Characterizing the $\gamma$-Ray Variability of Active Galactic Nuclei with the Stochastic Process Method

Haiyun Zhang, Dahai Yan, and Li Zhang

Department of Astronomy, Key Laboratory of Astroparticle Physics of Yunnan Province, Yunnan University, Kunming 650091, People’s Republic of China

1 Key Laboratory for the Structure and Evolution of Celestial Objects, Yunnan Observatories, Chinese Academy of Sciences, Kunming 650011, People’s Republic of China; yandahai@ynao.ac.cn

Received 2021 December 24; revised 2022 April 5; accepted 2022 April 13; published 2022 May 16

Abstract

Gamma-ray astronomy in the time domain has been advanced further as the variabilities of active galactic nuclei (AGNs) on different timescales have been reported in papers. We study the $\gamma$-ray variabilities of 23 jetted AGNs by applying a stochastic process method to the $\sim 12.7$ yr long-term light curve (LC) obtained from the Fermi-Large Area Telescope (Fermi-LAT). In this method, the stochastically driven damped simple harmonic oscillator (SHO) and the damped random-walk (DRW) models are used to model the long-term LCs. Our results show that the long-term variabilities of 23 AGNs can be characterized well by both SHO and DRW models. However, the SHO model is restricted in the overdamped mode, and the parameters are poorly constrained. The SHO power spectral densities (PSDs) are the same as those of the typical DRW PSD. In the plot of the rest-frame timescale that corresponds to the broken frequency in the PSD versus black hole mass, the intrinsic, characteristic $\gamma$-ray timescales of 23 AGNs occupy almost the same space with the optical variability timescales obtained from the accretion disk emission. This suggests a connection between the jet and the accretion disk. As with the optical variability of the AGN accretion disk, the $\gamma$-ray timescale is consistent with the thermal timescale caused by the thermal instability in the standard accretion disk of AGNs.

Unified Astronomy Thesaurus concepts: Active galactic nuclei (16); Gamma-rays (637); Jets (870); Light curves (918); Time series analysis (1916)

1. Introduction

High-energy (HE; $\geq 100$ MeV) $\gamma$-ray observations suggest that the emissions from active galactic nuclei (AGNs) dominate the extragalactic $\gamma$-ray sky (Abdollahi et al. 2020). The strongly Doppler-boosted blazars, an extreme class of AGNs whose emissions are mainly from nonthermal relativistic jets, are dominant in these powerful emitters. Blazars are classified into BL Lac objects (BL Lacs) and flat-spectrum radio quasars (FSRQs) according to the strength of their optical emission lines (Ajello et al. 2020). FSRQs have strong, broad emission lines, while BL Lacs have weak, narrow, or no such lines.

AGN variability has already been detected at entire electromagnetic wavelengths with timescales covering from decades down to minutes. Radio-loud AGNs are highly variable $\gamma$-ray emitters. This not only applies to blazars, which have strong and incessant flux variability, but also to misaligned jet sources such as radio galaxies (MAGIC Collaboration et al. 2018; Ait Benkhali et al. 2019). The underlying physical processes can be investigated by characterizing the variabilities (e.g., H. E. S. S. Collaboration et al. 2017; Yan et al. 2018; Rieger 2019; Bhatta & Dhital 2020).

The observations of the Fermi-Large Area Telescope (Fermi-LAT) have advanced the studies in the HE time domain. An attractive phenomenon is the $\gamma$-ray quasi-periodic oscillations (QPOs) detected in the LAT data of blazars (e.g., Ackermann et al. 2015; Sandrinelli et al. 2016; Zhou et al. 2018; Peñil et al. 2020). However, the reliability of these QPOs is always questionable (e.g., Covino et al. 2019; Ait Benkhali et al. 2020; Yang et al. 2021). Another interesting phenomenon is the fast $\gamma$-ray flares on timescales of a few minutes detected in the LAT data of FSRQs (Ackermann et al. 2016; Meyer et al. 2019; Shukla & Mannheim 2020). Besides, the statistical characteristics of $\gamma$-ray variability in AGNs have been extensively investigated. The commonly used methods are the analyses of the power spectral density (PSD) and flux distribution (e.g., Abdo et al. 2010; Shah et al. 2018; Meyer et al. 2019). Abdo et al. (2010) presented $\gamma$-ray variabilities of 106 AGNs systematically, using the first 11 months of the Fermi survey. They reported that more than 50% of the sources are found to be variable with a power-law (PL) PSD, and an underlying random-walk mechanism was reflected in some blazars. Nakagawa & Mori (2013) reported a characteristic timescale of $\sim 7.9$ days in the PSD of 3C 454.3 by analyzing the first 4 yr of Fermi-LAT data, and they used an internal shock model to interpret this timescale. Meyer et al. (2019) presented a detailed analysis of LAT LCs of bright $\gamma$-ray FSRQs and put strong constraints on blazar jet physics accordingly.

A stochastic process model has been widely used to describe the optical variability of AGN accretion disks (e.g., Kelly et al. 2009; MacLeod et al. 2010; Zu et al. 2013; Rakshit & Stalin 2017; Li & Wang 2018; Zhang et al. 2018; Hu et al. 2019; Burke et al. 2021). Generally, the damped random-walk (DRW) model can provide a successful fit to the long-term variability of AGN accretion disks. It has been proven that such a stochastic process model provides a powerful tool to extract information from AGN variability (e.g., Kasliwal et al. 2017; Burke et al. 2021). In the past few years, the stochastic process model has been applied to $\gamma$-ray variabilities of AGNs in several papers (Sobolewska et al. 2014; Goyal et al. 2018; Ryan et al. 2019; Tarnopolski et al. 2020; Covino et al. 2020; Yang...
et al. 2021; Zhang et al. 2021). Based on the stochastic model developed by Kelly et al. (2009), Sobolewska et al. (2014) modeled the $\gamma$-ray LCs of 13 blazars observed during the first 4 yr of the Fermi sky survey with the Ornstein–Uhlenbeck (OU) process (also called DRW) and a linear superposition of the OU processes (sup-OU). They showed that 10 of the 13 blazars prefer the sup-OU process over the OU process. The continuous-time autoregressive moving average (CARMA) method (Kelly et al. 2014), a generalized stochastic model that can be applied to astronomical time series, is flexible enough to capture the features of flux variability and to produce more accurate PSDs. Applying this method to the 9.5 yr LAT data of the same 13 blazars in Sobolewska et al. (2014), Ryan et al. (2019) reported that the DRW model is good enough to describe the $\gamma$-ray variability of the 13 blazars. In addition to CARMA, celerite is a newly developed method for modeling LC with the stochastic process model (Foreman-Mackey et al. 2017). It was applied to the LAT data of AGNs to examine the significance of $\gamma$-ray QPOs (Yang et al. 2021; Zhang et al. 2021).

