A Measurement of the QCD Colour Factors using Event Shape Distributions at $\sqrt{s} = 14$ to 189 GeV

S. Kluth, P. A. Movilla Fernández, S. Bethke, C. Pahl and P. Pfeifenschneider
Max-Planck-Institut für Physik,
Werner-Heisenberg-Institut
Föhringer Ring 6
80805 Munich, Germany

Abstract
Measurements of the QCD colour factors $C_A$ and $C_F$ and of the number of active quark flavours $n_f$ in the process $e^+e^- \rightarrow$ hadrons at high energy are presented. They are based on fits of $O(\alpha_s^2)+$NLLA QCD calculations to distributions of the event shape observables $1 - T$, $T$, $B_T$ and $B_W$ measured at centre-of-mass energies from 14 GeV to 189 GeV. Hadronisation effects are approximated with power correction calculations which also depend on the QCD gauge structure. In this approach potential biases from hadronisation models are reduced. Our results for individually measured quantities obtained from $1 - T$ and $C$ are

$$n_f = 5.64 \pm 1.35, \quad C_A = 2.88 \pm 0.27 \quad \text{and} \quad C_F = 1.45 \pm 0.27$$

in good agreement with QCD based on the SU(3) symmetry group where $n_f = 5$ for the energies considered here, $C_A = 3$ and $C_F = 4/3$. From simultaneous fits of $C_A$ and $C_F$ with $1 - T$ and $C$ we find

$$C_A = 2.84 \pm 0.24 \quad \text{and} \quad C_F = 1.29 \pm 0.18,$$

which is also in good agreement with the QCD expectation.

(Submitted to European Physical Journal C)
1 Introduction

The theory of strong interactions, Quantum Chromo Dynamics (QCD), gives a successful description of most aspects of hadronic final states in $e^+e^-$ annihilation and other high energy particle collision processes over a large range of centre-of-mass (cms) energies, see e.g. [1–10]. In particular, the strong coupling $\alpha_s$, the only free parameter in the theory, can be measured to an accuracy of about 3% [11].

Standard QCD contains three distinct colour charges which are carried by the quarks and obey the SU(3) symmetry group, see e.g. [12, 13]. Requiring local gauge invariance of the theory generates eight massless vector fields, the gluons, each carrying a colour charge and an anticolour charge. Since the gluons are colour charged QCD has more fundamental vertices than QED, where the gauge boson, the photon, is electrically neutral. The fundamental vertices are i) the radiation of a gluon from a quark line (gluon bremsstrahlung), ii) the conversion of a gluon into a quark-antiquark pair (quark pair production), iii) the conversion of a gluon into two gluons (triple gluon vertex) and iv) the conversion of a gluon into three gluons (quartic gluon vertex). The quartic gluon vertex is in $e^+e^-$ annihilation of order $O(\alpha_s^4)$ and thus inaccessible to the $O(\alpha_s^2)$ analysis presented here (see section 2).

The relative weights of each of the vertices are given by the QCD colour factors $C_F$, $T_F$ and $C_A$ corresponding to vertices i), ii) and iii), respectively [12]. The colour factor $T_F$ for vertex ii) contributes for each of $n_f$ active quark flavours, such that the product $T_F n_f$ is the weight for vertex ii).

The colour factors can be calculated from the generators of gauge transformations in the fundamental and adjoint representation once a particular symmetry group has been chosen. In standard QCD the symmetry group is SU(3) and the corresponding values of the colour factors are $C_F = 4/3$, $T_F = 1/2$ and $C_A = 3$. Perturbative QCD calculations, e.g. for cross sections, depend on the symmetry group only through the colour factors. It is thus straightforward to modify the prediction for a different symmetry group by recalculating the colour factors.

From an experimental point of view one can ask the question if SU(3) is indeed the correct symmetry group for QCD. This question may be tested experimentally by allowing the colour factors to vary in comparisons of QCD predictions with data. Several such studies have been performed by the LEP experiments based on correlations of angles in 4-jet final states [14–20]. In these studies the colour factors enter at leading order ($O(\alpha_s^2)$) of the QCD calculations and thus higher order corrections have not been taken into account. Measurements of the ratios $n_f/C_F$ and $C_A/C_F$ by this method have been in agreement with the QCD expectations.

A complementary approach has been to employ distributions of event shape observables measured at LEP 1 and $O(\alpha_s^2)$+NLLA QCD predictions [21, 22]. Here the sensitivity to the colour factors also enters at $O(\alpha_s^2)$ but higher order corrections are partially taken care of by the inclusion of the NLLA terms. More recent works combined the two analysis techniques outlined above [23] and also made use of recently calculated higher order corrections to the 4-jet angular correlations [24]. In both cases results have been
consistent with QCD predictions and systematic uncertainties were reduced.

It is also possible to use the dependence of QCD predictions on the energy scale of the hard process to gain sensitivity to the colour factors. The dependence of QCD predictions on the energy scale enters through the running of the strong coupling \( \alpha_s \) which in turn is a function of the colour factors.

A common property of all studies discussed so far is that they rely on Monte Carlo models of the hadronisation process, typically JETSET, HERWIG or ARIADNE. Dependence on these hadronisation models in studies of the QCD gauge structure may be viewed as a disadvantage, because the models assume standard QCD with the SU(3) symmetry group to be valid. No attempt was made to study the effects of non-standard colour factors on the hadronisation corrections. However, such effects could be significant, because the parton shower stages in the Monte Carlo models are based on QCD calculations which in turn depend on the QCD gauge symmetry.

Recently an analytic model of the hadronisation process has become available commonly referred to as power corrections. It is based on the analysis of the power-behaviour of non-perturbative effects, i.e. the growth of non-perturbative effects with inverse powers of the scale of the hard process. In this model approximate predictions for the non-perturbative effects on mean values and distributions of some event shape observables have been made and their colour structure is explicitly given. Comparison with data has been reasonably successful.

In this study we use fits of \( O(\alpha_s^2) + \text{NLLA} \) QCD predictions with power corrections to differential distributions of the event shape observables Thrust, C-parameter, Total and Wide Jet Broadening measured in \( e^+e^- \) annihilation at CMS energies from 14 GeV to 189 GeV to investigate the gauge structure of QCD. Section 2 introduces the observables and summarises the QCD predictions. Section 3 gives a description of the data sets used in the analysis and of the fit results. Finally in section 4 a summary and conclusions are given.

## 2 QCD Predictions

### 2.1 Event Shape Observables

We use the following event shape observables:

**Thrust** \( T \) This observable is defined by the expression

\[
T = \max \left( \sum_i \frac{|\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right)
\]

where \( \vec{p}_i \) is the momentum of reconstructed particle \( i \) in an event. The thrust axis \( \vec{n}_T \) is the direction for which the maximum occurs. We will use the form \( 1 - T \) here since its distribution is in this form more similar to those of the other observables.

---

2 The error treatment of the results of [23] have been criticised [25].
C-parameter $C$ The definition of this observable [43, 44] requires the introduction of the linearised energy-momentum tensor

$$\Theta^{\alpha\beta} = \frac{\sum_i (p_i^\alpha p_i^\beta) / |\vec{p}_i|}{\sum_i |\vec{p}_i|}, \quad \alpha, \beta = 1, 2, 3$$

and its three eigenvalues $\lambda_k$, $k = 1, 2, 3$. These define $C$ via

$$C = 3(\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1).$$

Note that $C$ may equivalently be calculated by

$$C = \frac{3}{\sum_i |\vec{p}_i|} \sum_{i<j} |\vec{p}_i| |\vec{p}_j| \sin^2(\theta_{ij})$$

where $\theta_{ij}$ is the angle between particles $i$ and $j$.

Jet Broadening The definitions of the jet broadening observables [45] employ a plane through the origin perpendicular to the thrust axis $\vec{n}_T$ to divide the event into two hemispheres $S_1$ and $S_2$. The Total and the Wide Jet Broadening $B_T$ and $B_W$ are defined as

$$B_i = \frac{\sum_{p \in S_i} |\vec{p} \times \vec{n}_T|}{2 \sum_j |\vec{p}_j|}, \quad B_T = B_1 + B_2, \quad B_W = \max(B_1, B_2).$$

For these observables complete $O(\alpha_s^2)$+NLLA QCD predictions as well as power correction calculations for their differential distributions are available.

