The analysis of the implementation of problem-based learning students’ generalization skills in obtaining the resolving total domination number

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Abstract. Problem Based Learning (PBL) is a way to form and teach using problems as a stimulus and focus for student activities. Student generalization skills are one of the skills that can be improved using PBL. Mathematical generalization is a competency achieved by students, which includes general perception (identification pattern), general expression (describing patterns), generality symbolic expression (formulating patterns), generality manipulation (using generalization results). This research was conducted to see the effect of PBL instrument development on students' ability to solve problems solving the total dominating set. The study used a mixed method, combining quantitative and qualitative methods. The research subjects consisted of two classes each with \(n = 24\) for control class, and \(n = 23\) for experimental class. In the control class and the experimental class PBL model is used, but only in the experimental class applied the student worksheet. The homogeneity test of the pre-test results for both classes showed a \(p\)-value of 0.2199 (\(p \geq 0.05\)), so that the average difference between the two classes was not significant (homogeneous). Based on the independent t-test on the post test scores of the two classes, the \(p\)-value was 0.01682 (\(p \leq 0.05\)), so it can be concluded that there is an effect of PBL instrument application on students' generalization skills.

1. Introduction

One of the disciplines that is very important in civilization is mathematics. Mastery of mathematical skills is important to achieve in order to compete in the rapid progress of the times. This is because mathematics is a science that can cover and develop almost all abilities that are needed by individuals in dealing with developments in all aspects of life.

One of the standards in the Principle and Standards for School Mathematics, apart from problem solving, representation, communication, and connection skills, are reasoning skills. In mathematical reasoning skills, one of the most important aspects is generalization skills. Generalization skills are a process of how students can draw conclusions in general or more simply from a problem. They tend to start from specific things to more general things. According to Tall [1] from a mathematical point of view, generalization is defined as the search for a larger image. According to Bassham [2] generalization is a statement about all or most of the members of a set. Generalization is an argument that relies on the characteristics of a population sample to make statements about the population as a whole [3]. The indicators in mathematical generalization skills are given in Table 1 [4].
### Table 1. Indicators of mathematical generalization skills.

| Indicator                           | Sub-Indicator                                      |
|------------------------------------|----------------------------------------------------|
| Perception of generality           | - Students are able to recognize a graph           |
| Expression of generality           | - Students are able to give symbols on graphs      |
|                                    | - Students are able to determine the cardinality on a graph |
| Symbolic expression of generality  | - Students are able to determine the dominator on the graph |
|                                    | - Students are able to represent vertices on a graph   |
| Manipulation of generality         | - Students are able to determine the resolving of a graph |
|                                    | - Students are able to determine resolving cardinality of a graph |

In learning mathematics, the ability to generalize mathematically is in solving problems. One of the learning models in accordance with generalizations is the Problem Based Learning (PBL) model. With PBL, students are more active in thinking and understanding the material in groups looking for and investigating real problems so that they get a deep impression and make more sense of what they are learning.

In discrete mathematics, there is a study that has the most application, namely graph theory. One of the topics in graph theory is total dominating set and resolving set. The definition of the resolving total dominating set is based on total dominating set and resolving set definitions. Thus, resolving total dominating set is the distance between the vertices $x$ and $y$ of graph or the representation of vertices with respect to total dominator. The set of vertex $D_t \subseteq V(G)$ is resolving total dominating set from $G$ if every two vertex $u, v \in V(G)$ and $w \in D_t$ such that $r(u|D_t) \neq r(v|D_t)$. The smallest cardinality of resolving total dominating set in $G$ is shown by $\gamma_{rt}(G)$.

