Multiple Single-Centered Attractors

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Abstract

In this paper we study spherically symmetric single-centered attractors in \(N = 2\) supergravity in four dimensions. The attractor points are obtained by extremising the effective black hole potential in the moduli space. Both supersymmetric as well as non-supersymmetric attractors exist in mutually exclusive domains of the charge lattice. Though for the supersymmetric black holes the attractor point is unique, in the non-supersymmetric case, we find multiple single-centered attractors for a given set of charges. We construct explicit examples of two distinct non-supersymmetric attractors in type \(IIA\) string theory compactified on \(K3 \times T^2\) carrying \(D0 - D4 - D6\) charges. We compute the entropy of these attractors and analyze their stability in detail.

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1 Introduction

The attractor mechanism, discovered originally by the seminal work of Ferrara, Kallosh and Strominger [1] in studying supersymmetry preserving, static, spherically symmetric, magnetically charged black holes in four-dimensional $N = 2$ supergravity theory coupled to vector multiplets, plays a central role in understanding the origin of black hole entropy. For these supersymmetric black holes, as one approaches the horizon, the scalar fields run into a fixed point in the moduli space. The fixed point in the moduli space is solely determined by the black hole charges and is independent of the values of the scalar fields at spatial infinity. Though initially it used to be thought as a consequence of supersymmetry, it has been subsequently shown that, for the single-centered, static, spherically symmetric configurations, the attractor mechanism is a consequence of extremality of the black hole [2]. This gives rise to the possibility of exploring the properties of extremal black holes which are non-supersymmetric [3].

Though there has been extensive study of non-supersymmetric attractors in recent years (see Refs. [4, 5] for a review on the topic), there are still a number of issues which remain to be resolved. One such issue is the stability of such black holes. Unlike their supersymmetric counterparts which are guaranteed to be stable, these non-supersymmetric attractors can either be stable or unstable [6, 7]. For a large class of models, including the ones arising from string compactifications, they also possess flat directions [6]. Stringy corrections can make these flat directions either stable or unstable depending upon the charges of the corresponding black hole configurations [8–13].

The other issue which has not been explored much is the multiplicity of these attractors. It is well known that, for extremal black holes, the equations of motion become algebraic as one approaches the horizon. For the static, spherically symmetric case, the black hole is described in terms of the motion of a particle in an effective one dimensional theory. The attractor value in the moduli space is obtained by extremising the effective black hole potential, which is an algebraic function of the vector multiplet moduli [3, 14, 15]. For supersymmetry preserving black holes of a given charge configuration, which are obtained by extremising the central charge, there is a unique stable attractor point in the moduli space. For the non-supersymmetric case, one can analogously construct a fake superpotential which upon extremization gives rise to the attractor solution [16, 17]. However, not all non-supersymmetric attractors are obtained this way and hence there is a possi-
bility of obtaining multiple solutions for a given set of charges. This is the main focus of our present work. We consider the compactification of type IIA supergravity to four dimensions, and by restricting the Calabi-Yau manifold to be a product type we explicitly construct multiple attractors carrying the same values of charges.

The plan of the paper is as follows. In the next section we will review the basics of attractor mechanism in four dimensional $\mathcal{N} = 2$ theories. Subsequently we will discuss the multiple black hole configurations in §3. Here we will find the explicit solutions and discuss their stability. Finally in §4 we will summarize our findings. Some of the detailed calculations will be carried out in the appendices.

## 2 Background

In this section we will review the basics of attractor mechanism. We consider $\mathcal{N} = 2$ supergravity theory in four dimensions coupled to $n$ vector multiplets. Hypermultiplets do not play any role in our analysis. The bosonic part of the supergravity Lagrangian is given by:

$$L = -\frac{R}{2} + g_{ab} \partial_\mu x^a \partial_\nu \bar{x}^b h^{\mu\nu} - \mu_{\Lambda\Sigma} F^\Lambda_{\mu\nu} F^\Sigma_{\Lambda\rho} h^{\mu\lambda} h^{\nu\rho} - \nu_{\Lambda\Sigma} F^\Lambda_{\mu\nu} \ast F^\Sigma_{\Lambda\rho} h^{\mu\lambda} h^{\nu\rho}$$  \hspace{1cm} (2.1)

Our notations and conventions in this section are the same as in Ref [2]. The vector moduli are denoted by the $n$ complex scalars $x^a$ with moduli space metric $g_{ab}$. The vector fields $A^\Lambda, (\Lambda = 0, 1, \cdots, n)$ with the corresponding field strengths $F^\Lambda$ consists of the graviphoton as well as the gauge fields from the vector multiplets. $\mu_{\Lambda\Sigma}$ and $\nu_{\Lambda\Sigma}$ are the gauge couplings, $h_{\mu\nu}$ is the metric of the four dimensional space time with scalar curvature $R$. The moduli space metric as well as the gauge couplings are determined in terms of the $\mathcal{N} = 2$ prepotential $F$.

For static, spherically symmetric configurations the metric $h_{\mu\nu}$ is given by:

$$ds^2 = e^{2U} dt^2 - e^{-2U} \gamma_{mn} dy^m dy^n$$  \hspace{1cm} (2.2)

where, for the extremal black holes, the spatial part of the above is given by the Euclidean metric $\gamma_{mn} = \delta_{mn}$, and the warp factor $U$ depends only on the radial coordinate $r$. Substituting this ansatz, and an appropriate expression for the gauge field $A^\Lambda$ satisfying the Bianchi identity, in the field equations for the Lagrangian (2.1), we find that they are equivalent to the equations of motion of an effective one dimensional system whose Hamiltonian
is constrained to be zero. For regular black hole horizon, the effective potential of the one dimensional system must be extremized at the black hole horizon. Since the effective black hole potential is a function of the scalar fields as well as the black hole charges, generically the extremization fixes the scalar fields at the horizon and their values are determined by the black hole charges. For stable attractors the Hessian must admit positive eigenvalues.

Our focus in this paper is on $\mathcal{N} = 2$ supergravity theories arising from the compactification of type $IIA$ string theory on a Calabi-Yau manifold $\mathcal{M}$. In the large volume limit, the $\mathcal{N} = 2$ prepotential is given by

$$F = D_{abc} \frac{X^a X^b X^c}{X^0} \quad (2.3)$$

where $D_{abc}$ are the intersection numbers: $D_{abc} = (1/6) \int_{\mathcal{M}} \alpha_a \wedge \alpha_b \wedge \alpha_c$ with $\alpha_a \in H^2(\mathcal{M}, \mathbb{Z})$.

