Recently, topological semimetals have attracted considerable eyes of researchers. Compared to topological insulator, topological semimetals have gapless bulk states and topologically protected surface Fermi arc states. There exist different types of topological semimetals, such as Dirac semimetal (DSM) [1, 2], Weyl semimetal (WSM) [3–6], and nodal line semimetal (NLSM) [7–10]. WSM was proposed to separate a single Dirac node into two Weyl nodes by breaking either time reversal symmetry or inversion symmetry. The surface states of Weyl semimetal become Fermi arc between a pair of Weyl points with opposite chiralities. Moreover, Weyl semimetals have been found in experiments such as TaAs family [11–14]. Nodal line semimetal is a three-dimensional graphene-like system with low-energy relativistic excitations, but the band touches are closed loop in momentum space instead of points. The surface states of node-line semimetal have drumheadlike surface flat bands. The node-line semimetal is also realized in experiments (For example Ca$_3$P$_2$ [15] and Cu$_3$PdN [16]).

In addition, new types of WSMs are proposed which are called type-II Weyl semimetal [17] and Hybrid (type-1.5) Weyl semimetal [18, 19]. In these types of WSMs, Lorentz invariance of low-energy excitations is broken. As a result, the nodes are tilted along given directions (see FIG.1(a) and (b)) and the transport properties become anisotropic. There are many remarkable phenomena appearing in type-II WSMs, such as the anisotropic negative magnetoresistance effect caused by Landau level collapse [20, 21] and the existence of tilted surface states [13]. In Hybrid (type-1.5) WSM, because the remaining symmetry (inversion symmetry, time reversal symmetry or mirror symmetry) of two nodes is broken, one Weyl node belongs to type-I and the other Weyl node belongs to type-II. These new types of WSMs attracted plenty of studies in past two years.

In this paper, based on a tight-binding model, we point out that there exists a new type of NLSM named type-II NLSM. For type-II NLSM, the zero energy bulk states have a closed loop in momentum space but the (local) Weyl cones on nodal line become tilted (see FIG.1(c) and (d)). In sec.II and sec.III, we introduce a two-band tight-binding model that describes type-II NLSM. In sec.IV, we study the effect of magnetic field on type-II NLSM and show the Landau level collapse in this system. Next, we study the correlation effect on type-II NLSM and the interaction-induced magnetic order of NLSM is found in sec.V. An interesting result is at critical point between type-I NLSM and type-II NLSM, arbitrary tiny interaction induces ferromagnetic order (FM) due to a flat band at Fermi surface. Finally, we give the conclusion and propose an experimental realization in sec.VI.
II. THE NODAL LINE HAMILTONIAN IN REAL SPACE ON CUBIC LATTICE

Firstly, we start with a nodal line semimetal from a three dimensional (3D) tight-binding Hamiltonian on cubic lattice that is given by

\[
H_0 = t_{x/y/z} \sum_{i,a} (-1)^a \left( c_{i,a}^\dagger \hat{c}^\dagger_{i+\delta_{i/2,z},a} + h.c. \right) \\
- 2t_{xy} \sum_{i,a} (-1)^a \left( c_{i,a}^\dagger \hat{c}^\dagger_{i+\delta_{i,0},a} - 2t_{z0} \sum_{i,a} (-1)^a c_{i,a}^\dagger \hat{c}_{i,a} \right) \\
+ t'_{xxs} \sum_{i} \left( e^{-ik_{xy} \cdot \vec{r}_{i,1}} \hat{c}_{i+\vec{b}_{i,2}}^\dagger + e^{ik_{xy} \cdot \vec{r}_{i,1}} \hat{c}_{i-\vec{b}_{i,2}}^\dagger \right) \\
- e^{-ik_{xy} \cdot \vec{r}_{i,1}} \hat{c}_{i+\vec{b}_{i,2}}^\dagger \hat{c}_{i-\vec{b}_{i,2}}^\dagger + h.c. \right) \\
+ t'_{yys} \sum_{i} \left( -ie^{-ik_{xy} \cdot \vec{r}_{i,1}} \hat{c}_{i+\vec{b}_{i,2}}^\dagger - ie^{ik_{xy} \cdot \vec{r}_{i,1}} \hat{c}_{i-\vec{b}_{i,2}}^\dagger \right) \\
+ ie^{-ik_{xy} \cdot \vec{r}_{i,1}} \hat{c}_{i+\vec{b}_{i,2}}^\dagger \hat{c}_{i-\vec{b}_{i,2}}^\dagger + h.c. \right) \tag{1}
\]

