Computation of a New Error Bound for Tensor Complementarity Problem with P Tensor

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Abstract
We propose a new error bound for the solution of tensor complementarity problem TCP(q, A) given that A is a P-tensor and q is a real vector. We show that the proposed error bound is sharper than the earlier version of error bound available in the literature. We establish absolute and relative error bound for TCP(q, A) where A is an even order positive diagonal tensor.

Keywords: Tensor complementarity problem, P-tensor, global error bound, positively homogeneous operator.

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1 Introduction
In this article a new error bound sharper than the earlier version is introduced for the tensor complementarity problem with P-tensor. During last several years, the tensor complementarity problem attains much attraction and has been studied extensively with respect to theory, to solution methods and applications. In recent years, various tensors with special structures have been studied. For details, see [41] and [44]. The tensor complementarity problem was studied initially by Song and Qi [42]. The tensor complementarity problem is a subclass of the non-linear complementarity problems where the function involved in the non-linear complementarity problem is a special polynomial defined by a tensor in the tensor complementarity problem. The polynomial functions used in tensor complementarity problems have some special structures. For a given mapping $F : \mathbb{R}^n \mapsto \mathbb{R}^n$ the complementarity problem is to find a vector $x \in \mathbb{R}^n$ such that
\begin{equation}
x \geq 0, \quad F(x) \geq 0, \quad \text{and} \quad x^TF(x) = 0.
\end{equation}
If $F$ is nonlinear mapping, then the problem (1.1) is called a nonlinear complementarity problem [13], and if $F$ is linear function, then the problem (1.1) reduces to a linear complementarity problem [4]. The linear complementarity problem may be defined as follows: Given a matrix $M \in \mathbb{R}^{n \times n}$ and a vector $q \in \mathbb{R}^n$, the linear complementarity problem [4], denoted by $LCP(q, M)$, is to find a pair of vectors $w, z \in \mathbb{R}^n$ such that

$$z \geq 0, \quad w = Mz + q \geq 0, \quad z^Tw = 0.$$  \hspace{1cm} (1.2)

The algorithm based on principal pivot transforms namely Lemke’s algorithm, Criss-cross algorithm which are used to find the solutions of linear complementarity problem are studied extensively considering several matrix classes.

It is important that large number of formulations not only enrich the linear complementarity problem but also generate different matrix classes along with their computational methods. For details see [9], [32], [11], [12], [36], [20], [30], [35], [17], [21], [37], [7], [8]. For details of game theory see [27], [33], [39], [29], [28], [6], [34] and for details of QMOP see [26]. Even matrix classes arise during the study of Lemke’s algorithm as well as principal pivot transform. For details see [25], [24], [31], [5], [38], [17], [18], [21], [19]. Now we consider the case of $F(x) = Ax^{m-1} + q$ with $A \in T_{m,n}$ and $q \in \mathbb{R}^n$ then the problem (1.1) becomes

$$x \geq 0, \quad Ax^{m-1} + q \geq 0, \quad x^T(Ax^{m-1} + q) = 0$$  \hspace{1cm} (1.3)

which is called a tensor complementarity problem, denoted by the TCP($q, \mathcal{A}$). Denote $\omega = Ax^{m-1} + q$, then the tensor complementarity problem is to find $x$ such that

$$x \geq 0, \quad \omega = Ax^{m-1} + q \geq 0, \quad x^T\omega = 0.$$  \hspace{1cm} (1.4)

Motivated by the discussion on positive definiteness of multivariate homogeneous polynomial forms [2], [15], [22], Qi [40] introduced the concept of symmetric positive definite (positive semi-definite) tensors. Song and Qi [44] studied $P(P_0)$-tensors and $B(B_0)$-tensors. The equivalence between (strictly) semi-positive tensors and (strictly) copositive tensors in symmetric case were shown by Song and Qi [45]. The existence and uniqueness of solution of TCP($q, \mathcal{A}$) with some special tensors were discussed by Che, Qi, Wei [3]. The boundedness of the solution set of the TCP($q, \mathcal{A}$) was studied by Song and Yu [46]. The sparse solutions to TCP($q, \mathcal{A}$) with a $Z$-tensor and its method to calculate were obtained by Luo, Qi and Xiu [23]. The equivalent conditions of solution to TCP($q, \mathcal{A}$) were shown by Gowda, Luo, Qi and Xiu [14] for a $Z$-tensor $\mathcal{A}$. The global uniqueness of solution of TCP($q, \mathcal{A}$) was considered by Bai, Huang and Wang [1] for a strong $P$-tensor $\mathcal{A}$. The properties of TCP($q, \mathcal{A}$) was studied by Ding, Luo and Qi [10] for a new class of $P$-tensor. The properties of the several classes of $Q$-tensors were presented by Suo and Wang [16]. In this article we introduce column adequate tensor in the context of tensor complementarity problem and study different properties of this tensor.

