MODIFICATION OF ANGULAR VELOCITY BY INHOMOGENEOUS MAGNETOROTATIONAL INSTABILITY GROWTH IN PROTOPLANETARY DISKS

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ABSTRACT

We have investigated evolution of magnetorotational instability (MRI) in protoplanetary disks that have radially nonuniform magnetic field such that stable and unstable regions coexist initially, and found that a zone in which the disk gas rotates with a super-Keplerian velocity emerges as a result of the nonuniformly growing MRI turbulence. We have carried out two-dimensional resistive magnetohydrodynamic simulations with a shearing box model. We found that if the spatially averaged magnetic Reynolds number, which is determined by widths of the stable and unstable regions in the initial conditions and values of the resistivity, is smaller than unity, the original Keplerian shear flow is transformed to the quasi-steady flow such that more flattened (rigid rotation in extreme cases) velocity profile emerges locally and the outer part of the profile tends to be super-Keplerian. Angular momentum and mass transfer due to temporally generated MRI turbulence in the initially unstable region is responsible for the transformation. In the local super-Keplerian region, migrations due to aerodynamic gas drag and tidal interaction with disk gas are reversed. The simulation setting corresponds to the regions near the outer and inner edges of a global MRI dead zone in a disk. Therefore, the outer edge of dead zone, as well as the inner edge, would be favorable sites to accumulate dust particles to form planetesimals and retain planetary embryos against Type I migration.

Key words: accretion, accretion disks – instabilities – MHD – planetary systems: formation – turbulence

1. INTRODUCTION

The ubiquity of extrasolar planets strongly suggests that planet formation is a common process associated with star formation. However, two serious barriers are recognized in planet formation theory: meter-size and Type I migration barriers. Terrestrial planets or icy cores for gas giants that are embedded in protoplanetary disks tend to lose orbital angular momentum and migrate inward through tidal interaction with the disk gas (“Type I migration”). Linear calculations (e.g., Tanaka et al. 2002) predict that the planets spiral into the host stars on timescales $\lesssim 10^5$ years for the minimum-mass solar nebula model, which is much shorter than observationally inferred disk lifetime. This is the Type I migration barrier for survival of planets with an Earth mass or more.

The meter-size barrier is the barrier for formation of planetesimals. Since rotation speed of disk gas is slightly slower than dust grains or planetesimals due to negative radial pressure gradient, which is due to the global structure of the disk gas, and the motions of meter-sized bodies (boulders) are marginally coupled to gas motion through aerodynamical gas drag, the meter-sized boulders suffer the fastest inward orbital migration and the timescale to spiral into the protostar is $\sim$ a hundred orbits (Adachi et al. 1976; Weidenschilling 1977). This timescale would be too short compared to the sticking timescale for the boulders to form planetesimals of 1–10 km radius or more, the motions of which are decoupled from the gas motion, since the meter-sized boulders are expected to only poorly stick together (Benz 2000).

One way to bypass the meter-size barrier is formation of clumps through self-gravitational instability of a dust layer, which can occur on orbital periods (Safronov 1969; Goldreich & Ward 1973). However, local Kelvin–Helmholtz instability due to difference in rotation velocities between the dust-rich layer (Keplerian) and an overlaying dust-poor layer (sub-Keplerian) would prevent dust grains from settling to the midplane to form a thin enough layer for the self-gravitational instability (Weidenschilling & Cuzzi 1993). Furthermore, if global turbulence exists in the disk, it also stirs up the dust grains from the midplane. Commonly observed strong Ha emission from young stars implies that protoplanetary disks are viscously evolving accretion disks with turbulent viscosity. One of the most likely sources for the turbulence is magnetorotational instability (MRI) in weakly ionized disk gas (Balbus & Hawley 1991).

Although turbulence inhibits formation of the thin dust layer near the midplane, dust grains could be trapped into turbulent eddies. It is expected that meter-sized boulders are concentrated in anticyclonic eddies (e.g., Barge & Sommeria 1995; Chavanis 2000; Johansen et al. 2004; Inaba & Barge 2006). In particular, Johansen et al. (2007) carried out a simulation of evolution of self-gravitating boulders in an MRI turbulent disk and found that 1000 km sized clumps are formed in eddies. One of the biggest issues in this model is the lifetime of eddies. To form clumps, the eddies must persist until boulders are accumulated into the eddies and produce highly dust-rich regions. Since rapid formation of highly dust-rich regions is required, Johansen et al. (2007) assumed meter-sized boulders that show the most favorable behavior in the disks.

Trapping of dust grains is also possible at a locally high-pressure disk radius, because a positive radial pressure gradient induces super-Keplerian gas flow in which dust grains suffer tail winds, while a normal negative gradient induces head winds (e.g., Nakagawa et al. 1986; Klahr & Lin 2005). Rice et al. (2004) demonstrated that meter-sized boulders can be accumulated in high-density spiral arms of self-gravitating disks.

The pressure maximum also exists near the inner boundary of MRI “dead” zone. Innermost disk regions are thermally ionized, so that MRI is active there. In outer regions, X-rays from host stars and cosmic rays penetrate all the way to the...
midplane of the disk, and the ionization degree may be high enough to activate MRI. On the other hand, in the intermediate regions, only surface layers are MRI-active and the regions near the midplane may be inactive (“dead”; Gammie 1996; Sano et al. 2000). Assuming steady gas accretion through a disk, disk gas surface density jumps up at the inner boundary of the dead zone according to the change in effective viscosity due to turbulence, so pressure maximum emerges there.

