Comment on “Infrared and pinching singularities in out of equilibrium QCD plasmas”

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Abstract

Analyzing the dilepton production from out of equilibrium quark-gluon plasma, Le Bellac and Mabilat have recently pointed out that, in the reaction rate, the cancellation of mass (collinear) singularities takes place only in physical gauges, and not in covariant gauges. They then have estimated the contribution involving pinching singularities. After giving a general argument for the gauge independence of the production rate, we explicitly confirm the gauge independence of the mass-singular part. The contribution involving pinching singularities develops mass singularities, which is also gauge dependent. This “additional” contribution to the singular part is responsible for the gauge independence of the “total” singular part. We give a sufficient condition, under which cancellation of mass singularities takes place.

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In the past years, much effort has been made to incorporate quantum field theory with nonequilibrium statistical mechanics, among which, we quote those of Altherr and Seibert \[1\], Altherr \[2\], Baier et al \[3\], Le Bellac and Mabilat \[4\] and the present author \[5\]. Out of equilibrium, pinching singularities appear \[1\] in association with self-energy inserted propagator. It has been shown in \[2\] that resummation of self-energy part eliminates the pinching singularity (see also \[6\]). An application of this result to the rate of hard-photon production from nonequilibrium quark-gluon plasmas is made in \[3\]. A renormalization scheme of number density is introduced in \[5\], such that the pinching singularities disappear. Le Bellac and Mabilat \[4\] are the first who have explicitly analyzed the infrared and mass (collinear) singularities in “lepton-pair” production rate.

First of all, let us summarize the results of \[4\]. The production rate of a lepton pair from a quark-gluon plasma is proportional to

$$\Pi(Q) = -\Pi_{12}(Q),$$

where $\Pi_{12}(Q)$ is the $(1,2)$-component of the photon self-energy part in real-time massless QCD. Following \[4\], we deal with $\Pi(Q)$ of a scalar “photon.” To two-loop order, $\Pi(Q)$ receives two contributions. The one $\Pi_{\Sigma}$ comes from the diagram with self-energy inserted quark propagator and the one $\Pi_{V}$ comes from the diagram with photon-quark vertex correction,

$$\Pi(Q) = \Pi_{V}(Q) + \Pi_{\Sigma}(Q),$$

$$\Pi_{\Sigma}(Q) = 2ie^2 \int \frac{d^4P}{(2\pi)^4} \sum_{j,l=1}^{2} \text{Tr} \left[ S_{ij}(P) \Sigma_{jl}(P) S_{l2}(P) S_{21}(P - Q) \right],$$

$$\Pi_{V}(Q) = -\frac{4}{3} e^2 g^2 \int \frac{d^4K}{(2\pi)^4} \int \frac{d^4K}{(2\pi)^4} g_{\mu\nu}(\text{gauge}) \sum_{j,l=1}^{2} (-)^{j+l} \text{Tr} \left[ S_{ij}(P - K) \gamma^\mu S_{jl}(P) \right] \times S_{2l}(P - Q) \gamma^\nu S_{l1}(P - Q - K) ] \Delta_{lj}(K),$$

where

$$\Sigma_{jl}(P) = i\frac{4}{3} g^2 (-)^{j+l} \int \frac{d^4K}{(2\pi)^4} g^{(\text{gauge})}_{\mu\nu}(K) \gamma^\mu S_{jl}(P - K) \gamma^\nu \Delta_{lj}(K).$$

The (part of the) gluon propagator $\Delta_{lj}(K)$ takes the form \[4\]

$$\Delta_{11}(K) = \Delta_{22}(K) = (1 + f(K)) \Delta_{R}(K) + f(K) \Delta_{A}(K),$$

$$\Delta_{12}(K) = f(K) (\Delta_{R}(K) + \Delta_{A}(K)), $$

$$\Delta_{21}(K) = (1 + f(K)) (\Delta_{R}(K) + \Delta_{A}(K)).$$
where \( \Delta_{R,A}(K) = \pm i/(K^2 \pm i0^+ \epsilon(k_0)) \). The quark propagator \( S_{ij}(K) \) takes the form \( S_{ij}(K) = \hat{K} \hat{\Delta}_{ij}(K) \), where \( \hat{\Delta}_{ij}(K) \) is given by (3) with the substitution \( f \rightarrow -\tilde{f} \). \( f(\tilde{f}) \) is related to the distribution function of gluon \( n \) (quark \( \tilde{n} \)) through

