FIGURE ROTATION OF COSMOLOGICAL DARK MATTER HALOS

Jeremy Bailin1 and Matthias Steinmetz2
Steward Observatory, University of Arizona, 933 North Cherry Avenue, Tucson, AZ 85721; and Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany; jbailin@as.arizona.edu, msteinmetz@aip.de
Received 2004 May 21; accepted 2004 August 6

ABSTRACT

We have analyzed galaxy- and group-sized dark matter halos formed in a high-resolution ΛCDM numerical N-body simulation in order to study the rotation of the triaxial figure, a property in principle independent of the angular momentum of the particles themselves. Such figure rotation may have observational consequences, such as triggering spiral structure in extended gas disks. The orientation of the major and minor axes are compared at five late snapshots of the simulation. Halos with significant substructure or that appear otherwise disturbed are excluded from the sample. We detect smooth figure rotation in 288 of the 317 halos in the sample. The pattern speeds follow a lognormal distribution centered at \( \Omega_p = 0.148 \, h \, \text{km s}^{-1} \, \text{kpc}^{-1} \) with a width of 0.83. These speeds are an order of magnitude smaller than required to explain the spiral structure of galaxies such as NGC 2915. The axis about which the figure rotates aligns very well with the halo minor axis in 85% of the halos and with the major axis in the remaining 15% of the halos. The figure rotation axis is usually reasonably well aligned with the angular momentum vector. The pattern speed is correlated with the halo spin parameter \( \lambda \) but shows no correlation with the halo mass. The halos with the highest pattern speeds show particularly strong alignment between their angular momentum vectors and their figure rotation axes. The figure rotation is coherent outside 0.12\( r_{200} \). The measured pattern speed and degree of internal alignment of the figure rotation axis drops in the innermost region of the halo, which may be an artifact of the numerical force softening. The axis ratios show a weak tendency to become more spherical with time.

Subject headings: dark matter — galaxies: evolution — galaxies: formation — galaxies: individual (NGC 2915) — galaxies: kinematics and dynamics — galaxies: structure

Online material: color figures

1. INTRODUCTION

Although there have been many theoretical studies of the shapes of cosmological dark matter halos (e.g., Dubinski & Carlberg 1991; Warren et al. 1992; Cole & Lacey 1996; Jing & Suto 2002), there has been relatively little work done on how those figure shapes evolve with time, in particular whether the orientation of a triaxial halo stays fixed or whether the figure rotates. While the orientation of the halo can clearly change during a major merger, it is not known whether the orientation changes between cataclysmic events. Absent any theoretical prediction one way or the other, it is usually assumed that the figure orientation of triaxial halos remains fixed when in isolation (e.g., Subramanian 1988; Johnston et al. 1999; Lee & Suto 2003).

Early work at detecting figure rotation in simulated halos was done by Dubinski (1992, hereafter D92). While examining the effect of tidal shear on halo shapes, he found that in all 14 of his \((1-2) \times 10^{12} \, M_\odot \) halos the direction of the major axis rotated uniformly around the minor axis. The period of rotation varied from halo to halo and ranged from 4 Gyr at the fast end to 50 Gyr at the slow end, or equivalently the angular velocities ranged between 0.1 and 1.6 km s\(^{-1}\) kpc\(^{-1}\). It is difficult to draw statistics from this small sample size, especially since the initial conditions of this simulation were not drawn from cosmological models but were performed in a small isolated box with the linear tidal field of the external matter prescriptively superposed (Dubinski & Carlberg 1991). Given that the main result of D92 is that the tidal shear may have a significant impact on the shapes of halos, it is clearly important to do such studies using self-consistent cosmological initial conditions.

Recent studies of figure rotation come from Bureau et al. (1999, hereafter BFP99) and Pfitzner (1999, hereafter P99). P99 compared the orientation of cluster mass halos in two snapshots spaced 500 Myr apart in an SCDM simulation (\( \Omega = 1, \Lambda = 0, h = 0.5 \)). He detected rotation of the major axis in ~5% of them and argued that the true fraction with figure rotation is probably higher. BFP99 presented one of these halos, which was extracted from its cosmological surroundings and left to evolve in isolation for 5 Gyr. During that time, the major axis rotated around the minor axis uniformly at all radii at a rate of 60° Gyr\(^{-1}\), or about 1 km s\(^{-1}\) kpc\(^{-1}\).