The paper is organised as follows. Section 2 contains some basic notations and results. In Section 3, We propose a new error bound for the solution of tensor complementarity
problem TCP\((q, \mathcal{A})\) given that \(\mathcal{A}\) is a \(P\)-tensor and \(q\) is a real vector. We show that the proposed error bound is sharper than the earlier version of error bound available in the literature. We establish absolute and relative error bound for TCP\((q, \mathcal{A})\) where \(\mathcal{A}\) is an even order positive diagonal tensor. The results are illustrated with the help of a numerical example.

## 2 Preliminaries

We begin by introducing some basic notations used in this paper. We consider tensor, matrices and vectors with real entries. Let \(m\)th order \(n\) dimensional real tensor \(\mathcal{A} = (a_{i_1i_2...i_m})\) be a multidimensional array of entries \(a_{i_1i_2...i_m} \in \mathbb{R}\) where \(i_j \in [n]\) with \(j \in [m]\). \(T_{m,n}\) denotes the set of real tensors of order \(m\) and dimension \(n\). For any positive integer \(n\), \([n]\) denotes set of \(\{1, 2, ..., n\}\). All vectors are column vectors. Let \(\mathbb{R}^n\) denote the \(n\)-dimensional Euclidean space and \(\mathbb{R}^n_+ := \{x \in \mathbb{R}^n : x \geq 0\}\). For any \(x \in \mathbb{R}^n\), let \(x[m] \in \mathbb{R}^n\) with its \(i\)th component being \(x_i^m\) for all \(i \in [n]\). \(\|x\|_\infty = \max\{|x_i| : i \in [n]\}\) and \(\|x\|_2 = (\sum_{i=1}^n |x_i|^2)^{\frac{1}{2}}\). Then for a continuous, positively homogeneous operator \(T : \mathbb{R}^n \mapsto \mathbb{R}^n\) it is obvious that \(\|T\|_\infty = \max_{\|x\|_\infty = 1} \|T(x)\|_\infty\) is an operator norm of \(T\) and \(\|T(x)\|_\infty \leq \|T\|_\infty \|x\|_\infty\) for any \(x \in \mathbb{R}^n\). For \(\mathcal{A} \in T_{m,n}\) and \(x \in \mathbb{R}\), \(Ax^{m-1} \in \mathbb{R}^n\) is a vector defined by

\[
(Ax^{m-1})_i = \sum_{i_2, i_3, ..., i_m=1}^n a_{i_1i_2i_3...i_m} x_{i_2}x_{i_3} \cdots x_{i_m}, \quad \text{for all } i \in [n]
\]

and \(Ax^m \in \mathbb{R}\) is a scalar defined by

\[
Ax^m = \sum_{i_1, i_2, i_3, ..., i_m=1}^n a_{i_1i_2i_3...i_m} x_{i_1}x_{i_2} \cdots x_{i_m}.
\]

For any \(\mathcal{A} \in T_{m,n}\),

\[
\|\mathcal{A}\|_\infty = \max_{i \in [n]} \sum_{i_2, ..., i_m=1}^n |a_{i_1i_2,...,i_m}|.
\]

Song et al. \[44\] defined two operators. Let \(\mathcal{A} \in T_{m,n}\). For \(x \in \mathbb{R}^n\), the operator \(T_{\mathcal{A}} : \mathbb{R}^n \mapsto \mathbb{R}^n\) is defined by

\[
T_{\mathcal{A}}x = \begin{cases} \|x\|_2^{2-m}Ax^{m-1}, & x \neq 0, \\ 0, & x = 0. \end{cases} \tag{2.1}
\]

When \(m\) is even, for \(x \in \mathbb{R}^n\) another operator \(F_{\mathcal{A}} : \mathbb{R}^n \mapsto \mathbb{R}^n\) is defined by

\[
F_{\mathcal{A}}x = (Ax^{m-1})^{\left(\frac{1}{m-1}\right)}. \tag{2.2}
\]

Song et al. \[44\] introduced two important quantities

\[
\alpha(T_{\mathcal{A}}) = \min_{\|x\|_\infty = 1} \max_{i \in [n]} x_i(T_{\mathcal{A}}x), \tag{2.3}
\]
for any positive integer $m$ and

$$\alpha(F_A) = \min_{\|x\|_\infty = 1} \max_{i \in [n]} x_i(F_A)$$  \hspace{1cm} (2.4)

when $m$ is even. Since $P$-tensors are all even order, then $\alpha(T_A)$ and $\alpha(F_A)$ are both well defined for any $P$-tensor. The following result is necessary and sufficient conditions for a $P$-tensor in terms of $\alpha(T_A)$ and $\alpha(F_A)$.

**Lemma 2.1:** \cite{14} Let $A \in T_{m,n}$. Then
(a) $A$ is a $P$-tensor if and only if $\alpha(T_A) > 0$,
(b) When $m$ is even $A$ is a $P$-tensor if and only if $\alpha(F_A) > 0$.

Since every strong $P$-tensor is a $P$-tensor, the following corollary is obvious.

**Corollary 2.1:** \cite{37} Let $A \in T_{m,n}$. Then
(a) $\alpha(T_A) > 0$ if $A$ is a strong $P$-tensor,
(b) $\alpha(F_A) > 0$ if $A$ is a strong $P$-tensor.

