Aspects of numerical modeling of mixed convection over heated horizontal plate

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Abstract. Mixed convection is a common type of heat transfer for relatively slow impinging flows. When fluid flow impinges a transverse flat plate a Hiemenz flow appears. It is basically driven by buoyancy forces, which make it unstable and challenging to simulate. The present study is dedicated to these challenges in terms of computational mesh reduction. Several methods of mesh reduction were tested, including computational domain simplification, symmetry application and mesh coarsening. Each simplification method was tested against experimental data to evaluate proposed hypotheses about symmetry applicability and unconditional advantage of finer meshes. All considered cases were simulated with previously validated numerical model. Although simulated process was seemingly purely symmetrical, it didn’t respond well to implementation of symmetrical boundary conditions. Simplification of computational domain yielded generally expected results showing significant deterioration after truncation of essential part of domain. Mesh coarsening led to erroneous results as well, but dependency wasn’t linear. Statistical analysis proved normality of simulation errors distribution. The present study addresses preprocessing stage of simulation process, providing some insight into simplification methods of computational mesh.

1. Introduction
Mixed convection occurs in many cases of heat transfer. Even a turbulent flow of heat carrier can be easily slowed down near ribs, sharp turns and dimples. When buoyancy forces affect fluid flow as much as pressure forces, aspects of mixed convection should be considered.

Although effects of mixed convection on convective heat transfer have been observed for quite a while, complexity of the process’ dynamics is still fascinates many contemporary researchers. One of the earliest studies of mixed convection dated back to 1911, when Hiemenz [1] observed a uniform distribution of heat transfer for stagnation point flow. Hiemenz flow may occur when downward flow impinges on heated horizontal plate. Two-dimensional representation of this problem has been addressed in several studies [2, 3, 4] conducted over the last two decades. At the same time some researchers [5, 6, 7] show asymmetric nature of Hiemenz flow.

Effect provided by Hiemenz flow is commonly used for thin film formation in chemical reactors, but there are also a number of general-purpose heat exchangers working with Hiemenz flow. Maintaining of stagnation point flow in heat exchanger might be challenging and highly dependent on its geometry and operation mode. Numerical simulation of flow dynamics and heat transfer has a huge advantage in such complex cases. Ever growing availability of numerical simulation is accompanied by robustness and flexibility of this tool.
Accuracy of numerical results is highly dependent on taken assumptions and computational mesh quality. These aspects were evaluated in present study dedicated to numerical simulation of Hiemenz flow over horizontal heated plate. An applicability of computational domain simplification was assessed in respect to its essential part and symmetry conditions. Two hypotheses have been evaluated in present study: applicability of all available symmetries and inverse relation between accuracy of numerical results and mesh elements size. Proposed hypotheses were tested on previously developed numerical model of vertical flow impinging horizontal heated plate. Several computational domains with different simplification rate were considered alongside with various mesh sizes and imposed symmetry conditions. Better understanding of limitations of numerical simulation of Hiemenz flow can be useful for consequent modeling of relevant heat exchangers.

2. Methods

2.1. Numerical model

Numerical model used in present study was adopted from a previous one [8], where incorporation of Smagorinsky turbulence model yielded the most adequate results. Numerical model was developed with Code_Saturne – free proprietary software designated for computational fluid dynamics and based on finite volume approach.

All simulations were carried out in unsteady formulation. Water was used as working medium with temperature-dependent physical properties. Adequacy of developed numerical model was assessed by comparison with adopted experimental data [9].

2.2. Simplification methods

Simplification of computational mesh is a very effective way of reducing computational effort of solving a problem. This can be done in various ways considering the nature of modeled process and computational domain configuration. Three approaches will be described later in order of their implementation.

According to adopted experimental study [9] a square channel with confuser at inlet and transverse rectangular plate placed downstream was considered as basic computational domain (figure 1). Heated plate transversely located in the middle of vertical channel. Water flow moves downward and impinges this plate, producing Hiemenz flow over it.

![Figure 1: Computational domain.](image-url)
2.2.1. Computational domain simplification. Since stagnation point flow occurs over horizontal plate, downstream part of the channel was considered not essential and was excluded from computational domain. Further on remained part of the channel (150 mm height) was addressed as a full-height.

Four cases of computational domain simplification were considered: (1) full-height channel with confuser, (2) full-height channel, (3) half-height channel and (4) quarter-height channel. All cases were computed on structural meshes comprised of 2.78, 0.67, 0.38 and 0.23 million hexahedral cells respectively.

Mesh cells size was limited to 2 mm. Near heat transfer surface mesh was inflated to ensure \( y^+ \leq 1 \) and \( T^+ \leq 1 \). Considering flow velocity, first row of cells was limited to be 0.12 mm thick. Total thickness of inflated rows (\( \delta \)) was 5.11 mm.