The pressure maximum, in other words, the outer boundary of a local super-Keplerian region also halts inward Type I migration (Tanaka et al. 2002; Masset et al. 2006). If residual and/or secondary-generated dust grains maintain the dead zone when accretion of planetary embryos proceeds, the inner boundary of the dead zone is a favorable site to retain planetary embryos. However, the inner boundary is generally located well inside 1 AU. It cannot provide building blocks for terrestrial planets in habitable zones and ice planets or retain the cores to form gas giants around solar-type stars, although it would play an important role in the architecture of short-period extrasolar planets.

Kretke & Lin (2007), however, pointed out that a local dead zone can appear near the ice line and the ice line can be a spatial barrier for dust migration due to gas drag. Because dust grains are the most efficient agents for charge recombination, ionization rates, and thickness of the active layers rapidly decreases across the ice line due to condensation of icy dust grains. Correspondingly, in a range of disk accretion rates, a local dead zone appears near the ice line and at the outer boundary of the dead zone, large amount of icy grains can be accumulated and cores are retained to form gas giants.

Brauer et al. (2008) performed simulations of dust settling and coagulation with radial migration due to gas drag and the ice line effect. They found that the dust to gas ratio increases and formation of planetesimals may be efficient near the ice line. Ida & Lin (2008a) showed that if the ionization rate is order of magnitude larger than that predicted by Kretke & Lin (2007) due to dust growth (Kretke & Lin 2007 assumed that all the dust grains have μm sizes), cores stop migration at the ice line and they efficiently form gas giants without significant reduction of Type I migration speed. Note that in order for the ice line effect to actually work, disk accretion rate and dust population must be within some ranges of parameters such that the thickness of the active layer is comparable to that of dead zone near the ice line, although the ranges are not too restricted.

Here we show through magnetohydrodynamic (MHD) simulations that the pressure maximum associated with quasi-steady local super-Keplerian rotation may be created in the MRI marginal regions, such as the outer boundary of the global dead zone as well as its inner boundary, without requirements of persistent turbulent eddies or the ice line effect. The accumulation of dust grains and retention of planetary embryos at the outer boundary of the dead zone have a great importance for formation of terrestrial and Jovian planets. MRI is controlled by the vertical component of magnetic field ($B_z$) penetrating the disks as well as by ionization degree (e.g., Sano et al. 2000). As shown in later sections, nonuniform temporal MRI turbulence that occurs in the marginally stable regions transforms disk gas flow into quasi-steady flow that has local rigid-rotation regions. This flow pattern is sustained by nonuniform pressure gradient produced by mass transfer associated with the temporal turbulence. In the outer regions of the local rigid-rotation regions, gas rotation is super-Keplerian.

Since dead zones may shrink as dust grains grow and surface areas for charge recombination decrease (Sano et al. 2000), such marginal state sweeps from outer disk regions to inner regions. Furthermore, the density fluctuations due to MRI turbulence induced by the enhanced ionization degree lead to disruptive collisions of small planetesimals and the collisions reproduce dust grains (Ida et al. 2008). The grains lower the ionization degree, so that disk gas becomes marginally stable against MRI. Then grain growth starts again. Thus, marginally MRI-stable state could be maintained in significant regions at $\lesssim$ 10 AU until disk gas is depleted to some degree (Ida et al. 2008). Oishi et al. (2007) showed the random torques due to the density fluctuations in the surface active layers affect planetesimals in the dead zone near the disk midplane. This effect may result in more continuous dust production and help in maintenance of the marginally stable state. Thus, it is expected that broad regions at $\lesssim$ 10 AU may once experience such local super-Keplerian motions and accumulate dust grains to form planetesimals during disk evolution.

In Section 2, we describe the numerical method and the initial conditions. We use a local shearing box with nonuniform $B_z$. In Section 3.1, the results with a fiducial model are shown. In Section 3.2, the dependence of the results on initial settings is presented. Summarizing the numerical results, we show in Section 3.3 that creation of the rigid-rotation and super-Keplerian regions is regulated by spatially averaged magnetic Reynolds number. Section 4 presents conclusion and discussion.

2. NUMERICAL MODEL

2.1. Shearing Box Model

We carry out local two-dimensional MHD simulations of protoplanetary disks in the shearing box model (Wisdom & Tremaine 1988; Hawley et al. 1995). The coordinates are centered at $r_0$ from a host star and rotating with Keplerian angular velocity at $r_0$ ($\Omega_0$). The radial, azimuthal, and vertical directions are $x$, $y$, and $z$, respectively. Assuming uniformity in the $y$ direction, we simulate the two-dimensional flow in the $x$–$z$ plane. From the flow in the $x$–$z$ plane, the evolution of $v_z$ is calculated. We will discuss coordinate sizes in the $x$–$z$ directions ($L_x$ and $L_z$) later. Periodic boundary conditions are applied for the $x$ and $z$ directions. In the $x$ direction, Keplerian shear motion in the $y$ direction is taken into account in the boundary condition.

