Estimating the impact of recombination uncertainties on the cosmological parameter constraints from cosmic microwave background experiments

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Received **insert**; Accepted **insert**

ABSTRACT
We use our most recent training set for the RICO code to estimate the impact of recombination uncertainties on the posterior probability distributions which will be obtained from future CMB experiments, and in particular the PLANCK satellite. Using a Monte Carlo Markov Chain analysis to sample the posterior distribution of the cosmological parameters, we find that PLANCK will have biases of −0.7, −0.3 and −0.4 sigmas for ns, Ω_b h^2 and log(10^10 A_S), respectively, in the minimal six parameter ΛCDM model, if the description of the recombination history given by RICO is not used. The remaining parameters (e.g. τ or Ω_dm h^2) are not significantly affected. We also show, that the cosmology dependence of the corrections to the recombination history modeled with RICO has a negligible impact on the posterior distributions obtained for the case of the PLANCK satellite. In practice, this implies that the inclusion of additional corrections to existing recombination codes can be achieved using simple cosmology-independent ‘fudge functions’.

Finally, we also investigated the impact of some recent improvements in the treatment of hydrogen recombination which are still not included in the current version of our training set for RICO, by assuming that the cosmology dependence of those corrections can be neglected. In summary, with our current understanding of the complete recombination process, the expected biases in the cosmological parameters inferred from PLANCK might be as large as −2.3, −1.7 and −1 sigmas for ns, Ω_b h^2 and log(10^10 A_S) respectively, if all those corrections are not taken into account. We note that although the list of physical processes that could be of importance for PLANCK seems to be nearly complete, still some effort has to be put in the validation of the results obtained by the different groups.

The new RICO training set as well as the fudge functions used for this paper are publicly available in the RICO-webpage.

Key words: cosmic microwave background

1 INTRODUCTION
The cosmic microwave background (CMB) is nowadays an essential tool of theoretical and observational cosmology. Recent advances in the observations of the CMB angular fluctuations in temperature and polarization (e.g. [Hinshaw et al. 2009]) provide a detailed description of the global properties of the Universe, and the cosmological parameters are currently known with accuracies of the order of few percent in many cases. The experimental prospect for the PLANCK satellite ([The Planck Collaboration 2006]), which was launched on May 14th 2009, is to achieve the most detailed picture of the CMB anisotropies down to angular scales of ℓ ~ 2500 in temperature and ℓ ~ 1500 in polarization. This data will achieve sub-percent precision in

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many cosmological parameters. However, those high accuracies will rely on a highly precise description of the theoretical predictions for the different cosmological models. Currently, it is widely recognised that the major limiting factor in the accuracy of angular power spectrum calculations is the uncertainty in the ionization history of the Universe (see Hu et al. 1999, Seljak et al. 2003).

This has motivated several groups to re-examine the problem of cosmological recombination (Zeldovich et al. 1968, Peebles 1968), taking into account detailed corrections to the physical processes occurring during hydrogen (e.g. Dubrovich & Grachev 2005; Rubiño-Martín et al. 2006; Chluba & Sunyaev 2006; Kholupenko & Ivanikh 2006; Rubiño-Martín et al. 2006; Chluba et al. 2007; Chluba & Sunyaev 2007; Khashenboim & Ivanov 2008; Hirata 2008; Chluba & Sunyaev 2009a,b; Jentschura 2009; Labzowsky et al. 2009; Hirata & Forbes 2009) and helium recombination (e.g. Kholupenko et al. 2007; Wong & Scott 2007; Switzer & Hirata 2008a,b; Hirata & Switzer 2008; Rubiño-Martín et al. 2008; Kholupenko et al. 2008; Chluba & Sunyaev 2009a,b). Each one of the aforementioned corrections individually leads to changes in the ionization history at the level of 0.1%, in such a way that the corresponding overall uncertainty in the CMB angular power spectra exceeds the benchmark of 3/ℓ at large ℓ (for more details, see Pico v1.4.2), hereafter FCRW09), thus biasing any parameter constraints inferred by experiments like Planck, which will be cosmic variance limited up to very high multipoles.

The standard description of the recombination process is provided by the widely used RECFAST code (Seager et al. 1994), which uses effective three-level atoms, both for hydrogen and helium, with the inclusion of a conveniently chosen fudge factor which artificially modifies the dynamics of the process to reproduce the results of a multilevel recombination code (Seager et al. 2000).

The simultaneous evaluation of all the new effects discussed above make the numerical computations very time-consuming, as they currently require the solution of the full multilevel recombination code. Moreover, some of the key ingredients in the accurate evaluation of the recombination history (e.g. the problem of radiative transfer in hydrogen and the proper inclusion of two-photon processes) are solved using computationally demanding approaches, although in some cases semi-analytical approximations (see e.g. Hirata 2008) might open the possibility of a more efficient evaluation in the future.

In order to have an accurate and fast representation of the cosmological recombination history as a function of the cosmological parameters, two possible approaches have been considered in the literature. The first one consists of the inclusion of additional fudge factors to mimic the new physics, as recently done in Wong et al. 2009 (see RECFAST v1.4.2), where they include an additional fudge factor to modify the dynamics of helium recombination. The second approach is the so-called RICO code (FCRW09), which provides an accurate representation of the recombination history by using a regression scheme based on PICO (Pendo & Wandelt 2007). The RICO code smoothly interpolates the X_1(z; p) function on a set of pre-computed recombination histories for different cosmologies, where z is the redshift and p represents the set of cosmological parameters.

In this paper, we present the results for parameter estimations using RICO with the most recent training set presented in FCRW09. This permits us to accurately account for the full cosmological dependence of the corrections to the recombination history that were included in the multi-level recombination code which was used for the training of RICO (see Sect. 2.1 for more details). With this tool, we have evaluated the impact of the corrections on the posterior probability distributions that are expected to be obtained for the PLANCK satellite, by performing a complete Monte Carlo Markov Chain analysis. The study of these posteriors have shown that the impact of the cosmology dependence is not very relevant for those processes included into the current RICO training set. Therefore, by assuming that the cosmology dependence of the correction in general can be neglected, we have also investigated the impact of recent improvements in the treatment of hydrogen recombination (see Sect. 2.2). The basic conclusion is that, with our current understanding of the recombination process, the expected biases in the cosmological parameters inferred from Planck might be as large as 1.5-2.5 sigma for some parameters as the baryon density or the primordial spectral index of scalar fluctuations, if all these corrections to the recombination history are neglected.