2.2.2. Symmetry implementation. Location and geometry of heated plate as well as vertical channel allows implementation of symmetry conditions. Six cases were considered (figure 2): (1) without symmetry, (2-3) half-domain with transverse symmetry, (4) half-domain with lateral symmetry, (5-6) quarter-domain. All cases were applied to full-height computational domain. Described above algorithm was used to generate 0.34, 0.67 and 1.14 million cells meshes.

2.2.3. Mesh elements size variation. The third approach to mesh reduction was based on actual resizing of mesh cells. Five maximum cell sizes were considered (2...5 mm) providing 0.05, 0.1, 0.21 and 0.67 million cells meshes. Cell size variation was tested on full-height computational domain with transverse symmetry applied (figure 2 (b)). Similar inflation algorithm was used in all cases with fixed height of the first row of cells (0.12 mm). To preserve number of prismatic layers and ensure required dimensionless distances \( y^+ \leq 1, T^+ \leq 1 \), total thickness of inflated rows (\( \delta \)) varied in a range of 5.11...13.6 mm.

2.3. Data reduction
2.3.1. Residuals evaluation. Obtained results adequacy was assessed by temperature distribution over heated plate. Adopted experimental results [9] were set as a baseline values. Distribution of plate temperature was derived from a series of measurements along y-oriented lines (at \( x = \pm 1.5 \) mm and \( x = \pm 29.5 \) mm), which were discretized by 100 data points. Arithmetic mean of each line data over 20 time steps (40 ÷ 50 s) was considered. Spatial-temporal standard deviations were obtained for each dataset.

Residuals were calculated twice: to assess results adequacy (comparison to experimental data) and to test proposed hypothesis about applicability of all available symmetries. Both times residuals were expressed as an absolute deviation of obtained temperature from some
baseline value $T_0$: $|\Delta T| = \frac{|T-T_0|}{T_0} \cdot 100\%$. For adequacy assessment $T_0$ was set to experimental value. Symmetry applicability was tested with $T_0$ set to mean value of left-hand ($x < 0$) and right-hand ($x > 0$) sides of computational domain, e.g. $T_0 = \frac{T_{x=1.5\ mm}+T_{x=-1.5\ mm}}{2}$. The latter was tested for transverse symmetries only.

2.3.2. Statistical analysis. Since coefficient of determination ($R^2$) considered as not sufficient for assessment of model adequacy, graphical residual analysis was provided. To supplement presented assessment, normality of errors distribution was evaluated with normal probability plots. They were constructed by plotting the sorted values of the residuals versus the associated theoretical values from the standard normal distribution.

3. Results

Numerical data were processed with Salome platform to obtain temporal-spatial temperature distribution over heated plate. Obtained datasets were grouped according to proposed simplification methods.

3.1. Computational domain simplification

Computational domain simplification cases were tested on structured mesh with 2 mm cells. Each case was computed in the same manner. On figure 3 (a) temperature residuals ($|\Delta T|$) along two y-oriented lines ($x = 1.5\ mm$ and $x = 29.5\ mm$) are presented in order of mentioning in section 2.2.1. Experimental values of plate temperature were used as baseline ($T_0$) to obtain $|\Delta T|$. On figure 3 (b) one can see residuals from mean value of left-hand ($x < 0$) and right-hand ($x > 0$) temperatures. These residuals were intended to show asymmetry of the process.

![Figure 3: Temperature residuals for various simplification cases.](image)

3.2. Symmetry implementation

Six cases of symmetry implementation were considered in present study (see figure 2). Each case was computed on a similar meshes with 2 mm cells with the same numerical model and boundary conditions. Experimental values of plate temperature were used as a baseline for residuals on figure 4 (a). Temperature residuals were calculated along two y-oriented lines ($x = 1.5\ mm$ and $x = 29.5\ mm$) and presented in order of mentioning in section 2.2.2. Bar plot depicted on
figure 4 (a) has two y-axes to negotiate scale differences between datasets at $x = 1.5 \text{ mm}$ and $x = 29.5 \text{ mm}$.

Due to specifics of symmetry allocation only full domain and domains with transverse symmetries were evaluated for ability to reproduce symmetric solution (figure 4 (b)).

![Figure 4](image)

(a) based on experimental data. (b) based on spatial average.

Figure 4: Temperature residuals for various symmetry implementation cases.

3.3. Mesh elements size variation

The most straightforward way of computational mesh reduction was tested as a third method of simplification. According to considered cell sizes (2...5 mm) eight datasets were obtained: at $x = 1.5 \text{ mm}$ and at $x = 29.5 \text{ mm}$ (figure 5). In other respects utilized meshes were the same.

![Figure 5](image)

(a) based on experimental data. (b) based on spatial average.

