The $\Lambda(1405)$ resonance as a genuine three-quark or molecular state

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Abstract. The mechanism for the formation of the $\Lambda(1405)$ resonance is studied in a chiral quark model that includes quark-meson as well as contact (four point) interactions. The negative-parity $S$-wave scattering amplitudes for strangeness $-1$ and $1$ are calculated within a unified coupled-channel framework that includes the $KN$, $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, $K\Xi$, $\pi\Lambda$, and $\eta\Sigma$ channels and possible genuine three-quark bare singlet and octet states corresponding to $\frac{3}{2}^-$ resonances. It is found that in order to reproduce the scattering amplitudes in the $S_{01}$ partial wave it is important to include the pertinent three-quark octet states as well as the singlet state, while the inclusion of the contact term is not mandatory. The Laurent-Pietarinen expansion is used to determine the $S$-matrix poles. Following their evolution as a function of increasing interaction strength, the mass of the singlet state is strongly reduced due to the attractive self-energy in the $\pi\Sigma$ and $\bar{K}N$ channels; when it drops below the $KN$ threshold, the state acquires a dominant $KN$ component which can be identified with a molecular state. The attraction between the kaon and the nucleon is generated through the $KN\Lambda^*$ interaction rather than by meson-nucleon forces.

1 Introduction

The lowest state with negative parity in the strange sector, the $\Lambda(1405)$ resonance, has been receiving particular attention since its discovery six decades ago \cite{1}. The intriguing property of this resonance is that it lies some $100$ MeV below the corresponding negative parity state in the nonstrange sector, the $N(1535)$, which cannot be explained in the ordinary quark model involving only three quarks.

The quark model calculations in the $S_{01}$ partial wave, nonrelativistic or relativistic \cite{2,3,4,5}, assuming that the three lowest negative parity and strangeness $-1$ resonances correspond to the three-quark states in which one quark is excited to the $p$ orbit, indeed predict the mass of the lowest state to be around $1500$ MeV, while the masses of the other two states turn out to be consistent with the masses of the non-strange negative parity resonances. To resolve the discrepancy between the quark model prediction of the $\Lambda(1405)$ mass and the observed one, Arima et al. \cite{6} included explicit $\bar{K}N$ and $\pi\Sigma$ configurations and showed that the significant downward shift of the $\Lambda(1405)$ mass can be attributed to the large attractive self-energy due to these additional degrees of freedom.

The idea that the $\Lambda(1405)$ is predominantly a $\bar{K}N$ bound system without explicit quark degrees of freedom has been pursued by several groups starting with the work of Dalitz et al. \cite{31} based on a vector exchange model. Most of the calculations were performed in the coupled-channel framework in a chiral unitary approach \cite{10,11,12}. This approach leads to the two-pole picture of the $\Lambda(1405)$ resonance \cite{13,14,15,16,17,18,19,20,21,22} with a narrow pole just below the $KN$ threshold, and a wider pole at the mass either around $1380$ MeV, or close to the $\pi\Sigma$ threshold. According to Myint et al. \cite{23} only the first pole produces a peak in the observed spectrum while the second pole affects only the shape of the detectable spectrum.

A model that is able to incorporate both the genuine (bare) three-quark states as well as the baryon-mesons pairs in a consistent approach is the Cloudy Bag Model (CBM). In the $SU(3)$ extended version of the CBM \cite{24} with a bare three-quark singlet state representing the $\Lambda(1405)$ and the baryon-meson configurations corresponding to $\bar{K}N$ and $\pi\Sigma$ channels, the authors were able to obtain a resonance below the $KN$ threshold and dominated by a $\bar{K}N$ molecular state. The $KN$ system was studied in the same model in Ref. \cite{25}. Lattice calculations \cite{26,27}, interpreted in the framework of Hamiltonian effective theory \cite{32}, reveal a dominant $\bar{K}N$ component in the light quark-mass regime. The Graz group \cite{33} confirmed a non-negligible singlet three-quark component with an admixture of the octet states at a level of 15 % – 20 %. The calculation of Gubler et
al. [33] confirms the dominant singlet component for the lowest lattice $\frac{1}{2}^-$ state, and identifies the second lattice state with the $\Lambda(1670)$. Their calculation is analyzed using hadronic effective theory in a finite volume in Ref. [34]; for pion masses above 290 MeV the lowest lattice state is identified with only one of the two states predicted by the chiral unitary approach.

Partial wave analyses for $K^-p$ scattering have been performed by the Kent group [35,36], Kamano et al. [37] and the Bonn-Gatchina group [38,39]. Kamano et al. [40] predicted two poles below the $KN$ threshold, while the Bonn-Gatchina group [41] found only one physically convincing pole near the $KN$ threshold belonging to the $0^-$ leading Regge trajectory and therefore is most likely dominated by the ordinary three-quark configuration. Similarly Klempt et al. [42] conclude that one of the two states found as poles below the $KN$ threshold has to be assigned to the predicted quark-model state.

In order to investigate the dominant mechanism responsible for the resonance formation in the $S_{01}$ partial wave, we devise a model that incorporates dynamical generation as well as generation through a three-quark resonance. In a similar approach we have been able to show, for instance, that the Roper resonance evolves from a genuine three-quark state [41], while its $I = J = \frac{3}{2}$ partner, the $\Delta(1600)$, emerges as a purely dynamically generated resonance [43].

We use a coupled-channel formalism incorporating quasi-bound quark-model states to calculate the meson-production amplitudes in the $S_{01}$ and $S_{11}$ partial waves. The meson-baryon vertices and the contact (four point) interaction are determined in a chiral quark model; in the present approach we use an $SU(3)$ extended version of the CBM [29]. The method of including bare three-quark states in the coupled-channel formalism has been described in detail in our previous papers [44,45,46,47,48,49,50], where we have analyzed scattering and electro-production amplitudes in different partial waves in the non-strange sector. In the present work we use an $SU(3)$ extended version of the CBM [29].