2.2. Basic Equations

We adopt compressible resistive MHD equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla \left( P + \frac{\mathbf{B}^2}{8\pi} \right) + \frac{1}{4\pi \rho} (\mathbf{B} \cdot \nabla) \mathbf{B} - 2\Omega_0 \times \mathbf{v} + 3 \Omega_0^2 x \hat{x},$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [(\mathbf{v} \times \mathbf{B}) - \eta \nabla \times \mathbf{B}],$$

where $\mathbf{v}$, $P$, $\rho$, and $c_s$ are velocity in the rotating frame, pressure, density, and sound speed of the gas, respectively. The third
and fourth terms on the right-hand side of Equation (1) are the Coriolis force and tidal force (the difference between the centrifugal force and the gravitational force from the central star), in which \( \mathbf{x} \) is a unit vector in the \( x \) direction. \( \mathbf{B} \) and \( \eta \) are magnetic field and resistivity. For simplicity, we omit the vertical component of the gravity. We include ohmic dissipation in Equation (3), while we neglect ambipolar diffusion. Its effect is weaker than the ohmic dissipation in midplane of inner (\( \lesssim 100 \text{ AU} \)) disk regions (Jin 1996), although in the surface layer or disk inner edge, dissipation due to ambipolar diffusion may be important (Chiang & Murray-Clay 2007).

We scale length and time by disk scale height (\( H = c_s/\Omega_0 \)) and \( \Omega_0^{-1} \), respectively. Then the independent parameters in the equations are plasma beta (\( \beta \)) and magnetic Reynolds number (\( R_m \)) defined, respectively, by

\[
\beta = \frac{2 c_s^2}{v_A^2}, \quad \text{and} \quad R_m = \frac{v_A^2}{\eta \Omega_0},
\]

where \( v_A = B/\sqrt{4\pi \rho} \) is Alfvén velocity and \( v_Az \) is its \( z \)-component. We assume spatially uniform and time-independent \( c_s \) and \( \eta \).

We solve the resistive MHD equations using the CIP-MOCCT method (Yabe & Aoki 1991; Stone & Norman 1992a, 1992b) with grid sizes of 0.01\( H \) (for dependence on our results on resolution, see Appendix A). We usually integrate our evolution until \( \tau \approx 100 \Omega_0^{-1} \). Although most of the numerical studies on nonlinear stages of MRI have assumed ideal MHD, we consider low-ionization state with finite resistivity. Several numerical simulations (Fleming et al. 2000; Hawley et al. 1996) showed that the finite magnetic resistivity reduces growth rates of the MRI.

2.3. Initial Conditions

We assume that gas density is initially uniform (\( \rho_0 \)), so pressure is also uniform (\( P_0 \)) due to the assumption of isothermal gas (constant \( c_s \)). We also assign the initial value of plasma beta as \( \beta = 400 \) uniformly. According to the large \( \beta \), we set initial steady flow as a uniform Keplerian shear flow, \( v_r = -(3/2) x \Omega_0 \).

The remaining parameter is \( R_m \). This value determines the MRI growth rate (Jin 1996; Sano & Miyama 1999). For \( R_m \approx 1 \), short wavelength modes are stabilized by ohmic dissipation and the growth rate declines. For \( \Omega_0 \propto x^{-3/2} \), the most unstable wavelength is (Sano & Miyama 1999)

\[
\lambda_{m,n} \approx 2 \pi \frac{\eta}{v_A z} \approx \frac{2 \sqrt{2 \pi}}{3 H R_m} \approx \frac{0.44}{R_m} H.
\]

Since MRI would not occur in the disk if \( H \approx \lambda_{m,u} \), the stabilization condition is \( R_m \ll R_m^{\text{crit}} \approx 0.44 \).

We consider marginally stable state for MRI, in which stable and unstable regions coexist nonuniformly. In this study, we assume constant \( \eta \), and the nonuniformity of \( R_m \) is set by that of \( v_Az \) (equivalently, \( B_z \)). Since nonuniform MRI is an essential point for the emergence of the super-Keplerian flow, similar results are obtained in the case of nonuniform \( \eta \). We will show the results with nonuniform \( \eta \) in the next paper.

In order to represent the nonuniformity of \( B_z \) with uniform \( \beta \), we set initial \( \mathbf{B}_0 \) such that

\[
\mathbf{B}_0 = (0, B_0 \sin \theta, B_0 \cos \theta),
\]

with uniform \( B_0 \) and nonuniform \( \theta \). As shown in Figure 1, we set \( \theta = 0 \) (\( \cos \theta = 1 \)) in the middle region (unstable) with radial size \( L_m \) and \( \theta = 85 \text{ deg} \) (\( \cos \theta = 0.087 \)) in the side regions (stable) with individual radial size \( L_s \). The box size in the horizontal direction is given by \( L_x = L_m + 2L_s \). Transition zones between \( \theta = 0^\circ \) and \( 85^\circ \) are set to avoid numerical instability. We include nonzero azimuthal magnetic component \( B_z \) because the plasma beta and therefore the magnetic pressure are set to be spatially uniform to establish the initial equilibrium. The azimuthal component \( B_z \) is calculated even in these two-dimensional simulations on the \( x-z \) plane, but the assumption of axisymmetry excludes the occurrence of magnetic dynamo on the \( x-y \) plane due to reconnection of \( B_z \) and the results here do not change even if \( B_z \) is set to be zero. In our preliminary three-dimensional simulations, we find that the MRI growth due to reconnection of \( B_z \) in the side areas hardly affects the features of azimuthally averaged fluid motion. This is because the essential process of the transformation from Keplerian flow to quasi-steady nonuniform rotation flow is temporal MRI growth, and the stabilization of MRI due to established rigid rotation but not due to dynamo of magnetic field (see Section 3). The effects of three-dimensional flow will be addressed in detail in the next paper.

