NUCLEAR MATTER IN RELATIVISTIC MEAN FIELD THEORY
WITH ISOVECTOR SCALAR MESON

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Abstract:

Relativistic mean field (RMF) theory of nuclear matter with the isovector scalar mean field corresponding to the $\delta$-meson [$a_0(980)$] is studied. While the $\delta$-meson mean field vanishes in symmetric nuclear matter, it can influence properties of asymmetric nuclear matter in neutron stars. The RMF contribution due to $\delta$-field to the nuclear symmetry energy is negative. To fit the empirical value, $E_s \approx 30\,\text{MeV}$, a stronger $\rho$-meson coupling is required than in the absence of the $\delta$-field. The energy per particle of neutron matter is then larger at high densities than the one with no $\delta$-field included. Also, the proton fraction of $\beta$-stable matter increases. Splitting of proton and neutron effective masses due to the $\delta$-field can affect transport properties of neutron star matter.

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RMF models of nuclear matter [1] play an important role in dense matter calculations, where relativistic effects increase with density. They represent a relativistic Hartree
approximation to the one-boson exchange (OBE) theory of nuclear interactions, with appropriately readjusted couplings to fit the saturation properties of nuclear matter. In the OBE description of nucleon-nucleon scattering, isoscalar mesons, $\sigma$, $\eta$, and $\omega$, and isovector mesons, $\pi$, $\rho$, and $\delta$, are involved.

In the RMF approximation, mean fields of pseudoscalar mesons $\eta$ and $\pi$ vanish in normal nuclear matter. The standard RMF model of nuclear matter [1] includes isoscalar mesons $\sigma$ and $\omega$, and the isovector $\rho$-meson. However, no $\delta$-meson contribution is present, although for asymmetric nuclear matter the density to which the $\delta$-field can couple does not vanish, $<\bar{\psi}\tau_3\psi> \neq 0$.

The $\delta$-field is not expected to play an important role for nuclei, whose isospin asymmetry is not large. Also, the short range of the $\delta$-meson exchange justifies neglecting its contribution at saturation density. However, for strongly isospin-asymmetric matter at high densities in neutron stars the contribution of the $\delta$-field should be considered. It is our aim to include the $\delta$-meson into the RMF model and study consequences of this generalization of the RMF theory. To our knowledge, such an extension of RMF models has not been studied.

We assume here that all the meson fields have the Yukawa couplings to nucleons. The interaction lagrangian reads

$$L_{\text{int}} = g_\sigma \bar{\psi}\gamma_\mu \gamma^\mu \psi - g_\omega \bar{\psi}\gamma_\mu \gamma^\mu \gamma_5 \psi - \frac{1}{2} g_\rho \bar{\psi}\gamma_\mu \gamma^\mu \tau \psi + g_\delta \bar{\psi}\gamma_5 \tau \psi. \quad (1)$$

Here $\vec{\delta}$ is the isovector scalar field of the $\delta$-meson. The free field lagrangians for $\psi, \sigma, \omega$, and $\rho$ fields are the same as in Ref.[1]. For the $\delta$-field we use the standard lagrangian

$$L_\delta = \frac{1}{2} \partial_\mu \vec{\delta} \partial^\mu \vec{\delta} - \frac{1}{2} m_\delta^2 \vec{\delta}^2. \quad (2)$$

For the $\sigma$-field we adopt the potential energy term of Boguta and Bodmer [2],

2
\[ U(\sigma) = \frac{1}{3} bm\sigma^3 + \frac{1}{4}c\sigma^4, \]  

(3)

where \( m \) is the bare nucleon mass.

