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Special Section: Nonlinear Systems in Geophysics: Past Accomplishments and Future Challenges

Key Points:
- For a nonlinear empirical mathematical model, solar forcing explained all of the observed global warmings.
- For doubling of the annual atmospheric CO₂ concentration, the global mean temperature (GMT) increased by 1.4°C and the sea level by 0.37 m.
- The empirical model for the 30 yr GMT moving trends predicts deceleration to less than 0.1°C/decade for period 2007–2037.

Supporting Information:
Supporting Information may be found in the online version of this article.

Correspondence to:
G. J. Orssengo, orssengo@lycos.com

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Determinant of the Sun-Climate Relationship Using Empirical Mathematical Models for Climate Data Sets

Girma J. Orssengo

1Independent Researcher, Perth, WA, Australia

Abstract
Previous studies have reported that human influences are required to explain the observed global warming using a linear model (LM) \( \Delta T = \lambda_{LM} \Delta F \) that relates change in solar forcing \( \Delta F \) to change in global mean temperature (GMT) \( \Delta T \). This model has the shortcoming of assuming a given \( \Delta F \) causes the same \( \Delta T \) irrespective of the value of the initial global warming rate \( (dT/dy)_o \).

Analysis of the GMT data showed that this warming rate has been increasing linearly since steady state \( (dT/dy)_o = 0 \) for year \( y_o = 1864.5 \) as given by \( dT/dy = (dT/dy)_o + a_f(y - y_i) \), where \( a_f = 7.1954 \times 10^{-3} \) °C/year is the secular GMT acceleration and \( y_i \) is the number of years of the change. The secular solar forcing due to 18% of the 11 yr solar cycle forcing of 0.19 W/m² (0.08% of Total Solar Irradiance) was expressed as \( \Delta F = 0.18 \times 0.19(y - y_i)/11 \). Defining the climate sensitivity as \( \lambda = \Delta(dT/dy)/\Delta F = a_f/(0.18 \times 0.19/11) = 0.023143 \) °C/year per W/m² removed the shortcoming of the LM and integration of the model for \( dT/dy \) above and then simplifying gave a secular GMT-solar forcing model given by \( \Delta T = (dT/dy)_o(y - y_i) + (\lambda^2/(2a_f))(\Delta F)^2 \) that explained all of the observed global warming and increase in atmospheric CO₂, sea level and ocean heat content. Therefore, for this nonlinear empirical model, invoking human influences to explain climate change was not required. The annual GMT model predicts a pause in global warming until 2040.

1. Introduction

The Intergovernmental Panel on Climate Change (IPCC) has reported that anthropogenic forcings are required to explain the observed global warming (IPCC, 2007, Section 9.4.1.2): “The fact that climate models are only able to reproduce observed global mean temperature (GMT) changes over the 20th century when they include anthropogenic forcings, and that they fail to do so when they exclude anthropogenic forces, is evidence for the influence of humans on global climate.”

Previous studies have reported that solar forcing alone does not explain most of the observed global warming using a linear model (LM) for the relationship between change in solar forcing \( \Delta F \) in W/m² and the corresponding change in GMT \( \Delta T \) in °C given by IPCC (2001, Section 6.2.1):

\[
\Delta T = \lambda_{LM} \Delta F
\]

where \( \lambda_{LM} = 0.5 \) °C per W/m² is the climate sensitivity for the LM.

IPCC (2001) noted that this equation “is defined for transition of the surface-troposphere system from one equilibrium state to another in response to an externally imposed radiative perturbation.” This definition of equilibrium climate sensitivity has been questioned in a report by Kerr (2013): “Equilibrium climate sensitivity is kind of an odd diagnostic, since it represents something that has never been, and will never be, observed in nature.”

Equation 1 has been widely used in the scientific literature to relate GMT to climate forcing. For example, Sévellec and Drijfhout (2018) assumed “globally averaged temperature responds linearly, with some lag, to the various forcing agents.”

Gregory et al. (2002) reported a similar linear relationship: “General circulation models indicate that the increase in global-average outgoing radiative flux when the climate is perturbed from steady state is proportional to the global-average surface temperature change \( \Delta T \).”

Lean and Rind (2008) assumed that “surface temperatures respond linearly (at some lag) to the various influences.” From data analysis of the GMT and climate forcings using the LM, they concluded that solar
forcing contributed only “10% of the warming in the past 100 yr.” However, the LM $\Delta T = \lambda_{LM} \Delta F$ does not take into account the contribution of the increase with time of the initial global warming rate $(dT/dy)$, where $y$ is time in year) on the observed increase in the secular GMT $\Delta T$ due to the thermal flywheel effect of the ocean (Hoffert et al., 1980) as given by $\Delta T_{flywheel} = (dT/du)(y - y_i)$.

For a given change in solar forcing $\Delta F$, the corresponding change in the secular GMT $\Delta T$ is expected to increase with increase in the initial global warming rate $(dT/du)$. This reasoning indicates a nonlinear relationship between $\Delta F$ and the corresponding $\Delta T$. In this paper, using nonlinear empirical mathematical models for the climate data sets, the solar forcing $F$ since steady state $(dT/du = 0)$ was found to explain all of the observed increases in the secular climate variables of the GMT, atmospheric $\text{CO}_2$, sea level, and ocean heat content.

2. Data and Methods

2.1. Data

In this paper, to determine the empirical mathematical relationships between the climate variables, publicly available climate data sets were used. The web addresses of these data sets are given in the Acknowledgment section.

2.2. The Climate Sensitivity $\lambda$

In contrast to previous studies that reported that solar forcing $\Delta F$ alone does not explain most of the observed global warming $\Delta T$ using the LM $\lambda_{LM} = \Delta T/\Delta F$ (Equation 1), with the unit of the climate sensitivity in °C per W/m$^2$, the aim of this paper is to present a nonlinear solar forcing model that, in addition to considering the direct effect of solar forcing $\Delta F$, takes into account the indirect thermal flywheel effect of the increase in the secular global warming rate $dT/du$ with time on the observed global warming. For this nonlinear model, the climate sensitivity $\lambda$, in °C/year per W/m$^2$, was defined as:

$$\lambda = \frac{\Delta(dT/du)}{\Delta F} = \frac{(dT/du)_i - (dT/du)_f}{F_i - F_f} = \frac{dT/du}{F}$$

(2)

This model indicates that each 11 yr secular solar forcing $\Delta F$ increases the secular GMT trend by $\Delta(dT/du)$ from its previous value, so the current secular GMT trend $dT/du$ and solar forcing $F$ are cumulative ($dT/du = \Sigma(\Delta dT/du)$ and $F = \Sigma(\Delta F)$) since steady state when $dT/du = 0$ and $F = 0$.

From the solar forcing model $dT/du = \lambda F$ (Equation 2), the change in secular GMT $\Delta T$ as a function of the secular solar forcing $F$ since steady state could be derived by integration, which gives:

$$\Delta T = \int \left( \frac{dT}{du} \right) du = \lambda \int F du$$

(3)

This nonlinear solar forcing model that explained all of the observed global warming defines the warming $\Delta T$ to be proportional to the integral of the solar forcing $F$ with respect to time. In contrast, the linear solar forcing model $\Delta T = \lambda_{LM} \Delta F$ (IPCC, 2001, Section 6.2.1) that solar forcing fails to explain all of the observed global warming defines the warming $\Delta T$ to be proportional to just the solar forcing $\Delta F$.

To determine the relationships between the climate variables, the climate data for the GMT, solar forcing, atmospheric $\text{CO}_2$, sea level, and ocean heat content (OHC) were approximated by their corresponding empirical mathematical models. In empirical mathematical modeling, the data for a given variable are approximated by a model given by (De Veaux et al., 2005, p. 149):

$$\text{Data} = \text{Model} + \text{Residual} = \text{Model} \pm 2\sigma$$

(4)

In the above relationship, the residual is the unexplained difference between data and model due to noise (random variation) in the data. Assuming that the distribution of the time-series of the residuals is approximately normal and their standard deviation is $\sigma$, about 95% of the residuals are expected to lie within two standard deviations ($\pm2\sigma$) relative to the modeled values (Figures 1b, 5, 8, 9c, 10, and 11) and all of the residuals within $\pm3.5\sigma$ (Figures 1a, 3, and 4).
Figure 1. Relationship between secular solar forcing $F$ and secular global mean temperature (GMT) $T$ was expressed as $dT/\text{dy} = \lambda F$. (a) Ninety-four year moving GMT trends data for the period from 1850–1944 to 1924–2018 define a straight line for the secular GMT trend model $dT/\text{dy}$. (b) Integration of the secular GMT trend $dT/\text{dy}$ in Figure 1a gives the secular GMT $T$ in Figure 1b. (c) From the cumulative solar forcing model (Equation 2) and the solar irradiance data in Figure 2, a linear secular solar forcing $F$ since steady state was derived.
The use of mathematical models as tools for the study of nature has been highlighted by Mandel (2012, p. 20): “Models, and especially mathematical ones, are our most powerful tool for the study of nature. They are, as it were, the “maps” from which we can read the interrelationships between natural phenomena.”

In this paper, to determine the climate sensitivity $\lambda = (dT / dy) / F$ (Section 3.2), the secular GMT trend $dT / dy$ (Section 3.1) was determined by using 94 yr moving GMT trends data and the secular solar forcing $F$ (Section 3.2) was determined using solar irradiance data (Coddington et al., 2016).

From the secular GMT trend $dT / dy$ (Section 3.1), the secular GMT $T$ (Section 3.3) was derived by integration. The empirical models for the relationships between the secular GMT $T$ and the other climate variables (annual atmospheric CO$_2$, Section 3.5; sea level, Section 3.6; and ocean heat content, Section 3.7) were determined by linear regression (Chou, 1975, p. 578). These secular empirical models were used to estimate the seasonal relationship between the climate variables (Section 3.8). Climate predictions based on the empirical models are given in Section 3.9. The summary and discussion of this paper are given in Section 4 and the conclusion in Section 5.

### 3. Results and Discussions

#### 3.1. Empirical Model for the Secular Global Mean Temperature Trend $dT / dy$

Knudsen et al. (2011), Swanson et al. (2009), and Wu et al. (2011) and references therein showed that the annual GMT data has multi-decadal oscillation (MDO). As a result, to determine the time-invariant climate sensitivity $\lambda$ in the model $dT / dy = \lambda F$ (Equation 2), this MDO shown in Figure 8 should be removed from the annual GMT data to obtain the secular GMT $T$ and its trend $dT / dy$. After removing the MDO from the annual GMT data, Wu et al. (2011) reported results for the secular GMT trends that are given in the last column of Table 1 and shown in Figure 1a.

From preindustrial control climate simulations, Swanson et al. (2009) found that the secular GMT $T$ since 1890 can be represented by a monotonically increasing curve that resembles a quadratic function. As a mathematical formula for the secular GMT trend $dT / dy$ as a function of time was not provided in either Swanson et al. (2009) or Wu et al. (2011), $dT / dy$ was derived in this study for the HadCRUT4 data (Morice et al., 2012). A quadratic function for the secular GMT $T$ (Swanson et al., 2009) means a linear function for the rate of change of the secular GMT (secular GMT trend) $dT / dy$ in °C/year that could be written, for any year $y$ after the steady state year $y_o$, as:

$$\frac{dT}{dy} = \left(\frac{dT}{dy}\right)_y + a_T (y - y_o) = a_T (y - y_o)$$  \hspace{1cm} (5)

where $a_T = 7.1954 \times 10^{-5}$ °C/year$^2$ is the secular GMT acceleration and $y_o = 1864.5$ is the year when the secular GMT trend started to increase from its steady state value of $(dT / dy)_o = 0$ as shown in Figure 1a.

The values of the two unknown parameters (De Veaux et al., 2005, p. 87) $a_T$ and $y_o$ of the secular GMT trend model $dT / dy = a_T (y - y_o)$ (Equation 5) were determined by fitting this model to 94 yr moving GMT trends data for trend period middle years from $y = 1897$ (trend period 1850–1944) to $y = 1971$ (trend period 1924–2018), as shown in Figure 1a and partly given in Table 1. The 94 yr trend period was selected because its moving GMT trends, for the GMT data since 1850, lie along a straight line that defines the secular GMT trend model $dT / dy$ with a coefficient of determination of $R^2 = 98.4\%$ and confirmed the corresponding results obtained using “empirical mode decomposition” by Wu et al. (2011).
The empirical model for the secular GMT trend derived above could also be verified by using the tropospheric temperature trend analysis result reported by Christy and McNider (2017): “We identify and remove the main natural perturbations (e.g., volcanic activity, El Niño Southern Oscillations (ENSOs)) from the global mean lower tropospheric temperatures ($LTT$) over January 1979–June 2017 to estimate the underlying, potentially human-forced trend. The unaltered value is $+0.155$ K dec$^{-1}$ while the adjusted trend is $+0.096$ K dec$^{-1}$. To verify the empirical model using this result, we use the above trend period’s middle year of $y = \frac{(1979 + 2017)}{2} = 1998$ in the model $dT/dy = 7.1954 \times 10^{-5}(y - 1864.5)$ (Equation 5), which gives a secular GMT trend of $dT/dy = 0.0095$ °C/year or 0.095 °C/decade, which is almost identical to the adjusted trend quoted above.

In this section, an empirical mathematical model for the secular GMT trend was derived and validated using previous results (Christy & McNider, 2017; Swanson et al., 2009; Wu et al., 2011). In the next section, this model is used to determine the climate sensitivity $\lambda$.

### 3.2. The Sun-Climate Relationship

To determine the sun-climate relationship as defined by the climate sensitivity given by $\lambda = \Delta(dT/\Delta y)/\Delta F$ (Equation 2), in addition to the model for the secular GMT trend $\Delta(dT/\Delta y) = a_T(y - y_i)$ (Equation 5), a model for the secular solar forcing $\Delta F$ is also required, and it is derived in this section using the solar irradiance data shown in Figure 2.

Previous studies have reported that the effect of the increase in Total Solar Irradiance (TSI) during the 11 yr solar cycle on the observed global warming is negligible. For example, J. C. Hall and Lockwood (2004) reported: “Foukal and Lean (1990), among others, have noted that the well-determined 0.1% TSI excursion from solar minimum to maximum ... appears quite insufficient to account for the observed rise in global temperature over the past century.”