The above choice indeed situates both stable and unstable regions in the simulation box. Substituting Equations (5) and (8) into Equation (6), we obtain

\[
R_m = \frac{v_A^2 \cos^2 \theta}{\eta \Omega_0^2} = \frac{2 \beta^{-1} H^2 \Omega_0 \cos^2 \theta}{\eta} = 2.5 \left( \frac{\eta}{0.002 H^2 \Omega_0} \right)^{-1} \cos^2 \theta.
\]

With a fiducial value \( \eta = 0.002 H^2 \Omega_0 \), MRI is initially activated (\( R_m > R_m^{\text{crit}} \)) in the middle region with \( \theta < \theta^{\text{crit}} \approx 65 \text{ deg} \). The exact dispersion relation (Appendix B) shows that wavelengths of all the growing modes exceed the vertical box size \( L_z \) for \( \theta < \theta^{\text{crit}} \approx 79 \text{ deg} \). The vertical size \( L_z \) is set to be \( \approx 0.5 H \) that is comparable to the most unstable wavelength \( \lambda_{m,u} \) (Equation (7)).

We have carried out 79 runs in total with various \( \eta, L_m, \) and \( L_s \). The detailed simulation parameters are given in Table 1. In some cases, we adopted larger values of \( \eta \), in which \( \lambda_{m,u} \) is larger (Equation (7)). Then we adopt \( L_z \) as large as \( \lambda_{m,u} \) with \( R_m \approx 1 \). In all runs, random fluctuations are initially given in the velocity field with a maximum amplitude of \( |\delta v_z| = 0.001 c_s \).
3. RESULTS

3.1. The Fiducial Model

We first present the detailed results from a fiducial model with \( \eta = 0.002 H^2 \Omega_0 \), \( L_0 = 1.43 H \), \( L_z = 4.05 H \), and \( L_z = 0.5 H \). Compared with the other models, this model initially has the largest stable regions. If the whole region has uniform magnetic field, MRI turbulence is self-sustaining. However, evolution of turbulence is quite different in the case of nonuniform magnetic field that we consider here. MRI turbulence appears only tentatively, but velocity field is transformed from Keplerian shear flow to another quasi-steady flow.

Time evolution of the magnetic field on the \( x-z \) plane is shown in Figure 2(a). In the panel of \( t \Omega_0 = 21 \), the field lines start to be stretched only in the initially unstable region. At \( t \Omega_0 = 27 \), MRI turbulence develops and the turbulence is transported into the initially stable regions. Accordingly, the magnetic perturbations become weaker and eventually vanish before they reach the boundary of the computing box (before they fully fill the stable region).

Figure 2(b) shows evolution of \( B_z \), averaged over \( z \). Turbulent diffusion smooths out \( B_z \), however, its level does not go below the critical value for stabilization of MRI (\( \sim 0.2 B_0 \)) in the initially unstable region (the middle region) even at \( t \Omega_0 = 70 \). Nevertheless, MRI turbulence ceases after \( t \Omega_0 \gtrsim 40 \). This is because in the middle region, rigid rotation is realized as a result of the angular momentum transfer during the MRI turbulence there (Figure 2(c)) and the free-energy source for MRI (differential rotation) is lost. In the other areas, \( B_z \) is smaller than the critical value and MRI does not grow. The stabilization due to rigid rotation is discussed with linear analysis in Appendix B.

As shown in Figure 2(a), the region of the rigid rotation expands as propagation of turbulence. This local rigid rotation is self-sustaining, at least, until the remaining \( B_z \) is diffused out by ohmic dissipation on timescale \( \sim L_z^2 / \eta \sim 10^3 \Omega_0 \) (see discussion in Section 4). As long as certain level of \( B_z \) is maintained, deviation from the rigid rotation excites MRI turbulence and this transfers angular momentum to bring the velocity profile back to the rigid-rotation state.

Figure 2(d) is the vertically averaged pressure distribution. The MRI turbulence radially transports mass, associated with the angular momentum transfer. Since in the unstable region, mass transfer is efficient and gas density is proportional to pressure in our isothermal model, pressure is decreased in the unstable region while it is increased in the stable regions adjacent

