Instrument Transformers Detection and Calibration with Synchronized Phasor Measurements

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Abstract: Instrument transformers with deviation can lead to biased measurements. It causes problems for various applications that use voltage and current measurements. This paper presents a method for detecting and calibrating instrument transformers using synchronized phasor measurements, while the validity of the proposed method is verified via simulation. It is shown that the proposed method can evaluate the instrument transformers and all the Ratio Correction Factors (RCFs) of instrument transformers can be calculated, which can improve the accuracy of measurements. The proposed approach is robust to system uncertainty and noise.

1. Introduction

Instrument transformers provide voltage and current signals to all the measuring, protection and control devices. For an ideal instrument transformer, the amplitude ratio of primary and secondary signals is equal to the conversion ratio, and both signals are in the same phase. But the nominal conversion ratios specified in the name plates may differ from the actual conversion ratios due to the influence of load, working conditions, age and other factors. This deviation is expressed as Ratio Correction Factor (RCF). RCF is a complex number, which is the ratio of measured phasor to real phasor. This factor can also be divided into magnitude correction factor (MCF) and phase angle correction factor (PACF). Due to the synchronous and accurate phasor measurements provided by Synchronized Phasor Measurement Units (PMUs), it is possible to calculate this factor.

IEEE C57.13 stipulates the deviation range of current transformers and voltage transformers corresponding to different accuracy classes. Instrument transformers installed in power systems should meet the requirements of corresponding standards, and their deviations should be within the required range. However, this allowable deviation can still affect the accuracy of measurements. Therefore, it is of great significance to study effective methods to calibrate instrument transformers to ensure the accuracy of the measurement data.

Field calibration is the most direct method to eliminate the introduction error of instrument transformer. In this method, the high-precision reference instrument transformer needs to be transported to the site together with other calibration equipment. It needs connections and disconnections of many instrument terminals. In addition, the instrument transformers need to stop running during calibration. Therefore, this method with heavy workload and high cost is impractical and difficult to carry out regular in the whole system. To solve this problem, remote calibration methods have been extensively studied in many papers. As introduced in reference [2]-[5], most methods need a calibrated voltage instrument transformer as reference. But how to set the reference transformer is still a problem. In addition, the line...
parameters and noise are also influencing factors that need to be considered. Reference [6] introduces the method based on open circuit test. This method does not involve the line parameters in the calculation process, so it is not affected by the error of the line parameters, but it needs calibrated voltage transformer and current transformer as reference, which is a stringent demand. Moreover, the main problem is that it is impossible to open the circuit at will in the actual system, so it is difficult to implement in practice. Reference [7] presents a novel approach for online calibration of PMU by using density-based spatial clustering. It can identify the overall bias errors introduced by both PMU and its instrumentation channel and does not require accurate system parameters. However, the influence of transformer deviation is not considered.

In this paper, a method of instrument transformer deviation detection and calibration based on synchronized phasor measurements is proposed. With the system parameters and measurements from PMUs, the instrument transformer deviation is evaluated, and those with large deviation are further calibrated to obtain RCF value, which realizes high-precision calibration. This method improves the robustness of calibration algorithm, and reduces the influence of system uncertainty and noise.

2. Materials and Methods

2.1. System model
Considering the two-bus system shown in Figure 1. The basic method will be developed based on this system and then extended to a larger network. If PMUs are placed on each end of the line, then measurements of two voltage phasor ($\tilde{V}_s$ and $\tilde{V}_r$) and two current phasor ($\tilde{I}_s$ and $\tilde{I}_r$) are obtained. Using Kirchhoff’s laws, the following equations can be given:

$$\tilde{I}_s - \frac{Y}{2} + \tilde{I}_r - \frac{Y}{2} = 0$$
$$\tilde{V}_s - Z \cdot (\tilde{I}_s - \frac{Y}{2}) - \tilde{V}_r = 0$$

(1)