2.2 Running of $\alpha_s$

The running of the strong coupling $\alpha_s$ is a direct consequence of the requirement that a complete QCD calculation, e.g. for a cross section $R$, should not depend on the choice of energy scale where the theory has been renormalised, formally expressed as the renormalisation group equation (RGE), see e.g. [13]:

$$\mu^2 \frac{dR}{d\mu^2} = \mu^2 \left( \frac{\partial}{\partial \mu^2} + \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s} \right) R = 0 .$$

The running of $\alpha_s$ is found by solving the following differential equation where $\beta(\alpha_s(\mu^2))$ is the beta function of QCD shown at two-loop accuracy [13]:

$$\mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = \beta(\alpha_s(\mu)) = -\beta_0 \alpha_s^2(\mu) - \beta_1 \alpha_s^3(\mu) ,$$

$$\beta_0 = \frac{11 C_A - 2n_f}{12 \pi}, \quad \beta_1 = \frac{17 C_A^2 - 5 C_A n_f - 3 C_F n_f}{24 \pi^2} .$$

The coefficients $\beta_0$ and $\beta_1$ are independent of the renormalisation scheme. The solution to equation (7) at this order is [13, 22]

$$\beta_0 \ln(x\mu^2) = \frac{1}{\alpha_s(\mu)} - \frac{1}{\alpha_s(Q)} + \frac{\beta_1}{\beta_0} \ln \left( \frac{\alpha_s(\mu)}{\alpha_s(Q)} \right) \frac{\beta_0 + \beta_1 \alpha_s(Q)}{\beta_0 + \beta_1 \alpha_s(\mu)} .$$
with $x_\mu = \mu/Q$. The quantity $x_\mu$ is commonly referred to as the renormalisation scale factor since it allows to study the effects of variations of the renormalisation scale in perturbative QCD predictions. It is usually varied to assess the effect of missing higher order in the perturbation series. Equation (7) may be solved numerically to obtain $\alpha_s(\mu)$ as a function of a reference $\alpha_s(Q)$. The QCD gauge structure enters via the coefficients $\beta_0$ and $\beta_1$ of the QCD $\beta$-function.

The perturbative evolution of $\alpha_s$ diverges for small scales, see e.g. [13]. The location of the divergence, referred to as the Landau pole, defines the parameter $\Lambda_{QCD}$ which in $O(\alpha_s)$ is given implicitly by $\alpha_s(Q) = 1/(\beta_0 \ln(Q^2/\Lambda_{QCD}^2))$. The value of $\Lambda_{QCD}$ is approximately 200 MeV and indicates the scale where perturbative evolution of $\alpha_s$ breaks down.

2.3 Perturbative QCD Calculations

The perturbative QCD prediction in $O(\alpha_s^2)$ for a normalised differential event shape distribution $dR_{PT}/dy$ of a generic observable $y$ measured at cms energy $\sqrt{s} = Q$ may be written as follows [21, 22]:

$$
\frac{dR_{PT}}{dy} = \frac{1}{\sigma_{tot}} \frac{d\sigma}{dy} = \frac{dA}{dy} C_F \hat{\alpha}_s(\mu) + \left( \pi \beta_0 \ln(x_\mu^2) - \frac{3}{4} C_F \right) \frac{2C_F dA}{dy} + \frac{dB}{dy} \hat{\alpha}_s^2(\mu) .
$$

Here $\sigma_{tot}$ is the cross section for the process $e^+e^- \rightarrow$ hadrons, $dA/dy$ and $dB/dy$ are the $O(\alpha_s)$ and the $O(\alpha_s^2)$ coefficients, respectively, and $\hat{\alpha}_s = \alpha_s/(2\pi)$. The coefficients $dA/dy$ are in fact a sum of three terms each proportional to a colour factor:

$$
\frac{dB}{dy} = C_F \left( \frac{dB}{dy} C_F + \frac{dB}{dy} C_A + \frac{dB}{dy} n_f \right) .
$$

The coefficients $dA/dy$ and $dB_i/dy$, $i = C_F, C_A, n_f$, are obtained by integrating the $O(\alpha_s^2)$ QCD matrix elements [46] in the $\overline{\text{MS}}$ renormalisation scheme using the program EVENT2 [47]. A QCD prediction in $O(\alpha_s^2)$ is expected to be valid in a region of phase space where the radiation of a single hard gluon dominates (3-jet region).

For the observables considered here calculations in NLLA have been performed [33, 48, 49]. The NLLA is valid in a region of phase space where multiple radiation of soft and collinear gluons from a system of two hard and back-to-back partons dominates (2-jet region). In the NLLA the cumulative normalised cross section $R_{NLLA}(y)$ is written as:

$$
R_{NLLA}(y) = \int_y^\infty \frac{1}{\sigma_{tot}} \frac{d\sigma}{dy} dy' = C(\alpha_s) e^{G(\alpha_s, L)}
$$

with $L = \ln(1/y)$. The functions $C(\alpha_s)$ and $G(\alpha_s, L)$ are known in NLLA as:

$$
C(\alpha_s) = 1 + C_1 \hat{\alpha}_s + C_2 \hat{\alpha}_s^2 \quad \text{and} \quad G(\alpha_s, L) = \sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \hat{\alpha}_s^n L^m \simeq L g_1(L\alpha_s) + g_2(L\alpha_s) .
$$

The coefficients $C_1$, $G_{11}$, $G_{12}$, $G_{22}$ and $G_{23}$ are known analytically with complete colour structure while analytical expressions for the coefficients $C_2$ and $G_{21}$ are not available.
The combination of the $O(\alpha_s^2)$ and the NLLA calculations is a procedure called matching. Several matching schemes have been used in measurements of $\alpha_s$, see e.g. [50]. We choose to employ the so-called ln($R$)-matching scheme with the following implementation [50]:

$$\ln R_{PT}(y) = Lg_1(L\alpha_s) + g_2(L\alpha_s) - (G_{11}L + G_{12}L^2)\hat{\alpha}_s - (G_{22}L^2 + G_{23}L^3)\hat{\alpha}_s^2 + A(y)\hat{\alpha}_s + (B(y) - \frac{1}{2}A(y)^2)\hat{\alpha}_s^2 \quad (12)$$

The subtraction of the $G_{nm}$ coefficients takes account of the contributions contained in the NLLA as well as in the $O(\alpha_s^2)$ calculations. The term $A(y)$ is defined by

$$A(y) = \int_0^y (dA/dy') dy'$$

and equivalently $B(y) = \int_0^y (dB/dy') dy'$. Our choice of the ln($R$)-matching scheme is motivated by two arguments, i) it is preferred theoretically [48] as well as experimentally [50] and ii) it depends only on $G_{nm}$ coefficients known analytically with full colour structure.

### 2.4 Non-perturbative QCD Calculations: Power Corrections

The model of Dokshitzer, Marchesini and Webber (DMW) [31] treats the effects of gluon radiation at low energy scales ($O(\Lambda_{QCD})$) where simple perturbative evolution of $\alpha_s$ breaks down due to the presence of the Landau pole. The model assumes that evolution of the strong coupling $\alpha_s$ down to energies around and below the Landau pole is possible. The form of $\alpha_s(\mu)$ at such low energy scales is a priori unknown and a non-perturbative parameter is introduced as the 0th moment over $\alpha_s(\mu)$:

$$\alpha_0 = \frac{1}{\mu_I} \int_0^\mu_I \alpha_s(k) dk \quad (13)$$

The quantity $\mu_I$ is referred to as the infrared matching scale where the non-perturbative and standard perturbative evolution of $\alpha_s$ are merged, generally taken to be 2 GeV.

