Dynamics of hot gauge theories*

Laurence G. Yaffe†

*aDepartment of Physics, University of Washington, Seattle, Washington 98195, USA

A brief overview is given of recent progress in understanding the dynamics of hot gauge theories.

1. Introduction

Much progress has been made on understanding static equilibrium properties of gauge theories at finite temperature. This includes perturbative results valid at asymptotically high temperature [1–4], as well as accurate results on phase structure, thermodynamics, correlation lengths and other observables from numerical simulations [5].

Far less progress has been made on dynamic properties of hot gauge theories, such as equilibration rates and transport properties, despite the fact that these properties are of direct interest in applications to both heavy ion collisions and early universe cosmology. The reason is simple. Static properties may be extracted directly from the Euclidean theory, to which the whole panoply of modern theoretical tools (numerical simulations, renormalization group methods, effective field theories, ...) may be applied. Dynamic properties require analytic continuation of thermal correlation functions back to Minkowski space, or equivalently a functional integral formulation with a complex measure involving a non-trivial contour in the complex time plane [6,7].

Nevertheless, considerable progress has been made in recent years in understanding dynamic processes in very hot, weakly coupled gauge theories.† The utility of theoretical results for asymptotically high temperature to data obtained at current or future heavy ion experiments is an open question. But at the very least, the asymptotically high temperature regime is an instructive theoretical laboratory which serves as a warm-up for efforts to understand more realistic non-equilibrium situations.

In the asymptotically high temperature regime where the running coupling $g(T)$ is small, one may disentangle a variety of phenomena which depend on different characteristic spatial or temporal scales. Dynamic phenomena display a richer set of characteristic scales than do static equilibrium properties. In a non-Abelian gauge theory like QCD, these include:

- $T^{-1}$ The energy of a typical quark or gluon is $O(T)$. Hence, $T^{-1}$ sets the scale for the de Broglie wavelength of typical excitations.

- $(gT)^{-1}$ Electric fields are Debye screened on this length scale. The corresponding frequency, $O(gT)$, is the scale of thermal corrections to the energy of typical excitations, as well as the plasma oscillation (plasmon) frequency.

- $(g^2T \ln 1/g)^{-1}$ This is the characteristic “color coherence length” — the maximum length over which a quark or gluon can be regarded as having a definite color. The factor of $1/g$ inside the logarithm comes from a ratio of the $(gT)^{-1}$ Debye length and the $(g^2T)^{-1}$ magnetic length.

- $(g^2T)^{-1}$ The amplitudes of magnetic field fluctuations on this length scale (or longer) are sufficiently large that their dynamics becomes non-perturbative.

- $(g^4T \ln 1/g)^{-1}$ This is the characteristic large angle scattering time for quarks or gluons — the mean time for their direction of motion

---

*Based on a talk presented at Lattice-2001, Berlin, August 2001.

†Supported, in part, by the U.S. Department of Energy under Grant No. DE-FG03-96ER40956.

1 “Very hot” means that the temperature is much larger than any other relevant mass scale. For QCD, this means $T$ is large compared to $\Lambda_{\text{QCD}}$ and the masses of all (active) quarks. For electroweak theory, this means $T \gtrsim M_W$. 
to change by $O(1)$. This is also the characteristic relaxation time for non-perturbative magnetic fluctuations whose wavelengths are $O([g^2T]^{-1})$.

For sufficiently small coupling $g$, the mean free path between scattering events which change a quark or gluon’s color is longer than their de Broglie wavelength by a parametrically large factor of $1/g^2 \ln g^{-1}$, and the mean free path between large angle scattering events is longer still by another factor of $1/g^2$. Consequently, one should regard quarks and gluons with typical “hard” momenta ($p \sim T$) as well-defined, weakly interacting quasiparticles.