\[
\begin{align*}
    f(K) &= -\theta(-k_0) + \epsilon(k_0)n(|k_0|, \epsilon(k_0)\hat{k}) \\
    \tilde{f}(K) &= \theta(-k_0) + \epsilon(k_0)\tilde{n}(|k_0|, \epsilon(k_0)\hat{k})
\end{align*}
\]

where \( \hat{k} = k/k \) with \( k = |k| \). Form of \( g^{(\text{gauge})}_{\mu\nu}(K) \) in (3) and (4) depends on the gauge choice:

\[
\begin{align*}
    g^{(\text{cov})}_{\mu\nu}(K) &= g_{\mu\nu} - \eta \frac{K_\mu K_\nu}{K^2} \quad \text{(covariant gauge)}, \\
    g^{(t)}_{\mu\nu}(K) &= g_{\mu\nu} - \frac{k_0}{k^2}(K_\mu n_\nu + n_\mu K_\nu) + \frac{K_\mu K_\nu}{k^2} \quad \text{(Coulomb gauge)}
\end{align*}
\]

where \( n^\mu = (1, 0) \). Observe that

\[
\sum_{j,l=1}^{2} S_{1j}(P)\Sigma_{jl}(P)S_{12}(P) = F^{(n)}(P) + F^{(p)}(P),
\]

\[
F^{(n)}(P) = -\tilde{f}(P)(\Delta_{R}^2(P) - \Delta_{A}^2(P))\hat{\Sigma} \Re \Sigma_{11}(P)\hat{\Sigma} \\
- \frac{1}{2} \tilde{f}(P)(\Delta_{R}^2(P) + \Delta_{A}^2(P))\hat{\Sigma}(\Sigma_{12}(P) - \Sigma_{21}(P))\hat{\Sigma},
\]

\[
F^{(p)}(P) = \Delta_{R}(P)\Delta_{A}(P)(1 - \tilde{f}(P))\Sigma_{12}(P) + \tilde{f}(P)\Sigma_{21}(P)\hat{\Sigma}.
\]

\( F^{(n)} \) is the “normal term,” which is the counterpart of the one that is present in equilibrium thermal field theory (ETFT), while \( F^{(p)} \) is the “pinch term,” which is absent in ETFT. Substituting (9) - (11) into (2), we have, with obvious notation,

\[
\Pi_{\Sigma}(Q) = \Pi^{(n)}_{\Sigma}(Q) + \Pi^{(p)}_{\Sigma}(Q).
\]

Le Bellac and Mabilat have shown that, in Coulomb gauge, the mass singularities cancel out both in \( \Pi^{(n)}_{\Sigma}(Q) \) and \( \Pi_{V}(Q) \), Eq. (3). While, in the covariant gauge, the cancellation holds only for \( \Pi^{(n)}_{\Sigma}(Q) \), and in \( \Pi_{V}(Q) \) there survives mass singularity. Then, the authors of have concluded that whether or not the singularity cancellation takes place is gauge dependent. [In Appendix A, we show in a gauge-independent manner how the cancellations of mass singularities take place in \( \Pi^{(n)}_{\Sigma}(Q) \), reconfirming the result of.]
We first verify that $\Pi(Q)$, Eq. (1), is gauge independent. Proof goes just as in vacuum ($T = 0$) theory. Consider the difference

$$\delta \Pi(Q) \equiv \Pi_{\Sigma}(Q)\bigg|_{\text{covariant}} - \Pi_{\Sigma}(Q)\bigg|_{\text{Coul}},$$

where "Coul" stands for Coulomb. Observe that, from (7) and (8), the difference $g_{\mu\nu}^{(\text{cov})}(K) - g_{\mu\nu}^{(t)}(K)$ is proportional to $K_\mu$ and/or $K_\nu$. Then, in evaluating $\delta \Pi(Q)$, we can use Ward-Takahashi relation,

$$S_{jk}(P - K)K S_{kl}(P) = -i(-)^k \delta_{kl} S_{jk}(P - K) + i(-)^j \delta_{jk} S_{kl}(P),$$

with no summation over repeated indices. After doing this, we see that, among many terms in the resultant expression for $\delta \Pi(Q)$, complete cancellations occur, so that $\delta \Pi(Q)$ vanishes and then is, of course, free from mass singularities.