In Figure 2, for solar cycles 21–23, the increase in TSI was $\Delta I_{11} = 1.1$ W/m$^2$, while for solar cycle 24 the TSI increase was a smaller value of $\Delta I_{11} = 0.8$ W/m$^2$. From these TSI data for the period 1976–2018, the mean increase in TSI for the nominal 11 yr solar cycle is $\Delta I_{11} = (3 \times 1.1 + 0.8)/(2018 - 1976) \times 11 = 1.1$ W/m$^2$. This TSI increase of $\Delta I_{11} = 1.1$ W/m$^2$ is only 0.08% of the TSI minimum of 1360.6 W/m$^2$ in Figure 2 and is related to the 11 yr solar cycle forcing $\Delta F_{11}$ using the formula (Haigh, 2003):

$$\Delta F_{11} = \frac{1 - a}{4} \Delta I_{11} = 0.175 \Delta I_{11} = 0.19 \text{ W/m}^2$$

Figure 2. Solar irradiance data for the increase in total solar irradiance (TSI) from solar activity minimum to maximum for solar cycles 21–24.
where $a = 0.3$ is the Earth’s albedo and 4 is the ratio of the Earth’s spherical surface area to the Earth’s circular area projected toward the sun.

The above 11 yr solar cycle increase in TSI of $\Delta I_{11} = 1.1 \text{ W/m}^2$ and the corresponding solar forcing of $\Delta F_{11} = 0.19 \text{ W/m}^2$ are identical to the values reported by Wigley and Raper (1990): “For the high frequency

$$\text{Figure 3.} \quad \text{Regression of the logarithm of the annual atmospheric CO}_2 \text{ concentration } \ln(C) \text{ on the secular global mean temperature (GMT) } T. \quad \text{The almost perfect correlation} \ (R^2 = 99.85\%) \text{ between data and model shown in this figure demonstrates that to obtain the time-invariant atmospheric CO}_2 \text{ doubling GMT of } T_x = 1.4 \ ^\circ\text{C}, \text{ the multi-decadal oscillating GMT data shown in Figure 8 should be removed from the annual GMT data shown in Figure 1b.}$$

$$\text{Figure 4.} \quad \text{Observed and modeled annual atmospheric CO}_2 \text{ concentrations} \ (R^2 = 99.86\%). \quad \text{The CO}_2 \text{ model gives a trend of} \ \frac{dC}{dy} = (325.96 - 314.53)/10 = 1.14 \text{ ppm/year for the period 1960–1970 and a trend of} \ \frac{dC}{dy} = (399.94 - 379.8)/10 = 2.01 \text{ ppm/year for the period 2005–2015. For the period 1965–2010 (45 yr), this increase in trend gives a constant acceleration for the relative rate of change of atmospheric CO}_2 \text{ concentration of} \ \frac{d(dC/dy)/C}{dy} = a_T \ln(2)/T_x = 3.55 \times 10^{-5} \text{ year}^{-2}, \text{ which is identical to the result} \ d(dC/dy)/C/dy = a_T \ln(2)/T_x = 3.55 \times 10^{-5} \text{ year}^{-2} \text{ from Equation 21.}$$
component, the maximum solar irradiance change is $\Delta I = 1.1$ W/m², which translates to a radiative perturbation at the top of the atmosphere of $\Delta Q = (1 - a) \Delta I / 4 = 0.19$ W/m², where $a$ [$a = 0.3$] is the planetary albedo.”

To relate the 11 yr solar cycle forcing of $\Delta F_{11} = 0.19$ W/m² (Equation 6) to the change in the secular GMT trend as defined by $\Delta (dT / dy) = \lambda \Delta F$ (Equation 2), the report by Wigley (1988) was used: “For solar irradiance variations on time scales of centuries or less, it is vitally important to distinguish the actual response of the climate from the equilibrium (or potential) response … for a 10 yr cyclic forcing the response is only 13%–23% of the equilibrium response. … This is a consequence of the oceans very large thermal inertia.” Using this report, the relationship between the central 11 yr secular solar forcing $\Delta F$ (that is related to “the actual response of the climate”) and the 11 yr solar cycle forcing of $\Delta F_{11} = 0.19$ W/m² (Equation 6) is given by:

$$\Delta F = 0.18 \Delta F_{11} = 0.18 \times 0.19 = 0.0342 \text{ W/m}^2$$

(7)
where 0.18 is the central damping factor for the oceanic thermal inertia and $\Delta F_{i1} = 0.19 \text{ W/m}^2$ is the observation based 11 yr solar cycle forcing (Equation 6).

Equation 7 gives the secular solar forcing over the 11 yr solar cycle period. Therefore, from this equation, the mean secular solar forcing per year is given by:

$$\frac{dF}{dy} = \frac{\Delta F}{\Delta y} = \frac{0.18 \times 0.19}{11} = 0.0031 \text{ W/m}^2/\text{year}$$  \hspace{1cm} (8)

Remarkably, this result for the secular solar forcing trend $dF/dy$ was found to be identical to that obtained from a linear correlation between coronal source magnetic flux and TSI reported by Lockwood and Stamper (1999): “we show that the 131% rise in the mean coronal source field over the interval 1901–1995 corresponds to a rise in the average TSI of $\Delta I = 1.65 \pm 0.23 \text{ W/m}^2$.” From this result, for the 94 yr period ($\Delta y = 94$), the mean secular solar forcing per year using Equation 6 is given by:

$$\frac{dF}{dy} = \frac{\Delta F}{\Delta y} = \frac{0.175 \Delta I}{\Delta y} = 0.0031 \text{ W/m}^2/\text{year}$$  \hspace{1cm} (9)

This proxy based result (Lockwood & Stamper, 1999) for the secular solar forcing trend $dF/dy$ is identical to the observation based result given by Equation 8 and was confirmed by Lockwood (2002) who also noted that this secular solar forcing trend agrees with previous reports. For example, for the last 150 yr period ($\Delta y = 150$), the above model gives a secular solar forcing of:

$$\Delta F = \frac{dF}{dy} \Delta y = 0.465 \text{ W/m}^2$$  \hspace{1cm} (10)

This proxy based solar forcing of $\Delta F = 0.465 \text{ W/m}^2$ for the last 150 yr is within the solar forcing range of 0.5 $\pm$ 0.3 W/m² since 1850 reported by Fröhlich and Lean (1998), within the solar forcing range of 0.4 $\pm$ 0.2 W/m² for the period 1850–2000 reported by Hansen et al. (2000), agrees with the solar forcing of 0.4 W/m² for the period 1860–2000 reported by Beer et al. (2000) and is identical to the secular solar forcing determined from the 11 yr solar cycle forcing given by Equation 8.

Two TSI variations are reported in the literature: The observation based TSI increase that results in the 11 yr solar cycle forcing of about 0.2 W/m² (Equation 6) with little discrepancies in the reported values that was used in this study and the proxy-based secular TSI trend with large discrepancies. Some reports consider the proxy based secular solar forcings (Equation 9) to be speculative (Foukal et al., 2006). Successive IPCC reports show lower solar forcings from 0.3 W/m² in IPCC (1996) to 0.05 W/m² in IPCC (2013). Regarding this discrepancy in the reported secular solar forcing, Dudok de Wit et al. (2017) noted: “weak secular variations in the TSI are hotly debated, as they may have a large implications on our understanding of the role of the Sun in climate change.” IPCC (2013) had also acknowledged these discrepancies: “All reconstructions rely on indirect proxies that inherently do not give consistent results. There are relatively large discrepancies among the models.”

Equation 7 gives the secular solar forcing for a single solar cycle period. As a result, since the nonlinear solar forcing model is cumulative (Equation 2), the secular solar forcing in W/m² for a series of 11 yr solar cycles is given by:

$$\Delta F = 0.18 \times 0.19 n_{\text{SolCyc}} = 0.18 \times 0.19 \frac{y - y_i}{11}$$  \hspace{1cm} (11)

where 0.18 is a central damping factor that takes into account the oceanic thermal inertia, 0.19 W/m² is the observation based 11 yr solar cycle forcing (Equation 6) and $n_{\text{SolCyc}} = (y - y_i)/11$ is the number of 11 yr solar cycles for the solar forcing period from year $y_i$ to year $y$.

For the nonlinear solar forcing model, to determine the climate sensitivity $\lambda = \Delta (dT/dy)/\Delta F$ (Equation 2) in °C/year per W/m², the secular GMT trend model $\Delta (dT/dy) = a_T (y - y_i)$ (Equation 5) and the secular solar forcing model $\Delta F = 0.18 \times 0.19(y - y_i)/11$ (Equation 11) were used, which on substitution gives:

$$\lambda = \frac{\Delta (dT/dy)}{\Delta F} = \frac{a_T}{0.18 \times 0.19/11} = 0.023143$$  \hspace{1cm} (12)

where $a_T = 7.1954 \times 10^{-5}$ °C/year² is the secular GMT acceleration (Figure 1a).
In this section, the secular solar forcing model \( \Delta F \) (Equation 11) and the time-invariant climate sensitivity \( \lambda \) (Equation 12) were derived. In the next section, these results are used to show that the secular solar forcing \( F \) shown in Figure 1c explains all of the observed global warming \( T \) in Figure 1b.

3.3. Empirical Models for the Change in Secular Global Mean Temperature \( \Delta T \)

In Section 3.1, an empirical model for the secular GMT trend \( dT/dy = (dT/dy)_i + a_T(y - y_i) \) (Equation 5) was derived. From this secular GMT trend model, for a period from year \( y_i \) to year \( y \) in Figure 1a, the change in secular GMT \( \Delta T \) in °C in Figure 1b could be derived by integration (Jeffrey & Dai, 2008, p. 154) as:

\[
\Delta T = \int \frac{dT}{dy}dy = \left[ \frac{dT}{dy} \right] (y - y_i) + a_T(y - y_i)^2/2 \tag{13}
\]

where \((dT/dy)_i\) is the initial global warming rate in °C/year for year \( y_i \) and \( a_T = 7.1954 \times 10^{-5} \) °C/year² is the secular GMT acceleration (Figure 1a). The above equation reduces to \( T = a_T(y - 1864.5)^2/2 \) relative to the steady state year of \( y_c = 1864.5 \) when \((dT/dy)_i = 0\). To distinguish the change in secular GMT relative to the steady state year of 1865 to other changes, this change is denoted by \( T \) instead of \( \Delta T \).

Equating the two secular GMT trend models \( \Delta (dT/dy) = a_T(y - y_i) \) (Equation 5) and \( \Delta (dT/dy) = \lambda \Delta F \) (Equation 12) and solving for the number of years from year \( y_i \) to year \( y \) gives the relationship \( y - y_i = \lambda \Delta F / a_T \).

Substituting this relationship into the last term of Equation 13 and simplifying gives:

\[
\Delta T = \left[ \frac{dT}{dy} \right] (y - y_i) + \lambda^2/2a_T(\Delta F)^2 = \left[ \frac{dT}{dy} \right] (y - y_i) + 3.7218(\Delta F)^2 \tag{14}
\]

where \( \Delta F \) in W/m² is the change in the secular solar forcing (Equation 11), \( \Delta T \) in °C is the corresponding change in the secular GMT, \( \lambda = 0.023143 \) °C/year per W/m² is the climate sensitivity, \( a_T = 7.1954 \times 10^{-5} \) °C/year² is the secular GMT acceleration and \((dT/dy)_i\) is the initial global warming rate for year \( y_i \) (Figure 1a).

For the period 1860–2000, Beer et al. (2000) reported a solar forcing of \( F = 0.4 \) W/m². Using this solar forcing in the LM \( T = 0.5F \) (Equation 1), they reported a solar caused global warming of only \( T = 0.2 \) °C, which indicates that the solar forcing of \( 0.4 \) W/m² explains only 33% of the observed global warming of about 0.6 °C of the 20th century (IPCC, 2001, Summary for Policymakers (SPMs), p. 2).

In contrast to the LM in the previous paragraph that indicates 67% of the observed global warming of about 0.6 °C of the 20th century is unexplained by the solar forcing of \( 0.4 \) W/m², the nonlinear empirical model \( T = 3.7218F^2 \) (Equation 14) since steady state \((dT/dy) = 0\) showed that this solar forcing explained the observed global warming. For example, by substituting the solar solar forcing of \( F = 0.4 \) W/m² (Beer et al., 2000) into this nonlinear solar forcing model gives:

\[
T = 3.7218F^2 = 0.6 \text{ °C} \tag{15}
\]

The above result shows that, for the nonlinear solar forcing model, the reported solar forcing of \( F = 0.4 \) W/m² explained all of the observed global warming of about 0.6 °C of the 20th century. As a result, it is unnecessary to invoke anthropogenic influences to explain the observed global warming. This result also shows that the solar forcing of \( F = 0.4 \) W/m² reported by Beer et al. (2000) is consistent with the solar irradiance data (Figure 2) that was used to derive the central secular solar forcing model \( \Delta F = 0.18 \times 0.19(y - y_i)/11 \) (Equation 11).

As the secular GMT \( T \) was derived by the integration of the secular GMT trend \( dT/dy \) and \( dT/dy \) has been linear (Figure 1a), the secular GMT \( T \), for any year \( y \) after the steady state year of 1864.5, could be determined from the area of the right-angled triangle under the secular GMT trend \( dT/dy \) line as given by:

\[
T = \frac{1}{2}(y - 1864.5)\frac{dT}{dy} \tag{16}
\]
where \( dT/\text{dy} \) is the secular GMT trend in °C/year for year \( y \) and 1864.5 is the steady state year when \( dT/\text{dy} = 0 \) and \( T = 0 \). For any year \( y \) after steady state, Equation 16 relates the secular GMT trend \( dT/\text{dy} \) in Figure 1a to the secular GMT \( T \) in Figure 1b.

