Dynamic multi-technology production-inventory problem with emissions trading

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ABSTRACT

We study a periodic-review multi-technology production-inventory problem of a single product with emissions trading over a planning horizon consisting of multiple periods. A manufacturer selects among multiple technologies with different unit production costs and emissions allowance consumption rates to produce the product to meet independently distributed random market demands. The manufacturer receives an emissions allowance at the beginning of the planning horizon and is allowed to trade allowances through an outside market in each of the following periods. To solve the dynamic multi-technology production-inventory problem, we virtually separate the problem into an inner layer and an outer layer. Based on the structural properties of the two layers, we find that the optimal emissions trading policy follows a target interval policy with two thresholds, whereas the optimal production policy has a composite base-stock structure. Our theoretical results show that no more than two technologies should be selected simultaneously at any state. However, different groups of technologies may be selected at different states. Our numerical tests confirm that it can be economically beneficial for a manufacturer to maintain multiple available technologies.

1. Introduction

In order to curb carbon dioxide (CO\textsubscript{2}) and sulfur dioxide (SO\textsubscript{2}) emissions to protect the environment, an increasing number of countries and regions have started to implement the Emissions Trading Scheme (ETS) as a market-based mechanism to encourage the adoption of green technologies in CO\textsubscript{2} and SO\textsubscript{2} emissions-intensive industries. With the ETS, manufacturers receive emissions allowances at the beginning of a multi-period planning horizon and are periodically allowed to trade their allowances through an outside market. To date, the ETS has been implemented in many industries, such as power stations, combustion plants, oil refineries, iron and steel makers, pulp and paper mills, and cement plants (European Commission, 2014).

In this article, we study a periodic-review multi-technology production-inventory problem with emissions trading over a multi-period planning horizon. A manufacturer produces a single product using technologies selected from multiple production technologies to meet independently distributed random market demands. Different technologies have different unit production costs and emissions allowance consumption rates. Under the ETS regulation, in each period, the manufacturer first observes its product inventory levels, allowance levels, and emissions trading prices and then determines the emissions allowance trading and selects appropriate production technologies with appropriate production quantities. Unmet demand in each period is completely backlogged. The objective of the manufacturer is to minimize the expected total discounted cost over the entire planning horizon.

Prior studies examined some special cases of our research problem. For example, Gong and Zhou (2013) successfully provide a solution for the case with a green technology and a traditional technology. However, in reality, manufacturers often have more than two available technologies. In many emissions-intensive industries, fuels such as coal, clean fuels, natural gas, and residual oil can be selected to generate heat in production processes. These fuels generating the same amount of heat have different unit production costs and consume different emissions allowances. In the cement industry, four different technologies (dry process with oil, dry process with coal, wet process with oil, and wet process with coal) can be adopted in production processes (Fort, 2005). These technologies hold different unit production costs and allowance consumption rates (Hendriks et al., 2002). The presence of multiple technologies gives manufacturers more options to pursue their business goals while satisfying emissions regulations. In the multi-technology case, the minimum production cost function (under given total production quantity and total allowance consumption) is only submodular instead of linear in the dual-technology case. Therefore, it is much more challenging to characterize structural properties of the optimal cost function and develop optimal policies for the multi-technology case.

To solve the multi-technology production-inventory problem with emissions trading, we virtually separate the problem into two layers: an inner layer where the shape of the minimum production cost function is determined and an outer layer...
where trading, total production quantity, and total allowance consumption decisions are made. For the inner layer, we first propose an algorithm to exclude the technologies that should never be selected and then derive the minimum total production cost function and structural properties of the optimal technology selection. For the outer layer, we first show structural properties of cost functions based on a preservation property of supermodularity and then fully characterize the optimal policies. Our results show that the optimal emissions trading policy follows a target interval policy with two thresholds, and the optimal production policy has a composite base-stock structure.

One popular myth about the multi-technology production-inventory problem is that even with many available technologies, a manufacturer usually adopts the best one or two technologies because of the maintenance costs for additional technologies. In this article, we theoretically confirm that when multiple technologies are available, no more than two technologies should be selected simultaneously in each period. However, we find that keeping more than two technologies available may bring significantly more benefits because additional technologies offer more options in the technology selection. Without considering the maintenance costs of technologies, our numerical tests show that keeping three technologies rather than two technologies may save a manufacturer up to 10% of the total operations cost.

The rest of this article is organized as follows. In Section 2, we review the related literature. In Section 3, we formulate the multi-technology production-inventory problem with emissions trading. In Section 4, we derive structural properties of the inner layer of our problem. In Section 5, we derive structural properties of the outer layer of our problem, and characterize the optimal policies. In Section 6, we conduct numerical tests to examine the benefits of keeping three technologies. In Section 7, we conclude and discuss the findings.

2. Literature review

This research studies the multi-technology production-inventory problem with emissions trading. It is strongly related to literature in two research streams: production problems with emissions regulations and dynamic production-inventory problems with capacity constraints.

Recently, production problems with emissions regulations have begun to draw considerable attention from researchers. Based on the Arrow-Karlin model, Dobos (2005) studies the effect of emissions trading on the optimal production-inventory strategy. Subramanian et al. (2007) investigate a three-stage game where a firm makes sequential decisions on emissions reduction investments, emissions allowance bidding, and production quantity. Drake et al. (2010) build a two-stage stochastic model to study the impacts of emissions tax and emissions trading regulations on a firm’s capacity and production technology selection decisions, respectively. Krass et al. (2013) examine the effect of environmental taxes on the adoption of green technologies. In their Stackelberg model, the environmental regulator sets a tax level first, based on the tax level, a profit-driven firm then makes its technology selection, production, and pricing decisions. However, none of the above studies considers the dynamic production-inventory problem.

Dynamic production-inventory problems with capacity constraints are extensively studied in the literature. Decroix and Arreola-Risa (1998) study a periodic-review multi-product production problem where products share a finite resource in each period. For homogeneous products, they identify the optimal policy with a modified base-stock structure; for general products, they propose a heuristic policy for the production planning. Yang et al. (2005) investigate a dual-sourcing inventory problem where in-house production is Markovian-capacitated and outsourcing may or may not be associated with a fixed cost. They find that a production-inventory-outsourcing policy is optimal. Yazlali and Erhun (2009) examine a dual-sourcing inventory problem where both suppliers have minimum and maximum order size constraints and find that the optimal policy has a modified dual base-stock structure. In the above literature, all capacity constraints are based on one production-inventory period, and the leftover capacity cannot be stored for future use.