There may be observational consequences to a dark matter halo whose figure rotates. BFP99 suggested that triaxial figure rotation is responsible for the spiral structure of the blue compact dwarf galaxy NGC 2915. Outside of the optical radius, NGC 2915 has a large \( \text{H} \)\( \text{I} \) disk extending to over 22 optical disk scale lengths (Murer et al. 1996). The gas disk shows clear evidence of a bar and a spiral pattern extending over the entire radial extent of the disk. BFP99 argue that the observed gas surface density is too low for a bar or spiral structure to form by gravitational instability and that there is no evidence of an interacting companion to trigger the pattern. They propose...
that the pattern may instead be triggered by a rotating triaxial halo.

Bekki & Freeman (2002) followed this up with smoothed particle hydrodynamics (SPH) simulations of a disk inside a halo whose figure rotates and showed that a triaxial halo with a flattening of $b/a = 0.8$ and a pattern speed of $3.84 \text{ km s}^{-1} \text{ kpc}^{-1}$ could trigger spiral patterns in the disk, or warps when the figure rotation axis is inclined to the disk symmetry axis. Masett & Bureau (2003, hereafter MB03) found that, in detail, the observations of NGC 2915 are better fitted by increasing the disk mass by an order of magnitude (for example, if most of the hydrogen is molecular, e.g., Pfenniger et al. 1994), but that a triaxial halo with $b/a \approx 0.85$ and a pattern speed of between 6.5 and 8.0 km s$^{-1}$ kpc$^{-1}$ also provides an acceptable fit.

MB03 concluded that if the halo were undergoing solid body rotation at this rate, its spin parameter would be $\lambda \approx 0.157$, which is extremely large (only $5 \times 10^{-3}$ of all halos have spin parameters at least that large). However, this argument may be flawed because the figure rotation is a pattern speed, not the speed of the individual particles that constitute the halo, and so it is in principle independent of the angular momentum; in some cases, the figure may even rotate retrograde to the particle orbits (Freeman 1966). Schwarzschild (1982) discusses in detail the orbits inside elliptical galaxies with figure rotation. He finds that models can be constructed from box and $X$-tube orbits, which have no net streaming of particles with respect to the figure (although they have prograde streaming at small radius and retrograde streaming at large radius) and so result in figures and particles with the same net rotation. He also constructs models that include prograde-streaming $Z$-tube orbits, which result in a figure that rotates slower than the particles. Stable retrograde $Z$-tube orbits also exist, but Schwarzschild (1982) did not attempt to include them in his models, so it may also be possible for the figure to rotate faster than the particles. While these results demonstrate the independence of the figure and particle rotation, it is not clear whether they can be translated directly to dark matter halos. Dark matter halos may have different formation mechanisms and may be subject to different tidal forces than elliptical galaxies, and the different density profile may also have a large effect on the viable orbital families (Gerhard & Binney 1985).

There are other consequences of triaxial figure rotation. A rotating potential introduces an oscillating force on particles moving within the potential. Disk stars that have orbital frequencies in resonance with this oscillating force may experience very large changes in their orbit due to the figure rotation. For instance, Tremaine & Yu (2000) examined the behavior of disks in halos with retrograde figure rotation. In these disks, stars can get trapped in the Binney resonance, where $\Omega_3 - \Omega_2 = \Omega_p$, for vertical and azimuthal frequencies $\Omega_2$ and $\Omega_3$, respectively, and a halo pattern speed of $\Omega_h$ (Binney 1981). If the pattern speed falls slowly toward zero, stars trapped in this resonance are pulled out of the disk and into polar orbits, while if the figure rotation smoothly proceeds from retrograde to prograde, the stars trapped in this resonance are flipped $180^\circ$ and end up on retrograde orbits. Figure rotation may also erase or modify any intrinsic alignments between the orientation of neighboring halos (J. Bailin & M. Steinmetz 2004, in preparation).

