Collisions of Shock Waves in General Relativity

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(Received )

We show that the Nariai–Bertotti Petrov type D, homogeneous solution of Einstein’s vacuum field equations with a cosmological constant describes the space–time in the interaction region following the head–on collision of two homogeneous, plane gravitational shock waves each initially traveling in a vacuum containing no cosmological constant. A shock wave in this context has a step function profile in contrast to an impulsive wave which has a delta function profile. Following the collision two light–like signals, each composed of a plane, homogeneous light–like shell of matter and a plane, homogeneous impulsive gravitational wave, travel away from each other and a cosmological constant is generated in the interaction region. Furthermore a plane, light–like signal consisting of an electromagnetic shock wave accompanying a gravitational shock wave is described with the help of two real parameters, one for each wave. The head–on collision of two such light–like signals is examined and we show that if a simple algebraic relation is satisfied between the two pairs of parameters associated with each incoming light–like signal then the space–time in the interaction region following the collision is a Bertotti space–time which is a homogeneous solution of the vacuum Einstein–Maxwell field equations with a cosmological constant.

§1. Introduction

Perhaps the best known 4–dimensional pseudo–Riemannian space–time manifold which is a Cartesian product of two 2–dimensional manifolds of constant Gaussian curvature is the Bertotti–Robinson\cite{1,2} solution of the vacuum Einstein–Maxwell field equations. The line–element is the sum of the two 2–dimensional line–elements corresponding to each manifold in the Cartesian product. It is well known that the Bertotti–Robinson solution of the vacuum Einstein–Maxwell field equations can be transformed into the Bell–Szekeres\cite{3} solution describing the space–time in the interaction region following the head–on collision of two homogeneous, plane electromagnetic shock waves (see\cite{3} p.399). As a special case, in the absence of electromagnetic fields, Bertotti\cite{1} has given a homogeneous solution of Einstein’s vacuum field equations with a cosmological constant. This solution was discovered earlier by Nariai\cite{4} (see also\cite{5}). Its importance for this paper is its association with the Bertotti–Robinson solution and since this association is due to Bertotti we will refer to the solution as the Nariai–Bertotti solution. In this paper we show that the Nariai–Bertotti solution describes the space–time in the interaction region following the head–on collision of two homogeneous plane gravitational shock waves. In addition we study the head–on collision of two homogeneous, plane, light-like signals each consisting of an electromagnetic shock wave accompanied by a gravitational shock wave. We show that if a simple relation is satisfied between the two pairs of
parameters associated with each incoming light–like signal then the space–time in
the interaction region following the collision is a Bertotti space–time.

The outline of this paper is as follows: In section 2 the gravitational shock
wave collision space–time is constructed and interpreted physically in section 3 in
which the region of the space–time coinciding with the Nariai–Bertotti space–time
is also identified. In section 4 the space–time model of a gravitational shock wave
accompanied by an electromagnetic shock wave is described. This is followed in
section 5 by the space–times describing the collision of two such light–like signals.
Properties of these space–times including the identification of the subregion in each
case which is a Bertotti space–time, are discussed in section 6.