Since the phase space of hard excitations is parametrically large compared to that of soft ($p \sim gT$) excitations, most physical observables are predominately sensitive to the properties of hard excitations. This includes bulk thermodynamic quantities like pressure or energy density, the Debye screening length, equilibration rates and transport coefficients, the photo-emission rate, and many others. But there are important exceptions. In the high temperature phase of electroweak theory, for example, the rate of baryon number violation [8], and the wall velocity of a bubble nucleated at a first order phase transition [9], depend on the low-frequency dynamics of non-perturbative magnetic fields.

### 2. Quasiparticle scattering

Understanding the various scattering processes which can affect a hard quasiparticle is a prerequisite for calculating any observable which probes quasiparticle dynamics. These scattering processes include the basic $2 \leftrightarrow 2$ particle processes of gauge boson exchange, Compton scattering, and pair annihilation illustrated in Fig. 1. For a hard scattering with momentum transfer $q \sim T$, the explicit $g^2$ in the amplitude, plus dimensional analysis, implies that the rate is of order $g^4T$. But a soft scattering, with momentum transfer $q \sim gT$, has a much larger $O(g^2T)$ rate due to the $1/q^2$ behavior of the exchanged gluon propagator, which reflects the long-range nature of Coulomb interactions. This enhancement of soft scattering is essentially cut off by Debye screening at the $gT$ scale. Note that a soft scattering event causes only a small $O(1/g)$ change in a quasiparticle’s direction, but will make an $O(1)$ change in its color.

The $O(g^4T \ln g^{-1})$ large angle scattering rate is the same as the two-body hard scattering rate except for the $\ln 1/g$ factor. This arises because an $O(1)$ change in angle need not be produced by a single hard scattering — it may also occur via a sequence of many soft scatterings each making a small change in the particle’s direction, which add incoherently to produce an $O(1)$ deflection. This gives rise to a log enhancement which is cut off at the soft $gT$ scale. And the $O([g^2 T \ln 1/g]^{-1})$ color coherence length is just the inverse of the two-body soft scattering rate again up to a log factor that arises from logarithmic sensitivity to transverse magnetic scattering with $q \ll gT$, which is only cut off by non-perturbative physics [10].

In addition to these $2 \leftrightarrow 2$ particle processes, it is also important to consider the bremsstrahlung and inelastic annihilation processes illustrated in Fig. 2 [11–13]. These amplitudes contain an ex-
licit factor of $g^3$, so one would naively expect an $O(g^6 T)$ rate which would be suppressed relative to the $g^4 T$ hard scattering rate. However, if one examines the near-collinear, soft-exchange region in which the momentum transfer $q \sim gT$ and the angle $\phi \sim g$, then one finds that these rates have a $1/g^4 T^4$ enhancement from the exchanged soft gluon propagator, a further $1/g^4 T^2$ enhancement from the nearly on-shell internal quark propagator, and a $g^2$ suppression from the near-collinear kinematics at the photon vertex. When combined with the explicit $g^8$ factors plus a $g^4 T^4$ phase space suppression from the restricted kinematics, one finds an $O(g^4 T)$ rate — exactly the same as the hard scattering rate.

Consequently, a hard quark (or gluon) moving through the plasma can “fission” into a nearly collinear pair of hard excitations, or “fuse” with another nearly collinear hard excitation, at a rate which is parametrically the same as the two body scattering rate. These processes may be regarded as $1 \leftrightarrow 2$ particle processes which are normally forbidden by kinematics, but which become kinematically allowed when accompanied by a soft transfer of momentum to other particles in the system. However, if one soft scattering with the rest of the system can take place during a near-collinear bremsstrahlung or annihilation process, so can two or more. The $1/g^4 T^2$ enhancement from the internal quark propagator is a sign that virtual intermediate states in these processes are living for a parametrically long time of order $1/g^2 T$ — which is not small compared to the color coherence time. Therefore, multiple soft scatterings occurring during these near-collinear processes cannot be neglected and will produce an $O(1)$ suppression in the resulting rate. This is known as the Landau-Pomeranchuk-Migdal (LPM) effect [14–17].

