A Novel Active Disturbance Rejection Control for the Quad Tilt Rotor in Conversion Process

Zhigang Wang* and Jianbo Li*
National Key Laboratory of Rotorcraft Aeromechanics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

*Corresponding author e-mail: ljb101@nuaa.edu.cn, wangzhigang@nuaa.edu.cn

Abstract. The quad tilt rotor (QTR) has complex dynamics characteristics, especially in conversion mode. It is difficult to model the QTR dynamics and the environmental factors have a great influence on it. To solve the problem of control in conversion mode of QTR, this paper carries out the design of the controller based on improved active disturbance rejection control (ADRC). According to the characteristics of flight in conversion mode, tracking differentiator (TD) is used to solve the problem of multiple integral when the system is high-order system. Extended state observer (ESO) with radial basis function (RBF) neural network is used to estimate and compensate for internal and external uncertainties, and the adaptive sliding model control in nonlinear state error feedback (NLSEF) is used to improve response speed of the controller and reduce the parameters which should be tuned. Through the flight control simulation of the QTR, the validity and rationality of the control system are verified.

1. Introduction
The quad tilt rotor (QTR) combines the advantages of multi rotor and fixed wing with good vertical takeoff/landing and cruising performance. It has the characteristics of long range and high cruising efficiency. Because of its multi rotor and fixed wing UAV flight characteristics, the design of its flight control system is the focus and difficulty [1-7].

Papachristos [8] exhibited the nonlinear/linearized dynamics and the corresponding attitude tracking controller designed by PID controller. However, due to the nonlinear coupling of longitude and latitude manipulation and the indeterminate complex variable mechanical structure with tilting of the nacelle [9, 10], the control effect of the PID controller is limited. Oner [11] designed a LQR controller for position control of the vehicle in vertical flight mode. The flight control method based on the optimal control needs to obtain an accurate linear model by linearizing the nonlinear model. However, for the nonlinear model, the control range of the controller is limited to the vicinity of the trimming point. References [12-14] combined intelligent algorithms with classical control algorithms, considering that the tilt quad rotor is high order, time varying and nonlinear system in the transition mode, Cheng Peng [15] introduced neural network into PID controller to improve controller robustness. But the controller structure is more complicated and the design process is difficult.

In this paper we had designed an attitude controller based on active disturbance rejection control (ADRC). Tracking differentiator (TD) arranges the transition process; extended state observer (ESO) estimates the system disturbance, and uses the nonlinear combination of errors as the control input to
realize the attitude control. To improve the response speed of the controller, sliding mode control is introduced into the nonlinear state error feedback (NLSEF) control law of the new ADRC. Finally, the simulation experiment and flight test were carried out to verify the rationality of the design of the control law.

2. Design of Flight Control Law

In this paper we design a novel ADRC structure, and introduces RBF neural network, sliding model control. The structure modification of ESO is carried out, and the disturbance observation value of RBF is used to replace the disturbance observation value of ESO, which is simple calculation, easy to implement and the problem of parameter trimming is solved to some extent. The sliding model control is introduced into the NLSEF to further improve the response speed and system robustness. The structure of the improved ADRC is shown in fig 1.

![Figure 1. The improved ADRC structure](image)

Radial Basis Function (RBF) neural network is a three-layer feedforward neural network with a single hidden layer. RBF neural network has the characteristics of universal approximation, and RBF neural network can be used to realize adaptive approximation of uncertain $f$.

RBF network is a three-layer feedforward network, the mapping from input to output is nonlinear, but the map- ping from implicit layer space to output space is linear. Thus, the learning speed is greatly accelerated and the local minima problem is avoided. The RBF network structure is shown in Fig 4.
The learning algorithm for the weight value $w_j(k)$ from the hidden layer to the output layer is

$$
\Delta w_j(k) = -\eta \frac{\partial J(k)}{\partial w_j(k-1)} = \eta e(k) h_j(x(k))
$$

(1)

$$
w_j(k) = w_j(k-1) + \Delta w_j(k) + \alpha (w_j(k-1) - w_j(k-2))
$$

(2)

Where, $\eta$ is learning speed ($\eta > 0$), $\alpha$ is momentum factor ($\alpha \in [0,1]$).