We consider dyonic charged black holes with electromagnetic charges $(p^A, q^A)$ arising due to $D$-branes wrapped on various cycles of the Calabi-Yau manifold. The effective black hole potential in this case can be expressed as [2]:

$$V = e^K \left[ g^{ab} \nabla_a W \nabla_b W + |W|^2 \right]. \quad (2.4)$$

Here, $K$ denotes the Kähler potential and $W$ is the superpotential. The Kähler covariant derivative is defined as $\nabla_a W = \partial_a W + \partial_a KW$ and the moduli space metric is given by $g_{ab} = \partial_a \partial_b K$. The superpotential $W$ is determined by the prepotential $F$ and the dyonic charges of the black hole:

$$W = \sum_{\Lambda=0}^{n} (q_\Lambda X^\Lambda - p^A \partial_\Lambda F), \quad (2.5)$$

where as the Kähler potential is given by:

$$K = - \log \left[ i \sum_{\Lambda=0}^{n} \overline{(X^\Lambda \partial_\Lambda F - X^\Lambda \overline{\partial_\Lambda F})} \right]. \quad (2.6)$$

The attractor values are determined by setting $\partial_a V = 0$, which in turn gives:

$$g^{bc} \nabla_a W \nabla_b \overline{W} + 2 \nabla_a W \overline{W} + \partial_a g^{bc} \nabla_b W \overline{\nabla_c W} = 0 \quad (2.7)$$

This is the necessary condition to have a regular horizon. This equation admits both supersymmetric as well as non-supersymmetric solutions. Supersymmetric solutions satisfy $\nabla_a W = 0$, where as the solutions for which $\nabla_a W \neq 0$ give rise to non-supersymmetric attractors.
The supersymmetry preserving attractors are guaranteed to be stable. However for the non-supersymmetric attractors this is not the case in general. In the later case the stability is ensured only when the mass matrix corresponding to the effective black hole potential admits non-negative eigenvalues \[3, 6\].

3 Multiple Attractors

In this section we will discuss the solutions of the attractor condition (2.7) in detail. For simplicity, we will consider the case where the non-vanishing intersection numbers are of the form $D_{abs}$, where the index $s$ is taking a fixed value $s = n$ and the indices $a, b = 1, 2, \cdots, (n - 1)$, i.e., topologically the Calabi-Yau manifold must be a product form of the type $\mathcal{M} = \mathcal{M}_4 \times \mathcal{M}_2$. To be more specific we consider the example of $K3 \times T^2$. However, our analysis will also be valid for $T^6$ since the intersection numbers satisfy the above property in this case as well. We consider the $\mathcal{N} = 2$ truncation of the type IIA string compactification on the above manifolds.

The prepotential (2.3) now takes the form:

$$F = C_{ab} \frac{X^a X^b X^s}{X^0}, \quad (3.1)$$

where $C_{ab}$ is the intersection matrix for a basis of $H_2(K3, \mathbb{Z})$: the integral homology of 2-cycles in $K3$. For convenience, we introduce $x^a = X^a/X^0, x^s = X^s/X^0$ and, exploiting the symplectic invariance, we set the gauge $X^0 = 1$ now on. We will consider black hole solutions arising due to the intersecting $D0 - D4 - D6$ configurations. The Kähler potential and the superpotential for this configuration are given by

$$K = - \log \left[ - iC_{ab}(x^a - \bar{x}^a)(x^b - \bar{x}^b)(x^s - \bar{x}^s) \right] \quad (3.2)$$

$$W = q_0 - 2p^a C_{ab} x^b x^s - p^s C_{ab} x^a x^b + p^0 C_{ab} x^a x^b x^s \quad (3.3)$$

The metric on the moduli space is found to be

$$g_{ss} = -\frac{1}{(x^s - \bar{x}^s)^2}, \quad g_{ab} = \frac{2}{M} \left( C_{ab} - \frac{2}{M} M_a M_b \right), \quad \text{and} \quad g_{as} = 0, \quad (3.4)$$

where we have introduced the notation $M_a = C_{ab}(x^b - \bar{x}^b)$ and $M = M_a(x^a - \bar{x}^a)$ for convenience. We will now outline some of the intermediate steps to carry out the equations.
of motion. Some of the individual terms contributing to the equations of motion are:

\[ \nabla_a W = 2(p^0 C_{ab} x^b x^s - D_a x^s - C_{ab} x^s p^s) - \frac{2M_a}{M} W, \]

\[ \nabla_s W = p^0 C_{ab} x^a x^b - 2D_a x^a - \frac{W}{x^s - \bar{x}^s}, \quad \nabla_s \nabla_s W = -2 \frac{\nabla_s W}{x^s - \bar{x}^s}. \]

\[ \nabla_a \nabla_b W = 2C_{ab}(p^0 x^s - p^s) - \frac{2}{M} \left( C_{ab} - \frac{4M_a M_b}{M} \right) W \]

\[ \quad - \frac{4}{M} \left[ M_a \left( p^0 C_{bc} x^c x^s - D_b x^s - C_{bc} x^s p^s \right) + M_b \left( p^0 C_{ac} x^c x^s - D_a x^s - C_{ac} x^s p^s \right) \right], \]

\[ \nabla_a \nabla_s W = 2 \left( p^0 C_{ab} x^b - D_a \right) - \frac{2}{x^s - \bar{x}^s} \left( p^0 C_{ab} x^b x^c - D_a x^s - C_{ab} x^b p^s \right) \]

\[ \quad - \frac{2M_a}{M} \left( p^0 C_{bc} x^b x^c - 2D_b x^b \right) + \frac{2M_a}{M} \frac{W}{x^s - \bar{x}^s}. \]

where, for convenience, we have introduced the notations \( D_a = C_{ab} p^b \), \( D = C_{ab} p^a p^b \) and \( C^{ab} C_{bc} = \delta^a_c \). To find the attractor point(s), we can now substitute the above expressions in the equations of motion:

\[ g^{bc} \nabla_a \nabla_b W \nabla_c W + 2 \nabla_a W W + \partial_a g^{bc} \nabla_b W \nabla_c W + g^{gs} \nabla_a \nabla_s W \nabla_s W = 0, \]

\[ g^{bc} \nabla_s \nabla_b W \nabla_c W + g^{gs} \nabla_s \nabla_a W \nabla_s W + 2 \nabla_s W W + \partial_s g^{gs} \nabla_s W = 0. \quad (3.5) \]

Substituting the expressions for the individual terms, the above equations become extremely complicated and it is hard to solve them in general without making any further assumption. Taking a clue from the supersymmetry preserving solutions we will first consider the simplest ansatz: \( x^s = p^0 t, x^s = p^t \) (with \( t = t_1 + it_2 \)) to solve the equation of motion (3.5). The solution has been carried out in Ref. [6]. In the following we will briefly summarize the results.

We notice that the equations take a particularly simple form after a rescaling of the quantities \( p^0, q_0 \) and \( t \) by \( \tilde{p}^0 \sqrt{Dp^s}, \tilde{q}_0/(\tilde{p}^0)^2 \) and \( i/\tilde{p}^0 \sqrt{M} \) respectively. The resulting equations depend only on a single parameter \((\tilde{q}_0)\) and we find:

\[ A_2 (B_1 - B_2) + B_2 (A_1 - A_2) = 0 \]

\[ A_2 (A_1 + A_2) - B_2 (B_1 + B_2) = 0 \quad (3.6) \]

where for convenience, we have used the following notations:

\[ A_1 = (\tilde{q}_0^2 - 3\tilde{t}_1^2 + \tilde{t}_3^2 + 3\tilde{t}_2^2 - 3\tilde{t}_1 \tilde{t}_2^2) \]

\[ 1 \text{ Our notations for } M_a, M, D_a \text{ and } D \text{ here are slightly different from the ones introduced in [6].} \]
The solution for which $\nabla_a W \neq 0$ is given by

$$\tilde{t}_1 = 1 - \frac{u^{1/3} + u}{1 + u^{1/3}}$$ (3.8)

$$\tilde{t}_2 = \frac{(u^2 - 1)}{u^{1/3}(1 + u^{1/3})}$$ (3.9)

where $u$ is defined by the relation $\tilde{q}_0 u + (u - 1)^2 = 0$.

