Bell-inequality violation with “thermal” radiation

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The model of a quantum-optical device for a conditional preparation of entangled states from input mixed states is presented. It is demonstrated that even thermal or pseudo-thermal radiation can be entangled in such a way, that Bell-inequalities are violated.

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I. INTRODUCTION

In last decades, the phenomenon of entanglement between two spatially separated photons was investigated both experimentally and theoretically mainly in order to show that quantum mechanics is not a local realistic theory [1]. As a counterpart to the particle-like behavior of photons, the entanglement of coherent states, that can be considered as the quantum analogue of deterministic light waves, was examined [2]. In both cases, the entangled states, that were used to test Bell inequalities, were usually considered to be pure states. Recently, the entanglement of mixed states has been analysed to understand, how the disorder influences on the amount of entanglement [3].

In this report, we examine a new situation, when an entangling device prepares the entangled states of radiation from mixed states (thermal or pseudo-thermal light) at the input. Similarly to the idea presented in Ref. [3], the entangling device can produce a four-mode entangled state with two mixed states and two vacuum states. It is shown, that even for very disordered states Bell inequalities can strongly be violated. If there is a narrow frequency portion of thermal radiation in the input of the entangling device then Bell inequalities are violated when the frequency of radiation is “low” and the temperature of thermal source is “high”. For a pseudo-thermal radiation the violation of Bell inequalities is even more significant. In addition, the violation can be enhanced for both the cases of radiations, if a lot of different modes are entangled with vacuum state. Thus almost the maximal Bell inequality violation can be achieved with such thermal states exhibiting a large entropy.

II. PREPARATION OF MIXED ENTANGLING STATE

We consider two separate systems A and B which consist locally of two modes A1, A2 and B1 and B2. All modes are initially unentangled. We further assume that the density matrices of these four modes are diagonal in orthonormal Fock (number-state) bases \{\ket{n}\} and that the modes A2 and B2 are in vacuum states,

\[
\hat{\rho}_A = \sum_n p_n |n\rangle_{A1} \langle n| \otimes |0\rangle_{A2} \langle 0|,
\]

\[
\hat{\rho}_B = \sum_m r_m |m\rangle_{B1} \langle m| \otimes |0\rangle_{B2} \langle 0|.
\] (1)

The density matrix of the total system has a factorized form \(\hat{\rho}_n = \hat{\rho}_A \otimes \hat{\rho}_B\). Now, one can consider a conditional operation which enables to prepare the following entangled states (for \(n \neq 0\) or \(m \neq 0\))

\[
|\psi_{nm}\rangle = \frac{1}{\sqrt{2}} (|n\rangle_{A1}|0\rangle_{A2}|B1|m\rangle_{B2} - |0\rangle_{A1}|n\rangle_{A2}|B1|0\rangle_{B2}).
\] (2)

The entangling device prepares, for each \(m, n\), the analogue of a singlet state, that was often employed to test Bell-type inequalities. Thus the initial density matrix \(\hat{\rho}_n\) is transformed into the form

\[
\hat{\rho}_{\text{out}} = N \sum_{nm} p_n r_m (1 - \delta_{n0}\delta_{m0}) |\psi_{nm}\rangle \langle \psi_{nm}|,
\] (3)

where \(N = \sum_{nm} p_n r_m (1 - \delta_{n0}\delta_{m0})^{-1} = (1 - p_0 r_0)^{-1}\).