§ 2. Gravitational Shock Wave Collision

The Nariai–Bertotti homogeneous solution of Einstein’s vacuum field equa-
tions with a cosmological constant is the Cartesian product of two 2–dimensional
manifolds of equal constant curvature, has a line–element which is the sum of the
two 2–dimensional line–elements and the equal constant curvatures can be positive
or negative. In local coordinates \( \{ x^\mu \} \), with \( \mu = 1, 2, 3, 4 \), it is a solution of
\[
R_{\mu \nu} = \Lambda g_{\mu \nu} ,
\]
where \( g_{\mu \nu} \) are the metric tensor components of the 4–dimensional manifold, \( R_{\mu \nu} \) are
the components of the Ricci tensor and \( \Lambda \) is the cosmological constant. We must
choose a sign for the cosmological constant and we begin by taking \( \Lambda < 0 \). Different
representations of the Nariai–Bertotti solution are, of course, possible. In addition
to those given in \[1\] and \[4\] there is the representation given by Ozsváth in which the
solution is derived as a member of the set of all homogeneous solutions of Einstein’s
vacuum field equations with a cosmological constant. A convenient representation
for our present purposes however is given by the line–element
\[
ds^2 = ds'^2 + ds''^2 ,
\]
with
\[
ds'^2 = d\xi^2 - \cos^2 \sqrt{-\Lambda \xi} \, dx^2 \quad \text{and} \quad ds''^2 = -d\lambda^2 - \cosh^2 \sqrt{-\Lambda \lambda} \, dy^2 .
\]
Labelling the coordinates \( \{ x^A \} \) with \( A = 1, 2 \) in either case, the Riemann tensor
components for each of these manifolds take the form
\[
R'_{ABCD} = -\Lambda (g'_{AC}g'_{BD} - g'_{AD}g'_{BC}) \quad \text{and} \quad R''_{ABCD} = -\Lambda (g''_{AC}g''_{BD} - g''_{AD}g''_{BC}) ,
\]
respectively, confirming that each 2–dimensional manifold has the same constant
curvature. In the line–element obtained by substituting (2.3) into (2.2) we write
\( \Lambda = -2ab \), where \( a \) and \( b \) are constants, and define new coordinates \( u \) and \( v \) in place
of \( \xi \) and \( \lambda \) via the transformations
\[
\sqrt{-\Lambda \xi} = au + bv ,
\]
\[
\sqrt{-\Lambda \lambda} = au - bv .
\]
This results in \( \text{(2.2)} \) taking the form

\[
\text{ds}^2 = -\cos^2(au + bv) \, dx^2 - \cosh^2(au - bv) \, dy^2 + 2 \, du \, dv .
\]  

(2.7)

For the case \( \Lambda > 0 \) we replace \( \text{(2.3)} \) by the line–elements

\[
\text{ds}^2' = -d\xi^2 - \cos^2\sqrt{\Lambda} \xi \, dx^2 \quad \text{and} \quad \text{ds}^2'' = d\lambda^2 - \cosh^2\sqrt{\Lambda} \lambda \, dy^2 .
\]

(2.8)

Now \( \text{(2.4)} \) is formally unchanged so that in this case the two 2–dimensional manifolds have equal constant curvatures but of the opposite sign to the equal constant curvatures of the manifolds with line–elements \( \text{(2.3)} \). The transformations \( \text{(2.5)} \) and \( \text{(2.6)} \) are now replaced by \( \sqrt{\Lambda} \xi = au + bv \) and \( \sqrt{\Lambda} \lambda = au - bv \) and these lead once again to \( \text{(2.7)} \) from \( \text{(2.2)} \) and \( \text{(2.8)} \).

In the space–time with line–element \( \text{(2.7)} \) there are two families of intersecting null hyperplanes, \( u = \text{constant} \) and \( v = \text{constant} \). This space–time describes the gravitational field in the interaction region \( u > 0, v > 0 \) following the head–on collision of two gravitational shock waves. To see this we replace \( u \) and \( v \) in \( \text{(2.7)} \) by \( u_+ = u\vartheta(u) \) and \( v_+ = v\vartheta(v) \) respectively, where \( \vartheta(u) \) is the Heaviside step function which is equal to unity if \( u > 0 \) and vanishes if \( u < 0 \) (similarly for \( \vartheta(v) \)). Now \( \text{(2.7)} \) is replaced by

\[
\text{ds}^2 = -(\theta^1)^2 - (\theta^2)^2 + 2 \theta^3 \theta^4 = g_{ab} \theta^a \theta^b ,
\]

(2.9)

with the 1–forms \( \{ \theta^a \} \), with \( a = 1, 2, 3, 4 \) given by

\[
\theta^1 = \cos(au_+ + bv_+) \, dx , \quad \theta^2 = \cosh(au_+ - bv_+) \, dy , \quad \theta^3 = du , \quad \theta^4 = dv .
\]

(2.10)

On the half–null tetrad given via these 1–forms the Ricci ten sor components are

\[
R_{ab} = -2ab \vartheta(u) \vartheta(v) g_{ab} - a \delta(u)(\tan bv_+ + \tanh bv_+)\delta_3^3 \delta_0^3
- b \delta(v)(\tan au_+ + \tanh au_+)\delta_4^4 \delta_0^4 ,
\]

(2.11)

where \( \delta(u) \) and \( \delta(v) \) are the Dirac delta functions singular on the null hyperplanes \( u = 0 \) and \( v = 0 \) respectively. The corresponding Newman–Penrose\[1\] components of the Weyl conformal curvature tensor are

\[
\Psi_0 = \frac{1}{2} a \delta(u)(\tan bv_+ - \tanh bv_+) + a^2 \vartheta(u) , \quad \Psi_1 = 0 ,
\]

