Reliability Analysis of a Solid Timber Column Subjected to Axial and Lateral Loading

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Abstract- In this paper, a reliability analysis of a solid timber column of square cross section subjected to axial and lateral loading in accordance with the design requirements of Eurocode 5 is carried out. Compression and bending were the two failure criteria considered in the reliability investigation. The First Order Reliability method was employed to solve the limit state functions formed from the two failure criteria and was coded in MATLAB for quick estimation of the reliability indices. The results obtained showed that both the load and slenderness ratios have effects on the reliability of a solid timber column. The results of the sensitivity analysis carried out on the stochastic variables showed that the reliability indices decreased with increase in slenderness ratio for 3m, 4m and 5m length of column considering both the compression and bending failure modes and decreased with increase in load ratio for 3m, 4m and 5m length of column considering both the compression and bending failure modes. The reliability indices also decreased with increase in length of column mode and decreased with increase in \( L/b \) ratio considering bending failure mode. The reliability indices were also found to decrease with increase in load ratio for varying values of axial and lateral loads at constant slenderness ratio and length of column considering compression and bending failure modes. The choice of adequate and suitable dimensions having a lower slenderness ratio will enhance the reliability of the column.

Keywords- Reliability analysis, solid timber column, Eurocode 5, failure criteria, slenderness ratio

1 INTRODUCTION

All civil engineering structures should serve the intended purpose for which they were designed for without failing (Afolayan, 2005; Abubakar, 2006; Akindahunsi and Afolayan, 2009; Ranganathan, 1990). The use of partial safety factor in the conventional design equations is not a guarantee for structural safety. This is because structural problems occurring in real world are stochastic rather than deterministic (Afolayan, 2002; Abejide, 2012). The use of conventional safety factors may lead to over-design or under-design of structural members or component due to lack of knowledge of the actual structural loadings (Afolayan & Opeyemi, 2008). The violation of ultimate and serviceability limit states of civil engineering structures may result to loss of lives and damage of properties (Sule, 2011). Consequently, there is a need for accurate determination of limit state to enhance efficient design.

However, attaining a limit state is always a difficulty due to uncertainties inherent in the design parameters such as material strength and geometrical properties. Structural reliability assessment therefore becomes a task of paramount importance. Fortunately, structures only rarely fail in a serious manner, but when they fail, it is often due to causes not directly related to the predicted normal loading or strength. Other causes such as human error, negligence, poor workmanship or neglected loading are most often involved (Melchers et al., 1999). To a large extent, these factors are foreseeable and predictable; their occurrence might be considered as the occurrence of imaginable events. However, not all possible reasons for structural failure are always imaginable (Ditlevsen, 1982a). Example of imaginable event includes the collapse of Tay Bridge in 1879 due mainly to underestimation of wind loadings in storm condition and Tacoma narrow bridge in 1940 due to wind excitation of the deck (Sibly and Walker, 1977).

The task of the structural engineer is to design and maintain the structure so that failed state is differed and in this task, he faces the problem inherent in the variability of engineering materials. However, a probabilistic approach always provides a rational way of dealing with such uncertainties that are inherent in structures by using statistical approach. Probabilistic concept may not provide solutions to all issues of structural uncertainties, but it has helped immensely in the reliability assessment of many civil engineering facilities (Abejide, 2014; Abubakar & Edache, 2007). In this paper, the reliability analysis of a solid timber column of square cross section subjected to both axial and horizontal loading is carried out in accordance with the design requirements of Eurocode 5 using First Order Reliability procedure. The limit state functions derived from the failure conditions in compression and bending were solved to obtain the reliability indices using a MATLAB code.

2 DERIVATION OF LIMIT STATE FUNCTIONS

The limit state functions are derived in accordance with the design requirements of Eurocode 5 for timber columns. The timber column (Fig. 1) considered in this study is two hinged with a square cross-section subjected to axial and lateral loading.

Fig. 1: A pin ended square timber column subjected to axial and lateral loadings

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2.1 Development of Limit State Equation for Compression Failure Condition

The design compressive stress in column parallel to grain is given by:

\[ \sigma_{c,d} = \frac{Q_1}{A} = \frac{Q_1}{b^2} \]  

(1)

Where: \( \sigma_{c,d} \) = Design stress, \( A \) = cross-sectional area, \( b \) = cross sectional dimension

\( Q_1 \) = axial load

\( Q_1 = P_1(1.4\alpha_a + 1.6) \)  

(2)

Where: \( \alpha_a \) = axial load ratio

Therefore, the failure condition in compression is given by:

\[ g(x) = f_{c,d} - \sigma_{c,d} \leq 0 \]  

(3)

According to Eurocode 5,

\[ f_{c,d} = \frac{K_{mod} f_{c,k}}{\gamma_m} \]  

(4)

Where: \( f_{c,d} \) = compressive strength parallel to the grain, \( K_{mod} \) = modification factor that accounts for the effect of the duration of load and moisture content, \( Q_1 \) = short term axial load.

\( \gamma_m \) = partial safety factor for the material property based on Eurocode 5, \( f_{c,k} \) = Characteristic value of the compressive strength.

