On the Secrecy Rate Region for the Interference Channel

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Abstract—This paper studies interference channels with security constraints. The existence of an external eavesdropper in a two-user interference channel is assumed, where the network users would like to secure their messages from the external eavesdropper. The cooperative binning and channel prefixing scheme is proposed for this system model which allows users to cooperatively add randomness to the channel in order to degrade the observations of the external eavesdropper. This scheme allows users to add randomness to the channel in two ways: 1) Users cooperate in their design of the binning codebooks, and 2) Users cooperatively exploit the channel prefixing technique. As an example, the channel prefixing technique is exploited in the Gaussian case to transmit a superposition signal consisting of binning codewords and independently generated noise samples. Gains obtained form the cooperative binning and channel prefixing scheme compared to the single user scenario reveals the positive effect of interference in increasing the network security. Remarkably, interference can be exploited to cooperatively add randomness into the network in order to enhance the security.

I. INTRODUCTION

In this work, we consider two-user interference channels with an external eavesdropper. Without the secrecy constraints, the interference channel is studied extensively in the literature. However, the capacity region is still not known except for some special cases [1]–[4]. Interference channels with confidential messages is recently studied by [5]–[7]. Nonetheless, the external eavesdropper scenario has not been addressed extensively in the literature yet. In fact, the only relevant work regarding the security of the interference channels with an external eavesdropper is the study of the secure degrees of freedom (DoF) in the K-user Gaussian interference channels under frequency selective fading models [7], where it is shown that positive secure DoFs are achievable for each user in the network.

In this work, we propose the cooperative binning and channel prefixing scheme for (discrete) memoryless interference channels with an external eavesdropper. The proposed scheme allows for cooperation in adding randomness to the channel in two ways: 1) Cooperative binning: The random binning technique of [8] is cooperatively exploited at both users. 2) Channel prefixing: Users exploit the channel prefixing technique of [9] in a cooperative manner. The proposed scheme also utilizes the message-splitting technique of [10] and partial decoding of the interfering signals is made possible at the receivers. The achievable secrecy rate region with the proposed scheme is given. For the Gaussian interference channel, the channel prefixing technique is exploited to inject artificially generated noise samples into the network, where we also allow power control at transmitters to enhance the security of the network.

The proposed scheme is closely related with that of [11]–[13]. [11] considered the relay-eavesdropper channel and proposed the noise-forwarding scheme where the relay node sends a codeword from an independently generated codebook to add randomness to the network in order to enhance the security of the main channel. [12] considered Gaussian multiple-access wire-tap channels and proposed the cooperative jamming scheme in which users transmit their codewords or add randomness to the channel by transmitting noise samples, but not both. The approach in this sequel, when specialized to the Gaussian multiple access channel with an external eavesdropper, generalizes and extends the proposed achievable regions given in [12], due to the implementation of simultaneous cooperative binning and jamming at the transmitters together with more general time-sharing approaches. This simultaneous transmission of secret messages and noise samples from transmitters is considered by [13]. In [13], authors proposed artificially generated noise injection schemes for multi-transmit antenna wire-tap channels, in which the superposition of a secrecy signal and an artificially generated noise is transmitted from the transmitter, where the noisy transmission only degrades the eavesdropper’s channel. For the single transmit antenna case, wire-tap channels with helper nodes is considered, in which helper nodes transmit artificially generated noise samples in order to degrade the eavesdropper’s channel. Remarkable, exploitation of the channel prefixing technique was transparent in these previous studies. The proposed scheme in this work shows that the benefit of cooperative jamming scheme of [12] and noise injection scheme of [13] originates from the channel prefixing technique. In addition, compared to [6], the proposed scheme allows for cooperation via both binning and channel prefixing techniques, whereas in [6] one of the transmitters is allowed to generate and transmit noise together with the secret signal and...
cooperation among network users as considered in this sequel was not implemented for the confidential message scenario.

The rest of this work is organized as follows. Section II introduces the system model. In Section III, the main result for discrete memoryless interference channels is given. Section IV is devoted to some examples of the proposed scheme for Gaussian channels. Finally, we provide some concluding remarks in Section V.