(2.12)

\[
\Psi_2 = \frac{1}{2} ab \vartheta(u) \vartheta(v) , \quad \Psi_3 = 0 , \quad \Psi_4 = \frac{1}{2} b \delta(v)(\tan au_+ - \tanh au_+) + b^2 \vartheta(v) .
\]

(2.13)

\[
\Psi_5 = \frac{1}{2} b \delta(v)(\tan au_+ - \tanh au_+) + b^2 \vartheta(v) .
\]

(2.14)

\[
\Psi_6 = \frac{1}{2} b \delta(v)(\tan au_+ - \tanh au_+) + b^2 \vartheta(v) .
\]

(2.15)

\[
\Psi_7 = \frac{1}{2} b \delta(v)(\tan au_+ - \tanh au_+) + b^2 \vartheta(v) .
\]

(2.16)

\section*{§3. The Nariai-Bertotti Space-Time}

In \( \text{(2.11)} \) the delta function terms represent light–like shells\[5\] on \( v > 0, u = 0 \) (the coefficient of a) and on \( u > 0, v = 0 \) (the coefficient of b) which form after the
collision at \( u = v = 0 \). The first term on the right hand side, which is non–zero for \( u > 0, v > 0 \), is the cosmological constant term \( \Lambda = -2a b \). We note that the Ricci tensor (2.11) vanishes for \( u < 0 \) and for \( v < 0 \) so that these pre–collision regions of space–time are vacuum regions (with the subregion \( u < 0 \text{ and } v < 0 \) Minkowskian space–time).

In (2.12)–(2.16) the delta function terms represent impulsive gravitational waves on \( u = 0, v > 0 \) (the coefficient of \( a \) in (2.12)) and on \( v = 0, u > 0 \) (the coefficient of \( b \) in (2.16)) which form after the collision. If \( u < 0 \) then the only surviving component of the Weyl tensor is \( \Psi_4 = b^2 \vartheta(v) \). This is a vacuum region of space–time and \( \Psi_4 \) is the Riemann tensor of an incoming gravitational shock wave. Similarly if \( v < 0 \) then the only non–vanishing Weyl tensor components are \( \Psi_0 = a^2, \Psi_2 = \frac{1}{2}a b, \Psi_4 = b^2 \) and this is a Petrov type D Weyl tensor describing the Nariai–Bertotti homogeneous, vacuum gravitational field with a cosmological constant.

It is remarkable that this head–on collision of gravitational shock waves generates a cosmological constant. It is well known that the energy in the incoming waves can be distributed in different forms following the collisions of gravitational or electromagnetic shock waves. Following the head-on collision of electromagnetic shock waves described by the Bell–Szekeres solution, for example, two impulsive gravitational waves are generated traveling away from each other (the delta function terms in (5.9) and (5.13) with \( g_0 = g_1 = 0 \) below). Following the head–on collision of gravitational shock waves described above two impulsive gravitational waves are also generated as well as two light–light shells of matter. The appearance of a cosmological constant can be interpreted as part of a redistribution of the energy in the incoming waves since it represents energy in the form of a perfect fluid with proper density \( \mu \) and isotropic pressure \( p \) satisfying the equation of state \( \mu + p = 0 \) of dark energy. An alternative way of viewing this is that the system contains two vacua, one with vanishing cosmological constant and one with a non–vanishing one, and the wave collision catalyzes a phase transition from one vacuum to another.

§4. In–Coming Waves

For convenience the space–time description of a light–like signal consisting of an electromagnetic shock wave accompanied by a gravitational shock wave can be written in a way that depends upon two parameters, a parameter \( e_0 \) associated with the electromagnetic shock wave and a parameter \( g_0 \) associated with the gravitational shock wave. If we wish to remove the electromagnetic shock wave we put \( e_0 = 0 \) and if we wish to remove the gravitational shock wave we put \( g_0 = 0 \). The expression for the metric tensor of the space-time depends upon the relative magnitudes of \( e_0^2 \) and \( g_0^2 \). If \( e_0^2 > g_0^2 \) then the line–element reads

