Simulation of maximum elastic deformations during flat grinding of low-rigidity prismatic workpieces

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Abstract. The flat grinding of low-rigidity prismatic workpieces side surfaces with the initial non-flatness of the base face is investigated. Mathematical models for determination of maximum elastic deformation of prismatic work-pieces when fixing and machining are presented. The theory of bars is used for determination of elastic deformations at the bending of prismatic workpieces. Maximal elastic deformation of prismatic workpieces at the bending is determined the method of More. Contact deformations of the surfaces of the workpiece and the machine table are taken into account. An experimental verification of the developed models was carried out. Keywords: elastic deformations, prismatic workpiece, small rigidity, flat grinding.

1. Introduction
The guides along which the moving parts of machines, measuring devices, robots and other machines move are one of the main structural elements and largely determine its capabilities and technical level, affecting the static and dynamic structural rigidity [1, 2]. Most of the guides have a prismatic shape with various types of cross sections. High demands on the quality of the machined side surface of the prismatic guides are usually ensured by flat grinding.

When grinding the side faces of prismatic workpieces of low rigidity, elastic deformations in the direction perpendicular to the surface of the machine table make it difficult to ensure the required quality, in particular, the specified tolerance of flatness, the machined surface. Under load, when fixing and grinding the workpiece, elastic deformations arise, comparable with the tolerances on the geometric parameters of the machined surface. After grinding and removing the magnetic field of the machine table, elastic deformations return a certain amount of deviation to geometric parameters, which can exceed specified requirements. The required tolerance on the geometric parameters of the machined surface is provided by additional transitions to grinding operations, which significantly increases the main grinding time. Significantly reduce grinding time is possible by limiting the amount of elastic deformation [3 - 5].

Objective of the research: developing an mathematical models of maximum elastic deformations of a prismatic workpiece of low rigidity during flat grinding by the periphery of the wheel, taking into account the cutting force, the attractive force of the magnetic field of the table and the change in the rigidity of the workpiece through the use of compensators; experimental verification of developed models.
2. Simulation of maximum elastic deformations

A significant part of the elastic deformation of the workpiece is determined by the deflection during bending of the workpiece under load, which, in turn, is due to the initial non-flatness of the surface of the workpiece in contact with the table surface. It is necessary to take into account contact deformation, a certain proportion of which may be plastic in real conditions of grinding workpieces. The determination of the maximum deflections of the workpieces is based on the theory of beam bending, therefore, the geometric parameters of the workpiece must satisfy the constraint: \( l_w/h > 10 \) (where \( l_w \) is the length of the workpiece, \( h \) is the largest cross-sectional size).

As studies have shown, the main factor determining the initial deviations from the flatness of the side faces of prismatic workpieces for grinding operations is the workpiece deformation as a result of its heat treatment and previous machining. In this regard, the workpiece receives a surface curvature with pronounced regular waves of macrodeviations. Based on the experimental studies of the surfaces of the workpieces before the grinding operation, the macrodeviation of the surface of the workpiece in contact with the surface of the machine table is modeled by a cylindrical surface with a guide in the form of a sinusoid with a characteristic wavelength \( l \) (figure 1) [6]. The length \( l \) depends on the design features of the workpiece, its bending stiffness, prior to grinding mechanical and heat treatment. With large lengths of the workpieces, several regular sinusoidal waves can be stacked in its length. The doubled amplitude of the sinusoid \( y_a \) is taken equal to the maximum height of the waves of macrodeviations (see figure 1).

![Figure 1](image.png)

**Figure 1.** The design scheme of the workpiece when securing the magnetic field of the table:
(a) is a diagram of the workpiece with the applied load; (b) - design scheme of the beam.

To determine the maximum deflections, the workpiece is modeled by a continuous beam with the number of spans \( n \) equal to the number of sine waves on the length of the workpiece \( n = l_w/l \). The load is modeled by a load of intensity \( q = q_c + q_m \) uniformly distributed along the central axis (from the action of the magnetic field attractive force of the machine table \( q_c \) and the workpiece mass \( q_m \)) and the concentrated force \( P_y \), the radial component of the cutting force (see figure 1).