where \(a = 1, 2\) is the orbital degree of freedom. \(\hat{c}_{i,a}\) is the annihilation operator of the electron at the site \(i\) with orbital degree of freedom. \(t_{x/y/z}\) are the nearest neighbor hoppings in \(x/y/z\) direction, \(t'_{xxs}/t'_{yys}\) are the orbital-flip hoppings in \(xoz/yoz\) plane. \(t_{xy}/t_{z0}\) are the effective Zeeman field. \(k_0\) is set to be unit. It is obvious that not only the inversion symmetry but also the time-reversal symmetry are broken.

Using Fourier transformation, we obtain the Hamiltonian in momentum space

\[
H_0 = \sum_{k} C_k^\dagger \mathcal{H}(k) C_k \tag{2}
\]

with

\[
\mathcal{H}(k) = \vec{h}(k) \cdot \vec{\sigma} \tag{3}
\]

where \(C_k = \left( C_{k,1}, C_{k,2} \right) \), \(\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)\) is the Pauli matrix, and \(\vec{h}(k) = (h_x(k), h_y(k), h_z(k))\) with

\[
h_x(k) = 4t'_{xz} \sin(k_x - k_{z0}) \sin(k_z) \\
h_y(k) = 4t'_{yz} \sin(k_y - k_{y0}) \sin(k_z) \\
h_z(k) = -2t_{x} \cos k_x - 2t_{y} \cos k_y - 2t_{z} \cos k_z + 2t_{x/y} (1 + \cos k_0) + 2t_{z0}
\]

Then, the spectrum for free fermions is derived as

\[
E_{k,\pm} = \pm \sqrt{h_x^2(k) + h_y^2(k) + h_z^2(k)} \tag{4}
\]

In the following parts of the paper, the hopping parameters are set to \(t_x = t_y = t_z = t_{x/y} = t_{z0} = t\).

FIG. 2: (Color online) (a) The illustration of the nodal line locates at \(k_x-k_y\) plane for \(k_z = 0\) with the radius of \(k_0 = \pi/2\). (b) The surface states with periodic boundary conditions along \(x\) and \(y\)-direction but open boundary condition along \(z\)-direction. There is a drumhead inside the nodal line. The parameters are \(k_0 = \pi/2, t'_{xz} = t'_{yz} = 0.5t\).

Next, we study the nodal line of the nodal line semimetal. In \(k_x-k_y\) plane, the nodal line satisfy the equation of

\[
\cos k_x + \cos k_y = 1 + \cos k_0 \tag{5}
\]

FIG. 2(b) shows the spectrum at \(k_z = 0\). For this case, the nodal line locates at \(k_x-k_y\) plane with the radius of \(k_0 = 0\). Comparing with FIG. 2(a), one can see that there exists a drumhead induced by the nodal line which is consistent with previous articles [15, 22], and the fermi surface like a disk in the core of the BZ.