If there are observational consequences to dark halo figure rotation, such as those found by Bekki & Freeman (2002) and Tremaine & Yu (2000), they can be used as a direct method to distinguish between dark matter and models such as modified Newtonian dynamics (MOND) that propose to change the strength of the force of gravity (Milgrom 1983; Sanders & McGaugh 2002). Many of the traditional methods of deducing dark matter cannot easily distinguish between the presence of a roughly spherical dark matter halo and a modified force or inertia law. However, a major difference between dark matter and MOND is that dark matter is dynamical, and so tests that detect the presence of dark matter in motion are an effective tool to discriminate between these possibilities. Among the tests that can make this distinction are the ellipticities of dark matter halos as measured using X-ray isophotes, the Sunyaev-Zeldovich effect, and weak lensing (Buote et al. 2002; Lee & Suto 2003, 2004; Hoekstra et al. 2004), the presence of bars with parameters consistent with being stimulated by their angular momentum exchange with the halo (Athanassoula 2002; Valenzuela & Klypin 2003), and spatial offset between the baryons and the mass in infalling substructure measured using weak lensing (Clowe et al. 2004). Rotation of the halo figure requires that dark matter is dynamic, and therefore observable structure triggered by figure rotation potentially provides another test of the dark matter paradigm.

In this paper, we use cosmological simulations to determine how the figures of $\Lambda$CDM halos rotate. The organization of the paper is as follows. Section 2 presents the cosmological simulations. Section 3 describes the method used to calculate the figure rotations, which are presented in § 4. Finally, we discuss our conclusions in § 5.

2. THE SIMULATIONS

The halos are drawn from a large high-resolution cosmological N-body simulation performed using the GADGET2 code (Springel et al. 2001). We adopt a “concordance” cosmology (e.g., Spergel et al. 2003) with $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_{\text{bar}} = 0.045$, $h = 0.7$, and $\sigma_8 = 0.9$. The only effect of $\Omega_{\text{bar}}$ is on the initial power spectrum, since no baryonic physics is included in the simulation. The simulation contains $512^3 = 134,217,728$ particles in a periodic volume $50 h^{-1}$ Mpc comoving on a side. All results are scaled into $h$-independent units when possible. The full list of parameters is given in Table 1.

A friends-of-friends algorithm is used to identify halos (Davis et al. 1985). We use the standard linking length of

$$b = 0.2\bar{n}^{-1/3},$$

where $\bar{n} = N/V$ is the global number density.

Measuring the figure rotation requires comparing the same halo at different times during the simulation. We analyze

| Parameter | Value |
|-----------|-------|
| $N$       | 512$^3$ |
| Box size ($h^{-1}$ Mpc comoving) | 50 |
| Particle mass ($10^7 h^{-1} M_{\odot}$) | 7.757 |
| Force softening length ($h^{-1}$ kpc) | 5 |
| Hubble parameter $h$ ($H_0 = 100 h$ km s$^{-1}$ Mpc$^{-1}$) | 0.7 |
| $\Omega_0$ | 0.3 |
| $\Omega_\Lambda$ | 0.7 |
| $\sigma_8$ | 0.9 |
| $\Omega_{\text{bar}}$ | 0.045 |
snapshots of the simulation at look-back times of approximately 1000, 500, 300, and 100 h⁻¹ Myr with respect to the z = 0 snapshot. The scale factor \( a \) of each snapshot, along with its corresponding redshift and look-back time, is listed in Table 2.

### Table 2: Snapshots Used to Calculate Figure Rotations

| Snapshot Name | Scale Factor (\( a \)) | Redshift (\( z \)) | Look-back Time (h⁻¹ Myr) |
|---------------|------------------------|-------------------|--------------------------|
| b090..         | 0.89                   | 0.1236            | 1108                     |
| b096..         | 0.95                   | 0.0526            | 496                      |
| b098..         | 0.97                   | 0.0309            | 296                      |
| b100..         | 0.99                   | 0.0101            | 98                       |
| b102..         | 1.00                   | 0.0              | 0                        |

3. METHODOLOGY

3.1. Introduction

The basic method is to identify individual halos in the final z = 0 snapshot of the simulation, to find their respective progenitors in slightly earlier snapshots, and to measure the rotation of the axes through their common plane as a function of time. Precisely determining the direction of the axes is crucial and difficult. When merely calculating axial ratios or internal alignment, errors on the order of a few degrees are tolerable. However, if a pattern speed of 1 km s⁻¹ kpc⁻¹, as observed in the halo of BFPM99, is typical, then a typical halo will only rotate by 4° between the penultimate and final snapshots of the simulation. Therefore, the axes of a halo must be determined more precisely than this in order for the figure rotation to be detectable. In fact, we should strive for even smaller errors to see whether slower rotating halos exist. It would have been detectable. In fact, we should strive for even smaller errors to more precisely than this in order for the figure rotation to be detectable.