II. SYSTEM MODEL

We consider a two-user interference channel with an external eavesdropper (IC-EE), comprised of two transmitter-receiver pairs and an additional eavesdropping node. The discrete memoryless IC-EE is denoted by

\[(X_1 \times X_2, p(y_1, y_2, y_e|x_1, x_2), Y_1 \times Y_2 \times Y_e),\]

for some finite sets \(X_1, X_2, Y_1, Y_2, Y_e\). Here the symbols \((x_1, x_2) \in X_1 \times X_2\) are the channel inputs and the symbols \((y_1, y_2, y_e) \in Y_1 \times Y_2 \times Y_e\) are the channel outputs observed at the decoder 1, decoder 2, and at the eavesdropper, respectively. The channel is memoryless and time-invariant.

We assume that each transmitter \(k \in \{1, 2\}\) has a secret message \(W_k\) which is to be transmitted to the respective receivers in \(n\) channel uses and to be secured from the external eavesdropper. In this setting, an \((n, M_1, M_2, P_{e,1}, P_{e,2})\) secret codebook has the following components:

1. The secret message sets \(W_k = \{1, \ldots, M_k\}\) for transmitter \(k = 1, 2\).
2. Encoding function \(f_k(\cdot)\) at transmitter \(k\) which maps the secret messages to the transmitted symbols, i.e., \(f_k : w_k \rightarrow X_k\) for each \(w_k \in W_k\) for \(k = 1, 2\).
3. Decoding function \(\phi_k(\cdot)\) at receiver \(k\) which maps the received symbols to estimate of the message: \(\phi_k(Y_k) = \hat{w}_k\) for \(k = 1, 2\).

Reliability of the transmission of user \(k\) is measured by \(P_{e,k}\), where

\[P_{e,k} = \frac{1}{M_1 M_2} \sum_{(w_1, w_2) \in W_1 \times W_2} \Pr \{ \phi_k(Y_k) \neq w_k | E_{w_1, w_2} \},\]

where \(E_{w_1, w_2}\) is the event that \((w_1, w_2)\) is transmitted from the transmitters.

For the secrecy requirement, the level of ignorance of the eavesdropper with respect to the secured messages is measured by the equivocation rate

\[\frac{1}{n} H(W_1, W_2|Y_e).\]

We say that the rate tuple \((R_1, R_2)\) is achievable for the IC-EE if, for any given \(\epsilon > 0\), there exists an \((n, M_1 = 2^{n R_1}, M_2 = 2^{n R_2}, P_{e,1}, P_{e,2})\) secret codebook such that,

\[\max \{P_{e,1}, P_{e,2}\} \leq \epsilon,\]

and

\[R_1 + R_2 - \frac{1}{n} H(W_1, W_2|Y_e) \leq \epsilon \tag{1}\]

for sufficiently large \(n\). The secrecy capacity region is the closure of the set of all achievable rate pairs \((R_1, R_2)\) and is denoted as \(C_{IC-EE}\).

A. The Gaussian Interference Channel with an External Eavesdropper in Standard Form

The Gaussian interference channel in standard form is given in [14]. We have the same transformation here for the Gaussian interference channel with an external eavesdropper (GIC-EE) model. We remark that the channel capacity will remain the same as the transformations are invertible. We represent the average power constraints of the transmitters as \(P_k\), where codewords should satisfy \(\frac{1}{n} \sum_{i=1}^{n} (X_k(i))^2 \leq P_k\) for \(k = 1, 2\).

Here the input-output relationship, i.e., \(p(y_1, y_2, y_e|x_1, x_2)\), changes to the following:

\[
\begin{align*}
Y_1 &= X_1 + \sqrt{c_1} X_2 + N_1, \\
Y_2 &= \sqrt{c_2} X_1 + X_2 + N_2, \\
Y_e &= \sqrt{c_e} X_1 + \sqrt{c_e} X_2 + N_e,
\end{align*}
\]

where \(N_k \sim \mathcal{N}(0, 1)\) for \(k = 1, 2, e\) as depicted in Fig. 1. The secrecy capacity region of the GIC-EE is denoted as \(C_{GIC-EE}\).

