Non-relativistic Supersymmetry

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We construct an $N = 1$ supersymmetric gauge theory from $z = 3$ Lifshitz field theory. By modifying the supersymmetry (susy) algebra based on the spacetime symmetry $SO(3) \times$ scaling symmetry, we get a supersymmetric Lagrangian with scalar, fermion and gauge fields, all of whom have the same limiting speed. This solves some naturalness problems of the original Lifshitz theory which is characterized by Lorentz symmetry violation. In order that the susy breaking does not introduce any disastrous terms into the theory, the susy breaking scale is required to be smaller than the scale of Lorentz symmetry violation.

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1. INTRODUCTION

Theory and experiments have been exploring possible violations of the Lorentz symmetry for decades. On the other hand, Lorentz symmetry has been very successful in the Standard Model of Particle Physics and in General Relativity. Low energy Lorentz violation possibly originates from high energy physics, such as quantum gravity and string theory. A well-known example is that non-commutative geometry in string theory due to a non-zero tensor $B_{\mu\nu}$ on a D-brane leads to Lorentz symmetry violation [16]. Recently, a new Lorentz symmetry violating theory of gravity, Hořava-Lifshitz Gravity [13], has been developed. In this theory, gravity is power-counting renormalizable and unitarity is maintained. More generally, it is possible to detect the hint of high energy physics from low energy effective field theory since the high energy physics leads to a modification of the dispersion relations of different particles [17]. Motivated by this, there have been many searches for signs of Lorentz symmetry violating phenomena in terrestrial, astrophysical and cosmological settings [1, 2, 11, 15, 18], and the constraints on the magnitude of such phenomena are becoming better and better.

The fact that different particles have different limiting speeds will open up the possibility of Cerenkov radiation for kinetic reasons, which is forbidden if Lorentz symmetry is kept. The most stringent constraint is from the highest energy cosmic rays. If Lorentz symmetry is broken, then there is no reason to keep the limiting speed the same in the dispersion relation of different particles. That the high energy cosmic rays which are possibly composed of hadrons, travelling astrophysical distances and times with energy of about $3 \times 10^{11}$ GeV gives the following constraint on the limiting speeds of protons $c_p$ and photons $c_\gamma$ [7]:

$$\frac{c_p - c_\gamma}{c_\gamma} < 10^{-23}. \quad (1)$$

Even if the two limiting speeds are the same at tree level, if there is no specific symmetry it is hard to keep them the same at all orders without fine-tuning. Moreover, dimension three operators which break Lorentz invariance are also highly constrained [8]. The requirement is that their coupling constants should be much less than the Lorentz violation scale from simple dimensional counting.

Here we introduce supersymmetry as the candidate to solve the naturalness problem mentioned above for theories with Lorentz symmetry violation. Supersymmetry is an extended symmetry of spacetime. In general, the algebra of supersymmetry is built on the Poincare
algebra. But here we are considering theories with Lorentz symmetry violation and thus the initial symmetry of space-time is not the Poincare symmetry. In the context of Hořava-Lifshitz gravity the initial symmetry is \( SO(3) \), the group of spatial rotations. Here, we will construct a new supersymmetry algebra starting from \( SO(3) \) rather than the Lorentz group \( SO(3,1) \).

In a supersymmetric theory the bosons and fermions which are in the same multiplet will by symmetry have the same dispersion relation. This will help us solve the naturalness problem (1). Therefore, if all the particle are in the same multiplet, such as in \( N = 8 \) supergravity or \( N = 4 \) super-Yang-Mills theory, the naturalness problem (1) should be completely absent. If not all particles are in the same multiplet, for example in the Minimal Supersymmetric Standard Model (MSSM), a gauge symmetry in combination with supersymmetry can keep the limiting speed of all particles the same.