On the light of the above observation, let us make a closer inspection of the results of [4]. As a covariant gauge, as in [4], we take the Feynman gauge ($\eta = 0$ in (7)) throughout in the sequel. We analyze $\Pi_{\Sigma}(Q)$, Eq. (2), which includes $G_{\mu\nu} \equiv \not{P} \gamma^\mu (\not{P} - \not{K}) \gamma^\nu \not{P}$. Simple algebra yields

$$g_{\mu\nu} G_{\mu\nu} = -2P^2 K^\mu,$$

$$\delta g_{\mu\nu} G_{\mu\nu} = 2P^2 \frac{k_0}{k^2} \left[ (2p_0 - k_0) \not{P} - p_0 \not{K} \right] + O((P^2)^2),$$

where $\delta g_{\mu\nu} \equiv g_{\mu\nu} - g_{\mu\nu}^{(t)}(K)$ and use has been made of $K^2 = (P - K)^2 = 0$. Thus, both $g_{\mu\nu} G_{\mu\nu}$ and $\delta g_{\mu\nu} G_{\mu\nu}$ are proportional to $P^2$. Now we notice that $P^2 \Delta_R(P) \Delta_\Lambda(P) = \not{P}/P^2$. This means that the pinching singularity in the "pinch" term $F^{(p)}(P)$, Eq. (11), turns out to be a mass singularity. The $(P^2)^2$ term in (11) does not lead to mass-singular contribution. As in [4], let us restrict our concern to singular contributions and ignore the $(P^2)^2$ term.

As far as mass-singular contributions are concerned, above observation $\delta \Pi(Q) = 0$ tells us that $\Pi_{\Sigma}^{\text{sing}}\bigg|_\text{Fey} = -\delta \Pi_{\Sigma}^{(p), \text{sing}}$, where "Fey" stands for Feynman. $\Pi_{\Sigma}^{\text{sing}}\bigg|_\text{Fey}$ with $Q = (q_0, 0, 0, 0)$ is explicitly evaluated in [4]. As a check, in Appendix B, we evaluate $\delta \Pi_{\Sigma}^{(p), \text{sing}}(q_0, 0, 0)$ and confirm $\delta \Pi_{\Sigma}^{(p), \text{sing}} = -\Pi_{\Sigma}^{\text{sing}}\bigg|_\text{Fey}$.

\footnote{There is a missing term in $\Pi_{\Sigma}^{\text{sing}}\bigg|_\text{Fey}$ in [4] (see Appendix B).}
The singular part of \( \Pi(q_0 \equiv 2\kappa, 0) \), being gauge independent, is also evaluated in Appendix B,

\[
\Pi^{\text{sing}} = \Pi^{(p), \text{sing}}_{\Sigma, \text{Cou}} - \Pi^{(p), \text{sing}}_{\Sigma, \text{Fey}} = \Pi^{(p), \text{sing}}_{V, \text{Fey}} + \Pi^{(p), \text{sing}}_{\Sigma, \text{Fey}} = -\frac{32}{3\pi} \alpha_s \kappa^2 \ln \frac{1}{\epsilon_y} \int \frac{d\Omega_k}{4\pi} \overline{n}(\kappa, \vec{k}) \overline{n}(\kappa, -\vec{k}) \left[ \int_{\epsilon_z}^1 dz \frac{(z - 1)^2 + 1}{2z} \right] \\
+ \int_0^{\infty} dz \frac{z^2 + 2}{z} n(\kappa z, \hat{k}) + \int_0^{\infty} dz \frac{z(z^2 + 1)}{z^2 - 1} \overline{n}(\kappa z, \hat{k}) \\
+ \frac{16}{3\pi} \alpha_s \kappa^2 \ln \frac{1}{\epsilon_y} \int \frac{d\Omega_k}{4\pi} \overline{n}(\kappa, -\hat{k}) \left[ \int_{\epsilon_z}^1 dz \frac{(z - 1)^2 + 1}{z} n(\kappa z, \hat{k}) \overline{n}(\kappa(1 - z), \hat{k}) \right] \\
+ \int_{\epsilon_z}^{\infty} dz \frac{(z - 1)^2 + 1}{z} n(\kappa z, \hat{k})(1 - \overline{n}(\kappa(1 - z), \hat{k}) \\
+ \int_{\epsilon_z}^{\infty} dz \frac{(z + 1)^2 + 1}{z} (1 + n(\kappa z, \hat{k})) \overline{n}(\kappa(1 + z), \hat{k}) \right].
\]