In our approach the scattering state in channel $\alpha$ which includes quasi-bound quark states $\Phi_{\alpha}$, $i = 1, \ldots, N_{r}$, assumes the form

$$|\Psi_{\alpha}\rangle = N_{\alpha} \left\{ a_{\alpha}^{\dagger}(k_{\alpha})|\Phi_{\alpha}\rangle + \sum_{i=1}^{N_{r}} c_{\alpha i} |\Phi_{i}\rangle \right\} + \sum_{\beta} \int \frac{dk \chi_{\alpha \beta}(k_{\alpha}, k)}{\omega(k_{\beta}) - E_{\beta}(k_{\beta}) - W} \left[ a_{\beta}^{\dagger}(k_{\beta})|\Phi_{\beta}\rangle \right].$$

where $\alpha (\beta)$ denote the channels and \[ \] stands for coupling to the appropriate total spin and isospin. The first term represents the bare meson and the baryon and defines the channel, the second term corresponds to the sum over $N_{r}$ bare three-quark resonant states, while the third term describes the meson cloud around the baryon in channel $\beta$. All quantities are written in the center-of-mass frame: $\omega(k_{\alpha})$ and $E_{\alpha}(k_{\alpha})$ are, respectively, the meson and the baryon off-shell energies in channel $\alpha$, the on-shell values are denoted as $k_{\alpha}$, $\omega_{\alpha} = \omega_{\alpha}(k_{\alpha})$ and $E_{\alpha} = E_{\alpha}(k_{\alpha})$, $W = \omega_{\alpha} + E_{\alpha}$ is the invariant energy, and $N_{\alpha} = \sqrt{\omega_{\alpha} E_{\alpha}/(\omega_{\alpha} W)}$ is the normalization factor. The integral is assumed in the principal value sense. The (half-on-shell) $K$ matrix is related to the scattering state by [40]

$$K_{\alpha \beta}(k_{\alpha}, k_{\beta}) = -\pi N_{\beta} |\Psi_{\beta}(|W_{\beta}(k_{\beta})|)\rangle,$$

with the property $K_{\alpha \beta}(k_{\alpha}, k_{\beta}) = K_{\beta \alpha}(k_{\beta}, k_{\alpha})$. The meson-baryon interaction $V$ in channel $\beta$ is explicitly written out in Appendix A. The $K$ matrix is proportional to the meson amplitude $\chi$ in Eq. (2),

$$K_{\alpha \beta}(k_{\alpha}, k_{\beta}) = \pi N_{\alpha} N_{\beta} \chi_{\alpha \beta}(k_{\alpha}, k_{\beta}).$$

The principal-value states $\{ |\Phi_{\alpha}(W)| \}$ are normalized as

$$\langle \Psi_{\alpha}(W) | \phi_{\beta}(W') \rangle = \delta(W - W') \left[ \delta_{\alpha \beta} + K^{2} \right].$$

They are not orthonormal; the orthonormalized states are constructed by inverting the norm. The amplitude $\chi$ satisfies a Lippmann-Schwinger type of equation:

$$\chi_{\alpha \gamma}(k, k_{\gamma}) = -\sum_{i} c_{\alpha i} V_{\alpha i}(k) + K_{\alpha \gamma}(k, k_{\gamma})$$

$$+ \sum_{\beta} \int dk' K_{\alpha \beta}(k, k') \chi_{\beta \gamma}(k', k_{\gamma}) \omega_{\beta}(k') + E_{\beta}(k') - W.$$

In our previous calculations we have included only one or two quasi-bound quark state; in the present work we allow for more quark states.
The explicit expression for the kernel $K_{\alpha\beta}$ will be discussed in the next subsection.

The meson amplitude can be written in terms of the resonant and nonresonant parts,

$$\chi_{\alpha\gamma}(k, k_\gamma) = \sum_i c_{\gamma i} V_{\alpha i}(k) + D_{\alpha\gamma}(k, k_\gamma),$$

such that (5) can be split into $N_r$ equations for the dressed vertices,

$$V_{\alpha i}(k) = V_{\alpha i}(k) + \sum_{\beta} \int dk' \frac{K_{\alpha\beta}(k, k')V_{\beta i}(k')}{\omega_{\beta}(k') + E_{\beta}(k') - W} ,$$

and an equation for the nonresonant amplitude,

$$D_{\alpha\gamma}(k, k_\gamma) = K_{\alpha\gamma}(k, k_\gamma) + \sum_{\beta} \int dk' \frac{K_{\alpha\beta}(k, k')D_{\beta\gamma}(k', k_\gamma)}{\omega_{\beta}(k') + E_{\beta}(k') - W} .$$

By requiring stationarity, $\langle \delta \Psi_{\alpha} | H - W | \Psi_{\alpha} \rangle = 0$, with respect to variation of the coefficients $c_{\alpha i}$ we get a system of equations:

$$\sum_j A_{ij} c_{\alpha j} = V_{\alpha i}$$

where

$$A_{ij} = (W - m_i^0) \delta_{ij} + \sum_{\beta} \int dk \frac{V_{\beta j}(k)V_{\beta i}(k)}{\omega_{\beta}(k) + E_{\beta}(k) - W} .$$

The matrix corresponding to this system is singular if $\det A(W) = 0$; at those $W$, the coefficients $c_{\alpha i}$ and consequently the $K$ matrix have poles. It is convenient to solve the system by first diagonalizing $A$:

$$U^T A U = \text{diag}[Z^{-1}_1(W - m_1), Z^{-1}_2(W - m_2), \cdots] ,$$

and then explicitly invert it:

$$(A^{-1})_{ji} = \sum_r U_{jr} U_{ri} Z_r / (W - m_r) .$$

The resulting resonant part of the $K$ matrix can be cast in the form

$$K_{\alpha\gamma}^{\text{res}} = \pi N_N N_N^\dagger \sum_r \frac{\tilde{V}_{\alpha r} \tilde{V}_{\gamma r}}{m_r - W} , \quad \tilde{V}_{\alpha r} = \sqrt{Z_r} \sum_i U_{ri} V_{\alpha i} .$$

Here $\sqrt{Z_r}$ is the wave-function renormalization while $U_{ri}$ are expansion coefficients of the physical resonance $r$ in terms of the bare three-quark states.

The $T$ matrix is finally obtained by solving the Heitler equation $T = K + iKT$.

2.2 The underlying quark model

The vertices are calculated in a version of the Cloudy Bag Model extended to the pseudo-scalar $SU(3)$ meson octet.

2.3 The kernel of the LS equation

Our previous calculation in the non-strange sector has shown that the background can be well described through the $u$-channel exchange processes alone. For $S$-partial waves, the contact interaction may also play an important role.