III. TYPE II NODAL LINE SEMIMETAL

In this part, a new type of NLSM named type-II NLSM is proposed. To get a typical type-II NLSM, we add a new term into the original model as

\[
\mathcal{H}(k) = \vec{h}(k) \cdot \vec{\sigma} + h_0 I \tag{6}
\]

with

\[
h_0(k) = C[-2t_x \cos(k_x) - 2t_y \cos(k_y) \\
+ 2t_{x/y} (1 + \cos(k_0))] \tag{7}
\]

\(C\) is a coefficient that determines the type of a NLSM. \(|C| = 1\) is a critical point: when \(|C| < 1\), the NLSM belongs to type-I nodal line SM; when \(|C| > 1\), the NLSM belongs to type-II nodal line SM. At the critical point
When $C > 0$, the tilting of the spectra towards to the center of the nodal line; while away from the center when $C < 0$. Numerical calculation of dispersions is shown in FIG.3, the coefficient $C > 0$ for (a)-(c), while $C < 0$ for (d)-(f). We can see clearly that the tilting of the nodal line towards to the center of the nodal line when $C > 0$; while away from the center when $C < 0$. The Fermion surface of the bulk system for $|C| = 0.6$, 1.0 and 1.5.

We then study the topological properties of Type-II nodal line SM. The topological protected surface state is a hallmark of topological system. In type-II nodal line semimetal, the surface states show similar behavior of the nodal states on bulk system – the surface states can also tilted and becomes ‘type-II’. In tilted NLSM, the surface states are shown in FIG.4 which are top views from z axis for lowest two bands near Fermi surface. In FIG.3 the coefficient $C > 0$ for (a)-(c), while $C < 0$ for (d)-(f). Due to the tilting effect for the case of $C \neq 0$, the drumhead-like surface flat band like FIG.2(b) disappears and instead by a dispersive one. Thus, the surface states in NLSM can also be tilted like nodal line in bulk, which is similar as type-II Weyl semimetal[17].

We discuss the evolution of Fermi surface of lowest energy band of bulk states. In type-I NLSM with $C = 0$, the Fermi surface of bulk states is a circle at $\mu = 0$ (here $\mu$ is the chemical potential). At the critical point $|C| = 1$, one band of NLSM becomes flat, which leads to a tilted surface state. While the Fermi surface of surface states is a disk at $\mu = 0$ when $C = 0$. At the critical point $|C| = 1$, it becomes a flat band with a hole in the center like FIG.3(h).

In addition, we also calculate the density of states (DOS). The expression for calculating DOS is

$$\rho(\omega) = -\frac{1}{\pi} \text{Im} \sum_{\sigma, k} G_{\sigma}(\omega, k)$$

where $G_{\sigma}(\omega, k)$ is Matsubara Green’s Function which are

$$G_{\uparrow}(\omega, k) = \frac{|E_{k,\pm}| + h_{z}}{2|E_{k,\pm}|} \frac{1}{\omega + i\eta - (h_{0} + E_{k,\uparrow})} + \frac{|E_{k,\pm}| - h_{z}}{2|E_{k,\pm}|} \frac{1}{\omega + i\eta - (h_{0} + E_{k,\downarrow})},$$

(8)

$$G_{\downarrow}(\omega, k) = \frac{|E_{k,\pm}| - h_{z}}{2|E_{k,\pm}|} \frac{1}{\omega + i\eta - (h_{0} + E_{k,\uparrow})} + \frac{|E_{k,\pm}| + h_{z}}{2|E_{k,\pm}|} \frac{1}{\omega + i\eta - (h_{0} + E_{k,\downarrow})}.$$

(9)

Here $\eta$ is an infinite small quantity and real, $\omega$ is the energy level. After considering the tilting effect on the spectra, the DOS changes correspondingly. In FIG.4(a) there always exists a sharp peak at $E = 0$ due to the flat band states for type-II NLSM. In FIG.4(b), for $k_{z} = 0$, owing to the existence of bulk flat band, there exists a sharp peak at $E = 0$ for the case of $|C| = 1$. 

