Collaborative transmit resource scheduling and waveform selection for target tracking in multistatic radar system

Yijie Wang\textsuperscript{1} \textsuperscript{©} | Chenguang Shi\textsuperscript{1,2} | Fei Wang\textsuperscript{1} \textsuperscript{©} | Jianjiang Zhou\textsuperscript{1}

\textsuperscript{1}Key Laboratory of Radar Imaging and Microwave Photonics, Ministry of Education, Nanjing University of Aeronautics and Astronautics, Nanjing, China
\textsuperscript{2}Science and Technology on Electro-Optic Control Laboratory, Luoyang, China

Abstract
This paper proposes a collaborative transmit resource scheduling and waveform selection (CTRSWS) strategy for target tracking in multistatic radar system which consists of one transmitter and an arbitrary number of receivers. The main mechanism of the proposed CTRSWS strategy is to exploit the optimisation technique to jointly optimise the illumination power, dwell time, waveform bandwidth, and pulse length under the constraints of several resource budgets and the predefined waveform library, aiming at improving the low probability of intercept (LPI) performance and target tracking accuracy of multistatic radar system simultaneously. The analytical expressions for the probability of intercept and the trace of the predicted error covariance matrix corresponding to the target state estimation are derived and adopted to evaluate the LPI performance and target tracking accuracy, respectively. Subsequently, the resulting non-convex and non-linear optimisation problem is resolved by an efficient and fast four-stage solution methodology. Several numerical results are provided to verify the effectiveness and superiority of the proposed CTRSWS scheme in terms of the achievable LPI performance and target tracking accuracy of multistatic radar system.

1 | INTRODUCTION

1.1 | Background and motivation

Multistatic radar system, which is also known as bistatic radar composed of multiple transmitters and multiple receivers at different sites [1–3], has attracted significant attention. It has been demonstrated that multistatic radar system has a number of performance advantages, for instance, electronic countermeasures [4], Inverse Synthetic Aperture Radar (ISAR) image optimisation [5], higher target tracking [6–8] and localization accuracy [9,10]. Meanwhile, extensive studies have been investigated to improve the performance of multistatic radar system, which can be grouped in two classical methods: resource scheduling and waveform selection.

Focusing on resource scheduling, extensive researches have been conducted in radar nodes, dwell time, illumination power and so on, for different optimisation objective [11–21]. For example, Yan et al. propose a cooperative target assignment and dwell allocation algorithm to optimise the target-to-radar assignment and the limited time resource for improving the multiple target tracking accuracy [11]. As implied in Ref. [12], Xie et al. jointly optimise the number of radar nodes and the transmitted power. Furthermore, to get more degree of freedom, both nodes and corresponding waveforms with individual nodes transmit variable waveforms are taken into consideration in Ref. [13]. The work of Ref. [14] researches joint power and bandwidth allocation optimisation to improve target tracking accuracy in clutter. In Ref. [15], Li et al. develop an effective solution for adaptive power and bandwidth allocation scheduling in centralized multiple target tracking. In Ref. [16], a joint time-space resource allocation and waveform selection optimisation model for the collocated MIMO radar is proposed, where the objective function which takes both the system resource and tracking precision into consideration is minimized under the guarantee of effective targets detection.
Meanwhile, adaptive waveform design has attracted a number of interests among scholars in target tracking. Early researches about target tracking have confirmed that the tracking performance can significantly be improved by applying adaptive transmit waveform optimisation and selection [17–19]. In Refs. [20,21], adaptive waveform selection was first considered for one-dimensional target tracking by static radar, and then it is extended to two-dimensional target tracking scenario in Refs. [17,18]. The mutual information between the actual target parameters and the corresponding radar-observed parameters is maximized in Ref. [22]. Adaptive waveform selection has been applied for many target tracking scenarios, such as clutter-free or cluttered environments [23]. Furthermore, Nijssen et al. propose a waveform optimisation algorithm for an adaptive multi-input-multi-output (MIMO) radar, which improves resolution as the increase of iterations [24,25]. The work of Ref. [26] shown that adaptive waveform selection can also provide a significant improvement in the tracking performance of multistatic radars. Nguyen et al. [7] propose a joint adaptive selection algorithm, which takes the advantages of both existing waveform-only and path-only optimisation techniques. In Ref. [27], the authors consider target tracking by a moving monostatic radar with both waveform selection and sensor motion strategies, where the pulse repetition interval (PRI) of the transmitted waveform and the Cramer-Rao lower bound (CRLB) are taken into consideration. A related problem is studied in Ref. [28], where the adaptive waveform design is synchronized with MIMO radar target tracking sensor scheduling. The work of Ref. [29] focuses on the issue of adaptive waveform selection to optimise the radar resolvability of closely located targets and proposes an adaptive waveform selection criterion aiming to maximize the practical resolution. In Ref. [30], a modified non-linear frequency modulated (NLFM) method is combined with a four-hidden-layer neural network to generate the required waveform.

However, the majority of the existing studies only concentrate on optimising target tracking accuracy for different radar systems, without paying much attention to the problem of low probability of intercept (LPI) [31,32], which has been a critical issue for radar system, especially in modern electronic warfare [33–36]. The work about LPI mainly focuses on waveform optimisation or power control. The work of Ref. [37] studies the resource allocation scheme for a wireless powered integrated radar and communications system, in which the integrated radar-communications waveform and energy beamforming are jointly optimised to minimize the total radiated energy of the integrated system. Shi et al. in Ref. [37] consider the problem of power minimization-based robust orthogonal frequency division multiplexing (OFDM) radar waveform design. In Ref. [38], Ahmed et al. optimise the radar power allocation according to the specified target positioning performance requirements and improve communication data rate by taking channel side information into account. In addition, Ref. [39] proposes a power minimization-based joint subcarrier assignment and power allocation algorithm for a radar and communication system based on LPI performance, which jointly optimises the available subcarriers and illumination power resource allocation to achieve an improved power-saving performance.

