Cosmological magnetic fields

Kerstin E Kunze

Departamento de Física Fundamental and IUFFyM, Universidad de Salamanca, Plaza de la Merced s/n, 37008 Salamanca, Spain
E-mail: kkunze@usal.es

Received 28 June 2013, in final form 25 September 2013
Published 28 November 2013
Online at stacks.iop.org/PPCF/55/124026

Abstract
Magnetic fields are observed on nearly all scales in the Universe, from stars and galaxies up to
galaxy clusters and even beyond. The origin of cosmic magnetic fields is still an open
question, however a large class of models puts its origin in the very early Universe. A
magnetic dynamo amplifying an initial seed magnetic field could explain the present day
strength of the galactic magnetic field. However, it is still an open problem how and when this
initial magnetic field was created.

Observations of the cosmic microwave background (CMB) provide a window to the early
Universe and might therefore be able to tell us whether cosmic magnetic fields are of a
primordial cosmological origin and at the same time constrain its parameters.

We will give an overview of the observational evidence of large-scale magnetic fields,
describe generation mechanisms of primordial magnetic fields and possible imprints in
the CMB.

1. Introduction
There is evidence for magnetic fields on small up to very large scales. Going beyond stars, there are observations of magnetic
fields in the interstellar medium of galaxies and clusters of
galaxies.

Important tracers of galactic and extragalactic magnetic
fields are diffuse synchrotron radiation and Faraday rotation.
Synchrotron radiation is emitted by electrons spiralling
around the magnetic field lines and the emissivity is determined
by the energy spectrum and number density of the electrons:
the frequency and the component of the magnetic field
perpendicular to the line of sight. Due to its high degree
of intrinsic linear polarization, the polarization of the diffuse
synchrotron radiation can be used to determine the structure
of the magnetic field. Its degree of polarization is determined
by the spectral index of the energy spectrum of the electrons.
For example, in the Milky Way, the degree of polarization
in a homogeneous magnetic field is 75% [2]. A lower
degree of polarization could indicate inhomogeneities in the
magnetic field or the electron distribution. The magnetic
field component along the line of sight, \(B_\|\), determines the
Faraday effect. The polarization plane of a linearly polarized
wave with wave length \(\lambda\) passing through a magnetized
medium is rotated by an angle \(RM\lambda^2\) where the rotation
measure \(RM\) is given by the integral over the path length,
\(RM \propto \int n_e B_\| ds \text{ (rad m}^{-2}\text{)}\) where \(n_e\) is the electron density
[1, 2]. Unless there is independent information about the
electron energy distribution, the magnetic field strength is
determined assuming equipartition. This means equalizing
the energy densities in the magnetic field and electrons. In
our own galaxy, this hypothesis can be tested. It is found
to be in good agreement with magnetic field estimates using
independent information of cosmic ray energy distributions
[3]. Zeeman splitting of spectral lines is another possibility,
though generally it is very limited due to the much larger line
width [1].

Over recent years, methods were developed to detect
truly cosmologically magnetic fields, not associated with any
stable, gravitationally bound structures. The spectral energy
distribution of some TeV blazars in the TeV and GeV range
hint at the presence of a cosmological magnetic field pervading
all space [4]. TeV blazars are a type of active galactic nucleus
(AGN) which produce \(\gamma\) ray photons in the TeV energy range.
These photons cannot travel very far from the source since
they interact with the extragalactic background light producing
electron–positron pairs. These particles interact with the
photons of the cosmic microwave background (CMB) via an
inverse Compton effect, thereby emitting secondary photons
in the GeV energy range. Thus an electromagnetic cascade
takes place. The trajectories of the electrons and positrons
in this cascade are deflected if the cascade takes place in a
magnetized medium. This could then lead to a time-delayed
observation of the GeV signal, the detection of an extended
emission of an initially point-like source or the absence of power in the GeV part of the energy spectrum of the source, since the charged particles have been deflected out of our line of sight. Observations of any of these effects imply a lower bound on any cosmological magnetic field. An interesting example is the TeV blazar 1ES0229+200. The observations of HESS and VERITAS telescopes indicate no significant time variability in the TeV energy flux over an observation time scale of three years. The corresponding Fermi/LAT data of this source show a lack of power in the GeV energy range. Depending on the particular model of the geometry and parameters of the electromagnetic cascades, the lower limit of the intergalactic magnetic field is estimated to be larger than \( B > 5 \times 10^{-15} \) G [5] or \( B > 10^{-18} \) G [6]. AGNs are thought to produce cosmic rays, in particular protons. These interact with the CMB photons as well as the EBL photons and produce secondary high-energy photons at energies above TeV [7]. In [8], it was suggested that the lack of timing correlations between observations in different energy bands of the same object, as observed for example in the blazar Markarian 421 [9], could be explained by the presence of cosmic rays and the subsequent generation of secondary photons. Including the cosmic ray contribution leads to a limit of \( 1 \times 10^{-17} \) G \(< B < 10^{-14} \) G [8].