![FIG. 4: (Color online) The dispersion of surface states of type-II nodal line SM. (a)-(c) are $C = 0.6$, $C = 1.0$ and $C = 1.5$; (d)-(f) are $C = -0.6$, $C = -1.0$ and $C = -1.5$. Due to the existence of the tilting term of $C \neq 0$, the drumhead-like surface flat bands like FIG.2(b) disappears. For the case of $|C| < 1$ ((a) and (d)), the Fermi surface is a circle (like (g)); at the critical point $|C| = 1$ ((b) and (e)), the Fermi surface changes into a flat band with a hole in the center (like (h)); for the case of $|C| > 1$ ((c) and (f)), the Fermi surface changes back into a circle one (like (i)).](image-url)
nates, we have line. After a unitary transformation between two coordi-
dial (∆first-order terms, considering the perturbation along ra-
ance along y = B
Landau level[21]. We now show that the collapsion of
in the prescribed direction is caused by the collapsion of
and

FIG. 5: (Color online) The DOS for Type-II nodal line SM. (a) for the whole Brillouin zone; (b) The DOS for ks = 0 where the nodal line locates.

IV. EFFECT OF MAGNETIC FIELD ON TYPE-II NODAL LINE SEMIMETAL

In type-II Weyl semimetal, the negative magnetic ef-
fect (NME) becomes anisotropic. The failure of NME
in the prescribed direction is caused by the collapse of
Landau level[21]. We now show that the collapse of
Landau level also appears in nodal line semimetal.

We add the magnetic field along x-direction, i.e.,
B = Bx and A = (0, Bz/2, −By/2), then use the usual
Peierls substitutions kx → k̂x − eBy/2, ky → k̂y + eBz/2.
We introduce the ladder operators

\[ a^\dagger = \hbar \partial_y + i \hbar \partial_z - \left( \frac{eBy}{2} + i \frac{eBz}{2} \right), \]

\[ a = \left[ -(\hbar \partial_y - i \hbar \partial_z) - \left( \frac{eBy}{2} - i \frac{eBz}{2} \right) \right], \]

where k̂x = −iℏ∂x, k̂y = −iℏ∂y. These operators rise
and fall the Landau levels of free electrons as

\[ a^\dagger |n, k_x\rangle = \sqrt{n+1} |n+1, k_x\rangle \]

and

\[ a |n, k_x\rangle = \sqrt{n}|n-1, k_x\rangle, \]

where |n, k_x\rangle is the free electrons Landau level wave-
function. When an electron occupies the state |n, k_x\rangle,
it rounds in circles in y-z plane. The translation invar-
ance along x-direction is preserved so that kx is still a
good quantum number.

We expanse Hamiltonian near nodal line and only keep
first-order terms, considering the perturbation along ra-
dial (∆kx) and tangential (∆ky) directions of the nodal
line. After a unitary transformation between two coordinates, we have

\[ \Delta k_x \sigma_x + \Delta k_y \sigma_y = \Delta k_T \sigma_T + \Delta k_R \sigma_R \]

where σx = σx cos θ − σy cos θ, σy = σx cos θ + σy sin θ
and \( k_T = k_x \sin \theta - k_y \cos \theta \), \( k_R = k_x \cos \theta + k_y \sin \theta \), and
θ is the intersection angle with x-axis in x-y plane. Then,
the Hamiltonian variation induced by the perturbation is

\[ \Delta H(k) = -2\Delta k_x (k_0 \sigma_R + \Delta k_T \sigma_T + \Delta k_R \sigma_R) \]

\[ + (2k_0 \Delta k_R + \Delta k_T^2) \sigma_x + 2Ck_0 \Delta k_R \sigma_0 \]

\[ - 2\Delta k_R \Delta k_T \sigma_R + 2k_0 \Delta k_R \sigma_T + 2Ck_0 \Delta k_R \sigma_0, \]

which is independent of kT because of there is no disper-
sion along nodal line. As magnetic field is applied along
x-direction, and AT (tangential directions of A) is irre-
levant, we focus tangent component of magnetic field
B sin θ. The corresponding Landau levels near the nodal
line becomes

\[ E_{n≥1} = \pm \nu_0 \sqrt{2n\alpha^2 e^2 B \sqrt{1 - (2k_x/\pi)^2}} \]