If there is at least one \(n > 0\) and one \(m > 0\) such that \(p_n \neq 0\) and \(r_m \neq 0\) then state (3) is entangled. It can be proved in a very straightforward way using the so called transposition criterion [4]. This criterion says that if operator \(\hat{\rho}_{T_B}\), obtained from \(\hat{\rho}\) by partial transposition in subsystem B, is not positive the state \(\hat{\rho}\) is entangled. Partial transposition of

\[
\hat{\rho}_{\text{out}} = \sum_{ijklmnst} \rho_{ijklmnst} |i_{A1}\rangle j_{A2}\rangle k_{B1}\rangle l_{B2}\rangle \langle m_{A1}\rangle n_{A2}\rangle s_{B1}\rangle t_{B2}|.
\]

in basis \(i_{A1}\rangle j_{A2}\rangle k_{B1}\rangle l_{B2}\rangle \equiv |i_{A1}|\rangle j_{A2}|k_{B1}|l_{B2}\rangle\) gives

\[
\hat{\rho}_{\text{out}}^{T_B} = \sum_{ijklmnst} \rho_{ijklmnst} |i_{A1}\rangle j_{A2}\rangle s_{B1}\rangle t_{B2}\rangle \langle m_{A1}\rangle n_{A2}|k_{B1}|l_{B2}|.
\]

Now, let us suppose vector...

*This paper is dedicated to Professor Jan Peřina in the occasion of his 65th birthday.
\[ |\phi_{mn}\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle_{A1}|m\rangle_{A2}|0\rangle_{B1}|n\rangle_{B2} + |m\rangle_{A1}|0\rangle_{A2}|n\rangle_{B1}|0\rangle_{B2} \right) \]

where \( m, n > 0 \) and calculate the following mean value

\[ \langle \phi_{mn} | \hat{\rho}_{\text{out}} | \phi_{mn} \rangle = -\frac{N_P m r_n}{2}. \tag{4} \]

If \( p_m \neq 0 \) and \( r_n \neq 0 \), this quantity is negative. On the other hand, the entanglement of discussed states can often be “masked” by the noise of original mixed states. E.g., conditional von Neumann entropy, \( S(\hat{\rho}_A) - S(\hat{\rho}) \), is positive for many particular cases here. Nevertheless, we will show that the entanglement is “strong” enough to violate CHSH-Bell inequality.

The proposed conditional operation can be, in principle, realized in the following way (see Fig. 1). Let us assume a Mach-Zehnder (M-Z) interferometer with equal-length arms and with one photon in its input. Into both the arms of the interferometer we insert a nonlinear Kerr medium effectively described by the following interaction Hamiltonian

\[ \hat{H}_{1,i} = \hbar \kappa \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a}_{1i}, \tag{5} \]

where \( \hat{a}^\dagger \) and \( \hat{a} \) are the creation and annihilation operators of the mode corresponding to the left (or right) arm in the M-Z interferometer, \( \hat{a}^\dagger_{1i} \) and \( \hat{a}_{1i} \), with \( i = A, B \), are the creation and annihilation operators of modes \( A1 \) (or \( B1 \)), and \( \kappa \) is a real interaction constant.

If there is a photon in the left arm of the central M-Z interferometer and the product \( \kappa \tau_{\text{int}} \), where \( \tau_{\text{int}} \) is an effective interaction time, is set to be equal exactly to \( \pi \) then the described device realizes the phase shift \( \pi \) in the left M-Z interferometer, \( A \), and effectively flips the modes \( A1 \) and \( A2 \) on the output. On the other hand, if there is no photon in the left arm then the states of modes \( A1 \) and \( A2 \) stay unchanged,

\[ \hat{U}_A |n\rangle_{A1}|0\rangle_{A2}|1\rangle = |0\rangle_{A1}|n\rangle_{A2}|1\rangle, \]
\[ \hat{U}_A |n\rangle_{A1}|0\rangle_{A2}|0\rangle = |n\rangle_{A1}|0\rangle_{A2}|0\rangle. \tag{6} \]