The distance between the contact points of the surfaces of the workpiece and the machine table \( y_n \) in sections with maximum deflections, with the coordinate \( x_{\text{max}} \) (see figure 1) is determined by the equation:

\[
y_n = 0.5 y_a \left[ 1 - \cos \left( 2\pi \cdot \frac{x_{\text{max}}}{l} \right) \right].
\]

The mathematical model of maximum elastic deformation when fixing the workpiece with the magnetic field of the table and grinding has the general form:

\[
w_{\text{max}} = w_{\text{qn}} + w_{\text{pn}} + w_{\text{cn}},
\]

where \( w_{\text{qn}}, w_{\text{pn}} \) is the maximum deflection of the workpiece during bending, respectively, under the action of the mass of the workpiece and the attractive force of the magnetic field of the machine table,
the radial component of the cutting force; \( w_{cn} \) - total contact deformation: \( w_{cn} = w_{cn} + w_{cpn} \), where \( w_{cpn} \) - contact deformation of the surface of the workpiece with the table plane under the action of the cutting force; \( w_{cpn} \) - contact deformation of the surface of the workpiece with the plane of the table under the action of the mass of the workpiece and the force of attraction of the magnetic field of the machine table.

A certain part of contact deformation under working conditions can be plastic. The magnitude of the plastic contact deformation is taken into account only to reduce the gap between the contacting surfaces of the table and the workpiece.

The conditions for ensuring the required flatness tolerance of the treated surface limit the maximum elastic deformation \( w_{max} \) to the allowable maximum elastic deformation \([\Delta]\) [4]:

\[
w_{max} \leq [\Delta],
\]

where \([\Delta] = \lambda \Delta - \Delta_a\), \(\lambda\) is coefficient precision; \(\Delta\) - flatness tolerances of the end surface during grinding process; \(\Delta_a\) - flatness tolerances when grinding hard workpieces.

When fixing the machine table with a magnetic field, a uniformly distributed load of intensity \(q\) acts on the workpiece (see Fig. 1). For \(n = 1\), we have a single-span beam. The maximum deflection from the action of the force of attraction of the magnetic field of the machine table and the dead weight of the workpiece \(w_{q1}\) and the coordinate of the maximum deflection \(x_{max1}\) will be equal to [7, 8]:

\[
w_{q1} = 13 \times 10^{-3} \frac{ql^4}{EI_z}; \quad x_{max1} = 0.5l,
\]

where \(E\) is the modulus of longitudinal elasticity of the workpiece material, \(I_z\) is the axial moment of inertia of the cross section of the workpiece relative to the central axis \(z\) located perpendicular to the bending plane.

The distance (gap) between the points of contact of the contacting surfaces of the workpiece and the table in the middle of the span is \(y_1 = y_a\) (see figure 1).

For \(n > 1\), the beam will be statically indeterminate and the maximum deflections \(w_{qn}\) and the coordinates of the maximum deflections \(x_{maxn}\) are determined by the initial parameter method by integrating the differential equation of the elastic line of the beam in the first span (see figure 1). Given the integration constants, the deflection of an arbitrary section of the first beam span will be equal to:

\[
w(x) = \frac{1}{EI_z} \left[ \frac{R_0}{6} \left( x^3 - l^2x \right) - \frac{q}{24} \left( x^4 - l^3x \right) \right].
\]

The coordinate of the section with the maximum deflection in the first span of the beam \(x_{maxn}\) is determined from the condition that the angle of rotation of the section is equal to zero, solving the cubic equation:

\[
\frac{R_0}{2} \left( x_{maxn}^2 - \frac{l^2}{3} \right) - \frac{q}{6} \left( x_{maxn}^3 - \frac{l^3}{4} \right) = 0.
\]

Substituting the coordinate \(x_{maxn}\) in (3), we obtain the dependences for the maximum deflections for a different number of spans — mathematical models of maximum deflections on the action of the magnetic field attractive force of the machine table and the dead weight of the workpiece — in the form:

\[
w_{qn} = a_nql^4 \times 10^{-3}/(EI_z).
\]

The values of the coefficients \(a_n\), the coordinates of the maximum deflections \(x_{maxn}\), the distances between the points of contact of the surfaces of the workpiece and the machine table \(y_n\) depending on the number of spans \(n\) are shown in table 1.
Table 1. Design parameters of the models of maximum deflections when fixing the workpiece.

