Diversity of rationality affects cooperation in spatial prisoner’s dilemma game

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In real world, individual rationality varies for the sake of the diversity of people’s individuality. In order to investigate how diversity of agent’s rationality affects the evolution of cooperation, we introduce the individual rationality proportional to the \( \beta \)th power of the each agent’s degree. Simulation results on heterogeneous scale-free network show that the dynamic process is greatly affected by the diversity of rationality. Both promotion and inhibition of cooperative behavior can be observed at different region of parameter \( \beta \). We present explanation to these results by quantitative and qualitative analysis. The nodes with middle degree value are found to play a critical role in the evolutionary processes. The inspiration from our work may provide us a deeper comprehension towards some social phenomenon.

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I. INTRODUCTION

In evolutionary biology, behavioral sciences, and more recently in economics, understanding conditions for the emergence and maintenance of cooperative behavior among selfish individuals becomes a central issue [1, 2]. Including cooperation and defection as the two competing strategies, the prisoner’s dilemma game (PDG) is regarded as a paradigm for studying this issue [3-6].

Based on a structured population [7-11], considerable efforts have been extended by allowing the players to voluntary participating [12], or introducing dynamic network model [13, 14], dynamic payoff matrices [15], dynamic preferential selection [16], and difference between interaction and learning neighborhoods [17]. Santos and Pacheco [18] have studied the PDG on heterogeneous scale-free networks, and observed that, when the underlying network is scale-free, cooperation can be greatly enhanced and becomes the dominating trait throughout the entire range of parameters of the game, due to the cooperators’ cluster forming nature [19].

In the mentioned works, individual particularity is not the main topic. However, particularity is ubiquitous among the individuals of social groups and animal species. Thus, the diversity of individuality inevitably appears between the players engaging in the evolutionary games. Instead of taking individual difference into account directly, some works concentrate on the individual similarity [20-22]. Recently, it has been directly proved that diversity of certain individual property can efficiently promote cooperative behavior in evolutionary games [23, 24]. The authors introduce social member’s extrinsically determined properties, like wealth or social status, to increase or decrease the fitness of a player depending on its location on the spatial grid. Different scaling factors are provided to different nodes, rescaling their payoff matrix in PDG [23].

Different from the diversity of extrinsically determined properties [23, 24], this paper concentrates on the diversity of the intrinsic property, individual rationality. Szabo’s stochastic evolutionary rule [25], especially the Fermi upgrading rule, has taken this vital and intrinsically determined property into account. In the formula of Fermi rule, the variable temperature indicates how rational the individual is, when making decision in the game. Just as temperature in statistical physics, this very variable, in former works [26-29] was viewed mainly as a stochastic noise. Phenomena like stochastic resonance [27] and second-order phase transition [28] are discovered. These works have considered the individual rationality to be at the same value for every player in the game. Therefore, this variable actually serves as a reflection of collective rationality belonged to the whole system. However, in real society, individual rationality depends on its intelligence, disposition, motivation, and circumstance, which differ from individual to individual. Serving as an intrinsic factor, different level of rationality determines different choice and correspondingly fosters different game result. Within our study, we regard the degree of a node as a rank of the individual’s certain social feature, for instance, social status, and directly relate this feature to the individual rationality. An agent’s rationality is set to be proportional to the \( \beta \)th power of the agent’s degree [30]. In this way, the diversity of rationality is associated with the diversity of degree. Networks with a heterogeneous topological structure are used in our work. We reported below that cooperation is enhanced in a certain region of parameter \( \beta \), but inhibited in other region.