The secular GMT \( T = \alpha_f(y - 1864.5)^2/2 \) (Equation 13) was derived by integrating the model for the secular GMT trend \( dT/\text{dy} = \alpha_f(y - 1864.5) \) (Equation 5). Alternatively, the secular GMT \( T \) could also be derived directly from the annual GMT data using the statistical property of moving averages (Chou, 1975, p. 708): “If the data show a uniform periodic fluctuation, a moving average of length equal to the period will completely eliminate the periodic variations.”

From their analysis of the GMT data, Wu et al. (2011) found that the GMT has an MDO period of about 65 yr, which means that a moving average of about 65 yr will “completely eliminate” the MDO from the annual GMT data. Data points for a 64 yr moving GMT averages were found to lie along the model for the Secular GMT \( T \) curve as shown in Figure 1b, with the model \( T \) almost perfectly explaining \( (R^2 = 99.97\%) \) the variability of the data. As a result, the 64 yr moving GMT averages data in Figure 1b confirmed the secular GMT \( T \) derived using the 94 yr moving GMT trends data in Figure 1a.

To simplify the empirical models, the origin of the secular GMT of \( T = 0 \) was taken to be for the year 1864.5, when the global warming rate started to increase from its steady state value of \( dT/\text{dy} = 0 \) as shown in Figure 1a. This was achieved by adding +0.356 °C to each of the annual GMT data from 1865 to 2018. The value of 0.356 °C was obtained from the vertical shift required to make the 64 yr moving GMT averages data coincide with the secular GMT \( T \) curve shown in Figure 1b.

With the 64 yr period for the MDO of the GMT determined, the empirical model for the GMT signal \( S \) that models the data for the annual GMT \( G \) could be derived as described in the next section.

### 3.4. Empirical Model for the Annual Global Mean Temperature Signal \( S \)

The secular GMT \( T \) shown in Figure 1b models the 64 yr moving GMT averages data with a correlation determination of \( R^2 = 99.97\% \), which demonstrates that the annual GMT data has an MDO of a 64 yr period (Chou, 1975, p. 708). To simultaneously model this MDO and the secular GMT, 22 yr moving GMT averages were used as shown in Figure 1b. The data for the 22 yr moving GMT averages were fitted with a model for the annual GMT signal \( S \) in °C, for any year \( y \) after the steady state year of 1864.5, given by:

\[
S = T + MDO = \frac{\alpha_f}{2}(y - 1864.5)^2 + b\sin\left[\frac{2\pi}{64}(y - 1862)\right]
\]  

(17)

where \( \alpha_f = 7.1954 \times 10^{-5}\text{C/year}^2 \) is the secular GMT acceleration (Figure 1a) and \( b = 0.09 \) °C is the amplitude of the MDO that describes the oscillation of \( S \) relative to \( T \) (Figures 1b and 8).

The annual GMT signal \( S \) given by Equation 17 explained 89.3% of the variation of the annual GMT data since 1865, which indicated a very high degree of correlation of +0.94 between data and model as shown in Figure 1b. The GMT signal \( S \) is expected to be valid until the secular GMT acceleration \( \alpha_f \) that has been constant since 1865 changes. However, the GMT signal \( S \) may not be extrapolated to the years before mid-19th century because the sea level, which is linearly proportional to the secular GMT as shown in Figure 5a, had been decelerating at that time (Jevrejeva et al., 2014, Figure 3). This observation indicates that the secular GMT \( T = \alpha_f(y - 1864.5)^2/2 \) (Equation 13) is a piecewise function that is only valid for the current constant secular GMT acceleration of \( \alpha_f = 7.1954 \times 10^{-5}\text{C/year}^2 \) since 1865 (Figure 1a).

In Figure 1b, the annual GMT residual given by \( G - S \) has the same standard deviation of 0.1 °C for two 76 yr periods 1865–1941 and 1942–2018. From this constant standard deviation, it is interesting to pose the question: Why has the GMT residual \( G - S \) in Figure 1b varied within the same two standard deviation uncertainty range of ±0.2 °C for the two periods?

To give a plausible answer to this question, we may identify the residual \( G - S \) in Figure 1b to be the annual global mean surface temperature \( G \) relative to the mixed ocean layer temperature \( S \), and the difference \( S - T \) to be the mixed ocean layer temperature \( S \) relative to the secular GMT \( T \) of the deeper ocean that only accumulates heat during global warming because it is a secular heat reservoir (Rossby, 1959, p. 12).
The mixed ocean layer temperature $S$ has an MDO that has existed for 8,000 yr with an oscillation period of 55–70 yr (Knudsen et al., 2011). This long-lived MDO, shown in Figure 8 since 1862, suggests it may be the mechanism for heat transfer from the mixed ocean layer to the deeper ocean as solar energy is accumulated in the mixed ocean layer during the warm phases (1921–1953 and 1985–2017, top horizontal axis label) and is sequestered in the deeper ocean during the cold phases (1889–1921 and 1953–1985). These heat accumulation in the mixed ocean layer and sequestration in the deeper ocean phases of the MDO result in multi-decadal anti-correlations between the surface and subsurface ocean temperatures (Zhang, 2007).

In addition to the MDO, there exists the interannual ENSO with El Niño warm phase and La Niña cold phase (McPhaden et al., 2006). The ocean and atmospheric circulations restrict the magnitude of the residual $G - S$ within the constant range of ±0.2 °C by heat transfer from the mixed ocean layer to the atmosphere during the warm El Niño events, resulting in drop in the OHC (Balmaseda et al., 2013) and from the mixed ocean layer to the deeper ocean during the cold La Niña events (England et al., 2014). The same uncertainty range of ±0.2 °C for the GMT residual $G - S$ for the two 76 yr periods 1865–1941 and 1942–2018 suggests no positive climate feedbacks because these residuals have been regulated by atmospheric and oceanic circulations to vary within this fixed range since steady state in 1865 (Figure 1b).

However, the secular GMT $T$ has been monotonically increasing since 1865 due to secular heat flow from the warmer mixed ocean layer to the colder deeper ocean. Regarding the deeper ocean, Rossby (1959) pointed out that it is thermally insulated from the atmosphere by the warmer mixed ocean layer: “These deeper layers are insulated from the atmosphere by stably stratified warmer water masses near to the sea surface and are not able directly to restore the radiation balance by means of an increased evaporation and cloud formation.” This observation indicates that we should make a distinction between the detrended annual global mean surface temperature given by $G - T$ in Figure 8 that does not include the secular increase in the GMT and the undetrended annual GMT $G$ in Figure 1b that includes the secular increase in the GMT due to heat energy storage in the deeper ocean.

Since the secular GMT $T$ has been monotonically increasing (Figure 1b) just like the annual atmospheric CO$_2$ concentration (Figure 4), regression analysis of these two monotonically increasing climate variables is expected to give a time-invariant CO$_2$ doubling secular GMT $T_{2x}$ as described in the next section.

### 3.5. Empirical Models for the Annual Atmospheric Carbon Dioxide Concentration $C$ and Its Trend $dC/dy$

Knutti and Hegerl (2008) and Wigley and Schlesinger (1985) and references therein have reported a linear relationship between the change in the secular GMT $\Delta T$ and the logarithm of the ratio of the corresponding final to the initial annual atmospheric carbon dioxide concentrations $\ln(C/C_i)$ given by:

$$\Delta T = \frac{T_{2x}}{\ln(2)} \ln\left(\frac{C}{C_i}\right)$$

(18)

In this equation, when the ratio $C/C_i = 2$, we have $\Delta T = T_{2x}$. As a result, the parameter $T_{2x}$ represents the CO$_2$ doubling secular GMT.

In Equation 18, the two unknown parameters $C_i$ and $T_{2x}$ could be determined by linear regression. For the linear regression analysis, the model $T = (T_{2x} / \ln(2)) \ln(C/C_i)$ (Equation 18) relative to steady state could be written as:

$$\ln(C) = \frac{\ln(2)}{T_{2x}} T + \ln(C_o)$$

(19)

where $C_o$ is the atmospheric CO$_2$ concentration corresponding to the reference secular GMT of $T = 0$ when the secular GMT trend was at its steady state ($dT/dy = 0$ in Figure 1a). To derive Equation 19 from 18, the logarithmic relationship $\ln(C/C_{i}) = \ln(C) - \ln(C_o)$ was used (Jeffrey & Dai, 2008, p. 122).

The parameters $T_{2x}$ and $C_o$ of the mathematical model $\ln(C) = (\ln(2)/T_{2x}) T + \ln(C_o)$ (Equation 19) were determined by linear regression of the logarithm of the annual atmospheric CO$_2$ concentration $\ln(C)$ on the secular GMT $T$ as shown in Figure 3, which gives $\ln(2)/T_{2x} = 0.49356 \, ^{\circ}C^{-1}$ for the slope of the...
regression line or the regression coefficient and \( \ln(C_a) = 5.58913 \) for the y-intercept. From these results, we obtain \( T_{2x} = \ln(2)/0.49356 = 1.4044 \, ^\circ\text{C} \) for the \( \text{CO}_2 \) doubling secular GMT and \( C_o = e^{5.58913} = 267.5 \, \text{ppm} \) for the atmospheric \( \text{CO}_2 \) concentration for the steady state year of 1864.5. In the above regression model, the secular GMT used was \( T = a_T(y - 1864.5)^2 / 2 \) (Figure 1b), where \( y \) is any year after the steady state, \( a_T = 7.1954 \times 10^{-5} \, ^\circ\text{C}/\text{year}^2 \) is the secular GMT acceleration (Figure 1a) and the atmospheric \( \text{CO}_2 \) concentrations were the annual values for the Mauna Loa data for the period 1959–2018 (Figure 4).

A proxy based result that is almost identical to the atmospheric \( \text{CO}_2 \) concentration of \( C_o = 267.5 \, \text{ppm} \) for mid-19th century (Figure 3) was reported by Stuiver (1978): “By using \( ^{13}\text{C} \) and \( ^{14}\text{C} \) isotopic data, an atmospheric \( \text{CO}_2 \) content of 268 ppm was obtained for mid-19th-century air.” Similar atmospheric \( \text{CO}_2 \) concentrations for mid-19th century was also reported by Elliott (1984): “Somewhat subjectively the group felt values between 260 and 280 ppmv were the most likely prevailing \( \text{CO}_2 \) concentrations during the mid-19th century.”

The almost perfect linear correlation \( (R^2 = 99.85\%) \) between the logarithm of the annual atmospheric \( \text{CO}_2 \) concentration and the secular GMT shown in Figure 3 is augmented by the results of Kuo et al. (1990) who found linear relationship between these two climate variables at many frequencies: “That both the \( \text{CO}_2 \) and global temperature data have positive slopes does not prove that the two series are related .... If it is found, however, that the fluctuations of the two detrended series are coherent over a band of frequencies, then it is more likely that the two series are related.” They then concluded: “The coherence results presented here provide significant evidence that the average global temperature and \( \text{CO}_2 \) concentration from 1958 to 1988 are linearly related at many frequencies.”

The above results are further supported by those reported by Keeling et al. (1995): “the decadal variations in temperature, and possibly in precipitation, almost directly correlate with the \( \text{CO}_2 \) concentration itself. If these decadal correlations are significant, it seems evident that the onset of a climate change, such as a warming trend, has a measurable influence on the atmospheric \( \text{CO}_2 \) concentration.”

All the above results indicate that the reported strong correlation between the cumulative industrial emission of \( \text{CO}_2 \) and the annual atmospheric \( \text{CO}_2 \) concentrations (Keeling et al., 1995, Figure 1b) is spurious because these two variables are not “linearly related at many frequencies.” This conclusion will further be confirmed if the observed \( \text{CO}_2 \) trend follows the predicted trend in Figure 4, irrespective of the actual magnitude of the global reduction in human emissions of \( \text{CO}_2 \) (Peters et al., 2017), demonstrating that the increase in the annual atmospheric \( \text{CO}_2 \) concentration has been predictable and could be explained by the increase in the secular GMT \( T \) as shown in Figure 3.

For the model \( \ln(C) = (\ln(2)/T_{2x})T + \ln(C_o) \) (Equation 19), once the two unknown parameters \( T_{2x} \) and \( C_o \) are determined by linear regression (Figure 3), the empirical model for the annual atmospheric \( \text{CO}_2 \) concentration \( C \) in ppm, for any year \( y \) after the steady state, could be written as:

\[
C = C_o \exp \left[ \frac{\ln(2)}{T_{2x}} T \right] = 267.5 \exp[0.49356T] = 267.5 \exp \left[ 1.7757 \times 10^{-5} (y - 1864.5)^2 \right]
\]  

(20)

where \( C_o = 267.5 \, \text{ppm} \) is the annual atmospheric \( \text{CO}_2 \) concentration for the steady state year of \( y = 1864.5 \), \( T_{2x} = 1.4044 \, ^\circ\text{C} \) is the \( \text{CO}_2 \) doubling secular GMT, \( T = a_T(y - 1864.5)^2 / 2 \) is the secular GMT, and \( a_T = 7.1954 \times 10^{-5} \, ^\circ\text{C}/\text{year}^2 \) is the secular GMT acceleration (Figure 3).

Almost all the variability \( (R^2 = 99.86\%) \) of the observed annual atmospheric \( \text{CO}_2 \) concentration for any year \( y \) from 1959 to 2018 for the Mauna Loa data is explained by the empirical model given by Equation 20, as shown in Figure 4.

An empirical model for the rate of change of the annual atmospheric \( \text{CO}_2 \) concentration \( (\text{CO}_2 \) trend \( dC/dy \) ), for any year \( y \) after steady state, could be derived by the differentiation of \( \ln(C) = (\ln(2)/T_{2x})T + \ln(C_o) \) (Equation 19) with respect to year \( y \), which after simplification gives:

\[
\frac{dC}{dy} = \frac{\ln(2)}{T_{2x}} \frac{dT}{dy} C = \frac{\ln(2)}{T_{2x}} a_T(y - 1864.5) C = 3.5513 \times 10^{-5} (y - 1864.5) C
\]  

(21)
where $T_{2x} = 1.4044 \, ^\circ C$ is the CO$_2$ doubling secular GMT (Figure 3), $dT/dy = a_T(y - 1864.5)$ is the secular GMT trend and $a_T = 7.1954 \times 10^{-5} \, ^\circ C/\text{year}^2$ is the secular GMT acceleration (Figure 1a). To derive Equation 21 from 19, the differentiation result $d(\ln(C))/dy = (dC/dy)/C$ was used (Jeffrey & Dai, 2008, p. 149).

