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Food donation management under supply and demand uncertainties in COVID-19: A robust optimization approach

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A B S T R A C T

COVID-19 pandemic and the associated lockdown have globally impacted the food-insecure low-income population. A community-based initiative of an NPO - working on malnutrition issues among the below poverty line children in an urban Indian setting - towards adoption of the slum- and shelter-dwelling children, faced challenges during the pandemic in tackling the twofold uncertainties regarding supply capacity reduction and demand surge. This paper extends earlier research on this NPO’s pre-COVID-time food assistance program by introducing a robust optimization-based mathematical programming model to determine the donors to engage, donor-to-beneficiary allocations, and the associated optimal donation flows while addressing these uncertainties. The issue of over-conservatism in robust optimization is addressed by parameterizing the decision-maker’s uncertainty budget. We present a detailed numerical study on a test problem along with interesting observations elicited by our sensitivity analyses.

1. Introduction and motivation

The unprecedented situation caused by the COVID-19 pandemic has aggravated the already severe food insecurity issue worldwide. While lockdowns and various social distancing norms implemented by different nations are essential to curb the spread of coronavirus, the impacts of these steps on the food-insecure low-income population are overwhelming. People relying on various food assistance programs, community kitchens, food banks, etc., suffered as those food aid-providing institutions operated at capacity during the lockdowns. Although the initial panic-buying and hoarding tendencies at groceries and supermarkets [11,27] - causing a substantial reduction in donation flows into the food-aid providing institutions - eventually subsided, various social distancing norms and exclusion of volunteers belonging to the vulnerable age groups reduced the workforce [24,27]. Along with the capacity reduction of various food-support providers, a demand surge during and after the lockdown can be attributed to diverse factors. For example, many previously employed adults, after substantial income reduction and job losses [3,24], became eligible to receive institutional food supports. Moreover, due to the closure of schools in the pandemic, many schoolchildren who previously received meals from their schools, increased the demand for immediate food support [1,30]. This paper introduces a problem regarding the sustenance of a community-driven food assistance program during the COVID-19 related lockdown in India and proposes an optimization technique-based solution approach.

Food Education and Economic Development (FEED) [13], a non-profit organization (NPO) operating in Asansol, a city in the state of West Bengal, India, addresses food insecurity, healthcare, and education-related issues of the slum- and roadside shelter-dwelling children up to 14 years of age. In one of its social awareness-building efforts, FEED proposes to engage some interested and resourceful private schools (attended by students from financially well-off families) to ensure a sustained food supply to a pre-identified set of slums and shelters. The specific plan was to ask these interested schools to adopt one or more of these slums. At regular intervals, these school students would make a certain amount of in-kind donations in the form of uncooked food items (cash donation is discouraged). The schools’ primary motivation behind participating in the program lies in building their goodwill through association in such social welfare programs.

Dalal [9] develop a two-stage stochastic programming (SP) model for strategically connecting the set of donor schools to a group of pre-identified beneficiaries, the children up to 14 years of age living at certain slums and shelters in that city. Representing the uncertainties in the supply capacity of each donor and demand of each beneficiary by introducing the capacity- and demand scenario sets, the model determines optimal assignment while satisfying various constraints. Some practical challenges discussed in Ref. [9] include: (i) a participating donor (school) must be connected to a beneficiary (slum); (ii) a...
beneficiary cannot be adopted by multiple donors directly; (iii) such one-donor restriction can induce unwanted shortage in the system, i.e., even if a donor has excess capacity, under certain circumstances, it may not be allowed to serve a beneficiary because another donor has already got connected to that beneficiary; (iv) while adopting a beneficiary, the donor must commit a minimum quantity to satisfy a certain proportion of the beneficiary’s demand.

This paper extends the problem of [9] to address the urgency faced by FEED in sustaining their food support program in the wake of the pandemic. Due to sudden job loss, most parents of the target age-group children (the primary beneficiaries of FEED) also became dependent on some food support for their survival. The author’s interactions with the NPO’s management revealed that the first five months of lockdown were most critical when almost threefold demand escalation occurred at their work areas. This observation is supported by similar claims in the literature and media. Since most of the slum-dwellers are daily wage earners and marginalized workers with inadequate social security, the nationwide lockdown reduced their earning to almost zero overnight [21]. Recognizing the gravity of the situation, the NPO decided: 

(1) to adopt the beneficiaries as was the case in Ref. [9] - we would use the generalization capacity that depends on the number of individual donors, inclination towards these social causes, and economic condition. However, due to the limited capacity of the NPO in terms of volunteers, transport resources, time to devote, etc., collecting donations from too many sources was not practical. Also, the long lockdown period experienced a varying degree of enthusiasm and response from the donors (i.e., diminishing return from the repeat donors) - resulting in supply capacity uncertainties.

In this backdrop, given the NPO’s limited operating capacities, we propose an optimization problem to help the decision-makers determine a fixed number of donors from a given potential donor-set to strategically engage in the food support program. However, unlike [9], we do not consider the SP framework because we believe it is difficult (if not impossible) to reliably estimate the occurrence probabilities of the supply and demand uncertainty scenarios, given the rarity of the event of pandemic and unavailability of adequate past data for meaningful probability estimation. Also, since the schools are no longer the only donating entities - as was the case in Ref. [9] - we would use the general terms ‘donor’ and ‘beneficiary’ hereafter. We adopt the robust optimization (RO) based approach in this work, where the uncertainty sets capture uncertainties related to supply and demand parameters. The robust counterpart (ROC) of our proposed problem becomes a mixed-integer programming (MIP) model that includes donor engagement, donor-to-beneficiary allocation, and the flow quantities as the decision variables. Since over-conservatism is a common criticism against RO, we adopt the uncertainty budget concept introduced by Bertsimas and Sim [7], and provide the decision-maker a flexibility lever for adjusting his/her extent of conservatism.

The contributions of this research are summarized as follows:

1. We extend the problem in Ref. [9] for strategic donor-beneficiary assignment problem in the food insecurity domain by considering demand and supply related uncertainties that emerged from the COVID-19 situation.
2. Acknowledging the challenges in constructing realistic scenarios and estimating their occurrence probabilities, we adopt an RO-based approach to represent the supply capacity and demand-related uncertainties.
3. Our detailed sensitivity analyses elicit interesting insights about the problem.

The rest of this paper is organized as follows. We discuss the relevant literature in §2. Our problem setting and mathematical model are introduced in §3. In §4 we present numerical study with detailed sensitivity analyses on a test problem to understand the impacts of various model parameters on the solution. Finally, we conclude with some highlights on possible future research directions in §5.

2. Related literature

This research focuses on a strategic food-donation distribution network design problem faced by an NPO under supply capacity and demand-related uncertainties, as was witnessed by various parts of the world during the COVID-19 pandemic. Since we present an RO-based approach to determine a subset of potential donors who would be strategically engaged and allocated to a set of food-insecure beneficiaries, we categorize the relevant papers as (a) works focusing on food insecurity due to the COVID-19 pandemic (published mid-2020 onward), (b) operations research/management science (OR/MS) application papers on NPO activities addressing food insecurity, (c) OR/MS papers implementing robust optimization techniques on problems from various domains.