The relevant components of meson fields are \( \sigma, \omega_0, \rho_0^{(3)} \), and for the \( \delta \)-field the isospin component \( \delta^{(3)} \). All remaining components vanish, in particular \( \delta^{(1)} = \delta^{(2)} = 0 \). The values of mean fields in the ground state are determined by proton and neutron densities. The field equations for vector meson fields give

\[ \bar{\omega}_0 = \left( g_\omega / m_\omega^2 \right) n_B \]  

and

\[ \bar{\rho}_0^{(3)} = \left( g_\rho / m_\rho^2 \right) (2x - 1)n_B, \]  

where \( n_B \) is the baryon density and \( x = n_P / n_B \) is the proton fraction. The field equations for scalar fields are non-trivial:

\[ m_\sigma^2 \bar{\sigma} + \frac{\partial U}{\partial \sigma} = g_\sigma (n_P^s + n_N^s), \]  

(4)

and

\[ m_\delta^2 \bar{\delta}^{(3)} = g_\delta (n_P^s - n_N^s). \]  

(5)

Here \( n_P^s \) and \( n_N^s \) is, respectively, proton and neutron scalar density:

\[ n_i^s = \frac{2}{(2\pi)^3} \int_0^{k_i} \frac{d^3k}{\sqrt{k^2 + m_i^2}}, \quad i = P, N. \]  

(6)

In Eq.(6) \( m_P \) and \( m_N \) is, respectively, proton and neutron effective mass,

\[ m_P = m - g_\sigma \bar{\sigma} - g_\delta \bar{\delta}^{(3)}, \]  

(7)

and

\[ m_N = m - g_\sigma \bar{\sigma} + g_\delta \bar{\delta}^{(3)}. \]  

(8)

The energy density of uniform nucleon matter is
\[
\epsilon = \frac{2}{(2\pi)^3} \left( \int_0^{k_P} d^3k \sqrt{k^2 + m_P^2} + \int_0^{k_N} d^3k \sqrt{k^2 + m_N^2} \right) + \frac{1}{2} C_\sigma^2 n_B^2 + \\
+ \frac{1}{2} C_\sigma^2 \frac{[m - \frac{m_P + m_N}{2}]^2}{2} + U(\bar{\sigma}) + \frac{1}{8} C_\rho^2 (2x - 1)^2 n_B^2 + \frac{1}{8} C_\delta^2 (m_N - m_P)^2. \tag{9}
\]

The model parameters in the isoscalar sector, \(C_\sigma^2 \equiv g_\sigma^2/m_\sigma^2, C_\omega^2 \equiv g_\omega^2/m_\omega^2, \bar{b} \equiv b/g_\sigma^3\), and \(\bar{c} \equiv c/g_\sigma^3\), are adjusted to fit the saturation properties of symmetric nuclear matter, the saturation density \(n_0 = 0.145 fm^{-3}\), the binding energy \(w_0 = -16 MeV\) per nucleon, and the compressibility modulus \(K_V \approx 280 MeV\). The fourth parameter, e.g. \(\bar{c}\), can be used to measure stiffness of the equation of state of symmetric nuclear matter. In the following we use two sets of parameters which reproduce the saturation properties but differ at higher densities. The soft equation of state is specified by the parameters \(C_\sigma^2 = 1.582 fm^2, C_\omega^2 = 1.019 fm^2, \bar{b} = -0.7188, \) and \(\bar{c} = 6.563\). For the stiff equation of state the parameters are \(C_\sigma^2 = 11.25 fm^2, C_\omega^2 = 6.483 fm^2, \bar{b} = 0.003825, \) and \(\bar{c} = 3.5 \times 10^{-6}\).