The result given by $(dC/dy)/C = (\ln(2)/T_{2x})a_T(y - 1864.5)$ (Equation 21) shows that the relative rate of change of the annual atmospheric CO$_2$ concentration $(dC/dy)/C$ has linearly increased with time, which indicates that the annual atmospheric CO$_2$ concentration has been predictable as shown in Figure 3 until the value of the secular GMT acceleration $a_T$ changes.

In Figure 3, assuming the constant secular GMT acceleration $a_T$ continues, the number of years it takes for the atmospheric CO$_2$ concentration to double from its steady state value of $C_s = 267.5 \, \text{ppm}$ for year 1864.5 could be determined by substituting the CO$_2$ doubling secular GMT of $T = T_{2x} = 1.4044 \, ^\circ C$ and the secular GMT acceleration of $a_T = 7.1954 \times 10^{-5} \, ^\circ C/\text{year}^2$ into the secular GMT model $T = a_T(y - 1864.5)^2/2$ (Equation 13), which gives $y_{2x} - 1864.5 = \sqrt{2T_{2x}/a_T} = 197.6 \, \text{years}$. This result gives a CO$_2$ doubling year of $y_{2x} = 1864.5 + 197.6 = 2062$ and a corresponding annual atmospheric CO$_2$ concentration of $C_{2x} = 2 \times 267.5 = 535 \pm 3.6 \, \text{ppm}$. Because of the reduced solar activity predicted to occur after about 2030 (Lean & Rind, 1998), the annual atmospheric CO$_2$ concentration for 2062 is expected to be less than the above CO$_2$ doubling value of $C_{2x} = 535 \, \text{ppm}$.

The radiative forcing for doubling of the annual atmospheric CO$_2$ concentration has been reported to be $F = 3.7 \, \text{W/m}^2$ (Knutti & Hegerl, 2008). This value is commonly calculated using the formula $F = 5.35\text{ln}(C/C_s)$ (Myhre et al., 1998), which for $C/C_s = 2$ gives $F = 3.7 \, \text{W/m}^2$.

This climate forcing of 3.7 W/m$^2$ could be estimated using the 11 yr solar cycle forcing of $\Delta F_{11} = 0.2 \, \text{W/m}^2$ (Fröhlich & Lean, 1998). From this solar forcing, the mean 11 yr solar cycle forcing per year is $\Delta F_{11}/\Delta y = 0.2/11 \, \text{W/m}^2$ per year. From this result, the 11 yr solar cycle forcing for doubling of the atmospheric CO$_2$ concentration since steady state in 1864.5 is given by:

$$\Sigma (\Delta F_{11})_{2x} = \frac{0.2}{11} (y_{2x} - 1864.5) = 3.6 \, \text{W/m}^2$$

(22)

where $y_{2x} - 1864.5 = 197.6 \, \text{yr}$ is the CO$_2$ doubling period since steady state calculated above.

The above comparison shows that almost all of the 3.7 W/m$^2$ climate forcing that has previously been attributed to the radiative forcing due to doubling of atmospheric CO$_2$ concentration appears instead to be due to the corresponding 11 yr solar cycle forcing since steady state (Equation 22).

In this section, the empirical models for the secular relationships between atmospheric CO$_2$ concentration, GMT, and solar forcing were presented. In the next section, the empirical models for the sea level are considered.

### 3.6. Empirical Models for the Secular Sea Level Rise SL and Its Trend $d(SL)/dy$

To determine the relationship between the sea level and the other climate variables, the sea level data was modeled. A linear regression of the annual sea level $SL$ in mm on the secular GMT $T$ in °C is shown in Figure 5a and is given by:

$$SL = K_{SL} T = K_{SL} a_T(y - 1864.5)^2/2 = 0.0192(y - 1864.5)^2/2$$

(23)

where $K_{SL} = 266.569 \, \text{mm/°C}$ is the sea level regression coefficient (Figure 5a), $T = a_T(y - 1864.5)^2/2$ is the secular GMT (Figure 1b), $a_T = 7.1954 \times 10^{-5} \, ^\circ C/\text{year}^2$ is the secular GMT acceleration (Figure 1a) and $y$ is any year after the steady state year of 1864.5.

The empirical sea level model $SL = K_{SL} T$ (Equation 23) explained 98.4% of the variability of the annual sea level for the tide gauges data as shown in Figure 5a. In this figure, the sea level model's uncertainty range of ±16 mm was based on the two standard deviations of the residuals (Equation 4).

For the secular sea level model $SL = K_{SL} T$ (Equation 23), its first derivative with respect to year $y$ gives the secular sea level trend $d(SL)/dy$ in mm/year as:
\[
\frac{d(SL)}{dy} = K_{SL} \frac{dT}{dy} = K_{SL} a_T(y - 1864.5) = 0.0192(y - 1864.5) \tag{24}
\]

where \( K_{SL} = 266.569 \text{ mm/}°\text{C} \) is the sea level regression coefficient (Figure 5a), \( \frac{dT}{dy} = a_T(y - 1864.5) \) is the secular GMT trend and \( a_T = 7.1954 \times 10^{-5} \text{C/year}^2 \) is the secular GMT acceleration (Figure 1a).

The empirical model for the secular sea level trend above could be verified by using the sea level rise observations reported in the literature. For example, IPCC (2013, SPM, p. 11) reported: “Over the period 1901–2010, global mean sea level rose by 0.19 [0.17–0.21] m.” For this trend period middle year of \( y = (1901 + 2010)/2 = 1955.5 \), the model \( \frac{d(SL)}{dy} = 0.0192(y - 1864.5) \) (Equation 24) gives a mean sea level trend of \( \frac{d(SL)}{dy} = 1.75 \text{ mm/year} \). For the period 1901–2010 (50 yr), this mean secular sea level trend results in a mean sea level rise of \( \Delta(SL) = 0.001(d(SL)/dy)\Delta y = 0.19 \text{ m} \), which is identical to the observed sea level rise quoted above.

As a second example, Fairbridge and Krebs (1962) reported: “An average curve for the world annual mean sea level for the century 1860–1960 has been obtained from a carefully selected world series of tide gauge records. The mean rise from 1900 to 1950 was 1.2 mm annually.” To verify the empirical model using this observation, we use the trend period’s middle year of \( y = (1900 + 1950)/2 = 1925 \) in the model \( \frac{d(SL)}{dy} = 0.0192(y - 1864.5) \) (Equation 24), which gives a mean secular sea level trend of \( \frac{d(SL)}{dy} = 1.2 \text{ mm/year} \), which is identical to the observed sea level trend quoted above.

The above agreements between the results of the secular sea level trend model \( \frac{d(SL)}{dy} = 0.0192(y - 1864.5) \) (Equation 24) and observations reported 51 yr apart, in 1962 and 2013, validate this model and show that this empirical model could be used for sea level prediction. The above model-observations agreements also show that the relationship between the secular sea level \( SL \) and the secular GMT \( T \) is linear as given by \( SL = K_{SL} T \) (Figure 5a), in contrast to a nonlinear model between these two climate variables given by \( d(SL)/dy = K_{SL}T \) reported by Rahmstorf (2007). A linear relationship between the secular sea level and GMT as shown in Figure 5a was also reported by Gornitz et al. (1982).

Differentiating the secular sea level trend \( d(SL)/dy = 0.0192(y - 1864.5) \) (Equation 24) with respect to year \( y \) gives a constant secular sea level acceleration of \( \frac{d^2(SL)}{dy^2} = 0.02 \text{ mm/year}^2 \), which is identical to the result reported by Jevrejeva et al. (2014): “We calculate an acceleration of 0.02±0.01 mm yr\(^{-2}\) in global sea level.”

The secular solar forcing \( F \) in W/m\(^2\) could be related to the sea level trend \( \frac{d(SL)}{dy} \) in mm/year by substituting \( \frac{dT}{dy} = (d(SL)/dy)/K_{SL} \) (Equation 24) into \( F = \frac{dT}{dy}/\lambda \) (Equation 12), which gives:

\[
F = \frac{1}{\lambda} \frac{dT}{dy} = \frac{1}{\lambda K_{SL}} \frac{d(SL)}{dy} = 0.1621 \frac{d(SL)}{dy} \tag{25}
\]

where \( \lambda = 0.023143 \text{ °C/year per W/m}^2 \) is the climate sensitivity (Equation 12) and \( K_{SL} = 266.569 \text{ mm/}°\text{C} \) is the sea level regression coefficient (Figure 5a).

Equation 25 relates solar forcing to the secular sea level trend and shows that an increase in solar forcing \( \Delta F \) is transformed into stored energy as an increase in the sea level trend \( \Delta(d(SL)/dy) \) or sea level acceleration. The model \( F = 0.1621d(SL)/dy \) (Equation 25) is used in Section 3.8 to estimate the seasonal solar forcing from the corresponding seasonal sea level change.

It is also possible to relate the change in the secular sea level to the change in the annual atmospheric CO\(_2\) concentration by substituting \( \Delta T = (T_{2x}/\ln(2))\ln(C/C_i) \) (Equation 18) into \( \Delta SL = K_{SL}\Delta T \) (Equation 23), which gives:

\[
\Delta SL = K_{SL}\Delta T = \frac{K_{SL}T_{2x}}{\ln(2)} \ln\left(\frac{C}{C_i}\right) = 540.1\ln\left(\frac{C}{C_i}\right) \tag{26}
\]

where \( K_{SL} = 266.569 \text{ mm/}°\text{C} \) is the sea level regression coefficient (Figure 5a) and \( T_{2x} = 1.4044 \text{ °C} \) is the CO\(_2\) doubling secular GMT (Figure 3).

In Figure 4, for the increase in the annual atmospheric carbon dioxide concentration from \( C_i = 314.53 \text{ ppm} \) for year 1960 to \( C = 389.57 \text{ ppm} \) for year 2010, the corresponding sea level rise could be calculated using the
model \( \Delta(SL) = 540.1 \ln(C/C_i) \) (Equation 26), which gives \( \Delta(SL) = 115.56 \text{ mm} \). This result is identical to the corresponding sea level rise for the period 1960–2010 in Figure 5a of \( \Delta(SL) = 203.03 - 87.47 = 115.56 \text{ mm} \). As a result, Equation 26 is the empirical model that relates the observed changes of the annual atmospheric CO\(_2\) concentration and sea level. If the annual atmospheric CO\(_2\) concentration doubles, say from \( C_i = 400 \text{ ppm} \) to \( C = 800 \text{ ppm} \), the above empirical model predicts a sea level rise of \( \Delta(SL)_{2x} = 0.54 \ln(2) = 0.37 \text{ m} \).

The empirical models for the sea level described above apply only to the secular variation. The sea level from tide gauges has an MDO as reported by Chambers et al. (2012): “We find that there is a significant oscillation with a period around 60 yr in the majority of the tide gauges examined during the 20th century.” This observed sea level MDO gives independent confirmation of the global mean surface temperature MDO shown in Figure 8.

In this section, the empirical models for the secular sea level, its trend, and acceleration for the tide-gauges data since steady state in 1865 were derived and validated using previous results (Fairbridge & Krebs, 1962; IPCC, 2013; Jevrejeva et al., 2014). In the next section, the empirical models for the OHC and its trend are presented.

### 3.7. Empirical Models for the Ocean Heat Content \( H \) and Its Trend \( dH/\text{dy} \)

In addition to explaining all of the observed increases in the GMT, atmospheric CO\(_2\) concentration and sea level presented in the previous sections, the solar forcing model \( dI/\text{dy} = \lambda F \) (Equation 12) also explained almost all of the observed increase in the ocean heat content, with the secular solar forcing \( F \) in W/m\(^2\) (Figure 1c) almost equal to the corresponding rate of change of the OHC \( Q \) in W/m\(^2\) as shown in this section.

Figure 5b shows a linear regression of the pentadal OHC \( H \) in J on the secular GMT \( T \) in °C given by (Levitus et al., 2005):

\[
H = C_{pw}m_{oce}T = C_{pw}(\rho_wA_{oce}d_{oce})T = 0.1546 \times 10^{22} d_{oce}T = K_HT 
\]  

(27)

where \( C_{pw} = 4187 \text{ J/(kg°C)} \) is the specific heat capacity of water (Antonov et al., 2004), \( m_{oce} \) is the mass in kg of the top ocean layer having depth \( d_{oce} \) in m and mean temperature \( T \) in °C, \( \rho_w = 1020 \text{ kg/m}^3 \) is the density of sea water, \( A_{oce} = 3.62 \times 10^{14} \text{ m}^2 \) is the world’s ocean surface area (Charette & Smith, 2010), \( K_H = 0.1546 \times 10^{22} d_{oce} = 51.288 \times 10^{22} \text{ J/°C} \) is the ocean heat capacity for the top 2,000 m layer based on the data from 1958 to 2012 (Figure 5b) that gives a mean top ocean layer depth of \( d_{oce} = K_H/(0.1546 \times 10^{22}) \approx 332 \text{ m} \) and \( T = 7.1954 \times 10^{-3}(y - 1864.5)^2/2 \) is the secular GMT for any year \( y \) after steady state in 1865 (Figure 1b).

For the top 2,000 m layer, 94.9% of the variability of the OHC data was explained by the empirical model \( H = K_HT \) (Equation 27) as shown in Figure 5b, which also shows the important empirical result that for a given ocean layer depth (300, 700, or 2,000 m) the ocean heat capacity \( K_H \) for each depth is a constant.

