Strong suppression of Coulomb corrections to the cross section of $e^+e^-$ pair production in ultrarelativistic nuclear collisions

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The Coulomb corrections to the cross section of $e^+e^-$ pair production in ultrarelativistic nuclear collisions are calculated in the next-to-leading approximation with respect to the parameter $L = \ln \gamma_A \gamma_B$ ($\gamma_{A,B}$ are the Lorentz factors of colliding nuclei). We found considerable reduction of the Coulomb corrections even for large $\gamma_A \gamma_B$ due to the suppression of the production of $e^+e^-$ pair with the total energy of the order of a few electron masses in the rest frame of one of the nuclei. Our result explains why the deviation from the Born result were not observed in the experiment at SPS.

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Electron-positron pair production in ultrarelativistic nuclear collisions is investigated intensively during almost two last decades, see recent reviews [3, 4]. This process is important in the problem of beam lifetime and luminosity of hadron colliders. It is also a serious background for many experiments because of its large cross section. For heavy nuclei, the effect of higher-order terms (Coulomb corrections) of the perturbation theory with respect to the parameters $Z_A \alpha$ and $Z_B \alpha$ can be very important ($Z_A$ and $Z_B$ are the charge numbers of the nuclei $A$ and $B$, $\alpha \approx 1/137$ is the fine-structure constant). However, no evidence of the Coulomb corrections has been found in the experiments [1, 2]. This circumstance stimulated considerable theoretical interest to this process. In the set of theoretical works [5, 6, 7] it was found that the exact in $Z_{A,B}\alpha$ cross section coincides with that obtained in the Born approximation in the ultrarelativistic limit. This statement was considered as an explanation of the experimental results [1, 2]. However, this conclusion contradicted to the result obtained in Ref. [8] with the help of the Weizsäcker-Williams approximation in the leading logarithmic approximation. This contradiction has been resolved in Ref. [9] (see also review [3]). Numerical evaluations in Refs. [5, 6, 7] in the Born approximation and in Refs. [19, 20, 21] with the Coulomb corrections taken into account. Using Eq. (3), the cross section $\sigma$ can be presented in the form:

$$\sigma_T = \sigma^0 + \sigma^A + \sigma^B + \sigma^{AB},$$

where $\sigma^0 \propto (Z_A \alpha)^2 (Z_B \alpha)^2$ is the Born cross section, $\sigma^A$ and $\sigma^B$ are the Coulomb corrections with respect to nucleus $A$ and $B$, respectively (containing the terms proportional to $(Z_B \alpha)^2 (Z_A \alpha)^{2n}$ and $(Z_B \alpha)^{2n} (Z_A \alpha)^2$, $n \geq 2$), and $\sigma^{AB}$ is the Coulomb corrections with respect to both nuclei (containing the terms proportional to $(Z_B \alpha)^n (Z_A \alpha)^l$ with $n, l > 2$). The cross section $\sigma^0$ coincides with the Born cross section of one pair production, which was calculated many years ago in Refs. [10, 11]. In the leading logarithmic approximation, the quantities $\sigma^{A,B} \propto L^2$ and $\sigma^{AB} \propto L$ were obtained in Refs. [8, 9] and Ref. [12], respectively.

The leading logarithmic approximation for $W(b)$ provides the factorization of $P_n(b)$ [13, 14, 15, 16], so that

$$P_n(b) = W^n(b) e^{-W(b)}. \quad (3)$$

The function $W(b)$ was calculated in Refs. [17, 18, 19, 20, 21] in the Born approximation and in Refs. [22, 23, 24, 25] with the Coulomb corrections taken into account. Using Eq. (3), the cross section $\sigma_1$ of one pair production can be represented as a sum of $\sigma_T$ and the unitarity correction $\sigma_{\text{unit}}$

$$\sigma_1 = \sigma_T + \sigma_{\text{unit}},$$

$$\sigma_{\text{unit}} = - \int d^2 b W(b) \left( 1 - e^{-W(b)} \right). \quad (4)$$

The existence of the unitarity correction was first recognized in Ref. [26] (see also review [3]). Numerical evaluation of this correction was performed in Refs. [20, 27]. The main contribution to $\sigma_1$ is given by the term $\sigma^0$ in...
The terms $\sigma^A$ and $\sigma^B$ in $\sigma_T$ also give important contributions to $\sigma_1$. In the leading logarithmic approximation, these terms have been derived in Refs. [8, 9]. The last two contributions, $\sigma^{AB}$ and $\sigma_{\text{unit}}$, to $\sigma_1$ are rather small, see Refs. [12, 20].

