Prediction of shear wave velocity in underground layers using Particle Swarm Optimization

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Abstract. Shear wave velocity ($V_s$) is considered a key soil parameter in the field of earthquake engineering. The time-averaged shear wave velocity in the upper 30 m ($V_{s30}$) layer of soil is used to classify seismic site class. In-situ $V_s$ test is sometimes unsuitable to the project’s need due to financial reasons, noisy environment on site or simply the lack of expertise. This paper attempts to develop a global prediction model for $V_s$ using Standard Penetration Resistance ($N_{spt}$), depth ($z$) and soil type ($s_t$) as the independent parameters. Two approaches to modelling would be taken: a multi-linear regression (MLR) model and an ensemble (EN-PSO) model. The EN-PSO model attempts to improve upon the accuracy of the MLR model prediction ability using the ensemble learning method. A dataset was compiled from literatures for this paper. 5 Base models were developed: MLR, Random Forest (RFR), Support Vector Machine (SVR), Artificial Neural Network (ANN) and k-Nearest Neighbor (KNN) which are combined into an ensemble model named EN-PSO. The weights for EN-SPO was then calculated using Particle Swarm Optimization (PSO). The performance of each models were then compared and it was shown that EN-PSO was the best in terms of: MAE (Mean Absolute Error) = 22.085, MAPE (Mean Absolute Percentage Error) = 9.1 \%, RMSE (Root Mean Square Error) = 31.741 and $R^2$ Coefficient of Determination) = 0.895. In addition, it was also shown that the EN-PSO model was able to improve upon the performance of the MLR model, which the most accurate among the Base models. Comparisons were also made between EN-PSO and other suggested Universal $V_s$ correlations and EN-PSO was shown to outperform the other correlation based on prediction using a modified Test set. Three new empirical correlations as alternative for the EN-PSO model was also presented.

1. Introduction
Shear wave velocity ($V_s$) is considered a key soil parameter in the field of earthquake engineering. The time-averaged shear wave velocity in the upper 30 m ($V_{s30}$) layer of soil is used by various building codes and design standards to classify seismic site class, as recommended by the National Earthquake Hazards Reduction Program (NEHRP) provisions [1]. Although directly performing in-situ $V_s$ test up to the depth of 30 m is the most robust way to calculate $V_{s30}$, it is oftentimes unsuitable to the project’s need due to financial reasons, noisy environment on site or simply the lack of expertise. The most common alternative to estimate $V_{s30}$ is through $V_s$ correlation with other soil parameters.
Previous researchers have tried to establish these correlations based on parameters such as soil penetration resistance ($N_{spt}$), depth ($z$), type of soil ($s$), geological age and overburden pressure. Out of these, $N_{spt}$ consistently exhibits the strongest correlation with $V_s$. As a consequence, $V_s$-$N_{spt}$ correlations can be found abundantly in the literature, usually presented in the form of $V_s = aN_{spt}^b$, where $a$ and $b$ are site-dependent coefficients. These types of correlations are popular due to the fact that $N_{spt}$ are widely used for soil investigation therefore $N_{spt}$ data can be easily obtained for most sites. However, most of these studies are done on a regional basis and only a few [2] [3] have tried to develop a global model for estimation of $V_{s30}$.

This paper attempts to develop a global prediction model for $V_s$ using $N_{spt}$, $z$ and $s$ as the independent parameters. Two approaches to modelling would be taken; a multi-linear regression (MLR) model and an Ensemble-Particle Swarm Optimization (EN-PSO) model. The EN-PSO model is an attempt to improve upon the accuracy of the MLR model prediction ability by utilizing the ensemble learning method. However, unlike the MLR approach, it will not yield an empirical equation due to the “black-box” nature of machine learning (ML) algorithm used as the base model for EN-PSO.

This paper would also compare the performance of these 2 models (MLR and EN-PSO) with global correlation models suggested by previous researchers [2] [3]. In order to fairly evaluate the performance of each of these models, a randomly picked subset of the $V_s$ dataset would be kept away (not utilized in training or model development) as the Test set.