The model and simulation result are presented in section II. To explore the mechanism for both promotion and inhibition of cooperation, we present a statistical analysis to the dynamic process in terms of microscopic arguments in section III. Final conclusion is to be drawn in section IV, as well as some sociological inspirations from our findings.
II. THE MODEL AND SIMULATION RESULT

To introduce the diversity of the intrinsic property, rationality, we consider an evolutionary two-strategy prisoner’s dilemma game with players located on vertices of a heterogeneous network. Each individual is allowed to interact with its nearest neighbors, and self-interactions are excluded. Players can adopt one of the two simplest strategies: “cooperate” (C) and “defect” (D). The strategy adoption mechanism is based on the rescaled version of the payoff matrix introduced by Nowak [31]:

\[
C \begin{pmatrix} C & D \\ D & (1 \ 0) \end{pmatrix} \text{ for } 1 < b < 2
\]  

(1)

During the evolutionary process, a player located on node \( i \) can follow the strategy of one of its randomly chosen neighbor at node \( j \), with the probability depending on the payoff difference \( \left(M_i - M_j\right) \),

\[
W_{ij} = \frac{1}{1 + \exp\left[(M_i - M_j)/T_i\right]}
\]

This is the Fermi updating rule [25], where \( T_i \) characterize the level of rationality pertinent to node \( i \). And \( T_i = 0 \) denotes complete rationality, where the individual always adopts the best strategy determinately; while \( T_i > 0 \), it introduces some irrational factor, that there is small possibility to select the worse one; \( T_i \to \infty \) denotes that the individual is completely irrational, and its decision is random. Within this study we consider the diversity of rationality defined by the following function [30]:

\[
T_i = N T_0 \frac{k_i^\beta}{\sum k_i^\beta}
\]

where \( N \) is the total number of nodes in the network, and \( k_i \) is the degree of node \( i \). Here we adopt the Barabási-Albert scale-free network [32 33]. \( T_0 \) denotes the average value of rationality. We use parameter \( \beta \) to tune the relationship between node \( i \)’s degree \( k_i \) and its rationality \( T_i \). While \( \beta < 0 \), nodes with larger degree gain lower value of rationality. While \( \beta = 0 \), rationality is uniformly distributed. \( \beta > 0 \) denotes a reversed situation as compared with the case \( \beta < 0 \): nodes with larger degree gain larger value of rationality. If \( \beta = 1 \), rationality is distributed by power-law. It is worth mentioning that significantly high value of \( T_i \) could induce substantially random behavior of an agent, though payoff difference may be large. On the contrary, very low \( T_i \) would heavily enhance agent’s sensitivity towards higher payoff.

The case of \( \beta = 0 \) is discussed in [18 19]. Although the updating mechanism we adopt is different from [18 19], when \( T \) is low, at a microscopic scale the following fact still exist: cooperators tend to occupy the hubs, since hubs are directly connected, if a defector occasionally takes over one hub, the probability that it gets reoccupied by a defector becomes essentially one. However, while using Fermi updating rule, this fact may be affected by high rationality value. Fig[1] is plotted to explore the influence of rationality in both cases, \( \beta = 0 \) and \( \beta \neq 0 \). The BA network is built with the initial number of nodes: \( m_0 = 2 \), the number of edges linked to the exiting nodes from the newly added node in each time step: \( m = 2 \), and the average degree \( \bar{k} = 4 \). The total number of nodes is \( N = 4225 \). Before the start of each game simulation, both strategies populate the spatial grid uniformly. We adopted a synchronous updating scheme. All the simulation results were obtained by averaging over 1000 generations after a transient time of 5000 generations. Each data is obtained by averaging over ten different network realizations with ten runs for each realization.