There is no particular reason for us to consider the above ansatz except for its simplicity.\footnote{For example, this ansatz plays no role in the case of non-supersymmetric attractors with vanishing central charge \cite{18}.}

However, in the present case, the scalar field $x^s$ plays a special role. Thus, it is natural to consider the ansatz $x^a = \rho^a t$ and $x^s = \rho^s j$. The previous equations can be obtained by setting $j = t$. In the following we will proceed by treating $t$ and $j$ independent. To simplify our analysis we will again rescale the charges as well as the variables as in the previous case. To this end we set $\rho^0, q_0, t$ and $j$ as $\tilde{\rho}^0 \sqrt{D \rho^s}, \tilde{q}_0/(\tilde{\rho}^0)^2, \tilde{t}/\tilde{\rho}^0 \sqrt{\tilde{\rho}^s}$ and $\tilde{j}/\tilde{\rho}^0 \sqrt{\tilde{\rho}^s}$ respectively. Once more we find that the equations are dependent only upon the charge $\tilde{q}_0$. After some simplification (as outlined in the appendix §A) we find the equations of motion to take the form:

$$X_2(Y_3 - Y_1) + Y_2(X_3 - X_1) = 0,$$

$$X_2(X_1 + X_3) - Y_2(Y_1 + Y_3) = 0,$$

$$2X_2Y_2 - (X_1Y_3 + X_3Y_1) = 0,$$

$$X_3^2 - Y_2^2 + X_1X_3 - Y_1Y_3 = 0,$$ (3.10)

where we have introduced the quantities $X_i, Y_i, (i = 1, 2, 3)$ as follows:

$$\tilde{p}_0^2 X_1 = \tilde{q}_0 - t_1^2 (j_1 - 1) + \tilde{t}_1 \{ \tilde{t}_1 (j_1 - 1) - 2 \tilde{j}_1 \} - 2 \tilde{t}_2 \tilde{j}_2 (\tilde{t}_1 - 1),$$

$$\tilde{p}_0^2 Y_1 = 2 \tilde{t}_2 \{ \tilde{t}_1 (\tilde{t}_1 - 1) - \tilde{t}_1 \} + j_2 \{ \tilde{t}_1 (\tilde{t}_1 - 2) - t_2^2 \},$$

$$\tilde{p}_0^2 X_2 = \tilde{q}_0 + t_2^2 (j_2 - 1) + \tilde{t}_1 \{ \tilde{t}_1 (j_1 - 1) - 2 \tilde{j}_1 \},$$

$$\tilde{p}_0^2 Y_2 = - \tilde{j}_2 \{ \tilde{t}_1 (\tilde{t}_1 - 2) + t_2^2 \},$$

$$\tilde{p}_0^2 X_3 = \tilde{q}_0 - t_2^2 (j_1 - 1) + \tilde{t}_1 \{ \tilde{t}_1 (j_1 - 1) - 2 \tilde{j}_1 \} + 2 \tilde{t}_2 \tilde{j}_2 (\tilde{t}_1 - 1),$$

$$\tilde{p}_0^2 Y_3 = \tilde{j}_2 \{ \tilde{t}_1 (\tilde{t}_1 - 2) - t_2^2 \} - 2 \tilde{t}_2 \{ \tilde{t}_1 (\tilde{t}_1 - 1) - \tilde{t}_1 \}.$$ (3.11)
As expected, we recover Eqs. (3.6) upon setting $\tilde{j} = \tilde{t}$. However, there is a possibility of obtaining genuinely new solutions from the above equations by treating $\tilde{j}$ and $\tilde{t}$ as independent variables. It is straightforward to solve Eq. (3.10) to find both supersymmetric as well non-supersymmetric solutions. The supersymmetric solution obtained from these equations are identical to the one obtained from Eq. (3.6). The non-supersymmetric solution is given by

$$\tilde{t}_1 = \frac{\tilde{q}_0}{2}, \quad \tilde{t}_2 = \frac{1}{2} \sqrt{\tilde{q}_0 (\tilde{q}_0 - 4)}, \quad \tilde{j}_1 = \frac{\tilde{q}_0 (\tilde{q}_0 - 3)}{2 + \tilde{q}_0 (\tilde{q}_0 - 4)}, \quad \tilde{j}_2 = \frac{\sqrt{\tilde{q}_0 (\tilde{q}_0 - 4)}}{(2 + \tilde{q}_0 (\tilde{q}_0 - 4))}.$$ (3.12)

In terms of the original variables before rescaling this solution takes the form:

$$t = \frac{p^0 q_0}{2 D p^s} + i \frac{\sqrt{q_0 (p^0 q_0^2 - 4 D p^s)}}{2 D p^s},$$ (3.13)

$$j = \frac{p^0 q_0 (p^0 q_0^2 - 3 D p^s)}{2 D^2 p^s - 4 D p^s p^0 q_0 + p^0 q_0^2} + i \frac{D p^s \sqrt{q_0 (p^0 q_0^2 - 4 D p^s)}}{2 D^2 p^s p^0 q_0 + p^0 q_0^2}.$$ (3.14)

Note that the above solution as well as the one given in Eq. (3.8) exist in the domain of the charge lattice for which $(p^0 q_0^2 - 4 D p^s q_0) > 0$. In contrast, the supersymmetric attractors exist in the mutually exclusive domain $(p^0 q_0^2 - 4 D p^s q_0) < 0$.

The entropy of the black hole can be computed from the formula $S = \pi V_0$ where $V_0$ is the value of the effective black hole potential at the attractor point. We find

$$S = \pi \sqrt{(p^0 q_0^2 - 4 D p^s q_0)},$$ (3.15)

which, interestingly, is also identical to the entropy of the non-supersymmetric attractor given in Eq. (3.8).

Clearly this is a distinct solution than the one described in Eq. (3.8) and hence we have multiple single centered attractors having the same charge configuration. The existence of multiple attractors with the same value of charges is not a surprise [19]. There exists even multiple multi-centered black holes [20], and one can transit from one configuration of multi-centered black hole to the other by crossing a wall of marginal stability. The walls of marginal stability are determined by the corresponding entropies [21]. What we find surprising is that the expression of entropy is the same for both the single centered solutions. Hence, in the present case, there exists no wall of marginal stability in the moduli space to determine the center to which the scalar field flow when one starts from a specific point in the moduli space.
One criteria which might distinguish the solutions form one another is stability. For the first solution \((3.8)\), there exist \((n+1)\) massive modes and the remaining \((n-1)\) fields become zero modes. For the solution \((3.12)\) we need to compute the mass matrix and diagonalize it to find the stable directions.