1.1. NEHRP Seismic Site Class

![NEHRP seismic site classification](image)

Figure 1 shows the site classes based on the value of $V_{s30}$. There are a total of 6 classes: Class A, Class B, Class C, Class D, Class E and Class F. The lower ranges of $V_{s30}$ are much more susceptible to ground motion when an earthquake occurs. Therefore, this site classification is essential for an engineer in order to determine the appropriate earthquake design’s safety factor. $V_{s30}$ is calculated using equation (1).

$$V_{s30} = \frac{\sum_{i=1}^{n} d_i}{\sum_{i=1}^{n} \sqrt{V_{si}}}$$

where $d_i$ is the thickness of layer $i$ and $V_{si}$ is the shear velocity in layer $i$.

1.2. Ensemble Learning

Ensemble learning refers to a set of procedures used to combine the output of multiple base models that would theoretically perform better than any single base models that constitutes the ensemble [4]. The essence of the method is that by combining the output of multiple model, the error of each base models are averaged out. Various empirical studies have shown ensemble models are oftentimes better in accuracy compared to its individual base models [5] [6] [7]. A few common types of ensemble method are bagging, boosting, averaging and stacking.
1.3. Particle Swarm Optimization

Particle swarm optimization (PSO) is a metaheuristic that can be used to find solutions to optimization problems through the process of stochastic optimization. PSO was presented with the original purpose of simulating social behaviors of organism’s group such as a swarm of bees [8]. Compared to other classic optimization method such as gradient descent and quasi-newton methods, PSO does not use gradients, therefore can be applied to problems which are not differentiable. A summary of PSO applications in engineering problems was reported by Tomasetti & Cagnina [9].

1.4. Suggested Universal Correlation

In order to validate the performance of the models developed in this study, four \( V_s-N_{sp} \) models by previous researchers were used as benchmark models. These correlations are selected as the benchmark due to the fact that it is specifically developed for estimating \( V_s \) regardless of region. Table 1 show the summary of the correlations.

| Source | [3] | [4] |
|--------|-----|-----|
| Model Name | AM-ALL | AM-SN-CL | AR-ALL | AR-SN-CL |
| Soil Type | All soil type, Cohesive, Cohesionless | All soil type, Sand, Clay |
| \( V_s \) | 77.13N_{sp}^{0.377} | 75.05N_{sp}^{0.388} | 91.87N_{sp}^{0.388} | 75.478N_{sp}^{0.3799} | 79.217N_{sp}^{0.3699} | 99.708N_{sp}^{0.3358} |

2. Methodology

2.1. Research Work Flow

Figure 2 shows the work flow for this research. The first phase consists of conducting an extensive literature review with the purpose of compiling suggested universal correlation and compiling data from published research paper. The suggested universal correlations chosen as the benchmark for this research were shown in Table 1 and were taken from two different papers [3] [4]. Each of those papers presented three different correlations categorized based on the soil type: all soil types, sand and clay.

These correlations were chosen specifically because they were developed for the purpose of estimating \( V_s \) regardless of region in comparison with most other correlations which were developed for a specific region. Meanwhile, the process of extracting and compiling data from published papers are described in section 2.2 and the list of papers can be referenced in Table 2. These data were then combined as the main dataset used for the purpose of training and testing the ML models.

After the main dataset is compiled, it was then split into three parts: Train set, Validation set and Test set. The Train set was used to train the base models, meanwhile the validation set was used for the purpose of evaluating the cost function of PSO during the process of optimizing the weights for equation (2). Finally, the Test set was used to calculate the performance metrics (Coefficient of Determination, Mean Absolute Error, Mean Absolute Percentage Error and Root Mean Square Error) for the purpose of comparison the performances of all the models and the benchmark correlations.
2.2. Compiled Dataset for Training and Testing

The dataset used to develop the models presented in this paper are compiled from various paper as shown in Table 2. It should be noted that some of the $V_s$ profile from these papers are not included in the dataset due to not having enough parameters of interest. Each $V_s$ measurement that was included in the dataset include information on $V_s$, $N_{sp}$, $z$ and $s_t$. The varied region of sources ensured that the developed models are not region dependent.
Table 2. $V_s$ measurement sources for each region

| Sources                        | No. of measurements | Region              |
|--------------------------------|---------------------|---------------------|
| Tran & Hiltunen (2008) [10]    | 48                  | United States of America |
| Dikmen (2009) [11]             | 12                  | Turkey              |
| Prakoso (2011) [12]            | 38                  | Indonesia           |
| Tsiambaos & Sabatakakis (2011) | 28                  | Greece              |
| Gautam (2016) [14]             | 98                  | Nepal               |