3.2. Halo Matching

To match up the halos at z = 0 with their earlier counterparts, we use the individual particle numbers provided by GADGET, which are invariant from snapshot to snapshot, and find which halo each particle belongs to in each snapshot. The progenitor of each z = 0 halo in a given z > 0 snapshot is the halo that contributes ≥90% of the final halo mass. Sometimes no such halo exists; in these cases, the halo has only just formed or underwent a major merger and so is not useful for our purposes. Figure 1 shows a histogram of the fraction of the final halo mass that comes from the b096 (\( z \approx 0.05 \)) halo that contributes the most mass. There are also some cases in which two nearby objects are identified as a single halo in an earlier snapshot but as distinct objects in the final snapshot. We therefore impose the additional constraint that the mass contributed to the final halo must also be ≥90% of the progenitor’s mass. In the longer time between the earliest snapshot b090 and the final snapshot b102, a halo typically accretes a greater fraction of its mass, and so a more liberal cut of 85% is used for this snapshot (see the dashed histogram in Fig. 1); 492 of the halos that satisfied the mass cut did not have a progenitor that satisfied these criteria in at least one of the z > 0 snapshots and so were eliminated from the analysis, leaving a sample of 940 matched halos.

3.3. Error in Axis Orientation

There are two sources of errors that enter into the determination of the axes: how well the principal axes of the particle distribution can be determined, and whether that particle distribution has a smooth triaxial figure. We here estimate the error assuming that it is not biased by substructure. The halos of P99 show that most of his halos had orientation errors of between 8° and 15°, corresponding to a minimum resolvable figure rotation of ~0.6 km s⁻¹ kpc⁻¹ for a 2 σ detection in snapshots spaced 500 Myr apart. A major difficulty in determining the principal axes so precisely is substructure. The orientation of a mass distribution is usually found by calculating the moment of inertia tensor \( I_{ij} = \sum_k m_k \delta \mathbf{r}_k \cdot \mathbf{r}'_{k,j} \) and then diagonalizing \( I_{ij} \) to find the principal axes. However, this procedure weights particles by \( r^2 \). Therefore, substructure near the edge of the halo (or of the subregion of the halo used to calculate the shape) can exert a large influence on the shape of nearly spherical halos, especially if a particular subhalo is part of the calculation in one snapshot but not in another, such as when it has just fallen in. This is particularly problematic because subhalos are preferentially found at large radii (Ghigna et al. 2000; De Lucia et al. 2004; Gill et al. 2004; Gao et al. 2004). Moving substructures can also induce a false measurement of figure rotation due to their motion within the main halo at approximately the circular velocity. To mitigate this, we first use particles in a spherical region of radius 0.6\( r_{vir} \) surrounding the center of the halo, rather than picking the particles from a density-dependent ellipsoid, as in Warren et al. (1992) or Jing & Suto (2002). We find that those methods allow substructure at one particular radius to influence the overall shape of the ellipsoid from which particles are chosen for the remainder of the calculation and therefore bias the results even when other measures are adopted to minimize their effect. The choice of a spherical region biases the derived axis ratios toward spherical values but does not affect the orientation. Second, the particles are weighted by \( 1/r^2 \) so that each mass unit contributes equally regardless of radius (Gerhard 1983). Both D92 and P99 take similar approaches, but using radii based on ellipsoidal shells. Therefore, we base our analysis on the principal axes of the reduced inertia tensor:

\[
I_{ij} = \sum_k m_k \delta \mathbf{r}_k \cdot \mathbf{r}'_{k,j} \]

In the majority of halos, the substructure is a small fraction of the total mass, usually less than 5% of the total mass within 60% of the virial radius (De Lucia et al. 2004, their Fig. 8), so its effect is much reduced. There are still some halos that have not yet relaxed from a recent major merger, in which case the “substructure” constitutes a significant fraction of the halo. To find these cases, the reduced inertia tensor is separately calculated enclosing spheres of radius 0.6, 0.4, 0.25, 0.12, and 0.06 times the virial radius to look for deviations as a function of radius (see § 3.4.1 for details). These radii are always with respect to the z = 0 value of \( r_{vir} \).

