Stepped Graphene-based Aharonov-Bohm Interferometers

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Aharonov-Bohm interferences in the quantum Hall regime are observed when electrons are transmitted between two edge states. Such a phenomenon has been extensively studied in 2D systems such as quantum point contacts, anti-dots and p-n junctions. Based on a theoretical investigation of the magnetotransport in stepped graphene, a new kind of Aharonov-Bohm interferometers is proposed herewith. Indeed, when a strong magnetic field is applied in a proper direction, oppositely propagating edge states can be achieved in both terrace and facet zones of the step, leading to the edge states equilibration and hence a nice Aharonov-Bohm oscillation in the conductance in the quantum Hall regime. Taking place in the unipolar regime, this Aharonov-Bohm interference is also predicted in stepped systems of other 2D layered materials.

The fascinating properties of quantum hall devices arise from their ideal 1D edge states formed in a 2D electron system when a high magnetic field is applied [1, 2]. These edge states are particularly attractive due to their large coherence lengths, which is mandatory for constructing electron interferometers. However, since the edge channels are spatially separated, a mechanism for creating the electron transmission between them is required to achieve the interference effects. In this regard, one explored technique consists in building constrictions (quantum point contacts) in a sample, where the interedge tunneling paths can occur [3–17]. Setups consisting of a pair of quantum point contacts with an internal cavity has been demonstrated to work well as quantum Hall, electronic Fabry-Pérot, and Aharonov-Bohm interferometers. Another mechanism has also been suggested in systems consisting of an antidot introduced between their edges [18–25]. Electronic currents encircling the antidot can be achieved and a similar Aharonov-Bohm interference is hence observed.

Graphene, a truly 2D material, is an ideal platform for investigating quantum Hall and interference effects. Remarkably, owning to an unique linear dispersion and Dirac-like fermions [26], Landau levels and a half-integer quantum Hall effect with an unusual quantized sequence compared to the conventional systems have been observed in graphene when a strong magnetic field (B-field) is applied [27, 28]. In addition, with its semimetal character, quantum Hall systems based on graphene can work in both the unipolar and bipolar regimes that can be generated and controlled by gate voltages [28–31]. Interestingly, in the bipolar regime the chiral edge states equilibration at the p-n interfaces in graphene has been observed, resulting in fractional quantum Hall conductance plateaus [31].

With its typically high carrier mobilities, graphene is also an ideal material to perform the investigation on interference effects, including the Aharonov-Bohm (AB) one. Several experimental and theoretical observations of the AB effect in graphene quantum rings have been reported (i.e., see Ref. [32] and references therein). Remarkably, graphene p-n junctions can also work as AB interferometers [33–35] in the quantum Hall regime. In particular, the oppositely propagating edge states are formed in two different doped zones and their equilibration at the p-n interface acquires conductance oscillations of the AB periodicity when a strong B-field is applied.

Motivated by such scientific context, a new kind of Aharonov-Bohm interferometers based on stepped graphene channels is proposed herewith. These non-planar systems have been actually achieved in several experimental situations. For example, the step bunching on the SiC surface is often observed in epitaxial graphene growth by thermal decomposition of SiC [36–

FIG. 1. Stepped graphene (a) with the ribbon width $W$ and the length of facet zone $L_F$. (b) Schematic of the side view illustrating the applied magnetic field (red dashed lines of arrows) and its normal components (green and blue arrows, respectively) in both terrace and facet zones. $\theta_B$ and $\theta_S$ are the angles of the field and of the facet zone relative to the terrace one (Ox axis), respectively.
a promising method for the production of large-area high-quality graphene. This kind of stepped graphene channels with terrace size of several μm and step height of tens nm can be controllably produced by varying the heating rate [42]. These non-planar systems have been also synthesized in an even better controllable way by draping graphene on pre-structured substrates [45–52].

In the present work, we demonstrate that a $B$-field applied to stepped graphene induces different effects on the electron motion in its terrace and facet zones, essentially resulting from the effects of non-uniform normal component of the field on the in-plane transport through the system. Actually, this feature has been observed in several non-planar 2D systems (including graphene), resulting in anisotropic magnetotransport pictures when the transport takes place in the directions aligned parallel or perpendicular to the step edge [36–40, 53–57]. Here, a novel phenomenon is predicted when tuning the direction of $B$-field applied to stepped graphene. In particular, a non-uniform profile containing alternatively opposite normal $B$-components along the channel can be created, inducing a beautiful AB interference picture in the quantum Hall regime. This interference is a direct result of interedge scatterings at the interface between the zones of opposite normal $B$-fields, in analogy with those observed in the graphene quantum Hall p-n devices [29–31, 33–35].

