Implication of a Quasi Fixed Point with a Heavy Fourth Generation: The emergence of a TeV-scale physical cutoff

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It has been shown in a recent paper that the Higgs quartic and Yukawa sectors of the Standard Model (SM) with a heavy fourth generation exhibit at a two-loop level a quasi fixed point structure instead of the one-loop Landau singularity and which could be located in the TeV region, a scale which is denoted by $\Lambda_{FP}$ in this paper. This provides the possibility of the existence of a TeV-scale physical cutoff endowed with several implications. In the vicinity of this quasi fixed point bound states and Higgs-like condensates made up of the 4th generation quarks and leptons get formed. It implies the possibility of a dynamical electroweak symmetry breaking generated by 4th generation condensates. The quasi fixed points also hint at at a possible restoration of scale symmetry at $\Lambda_{FP}$ and above and the emergence of a theory which could be deeper than the SM.

The Standard Model, with all of its successes, has some theoretical shortcomings which hamper its status as a fundamental theory. Perhaps the most serious issue with the SM is the existence of a fundamental scalar with its associated quadratic mass divergence problem, especially if the physical cutoff scale is the Planck mass. It is therefore a natural question to ask whether or not one can find a way to lower the physical cutoff to around the electroweak scale.

It has been argued that a theory is natural if it is stable under tiny variations of fundamental parameters. Mass corrections to the fundamental scalar such as the SM Higgs field are proportional to the physical cutoff if it exists. If the Planck mass is the physical cutoff, a fine tuning to one part in $10^{38}$ in the coupling is needed if one were to keep the Higgs mass at the electroweak scale. It is fair to say that this line of reasoning has led to important developments in supersymmetric, technicolor, extra dimensional and little Higgs models \cite{1}. From hereon, the term “hierarchy problem” will simply refer to the existence of two scales: the electroweak and Planck scales, and the aforementioned issue. This problem exists regardless of whether or not the SM is embedded into some grand unified theory.

Another line of thought is related to the possibility of a restoration of scale symmetry above a certain energy scale which could be taken as a physical cutoff scale \cite{2}. Notice that, at the quantum level, the trace of the energy momentum tensor is proportional to the $\beta$ function, i.e. $T^\mu_\mu \propto \beta(g)$, which indicates the breaking of scale invariance if $\beta(g) \neq 0$ even if there is scale symmetry at tree level. However, if the theory has a fixed point i.e. $\beta(g) = 0$ at some energy scale $\Lambda_{FP}$, that scale could be taken as a physical cutoff. Scale symmetry is restored at that energy. The physics at or below the physical cutoff scale will be insensitive to that above $\Lambda_{FP}$. The hierarchy issue might be be “resolved” if the energy scale where the fixed point is located is in the TeV region. In addition, this possibility could be further strengthened if the electroweak symmetry itself can be dynamically broken close to the fixed point.

In this manuscript, we would like to suggest that the existence of a quasi fixed point \cite{3} in the quartic and Yukawa sectors of the SM with a heavy fourth generation provides a natural candidate for a physical cutoff $\Lambda_{FP}$ which could be located in the TeV region. In fact, this quasi fixed point which appears at two loops in the $\beta$ functions of the Higgs-Yukawa sector provides a solution to the Landau pole problem which shows up at one loop. As shown in \cite{4}, the one-loop Landau singularity appears at approximately the same energy scale as the quasi fixed point when the fourth generation is sufficiently heavy (see Fig. 1). The transformation of the Landau pole into a quasi fixed point at a similar scale has

FIG. 1. The Landau pole(dotted lines) and the quasi fixed point(solid lines) of the Yukawa couplings of the fourth generation fermions and the top quark. For a heavy fourth generation (left side), both the Landau singularity from one-loop RGEs and the quasi fixed point from two-loop RGEs appear at about $2 \sim 3$ TeV, while for a light fourth generation (right side), their locations at the energy scale differ by two orders of magnitude.

