Sensor Head Geometry for Tomography from Limited Views

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Abstract. We study the performance of a Sinusoidal Hough Transform (SHT) algorithm for recovery of sinograms from a severely limited number of measurements. This problem is typical for sensors targeting limited access tomography, in most cases resulting in limited angular views. The aim of this work is to establish the variations in performance as a result of the chosen sensor geometry. The performance of SHT-aided reconstruction is evaluated by 3 different error metrics in the case of sensor heads with a decreasing number of line integral measurements – down to 28, which is relevant for demanding hard-filed Tomography applications in industry. The grouping of these measurements into parallel beam angular projections and robustness to noise is studied for several sensor head geometries, comprising of only 3 or 4 angular views. The SHT-based algorithm is tested on 3 model objects with variations in their definition (diffuse vs abrupt boundaries), as well as topology. The reported results indicate that with a fixed small number of measurements, the larger number of angular projections yields better performance compared to grouping a larger number of line integrals into a smaller number of angular projections.

1. Introduction
Demands for Tomography from limited views arise most typically in industry, because of limited physical access, limited time for data acquisition and limited resources for the storage and processing of data. Often all three factors above are affecting the imaging task simultaneously. A recently introduced solution is the recovery of a sparse sinogram by Sinusoidal Hough Transform [1]. To date, SHT algorithms have been introduced and verified by experimental data in the case of 32 path integral measurements [2]. The SHT approach for tomography imaging sensors allows the recovery of missing data samples by identifying the main constituents of the imaged frame and re-sampling the set of measurements for further use by standard data inversion routines. GPT was originally introduced[3] and successfully demonstrated for imaging of temperature[4]. The new Photonic GPT (PGPT) is based on the controlled change in the waveguiding properties of plastic optical fibers (POF), sensitized to external influence (mechanical, chemical, etc.) and the imaged parameter is reconstructed from simple light intensity measurements at the periphery of the surface, with low cost optoelectronic components. This allows the application in a variety of imaging and control scenarios, ranging from healthcare to structural and ambient monitoring.

2. The SHT algorithm
The details of the methodology for the implementation of SHT have been published elsewhere [2]. In brief, the algorithm uses SHT to map the 2-D sinogram $p(t_j,\varphi_i)$ to the 2-D Hough space $h(x,y)$. In Hough space, sinusoidal traces (corresponding to individual centers of mass) are indicated by peaks of
high count above a pre-defined threshold. Identifying all the sinusoids that meet the threshold criteria allows the reconstruction of their traces, and the sinogram re-sampling with the required precision. Next, the support of the sinogram is identified together with any null segments that it may contain. The final step of the applied SHT algorithm is to estimate the missing projection samples along the displacement coordinate in the sinogram. Here, the Piecewise Cubic Hermite Interpolation was used and each interpolated projection was normalized and then scaled by the average subject mass.

3. Model objects
The variations in the SHT algorithm’s performance with sensor head geometry is studied in the reconstruction of three model objects (see Fig.1):
1. *Gaus2* is a weighted superposition of a smooth circular-symmetric Gaussian function and an asymmetric Gaussian function, (Gaussians with circular and elliptic cross sections)
2. *GauCyl*, is a weighted superposition of the asymmetric Gaussian component of *Gaus2* and a cylinder, projecting in 2D as a homogenous disk with abrupt boundaries.
3. *KBO*, with two smoothly linked centers of mass, represents the type of subject often found in industrial applications where the objects are quasi-homogeneous with weakly defined boundaries.

![Fig. 1: Objects used for this study (top view in second row):](image)

(a) Gaus2: two Gaussians, one with elliptical cross-section; (b) GauCyl: an elliptical Gaussian and a cylinder with abrupt boundaries; (c) KBO: a realistic 3 bit object

4. Imaging geometries
The sensor head geometries considered in this study can be characterized as typical for severe undersampling of the Radon transform, since they provide a very small number of line integrals (LI). The general approach in varying the geometry was that as the model objects are kept the same, the varying number of LIs changes the beam density, which is used as a parameter in the evaluation stage. Table 1. shows the parameters of the main sensor geometries that were realised. Additional variations within the same number of LIs are not shown in the table, but were included in the analyses.

| LIs     | 52 | 44 | 41 | 40 | 36 | 33 | 28 |
|---------|----|----|----|----|----|----|----|
| projections | 4  | 4  | 3  | 4  | 4  | 3  | 4  |
| LI density [m⁻²] | 1650 | 1327 | 1369 | 1006 | 974 | 1188 | 714 |
Table 1: main sensor geometries listed by number of line integrals (LI), number of projections and beam density.

