Energy loss at NLO in a high-temperature Quark-Gluon Plasma

Jacopo Ghiglieri

Institute for Theoretical Physics, Albert Einstein Center, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland

Abstract

We present an extension of the Arnold-Moore-Yaffe kinetic equations for jet energy loss to NLO in the strong coupling constant. A novel aspect of the NLO analysis is a consistent description of wider-angle bremsstrahlung (semi-collinear emissions), which smoothly interpolates between 2 ↔ 2 scattering and collinear bremsstrahlung. We describe how many of the ingredients of the NLO transport equations (such as the drag coefficient) can be expressed in terms of Wilson line operators and can be computed using a Euclidean formalism or sum rules, both motivated by the analytic properties of amplitudes at light-like separations. We conclude with an outlook on the computation of the shear viscosity at NLO.

Keywords:
Quark-Gluon Plasma, jet quenching, energy loss, transport coefficients, higher-order

1. Introduction

Two main avenues for the investigation of the medium produced in heavy-ion collision are the study of its bulk properties on one hand and the analysis of hard probes on the other. On the theory side, the former is mostly studied through an effective hydrodynamic description, which kicks in after at some initial time \( \tau_0 \sim 1 fm/c \), after a rapid thermalization process has taken place. For what concerns hard probes, considerable activity is dedicated to the investigation of jet quenching. Theory overviews of hydrodynamics, thermalization and jet quenching formalisms have been presented at this conference in [1], [2, 3], [4], with experimental reviews of flow and jet data in [5, 6]. In this contribution we will concentrate on a weak-coupling theory approach that is well suited to compute in-medium jet propagation, thermalization and the transport coefficients of the QCD medium. It is the effective kinetic theory derived by Arnold, Moore and Yaffe (AMY) [7] and used for the leading-order computation of the transport coefficients, such as the shear viscosity, in [8]. In this contribution we will show how the version of this kinetic theory suited to the study of jet propagation can be extended to the next order in the strong coupling \( g = \sqrt{\frac{4\pi}{\alpha_s}} \), effectively giving a summary of the results presented in detail in [9] and introduced more pedagogically in [10]. One important motivation for this extension to NLO is to gauge the stability of perturbation theory, which requires \( g \ll 1 \) at finite temperatures, when extrapolated to temperatures where \( \alpha_s \sim 0.3 \).

This contribution is organized as follows: in Sec. 2 the LO kinetic theory is reviewed and a useful reorganization of its collision operator is introduced and extended to NLO in the case of jet quenching, while Sec. 3 contains an outlook on the extension to transport coefficients and a brief conclusion.
2. Reorganization of the collision operator

The AMY kinetic theory can be written as

\[ \partial_t + v \cdot \nabla_x f(p) = C_{2 \leftrightarrow 2} + C_{1 \leftrightarrow 2}, \]

where the l.h.s. is the typical one for a Boltzmann equation in the absence of external forces, while the r.h.s. is the collision operator, written as a sum of 2 ↔ 2 and 1 ↔ 2 components. The former are the standard elastic scatterings of a gauge theory, complemented by Hard Thermal Loop (HTL) resummation [11] for IR finiteness. 1 ↔ 2 labels 1 + n ↔ 2 + n processes, i.e. the collinear splittings/joinings of one particle into two other, induced by n ≥ 1 soft scatterings with medium constituents. The coherent, destructive interference of these scatterings gives rise to the well-known Landau-Pomeranchuk-Migdal (LPM) effect. For ease of illustration, we will omit quarks entirely in this contribution, i.e. we consider energetic gluons propagating through an equilibrated gluon plasma at a temperature \( T \). The phase space distribution of the jet gluons and the collision operator can be linearized in \( f \), i.e. only one of the 3 or 4 particles in a 1 ↔ 2 or 2 ↔ 2 process is a jet parton, while all others are thermal and characterized by the equilibrium distribution Bose-Einstein \( \frac{\rho}{n_0} \).