Several authors express their concerns about the escalation of food insecurity issues at different parts of the world due to pandemics and the authority-imposed extreme social distancing measures like regional or national level lockdowns [11,18,20]. The effects of such extreme measures on developing countries like India, where a vast number of daily wage earners had to choose between health risk and hunger, is reflected in Ref. [21]. On the other hand, a study by Principato et al. [29] highlights some positive effects of pandemic during the lockdown in Italy in terms of increased awareness about food waste reduction and the need for efficient household-level food management. While some authors [20, 22, 27] highlight the vulnerability of the food system in the UK, works of [1,30] focus on child obesity issues due to school closure and the reduced school-meal availability in the USA. Hobbs [17] provides an early assessment of the implication of the pandemic on food supply chains in Canada, which experience demand-side uncertainties due to panic buying, altered food consumption patterns (e.g., increase of home-cooked food in the western countries) as well as supply-side uncertainties resulting from reduced workforce and logistics network’s disruption.

As the COVID-19 pandemic spread globally, several authors focused on conducting surveys of different target populations with various research questions to understand its multi-dimensional impact on human lives. Niles et al. [24] conduct a population-level survey in Vermont, USA, to measure the effect of the government’s stay-at-home order on food insecurity at an early stage of the pandemic. Their survey indicates a statistically significant rise in the food-insecure household due to various reasons, including employment loss, pay cut, reduced physical access to food for public-transit dependent population, reduction in the choice of food, etc., to mention a few. Adams et al. [1] conduct an online survey on 584 US families to understand the pandemic’s impact on the home food environment by capturing the parent-reported food insecurity-related information that includes changed child feeding patterns. Ahn and Norwood [3] conduct an internet-based household-level survey in the USA to examine whether, in the wake of the pandemic, the increased unemployment level also increases food insecurity, or the government’s various financial countermeasures can put a restraint on it. As a part of Feeding America’s [14] ‘Map the Meal Gap’ (MMG) program, Gundersen et al. [16] conduct a survey to measure the county-level food insecurity in the wake of the pandemic. While masking the underlying geographic diversity, their study projects an increase of 17 million food-insecure US-population due to increased unemployment and poverty.
We now discuss the works that use OR/MS-based approaches in NPO operations focusing on food insecurity. Berenguer and Shen [5] present a recent review of various operations management challenges faced by NPOs. Several researchers consider optimizing various NPO operations such as the assignment of agencies to food banks, food donation collection, excess food rescue, and timely distribution of food to needy people. Since total donated food supply is, in general, inadequate to fulfill total demand, some works focus on the critical issue of maintaining equity or fairness in distributing scarce resources. Falasca and Zobel [12] address the challenge of volunteer assignments in humanitarian organizations, acknowledging the importance of including both the abilities and preferences of volunteers. The authors consider a multi-criteria decision-making framework with the objectives of reducing total shortage costs due to non-completion of tasks as well as minimizing the number of undesired assignments. For food banks operating under strict budget constraints, Davis et al. [10] present a set covering model to optimize donation collection and delivery schedules while considering constraints related to consecutive collection days, perishability of collected food, fleet capacity, etc. Nair et al. [23] present a variant of the vehicle routing problem (VRP) for food rescue and redirection. Instead of using cost minimization objective as done in a commercial model, the authors stress fairness by considering egalitarian (maximize the minimum utility) and utilitarian (maximize total utility) approaches in a goal programming framework. Granillo-Macias [15] present an optimization problem regarding food distribution and delivery logistics for a school meal support program in one state of Mexico. The author considers optimization problems to determine distribution center locations and cost-effective routes from those centers to reach the schools. They devise a variant of the Adaptive Large Neighborhood Search (LNS) algorithm to solve the VRP with a time window. A recent study by Dalal [9] focuses on the problem faced by an NPO working in the food insecurity domain for below-poverty level children in an urban Indian setting. The NPO attempts to connect a set of resourceful private schools to the malnourished slum-dwelling children via a social welfare program. In the presence of both supply-side (donation volume) and demand-side uncertainties (number of children), the paper introduces a two-stage stochastic programming (SP) model to strategically connect the donor schools to the beneficiary slums while satisfying several practical constraints. Our paper extends this problem by introducing a robust optimization-based model to address the aggravated supply and demand uncertainties in the wake of COVID-19.

We finally present a brief discussion of the robust optimization (RO) literature and RO-based models in the food insecurity domain. For an extensive theoretical background of RO, we refer to Refs. [4,6]. In the food insecurity domain, Orgut et al. [25] introduce a RO model to ensure equitable and effective distribution of donated food under capacity-related uncertainty of a food bank. The authors use a robustness control parameter or uncertainty budget as suggested in Ref. [7]. We also adopt this uncertainty budget concept of Bertsimas and Sim [7] to avoid obtaining an over-conservative and costly solution from our RO-based model. Although we do not find any other food insecurity-related research where RO framework is applied, a recent paper by Paul and Wang [26] in the humanitarian logistics domain uses the uncertainty budget to represent demand and response time-related uncertainties. We adopt their approach in our model’s uncertainty representation as illustrated later in §3.1.

In summary, we find many survey-based research and opinion papers discussing the effects of the pandemic on food insecurity. There also exist several OR/MS-based works in food insecurity and food waste that address problems related to service facility location, distribution logistics, equity, and effectiveness in distribution strategy, etc., either in a deterministic or stochastic setting. Therefore, our paper introduces a new RO-based model for optimally selecting the donors to engage and allocate to beneficiaries while avoiding the over-conservatism issue of RO using uncertainty budgets for supply and demand-side uncertainties.

3. Problem setting and model formulation

We consider a three-tier system comprising potential donors, beneficiaries, and one warehouse at the intermediate tier (see schematic diagram in Fig. 1). We want to determine which potential donors to engage and which donor-to-beneficiary assignments to make so that these donors’ supply capacities are effectively utilized to satisfy these beneficiaries’ demands while optimizing specific performance criteria and meeting several practical restrictions. The beneficiary set J comprises a pre-identified group of slums and shelters in the region under consideration by the NPO. The demand at a beneficiary node is generally estimated by the number of food-insecure children of up to 14 years of age. However, since in the wake of the pandemic, the NPO extended food assistance to these children’s adult family members, the total population of a slum or shelter is now used to estimate the nominal demand ($\bar{Q}_j$) at the beneficiary node $j \in J$. We use the number of meals per year as the aggregate level demand measure for each $j \in J$; however, we assume this demand to be uncertain due to future situations that the pandemic may unfold. For example, depending on the extension or shortening of the lockdown period, demands can change significantly. The pandemic severely affected the NPO’s service-providing capacity in terms of the non-availability of many senior volunteers (belonging to the COVID-vulnerable age group), lack of logistical support, and time to devote to social work, etc. Therefore, collecting donations from many small donors to accumulate operational expenses was not practical. To address this issue, various organizations, civil societies, local communities, clubs, corporate houses, etc., are considered as the consolidated donor entities, which, together, represent the potential donor-node set $I$. We are interested in finding which subset of $I$ should be engaged for running the NPO’s program. For every potential donor $i \in I$, we assume a nominal donation capacity ($\bar{Q}_i$) in terms of the number of meals per year (an aggregate unit) that can be supplied from their contribution. However, this supply capacity is subject to uncertainty due to many practical issues such as changes in the number of donors, contribution quantities, reduced motivation for repeated donation requirements, etc.

Among various approaches of uncertainty handling in optimization, stochastic programming (SP) and robust optimization (RO) are extensively adopted in the literature. The SP approach optimizes an expected measure (e.g., minimize expected cost, maximize expected profit),
On the other hand, the RO approach does not require any such occurrence probability estimation. RO ensures obtaining a feasible solution for any realization of the uncertain parameters as long as their values belong to the uncertainty set considered for the model. In our work, we adopt RO to address uncertainties regarding the supply capacity and demand values. Since the rare global event of the COVID-19 pandemic lacks past data that might help in realistically constructing scenarios with meaningful occurrence probabilities, we claim that the RO approach is better suited for modeling our decision problem with uncertainties.