**Definition 2.1.** Let $e: \mathbb{R}^n \mapsto \mathbb{R}_+^n$ be a function. Assume that TCP($q, A$) has a nonempty solution set.
(a) We say that $e(x)$ is a residual function of TCP($q, A$), if $e(x) \geq 0$, and $e(x) = 0$ if and only if $x$ solves TCP($q, A$).
(b) We say that a residual function $e(x)$ is a lower global error bound for TCP($q, A$), if $\exists$ some constants $c_1 > 0$ such that for each $x \in \mathbb{R}^n$ and any solution $\bar{x}$, $c_1 e(x) \leq \|x - \bar{x}\|$.  
(c) We say that a residual $e(x)$ is an upper global error bound for the TCP($q, A$) if there exists some constant $c_2 > 0$ such that for each $x \in \mathbb{R}^n$ and any solution $\bar{x}$, $\|x - \bar{x}\| \leq c_2 e(x)$.

**Definition 2.1:** \cite{46} Given $A = (a_{i_1i_2...i_m}) \in T_{m,n}$ and $q \in \mathbb{R}^n$, a vector $x$ is said to be (strictly) feasible, if $x(>0)$ and $Ax^{m-1} + q(>) \geq 0$.
TCP($q, A$), defined by equation $(1.3)$ is said to be (strictly) feasible if a (strictly) feasible vector exists.
TCP($q, A$) is said to be solvable if there is a feasible vector $x$ satisfying $x^T(Ax^{m-1} + q) = 0$ and $x$ is the solution.

**Definition 2.2:** \cite{44} A tensor $A = (a_{i_1i_2...i_m}) \in T_{m,n}$ is said to be a $P$-tensor, if for each $x \in \mathbb{R}^n \setminus \{0\}$, there exists an index $i \in [n]$ such that $x_i \neq 0$ and $x_i(Ax^{m-1})_i > 0$.

**Definition 2.3:** \cite{1} A tensor $A \in T_{m,n}$ is said to be strong $P$-tensor if for any two different $x = (x_i)$ and $y = (y_i)$ in $\mathbb{R}^n$, $\max_{i \in [n]}(x_i - y_i)(Ax^{m-1} - Ay^{m-1})_i > 0$.

**Theorem 2.1:** \cite{1} For any $q \in \mathbb{R}^n$ and a $P$-tensor $A \in T_{m,n}$, the solution set of TCP($q, A$) is nonempty and compact.

**Theorem 2.2:** \cite{1} Let $A \in T_{m,n}$ be a strong $P$-tensor. Then the TCP($q, A$) has the property of global uniqueness and solvability property.
**Theorem 2.3:** Let $A \in T_{m,n}$ ($m \geq 0$) be a $P$-tensor, and let $x$ be a solution of TCP($q, A$). If $m$ be even, then

$$
\frac{||(-q)_+||_m^{\frac{1}{m-1}}}{||A||_\infty^{\frac{1}{m-1}}} \leq ||x||_\infty \leq \frac{||(-q)_+||_\infty^{\frac{1}{m-1}}}{\alpha(F_A)}.
$$

(2.5)

**Theorem 2.4:** Given $q \in \mathbb{R}^n$, $A \in T_{m,n}$ with $A$ being a $P$-tensor and $\alpha(F_A)$ is defined by (2.4). For any $u \in \mathbb{R}^n$, let $x$ be a solution of TCP($q, A$). Suppose the residue function $\tilde{v}$ is defined as $\tilde{v} = \min\{u, [(A(u-z)^m)^{\frac{1}{m-1}}] + (A^m q^{\frac{1}{m-1}}]\}$. Then for any $u \in \mathbb{R}^n$,

$$
\frac{1}{1 + ||A||_{\infty}^{\frac{1}{m-1}}} ||\tilde{v}||_\infty \leq ||u - x||_\infty \leq \frac{1 + ||A||_{\infty}^{\frac{1}{m-1}}}{\alpha(F_A)} ||\tilde{v}||_\infty
$$

(2.6)

### 3 Main results

We begin with a theorem which provides a solution bound for TCP($q, A$) involving $P$-tensor.

**Theorem 3.1:** Let $A \in T_{m,n}$ be a $P$-tensor where $m$ is even and $x$ be a solution of TCP($q, A$). Then

$$
\frac{||(-q)_+||_m^{\frac{1}{m-1}}}{||A||_\infty^{\frac{1}{m-1}}} \leq ||x||_\infty \leq \frac{||(-q)_+||_\infty^{\frac{1}{m-1}}}{\alpha(F_A)}.
$$

(3.1)

**Proof.** Note that from Theorem 2.3 we obtain the right hand inequality. We prove the left hand inequality. Since $x$ is a solution of TCP($q, A$) we have $Ax^m + q \geq 0$. Which implies

$$
Ax^m = -q.
$$

(3.2)