Then the learning algorithm for $b_j(k), c_{ji}(k)$ is:

$$
\Delta b_j(k) = -\eta \frac{\partial J(k)}{\partial b_j(k-1)} = \eta e(k) w_j(k-1) h_j(x(k)) \frac{\|x(k) - c_j(k-1)\|^2}{b_j^2(k-1)}
$$

(3)

$$
b_j(k) = b_j(k-1) + \Delta b_j(k) + \alpha (b_j(k-1) - b_j(k-2))
$$

(4)

$$
\Delta c_{ji}(k) = -\eta \frac{\partial J(k)}{\partial c_{ji}(k-1)} = \eta e(k) w_j(k-1) h_j(x(k)) \frac{x_j(k)-c_{ji}(k-1)}{b_j^2(k-1)}
$$

(5)

$$
c_{ji}(k) = c_{ji}(k-1) + \Delta c_{ji}(k) + \alpha (c_{ji}(k-1) - c_{ji}(k-2))
$$

(6)

Using neural network system $\hat{f}(x)$ replace $f(x)$, to realize the compensation of ESO. Approximation of $f(x)$ by RBF network, the input of the network is $x = [e \hspace{1cm} \hat{e}]^T$, and the output of RBF neural network is:

$$
\hat{f}(x) = \hat{W}^T h(x)
$$

(7)
The new ESO can be written as:

\[
\begin{align*}
e &= z_1 - y \\
\dot{z}_1 &= z_2 - k_1 e \\
\dot{z}_2 &= z_3 - k_2 \hat{f}(x) + g(x)u \\
\dot{z}_3 &= \hat{f}(x)
\end{align*}
\]

(8)

In order to further improve the response speed of the controller, sliding mode control is introduced into the non-linear feedback control law of the advanced ADRC in this paper. Design sliding mode switching function:

\[ s = \dot{e} - ce \]

(9)

Where, \( c > 0 \), thus:

\[ \dot{s} = \ddot{e} + ce = \ddot{y}_d - \dot{y} + ce = \ddot{y}_d - f(x) - g(x)u + ce \]

(10)

Define Lyapunov function:

\[ L = \frac{1}{2} s^2 \]

(11)

\[ \dot{L} = ss = s(\ddot{y}_d - f(x) - g(x)u + ce) \]

\[ = s(-f(x) - \hat{f}(x) + \ddot{y}_d + ce + \eta \text{sgn}(s)) + ce \]

\[ = s(-\eta \text{sgn}(s)) \]

\[ = -\eta |s| \]

(12)

Where, \( \eta \geq D \), thus:

\[ \dot{L} = -\eta |s| \leq 0 \]

(13)

To eliminate chattering in sliding mode control, using saturation function \( \text{sat}(s) \) instead of symbolic function \( \text{sgn}(s) \).

\[ \text{sat}(s) = \begin{cases} 
1 & s > \Delta \\
k s & |s| \leq \Delta, k = 1/\Delta \\
-1 & s < -\Delta 
\end{cases} \]

(14)

Where, \( \Delta \) is “boundary layer”.

Selecting the appropriate function \( g(e) = -\eta \text{sgn}(s) \) to replace the nonlinear synthetic control function \( f_{al}(e, \alpha, \delta) \).

\[
\begin{aligned}
e_1 &= v_1 - z_1 \\
e_2 &= v_2 - z_2 \\
u &= \beta_1 g(e_1) + \beta_2 g(e_2)
\end{aligned}
\]  

The expression of the new NLSEF are:

\[
\begin{aligned}
e_1 &= v_1 - z_1 \\
e_2 &= v_2 - z_2 \\
u &= -\beta \eta \text{sat}(\dot{e}_1 + c_1 e_1) - \beta \eta \text{sat}(\dot{e}_2 + c_2 e_2)
\end{aligned}
\]  