We present the uncertain supply capacity (a random variable) \( \tilde{Q}_i \) of a potential donor node \( i \in I \) using: (i) nominal supply capacity \( \bar{Q}_i \) and (ii) maximum allowed deviation \( \Delta Q_i \) from the nominal. Specifically, we assume a symmetric uncertainty interval \( [\tilde{Q}_i - \Delta Q_i, \tilde{Q}_i + \Delta Q_i] \), around the \( \bar{Q}_i \) value, i.e., the actual capacity realization of donor \( i \) would lie within this interval. We assume \( \bar{Q}_i \) to be a constant fraction of \( \tilde{Q}_i \), and vary this fraction systematically for conducting sensitivity analyses in §4. A similar approach has been adopted for handling the beneficiary demand uncertainty, assuming the random demand \( \tilde{D}_j \) to lie in the interval \( [\tilde{D}_j - \Delta D_j, \tilde{D}_j + \Delta D_j] \), where \( \tilde{D}_j \) and \( \Delta D_j \) (meals per year) represent nominal demand and maximum allowed deviation from the nominal value, respectively.

Since the RO approach is often criticized for its conservatism, we adopt the concept of uncertainty budget introduced by Bertsimas and Sim [7] to provide the decision-maker with a tool to explore the trade-off between conservatism and the performance measure. Specifically, instead of allowing all uncertain demand and supply parameters to deviate from their respective nominal values, we adopt cardinality constrained uncertainty of Bertsimas et al. [6], in which the maximum number of parameters allowed to deviate simultaneously is restricted. We discuss the details of our RO approach in §3.2 and present a detailed analysis in §4.2.

We now discuss the detailed setting of our decision problem. Being an optimization problem having a humanitarian perspective, profit maximization or cost minimization need not be our sole objective. However, as the NPO operates under a tight budget, cost reduction is an important concern while also keeping the unmet demand low. Since the economic estimation of the effort of engaging a donor node in this endeavor is difficult, we use as a proxy the maximum number of potential donor nodes that can be engaged (parameter \( P \)) in this program. Thus a large value of \( P \) represents the NPO’s adequate capability of fulfilling the logistical needs to run the food support operation with many donors.

The optimization problem involves strategic decision of engaging at most \( P \) potential donors (decision variable \( y_i \)), donor–beneficiary assignment (decision variable \( u_{ij} \)), and donation flows using a direct (solid arrows in Fig. 1; flow variable \( x_{ij} \)) or an indirect (dotted arrows in Fig. 1; flow variables \( v_i, w_j \) mode [the latter via an warehouse]. Of course, these flow values must obey the demand and supply capacity related constraints for each beneficiary- and donor node.

In our problem setting, a beneficiary cannot be adopted directly by multiple donors since such engagement is discouraged to avoid conflicts and complexities in NPO operations. However, the opposite is possible, i.e., a donor with a large capacity may adopt multiple beneficiaries directly. Typically, in our problem domain, even in the absence of uncertainties, total demand often overwhelms total supply causing a shortage in the system, therefore, a combined effect of supply and demand uncertainties would only aggravate the shortage. In addition to that, the ‘one donor for one beneficiary’ condition may become more restrictive, leading to additional (however, avoidable) shortages. To address this issue, we introduce an intermediate warehouse (an approach also adopted in Ref. [9]) that can accept inflows from multiple donors and distribute them to one or more beneficiaries based on the requirement. However, instead of considering an explicit warehouse opening decision like [9], we do not associate any capacity limit, holding cost, or fixed cost to warehouse node, because we treat it as a mechanism to achieve pooling benefits. Also, the warehouse being a transshipment node, its total inflow must equal total outflow. If need arises, indirect flows (dashed arrows in Fig. 1) via warehouse can reach a beneficiary node, thus, providing a mix of direct and indirect donations.

We adopt some other conditions of Dalal [9] to our current problem setting. Note that since an optimal solution may allow some donors to fulfill a small proportion of a beneficiary’s demand, that donor can claim to have adopted that beneficiary (which would be unfair for other donors). To avoid such a conflict-raising situation, we impose a condition regarding ensuring the minimum direct donation flow. In other words, if a donor sends the direct flow to a beneficiary, it must satisfy at least a predefined fraction (parameter \( \eta \)) of the beneficiary’s demand. A high value of \( \eta \) would impose a more stringent condition on a donor for adopting a beneficiary.

We now explain the rationale behind our logistical cost considerations for the direct and indirect flows. First, note that the nonprofit operations literature has discussed the benefits of earmarking donations, however, at the cost of sacrificing flexibility of utilizing the fund [2,5]. In our problem setting, the direct flow represents donation earmarked for specific beneficiaries, while the indirect flow represents the non-earmarked donation. Since earmarking is known to positively impact donation volume, to manage the NPO’s operations during a challenging time of the pandemic, we prefer to adopt the strategy that would secure more donations. Therefore, we assume that the direct donor–beneficiary flow is preferred over the indirect flow. In the case of direct flow, a donor–beneficiary association is maintained, which gets lost in the indirect one. To establish this preference, in our parameter setting for the mathematical model, we set the unit cost for indirect flow (\( c_{ij} \) or \( c_j \)) as five times the direct flow (\( c_{ij} \)). While we do not ensure complete demand fulfillment of all beneficiaries, to reduce the possibility of incurring any avoidable shortage just for saving on logistical costs, we charge a large penalty for unmet demand at beneficiary nodes. One can set different values for this penalty cost parameter \( \Pi_j \) depending on appropriate socio-economic conditions at every beneficiary node \( j \); however, we keep \( \Pi_j = \Pi, \forall j \in J \), since all the beneficiary locations are comparable (all within a city limit) in our problem. With the above setting, we present an optimization problem to minimize the sum of donation distribution and shortage cost.

3.1. Model formulation

We present all notation in Table 1 and also refer the reader to Fig. 1. Then, we formulate our robust mathematical model (P-RO) as follows:

\[
\text{Min } Z = \sum_{i \in I} \sum_{j \in J} c_{ij}x_{ij} + \sum_{i \in I} \sum_{j \in J} c_jv_i + \sum_{j \in J} \sum_{i \in I} c_jw_j \tag{1}
\]

subject to

\[
\sum_{i \in I} y_i \leq P \tag{2}
\]

\[
\sum_{i \in I} u_{ij} \leq 1 \quad \forall j \in J \tag{3}
\]

\[
u_{ij} \leq y_i \quad \forall i \in I, j \in J \tag{4}
\]

\[
\sum_{i \in I} u_{ij} \geq y_i \quad \forall i \in I \tag{5}
\]

\[
x_{ij} \leq M_iu_{ij} \quad \forall i \in I, j \in J \tag{6}
\]
\[\sum_{j \in J} w_j + w_i \leq \tilde{Q}_i \quad \forall i \in I \tag{7}\]
\[\sum_{i \in I} x_i + w_j + s_j \geq \tilde{D}_j \quad j \in J \tag{8}\]
\[\sum_{j \in J} w_i = \sum_{i \in I} v_i \tag{9}\]
\[v_i \leq M_j y_i \quad \forall i \in I \tag{10}\]
\[w_j \leq M_i \sum_{i \in I} u_i \quad \forall j \in J \tag{11}\]
\[x_0 \geq \eta D_i u_i \quad \forall i \in I, j \in J \tag{12}\]
\[y_i, u_i \in \{0, 1\} \quad \forall i \in I, j \in J \tag{13}\]
\[x_0, v_i, w_j, s_j \geq 0 \quad \forall i \in I, j \in J \tag{14}\]

