Recent experimental results for the $\pi^+$ electric form factor from Jefferson Lab

G.M. Huber
University of Regina, Regina, Saskatchewan S4S 0A2, Canada

Abstract. The pion holds a special role in our quest to understand hadrons in terms of the underlying degrees of freedom of QCD. It is the lightest mass QCD system and is used as a test case for all models of hadronic structure. While the $\pi^+$ form factor ($F_{\pi}$) can be calculated using perturbative QCD at large $Q^2$, its calculation in the intermediate $Q^2$ transition region is more complex and model-dependent. Over the past several years, two experiments have been mounted at Jefferson Lab to measure $F_{\pi}$ over the range of $Q^2=0.6$ to 2.45 GeV$^2$. The technique utilizes a precision Rosenbluth separation of the $p(e,e'\pi^+)n$ reaction at low $-t$. $F_{\pi}$ is then extracted from the separated longitudinal response function ($\sigma_L$) with the aid of a model. The recent Jefferson Lab data are reviewed, with special emphasis on the extraction of the form factor from the $\sigma_L$ data.

1. Introduction

The electric charge form factor, $F_{\pi}$, of the pion is of key interest in the study of the quark-gluon structure of hadrons. One of the reasons is that the valence structure of the pion $\langle q\bar{q}\rangle$ is relatively simple. The hard part of the $\pi^+$ form factor can be calculated within the framework of perturbative QCD (pQCD) as the sum of logarithms of $Q^2$ and powers of $Q^2$ [1]. As $Q^2 \to \infty$, only the leading-order term remains

$$F_{\pi}(Q^2 \to \infty) \to \frac{16\pi\alpha_s(Q^2)f_{\pi}^2}{Q^2}.$$  

Thus, in contrast to the nucleon case, the asymptotic normalization of the pion function is known from the decay of the pion ($f_{\pi}^2$).

At experimentally accessible $Q^2$, the pion charge form factor is also an important observable because it can be calculated in a wide variety of theoretical approaches. In this regime, the prediction for $F_{\pi}(Q^2)$ is less certain, since the calculation of the “soft” contributions, such as gluonic and quark-sea effects, is difficult and model-dependent. Special attention must be applied theoretically to the interplay of “soft” and “hard” contributions at intermediate momentum transfer. A general characterization of these approaches is that they are consistent with the measured $\pi^+$ charge radius at low $Q^2$, and then progressively diverge at increasing $Q^2$. Reliable experimental data are necessary at intermediate $Q^2$ to guide these theoretical investigations. The $F_{\pi}$ experimental program at Jefferson Lab is specifically intended to address this question, the description of $F_{\pi}(Q^2)$ in the gap between the “soft” and “hard” regions.
2. Experimental Method

The experimental measurement of the pion form factor poses special challenges. At low $Q^2$, $F_\pi$ can be determined in a model-independent manner from the scattering of high-energy pions by atomic electrons, yielding form factor values governed by the charge radius of the pion [2]. To access the region above $Q^2 > 0.3$ GeV$^2$, one must employ high-energy electroproduction of pions on the proton. For selected kinematics near the pion pole at $t = m_\pi^2$, this process can be described as quasi-elastic scattering of the electron from a virtual $\pi^+$ in the proton. The physical region for $t$ in pion electroproduction is negative, so measurements should be performed at the smallest attainable values of $-t$. To minimize background contributions, it is also necessary to separate out the longitudinal cross section $\sigma_L$, via a Rosenbluth L/T/LT/TT separation. The value of $F_\pi(Q^2)$ is then determined by comparing the measured longitudinal cross section at small values of $-t$ to the best available electroproduction model. The obtained $F_\pi$ values are in principle dependent upon the model used, although one anticipates this dependence to be reduced at sufficiently small $-t$.