For the kernel of the LSE we assume, apart from the contact term, a term that reduces to the $u$-channel exchange term when evaluated (half) on-shell:

$$K_{\alpha\beta}(W, k, k') = V_{\alpha\beta}^{\text{res}}(k, k') + \sum_i f_{\alpha\beta}^i V_{i\beta}^{\text{res}}(k)V_{i\alpha}^{\text{res}}(k') - \frac{1}{\omega_{\beta}(k') + \omega_{\alpha}(k) + E_1 - W} .$$

Here $E_1 \equiv E_1(k + k') \approx \sqrt{m_1^2 + k^2 + k'^2}$ is the energy of the exchange baryon for which the resonances ($X^*$ and $S^*$ listed in table [3] and [4]) may be considered. For the $s$-wave mesons only the isospin quantum numbers of baryons ($I_{\alpha(\beta)}$, $I_1$) and meson ($t_{\alpha(\beta)}$) are involved:

$$f_{\alpha\beta}^i = \sqrt{(2I_1 + 1)(2I_\beta + 1)} W(t_{\alpha I_3 t_{\beta I_3} I_1}) .$$
where \( W(\cdots) \) are the Racah coefficients. In our previous calculations we have used a separable approximation for the \( u \)-exchange potential of the kernel \([13]\):

\[
K_{\alpha\beta}^{\text{sep}}(W, k, k') = \sum_i f_{i\alpha\beta} \frac{m_i}{E_{i\alpha}} (\omega_i + \epsilon_i^\alpha) \times \left( \frac{V_{i\beta}(k)}{2} \frac{V_{i\alpha}(k')}{2} \right), \tag{15}
\]

\[
\epsilon_i^\beta = \frac{m_i^2 - m_a^2 - m_\beta^2}{2E_{i\alpha}}. \tag{16}
\]

Here \( \omega_i, \omega_i, E_{i\alpha}, \epsilon_i^\alpha, \epsilon_i^\beta \) are evaluated on-shell. When one of the mesons is on-shell, both forms reduce to the same expression. In the present approach all baryons appearing in the channels are stable and the LSE can be solved numerically rather easily; since in this partial wave the meson-baryon couplings are relatively small, both full (unseparable) and separable form yield very similar results. However, since the contact term cannot be written in a separable form, the LSE has to be anyway solved numerically.

### 3 Solving the scattering equation

#### 3.1 Solution for \( KN \) scattering

For strangeness \( S = 1 \) there are no baryon resonances and only background processes govern \( KN \) scattering. Furthermore, for isospin \( I = 0 \) there is no contribution from the contact term. This gives us an opportunity to examine the validity of our approximation for the background term which stems from the \( u \)-channel exchange potential. The scattering amplitudes are obtained by solving Eq. \( [15] \).

It turns out that the main contribution comes from the exchange of the singlet \( A_1^* \) baryon which can be identified with the \( \Lambda(1405) \) resonance and the octet \( \Lambda_8^* \) baryon identified with the \( \Lambda(1800) \) resonance. The identification of the octet and decuplet \( \Sigma^* \) is less clear and we do not consider their contribution here.

Using our choice of \( R = 0.83 \) fm and \( f_\pi = 73 \) MeV, as well as the coupling constants predicted by the quark model (table \[5\]), we underestimate the experimental \( KN \) amplitudes (fig. \[1\] short dashes). Multiplying the coupling constants by a renormalization factor \( f_a = 1.35 \) we are able to reproduce the amplitudes at low and intermediate energies (solid line). The agreement is improved if we choose a slightly smaller bag radius, \( R = 0.78 \) fm, and \( f_a = 1.30 \) (dashes).

In the following calculation we use the form and the renormalization factor derived above; we retain our standard choice of the bag radius, \( R = 0.83 \) fm, which, as we see in the following, yields the most consistent results in other sectors.

In the isospin \( I = 1 \) channel (fig. \[2\]) the contact interaction is present and dominates the amplitude; the exchange potential is relatively small and has the opposite sign with respect to the \( I = 0 \) case as well as with respect to the contact potential. We obtain a good agreement with the contact potential alone using the rather standard choice of \( R = 0.83 \) fm and the experimental value \( f_\pi = 93 \) MeV (dashes); adding the exchange potential the experimental data are equally well reproduced by taking \( R = 0.90 \) fm and \( f_\pi = 85 \) MeV (solid line). Similarly as in the \( I = 0 \) case the data support bag radii close to those used to describe baryons. Larger radii, e.g. \( R = 1 \) fm (short dashes), underestimate the \( \text{Im} T \) part of the amplitude at higher \( W \).

#### 3.2 Dynamically generated \( \Lambda(1405) \)

Switching to the strangeness \( S = -1 \) sector, we first consider the case with no bare three-quark states. We solve the LSE \( [3] \) by using two channels, \( \pi\Sigma \) and \( KN \), and assuming only contact interaction. We first use our standard choice of model parameters, \( R = 0.83 \) fm and \( f_\pi = 73 \) MeV. The position of the pole in the complex \( W \) plane is calculated by using the Laurent-Pietarinen expansion \([52, 53, 54]\). We obtain two poles: the upper pole very close to

![Fig. 1.](image1.png)  
Fig. 1. \( T \)-matrix amplitudes for the reaction \( KN \to KN \) in the \( S_{01} \) partial wave for different bag radii and interaction strengths. (See text for curve assignments.) Experimental data are from \([51]\).

![Fig. 2.](image2.png)  
Fig. 2. Same as fig. \[1\] but for the \( S_{11} \) partial wave.
the $KN$ threshold and the lower one approaching the $\pi\Sigma$ threshold (table 1). Reducing the strength of the interaction by assuming slightly larger values for $f_\pi$, the mass of the lower pole rises to the nominal value given in [55], while the upper pole remains close to the $KN$ threshold. The width of the upper pole is consistent with the PDG value while the width of the lower pole seems to be underestimated by a factor of two. Let us note that in this case there is only one pole of the $K$ matrix, which lies close to its nominal Breit-Wigner mass (fig. 3).

If we further reduce the strength of the contact term, only one pole remains, with a mass slightly below the $KN$ threshold. This pole corresponds to the pole found in the same model in Ref. [26].

