A Direct and Generalized Construction of Polyphase Complementary Set With Low PMEPR

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Abstract—A salient disadvantage of orthogonal frequency division multiplexing (OFDM) systems is the high peak-to-mean envelope power ratio (PMEPR). The PMEPR problem can be solved by using complementary sequences with low PMEPR. In this paper, we present a new construction of complementary set (CS) by using generalized Boolean functions (GBFs), which generalizes the constructions given by Davis et al., Paterson and Schmidt. The proposed CS provides lower PMEPR upper bound as compared to Schmidt’s method for the sequences corresponding to higher order (\(\geq 3\)) GBFs. We obtain complementary sequences with maximum PMEPR of \(2^{k+1}\) and \(2^{k+2} - 2M\) where \(k, M\) are non-negative integers that can be easily derived from the GBF associated with the CS.

Index Terms—Complementary set (CS), Golay complementar pair (GCP), generalized Boolean function (GBF), orthogonal frequency-division multiplexing (OFDM), peak-to-mean envelope power ratio (PMEPR), Reed-Muller (RM) code.

I. INTRODUCTION

A major problem of orthogonal frequency-division multiplexing (OFDM) is its large peak-to-mean envelope power ratio (PMEPR) for the uncoded signals. PMEPR reduction through a coding technique can be achieved by designing a large codebook whose codewords, e.g., in the form of sequences, have low PEMP values. This paper is focused on PMEPR reduction using codebooks from complementary sequences which to be introduced in the sequel.

The concept of Golay complementary pair (GCP) was introduced by M. J. E. Golay in [1], where either sequence from a GCP is known as Golay sequence. By definition, the aperiodic autocorrelation (AAC) sidelobes of two constituent sequences in a GCP sum to zero everywhere except for the in-phase time shift position. GCP was extended to complementary sets (CSSs) by Tseng and Liu in [2] where each CS consists of two or more constituent sequences, called complementary sequences, and with zero AAC sum property. A PMEPR reduction method was introduced by Davis and Jedwab in [3] to construct Golay sequences of length \(2^m\) (\(m\) is a positive integer) by using second-order generalized Boolean function (GBF). Subsequently, Paterson employed complementary sequences to enlarge the code rate by relaxing the PMEPR of OFDM signal in [4]. Specifically, Paterson showed that each coset of generalized first-order Reed-Muller (RM) codes \(RM_1(1, m)\) inside second-order generalized RM codes \(RM_2(2, m)\) \(q\) is an even number no less than 2) can be partitioned into CSs of size \(2^{k+1}\) (where \(k\) is a non-negative integer depending only on \(G(Q)\), a graph naturally associated with the quadratic form \(Q\) in \(m\) variables which defines the coset) and provided an upper bound on the PMEPR of arbitrary second-order cosets of \(RM_q(1, m)\). The construction given in [4, Th. 12] was unable to provide a tight PMEPR bound for all the cases. By giving an improved version of [4, Th. 12] in [4, Th. 24], Paterson raised the following question:

“What is the strongest possible generalization of [4, Th. 12]?”

In [4, Th. 24], it is shown that after deleting \(k\) vertices in \(G(Q)\), if the resulting graph contains a path and one isolated vertex, then \(Q + RM_q(1, m)\) can be partitioned into CSs of size \(2^{k+1}\) instead of \(2^{k+2}\), i.e., there is no need to delete the isolated vertex. Later, a generalization of [4, Th. 12] was made by Schmidt in [5] to establish a construction of complementary sequences that are contained in higher-order generalized RM codes. Schmidt shows that a GBF also can be used to generate a CS of a given size if it can be expressed by a path when it is restricted to certain variables. However, Schmidt’s construction is not applicable when the graphs of some or all restricted Boolean functions are not path. As a result, a reasonable number of sequences with low PMEPR were excluded from the Schmidt’s construction. Hence, an improved version of [5, Th. 5] or, a more generalized version of [4, Th. 12] is expected to extend the range of coding options with good PMEPR bound for practical applications of OFDM.