The same is true about the right arm of the central M-Z interferometer and modes \( B1 \) and \( B2 \). These unitary transformations \( \hat{U}_i, i = A, B \), can be expressed as

\[ \hat{U}_i = \hat{U}_{BS,i}^\dagger \hat{U}_{I,i} \hat{U}_{BS,i}, \tag{7} \]

where \( \hat{U}_{BS,i} \) is the 50:50 beam splitter transformation and \( \hat{U}_{I,i} \) accounts for the nonlinear interaction in Kerr medium,

\[ \hat{U}_{BS,i} = \exp \left[ \frac{\pi i}{4} (\hat{a}^\dagger_{1i} \hat{a}_{1i} - \hat{a}^\dagger_{1i} \hat{a}_{1i}) \right], \]
\[ \hat{U}_{I,i} = \exp (i \kappa \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a}_{1i}). \tag{8} \]

So, if the photon goes through the left arm the modes \( A1 \) and \( A2 \) are flipped while the state of system \( B \) is unchanged. Completely symmetrical situation occurs, if the photon goes through the right arm.

Due to the path uncertainty of the photon in the interferometer the state of the whole system after the interaction is given by the formula

\[ |\Psi \rangle = \frac{1}{\sqrt{2}} \left( |0\rangle_{A1}|n\rangle_{A2}|0\rangle_{B1}|m\rangle_{B2} + i |1\rangle_{A1}|0\rangle_{A2}|m\rangle_{B1}|0\rangle_{B2} \right), \tag{9} \]

where the kets without any subscript denote possible states of the photon inside the M-Z interferometer situated in the center. Which-way information is finally erased by a beam splitter with amplitude reflectivity \( i/\sqrt{2} \) (the last one in the M-Z interferometer) followed by two photodetectors \( D_+ \) and \( D_- \) (see Fig. 1). Depending on which one of these two detectors fires we obtain one of two possible output states of modes \( A1, A2, B1, \) and \( B2 \). Detector \( D_+ \) fires with probability \( w_+ = (1 + \delta_{n0}\delta_{m0})/2 \) and if it clicks the following state is obtained

\[ |\Psi_+ \rangle = \frac{1}{\sqrt{2}} \left( |n\rangle_{A1}|0\rangle_{A2}|0\rangle_{B1}|m\rangle_{B2} + |0\rangle_{A1}|n\rangle_{A2}|m\rangle_{B1}|0\rangle_{B2} \right). \tag{10} \]

Similarly, detector \( D_- \) clicks with probability \( w_- = (1 - \delta_{n0}\delta_{m0})/2 \) and when it fires one obtains the state

\[ |\Psi_- \rangle = \frac{1}{\sqrt{2}} \left( |0\rangle_{A1}|n\rangle_{A2}|m\rangle_{B1}|0\rangle_{B2} - |n\rangle_{A1}|0\rangle_{A2}|0\rangle_{B1}|m\rangle_{B2} \right), \tag{11} \]

which is exactly the considered state \( |\bar{2} \rangle \).

1 Quantity \( \tau_{\text{int}} \) has the meaning of the parameter of the device. It is not a usual time variable. It represents the effective expression of the fact that the nonlinear medium has finite dimensions.
III. BELL-INEQUALITY VIOLATION

In order to demonstrate the violation of Bell inequalities one needs local operations analogous to spin rotations. In our particular case the following operations do the job

$$|n\rangle_1|0\rangle_2 \rightarrow \cos \theta |n\rangle_1|0\rangle_2 + \sin \theta |0\rangle_1|n\rangle_2 \text{ for } n \neq 0,$$

$$|0\rangle_1|n\rangle_2 \rightarrow -\sin \theta |n\rangle_1|0\rangle_2 + \cos \theta |0\rangle_1|n\rangle_2 \text{ for } n \neq 0,$$

$$|0\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|0\rangle_2,$$  \hspace{1cm} (12)

where $\theta$ is the parameter of transformation, it does not depend on $n$.