The considered systems consist in graphene nanoribbons (GNRs) in the step geometry as illustrated in Fig.1. In principle, a small in-plane strain can occur in the bent zone of the step, however negligible due to the mechanical robustness of graphene layers [45]. Therefore, the $p_z$ tight-binding Hamiltonian [26] is still a good approach to compute the electronic transport in these GNRs. When a $B$-field is applied, the system Hamiltonian reads

$$H = \sum_n U_n c_n^\dagger c_n + t_0 \sum_{(n,m)} e^{i\phi_{nm}} c_n^\dagger c_m$$

(1)

where $U_n$ represents the potential energy at the $n^{th}$ site, $t_0 = -2.7$ eV corresponds the hopping energy between nearest-neighbor $n^{th}$ and $m^{th}$ atoms, and $\phi_{nm} = \frac{1}{\hbar} \int_{r_n}^{r_m} A(r) dr$ is the Peierls phase describing the effects of the $B$-field. Here, the effects of the magnetic field $B = B(\cos \theta_B, 0, \sin \theta_B)$ are considered by introducing the vector potential $A(r) = -B(y \sin \theta_B, z \cos \theta_B, 0)$. The above Hamiltonian is solved using the Green’s-function technique [38], allowing for the calculation of the quantities of electronic transport perpendicular to the step edge, i.e., along the Ox axis shown in Fig.1.

Fig.2a displays the dependence of conductance on $B$-field applied in different directions in a step constituted by an armchair GNR. As mentioned, the applied $B$-field induces different normal components ($B_S$) in the terrace and facet zones, i.e., $B_N = B \sin \theta_B$ (green arrows) and $-B \sin(\theta_S - \theta_B)$ (blue arrow), respectively (see Fig.1b). First, for $\theta_B > \theta_S$, although they have different amplitudes, these $B_N$-fields are parallel, thus inducing the same propagating edge states in the two zones as illustrated in Fig.2b. Therefore, when a large $B$-field is applied, a conventional Landau quantization is still obtained, i.e., the conductance represents quantized values as shown in Fig.2a for $\theta_B = 90^\circ$. For $\theta_B = \theta_S$, the $B_N$-component in the facet zone is canceled and hence the $B$-field has no effect on the in-plane transport in this zone. Consequently, the system behaves as a magnetic-field heterojunction consisting of finite- and zero-field zones and the scatterings at their interface basically explain the reduction of conductance obtained for $\theta_B = 60^\circ \equiv \theta_S$ displayed in Fig.2a.

Most interestingly, when $0 < \theta_B < \theta_S$, two opposite $B_N$-components alternate in the terrace and facet zones, as discussed above. Similarly to the effects of $B$-field in different doped zones of graphene $p$-$n$ junctions.
FIG. 3. Left-injected local density of states at $B = 25.8 \, T$ (a) and $27.4 \, T$ (b), corresponding to conductance peak and valley, respectively (see Fig. 2a for $\theta_B = 30^\circ$).

[31, 33, 34], opposite edge states in the terrace and facet zones are created, thus inducing the full equilibration at their interface as illustrated in Fig.2c. As an important consequence, a very nice AB interference picture is observed, i.e., the conductance as a function of $B$-field represents a strong oscillation in the quantum Hall regime (see the case of $\theta_B = 30^\circ$ in Fig.2a). This result is essentially due to the interedge backscatterings diagrammatically described in Fig.2c and is further demonstrated by analyzing the computed left-injected local density of states in Fig.3, that illustrates the left-to-right electron-wave propagation. Indeed, backscatterings are almost absent (Fig.3a) when the phase coherence condition is satisfied, leading to conductance peaks. In the phase incoherence condition, strong interedge backscatterings (Fig.3b) and hence a low conductance are achieved.