The computation of the mass matrix has been discussed in detail in appendix §B. Using the explicit expressions for the elements of the matrices \(\Sigma_0, \Sigma_1, \Sigma_2\) and \(\Sigma_3\), the mass matrix is found to have the form:

\[
M = e^{K_0} 8 \left( p^{02} q_0^2 - 4 D p^s q_0 \right) \frac{1}{\alpha D} \left( \begin{array}{ccc}
\frac{4 D_a D_d}{D} & 2 D_a & \beta D_a p^s \\
\frac{2D_a}{p^s} & D \frac{\alpha^2}{4p^s} & 0 \\
0 & 2 \left( \frac{2D_a D_d}{D} - C_{ad} \right) & 0 \\
\beta D_a p^s & 0 & D \frac{\alpha^2}{4p^s} 
\end{array} \right), \tag{3.16}
\]

where we have used the notations:

\[
\alpha = \frac{2D^2 p^s - 4 D p^s p^{04} q_0^2}{(D p^s)^2} \quad \text{and} \quad \beta = \frac{(p^{02} q_0^2 - 2 D p^s) \sqrt{(p^{04} q_0^2 - 4 D p^s p^{02} q_0^2)}}{(D p^s)^2}.
\]

Note that, for the non-supersymmetric solution, the numerator in the pre-factor of the mass matrix is positive (which also implies that \(\alpha > 0\)). We now need to find the eigenvalues of the above matrix. By a trivial change of basis the above mass matrix can be brought into a block diagonal form of the type

\[
8 e^{K_0} \left( p^{02} q_0^2 - 4 D p^s q_0 \right) \frac{1}{\alpha D} \left( \begin{array}{cc}
M_u & 0 \\
0 & M_d
\end{array} \right)
\]

where the \((n-1) \times (n-1)\) matrix \((1/\alpha D)M_u\) is given by a positive multiple of the moduli space metric of \(K3\) and hence it possesses \((n-1)\) positive eigenvalues for any smooth \(K3\) surface. On the other hand, the \((n+1) \times (n+1)\) matrix \(M_d\) is given by

\[
M_d = \left( \begin{array}{ccc}
\frac{4 D_a D_d}{D} & 2 D_a & \beta D_a p^s \\
\frac{2D_a}{p^s} & D \frac{\alpha^2}{4p^s} & 0 \\
\beta D_a p^s & 0 & D \frac{\alpha^2}{4p^s}
\end{array} \right). \tag{3.17}
\]

We will now obtain the eigenvalues of this matrix. Consider first a vector of the form:

\[
\begin{pmatrix}
0 \\
\varphi_1 \\
\varphi_2
\end{pmatrix}
\]
This will be an eigenvector with eigenvalue \( D(\alpha/2p^s)^2 \) provided \( 2\varphi_1 + \beta\varphi_2 = 0 \). Vectors of the form
\[
\begin{pmatrix}
\omega_a \\
0 \\
0
\end{pmatrix}
\]
will be eigenfunctions with zero eigenvalues, provided \( D_a\omega_a = 0 \). Since \( a = 1, \ldots, (n - 2) \), there will be \((n - 2)\) such linearly independent vectors with zero eigenvalue. Finally, consider vectors of the form
\[
\begin{pmatrix}
\omega_a \\
\varphi_1 \\
\varphi_2
\end{pmatrix}
\]
Vectors of this type with \( \varphi_1 \neq 0, \varphi_2 \neq 0 \) will satisfy the eigenvalue equation with eigenvalue \( \lambda \) provided
\[
\begin{align*}
\omega_a &= \frac{D_a}{D}D_aD_a + \frac{1}{p^s}(2\varphi_1 + \beta\varphi_2) \\
\lambda &= \frac{\alpha}{2p^s}D_aD_a + \frac{\varphi_1}{p^s}(2\varphi_1 + \beta\varphi_2) \\
\frac{\beta}{p^s}D_aD_a + D \left( \frac{\alpha}{2p^s} \right)^2 \varphi_2 &= \frac{4\varphi_2}{D}D_aD_a + \frac{\varphi_2}{p^s}(2\varphi_1 + \beta\varphi_2)
\end{align*}
\] (3.18)
The last two of the above equations are compatible with each other if and only if \( \varphi_2 = (\beta/2)\varphi_1 \). Substituting for the above value of \( \varphi_2 \) we get a quadratic equation for \( \varphi_1 \) which admits the solutions
\[
\varphi_{\pm} = -\frac{8p^s}{D\alpha^2}D_aD_a, \frac{D}{2p^s},
\]
with the respective eigenvalues \( \lambda_\mp = 0, ((4/D)D_aD_a + D(\alpha/2p^s)^2) \). Thus the matrix \((1/\alpha D)M_d\) has \((n - 1)\) zero eigenvalues and two nonzero eigenvalues: \((\alpha/(2p^s))^2\) and \((\alpha/\alpha)(2/D)^2D_aD_a + (\alpha/2p^s)^2\), both positive. To summarise, we observe that for the solution (3.12) the mass matrix admits \((n + 1)\) positive eigenvalues and \((n - 1)\) zero eigenvalues. Thus, neither the entropy nor the number of zero modes distinguishes these two attractors from one another.

Before closing this session, we would like to point out that the appearance of multiple non-supersymmetric attractors seems to be a distinctive feature of the \( D0 - D4 - D6 \) configuration. Upon setting the \( D6 \) charge to zero in Eqs.(3.8) and (3.12), we find identical expression for the \( D0 - D4 \) solution. On the other hand, while the solution (3.8) gives a smooth \( D0 - D6 \) solution in the limit \( p^a, p^s \to 0 \), Eqs.(3.12) becomes singular in this limit. Thus the solution (3.12) exists only for finite nonzero values of \( D4 \) charges.
4 Conclusion

In this paper we have studied non-supersymmetric attractors in $\mathcal{N} = 2$ supergravity obtained from the compactification of type $IIA$ string theory on a Calabi-Yau manifold. By making some specific assumption on the intersection numbers we studied the attractor equations for spherically symmetric, extremal black holes. We found two distinct single centered attractors with the same charge configurations. These multiple solutions with the same charge configurations share many common properties. In particular, we found that the entropy for the corresponding black holes are the same and also they share the same number of zero modes.

In the case of (supersymmetric) multi-centered black holes [10,20], the moduli space is divided in to several domains which are separated by walls of marginal stability with each of the domains corresponding to a particular multi-centered black hole [21]. The expression for the entropy determines the walls of marginal stability. Thus, it is surprising to find that the expression for the entropy is the same for both the attractor solutions. In the present case, in order to understand the domains in the moduli space corresponding to each of the centers, we need to obtain the full radial flow for both the solutions. There are few more issues that deserves future study. It would be interesting to explore the existence of multiple attractor points for a more general Calabi-Yau manifold without imposing any restriction on the intersection numbers. A related issue is to understand the full set of non-supersymmetric attractor points for a given charge configuration in a particular Calabi-Yau compactification. It would also be interesting to see what happens to the new solution we find when stringy corrections are included. Even for the simplest case of $D0 - D4$ black holes the stringy correction introduces richer space of attractor solutions [8]. The stability conditions also change in an interesting way [9,10]. Adding $D6$ branes to this configuration will certainly enhance this already rich structure of non-supersymmetric attractors as we now have a new ansatz to explore the solutions. We hope to study some of these issues in near future.