The paper is organized as follows. Sect. 2 takes the current training set for RICO, and provides an updated list of physical processes during recombination which were not included in FCRW09. Sect. 3 presents the impact of the recombination uncertainties on cosmological parameter estimation, focusing on the case of PLANCK satellite. Sect. 4 further extends this study to account for the remaining recombination uncertainties described in Sect. 2. Sect. 5 presents the analysis of present-day CMB experiments, for which the effect is shown to be negligible. Finally, the discussion and conclusions are presented in sections 6 and 7 respectively.

### 2 UPDATED LIST OF PHYSICAL PROCESSES DURING RECOMBINATION

In this Section we provide an updated overview on the important physical processes during cosmological recombination which have been discussed in the literature so far. We start with a short summary of those processes which are already included into the current training set (FCRW09) of RICO (Sect. 2.1). The corresponding correction to the ionization history close to the maximum of the Thomson visibility function is shown in Fig. 1.

We then explain the main recent advances in connection with the radiative transfer calculations during hydrogen recombination (Sect. 2.2), which lead to another important correction to the cosmological ionization history (see Fig. 1) that is not yet included into the current training set of RICO. However, as we explain below (Sect. 3.4) it is possible to take these corrections into account (Sect. 4), provided that their cosmology dependence is negligible. Our computations show that this may be a valid approximation (Sect. 3.4). We end this section mentioning a few processes that have been recently addressed but seem to be of minor importance in connection with parameter estimations for PLANCK. Overall it seems that the list of processes that could be of importance in connection with PLANCK is nearly completed.
Recombination uncertainties and CMB experiments

Figure 1. Recombination uncertainties with respect to the standard RECFAST v1.4.2 code. For the fiducial cosmological model, it is shown the correction to the recombination history which is incorporated in the new Rico training set (dotted line); the correction due to Ly radiative transfer effects (Chluba & Sunyaev 2009a,c,e) (dot-dashed); the effect of Raman scattering (Hirata 2008) (dashed); and the combination of all previous effects (solid line).

However, still some effort has to go in cross-validation of the results obtained by different groups.

2.1 The current training set for Rico

As demonstrated in FCRW09, Rico can be used to represent the recombination history of the Universe, accurately capturing the full cosmology dependence and physical model of the multilevel recombination code that was used in the computations of the Rico training set.

For the current Rico training set we ran our full recombination code using a 75-shell model for the hydrogen atom. The physical processes which are included during hydrogen recombination are described in detail in FCRW09: the induced 2s-1s two-photon decay (Chluba & Sunyaev 2006b); the feedback of Lyman α photons on the 1s-2s absorption rate (Kholupenko & Ivanchik 2006); the non-equilibrium populations in the angular momentum substates (Rubino-Martín et al. 2006; Chluba et al. 2007); and the effect of Lyman series feedback (Chluba & Sunyaev 2007). For helium recombination we took into account: the spin-forbidden He i 2P1 − 1S0 transition (Dubrovich & Grachev 2003); and the acceleration of helium recombination by neutral hydrogen (Switzer & Hirata 2008a). Furthermore, we also updated our physical constants according to the NIST database, including the new value of the gravitational constant and the helium to hydrogen mass ratio (Wong & Scott 2007).

A more detailed description of the physical processes that were taken into account in the current Rico training set can be found in FCRW09. This Rico training set is now publicly available at http://cosmos.astro.uiuc.edu/rico.

Figure 2. Same as figure 1 but for the angular power spectrum (APS). For the fiducial cosmological model, it is shown the correction to the APS due to the recombination history which is incorporated in the new Rico training set (gray lines), and the correction due to the additional inclusion of Ly radiative transfer effects (Chluba & Sunyaev 2009a,c,e) and Raman scattering (Hirata 2008) (dark lines).

2.2 Updated radiative transfer calculations during hydrogen recombination

As already pointed out earlier (e.g. Chluba & Sunyaev 2008b) in particular a detailed treatment of the hydrogen Lyman α radiative transfer problem including two-photon corrections is expected to lead to an important additional modification during hydrogen recombination. An overview of the relevant physical aspects in connection with this problem was already given in Sect. 2.3.2 and 2.3.3 of FCRW09. However, at that time the problem was still not solved at full depth, but recently several important steps were taken, which here we now want to discuss briefly (for additional overview see also Sunyaev & Chluba 2009).

2.2.1 Partial frequency redistribution due to line scattering

Recently, the effect of partial frequency redistribution on the Lyman α escape rate and the ionization history during hydrogen recombination was independently studied in detail by Chluba & Sunyaev (2009a) and Hirata & Forbes (2009). As shown by Chluba & Sunyaev (2009a), the atomic recoil effect leads to the dominant contribution to the associated correction in the ionization history, and the result for this process alone seems to be in good agreement with the one obtained earlier by Grachev & Dubrovich (2008), leading to \( \Delta N_e / \Delta N_e \sim -1.2\% \) at \( z \sim 900 \). Also the computations by Hirata & Forbes (2009) seem to support this conclusion.

However, Chluba & Sunyaev (2009a) also included the effect of Doppler broadening and Doppler boosting which was neglected in the analysis of Grachev & Dubrovich (2008). Doppler boosting acts in the opposite direction as atomic recoil and therefore decelerates recombination, while

\[^1\] http://www.nist.gov/, 2008 May.

\[^2\] They used a Fokker-Planck approximation (e.g. Rybicki 2006) for the frequency redistribution function.
the effect of Doppler broadening can lead to both an increase in the photons escape probability or a decrease, depending on the initial frequency of the photons (see Chluba & Sunyaev 2009a, for more detailed explanation). The overall correction to the recombination history due to line scattering amounts to $\Delta N_e/\Delta N_\alpha \sim -0.6\%$ at $z \sim 900$. The results of Chluba & Sunyaev (2009a) seem to be rather similar to those of Hirata & Forbes (2003), however a final comparison will become necessary in order to reach full agreement on the final correction. In the computations presented below we will use the results of Chluba & Sunyaev (2009a).

2.2.2 Two-photon transitions from higher levels

Initially, the problem of two-photon transitions from highly excited levels in hydrogen and helium was proposed by Dubrovich & Grachev (2005). However, for hydrogen recombination only very recently this problem has been solved convincingly by Hirata (2008) and Chluba & Sunyaev (2009c), using two independent, conceptually different approaches. Also until now, Chluba & Sunyaev (2009c) took the main contribution from the 3d-1s and 3s-1s two-photon profile corrections into account, while Hirata (2008) also included the ns-1s and nd-1s two-photon profile corrections for larger $n$.