The following inequation is proved by Zheng et al [47] in Theorem 3.2. However for the sake of completeness we give the details. We have

$$
||Ax||_{\infty}^{\frac{1}{m-1}} = \left(\max_{i \in [n]} \left\{ |(Ax^{m-1})_i| \right\}\right)^{\frac{1}{m-1}}
$$

$$
\leq \left(\max_{i \in [n]} \left\{ \sum_{i_2, \ldots, i_m=1}^n a_{i_2 \ldots i_m} x_{i_1} x_{i_2} \ldots x_{i_m} \right\}\right)^{\frac{1}{m-1}}
$$

$$
\leq \left(\max_{i \in [n]} \left\{ \sum_{i_2, \ldots, i_m=1}^n |a_{i_2 \ldots i_m}| ||x||^{m-1} \right\}\right)^{\frac{1}{m-1}}
$$

$$
= \left(||A||_\infty ||x||^{m-1} \right)^{\frac{1}{m-1}}
$$

$$
= ||A||_{\infty}^{\frac{1}{m-1}} ||x||_\infty.
$$
Therefore,
\[ ||A||_{\infty}^\frac{1}{m-1} ||x||_\infty \geq ||Ax||_{\infty}^\frac{1}{m-1} \geq ||(Ax)_+||_{\infty}^\frac{1}{m-1} \geq ||(-q)_+||_{\infty}^\frac{1}{m-1} \]
using Equation 3.2

Hence the left hand inequality.

Here we investigate the global error bound for the TCP(q, A).

**Theorem 3.2:** Let \( A \in T_{m,n} \) be a \( P \)-tensor, \( z \in \mathbb{R}^n \) be a solution of TCP(q, A) and \( u \) be an arbitrary vector in \( \mathbb{R}^n \). Then

\[
\frac{||v||_\infty(1 + ||A||_{\infty}^\frac{1}{m-1}) - \sqrt{D}}{2\alpha(F_A)} \leq ||z - u||_\infty \leq \frac{||v||_\infty(1 + ||A||_{\infty}^\frac{1}{m-1}) + \sqrt{D}}{2\alpha(F_A)}, \quad \forall \ u \in \mathbb{R}^n,
\]

where

\[
v = u - \max\{0, u - \left( (A(u - z)^{m-1})^\frac{1}{m-1} + (Az^{m-1} + q)^{\frac{1}{m-1}} \right) \},
\]

and

\[
D = ||v||_\infty^2 (1 + ||A||_{\infty}^\frac{1}{m-1})^2 - 4\alpha(F_A)v_t^2 \geq 0,
\]

the quantity \( \alpha(F_A) \) is defined by (2.4) and \( t \) satisfies \( v_t \neq 0 \),

\[
\max_{i \in [n]} \{ (u - z)_i (A(u - z)^{m-1})_i \} = (u - z)_t (A(u - z)^{m-1})_t.
\]

**Proof.** Consider the TCP(q, A), which is to find \( z \in \mathbb{R}^n \) such that,

\[
z \geq 0, \quad Az^{m-1} + q \geq 0, \quad z^T(Az^{m-1} + q) = 0.
\]

Let the solution set of TCP(q, A) be denoted by \( SOL(A, q) = \{ z \in \mathbb{R}^n : z \geq 0, \ Az^{m-1} + q \geq 0, \ z^T(Az^{m-1} + q) = 0 \} \). Let \( z \in SOL(A, q) \). Then for \( w = (Az^{m-1} + q)^{\frac{1}{m-1}} \geq 0 \), we have

\[
z \geq 0, \quad w \geq 0, \quad z^Tw = 0.
\]

Let \( v = u - \max\{0, u - \left( (A(u - z)^{m-1})^\frac{1}{m-1} + (Az^{m-1} + q)^{\frac{1}{m-1}} \right) \} \). Now we consider the vector \( y = u - v \) and \( x = (A(u - z)^{m-1})^\frac{1}{m-1} + (Az^{m-1} + q)^{\frac{1}{m-1}} - v \). Then

\[
y = u - v = \max\{0, u - \left( (A(u - z)^{m-1})^\frac{1}{m-1} + (Az^{m-1} + q)^{\frac{1}{m-1}} \right) \} \geq 0.
\]

Again,

\[
x = (A(u - z)^{m-1})^\frac{1}{m-1} + (Az^{m-1} + q)^{\frac{1}{m-1}} - v
= (A(u - z)^{m-1})^\frac{1}{m-1} + (Az^{m-1} + q)^{\frac{1}{m-1}} - u
+ \max\{0, u - \left( (A(u - z)^{m-1})^\frac{1}{m-1} + (Az^{m-1} + q)^{\frac{1}{m-1}} \right) \}
= \max\{0, \left( (A(u - z)^{m-1})^\frac{1}{m-1} + (Az^{m-1} + q)^{\frac{1}{m-1}} \right) - u \} \geq 0.
\]
Also by the construction of the vectors $y$ and $x$ we have $y_i, x_i = 0$, $\forall i \in [n]$. Thus the vectors $y$ and $x$ satisfy the following inequalities and complementarity condition.