The heat capacity of \( K_H = 51.288 \times 10^{22} \text{ J/°C} \) for the top 2,000 m ocean layer (Figure 5b) is very much greater than the heat capacity of the atmosphere or the mixed ocean layer. For example, the heat capacity of the atmosphere is \( K_{H,\text{atm}} = C_{pw}m_{\text{atm}} = 0.53 \times 10^{22} \text{ J/°C} \), where \( C_{pw} = 1000 \text{ J/(kg °C)} \) is the specific heat capacity of dry air and \( m_{\text{atm}} = 5.3 \times 10^{18} \text{ kg} \) is the mass of the atmosphere (Levitus et al., 2005). From \( K_{H,\text{oce}} = 0.1546 \times 10^{22} d_{oce} \) (Equation 27), the heat capacity of the mixed ocean layer is \( K_{H,\text{ml}} = 0.1546 \times 10^{22} d_{\text{ml}} = 15.46 \times 10^{22} \text{ J/°C} \), where \( d_{\text{ml}} = 100 \text{ m} \) is the depth of the mixed ocean layer (Hoffert et al., 1980). From these results, the ratio of the heat capacity of the mixed ocean layer to that of the atmosphere is given by:

\[
\frac{K_{H,\text{ml}}}{K_{H,\text{atm}}} = \frac{15.46 \times 10^{22}}{0.53 \times 10^{22}} = 29
\]  

(28)

This result shows that the heat capacity of the atmosphere is very small compared to that of the mixed ocean layer. This relationship is used to direct the direction of the seasonal heat flow between the mixed ocean layer and the atmosphere in Figure 6.
To compare the OHC rate $dH/dt$ in J/s (W) to solar forcing, it is expressed per meter square of the world’s ocean surface area $A_{oce}$ (or W/m²) using the relation $Q = (dH/dt)/A_{oce}$ (Dwyer et al., 2012). Substituting the relationship between time in second $dt$ and the corresponding time in year $dy$ given by $dt = 31.536 \times 10^6 dy$ and the ocean surface area of $A_{oce} = 3.62 \times 10^{14}$ m² into the above relationship $dH/dt = A_{oce}Q$ and simplifying gives:

$$\frac{dH}{dy} = 31.536 \times 10^6 A_{oce}Q = 1.1416 \times 10^{22} Q$$  \hspace{1cm} (29)

where $Q$ is the OHC rate in W/m² and $dH/dy$ is the rate of change of the OHC (OHC trend) in J/year.

The OHC rate $Q$ could be related to the secular GMT trend $dT/dy$. Since $K_H$ is a constant for a given ocean layer depth (Figure 5b), substituting $H = K_H T$ (Equation 27) into $Q = (dH/dy)/(1.1416 \times 10^{22})$ (Equation 29) gives:

$$Q = \frac{1}{1.1416 \times 10^{22}} \frac{dT}{dy} = \frac{K_H}{1.1416 \times 10^{22}} \frac{dT}{dy} = \frac{1}{0.0223} \frac{dT}{dy}$$  \hspace{1cm} (30)

where $K_H = 51.288 \times 10^{22}$ J/°C is the ocean heat capacity for the top 2,000 m layer (Figure 5b) and $dT/dy$ is the secular GMT trend in °C/year (Figure 1a).

Dividing the OHC rate $Q = (dT/dy)/0.0223$ (Equation 30) by the seasonal solar forcing model $F = (dT/dy)/0.0231$ (Equation 12) shows that they are approximately equal as given by the relation:

$$\frac{Q}{F} = 1.04 \approx 1$$  \hspace{1cm} (31)

This result shows that the seasonal solar forcing $F$ (Figure 1c) explains nearly all of the OHC rate $Q$ for the top 2,000 m ocean layer. In Equation 7, when calculating the secular solar forcing from the 11 yr solar cycle forcing, a central damping factor of 0.18 was used, which has the range of values from 0.13 to 0.23 (Wigley & Raper, 1990). If the value of 0.187 instead of the central value of 0.18 were used for this damping factor in Equation 7, the solar forcing $F$ and the OHC rate $Q$ in Equation 31 would have been identical. The nonlinear mathematical relationship between the OHC rate $Q$ in W/m² and the secular GMT $T$ given by $dT/dy = 0.0223 Q$ (Equation 30) supports the nonlinear model of this study between secular solar forcing $F$ in W/m² and secular GMT $T$ given by $dT/dy = 0.0231 F$ (Equation 2) instead of the LM $\Delta T = \lambda_{LM} \Delta F$ (Equation 1).

The empirical models for the OHC and its trend derived above could be verified by using the OHC analysis results for the top 2,000 m ocean layer (OHC2000) reported by Levitus et al. (2012): “The global linear trend of OHC2000 is $0.43 \times 10^{22}$ J yr⁻¹ for 1955–2010 which corresponds to a total increase in heat content of $24.0 \pm 1.9 \times 10^{22}$ J (± 2 S.E.), and a median increase of temperature of 0.09°C. This represents a rate of 0.39 W m⁻² per unit area of the World Ocean.”

For the above OHC analysis period’s middle year of $y = (1955 + 2010)/2 = 1983$, Figure 1a gives a secular GMT trend of $dT/dy = 0.0085$ °C/year. Substituting this value into $Q = (dT/dy)/0.0223$ (Equation 30) gives an OHC rate of $Q = 0.38$ W/m². Substituting $Q = 0.38$ W/m² into $dH/dy = 1.1416 \times 10^{22} Q$ (Equation 29) gives an OHC trend of $dH/dy = 0.43 \times 10^{22}$ J/year. For the period 1955–2010 in Figure 5b, the increase in the OHC for the top 2,000 m layer is $\Delta H = (39.1 - 15.1) \times 10^{22} = 24 \times 10^{22}$ J. These empirical model results are almost identical to the OHC analysis results quoted in the previous paragraph.

Levitus et al. (2012) also reported a mean temperature increase of 0.09 °C for the top 2,000 m ocean layer for the period 1955–2010. For this period, from the previous paragraph, the mean OHC increase for the top
2,000 m ocean layer is \( \Delta H = 24 \times 10^{22} \) J. If the temperature in this ocean layer were distributed uniformly, using the empirical model \( \Delta H = 0.1546 \times 10^{22} d_{oce} \Delta T \) (Equation 27), the mean temperature increase \( \Delta T_{oce} \) for the top ocean layer depth of \( d_{oce} = 2,000 \) m would be:

\[
\Delta T_{oce} = \frac{\Delta H}{0.1546 \times 10^{22} d_{oce}} = 0.08 \, ^\circ\text{C}
\]  

(32)

For the OHC trend period 1955–2010, the mean ocean temperature increase of 0.08 °C for the top 2,000 m layer (Equation 32) is almost equal to the corresponding value of 0.09 °C reported by Levitus et al. (2012). The above agreements with the reported observations validate the empirical models for the OHC and its trend presented in this section.

The OHC increases because heat is removed from the seasonally solar heated high-temperature atmosphere and is stored in the ocean at low temperature because the heat capacity of the ocean is much greater than that of the atmosphere. For example, in Equation 32, the OHC increase of \( \Delta H = 24 \times 10^{22} \) J resulted in a mean temperature increase of only 0.08 °C for the top 2,000 m ocean layer. The effective cooling of the atmosphere that resulted in the observed increase in the OHC of \( \Delta H = 24 \times 10^{22} \) J could be estimated as:

\[
\Delta T_{atm} = \frac{\Delta H}{K_{H,atm}} = \frac{24 \times 10^{22}}{0.53 \times 10^{22}} = 45 \, ^\circ\text{C}
\]  

(33)

where \( K_{H,atm} = 0.53 \times 10^{22} \) J/°C is the heat capacity of the atmosphere (Equation 28).

For the period 1955–2010, the above results show that heat energy corresponding to an effective 45 °C warming (Equation 33) had been removed from the atmosphere and was stored in the top 2,000 m ocean layer causing a mean temperature rise of only 0.08 °C (Equation 32). This quantitative example demonstrates the heat reservoir property of the ocean described by Zemansky et al. (1975, p. 95): “The ocean and the outside air are approximate examples of an ideal body called a heat reservoir. A heat reservoir is a body of such a large mass that it may absorb or reject an unlimited quantity of heat without suffering an appreciable change in temperature or in any other thermodynamic coordinate.”

In this section, empirical models for the OHC and its trend were derived and validated using previous results (Levitus et al., 2012). In the next section, the relationship between the secular climate trends, and their asymmetric seasonal oscillations are presented.

3.8. Relationships Between Secular Trends and Seasonal Oscillations of Climate Variables

In this section, the observed secular increases in the climate variables shown in Figures 1b, 3, and 5 were found to be the result of the seasonal solar ocean heating rate on average being greater than the seasonal ocean cooling rate. This seasonal asymmetry could be explained by heat energy storage as a result of heat flow from the warmer mixed ocean layer to the colder deeper ocean as shown schematically in Figure 6, which satisfies the second principle of thermodynamics that “heat must flow downhill on the temperature scale” (Holman, 1981, p. 2).

Figure 6 describes the direction of the seasonal heat flow between the atmosphere and mixed ocean layer that explains the seasonal changes in the global sea level shown in Figure 7. The seasonal global mean surface air temperature increases by 3.7 °C from January to July (Jones et al., 1999), which means that this seasonal increase in air temperature leads by two months the seasonal increase in global sea level from March to September shown in Figure 7. These observations indicate seasonal heat removal from the hotter atmosphere and storage in the relatively colder mixed ocean layer that increases the OHC and the sea level.

From Figure 7, energy balance for the seasonal changes gives:

\[
F = F_{\text{SeaHeat}} = F_{\text{SeaCool}}
\]  

(34)

where \( F \) is the secular solar forcing, \( F_{\text{SeaHeat}} \) is the seasonal solar ocean heating rate that causes increases in the seasonal climate variables and \( F_{\text{SeaCool}} \) is the seasonal ocean cooling rate that causes decreases in the seasonal climate variables (Figure 7).
Equation 34 is consistent with the conclusion of Ellis et al. (1978): “The world oceans apparently store and release heat in phase with the annual variation in the net radiation balance.”

The empirical model \( F \frac{d SL}{dy} = 0.1621 \) (Equation 25) relates the secular solar forcing \( F \) to the secular sea level trend \( d SL/dy \), which means that if the value of one of these two climate variables is known, the value of the other could be estimated. As a result, in Figure 7, \( F_{\text{SeasHeat}} \) and \( F_{\text{SeasCool}} \) could be estimated from the observed mean seasonal changes in the sea level \( \Delta(SL)_{\text{rise}} \) and \( \Delta(SL)_{\text{fall}} \) as \( \Delta(SL) = \Delta(SL)_{\text{rise}} - \Delta(SL)_{\text{fall}} \).

However, before using the secular solar forcing model \( F = 0.1621 \frac{d(SL)}{dy} \) (Equation 25) to determine the seasonal relationship between solar forcing and sea level trend, it is important to first verify whether the secular empirical models presented in this paper could also be used as seasonal empirical models.

### 3.8.1. Relationship Between the Seasonal Amplitudes of Atmospheric CO₂ Concentration and Sea Level Determined Using Their Secular Empirical Model

To show that the secular empirical models presented in this paper could also be used as seasonal models, the strong correlation \( (R^2 = 97.8\%) \) between the sea level and atmospheric CO₂ concentration, after removing a six-month lag in the CO₂ concentration, shown in Figure 7 was used. This figure shows that the standardized atmospheric CO₂ concentration and sea level are almost identical, which indicates that a mathematical relationship exists between these two climate variables as described in this section.

The lag of about six months between the seasonal oscillations of the sea level and atmospheric CO₂ concentration is probably related to the seasonal oscillation of the sea level pressure (Hsu & Wallace, 1976) and Henry’s Law (Zumdahl, 1989, p. 482): “the amount of a gas dissolved in a solution is directly proportional to the pressure of the gas above the solution.” This law indicates that when the seasonal sea level pressure increases, the amount of dissolved CO₂ in the mixed ocean layer also increases that decreases the atmospheric CO₂ concentration. For the subtropical and mid-latitude oceans of the Northern Hemisphere, the sea level pressure is maximum in summer (Hsu & Wallace, 1976), which makes the corresponding atmospheric CO₂ concentration at Mauna Loa minimum at the end of summer in October.
It is possible to relate the secular sea level trend \( \frac{d \text{SL}}{dy} \) in mm/year to the annual atmospheric \( \frac{d \text{CO}_2}{dy} \) trend in ppm/year by substituting the secular GMT trend model \( \frac{dT}{dy} \) into \( \frac{d \text{SL}}{dy} = K_{\text{SL}} \left( \frac{dT}{dy} \right) \) (Equation 21), which gives:

\[
\frac{d \text{SL}}{dy} = K_{\text{SL}} \left( \frac{dT}{dy} \right) = K_{\text{SL}} \frac{T_{2x}}{\ln(2)} \left( \frac{d \text{CO}_2}{dy} \right) = \frac{540.1}{C} \frac{d \text{CO}_2}{dy} \]  \hspace{1cm} (35)
\]

where \( K_{\text{SL}} = 266.569 \text{ mm/°C} \) is the sea level regression coefficient (Figure 5a) and \( T_{2x} = 1.4044 \) °C is the \( \text{CO}_2 \) doubling secular GMT (Figure 3).

Equation 35 relates the secular atmospheric \( \text{CO}_2 \) concentration trend \( \frac{d \text{CO}_2}{dy} \) in Figure 4 to the secular sea level trend \( \frac{d \text{SL}}{dy} \) in Figure 5a. For example, for the period 2000–2010 in Figure 4, \( \frac{d \text{CO}_2}{dy} = (389.57-370.61)/10 = 1.9 \text{ ppm/year} \). Substituting this \( \text{CO}_2 \) trend and the annual \( \text{CO}_2 \) concentration of \( C = 400 \text{ ppm} \) for the trend period's middle year of 2005 (Figure 4) into \( \frac{d \text{SL}}{dy} = \frac{540.1}{400} \frac{d \text{CO}_2}{dy} \) (Equation 35) gives \( \frac{d \text{SL}}{dy} = 2.7 \text{ mm/year} \). This result could be verified by using the observed sea level trend in Figure 5a for the period 2000–2010 of \( \frac{d \text{SL}}{dy} = (203.03-176.08)/10 = 2.7 \text{ mm/year} \), which is identical to the sea level trend determined from the \( \text{CO}_2 \) trend using Equation 35. As a result, this equation is the empirical mathematical model that relates the observed secular trends of atmospheric \( \text{CO}_2 \) concentration and sea level.