Meanwhile, Figure 3 shows the histograms of the parameters included in the dataset. Most of the $V_s$ measurements are in the range of 176-273 m/s with a total of 105 measurements as shown in Figure 3(a). The mean ($\mu$) of the $V_s$ range is 237.898 m/s with a standard deviation ($\sigma$) of 108.5. Figure 3(b) shows that an overwhelming majority of the $N_{spt}$ measurements are in the range of 1-13 blows per 300 mm, which make sense considering that most of the shallow $N_{spt}$ measurements are in this range while deep $N_{spt}$ measurements are scattered inconsistently depending on the site. The $\mu$ of the $N_{spt}$ measurements is 20 blows per 300 mm, meanwhile the $\sigma$ is 19.4. Meanwhile, figure 3(c) shows that most of the measurements are taken in the range of depth of 1-30 m, which can be explained due the fact that most of the investigation are most interested in top 30 m soil layer. Figure 3(d) shows that most common soil type in the dataset are of clay type followed by sandy soil.

![Figure 3](image)

2.3. Base Models for Ensemble Averaging

There are five base models which constitutes the EN-PSO model: Multi-Linear Regression model (MLR), Random Forest model (RFR), Support Vector Machine model (SVR), Artificial Neural Network model (ANN) and k-Nearest Neighbor Model (KNN). All of these models are trained using the Train
The models were programmed using Scikit-Learn [15] module in Python. The final output of the EN-PSO model are calculated using equation (2).

\[ \frac{\sum w_i + \text{Out}_i}{\sum w_i} + w_B \]  

where, \( w_i \) is the weight for model \( i \), \( \text{Out}_i \) is the output for model \( i \) and \( w_B \) is the bias term.

\( w_i \), \( w_A \) and \( w_B \) are calculated using PSO. The cost function used by PSO in this case is the mean absolute error of the prediction of EN-PSO on the Validation set. The Validation set was used instead of the Train set in order to avoid overfitting which would cause the model to not generalize well. The parameters of the PSO algorithm were set to 30 particles, \( c_1 = c_2 = 0.25 \), \( w = 0.9 \), boundary of (-10, 10) and a total of 20 runs of 140 iterations.

3. Result and Discussions

This section summarizes the discussion and results of this study. Subsection 3.1 shows the comparison between the performance of each Base Models (MLR, RFR, SVR, ANN, KNN) on the Test set. Meanwhile, subsection 3.2 will present the results of using PSO to find the optimal weights for equation 2. Afterwards, subsection 3.3 will discuss the difference in performance between MLR and EN-PSO using the test set, thus concluding whether or not the EN-PSO model was able to improve upon the performance of the MLR model. Next, subsection 3.4 will compare the performance between EN-PSO and the suggested Universal Correlation (refer subsection 1.4). The final subsection, subsection 3.5, would present the empirical equations derived from the MLR model and discuss the suitability of the equations for different soil types.

3.1. Performance of each Base Models

![Figure 4. Performance of each Base Models on the Test set](image_url)

It can be seen from figure 4 the MLR model outperforms the other Base Models, scoring the lowest in terms of mean average error (MAE), root mean squared error (RMSE) and mean absolute percentage error (MAPE) while also scoring the highest for coefficient of determination (R²). It should be noted that the score for each performance metrics was normalized to values in the range of 0.1-0.9 in order to enable comparisons across the four categories. Table 3 shows the unscaled value for the performance metrics score for each of the Base Models.
Table 3. Performance Metrics Scores for each Base Models

|          | MLR  | ANN  | KNN  | RFR  | SVR  |
|----------|------|------|------|------|------|
| MAE      | 24.80| 39.80| 34.05| 31.87| 34.31|
| RMSE     | 32.43| 49.66| 38.21| 44.04| 40.27|
| MAPE     | 0.101| 0.155| 0.157| 0.143| 0.143|
| $R^2$    | 0.89 | 0.78 | 0.85 | 0.81 | 0.85 |