Magnetic fields play an important role in the physics of star formation as they allow one to reduce the angular momentum of the protostellar cloud during collapse. Moreover, the magnetic pressure acts against gravitational collapse. Thus the presence of magnetic fields has an important effect on the distribution of stellar masses as well as density perturbations, which will be discussed in more detail in section 3.

In general, magnetic fields in spiral galaxies have a regular component and a random component. Depending on the location within the galaxy, the regular or the random component can be dominant. Typically, the field strength of the regular component in spiral galaxies is of the order of 1–5 \( \mu \)G [3]. There are examples of galaxies with much stronger regular magnetic fields of up to 15 \( \mu \)G, such as in the interarm region of NGC 6946 [10]. In general, the magnetic field in the spiral arms is dominated by the random component, which is due to the star formation and expansion of supernovae remnants, leading to turbulence in the interstellar medium, thereby entangling the magnetic field lines. The total magnetic field strengths, including the regular and random components, is on average of the order of 9 \( \mu \)G; however, in the prominent spiral arms of M51, it is of the order of 30–35 \( \mu \)G [3].

Over recent years, the number of Faraday rotation measures RM to estimate the magnetic field in our own Galaxy has increased significantly thanks to, e.g., the National Radio Astronomy Observatory VLA Sky Survey (NVSS) of polarized radio sources. In [11], 37 543 RMs have been derived. The properties of the galactic magnetic field vary with location. The central region of the galaxy is endowed with a highly regular magnetic field of a strength up to milligauss concentrated in filaments [12, 13]. The Galactic halo magnetic field in the solar neighbourhood is found to be of the order of \( 10^{-1} \) \( \mu \)G. The local regular disc magnetic field is \( 4 \pm 1 \) \( \mu \)G and the total magnetic field strength \( \langle B_0 \rangle = 6 \pm 2 \) \( \mu \)G [3]. Multiple field reversals are observed in the galactic magnetic field. This is rather unusual and has not been observed in other spiral galaxies. However, it might actually be the result of us observing from within the galaxy, meaning that observations trace different volumes, or due to large-scale anisotropic field loops [3, 12].

Observations of clusters of galaxies indicate magnetic fields of the order \( \mu \)G. Faraday rotation measurements of the Coma cluster yield a field strength of 2 \( \mu \)G [14]. The structure of the magnetic field depends on the type of cluster. The random component is of the order of \( \mu \)G for non-cooling flow clusters, such as the Coma cluster with larger correlation length (10–30 kpc) and can reach several \( \mu \)G for cooling flow clusters, such as the Hydra A cluster and shorter correlation lengths [15].

There are also observations of magnetic fields associated with high-redshift galaxies. In particular in [16], Faraday rotation measures of 268 quasars and radio galaxies up to a redshift \( z \sim 3.7 \) were determined. It was found that these objects are endowed with \( \mu \)G-level magnetic fields, indicating that they were generated very quickly at early cosmological epochs.

### 2. Generation of primordial magnetic fields

The wealth of observations of magnetic fields in the Universe naturally leads to the question of their origin. Clearly, the presence of magnetic fields in high-redshift objects puts their time of generation long before the present epoch. Indications of void magnetic fields not associated with any gravitationally bound structures add a novel aspect since, in this case, the generation mechanism cannot rely directly on, say, amplification mechanisms during gravitational collapse or dynamo mechanisms.

