A Review of Backtesting VaR Models

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Abstract: Given the circumstances of huge macroeconomic changes, coupled with lack of liquidity and irrational market behaviour, it becomes very crucial to measure the level of risk (which cannot be diversified) for any potential investor or agent, in order to survive in the global competitive market, while simultaneously acknowledging the frequency and severity of such outcomes. Measurement of risk can help in knowing the level of risk that needs to be diversified by enabling one to make better asset-liability management decisions. Besides, a trader requires a comprehensive model of the short-term risks with a high level of accuracy. A good risk manager should allow one to effectively constrain the threats to an acceptable level, thereby, increasing the confidence. So the VaR no doubt is an easy method to measure market risk, but there is no one method to compute VaR. This is a grave issue as different methods yield results which can be significantly different. Such differences result in poor approximation of risks. The risk of measuring risk inaccurately owing to deficiencies of the underlying method or model of VaR is called Model Risk, and is considered an important issue in risk management.

To know the accuracy of our forecasted VaR we need to track the performance of the model in the future using operational criteria. So this is the methodology that we have adopted to check how the values have performed in the past. This can be done through a joint test of Kupiec & Peter Christoffersen Test and VaR Violation Ratio Test. The data for this study has been taken top 5 Nifty Sectoral Indices of the NSE. The data is Nifty Sectoral indices daily closing Prices from 01st April 2007 to 31st March 2017.

Keywords: Value at Risk, NIFTY Sectorial Indices

I. INTRODUCTION

It is necessary for the trader to apply a risk factor mapping, allowing the senior management to define future objectives by decomposing the total portfolio risk into separate components. It is often desirable to foretell and calculate the future to a reasonable degree of certainty. Amongst all methods and procedures, the dependable ones utilised to make forecasts are refined from the theory of probability and applications of statistics. Within the world of financial transactions, it is considered prudent to mix quantitative engineering techniques involving the likes of operations research and quantum physics to foresee the shifts of liquidity price in stock markets. Stock Market regulations support economic stability of both global and national financial architecture, providing a sustainable growth in future without stagnating present financial situation. Thus, regulators are chiefly concerned with threats to the financial architecture on macro levels such as risks that provide a systematic threat as well as potential for contagion. We can categorize the above risks as: Credit Risk, Legal Risk, Liquidity Risk, Market Risk, Operational risk etc.

While the risks listed are relevant to market stability, widespread use of highly leveraged derivatives instruments and the resulting globalization of financial markets have clearly established Market risk as one of the chief concerns for regulators and market stakeholders alike. Primarily, this fear has been a driving force for establishing internationally accepted regulations, norms and techniques for calculating, monitoring, checking and accounting market risk. The models or methodologies for measuring market risk are primarily of three categories: Parametric VaR, Non-Parametric VaR & Semi-Parametric VaR. A Parametric VaR model has become the pre-eminent international standard for measuring market risk. However, estimating these models has had its own limitations as the market behaviour is far from the assumptions made by normality. Thus to envisage VaR, with a certain degree of certainty, the assumption of normality needs to be suspended and there arises a need to estimate and evaluate through Non-Parametric & Semi-Parametric models.

II. LITERATURE REVIEW

Lopez (1998) evaluated the two most prominent methods available for estimating VaR estimates viz. Binomial method and Interval estimates. This study studies that among these two hypothesis-testing methods, which one has greater statistical power. Statistical power means the ability of the method to reject null hypothesis when it is incorrect. Moreover, the study also suggests a new method for evaluating VaR estimates based on standard forecast evaluation technique and tests VaR estimates for four types of portfolio return series: normal distribution, t-distribution with six degrees of freedom and two GARCH models. In total, there are eight VaR models that are studied in this paper.
The study concludes that the loss function based evaluation criteria which adapt hypothesis-testing methods are reasonably accurate. This is so because it can distinguish between different VaR models. Furthermore, all the three evaluation methods provide complementary results, and hence, any of them may be used to evaluate VaR estimates.