In the spirit of RMF models, the parameters \(C_\rho^2 \equiv g_\rho^2/m_\rho^2\) and \(C_\delta^2 \equiv g_\delta^2/m_\delta^2\) of the isovector sector should be constrained to fit the nuclear symmetry energy, \(E_s = 31 \pm 4 MeV\) [3]. In terms of the model parameters, the symmetry energy is

\[
E_s = \frac{1}{8} C_\rho^2 n_0 + \frac{k_0^2}{6 \sqrt{k_0^2 + m_0^2}} - C_\delta^2 \frac{m_0^2 n_0}{2(k_0^2 + m_0^2)(1 + C_\delta^2 A(k_0, m_0))}, \tag{10}
\]

where

\[
A(k_0, m_0) = 4 \left( \frac{2\pi}{3} \right)^3 \int_0^{k_0} \frac{d^3 pp^2}{(p^2 + m_0^2)^{3/2}} \tag{11}
\]

is a function of the Fermi momentum, \(k_0 = k_P = k_N\), and the effective mass, \(m_0 = m_P = m_N\), of symmetric nuclear matter at saturation density. The constraint (10) gives \(C_\rho^2\) as a function of \(C_\delta^2\), \(C_\rho^2 \equiv C_\rho^2(C_\delta^2)\), which is shown in Fig.1. Hence by demanding to fit the bulk properties of nuclear matter we are not able to fix independently \(C_\rho^2\) and \(C_\delta^2\). Instead,
we explore here a range of values of $C^2_\delta$ corresponding to the Bonn potentials A, B and C [4]. The maximum value $C^2_\delta = 2.6 fm^2$ corresponds to the parameter $g_\delta = 8.0$ of the Bonn potential C [4].

The fact that the constraint (10) gives $C^2_\rho$ as a monotonically increasing function of $C^2_\delta$ plays a crucial role in our analysis. The nuclear symmetry energy, Eq.(10), is derived in the RMF approximation which neglects exchange contributions. Inclusion of the Fock terms in relativistic nuclear matter calculations gives a sizable correction to the nuclear symmetry energy [5]. We expect, however, that adding the Fock contribution will not change the general behaviour of the curves in Fig.1, although it can affect their intercept and slope. This topic will be discussed in detail elsewhere [6].

To estimate the effect of the $\delta$-field we first consider the case when the coupling constant $C^2_\rho$ is adjusted with no $\delta$-field present ($C^2_\delta = 0$). For the soft and stiff equation of state we find, respectively, $C^2_\rho = 5.0 fm^2$ and $C^2_\rho = 4.29 fm^2$. In Fig.2 we show corresponding energies of pure neutron matter. When the $\delta$-field is switched on, the energy/particle becomes smaller. In Fig.2 we show curves for a few values of $C^2_\delta$ from the range compatible with the Bonn potentials A, B and C [4].

The contribution due to the $\delta$-field is attractive and quite large. In case of the stiff equation of state the neutron matter becomes selfbound for $C^2_\delta \geq 1.1 fm^2$. Already a smaller value, $C^2_\delta \approx 1.0 fm^2$, lowers the symmetry energy to about $20 MeV$. One should note that for $C^2_\rho = 2.5 fm^2$ the neutron matter becomes as strongly bound as the symmetric nuclear matter. For the soft equation of state the binding is stronger.

To avoid such an unphysical situation, the increased binding due to the $\delta$-field has to be balanced by the higher repulsion due to the $\rho$-field. In particular, if we demand the symmetry energy to be reproduced, the formula (10) shows that the parameter $C^2_\rho$ has its lowest value for $C^2_\delta = 0$. For any finite value of the $\delta$-coupling, $C^2_\delta > 0$, the strength of the $\rho$-coupling, $C^2_\rho$, increases. In this case inclusion of the $\delta$-meson results in higher energy/particle at high densities, where the contribution of vector mesons dom-
inates. The value $C_\rho^2(0)$ should be regarded as the lower bound, with the actual value larger, corresponding to some finite $C_\delta^2$. Similarly, the energy/particle of neutron matter is bounded from below by values obtained for $C_\rho^2(0)$. Actual energies, for finite $C_\delta^2$, are higher. Below we show results for $C_\delta^2 = 2.5 f m^2$ which is close to the value of the ratio $g_\delta^2/m_\delta^2$ corresponding to the Bonn potential C [4].