(15)

Here \( d\Omega_k \) is the element of the solid angle in the \( k \)-space. The cutoff factor \( \epsilon_y \) is defined by \( 1 - |\hat{p} \cdot \hat{k}| \geq \epsilon_y \) (cf. Eq. (B.1) in Appendix A) and \( \epsilon_z \) is the infrared cutoff \( k \geq \epsilon_z \kappa \).

Let us clarify the relation between the present result and the result of [4]. We start with picking out from \( F^{(p)}(P) \),

\[
\overline{\Sigma}(P) \equiv (1 - \bar{f}(P)) \Sigma_{12}(P) + \bar{f}(P) \Sigma_{21}(P).
\]

(16)

For the purpose of estimating \( \Pi^{(p)}_{\Sigma} \), Le Bellac and Mabilat [4] have analyzed \( \overline{\Sigma}(P) \) within the hard-thermal-loop resummation scheme. The net production rate of an (anti)quark is given by \( \Gamma(P) = \text{Tr}[\bar{\Sigma}(P) \bar{P}]/(4p) \), with \( P = (p, p) \) for quark and \( P = (-p, -p) \) for antiquark. Arguing that \( \Gamma(P) \) on the mass shell \( P^2 = 0 \), being gauge independent, is relevant to \( \Pi(Q) \), the authors of [4] have concluded that \( \Pi(Q) \) is gauge dependent since \( \Pi^{\text{sing}}(Q) \) is. It is clear from the above argument that this is not the case. As has been discussed above in conjunction with (14), \( \bar{P} \delta \Sigma(P) \bar{P} \) (as well as \( \bar{P} \Sigma(P) \bar{P} \bigg|_{\text{Fey}} \)) vanishes on the mass shell \( P^2 = 0 \). [As a matter of fact \( \Sigma_{12(21)}(P) \) in (13) vanishes on the mass shell \( P^2 = 0 \), since \( P^2, K^2 \) and \( (P - K)^2 \) cannot vanish simultaneously.] However, as noted above, in calculating \( \delta F^{(p)} \) (cf. Eq. (11)), the factor \( P^2 \), Eq. (14), “eliminates” one \( \Delta \) and the mass-singular contribution \( \delta \Pi^{(p), \text{sing}}_{\Sigma} \) (as well as \( \Pi^{(p), \text{sing}}_{\Sigma, \text{Fey}} \)) emerges. Thus we have learned that the mass-
singular contribution does not come from the gauge-independent quantity,
\[ \bar{\mathcal{P}} \Sigma(P) \bar{\mathcal{P}} \bigg|_{P^2 = 0} = -2ip\Gamma(P) \mathcal{P} \bigg|_{P^2 = 0} \left( = 0 \right), \]
but comes from the gauge-dependent quantity
\[ d\bar{\mathcal{P}} \Sigma(P) \mathcal{P} / dP^2 \bigg|_{P^2 = 0} = \mathcal{P} \Sigma(P) \mathcal{P} / P^2 \bigg|_{P^2 = 0}. \]

The observation made above in conjunction with (14) applies to the contribution (to \( \delta F^{(p)} \)) from the soft-\( K \)-region (cf. (11) with (4)), in which \( K^2 \neq 0 \). [The soft \((P-K)\)-region is not important, at least, for the system, which is not far from thermal and chemical equilibrium.] Above observation on \( \delta \Pi^{(p), \text{sing}}_{\Sigma} \) holds as it is, except that \( \Sigma_{12(21)}(P) \) does not vanish on the mass shell. It is to be noted in passing that \( \Pi^{(p)}_{\Sigma} \big|_{\text{Fey}} \) develops pinch singularity. This is because, in the present case, \( K^2 \neq 0 \), we have, in place of (13), \( g_{\mu\nu} G^{\mu\nu} = -2P^2 K + 2K^2 \mathcal{P} \). The second term on the right-hand side leads to pinching singularity in \( \Pi^{(p)}_{\Sigma} \). Because of the factor \( K^2 \), which is small, the “residue” of the pinching contribution is relatively small.