$\beta$ energy momentum tensor is proportional to the trace of the energy momentum tensor which could be taken as a physical cutoff scale \cite{2}. Notice that, at the quantum level, the trace of the energy momentum tensor is proportional to the $\beta$ function, i.e. $T^\mu_\mu \propto \beta(g)$, which indicates the breaking of scale invariance if $\beta(g) \neq 0$ even if there is scale symmetry at tree level. However, if the theory has a fixed point i.e. $\beta(g) = 0$ at some energy scale $\Lambda_{FP}$, that scale could be taken as a physical cutoff. Scale symmetry is restored at that energy. The physics at or below the physical cutoff scale will be insensitive to that above $\Lambda_{FP}$. The hierarchy issue might be be “resolved” if the energy scale where the fixed point is located is in the TeV region. In addition, this possibility could be further strengthened if the electroweak symmetry itself can be dynamically broken close to the fixed point.

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important implications: (1) It provides the appearance of a TeV-scale physical cutoff \( \Lambda_{FP} \); (2) It provides the possibility of an interesting theory beyond the SM which lies above the physical cutoff coming from the possibility of the restoration of scale symmetry. Furthermore, near this “cutoff”, the Yukawa couplings of the 4th generation are large enough for condensates to form and spontaneously break the electroweak symmetry. Such ultraviolet fixed points in the Higgs-Yukawa sector were previously found and studied in the context of SU(5) gauge coupling unification [4] with values comparable to those found in [3].

Although our results are based on the existence of a quasi fixed point at the two-loop level, they provide a strong hint at a possible approach to the hierarchy problem. In particular, it is also possible that the \( \beta \) functions in the gauge sector might exhibit a quasi fixed point at the two-loop level thus providing an additional argument in favor of our suggestion. Such gauge quasi fixed had been contemplated some time ago, along with its implications, in a context in which the SM merges into a larger scale-invariant gauge group and that scale symmetry is broken spontaneously [5].

For heuristic purpose, we first present a simplified discussion of the dependence of the scalar mass on the physical cutoff, the so-called quadratic divergence. A very nice way to treat this “quadratic divergence” is to use the intuitive approach of Wilson [6] where one divides the momentum integration into “slices”. Let us suppose that there is a physical cutoff scale which we will denote by \( \Lambda_{\max} \). As mentioned above, in [3] we showed the evolution of the couplings at one and two loops (1) and one can see that, for a heavy 4th generation, the one-loop Landau singularity appears at a scale similar to that of the two-loop quasi fixed point. This is the scale that we referred to as \( \Lambda_{FP} \) which will be identified with \( \Lambda_{\max} \) in this manuscript.

Let us now divide the separation between the electroweak scale \( \Lambda_{EW} \) and the cutoff scale \( \Lambda_{\max} \) into \( n \) equal slices, each of size \( \delta q \), i.e. \( \Lambda_{EW} + n \delta q = \Lambda_{\max} \), with \( \delta q \ll \Lambda_{EW} \). When \( n \to \infty \), one of course recovers \( \Lambda_{\max} \to \infty \). Each momentum slice will be characterized by a “constant” value (within that slice) of the (Yukawa or quartic) coupling. The Wilsonian way to look at the loop integration is to consider the contribution from an \( n \) set of theories (for \( n \) slices), each characterized by the same Lagrangian but endowed with a different coupling constant. (The usual divergence encountered in field theory with an infinite cutoff can be viewed as coming from the contribution of an infinite set of such theories.) Let us first look at the fermion loop correction to the scalar mass. Schematically, one writes,

\[
\delta m_H^2 = m_H^2(\Lambda_{EW}) - m_{H,0}^2,
\]

with \( \delta m_H^2 = m_H^2(\Lambda_{EW}) - m_{H,0}^2 \),

\[
\delta m_H^2 \approx c \frac{g_2^2(\Lambda_{EW})}{\Lambda_{EW}^{2\max}} \int_{\Lambda_{EW}}^{(\Lambda_{EW}+\delta q)^2} dk^2 + \cdots + c \frac{g_2^2(\Lambda_{EW}+(n-1)\delta q)}{(\Lambda_{EW}+(n-1)\delta q)^2} \int_{\Lambda_{EW}+(n-1)\delta q}^{(n\delta q)^2} dk^2,
\]

(1)

where \( m_{H,0} \) is the tree-level value of the scalar mass of \( O(\Lambda_{EW}) \) and \( c \sim O(1/16\pi^2) \). A few words are in order at this point. Eq. (1) can be transformed into a Renormalization Group equation relating \( \delta m_H^2 \) at one energy scale to another at a different scale. It involves the running of the dimensionless coupling \( g_2^2 \) as one can explicitly see in (1). Notice that the usual discussion of the quadratic divergence assumes a constant coupling e.g. a constant Yukawa coupling. The behavior of \( g_2^2 \) can greatly influence the value of \( \delta m_H^2 \). Below we will present the importance of the TeV-scale physical cutoff and the existence of a quasi fixed point as opposed to a Landau pole.