\[ E_{n=0} = 0 \]

where \( \nu_0 = 2k_0 \), \( \alpha = \sqrt{1 - \beta^2} \), \( \beta = C \), e is elementary
charge, \hbar is Planck constant. In type-I region, the zeroth
level \( E = 0 \) is maintained; in type-II region |C| > 1,
\( 1 - \beta^2 < 0 \), so that \( \alpha \) is imaginary and the expression is
invalid. This corresponds to collapsing of Landau levels
mentioned in Ref. [21]. The zeroth Landau level also
disappears.

FIG. 6: (Color online) Dispersion of type-II NLSM in a mag-
etic field along x-direction with finite tilting strength

In FIG 6 we also give the numerical results with differ-
et tilting strengths C. There are two flat bands near
nodal line when |C| < 1, which correspond to zeroth
Landau level. When |C| > 1, the flat bands disappears,
and the system becomes metal which is similar to Weyl
semimetal[17–19].

V. CORRELATION EFFECT ON TYPE-II NODAL LINE SEMIMETAL

In this part, we study the correlation effect on type-
II NLSM by considering an on-site repulsive interaction.
Then the Hamiltonian is rewritten as
\[ H = H_{0,\uparrow} + H_{0,\downarrow} + U \sum_{i,a} \hat{n}_{i,\uparrow,a}\hat{n}_{i,\downarrow,a} - \mu \sum_{i,\tau,a} \hat{c}_{i,\tau,a}^{\dagger}\hat{c}_{i,\tau,a} \] (15)
where \( H_{0,\uparrow} \) and \( H_{0,\downarrow} \) are the Hamiltonians of Eq. (11) after considering the spin degree of freedom. \( \hat{n}_{i,\tau,a} = \hat{c}_{i,\tau,a}^{\dagger}\hat{c}_{i,\tau,a} \) is the operator of particle number with two spin degrees of freedom \( \tau \) and two orbital degrees of freedom \( a \), \( U \) is the on-site Coulomb repulsive interaction strength and \( \mu \) is the chemical potential.

After Fourier transformation, the self-consistent equations in momentum space can be rewritten as
\[ M_F = \frac{1}{2N} \sum_{k} \theta (-E_{1\uparrow}) + \theta (-E_{2\uparrow}) - \theta (-E_{1\downarrow}) - \theta (-E_{2\downarrow}) \] (19)
\[ 1 = \frac{1}{2N} \sum_{k} \left[ \theta (-E_{1\uparrow}) + \theta (-E_{2\uparrow}) + \theta (-E_{1\downarrow}) + \theta (-E_{2\downarrow}) \right] \] (20)
where \( \theta (x) \) is a step-up function and \( \theta (x) = 1 \) for \( x > 0 \) and \( \theta (x) = 0 \) for \( x < 0 \), \( N \) is the number of the unit cells and

\[ E_{1\uparrow} = h_0 - \frac{U M_F}{2} - \mu_{\text{eff}} - E_k; \]
\[ E_{2\uparrow} = h_0 - \frac{U M_F}{2} - \mu_{\text{eff}} + E_k; \]
\[ E_{1\downarrow} = h_0 + \frac{U M_F}{2} - \mu_{\text{eff}} - E_k; \]
\[ E_{2\downarrow} = h_0 + \frac{U M_F}{2} - \mu_{\text{eff}} + E_k, \]
with \( \mu_{\text{eff}} = \mu - \frac{U}{2} \).

