Investigating the process of rock destruction by an indenter in a well through the theory of contact problems

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Abstract. The article describes the results of the study of the process of destruction of rocks by contact. The indentation of the indenter into the well wall was considered. A body with a cylindrical surface was used as an indenter. The authors of the article applied the theory of contact interaction of G. Hertz. The strength theory was used as a criterion for destruction. K. Mohr. The dependence obtained by the authors was tested using mathematical modeling. The results of mathematical modeling have confirmed the possibility of practical application of the resulting relationship.

1. Introduction
The theory of contact problems is now widely used in mechanical engineering. The proposed methods allow her to find the places of stress concentrations in a solid. This allows the assessment of the state of various parts under load.

Machine parts can be represented as elastic bodies. The use of the laws of the theory of elasticity for rocks is difficult. This is due to their considerable heterogeneity and anisotropy. Therefore, the calculations of their deformation and destruction, in the case of their representation by elastic bodies, can only be approximate.

Despite this, the theory of elasticity has found its application for solving problems of geomechanics [1]. The results obtained in this case allow the state of the rock massifs to be assessed with sufficient accuracy for mining practice. The theory of contact problems can also be used in mining. It can be used to assess the effectiveness of the impact of rock tools and indenters on the rock. The results of such calculations can reveal the main regularities of rock destruction processes.

2. Express method for determining the mechanical properties of rocks
In this article, the authors attempt to use the theory of contact problems to study the process of rock destruction in a well as an indenter. The results can be used to assess the properties of rocks by express methods. One of these methods was developed jointly with L.T. Dvornikov [2].

The method can be successfully used for studying the properties of rocks both in samples and massif outcrops and in wells drilled in the massif. In accordance with this method, the strength properties of rocks are determined by the magnitude of the effort required for brittle fracture of the rock by the indenter of a special shape.

The indenter used in the method is shown in figure 1, where: 1 – the body of the indenter made in the form of a straight circular cylinder with diameter D, equal to its length L; 2 – ends of the indenter having roundings with radius R equal to the radius of the straight circular cylinder forming the body 1.
of the indenter, 3 – the rock under test, P – the force applied to the indenter. The action of the indenter on the rock 3 to be destroyed is carried out along the generatrix of the quadric cylinder forming the body 1 of the indenter. When measuring in wells, the indenter is positioned in such a way that the generatrix of the cylinder constituting its body is directed parallel to the generatrix of the cylindrical surface of the well.

Figure 1. The indenter used in the proposed method [2].

Determination of the rocks properties of samples in accordance with the developed method is supposed to be carried out on a manual hydraulic press. To perform the research in the well conditions, the authors proposed a special device Strength Tester PSSH-1 (“PSSH-1”) [3]. More details about the method are written in [4].

3. Solution of the contact problem

Contact interaction of two cylindrical bodies can be described by the theory of H. Hertz. Let us take an indenter with a cylindrical contact surface as one body, and a rock with a drilled well as another body [5]:

\[
P_0 = 0.5642 \left( \frac{q}{\eta} \frac{R_2 - R_1}{R_1} \right)^{3/2}
\]

In the formula (1) indicated: \( P_0 \) is the maximum pressure between the indenter and the surface of the well; \( q \) is the load per unit length of the indenter; \( R_i \) is the indenter radius; \( R_j \) is the well radius; \( \eta \) is the elastic constant of bodies in contact.

\[
\eta = \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2}
\]

In the formula (2) indicated: \( \mu_i \) is the Poisson's ratio of the material from which the indenter is made; \( E_i \) is the modulus of elasticity of the material from which the indenter is made; \( \mu_r \) is the Poisson's ratio of the rock; \( E_r \) is the modulus of elasticity of the rock.

Relationship (1) allows you to determine the maximum pressure that occurs in the contact area. According to the theory [6], the most dangerous point in the rock to be destroyed will be under the indenter at a depth of 0.4b, where b is the width of the contact strip. The main stresses occurring at this point will be \( \sigma_1 = 0.78P_0 \), \( \sigma_2 = 0.288P_0 \), \( \sigma_3 = 0.18P_0 \). The maximum shear stress will be \( \tau_{\text{max}} = 0.3P_0 \) [6].