### Table 1. The pole parameters obtained from the $S_{01}$ partial wave amplitudes in the $\pi\Sigma$ channel using the Laurent-Pietarinen expansion

| Resonance | $ReW$ [MeV] | $-2ImW$ [MeV] | Module [MeV] | $R$ [fm] | $f_\pi$ [MeV] |
|-----------|-------------|--------------|-------------|---------|-------------|
| $\Lambda(1380)$ | 1348 | 33 | 16 | 0.83 | 73 |
| $\Lambda(1405)$ | 1433 | 20 | 1 | 0.83 | 73 |

Fig. 3. The real and imaginary part of the $\pi\Sigma \rightarrow \pi\Sigma$ amplitude for $R = 0.83$ fm and $f_\pi = 78$ MeV (solid line), and for $R = 1.1$ fm and $f_\pi = 93$ MeV (dashes). Since below the $KN$ threshold only the $\pi\Sigma$ channel is open, $ImT$ is proportional to the invariant mass distribution of $\pi\Sigma$ pairs and is compared to the experimental points in [55].

Our results with smaller values of $R$ and $f_\pi$ are consistent with the predictions of the chiral unitary theory and seem to suggest that no bare three-quark states are needed to reproduce at least the lowest resonance in the $S_{01}$ partial wave. There is a caveat, however: if we want to reproduce the subsequent two $I = 0$ resonances, the $\Lambda(1670)$ and $\Lambda(1800)$, we have to assume the existence of at least two genuine quark model states. Yet carrying out such a calculation we find that the required strength of the contact potential, necessary to support dynamically generated resonances below the $KN$ threshold, is much too large in order to reproduce the experimental scattering amplitudes in the region of the upper two $S_{01}$ resonances. In fact, Kamano et al. [37], who have done a rather extensive analysis of the partial-wave amplitudes for $K^{-}\pi$ scattering, have found a better agreement for the $S_{01}$ case without including the contact interaction.

### 3.3 Including bare three-quark states in the $S_{01}$ partial wave

For the $S_{01}$ partial wave we include the quark model states corresponding to the lowest three resonances, assuming one quark is promoted from the $s$ orbit to the $p_{1/2}$ orbit. We further assume one singlet configuration, that can be identified as the $\Lambda(1405)$, and two octet configurations, with internal spin $S = \frac{1}{2}$ (doublet) and $S = \frac{3}{2}$ (triplet) that can be identified as the $\Lambda(1670)$ and $\Lambda(1800)$. We use the $j$-$j$ coupling scheme identical to the one used for the non-strange $S_{11}$ resonances [48]. The bare mass of the singlet state has been fixed by requiring that a pole of the $K$ matrix lies at $W = 1405$ MeV; the massess and a possible mixing angle of the bare octet states are free parameters.

We consider four channels: $\pi\Sigma$, $KN$, $\eta\Lambda$, and $K\Xi$, and assume that the physical $\eta(548)$ implies $\eta = \eta_k \cos \theta_P - \eta_\pi \sin \theta_P$ with $\theta_P = -11.3^\circ$ [55]. We are not interested in obtaining the best fit to the experimental amplitudes but rather to investigate to what extent the quark model is able to reproduce the main features of the scattering amplitudes. We therefore retain the quark-model values in table 4 for the first two channels, as well as for $\eta_\pi \Lambda_0^*$. The measured cross-section for $K^- + p \rightarrow \eta_\Lambda$ [57] imposes a rather strong constraint on $\eta_\Lambda$ coupling to the octet $A_2$ and suggests a much smaller value for this coupling than the one predicted by the quark model; similarly we take smaller values for $g_{K\Xi A_2^*}$. We further assume our standard choice for the bag radius of $R = 0.83$ fm for all pertinent baryons as well as for the decay constants $f_\pi = f_\eta = f_K = 73$ MeV.

The background potential (and the kernel entering the LSE) consists of the $u$-channel exchange potential and the contact potential. Based on our discussion of $KN$ scattering for $I = 0$ we keep beside the nonstrange baryons in table 6 only the $\Lambda^*$ as the exchange baryons with $S = -1$, and further assume a renormalization of the coupling constants in table 3 by a factor of $f_\pi = 1.35$ for the $KN$ as well as for the $\pi\Sigma$ channels. We control the strength of the contact term by adjusting the value of $f_\pi$ which is allowed to differ from the (fixed) value used in the meson-baryon interaction.
As we have mentioned in the previous section, using the strength of the contact interaction that produces two poles below the $KN$ threshold results in the scattering amplitudes which strongly disagree with experiment. Only when the contact interaction is reduced to less than 10% of that strength, the calculated amplitudes start to exhibit the typical pattern seen in the $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \pi\Sigma$ reactions. In fact, we have found the best overall agreement in this partial wave without including the contact interaction.

The results for scattering in the $S_{01}$ partial wave are displayed in fig. 4; the real and the imaginary parts of the $T$ matrix are compared to the results for $\bar{K}N \rightarrow \bar{K}N$ and $K\eta$ from the single-energy partial-wave analysis [35], as well as to the analysis of the Bonn-Gatchina group [38]. Furthermore, the calculated cross-section for $K^-p \rightarrow \eta\Lambda$ is compared to the measured one [57], and in addition, the imaginary part of $T(\pi\Sigma \rightarrow \pi\Sigma)$ is confronted with the $\pi\Sigma$ invariant mass spectrum [50]. The bare masses used in the calculation are displayed in table 2; we assume no mixing between the two bare octet configurations, the $K\Sigma$ couplings to all $\Lambda^*$ are reduced to 30% of the QM value, while the $\eta\Lambda$ coupling to the octet $\Lambda^*$ even to 10% of the QM value.

Our calculation shows that the scattering amplitudes are dominated by the resonant terms and that the background potential plays a rather minor role. Furthermore, evaluating the amplitudes without solving the LSE, i.e. by keeping only the leading terms in Eqs. (7) and (8), and readjusting slightly the bare masses, the result of the full

![Fig. 4. The scattering amplitudes for reactions $\bar{K}N \rightarrow \bar{K}N$, $\bar{K}N \rightarrow \pi\Sigma$ and $\pi\Sigma \rightarrow \pi\Sigma$, and the cross-section for $K^-p \rightarrow \eta\Lambda$. “LSE” stands for solving the Lippmann-Schwinger equation, “lead” for the leading order, “2res+2ch” for only two channels and two resonances, “BG” is the Bonn-Gatchina solution [38]. Experimental data from [35] for the amplitudes, [57] for the cross-section and [50] for the invariant mass distribution.](image-url)
calculation changes only insignificantly (long dashes vs. solid line for the LSE in fig. 3). We have performed the Laurent-Pietarinen expansion to determine the positions of the poles in the complex W plane (table 2). The mass and the width of the lowest pole determined in the \( \pi \Sigma \) channel are consistent with the PDG values for the \( \Lambda(1405) \). The values for the second pole are calculated from the amplitudes for three different reactions, and all three give values consistent with the PDG result for the \( \Lambda(1670) \). For the third pole the values from \( \bar{K}N-\bar{K}N \) differ more substantially from the preferred PDG values, while the results for \( K\bar{N}-\pi \Sigma \) agree well with the PDG values. These results change only marginally in the leading order.