$$y \geq 0, \quad x \geq 0, \quad y^T x = 0. \quad (3.7)$$

Then for $i \in [n]$,

$$(y - z)_i(x - w)_i = y_i x_i - z_i x_i - y_i w_i + z_i w_i = -z_i x_i - y_i w_i \leq 0. \quad (3.8)$$

Again

$$(y - z)_i(x - w)_i = (u - v - z)_i((\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1} - v)_i$$

$$= (u - z)_i((\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1})_i - v_i((\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1})_i$$

$$- (u - z)_i v_i + v_i^2$$

$$\leq 0 \quad \text{by } (3.8).$$

Hence for each $i$ we obtain

$$(u - z)_i((\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1})_i \leq v_i((\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1})_i + (u - z)_i v_i - v_i^2. \quad (3.9)$$

As $\mathcal{A}$ is a $P$-tensor, $\max_{i \in [n]} \{x_i(\mathcal{A}x^{m-1})_i \} > 0$, $\forall x \in \mathbb{R}^n \backslash \{0\}$. Let $t$ be the particular index for which

$$\max_{i \in [n]} \{(u - z)_i((\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1})_i \} = (u - z)_i((\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1})_t \geq 0. \quad (3.10)$$

Now using from (3.3) we have

$$||\mathcal{A}(u - z)^{m-1}||^\frac{1}{m-1} \leq ||\mathcal{A}||^\frac{1}{m-1} ||u - z||_{\infty}. \quad (3.11)$$

Again from Equation 14 of [17] we have,

$$\alpha(F_A)||z - u||^2_{\infty} \leq \max_{i \in [n]} \{(u - z)_i(\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1}_i \}. \quad (3.12)$$

Then from (3.9), (3.10), (3.11), and (3.12) we obtain,

$$\alpha(F_A)||z - u||^2_{\infty} \leq \max_{i \in [n]} \{(u - z)_i(\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1}_i \}$$

$$\leq (u - z)_i v_i + v_i((\mathcal{A}(u - z)^{m-1})^\frac{1}{m-1})_t - v_i^2$$

$$\leq ||u - z||_{\infty} ||v||_{\infty} + ||\mathcal{A}(u - z)^{m-1}||^\frac{1}{m-1} ||v||_{\infty} - v_i^2$$

$$\leq ||v||_{\infty} ||u - z||_{\infty} + ||\mathcal{A}||^\frac{1}{m-1} ||u - z||_{\infty} ||v||_{\infty} - v_i^2$$

$$= (1 + ||\mathcal{A}||^\frac{1}{m-1}) ||v||_{\infty} ||u - z||_{\infty} - v_i^2.$$

Therefore,

$$\alpha(F_A)||z - u||^2_{\infty} \leq (1 + ||\mathcal{A}||^\frac{1}{m-1}) ||v||_{\infty} ||u - z||_{\infty} - v_i^2 \quad (3.13)$$
From (3.13) we observe that if \( v_t = 0 \) then \( u = z \), which means \( u \) is the true solution. Thus if \( v_t \neq 0 \), then from (3.13) we obtain,

\[
\alpha(F_A)||z - u||^2_\infty - (1 + ||A||^{\frac{1}{\infty-1}})||v||_\infty||u - z||_\infty + v_t^2 \leq 0. \tag{3.14}
\]

Solving the above inequality we obtain the desired relation.

Here we propose a relative error bound for the approximate solution for TCP\((q, A)\) provided that the involved tensor \( A \) is a P-tensor.

**Theorem 3.3:** Let \( A \in T_{m,n} \) be a P-tensor and \( 0 \neq z \in \mathbb{R}^n \) be a solution of TCP\((q, A)\) and \( u \) be an arbitrary vector in \( \mathbb{R}^n \). If \((-q)_+ \neq 0\), we have

\[
\frac{||v||_\infty(1 + ||A||^{\frac{1}{\infty-1}}) - \sqrt{D}}{2||(q)_+||^{\frac{1}{\infty-1}}_\infty} \leq \frac{||z - u||_\infty}{||z||_\infty} \leq \frac{||A||^{\frac{1}{\infty-1}}_\infty[||v||_\infty(1 + ||A||^{\frac{1}{\infty-1}}) + \sqrt{D}]}{2\alpha(F_A)||(q)_+||^{\frac{1}{\infty-1}}_\infty} \tag{3.15}
\]

where \( v = u - \max\{0, u - [(A(u - z)^{m-1})^{\frac{1}{m-1}} + (Az^{m-1} + q)^{\frac{1}{m-1}}]\} \),

\( D = ||v||^2_\infty(1 + ||A||^{\frac{1}{\infty-1}})^2 - 4\alpha(F_A)u_t^2 \geq 0 \) and \( t \) satisfies \( v_t \neq 0 \),

\[
(u - z)_t(A(u - z)^{m-1})_t = \max_{i \in [n]}\{(u - z)_i(A(u - z)^{m-1})_i\}.
\]

**Proof.** From Theorem 3.1 we have inequation (3.1). Since \((-q)_+ \neq 0\), and \( z \neq 0 \) the inequation (3.1) gives

\[
\frac{||A||^{\frac{1}{m-1}}_\infty}{||(q)_+||^{\frac{1}{m-1}}_\infty} \leq \frac{1}{||z||_\infty} \leq \frac{\alpha(F_A)}{||(q)_+||^{\frac{1}{m-1}}_\infty}. \tag{3.16}
\]

From Theorem 3.2 we have inequation 3.4. Now combining inequations (3.4) and (3.16) we obtain the desired result.

