Gravity Waves as a Probe of Hubble Expansion Rate During An Electroweak Scale Phase Transition

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Just as big bang nucleosynthesis allows us to probe the expansion rate when the temperature of the universe was around 1 MeV, the measurement of gravity waves from electroweak scale first order phase transitions may allow us to probe the expansion rate when the temperature of the universe was at the electroweak scale. We compute the simple transformation rule for the gravity wave spectrum under the scaling transformation of the Hubble expansion rate. We then apply this directly to the scenario of quintessence kination domination and show how gravity wave spectra would shift relative to LISA and BBO projected sensitivities.

1. INTRODUCTION

The detection of gravity waves (GWs) generated during a scalar sector’s first order phase transition (PT) represents an interesting future possibility [1–26]. During a first order PT, bubbles of true vacuum nucleate, stir up the cosmological fluid, and collide, producing GWs. First order PTs at the electroweak scale in particular have received much attention because of their possible connection to a well motivated electroweak baryogenesis scenario in which the baryon asymmetry is generated during an electroweak phase transition (EWPT) [27, 28]. Studies of this scenario are particularly timely given that Tevatron and the LHC are actively probing the Higgs sector responsible for electroweak symmetry breaking. If the Higgs boson or any degree of freedom that can be responsible for a first order PT at the electroweak scale is found in the ongoing experiments, future experiments may eventually be able to measure all of the parameters necessary to give an accurate prediction for the GWs. It is important to emphasize that even if the electroweak symmetry breaking is not a first order PT, a typical beyond the standard model scalar sector has multiple degrees of freedom, and some of these can undergo a first order PT at a temperature near the electroweak scale.

Just as the relative isotope abundance measurements have led to a constraint on the expansion rate during big bang nucleosynthesis, the measurement of GWs may allow us to constrain any non-standard expansion rate during the time of EWPT. Several detailed computations of the gravity wave spectrum exist, and each give varying degrees of dependence on the Hubble expansion rate. However, to our knowledge, previous work does not sufficiently discuss the general dependence of the Hubble expansion rate to directly answer the following question: how would the observed gravity wave spectrum change if the Hubble expansion during the EWPT was changed from that of pure relativistic degrees of freedom?

We compute a simple transformation rule for the gravitational wave spectrum in terms...
of $\xi \equiv H_\ast / H_U$, where $H_U$ is the expansion rate which assumes radiation domination and $H_\ast$ is the actual expansion rate:

$$\frac{d\rho_{GW}(k)}{d\ln k} \rightarrow \frac{1}{\xi^2} \frac{d\rho_{GW}(k/\xi)}{d\ln k}. \quad (1)$$

where $\frac{d\rho_{GW}(k)}{d\ln k}$ is the spectrum computed assuming radiation domination. This immediately implies the following: 1) The peak frequency of the spectrum will shift from the standard scenario frequency $f_p$ as $f_p \rightarrow \xi f_p$, and 2) the peak amplitude of the spectrum will be suppressed from the standard scenario amplitude $A_p$ as $A_p \rightarrow A_p/\xi^2$. The intuition for the amplitude is that less source contributes to the gravitational wave at a typical spacetime point today because of the smaller intersection of the past null boundary with the approximately compact time support of the source. The intuition for the frequency shift is that all conformal symmetry breaking scales relevant for the observable frequency range is controlled by the Hubble expansion rate.

We then apply this scaling relationship to the results of [14, 16] to compute how the gravity wave spectrum will transform due to the assumption of the existence of a quintessence kination dominated phase [29–35]. Such assumptions are interesting because as pointed out by [32] (related scenarios were also suggested before by [31, 36]), the freeze-out abundance of thermal relics can be strongly enhanced in scenarios in which the energy density is dominated by the kinetic energy of the quintessence field (kination domination) during the time of freeze-out, but dilutes away by the time of big bang nucleosynthesis (BBN). Such kination dominated freeze-out scenarios are then consistent with standard cosmology and predict that the standard relic abundance computed from the parameters extracted from collider measurements will be mismatched from the relic abundance deduced by observational cosmology. Thus, this scenario has interesting implications for physics models with thermal dark matter candidates (e.g. models with low energy supersymmetry such as the minimal supersymmetric extension of the SM (MSSM), technicolor models, models with large/warped extra dimensions, or certain classes of little Higgs models), which will be probed at the LHC and other experiments in the foreseeable future (for collider implications of this class of models, see for example [37]). Furthermore, as pointed out by [38, 39], large annihilation cross sections compatible with the dark matter explanation of the excess positrons [40, 41] can be compatible with the right thermal relic abundance since the effective boost factor coming from the kination scenario can easily be as large as $10^3$. Finally, since the measurement of CMB B-mode polarization can almost model independently falsify this scenario, this scenario can be nontrivially checked with a variety of probes. In particular, the EWPT gravity wave probe in conjunction with dark matter cosmology can represent a smoking gun probe of the scenario if the gravity wave is measurable [35] and colliders can eventually measure the requisite short-distance parameters with sufficient accuracy.

The order of presentation is as follows. In the next section, we present the main analytic result of this paper. Sec. 3 focuses on checking explicit consistency of our result with the existing literature on explicit gravity wave spectrum computations. In Sec. 4 we apply the transformation relation to the quintessential kination scenario and give plots showing how the transformed spectra look relative to the projected sensitivities of LISA and BBO. Sec 5 discusses all of the caveats associated with the analytic result. We then conclude with a summary. The appendices contain some of the details used throughout the paper.

As far as conventions are concerned, we use the reduced Planck’s constant $M_p = 2.4 \times 10^{18}$
GeV and also assume a flat FRW background metric \( ds^2 = a^2(t)(dt^2 - |d\vec{x}|^2) \).

2. GENERAL ANALYTIC ARGUMENTS

In this section, we compute how the gravitational wave spectrum will transform under the situation that during the last part of the PT, the Hubble expansion rate is \( \xi H_U \) where \( H_U \) is what the expansion rate would be in the “usual” radiation domination epoch. Because the main arguments rely only on the general form of the gravity wave equation and dimensional analysis, the transformation results will be very robust and nearly model independent. In a latter section, we check the consistency of the transformation rules with explicit model dependent computations in the literature.

Consider the transverse-traceless perturbation \( h_{ij}^{TT} \) about the background metric:

\[
\begin{align*}
ds^2 &= a^2(t)\left[dt^2 - (\delta_{ij} + h_{ij}^{TT})dx^i dx^j\right].
\end{align*}
\]

Using the pseudotensor expression for the energy density in gravity waves, the energy density in GWs can be expressed as

\[
\rho_{GW} = \frac{M_p^2}{4a^2} \langle \partial_0 h_{ij}^{TT} \partial_0 h_{ij}^{TT}\rangle.
\]

During a phase transition (PT) the energy density in the gravity wave can be written as

\[
\rho_{GW}(\vec{x}, t) = \frac{1}{M_p^2} \left(\frac{a_s}{a}\right)^4 \left\langle \left\{ \frac{\partial}{\partial t} \left[ \Lambda_{ij,lm} \int d^4x' G_{ret}(x; x') T_{lm}(x') \right] \right\}^2 \right\rangle |_{PT}
\]

\[
\sim \frac{1}{M_p^2} \left(\frac{a_s}{a}\right)^4 \left\langle \frac{\partial}{\partial t} \left( \frac{1}{\Box} T_{ij} \right) \frac{\partial}{\partial t} \left( \frac{1}{\Box} T_{ij} \right) \right\rangle |_{PT}
\]

where \( a_s \) is the scale factor at the beginning of the PT, \( G_{ret}(x, t; x', t') \) is the Minkowski retarded Green’s function

\[
\Box G_{ret}(x; x') = \left[ \frac{\partial^2}{\partial t^2} - \nabla^2 \right] G_{ret}(x; x') = \delta^4(x - x')
\]

and the transverse traceless non-local projection operator is defined as

\[
\Lambda_{ij,lm} = P_{il} P_{mj} - \frac{1}{2} P_{ij} P_{lm}
\]

\[
P_{ij} = \delta_{ij} - \frac{1}{\nabla^2} \partial_i \partial_j
\]

where \( T_{ij} \) is the conformal coordinate stress tensor and the symbol \( |_{PT} \) represents the evaluation at the end of the phase transition. Evaluation of the right hand side is accomplished through the power spectrum \( P(k_1, t_1', t_2') \) written as

\[
\langle \tilde{T}_{ij}(t'_1, \vec{k}_1) \tilde{T}_{ij}^*(t'_2, \vec{k}_2) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 - \vec{k}_2) P(k_1, t_1', t_2') \left[ \rho_f^{\text{rest}} \gamma_{ij} v_f^2 \right] a_s^2
\]

which incorporates spatial translational invariance and a suggestive normalization of bubble pressure squared that assumes that the correlator is dominated by disconnected diagrams.
(i.e. bubble interactions are neglected). Here, $\rho_f^{\text{rest}}$ is a fiducial energy density of the fluid measured by an observer at rest with the fluid at the location of the bubble wall, $\gamma_{\nu f}$ is the usual Lorentz contraction factor associated with the fluid velocity $v_f$ behind the bubble wall. It is related to the bubble wall velocity $v_w$ by $v_f = (v_w - c_s)/(1 - v_w c_s)$, where $c_s$ is the speed of sound and $v_w$ is taken to be a constant. We therefore find the gravity wave spectrum as

$$
\frac{d\rho_{GW}}{d\ln k} = \frac{1}{(2\pi)^2 M_p^2} \left( \frac{a_*}{a} \right)^4 \left[ \rho_f^{\text{rest}} \gamma_{\nu f}^2 v_f^2 \right] \int d\ell_1' d\ell_2' \cos[k(\ell_1' - \ell_2')] \left[ k^3 P(k, \ell_1', \ell_2') \right] \tag{10}
$$