For the observables considered here the power corrections to the differential distributions have been calculated up to two loops [36,51]. It turns out that a distribution $dR/dy$ measured at cms energy $\sqrt{s} = Q$ can be described by the shifted perturbative prediction $dR_{PT}/dy$:

$$\frac{dR}{dy} = \frac{dR_{PT}}{dy}(y - PD_y) \quad (14)$$

The factor $P$ is universal [51] and depends mainly on the non-perturbative parameter $\alpha_0$, the infrared matching scale $\mu_I$ and the cms energy $Q$:

$$P = \frac{4C_F}{\pi^2} \mathcal{M} \mu_I \left( \alpha_0(\mu_I) - \alpha_s(Q) - 2\beta_0\alpha_s^2(Q) \left( \ln \frac{Q}{\mu_I} + K_{\beta_0} + 1 \right) \right) \quad (15)$$

with $K = (67/18 - \pi^2/6)C_A - (5/9)n_f$ for the $\overline{\text{MS}}$ renormalisation scheme. The negative terms containing $\alpha_s(Q)$ are necessary for a consistent merging of the strong coupling in the non-perturbative and the perturbative region. The quantity $\mathcal{M}$ (Milan factor) stems from two-loop effects and is given by [34]

$$\mathcal{M} = 1 + (1.575C_A - 0.104n_f)/(4\pi\beta_0) \quad (16)$$
Its numerical value is 1.49 in standard QCD with $n_f = 3$ with an estimated theoretical uncertainty of 20% [51]. The factor $D_y$ in equation (14) is observable specific and is given by:

$$
D_{1-T} = 2 \\
D_C = 3 \pi \\
D_{B_T,B_W} = \frac{1}{2} \ln \frac{1}{y} + F_y(y, \alpha_s(yQ)) .
$$

The functions $F_y, y = B_T, B_W$, with known colour structure, describe additional changes to the distributions referred to as squeeze [36, 52].

3 Analysis of the Data

3.1 Event Shape Data

We use all published data on distributions of our observables $T, C, B_T$ and $B_W$ which include experimental systematic uncertainties in their errors. The data available in the fits are listed in table 1 with references and the ranges which are considered in the fits. The fit ranges are determined by the following criteria:

Data with $\sqrt{s} \geq M_{Z^0}$ We choose to follow the approach of the OPAL collaboration [2–4, 50] because we use the same $\mathcal{O}(\alpha_s^2)$+NLLA calculations with the ln($R$)-matching. The OPAL collaboration required the experimental and hadronisation corrections to be reasonably uniform and not strongly model dependent within the fit ranges. In addition, bins at the edges of fit ranges with large $\chi^2$ contributions were removed. The fit ranges of distributions of the other experiments are adjusted to match the ones of OPAL as closely as possible.

Data with $\sqrt{s} < M_{Z^0}$ We use the studies of the recently reanalysed JADE data as a guideline [3, 18]. However, we require in addition that i) the fit ranges should extend less far into the 2-jet region than those used in [50] and ii) the distance between the extreme 2-jet region and the fit ranges should increase with decreasing $\sqrt{s}$. These requirements reduce the sensitivity of the analysis to hadronisation corrections which are generally larger in the 2-jet region than in the 3-jet region. We also demand that no single bin at the edge of a fit range should have a large $\chi^2$ contribution.

Within these ranges experimental corrections for limited acceptance and resolution as well as non-perturbative effects as estimated with Monte Carlo models are well under control.

3.2 Fit Procedure

The fits are based on equation (7) for the running of $\alpha_s$, on equation (12) for the perturbative QCD prediction and on equation (14) for the power corrections. We vary the strong coupling $\alpha_s(M_{Z^0})$ with the mass of the $Z^0$ boson as a reference scale, one colour factor, i.e. $n_f$, $C_A$ or $C_F$, and optionally the non-perturbative parameter $\alpha_0$ in the fits.
We also investigate fits where $\alpha_s(M_Z^0)$, the two colour factors $C_A$ and $C_F$ and optionally $\alpha_0$ are free parameters. Fixed parameters are always set to the values as expected in standard QCD, i.e. $n_f = 5$, $C_A = 3$ and $C_F = 4/3$. To avoid unphysical results and to stabilise the fits the free parameters are bounded by $0.01 < \alpha_s(M_Z^0) < 0.3$, $0 < n_f < 20$, $0 < C_A < 10$, $0 < C_F < 10$ and $0 < \alpha_0 < 10$.

The fit procedure minimises a $\chi^2$ constructed from the difference in bin $i$ between the data value $d_i$ and the theoretical prediction $t_i$ divided by the total error $\sigma_i$:

$$\chi^2 = \sum_{i \in \text{fit ranges}} \left( \frac{d_i - t_i}{\sigma_i} \right)^2.$$  \hspace{1cm} (18)

Possible correlations between bins of a given distribution or between different distributions are neglected. With the exception of the OPAL data for $1 - T$, $B_T$ and $B_W$ at $\sqrt{s} = M_Z^0$ the data are measured in bins which are wider than the typical experimental resolution in order to reduce bin-to-bin migrations. Many of the distributions used in this analysis were measured by the same experiments such that the presence of some correlation between distributions may be expected. There may also be correlations between the data points of any given distribution due to common systematic uncertainties. However, information about such correlations is not available in the references for the data (see table I).

The errors on the fitted parameters are calculated in the fit procedure from the diagonal elements of the covariance matrix after the fit has converged (“fit error”). These errors contain the contributions from statistical fluctuations and systematic uncertainties quoted by the individual experiments.

We follow two alternatives to study the influence of non-perturbative effects on our fits. Firstly, we fix the non-perturbative parameter $\alpha_0$ at a previously measured value and repeat the fits with $\alpha_0$ varied by its total errors. Secondly, we allow $\alpha_0$ to vary in the fit. The fixed value of $\alpha_0$ is taken from [6], $\alpha_0 = 0.473^{+0.058}_{-0.041}$. However, some modifications had to be applied. The measurement of [6] used an erroneous value of the Milan factor, $\mathcal{M} = 1.794$ instead of 1.49, both for $n_f = 3$. From equation (15) we infer that it is sufficient to scale $\alpha_0$ by the ratio of the two values for $\mathcal{M}$ as a correction. In our study we vary the colour factor $n_f$ and it would therefore be inconsistent to use two different values of $n_f$, i.e. $n_f = 5$ in the perturbative part and $n_f = 3$ in the non-perturbative part of the QCD prediction for massless quarks. To find our standard value of $\alpha_0$ we scale it by the ratio of values for the Milan factor determined with $n_f = 3$ and $n_f = 5$, respectively. Our final value is $\alpha_0 = 0.543 \pm 0.058$ with symmetrised errors.

### 3.3 Effects of $b\bar{b}$ events at low $\sqrt{s}$

The presence of events from the reaction $e^+e^- \rightarrow b\bar{b}$ at low cms energies $\sqrt{s}$ can distort the event shape distributions, because the effects of weak decays of heavy B-hadrons on the topology of hadronic events cannot anymore be neglected. An additional potential problem arises from comparing QCD calculations based on massless quarks with data containing massive quarks at $\sqrt{s}$ close to the production threshold.

At $\sqrt{s} \ll M_Z^0$ $b\bar{b}$ events constitute about 9% of the total event samples. Ideally one would correct the data experimentally by identifying $b\bar{b}$ events and removing them from...
the sample. However, since we have only published event shape data without information on specific quark flavours we resort to a correction based on Monte Carlo simulations. We generate samples of $10^6$ events at each $\sqrt{s}$ with the JETSET 7.4 program [28] with the parameter set given in [53]. For each event shape observable we build the ratio of distributions calculated with u, d, s and c quark events to those calculated with all events. This ratio is multiplied with the bin contents of the data to obtain corrected distributions. This procedure is applied to all data at $\sqrt{s} < M_{Z^0}$. It was verified that the simulation provides an adequate description of the data at all values of $\sqrt{s} < M_{Z^0}$.

