Self-Adaptive PID Control Based on RBF Network for Trajectory Tracking of Dual-Mass Servo System

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Abstract. This paper presents a state-of-the-art algorithm of self-adaptive RBF network PID control and aims at achieving the precise and fast trajectory tracking for a dual-mass servo system. To increase the robustness of servo system parameter varying and external disturbances, a classical PID algorithm with enhanced structure based on RBF neural network is proposed. Extensive simulations show that it practically validates the superiority of the proposed RBF adaptive PID controller. The experiment was simulated respectively under the external disturbance, which indicates that accurate tracking performance of the servo system with dual-mass load has been achieved and also verifies the effectiveness of self-adaptive PID control strategy.

1. Introduction

The PID control extensively applies in control strategy for servo drive system. Lukichev, D.V. adopted fuzzy adaptive PID method for the elasticity and friction in the dual-mass servo system[1]. Groups of Chen J.H. adopted the network into nonlinear system realizing the on-line adjusting PID parameters effectively [2]. However, classical PID controller can’t better adapt features of time-varying. To improve the control performance, several schemes of self-tuning PID controllers were studied in many papers (see for example [3, 4]). Serkies et al., in [5] applied the MPC to the position control for dual-mass system, and obtained the better performance. Sliding model control and iterative learning strategy were also improved in [6] and [7] which all obtained the satisfying results.

A lot of control methods have been proposed, such as e.g. resonance ratio control [8], sliding mode control, adaptive or predictive control [9-13]. These studies have made indelible contributions to this problem. However, the purpose of this paper is to verify and improve the tracking performance of position/speed servo system whether the action of friction forces or not. Namely, the control objective was a dual-mass servo system with tricyclic structure. The proposed adaptive PID control based on RBF network uses the self-learning ability of RBF to automatically tune and modify the PID parameters on-line so as to track the trajectory of position/speed servo system effectively. Extensive simulations show that the proposed controller gives satisfactory results.

2. Model of Dual-Mass Servo System and Control Structure Design

2.1. Design the Model Diagram of Servo System
This paper focuses on the dual-mass servo system which simplifies from the three-inertia servo system consisted of DC motor, pure inertial load and the equivalent drive shaft that links both motor and load as shown in figure 1. The first part of this model is an equivalent circuit for DC motor, where the rotary inertia of motor, load and shaft is denoted as $J_m$, $J_L$ and $J_S$ respectively.

![Figure 1. Motor-Shaft-Load equivalent model diagram.](image1)

where $u(t)$ is the armature voltage; $\theta_L(t)$ is load angular position; $R_m$ and $L_m$ are the resistance and inductance of the motor armature winding in the motor, respectively. $V_{emd}$ denotes the electromotive force; $\omega_m$, defined by $\dot{\theta}_m(t)$, denotes the motor angular velocity; $\dot{\theta}_L(t)$ denotes the load speed; $b_m$ and $b_L$ are the viscosity damp coefficient of motor and load, respectively. $T_{ml}$ is the output torque on the load side. $K_i$ is defined as the coefficient of current feedback. $K_m$ and $K_L$ are defined as the moment coefficient. $J_m$ and $J_L$ are the equivalent moment of inertia of motor and load with frame. Assuming that the parameters for dual-servo system show as table1.

![Figure 2. (a) Block diagram of Servo system motor diagram; (b) Block diagram of load.](image2)

| R_m (Ω) | L_m (H) | K_i | J_m (kg·m^2) | b_m | K_m | K_L | J_L (kg·m^2) | b_L |
|---------|---------|-----|--------------|-----|-----|-----|--------------|-----|
| 1.0     | 0.001   | 0.001 | 0.005        | 0.010 | 10.0 | 5.0 | 0.15         | 8.0 |

### 2.2. Mathematical Model of Three-Mass Servo System

Based on the Newton's law and Kirchhoff's law, electrical equation and dynamic equation of three-mass servo system are written as equation (1) which can be finished to equation (2).

$$u(t) = R_m i(t) + L_m \frac{di(t)}{dt} + K_m i(t) + V_{emd}(t)$$

$$V_{emd}(t) = C_e \frac{d\theta_m(t)}{dt}$$

$$u(t) = R_m i(t) + L_m \frac{di(t)}{dt} + K_m i(t) + C_e \frac{d\theta_m(t)}{dt}$$

The first derivative of armature current $i(t)$ is simplified as:
\[
\frac{di(t)}{dt} = -\left(\frac{R_m + K_L}{L_m}\right)i(t) - \left(\frac{C_e}{L_m}\right)\frac{d\theta_m(t)}{dt} + \left(\frac{1}{L_m}\right)u(t) \quad (3)
\]

Where \(C_e\) is the back EMF constant. The equations for the mechanical side of the system are showed as equation (4) that can be finished to equation (5).

\[
J_m \frac{d^2\theta_m(t)}{dt^2} + b_m \frac{d\theta_m(t)}{dt} = T_m - K_L (\theta_m(t) - \theta_L(t)) \quad T_m = K_m i(t) \quad (4)
\]

\[
J_m \frac{d^2\theta_m(t)}{dt^2} + b_m \frac{d\theta_m(t)}{dt} = K_m i(t) - K_L (\theta_m(t) - \theta_L(t)) \quad (5)
\]