For the sake of argument, let us first assume that that \( g_2^2 \) is slowly varying between \( \Lambda_{EW} \) and \( \Lambda_{EW} + (n-1)\delta q \). (See Figs. [3] [3]) It means that \( g_2^2(\Lambda_{EW}) \) is not too different from \( g_2^2(\Lambda_{EW} + (n-1)\delta q) \). There will not be a gross error by making the approximation \( g_2^2(\Lambda_{EW} + (n-1)\delta q) \approx \cdots \approx g_2^2(\Lambda_{EW}) \). With that approximation and using \( \delta q \ll \Lambda_{EW} \), the mass correction from the fermion loop is approximately given by

\[
\delta m_H^2 \approx c g_2^2(\Lambda_{EW}) (2\Lambda_{EW} n\delta q + (n^2 - 2n)(\delta q)^2) \\
\approx c g_2^2(\Lambda_{EW}) \Lambda_{max}^2(1 - \frac{\Lambda_{EW}}{\Lambda_{max}})^2,
\]

(2)

where we have made use of \( n^2(\delta q)^2 = (\Lambda_{max} - \Lambda_{EW})^2 \) and have kept the dominant term in the second line of (2). One can see from (2) that \( \delta m_H^2 \) becomes very large when \( \Lambda_{max} \gg \Lambda_{EW} \), i.e. \( \delta m_H^2 \propto \Lambda_{max}^2 \gg \Lambda_{EW}^2 \). In particular, if \( g_2^2 \) is still varying when one reaches the Planck scale then it is this scale which provides a physical cutoff. This is the case with three generations where the top quark Yukawa coupling is the dominant one as can be seen from Fig. [2].

If, on the other hand, \( \Lambda_{max} \sim \Lambda_{FP} \sim O(\Lambda_{EW}) \) as is the case with a heavy fourth generation discussed above, we are faced with a couple of options- and this is where the solution to the Landau pole comes in. Although the cutoff scale can be of \( O(\text{TeV}) \), the correction \( \delta m_H^2 \) also depends on the value of the coupling at that scale. If we stay with the one-loop result in the \( \beta \) functions, as we can see from Fig. [3] \( g_2^2 \) blows up at the cutoff (Landau pole) and \( \delta m_H^2 \) would be out of control even if the cutoff is finite. On the other hand, with the quasi fixed point now being the solution to the Landau pole problem, \( g_2^2 \) has a finite value and, as a consequence, \( \delta m_H^2 \propto \Lambda_{FP}^2 \). The mass correction coming from the two-loop quartic
contribution proportional to \( \lambda^2 \) can be treated in a similar fashion with \( g_Y^2 \rightarrow \lambda^2 \) and \( c \sim O((1/16\pi^2)^2) \) yielding a similar conclusion.

One may wonder whether the above argument based on the Higgs-Yukawa sector is sufficient for the statement \( \delta m_H^2 \propto \Lambda_{FP}^2 \) to be correct when we turn on the gauge couplings. However, if the gauge sector also has a fixed point around \( \Lambda_{FP} \), it will imply that this might be the true physical cutoff scale of the SM. As mentioned above, one might have situations in which the gauge sector exhibits a quasi fixed point at the two-loop level e.g. the scenario described in [3]. Another possibility is the model involving a heavy fourth generation described in [2]. As it is emphasized in [3], when \( \Lambda_{FP} \) is reached, one is no longer justified in evolving the gauge couplings beyond that scale. In fact, a look at Fig. 3 reveals that, as one approaches the quasi fixed point (to be shown subsequently) from below, there is a region very close to it where bound states and condensates- many of which carrying the SM quantum numbers- get formed [3] and interact with the gauge bosons. The naive gauge coupling evolution using the two-loop \( \beta \) functions can obviously not be trusted. In fact, it may happen that these extra composite degrees of freedom can lead to a quasi fixed point in the gauge sector i.e. one may have \( \beta(g_i) = 0 \) at the two-loop level, where \( g_i \) refers to the three SM gauge couplings. The same remarks apply to the light fermion Yukawa couplings besides the possibility that they reach a fixed point close to the ones mentioned above. For this reason, we will assume from hereon that \( \Lambda_{max} \) (or \( \Lambda_{FP} \) as discussed below) provides the true physical cutoff scale.