In the analysis of Chluba & Sunyaev (2009c), three independent sources for the corrections in connection with the two-photon picture were identified (we will discuss the other two processes in Sect. 2.2.4 and 2.2.5). As they explain, the total modification coming from purely quantum mechanical aspects of the problem (i.e. corrections due to deviations of the line profiles from the normal Lorentzian shape, as pointed out by Chluba & Sunyaev (2008a)) leads to a change in the free electron number of $\Delta N_e/\Delta N_\alpha \sim -0.4\%$ at $z \sim 1100$. It also seems clear that the remaining small difference (at the level of $\Delta N_e/\Delta N_\alpha \sim 0.1\%-0.2\%$) between the results of Hirata (2008) and Chluba & Sunyaev (2009c) for the total correction related to the two-photon decays from excited states is because Chluba & Sunyaev (2009c) only included the full two-photon profiles of the 3s-1s and 3d-1s channels. Below we will use the results of Chluba & Sunyaev (2009c) for this process.

2.2.3 Time-dependent aspects in the emission and absorption of Lyman $\alpha$ photon

One of the key ingredients for the derivation of the escape probability in the Lyman $\alpha$ resonance using the Sobolev approximation (Sobolev 1963) is the quasi-stationarity of the line transfer problem. However, as shown recently (Chluba & Sunyaev 2009a), at the percent-level this assumption is not justified during the recombination of hydrogen, since (i) the ionization degree, expansion rate of the Universe and Lyman $\alpha$ death probability change over a characteristic time $\Delta z/z \sim 10\%$, and (ii) because a significant contribution to the total escape probability is coming from photons emitted in the distant wings (comparable to $10^2 - 10^3$ Doppler width) of the Lyman $\alpha$ resonance. Therefore, one has to include time-dependent aspects in the emission and absorption process into the line transfer problem, leading to a delay of recombination by $\Delta N_e/\Delta N_\alpha \sim +1.2\%$ at $z \sim 1000$. Below we will use the results of Chluba & Sunyaev (2009a).

2.2.4 Thermodynamic asymmetry in the Lyman $\alpha$ emission and absorption profile

As explained in Chluba & Sunyaev (2009c), the largest correction related to the two-photon formulation of the Lyman $\alpha$ transfer problem is due to the frequency-dependent asymmetry between the emission and absorption profile around the Lyman $\alpha$ resonance. This asymmetry is given by a thermodynamic correction factor, which has an exponential dependence on the detuning from the line center, i.e. $f_\nu \propto \exp[(\nu - \nu_\alpha)/kT_\alpha]$, where $\nu_\alpha$ is the transition frequency for the Lyman $\alpha$ resonance. Usually this factor can be neglected, since for most astrophysical problems the main contribution to the number of photons is coming from within a few Doppler width of the line center, where the thermodynamic factor indeed is very close to unity. However, in the Lyman $\alpha$ escape problem during hydrogen recombination contributions from the very distant damping wings are also important (Chluba & Sunyaev 2009c), so that there $f_\nu \neq 1$ has to be included.

As explained in Chluba & Sunyaev (2008c), the thermodynamic factor also can be obtained in the classical picture, using the detailed balance principle. However, in the two-photon picture this factor has a natural explanation in connection with the absorption of photons from the CMB blackbody ambient radiation field (Chluba & Sunyaev 2009c). This process leads to a $\sim 10\%$ increase in the Lyman $\alpha$ escape probability, and hence accelerates hydrogen recombination. For the correction to the ionization history, Chluba & Sunyaev (2009d) obtained $\Delta N_e/\Delta N_\alpha \sim -1.9\%$ at $z \sim 1100$. Note also that in the analysis of Chluba & Sunyaev (2009c) the thermodynamic correction factor was included for all ns-1s and nd-1s channels with $3 \leq n \leq 10$. Below we will use the results of Chluba & Sunyaev (2009d) for this process.

2.2.5 Raman scatterings

Hirata (2008) also studied the effect of Raman scatterings on the recombination dynamics, leading to an additional delay of hydrogen recombination by $\Delta N_e/\Delta N_\alpha \sim 0.9\%$ at $z \sim 900$. Here in particular the correction due to 2s-1s Raman scatterings is important. Again it is expected that a large part of this correction can be attributed to time-dependent aspects and the correct formulation using detailed balance, and we are currently investigating this process in detail. In the computations presented below we will use the results of Hirata (2008) for the effect of Raman scatterings.

2.3 Additional processes

There are a few more processes that here we only want mention very briefly (although with this the list is not meant to be absolutely final or complete), and which we did not account for in the computations presented here. However, it is expected that their contribution will not be very important.
2.3.1 Effect of electron scattering

The effect of electron scattering during hydrogen recombination was also recently investigated by Chluba & Sunyaev (2009c) using a Fokker-Planck approach. This approximation for the frequency redistribution function may not be sufficient towards the end of hydrogen recombination, but in the overall correction to the ionization history was very small close the maximum of the Thomson visibility function, so that no big difference are expected when more accurately using a scattering Kernel-approach. Very recently Haimoud et al. (2009) showed that this statement indeed seems to be correct.

2.3.2 Feedback of helium photons

Very recently Chluba & Sunyaev (2009a) investigated the feedback problem of helium photons including the processes of $\gamma$(HeI) $\rightarrow$ HeI, $\gamma$(HeII) $\rightarrow$ H I, $\gamma$(HeII) $\rightarrow$ He I and $\gamma$(HeII) $\rightarrow$ H I feedback. They found that only $\gamma$(HeI) $\rightarrow$ HeI feedback leads to some small correction ($\Delta N_e/N_e \sim +0.1\%$ at $z \sim 2300$) in the ionization history, while all the other helium feedback induced corrections are negligible. This is because the $\gamma$(HeI) $\rightarrow$ H I, $\gamma$(HeI) $\rightarrow$ He I and $\gamma$(HeII) $\rightarrow$ H I feedback processes all occur in the pre-recombination epochs of the considered species, where the populations of the levels are practically in full equilibrium with the free electrons and ions.

The $\gamma$(HeI) $\rightarrow$ HeI feedback process was already studied by Switzer & Hirata (2008a), but the result obtained by Chluba & Sunyaev (2009c) seems to be smaller. However, it is clear that any discrepancy in the helium recombination history at the 0.1% - 0.2% level will not be very important.

We would also like to mention, that although the $\gamma$(HeI) $\rightarrow$ H I, $\gamma$(HeII) $\rightarrow$ He I and $\gamma$(HeII) $\rightarrow$ H I feedback processes do not affect the ionization history, they do introduce interesting changes in the recombinational radiation, increasing the total contribution of photons from helium by 40% - 70% (Chluba & Sunyaev 2009c).