Note that the integration is assumed to be over all time, but $k^3 P(k, \ell_1', \ell_2')$ will have support dominantly over a time period $\Delta t$ surrounding the time of the PT. As we will check explicitly later, this turns out to be reasonable even for relatively longer lived turbulent sources. The characteristic sizes governing $k^3 P$ include the duration of the last part of the PT $a_* \Delta t < \frac{1}{H_*}$ (where $H_*$ is the Hubble expansion rate at the time of the phase transition), the time $t_*$ at which the PT occurs, and the size of the typical bubble $R$. Since

$$
\frac{1}{R} \sim \frac{1}{v_w a_* \Delta t} \tag{11}
$$

if we assume that these are the only important scales in the spectrum and that $t_*$ only enters as $t'_1 - t_*$, we can write

$$
F_{k\Delta t}((t'_1 - t_*)/\Delta t, (t'_2 - t_*)/\Delta t) \equiv k^3 P(k, t'_1, t'_2). \tag{13}
$$

Caveats to this assumption will be discussed in Sec. 5, but the general conclusions there will be that this assumption is robust. This gives us

$$
\frac{d\rho_{GW}}{d\ln k} = \frac{a_*^2}{(2\pi)^2 M_p^2} \left( \frac{a_*}{a} \right)^4 \left[ \rho_f^{\text{rest}} \gamma_{\nu f}^2 v_f^2 \right] \int d\ell_1' d\ell_2' \cos[k(\ell_1' - \ell_2')] \ F_{k\Delta t}(t'_1 - t_*, t'_2 - t_*) \tag{14}
$$

where $q_i$ is integrated over $(-\infty, \infty)$ and is dimensionless. Hence, the only conformal symmetry breaking scale in the integrand is $\Delta t$.

To proceed with the analysis, we need to determine what sets the mass dimension scale of $\Delta t$. As is well known, the nucleation rate of the PT bubble per unit volume per unit time at temperature $T$ is

$$
\gamma = C_1 T^4 \exp \left[ \left( -S^{(3)}_* - (t - t_*) \frac{dS^{(3)}}{dt} \big|_{t_*} \right) / T \right] \tag{16}
$$

where $S^{(3)}$ corresponds to the appropriate bounce action at finite temperature and $C_1$ is assumed to be $\mathcal{O}(1)$. Hence,

$$
\left. \frac{dS^{(3)}}{dt} \right|_{t_*} = \frac{\dot{T}}{T} \left. \frac{dS^{(3)}}{d\ln T} \right|_{t_*} = - \left[ \frac{a(t_*) H_*}{1 + \frac{1}{3} \frac{d\ln g_{*s}}{d\ln T}} \right] \left. \frac{dS^{(3)}}{d\ln T} \right|_{t_*} \tag{17}
$$

where $H_*$ is the expansion rate at the time $t_*$ of the PT and we have assumed that the entropy whose density proportional to $g_{*S} T^3 a^3$ is conserved where $a$ is the scale factor and $g_{*S}$ counts the entropy degrees of freedom. As given by Eq. (A13) of Appendix A the completion of the PT occurs during a time interval of

$$a_* \Delta t \propto \frac{1}{H_*}$$

for $H_*=a_*\Delta t < 1$. Hence, the only Hubble expansion rate dependence in the integrand

$$\cos [k\Delta t(q'_1 - q'_2)] F_{k\Delta t}(q'_1, q'_2)$$

is in $k\Delta t$.

Explicitly, with the definition

$$\xi \equiv \frac{H_*}{H_U},$$

where the $U$ subscript stands for “usual” radiation domination scenario, the integral

$$\int dq'_1 dq'_2 \cos [k\Delta t(q'_1 - q'_2)] F_{k\Delta t}(q'_1, q'_2)$$

is invariant under the transformation

$$\Delta t \rightarrow \Delta t/\xi$$

$$k \rightarrow k\xi$$

Hence, with the present assumptions, the spectrum changes under the transformation $H_U \rightarrow H_U\xi$ as

$$\frac{1}{(2\pi)^2} \frac{1}{M_p^2} \left(\frac{a_*}{a}\right)^4 \left[\rho_{\text{rest}}^2 \gamma_{ij} \nu_i^2 \nu_j^2\right]^2 (a_*\Delta t)^2 \int dq'_1 dq'_2 \cos [k\Delta t(q'_1 - q'_2)] F_{k\Delta t}(q'_1, q'_2) \rightarrow$$

$$\frac{1}{(2\pi)^2} \frac{1}{M_p^2} \left(\frac{a_*}{a}\right)^4 \left[\rho_{\text{rest}}^2 \gamma_{ij}^2 \nu_i^2 \nu_j^2\right]^2 (a_*\Delta t/\xi)^2 \int dq'_1 dq'_2 \cos [k\Delta t(q'_1 - q'_2)/\xi] F_{k\Delta t}(q'_1, q'_2)$$

or equivalently

$$\frac{d\rho_{\text{GW}}(k)}{d\ln k} \rightarrow \frac{1}{\xi^2} \frac{d\rho_{\text{GW}}(k/\xi)}{d\ln k}.$$ 

This is the main analytic result of this paper. Although this result in some sense has been reported in the literature indirectly before (as we survey below), one of the points of this study is to examine the robustness of the relationship and to spell out the assumptions necessary.

From this equation, we can thus extract two easy to remember features:

1. If the peak frequency without quintessence is $k_p$, the new peak frequency is at $k_p\xi$. This is intuitive from recognizing that a smaller bubble length scale results if the expansion is faster (corresponding to $\xi > 1$) since the bubble length scale has a smaller time to grow before the PT is completed.
2. The amplitude of the spectrum at the peak should decrease as \(1/\xi^2\) compared to case with \(\xi = 1\). The intuition for this result is that less source contributes to the gravitational wave at a fixed point because of the past Minkowski null boundary and the approximately compact time support of the source.

Eq. (25) is essentially a classical dimensional analysis coming from the assumptions summarized by Eqs. (11) and (12), and the result is useful in allowing us to read off how the gravity wave spectrum depends on the general assumptions of the Hubble expansion during the PT. As we shall see in the literature survey, the details of the spectrum for frequencies larger than the peak frequency is difficult to compute and very uncertain. Nonetheless, we will show in sections 3 and 5 that our scaling arguments above are robust, and Eq. (25) will most likely apply to improved spectra that will be derived by all future more accurate computational techniques.

Note that the amplitude dependence on the fluid energy \(\rho_{f}^{\text{rest}}\) written in Eq. (24) in a very intuitive form is a bit misleading since it naively looks as one can increase the amplitude of the gravity wave by increasing this quantity. However, since \(\rho_{f}^{\text{rest}}\) scales as the radiation energy density, when the gravity wave energy density is compared with the radiation energy density, one power of it is normalized away. The second power of \(\rho_{f}^{\text{rest}}\) actually represents the clock units with which to measure \((a_* \Delta t)^2\) since when divided by \(M_p^2\), it represents the approximate expansion rate squared of a radiation dominated universe.

Hence, it is not really the absolute magnitude of \(\rho_{f}^{\text{rest}}\) that is important for increasing the measurable gravity wave amplitude but the dimensionless quantity \((a_* \Delta t)^2 \rho_{f}^{\text{rest}} / M_p^2\). As we will see explicitly in Sec. 4, obtaining a spectrum observable at the LISA experiment will require the duration \(a_* \Delta t\) of the PT to be of the same order of magnitude as the expansion rate determined by the radiation energy density during the PT. Physically, this effectively corresponds to a sufficiently large potential barrier suppressing bubble nucleation such that a non-negligible supercooling occurs before the PT completes. Although somewhat tangential to the point of our paper, we discuss this issue a bit further in Sec. 4 and Appendix D.

Although our next goal is to utilize Eq. (25) to make predictions for the quintessence kination dominated scenario, we will in the next section first check the consistency of our result with some of the explicit computations in the literature. There, we will also consider turbulence contributions to the gravity wave production and show that even when the turbulent source is long lived, its dominant contribution will be from the phase transition time period, allowing Eq. (25) to remain a good approximation. If the reader is not interested in the consistency check, the reader is encouraged to skip to Sec. 4.