Under what conditions could the Higgs-Yukawa sector give rise to a TeV-scale physical cutoff? From the above discussion, one can deduce that this would happen if the couplings in the Higgs-Yukawa sector reach some quasi fixed points at a TeV scale. The next question concerns whether or not such quasi fixed points exist in the SM.

In Fig. 2 we show the evolution of the Higgs quartic and top Yukawa couplings at two loops as a function of energy for two initial Higgs masses. Dashed line: 170 GeV; Solid line: 500 GeV

![Fig. 2. The Higgs quartic and top Yukawa couplings at two loops as a function of energy for two initial Higgs masses. Dashed line: 170 GeV; Solid line: 500 GeV](image1)

and top Yukawa couplings at two loop for two initial values of the Higgs mass: 170 GeV and 500 GeV. It is amusing to note that quasi fixed points also seem to exist at two loops in the three generation case. However, one can see from Fig. 2 that such a fixed point is either around the Planck scale (heavy Higgs case) or beyond it (light Higgs case), in which case the cutoff is the Planck scale itself. This is the classic hierarchy problem of the SM with three generations. The most economical way to lower the cutoff scale would be to modify the particle content, e.g. by adding a fourth generation [3] or by adding extra chiral doublets. Let us then start with the SM endowed with four generations. Studies performed in the past few years have shown that precision data do not exclude the existence of the fourth generation [7]. Furthermore, if the fourth generation were to exist, experimental constraints (under a certain assumption) from the Tevatron put a lower bound on the mass at around 338 – 385 GeV [8].

The two-loop renormalization group equations (RGE)
for the Higgs-Yukawa sector with four generations are given by
\[ 16\pi^2 \frac{dY}{dt} = \beta_Y , \]
where \( Y \) represents the quartic coupling \( \lambda \), the Yukawa couplings \( g_t^2, g_b^2, g_\tau^2 \) for the top quark and the fourth quark and lepton respectively, and the gauge couplings \( g_i^2, i = 1, 2, 3 \). Explicit expressions for \( \beta_Y \) up to two loops can be found in [3].

As it was done in [3], we first set the \( \beta \) functions of the Higgs-Yukawa sector to be equal to zero to find the fixed points (at two-loop level), namely
\[
\beta_Y \big|_{g_{1,2,3} = \text{const.}} = 0, \quad \text{for } Y = \lambda, g_t^2, g_b^2, g_\tau^2. \tag{4}
\]
The roots of (4) yield the values of the fixed points:
\[
\lambda^* \approx 17, \quad g_t^{2*} \approx 31, \quad g_b^{2*} \approx 52, \quad g_\tau^{2*} \approx 54 \tag{5}
\]
which correspond to the \( M_S \) masses (using \( m_H = v\sqrt{2\lambda} \) and \( m_f = v\sqrt{g_i^2/2}, v = 246 \text{ GeV} \) ) \( m_H = 1.44 \text{ TeV}, m_t^* = 0.97 \text{ TeV}, m_b^* = 1.26 \text{ TeV}, m_\tau^* = 1.28 \text{ TeV} \), where the asterisks refer to the values of the masses at the fixed points. Notice that \( (g_i^2, \lambda)/16\pi^2 \) are typical expansion parameters and, with the fixed point values given above, these parameters are estimated to be \( g_t^{2*}/16\pi^2 \approx 0.2, g_b^{2*}/16\pi^2 \approx 0.33, g_\tau^{2*}/16\pi^2 \approx 0.34 \) plus \( \lambda^*/16\pi^2 \approx 0.11 \), which are not large. The fixed point values given in [5] are comparable to those found in [4] but which were used in a different context, that of SU(5) gauge coupling unification. A remark is in order here: we have neglected the b-quark and \( \tau \)-lepton Yukawa couplings in the RGEs but the structure of the \( \beta \) functions suggest that they also reach a quasi fixed point [12]. Also the \( \beta \) functions of the gauge sector could reach a quasi fixed point at the two-loop level when additional composite degrees of freedom are included, as we have mentioned above. Notice the comments concerning the light fermion Yukawa couplings made above. The energy scales where the quasi fixed points appear which are presented here and in [3] depend primarily on the fourth generation.

