Mechanical Characteristics Analysis of Surrounding Rock on Anchor Bar Reinforcement

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Abstract: Through the homogenization method, the composite of rock and anchor bar is considered as the equivalent material of continuous, homogeneous, isotropic and strength parameter enhancement, which is defined as reinforcement body. On the basis of elasticity, the composite and the reinforcement are analyzed. Based on strengthening theory of surrounding rock and displacement equivalent conditions, the expression of reinforcement body strength parameters and mechanical parameters is deduced. The example calculation shows that the theoretical results are close to the results of Jia-mei Gao⁹, however, closer to the results of FLAC³D numerical simulation, it is proved that the model and surrounding rock reinforcement body theory are reasonable. The model is easy to analyze and calculate, provides a new way for determining reasonable bolt support parameters, can also provides reference for the stability analysis of underground cavern bolting support.

1. Introduction
Due to complex and varied geological conditions of roadway, properties of surrounding rock differ in thousands ways, stability of roadway surrounding rock under the condition of rock bolting is a hot research which has been taken seriously by many scholars. Li and Stillborg¹, Cai², Fahimifar and Soroush³ has analyzed the interaction between the bolt and the surrounding rock from the Angle of mechanical coupling, they vigorously analyzed the shear and axial force of the bolt itself. Pelizza et al⁴ considered that the rock bolting improved the mechanical parameters of surrounding rock, increased the cohesion of surrounding rock and had less influence on the internal friction angle surrounding rock. The above research has achieved great breakthrough, therefore, Rock bolting has been developed rapidly and widely used.

However, there are still a few shortcomings such as the model calculation and the analytical solution expression are relatively complex, and the control theory of roadway surrounding rocks is not completed. Based on the elastic theory, this article analyzes the synergy of surrounding rock and anchor, and obtains the displacement and stress analytical solutions of surrounding rock and reinforcement. Homogenization method is used to establish a model of interaction between surrounding rock and reinforcement. According to the strength enhancement theory of surrounding rock and the displacement equivalent conditions, the expressions of the strength parameters and mechanical parameters of the reinforcement are deduced, and the methods of evaluating the stability of the surrounding rock are proposed. This model is simple and easy to calculate.

2. The concept of Surrounding rock reinforcement body
Excavation of the cavern changes the initial stress state of the surrounding rock and makes the stress
of the surrounding rock redistribute. The anchor bar needs to be placed in the surrounding rock in time to play the role of actively supporting the surrounding rock. In order to analyze the stability of surrounding rock more quickly and conveniently, through homogenization method, the composite of rock and anchor under the high density supporting model of anchor bar is considered as the equivalent material of continuous, homogeneous, isotropic and enhanced strength parameters, which is defined as the surrounding rock reinforcement body, and coordinate deformation with the deep surrounding rock. The surrounding rock is affected by the axial stress and shear stress of the anchor bar. The axial stress of the anchor bar changes the surrounding rock in the anchorage zone from two-way stress state to three-way stress state, which increases the strength parameters of the reinforcement body. Horizontal connection of the anchor bar with the surrounding rock in the anchorage zone can bear the shear force and the bending moment, which improves the mechanical parameters of the reinforcement body.

3. Determine the strength parameters of the reinforcement body

3.1. Basic assumptions
When analyzing the deformation of surrounding rock, the following assumptions are made.

(1) the roadway is round.
(2) the original rock stress is equal to the isostatic pressure (hydrostatic pressure) state.
(3) Surrounding rocks are continuous, homogeneous, isotropic, creep-free or viscous behavior.