For galactic magnetic fields, it is generally assumed that a galactic dynamo is operating thereby amplifying an initial seed field. Depending on the efficiency of the dynamo, the magnetic seed field can be as small as \( B_{\text{seed}} = 10^{-20} \) G [17]. Taking into account the contribution of dark energy observed in our Universe significantly relaxes the lower bound on the field strength to explain the present day galactic magnetic field of \( \mu \)G level, lowering it to \( B_{\text{seed}} \sim 10^{-30} \) G [18]. The standard mechanism is the \( \alpha-\Omega \) mean field dynamo theory. The key equation is [19]

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{\eta c^2}{4\pi} \nabla^2 \vec{B} \tag{1}
\]

where \( \eta \) is the resistivity of the plasma. \( \nabla^2 \vec{B} \) can be approximated by \( \vec{B}/L^2 \), so that the last term can be written in terms of a decay time \( \tau_{\text{decay}} \) as \( \vec{B}/\tau_{\text{decay}} \). It turns out that in the galaxy, the last term can be neglected since for a typical gas temperature of \( 10^4 \) K, the resistive term \( \eta c^2/4\pi \simeq 10^4 \) cm\(^2\) s\(^{-1}\) [19] implies a decay time of \( 10^{16} \) L/(1 pc\(^2\))\(^{-1}\) years where \( L \) is the correlation length of the magnetic field. Neglecting the resistivity term in equation (1), the mean field \( \alpha-\Omega \) dynamo is determined by splitting the velocity as well as the magnetic field in a mean part, indicated by an index 0 and a random part, indicated by a \( \delta \). Ensemble averaging over the fluctuations
yields \[19\]
\[\frac{\partial \vec{B}_0}{\partial t} = \nabla \times (\vec{v}_0 \times \vec{B}_0) + \nabla \times \left( (\delta \vec{v} \times \delta \vec{B}) \right). \tag{2}\]

Solving the corresponding evolution equation for the magnetic field fluctuation \( \delta \vec{B} \) in terms of the turbulent velocity \( \delta \vec{v} \) in the quasilinear expansion, leads to \[19\]
\[\langle \delta \vec{v} \times \delta \vec{B} \rangle = \alpha \vec{B}_0 - \beta \nabla \times \vec{B}_0 \tag{3}\]
where \( \alpha = -\frac{1}{3} \tau_c (\delta \vec{v} \cdot (\nabla \times \delta \vec{v})) \), where \( \tau_c \) is the correlation time. Hence \( \alpha \) is determined by the kinetic helicity of the random velocity field. \( \beta = \frac{1}{3} \tau_c (\delta v^2) \), which determines the diffusion that smooths out the turbulent magnetic field component \[20\].

Finally, the mean field dynamo equation reads \[19\]
\[\frac{\partial \vec{B}_0}{\partial t} = \nabla \times (\vec{v}_0 \times \vec{B}_0) + \nabla \times (\alpha \vec{B}_0) + \beta \nabla^2 \vec{B}_0. \tag{4}\]

Applied to the galactic disc, it is appropriate to use cylindrical coordinates; the mean velocity is expressed in terms of the galactic rotation. The key ingredients for the mean field dynamo to work are the differential rotation of the galaxy (\( \Omega \)) and the turbulent motion (\( \alpha \)). There are certain problems with the \( \alpha-\Omega \) mean field dynamo, such as achieving enough turbulence and hence amplification, as discussed, e.g., in \[19, 21\].

The problem of cluster magnetic fields is still much less understood than that of galactic magnetic fields. As mentioned above, clusters of galaxies are endowed with magnetic fields at the \( \mu \)G level and coherence lengths of order \( 10 \) kpc. The source of the cluster magnetic fields could be, for example, outflows from active galaxies thereby transporting bubbles of magnetized plasma into the intergalactic medium; although this could only act as seed fields for the cluster fields and would need further amplification. However, in this case, the \( \alpha-\Omega \) mean field dynamo does not work since galaxy clusters are found to have only very weak rotation. Small-scale turbulent dynamo action is a possibility, though the origin of the turbulence in clusters is still not resolved \[21\].