More constructions of CS with low PMEPR can be found in [6]–[9]. In [6], a framework is given to identify known Golay sequences and pairs of length \(2^m\) (\(m > 4\)) over \(\mathbb{Z}_{2^h}\) (\(h\) is a positive integer) in explicit algebraic normal form. Nowadays, besides polyphase complementary sequences, the design of quadrature amplitude modulation (QAM) complementary sequences with low PMEPR is also an interesting research topic. In [8], QAM Golay sequences were introduced by Li, based on quadrature phase shift keying (PSK) GDJ-code. Later, Liu et al. constructed QAM Golay sequences by using properly selected Gaussian integer pairs [9]. Recently, some constructions on complementary or near-complementary sequences have been reported in [10]–[14]. These sequences may also be applicable in OFDM system to deal with PMEPR problem, in addition to their applications in scenarios such as...
asynchronous communications and channel estimation.

In this paper, we propose a novel construction to generate CS with low PMEPR. It uses higher order GBFs and graphical properties of restricted Boolean functions. By taking advantage of certain graphical properties related to both Path and isolated vertices associated to higher order GBFs, we show that the PMEPR upper bound can be further tightened. By moving to higher order RM code, we give a partial answer to the aforementioned open question raised by Paterson on strongest generalization of [4, Th. 12]. It will be shown that our proposed construction includes the previous ones by Davis and Jedwab [3], Paterson [4], and Schmidt [5] as special cases.

The remaining paper is organized as follows. In Section II, some useful notations and definitions are given. In Section III, a generalized construction of CS is presented. Section IV contains a discussion. Finally, concluding remarks are drawn in Section V.

II. PRELIMINARY

A. Definitions of Correlations and Sequences

Let $a = (a_0, a_1, \ldots, a_{L-1})$ and $b = (b_0, b_1, \ldots, b_{L-1})$ be two complex-valued sequences of equal length $L$ and let $\tau$ be an integer. Define

$$C(a, b)(\tau) = \begin{cases} \sum_{i=0}^{L-1} a_i \alpha^i b_{\tau+i}^*, & 0 \leq \tau < L, \\ \sum_{i=0}^{L-1} a_i \alpha^{i+\tau} b_{-\tau+i}^*, & -L \leq \tau < 0, \\ 0, & \text{otherwise}, \end{cases}$$

and $A(a)(\tau) = C(a, a)(\tau)$. The above mentioned functions are called the aperiodic cross-correlation function between $a$ and $b$ and the AAC function of $a$, respectively.

**Definition 1:** A set of $n$ sequences $a^0, a^1, \ldots, a^{n-1}$, each of equal length $L$, is said to be a CS if

$$A(a^0)(\tau)+A(a^1)(\tau)+\cdots+A(a^{n-1})(\tau) = \begin{cases} nL, & \tau = 0, \\ 0, & \text{otherwise}. \end{cases}$$

A CS of size two is called GCP. Sequences lying in a CS of size $n \geq 4$ is called complementary sequences in this paper.

B. PMEPR of OFDM signal

For $q$-PSK modulation, the OFDM signal for the word $a = (a_0, a_1, \ldots, a_{L-1})$ (where $a_i \in \mathbb{Z}_q$) can be modeled as the real part of

$$S(a)(t) = \sum_{j=0}^{L-1} a_j e^{2\pi i (f_0 + j f_s) t},$$

where $\omega_q = \exp(2\pi \sqrt{-1}/q)$ is a complex $q$th root of unity and $f_0 + j f_s \ (0 \leq j < L)$ is $j$th carrier frequency of the OFDM signal. We define the instantaneous envelope power of the OFDM signal as [4]

$$P(a)(t) = |S(a)(t)|^2.$$

From the above expression, it is easy to derive that

$$P(a)(t) = \sum_{\tau=0}^{L-1} A(a)(\tau) \exp(2\pi \sqrt{-1} \tau f_s t)$$

$$= A(a)(0) + 2 \cdot \text{Re} \left\{ \sum_{\tau=1}^{L-1} A(a)(\tau) \exp(2\pi \sqrt{-1} \tau f_s t) \right\},$$

(3)

where $\text{Re}\{x\}$ denotes the real part of a complex number $x$.