III. THE DISCRETE MEMORYLESS INTERFERENCE CHANNEL WITH AN EXTERNAL EAVESDROPPER

In this section, we introduce the proposed cooperative binning and channel prefixing scheme for the IC-EE model. With this scheme, transmitters design their secrecy codebooks using the random binning technique [8]. This binning structure in the codebook let a transmitter to add randomness in its own signals. However, the price of adding extra randomness to secure the transmission appear as a rate loss in the achievable
rate expressions. In our scenario, the proposed strategy allows for cooperation in design of these binning codebooks, and allows for cooperation in prefixing the channel as we utilize the channel prefixing technique of [9] at both users. Hence, users of the interference channel will add only sufficient amount of randomness as the other user will help to increase the randomness seen by the eavesdropper. The achievable secure rate region with this scheme is described below.

First consider auxiliary random variables $Q$, $C_1$, $S_1$, $Q$, $C_2$, $S_2$, and $O_2$ defined on arbitrary finite sets $Q$, $C_1$, $C_2$, $S_1$, $S_2$, and $O_2$, respectively. Now, let $P$ be the set of all joint distributions of the random variables $Q$, $C_1$, $S_1$, $C_2$, $S_2$, $O_2$, $X_1$, $X_2$, $Y_1$, $Y_2$, and $Y_e$ that factors as $p(q, c_1, s_1, o_1, c_2, s_2, o_2, x_1, x_2, y_1, y_2, y_e) = p(q) p(c_1|q) p(s_1|q) p(o_1|q) p(c_2|q) p(s_2|q) p(o_2|q) p(x_1|c_1, s_1, o_1, q) p(x_2|c_2, s_2, o_2, q) p(y_1|y_e, c_1, s_1, o_1, q) p(y_2|y_e, c_2, s_2, o_2, q).$ Here, the variable $Q$ serves as a time-sharing parameter. See, for example, [10], [15] for a discussion on time-sharing parameters. The variable $C_1$ is used to construct the common secured signal of transmitter 1 that has to be decoded at both receivers, where the random binning technique of [8] is used for this construction. The variable $S_1$ is used to construct the self secured signal that has to be decoded at receiver 1 but not at receiver 2, where the random binning technique of [8] is used for this construction. The variable $O_1$ is used to construct other signal of transmitter 1 that has to be decoded at receiver 2 but not at receiver 1, where the conventional random codebook construction, see for example [15], is used for this signal, i.e., no binning is implemented. Similarly, $C_2$, $S_2$, and $O_2$ are utilized at user 2. Finally, it is important to remark that the channel prefixing technique of [9] is exploited with this construction as we transformed the channel $p(y_1, y_2, y_e|x_1, x_2)$ to $p(y_1, y_2, y_e|x_1, s_1, o_1, c_1, s_1, o_1, q)$ using the prefixes $p(x_1|c_1, s_1, o_1, q)$ and $p(x_2|c_2, s_2, o_2, q)$.

To ease the presentation, we first state the following definitions. We define $T_1 \triangleq C_1$, $T_2 \triangleq S_1$, $T_3 \triangleq O_1$, $T_4 \triangleq C_2$, $T_5 \triangleq S_2$, $T_6 \triangleq O_2$ and corresponding rates $R_{T_1}$ and $R_{T_6}$. Note that we choose $R_{O_1} = R_{O_2} = 0$ below. Also, we define $T_S \triangleq \{ T_i | i \in S \}$.

**Definition 1:** $\mathcal{R}(p)$ is the set of all tuples $\{(R_{C_1}, R_{C_2}, R_{S_1}, R_{S_2}, R_{C_1}, R_{C_2}, R_{O_1}, R_{O_2})\}$ satisfying
\[
\sum_{i \in S} R_{T_i} + R_{T_6} \leq I(T_S; Y_1 | T_{S'}, Q), \forall S \subseteq \{1, 2, 4, 6\},
\]
for a given joint distribution $p$.

