Radiative mass in QCD at high density

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Abstract

We show that radiatively generated Majorana mass for antiquarks is same as the Cooper-pair gap. We then calculate the electromagnetic corrections to mass of particles in the color-flavor locking phase of QCD at high density. The mass spectrum forms multiplets under $SU(2)_V \times U(1)_V$. The charged pions and kaons get the electromagnetic mass which is proportional to the Cooper-pair gap.

PACS numbers: 12.38.Aw, 11.30.Rd, 12.20.Ds

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It has been known for more than two decades that the ground state of dense matter is color superconductor [1]. However, intense study on color superconductivity has begun quite recently, after realizing that it may lead to a large superconducting gap [2,3], which can be explored by heavy ion collision experiments or in the core of neutron stars. Superconducting quark matter has two different phases, depending on density. At intermediate density, the Cooper pair is color anti-triplet but flavor singlet, breaking only the color symmetry down to a subgroup, $SU(3)_c \rightarrow SU(2)_c$. For high density where the strange quark is light, compared to the chemical potential, $m_s < \mu$, the Cooper-pair condensate is predicted to take a so-called color-flavor locking (CFL) form [4]:

\[
\langle \psi_{Li}^a(\vec{p}) \psi_{Lj}^b(-\vec{p}) \rangle = -\langle \psi_{Ri}^a(\vec{p}) \psi_{Rj}^b(-\vec{p}) \rangle = k_1 \delta_i^a \delta_j^b + k_2 \delta_i^b \delta_j^a,
\]

where $i,j = 1,2,3$ and $a,b = 1,2,3$ are flavor and color indices, respectively. At much higher density ($\mu \gg \Lambda_{\text{QCD}}$), $k_1(\equiv \Delta_0) \approx -k_2$ and color-flavor locking phase is shown to be energetically preferred [5,6].

Recently, the meson mass in the CFL phase has been calculated by several groups [7–11]. It is found that the (pseudo) Nambu-Goldstone boson mass due to the current quark mass vanishes at high density, since the Dirac mass connects quark with anti-quark and is thus a subleading operator in $1/\mu$ expansion, provided that the Majorana mass for anti-quarks, called antigap, is not too large. In this letter, we calculate the Majorana mass (or antigap) of antiquarks to implement the above results and then we calculate the electromagnetic corrections, which is a leading order in $1/\mu$ expansion, to the particle mass in the CFL phase, using the high density effective theory developed in [12,6].

At low energy, $E < \mu$, the quarks near the Fermi surface are relevant degrees of freedom while anti-quarks, the holes deep in the Dirac sea, are irrelevant and decoupled, since the energy of quarks near the Fermi surface in perturbative QCD is $E_+ = -\mu + \sqrt{\vec{p}^2 + m^2} \approx \vec{l} \cdot \vec{v}_F < \mu$, while anti-quarks have energy $E_- = \mu + \sqrt{\vec{p}^2 + m^2} \approx 2\mu + \vec{l} \cdot \vec{v}_F > \mu$, where we decompose the quark momentum as $\vec{p} = \vec{p}_F + \vec{l}$ with $|\vec{l}| < \mu$, and $\vec{v}_F \equiv \vec{p}_F/\mu$ is the Fermi velocity.

Since an arbitrarily small energy can create a pair of quarks and holes near the Fermi surface, quarks and holes can form pairs and condense to gain energy if any attraction is provided. It turns out that color anti-triplet diquark condensates are energetically more preferred, compared to particle-hole condensates [13]. Since magnetic gluons are not screened in dense quark matter, the (long-range) magnetic gluon exchange interaction leads to a much bigger Cooper-pair gap, which is in hard-dense loop (HDL) approximation given as [14–18],

\[
\Delta_0 \sim \frac{\mu}{g_s^2} \exp \left( -\frac{3\pi^2}{\sqrt{2}g_s} \right).
\]

