Pion Breather States in QCD

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Abstract

We describe a class of pionic breather solutions (PBS) which appear in the chiral lagrangian description of low-energy QCD. These configurations are long-lived, with lifetimes greater than $10^3$ fm/c, and could arise as remnants of disoriented chiral condensate (DCC) formation at RHIC. We show that the chiral lagrangian equations of motion for a uniformly isospin-polarized domain reduce to those of the sine-gordon model. Consequently, our solutions are directly related to the breather solutions of sine-gordon theory in 3+1 dimensions. We investigate the possibility of PBS formation from multiple domains of DCC, and show that the probability of formation is non-negligible.

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1 Introduction

In recent work [1], we studied the evolution of domains of disoriented chiral condensate (DCC) [2, 3] using the chiral lagrangian as a controlled, long-wavelength description. (For recent reviews of work on DCC, see [4, 5]. Other studies of DCC behavior are described in [6]–[17].) Our main interest in [1] was the effect of multiple domain interactions on isospin fluctuations and the experimental signatures of DCC. As a byproduct of that investigation, we discovered a class of long-lived classical pion configurations which we shall refer to as Pion Breather States (PBS) or Pion Balls. In this letter we report in more detail on the properties of PBS and investigate further the possibility of their formation in heavy ion collisions. PBS formation would likely lead to a variety of dramatic signals at colliders such as RHIC, and provide striking evidence for coherent, long-wavelength phenomena involving the QCD vacuum.

We also note the equivalence between the chiral lagrangian dynamics of a uniformly isospin-polarized domain and the sine-gordon model. This equivalence implies that our PBS configurations are related to previously known 3+1 sine-gordon breathers [18].

2 Pion Dynamics

The evolution of soft pions is described by the chiral lagrangian, the unique low-energy effective lagrangian with the symmetries of QCD. To order $O(p^2)$ the lagrangian is

$$\mathcal{L} = \frac{F_\pi^2}{4} tr(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) + F_\pi^2 m_\pi^2 tr \Sigma^\dagger + F_\pi^2 m_\pi^2 tr \Sigma,$$

where

$$\Sigma = e^{i\bar{\pi} \cdot \vec{\tau}/F_\pi},$$

$\bar{\pi}$ represents the three pion fields, $\vec{\tau}$ are the three Pauli matrices, and $F_\pi$ is the pion decay constant. An explicit chiral symmetry breaking term has been included to give the three pion types nonzero but degenerate masses. We neglect isospin breaking and electromagnetism in what follows.

Since higher order terms and quantum loop effects are suppressed by powers of the spacetime derivatives, the behavior of soft semiclassical configurations can be approximated by the classical equations of motion derived from this lagrangian. In [1] we described a numerical technique which avoids the constrained equations of motion resulting from (1) by using the linear sigma model in the limit of large self-coupling. The same technique will be used for simulations of multi-pion evolution in this paper.
For pion configurations which are uniformly isospin-polarized, the dynamics simplifies dramatically. Using the identity
\[ e^{i \vec{A} \cdot \vec{r}} = \cos(|A|) + i \frac{\vec{A} \cdot \vec{r}}{|A|} \sin(|A|) \] (3)
equation (1) can be written
\[ \mathcal{L} = \frac{1}{2} S(|\vec{\pi}|)^2 \partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} + \frac{1}{2} \left( 1 - S(|\vec{\pi}|)^2 \right) \frac{(\vec{\pi} \cdot \partial^\mu \vec{\pi})^2}{\pi^2} + F_\pi^2 m_\pi^2 \cos \left( \frac{|\vec{\pi}|}{F_\pi} \right) \] (4)where
\[ S(|\vec{\pi}|) = \frac{F_\pi}{|\vec{\pi}|} \sin \left( \frac{|\vec{\pi}|}{F_\pi} \right). \]
It is easy to see that if any two components of the pion field are set to zero, the equations of motion guarantee that they remain zero. Thus a perfectly isospin-polarized domain remains so under dynamical evolution. Without loss of generality we can consider a configuration which is polarized in the \( \pi_0 \) direction, yielding
\[ \mathcal{L} = \frac{1}{2} \partial^\mu \pi_0 \partial_\mu \pi_0 + F_\pi^2 m_\pi^2 \cos \left( \frac{\pi_0}{F_\pi} \right), \] (5)
which is simply the sine-gordon lagrangian. Exact breather solutions are known for this model in 1+1 dimensions [19], and their 3+1 dimensional counterparts have been studied numerically [18]. Of course, we do not expect to find completely isospin-polarized configurations in realistic situations. The study of multiple domain configurations require that we retain (1) or (4). Below we study PBS formation from single as well as multiple domain initial conditions.

3 Single Domain Evolution

We studied the evolution of single domains with spherical as well as cylindrical symmetry. The spherical iniital conditions were taken as
\[ |\vec{\pi}(r, z)| = A \, g(r - a), \] (6)
where \( g(x) \) is a smoothed step function given by
\[ g(x) = 1 / \left( 1 + e^{k_1(x)} \right) \] (7)
or
\[ g(x) = A \left( \frac{2}{\pi} \right) \tan^{-1}(e^{-x}) \]. \tag{8}

Both cases were used, without appreciably affecting the final results. The value of \( a \) was chosen to fix the configuration size at 10 fm, and the value of \( k_1 \) was chosen to fix the 'skin' size at approximately 1 fm. In the cylindrical case we used the functional form
\[ |\vec{\pi}(r, z)| = A |z - b| g(r - a) \]. \tag{9}

For sufficiently low initial strengths \( s \), the pions in the initial configuration are essentially non-interacting, and the energy disperses on a timescale of order the initial domain size. However, at higher initial densities we observed PBS formation. Figure 1 shows a plot of the central energy vs. time (we define central energy in this paper as the energy within a fiducial volume of radius 20 fm centered at the origin) for varying overal initial field strengths. As predicted, for small field strengths the central energy drops to a small fraction of its initial value on time scales of 100 fm, a bit larger than the initial configuration size.

