Nonsymmetric Gravity has Unacceptable Global Asymptotics

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Abstract

We analyze the radiative aspects of nonsymmetric gravity theory to show that, in contrast to General Relativity, its nonstationary solutions cannot simultaneously exhibit acceptable asymptotic behavior at both future and past null infinity: good behavior at future null infinity is only possible through the use of advanced potentials with concomitant unphysical behavior at past null infinity.
1. Introduction

Recent examinations of the nonsymmetric gravitational theory (NGT) uncovered a number of fundamental deficiencies: The gauge invariance present in the linearization of NGT about flat space cannot be deformed to the full theory, resulting in vector ghost excitations via curvature coupling. Also, generic solutions suffered unacceptable asymptotic behavior at future null infinity ($I^+$).

Our main purpose here is to consider more explicitly the mechanism underlying asymptotic failure of NGT. The central criterion for the NGT field equations to admit physically acceptable solutions appropriate for the description of bounded radiating systems is, at a minimum, that the metric have – in a suitable (Cartesian) coordinate system – an expansion in powers of $1/r$ about the Minkowski flat metric, $\eta_{\mu\nu}$. The key point in this requirement is that this expansion be valid not only at spatial infinity ($r \to \infty$, $t$ fixed) but, most importantly, at both $I^+$ ($r \to \infty$, $u = t - r$ fixed) and $I^-$ ($r \to \infty$, $v = t + r$ fixed). We shall establish here that acceptable asymptotic behavior at $I^+$ ($I^-$) is obtainable, in particular solutions, only at the cost of diverging physical quantities at $I^-$ ($I^+$). We note that the asymptotic behavior at $I^+$ of such particular solutions has recently been claimed to demonstrate the consistency of the model. In particular, then, our result displays the physical flaws in those analyses.

In Section 2 we present the complete argument, after carefully defining the problem at hand. The main result follows from a simple lemma which implies that requiring good asymptotics at $I^+$ forces the use of advanced potentials in constructing the solution. In an appendix, a simplified but faithful analogue vector model is discussed, which exhibits our main points regarding NGT with a minimum of technicality.

2. Asymptotic Analysis of the Field Equations

For the present purpose it suffices to analyze the NGT field equations in an expansion in powers of the antisymmetric component $B_{\mu\nu} = -B_{\nu\mu}$ of the total metric variable $g_{\mu\nu} \equiv G_{\mu\nu} + B_{\mu\nu}$.[5, 1] Indeed, we will only need the field equations to first order in $B$ to exhibit the radiative instability of NGT; the higher order powers can only exacerbate the problems. We rewrite the NGT field equations (Eqs. (3) and (4) of [1]) as propagation equations for the three independent fields $G_{\mu\nu}$, $B_{\mu\nu}$, and $\Gamma_{\mu}$,

$$R_{\mu\nu}(G) = \mathcal{O}(B^2, \Gamma B),$$  \hspace{1cm} (2.1)
\[ \Box \Gamma_\nu = -3 \nabla^\mu (R^\alpha_\alpha^\beta_\mu B_{\alpha\beta}) + O(B^3, \Gamma B^2), \tag{2.2} \]

\[ \Box B_{\mu\nu} = \frac{4}{3} (\partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu) + 2 R^\alpha_\mu^\beta_\nu B_{\alpha\beta} + O(B^3, \Gamma B^2), \tag{2.3} \]

together with the two subsidiary conditions

\[ \nabla^\nu \Gamma_\nu = 0, \tag{2.4} \]

\[ \nabla^\nu B_{\mu\nu} = O(B^3). \tag{2.5} \]