Here we consider a positive diagonal tensor of even order which is eventually a P-tensor, and find a property of the quantity \( \alpha(F_A) \).

**Lemma 3.1:** Let \( A \in T_{m,n} \) be a positive diagonal tensor, where \( m \) is even. Then \( \alpha(F_A) = \min_{i \in [n]}\{(a_{ii...i})^{\frac{1}{m-1}}\} \). Where \( a_{ii...i} \) denote the main diagonal elements of \( A \), for \( i = 1, 2, ..., n \).

**Proof.** Let \( x \in \mathbb{R}^n \) be an arbitrary vector satisfied with \( ||x||_\infty = 1 \). Then we have

\[
\max_{i \in [n]}\{x_i(F_A(x),i)\} = \max_{i \in [n]}\{x_i(Ax^{m-1})^{\frac{1}{m-1}}\}
\]

\[
= \max_{||x||_\infty = 1}\{x_i(a_{ii...i}x^{m-1})^{\frac{1}{m-1}}, i = 1, 2, \ldots n\}
\]

\[
= \max_{||x||_\infty = 1}\{(a_{ii...i})^{\frac{1}{m-1}}x^2, i = 1, 2, \ldots n\}
\]

\[
\geq \min_{i \in [n]}\{(a_{ii...i})^{\frac{1}{m-1}}\} > 0.
\]
Thus
\[ \alpha(F_A) = \min_{||x||_\infty=1} \max_{i \in [n]} \{x_i(F_A(x))_i\} \geq \min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}. \quad (3.17) \]

On the other hand by Theorem 4.2 of [43] we have
\[ \alpha(F_A) \leq \min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}. \quad (3.18) \]

Thus from (3.17) and (3.18) we have,
\[ \alpha(F_A) = \min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}. \quad (3.19) \]

Here we present solution bound, absolute and relative error bound of the solution of TCP\((q, A)\), where \(A\) is an even order positive diagonal tensor.

THEOREM 3.4: Let \(A \in T_{m,n}\) be a positive diagonal tensor and \(m\) be even. Let \(z \in \mathbb{R}^n\) be a solution of the TCP\((q, A)\) and \(u\) be an arbitrary vector in \(\mathbb{R}^n\). Then,
\[
\frac{||v_1||_\infty(1 + (\max_{i \in [n]} \{a_{i i - i}\})^{\frac{1}{m-1}}) - \sqrt{D_1}}{2 \min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}} \leq ||z - u||_\infty \leq \frac{||v_1||_\infty(1 + (\max_{i \in [n]} \{a_{i i - i}\})^{\frac{1}{m-1}}) + \sqrt{D_1}}{2 \min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}}
\]

and
\[
\frac{||(q)_+||_\infty^{\frac{1}{m-1}}}{(\max_{i \in [n]} \{a_{i i - i}\})^{\frac{1}{m-1}}} \leq ||z||_\infty \leq \frac{||(q)_+||_\infty^{\frac{1}{m-1}}}{\min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}}
\]

where \(v_1 = u - \max\{0, u - [(A(u - z)^{m-1})^{\frac{1}{m-1}} + (A z^{m-1})^{\frac{1}{m-1}}]\}, D_1 = ||v_1||^2_\infty(1 + (\max_{i \in [n]} \{a_{i i - i}\})^{\frac{1}{m-1}})^2 - 4(\min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\})^2 t^2 \geq 0\) and \(t\) satisfies \(\max_{i \in [n]} \{(u - z)_i (A(u - z)^{m-1})_i\} = (u - z)_t (A(u - z)^{m-1})_t \geq 0\) and \(v_t \neq 0\).

Proof. \(A\) is positive diagonal tensor of even order then \(a_{i i - i} > 0, \forall i \in [n]\) and all other entries of \(A\) are zeros. Then \(A\) is a \(P\)-tensor and \(||A||_\infty = \max_{i \in [n]} \{a_{i i - i}\}\) also from Lemma 3.1, \(\alpha(F_A) = \min_{i \in [n]} \{(a_{i i - i})^{\frac{1}{m-1}}\}\). In Theorem 3.1 putting the values of \(\alpha(F_A)\) and \(||A||_\infty\) in (3.1) we obtain
\[
\frac{||(q)_+||_\infty^{\frac{1}{m-1}}}{(\max_{i \in [n]} \{a_{i i - i}\})^{\frac{1}{m-1}}} \leq ||z||_\infty \leq \frac{||(q)_+||_\infty^{\frac{1}{m-1}}}{\min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}}.
\]