In Fig[1] in the case of \( \beta = 0 \), when \( T_0 = 1 \), cooperation is dominating over the entire range of the temptation to defect \( b \), but sharp decrease of the frequencies of cooperators \( \rho_C \) can be measured when \( T_0 \) becomes large. This indicates that the cooperation promoted by hubs is indeed sensitive to the value of rationality. At the same time, in the case of \( \beta = -1 \), the value of \( \rho_C \) is not evidently affected by intense variation of \( T_0 \), and there always exists a broad range in the parameter space within which cooperation rule completely. Thus, there must be other factors contributing to the facilitation of cooperation.
To further investigate how cooperation is influenced by the parameter $\beta$, the variation of $\rho_C$ versus $\beta$ is demonstrated in Fig. 2. It is observed that $\rho_C$ and $\beta$ have apparently non-monotonous relationship. The shapes of the curves are similar to a gorge located in a plateau. This shape indicates that diversity of rationality can either promote or inhibit cooperation, depending on the value of $\beta$. To highlight the sharp contrast between the effective promotion and serious inhibition, in this paper the gorge promote or inhibit cooperation, depending on the value of $b$. The average value of rationality $T_0 = 1$. Middle: The average value of rationality $T_0 = 0.1$, $0.2$, $0.3$, $0.4$, and $0.5$. Right: The temptation to defect $b = 1.8$. The average value of rationality $T_0 = 0.1$, $0.2$, $0.3$, and $0.4$.

FIG. 2: The frequencies of cooperators $C$ versus $\beta$. Left: The temptation to defect $b = 1.04$, $1.08$, $1.6$, and $1.75$. The average value of rationality $T_0 = 1$. Middle: The temptation to defect $b = 1.4$. The average value of rationality $T_0 = 0.1$, $0.2$, $0.3$, $0.4$, and $0.5$. Right: The temptation to defect $b = 1.8$. The average value of rationality $T_0 = 0.1$, $0.2$, $0.3$, and $0.4$.

III. ANALYSIS AND DISCUSSION

When $\beta = 0$, in [18, 19], the prevalence of cooperation is because that on the heterogeneous network topology, hubs can stick together the cooperator cycles that would otherwise be disconnected, and form stable cooperative clusters (C cluster) [19]. In [19], a cooperative cluster, namely a cooperator core, is a connected component fully and permanently occupied by pure cooperators. While invaded by defectors, the local structure of a C cluster can be viewed as a C strategy hub surrounded by a number of periphery neighbors, most of which are cooperators. Then whether C clusters are stable or not when $\beta \neq 0$ is to be analyzed. Two opposite effects generated by the four crucial dynamic processes determine the fluctuation number of cooperators and defectors in a C cluster in every next time step, and thus determine the stability of the C cluster:

Effect1: Corruption of C cluster

Process (A): the hub node of a C cluster adopts the strategy of a periphery defector, and then transits to D strategy;

Process (D): a periphery cooperator adopts the strategy of the hub defector, and then transits to D strategy.

Effect2: Consolidation of C cluster

Process (B): a hub defector adopts the strategy of a periphery cooperator, and then transits to C strategy;

Process (C): a periphery defector adopts the strategy of the hub node of a C cluster, and then transits to C strategy.

The four processes are corresponding to four kinds of strategy transition probability according to the Fermi updating rule. Based on mean-field approximation, imaging a localized block in the network, a node is surrounded by $k$ neighbors among which the cooperators have a proportion of $\mu$ while the defectors have the rest fraction $(1 - \mu)$. The payoff difference between a cooperator and a defector can be denoted by $\mu(k_i - k_j b)$ or $\mu(k_i b - k_j)$. Because the mean-field hypothesis is not always fit for the evolutionary games on networks, the following analysis can only be qualitative. From equation (2) and (3), we gain the four kinds of transition probability:

\[ W_{H \rightarrow P}^{C \rightarrow D} = \frac{1}{1 + \exp\left(\frac{\mu(k_i - k_j b) - \mu(k_i b - k_j)}{k_\beta}\right)} \]  
\[ W_{C \rightarrow D}^{P \rightarrow H} = \frac{1}{1 + \exp\left(\frac{\mu(k_i - k_j b) - \mu(k_i b - k_j)}{k_\beta}\right)} \]  
\[ W_{D \rightarrow C}^{H \rightarrow P} = \frac{1}{1 + \exp\left(\frac{\mu(k_i - k_j b) - \mu(k_i b - k_j)}{k_\beta}\right)} \]  
\[ W_{D \rightarrow C}^{P \rightarrow H} = \frac{1}{1 + \exp\left(\frac{\mu(k_i - k_j b) - \mu(k_i b - k_j)}{k_\beta}\right)} \]
The upper scripts $H$ and $P$ denote hub and periphery respectively. Through these two formulas, the term $(k^\beta/k^\beta)$ remodifies and extends the Fermi rule.