There are also some production-inventory papers that consider the problems where the leftover capacity can be stored for future use. Bassok and Anupindi (1997) study a single-sourcing inventory problem where the cumulative order quantities over the planning horizon must be larger than a minimum total commitment. They show that the optimal policy has a dual base-stock structure. Chao et al. (2008) examine a dynamic inventory problem where the firm’s replenishment decisions are constrained by cash flow. They identify the structure of the optimal policy and provide an algorithm to compute policy parameters. Yang (2004) investigates a production-inventory problem where raw materials can be stored and traded. He finds that the optimal policy is a combination of two base-stock policies for the raw material and for the finished product, respectively. In the above literature, various capacities are consumed by a single technology. However, in our problem, there are multiple technologies, which consume the emissions allowance at different rates, and the manufacturer has an additional decision on the technology selection.

The closest literature to this article is Gong and Zhou (2013). Based on a preservation property of supermodularity, Gong and Zhou (2013) successfully characterize the optimal emissions trading policy and the optimal production policy for the dual-technology problem. However, for the multi-technology case studied in this article, the minimum production cost function is only submodular instead of linear in the dual-technology case. Therefore, the preservation property in Gong and Zhou (2013) cannot be applied to solve our problem. The presence of three or more technologies significantly complicates the analysis of optimal policies.

3. Model

We consider a periodic-review multi-technology production-inventory problem with emissions trading. Over a planning horizon consisting of $T$ periods (indexed by $t = 1, \ldots, T$), a manufacturer produces a single product to satisfy independently distributed random market demands $D_1, \ldots, D_T$. We assume that the unmet demand in each period is completely backlogged.
At the beginning of the entire planning horizon, the manufacturer receives $z_1$ emissions allowances and is allowed to periodically trade the allowances through an outside market. At the end of the entire planning horizon (i.e., the terminal period), the manufacturer is heavily fined with unit penalty cost $\pi$ for any extra emissions quantity over the received allowances.

Denote by $\tilde{K}_i$ and $\tilde{k}_i$ the unit allowance purchasing and selling random prices, respectively, in period $t \in \{1, \ldots, T\}$, where $\gamma \in (0, 1]$ is the one-period discount factor. Let $\tilde{K}_t = (\tilde{K}_i, \tilde{k}_i)$ be the emissions trading price vector in period $t$ and $\tilde{K}_1 = (K_1, k_1)$ be its realization, where $K_i$ and $k_i$ are the unit allowance purchasing and selling realized prices, respectively. We assume that $\Pr(0 \leq k_i \leq K_i \leq \gamma^{T-t+1}\pi) = 1$, and random trading price vectors $\tilde{K}_1, \ldots, \tilde{K}_T$ follow a Markov process. That is, the distribution of $\tilde{K}_i$ is only determined by $\tilde{K}_{i-1}$. In addition, we assume that $D_t$ and $\tilde{K}_t$ are independently distributed.

To avoid the speculation that a manufacturer buys (sells) an infinite amount of allowances in a period and then sells (buys) them in a future period, we assume that the allowance purchasing (selling) price in each period is higher (lower) than the discounted conditional expected selling (purchasing) price in a future period; that is,

$$K_i \geq \gamma^t E[\tilde{K}_{t+1} | \tilde{K}_i] \quad \text{and} \quad k_i \leq \gamma^t E[\tilde{K}_{t+1} | \tilde{K}_i]$$

for $t = 1, \ldots, T$, $i = 1, \ldots, T - t$, and all $\tilde{K}_i$.

The manufacturer selects technologies from an available technology set $\tilde{M}$, which contains more than two technologies (in Section 5, we will show the results for the special case where only one technology is available). We index the available technologies as $\tilde{1}, \ldots, \tilde{M}$ to distinguish them from the effective technologies that we will discuss later in this article. The $\tilde{M}$ available technologies are different from each other in terms of their unit production costs and allowance consumption rates. Denote $c_i$ and $\mu_i$ as technology $\tilde{i}$’s ($\tilde{i} = \tilde{1}, \ldots, \tilde{M}$) unit production cost and allowance consumption rate, respectively. Without loss of generality, we assume that $\mu_1 > \cdots > \mu_{\tilde{M}} \geq 0$ and $0 < c_1 < \cdots < c_{\tilde{M}}$. That is, no technology dominates another technology in both unit production cost and allowance consumption rate.

The sequence of the events in period $t \in \{1, \ldots, T\}$ is as follows.

1. The manufacturer checks the current inventory level $x_t$ and the allowance level $z_t$ and observes the trading price $\tilde{K}_t = (\tilde{K}_i, \tilde{k}_i)$ in the outside market.
2. The manufacturer trades (buys or sells) emissions allowances, and its allowance level after the trading is $\tilde{z}_t$. The manufacturer selects from the $\tilde{M}$ available technologies and determines their production quantities $q_{\tilde{1}t}, \ldots, q_{\tilde{M}t}$. After the decisions are made, the post-production inventory level and the ending allowance level are $y_t = x_t + \sum_{i=1}^{\tilde{M}} q_{it}$ and $w_t = \tilde{z}_t - \sum_{i=1}^{\tilde{M}} \mu_i q_{it}$, respectively.
3. Market demand $D_t$ occurs. Leftover inventory is carried over to the next period with unit holding cost $h$, and unmet demand is backlogged to the next period with unit backlogging cost $b$.

Please note that the allowance levels $z_t$, $\tilde{z}_t$, and $w_t$ can be negative; i.e., $z_t$, $\tilde{z}_t$, $w_t \in \mathbb{R}$, because the manufacturer is allowed to owe emissions allowances before the terminal period. The objective of the manufacturer is to minimize the expected total discounted cost over the entire planning horizon. The total cost includes the production cost, the emissions trading cost (which can be negative), and the holding–backlogging cost in each period. We denote the emissions trading cost in period $t \in \{1, \ldots, T\}$ as $C_t$ and the holding–backlogging cost in period $t$ as $G_t$. Then,

$$C_t(\tilde{z}_t - z_t, K_t) = K_t(\tilde{z}_t - z_t)^+ - k_t(z_t - \tilde{z}_t)^+,$$

$$G_t(y_t) = h E_{D_t}[ (y_t - D_t)^+ ] + b E_{D_t}[ (D_t - y_t)^+ ],$$

where $u^+ = \max\{u, 0\}$.

Following the sequence of events, the dynamics of the inventory level and the allowance level are

$$x_{t+1} = y_t - D_t = x_t + \sum_{i=1}^{\tilde{M}} q_{it} - D_t$$

and

$$z_{t+1} = w_t = \tilde{z}_t - \sum_{i=1}^{\tilde{M}} \mu_i q_{it},$$

respectively.