Finally, for an instructive purpose, we derive a sufficient condition for the absence of mass singularities, i.e., the condition under which (15) vanishes. The condition is
\[
[1 + f(K)] [1 - \tilde{f}(P - K)] = [1 - \tilde{f}(P)][1 + f(K) - \tilde{f}(P - K)], \tag{17}
\]
which holds for equilibrium case. For simplicity of presentation, we take the self-interacting scalar theory counterpart (4) to (17),
\[
[1 + f(K)] [1 + f(P - K)] = [1 + f(P)][1 + f(K) + f(P - K)]. \tag{18}
\]
The same conclusion, obtained below, applies to (17).

We start with the following observation. It is well known that the reaction rates are free from mass singularities if all the (energy) degenerate states are prepared in the initial state (see e.g., [7])). Mass singularities arise from collinear configurations of massless particles. For convenience, we enclose the system into a box of volume \( V (= L^3) \) and employ a periodic boundary condition. In the interaction representation, the scalar field may be expressed as a superposition of plane waves, whose “coefficients” \( a_{\mathbf{k}j} \) and \( a_{\mathbf{k}j}^\dagger \) are the annihilation and creation operators, respectively. Here \( \mathbf{k} \) is the
momenta of a particle with mode \( j \). The above observation tells us that for the system characterized by the density operator

\[
\rho \left( \sum_j g(\hat{k}_j) k_j a^\dagger_{k_j} a_{k_j} \right) \tag{19}
\]

reaction rates are free from mass singularities. In (19), \( k_j = |k_j| \) is the energy of a particle with \( k_j \) and \( f(\hat{k}_j) \) is a (smooth) function. If \( f(\hat{k}_j) = 1 \), the argument of \( \rho \) in (19) is essentially an energy operator in the initial (remote past) state. It should be remarked that, even in the present context, the form of the density operator takes much more complicated form, since the system is in general not uniform in space-time. However, as far as a reaction rate taking place in the system is concerned, all the propagators describing the reaction rate have \([4, 6]\) common center-of-mass or macroscopic coordinates of the space-time region, where the reaction takes place. Then it is sufficient to use (19).

Let us show that (19) leads to (18). We start with computing \( \text{Tr} \rho \)

\[
\text{Tr} \rho = \sum_{\{n_j\}} \rho \left( \sum_j k_j g(\hat{k}_j) n_{k_j} \right) = \int dE \rho(E) \sum_{\{n_j\}} \delta \left( E - \sum_j k_j g(\hat{k}_j) n_{k_j} \right) = \int dE \rho(E) \sum_{\{n_j\}} \int_{-\infty}^\infty d\xi \frac{e^{i\xi E} e^{-i\xi \sum_j k_j g(\hat{k}_j) n_{k_j}}}{2\pi} = \int dE \rho(E) \int_{-\infty}^\infty d\xi \frac{e^{i\xi E} G}{2\pi},
\]

where

\[
G \equiv \prod_j \frac{1}{1 - e^{-i\xi k_j g(\hat{k}_j)}}. \tag{40}
\]

Now, the number density is evaluated as

\[
n(k_l, \hat{k}_l) \equiv \text{Tr} n_{\hat{k}_l} \rho = \int dE \rho(E) \int \frac{d\xi}{2\pi} e^{i\xi E} \frac{1}{e^{i\xi k_l g(\hat{k}_l)} - 1} G. \tag{41}
\]