3.2. Coordinate deformation of surrounding rock and anchorage body
When the roadway is excavated, the stress redistribution of surrounding rock due to excavation will cause the surrounding rock to squeeze into the cavern, and the deformation of surrounding rock will be inevitably bound by anchor bar, at the same time, the anchor will generate the internal force affected by the surrounding rock. Xian-chun Yao, Ning Li and Yun-sheng Chen took the stress of roadway surrounding rock as the parameter to establish the model, based on the neutral point theory and Mindlin solution, the shear stress and axial force distribution of the anchor bar are obtained[6].

\[
\tau_a(r) = \frac{-\varepsilon S d_b P}{4G R_0} \left(1 - \frac{r - R_0}{L} e^\alpha\right) \quad R_0 < r < R_i
\]
\[
\tau_a(r) = \frac{-\varepsilon S d_b P R_0^2}{4G r^3} + \frac{N}{\pi d_b} \left[t(r - R_i)\right] \cdot \exp\left(-\frac{1}{2} t(r - R_i)^2\right) \quad R_i < r < R_0 + L
\]

Wherein, \(P\) is the initial ground stress, \(R_0\) is the radius of the cavern, \(G_i\) is the shear modulus of rock mass, \(\varepsilon\) is Poisson ratio and elastic modulus of surrounding rock, \(\alpha\) is the attenuation coefficient, the value is \(3E_u d_b / E\), \(L\) is the distance from the anchor neutral point to the center of the cavern, the value is \(R_0 + Le^{-\alpha}\), \(r\) is the distance from the center of the cavern, \(E_i\) is the elastic modulus of anchor bar, \(d_b\) is the anchor bar diameter, \(\alpha\) is the cross-sectional area of the anchor bar, the value is \(\pi d_b^2 / 4\), \(S\) is the influenced range of anchor bar, the value is \(r S, S_1, S_2\) is the axial spacing of the anchoring arrangement, \(S_1\) is the circumferential spacing of the anchor arrangement, \(N\) is the axial force of neutral point of anchor bar, and

\[
N = \int_{R_0}^{R_i} \pi d_b |\tau_a(r)| dr
\]
\[
\tau_c(r) = \frac{P t}{\pi d_b} \left(r - R_0\right) \exp\left[-\frac{1}{2} t(r - R_0)^2\right] \quad F(r) = P t \exp\left[-\frac{1}{2} t(r - R_0)^2\right]
\]

Based on the Mindlin problem and the displacement solution of Kelvin's problem respectively, the elastic solutions of the shear stress and the axial force distribution of the anchored section of a full-length bonded anchor bar are deduced[6], as follows.

\[
\tau_c(r) = \frac{P t}{\pi d_b} \left(r - R_0\right) \exp\left[-\frac{1}{2} t(r - R_0)^2\right] \quad F(r) = P t \exp\left[-\frac{1}{2} t(r - R_0)^2\right]
\]
Where, $P_j$ is the pretension force of anchor bar.

Through the above analysis, we can see that the interfacial shear stress of the anchor bar placed in the surrounding rock and subjected to the pretension force is composed of two parts, one is caused by the deformation of surrounding rock, the other is caused by the pretension force of anchor bar. The interfacial shear stress distribution of the anchor bar can be expressed as,

$$
\begin{align*}
\tau_i(r) &= \tau_a(r) + \tau_c(r) \\
\tau_e(r) &= \tau_a(r) + \tau_c(r) \\
&\text{for } R_0 < r < R_i \\
\end{align*}
$$

(3)

It’s assumed that the anchor bar is distributed symmetrically along the roadway, and apply the support force of the anchor bar on the surrounding rock to anchorage zone in the form of additional volume force, the support force of anchor bar to the surrounding rock is simplified to the axial symmetrical radial bulk force $f(r)$. The volume force of surrounding rock can be obtained by the ratio of the resultant force on the interface of the anchor bar and the volume of this micro segment.

$$
\begin{align*}
f_i(r) &= -\frac{dQ}{dV} = -\frac{nA_s \tau_i(r)}{rS_s L_s} \\
f_e(r) &= -\frac{dQ}{dV} = -\frac{nA_s \tau_e(r)}{rS_s L_s} \\
&\text{for } R_0 < r < R_i \\
&\text{for } R_i < r < R_o + L
\end{align*}
$$

(4)

The support force of the anchor bar to the surrounding rock is equivalent to the additional volume force of the surrounding rock so as to establish the computational model, which is shown in figure 1.

The support force of the anchor bar to the surrounding rock is equivalent to the additional volume force of the surrounding rock so as to establish the computational model, which is shown in figure 1. The equilibrium differential equation, the physical equation and the geometric equation are established for the surrounding rock and the anchorage body respectively. Therefore, the expressions of stress and displacement of each point can be solved according to the boundary conditions, finally, the radial displacement and circumferential stress at the wall are obtained.