| n  | \( a_n \) | \( x_{\text{max},n} \) | \( y_{\text{max},n} \) | \( y_{\text{max},nk} \) | \( x_{\text{max},nk} \times l \) | \( y_{\text{max},nk} \times y_a \) |
|----|--------|----------------|----------------|----------------|----------------|----------------|
|    |        |                |                |                |                |                |
| n = 1 | 13,00  | 0,500          | 1,000          | -              | -              | -              |
| n = 2 | 5,42   | 0,422          | 0,941          | -              | -              | -              |
| n = 3 | 6,99   | 0,450          | 0,976          | -              | -              | -              |
| n = 4 | 6,51   | 0,437          | 0,961          | -              | -              | -              |
| n = 5 | 6,56   | 0,440          | 0,965          | -              | -              | -              |
| n ≥ 6 | 6,53   | 0,440          | 0,965          | -              | -              | -              |

The maximum deflections \( w_{qk} \), their coordinates \( x_{\text{max},nk} \) and the distances between the contact points of the surfaces of the workpiece and the table \( y_{\text{nk}} \) are determined using equations (1), (3), (4), taking into account the reduction in the span length. The maximum number of pairs of compensator bars on the characteristic length is limited to five of reasons of cutting-down of auxiliary time for grinding operations. When installing two pairs of compensator bars, we obtain the design scheme of the three span beams \( n = 3 \) (figure 2).

For increase in rigidity of workpiece at a bend when fixing with magnetic field of table of the machine it is recommended to use compensator bars [5]. The pairs of compensators are set with a splitting of the characteristic length \( l \) into equal parts \( l/2, l/3, l/4, l/5, l/6 \). When installing two pairs of compensators, we obtain the design scheme of the three span beams \( n = 3 \) (figure 2).

Figure 2. The design scheme of the workpiece when fixing the magnetic field of the machine table with two pairs of compensator bars.

The maximum deflections \( w_{q3k} \), their coordinates \( x_{\text{max},3k} \) and the distances between the contact points of the surfaces of the workpiece and the table \( y_{3k} \) are determined using equations \( 1, 3, 4 \), taking into account the reduction in the span length. The maximum number of pairs of compensator bars on the characteristic length is limited to five of reasons of cutting-down of auxiliary time for grinding operations. When installing two pairs of compensator bars (see figure 2) we get:

\[
y_{32} = 0,2066y_a;
\]

\[
w_{q3k} = 6,99 \times 10^{-3} \frac{ql^4}{EI_z} \left( \frac{l}{3} \right)^4 = 0,0863 \times 10^{-3} \frac{ql^4}{EI_z} = a_{32} \times 10^{-3} \frac{ql^4}{EI_z}; x_{\text{max},32} = 0,45 \left( \frac{l}{3} \right) = 0,15l.
\]

The values of the coefficients \( a_{nk} \), coordinates \( x_{\text{max},nk} \) and the distance between the points of contact of the surfaces of the workpiece and the table \( y_{nk} \) depending on the number of pairs of compensator bars \( k \) are given in table 1.
Mathematical models of maximum deflections from the action of the magnetic field attractive force of the machine table and the dead weight of the workpiece with expansion joints are presented in the form:

\[ w_{qnk} = a_{nk} q t^4 \times 10^{-3} / (EI_z). \]

When using more than two pairs of compensator bars closing of spacing between the surfaces of workpiece and table of the machine (contact of surfaces at deformation of bend) will consistently come from the first flight of beam to average. If the size of the maximum deflection in the first flight of beam is more or is equal to spacing between the surfaces (contact points) of workpiece and table:

\[ w_{qn2} \geq y_{n2}; \quad w_{qn3} \geq y_{n3}; \quad w_{qn4} \geq y_{n4}; \quad w_{qn5} \geq y_{n5}, \]

and amount of clearance will be less or is equal to the allowed maximum elastic deformation of the workpiece \([\Delta]\); \(y_{n2} \leq [\Delta]; \quad y_{n3} \leq [\Delta]; \quad y_{n4} \leq [\Delta]; \quad y_{n5} \leq [\Delta], \) that further calculations of size of the maximum deflection needs to be continued on average flight of beam (see figure 2) \([5]\).

When determining the maximum deflection in the average span \(w'_{qnk}\) in equations (3), (4), terms with the reactions of the supports to the middle span will be added. Mathematical models of maximum deflections in the average span from the action of the force of attraction of the magnetic field of the machine table and the dead weight of the workpiece with compensators are presented in the form:

\[ w'_{qnk} = a'_{nk} q t^4 \times 10^{-3} / (EI_z). \]

The values of the coefficients \(a'_{nk}, \) the coordinates \(x'_{\text{maxnk}}\) and the distances between the points of contact of the surfaces of the workpiece and the table \(y'_{nk}\) in the average span, depending on the number of pairs of compensators \(k,\) are given in table 1.