All dynamo mechanisms have in common the need for an initial seed magnetic field. Broadly speaking, there are two different classes of generation mechanisms \[22, 23\]. One class of models puts the generation of large-scale magnetic fields in the very early Universe when it was expanding extremely rapidly (generically, exponentially), a stage known as inflation. During inflation, quantum fluctuations in matter fields or other fields, such as the electromagnetic field, are stretched beyond the horizon to very large scales, i.e. super horizon scales, where they become classical and their amplitude could be amplified. The generation of magnetic fields during inflation was first proposed in \[17\]. However, it was immediately clear that within standard electrodynamics, the field strength of the generated magnetic fields falls short of the required minimal magnetic field strength \( B_{\text{seed}} \sim 10^{-20} \) G and even of the less stringent bound in the presence of a cosmological constant, in order to seed the galactic dynamo. It is common to use the ratio of magnetic field energy density over photon energy density \( r \equiv \frac{\rho_B}{\rho_r} \) for a frozen-in magnetic field \( \rho_B \propto a^{-\gamma} \) where \( a \) is the scale factor. Since the photon energy density has the same scaling with \( a, r \) is a constant in this case. \( B_r \approx 10^{-20} \) G corresponds to \( r \approx 10^{-37} \) and \( B_r \approx 10^{-30} \) G to \( r \approx 10^{-57} \). For a stochastic magnetic field, \( r \) is calculated using the energy density stored in the mode with comoving wave number \( k \), that is \( \rho_B = k^D n_B \). Following \[17\], we assume that the energy density stored in the mode with comoving wave length \( \lambda \) is of the order of the energy density in a thermal bath at the Gibbons–Hawking temperature of de Sitter space. Thus, the first horizon crossing the magnetic energy density is given by
\[\rho_B(a_2) \approx H^4 \approx \left( \frac{M^4}{\lambda^2 M_p^2} \right)^{\frac{1}{2}} \tag{5}\]
where the constant energy density during inflation is given by \( M^4 \) and \( a_2 \) is the scale factor at the time when the comoving length scale \( \lambda \) was crossing the horizon during inflation. \( M_p \) is the Planck mass. At the end of inflation, this implies
\[r(a_1) \approx 10^{-104} \left( \frac{\lambda}{1 \text{ Mpc}} \right)^{-4} \left( \frac{M}{T_{\text{RH}}} \right)^{10/3} \tag{6}\]
Thus at a galactic scale, \( \lambda = 1 \) Mpc, and typical values for \( M \) and the reheat temperature \( T_{\text{RH}} \), say \( M = 10^{17} \) GeV and \( T_{\text{RH}} = 10^9 \) GeV, \( r \) is of the order of \( r \approx 10^{-80} \), which is far below the required minimal value even in the presence of a cosmological constant.

Therefore, in the case of a flat background, it is necessary to go beyond the standard model. The key point is to effectively change the amplification on super horizon scales. There are different possibilities of modifying the standard four-dimensional electromagnetic Lagrangian, such as coupling to curvature terms, coupling to a scalar field or extra dimensions. There are models for which cosmologically relevant magnetic fields can be generated during inflation, for an extensive review see \[22\]. However, it has to be ensured that there are no problems with back reaction or strong coupling \[24\] nor with the generation of the required curvature perturbations for standard \( \Lambda \)CDM cosmology \[25\]. For open universes, it has been argued that standard electrodynamics is sufficient \[26\] though recently this has been questioned \[27\]. In general, the correlation length is not a problem for magnetic fields generated during inflation but rather the field strength. This is exactly the opposite in the second class of generation mechanisms of large-scale magnetic fields. For this type of model, magnetic fields are generated after inflation during some phase transition, such as the electroweak or QCD phase transition. The underlying idea is charge separation, as in a battery mechanism. Magnetic fields generated during phase transitions have non-vanishing magnetic helicity, enabling inverse cascade processes. The correlation lengths of magnetic fields generated, e.g., during the electroweak phase transitions would have correlation lengths of the order of 1 AU. However, this could be increased by an inverse cascade, transferring power from smaller to larger scales (see, e.g., \[22\]).
3. Imprints of primordial magnetic fields on the CMB

Magnetic fields are determined by their energy density and pressure, as well as their anisotropic stress. These are additional contributions that have to be taken into account when describing the evolution of perturbations in the primordial plasma, which later on in the evolution of the Universe provide the inhomogeneities in the gravitational potential necessary for galaxy formation and large-scale structure in general. Moreover, magnetic fields also change the evolution of the baryon velocity due to the contribution of the Lorentz force term, long before recombination electrons and baryons are tightly coupled with the photons, due to Thomson scattering of photons off free electrons. Thus long before decoupling, the Lorentz term also effects the evolution of the photons. All of this leads to important effects of primordial magnetic fields present before decoupling on the angular power spectrum of the temperature anisotropies and polarization of the CMB, as well as the matter power spectrum [28–30].