![Figure 1: Energy in central region vs. time](image)

As the initial field strength is increased, PBS are formed, and their lifetime increases rapidly to a maximum of 1200 fm. During their lifetime the central energy remains almost constant at \( \approx 26 \) GeV. Inspection of the field evolution reveals an initial chaotic period during which packets of energy are radiated away, finally leaving a central region with nearly periodic oscillations of unique pattern and characteristic shape (see figure 2). The PBS can oscillate through of order 100 cycles of period \( \sim 10 \) fm, amplitude \( 5.4F_\pi \) and radius \( \approx 4 \) fm.
more or less unchanged. The configuration breaks apart when small perturbations in the oscillation pattern suddenly grow causing the shape to fall apart quickly, again on timescales of order of 100 fm. The decay generates approximately 200 pions.

![Ball 1: Downswing](image)

**Figure 2: Evolution of the lower energy PBS**

The ability of an initial configuration to form a PBS seems unpredictable, aside from the obvious condition that the initial energy be greater than that of the PBS. Lifetimes of resultant PBS are small just above threshold and grow with increasing energy to the maximum value. At higher energies we observed a second more energetic PBS with energy \( \approx 112 \, \text{GeV} \) and lifetime \( \approx 4000 \, \text{fm} \). This configuration has a frequency of oscillation of \( 0.0619 \) cycles/fm, amplitude \( 12.9 F_{\pi} \) and approximate radius of 5 fm (see figure 3).

The outcome of any given run is to either create one of these two PBS states, or to disperse the energy by around 100 fm. In some cases, the more energetic PBS may decay to the other before the energy fully escapes. We searched for novel behavior in larger and more energetic configurations, but none was found, suggesting the absence of a third type of PBS with energy less than 4000 GeV. Low energy cylindrically symmetric simulations were performed and yielded results similar to the spherical case, with the less energetic PBS observed.
4 Multiple Domains

To get a better understanding of PBS formation in more realistic circumstances we studied multiple domain initial conditions. For simplicity, we restricted ourselves to two and three domain configurations with cylindrical symmetry. Each configuration consists of two or three cylinders, stacked along the $\hat{z}$ axis, of radius 5.5 fm and height 11 fm. The isospin orientation within each cylinder is uniform, interpolating to the vacuum orientation at large $r$ and $|z|$. The isospin orientation varies between domains as

$$\hat{\pi}(r, z) = \frac{2}{\alpha} \arctan(k_2 z) .$$  

Note that isospin symmetry allows the configuration to be rotated so that the two domains fall only in the $\pi_0 - \pi_+\pi_-$ plane, so only one angle need be specified. With multiple domains this generalizes in an obvious manner, by summing step functions with different offsets.

We performed simulations in the two domain case, with angle $\alpha$ between the directions of the domains in pion isospin space, and $A$ the overall field strength. The central energy vs. field strength at time 70 fm is shown in figure 4. 70 fm was chosen as a time by which the no-PBS case will have dispersed, but the PBS will not have. In the single domain case the PBS is formed sporadically but consistently when the initial energy is above threshold. However, even a slight misalignment in the two domain case suppresses PBS formation for energies much greater than threshold. The remaining peak of formation exists for (roughly)
\( \alpha < 90^\circ \), in about 50\% of the cases. For three domains, we continued to find PBS formation for a significant fraction of the cases we sampled, roughly 25\%. Again, an approximate rule of thumb was that the PBS formation peak exists when the total change in the isospin orientation is less than 90\(^\circ\) over all the domains.

![PBS 'resonance' plots for varying angle](image)

**Figure 4:** PBS formation vs. initial field strength

Finally, we tested the effect of incoherent contamination on the initial conditions. To generate incoherent contamination we superposed sinusoidal field configurations with random phases, multiplied by the same shape function of our coherent initial condition. We found that the incoherent contamination is very destructive, decreasing significantly the regions of phase space that led to PBS formation. Nevertheless, significant regions of PBS formation were found for contamination of order 10 percent. We intend to investigate these effects in more detail in future work [20].

5 Discussion

Our investigations indicate the following PBS properties: they are long-lived, can form from a robust variety of initial conditions, and could provide a dramatic signal of non-linear QCD vacuum dynamics at relativistic heavy-ion colliders. We intend to report in more detail on these properties in future work [20]. However, several questions of a theoretical nature remain unanswered. A better, analytical, understanding of solutions to the sine-gordon equations in 3+1 dimensions would be welcome. In particular, we would like to know whether exactly
stable solutions with infinite lifetimes exist. One might imagine that such solutions could be related to previously known time-dependent, non-dissipative configurations [21, 22]. The criteria for stability of such configurations is generically

\[ E < Qm , \]  

where \( E \) is the total energy, \( Q \) a conserved charge (in our case, isospin) and \( m \) the mass of an asymptotic particle state. However, one can show [20] that with the chiral lagrangian (1) the inequality (11) is never satisfied, so any non-dissipative solutions are at best metastable.

Our attempts to find analytical solutions have so far have been unsuccessful. In particular, we applied the technique described in [23] to generate the set of continuous symmetries for our differential equations. This yielded that the only continuous symmetry of the spherically symmetric 3+1 Sine-Gordon equation is time translation, which corresponds to no non-trivial solution.

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