Algebraic thinking characteristics of eighth grade junior high school students based on Superitem Test of SOLO model

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Abstract. The study aimed to know the characteristic of algebraic thinking of eighth grade junior high school students based on identification result using Superitem test of SOLO model. This study used descriptive qualitative method. The subject of this study is eighth grade junior high school students consisting of 32 students. The data collection method used in this study is superitem test of SOLO model and interview. The results showed there were differences of algebraic thinking characteristics of students based on levels of Superitem test of SOLO model. Subjects at the unistructural level have algebraic thinking characteristics as: arithmetics generalizations. Subjects at the multistructural level have algebraic thinking characteristics as: representation, mathematics language, mathematics modeling, and arithmetic generalization. Subjects at the relational level have algebraic thinking characteristics as: mathematics language, mathematics modeling, representation, arithmetic generalization, and problem solving. Subjects at the extended abstract level have algebraic thinking characteristics as: representation, mathematics language, mathematics modeling, arithmetic generalization, problem solving, and quantitative reasoning.

1. Introduction

One of the scopes of topics in mathematics that can be used in daily life and capable of helping humans in advancing science and technology is algebra. Algebra is a very important material to be mastered by students because both implicit and explicit algebra are used in the activities of daily life [1]. The concept of algebra is used in all areas of mathematics [2]. Based on this, it appears that algebra cannot be separated from mathematics, in mathematics, it must contain algebraic topics, even in other topics in mathematics it must contain algebra, as in geometry, although geometry discusses the science of either plane or field space, still in it will contain algebra in problem solving. Given that algebra is a very important topic, it becomes a necessity for students to be able to think algebraically in mathematics learning. Algebraic thinking is a crucial and fundamental element of mathematical thinking and reasoning. It initially involves recognising patterns and general mathematical relationships among numbers, objects and geometric shapes [3]. Algebraic thinking can be used as a cognitive support of students in learning algebra in school [4]. Algebraic thinking becomes an important thing in learning mathematics that will prepare students in training critical thinking skills in order to be able to participate fully in the life of society and to be able to achieve better learning in algebra.

Kriegler explains the components of algebraic thinking, they are 1) the development of mathematical thinking tools, and 2) the study of fundamental algebraic ideas. The mathematical thinking tool consists
of three characteristics: a) problem solving skills consists of using problem solving strategies and exploring multiple approaches/ solutions, b) representation skills consists of displaying relationships visually, symbolically, numerically, verbally; translating among different representations; interpreting information within representations; and c) quantitative reasoning skills consists of analyzing problems to extract and quantify essential features and inductive and deductive reasoning [5]. The fundamental algebraic ideas is algebra as: a) generalized arithmetics consists of conceptually based computational strategies; ratio and proportion; estimation, b) algebra as the language of mathematics consists of meaning of variables and variable expressions; meaning of solutions; understanding and using properties of the number system; reading, writing, manipulating numbers and symbols using algebraic conventions; using equivalent symbolic representations to manipulate formulas, expressions, equations, inequalities; and c) algebra as a tool for functions and mathematical modeling consists of seeking, expressing, generalizing patterns and rules in real-world contexts; representing mathematical ideas using equations, tables, graphs, or words; tasking with input/output patterns; developing coordinate graphing skills [5]. On the other hand, Lew describes six types of mathematical thinking, they are: generalization, abstraction, analytical thinking, dynamic thinking, modeling, and organization [6].

But in fact, the topic of algebra itself becomes a dislike topic by students, even becoming one of the feared mathematics topics by students at school. This was also stated by Radford in his research that algebra is one of the most frightening branches of school mathematics [7]. Because students have difficulties in understanding abstract concepts in algebra. The following results of previous studies related to the student’s difficulties in algebra, that difficulties was experienced by seventh grade junior high school students in solving algebra problems related to the concept and principles, they are a) in understanding of concept, medium and low ability students still have difficulty in using symbols to represent concepts, and it did not happen to the high ability students, b) errors made by students in understanding of the concept is when students have to group similar term and replace variables with numbers, and c) in the mastery of principles, they still have difficulty in applying the role of principles in mathematics, it was still happened to to the high, medium, and low ability students, because they did not understand mathematics concepts in previous lessons [8].

Therefore, it is required to be more attention to know the students’ thinking process in depth, particularly the algebraic thinking process. One way to know the students’ thinking process in depth is using the Superitem test of SOLO model (Structure of Observed Learning Outcome). The SOLO model can be used in assessing students' cognitive outcomes in several skills and scope of mathematics including statistics, algebra, probability, geometry, error analysis and problem solving [9]. According to Collis, Romberg & Jurdak, Lam & Foong, Wilson & Inventosh, Superitem test of SOLO model is a test developed based on the SOLO taxonomy used as an alternative assessment tool to monitor the students' cognitive developmental abilities in solving mathematics problems consisting of four levels, i.e. Unistructural level, Multistructural level, Relational level, and Extended Abstract level [9]. The taxonomic model requires the ability of students to provide some alternative answers or solutions and can relate some answers or solutions [10]. Therefore, the researcher want to know the process or algebraic thinking the characteristic of eighth grade junior high school students in solving Superitem test of SOLO model.

