Protecting information in a parametrically driven hybrid quantum system

Siddharth Tiwary\textsuperscript{1,2,*} and Himadri Shekhar Dhar\textsuperscript{1,3,†}

\textsuperscript{1}Department of Physics, Indian Institute of Technology Bombay, Mumbai 400076, India
\textsuperscript{2}Department of Physics, University of California, Berkeley, CA 94720, USA
\textsuperscript{3}Centre of Excellence in Quantum Information, Computation, Science and Technology, Indian Institute of Technology Bombay, Mumbai 400076, India

The transfer and storage of quantum information in a hybrid quantum system, consisting of an ensemble of atoms or spins interacting with a cavity, is adversely affected by the inhomogeneity of the spins, which negates the coherent exchange of excitations between the physical components. Using a full quantum treatment based on variational renormalization group, we show how quantum information encoded in the states of a parametrically driven hybrid system is strongly protected against any decoherence that may arise due to the inhomogeneity in the spin-ensemble.

\section{I. INTRODUCTION}

The development of quantum devices must often accommodate conflicting requirements, ranging from readily interacting qubits for information processing and communication to stable, protected states for storage of information [1]. An important cog in the contemporary quantum ecosystem that can overcome such conflicts is a hybrid system [2, 3], where the synergy between different physical systems is achieved by delegating tasks to components that offer specific advantages. For instance, while superconducting circuits are amenable for information processing [4], an ensemble of atoms or spins can offer longer coherence times for storage [5, 6]. Such ensembles can then be coupled to resonators that readily allow for transfer of information [7–10]. As such, hybrid systems based on spin-ensembles coupled to microwave cavities have gained significant traction in quantum information processing [8–13].

A key aspect of working with an ensemble of two-level atoms or spin-1/2 particles is that the transition frequencies for each particle may not be identical, a feature known as inhomogeneous broadening. Such broadening has the detrimental effect of dephasing any information stored or transmitted from the ensemble to a coupled cavity, thus limiting the performance of any information processing protocol [14]. Over the years, several approaches have been proposed and implemented to overcome the effect of inhomogeneous broadening. A very natural recourse is the effect of “cavity protection”, whereby the coupling of the spin-ensemble with the cavity is significantly increased to suppress the dephasing arising due to the broadening [15–17]. Spin-ensemble based hybrid systems already offer the advantage of operating in the strong-coupling regime, due to the enhanced collective coupling strength of a large number of spins [8–10]. However, increasing this coupling further to suppress decoherence may require operational regimes, such as high-Q cavities or increased spin density, which may either be difficult to implement or introduce unwanted spin interactions. Other approaches rely on refocusing mechanisms using spin-echo [18] or sophisticated engineering of the spectral distribution of the ensemble based on either optimal selection [19], dynamical decoupling [20] or hole-burning [21, 22]. While each method has its advantages, they often rely on complex control protocols or challenging experimental schemes to manipulate the spins, which may impede the performance of the hybrid system.

In this work, we propose a radically different method to overcome decoherence due to inhomogeneous broadening – one that does not involve altering the intrinsic spectral distribution of the spins but focuses on how information is encoded in the hybrid quantum system. In this approach, the quantum cavity is subjected to a parametric, two-photon drive, which exponentially enhances the coupling [23, 24] between the states of the cavity in the squeezed frame and the inhomogeneously broadened spin-ensemble. Therefore, any information now encoded in this new cavity frame experiences an implicit cavity protection effect. A clear advantage here is that by tuning the parametric driving strength, the effective coupling can be increased without resorting to the design of expensive cavities or the use of a high spin-density ensemble. Such enhancement in coupling has been studied in other contexts, including ultrastrong coupling between a few qubits and the cavity [25, 26], creation of entanglement [27, 28], superradiant phase transition [29, 30] and squeezed lasing [31].

To analyze the decoherence and subsequent cavity protection effect during the temporal evolution of the hybrid quantum system, we first look at the semiclassical equations of motion for the average photonic and spin excitations [32–34]. However, the study of protection of quantum information encoded in the photonic states requires a full quantum treatment, which is done using a variational renormalization method that captures the temporal dynamics of a driven-dissipative, spin-ensemble-cavity based hybrid system [35]. This then allows us to explicitly investigate any potential loss of information or fidelity of the encoded quantum state.

