Geometric Suppression of Single-Particle Energy Spacings in Quantum Antidots

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Abstract
Quantum Antidot (AD) structures have remarkable properties in the integer quantum Hall regime, exhibiting Coulomb-blockade charging and the Kondo effect despite their open geometry. In some regimes a simple single-particle (SP) model suffices to describe experimental observations while in others interaction effects are clearly important, although exactly how and why interactions emerge is unclear. We present a combination of experimental data and the results of new calculations concerning SP orbital states which show how the observed suppression of the energy spacing between states can be explained through a full consideration of the AD potential, without requiring any effects due to electron interactions such as the formation of compressible regions composed of multiple states, which may occur at higher magnetic fields. A full understanding of the regimes in which these effects occur is important for the design of devices to coherently manipulate electrons in edge states using AD resonances.

Key words: Antidot, Edge-states, Aharonov-Bohm
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Several exciting developments in the field of mesoscopic physics have recently emerged from the study of coherent electronic devices [1]. By utilising coherent edge states in quantum Hall systems, devices can be constructed which coherently manipulate electron spin, charge, and phase, offering important prospects for the study of quantum information in the solid state. Quantum antidots in particular offer spin-selective coherent control of these states, making them potentially important in future applications [2]. An antidot (AD) is simply a potential hill in a two-dimensional electron system, which in a perpendicular magnetic field (≥ 0.3 T) gains a set of quasibound states quantised by the Aharonov-Bohm (AB) effect. When placed at the centre of a constriction, these states couple to extended edge channels, and their structure may then be investigated through the scattering processes which determine the conductance. In the simplest model, these noninteracting single-particle (SP) states adjust with changing magnetic field in order to maintain their enclosed flux, producing a set of magnetococonductance oscillations as they pass through the Fermi energy (EF) in the leads. Many details of AD behaviour may be understood within this SP picture, particularly at low fields.
(≤ 3 T) [3]. Other effects, such as Coulomb Blockade [4] and the Kondo Effect [5] require a self-consistent model including electron interactions [6]. There has been some debate over the precise nature of these effects [7,8,9], particularly concerning the presence or absence of compressible regions (CRs) composed of multiple partially-occupied SP states within a few \( k_B T \) of \( E_F \). Certainly, in the limit of large AD size and high field, CRs are expected to form in order to minimise the Coulomb energy associated with abrupt changes in electron density [10], but these may be suppressed in other regimes by the AB quantisation and exchange effects. Recent numerical calculations accounting for both Coulomb and spin-dependent interactions confirm this expectation [11]; for an AD of radius 200 nm in the integer quantum Hall regime with filling factor \( \nu = 2 \), CRs are significantly quenched for fields below ≲ 4 T, and then a CR only forms for the outer spin state.

We present a series of measurements designed to explore the evolution of SP states with magnetic field. Our device[2] consists of an AD gate (200 nm in diameter) centered in a 1-µm channel, contacted by a metal gate isolated from the split gate by an insulating layer of crosslinked polymer, allowing us to control the voltage on the AD independently. As a function of \( B \) and source-drain bias, the conductance forms a complicated pattern of resonances not unlike that observed in quantum dots. From these data, we can extract the various transition energies of the system — the Coulomb energy \( E_C \), the Zeeman spin-splitting \( E_Z \), and the SP energy \( \Delta E_{\text{SP}} \). As was noted in [12], at higher fields we observe a clear suppression of \( \Delta E_{\text{SP}} \) below the expected \( 1/B \) dependence for circular AD states, and it was then suggested that this could reflect the formation of CRs. We have investigated this further, however, and find that this suppression occurs systematically on the high-\( B \) end of the \( \nu = 2 \) plateau, coinciding with the transition of AB resonances from transmission peaks to reflection valleys. We have also seen this effect at relatively low fields ≲ 2 T (see Fig. 1) where CRs are less likely to form. We therefore propose an alternative explanation for this effect, namely that the presence of the split-gate spreads out the AB states in the constrictions, leading to a suppression of \( \Delta E_{\text{SP}} \) for the lowest SP states in a Landau level.