**Definition 2:** $\mathcal{R}_{p}(2)$ is the set of all tuples $\{(R_{C_2}, R_{C_2}, R_{S_2}, R_{S_2}, R_{C_1}, R_{C_2}, R_{O_1}, R_{O_2})\}$ satisfying
\[
\sum_{i \in S} R_{T_i} + R_{T_6} \leq I(T_S; Y_2 | T_{S'}, Q), \forall S \subseteq \{1, 3, 4, 5\},
\]
for a given joint distribution $p$.

**Definition 3:** $\mathcal{R}_{c}(p)$ is the set of all tuples $\{(R_{C_1}, R_{S_1}, R_{O_1}, R_{C_2}, R_{S_2}, R_{O_2})\}$ satisfying
\[
\sum_{i \in \{2, 3, 4, 5, 6\}} R_{T_i} \leq I(T_S; Y_e | T_{S'}, Q), \forall S \subseteq \{1, \cdots, 6\},
\]
\[
\sum_{i \in \{1, 2, 3, 4, 5, 6\}} R_{T_i} = I(T_1, T_2, T_3, T_4, T_5, T_6; Y_e | Q),
\]
for a given joint distribution $p$.

**Definition 4:** $\mathcal{R}(p)$ is the closure of all $(R_1, R_2)$ satisfying
\[
R_1 = R_{C_1} + R_{S_1},
R_2 = R_{C_2} + R_{S_2},
(R_{C_1}, R_{C_2}, R_{S_1}, R_{S_2}, R_{C_1}, R_{C_2}, R_{O_1}, R_{O_2}) \in \mathcal{R}_1(p),
(R_{C_2}, R_{C_2}, R_{S_2}, R_{S_2}, R_{C_1}, R_{C_2}, R_{O_1}, R_{O_2}) \in \mathcal{R}_2(p),
(R_{C_1}, R_{S_1}, R_{O_1}, R_{C_2}, R_{S_2}, R_{O_2}) \in \mathcal{R}_c(p),
\]
and
\[
R_{C_1} \geq 0, R_{C_2} \geq 0, R_{S_1} \geq 0, R_{S_2} \geq 0, R_{O_1} \geq 0, R_{O_2} \geq 0,
\]
for a given joint distribution $p$.

We now state the main result of the paper. The achievable secrecy rate region using the cooperative binning and channel prefixing scheme is as follows.

**Theorem 5:** $\mathcal{R}^\text{IC-EE} \triangleq \{ \bigcup_{p \in P} \mathcal{R}(p) \} \subseteq \mathcal{R}^\text{IC-EE}.$

**Proof:** The proof is omitted and will be provided in the journal version of this work.

IV. THE GAUSSIAN INTERFERENCE CHANNEL WITH AN EXTERNAL EAVESDROPPE

In this section, we provide some examples of the proposed coding scheme for Gaussian interference channels and show that the proposed scheme provides gains in securing the network by exploiting cooperative binning, cooperative channel prefixing, and time-sharing techniques.

Firstly, we describe how the channel prefixing can be implemented in this Gaussian scenario. Here, one can independently generate and transmit noise samples for each channel use from the transmitters (without constructing a codebook and sending one of its messages) to enhance the security of the network. As there is no design of a codebook at the interfering user for this noise transmission, receivers and the eavesdropper can only consider this transmission as noise. Accordingly, transmitter $k \in \{1, 2\}$ uses power $P_k^b$ for the construction of its (binning) codewords, which are explained in the previous section, and obtains, somehow, the signal $X^b_k \sim \mathcal{N}(0, P_k^b)$. In addition, it uses power $P_k^j$ for its jamming signal and generates i.i.d. noise samples represented by $X^j_k \sim \mathcal{N}(0, P_k^j)$, where we choose $P_k^b + P_k^j \leq P_k$. Then, it sends $X^b_k + X^j_k$ to the channel, instead of just sending $X^b_k$.