### Table 1

Simulation Parameters and Results for 91 Runs

| \( \eta/\Omega_0 H^2 \) | \( L_0/H \) | \( L_z/H \) | \( R_{m,ave} \) | Result | \( \Delta v/v \) |
|------------------------|-----------|-----------|-------------|--------|-------------|
| 0.0012                 | 1.5       | 6.5, 6.0, 5.0, 4.0 | 0.10, 0.11, 0.13, 0.16 | A, B, B | 0.82, 1.2, 0.91, 0.86 |
| 1.0, 0.51              |           | 0.61, 1.1 | C, C       |        | 0.66, 0.51  |
| 2.0                    | 7.5, 6.0, 4.0 | 0.13, 0.16, 0.23 | B, B, B | 1.7, 1.1, 0.98 |
| 1.0, 0.51              |           | 0.89, 1.4 | C, D       |        | 0.68, 0.68  |
| 2.5                    | 0.51      | 1.7       | D          |        | …           |
| 0.002                  | 1.0       | 1.1, 0.54 | 0.23, 0.42 | B, B   | 0.40, 0.38  |
| 1.2                    | 3.1       | 0.10      | A          | 0.57   |             |
| 1.4                    | 4.0, 3.1, 1.1, 0.55 | 0.096, 0.13, 0.37, 0.64 | A, B, B, C | 0.73, 0.55, 0.48, 0.41 |
| 0.15, 0.10, 0.05 | 1.2, 1.4, 1.5 | D, D, D | ……       |        |             |
| 1.9                    | 6.0, 4.0, 3.0 | 0.094, 0.14, 0.19 | A, B, B | 0.97, 0.88, 0.54 |
| 2.0, 1.0, 0.35         | 0.28, 0.53, 1.1 | B, C, C | 0.87, 0.72, 0.43 |
| 2.4                    | 2.0, 0.35 | 0.38, 1.3 | B, D       | 1.2, … |             |
| 2.9                    | 10.0, 8.0, 1.0 | 0.092, 0.11, 0.81 | A, B, C | 1.2, 1.4, 0.99 |
| 3.4                    | 12.0, 8.0, 4.0, 0.24 | 0.090, 0.14, 0.28, 1.8 | A, B, B, D | 1.0, 1.2, 1.1, … |
| 3.9                    | 3.1       | 0.44      | B          | 0.77   |             |
| 0.0028                 | 1.4       | 5.1, 4.0, 2.1, 0.57 | 0.15, 0.069, 0.14, 0.45 | A, A, B, C | 0.35, 0.45, 0.28, 0.23 |
| 0.37, 0.17, 0.11 | 0.61, 0.87, 1.00 | C, C, D | 0.34, 0.14, … |        |             |
| 1.9                    | 4.0, 3.1, 2.1, 1.1 | 0.099, 0.14, 0.20, 0.38 | A, A, B, B | 0.48, 0.31, 0.49, 0.42 |
| 0.57, 0.17, 0.11 | 0.61, 1.05, 1.2 | C, C, D | 0.40, 0.25, … |        |             |
| 2.4                    | 6.1, 5.1, 4.0, 1.1 | 0.088, 0.11, 0.13, 0.49 | A, A, B, B | 0.50, 0.62, 0.66, 0.53 |
| 0.57, 0.37, 0.11 | 0.75, 0.92, 1.25 | C, C, D | 0.46, 0.28, … |        |             |
| 2.9                    | 7.1, 6.1, 5.1, 2.1 | 0.093, 0.11, 0.12, 0.47 | A, A, B, B | 0.60, 0.42, 0.50, 0.48 |
| 1.1, 0.57, 0.27, 0.17 | 0.58, 0.85, 1.13, 1.26 | C, C, D, D | 0.56, 0.37, … |        |             |
| 3.4                    | 8.1, 7.1, 6.1, 2.1 | 0.0097, 0.11, 0.13, 0.39 | A, A, B, B | 1.00, 0.79, 0.58, 0.78 |
| 1.1, 0.37, 0.27 | 0.66, 1.1, 1.3 | C, C, D | 0.48, 0.61, … |        |             |
| 3.9                    | 8.1, 2.1, 1.1 | 0.11, 0.45, 0.73 | B, B, B | 0.46, 0.43, 0.49 |
| 0.0036                 | 1.8       | 4.1       | 0.077      | A      | 0.28        |
| 0.0044                 | 1.7       | 4.1       | 0.063, 0.085 | A, A | 0.17, 0.18  |
| 2.1, 1.1, 0.43 | 0.13, 0.24, 0.50 | B, B, B | 0.20, 0.18, 0.18 |
| 0.0052                 | 2.3       | 1.3       | 0.24       | B      | 0.15        |
| 0.0060                 | 2.4       | 3.3, 2.3, 1.3, 0.32 | 0.084, 0.36, 0.21, 0.50 | A, B, B | 0.22, 0.17, 0.21, 0.20 |

Notes. \( \eta \), \( L_0 \), \( L_z \), \( R_{m,ave} \), and \( \Delta v/v \) are magnetic resistivity, radial width of unstable region, that of stable regions, initial averaged magnetic Reynolds number, and the maximum deviation from Kepler velocity, respectively. For highly turbulent cases (marked “D”), \( \Delta v/v \) is omitted. The “result” in the fifth column indicates classification of the results (Section 3.3).

Multiple values in the columns correspond to different runs. (1) fiducial model; (2) model-s11; (3) model-s055; (4) model-s005; (5) model-u34; (6) model-p0
respectively. The unstable region is initially set between the two vertical dotted lines. In Figure 3, we plot radial components with the Coriolis and tidal forces raised by a deviation from the boundaries between the stable and unstable regions equilibrates to the unstable region. The resultant pressure gradients near the boundary between the stable and unstable regions equilibrates with the Coriolis and tidal forces. Thus, the rigid rotation with pressure variation caused by the MRI turbulence is quasi-steady.

As stated in Section 1, dust grains can be trapped in outer edge of local super-Keplerian regions, because dust grains suffer tail wind and migration outward in the super-Keplerian regions (see Sections 1 and 4), while they suffer head wind in the other sub-Keplerian regions. In the result of Figure 2, the super-Keplerian region exists at $0.0 < x / H \lesssim 2.0$ in the quasi-steady state.

### 3.2. Dependence on Widths of Stable and Unstable Regions

In the limit of $L_a \to 0$ or $L_a \to \infty$, the system would have uniform turbulence and would not acquire the local rigid rotation. To derive the condition for establishment of the local rigid rotation, we perform runs with different values of $L_a$ and $L_u$.

