Emergence of moving pattern in a collective game

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Abstract. In this paper, we propose a collective game where agents, distributed in one-dimensional space, seek for their best positions through adaptive learning. The movements of agents are driven by a simple rule: they are attracted by others with higher status and yet restricted by their initial positions, according to which a benefit function is formulated. It is found that some fascinating complex patterns arise from such simple rules. Aggregations of agents emerge in the form of clusters along this position line after a dozen rounds. While agents strive to move forward, a chain of clusters shift backward and ultimately collapse one by one at an almost fixed point. The trail of shifting behaves like an attractor which attracts different groups of agents to join in. Such intricate moving pattern turns out quite sharp and clear when agents have a uniform pace length.

1. Introduction

It is commonly believed that complexity originates in simplicity. One of the most fundamental principles of Stephen Wolfram’s *A New Kind of Science* lies in the insistence that more complex rules do not necessarily lead to greater complexity in overall behaviour. In other word, simple programs with straightforward rules, such as elementary cellular automata, are sufficient to capture the essence of almost any complex process [1]. Wolfram’s research as well as other pioneers’ endeavours such as John Conway’s Game of Life [2] reveal the computational nature of social sciences and promote the development of the discipline known as “computational social science”.

Parallel with cellular automata, agent-based modeling (ABM) is another effective means to cope with problems of sociology and the complexity of social dynamics [3-5]. With a bottom-up approach, agent-based models are a kind of microscopic model which attempts to investigate and sometimes predict agents’ collective behaviour, broadly known as occurrence of emergence [6]. Within the last decades, ABM has expanded in social science studies and scored remarkable achievements, including some renowned models such as *Boids* simulating the flocking behaviour of birds [7] and *Sugarscape* exploring social phenomena like seasonal migrations [8].

A focus issue within social sciences never fails to fascinate researchers: the causes of aggregation. Aggregation is ubiquitous in nature, such as a flock of birds or a school of fish. In human society, it is also a common social phenomenon that people with different social status, political power, and religious belief are inclined to form separate communities, broadly referred to as social stratification [9]. Substantial endeavours have been made to search for explanatory insight into the formation. There is research verifying that aggregation results from collaborative interactions [10,11] and human evolved psychological processes [12].

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On the basis of previous studies, we regard an agent-based model with simple interaction rules and adaptive learning players as a valid approach to understanding aggregate phenomena. Such a model was proposed in our recent research where agents distributed in one dimensional space are driven by benefits obtained from higher-status ones, the static pattern of which has exhibited a fractal characteristic [13]. To simulate human society, the social climbing phenomenon in which people prefer to forge links with others with higher social status has attracted attention [14,15]. In our previous work we assumed that people are not only catching up with others with higher status and yet restricted by their initial positions. In this work, we further investigate such a simple model and astonishingly unearth even more complicated and enchanting dynamic processes.

The remaining parts of this paper are organized as follows. Section 2 of this paper elaborates on how the model runs. An overall evolution of agents’ location distribution over time is displayed. In-depth analysis of the formation of moving patterns is offered in Section 3. The short-term and long-term movements of a group of agents who aggregate at a specific time point are examined. The moving trails of all agents, grouped by pace length and initial position, are exhibited. A particular case is discussed where pace lengths are unified. In Section 4 we conclude.

2. Model description

To explore the generation mechanism of aggregations, we design such an agent-based model where agents locate on a line and adjust their locations to seek for the maximum benefits. The interacting rules of the model and some preliminary results are presented in this section.

2.1. Interacting rules

At the very beginning, each agent is endowed with an initial position randomly distributed in this one-dimensional space of finite length. Then the position adjustments are made at each time step by every agent in order to maximize his benefit. The benefit function for one agent involves his current position and those of other agents with higher status, as well as his initial position. Specifically, the benefit gained by agent $i$ from agent $j$ at time $t$ is formulated as follows,

$$B_{ij}(t) = \begin{cases} 0, & \text{if } x_j(t) < x_i(t); \\ \frac{1}{2} [x_i(t) + x_j(t)] - x_i(0), & \text{if } x_j(t) \geq x_i(t), \end{cases}$$

where $x_i(t)$ represents agent $i$’s position at time $t$, and hence $x_i(0)$ is his initial position. Only people ahead make contributions to his benefit, whereas those behind do not. By summing up all contributions from others, we express expected benefit of agent $i$ in the form of the distribution of all agents’ current positions as well as his own initial position:

$$B_i(t) = \sum_{j \neq i}^{N} B_{ij}(t).$$

Since agent $i$ is assumed to be an adaptive actor, he has only three choices of next move: staying at current position, moving upward and moving downward. There is always a trade-off when he makes his choice of movement. Suppose he moves forward, the higher position will bring him more benefits but the number of agents ahead would be reduced and his benefit will then be lessened. He makes his decision based on the comparison of the respective benefits induced by the three options. The shift in location is given by

$$x_i(t + 1) = x_i(t) + \text{sign} \cdot \delta_i,$$

where $\text{sign} = -1, 0, 1$ and $\delta_i$ denotes the pace length of agent $i$. The agent makes the decision by calculating and comparing the benefits of these three possible positions at time $t + 1$ while expecting others to retain their positions at time $t$. Each move always guarantees the agent the maximum benefit at the current time point.

2.2. Simulation results

In the beginning, 500 agents are positioned randomly from 0 to 1 on a number axis, and they will be restricted within this interval hereafter. Then, they will adjust their locations to seek for higher benefit.
Their moving abilities are set to be diverse and each agent is endowed with a certain pace length which is randomly sampled from a uniform distribution from 0 to 0.01. Given these parameters, we ran the model and recorded the evolution of the locations of all agents over a period of 200 time steps.