The objective function (1) minimizes sum of distribution- and shortage cost incurred across the system. First, constraint (2), by ensuring engagement of at most P donors in the program, puts an implicit budget constraint or capacity limitation of the NPO. Constraint (3) ensures that for a particular beneficiary j, at most one direct donation inflow would occur. Next, by constraints (4) and (5) together, we establish the relationship between engagement of a donor node and donor-to-beneficiary assignment. While \(y_i = 0\) ensures that no assignment of donor i would be possible (since all \(w_i = 0\), constraint (5) tells that if a donor is engaged (i.e., \(y_i = 1\)), at least one beneficiary must be assigned to it. We next add a big-M based forcing constraint (6) to connect the direct donation flow \(x_0\) and the corresponding binary indicator variable \(u_i\). Supply capacity and demand constraints are represented by (7) and (8), respectively. Equation (9) represents flow balance constraint at the transshipment node (warehouse). Another forcing constraint (10) ensures that no donor-to-warehouse flow is possible if the donor is not engaged. Constraint (11) includes an important condition: if a beneficiary j does not receive any direct donation from any i, then it is also not allowed to receive indirect donation from the warehouse. Next, constraint (12) implements the condition regarding minimum donation quantity in the case of a direct donation flow from i to j by stating that: if a direct flow occurs, its quantity must be at least \(\eta\) fraction of the effective demand \(\tilde{D}_j\) of the beneficiary j. Finally, constraints (13)–(14) list the decision variables.

### 3.1.1. Difference between the model in Ref. [9] and its extension P-RO

As we extend this P-RO model from Ref. [9], we highlight below the critical differences between these two works. First [9], present a two-stage stochastic programming (SP) model where the supply-demand uncertainties are represented by scenarios, and therefore, all constraints and flow variables therein are augmented with a scenario index. The scenario probabilities are critical inputs for the SP model, which are difficult to estimate in our current problem’s context (no historical data on the pandemic-induced supply/demand uncertainties). The P-RO model and its robust counterpart (ROC) that we derive next in §3.2, do not need probability inputs. Moreover, we use uncertainty budget parameters regarding supply capacity and demand values to control the level of conservatism. Secondly, in addition to the strategic decisions involving connection of donors to beneficiaries as also taken in Ref. [9], P-RO model includes an additional level of complexity in terms of decision variable \(y_i\), i.e., deciding which donors to engage. The upper limit \(P\) on the number of such donors represents the NPO’s capacity/budget limitation. Thirdly, the constraints (4), (5), (10) involving \(y_i\) are not present in Ref. [9]. The remaining constraints, representing various conditions faced by the NPO are taken from the SP setting in Dalal [9], however, we make necessary adjustments to those while deriving their ROCs.

#### 3.1.2. Estimating big-M values

Since it is well-known that the use of some arbitrarily large big-M values may lead to issues regarding precision and numerical instabilities, we use some better estimated values for \(M_1\), \(M_2\), and \(M_3\) for the three big-M parameters in constraints (6), (10), and (11). Acknowledging that the maximum direct flow between an (i, j) pair cannot exceed the minimum value between i) maximum supply capacity of i, and ii) maximum demand of j, we set \(M_1 = \min(\tilde{Q}_i, \tilde{D}_j)\). Using similar observation, we set \(M_2 = (\tilde{Q}_i + \tilde{Q}_j)\) and \(M_3 = (\tilde{D}_j + \tilde{D}_j)\).

#### 3.1.3. Uncertainty representation in RO model

In the above model P-RO, we consider two uncertain parameters, namely, supply capacity of donor \(i \in I\) and demand of beneficiary \(j \in J\), that are represented by random variables \(\tilde{Q}_i\) and \(\tilde{D}_j\), respectively. Values of these random variables are contained in uncertainty sets. We first present the set \(\Phi^0\) as follows:

\[\Phi^0 = \left\{ \tilde{Q}_i \in \mathbb{R} : \frac{\tilde{Q}_i - \tilde{Q}_i^0}{\tilde{Q}_i} \leq \frac{\Gamma}{|\Gamma|} \quad \forall i \in I \right\}\]

The ratio \(\tilde{Q}_i^0 / |\Gamma|\) controls the extent of actual deviation (the maximum allowed deviation is \(\tilde{Q}_i^0\) from its nominal value \(\tilde{Q}_i\). The denominator in this ratio represents the maximum number of \(\tilde{Q}_i\) parameters that can deviate from their nominal values. Thus, the random parameter \(\tilde{Q}_i\) assumes a value from the range \([\tilde{Q}_i - (\frac{\Gamma}{|\Gamma|}) \tilde{Q}_i, \tilde{Q}_i + (\frac{\Gamma}{|\Gamma|}) \tilde{Q}_i]\).

Similarly, we define the demand uncertainty set as follows:

\[\Phi^0 = \left\{ \tilde{D}_j \in \mathbb{R} : \frac{\tilde{D}_j - \tilde{D}_j^0}{\tilde{D}_j} \leq \frac{\Gamma^0}{|\Gamma^0|} \quad \forall j \in J \right\}\]

The random parameter \(\tilde{D}_j\) would assume a value from the range \([\tilde{D}_j - (\frac{\Gamma^0}{|\Gamma^0|}) \tilde{D}_j, \tilde{D}_j + (\frac{\Gamma^0}{|\Gamma^0|}) \tilde{D}_j]\).

Note that an increase in the uncertainty budget via the parameters \(\Gamma^Q\) and \(\Gamma^D\) would lead to an expansion of the corresponding uncertainty sets. A larger uncertainty set represents increased level of conservatism and a costlier solution. If we consider all donors’ capacities and all beneficiaries’ demands to change simultaneously from their corresponding nominal values (an extreme situation hardly expected in reality), then \(\Gamma^Q = |\Gamma|\) and \(\Gamma^D = |\Gamma|\), i.e., the ratios \(\tilde{Q}_i^0 / |\Gamma|\) and \(\tilde{D}_j^0 / |\Gamma|\) become 1, and we reach the most conservative case (Soyster [31]). On the other hand, setting uncertainty budgets \(\Gamma^Q\) and \(\Gamma^D\) to zero implies that all uncertain parameters would take their respective nominal values, and we obtain the deterministic model. Thus, by adjusting the uncertainty budget, the decision-maker can control the level of conservatism and obtain a practically useful solution with a reasonable cost implication.

### 3.2. Robust counterpart derivation

In the constraints (7), (8), and (12) of P-RO model, the presence of uncertain parameters represented by random variables \(\tilde{Q}_i\) and \(\tilde{D}_j\) require us to develop robust counterparts (ROC) of those constraints. We would illustrate this derivation in detail, and present the complete P-ROC, the robust counterpart of P-RO at the end of this section.

To determine the ROCs, we adopt the concept of uncertainty budget introduced in Ref. [7]. However, we illustrate below how a straightforward application of the approach of Bertsimas and Sim [7] that uses dualization of an inner maximization problem, on our constraints (7), (8), and (12) produces an extremely conservative model similar to Soyster [31] where the highest protection is admitted by considering simultaneous worst-case change in all the uncertain model parameters. Therefore, we adopt the approaches of Liu et al. [19], Paul and Wang [26] that suggest creating a common uncertainty budget for a group of...
We mitigate the same using a common uncertainty budget.

