Quasi-particle perspective on QCD matter and critical end point effects

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Abstract
Our quasi-particle model is compared with recent lattice QCD data at finite temperature and baryon number density with emphasis on the coefficients in the Taylor series expansion of thermodynamic observables. The inclusion of static critical end point effects into the equation of state is discussed.

1 Introduction
The QCD phase diagram exhibits an astonishingly rich phase structure. The (pseudo-) critical line, which separates the phase dominated by quark and gluon degrees of freedom at large temperature $T$ and baryochemical potential $B$ from the one dominated by hadrons and resonances, has been investigated by means of lattice gauge theory [1, 2, 3, 4]. From theoretical reasoning [5, 6] for finite quark masses, the phase transition is of first order at finite $T$ and large $B$ ending in a critical point of second order. For smaller $B$, thermodynamic observables change rapidly but continuously indicating a crossover regime. There, the equation of state (EoS) has been computed for $N_f=2$ quark flavours [7, 8]. While the location of the critical end point (CEP) showing a strong quark mass dependence [3, 9] was determined by first principle QCD evaluations [4, 10], the extension of the critical region is fairly unknown. Lattice QCD studies of the volume dependence of the Binder cumulant [3, 11] indicate that the CEP belongs to the static universality class of the 3D Ising model. At present, many investigations aim to study implications of such a fundamental issue of QCD. In particular, observable consequences of the occurrence of the CEP as novel feature of QCD are discussed [12]. Having in mind the successful hydrodynamical description of the expansion stage in heavy-ion collisions, one intriguing problem concerns the manner the EoS of strongly interacting matter becomes modified by the CEP. This is the subject of the present contribution.

In [2], the quasi-particle model is shortly reviewed and compared with recent lattice QCD results. We focus on the Taylor series expansion coefficients [7, 8]. In [13], CEP effects on the EoS are discussed for a toy model and for our QCD based quasi-particle model. The results are summarized in [4].

2 Taylor series expansion of the EoS
Thermodynamic observables can be expressed as Taylor series expansions in powers of $B=0$. Accordingly, the pressure is decomposed into

$$ p(T; B) = \sum_{n=0}^{\infty} \frac{B^n}{3^n} c_n(T) ; $$

(1)

where $c_0(T) = p(T; B=0)=T^4$ and $c_n(T) = \delta^n p=0 \int_0^{\infty} \frac{T^k}{k!} d^k \delta x^n$ with $B=3$. In [7, 8], the Taylor expansion coefficients up to $c_6(T)$ have been presented basing on first principle QCD evaluations.

Achieving a flexible parametrization of the EoS, we formulated a model which describes the quark-gluon fluid in terms of quasi-particle excitations [13, 14]

$$ p(T; B) = \prod_{i=qg} \left[ \prod_{l=1}^{\infty} \frac{B(T; l)}{B(T; 0)} \right] ; $$

(2)
Here, $p_i$ denote thermodynamic standard expressions for quarks and transverse gluons with dynamically generated self-energies $\gamma_i$ and a non-perturbative effective coupling $G^2(\tau; )$ as essential input. Thermodynamic self-consistency is ensured through the stationarity conditions $p=\gamma_i=0$, imposing in turn conditions onto $B(\tau; )$. From Maxwell’s relation for $p$, a flow equation for $G^2(\tau; )$ follows [13]

$$\frac{a}{\Theta} \frac{\partial G^2}{\partial \Theta} + a_T \frac{\partial G^2}{\partial T} = b$$

Knowing $G^2$ on an arbitrary curve in the $T -$ plane, (3) can be solved as a Cauchy problem. For convenience, we adjust $G^2(\tau; = 0)$ to lattice data at $\tau = 0$ enabling a mapping into the finite chemical potential region via (4). The Taylor expansion coefficients follow straightforwardly from (2) as integral expressions involving $G^2$ and higher order derivatives of the effective coupling at vanishing chemical potential. The latter can be computed by exploiting the flow equation (3) (cf. [14] for details). In Fig. 1, a fairly good agreement between Taylor expansion coefficients from the quasi-particle model and lattice results is shown. In particular, the peak in $c_4(\tau)$ and the dipole structure in $c_6(\tau)$ are reproduced. Adjusting the parametrization of the effective coupling onto $c_0(\tau)$ dictates a change in $G^2(\tau; = 0)$ at $T_c = 170$ MeV from a regularized logarithmic dependence (resembling the perturbative behaviour at large $\tau$ ) into a linear dependence. This change in the curvature of $G^2$ can be considered as implemented phase transition and is responsible for the pronounced structures in $c_4, c_6(\tau)$. In fact, these structures disappear when neglecting higher order derivatives of the effective coupling in the integral expressions of $c_4, c_6(\tau)$ serving for a test of the flow equation (3).

