Universality Class of Confining Strings

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A recently proposed model of confining strings has a non-local world-sheet action induced by a space-time Kalb-Ramond tensor field. Here we show that, in the large-$D$ approximation, an infinite set of ghost- and tachyon-free truncations of the derivative expansion of this action all lead to $c = 1$ models. Their infrared limit describes smooth strings with world-sheets of Hausdorff dimension $D_H = 2$ and long-range orientational order, as expected for QCD strings.
1. Introduction

Nonwithstanding the large amount of evidence suggesting the possibility of a string description of quark confinement, a consistent model of non-critical strings has yet to be found. The simplest possibility, provided by the Nambu-Goto string, can be consistently quantized only in $D = 26$ or $D \leq 1$ dimensions due to the conformal anomaly. Large world-sheets in Euclidean space crumple and the model is inappropriate to describe the expected smooth strings dual to QCD. In an attempt to cure this problem, a naively marginal term proportional to the square of the extrinsic curvature was added to the action. The resulting rigid string is, however, different from the Nambu-Goto string only in the ultraviolet region, since the new term turned out to be infrared irrelevant. Thus the rigidity did not really help in preventing crumpling.

Recent progress in this field is based on two types of actions. A first model of confining strings is based on an induced string action which can be explicitly derived for compact QED and for Abelian-projected $SU(2)$. In its non-local formulation, the model was independently proposed in [11]. A second proposal, analyzed further in [13], is based on a string action in a five-dimensional curved space-time with the quarks living on a four-dimensional horizon. Both proposals, whose interrelation has been investigated in [14], enjoy the necessary zigzag invariance of QCD strings.

In its world-sheet formulation, the induced string possesses a non-local action with negative stiffness just as the world-sheets of magnetic strings of the Abelian Higgs model in the London limit of infinite Higgs mass. Such an action may be brought to a quasi-local form via a derivative expansion of the interaction between the surface elements. For a conventional renormalization group study of the geometric properties of the fluctuating world-sheets we truncate this derivative expansion. This makes the model non-unitary, but in a spurious way. In the truncated action the stiffness is negative, so that a stable truncation must include at least a sixth-order term in the derivatives. In [16] we have shown that this term has the desired properties of solving the infrared problem of Nambu-Goto and rigid strings in the large-$D$ approximation. While perturbatively irrelevant, it becomes relevant in the large-$D$ approximation, a phenomenon familiar from 3D Gross-Neveu model. It suppresses crumpling and the model has an infrared-stable fixed point corresponding to a tensionless smooth string whose world-sheet has Hausdorff dimension $D_H = 2$. The corresponding long-range orientational order is caused by a frustrated antiferromagnetic interaction between normals, a mechanism first recognized in [18] and confirmed by recent numerical simulations.
The purpose of this paper is to determine the universality class of confining strings determined by the finite-size scaling of the Euclidean effective action of the model [16] on a cylinder of (spatial) circumference $R$. In the limit of large $R$ this takes the form

$$\lim_{\beta \to \infty} \frac{S_{\text{eff}}}{\beta} = TR - \frac{\pi c(D - 2)}{6R} + \ldots ,$$

for $(D - 2)$ transverse degrees of freedom, the universality class being encoded in the pure number $c$. This suggests that the effective theory describing the infrared behaviour is a conformal field theory (CFT) with central charge $c$ [21]. In this case the number $c$ also fixes the Lüscher term [22] in the quark-antiquark potential:

$$V(R) = TR - \frac{\pi c(D - 2)}{24R} + \ldots .$$

By interchanging $R$ in (1.1) with the inverse temperature $\beta$ we obtain immediately the low-temperature behaviour of the model. We shall give an estimate of the deconfinement temperature as well as its range of validity.

Finally we shall generalize the results of [16] to higher truncations of the original non-local action and show that the universality class and the geometric properties of world-sheets are largely independent of the level of the truncation, implying the irrelevance of the truncation and the spurious non-unitarity deriving from it altogether.