\[
\text{ds}^2 = -\cos^2 \sqrt{e_0^2 + g_0^2} u_+ \text{d}x^2 - \cos^2 \sqrt{e_0^2 - g_0^2} u_+ \text{d}y^2 + 2\text{du} \text{dv} ,
\]
where $u_+ = u \vartheta(u)$ and again $\vartheta(u)$ is the Heaviside step function which equals unity if $u > 0$ and vanishes if $u < 0$. On the other hand if $e_0^2 < g_0^2$ then the line–element reads

$$ds^2 = -\cos^2 \sqrt{g_0^2 + e_0^2} u_+ \, dx^2 - \cosh^2 \sqrt{g_0^2 - e_0^2} u_+ \, dy^2 + 2du \, dv \, . \quad (4.2)$$

We note the obvious fact that (4.1) does not permit the special case $e_0 = 0$ while (4.2) does not permit the special case $g_0 = 0$. Both of these line–elements take the form

$$ds^2 = -(\theta^1)^2 - (\theta^2)^2 + 2\theta^3 \theta^4 = g_{ab} \theta^a \theta^b \, , \quad (4.3)$$

with the 1–forms $\theta^1 = \cos \sqrt{e_0^2 + g_0^2} u_+ \, dx$, $\theta^2 = \cos \sqrt{e_0^2 - g_0^2} u_+ \, dy$, $\theta^3 = du$ and $\theta^4 = dv$ in the case of (4.1), and the 1–forms $\theta^1 = \cos \sqrt{g_0^2 + e_0^2} u_+ \, dx$, $\theta^2 = \cosh \sqrt{g_0^2 - e_0^2} u_+ \, dy$, $\theta^3 = du$ and $\theta^4 = dv$ in the case of (4.2), defining a half–null tetrad. The constants $g_{ab}$ are the tetrad components of the metric tensor and tetrad indices are raised and lowered using $g^{ab}$ (the components of the inverse of $g_{ab}$) and $g_{ab}$ respectively. We note that the hypersurfaces $u = \text{constant}$ and $v = \text{constant}$ are intersecting null hyperplanes in the space–times with line–elements (4.1) and (4.2). A calculation of the Ricci tensor components $R_{ab}$ on the half–null tetrad in either case reveals that

$$R_{ab} = -2 e_0^2 \vartheta(u) \delta_a^3 \delta^3_0 \, . \quad (4.4)$$

These are the vacuum Einstein–Maxwell field equations $R_{ab} = 2 E_{ab}$ with the electromagnetic energy tensor $E_{ab} = F_{ac} F_b^c - \frac{1}{4} g_{ab} F_{cd} F^{cd}$ derived from the Maxwell 2–form

$$F = \frac{1}{2} F_{ab} \theta^a \wedge \theta^b = e_0 \vartheta(u) \theta^1 \wedge \theta^3 \, . \quad (4.5)$$

It is easily checked that this is a solution of Maxwell’s vacuum field equations ($dF = 0 = d^* F$, where $d$ denotes exterior differentiation and the star denotes the Hodge dual). In addition (4.5) is an algebraically special Maxwell field and is thus purely radiative with propagation direction in space–time that of the vector field $\partial/\partial v$. The wave profile is the step function and so we have here an electromagnetic shock wave. We also find that in the case of (4.1) or (4.2) the Newman–Penrose components $\Psi_A$ ($A = 0, 1, 2, 3, 4$) of the Weyl conformal curvature tensor vanish except for

$$\Psi_0 = g_0^2 \vartheta(u) \, . \quad (4.6)$$

This is a Petrov type N Weyl tensor with degenerate principle null direction $\partial/\partial v$ and it therefore represents a pure gravitational radiation field. The radiation has the step function profile and is therefore a shock wave. This shock wave accompanies the electromagnetic shock wave (4.5).

If we wish to consider similar light–like signals to those above but propagating in the opposite direction then labeling them with the pair of parameters $e_1, g_1$ the corresponding line–elements are: if $e_1^2 > g_1^2$ then

$$ds^2 = -\cos^2 \sqrt{e_1^2 + g_1^2} v_+ \, dx^2 - \cosh^2 \sqrt{e_1^2 - g_1^2} v_+ \, dy^2 + 2du \, dv \, , \quad (4.7)$$
and if \( e_1^2 < g_1^2 \) then

\[
ds^2 = -\cos^2 \sqrt{g_1^2 + e_1^2} v_+ dx^2 - \cosh^2 \sqrt{g_1^2 - e_1^2} dy^2 + 2 du dv \, ,
\]

(4.8)

with \( v_+ = v \psi(v) \). In principle one should be able to obtain the space–time following the collision (at \( u = v = 0 \)) of either of (4.1) or (4.2) with either of (4.7) or (4.8).