In Dini [5] regarding the effect of the application of research based learning on student generalization skills. It shows the level of thinking of student generalization in solving local H-decomposition antimagic total coloring problems. It shown that 7% of research subjects who fall into very good criteria, 28% in research subjects who fall into the good criteria, 53% for the criteria is good enough, and 12% for those who are not good. Harjito [6] showed in his study about problem based learning implementation and its influence to the students generalization thinking skills on solving r-dynamic vertex coloring. It shown that 56% were in the very high category, 23% were in the high category, 13% were in the average category, and 8% were in low category which closed to Dini’s result. Monalisa [7] have finished presented the result finding of the implementation of research based learning in developing the students mathematical generalization thinking skills in solving a paving blocks design problem. The results show, after the implementation of research based learning, 23% students are categorized in low level of mathematical generalization thinking skills, and 27% are categorized good level of mathematical generalization thinking skills and 50% are categorized in high level of mathematical generalization thinking skills.

Based on the description above, the researcher wants to analyze the effect of problem based learning development on students' mathematical generalization skills in solving problems resolving total dominating set. The learning tool will be focused on discrete modeling with the material, resolving total dominating set. Resolving total dominating set is one of the materials of graph theory where students are required to find the dominating set, then analyze the pattern and distance (resolving) and its domination number of the specified graph. Therefore, it is very necessary for researchers to make devices related to their research.
2. Resolving total dominating set

In graph theory, the minimum cardinality of \( D \subseteq V(G) \) such that all other vertex are determined by their distance to a vertex on \( S \) is called a metric dimension. The set of ordered vertex \( D = w_1, w_2, \ldots, w_k \) and a vertex \( v \in G \), namely \( d(v, w) \) with the representation of the distance between \( v \) and \( w \in D \) is \( \text{r}(v|D) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k)) \). If the representation of any two vertex on \( G \) is different, then \( D_t \subseteq V(G) \) is a resolving set. The set of vertex \( D_t \subseteq V(G) \) is resolving total dominating set from \( G \) if the vertex representation \( u, v \in V(G) \) with respect to \( x \in D_t \) is \( \text{r}(v|D_t) \) so that \( \text{r}(v|D_t) \neq \text{r}(u|D_t) \). The resolving set of graphs has been introduced by Slater and Harary [8] [9]. Research on resolving set can be seen in [10].

The definition of the resolving total dominating set is based on the definitions described above regarding the dominating set, the total dominating set, and the Resolving set. Thus, the set of vertex \( D_t \subseteq V(G) \) is resolving total dominating set from \( G \) if every two vertex \( u, v \in V(G) \) and \( w \in D_t \) such that \( \text{r}(u|D_t) \neq \text{r}(v|D_t) \). The smallest cardinality of Resolving total dominating set in \( G \) is shown by \( \gamma_{rt}(G) \). An example of the resolving total dominating set in \( P_7 \) can be seen in Figure 1 where the blue dot is the dominator.

![Figure 1. Total dominating set on \( P_7 \).](image1)

![Figure 2. Resolving total dominating set on \( P_7 \).](image2)

Figure 1 is an illustration of the total dominating set which the black dot with the blue square is a total dominating set. At least one other dominator vertex is connected to each other. Figure 2 illustrates the resolving total dominating set of \( P_7 \) and its representation.

3. Research method

This study is a form of mixed-method research. In this research, the impact on student generalization skills, a problem-based learning model was developed. In [11] a mixed method research method is a combination method between quantitative research methods and qualitative research methods. The research design used is Sequential Exploratory, which is a combination research with the first stage of qualitative data collection and analysis, while the second stage is followed by data collection and quantitative data analysis to conclude the research results of the first stage. This study produces mathematics learning equipment products in the form of Student Worksheets and monographs (in other paper). This research procedure refers to the Thiagarajan 4-D development model which consists of four stages, namely the define stage, the design stage, the developmental stage, and the dissemination stage [12].
2.1 Population

This research was conducted in the academic year 2020/2021 to be exact, the odd semester. The place of research is Mathematics Education Study Program, Teacher Training and Education Faculty, Jember University.