Over the past decade, the $F_\pi$ Collaboration has measured the $p(e,e'\pi^+)n$ reaction at the Thomas Jefferson National Accelerator Facility (JLab) in order to study the pion form factor at intermediate $Q^2$. Because of the excellent properties of the JLab electron beam and experimental setup, L/T separated cross sections were determined with high accuracy. We have completed two experiments: $F_\pi$ I, which took $Q^2 = 0.6 - 1.6$ GeV$^2$ data at $W = 1.95$ GeV using 4 GeV electron beam, and $F_\pi$ II, which extended these measurements to the highest $Q^2$ possible with a 6 GeV electron beam. This second experiment, taken above the resonance region, $W = 2.22$ GeV, allowed new $Q^2 = 1.6$ GeV$^2$ data to be obtained 30% closer to the $t = m_\pi^2$ pole with significantly reduced model uncertainties in $F_\pi$, as well as new data at $Q^2 = 2.45$ GeV$^2$ to be acquired.

In both experiments, the JLab electron beam was brought upon a liquid hydrogen target, and coincidences were recorded between charged pions in the 7 GeV/c High Momentum Spectrometer (HMS) and electrons in the 1.7 GeV/c Short Orbit Spectrometer (SOS) of Hall C. Charged pions in the HMS were identified by requiring no signal in the gas Čerenkov detector, and by using time-of-flight (TOF) between the two scintillator hodoscope planes. However, in the second experiment, the hadron momentum was too high for TOF to yield a clean pion identification, and so an aerogel Čerenkov detector was constructed and installed at the HMS focal plane to supplement the TOF [3]. Electrons in the SOS were identified by using the combination of a lead glass calorimeter and gas Čerenkov detector. Any remaining contamination from real electron-proton coincidences was eliminated with a coincidence time cut of $\pm 1$ ns.

The momenta of the scattered electron and the pion at the target vertex were reconstructed from the wire chamber information of the spectrometers, correcting for energy loss in the target. From these, the values of $Q^2$, $W$, $t$, and the missing mass were reconstructed. A cut on the latter was used to select the neutron exclusive final state, excluding additional pion production. Experimental yields were calculated after correcting for several inefficiencies, the dominant sources in $F_\pi$ II being particle tracking efficiency (3-4%), pion absorption (4.8%), and computer dead time (1-11%). Background from aluminium target cell walls (2-4% of the yield) and random coincidences ($\sim 1\%$) were subtracted from the charge normalized yields.

Calibrations with the over-determined $p(e,e'p)$ reaction were essential to performing the L/T separation. The beam momentum and the spectrometer central momenta were determined absolutely to better than 0.1%, while the incident beam angle and spectrometer central angles were determined with an absolute accuracy of about 0.5 mrad. The spectrometer acceptances were checked by comparison of the data to MC simulations. Finally, the overall absolute cross section normalization was checked. The calculated yields for $e + p$ elastics agreed to better than 2% with the established values.

The unpolarized pion electroproduction cross section can be written as the product of a
virtual photon flux factor and a virtual photon cross section,

$$\frac{d^5 \sigma}{d \Omega_e dE_e' d \Omega_{\pi}} = J(t, \phi \rightarrow \Omega_{\pi}) \Gamma_v \frac{d^2 \sigma}{dt d\phi}, \quad (2)$$

where $J(t, \phi \rightarrow \Omega_{\pi})$ is the Jacobian of the transformation from $dt d\phi$ to $d \Omega_{\pi}$, $\phi$ is the azimuthal angle between the scattering and the reaction plane, and $\Gamma_v = \frac{\alpha}{2\pi} \frac{E_e'}{E_e} \frac{1}{1-\epsilon} \frac{W^2-M^2}{2M}$ is the virtual photon flux factor. The virtual photon cross section can be expressed in terms of contributions from transversely and longitudinally polarized photons,

$$2\pi \frac{d^2 \sigma}{dtd\phi} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{dt} \cos \phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos^2 \phi. \quad (3)$$

Here, $\epsilon = \left(1 + 2 \frac{|q^2|}{Q^2} \tan^2 \frac{\theta}{2}\right)^{-1}$ is the virtual photon polarization, where $q^2$ is the square of the three-momentum transferred to the nucleon and $\theta$ is the electron scattering angle. The individual components in Eqn. 3 were determined from a simultaneous fit to the $\phi$-dependence of the measured cross sections, $\frac{d^2 \sigma}{dtd\phi}$, at two values of $\epsilon$. A representative example from the $F_{\pi}^{II}$ experiment [4] as a function of $\phi$ is shown in Fig. 1.