This remains an open question. In the next section we give the space–time following the collision of the signal described by (4.1) with the signal described by (4.7) and the space–time following the collision of the signal described by (4.2) with the signal described by (4.8), and this only if the simplifying assumption

\[
e_0 g_1 = e_1 g_0 \, ,
\]

(4.9)

is satisfied by the parameters involved. In both of these cases the space–time in the interaction region following the collision is a Bertotti space–time.

§5. Post Collision Space-Times

If \( e_0^2 > g_0^2 \) and (4.9) is satisfied (and thus \( e_1^2 > g_1^2 \)) then the solution of the collision problem we propose is described by the line–element

\[
ds^2 = -\cos^2 \Psi dx^2 - \cos^2 \phi dy^2 + 2 du dv \, ,
\]

(5.1)

where

\[
\Psi = \sqrt{e_0^2 + g_0^2} u_+ + \sqrt{e_1^2 + g_1^2} v_+ \, ,
\]

(5.2)

\[
\phi = \sqrt{e_0^2 - g_0^2} u_+ - \sqrt{e_1^2 - g_1^2} v_+ \, .
\]

(5.3)

If \( e_0^2 < g_0^2 \) and (4.9) is satisfied (and thus \( e_1^2 < g_1^2 \)) the solution we propose is given by the line–element

\[
ds^2 = -\cos^2 \Psi dx^2 - \cosh^2 \chi dy^2 + 2 du dv \, ,
\]

(5.4)

where \( \Psi \) is given in (5.2) and

\[
\chi = \sqrt{g_0^2 - e_0^2} u_+ - \sqrt{g_1^2 - e_1^2} v_+ \, .
\]

(5.5)

In the case of (5.1) or (5.4) the line–element can again be written in the form (4.3) with now the 1–forms given either by \( \theta^1 = \cos \Psi dx, \theta^2 = \cos \phi dy, \theta^3 = du \) and \( \theta^4 = dv \) or by \( \theta^1 = \cos \Psi dx, \theta^2 = \cosh \chi dy, \theta^3 = du \) and \( \theta^4 = dv \) respectively. In the case of (5.1), the Ricci tensor components on the half–null tetrad given via the 1–forms can be written in the form

\[
R_{ab} = A \vartheta(u) \vartheta(v) g_{ab} + 2 E_{ab} + \left\{ \sqrt{e_0^2 - g_0^2} \tan \sqrt{e_1^2 - g_1^2} v_+ \right\} \delta(u) \delta^3 \delta^3 + \left\{ \sqrt{e_0^2 - g_0^2} \tan \sqrt{e_1^2 - g_1^2} u_+ \right\} \delta(v) \delta^4 \delta^4 ,
\]

(5.6)
where
\[ \Lambda = -2g_0 g_1 , \] (5.7)
and the components \( E_{ab} = E_{ba} \) are identically zero except for
\[ E_{11} = - E_{22} = e_0 e_1 \vartheta(u) \vartheta(v) , \quad E_{33} = - e_0^2 \vartheta(u) , \quad E_{44} = - e_1^2 \vartheta(v) . \] (5.8)

The Newman–Penrose components of the Weyl conformal curvature tensor for (5.1) are
\[ \Psi_0 = \frac{1}{2} \left\{ \sqrt{e_0^2 - g_0^2} \tan \sqrt{e_1^2 - g_1^2} \vartheta(u) \right\} \delta(u) + g_0^2 \vartheta(u) , \] (5.9)
\[ \Psi_1 = 0 , \] (5.10)
\[ \Psi_2 = \frac{1}{3} g_0 g_1 \vartheta(u) \vartheta(v) , \] (5.11)
\[ \Psi_3 = 0 , \] (5.12)
\[ \Psi_4 = \frac{1}{2} \left\{ \sqrt{e_1^2 + g_1^2} \tan \sqrt{e_0^2 + g_0^2} \vartheta(u) \right\} \delta(v) + g_1^2 \vartheta(v) . \] (5.13)