Longin (2000) gave emphasis to foreign exchange crisis or bond market breakdown, stock market crashes and market corrections amidst normal conditions relate to extreme price movements and financial markets. He creates an approach in view of extreme values to calculate the VaR in a way that covers market conditions extending from the usual environment on which the current VaR methods are used to the financial crisis which are due to the concentration of stress testing. Univariate Extreme Value Theory is utilized to calculate the VaR of a fully aggregated place with little quantum of assets and equilibrate composition which manages the problem of tail modelling while Multivariate Extreme Value Theory is helpful in calculating the VaR of decomposed position of risk factors with many assets and time changing composition that tends to the issue of risk-clustering of assets from various markets. Sarma (2003) implemented a two-stage model selection criteria for choosing appropriate VaR model among many alternatives. While the first stage involves conducting tests to determine statistical accuracy of a model, the second stage uses subjective loss functions.

The study computes VaR using RiskMetrics, historical simulation and GARCH methods for 95% and 99% level of confidence on the S&P 500 stock index and the NIFTY 50 Stock index. In total, 15 competing VaR models were chosen. The first stage involves testing the 15 chosen models for conditional coverage. The study draws on a previous study (Christofferson, 1998) which suggests a three step testing process: a test for correct unconditional coverage, a test for independence and a test for correct conditional coverage.

All the tests involve hypothesis testing. The second stage involves ascertaining loss functions. It is a utilitarian method for managers. The study uses two loss functions: Regulatory loss function, and Firm loss function. The study concludes that models pass the scrutiny of statistical accuracy but only few stand strong against the Loss Function tests.

Skiadopoulos, Lambadiaris, Papadopoulou & Zoulis (2003) pointed out that the accuracy of the historical and the Monte Carlo simulation in two different markets is diverse from each other. In the case of stock portfolio, he came to a conclusion that the best performing method depends on the chosen backtesting measure and the confidence level in the bond case and in the case of the stock portfolio, the historical simulation is not the suitable method for a risk manager to calculate the daily VaR.

Blanco, Maksim, & Oks (2003) estimated the accuracy of risk measurement models. They presented an overview of the backtesting methods and nailed the importance of conducting backtests on regular basis on various risk models which are used. They also suggested and alternative to measure VaR using a “macro” approach. This tool is complementary to traditional risk methodologies.

They put their efforts to answer the problem “How can one assess the performance and accuracy of a VaR model?” Their definition of accuracy in this context is as follows:

A. How accurately does the VaR model quantamize a particular percentile of the entire profit-and-loss distribution?
B. How well does the model envisage the magnitude and frequency of losses?

In this article various tests like the P&L test of VaR, the Christofferson test of fat tails, qualitative methods like forecast evaluation are used for estimating the possibility of successful VaR data at any percentile levels.

Nath & Samanta (2004) studied 31 government securities and 2 hypothetical fixed income portfolios of banks and Primary Dealers (PDs). The study uses variance-Covariance, historical simulation and Hill’s estimator, a tail index method, to compute VaR. For the purpose of evaluating the accuracy of estimated VaR, this study uses many methods including Backtesting, Kupiec’s test, Christofferson’s Test for conditional coverage and loss functions like regulatory loss function and firm loss function.

Empirical results from this study are that RiskMetrics underestimates VaR while historical simulation and Hill’s estimator methods yield conservative estimate of VaR. The authors have suggested that it would be appropriate to search for better loss functions for evaluating VaR.

Samanta & Thakur (2006) assessed the accuracy of VaR estimations obtained through the application of tail-index. The data has been taken for two stock price indices on daily basis that is BSE Sensex and BSE 100 from 1999 to 2005. They discovered that tail-index based methods provide comparatively more traditional estimates of VaR and have high probability of making it through the regulatory bank tests.
III. METHODOLOGY

The Top 5 NIFTY 50 Sectoral Indices considered are Nifty Banking, Nifty Information Technology, Nifty Private Bank, Nifty Fast Moving Consumer Goods and Nifty Financial Closing prices data are obtained from the website https://www.nseindia.com. We all know that investors do not depend on the prices for measuring the performance of the portfolio, the investors look at the returns of the portfolio. So in this study returns are calculated using the following formula:

\[ r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \]

Where \( r_t \) = returns from the portfolio, \( P_t \) and \( P_{t-1} \) are the closing prices of the portfolio at \( t^{th} \) and \((t-1)^{th}\) period. And have reviewed the Backtesting by investing the VaR Models.