In the present paper, we calculate the leading corrections to $\sigma^{AB}$ (which are also the corrections to $\sigma_1$). We show that these corrections essentially diminish the magnitude of $\sigma^{AB}$ even for the parameters of LHC ($\gamma_A = \gamma_B \approx 3000$, $Z_A = Z_B = 82$). It is convenient to calculate $\sigma^A$ in the rest frame of the nucleus A, where the nucleus B has the Lorenz factor $\gamma = 2\gamma_A\gamma_B$ at $\gamma_A, \gamma_B > 1$. Note that $\sigma^A$, being proportional to $(Z_B\alpha)^2$, can be directly calculated as the Coulomb corrections to $\sigma_1$ with respect to the parameter $Z_A\alpha$, so that it can be represented as

$$\sigma^A = \int_{2m}^{\infty} d\omega \int_{(\omega/\gamma)^2}^{\infty} dQ^2 \left[ \frac{dn_\perp(\omega, Q^2)}{d\omega dQ^2} \sigma_\perp(\omega, Q^2) \right]$$

$$+ \frac{dn_\parallel(\omega, Q^2)}{d\omega dQ^2} \sigma_\parallel(\omega, Q^2)$$

(5)

Here

$$dn_\perp(\omega, Q^2) = \frac{Z_B^2\alpha}{\pi} \left( 1 - \frac{(\omega/\gamma)^2}{Q^2} \right) \frac{d\omega dQ^2}{\omega Q^2},$$

$$dn_\parallel(\omega, Q^2) = \frac{Z_B^2\alpha}{\pi} \frac{d\omega dQ^2}{\omega Q^2}$$

(6)

are the numbers of virtual photons $\gamma^*_\perp, \gamma^*_\parallel$ with the energy $\omega$, the virtuality $-Q^2 < 0$, and the transverse and longitudinal polarizations, respectively. The quantities $\sigma_\perp(\omega, Q^2)$ and $\sigma_\parallel(\omega, Q^2)$ are the Coulomb corrections to the cross sections of the processes $\gamma^*_\perp, \gamma^*_\parallel A \rightarrow e^+e^-A$.

Let us discuss the contributions to $\sigma^A$ of different regions of integration with respect to $\omega$ and $Q^2$.

The leading logarithmic contribution $\propto L^2$ comes from the integration of $\sigma_\perp$ over the region

$$I. \ m \ll \omega \ll m\gamma, \ \ (\omega/\gamma)^2 \ll Q^2 \ll m^2.$$  

The leading correction $\propto L$ comes from the following regions:

$$II. \ Q^2 \sim m^2, \ m \ll \omega \ll \gamma m$$  

$$III. \ Q^2 \sim (\omega/\gamma)^2, \ m \ll \omega \ll \gamma m$$  

$$IV. \ \omega \sim m, \ \ (m/\gamma)^2 \ll Q^2 \ll m^2$$

(7)

Note that the cross section $\sigma_\parallel$ gives logarithmically enhanced contribution only in region II. Therefore, since we are going to keep the terms proportional to $L^2$ or $L$, we can write $\sigma^A$ as

$$\sigma^A = \sigma^A_{\text{as}} + \delta\sigma^A,$$

$$\sigma^A_{\text{as}} = \int_{2m}^{\infty} d\omega \int_{(\omega/\gamma)^2}^{\infty} dQ^2 \left[ \frac{dn_\perp(\omega, Q^2)}{d\omega dQ^2} \sigma_\perp(\infty, Q^2) \right]$$

$$+ \frac{dn_\parallel(\omega, Q^2)}{d\omega dQ^2} \sigma_\parallel(\infty, Q^2),$$

$$\delta\sigma^A = \int d\omega \int_{(\omega/\gamma)^2}^{\infty} dQ^2 \delta\sigma_\perp(\omega, Q^2)$$