Figure 1 Two-bus system model

Considering the RCFs of instrument transformers, voltage and current phasor can be written as:

$$\tilde{V}_s = \tilde{R}_{V_s} \cdot \tilde{V}_{sm}, \quad \tilde{I}_s = \tilde{R}_{I_s} \cdot \tilde{I}_{sm}$$
$$\tilde{V}_r = \tilde{R}_{V_r} \cdot \tilde{V}_{rm}, \quad \tilde{I}_r = \tilde{R}_{I_r} \cdot \tilde{I}_{rm}$$

(2)

In (2), $\tilde{R}_{V_s}, \tilde{R}_{V_r}, \tilde{R}_{I_s}$ and $\tilde{R}_{I_r}$ are the RCFs of voltage transformers and current transformers. The subscript $m$ represents the measured values, which are the phasor measurements provided by PMUs.

2.2. Detection method of deviation
The line parameters can be obtained from (1):
The true values of voltage and current in (3) can be replaced by measured values and RCFs according to (2). Substituting each phasor in (3) with its magnitude and phase angle, and setting the phase angle of current at the r end as the reference, the line parameters can be derived:

\[
R = \text{real} \left( \frac{R_{Vr}^2 V_s^2 e^{i(\theta_{Vr} + \theta_{Rr})} - R_{Vs}^2 V_r^2 e^{i(\theta_{Vs} + \theta_{Rr})}}{R_{Vr} R_{Vs} I_s V_r e^{i(\theta_{Vr} + \theta_{Rs} + \theta_{Rr})} - R_{Vs} R_{Vr} I_s V_r e^{i(\theta_{Vs} + \theta_{Rs} + \theta_{Rr})}} \right)
\]

\[
X = \text{imag} \left( \frac{R_{Vr}^2 V_s^2 e^{i(\theta_{Vr} + \theta_{Rr})} - R_{Vs}^2 V_r^2 e^{i(\theta_{Vs} + \theta_{Rr})}}{R_{Vr} R_{Vs} I_s V_r e^{i(\theta_{Vr} + \theta_{Rs} + \theta_{Rr})} - R_{Vs} R_{Vr} I_s V_r e^{i(\theta_{Vs} + \theta_{Rs} + \theta_{Rr})}} \right)
\]

\[
B = \text{imag} \left( \frac{R_{Vr} I_s e^{i(\theta_{Vr} + \theta_{Rs})} + R_{Vs} I_s e^{i(\theta_{Vs} + \theta_{Rs})}}{R_{Vr} V_s e^{i(\theta_{Vr} + \theta_{Rs})} + R_{Vs} V_r e^{i(\theta_{Vs} + \theta_{Rs})}} \right)
\]

To study the sensitivity of line parameters to RCFs, partial derivatives are taken and the following equation can be derived:

\[
\begin{bmatrix}
\frac{\partial R}{\partial X} \\
\frac{\partial R}{\partial B}
\end{bmatrix} =
\begin{bmatrix}
A_R & B_R & C_R & D_R & E_R & F_R & G_R & H_R \\
A_X & B_X & C_X & D_X & E_X & F_X & G_X & H_X \\
A_B & B_B & C_B & D_B & E_B & F_B & G_B & H_B
\end{bmatrix}
\begin{bmatrix}
\frac{\partial R_{Vs}}{\partial X} \\
\frac{\partial R_{Vr}}{\partial R_{Vs}} \\
\frac{\partial R_{Ir}}{\partial R_{Vs}} \\
\frac{\partial \theta_{Rs}}{\partial R_{Vs}} \\
\frac{\partial \theta_{Rs}}{\partial \theta_{Rs}} \\
\frac{\partial \theta_{Rs}}{\partial R_{Vs}}
\end{bmatrix}
\]

(5)

In (5), the factors A ~ H are the partial derivatives of the line parameters for each quantity to be investigated. Taking R as an example, these factors are:

\[
\frac{\partial R}{\partial X} = A_R,
\frac{\partial R}{\partial B} = B_R,
\frac{\partial R}{\partial C} = C_R,
\frac{\partial R}{\partial D} = D_R,
\frac{\partial R}{\partial E} = E_R,
\frac{\partial R}{\partial F} = F_R,
\frac{\partial R}{\partial G} = G_R,
\frac{\partial R}{\partial H} = H_R
\]