The net result is that a leading-order calculation of any observable which is sensitive to the dynamics of hard quasiparticles must correctly incorporate both the appropriate two-body scattering processes as well as LPM-suppressed near-collinear emission processes.

\[ \frac{d\Gamma}{dk} = B(k) \left[ \ln(T/m_\infty) + C(k/T) + O(g_s) \right], \]

where $B(k) = \frac{1}{2} \alpha_{EM} \alpha_s T^2 \left( \sum_s q_s^2 \right) k/(e^{k/T}+1)$, $q_s$ is the charge assignment of quark species $s$, and $m_\infty^2 \equiv g_s^2 T^2/3$ is the asymptotic thermal quark mass. The function $C(k/T)$, shown in Fig. 3, is the “constant under the log”; it is a non-trivial function of $k/T$ but it is independent of the strong coupling $g_s$, whereas $\ln(T/m_\infty) \sim \ln(1/g_s)$ since $m_\infty = O(g_s T)$. The lowest order two body processes $q\bar{q} \to g\gamma$ and $qg \to q\gamma$ generate the $\ln(T/m_\infty)$ term and part of $C(k/T)$; these contributions were evaluated in [18–19]. The near-
collinear processes only affect the non-logarithmic term $C(k/T)$. Their contribution to the leading-order emission rate was recently shown to involve the solution of a non-trivial integral equation which incorporates the effects of multiple soft scatterings during the emission process, dynamical screening in the plasma, and thermal corrections to the quasiparticle dispersion relations \[28\].

4. Transport coefficients

Transport coefficients, such as shear viscosity or flavor diffusion constants, are physical observables of obvious interest which are sensitive to the dynamics of quasiparticles. Specifically, they depend on the rates at which non-equilibrium fluctuations in the phase space distribution of quasiparticles relax back toward equilibrium. And these rates depend on exactly the same scattering processes depicted in Figs. 1 and 2. Parametrically, these transport coefficients are proportional to the large angle scattering time and hence scale as $(g^4 \ln 1/g)^{-1}$. The fact that transport coefficients grow linearly with mean free time is well known from the simple Drude model of conductivity; the current induced by an applied field is proportional to the length of time between collisions during which charges are accelerated by the field. It is the large angle scattering rate which is relevant, not the much faster rate of soft scattering, because these transport coefficients all involve the flux of gauge invariant conserved densities (i.e., momentum density, isospin density, etc.), and the contribution of a quark or gluon to these fluxes is not significantly changed if it undergoes a scattering with tiny momentum transfer\[4\].

These transport coefficients may be formally defined by Kubo formulas relating them to the zero frequency limit of a current-current or stress tensor-stress tensor Wightman correlation function. The inverse dependence of the coupling constant is an obvious sign than an infinite number of Feynman diagrams must contribute to the leading order result\[4\]. However, the complicated set of diagrams which contribute to the leading-order result may be summed up by a suitable integral equation, which is precisely a linearized Boltzmann equation (projected onto a particular symmetry channel) for small perturbations in the quasiparticle phase space densities.

As a specific example, the flavor diffusion constant characterizing the relaxation of fluctuations in isospin or strangeness density (in the high temperature limit where quark masses are negligible) has the form

$$D = A/\{g_s^4 T [\ln g_s^{-1} + O(1)]\}, \tag{2}$$

where $A$ is a pure number (depending on the number of active quark flavors). Just as for the photon emission rate, the leading-log coefficient $A$ is determined by $2 \leftrightarrow 2$ scattering processes, while the $O(1)$ constant “under” the log also depends on near-collinear emission and absorption processes in the presence of multiple soft scatterings. Quite a few efforts have been made to evaluate QCD transport coefficients like $D$ in a “leading-log” approximation (where one pretends that $\ln 1/g_s \gg 1$) by solving a linearized Boltzmann equation incorporating appropriate $2 \leftrightarrow 2$ particle scattering rates \[29\]-\[35\]. Nevertheless, until recently almost all reported results were incorrect due to a failure to appreciate that Compton scattering and pair annihilation processes, in addition to gluon exchange, contribute to the leading-log result. This has been remedied in recent work \[36\] which found

$$A \simeq \frac{2^4 3^6 \zeta(3)^2 \pi^{-3}}{24 + 4N_f + \pi^2}, \tag{3}$$

for QCD with $N_f$ flavors\[4\]. The three terms in the denominator arise, in order, from $t$-channel gluon exchange between a quark and a gluon, $t$-channel gluon exchange between two quarks, and Compton scattering or pair annihilation to gluons.