Observations exclude the presence of any homogeneous magnetic field on large scales of a present day field strength larger than $10^{-11}$ G [2]. Therefore, assuming a Gaussian random field, it is completely determined by its two point function in Fourier space (e.g. [30]),

$$\langle B^*_j(\vec{k})B_j(\vec{k}') \rangle = \delta_{kk'} P_S(k) \left( \delta_{ij} - \hat{k}_i \hat{k}_j \right) + \delta_{k_i k_j} P_A(k) \epsilon_{ijm} \delta_{lm},$$

(7)

where $P_S(k)$ is the power spectrum of the symmetric part related to the magnetic energy density, $P_A(k)$ is the power spectrum of the asymmetric part related to the magnetic helicity and a hat indicates a unit vector. Typically, for these power spectra, a power law is assumed. Moreover, since magnetic fields suffer viscous damping before decoupling [31], there is an upper cut-off $k_m$. In addition to the parity, even modes helical magnetic fields induce odd parity modes in the CMB, such as cross correlations between the $E$ and $B$ polarization, as well as between the temperature and the polarization $B$ mode [30, 32]. Using data from the CMB experiments WMAP, QUaD and ACBAR [33] put an upper limit on a non-helical magnetic field of 6.4 nG at a scale of 1 Mpc.

There is a characteristic signature of magnetic fields in the linear matter power spectrum at small scales. The origin is in the Lorentz term in the baryon velocity equation, which leads to the evolution of the total matter perturbation $\Delta_m$ [29, 33]

$$\ddot{\Delta}_m + \mathcal{H} \dot{\Delta}_m - \frac{3}{2} \mathcal{H}^2 \Delta_m = \mathcal{H}^2 \Omega_r \Delta_B - \frac{k^2}{3} \Omega_B L,$$

(8)

where $\mathcal{H} = \dot{a}/a$, $\Delta_B$ is the magnetic energy density in terms of the photon energy density and $L = \Delta_B - \frac{2}{3} \pi_B$ is the Lorentz term where $\pi_B$ is the magnetic anisotropic stress. During matter domination on small scales, the resulting linear matter power spectrum then behaves as $P_{\Delta_m} \propto k^4 P_l$. Therefore, the magnetic field adds power on small scales, which is absent in the standard $\Lambda$CDM model without a magnetic field.

4. Summary

There is observational evidence for large-scale magnetic fields in the Universe on a large range of scales. Over decades, there has been a multitude of detections of magnetic fields in galaxies, including our own Milky Way and clusters of galaxies. These are typically in the range of $\mu$G. In recent years, observations of TeV blazars point towards the presence of a truly cosmologically, that is void, magnetic field in the femtogauss range. Observations of the CMB put an upper limit in the nG range.

It is generally assumed that a dynamo mechanism amplifies an initial seed magnetic field to the present day $\mu$G level. The origin of the initial seed magnetic field is still an open problem. There is a range of proposed mechanisms that take place during inflation in the very early Universe. In a flat Universe, it requires breaking conformal invariance of standard electrodynamics. The problem here is to achieve strong enough magnetic fields but the correlation length is not a problem. On the contrary, magnetic fields generated after inflation, during a phase transition, can be very strong however, their correlation length is very small since it is limited by the horizon size at the time of generation. This problem could be alleviated by using an inverse cascade, operating due to the helical nature of the fields. Magnetic fields present before decoupling have an influence on the CMB as well as the matter power spectrum, which will offer new possibilities of constraining a truly cosmological magnetic field with future experiments.

Acknowledgments

I would like to thank the organizers of the 40th EPS Conference on Plasma Physics for the invitation and the EPS for financial support. Spanish Science Ministry grants FPA2009-10612, FIS2012-30926 and CSD2007-00042 are gratefully acknowledged.