The separated cross sections are determined at fixed values of $W$, $Q^2$ and $t$, common for both high and low values of $\epsilon$. However, the experimental acceptances in $W$, $Q^2$ and $t$ are correlated and cover a range in each of these quantities. Thus, the measured yields represent an average over that range, and each $t$-bin has a different average value of $Q^2$, $W$. In order to minimize errors resulting from averaging, a phenomenological cross section model in the MC simulation was used. It was determined by fitting the different cross sections $\sigma_X$ of Eqn. 3 globally to the data as a function of $Q^2$ and $t$. The dependence of the cross section on $W$ was assumed to follow the phase space factor $(W^2 - M^2)^{-2}$, which is supported by previous data [5]. By combining for every $(Q^2, t)$-bin the $\phi$-dependent cross sections measured at two values of the incoming electron energy, and thus of $\epsilon$, the experimental values of $\sigma_L$, $\sigma_T$, $\sigma_{LT}$ and $\sigma_{TT}$ can be determined by fitting the $\phi$ and $\epsilon$-dependences. Separated cross sections, $\sigma_L$ and $\sigma_T$, for the $F_{\pi}^{II}$ experiment are shown in Fig. 2.

Figure 1. Representative example of the measured $F_{\pi}^{II}$ cross sections, $\frac{d^2 \sigma}{dt d\phi}$, as a function of $\phi$ at $Q^2=1.59$ GeV$^2$ for two values of $\epsilon$ [4]. The curves shown (with line segments connecting the points) represent the model cross section used in the Monte Carlo (MC) simulation.
3. Extraction of $F_\pi(Q^2)$ from the Data

It is worth repeating that the $\sigma_L$ cross sections versus $t$ over some range of $Q^2$ and $W$ are the actual observables measured by the experiment. There are a number of approaches for the extraction of the pion form factor from these cross sections, each with their associated uncertainties.

Frazer [6] originally proposed that $F_\pi$ be extracted from $\sigma_L$ via a kinematic extrapolation to the pion pole, and that this be done in an analytical manner (Chew-Low method). However, this extrapolation procedure fails to produce a reliable answer, since different polynomial fits, each of which are equally likely in the physical region, differ considerably when continued to $t = m^2_\pi$. Some attempts were made [7] to reduce this uncertainty by providing some theoretical constraints on the behavior of the pion form factor in the unphysical region, but none proved adequate.

A more reliable approach is to use a model of the longitudinal electroproduction cross section incorporating the $\pi^+$ production mechanism and spectator neutron effects to extract $F_\pi(Q^2)$ from the $\sigma_L$ data. In principle, the experimentalist would like to use a variety of models to extract $F_\pi(Q^2)$, so that the model-dependence of the extracted pion form factor value can be better understood. At present, the Regge model by Vanderhaeghen, Guidal and Laget (VGL, Ref. [8]) is the only reliable model for our use. However, our philosophy remains to publish the experimentally measured $\sigma_L$, so that updated values of $F_\pi(Q^2)$ could be extracted in the future, should even more sophisticated models for the $p(e,e'\pi^+n)$ reaction become available. The experimental result is not permanently 'locked-in’ to a specific model.

In the VGL Regge model, the exchange of high-spin, high-mass particles is taken into account by replacing the pole-like Feynman propagators of Born term models with Regge propagators. Many of the model’s free parameters were fitted to pion photoproduction, with the result giving a good fit to existing data for $-t$ below 1 GeV$^2$. For electroproduction, the pion form factor and the $\rho\pi\gamma$ form factor are added as adjustable parameters, parameterized with a monopole form

$$F_\pi(Q^2) = \left[1 + Q^2/\Lambda^2_\pi\right]^{-1}. \quad (4)$$

Over the range of $-t$ covered by this work, $\sigma_L$ is completely determined by the $\pi$ trajectory,
whereas $\sigma_T$ is also sensitive to the $\rho$ exchange contribution. $\Lambda^{2}_{\pi}$ is determined by fitting the model to the $\sigma_L(t)$ data.