A. Kupiec’s Test and Peter Christoffersen Test

It is a statistical tool to measure the accuracy of a VaR model. Ideally, the failure rate must converge at the level of confidence (p) at which the VaR is calculated. However, it may be observed that violations occur only marginally beyond the p value. This can be attributed to bad luck, and not because of the inability of chosen method. This is so called Failure rate which is defined as the proportion of times actual loss is exceeded by the estimated VaR. If \( N \) denotes the number of times actual losses exceeded the estimated VaR, over an observation period of \( T \) days, then \( N/T \) is the failure rate. If VaR is calculated at p level of confidence, then ideally \( N/T \) should converge to p.

In order to evaluate if the observed violations are statistically significant from the p value, the null hypothesis (H0) \(-p=N/T\) is tested against the alternate hypothesis (H1) \(-p ≠ N/T\). Kupiec (1995) suggested using likelihood ratio statistic to measure accuracy of VaR models. The statistic is as follows

\[ LR = -2 \log \left( \frac{1}{p} \right)^N p^N + 2 \log \left( 1 - (N/T) \right)^T (N/T)^N \]

The statistic is a Chi-square test with one degree of freedom. If observed value of chi-square test is greater than \( N/T \), then null hypothesis is rejected.

There are some points which we must make note of to ensure smooth progress. Firstly, while allowing a reasonable quantity of exceptions, we must check to ensure that those observations which are in excess of VaR levels are labelled as serially independent. That is, we must ensure that they are spread out comprehensively over time. A reliable indicator of how efficient a model is its capability to get past exception clustering by responding with speed to any changes or fluctuations in correlations and instrument volatilities. In skimming through these tests, we must understand how the exceptions are independent as put forth by Christofferson (1998) and Haas (2001).

B. Limitations of Kupiec and Christofferson Test

When checking back to conform to the present regulatory framework of one month, we can identify a limitation in the underlying fact that the sample sizes are consistent which lead up to the unfavourable proposition of a statistically weak test. Firstly, the frequency of losses has already been considered which has a consequence of a set of clustered exceptions which may lean towards to a rejection of the model. This has taught us a vital lesson with regards to the backtesting model on how we should not only depend upon the unconditional coverage test but also consider the coverage test.

C. A mixed test of Kuiper and Peter Kristofferson Test

Peter Kristofferson and Kuiper put forth the trailblazing concept of checking whether or not the given exceedances are correct and also as to whether or not they are independent. The combined test is instrumental in allowing backtesting within the domain of volatility clustering. As we rely upon the likelihood test as the fulcrum, the test statistic is as follows:

\[ LR(\text{joint}) = LR(\text{coverage}) + LR(\text{independence}) \sim \chi^2 \]

We take charge of three variables, namely \( H_0 \), \( H_1 \), from the first part of the joint test and \( H_2 \) from the later part. \( H_1 \) represents and shows that the exceedances are correct while \( H_2 \) reverses the approach by claiming that the exceedances are not correct and independent of each other.

It is important to note that a 1% VaR is equivalent to a violation of VaR by 1%. An alternative outlook would be to observe violations exceeding 5% of the time which would be overestimating. Excessive grouping of violation in VaR possesses the potential of posing a perilous problem. Thus, we arrive at the natural conclusion that an independence test must be conducted. If VaR violation clustering is in existence, it would be possible to forecast and make predictions regarding today’s violation just on the basis of data from yesterday’s violation.
D. VaR Violation Ratio

In performing an evaluation of VaR performance with the intent of foretelling risk. It is essential to consider the VaR violation ratio. Formulating the possibility of a violation in VaR is given by the condition

\[ \eta_t = \begin{cases} 1, & \text{if } y_t \leq -V\!aR_t \\ 0, & \text{if } y_t > -V\!aR_t \end{cases} \]