Denote by $V_t(x_t, z_t, K_t)$ the optimal expected total discounted cost from period $t$ to the end of the planning horizon under the given state $(x_t, z_t, K_t)$. Then, $V_t(x_t, z_t, K_t) \ (t = 1, \ldots, T)$ satisfies the following recursion equation:

$$V_t(x_t, z_t, K_t) = \min_{\tilde{q}_t, q_{\tilde{1}t}, \ldots, q_{\til{M}t}} \left\{ \sum_{i=1}^{\til{M}} c_i \til{q}_i + C_t(\til{z}_t - z_t, K_t) + G_t \left( x_t + \sum_{i=1}^{\til{M}} q_{it} \right) + \gamma E_t \left[ V_{t+1} \left( x_{t+1} + \sum_{i=1}^{\til{M}} q_{it} - D_t, \til{z}_t - \sum_{i=1}^{\til{M}} \mu_i q_{it} \right) \right] \right\},$$

where $E_t[ \cdot ]$ is short for $E_{D_t}[E_{K_t} [ \cdot | K_t ]]$ (recall that the distribution of $\tilde{K}_{t+1}$ only depends on $K_t$); $\Xi = \{ (\til{z}_t, q_{\til{1}t}, \ldots, q_{\til{M}t}) : \til{z}_t \in \mathbb{R}, q_{\til{1}t}, \ldots, q_{\til{M}t} \geq 0 \}$ is the region of feasible decisions.

At the end of the planning horizon, the cost function is

$$V_{T+1}(x_{T+1}, z_{T+1}, K_{T+1}) = p(-x_{T+1})^+ + pD(-x_{T+1})^+ + \pi(-x_{T+1})^+,$$

where $p$ and $pD$ are the unit penalty cost and unit salvage cost of the product, respectively. It is intuitive that $p \geq pD$. We assume that there is no salvage value for the leftover emissions allowances at the end of the planning horizon. If it is incorporated, all subsequent analyses and results remain valid. In the following discussion, we omit the subscripts $t$ and $T+1$ when there is no confusion.

Define $F(q, r)$ as the minimum production cost under given $(q, r)$, where $q = \sum_{i=1}^{\til{M}} q_{it}$ and $r = \sum_{i=1}^{\til{M}} \mu_i q_{it} = \til{z} - w$ are the total production quantity and the total allowance consumption, respectively. With respect to $\Xi$, the domain of $F(q, r)$ is
\[ \Theta = \{ (q, r) : r \geq 0, r/\mu_1 \leq q \leq r/\mu_M \}. \] Note that \( r/\mu_M \) is set as \( \infty \) if \( \mu_M = 0 \). It is easy to verify that \( \Theta \) is a lattice. \( F(q, r) \) can be determined by solving the following problem:

\[
F(q, r) = \min_{q_1, \ldots, q_M \geq 0} \left\{ c_i q_i + \cdots + c_M q_M : \sum_{i=1}^M q_i = q, \sum_{i=1}^M \mu_i q_i = r \right\}.
\] (5)

Problem (5) is a two-constraint multi-variable linear programming problem with knapsack-like features. Following problem (5), problem (3) can be transformed into

\[
V_t(x, z, K) = \min_{z \in \mathbb{R}} \{ C_t(\tilde{z} - z, K) + W_t(x, \tilde{z}, K) \},
\]

\[
W_t(x, \tilde{z}, K) = \min_{(q, r) \in \Theta} \{ F(q, r) + H_t(x + q, \tilde{z} - r, K) \},
\]

where \( H_t(y, w, K) = G_t(y) + \gamma E_t[V_{t+1}(y - D_t, w, \tilde{K}_{t+1})] \).

By introducing the minimum production cost function \( F(q, r) \), we virtually separate the production-inventory problem into an inner layer and an outer layer. The inner layer includes problem (5), where the shape of \( F(q, r) \) is determined via the optimal technology selection; the outer layer includes problems (6) and (7), where decisions on trading, total production quantity, and total allowance consumption are made based on the determined shape of \( F(q, r) \). In the following analysis, we first study structural properties of the inner layer, then derive structural properties of the outer layer, and finally characterize the optimal policies.

4. Structural properties of the inner layer

Within the available technology set \( M \), some technologies should never be selected due to their higher unit production costs or emissions consumption rates. We define them as ineffective technologies. The other technologies are thus considered as effective technologies. In this section, we first propose an algorithm to determine the effective technology set by excluding ineffective technologies, then derive the minimum production cost function, and finally characterize structural properties of the optimal technology selection.

4.1 Effective technology set

Intuitively, a technology may be dominated by a combination of two other technologies, although it cannot be dominated by another technology under the assumptions \( \mu_1 > \cdots > \mu_M \geq 0 \) and \( 0 < c_1 \leq \cdots \leq c_M \). Therefore, we make the following definition.

**Definition 1.** For a technology \( \tilde{t} \), if there exist technologies \( \tilde{i} \) and \( \tilde{j} \) such that \( \mu_{\tilde{t}} = \lambda_{\tilde{t}} \mu_{\tilde{i}} + (1 - \lambda_{\tilde{t}}) \mu_{\tilde{j}} \) and \( c_{\tilde{t}} \geq \lambda_{\tilde{t}} c_{\tilde{i}} + (1 - \lambda_{\tilde{t}}) c_{\tilde{j}} \) with a \( \lambda_{\tilde{t}} \in (0, 1) \), we define technology \( \tilde{t} \) as an ineffective technology; otherwise, we define technology \( \tilde{t} \) as an effective technology.

Clearly, producing with ineffective technology \( \tilde{t} \) incurs the same amount of allowance consumption as, but a larger or equal production cost than, producing a \( \lambda_{\tilde{t}} \) portion with technology \( \tilde{i} \) and a \( 1 - \lambda_{\tilde{t}} \) portion with technology \( \tilde{j} \). Hence, to minimize the production cost, ineffective technologies should never be selected for the production; only effective technologies should be considered.

Next, we propose an algorithm that excludes ineffective technologies and determines the effective technology set (denoted by \( \mathbb{S} \)). Before introducing the idea of the algorithm, we first derive a helpful lemma from problem (5).

**Lemma 1.** It is optimal to produce with no more than two technologies for any \( (q, r) \in \Theta \).

Proofs of this lemma and the subsequent results can be found in the online supplement. Lemma 1 tells us that no more than two technologies should be selected to minimize the production cost. Based on this result, we can sequentially determine the effective technologies.