2.3.3 Other small correction

Recently, several additional processes during hydrogen recombination were discussed. These include: the overlap of Lyman series resonances caused by the thermal motion of the atoms (Haimoud et al. 2009); quadrupole transitions in hydrogen (Grin et al. 2009); hydrogen deuterium recombination (Fung & Chluba 2003; Kholupenko 2009); and 3s-2s and 3d-2s two-photon transitions (Chluba & Sunyaev 2008h). All these processes seem to affect the ionization history of the Universe at a level (well) below 0.1%.

Furthermore, one should include the small re-absorption of photons from the 2s-1s two-photon continuum close to the Lyman $\alpha$ resonance, where our estimates show that this leads to another $\Delta N_e/N_e \sim 0.1\% - 0.2\%$ correction.

2.4 Overall correction

Figure 1 shows the current best-estimate of the remaining overall correction to the recombination history, while Fig. 2 translates these corrections into changes of the angular power spectrum. To describe the Ly-$\alpha$ transfer effects, we adopt the curve presented in Chluba & Sunyaev (2009b), which includes the results of the processes investigated by Chluba & Sunyaev (2009c) and Chluba & Sunyaev (2009d). We also include the effect of Raman scattering on hydrogen recombination as described in Hirata (2008).

It seems that the remaining uncertainty due to processes that were not taken into account here can still exceed the 0.1% level, but likely will not lead to any significant addition anymore. However, it is clear that a final rigorous comparison of the total result from different independent groups will become necessary to assure that the accuracy required for the analysis of PLANCK data will be reached. Such detailed code comparison is currently under discussion.

3 IMPACT OF RECOMBINATION UNCERTAINTIES ON COSMOLOGICAL PARAMETER ESTIMATION

Here, the Rico code together with the latest training set described above is used to evaluate the impact of the recombination uncertainties in the cosmological parameter constraints inferred from CMB experiments.

All our analyses use the software packages CAMB (Lewis et al. 2000) and CosmoMC (Lewis & Bridle 2002). CAMB is used to calculate the linear-theory CMB angular power spectrum, and has been modified here to include the recombination history as described by Rico. CosmoMC uses a Monte Carlo Markov Chain (MCMC) method to sample the posterior distribution of the cosmological parameters from a given likelihood function which describes the experimental constraints on the CMB angular power spectrum.

The default parametrisation which is included inside CosmoMC exploits some of the intrinsic degeneracies in the CMB angular power spectrum (see e.g. Kosowsky et al. 2002), and uses as basic parameters the following subset:

$$\vec{\mu}_{\text{std}} = \{\Omega_b h^2, \Omega_{\text{adm}} h^2, \theta, \tau, n_S, \log(10^{10} A_S)\}. \tag{1}$$

Here, $\Omega_b h^2$ and $\Omega_{\text{adm}} h^2$ are the (physical) baryon and dark matter densities respectively, where $h$ stands for the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$; $\theta$ is the acoustic horizon angular scale; $\tau$ is the Thomson optical depth to reionization; and $n_S$ and $A_S$ are the spectral index and the amplitude of the primordial (adiabatic) scalar curvature perturbation power spectrum at a certain scale $k_0$. For the computations in this paper, we will use $k_0 = 0.05$ Mpc$^{-1}$.

It is important to note that the above parametrisation $\vec{\mu}_{\text{std}}$ makes use of an approximate formula for the sound horizon ($H_u$ & Sugiyama 1996), which used in its derivation some knowledge on the recombination history provided by RECFAST. Therefore, the use of this parameter is not appropriate if we are changing the recombination history, as this might introduce artificial biases. For this reason, in this paper we have modified CosmoMC to use a new parametrisation $\vec{\mu}_{\text{new}}$, which is defined as

$$\vec{\mu}_{\text{new}} = \{\Omega_b h^2, \Omega_{\text{adm}} h^2, H_0, \tau, n_S, \log(10^{10} A_S)\}. \tag{2}$$

3 http://camb.info/
4 http://cosmologist.info/cosmomc/
where we have replaced $\theta$ by $H_0$ as a basic parameter. In this way, we still exploit some of the well-known degeneracies (e.g. that between $\tau$ and $A_S$), but we eliminate the possible uncertainties which may arise from the use of $\theta$, at the expense of decreasing slightly the speed at which the chain converges. As we show below, both the shape of the posteriors and the confidence levels for the rest of the parameters is practically unaffected by this modification in the parametrization, despite of the fact that the CosmoMC code now assumes a flat prior on $\theta$, while in the parameter estimation of the Planck data we use not a simulation but the actual data we use. We eliminate the foreground contamination from this reference mock data is then produced using the cosmological recombination history as computed with our complete multi-level recombination code for this particular cosmology ($n = 75$ shells to model the hydrogen atom). The shape of the likelihood function adopted here corresponds to the exact full-sky Gaussian likelihood function given in Lewis (2002), which is implemented in CosmoMC using the all_l_exact subroutine in CosmoMC, but a flat prior on $H_0$ instead of a flat prior on $\theta$. A modified version of the params.f90 subroutine inside CosmoMC which uses this new parametrization is also available in the RTco webpage.

Finally, concerning the convergence of the chains, all computations throughout this paper were obtained using at least five independent chains; those chains have been run until the Gelman & Rubin (1992) convergence criterion $R - 1$ yields a value smaller than 0.005 for the minimal (6-parameter) case, and values smaller than 0.02–0.2 for those cases with a larger number of parameters.

3.1 Impact on parameter estimates for Planck alone

For the case of Planck, the mock data is prepared as follows. We assume a Gaussian symmetric beam, and the noise is taken to be uniform across the sky. For definiteness, we have adopted the nominal values of the beam and pixel noise which correspond to the 143 GHz Planck band, as described in the Planck Collaboration (2009). Thus, we have $\theta_{\text{beam}} = 7.1', w_T = 1.53 \times 10^{-4} \mu\text{K}^2$ and $w_P = 5.59 \times 10^{-4} \mu\text{K}^2$, where $w_T$ and $w_P$ indicate the intensity and polarization sensitivities, respectively. Note that using a single frequency channel is implicitly assuming that the remaining Planck frequencies have been used to fully remove the foreground contribution from this reference 143 GHz channel. For a complete discussion on which combination of Planck channels is more appropriate in terms of the parameter constraints, see Colombo et al. (2009).