We define the PMEPR of the signal $S(a)(t)$ as

$$\text{PMEPR}(a) = \frac{1}{n} \sup_{0 < t < 1} P(a)(t).$$

The largest value that the PMEPR of an $n$-subcarrier OFDM signal is $n$. It is also noted that the PMEPR of a sequence lying in a CS of size $K$ (where, $K$ is a positive integer no less than 2) is upper bounded by $K$.

C. Generalized Boolean Functions

Let $f$ be a function of $m$ variables $x_0, x_1, \ldots, x_{m-1}$ over $\mathbb{Z}_q$. A monomial of degree $r$ is defined as the product of any $r$ distinct variables among $x_0, x_1, \ldots, x_{m-1}$. There are $2^m$ distinct monomials over $m$ variables listed below:

$$1, x_0, x_1, \ldots, x_{m-1}, x_0 x_1, x_0 x_2, \ldots, x_{m-2} x_{m-1}, \ldots, x_0 x_1 \cdots x_{m-1}.$$

A function $f$ is said to be a GBF of order $r$ if it can be uniquely expressed as a linear combination of monomials of degree at most $r$, where the coefficient of each monomial is drawn from $\mathbb{Z}_q$. A GBF of order $r$ can be expressed as

$$f = Q + \sum_{i=0}^{m-1} g_i x_i + g',$$

(4)

where

$$Q = \sum_{p=2}^{r} \sum_{0 \leq a_0, a_1, \ldots, a_{p-1} \leq q} a_{a_0, a_1, \ldots, a_{p-1}} x_{a_0} x_{a_1} \cdots x_{a_{p-1}},$$

(5)

and $g_i, g', a_{a_0, a_1, \ldots, a_{p-1}} \in \mathbb{Z}_q$.

D. Quadratic Forms and Graphs

Let $f$ be a $r$th order GBF of $m$ variables over $\mathbb{Z}_q$. Assume $x = (x_{j_0}, x_{j_1}, \ldots, x_{j_{k-1}})$ and $e = (c_0, c_1, \ldots, c_{k-1})$. Then $f \mid_{x,e}$ is obtained by substituting $x_{j_a} = c_a \ (a = 0, 1, \ldots, k-1)$ in $f$. If $f \mid_{x,e}$ is a quadratic GBF, then $G(f \mid_{x,e})$ denotes a graph with $V = \{x_0, x_1, \ldots, x_{m-1}\} \setminus \{x_{j_0}, x_{j_1}, \ldots, x_{j_{k-1}}\}$ as the set of vertices. The $G(f \mid_{x,e})$ is obtained by joining the vertices $x_{a_1}$ and $x_{a_2}$ by an edge if there is a term $q_{a_1 a_2} x_{a_1} x_{a_2} \ (0 \leq a_1 < a_2 < m, x_{a_1}, x_{a_2} \in V)$ in the GBF $f \mid_{x,e}$ with $q_{a_1 a_2} \neq 0 \ (q_{a_1 a_2} \in \mathbb{Z}_q)$. For $k = 0$, $G(f \mid_{x,e})$ is nothing but $G(f)$. Throughout this paper
E. Sequence Corresponding to a Boolean Function

Corresponding to a GBF \( f \), we define a complex-valued vector (or, sequence) \( \psi(f) \), as follows.

\[
\psi(f) = (\omega_q^{f_0}, \omega_q^{f_1}, \ldots, \omega_q^{f_{m-1}}),
\]

(6)

where \( f_i = f(i_0, i_1, \ldots, i_{m-1}) \) and \( (i_0, i_1, \ldots, i_{m-1}) \) is the binary vector representation of integer \( i \) (i.e., \( \sum_{\alpha=0}^{2^m} i_\alpha 2^\alpha \)).