Assuming that N sets of PMU data under different load conditions are given, the RCFs of the instrument transformer can be estimated using least squares algorithm. However, due to the coupling between the instrument transformer parameters in the equation, only one local optimal solution can be obtained here, and there is an overall offset, so the required quantity is treated as follows:
\[
R'_{Vs} = \frac{R_{Vs}}{R_t}, \quad R'_{Vr} = \frac{R_{Vr}}{R_t}, \quad R'_{Is} = \frac{R_{Is}}{R_t}
\]
\[
\theta'_{R_{Vs}} = \theta_{R_{Vs}} - \theta_{R_{Vr}}, \quad \theta'_{R_{Vr}} = \theta_{R_{Vr}} - \theta_{R_{Is}}, \quad \theta'_{R_{It}} = \theta_{R_{It}} - \theta_{R_{t}}
\]

Substitute it into the system model again, and the following equation can be obtained:

\[
\begin{bmatrix}
\frac{\partial R}{\partial X} \\
\frac{\partial X}{\partial B}
\end{bmatrix} = \begin{bmatrix}
A_R & B_R & C_R & D_R & E_R & F_R \\
A_X & B_X & C_X & D_X & E_X & F_X \\
A_B & B_B & C_B & D_B & E_B & F_B
\end{bmatrix} \begin{bmatrix}
\frac{\partial R'_{Vs}}{\partial X} \\
\frac{\partial R'_{Vr}}{\partial X} \\
\frac{\partial R'_{Is}}{\partial X} \\
\frac{\partial \theta'_{R_{Vs}}}{\partial X} \\
\frac{\partial \theta'_{R_{Vr}}}{\partial X} \\
\frac{\partial \theta'_{R_{It}}}{\partial X}
\end{bmatrix}
\]

The relation of magnitude ratio and phase angle difference of RCFs can be obtained using N sets of measurements under different load conditions. The magnitude parameter can be obtained after correction with the offset obtained from the ratio relation. The phase angle parameter can be obtained by selecting one instrument transformer as the phase angle reference.

If the accurate line parameters are known, the deviation of the instrument transformer can be obtained directly. However, accurate line parameters can not be obtained in practice, so the method described above can not get correct results. Considering the line parameter error, an error band factor \( \alpha \) is introduced, and the following constraints are obtained:[7]

\[
\begin{align*}
(1 - \alpha)R_0 & \leq R \leq (1 + \alpha)R_0 \\
(1 - \alpha)X_0 & \leq X \leq (1 + \alpha)X_0 \\
(1 - \alpha)B_0 & \leq B \leq (1 + \alpha)B_0
\end{align*}
\]

The feasible region for transmission line parameters is shown in Figure 2. The deviation values corresponding to each point in the region are clustered using DBSCAN to get the number of core points and the searching distance. The point with maximum number of core points and minimum searching distance can be obtained using the data filter[7]. This point corresponds to the deviation of the line parameters, and its corresponding calculated data are the result of deviation detection.

![Figure 2 Feasible region for transmission line parameters](image-url)
2.3. Calibration method of instrument transformers
Rewrite (1) as follows:

\[
\begin{align*}
I_r &= V_r \cdot \frac{Y}{2} + (V_r - \bar{V}_r) \cdot \frac{1}{Z} \\
\bar{I}_r &= \bar{V}_r \cdot \frac{Y}{2} - (\bar{V}_r - \bar{V}_r) \cdot \frac{1}{Z} 
\end{align*}
\]  

(9)

On multiplying both equations of (9) by \(Z\), and on multiplying the first equation of (9) by \(W\) where \(W = (1 + \frac{Y}{2}Z)\) and rearranging we get:

\[
\begin{align*}
W^2\bar{V}_r - W\bar{V}_r - WZ\bar{I}_r &= 0 \\
W\bar{V}_r - Z\bar{I}_r &= V_s
\end{align*}
\]