The fiducial cosmological model used for these computations corresponds to the WMAP5 cosmology (Hinshaw et al. 2009), and has parameters $\Omega_b h^2 = 0.02273$, $\Omega_{\text{dm}} h^2 = 0.1099$, $\Omega_c h^2 = 0.0876$, $n_S = 0.963$ and an amplitude $2.41 \times 10^{-9}$ at $k = 0.002 \text{ Mpc}^{-1}$ (or equivalently $A_S = 2.14 \times 10^{-9}$ at $k = 0.05 \text{ Mpc}^{-1}$, or $\log(10^{10}A_S) = 3.063$). The mock data is then produced using the cosmological recombination history as computed with our complete multi-level recombination code for this particular cosmology (using $n = 75$ shells to model the hydrogen atom). The shape of the likelihood function adopted here corresponds to the exact full-sky Gaussian likelihood function given in Lewis (2002), which is implemented in CosmoMC using the all_l_exact subroutine in CosmoMC. We note that for the mock Planck data we use not a simulation but the actual (fiducial) angular power spectrum.

We run CosmoMC for the case of Planck mock data alone and the minimal model with six free parameters. For each set of runs, we consider two cases for the recombination history, one is the current version of RECFAST (v1.4.2), and the second one is the RTco code with our latest training set. The main result are summarized in table I and the corresponding posterior distributions are shown in Fig. 3. We note that the sizes of the error bars on these parameters are similar to those obtained for Planck by other authors (e.g. Bond et al. 2004; Colombo et al. 2009). For this six parameter case, the largest biases do appear in $n_S$ ($-0.7$ sigmas), $\Omega_b h^2$ ($-0.3$ sigmas) and $\log(10^{10}A_S)$ ($-0.4$ sigmas).

The sign of the correction in $n_S$, which is the parameter having the largest bias, can be understood as follows. The physical effects to the recombination history included in RTco produce a slight delay of the recombination around the peak of the visibility function, i.e. an excess of electrons with respect to the standard computation (see Fig. 1). This in turn produces a slightly larger Thomson optical depth, which increases the damping of the anisotropies at high multipoles. In order to compensate this excess of damping, the analysis which uses the standard RECFAST code gives a lower value of $n_S$.

The biases on the other three parameters, i.e. $\tau$ (which is well-constrained by large scale polarization measurements), $\Omega_{\text{dm}} h^2$ and $H_0$, are negligible (i.e. less than 0.1 sigmas). However, if the analysis is repeated with the standard parametrization using $\theta$ as basic parameter, then we would find a +1.8 sigma bias in $\theta$, while for the rest of the parameters the posteriors remain the same as before. For illustration of these facts, Fig. 4 shows a comparison of the posteriors for $n_S$ obtained with the two parametrizations, as well as the posteriors obtained for $\theta$ if using the standard parametrization.

Finally, we have checked that the inclusion of lensed angular power spectra on the complete procedure modifies neither the shape of the posteriors nor the biases. Therefore, for the rest of the paper we perform the computations in the case without lensing. This decreases the computational time by a significant factor.

3.2 Importance of the cosmology dependence of the correction

One of the questions that can be explored with RTco and the new training-set is the importance of the cosmology dependence of the corrections to the recombination history. In order to obtain a simplified description of the recombination history, it is important to evaluate if the cosmology dependence of the corrections plays a significant role in determining the final shape of the posteriors. To explore this issue, we have modified the standard RECFAST code by introducing the following (cosmology independent) correction:

$$x_e^{\text{new}}(z; \{\text{cosmology}\}) = x_e^{\text{RECFAST}}(z; \{\text{cosmology}\}) f(z)$$

where this function $f(z)$ is computed as

$$f(z) = 1 + \frac{\Delta x_e}{x_e}$$

for a certain fiducial cosmological model. By introducing this modification inside CosmoMC, we have compared the posterior distributions obtained in this case with those obtained using the full RTco code with the new training set.
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Figure 3. Impact on parameter estimates for Planck satellite, in the six-parameter case. Dark solid line shows the 1-D and 2-D posterior distributions which are obtained when the Rico code is used to describe the recombination history. Light dashed solid line shows the same posteriors obtained with the Recfast code. The biases are evident in $n_S$, $\Omega_b h^2$ and $\log(10^{10} A_S)$.

Table 1. Impact of recombination uncertainties on the confidence limits of the cosmological parameters for the Planck satellite. Confidence intervals are derived as the 0.16, 0.5 and 0.84 points of the cumulative probability distribution function, in such a way that our parameter estimate is the median of the marginalised posterior probability distribution function, and the confidence interval encompasses 68 percent of the probability. The biases correspond to the case of using the Rico training set.

| Parameter | With Rico | With Recfast (v1.4.2) | Bias (in sigmas) | Fiducial model |
|-----------|-----------|-----------------------|------------------|----------------|
| $\Omega_b h^2 (\times 10^2)$ | 2.273 ± 0.014 | 2.269 ± 0.014 | -0.31 | 2.273 |
| $\Omega_{\text{dm}} h^2$ | 0.1098 ± 0.0013 | 0.1099 ± 0.0012 | 0.06 | 0.1099 |
| $H_0$ | 71.9 ± 0.7 | 72.0 ± 0.6 | 0.11 | 71.9 |
| $\tau$ | 0.087 ± 0.006 | 0.087 ± 0.006 | -0.04 | 0.087 |
| $n_S$ | 0.9632 ± 0.0038 | 0.9606 ± 0.0037 | -0.74 | 0.963 |
| $\log(10^{10} A_S)$ | 3.063 ± 0.008 | 3.059 ± 0.009 | -0.42 | 3.063 |

The results of this comparison are shown in Figure 5. The fact that the differences in the posterior are negligible indicates that there is no significant cosmological dependence in the corrections to the ionization history included in the RICO training set. Therefore, and for the case of the PLANCK satellite, one can in principle use cosmology independent ‘fudge functions’, as the one presented in Eq. (4), to accommodate additional corrections to the recombination history.

As a final check in this section, we have explored the sensitivity of the fudge function $f(z)$ to the cosmological model which is used as fiducial model for the full computation of the recombination history. Taking as a reasonable range of variation the two-sigma confidence interval which is obtained of the analysis of WMAP5 data, we have compared the $f(z)$ functions obtained from cosmological models which differ two-sigmas with respect to the actual fiducial model which is used in this paper. The result is that the changes in $f(z)$ with respect to the solid curve presented in Fig. 4 are below one percent, thus giving a negligible correction to the main effect included in $f(z)$. 
Figure 4. Top: Posterior distributions for $n_S$ obtained with two different parametrizations. The solid line corresponds to the marginalised 1-d posterior distribution obtained using Rico while the dashed line corresponds to the case of using Recfast, both with the new parametrization. The dotted lines show in each case the posteriors obtained when the standard parametrization is used (i.e. $\theta$ instead of $H_0$ as fundamental parameter). Bottom: Bias recovered on $\theta$ parameter if the standard parametrization is used. This case corresponds to the same case as Fig. 3, i.e. six parameter case.