3. Stability analysis

The design adaptive law is:

\[
\dot{W} = -\gamma E^T P b h(x)
\]

Closed loop dynamic equation of the system can be written as:

\[
\dot{e} = -K^T E + [\hat{f}(x) - f(x)]
\]

Set:

\[
A = \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

Then, the dynamic equation can be written as a vector pattern:

\[
\dot{E} = AE + b[\hat{f}(x) - f(x)]
\]

Set the optimal parameter is:

\[
W^* = \arg \min_{w \in \Omega} \sup_{x \in \Omega} |\hat{f}(x) - f(x)|
\]

Where, \( \Omega \) is the set of \( W \). The minimum approximation error is defined as:

\[
\omega = \hat{f}(x|W^*) - f(x)
\]
The Eq. (20) can be written as:

$$\dot{E} = AE + b \left\{ \begin{array}{c} \hat{f}(x) - \hat{f}(x|W^*) \\ + \omega \end{array} \right\}$$

(23)

The closed-loop dynamic equation:

$$\dot{E} = AE + b \left( \hat{W} - W^* \right)^T h(x) + \omega$$

(24)

Define lyapunov function:

$$V = \frac{1}{2} E^T PE + \frac{1}{2\gamma} \left( \hat{W} - W^* \right)^T \left( \hat{W} - W^* \right)$$

(25)

Where, $\gamma$ is positive constant, $P$ is a positive definite matrix and satisfies the Lyapunov equation:

$$A^T P + PA = -Q$$

(26)

Set, $V_1 = \frac{1}{2} E^T PE$, $V_2 = \frac{1}{2\gamma} \left( \hat{W} - W^* \right)^T \left( \hat{W} - W^* \right)$ and $M = b \left[ \left( \hat{W} - W^* \right)^T h(x) + \omega \right]$, the Eq. (24) can be written as:

$$\dot{E} = AE + M$$

(27)

Thus,

$$\dot{V}_1 = \frac{1}{2} \dot{E}^T PE + \frac{1}{2} E^T \dot{P} E$$

$$= \frac{1}{2} \left( E^T A^T + M^T \right) PE + \frac{1}{2} E^T P \left( AE + M \right)$$

$$= \frac{1}{2} E^T \left( A^T P + PA \right) E + \frac{1}{2} M^T PE + \frac{1}{2} E^T PM$$

(28)

$$= -\frac{1}{2} E^T Q E + \frac{1}{2} \left( M^T PE + E^T PM \right)$$

$$= -\frac{1}{2} E^T Q E + E^T PM$$

Substitute $M$ into Eq. (28), and considering $E^T P b \left( \hat{W} - W^* \right)^T h(x) = \left( \hat{W} - W^* \right)^T \left[ E^T P bh(x) \right]$, derived:
The derivative of $V$ is:

$$
\dot{V}_1 = -\frac{1}{2} E^T Q E + E^T P b (\dot{W} - W^*)^T h(x) + E^T P b \omega \\
= -\frac{1}{2} E^T Q E + \left(\dot{W} - W^*\right)^T E^T P b h(x) + E^T P b \omega
$$

(29)

$$
\dot{V}_2 = \frac{1}{\gamma} (\dot{W} - W^*)^T \dot{W}
$$

(30)

The derivative of $V$ is:

$$
\dot{V} = \dot{V}_1 + \dot{V}_2 = -\frac{1}{2} E^T Q E + E^T P b \omega + \frac{1}{\gamma} (\dot{W} - W^*)^T \left[ \dot{W} + \gamma E^T P b h(x) \right]
$$

(31)

So,

$$
\dot{V} = -\frac{1}{2} E^T Q E + E^T P b \omega
$$

(32)

Since, $-\frac{1}{2} E^T Q E \leq 0$, by selecting a neural network with a very small minimum approximation error $\omega$ to achieve $\dot{V} \leq 0$.