In the fiducial case, $L_u = 1.4H$ and $L_a = 4.0H$. We first present the results of a series of runs with various values of $L_u$ and the fixed $L_a$. In model-s11, model-s055, and model-005, $L_u = 1.1H$, $0.55H$, and $0.05H$, respectively. The other parameters are the same as those in the fiducial model. Figure 4 describes the evolution of the magnetic field in model-s11. Since the stable region is narrower than the fiducial case, the magnetic perturbations that arise in the unstable region propagate to the boundary of the computational box before they are dissipated. The whole region including the initially stable region becomes temporarily turbulent. However, the perturbations that reach the boundary are weakened and do not have enough momentum to go into the unstable region again. The turbulence ceases after $r \Omega_0 \lesssim 40$ and the quasi-steady rigid rotation is obtained as the result, although the pressure contrast is smaller than that in the fiducial model (the minimum pressure is $\simeq 0.65P_0$, while it is $\simeq 0.33P_0$ in the fiducial model).

The evolution of the MRI in model-s055 is shown in Figure 5. The narrower stable region allows the magnetic perturbations to come back to the unstable region because of the periodic boundary condition. The turbulence lives longer, but it eventually vanishes by $r \Omega_0 \sim 70$. The quasi-steady velocity profile is more flattened than the Keplerian is established, although it is not as distinctive as the rigid rotation in the fiducial model.

Figure 6 shows the result of model-s005. Because of the small stable regions, turbulence expanded to the entire region does not cease and uniform turbulence is maintained. Efficient angular momentum transfer in the entire region prevents velocity field from having a quasi-steady super-Keplerian region.

So far, we have changed values of $L_u$ while $L_a$ is constant. We also performed a run (model-u34) with $L_a = 3.4H$ that is enlarged from the fiducial model. The other parameters are the same as those in the fiducial model. The results are shown in Figure 7. In the enlarged unstable region, the magnetic field is stretched enough for reconnection. After the reconnection, long-lived magnetic perturbations are sustained around the boundary between unstable and stable regions (the panel at $r \Omega_0 = 58.5$). The magnetic perturbations reach the computational boundary, because the conversion from magnetic energy to kinetic energy during the magnetic reconnection adds horizontal motion to the fluid element. However, the magnetic perturbations are not strong enough to pass through the stable region and come back to the unstable region. As a result, the evolution is similar to model-s11, except that the rigid rotation is established in the regions near the boundary between unstable and stable regions. The panels at $r \Omega_0 = 61.5$ and $r \Omega_0 = 99.0$ show that this flow pattern is also quasi-steady.

These results suggest that dissipation of magnetic perturbations during the passage through stable regions plays an essential role in creation of rigid-rotation regions. We performed additional series of runs with a larger value of the resistivity,
Figure 3. Time variation of forces exerted on fluid at $x/H = 0.7$ and $z/H = 0.25$, at which super-Keplerian flow is established. The dotted, dash-dotted, thin dashed, and dashed lines express gas radial components of pressure gradient, magnetic pressure gradient, magnetic tension, sum of gravity, and Coriolis forces on the right-hand side of the equation of motion (Equation (1)), respectively. The thick solid line is total force. The unit of the forces is $H\Omega_0^2$. The pressure gradient and the gravity/Coriolis forces are dominated and approximately equilibrate with each other.

Figure 4. Results of model-s11. (a) Time evolution of the magnetic field (solid lines) and angular velocity $v_y$ (contours) on the $x$–$z$ plane. (b) Time evolution of vertically averaged angular velocity ($v_y$). The bold, dashed, and thin solid lines express the snapshots at $t\Omega_0 = 0.0, 40.0$, and 70.0, respectively. The meanings of lines are the same as in Figure 2.

Figure 5. Same plots as Figure 4 except for model-s055.

established for these values of $\eta$. We discuss the dependence on the resistivity in the next section.

3.3. Condition for the Rigid Rotation

As we have shown, establishment of the quasi-steady rigid rotation depends on $L_u$, $L_s$, and $\eta$. Through the results with various $L_u$, $L_s$, and $\eta$ as listed in Table 1, we found that the results are summarized by a single parameter, a spatially averaged magnetic Reynolds number, defined by (Sano & Miyama 1999; Sano et al. 2004)

$$R_{m,ave} = \frac{v_{Az,ave}^2}{\eta\Omega_0},$$

where $v_{Az,ave}$ is evaluated by a spatially averaged vertical
magnetic field \((B_{z,\text{ave}})\) at the initial stage. In our simulation setting, \(B_{z,\text{ave}} \approx B_0(L_u + 2 \cos 85° L_v)/(L_u + 2L_v)\).

In many cases, the initial distributions of vertical magnetic field are spatially smoothed out by turbulent diffusion after \(\sim 10\) orbits. If the remaining magnetic field \((\sim B_{z,\text{ave}})\) is still large enough to globally cause MRI turbulence, the transition to the quasi-steady state does not occur, as seen in the results of model-s005. Linear theory shows that the critical magnetic Reynolds number for occurrence of MRI is \(R_m \approx 1\) (Sano & Miyama 1999; Sano et al. 2004). Thus, \(R_m,\text{ave}\) regulates the establishment of the quasi-steady rigid rotation in our simulations.

We summarize the results of 91 runs with different parameters in Figure 9 and found that evolution of the magnetic field is indeed classified into four types by the values of \(R_m,\text{ave}\) as follows.

1. **Type A** \((R_m,\text{ave} \lesssim 0.1)\). Local MRI turbulence generated in the initially unstable region propagates both inward and outward, but the magnetic perturbations are dissipated before they reach the boundary of the simulation box. After a few tens of orbits, the turbulence vanishes in the entire region and the quasi-steady flow is established. In the initially unstable region, rigid-rotation flow is resulted in by angular momentum transfer due to the MRI turbulence. This class is represented by filled circles in Figure 9 and it includes the fiducial model and model-\(\eta 0\).