3. SURVEY OF EXPLICIT COMPUTATIONS AND DETAILED ANALYSIS

In this section, we will survey the literature which computes the gravity wave spectrum using both simulations and analytic techniques [3, 5, 10, 13, 14, 16, 18, 19, 44]. Our aim is to show that our scaling assumptions resulting in Eq. (25) are consistent with the existing explicit computations.

References [13, 14, 16, 18] find the following generic result regarding GWs generated from bubble collision. The spectrum is found to have a rising and a falling shape, where the increasing side of the spectrum scales as \(k^3\) and the peak position is at a wave vector of order of \(1/R_\ast\) (\(R_\ast\) is the typical bubble size at the end of the PT). Physically, one can
attribute the $k^3$ scaling law to the compactness of the sources and the spatial homogeneity of their distribution. At the same time, one can understand the appearance of the conformal symmetry breaking scale $1/R_*$ as being the only identifiable classical length scale in the problem. While there is no well known uncertainty about the rising part of the spectrum [18], there is a large uncertainty associated with the falling part (UV part) of the spectrum. Direct simulations find that it scales as $k^{-1}$ [16], while the analytic calculations give model-dependent results [14, 18].

In addition to bubble collisions, turbulent motion of the fluid during the EWPT will also generate GWs [7–10, 19]. One can estimate the turbulence spectrum with either dimensional analysis or the velocity correlation function [45]. It is possible that fluid turbulence and magnetohydrodynamic (MHD) turbulence can contribute to GW production as much as bubble collisions.

We now consider the details below.

### 3.1. Simulation for bubble collision

GW generation from bubble collisions in zero temperature vacuum has been extensively studied by Kosowsky et al. [3]. Each bubble is represented as an $O(3,1)$ kink solution to the scalar field $(\phi)$ equation of motion. The latent energy released in the phase transition is turned into the bubble wall’s gradient energy $(\nabla \phi)^2$ and kinetic energy $(\partial_t \phi)^2$. Using numerical simulations, it is found that the GW spectrum depends primarily on the large scale features of the source. Specifically, a model of the source’s stress energy tensor which ignores the collision region between the two bubbles still results in a GW spectrum that is almost identical to the one from a full simulation. This “envelope approximation” when applied to many bubbles collisions [$N \sim 20–200$] results in a GW spectrum characterized by a peak frequency and a ratio of GW total energy over the released latent energy as

$$\omega_{\text{max}} = 1.6\beta, \quad E_{\text{GW}}/E_{\text{vac}} = 0.06(H_*/\beta)^2$$ (26)

where $\beta$ is a parameter controlling the bubble nucleation rate $\gamma$ as in $\gamma \propto \exp(\beta a_* t)$, and $\beta$ is related to the PT duration by $\beta \sim (a_* \Delta t)^{-1}$. Upon going through their arguments, one can show that $H_*$ (Hubble expansion rate at the time of the PT) in this equation can be seen to be a reparameterization of radiation temperature $T_*$ rather than the expansion rate of the universe itself.\(^1\) Hence, Eq. (26) is consistent with our result that the fraction of the energy of the GW is proportional to the PT duration $(a_* \Delta t)^2$. A statistical approach is also considered in [4], where the gravity wave spectrum is computed as an incoherent sum over individual bubbles weighted by the bubble size distribution function. In the many bubble case, the high frequency part of the spectrum is enhanced by the multiplicity of small bubbles.

Building on the zero temperature work, the finite temperature situation is studied in [5]. Finite temperature modifies the Higgs boson’s effective potential and introduces a fluid dynamical degree of freedom. If the PT produces large latent heat, the bubble wall velocity

\(^1\) Even though their bubble computations are based on zero temperature vacuum, they add in the expansion of the universe by hand to scale the energy density appropriately for the physical values today. This is the source of the $T_*$ dependence.
would be supersonic, leading to a process called detonation. As the detonation front expands, the fluid that is swept by the front is being compressed and dragged along (see [1] for details of computing the single bubble’s velocity profile and temperature profile). A full simulation of two bubble collision is computationally difficult, due to the chaotic fluid motion and the wide range of length scales involved. To circumvent this difficulty, “envelope approximation” is again applied, resulting in the following total GW energy fraction and peak frequency [5]:

$$\Omega_{GW}\hbar^2 \approx 1.1 \times 10^{-6}\kappa^2 \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{v_w^3}{0.24 + v_w^3}\right) \left(\frac{100}{g_*}\right)^{1/3}$$

$$f_{max} \approx 5.2 \times 10^{-8}\text{Hz} \left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{1\text{ GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6}$$

where $v_w$ is the detonation front’s velocity, $T_*$ is the phase transition temperature, and $g_*$ is the relativistic degree of freedom during the EWPT. Here $\alpha = \rho_{vac}/\rho_{rad}$ is the ratio of vacuum energy density to the radiation energy density, which characterizes the strength of the PT. The variable $\kappa \approx \rho_{fluid}/\rho_{vac}$ is an efficiency factor quantifying the fraction of the available vacuum energy that goes into the kinetic motion of the fluid. Evidently, the GW spectrum’s amplitude and peak position have the same scaling dependence on $\beta$ as the zero temperature case, which is consistent with our scaling result since $\beta$ quantifies $1/(a_0\Delta t) \propto H$ while $H_*^2$ here represents the radiation energy density and not the total expansion rate. (Even though $H_*$ here corresponds to $H_U$ in other sections of the paper, we maintain the original literature’s notation in this survey section to emphasize the non-transparency of the scaling relationship in the literature.) Hence, if exotic fluid component contributions to the stress tensor increase the expansion rate during the PT keeping the temperature fixed, then the scaling can be read off from the existing literature by scaling with $\beta$. In that sense, the change in the peak position and amplitude resulting from our Eq. (25) is not particularly new. On the other hand, the robustness of this simple scaling relationship for the entire observable spectrum and its application to the kination dominated quintessence scenario have not been explored before this paper, to our knowledge. Generalizing the two bubble collision simulation to many bubbles ($N \sim 100$) [16], the GW spectrum’s high frequency part is found to be enhanced from $k^{-1.8}$ to $k^{-1}$, again due to the small bubble effect at the end of phase transition.

### 3.2. Analytic calculation for Bubble Collision

Reference [14] uses the stochasticity of the source to estimate the gravity wave’s spectrum. Two assumptions are made about the velocity field in the bubble collision:

1. The velocity field’s distribution is approximately Gaussian, i.e. the four points correlator $\langle vvvv \rangle$ is determined by the two point correlator $\langle vv \rangle$.

2. The two point correlator $\langle v(x)v(y) \rangle$ is nonzero if $x$ and $y$ can belong to the same bubble.
The following spectrum is thereby obtained:

\[
\frac{d\Omega(k, \eta_0) h^2}{d \ln k} \approx \frac{3}{2\pi^3} \left( \frac{g_0}{g} \right)^{\frac{1}{3}} \Omega_{rad} h^2 \left( \frac{\Omega^*_{kin}}{\Omega^*_{rad}} \right)^2 \left( \frac{H_*}{\beta} \right)^2 \frac{(1 - s^3)^2}{s^4} \times \frac{0.21 \left( \frac{Z}{Z_m} \right)^3}{1 + \left( \frac{Z}{Z_m} \right)^2 + \left( \frac{Z}{Z_m} \right)^{4.8}}
\]

where \( s \) is the ratio of bubble wall thickness to radius and \( Z/Z_m \sim k/k_p \) is a dimensionless wave-number. ² Since the amplitude contains the factor \( \left( \frac{H_*}{\beta} \right)^2 \), and the frequency dependence is through a function of \( Z = kL_* \) (where \( L_* \) is the typical length scale at the end of the stirring phase), the spectrum’s parametric scaling is consistent with our scaling rule result in Eq. (25).

### 3.3. Scaling of the Turbulence Generated Gravity Wave Spectrum

As the thermal bubbles percolate, the fluid within the bubbles collides and generates turbulence. Even though a detailed simulation of turbulence’s evolution is difficult, some statistical features can be derived from dimensional analysis and intuition. Generally, the turbulence contains eddies of different sizes, as the larger ones break down to smaller ones, energy is also cascaded down to smaller scales. In a fully developed turbulence, the standard intuitive assumptions are that the energy cascade rate is a constant over time and is also a constant for different scales [5, 45]. For example, if the cascade rate \( \epsilon \) were not constant on different scales, there cannot be a steady state since energy would be building up at a particular length scale. However, as far as eddy sizes are concerned, since a largest scale \( L \) fixed by the bubble size and a smallest scale fixed by the viscosity exist, the energy cascade rate cannot be exactly scale invariant. Assuming that a turbulent eddy of size \( l \) with velocity \( v_l \) breaks down in a few turnover times \( \tau_l \sim l/v_l \) (which is true by dimensional analysis assuming that viscosity plays a negligible role), the energy cascade rate per unit mass for non-relativistic eddies is

\[
\epsilon \sim \frac{v_l^2}{l/v_l} \sim v_l^3/l, \text{ or equivalently } \nu \sim (\epsilon l)^{1/3}.
\]