At the mean field level, we can also define other long range orders: the antiferromagnetic (AF) order of spin degree of freedom for bulk states
\[ \langle n_{i,\tau} \rangle = \frac{1}{2} \left[ n + (-1)^{i} \tau M_{AF} \right] \] (21)
where \( M_{AF} \) is the AF order parameter of spin degree of freedom; the ferromagnetic (FM) order of orbital degree of freedom for bulk states
\[ \langle n_{i,a} \rangle = \frac{1}{2} \left[ n + (-1)^{a} M_F' \right] \] (22)
where \( M_F' \) is FM order parameter of orbital degree of freedom; the antiferromagnet (AF) order of orbital degree of freedom for bulk states
\[ \langle n_{i,a} \rangle = \frac{1}{2} \left[ n + (-1)^{i} (-1)^{a} M_{AF}' \right] \] (23)
where \( M_{AF}' \) is AF order parameter of orbital degree of freedom. These numerical calculations are the same as the FM case of spin degree of freedom.

Then by using mean field approach, we obtain the global phase diagram for different NLSMs with different tilting strengths \( C \) in FIG 7. In FIG 7 there exist six phases: nodal line SM without any long range order, nodal line SM with FM order of spin degree of freedom (FM-SM), metal with AF order of spin degree of freedom (Spin AFM-M), insulator with AF order of spin degree of freedom (Spin AFM-I), insulator with Ferrimagnetic order of orbital degree of freedom (Orbital Ferrimagnetic-I) and nodal line insulator with FM order of spin degree of freedom (FM-I).

![FIG. 7: (Color online) Global phase diagram for different tilting strengths C. There exist six phases: nodal line SM without any magnetic order, nodal line SM with FM order of spin degree of freedom (FM-SM), metal with AF order of spin degree of freedom (Spin AFM-M), insulator with AF order of spin degree of freedom (Spin AFM-I), insulator with Ferrimagnetic order of orbital degree of freedom (Orbital Ferrimagnetic-I) and nodal line insulator with FM order of spin degree of freedom (FM-I). In the global phase diagram, there are two kinds of quantum phase transitions: one is the quantum phase transition between a long range ordered state and a phase without the long range order, the other is metal-insulator transition that is characterized by the condition of zero fermion’s energy gaps. Because the orbital SU(2) rotation symmetry is broken, when considering the repulsive interaction, magnetic order of spin degree of freedom may appears and the corresponding spin SU(2) rotation symmetry is spontaneously broken. By the mean field theory, the ferromagnetic (FM) order of spin degree of freedom for bulk states is denoted by
\[ \langle n_{i,\tau} \rangle = \frac{1}{2} \left[ n + \tau M_F \right] \] (16)
where \( n \) is the number of particles, and we only consider the half-filling case for \( n = 1 \). \( \tau = 1 \) represents spin up and \( \tau = -1 \) represents spin down. \( M_F \) is the FM order parameter of spin degree of freedom. We can write the self-consistent equations as
\[ \langle n_{i,\uparrow} \rangle + \langle n_{i,\downarrow} \rangle = 1, \] (17)
\[ \langle n_{i,\uparrow} \rangle - \langle n_{i,\downarrow} \rangle = M_F. \] (18)
quantum phase transition between a long range ordered state and a phase without the long range order, the other is metal-insulator transition that is characterized by the condition of zero fermion’s energy gaps.

In the global phase diagram, a remarkable result is about the magnetic phase transition at \( C = 1 \). For the case of \( C = 1 \), there exists a flat band Fermi surface (See FIG. 8[1]). As a result, a very tiny repulsive interaction will induce an FM order of spin degree of freedom (See the result in FIG. 7). In FIG. 8 we also plot the magnetization, the energy gap and the ground state energy for the cases \( C = 0.6 \), \( C = 1.0 \) and \( C = 1.7 \), respectively. The first, second and third rows represent magnetization, the energy gap and the ground state energy respectively. Different columns represent different tilting strengths. In these figures, we use different colored lines to distinguish different magnetic order phases.

Next, we consider the correlated effect on surface states and show the interaction-induced surface orders in the NLSMs. Because the orbital SU(2) rotation symmetry is broken and the antiferromagnetic order of spin degree of freedom for surface states is not well defined, we focus on ferromagnetic order of spin degree of freedom for surface states.