Systematic effects due to uncertainties in the Monte Carlo parameters are expected to be small for the ratio except for those parameters which only affect the b$\bar{b}$ events in the samples. The most important such parameter is the value of $\epsilon_b$ in the Peterson fragmentation function [54] which controls the fragmentation of b quarks in the simulation. Threshold effects at low $\sqrt{s}$ which depend on the value of the b-quark mass in the simulation are found to be negligible for the fit results.

### 3.4 Systematic Variations

As systematic variations we consider the following changes in the analysis:

**Renormalisation scale** The standard fits are carried out with the renormalisation scale parameter $x_\mu = 1$. The dependence on the renormalisation scale is investigated by repeating the fits with $x_\mu = 0.5$ and $x_\mu = 2.0$ [50]. Deviations of the fit results w.r.t. the standard fits are taken as asymmetric uncertainties.

**Power corrections ($\alpha_0$ fixed)** The standard fits with fixed $\alpha_0$ employ $\alpha_0 = 0.543$, as explained above. The fits are repeated with $\alpha_0$ varied by its error, i.e. $\alpha_0 = 0.485$ and $\alpha_0 = 0.601$. Again, deviations of the fit results w.r.t. the standard fits are taken as asymmetric uncertainties.

**Power corrections ($\alpha_0$ free)** The standard fits use the Milan factor as given by equation (16). The fits are repeated with the Milan factor scaled by a factor of 0.8 and 1.2, respectively, and deviations w.r.t. results of the standard fits are considered as asymmetric uncertainties. We also change the value of the infrared matching scale $\mu_I$ from its standard value of 2 GeV to 1 GeV and 3 GeV, respectively, and repeat the fits. Deviations w.r.t. the standard results are counted as asymmetric uncertainties, except for $\alpha_0$ where changing $\mu_I$ corresponds to a redefinition of the non-perturbative parameter $\alpha_0$.

**Fragmentation of b quarks** The standard analysis is carried out with corrected data at $\sqrt{s} < M_{Z^0}$ based on the JETSET tuning of [53] as explained in section 3.3. The value of the JETSET parameter $\epsilon_b$ is varied around its central value $\epsilon_b = 0.0038 \pm 0.0010$ by adding or subtracting its error and the analysis including correction of the data at $\sqrt{s} < M_{Z^0}$ is repeated. Deviations w.r.t. the standard results are considered as asymmetric uncertainties.

**Experimental uncertainties** We change the composition of the data sets for the fits in two ways, i) the data measured at $\sqrt{s} = M_{Z^0}$ are removed and ii) only data measured
at $\sqrt{s} \geq M_{Z^0}$ are used. The larger of the two deviations w.r.t. the standard result observed with the data set i) or ii) is considered as the systematic uncertainty from variations in the input data.

The systematic uncertainties from the renormalisation scale, the power corrections with $\alpha_0$ either fixed or free, the fragmentation of $b$ quarks and from the input data are added in quadrature with the error from the fit to arrive at the total error.

### 3.5 Fit Results

The tables 4, 5 and 6 present the results of two-parameter fits to $\alpha_s(M_{Z^0})$ and $n_f$, $C_A$ or $C_F$, respectively, with $\alpha_0$ fixed. The results of three-parameter fits to $\alpha_s(M_{Z^0})$, $C_A$ and $C_F$ are given in table 7. The results of three-parameter fits to $\alpha_s(M_{Z^0})$, $\alpha_0$ and one of the colour factors $n_f$, $C_A$ or $C_F$ are shown in tables 8, 9 and 10 respectively, while table 11 displays the results of four-parameter fits to $\alpha_s(M_{Z^0})$, $\alpha_0$, $C_A$ and $C_F$. The rows labelled “hadr.” contain the quadratic sum of the uncertainties from the power corrections and the fragmentation of $b$-quarks. The power correction uncertainties are the dominating contribution in all cases. The tables show symmetrised errors except for the fit to $\alpha_s(M_{Z^0})$ and $n_f$ where the variation of $\alpha_0$ leads to a large difference between the positive and negative error. Figures 1, 2 and 3 give a graphic display of the results.

We observe stable fits in all cases for the observables $1 - T$ and $C$. The fit results from these observables for the colour factors are generally in good agreement with the expectations from standard QCD within their total uncertainties. The results for $\alpha_s(M_{Z^0})$ and $\alpha_0$ are also generally consistent with previous measurements within their total uncertainties 1, 2. The fit of $n_f$ and $\alpha_s(M_{Z^0})$ with $1 - T$ is consistent with an earlier analysis using similar data 20. The fit of $\alpha_s(M_{Z^0})$, $\alpha_0$, $C_A$ and $C_F$ with $C$ converges but has such large fit errors that sensitivity to the colour factors is essentially lost; consequently we do not show the results.

With the other two observables, $B_T$ and $B_W$, we also find generally consistent results with standard QCD and previous measurements. However, fits of $B_T$ to $\alpha_s(M_{Z^0})$ and $n_f$ and fits of of $B_W$ to $\alpha_s(M_{Z^0})$, $C_A$ and $C_F$ as well as to $\alpha_s(M_{Z^0})$ and single colour factors with $\alpha_0$ free are unstable and we do not show these results. Our interpretation is that the QCD calculations with power corrections for $1 - T$ and $C$ provide an adequate description of the data while there are still effects in the data for $B_T$ and $B_W$ which are not well described by the $\mathcal{O}(\alpha_s^2) +$NLLA QCD calculations with power corrections 21, 22, 23, 24.

The values of $\chi^2/d.o.f.$ are smaller than one in all cases, except the fit of $\alpha_s(M_{Z^0})$ and $C_A$ with $B_T$ where $\chi^2/d.o.f. = 1.01$. The small values of $\chi^2/d.o.f.$ are consistent with the good description of the data by the fitted predictions and probably indicate the presence of correlations between the data points used in a given fit.

We find that in all fits with $\alpha_0$ free the variation of the Milan factor leads to significantly smaller systematic uncertainties than the variation of $\alpha_0$ in fits with $\alpha_0$ fixed. The fitted values of $\alpha_0$ vary over ranges compatible with the total error of the fixed value of $\alpha_0$ when the Milan factor is changed. This is consistent with 21 where the error on $\alpha_0$ was dominated by the variation of the Milan factor. We conclude that the ratio of non-perturbative and perturbative contributions is well constrained by the data such that
changes in the Milan factor are compensated by similar changes with the opposite sign in \( \alpha_0 \). As a consequence we find that the hadronisation systematic uncertainties of fits to the colour factors are reduced when \( \alpha_0 \) is a free parameter while the fit errors increase due to the presence of an extra free parameter in the fit.

We observe that the experimental uncertainties estimated by repeating the fits with reduced data sets are larger in the fits with \( \alpha_0 \) free. This effect is likely to be a reflection of the increased fit errors. We choose to keep the larger uncertainties since we cannot rule out the possibility that there are systematic effects in the data to which the fits with \( \alpha_0 \) free are more sensitive than the fits with \( \alpha_0 \) fixed.

In simultaneous fits with several free parameters correlations between them may influence the results. In table 10 we show the correlations observed in the standard fits with the observable \( 1 - T \). In addition in the first row of table 10 the correlation observed in a fit with only \( \alpha_s(M_{Z^0}) \) and \( \alpha_0 \) as free parameters is displayed. The presence of large correlations in some fits causes increased fit errors and may also contribute to the experimental uncertainties. We find a similar pattern of correlations with the other observables \( C, B_T \) and \( B_W \).