Analogous leading-log results for shear viscosity and electrical conductivity may also be found in \[36\].

\[2\]This is not true for the “color conductivity” of a non-Abelian plasma, which is sensitive to very soft scattering processes.

\[3\]The usual loop expansion breaks down because one is evaluating the correlation function at an exceptional point in momentum space, namely $\omega, k \rightarrow 0$.

\[4\]This is an approximate form, accurate to within a fraction of a percent, which results from using a one-term variational ansatz in the relevant integral equation \[36\].
Of course, a leading-log result, in which the constant under the log is totally undetermined (and hence relative corrections suppressed only by $1/\ln g_s^{-1}$ are neglected) is unlikely to provide a useful prediction in any realistic theory. At the very least, one would like to obtain a complete leading-order result, in which neglected effects are suppressed by at least a factor of the coupling $g_s$. Doing so requires the correct inclusion of near-collinear gluon emission or absorption processes analogous to the near-collinear processes relevant for photon emission. It should be possible to augment the linearized Boltzmann equation with effective $1 \leftrightarrow 2$ particle scattering terms which correctly reproduce these near-collinear reactions, and whose transition rates incorporate the correct LPM suppression effects. (However, this will require solving a non-trivial integral equation just to determine the kernel to be used in another integral equation.) Such work is currently in progress.

5. Soft gauge field dynamics

As noted earlier, a few important observables like the rate of baryon number violating transitions (also known as the “topological transition rate”) are not primarily sensitive to the dynamics of hard quasiparticles, but instead probe low frequency, long wavelength gauge field dynamics. The starting point for understanding this regime is the observation that the relevant degrees of freedom, namely modes of the gauge field with $k, \omega \ll T$, will have parametrically large occupation numbers since the Bose distribution $n_b(\omega) \sim T/\omega$ for $\omega \ll T$. Consequently, these modes may be viewed as classical fields [37]. But these soft modes of the gauge field are driven by the color current generated by all the hard quasiparticles in the plasma. If one splits the theory into hard and soft degrees of freedom, one may formulate a Boltzmann-Vlasov kinetic theory which describes the propagation of ultrarelativistic hard excitations in the background of a long wavelength classical gauge field, together with the non-Abelian Maxwell equation $D_\mu F^{\mu \nu} = j^\nu$ characterizing the reaction of the hard quasiparticles back on the soft gauge field. This kinetic theory (linearized in the deviation of quasiparticle distributions away from equilibrium) is a formulation of the well-known hard thermal loop (HTL) effective theory [38–40].

As shown by Bödeker [41], one may systematically integrate out the effects of fluctuations down to a scale $\mu \ll gT$, and derive an effective theory for the soft gauge field alone. Of course, since hard quasiparticles propagate as nearly free particles, with definite color, over distances up to the color coherence length $\gamma^{-1} = O[(g^2 T \ln 1/g)^{-1}]$, this effective theory will be non-local on the scale of $\gamma^{-1}$. But if one restricts attention to distances large compared to the color coherence length, or wavenumbers $k \ll \gamma$, then one may formulate a consistent local effective theory. The result is a stochastic theory which simply combines Ampere’s law, Ohm’s law, and the fluctuation-dissipation theorem [41–42].

$$D \times B = \sigma E - \zeta.$$  \hspace{2cm} (4)

The single parameter $\sigma$ is the color conductivity, and $\zeta$ is Gaussian noise with a variance $\langle \zeta(x)\zeta(y) \rangle = 2\sigma T \delta^4(x-y)$.