A visual representation of how the distribution of agents along the position line evolves with time is shown in figure 1, where horizontal and vertical axes represent time and space respectively. The colours in this figure specify the corresponding densities of population – a darker pixel corresponds to a higher density here and now.

When the simulation started, the agents are distributed randomly in the space, so the colours at the beginning are light and nearly even. After about 20 rounds, darker pixels come into being which means several clusters of agents emerge. The right-downward dark stripes can be obviously observed in this figure, and they disappear at almost a same position around 0.2. In other word, the clusters shift downward as time goes by and ultimately collapse at one point.

3. Formation of moving patterns

3.1. Moving tracks of a group of agents
In order to further analyze the complicated pattern arising from interaction among a group of benefit seekers, whose ability of movement and initial position are different in our simulation settings, we
carried out the following analysis. Firstly, we narrow our focus to one single cluster by observing movement of agents who aggregate to form a cluster at a specific time point. To be more specific, we choose to investigate the cluster indicated by the black arrow in figure 1. We trace the movement of the agents who are included in this cluster at time 55 in the short term and long term.

Figure 2(a) plots the movement trails of this group of agents in the short run (from time 51 to time 60) and each curve corresponds to one individual agent. It is observed that some of the agents keep pace with each other and remain in this cluster throughout the short period, while others just pass by and leave the cluster after the short encounter. Looking into the trails of this group during the whole process, we can get more interesting findings, as shown in figure 2(b). In the long term, most agents in this group move upward and downward in coordination with each other. In most cases, they go upward successively, but downward synchronously, this is the reason for the formation of the clusters. One possible explanation is that once the agent with the highest status in this group moves downward, he would mobilize more agents to follow immediately for fear of benefit loss.

3.2. Moving tracks of all agents

We can explore all agents’ movements by plotting the changes in positions of all agents during the whole process in figure 3. It is found that it is not the same group of agents who accomplish the shift of a cluster mentioned above, but a combination of several different groups of agents, who move around different centre points. It appears that different groups are playing a relay game: one group of people relay the downward-shifting cluster to the next group and then move upward themselves,
resulting in those amazing dark stripes in figure 1. It is difficult to interpret that the shifts of clusters behave like an attractor which can connect several downward segments perfectly to form a completed path.

By selecting those with little pace lengths (less than 0.002 here) (see figure 4(a)), it is found that these people are hardly influenced by others due to their limited moving capacities. They merely keep moving forward at their own speed all the time. By selecting those with larger pace lengths (more than 0.003 here) (see figure 4(b)), the structure of the aggregation pattern is presented more clearly.

To examine the distinctions among movements of agents with different levels of initial positions, we divide all agents into three groups, one with lower initial positions (from 0 to 0.33), one with medium initial positions (from 0.33 to 0.66), and another one with higher initial positions (from 0.66 to 1). In figure 5, movements of the three groups are displayed respectively. Almost every agent will finally improve his status by moving upward, but he cannot improve it significantly due to the limitation of initial status. Some of lower-level agents break the bonds and intrude into the medium area; a smaller proportion of medium-level agents promote their status to achieve the high level; none of the agents with initial higher status ever leaves the “high zone”.

![Figure 5](image1.png)

**Figure 5.** Moving tracks of agents with (a) lower initial positions (from 0 to 0.33); (b) medium initial positions (from 0.33 to 0.66); (c) higher initial positions (from 0.66 to 1).

3.3. Uniform pace length

In the particular case where all agents have the same pace length (0.005 here), which means the sole randomness remained comes from the random initial positions, the intricate aggregation pattern still appears and displays a clearer structure, as shown in figure 6. The relays of shifting clusters constitute a beautiful carpet pattern which is both intriguing and bewildering. What factors prompt the groups of ascending agents to begin to turn back and the descending ones to begin to gain higher status remain ambiguous to us.

![Figure 6](image2.png)

**Figure 6.** Moving tracks of agents with uniform pace length.
4. Conclusion
Complexity arises from simple rules, which is a common belief in complexity science. Recently, W. Brian Arthur argued that “an economy that is not dead, static, timeless, and perfect, but one that is alive, ever-changing, organic, and full of messy vitality” [16]. In this paper, a vivid demonstration of this conception is exhibited in a collective best-position searching model. The agents adaptively adjust their positions in order to get more benefit. The benefit for one agent obtained from others with higher status is formulated by the sum of differences between the mean value of his and another one’s current positions, and his initial position. The simulation results of the model show that the agents will aggregate as clusters when they are moving upward separately for larger benefits. At first glance, the clusters are generated by synchronized downward move in one group. Actually, from an overall perspective, the clusters shift in the form of a “relay race” accomplished by different groups of people successively. The moving aggregation pattern becomes more distinct when we assume all agents possess the same movement ability. However, how this complex pattern is derived from those simple rules remains ambiguous and incomprehensible so far.

This model provides a perspective to understand the formation mechanism of some social norms, that is to say, individual behaviour of each agent is motivated by both his inner psychological desire and the social circumstance or situation he is facing, while the circumstances are co-created by all individuals. In the course of interpersonal interaction, people form spontaneous communities within the whole society since collective actions undoubtedly offer protection and benefit. Following this perspective, we believe that many other complicated sociological phenomena can be understood by taking beneficial interaction and mobility of agents into account.

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