2.2 Instrument

The data collection method used is interviews, observation, and test method. The experimental research design used two classes, namely the experimental class and the control class. The experimental class is a class that uses PBL instrument (student worksheet) with mathematical generalization indicators. The control class is a class that uses a problem based learning model without student worksheet. In this experimental research, it will be compare the mathematical generalization skills of the experimental class with the control class. The research design used was in the form of a non-equivalent control group design with a scheme such as Table 2 [13].

Table 2. Non-equivalent control group design scheme.

|          | $R_1$ | $X$ | $R_2$ |
|----------|-------|-----|-------|
| Control class | $R_3$ |     | $R_4$ |

Note that $R_1$ and $R_3$ are the score of pre-test while $R_2$ and $R_4$ are the score of post-test. $X$ is treatment in the experimental class was in the form of PBL learning tools.

Collecting data in this study is to select the sample class used as research subjects in the form of the experimental class and the control class. The pre-test is carried out at the beginning of the lesson and the post-test is carried out at the end of the lesson. If the data in this study were normally distributed and homogeneous, then using data analysis techniques in the form of independent t-test. Measuring the level of reliability of data collection tools in this study using Alpha Cronbach. The homogeneity test of variance uses R-Shiny, if the significance value is > 0.05, it can be concluded that the variance of the data group is the same. The independent t-test in this study used R-Shiny by entering the pre-test and post-test data for the class used experimentally. The hypothesis are formulated in the form of a pair of null hypotheses ($H_0$)
and alternative hypotheses \((H_1)\). With the test criteria accept \(H_0\) if the \(p\)-value > 0.05 then \(H_0\) is accepted and \(H_1\) if the \(p\)-value 0.05 then \(H_0\) is rejected.

4. **Result and discussion**

The following are the results of our research related to the analysis of students’ mathematical generalization thinking skills in solving problems in the application of problem-based learning. We give pre-test and post-test for both classes, either the experimental class or the control class. The student worksheet is also distributed to research respondents, especially only the experimental class.

4.1 **Instrument validity**

Before we start the data analysis with normality and homogeneity test, pre-test and post-test which applied on both class must be valid and reliable. Thus, we will show the validity and reliability of the test given on the Table 3.

**Table 3.** The test result of the instrument validity.

|       | Item_1                  | Item_2                  | Item_3                  | Item_4                  | SCORE                  |
|-------|-------------------------|-------------------------|-------------------------|-------------------------|------------------------|
| Item_1| **Pearson Correlation** | .487**                  | .025                    | .059                    | .454**                 |
|       | Sig. (2-tailed)         | .001                    | .867                    | .695                    | .001                   |
|       | **N**                   | 47                      | 47                      | 47                      | 47                     |
| Item_2| **Pearson Correlation** | .487**                  | 1                       | .503**                  | .159                   | .852**                 |
|       | Sig. (2-tailed)         | .001                    | .000                    | .284                    | .000                   |
|       | **N**                   | 47                      | 47                      | 47                      | 47                     |
| Item_3| **Pearson Correlation** | .025                    | .503**                  | 1                       | .174                   | .722**                 |
|       | Sig. (2-tailed)         | .867                    | .000                    | .243                    | .000                   |
|       | **N**                   | 47                      | 47                      | 47                      | 47                     |
| Item_4| **Pearson Correlation** | .059                    | .159                    | .174                    | 1                      | .536**                 |
|       | Sig. (2-tailed)         | .695                    | .284                    | .243                    | .000                   |
|       | **N**                   | 47                      | 47                      | 47                      | 47                     |
| **SCORE** | **Pearson Correlation** | .454**                  | .852**                  | .722**                  | .536**                 | 1                      |
|       | Sig. (2-tailed)         | .001                    | .000                    | .000                    | .000                   |
|       | **N**                   | 47                      | 47                      | 47                      | 47                     |

**. Correlation is significant at the 0.01 level (2-tailed).**

Based on the table, it can be seen that the value of \(r_{count}\) from number 1 was 0.459, number 2 was 0.833, number 3 was 0.726, and number 4 was 0.547. We can see that all items was \(r_{count} > r_{table}\) with item \(N = 47\), so all items were valid.

