Spin and transverse momentum dependent Fracture Function in SIDIS

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Summary. — The recently developed leading twist formalism for spin and transverse-momentum dependent fracture functions is shortly described. We demonstrate that the process of double hadron production in polarized SIDIS – with one spinless hadron produced in the current fragmentation region (CFR) and another in the target fragmentation region (TFR) – would provide access to all 16 leading twist fracture functions. Some particular cases are presented.

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1. – Introduction

So far most SIDIS experiments were studied in the CFR, where an adequate theoretical formalism based on distribution and fragmentation functions has been established (see for example Ref. [1]). However, for a full understanding of the hadronization process after the hard lepton-quark scattering, also the factorized approach to SIDIS description in the TFR has to be explored. The corresponding theoretical basis – the fracture functions formalism – was established in Ref. [2] for hadron transverse momentum integrated unpolarized cross-section. Recently this approach was generalized [3] to the spin and transverse momentum dependent case (STMD).

We use the standard DIS notations and in the $\gamma^* - N$ c.m. frame we define the z-axis along the direction of $q$ (the virtual photon momentum) and the x-axis along $\ell_T$, the lepton transverse momentum. The kinematics of the produced hadron in the TFR is defined by the variable $\zeta = P_h^-/P^+ \simeq E_h/E$ and its transverse momentum $P_{h\perp}$ (with magnitude $P_{h\perp}$ and azimuthal angle $\phi_h$). The azimuthal angle of the nucleon transverse polarization is denoted as $\phi_S$. 
The STMD fracture functions $\mathcal{M}$ has a clear probabilistic meaning: it is the conditional probability to produce a hadron $h$ in the TFR when the hard scattering occurs on a quark $q$ from the target nucleon $N$.

The most general expression of the LO STMD fracture functions for unpolarized ($\mathcal{M}^{[\gamma^{-}]})$, longitudinally polarized ($\mathcal{M}^{[\gamma^{-};\gamma^+]}$) and transversely polarized ($\mathcal{M}^{[\pi^{-};\gamma^+]}$) quarks are introduced in the expansion of the leading twist projections as [3, 4]:

$$
\mathcal{M}^{[\gamma^{-}]} = \hat{u}_1 + \frac{P_{h\perp} \times S_{\perp}}{m_h} \hat{u}^h_{1T} + \frac{k_{\perp} \times S_{\perp}}{m_N} \hat{u}^h_{1T} + \frac{S_{\parallel} (k_{\perp} \times P_{h\perp})}{m_N m_h} \hat{u}^h_{1L}
$$

$$
\mathcal{M}^{[\gamma^{-};\gamma^+]} = S_{\parallel} \hat{t}_{1L} + \frac{P_{h\perp} \cdot S_{\perp}}{m_h} \hat{t}_{1h} + \frac{k_{\perp} \cdot S_{\perp}}{m_N} \hat{t}_{1h} + \frac{k_{\perp} \times P_{h\perp}}{m_N m_h} \hat{t}_{1h}
$$

$$
\mathcal{M}^{[\pi^{-};\gamma^+]} = S_{\parallel} \hat{t}_{1L} + \frac{P_{h\perp} \cdot S_{\perp}}{m_h} \hat{t}_{1h} + \frac{S_{\parallel} k_{\perp}^1}{m_N} \hat{t}_{1L} + \frac{(P_{h\perp} \cdot S_{\perp}) P_{h\perp}^i}{m_h} \hat{t}_{1h} + \frac{(k_{\perp} \cdot S_{\perp}) P_{h\perp}^i - (P_{h\perp} \cdot S_{\perp}) k_{\perp}^i}{m_N m_h} \hat{t}_{1h} + \frac{\epsilon^{ij}_P h_{ij}}{m_h} \hat{t}_{1h} + \frac{\epsilon^{ij}_P h_{ij}}{m_N} \hat{t}_{1h},
$$

where $k_{\perp}$ is the quark transverse momentum and by the vector product of two-dimensional vectors $a$ and $b$ we mean the pseudo-scalar quantity $a \times b = \epsilon^{ij} a_i b_j = ab \sin(\phi_b - \phi_a)$. All fracture functions depend on the scalar variables $x_B, k_{\perp}^2, \zeta, P_{h\perp}^2$ and $k_{\perp} \cdot P_{h\perp}$.

The single hadron production in the TFR of SIDIS does not provide access to all fracture functions.

2. – Double hadron leptoproduction (DSIDIS)

In order to have access to all fracture functions one has to "measure" the scattered quark transverse polarization, for example exploiting the Collins effect [5] – the azimuthal correlation of the fragmenting quark transverse polarization, $s'_T$, with the produced hadron transverse momentum, $p_{\perp}$:

$$
D(z, p_{\perp}) = D_{1}(z, p_{\perp}^2) + \frac{p_{\perp} \times s'_T}{m_h} H_{1}^\perp(z, p_{\perp}^2),
$$

where $s'_T = D_{nn}(y) s_T$ and $\phi_{s'} = \pi - \phi_s$ with $D_{nn}(y) = [2(1-y)]/[1 + (1-y)^2]$.