So far, Fermi-LAT collecting data for more than 12 yr has provided an excellent opportunity to study the long-term $\gamma$-ray variability in AGNs. In this paper, we apply the celerite model to 12.7 yr Fermi-LAT LCs of 23 bright LAT AGNs, including 10 BL Lac objects (BLLs), 12 FSRQ objects, and 1 radio galaxy (RDG). We aim to investigate the $\gamma$-ray variabilities of AGNs on long-term timescales. The format of this paper is as follows. In Section 2, we briefly introduce the Fermi-LAT data-processing method. In Section 3, the stochastic process models are briefly described. In Section 4, we show the modeling results for the LCs of 23 AGNs with the celerite method. In Section 5, we focus on the variability characteristic timescales in the jets and compare them with the optical results obtained from AGN accretion disk emissions. In Section 6, we discuss our results. Finally, a summary is presented in the Section 7.

2. Fermi-LAT Data Analysis

We collect 23 bright $\gamma$-ray AGNs with significant variability in the LAT data. The information of these sources are listed in the Table 1.

All data analyzed here come from the LAT 8 yr Source Catalog (4FGL; Abdollahi et al. 2020), spanning the time range of MJD 54682–59332 which gives in total ~12.7 yr of data in the energy range of 0.1–500 GeV. We consider only SOURCE class events (evclass = 128) and event type three (evttype = 3) from the region of interest (ROI) at 15° for each source. The maximum zenith angle is set to 90° to avoid contamination from Earth’s limb. The DATAQUAL > 0 and LATCONFIG == 1 options are chosen to ensure good data quality and the proper time intervals. The instrument response function P8R3_SOURCE_V3 is applied in the analysis. We use the Galactic (gll_iem_v07.fits) and the extragalactic (iso_P8R3_SOURCE_V3_v1.txt) diffuse emission models, which are the latest Pass 8 background models. We use the binned maximum-likelihood analysis5 (Abdo et al. 2009),

5 https://fermi.gsfc.nasa.gov/ssc/data/analysis/scitools/binned_likelihood_tutorial.html
which is the preferred method for most types of LAT analysis. It is a three-dimensional maximum-likelihood algorithm, i.e., events are binned into channels of energy and position in the sky (Abdo et al. 2009), and a maximum-likelihood optimization technique is performed to determine the best-fit parameters and the Test Statistic $TS = 2A\Delta \log(\text{likelihood})$ between models with and without the source (Mattox et al. 1996). In the fitting, we use the spectral model of the LogParabola (LP) form $(dN/dE = N_0(E/E_0)^{-\alpha + \beta \log(E/E_0)}$).

3. Stochastic Process Method

We use the stochastic process method implemented in the celerite package (Foreman-Mackey et al. 2017). In the celerite package, a specific and stationary kernel function (i.e., covariance function) is required, which can be defined by users.

3.1. DRW Model

The DRW process is described by a first-order stochastic differential equation (see details in Kelly et al. 2009; Moreno et al. 2019). It represents a competition between a process trying to maintain an equilibrium state and a perturbation moving the system out of stability. It is sometimes also written as a Langevin equation of the form

$$\frac{d}{dt} + \frac{1}{\tau_{\text{DRW}}} y(t) = \sigma_{\text{DRW}} \epsilon(t), \quad (1)$$

where $\tau_{\text{DRW}}$ is the damping timescale and $\sigma_{\text{DRW}}$ is the amplitude of random perturbations.

Following the settings in the celerite package, the covariance function for the DRW is written as

$$k(t_{nm}) = a \cdot \exp(-t_{nm}c), \quad (2)$$

where $t_{nm} = |t_n - t_m|$ is the time lag between measurements $m$ and $n$, $a = 2\pi^2\tau_{\text{DRW}}$, and $c = 1/\tau_{\text{DRW}}$. The PSD is written as (Foreman-Mackey et al. 2017)

$$S(\omega) = \frac{2}{\pi} \frac{a}{c + (\omega/c)^2}. \quad (3)$$

The DRW PSD is a broken PL form, and the index changes from 0 at low frequencies to $-\Delta$ at high frequencies. The broken frequency $f_b$ corresponds to the damping timescale $\tau_{\text{DRW}} = 1/(2\pi f_b)$.

3.2. SHO Model

The dynamics of a stochastically driven damped simple harmonic oscillator (SHO) provides a physically motivated model as it can describe the variability driven by noisy physical processes, which grows most strongly at the characteristic timescale but is also damped owing to dissipation in the system (Foreman-Mackey et al. 2017). The differential equation for this system is

$$\frac{d^2}{dt^2} + \frac{\omega_0}{Q} \frac{d}{dt} + \omega_0^2 y(t) = \epsilon(t), \quad (4)$$

with the frequency of the undamped oscillator $\omega_0$, the quality factor of the oscillator $Q$, and a stochastic driving force $\epsilon(t)$. When $\epsilon(t)$ is white noise, the PSD of this process is

$$S(\omega) = \frac{2}{\pi} \frac{S_0 \omega_0^4}{(\omega^2 - \omega_0^2)^2 + \omega^2 \omega_0^4 / Q^2}. \quad (5)$$

where $S_0$ is proportional to the power at $\omega = \omega_0$. The SHO PSD is complex. For the overdamped SHO ($Q < 0.5$), it is also a broken PL at low frequencies, very similar to the DRW PSD, while at high frequencies, the index can be as small as $\sim -4$ (Kasliwal et al. 2017; Moreno et al. 2019). For the underdamped SHO ($Q > 0.5$), a Lorentzian appears in the PSD, i.e., a QPO signal (Foreman-Mackey et al. 2017; Moreno et al. 2019).