### 3.6 Combination of Results

We construct combined results only from fits to \( 1 - T \) and \( C \), because we found these to be stable in all cases. We choose to base the combination for individual colour factors on the fits with \( \alpha_0 \) free since these results should be less biased by input values measured assuming standard QCD. They also have smaller hadronisation uncertainties as discussed above. We build unweighted averages of the fit results for the standard analysis and for all systematic variations. As the fit error we choose the smaller of the two individual fit errors and we then construct the total error in the same way as for an individual observable. In this way correlations between systematic variations of the observables are taken into account. We find as our final results for the individually measured colour factors:

\[
\begin{align*}
n_f & = 5.64 \pm 0.79(\text{fit}) \pm 0.32(\text{scale}) \pm 0.48(\text{had.}) \pm 0.93(\text{exp.}) \quad (19) \\
& = 5.64 \pm 1.35, \\
C_A & = 2.88 \pm 0.16(\text{fit}) \pm 0.06(\text{scale}) \pm 0.10(\text{had.}) \pm 0.18(\text{exp.}) \\
& = 2.88 \pm 0.27 \quad \text{and} \\
C_F & = 1.45 \pm 0.21(\text{fit}) \pm 0.04(\text{scale}) \pm 0.04(\text{had.}) \pm 0.16(\text{exp.}) \\
& = 1.45 \pm 0.27.
\end{align*}
\]

The total uncertainties for all three colour factors are dominated by the fit error and the experimental uncertainties. The results are in good agreement with the expectation from standard QCD with five active quark flavours. The results for \( \alpha_s(M_{Z^0}) \) and \( \alpha_0 \) from the combination are consistent with the individual fits and with previous measurements \([6,11]\); see table 11.

In the case of \( n_f \) the combined result has a slightly larger total uncertainty than the individual result based on the observable \( 1 - T \) shown in table 4. This is mainly due to
the increased experimental uncertainty of the combined result which in turn is dominated by the experimental uncertainty of the fit with \( C \). For the other colour factors \( C_A \) and \( C_F \) the total uncertainties of the combined results are somewhat smaller than the total uncertainties of the individual results. These observations are consistent, since we do not expect the total uncertainties to be greatly reduced by a combination of individual results due to the large correlations between the event shape distributions.

A combined result for the results of the simultaneous fits of \( C_A \) and \( C_F \) may only be constructed from the fits with \( \alpha_0 \) fixed, because in the fits with \( \alpha_0 \) free we obtained usable fits only for the observable \( 1 - T \). Applying the same procedure as outlined above yields the following results for combining the results of the fits with the observables \( 1 - T \) and \( C \) shown in table 5:

\[
\begin{align*}
C_A &= 2.84 \pm 0.13(\text{fit}) \pm 0.06(\text{scale}) \pm 0.11(\text{had.}) \pm 0.15(\text{exp.}) \\
&= 2.84 \pm 0.24 \\
C_F &= 1.29 \pm 0.07(\text{fit}) \pm 0.17(\text{had.}) \pm 0.02(\text{exp.}) \\
&= 1.29 \pm 0.18.
\end{align*}
\]

The scale uncertainty of the combined result for \( C_F \) is smaller than 0.01 consistent with the small scale uncertainties of the individual results, see table 5. The results for \( C_A \) and \( C_F \) are in good agreement with the combined results discussed above, see equation (19).

The averaged correlation coefficient from the fits is \( \rho_{C_A-C_F} = -0.89 \). However, we observe that variations of the results for \( C_A \) and \( C_F \) are positively correlated when \( \alpha_0 \) is changed. In order to find a conservative estimate of the correlation coefficient we construct a covariance matrix by summing i) the covariance matrix from the fit and ii) one covariance matrix for each systematic uncertainty. The covariance matrices ii) are constructed from the systematic uncertainties, symmetrised if necessary, and \( \rho_{C_A-C_F} = -0.89 \), except for the systematic uncertainty from the variation of \( \alpha_0 \) where \( \rho_{C_A-C_F} = 1.0 \) is used. The resulting total correlation coefficient is \( \rho_{C_A-C_F} = 0.19 \). Figure 4 presents the combined results which are in good agreement with standard QCD within the uncertainties. Some other possible gauge groups are indicated including \( U(1)^3 \), an abelian QCD with three quark colours and colour neutral gluons. Our results exclude all shown alternatives to \( SU(3) \) as the gauge group of QCD at more than 95% confidence level.

4 Summary and Conclusions

We have studied fits of \( \mathcal{O}(\alpha_s^2) \)+NLLA QCD predictions for distributions of the event shape observables \( 1 - T, C, B_T \) and \( B_W \) with power correction calculations to model hadronisation effects to data measured at cms energies ranging from 14 GeV to 189 GeV. In these fits we varied simultaneously the strong coupling \( \alpha_s(M_Z^0) \), one of the QCD colour factors \( C_A \) or \( C_F \), or the the number active quarks \( n_f \), and in some cases also \( \alpha_0 \), the free parameter of the power correction calculations. We investigated in addition fits, where \( \alpha_s(M_Z^0) \), \( C_A \) and \( C_F \) and optionally \( \alpha_0 \) were varied.
We found stable fits in all cases with the observables $1 - T$ and $C$ while some fits with $B_T$ and in particular $B_W$ appear unreliable. We take this as an indication that the current $O(\alpha_s^2) + \text{NLLA QCD}$ calculations with power corrections for $B_T$ and $B_W$ describe the data not as well as the same calculations for $1 - T$ and $C$.

We observed that the variation of the Milan factor $M$ and the infrared matching scale $\mu_I$ in the fits with $\alpha_0$ free gave rise to smaller systematic uncertainties than the systematic variation of $\alpha_0$ in the fits with $\alpha_0$ fixed. Our conclusion is that the relative contributions of the perturbative $O(\alpha_s^2) + \text{NLLA QCD}$ calculations and the power correction terms to the total predictions are quite well constrained by the data.

A combination of the individual results of the fits with $\alpha_s(M^0)$, an individual colour factor and $\alpha_0$ as free parameters with the observables $1 - T$ and $C$ yields our final results:

- $n_f = 5.64 \pm 1.35$,
- $C_A = 2.88 \pm 0.27$ and
- $C_F = 1.45 \pm 0.27$.

The combination of results from simultaneous fits of $C_A$, $C_F$ and $\alpha_s(M^0)$ for $\alpha_0$ fixed with the observables $1 - T$ and $C$ gives

$$C_A = 2.84 \pm 0.24 \quad \text{and} \quad C_F = 1.29 \pm 0.18$$

with total correlation coefficient $\rho_{C_A-C_F} = 0.19$. These results are in good agreement with the expectation from standard QCD based on the SU(3) symmetry group for $e^+e^-$ annihilation data at high energies, i.e. $n_f = 5$ with $T_F = 1/2$, $C_A = 3$ and $C_F = 4/3$. There is also good agreement between the individual and the simultaneous measurements of $C_A$ and $C_F$. All combined results for $\alpha_s(M^0)$ and $\alpha_0$ are in agreement with previous measurements. We found that the total uncertainties of the combined results are approximately of the same size as the total uncertainties of the individual results. This is consistent, because the event shape distributions are correlated with each other and the total uncertainties are dominated by the fit errors and experimental uncertainties.

We present our final conclusions from two points of view. Firstly, we assume that the power correction calculations are a good model of hadronisation effects in event shape distributions. In this case we have performed a complementary test of the gauge structure of QCD with competitive uncertainties on the measurements of the QCD colour factors compared to other analyses [19,20,23,24]. Secondly, under the assumption that QCD with SU(3) group symmetry is the correct theory of strong interactions, our analysis provides a successful consistency check of the power correction calculations.