**Table 4.** The test result of the instrument reliability.

| Reliability Statistics |
|------------------------|
| **Cronbach’s Alpha**   | **N of Items**           |
| .544                   | 4                        |
Based on the table, it can be seen that the value of whole reliability was 0.544 and \( r_{table} \) from the significance level of 1% with \( df = N - 2 = 47 \), \( r_{table} = 0.3683 \). Therefore, \( r_{count} > r_{table} \) then the instrument items were reliable.

### 4.2 Main Result

This study result based on analysis of the generalization thinking skill of student in solving resolving total dominating set with problem based learning tools. Both experimental and control class was given the pretest in the beginning and post test at the end. The pretest result of 48 students show that the generalization thinking skill of 24 students in the control class are 21% students on high generalization thinking skill, 34% students on medium generalization thinking skill, and 45% students on low generalization thinking skill. The pretest result of 23 students in the experimental class are 25% on high generalization thinking skill, 40% on medium generalization thinking skill, and 35% on low generalization thinking skill. We show the percentage on the diagram below as follow:

**Figure 4.** The pre test result of generalization thinking skill.

The pre-test data results were analyzed by using quantitative statistics of variance to find out the differences in the students' generalization thinking skill before the implementation of problem based learning and student worksheet. The data analysis used was the R-shiny application of normality test analysis. It was done to find out whether or not the data distribution was normally distributed. The data distribution was significant if the value was \( \geq 0.05 \).
Table 5. The test result of the pre-test normality.

| statistic | p.value | method                |
|-----------|---------|-----------------------|
| 0.96      | 0.11    | Shapiro-Wilk normality test |
| 0.96      | 0.11    | Shapiro-Wilk normality test |

According to the Table 5, the significant value obtained was 0.11. It revealed that the data of both classes were in normally distribution. After the data was normal, then we take homogeneous test and independent t-test given in the Figure 5.

Figure 5. The homogeneous test and independent t-test of pre-tes using R-shiny.

Based on the Figure 3, we can see that $p = 0.2199$ that mean $p > 0.05$ and both of class showed the data was homogeneous. The $p$-value of independent t-test showed $p = 0.2682$ that mean $p > 0.05$ sig value (2-tailed). It means that $H_0$ was accepted then there were no differences in the mean scores of pre-test in both class.

The next step of this study was take the control class using a problem based learning method without student worksheet and the experimental class using student worksheet with problem based learning method. At the end of class, both of class given the post-test. The result of post-test on the control class which have 24 students show that 23% students on high generalization thinking skill, 51% on medium generalization thinking skill, and 26% on low generalization thinking skill. On the experimental class, there are 29% of 23 students on high generalization thinking skill, 55% on medium generalization thinking skill, and 16% on low generalization thinking skill. The result of post-test diagram, we can see on the figure 4 below, as follow.

Figure 6. The post-test result of generalization thinking skill.
The post-test data results were analyzed by using quantitative statistics of variance to find out the differences in the students' learning outcomes. The data analysis used was the R-shiny application of normality test analysis. It was done to find out whether or not the data distribution was normally distributed. The data distribution was significant if the value was $\geq 0.05$.

Table 6. The test result of the post-test normality.

| statistic | p.value | method                  |
|-----------|---------|-------------------------|
| 0.96      | 0.10    | Shapiro-Wilk normality test |
| 0.96      | 0.10    | Shapiro-Wilk normality test |

According to the Table 6, the significant value obtained was 0.10. It revealed that the data of both classes were in Normal distribution. After the data was normal, then we independent t-test as follow.

Uji-T 2-Kelompok Bebas: Data= IMPOR Y= Score G= Class  
Ha: two.sided Varians Sama: TRUE  
Two Sample t-test

data: datasetInput(), input$var.yt2b by datasetInput(), input$var.g1  
t = -2.4963, df = 45, p-value = 0.01628  
alternative hypothesis: true difference in means is not equal to 0  
95 percent confidence interval:  
-2.1799678 -0.2330756  
sample estimates:  
mean in group 1 mean in group 2  
12.75000 13.95652

Figure 8. The independent t-test of post-test using R-shiny.