3.3. Model Selection

The Akaike information criterion (AIC) estimates the relevant information that is lost when models are used to represent the underlying processes that generate the data. It is an estimator of the relative quality of models for a given set of data: The less information a model loses, the higher the quality of that model. We use the corrected AIC ($\text{AIC}_c$) to perform the model selection, which is given by

$$\text{AIC}_c = 2k - 2 \log L + \frac{2k(k+1)}{n-k-1}. \quad (6)$$

where $k$ is the number of model parameters, $L$ is the maximum likelihood, and $n$ is the number of data points. A preferred model is one that minimizes $\text{AIC}_c$. It is accepted that $\Delta(\text{AIC}_c) \geq 10$ is a difference substantial enough to prefer the model with smaller $\text{AIC}_c$ (Burnham & Anderson 2004; Sobolewska et al. 2014).

4. Results

In the fitting, Markov Chain Monte Carlo (MCMC), implemented in the package emcee4 (Foreman-Mackey et al. 2013), is used to sample the posterior probability density in our analysis. The priors for the parameters are assumed to be flat. We run the MCMC sampler for 50,000 steps, of which the first 20,000 steps are taken as burn-in. We calculate the maximum likelihood for optimization, which is executed 100 times to resolve the possible instability of the algorithm L-BFGS-B, and then calculate $\text{AIC}_c$ by using the maximum value among the 100 values.

For the data from MJD 54682 to 59332, the 15 day binning LCs of the 23 AGNs are structured by performing the binned likelihood method for each time bin. The time bins having a TS value of $\geq 25$ are selected here in order to get reliable and high signal-to-noise ratio results (e.g., Kapanadze et al. 2020). In Table 2, we report the mean cadence for each LC.

Each LC is fitted by the SHO and DRW models, respectively. The goodness of the fit is assessed by analyzing the probability densities of the standardized residuals and the autocorrelation function (ACF) of the standardized residuals. The distribution of the standardized residuals is fitted by a normal distribution. The parameters and the reduced $\chi^2 (\chi^2_{\text{red}})$ are given in Table 2. It is shown that the distribution of the standardized residuals is in good agreement with the normal distribution with the mean value close to zero and the standard deviation less than one. In Figure 1, we show the fitting results for 3C 454.3 and 3C 279 as an example. The ACFs of the residuals are randomly distributed.

4. https://github.com/dper/merce
Table 2
Modeling Results for 23 AGNs

| Name         | Mean Cadence (days) | Model | AICc | Normal Distribution Fitting | μ (5) | σ (6) | $\chi^2_{\text{red}}$ (7) |
|--------------|---------------------|-------|------|----------------------------|-------|-------|---------------------------|
| 3C 454.3     | 16.37               | SHO   | 2341.31 | $-0.19 \pm 0.03$ | 0.33 ± 0.02 | 1.904               |
| 3C 279       | 15.71               | DRW   | 2338.69 | $-0.16 \pm 0.03$ | 0.41 ± 0.02 | 1.401               |
| PKS 1424−41  | 16.01               | SHO   | 2078.40 | $-0.18 \pm 0.02$ | 0.25 ± 0.02 | 2.320               |
| PKS 1510−089 | 15.66               | DRW   | 2076.14 | $-0.18 \pm 0.02$ | 0.25 ± 0.02 | 2.320               |
| PKS 1502+106 | 19.19               | SHO   | 873.39  | $-0.11 \pm 0.06$ | 0.68 ± 0.05 | 1.377               |
| BL Lac       | 16.09               | SHO   | 1284.71 | $-0.10 \pm 0.05$ | 0.68 ± 0.05 | 1.154               |
| PKS 2155−304 | 15.35               | SHO   | 293.01  | $-0.09 \pm 0.04$ | 0.57 ± 0.03 | 1.145               |
| PKS 0537−441 | 16.43               | SHO   | 584.69  | $-0.09 \pm 0.05$ | 0.76 ± 0.04 | 0.939               |
| 3C 66A       | 16.09               | SHO   | 271.57  | $-0.09 \pm 0.05$ | 0.83 ± 0.05 | 1.010               |
| NGC 1275     | 15.20               | SHO   | 972.44  | $-0.08 \pm 0.05$ | 0.78 ± 0.04 | 0.875               |
| CTA 102      | 18.32               | SHO   | 1727.43 | $-0.07 \pm 0.05$ | 0.79 ± 0.04 | 0.853               |
| PKS 0454−234 | 16.15               | SHO   | 842.94  | $-0.07 \pm 0.05$ | 0.74 ± 0.05 | 1.262               |
| 4C+21.35     | 24.13               | SHO   | 993.40  | $-0.09 \pm 0.06$ | 0.69 ± 0.05 | 1.582               |
| 4C+38.41     | 17.22               | SHO   | 991.29  | $-0.09 \pm 0.06$ | 0.73 ± 0.04 | 1.244               |
| Ton 599      | 22.04               | SHO   | 733.25  | $-0.09 \pm 0.06$ | 0.72 ± 0.04 | 1.386               |
| 3C 273       | 22.35               | SHO   | 1083.29 | $-0.09 \pm 0.06$ | 0.82 ± 0.04 | 1.418               |
| B2 1520+31   | 22.15               | SHO   | 1081.02 | $-0.10 \pm 0.05$ | 0.73 ± 0.04 | 1.206               |
| Mkn 421      | 15.15               | SHO   | 605.98  | $-0.16 \pm 0.06$ | 0.69 ± 0.05 | 1.297               |
| PKS 0426−380 | 15.76               | SHO   | 709.48  | $-0.09 \pm 0.05$ | 0.77 ± 0.04 | 1.471               |
| S5 0716+71   | 16.37               | SHO   | 758.54  | $-0.19 \pm 0.06$ | 0.85 ± 0.04 | 1.630               |
| PG 1553+113  | 15.15               | SHO   | 756.39  | $-0.08 \pm 0.05$ | 0.85 ± 0.04 | 1.612               |
| OJ 287       | 26.34               | SHO   | 311.75  | $-0.02 \pm 0.05$ | 0.54 ± 0.05 | 1.475               |
| TXS 0506+056 | 23.26               | SHO   | 246.85  | $-0.12 \pm 0.08$ | 0.77 ± 0.07 | 1.891               |

Note. (1) source name, (2) the mean cadence of LC, (3) model, (4) AICc, (5)−(7) the results and the reduced $\chi^2$ of normal distribution fitting to the standard residuals.

around zero and are almost inside the 95% confidence limits of the white noise. It indicates that the model has captured the characteristics of the time series. It is notable that the standardized residuals corresponding to the two or three highest fluxes are large. The DRW and SHO models likely fail to describe the brightest flares. When we fit the LC excluding the two or three highest flux points, the modeling results are unchanged.