In Theorem 3.2 putting the values of \(\alpha(F_A)\) and \(||A||_\infty\) in (3.2) we obtain
\[
\frac{||v_1||_\infty(1 + (\max_{i \in [n]} \{a_{i i - i}\})^{\frac{1}{m-1}}) - \sqrt{D_1}}{2 \min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}} \leq ||z - u||_\infty \leq \frac{||v_1||_\infty(1 + (\max_{i \in [n]} \{a_{i i - i}\})^{\frac{1}{m-1}}) + \sqrt{D_1}}{2 \min_{i \in [n]} \{(a_{ii})^{\frac{1}{m-1}}\}}
\]
where \( v_1 = u - \max \{0, u - [(A(u-z)^{m-1})^{\frac{1}{m-1}} + (A z^{m-1} + q)^{\frac{1}{m-1}}]\}, D_1 = ||v_1||_\infty^2 (1 + (\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}})^2 - 4(\min_{i \in [n]} \{(a_{ii-i})^{\frac{1}{m-1}}\}) v_t^2 \geq 0 \) and \( t \) satisfies \( \max_{i \in [n]} \{(u - z)_i (A(u-z)^{m-1})_i \} \geq 0 \) and \( v_t \neq 0 \).

**Theorem 3.5:** Let \( \mathcal{A} \in T_{m,n} \) be a positive diagonal tensor of even order, \( 0 \neq z \in \mathbb{R}^n \) be a solution of TCP(\( q, \mathcal{A} \)) and u be an arbitrary vector in \( \mathbb{R}^n \). If \((-q)_+ \neq 0\) then

\[
||v_1||_\infty (1 + (\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}}) - \sqrt{D_1}
\]

\[
\leq \frac{||z - u||_\infty}{||z||_\infty}
\]

\[
\leq \frac{2(\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}} (||v_1||_\infty (1 + (\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}}) + \sqrt{D_1})}{2 \min_{i \in [n]} \{(a_{ii-i})^{\frac{1}{m-1}}\}} ||(-q)_+||_\infty
\]

where \( v_1 = u - \max \{0, u - [(A(u-z)^{m-1})^{\frac{1}{m-1}} + (A z^{m-1} + q)^{\frac{1}{m-1}}]\}, D_1 = ||v_1||_\infty^2 (1 + (\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}})^2 - 4(\min_{i \in [n]} \{(a_{ii-i})^{\frac{1}{m-1}}\}) v_t^2 \geq 0 \) and \( t \) satisfies \( \max_{i \in [n]} \{(u - z)_i (A(u-z)^{m-1})_i \} \geq 0 \) and \( v_t \neq 0 \).

**Proof.** Let \( \mathcal{A} \in T_{m,n} \) be a positive diagonal tensor and \( z \in \mathbb{R}^n \) be a solution of TCP(\( q, \mathcal{A} \)) and let \( u \) be an arbitrary vector in \( \mathbb{R}^n \). Then by Theorem 3.4, the inequalities (3.20) and (3.21) hold. Since \((-q)_+ \neq 0\) and \( z \neq 0 \) from (3.20) we have

\[
\frac{(\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}}}{||(-q)_+||_\infty} \leq \frac{1}{||z||_\infty} \leq \frac{\min_{i \in [n]} \{(a_{ii-i})^{\frac{1}{m-1}}\}}{||(-q)_+||_\infty}.
\]

(3.23)

Now combining inequalities (3.21) and (3.23) we obtain

\[
||v_1||_\infty (1 + (\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}}) - \sqrt{D_1}
\]

\[
\leq \frac{||z - u||_\infty}{||z||_\infty}
\]

\[
\leq \frac{(\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}} (||v_1||_\infty (1 + (\max_{i \in [n]} \{a_{ii-i}\})^{\frac{1}{m-1}}) + \sqrt{D_1})}{2 \min_{i \in [n]} \{(a_{ii-i})^{\frac{1}{m-1}}\}} ||(-q)_+||_\infty
\]

(3.26)

\( \square \)

Here we give an numerical example to illustrate the result.