The strategy transition of a hub or periphery node is determined by the four processes. For a hub node, both process (A) and (B) could happen; for a periphery node, both process (C) and (D) could happen. If occurrence rates of process (A) are higher than of (B), and of (D) are higher than of (C), separately, then C clusters are unstable, and the whole system will asymptotically be meshed in a absorbing state of D. We can regard the strategy transition probability in a certain process as the occurrence rate of this process.

To calculate the four kinds of transition probability of nodes with different degree as process, we approximate classify the nodes in the following way: (1)Nodes with small value of degree: $m \leq k_S \leq \bar{k}$; (2)Nodes with middle value of degree: $\bar{k} < k_m < k'$; (3)Nodes with large value of degree: $k' \leq k_L \leq k_{max}$. Fig 3 shows the strategy transition probability of the high degree nodes (the upper scripts $L$, $M$, and $S$ respectively denote nodes with large, middle, and small value of degree). Sharp increase can be observed from the region of cooperation crisis (see Fig 2), and the probability of process (A) is slightly higher than that of process (B), but they both become equivalent to 0.5 when $\beta$ gets larger. However, in Fig 4 curves concerning the strategy transition probability of the middle degree nodes present a symmetrical fashion. More importantly, large variation of $W$ only exists in the region of crisis, where process (A) always obtains larger occurrence rate than process (B), and process (D) always obtains larger occurrence rate than process (C). These results indicate the advantage of defectors, especially while middle degree nodes participating in the game. In our calculation, we build a BA network with the largest degree $k_{max} = 172$ and the smallest degree $k_S = m = 2$. For simplicity, degrees for the three classes of nodes are confined to isolated values. For example, here we set $k' = 50$; high degree value: $k_{L1} = 172$ and $k_{L2} = 100$; middle degree value: $k_{M1} = 12$ and $k_{M2} = 30$; low degree value: $k_S = 2$; and $\mu = 0.75$. Small value change brings no impact on the qualitative results.

To explore the roots of the agents’ diverse behavior, Fig 4 is plotted to examine the cooperation crisis versus $\beta$ for the three classes of nodes. The rationality of the nodes with smallest and largest degree displays a monotonous decrease and increase respectively, while that of nodes with other degree value varies in a non-monotonous fashion. Mathematical explanation to the numerical results is not complicated. It is crucial to note that the peak values of $T_M$ are all around the region where the cooperation crisis takes place. These peaks nicely explain the large variation in Fig 4. Furthermore, the monotonous increase of rationality of the nodes with the largest degree results in the asymmetrical fashion in Fig 3 as well as in Fig 2. Indeed, rationality value change contributes to agents’ behavior change.
In the region where the rationality of middle degree nodes reach their peak, a node with relatively larger degree becomes rather irrational, and thus gains a much higher probability to adopt the strategy of a node with relatively lower payoff and lower degree. Simultaneously, nodes with relatively lower degree, for their irrationality, are not inclined to imitate their larger degree and higher payoff neighbors. Consequently, this mechanism deteriorates the validity of the cooperation-facilitating mechanism reported in [18, 19]. As a result of the predominance of process (A) and (D) against (B) and (C), as demonstrated in Fig. 4 defectors, though initially may be the minority in a C cluster, do not only obtain a great chance to survive, but also propagate fast and asymptotically dominate the whole network. This is why the C clusters fail to maintain their stability and why cooperation crisis occurs. The peak rationality values of middle degree nodes play a key role. Outside this region, cooperative behavior is promoted.