From table 3, it can be seen that the MLR model scored the lowest in the MAE category. It is the only model that was able to score a value of under 25. Meanwhile, the RMSE category shows a similar trend. Again, the MLR model was the only model capable of reaching a score under the value of 35. In conjunction with that, the MAPE for the ML models was 0.101 which means that the error for the MLR model was 10.1%, the lowest compared to all of the Base Models. Finally, in the $R^2$ category, the MLR model scored the highest in the category with a value of 0.89. This shows that the MLR model was able to fit well over the distribution of the Test set. Subsection 3.3 would later discuss whether or not the EN-PSO model was able to improve upon the performance of the MLR model.

3.2. Optimal weights for EN-PSO outputs

Table 4. Results of using PSO to find the optimal weights (Top five)

| Run | Cost | $w_{ANN}$ | $w_{RFR}$ | $w_{SVR}$ | $w_{KNN}$ | $w_{MLR}$ | $w_B$ |
|-----|------|-----------|-----------|-----------|-----------|-----------|-------|
| 6   | 22.0714 | 0.02673 | 2.26251 | 3.08802 | -1.3702 | 8.61801 | -3.0508 |
| 19  | 22.073 | 0.07413 | 2.32814 | 3.22106 | -1.5117 | 9.41667 | -2.7688 |
| 5   | 22.075 | 0.02286 | -2.6641 | -3.5855 | 1.49519 | -9.5091 | -3.3626 |
| 17  | 22.0751 | -0.0299 | 2.59001 | 3.47864 | -1.4366 | 9.15359 | -3.4105 |
| 10  | 22.0752 | 0.02684 | -2.0346 | -2.7294 | 1.12121 | -7.1507 | -3.4378 |

Table 4 shows the top five result for the optimal weights using PSO. The first column denotes the run no. on which the weights were found and the rows are sorted based on the value of the final cost (lowest is the best). There were a total of 20 runs of PSO simulations run and these five runs are the best five. The cost was calculated based on the prediction value of equation 2 on the Validation set.

It is shown in table 4 that the optimal weights were found during run no. 6, with $w_{ANN} = 0.02673$, $w_{RFR} = 2.26251$, $w_{SVR} = 3.08802$, $w_{KNN} = -1.3702$, $w_{MLR} = 8.61801$ and $w_B = -3.0508$. It can be seen that PSO algorithm consistently places most of the weights on $w_{MLR}$, showing agreement with the result of the previous section where the MLR model was seen to be the best in all category. Meanwhile, the weights for $w_{ANN}$ seems to suggest that the ANN model is almost redundant. However, the ANN models was still kept as one of the Base Models due to little difference in computation cost.

3.3. Comparison between performance of MLR versus EN-PSO

Table 5. Percentage of improvement for EN-PSO over MLR model

|          | MLR  | EN-PSO | Percentage of improvement (%) |
|----------|------|--------|-------------------------------|
| MAE      | 24.80| 22.085 | 10.95                         |
| RMSE     | 32.43| 31.741 | 2.13                          |
| MAPE     | 0.101| 0.091  | 9.54                          |
| $R^2$    | 0.890| 0.895  | 0.56                          |
Table 5 shows the performance metrics score of the MLR and EN-PSO model. It can be seen that the EN-PSO model improved upon the MLR model across all four categories. EN-PSO showed the most improvement over the MLR model in the MAE category, improving from 24.8 to 22.085 which is a 10.95 % improvement. Meanwhile, the RMSE was improved by a modest 2.13 % from 32.433 to 31.741. Using the ensemble method of EN-PSO, the score for MAPE improved by 9.54 % with the MAPE score of EN-PSO being 9.1 % which is less than 10 %. Finally, the $R^2$ was improved by a modest 0.56 % increasing from 0.890 to 0.895. Noting on these improvements, it is clear that the ensemble method was able to improve upon the performance of the MLR model. Figure 5 shows the percentage improvement for the EN-PSO model compared to the MLR model.