5 Acknowledgment

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A The Attractor Equation

In this appendix, we will outline some of the computational details required to find the non-supersymmetric attractor when we use the ansatz:

\[ x^a = p^a t, \quad x^s = p^s j. \]  

(A.1)

We denote \( t_1, j_1 \) to be the real parts and \( t_2, j_2 \) to be the imaginary parts of \( t, j \) respectively. With this ansatz, the derivatives of the Kähler potential are given by:

\[ \partial_a K = iD_a \partial_{t_2}, \quad \partial_s K = i \frac{t_2}{2p^s j_2} . \]  

(A.2)

The metric on the moduli space can be shown to have the form

\[ g_{ab} = \frac{1}{2D_{t_2}^2} \left( 2 \frac{D_a D_b}{D} - C_{ab} \right), \]  

(A.3)

\[ g_{ss} = \frac{1}{(2p^s j_2)^2}, \quad g_{as} = 0, \]  

(A.4)

with its inverse

\[ g^{a\bar{b}} = 2t_2^2 D \left( \frac{2}{D} p^a p^b - C^{ab} \right), \]  

(A.5)

\[ g^{s\bar{s}} = (2p^s j_2)^2, \quad g^{a\bar{s}} = 0. \]  

(A.6)

In the above we have introduced the notation \( D_a = C_{ab} p^b, \quad D = C_{ab} p^a p^b \) and \( C^{ab} C_{bc} = \delta^a_c \). We also compute derivatives of the metric, which will be of use in evaluating the equations of motion:

\[ \partial_a g^{bc} = 2it_2 (D_a C^{bc} - \delta^b_a \delta^c p^b - \delta^c_a \delta^b p^b) \]  

(A.7)

\[ \partial_s g^{s\bar{s}} = -4ip^s j_2. \]  

(A.8)

Using the above mentioned ansatz, we can simplify the superpotential and its covariant derivatives and express them in terms of the rescaled variables. We find:

\[ W = X_1 + iY_1, \]

\[ \nabla_a W = \frac{p^a p^s}{2t_2 \sqrt{D p^s}} D_a (Y_2 + iX_2), \]

\[ \nabla_s W = \frac{p^s}{2t_2 \sqrt{D p^s}} D(Y_3 + iX_3), \]

\[ \nabla_a \nabla_b W = \frac{(p^0)^2 p^s}{2t_2^2} \left[ \left\{ 2 \left( C_{ab} - \frac{2D_a D_b}{D} \right) X_2 - C_{ab} X_3 \right\} - i \left\{ 2 \left( C_{ab} - \frac{2D_a D_b}{D} \right) Y_2 + C_{ab} Y_3 \right\} \right], \]

\[ \nabla_a \nabla_s W = -\frac{(p^0)^2}{2t_2 j_2} D_a (X_2 + iY_2), \]

\[ \nabla_s \nabla_s W = -\frac{(p^0)^2}{2p^s j_2^2} D(X_3 - iY_3). \]  

(A.9)
The expressions for $X_i$ and $Y_i, (i = 1, \cdots, 3)$ are defined in Eq. (3.11). The individual terms in
the equation of motion can now be computed as follows:

\[
g^{bc} \nabla_a \nabla_b W \nabla_c \overline{W} = -\frac{g^{0p}p^sD_a}{t_2\sqrt{dp^s}}(Y_2 - iX_2) \left[(2X_2 + X_3) - i(2Y_2 - Y_3)\right],
\]

\[
2\nabla_a W \nabla_b W = \frac{2g^{0p}p^sD_a}{t_2\sqrt{dp^s}}(Y_2 + iX_2)(X_1 - iY_1),
\]

\[
\partial_a g^{bc} \nabla_b W \nabla_c W = \frac{-2i}{t_2\sqrt{dp^s}}(X_2^2 + Y_2^2),
\]

\[
g^{ss} \nabla_a \nabla_s W \nabla_s \overline{W} = \frac{-g^{0p}p^sD_a}{t_2\sqrt{dp^s}}(X_2 + iY_2)(Y_3 - iX_3);
\]

and

\[
g^{bc} \nabla_s \nabla_b W \nabla_c \overline{W} = \frac{i}{t_2\sqrt{dp^s}}(X_2 + iY_2)^2,
\]

\[
g^{ss} \nabla_s \nabla_s W \nabla_s \overline{W} = \frac{i}{t_2\sqrt{dp^s}}(X_3^2 + Y_3^2),
\]

\[
2\nabla_s W \nabla_s W = \frac{2g^{0p}p^sD_a}{t_2\sqrt{dp^s}}(X_3 - iY_3)(X_1 - iY_1),
\]

\[
\partial_s g^{ss} \nabla_s W \nabla_s \overline{W} = \frac{-i}{t_2\sqrt{dp^s}}(X_3^2 + Y_3^2).
\]

Adding the above terms and simplifying we find the equations of motion as given in (3.10).

**B The Mass Matrix**

In this appendix we will evaluate the mass matrix for our attractor solution. Expanding the
effective black hole potential around the attractor point, we find the quadratic terms to be of the form:

\[
2\partial_a \partial_d V \left(y_1^a y_1^d + y_2^a y_2^d\right) + 4Re(\partial_a \partial_s V) \left(y_1^a y_1^s + y_2^a y_2^s\right) - 4Im(\partial_a \partial_s V) \left(y_2^a y_1^s - y_1^a y_2^s\right)
\]

\[
+ 2\partial_s \partial_d V \left((y_1^s)^2 + (y_2^s)^2\right) + 2Re(\partial_s \partial_d V) \left(y_1^a y_1^d - y_2^a y_2^d\right) - 2Im(\partial_s \partial_d V) \left(y_2^a y_1^d + y_1^a y_2^d\right)
\]

\[
+ 4Re(\partial_a \partial_s V) \left(y_1^a y_1^s - y_2^a y_2^s\right) - 4Im(\partial_a \partial_s V) \left(y_2^a y_1^s + y_1^a y_2^s\right),
\]

(B.1)

where, we set $x^a = x_0^a + y_1^a + iy_2^a$ and $x^s = x_0^s + y_1^s + iy_2^s$. From the above we notice that the
mass matrix can be recast as

\[
M = I \otimes \Sigma_0 - \sigma_1 \otimes \Sigma_1 + i\sigma_2 \otimes \Sigma_2 + \sigma_3 \otimes \Sigma_3,
\]

(B.2)
where, $\sigma_1$, $\sigma_2$ and $\sigma_3$ are the Pauli matrices and the $n \times n$ matrices $\Sigma_i$ $(i = 0, \cdots, 3)$ are given by

$$
\Sigma_0 = \begin{pmatrix}
2\partial_b\partial_dV & 2\text{Re}(\partial_b\partial_sV)
\end{pmatrix},
\Sigma_1 = \begin{pmatrix}
2\text{Im}(\partial_a\partial_dV)
\end{pmatrix},
\Sigma_2 = \begin{pmatrix}
0
\end{pmatrix},
\Sigma_3 = \begin{pmatrix}
2\text{Re}(\partial_b\partial_dV)
\end{pmatrix}.
$$