In [4, 5, 12], the authors studied Lorentz violation in supersymmetric models in which the superalgebra itself did not violate the Lorentz symmetry, which is motivated from the low energy effective field theory point of view. At high energy, the superalgebra needs to be changed because the spacetime symmetry explicitly breaks Lorentz invariance. To study this problem, we need to work in the context of a specific theory with Lorentz violation at high energy, which is the reason that we here use Hořava-Lifshitz theory to construct supersymmetry. This theory is complete in the ultraviolet (UV), and in the infrared (IR) the Lorentz symmetry is emergent. The supersymmetry constructed here can be extended to other Lorentz violating theories.

We shall start with the spacetime symmetry and scaling symmetry of the Lorentz violating Lifshitz theory in Sec. 2. We derive the supersymmetry generators and their commutators in Sec. 3 from the free field Lagrangian. Then we will use superspace language to construct the susy lagrangian and interactions in Sec. 4. So far, the discussion does not contain gauge fields. By introducing a gauge symmetry in Sec. 5, we can ensure that different particles in different multiplets also have the same limiting speed, which solve the naturalness problem. In Sec. 6, we will discuss the relationship between susy breaking and Lorentz violation, and we will give the conclusion in the last section.
2. \textit{SO(3) \times SCALING SYMMETRY}

Lifshitz theory explicitly breaks Lorentz invariance, and the remaining symmetry is \textit{SO(3)\times Scaling Symmetry}, spatial rotations and the anisotropic scaling between space and time introduced in [13]. With these symmetries, there is no reason that the limiting velocities of all species be the same. Demanding agreement with the cosmic ray experiments would require a fine-tuning of the parameters in the model to unbelievable accuracy. In order to avoid having to do this fine-tuning, we try to extend the \textit{SO(3)} symmetry by adding supersymmetry.

The Lorentz symmetry \textit{ISO(3,1)} algebra is composed of commutators of the translation generator \(P_\mu\) and the spacetime rotation generator \(M_{\mu\nu}\),

\[
[P_\mu, P_\nu] = 0 \quad (2)
\]

\[
[M_{\mu\nu}, P_\lambda] = i (\eta_{\nu\lambda} P_\mu - \eta_{\mu\lambda} P_\nu) \quad (3)
\]

\[
[M_{\mu\nu}, M_{\lambda\sigma}] = i (\eta_{\nu\lambda} M_{\mu\sigma} + \eta_{\mu\sigma} M_{\nu\lambda} - \eta_{\mu\lambda} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\lambda}) \quad (4)
\]

To introduce supersymmetry, we add two-component Weyl spinor generator \(Q_\alpha\) and \(\bar{Q}_{\dot{\alpha}}\) to compose the extended algebra,

\[
[M_{\mu\nu}, Q_\alpha] = -i (\sigma_{\mu\nu})^\beta_\alpha Q_\beta \quad (5)
\]

\[
[M_{\mu\nu}, \bar{Q}_{\dot{\alpha}}] = -i (\bar{\sigma}_{\mu\nu})^\dot{\beta}_{\dot{\alpha}} \bar{Q}^{\dot{\beta}} \quad (6)
\]

\[
[P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_{\dot{\alpha}}] = 0 \quad (7)
\]

\[
\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \quad (8)
\]

\[
\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2 \sigma^\mu_{\alpha\dot{\beta}} P_\mu \quad (9)
\]

In our case we do not have boost invariance. Hence, we set the boost generator

\[
M_{0i} = 0 \quad ,
\]

which reduces the spacetime symmetry to \textit{SO(3)}. The symmetry group \textit{SO(3)} is equivalent to \textit{SU(2)}, while \textit{SO(3,1)} is equivalent to \textit{SU(2) \times SU(2)}. To obtain a supersymmetric generator algebra in the case of \textit{SO(3)} the two spinors can be identified as

\[
Q_\alpha^* = \bar{Q}_{\dot{\alpha}} \quad .
\]
The other symmetry that we mentioned in Lifshitz theory is scaling symmetry, which posits that the Lagrangian is invariant under the following anisotropic scaling of space and time:
\[
 x \rightarrow e^{-\Omega}x, \quad t \rightarrow e^{-z\Omega}t,
\]
where \( z \) is taken to be an integer chosen as \( z = 3 \) to obtain a theory of gravity in 4 spacetime dimension. If we consider the momentum conjugate of the spacetime coordinate, the relationship between energy and momentum is as follows,
\[
 (\Delta p)^z \sim \Delta E,
\]
where \( p \) is the spatial momentum. This relationship will be reflected in the dispersion relation.