And the kinetic equation of transmission shaft is written as:

\[
J_s \left(\dot{\theta}_m(t) - \dot{\theta}_L(t)\right) = K_L \left(\theta_m(t) - \theta_L(t)\right) - T_{ml} \quad (6)
\]

The load dynamic equation assuming without regard to friction is derived as:

\[
J_L \ddot{\theta}_L(t) = T_{ml} - b_L \dot{\theta}_L(t) \quad (7)
\]

However, the inertia of transmission shaft \(J_s\) relative to \(J_L\) is extremely small and its quality mainly distributes over the length so that it can be approximate to zero in this study. Therefore, equation (6) can be simplified as follows.

\[
K_L \left(\theta_m(t) - \theta_L(t)\right) - T_{ml} = 0 \quad (8)
\]

2.3. Describing System by the State Equations

Five derivative terms in this system model are expressed as in equations (i.e., (3), (4), (5), (6), (7)). So there are 5 state variables in the state space model, which can be defined as the outputs of the integrators with \(x_1(t)\) being \(i(t)\), \(x_2(t)\) being \(\theta_m(t)\), \(x_3(t)\) being \(\dot{\theta}_m(t)\), \(x_4(t)\) being \(\theta_L(t)\), \(x_5(t)\) being \(\dot{\theta}_L(t)\) and \(y(t) = \theta_L(t)\). Equations (i.e., (5), (7)) are simplified, in order to get the second derivatives of \(\dot{\theta}_m(t)\) and \(\dot{\theta}_L(t)\) which are derived as shown as:

\[
\frac{d^2\theta_m(t)}{dt^2} = -\left(\frac{b_m}{J_m}\right)\frac{d\theta_m(t)}{dt} + \left(\frac{K_m}{J_m}\right)i(t) - \left(\frac{b_L}{J_L}\right)\frac{d\theta_L(t)}{dt} \quad (9)
\]

\[
\frac{d^2\theta_L(t)}{dt^2} = \left(\frac{K_m}{J_L}\right)\left(\theta_m(t) - \theta_L(t)\right) - \left(\frac{b_L}{J_L}\right)\frac{d\theta_L(t)}{dt} \quad (10)
\]

The state space description of linear time-invariant system is formed as:

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t) \\
\dot{x}_3(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t)
\end{bmatrix} =
\begin{bmatrix}
\frac{R_m + K_L}{L_m} & 0 & -\frac{C_e}{L_m} & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\frac{K_m}{J_m} & -\frac{K_L}{J_m} & -\frac{b_m}{J_m} & \frac{K_i}{J_m} & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & \frac{K_i}{J_L} & 0 & -\frac{K_i}{J_L} & \frac{b_l}{J_L}
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t) \\
x_4(t) \\
x_5(t)
\end{bmatrix} +
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} u(t)
\]

(11)
\[ y(t) = [0 \ 0 \ 0 \ 1 \ 0] x(t) \]  

where \( x(t) = [x_1(t), x_2(t), x_3(t), x_4(t), x_5(t)] \) is a vector as in equation (12).

3. Adaptive PID Controller Based on RBF Network

In this paper, structure of this controller is showing as figure 3. There are double outputs which are Out1 representing the load angular position \( \dot{\theta}_L(t) \) and Out2 showing the load angular speed \( \dot{\dot{\theta}}_L(t) \).

![Figure 3. The structure of adaptive PID controller.](image)

Theoretically, conventional PID control law can be configured for such servo system to realize the better tracking property:

\[ u(t) = u_{k-1}(t) + K_p e(t) + K_i \int_0^t e(t) dt + K_d de(t) \]  

where \( u(t) \) and \( u_{k-1}(t) \) are the \( k \)th and \((k-1)\)th control inputs, respectively; \( K_p, K_i, K_d \) are proportion coefficient, integral coefficient and differential coefficient, respectively; \( e(t) \) is the angular position deviation of trajectory tracking for this servo system, which can be given as:

\[ e(t) = y_d(t) - y(t) \]  

The outputs of the adaptive controller is represented as:

\[ u_k(t) = u_{k-1}(t) + [K_{pid} + e(t) \cdot d_{jac} \cdot eta_{pd}] \cdot x \]  

Where \( K_{pid} = [K_p, K_i, K_d] \) is a 3-dimension vector which adopts the gradient descent method for tuning; \( d_{jac} \) represents the Jacobi matrix, called sensitivity; \( eta_{pd} \) represents the learning ratio for tuning PID parameters in real-time; these variables are defined for programming in Matlab easily.