Equation 35 was also found to relate the observed seasonal changes of the sea level and atmospheric \( \text{CO}_2 \) concentration reported in the literature. For example, Zhao and Zeng (2014) reported: “Modern measurements at Mauna Loa… have shown an increase in atmospheric \( \text{CO}_2 \) concentration from <320 ppm in 1958 to 400 ppm in 2013. There is also a mean seasonal cycle that is characterized with a 5-months decrease (minimum in October) and a 7-months increase (maximum in May). The peak-to-trough amplitude of this seasonal cycle is approximately 6.5 ppm.” From this report, to estimate the seasonal peak-to-trough amplitude of the sea level using \( \frac{d \text{SL}}{dy} = \frac{540.1}{400} \frac{d \text{CO}_2}{dy} \) (Equation 35), the values to be used are \( C = 400 \text{ ppm} \) and \( \frac{d \text{CO}_2}{dy} = 6.5 \text{ ppm/year} \), which on substitution gives:
Figure 9. Relationship between secular solar forcing and global mean temperature (GMT). (a) Global warming for the period 1990–2016 is a sum of the thermal flywheel effect of 0.234 °C due to the initial global warming rate of 0.009 °C/year for year 1990 and due to the solar effect of 0.024 °C (b) Solar forcing due to 18% of the 11 yr solar cycle forcing of 0.19 W/m² since 1864.5 that is assumed to reverse after 2030. (c) Constant global warming acceleration for the period 1865–2030 due to the 11 yr secular solar forcing of 0.18 × 0.19 W/m² followed by deceleration due to the assumed reversal of solar activity. This empirical model predicts a pause in the annual GMT for the period 2016–2040, with all the values less than that for 2016.
This empirical model result is identical to the satellite altimetry observation of 8.8 mm for the central peak-to-trough total sea level amplitude for the Envisat mission reported by Leuliette and Miller (2009, Table 1). This agreement between model and observations indicates that the secular empirical models presented in this paper could also be used as seasonal empirical models.

C. A. S. Hall et al. (1975) attributed the seasonal changes in atmospheric CO₂ concentration to the biosphere: “We, and others…, attribute the seasonal variation in atmospheric CO₂ (about 6 ppm) principally to seasonal changes in net photosynthesis and respiration of the biosphere, although purely physical and chemical criteria, such as annual variations in sea surface temperatures, may contribute slightly.”

This empirical model result is identical to the satellite altimetry observation of 8.8 mm for the central peak-to-trough total sea level amplitude for the Envisat mission reported by Leuliette and Miller (2009, Table 1). This agreement between model and observations indicates that the secular empirical models presented in this paper could also be used as seasonal empirical models.

As a result, in this study, the causes of the seasonal oscillations of all of the climate variables are considered to be the seasonal solar ocean heating rate $F_{\text{Seas Heat}}$ and the seasonal ocean cooling rate $F_{\text{Seas Cool}}$ shown in Figure 7. $F_{\text{Seas Heat}}$ and $F_{\text{Seas Cool}}$ could be estimated from the observed
asymmetric seasonal changes in the sea level as described in the following sections. This asymmetry could be explained by seasonal heat storage in the colder deeper ocean because of heat flow from the warmer mixed ocean layer.

3.8.2. Estimation of the Mean Seasonal Solar Ocean-Heating Rate $F_{\text{SeaHeat}}$ From the Mean Seasonal Global Sea Level Rise $\Delta(SL)_{\text{Rise}}$

To estimate the value of $F_{\text{SeaHeat}}$ shown in Figure 7 using the empirical model $F = 0.1621 \frac{d(SL)}{dy}$ (Equation 25), the mean global sea level rise $\Delta(SL)_{\text{Rise}}$ from March to September was required.

From analysis of the seasonal sea level oscillations of the Envisat satellite altimetry data for the period 2004–2007, Leuliette and Miller (2009, Table 1) reported a peak-to-trough amplitude of 8.8 mm and a secular sea level trend of 2.7 mm/year. From these values, $\Delta(SL)_{\text{Rise}}$ and $\Delta(SL)_{\text{Fall}}$ shown in Figure 7 could be estimated.

From this figure, we have $\Delta(SL) = \Delta(SL)_{\text{Rise}} - \Delta(SL)_{\text{Fall}} = 2.7$ mm. The mean of $\Delta(SL)_{\text{Rise}}$ and $\Delta(SL)_{\text{Fall}}$ gives the peak-to-trough amplitude, which could be written as $(\Delta(SL)_{\text{Rise}} + \Delta(SL)_{\text{Fall}}) / 2 = 8.8$ mm. Solving these two equations simultaneously gives $\Delta(SL)_{\text{Rise}} = 10.15$ mm and $\Delta(SL)_{\text{Fall}} = 7.45$ mm. As shown in Figure 7, since the seasonal sea level rise $\Delta(SL)_{\text{Rise}}$ and fall $\Delta(SL)_{\text{Fall}}$ repeat each and every year, their unit is mm/year. As a result, the difference between the seasonal mean sea level rise rate of $(d(SL)/dy)_{\text{Rise}} = 10.15$ mm/year and fall rate of $(d(SL)/dy)_{\text{Fall}} = 7.45$ mm/year results in the observed mean secular sea level trend of $d(SL)/dy = (d(SL)/dy)_{\text{Rise}} - (d(SL)/dy)_{\text{Fall}} = 2.7$ mm/year, which is identical to the secular sea level trend $d(SL)/dy = (203.03–176.08)/10 = 2.7$ mm/year in Figure 5a for the period 2000–2010.

Substituting $(d(SL)/dy)_{\text{Rise}} = 10.15$ mm/year into $F = 0.1621 \frac{d(SL)}{dy}$ (Equation 25) gives:

$$\frac{d(SL)}{dy}_{\text{Rise}} = 0.1621 \left( \frac{d(SL)}{dy} \right)_{\text{Rise}} = 1.65 \text{ W/m}^2$$  (37)

This seasonal solar ocean heating rate of $F_{\text{SeaCool}} = 1.65 \text{ W/m}^2$ for year 2005 agrees with the radiative forcing for atmospheric CO$_2$ concentration of +1.66 W/m$^2$ reported in IPCC (2007, Figure SPM2). This comparison indicates that the warming climate forcing of 1.66 W/m$^2$ that has previously been attributed to the observed increase in the annual atmospheric CO$_2$ concentrations appears instead to be due to the natural seasonal solar ocean heating rate.

Using the model $dT/\text{dy} = \lambda F$ (Equation 12) and the seasonal solar ocean heating rate of $F_{\text{SeaHeat}} = +1.65 \text{ W/m}^2$ (Equation 37), the seasonal GMT warming rate is:

$$\frac{dT}{\text{dy}}_{\text{SeaWarm}} = \lambda F_{\text{SeaHeat}} = 0.0382 \text{ °C/year} = 3.82 \text{ °C/century}$$  (38)

where $\lambda = 0.023143 \text{ °C/year per W/m}^2$ is the climate sensitivity (Equation 12).

The above seasonal increase in GMT of 0.0382 °C (Equation 38) is two orders of magnitude less than the seasonal increase in the surface air temperature of 3.7 °C (Jones et al., 1999) because the heat capacity of the ocean is very large compared to that of the atmosphere, which indicates seasonal heat removal from the solar heated high-temperature atmosphere of 3.7 °C from January to July and heat storage at low temperature of 0.0382 °C in the ocean from March to September that increases the sea level by 10.15 mm (Figure 7).

The seasonal GMT warming rate of $(dT/\text{dy})_{\text{SeaWarm}} = 3.82 \text{ °C/century}$ (Equation 38) is about six-times the observed global warming of about 0.6 °C of the 20th century, which is because the observed global warming is related to the difference between the mean seasonal warming and cooling rates as shown in the next section.

3.8.3. Estimation of the Mean Seasonal Ocean-Cooling Rate $F_{\text{SeaCool}}$ From the Mean Seasonal Global Sea Level Fall $\Delta(SL)_{\text{Fall}}$

During the seasonal cooling from September to March, heat flows from the relatively warmer mixed ocean layer to the colder atmosphere that results in global sea level fall shown in Figure 7. Substituting the mean seasonal sea level fall rate of $(d(SL)/dy)_{\text{Fall}} = 7.45$ mm/year (Section 3.8.2) into $F = 0.1621 \frac{d(SL)}{dy}$ (Equation 25) gives a mean seasonal ocean cooling rate of:
This seasonal ocean cooling rate of $F_{\text{SeaCool}} = 1.21 \text{ W/m}^2$ for year 2005 agrees with the total aerosol radiative forcing of $-1.2 \text{ W/m}^2$ reported in IPCC (2007, Figure SPM2). This comparison indicates that the cooling climate forcing of about $-1.2 \text{ W/m}^2$ has previously been attributed to total aerosol forcing appears instead to be due to the natural seasonal ocean cooling rate.

Using the model $dT/\text{dy} = \lambda F$ (Equation 12) and the seasonal ocean cooling rate of $F_{\text{SeaCool}} = 1.21 \text{ W/m}^2$ (Equation 39), the seasonal GMT cooling rate is:

$$\left(\frac{dT}{\text{dy}}\right)_{\text{SeaCool}} = 0.028 \text{ °C/year} = 2.8 \text{ °C/century}$$

where $\lambda = 0.023143 \text{ °C/year per W/m}^2$ is the climate sensitivity (Equation 12).

The above results show that the observed recent secular trends in the climate variables have been due to the secular solar forcing of $F = 0.44 \text{ W/m}^2$ (Figure 1c), which is the difference between the seasonal solar ocean heating rate of $F_{\text{SeaHeat}} = 1.65 \text{ W/m}^2$ (Equation 37) and seasonal ocean cooling rate of $F_{\text{SeaCool}} = 1.21 \text{ W/m}^2$ (Equation 39). This asymmetry in the seasonal heating and cooling rates was attributed to the seasonal heat storage due to heat flow from the warmer mixed ocean layer to the colder deeper ocean since 1865.

### 3.9. Climate Predictions

As shown in Figure 3, since the empirical model for the annual atmospheric CO$_2$ concentration almost perfectly explained ($R^2 = 99.85\%$) the observed values since 1859 and since this model also correctly estimated the reported central atmospheric CO$_2$ concentration of about 270 ppm for the mid-19th century (Elliott, 1984; Suiver, 1978), this model was used for the prediction of the atmospheric CO$_2$ concentration for year 2100. Since the climate variables are related to each other (Figures 1, 3 and 5), the secular sea level rise and global warming for year 2100 were estimated from the predicted value of the annual atmospheric CO$_2$ concentration. Decadal predictions of the surface GMT are also presented.

Assuming that the observed constant secular GMT acceleration since 1865 (Figure 1a) continues, the annual atmospheric CO$_2$ concentration could be calculated using the model $C = 267.5 \text{ exp} [1.7757 \times 10^{-5} (y - 1864.5)^2]$ (Equation 20), which for the year $y = 2100$ gives $C = 716.18$ ppm. This result is near the middle of the projected range reported in IPCC (2001, SPM, p. 12): “By 2100, carbon cycle models project atmospheric CO$_2$ concentrations of 540–970 ppm for the illustrative Special Report on Emission Scenarios (SRES).” Unlike the GMT and sea level, the annual atmospheric CO$_2$ concentration does not have MDO because it depends only on the secular GMT $T$ as shown in Figure 3. As a result, the annual atmospheric CO$_2$ concentration is much easier to predict assuming that the observed constant secular GMT acceleration of $dT_y = 7.1954 \times 10^{-5} \text{ °C/year}^2$ continues (Figure 1a).

For the increase in the annual atmospheric CO$_2$ concentration from $C_y = 353.83$ ppm for year 1990 (Figure 3) to $C = 716.18$ ppm for year 2100 calculated above, the corresponding sea level rise could be calculated using the model $\Delta SL = 540.11 \text{ln}(C/C_y)$ (Equation 26), which gives a sea level rise of $\Delta SL = 0.54 \text{ln}(C/C_y) = 0.38 \pm 0.016$ m. This result is near the middle of the projected range reported in IPCC (2001, SPM, p. 16): “Global mean sea level is projected to rise by 0.09–0.88 m between 1990 and 2100, for the full range of SRES scenarios.”
For the increase in the annual atmospheric CO₂ concentration from \(C_i = 353.83\) ppm for year 1990 to \(C = 716.18\) ppm for year 2100, the corresponding increase in the secular GMT could be calculated using the model \(\Delta T = \ln(C / C_i) / 0.49356\) (Figure 3), which gives a global warming of \(\Delta T = 1.43 \pm 0.2^\circ C\). This result is at the minimum end of the projected global warming range reported in IPCC (2001, SPM, p. 13): “The globally averaged surface temperature is projected to increase by 1.4–5.8°C ... over the period 1990–2100.” For the increase in the annual atmospheric CO₂ concentration from \(C_i = 267.5\) ppm for the steady state year 1864.5 (Figure 3) to \(C = 716.18\) ppm for year 2100, the projected global warming is \(\Delta T = \ln(C / C_i) / 0.49356 = 2 \pm 0.2^\circ C\) (Figure 9c), which is identical to the goal of the Paris Agreement (Schleussner et al., 2016).

In addition to the secular global warming discussed above, the annual GMT data also has an MDO as shown in Figure 8. This figure, for the period 1862–2018, shows the MDO obtained from the annual GMT and the 22 yr moving GMT averages after detrending the data by removing the secular GMT \(T = 7.1954 \times 10^{-5} (y - 1864.5)^2 / 2\) (Equation 13). The MDO represented by the detrended 22 yr moving GMT averages shown in Figure 8 was found to correlate with the warming and cooling phases of the global mean surface temperature reported at that time in the scientific literature.

For example, for the period 1921–1953 in Figure 8 (top horizontal axis label), the 22 yr moving GMT averages were in a warming phase, which correlates with the arctic warming reported at that time by Landsberg (1958): “The recent warming of the arctic has been particularly notable at the edges of the forested regions both in North America and Eurasia. The tree line has been advancing gradually northward. In some areas which have been resurveyed, the forest has advanced 2 miles northward over the last 30 yr ... There is little doubt that the past two or three decades, taken over-all, have been among the warmest in centuries.”