Bell-type experiment consists of two “rotations” according to recipe (12), performed by two possibly space-like separated observers, followed by realistic yes–no detection on each mode. Each such detection has only two possible outcomes (detector either fires or it does not), that can be described by projectors $|0\rangle\langle 0|$ (for “no”) and $1 - |0\rangle\langle 0|$ = $\sum_{n=1}^{\infty} |n\rangle\langle n|$ (for “yes”). Let us assign the following values to these outcomes: $z_i=0$ if the detector (in mode $i$) is quiet and $z_i=1$ if it clicks. Then the results $X$ and $Y$ of local two-mode measurements (including “rotations”) performed by the first and the second observer, respectively, can be expressed as

$$X(\theta) = z_{A1}(\theta) - z_{A2}(\theta),$$

$$Y(\theta) = z_{B1}(\theta) - z_{B2}(\theta).$$  \hspace{1cm} (13)

After the experiment is repeated many times and our two observers compare their results, the mean value of Bell operator (for CHSH inequalities) can be estimated,

$$\mathcal{B} = |C(\theta_A, \theta_B) + C(\theta_A, \theta_B') + C(\theta_A', \theta_B) - C(\theta_A', \theta_B')|,$$  \hspace{1cm} (14)

where correlation function

$$C(\theta_1, \theta_2) = \sum_{j,k} X_j Y_k p(X_j, Y_k | \theta_A, \theta_B)$$  \hspace{1cm} (15)

(summations go over all possible results). Every local-realistic theory must fulfill the following inequality $\mathcal{B} \leq 2$ \[3\]. However, it follows from straightforward quantum-mechanical calculations that for state \[3\] the correlation function \[3\] reads

$$C(\theta_A, \theta_B) = -\cos [2(\theta_A - \theta_B)] \frac{(1 - p_0)(1 - r_0)}{1 - p_0 r_0}.$$  \hspace{1cm} (16)

Therefore the results of the above mentioned local measurements performed on state \[3\] violate inequality $\mathcal{B} \leq 2$ in principal. Maximal violation,

$$\mathcal{B}_{\text{max}} = 2\sqrt{2} \frac{(1 - p_0)(1 - r_0)}{1 - p_0 r_0},$$  \hspace{1cm} (17)

occurs for the angles

$$\theta_A = 0, \quad \theta_A' = \frac{\pi}{4}, \quad \theta_B = \frac{\pi}{8}, \quad \theta_B' = -\frac{\pi}{8}.$$  \hspace{1cm} (18)

If both the mixed states have the same overlap with vacuum state $p_0 = r_0$, the condition for the violation of Bell inequality for the considered angles is given in a simple form

$$p_0 < \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \approx 0.1716.$$  \hspace{1cm} (19)

As one can see the maximum value of $\mathcal{B}$ depends on the probability of the presence of the vacuum state in the input density matrices. Thus, if the input density matrices of systems $A1$ and $B1$ do not contain the vacuum state the maximal violation of CHSH-Bell inequality is the same as for the pure EPR maximally entangled state of two spin-half particles. In the opposite case, the mean value of Bell operator decreases as the contribution of the vacuum state increases in the mixtures. It should be noticed that for properly chosen local measurements the violation of CHSH-Bell inequality does not depend on the randomness contained in the mixture but only on the overlaps of the vacuum state and the input density matrices.

IV. THERMAL AND PSEUDO-THERMAL RADIATION

There are two mixed states of special interest, namely thermal radiation, exhibiting Bose-Einstein statistics, and pseudo-thermal radiation, exhibiting Poissonian statistics. Let us study now the entangled states prepared by the device proposed in Sec. II when thermal and pseudo-thermal states are at the input.