When grinding, the amount of elastic displacement (squeezing) of the workpiece from the action of the radial component of the cutting force \(P_y\) will depend on the ratio of the stiffnesses of the technological system (machine spindle - attachment points of the machine spindle) - \(j_c\) and workpiece - \(j_{wn}: \) \(c_n = j_c / (j_{wn} + j_c).\) In this case, the rigidity of the workpiece \(j_{wn}\) will depend on the number of spans of the simulated continuous beam \(n.\) The rigidity of surface grinding machines with a rectangular table is determined according to GOST 13135-90 \([3]\).

For \(n = 1,\) we have a single-span beam. The maximum deflection under the action of the radial component of the cutting force \(w_{p1}\) and the coordinate of the maximum deflection \(x_{\text{max1}}\) will be equal to \([8]:\)

\[ w_{p1} = 20,8 \times 10^{-3} P_y l^3 / EI_z; \quad x_{\text{max1}} = 0,5l. \]

The distance (gap) between the contact points of the contacting surfaces of the workpiece and the table is \(y_1 = y_a\) (see figure 1).

For \(n > 1,\) the beam will be statically indeterminate and the maximum deflections \(w_{pn}\) and the coordinates of the maximum deflections \(x_{\text{maxn}}\) are determined by the initial parameter method by integrating the differential equation of the elastic line of the beam in the first span (figure 3) \([8].\)

To obtain the differential equation of the elastic line of the beam, it is necessary to reveal the static indeterminacy and determine the reaction of the zero support \(R_0 = X_1\) (see figure 3). The position of the force \(P_y\) corresponding to the maximum deflection \(x_{\text{maxn}}\) (see figure 3) is determined from the condition of the extremum of the deflection in the first span — the angle of rotation of the cross section to zero. The solution is implemented numerically using the Mathcad program.

Mathematical models of maximum deflections from the action of the radial component of the cutting force are presented in the form:

\[ w_{pn} = b_n c_n P_y l^3 / (EI_z). \]

The values of the coefficients \(b_n,\) the coordinates \(x_{\text{maxn}}\) and the distances between the points of contact of the surfaces of the workpiece and the table \(y_n\) depending on the number of spans \(n\) are shown in table 2.
Figure 3. The design schemes of the workpiece under the action of the radial component of the cutting force: \( a \) - two span beam \( n = 2 \); \( b \) - three span beam \( n = 3 \)

Table 2. Design parameters of the models of maximum deflections when grinding the workpiece.

| The number of spans, \( n \) | The number of pairs of compensators, \( k \) | \( a_n \) | \( b_n \) | \( \chi_{\max_n} \times x_j \) | \( y_{\max_n} \times y_a \) | \( a'_{nk} \) | \( b'_{nk} \) | \( \chi'_{\max_{nk}} \times x_j \) | \( y'_{\max_{nk}} \times y_a \) |
|-------------------------|---------------------------------|--------|--------|-----------------|-----------------|--------|--------|-----------------|-----------------|
| without compensator bars |
| \( n = 1 \)              | 20.8                            | 13.00  | 0.500  | 1,000           | -               | -      | -                | -               |
| \( n = 2 \)              | 15.1                            | 5.34   | 0.469  | 0.991           | -               | -      | -                | -               |
| \( n = 3 \)              | 17.0                            | 6.82   | 0.481  | 0.964           | -               | -      | -                | -               |
| \( n = 4 \)              | 17.9                            | 6.43   | 0.486  | 0.998           | -               | -      | -                | -               |
| \( n = 5 \)              | 18.5                            | 6.47   | 0.490  | 0.999           | -               | -      | -                | -               |
| \( n \geq 6 \)           | 18.9                            | 6.43   | 0.492  | 0.999           | -               | -      | -                | -               |

| with compensator bars   |
|-------------------------|---------------------------------|---------|--------|-----------------|-----------------|--------|--------|-----------------|-----------------|
| \( k = 1 \)             | 1.890                           | 0.33400 | 0.235  | 0.4510         | -               | -      | -                | -               |
| \( k = 2 \)             | 0.630                           | 0.08420 | 0.160  | 0.2350         | 0.428           | 0.00643 | 0.500  | 1,000           |
| \( k = 3 \)             | 0.280                           | 0.02510 | 0.122  | 0.1390         | 0.198           | 0.00715 | 0.371  | 0.845           |
| \( k = 4 \)             | 0.148                           | 0.01040 | 0.098  | 0.0918         | 0.113           | 0.00515 | 0.500  | 1,000           |
| \( k = 5 \)             | 0.088                           | 0.00496 | 0.082  | 0.0649         | 0.069           | 0.00245 | 0.416  | 0.931           |