The paper is arranged in the following way. In Sec. II, we look at the model that describes the dynamics of the

---

\* siddharth110200@gmail.com
\† himadri.dhar@iitb.ac.in
hybrid system consisting of an ensemble of spins inside a cavity under parametric driving. In Sec. III, we derive the semiclassical equations for the photon and spin excitation operators and study the transition from weak to strong coupling regime. For full quantum solutions, we present the variational renormalization group method for driven-dissipative systems in Sec. IV, and show the protection effect under increasing driving strength. Finally, we end with a discussion of the main results in Sec. V.

II. THE EFFECTIVE HAMILTONIAN

The dynamics of a hybrid quantum system, consisting of an ensemble of $N$ two-level atoms or spin-1/2 particles interacting with a quantum cavity, as shown in Fig. (1), is governed by the Tavis-Cummings model [36]. For a system subjected to a parametric, two-photon drive with a frequency $\omega_d$, the corresponding Hamiltonian ($h = 1$) under the rotating wave approximation (RWA) is given by,

$$
\mathcal{H}_0 = \frac{1}{2} \sum_{k=1}^{N} \Delta_k \sigma_k^z + \Delta_c \hat{a}^\dagger \hat{a} + \sum_{k=1}^{N} g_k \left( \sigma_k^z \hat{a}^\dagger + \sigma_k^z \hat{a} \right) - \eta \left( \hat{a}^2 + \hat{a}^\dagger 2 \right),
$$

where the system is in a frame rotating with half the driving frequency, $\omega_d/2$. The cavity and the spin transition frequencies are given by $\omega_c$ and $\omega_k$ for $k = 1, \ldots, N$. The corresponding frequency detunings with respect to $\omega_d/2$ are $\Delta_{c,k} = \omega_{c,k} - \omega_d/2$. As usual, $\hat{a}$ is the photon annihilation operator, $\{ \sigma_k^z, \sigma_k^\dagger \}$ are the spin operators given by the Pauli matrices, $g_k$ is the coupling between the $k$th spin and the cavity field, and $\eta$ is the strength of the parametric drive.

For this study, the fundamental quantity of interest is the inhomogeneity of the spin ensemble, given by the distribution of the spin transition frequencies $\omega_k$ and the spin-photon coupling $g_k$. The frequency distribution is taken to be Gaussian (cf. [37, 38]) with a standard deviation or width given by $\delta$. For simplicity, the coupling is taken to be identical i.e., $g_k = g, \forall k$, which results in an effective coupling strength of $\Omega = \sqrt{\sum_{k=1}^{N} g_k^2} = \sqrt{N} g$. This demonstrates an enhancement of the coupling by a factor of $\sqrt{N}$ compared to that of a single spin. Hence, ensembles with a high number of atoms or spins readily exhibit phenomena related to strong light-matter coupling even when individual particles couple only weakly to the cavity [9, 10].

Importantly, the effective spin-photon coupling can be further enhanced by parametrically driving the system. Using the unitary operator $\mathcal{U} = \exp \left[ r (a^2 - a^\dagger 2)/2 \right]$, where $r = 1/2 \tanh^{-1}(\eta/\Delta_c)$ is the squeezing parameter, the cavity states can be transformed to a squeezed frame. The transformed spin-ensemble-cavity Hamiltonian is then given by $\mathcal{H}_{sq} = \mathcal{U} \mathcal{H}_0 \mathcal{U}^\dagger$, such that

$$
\mathcal{H}_{sq} = \Delta_c \hat{a}^\dagger \hat{a} + \frac{1}{2} \sum_{k=1}^{N} \left( \Delta_k \sigma_k^z + g_k e^{r} \left( \hat{a} + \hat{a}^\dagger \right) \left( \sigma_k^z + \sigma_k^\dagger \right) - g_k e^{-r} \left( \hat{a} - \hat{a}^\dagger \right) \left( \sigma_k^z - \sigma_k^\dagger \right) \right),
$$