The separation between 1 ↔ 2 and 2 ↔ 2 processes ceases to be well defined beyond leading order. In order to have a collision operator that is more easily extended to NLO and which makes more transparent the introduction of effective Wilson line descriptions for soft scattering processes, we introduce the following reorganization

\[ \partial_t + v \cdot \nabla_x f(p) = C_{\text{large}}[\mu_\perp] + C_{\text{diff}}[\mu_\perp] + C_{\text{coll}}, \]

where the three processes are large-angle processes, diffusion processes and collinear processes. A large-angle process is a 2 ↔ 2 process with an \( \mathcal{O}(1) \) angular deflection, which translates into a large momentum transfer \( Q \gg T \), enforced by an infrared cutoff \( \mu_\perp \). An example is depicted on the left in Fig. 1. A collinear process is instead a 1 ↔ 2 process with strictly collinear kinematics: at leading order it coincides with 1 ↔ 2 processes, but at NLO a subtraction of the limits where it blurs with other processes is required. It is represented on the right in Fig. 1. Finally, the region of soft momentum exchanges in 2 ↔ 2 processes is described in a diffusion picture, i.e.

\[ C_{\text{diff}}[\mu_\perp] \equiv -\frac{\partial}{\partial p^i} \left[ \eta_0(p)p^i f(p) \right] - \frac{1}{2} \frac{\partial^2}{\partial p^i \partial p^i} \left[ \left( \hat{p}^i \hat{q}_L + \frac{1}{2} (\delta^i - \hat{p}^i \hat{q}^j) \hat{q}_L \right) f(p) \right], \]

where the three coefficients entering in this Fokker-Planck equation are the drag \( \eta_0 \) and the longitudinal and transverse momentum broadening coefficients \( \hat{q}_L \) and \( \hat{q} \). These two can be defined as effective force-force correlators on Wilson lines along the classical trajectories of the particles, i.e.

\[ \hat{q}^{ij} \equiv \int_{-\infty}^{\infty} dt' \left\langle F^i(t) F^j(0) \right\rangle, \quad \hat{q}^{i}(x^+) \equiv U^i(x^+, -\infty) g F^{ij}(x^+) v_j U(x^+, -\infty). \]

where \( U \) is an adjoint Wilson line in the \( x^+ \equiv (x^0 + x^+) / 2 \) light-cone direction \( (x^- \equiv \frac{x^0 - x^+}{2}) \) in which we have taken the energetic jet particle to be. Similarly, \( v^\mu = (1, 0, 0, 1) \) is a null vector pointing in that same
direction. At leading order this operator takes the form depicted in the first diagram on the left in Fig. 2. The LO $\hat{q}$ can be easily evaluated using the mapping to the three-dimensional Euclidean theory introduced by Caron-Huot [12] and reviewed in [10], yielding

$$\hat{q} = g^2 C_A \int d^2 q_\perp (2\pi)^2 \int dq_\perp^+ \left( F^{-1}(Q) F^{-} \right)_{q^+ = 0} = g^2 C_A T \int d^2 q_\perp (2\pi)^2 \frac{m_D^2}{q_\perp^2 + m_D^2} = \frac{g^2 C_A T m_D^2}{2\pi} \ln \frac{\mu_\perp}{m_D}, \quad (5)$$

where $m_D^2 = N_c g^2 T^2 / 3$ is the Debye mass. The longitudinal one can instead be evaluated using a new sum rule, based on the analytical properties of amplitudes at space- and light-like separations [9], yielding

$$\hat{q}_L = g^2 C_A \int d^2 q_\perp (2\pi)^2 \int dq_\perp^+ \left( F^{-1}(Q) F^{-} \right)_{q^+ = 0} = g^2 C_A T \int d^2 q_\perp (2\pi)^2 \frac{m_\infty^2}{q_\perp^2 + m_\infty^2} = \frac{g^2 C_A T m_\infty^2}{2\pi} \ln \frac{\mu_\perp}{m_\infty}, \quad (6)$$

where $m_\infty^2 = m_D^2 / 2$ is the asymptotic mass of gluons. Finally, $\eta_D$ can be determined from the other two

through an Einstein-like relation, obtained by imposing that the Fokker-Planck picture be equivalent to the Boltzmann one for $Q \gg gT$ and that it show a fixed point at equilibrium. The UV logarithmic dependence of the diffusion sector cancels with the IR one in large-angle scattering processes.

When considering higher-order terms, soft gluon loops, thanks to the Bose enhancement $n_B(gT) \sim 1/g$, are only suppressed by $g$, rather than $g^2$. This implies that the collinear and diffusion sector receive $O(g)$ corrections from the inclusion of soft gluon loops. Furthermore, a new, semi-collinear process has to be considered at NLO.

In the collinear sector, the soft scattering rate $d\Gamma/(d^2 q_\perp)$ inducing the splitting process, receives $O(g)$ corrections from these loops. The Euclidean mapping mentioned before was indeed first applied to the computation of this correction [12]. A similar soft correction to the dispersion relation [13] of the collinear particles needs also to be considered.