4 ESTIMATING THE CORRECTIONS FROM THE REMAINING RECOMBINATION UNCERTAINTIES

Although a full recombination code which includes all the physical effects discussed in Sect. 2 is still not available, there is a good agreement in the community about the list of relevant physical processes that have to be included. Moreover, all those effects have been already discussed in the literature by at least one group, so we have estimates of the final impact of these corrections on the recombination history (see Fig. 1).

Based on these estimates, we have quantified the impact of future corrections to the recombination history using the approximation described in Eq. 3, where $f(z)$ is taken from Fig. 1 as described in Eq. 4. Using this function, we have obtained the posterior distributions for the same case discussed above (nominal PLANCK satellite sensitivities and a six parameter analysis). The basic results are shown in Table 2 and Fig. 5. The biases in the different parameters increase very significantly, as one would expect from the inspection of Fig. 1, specially for $n_S$ ($-2.3$ sigmas), $10h^2$ ($-1.65$ sigmas) and $\log(10^{10}A_s)$ ($-1$ sigmas). Therefore, as pointed out by several authors (see e.g. Chluba & Sunyaev 2008b; Hirata 2008), the detailed treatment of the hydrogen Lyman $\alpha$ radiative transfer problem constitutes the most significant correction to our present understanding of the recombinational problem. If this effect is not taken into account when analysing PLANCK data, the final constraints could be significantly biased.

4.1 Extended cosmological models

In this subsection we describe to what extent the full set of corrections to the recombination history may affect the cosmological constraints on some extensions to the (minimal) six-parameter model which was used in this work. Throughout this subsection, we compare the complete recombination history (which includes the additional corrections shown in Fig. 1) with the constraints that would be inferred using Recfast v1.4.2.

4.1.1 Tensor perturbations

We first consider the case of including tensor perturbations in addition to the previous model. For these computations, we consider an 8-parameter model, by including the spectral index of the primordial tensor perturbation ($n_T$) and the tensor-to-scalar ratio $r$, in addition to the parameters in equation 2. As pivot scale, we are using $k_0 = 0.05 \text{ Mpc}^{-1}$.

Fig. 7 shows the impact of recombination uncertainties on the $r$-$n_S$ plane. The constraints on $r$ are determined by large-scale information, and therefore the modifications of the recombination history do not bias this parameter. However, due to the important bias on $n_S$, the 2-D contours on this plane are significantly shifted.

4.1.2 Scale dependence of spectral index

The running of the spectral index is a possible extension to the simple $\Lambda$CDM model which is under debate in the liter-
Table 2. Estimate of the impact of remaining recombination uncertainties on the confidence limits of the cosmological parameters for the Planck satellite. Confidence intervals are derived as in table 1 so the confidence interval encompasses 68% per cent of the probability.

|                              | With $f(z)$ from Fig. 1 | With Recfast (v1.4.2) | Bias (in sigmas) | Fiducial model |
|------------------------------|--------------------------|-----------------------|------------------|----------------|
| $\Omega_b h^2 \times 10^2$  | 2.274$^{+0.014}_{-0.015}$ | 2.250$^{+0.015}_{-0.013}$ | -1.65            | 2.273          |
| $\Omega_{cdm} h^2$          | 0.1098$^{+0.0013}_{-0.0012}$ | 0.1098$^{+0.0012}_{-0.0011}$ | 0.02             | 0.1099         |
| $H_0$                       | 71.9 ± 0.6               | 71.7 ± 0.6            | -0.41            | 71.9           |
| $\tau$                      | 0.087 ± 0.006            | 0.086 ± 0.006         | -0.18            | 0.087          |
| $n_S$                       | 0.963$^{+0.0037}_{-0.0036}$ | 0.955$^{+0.0035}_{-0.0034}$ | -2.27            | 0.963          |
| $\log(10^{10} A_S)$         | 3.064 ± 0.009            | 3.055 ± 0.009         | -0.99            | 3.063          |

Figure 6. Posterior distributions obtained with the remaining recombination uncertainties (see text for details). Solid line corresponds to the posteriors using the correct description of the recombination history, while the dashed lines represent the case in which the current description (Recfast v1.4.2) is used.

Figure 7. Posterior distributions for the running of the spectral index.

Figure 8. Posterior distributions for the spatial curvature, $\Omega_K$. The inclusion of this additional parameter introduces a practical complication, since in our new parametrization $\Omega_K$, the $H_0$ parameter is highly correlated with $\Omega_K$. Fig. 9 presents the posterior distributions for this case, in which the degeneration between $H_0$ and $\Omega_K$ is clearly visible. There are two things to note. First, there is no significant bias to $\Omega_K$ due to the inclusion of recombination uncertainties in light of WMAP observations (see e.g. Spergel et al. 2007, which gives $n_{\text{run}} = dn_S/d\ln k = -0.055 \pm 0.030$). Fig. 8 illustrate the impact of the recombination uncertainties on the determination of the running of the spectral index. For this computation, we considered a 7-parameter model by adding $n_{\text{run}} = dn_S/d\ln k$, which again is computed at the same pivot scale of $k_0 = 0.05$ Mpc$^{-1}$. When studying the one-dimensional posterior distributions for all parameters, we find that the inclusion of $n_{\text{run}}$ does not affect the shape of the rest of the posteriors. The bias on $n_{\text{run}}$ due to recombination uncertainties is not very significant (changes from $n_{\text{run}} = -0.0012 \pm 0.0050$ to $n_{\text{run}} = -0.0034 \pm 0.0050$, i.e. $0.4$ sigmas) but has to be taken into account.

For completeness, we have run a 9-parameter case, in which we allow to vary simultaneously $r$, $n_T$ and $n_{\text{run}}$ in addition to the other six parameters. In this case, we have checked that the contours shown in Fig. 7 and Fig. 8 are not affected by the inclusion of the other parameter.

4.1.3 Curvature

Another possible extension of the minimal six-parameter model is to constrain simultaneously the spatial curvature, $\Omega_K$. The inclusion of this additional parameter introduces a practical complication, since in our new parametrization $\Omega_K$, the $H_0$ parameter is highly correlated with $\Omega_K$. Fig. 9 presents the posterior distributions for this case, in which the degeneration between $H_0$ and $\Omega_K$ is clearly visible. There are two things to note. First, there is no significant bias to $\Omega_K$ due to the inclusion of recombination uncertainties...
but allowing for a different normalization through the $A_{SZ}$
parameter.

When the parameter constraints are inferred with the inclusion of this additional parameter, we find that (neglecting the intrinsic bias due to the degeneracy between $n_s$ and $A_{SZ}$) the relative bias is practically not affected. In particular, we obtain a bias of -1.42, -0.44, -2.09 and -1.03 sigmas for $\Omega_m h^2$, $H_0$, $n_s$ and $\log(10^{10} A_s)$, respectively. Those numbers are comparable to the net biases which have been obtained for the minimal model in Table 2.