2. Finite-size scaling

The truncated world-sheet model of confining strings proposed in [16] is defined in Euclidean space by the action

$$S = \int d^2 \xi \sqrt{g} \, g^{ab} \, D_a x_\mu \left( t - s D^2 + \frac{1}{m^2} D^4 \right) D_b x_\mu ,$$

where $g$ and $D_a$ represent, respectively, the determinant and the covariant derivatives with respect to the induced metric $g_{ab} = \partial_a x_\mu \partial_b x_\mu$ on the world-sheet $x(\xi_0, \xi_1)$. The first term represents a bare surface tension $2t$, while the second accounts for rigidity with a stiffness parameter $s$ which is negative when generated dynamically by a tensor field in four-dimensional space-time [14,15]. The last term ensures the stability of the model. Since it contains the square of the gradient of the extrinsic curvature matrices it suppresses the formation of spikes on the world-sheet. In the large-$D$ approximation it generates a string
tension proportional to the square mass $m^2$ which takes control of the fluctuations where the orientational correlation die off. For $t, s, m \to 0$, one reaches an infrared fixed-point describing tensionless smooth strings with long-range orientational order [16]. While the model (2.1) is a toy version of the action induced by an antisymmetric tensor field, it is known [23] that QCD strings possess a curvature expansion of exactly this type.

In this paper we shall analyze the leading large-$D$ behaviour of the effective action on a cylinder of (spatial) circumference $R$. This is the extension to our model of the calculations [24] for the Nambu-Goto string and [25,26] for the rigid string. Contrary to these papers, however, we consider periodic boundary conditions as in [27] in order to avoid the problem of a non-uniform saddle-point metric pointed out in [24,25]. In order to simplify analytic computations we shall moreover equate the stiffness to its fixed-point value from the beginning by setting $s = 0$ in (2.1).

The large-$D$ calculation requires the introduction of a Lagrange multiplier matrix $\lambda^{ab}$ enforcing the constraint $g_{ab} = \partial_a x_\mu \partial_b x_\mu$. The action (2.1) is thus extended to

$$S \to S + \int d^2 \xi \sqrt{g} \lambda^{ab} (\partial_a x_\mu \partial_b x_\mu - g_{ab}) .$$

The world-sheet is parametrized in a Gauss map as $x_\mu(\xi) = (\xi_0, \xi_1, \phi_i(\xi))$ with $i = 2, \ldots, D - 1$. Here $-\beta/2 \leq \xi_0 \leq \beta/2$ and $-R/2 \leq \xi_1 \leq R/2$ and $\phi_i(\xi)$ describe the $D - 2$ transverse fluctuations. We look for a saddle-point with diagonal metric $g_{ab} = \text{diag} (\rho_0, \rho_1)$ and Lagrange multiplier $\lambda^{ab} = \lambda g^{ab}$. With this ansatz the extended action becomes

$$S = A_{\text{ext}} \sqrt{\rho_0 \rho_1} \left[ (t + \lambda) \frac{\rho_0 + \rho_1}{\rho_0 \rho_1} - 2\lambda \right] + \int d^2 \xi \sqrt{g} g^{ab} \partial_a \phi^i \left( t + \lambda + \frac{1}{m^2} D^4 \right) \partial_b \phi^i ,$$

where $A_{\text{ext}} = \beta R$ is the extrinsic, projected area in coordinate space. By integrating over the transverse fluctuations we get, in the limit $\beta \to \infty$, an effective action

$$S_{\text{eff}} = S_0 + S_1 ,$$

$$S_0 = A_{\text{ext}} \sqrt{\rho_0 \rho_1} \left[ (t + \lambda) \frac{\rho_0 + \rho_1}{\rho_0 \rho_1} - 2\lambda \right] ,$$

$$S_1 = \frac{D - 2}{4\pi} \beta \sqrt{\rho_0} \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dp_0 \ln \left[ p^2 \left( t + \lambda + \frac{p^4}{m^2} \right) \right] ,$$