AB oscillation period observed in quantum rings with area $S$ is known to be given by $\Delta B = h/eS$ [59]. To examine this point in the considered systems (for $\theta_B < \theta_S$), the above formula should be rewritten as

$$\Delta B = \frac{h}{eS} \frac{1}{|\sin(\theta_S - \theta_B)|}$$

(2)

where $S$ is the area of the surface enclosed by the edge channel in the facet zone. Actually, the oscillation periods $\Delta B \approx 3.55 \, T$ and $7.3 \, T$ are obtained in the high field regime with the facet zones of $\approx 3000 \, \text{nm}^2$ and $1500 \, \text{nm}^2$, respectively (see Fig.4a and additionally Figs.2a and 4b). Indeed, the formula (2) predicts quite well these results of $\Delta B$ if $S \approx 2340 \, \text{nm}^2$ and $1156 \, \text{nm}^2$ are considered, which are about 22% smaller than the area of the corresponding facet zones. Note that here, the edge states are formed inside the facet zone (see Fig.3) and hence the value of $S$ in Eq.(2) to estimate $\Delta B$ is basically proportional to but smaller than the area of the facet zone. Thus, the origin of observed conductance oscillation is really the AB interference due to the equilibration of edge states in both terrace and facet zones of the system.

The observed AB interference is also found to be sensitive to some other structural parameters. First, perfect armchair GNRs can be divided in two main classes with significantly different electronic properties, depending on the number of dimer lines $N_a$ across the ribbon width: $N_a \neq 3p + 2$ and $N_a = 3p + 2$ [60]. The former class exhibits a semiconducting behavior while the latter one can be considered to be quasi-metallic with negligible bandgap. Moreover, in contrast to semiconducting ribbons, the first subband of metallic GNRs is

FIG. 4. (a,b) Conductance as a function of $B$-field at $E_F = 75 \, \text{meV}$ computed for semiconducting ($N_a = 324$) and metallic ($N_a = 326$) GNR systems, respectively, with $\theta_S = 60^\circ$, $\theta_B = 30^\circ$, and $W \approx 40 \, \text{nm}$. Two values $L_F \approx 37.5 \, \text{nm}$ and $75.0 \, \text{nm}$ are studied in (a) while $L_F \approx 75.0 \, \text{nm}$ in (b). Except for the perfect edge case in (b), edge disorders with $\approx 15\%$ of removed edge atoms are applied.
is almost invisible for perfect edges but the effect is much more pronounced when edge disorder is introduced.

In Fig. 5, the dependence of the conductance on both Fermi energy $E_F$ and $B$-field is presented. Basically, two typical zones, $E_F \leq E_1$ and $E_1 > E_F$, are specified where $E_1 = \sqrt{2e\hbar v_F^2}\left|B_N\right|$ is the first Landau level and $v_F$ is the Fermi velocity in graphene [27]. In particular, beautiful AB oscillations are predicted for $E_F \leq E_1$ whereas the interference picture is blurred in the high energy zone. Similarly, this feature has been previously demonstrated in quantum ring systems [62, 63] that the AB interference can be perfectly observed if only a single subband of GNRs contributes to the transport, otherwise the picture is significantly disturbed by the multi-subband contribution. Note that because of its different $B_N$-components in the terrace and facet zones, the minimum value of $B$-field to achieve the strong AB interference is determined by $B_{\min} = B_0/\min\{\sin\theta_B, |\sin(\theta_S - \theta_B)|\}$ with $B_0 = E_F^2/2e\hbar v_F^2$, i.e., only the first subband is present at $E = E_F$ in both terrace and facet zones.

Finally, it is very worth noting that differently from the graphene $p$-$n$ junctions, the edge states equilibration in the considered stepped systems is achieved in the unipolar regime. This suggests that the effect can be observed not only in graphene (similarly, other semi-metallic materials) but also in systems made of 2D semiconducting materials. Indeed, a similar AB interference picture is obtained in monolayer phosphorene nanoribbon systems by tight-binding calculations [64] and is presented in Fig. 6. Thus, compared to the effect in graphene $p$-$n$ junctions [31, 33, 34], the present prediction represents a much larger range of possibilities to design AB interferometers using 2D layered materials.

To conclude, the magnetotransport through stepped
graphene was investigated using atomistic tight-binding calculations. By applying the $B$-field in a proper direction, opposite normal components of the field can be created, thus inducing opposite edge states, in the terrace and facet zones. The equilibration of these edge states was observed, leading to a nice Aharonov-Bohm interference picture in the quantum Hall regime. The properties of this interference, depending on the carrier energy and structural parameters, were systematically clarified. Moreover, since it is observed in the unipolar regime, this Aharonov-Bohm effect can be also achieved in stepped systems made of other (both semimetallic and semiconducting) 2D layered materials.

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