In above formula, \( V_1 \) refers to the count of \( n \), and \( V_0 \) refers to the count of \( n = 0 \). The above is easily obtained \( \eta = \sum \eta_t \) and \( V = W - V_1 \). We are given an understanding of the VaR Violation ratio defined as the proportion or ratio amongst an observed number of violations versus the expected number of violations. In the event or possibility that the ration with regard to observed violations turns out to be less than one, it results in an attempt at over-forecasting risk. On the other hand, is the ration lies in the region above one, then it reverses and under-forecasts risk. It highlights the need of carrying out a test of significance with respect to the violation ratio apart simple violation ratio analysis to make it more appropriate.

E. Berkowitz Test

The Duration of time between two VaR violation should be ideally independent and not to be clustered i.e. i.i.d. and it will follow exponential distribution which is having memoryless property. Here we are using exponential distribution for VaR Duration test with the following density function as

\[ f(x) = \lambda e^{-\lambda x} \]

So let us take the \( H_0 \) of the first part of the duration test. \( H_0 \) denotes the duration which is exponential distributed. For any distribution which enframes the exponential as a constrained case, and we allow a likelihood ratio test to see whether the case holds.

F. Backtesting results of VaR using VaR Exceedances Test

The calculated VaR using the methodologies prerequisite to be judged based on the performance using VaR exceedances test. The instrument of VaR violation ratio is also used for the validation of the model. The critical values for the VaR exceedances test at 99% and 95% confidence levels are given below:

- Unconditional coverage test:-\((6.63489, 3.841459)\)
- Conditional coverage test:-\((9.21034, 5.991465)\)

The values of VaR violation ratio and VaR exceedances test results are given below. Some important abbreviations:

1) EE= Expected exceedances
2) AE= Actual exceedances
3) uc.H0 =Null hypothesis of the unconditional coverage test
4) uc.LRstat = Estimated Likelihood Ratio of the unconditional coverage test
5) uc.LRp = p-value of the Estimated Likelihood Ratio of the unconditional coverage test
6) uc.Decision = Decision on the unconditional coverage test
7) cc.H0 = Null hypothesis of the conditional coverage test
8) cc.LRstat = Estimated Likelihood Ratio of the conditional coverage test
9) cc.LRp = p-value of the Estimated Likelihood Ratio of the conditional coverage test
10) cc.Decision = Decision on the unconditional coverage test
11) VR is nothing but VaR violation ratio

For selecting a model that is performing well, there are conditions to be met for a good model:

a) VaR Exceedances entail to be correct according to unconditional coverage test results.

b) VaR Exceedances requisite to be correct as well as independent according to the conditional coverage test results.

c) VR= 1, it is difficult to get the value of 1 in reality. So the good VR lies between 0.8 to 1.2.

G. Understandings from the Backtesting Results

The results of the VaR computation based on Filtered Historical Simulation, t Copula, Gaussian Copula, Monte-Carlo Simulation, EGARCH (1,1) and GARCH (1,1) at 99% and 95% confidence level suggests, for 1% VaR and 5% VaR, portfolio of NIFTY 50 Sectoral Indices, the VaR exceedances are correct according to the unconditional coverage test. Besides, the VaR violation ratio is within the acceptable range; for the Nifty Sectoral portfolio indices at their respective \( \alpha \) values. But based on the conditional
coverage test, portfolio of Nifty Sectoral Indices, for 1% VaR and 5% VaR the conditions are not performing well since violations are binary so they exhibit Nan values in the Mixed Kupiec test and there is not enough information to take the decision for Hypothesis testing.

The outcomes of VaR computation based on at 99% and 95% confidence level relieves that, for all the 0.05 and 0.01 values of α, no indices are doing well in terms of the VaR violation ratio, unconditional coverage test and conditional coverage test.

Therefore, VaR computed through Exponential Weighted Moving Average, Historical Simulation, Filtered Historical Simulation, Monte-Carlo Simulation and Gaussian Copula relatively over-predicts the risk under the assumed estimation window of 500.