Here \( G_{\mu\nu} \) is the Riemannian metric that acts as the background for the remaining equations and is used for all covariant operations (e.g., to define the wave operator \( \Box \)); the vector \( \Gamma_\mu \), which originated as a contracted torsion, is a propagating field despite its role as a Lagrange multiplier ensuring condition \( (2.5) \). The constraint \( (2.4) \) represents a gauge choice on \( \Gamma_\mu \). Consistency between the propagation and subsidiary equations is ensured by differential identities; once \( (2.4), (2.5) \) are satisfied on a Cauchy hypersurface they remain valid by virtue of \( (2.1) - (2.3) \). These equations may now be solved order by order in an expansion \( g_{\mu\nu} = G_{\mu\nu}^{(0)} + (G_{\mu\nu}^{(1)} + B_{\mu\nu}^{(1)}) + \ldots \) about a given Einstein space metric \( G_{\mu\nu}^{(0)} \). A point to keep in mind is that, besides the effective source terms on the RHS of eqs \( (2.2) \) and \( (2.3) \), there will necessarily appear additional localized source terms representing the matter couplings of the NGT sector. The arguments given below apply to the total effective + matter source terms for the NGT variables. We now turn to the discussion of radiative solutions of these equations.

In broad terms, a radiative (asymptotically flat) solution in any classical field theory is one in which all the fields (including the gravitational one) exhibit a \( 1/r \) decay at both future and past null infinity, \( \text{i.e.} \) in which the deviations between the fields and their (constant) asymptotic values behave, at leading order, as \( f_{\text{out}}(u, \theta, \phi)/r \) on \( I^+ \) and \( f_{\text{in}}(v, \theta, \phi)/r \) on \( I^- \). Roughly speaking, \( f_{\text{out}} \) (\( f_{\text{in}} \)) measures the amount of outgoing (incoming) radiation present in the solution. Among such general radiative solutions, classical field theory further selects the “retarded” ones which contain no incoming radiation on \( I^- \) \( (f_{\text{in}}(v, \theta, \phi) = 0) \) but only outgoing radiation on \( I^+ \), associated to the time dependence of some source. A convenient way of ensuring that one has selected the physically correct solution is to consider sources that are stationary for all times \( t \leq t_0 \) and turn on after \( t = t_0 \). Then all fields (notably the gravitational one) must be stationary up to the time \( t_0 \). We

\[ ^1 \text{In the case of gauge fields with associated conserved charges, one must subtract their associated static potentials when defining the wave amplitudes } f_{\text{in}} \text{ and } f_{\text{out}} \text{ then (e.g. remove the Schwarzschild } M/r \text{ term when defining the gravitational wave amplitude } f_{ij}^{TT}/r). \]
refer to such a solution as a "causal" one. [When iteratively solving the field equations by means of propagators and covariant "spin-projection" operators, the causal nature of the solutions is enforced by consistently using retarded propagators. This point is further discussed in the Appendix, in the context of a simplified model of NGT. We show there the equivalence of the spin-projection technique with the more direct approach used in the text for exhibiting the asymptotic failure of NGT.]

For General Relativity, radiative solutions [6], as well as more specific causal ones [7], are known to exist and to contain a wave zone in which the metric approaches (in suitable coordinate systems) its Minkowski value as \( f(t \pm r, \theta, \phi)/r \) at \( \mathcal{I}^\pm \). Thus in GR, not only can one exhibit solutions with good radiative behavior at either \( \mathcal{I}^+ \) or \( \mathcal{I}^- \), but this behavior does not entail any radiation (let alone rising behavior in \( r \)) at the other null infinity. In contrast, we will see that NGT can at best be "tuned" to obtain good "radiative" behavior at one end only at the cost of totally unphysical properties of the geometry at the other. Already in [1] it was shown for the NGT field equations (2.1 - 2.3) that good fall-off behavior at \( \mathcal{I}^+ \) was in fact inconsistent with the retarded solution for \( \Gamma_\mu \) in (2.2), the root of the problem being the slow decay at \( \mathcal{I}^+ \) of the effective source term \( \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu \) on the RHS of Eq.(2.3). We now show that if one demands that the NGT variables fall off at \( \mathcal{I}^+ \) faster than their normal rate (in an attempt to ensure the desired outgoing wave behavior of \( G_{\mu\nu} \)), then the retarded solution for \( \Gamma_\mu \) must be replaced by the advanced one.