Mohr's theory of strength is widely used in mining science. In accordance with it, the ultimate stress state of the rock at the time of destruction [7]:

...
In the formula (3) indicated: \( \tau \) is the shear stress in the elementary volume of the rock at destruction; \( \sigma_t \) is the ultimate strength of the rock under tension; \( \sigma_c \) is the ultimate strength of rock under compression; \( \sigma \) is the stress in the elementary volume of the rock at destruction.

The destruction of the rock will begin at a dangerous point indicated earlier. With this in mind, \( \tau = \tau_{\text{max}} = 0.3P_\alpha \). The value of \( \sigma \) can be determined by the expression [8]:

\[
\sigma = \sigma_{\text{max}} \cos^2 \alpha + \sigma_{\text{min}} \sin^2 \alpha .
\]

In the formula (4) indicated: \( \sigma_{\text{sec}} = \sigma_t, \, \sigma_{\text{min}} = \sigma_c, \, \alpha = 45^\circ \).

After performing the calculations, we can determine, that \( \sigma = 0.48P_\alpha \).

It is known, that \( \sigma_t = 10\sigma_c \) [9] and \( \tau_{\text{max}} = 0.3P_\alpha \), \( \sigma = 0.48P_\alpha \), \( \sigma_c = 10\sigma_c \). Then the relationship (3) can be simplified and written:

\[
P_\alpha^2 - 2.8622P_\alpha\sigma_c - 0.59677\sigma_c^2 = 0 .
\]

The relationship (5) is a quadratic equation with variable \( P_\alpha \). We solve it. \( P_{ii} = 3.0573\sigma_c \), \( P_{ii} = -0.1950\sigma_c \).

It’s obvious that \( P_{ii} > 0, \, \sigma_c > 0 \). Then, the only solution satisfying the conditions is \( P_{ii} = 3.0573\sigma_c \). We substitute it in dependence (1) and express \( q \).

\[
q = \left( \frac{\sigma_c}{0.1845} \right)^2 \frac{\eta R_c R_i}{R_c - R_i} .
\]

It is known, that

\[
q = \frac{F}{L} .
\]

In the formula (7) indicated: \( F \) is the force acting on the indenter; \( L \) is the length of the indenter. We substitute relationship (7) into dependence (6) and express \( F \).

\[
F = \left( \frac{\sigma_c}{0.1845} \right)^2 \frac{\eta R_c R_i L}{R_c - R_i} .
\]

4. Results and discussion
Relationship (8) is a dependency describing the process of rock destruction. To verify it, the authors of the article conducted mathematical modeling in a computer program T-Flex. This software environment is used to calculate the destruction of solids by the finite element method.

Figure 2 shows a three-dimensional model of the interaction of the indenter and the rock. Limestone from the Talnakh deposit was imitated as a rock. The properties of limestone were determined in [9].

The contact surface of the indenter was in the form of a straight circular cylinder. The cylinder diameter was 8 mm. The length of the cylinder was equal to the diameter. The material of the indenter is the tungsten-cobalt alloy VK-15 [10]. The sizes of sample of limestone are shown in Figure 2. The diameter of the well in the limestone was set to 56 mm. The material properties of the indenter and rock are given in Table 1. The impact force on the indenter was calculated in accordance with dependence (8). It was 262 N.
The results of mathematical modeling allowed us to calculate the force on the indenter required for the destruction of the rock. Its value was 9% lower than the value calculated by the formula (8). In the process of modeling, the minimum possible finite element mesh was specified. For this, the model considered only the contact area and the adjacent surface. The dimensions of the adjacent surface were at least two diameters of the indenter. The accuracy of the calculations was 0.001 N.

5. Conclusion
The research results indicate the possibility of using the theory of contact problems to study the processes of rock destruction by indenters and mining tools. It should be noted that the results obtained must be confirmed by field experiments. According to the results of their implementation, the dependence (8) can be adjusted. Perhaps, it will require the introduction of a coefficient that takes into account the heterogeneity of the rock.

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