A single mode of thermal radiation has the density matrix

$$\hat{\rho} = \sum_n \frac{\langle n \rangle^n}{(1 + \langle n \rangle)^{1+n}} |n\rangle\langle n|,$$  \hspace{1cm} (20)

where

$$\langle n \rangle = \frac{1}{\exp(\frac{\hbar \omega}{k_B T}) - 1}.$$  \hspace{1cm} (21)

For example, if the temperature of a radiation source (e.g., incandescent lamp) $T \approx 3000 K$ and the optical frequency $\omega \approx 2.5 \times 10^{15} Hz$, the mean value of photon number is $\langle n \rangle \approx 1.77 \times 10^{-3}$. The probability of the vacuum state in the mixture is

$$p_0 = 1 - \exp \left(-\frac{\hbar \omega}{k_B T}\right) = \frac{1}{1 + \langle n \rangle},$$  \hspace{1cm} (22)

what leads to the value $p_0 \approx 0.9982$ for the above given data. Thus in the optical region, the overlap of vacuum and thermal light is too large and the Bell-inequality violation does not occur.
The dependence of the maximal Bell-inequality violation on the parameter $\beta_i = \hbar \omega_i/k_B T_i$, $i = A, B$ of particular modes $A1, B1$ can be simply evaluated:

$$
E_{\text{max}} = 2\sqrt{2} \frac{1}{\exp(\beta_A) + \exp(\beta_B) - 1} = 2\sqrt{2} \frac{1}{1 + \langle n \rangle_A^{-1} + \langle n \rangle_B^{-1}}
$$

(23)

and it is displayed in Fig. 2. Only for very small $\beta_A$ and $\beta_B$, i.e., for high temperatures and small frequencies, CHSH-Bell inequality is violated. Thus for the given temperature $T$ of both the thermal sources the infrared component of radiation gives better results than the ultra-violet one. On the other hand, for the fixed frequency $\omega$ of both the sources the higher temperature leads to the stronger violation of Bell inequality. If both the sources are identical the Bell-inequality violation occurs only if the dimensionless parameter $\beta$ satisfies relation $\beta < \ln (\sqrt{2} + 1) \approx 0.1882$ or the mean number $\langle n \rangle$ is sufficiently large $\langle n \rangle > 2(\sqrt{2} + 1) \approx 4.828$. Consequently, for the visible component of radiation the thermal sources must have an “astronomical” temperature $T \approx 101000K$, whereas for the infrared component with $\omega \approx 5 \times 10^{13}$Hz, temperature $T > 2021K$ is sufficient to obtain Bell-inequality violation.

Another interesting kind of mixed state is that corresponding to pseudo-thermal light [10]. Its density matrix can be written as

$$
\hat{\rho} = \sum_n \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle} |n\rangle \langle n|.
$$

(24)

Pseudo-thermal radiation can be obtained from an intensity-stabilized single mode laser with the phase uniformly distributed in the interval $(0, 2\pi)$. In contrast to thermal radiation, the maximally probable state in the mixture (24) is not vacuum state but it is state $|n\rangle$, where $n$ corresponds approximately to the mean number of photons $\langle n \rangle$. Thus the overlap of pseudo-thermal light with the vacuum state is much less than for thermal light. The probability of the vacuum state in the density matrix (24) is $p_0 = \exp(-\langle n \rangle)$. This leads to maximal Bell-inequality violation

$$
E_{\text{max}} = 2\sqrt{2} \frac{[1 - \exp(-\langle n \rangle)_A][1 - \exp(-\langle n \rangle)_B]}{1 - \exp(-\langle n \rangle_A + \langle n \rangle_B)}.
$$

(25)

From Fig. 3 one can see that in the case of pseudo-thermal light the Bell-inequality violation is achieved for less $\langle n \rangle_A$ and $\langle n \rangle_B$ than in the case of thermal light. If one considers two identical pseudo-thermal sources, then the Bell inequality is violated if $\langle n \rangle > \ln (\sqrt{2} + 1)$. As the pseudo-thermal light with such a mean photon number can be experimentally achieved from the laser light in optical frequencies, the violation can be obtained more simply than for the thermal light.

FIG. 2. The maximal violation of the CHSH-Bell inequality for thermal light as the function of parameters $\beta_A = \hbar \omega_A/k_B T_A$ and $\beta_B = \hbar \omega_B/k_B T_B$.