More precisely, we demand that when \( r \to \infty \) with \( u = t - r \) fixed,

\[
\Gamma_\mu = \mathcal{O}(1/r^{1+\epsilon}) , \quad R^\alpha_{\mu \nu} B_{\alpha \beta} = \mathcal{O}(1/r^{3+\epsilon}) , \quad \epsilon > 0 .
\]

The consequences of \( \Gamma_\mu \) propagating by the advanced Green’s function are quite drastic: firstly, at all past times \( \Gamma_\mu \) then acts as a time-dependent source in (2.1) and (2.3), making it impossible to have causal solutions where the geometry is stationary before the time when the matter sources turn on; secondly and more globally, time-reversing the above argument immediately provides a contradiction with the required physical behavior on \( \mathcal{I}^- \). The underlying physics is illustrated by a simpler model deforming Maxwell theory to provide a simplified but faithful version of NGT; it is given in the Appendix.

Our demonstration of this result uses the following simple lemma, well-known in its time-reversed formulation (see, e.g., [8]), which we prove here for the sake of convenient reference:

Consider, in an asymptotically flat spacetime, a time-dependent source, \( \rho(x) \), falling off as
\( \mathcal{O}(1/r^{2+\epsilon}) \) (\( \epsilon > 0 \)) near \( \mathcal{I}^+ \). If \( \phi(x) \) is a solution of the inhomogeneous wave equation\(^2\)

\[
\Box \phi(x) = \rho(x),
\]

that falls off as \( \mathcal{O}(1/r^{1+\epsilon}) \) at \( \mathcal{I}^+ \), then it is necessarily the advanced solution,

\[
\phi(x) = \int d^4x' G_{\text{adv}}(x, x') \rho(x').
\]

The proof of this lemma follows from a straightforward application of Green’s theorem (see, e.g. \cite{4}). Integrating Green’s identity over the spacetime region \( \mathcal{R} = \mathcal{R}(x, \bar{r}) = \{x' : t' - t \leq \bar{r}\} \), for given \( t = x^0 \) and \( \bar{r} > 0 \), we have

\[
\phi(x) - \int_{\mathcal{R}} dV' G_{\text{adv}}(x, x') \rho(x') = \int_{\partial \mathcal{R}} d\Sigma' [\partial'_\mu G_{\text{adv}}(x, x') \phi(x') - G_{\text{adv}}(x, x') \partial'_\mu \phi(x')].
\]  

(2.9)

where \( dV' \) is the (curved) spacetime volume element, \( \partial \mathcal{R} \) the boundary of \( \mathcal{R} \), i.e. the spacelike hypersurface \( t' = t + \bar{r} \), and \( d\Sigma'^\mu = d\Sigma' n'^\mu \) the surface element of \( \partial \mathcal{R} \). For large enough \( \bar{r} \) we may replace \( G_{\text{adv}} \) on \( \partial \mathcal{R} \) by its flat space limit \( G_{\text{adv}}^{(0)}(x, x') \), where

\[
G_{\text{adv}}^{(0)}(x, x') = -\frac{1}{2\pi} \theta(t' - t) \delta((x - x')^2) = -\frac{1}{4\pi} \frac{\delta(t - t' + |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|}.
\]

(2.10)

The RHS of (2.9) then only has a contribution from the intersection of the support of \( G_{\text{adv}}^{(0)} \) with the hypersurface \( \partial \mathcal{R} \), i.e., on the 2-sphere \( \bar{r} = t' - t = |\vec{x} - \vec{x}'| \). In fact (\( R' \equiv |\vec{x} - \vec{x}'| \)),

\[
\phi(x) - \int_{\mathcal{R}} dV' G_{\text{adv}}(x, x') \rho(x') = \int_{\vec{t}' = t + \bar{r}} d\Sigma' [\partial'_\mu G_{\text{adv}}(x, x') \phi(x') - G_{\text{adv}}(x, x') \partial'_\mu \phi(x')]
\]

\[
= \frac{1}{4\pi} \int_{t' = t + \bar{r}} d\Omega' dR' \left[ -\partial_{R'} \delta(t - t' + R') \phi(x') - \delta(t - t' + R') \partial'_{R'} \phi(x') \right]
\]

\[
= \frac{1}{4\pi} \int_{R' = \bar{r}} d\Omega' \left[ \partial_{R'} (R' \phi(x')) - \partial'_{R'} (R' \phi(x')) \right]|_{t' = t + \bar{r}}.
\]

(2.11)

The boundary conditions \( \phi = O(1/r^{1+\epsilon}) \), \( \rho = O(1/r^{2+\epsilon}) \) at \( \mathcal{I}^+ \) allow one to take the limit \( \bar{r} \to \infty \).