Suppose that it is optimal to produce with technologies \( \tilde{i} \) and \( \tilde{j} (\mu_{\tilde{i}} > \mu_{\tilde{j}}) \) under a given \( (q, r) \). We can conclude that both technologies \( \tilde{i} \) and \( \tilde{j} \) are effective, and the total production quantity satisfies \( r/\mu_1 \leq q \leq r/\mu_j \). In this case, it is easy to find that the production quantities of technologies \( \tilde{i} \) and \( \tilde{j} \) are \( q_i = (r - \mu_j q)/(\mu_i - \mu_j) \) and \( q_j = (\mu_i q - r)/(\mu_i - \mu_j) \), respectively, from \( q_i + q_j = q \) and \( \mu_i q_i + \mu_j q_j = r \). As a result, the production cost is

\[
c_i(r - \mu_j q)/(\mu_i - \mu_j) + c_j(\mu_i q - r)/(\mu_i - \mu_j)
\]

\[
= (\mu_i q - r)(c_j/(\mu_i - \mu_j)) + c_q q.
\]

This indicates that given the effective technology \( \tilde{i} \), the effective technology \( \tilde{j} \) should be the one with the minimum of \( (c_j - c_i)/(\mu_i - \mu_j) \) (to minimize the production cost) and the minimum of \( \mu_j \) (to achieve a large total production quantity). In fact, any technology \( \tilde{t} \) with \( \mu_i < \mu_t < \mu_j \) is ineffective. This can be verified as follows. Let \( \lambda_{\tilde{t}} = (\mu_j - \mu_t)/(\mu_j - \mu_i) \), then \( 0 < \lambda_{\tilde{t}} < 1 \) and \( \mu_{\tilde{t}} = \lambda_{\tilde{t}} \mu_i + (1 - \lambda_{\tilde{t}}) \mu_j \). Producing with technology \( \tilde{t} \) is dominated by producing a \( \lambda_{\tilde{t}} \) portion with technology \( \tilde{i} \) and a \( 1 - \lambda_{\tilde{t}} \) portion with technology \( \tilde{j} \) because

\[
\lambda_{\tilde{t}} c_i + (1 - \lambda_{\tilde{t}}) c_j = c_i + (\mu_j - \mu_{\tilde{t}})(c_j - c_i)/(\mu_j - \mu_i)
\]

\[
\leq c_i + (\mu_j - \mu_{\tilde{t}})(c_j - c_i)/(\mu_j - \mu_i) = c_j.
\]

The above discussion suggests that giving an effective technology, the next effective technology must be the one with the minimum of \( (c_j - c_i)/(\mu_j - \mu_j) \) and the minimum of \( \mu_j \) (or the maximum of \( \tilde{j} \) because \( \mu_j > \cdots > \mu_M \)). It is clear that technology \( \tilde{1} \) is effective because it will be selected when \( q = r/\mu_j \). Now, we are ready to introduce the algorithm to exclude ineffective technologies and determine the effective technology set \( \mathbb{S} \).

Algorithm that determines the effective technology set \( \mathbb{S} \):

**Step 1:** Set \( M = \{ 1, \cdots, M \} \), \( \mathbb{S} = \emptyset \).

**Step 2:** Add technology 1 with \( c_1 = c_i \) and \( \mu_1 = \mu_i \) into \( \mathbb{S} \), and delete technology \( i \) from \( M \).

**Step 3:** Add technology \( i+1 \) with \( c_{i+1} = c_j \) and \( \mu_{i+1} = \mu_j \) into \( \mathbb{S} \), and delete technology \( i \) from \( M \). where \( j = \max \{ \arg \min (c_j - c_i)/(\mu_j - \mu_i) : \mu_t < \mu_i, \tilde{t} \in M \} \).

**Step 4:** Let \( i \mapsto i+1 \), if \( M \neq \emptyset \), return to Step 3, otherwise, stop.
Denote by $m$ the number of the technologies in the effective technology set $\mathbb{S}$. From the above algorithm, it can be verified that $m \geq 2$, and technology $m$ in $\mathbb{S}$ must be the available technology $\hat{M}_i$; i.e., $c_m = \epsilon_{\hat{M}_i}$ and $\mu_m = \mu_{\hat{M}_i}$. Recall that the first effective technology is technology 1. Therefore, the domain of the minimum production cost function $F(q, r)$ can be rewritten as $\Theta = \{(q, r) : r \geq 0, r/\mu_1 \leq q \leq r/\mu_m\}$. In addition, the algorithm implies that the parameters of effective technologies satisfy $\mu_1 > \cdots > \mu_m \geq 0$ and $0 < c_1 < \cdots < c_m$ because $\mu_1 > \cdots > \mu_m \geq 0$ and $0 < c_1 < \cdots < c_m$. To show how the above algorithm works, we give a simple example. Consider three technologies with $c_1 = 1, c_2 = 2, c_3 = 3$, $\mu_1 = 1, \mu_2 = 2, \mu_3 = 3$, and $\epsilon_{\hat{M}_i} = 0$. The above algorithm works as follows. Initially, $\mathbb{M} = \{1, 2, 3\}$ and $\mathbb{S} = \emptyset$. Following Step 2, technology 1 is indexed as technology 1 in $\mathbb{S}$, and $\mathbb{M} = \{2, 3\}$. For technologies 2 and 3, we have $(c_2 - c_1)/(\mu_1 - \mu_2) = 1$ and $(c_3 - c_1)/(\mu_1 - \mu_3) = 2$. Following Step 3, technology 3 is indexed as technology 2 in $\mathbb{S}$, and technology 2 $(\mu_2 > \mu_3)$ and technology 3 are deleted from $\mathbb{M}$, thus $\mathbb{M} = \emptyset$. The algorithm stops at Step 4. The above process indicates that technology 2 is ineffective, and the number of effective technologies is smaller than the number of available technologies.

### 4.2 Minimum production cost function

In this subsection, we derive the minimum production cost function and study its structural properties. Define

$$v_i = (c_i - c_{i-1})/ (\mu_{i-1} - \mu_i) \text{ for } i = 2, \ldots, m. \quad (8)$$

$v_i$ is the ratio of unit production cost difference over allowance consumption rate difference of technologies $i$ and $i-1$. Note that $v_i > 0$ for $i = 2, \ldots, m$ because $\mu_1 > \cdots > \mu_m$ and $c_1 < \cdots < c_m$. Let $\omega_i = (c_{i+1} - c_i)/(\mu_i - \mu_{i+1})$, then we have $
abla i = c_{i-1} + \mu_{i-1}v_i = c_i + \mu_i,v_i$. The following lemma characterizes the minimum production cost function:

**Lemma 2.** If the total production quantity $q \in [r/\mu_{i-1}, r/\mu_i]$ $(i = 2, \ldots, m)$, then it is optimal to produce $(r - \mu_i q)/(\mu_i - \mu_{i-1})$ units with effective technology $i$ and $(\mu_{i-1} q - r)/(\mu_i - \mu_{i-1})$ units with effective technology $i-1$, and the minimum production cost is

$$F(q, r) = \omega_i q - v_i r, \quad q \in [r/\mu_{i-1}, r/\mu_i], \quad i = 2, \ldots, m, \quad (9)$$

where $0 < v_2 < \cdots < v_m$ and $0 < \omega_2 < \cdots < \omega_m$.