The results of the VaR computation based on Monte Carlo simulation using Brownian motion and Historical simulation at 99% and 95% confidence level reveals that, for 1% VaR and 5% VaR, the portfolio of Nifty Sectoral Indices are correct as per the unconditional coverage test and for the VaR violation ratio is also performing better. For 1%, 5% VaR, is performing well based on the condition of VR.

But none if the indices are independent of their previous exceedances indicating the failure of conditional coverage test.

The result of the Gaussian Copula and Monte Carlo simulation at 99%, 95% confidence level suggests that, for alpha equal to 0.01 and 0.05, even though the unconditional coverage test accomplishes but the VaR Violation doesn’t lie in the acceptable range.

The VaR violation ratio also is showing better results for Monte Carlo Simulation using Brownian motion and each time when I ran the simulation trails of 1,00,000.

The study shows that the VaR violation result differs and lie in the range of 0.8 to 1.8 satisfying the all the three requirements when alpha is 0.01 and 0.05 respectively.

However, there is no best method for the Nifty Sectoral portfolio indices if the study uses EGARCH (1,1), GARCH (1,1), EWMA and the Gaussian Copula. In the case of the previous, inferring from the VaR violation ratios, for 1% and 5% VaR, these models are not performing moderately well.

Nevertheless, the conventional methods hold moderately good only for 1% and 5% VaR. All these results have compiled with a good conclusion based on the unconditional coverage test, so, there is no unique method that is performing well for the Nifty Sectoral portfolio indices.

Though the Nifty Sectoral portfolio indices are relatively better in forecasting the risk under Historical simulation and Monte Carlo simulation using Brownian motion respectively under both 99 as well as 95% confidence level. The Nifty Sectoral portfolio indices are moderately well in terms of correct exceedances under Monte Carlo simulation and the Exponential Weighted Moving Average at 0.01 significant level.

The Nifty Sectoral portfolio indices is also doing satisfactory under Variance-Covariance Approach at significance levels of 0.01 and 0.05 though it is underestimating the risk.

The findings of the backtested VaR model based on GJR GARCH-EVT-Copula, Filtered Historical Simulation, t Copula and Generalized Extreme Value Distribution at 99 % and 95% confidence level suggests that the model is good under the unconditional coverage test for the Nifty Sectoral Portfolio Indices for α equal to 0.01 and 0.05 and are even showing VR violation ratio of 0 unlike Filtered Historical Simulation showing VaR violation ratio under the acceptable range of 0.05263 and 0.08163 which is close to 0.

The implication of that of the results on the 99% and 95% confidence level indicates that the losses doesn’t exceed the VaR and due to violations are binary there is little information to make hypothesis testing decision.

The results of the study show that NIFTY 50 Sectorial Indices portfolio perform best in case of Evt-Copula-GJR GARCH, t copula, Generalised Extreme Value approach and Filtered Historical Simulation and perform better in the case of Monte-Carlo simulation using Brownian motion, Variance-covariance approach and perform moderately good in case of Historical simulation methods across all evaluation criteria at 99% & 95% confidence level.

The Empirical results clearly validates that the maximum loss and gain of GJR GARCH model with the copula-EVT based approach performs best followed by t copula outperforms traditional VaR.
VaR Test results for various VaR Technique at 99%, 95% Confidence levels for Nifty Sectoral Indices