As noted in the introduction, the color coherence length $\gamma^{-1}$ is parametrically smaller than the non-perturbative magnetic length by one factor of $1/\ln g^{-1}$. Consequently, if one applies this effective theory to non-perturbative $g^2 T$ scale physics, then corrections to this effective theory will be suppressed by powers of $1/\ln g^{-1}$. In fact, it is possible to show [43] that corrections are suppressed by two powers of $1/\ln g^{-1}$ provided one determines the correct value of the color conductivity by appropriately matching the long distance effective theory to the underlying HTL theory which in turn is derived straight from hot QCD. One finds (for an SU($N_c$) theory)

$$\sigma^{-1} = \frac{3N_c \alpha T}{m_D^2} \left[ \ln \left( \frac{m_D}{\gamma(\mu)} \right) + \mathcal{C} \right],$$ \hspace{2cm} (5)

where $m_D$ is the leading-order Debye mass (inverse screening length), $\gamma(\mu) = N_c \alpha T \ln(m_D/\mu)$, $\mu$ is a renormalization scale which should be chosen to be of order $\gamma$, and $\mathcal{C} = 3.041$. The leading-log coefficient was found in [42–44], while the constant under the log, $\mathcal{C}$, is the result of the very tricky matching calculation in [45].
In the effective theory (4) pure dimensional analysis shows that
the topological transition rate (per unit volume) has the form

$$\Gamma = \kappa (\alpha T)^5 / \sigma$$  \hspace{1cm} (6)

with $\kappa$ a dimensionless pure number which depends on the
non-perturbative dynamics of the theory. But this effective theory is
a super-renormalizable, UV finite theory which may be
discretized on a spatial lattice and numerically simulated in a completely unambiguous fashion.
(In fact, in $A_0 = 0$ gauge, the effective theory (4) is precisely the
stochastic quantization of 3d Yang Mills theory.) Such a numerical simulation
was performed in [46], with a result that
$\kappa = 22.6 \pm 1.5$.

The topological transition rate has also been
extracted from real time simulations [47,48] of
two different more microscopic (i.e., less “effective”) theories which are lattice versions of the
Boltzmann-Vlasov theory described earlier. Surprising good agreement was found between the
results obtained using the long distance effective theory (4) and both of these more microscopic
formulations. As a result, the non-perturbative high temperature baryon violation rate is now a
satisfyingly well determined quantity [48].

6. Open questions

Many aspects of hot gauge field dynamics offer opportunities for further progress. Examples of
problems which can be addressed using perturbative methods include the following.

1. Perform complete leading-order evaluations of QCD transport coefficients such as shear viscosity, flavor diffusion, and electrical conductivity.

2. Evaluate bulk viscosity in hot QCD. Bulk viscosity does not receive a leading-log con-
tribution from $2 \leftrightarrow 2$ particle processes, and at present there are no published results de-
riving even the parametric dependence on coupling in a gauge theory.

3. Compute any transport coefficient beyond leading order, even in a non-gauge theory.

Can a valid effective theory still take the form of kinetic theory?

Other natural topics involve the development or improvement of methods for studying real time
dynamics outside of perturbation theory. To list just a few goals:

1. Test the domain of utility of leading order, or leading-log perturbative results for ob-
servables like transport coefficients. This has been accomplished, so far, only in one
special case, namely the $N_f \to \infty$ limit [49],
where the next-to-leading log approxima-
tion works surprisingly well as long as the
coupling $g^2 N_f$ is not so strong that its scale
dependence becomes a dominant effect.

2. Develop a practical scheme for extracting transport coefficients from Euclidean lattice
gauge theory simulations. This would in-
volve extracting an estimate for the zero
frequency slope of a spectral density from
knowledge of a Euclidean correlator.