References

[1] Zweibel E and Heiles C 1997 Nature 385 131
[2] Wielebinski R and Beck R (ed) 2005 Cosmic Magnetic Fields (Lecture Notes in Physics) (Berlin: Springer)
[3] Beck R 2003 Magnetic fields in the Milky Way and other spiral galaxies (arXiv:astro-ph/0310287)
[4] Neronov A and Vovk I 2010 Science 328 73
[5] Tavecchio F, Ghisellini G, Foschini L, Bonnoli G, Ghirlanda G and Coppi P 2010 Mon. Not. R. Astron. Soc. 406 L70
[6] Dermer C D, Cavadini M, Razzaque S, Finkbeiner D and Lott B 2011 Astrophys. J. 733 L21
[7] Essey W, Kalashev O E, Kusenko A and Beaumont J F 2010 Phys. Rev. Lett. 104 141102
[8] Essey W, Ando S and Kusenko A 2011 Astropart. Phys. 35 135
[9] Horan D et al 2009 Astrophys. J. 695 596
[10] Beck R and Hoernes P 1996 Nature 379 47
[11] Taylor A R, Stil J M and Sunstrum C 2009 Astrophys. J. 702 1230
[12] Beck R 2001 Space Sci. Rev. 99 243
[13] Han J 2012 Magnetic fields in our Milky Way galaxy and nearby galaxies (arXiv:1212.5464)
[14] Kim K-T, Kronberg P P, Dewdney P E and Landeckker T L 1990 Astrophys. J. 355 29
[15] Ensslin T, Vogt C and Pfommer C 2005 Magnetic fields in clusters of galaxies (arXiv:astro-ph/0501338)
[16] Kronberg P P, Bernet M L, Miniati F, Lilly S J, Short M B and Higdon D M 2008 Astrophys. J. 676 70
[17] Turne M S and Widrow L M 1988 Phys. Rev. D 37 2743
[18] Davis A-C, Lilley M and Tornkvist O 1999 Phys. Rev. D 60 021301
[19] Kuala R M and Zweibel E G 2008 Rep. Prog. Phys. 71 046091
[20] Kuala R M 2005 Cosmic Magnetic Fields (Lecture Notes in Physics) ed R Wielebinski and R Beck (Berlin: Springer) Kuala R M 2005 Plasma Physics for Astrophysics (Princeton, NJ: Princeton University Press)
[21] Brandenburg A and Subramanian K 2005 Phys. Rep. 417 1
[22] Kuala A, Kuala K E and Tsagas C G 2011 Phys. Rep. 505 1
[23] Grasso D and Rubinstein H R 2001 Phys. Rep. 348 163 Widrow L M 2002 Rev. Mod. Phys. 74 775 Giovannini M 2004 Int. J. Mod. Phys. D 13 391
[24] Martin I and Yokoyama J 2008 J. Cosmol. Astropart. Phys. JCAP0801(2008)025 Demozzi V, Mukhanov V and Rubinstein H 2009 J. Cosmol. Astropart. Phys. JCAP0908(2009)025 Himmetoglu B, Contaldi C R and Peloso M 2009 Phys. Rev. D 80 123530 Bartolo N, Matarrese S, Peloso M and Ricciardone A 2013 Phys. Rev. D 87 023504 Ferreira R J Z, Iain R K and Sloth M S 2013 Inflationary magnetogenesis without the strong coupling problem (arXiv:1305.7151 [astro-ph.CO])
[25] Kuala K E 2013 Phys. Rev. D 87 063505 Caldwell R R, Motta L and Kamionkowski M 2011 Phys. Rev. D 84 123525 Kuala K E 2010 Phys. Rev. D 81 043526
[26] Tsagas C G and Kuala A 2005 Phys. Rev. D 71 123506 Barrow J D and Tsagas C G 2008 Phys. Rev. D 77 107302 Barrow J D and Kuala C G 2008 Phys. Rev. D 77 109904 (erratum) Barrow J D, Kuala C G and Yamamoto K 2012 Phys. Rev. D 86 023533
[27] Adamek J, de Rham C and Durrer R 2012 Mon. Not. R. Astron. Soc. 423 2705 Shtanov Y and Sahni V 2013 J. Cosmol. Astropart. Phys. JCAP1301(2013)008
[28] Yamazaki D, Ichiki K, Kajino T and Mathews G J 2006 Astrophys. J. 646 719 Giovannini M and Kuala K E 2008 Phys. Rev. D 77 063003 Paoletti D, Finelli F and Paci F 2009 Mon. Not. R. Astron. Soc. 396 523 Shaw J R and Lewis A 2010 Phys. Rev. D 81 043517
[29] Kuala K E 2011 Phys. Rev. D 83 023506 Kuala K E 2012 Phys. Rev. D 85 083504
[30] Jedamzik K, Katalinic V and Olinto A V 1998 Phys. Rev. D 57 3264 Subramanian K and Barrow J D 1998 Phys. Rev. D 58 083502
[31] Shaw J R and Lewis A 2012 Phys. Rev. D 86 043510