In this limit the RHS of Eq.(2.11) vanishes, and the volume integral on the LHS converges to the RHS of Eq.(2.8), thereby proving our lemma.

The demonstration now proceeds by applying our lemma to (2.2) which is an inhomogeneous wave equation for \( \Gamma_\mu \), taking into account the boundary conditions (2.6), the second of which implies the needed fall-off of the effective source term \( \nabla^\mu (R_{\mu \nu}^\alpha B_{\alpha \beta}) = O(1/r^{2+\epsilon}) \). Since \( \Gamma_\mu \) is a solution

\(^2\)For simplicity of the following argument we consider a scalar wave equation, but it of course applies to tensor fields as well.
of the inhomogeneous wave equation (2.2), decaying faster than \( r^{-1} \) at \( \mathcal{I}^+ \), the lemma proven above implies that it is the \textit{advanced potential} solution, \textit{i.e.}, the convolution of the total (effective + matter) source with the – unique – advanced propagator. In the time-dependent situation of interest, this implies that \( \Gamma_\mu \) – and hence the curl, \( \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu \) – is non-zero at all past times. But the curl is just the effective source for \( B \) in (2.3), which must therefore be nonvanishing at all times in the past, and from (2.1) we see the same must be true for \( G_{\mu\nu} \). Clearly this is inconsistent with the proposition that the solution with acceptable asymptotic behavior on \( \mathcal{I}^+ \) describes in a causal manner gravitational radiation associated to a matter source turned on at some time \( t_0 \).

One might attempt to salvage – if not a causal solution properly so-called – at least a geometry whose good asymptotics on \( \mathcal{I}^+ \) can coexist with good fall-off behavior on \( \mathcal{I}^- \). However the time-reverse of the simple argument in [1] shows that this is simply not possible. The assumption of good fall-off for \( G \) and \( B \), and the discussion above, imply that the source, \( \nabla(R \cdot B) + \text{matter contribution} \), generates \textit{via the advanced Green’s function} (at fastest) a \( 1/r \) fall-off for \( \Gamma_\mu \) on \( \mathcal{I}^- \). This makes it obvious that a source term \( \sim \rho_n(v, \theta, \phi)/r^n \) implies at fastest a \( \sim \phi_{n-1}(v, \theta, \phi)/r^{n-1} \) fall-off for the solution (in the special case \( n = 2 \), the solution falls off as \( \log(r)/r \) at best). The second is obtained by noticing that, from Eqs. (2.2) and (2.3), the 2-form \( B_{\mu\nu} \) satisfies an equation of the form (with indices suppressed) \( \Delta^2 B = \nabla\nabla(R \cdot B) + \mathcal{O}(B^3, \Gamma B^2) \), where \( \Delta \) denotes the Hodge-De Rham wave operator. Like \( \Delta \), the iterated operator \( \Delta^2 \) is a hyperbolic differential operator which possesses uniquely defined retarded and advanced Green’s functions. Generalizing the lemma given above, we see that if one demands that \( B \) and \( R \cdot B \) fall off sufficiently fast at \( \mathcal{I}^+ \), \( B(x) \) must necessarily be given by the convolution of the total source term, \( \nabla\nabla(R \cdot B) + \text{matter contribution} \), with the \textit{advanced} Green’s function of \( \Delta^2 \), say \( H_{adv}^{(0)}(x, x') \). In the flat spacetime limit the latter Green’s function reads

\[
H_{adv}^{(0)}(x, x') = \frac{1}{8\pi} \theta(t' - t - |\vec{x} - \vec{x}'|),
\]

and does not fall off with \( |\vec{x} - \vec{x}'|^{-1} \). The matter contribution to the source of \( \Delta^2 \) will generate an advanced \( B \) wave behaving as \( r^0 \) on \( \mathcal{I}^- \), while the extended effective source can even give a worse
fall-off if $\triangledown \triangledown (R \cdot B)$ does not decay fast enough at infinity. [Let us note for completeness that in the pure vacuum NGT case our argument must be phrased somewhat differently: good fall-off for $G_{\mu \nu}$ necessitates rapid fall-off for $\Gamma$ and $B$ (such as (2.6)) at both ends. In turn this rapid fall-off excludes the presence of both incoming and outgoing NGT radiation (exclusion at one end only would suffice for the argument to go through). Then the unique solution of the hyperbolic system satisfied by $\Gamma$ and $B$ must be the trivial (pure GR) one, $\Gamma = B = 0$ everywhere.]