(10)

Assuming that the voltage transformer on the s end is unbiased and other instrument transformers are biased, the following equation can be obtained:

\[
\begin{align*}
W^2\bar{V}_{sm} - W\bar{R}_{vr}\bar{V}_{rm} - WZ\bar{R}_{ls}\bar{I}_{sm} &= 0 \\
W\bar{R}_{vr}\bar{V}_{rm} - Z\bar{R}_{lr}\bar{I}_{rm} &= \bar{V}_{sm}
\end{align*}
\]

(11)

With \(N\) sets of measurements under different load conditions, it can be obtained:

\[
\begin{bmatrix}
\bar{V}_{sm1} - \bar{V}_{rm1} - \bar{I}_{sm1}  & 0 \\
0 & \bar{V}_{rm1} & 0 & \bar{I}_{rm1} \\
\bar{V}_{sm2} - \bar{V}_{rm2} - \bar{I}_{sm2}  & 0 \\
0 & \bar{V}_{rm2} & 0 & \bar{I}_{rm2} \\
.. & .. & .. & .. \\
\bar{V}_{smN} - \bar{V}_{rmN} - \bar{I}_{smN}  & 0 \\
0 & \bar{V}_{rmN} & 0 & \bar{I}_{rmN}
\end{bmatrix}
\begin{bmatrix}
W^2 \\
W\bar{R}_{vr} \\
WZ\bar{R}_{ls} \\
Z\bar{R}_{lr}
\end{bmatrix}
= \begin{bmatrix}
0 \\
\bar{V}_{sm1} \\
0 \\
\bar{V}_{sm2} \\
.. \\
.. \\
\bar{V}_{smN}
\end{bmatrix}
\]

(12)

The RCF of instrument transformers can be obtained by solving this equation. But this result will be affected by system uncertainty and noise. To solve this problem, H-infinity filter is introduced. Aiming at a dynamic system, the H-infinity filter enhances the anti-interference ability to unmodeled noise by considering the worst estimation error and minimizing it. The model studied in this paper is transformed into a dynamic system model, and the quantity to be solved in (12) is taken as the state quantity of the dynamic system. Take (12) as the system measurement equation. Applying the H-infinity filter, the following equation can be obtain:

\[
\begin{align*}
K_k &= P_k[I - \theta P_k + H_k^T R_k^{-1}H_k P_k]^{-1}H_k^T R_k \\
\tilde{x}_{k+1} &= \tilde{x}_k + K_k(y_k - H_k\tilde{x}_k) \\
P_{k+1} &= P_k[I - \theta P_k + H_k^T R_k^{-1}H_k P_k]^{-1} + Q_k
\end{align*}
\]

(13)

By solving this problem, the RCFs of the instrument transformers can be obtained.
3. Results & Discussion
This section introduces the simulation results to prove the effectiveness of the proposed method. In each simulation, the power flow solution of each load condition is taken as the real value, multiplying these values by the assigned RCFs, and then adding appropriate noise to generate phasor values. The RCFs of instrument transformers are randomly selected within the range specified in the standard.

3.1. Detection method performance

3.1.1. No error in line parameters
In this case, there is no error in the line parameters, different combinations of deviation are added to the instrument transformers and the proposed method is used to detect them. The results for four cases are listed in Table 1. The results show that the true value of the instrument transformer deviation is consistent with the calculated value, which proves the correctness of the proposed method under the ideal condition that the line parameters are accurately known.