A comparison of the $F_{\pi}^{II}$ data to the VGL model [8] prediction is shown in Fig. 2. The $t$-dependence of the longitudinal cross section is well-described at both central values of $Q^2$. However, the transverse cross section is underpredicted systematically. The value of $F_{\pi}$ was determined from a least squares fit of the Regge model prediction to the data. The extracted values of $\Lambda^{2}_{\pi}$ are insensitive ($\sim 1\%$) to the $t$-bin used in the fit.

4. The $F_{\pi}^{I}$ Data

The result of the $F_{\pi}^{I}$ experiment was originally published in [9]. Since then, the whole analysis chain has been re-examined, and special attention paid to the contribution of various systematic uncertainties to the final uncertainty of the separated cross sections. Furthermore, the method to determine $F_{\pi}$ from the longitudinal cross sections was critically reconsidered. Compared to the analysis of Ref. [9], small adjustments were made in the values of cuts and efficiencies, and a small mistake in the calculation of $\theta$ was corrected. Also, a more direct method to separate the respective cross sections was developed, which was subsequently used also in the $F_{\pi}^{II}$ analysis.

The differences in the extracted unseparated cross sections are small, but the L/T separation magnifies them. Except for a few cases, the difference with the previous values is well within the total uncertainty quoted in Ref. [9]. On average, $\sigma_L$ is 6% smaller than in Ref. [9] and $\sigma_T$ is 3% larger. The largest differences occur for $Q^2 = 1.0 \text{ GeV}^2$, where $\sigma_L$ is 14% smaller and $\sigma_T$ is 10% larger. These cross sections are available from Ref. [10].

The VGL model [8], evaluated with certain $\Lambda^{2}_{\pi}$ and $\Lambda^{2}_{\rho}$, is compared to the $F_{\pi}^{I}$ data in Fig. 3. As for the $F_{\pi}^{II}$ data in Fig. 2, the model strongly underestimates $\sigma_T$ for any value of $\Lambda^{2}_{\rho}$ used. The VGL model calculation for $\sigma_L$ gives the right magnitude, but the $t$-dependence of the data is somewhat steeper than that of the calculations. This is most visible at $Q^2 = 0.6 \text{ GeV}^2$. This discrepancy between the data and VGL model may possibly be attributed to resonance contributions which are not included explicitly in the Regge model. This is supported by the fact that the discrepancy is strongest at the lowest $Q^2$ value, at higher $Q^2$ the resonance form factor reduces such contributions.
Given the discrepancy between the $t$-dependence predicted by the VGL Regge model and the $F_\pi$ data, what is the best way to determine $F_\pi(Q^2)$ from the measured $\sigma_L$ cross sections? In our original $F_\pi$ analysis [9] some choices were made for the ‘missing background’ which have been criticized. The difficulty is that there is no theoretical guidance for the assumed interfering background. This applies even if one assumes that the background is due to resonances: virtually nothing is known about the L/T character of resonances at $W = 1.95$ GeV, let alone their influence on $\sigma_L$ (via interference with the VGL amplitude). Given that no information is available on the background, one is forced to make some assumptions in extracting $F_\pi$ from the $F_\pi$ data. Our guiding principle here is to minimize these assumptions to the greatest extent possible. Our revised form factor extraction method relies on a single assumption, that the contribution of the background is small at the kinematic endpoint $t_{\text{min}}$.

Our best estimate for $F_\pi$ is determined as follows. Using the value of $\Lambda_\pi^2$ as a free parameter, the VGL model was fitted to each $t$-bin separately, yielding $\Lambda_\pi^2(Q^2, W, t)$ values as shown in Fig. 4. $\Lambda_\pi^2$ tends to decrease as $-t$ increases, presumably because of an interfering background not included in the model. One expects the effect of this background to be smallest at the smallest value of $|t|$ allowed by the experimental kinematics, $|t_{\text{min}}|$, so an extrapolation of $\Lambda_\pi^2$ to this physical limit is used to obtain our best estimate of $F_\pi$.