We find that only halos with at least \( 4 \times 10^3 \) particles, or masses of at least \( 3 \times 10^{11} h^{-1} M_\odot \), have sufficient resolution for the orientation of the principal axis to be determined at sufficient precision (see § 3.3). There are 1432 halos in the z = 0 snapshot satisfying this criterion, with masses extending up to \( 2.8 \times 10^{14} h^{-1} M_\odot \).
for which this assumption does not hold will become apparent later in the calculation.

For a smooth triaxial ellipsoid represented by \( N \) particles, the error is a function of \( N \) and of the intrinsic shape: as the axis ratio \( b/a \) or \( c/b \) approaches unity, the axes become degenerate. To quantify this, we have performed a bootstrap analysis of the particles in a sphere of radius \( 0.6 r_{\text{vir}} \) of each \( z = 0 \) halo (Heyl et al. 1994). If the sphere contains \( N \) particles, then we resample the structure by randomly selecting \( N \) particles from that set allowing for duplication and determine the axes from this bootstrap set. We do this 100 times for each halo. The dispersion of these estimates around the calculated axis is taken formally as the “1 \( \sigma \)” angular error and is labeled \( \theta_{\text{boot}} \).

As expected, the two important parameters are \( N \) and the axis ratio. We focus here on the major axis, for which the important axis ratio is \( b/a \) (the results for the minor axis are identical with the minor-to-intermediate axis ratio \( c/b \) replacing \( b/a \)). The top panels of Figures 2 and 3 show the dependence of the bootstrap error on \( N \) and \( b/a \), respectively, for all halos with \( M > 10^{11} h^{-1} M_\odot \). The solid lines are empirical fits:

\[
\theta_{\text{err}, N} = \frac{2}{\sqrt{N}},
\]

and

\[
\theta_{\text{err}, b/a} = 0.005 \frac{\sqrt{b/a}}{1 - b/a}.
\]

The form of equation (3) is not surprising; if a smooth halo was randomly sampled, we would expect the errors to be Poissonian with an \( N^{-1/2} \) dependence. However, the cosmological halos are not randomly sampled. Individual particles “know” where the other particles are, because they have acquired their positions by reacting in the potential defined by those other particles. Therefore, the errors may be less than expected from a randomly sampled halo. To test this, we construct a series of smooth prolate NFW halos (Navarro et al. 1996) with \( b/a \) axis ratios ranging from 0.5 to 0.9, randomly sampled with between \( 3 \times 10^5 \) and \( 3 \times 10^5 \) particles, and perform the bootstrap analysis identically for each of these halos as for the cosmological halos. Because the method for calculating axis ratios outlined in

\textit{Fig. 1.}—Histogram of the fraction of the final mass that comes from the \( b096 \) (\( \approx 0.05 \); solid line) and \( b090 \) (\( \approx 0.12 \); dashed line) halo that contributes the most mass.

\textit{Fig. 2.}—Angular bootstrap error \( \theta_{\text{boot}} \) as a function of the number of particles \( N \) within the central \( 0.6 r_{\text{vir}} \) of each halo. Points are the cosmological halos, and asterisks are randomly sampled smooth NFW halos. Top: Angular error \( \theta_{\text{boot}} \). The solid line is the fit \( \theta_{\text{err}, N} \) from eq. (3). Middle: Ratio between the angular error and the error expected for the halo given its axis ratio \( b/a \), i.e., \( \theta_{\text{boot}}/\theta_{\text{err}, b/a} \). The solid line is \( \theta_{\text{err}, b/a} \) from eq. (4) renormalized to the typical error of 0.02 rad. Bottom: Ratio between the angular error and the analytic estimate \( \theta_{\text{an}} \) from eq. (5). [See the electronic edition of the Journal for a color version of this figure.]

\textit{Fig. 3.}—Angular bootstrap error \( \theta_{\text{boot}} \) as a function of the axis ratio \( b/a \) of each halo. Points are the cosmological halos, and asterisks are randomly sampled smooth NFW halos. Top: Angular error \( \theta_{\text{boot}} \). The solid line is the fit \( \theta_{\text{err}, b/a} \) from eq. (4). Middle: Ratio between the angular error and the error expected for the halo given the number of particles \( N \), i.e., \( \theta_{\text{boot}}/\theta_{\text{err}, N} \). The solid line is \( \theta_{\text{err}, b/a} \) from eq. (4) renormalized to the typical error of 0.02 rad. Bottom: Ratio between the angular error and the analytic estimate \( \theta_{\text{an}} \) from eq. (5). [See the electronic edition of the Journal for a color version of this figure.]
apparent an extremely tight relation between the residual and \( N \), while in the middle panel of Figure 3 we have divided out the dependence of \( \theta_{\text{boot}} \) on \( N \), showing the equally tight relation between the residual and \( b/a \). It is apparent from comparing the points and asterisks that the errors in the cosmological halos are slightly smaller than for randomly sampled smooth halos.