The results of the independent t-test, as presented in the Figure 6, showed that variance was obtained with sig. value (2-tailed) was 0.01628 < 0.05. Thus, it can be concluded that the results of the post-test gained from experimental and control classes had significant differences. The mean score of the control class was 12.75, while the experimental class was 13.96. It revealed that the average of students' generalization thinking skills in the experimental class was significantly higher than the control one. It proved that learning by using student worksheet and problem-based learning tools affected their generalization thinking skills significantly.
4.3 Phase Portait
In this study, the student phase portrait is based on the flow of students' generalization abilities in completing a resolving total dominating set study based on problem based learning. Three subjects were described to represent the level of generalization skill as the illustrations. We found their thoughts for solving resolving total dominating set through the interview.

\[
\begin{align*}
\text{Figure 9. The Results of S01.} & \\
\text{The graph resulting from the shadow operation } D_2(P_n). \quad & \\
\text{The vertices and edges’ minimum cardinality:} & \\
V &= \{x_i : 1 \leq i \leq n\} \cup \{x_i^1 : 1 \leq i \leq n\} \\
|V| &= 2n \\
E &= \{x_i, x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_i, x_i^1 : 1 \leq i \leq n-1\} \\
|E| &= 4n - 4
\end{align*}
\]

Based on Figure 4, it can be seen that S01 were able to recognize a graph. S01 correctly labeled each vertex and edge on the centipede graph. Then, S01 correctly showed the vertex and edge functions, and the cardinality of the vertices and edges in the centipede graph. So that, S01 fullfill the indicator 1 and indicator 2 perfectly.

\[
\begin{align*}
\text{Figure 10. The Results of S01.} & \\
\text{Total dominating set’s cardinality:} & \\
D_{D_2}(P_n) &= \{x_i : 1 \leq i \leq n-1\} \\
\gamma_t(D_{D_2}(P_n)) &= n - 1
\end{align*}
\]

Go on the Figure 5, it can be seen that S01 were able to specify total dominating set with the minimum total dominator. We can be seen that S01 correctly determine the minimum cardinality of total dominator set. So that, S01 fullfill the indicator 3 perfectly.

- Vertice’s representation
  \[
  \begin{align*}
  r(x_1|D_2(P_n)) &= (0, 1, 2, \ldots) \\
r(x_2|D_2(P_n)) &= (1, 0, 1, 2, \ldots) \\
r(x_3|D_2(P_n)) &= (2, 1, 0, 1, 2, \ldots) \\
r(x_4|D_2(P_n)) &= (3, 1, 2, 0, 1, \ldots) \\
r(x_n|D_2(P_n)) &= (n - 1, n - 2, \ldots, 0, 1, 2, \ldots, n) \\
r(x_1|D_2(P_n)) &= (2, 1, 2, \ldots) \\
r(x_2|D_2(P_n)) &= (1, 2, 1, \ldots) \\
r(x_3|D_2(P_n)) &= (2, 1, 2, 1, \ldots) \\
r(x_4|D_2(P_n)) &= (3, 2, 1, 2, 1, \ldots) \\
r(x_n|D_2(P_n)) &= (n - 1, 2, 1,\ldots, n) \\
r(x_n|D_2(P_n)) &= (n - 1, 2, 1,\ldots, n)
  \end{align*}
  \]

- Because the representation of vertices on $D_2(P_n)$ to $D_{D_2}(P_n) = \{x_i : 1 \leq i \leq n - 1\}$ is resolving total dominating set. Then, $\gamma_t(D_2(P_n)) = n - 1$. 

\[
\text{Figure 11. The Results of S01.}
\]
For the last indicator, it can be seen that S01 perfectly showed the representation of vertices on graph. S01 showed the representation applies until n too. Then, S01 correctly showed the resolving total domination number in the minimum state. So that, S01 fullfill the indicator 4.