Armed with the above preliminary, we take (18) with \( p_0 > k_0 > 0 \). The right-hand
side of (18) reads
d_\mathcal{E} \rho(E) \int_{-\infty}^{\infty} \frac{d\xi}{2\pi} e^{i\xi E} [1 + \mathcal{N}(\xi, \hat{k}_n) + \mathcal{N}(\xi, \hat{k}_j) + \mathcal{N}(\xi, \hat{k}_n - \hat{k}_j)] G, \tag{20}

where \mathcal{N}(\xi, \mathbf{k}) \equiv \frac{1}{e^{i\xi \hat{k}} g(\hat{k}) - 1}. Since we are interested in collinear configuration, \hat{k}_n \simeq \hat{k}_j \simeq \hat{k}_n - \hat{k}_j and |k_n - k_j| \simeq k_n - k_j. Then we have

\text{Eq. (20)} \simeq \int dE \rho(E) \int_{-\infty}^{\infty} \frac{d\xi}{2\pi} e^{i\xi E} e^{i\xi k_n g(\hat{k})} \mathcal{N}(\xi, \hat{k}_j) \mathcal{N}(\xi, \hat{k}_n - \hat{k}_j) G,

which is the left-hand side of (18). For other regions of (p_0, k_0) than the one studied above, one can similarly prove (18).

It should be noted that, since \rho(E) and g(\hat{k}) are arbitrary functions, the resultant form for \mathcal{N}(\mathbf{k}, \hat{\mathbf{k}}) “covers” a wide class of functions.

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**Appendix A Absence of mass singularity in \Pi^{(n)}_{\Sigma}(Q)**

In this Appendix, we show that \Pi^{(n)}_{\Sigma}(Q) is free from mass singularities, reconfirming the result in [4]. Manipulation goes as follows. Substitute \text{Eq. 10}, into (2) with (4). Use the form of \text{g(gauge)}, Eq. (7) or Eq. (8), and forms for \Delta_{lj} and S_{lj} (cf. Eq. 5). The resultant expressions may be rearranged as

\[ \Pi^{(n)}_{\Sigma} = \frac{8}{3\pi^2} \alpha_\alpha \int \frac{d^4 P}{2\pi} \int \frac{d^4 K}{2\pi} \text{Tr} \left[ G^{\mu\nu}(\mathcal{P} - \mathcal{Q}) \right] \]

\[ \text{For the mode-overlapping “points”, } k_n = k_j, k_n = 0, k_j = 0, \text{ (20) does not hold in general. The contributions from such “points” are } O(1/V) \text{ smaller than the “reference contribution” and vanish in the limit } V \to \infty. \text{ Incidentally, for the case of equilibrium system, (20) holds [8].} \]
\[ \times \hat{g}^{(\text{gauge})}_{\mu\nu} \tilde{f}(P)(1 - \tilde{f}(P - Q))\delta_i((P - Q)^2) \]
\[ \times \left[ \left( \frac{1}{2} + f(K) \right) \delta_i(K^2) \left( \Delta^2_{\delta_i}(P)\Delta_R(P - K) + \Delta^2_{\delta_i}(P)\Delta_A(P - K) \right) \right. \]
\[ + \left. \left( \frac{1}{2} - \tilde{f}(P - K) \right) \delta_i((P - K)^2) \left( \Delta^2_{\delta_i}(P)\Delta_R(K) + \Delta^2_{\delta_i}(P)\Delta_A(K) \right) \right] , \]
(A.1)

where \( G^{\mu\nu} \) is as in (13) and \( \delta_i(k^2) \equiv \epsilon(k_0)\delta(K^2) \) etc. For the Coulomb gauge, \( \hat{g}^{(\text{gauge})}_{\mu\nu} = g^{(\text{t})}_{\mu\nu} \). For the covariant gauge, \( \hat{g}^{(\text{gauge})}_{\mu\nu} = g_{\mu\nu} + \eta K_{\mu} K_{\nu}(\partial/\partial K^2) \), where \( \partial/\partial K^2 \) applies to \( \delta(K^2) \), \( \Delta_R(K) \) and \( \Delta_A(K) \). Equation (A.1) is manifestly free from mass singularities. Mass singularity arises from the terms \( \Delta^2_{\delta_i}(P)\Delta_A(K) \) and \( \Delta^2_{\delta_i}(P)\Delta_A(K) \). In obtaining (A.1), cancellations occur between those terms.