where $\Delta_c = \Delta_c/\cosh(2r)$. For large $r$ (i.e., $e^{-r} \to 0$), the Hamiltonian $\mathcal{H}_{sq}$ is nothing but the inhomogeneous Dicke model or the $N$-spin Rabi model in the squeezed frame, but with an exponentially enhanced coupling $\tilde{g}_k = g_k e^{r}/2$. Moreover, $\mathcal{H}_{sq}$ reduces to the Tavis-Cummings model in certain regimes where $|\Delta_c - \Delta_k| \ll \Delta_c + \Delta_k$ and $\tilde{g}_k \ll \Delta_c$, which ensures that RWA is valid for the transformed Hamiltonian and the system is not in the ultra-strong coupling regime [39, 40]. Therefore, the effective spin-cavity coupling in the transformed frame can be readily controlled simply by changing the parametric driving strength $\eta$.

III. DYNAMICS IN THE SEMICLASSICAL REGIME

A key indicator to investigate the protection of information encoded in hybrid quantum systems is to investigate how quickly the average spin excitation or photon number in the cavity is lost as the system evolves. These quantities can be readily calculated using semiclassical equations of motion [32–34]. Now, in addition to inhomogeneity in the ensemble, the spins and the cavity in a hybrid system are also intrinsically lossy, i.e., they naturally lose coherence with time. Thus, the dynamics of a hybrid system is best described by a Lindblad master equation (ME), given by

$$
\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] + \kappa \mathcal{L}_{\hat{a}}[\rho] + \sum_{k} \{ \gamma_k \mathcal{L}_{\sigma_k^{-}}[\rho] + \gamma_p \mathcal{L}_{\sigma_k^{\dagger}}[\rho] \},
$$

where $\rho$ is the density matrix of the system and the Lindblad operators are $\mathcal{L}_{\hat{a}}[\rho] = \hat{a} \rho \hat{a}^\dagger - \frac{1}{2} \{ \hat{a} \hat{a}^\dagger \rho, \rho \hat{a}^\dagger \}$. The photon...
loss rate is \( \kappa \), the rate of radiative decay and dephasing of spins is \( \gamma_a \) and \( \gamma_p \). The equations of motion for the photonic and spin excitations, \( \langle \hat{a} \rangle \), \( \langle \sigma_k^- \rangle \) and \( \langle \sigma_k^z \rangle \) are calculated using Eq. (3), for the Hamiltonian \( \mathcal{H} = \mathcal{H}_0 \). Using the semiclassical approximation for large \( N \) i.e., \( \langle \sigma_k^- \hat{a} \rangle \approx \langle \sigma_k^- \rangle \langle \hat{a} \rangle \), these are given by

\[
\begin{align*}
\frac{d \langle \hat{a} \rangle}{dt} &= - (\kappa + i \Delta_c) \langle \hat{a} \rangle - i \sum_k g_k \langle \sigma_k^- \rangle + i \eta \langle \hat{a} \rangle^* , \\
\frac{d \langle \sigma_k^- \rangle}{dt} &= - (\gamma_a + 2 \gamma_p + i \Delta_k) \langle \sigma_k^- \rangle + i g_k \langle \sigma_k^z \rangle \langle \hat{a} \rangle , \\
\frac{d \langle \sigma_k^z \rangle}{dt} &= - 2 \gamma_a (1 + \langle \sigma_k^z \rangle) + 2 i g_k (\langle \sigma_k^- \rangle \langle \hat{a} \rangle^* - \langle \sigma_k^+ \rangle \langle \hat{a} \rangle) ,
\end{align*}
\]

where, \( \langle a^\dagger \rangle = \langle a \rangle^* \) and \( \langle \sigma_k^+ \rangle = \langle \sigma_k^- \rangle^* \). By solving the equations above, we study the dynamics of a hybrid system consisting of an inhomogeneous ensemble of \( N = 10^4 \) spins. This ensemble is sufficiently large to use the semiclassical approximation in the stable region \([34]\). The key parameters used in the calculation are cavity detuning \( \Delta_c = 70 \text{ GHz} \), and the width of the spectral distribution, \( \delta = 50 - 80 \text{ MHz} \). The collective coupling is taken to be \( \Omega = 40 \text{ MHz} \) and the parametric driving strength \( \eta = \Delta_c \tan h(2r) \), with squeezing parameter \( r = 0.0 - 2.4 \). While the effective spin-cavity coupling can be enhanced by increasing the driving strength \( \eta \), care must be taken to ensure that the dynamics is not driven to the nonlinear regime, where the information is no longer protected. Hence, the parameters are chosen to be as close to the RWA as possible, i.e., \( \Delta_c \approx \Delta_k \) and \( \delta g_k \ll \Delta_c \). Moreover, to better characterize the effect of inhomogeneous broadening in the semiclassical limit, the photon and spin losses are taken to be negligible as compared to the coupling and width of the distribution.