It is interesting that the assumptions of \( \epsilon \) being a constant over different length scales and viscosity being unimportant until very short length scales fix the velocity spectrum by dimensional analysis. As \( l \) decreases (i.e. for smaller eddies), the dissipation effect becomes increasingly important, and the energy dissipation rate per unit mass is given by

\[
\nu (\nabla v)^2 \sim \nu \left( \frac{v_l}{l} \right)^2
\]

where \( \nu \) is the kinematic viscosity. If \( l \) is decreased to the point that the energy dissipation rate equals the energy cascade rate, the turbulence would cease to exist. This scale is the Kolmogorov microscale \( \lambda \):

\[
\epsilon \sim \nu \left( \frac{v_l}{\lambda} \right)^2 \implies \lambda \sim \frac{L(Lv_L/\nu)^{-3/4} \sim L(Re)^{-3/4}}
\]

² We use \( k \) and \( \omega \) interchangeably in the GW spectrum.
where the dimensionless Reynolds number $R_e \equiv L v_L / \nu$ relate the largest scale $L$ and the smallest scale $\lambda$. In the case of EWPT, the Reynolds number is usually on order of $10^{13}$, causing $\lambda$ to be negligible compared to $L$, indeed. Given these two parameters $L$ and $v_L$, the energy spectrum of a fully developed turbulence can be estimated as

$$E(k) \sim \epsilon^{2/3} k^{-5/3}, \text{ for } \lambda < k^{-1} < L$$

(33)

which is used in [5] to estimate a GW spectrum

$$\frac{\omega \, d\rho_{GW}}{\rho} d\omega \approx (\frac{H_s}{\beta})^2 v v_0^6 \left( \frac{\omega}{\omega_0} \right)^{-9/2}.$$  

(34)

In the formula, the parameter $v_0$ is the typical fluid velocity at the length scale of the largest bubble size $L$, not to be confused with the bubble boundary velocity $v$, and $\omega_0 \sim \tau_L^{-1} \sim \beta v^{-1} v_0$. This spectrum is valid up to the smallest scale of turbulence, i.e. $k^{-1} \sim \lambda$. The factors $(\beta)^{-2}$ and $\omega / \omega_0$ in the spectrum indicate that the turbulence generated gravity wave spectrum also is consistent with our scaling rule equation Eq. (25). Just as for all the previous examples, $H_s$ should be viewed as parameterizing the critical temperature $T_s$ and not the true expansion rate of the scale factor.

Reference [19] also considers the initial stirring phase and the final decay phase of the turbulence. The velocity correlator function of the turbulence is used to find the stress-energy correlation functions. It is found that both the MHD and fluid turbulence last for a long time after the PT has ended, in contrast with the bubble collision case. The gravity wave spectrum is found to be

$$\frac{d\Omega_{GW} h_0^2}{d \log k} = 12(2\pi)^2 C_s^2 \Omega_{rad,0} h_0^2 \left( \frac{g_0}{g_{fin}} \right)^{1/3} \left( \frac{\Omega_{S*}}{\Omega_{rad*}} \right) K_*^3$$

$$\times \left[ \int_0^{y_{top}} dy \frac{y^{\gamma_k+2}}{y + \frac{y_{fin}}{\tau_L}} I_s(K_s, y, y) \int_y^{y_{top}} \frac{dz}{z + \frac{y_{fin}}{\tau_L}} \cos \left( \frac{\pi K_s}{v_L} (z - y) \right) \right]$$

(35)

$$+ \int_{y_{fin}}^{y_{top}} dy \frac{y^{-\gamma} y_{fin}}{y + \frac{y_{fin}}{\tau_L}} I_s(K_s, y, y) \int_y^{y_{top}} \frac{dz}{z + \frac{y_{fin}}{\tau_L}} \cos \left( \frac{\pi K_s}{v_L} (z - y) \right).$$

In this spectrum, the subscript 0 denotes the present time, * denotes the end of PT, and $fin$ denotes the end of the turbulence. The subscript $s$ in $C_s$ can be either $v$ or $b$, which stands for the fluid turbulence or magnetic turbulence. $C_s$ is a numerical factor and $\Omega_s$ is the corresponding source’s energy fraction of the total energy density. The wave-vector $k$ is rendered dimensionless as $K_* = k L_*$. The variable $y$ is a time variable normalized by the largest eddy turn over time $\tau_L$: $y = \frac{t_{fin}}{t_{in}}$ where $t_{in}$ is the beginning time of the stirring phase. The finish time of the turbulence is denoted as $y_{fin}(k)$, the $k$-dependence indicates that different modes end at different times. The integral of $z$’s upper limit $y_{top} \equiv \min[y_{fin}, y + \frac{\gamma_k y_{fin}}{\pi K_*}]$ serves as a cut-off of correlation between sources at different time. $I_s(K_s, y, y)$ is the normalized dimensionless equal-time correlator of the source. The index $\gamma$ is explained below.

The first line represents an overall normalization, controlled by the amplitude of the source. The second line with time integral $\int_0^1 dy ...$ represents the contribution to GW from turbulence during the stirring up time period, i.e. the PT period. The last line with time integral $\int_{y_{fin}}^{y_{top}} dy ...$ represents the contribution to GW during the decay of the turbulence. We
shall show the evolution of the turbulence’s stress energy tensor only depends on one scale, i.e. the PT duration $\Delta t$, which is also the largest eddy turn over time $\tau_L$. In the stirring up part, $\tau_L$ is clearly the only scale. In the free decay part, there might be a new time scale controlling the decay, but we shall see there is none. The decay of the turbulence is modeled as

$$\frac{\Omega_T}{\Omega_{rad}} \sim \left( \frac{t - t_m}{\tau_L} \right)^{-5 \gamma}.$$  (36)

This power law relation with time is a scale free relation. Therefore, both parts of the evolution of the turbulence contain at most one scale. On the other hand the correlation between sources at different times is also modeled in a scale free way. It is assumed that the source with a certain wave vector $k$ at two different times $t_1$ and $t_2$ are uncorrelated if the time lapse is larger than a few oscillation time, i.e.

$$\langle \tilde{T}(k, t_1) \tilde{T}(k, t_2) \rangle = 0, \text{ if } |t_1 - t_2| \gtrsim \frac{x_c}{k}, x_c \sim O(1)$$  (37)

Thus the evolution and correlation of the source contains no other scale than $\tau_L$. Finally, one can schematically put the above Eq. (35) into the following form:

$$\rho_{src} \times \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{+\infty} dt_2 P(kL, t_1, t_2) \propto (\Delta t)^2 \int_{-\infty}^{+\infty} \frac{d t_1}{\Delta t} \int_{-\infty}^{+\infty} \frac{d t_2}{\Delta t} \tilde{P}(kL, t_1, t_2)$$

where $\tilde{P}(kL, t_1, t_2)$ is a scale free formula. Thus, the GW spectrum in the long lasting turbulence case is still consistent with our assumptions leading to our scaling rule equation Eq. (25).

4. EXAMPLE: KINATION DOMINATION PHASE OF QUINTESENCE

In this section we will apply Eq. (25) to the results of [14, 16] to compute how the gravity wave spectrum will shift due to the assumption of the existence of a quintessence kination dominated phase [29–35]. The class of models that we are interested in can be described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial q)^2 - V(q) + \mathcal{L}_M - \frac{M_p^2}{2} R$$  (38)

where the real scalar field $q$ couples only to gravity (described by the Ricci scalar $R$) and the matter sector $\mathcal{L}_M$ (which must contain the Standard Model sector) through the minimal metric coupling. As discussed for example in [38], the quintessence energy density scaling with $a$ is not strongly constrained during the early universe if one is willing to tune $V(q)$. If the kinetic energy dominates, the phase of the $q$ field is said to be in a kination dominated phase, and the energy density behaves as

$$\rho_q \equiv \frac{1}{2} q^2 + V(q) \propto a^{-6}$$  (39)

with an equation of state $w = 1$. Starting from this phase, when the kinetic energy has decayed away, $P/\rho$ can behave as $w \approx -1$ equation of state fluid during cosmological periods.
for which we have empirical evidence for the existence of dark energy. If \( V(q) \) participates such that it gently pushes \( q \) to compensate for the Hubble friction, then \( \rho_q \) can decrease less quickly than \( a^{-6} \). Instead of focusing on the details of the finely tuned \( V(q) \) that can realize different scaling behavior with \( a \), we will simply parameterize the quintessence energy decay as\(^3\)

\[
\rho_q \propto a^{-n}
\]
with \( n \in \{4, 5, 6\} \). The results can also be used to understand situations in which the faster expansion rate does not arise from quintessence but rather other exotic fluid components contributing to the stress-energy tensor \(^{36}\).

A large family of quintessence models can be described by two parameters \((n, \eta)\) characterizing the energy density’s scaling behavior (defined above) with Hubble expansion and the relative relic density at the time of BBN, respectively. The latter is defined (e.g. \(^{37, 38}\)) as

\[
\eta \equiv \frac{\rho_q(t_{BBN})}{\rho_\gamma(t_{BBN})}. \tag{41}
\]

Current projection of the collider sensitivity to \( \eta \) if dark matter is an MSSM LSP is \( 10^{-4} \) for the LHC (upper bound only) and \( 10^{-6} \) for the ILC \(^{37}\). As we will now see, the gravity wave probe sensitivity is even greater.