In order to obtain the mass matrix, we need to evaluate each of these terms at the attractor point. In the following we will outline some of the intermediate steps in evaluating the mass matrix. A straightforward differentiation gives the second derivative terms of the effective black hole potential as [6]:

$$
e^{-K_0}\partial_a\partial_dV = g^{bc}\nabla_b\nabla_c W \nabla_d W + g^{ss}\nabla_s \nabla_s W \nabla_s W + \partial_a g^{bc}\nabla_b \nabla_c W \nabla_d W 
+ \partial_d g^{bc}\nabla_b \nabla_c W + 3\nabla_a \nabla_d W \nabla_s W + \partial_a \partial_d g^{bc}\nabla_b \nabla_s W \nabla_s W,
$$

$$
e^{-K_0}\partial_a\partial_sV = g^{bc}\nabla_b \nabla_s W \nabla_a W + g^{ss}\nabla_s \nabla_s W \nabla_s W + \partial_a g^{bc}\nabla_b \nabla_s W \nabla_s W 
+ \partial_s g^{bc}\nabla_b \nabla_s W \nabla_s W + 3\nabla_a \nabla_s W \nabla_s W,
$$

$$
e^{-K_0}\partial_b\partial_dV = g^{bc}\nabla_b W \nabla_c W \nabla_d W + g^{ss}\nabla_s \nabla_s W \nabla_s W + 2|W|^2 g_{ad} 
+ \partial_a g^{bc}\nabla_b \nabla_c W \nabla_d W 
+ \partial_d g^{bc}\nabla_b \nabla_c W + 3\nabla_a \nabla_d W \nabla_s W + \partial_a \partial_d g^{bc}\nabla_b \nabla_s W \nabla_s W,
$$

$$
e^{-K_0}\partial_b\partial_sV = g^{bc}\nabla_b \nabla_s W \nabla_c W + g^{ss}\nabla_s \nabla_s W \nabla_s W + \partial_a g^{bc}\nabla_b \nabla_s W \nabla_s W 
+ \partial_s g^{bc}\nabla_b \nabla_s W \nabla_s W + 3\nabla_a \nabla_s W \nabla_s W,
$$

$$
e^{-K_0}\partial_s\partial_dV = g^{bc}\nabla_s \nabla_b W \nabla_c W \nabla_d W + g^{ss}\nabla_s \nabla_s W \nabla_s W + 2|W|^2 g_{ss} 
+ \partial_a g^{ss}\nabla_s \nabla_s W \nabla_s W + \partial_s g^{ss}\nabla_s \nabla_s W \nabla_s W + 4\nabla_s \nabla_s W \nabla_s W + \partial_s \partial_s g^{ss}\nabla_s \nabla_s W \nabla_s W,
$$

$$
e^{-K_0}\partial_s\partial_sV = g^{bc}\nabla_s \nabla_s W \nabla_c W + g^{ss}\nabla_s \nabla_s W \nabla_s W + 2\partial_a g^{ss}\nabla_s \nabla_s W \nabla_s W 
+ 3\nabla_s \nabla_s W + \partial_a \partial_a g^{ss}\nabla_s \nabla_s W \nabla_s W - g^{ss}\nabla_s \nabla_s W \nabla_s W
$$

(B.3)

where $K_0$ is the value of the Kähler potential at the attractor point.

We will now evaluate each of the above expressions separately. The individual terms in
\[ e^{-K_0 \partial_a \partial_d V} \text{ are given by} \]

\[
g^{bc} \nabla_a \nabla_b \nabla_d W \nabla_c W = 2 \left( \frac{(p^0)^2 p^s}{t_2^2} \right) (Y_2 - iX_2) \left[ \left\{ \frac{1}{2} C_{ad}(Y_3 + 3Y_2) + \frac{D_aD_d}{D} (Y_3 - 3Y_2) \right\} \right.
\]

\[
- i \left\{ \frac{1}{2} C_{ad}(X_3 - 3X_2) + \frac{D_aD_d}{D} (X_3 + 3X_2) \right\},
\]

\[
g^{gs} \nabla_a \nabla_s \nabla_d W \nabla_s W = - \frac{1}{2} \left( \frac{(p^0)^2 p^s}{t_2^2} \right) (Y_3 - iX_3) \left[ \left\{ 2 \left( C_{ad} - 2 \frac{D_aD_d}{D} \right) Y_2 + C_{ad}Y_1 \right\} \right.
\]

\[
- i \left\{ 2 \left( C_{ad} - 2 \frac{D_aD_d}{D} \right) X_2 - C_{ad}X_1 \right\},
\]

\[
\partial_a g^{bc} \nabla_b \nabla_d W \nabla_c W = \partial_d g^{bc} \nabla_b \nabla_a W \nabla_c W = - \left( \frac{(p^0)^2 p^s}{t_2^2} \right) (X_2 + iY_2)
\]

\[
\left[ \left\{ 2 \left( C_{ad} - 2 \frac{D_aD_d}{D} \right) X_2 - C_{ad}X_3 \right\} - i \left\{ 2 \left( C_{ad} - 2 \frac{D_aD_d}{D} \right) Y_2 + C_{ad}Y_3 \right\} \right],
\]

\[
3 \nabla_a \nabla_d W \nabla_c W = \frac{3}{2} \left( \frac{(p^0)^2 p^s}{t_2^2} \right) (X_1 - iY_1) \left[ \left\{ 2 \left( C_{ad} - 2 \frac{D_aD_d}{D} \right) X_2 - C_{ad}X_3 \right\} \right.
\]

\[
- i \left\{ 2 \left( C_{ad} - 2 \frac{D_aD_d}{D} \right) Y_2 + C_{ad}Y_3 \right\},
\]

\[
\partial_a \partial_d g^{bc} \nabla_b W \nabla_c W = \frac{(p^0)^2 p^s}{t_2^2} \left( C_{ad} - 2 \frac{D_aD_d}{D} \right) (X_2^2 + Y_2^2),
\]

\[
- g^{bc} \partial_a g_{de} \nabla_b W \nabla_c W = i \frac{(p^0)^2 p^s}{t_2^2} \left( C_{ad} - 2 \frac{D_aD_d}{D} \right) (Y_2 + iX_2) (X_1 - iY_1). \tag{B.4}
\]