Although the above studies provide us a guidance to deal with the problem of transmit resource scheduling and waveform selection optimisation problem, not all transmit resource and waveform parameters are taken into account collaboratively in the optimisation problem. Hence, in the current work, we extend the existing researches and develop a collaborative transmit resource scheduling and waveform selection (CTRSWS) strategy in multistatic radar system which consists of one transmitter and an arbitrary number of receivers. Of our knowledge, there are still no published references that take both LPI performance and target tracking accuracy into consideration collaboratively for multistatic radar system. This gap motivates this work.

1.2 Our contributions

The main idea of the current study is to enhance the LPI performance and target tracking accuracy of multistatic radar system at the same time while satisfying several system constraints, which is formulated as a non-convex and non-linear optimisation problem. Then, an efficient four-stage solution approach is proposed to solve the underlying problem.

The major contributions and results of this paper can be summarized in the following:

- The analytical expressions for the probability of intercept and the trace of the predicted error covariance matrix corresponding to the target state estimation are derived and adopted to assess the LPI performance and target tracking accuracy, respectively.
- By integrating the transmit resource scheduling and waveform selection frameworks into a coherent one, a CTRSWS strategy for target tracking in a multistatic radar system is proposed, which is subsequently described as a mathematical optimisation problem. Mathematically speaking, the CTRSWS scheme aims to enhance the LPI performance and target tracking accuracy of the underlying system at the same time under the constraints of system resource budgets and predetermined waveform library. In such scenario, the developed CTRSWS strategy can be formulated as a problem of minimizing the probability of intercept and the trace of the predicted error covariance matrix of the overall system simultaneously, which collaboratively optimises the illumination power, dwell time, waveform bandwidth, and pulse length in the view of several system constraints.
- An efficient and fast four-stage solution methodology is proposed to solve the resulting non-convex and non-linear optimisation problem. In order to tackle the underlying CTRSWS problem in real time, we develop an efficient and fast four-stage solution technique. Step (1) The optimisation problem is equivalently recast as two sub-problems. Step (2) The illumination power and dwell time are optimised. Step (3) The waveform bandwidth and pulse length are
optimised. Step (4) The measurement for next time slot is obtained to finish target tracking.

- Numerical simulation results are provided to validate the effectiveness and superiority of our proposed CTRSWS strategy, in terms of the achievable LPI performance and target tracking accuracy of multistatic radar system. To be specific, the presented CTRSWS strategy is able to achieve the better trade-off between LPI performance and target tracking accuracy of multistatic radar system when compared to other existing methods. Additionally, we consider three different scenarios, where our proposed CTRSWS strategy still exhibits lower probability of intercept, lower root mean square error (RMSE) and lower average RMSE (ARMSE).

1.3 Organization of the paper

The rest of the current study is organized as follows: Section 2 introduces the considered multistatic radar system and target tracking scenario. The CTRSWS strategy is presented in developed in Section 3. In Section 3.1, the basis of the proposed strategy is introduced. Sections 3.2 and 3.3 derive the analytical expressions for the probability of intercept and the predicted error covariance matrix corresponding to the target state estimation, respectively. The underlying non-convex and non-linear optimisation problem is formulated and resolved by employing a developed efficient four-stage solution technique in Sections 3.4 and 3.5, respectively. Numerical simulations are provided in Section 4 to demonstrate the effectiveness and advantages of the proposed CTRSWS algorithm. Finally, the concluding remarks are made in Section 5.

2 SYSTEM MODEL

2.1 Problem scenario

Let us consider a multistatic radar system, which consists of one dedicated radar transmitter and \( N \) radar receivers. The underlying system can measure time delay, Doppler shift by using a bank of matched filters and square-law envelope detectors and arrival angle can be estimated using a phased array antenna. Thus, for each radar receiver, the time delay, Doppler shift and arrival angle can be measured. Then multistatic radar system tracks the single target using extended Kalman filtering (EKF) algorithm to update the position of each time. Note that radar transmitter and \( N \) receivers are located at different position, which communicate by a data link. As a result, the radar transmitter optimises transmit resource scheduling and waveform selection adaptively at each time slot according to measurements from different radar receivers, which includes the illumination power, dwell time, waveform bandwidth, and pulse length. Thus, the problem of CTRSWS is critically important for the LPI performance and target tracking accuracy of the considered system.

To begin with, we assume the radar transmit signal is narrowband. Thus, radar transmit pulse can be represented by [20]:

\[
s_i(t) = \sqrt{2E_i} \sqrt{E_i} e^{j2\pi f_i t} + n(t) e^{j2\pi f_i t},
\]

where \( f_c \) denotes the carrier frequency and \( E_i \) is the energy of the transmit pulse. In order to ensure a fair comparison among waveforms, the complex envelop of transmit pulse is assumed to be normalized so that:

\[
\int_{-\infty}^{\infty} |\tilde{s}(t)|^2 dt = 1.
\]

The received signal reflecting by a single target at the \( i \)-th receiver can be given by:

\[
s_i(t) = \sqrt{2E_i} \sqrt{E_i} e^{j\phi} \tilde{s}(t - \tau_i) e^{j2\pi v_i t} + n(t) e^{j2\pi f_i t},
\]

where \( E_r \) is the energy of received signal, \( \phi \) is random phase shift, and \( n(t) \) is additive zero-mean complex white Gaussian noise with power spectral density \( N_0/2 \). In this scenario we considered, the speed of radar waveform propagation is very large so that time compression due to Doppler shift can be ignored. Target time delay \( \tau_i \) and Doppler shift \( v_i \) can be given respectively by \( \tau_i = (r_0 + r_i)/c \), \( v_i = -(2\pi f_0/c)f_c \), where \( r_0 \) is the distances from target to the transmitter, \( r_i \) is the distances from target to the \( i \)-th receiver, \( c \) is the speed of waveform propagation and \( f_0 \) is the speed of target motion [40].