Being irrelevant modes and thus decoupled from the low energy dynamics, anti-quarks do not get dynamical Majorana mass (namely, do not form condensates) in dense matter, since it is energetically not preferred. However, they will get a Majorana mass radiatively once a Cooper-pair gap opens for quarks (or holes) near the Fermi surface, since all symmetries that forbid Majorana mass for anti-quarks are then broken. The Cooper-pair gap, $\Delta$, can be transmitted to anti-quarks through their couplings to quarks near the Fermi surface or to gluons. In the leading order in loop expansion, there are two diagrams that contribute
to the anti-quark Majorana mass, denoted as $\bar{\Delta}_0$, shown in Fig. 1. From the first diagram [Fig. 1(a)], we get

$$\bar{\Delta}_0 = (-ig_s)^2 \frac{d^4l}{(2\pi)^4} \int T^A \Delta T^A \gamma_\perp \frac{\gamma_\perp^\dagger}{\bar{l}^2 - \Delta^2(l_\parallel)} \gamma_\perp D_{\mu\nu}(l), \tag{3}$$

where $\gamma_\perp = \gamma_\mu - \gamma_\parallel^\mu$ with $\gamma_\parallel^\mu = (\gamma^0, \vec{v}_F \cdot \vec{v}_F)$ and $D_{\mu\nu}$ is the (Higgsed) gluon propagator. Note that antiquarks interact with quarks near the Fermi surface via the magnetic gluons only, whose propagator is given Euclidean space as $[17]$.

$$D_{\mu\nu}^m(l) = \frac{P_{\mu\nu}^T}{|l|^2 + M_0^2 \Delta_0 + \pi M^2 l_\parallel/2}, \tag{4}$$

where the polarization tensor $P_{ij}^T = \delta_{ij} - l_i l_j/|l|^2$ and $P_{00}^T = 0 = P_{0i}^T$, the Meissner mass $M_0^2 = \alpha_s \mu^2/2$ and the Debye screening mass $M^2 = 3g_s^2 \mu^2/(4\pi^2)$. Taking the trace over gamma matrices in Eq. (3), we get in Euclidean space

$$\bar{\Delta}_0 = \frac{2g_s^2}{3} \frac{d^4l}{(2\pi)^4} \int \frac{|l|}{|l|^2 + M_0^2 \Delta_0 + \pi M^2 l_\parallel/2} \cdot \Delta(l_\parallel), \tag{5}$$

which is nothing but the Cooper-pair gap at zero external momentum except that here we have Higgsed magnetic gluons only, instead of full gluons. If we take $\Delta(l_\parallel) \approx \bar{\Delta}_0$, we get

$$\bar{\Delta}_0 = \Delta_0 \cdot \frac{g_s^2}{72\pi^2} \ln \left( \frac{4\mu^2}{\Delta_0^5} \right) \cdot \ln \left( \frac{48\mu^3}{5\pi M^2 \Delta_0} \right) = \Delta_0 (1 + O(g_s)), \tag{6}$$

where we used in the logarithm $\Delta_0 \sim \mu g_s^{-5} \exp (-6\pi/g_s)$, the solution to the gap equation in the constant gap approximation $[13,17]$. The contribution from the second diagram in Fig. 1(b) is $\sim g_s^2 \Delta_0 \ln(\mu^2/\Delta_0^5)$, which is suppressed by $O(g_s)$. Therefore, we find that in the leading order the Majorana mass for the anti-quarks is same as that of quarks and holes near the Fermi surface. Because of this Majorana mass term of antiquarks, additional corrections will be generated to the Dirac mass term. As in $[12,6]$, if we introduce the charge conjugated field $\bar{\psi}_c = C \bar{\psi}^T$ with $C = i\gamma^0 \gamma^2$ and decompose the quark field into states ($\psi_+$) near the Fermi surface and the states ($\psi_-$) deep in the Dirac sea, the Dirac mass term can be rewritten as

$$m_q \bar{\psi} \psi = \frac{1}{2} m_q \left( \bar{\psi}_+ \psi_- + \bar{\psi}_- \psi_+ \right) + \frac{1}{2} m_q^T \left( \bar{\psi}_c+ \psi_c^- + \bar{\psi}_c^- \psi_c^+ \right). \tag{7}$$

