Combining scenario approach and Choquet integral in decision making

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Abstract. Scenario-based approach is applied to evaluate alternatives. Using the ideas of Dempster-Schaeffer a capacity distribution is defined over scenarios. This allows describing possible future developments in terms of individual and integrated expert judgments and transforming expert judgments into scenario assessments. To get a final numerical estimate of alternatives, we use integration over non-additive measures applying the Choquet integral.

1. Introduction

In many cases, the evaluation and choice of alternatives should be made taking into account the external environment. However, it may be difficult to reduce the uncertainty inherent in the external environment to a single risk parameter. The uncertainty of the external environment is in complex interaction with the result of choice, influencing it in the context of global and local events. Changes in the external environment can lead to a fairly radical change in the entire landscape in which the choice is implemented, and make optimal decision-making strategies ineffective. There are approaches to decision-making under uncertainty involving scenarios.

The scenario approach has a long history. The scenario approach received its methodology about three decades ago [1], [2] (see also [3]). Scenario planning is widely used in decision-making in strategic management in order to facilitate consideration of uncertainty and uncontrolled factors [4]. A discussion of the application of the scenario approach to decision theory can be found in [5]. In [6] the synergistic effect of combining scenario approach and quantitative decision models is discussed. In particular, in [6], following [7], the author considers four spaces in the context of decision-making: known, knowable, complex, and chaotic. The most relevant for scenario approach is a complex space. Decision-making in complex systems involves taking into account a large number of interacting factors. The most adequate representation of complex systems can be obtained using soft models.

The scenario approach is often considered as an alternative to the forecast-based approach. Forecasts are usually based on the current state of affairs, and therefore in some cases lead to incorrect decisions when the environment changes radically. In contrast to forecasts, scenarios are consistent stories about an alternative future. Scenarios can help to develop a common understanding of possible events, decisions, and actions. Scenarios are primarily designed to help critically evaluate their assumptions, develop strategies, and test plans. Using a scenario approach, it makes sense to talk not about the probability of events, but to apply the concepts of soft computing to describe uncertainty in terms of “possibility”, “plausibility”, “necessity” etc.
It seems that the scenario approach is appropriate in models with greater uncertainty than the probabilistic one. The use of the scenario approach in stochastic models, as is done, for example, in [8], is rather technical in nature. However, in [9], it is shown that the scenario approach in decision-making can be combined with an understanding of risk that is close to the classical one.

Despite the fact that the general approaches to scenario modeling have been developed for a long time, the exact boundaries and methods have not yet been outlined, and researchers apply certain approaches, referring them to scenario modeling, focusing primarily on the specifics of the problems being solved (see [10], [11]).

The scenario approach avoids the direct application of probabilities. At the same time, to obtain quantitative estimates, probability in one form or another is embedded in quantitative models. In recent years, the use of the Choquet integral has become popular in decision theory. The Choquet integral was introduced into mathematical practice almost seven decades ago, but it became systematically applied in decision theory after the fundamental works of Schmeidler [12], [13]. The main idea is to set the distribution of capacity on the set of criteria, and then get an integrated criterion using the Choquet integral.

The formalization of the scenario approach proposed in this paper is based on a combination of Dempster-Schaeffer ideas and Choquet integration. We do not stop at the methodological issues of scenario construction and consider the family of scenarios to be predetermined. Moreover, we suppose that a criterion is defined in the space of alternatives that allows us to evaluate the alternative. In this paper, we propose a method for constructing an aggregated criterion that takes into account estimates for the entire set of scenarios.

Using the Dempster-Schaeffer technique, we construct a capacity distribution that allows us to describe possible future developments in terms of individual and integrated expert judgments and transform expert judgments into scenario estimates. According to the classification proposed in [14], this approach is based on the definition of scenarios as a representative sample of future states.

The paper is organized as follows. In section 2 we give the necessary definitions and describe the main construction. Section 3 provides a brief description of the Dempster-Schaeffer design. In section 4 the proposed method is illustrated by a numerical example. Section 5 concludes.

2. Events and scenarios. Belief and plausibility measures

We suppose that alternatives are evaluated by $m$ criteria. Denote by $A_i$ the set of possible values of criterion $i$. Alternative $a$ is represented by a point $\langle a_i \rangle$ in space $A = \prod_{i=1}^{m} A_i$, where $a_i$ is the value of criterion $i$ on alternative $a$. We assume that a family of functionals $F_w : A \rightarrow \mathbb{R}$, $w \in W$, is given on $A$, where $W$ can be considered as a set of methods for evaluating alternatives. So, $F_w(a)$ can be treated as an evaluation of alternative $a$ using method $w$.

