Design and Modeling of Parallel Two-degree-of-freedom Variable Stiffness Actuator

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Abstract. In this paper, a new 2-DOF variable stiffness actuator based on a 2-DOF spherical wrist parallel mechanism is proposed. A 2-DOF VSA based on a parallel ball wrist mechanism and a parallel arrangement of leaf spring sets is used. Control its stiffness adjustment, analysis its variable stiffness principle and actuator mechanical structure. The parallel arrangement of the leaf spring groups reduces the mass and volume of the joints, enabling simultaneous adjustment of the stiffness. The analytical formula of the rotational stiffness of the actuator is established according to the geometric nonlinearity of the large deflection of the leaf spring.

1. Introduction
With the improvement of robot application environment and performance requirements, robots and humans are getting closer and closer. How to improve the safety of humans and robots becomes the primary factor in robot design in the future. The future robot itself needs to be safe, and it should not be more dangerous than a conscious person in human-computer interaction. This requires future robots to have similar size, function, speed and flexibility to humans\cite{1-3}.

In the design of AwAS\cite{4}, one motor connected to the harmonic reducer controls the joint output position, while the other linear motor control adjusts the flexibility of the joint. In 2011, Professor Caldwell D G proposed another method to change the flexibility, a new variable stiffness structure (AwAS-II)\cite{5}. After that, the optimized design was carried out on the basis of AsAW-I and AsAW-II, and the linear motors and ball screws in the first two generations of variable stiffness actuators were replaced by rack and pinion, and a large range of springs could be realized with a small spring. Variable stiffness, resulting in a new variable stiffness actuator (CompAct-VSA)\cite{6,7}.

Changing the physical properties of the cantilever beam to achieve stiffness adjustment is a structural control stiffness. The core component of the stiffness adjustment in the robot joint is the elastic element, which can be obtained by changing the elastic modulus of the elastic element, the moment of inertia and the effective length of the elastic element. Junho Choi et al. achieved joint stiffness changes by varying the effective length of the leaf spring\cite{8}.

2. Variable stiffness actuator mechanism design
In this design, two mechanically coupled parallel mechanisms are used to form a 2-DOF ball and socket parallel mechanism. The leaf spring are arranged in parallel, and the single motor is used to control the
stiffness adjustment, which greatly reduces the volume of the variable stiffness actuator. The rotating link acts as an output end through the ball and socket parallel mechanism, and the stiffness is controlled by the effective length of the leaf spring by the rolling roller on the movement of the leaf spring.

2.1 Two degrees of freedom variable stiffness principle
The end of the variable stiffness actuator uses two mechanically coupled 2-DOF parallel mechanisms as shown in Figure 1, which can behave like a human wrist. The end is mainly composed of four parts: a rotating joint 1, a rotating joint 2, a connecting rod and a base. These four sections are connected by a rotating shaft whose links are at the center of the mechanism and intersect.

![Figure 1. Parallel structure and variable stiffness unit](image)

The slider on the moving support clamps the leaf spring in the middle to restrict its lateral movement, and moves along the length of the leaf spring to achieve a change in the effective length $l_j$. The distance from the point of application of the load to the center of rotation is $R$. The effective length $l_j$ does not change when the center of rotation is rotated by a certain angle relative to the original position under the action of the external moment $M$, and the length changes. The joint position motor and the moving support motor independently control the position of the joint and the joint stiffness change.

2.2 Actuator mechanical structure design
The unit leaf spring set is arranged in parallel, and the relative position of the rollers is used to achieve a change in stiffness. In Fig. 2, the adjusting motor is installed at the center of the base to transmit the motor power to the lead screws 1 and 2 through the meshing of the gears and the gears 1, 2, and the utility model can provide a large axial force while using a trapezoidal screw. Achieve self-locking in the axial direction. The rotation of the motor is adjusted, and the rotary motion is converted into a linear motion by the lead screw. The linear motion of the rollers ensures that the movement of the rollers on the leaf springs results in a change in the effective length of the leaf springs. The variable stiffness actuator stiffness $K$ can be adjusted by adjusting the relative position $l_j$ of the roller by actual demand.