Adding all these terms, we find a simple expression for \( e^{-K_0 \partial_a \partial_d V} \) which is listed towards the end of this section. Similarly, we will now evaluate the remaining terms in the second derivatives of the potential. Individual terms in \( e^{-K_0 \partial_a \partial_s V} \) are evaluated to be

\[
g^{bc} \nabla_a \nabla_b \nabla_s W \nabla_c W
\]

\[
= - \left( \frac{(p^0)^2}{2t_2j_2} \right) D_a \left[ \{2 (X_2^2 - Y_2^2) + X_1X_2 + Y_1Y_2 \} + i\{4X_2Y_2 + X_1Y_2 - X_2Y_1 \} \right],
\]

\[
g^{gs} \nabla_a \nabla_s \nabla_d W \nabla_s W = - \left( \frac{(p^0)^2}{t_2j_2} \right) D_a \left[ X_2X_3 - Y_2Y_3 + i \right. 
\]

\[
\left. \left( X_2Y_3 + X_3Y_2 \right) \right],
\]

\[
\partial_a g^{bc} \nabla_b \nabla_s W \nabla_c W = \left( \frac{(p^0)^2}{t_2j_2} \right) D_a \left( X_2^2 - Y_2^2 + 2iX_2Y_2 \right),
\]

\[
\partial_s g^{gs} \nabla_s \nabla_a W \nabla_s W = \left( \frac{(p^0)^2}{t_2j_2} \right) D_a \left[ (X_2X_3 - Y_2Y_3) + i \right. 
\]

\[
\left. (X_2Y_3 + X_3Y_2) \right],
\]

\[
3 \nabla_a \nabla_s W = - \frac{3}{2} \left( \frac{(p^0)^2}{t_2j_2} \right) D_a \left[ (X_1X_2 + Y_1Y_2) + i \right. 
\]

\[
\left. (X_1Y_2 - X_2Y_1) \right]. \tag{B.5}
\]

Addition of all these terms gives the value of \( e^{-K_0 \partial_a \partial_s V} \). Individual terms in \( e^{-K_0 \partial_s^2 V} \) are given
by
\[ g^{bc} \nabla_b \nabla_c W \nabla_d W = \frac{(\tilde{p}^0)^2 D}{p^* j_2^2} (Y_2^2 - X_2^2 - 2iX_2Y_2), \]
\[ g^{ss} \nabla_s \nabla_s W \nabla_s W = -3 \frac{(\tilde{p}^0)^2 D}{2 p^* j_2^2} (X_3^2 + Y_3^2), \]
\[ 2 \partial_s g^{ss} \nabla_s \nabla_s W \nabla_s W = 2 \frac{(\tilde{p}^0)^2 D}{p^* j_2^2} (X_3^2 + Y_3^2), \]
\[ 3 \nabla_s \nabla_s W \nabla_s W = -3 \frac{(\tilde{p}^0)^2 D}{p^* j_2^2} [(X_1X_3 - Y_1Y_3) - i (X_1Y_3 + X_3Y_1)], \]
\[ \partial_s^2 g^{ss} \nabla_s \nabla_s W \nabla_s W = - \frac{(\tilde{p}^0)^2 D}{2 p^* j_2^2} (X_3^2 + Y_3^2), \]
\[ -g^{ss} \partial_s g_{ss} \nabla_s W \nabla_s W = \frac{(\tilde{p}^0)^2 D}{2 p^* j_2^2} [(X_1X_3 - Y_1Y_3) - i (X_1Y_3 + X_3Y_1)]. \] (B.6)

From the above we find that \( e^{-K_0} \partial_d V \) vanished upon using the equations of motion. \( e^{-K_0} \partial_d V \) contains the following terms

\[ g^{bc} \nabla_b \nabla_c \nabla_b W \nabla_c W \nabla_d W = \frac{(\tilde{p}^0)^2 p^s}{2t^2} \left[ - \left( C_{ad} - 2 \frac{D_a D_d}{D} \right) \left( 4X_2^2 + 4Y_2^2 + X_3^2 + Y_3^2 \right) + 4C_{ad} (X_2X_3 - Y_2Y_3) \right], \]
\[ g^{ss} \nabla_s \nabla_s \nabla_s W \nabla_s W \nabla_s W = \frac{(\tilde{p}^0)^2 p^s}{t^2} \left( C_{ad} - 2 \frac{D_a D_d}{D} \right) (X_2^2 + Y_2^2), \]
\[ 2 |W|^2 g_{ad} = - \frac{(\tilde{p}^0)^2 p^s}{t^2} \left( C_{ad} - 2 \frac{D_a D_d}{D} \right) (X_1^2 + Y_1^2), \]
\[ g^{bc} \nabla_b W \nabla_c W \nabla_d W = - \frac{(\tilde{p}^0)^2 p^s}{t^2} \left( C_{ad} - 2 \frac{D_a D_d}{D} \right) (X_2^2 + Y_2^2), \]
\[ g^{ss} \nabla_s W \nabla_s W \nabla_s W = - \frac{(\tilde{p}^0)^2 p^s}{2t^2} \left( C_{ad} - 2 \frac{D_a D_d}{D} \right) (X_3^2 + Y_3^2), \]
\[ \partial_a g^{bc} \nabla_b W \nabla_c W \nabla_d W = \frac{(\tilde{p}^0)^2 p^s}{t^2} \left[ 2 \left( C_{ad} - 2 \frac{D_a D_d}{D} \right) (X_2^2 + Y_2^2) - C_{ad} (X_2X_3 - Y_2Y_3) \right] \]
\[ + iC_{ad} (X_2X_3 + X_3X_2), \]
\[ \partial_d g^{bc} \nabla_a W \nabla_b W \nabla_c W = \frac{(\tilde{p}^0)^2 p^s}{t^2} \left[ 2 \left( C_{ad} - 2 \frac{D_a D_d}{D} \right) (X_2^2 + Y_2^2) - C_{ad} (X_2X_3 - Y_2Y_3) \right] \]
\[ - iC_{ad} (X_2X_3 + X_3X_2), \]
\[ 3 \nabla_a W \nabla_d W = 3 \frac{(\tilde{p}^0)^2 p^s D_a D_d}{D} (X_2^2 + Y_2^2), \]
\[ \nabla_a \nabla_d g^{bc} \nabla_b W \nabla_c W = - \frac{(\tilde{p}^0)^2 p^s}{t^2} \left( C_{ad} - 2 \frac{D_a D_d}{D} \right) (X_2^2 + Y_2^2). \] (B.7)
Summing up the above terms and using equations of motion, we obtain a simple expression for \( e^{-K_0 \partial_a \partial_d V} \). Similarly, the second derivative \( e^{-K_0 \partial_a \partial_b V} \) has the following terms

\[
g^{bc} \nabla_a \nabla_b W \nabla_c \nabla_s W = \frac{(p^0)^2}{t_{2,j2}} D_a \left[ \left\{ 2 \left( X_2^2 - Y_2^2 \right) + X_2 X_3 + Y_2 Y_3 \right\} - i \left( 4X_2 Y_2 - X_2 Y_3 + X_3 Y_2 \right) \right],
\]

\[
g^{ss} \nabla_a \nabla_s W \nabla_s \nabla_s W = \frac{(p^0)^2}{j_{2,p}^s} D_a \left[ \left\{ X_2 X_3 - Y_2 Y_3 \right\} + i \left( X_2 Y_3 + X_3 Y_2 \right) \right],
\]

\[
\partial_a g^{bc} \nabla_b W \nabla_c \nabla_s W = \frac{(p^0)^2}{t_{2,j2}} D_a \left( Y_2^2 - X_2^2 + 2iX_2 Y_2 \right),
\]

\[
\partial_s g^{ss} \nabla_a \nabla_s W \nabla_s W = -\frac{(p^0)^2}{j_{2,p}^s} D_a \left[ \left\{ X_2 X_3 - Y_2 Y_3 \right\} + i \left( X_2 Y_3 + X_3 Y_2 \right) \right],
\]

\[
3 \nabla_a W \nabla_s W = \frac{3(p^0)^2}{2} D_a \left[ \left\{ X_2 X_3 + Y_2 Y_3 \right\} + i \left( X_2 Y_3 - X_3 Y_2 \right) \right]. \tag{B.8}
\]