(11)

$$\delta\sigma_\perp(\omega, Q^2) = \sigma_\perp(\omega, Q^2) - \sigma_\perp(\infty, Q^2).$$

(12)

The quantities $\sigma_\perp(\infty, Q^2)$ can be calculated within the quasiclassical approximation. Following the method described in detail in Ref. [28], we obtain

$$\sigma_\perp(\infty, Q^2) = \frac{\alpha}{\omega} \Re \int d\varepsilon S_{\varepsilon} \int d\mathbf{r}_1 d\mathbf{r}_2 e^{-i\mathbf{k}\cdot\mathbf{r}}$$

$$\times \left[ \left( 2e \cdot \mathbf{p}_2 + \mathbf{k}\mathbf{e} \right) D_- \right] \left[ \left( 2e^* \cdot \mathbf{p}_1 - \mathbf{k}^*\mathbf{e} \right) D_+ \right],$$

$$D_- = D(\mathbf{r}_2 - \mathbf{r}_1)\varepsilon, \ D_+ = D(\mathbf{r}_1, \mathbf{r}_2)\varepsilon - \omega,$$  

(13)

where $D(\mathbf{r}_2, \mathbf{r}_1)\varepsilon$ is the quasiclassical Green function of the squared Dirac equation, $\varepsilon = (0, 1, 0, 0)$ for $\sigma_\perp$ and $\varepsilon = (0, 0, 0, Q/\omega)$ for $\sigma_\parallel$ in the frame where $\mathbf{k}$ is directed along $z$ axis. Using the explicit expressions for the Green functions from Ref. [28], we obtain the results for these cross sections:

$$\sigma_\perp(\infty, Q^2) = \alpha N \int_0^1 \frac{dy}{y} \frac{1 + 2(1 - 2y\bar{y}) (1 + y\bar{y}Q^2/m^2)}{(1 + y\bar{y}Q^2/m^2)^2},$$

$$\sigma_\parallel(\infty, Q^2) = 4\alpha N \int_0^1 \frac{y\bar{y}Q^2/m^2}{(1 + y\bar{y}Q^2/m^2)^2},$$

$$N = \frac{4(Z_A\alpha)^2}{3m^2} \Re[\psi(1 + iZ_A\alpha) - \psi(1)],$$

$$\bar{y} = 1 - y,$$  

(14)

where $\psi(x) = d\ln \Gamma(x)/dx$. These formulas agree with the result of Ref. [29] if one takes into account the missing factor $y\bar{y}$ in $\sigma_\parallel$ pointed out in Ref. [30]. Substituting Eq. (13) in Eq. (11) and taking the integrals over $\omega$ and $Q^2$, we obtain within the logarithmic accuracy

$$\sigma^A_{\text{as}} = \frac{7(Z_B\alpha)^2 N}{3\pi} \left[ L^2 + \frac{20}{21} L \right].$$  

(15)

We remind that $L = \ln(\gamma_A, \gamma_B) = \ln(\gamma/2)$. The result (15) is in agreement with those obtained in Refs. [30, 31].

Let us pass to the contribution $\delta\sigma^A$. Eq. (12), which was not considered so far. In Ref. [31] it was conjectured that the term $\delta\sigma^A$ can be safely omitted. We show below that this guess is completely wrong. The function $\delta\sigma_\perp(\omega, Q^2)$ in the integrand provides convergence of the integral over $\omega$ in the region $\omega \sim m$. The logarithmically enhanced contribution is given by the region $(m/\gamma)^2 \ll Q^2 \ll m^2$ of integration over $Q^2$. Since
increases, so that numerical results in Refs. [32, 33] were difficult in computations. The difficulties grow as this region the ratio $F$ or our purpose, this difference is not important because in comparison with unity .