Note that unlike in Ref. [7] where the uncertain parameters appear at the left-hand-side of an inequality, we have \( Q_i \) parameters at the right-hand-side of (7). However, we can consider a variable \( \mu_i \) to be multiplied to \( Q_i \), whose value equals to 1, i.e., we add an explicit constraint (fixing the variable) \( \mu_i = 1 \), \( \forall i \in I \) and rewrite (7) as:

\[
\sum_{i \in J} x_{ij} + v_i - \tilde{Q}_i \mu_i \leq 0,
\]

\( \mu_i = 1 \) \( \forall i \in I \).

Now, as per the dualization-based approach of [7], the ROC of (7) becomes:

\[
\sum_{i \in J} x_{ij} + v_i - \tilde{Q}_i \mu_i + \beta_i(\Gamma^Q_{\mu_i} \tilde{\mu}_i) \leq 0
\]

\( \forall i \in I \).

Here, \( \Gamma^Q_{\mu_i} \) is the uncertainty budget, i.e., the number of parameters in the constraint \( i \in I \) whose values can change from their corresponding nominal values simultaneously. With dual variables \( \varphi_{\mu_0} \) and \( \psi_{\mu_0} \), the corresponding minimization problem for each \( i \in I \) becomes:

Minimize \( \varphi_{\mu_0} + \Gamma^Q_{\mu_i} \varphi_{\mu_0} \)

subject to

\[
\varphi_{\mu_0} + \psi_{\mu_0} \geq \tilde{Q}_i \tilde{\mu}_i
\]

\( \varphi_{\mu_0}, \psi_{\mu_0} \geq 0 \).

Note that in each constraint of type (7), since there is only one uncertain parameter \( Q_i \), we have \( \Gamma^Q_{\mu_i} = 1 \). Consequently, the above dual problem’s objective becomes \( \varphi_{\mu_0} + \psi_{\mu_0} \), which is identical to the left-hand-side of the \( \tilde{Q}_i = \mu_i \) constraint. Hence, each minimization problem \( i \) solves trivially with the optimal value equal to \( \tilde{Q}_i \tilde{\mu}_i = Q_i \) (since \( \tilde{\mu}_i = 1 \) \( \forall i \in I \)). This makes the ROC of (7), where we consider a simultaneous worst-case deviation of all uncertain parameters \( Q_i \) from their nominal values:

\[
\sum_{i \in J} x_{ij} + v_i \leq \tilde{Q}_i - \tilde{Q}_i \quad \forall i \in I.
\]

But, as we do not want such extreme conservatism, we need to devise a way to adjust the level of protection. To this end, we consider a common uncertainty budget \( \Gamma^Q \in [0, |I|] \) (index \( i \) is dropped in \( \Gamma^Q \)) to indicate how many \( Q_i \) parameters can deviate simultaneously from \( \tilde{Q}_i \), their respective nominal values. Then, we present the ROC of (7) as follows:

\[
\sum_{i \in J} x_{ij} + v_i \leq \tilde{Q}_i - \tilde{Q}_i \times \left( \frac{\Gamma^Q}{|I|} \right) \quad \forall i \in I.
\]

A similar approach produces the ROC of demand constraint (8) as follows:

\[
\sum_{i \in J} x_{ij} + w_j \leq \tilde{D}_j + \tilde{D}_j \times \left( \frac{\Gamma^D}{|J|} \right) \quad \forall j \in J,
\]

where the uncertainty budget parameter \( \Gamma^D \in [0, |J|] \) indicates the number of \( D_j \) parameters that can deviate simultaneously from their respective nominal values \( \tilde{D}_j \). Using the same approach and \( \Gamma^D \in [0, |J|] \), we derive the ROC of (12) as follows:

\[
x_{ij} \geq \eta \left[ \tilde{D}_j + \tilde{D}_j \times \left( \frac{\Gamma^D}{|J|} \right) \right] u_{ij} \quad i \in I, j \in J.
\]

The detailed derivations of (16) and (17) are presented in Appendix A.

3.3. The ROC model

With ROCs of certain constraints as discussed above, the complete version of robust counterpart P-ROC is presented below.

\[
(P - ROC) \quad \text{Min} \quad Z = \sum_{i \in J} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in J} c_i v_i + \sum_{j \in J} c_j w_j + \sum_{j \in J} \Pi D_j
\]

subject to

\[
(2) - (6), (9) - (11), (13), (14), (15) - (17).
\]

4. Numerical study

In this section, we first discuss our approach of constructing a test case with randomly generated data. Our analyses on impacts of varying different model parameters are discussed next. Then, we illustrate a stochastic programming (SP)-based formulation of our problem.

4.1. A test case

To illustrate various trade-offs and effects of changing certain parameters on optimal solution, we construct a test case with randomly generated parameters. The problem instance consists of 12 potential donors, 8 beneficiaries, and 1 warehouse. As all locations are within the city limit, to represent those in our model, we take a 100 x 100 grid and randomly generate the (X,Y) coordinates for the locations of these 21 nodes. We use Euclidean distance between the node pairs as a proxy of distance. The implementation is done using Concert Technologies (Java with CPLEX 12.7). All instances are solved to optimality within a minute on a laptop with Intel (R) Core (TM) i5-4300 U CPU @ 1.90 GHz processor and 16 GB RAM.

We group these donor and beneficiary nodes into high and low categories, based on their nominal annual supply capacities \( Q_i \) and demands \( D_j \). Specifically, we use for the high capacity donor: \( Q_i \sim \text{Uniform}[20,000; 30,000] \), and for the low capacity donor: \( Q_i \sim \text{Uniform}[10,000; 16,000] \). For the beneficiary nodes, for high demand we use \( D_j \sim \text{Uniform}[15,000; 25,000] \), and for low demand we use: \( D_j \sim \text{Uniform}[7,000; 12,000] \). The maximum allowed deviations from nominal parameter values, i.e., \( Q_i \) and \( D_j \) are calculated as \( Q_i = f_i \times \tilde{Q}_i \) and \( D_j = g_j \times \tilde{D}_j \), respectively. In the numerical analyses that we discuss next, we systematically vary the fractions \( f_i \) and \( g_j \) with four specific values: 5%, 10%, 15%, and 20%.

Next, assuming direct and indirect distribution costs to be linear functions of the Euclidean distances between the node pairs, we calculate \( c_{ij} = a_i \times d_{ij} \), \( c_i = a_2 \times d_{ij} \), and \( c_j = a_2 \times d_{ij} \), where \( d_{ij} \) and \( d_{ij} \) represent distances between donor \( i \) to warehouse, and warehouse to beneficiary \( j \), respectively. While the unit direct transportation cost is set as \( a_1 = 0.1 \), the unit indirect cost is considered as \( a_2 = 0.5 \) to discourage the indirect donation flows.

With the above settings, we conduct sensitivity analyses of parameters regarding the minimum demand fulfillment commitment \( \eta \), uncertainty budgets on supply capacities and demands \( \Gamma^Q, \Gamma^D \), maximum allowed deviations \( Q_i, D_j \), and the NPO’s capacity as captured by the number of donors to engage \( P \). The detailed discussions on these analyses are presented in §4.2. We also discuss in §4.3 the SP-based programming approach if the supply and demand uncertainties cannot be represented by scenario sets and their occurrence probabilities can be estimated realistically.
4.1.1. A note on estimating the model parameters in practice

While we present a decision-making problem motivated by the practical challenges faced by an NPO during the nation-encouraged lockdown in the wake of the COVID-19 pandemic, our model parameters are randomly generated. We now suggest some possible ways of incorporating actual data into the model. We first note that the number of donor entities to engage in the food assistance program (P value) is primarily a call that the NPO needs to make by assessing its capability. To this end, the NPO should consider staffing, time, effort, physical distances from the NPO’s base location, etc. While a larger P value would improve responsiveness, it would demand more resources.