### 3 Critical end point

Starting from a thermodynamic potential, e. g. Gibbs free enthalpy, it can be decomposed into an analytic and a non-analytic part where the latter is related to phase transitions and critical phenomena [15]. Accordingly, the EoS formulated in terms of the entropy density is given through $s = s_h + s_n$. Here, the an-
coefficients. The entropy density contributions are given by the same ansatz for the adiabatic expansion of the system and its related cooling, both chosen such that standard thermodynamic consistency conditions are satisfied. Accordingly, during the transition. By shrinking the critical region to the CEP acts as an attractor on trajectories on the left whereas on the right a repulsive impact is found. Evidently, the curves on the right side of the CEP display the existence of the first order phase transition. A convenient formulation of the entropy density contribution can be found in the pioneering work we rely on. In the following, we estimate the phase border line to be given by $T_c(B) = T_c - \frac{1}{2} d(B = 3T_c)^2$ with $d = 0.22$ according to and locate the CEP at $B = 360$ MeV in agreement with.

As a simple toy model, let us employ the first terms in (1), however, with constant expansion coefficients. The entropy density contributions are given by

$$s_n(T; B) = 4c_0 T^3 + \frac{2}{9} c_2 T^2 B; \quad s_n(T; B) = \frac{2}{9} c_2 T A \tanh \left( S_c(T; B) \right)$$

with $c_0 = (32 + 21N_f)^2 = 180$, $c_2 = N_f - 2$ and $N_f = 2$. $n_B$ follows from via standard thermodynamic relations (cf. [17]) with integration constant $n_B(0; B) = \frac{4}{3} c_4 (B = 3)^2$ where $c_4 = N_f - 4$. The ansatz for $s_n$ has been chosen such that $s_n = 0$ for $T > 0$ and the net baryon density vanishes at $B = 0$. The parameter $A$ describes the strength of the non-analytic contribution in the EoS. We apply the same $S_c(T; B)$ as in assuming a fairly large critical region parametrized by $T = 100$ MeV, $B = 200$ MeV and a stretch factor $D = 0.25$. Hence, CEP effects on the EoS and in particular on isentropic trajectories $s = n_B = \text{const.}$ in the $T - B$ plane can be demonstrated. In Fig. 2, the influence of the strength parameter $A$ on the behaviour of isentropic trajectories is exhibited. For increasing $A > 0$, the trajectories for large $s = n_B$ tend to be attracted towards larger $B$ due to the presence of the CEP. In fact, the CEP acts as an attractor on trajectories on the left whereas on the right a repulsive impact is found. Evidently, the curves on the right side of the CEP display the existence of the first order phase transition. By shrinking the critical region to $T = 10$ MeV, $B = 10$ MeV and $D = 0.06$, the influence of the CEP decreases (right panel of Fig. 2) in comparison with the results obtained for a large critical region (left panel of Fig. 2). In particular, the sections on the hadronic side become less affected when decreasing the extension of the critical region.

The parameters in the non-analytic entropy density contribution and in particular $A$ have to be chosen such that standard thermodynamic consistency conditions are satisfied. Accordingly, during the adiabatic expansion of the system and its related cooling, both $n_B$ and $s$ must decrease. For $A < 0$
4 Conclusion

Our quasi-particle model without implemented CEP was successfully compared with recent lattice QCD results of the Taylor series expansion coefficients $c_0(T)$ and $c_2(T)$. Accordingly, our extrapolation procedure into the finite chemical potential region was tested. We considered simple models including phenomenologically the QCD critical end point and studied the effects on isentropic trajectories. We followed [17] and looked for indications of the CEP acting generically as attractor or repulsor. In fact, this is of interest with respect to the question whether CEP effects show up in heavy ion experiments only
in a very narrow beam energy range. Clearly, appropriate dynamical simulations are needed to account properly for such questions. The study of the pattern of isentropic trajectories is only a first step towards elucidating possible implications of the very existence of the CEP in QCD.

Inspiring discussions with M. Asakawa and F. Karsch are greatly acknowledged. The work is supported by BMBF, GSI, EU-I3HP.

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