To summarize, we have shown that NGT necessarily displays unphysical asymptotic behavior in its time-dependent solutions, and in particular that the asymptotic (but near $\mathcal{I}^+$ only) solutions proposed in [4] behave catastrophically at past null infinity. This loss of correct asymptotic behavior can ultimately be traced back to the fundamental shortcoming of NGT that it violates the gauge invariance of the long-range $B_{\mu \nu}$ field. Indeed, the gauge invariant (string-generated) massless $B$ theory coupled to Einstein gravity does not suffer from these defects. We mention that the other remedy (which has the further virtue of allowing phenomenologically interesting matter couplings) is to endow $B$ with a finite range [1].

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Appendix – A Simple Vectorial Model of NGT

We consider here the simplest analogue of the $B$-field system, which describes a vector field $A_\mu$ coupled to an external nonconserved localized source $J_\mu$. The Lagrangian is $\mathcal{L} = (-1/4) F_{\mu \nu}^2 - \partial^\mu S A_\mu + J_\mu A_\mu$, while the field equations read

$$\partial^\alpha F_{\mu \alpha} = -\partial_\mu S + J_\mu, \quad \partial^\mu A_\mu = 0. \quad (A.1)$$

But for the loss of gauge invariance implied by current nonconservation, this system would simply be a gauge fixing of the Maxwell theory, in which $S$ enters as a Lagrange multiplier enforcing the condition $\partial^\mu A_\mu = 0$. It is clear that this quasi-Maxwellian vector model closely mimics NGT: $S$ has the role played by $\Gamma_\mu$ in NGT, while the nonconserved source $J_\mu$ can either be thought of as replacing the $RB$ term in (2.3), or as one of the local matter sources in the more general case of non-vacuum NGT. [Note that there is no natural reason to require conservation of such sources since there is no corresponding gauge invariance.]
Despite its origin as a Lagrange multiplier, $S$ is a relevant excitation because Eq. (A.1) implies
\[ \Box S = \partial_\mu J^\mu \neq 0, \]
and it is no longer consistent to set it to zero as it would be in ordinary gauge-fixed electrodynamics. In turn, $S$ excites the – nonpositive energy – longitudinal modes. In fact, from the Lagrangian one easily computes the corresponding energy momentum tensor
\[ T_{\mu\nu} = F_\mu^\alpha F_{\nu\alpha} - \frac{1}{4} \eta_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} + \partial_\mu S A_\nu + \partial_\nu S A_\mu - \eta_{\mu\nu} \partial^\alpha S A_\alpha. \]  
(A.3)
The expression for the energy
\[ T_{00} = \frac{1}{2} (F_{0i})^2 + \frac{1}{4} (F_{ij})^2 + \partial_0 S A_0 + \partial_i S A_i \]
(A.4)
is clearly not positive definite, reflecting the presence of negative-energy modes. Indeed, the canonical analysis for the $S$/longitudinal sector (i.e., under the temporally local orthogonal decomposition $A_i = A_i^T + A_i^L$, where $A_i^L = \partial_i a / \sqrt{-\nabla^2}$) in the absence of sources reveals a dipole ghost structure [10]: the action takes the form
\[ I_0 = \int d^4x \left[ \frac{1}{2} (\dot{a} - (\sqrt{-\nabla^2} A_0))^2 + S(\dot{A}_0 + (\sqrt{-\nabla^2} a)) \right], \]
(A.5)
and eliminating the $S$-constraint leaves a manifestly dipole-ghost action for the remaining mode:
\[ I_{gh} = \int d^4x A^2, \quad \Phi(x) = (\frac{1}{\sqrt{-\nabla^2}} A_0)(x). \]
(A.6)
This fact alone would of course rule out the model as unphysical; however, the problems in fact occur at the more fundamental level of asymptotics. For a local source $J$ with compact support in space time, we may look for solutions corresponding to purely outgoing radiation in the wave zone. It is impossible for all the propagating fields to have the usual $1/r$ decay, i.e., the retarded form $f(x) = f(t-r)/r$. The argument here is of course precisely that in the text: from (A.2) for a localized source we find that the retarded solution for $S$ falls off as $1/r$ at $T^+$. As a consequence the $A_\mu$ wave equation has a source which falls off too slowly and $1/r$ fall-off for $A$ itself is inconsistent. As proven in the text, the only way out would be to use advanced solutions and thereby sacrifice the interpretation as a causal solution, and more generally to force bad asymptotics at $T^-$. As a fallacious argument based on covariant spin-projection operators has been adduced in the last three references in [4], let us indicate here how our result on the bad fall-off of $A$ can
be recovered by introducing the two orthogonal projection operators $P^T$ and $P^L$ (sufficient in our quasi-Maxwellian example). Formally these are defined as projection operators such that $P^T + P^L = 1$, where $P^L_{\mu\nu}(x, x') = \partial_\mu \Box^{-1}(x, x') \partial_\nu'$. One should note immediately that, in contrast to the temporally local projection operators used earlier (which rely only on assuming the usual spatial boundary conditions, i.e., sufficiently rapid fall-off at spatial infinity), projection operators based on $\Box^{-1}$ require global spacetime boundary conditions. To generate a consistent operator algebra these boundary conditions must be taken to be the same for all fields, and of course for the present problem we must impose causal boundary conditions; namely, all fields and sources must vanish (or become static) before some time $t_0$. Then one can consistently define an algebra of projection operators based on the retarded Green’s function $\Box^{-1}_{ret}$ (in mathematical language, such retarded projection operators form a convolution algebra). In that case the longitudinal projection of (A.1) gives, unambiguously,