By themselves, the above quasi fixed points do not tell us about the energy scales where they appear since these values depend mainly on group-theoretical coefficients which enter the RGEs and on the initial values of the couplings at the electroweak scale. It goes without saying that it is the dynamics of the SM which would determine the values of these scales. Intuitively speaking, one expects that the larger the initial values of quartic and Yukawa couplings at the electroweak scale are, the “faster” they get to the fixed points. In [3], we show that the Higgs-Yukawa sector is almost decoupled from the gauge sector and that the quasi fixed points which are the roots of (4) are affected very little by the presence of the gauge couplings.

At this point, it is worthwhile to reiterate the following point referred to above. As shown in Fig. [1], the energy scale where the one-loop Landau singularity appears more or less “coincides” with that where the two-loop quasi fixed point appears for a heavy 4th generation. In this respect, it is the energy scale \( \Lambda_{FP} (= \Lambda_{max} \text{ here}) \) that is crucial rather than the actual values of the couplings at the quasi fixed point. In fact, if one were to include (unknown) higher orders beyond two loops to the RGE’s, it might happen that the actual values of the couplings at \( \Lambda_{FP} \) might be lower while preserving the fixed point structure i.e. \( \beta = 0 \). As we referred to earlier, if scale symmetry is restored at \( \Lambda_{FP} \) as hinted from the two-loop result, the physics at \( \Lambda_{FP} \) or below will not depend on the physics above it.

The energy scales where the fixed points are located can be found by numerically integrating the RGEs [3]. The results are shown in Fig. [3] for two widely separated values of the 4th generation masses. The experimentally disallowed case with the smaller 4th generation quark mass \( M_q = 120 \text{ GeV} \) is shown only for comparison and to illustrate the naturalness issue discussed below. There are several implications we would like to present concerning Fig. [3]

1) The quasi fixed points obtained by the RGE evolution agree well with those obtained by setting \( \beta_Y = 0 \). They are \( m_H^{FP} = 1.446 \text{ TeV}, m_t^{FP} = 0.965 \text{ TeV}, m_\tau^{FP} = 1.260 \text{ TeV}, m_l^{FP} = 1.282 \text{ TeV} \).

2) The locations in energy scale of the quasi fixed points for the two illustrated examples can be read from Fig. [3]. For \( M_q = 450 \text{ GeV}, M_l = 350 \text{ GeV} \), one has \( \Lambda_{FP} \approx 3 \text{ TeV} \). For \( M_q = 120 \text{ GeV}, M_l = 100 \text{ GeV} \), one has \( \Lambda_{FP} \approx 10^{16} \text{ GeV} \). One cannot fail but to notice that the heavier the fourth generation is the lower the fixed point \( \Lambda_{FP} \) becomes. As we have argued in the beginning of the paper, \( \Lambda_{FP} \) could be considered to be a physical cutoff and that the mass correction to the Higgs scalar is proportional to the square of that cutoff. From this, one can infer that, not only a light 4th generation such as the \( M_q = 120 \text{ GeV}, M_l = 100 \text{ GeV} \) case is ruled out by experiment, it is also “disfavored” from a theoretical viewpoint. This leaves us with a “heavy” fourth generation scenario with a TeV-scale physical cutoff scale \( \Lambda_{FP} \) [3]. This, as we claimed above, might be a possible solution to the hierarchy problem, in addition to being the solution to the Landau pole problem present at the one-loop level.

3) Although our results were obtained with a heavy fourth generation, one can envision a situation in which a fourth generation with mass around 400 – 500 GeV and endowed with TeV-scale physical cutoff scale \( \Lambda_{FP} \) is replaced by several chiral doublets with lower masses such as the mirror fermions which are used in the model of electroweak-scale right-handed neutrinos [10]. In fact, bound states and condensates get formed as one approaches \( \Lambda_{FP} \) and this necessitates a non-perturbative
The appropriate framework for such non-perturbative treatment is to put the SM on a lattice. A gauge-invariant lattice formulation of the SM is possible only if one introduces mirror fermions [11].