where

$$p^2 \equiv p_0^2 + \omega_n^2 , \quad \omega_n \equiv \frac{2\pi}{R \sqrt{\rho_1}} n .$$
By introducing the mass scale \( \mu = \sqrt{m^2 + \lambda} \) we can rewrite the sums and integrals in the one-loop contribution \( S_1 \) as

\[
S_1 = \frac{D - 2}{4\pi} \beta \sqrt{\rho_0} \left( S_1^0 + 2 \text{Re} \ S_1^\mu \right), \\
S_1^0 = \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx \ m \ln \left( x^2 + \frac{\omega_n^2}{m^2} \right), \\
S_1^\mu = \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx \ \ln \left( x^2 + \omega_n^2 + i\mu^2 \right),
\]

where \( \text{Re} \) denotes the real part. We shall dispose of the ultraviolet divergences in these quantities by analytic regularization. Defining first the logarithm as \( \ln x = \left[ -(d/d\beta) x^{-\beta} \right]_{\beta=0} \), and using the analytic interpolation of the integral \[28\]

\[
\int_{-\infty}^{+\infty} dx \ 1 \ (x^2 + q^2)^n = q^{1-2n} \Gamma \left( \frac{1}{2} \right) \Gamma \left( n - \frac{1}{2} \right) / \Gamma(n)
\]

to any real \( n \), leads to the following formula for the regularized integrals:

\[
\int_{\text{reg}} dx \ \ln \left( x^2 + a^2 \right) = 2\pi a.
\]

The sums are then regularized by analytic continuation of the formula \( \sum_{n=1}^{\infty} n^{-z} = \zeta(z) \) for the Riemann zeta function \[28\]. Using \( \zeta(-1) = -1/12 \) one obtains immediately

\[
S_1^0 = -\frac{2\pi^2}{3R\sqrt{\rho_1}},
\]

which leads to the well known results for the Nambu-Goto \[24,29\] and the rigid \[27\] strings.

The computation of \( S_1^\mu \) is a bit more involved. First we represent the right-hand side of \( (2.7) \) by the analytic continuation of the following integral representation \[28\] of the gamma function,

\[
\frac{1}{(x^2 + q^2)^s} = \frac{1}{\Gamma(s)} \int_0^{\infty} dt \ t^{s-1} \exp \left[ -(x^2 + q^2) t \right].
\]

We then substitute the sum in \( S_1^\mu \) by an equivalent expression by means of the duality relation \[30\]