It is also obvious from (A.1) that \( \Pi_S^{(n)} \) is free from divergence due to infrared singularities, provided that, as \( k \to 0^+ \), \( f(k, \hat{k}) \propto k^{-n} \) with \( n < 2 \) and \( \tilde{f}(k, \hat{k}) \propto k^{-n'} \) with \( n' < 2 \). In actual computation of \( \Pi \), for a propagator with soft momentum, one should use hard-thermal-loop resummed effective one.

**Appendix B Computation of the singular part of \( \Pi_S^{(p)}(Q) \)**

Here, we compute the singular part of the “pinch” contribution \( \Pi_S^{(p)}(q_0, q = 0) \). Substituting (11) into (2) with (4) and using (13) and (14), and the forms for \( \Delta_{ij} \) and \( S_{ij} \), we have

\[ \Pi_S^{(p), \text{sing}} = -\frac{128}{3\pi} \alpha_s Q^2 \int \frac{d^4 P}{2\pi} \int \frac{d^4 K}{2\pi} G^{(\text{gauge})}(1 - \tilde{f}(P - Q))\delta_i(K^2)\delta_i((P - K)^2) \]
\[ \times \delta_i((P - Q)^2) \frac{1}{P^2} \left[ f(K)\tilde{f}(P - K) - \tilde{f}(P)\delta_i((P - K)^2) \right] , \]

where

\[ G^{(\text{Fey})} = \frac{k_0}{q_0} , \]

\[ G^{(\text{Cou})} = \frac{(q_0 - k_0)^2 + k_0^2}{2q_0 k_0} . \]
Here use has been made of $p_0 = q_0/2$, which comes from $(P - Q)^2 = 0$. Making the change of variable $P \rightarrow P + K$, we extract the mass-singular part,

$$
\delta(P^2) \int d^4K \delta(K^2) P \frac{1}{(P + K)^2}
$$

$$
= \pi \delta(P^2) \int dk k^2 \int dk_0 \delta(K^2) \int_{-1+\epsilon_y}^{1-\epsilon_y} d(\hat{p} \cdot \hat{k}) \frac{1}{p_0 k_0 - \hat{p} \cdot \hat{k}}
$$

$$
\rightarrow \frac{\pi}{p} \delta(P^2) \int dk k \int dk_0 \epsilon(p_0 k_0) \delta(K^2) \ln \frac{1}{\epsilon_y}.
$$

(B.1)

Using (B.1), cutting off the infrared region, $k \equiv \kappa z \geq \epsilon_z \kappa$, and changing the integration variable suitably, we arrive at the final form. As it should be, $\delta \Pi^{(p),\text{sing}} = \Pi^{(p),\text{sing}}_{\text{Fey}} - \Pi^{(p),\text{sing}}_{\text{Cou}}$ is equal to $-\Pi^{\text{sing}}_V$, which has been evaluated in [4]. There is a missing term though in $\Pi^{\text{sing}}_V$ in [4],

$$
-\frac{32}{3\pi} \alpha_s \kappa^2 (\bar{n}(\kappa))^2 \ln \frac{1}{\epsilon_y} \int_{\epsilon_y}^1 dz \frac{1-z}{z},
$$

where $\kappa = q_0/2$. The form for $\Pi^{\text{sing}}$, being gauge independent, reads (15) in the text.
References

[1] T. Altherr and D. Seibert, Phys. Lett. B 333 (1994) 149

[2] T. Altherr, Phys. Lett. B 341 (1995) 325

[3] R. Baier, M. Dirks, K. Redlich and D. Schiff, Phys. Rev. D 56 (1997) 2548

[4] M. Le Bellac and H. Mabilat, Zeit. Phys. C 75 (1997) 137

[5] A. Niégawa, to be published in Phys. Lett. B.

[6] K.-C. Chou, Z.-B. Su, B.-L. Hao and L. Yu, Phys. Rep. 118 (1985) 1

[7] A. Niégawa, Phys. Rev. Lett. 71 (1993) 3055

[8] A. Niégawa, hep-ph/9710428, to be published in Phys. Rev. D