Let $X$ be a set of scenarios. Depending on the scenario, the values of the criteria on alternative $a$ may change. Let us denote by $a(x) = \langle a_i(x) \rangle \in A$ the set of values for the criteria $i = 1, \ldots, m$ on alternative $a$ under scenario $x$. The method of evaluating also depends on the scenario. The evaluation of alternative $a$ when implementing scenario $x$ will be denoted by $F_w(a(x))$.

We assume that scenarios are implemented depending on what events occur. We will assume that there is a finite set of incompatible events $S$. For each event $s \in S$ probability $Pr(s)$ is specified such that $\sum_{s \in S} Pr(s) = 1$.

The basic structure for defining the capacity function in the scenario space is a compatibility relation that links events and scenarios. Specifically, the compatibility relation links event $s \in S$ to the scenarios that are possible when this event occurs. Let $G \subseteq S \times X$ be a compatibility relation. If $s \in S$ then $G(s) \subseteq X$ is the set of scenarios that can occur when $s$ occurs.

Let us define basic probabilities in $X$. Given $Y \subseteq X$ we put
So the basic probability of \( Y \subseteq X \) is obtained by summing up the probabilities of all those events with respect to which exactly all scenarios from \( Y \) are possible. If \( m(Y) \neq 0 \) then \( Y \) is called focal.

Using basic probabilities, it is possible to define belief and plausibility measures:

\[
Bel(Y) = \sum \{m(Y') | Y' \subseteq Y\}; \quad Pl(Y) = \sum \{m(Y') | Y' \cap Y \neq \emptyset\}.
\]

Belief and plausibility measures can be directly used to evaluate alternatives. The alternative \( a \) estimate \( V_a(x) = F_{w_{(x)}}(a(x)) \) in this case is interpreted as a "random variable" with respect to the belief (plausibility) measure. For example

\[
Bel(V_a > \alpha) = Bel\{x \in X | F_{w_{(x)}}(a(x)) > \alpha\}.
\]

The problem of the optimal alternative choice can be obtained by reformulation of the problem of stochastic programming. We have to choose \( a \) such that

\[
Bel(V_a > \alpha) \rightarrow \max \text{ subject to } \alpha \geq h,
\]

either

\[
\alpha \rightarrow \max \text{ subject to } Bel(V_a > \alpha) \geq h
\]

where \( h \) is a given threshold.

For \( Y \subseteq X \) let \( \bar{Y} = X \setminus Y \). Then

\[
Bel(Y) + Pl(\bar{Y}) = 1 \quad \text{and} \quad Bel(\bar{Y}) + Pl(Y) = 1.
\]

It can be easily seen that

\[
Bel(Y) + Bel(\bar{Y}) \leq 1 \quad \text{and} \quad Pl(Y) + Pl(\bar{Y}) \geq 1.
\]

Taking into account these relations, the belief measure and the plausibility measure are called the lower and upper probability, respectively. If all the focal sets are single-point then \( Bel \) and \( Pl \) coincide and turn out to be just probabilistic measures. In general, they serve as an estimate of a probability measure. This statement can be given precise meaning if we consider the expected value of a numerical function given on a set of scenarios with respect to \( Bel \) and \( Pl \).

Using Lebesgue-Stieltjes integral let

\[
E^* (V_a) = \int_{-\infty}^{+\infty} r \cdot dBel((x|V_a(x) \leq r)) = \sum \{m(Y) \cdot \max_{x \in Y} V_a(x) | Y \subseteq X\};
\]

\[
E_*(V_a) = \int_{-\infty}^{+\infty} r \cdot dPl((x|V_a(x) \leq r)) = \sum \{m(Y) \cdot \min_{x \in Y} V_a(x) | Y \subseteq X\}.
\]

Denoting by \( \Pi \) the set of all probability measures \( Q \) such that \( Bel(Y) \leq Q(Y) \leq Pl(Y) \) for all \( Y \subseteq X \) we get

\[
E^* (V_a) = \sup \{E_Q (V_a) | Q \in \Pi\}; \quad E_*(V_a) = \inf \{E_Q (V_a) | Q \in \Pi\}.
\]

To obtain an aggregated evaluation of the alternatives it can also be used the Choquet integral. The necessary definitions are provided below.