We can be seen that S01 fullfill the indicator 1 until indicator 4 perfectly. So that, S01 categorized as high level of generalization thinking skill. After that, we will showed potrait phase of S01 as followed.

**Figure 12. Potrait Phase of S01.**

We can see on the Figure 8 that generalization thinking skill from S01 was perfectly complete. S01 can recognize the graph as the first thing to do. After that, S01 goes to labelled the graph on 2a and count the cardinality of vertices and edge on 2b for the next. Forward from 2b, S01 goes to 3a that S01 were able to specify total dominating set and the minimum cardinality of total domination number on 3b. Then S01 goes to 4a that S01 determine the representation of vertices for show the resolving total dominating set. The last, S01 goes to 4b that S01 determine the minimum cardinality of resolving total domination number of the graph.

**Figure 13. The Results of S02.**

Based on Figure 9, it can be seen that S02 were able to recognize a graph. S02 correctly labeled each vertex and edge on the centipede graph. Then, S02 correctly showed the vertex and edge functions, and the cardinality of the vertices and edges in the centipede graph. So that, S02 fullfill the indicator 1 and indicator 2 perfectly.

**Figure 14. The Results of S02.**
Go on the Figure 10, it can be seen that S02 were able to specify total dominating set with the minimum total dominator. We can be seen that S02 correctly determine the minimum cardinality of total dominator set. So that, S02 fulfill the indicator 3 perfectly.

- **Vertice’s representation**
  - \( r(x_1|D_{c_n}) = (0, 1, 2, \ldots, n - 1) \)
  - \( r(x_2|D_{c_n}) = (1, 0, 1, 2, \ldots, n - 2) \)
  - \( r(x_3|D_{c_n}) = (2, 1, 0, 1, 2, \ldots, n - 3) \)
  - \( r(x_4|D_{c_n}) = (3, 1, 2, 0, 1, \ldots, n - 4) \)
  - \( r(x_i|D_{c_n}) = (n - 1, n - 2, 0, 1, 2, \ldots, n - i) \)

- **Because the representation of vertices on** \( D_2(G_n) \) **to** \( D_{c_n} = \{x_i : 1 \leq i \leq n - 1\} \) **is resolving total dominating set. Then,** \( y_{rt}(D_2(G_n)) = n - 1 \).

**Figure 15.** The Results of S02.

For the last indicator, it can be seen that S02 perfectly showed the representation of vertices on graph. S01 showed the representation applies until \( n \) too. However, S02 didn’t show the minimum cardinality of resolving total domination number. So that, S02 didn’t fulfill indicator 4. We can be seen that S02 fulfill the indicator 1 until indicator 3 but not indicator 4. So that, S02 categorized as medium level of generalization thinking skill. After that, we will showed the portrait phase of S02 as followed.

**Figure 16.** The Results of S02.

We can see on the Figure 12 that generalization thinking skill from S02 was not complete. S01 can recognize the graph as the first thing to do. After that, S02 goes to labelled the graph on 2a and count the cardinality of vertices and edge on 2b for the next. Forward from 2b, S02 goes to 3a that S02 were able to specify total dominating set but S02 jump to 4a that mean S02 determine the representation of the vertices before specify the minimum cardinality of total domination number on 3b. Then S02 backward to 3b and 4a for the last thing to do without choose 4b for determine the minimum cardinality of resolving total domination number of the graph.
Based on Figure 13, it can be seen that S03 were able to recognize a graph. S03 correctly labeled each vertex and edge on the centipede graph. So that, S03 fulfill the indicator 1 perfectly. It can be seen that S03 were able showed the vertex and edge functions, and the cardinality of the vertices and edges in the centipede graph. So that, S03 fulfill the indicator 2 perfectly.

For the indicator 3 and indicator 4, S03 didn’t show the representation of vertices and the minimum cardinality of resolving total domination number. So that, S03 categorized as low level of generalization thinking skill. After that, we will showed the portrait phase of S03 as followed.

