Effect of Zero Mode on the Response of Trapped Bose-Condensed Atoms

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Abstract. The response of the trapped Bose-Einstein condensate (BEC) is investigated. We regard the BEC as a manifestation of the spontaneous breakdown of the global phase symmetry. Then, the Goldstone theorem leads to the existence of the zero energy excitation mode (zero-mode). We calculate the effect of the zero-mode to the response frequency and show that the contribution of the zero-mode to the first excitation mode becomes dominant as the temperature and/or the coupling constant are increased.

Since the first experimental realization of Bose–Einstein Condensation (BEC) of trapped neutral alkali-metal atoms, a large number of experimental as well as theoretical studies have been achieved. These experiments are controlled very well; we can change not only the shape of trapping potential but also the sign and magnitude of the interaction strength. It follows that the BEC offers a good opportunity for testing the quantum many-body theories.

According to quantum field theory, the BEC is recognized as a manifestation of the spontaneous breakdown of the global phase symmetry. Then, the Goldstone theorem leads to the existence of the zero energy excitation mode (zero-mode). However, the consistent quantization scheme is not easy to construct, because of, for instance, the problem of the infrared divergence. The widely used Bogoliubov prescription ignores the zero-mode, so it is not a consistent quantization scheme because we cannot satisfy the canonical commutation relation.

Recently, the consistent quantization scheme was proposed in [1]. They introduced the infinitesimal breaking term of the global phase symmetry. Here the breaking term plays a role of a regulator of infrared divergences and enables us to construct the generalized Bogoliubov transformation including the zero-mode[1]. In this work, we apply this consistent scheme and discuss the experimental observability of the effect of the zero mode.

We focus on the experiment of collective excitation at finite temperature observed in JILA [2], where the condensate and non-condensate oscillate collectively under the time dependent perturbation.

For this system, it is already known that the response frequencies, calculated under the Hartree–Fock–Bogoliubov–Popov (HFBP) approximation[3] or under the assumption of low temperature and spherical symmetry[4], coincide with the first quasi-particle energy calculated under the same approximation and assumptions. So in this study we numerically calculate the
first quasi-particle energy including zero-mode according to the formalism established in [1]. This time we assume one-dimensional system for simplicity. As an extension of our previous study[4], we numerically calculate the above quantity for the various coupling constant and over a wide range of the temperature. We also make the HFBP approximation. To be consistent with the experimental setup[2], the parameters are fixed so as to reproduce the trapped atoms of $^{85}\text{Rb}$, where $m = 1.42 \times 10^{-25}$kg, with the trapping frequency $\omega = 200 \times 2\pi$Hz. The coupling constant is a changeable parameter here, because experimentalists can change it using the technique of Feshbach resonance.

Figure 1 shows the first quasi-particle energy for various values of the temperature and coupling constant, including the zero-mode. One can see that the first quasi-particle energy increases as the temperature and/or the coupling constant are increased[5]. To extract the effect of the zero mode explicitly, we show the “difference” for various values of the temperature and coupling constant in Figure 2. The quantity “difference” is given by the quasi-particle energy calculated with zero-mode subtracted by the one without zero-mode. One can see that the effect of the zero mode is drastically enhanced as the temperature and/or coupling constant are increased[5]. The difference is still small to measure. However, the result implies the possibility of the experimental observation under the high temperature and/or the large coupling constant.

![Figure 1](image1.png)  
**Figure 1.** Temperature and coupling dependence of the first quasi-particle energy.

![Figure 2](image2.png)  
**Figure 2.** Temperature and coupling dependence of the effect of the zero mode.

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