$$\partial_\mu S(x) = J^L_\mu = \partial_\mu \int d^4x' \Box_{ret}^{-1}(x, x') \partial^\nu J_\nu(x') ,$$  

(A.7)

and hence

$$S(x) = \int d^4x' \Box_{ret}^{-1}(x, x') \partial^\nu J_\nu(x') .$$  

(A.8)

The transverse projection of (A.1) (note that the second equation in (A.1) just gives $A_\mu = A^T_\mu$) implies $\Box A^T_\mu = -J^T_\mu$, whose unique causal solution is

$$A^T_\mu = - \int d^4x' \Box_{ret}^{-1}(x, x') J_\mu(x') + \partial_\mu \int d^4x' \Box_{ret}^{-2}(x, x') \partial^\nu J_\nu(x') ,$$  

(A.9)

where the second term involves the retarded version of the non-decaying Green’s function (2.12). Not surprisingly, we have simply recovered the results discussed above, with bad fall-off$^3$ for $A$.

The fallacy in the arguments of [4] consisted in simply ignoring the crucial, localized, source terms in (A.1) and (A.9). The problems discussed in this simple model serve to elucidate the fatal flaws undermining the asymptotic – near $I^+$ only – solutions of [4]. Good asymptotic behavior of $G_{\mu\nu}$ requires rapid $\Gamma$ fall-off; the presence of sources, however, makes such behavior impossible near $I^+$ except by the

\footnote{Let us recall that in the second paper of [1] we verified explicitly that – in the usual matter-coupled version of NGT, considered at the post-Newtonian approximation – the analogue of the last term in (A.9) (which also contains a divergence) generates non-decaying behavior for $B_{\mu\nu}$ with the dipole moment of the particle number density playing the role of a macroscopic source there.}

The fallacy in the arguments of [4] consisted in simply ignoring the crucial, localized, source terms in (A.1) and (A.9). The problems discussed in this simple model serve to elucidate the fatal flaws undermining the asymptotic – near $I^+$ only – solutions of [4]. Good asymptotic behavior of $G_{\mu\nu}$ requires rapid $\Gamma$ fall-off; the presence of sources, however, makes such behavior impossible near $I^+$ except by the
use of advanced potentials in solving for $\Gamma$, thus removing the possibility of acceptable behavior
near $I^-$. No consistent causal solutions are possible in NGT, unlike in GR.

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