Combining equations (3) and (4), and noting that the typical error is \( \theta_{\text{boot}} \approx 0.02 \) rad, we find the bootstrap error is well fitted by

\[
\theta_{\text{err}} = \frac{1}{2N} \frac{\sqrt{b/a}}{1 - b/a}.
\]

The bottom panels of Figures 2 and 3 show the residual ratio between the bootstrap error \( \theta_{\text{boot}} \) and the analytic estimate \( \theta_{\text{err}} \). The vast majority of points lie between 0.8 and 1.0, indicating that \( \theta_{\text{err}} \) overestimates the error by \( \sim 10\% \). Equation (5) breaks down as \( b/a \) approaches unity; these halos are nearly oblate and do not have well-defined major axes. It also becomes inaccurate at very low \( b/a \) because of low-mass, poorly resolved halos. Even in these cases, the error estimate is conservative, but to be safe we have eliminated axes with \( b/a < 0.35 \) or \( b/a > 0.95 \) from the subsequent analysis, regardless of the nominal error. The randomly sampled smooth halos follow equation (5) extremely well, so the non-Poissonianity of the sampling in simulated halos reduces the errors by 10%.

Calculating the bootstraps is computationally expensive, so equation (5) is used for the error in all further computation. Because this estimate is expected to be correct for smooth ellipsoids, cases in which the error is anomalous are indications of residual substructure.

3.4. Figure Rotation

Ideally one would fit the figure rotation by comparing the orientation of each of the axes at each snapshot to that of a unit vector rotating uniformly along a great circle and minimize the \( \chi^2 \) to find the best-fit uniform great circle trajectory. However, this requires minimizing a nonlinear function in a four-dimensional parameter space, a nontrivial task.

We adopt two simpler and numerically more robust methods for measuring the figure rotation. The first method, referred to as the “plane method,” involves fitting the major or minor axis measurements at all five snapshots to a plane and then measuring the rotation of the axis along the plane. This fully takes the errors and measurements at all snapshots into account. However, it presupposes that the figure rotation axis is perpendicular to the plane containing the major or minor axis. The second method, referred to as the “quaternion method,” involves comparing all of the axes at two snapshots to find the axis through which the figure has rotated. This method gives a figure rotation axis that is not constrained to have any particular relation to the major or minor axis. However, by construction it measures the rotation from a single reference frame to another single reference frame and therefore can only include information from two snapshots at a time. It is also not possible to take into account the errors in the axis determinations; in particular, for prolate halos, where the error in the determination of the intermediate and minor axes are much larger than the error in the major axis, physical rotation of the major axis can be masked by spurious fluctuations in the two degenerate axes.

The strengths and weaknesses of these two methods complement each other well. We adopt the plane method as our primary method of measuring the figure rotation. The quaternion method is used to check for bias in the derived figure rotation axes.

3.4.1. Plane Method

For the plane method, we first solve for the plane \( z = ax + by \) that fits the major axis measurements of the halo best at all time steps, assuming the error is negligible. The change of the phase of the axes in this plane as a function of time are then fitted by linear regression. A schematic diagram of this process is shown in Figure 4. We follow the same procedure for the minor axes when appropriate, as discussed in § 3.4.

The degree to which the axes are consistent with lying in a plane is checked by calculating the out-of-plane \( \chi^2 \):

\[
\chi^2_{\text{oop}} = \frac{1}{\nu} \sum_i \frac{\Delta \theta_i^2}{\theta_{\text{err}, i}^2},
\]

where \( \nu \) is the number of degrees of freedom and \( \Delta \theta_i \) is the minimum angular distance between the major axis at time step \( i \) and the great circle defined by the best-fit plane.