Let us consider a double hadron production process (DSIDIS)

$$
l(\ell) + N(P) \rightarrow l(\ell') + h_1(P_1) + h_2(P_2) + X
$$

with (unpolarized) hadron 1 produced in the CFR ($x_{F1} > 0$) and hadron 2 in the TFR ($x_{F2} < 0$), see Fig. 1. For hadron $h_1$ we will use the ordinary scaled variable $z_1 = P_{1T}/k^+ \simeq P_1/P_q$ and its transverse momentum $P_{1\perp}$ (with magnitude $P_{1\perp}$ and azimuthal angle $\phi_1$) and for hadron $h_2$ the variables $\zeta_2 = P_2^- / P^- \simeq E_2/E$ and $P_{2\perp}$ ($P_{2\perp}$ and $\phi_2$).

The LO expression for the DSIDIS cross-section includes all fracture functions:

$$
\frac{d^2 \sigma}{d^2 \ell d^2 \zeta_2 d^2 p_{1\perp} d^2 p_{2\perp} d\phi_2} = \frac{Q^4 y}{x_B} \frac{Q^4 y}{x_B} [1 + (1-y)^2] \times
$$
Fig. 1. – DSIDIS description in factorized approach at LO.

\[
\left( M_{h_2}^{[\gamma]} \otimes D_{1q}^{h_1} + \lambda D_{U}(y) M_{h_2}^{[\gamma \gamma_5]} \otimes D_{q}^{h_1} + M_{h_2}^{[\sigma^{-\gamma_5}]} \otimes \frac{P_{1} \times \sigma_{T}}{m_{h_1}} H_{1q}^{h_1} \right) = \frac{\alpha^2 x_B}{Q^2 y} \left[ 1 + (1 - y)^2 \right] \left( \sigma_{UU} + S_{\|} \sigma_{UL} + S_{\perp} \sigma_{UT} + \lambda D_{U} \sigma_{LU} + \lambda S_{\|} D_{U} \sigma_{LL} + \lambda S_{\perp} D_{U} \sigma_{LT} \right),
\]

where \( D_{U}(y) = y(2 - y)/(1 + (1 - y)^2) \).

3. – Examples of unintegrated cross-sections: beam spin asymmetry

We show here explicit expressions only for \( \sigma_{UU} \) and \( \sigma_{LU} \) \(^{(1)}\)

\[
\sigma_{UU} = F_{0}^{\Delta_{1}} - D_{n} \left[ \frac{P_{2}^2}{m_{1} m_{N}} F_{kp1}^{\perp \cdot H_{1}^{+}} \cos(2\phi_{1}) + \frac{P_{1} \cdot P_{2}^{h}}{m_{1} m_{2}} F_{p_1}^{h} \cdot H_{1}^{+} \cos(\phi_{1} + \phi_{2}) \right.
\]

\[
\left. + \left( \frac{P_{2}^2}{m_{1} m_{N}} F_{kp2}^{\perp \cdot H_{1}^{+}} + \frac{P_{2}^2}{m_{1} m_{2}} F_{p_2}^{\perp \cdot H_{1}^{+}} \right) \cos(2\phi_{2}) \right].
\]

\[
\sigma_{LU} = -\frac{P_{1} \cdot P_{2}}{m_{2} m_{N}} F_{k1}^{h} \cdot D_{1} \sin(\phi_{1} - \phi_{2}),
\]

where the structure functions \( F_{\cdots} \) are specific convolutions \([6, 7]\) of fracture and fragmentation functions depending on \( x, z_{1}, \zeta_{2}, P_{1}, P_{2}^{\perp}, P_{1}, P_{2} \).

We notice the presence of terms similar to the Boer-Mulders term appearing in the usual CFR of SIDIS. What is new in DSIDIS is the LO beam spin SSA, absent in the CFR of SIDIS. We further notice that the DSIDIS structure functions may depend in principle on the relative azimuthal angle of the two hadrons, due to presence of the last term among their arguments: \( P_{1} \cdot P_{2} = P_{1} P_{2} \cos(\Delta \phi) \) with \( \Delta \phi = \phi_{1} - \phi_{2} \). This term arise from \( k_{\perp} \cdot P_{\perp} \) correlations in STMD fracture functions and can generate a long range correlation between hadrons produced in CFR and TFR. In practice it is convenient to chose as independent azimuthal angles \( \Delta \phi \) and \( \phi_{2} \).

Let us finally consider the beam spin asymmetry defined as

\[
A_{LU}(x, z_{1}, \zeta_{2}, P_{1}, P_{2}^{\perp}, \Delta \phi) = \frac{\int d\phi_{2} \sigma_{LU}}{\int d\phi_{2} \sigma_{UU}} = \frac{P_{1} \cdot P_{2}}{m_{2} m_{N}} F_{k1}^{h} \cdot D_{1} \sin(\Delta \phi).
\]

\(^{(1)}\) Expressions for other terms are available in \([6]\).
If one keeps only the linear terms of the corresponding fracture function expansion in series of $P_1 \cdot P_2$ one obtains the following azimuthal dependence of DSIDIS beam spin asymmetry:

\[ A_{LU}(x, z_1, z_2, P^2_1, P^2_2) = a_1 \sin(\Delta \phi) + a_2 \sin(2\Delta \phi) \]

with the amplitudes $a_1, a_2$ independent of azimuthal angles.

In Fig. 2 we present the first preliminary results [8] for $A_{LU}$ asymmetry from CLAS experiment at JLab with $\pi^+$ produced in CFR and $\pi^-$ in TFR. The nonzero effect were observed!

![Fig. 2. – The preliminary results for $A_{LU}$ asymmetry from CLAS experiment at JLab.](image)

We stress that the ideal opportunities to test the predictions of the present approach to DSIDIS, would be the future JLab 12 upgrade, in progress, and the EIC facilities, in the planning phase.

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