3. Extract more physics from real-time classical lattice gauge theory simulations. It
should be possible to exhibit the presence of
over-damped low frequency gauge field dy-
namics in observables other than the topo-
logical transition rate. Current efforts in
this direction have found somewhat per-
plexing results [50].

4. And last but not least, develop better meth-
ods for exploring the real time dynamics
of systems with substantial departures from
equilibrium. A couple of recent steps in this
direction include [51,52].
REFERENCES

1. K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B 425, 67 (1994) [hep-ph/9404201].
2. C. x. Zhai and B. Kastening, Phys. Rev. D 52, 7232 (1995) [hep-ph/9507380].
3. P. Arnold and C. x. Zhai, Phys. Rev. D 51, 1906 (1995) [hep-ph/9403630]; Phys. Rev. D 50, 7603 (1994) [hep-ph/9408276].
4. E. Braaten and A. Nieto, Phys. Rev. D 53, 3421 (1996) [hep-ph/9510408].
5. See, for example, G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier and B. Petersson, Nucl. Phys. B 469, 419 (1996) [hep-lat/9602007]; F. Karsch, Nucl. Phys. Proc. Suppl. 83, 14 (2000) [hep-lat/9909006]; F. Karsch, E. Laermann and A. Peikert, Phys. Lett. B 478, 447 (2000) [hep-lat/0002003]; and references therein.
6. E. M. Lifshitz and L. P. Pitaevskii, *Physical kinetics*, Pergamon, 1981.
7. M. Le Bellac, *Thermal field theory*, Cambridge, 1996.
8. P. Arnold, D. Son and L. G. Yaffe, Phys. Rev. D 55, 6264 (1997) [hep-ph/9609461]. See also P. Huet and D. T. Son, Phys. Lett. B 393, 94 (1997) [hep-ph/9610259]; P. Arnold, Phys. Rev. D 55, 7781 (1997) [hep-ph/9701393].
9. G. D. Moore, JHEP 0003, 006 (2000) [hep-ph/0001274].
10. R. D. Pisarski, Phys. Rev. Lett. 63, 1129 (1989); Phys. Rev. D 47, 5589 (1993).
11. P. Aurenche, F. Gelis, R. Kobes and H. Zaraket, Phys. Rev. D 58, 085003 (1998) [hep-ph/9804224].
12. P. Aurenche, F. Gelis and H. Zaraket, Phys. Rev. D 61, 116001 (2000) [hep-ph/9911367].
13. P. Aurenche, F. Gelis and H. Zaraket, Phys. Rev. D 62, 096012 (2000) [hep-ph/0003326].
14. L. D. Landau and I. Pomeranchuk, Dokl. Akad. Nauk Ser. Fiz. 92 (1953) 535.
15. L. D. Landau and I. Pomeranchuk, Dokl. Akad. Nauk S.S.S.R. 105, 77 (1955).
16. A. B. Migdal, Dokl. Akad. Nauk S.S.S.R. 105, 77 (1955).
17. A. B. Migdal, Phys. Rev. 103, 1811 (1956).
18. J. Kapusta, P. Lichard and D. Seibert, Phys. Rev. D 44, 2774 (1991) [Erratum—ibid. D 47, 4171 (1993)].
19. R. Baier, H. Nakkagawa, A. Niegawa and K. Redlich, Z. Phys. C 53, 433 (1992).
20. M. G. Mustafa and M. H. Thoma, Phys. Rev. C 62, 014902 (2000) [hep-ph/0001230].
21. D. K. Srivastava, Eur. Phys. J. C 10, 487 (1999) [Erratum-ibid. C 20, 399 (1999)].
22. F. D. Steffen and M. H. Thoma, Phys. Lett. B 510, 98 (2001) [hep-ph/0103044].