Again, we define \( \psi(f \mid x_e) \) as a complex-valued sequence with \( \omega_q^{f(i_0, i_1, \ldots, i_{m-1})} \) as \( \alpha \)th component if \( i_j = \alpha \) for each \( 0 \leq \alpha < k \) and equal to zero otherwise.

Next, we present a lemma which will be used in our proposed construction.

Lemma 1 ([15]): Suppose that there are two GBFs \( f \) and \( f' \) of \( m \)-variables \( x_0, x_1, \ldots, x_{m-1} \) over \( \mathbb{Z}_q \), such that for some \( k \) \((\leq m - 3)\) restricting variables \( x = (x_{j_0}, x_{j_1}, \ldots, x_{j_{k-1}}) \), \( f \mid_{x_e} \) and \( f' \mid_{x_e} \) is given by

\[
f \mid_{x_e} = P + L + g_i x_i + g,
\]

\[
f' \mid_{x_e} = P + L + g_i x_i + \frac{q}{2} x_a + g,
\]

(7)

where \( L = \sum_{a=0}^{m-k-2} g_a x_a \), \( P \) is the quadratic form present in \( f \mid_{x_e} \) or, \( f' \mid_{x_e} \), both \( G(f \mid_{x_e}) \) and \( G(f' \mid_{x_e}) \) consist of a path having the identical weight \( q/2 \) over \( m-k-1 \) vertices, given by \( P \), \( x_i \) be the either end vertex, \( x_1 \) be an isolated vertex, and \( g_i, g \in \mathbb{Z}_q \). Then for fixed \( e \) and \( d_1 \neq d_2 \)

\[
C(f \mid_{x_{j_1}=e_{d_1}, x_{j_2}=e_{d_2}}) = \begin{cases} 
\omega_q^{f(d_1-d_2)g} 2^{-m-k}, & \tau = (d_2 - d_1)q^2 \\
0, & \text{otherwise.}
\end{cases}
\]

(8)

Throughout this paper, the number \( q \) will be taken as even and no less than 2.

III. PROPOSED CONSTRUCTION

In this section, we present a generalized construction of CS. For ease of presentation, whenever the context is clear, we use \( C(f, g)(\tau) \) to denote \( C(\psi(f), \psi(g))(\tau) \) for any two GBFs \( f \) and \( g \). Similar changes are applied to restricted Boolean functions as well.

Theorem 1: Let \( f \) be a GBF of \( m \) variables over \( \mathbb{Z}_q \) with the property that there exist \( M \) number of such \( e \) for which \( G(f \mid_{x_e}) \) is a path over \( m-k \) vertices and there exist \( N_i \) number of such \( e \) for which \( G(f \mid_{x_e}) \) consists of a path over \( m-k-1 \) vertices and one isolated vertex \( x_i \) such that \( M, N_i \geq 0, M + \sum_{i=1}^{N_i} = 2^k \). Suppose further that all the relevant edges in \( G(f \mid_{x_e}) \) (for all \( e \)) have identical weight of \( q/2 \). Then for any choice of \( g_j, g' \in \mathbb{Z}_q \) (where, \( g_j \) is the coefficient of \( x_j \) and \( g' \) is a constant term present in \( f \), \( j = 0, 1, \ldots, m-1 \)), \( \psi(f) \) lies in a set \( S \) of size \( 2^{k+1} \) with the following aperiodic auto-correlation property.