References

[1] JADE and OPAL Coll., P. Pfeifenschneider et al.: Eur. Phys. J. C 17 (2000) 19–51
[2] OPAL Coll., G. Abbiendi et al.: Eur. Phys. J. C 16 (2000) 185–210
[3] OPAL Coll., K. Ackerstaff et al.: Z. Phys. C 75 (1997) 193–207
[4] OPAL Coll., G. Alexander et al.: Z. Phys. C 72 (1996) 191–206

[5] JADE Coll., P. A. Movilla Fernández, O. Biebel, S. Bethke, S. Kluth, P. Pfeifenschneider et al.: Eur. Phys. J. C 1 (1998) 461–478

[6] JADE Coll., O. Biebel, P. A. Movilla Fernández, S. Bethke et al.: Phys. Lett. B 459 (1999) 326

[7] H1 Coll., C. Adloff et al.: DESY-00-145, (2000), Sub. to Eur. Phys. J. C

[8] ZEUS Coll., M. Derrick et al.: Z. Phys. C 72 (1996) 399–424

[9] D0 Coll., B. Abbott et al.: Phys. Rev. Lett. 82 (1999) 2451–2456

[10] O. Biebel: Phys. Rep. 340 (2001) 165–289

[11] S. Bethke: J. Phys. G 26 (2000) R27

[12] T. Muta: Foundations of Quantum Chromo Dynamics. Vol. 5 of World Scientific Lecture Notes in Physics, World Scientific (1987)

[13] R.K. Ellis, W.J. Stirling and B.R. Webber: QCD and Collider Physics. Vol. 8 of Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, Cambridge University Press (1996)

[14] L3 Coll., B. Adeva et al.: Phys. Lett. B 248 (1990) 227

[15] OPAL Coll., M. Z. Akrawy et al.: Z. Phys. C 49 (1991) 49–57

[16] DELPHI Coll., P. Abreu et al.: Phys. Lett. B 255 (1991) 466

[17] ALEPH Coll., D. Decamp et al.: Phys. Lett. B 284 (1992) 151

[18] DELPHI Coll., P. Abreu et al.: Z. Phys. C 59 (1993) 357

[19] OPAL Coll., R. Akers et al.: Z. Phys. C 65 (1995) 367–377

[20] DELPHI Coll., P. Abreu et al.: Phys. Lett. B 414 (1997) 401–418

[21] N. Magnoli, P. Nason and R. Rattazzi: Phys. Lett. B 252 (1990) 271–281

[22] OPAL Coll., R. Akers et al.: Z. Phys. C 68 (1995) 519–530

[23] ALEPH Coll., R. Barate et al.: Z. Phys. C 76 (1997) 1–14

[24] B. Dienes: forthcoming, Presented at 30th International Symposium on Multiparticle Dynamics, Tihany, Hungary, October 9 - 15, 2000

[25] G. R. Farrar: In: 11ème Rencontre de Physique de la Vallée d’Aoste: Results and Perspectives in Particle Physics, La Thuile, Italy, 2-8 Mar 1997, M. Greco (ed.), Frascati physics series 9, pages 779–792. INFN Frascati, 1997
[26] T. Hebbeker: Z. Phys. C 60 (1993) 63–70
[27] L3 Coll., M. Acciarri et al.: Phys. Lett. B 489 (2000) 65–80
[28] T. Sjöstrand: Comput. Phys. Commun. 82 (1994) 74
[29] G. Marchesini et al.: Comput. Phys. Commun. 67 (1992) 465–508
[30] L. Lönnblad: Comput. Phys. Commun. 71 (1992) 15–31
[31] Yu.L. Dokshitzer, G. Marchesini and B.R. Webber: Nucl. Phys. B 469 (1996) 93–142
[32] Yu. L. Dokshitzer and B. R. Webber: Phys. Lett. B 352 (1995) 451–455
[33] Yu. L. Dokshitzer and B. R. Webber: Phys. Lett. B 404 (1997) 321–327
[34] Yu. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam: Nucl. Phys. B 511 (1998) 396–418
[35] S. Catani and B. R. Webber: Phys. Lett. B 427 (1998) 377–384
[36] Yu. L. Dokshitzer, G. Marchesini and G. P. Salam: Eur. Phys. J. direct C 3 (1999) 1–45
[37] Yu. L. Dokshitzer: [hep-ph/9911299](hep-ph/9911299) (1999), Invited talk at 11th Rencontres de Blois: Frontiers of Matter, Chateau de Blois, France, 28 Jun - 3 Jul 1999
[38] DELPHI Coll., P. Abreu et al.: Phys. Lett. B 456 (1999) 322–340
[39] H1 Coll., C. Adloff et al.: Eur. Phys. J. C 14 (2000) 255–269
[40] P. A. Movilla Fernández, O. Biebel and S. Bethke: PITHA 99/21, [hep-ex/9906033](hep-ex/9906033) (1999), Contributed to International Europhysics Conference on High-Energy Physics (EPS-HEP 99), Tampere, Finland, 15-21 July
[41] S. Brandt, Ch. Peyrou, R. Sosnowski and A. Wroblewski: Phys. Lett. 12 (1964) 57–61
[42] E. Fahri: Phys. Rev. Lett. 39 (1977) 1587–1588
[43] G. Parisi: Phys. Lett. B 74 (1978) 65
[44] J. F. Donoghue, F. E. Low and S.Y. Pi: Phys. Rev. D 20 (1979) 2759
[45] S. Catani, G. Turnock and B.R. Webber: Phys. Lett. B 295 (1992) 269
[46] R.K. Ellis, D.A. Ross and A.E. Terrano: Nucl. Phys. B 178 (1981) 421
[47] S. Catani and M.H. Seymour: Phys. Lett. B 378 (1996) 287–301
[48] S. Catani, L. Trentadue, G. Turnock and B.R. Webber: Nucl. Phys. B 407 (1993) 3
[49] Yu. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam: JHEP 1 (1998) 11
[50] OPAL Coll., P. D. Acton et al.: Z. Phys. C 59 (1993) 1–19
[51] Yu. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam: JHEP 5 (1998) 3
[52] P. A. Movilla Fernández: Nucl. Phys. Proc. Suppl. 74 (1999) 384–387
[53] OPAL Coll., G. Alexander et al.: Z. Phys. C 69 (1996) 543–560
[54] C. Peterson, D. Schlatter, I. Schmitt and P. Zerwas: Phys. Rev. D 27 (1983) 105
[55] ALEPH Coll., D. Busculic et al.: Z. Phys. C 73 (1997) 409–420
[56] ALEPH Coll., D. Buskulic et al.: Z. Phys. C 55 (1992) 209–234
[57] DELPHI Coll., P. Abreu et al.: Z. Phys. C 73 (1996) 11–60
[58] L3 Coll., B. Adeva et al.: Z. Phys. C 55 (1992) 39–62
[59] OPAL Coll., P. D. Acton et al.: Z. Phys. C 55 (1992) 1–24
[60] SLD Coll., K. Abe et al.: Phys. Rev. D 51 (1995) 962
[61] AMY Coll., Y.K. Li et al.: Phys. Rev. D 41 (1990) 2675–2688
[62] TASSO Coll., W. Braunschweig et al.: Z. Phys. C 47 (1990) 187–198
[63] HRS Coll., D. Bender et al.: Phys. Rev. D 31 (1985) 1
[64] MARK II Coll., A. Petersen et al.: Phys. Rev. D 37 (1988) 1
| $\sqrt{s}$ | Experiment | $1 - T$ | $C$     | $B_T$   | $B_W$   |
|----------|------------|---------|---------|---------|---------|
| 189      | L3 [27]    | 0.05-0.30 | 0.15-0.60 | 0.08-0.26 | 0.045-0.195 |
|          | OPAL [2]   | 0.04-0.30 | 0.18-0.60 | 0.075-0.25 | 0.05-0.20   |
| 183      | DELPHI [38]| 0.04-0.28 | 0.16-0.64 | 0.07-0.24  | 0.05-0.20   |
|          | L3 [27]    | 0.05-0.30 | 0.15-0.60 | 0.08-0.26  | 0.045-0.195 |
|          | OPAL [2]   | 0.04-0.30 | 0.18-0.60 | 0.075-0.25 | 0.05-0.20   |
| 172      | DELPHI [38]| 0.04-0.24 | 0.16-0.64 | 0.08-0.27  | 0.04-0.17   |
|          | L3 [27]    | 0.05-0.30 | 0.15-0.60 | 0.08-0.26  | 0.045-0.195 |
|          | OPAL [2]   | 0.04-0.30 | 0.18-0.60 | 0.075-0.25 | 0.05-0.20   |
| 161      | DELPHI [38]| 0.04-0.24 | 0.16-0.64 | 0.08-0.27  | 0.04-0.17   |
|          | L3 [27]    | 0.05-0.30 | 0.15-0.60 | 0.08-0.26  | 0.045-0.195 |
|          | OPAL [2]   | 0.04-0.30 | 0.18-0.60 | 0.075-0.25 | 0.05-0.20   |
| 133      | ALEPH [52] | 0.04-0.30 |          |          |          |
|          | DELPHI [38]| 0.04-0.24 | 0.16-0.64 | 0.08-0.27  | 0.04-0.17   |
|          | L3 [27]    | 0.05-0.25 | 0.15-0.64 | 0.08-0.26  | 0.045-0.195 |
|          | OPAL [4]   | 0.04-0.30 | 0.18-0.60 | 0.075-0.25 | 0.05-0.20   |
| 91       | ALEPH [56] | 0.06-0.30 | 0.20-0.64 |          |          |
|          | DELPHI [57]| 0.06-0.30 | 0.20-0.64 | 0.09-0.24  | 0.07-0.17   |
|          | L3 [28]    | 0.065-0.29| 0.22-0.64 |          |          |
|          | OPAL [50, 59]| 0.06-0.30| 0.20-0.64 | 0.09-0.23  | 0.07-0.17   |
|          | SLD [60]   | 0.06-0.26 | 0.24-0.64 | 0.08-0.26  | 0.08-0.20   |
| 55       | AMY [61]   | 0.10-0.30 |          |          |          |
| 44       | JADE [5]   | 0.08-0.30 | 0.24-0.58 | 0.10-0.24  | 0.08-0.18   |
|          | TASSO [62] | 0.08-0.28 |          |          |          |
| 35       | JADE [5]   | 0.08-0.30 | 0.24-0.58 | 0.10-0.24  | 0.08-0.18   |
|          | TASSO [62] | 0.08-0.28 |          |          |          |
| 29       | HRS [62]   | 0.10-0.30 |          |          |          |
|          | MARKII [4] | 0.10-0.30 |          |          |          |
| 22       | TASSO [62] | 0.10-0.28 |          |          |          |
| 14       | TASSO [62] | 0.12-0.28 |          |          |          |

