Optical phase retrieval with the image of intensity in the focal plane based on the convolutional neural networks

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Abstract. One of the most important factors for improving the resolution of optical systems is to compensate for the aberrations (distortions) of the wave front. As a rule, whether special measuring devices (wavefront sensors) are used for such compensation or adaptive mirrors that perform iterative correction of the wavefront. However, often (for reasons of compactness or weight reduction), it is not possible to use the special equipment for measuring aberrations. To obtain certain information on the wave front, one can use the measured point spread function (PSF) or the intensity pattern in the focal plane. Methods of processing two PSFs (focal and nonfocal) with the help of neural networks are known. In this paper, we investigate the possibility of recognizing the wave front from a single intensity pattern in the focal plane. The technology of deep machine learning - convolutional neural network is chosen as the way for implementation. The idea of this technology lies in the alternation of convolutional and subsampling layers, for the purpose of efficient image recognition. Such approach will allow to optimize the process of compensation of optical system aberrations and to reduce the amount of required input data.

1. Introduction

The presence of aberrations in optical systems leads to a deterioration of resolution in images, which is a fundamental problem in many applications from space exploration to vision improving [1-7]. Different methods and approaches were designed to reduce aberrations and increase resolution. One of the ways is apodization of the optical system based on the addition of the system with amplitude and / or phase optical elements, including asymmetric ones [8–13]. However, as a rule, apodization allows not only reducing the size of the central light spot, but also leads to a distortion of the point spread function (PSF) and the appearance of side lobes [14-16], which degrade the imaging properties of the optical system and require additional digital processing [17, 18]. Another approach to improving image quality is to compensate for wavefront aberrations using adaptive optics [3, 4, 19, 20]. Usually, adaptive systems are complex optical instruments that include a wavefront sensor that measures distortion (aberration), a wavefront corrector, and control system that implements the connection between the sensor and the corrector. The wavefront sensor is one of the main elements in the system of adaptive optics, because the information received by this sensor serves as the initial for the functioning of the corrective elements of the system. So that the error of the sensor operation determines the error of the entire system as a whole. Today there is a wide variety of wavefront
sensors, including interference [21], however, the most common is the Shack-Hartman sensor [22, 23]. The advantage of Shack-Hartman sensor is that it is completely achromatic (there is no dependence on the wavelength), and also capable of performing measurements in beams with partial coherence, i.e. works with non-point (extended) radiation sources. However, the Shack-Hartman sensor has one major drawback: it produces the crosstalk on the CCD. The crosstalk arises when a sufficiently distorted wavefront falls on the CCD matrix, since due to the strong deviations the wavefront can go beyond the limits of its subarray and get to the next the CCD matrix. Wavefront can also be analyzed on the basis of mode decomposition. Notice that the generally accepted representation of the wavefront is a series of Zernike polynomials [24-27], which correspond to different wavefront aberrations. The expansion coefficients of the wavefront in orthogonal Zernike polynomials allow us to determine rms error of deviation from the ideal front. High absolute values of coefficients indicate that those aberrations are greatest and, thus, allows to significantly accelerate and simplify the analysis. For direct optical measurement of the amplitudes of the expansion coefficients of the wavefront in orthogonal Zernike polynomials were proposed multi-order diffractive optical elements [26, 28, 29]. These elements have been used successfully for analyzing and reconstructing the wavefront [26, 29], however, an unambiguous restoration possible only for small aberrations [29]. In connection with the mentioned disadvantages of various wavefront sensors, other methods are being developed for determining aberrations/ In particular, by measuring PSF outside the focal plane [30, 31]. In this case, the iterative procedure is applied the convergence of which to the correct solution essentially depends on the initial approximation [32]. In [7], an algorithm was developed for recognizing aberrations by a set of tabulated PSF for discrete values of the five parameters based on the enumeration of all possible combinations. To process such a large amount of data the involvement of the computational cluster Keldysh (MSU) was required. This paper discusses a different approach for analyzing of the various aberrations - based on convolutional neural networks [33]. It should be noted that such neural networks are already successfully used to improve image quality obtained using diffractive lenses [34, 35].

2. Theoretical foundations

2.1. Wave aberrations

Wave aberration is the distortion of the wavefront shape (deviation of wavefront shapes from ideal). The point spread function (PSF) describes the response of an imaging system to a point source or point object. A more general, the PSF is a system's impulse response. The generally accepted representation of the wavefront, as well as its aberrations, is the basis of Zernike polynomials [24-29]. We consider circular Zernike polynomials, which are a complete set of orthogonal functions with angular harmonics in a circle of radius $R$ [36]:

$$\psi_{m,n}(r,\phi) = \frac{n+1}{\pi R^2}Z^m_n(r)\begin{cases} \cos\left(\frac{m\phi}{n}\right) & \text{if } m \text{ is even;} \\ \sin\left(\frac{m\phi}{n}\right) & \text{if } m \text{ is odd;} \end{cases}$$

(1)

where $Z^m_n(r)$ are Zernike radial polynomials ($|m| \leq n$):

$$Z^m_n(r) = \sum_{p=0}^{(n-m)/2} \frac{(-1)^p (n-p)!}{p!(n+m/2 - p)!(n-m/2 - p)!} \left(\frac{r}{R}\right)^{n-2p}$$

(2)

The decomposition of the light field with a complex amplitude $E(r,\phi)$ in a series of functions (1) can be written as

$$E(r,\phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} C_{m,n} \psi_{m,n}(r,\phi)$$

(3)