We give the posterior probability density distribution of the parameters resulting from the SHO and DRW modelings in Figure 2. The values of the model parameters are given in Table 3. One can see that the model parameters in the DRW model are constrained well, although the two parameters are degenerate while in the SHO model, the strong degeneracy between $\omega_0$ and $Q$ leads to large uncertainties in the two parameters. In some cases, the large uncertainties on $Q$ cause the unilateral distribution of $\omega_{\text{up}}$, and the upper limit for $\omega_0$ is meaningless. For all sources, the SHO model is constrained in the overdamped mode ($Q < 0.5$).

In Figure 3, we show the PSDs for 3C 454.3 and 3C 279 constructed from the modeling results with the SHO and DRW models. The PSDs for SHO and DRW are almost the same. The index changes from 0 at frequencies below $f_b$ to $-2$ at frequencies above $f_b$. 


The difference between the AICc for SHO and DRW is small (Table 2), indicating that the fits of the two models are comparable. However, the poorly constrained \( \omega_0 \) and \( Q \) suggest that the DRW model is preferred over the SHO model.

There is no significant difference between the forms of the PSDs from different types of AGNs (Figure 4). The PSDs for the 23 sources are typical DRW PSDs, and the broken frequencies are between 0.001 \( \text{day}^{-1} \) and 0.01 \( \text{day}^{-1} \).

5. Variability Characteristic Timescales in Relativistic Jets

Characteristic timescale is an important parameter in source variability. Our results show that the DRW model can describe the \( \gamma \)-ray variability of the 23 targets successfully, and the higher-order model SHO is unnecessary. From the fittings with the DRW model, we can obtain the characteristic timescales for the 23 sources. The characteristic (damping) timescales with errors in \( \tau \) are given in Table 3. One can see that the timescales are between 20 days and 250 days. Note that the measurement of the damping timescale can be biased by the insufficient length of the LC (Kozlowski 2017; Suberlak et al. 2021). Moreover, Burke et al. (2021) found that the measurement of the damping timescale is reliable when it is larger than the typical cadence of the LC. We use the following criteria to select the reliable measurements of the damping timescale: (1) The length of the LC should be 10 times that of the timescale at least, and (2) the derived timescale should be larger than the mean cadence of the LC. It is found that all the characteristic \( \gamma \)-ray timescales for the 23 sources are reliable.

The LCs are fitted in the observed frame, and the values of the damping timescales in Table 3 are in the observed frame. In order to get the timescale in the rest frame (\( \tau_{\text{damping}}^{\text{rest}} \)), the timescale should be corrected by the cosmological time dilation and Doppler beaming effect,

\[
\tau_{\text{damping}}^{\text{rest}} = \frac{\tau_{\text{damping}} \delta_D}{1 + z},
\]

where \( \delta_D \) is the Doppler factor. The Doppler factor for the \( \gamma \)-ray emission region in an AGN jet is difficult to measure. It is estimated by different methods based on, for example, modeling the broadband spectral energy distributions (Chen 2018; Pei et al. 2020), the opacity of the \( \gamma \)-rays, and the brightness temperature of the radio flare (Liodakis et al. 2017). In Table 4, we list the average values of \( \delta_D \) for different types of AGNs estimated by three recent papers (Liodakis et al. 2017; Chen 2018; Pei et al. 2020). One can see that the uncertainties on the Doppler factor are very large. We use the average results of the three papers to correct the timescale. The average \( \delta_D \) for blazars is 10. It is found that \( \tau_{\text{damping}}^{\text{rest}} \) is between 100 days and 1500 days, and the average \( \tau_{\text{damping}}^{\text{rest}} \) is \( \approx 510 \) days.

The characteristic timescale in the optical variability of the AGN accretion disk has been extensively studied (e.g., Collier & Peterson 2001; Kelly et al. 2009; MacLeod et al. 2010; Simm et al. 2016; Suberlak et al. 2021). Recently, Burke et al. (2021) reported a correlation between the optical characteristic timescale and the black hole mass. We also show our results in the plot of \( \tau_{\text{damping}}^{\text{opt}} - M_{BH} \) (Figure 5), together with the results in Burke et al. (2021). It can be seen that the \( \gamma \)-ray variability timescales of AGNs occupy the same space as the optical variability timescales. Namely, in the same range of black hole masses, the \( \gamma \)-ray variability timescales are consistent with the optical variability timescales within the errors. There is no correlation between the \( \gamma \)-ray variability timescale of AGNs and the black hole mass. This is probably due to the small dynamic range of the black hole masses in the sample of \( 10^{10} - 10^{9} M_\odot \) (with the exception of NGC 1275), making any correlation difficult to identify. We have searched for \( \gamma \)-ray AGNs with a smaller black hole mass. However, they are not bright enough to perform the variability analysis. In the Appendix, we use the \( \gamma \)-ray flares from the Crab Nebula to extend the mass of the central engine to a much smaller range.

The \( \gamma \)-ray \( \tau_{\text{damping}}^{\text{rest}} \) values slightly and systematically lie above the optical relation of Burke et al. (2021). In addition, we fit our \( \gamma \)-ray results and the optical results together, resulting in the best-fit relation

\[
\tau_{\text{damping}}^{\text{rest}} = 154.22_{-13.75}^{+14.76} \left( \frac{M_{BH}}{10^8 M_\odot} \right)^{0.43_{-0.04}^{+0.04}},
\]
with an intrinsic scatter of $0.23 \pm 0.03$ dex and Pearson correlation coefficient $r = 0.80$. It is similar to the optical result.