In order to calculate the function $G(ZA\alpha)$ it is necessary to know the magnitude of the Coulomb corrections $\sigma_{\gamma\alpha}(\omega)$ in the energy region where the produced $e^-e^-$ pair production is not ultrarelativistic. The formal expression for it, exact in $ZA\alpha$ and $\omega$, was derived in Ref. [32]. This expression has a very complicated form causing severe difficulties in computations. The difficulties grow as $\omega$ increases, so that numerical results in Refs. [32, 33] were obtained only for $\omega < 5$ MeV. In a set of later publications [34, 35, 36, 37] (see also reviews [38, 39]) the magnitude of $\sigma_{\gamma\alpha}(\omega)$ has been obtained for higher values of $\omega$ and several $ZA$. In the high-energy region $\omega \gg m$, the consideration is greatly simplified. As a result, a rather simple form of the Coulomb corrections was obtained in [40] in the leading approximation with respect to $m/\omega$ and in Ref. [28] in the next-to-leading approximation. In Ref. [42], a simple formula, which correctly reproduces the low-energy results and the high-energy limit, was suggested. This “bridging” expression has high accuracy at intermediate energies and differs from the exact result for $\sigma_{\gamma\alpha}(\omega)$ only in the region close to the threshold $\omega = 2m$.

For our purpose, this difference is not important because in this region the ratio $\sigma_{\gamma\alpha}(\omega)/\sigma_{\gamma\alpha}(\infty)$ can be neglected in comparison with unity.

The function $G(Z\alpha)$ is shown in Fig. 1. It is seen that $G(Z\alpha)$ varies slowly from $-6.6$ for $Z = 1$ to $-6.14$ for $Z = 100$ being large for all interesting values of $Z$. The large value of $G$ leads to a big difference between $\sigma^A$ from Eq. (17) and its leading logarithmic approximation $\sigma^A_{LA} = 7(ZB\alpha)^2NL^2/(3\pi)$ even for very large $\gamma$. This statement is illustrated in Fig. 2 where the ratio $\sigma^A/\sigma^A_{LA}$ is shown as a function of $\gamma$ (solid curve). If one omits the contribution $\delta \sigma^A$ and use $\sigma^A_{LA}$, as an approximation to $\sigma^A$, then the contribution of linear in $L$ term becomes much less important, see the dashed curve in Fig. 2. Note that for Pb-Pb collisions at LHC one has

$$\gamma \approx 1.8 \times 10^7$$ and $\sigma^A/\sigma^A_{LA} \approx 0.66$. For Au-Au collisions at RHIC one has $\gamma \approx 2.3 \times 10^4$ and $\sigma^A/\sigma^A_{LA} \approx 0.42$.

For the experiments at SPS [1, 2], the Lorentz factor was $\gamma \approx 200$. Naturally, we can not use the result (17) obtained in the logarithmic approximation in the region $\gamma \lesssim 500$ where the logarithmic correction to $\sigma^A$ becomes larger than the leading term $\sigma^A_{LA}$. However, we can claim that, due to the strong compensation between the leading term and the correction, the Coulomb corrections $\sigma^A$ are much smaller than $\sigma^A_{LA}$ at $\gamma \lesssim 500$. Therefore, this naturally explains why there was no evidence of the Coulomb corrections in the experiments [1, 2].

Let us discuss now the importance of the Coulomb corrections $\sigma^A$ in comparison with the Born cross section $\sigma^0$. The ratio $\sigma^A/\sigma^0$ is shown in Fig. 3. In the next-to-leading approximation for $\sigma^A$ this ratio (solid curve) is small ($\lesssim 5\%$), while the same ratio obtained with $\sigma^A$ approximated by $\sigma^A_{LA}$ reaches $20\%$ at $\gamma \sim 1000$.

To summarize, we have calculated the Coulomb corrections $\sigma^A$ to $e^-e^-$ pair production in the next-to-leading logarithmic approximation. After the account of the next-to-leading term, the magnitude of $\sigma^A$ becomes small in comparison with the Born cross section, in contrast to the leading term $\sigma^A_{LA}$. The big difference between our result and previously suggested one has a simple explanation. The latter was based on the use of the high-energy asymptotics for the Coulomb corrections.
The ratio $\sigma^A/\sigma^0$ (solid curve) as a function of $\gamma$ for $Z_A = 82$. Dashed curve shows the ratio $\sigma^A_{\Lambda}/\sigma^0$.

![Graph showing the ratio of photoproduction cross sections]

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