Figure 5: Temperature residuals for mesh size evaluation.
On figure 5 (a) one can see correlation of cell size and corresponding temperature residuals, when experimental data are set as baseline values. Figure 5 (b) depicts the same correlation with an averaged temperature of left-hand \((x < 0)\) and right-hand \((x > 0)\) sides of the plate as a baseline values.

### 3.4. Statistical analysis
To check whether errors inherent to obtained numerical results are random, normal probability plots were made. Probability plots for full-height domain were presented on figure 6 for two datasets. Linear regression was added to aid visualize probability distribution. For these plots, structural mesh comprised of 2 mm cells with transverse symmetry (figure 2 (b)) was used.

![Figure 6: Normal probability plots for mesh of full-height domain with 2 mm cells.](image)

\[ (a) \text{ at } x = 1.5 \text{ mm.} \]

\[ (b) \text{ at } x = 29.5 \text{ mm.} \]

This case was considered for statistical analysis because it produced the most adequate results based on residuals analysis that was discussed above and in previous study [8].

### 4. Discussion
Hiemenz flow, which occurs in several types of heat exchangers, was simulated with various assumptions to evaluate their applicability. Each assumption tolerated a corresponding simplification method concerning geometry of computational domain, symmetry conditions or cell size. Each simplification method was tested against experimental data to evaluate proposed hypotheses about symmetry applicability and unconditional advantage of finer meshes.

Numerical data prove that computational domain simplification may yield erroneous results and lead to unstable solution, but is very efficient method of mesh reduction to some extent. At the same time symmetry conditions are as much efficient in mesh reduction, while affect results adequacy less dramatically. Thus proposed hypothesis about symmetry applicability is partially supported, because Hiemenz flow appeared to be slightly asymmetric.

Obtained data also show deterioration of results adequacy with coarsening of computational mesh. This basically supports proposed hypothesis about correlation between mesh size and results conformity, although this correlation is not linear.

Simplification of computational domain affects results dramatically. Obtained results confirm expected inadequacy as a result of domain truncation: note residuals increase on figure 3 (a) for half-height and quarter-height domains. At the same time, incorporation of confuser into computational domain produces significant deviation from expected value. This contradiction might be explained by specifics of numerical model that was developed in anticipation of uniform inlet flow and not a confuser-formed profile.
Spatial variance of temperature at ±x is negligible for all cases of computational domain simplification (figure 3 (b)). However, domain with confuser produces spatial variances, which are 9.5 times apart. This indicates significant asymmetry exclusively at the edges of the plate, which was not observed during experimental study.

Symmetry conditions produce a reasonable error (under 8 %) while significantly reducing computational effort. Obtained results clearly indicate undesirable effect from implementation of lateral symmetry for considered domain (see cases with lateral and double symmetry on figure 4 (a)). It should be noted that transverse symmetry does not provide more adequate results than full domain. Lower residuals for cases with transverse symmetry appeared as a result of lacking inflation of mesh near vertical walls. Thus near-wall region should be considered as error source. This explains why the most accurate results obtained for domain with just two vertical walls (case "1/2 TM" on figure 4 (a) and (b)).

Clearly, finer meshes produce more accurate results. Unfortunately, considered range of cell sizes is unable to illustrate the minimal reasonable cell size, beyond which no distinguishable gain of accuracy can be achieved. However, on figure 5 (b) one can see a distinct leap of spatial variance at cell size 4 mm, which indicate strongly unstable solution.

Statistical analysis of particular case (2 mm cells, full-height domain with transverse symmetry – figure 2 (b)) shows fairly normal distribution of errors (figure 6). Both datasets have apparent linear trend, except in the extreme tails. Thus used in this study numerical model should be considered adequate.

Ongoing research of Hiemenz flow with innovative modeling approaches in recent studies [2, 4] has a strong potential of practical use. The present study addresses preprocessing stage of simulation process, providing some insight into simplification methods of computational mesh.

Conclusions
The present study is dedicated to aspects of Hiemenz flow simulation regarding simplification of computational mesh. Initially two hypotheses were proposed: about symmetry applicability and unconditional advantage of finer meshes.

Based on adopted experimental study of vertical water flow impinging horizontal heated plate, two sets of computational domains were created. The first set of four domains was dedicated to mesh simplification method based on evaluation of essential part of geometry. The second set was comprised six domains and was used to test symmetry applicability.

Although simulated process was seemingly purely symmetrical, it didn’t respond well to implementation of symmetrical boundary conditions. Simplification of domain yielded generally expected results showing significant deterioration after truncation of essential part of domain.

The third method of mesh simplification was based on direct variation of cell size. Mesh coarsening expectedly led to erroneous results, but dependency wasn’t linear. Statistical analysis proved normality of simulation errors distribution. The present study provides some insight into simplification methods of computational mesh.

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