| Column | alpha | EE | AE | uc.Ho | ucl.stat | ucl.critical | uc.LRP | uc.LRstat | cc.HO | ccLRstat | cc.LRP | ccLRstat | cc.Decision | VR  |
|--------|-------|----|----|-------|----------|-------------|--------|------------|-------|----------|--------|----------|------------|-----|
| E-WMA  | 0.01  | 19 | 10 | Correct Exceedances | 5.957039 | 6.634897 | 0.0145767 | Fail to Reject H0 | NaN | 9.21034 | NaN | NaN | NaN | 0.52631579 |
| GARCH(1,1)-norm, std, ged | 0.05 | 98 | 42 | Correct Exceedances | 43.5649 | 3.841459 | 4.10E-11 | "Reject H0" | NaN | 5.991465 | NaN | NaN | NaN | 0.42857143 |
| Variance-Covariance Approach | 0.01 | 19 | 84 | Correct Exceedances | 116.6419 | 6.634897 | 0 | "Reject H0" | NaN | 9.21034 | 0 | "Reject H0" | 4.42105263 |
| E-GARCH(1,1)-ged | 0.05 | 98 | 181 | Correct Exceedances | 58.2284 | 3.841459 | 2.331E-14 | "Reject H0" | 119.5141 | 9.21034 | 2.24E-14 | "Reject H0" | 1.84693878 |
| E-GARCH(1,1)-std | 0.01 | 19 | 30 | Correct Exceedances | 4.604965 | 6.634897 | 3.18E-02 | Fail to Reject H0 | 5.105716 | 9.21034 | 0.0770326 | "Fail to Reject H0" | 1.57094737 |
| Historical Simulation | 0.05 | 98 | 50 | Correct Exceedances | 32.76398 | 6.634897 | 1.04E-08 | "Reject H0" | NaN | 9.21034 | NaN | NaN | NaN | 2.63157895 |
| Monte-Carlo Simulation using Brownian Motion | 0.01 | 19 | 54 | Correct Exceedances | 40.62728 | 6.634897 | 1.84E-10 | "Reject H0" | NaN | 9.21034 | NaN | NaN | NaN | 2.84210526 |
| Monte-Carlo Simulation using Brownian Motion | 0.05 | 98 | 54 | Correct Exceedances | 26.51089 | 3.841459 | 4.40E-07 | "Reject H0" | NaN | 5.991465 | NaN | NaN | NaN | 0.55102041 |
| Historical Simulation | 0.01 | 19 | 13 | Correct Exceedances | 2.970676 | 6.634897 | 1.022125 | "Fail to Reject H0" | NaN | 9.21034 | NaN | NaN | NaN | 0.68421053 |
| Monte-Carlo Simulation using Brownian Motion | 0.05 | 98 | 72 | Correct Exceedances | 8.47145 | 3.841459 | 0.0038676 | "Reject H0" | 11.98952 | 5.991465 | 0.00249777 | "Reject H0" | 0.73469388 |
| Monte-Carlo Simulation using Brownian Motion | 0.01 | 19 | 20 | Correct Exceedances | 0.002462612 | 6.634897 | 9.60E-01 | Fail to Reject H0 | 0.4112567 | 9.21034 | 0.8141238 | Fail to Reject H0 | 1.05263158 |
| Monte-Carlo Simulation using Brownian Motion | 0.05 | 98 | 100 | Correct Exceedances | 0.01283352 | 6.634897 | 9.10E-01 | Fail to Reject H0 | 2.841788 | 9.21034 | 0.241488 | Fail to Reject H0 | 1.02040816 |
### VaR Test results for various VaR technique at 99%, 95% Confidence levels for Nifty Sectoral Indices