Now, we can use the scheme proposed in the previous section for the design of the signals $X^b_k$. Below we will use superposition coding to construct this signal. But first, for a rigorous presentation, we provide some definitions. Let $A$ denote the set of all tuples $\{(P_1^b(q), P_1^j(q), P_1^c(q), P_2^b(q), P_2^c(q), P_1^j(q), P_2^j(q))\}$ satisfying $P_k^b(q) \triangleq P_k^b(q) + P_k^c(q) + P_k^j(q)$ and
\[
\sum_{q \in \mathcal{Q}} (P_k^b(q) + P_k^j(q)) \leq P_k, \text{ for } k = 1, 2.
\]
Now, we define a set of joint distributions $\mathcal{P}_1$ as follows.
clusions of the achievable region given by Corollary 6.

B. Numerical Results and Discussion

In this section we provide numerical results for the following subregions of the achievable region given by Corollary \ref{cor:rdma}

1) $R_{GIC}$: This region is provided above, where we utilize both cooperative binning and channel prefixing.

2) $R_{GIC}^{(b \text{ or } cp)}$: Here we utilize either cooperative binning or channel prefixing scheme at a transmitter, but not both.

3) $R_{GIC}^{(ncp)}$: Here we only utilize cooperative binning. Accordingly, jamming powers are set to zero.

4) $R_{[C-TDMA]}$: This region is an example of utilizing both time-sharing and cooperative channel prefixing. No cooperation is used.

5) $R_{[C-TDMA]}^{(ncp)}$: Here we do not allow transmitters to jam the channel during their dedicated time slots and call this case no self channel prefixing (nscp).

6) $R_{[C-TDMA]}^{(ncp)}$: Here no channel prefixing is implemented. This case refers to conventional TDMA scheme, in which users are allowed to transmit during only their assigned slots. Hence, this scheme only utilizes time-sharing.

Numerical results are provided in Fig. 2 and Fig. 3. The first scenario depicted in Fig. 2 shows the benefits of cooperative binning technique. Also, cooperative channel prefixing does not help to enlarge the secure rate region in this scenario. Secondly, in Fig. 3, we consider an asymmetric scenario, in which the first user has a weak channel to the eavesdropper but the second user has a strong channel to the eavesdropper.
Here, the second user can help the first one to increase its secrecy rate. However, channel prefixing and time-sharing does not help to the second user as it can not achieve positive secure rate without an implementation of cooperative binning. Remarkable, cooperative binning technique helps the second user to achieve positive secure transmission rate in this case. These observations suggest the implementation of all three techniques (cooperative binning, cooperative channel prefixing, and time-sharing) as considered in our general rate region, i.e., $R^{GIC-EE}$.

![Fig. 2. Numerical results for GIC-EE with secrecy rate. However, channel prefixing and time-sharing does not help to the second user as it can not achieve positive secure rate without an implementation of cooperative binning. Remarkable, cooperative binning technique helps the second user to achieve positive secure transmission rate in this case. These observations suggest the implementation of all three techniques (cooperative binning, cooperative channel prefixing, and time-sharing) as considered in our general rate region, i.e., $R^{GIC-EE}$.

C. Some Implications of the Proposed Scheme

It can be shown that the proposed scheme reduces to the noise-forwarding scheme of [11] for the discrete memoryless relay-eavesdropper channel. Remarkable, the channel prefixing technique can be exploited in this scenario to increase the achievable secure rates. For example, for the Gaussian channel, injecting i.i.d. noise samples can increase the achievable secure transmission rates as shown in [16]. Our result here shows that the gain resulting from the noise injection comes from the exploitation of the channel prefixing technique. In addition, the proposed scheme, when specialized to a Gaussian multiple-access scenario, results in an achievable region that generalizes and extends the proposed regions given in [12] due to the implementation of simultaneous cooperative binning and channel prefixing at the transmitters together with more general time-sharing approaches.

V. CONCLUSION

In this work, we have considered two-user interference channels with an external eavesdropper. We have proposed the cooperative binning and channel prefixing scheme that utilizes random binning, channel prefixing, and time-sharing techniques and allows transmitters to cooperate in adding randomness to the channel. For Gaussian interference channels, the channel prefixing technique is exploited by letting users to inject independently generated noise samples to the channel. The most interesting aspect of our results is, perhaps, the unveiling of the role of interference in cooperatively adding randomness to the channel to increase the secrecy rates of multi-user networks.

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