2. Research Method
This study used descriptive qualitative method. This study is located at one junior high school in Bandung, West Java. Subjects of this study are eighth grade students consisting of 32 children in one class at the school. The data collection method used in this study is 1) Superitem test of SOLO model consisting of four questions with each question consisting of four item level questions (unistructural level, multistructural, relational, and extended abstract), and 2) interview. The four problems use algebraic thinking indicator according to Kriegler, with the first question being a matter related to generalization, the second question is a matter related to mathematical modeling, the third question is a matter related to problem solving, and the fourth question is a matter related to quantitative reasoning. Here is one of the four questions given to students.
Dad bought Hasan a triangular toy made of wood that could be disassembled. The triangle is an equilateral triangle with a side length of 10 cm. Then, Hasan tried to arrange the triangle in a row to form a rectangle like in the picture below. If the number of triangles is 3, the perimeter of the toy is 50 cm.

![Figure 1](image1.png)

**Figure 1.** Triangular toys.

Unistructural Level:
a. How many the formed toys perimeter if the number of triangles assembled is 4?

Multistructural Level:
b. How many the formed toys perimeter if the number of triangles assembled is 6 and 15?

c. How many the formed toys perimeter if the number of triangles assembled is x? Then, write a formula to determine the formed toys perimeter by considering \( p \) is the perimeter of the toys.

d. From the formula, apply to solve how many the number of triangles needed if the toys perimeter is 100 cm.

Extended Abstract Level:
e. Can you form a formula to express the formed perimeter \( (r) \) of any length of toy \( (s) \) if the triangular toy forms another quadrilateral like the image below.

![Figure 2](image2.png)

**Figure 2.** Triangular toys with Extended Abstract Level.

3. Results and Discussion

Based on the analysis that has been done by the researcher on the result of the students' task in solving the Superitem test of SOLO Model, eight students are obtained, they are two students are at unistructural level, two students are at multistructural level, two students are at relational level, and two students are at extended abstract level. These eight students who as the subject of study were interviewed by the researchers.

From each item in each question, students are identified the algebraic thinking characteristics based on algebraic thinking characteristics according to Kriegler above. The following presents the algebraic thinking characteristics experienced by students in solving the Superitem test of SOLO model in Table 1.

| Level | Problem Solving | Representation | Quantitative Reasoning | Arithmetic Generalization | Mathematics Language | Modeling/Function |
|-------|-----------------|----------------|------------------------|--------------------------|---------------------|------------------|
| U     | -               | -              | -                      | 62.5%                    | -                   | -                |
| M     | -               | 5.5%           | -                      | 56.25%                   | 21.88%              | 21.88%           |
| R     | 30%             | 12.5%          | -                      | 30%                      | 57.5%               | 57.5%            |
| E     | 54.17%          | 16.67%         | 25%                    | 30%                      | 54.17%              | 54.17%           |

**Table 1.** Algebraic thinking characteristics of students based on superitem test of SOLO model
Table 1 above shows that that 62.5% of students at a structural level (U) show characteristics as arithmetic generalizations. This is obtained from the number of students who answered correctly as shown in table 4.1 above, it turns out that all level U students showed the same characteristics so that 62.5% showed characteristics as arithmetic generalizations. Furthermore, the table also shows that not all levels provide overall algebraic thinking characteristics such as at a unistructural level (U) or students who reach level of U only show algebraic thinking as arithmetic generalizations. Here is one of the multistructural and extended abstract subject answers presented in below.

![Multistructural answer](image1)

![Extended Abstract answer](image2)

**Figure 3.** Students’ answer for number 1, a) multistructural subject, and b) extended abstract subject

In Figure 3 above, it shows that there are differences in algebraic thinking characteristics of multistructural level and extended abstract level subjects. Subject at the multistructural level shows algebraic thinking characteristics as a modeling/ function in solving the question of the number 1 point of a and b because these multistructural subject begins to search the pattern of formed toys perimeter, but the subject can’t generalize or represent the situation into algebraic forms. Meanwhile, the extended abstract subject shows the algebraic thinking characteristics as a modeling/ function in solving the problem of the number 1 point of a and b because extended abstract subject generalizes the pattern with symbolic formulas which in this case, these subject uses arithmetic formulas. Then, another characteristics is mathematics language, because extended abstract subject uses algebra expressions and standard notation. In addition, at the next level question, the extended abstract subject can solve it well, unlike multistructural subjects. At relational and extended abstract level in question of number 1, the extended abstract subject also shows the algebraic thinking characteristics as modeling/ function because extended abstract subject is able to find, use, generalize rules in algebra form i.e. equations. The characteristics of algebraic thinking as a mathematics language is shown when the extended abstract subject can use variables as the known elements and manipulate algebra into equations. Furthermore, the discussion of algebraic thinking characteristics of students based on SOLO model level is described as follows.

a. Algebraic thinking characteristics of unistructural level subject.