2.2 Target dynamic model

Let \( X_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^T \) represents the target state at time slot \( k \), where \([x_k, y_k] \) is the target position at time slot \( k \) and \([\dot{x}_k, \dot{y}_k] \) is the target velocity at time slot \( k \). The equation of target motion state can be given by:

\[
X_{k+1} = FX_k + W_k,
\]

where \( W_k \sim \mathcal{N}(0, Q) \) denotes the process noise matrix, \( F \) is the state transition matrix, the matrix \( Q \) is given by:

\[
Q = q \begin{bmatrix}
\Delta T^4 & \Delta T^3 & 0 & 0 \\
\Delta T^3 & \Delta T^2 & 0 & 0 \\
0 & 0 & \Delta T^4 & \Delta T^3 \\
0 & 0 & \Delta T^3 & \Delta T^2
\end{bmatrix},
\]

where \( \Delta T \) is the sampling interval and \( q \) is a constant corresponding to the level of target maneuverability.
2.3 Measurement model

The measurement obtained by multistatic radar system is given by:

\[ Z_{k+1} = h(X_{k+1}) + N_{k+1}, \]  

(6)

where \( Z_{k+1} \) is measurement vector of multistatic radar system at time slot \( k+1 \), \( N_{k+1} = [N_{1,k+1}, \ldots, N_{N,k+1}]^T \) denotes process noise matrix of multistatic radar system, \( N_{i,k+1} \sim \mathcal{N}(0, R_{i,k+1}) \) denotes process noise matrix of the \( i \)-th radar receiver, \( R_{i,k+1} \) is measurement error covariance matrix of the \( i \)-th radar receiver for time slot \( k+1 \), \( h(X_{k+1}) \) denotes the measurement function, the measurement vector \( Z_{k+1} \) can be written as:

\[
Z_{k+1} = \begin{bmatrix}
R_{1,k+1} \\
\psi_{1,k+1} \\
\theta_{1,k+1} \\
\vdots \\
R_{N,k+1} \\
\psi_{N,k+1} \\
\theta_{N,k+1}
\end{bmatrix} + N_{k+1} = \begin{bmatrix}
b_{R_1}(X_{k+1}) \\
b_{\psi_1}(X_{k+1}) \\
b_{\theta_1}(X_{k+1}) \\
\vdots \\
b_{R_N}(X_{k+1}) \\
b_{\psi_N}(X_{k+1}) \\
b_{\theta_N}(X_{k+1})
\end{bmatrix} + N_{k+1},
\]

(7)

where \( R_{i,k+1}, \psi_{i,k+1} \) and \( \theta_{i,k+1} \) denote the actual measurement distance, velocity and arrival angle at the \( i \)-th radar receiver, respectively. With the radar transmitter located at \((x_{i,k+1}, y_{i,k+1})\) and the \( i \)-th radar receiver located at \((x_{i,k+1}, y_{i,k+1})\) for time slot \( k+1 \). The transfer function \( b_{R_i}(X_{k+1}) \), \( b_{\psi_i}(X_{k+1}) \) and \( b_{\theta_i}(X_{k+1}) \) at the \( i \)-th radar receiver can be given by:

\[
b_{R_i}(X_{k+1}) = \sqrt{(x_{k+1} - x_i)^2 + (y_{k+1} - y_i)^2} \\
+ \sqrt{(x_{k+1} - x_i, i,k+1)^2 + (y_{k+1} - y_i, i,k+1)^2},
\]

(8)

\[
b_{\psi_i}(X_{k+1}) = \frac{\dot{x}_{k+1}(x_{k+1} - x_i) + \dot{y}_{k+1}(y_{k+1} - y_i)}{\sqrt{(x_{k+1} - x_i)^2 + (y_{k+1} - y_i)^2}} \\
+ \frac{\dot{x}_{k+1}(x_{k+1} - x_i, i,k+1) + \dot{y}_{k+1}(y_{k+1} - y_i, i,k+1)}{\sqrt{(x_{k+1} - x_i, i,k+1)^2 + (y_{k+1} - y_i, i,k+1)^2}},
\]

(9)

\[
b_{\theta_i}(X_{k+1}) = \arctan \left( \frac{y_{k+1} - y_i, i,k+1}{x_{k+1} - x_i, i,k+1} \right).
\]

(10)

2.4 EKF algorithm

The key of radar target tracking is to predict the target's current state of motion and form the target trajectory, according to the predicted value of the moving target's previous state of motion and the observed value of the current state of motion. Taking the uncertainty of the target motion into consideration, we use the EKF algorithm for target tracking.

The extended Kalman filter algorithm is an improved algorithm based on the standard Kalman filter algorithm, which is suitable for handling non-linear scenarios. The EKF algorithm flow can be described as follows [41]:

\[
\begin{align*}
P_{k+1|k} &= FP_{k|k}F^T + Q_k, \\
P_{k+1|k}^* &= (I - K_kP_{k+1|k})P_{k+1|k},
\end{align*}
\]

(11)

\[
\begin{align*}
\hat{X}_{k+1|k} &= H_{k+1}X_{k+1|k} + R_{k+1}, \\
\hat{X}_{k+1|k} &= X_{k+1} - K_k(S_{k+1} - h_{k+1}),
\end{align*}
\]

(12)

\[
\begin{align*}
P_{k+1|k+1} &= [I - K_kS_{k+1}]P_{k+1|k},
\end{align*}
\]

(16)

where \( P_{k+1|k} \) and \( \hat{X}_{k+1|k} \) are priori estimate and covariance matrices of the state vector, respectively. \( k_{k+1} \) is the innovation covariance, and \( K_{k+1} \) is the filter gain matrix. \( I \) is an identity matrix. \( P_{k+1|k+1} \) and \( \hat{X}_{k+1|k+1} \) are posteriori estimate and covariance matrices of the state vector, respectively. \( R_{k+1} = \text{diag}[R_{1,k+1}, R_{2,k+1}, \ldots, R_{N,k+1}] \) is measurement error covariance matrix of multistatic radar system. \( H_{k+1} \) is a Jacobian of \( h(X_{k+1}) \) given by:

\[
H_{k+1} = \begin{bmatrix}
\frac{\partial h(x_i)}{\partial x_i} \\
\frac{\partial h(x_i)}{\partial y_i}
\end{bmatrix}_{|x_i = \hat{x}_{k+1|k}}.
\]