\[
A(S)(\tau) = \begin{cases} 
\sum_{i=1}^{m} N_i + 2^{m+1} M, & \tau = 0, \\
\omega_q^{g_i} 2m \sum_{e \in S_{N_i}} \omega_q^{-L_i}, & \tau = 2^i, i=1, 2, \ldots, p, \\
\omega_q^{-g_i} 2m \sum_{e \in S_{N_i}} \omega_q^{-L_i}, & \tau = -2^i, i=1, 2, \ldots, p, \\
0, & \text{otherwise.}
\end{cases}
\]

(9)

where, \( g_i, g' \in \mathbb{Z}_q \), \( i = 1, 2, \ldots, p \) is the coefficient of \( x_i \) in \( f \), \( S_{N_i} \) contains all those \( e \) for which \( G(f \mid_{x_e}) \) consists of a path over \( m-k-1 \) vertices and one isolated vertex labeled \( i \) (\( i \in \{0, 1, \ldots, m-1\} \) \( \setminus \{j_0, j_1, \ldots, j_{k-1}\} \) and \( l_1, l_2, \ldots, l_p \) are all distinct),

\[
L_i = \sum_{r=1}^{k} \sum_{1 \leq i_1 < i_2 < \cdots < i_r < k} q_i, \quad (q_i, q_{i_2}, \ldots, q_{i_r}) \in \mathbb{Z}_q \),
\]

and \( q_i \) is the coefficient of \( x_{j_1} x_{j_2} \cdots x_{j_r} x_i \) in \( f \).

Proof: Due to page limitation, we cannot provide the proof here. The details of the proof can be found in [16].

We have introduced \( M \) and \( N_i \) (\( i = 1, 2, \ldots, p \)) in Theorem 1 with the condition \( M + \sum_{i=1}^{N_i} = 2^k \). \( M, N_i \geq 0 \). Therefore, \( M \) and \( N_i \)’s range from 0 to \( 2^k \).

Note 1 (Construction of GBFs as Defined in Theorem 1): The GBFs \( f \), as defined in Theorem 1, can be expressed as

\[
\frac{q}{2} \sum_{e \in S_M} \sum_{i=0}^{m-k-2} x_{\pi_e(i)} x_{\pi_{i+1}(i)} x_{\pi_{i+2}(i+1)} \cdots x_{\pi_{i+k-1}(i+k-1)} \prod_{\alpha=0}^{k-1} x_{\pi_{i+\alpha}(i+\alpha)} (1 - x_{j_{\alpha}})^{1-c_{\alpha}}
\]

\[
+ \frac{q}{2} \sum_{d=1}^{p} \sum_{i=0}^{m-k-3} x_{\pi_{d}(i)} x_{\pi_{i+1}(i+1)} \cdots x_{\pi_{i+k-2}(i+k-2)} \prod_{\alpha=0}^{k-1} x_{\pi_{i+\alpha}(i+\alpha)} (1 - x_{j_{\alpha}})^{1-c_{\alpha}}
\]

\[
\sum_{r=1}^{k} \sum_{1 \leq i_1 < i_2 < \cdots < i_r < k} q_i, \quad (q_i, q_{i_2}, \ldots, q_{i_r}) \in \mathbb{Z}_q \),
\]

(10)

where \( q_i \) are \( N_i \) permutations of \( \{0, 1, \ldots, m-1\} \) \( \setminus \{j_0, j_1, \ldots, j_{k-1}\} \) and \( \alpha_{i_1, i_2, \ldots, i_r} \)’s belong to \( \mathbb{Z}_q \).
Remark 1: Let $f$ be a quadratic GBF with the property that for all $c \in \{0,1\}^k$, $G(f \mid x=c)$ is a path in $m-k$ vertices. Then from Theorem 1, we have $M = 2^k$ and

$$A(S)(\tau) = \begin{cases} 
2m+k+1, & \tau = 0, \\
\sum_{\omega_{q_i}^L} \omega_{q_i}^L, & \tau = 2^i, \\
\omega_{q_i}^L \sum_{\omega_{q_i}^L}, & \tau = -2^i, \\
0, & \text{otherwise,}
\end{cases}$$

(11)

Hence, $S$ is a CS of size $2^{k+1}$ and therefore, Paterson’s construction [4, Th. 12] turns to be a special case of our proposed one.