In our approach only the \( \pi \Sigma \) channel provides the information about the poles below the \( KN \) threshold. Still, some information can be obtained also from the \( K\bar{N} \) amplitude by expanding the \( T \) matrix for small \( k \) [40].

\[
T = k (a^{-1} - ik + \frac{1}{r}k^2)^{-1},
\]

which yields the scattering length \( a_{\text{LSE}} = (-1.32 + i 1.00) \) fm and \( a_{\text{lead}} = (-1.42 + i 0.71) \) fm, and the pole at \( W_{\text{LSE}} = (1434.8 - i 22.1) \) MeV and at \( W_{\text{lead}} = (1425.6 - i 21.5) \) MeV. Both values of the scattering length are inside the allowed region established in Ref. [58] and deduced from the SIDDHARTA measurements [59]. While the width is consistent with the values in table 2 for the \( \pi \Sigma \) channel, the mass comes much too close to the threshold value in this approximation.

Next we have considered the case with only two channels, \( \pi \Sigma \) and \( K\bar{N} \), and two bare three-quark states, the \( \Lambda_1^\ast \) and \( \Lambda_2^\ast \). Again we obtain a similar behaviour of amplitudes as in the full calculation in a wide energy range except for the interval in the vicinity of the second resonance \( \Lambda(1670) \), neglected in this approximation.

We shall discuss the nature of the lowest resonance in Sec. 4.

Let us comment here on similar calculations in the framework of the same model in Refs. [26] and [60] where the contact interaction, a satisfactory agreement — at least for \( W \) below \( 1750 \) MeV — is reached by using \( f_\pi = 128 \) MeV for the contact interaction. In addition, we assume that the strength of the \( \pi \Sigma \Sigma \pi \) coupling constant is reduced by 30% with respect to the quark-model value, and a mixing angle of 20° is used already at the level of bare \( 2 \Sigma^\ast \) and \( 4 \Sigma^\ast \) states. For the channels \( K\bar{N}, \pi\Lambda, K\Xi \) we fix the coupling constants to their quark-model values, while for the \( \eta \Sigma \) channel we use the same prescription as for the \( \eta \Lambda \) channel in the \( S_{01} \) partial wave. The optimal masses of the bare quark states remain close to their nominal values 1750 MeV and 1900 MeV, i.e. \( m(2 \Sigma_8^\ast) = 1750 \) MeV and \( m(4 \Sigma_8^\ast) = 1876 \) MeV.

In fig. 5 we compare the full calculation by solving LSE with all five channels, the full calculation which includes only \( \pi \Sigma, K\bar{N}, \pi\Lambda \) channels, and the calculation in the leading order (without solving LSE). As expected, the first three channels dominate at lower \( W \), but in contrast to the \( S_{01} \) partial wave, the leading order solution differs considerably from the full solution as a consequence of a much stronger potential that enters the LSE.

The positions of the poles in the complex \( W \) plane are displayed in table 3. While the lower pole is located at too large \( \text{Re}W \) and too small \( \text{Im}W \) compared to the PDG values, the upper pole is better reproduced in the case of five channels. The scattering length in the case of five (three) channels is \( a_1 = (0.54 - i 0.39) \) fm (\( a_1 = (0.52 - i 0.41) \) fm); while the real parts are well within the allowed region advocated in Ref. [58], the imaginary parts seems to be slightly too low.

### Table 3. The pole parameters obtained in the \( S_{11} \) partial wave by using five or three channels.

| res. react. | \( \text{Re}W \) [MeV] | \( -\text{Im}W \) [MeV] | modul. QM | assign. [MeV] |
|---|---|---|---|---|
| \( \Sigma(1750) \) 5 & 3 | 2\( ^8\)70 | 1750 |
| \( \bar{K}N-\bar{K}N \) 5 | 1738 | 48 | 1.7 |
| 3 | 1784 | 75 | 7.8 |
| \( K\bar{N}-\pi\Sigma \) 5 | 1786 | 54 | 4.8 |
| 3 | 1785 | 75 | 13.8 |
| \( K\bar{N}-\pi\Lambda \) 5 | 1788 | 49 | 2.1 |
| 3 | 1785 | 73 | 5.1 |
| \( \Sigma(1900) \) 5 & 3 | 4\( ^8\)70 | 1876 |
| \( \bar{K}N-\bar{K}N \) 5 | 1924 | 96 | 18.1 |
| 3 | 1914 | 61 | 21.5 |
| \( K\bar{N}-\pi\Sigma \) 5 | 1925 | 123 | 10.7 |
| 3 | 1914 | 60 | 3.0 |
| \( K\bar{N}-\pi\Lambda \) 5 | 1924 | 81 | 10.3 |
| 3 | 1914 | 61 | 12.9 |

From both partial waves we can construct the amplitudes for the decay of the \( \Lambda(1405) \) into \( \Sigma^+\pi^- \), \( \Sigma^-\pi^+ \), and \( \Sigma^0\pi^0 \), and compare them to those extracted in the
In order to obtain a deeper insight into the mechanism of resonance formation in the presence of genuine three-quark states in the $S_{01}$ partial wave we follow the evolution of the $S$-matrix poles in the complex energy plane as a function of the interaction strength by performing the Laurent-Pietarinen expansion. We start from the genuine three-quark states and calculate the scattering amplitudes by gradually increasing the meson-baryon coupling constants in all channels by a factor $g$, $0 < g \leq 1$, to finally reach the physical values used in the previous Section. This approach has been used in our previous work \cite{4,45} to show that the Roper resonance evolves from the genuine three-quark state, while the $\Delta(1600)$ emerges as a dynamically generated state.