Finally, we evaluate terms in \( e^{-K_0 \partial_s \partial_s V} \):

\[
g^{bc} \nabla_s \nabla_b W \nabla_c \nabla_s W = \frac{(p^0)^2}{2 j_{2,p}^s} D \left( X_2^2 + Y_2^2 \right),
\]

\[
g^{ss} \nabla_s \nabla_s W \nabla_s \nabla_s W = \frac{(p^0)^2}{j_{2,p}^s} D \left( X_3^2 + Y_3^2 \right),
\]

\[
2|W|^2 g_{ss} = \frac{(p^0)^2}{2 j_{2,p}^s} D \left( X_1^2 + Y_1^2 \right),
\]

\[
g^{bc} \nabla_b W \nabla_c W g_{ss} = \frac{(p^0)^2}{2 j_{2,p}^s} D \left( X_2^2 + Y_2^2 \right),
\]

\[
\partial_a g^{ss} \nabla_s \nabla_s W \nabla_s W = -\frac{(p^0)^2}{j_{2,p}^s} D \left( X_2^2 + Y_2^2 \right),
\]

\[
\partial_s g^{ss} \nabla_a \nabla_s W \nabla_s W = -\frac{(p^0)^2}{j_{2,p}^s} D \left( X_3^2 + Y_3^2 \right),
\]

\[
4 \nabla_s W \nabla_s W = \frac{(p^0)^2}{j_{2,p}^s} D \left( X_3^2 + Y_3^2 \right),
\]

\[
\partial_a \partial_s g^{ss} \nabla_s W \nabla_s W = \frac{(p^0)^2}{2 j_{2,p}^s} D \left( X_3^2 + Y_3^2 \right). \tag{B.9}
\]

Adding all these terms and using equations of motion, we get an expression for \( e^{-K_0 \partial_a \partial_d V} \).

To summarize, the various terms in the mass matrix are found to be

\[
e^{-K_0 \partial_a \partial_d V} = -2 \frac{(p^0)^2}{t_{2,j2}} p^{s} C_{ad} \left[ \left\{ X_1 X_3 + Y_1 Y_3 \right\} - i \left( X_3 Y_1 - X_1 Y_3 \right) \right].
\]

\[
e^{-K_0 \partial_a \partial_b V} = -2 \frac{(p^0)^2}{t_{2,j2}} D_a \left[ \left\{ X_1 X_2 + Y_1 Y_2 \right\} + i \left( X_1 Y_2 - X_2 Y_1 \right) \right].
\]

\[
e^{-K_0 \partial_s \partial_s V} = 0
\]

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\[ e^{-K_0 \partial_a \partial_d V} = -2\left(\frac{\tilde{p}^0}{t_2}\right)^2 p^s \left(C_{ad} - 4 \frac{D_a D_d}{D}\right) (X_1^2 + Y_1^2). \]
\[ e^{-K_0 \partial_a \partial_s V} = 2\left(\frac{\tilde{p}^0}{t_2 j_2}\right)^2 D_a [(X_2 X_3 + Y_2 Y_3) + i (X_2 Y_3 - X_3 Y_2)]. \]
\[ e^{-K_0 \partial_a \partial_S V} = 2\left(\frac{\tilde{p}^0}{j_2^2 p^s}\right) D (X_1^2 + Y_1^2). \] (B.10)

Note that, for the new solution (3.12), for which we are interested to find the mass matrix, we have \( X_1 = -X_2 = -X_3 \) and \( Y_1 = Y_2 = -Y_3 \). Using this we simplify the above equations to find:

\[ e^{-K_0 \partial_a \partial_d V} = 2\left(\frac{\tilde{p}^0}{t_2}\right)^2 C_{ad} (X_1^2 + Y_1^2), \]
\[ e^{-K_0 \partial_a \partial_s V} = 2\left(\frac{\tilde{p}^0}{t_2 j_2}\right)^2 D_a [(X_1^2 - Y_1^2) - 2i X_1 Y_1], \]
\[ e^{-K_0 \partial_a \partial_s V} = 0, \]
\[ e^{-K_0 \partial_a \partial_d V} = -2\left(\frac{\tilde{p}^0}{t_2 j_2}\right)^2 \left(C_{ad} - 4 \frac{D_a D_d}{D}\right) (X_1^2 + Y_1^2). \]
\[ e^{-K_0 \partial_a \partial_s V} = 2\left(\frac{\tilde{p}^0}{t_2 j_2}\right)^2 D_a [(X_1^2 - Y_1^2) + 2i X_1 Y_1], \]
\[ e^{-K_0 \partial_a \partial_S V} = 2\left(\frac{\tilde{p}^0}{j_2^2 p^s}\right) D (X_1^2 + Y_1^2). \] (B.11)

Using the solution (3.12), finally we get

\[ e^{-K_0 \partial_a \partial_d V} = 4\tilde{q}_0 \frac{p_s}{(\tilde{p}^0)^2} \left(\frac{\tilde{q}_0 - 4}{2 + \tilde{q}_0(\tilde{q}_0 - 4)}\right) C_{ad} \]
\[ e^{-K_0 \partial_a \partial_s V} = 4\tilde{q}_0 \frac{p_s}{(\tilde{p}^0)^2} \left(\frac{\tilde{q}_0 - 4}{2 + \tilde{q}_0(\tilde{q}_0 - 4)}\right) \left(1 - \frac{i}{2} \sqrt{\tilde{q}_0(\tilde{q}_0 - 4)}\right) D_a \]
\[ e^{-K_0 \partial_a \partial_s V} = 0 \]
\[ e^{-K_0 \partial_a \partial_d V} = 4\tilde{q}_0 \frac{p_s}{(\tilde{p}^0)^2} \left(\frac{\tilde{q}_0 - 4}{2 + \tilde{q}_0(\tilde{q}_0 - 4)}\right) \left(4 \frac{D_a D_d}{D} - C_{ad}\right) \]
\[ e^{-K_0 \partial_a \partial_s V} = 4\tilde{q}_0 \frac{p_s}{(\tilde{p}^0)^2} \left(\frac{\tilde{q}_0 - 4}{2 + \tilde{q}_0(\tilde{q}_0 - 4)}\right) \left(1 + \frac{i}{2} \sqrt{\tilde{q}_0(\tilde{q}_0 - 4)}\right) D_a \]
\[ e^{-K_0 \partial_a \partial_S V} = \frac{\tilde{q}_0}{(\tilde{p}^0)^2 p^s}(\tilde{q}_0 - 4)(2 + \tilde{q}_0(\tilde{q}_0 - 4))D \] (B.12)

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