We get the Eddington ratio (the ratio between the accretion disk luminosity and the Eddington luminosity) for 15 out of 23 AGNs from Xiong et al. (2015), which are listed in Table 1. Except for Mrk 421, BL Lac, and OJ 287, the remaining 12 sources have an Eddington ratio between 0.01 and 1. From Figure 6, one can see that the 12 sources have a similar Eddington ratio to the normal quasars in Burke et al. (2021), and the characteristic timescale is independent of the Eddington ratio.

6. Discussion

The stochastic process models are increasingly used to model the $\gamma$-ray variability of blazars, providing an effective tool to study the statistical properties of the variability. We model the $\gamma$-ray variability of 23 bright LAT AGNs with the DRW and SHO models by performing the celerite package. It is found that the DRW model with two parameters can fit the $\gamma$-ray variability of 23 AGNs successfully, and the fits with the SHO model with three parameters for the 23 sources are not improved. The SHO is constrained in the overdamped mode ($Q < 0.5$), and the strong degeneracy between $\omega_0$ and $Q$ leads
Table 3

| Name            | Model | Parameter of SHO  | Parameter of DRW  | Damping Timescale |
|-----------------|-------|-------------------|-------------------|-------------------|
|                 | (1)   | ln $S_0$ (2)      | ln $Q$ (3)        | ln $\omega_0$ (4) |
|                 |       | ln $\sigma_{\text{DRW}}$ (5) | ln $\tau_{\text{DRW}}$ (6) | (7) |
| 3C 454.3        | SHO   | 10.60 $^{+0.41}_{-0.34}$ | $-1.75^{+0.43}_{-1.08}$ | $-2.44_{-0.27}$ |
|                 | DRW   | ...               | ...               | ...               |
| 3C 279          | SHO   | 7.45 $^{+0.37}_{-0.27}$  | $-2.07_{-0.69}$   | $-0.98_{-0.85}$   |
|                 | DRW   | ...               | ...               | ...               |
| PKS 1424–41     | SHO   | 7.28 $^{+0.38}_{-0.45}$  | $-3.28_{-0.79}$   | $-1.47_{-0.76}$   |
|                 | DRW   | ...               | ...               | ...               |
| PKS 1510–089    | SHO   | 7.57 $^{+0.31}_{-0.27}$ | $-1.46_{-0.96}$   | $-2.14_{-0.88}$   |
|                 | DRW   | ...               | ...               | ...               |
| PKS 1502+106    | SHO   | 7.91 $^{+0.08}_{-0.68}$ | $-3.89_{-1.02}$   | $-1.54_{-0.82}$   |
|                 | DRW   | ...               | ...               | ...               |
| BL Lac          | SHO   | 6.74 $^{+0.44}_{-0.56}$ | $-3.22_{-0.93}$   | $-1.02_{-0.80}$   |
|                 | DRW   | ...               | ...               | ...               |
| PKS 2155–304    | SHO   | 2.66 $^{+0.34}_{-0.54}$ | $-2.58_{-0.73}$   | $-1.56_{-0.76}$   |
|                 | DRW   | ...               | ...               | ...               |
| PKS 0537–441    | SHO   | 6.35 $^{+0.93}_{-0.64}$ | $-4.01_{-0.73}$   | $-1.48_{-0.80}$   |
|                 | DRW   | ...               | ...               | ...               |
| 3C 66A          | SHO   | 3.15 $^{+0.50}_{-0.40}$ | $-3.11_{-0.64}$   | $-1.40_{-0.72}$   |
|                 | DRW   | ...               | ...               | ...               |
| NGC 1275        | SHO   | 5.51 $^{+0.35}_{-0.56}$ | $-3.00_{-0.56}$   | $-1.28_{-0.64}$   |
|                 | DRW   | ...               | ...               | ...               |
| CTA 102         | SHO   | 9.31 $^{+0.43}_{-0.35}$ | $-2.79_{-0.76}$   | $-1.56_{-0.67}$   |
|                 | DRW   | ...               | ...               | ...               |
| PKS 0454–234    | SHO   | 5.28 $^{+0.43}_{-0.35}$ | $-1.84_{-1.17}$   | $-2.44_{-0.84}$   |
|                 | DRW   | ...               | ...               | ...               |
| 4C+21.35        | SHO   | 7.41 $^{+0.36}_{-0.36}$ | $-2.71_{-0.54}$   | $-1.32_{-0.57}$   |
|                 | DRW   | ...               | ...               | ...               |
| 4C+38.41        | SHO   | 6.57 $^{+0.42}_{-0.42}$ | $-2.95_{-0.59}$   | $-1.60_{-0.63}$   |
|                 | DRW   | ...               | ...               | ...               |
| Ton 599         | SHO   | 5.72 $^{+0.35}_{-0.39}$ | $-1.84_{-1.17}$   | $-2.39_{-1.09}$   |
|                 | DRW   | ...               | ...               | ...               |
| 3C 273          | SHO   | 6.14 $^{+0.39}_{-0.39}$ | $-2.43_{-0.59}$   | $-1.00_{-0.47}$   |
|                 | DRW   | ...               | ...               | ...               |
| B2 1520+31      | SHO   | 5.19 $^{+0.34}_{-0.38}$ | $-2.95_{-0.56}$   | $-1.27_{-0.64}$   |
|                 | DRW   | ...               | ...               | ...               |
| Mkn 421         | SHO   | 3.20 $^{+0.34}_{-0.29}$ | $-1.98_{-0.93}$   | $-1.88_{-0.90}$   |
|                 | DRW   | ...               | ...               | ...               |
| PKS 0426–380    | SHO   | 5.25 $^{+0.41}_{-0.41}$ | $-2.76_{-0.55}$   | $-1.89_{-0.77}$   |
|                 | DRW   | ...               | ...               | ...               |
| SS 0716+71      | SHO   | 3.23 $^{+0.26}_{-0.30}$ | $-2.08_{-0.53}$   | $-1.09_{-0.41}$   |
|                 | DRW   | ...               | ...               | ...               |
| PG 1553+113     | SHO   | 2.02 $^{+1.15}_{-0.76}$ | $-3.49_{-1.27}$   | $-2.19_{-1.20}$   |
|                 | DRW   | ...               | ...               | ...               |
| OJ 287          | SHO   | 2.80 $^{+0.36}_{-0.52}$ | $-1.96_{-0.66}$   | $-1.76_{-0.67}$   |
|                 | DRW   | ...               | ...               | ...               |
| TXS 0506+56     | SHO   | 3.44 $^{+0.42}_{-0.30}$ | $-3.29_{-0.70}$   | $-1.45_{-0.78}$   |
|                 | DRW   | ...               | ...               | ...               |

Note. (1) source name, (2) model, (3)–(5) posterior parameters of modeling LCs with the SHO model, (6)–(7) posterior parameters of modeling LCs with the DRW model, and (8) damping timescale. The uncertainties of the model parameters and damping timescales represent 1σ confidence intervals.

to poor constraints on the two parameters. The PSDs of the 23 sources are of typical DRW form with $f_0$ between 0.001 day$^{-1}$ and 0.01 day$^{-1}$.