We take the gravity wave spectrum from the recent literature and compute the shifted spectrum due to quintessence. In Fig. 1, the plots in the left column are based on the formula from numerical simulations \(^{16}\). Starting from the nMSSM Higgs model with superpotential

\[
W_{nMSSM} = \lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 - \frac{m^2_{12}}{\lambda} \hat{S} + W_{MSSM} \tag{42}
\]

the result reported in \(^{13}\) involved a scan over the model parameter space looking for regions that give rise to a strong first order PT. One set of PT parameters consists of \( \{\alpha = 0.2, T_* = 70 \text{ GeV}, \beta/H_* = 30\} \), taken from set (6) in Table I in \(^{16}\). We shall use this set of parameters for all the plots in Fig. 1. To make the plots on the left column of Fig. 1, we use

\[
\Omega_{GW}(f) = \tilde{\Omega}_{GW} \frac{(a + b) \tilde{f}^b f^a}{b f^{(a+b)} + a f^{(a+b)}} \tag{43}
\]

\(^3\) See appendix C for a formal mapping to potential.
Figure 1: These plots illustrate the gravitational wave spectrum change due to the effects of a kination phase of quintessence. The phase transition parameters are chosen as $\alpha = 0.2, T_s = 70 \text{ GeV}, \beta/H_U = 30$. On the left are plots based on the simulation of Konstandin et al. [16] and on the right are plots based on that of Caprini et al. [14]. From the top row downward we have $n = 4, 5, 6$. In each plot, we have the detector sensitivity lines, the original spectrum line without quintessence and the lines for $\eta = 10^{-8}, 10^{-4}, 1$. 
where

\[
\tilde{f} = 16.5 \times 10^{-3} \text{mHz} \left( \frac{f_*}{\beta} \right) \left( \frac{\beta}{H_*} \right) \left( \frac{T_*}{100 \text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6}
\] (44)

\[
\frac{\tilde{f}_*}{\beta} = \frac{0.62}{1.8 - 0.1 v_b + v_b^2}
\] (45)

\[
\tilde{\Omega}_{GW} = 1.67 \times 10^{-5} h^{-2} \frac{0.11 v_b^3}{0.42 + v_b^6} \kappa^2 \left( \frac{H_*}{\beta} \right)^2 \left( \frac{\alpha}{\alpha + 1} \right)^2 \left( \frac{100}{g_*} \right)^{1/3}
\] (46)

\[
v_b(\alpha) = \sqrt{1/3 + \sqrt{\alpha^2 + 2\alpha/3}}
\] (47)

\[
\kappa(\alpha) = \frac{1}{1 + 0.715\alpha} [0.715\alpha + \frac{4}{27} \sqrt{3\alpha}].
\] (48)

All the variables we do not explicitly discuss below have been defined in an earlier part of this paper. The quantities with tildes correspond to the quantities evaluated at the peak of the GW spectrum, while those with a star subscript refer to the values defined at the time of the phase transition. The parameters \(a\) and \(b\) in the above first formula correspond to the absolute values of the slopes of the increasing and decreasing regions of the gravity wave spectrum, and they are fit from the numerical simulations to be \(a = 3\) and \(b = 1\) [16]. The modified spectra are plotted by shifting the original spectrum according to Eq. (25), where \(\xi\) is computed for each curve from the corresponding \((n, \eta)\) as

\[
\xi = \sqrt{\frac{\rho_{rad}(t_{BBN}) \eta \left( \frac{a(t_{BBN})}{a(t_{EWPT})} \right)^n + \rho_{rad}(t_{EWPT})}{\rho_{rad}(t_{EWPT})}}, \text{ where } \rho_{rad}(g, T) = g \pi^2 T^4
\] (49)

To show that our scaling of the spectra can be applied independently of the details of the PT computation, we plot on the right column of Fig. 1 spectra based on analytic estimation of [13]:

\[
\frac{d\Omega(k, \eta_0) h^2}{d \ln k} \approx \frac{3}{2\pi^3} \left( \frac{g_0}{g_*} \right)^{1/2} \Omega_{\text{rad}} h^2 \left( \frac{\Omega_{\text{kin}}}{\Omega_{\text{rad}}} \right)^2 \left( \frac{H_*}{\beta} \right)^2 \left( 1 - s^3 \right)^2 \times \frac{0.21 \left( \frac{Z}{Z_m} \right)^3}{1 + \left( \frac{Z}{Z_m} \right)^2 + \left( \frac{Z}{Z_m} \right)^{4.8}}
\] (50)

\[
\frac{\Omega_{\text{kin}}}{\Omega_{\text{rad}}} = \frac{4}{3} \frac{(sv_f)^2}{1 - (sv_f)^2}
\] (51)

\[
s = c_s/v_b
\] (52)

\[
v_f = (v_b - c_s)/(1 - v_b c_s)
\] (53)

\[
\Omega_{\text{rad}} h^2 = 4.15 \times 10^{-5}
\] (54)

where \(Z = k v_b/(a \beta), Z_m = 3.8, c_s = \sqrt{1/3}, g_0 = 3.75, g_* = 106.75,\) and detonation front velocity \(v_b\) is related to \(\alpha\) as usual. As mentioned above, the PT parameters are chosen to be the same as those for the left column plots.
The three rows correspond to the parameters \( n = 4, 5, 6 \) respectively, and the four curves within each plot from top to bottom correspond to \( \eta = 0, 10^{-8}, 10^{-4}, 1 \) respectively. According to the bottom row plots, if the underlying Higgs physics is determined sufficiently accurately at colliders, a measurement of gravity wave spectrum by the projected BBO experiment matching the properties of the Higgs model would rule out any kination dominated scenario explanation of \( \mathcal{O}(1\%) \) discrepancy between collider determination of thermal relic density and cosmological measurements. More specifically, for \( \eta \ll 1 \), the discrepancy caused by the kination phase can be expressed as

\[
\frac{\Delta \Omega^{(K)}}{\Omega^{(U)}} \sim 10^3 \eta \left( \frac{m_\chi}{100 \text{ GeV}} \right)^2
\]

for a dark matter particle of mass of order 100 GeV \[37\]. As expected, it is clear from the Fig. 1 that GWs have much stronger sensitivity to \( \eta \) and \( n \) than collider/dark matter combination of measurements. Since the kination scenario effectively allows a large (up to \( 10^3 \)) boost factor reconciling enhanced galactic annihilations (such as those relevant for PAMELA \[40\]) with dark matter abundance \[38\], such measurements from BBO can strongly constrain scenarios reconciling collider physics, cosmological DM abundance, and indirect dark matter signals. Even more optimistically, if one can obtain the peak position (and the right column plot happens to give the correct picture), then one may be able to measure both \( \eta \) and \( n \), which can be overconstrained by the possible dark matter data which also is sensitive to \( \eta \) and \( n \).

Given that the parameters used for Fig. 1 does not lead to a GW spectrum that LISA can measure, one might ask “What kind of underlying scalar sector parameters will lead to observable gravity waves for LISA?” Naively, one might suspect that since \( (\rho f)_{\text{rest}}^2 \) appears in front of Eq. (24), one simply would have to make this large. However, as we discussed at the end of Sec. 2, the duration \( a_\ast \Delta t \) of the phase transition uses \( \rho f^\text{rest} \) as a clock and the two quantities are not independent. For example, in the absence of exotic fluid element like quintessence, \( (a_\ast \Delta t)^2 \) scales inverse proportionally with \( \rho f^\text{rest} \). Hence, the combination \( \rho f^\text{rest} a_\ast \Delta t \) does not increase even if one increases \( \rho f^\text{rest} \) by itself. Hence, to make the relevant combination large, one actually needs a more difficult to achieve model building ingredient of making \( dS^{(3)}/dT \) as small as possible. Typically, this requires small thermal corrections to the relevant scalar field near the critical temperature. Keeping other parameters fixed, this corresponds to a large \( T_c \) (this is the temperature at which the PT begins and not when most of the PT completes). Qualitatively, this corresponds to a situation in which the system is closer to having supercooling rather than not. A semi-quantitative discussion within the context of a toy model is presented in Appendix D.

Although we provide no underlying physics model, we plot in Fig. 2 a hypothetical spectrum that would be generated by a “long” PT to demonstrate the scaling effect on a gravity wave spectrum measurable by LISA. Because we are setting \( \beta/H = 1 \) for Fig. 2, this is at the edge of the validity of the scaling relationship which assumed that \( a_\ast H_\ast \Delta t < 1 \). On the other hand, the qualitative behavior of the scaling relationship is accurate. Note that a typical beyond the SM Higgs sector extended by singlets go through many PTs before the EWPT, some of which may lead to the spectrum as shown in Fig. 2. The effect of superposing such spectra is beyond the scope of this work.
Figure 2: This figure is an analog of Fig. 1 except with the phenomenological parameters of the phase transition tuned to give a larger amplitude. The parameters are $\beta/H_U = 1$, $\alpha = 1$, $T_\ast = 70$ GeV. Physically, this corresponds to a phase transition with a long duration ($\beta/H_U$ is not large) and a large ratio of vacuum energy density to radiation energy density ($\alpha = 1$ instead of $\alpha = 0.2$).