Figure 2 captures the dynamics of a hybrid system under parametric driving. The semiclassical equations are derived using the Hamiltonian \( \mathcal{H}_0 \), such that the average photon number in the squeezed frame is given

\[
\begin{align*}
n(t) &= \langle |\hat{a}|^2 \rangle_s = \frac{1}{2} (\cosh(2r) \langle |\hat{a}|^2 \rangle - \sinh(2r) \Re(\hat{a}^2)) ,
\end{align*}
\]

where \( r \) is the squeezing parameter. Similar results are obtained if one instead studies the equations of motion in the transformed frame by using \( \mathcal{H}_s q \). In Fig. 2(a), the temporal evolution of \( n(t) \) is shown for different values of squeezing parameter \( r \), and a fixed width of the spin frequency distribution, \( \delta = 60 \text{ MHz} \). It is assumed that the hybrid system is initialized such that the cavity has photon number equal to unity i.e., \( n(0) = 1 \).

The figure shows that for \( r = 0 \), the average photon number quickly decoheres with time due to the inhomogeneity in the spin ensemble (other sources of dissipation have been ignored at this point). However, these excitations are sustained for higher values of \( r \), which shows that boosting the parametric drive leads to a reduced decay rate and a longer lifetime for the photon. The rate at which the cavity excitation \( n(t) \) is dephasing can be captured by fitting the peaks of the Rabi oscillations using the relation, \( n(t) = n(0)e^{-\zeta \omega_0 t} \), where \( \zeta \) is a dimensionless quantity, and \( \omega_0 = 10 \text{ MHz} \) is a reference frequency. So, higher \( \zeta \) implies faster decoherence of the photonic excitations in the hybrid quantum system. Figure 2(b) shows the variation of the rate of decay \( \zeta \) as a function of squeezing parameter \( r \), for spin ensembles of different width \( \delta \). The plots clearly show that loss of excitation and decoherence in the system is more severe for ensembles with more inhomogeneity (larger width \( \delta \)). Importantly, parametric drives of increasing intensities, which give us higher values of \( r \), are able to reduce the decay rate \( \zeta \) successfully even for significantly broadened spin ensembles, thus giving an overall “cavity protection” effect. In fact, the protection is also observed when the inhomogeneity is larger than the effective coupling, i.e., \( \delta > \Omega \), albeit in the absence of any other dissipation in the system.

IV. PROTECTING INFORMATION IN THE HYBRID SYSTEM

While semiclassical equations highlight the revival of macroscopic quantities such as the average photon number in the cavity, to study protection of information encoded in any superposed quantum state, the dynamics of the full quantum system is necessary. This is a computationally challenging task due to the large Hilbert space of the spin-ensemble-cavity system. To overcome this, we use a variational renormalization group method to study driven-dissipative dynamics \([35]\), which is a tensor-network method similar to density matrix renormalization group and matrix-product states \([41]\). The method employs a time-adaptive approach to temporally evolve the density matrix \( \rho \) of a spin-cavity system of about hundred spins, using the Lindblad ME formalism. To achieve this, all states \( \rho \) and operators \( \hat{O} \) are mapped to a higher
dimensional space as superkets $|\rho\rangle$ and superoperators $\hat{O}$ i.e., $\rho \rightarrow \text{vec}(\rho) = |\rho\rangle$ and $\hat{O}\rho \rightarrow (\hat{O} \otimes I)|\rho\rangle = \hat{O}|\rho\rangle$. This allows the master equation in Eq. (3) to be mapped to a Schrödinger like equation, $d|\rho\rangle/dt = \hat{L}|\rho\rangle$, in the superoperator space, where