Spatial spectrum that corresponds to the Fourier transform of the field $E(r,\phi)$ also can be represented as a decomposition in the Zernike basis (1):
where $W_{nm}(\rho)$ are the functions calculated as the Fourier-Hankel transform of the Zernike functions and are defined as follows [24]:

$$W_{nm}(\rho) = \int_0^R Z_n^m(r) J_n^m\left(\frac{2\pi}{\lambda f} \rho r\right) r \, dr = (-1)^{(m-n)/2} R^3 J_{n+1}^m\left(\frac{2\pi}{\lambda f} R \rho\right) \left(\frac{2\pi}{\lambda f} \rho\right)^{-1}$$

Detection of one or another aberration in the wavefront is linear proportional to the intensity of the corresponding correlation peak and can be realized only for small phase aberrations, i.e. when the approximation is valid:

$$A(r,\phi) = \exp[i\alpha\psi(r,\phi)] = 1 + i\alpha\psi(r,\phi) - \frac{\alpha^2\psi^2(r,\phi)}{2} + ... \approx 1 + i\alpha\psi(r,\phi)$$

For arbitrary phase distributions, the ratio between the detected aberration and magnitude of the correlation peak may be non-linear. Table 1 shows the results of detecting a coma aberration corresponding to the Zernike polynomial $(n, m) = (3, 1)$, with different values of $\alpha$. As can be seen, when the magnitude of the aberrations $\alpha = 0.6$, one can perform confident detection and restoration of phase aberrations based on expansions in Zernike functions, but with further increase in the magnitude of the distortion detection becomes problematic.

Table 1. Detection of coma at different values of $\alpha$ based on the decomposition of the wavefront with Zernike functions.

| Aberration | PSF | Expansion coefficients | Phase restoration |
|------------|-----|------------------------|-------------------|
| $\alpha = 0, 2\pi$ | | | |
| $\alpha = 0, 4\pi$ | | | |
| $\alpha = 0, 6\pi$ | | | |
| $\alpha = 0, 8\pi$ | | | |

Table 1 also shows that there is a significant correlation between type and value of aberration and the image of PSF. Thus, it can be assumed that the use of convolutional neural network will allow to effectively solve the problem of recognition of aberrations, even with large values of wavefront distortion.

2.2. Neural network

The neural network is an artificial model consisting of neurons and synapses between them. These networks are capable of solving various classification problems by training on large amounts of data.
In our case, the network will try to determine (predict) the phase of the given PSF. Based on the task, it would be best to choose a convolutional neural network [37] as a solution.

Convolutional neural networks (CNN) is a special neural network architecture, a deep learning technology that focuses primarily on image recognition. The basic idea of the CNN is to use convolution layers and sub-sampling (pooling) layers. This type of network has the greatest success in computer vision tasks.

3. Training data
To train the neural network, 10,000 pairs of images were generated. Each pair consists of: a wavefront aberration based on Zernike polynomials and the corresponding PSF calculated using the Fourier transform. The pairs were generated using formula (3) by changing the coefficients $C_{nm}$ and Zernike functions $Ψ_{nm}(r, φ)$. The script for the formation of training data was implemented using the software MATLAB.

The data were divided into three sets - training, validation and test set, in the ratio of 89%, 9% and 2%, respectively. The training set was used to train the neural network, the validation to check the accuracy at each “epoch” of training and the test to further prevent the problem of overfitting.

The Figure 1 shows an example of the data obtained for training the network.

![Figure 1. Training data example (PSF and the corresponding phase).](image)

4. Neural network implementation
To implement the neural network, Python’s library named “keras” was used. This library is mainly created for machine learning tasks. The ADAM optimizer was also used. The LOSS function is given below:

$$ Error = Σ_{i=0}^{n}(p_i - t_i)^2 $$

(7)

4.1. Architecture optimization
The main purpose of this stage is to find the required number of layers of the neural network and the number of nodes in each layer. Architecture that is too simple will lead to low accuracy of predictions, too complicated - to the problem of “overfitting”. The second case is threatened by the fact that the network will show close to one hundred percent result on the training data but when switching to new data, the accuracy will rapidly fall. To avoid this, the dropout technology was used in this work [38]. This method allows you to "turn off" some network nodes at each workout step, this will allow the network to constantly change its architecture during the workout and thus partially avoid the problem of "overtraining".

4.2. Final result
After iterating through various architectures and hyperparameters, a combination with the best result of prediction accuracy was selected.

The accuracy of prediction on this architecture with selected hyper parameters approaches 97%.
Figure 2. Example of overfitting problem (loss/test graph on the left, loss/validation on the right).

(a) Standard Neural Network
(b) Network after Dropout

Figure 3. Dropout technology example.

Table 2. The architecture of the final version of the neural network.

| Layers               |
|----------------------|
| Input                |
| Convolutional 30 3x3 |
| Convolutional 30 3x3 |
| Pooling 2x2          |
| Convolutional 60 3x3 |
| Convolutional 60 3x3 |
| Pooling 2x2          |
| Convolutional 100 3x3|
| Convolutional 100 3x3|
| 1000 Fully-connected |

Figure 4. Prediction example.

In the example above (fig. 4), the trained neural network receives an image of an arbitrary PSF and makes a prediction about its phase, trying to “guess” the numbers of the Zernike functions and coefficients with them.
5. Results
In this paper, the wavefront with aberrations in the form of a superposition of Zernike functions was calculated. In the superposition, both the functions themselves and their weights are varied. Based on the calculated wavefront aberrations, the corresponding point spread functions are obtained using the fast Fourier transform. A data set was formed to solve the problem of aberration recognition by the TRF using a convolutional neural network, which was implemented based on the MNIST network template. As a result of this work, it was proved that it is possible to achieve the accuracy, required for use in some areas, using deep learning technologies to compensate for wavefront aberrations from the measured point dispersion function. Development in this direction will simplify the existing optical systems and open up new possibilities for obtaining images.

6. References
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