\[
\sum_{n=-\infty}^{+\infty} \exp (-n^2 t) = \sqrt{\frac{\pi}{t}} \sum_{n=-\infty}^{+\infty} \exp \left( -\frac{\pi^2 n^2}{t} \right).
\]
Separating out the $n = 0$ term in the sum and using the representation
\[
K_\nu \left( 2\sqrt{\beta \gamma} \right) = \frac{1}{2} \left( \frac{\gamma}{\beta} \right)^{\frac{\nu}{2}} \int_0^\infty dx \ x^{\nu-1} \exp \left( -\gamma x - \frac{\beta}{x} \right)
\]
(2.12)
of the modified Bessel function \[28\] in the remainder we find
\[
S_1^\mu = \frac{\pi \mu^2 R \sqrt{\rho_1}}{4} - \sum_{n=1}^\infty \frac{4 \sqrt{\mu^2}}{n} K_1 \left( R_n \sqrt{i \mu^2 \rho_1} \right).
\]
(2.13)
Altogether, we obtain the effective action on the cylinder:
\[
S_{\text{eff}} = \beta R \sqrt{\rho_0 \rho_1} \left[ (t + \lambda) \frac{\rho_0 + \rho_1}{\rho_0 \rho_1} - 2\lambda + \frac{D - 2}{2} \frac{\mu^2}{4} \right] - \frac{D - 2}{2} \frac{\beta \sqrt{\rho_0}}{2\pi} \left[ \frac{2\pi^2}{3R \sqrt{\rho_1}} + \text{Re} \left[ \sum_{n=1}^\infty \frac{8 \sqrt{\mu^2}}{n} K_1 \left( nR \sqrt{i \mu^2 \rho_1} \right) \right] \right].
\]
(2.14)
Being interested only in the large-$R$ behaviour, we may neglect the exponentially small terms arising from the Bessel functions and arrive at the relevant approximation to $S_{\text{eff}}$ to be used in the remaining computation:
\[
S_{\text{eff}} = \beta R \sqrt{\rho_0 \rho_1} \left[ (t + \lambda) \frac{\rho_0 + \rho_1}{\rho_0 \rho_1} - 2\lambda + \frac{D - 2}{2} \frac{\mu^2}{4} \right] - \frac{D - 2}{2} \frac{\pi \beta}{3R} \frac{\rho_0}{\rho_1}.
\]
(2.15)
The factor $(D - 2)$ in $S_{\text{eff}}$ ensures that, for large $D$, the fields $\lambda$, $\rho_0$ and $\rho_1$ are extremal and satisfy thus the saddle-point (“gap”) equations
\[
\frac{\rho_0 + \rho_1}{\rho_0 \rho_1} - 2 + \frac{D - 2}{2} \frac{\mu^2}{8(t + \lambda)} = 0,
\]
(2.16)
\[
\frac{t + \lambda}{2} \left( \frac{1}{\rho_1} - \frac{1}{\rho_0} \right) - \lambda + \frac{D - 2}{2} \frac{\mu^2}{8} - \frac{D - 2}{2} \frac{\pi}{6R^2 \rho_1} = 0,
\]
(2.16)
\[
\frac{t + \lambda}{2} \left( \frac{1}{\rho_0} - \frac{1}{\rho_1} \right) - \lambda + \frac{D - 2}{2} \frac{\mu^2}{8} + \frac{D - 2}{2} \frac{\pi}{6R^2 \rho_1} = 0.
\]
(2.16)
Substituting the second of these equations in (2.15) we obtain the simplified form of the effective action
\[
S_{\text{eff}} = \beta R \frac{T}{\sqrt{\rho_1 / \rho_0}},
\]
(2.17)
with $T \equiv 2(t + \lambda)$ being the physical string tension.

The saddle-point equations are easily solved as follows. The sum of the last two equations yields an equation for $\lambda$ alone,
\[
\lambda - \frac{D - 2}{2} \frac{\mu^2}{8} = 0.
\]
(2.18)
Using $\mu^2 = m\sqrt{t + \lambda}$ this leads to the following solution for the string tension $T = 2(t + \lambda)$:

$$T = \frac{a^2}{32} \left( \frac{D - 2}{2} \right)^2 m^2,$$

$$a^2 \equiv \frac{1 + 128 \left( \frac{2}{D-2} \right)^2 \frac{t}{m^2} + \sqrt{1 + 256 \left( \frac{2}{D-2} \right)^2 \frac{t}{m^2}}}{2},$$

reproducing the result of [16]. In a second step we subtract the second equation from the third, and multiply the result by $2\rho_1/T$, obtaining

$$\frac{\rho_1}{\rho_0} = 1 - \frac{\pi(D - 2)}{3TR^2}. \quad (2.20)$$

Expanding the square-root of this expression and multiplying by $\beta RT$ we obtain the final result

$$\frac{S_{\text{eff}}}{\beta} = TR - \frac{\pi(D - 2)}{6R} + \ldots. \quad (2.21)$$

Thus we conclude that confining strings are characterized by $c = 1$. Although they share the same value of $c$, confining strings are clearly different $c = 1$ theories than Nambu-Goto or rigid strings. Indeed the former are smooth strings on any scale while the latter crumple and fill the ambient space, at least in the infrared region. Our result $c = 1$ is in agreement with recent precision numerical determinations [31] of this constant.