| Column | alpha | EE | AE | uc.Ho | uc.LRstat | uc.critical | uc.LRp | uc.Decision | cc.Ho | cc.Lrstat | cc.critical | cc.LRp | cc.Decision | VR |
|--------|-------|----|----|-------|-----------|-------------|--------|-------------|-------|-----------|-------------|--------|-------------|----|
| Evt-Copula-GJR GARCH | 0.01 | 19 | 0 | Correct Exceedances | NaN | 6.634897 | NaN | NaN | Correct Exceedances & Independent | NaN | 9.21034 | NaN | NA | 0 |
| | 0.05 | 98 | 0 | Correct Exceedances | NaN | 3.841459 | NaN | NaN | Correct Exceedances & Independent | NaN | 5.991465 | NaN | NA | 0.05263 |
| Filtered Historical Simulation | 0.01 | 19 | 1 | Correct Exceedances | 31.77019 | 6.634897 | 1.74E-08 | “Reject H0” | Correct Exceedances & Independent | NaN | 9.21034 | NaN | NA | 0.08163 |
| | 0.05 | 98 | 8 | Correct Exceedances | 145.8033 | 3.841459 | 0 | “Reject H0” | Correct Exceedances & Independent | NaN | 5.991465 | NaN | NA | 0 |
| tCopula | 0.01 | 19 | 0 | Correct Exceedances | NaN | 6.634897 | NaN | NaN | Correct Exceedances & Independent | NaN | 9.21034 | NaN | NA | 0 |
| | 0.05 | 98 | 1 | Correct Exceedances | 191.626 | 3.841459 | 0 | “Reject H0” | Correct Exceedances & Independent | NaN | 5.991465 | NaN | NA | 0.0102 |
| Generalised Extreme Value Distribution | 0.01 | 19 | 0 | Correct Exceedances | NaN | 6.634897 | NaN | NaN | Correct Exceedances & Independent | NaN | 9.21034 | NaN | NA | 0 |
| | 0.05 | 98 | 0 | Correct Exceedances | NaN | 3.841459 | NaN | “Reject H0” | Correct Exceedances & Independent | NaN | 5.991465 | NaN | NA | 0 |
| Monte-Carlo Simulation | 0.01 | 19 | 7 | Correct Exceedances | 11.10057 | 6.634897 | 0.000803 | “Reject H0” | Correct Exceedances & Independent | NaN | 9.21034 | NaN | NA | 0.36842 |
| | 0.05 | 98 | 45 | Correct Exceedances | 38.48128 | 3.841459 | 5.59E-10 | “Reject H0” | Correct Exceedances & Independent | 43.82509 | 5.991465 | 3.04E-10 | “Reject H0” | 0.45918 |
| Gaussian Copula | 0.01 | 19 | 4 | Correct Exceedances | 18.8908 | 6.634897 | 1.38E-05 | “Reject H0” | Correct Exceedances & Independent | NaN | 9.21034 | NaN | NA | 0.21053 |
| | 0.05 | 98 | 4 | Correct Exceedances | 168.8515 | 3.841459 | 0 | “Reject H0” | Correct Exceedances & Independent | NaN | 5.991465 | NaN | NA | 0.04082 |

#### H. Backtesting results of VaR using VaR Duration Test

The calculated VaR using the methodologies prerequisite to be judged based on the performance using VaR duration test. The instrument of VaR violation ratio is also used for the validation of the model. Some important abbreviations:

- LRp = The Likelihood Ratio Test Statistic.
- uLL = The unrestricted Log-Likelihood value
- b = The estimated Weibull parameter which when restricted to the value of 1 results in the Exponential distribution.
- rLL = The restricted Log-Likelihood value
- H0 = The Null Hypothesis decision for given the confidence level.

For selecting a model that is performing well, there are conditions to be met for a good model:

- The duration of time between VaR violations (no-hits) should ideally be independent and not cluster.
- Under H0, VaR violation duration should have no memory.

#### I. Understanding from the Backtesting Results

The findings of the backtested VaR model based on the GJR GARCH model with the Copula-EVT based approach, t Copula, Generalised Extreme Value Approach and Filtered Historical Simulation at 99% and 95 % confidence level suggests that the model is good as the VaR Duration between Exceedances have no memory. In the project the VaR violation is taken for monthly basis. The probability that the VaR violation occurs less than or more than 5 times in a year for 95 % confidence level is independent of its history that is it doesn’t depend on its past.
VaR Duration test for various VaR technique at 99%, 95% Confidence level for Nifty Sectoral Indices