$$\hat{L} = -i(\mathcal{H} \otimes I - I \otimes \mathcal{H}^T) + \kappa \hat{L}_{a} + \sum_k \left( \gamma_{h}\hat{L}_{\sigma_k^-} + \gamma_{p}\hat{L}_{\sigma_k^z} \right),$$

and $\hat{L}_k = \hat{x} \otimes \hat{x}^* - \frac{1}{2} \hat{x}^\dagger \hat{x}^T \otimes I - \frac{1}{2} I \otimes \hat{x}^T \hat{x}$. At this point, the variational renormalization has two key parts. First, finding the renormalized representation for the superket $|\rho\rangle$ in a significantly reduced subspace, and second, evolving the initial state $|\rho\rangle$ in a time-adaptive manner. As with most tensor-network methods, the renormalized space is obtained by eliminating the null space or those with marginal singular values. The time evolution is then governed using the Lindblad ME in Eq. (5), using the transformed Hamiltonian $\mathcal{H} = \mathcal{H}_{aq}$, and evolved adaptively for small intervals $\Delta t$, such that $|\rho(t + \Delta t)\rangle = e^{\hat{L}\Delta t}|\rho(t)\rangle$. Now, $\hat{L} = \sum_k \hat{L}_k$, and the second-order Suzuki-Trotter decomposition helps expand $e^{\hat{L}\Delta t}$ into a product of single spin-cavity evolution: $e^{\hat{L}\Delta t} = e^{\hat{L}_1\Delta t} \cdots e^{\hat{L}_N\Delta t}$ (see Refs. [35, 42]). The computation then follows an iterative sweeping protocol as used in other time-adaptive tensor-network methods [43, 44].

For the full quantum dynamics of the hybrid system using variational renormalization group, we consider an ensemble of $N = 100$ spins inside the cavity. For consistency with semiclassical results, we use the same set of parameters, including the collective coupling strength $\Omega = 40$ MHz. However, we now consider other dissipative terms, such as the photon loss rate $\kappa = 7$ MHz, and radiative loss and dephasing for spins equal to $\gamma_{h} = \kappa/16$ and $\gamma_{p} = \kappa/16$, respectively. Due to additional losses, the spin distribution width $\delta = 30$ MHz is taken to be slightly smaller than in the semiclassical case. At the start, the quantum information is encoded in the effective state of the cavity in the squeezed frame, which at time $t = 0$ is given by $|\psi(0)\rangle$. As the dynamics is governed by the Lindblad ME, the cavity state at time $t$ is given by the density matrix $\rho_{c}(t)$. To study the effect of decoherence and protection of information, we look at two distinct figures of merit, viz. quantum fidelity and the Wigner function. The quantum fidelity $\mathcal{F}(t)$ is taken between the state $\rho_{c}(t)$ at $t$ and the initial state encoded in the system $|\psi(0)\rangle$, i.e., $\mathcal{F}(t) = \sqrt{\langle \psi(0)|\rho_{c}(t)|\psi(0)\rangle}$. The Wigner function allows us to study the phase-space properties of the encoded information, which is not captured by the fidelity, and is defined as $W(p,q) = \frac{1}{\pi} \int e^{2\pi i(qy - px)} \rho_{c}(t) \cos y dy$. Figure 3 shows the maximal fidelity of the information encoded in the initial state and the temporally evolving photon density matrix $\rho_{c}(t)$, as well as its quasiprobability distribution in terms of $W(p,q)$. The cavity at $t = 0$ encodes the superposition state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$, where $|1\rangle$ and $|2\rangle$ are the Fock states in the transformed basis. In Fig. 3(a), the plot shows the best-fit envelope of the maximal fidelity of the density matrix at $t$ with the initially prepared superposition state that encodes the information [45]. The decrease in $\mathcal{F}(t)$ corresponds to decoherence or loss of information, as the initially encoded state is no longer accurately accessible. This may take place due to inhomogeneous broadening, as well as other losses in the system. However, under increased parametric driving (higher values of $r$), the loss of fidelity is significantly minimized compared to the undriven system, thus highlighting a robust protection effect on the information encoded in the system. This is also evident in the phase-space properties of the information stored in the cavity as shown in Figs. 3(b)-(d), in terms of the Wigner function of $\rho_{c}(t)$. While Fig. 3(b), shows the $W(p,q)$ for the initially encoded state, Figs. 3(c) and (d) correspond to density matrices at time $t$ for the undriven ($r = 0$) and the parametrically driven ($r = 2$) system, respectively. These plots clearly show that the phase coherence is better preserved for higher values of $r$.