2. **Type B** \((0.1 \lesssim R_m,\text{ave} \lesssim 0.5)\). The magnetic perturbations reach the boundary, but they do not intrude back into the original unstable region. The quasi-steady rigid-rotation region appears as in Type A. This class is represented by triangles in Figure 9 and it includes model-s11 and model-u34.

3. **Type C** \((0.5 \lesssim R_m,\text{ave} \lesssim 1.0)\). The magnetic perturbations intrude the unstable region after the passage through the stable regions. However, the diffused \(B_z\) is not large enough to globally maintain turbulence and the quasi-steady rigid-rotation region is still formed, although their locations are not necessarily the same as in Type A and Type B. This class is represented by daggers in Figure 9 and it includes model-s055.

4. **Type D** \((1.0 \lesssim R_m,\text{ave})\). Even after the turbulent diffusion, \(B_z\) is able to maintain the turbulence in the entire region. Because of the uniform turbulent state, the quasi-steady rigid-rotation state is not generated. This class is represented by crosses in Figure 9 and it includes model-s005.

4. **CONCLUSION AND DISCUSSION**

We have investigated evolution of patchy MRI due to radially nonuniform magnetic field and found that, under some conditions, the original Keplerian shear flow is transformed into quasi-steady profile involving local rigid-rotation regions. The outer parts of the rigid-rotation regions are generally super-Keplerian. Such a situation would arise in the outer boundary of MRI dead zone as well as the inner boundary in a protoplanetary disk, as discussed in Section 1.

Assuming uniformity in the azimuthal direction of disks, we have carried out two-dimensional resistive MHD simulations in a shearing box model with periodic boundary conditions. We set up both stable and unstable regions in the box, changing direction of the vertical seed magnetic field \((B_z)\) nonuniformly. In the initially unstable region, MRI turbulence is generated locally and magnetic perturbations propagate both radially inward and outward by the turbulent diffusion. If the unstable region is sufficiently large compared with the stable region, the turbulence eventually covers the entire region and the initial nonuniformity
vanishes. However, if the stable region is relatively large, diffused magnetic perturbations no more maintain MRI turbulence. After the turbulence ceases, the initial flow of uniform Keplerian shear is transformed into a different quasi-steady state. In the quasi-steady state, rigid rotation is established locally. The deviation from Keplerian shear motion is supported by pressure gradient that has been produced also by mass transport associated with the tentative turbulence. Through simulations with various initial conditions, we found that the quasi-steady rigid rotation is established if the spatially averaged magnetic Reynolds number satisfies $R_{m,ave} \lesssim 1$ in the initial state.

Because the center of the local rigid rotation is often Keplerian, super-Keplerian flow appears in the outer parts of the rigid-rotation region. As explained in Section 1, dust grains and planetary embryos can be trapped in the boundary between regions of sub- and super-Keplerian motion through radial migration induced by aerodynamic gas drag and Type I migration. The boundary is coincident with pressure maxima in the quasi-steady state.

The effect of global pressure gradient is included by shifting the initial gas velocity from the pure Keplerian speed to slight sub-Kepler. The sift, which is the velocity difference between the disk gas and the dust grains, is (e.g., Adachi et al. 1976; Weidenschilling 1977)

$$\Delta v_r = \frac{c_s^2}{2v_K^2} \frac{d \ln P}{d \ln r} v_K \simeq -5 \times 10^{-2} \left( \frac{r}{1 \text{ AU}} \right)^{3/4} c_s,$$  \hspace{1cm} (11)

where $d \ln P/d \ln r$ is the global pressure gradient and the temperature distribution in the limit of optically thin disks around solar-luminosity stars, $T = 280(r/1 \text{ AU})^{-1/2}$, is assumed. Since maximum values of $\Delta v_r$ in the super-Keplerian regions are $\gtrsim 0.4c_s$, in our results, the MRI effects can easily surpass the global pressure gradient effects and super-Keplerian regions can emerge even when the initial downshift in the gas rotation velocity is present.

If the super-Keplerian region is sustained long enough for dust grains to accumulate, planetesimals can be formed through self-gravitational instability (Youdin & Shu 2002; Johansen et al. 2006). In the case of $R_{m,ave} \lesssim 1$ in which MRI turbulence ceases in the entire region after a few tens of orbits, we found that $B_z$ is still large enough in the rigid-rotation region. In that region, MRI is suppressed by disappearance of shear motion but not by dissipation of $B_z$ (in the regions other than the rigid-rotation region, diffused-out $B_z$ is smaller than the value for MRI to occur). If the rigid rotation tries to go back to the original Keplerian shear motion, MRI turbulence again occurs and it transfers angular momentum to recover the rigid rotation. Thus, the rigid rotation and hence the associated super-Keplerian rotation are self-sustaining. When the remaining $B_z$ is diffused out by ohmic dissipation on timescale $\sim L_u^2/\eta \sim 10^3 \Omega_0^{-1}$, such stabilization mechanism is no more effective. Then the rigid rotation can go back to the original Keplerian shear motion by the residual uniform viscosity. However, since MRI no more occurs, the residual viscosity would be very small. Thus, it is expected that the super-Keplerian regions would survive long enough for accumulation of dust grains and formation of planetesimals. We also did a calculation starting from the end result of the super-Keplerian rotation state, artificially modifying $B_z$ to uniform distribution. However, we do not see any relaxation of the velocity field back to Keplerian rotation within the timescales of 40 orbits.