Since now the states in the Dirac sea can propagate into their charge conjugated states via the radiatively generated Majorana mass term, the Dirac mass term becomes, if one integrates out $\psi_-$ fields,

$$m_q \bar{\psi} \psi = \frac{m_q^2}{2\mu} \bar{\psi}_+ \left( 1 - i\partial \cdot V \right) \psi_+ + \frac{m_q m_q^T}{4\mu^2} \psi_+ \Delta_0 \psi_c^+ + \cdots, \tag{8}$$
where $V^\mu = (1, \bar{\nu}_F)$ and the ellipsis denotes the terms higher order in $1/\mu$. Then, the vacuum energy shift due to the Dirac mass term is $\sim m^2_\psi \Delta^2 \ln(\mu^2/\Delta^2)$ in the leading order, which shows the meson mass due to the Dirac mass indeed vanishes at asymptotic density, $m^2_\psi \sim m^2_\psi \Delta^2 / \mu^2 \cdot \ln(\mu^2/\Delta^2)$, since the vacuum energy in the meson Lagrangian is $m^2_\psi F^2$ with the pion decay constant $F \sim \mu$.\[1\]

Now, we calculate the electromagnetic mass of particles in the CFL phase. Since the color and flavor are locked in the CFL phase, simultaneous rotations of color and flavor space are only unbroken, leading to an interesting symmetry-breaking pattern,

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_V \times Z_2,$$

where $SU(3)_V$ is the diagonal subgroup of three $SU(3)$’s. The particle spectrum of the CFL phase is then as following: There are 8 massive gluons of mass, $m_g \sim g_s \mu$ by Higgs mechanism, which form an vector meson octet under $SU(3)_V$ and there are 9 massive quarks, $\psi_q^a$, which are octet and singlet under $SU(3)_V$. The octet, $\psi_8 = P_8 \psi$, has mass $k_2$ and the singlet, $\psi_S = P_1 \psi$, has mass $3k_1 + k_2$, which can be seen if we project out the gap as

$$k_1 \delta^a_i \delta^b_j + k_2 \delta^a_i \delta^b_j = k_2 Q_{ab}^{ij} + (3k_1 + k_2) P_{ij}^{ab}$$

where the projectors are $Q_{ab}^{ij} = \delta^a_i \delta^b_j - 1/3 \delta^a_i \delta^b_j$, $P_{ij}^{ab} = 1/3 \delta^a_i \delta^b_j$, and $Q^2 = P_8$.[19] Finally, we have nine Nambu-Goldstone (NG) bosons, consisting of meson octet and singlet, associated with chiral symmetry breaking and baryon superfluidity, respectively.

Since the Cooper-pair carries electric charge, there will be mixing between gluons and photons through the Cooper-pair gap. A linear combination of two fields, called a modified photon, $\tilde{A}_\mu = A_\mu \cos \theta + A^V_\mu \sin \theta$, remain massless and couples to fields with strength $\tilde{e} = e \cos \theta$, where $\tan \theta = e / g_s$ and $A^V_\mu$ is the gluon field corresponding to the color hypercharge, $Y = \text{diag}(-2/3, 1/3, 1/3)$. Once we turn on the electromagnetism, the global symmetry $SU(3)_V$ will break explicitly down to $SU(2)_V \times U(1)_V$, since the electric charge $Q = \text{diag}(2/3, -1/3, -1/3)$ is not flavor singlet or equivalently the charge of $U(1)_{\tilde{Q}}$, modified electromagnetism is not identity but given in the color-flavor locked space as

$$\tilde{Q}_{ai} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$  

Under $SU(2)_V$, the octets of $SU(3)_V$ are decomposed as $8 = 3 \oplus 2 \oplus \bar{2} \oplus 1$, among which the doublet has $\tilde{Q} = +1$ and the anti-doublet has $\tilde{Q} = -1$ and all others together with $SU(3)_V$ singlets are neutral under $U(1)_{\tilde{Q}}$. All particles with $U(1)_{\tilde{Q}}$ charge will get (modified) electromagnetic masses.

We first calculate the electromagnetic mass to the mesons. The leading-order contribution to the vacuum energy, due to the (modified) electromagnetic interaction, is given by a diagram, shown in Fig. 2, which is

\[1\]In the Erratum[7], Son and Stephanov also found the $1/\mu$ dependence of the meson mass. But, they claim that there is no logarithmic correction to the meson mass, contrary to all others’ results[8-11]. In the effective theory point of view, the operator, which corresponds to the cross diagram they claim to cancel the logarithm, does not exist in the leading order.