In the following section we will discuss how the scaling symmetry impacts on the supersymmetry from which we can derive the right form of the spinor generator and of the renormalizable supersymmetric Lagrangian. It will modify the susy algebra (9).

3. SUPERSYMMETRIC LAGRANGIAN

In this section, we will from the most basic boson and fermion Lagrangian derive the explicit form of the spinor generator. The simplest free boson Lagrangian with two scalar fields is
\[
 \mathcal{L}_s = \partial_0 \phi^* \partial_0 \phi - \partial_i \partial^2 \phi^* \partial_i \partial^2 \phi.
\]
As a first step, we neglected other kinetic terms which are super-renormalizable in the \( z = 3 \) Lifshitz theory. These additional terms are important when discussing low-energy phenomenology. However, just from the simplified case taken above, we can clearly see the physics. Given the above bosonic Lagrangian, the Lagrangian for the two Weyl fermions which are the supersymmetric partners of the bosons is as follows
\[
 \mathcal{L}_f = i \psi^\dagger \sigma^0 \partial_0 \psi + i \psi^\dagger \sigma^i \partial_i \partial^2 \psi,
\]
where \( \sigma^0 \) is the identity matrix and \( \bar{\sigma}^i \) is conjugate of \( \sigma^i \)

The susy transformation should change \( \phi \) to \( \psi \),
\[
 \delta \phi = \epsilon \psi
\]
\[ \delta \phi^* = \epsilon^\dagger \psi^\dagger, \]  \hspace{1cm} \text{(17)}

while the transformation from fermions to bosons can be derived from the requirement that the symmetry is obeyed by the action. The result is

\[ \delta \psi = -i \epsilon^\dagger \partial_0 \phi - i \sigma^i \epsilon^\dagger \partial_i \partial^2 \phi \]  \hspace{1cm} \text{(18)}

\[ \delta \psi^\dagger = i \epsilon \partial_0 \phi^* + i \sigma^i \partial_i \partial^2 \phi^*. \]  \hspace{1cm} \text{(19)}

The whole action is invariant under these transformation up to some total derivatives. The commutator of the supersymmetric spinor generators is derived from the commutator \( \delta_1 \delta_2 - \delta_2 \delta_1 \).

\[ \{ Q_\alpha, \bar{Q}_\beta \} = 2 \sigma^0 P_0 + 2 \sigma^i P^2 P_i, \]  \hspace{1cm} \text{(20)}

where \( P^2 = P^i P_i \).

If supersymmetry is not broken, it is easy to see that the speed of light of the two superpartners are the same, because the supersymmetric transformation fixes the coefficients in front of \( p^3 \) for fermions and of \( p^6 \) for bosons.

### 4. SUPERSPACE AND SUPERDERIVATIVES

In this section, we will derive the superspace formulation of the modified supersymmetry proposed in the previous section. Because the dimension of the superfield is zero in the \( z = 3 \) Lifshitz theory, the D-term and F-term in the theory are different from the case of regular supersymmetry. In the end, we will give the interaction Lagrangian which includes some new interactions between bosons and fermions.

Constructing supersymmetry can proceed by two methods, one is by direct construction as above, and the other is from the superspace formalism making use of general superfields. In the following we illustrate this superfield method, a method which makes it easy to see why the “speed of light” is the same for different species, and which also makes it easy to deal with interactions. Below, \( \theta \) denotes the superspace variable.