3. The deconfinement temperature

By changing $R$ into $\beta$ and $\rho_0$ into $\rho_1$ in the above formulas we obtain the behaviour of the model (2.1) at temperature $T = 1/k_B\beta$. Having neglected the contribution of the Bessel functions in (2.14), however, we can only study low temperatures, with $\beta\mu\sqrt{\rho_0} > 1$. Using (2.17) and (2.20) we get

$$\left( \frac{S_{\text{eff}}}{R} \right)^2 = \beta^2 T^2 - \frac{\pi(D - 2)T}{3}. \quad (3.1)$$

Raising the temperature, this quantity, representing the square mass of the lowest state, crosses zero at an inverse temperature

$$\beta_{\text{dec}} = \sqrt{\frac{\pi(D - 2)}{3T}} = \frac{1}{m} \sqrt{\frac{128\pi}{3(D - 2)a^2}}, \quad (3.2)$$
which specifies the deconfinement temperature of the model. Note that this result coincides with the corresponding one for Nambu-Goto \cite{29} and rigid \cite{27} strings when expressed in terms of the string tension.

In order to establish the range of validity of this result we need to know the value of $\sqrt{\rho_0}$. This is obtained by substituting (2.20) into the first of the saddle-point equations (2.16), yielding

$$
\rho_0 = \frac{2a \left( 1 - \frac{\pi (D-2)}{6 \beta^2} \right)}{2a - 1}.
$$

(3.3)

At the deconfinement temperature this becomes

$$
(\rho_0)_{\text{dec}} = \frac{a}{2a - 1}.
$$

(3.4)

The value (3.2) of the deconfinement temperature is consistent with our approximation only if the equation $\beta_{\text{dec}} \sqrt{\rho_{0\text{dec}}} > 1$ is satisfied. Only then can we neglect the Bessel functions down to the deconfinement transition. Using the above value of $\beta_{\text{dec}}$ and $\rho_{0\text{dec}}$ this condition translates into

$$
a < \left( \frac{8\pi}{6} - \frac{1}{2} \right) \simeq 3.5.
$$

(3.5)

Thus, formula (3.2) for the deconfinement temperature is reliable in the region of small $(t/m^2)$ where $a \simeq 1$. Otherwise there are sizable corrections from the sum over $n$ in (2.14).

4. Generalization to higher truncations

Having established that the model (2.1) describes smooth strings with $c = 1$, the question arises as to how much these results depend on the truncation of the original non-local action after the $D^4$ term. To answer this question let us consider instead of (2.1) an arbitrary truncation

$$
S|_n = \int d^2 \xi \sqrt{g} g^{ab} D_a x_\mu \, V_n(D^2) \, D_b x_\mu,
$$

(4.1)

$$
V_n(D^2) = (\alpha_0 + \lambda) \Lambda^2 + \sum_{k=1}^{2n} \frac{\alpha_k}{\Lambda^{2k-2}} D^{2k}.
$$

Here $\Lambda$ represents the fundamental mass scale in the model, to be identified with the QCD mass scale, and we have already included in the action the Lagrange multiplier $\lambda$ arising from (2.2) (note that here we have defined $\lambda$ as a dimensionless quantity). Since all
expansion coefficients $\alpha_k$ are positive, the series is alternating in momentum space, with all terms with odd index $k$ being negative \cite{11,18}. Thus, stable truncations must end with an even $k = 2n$.

Following \cite{16}, the only condition we shall impose on the coefficients $\alpha_k$ is the absence of both tachyons and ghosts. This requires that the Fourier-transform $V_n(p^2)$ has no zeros on the real $p^2$-axis. The polynomial $V_n(p^2)$ has thus $n$ pairs of complex-conjugate zeros in the complex $p^2$-plane.