4
\[ \delta E = \frac{-\tilde{e}^2}{2} \sum_A \int d^4x \tilde{D}^{\mu\nu}(x) \langle \Omega | T J^A_\mu(x) J^A_\nu(0) | \Omega \rangle, \]  

(12)

where \( | \Omega \rangle \) denotes the CFL vacuum and \( \tilde{D}_{\mu\nu} \) is the modified photon propagator in HDL approximation. At low energy, modified photons will not interact with color superconductors. But at high energy, modified photons of energy larger than twice of the gap will get screened and Landau damped, with a screening mass \( \tilde{M}^2 \simeq (8/9) \sin^2 \theta M^2 \), since it can break Cooper-pairs and scatter with the gaped quarks in the Fermi sea.

Since the main contributions to the loop integration in Eq. (12) come from photon energy larger than the gap, we get

\[ V_{em} = 4 \frac{\tilde{e}^2}{8\pi} \Delta_0^2 \mu^2 \ln \left( \frac{\mu^2}{\Delta_0^2} \right) \cdot \left[ \ln \left( \frac{16\mu^3}{\pi M^2 \Delta_0} \right) \right]^2 \simeq 18\pi \left( \frac{\tilde{e}}{g_s} \right)^2 \Delta_0^2 \mu^2 \ln \left( \frac{\mu^2}{\Delta_0^2} \right), \]  

(13)

where the gap in the second logarithm is traded with the coupling \( g_s \) by Eq. (2). The mass of (pseudo) Nambu-Goldstone bosons is then just the curvature of the vacuum energy at the origin of the NG boson manifold \([20]\),

\[ m_{ab}^2 = \frac{1}{F^2} \frac{\partial^2}{\partial \alpha_a \partial \alpha_b} \delta E(U)|_{U=1}, \]  

(14)

where \( U \) is a unitary matrix in the chiral group and \( \alpha_a \)'s are its coordinates in the group space. We therefore find the charged NG bosons get the (modified) electromagnetic mass

\[ m_{\pi}^2 = \frac{9\pi}{4} \left( \frac{\tilde{e}}{g_s} \right)^2 \Delta_0^2 \frac{\mu^2}{F^2} \ln \left( \frac{\mu^2}{\Delta_0^2} \right). \]  

(15)

Since the pion decay constant \( F \simeq 0.209\mu \) \([7]\), the charged NG bosons get the electromagnetic mass \( m_{\pi} \simeq 12.7 \sin \theta \Delta_0 [\ln(\mu^2/\Delta_0^2)]^{1/2} \).

Finally, we calculate the (modified) electromagnetic mass of charged gluons and quarks. Since the (modified) photon contributes to the self energy of charged gluons and quarks, we just calculate the one-loop corrections due to the (modified) photon to the self energy to get electromagnetic mass for the charged gluons and quarks

\[ \delta m_g^2 \sim \tilde{e}^2 \Delta_0^2 \ln \left( \frac{\mu^2}{\Delta_0^2} \right) \quad \text{and} \quad \delta m_\psi \sim \tilde{e}^2 \Delta_0 \ln \left( \frac{\mu^2}{\Delta_0^2} \right). \]  

(16)

To conclude, we show that the anti-quarks, holes deep in the Dirac sea, get Majorana mass radiatively, which is same as the Cooper-pair gap. We calculate the (modified) electromagnetic mass of particles in the CFL color superconductor. As the modified electromagnetic interaction breaks explicitly the global \( SU(3)_V \) symmetry down to \( SU(2)_V \times U(1)_V \), the degeneracy in the mass spectrum is lifted such that among octets the \( SU(2)_V \) doublets become more massive. Especially, the charged pions and kaons get electromagnetic mass, of order of the Cooper-pair gap, and do not vanish at high density, while neutral mesons become massless at asymptotic density.
ACKNOWLEDGMENTS

The author wishes to thank T. Lee, D.-P. Min, and K. Rajagopal for useful comments and the Institute for Nuclear Theory at the University of Washington, where part of this work was done, for its hospitality. This work was supported by the academic research fund of Ministry of Education, Republic of Korea, Project No. BSRI-99-015-DI0114.
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FIG. 1. The radiative correction to the Majorana mass of antiquarks. The double solid line denotes antiquarks and the single solid line denotes the quarks near the Fermi surface.

FIG. 2. The (modified) electromagnetic contribution to the vacuum energy in the leading order. The wiggly line denotes the modified photon in the HDL approximation.