The brightest flares ($\geq 10^{-5}$ ph cm$^{-2}$ s$^{-1}$) in 3C 279 and 3C 454.3 are poorly fitted by both DRW and SHO models. Indeed, the extreme γ-ray flares in 3C 279 seem special (e.g., Shukla & Mannheim 2020). The γ-ray photon index in the extreme flare on 2012 December 16 is 1.7 (Hayashida et al. 2015), significantly smaller than the typical γ-ray photon index of 2.4 for FSRQs (Abdollahi et al. 2020). Minute-scale GeV γ-ray variability from 3C 279 was observed in an extreme flare on 2016 June 15 (Ackermann et al. 2016). Nalewajko (2013)
The individual γ-ray flares of 3C 454.3 with the flux above $0.71 \times 10^{-5}$ ph cm$^{-2}$ s$^{-1}$ and found that the γ-ray flares of 3C 454.3 have more complex LCs than other blazars. The extreme flares may have a different physical mechanism than the underlying long-term stochastic variability.

These brightest flares are expected to have an impact on the slope of the PSD at high frequencies (Ryan et al. 2019). We have examined how the brightest flares in 3C 279 and 3C 454.3 do not affect the modeling results for long-term variabilities. It is worth performing a further and careful study of the brightest flares using an adaptive binning algorithm.

The theoretical PSD expected from the one-zone leptonic emission model has been investigated (Finke & Becker 2014, 2015; Chen et al. 2017). Thiersen et al. (2022) simulated the multiwavelength variability of blazars from a purely numerical approach by using a time-dependent one-zone leptonic emission model. They showed that a PL PSD for the emission variability is produced by introducing stochastic variations for model parameters in the emission region, and the PSD is similar to the underlying PL of the model parameter variation. No spectral break is found in their produced PSDs. The results of Thiersen et al. (2022)

Table 4

| Source Type | BL Lac | RDG | Ref. |
|------------|-------|-----|------|
| FSRQ       | 14    | ... | 1    |
| 15 ± 9     | 9 ± 8 | 1.4 ± 0.8 | 2 |
| 7 ± 4      | 4 ± 3 | ... | 3 |

Note. Doppler factors with the errors for different types of AGNs taken from the references: (1) Chen (2018), (2) Liodakis et al. (2017), and (3) Pei et al. (2020).
indicate that in the frame of the one-zone emission model, physical processes associated with electron cooling, light-crossing, and electron escape would not produce a break in the PSD. The broken frequencies we obtained are between $10^{-8}$ Hz and $10^{-7}$ Hz. The corresponding intrinsic timescale is several hundred days at least, which cannot be the timescale corresponding to electron cooling or acceleration process.

The $\gamma$-ray timescales of the AGNs we obtained are very close to the optical timescales obtained from modeling AGN accretion disk emissions in Burke et al. (2021). Burke et al. (2021) speculated that the optical timescales could be associated with the thermal timescales expected in the AGN standard accretion disk theory, and the optical variability may be driven by the thermal instability of the accretion disk. The similarity between the $\gamma$-ray and optical characteristic timescales could imply a connection between the jet and accretion disk. The

\[
R_{\text{th}} = \frac{\alpha}{M_{\text{BH}}^{0.01}} \frac{c}{\Sigma^{1/2}} \text{yr},
\]

where $R$ is the emission distance on the accretion disk from the central black hole, $R_{\text{th}} = 2GM_{\text{BH}}/c^2$ is the Schwarzschild radius, and $\alpha$ is the standard disk viscosity parameter.
thermal instability may also cause the $\gamma$-ray variability in the jet. However, the detailed mechanism that connects the accretion disk and the jet is unclear. The $\gamma$-ray timescales are slightly larger than the optical timescales of normal quasars. This may be due to the distance from the $\gamma$-ray emission region to the accretion disk extending the intrinsic timescale slightly larger than the optical timescales of normal quasars. $\gamma$

Ruan et al. (2012) modeled the nonthermal optical variabilities of 51 $\gamma$-ray blazars and found that the blazar optical $\tau^\text{rest}_{\text{damping}}$ peaks at $\sim$1000 days (assuming a typical Doppler factor of 10), which is systematically larger than the $\gamma$-ray $\tau^\text{rest}_{\text{damping}}$ in this work. The discrepancy between the blazar $\gamma$-ray and optical $\tau^\text{rest}_{\text{damping}}$ may imply that the $\gamma$-ray and optical emissions are produced in different regions. The $\gamma$-ray emission region is closer to the accretion disk than the optical emission region. Ruan et al. (2012) found that nonthermal optical characteristic timescales of blazars are $\sim$4 times smaller than those of normal quasars. They considered that the discrepancy between the optical characteristic timescales for blazars and normal quasars could be caused by the Doppler effect if the jet variability and accretion disk variability have the same origin. Combining with our $\gamma$-ray results, we suppose that the discrepancy between the characteristic timescales for blazars and normal quasars is not only caused by the Doppler effect but also related to the location of the jet emission region (the distance from the accretion disk). The jet’s long-term variability may be the convolution of the accretion disk variability with a transfer function that is related to the Doppler factor and the distance from the jet emission region to the accretion disk at least.

7. Summary

We have applied a stochastic process method to the $\sim$12.7 yr Fermi-LAT LCs of 23 jetted AGNs in order to investigate $\gamma$-ray variability properties. The SHO and DRW models are both used to model long-term LCs. Our main results are as follows.