Because the nodal line locates at \( k_x-k_y \) plane, we con-

FIG. 8: (color online) The first, second and third rows represent magnetization, the energy gap and the ground state energy respectively. Different columns represent different tilting strengths. In these figures, we use different colored lines represent different phases, like blue line represents nodal line SM-FM, red line represents Spin AFM-M, cyan line represents Spin AFM-I, green line represents Orbital Ferrimagnetic-I and magenta line represents nodal line FM-I. We use black dotted lines to distinguish different magnetic order phases.

FIG. 9: (Color online) The illustration of the OBC for correlated effect on surface states. The system with periodic boundary conditions along x and y-direction, but open boundary conditions along z-direction. \( M_1-M_{10} \) are ferromagnetic orders for different levels of system.

FIG. 10: (color online) The ferromagnetic order \( M \) of the system with open boundary condition for the case of \( C = 0.1 \). The black dash line represents the magnetic phase transition from \( M = 0 \) to \( M \neq 0 \). (a)-(e) are the FM orders of different sites. One can see that the surface FM order is more robust than the bulk FM order.
FIG. 11: (color online) Global phase diagram for different tilting strengths $C$ on surface states of NLSMs. There are three phases: surface nodal line SM without any magnetic order, surface nodal line SM with FM order of spin degree of freedom (FM-SM), and surface nodal line insulator with FM order of spin degree of freedom (FM-insulator). There are two phase transition: the magnetic phase transition and the metal-insulator phase transition.

Consider a system with periodic boundary conditions (PBC) along $x$ and $y$-direction, but open boundary conditions (OBC) along $z$-direction. Now, due to SU(2) spin rotation symmetry, the ansatz of FM order of spin degree of freedom is the same as Eq. (16). Along $z$-direction, the system have 10 lattice site like Fig. 9. Because there is no translation symmetry along $z$-direction, we must calculate the mean field ansatz of FM order site-by-site. After considering inverse symmetry, there are five different cases to calculate. In Fig. 10 (a)-(e) are the FM orders of on different lattice sites along $z$-direction.

After numerical calculations, we get the global phase diagram for different types of NLSMs with OBC in Fig. 11. Comparing with Fig. 7, there exist three phases: surface nodal line SM without any magnetic order, surface nodal line SM with FM order of spin degree of freedom (FM-SM), and surface nodal line insulator with FM order of spin degree of freedom (FM-insulator). There are two phase transition: the magnetic phase transition and the metal-insulator phase transition. Due to the effect of OBC, the results are different from Fig. 7. When we tune the strength of repulsive interaction, the bulk FM order appears earlier than the surface FM order for different types of NLSMs.

Beyond the critical tilting point $C = 1$, one of the energy bands of surface states reverses. See Fig. 12. For different tilting strengths, with the increase of interaction, the shape of Fermi surface for surface states changes, and finally the system becomes an insulator.

FIG. 12: (color online) Fermi surface for surface states with different tilting strength $C$.

VI. CONCLUSION

In this paper, we pointed out that there exists a new type of node-line semimetal - type-II NLSM based on a two-band cubic lattice model. We studied the effect of magnetic field on type-II NLSM and found the Landau level collapse in this system. After considering repulsive interaction and additional spin degree of freedom, different magnetic orders appear in the bulk states and ferromagnetic order exist in surface states. At critical point between type-I NLSM and type-II NLSM, arbitrary tiny interaction induces ferromagnetic order due to a flat band at Fermi surface.

Finally, we propose an experimental setup to realize the NLSM on optical lattice. The model discussed in this paper includes complex-valued nearest and next nearest neighbor hopping in cubic lattice. Hopefully this can be realized in a three-dimensional optical lattice with two components of Fermi atoms such as $^6$Li and $^{40}$K. The real-valued hopping can be induced by kinetic which could be tuned by change the potential depth and the imaginary-valued hopping could be induced by a two-photon Raman process or shaking lattice. Similar system in one dimension and two dimensions had been realized recently [23, 24].

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