On the left side of crisis, especially when $\beta < 0$, large and middle degree nodes are very rational. For high and middle-ranking defectors, when severely weakened by the low-ranking neighbors who follow their defective strategies, low rationality value will result in much greater sensitivity towards payoff, then a little higher payoff of a neighbor is enough attractive for them to imitate this neighbor, even if the degree of which is much lower. Thereby, they gain much greater chance to transit to C strategy than in the case of $\beta = 0$, in which low-ranking players could hardly influence the high and middle-ranking ones. Clearly, when $\beta < 0$, the efficiency of cooperation promotion is largely enhanced, even though $T_0$ is significantly large.

On the right side of crisis, the decision of a node with high degree becomes random, and its strategy transition probability only depends on the proportion between its neighboring cooperators and defectors. This irrational hub is surrounded by large number of rational nodes with middle and small degree, and these nodes can quickly form a obedient domain around it and leave it with few defective neighbors. On the other hand, highly rational neighbors of a hub defector would first adopt the hub’s strategy, simultaneously resulting in a sudden drop of the hub’s payoff, then abandon this strategy, for the sake that the low payoff hub hardly affects them and their cooper- ator neighbors with a little higher payoff could overturn their D strategies. After that, the hub defectors will gain much more cooperative neighbors, and thus much greater probability to transit into a cooperator. These facts exist at high $T_0$, too. Notably, middle degree nodes are rational outside crisis, but irrational in crisis.

Based on the above discussion, as parameter $\beta$ varies from negative value to a large positive value, the system experiences successive sorts of dynamic processes, corresponding to the different parts and different shapes of the curves ($\rho_C$ versus $\beta$, show as Fig. 2). When $\beta$ is small or negative, $\rho_C$ is on the plateau. For nodes with large and middle degree, process (A), (B), (C), and (D) have approximately the same occurrence rate, and the whole system is globally dominated by cooperation. When $\beta$ gets larger, process (A) and (D) begin to show considerable predominance over process (C) and (B), and thus, the stability of C clusters is severely disturbed. When $\beta$ is further increased, process (A) and (D) becomes overwhelming, and C clusters are totally destroyed. $\rho_C$ decays abruptly into the valley, and the crisis comes. When $\beta$ continues to increase, the predominance of process (A) and (D) starts to decline, and C clusters begin to res- urrect. Finally, cooperative behavior holds global prevalence, and the probability of the four process return to a similar value. $\rho_C$ arrives at another plateau.

IV. CONCLUSION

To sum up, in this paper, we introduce a set of rationality distributions to investigate the effect of rationality diversity on evolutionary prisoner’s dilemma game on BA Scale-Free network. Our model modifies the Fermi updating rule, and our results largely extend the results in [18, 19]. Our work reveals that diversity of individual rationality heavily influences the evolutionary process. Two routes, produced through two sorts of rationality distributions (on the two sides of crisis), promote cooperation, even while the average value of rationality is high. On the contrary, severe deterioration of cooperation, namely the cooperation crisis, also appears in another sort of rationality distribution. By analyzing the
stability of C clusters, causation of the routes of cooperation promotion and deterioration is interpreted.

The crucial contribution made by nodes with middle degree value may provide some sociological inspiration. Degree is often viewed as a certain rank of game players. Perhaps we could analogize it to some social rank of individuals, then middle degree nodes might be related to the middle class, which is neither the most powerful class nor the most populous class. Middle class could serve as social stabilizer, pointed out by Samuel P. Huntington [34]. However, middle class could also display subversive function, argued by Huntington’s opponents. Individual rationality could be affected by political or economical factors, and are not unchangeable. Probably, since the organization of society largely depends on the emergence of cooperation [35-37], such two contrary functions could be relevant to two rationality level of middle class in two sorts of rationality distribution. Further investigation on the diversity of rationality might yield new insights towards complex social phenomenon.

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