5. CAVEATS

In this section, we explore the limitations of the assumptions leading to the scaling Eq. (25). The most important assumption is that there is only one relevant dimensionful scale, the Hubble expansion rate, during the generation of GWs. Therefore, the existence of any other conformal symmetry breaking physics relevant within the kinematic regime of our interest may invalidate our argument. The possible scales include the bubble wall thickness, the correction to the energy density due to bubble interactions, and the dissipation
scale associated with turbulence.

The bubble wall thickness is usually defined near the phase boundary where the energy is concentrated. In the vacuum bubble case, the most prevalently nucleated Higgs bubble solutions are believed to be described by $O(3,1)$ symmetric solutions $\phi(t, r)$ which are functions of $t^2 - r^2$. In the lab frame the bubble wall’s thickness will contract towards zero as the bubble wall velocity approaches the speed of light. Thus the bubble wall thickness is very small compared to the bubble radius. In the thermal bubble case, the energy is modeled to be contained in the fluid’s kinetic energy rather than in the Higgs field’s profile. Here the bubble wall is defined by the portion of fluid having large enough velocity to give an appreciable contribution to GW. Steinhardt [11] showed that there is a large class of solutions in which the fluid’s speed only depends on parameter $r/t$, i.e. as the bubble expands the velocity profile would expand accordingly. Therefore the bubble wall thickness is proportional to the bubble size, and in particular, it is not a scale independent of the Hubble rate.

Next we would like to consider the effect of small scale physics in the bubble collision case. We will show below that the dimensionless gravitational wave spectrum

$$k^3P(k, t_1, t_2)$$

in the $k$ range of interest for BBO and LISA depends appreciably only on the dimensionless combinations

$$\{k\Delta t, \frac{t_i - t_s}{\Delta t}, \frac{\bar{M}}{T_*(\bar{M})}\}$$

where $t_s$ is the time of the PT, $T_*$ is the temperature at the time of the PT which is assumed to depend only on the short distance physics parameters $\bar{M}$ as far as the conformal symmetry breaking parameters are concerned, and $a_s\Delta t \propto 1/H_*$ is the duration of the PT proportional to the inverse Hubble expansion rate at the time of the phase transition. Since the last of these parameters do not scale with the Hubble expansion rate, the conclusion of our paper is robust if Eq. [57] are the only relevant conformal symmetry breaking parameters.

To show this, we start with the effective classical description of the stochastic process of bubble creation and stress tensor evolution. Suppose we make the reasonable approximation that scalar field evolves classically except with a stochastic source term that can create bubbles. Hence, we write the equation of motion for the scalar sector as

$$D_\mu D^\mu \phi_i + \frac{\partial V}{\partial \phi_i^*} - \frac{\partial}{\partial \phi_i^*} L_I = J_i(x)$$

where $L_I$ is a short distance physics governed non-derivative interaction Lagrangian, $V$ is a scalar sector potential, and

$$J_i(x) = \int d^4 y \pi(y) j_i(x - y)$$

is a stochastic distribution which accounts for thermal/quantum fluctuation induced bubble creation. The stochastic function $\pi(y)$ is the number of bubbles created per unit time per unit volume at a small cell volume$^4$ centered at $y$ and $j_i(x - y)$ is a non-stochastic fixed

---

$^4$ Bubble nucleation from a classical “appearance” point of view is a collective field process typical of soliton creation since a bubble of radius larger than a critical radius is required to appear for it to expand classically.
function\textsuperscript{5} representing the effective classical source for a single bubble. Let $P(\pi)$ be the probability of obtaining a particular function $\pi$ for a single realization of the universe. It is in principle fixed by the path integral computation of the bubble nucleation, but the details will not be important for the scaling arguments that we present if the conditions we later discuss are satisfied. The ensemble average $\langle T_{ij}(t'_1, \vec{x}_1)T_{ij}(t'_2, \vec{y}) \rangle$ can be written as

\[ \langle T_{ij}(t'_1, \vec{x}_1)T_{ij}(t'_2, \vec{x}_2) \rangle = \int \mathcal{D}\pi P(\pi) \{ T_{ij}(t'_1, \vec{x}_1)\rvert_\pi T_{ij}(t'_2, \vec{x}_2)\rvert_\pi \}. \]  

(60)

where $T_{ij}$ (a functional of $\vec{\phi}$) is implicitly dependent on $\pi$ through Eqs. (58) and (59):

\[ \phi_i(x) = \phi_i[\pi](x). \]  

(61)

Note that the functions $\pi$ that generate non-vanishing inhomogeneities are the only important contributions, even if they may not be at the peaks of the probability functional $P(\pi)$.

Now, we impose 3 conditions which will limit the number of conformal breaking parameters that the system depends on:

1. The value of $\langle T_{ij}(t'_1, \vec{x}_1)T_{ij}(t'_2, \vec{x}_2) \rangle$ needs to be known only in the interval $t_* < t'_i < t_* + \Delta t$ (where $a_*\Delta t < 1/H_*$) to obtain an order of magnitude accurate gravity wave spectrum since it falls off rapidly outside of that time interval. As shown in Appendix A, the effective duration of the PT is

\[ a_*\Delta t \approx \frac{1}{H_*} F\left( \frac{\bar{M}}{T_*(\bar{M})} \right) \]  

(62)

where $F \ll 1$ is a dimensionless function of short distance physics conformal symmetry breaking parameters $\bar{M}$ and the critical temperature $T_*$. To make the argument of the dimensionless function dimensionless, $\bar{M}$ has been scaled by the critical temperature $T_*$ which itself is assumed to be a function of $\bar{M}$ only and not $H_*$. The dominance of the correlator within this short window of time is a reasonable generic assumption since the completion of the PT at $t_* + \Delta t$ corresponds to a homogeneous and isotropic scalar field phase which cannot source GWs. On the other hand, the plasma inhomogeneities can grow (albeit slowly because of the relativistic pressure) through gravitational clustering. As a first guess, this effect should be most pronounced on short distance physics scales (characterized by $\bar{M}$) for which the inhomogeneities leading to gravitational potential would be the largest. If this is true we can neglect these complications as we discuss further below. Another caveat is that $T_*$ is not necessarily independent of $H_*$ since a very large $H_*$ can lead to the decoupling of a particular relativistic species if that species has interaction rate $\Gamma < H_*$. In that case, the temperature of that species, call it $T_1$, can evolve differently from the rest of the thermal plasma. If a finite Higgs VEV can lead to interactions with this species at the time $t_*$ of the PT, one can have in addition to $T_*$ another temperature $T_1(t_*)$ which now does depend on $H_*$. Note that in such situations, $T_*$ and $T_1(t_*)$ can be coupled and the dependence on $H_*$ enters through $T_1(t_*)$. This is highly model dependent and as long as the number of species

\textsuperscript{5} Strictly speaking, it is a distribution since it may contain Dirac delta functions.
There are any physically relevant conformal symmetry breaking scales that we missed. We will in effect be reanalyzing some of the steps in Sec. 2 to obtain a sense of what kind of details can invalidate the scaling result of Eq. (25).

These conditions will now be used for a dimensional analysis aimed at checking whether the beginning of PT time depends on these parameters, we must have

$$\delta \propto \frac{1}{10^{-13}, 10^{-7}} \left( \frac{g_{s}(t_{0})}{3.9} \right)^{-1/3} \left( \frac{g_{s}(t_{s})}{10^{2}} \right)^{1/3} \left( \frac{T_{s}}{10^{2} \text{ GeV}} \right) \text{ GeV}$$

(corresponding to $10^{-4}$ to $10^{2}$ Hz range detector frequencies). Since we expect $F_{2} \sim \mathcal{O}(1)$ as noted above, for unsuppressed dependence on these parameters, we must have $\mathcal{O}(1)$ parametric possibilities for these dimensionless parameters on which $F_{2}$ depends. For example, for $M_{i}a_{s}\Delta t \sim 10^{14}$ to produce an $\mathcal{O}(1)$ number consistent with $F_{2}$, we could have a term proportional to

$$(M_{i}a_{s}\Delta t)^{-1/14}$$

in which case any scaling associated with $\Delta t$ will lead to a suppressed change in $F_{2}$. Another possibility is an extreme fine tuning of parameters

$$1/(10^{-14}M_{i}a_{s}\Delta t).$$
Table I: Dimensionless parameters with $a_s \Delta t \sim 10^{-2}/H_* \sim 10^{12}\text{GeV}^{-1}$ and $T_* \sim M_* \sim 100\text{ GeV}$.

Since we expect $F_2 \sim \mathcal{O}(1)$, for unsuppressed dependence on these parameters, we must have $\mathcal{O}(1)$ parametric possibilities for these dimensionless parameters on which $F_2$ depends if there is no fine tuned dimensionless parameters in the theory.