4. Simulation results and analysis

In this paper, the tilt quad rotor transition mode in taken as the control object, and the numerical simulation verification is carried out for the nacelle tilt angle is $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$ respectively. The model parameters of the TQR system in the simulation are shown in Tab 1:

![Tilt quad rotor diagram](image)

Figure 3. Tilt quad rotor
Table 1. Parameters of the tilt quad rotor system

| Parameters            | Value | Unit |
|-----------------------|-------|------|
| Takeoff weight        | 20    | kg   |
| Front wing area       | 0.278 | m²   |
| Front wing length     | 1.4   | m    |
| Rear wing area        | 0.5   | m²   |
| Rear wing length      | 1.8   | m    |
| Vertical tail area    | 0.07  | m²   |
| Propeller diameter    | 0.51  | m    |

The command signal of the three attitude angles is step signal with amplitude of 1, and the external disturbance signal is $1.2\sin(t)$. The simulation results under the traditional ADRC, improved ADRC and PD controller are shown as following:

![Figure 4. Tilt angle is 0°](image-url)
Figure 5. Tilt angle is 30°
Figure 6. Tilt angle is 60°
From the Fig 4(a) we can see that the system overshoot of improved ADRC is about 1%, the adjustment time is about 0.6s. And the system overshoot of traditional ADRC is about 15%, and the adjustment time is about 2s. The adjustment time of PD controller is about 3s. Fig 4(b) shows that the improved ADRC almost has no system overshoot, and the adjustment time of improved ADRC is about 0.5s, the system overshoot of traditional ADRC is about 10%. However, the adjustment time of traditional ADRC and PD controller is about 1s and 3s. Fig 4(c) shows that the system overshoot of improved ADRC is about 2%, and the adjustment time of improved ADRC is about 0.6s, the system overshoot of traditional ADRC is about 30%. However, the adjustment time of traditional ADRC and PD controller is about 0.6s and 2.7s. Fig 5(a) shows that the system overshoot of improved ADRC is about 0%, and the adjustment time of improved ADRC is about 0.5s the system overshoot of traditional ADRC is about 20%. However, the adjustment time of traditional ADRC and PD controller is about
0.5s and 2.8s. Fig (5b) shows that the adjustment time of improved ADRC is about 0.5s, the system overshoot of traditional ADRC is about 18%. However, the adjustment time of traditional ADRC and PD controller is about 0.5s and 2.5s. Fig 5(c) shows that the adjustment time of improved ADRC is about 0.7s, the system overshoot of traditional ADRC is about 19%. However, the adjustment time of traditional ADRC and PD controller is about 0.7s and 2.9s. Fig 6(a) shows that the adjustment time of improved ADRC is about 0.6s, the system overshoot of traditional ADRC is about 15%. However, the adjustment time of traditional ADRC and PD controller is about 0.6s and 2.8s. Fig 6(b) shows that the adjustment time of improved ADRC is about 0.6s, the system overshoot of traditional ADRC is about 16%. However, the adjustment time of traditional ADRC and PD controller is about 0.6s and 2.9s. Fig 6(c) shows that the adjustment time of improved ADRC is about 0.7s, the system overshoot of traditional ADRC is about 20%. However, the adjustment time of traditional ADRC and PD controller is about 0.6s and 2.9s. Fig 7(a) shows that the adjustment time of improved ADRC is about 0.5s, the system overshoot of traditional ADRC is about 8%. However, the adjustment time of traditional ADRC and PD controller is about 0.5s and 2.6s. Fig 7(b) shows that the adjustment time of improved ADRC is about 0.5s, the system overshoot of traditional ADRC is about 10%. However, the adjustment time of traditional ADRC and PD controller is about 0.5s and 2.9s. Fig 7(c) shows that the adjustment time of improved ADRC is about 0.5s, the system overshoot of traditional ADRC is about 20%. However, the adjustment time of traditional ADRC and PD controller is about 0.6s and 3.2s.