![Figure 2. Actuator variable stiffness element](image)
3. Theoretical analysis of 2-DOF variable stiffness actuator

3.1 Variable stiffness analysis based on small deflection theory

![Figure 3. Force analysis of spring plate](image)

The effect of the effective length on the axis of rotation when the leaf spring begins to produce a slight deformation is shown in Figure 3 above.

According to the Bernoulli-Euler equation, the bending moment is proportional to the curvature of the beam, i.e.

$$ M = EI \frac{d\theta}{ds} $$

(1)

Variable stiffness actuator output torque:

$$ M = \frac{3R \sin \varphi EI (R \cos \varphi - r \sin \varphi)}{L^3} $$

(2)

Get variable stiffness actuator stiffness:

$$ K = \frac{3EI (R^2 \cos(2\varphi) - Rr \sin(2\varphi))}{L^3} $$

(3)

3.2 Stiffness analysis based on large deflection deformation

![Figure 4. Principle of large deflection deformation](image)

Figure 4 shows based on the important relationship formula of the cantilever beam bending theory, the formula represents the deflection of the beam, which is the size of the beam bending. The relationship between the free end rotational load $P$ of the cantilever beam and the moment-curvature of the cantilever beam is as follows:
\[ EI \frac{d\phi}{ds} = P \cos \{ \beta \phi(0) \} (X - X_w) + P \sin \{ \beta \phi(0) \} (Y - Y_w) \]  

(4)

You can write the above form in the following form:

\[ \sqrt{\frac{P}{EI}} l = F(k) - F(k, \phi) \]  

(5)

In the mathematical formula:

\[ F(k) = \int_0^\pi \frac{dt}{\sqrt{1 - k^2 \sin^2 t}}, F(k, \phi) = \int_0^\phi \frac{dt}{\sqrt{1 - k^2 \sin^2 t}} \]  

(6)

The values for the elliptic integrals \( F(k) \) and \( F(k, \phi) \) can be found in the elliptic integral table. The second integral is the second type of elliptic integral, which is recorded as:

\[ \begin{align*}
E(k) &= \int_0^\pi \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \\
E(k, \phi) &= \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}
\end{align*} \]  

(7)

Numerical solution of \( E(k) \) and \( E(k, \phi) \) using elliptic integral table:

\[ f_B = l - 2 \sqrt{\frac{EI}{P}} \left[ E(k) - E(k, \phi) \right] \]  

(8)

Variable stiffness actuator input and output twist angle \( \theta \) and spring blade free end deformation deflection \( f_B \) have the following geometric relationship:

\[ R \sin \theta = f_B \]  

(9)

The force of the force \( P \) acting on the roller:

\[ M = P (R \cos \theta - r \sin \theta) \]  

(10)

Obtain an variable stiffness actuator stiffness value:

\[ K = \frac{dM}{d\theta} \]  

(11)

4. Performance analysis of variable stiffness actuator principle

The stiffness of the variable stiffness actuator is only affected by the effective length \( L \). According to the selection of the leaf spring material, \( E=208 \text{Gpa} \), \( a=10 \text{mm} \), \( b=0.6 \text{mm} \) can be obtained. \( L= 45 \text{mm} \). The variable stiffness curve is shown in the figure 5 below.
When the effective length of the leaf spring takes a fixed value, the stiffness of the actuator decreases as the passive corner increases. When the passive deflection angle is 0, it can be seen that the actuator stiffness value reaches an infinite value when the effective length is close to 0, and the actuator becomes completely rigid.

When the physical properties of the leaf spring are determined, the torque of the actuator changes according to the effective length of the leaf spring and the actuator's passive deflection angle $\phi$. It can be seen from the figure that the torque of the actuator changes nonlinearly when the effective length of the leaf spring becomes small, and the torque reaches the maximum when the effective length is close to 0.

5. Conclusion
A 2-DOF variable stiffness actuator with variable stiffness is achieved by varying the effective working length of the spring element. The leaf spring are arranged in parallel, and a single motor is used to adjust the stiffness variation of the actuator, which greatly reduces the mass and volume of the joint. In this paper, the mechanism principle of the variable stiffness actuator is introduced. When the large deflection of the leaf spring is deformed, the Jacob elliptic function is used to obtain an accurate analytical model of the large deflection bending stiffness of the cantilever beam at the free end.

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