Notably, for $r = 0$, the maximal fidelity drops below 1/2, which is the probability of randomly guessing the stored information. However, under parametric driving, $\mathcal{F}$ decoheres at a rate of $\kappa/4 \cdot 10^{-3}$, which is far smaller than all decoherence rates arising from any inhomogeneity or losses in the system. The lifetime of the quantum
information stored in the parametrically driven hybrid system increases by a factor of $10^3$, as compared to information stored in the photonic states of the bare cavity, thus highlighting the strong protection experienced by the information stored in the hybrid quantum system.

V. DISCUSSION

Hybrid quantum systems based on spin ensembles interacting with a cavity are ubiquitous in the design of quantum computing and communication architecture. As such, quantum information stored or transferred using these systems needs to be protected from decoherence arising due to inhomogeneity in the system. While there exist several sophisticated techniques to regain coherence, based on engineering of the spin distribution or using expensive cavities, the present results show that by simply using a parametric drive on the hybrid system and encoding information in the transformed states, an effective protection protocol can be achieved. It is to be noted that this protocol is significantly different from other applications of squeezed light such as its use for enhanced precision measurements [46, 47] or encoding information in continuous variable information processing [48–50].

From an experimental perspective, the use higher-order interactions in hybrid systems can be naturally adapted in the design of qubits, registers and quantum memories across a wide range of platforms, ranging from implementation of parametric driving in superconducting circuits [51, 52] and optomechanical devices [53] to the coupling of microwave resonators to ensembles based on electron spins [8], nitrogen-vacancy centers [9], and semiconductor qubits [54]. In recent studies, parametric modulation of potential has been used to observe enhanced effective interaction in trapped ions [55], while superradiant phase transition and strong entanglement has been observed in a nuclear magnetic resonance quantum simulator using antisqueezing effects [56].

Therefore, the protocol can be a powerful mechanism for designing versatile devices with significantly reduced error in information processing, and with much simpler engineering and low experimental overheads. Notably, the parametrically driven system can also be used to experimentally investigate regimes with ultrastrong coupling in ensembles, which will not only allow for the study of interesting protocols in quantum information but also throw more light on fundamental physics related to collective phenomena, many-body entanglement and nonequilibrium, driven-dissipative dynamics.

ACKNOWLEDGMENTS

We thank Sudipto Singha Roy for helpful suggestions and acknowledge the use of SpaceTime, the high-performance computing facility at IIT Bombay, for a part of the quantum simulations. HSD acknowledges financial support from the Industrial Research & Consultancy Centre, IIT Bombay via grant RD/0521-IRCCSH0-001.