This latter possibility cannot be excluded based on dimensional analysis of the form here, and our scaling analysis can break down if there are extremely large or small parameters. The extreme level of fine tuning must be at least at the level of 1 part in $10^{10}$ even when we allow for dynamically generated dimensionless numbers of order $10^4$. Finally, we can divide those quantities in the table that are large with each other to give an $\mathcal{O}(1)$ number. Hence, from the table, barring an unlikely dynamically generated fine tuning that we discussed, we conclude that over the frequency range of interest for gravitational wave detectors

$$F_2 \approx F_3(k \Delta t, t'_i - t_*, \frac{\bar{M}}{T_*(\bar{M})}).$$

Since the last factor $\bar{M}/T_*(\bar{M})$ is independent of $H_*$ we arrive at our robust approximation that the dimensionless spectrum $k^3P(k)$ only depends on the form given in Eq. (13).

Next, we consider another possible scale, the turbulence’s microscale $\lambda$, which is related to the largest scale as $\lambda = (Re)^{-3/4}L \sim 10^{-10}L$. In the short-lasting source’s model [5, 10], this scale serves as a cut-off to the gravity wave spectrum. Since this scale is $10^{10}$ higher than the peak scale, we can follow the same reasoning about the short distance scale physics to show that it would not be significant for the GW detector LISA or BBO. In the recent work [19] where long-lasting source is considered, the microscale can affect the duration of the turbulence’s free decay. This duration of turbulence may appear as an independent time-scale, but as we show in Appendix B, the decay duration is sufficiently long that the turbulence has depleted most of its energy towards the end of this duration, and the exact ending time, to an excellent approximation, does not enter the GW spectrum.

Another issue that we did not address are classical scaling violations coming from quantum radiative corrections. If the $\xi$ scaling is many orders of magnitude, the anomalous dimension effect may give (depending on what renormalization prescription is chosen) an $\mathcal{O}(1)$ correction to the results presented here. However, in that case, GWs from first order PTs are much less likely to be measurable and therefore are unlikely to be of practical interest.

Before we conclude the caveat section, we would like to comment on the velocity dependence of the GW spectrum. In [5], the GW spectrum’s magnitude is given by Eq. (27). The
GW spectrum depends on the velocity $v_w$ implicitly through $\alpha$ as for example in

$$\kappa^2 \left( \frac{\alpha}{1 + \alpha} \right)^2 \left( \frac{v_w^3}{0.24 + v_w^3} \right),$$

(69) since $v_w$ and $\kappa$ are functions of $\alpha$ as in Eq. (47) and (48). In the weak detonation limit of $\alpha \to 0$, we have $v_w \to \sqrt{1/3}$ and $\kappa \to \frac{4}{27} \sqrt{3/3} \alpha$, causing the prefactor to scale as $\alpha^3$. In Caprini et al. [14], the velocity dependence goes as

$$v_f^4 \frac{(1 - s^3)^2}{(1 - (sv_f)^2)^4},$$

(70) which in the weak detonation limit scales as $\alpha^5$ (since $v_f \to \sqrt{3\alpha/2}$, $s \to 1 - \sqrt{2\alpha}$). For other region of parameters, as is shown in Fig.(13b) of [14], the two approaches can give peak amplitudes that differ by one order of magnitude. It is clear that the velocity dependence of the GW spectrum is both uncertain and can be of high polynomial power, making the current numerical uncertainty in the bubble wall velocity (see e.g. [42]) a significant source of overall GW spectrum uncertainty relevant for assessing the measurability of the GW. It is comforting however to know that our scaling rule is largely independent of this uncertainty.

6. SUMMARY

In this paper, we have presented an analytic transformation rule Eq. (25) that is useful for understanding how the gravitational wave spectrum generated through an electroweak scale first order phase transition at a fixed temperature $T_*$ would change if the expansion rate of the universe during the phase transition were different from that inferred from the assumption of pure radiation domination. We have explored the remarkable robustness of the scaling relationship with respect to many computational uncertainties in the gravity wave spectrum.

We apply this transformation rule to the example of a universe having a quintessential kination dominated phase and find as expected a strong sensitivity to the single phenomenological parameter controlling this scenario. In principle, this scaling relationship together with dark matter properties measured by colliders can be used to overconstrain this single phenomenological parameter. Unfortunately, if the current technology of gravity wave computations is correct, then we find that any measurement of the gravity wave spectrum at the level of the projected BBO sensitivity can rule out any appreciable boost factors relevant for reconciliations between various sets of data such as that between colliders and cosmology and/or indirect detection (such as that relevant [38] for PAMELA data [40]).

Nonetheless, using the results of this work, any future gravity wave detection experiments measuring phase transition induced gravity waves can understand their measurement’s sensitivity to the expansion rate of the universe. It would indeed be exciting to have an observational anchor on the expansion rate of the universe when the universe is as hot as 100 GeV, just as isotope abundance measurements allow us to have an observational anchor on the expansion rate at a temperature of 1 MeV in the context of big bang nucleosynthesis.
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Appendix A: Bubbles Filling Space

Here we review a well known argument [46] about how first order PT bubbles fill space. For this section, we will use the metric parameterization $ds^2 = dt^2 - a^2(t)|d\vec{x}|^2$. Let $\gamma(t)$ denote the probability per volume per time of bubble formation. Assuming that the bubble wall is not accelerating in the locally inertial frame (which is what is typically done in the literature when the bubble wall velocity $v_w$ is taken to be a constant), we have

$$\frac{d\vec{x}}{d\tau} \propto \frac{1}{a}. \quad (A1)$$

Hence, we have

$$\frac{1}{\sqrt{1 - a^2 (\frac{dr}{dt})^2}} \frac{dr}{dt} = \frac{K}{a} \quad (A2)$$

where the constant $K$ is to be determined by a boundary condition. Setting the boundary condition

$$a_i \frac{dr}{dt} \bigg|_{t_i} = v_w, \quad (A3)$$

we find

$$K = \frac{v_w}{\sqrt{1 - v_w^2}} \quad (A4)$$

and

$$\frac{dr}{dt} = \frac{v_w}{a} \quad (A5)$$

Hence, a bubble nucleated at time $t_i$ fills a comoving volume

$$V_3(t_i, t) = \frac{4\pi}{3} \left[ v_w \int_{t_i}^{t} \frac{dt'}{a(t')} \right]^3. \quad (A6)$$

Next, we compute $P$, the probability that a point in comoving space is in false vacuum. The probability that at time $t + dt$ the vacuum at a point is still in false vacuum given that it is in false vacuum at time $t$ is

$$P(t + dt) = P(t)[1 - P_c] \quad (A7)$$

where $P_c$ is the probability of nucleating a bubble within the past causal cone surface volume of thickness $dt$ with the causal signal propagation speed given by $v_w$ since it is the bubble
wall that needs to reach the given point in consideration. Since the relevant surface volume of the causal cone can be easily computed to be

\[ a^3V_3(t_i, t)dt, \tag{A8} \]

we can multiply this by \( \gamma \) to find \( P_c \) to arrive at

\[ \frac{dP}{dt} = -Pa^3V_3(t_i, t)\gamma(t). \tag{A9} \]

Solving for \( P \), we find

\[ P(t) = P(t_i) \exp \left( -\int_{t_i}^t dt' \gamma(t')V_3(t_i, t')a^3(t') \right). \tag{A10} \]

Using Eq. [16], we find

\[ P(t) = P(t_i) \exp \left( -C_1 \int_{t_i}^t dt' \exp \left[ \left( -S_{s}^{(3)} + \frac{(t' - t_*) H_*}{1 + \frac{1}{3} \frac{d\ln q_{s_S}}{d\ln T}|_{t_*}} \right) / T(t') \right] T^4(t')V_3(t_i, t')a^3(t') \right). \tag{A11} \]

When \( t < t_c \) where

\[ t_c \sim t_* + \left[ \frac{H_*}{1 + \frac{1}{3} \frac{d\ln q_{s_S}}{d\ln T}|_{t_*}} \right]^{-1}, \tag{A12} \]

\( S_{s}^{(3)}/T \) is large such that the exponential suppression in the integrand makes the integral in front of \( C_1 \) negligible. After that time scale, probability of not being in the false vacuum is \( O(1) \), and the PT is assumed to be completed. Hence, we can conclude that the duration of the PT scales as

\[ \Delta t_{\text{proper}} \equiv t_c - t_* \propto \frac{1}{H_*}. \tag{A13} \]

In terms of conformal time, when \( H_*\Delta t_{\text{proper}} \ll 1 \), we have

\[ \Delta t_{\text{conformal}} \approx \Delta t_{\text{proper}}/a(t_*) \propto \frac{1}{H_*}. \tag{A14} \]

The variable \( \Delta t_{\text{conformal}} \) corresponds to the variable \( \Delta t \) in Eq. [18].