We set the unit costs for direct and indirect modes of distribution to establish a stronger preference for the direct mode. The underlying reason has already been discussed before in §3. However, we need not minutely estimate the cost implications for using these two modes. A decision-maker can either accept the values we use or choose any other appropriate for their parameter setting. Also, since our study region is restricted to a small city, we use the Euclidean distance between any node pair to give a reasonable distance estimate. But, if the model is adopted for larger geography, e.g., state- or nation-wide network, then the actual transportation network distance from a Geographic Information System (GIS) would be more appropriate.

As we consider a 365-day meal-providing plan, the annual demand at a beneficiary is calculated as: Annual Demand (meals/year) = (Number of beneficiaries × Number of meals per day) × 365. The particular NPO we collaborated with provides a single meal during the regular time; however, during the pandemic, they provided two meals per person daily at some extremely backward areas. Identification of potential community-beneficiaries restricted to a small city, we use the Euclidean distance between any node pair to give a reasonable distance estimate. But, if the model is adopted for larger geography, e.g., state- or nation-wide network, then the actual transportation network distance from a Geographic Information System (GIS) would be more appropriate.

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4.2. Numerical analyses

We now discuss different experiments conducted to highlight the impacts of varying the minimum fractional demand to be satisfied by direct donation (η), the number of donors to engage (P), uncertainty budgets (Γ^D and Γ^P), and allowed maximum supply and demand deviations from the nominals (Q̃, D̃). We also investigate the effects of not allowing pooling via warehouse. In the numerical analysis, we explain the effect of changing one parameter at a time, while keeping all other parameters unchanged. Specifically, on a network with |J| = 12, |I| = 8, and one warehouse, we set η to 0.25, and 0.5; vary parameter P from 2 to 6; Γ^D ∈ [0, |I|] and Γ^P ∈ [0, |J|]. We present partial results along with our following discussions on various parameter changes.

4.2.1. Effects of η and P

As parameter η - the minimum proportion of effective demand of a beneficiary node (D̃) that must be satisfied via direct flow - increases, the constraint (17) becomes stricter. Consequently, the objective function would either remain unchanged or deteriorate (i.e., increase in our minimization problem). The same is observed as η changes from 0.25 to 0.50, keeping all other model parameters fixed.

To illustrate the η-effect, we fix P and Γ^P, switch between η = 0.25 and 0.5, while gradually increasing Γ^D. With Γ^D = 0 (i.e., no capacity-related uncertainty), for Γ^P = 0, ... , 3 (integer values only), we do not observe any change in the objective function value (OFV) between η = 0.25 and 0.5 settings. As the increase in Γ^P = 4, ... , 8, we observe an increase in ∆OFV, (the difference between OFVs for η = 0.25 and 0.5), however, with P = 2, ... , 4. As the engagement of fewer donors (lower P) already causes shortages in the system, (costlier) indirect flows are discouraged. Hence, for P = 2, ... , 4, with the increase of η, the increase in OFV is primarily due to direct flow, and no change in the indirect cost or shortage penalty have been observed. However, as P is further increased, engagement of more donors help in reducing shortage. Then, a trade-offs between shortage penalty and indirect flow (5-time costlier than the direct flow) dictates whether the system’s flexibility would be exploited by connecting multiple beneficiaries from one donor via the warehouse. Table 2 exhibits a representative sample result, with Γ^P = 6, Q̃ = D̃ = 5% of nominal values, P = 2 and 5. Table 2 also illustrates that the increase of P (more donors) causes a significant decreases of shortage, resulting in overall decrease of the OFV.

The impact of P is further illustrated in Table 3, where we show the changes in OFV, shortage- and distribution costs, as P increases. We set η = 0.25, Q̃ = D̃ = 5% of the respective nominal parameter values, and consider three uncertainty budget pair values (Γ^P, Γ^D) = (0, 0), (6, 4) and (12, 8), representing the deterministic (no uncertainty budget, i.e., nominal), medium uncertainty budget, and the most conservative

![Table 1](https://example.com/table1.png)

| Table 1: List of notations. |
|-----------------------------|
| **Sets & Indices**          |
| i                          | set of potential donors, i ∈ I |
| J                          | set of beneficiaries, j ∈ J |
| **Parameters**             |
| P                          | maximum number of donors that can be engaged |
| J^i                        | unit distribution cost from donor i ∈ I to warehouse |
| J^j                        | unit distribution cost from warehouse to beneficiary j ∈ J |
| D^i_j                      | nominal aggregated demand of j ∈ J (of meals/year) |
| D̃_j                       | maximum allowed deviation from D^i_j (of meals/year) |
| D̃_j^eff                    | effective demand at j ∈ J (of meals/year) |
| Q̃_j^eff                   | nominal aggregated supply from i ∈ I (of meals/year) |
| Q̃_j                       | maximum allowed deviation from Q̃_j (of meals/year) |
| I^P                        | effective supply at i ∈ I (of meals/year) |
| Π^P                        | penalty for unmet demand at j ∈ J (set Π^P = Π, j ∈ J) |
| η^<0                       | minimum demand-fraction to be fulfilled by direct flow |
| Γ^P                        | uncertainty budget on supply (of donors whose effective capacities can simultaneously deviate from Γ^P) |
| M_j                        | i = 1, ... , 3; very large constants (big M’s) |
| **Decision Variables**      |
| y_i                        | 1, if donor i ∈ I is engaged, 0 otherwise |
| x_i^j                      | donation flow from donor i ∈ I to beneficiary j ∈ J |
| u_j                        | 1, if donor i ∈ I adopts beneficiary j ∈ J, 0 otherwise |
| v_j                        | flow from donor i ∈ I to warehouse |
| w_j                        | flow from warehouse to beneficiary j ∈ J |
| s_j                        | shortage (unmet demand) at beneficiary j ∈ J |
and gradual reduction in the distribution cost. Although for our numerical analyses we vary $\eta = 0.25$, $\bar{P}_i = \bar{D}_i = 5\%$ of nominals.

### Table 2

Effect of varying $\eta$ on OFV; Parameter settings: $\Gamma^Q = 6$, $\bar{Q}_i = \bar{D}_i = 5\%$ of nominals.

| $\eta$ | $\Gamma^D$ | $P$ | OFV | $\Delta$OFV | $P$ | OFV | $\Delta$OFV |
|--------|------------|-----|-----|-------------|-----|-----|-------------|
| 0.25   | 1          | 2   | 7,351,333 | 5  | 624,873 | 101210 |
| 0.50   | 1          | 2   | 7,351,753 | 420 | 624,873 | 0 |
| 0.25   | 2          | 2   | 7,428,559 | 5  | 634,583 | 0 |
| 0.50   | 2          | 2   | 7,429,219 | 660 | 634,583 | 0 |
| 0.25   | 3          | 2   | 7,505,784 | 5  | 687,413 | 0 |
| 0.50   | 3          | 2   | 7,506,684 | 900 | 687,413 | 0 |
| 0.25   | 4          | 2   | 7,583,010 | 5  | 759,683 | 0 |
| 0.50   | 4          | 2   | 7,584,150 | 1140 | 759,683 | 0 |
| 0.25   | 5          | 2   | 7,660,236 | 5  | 831,863 | 0 |
| 0.50   | 5          | 2   | 7,661,616 | 1380 | 831,863 | 0 |
| 0.25   | 6          | 2   | 7,737,461 | 5  | 904,088 | 0 |
| 0.50   | 6          | 2   | 7,739,081 | 1620 | 904,088 | 0 |
| 0.25   | 7          | 2   | 7,814,687 | 5  | 971,750 | 0 |
| 0.50   | 7          | 2   | 7,816,547 | 1860 | 971,750 | 0 |
| 0.25   | 8          | 2   | 7,891,913 | 5  | 1,033,988 | 0 |
| 0.50   | 8          | 2   | 7,893,908 | 1995 | 1,033,988 | 0 |