For each sensor head design and object, datasets were generated without noise and with two values of the signal-to-noise ratios (SNR), 30dB and 20dB. Only electronic noise was considered as a source of random errors in the measurements. Its effect is simulated on the forward transformed data, assuming that the random measurement noise is uncorrelated for any two independent LIs. Example reconstructions of Gaus2, involving sinogram processing by SHT, are shown in Fig. 2.

![Example reconstructions of Gaus2](image)

Fig. 2: (a) Sketch of three PGPT sensor heads with number of path integrals = 52, 28 (4 angles) and 41 (3 angles). Reconstructions of the object in Fig 1.a: (b) without noise, (c) SNR=30dB (d) SNR=20dB

5. Sensor head performance

Three of the most popular image quality measures[5] are applied for the quantitative evaluation on the reconstructions against each of the geometries, reconstructed objects and noise levels:

1. Average difference:

\[
\text{av err} = \frac{\sum_{x,y} |f(x,y) - h(x,y)|}{MN},
\]

where \(f\) and \(h\) are the reference and reconstructed images, respectively, with dimensions \(M \times N\).

2. Normalized \(rms\) error:

\[
\text{norm rms} = \left\{ \frac{\sum_{x,y} [f(x,y) - h(x,y)]^2}{\sum_{x,y} [f(x,y) - f]^2} \right\}^{1/2},
\]
where $f'$ is the average value of all the phantom pixels. Eq (2) is normalized against the image variance, the error does not depend on the level of activity in an image.

3. Normalized absolute error:

$$\text{norm}_{\text{abs}} = \frac{\sum |f(x,y) - h(x,y)|}{\sum |f(x,y)|}.$$  (3)

It is important to clarify here that while all three are common objective criteria, their correlation with subjective image perception is not guaranteed. All objects have been normalized to unity; therefore it is possible to compare e.g. between model objects, between geometries with the same number of projections, etc.. The robustness of the tested geometries to noise, up to the level of SNR=20dB, is demonstrated for all cases, as shown in Fig. 2 for Gauss2 and some representative geometries. With this in mind, sets of data were collected and analyzed for the 3 model objects, the 3 levels of noise and the 9 different sensor head geometries.

6. Results and discussion

The complete set of results was studied for trends related to the parameters in Table 1. Figures 3 and 4 show plots of the calculated different quality measures against the LI density and the total number of LIs respectively. Fig. 3 shows that for comparatively close values for density, $\text{norm rms}$, while $\text{norm abs}$ and average $\text{diff}$ are scattered over a much larger range of error values. This can be interpreted along the lines that the density of sampling, as an intuitive parameter, is not a good measure for the optimal design of the sensor head: the scatter in that plot suggests the reduced significance of the sampling beam density when compared to Fig.4. The latter shows a well behaved dependence from the total number of LIs. Furthermore, no substantial increase for the lowest numbers is manifested in Fig.4. Another important observation on that figure is the grouping of errors in the case of 3-angle geometry (area encircled by the red dashed line) and 4-angle geometry, (blue dashed area). It implies that the number of angular views overrides both, the sampling density and the total number of LIs. The robustness to noise over 2 orders of magnitude was demonstrated by the errors.

**Fig. 3: Errors at varying LI density**
increasing by less than 4% (and typically around 2.5%) over that range of SNR.

![Graph showing error vs. number of path integrals]

**Fig. 4: Errors at varying number of LIs**

### 7. Conclusion

Various realistic sensor head geometries were tested, with a set of model objects, for the suitability of SHT sinogram recovery in the case of severely undersampled Radon Transform. The results indicate that the impact of the number of projections is highest, while there is no obvious dependence on the density of spatial sampling by the LIs. As with a strongly reduced number of LIs the implementation of parallel beam projections becomes less feasible (which can also be dictated by access limitations), an interesting future application of the SHT algorithm would be the case of random beam distributions. Furthermore, the SHT approach shows good robustness to noise. The obtained results can be utilised for the design of tomography sensor heads and imply that the SHT performance allows design variations to meet the geometrical requirements in a restrictive environment. Most notably, because the SHT approach can identify the location of the main constituent objects without performing the traditional inverse Radon transform, it can be a very efficient tool when further detail is not of interest.

### References

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