23. D. Dutta, S. V. Sastry, A. K. Mohanty, K. Kumar and R. K. Choudhury, hep-ph/0104134.
24. J. Alam, P. Roy, S. Sarkar and B. Sinha, nucl-th/0106038.
25. R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B 478 (1996) 577 [hep-ph/9604327].
26. B. G. Zakharov, JETP Lett. 63 (1996) 952 [hep-ph/9607440]; Pisma Zh. Eksp. Teor. Fiz. 64 (1996) 737 [JETP Lett. 64 (1996) 781] [hep-ph/9612431].
27. P. Arnold, G. Moore, and L. Yaffe, *Photon Emission from Quark-Gluon Plasma: Complete Leading Order Results*, in preparation.
28. P. Arnold, G. D. Moore and L. G. Yaffe, hep-ph/0109064.
29. G. Baym, H. Monien, C. J. Pethick and D. G. Ravenhall, Phys. Rev. Lett. 64, 1867 (1990); Nucl. Phys. A525, 415C (1991).
30. H. Heiselberg, Phys. Rev. D 49, 4739 (1994) [hep-ph/9401309].
31. H. Heiselberg, Phys. Rev. Lett. 72, 3013 (1994) [hep-ph/9401317].
32. G. Baym and H. Heiselberg, Phys. Rev. D56, 5254 (1997) [astro-ph/9704214].
33. M. Joyce, T. Prokopec and N. Turok, Phys. Rev. D53, 2930 (1996) [hep-ph/9410281].
34. G. D. Moore and T. Prokopec, Phys. Rev. D52, 7182 (1995) [hep-ph/9506475].
35. M. Joyce, T. Prokopec and N. Turok, Phys. Rev. D53, 2958 (1996) [hep-ph/9410282].
36. P. Arnold, G. D. Moore and L. G. Yaffe, JHEP 0011, 001 (2000) [hep-ph/0010177].
37. D. Y. Grigoriev and V. A. Rubakov, Nucl. Phys. B 299, 67 (1988); D. Y. Grigoriev, V. A. Rubakov and M. E. Shaposhnikov, Nucl. Phys. B 326, 737 (1989). For a review,
see E. Iancu, hep-ph/9807299.
38. E. Braaten and R. D. Pisarski, Nucl. Phys. B337 (1990) 569.
39. J. Frenkel and J. Taylor, Nucl. Phys. B334, 199 (1990).
40. J. Taylor and S. Wong, Nucl. Phys. B346, 115 (1990).
41. D. Bodeker, Phys. Lett. B 426, 351 (1998) hep-ph/9801430.
42. P. Arnold, D. T. Son and L. G. Yaffe, Phys. Rev. D 59, 105020 (1999) hep-ph/9810216; Phys. Rev. D 60, 025007 (1999) hep-ph/9901304.
43. P. Arnold and L. G. Yaffe, Phys. Rev. D 62, 125013 (2000) hep-ph/9912305.
44. A. Selikhov and M. Gyulassy, Phys. Lett. B 316, 373 (1993) nucl-th/9307007.
45. P. Arnold and L. G. Yaffe, Phys. Rev. D 62, 125014 (2000) hep-ph/9912306.
46. G. D. Moore, Nucl. Phys. B 568, 367 (2000) hep-ph/9810313.
47. G. D. Moore, C. r. Hu and B. Muller, Phys. Rev. D 58, 045001 (1998) hep-ph/9710436; G. D. Moore and K. Rummukainen, Phys. Rev. D 61, 105008 (2000) hep-ph/9906259; D. Bodeker, G. D. Moore and K. Rummukainen, Phys. Rev. D 61, 056003 (2000) hep-ph/9907545.
48. G. D. Moore, hep-ph/9902464.
49. G. D. Moore, JHEP 0105, 039 (2001) hep-ph/0104121.
50. J. Ambjorn, K. N. Anagnostopoulos and A. Krasnitz, JHEP 0106, 069 (2001) hep-ph/0101309; hep-lat/0110092.
51. R. Baier, A. H. Mueller, D. Schiff and D. T. Son, Phys. Lett. B 502, 51 (2001) hep-ph/0009237.
52. J. Berges, hep-ph/0105311.