The stress and displacement expressions of surrounding rock are as follows:

$$
\begin{align*}
\sigma_r' &= \frac{A'}{r^2} + 2C' \\
\sigma_\theta' &= \frac{A'}{r^2} + 2C' \\
u_r &= \frac{1}{E_s} \left[ -\frac{A'}{r} + 2(1-2u_e)C' \right]
\end{align*}
$$

(5)

The stress and displacement expressions of the anchorage body are as follows:

$$
\begin{align*}
\sigma_r &= \frac{X_i}{(1-u_e)r^2} + \frac{Y_i}{2r^2} + \frac{D_i}{r^2} \\
\sigma_\theta &= \frac{X_i}{(1-u_e)r^2} + \frac{Y_i}{2r^2} + \frac{Z_i}{(1-2u_e)c_i} - \frac{D_i}{r^2} \\
u_r(r) &= \frac{r(1+u_e)}{E_s} \left[ \frac{X_i}{(1-u_e)r^2} + \frac{Y_i}{2r^2} + \frac{Z_i}{(1-2u_e)c_i} - \frac{D_i}{r^2} \right]\end{align*}
$$

(6)

Where $i=1$ represents the state of the wall to the neutral point, $i=2$ represents the state of the neutral point to the distal end of the anchor bar.

$$
\begin{align*}
X_i &= \int r^2 f_i(r) dr \\
Y_i &= \int r^2 f_i(r) dr \\
Z_i &= \int f_i(r) dr
\end{align*}
$$

The boundary condition can be expressed as,
\[
\left\{ \begin{array}{c}
\sigma_{r}'_{j-r_0} = -P \quad (\sigma_{r_2})_{j-R_0} = 0 \quad (\sigma_{r_2})_{j-R_1} = (\sigma_{r_1})_{j-R_1} \\
\sigma_{r}'_{j-R_1} = (\sigma_{r_2})_{j-R_0+L} \quad (u_{r_2})_{j-R_0+L} = (u_{r_1})_{j-R_0+L}
\end{array} \right.
\] (7)

The radial displacement and circumferential stress of the cava wall are solved by the equation (5), (6), (7).

\[
u_i(R_0) = \frac{R_0(1+u_i)}{E_s}\left[2P(u_i-1) + \int_{R_0}^{R_1} f_1(r)dr + \int_{R_0}^{R_1} f_2(r)dr \right]
\]

\[
\sigma_{r1}(R_0) = -2P + \frac{1}{1-u_i}\left[\int_{R_0}^{R_1} f_1(r)dr + \int_{R_0}^{R_1} f_2(r)dr \right]
\] (8)

3.3. Coordinate deformation of surrounding rock and reinforcement body

The common deformation model of surrounding rock and reinforcement is established to calculate the displacement and stress expression of surrounding rock and reinforcement, as shown in figure 2.

The stress and displacement expressions of the reinforcement body can be expressed as,

\[
\sigma_{i} = \frac{A_i}{r^2} + 2C_j \quad \sigma_{i} = -\frac{A_i}{r^2} + 2C_j \quad u_{i} = \frac{1+u_i}{E_j} \left[\frac{A_i}{r} + 2(1-2u_i)C_j r \right] \]

(9)

Where \( j = 3 \) represents the state of the surrounding rock, \( j = 4 \) represents the state of the reinforcement body, and \( E_3 = E_r, \ E_4 = E_s, \ u_3 = u_r, \ u_4 = u_s \).