Because of the arguments given above, these are our final estimate of $F_\pi$ from the $F_\pi$ data using the VGL Regge model. They are based on a simpler and more transparent $F_\pi$ extraction procedure than our original analysis. Since the actual observables of our experiment are the $\sigma_L$ cross sections, other (better) values of $F_\pi$ may be extracted from them when improved models for the $p(e, e'\pi^+)n$ reaction become available.

The $F_\pi$ values determined in this manner correspond to the true ‘VGL model’ $F_\pi$ values if and only if the ‘background’ vanishes at $t' = 0$. Because of the uncertainty inherent in this assumption, we also estimate a ‘model uncertainty’ due to the fact that the VGL model does not alone account for the $t$-dependence of the $F_\pi \sigma_L$ data. Lacking a model for the background, one can only try to make a fair estimate of this uncertainty. This was done by looking at the variation in the fitted values of $\Lambda_\pi^2$ when using two different assumptions for the background.

(i) The first case assumes a $t$-independent negative background in addition to the VGL model.
That is, when fitting the value of $\Lambda^2_\pi$ (and hence $F_\pi$) to the five $t$-bins at each $Q^2$, a flat background that yields a negative contribution to $\sigma_L$ is included as an additional free parameter in the fit. The fitted contribution of the background is found to drop strongly as $Q^2$ is increased from 0.6 to 1.6 GeV$^2$. The uncertainty assigned is one half the difference with the $t_{min}$ extrapolation result for $\Lambda^2_\pi$, and drops from 15% at $Q^2 = 0.6$ GeV$^2$ to 2% at $Q^2 = 1.6$ GeV$^2$.

(ii) The second case assumes an interfering background amplitude with a $t$-independent magnitude and phase. Besides the VGL amplitude, this interfering background amplitude is fit together with the value of $\Lambda^2_\pi$. Although the fitting uncertainties are very large, the $\sigma_L$ data weakly support an interfering amplitude whose magnitude decreases monotonically with increasing $Q^2$, but whose phase with respect to the VGL amplitude does not necessarily result in a net negative cross section contribution to $\sigma_L$, as assumed above. The resulting $\chi^2/\text{d.o.f.} = 1$ variation in $\Lambda^2_\pi$ is taken as the uncertainty here. It is of comparable magnitude to the first estimate above, but is not as strongly $Q^2$-dependent.

The larger of these two uncertainty estimates at each $Q^2$ is taken as the ‘model uncertainty’ in our extracted $F_\pi$ result. Please note that our collaboration is still discussing the best way to estimate the model uncertainty, and so the final values we quote may differ from those presented here.

Analysis of other data at higher $W$ indicate that the discrepancy with the $t$-dependence of the VGL calculation is much smaller there. The data from Brael et al. [5], taken at $Q^2 = 0.70$ GeV$^2$ and a value of $W=2.19$ GeV, were reanalyzed using the $F_\pi$ extraction method presented here. The result is 0.4% higher than that obtained using our earlier $F_\pi$ extraction method of Ref. [9]. These indicate that our $F_\pi$ extraction methods are robust when the ‘background’ contribution is small, as appears to the case at this higher value of $W$. This is also consistent with our experience from the $F_\pi$II experiment [4] at $W=2.22$ GeV, again indicating that the ‘background’ contributions for $\sigma_L$ are smaller at higher $W$, even though the model strongly underpredicts the magnitude of $\sigma_T$.

5. Results and Outlook
Fig. 5 compares the recent $F_\pi$I and $F_\pi$II data to a variety of QCD-based calculations. The revised $F_\pi$I result at $Q^2 = 1.6$ GeV$^2$ agrees well with that from the second experiment, taken at higher $W$ and 30% closer to the $\pi^+$ pole. The excellent agreement between these two results, despite their significantly different $t_{min}$ values, indicates the reliability of the $F_\pi$ extraction procedure.