Substituting equations (1) and (4) in equation (3) yields:

\[ g(x) = \frac{K_{mod} f_{c,k}}{\gamma_m} - \frac{Q_1}{b^2} \]  

(5)

From mechanics of engineering materials,

\[ \lambda^2 = \frac{L^2}{r^2} \]  

(6)

\[ r^2 = \frac{I}{A} \]  

(7)

Where: \( I = \frac{b^4}{12} \) and \( A = b^2 \)

Therefore,

\[ r^2 = \frac{b^2}{12} \]  

(8)

Substituting for \( r^2 \) in equation (6) yields:

\[ \lambda^2 = \frac{12L^2}{b^2} \]  

(9)

Therefore,

\[ \lambda = \frac{3.464L}{b} \]  

(10)

Where: \( \lambda \) = derived slenderness ratio of a solid square timber column

Multiplying the second term of equation (5) by \( 3.464L \) and dividing by the same and substituting for \( \lambda \) in equation (10) transforms equation (5) to:

\[ g(x) = \frac{K_{mod} Q_{m}}{\gamma_m} - \frac{Q_1 \lambda}{3.464bL} \]  

(11)

Using equation (2), equation (11) becomes:

\[ g(x) = \frac{K_{mod} Q_{m}}{\gamma_m} - \frac{P_1(1.4\alpha_a + 1.6)\lambda}{3.464bL} \]  

(12)

\( P_1 \) = short term imposed axial and load

Equation (12) represents the limit state function for a solid square timber column subjected to an axial load.

2.2 Development of Limit State Function for Bending Failure Condition

The design bending stress parallel to grain is given by:

\[ \sigma_{m,d} = \frac{M}{Z} \]  

(13)

Where:

\[ M = \frac{Q_2L^2}{8} \]  

(14)

and

\[ Z = \frac{b^3}{6} \]  

(15)

Using equations (14) and (15), equation (13) becomes:

\[ \sigma_{m,d} = \frac{0.75Q_2L^2}{b^2} \]  

(16)

Where: \( Q_2 \) = short term lateral load

Similarly,

\[ Q_2 = P_2(1.4\alpha_a + 1.6) \]  

(17)

According to Eurocode 5, the design bending strength parallel to grain is given by:

\[ f_{m,d} = \frac{K_{mod} f_{m,k}}{\gamma_m} \]  

(18)

Where: \( f_{m,k} \) = characteristic value of the bending strength

The bending failure condition is given by:

\[ g(x) = f_{m,d} - \sigma_{m,d} \]  

(19)
Using equations (16) and (18), equation (19) becomes:

\[ g(x) = \frac{K_{mod} f_{m,k}}{\gamma_m} - \frac{0.75Q_2L^2}{b^3} \]  (20)

Again, substituting for \( Q_2 \) using equation (17) transforms equation (20) to:

\[ g(x) = \frac{K_{mod} f_{m,k}}{\gamma_m} - \frac{0.75P_1(1.4\alpha + 1.6)L}{b^3} \]  (21)

Multiplying the second term of equation (20) by 3.464L and dividing by the same and using equation (10) transforms equation (21) to:

\[ g(x) = \frac{K_{mod} f_{m,k}}{\gamma_m} - \frac{0.75P_1(1.4\alpha + 1.6)L}{3.464b^2} \]  (22)

Where: \( P_1, \alpha \) = short term imposed lateral load and lateral load ratio respectively.

Equation (22) is the limit state equation for a solid square timber column subjected to lateral load.

3 First Order Reliability Method

The First Order Reliability method is an approximate method of computing the probability failure of a system with random variables. The vector of random variables \( X=(X_1,X_2,...,X_n) \) are the basic variables with joint probability function:

\[ F_X(X) = P\left(\bigcap_{i=1}^{n}\{X_i \leq x_i\}\right) \]  (23)

For First Order Reliability Method, \( F_X(X) \) is continuous and differentiable with respect to the basic variables implying that the probability density of \( F_X(X) \) exists. The performance function, \( g(X) \) of a structure at a limit state is usually a function of the basic variables. Mathematically, the performance function \( g(X) \) represents the limit state and \( g(X)=0 \) represents failure domain.

First order approximation to probability of failure is given by:

\[ P_f = P(X \in F_g) = P(g(X) \leq 0) = \int_{D_g} f_g = \phi(-\beta)g(X) \leq 0 \]  (24)

Where: \( \beta = \) reliability index which represents the minimum distance between the origin and the failure surface and it is given by:

\[ \beta = \min \|X\| \quad \text{for} \quad \{X : g(X) < 0\} \]  (25)

Where: \( \beta = \) reliability index.

The statistics of the basic variables are shown in Table 1. The characteristic values of bending and compressive strength parallel to grain used in this study correspond to softwood specie of Strength Class C24 obtained from EN 338 (2009).

4 Results and Discussion

Reliability indices were obtained using First Order Reliability Method coded in MATLAB and the various plots of reliability indices against varying load ratios are as shown in Figures 2 to 4 for compression failure mode; Figures 5 to 7 for bending failure mode; Figures 8 to 10 at varying load ratio and at constant slenderness ratio of 34.5, 40 and 45.5.