The presence of the mean $\delta$-field leads to splitting of proton and neutron effective masses, Eqs.(7) and (8). In Fig.3 and Fig.4 we show the effective masses $m_P$ and $m_N$ as functions of baryon density for a few values of the proton fraction $x$. It is interesting to note that for the stiff equation of state, Fig.3, the proton and neutron effective masses in pure neutron matter both decrease monotonically with density. For the soft equation of state, Fig.4, the proton effective mass in pure neutron matter increases with density, approaching asymptotically the value $m_P = 2m$ for the largest couplings $C_\delta^2$. In contrast, the neutron effective mass decreases monotonically. Such a splitting of proton and neutron mass can affect the transport properties of dense matter.

The energy per particle is presented in Fig.5 where we show results for pure neutron matter for both soft and stiff equations of state. For comparison, curves for neutron matter with no $\delta$-field included, and for symmetric nuclear nuclear matter, are also shown. One can notice that the energy/particle with the $\delta$-contribution included increases more rapidly with neutron matter density for both equations of state than the energy for $C_\delta^2 = 0$.

The proton fraction of $\beta$-stable neutron star matter, which satisfies the condition

$$\mu_N - \mu_P = \sqrt{k_N^2 + m_N^2} - \sqrt{k_P^2 + m_P^2} + \frac{1}{2} C_\rho^2 (1 - 2x)n_B = \mu_e, \quad (12)$$

where the electron chemical potential is $\mu_e = (3\pi^2 xn_B)^{1/3}$, is shown in Fig.6. One can notice that for $C_\delta^2 = 2.5 f m^2$ the proton fraction at high densities is larger than for $C_\delta^2 = 0$ for both equations of state. It exceeds the critical value for the direct URCA process to be of importance in the cooling of neutron stars, which is $x_{URCA} \approx 0.11$.

In conclusion, the RMF theory with the contribution due to the $\delta$-meson mean field,
constrained to fit bulk properties of nuclear matter, predicts higher energy/particle of
the neutron matter at a given baryon density than in absence of the $\delta$-field. Although
the $\delta$-field provides an additional binding, fitting of the symmetry energy, $E_s \approx 30\text{MeV}$,
requires a stronger $\rho$-meson coupling which dominates at higher densities. The proton
fraction of the $\beta$-stable neutron star matter is considerably higher than the one with no
$\delta$-field contribution. Proton and neutron effective masses are split, an effect which can
modify the transport properties of dense matter.

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The $\rho$-meson coupling, $C_\rho^2$, required to fit the empirical value of the nuclear symmetry energy, $E_s \approx 30\text{MeV}$, as a function of the $\delta$-meson coupling, $C_\delta^2$. 
Fig. 2

The energy per particle of pure neutron matter for a few values of the $\delta$-meson coupling, $C_\delta^2$. For $C_\rho^2$ the same value is used for all curves; it is the value which fits the nuclear symmetry energy with no $\delta$-field. Solid and dashed curves correspond to the soft and stiff equation of state, respectively. For comparison, the energy per particle of symmetric nuclear matter is shown (dash-dotted curves).
Effective masses of protons and neutrons for a few values of the proton fraction, for the stiff equation of state and for $C_2^2 = 2.5 \, fm^2$. 

Fig. 3
The same as Fig. 3 for the soft equation of state.
Fig. 5

The energy per particle of neutron matter for $C^2_\delta = 2.5 \text{fm}^2$, corresponding to the Bonn potential C, and with no $\delta$-meson contribution. The parameter $C^2_\rho$ is adjusted to fit the nuclear symmetry energy in both cases. Solid and dotted curves are, respectively, for the soft and stiff equation of state. The energy per particle of symmetric nuclear matter is also shown.
Proton fraction of neutron star matter in the presence of the $\delta$-field and with no $\delta$-field. Solid and dotted curves correspond to the soft and stiff equation of state, respectively.