Remark 2: From Remark 1, for $k = 0$, $S$ is a CS of size 2, i.e., $S$ is a GCP and thus the GDJ code in [3] is also a special case of Theorem 1.

Remark 3: Let $f$ be a quadratic GBF with the property that for all $c \in \{0,1\}^k$, $G(f \mid x=c)$ contains a path in $m-k-1$ vertices and one isolated vertex $x_i$. We also assume that all edges in the original graph between the isolated vertex and the $k$ deleted vertices are weighted by $q/2$. Then, from Theorem 1, we have $N_1 = 2^k$, $S_{N_1} = \{0,1\}^k$, $L_{N_1} = 2 \sum_{\alpha=0}^{k-1} c_\alpha$, and

$$A(S)(\tau) = \begin{cases} 
2m+k+1, & \tau = 0, \\
\sum_{\omega_{q_i}^L} \omega_{q_i}^L, & \tau = 2^i, \\
\omega_{q_i}^L \sum_{\omega_{q_i}^L}, & \tau = -2^i, \\
0, & \text{otherwise,}
\end{cases}$$

(12)

Therefore, $\psi(f)$ lies in a CS of size $2^{k+1}$ and the result given by Paterson in [4, Th. 24] is a special case of Theorem 1.

Remark 4: Let $f$ be a GBF with the property that for all $c \in \{0,1\}^k$, $G(f \mid x=c)$ is a path in $m-k$ vertices. Then from Theorem 1, we have $M = 2^k$ and

$$A(S)(\tau) = \begin{cases} 
2m+k+1, & \tau = 0, \\
0, & \text{otherwise.}
\end{cases}$$

(13)

From (13), it is clear that $\psi(f)$ lies in a CS of size $2^{k+1}$ and hence the PMEPR of $\psi(f)$ is almost $2^{k+1}$. Therefore, the result given by Schmidt in [5, Th. 5] is a special case of Theorem 1.

Note 2: Let $f$ be GBF as defined in Theorem 1. If $\sum_{\omega_{q_i}^L} \omega_{q_i}^L \neq 0$, $\sum_{\omega_{q_i}^L} \omega_{q_i}^L \neq 0$ (i.e., 1, 2, ..., $p$), $\psi(f)$ lies in a CS, $S'$, of size $2^{k+2}$ with at most PMEPR $2^{k+2} - 2M$. The set $S'$ can be expressed as

$$\left\{ f + \frac{q}{2} \left( \sum_{i=1}^{p} x_i + dx_c \right) : d \in \{0,1\}^k, d', \in \{0,1\} \right\}$$

(14)

where

$$S = \left\{ f + \frac{q}{2} \left( d \cdot x + dx_c \right) : d \in \{0,1\}^k, d, \in \{0,1\} \right\},$$

$$S_1 = \left\{ f + \frac{q}{2} \left( d \cdot x + \sum_{i=1}^{p} x_i + dx_c \right) : d \in \{0,1\}^k, d, \in \{0,1\} \right\}.$$
same $f$. Therefore, the PMEPR of $\psi(f)$ is at most 6 and from Schmidt’s construction the PMEPR upper bound of $\psi(f)$ is at most 32.