For the period 1953–1985 in Figure 8 (top horizontal axis label), the 22 yr moving GMT averages were in a cooling phase, which correlates with the global cooling reported at that time by Kukla and Matthews (1972): “The present global cooling, which reversed the warm trend of 1940s, is still underway. ... The present cooling is especially demonstrable in certain key regions in arctic and subarctic latitudes. Thus, snowbanks today cover areas of Baffin Island that were seasonally free of snow for the 30 or 40 years preceding the present summer cooling.”

For the period 1985–2017 in Figure 8, the 22 yr moving GMT averages were in a warming phase, which correlates with the decrease in arctic sea ice cover reported by Comiso et al. (2008): “Each of the years after 2004 shows less ice throughout the year than any of the five-year averages going back to 1980–1984.”

For the period 1953–1985 in Figure 8, the 22 yr moving GMT averages were in a cooling phase and arctic sea ice cover was increasing (Kukla & Matthews, 1972), while the secular GMT \(T\) was increasing as shown in Figure 1b. This observation indicates that the secular GMT \(T\) is uncorrelated with the global mean surface temperature. As a result, the observed increases in the global mean surface temperature of the 20th century appear to be caused by the warming phases of the 22 yr moving GMT averages shown in Figure 8, instead of the monotonically increasing secular GMT \(T\) since mid-19th century shown in Figure 1b. This observation regarding the secular GMT \(T\) or the deeper ocean secular warming is consistent with the report by Rossby (1959, p. 12): “These deeper layers are insulated from the atmosphere by stably stratified warmer water masses near to the sea surface and are not able directly to restore the radiation balance by means of an increased evaporation and cloud formation.”

Since 1862, the above agreements between the reported multi-decadal warming and cooling phases of the global mean surface temperature and the 22 yr moving GMT averages model result shown in Figure 8 indicates that this model could be used for prediction. Assuming that this model’s sinusoidal pattern \((R^2 = 96.9\%)\) continues, a cooling phase is predicted to occur for the period 2017–2049 (top horizontal axis label).

4. Summary and Discussion

Previous studies have reported that solar forcing alone does not explain most of the observed global warming using the LM for the relationship between the change in solar forcing \(\Delta F\) and the corresponding change in GMT \(\Delta T\) given by (IPCC, 2001, Section 6.2.1):
\[ \Delta T = \lambda_{LM} \Delta F \]  

(41)

where \( \lambda_{LM} = 0.5 \text{ °C per W/m}^2 \) is the equilibrium climate sensitivity for the LM.

The above model has been widely used in the scientific literature to relate GMT to climate forcing. For example, Sévellec and Drijfhout (2018) assumed “globally averaged temperature responds linearly, with some lag, to the various forcing agents.”

Similarly, Lean and Rind (2008) assumed “surface temperature responds linearly (at some lag) to the various influences” and from analysis of climate data using the LM, they reported that solar forcing contributed only “10% of the warming in the past 100 yr.”

The definition of a time-invariant climate sensitivity \( \lambda_{LM} \) for the LM \( \lambda_{LM} = \Delta T / \Delta F \) indicates that for a given solar forcing \( \Delta F \), the observed change in GMT \( \Delta T \) is the same for any initial global warming rate, which is the main shortcoming of the LM. It is reasonable to expect that for a given solar forcing \( \Delta F \), the observed global warming \( \Delta T \) increases with increase in the initial global warming rate \((dT / dy)\), which indicates a nonlinear relationship between these two climate variables.

In this study, to overcome the above shortcoming of the LM \( \Delta T = \lambda_{LM} \Delta F \), an empirical nonlinear relationship between change in secular solar forcing \( \Delta F \) in W/m² and the corresponding change in the secular GMT \( \Delta T \) in °C was derived that takes into account the effect of the initial global warming rate \((dT / dy)\), as given by (Equation 14):

\[
\Delta T = \left( \frac{dT}{dy} \right) (y - y_i) + \frac{\lambda^2}{2\alpha_T} (\Delta F)^2 = \left( \frac{dT}{dy} \right) (y - y_i) + 3.7218 (\Delta F)^2
\]

(42)

where \( \lambda = 0.023143 \) °C/year per W/m² is the climate sensitivity, \( \alpha_T = 7.1954 \times 10^{-5} \) °C/year² is the secular GMT acceleration and \((dT / dy)\), is the initial global warming rate for year \( y_i \).

For the above empirical model for the secular GMT \( \Delta T \), from an analogous model for the motion of a particle acted upon by a constant force (Timoshenko & Young, 1956, p. 267), the initial global warming rate \((dT / dy)\), represents the “heredity” of the secular GMT \( \Delta T \), while the solar forcing \( \Delta F \) represents its “environment.”

As a result, in Equation 42, since the component of the secular global warming related to the initial global warming rate given by \( \Delta T_{flywheel} = (dT / dy) (y - y_i) \) is due to heredity, this secular global warming during the subsequent period was explained without any climate forcing, so this heredity component of global warming was attributed to the “thermal flywheel” effect of the ocean (Hoffert et al., 1980) in this study, instead of to anthropogenic greenhouse gasses and aerosols assumed by the LM.

The solar forcing \( \Delta F \) to be used in the above model was derived from a fraction (18%) of the 11 yr solar cycle forcing of \( \Delta F_{11} = 0.19 \) W/m² and is given by (Equation 11):

\[
\Delta F = 0.18 \Delta F_{11} n_{SolCyc} = 0.18 \times 0.19 n_{SolCyc} = 0.18 \times 0.19 \left( \frac{y - y_i}{11} \right)
\]

(43)

where 0.18 is a central damping factor that takes into account the oceanic thermal inertia, 0.19 W/m² is the observation based 11 yr solar cycle forcing (Equation 6) and \( n_{SolCyc} = (y - y_i) / 11 \) is the number of 11 yr solar cycles for the solar forcing change period from year \( y_i \) to year \( y \).

In the secular solar forcing model \( \Delta F \) above, the number of 11 yr solar cycles \( n_{SolCyc} \) was considered because the nonlinear solar forcing model is cumulative (Equation 2). For a single 11 yr solar cycle \( (n_{SolCyc} = 1) \), the above solar forcing model \( \Delta F = 0.18 \times 0.19 n_{SolCyc} \) gives \( \Delta F = 0.18 \times 0.19 \) W/m². Using this solar forcing in the LM \( \Delta T_{LM} = 0.65 \Delta F \), Wigley and Raper (1990) reported a central global warming of \( \Delta T_{LM} = 0.02 \) °C due to the 11 yr secular solar forcing. In contrast, using the same solar forcing \( \Delta F \) in the nonlinear model \( \Delta T = 3.7218 (\Delta F)^2 \) (Equation 42) gives a global warming of only \( \Delta T = 0.0044 \) °C, which is an order of magnitude less than the result from the LM of \( \Delta T_{LM} = 0.02 \) °C. As a result, the nonlinear model result above confirmed the conclusion of Wigley and Raper (1990): “These values are so small that the solar effect would be virtually undetectable.”

However, because the empirical global warming model \( T = 3.7218 F^2 \) (Equation 42) is nonlinear, the change in the secular GMT for a single 11 yr secular solar forcing is very small but becomes significant
with increases in the number of 11 yr solar cycle forcings. For example, for year 2016, the number of 11 yr solar cycles since steady state in 1864.5 is \( n_{\text{SolCyc}} = (2016 - 1864.5) / 11 = 13.77 \). Substituting this value into \( F = 0.18 \times 0.19 n_{\text{SolCyc}} \) (Equation 43) gives a secular solar forcing of \( F = 0.471 \text{ W/m}^2 \) for year 2016 as shown in Figure 9b. Using this solar forcing in the secular GMT model \( T = 3.7218 F^2 \) (Equation 42) gives a global warming of \( T = 0.83 ^\circ C \) as shown in Figure 9c. This solar forcing of \( F = 0.471 \text{ W/m}^2 \) that explained all of the observed global warming is within the solar forcing range of \( 0.5 \pm 0.3 \text{ W/m}^2 \) since 1850 reported by Fröhlich and Lean (1998), within the solar forcing range of \( 0.4 \pm 0.2 \text{ W/m}^2 \) for the period 1850–2000 reported by Hansen et al. (2000), agrees with the solar forcing of \( 0.4 \text{ W/m}^2 \) for the period 1860–2000 reported by Beer et al. (2000) and is identical to the secular solar forcing determined from the long-term drift of the coronal source magnetic flux (Lockwood & Stamper, 1999) given by Equation 9.

In a solar forcing paper, Lockwood and Fröhlich (2007) stated: “Here we show that over the past 20 yr, all the trends in the Sun that could have had an influence on the Earth’s climate have been in the opposite direction to that required to explain the observed rise in GMTs.” How does the nonlinear model of this study explain this recent observation of oppositely directed trends in solar forcing and GMT?

To answer this question, the nonlinear solar forcing model above (Equation 42) was used to decompose the observed secular increase in GMT of \( 0.26 ^\circ C \) from 0.57 \( ^\circ C \) to 0.83 \( ^\circ C \) for the 26 yr period from 1990 to 2016 (Figure 9c) into thermal flywheel effect (related to the initial secular GMT trend \( (dT / dy)_i \)) and solar effect (related to the secular solar forcing \( \Delta F \)).

For year \( y = 1990 \), the secular solar forcing model \( F = 0.18 \times 0.19(y - 1864.5) / 11 \) (Equation 43) gives \( F = 0.3902 \text{ W/m}^2 \) since steady state as shown in Figure 9b. Using this solar forcing in the nonlinear model \( T = 3.7218 F^2 \) (Equation 42) gives a solar caused global warming of \( T = 0.57 ^\circ C \) for year 1990 as shown in Figures 9a and 9c.

In addition to increasing the secular GMT to 0.57 \( ^\circ C \) for 1990, solar forcing has also increased the thermal flywheel effect of the ocean by increasing the global warming rate \( dT / dy \). Using the above solar forcing of \( F = 0.3902 \text{ W/m}^2 \) in the model \( dT / dy = \lambda F \) (Equation 12), where \( \lambda = 0.023143 \text{ C/year per W/m}^2 \) is the climate sensitivity, gives a global warming rate of \( dT / dy = 0.009 ^\circ C/\text{year} \) for year 1990 as shown in Figure 9a.

From Figure 9b, the secular solar forcing for the period 1990–2016 is only \( \Delta F = 0.471 - 0.3902 = 0.0808 \text{ W/m}^2 \). Using this solar forcing in the model \( \Delta T = 3.7218 (\Delta F)^2 \) gives a direct solar caused global warming of \( \Delta T_{\text{solar}} = 0.024 ^\circ C \), which is only about 9% of the observed global warming of 0.26 \( ^\circ C \) for the period 1990–2016 shown in Figure 9c. As a result, to explain all of the observed global warming using the nonlinear model, the indirect thermal flywheel effect of the ocean given in Equation 42 should be included. For the initial global warming rate of \( (dT / dy)_i = 0.009 ^\circ C/\text{year} \) for year 1990 (Figure 9a), the global warming for the period 1990–2016 (\( \Delta y = 26 \)) due to this initial global warming rate alone is \( \Delta T_{\text{flywheel}} = (dT / dy) \Delta y = 0.234 ^\circ C \). Adding the above global warmings due to the solar effect and the thermal flywheel effect to the secular GMT of 0.57 \( ^\circ C \) for 1990, we get for year 2016 a secular GMT of:

\[
T_{2016} = T_{1990} + \Delta T_{\text{solar}} + \Delta T_{\text{flywheel}} = 0.57 + 0.024 + 0.234 = 0.83 ^\circ C
\]

(44)

which is identical to the secular GMT for 2016 shown in Figure 9c. The relative magnitudes of the three decomposed secular GMTs in the above equation for the global warming of 0.83 \( ^\circ C \) for year 2016 are shown in Figure 9a as areas in the secular GMT trend diagram.

For the period 1990–2016, Equation 44 shows that solar forcing explained only 9% (\( = 0.024 \times 100 / (0.024 + 0.234) \)) of the observed global warming of 0.26 \( ^\circ C \). As a result, most of the warming that is unexplained by solar forcing and that has been attributed to anthropogenic greenhouse gasses by the LM appear to instead be due to the thermal flywheel effect of the ocean.

From the above result, since the solar effect on couple of decades time scale is very small, the effect of decline in solar activity could not be detected. This is because the short-term change in the secular GMT is dominated by the accelerating thermal flywheel effect of the ocean. For the global cooling to start as a result of reduced solar activity, the ocean has to first decelerate back to its steady state \( (dT / dy = 0) \) and that takes more than a century as described next.
Lean and Rind (1998) have argued that lower solar activity is expected to start around 2030: “Evidence from both cosmogenic isotopes and sun like stars points to the likelihood of future solar activity falling to lower levels, rather than increasing. Extrapolation of periodicities present in cosmogenic isotope data infer that this decrease may commence around 2030.” For this scenario, assuming the constant secular GMT acceleration since steady state in 1865 reverses into deceleration of the same magnitude after 2030 as shown in Figure 9a, since it has taken the secular GMT trend 165 yr (= 2030 – 1865) to reach its peak from its steady state, the secular GMT trend is expected to return back to it steady state (dT/dy = 0) by 2030 + 165 = 2195 as shown. In Figure 9a, since the secular GMT trend is positive (dT/dy > 0) for the period 2030–2195, secular global warming is expected to continue for 165 more years but at a decelerating rate as shown in Figure 9c.

This estimate of 2195 for the start year of secular global cooling, assuming reduced solar activity after 2030, could be calculated by setting to zero the secular GMT trend dT/dy (Equation 5) as given by
dT/dy = (dT/dy) + aT(y - 2030) = 0, where aT = -7.1954 x 10^-5 °C/year^2 is the assumed secular GMT deceleration after 2030 due to reduced solar activity. For the initial global warming rate of (dT/dy), = 0.0119 °C/ year for 2030 (Figure 9a), the year for the start of secular global cooling is y = -(dT/dy)/aT + 2030 = 2195, which is identical to the result given in the previous paragraph. This result shows that assuming reduced solar activity after 2030, secular global warming would continue until year 2195 because the thermal flywheel effect of the ocean (dT/dy) should first return to steady state before secular global cooling could start.