### Table 3

Effect of varying $P = 2, \ldots, 8$; Parameter settings: $\eta = 0.25$, $\bar{Q}_i = \bar{D}_i = 5\%$ of nominals.

| $(\Gamma^Q, \Gamma^D)$ | $P$ | OFV | Shortage Cost | Distribution Cost |
|------------------------|-----|-----|---------------|-------------------|
| (0, 0)                 | 2   | 7,143,000 | 6,900,000 | 243,000 |
|                        | 3   | 4,723,000 | 4,400,000 | 323,000 |
|                        | 4   | 2,358,000 | 1,900,000 | 438,000 |
|                        | 5   | 534,400   | 534,400   | 534,400 |
|                        | 6   | 450,600   | –         | 450,600 |
|                        | 7   | 421,800   | –         | 421,800 |
|                        | 8   | 421,400   | –         | 421,400 |
| (0, 4)                 | 2   | 7,583,010 | 7,347,500 | 235,510 |
|                        | 3   | 5,223,510 | 4,910,000 | 313,510 |
|                        | 4   | 2,897,440 | 2,472,500 | 424,940 |
|                        | 5   | 759,638   | 132,500   | 627,138 |
|                        | 6   | 486,878   | –         | 486,878 |
|                        | 7   | 460,800   | –         | 460,800 |
|                        | 8   | 450,415   | –         | 450,415 |
| (12, 8)                | 2   | 8,022,325 | 7,795,000 | 227,325 |
|                        | 3   | 5,723,325 | 5,420,000 | 303,325 |
|                        | 4   | 3,455,200 | 3,045,000 | 410,200 |
|                        | 5   | 1,317,825 | 765,000   | 552,825 |
|                        | 6   | 547,870   | –         | 547,870 |
|                        | 7   | 485,275   | –         | 485,275 |
|                        | 8   | 476,980   | –         | 476,980 |

settings, respectively. Note that along with the reduction in shortage cost (that eventually becomes zero for large $P$), a gradual increase in the distribution cost is observed as $P$ increases from 2 to 5. This is attributed to an increase in the flow from more donors, primarily using direct flow. As $P$ further increases, the donor-beneficiary distance reduction causes a gradual reduction in the distribution cost. Although for our numerical analyses we vary $P$ from 2 to 6, to specifically highlight this distribution cost changing pattern, we took the relevant runs with two more values, i.e., $P = 7, 8$ and present the OFV, shortage- and distribution cost values in Table 3. A comparison of costs corresponding to the three $(\Gamma^Q, \Gamma^D)$ values also highlights the impact of uncertainty budget on the solution. Table 3 also shows that for a particular $P$, a higher OFV and shortage cost are incurred with a larger uncertainty budgets $(\Gamma^Q, \Gamma^D)$. The general patterns of change in different cost components has been observed to be the same for each $(\Gamma^Q, \Gamma^D)$ pair.

#### 4.2.2. Varying uncertainty budgets $(\Gamma^Q, \Gamma^D)$ and maximum deviations $(\bar{Q}_i, \bar{D}_i)$

As we consider uncertainties in both supply capacities at donor nodes and demands at beneficiary nodes, changes in uncertainty budgets $(\Gamma^Q$ and $\Gamma^D)$ would impact the optimal solution. With this change, if the maximum allowed deviations from the nominal parameter values are also altered, an additional impact would be observed. In Fig. 2, we highlight these effects by presenting our model’s solutions with $\Gamma^Q = 0, 4, 8,$ and 12, while systematically varying $\Gamma^D \in \{0, |J|\} = \{0, 8\}$. We fix other parameters as $\eta = 0.25$, $P = 3$, and set the maximum allowed deviations $\bar{Q}_i$ and $\bar{D}_i$ to 10% (top graph) and 20% (bottom graph) of the respective nominal values, i.e., $\bar{Q}_i$ and $\bar{D}_i$. We discuss the important observations below.

In both sides of Fig. 2, each line corresponding to a specific $(\Gamma^Q, \Gamma^D)$ on shortage, expressed as the percentage change over its base value, i.e, the shortage incurred with $(\Gamma^Q = \Gamma^D = 0$ setting. For both 10% (Fig. 3 - top) and 20% (Fig. 3 - bottom) allowed deviations from the nominals, an increase in shortage is experienced with the increase of $\Gamma^D$. However, the highest percentage increase in shortage is observed in case of $\Gamma^Q = 0$ and it gradually decreases for $\Gamma^Q = 4, 8, 12$. This pattern is observed due to the gradual increase in the base value of shortage as $\Gamma^Q$ value increases, thereby reducing its percentage increase. Although we explain this for the $P = 3$ and $\eta = 0.25$ settings used in Fig. 3, the same argument is applicable for the other values of $P$ and $\eta$ considered in our numerical analysis.

![Fig. 2. Effects of uncertainty budgets and maximum deviations on OFV ($\eta = 0.25, P = 3$).](image)
amount of data, the possibility of constructing realistic scenarios cannot be overlooked. Therefore, in this section, we discuss how with a scenario-based uncertainty representation we can use a two-stage stochastic programming (SP) approach in our problem.

Note that the purpose of this section is to illustrate how SP can be used to model our problem in the presence of adequate and reliable data along with their associated probability distributions. We present below the new notations and then, our SP model.

In addition to previously defined notations, we introduce some new ones for the SP model:

| Symbol | Description |
|--------|-------------|
| \( \Omega \) | set of scenarios, \( \omega \in \Omega \) |
| \( p_\omega \) | probability of scenario \( \omega \in \Omega \) to occur assuming \( p_\omega = \frac{1}{|\Omega|} \) \( \forall \omega \in \Omega \) |
| \( D_{\omega j} \) | demand at \( j \in J \) in scenario \( \omega \in \Omega \) |
| \( Q_{\omega i} \) | supply capacity of \( i \in I \) in scenario \( \omega \in \Omega \) |

All the flow-related variables such as \( x_{ij}, v_i, w_j, s_j \) are now augmented with scenario index \( \omega \in \Omega \), and become \( x_{ij\omega}, v_{i\omega}, w_{j\omega}, \) and \( s_{j\omega} \) respectively. There is no change in the binary decision variables and other model parameters. While these binary variables represent strategic, first-stage decisions, the augmented flow variables represent the second stage recourse actions depending on specific scenario-realization.