5 IMPACT ON PARAMETER ESTIMATES USING PRESENT-DAY CMB EXPERIMENTS

One would expect that the order of magnitude of the corrections to the recombination history discussed in previous sections (at the level of 1%-2%) would have a negligible impact on the parameters constraints that we would infer from present-day CMB experiments. As shown in FCRW09, the changes introduced in the power spectra (both temperature and polarization) are significant at high multipoles, in the sense that they are larger than the benchmark level estimated as $\pm 3/\ell$ (see Seljak et al. 2003).

To quantify this fact, we have obtained the posterior distributions for the case of the minimal model with six free parameters, combining the CMB information from WMAP5 (Hinshaw et al. 2009), ACBAR (Kuo et al. 2007), CBI (Sievers et al. 2007) and Boomerang (Jones et al. 2006; Montroy et al. 2006), together with measurements on the linear matter power spectrum based on luminous red galaxies from SDSS-DR4 (Tegmark et al. 2006). Figure 10 presents the results for the case of using the standard Recfast recombination history, together with the case of using our most complete description of the recombination history, as presented in the previous section. As expected, the modifications on the shape of the posteriors are very small and no biases are seen in the parameters except for $n_s$ and $\log(10^{10} A_s)$, which are slightly biased. Our analysis including the full description of the recombination history gives $n_s = 0.970 \pm 0.013$ and $\log(10^{10} A_s) = 3.075 \pm 0.038$, while the result using Recfast 1.4.2 gives $n_s = 0.967^{+0.013}_{-0.012}$ and $\log(10^{10} A_s) = 3.066^{+0.038}_{-0.036}$. In other words, this is a ~0.25 and $\sim -0.22$ sigma bias on $n_s$ and $\log(10^{10} A_s)$, respectively. For completeness, we have run also the MCMC for the case of using WMAP5 data alone. In that case, the bias decreases to $\lesssim -0.15$ sigmas for those two parameters.

6 DISCUSSION

In this section, we now discuss the results presented in this paper focusing in three particular aspects. On one hand, we discuss the robustness of our results against possible modifications of the physical description of the recombination process. Second, we also consider the dependence of the obtained biases if additional parameters are included in the MCMC analysis. Finally, we discuss the possible impact of recombination uncertainties on the results obtained from other cosmological probes different from CMB anisotropies.
Recombination uncertainties and CMB experiments

6.1 Dependence of the results on the description of the recombination process

As discussed above (Sect. 2), there is a wide agreement in the community about the list of physical processes which should be included in the description of the cosmological recombination process. In many cases, these physical processes have been treated separately by at least two separate groups, and the agreement on the signs and amplitudes of the corrections is excellent in most of the cases (e.g. see the compilation of uncertainties in the physics of recombination in Table 2.1 of Wong 2008). Although an agreement at the level of \( \lesssim 0.1\% \) is still not reached, we are almost there, as the remaining uncertainties seem to be at the level of 0.1%-0.3% between the different groups. In this sense, one would expect that a code which includes self-consistently all those processes should obtain essentially the same biases that have been described in Sect. 4 and have been reported in Table 2.

However, apart from the processes described in section 2.3, there are still some possible uncertainties which might lead to measurable biases on the cosmological parameters. Below we now briefly address them.

6.1.1 Hydrogen recombination

One effect which might lead to additional biases on the cosmological parameters is the inclusion of very high-n states in the cosmological hydrogen recombination. The computations in this paper are based on a training set which uses \( n = 75 \) shells to describe the hydrogen atom. To explore the dependence of higher number of shells, we have repeated the standard six-parameter computation for the mock PLANCK data presented in Sect. 3.1 and Sect. 4 but taking as a reference model the one computed using \( n = 110 \) hydrogen shells, and trying to recover it with Rico (which uses \( n = 75 \) hydrogen shells). The recovered posteriors using the Rico code in this case do not show any appreciable bias in any of six parameters of the minimal model. This is illustrated in Fig. 11 where we present the posterior distribution for the \( n_S \) parameter, which is the one having the largest bias.

6.1.2 Helium recombination

As mentioned in Sect. 2.3 and discussed in FCRW09, we do not expect major changes in our current understanding of the helium recombination, at a level which might be relevant for the computation of CMB anisotropies. This statement assumes that the uncertainties in the modelling of the helium atom are well controlled, but, as described in Rubiño-Martín et al. (2008), we still lack an accurate description of the photoionization cross-sections, energies or transition rates for the Helium atom, which might lead to small changes in these results. One may also wonder whether the small differences found for the correction caused by Helium feedback processes (see Sect. 2.3.2) could matter.

By far, the most relevant correction to the helium re-
combination history is caused by the absorption of He\textsuperscript{i} photons by neutral hydrogen, although the inclusion of the 2\textsuperscript{3}P\textsubscript{1} - 1\textsuperscript{1}S\textsubscript{0} intercombination line also gives some contribution. These two effects are already included in the codes, both in \texttt{Rico} (FCR\textsubscript{W09}) and in \texttt{Recfast v1.4.2} (Wong et al. 2008). For completeness, we have also quantified for this paper the impact that these two corrections to the helium recombination would have on the recovered cosmological parameters. Our computations show that, if these corrections are not included (which in practice corresponds to setting \texttt{RECFAST_Heswitch} = 0 in \texttt{Recfast v1.4.2}, or equivalently, using \texttt{Recfast v1.3}), then the resulting biases on the parameters are found to be -3.2, -2.0, -1.2 and -0.7 sigmas for $n_s$, $\Omega_b h^2$, log(10\textsuperscript{10}A\textsubscript{S}) and $H_0$, respectively. For illustration, we show in Fig. 12 a comparison of the posterior distribution obtained for the $n_s$ parameter when using the \texttt{Recfast} code with (\texttt{RECFAST_Heswitch} = 6) and without (\texttt{RECFAST_Heswitch} = 0) all the corrections to the helium recombination.

This simple computation shows that additional corrections to the helium recombination history at the $\sim 0.1\%$ level will not matter much to the analysis of future \texttt{Planck} data. Therefore, the physics of helium recombination already seems to be captured at a sufficient level of precision, when including the acceleration caused by the hydrogen continuum opacity and the 2\textsuperscript{3}P\textsubscript{1} - 1\textsuperscript{1}S\textsubscript{0} intercombination line, which together lead to a $\sim -3\%$ correction to $X_e(z)$ at $z \sim 1800$ (Fendt et al. 2009).