When installing compensator bars, the workpiece is also modeled by a continuous beam. The maximum deflections \( w_{p_nk} \), their coordinates \( \chi_{\max_n} \times x_j \) and the distances between the contact points of the surfaces of the workpiece and the table \( y_{nk} \) are determined taking into account the reduction in the span length. The calculation results are shown in Table 2. With the simultaneous action of the forces of attraction of the magnetic field of the machine, the dead weight of the workpiece, the cutting force, the maximum deflection from the action of the cutting force prevails, therefore, the total maximum deflection of the workpiece is determined in the section where the maximum deflection from the action of the cutting force occurs. The calculation results are shown in Table 2.

The real surface of the workpiece has a longitudinal and transverse undulation and roughness, the parameters of which are measured experimentally. Contact deformation consists of three components:

\[
 w_{cm} = w_{cm1} + w_{cm2} + w_{cm3},
\]

where \( w_{cm1} \) - contact deformation due to deformation of microroughnesses; \( w_{cm2} \) - contact deformation
due to wave deformation; \( w_{\text{wk3}} \) - contact deformation due to deformation of macrodeviations.

The calculation formulas for calculating contact deformations are presented in [9].

3. Experimental studies

To study the elastic deformations that occur during fixing and grinding of the workpiece, the guide of the LRX 6/350 single-row roller bearing is selected (figure 4).

![Figure 4. The investigated guide bearing LRH 6/350](image)

The bearing guide made of steel 20Kh (GOST 4543 - 71) is subjected to cementation to obtain a cemented layer with a depth of 0,4 – 0,8 mm. Before the operation of flat grinding, transverse and longitudinal holes are drilled in the workpiece, and longitudinal grooves are processed (see figure 4) [5]. The deviations from the flatness of the surfaces of the side faces of the workpiece were measured by the indicator head during the longitudinal movement of the table with the workpiece. The deviations were measured in increments of 16 mm when moving from left to right and indented from the edge of the workpiece 2 mm for the surface of face C (see figure 4). Each measurement was performed 6 times. Comparison of variances by the Cochren criterion showed that in all cases the variances differ insignificantly. Schemes for determining the parameters of a sinusoid: a characteristic wavelength \( l \), doubled amplitude \( y_a \), are shown in figure 5. The macrodeviation from flatness is approximated by a cylindrical surface with a guide in the form of a sinusoidal wave with a characteristic length \( l = 245 \text{ mm} \).

Experimental studies of the maximum elastic deformations were carried out under the action of the specific force of attraction of the magnetic field of the machine table \( p = 0,02 \text{ MPa} \). The experimentally determined deviations for face \( A \) in the absence of the force of attraction of the magnetic field of the machine table \( (p = 0) \) and under the action of a magnetic field \( (p = 0,02 \text{ MPa}) \) are shown in figure 5.

The workpiece guide is examined with the state of the surfaces before the grinding operation. The blank lies on the magnetic table of the machine with face \( A \).

The value of the maximum elastic deformation at the intensity of a uniformly distributed load from the action of the magnetic field \( q_c = p \cdot b = 0,02 \times 15 = 0,30 \text{ N / mm} \), of its own weight
Figure 5. Deflections of face $A$ with a single span design.

$q_m = 0.5 \cdot 9.81 / 350 = 0.014$ N / mm, with a length $l = 245$ mm, in accordance with formula (2) will be equal to: $w_{\text{max}} = w_{q1} + w_{p1} + w_{c1} = 20.3 + 0 + 0.27 = 20.6$ μm.

Experimental and theoretical results were processed only in the span between the points of contact of face $A$ with the table surface (see figure 5). The experimentally measured maximum deviations of face $A$ under the action of the specific force of attraction of the magnetic field of the machine table $p = 0.02$ MPa (see figure 5) have relative errors from theoretical deviations within 17%. The diagrams in figure 5 are based on the average deviations.

4. Conclusions

Mathematical models of the maximum elastic deformations of a prismatic workpiece of low stiffness during flat grinding by the periphery of the wheel are developed taking into account the cutting force, the attractive force of the magnetic field of the table and the change in the rigidity of the workpiece due to the use of compensator bars.

Experimental verification confirms the performance of the developed models.

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