Lemma 2 indicates that (i) it is optimal to produce with a single technology only when $q = r/\mu_{i-1}$ $(j = 1, \ldots, m)$ and (ii) if two technologies are selected simultaneously, they should be successive technologies in the effective technology set $\mathbb{S}$.

From Lemma 2, we know that the minimum production cost function $F(q, r)$ is linear when $m = 2$. However, it becomes nonlinear when $m \geq 3$ (see Fig. 1). To study structural properties of $F(q, r)$, we use the concept of submodularity. Suppose that $f(x, y)$ is a real-valued function on a lattice $L$. If

$$f(x^1, y^1) + f(x^2, y^2) \geq f(\max\{x^1, x^2\}, \max\{y^1, y^2\}) + f(\min\{x^1, x^2\}, \min\{y^1, y^2\}),$$

for all $(x^1, y^1), (x^2, y^2) \in L$, then $f(x, y)$ is submodular on $L$. If $f$ is supermodular, then $-f$ is supermodular. Please refer to T domin (1998) for the properties of submodular functions. Here we omit them for brevity.

It can be proved that $F(q, r)$ has the following properties (recall that the domain of $F(q, r)$—i.e., $\Theta$—is a lattice).

**Lemma 3.** The minimum production cost function $F(q, r)$ is convex and submodular over its domain $\Theta = \{(q, r) : r \geq 0, r/\mu_1 \leq q \leq r/\mu_m\}$.

When $m = 2$ the linear function $F(q, r)$ is not only submodular but also supermodular. Lemma 3 indicates that $F(q, r)$ is just submodular but not supermodular when $m > 2$. This implies that the presence of more than two technologies makes the analysis of optimal policies much more complicated.

### 4.3 Structural properties of the optimal technology selection

Lemma 2 tells us that it is optimal to produce with a single technology or two successive technologies from the effective technology set $\mathbb{S}$. To solve our problem, an additional question needs to be answered: What are the conditions for trading allowances?

Consider the case of producing with two technologies $i-1$ and $i$, the minimum production cost can be rewritten as

$$\omega_i q - v_i r = c_{i-1} q + v_i (\mu_i q - r) = c_i q - v_i (r - \mu_i q),$$

where $\mu_i q \leq r \leq \mu_{i-1} q$. Equation (10) indicates that producing with technologies $i-1$ and $i$ simultaneously can be viewed as (i) producing exclusively with technology $i-1$ and purchasing $\mu_{i-1} q - r$ units of allowances at price $v_i$ or (ii) producing exclusively with technology $i$ and selling $r - \mu_i q$ units of allowances at price $v_i$. Accordingly, we define $v_i$ as the virtual allowance selling price of technology $i$ relative to technology $i-1$ or the virtual allowance purchasing price of technology $i-1$ relative to technology $i$, or simply the virtual allowance trading price. Consequently, we can easily generate the following property.
Lemma 4. Producing with technologies \( i - 1 \) and \( i \) simultaneously is equivalent to producing exclusively with technology \( i - 1 \) (or \( i \)) with virtual allowance purchasing (or selling) at the price \( v_i \).

Given the emissions trading price \( K = (K, k) \), Lemma 4 tells us that (i) if \( K \leq v_i \) (or \( K > v_i \)), then producing exclusively with technology \( i - 1 \) while purchasing allowances costs less (or more) than producing with technologies \( i - 1 \) and \( i \) simultaneously and (ii) if \( k \geq v_i \) (or \( k < v_i \)), then producing exclusively with technology \( i \) while selling allowances costs less (or more) than producing with technologies \( i - 1 \) and \( i \) simultaneously. Consequently, we can generate the following results.

Corollary 1. No allowance should be traded when technologies \( i - 1 \) and \( i \) are selected simultaneously. Allowances should be traded only if a single technology is selected.

Lemma 4 and Corollary 1 also imply that the virtual emissions trading price \( v_i \) has a strong impact on the optimal policies, as will be shown in the next section.

5. Structural properties of the outer layer and the optimal policies

In this section, we first characterize structural properties of the outer layer based on a new preservation property of supermodularity and then characterize the optimal emissions trading policy and the optimal production policy accordingly.

By establishing a preservation property of supermodularity for the linear production cost function (see Gong and Zhou (2013), Lemma 1, page 913), Gong and Zhou (2013) successfully solve the dual-technology problem. However, the minimum production cost function \( F(q, r) \) in our problem is just submodular. Consequently, the preservation property established in Gong and Zhou (2013) cannot be applied to solve our problem. To study structural properties of the outer layer, we establish the following preservation property of supermodularity.

Lemma 5. If \( F: \Theta \rightarrow \mathbb{R} \) is submodular and convex and \( H: \mathbb{R}^2 \rightarrow \mathbb{R} \) is supermodular and convex, then \( W(x, z) = \min_{(q, r) \in \Theta} \{F(q, r) + H(x + q, z - r)\} \) is supermodular and convex in \( \mathbb{R}^2 \).

Based on the preservation property in Lemma 5, we can prove the following structural properties of the outer layer:

Proposition 1. For all periods, \( H_t(y, w, K) \), \( W_t(x, z, K) \) and \( V_t(x, z, K) \) are supermodular and convex in \( \mathbb{R}^2 \) under any given \( K \).

Supermodularity and convexity can be applied to determine the optimal solutions for the outer layer. In problem (6), the convexity of \( W_t(x, z, K) \) indicates that there exist one threshold for purchasing allowances and another threshold for selling allowances, respectively. In problem (7), if it is optimal to select a single technology at all possible states, then the convexity of \( H_t(y, w, K) \) is sufficient to characterize the optimal production policy; if it is optimal to select single or multiple technologies at all possible states, then the supermodularity of \( H_t(y, w, K) \) can be applied to determine the region of the states where the manufacturer produces with a single technology, and the convexity of \( H_t(y, w, K) \) can be applied to determine the region of the states where the manufacturer produces with two successive technologies.

We next characterize the optimal emissions trading policy and the optimal production policy. Lemma 4 clearly indicates that the optimal technology selection is determined by the relationship between virtual trading prices \( v_2, \ldots, v_m \) and the realized trading price \( K = (K, k) \) in the outside market. Since \( v_2 < \cdots < v_m \) and \( k \leq K, \) the relationship between \( v_2, \ldots, v_m \) and \( K \) should be one of the following four types.

