We study the effect of interaction on the temperature change in the process of adiabatic mixing of two components of fermi gases by the real-space Bogoliubov-de Gennes (BdG) method. We find that the competition of the adiabatic expansion and the attractive interaction make it possible to cool or heal the system depending on strength of interactions and the initial temperature. The change of temperature in a bulk system and a trapped system have been investigated respectively.

Exciting development in ultracold atom systems has opened a new possibility to stimulate the many-body Hamiltonian that have been used to study strongly correlated systems in condensed matter physics. Among the most exciting breakthroughs are experimental realization of the quantum phase transition from the superfluid to Mott-insulating phase in bosonic system [1], and the metal-insulator transition with fermionic atoms [2, 3] in optical lattice. Many of these many-body phenomena in ultracold atom systems are sensitive to temperature, for an example, a change in temperature due to adiabatic or non-adiabatic tuning of the parameters in the many-body Hamiltonian may hide the signature of the quantum phase transition, replacing it with a thermal transition instead. Adiabatic process, which keeps the entropy of the system a constant, is known to play an important role in experimental manipulation of the ultracold atoms, especially in cooling the many-body systems. Therefore, how temperature changes in the process of adiabatic tuning the many-body Hamiltonian is a question of great interest and has important practical application in the experiment due to its potential relation with cooling the ultracold atoms. In this process, the interaction have been known to play a key role in determining the temperature changing [4, 12], and may lead to anomalous phenomena [12].

In this paper, we study the effect of interaction on the temperature change in the process of adiabatic mixing of two components of fermi gases. Initially, the spin↑ and spin↓ fermions are well separated, and the gases are completely mixed finally. We introduce the s-wave interaction between the spin up and down fermions, which could be adjusted by s-wave Feschbach resonance [13, 16]. We assume the interaction is attractive thus we can safely use real-space BdG method to analyze this question. Without the interaction, the adiabatic mixing of the two component fermions would definitely cool the system. Below we focus on the effect of the interaction. Whether the process of the adiabatic mixing in this interacting many-body system would cool or heating the system? We would show below that the answer to this question not only depends on the strength of the interaction, but also on the initial temperature of our system.

The paper is organized as follows: firstly, we study the thermodynamic properties of an ideal situation and calculate the change of temperature from an ideal initial configurations (a): a completely separate fermi gases to an ideal final configuration (b): a mixture of attractive fermions (as shown in Fig.1). Further more, we propose an experimental setup to realize this adiabatic mixing process by introducing a gradient magnetic field, which recently been applied as a super cool atom thermometer [17]. We show that by tuning the gradient of the magnetic field from a large value to zero (as shown in Fig.3) slowly enough, Configuration (a) and (b) in Fig.1 can be connected adiabatically. Using the real space self-consistent Bogoliubov-de Gennes (BdG) method, we study the temperature change following the isentropic process, and observe the variation of the real-space distribution of the particle number as well as the superconductor order parameters in the process of adiabatic mixing. In the final part of the paper, we briefly discuss the possible application and relations of our result on recent experiments about the mixing of two-component fermi gases.

Firstly, we study an ideal situation. Considering two situations: one is the two-component fermions separated completely and there is no interaction between them, as shown in Fig.1(a); The other is fermi gases fully mixed and the attractive interaction between spin up and spin down fermions lead to pairing between them, as shown in Fig.1(b). Suppose there is a isentropic process from Configuration (a) to Configuration (b), we address the question how do the temperature change? In the isentropic process, it is well known that the expansion of the fermions would cool the system [12, 13], however, the mixing of attractive spin up and spin down fermions leads to pairing between them, which would heat the system considering the entropy of the fermionic superfluidity is lower than the normal state in the same temperature. The temperature change obviously depends on the strength of the interaction since we are dealing with a many-body systems. Further more, we find that it also depends on the

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initial temperature of the well-separated noninteracting state. In the region of very low temperature, the pairing effect on the temperature change overwhelm that of the adiabatic expansion thus the temperature of the system would increase in this isoentropic process. When the temperature is close to the critical temperature $T_c$, the pairing effect is not important and the temperature change mainly determined by the adiabatic mixing, which leads to the cooling of the system.