4) The “dips” in Fig. 3 correspond to a minimal value of \( \lambda \) at the electroweak scale for which the vacuum stability \((\nabla > 0)\) is satisfied. They are located at \( \Lambda_{FP} \sim 3 \text{ TeV} \), \( 10^{16} \text{ GeV} \) corresponding to \( M_q = 450 \text{ GeV} \), \( M_l = 350 \text{ GeV} \) (heavy) and \( M_q = 120 \text{ GeV} \), \( M_l = 100 \text{ GeV} \) (light) respectively. However, numerical calculations have shown that, in order for \( \lambda_{FB} \sim 0 \) at the dips and to ensure vacuum stability, one needs to fine-tune the initial value of \( \lambda \) to 2 decimal places for \( \Lambda_{FP} \sim 3 \text{ TeV} \) and to more than eight decimal places for \( \Lambda_{FP} \sim 10^{16} \text{ GeV} \). In the latter case, if one fine-tunes the initial \( \lambda \) to less than eight decimal places, it will turn negative and the vacuum will be unstable. It is not so with the "heavy" case. It is amusing to note that the "light" fourth generation is not only ruled out by experiment but also theoretically by the hierarchy and naturalness problems. This vacuum stability naturalness issue is deeply linked to the hierarchy problem: A heavy fourth generation might provide a solution to the hierarchy problem and, at the same time, is devoid of the naturalness problem.

5) Around the dip and its vicinity i.e. near or at the fixed point, the Yukawa couplings of the fourth generation quarks and leptons become large and lead to the formation of bound states. This is studied in details in the companion paper [3] and we will summarize our results here. To gain insight into the bound state formation, we start with the range of \( \lambda \) near the fixed point where one can use the Schrödinger equation with a Yukawa potential of the form \( V(r) = -\alpha_Y(r)(e^{-m_H(r)r}/r) \) where \( \alpha_Y = \frac{m_{Q_F}^2}{4\pi^2 v^2} \) with \( v = 246 \text{ GeV} \). The bound state condition of a non-vanishing binding energy translates into the constraint on \( K_f \equiv \frac{\alpha_Y m_Q}{m_H} = \frac{g_f^2}{16\pi^3\lambda} \) which is \( K_f \geq 2 \) and \( K_f \geq 1.68 \) using a Rayleigh-Ritz variational technique and a numerical integration respectively. Using the fixed-point values for the fourth generation and the top quark, we obtain \( K_q = 1.82 \), \( K_l = 1.92 \), and \( K_t = 0.82 \) which imply that fourth-generation bound states are rather loose and there are no top-quark bound states in this region. As shown in Figs. 3, the quartic coupling decreases rapidly (with the Yukawa couplings being nearly constant) as one moves away from the fixed point value and the Yukawa interactions become increasingly long-range. The correlation length \( \xi_H \sim 1/m_H \) goes from a short-range correlation (small \( \xi \)) to an infinite-range correlation \((\xi = \infty)\) at the “dip” where one expects condensates and tight bound states to be formed, e.g., \( \langle \bar{Q}_L Q_R \rangle \sim -c A_{Q_F}^2 \) where \( c \) is a constant which depends on the details of the dynamics. These condensates which contribute to the spontaneous breakdown of the SM come from extra composite Higgs doublets formed from the quarks and leptons of the fourth generation. (From the previous discussion, it is not clear whether or not there could be top condensates.) One expects also bound states with various spins of the form \( QQ, LL \) and even “lepto-quarks” \( QQ +H.c. \) to form due to the strong Yukawa interactions near the “dip”. These issues will be presented in [12].