[1] K. Mølmer, Needle in a haystack, Nature Phys. 10, 707 (2014).
[2] Z.-L. Xiang, S. Ashhab, J.Q. You, and F. Nori, Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems, Rev. Mod. Phys. 85, 623 (2013).
[3] G. Kurizki, P. Bertet, Y. Kubo, K. Mølmer, D. Petrosyan, P. Rabl, and J. Schmiedmayer, Quantum technologies with hybrid systems, Proc. Natl. Acad. Sci. U.S.A. 112, 3866 (2015).
[4] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Coherent control of macroscopic quantum states in a single-Coooper-pair box, Nature 398, 786 (1999).
[5] B. Jølsgaard, J. Sherson, J.I. Cirac, J. Fiurášek, and E.S. Polzik, Experimental demonstration of quantum memory for light, Nature 432, 482 (2004).
[6] C. Grezes, B. Jølsgaard, Y. Kubo, M. Stern, T. Umeda, J. Isoya, H. Sumiya, H. Abe, S. Onoda, T. Oshihama, V. Jacques, J. Esteve, D. Vion, D. Esteve, K. Mølmer, and P. Bertet, Multimode storage and retrieval of microwave fields in a spin ensemble, Phys. Rev. X 4, 021049 (2014).
[7] J. Verdú, H. Zoubi, Ch. Koller, J. Majer, H. Ritsch, and J. Schmiedmayer, Strong Magnetic Coupling of an Ultra-cold Gas to a Superconducting Waveguide Cavity, Phys. Rev. Lett. 103, 043603 (2009).
[8] D.I. Schuster, A.P. Sears, E. Ginossar, L. DiCarlo, L. Frunzio, J.J.L. Morton, H. Wu, G.A.D. Briggs, B.B. Buckley, D.D. Awschalom, and R.J. Schoelkopf, High-Cooperativity Coupling of Electron-Spin Ensembles to Superconducting Cavities, Phys. Rev. Lett. 105, 140501 (2010).
[9] Y. Kubo, F.R. Ong, P. Bertet, D. Vion, V. Jacques, D. Zheng, A. Dréau, J.-F. Roch, A. Aufeves, F. Jelezko, J. Wrachtrup, M.F. Barthe, P. Bergonzo, and D. Esteve, Strong Coupling of a Spin Ensemble to a Superconducting Resonator, Phys. Rev. Lett. 105, 140502 (2010).
[10] R. Amsüss, Ch. Koller, T. Nöbauer, S. Putz, S. Rotter, K. Sandner, S. Schneider, M. Schramböck, G. Steinhauser, H. Ritsch, J. Schmiedmayer, and J. Majer, Cavity QED with Magnetically Coupled Collective Spin States, Phys. Rev. Lett. 107, 060502 (2011).
[11] J.H. Wesenberg, A. Ardavan, G.A.D. Briggs, J.J.L. Morton, R.J. Schoelkopf, D.I. Schuster, and K. Mølmer, Quantum Computing with an Electron-Spin Ensemble, Phys. Rev. Lett. 103, 070502 (2009).
[12] A.A. Clerk, K.W. Lehnert, P. Bertet, J.R. Petta, and Y. Nakamura, Hybrid quantum systems with circuit quantum electrodynamics, Nature Phys. 16, 257 (2020).
[13] A. Blais, A.L. Grimsnmo, S.M. Girvin, and A. Wallraff, Circuit quantum electrodynamics, Rev. Mod. Phys. 93, 025005 (2021).
[14] Z. Kurucz, J.H. Wesenberg, and K. Mølmer, Spectroscopic properties of inhomogeneously broadened spin ensembles in a cavity, Phys. Rev. A 83, 053852 (2011).
[50] K. Park, J. Hastrup, J.S. Neergaard-Nielsen, J.B. Brask, R. Filip, and U.L. Andersen, Slowing quantum decoherence of oscillators by hybrid processing, npj Quantum Inf. 8, 67 (2022).

[51] B. Royer, S. Puri, and A. Blais, Qubit parity measurement by parametric driving in circuit QED, Science Advances 4, eaau1695, (2018).

[52] A. Eddins, J.M. Kreikebaum, D.M. Toyli, E.M. Levenson-Falk, A. Dove, W.P. Livingston, B.A. Levitan, L.C.G. Govia, A.A. Clerk, and I. Siddiqi, High-Efficiency Measurement of an Artificial Atom Embedded in a Parametric Amplifier, Phys. Rev. X 9, 011004 (2019).

[53] M. Aspelmeyer, T.J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. 86, 1391 (2014).

[54] X. Mi, J.V. Cady, D.M. Zajac, P.W. Deelman, and J.R. Petta, Strong coupling of a single electron in silicon to a microwave photon, Science 355, 156 (2017).

[55] S. C. Burd, R. Srinivas, H. M. Knaack, W. Ge, A. C. Wilson, D. J. Wineland, D. Leibfried, J. J. Bollinger, D. T. C. Allcock, and D. H. Slichter, Quantum amplification of boson-mediated interactions Nature Physics 17, 898 (2021).

[56] X. Chen, Z. Wu, M. Jiang, X.-Y. Lü, X. Peng, and J. Du, Experimental quantum simulation of superradiant phase transition beyond no-go theorem via antisqueezing, Nature Commun. 12, 6281 (2021).