(17)

3 CTRSWS STRATEGY

3.1 Basis of the technique

Mathematically speaking, the proposed CTRSWS scheme can be formulated as a problem of collaboratively optimising the adaptable parameters, such as the illumination power, dwell time, waveform bandwidth, and pulse width, to simultaneously enhance the LPI performance and target tracking accuracy of multistatic radar system subject to several system constraints. The closed-form expressions for the probability of intercept and the trace of the predicted error covariance matrix corresponding to the target state estimation are derived and employed to gauge the LPI performance and target tracking accuracy, respectively. We are then in a position to optimise the transmit scheduling and waveform selection in order to achieve the improved LPI performance and target tracking accuracy for multistatic radar system in the meantime.

The general steps of the CTRSWS strategy are detailed as follows.
3.2 | Metric for LPI performance

In this section, the probability of intercept for time slot \( k + 1 \) is defined and employed as metric for LPI performance. For electronic countermeasures, the problem of radar signal interception by intercept receiver is a multidimensional problem including time domain, frequency domain, airspace domain and power domain.

To begin with, some moderate assumptions are summarised to simplify the considered problem as follows:

- **Assumption 1**: The intercept receiver is mounted on a moving target with an omni-directional antenna.

- **Assumption 2**: Intercept receiver uses step-by-step search method.

- **Assumption 3**: The radar signal main beam is aligned with the intercepting receiver antenna.

The probability of intercept by intercept receiver \( p_t \) can be expressed by:

\[ p_t = p_s \cdot p_t \cdot p_i \cdot p_d', \]  
\[ (18) \]

where \( p_s, p_t, p_i \) and \( p_d' \) denote the probability of intercept in airspace domain, frequency domain, time domain and power domain, respectively.

The probability of at least one intercept in airspace domain, frequency domain and time domain during dwell time \( T_d \) can be expressed by [42,43]:

\[ p_s \cdot p_t \cdot p_i = 1 - K_0 \exp\left( -\frac{T_d(T_d + \tau_1)}{T_i \Delta T} \right), \]  
\[ (19) \]

where \( T_i \) is total search time of intercept receiver, \( \tau_1 \) is the duration of frequency in each step bandwidth, \( K_0 \) denotes the probability that one intercept occurs at the beginning. Because of \( \tau_1 \ll T_i, T_d \ll \Delta T \), we can get \( K_0 \approx 1 \).

The transmit signal is intercepted by intercept receiver together with the noise signal. Due to uncertainty of noise signal, the probability of intercept in power domain \( p_d' \) is the detection probability of intercept receiver to the radar signal under given false alarm probability \( p'_f_a \), which is given by:

\[ p_d' = \frac{1}{2} \text{erfc}\left( \sqrt{-\ln p'_f_a} - \sqrt{\text{SNR}_I + 0.5} \right), \]  
\[ (20) \]

where \( \text{erfc}(\cdot) \) is error complementary function, \( \text{SNR}_I \) is the single pulse signal-to-noise ratio from intercept receiver output, which can be expressed by:

\[ \text{SNR}_I = \frac{P_R G_R \lambda^2 G_{IP}}{(4\pi)^2 R_i^2 k T_0 B_i F_i}, \]  
\[ (21) \]

where \( P_R \) is radar illumination power, \( G_R \) is transmitting antenna gain of radar, \( G_{IP} \) is receiving antenna gain of intercept receiver, \( \lambda \) is wavelength of radar transmit signal, \( G_{IP} \) is processing gain of intercept receiver, \( k \) is Boltzmann constant, \( T_0 \) is system noise temperature of intercept receiver, \( B_i \) is system bandwidth of intercept receiver, \( F_i \) is noise factor of intercept receiver, \( R_i \) denotes the distance from the radar transmitter to the target.

Thus, under given false alarm probability \( p'_f_a \), the probability of intercept for multistatic radar system under the target tracking scenario is given by:

\[ p_t = p_s \cdot p_t \cdot p_i \cdot p_d' \]
\[ = \left[ 1 - \exp\left( -\frac{T_d(T_d + \tau_1)}{T_i \Delta T} \right) \right] \cdot \frac{1}{2} \text{erfc}\left( \sqrt{-\ln p'_f_a} \right) - \frac{P_R G_R \lambda^2 G_{IP}}{(4\pi)^2 R_i^2 k T_0 B_i F_i + 0.5} \]  
\[ \approx \frac{T_d(T_d + \tau_1)}{2T_i \Delta T} \cdot \text{erfc}\left( \sqrt{-\ln p'_f_a} \right) - \frac{P_R G_R \lambda^2 G_{IP}}{(4\pi)^2 R_i^2 k T_0 B_i F_i + 0.5} \]  
\[ (22) \]

One can observe from Equation (22) that the probability of intercept for multistatic radar system is related to dwell time and illumination power as part of radar transmit resource. Moreover, it is noteworthy that the function of probability of intercept corresponding to dwell time and illumination power is not non-linear function. To satisfy the real-time requirement of the algorithm, we used an efficient and fast optimisation technique to solve this introduced problem introduced in detail in the following section.

3.3 | Metric for target tracking accuracy

Before establishing CTRSWS optimisation problem in multi-static radar system, the metric to evaluate target tracking accuracy should be given. We utilized the trace of predicted error covariance matrix in the EKF algorithm to evaluate target tracking accuracy.