In the present case we deal with an evolution of three resonances that lie relatively close to each other and strongly mix, particularly in the region of intermediate coupling strengths. This represent a serious difficulty in identifying reactions $p+p \rightarrow \Sigma^\pm + \pi^0 + K^0 + p$ by the HADES collaboration \cite{61}, and $\gamma + p \rightarrow K^+ + \Sigma^0 + \pi$ by the CLAS Collaboration \cite{62}. The corresponding $|T_{\Sigma^- \pi^+}|^2$, $|T_{\Sigma^0 \pi^0}|^2$ and $|T_{\Sigma^+ \pi^-}|^2$ can be straightforwardly expressed in terms of the $T$-matrix amplitudes for the $S_{01}$ and $S_{11}$ partial waves (assuming no contribution from $I = 2$) and related to the cross-section for the above reactions. We shall not attempt to write down the explicit expression for the cross-section but rather compare the qualitative behaviour of $|T_{\Sigma\pi}|^2$ with the mass distribution of the model by Bayar et al. \cite{63} based on HADES data. Comparing our fig. 6 with their fig. 6 we notice that the positions of peaks for the $\pi^+$, $\pi^0$ and $\pi^-$ distributions are similar, also the $\pi^+$ distribution is dominant in both cases. Let us note that below the $KN$ threshold the $\pi^0$ distribution is one third of the $\text{Im}T_{\Sigma^- \pi^+ \Sigma}$ amplitude and is in our case peaked around the nominal mass of the resonance at $m = 1405$ MeV which corresponds to one of the free parameters in our model. Such a value is supported also by the analysis of HADES data by Hassanvand et al. \cite{64}. A similar comparison of mass distributions in different channels to those obtained by \cite{22} and \cite{65} (both based on CLAS data), is less conclusive.

\section{4 The structure of the resonances}

\subsection{4.1 Evolution of poles in the complex energy plane}

In order to obtain a deeper insight into the mechanism of resonance formation in the presence of genuine three-quark states in the $S_{01}$ partial wave we follow the evolution of the $S$-matrix poles in the complex energy plane as
the poles belonging to individual resonances, which may overlap or even cross. Furthermore, the presence of the $KN$ and $\eta\Lambda$ thresholds may strongly influence the poles in their vicinity. It turns out that the procedure is more stable if one uses the leading solution due to smaller widths of the resonances. Yet the final solution stays close to the threshold value, has a pole which can unmistakably be attributed to the physical pole in the vicinity. It turns out that the procedure is more stable if one uses the leading solution due to smaller widths of the resonances.

The evolution for the reactions $\bar{K}N \rightarrow \bar{K}N$, $\bar{K}N \rightarrow \pi\Sigma$, and $\pi\Sigma \rightarrow \pi\Sigma$ is shown in fig. 7. We do not show $\bar{K}N \rightarrow \eta\Lambda$ since the relevant pole lies very close to the threshold and its determination is less reliable.

The evolution of the lower resonance (left panels) starts with the three-quark octet configuration. The evolution for $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \pi\Sigma$ stops at $g = 0.6$ and $g = 0.7$, respectively, as the widths and moduli vanish. This does not happen in the $\pi\Sigma$ channel, the evolution rather continues away from the real axis. Beyond $g > 0.5$ another branch appears and evolves toward the pole that can be identified as $\Lambda(1405)$; at larger $g$ this branch can also be obtained by using the small-$k$ expansion for $T_{\bar{K}N \bar{K}N}$. From our analysis it is unclear whether (i) this branch emerges at the threshold and evolves independently of the upper branch or (ii) it smoothly evolves from the genuine three-quark state, in which case there would exist a bifurcation for $\pi\Sigma \rightarrow \pi\Sigma$ in the intermediate regime of $g$. Though the curves presented in fig. 7 favor the first possibility, we should mention that the determination of the pole in the vicinity of the threshold is unreliable and its position very close to the threshold may be an artifact. Plotting the (real) pole of the $K$ matrix as a function of $g$ supports the second possibility since it exhibits a rather rapid transition from the bare value to values below the threshold.

The branch above $g = 0.80$, where the mass of the pole starts moving away from the threshold value, has a clear physical interpretation, which will be discussed in the following.

The evolution of the middle resonance (central panels) starting with the three-quark octet configuration with spin $S = \frac{3}{2}$ is smooth except around $g = 0.6$ in the $\bar{K}N$ channel where the pole pertaining to the $\Lambda(1405)$ (left panel) in this channel disappears. All three evolutions end up at the pole which can unmistakably be attributed to the physical $\Lambda(1670)$ resonance, and confirm the assignment given in table 2.

The evolution of the upper resonance (right panels) starting with the three-quark octet configuration with spin $S = \frac{1}{2}$ is smooth and in $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \pi\Sigma$ evolves to the resonance that can be identified as $\Lambda(1800)$, though in the $KN$ channel it terminates at too large ReW and ImW. The $\pi\Sigma$ system is only weakly coupled to the bare state and above $g = 0.5$ becomes too weak to be detected.

4.2 Structure of the $\Lambda(1405)$ resonance

Let us observe the evolution of the $KN$ channel below the $KN$ threshold as $W$ approaches $m_\Lambda$, the lowest pole of the $K$ matrix. If we normalize the corresponding channel state by inverting the norm and taking into account that both terms are dominated by $c_{\bar{K}N\Lambda} \propto (W - m_\Lambda)^{-1}$, the
channel state can be cast in the form

$$|\Psi_{KN}\rangle = \sqrt{Z} \left( |\Phi^{0}_{A^*}\rangle + \int \frac{dk}{\omega_{k} + E_{N}(k) - W} \left[ a_{K}^{\dagger}(k)|\Phi_{N}\rangle \right] + \int \frac{dk}{\omega_{k} + E_{\Sigma}(k) - W} \left[ a_{\pi}^{\dagger}(k)|\Phi_{\Sigma}\rangle \right] + \ldots \right) . \quad (17)$$

where \( k \) refers to the kaon momentum. The terms involving the \( \eta A \) and \( K\Sigma \) components, as well as the octet admixtures to singlet three-quark state, are small and will be neglected in the following. The norm then reads

$$Z^{-1} = 1 + \int \frac{dk}{\omega_{k} + E_{N}(k) - W^2} - \frac{d}{dW} \Sigma_{\pi\Sigma} (W) . \quad (18)$$

We notice that for \( W \) close to the threshold, the second term in Eq. (18) strongly dominates and the system is very loosely bound (see fig. 8), however, at the physical value of the coupling strength it becomes comparable to the weight of the bare three-quark component (see fig. 9). The contribution from the \( \pi\Sigma \) component (expressed in terms of the derivative of the self-energy) remains very small. On the other hand, the contribution to the energy is dominated by both the \( KN \) as well as the \( \pi\Sigma \) self-energies, which are responsible to push the mass of the physical \( \Lambda(1405) \) below the \( KN \) threshold; this mechanism is similar to the one proposed by Arima et al. \[7\].