1. Allowance trading prices in the outside market are relatively low compared to the virtual trading prices, i.e., \( v_2 \geq K \).
2. Allowance trading prices in the outside market are relatively high compared to the virtual trading prices, i.e., \( v_m \leq k \).
3. Allowance trading prices in the outside market are comparable to the virtual trading prices, and the difference between the allowance selling price and purchasing price is small; i.e., there exists \( i_0 \in \{2, \ldots, m - 1\} \) such that \( v_{i_0} \leq k \leq v_{i_0 + 1} \).
4. Allowance trading prices in the outside market are comparable to the virtual trading prices, and the difference between the allowance selling price and purchasing price is large; i.e., there exist \( i_0 \in \{1, \ldots, m - 1\} \) and \( j_0 \geq i_0 + 1 \) such that \( k < v_{i_0 + 1} < \cdots < v_{j_0} < K \) (note that \( v_{i_0} \leq k \) when \( i_0 > 1 \) and \( v_{i_0} \leq k \) when \( j_0 < m \)).

For relationship types 1 to 3, it is optimal to produce with a single technology with or without emissions trading. For relationship type 4, it may be optimal to produce with two successive technologies at some states.

We first consider the relationship types 1, 2, and 3. For the case of producing with a single technology \( i \) \((i = 1, \ldots, m)\) without emissions trading, we define

\[
s_i(y, x + z, K) = \arg \min_{y \in \mathbb{R}} \{c_i y + H_i(y, \mu_i x + z - \mu_i y, K)\},
\]

which denotes the optimal post-production inventory level at the state \((x, z, K)\). Furthermore, if it is optimal to purchase allowances from the outside market, we define (recall that the domain of the ending allowance level \( w \) is \( \mathbb{R} \)):

\[
w_i^T(y, K) = \arg \min_{w \in \mathbb{R}} \{K w + H_i(y, w, K)\},
\]

\[
(\hat{S}^T_i(K), \hat{u}^T_i(K)) = \arg \min_{y, w \in \mathbb{R}} \{(c_i + \mu_i K) y + kw + H_i(y, w, K)\},
\]

where \( w_i^T(y, K) \) denotes the optimal ending allowance level under a given inventory level \( y \). \( \hat{S}^T_i(K) \) denotes the optimal post-production inventory level, and \( \hat{u}^T_i(K) = w_i^T(\hat{S}^T_i(K), K) \). Similarly, if it is optimal to sell allowances in the outside market, we define

\[
w_i^U(y, K) = \arg \min_{w \in \mathbb{R}} \{kw + H_i(y, w, K)\},
\]

\[
(\hat{S}^U_i(K), \hat{u}^U_i(K)) = \arg \min_{y, w \in \mathbb{R}} \{(c_i + \mu_i k) y + kw + H_i(y, w, K)\},
\]

where \( w_i^U(y, K) \) denotes the optimal starting allowance level under a given inventory level \( y \).

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where \( w^I_U(y, K) \) denotes the optimal ending allowance level under a given inventory level \( y \), \( \hat{S}^*_U(K) \) denotes the optimal post-production inventory level, and \( \hat{w}^I_U(K) = w^I_U(\hat{S}^*_U(K), K) \). Note that the supermodularity of \( H_t(y, w, K) \) implies that \( w^I_U(y, K) \) and \( w^I_U(y, K) \) are decreasing in \( y \).

The optimal policies for relationship types 1 to 3 can be described in the following theorem.

**Theorem 1.** In period \( t \) \((t = 1, \ldots, T)\), if: (i) \( v_2 \geq K \); (ii) \( v_{m-1} \leq k \); or (iii) \( v_m \leq k \leq K \leq v_{m+1} \) \((i_0 \in \{2, \ldots, m - 1\})\), it is optimal to produce with a single technology \( i \), where (i) \( i = 1 \) if \( v_2 \geq K \); (ii) \( i = m \) if \( v_m \leq k \); and (iii) \( i = 0 \) if \( v_m \leq k \leq K \leq v_{m+1} \). Define

\[
L_a(x, K) \triangleq \mu_i [\hat{S}^*_U(K) - x]^+ + w^I_U(\max{\hat{S}^*_U(K), x}, K)
\]

and

\[ U_a(x, K) \triangleq \mu_i [\hat{S}^*_U(K) - x]^+ + w^I_U(\max{\hat{S}^*_U(K), x}, K), \]

then \( L_a(x, K) \) and \( U_a(x, K) \) are decreasing in \( x \), and the optimal policies are as follows.

1. When \( z \geq U_a(x, K) \), sell allowances down to \( \hat{z}^* = U_a(x, K) \), and then produce with technology \( i \) up to \( y^* = \max{\hat{S}^*_U(K), x} \).
2. When \( L_a(x, K) < z < U_a(x, K) \), do not trade allowances, and then produce with technology \( i \) up to \( y^* = \max{s_i(\mu x + z, K), x} \).
3. When \( z \leq L_a(x, K) \), purchase allowances up to \( \hat{z}^* = L_a(x, K) \), and then produce with technology \( i \) up to \( y^* = \max{\hat{S}^*_U(K), x} \).

Theorem 1 illustrates that (i) the optimal emissions trading policy follows a target interval policy with two thresholds \( U_a(x, K) \) and \( L_a(x, K) \); (ii) it is optimal to exclusively produce with a single technology at all possible states for relationship types 1 to 3; and (iii) the optimal production policy has a composite base-stock structure. The base-stock levels \( \hat{S}^*_U(K) \) and \( \hat{S}^*_U(K) \) are independent of \( x \) and \( z \), but the base-stock level \( s_i(\mu x + z, K) \) depends on \( \mu, x + z \). This occurs because producing with a single technology and trading allowances affects the inventory level and the allowance level independently. In contrast, producing with a single technology without emissions trading affects the inventory level and the allowance level simultaneously, since the allowance consumption is always \( \mu_i \) times the production quantity. Furthermore, the base-stock levels are determined by the emissions trading price in the current period, since the trading price in the next period is influenced by that in the current period. In particular, the optimal production policy degenerates into a base-stock policy when \( k = K \) because \( U_a(x, K) = L_a(x, K) = \hat{S}^*_U(K) = \hat{S}^*_U(K) \).

Figure 2 illustrates the optimal policies for relationship types 1 to 3 proposed in Theorem 1. Specifically, in Region I, it is optimal to sell allowances down to \( U_a(x, K) \) and then produce with technology \( i \) and increase the inventory level up to \( \max{\hat{S}^*_U(K), x} \). This situation matches case 1 in Theorem 1. In Region III, it is optimal to purchase allowances up to \( L_a(x, K) \), and then produce with technology \( i \) and increase the inventory level up to \( \max{\hat{S}^*_U(K), x} \); this situation matches case 3 in Theorem 1. In Regions II and IV, it is optimal not to trade allowances but only to produce with technology \( i \) and increase the inventory level up to \( \max{\hat{S}^*_U(K), x} \); this situation matches case 2 in Theorem 1.