A superfield transforms as follows under the action of the extended spacetime symmetry,

\[ S(x^\mu, \theta, \theta^\dagger) \rightarrow \exp(i(\xi Q + \xi^\dagger Q^\dagger - a^\mu P_\mu)) S(x^\mu, \theta, \theta^\dagger) \]  \hspace{1cm} \text{(21)}

where the generators are

\[ P_\mu = i \partial_\mu \]  \hspace{1cm} \text{(22)}
\[ iQ_\alpha = \frac{\partial}{\partial \theta_\alpha} - i\tilde{\sigma}^{\mu}_{\alpha \beta} \theta^\beta \partial_\mu \] (23)

\[ iQ^\dagger_\beta = \frac{\partial}{\partial \theta^\dagger_\alpha} - i\theta^\alpha \tilde{\sigma}^{\mu}_{\alpha \beta} \partial_\mu . \] (24)

\( \tilde{\sigma} \) is a newly defined sigma matrix in the Lifshitz theory,

\[ \tilde{\sigma}^0 = \sigma^0 \] (25)

\[ \tilde{\sigma}^i = \sigma^i f(P) . \] (26)

The information about the scaling symmetry of Lifshitz theory is hiding in the redefined sigma matrix \( \tilde{\sigma}^\mu \). After the redefinition, the susy algebra and superfield are written in the same form as the regular supersymmetry. \( f(P) \) is a function of the magnitude of the spatial momentum. In the \( z = 2 \) case, the spinor generator algebra is simpler, \( \{ Q_\alpha, \tilde{Q}_\beta \} = 2P^2 \). In Sec. 3, we considered the marginal kinetic operator, \( f(p) = P^2 \). Including some super-renormalizable kinetic operators, \( f(P) = 1 + P^2 \), and in the low energy limit, the supersymmetry algebra returns to the algebra with Lorentz symmetry.

From the spinor generator algebra,

\[ \{ Q_\alpha, Q^\dagger_\beta \} = 2\tilde{\sigma}^{\mu} P_\mu , \] (27)

we can derive the fermionic derivatives \( D_\alpha \) and \( D^\dagger_\beta \), which anticommute with the supersymmetric generator, and are written as follows

\[ D_\alpha = \frac{\partial}{\partial \theta_\alpha} + i\tilde{\sigma}^{\mu}_{\alpha \beta} \theta^\beta \partial_\mu \] (28)

\[ D^\dagger_\beta = -\frac{\partial}{\partial \theta^\dagger_\alpha} - i\theta^\alpha \tilde{\sigma}^{\mu}_{\alpha \beta} \partial_\mu . \] (29)

Applying the superderivatives as a constraint, the chiral superfield \( \Phi \) can be defined, which satisfies

\[ D^\dagger_\beta \Phi = 0 . \] (30)

Almost all the formulas are the same as in the case of regular susy if we use the redefined \( \tilde{\sigma} \). In particular, the superfield can be expanded in superspace as

\[ \Phi(x^\mu, \theta, \theta^\dagger) = \phi + \sqrt{2}\theta \psi + \theta \theta F + i\theta \tilde{\sigma}^{\mu} \theta^\dagger \partial_\mu \phi + \frac{i}{\sqrt{2}} \theta \theta \theta^\dagger \tilde{\sigma}^{\mu} \partial_\mu \psi \]

\[ -\frac{1}{4}(\partial^0 \partial^0 + \partial_i \partial^i P^4)\phi \theta \theta \theta^\dagger \theta^\dagger , \] (31)
where
\[ \partial_0 \partial^0 + \partial_i \partial^i P^4 \equiv \hat{\partial}_\mu \hat{\partial}^\mu \] (32)
and where \( F \) is the auxiliary field.