It's worth noting that \( \mathbf{R}_{k,k+1} \) is characterized by its waveform parameters [20], which can be calculated by:

\[ \mathbf{R}_{k,k+1} = \mathbf{T} \mathbf{C}_{k+1} \mathbf{T}^\top, \]  
\[ (23) \]
where $\mathbf{T} = \text{diag}[c/2, c/2f_c, 1]$ is transfer matrix, $\mathbf{C}_{k+1}$ is CRLB related to bandwidth, pulse length and arrival angle. For Gaussian-LFM signal, the CRLB can be expressed as:

$$
\mathbf{C}_{k+1} = \frac{1}{\text{SNR}_{i,k+1|k}} 
\begin{bmatrix}
    2\lambda_{k+1}^2 & -4b_{k+1}\lambda_{k+1}^2 & 0 \\
    -4b_{k+1}\lambda_{k+1}^2 & \frac{1}{2\pi^2\lambda_{k+1}^2} + 8b_{k+1}\lambda_{k+1}^2 & 0 \\
    0 & 0 & \sigma^2_b
\end{bmatrix}
$$

where $b_{k+1} = W_{k+1}/2T_s$, $W_{k+1}$ is bandwidth of transmit signal for time slot $k + 1$, $T_s$ is the effective pulse length, $\lambda_{k+1}$ is the Gaussian pulse length parameter for time slot $k + 1$ which is determined the value of $\lambda_{k+1} = T_s/7.4338, \sigma_b$ is a constant, the predicted signal to noise ratio SNR$_{i,k+1|k}$ at $i$-th radar receiver for time slot $k + 1$ can be calculated by:

$$
\text{SNR}_{i,k+1|k} = \frac{T_{d,k+1}P_{r,k+1}G_iG_{rx}\lambda_{k+1}^2G_{RP}}{(4\pi)^2T_iR_{i,k+1}^2kT_BF_i}
$$

Thus, the resulting CTRSWS optimisation problem can be developed as:

$$
\begin{align*}
\min_{T_{d,k+1}, P_{r,k+1}} & \quad p_{l,k+1}, \\
\text{s.t.} & \quad \text{SNR}_{\text{net},k+1|k} = \text{SNR}_{\text{min}},
\end{align*}
$$

where $\text{SNR}_{\text{net},k+1|k} = \text{SNR}_{\text{min}}$ is target tracking SNR threshold. $p_{l,k+1}$ denotes the probability of intercept for time slot $k + 1$ as follow:

$$
p_{l,k+1} = \frac{T_{d,k+1}(T_{d,k+1} + \Delta T)}{2T_i\Delta T} \cdot \text{erfc} \left( \frac{-\ln p_a}{2T_i\Delta T} \right) - \frac{P_{l,k+1}G_iG_{rx}\sigma_b^2G_{RP}}{(4\pi)^2T_iR_{i,k+1}^2kT_BF_i} + 0.5
$$

In the optimisation problem (Equation 26), the first constraint on $P_{l,k+1}$ implies that the illumination power is limited while the second one stands that the dwell time is also constrained. The third one on $\text{SNR}_{\text{net},k+1|k}$ takes the predicted SNR of multistatic radar system for time slot $k + 1$ into consideration, whose objective is to ensure target tracking steadily with change of target position. Thus, the $\text{SNR}_{\text{net},k+1|k}$ can be calculated by:

$$
\text{SNR}_{\text{net},k+1|k} = \sum_{i=1}^{N} \frac{T_{d,k+1}P_{l,k+1}G_iG_{rx}\sigma_b^2G_{RP}}{(4\pi)^2T_iR_{i,k+1}^2kT_BF_i}
$$

### 3.4 Problem formulation

The primary objective is to formulate the CTRSWS optimisation problem, whose purpose is to achieve enhanced LPI performance and target tracking accuracy. To be specific, we concentrate on CTRSWS optimisation problem to minimize the LPI performance metric and target tracking accuracy metric by optimising dwell time and illumination power, waveform bandwidth and pulse length under the constraints of several resource budgets and the predefined waveform library.

### 3.5 Solution technique

As we analysis above, the probability of intercept is characterized by dwell time and illumination power, while the trace of predicted error covariance matrix is characterized by transmit resource and waveform parameters including dwell time, illumination power, waveform bandwidth and pulse length. It is hard to solve problem (Equation 26) by intelligence algorithm, as its objective function has different number of variables. To solve this problem, we develop
an efficient and fast four-stage solution technique: Step (1) The optimisation problem is equivalently recast as two sub-problems. Step (2) The illumination power and dwell time are optimised. Step (3) The waveform bandwidth and pulse length are optimised. Step (4) The measurement for next time slot is obtained to finish target tracking.

- **Recasting problem as two sub-problems:** In electronic countermeasures, the precondition of ensuring target tracking accuracy is that radar transmitter is in a safe position, where the probability of intercept by intercept receiver is as low as possible. Thus, we view the LPI performance as first optimisation objective and target tracking accuracy as second optimisation objective. Then, the optimisation problem (26) can equivalently be recast as the following two sub-problems:

\[
\begin{align*}
&\text{min}_{T_{d,k+1}, P_{t,k+1}} P_{1,k+1}, \\
\text{s.t.:} & \quad P_{\min} \leq P_{t,k+1} \leq P_{\max}, \\
& \quad T_e \leq T_{d,k+1} \leq T_{\max}, \\
& \quad \text{SNR}_{wet,k+1}|k \geq \text{SNR}_{\min},
\end{align*}
\]

(29)

and

\[
\begin{align*}
&\text{min}_{T_{d,k+1}, P_{t,k+1}, W_{k+1}|k+1} \text{tr}(\hat{P}_{k+1}|k+1), \\
\text{s.t.:} & \quad P_{\min} \leq P_{t,k+1} \leq P_{\max}, \\
& \quad T_e \leq T_{d,k+1} \leq T_{\max}, \\
& \quad [W_{k+1}, \lambda_{k+1}] \in \Omega, \\
& \quad \text{SNR}_{wet,k+1}|k \geq \text{SNR}_{\min}.
\end{align*}
\]