In Figure 9c, for the secular GMT T for the period 1865–2195, the first steady state year is 1865 (T = 0 °C), the inflection point year is 2030 (T_2030 = 1 °C) when the constant secular GMT acceleration since steady state is expected to reverse into deceleration, and the second steady state year is 2195 (T_2195 = 2T_2030 = 2 °C). In Figure 9c, the annual GMT noises for the period 2019–2195 were computer generated as random real numbers within the range of ±0.2 °C, which are within two standard deviations of the annual GMT residuals. As shown in Figure 9c, the global warming difference for the period 1865–2100 between continued secular GMT acceleration to 2 °C and deceleration after 2030 to 1.64 °C due to the predicted lower solar activity (Lean & Rind, 1998) is only 0.36 °C. Remarkably, the secular global warming prediction range of this study from 1.64 to 2 °C for year 2100 since steady state in 1865 shown in Figure 9c is almost identical to the “long-term temperature goal” of The Paris Agreement (Schleussner et al., 2016): “The Paris Agreement sets a long-term temperature goal of holding the global average temperature increase to well below 2 °C, and pursuing efforts to limit this to 1.5 °C above pre-industrial levels.”

In addition to the secular GMT T (Figure 9c) discussed above, the annual GMT data also has an MDO superimposed on the secular GMT. The mechanisms of the MDO in the GMT data have been described in several previous studies. For example, Chen and Tung (2014) reported: “The latter part of the 20th century saw rapid global warming as more heat stayed near the surface. In the 21st century, surface warming slowed as more heat moved into deeper oceans.” Knudsen et al. (2011) found that the MDO has existed for the last 8,000 yr. These results indicate that the MDO in the GMT data is due to accumulation of absorbed solar energy in the mixed layer during the warm phases of the MDO and sequestration of this heat energy in the deeper ocean during the cold phases. Zhang (2007) found that the surface and subsurface temperatures of the North Atlantic Ocean are anti-correlated, which indicates that the MDO is driven by the temperature difference between the surface and subsurface ocean.

The thermal flywheel effect of the deeper ocean as represented by the secular GMT trend dT/ dy could be distinguished from the thermal flywheel effect of the mixed ocean layer by modeling the rate of change of the GMT of the latter as shown in Figure 10. This figure shows the 30 yr GMT moving trends data from 1850–1880 to 1988–2018 and their empirical model dS/ dy. In this figure, the model dS/ dy is the sum of the secular GMT trend dT/ dy of the deeper ocean and the MDO GMT trend of the mixed ocean layer, so the thermal flywheel effect of the mixed ocean layer could be modeled by the MDO GMT trend given by

dS/ dy = 0.009°C/yr [2π(y – 1862)/64] in °C/year, where y is the middle year of a 30 yr trend period. The amplitude of this sinusoidal function gives a peak 30 yr GMT trend of 0.09 °C/decade due to the MDO alone. As a result, since the model dS/ dy in Figure 10 gives a peak 30 yr GMT trend of 0.18 °C/decade for year 2005 (top horizontal axis label), the contribution of the warm phase of the MDO to the observed 30 yr GMT trend for the period 1975–2005 is 0.09/0.18 x 100 = 50%, or half of the observed 30 yr GMT trend. From these results, the secular GMT trend for the period 1975–2005 is dT/ dy = 0.18 – 0.09 = 0.09 °C/decade, which is identical to the secular GMT trend for the trend period middle year of y = 1990 shown in Figure 9a.
The 30 yr moving GMT trends data and their model shown in Figure 10 could be compared with the projection in IPCC (2007, SPM, p. 12): "For the next two decades, a warming of about 0.2 °C per decade is projected for a range of SRES. ... Since IPCC's first report in 1990, assessed projections have suggested global average temperature increases between about 0.15 °C and 0.3 °C per decade for 1990–2005. This can now be compared with observed values of about 0.2 °C per decade, strengthening confidence in near-term projections". The comparison in Figure 10 shows that the 30 yr GMT moving trends have been slowly decelerating since year 2005 (top horizontal axis label) from the observed peak value of 0.2 °C per decade toward the minimum projected value of 0.15 °C per decade in IPCC (2007). As shown in Figure 10, since the model \( ds/dy \) explained 97.2% of the variability of the 30 yr GMT moving trends data for the period 1880–2018 and since the residuals (Equation 4), without the outliers, for this model were found to be normally distributed as shown in Figure 11, it is reasonable to assume that the 139 yr long pattern of this model will continue for the next 20 yr. As a result, the empirical model in Figure 10 predicts, before the end of this decade of the 2020s, deceleration of the 30 yr GMT moving trend from the observed peak value of 0.2 °C/decade for year 2005 (top horizontal axis label) to less than the minimum projected value of 0.15 °C/decade in IPCC (2007).

The verification of the above prediction, for a 30 yr GMT trend of less than 0.15 °C/decade before the end of this decade of the 2020s, would validate the interpretations that a climate shift had already occurred in 2005 from acceleration to deceleration of the 30 yr GMT trends as shown by the empirical model \( ds/dy \) in Figure 10 and that the 139 yr long pattern identified by this model is due to the thermal flywheel effect of the ocean that causes a predictable MDO in the GMT trends data. For example, the model peak in year 1941 (top horizontal axis label) repeated 64 yr later in 2005. Furthermore, the trough in 1909 repeated 64 yr later in 1973, indicating multi-decadal climate change caused by an ocean cycle (instead of greenhouse gases and aerosols) that will be confirmed if the next 30 yr GMT trend trough of less than 0.1 °C/decade occurs for the period 2007–2037.

For the nonlinear empirical solar forcing model, the main conclusion of this paper that solar forcing explained all of the observed global warming is contrary to the current zeitgeist and the reported scientific consensus (Cook et al., 2013) on the anthropogenic global warming (AGW) theory that human emissions of CO₂ explains the observed global warming. Currently, a person holding a view contrary to this scientific consensus is labeled a “climate change denier” by mainstream exponents (Van Rensburg, 2015).

The AGW theory is based on the greenhouse effect that is explained by Houghton (1997, p. 10) as: “The basic principle of global warming can be understood by considering the radiation energy from the sun which warms the Earth’s surface and the thermal radiation from the Earth and the atmosphere which is radiated out to space. On average these two radiation streams must balance. If the balance is disturbed (for instance by an increase in atmospheric carbon dioxide) it can be restored by an increase in the Earth’s surface temperature.”

In the above explanation of the greenhouse effect, the incoming solar radiation and the outgoing thermal radiation are assumed on average to balance. This assumption was found to be inconsistent with the second principle of thermodynamics that states: “heat must flow downhill on the temperature scale” (Holman, 1981, p. 2) from the warmer mixed ocean layer to the colder deeper ocean during global warming that makes the incoming solar radiation greater than the outgoing thermal radiation because of heat energy storage in the deeper ocean.

The solar forcing model \( F = \lambda^{-1}(dT/dy) \) (Equation 12) was found to be analogous to the model for the voltage \( V \) across an inductor given by \( V = L(di/dt) \). The energy storage and retrieval characteristics of an inductor have been described as (Alexander & Sadiku, 2009, p. 228): “the ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time... For current \( i \) through an inductor, an abrupt change is not possible.” The energy storage and retrieval characteristics of the world oceans have been described as (Ellis et al., 1978): “The world oceans apparently store and release heat in phase with the annual variation in the net radiation balance.” This inductor-ocean analogy indicates a nonlinear relationship between GMT and solar forcing because of heat energy storage in the ocean as given by the model of this study \( \Delta(dT/dy) = \lambda \Delta F \) (Equation 2) and that for the secular GMT \( T \) “an abrupt change is not possible” because of the enormous thermal inertia of the deep ocean.
From the above inductor-ocean analogy, the globe warms not to restore balance between incoming solar and outgoing thermal radiation as assumed by the AGW theory, but to store heat energy in the deeper ocean during secular global warming (e.g., from the Dark Age Cold Period to the Medieval Warm Period [Ljungqvist, 2010, Figure 3]) that is “retrieved at a later time” during secular global cooling from the Medieval Warm Period to the Little Ice Age. As a result, the stored heat energy in the ocean is there to ameliorate atmospheric cooling (at night, in winter, and during cold periods like the little ice age) due to heat flow downhill on the temperature scale from the relatively warmer ocean to the colder atmosphere. The direction of this net heat flow reverses during atmospheric heating (during the day, in summer, and warm periods like the Medieval Warm Period) from the warmer atmosphere to the relatively colder ocean.

From empirical analysis of climate data, Lean and Rind (2008) reported a solar caused secular GMT trend of 0.007 °C/decade. For the 15.15 decades for the period 1864.5–2016, this warming rate gives a global warming of only ΔT = 0.007 × 15.15 = 0.11 °C. As a result, based on the LM, solar forcing does not explain most of the observed secular global warming of 0.83 °C for the period 1864.5–2016 (Figure 9c). For the LM ΔT = ΔF (Equation 1), solar forcing changes the secular GMT T. In contrast, for the nonlinear model Δ(dT/dy) = λΔF (Equation 2), solar forcing changes the secular GMT trend dT/dy. Therefore, the above solar caused global warming rate result of 0.007 °C/decade could be interpreted as the change in the secular GMT trend per decade, which gives a solar caused global warming acceleration of 0.007 °C/decade² or $a_T = 7 \times 10^{-5} \text{ °C/year}^2$. For the period 1864.5–2016 ($\Delta y = 151.5$), this constant acceleration results in a secular global warming of $T = a_T (\Delta y)^2/2 = 0.80 \text{ °C}$, which shows that the solar caused global warming rate result of 0.007 °C/decade by Lean and Rind (2008) explained almost all of the observed secular global warming of 0.83 °C when the nonlinear model’s interpretation was applied (i.e., for the nonlinear model, solar forcing increased the secular GMT trend by 0.0072 °C/decade per decade as shown in Figure 9a by the linearly increasing secular GMT trend data and model since steady state in 1864.5).

The nonlinear empirical solar forcing model of this study given by $\Delta T = (dT/dy)(y - y_1) + 3.7218(\Delta F)^2$ (Equation 42) was an implementation of the thermal flywheel effect of the deep ocean on climate change described by Hoffert et al. (1980): “There has existed for some time the impression that the deep oceans of the world, by virtue of their extremely high heat capacity relative to the atmosphere, may act as a kind of thermal flywheel, providing temporary heat storage during periods of climatic change and affecting the rate of change of global surface temperatures over timescales of years to centuries.” In the above empirical model, by including the effect of the accelerating initial global warming rate (dT/dy), on change in the secular GMT as given by $\Delta T_{flywheel} = (dT/dy)(y - y_1)$, the thermal flywheel effect of the deep ocean on change in the secular GMT was taken into account as shown in Figure 9c.

5. Conclusion

For the linear climate forcing model $\Delta T = 0.5\Delta F$ (Equation 1), the solar forcing of $\Delta F = 0.471 \text{ W/m}^2$ for the period 1865–2016 (Figure 9b) explains a global warming of only 0.24 °C, so most of the corresponding observed global warming of 0.83 °C (Figure 9c) is unexplained by solar forcing for the LM. In contrast, for the nonlinear empirical mathematical model $\Delta T = 3.7218(\Delta F)^2$ (Equation 42), the above solar forcing was found to explain all of the observed global warming. This global warming in turn explained all of the observed increases in the annual atmospheric carbon dioxide concentration (Figure 3), sea level (Figure 5a) and OHC (Figure 5b). For doubling of the annual atmospheric carbon dioxide concentration from 267.5 ppm for year 1865 to 535 ppm for 2062, the corresponding increase for the secular GMT was found to be 1.4 ± 0.2 °C (Figure 3) and 0.37 ± 0.016 m for the sea level (Equation 26), assuming the observed constant secular GMT acceleration of $a_T = 7.1954 \times 10^{-5} \text{ °C/year}^2$ since steady state in 1865 continues (Figure 9c). The empirical mathematical model dS/dy in Figure 10 predicts, before the end of this decade of the 2020s, the deceleration of the 30 yr GMT trend from the observed peak value of 0.2 °C/decade for 2005 (top horizontal axis label) to less than the minimum projected value of 0.15 °C/decade in IPCC (2007, SPM, p. 12). In contrast to this deceleration prediction, the model dT/dy for the 94 yr GMT moving trends (Table 1 and Figure 1a), which smoothes out the 30 yr GMT trends, predicts acceleration from the observed global warming trend of 0.68 °C/century for the period 1911–2005 to within the range of 0.75–0.89 °C/century for the period 1931–2025 and to within the range of 0.78–0.92 °C/century for the period 1936–2030. Similarly, the
annual atmospheric CO₂ concentration model (Figure 4) predicts acceleration from the observed value of 379.8 ppm for year 2005 to within the range of 419.05–426.25 ppm for year 2025 and to within the range of 431.46–438.66 ppm for year 2030. The verifications of the above short-term predictions would support the long-term predictions and interpretations presented in this paper.

Conflict of Interest

The author declares no conflicts of interest relevant to this study.

Data Availability Statement

In this paper, the following publicly available climate data sets were used: Global mean temperature (https://www.metoffice.gov.uk/hadobs/hadcrut4/data/current/time_series/HadCRUT.4.6.0.0_annual_ns_avg.txt), Total Solar Irradiance (https://lasp.colorado.edu/lisird/data/nrl2_tsi_P1Y/), Atmospheric carbon dioxide (ftp://afpt.cmdl.noaa.gov/products/trends/co2/co2_annmean_mlo.txt), Sea level (http://www.cmar.csiro.au/sealevel/sl_data_cmar.html) and Ocean heat content (https://data.nodc.noaa.gov/woa/DATA_ANALYSIS/3M_HEAT_CONTENT/DATA/basin/pentad/pent_h22-w0-2000m.dat). The Mathematica codes used to model these data sets and draw the figures presented in this paper are given in the supporting information (2019EA001015RR-SupInfo-Codes01.pdf). Deep appreciation is expressed to those who develop and maintain the climate data sets and to the organizations (CSIRO, MetOffice, NOAA and University of Colorado) that have made them freely available.

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