The dimension of all the operators in the superfield can be derived by power counting,

\[
\begin{array}{c|cccc}
\text{dim} & \phi \text{ and } \Phi & \psi & \theta & F \\
\hline
& \frac{3-z}{2} & \frac{3}{2} & -\frac{z}{2} & z
\end{array}
\]

The kinetic term comes from the D term of \( \Phi^\dagger \Phi \)

\[
\left[ \Phi_i^\dagger \Phi_j \right]_D = \left[ \Phi_i^\dagger \Phi_j \right]_{\text{coeff. of } \theta^\dagger \theta^\dagger \theta \theta} \\
= F_i^\dagger F_j + \frac{1}{2} \hat{\partial}_\mu \phi_i^\dagger \hat{\partial}^\mu \phi_j - \frac{1}{4} \phi_i^\dagger \hat{\partial}_\mu \hat{\partial}^\mu \phi_j - \frac{1}{4} \hat{\partial}_\mu \hat{\partial}^\mu \phi_i^\dagger \phi_j \\
+ i \frac{1}{2} \psi_i^\dagger \hat{\partial}^\mu \hat{\partial}_\mu \psi_j + i \frac{1}{2} \hat{\partial}_\mu \psi_i^\dagger \hat{\partial}^\mu \psi_j.
\]
(33)

There are new terms in the Lagrangian of the form,

\[ \mathcal{L}_{\text{NEW}} = [(\Phi^\dagger \Phi)^n]_D, \]
(34)

where \( n \) is an integer, and in the limit of low energies, the smaller \( n \) terms dominate, since the scale of Lorentz violation suppresses these terms by a factor \( (\frac{\phi}{M})^n \). And there are other possible terms \( [\Phi^\dagger \Phi \Phi^n]_D + h.c. \) in the lagrangian which are forbidden by \( U(1) \) or other symmetries.

The superpotential is constructed by holomorphic function of superfield,

\[ W(\Phi) = \sum_{i=1}^{\infty} g_n \Phi^n \]
(35)

and yields the interaction Lagrangian,

\[ \mathcal{L}_{\text{int}} = \int \text{d}^4 x \left[ W \right]_F . \]
(36)

\[ \mathcal{L}_{\text{int}} = - \left( \frac{\partial W(\phi)}{\partial \phi_i} \right) \cdot F - \frac{1}{2} \frac{\partial W(\phi)}{\partial \phi_i \phi_j} \psi_i \psi_j + h.c. \]
(37)

The first term includes \( \left( \frac{\partial W(\phi)}{\partial \phi_i} \right)^2 \), since there is a \( F^2 \) term from the kinetical D-term. However, in Lifshitz theory, we introduce many new D-terms which contain other terms depending on \( F \), for example, the term \([\lambda (\Phi^\dagger \Phi)^2]_D \). There are no four fermion terms from the

\[^1\text{Please note the different partial derivative operators for fermions and bosons in the formula, one of which has a hat, while the other one does not have a hat.}\]
F-term due to the fact that $\theta^4 = 0$, but the newly introduced D term will give a four-fermion interaction. We write down the terms having F and four fermion interaction,
\[
\left[\lambda (\Phi^\dagger \Phi)^2\right]_D \supset \lambda (2\phi^\dagger \phi F^\dagger F - 2F^\dagger \psi \phi^\dagger - 2F \bar{\psi} \psi \phi + \lambda \bar{\psi} \psi \bar{\psi} \psi) \quad (38)
\]
After some calculation, we obtain the interaction terms written without the auxiliary field $F$,
\[
\mathcal{L}_{int} = -\left(\frac{\partial W}{\partial \phi} - 2\lambda \phi \bar{\psi} \psi\right) \left(\frac{\partial W}{\partial \phi^\dagger} - 2\lambda \phi^\dagger \psi \psi\right) + \lambda \bar{\psi} \psi \bar{\psi} \psi - (\frac{1}{2} \frac{\partial W}{\partial \phi_i \phi_j}) \psi_i \psi_j + h.c) \quad (39)
\]
This potential could be from usual scalar field vacuum expectation value, and also possibly from fermion condensation. Also, from the interaction Lagrangian, there are new phenomena which follow from the low energy effective field theory and which depend on the form of the superpotential.