(30)

- **Optimisation of illumination power and dwell time:** From the sub-problem (Equation 29), it is obvious that illumination power and dwell time can be converted each other when ‘=’ in third constraint \(\text{SNR}_{wet,k+1}|k \geq \text{SNR}_{\min}\) holds. Since when the predicted SNR of multistatic radar system is equal to \(\text{SNR}_{\text{min}}\), the minimum illumination power is achieved, this given:

\[
\sum_{i=1}^{N} \frac{T_d \cdot T_{d+1} \cdot P_t \cdot G_i \cdot G_{t,i} \cdot \sigma_i^2 \cdot G_{RP}}{(4\pi)^3 T_e \cdot R_{k+1} \cdot [R_i \cdot kT_0 B_i F_i]} = \text{SNR}_{\text{min}}.
\]

(31)

Rearranging terms yields:

\[
P_{t,k+1} = \frac{\text{SNR}_{\min} \cdot (4\pi)^3 T_e \cdot R_{k+1}^2 \cdot [kT_0 B_i F_i]}{T_d \cdot T_{d+1} \cdot G_i \cdot G_{t,i} \cdot \sigma_i^2 \cdot G_{RP}} \cdot \frac{1}{\sum_{i=1}^{N} [R_i \cdot kT_0 B_i F_i]} = P_{1,k+1}.
\]

(32)

Substituting Equation (32) into Equation (27), Equation (27) can be rewritten as:

\[
P_{1,k+1} = \frac{T_d \cdot (T_{d+1} + T)}{2T_1 \Delta T} \cdot \text{erfc} \left( \sqrt{-\ln \rho_k} \right) - \frac{4\pi \text{SNR}_{\min} T_i B_{iF_i} G_{i,t} G_{t,i} \sigma_{i,F_{i,t}} G_{RP}}{T_{d+1} B_{iF_i} G_{i,t} G_{t,i} \sigma_{i,F_{i,t}} G_{RP}} \cdot \frac{1}{\sum_{i=1}^{N} [R_i \cdot kT_0 B_i F_i]} + 0.5
\]

(33)

As indicated in Ref. [44], the probability of intercept corresponding to dwell time is upper convex function. That is to say, optimal solution of dwell time is always at the boundary:

\[
T_{d,k+1} = \arg \min \{P_{1,k+1}(T_i), P_{1,k+1}(T_{\max})\}.
\]

(34)

After getting optimal dwell time for time slot \(k+1\), the illumination power can also be calculated by Equation (32).

- **Optimisation of waveform bandwidth and pulse length:** After optimisation of illumination power and dwell time, this optimal solution \(P_{1,k+1} \) and \(T_{d,k+1} \) is applied to solve the remaining sub-problem (Equation 30), which can be rewritten as:

\[
\begin{align*}
&\text{min}_{W_{k+1}, \lambda_{k+1}} \text{tr}(\hat{P}_{k+1}|k+1), \\
\text{s.t.:} & \quad [W_{k+1}, \lambda_{k+1}] \in \Omega.
\end{align*}
\]

(35)

For each possible waveform bandwidth and pulse length in waveform library, the optimal waveform bandwidth and pulse length is selected for time slot \(k+1\) by minimizing the trace of predicted error covariance matrix.

- **Finish tracking for next time:** After determining the illumination power, dwell time, waveform bandwidth and pulse length for time slot \(k+1\), the radar transmitter illuminates the target and produce target measurement for time slot \(k+1\), which is used to finish target tracking for time slot \(k+1\). Then, according to EKF algorithm, we update the target state vector and error covariance as predicted initial value for next time slot.

In general, the general steps of the proposed CTRSWS strategy for target tracking in multistatic radar system are summarized in **Algorithm 1**.

**Algorithm 1:** The General Steps of the CTRSWS Strategy for Target Tracking in Multistatic Radar System

1. **Step 1:** Given model probability \(e_{jk}\), predicted state matrix \(\hat{X}_{j,k+1}|k\) and error covariance matrix \(P_{j,k+1}|k\);

2. **Step 2:** Use the EKF algorithm to calculate predicted state value \(\hat{X}_{k+1}|k\) and calculate predicted distance.
$R_{k+1}$ from the radar transmitter to the target for time slot $k + 1$.

**Step 5:** Recast optimisation problem as two sub-problems and view LPI performance as first objective;

**Step 4:** Optimise LPI performance, and determine dwell time $T_{d,k+1}$ for time slot $k + 1$ according to Equation (34). Subsequently, the illumination power $P_{i}$ for time slot $k + 1$ is determined.

**Step 5:** With dwell time and illumination power determined, optimise target tracking accuracy. The optimal waveform bandwidth and pulse length is selected from the waveform library $\Omega$.

**Step 6:** Illuminates the target with optimal transmit resource and produce target measurement to finish target tracking for time slot $k + 1$;

**Step 7:** Initialize the target’s state information in Step 1;

**Step 8:** $k + 1 \rightarrow k$ and repeat Step 1–6 until whole target tracking process is over.

### 4 | SIMULATION RESULTS AND PERFORMANCE EVALUATION

#### 4.1 | Numerical setup

In this subsection, several numerical results are provided to demonstrate the effectiveness and superiority of the CTRSWs strategy for target tracking in multistatic radar system. It is assumed that the initial position and velocity of the target are $[60, 80]$ km and $[0.15, 0.26]$ km/s, respectively. We also take waveform library $\Omega$ into consideration based on Gaussian linear frequency modulation (GLFM) signal, which consists of one hundred waveforms with $\lambda = [9.1, 9.2, \ldots, 9.9, 10]$ $\mu$s and $\mu = [1, 2, \ldots, 9, 10]$ MHz [6]. The other parameters used in the simulations are summarized in Table 1, while Table 2 describes the detailed description of target motion.