We can see from Fig 4-Fig 7 that the simulation result of improved ADRC is much better than that of Traditional ADRC and PD controller. The improved ADRC can quickly follows the transient signal, and almost has no system overshoot. The traditional ADRC can quickly follows the transient signal as well, however, the system overshoot is large, and it is oscillation. The effect of disturbance compensation is not as good as the improved ADRC. The PD controller adjustments time is much slower that the two ADRC’s.

5. Conclusion
In this paper, based on the problems of strong coupling, nonlinear and accurate modeling of tilt quad rotor, a control system based on improved ADRC is designed.

(1) The RBF neural network can approximate any function infinitely. The uncertainty of ESO is approximated by RBF neural network. The novel ESO can better observe and compensate the internal and external disturbance of the system.

(2) By using the sliding mode and the adaptive control in the NLSEF, the system respond speed is improved, and the number of parameters of the controller which should be set is reduced.

(3) The stability of improved control algorithm is proved by Lyapunov equation, and the simulation result shows that the whole system of the improved controller can better observe and compensate disturbance, simplify parameter setting and respond is more quickly.

Acknowledgments
A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

References
[1] Marr RL, Blackman S and Weiberg JA. Wind tunnel and flight test of the XV-15 Tilt Rotor Research Aircraft. J Aircr 1979; 19: 1005–1011.
[2] Wang Q and Wu W. Modelling and analysis of tiltrotor aircraft for flight control design. Inf Technol J 2014; 13: 855 – 894.
[3] Chun-hua LI and Guo-hua XU. Modeling and analytical investigation on rotor/wing aerodynamic interaction for tiltrotor aircraft. Acta Aerodyn Sin 2008; 3: 109 – 117.
[4] Guo J, Song Y and Xia P. Full envelope flight control method for small unmanned tilt rotor aircraft. J Nanj Univ Aeronaut Astronaut 2009; 41: 439 – 444.
[5] Song Y. Design of flight control system for a small unmanned tilt rotor aircraft. Chin J Aeronaut
2009; 22: 250 – 256.

[6] Li H, Qu X and Wang W. Multi-body motion modeling and simulation for tilt rotor aircraft. Chin J Aeronaut 2010; 23: 415 – 422.

[7] Rysdyk RT and Calise AJ. Adaptive model inversion flight control for tilt-rotor aircraft. J Guid Control Dyn 2012; 22: 402 – 407.

[8] Papachristos C, Alexis K and Tzes A. Design and experimental attitude control of an unmanned tiltrotor aerial vehicle. In: International conference on advanced robotics, 2011. Piscataway: IEEE.

[9] Sato M and Muraoka K. Flight controller design and demonstration of quad-tilt-wing unmanned aerial vehicle. J Guid Control Dyn 2015; 38; 1 – 12.

[10] Oner KT, Oner KT and C¸ etinsoy E. Modeling and position control of a new quad-rotor unmanned aerial vehicle with tilt-wing mechanism. International Conference on Control, Automation, Robotics and Vision (ICCARV’08), Laval, France. 2008.

[11] Oner KT, Cetinsoy E and Unel M. Dynamic model and control of a new quadrotor unmanned aerial vehicle with tilt-wing mechanism. In: Proceedings of World Academy of Science Engineering & Technology, No. 58, 2008. 2 (9) 1008 - 1013.

[12] Flores G, Lugo I and Lozano R. 6-DOF hovering controller design of the Quad Tiltrotor aircraft: simulations and experiments. In: Decision & control, 2014, pp.6123, 6128. Piscataway: IEEE.

[13] Niu ZG and Zhang JM. Method of smooth-switch fuzzy PID for linear motor control. Proc CSEE 2006; 26: 132 – 136.

[14] Teng W, Pan H and Ren J. Neural network PID decoupling control based on chaos particle swarm optimization. In: Control conference, 2014, pp.5017, 5020. Piscataway: IEEE.

[15] Peng C, Wang XM and Chen X. Design of tiltrotor flight control system in conversion mode using improved neutral network PID. Adv Mat Res 2014; 850 – 851: 640 – 643.