The combined data sets are consistent with a variety of models. Up to $Q^2 = 1.5$ GeV$^2$, the Dyson-Schwinger calculation of Ref. [11], the QCD sum-rule calculation of Ref. [12], and the light front quark model calculation of Ref. [13] are nearly identical, and are all very close to the monopole form factor constrained by the measured pion charge radius [2]. The JLab data are below the monopole curve. A significant deviation from this curve would indicate the increased role of perturbative components at moderate $Q^2$, which provide in the $Q^2 \approx 2$ GeV$^2$ region a value of $Q^2F_\pi \approx 0.15 - 0.20$ only [14]. The dispersion relation calculation of Geshkenbein et al. [15] is closer to our results in the $Q^2 = 0.6\text{-}1.6$ GeV$^2$ region, while still describing the low $Q^2$ data used for determining the pion charge radius. The quark-hadron duality calculation by Melnitchouk [16] is not expected to be valid below $Q^2 = 2.0$ GeV$^2$. This is reflected in its significant deviation from the monopole curve at low $Q^2$. To better distinguish between these different models, it is clear that especially higher $Q^2$ data, as well as higher quality data in the $Q^2 = 0.6\text{-}1.6$ GeV$^2$ region, are needed. Plans are underway to address both of these at JLab.
Figure 5. JLab $Q^2 F_\pi$ data, compared to previously published data. The solid Brauel et al. [5] point has been reanalyzed using the $F_\pi$ extraction method of this work. The outer error bars of the $F_\pi I$ data and the reanalyzed Brauel et al. data include all experimental and model uncertainties, added in quadrature, while the inner error bars reflect the experimental uncertainties only. Also shown are the Dyson-Schwinger [11] (solid), quark-hadron duality [16] (short-dash), QCD sumrule [12] (dot), light front quark model [13] (dash-dot), and dispersion relation [15] (long-dash) calculations.

Acknowledgments
The author would like to thank the $F_\pi$ Collaboration, and in particular Henk Blok, Dave Gaskell, Tanja Horn, Dave Mack, and Vardan Tadevosyan for their very productive and enjoyable scientific collaboration over the course of this work. The author would also like to thank Drs. Guidal, Laget and Vanderhaeghen for stimulating discussions and for modifying their computer program for our needs. This work is supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC).

References
[1] G.P. Lepage, S.J. Brodsky, Phys. Lett. 87B (1979) 359.
[2] S. R. Amendolia, et al., Nucl. Phys. B277 (1986) 168.
[3] R. Asaturyan, et al., Nucl. Instrum. Meth. A548, 364-374 (2005).
[4] T. Horn, et al., Phys. Rev. Lett. 97 (2006) 192001.
[5] P. Brauel, et al., Z. Phys. C3 (1979) 101.
[6] W.R. Frazer, Phys. Rev. 115 (1959) 1763.
[7] B.H. Kellett, C. Verzegnassi, Nuo. Cim. 20A (1974) 194.
[8] M. Vanderhaeghen, M. Guidal and J.-M. Laget, Phys. Rev. C 57 (1998) 1454; Nucl. Phys. A627 (1997) 645.
[9] J.Volmer, et al., Phys. Rev. Lett. 86 (2001) 1713.
[10] V. Tadevosyan, et al., Preprint nucl-ex/0607007.
[11] P. Maris, P.C. Tandy, Phys. Rev. C 62 (2000) 055204.
[12] V.A. Nesterenko, A.V. Radyushkin, Phys. Lett. 115 B (1982) 410.
[13] C.-W. Hwang, Phys. Rev. D 64 (2001) 034011.
[14] A.P. Bakulev, et al., Phys. Rev. D 70 (2004) 033014.
[15] B.V. Geshkenbein, Phys. Rev. D 61 (2000) 033009.
[16] W. Melnitchouk, private communication 2006 and Eur. Phys. J. A 17 (2003) 233.