From the various plots obtained from the reliability analysis, it can be seen that:

- The reliability index decreased with increase in slenderness ratio and load ratio for 3m, 4m and 5m length of column considering compression and bending failure modes at (Figures 2 to 7). This is because the increase in slenderness ratio of a column makes a column vulnerable to failure by buckling due to reduced column stiffness and this reduces the reliability index.
- Reliability index decreased with increase in length of column considering bending failure mode at constant slenderness ratio of 34.5 (Figures 5 to 7).
- Reliability index decreased with increase in \( L/b \) ratio considering bending failure mode at constant slenderness ratio (Figures 8 to 10).
- Both the load and slenderness ratios have effects on the reliability of a solid timber column (Figures 2 to 10).
- The lower value of slenderness ratio yielded a higher value of reliability index (Figures 2 to 7).
- The vertical load ratio has no effect on the reliability indices of a timber column in compression (Figures 2 to 4).
- The lateral load ratio has no effect on the reliability indices of a timber column in compression (Figures 5 to 7).
- The reliability index decreased with increase in load ratio for varying values of axial and lateral loads at constant slenderness ratio and length of column considering compression and bending failure modes.

![Table 1. Statistical Characteristics of the Basic Variables (Benu & Sule, 2012)](image)

| S/N | Parameter | Probability Distribution | Mean | Standard Deviation | Coefficient of Variation |
|-----|-----------|--------------------------|------|--------------------|-------------------------|
| 1   | \( P_t \) | Gumbel                   | 65,000N | 1950N              | 0.030                  |
| 2   | \( P_t \) | Gumbel                   | 3.25N/mm m m | 0.975N/mm m m | 0.30                |
| 3   | \( K_{mod} \) | Lognormal                | 0.90 | 0.135              | 0.15                |
| 4   | L         | Normal                   | 300mm | 30mm               | 0.01                |
| 5   | \( \alpha \) | Normal                   | 300mm | 3mm                | 0.01                |
| 6   | \( f_{m,k} \) | Lognormal                | 24N/mm m m | 3.6N/mm m m | 0.15                |
| 7   | \( f_{k} \) | Lognormal                | 21N/mm m m | 3.15N/mm m m | 0.15                |
| 8   | \( \gamma_m \) | Lognormal                | 1.30 | 0.195              | 0.15                |
| 9   | \( L/b \) | Varying                   | Varying | Varying            | 0.32                |
| 10  | \( \lambda \) | Normal                   | Varying - | Varying            | 0.115               |
| 11  | \( \alpha_v \) | Varying                   | Varying | Varying            | 0.30                |
| 12  | \( \alpha_n \) | Varying                   | Varying | Varying            | 0.30                |

Figure 1. Probability density function of \( g(X) \) for different values of \( \alpha \), \( \gamma_m \) and \( \lambda \).
(Figures 11 to 12). This trend is expected because the increase in values of the vertical and lateral loads reduced the load bearing capacity of the column thereby reduction of values of reliability indices.

- The timber column is safe considering the target reliability index of 2.5 (Melchers, 1999) required for timber members for all the failure modes and geometric properties considered. However, the reliability of the timber column can be improved if adequate and suitable dimensions are chosen to have a lower slenderness ratio (Benu & Sule, 2012).

![Fig. 2: Reliability index - load ratio, 3m length of column (compression)](image)

![Fig. 3: Reliability index - load ratio, 4m length of column (compression)](image)

![Fig. 4: Reliability index - load ratio, 5m length of column (compression)](image)

![Fig. 5: Reliability index - load ratio, 3m length of column (bending)](image)

![Fig. 6: Reliability index - load ratio, 4m length of column (bending)](image)

![Fig. 7: Reliability index - load ratio, 5m length of column (bending)](image)

![Fig. 8: Reliability index - load ratio, slenderness ratio = 34.5, column length = 3m (bending)](image)

![Fig. 9: Reliability index - load ratio, slenderness ratio = 40, column length = 3m (bending)](image)
5 CONCLUSION
The results of reliability analysis of a solid square timber column for varying load ratios and varying slenderness ratios for 3m, 4m and 5m length of column for both compression and bending failure modes have been presented. The results of the reliability analysis showed that the reliability index decreased with increase in slenderness ratio and load ratio for 3m, 4m and 5m length of column considering compression and bending failure modes. It was shown that the reliability index increased with increase in column length for compression failure mode at constant slenderness ratio.

It was found that the reliability index decreased with increase in length of column considering bending failure mode at constant slenderness ratio. It was also found that the reliability index decreased with increase in L/b ratio considering bending failure mode at constant slenderness ratio. It was also found that both the load and slenderness ratios have effects on the reliability of a solid timber column. It was found that the lower value of slenderness ratio yielded higher values of reliability indices. It was also found that the vertical load ratio has no effect on the reliability indices of a timber column in bending and the lateral load ratio has no effect on the reliability indices of a timber column in compression. It was also found that the reliability index decreased with increase in load ratio for varying values of axial and lateral loads at constant slenderness ratio and length of column considering compression and bending failure modes.

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