Table 1 Detection results under ideal condition

| Deviation | True | Calculated | True | Calculated | True | Calculated | True | Calculated |
|-----------|------|------------|------|------------|------|------------|------|------------|
| $\partial R_{V_s}$ | 0 | 0.0032 | -10 | -9.9997 | -20 | -20.0684 | -20 | -19.9904 |
| $\partial R_{V_p}$ | 0 | 0.0030 | 10 | 9.9998 | 10 | 10.0731 | 20 | 19.9935 |
| $\partial R_{I_s}$ | -10 | -10.2 | 0 | 0.0014 | -10 | -10.0231 | 10 | 10.0029 |
| $\partial R_{I_p}$ | 0 | 0.0052 | 0 | 0.0013 | 0 | 0.0261 | -10 | -9.9953 |
| $\partial \theta'_{R_0}$ | 0 | -0.0189 | -17.5 | -17.5189 | 0 | -0.0186 | 17.5 | 17.4811 |
| $\partial \theta'_{R_0}$ | -17.5 | -17.5194 | 0 | -0.0194 | -17.5 | -17.5194 | -17.5 | 17.5194 |
| $\partial \theta'_{R_0}$ | 0 | -0.0163 | 0 | -0.0163 | 0 | -0.0163 | 17.5 | 17.4837 |

3.1.2. One line parameter has error
In this case, a 2% error is added to the line resistance, and the instrument transformer deviation is added to the voltage transformer on the s end. The result of DBSCAN and the relationship between detection results and error in $R$ are shown in Figure 3 and Figure 4. Table 2 gives the results of instrument transformer detection. It can be seen from the results that this method can evaluate the instrument transformer deviation when there are errors in the system line parameters, and provide reference for the subsequent calibration of instrument transformers.

Table 2 Detection results with error in $R$

| Deviation | True | Calculated |
|-----------|------|------------|
| $\partial R_{V_s}$ | -30 | -29.3218 |
| $\partial R_{V_p}$ | 0 | 0.0243 |
| $\partial R_{I_s}$ | 0 | 0.0329 |
| $\partial R_{I_p}$ | 0 | 0.0297 |
| $\partial \theta'_{R_0}$ | 0 | -0.0278 |
| $\partial \theta'_{R_0}$ | 0 | 0.0272 |
| $\partial \theta'_{R_0}$ | 0 | -0.0158 |
3.2. Calibration method performance

3.2.1. Case 1: Gaussian white noise in the measurements
In this case, 60dB Gaussian white noise is added to the measurements, and the voltage transformer on s end is set as a reference. Calibration is carried out using the method of reference [2] (M1) and the method proposed in this paper (M2). The results are shown in Figure 5 and Figure 6.
It can be seen that the results with M1 fluctuate greatly under the influence of noise. Although it can be suppressed by mean filtering, there are still some fluctuations, and it needs more data points to achieve better result. However, the results with the method proposed in this paper fluctuate very little and can converge to the true value quickly and stably.

3.2.2. Case 2: Other kinds of noise in the measurements

When there are other kinds of noise in the system, the distinction will be more obvious. In this case, the noise with student’s-t distribution is added to the measurements, and the voltage transformer on s end is set as a reference. Calibration is carried out using M1 and M2. The results are shown in Figure 7 and Figure 8.
The results show that, under other noises, the mean filtering can't handle them well. It can be seen that the fluctuation and error of the calculation results with M1 become larger, while the calculation results can still converge stably with the method proposed.

3.2.3. Influence of different noise strength
Add noise with different strength to the measurements and compare the calibration results with the two methods, it is shown in Figure 9.

![Figure 9 Calibration results under different strength of noise](image)

It can be seen that the result with M1 is greatly affected by the noise strength. As the noise strength increases, the results get worse, while the result with the method proposed is basically stable under different noise strength.

The simulation above show that the method proposed in this paper can calibrate instrument transformers with high precision and has strong robustness to system noise.

4. Conclusions
In this paper, a method of detection and calibration of instrument transformer deviation based on PMU measurements is proposed. This method doesn't need a specific model of instrument transformer, and a calibrated accurate instrument transformer as a reference. It can evaluate the status of the instrument transformers running in power system through deviation detection, and provide a basis and reference for subsequent instrument transformer calibration. In the process of calibration, it can eliminate the influence of system uncertainty and noise, and has strong robustness.

As a remote calibration method, it can be often implemented as needed to monitor the status of instrument transformers, adjust parameters in time and effectively improve the accuracy and reliability of measurements. Having been tested by simulation, it remains to be seen how the method performs in actual system.

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