For diffusion, the two-loop diagrams depicted in Fig. 2 need to be computed to obtain the $O(g)$ corrections to $\hat{q}$ and $\hat{q}_L$. The former (related to $d\Gamma/(d^2 q_\perp)$) was done in [12] using the Euclidean mapping, while the latter [9] simplifies greatly using the aforementioned sum rule, resulting in the replacement of $m_\infty^2$ in Eq. (6) with $m_\infty^2 + \delta m_\infty^2$, where $\delta m_\infty^2$ is the $O(g)$ correction to the asymptotic mass.

Finally, semi-collinear processes can be seen as collinear processes with larger opening angles. This reduces the collinear enhancement, making them subleading. Furthermore, the relaxed kinematical constraints also allow the interactions with plasmons, besides the usual space-like soft scatterings, as shown in Fig. 3.

The evaluation of these diagrams proceeds similarly to that of the collinear ones in the single-scattering (Bethe-Heitler) limit, where LPM interference is suppressed. The rate turns out to be proportional to the
DGLAP splitting kernel multiplying a generalized $\tilde{q}$ which keeps track of the component of the soft gluon momentum in the other light-cone direction. We call it $\tilde{q}(\delta E)$ and it reads

$$\tilde{q}(\delta E) = g^2 C_A \int^{\infty}_{-\infty} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{d q^+}{2\pi} \left\langle F^{-1}(Q) F_+^{\perp} \right\rangle_{q_\perp}^{\perp} = \delta E.$$  

(7)

It too can be evaluated using the Euclidean mapping. An IR log divergence in these processes cancels exactly an UV one in diffusion processes.

In order to establish the quantitative effect of the NLO corrections we have just obtained, a numerical implementation in a Monte Carlo generator such as MARTINI [14], which currently implements $1 \leftrightarrow 2$ and $2 \leftrightarrow 2$ processes at LO, is underway.

3. Outlook on transport coefficients and conclusions

The techniques we have briefly illustrated, which allow to cast all the intricate soft dynamics into a few effective operators evaluated using Euclidean mappings or sum rule, can in principle be applied to transport coefficients as well. There is however one extra major difficulty in that case. Consider the computation of the shear viscosity $\eta$: it requires knowing how a disturbance in the energy-momentum tensor $T^{ij}$ sources a second $T^{ij}$ disturbance. In other words, one needs a linearized kinetic theory where two off-equilibrium distributions are considered within the same process, rather than one. In the case of a soft $2 \leftrightarrow 2$ scattering, one has then the two possibilities in Fig. 4. In the leftmost case, the two $T^{ij}$ insertions are on the same side of the soft gluon. Their momenta are thus strongly correlated, as they differ by $Q \sim gT$ and a diffusion picture similar to the one we have introduced is applicable. In the rightmost case, on the other hand, the momenta are uncorrelated and the diffusion picture is not applicable. An inspection of the leading-order calculation shows however that such terms are UV finite, making the prospect of a direct, “brute-force” NLO calculation in the HTL theory slightly less daunting.

In conclusion, the reorganization of the kinetic theory, which recasts all soft contributions in light-front Wilson-line operators, is an extremely useful tool, which is now generalized to NLO for jet propagation. The extension to transport coefficient is more problematic, but a reliable estimate should be feasible with the current technology.

Acknowledgements I thank G. Moore and D. Teaney for collaboration. My work is supported by the Swiss National Science Foundation (SNF) under grant 200020_155935.

References

[1] G. Denicol, these proceedings.
[2] A. Kurkela, these proceedings, arXiv:1601.03283.
[3] P. Chesler, these proceedings.
[4] Y. Mehtar-Tani, these proceedings.
[5] S. Mohapatra, these proceedings.
[6] M. Nguyen, these proceedings.
[7] P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 0301 (2003) 030. arXiv:hep-ph/0209353.
[8] P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 0305 (2003) 051. arXiv:hep-ph/0302165.
[9] J. Ghiglieri, G. D. Moore, D. Teaney, arXiv:1509.07773.
[10] J. Ghiglieri, D. Teaney, Int. J. Mod. Phys. E24 (2015) 1530013, to appear in QGP5, ed. X-N. Wang. arXiv:1502.03730.
[11] E. Braaten, R. D. Pisarski, Nucl. Phys. B337 (1990) 569.
[12] S. Caron-Huot, Phys. Rev. D79 (2009) 065039. arXiv:0811.1603.
[13] S. Caron-Huot, Phys. Rev. D79 (2009) 125002. arXiv:0808.0155.
[14] B. Schenke, C. Gale, S. Jeon, Phys. Rev. C80 (2009) 034913. arXiv:0909.2037.