![Graph of maximal violation of CHSH-Bell inequality](image)

FIG. 3. The border of violation of CHSH-Bell inequality for thermal and pseudo-thermal light in dependence on mean photon numbers $\langle n \rangle_A$ and $\langle n \rangle_B$.

Real light sources emit to a large amount of different independent modes. The density matrix of this multi-mode state is given in the following form

$$
\hat{\rho} = \prod_{\mu} \sum_{n_{\mu}=0}^{\infty} \frac{(\langle n_{\mu} \rangle)^{n_{\mu}}}{(1 + \langle n_{\mu} \rangle)^{1/n_{\mu}}} |n_{\mu}\rangle \langle n_{\mu}|,
$$

(26)

where $n_{\mu}$ is photon number for particular mode $\mu$ and $|n_{\mu}\rangle$ is the Fock state of the corresponding mode. Let us suppose that this multi-mode thermal state is feeded us suppose that this multi-mode thermal state is feded to the inputs $A1$ and $B1$ and the multi-mode vacuum states are present in the inputs $A2$ and $B2$. The analysis presented in Sec. III may be generalized to multi-mode

\[\text{2}\text{In reality there could be a problem to set the proper parameters of Kerr interaction for all the frequency components together.}\]
light in a straightforward way. We define the “rotations” of the multi-mode vacuum \(|\{0\}\rangle\) and any excited multi-mode state \(|\{n\}\rangle\) as follows,

\[
\begin{align*}
|\{n\}\rangle_2 & \rightarrow \cos \theta |\{n\}\rangle_1 |\{0\}\rangle_2 + \sin \theta |\{0\}\rangle_1 |\{n\}\rangle_2, \\
|\{0\}\rangle_1 |\{n\}\rangle_2 & \rightarrow - \sin \theta |\{n\}\rangle_1 |\{0\}\rangle_2 + \cos \theta |\{0\}\rangle_1 |\{n\}\rangle_2,
\end{align*}
\]

(27)

for \(|\{n\}\rangle \neq 0\), and for multi-mode vacuum in both the modes: \(|\{0\}\rangle_1 |\{0\}\rangle_2 \rightarrow |\{0\}\rangle_1 |\{0\}\rangle_2\). Detection that discriminates between the field vacuum and other states has two possible outcomes described by projectors \(|\{0\}\rangle\langle\{0\}|\) and \(1 - |\{0\}\rangle\langle\{0\}|\). It can be shown that the maximal violation of Bell inequality exhibits the same form (17) as in the case of single-mode radiation, but with the following notation

\[
p_0 = \prod_\mu p_{0,\mu}, \quad r_0 = \prod_\mu r_{0,\mu}.
\]

V. CONCLUSION

An entangling device employing non-linear dynamics and postselection has been proposed and it has been shown that two mixed states can be entangled in such a way that the entanglement of the resulting state is strong enough to violate Bell inequalities (when proper local measurements are chosen). The disorder due to the statistical nature of the density matrices of input states is irrelevant – it does not influence the violation of Bell inequality. The only parameters affecting the maximum of the mean value of Bell operator are overlaps \(p_0 = \langle 0 | \rho_{A1} | 0 \rangle\) and \(r_0 = \langle 0 | \rho_{B1} | 0 \rangle\). This is also the reason of a contra-intuitive behavior when the entanglement increases as the input thermal state becomes more ‘classical’ (\(\beta \rightarrow 0\)), whereas in the ‘quantum’ limit (\(\beta \rightarrow \infty\)) the entanglement vanishes. Another contra-intuitive aspect of this phenomena appears if the multi-mode thermal radiation is considered. Since the overlap with multimode vacuum becomes smaller as the number of modes increases, the multi-mode thermal radiation can violate Bell inequality more notably, irrespective of its larger entropy. Thus this “classical-like” radiation can be strongly entangled in the ideal case and even exhibit the pronounced quantum nonlocality. Unfortunately, like the other kinds of mesoscopic states, the described quantum superpositions are very sensitive to the destructive influence of decoherence and losses.

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