**Gong and Zhou (2013)** show the optimal policies for the problem where only one technology is available (please refer to Corollary 1 in Gong and Zhou (2013), page 917). This single-technology problem can also be solved by our approach. By introducing a virtual technology with a zero unit-production cost and infinite allowance consumption rate, the single-technology problem can be transformed into a dual-technology problem with a zero virtual allowance trading price. Since \( 0 \leq k \), the optimal policies for the single-technology problem has the same structure as those shown in Theorem 1 and Fig. 2, where technology \( i \) is the available technology.

We now consider the much more complex relationship type 4. From Corollary 1, we know that no allowance should be traded if two successive technologies are selected simultaneously. Define the following additional parameters for production with technologies \( i - 1 \) and \( i \) simultaneously:

\[ w_{ij}(y, K) = \min\{ \nu_i y + v_j w + H_t(y, w, K), \}
\]

\[
\hat{w}_{ij}(K) \triangleq \min_{y, w \in \mathbb{R}} \{ \omega_i y + v_j w + H_t(y, w, K),
\]

where \( w_{ij}(y, K) \) denotes the optimal ending allowance level under a given inventory level \( y \), \( \hat{S}^*_U(K) \) denotes the optimal post-production inventory level, and \( \hat{w}_{ij}(K) = w_{ij}(\hat{S}^*_U(K), K) \). It can be verified that \( w_{ij}(y, K) \) is decreasing in \( y \) from the supermodularity of \( H_t(y, w, K) \).

Then, the optimal policies for relationship type 4 can be characterized in the following theorem.

**Theorem 2.** In period \( t \) \((t = 1, \ldots, T)\), if there exist \( i_0 \in \{1, \ldots, m - 1 \} \) and \( j_0 \geq i_0 + 1 \) \((j_0 \in \{2, \ldots, m \})\) such that \( k < v_{i_0+1} < \cdots < v_{j_0} < K \), and we define

\[
L_{ij}(x, K) \triangleq \mu_i [\hat{S}^*_U(K) - x]^+ + w_{ij}(\max{\hat{S}^*_U(K), x}, K),
\]

\[
u_{ij}(x, K) \triangleq \mu_{i-1} [\hat{S}^*_U(K) - x]^+ + w_{ij}(\max{\hat{S}^*_U(K), x}, K),
\]

\[ (i = i_0 + 1, \ldots, j_0), \]

\[
L_{ij}(x, K) \triangleq \mu_j [\hat{S}^*_U(K) - x]^+ + w_{ij}(\max{\hat{S}^*_U(K), x}, K),
\]

\[ \text{and} \]

\[
u_{ij}(x, K) \triangleq \mu_{j-1} [\hat{S}^*_U(K) - x]^+ + w_{ij}(\max{\hat{S}^*_U(K), x}, K),
\]

then \( L_{ij}(x, K) \) and \( U_{ij}(x, K) \) are decreasing in \( x \), and the optimal policies areas follows.
(1) When \( z \geq U_{ij} (x, K) \), sell allowances down to \( z^* = U_{ij} (x, K) \) and then produce exclusively with technology \( i_0 \) up to \( y^*_i = \max \{ S_{ij} (K) \}, x \).

(2) When \( u_{i+1} (x, K) \leq z < U_{ij} (x, K) \), do not trade allowances and then produce exclusively with technology \( i_0 \) up to \( y^*_i = \max \{ S_{ij} (\mu_i x + z, K), x \} \).

(3) When \( l_i (x, K) < z < u_i (x, K) \) \((i = i_0 + 1, \ldots, j_0)\), do not trade allowances and then produce \( \mu_i x + z - \mu_i S_{ij} (K) - \mu_i (\mu_i - 1) \) units with technology \( i-1 \) and \( \mu_i (\mu_i - 1) / (\mu_i - 1) \) units with technology \( i \).

(4) When \( u_{i+1} (x, K) \leq z \leq l_i (x, K) \) \((i = i_0 + 1, \ldots, j_0 - 1)\), do not trade allowances and then produce exclusively with technology \( i_0 \) up to \( y^*_i = \max \{ 0, \mu_i x + z, K \}, x \).

(5) When \( L_{ij} (x, K) < z \leq L_{ij} (x, K) \), do not trade allowances and then produce exclusively with technology \( j_0 \) up to \( y^*_i = \max \{ S_{ij} (\mu_i x + z, K), x \} \).

(6) When \( z \leq L_{ij} (x, K) \), purchase allowances up to \( z^* = L_{ij} (x, K) \) and then produce exclusively with technology \( j_0 \) up to \( y^*_i = \max \{ S_{ij} (K), x \} \).

Theorem 2 indicates that the optimal emissions trading policy and the optimal production policy for relationship type 4 have similar properties as those for relationship types 1 to 3. However, the optimal technology selection for relationship type 4 is much more complicated. Specifically, although only a single technology or two successive technologies in \( S \) should be selected at any state, different groups of technologies can be optimal at different states. In Theorem 2, the base-stock levels \( S_{ij+1} (K) \), \( \ldots, S_{ij} (K) \) for the production with two technologies are also independent of \( x \) and \( z \) because production with two technologies affects the inventory level and the allowance level independently.

Figure 3 illustrates the optimal policies when \( k < v_3 < v_4 < K \) (i.e., \( i_0 = 2 \) and \( j_0 = 4 \)). Specifically, in Region I, it is optimal to sell allowances down to \( U_{ij} (x, K) \) and then produce exclusively with technology 2 and increase the inventory level up to \( y^*_i = \max \{ S_{ij} (K), x \} \); this situation matches case 1 in Theorem 2. In Region V, it is optimal to purchase allowances up to \( L_{ij} (x, K) \) and then produce exclusively with technology 4 and increase the inventory level up to \( y^*_i = \max \{ S_{ij} (K), x \} \); this situation matches case 6 in Theorem 2. In Regions II, III, and IV, it is optimal to produce exclusively with technologies 2, 3, and 4, to increase the inventory level up to \( \max \{ S_{ij} (\mu_i x + z, K), x \} \), \( \max \{ S_{ij} (\mu_i x + z, K), x \} \), and \( \max \{ S_{ij} (\mu_i x + z, K), x \} \), respectively; these situations match cases 2, 4, and 5 in Theorem 2 when \( S_{ij} (\mu_i x + z, K) > x \) for \( i = 2, 3, 4 \) respectively. In Region VI, it is optimal to produce with technologies 3 and 4 simultaneously and increase the inventory level up to \( S_{ij} (K) \) while consuming \( z - \bar{w}_{ij} (K) \) units of allowances; in Region VII, it is optimal to produce with technologies 3 and 4 simultaneously and increase the inventory level up to \( S_{ij} (K) \) while consuming \( z - \bar{w}_{ij} (K) \) units of allowances; in Region VIII, it is optimal to produce and trade nothing; the situation matches cases 2, 4, and 5 in Theorem 2 when \( S_{ij} (\mu_i x + z, K) \leq x, i = 2, 3, 4 \). Consequently, different groups of technologies will be adopted at different states. As the allowance \( z \) increases, technology 4 will be replaced gradually by technology 3 and then technology 3 will be replaced gradually by technology 2.