6) Last but not least, below \( \Lambda_{FP} \) (but close to it) dynamical electroweak symmetry breaking occurs [12] at values of the Yukawa couplings which are smaller than the quasi fixed point values as can be seen from Fig. 3 at the location of the “dip”. Above \( \Lambda_{FP} \), we conjecture that \( \beta = 0 \) is preserved even (unknown) higher orders are included in the RGE equations and scale symmetry is restored. If that is the case, the physics below \( \Lambda_{FP} \) will be independent of what goes on above it. The possibility of the existence of a TeV-scale physical cutoff leads to interesting implications: (1) The dynamical breaking of the electroweak symmetry below \( \Lambda_{FP} \) and (2) The restoration of scale symmetry above \( \Lambda_{FP} \) possibly leading to an interesting scale-invariant theory beyond the SM.

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[1] The list of references related to these topics is extensive and it is impossible to put it in this note.

[2] M. Shaposhnikov and D. Zenhausern, Phys. Lett. B 671, 162 (2009) [arXiv:0809.3406] [hep-th]. H. Gies, S. Rechenberger and M. M. Scherer, Eur. Phys. J. C 66, 403 (2010) [arXiv:0907.0327] [hep-th].

[3] P. Q. Hung and Chi Xiong, “Renormalization Group Fixed Point with a Fourth Generation: Higgs-induced Bound States and Condensates” [arXiv:0911.3890] [hep-ph].

[4] P. Q. Hung, “Minimal SU(5) resuscitated by Higgs coupling fixed points,” [arXiv:hep-ph/9710297] P. Q. Hung, Phys. Rev. Lett. 80, 3000 (1998) [arXiv:hep-ph/9712338].

[5] P. Q. Hung, “Prediction of the number of generations and Higgs doublets in the Standard Model,” PRINT-88-0529-Ecole Polytechnique (May 1988).

[6] K. G. Wilson, Rev. Mod. Phys. 55, 583 (1983).

[7] G. D. Kribs, T. Pfehn, M. Spannowsky and T. M. P. Tait, Phys. Rev. D 76, 075016 (2007) [arXiv:0706.3718] [hep-ph]; P. Q. Hung and M. Sher, Phys. Rev. D 77, 037302 (2008) [arXiv:0711.4353] [hep-ph].

[8] The most recent CDF bounds are \( m_{t'} > 338 \text{ GeV} \) [arXiv:0912.1067] [hep-ex] and \( m_{t'} > 385 \text{ GeV} \) (CDF talk at the 2010 ICHEP) remembering that the \( b' \) mass bound e.g. came from a particular assumption on the branching ratio (see e.g. the discussion of P. Q. Hung and M. Sher, Phys. Rev. D 77, 037302 (2008) [arXiv:0711.4353] [hep-ph]).

[9] The fact that naturalness points towards a sufficiently heavy fourth family has been discussed earlier in B. Holdom, JHEP 0608, 076 (2006) [arXiv:hep-ph/0606146]. See also references included therein. Other
works on aspects of electroweak symmetry breaking involving a fourth generation include: B. Holdom, Phys. Rev. Lett. 57, 2496 (1986), [Erratum-ibid. 58, 177 (1987)]; W. A. Bardeen, C. T. Hill and M. Lindner, Phys. Rev. D 41, 1647 (1990); C. T. Hill, M. A. Luty and E. A. Paschos, Phys. Rev. D 43, 3011 (1991); T. Elliott and S. F. King, Phys. Lett. B 283, 371 (1992); S. W. Ham, S. K. Oh and D. Son, Phys. Rev. D 71, 015001 (2005) [arXiv:hep-ph/0411012]; M. S. Carena, A. Megevand, M. Quiros and C. E. M. Wagner, Nucl. Phys. B 716, 319 (2005) [arXiv:hep-ph/0410352]; R. Fok and G. D. Kribs, Phys. Rev. D 78, 075023 (2008) [arXiv:0803.4207 [hep-ph]]; Y. Kikukawa, M. Kohda and J. Yasuda, arXiv:0901.1962 [hep-ph]. For an extensive list of references, see G. Cvetic, Rev. Mod. Phys. 71, 513 (1999) [arXiv:hep-ph/9702381].

[10] P. Q. Hung, Phys. Lett. B 649, 275 (2007) [arXiv:hep-ph/0612004].

[11] I. Montvay, Phys. Lett. B 199, 89 (1987). See remarks made along this line in P. Q. Hung, Proceedings of the XXI Rencontres de Blois, Blois, France, June 21-26, 2009.

[12] P. Q. Hung and Chi Xiong, in preparation.