The boundary condition can be listed as,

\[
(\sigma_{r_3})_{j-R_0} = 0 \quad (\sigma_{r_4})_{j-R_0+L} = (\sigma_{r_4})_{j-R_0+L} = (u_{r_3})_{j-R_0+L} = (u_{r_4})_{j-R_0+L}
\] (10)

The radial displacement and circumferential stress of the cava wall are solved by the equation (9), (10).

\[
\sigma_{r1}(R_0) = \frac{4PE(L+R_0)^3}{\left[(-2u^2-u+1)E_r + (1+u)E_s \right] \left[(L^2+2LR_0)+(2-2u^2)E_s R_0^2 \right]}
\]

\[
u_{r1}(R_0) = \frac{-4PR_0(L+R_0)^3}{\left[(-2u^2-u+1)E_r + (1+u)E_s \right] \left[(L^2+2LR_0)+(2-2u^2)E_s R_0^2 \right]}
\] (11)

Under the same initial stress, based on the same deformation of the wall caused by the coordinated deformation of the surrounding rock and the anchorage body and the coordinated deformation of the surrounding rock and the reinforcement body, the elastic modulus and Poisson ratio of surrounding rock reinforcement body can be obtained by establishing equivalent conditions.

\[
\sigma_{r1}(R_0) = \sigma_{r1}(R_0) \quad \sigma_{r1}(R_0) = \sigma_{r1}(R_0)
\] (12)

4. Determine the mechanical parameters of the reinforcement body

The internal friction angle of the reinforcement body is approximately equal to the internal friction angle of the rock mass\[7\], that is \( \phi = \phi_r, \) \( \phi_r \) is the internal friction angle of surrounding rock.

The increase of the cohesion of the reinforcement body is caused by two aspects, one is the transverse action of the anchor bar to increases the shear strength of the fracture surface, The other part is the preload of the anchor bar to exert certain pressure on the surrounding rock, improve the stress state of the rock mass. The maximum principal stress direction of the anchorage rock mass is perpendicular to the anchor bar, and the angle between the main fracture plane of the anchorage body and the direction of the maximum principal stress is \( \beta = \pi/4 - \phi_r/2 \)\[8\].

In the wedge element of anchor bar and surrounding rock, the cohesion provided by the transverse action of the anchor can be expressed as \( c_n \), the cohesion provided by the preload of the anchor bar can be expressed as \( c_n \).
\[ c_m = \frac{\sigma_y a l^2}{4\sqrt{3}S}, r \cos \beta \]  
\[ c_n = \frac{F(r)}{S}, r \cos \beta \tan \phi \]

Where, \( \sigma_y \) is the anchor bar yield strength.

it can be concluded that the equivalent cohesive force of reinforcement body can be expressed as,

\[ c = \frac{1}{L} \int_{r_0}^{r_0+L} (c_s + c_m + c_n) dr \]

Where, \( c_s \) is the cohesion of surrounding rock.

5. Example analysis

| Table 1. Analysis of parameters |
|-----------------------------|-----------------------------|
| Parameter | Value | Parameter | Value |
| \( P_0 \)/MPa | 8.76 | \( E_b \)/MPa | 206 000 |
| \( R_0 \)/m | 3 | \( d_0 \)/m | 0.020 |
| \( E_s \)/MPa | 1 380 | \( \mu_c \) | 0.6 |
| \( u_s \) | 0.42 | \( S \)/rad | \( \pi/10 \) |
| \( c_s \)/MPa | 1.568 | \( L \)/m | 2 |
| \( \phi_s \)/(°) | 35 | \( P \)/KN | 0 |

Table 2. The physical and mechanical parameters of reinforcement body

| Parameter | Value | Parameter | Value |
|-----------------------------|-----------------------------|
| \( E_s \)/MPa | 1 380 | \( E \)/MPa | 1 456.9 |
| \( u \) | 0.42 | \( \mu \) | 0.361 |
| \( c_s \)/MPa | 1.568 | \( c \) | 1.633 |
| \( \phi \)/(°) | 35 | \( \phi \) | 35 |

A example is given, through comparison with the Jia-mei Gao[9] and FLAC3D numerical simulation to prove that the model is correct. The analysis parameters are shown in table 1. Taking the parameters of table 1 into the 3 and 4 sections derived formula, the equivalent strength and mechanical parameters of the reinforcement body can be obtained as shown in table 2.