**Example 2:** Let $f$ be GBF of 5 variables over $\mathbb{Z}_4$, given by

$$f(x_0, x_1, x_2, x_3, x_4) = 2(x_0 x_1 x_2 + x_0 x_1 x_3 + x_1 x_3 + x_3 x_2 + x_0 x_4) + x_1 + 2x_2 + 2x_3 + 2x_4 + 3.$$  \hspace{1cm} (20)

From (20), it is observed that $G(f \mid x_0 = 0)$ and $G(f \mid x_0 = 1)$ both contain a path over the vertices $x_1, x_2, x_3$ and one isolated vertex $x_4$. Therefore, $p = 1$, $N_1 = 2$, $S_{N_1} = \{0, 1\}$, $g_0 = 2$, $g_1 = 1$, $g_2 = g_3 = g_4 = 2$, and $g' = 3$. Since, $\sum_{c \in S_{N_1}} \omega_c^4 = \sum_{c \in S_{N_1}} \omega_c^{L_c^4} = 0$, by using Theorem 1

$$S = \{2(x_0 x_1 x_2 + x_0 x_1 x_3 + x_1 x_3 + x_3 x_2 + x_0 x_4) + x_1 + 2x_2 + 2x_3 + 2x_4 + 3 \} : \begin{array}{l} d = 0, d \in \{0, 1\}. \end{array}$$

(21)

is a CS of size 4. Therefore, the PMEPR of $\psi(f)$ is at most 4 and from Schmidt’s construction, the PMEPR upper bound of $\psi(f)$ is 8.

**IV. DISCUSSION:**

In this section, we discuss the efficiency of our construction as compare to Schmidt’s construction.

The PMEPR upper bound of a sequence $\psi(f)$ corresponding to a GBF $f$ having the graphical property as stated in Theorem 1 is $2^{k+2} - 2M$ when $\sum_{c \in S_{N_1}} \omega_q^L_c \neq 0$, $\sum_{c \in S_{N_1}} \omega_q^{-L_c} \neq 0$. For the same $\psi(f)$, the PMEPR upper bound from Schmidt’s construction is $2^{k+p+1} \{2^{k+2} - 2M\}$ only when $G(f \mid x^c = c')$ is a path for all $c \in \{0, 1\}^k$, $c' \in \{0, 1\}^p$, where $x' = (x_1, x_2, \ldots, x_k)$. $x^c$ denotes the concatenation of $x$ and $x'$, and $c c'$ denotes the concatenation of $c$ and $c'$. Otherwise, from Schmidt’s construction, the PMEPR upper bound for $\psi(f)$ is strictly greater than $2^{k+p+1}$. Specifically, as shown in Example 1, our proposed construction implies that the PMEPR upper bound for $\psi(f)$ is 6, whereas from Schmidt’s construction the PMEPR upperbound for same $\psi(f)$ is 32. In (10), if $p = 1$, $M = 0$, $g_1^1, \ldots, g_1^n = 0$ ($r = 2, 3, \ldots, k$, and $g_1^i \in \{0, \frac{1}{2}\}$ (but $g_1^1, g_1^1, \ldots, g_1^n$ all cannot be zero at the same time), then from Theorem 1, $\sum_{c \in S_{N_1}} \omega_q^{L_c^1} = \sum_{c \in S_{N_1}} \omega_q^{-L_c^1} = 0$, and therefore the PMEPR upper bound for $\psi(f)$ is $2^{k+1}$ as in Example 2, we can see the PMEPR upper bound for $\psi(f)$ is at most 4 whereas Schmidt’s construction provides a PMEPR bound of 8 for the same $\psi(f)$. As described above, our construction can provide tighter PMEPR upper bound as compare to Schmidt’s construction.

**V. CONCLUSIONS**

In this paper, we have proposed a direct and generalized construction of polyphase CS by using higher GBFs whose restricted forms lead to graphs with isolated vertices. The proposed construction provides tighter PMEPR upper bound as compared to Schmidt’s construction. By having a less restrictive property than Paterson’s and Schmidt’s construction it is found that our proposed construction can provide a better code rate and the full results of this research which can be found in [16], will be submitted out as a journal paper. We have shown that our proposed construction generates sequences with maximum PMEPR upper bound 4 and 8. In addition to that we have obtained sequences with maximum PMEPR upper bound 6. The constructions given by Davis et al., Paterson and Schmidt appear as special cases of our proposed construction.

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