\[
\begin{align*}
\text{(SP)} \quad \text{Min } &= \sum_{\omega \in \Omega} \left[ \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij\omega} + \sum_{i \in I} c_i v_{i\omega} + \sum_{j \in J} c_j w_{j\omega} + \sum_{j \in J} \Pi_j s_{j\omega} \right] \\
\text{subject to} & \quad \sum_{\omega \in \Omega} p_\omega \leq P \\
& \quad \sum_{\omega \in \Omega} u_{ij} \leq 1 \quad \forall j \in J \\
& \quad u_{ij} \leq y_i \quad \forall i \in I, j \in J \\
& \quad \sum_{\omega \in \Omega} u_{ij} \geq y_i \quad \forall i \in I \\
& \quad x_{ij\omega} \leq M_i u_{ij} \quad \forall i \in I, j \in J, \omega \in \Omega \\
& \quad \sum_{j \in J} v_{i\omega} + y_i \leq Q_{\omega i} \quad \forall i \in I, \omega \in \Omega \\
& \quad \sum_{j \in J} w_{j\omega} + s_{j\omega} \geq D_{j\omega} \quad j \in J, \omega \in \Omega \\
& \quad \sum_{j \in J} w_{j\omega} = \sum_{j \in J} v_{i\omega} \quad \omega \in \Omega \\
& \quad v_{i\omega} \leq M_i y_i \quad \forall i \in I, \omega \in \Omega \\
& \quad w_{j\omega} \leq M_j s_{j\omega} \quad \forall j \in J, \omega \in \Omega \\
& \quad x_{ij\omega} \geq \eta D_{ij\omega} \quad \forall i \in I, j \in J, \omega \in \Omega \\
& \quad y_i, u_{ij}, v_{i\omega}, w_{j\omega}, s_{j\omega} \geq 0 \quad \forall i \in I, j \in J, \omega \in \Omega.
\end{align*}
\]

The above SP model minimizes the expected distribution- and shortage cost over all scenarios \( \omega \in \Omega \). Note that except for the first four constraints that involve only the strategic decision variables, all other constraint sets become augmented with \( \omega \in \Omega \). Thus, while these constraints represent the same conditions as discussed before in §3.1, they now should hold for all \( \omega \in \Omega \).

We note here that a straightforward comparison between our RO and SP model would not be meaningful since they use two different representations of the uncertainties. While RO model has specific intervals for the uncertain parameters and uses an uncertainty budget to decide how many of those parameters may simultaneously assume the extreme values, the SP model uses discrete scenarios for demand and supply capacity values at the beneficiary and donor nodes along with their occurrence-probabilities - another extrinsic input for the SP model.

4.4. Managerial insights

We conclude this section by highlighting the major insights elicited from our analyses of the model. We observe certain interactions among (i) the NPO’s capacity of engaging donors, (ii) minimum demand
fulfillment requirement for direct flow to a beneficiary, and (iii) the supply-demand related uncertainty budget. If the NPO is low on capability, the system-level shortage would naturally increase. In this situation, the pooling benefit (indirect donation flow via warehouse) would not materialize since only the direct flows would be encouraged. Furthermore, an increase in the minimum fraction of donation flow ($q$) would adversely impact the direct flow pattern, forcing to send the flow from a donor to a distant beneficiary, thereby increasing the distribution cost. With the increase in supply-demand uncertainty budgets, total cost would increase as more shortages would result in. Once the NPO’s capability of engaging multiple donors increases, an attempt for shortage reduction is observed by means of indirect flows through the warehouse. In short, the flexibility lever gets activated only if the system has some flexibility in terms of supply capacity.

5. Conclusion and future research

In this paper, we extend the problem of Dalal [9] for establishing strategic donor-beneficiary assignment in a food assistance program of an India-based NPO. While their two-stage stochastic programming model determines optimal connection between existing sets of donors and beneficiaries under scenario-represented supply and demand uncertainties, the unprecedented COVID-19 circumstances escalate such uncertainties to a new level, where the pre-COVID set of donors’ capacities may prove inadequate. Therefore, we introduce a new robust optimization based model to determine: which donors should be engaged from a set of potential donors, whom would they connect, and how much donation flow would occur, in order to obtain an optimal solution for any demand and supply uncertainty realization, as long as they are contained within a pre-defined uncertainty set. Our model provides the decision maker the flexibility to adjust the level of conservatism by fine-tuning his/her uncertainty budget. We conduct a detailed sensitivity analysis on our test instance and capture various interesting insights.

We conclude by mentioning some future research direction. Since our ROC model, having both location and assignment decisions as binary variables, is a difficult mixed-integer programming (MIP) problem, a realistic size instance might pose computational challenges. Therefore, we intend to develop efficient solution algorithm in the future. While our paper addresses a strategic assignment problem with an aggregate view on supply and demand uncertainties, we plan to develop a tactical model to make periodic decisions in the presence of frequent periodic fluctuations (monthly or weekly) of supply and/or demand parameters. We would also like to study the potential adverse effect of too frequent change in the donor-beneficiary allocations. Another possible research direction can focus on inclusion of equity or fairness in the system where demand typically overpowers the supply. A ground-level implementation of the decisions from our strategic model would require a close attention to the detailed logistical arrangements, particularly if physical delivery of cooked, perishable food item is involved. A variant of routing problem can evolve, containing appropriate constraints from the problem context.

Credit authorship contribution statement

Jyotirmoy Dalal: Conceptualization, Methodology, Analysis, Software, Writing - review & editing.

Appendix

A. Derivation of ROCs of two “≥” type constraints

If we go for a straightforward adoption of the approach in Ref. [7], the ROC of (8) becomes:

$$-\sum_{i,j} x_{ij} - w_j - s_j + \psi_j (D_j^0, \theta_j^0) \leq 0 \quad \forall j \in J$$

and $\theta_j = 1 \forall j \in J$ where the protection factor, $\psi_j (D_j^0, \theta_j^0)$ comes from the inner maximization problem $\forall j \in J$:

$$\beta_j (\Gamma_j^0, \Delta_j^0) = \text{Maximize} \left[ \bar{D}_j \theta_j q_j \right]$$

subject to

$$0 \leq q_j \leq 1$$

$$q_j \leq \Gamma_j^0.$$  

$\Gamma_j^0$ represents the uncertainty budget. With variables $\varphi_j (\cdot)$ and $\psi_j (\cdot)$, we dualize the problem for each $j \in J$:

Minimize $\varphi_j (\cdot) + \Gamma_j^0 \psi_j (\cdot)$

subject to

$$\varphi_j (\cdot) + \psi_j (\cdot) \geq \bar{D}_j \theta_j$$

$$\varphi_j (\cdot) \geq 0.$$  

Since in each constraint $j$ of type (8) only one $\bar{D}_j$ parameter is uncertain, we have $\Gamma_j^0 = 1$. Hence, the dual problem’s objective becomes $\varphi_j (\cdot) + \psi_j (\cdot)$, which is same as the left-hand-side of our “≥” constraint. Now, the problem $j \in J$ solves trivially with the optimal objective value equal to $\bar{D}_j \theta_j$ (since $\bar{D}_j = 1 \forall j \in J$). This makes the ROC of (8):

$$\sum_{j \in J} x_{ij} + w_j + s_j \geq D_j + \bar{D}_j \quad \forall j \in J,$$  

To avoid this extreme conservatism and adjust the level of protection, we consider an overall uncertainty budget $\Gamma^0 \in [0, |J|]$ (index $j$ dropped in $\Gamma^0$) to indicate how many $D_j$ parameters can deviate simultaneously from their respective $\bar{D}_j$ values, and obtain the ROC of (8) as follows:
\[
\sum_{i \in I} x_i + w_j + s_j \geq \bar{D}_j + \tilde{D}_j \times \left( \frac{u^j}{|J|} \right) \quad \forall j \in J.
\]

We note here that the derivation of ROC of (12), another “\( \geq \)” type constraint, follows the very similar steps as shown above, hence, we omit the details.

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