Table 1: The sources of the data and the fit ranges for the observables $1 - T$, $C$, $B_T$ and $B_W$ are shown. The cms energy $\sqrt{s}$ at which the experiments analysed their data is given in GeV.
Table 2: Results are shown for fits with the observables $1 - T$, $C$ and $B_W$ to $\alpha_s(M_{Z^0})$ and $n_f$.

|         | $1 - T$                  | $C$                 | $B_W$            |
|--------|--------------------------|---------------------|------------------|
|        | $n_f$ | $\alpha_s(M_{Z^0})$ | $n_f$ | $\alpha_s(M_{Z^0})$ | $n_f$ | $\alpha_s(M_{Z^0})$ |
| fit result | 7.32 | 0.124         | 5.82 | 0.114         | 2.06 | 0.093         |
| fit error  | $\pm 0.49$ | $\pm 0.002$ | $\pm 0.52$ | $\pm 0.002$ | $\pm 1.04$ | $\pm 0.003$ |
| $\chi^2$/d.o.f. | 130.8/236 | 104.0/155 | 109.0/121 |
| ren. scale | $\pm 0.11$ | $\pm 0.003$ | $\pm 0.55$ | $\pm 0.002$ | $\pm 1.33$ | $\pm 0.002$ |
| hadr. | $\pm 0.07$ | $\pm 0.000$ | $\pm 2.30$ | $\pm 0.006$ | $\pm 1.47$ | $\pm 0.003$ |
| exp. | $\pm 0.42$ | $\pm 0.003$ | $\pm 1.95$ | $\pm 0.006$ | $\pm 2.89$ | $\pm 0.010$ |
| tot. error | $\pm 0.08$ | $\pm 0.008$ | $\pm 3.14$ | $\pm 0.009$ | $\pm 3.66$ | $\pm 0.011$ |

Table 3: Results are shown for fits with the observables $1 - T$, $C$, $B_T$ and $B_W$ to $\alpha_s(M_{Z^0})$ and $C_A$.

|         | $1 - T$                  | $C$                 | $B_T$            | $B_W$            |
|--------|--------------------------|---------------------|------------------|------------------|
|        | $C_A$ | $\alpha_s(M_{Z^0})$ | $C_A$ | $\alpha_s(M_{Z^0})$ | $C_A$ | $\alpha_s(M_{Z^0})$ |
| fit result | 2.62 | 0.123         | 2.88 | 0.113         | 3.82 | 0.101         |
| fit error  | $\pm 0.08$ | $\pm 0.002$ | $\pm 0.07$ | $\pm 0.002$ | $\pm 0.10$ | $\pm 0.002$ |
| $\chi^2$/d.o.f. | 128.8/236 | 103.7/155 | 139.6/138 | 105.9/121 |
| ren. scale | $\pm 0.03$ | $\pm 0.004$ | $\pm 0.08$ | $\pm 0.003$ | $\pm 0.21$ | $\pm 0.003$ |
| hadr. | $\pm 0.42$ | $\pm 0.006$ | $\pm 0.49$ | $\pm 0.008$ | $\pm 0.36$ | $\pm 0.004$ |
| exp. | $\pm 0.07$ | $\pm 0.003$ | $\pm 0.31$ | $\pm 0.005$ | $\pm 0.23$ | $\pm 0.003$ |
| tot. error | $\pm 0.43$ | $\pm 0.008$ | $\pm 0.59$ | $\pm 0.010$ | $\pm 0.49$ | $\pm 0.006$ |

Table 4: Results are shown for fits with the observables $1 - T$, $C$, $B_T$ and $B_W$ to $\alpha_s(M_{Z^0})$ and $C_F$.
| 1 – T | C |
|-------|---|
| \( C_A \) | \( C_F \) | \( \alpha_s(M_Z^0) \) | \( C_A \) | \( C_F \) | \( \alpha_s(M_Z^0) \) |
| fit result | 2.72 | 1.28 | 0.124 | 2.96 | 1.30 | 0.114 |
| fit error | ±0.13 | ±0.07 | ±0.002 | ±0.18 | ±0.08 | ±0.002 |
| \( \chi^2/\text{d.o.f.} \) | 127.9/235 | | 103.6/154 | |
| ren. scale | ±0.04 | ±0.00 | ±0.004 | ±0.08 | ±0.00 | ±0.003 |
| hadr. | ±0.10 | ±0.17 | ±0.010 | ±0.12 | ±0.16 | ±0.009 |
| exp. | ±0.11 | ±0.13 | ±0.005 | ±0.40 | ±0.10 | ±0.004 |
| tot. error | ±0.20 | ±0.22 | ±0.012 | ±0.46 | ±0.21 | ±0.011 |

| \( B_T \) |
|-------|---|
| \( C_A \) | \( C_F \) | \( \alpha_s(M_Z^0) \) |
| fit result | 3.39 | 1.58 | 0.097 |
| fit error | ±0.21 | ±0.06 | ±0.002 |
| \( \chi^2/\text{d.o.f.} \) | 128.0/137 | |
| ren. scale | ±0.10 | ±0.01 | ±0.002 |
| hadr. | ±0.16 | ±0.19 | ±0.008 |
| exp. | ±0.48 | ±0.02 | ±0.005 |
| tot. error | ±0.56 | ±0.20 | ±0.010 |

Table 5: Results are shown for fits with the observables 1 – T, C and \( B_T \) to \( \alpha_s(M_Z^0) \), \( C_A \) and \( C_F \).