In the next paper, we will demonstrate the accumulation of dust grains and discuss the effect of the azimuthal magnetic field, especially in stable region by three-dimensional simulation. We will also show the results of nonuniform resistivity case with constant $B_z$, which may be more likely to occur at the outer edge of a dead zone. We find similar appearance of super-Keplerian regions, because intrinsic physics to transform initial Keplerian flow to quasi-steady nonuniform rotation flow is temporal generation of MRI, and the stabilization of the MRI due to the established rigid rotation but not due to dynamo of magnetic field (see Section 3).
The appearance of the super-Keplerian region also halts inward Type I migration of planetary embryos (Tanaka et al. 2002; Masset et al. 2006). Since the process we found also works at the outer boundary of a dead zone and the outer boundary migrates from ≈10 AU to the proximity of the central star, this process may also help the formation of cores massive enough to onset runaway gas accretion and retain terrestrial planets against Type I migration. This may play an important role in frequency convergence with increasing resolution. This could be because a future study, one may reasonably expect the possibility of dust concentration at the outer edge of the super-Keplerian area. Fromang & Papaloizou (2007) included only the numerical resistivity and ohmic dissipation (resistivity) is kept constant in our resistive MHD simulations, while the simulation of Fromang & Papaloizou (2007) showed the decrease of Maxwell stress with increasing resolution and stated that the MRI turbulence activity is notoriously ill behaved in high-resolution calculation. Our resolution test, however, shows the convergence with increasing resolution. This could be because of the ohmic dissipation (resistivity) is kept constant in our resistive MHD simulations, while the simulation of Fromang & Papaloizou (2007) only included the numerical resistivity and it decreases with increasing resolution. However, more detailed study is needed to clarify the difference in convergence between our simulation and the simulation of Fromang & Papaloizou (2007), which is left to our future study.

This fact will be important when we investigate the motion of dust particles. The degree of particle concentration may be dependent on the turbulence activity. While the details are left for a future study, one may reasonably expect the possibility of dust concentration at the outer edge of the super-Keplerian area.

Figure 10(b) also shows the eligibility of the integration time. The saturation level varies only slightly in tΩ0 > 60.0 in the two high-resolution cases. The quasi-steady state has already been created by this time. These indicate that our choice of the integration time tΩ < 100.0 is validated.

APPENDIX B

DISPERSION RELATIONS

Our simulations show the MRI stabilization in strong magnetic field with nearly rigid rotation, which is consistent with the linear analyses. The linear analysis using ideal MHD equations in Balbus & Hawley (1991) gave the critical wavelength (Equation (2.14b) in their paper, neglecting the Brunt–Väisälä frequency),

\[
|\lambda_{z,\text{crit}}| = \frac{v_{\lambda c}}{\Omega} \left| \frac{d\Omega}{d \ln r} \right|^{-1/2} = \frac{v_{\lambda c}}{\Omega} \frac{1}{2 \langle q \rangle}^{-1/2}, \tag{B1}
\]

where \( q \) is defined as \( \Omega(r) \propto r^{-q} \). The perturbations with wavelength shorter than \( \lambda_{z,\text{crit}} \) are stable. When the rotation becomes rigid rotation (\( q \to 0 \)), \( \lambda_{z,\text{crit}} \) becomes large. If \( \lambda_{z,\text{crit}} \) is larger than the scale height of a disk, the system is stable, irrespective of magnetic field strength.

With the effect of ohmic dissipation, the dispersion relation was obtained by Jin (1996) and Sano & Miyama (1999) as

\[
s^4 + 2\xi \sigma^3 + \left( 2q^2 + \xi^2 + \frac{\kappa^2}{\Omega^2} \right) \sigma^2 + 2\xi \left( q^2 + \frac{\kappa^2}{\Omega^2} \right) \sigma
+ \left( -4 + \frac{\kappa^2}{\Omega^2} \right) q^2 + q^4 + \frac{\kappa^2}{\Omega^2} \xi^2 = 0, \tag{B2}
\]

where \( q \) is a growth rate in units of orbital frequency, \( q = k_z v_{\lambda c}/\Omega, \xi = k_z^2 \eta/\Omega, \) and \( \kappa \) is an epicyclic frequency defined by

\[
\kappa^2 = \frac{2 \Omega}{r} \frac{d(r^2 \Omega)}{dr} = (2 - q) 2 \Omega^2. \tag{B3}
\]

This dispersion relation is derived with the assumption of uniform density and negligible Brunt–Väisälä frequency. Since the density is almost uniform in the rigid rotation in our simulation, we apply this dispersion relation to calculate the predicted \( \sigma \) with the quantities obtained by our simulation. We plot the temporally and vertically averaged \( q = -d\dot{v}_z/dx \), Maxwell stress \((-\langle B_z B_z \rangle/4\pi \rho_0)\), and the evaluated growth rate \( \sigma \) in Figure 11. The growth rate is very small in the middle region of nearly rigid rotation (\( q \ll 1 \)), although Maxwell stress is not small there. Thus we conclude that MRI is suppressed by established nearly rigid rotation, but not by dissipation of magnetic field.
Figure 11. Temporally and vertically averaged shear rate $q = -dv_x/dt$ (top), Maxwell stress normalized by initial pressure $-\langle B_x B_y \rangle/4\pi P_0$ (middle), and the growth rate estimated by the dispersion relation (Equation (B2) in Appendix B) with the simulated values (bottom) of the fiducial model. Bold lines are values temporally averaged over $t\Omega_0 = 45.0$–70.0 and dashed lines mean the initial values.

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