(i) The long-term variability of 23 sources in our sample can be described well by both the SHO and DRW models. However, modeling with the SHO is not improved, and the parameters $\omega_1$ and $Q$ are poorly constrained. This suggests that the DRW model is preferred over the SHO model for the long-term $\gamma$-ray variability of AGNs. The PSDs for the 23 sources are in the typical DRW PSD form.

(ii) The intrinsic characteristic timescale of AGNs extracted from modeling the $\gamma$-ray variability is between 100 days to 1500 days. Such a long timescale cannot be produced in a one-zone leptonic emission model within the typical parameter space. In the plot of $\tau^\text{rest}_{\text{damping}} - M_{\text{BH}}$, the $\gamma$-ray timescales obtained from jet emissions occupy almost the same space as the optical timescales obtained from the accretion disk emissions. Both the $\gamma$-ray and optical timescales are consistent with the thermal timescale expected from the AGN standard accretion disk. It may indicate a connection between the jet and the accretion disk.

In conclusion, our results suggest that the origin of the $\gamma$-ray variability could be related to the thermal instability in the accretion disk; however, the detailed process that drives the variability is unclear.

We thank the referee for valuable suggestions and Dr. Xiaoyuan Huang (PMO) for providing the $\gamma$-ray flare data of the Crab Nebula. This work is partially supported by the National Key R & D Program of China under grant No. 2018YFA0404204 and the National Natural Science Foundation of China (U1738211 and 11803081). H.Y.Z. acknowledges financial support from the Scientific Research Fund project of the Yunnan Education Department (2022Y053) and the Graduate Research innovation project of Yunnan University (2021Y034). The work of D.H.Y. is also supported by the CAS Youth Innovation Promotion Association and Basic Research Program of Yunnan Province (202001AW070013).

Facility: Fermi(LAT).

Software: Fermi-tools-conda, celerite (Foreman-Mackey et al. 2017), emcee (Foreman-Mackey et al. 2013), NumPy (Harris et al. 2020), Matplotlib (Hunter 2007), Astropy (Astropy Collaboration et al. 2013, 2018), SciPy (Virtanen et al. 2020).

Appendix

A.1. Crab Nebula $\gamma$-Ray Flare

$\gamma$-ray flares from the Crab Nebula were observed by AGILE (Tavani et al. 2011) and Fermi-LAT (Abdo et al. 2011). The central pulsar, PSR B0531+21, has a mass of 1.4 solar mass. To extend the $\gamma$-ray $\tau^\text{rest}_{\text{damping}}$–mass relation to a much smaller mass, we consider the $\gamma$-ray flares from the Crab Nebula. Huang et al. (2021) identified 17 flares in the $\gamma$-ray emission from the Crab Nebula. The flare during MJD 55654.65–55678.65 has a good sample, and the flux uncertainties are relatively small. We use the DRW model to model 4 hr binning LCs during MJD 55654.65–55678.65. The fitting results, PSD, and posterior probability densities of the parameters are shown in Figures 7 and 8. We obtain the characteristic timescale $1.8^{+0.7}_{-1.3}$ days. This timescale is less than 1/10 of the length of the LC and larger than the mean cadence (0.36 days), which is reliable. We use the result to extend the $\gamma$-ray $\tau^\text{rest}_{\text{damping}}$–mass relation (Figure 9). There is a correlation (Pearson correlation coefficient $r = 0.90$) between the $\gamma$-ray characteristic timescale and mass when adding the result of the Crab Nebula, i.e.,

$$\tau^\text{rest}_{\text{damping}} = 257.52^{28.49}_{33.21} \left( \frac{M_{\text{BH}}}{10^8 M_\odot} \right)^{0.26^{0.04}_{0.04}},$$

with an intrinsic scatter of 0.21 ± 0.05 dex.

The Astrophysical Journal, 930:157 (13pp), 2022 May 10

Zhang, Yan, & Zhang
Figure 7. DRW modeling results of $\gamma$-ray flare (MJD 55654.65–55678.65) from the Crab Nebula. The symbols and lines are the same as those in Figure 1.

Figure 8. Left panel: posterior probability densities of DRW parameters for the Crab Nebula. The symbols and lines are the same as those in Figure 2. Right panel: DRW PSD of the $\gamma$-ray LC of the Crab Nebula. The corresponding color region denotes the 1σ confidence interval.
Figure 9. Variability damping timescale (in the rest frame) as a function of the mass of the central engine. The gray data, lines, area as well as the crosses represent the optical results taken from Burke et al. (2021). The data in color are our results from the γ-ray LCs of the AGNs and the Crab Nebula. The orange line and shaded band are the best-fit relation and 1σ uncertainty for 23 AGNs and the Crab Nebula.

ORCID iDs

Dahai Yan (闫力大的) © https://orcid.org/0000-0003-4895-1406
Li Zhang (张力大) © https://orcid.org/0000-0002-5880-8497