Figure 3 and 4 are the displacement and stress curves of surrounding rock after rocking bolting, the theoretical values calculated in this paper are compared with the results of the Jia-mei Gao[9] and the numerical simulation of FLAC3D. After rock bolting, it is calculated that the radial displacement and circumferential stress of the roadway wall are 26.12mm and 16.67MPa respectively through this model, the results calculated by Jia-mei Gao are 27.03mm and 17.15MPa, and the results of FLAC3D software are 21.53mm and 15.97MPa, respectively. It can be seen that the theoretical solution of this paper is close to the result of Jia-mei Gao, but compared with that, the theoretical solution obtained by this paper is closer to the result of FLAC3D numerical simulation, and the rationality of the model is explained. Figure 3 and 4 shows that the displacement and stress of surrounding rock near the wall are larger, the farther away from the wall, the smaller the change rate is, and finally tends to be stable. In figure 4, the shear stress curve calculated in this paper fluctuate at a distance of 2m from the cava wall,
this is because the anchorage zone is regarded as a homogeneous reinforcement body, and when the surrounding rock coordinate deformation with reinforcement body, the material properties of the two materials are different which do not accord with the assumption of uniformity and belong to the contact problem. The reasonable mechanical transfer model of anchor bar and surrounding rock shows that the rock bolting improves the stress state of surrounding rock, reduces the surrounding rock circumferential stress and radial displacement, reduces the maximum shear stress in the roadway surrounding rock, improves the stability of surrounding rock, and is conducive to the stability of the roadway engineering.

6. Conclusion
(1) Based on the theory of elasticity, the common deformation of surrounding rock and anchor is analyzed, and displacement and stress expressions of surrounding rock and anchor in circular roadway are obtained. Finally, the radial displacement and Circumferential stress at the roadway wall are calculated.

(2) Through the homogenization method, the composite of rock and anchor bar is considered as reinforcement body. On this basis, the coordinated deformation model of surrounding rock and reinforcement body is established. According to the displacement equivalent conditions, the expressions of elastic modulus and Poisson ratio of reinforcement body are derived. According to the theory of strength strengthening of rock bolting, the expression of cohesion and internal friction angle of added solid are derived.

(3) Combined with the calculated example, the theoretical solution of this paper is compared with the results of the solution of the Jia-mei Gao and the FLAC3D numerical simulation, and the correctness of the model and the reinforcement body method of surrounding rock is verified. Strictly speaking, the deformation of the rock mass is rarely linear elastic. However, the analytical solution under the elastic state in this paper can be used as the starting point to explore the reinforcement of rock bolting to roadway surrounding rock, the regularity obtained from it has certain significance, which lays a foundation for the elastic-plastic and viscoelastic analysis of the roadway rock bolting.

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References
[1] Li C and Stillborg B 1999 Analytical models for rock bolts International Journal of Rock Mechanics and Mining Sciences. 36 1013-29
[2] Cai Y, Esaki T and Jiang Y J 2004 An analytical model to predict axial load in grouted rock bolt for soft rock tunneling Tunneling and Underground Space Technology. 19 607-18
[3] Fahimifar A and Sorουsh H 2005 A theoretical approach for analysis of the interaction between grouted rock bolts and rock masses Tunneling and Underground Space Technology. 20 333-43
[4] Sebastiano Pelizza, Sang-Hwan Kim and Jong-Soo Kim 2006 A study of strength parameters in the reinforced ground by rock bolts Tunnelling and Underground Space Technology. 21 378-9
[5] Xian-chun Yao, Ning LI and Yun-sheng Chen 2005 Theoretical solution for shear stresses on interface of fully grouted bolt in tunnels Chinese Journal of Rock Mechanics and Engineering. 24(13) 2273-75
[6] Chun-an You and Yu-bao Zhan 2005 Distributing characters and analysis of stresses in prestressed cable Chinese Journal of Rock Mechanics and Engineering. 24(6) 925-28
[7] Shuang-suo Yang and Bai-sheng Zhang 2003 The influence of bolt action force to the mechanical property of rocks Rock and Soil Mechanics. 24(10) 280-82
[8] Chao-jiong Hou 2017 Key technologies for surrounding rock control in deep roadway Journal of University of Mining & Technology. 46(5) 972-76
[9] Jia-mei Gao and Zhi-lin 1989 A linear elastic analysis of stress and displacement in wall rocks under full-column cemented rock bolting conditions *Quarterly of CIMR*. 9(3) 55-64