In this subsection, several numerical results are provided to demonstrate the effectiveness and superiority of the CTRSWs strategy for target tracking in multistatic radar system. It is assumed that the initial position and velocity of the target are $[60, 80]$ km and $[0.15, 0.26]$ km/s, respectively. We also take waveform library $\Omega$ into consideration based on Gaussian linear frequency modulation (GLFM) signal, which consists of one hundred waveforms with $\lambda = [9.1, 9.2, \ldots, 9.9, 10]$ $\mu$s and $\mu = [1, 2, \ldots, 9, 10]$ MHz [6]. The other parameters used in the simulations are summarized in Table 1, while Table 2 describes the detailed description of target motion.

**Table 1** Simulation parameters

| Symbol | Value | Symbol | Value |
|--------|-------|--------|-------|
| $f_c$  | 12 GHz| $T$    | 100 s |
| $\Delta t$ | 1 s | $G_x$ | 30 dB |
| $G_{c}(\forall i)$ | 30 dB | $G_i$ | 6 dB |
| $G_{ip}$ | 3 dB | $B_i$ | 40 GHz |
| $F_i$ | 6 dB | $\lambda_i$ | 0.025 m |
| $T_i$ | $5 \times 10^{-4}$ s | $T_{min}$ | $5 \times 10^{-3}$ s |
| $G_{ip}$ | 16.5 dB | $F_i$ | 3 dB |
| $B_i$ | 1 MHz | $\sigma_i$ | 20 m$^2$ |
| $T_{min}$ | 0 | $T_{max}$ | 14 kW |
| SNR$_{min}$ | 19 dB | $p_{fi}$ | $10^{-8}$ |

**Table 2** The description of target motion

| Time Slots | Target Motion |
|------------|---------------|
| 1 – 30s    | Constant velocity |
| 31 – 50s   | Right turn($\omega = 3$ rad) |
| 51 – 80s   | Constant velocity |
| 81 – 100s  | Right turn($\omega = 3$ rad) |

**Figure 1** Target trajectory with respect to the multistatic radar in Scenario 1

**Figure 2** Dwell time optimisation results in Scenario 1
section (RCS) model. In Scenario 1, the radar receivers are deployed in the four vertices of a square, while the radar receivers are distributed in a straight line in Scenario 2. The deployment of the radar receivers in Scenario 3 is same as Scenario 1 but with RCS changing constantly. Furthermore, for each scenario, we compare the proposed strategy in terms of LPI performance and target tracking accuracy of multistatic radar system with the following three benchmarks:

- Fixed Dwell Time and Waveform Selection (FDTWS): The transmit power, waveform bandwidth and pulse length are optimally adjusted by utilizing the proposed solution technique, and the dwell time is fixed and set as $2.5 \times 10^{-3}$ s.
- Transmit Resource Scheduling and Fixed Waveform (TRSFW): The transmit power and dwell time are optimally adjusted by utilizing the proposed solution technique, while waveform parameters are fixed with waveform bandwidth $\lambda = 9.1 \mu s$ and pulse length $W = 1$ MHz.

### 4.2 Scenario one

In this scenario, a multistatic radar system with one radar transmitter and four radar receivers is considered as shown in Figure 1. Here, we consider the target tracking scenario, where the radar transmitter is located at the origin $[0, 0]$ km, and four radar receivers are located at $[30, 35]$ km, $[20, 25]$ km, $[20, 35]$ km and $[30, 25]$ km, respectively. Meanwhile, the target RCS model is uniform with RCS keeping $3 \, m^2$. 

---

**FIGURE 3** Transmit power optimisation results in Scenario 1

**FIGURE 4** Pulse length optimisation results in Scenario 1

**FIGURE 5** Waveform bandwidth optimisation results in Scenario 1

**FIGURE 6** The probability of intercept in Scenario 1
The following results are the average results from 200 Monte-Carlo trials. Figures 2 and 3 show transmit resource scheduling optimisation results about dwell time and illumination power at different time slots, respectively. For time slot \( k < 38 \), the radar transmitter selects the largest dwell time, while the shortest dwell time is selected for time slot \( k \geq 38 \). It is noteworthy that the illumination power for time slot \( k < 38 \) in Figure 3 is close to 150 W, which is related with the largest dwell time. Figures 4 and 5 show waveform selection results at different time slots. It can be clearly seen from the above results that the proposed CTRSWS strategy can adaptively adjust radar transmit resource and waveform parameters according to target tracking state at different time slots. To better examine the effectiveness of the proposed strategy, Figures 6 and 7 present the probability of intercept and RMSEs in target tracking, respectively, where the root mean square error (RMSE) for time slot \( k \) is defined as:

\[
\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}
\]

(a) (b) (c) (d)

**Figure 7** The RMSE in Scenario 1

**Figure 8** Comparison of the probability of intercept by employing different strategies in Scenario 1 (a) CTRSWS; (b) FDTWS; (c) TRSFW; (d) FDTFW

**Figure 9** Comparison of RMSE by employing different strategies in Scenario 1
\( \text{ARMSE}_k = \sqrt{\frac{1}{M_c} \sum_{m=1}^{M_c} [(x_k - \hat{x}_{m,k})^2 + (y_k - \hat{y}_{m,k})^2]} \)  \hfill (36)

where \( M_c \) is the number of Monte-Carlo trials, \([x_k, y_k]\) is the target actual position for time slot \( k \), while \([\hat{x}_{m,k}, \hat{y}_{m,k}]\) is the target state estimate at the \( m \)-th trial for time slot \( k \). Therefore, we can conclude the effectiveness of the proposed CTRSWS strategy.