Let \( c \in \mathbb{R} \) be such that \( V_a(x) + c \geq 0 \) for all \( x \in X \). Suppose that the values of \( V_a(x) + c \) are ordered so that \( v_1 > v_2 > \ldots > v_m \geq 0 \) is the full list of values of \( V_a(x) + c \). Let

\[
Y_i = \{x|V_a(x) + c = v_i\}, \quad i = 1, \ldots, m
\]

be a partition of \( X \).

Put

\[
\Delta Bel_i = Bel(Y_i); \quad \Delta Bel_i = Bel(Y_i \cup \ldots \cup Y_i) - Bel(Y_i \cup \ldots \cup Y_{i-1}), \quad i = 2, \ldots, m.
\]

Now, using the Choquet integral, it is possible to find the mathematical expectation of \( V_a \) with respect to the belief measure:
\[
E_{Bel}(V_a) = v_1 \cdot \Delta Bel_1 + \ldots + v_m \cdot \Delta Bel_m - c.
\]

Putting \(v_{m+1} = 0\) we can calculate the same value as follows:

\[
E_{Bel}(V_a) = \sum_{i=1}^{m}(v_i - v_{i+1})\text{Bel}(Y_i \cup \ldots \cup Y_j) - c.
\]

In the same way, mathematical expectation of \(V_a\) can be calculated with respect to a plausibility measure:

\[
E_{Pl}(V_a) = \sum_{i=1}^{m}(v_i - v_{i+1})\text{Pl}(Y_i \cup \ldots \cup Y_j) - c.
\]

Values \(E_{Bel}(V_a)\) and \(E_{Pl}(V_a)\), as in the case of a probabilistic situation, can serve as numerical estimates of alternative \(a\). These estimates should be used with caution since they may be non-additive. Generally, \(E_{Bel}(V_a + V_b)\) need not be the same as \(E_{Bel}(V_a) + E_{Bel}(V_b)\). Equality occurs when functions \(V_a, V_b\) are comonotonic, i.e., inequalities \(V_a(x) \leq V_a(y)\) and \(V_b(x) \leq V_b(y)\) are equivalent for any pair of scenarios \(x \text{ and } y\). With respect to a constant, any function is comonotonic. With this in mind, integrals \(E_{Bel}(V_a)\) and \(E_{Pl}(V_a)\) do not depend on the choice of \(c\) in the definition of the integral.

3. Combining expert evaluations

Assume that the set of scenarios \(X\) is fixed, and experts 1 and 2 have specified their event spaces \(S_1\) and \(S_2\), as well as the compatibility relations \(G_1 \subseteq S_1 \times X\) and \(G_2 \subseteq S_2 \times X\). Using the Dempster-Schaeffer method, it is possible to combine expert evaluations.

Let \(m_1\) and \(m_2\) be basic probabilities in \(X\) corresponding to experts 1 and 2. Put

\[K = 1 - \sum \{m_1(Y_1) \cdot m_2(Y_2) | Y_1 \cap Y_2 = \emptyset\}.
\]

If \(K = 0\) then the expert evaluations are incompatible and cannot be aggregated. If \(K \neq 0\) then the combined basic probability \(m = m_1 \oplus m_2\) can be defined as follows:

\[m(Y) = \frac{1}{K} \sum \{m_1(Y_1) \cdot m_2(Y_2) | Y_1 \cap Y_2 = Y\}.
\]

Operation \(\oplus\) is commutative and associative. This allows using it to combine estimates of any finite number of experts. It is worth noting that the order of experts does not matter.

4. Numerical example

Consider the application of the proposed approach to evaluation of cash flows. Cash flow \(CF\) can be presented by a finite sequence \(CF = (CF_0, CF_1, \ldots)\) where \(CF_i\) is a payment at time \(i\). The cash flows form in \(I^\infty\) a subspace that will be denoted by \(C\). If the preference relation in \(C\) satisfies natural conditions and is invariant with respect to the time shift operator then any functional representing this preference relation has the following form:

\[F_w(CF) = \sum_{i=1}^{\infty} CF_i (1 + w)^{-i},\]

where \(w \geq 0\) is a discount rate (see [15]). Such a functional is called the net present value and denoted by \(NPV\).