For (5.4) the Ricci tensor components on the half–null tetrad are given by
\[ R_{ab} = \Lambda \vartheta(u) \vartheta(v) g_{ab} + 2 E_{ab} - \left\{ \sqrt{g_0^2 + e_0^2} \tan \sqrt{g_1^2 + e_1^2} \vartheta(u) \right\} \delta(u) \delta_a^3 \delta_b^3 - \left\{ \sqrt{g_1^2 + e_1^2} \tan \sqrt{g_0^2 + e_0^2} \vartheta(u) \right\} \delta(v) \delta_a^4 \delta_b^4 , \] (5.14)
with \( \Lambda \) and \( E_{ab} \) as in (5.7) and (5.8) and the Newman–Penrose components of the Weyl conformal curvature tensor are
\[ \Psi_0 = \frac{1}{2} \left\{ \sqrt{g_0^2 + e_0^2} \tan \sqrt{g_1^2 + e_1^2} \vartheta(u) \right\} \delta(u) + g_0^2 \vartheta(u) , \] (5.15)
\[ \Psi_1 = 0 , \] (5.16)
\[ \Psi_2 = \frac{1}{3} g_0 g_1 \vartheta(u) \vartheta(v) , \] (5.17)
\[ \Psi_3 = 0 , \] (5.18)
\[ \Psi_4 = \frac{1}{2} \left\{ \sqrt{g_1^2 + e_1^2} \tan \sqrt{g_0^2 + e_0^2} \vartheta(u) \right\} \delta(v) + g_1^2 \vartheta(v) . \] (5.19)
In the case of (5.1) and (5.4), $E_{ab}$ given by (5.8) is the electromagnetic energy tensor of a Maxwell 2–form

$$F = e_0 \theta(u) \theta^1 \wedge \theta^3 + e_1 \theta(v) \theta^1 \wedge \theta^4.$$  \hspace{1cm} (5.20)

It is easily checked that on account of (4.9) this satisfies Maxwell’s vacuum field equations in the space–times with line–elements (5.1) and (5.4) with the appropriate choice of 1–forms in each case (indicated following (5.5) above).

§6. The Bertotti Space–Time

In the space–times with line–elements (5.1) and (5.4) the (overlapping) regions prior to the collision of the light–like signals correspond to $u < 0$ with line–element (4.7) or (4.8) and $v < 0$ with line–element (4.1) or (4.2). The post–collision region of the space–times with line–elements (5.1) and (5.4) is $u > 0, v > 0$. We see from the Ricci tensor components (5.6) or (5.14) that the boundaries of this post–collision region, $v = 0$ with $u > 0$ and $u = 0$ with $v > 0$, are the histories of plane, light–like shells of matter (e.g. bursts of neutrinos)8) traveling away from each other with the speed of light in the directions of the incoming signals. These objects are described by the delta function terms in (5.6) and (5.14). The delta function terms in the Weyl tensor components (5.9) and (5.13) or (5.15) and (5.19) describe plane, impulsive gravitational waves8) accompanying these light–like shells. The Maxwell 2–form (5.20) describes the two incoming electromagnetic shock waves and also gives the resulting electromagnetic field in the region $u > 0, v > 0$. We see that in this post–collision region of space–time (5.6) and (5.14) both simplify to

$$R_{ab} = \Lambda g_{ab} + 2 E_{ab},$$  \hspace{1cm} (6.1)

with $\Lambda$ given by (5.7) and $E_{ab}$ by (5.8) when $u > 0$ and $v > 0$. Thus $\Lambda$ is a cosmological constant. This homogeneous space–time with line–element (5.1) or (5.4) with $u > 0, v > 0$ is thus a solution of the Einstein–Maxwell vacuum field equations with a cosmological constant and is a Bertotti1) space–time. Finally we note that (5.1) specialized by taking $g_0 = 0 = g_1$ yields the Bell–Szekeres3) line–element while (5.4) specialized by taking $e_0 = 0 = e_1$ yields the line–element given in (2.9) and (2.10).

An alternative interpretation of the post collision region, mentioned at the end of section 3, is that dark energy in the form of a perfect fluid with isotropic tension equal to proper energy density is present. Thus the collisions described in this paper could be interpreted as a source of this special kind of matter.

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