![Percentage of improvement for EN-PSO model over the MLR model](image)

**Figure 5.** Percentage of improvement for EN-PSO model over the MLR model

3.4. *Comparison between EN-PSO versus suggested Universal Correlation*

![Shear wave velocity vs Data point](image)

(a) Observed vs AM-ALL (b) Observed vs AR-ALL (c) Observed vs AM-SN-CL (d) Observed vs AR-SN-CL
In order to enable an unbiased comparison between the various models, a slightly modified Test set was used. This was due to the fact that model AR-SN-CL and AM-SN-CL are intended for use on sand and clay only; therefore, this modified Test set only contains \( V_s \) measurements in soil of type sand and clay only. Each graph in Figure 6 is a comparison between the prediction distribution of each model compared to real observation of \( V_s \) measurements in the modified Test set.

It can be seen from Figure 6(a) and 6(b) that both AM-ALL and AR-ALL, which are both developed for use of all soil types, both have a tendency to underpredict the value of \( V_s \). The error value for both these models shows an increasing trend as it goes into the higher \( V_s \) range value. Meanwhile, Figure 6(c) and 6(d) shows better performance for AM-SN-CL and AR-SN-CL although like AM-ALL and AR-ALL, it’s error values increase in the higher \( V_s \) range value. This also serves to highlight the importance of taking the soil type into account when predicting \( V_s \) value since that is the main difference between these two groups of models. Finally, Figure 6(e) shows that the EN-PSO’s prediction distribution was able to follow the real distribution closely in average. Unlike the previous four models, EN-PSO was able to follow the real distribution even in the higher ranges of \( V_s \) values. Table 6 and Figure 7 further shows that when comparing across all four performance metrics, the EN-PSO model was the best performing model.

**Figure 6.** Distribution of prediction on Test set for: (a) AM-ALL, (b) AR-ALL, (c) AM-SN-CL, (d) AR-SN-CL and (e) EN-PSO

|          | AM-ALL | AM-SN-CL | AR-ALL | AR-SN-CL | EN-PSO |
|----------|--------|----------|--------|----------|--------|
| MAE      | 49.68  | 44.17    | 50.76  | 43.74    | 24.26  |
| RMSE     | 67.50  | 62.72    | 68.53  | 61.48    | 33.56  |
| MAPE     | 0.172  | 0.156    | 0.177  | 0.155    | 0.096  |
| \( R^2 \) | 0.58   | 0.61     | 0.58   | 0.61     | 0.88   |
3.5. Empirical Equation

Even though the EN-PSO model has been proven to perform better than the MLR model, a disadvantage of the EN-PSO model is that there is no simple correlation that can be derived from the model compared to the MLR model. Considering that the performance of the MLR model is not far behind that of the EN-PSO model, this paper will present three empirical correlations as alternative to the EN-PSO model for cases where the use of simple correlations is preferred. Technically, there are five (one for each soil type) empirical correlations that can be derived from the MLR model. However, this paper will only present three (for sand, silt and clay) empirical correlations due to the low number of gravel and rock samples in the dataset. The following shows: equation (3) (for clay soil type), equation (5) (for sand soil type) and equation (6) (for silt soil type):

\[ V_s(\text{Clay}) = 0.66 N_{SPT} + 4.39 z + 101.10 \]  
(3)

\[ V_s(\text{Sand}) = 0.66 N_{SPT} + 4.39 z + 135.27 \]  
(4)

\[ V_s(\text{Silt}) = 0.66 N_{SPT} + 4.39 z + 253.28 \]  
(5)

4. Conclusion

A \( V_s \) measurements dataset was compiled from five papers (refer table 2) to be used to develop prediction models for this paper. 5 Base models were developed using a Train set randomly selected from this dataset: MLR, RFR, SVR, ANN and KNN. These Base models were then combined into an ensemble model named EN-PSO. The weights for Equation (2) was then calculated using PSO. The performance of each models were then compared and it was shown that EN-PSO was the best in all four performance metrics: MAE = 22.085, MAPE = 9.1 %, RMSE = 31.741 and \( R^2 = 0.895 \). In addition, it was also shown that the EN-PSO model was able to improve upon the performance of the MLR model, which the most accurate among the Base models. Comparisons were also made between EN-PSO and other suggested Universal \( V_s \) correlations and EN-PSO was shown to outperform the other correlation based on prediction using a modified Test set. Three new empirical correlations as alternative for the EN-PSO model was also presented.
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