Consider an investment cash flow with \(I = -CF_0\) being the value of the initial investment. Let \(X\) consists of seven scenarios \(x_1, x_2, \ldots, x_7\) related to the implementation of the investment project. For each scenario, the project is described by the amount of initial investment, the net flow of payments over the years, and the discount rate (table 1).
Table 1. Scenarios.

|   | 1   | 2   | 3   | %  | NPV |
|---|-----|-----|-----|----|-----|
| x1| -1200| 400 | 600 | 15%| 127 |
| x2| -1000| 300 | 400 | 10%| 54  |
| x3| 1000 | 700 | 500 | 10%| 50  |
| x4| 1000 | 700 | 500 | 15%| 16  |
| x5| 1000 | 700 | 500 | 15%| -13 |
| x6| 1000 | 700 | 500 | 20%| -67 |
| x7| 1000 | 300 | 900 | 20%| -125|

The event space contains five events, the probabilities of which are shown in table 2.

Table 2. Probabilities of events.

| Event | s1  | s2  | s3  | s4  | s5  |
|-------|-----|-----|-----|-----|-----|
| Probability | 0.1 | 0.2 | 0.4 | 0.2 | 0.1 |

The compatibility relation is shown in table 3 ("1" means compatibility, "0" means incompatibility).

Table 3. The compatibility relation.

|   | s1  | s2  | s3  | s4  | s5  |
|---|-----|-----|-----|-----|-----|
| x1 | 0   | 0   | 1   | 0   | 0   |
| x2 | 1   | 0   | 0   | 1   | 1   |
| x3 | 1   | 1   | 1   | 0   | 0   |
| x4 | 1   | 0   | 1   | 0   | 1   |
| x5 | 0   | 1   | 0   | 0   | 0   |
| x6 | 0   | 1   | 0   | 1   | 0   |
| x7 | 0   | 0   | 0   | 1   | 0   |

Table 4. Integration.

| j  | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   |
|----|-----|-----|-----|-----|-----|-----|-----|-----|
| v_j | 327 | 254 | 250 | 216 | 187 | 133 | 75  | 0   |
| v_j - v_{j+1} | 73  | 4   | 34  | 29  | 54  | 58  | 75  | 0   |
| Y_j \cup \ldots \cup Y_f | x1  | x1, x2 | x1, x2, x3 | x1, x2, x3, x4 | x1, x2, x3, x4, x5 | X  |
| Bel(Y_j \cup \ldots \cup Y_f) | 0.0 | 0.0 | 0.0 | 0.6 | 0.6 | 0.8 | 1.0 |
| Pl(Y_j \cup \ldots \cup Y_f) | 0.4 | 0.8 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |

Using table 4 we calculate the expected value of NPV:

\[ E_{Bel}(NPV) = 29 \cdot 0.6 + 54 \cdot 0.6 + 58 \cdot 0.8 + 75 \cdot 1.0 - 200 = -28.8; \]

\[ E_{Pl}(NPV) = 73 \cdot 0.4 + 4 \cdot 0.8 + 34 \cdot 1.0 + 58 \cdot 1.0 + 75 \cdot 1.0 - 200 = 82.4. \]

Thus, the project should be considered ineffective with respect to Bel, and effective with respect to Pl (in a deterministic case a project is effective if \( NPV > 0 \)). To obtain an aggregated criterion, we
can use the Hurwitz criterion with the constant of optimism-pessimism \( \gamma \) and present the aggregated criterion as follows:

\[
C_\gamma = (1 - \gamma)E_{\text{bel}}(NPV) + \gamma E_{\text{pl}}(NPV).
\]

The project is effective if \( \gamma \geq 0.26 \).

Note that

\[
\text{Bel}(NPV > 0) = 0.6 ; \text{Pl}(NPV > 0) = 1.0
\]

The latter equalities can be interpreted as follows: the project may well be effective, but confidence in this is estimated at the level of 0.6.

5. Conclusion

The approach presented in this paper was inspired by the analysis of investment decisions. Investment projects are usually evaluated using numerical indicators, primarily \( NPV \). Thus, the problem of decision making becomes essentially a problem of adequate measurement under uncertainty. The approach proposed in this paper can also be used in measurement, when the measurement results significantly depend on the conditions (scenarios for performing measurements). Events in this context can be considered as factors that affect the measurement conditions. Splitting a probability measure into a belief and plausibility measures serves as a formalized manifestation of uncertainty higher than the probabilistic one.

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