**Appendix B: The decay duration of turbulence**

In the GW spectrum formula Eq. (35) in reference [19], the free decay part of turbulence gives a contribution that proportional to the integral

\[ \int_{1}^{y_{\text{fin}}} dy \frac{y^{-7\gamma_*}}{y + \frac{t_{in}}{\tau_{L}}} I_s(K_*, y, y) \int_{y}^{y_{\text{top}}} \frac{dz}{z + \frac{t_{in}}{\tau_{L}}} \cos \left( \frac{\pi K_*}{v_L}(z - y) \right) \tag{B1} \]

Here, \( y \) and \( z \) are dimensionless time variables: e.g. \( y = (t - t_{in})/\tau_{L} \) where \( t_{in} \) is the beginning time of the stirring phase and \( \tau_{L} \) is the largest eddy turn over time. We now show that the
ending time of turbulence as a function of the scale length $y_{fin}(k)$ is very large for the peak position ($y_{fin}(k_{\text{peak}}) \sim 10^4$) and therefore the exact ending time is irrelevant to GW spectrum at the peak position. We shall consider the behavior of the integrand as $y \to y_{fin}$, and take $\gamma_* = 2/7$ for concreteness:

$$
\int_1^{y_{fin}} dy y^{-7\gamma} I_s(K_*, y, y) \int_y^{y_{top}} \frac{dz}{z + \frac{\ln z}{\tau_L}} \cos \left( \frac{\pi K_*}{v_L} (z - y) \right) \sim \int_1^{\infty} dy y^{-5} - O(y_{fin}^{-4}) \tag{B2}
$$

where we have used the following estimation

$$
I_s(K_*, y, y) \to \begin{cases} 
  y^{-1.05} & \text{MHD turbulence} \\
  y^{-1} & \text{fluid turbulence} \sim y^{-1}
\end{cases} \tag{C1}
$$

$$
y_{top} - y = \min[y_{fin} - y, \frac{x_c v_L}{\pi K_*}] \sim v_L/K_* \sim O(1)
$$

Therefore the GW’s relative dependence on $y_{fin}$ is as weak as $y_{fin}^{-4}$. For the ending time of turbulence at the stirring scale $K_* = kL_* \sim 1$, one can use Eq. (72) of [19]

$$
y_{fin} \sim \frac{t_{fin}(k)}{\tau_L} \approx \left( \frac{2 \times 10^{-14}}{K_*} \right)^{\frac{28}{51}}, \text{ for } K_* > 0.07 \quad \tag{B3}
$$

which shows that $y_{fin}(K_* = 1) \sim 10^4$. If one take $y_{fin} \to \infty$ in the above integral, the error introduced is on the order of $O(10^{-16})$, very small compared to the integral itself which is at least $O(1)$.

### Appendix C: A Formal Map between $V(q)$ and $n$

Here we give a formal map between the quintessence potential $V(q)$ and the quintessence energy density dilution behavior $a^{-n}$.

The equation of motion of the quintessence can be written using the scale factor $a$ as a time variable assuming that the only other component during the era of interest is radiation which dilutes as $a^{-4}$:

$$
\frac{1}{a^3} \frac{\sqrt{2} \sqrt{V(q)/\rho_{R0} + \left( \frac{a_0}{a} \right)^4}}{\sqrt{6 - \frac{1}{M_p^2} \left( \frac{d q}{d \ln a} \right)^2}} d \ln a \left( \frac{\sqrt{2} \sqrt{V(q)/\rho_{R0} + \left( \frac{a_0}{a} \right)^4}}{\sqrt{6 - \frac{1}{M_p^2} \left( \frac{d q}{d \ln a} \right)^2}} d \ln a \right) + \frac{M_p V'(q)}{\rho_{R0}} = 0. \quad \tag{C1}
$$

The solution to this equation defines a functional

$$
q_s[a, V(q)]. \quad \tag{C2}
$$

Note that the equation has been normalized to be dimensionless: $V$ is measured in units of initial radiation energy density $\rho_{R0}$ and $q$ is measured in units of $M_p$. Next, one can solve $q = q_s[a, V(q)]$ for $a[q, V(q)]$. This can be put into the energy density scaling equation for the quintessence energy density, yielding the equation

$$
\frac{\rho_{q0}}{\rho_{R0}} \left( \frac{a_0}{a[q, V(q)]} \right)^n = \frac{1}{2} \left[ \frac{\sqrt{2} \sqrt{V(q)/\rho_{R0} + \left( \frac{a_0}{a[q, V(q)]} \right)^4}}{\sqrt{6 - \left( \frac{d q_s[a, V(q)]/M_p}{d \ln a} \right)^2}} \right]^2 \frac{d q_s[a, V(q)]/M_p}{d \ln a} a=a[q, V(q)] + \frac{V(q)}{\rho_{R0}} \quad \tag{C3}
$$

which can in principle be solved for $V(q)$.
Appendix D: An Estimate of Temperature Dependence of $S^{(3)}$

In this section, we will give a semi-quantitative argument of which parameter choices of effective potential governing the PT will lead to an enhanced gravity wave amplitude. Consider the high temperature expansion of the effective potential for a single real field in the form

$$V(\phi) = \frac{1}{2}(\mu^2 + cT^2)\phi^2 - E\phi^3 + \frac{\lambda}{4}\phi^4$$  \hspace{1cm} (D1)

where $\mu^2 < 0$ and $c$ is a thermal correction dependent parameter. Equation of motion yields

$$0 = \frac{1}{L^2}\phi' + (\mu^2 + cT^2)\phi - 3E\phi^2 + \lambda\phi^3.$$  \hspace{1cm} (D2)

where we have estimated $\nabla^2 \phi \sim \frac{1}{L^2}\phi$. In solving for $L$, we need a characteristic value for $\phi$ which we will call $\phi_c$. We can set this characteristic value to be between the local maximum $\phi_u$ and the minimum $\phi_*$ (not the one at the origin) of the effective potential. (Recall that the cubic term $-E\phi^3$ is responsible for there being a bump in the potential giving rise to a local maximum.) Explicitly solving $V'(\phi) = 0$, we find

$$\phi_u(T) = \frac{3E}{2\lambda} \left( 1 - \sqrt{1 - \frac{4\lambda}{9E^2}(\mu^2 + cT^2)} \right)$$  \hspace{1cm} (D3)

and

$$\phi_*(T) = \frac{3E}{2\lambda} \left( 1 + \sqrt{1 - \frac{4\lambda}{9E^2}(\mu^2 + cT^2)} \right).$$  \hspace{1cm} (D4)

Hence, if we make a somewhat arbitrary but reasonable definition for the characteristic value to be

$$\phi_c \equiv \phi_u(T) + \frac{\phi_*(T) - \phi_u(T)}{2},$$  \hspace{1cm} (D5)

we find

$$\phi_c = \frac{3E}{2\lambda}$$  \hspace{1cm} (D6)

independently of the temperature except through $E$ and $\lambda$ which we assume to be dominated by the non-thermal contribution ($E$ coefficients that rely on thermal corrections do not yield strong phase transitions typically anyway). This yields the length scale associated with the bubble action to be

$$L = \frac{1}{\sqrt{\frac{9E^2}{4\lambda} + |\mu^2| - cT^2}}$$  \hspace{1cm} (D7)

where we have displayed our assumption of $\mu^2 < 0$ manifestly. Hence, we have

$$S^{(3)} \sim L^3 \left( \left( \frac{\phi_c}{L} \right)^2 + V(\phi_c) \right)$$  \hspace{1cm} (D8)

$$= \frac{\phi_c^2}{\sqrt{\frac{9E^2}{4\lambda} + |\mu^2| - cT^2}} + \frac{1}{2}(\mu^2 + cT^2)\phi_c^2 - E\phi_c^3 + \frac{\lambda}{4}\phi_c^4 \left( \frac{\frac{9E^2}{4\lambda} + |\mu^2| - cT^2}{2} \right)^{3/2}. \hspace{1cm} (D9)$$
Computing the temperature derivative at the critical temperature of

\[ T = T_c = \frac{\sqrt{2E^2 + \lambda |\mu|^2}}{\sqrt{c\lambda}} \]  

(D11)

(which is obtained by setting \( V(\phi_a) = 0 \) and solving for \( T \)) we find

\[ \frac{dS^{(3)}}{dT} \bigg|_{T=T_c} \sim 50 \frac{\sqrt{c}}{\lambda} \sqrt{2 + \frac{\lambda |\mu|^2}{E^2}}. \]  

(D12)

Hence, we conclude

\[ \frac{\beta}{H_*} \sim \frac{50}{1 + \frac{1}{3} \frac{d\ln g_*}{d\ln T}} \frac{\sqrt{c}}{\lambda} \sqrt{2 + \frac{\lambda |\mu|^2}{E^2}}. \]  

(D13)

To make this order unity (appropriate for an enhanced gravity wave amplitude), we can consider \( \sqrt{c} \ll 1 \). Assuming \( \lambda \sim O(1) \), we find

\[ c \lesssim 10^{-2}. \]  

(D14)

This leads to a PT temperature of

\[ T_c \gtrsim \text{TeV} \left( \frac{|\mu|}{100 \text{ GeV}} \right) \left( \frac{\sqrt{2E^2/|\mu|^2 + \lambda}}{\sqrt{\lambda}} \right) \]  

(D15)

corresponding to a high temperature PT of a weakly coupled scalars.

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