For situation (a), the entropy and particle number at finite temperature in the non interacting fermionic gases are: are as follows:

$$N = N_a + N_b = \sum_k 2k_B f(\epsilon_k - \mu_a, T)$$  \hspace{1cm} (1)

$$S_a = -2k_B \sum_k [f(\epsilon_k - \mu_a, T) \ln f(\epsilon_k - \mu_a, T) + f(-\epsilon_k - \mu_a, T) \ln f(-\epsilon_k - \mu_a, T)]$$  \hspace{1cm} (2)

while for situation (b), the corresponding thermodynamic properties are:

$$N = \sum_k (1 - \frac{\epsilon_k - \mu_b}{E_k}) \tanh \frac{E_k}{2k_BT}$$  \hspace{1cm} (3)

$$S_b = -2k_B \sum_k [f(E_k, T) \ln f(E_k, T) + f(-E_k, T) \ln f(-E_k, T)]$$  \hspace{1cm} (4)

$$\frac{m}{4\pi h^2} = \sum_k \frac{1}{2} \left( \frac{E_k}{E_k} - \frac{1}{\epsilon_k} \right)$$  \hspace{1cm} (5)

where $\mu_i$ is chemical potential and $T$ is temperature, while $S$ is the entropy of the system and $N$ is the total particle number. $f(x, T)$ is the function of fermi distribution at temperature $T$. $a$ is the scattering length of attractive interaction. $\epsilon_k = \frac{\hbar^2 k^2}{2m}$, $E_k = \sqrt{(\epsilon_k - \mu_b)^2 + \Delta^2}$, where $\Delta$ is the pairing parameter for BCS state. The fermi momentums of free particle in two above situations have a relationship $\frac{\epsilon_F}{\mu} = k_F^3 = k_F^3$. In our calculation, $k_F$ and $\epsilon_F$ is used as scaling quantity of momentum and energy, respectively. The curves of entropy vs temperature of such two situations are shown in Fig. 2(a).

![FIG. 1: Two configurations of two component fermi gases in homogenous bulk system. (a) Separate noninteracting fermi gases; (b) Fully-mixed fermionic superfluidity.](image-url)

![FIG. 2: (a) S-T curves in the homogenous bulk system for separated noninteracting fermi gases and fully mixed fermionic superfluidity. The interaction parameter is set as $\kappa/\alpha = -0.6$. Two dashed lines denote two isoentropic processes. $\Delta T/T_F$ is the variation of temperature during the adiabatic process (blue is cooling while red is heating process). (b) Dependence of the temperature change in the isoentropic process on the initial temperature $T$ as well as the strength of the interaction.](image-url)
cally. The dashed lines in Fig. 2(a) denote two isentropic processes with entropy fixed at $S_1$ and $S_2$, and the first one is cooling while the second is heating. Fig. 2(b) has shown the change of temperature in the adiabatic processes with different interaction strength, from which we can clearly see the heating and cooling regions respectively.

Now we discuss the experimental realization of this adiabatic mixing process by applying a gradient magnetic field, which was recently used as a super cool atom thermometer [17]. The gradient of the magnetic field is along the x-direction $B(x) = B(x) \hat{z}$. By tuning the gradient of the magnetic field from a large value to zero, configuration (a) and (b) in Fig. 3 are connected adiabatically, as shown in Fig. 3, we assume that there is a hard constraint along x-direction while fermions are free along $y$ and $z$ directions:

$$V_{\text{trap}}(x) = \begin{cases} \infty, & (x \in [-L/2, L/2]); \\ 0, & \text{otherwise}. \end{cases}$$ (6)

To determine the thermodynamic properties such as the entropy and distribution of the particles in our system, we use the real-space BdG method to study this fermionic superfluidity under this gradient magnetic field. The hamiltonian in this case can be written as:

$$H = \sum_\sigma \int dx \Psi_\sigma^\dagger(x)(-\frac{\hbar^2}{2m} \nabla^2 - \mu_\sigma + V_\sigma(x))\Psi_\sigma(x) + g \int dx \Psi_\uparrow^\dagger(x)\Psi_\downarrow^\dagger(x)\Psi_\downarrow(x)\Psi_\uparrow(x)$$ (4)

$\mu_\sigma$ is the chemical potential of spin $\sigma$. $V_\sigma(x)$ include trap potential $V_{\text{trap}}$ and Zeeman shift $V_{z\text{ee}}^\sigma$. Here, we assume Zeeman shift of two components has form as $V_{z\text{ee}}^\sigma(x) = \sigma \lambda x$ ($\lambda > 0$). Due to opposite energy shift, fermions with different spin are pulled to opposite direction along $x$ axis. Since there is not extra gradient potential and constraint along $y,z$ directions, which means

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In order to show the density distribution in this inhomogenous system, we adopt to solve BdG equations in real space [18, 20]. Within mean-field approximation, the pairing gap and density are defined as $\Delta(x) = -g\langle \Psi_\uparrow(x)\Psi_\downarrow(x) \rangle$, where $g$ is the bare interaction parameter, $n_\sigma(x) = \langle \Psi_\sigma^\dagger(x)\Psi_\sigma(x) \rangle$. We use Bogoliubov transformation $\Psi_\sigma(x) = \sum_l (u_{l\sigma}(x)c_{l\sigma} + \sigma v_{l\sigma}^\dagger e^{iF_{l\sigma}})$ to diagonalize the hamiltonian and get BdG equation:

$$\begin{bmatrix} H_\sigma & u_\sigma & u_\sigma \\ \Delta^*(x) & -H_\sigma + \mu_\sigma & \Delta(x) \\ -u_\sigma & v_\sigma & -\mu_\sigma \end{bmatrix} = E_{l\sigma} \begin{bmatrix} u_\sigma \\ v_\sigma \\ u_\sigma \\ v_\sigma \end{bmatrix}$$ (5)