6. Numerical study

In this section, we conduct numerical experiments to examine the economic benefits of tri-technology over dual-technology in a two-period system. We check how the received allowance \( z_3 \) and the emissions trading price \( K \) influence the benefits of tri-technology over dual-technology.

We first assume that the market demand in each period follows a truncated negative binomial distribution with mean = 3 and variance = 6. The maximum demand is set as 10. We further assume that emissions trading prices in the two periods follow a two-state Markov process with \( K_1 = (15, 54) \) and \( K_2 = (16, 55) \). The transition probabilities are \( Pr_{ij}^1 = 0.6 \), \( Pr_{ij}^2 = 0.4 \), \( Pr_{ij}^3 = 0.7 \), and \( Pr_{ij}^4 = 0.3 \), where \( Pr_{ij}^1 \) and \( Pr_{ij}^2 \) shows the probability for the current trading price \( K_i \) to change to \( K_j \) in the next period. For example, if the current trading price is \( K_1 \) in period 1, then with the probability of \( p_{i1} = 0.4 \), it changes to \( K_2 \) in period 2. Other parameters are set as \( h = 1.2, b = 30, p = 40, pD = 3, \pi = 60 \), and \( \gamma = 0.97 \).

We assume that there are three technologies: \( L, M, \) and \( H \). Their unit production costs and allowance consumption rates are

\[
(c_L, c_M, c_H) = (3, 9, 18), \quad \text{and} \quad (\mu_L, \mu_M, \mu_H) = (3, 2, 1),
\]

respectively. Note that technology \( L \) has the lowest unit production cost but the highest allowance consumption rate, and technology \( H \) has the highest unit production cost but the lowest allowance consumption rate. We can easily verify that all three technologies are effective, the virtual allowance selling price of technology \( M \) (to technology \( L \)) is six, and the virtual allowance purchasing price of technology \( M \) (to technology \( H \)) is nine.

Denote \( V_i (x, z, K) \) as the optimal cost of the system when all three technologies \( L, M, \) and \( H \) are available, given the initial state \( (x_1, z_1, K) \) and denote \( V_i (x, z, K) \) as the optimal cost of the system when technology \( i \) is removed from the effective technology set \( S \), given the same initial state. Therefore, the benefits of technology \( i \) \((i = L, M, H)\) under a given
received allowance $z_1$ can be calculated as
\[
\rho^i(z_1, K) = \frac{\text{Average}_{x_i \in [-6, 6]} \left[ V^i_1(x_i, z_1, K) - V_1(x_i, z_1, K) \right]}{|V^i_1(x_i, z_1, K)| \times 100\%}.
\]

We further denote $\rho^i_{\text{max}}(K) = \max_{z_1 \in [0, 18]} \{\rho^i(z_1, K)\}$ as the maximum level of the benefits of technology $i$.

Figure 4 shows how the received allowances influence the benefits of three technologies. As shown in Fig. 4, the benefit of technology $H$ is quite high when the received allowance level $z_1$ is very low. The benefits of technologies $L$ and $M$ are quite low when the received allowance level $z_1$ is low, and the benefits start to significantly increase as the received allowance level increases. The maximum benefits for the three technologies are $\rho^H_{\text{max}}(K^1) = 10.12\%$, $\rho^L_{\text{max}}(K^1) = 8.97\%$, and $\rho^M_{\text{max}}(K^1) = 9.86\%$, respectively.

The above statement occurs because technology $H$ has the lowest allowance consumption rate. From the optimal policies shown in Theorem 2, we know that it is optimal to produce with technology $H$ when the received allowance $z_1$ is low. Therefore, keeping technology $H$ is most beneficial when the received allowance is low. On the other hand, technologies $L$ and $M$ have lower costs but have higher allowance consumption rates than technology $H$. Therefore, keeping technology $M$ ($L$) becomes beneficial when the system has medium (high) received allowances to spare.

Next, we examine the impact of emissions trading prices on the benefits of tri-technology. We set $K^1 = (15 + \Delta, 5.4 - \Delta)$, $K^2 = (16 + \Delta, 5.5 - \Delta)$ and change $\Delta$ from zero to 2.5 with step 0.5 (note that all technologies are effective when $\Delta$ varies from zero to 2.5). A larger value of $\Delta$ implies a larger difference between the allowance purchasing price and selling price. The maximum levels of the benefits of the three technologies are shown in Fig. 5.

As shown in Fig. 5, the maximum levels of the benefits of technologies $L$, $M$, and $H$ have no monotonic property with respect to $\Delta$. The maximum of $\rho^L_{\text{max}}(K^1)$, $\rho^M_{\text{max}}(K^1)$, and $\rho^H_{\text{max}}(K^1)$ are 10.53%, 10.07%, and 10.55%, respectively.

Figure 5 also indicates that, without considering the maintenance costs of technologies, keeping three technologies rather than two technologies can save a manufacturer up to 10% of the total operations cost. This fits the situations where maintenance costs do not exist or are negligible; e.g., the technologies with coal, clean fuels, and residual oil, respectively. In other situations where the maintenance costs might be significant, the benefits of keeping three technologies will decrease.

7. Conclusions and discussion

By introducing a minimum production cost function, we virtually separate the periodic-review multi-technology production-inventory problem with emissions trading into two layers: an inner layer where the minimum production cost function is determined based on the effective technology set and an outer layer where the optimal trading, total production quantity, and total allowance consumption are determined based on structural properties of the cost function. We characterize the structural properties of the technology selection in the inner layer. For the outer layer, we find that the optimal emissions trading policy follows a target interval policy with two thresholds, and the optimal production policy has a composite base-stock structure.

In addition to solving the problem, we generate some interesting results. In particular, we prove that when multiple technologies are available, no more than two technologies should be selected simultaneously in any period. Inspired by this finding, we further check the benefits of tri- versus dual-technology through numerical tests. Without considering maintenance cost, our tests show significant benefits of three available technologies versus two technologies.

Our results can be extended to problems with non-stationary production, holding, and backlogging costs and allowance consumption rates; they can also be extended to problems with identical lead times by replacing the inventory level with the inventory position. Although the emissions allowance in our article is one kind of resource that can be negative, our results can also be extended to the problems with resource constraints that cannot be negative.

In this article, we assume that each technology is uncapacitated, and there are no switching costs among technologies. For production-inventory problems with capacity constraint and switching costs, structural properties of optimal technology selection, and the convexity and supermodularity of cost functions no longer hold. Future research can be conducted to solve these problems.
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