**3.0.1 Numerical Example**

Consider the tensor \( \mathcal{A} \in T_{4,2} \) such that \( a_{1111} = 1, \ a_{2222} = 8 \). For \( z = \left( \begin{array}{c} z_1 \\ z_2 \end{array} \right) \in \mathbb{R}^2 \), \( \mathcal{A} z^2 = \left( \begin{array}{c} z_1^2 \\ 8z_2^2 \end{array} \right) \). Then \( \mathcal{A} \) is a P-tensor. Let \( q = \left( \begin{array}{c} 1 \\ -1 \end{array} \right) \). Then the TCP(\( q, \mathcal{A} \)) is to
find \( z \in \mathbb{R}^2 \) such that
\[
z \geq 0, \quad A z^3 + q \geq 0, \quad z^T(A z^3 + q) = 0. \quad (3.27)
\]
Solving (3.27) we have \( z = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \). Then \( A z^3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \), \( A z^3 + q = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and \((A z^3 + q)^\frac{1}{2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \). Now \( ||A||_\infty = \max \{1, 8\} = 8 \), therefore \( ||A||_{\infty}^{\frac{1}{2}} = 2 \). Consider \( u \in \mathbb{R}^n \) such that \( u = \begin{pmatrix} 0.5 \\ 0.4 \end{pmatrix} \). Then \( u - z = \begin{pmatrix} 0.5 \\ 0.4 \end{pmatrix} \), \( A(u - z)^3 = \begin{pmatrix} 0.125 \\ 0.008 \end{pmatrix} \) and \((A(u - z)^3)^\frac{1}{2} = \begin{pmatrix} 0.5 \\ 0.8 \end{pmatrix} \). Then \( v = u - \max \{0, (A(u - z)^3)^\frac{1}{2} + (A z^3 + q)^\frac{1}{2} - u\} = \begin{pmatrix} 0.5 \\ 0.4 \end{pmatrix} \) and \( ||v||_\infty = \max \{0.5, 0.4\} = 0.5 \). Now
\[
\max_{i \in [2]} \{(u - z)_i(A(u - z)^3)_i\} = \{0.625, 0.0008\} = 0.625, \quad (3.28)
\]
and the maximum occurs at \( i = 1 \). Therefore \( t = 1 \) and \( v_i = v_1 = 0.5 \not= 0 \). Also
\[
F_A(z) = (A z)^\frac{1}{2} = \begin{pmatrix} z_1 \\ 2z_2 \end{pmatrix}.
\]
Therefore \( \alpha(F_A) = \min \{1, 2\} = 1 \). Now
\[
D = (0.5)^2 \cdot (1 + 2)^2 - 4 \cdot 1 \cdot (0.5) = 0.25 > 0.
\]
Then upper bound of the error is \( U B_{AKD} = \frac{0.5 \cdot (1 + 2) \cdot \sqrt{0.25}}{2^\frac{1}{2}} = 1 \) and the lower bound is \( LB_{AKD} = \frac{0.5 \cdot (1 + 2) \cdot \sqrt{0.25}}{2^\frac{1}{2}} = 0.5 \). But according to Theorem 2.4 proposed by Zheng et al. in [47] the upper bound is \( UB = \frac{(1 + 2)^2}{0.5} = 1.5 \) and the lower bound is \( LB = \frac{1}{1 + 2} \cdot (0.5) = 0.1667 \). Thus we see that \( UB_{AKD} < UB \) and \( LB < LB_{AKD} \). Hence our error bound is improved.
Obviously when \( D \rarr 0 \) the error bounds get better. Now we show that the error bound in Theorem 3.2 is better than that of in Theorem 2.4.
Here we are showing that our proposed error bound is sharper that the error bound proposed by Zheng et al. [47].

### 3.1 Comparison of upper bounds:

The upper bound of error from inequation (2.10) is \( UB = ||\tilde{v}||_\infty \cdot \frac{1 + ||A||_{\infty}^{-1}}{\alpha(F_A)} \) and the upper bound of error from inequation (3.4) is \( UB_{AKD} = \frac{||v||_\infty (1 + ||A||_{\infty}^{-1})^2 - 4\alpha(F_A)v_i^2}{2\alpha(F_A)} \). Now we have \( D = ||v||_\infty^2 (1 + ||A||_{\infty}^{-1})^2 - 4\alpha(F_A)v_i^2 \geq 0 \) i.e., \( 4\alpha(F_A)v_i^2 = ||v||_\infty^2 (1 + ||A||_{\infty}^{-1})^2 - D \geq 0 \), since \( \alpha(F_A) \geq 0 \). Therefore
\[
(1 + ||A||_{\infty}^{-1}) \geq \sqrt{\frac{D}{||v||_\infty^2}}. \quad (3.29)
\]
Note that by construction we have $||v|| \leq ||\tilde{v}||$. By taking the ratio of the upper bounds we obtain,

$$\frac{UB_{AKD}}{UB} = \frac{\frac{||v||_\infty (1 + ||A||_\infty^{-1}) + \sqrt{D}}{2\alpha(F_A)}}{||\tilde{v}||_\infty \cdot \frac{1+||A||_\infty^{-1}}{\alpha(F_A)}} \leq \frac{(1 + ||A||_\infty^{-1}) + \sqrt{D}}{2(1 + ||A||_\infty^{-1})} \leq \frac{(1 + ||A||_\infty^{-1}) + (1 + ||A||_\infty^{-1})}{2(1 + ||A||_\infty^{-1})} = 1$$

by (3.29).

4 Conclusion

In this article we introduce an error bound for the solution of tensor complementarity problem TCP($q, A$) given that $A$ is a $P$-tensor and $q$ is a real vector. We establish a relative error bound for TCP with $P$-tensor. We find the value of $\alpha(F_A)$ for even order positive diagonal tensor. We establish absolute and relative error bound for TCP($q, A$) where $A$ is an even order positive diagonal tensor. We prove that our proposed upper bound is sharper than the earlier upper bound of absolute error available in the literature. One numerical example is illustrated to support our result.

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