Here, $H_\sigma = -\frac{\hbar^2}{2m} \nabla^2 + gn_\sigma(x) + \sigma \lambda x$ and $l$ denotes different eigenstate of quasi particle. Since there is some symmetry between spin up and down, $\begin{bmatrix} u_{l\uparrow}(x) \\ u_{l\downarrow}(x) \end{bmatrix} = \begin{bmatrix} -v_{l\uparrow}(x) \\ v_{l\downarrow}(x) \end{bmatrix}$

and $E_{l\uparrow} = -E_{l\downarrow}$. We can drop $\sigma$ in the above matrix equation. Pairing gap and density can be expressed by the quasi particle wave function $u_{l\uparrow}(x)$ and $v_{l\downarrow}(x)$: $\Delta(x) = g\sum_l |u_{l\uparrow}(x)|^2 f(E_{l\uparrow})$, $n_{l\uparrow}(x) = \sum_l |u_{l\uparrow}(x)|^2 f(E_{l\uparrow})$ and $n_{l\downarrow}(x) = \sum_l |v_{l\downarrow}(x)|^2 f(-E_{l\downarrow})$, where $f(x)$ is Fermi-Dirac function at temperature $T$. We have adopt the hybrid process introduced in [18, 19]. Besides the discrete spectra, the high energy part above an enough energy cutoff is included by local density approximation. The bare interaction $g$ is replaced by a position-dependent effective interaction $g_{eff}(x)$. Since the particle number of each component should conserve, we obtain the distribution of pairing parameter $\Delta(x)$ and density by self-

FIG. 3: (a) For the magnetic field with large gradient, fermions with different spin are well separated; (b) By adiabatically tuning the gradient of the magnetic field to zero, the fermions are fully mixed.

FIG. 4: Distribution of density and pairing parameter. The total number of particle $N = 200$ and the bare interaction parameter $g$ is set as $-1.937$. We choose the temperature $T = 2 \times 10^{-6} T_F$, and (a)$\lambda=0$, (b)$\lambda=0.016$, (c)$\lambda=0.0286$, (d)$\lambda=0.0446$. 
FIG. 5: (a) S-T curves in the trapped system with different magnetic field gradient $\tilde{\lambda} = 0, 0.016, 0.0286$. The bare interaction parameter $g$ is set as -1.937. (b) contour of temperature change of the trapped system while the magnetic field gradient is tuned from $\tilde{\lambda} = 0.0286$ to $\tilde{\lambda} = 0$ adiabatically.

consistent calculation. As shown in Fig. 4, where we can find fermions with different spin are separated gradually while the scale and amplitude of $\Delta(x)$ decreases rapidly with the increasing of magnetic gradient.

Now we return to address the same question proposed above: how do the temperature change in this adiabatic process. By using the equation of entropy $S = -k_B \sum_l \left[ f(E_l) \ln f(E_l) + f(-E_l) \ln f(-E_l) \right]$, we can calculate the physical quantities and we can draw the curves of entropy vs temperature with different magnetic gradient $\tilde{\lambda} = 0, 0.016, 0.0286$. (Notice that Fig.1(a) corresponding two extreme condition of this case) Heating and cooling during the isoentropic processes are denoted by two dashed lines in the figure. The change of temperature in the adiabatic mixing process as a function of initial state temperature $T$ and strength of interaction $g$ is shown in Fig. 5(b), where we set the gradient of the magnetic field is tuned from $\tilde{\lambda} = 0.0286$ to $\tilde{\lambda} = 0$ adiabatically. As shown in the figure, both healing and cooling occur in our case.

In conclusion, we have shown the change of temperature for attractive fermi gases during the adiabatic mixing process in a homogenous bulk system and a trapped system. The competition between the effect of the adiabatic mixing and the interaction on the entropy lead to interesting cooling and healing process. We focus on the attractive case where we can safely use mean-field method. Recently, the itinerant ferromagnetism has been discovered in a two-component fermi gases with repulsive interactions [21]. More recently, the mixing of two spin components of a strongly interacting Fermi gas have been realized experimentally [22]. However, for the repulsive interaction, the mean-field result may be unreliable and the similar question how do the temperature change in the process of adiabatic mixing of fermi gases with repulsive interaction may be more interesting and deserve further investigation via other numerical method such as dynamic mean-field theory (DMFT) [23].

Acknowledgments: This work was funded by National Natural Science Foundation of China (Grant No. 10574022 and Grant No. 60878059) and “Hundreds of Talents ” program of the Chinese Academy of Sciences.

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