Subsequently, we compare the proposed CTRSWS strategy in terms of the probability of intercept, RMSE and average RMSE (ARMSE) of multistatic radar system with FDTWS, TRSFW and FDTFW, which are shown in Figures 8–10, respectively. The ARMSE can be calculated as follows:

\[
\text{ARMSE} = \frac{1}{T} \sum_{k=1}^{T} \sqrt{\frac{1}{M_c} \sum_{m=1}^{M_c} [(x_k - \hat{x}_{m,k})^2 + (y_k - \hat{y}_{m,k})^2]} \hfill (37)
\]

where \( T \) is total time of target tracking. From Figure 8 to Figure 10, it can be seen that FDTWS strategy exhibits much higher probability of intercept, while the TRSFW strategy exhibits much higher RMSE and ARMSE compared with the proposed strategy. FDTFW strategy is applicable to fixed dwell time scenarios, while FRSFW strategy is applicable to fixed waveform. In the optimization process for the FDTWS strategy, it is unavoidable that the FDTWS strategy shows higher probability of intercept for merely waveform selection. That is to say, FRSFW strategy only optimize waveform
selection parameters, which results low RMSE and ARMSE but high probability of intercept. Meanwhile, the TRSFW strategy just optimize resource scheduling, leading to LPI but high RMSE and ARMSE. With comparison, the proposed CTRSWS strategy takes both transmit resource scheduling and waveform selection into account, which presents lower probability of intercept, lower RMSE and ARMSE. In summary, the LPI performance and target tracking accuracy can be significantly enhanced by employing the proposed CTRSWS strategy in multistatic radar system.

4.3 Scenario 2

To verify that the proposed CTRSWS strategy can adapt to various scenarios, we simulate a scenario where the radar receivers are distributed in a straight line in Figure 11. Herein, we consider the target tracking scenario where the radar transmitter is located at the origin [0, 0] km, and four radar receivers are located at [0, 60] km, [20, 40] km, [40, 20] km and [60, 0] km. Meanwhile, the target RCS model still is uniform with RCS keeping $3 \text{ m}^2$.

Similar to Scenario 1, Figures 12 and 13 show transmit resource scheduling optimisation results about dwell time and illumination power, while Figures 14 and 15 show waveform selection results at different times. One can observe that the dwell time changes from $2.5 \times 10^{-2}$ s to $5 \times 10^{-4}$ s for time slot 28, meanwhile illumination power changes correspondingly. Different waveform parameters are selected at each step of target tracking. The probability of intercept and RMSE change over times during target tracking in Figures 16 and 17. It is conclusion that CTRSWS strategy can process transmit.
resource scheduling and waveform selection effectively in Scenario 2.

The probability of intercept, RMSE and ARMSE achieved with the CTRSWS strategy are again used as metrics to compare with the three other strategies in Figures 18–20, respectively. As expected, FRSFW strategy exhibits low RMSE and ARMSE but high probability of intercept, while TRSFW strategy exhibits LPI but high RMSE and ARMSE under the scenario where the location of radar receivers changes. Our proposed CTRSWS strategy still exhibits lower probability of intercept, lower RMSE and ARMSE compared with FRSFW and TRSFW strategy, which verify the superiority of CTRSWS strategy.

### 4.4 Scenario 3

In order to better evaluate the effect of the target RCS on the CTRSWS optimization results, we then expand our simulation with the consideration of the more complicated scenario where target RCS changes over times. Figure 21 shows the target RCS observed by four radar receivers. The distribution of the radar
receivers is the same as Scenario 1, where the radar transmitter is located at the origin \([0, 0]\) km, and four radar receivers are located at \([30, 35]\) km, \([20, 25]\) km, \([20, 35]\) km and \([30, 25]\) km, respectively. Figures 22 and 23 show transmit resource scheduling optimisation results about dwell time and illumination power at different time slots, respectively. Figures 24 and 25 show waveform selection results at different time slots. Compared with Scenario 1, the results reveal that the variations of dwell time and illumination power are more severe related to the target RCS. Moreover, Figures 26 and 27 illustrate the probability of intercept and RMSE in target tracking. From the above results, we can conclude the effectiveness of the proposed CTRSWS strategy in complex scenario.

Subsequently, we compare our proposed strategy with the three other strategies as well in Figures 28–30, respectively. As expected, our proposed CTRSWS strategy still exhibits lower probability of intercept, lower RMSE and lower ARMSE. It is worth noting that the jitters of the target RCS are intense, the RMSE of CTRSWS strategy still have a great stableness compared with the RMSE of TRSFW and FDTFW strategy. Therefore, it can be concluded that the proposed CTRSWS strategy is able to adjust the illumination power, dwell time,
waveform bandwidth, and pulse length to deal with the change in target RCS, hence leading to the optimum LPI performance and target tracking accuracy.

In conclusion, we compare the results under three different scenarios. As introduced above, the radar receivers are deployed in the four vertices of a square in Scenario 1, while the radar receivers are distributed in a straight line in Scenario 2. Compared Figure 3 with Figure 13, the illumination power in Scenario 2 is lower than one in Scenario 1. Meanwhile, dwell time and waveform selection exist differences under two scenarios. The results also reveal that the location of radar receivers can influence illumination power, dwell time, waveform bandwidth and pulse length, further affecting for every time slot. More complicated factor with RCS changing constantly is considered in Scenario 3. Although the illumination power, dwell time, waveform bandwidth and pulse length vary with target RCS, the RMSE maintain a steady trend in Scenario 3 compared with Scenario 1 and Scenario 2.

5 | CONCLUDING REMARKS

A CTRSWS algorithm is presented for target tracking in a multistatic radar system, with the primary objective of simultaneously enhancing the LPI performance and target tracking accuracy of the underlying system by collaboratively optimising the adaptable parameters. It is noteworthy that the considered multistatic radar system consists of one transmitter and an arbitrary number of receivers. To facilitate the optimisation, an efficient and fast four-stage solution technique is developed to solve the resulting non-convex and
non-linear problem. Numerical results illustrate that the proposed strategy can enhance the LPI performance and target tracking accuracy of multistatic radar system efficiently. It is also shown that the proposed CTRSWS strategy can offer unique advantage in terms of LPI performance and target tracking accuracy enhancement compared to other existing approaches. In potential future research, we will take the transmitter and receiver path optimisation into account and jointly optimise the available probing resources to further generalize the presented scheme.

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