5. GAUGE FIELD AND GAUGINO

Bosons and fermions in the same supermultiplet have the same limiting speed because of supersymmetry. It is essential to see whether we can arrange for particles in different multiplets to have identical limiting speed by introducing a gauge symmetry and relating particles in different supermultiplets by gauge transformations. If two supermultiplets are totally decoupled, then there is no reason that keep the coefficients of the kinetic terms the same.

First of all, consider the kinetic term of a chiral superfield which is derived from $\mathcal{L}_{kin} = \left[\Phi^\dagger \Phi\right]_D$. The supersymmetry transformation $Q$ fixes the form of $D_\alpha$, and then the form of the chiral supermultiplets is the same across all multiplets, and therefore the limiting speed of the two chiral supermultiplets are the same. We will see the coefficients of vector supermultiplets are also the same as that of the chiral supermultiplet since the gauge symmetry relates the two types of supermultiplets.

A general vector supermultiplet can be written as
\[
V(x, \theta, \theta^\dagger) = \mathcal{L} + i\theta \chi - i\theta^\dagger \chi^\dagger + \frac{1}{2} i\theta \theta \left[M + iN\right] - \frac{1}{2} i\theta^\dagger \theta^\dagger \left[M - iN\right]
\]
\[
\theta \tilde{\sigma}_\mu^I \theta^I \partial_{\mu} \chi + i\theta \theta \theta^\dagger \left[\lambda^I + i\tilde{\sigma}_\mu^I \partial_{\mu} \chi\right] - i\theta^\dagger \theta^I \theta \left[\lambda + i\tilde{\sigma}_\mu^I \partial_{\mu} \chi\right]
\]
\[
+ \frac{1}{2} i\theta \theta \theta^\dagger \theta^\dagger \left[D - \frac{1}{2} \tilde{\sigma}_\mu^I \tilde{\sigma}^\mu \chi\right], \quad (40)
\]
where $\tilde{\sigma}_{1\sigma 2}$ is some newly defined sigma matrix which only obeys the $SO(3)$ spatial symmetry. And $C, M, N, D$ are scalar fields, $\chi, \lambda$ are Weyl spinor, and $V_\mu$ is a vector field. A gauge transformation leads to the following change in the form of a superfield:

$$V(x, \theta, \theta^\dagger) \to V(x, \theta, \theta^\dagger) + i(\Phi(x, \theta, \theta^\dagger) - \Phi^\dagger(x, \theta, \theta^\dagger)),$$

where

$$\Phi(x^\mu, \theta, \theta^\dagger) = (\phi - \phi^\dagger) + \sqrt{2}(\theta\psi - \theta^\dagger\bar{\psi}) + \theta(\bar{F} - F^\dagger) + i\theta\hat{\sigma}^\mu\theta^\dagger \partial_\mu(\phi - \phi^\dagger)$$

$$+ \frac{i}{\sqrt{2}}\theta\theta^\dagger\hat{\sigma}^\mu\partial_\mu\psi - \frac{i}{\sqrt{2}}\theta^\dagger\theta^\dagger\hat{\sigma}^\mu\partial_\mu\bar{\psi} - \frac{1}{4}\theta\theta^\dagger\theta^\dagger\partial_\mu(\phi - \phi^\dagger).$$

The way to construct a gauge symmetry is to introduce a chiral superfield, by which the sigma term is fixed, $\tilde{\sigma} = \hat{\sigma}$. Therefore, if the gauge field is coupled to the chiral supermultiplet, the coefficients of the kinetic terms are the same for all particles.

The component $V_\mu$ in the vector supermultiplet is a regular gauge field, and the field strength is derived from the field strength superfield $W_\alpha = D^{\dagger 2}D_\alpha V$,

$$W_\alpha(y, \theta) = i\lambda_\alpha - \frac{1}{2}(\hat{\sigma}^\mu\hat{\sigma}^\nu)_{\alpha\beta}V_{\mu\nu}(y) \theta_\beta + \theta^2\hat{\sigma}_{\alpha\beta}\partial\lambda^{\dagger\beta}$$

where $V_{\mu\nu}$ is the field strength

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu.$$

The Lagrangian of the Super-Yang-Mills field is the F-term of $W_\alpha W^\alpha$,