Our model therefore confirms the picture in which the \( \Lambda(1405) \) is predominantly a \( KN \) molecular state; however, in our model the binding mechanism is not the contact interaction that would generate attraction between the (anti)kaon and the nucleon but rather the \( KN A^* \) interaction which implies the presence of a bare three-quark configuration with the quantum numbers of the resonance; the presence of the \( \pi\Sigma \) channel is also necessary to ensure the binding. Let us mention that the \( \Lambda(1405) \) is not a Feshbach resonance since the energy of the state \[17\] alone is well above the \( KN \) threshold.

A similar model with a bare state and the kaon cloud around the nucleon was proposed long ago by Thomas et al. \[20\] in the framework of the same model; in our approach we have extended the model by inclusion of other channels and resonances, but also showing that the presence of the contact term is not mandatory. Our picture of the resonance can also be related to the state found on the lattice \[29,30\] and interpreted in the framework of Hamiltonian effective theory \[31\].

5 Conclusion

As we have shown in Sec. 3 our model is able to generate a \( \frac{1}{2}^{-} \) resonance — or even two resonances — below the \( \bar{K}N \) threshold either as a molecular state or a genuine three-quark state dressed with baryon-meson pairs. We believe that in order to give credence to either of the two approaches it is important to carry out the calculation of the \( S_{01} \) partial wave amplitudes in the relevant channels by treating all three resonances in a unified framework.

The main and, admittedly, rather surprising, conclusion of our investigation is that the scattering amplitudes are dominated by the quark degrees of freedom rather than by non-linear dynamics of the baryon-meson systems. By using our standard choice of model parameters which had successfully reproduced the scattering as well as electroproduction amplitudes in the nonstrange sector we have been able to obtain a satisfactory result already in the leading order; solving the LSE only marginally improves the results. Furthermore, as in the calculation of Kamano et al. \[37\], we have found that including the contact interaction does not improve the results. Our results therefore confirm that the main mechanism to lower the mass of the \( \Lambda(1405) \) by \( \approx 250 \) MeV with respect to its bare value is the one suggested by Arima et al. \[7\], that is, the attractive self-energy term in the \( \pi\Sigma \) and \( KN \) channels, with the latter term being strongly enhanced due to the presence of the \( KN \) threshold. Nonetheless, even without includ-
ing the contact interaction, we have been able to observe
the formation of the \( KN \) molecular state. By gradually in-
creasing the strength of all meson-baryon couplings, a res-
onant state, strongly dominated by a weakly bound \( KN \)
component and an almost negligible admixture of the bare
three-quark and \( \pi\Sigma \) components, emerges slightly below
the \( KN \) threshold. At the physical strength, at which this
state can be identified with the \( \Lambda(1405) \) resonance, the
molecular component becomes weaker but still dominates
over the bare quark state which, in turn, is dominated by
the singlet component. There is, however, an important
difference between our state and the molecular state of the
chiral unitary approach; in our approach the attraction is
over the bare quark state which, in turn, is dominated by
the singlet component. There is, however, an important
difference between our state and the molecular state of the
chiral unitary approach; in our approach the attraction is
generated through the \( KN\Lambda^* \) coupling and therefore the
presence of the singlet state is mandatory.

Regarding the two higher lying resonances, the \( \Lambda(1670) \)
is well reproduced and assigned to \( S = \frac{1}{2} \); with the \( \Lambda(1800) \)
there remains some ambiguity about the determination of
its mass in different channels, which signals that our model
becomes less reliable at higher \( W \).

In the \( S_{11} \) partial wave it turns out that the inclusion of
the contact interaction is important; we reproduce rea-
sonably well the scattering amplitudes in the energy range
from the threshold up to around 1750 MeV using either
all five channels or solely the \( \pi\Sigma, KN, \) and \( \pi\Lambda \) channels.

**A The Cloudy Bag Model meson-quark vertices and coupling constants**

The \( s \)-wave quark-meson vertices \( \hat{V}(k) \) in Eq. (11) are
evaluated in the Cloudy Bag Model assuming that in the
resonant state one of the three quarks is excited from the
\( 1s \) state to the \( 1p_{1/2} \) state.

For the quark part of the quark-pion, quark-eta meson
and quark-kaon interaction we obtain

\[
\hat{V}_i^{\pi}(k) = \sum_{i=1}^{3} \tau_i(i) \mathcal{P}_{sp}(i),
\]

\[
\hat{V}_i^{\eta}(k) = \sum_{i=1}^{3} \lambda_i(i) \mathcal{P}_{sp}(i),
\]

\[
\hat{V}_i^{K}(k) = \mathcal{V}^{K}(k)
\]

Here \( \mathcal{P}_{sp} = \sum_{m_j} |sm_j\rangle\langle p_{1/2}m_j\rangle \), \( \omega_s = 2.043 \), \( \omega_{p_{1/2}} = 3.811 \), \( X(\bar{K}^0) = -(\lambda_0-i\lambda_\gamma)/\sqrt{2} \), \( X(\bar{K}^0) = -(\lambda_0+i\lambda_\gamma)/\sqrt{2} \), \( X(\bar{K}^+) = -(\lambda_4-i\lambda_\gamma)/\sqrt{2} \), \( X(\bar{K}^-) = -(\lambda_4+i\lambda_\gamma)/\sqrt{2} \). Assuming \( f_\eta = f_K = f_\pi \), the form-factors of the surface part
and of the volume part take the form

\[
\mathcal{V}^{\pi}(k) = \mathcal{V}^{\eta}(k) = \mathcal{V}^{K}(k)
\]

\[
V^{\pi}(k) = V^{\eta}(k) = V^{K}(k) = \frac{1}{2f_\pi} \sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}} \frac{1}{2\pi} \sqrt{\omega_k} \frac{j_0(kR)}{kR}. \]

\[
\times \int_0^R dr \ r^2 [u_s(r)u_p(r) + v_s(r)v_p(r)] j_0(kr). \]

For the physical \( \eta \) we assume \( \eta = \cos \theta_\pi \eta_8 - \sin \theta_\pi \eta_1 \), for the singlet \( \eta_1 \), \( \eta_8 \) is replaced by \( \Lambda_1 \) in Eq. (20).

The coupling constants for the \( s \)-channel exchange po-
tential are collected in table 4, those for the \( u \)-channel exchange potential involving strange baryons in table 5,
and for exchange of nonstrange baryons in table 6.