We can see on the Figure 15 that generalization thinking skill from S03 was very not complete. S03 can recognize the graph as the first thing to do. After that, S03 goes to labelled the graph on 2a and count the cardinality of vertices and edge on 2b for the next. Forward from 2b, S03 goes to 3a that S03 were able to specify total dominating set but S03 jump to 4a that mean S03 determine the representation of the vertices before specify the minimum cardinality of total domination number on 3b. 4a was the last thing to do for S03 without choose 3b and 4b for determine the minimum cardinality of total domination number and resolving total domination number of the graph.

4.4 Discussion
This research was done to analyze the generalization thinking skills in the implementation of problem-based learning instrument. Generalization thinking skills fulfill several indicators; they are perception of generality, expression of generality, symbolic expression of generality and manipulation of generality. The subjects used were 24 students in the control class and 23 students in the experimental class. Problem-based learning was conducted on both class but especially on the experimental class we use student worksheet for determine generalization thinking skill. In the experimental class, it was found that there was an effect of student worksheet and problem-based learning on generalization thinking skills. We can see generalization thingking skill on experimental class on students answer sheet on post-
test. Student with higher generalization thinking skill (S01) has fulfilled all indicators of generalization thinking skill. The medium one (S02) only fulfilled 3 indicators of generalization thinking skill. Student with low generalization thinking skill (S03) only fulfilled 2 indicators of generalization thinking skill. Besides the student answer sheet, we can find out how their thinking flow through the portrait phase. S01 shown that the flow of his thought was in accordance with the indicator of generalization thinking skill. He solved resolving total dominating set coherently. S02 portrait phase shown that he past the 4b, jumped from 3a to 4a, and then backward from 3b to 4a. Thus, S02 not solved resolving total dominating set coherently. S03 portrait phase shown that he past the 3b and 4b, and jumped from 3a to 4a. Thus, S03 not solved resolving total dominating set coherently.

The results of the independent sample t-test on the pre-test questions of the control and the experimental class obtained \( p = 0.2682 \) and if \( p \)-value \( > 0.05 \), then there was no significant difference between the learning outcomes of the control class and the experimental class. The post-test results, the independent sample t-test conducted in the experimental and control class obtained Sig. (2-tailed) \( p = 0.01628 \) was \( p < 0.05 \). It can be concluded that the result of the post-test gained from and control classes had significant differences. The mean score of the control class was 12.75, while the experimental class was 13.96. It revealed that the average of students’ generalization thinking skills in the experimental class was significantly higher than the control one. This proves that the two classes represent differences in terms of students’ learning outcomes after the implementation student worksheet.

This research used a problem-based learning and student worksheet to improve students’ generalization thinking skills and it had been proved that it affected the student’s learning process. Before using problem-based learning and student worksheet, students had difficulty in finding ideas and class lectures. After conducted problem-based learning and student worksheet, they were active in finding the patterns of resolving total dominating set. Also assessed generalization thinking skills in their research. This research was done to analyze generalization thinking skills and the application of PBL and student worksheet. The PBL model provided opportunities for students to be more active in the learning process; beside that, the implementation of PBL and student worksheet improved generalization thinking skills. Finally we recommend the use of problem based learning in every advance subject courses.

5. Conclusion
The research results show that the implementation of Problem-Based Learning (PBL) student worksheet had a significant effect on students’ generalization thinking skills in the experimental class. The independent t-test of post-test score showed that the mean score of experimental class larger than the control class. The results show that there were improvements on experimental class of generalization thinking skills that depicted from the post-test. There were students with high generalization thinking skill were gain from 25% to 29%, the medium generalization thinking skill were gain from 40% to 55%, and the low generalization thinking skill decreased from 35% to 16%. Thus, the implementation of PBL student worksheet can improve the generalization thinking skill. However, we need to develop this research, so we offer to open problem to determine the implementation of Problem Based Learning instrument development to another thinking skill and vice versa.

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