$$\mathcal{L} = \frac{1}{4}[W_\alpha W^\alpha]_F + h.c.$$

$$= -\frac{1}{4}\hat{V}_{\mu\nu}\hat{V}^{\mu\nu} + i\lambda\hat{\sigma}^\mu\partial_\mu\lambda^\dagger - \frac{1}{4}\hat{V}_{\mu\nu}(\ast\hat{V}_{\mu\nu}),$$

where the redefined field strength is

$$\hat{V}_{\mu\nu} = \hat{\partial}_\mu V_\nu - \hat{\partial}_\nu V_\mu,$$

and all the derivative terms in the chiral field change from being normal partial derivatives to gauge covariant derivatives, but note that $\hat{\sigma}^\mu$ does not change. The covariant kinetic term of chiral superfield is as follows,

$$\int d^4\theta\Phi^\dagger e^{\phi^V}\Phi.$$
Note that our construction of the gauge theory is different from the suggestion of [6]. In order to make the Lagrangian gauge invariant, it follows from the gauge transformation of a derivative term of a matter field that we need introduce the gauge field with the infinitesimal transformation rule,

$$\delta A_\mu = \hat{\partial}_\mu \epsilon + if\epsilon A_\mu ,$$

where $f$ is the structure constant. In the Abelian gauge field case, $f = 0$.

### 6. SUSY BREAKING

The $N = 0$ Lagrangian contains terms such as $\partial_i^2 \phi \phi^A$, which is renormalizable in the $z = 3$ case, so the loop corrections will make the speed of light run with the scale. In the supersymmetric Lifshitz theory, there is the same interaction term containing derivatives in this way. This will lead to a quantum correction of the speed of light, but the supersymmetry will cancel the loop effect between bosons and fermions. Hence, this renders the theory free of the need of fine-tuning the speed of light of different particles.

If SUSY is broken, there is a soft supersymmetry breaking term introduced which make the speed of light of particles different. One way to keep the difference small or absent is to break supersymmetry at a low energy scale (Lorentz symmetry is emergent which is equivalent to $z = 1$ Lifshitz theory), and then the correction to the speed of light is not a soft term any more, and cannot yield different corrections to fermions and bosons from the Hidden Sector. On the other hand, if supersymmetry breaks around or above the scale of Lorentz symmetry violation, the speed of light becomes related to a soft term, which makes it suffer from the fine-tuning problem to obtain agreement with the results from the cosmic ray experiments. Therefore, we obtain the constraint that the susy breaking scale should be smaller than the Lorentz violation scale.

### 7. DISCUSSION

In this paper, we have constructed an $N = 1$ supersymmetric theory with scalar fields, gauge fields and fermion fields from a UV complete theory, the $z = 3$ Lifshitz theory. The different particles have the same kinetic Lagrangian and hence the same limiting speed, which can solve the naturalness problem in theories with Lorentz symmetry violation.
Our starting point is a supersymmetry algebra which is different from what is presented in previous papers on supersymmetric Lorentz violating theory [3, 4, 12]. However, in the low energy limit, the supersymmetry algebra will be the same. Concerning the dimension three Lorentz violating operators, they do not arise in the Lifshitz theory, and thus this is also true in the supersymmetric case.

Since supersymmetry will make the loop calculations much different from the $N = 0$ field theory case, it is important to calculate the renormalization group and find the fixed point of the Lifshitz theory in the supersymmetric framework. In the original paper on Hořava-Lifshitz theory [13], a power-counting renormalizable theory of gravity, the author pointed that in the UV, and in the $z = 3$ case, there is a free-field fixed point. In [9, 10], the authors studied the $z = 3$ Lifshitz theory with fermions and gauge fields and calculated its renormalization group. And in [14], the authors studied the one-loop renormalization of scalar field in Lifshitz theory. By introducing supersymmetry, there should be some changes in the renormalization group equations and in the conclusions concerning fixed points.

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