| Column 1          | alpha | stdLL | stdLl | Ulp   | H0           | Decision          |
|-------------------|-------|-------|-------|-------|--------------|-------------------|
| EWMA              | 0.01  | 0.7655983 | -56.99677 | -57.53355 | 0.3001399 | Duration Between Exceedances have no memory (Weibull b=1 = Exponential) Fail to Reject H0 |
| Filtered Historical Simulation | 0.05  | 0.7857101 | -197.9488 | -199.927 | 0.02754354 | Duration Between Exceedances have no memory (Weibull b=1 = Exponential) Fail to Reject H0 |
| Gaussian Copula   | 0.01  | 0.8261413 | -46.32153 | -46.50752 | 0.5419269 | Duration Between Exceedances have no memory (Weibull b=1 = Exponential) Fail to Reject H0 |
| GEV Distribution  | 0.01  | 0.001  | -39.81745 | -22.47369 | 0.7349369 | Duration Between Exceedances have no memory (Weibull b=1 = Exponential) Fail to Reject H0 |
| Historical Simulation | 0.01  | 0.001  | -135.9654 | -73.25922 | 1         | Duration Between Exceedances have no memory (Weibull b=1 = Exponential) Fail to Reject H0 |
| Monte Carlo Simulation | 0.05  | 0.7528866 | -300.4644  | -307.2285 | 0.000235016 | Duration Between Exceedances have no memory (Weibull b=1 = Exponential) Fail to Reject H0 |
| t-copula          | 0.01  | 0.001  | -61.87365 | -34.909202 | 1         | Duration Between Exceedances have no memory (Weibull b=1 = Exponential) Fail to Reject H0 |
| t-Copula          | 0.05  | 0.6804513 | -218.9384 | -226.4947 | 0.000101063 | Duration Between Exceedances have no memory (Weibull b=1 = Exponential) Fail to Reject H0 |

Source: Author’s Calculation

IV. SUMMARY AND CONCLUSIONS

Risk measurement and management of any financial organization eliminates the susceptibility to any plunges in stock prices and noticeable increases in uncertainties about the value of financial assets. Particularly during a financial crisis, the risks associated with investments increase significantly to the level that may create panic in stock markets all over the world. One such recent example is the financial crisis of 2008-09 that has left a remarkable impression on economic and financial structure worldwide, leaving entire generation of investors perplexed with the situations in the market. This calls for a good accurate measure of risk and from the finance literature, no other measure qualifies to be more accurate than VaR.

In this analysis, the effectiveness of the Value at Risk methods are evaluated based on various methodologies. First, it is not always possible to compare the VaR by various VaR methods and may often be fairly different, as demonstrated by numerous studies. Therefore, using the VaR in practice requires re-evaluation of the accuracy of the estimated VaR. We obtained suitable inferences through empirical testing of VaR models based on considered methods and backtesting. The Monte Carlo simulation method, Gaussian Copula, Exponential Weighted Moving Average Approach, GARCH (1,1)-norm, GARCH (1,1)-std, GARCH (1,1)-ged, E-GARCH (1,1)-ged, E-GARCH (1,1)-std were found not suitable for any of the alpha value for Nifty Sectoral portfolio. For the Nifty Sectoral indices, the Variance-Covariance approach, Historical Simulation and the Monte-Carlo simulation using Brownian motion were found to be comparatively better at both 99 as well as 95% confidence levels with corresponding ratios of 1.58, 1.21; 0.68, 0.74 and the range of the estimates lies between 0.8 to 1.8 for each random hypothetical returns respectively. The Nifty Sectoral Indices perform moderately good in this case. The conclusion that can be drawn from the VaR violation ratio is that GJR GARCH model with the Copula-EVT based approach, t Copula, Generalised Extreme Value Approach and Filtered Historical Simulation are more reliable for forecasting the risk. The indices of most of these models satisfy the Unconditional Coverage test results but fail to satisfy the Conditional Coverage test that considers only the frequency of losses and not the time of occurrence. As a result, these tests may fail to reject a model that produces clustered exceptions since the test is based on violations that are binary. The size of the forecasting window was assumed to be 500 but this little information is not enough to perform the hypothesis testing, giving NaN values in the Mixed Kupiec test. So eliminating Nan values by increasing the forecasting window or performing a VaR Duration Test is absolutely necessary. The null hypothesis says that the Duration between Exceedances have no memory. So at 1% & 5% level of significance, there is no sufficient evidence to reject null hypothesis. It has been observed that the hybrid method with GJR GARCH–EVT-t Copula, performed better when compared to other methods.
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