\[ \Delta E_{\text{SP}} = -\frac{h}{eB} \int_0^{2\pi} \left( \frac{dU}{dr} \right)_{(\zeta, \theta)}^{-1} \mathcal{C}(\theta) d\theta^{-1}, \]

where \( \mathcal{C}(\theta) \) is the contour at the Fermi energy defined by \( U_{\text{eff}}(\mathcal{C}, \theta) = E_F \), using the effective potential \( U(r, \theta) + E_{\text{cyc}} + E_Z \), where \( E_{\text{cyc}} \) and \( E_Z \) are the cyclotron and Zeeman energies for states in the constrictions. For a circularly symmetric potential this gives the \( 1/B \) dependence mentioned above [12], but it is clear

\[ \text{Fig. 1. Top panels: Single-particle energy } \Delta E_{\text{SP}} \text{ (circles)} \text{ extracted from DC-bias measurements in different ranges of magnetic field (with different gate voltages). A } 1/B \text{ fit (dashed curve) fails to match the data while our model (solid curve) predicts the reduction of } \Delta E_{\text{SP}} \text{ at higher fields. Bottom panels: Conductance (c) in units of } 2e^2/h \text{ as a function of } B \text{ and } E_F, \text{ calculated from the full non-interacting Green’s function computed using an iterative procedure [13], and the corresponding energy spacing (d), calculated at } E_F \text{ = 11.7 mV. As in the experimental data, our model (solid curve) accounts for the discrepancy from the } 1/B \text{ dependence (dashed curve).} \]
that a reduction in the slope of the potential over any region of the contour results in a suppressed $\Delta E_{\text{SP}}$. It is worth noting that the magnetic flux does not solely determine the location of AD states, since the nonuniform AD potential contributes an additional phase factor to the wavefunctions beyond the magnetic AB phase.\footnote{This also means that AD states do not enclose integer multiples of the flux quantum $h/e$ as commonly assumed, but rather are pushed outwards by the AD potential.} but for a potential that varies sufficiently slowly, the dominant contribution to the difference between adjacent states arises from the change in magnetic flux, and so a model considering only differences in area is appropriate.

Using the bare electrostatic potential\footnote[14]{resulting from the gates on our device, we can use Eqn. 1 to calculate $\Delta E_{\text{SP}}$ as a function of $B$ to compare with the data. In Fig. 1 (a and b) we show the results of this calculation for two data sets with different experimental parameters, and a comparative best-fit curve $\propto 1/B$. The model itself has no free parameters; the potential} resulting from the gates on our device, we can use Eqn. 1 to calculate $\Delta E_{\text{SP}}$ as a function of $B$ to compare with the data. In Fig. 1 (a and b) we show the results of this calculation for two data sets with different experimental parameters, and a comparative best-fit curve $\propto 1/B$. The model itself has no free parameters; the potential is completely determined by the arrangement of gates, the measured AB period (related to the AD area by $A\Delta B = h/e$), and $\Delta E_{\text{SP}}$ at low field, and we can estimate $E_F$ from the field at which the $\nu = 2$ state is depopulated in the channel. We have included no effects of tunnelling between SP states and the leads in this calculation, which results in an artificial drop to zero as the saddle point reaches $E_F$ and closed orbits no longer exist. For additional comparison with this essentially classical model, we have calculated the full (noninteracting) Green’s function for an AD + split-gate geometry using an iterative procedure\footnote[13]{This also means that AD states do not enclose integer multiples of the flux quantum $h/e$ as commonly assumed, but rather are pushed outwards by the AD potential.}. Fig. 2 shows the calculated local density of states for an AD device at a reflection resonance, and Fig. 1c shows the calculated conductance as a function of $B$ and $E_F$. From the conductance we can calculate $\Delta E_{\text{SP}}$, as shown in Fig. 1d, and we find a nearly identical suppression at higher fields which is well matched by our geometric model. Since this calculation includes no electron interaction effects, we can be sure that CRs are not required to produce this behaviour.

We therefore conclude that the observed suppression of $\Delta E_{\text{SP}}$ is a simple result of the potential profile in our experimental geometry, rather than a signature of a reorganisation of states into a CR. Although we know that interaction effects become essential for an understanding of AD resonances at high fields, this study demonstrates the capacity of the SP model to explain relatively complicated features of the excitation spectrum of ADs in the low-field regime. An understanding of this structure is critical in the design of AD-based applications which seek to utilise processes in a specific regime. By choosing AD sizes and fields appropriately, it may be possible to construct devices which use ADs in different regimes (even on a single chip) to manipulate spin or charge for different purposes. These avenues of research remain largely unexplored, and may hold many exciting advances in the study of quantum-coherent electronics.

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