6.2 Recombination uncertainties and other extended cosmological models

In addition to those extended models described in Sect. 4.1 there is a number of possible non-standard models for which the inclusion of refined recombination physics might be of importance. For example, when using current CMB data to constrain the presence of hypothetical sources of Ly\textsc{a} resonance radiation or ionizing photons at high redshifts (e.g. Peebles et al. 2000; Bean et al. 2003, 2007); or to probe dark matter models with large annihilation cross-section (e.g. Padmanabhan & Finkbeiner 2005; Galli et al. 2004; Huetsi et al. 2009); or energy release by long-lived unstable particles (Chen & Kamionkowski 2004; Zhang et al. 2007); or when exploring the variation of fundamental constants with time (see e.g. Galli et al. 2009, for Newton’s gravitational constant, or Landau et al. 2008, for the fine structure constant and the Higgs vacuum expectation value), it is obvious that neglecting physically well understood additions to the recombination model, as described in Sect. 4, could lead to spurious detections or confusion, in particular, when the possible effects are already known to be rather small (e.g. see Galli et al. 2004, in the case of dark matter annihilations).

More generally speaking, given that the largest re-
6.3 Recombination modelling and other cosmological probes

The combination of CMB data with other datasets usually helps to improve the parameter constraints, in some cases by breaking internal degeneracies which are inherent to the CMB data alone. One of the commonly used external datasets is the Baryon Acoustic Oscillations (BAOs). de Bernardis et al. (2009) have recently shown that a possible delay of recombination (Peelies et al. 2000) by extra sources of ionizing or exciting photons leads to biases on the constraints from BAOs, because they largely rely on the determination of the size of the acoustic horizon at recombination. Their conclusions can be directly translated here, stressing that a fully consistent combination of the constraints from CMB and BAOs should be done by using the same recombination history in both cases.

In addition, we would like to point out that in principle one could reduce the uncertainty in our knowledge of the recombination epoch and its possible non-standard extensions (e.g. due to annihilating dark matter Padmanabhan & Finkbeiner 2003) in two ways. On one hand, one could search for the imprint of the cosmological hydrogen recombination lines on the CMB angular power spectrum, Růžička-Martin et al. (2006); Hernández-Monteagudo et al. (2007), which arises due to the resonant scattering of CMB photons by hydrogen atoms at each epoch (Basu et al. 2004). On the other hand, one could also try to directly observe the photons that are emitted during the recombination epoch. Today these photons should still be visible as small distortion of the CMB energy spectrum in the mm, cm and dm spectral bands (e.g. see Dubrowski 1972; Dubrovich & Stoliarov 1977; Růžička-Martín et al. 2006; Chluba & Sunyaev 2006a; Chluba et al. 2007; Růžička-Martín et al. 2008; Chluba & Sunyaev 2009b). These observations not only would open another possibility to determine some of the key cosmological parameters, such as the primordial helium abundance, the number density of baryons and the CMB monopole temperature at recombination (e.g. Chluba & Sunyaev 2008), but they would also allow us to directly check our understanding of the recombination process and possible non-standard aspects (e.g. see Sunyaev & Chluba 2009 for an overview), for example, in connection with early energy release (Chluba & Sunyaev 2009a), or dark matter annihilations (Chluba 2009).

7 CONCLUSIONS

In this paper, we have performed a MCMC analysis of the expected biases on the cosmological constraints to be derived from the upcoming Planck data, in the light of recent developments in the description of the standard cosmological recombination process. Our main conclusions are:

- An incomplete description of the cosmological recombination process leads to significant biases (of several sigmas) in some of the basic parameters to be constrained by Planck satellite (see Table 2), and in general, by any future CMB experiment. However, these corrections have a minor impact for present-day CMB experiments; for instance, using WMAP5 data plus other cosmological datasets, we find combination uncertainties are obtained for $n_s$, $\Omega_b h^2$ and $\log(10^{10} A_s)$, one can say that any additional parameter showing a strong correlation with those three might be biased if an incomplete description of the recombination physics is used. In this sense, neglecting the refinements to the recombination model could be as important as not taking into account, for instance, uncertainties in the beam shapes, which also have been shown to compromise our ability to measure $n_s$ (Colombo et al. 2007); or the combined effect of beam and calibration uncertainties, which introduce significant biases to $n_s$ and $\Omega_b h^2$ (Bridle et al. 2002), although other parameters (like $\Omega_{\Lambda}$) are essentially not affected because they are basically constrained by the position of the peaks, and not by their amplitudes.
a \sim -0.25 and \sim -0.22\,sigma bias on $n_S$ and log(10^{10}\,A_S), respectively, while the rest of the parameters remain unchanged.

- Today, it seems that our understanding of cosmological recombination has reached the sub-percent level in $X_e$ at redshifts $5 \lesssim z \lesssim 1600$. However, it will be important to cross-validate all of the considered corrections in a detailed code comparison, which currently is under discussion among the different groups.

- Given the range of variation of the relevant cosmological parameters, it is possible to incorporate all the new recombination corrections by using (cosmology independent) fudge functions. Here we described one possibility which uses a simple correction factor to the results obtained with RecFast (see Sect. 5.2). We provide the function $f(z)$ on the Rico-webpage.

- The physics of helium recombination already seems to be captured at a sufficient level of precision, when including the acceleration caused by the hydrogen continuum opacity and the $^2P_1-^1S_0$ intercombination line. The biases caused by neglecting only these corrections are -0.8 and -0.4\,sigma, for $n_S$ and $\Omega_b h^2$, respectively.

- When allowing for more non-standard additions to the recombination model (e.g. related to annihilating dark matter), the biases introduced by an inaccurate recombination model could lead to spurious detections or additional confusion (see Sect. 6.2).

ACKNOWLEDGEMENTS

JAR-M and JC are grateful to R. A. Sunyaev for useful comments and discussion. Furthermore, they would like to thank M. Bucher, J. Fung, S. Galli, D. Grin, Y. Haimoud, C. Hirata, U. Jentschura, S. Karshenboim, E. Kholupenko, L. Labzowsky, D. Scott, and D. Solovyev for stimulating and very friendly discussion during the recombination workshop held in July 2009 in Orsay/Paris. The authors are also very grateful to E. Switzer for comments and useful discussions.

This work has been partially funded by project AYA2007-68058-C03-01 of the Spanish Ministry of Science and Innovation (MICINN). JAR-M is a Ramón y Cajal fellow of the MICINN. WAF was supported through a fellowship from the Computational Science and Engineering program at the University of Illinois. Some of the calculations in this work used the Linux cluster in the University of Illinois Department of Physics.

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