References

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009, ApJS, 183, 46
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010, ApJ, 722, 520
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2011, Sci, 331, 739
Abdollahi, S., Acero, F., Ackermann, M., et al. 2020, ApJS, 247, 33
Ackermann, M., Albert, A., Angioni, R., Axelsson, M., et al. 2020, ApJ, 892, 105
Acknowledgement, Price-Whelan, A. M., Sipőcz, B. M., et al. 2013, A&A, 558, A33
Chen, L. 2018, ApJS, 235, 39
Chen, X., Pohl, M., Böttcher, M., & Gao, S. 2016, MNRAS, 458, 3260
Collier, S., & Peterson, B. M. 2001, ApJ, 555, 775
Covino, S., Landoni, M., Sandrinelli, A., & Treves, A. 2020, ApJ, 895, 122
Covino, S., Sandrinelli, A., & Treves, A. 2019, MNRAS, 482, 1270
Fan, Z.-H., & Cao, X. 2004, ApJ, 602, 103
Ferrarese, L., Remillard, R. A., Peterson, B. M., et al. 2001, ApJL, 555, L79
Finke, J. D., & Becker, P. A. 2014, ApJL, 791, 21
Finke, J. D., & Becker, P. A. 2015, ApJ, 809, 85
Foreman-Mackey, D., Agol, E., Ambikasaran, S., & Anglus, R. 2017, AJ, 154, 220
Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306
Gebhardt, K., Kormendy, J., Ho, L. C., et al. 2000, ApJL, 543, L5
Ghisellini, G., Tavecchio, F., Foschini, L., et al. 2010, MNRAS, 402, 497
Goyal, A., Stawarz, L., Zola, S., et al. 2018, ApJ, 863, 175
Gupta, S. P., Pandey, U. S., Singh, K., et al. 2012, NewA, 17, 8
H. E. S. S. Collaboration, Abdalla, H., Abramowski, A., et al. 2017, A&A, 598, A39
Harris, C. R., Millman, K. J., van der Walt, S. J., et al. 2020, Natur, 585, 357
Hayashida, M., Nalewajko, K., Madejski, G., et al. 2015, ApJ, 807, 79
Huang, X., Yuan, Q., & Fan, Y.-Z. 2021, ApJ, 908, 65
Hunter, J. D. 2007, CSE, 9, 90
Kapanadze, B., Gurchumelia, A., Dorner, D., et al., 2020, ApJ, 247, 27
Kasliwal, V. P., Vogeley, M. S., & Richards, G. T. 2017, MNRAS, 470, 3027
Kaur, N., Ballytan, K. S., Chandra, S., Sameer, & Ganesh, S. 2018, AJ, 156, 36
Kaur, N., Sameer, Ballytan, K. S., & Ganesh, S. 2017, MNRAS, 469, 2305
Kelly, B. C., Bechtold, J., & Siemiginowska, A. 2009, ApJ, 698, 895
Kelly, B. C., Becker, A. C., Sobolewska, M., Siemiginowska, A., & Uttley, P. 2014, ApJ, 788, 33
Kozlowski, S. 2017, A&A, 597, A128
Li, Y.-R., & Wang, J.-M. 2018, MNRAS, 476, L55
Liodakis, I., Marchili, N., Angelakis, E., et al. 2017, MNRAS, 466, 4625
Liu, Y., Jiang, D. R., & Gu, M. F. 2006, ApJ, 637, 669
Lu, K.-X., Huang, Y.-K., Zhang, Z.-X., et al., 2019, ApJ, 877, 23
MacLeod, C. L., Ivezic, Z., Kochanek, C. S., et al., 2010, ApJ, 721, 1014
MAGIC Collaboration, Ansoldi, S., Antonelli, L. A., et al. 2018, A&A, 617, A91
Mattos, J. R., Becerich, D. L., Chiang, J., et al. 1996, ApJ, 461, 396
Mcrelle, R. J., & Dunlop, J. S. 2001, MNRAS, 327, 199
Meyer, D., Scargle, J. D., & Blandford, R. D. 2019, ApJ, 877, 39
Moreno, J. L., Siemiginowska, A., Richards, G. T., & Yu, W. 2019, PASP, 131, 063001
Nakagawa, K., & Mori, M. 2013, ApJ, 773, 177
Nalewajko, K. 2013, MNRAS, 430, 1324
Padovani, P., Oikonomou, F., Petropoulou, M., Giommi, P., & Resconi, E. 2019, MNRAS, 484, L104
Paliya, V. S., Marcotulli, L., Ajello, M., et al. 2017, ApJ, 851, 33
Peñal, P., Domínguez, A., Buson, S., et al., 2020, ApJ, 896, 134
Pei, Z., Fan, J., Yang, J., & Bastieri, D. 2020, PASA, 37, e043
Rickert, S., & Stalins, C. S. 2017, ApJ, 842, 96
Rieger, F. 2019, Galax, 7, 28
Ruan, J. J., Anderson, S. F., MacLeod, C. L., et al. 2012, ApJ, 760, 51
Ryan, J. L., Siemiginowska, A., Sobolewska, M., & Grindlay, J. 2019, ApJ, 885, 12
Sandrinelli, A., Covino, S., Dotti, M., & Treves, A. 2016, AJ, 151, 54
Sani, E., Ricci, F., La Franca, F., et al. 2018, FrASS, 5, 2
Sharrato, T., Ghisellini, G., Maraschi, L., & Colpi, M. 2012, MNRAS, 421, 1764
Shah, Z., Mankuzhiyil, N., Sinha, A., et al. 2018, RAA, 18, 141
Shaw, M. S., Romani, R. W., Cotter, G., et al. 2012, ApJ, 748, 49
Shukla, A., & Mannheim, K. 2020, NatCo, 11, 4176
Sirun, T., Salvato, M., Saglia, R., et al. 2016, A&A, 585, A129
Sobolewska, M. A., Siemiginowska, A., Kelly, B. C., & Nalewajko, K. 2014, ApJ, 786, 143
Suberlak, K. L., Ivezic, Z., & MacLeod, C. 2021, ApJ, 907, 96
Tarnopolski, M., Zywicka, N., Marchenko, V., & Pascual-Granado, J. 2020, ApJS, 250, 1
Tavani, M., Bulgarelli, A., Vittorini, V., et al. 2011, Sci, 331, 736
Thiersen, H., Zacharias, M., & Böttcher, M. 2022, ApJ, 925, 177
Tremaine, S., Gebhardt, K., Bender, R., et al. 2002, ApJ, 574, 740
Virtanen, P., Gommers, R., Oliphant, T. E., et al. 2020, NatMe, 17, 261
Wang, J.-M., Luo, B., & Ho, L. C. 2004, ApJL, 615, L9
Xiong, D., Zhang, X., Bai, J., & Zhang, H. 2015, MNRAS, 450, 3568
Yan, D., Yang, S., Zhang, P., et al. 2018, ApJ, 864, 164
Yang, S., Yan, D., Zhang, P., Dai, B., & Zhang, L. 2021, ApJ, 907, 105
Zhang, H., Yan, D., Zhang, P., Yang, S., & Zhang, L. 2021, ApJ, 919, 58
Zhang, H., Yang, Q., & Wu, X.-B. 2018, ApJ, 853, 116
Zhang, P.-f., Yan, D.-h., Zhou, J.-n., Wang, J.-c., & Zhang, L. 2020, ApJ, 891, 163
Zhou, J., Wang, Z., Chen, L., et al. 2018, NatCo, 9, 4599
Zu, Y., Kochanek, C. S., Kozlowski, S., & Udalski, A. 2013, ApJ, 765, 106