![Unistructural answer](image3)

**Figure 4.** Students’ answer of unistructural for number 1.
Figure 4 above shows students at unistructural level have algebraic thinking as arithmetic generalizations. This shows also that students are still relatively low because students at this level only answer based on simple methods/ calculation strategies. They have not been able to generalize or represent the situation well into algebraic forms. This is in line with the results of previous studies by Lian, Yew, & Idris which states that low-ability students (unistructural and multistructural) show their more ability in the calculation method where the given task is clearly shown and can be understood sequentially, but they failed to link it to the linear equation given in the question due to a lack of understanding of the algebraic concept, especially the unknown and linear equations [9].

b. Algebraic thinking characteristics of multistructural level subject

Based on the explanation above, the multistructural subject shows the algebraic thinking characteristics as a modeling/function in solving the problem of the numbers 1 of point a and b. In the second and third questions, multistructural subject also shows modeling/function characteristics, because these subject can state a problem situation in matter into mathematics model, i.e. a linear equation. It also means the multistructural subject shows characteristics as a mathematics language, because these uses variables. In addition, multistructural subject shows the characteristics of representation, because in second question these subject can make a form of representational of given information on the task, i.e. tables and graphs. Then, on the fourth question, multistructural subject shows the algebraic thinking characteristics as arithmetics generalizations, because in solving unistructural and multistructural questions is based on a conceptual calculation strategy.

c. Algebraic thinking characteristics of relational level subject.

Figure 5 above shows the relational subject can search, find, and generalize the pattern into algebraic form and state the problem situation into the mathematics equations. This also means that the relational subject shows the characteristics as a mathematics language, because these uses variables in solving it. In the second question, the relational subject shows the algebraic thinking characteristics as representation, because this subject makes the representational form of given information on the task, the forms of representation are the table and the graph. Another algebraic thinking characteristics shown by relational subject is arithmetic generalization, because the relational subject can read, write, manipulate numbers and symbols using algebraic rules. Also, in solving the problem, the relational subject also shows the characteristics of problem solving. From these characteristics also shows that students are able to generalize and represent problem situations into algebraic forms. This is similar with the results of previous research by Lian, Yew, & Idris which states that relational students appear to be able to find, generalize into linear equations and identify linear relationships between variables [9].
d. Algebraic thinking characteristics of extended abstract level subject.

![Figure 6. Students’ answer for number 2b](image)

Based on explanation on the figure 3 above, the extended abstract subject shows the algebraic thinking characteristics as modeling/function and mathematics language. It was seen in completing the second to the fourth question, because the extended abstract subject can state the problem situation into the mathematics model, i.e. the equations. In the second question, the extended abstract subject shows the algebraic thinking characteristics as representation. It was seen at figure 6 above, these subject makes the representational form of given information on the task, i.e. table and the graph. Also, the extended abstract subject shows algebraic thinking characteristics as problem solving and quantitative reasoning. As problem-solving, the extended abstract subject reveals a problem-solving strategy and solves the problem on any number from unistructural level until extended abstract level. Then, quantitative reasoning, because the extended abstract subject uses the reasoning in solving the second, third, and fourth question. Students at extended abstract level show the highest achievement of thinking because of the ability of the narrative that was shown in solving problems. This was stated by Levins and Mooney that students who show the highest level of thinking are students who reach an extended abstract level because students are able to integrate and generalize all relevant concepts and ideas that are unknown from the problem, and they also show reasoning ability and make hypotheses for the given situation [11,12].

4. Conclusion
Based on the discussion described above, it can be concluded that subjects at the unistructural level in solving the Superitem test of SOLO model have algebraic thinking characteristics as arithmetic generalization. Algebraic thinking characteristics of subjects at the multistructural level is modeling/ function, mathematics language, representation, and arithmetic generalizations. Then, subjects at the relational level is modeling/ function, mathematics language, representation, arithmetic generalization, and problem solving. And last, subjects at the extended abstract level in solving the Superitem test of SOLO model have algebraic thinking characteristics as modeling/ function, mathematics language, representation, arithmetic generalization, problem solving, and quantitative reasoning.

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