---

**Table 4. Reduced \( X^* \to MB \) matrix elements**

| \( X^*/BM \) | \( \pi\Sigma \) | \( \pi\Lambda \) | \( \eta_1 \Lambda \) | \( \eta_8 \Lambda \) | \( \eta_1\Sigma \) | \( \eta_8\Sigma \) | \( KN \) | \( K\Xi \) |
|----------------|----------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| \( A_1^* \)    | \( \sqrt{3} \) | 0              | 0              | 1               | 0              | 0              | \( \sqrt{2} \) | \( \sqrt{2} \) |
| \( A_2^* \)    | \( -\frac{2\pi}{\sqrt{3}} \) | 0              | 0              | 0               | \( \frac{\sqrt{2}}{\sqrt{3}} \) | \( -\frac{2\pi}{\sqrt{3}} \) |
| \( A_3^* \)    | \( \frac{2\pi}{\sqrt{3}} \) | 0              | 0              | 0               | -1             | -1             | \( \frac{\sqrt{2}}{\sqrt{3}} \) | \( \frac{2\pi}{\sqrt{3}} \) |

**Table 5. Reduced \( B \to MX^* \) matrix elements**

| \( X^*/BM \) | \( \Sigma \pi \) | \( \pi\Lambda \) | \( \eta_1 \Lambda \) | \( \eta_8 \Lambda \) | \( \eta_1\Sigma \) | \( \eta_8\Sigma \) | \( NK \) | \( \Xi\bar{K} \) |
|----------------|----------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| \( A_1^* \)    | 1              | 0              | 0              | -1             | 0              | 0              | 1              | 1              |
| \( A_2^* \)    | \( \frac{1}{3} \) | 0              | \( \frac{2\sqrt{2}}{3} \) | \( -\frac{1}{3} \) | 0              | 0              | 1              | \( -\frac{2}{3} \) |
| \( A_3^* \)    | \( -\frac{2}{3} \) | 0              | \( \frac{2\sqrt{2}}{3} \) | \( \frac{2}{3} \) | 0              | 0              | 0              | \( -\frac{2}{3} \) |

**Table 6. Reduced \( B \to MS^* \) matrix elements for non-strange**

| \( S^*/BM \) | \( \Sigma \bar{K} \) | \( N\pi \) | \( \Lambda\bar{K} \) | \( \eta_1 \bar{K} \) | \( \eta_8 \bar{K} \) |
|---------------|-----------------|-----------|-----------------|-----------------|-----------------|
| \( S_{112} \)  | \( \frac{1}{3} \sqrt{2} \) | \( -\frac{2}{3} \) | \( -\sqrt{2} \) | \( \frac{2\sqrt{2}}{3} \) | \( \frac{2}{3} \) |
| \( S_{114} \)  | \( -\frac{2}{3} \sqrt{2} \) | \( \frac{2}{3} \) | 0               | \( \frac{2\sqrt{2}}{3} \) | \( \frac{2}{3} \) |
| \( S_{31} \)   | \( \frac{2}{3} \sqrt{2} \) | 0         | 0               | 0               | 0               |
B Contact interaction

For the $s$-wave mesons the contact interaction can be cast in the form

$$V_{\alpha\beta}^c(k, k') = \frac{g_{\alpha\beta}}{2f_\pi^2} \frac{kk'}{2\pi^2 \sqrt{2\omega_\alpha(k) \sqrt{2\omega_\beta(k')}}} \times [\omega_\alpha(k) + \omega_\beta(k')] \int_0^R dr r^2 [u_\alpha^2(r) + v_\beta^2(r)] j_0(kr) j_0(k'r).$$

(23)

Here $g_{\alpha\beta}$ can be identified with $-2f_\pi^2\Lambda_{\alpha\beta}$ of [26] and $\frac{1}{2}D_{\alpha\beta}$ of [11] and are collected in table 7.

Table 7. $g_{\alpha\beta}$ for isospin 0 and 1.

| $g_{\alpha\beta}$ | $KN$ | $\bar{KN}$ | $\pi\Sigma$ | $\eta\Lambda$ | $K\Xi$ |
|------------------|-------|------------|-------------|--------------|-------|
| $I = 0$          |       |            |             |              |       |
| $K\bar{N}$       | $\frac{1}{2}$ | $\frac{3\sqrt{2}}{\sqrt{6}}$ | $\frac{3\sqrt{2}}{\sqrt{6}}$ | 0 |
| $\pi\Sigma$      | $\frac{3\sqrt{2}}{\sqrt{6}}$ | 2 | 0 | $\frac{3\sqrt{2}}{\sqrt{6}}$ |
| $\eta\Lambda$    | $\frac{1}{\sqrt{2}}$ | 0 | 0 | $-\frac{3\sqrt{2}}{\sqrt{6}}$ |
| $K\Xi$           | $0$ | $\frac{\sqrt{2}}{\sqrt{6}}$ | $-\frac{3\sqrt{2}}{\sqrt{6}}$ | $\frac{3}{2}$ |

| $g_{\alpha\beta}$ | $K\bar{N}$ | $K\Sigma$ | $\pi\Lambda$ | $K\Xi$ | $\eta\Sigma$ |
|------------------|------------|-----------|--------------|-------|--------------|
| $I = 1$          |            |           |              |       |              |
| $K\bar{N}$       | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{3\sqrt{2}}{\sqrt{6}}$ | 0 | $-\frac{3\sqrt{2}}{\sqrt{6}}$ |
| $\pi\Sigma$      | $\frac{1}{2}$ | 1 | 0 | 1 | 0 |
| $\pi\Lambda$     | $-\frac{3\sqrt{2}}{\sqrt{6}}$ | 0 | 0 | $-\frac{3\sqrt{2}}{\sqrt{6}}$ | 0 |
| $K\Xi$           | 0 | 1 | $-\frac{3\sqrt{2}}{\sqrt{6}}$ | 1 | $-\frac{3\sqrt{2}}{\sqrt{6}}$ |
| $\eta\Sigma$    | $-\frac{3\sqrt{2}}{\sqrt{6}}$ | 0 | 0 | $-\frac{3\sqrt{2}}{\sqrt{6}}$ | 0 |

Oset [11] has an opposite sign for $K\bar{N} \leftrightarrow \pi\Sigma$, which is compensated by changing the sign for $\Sigma^* \rightarrow \pi\Sigma$. 
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