The second term of equation (15) is equivalent to \( \Delta u(t) \). Dropping down the property function \( E(k) \) to zero is set as the tuning indicator of dynamic controller, showing as:

\[ E(k) = \frac{1}{2}(y_d(t) - y(t))^2 = \frac{1}{2} e^2(t) \]  

However, The desired tracking property under the nominal plant model can’t be guaranteed when you add some external disturbances or set different values for parameters of this plant model.
4. Verifications Of Trajectory Tracking
Considering the true situation in this paper, the structure of RBF sets as 3-5-1. The state equation of
plant as shown in equation (13). The initial parameters gives as table 2.

| Table 2. Initial parameters for adaptive controller |
| --- | --- | --- | --- | --- |
|   |   |   |   |   |
| $K_p$ | $K_i$ | $K_d$ | $\eta$ | $\alpha$ | $\beta$ | $w$ |
| 430 | 11 | 75 | 0.25 | 0.08 | 0.01 | 0.5 |

Where $\eta$ is learning ratio, $\alpha$ and $\beta$ represent the momentum factor. Experimental simulation is
carried out according to whether disturbance is added or not.

4.1. Experiment with Disturbance by Conventional PID Control Algorithm
4.1.1. Step Signal Trajectory Tracking
Figure 4 is shown the tracking results of a step signal trajectory of position whose amplitude is 1 mm
triggering at time 0 with disturbance. The range of the amplitude of the random disturbance for the
feed-forward channel is setting as $[-0.1, 0.1]$. The noise power of white noise for the feedback
channel is setting as 0.001. This trajectory with disturbance from figure 4 has a bigger overshoot and
longer rising time, compared with effect of no disturbance.

![Figure 4](image)

Figure 4. The position tracking results of step signal trajectory by conventional PID control method.

4.1.2. Sine Wave Trajectory Tracking
Due to the disturbance, the tracking effect is shown in figure 6 as inputting the sine wave whose
amplitude is 1mm, its frequency setting as $\pi/2$ Hz. The setting of disturbance et al., are same as above.

![Figure 5](image)

Figure 5. The last tracking results of Sine wave trajectory by conventional PID control algorithm: (a)
position tracking; (b)speed tracking; (c)position tracking error; (d) speed tracking error.
4.1.3. Rectangular Wave Trajectory Tracking

Figure 6 is shown the tracking results of a square wave whose frequency is $\pi/2$ Hz, duty cycle ratio is 1/2, and its amplitude is 1mm. The tracking effect both position and speed decreases gradually with the increase of the simulating time. The setting of disturbance et al., are same as above. This trajectory with disturbance from figure 6 (c) also has a bigger tracking error.

![Figure 6](image)

Figure 6. The last tracking results of square signal trajectory by conventional PID control algorithm: (a) position tracking; (b) speed tracking; (c) position tracking error; (d) speed tracking error.

4.2. Experiment with Disturbance by adaptive PID controller

4.2.1. Rectangular Wave Trajectory Tracking

From figure 7 we can see that the actual position trajectory under the disturbance fluctuates around desired value by the adaptive PID algorithm. The setting of disturbance et al., are same as above.

![Figure 7](image)

Figure 7. The tracking results of step signal trajectory by adaptive method.

It can be seen from the partial enlarged drawing of figure 7 that the actual trajectory under the disturbance has an overshoot about 1.2%, which is much less than 2%~5%. Its rising time is about 20ms. In short, the real position output trajectory can quickly track the expecting signal with a little overshoot.
4.2.2. Sine Wave Trajectory Tracking
We can observe from figure 8(c) that the actual position trajectory error under the disturbance fluctuates over much smaller range about ±0.006mm. By the way, the speed tracking error from figure 8 (d) also fluctuates around desired value about ±0.5mm/s. The setting of the random disturbance and the noise power are the same as above.

Figure 8. The last tracking results of Sine wave trajectory by adaptive PID control algorithm: (a) position tracking; (b)speed tracking; (c)position tracking error; (d) speed tracking error.

4.2.3. Rectangular Wave Trajectory Tracking
This paper mainly research the position trajectory tracking which is paid more attention. Figure 9 shows that the amplitude of the square wave is 1 mm, and its frequency is $\pi/2$ Hz. As shown above in figure 9 (a), actual position trajectory of square wave quickly approaches to the expected value under the disturbance. The position tracking error remains ±0.5 mm, which is desired value. The setting of the random disturbance and the noise power are the same as above. Figure 9 (c) and (d) are also shown the tracking error.

Figure 9. The last tracking results of square signal trajectory by adaptive PID control algorithm: (a) position tracking; (b)speed tracking; (c)position tracking error; (d) speed tracking error.

As shown above, the sine wave is faster in convergent radio and less tracking error fluctuating over ±0.006mm, compared with square wave. This slight difference may be caused by a different characteristics between square wave and sine signal.
5. Conclusion
This paper studied the trajectory tracking of dual-mass servo system with Adaptive PID control algorithm based on the RBF network. This adaptive control strategy is adopted to verify and improve the robustness of this servo system and realize the precise and fast trajectory tracking for the dual-mass motor servo system. Extensive simulation results indicate that the angular position and speed tracking errors approximate to zero near under the disturbance. In the next study, it will be improved for the undesired speed tracking error of square wave to make it approach to zero approximately.

Acknowledgements
This work was supported by National Natural Science Foundation of China (Grant No. 52065060). Supported by the National Natural Science Foundation of China "Research on Electromagnetic Topological Structure Optimization Technology of High Energy Efficient Maglev Plane Motor (52065060)".

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