| 1 – T | C |
|-------|---|
| \( n_f \) | \( \alpha_s(M_Z^0) \) | \( \alpha_0 \) | \( n_f \) | \( \alpha_s(M_Z^0) \) | \( \alpha_0 \) |
| fit result | 6.39 | 0.121 | 0.521 | 4.88 | 0.111 | 0.526 |
| fit error | ±0.79 | ±0.003 | ±0.015 | ±1.34 | ±0.004 | ±0.023 |
| \( \chi^2/\text{d.o.f.} \) | 127.9/235 | 103.4/154 | |
| ren. scale | ±0.21 | ±0.004 | ±0.001 | ±0.42 | ±0.003 | ±0.005 |
| hadr. | ±0.47 | ±0.001 | ±0.057 | ±0.48 | ±0.001 | ±0.005 |
| exp. | ±0.69 | ±0.004 | ±0.010 | ±2.55 | ±0.008 | ±0.038 |
| tot. error | ±1.17 | ±0.006 | ±0.060 | ±2.96 | ±0.010 | ±0.079 |

| \( B_T \) |
|-------|---|
| \( n_f \) | \( \alpha_s(M_Z^0) \) | \( \alpha_0 \) |
| fit result | 3.77 | 0.108 | 0.635 |
| fit error | ±1.12 | ±0.003 | ±0.019 |
| \( \chi^2/\text{d.o.f.} \) | 132.1/137 | |
| ren. scale | ±0.46 | ±0.004 | ±0.006 |
| hadr. | ±0.33 | ±0.001 | ±0.089 |
| exp. | ±2.97 | ±0.009 | ±0.002 |
| tot. error | ±3.22 | ±0.010 | ±0.091 |

Table 6: Results are shown for fits with the observables 1 – T, C and \( B_T \) to \( \alpha_s(M_Z^0) \), \( \alpha_0 \) and \( n_f \).
|       | $1 - T$       | $C$                          |
|-------|---------------|------------------------------|
|       | $C_A$         | $\alpha_s(M_{Z^0})$ | $\alpha_0$ | $C_A$         | $\alpha_s(M_{Z^0})$ | $\alpha_0$ |
| fit result | 2.73          | 0.121            | 0.528     | 3.03          | 0.111            | 0.528     |
| fit error  | ±0.16         | ±0.003            | ±0.020    | ±0.25         | ±0.004            | ±0.029    |
| $\chi^2$/d.o.f. | 128.1/235      |                       |           | 103.4/154      |                       |           |
| ren. scale | ±0.05         | ±0.004            | ±0.001    | ±0.08         | ±0.003            | ±0.002    |
| hadr.     | ±0.10         | ±0.001            | ±0.057    | ±0.09         | ±0.001            | ±0.064    |
| exp.      | ±0.14         | ±0.004            | ±0.012    | ±0.50         | ±0.008            | ±0.053    |
| tot. error | ±0.24         | ±0.006            | ±0.061    | ±0.57         | ±0.009            | ±0.089    |

Table 7: Results are shown for fits with the observables $1 - T$, $C$ and $B_T$ to $\alpha_s(M_{Z^0})$, $\alpha_0$ and $C_A$.

|       | $1 - T$       | $C$                          |
|-------|---------------|------------------------------|
|       | $C_F$         | $\alpha_s(M_{Z^0})$ | $\alpha_0$ | $C_F$         | $\alpha_s(M_{Z^0})$ | $\alpha_0$ |
| fit result | 1.42          | 0.113            | 0.478     | 1.49          | 0.105            | 0.490     |
| fit error  | ±0.21         | ±0.009            | ±0.041    | ±0.31         | ±0.011            | ±0.074    |
| $\chi^2$/d.o.f. | 129.4/137      |                       |           | 103.2/154      |                       |           |
| ren. scale | ±0.03         | ±0.006            | ±0.013    | ±0.10         | ±0.006            | ±0.020    |
| hadr.     | ±0.03         | ±0.001            | ±0.053    | ±0.06         | ±0.002            | ±0.059    |
| exp.      | ±0.21         | ±0.011            | ±0.022    | ±0.10         | ±0.004            | ±0.031    |
| tot. error | ±0.30         | ±0.015            | ±0.072    | ±0.35         | ±0.014            | ±0.103    |

Table 8: Results are shown for fits with the observables $1 - T$, $C$, $B_T$ and $B_W$ to $\alpha_s(M_{Z^0})$, $\alpha_0$ and $C_F$. 
Table 9: Results are shown for fits with the observables $1 - T$ to $\alpha_s(M_{Z^0})$, $\alpha_0$, $C_A$ and $C_F$.

| fit result | $C_A$ | $C_F$ | $\alpha_s(M_{Z^0})$ | $\alpha_0$ |
|------------|-------|-------|---------------------|------------|
| fit error  | ±0.19 | ±0.26 | ±0.016             | ±0.093     |
| $\chi^2$/d.o.f. | | | | |
| ren. scale | ±0.03 | ±0.02 | ±0.005             | ±0.006     |
| hadr.      | ±0.16 | ±0.14 | ±0.010             | ±0.042     |
| exp.       | ±0.17 | ±0.35 | ±0.025             | ±0.164     |
| tot. error | ±0.30 | ±0.46 | ±0.032             | ±0.194     |

Table 10: The correlations between the fit parameters for all types of standard fits to the $1 - T$ distributions are shown.

| fit type | $\alpha_0$ | $n_f$ | $C_A$ | $C_F$ |
|----------|------------|-------|-------|-------|
| $\alpha_s(M_{Z^0})-\alpha_0$ | $\alpha_s$ | -0.87 | | |
| $\alpha_s(M_{Z^0})-n_f$ | $\alpha_s$ | 0.985 | | |
| $\alpha_s(M_{Z^0})-C_A$ | $\alpha_s$ | -0.981 | | |
| $\alpha_s(M_{Z^0})-C_F$ | $\alpha_s$ | | -0.997 | |
| $\alpha_s(M_{Z^0})-C_A-C_F$ | $\alpha_s$ | | 0.20 | -0.68 |
| $\alpha_s(M_{Z^0})-\alpha_0-n_f$ | $\alpha_s$ | 0.69 | 0.97 | 0.82 |
| $\alpha_s(M_{Z^0})-\alpha_0-C_A$ | $\alpha_s$ | 0.78 | -0.97 | -0.89 |
| $\alpha_s(M_{Z^0})-\alpha_0-C_F$ | $\alpha_s$ | 0.96 | | -0.998 |
| $\alpha_s(M_{Z^0})-\alpha_0-C_A-C_F$ | $\alpha_s$ | 0.988 | -0.72 | -0.985 |

Table 11: The combined results based on $1 - T$ and $C$ are shown with total errors. The results for $C_A$, $C_F$ and $\alpha_s(M_{Z^0})$ are from fits with $\alpha_0$ fixed.

| | $n_f$ | $\alpha_s(M_{Z^0})$ | $\alpha_0$ | $C_A$ | $\alpha_s(M_{Z^0})$ | $\alpha_0$ |
|---|------|---------------------|------------|-------|---------------------|------------|
| result | 5.64 | 0.116              | 0.524      | 2.88  | 0.116              | 0.526      |
| error  | ±1.35| ±0.005             | ±0.064     | ±0.27 | ±0.005             | ±0.067     |

| | $C_F$ | $\alpha_s(M_{Z^0})$ | $\alpha_0$ | $C_A$ | $C_F$ | $\alpha_s(M_{Z^0})$ |
|---|------|---------------------|------------|-------|------|---------------------|
| result | 1.45 | 0.109              | 0.484      | 2.84  | 1.29 | 0.119               |
| error  | ±0.27| ±0.013             | ±0.075     | ±0.24 | ±0.18 | ±0.10               |
Figure 1: The figures present results of fits to $\alpha_s(M_{Z^0})$ and one of the colour factors $n_f$, $C_A$ or $C_F$ with observables as given on the vertical axis. The vertical dotted lines indicate the expectation from standard QCD for the colour factor.
Figure 2: The figure presents results from fits to $\alpha_s(M_Z)$ and the colour factors $C_A$ and $C_F$ with observables as given on the vertical axis. The error bars show total uncertainties. The vertical dotted lines indicate the expectations from standard QCD for the colour factors.
Figure 3: The figures present results from fits to $\alpha_s(M^0)$, $\alpha_0$ and one of the colour factors $n_f$, $C_A$ or $C_F$ with observables as given on the vertical axis. The error bars show total uncertainties. The vertical dotted lines indicate the expectation from standard QCD for the colour factor.
Figure 4: The figure presents the combined results for the colour factors $C_A$ and $C_F$ from fits to $\alpha_s(M_Z^0)$, $C_A$ and $C_F$ based on the observables $1 - T$ and $C$. The square and triangle symbols indicate the expectations for $C_A$ and $C_F$ for different symmetry groups.