For simplicity of computation we shall set all coefficients with odd $k$ to zero, $\alpha_{2m+1} = 0$ for $0 \leq m \leq n - 1$. This, however, is no drastic restriction since, as we shall now demonstrate, this is their value at the infrared-stable fixed point anyhow. With this simplification all pairs of complex conjugate zeros of $V_n(p^2)$ lie on the imaginary axis and we can represent $V_n(p^2)$ as

$$\Lambda^{4n-2} \frac{\alpha_{2n}}{\alpha_{2n}} V_n(p^2) = \prod_{k=1}^{n} (p^4 + \gamma_k^2 \Lambda^4), \quad (4.2)$$

with purely numerical coefficients $\gamma_k$. This expression substitutes $[p^4 + m^2(t + \lambda)]$ inside the logarithm in the one-loop contribution \cite{2.4}, which becomes

$$S_1 = \frac{D - 2}{4\pi} \frac{\beta \sqrt{\rho_0}}{\rho_0} \left( \sum_{k=1}^{n} 2 \text{Re} S_k^2 \right),$$

$$S_k^2 = \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx \ln \left( x^2 + \omega_l^2 + i\gamma_k \Lambda^2 \right). \quad (4.3)$$

Using \cite{2.13} and neglecting as before the Bessel functions for large $R$, we see that the only modification to \cite{2.15}, \cite{2.16} and \cite{2.17} due to the higher-order truncation is the substitution

$$\mu^2 \rightarrow \sum_{k=1}^{n} \gamma_k \Lambda^2. \quad (4.4)$$

The Lagrange multiplier $\lambda \Lambda^2$ and the string tension $T = 2(\alpha_0 + \lambda) \Lambda^2$ are now determined by the new saddle-point equation

$$\lambda - \frac{D - 2}{16} \sum_{k=1}^{n} \gamma_k = 0. \quad (4.5)$$

The value $c = 1$ for the universal term in \cite{2.21}, however, remains unchanged.
The new saddle-point equation for $\lambda$ is still polynomial, although of higher-order. The requirement that this polynomial “gap” equation has at least one solution on the real axis with $(\alpha_0 + \lambda) \geq 0$ provides the condition on the coefficients $\alpha_{2m}$, $0 \leq m \leq n$, that defines the universality class of confining strings at level $n$.

Note that with all $\alpha_{2m+1} = 0$ for $0 \leq m \leq n - 1$, no normalization scale needs to be introduced to define the one-loop term $S_1$. In other words, a scale introduced to properly define the logarithm in (2.4) would drop out at the end of the computation since the result does not contain logarithms. As a consequence, in a renormalization analysis as in [16], there are no anomalous dimensions and the infrared limit $\Lambda^2 \to 0$ of vanishing string tension is characterized by $\beta(\alpha_{2m}) = 0$ for $0 \leq m \leq n$. The point $\Lambda = 0$ is thus again an infrared-stable fixed point characterized by $\alpha_{2m+1} = 0$ for $0 \leq m \leq n - 1$, and $n + 1$ renormalization group invariant numerical coefficients $\alpha_{2m}$, $0 \leq m \leq n$, varying in a range where there exists a real solution to the “gap” equation.

The geometric properties of world-sheets in the vicinity of this point can be easily obtained by decomposing

$$\frac{1}{V_n(p^2)} = \frac{\Lambda^2}{\alpha_{2n}} \sum_{k=1}^{n} \frac{\eta_k}{p^4 + \gamma_k^2 \Lambda^2}.$$ (4.6)

This decomposition is always possible since it is determined by a linear system of $n$ equations for the $n$ numerical coefficients $\eta_k$. At this point we can simply apply to each term in the above decomposition the discussion of [16] and conclude that the infrared point of vanishing tension is characterized by long-range orientational order and Hausdorff dimension $D_H = 2$ of world-sheets.

We have thus shown that $c$ and the smooth geometric properties are independent of an infinite set of truncations, provided that a solution for the polynomial “gap” equation exists. These properties are presumably common to a large class of non-local world-sheet interactions.
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