Because the axes have reflection symmetry, it is impossible to measure a change in phase of more than \( \pi/2 \). The phases are adjusted by units of \( \pi \) such that the difference in phase between adjacent snapshots is always less than \( \pi/2 \). If the figure were truly rotating by \( 90^\circ \) or more between the snapshots, it would be impossible to accurately measure this rotation since the angular frequency would be larger than the Nyquist frequency of our sampling rate. Any faster pattern speeds would be aliased to lower angular velocities, with an aliased angular velocity of \( \Omega_{\text{Nyq}} - (\Omega_p - \Omega_{\text{Nyq}}) \), where \( \Omega_p \) is the intrinsic angular velocity of the pattern and \( \Omega_{\text{Nyq}} \) is the Nyquist frequency. For snapshots spaced 500 \( h^{-1} \) Myr apart, the maximum time between the snapshots we analyze, the maximum detectable angular velocity is 3.8 \( \text{km s}^{-1} \text{kpc}^{-1} \). We do not expect the figure to change so dramatically, as we have excluded major mergers. However, this can be checked post facto by checking whether the distribution of measured angular velocities extends up to the Nyquist frequency; if so, then there are likely even more rapidly rotating figures whose angular frequency is aliased into the detectable range, fooling us into thinking
they are rotating slower. If the measured distribution does not extend to the Nyquist frequency, then it is unlikely that there are any figures rotating too rapidly to be detected (see § 4).

The best-fit linear relation for the phase as a function of time is found by linear regression. Because the component of an isotropic angular error projected onto a plane is half of the isotropic error, we divide the error of equation (5) by 2 before we perform the regression. The slope of the linear fit gives the pattern speed $\Omega_p$ of the figure rotation. The error is the 1 $\sigma$ limit on the slope.

Once we have calculated the pattern speed for each halo, we can eliminate the cases in which substructure has severely impacted the results. In these cases, the signal is dominated by a large subhalo at a particular radius, so the derived pattern speed will be significantly different when the sphere is large enough to include the subhalo from when the subhalo is outside the sphere. We have calculated the pattern speed using enclosing spheres of radius 0.6, 0.4, 0.25, 0.12, and 0.06 of the virial radius. The fraction of mass in subhalos can be estimated via the spheres of radius 0.6, 0.4, 0.25, 0.12, and 0.06, and adopt the largest value of $\Omega_p$ at adjacent radii. Because the reduced inertia tensor is mass-weighted, the figure rotation of a sphere with a smooth component rotating at $\Omega_{p,\text{smooth}}$ plus a subhalo containing a fraction $f_s$ of the total mass moving at the circular velocity $v_c$ at radius $R_{\text{vir}}$ is approximately

$$\Omega_p \approx (1 - f_s)\Omega_{p,\text{smooth}} + f_s \frac{v_c}{R_{\text{vir}}},$$

where the difference due to the presence of the subhalo is

$$\Delta \Omega_p = f_s \frac{v_c}{R_{\text{vir}}} = f_s \sqrt{\frac{GM(<R_{\text{vir}})}{R^3}}.$$  

If the density profile is roughly isothermal, the enclosed mass is

$$M(<R_{\text{vir}}) = \frac{4}{3} \pi \Delta \rho_{\text{crit}} R_{\text{vir}}^3.$$ 

Solving equations (8) and (9) gives an expression for the fraction of the mass in substructure given a jump of $\Delta \Omega_p$ in the measured pattern speed when crossing radius $R_{\text{vir}}$:

$$f_s = \frac{\Delta \Omega_p R}{\sqrt{4/3} \pi G \Delta \rho_{\text{crit}}}.$$ 

The term in the square root is equal to 0.72 h km s$^{-1}$ kpc$^{-1}$. For each halo, we compute the value of $f_s$ due to the jump $\Delta \Omega_p$ between each set of adjacent radii, i.e., for $R = 0.4, 0.25, 0.12,$ and 0.06, and adopt the largest value of $f_s$ as the substructure fraction of the halo. Figure 5 shows log-weighted projected densities of four halos in the mass range $(2-3) \times 10^{12}$ $h^{-1} M_\odot$, with a variety of values of $f_s$ ranging from 0.166 at the top left to 0.016 at the bottom right. After examining a number of halos spanning a range of $f_s$, we adopt a cutoff of $f_s < 0.05$ for undisturbed halos. This eliminates 289 of the 940 halos, leaving 651 undisturbed halos.

A further 158 halos were eliminated because the angular error approached $\pi/2$ in at least one of the snapshots. This includes the halos with $b/a < 0.35$ or $b/a > 0.95$ discussed in § 3.3. We also eliminate cases in which the reduced $\chi^2$ from the linear fit of phase versus time indicates that the intrinsic error of the direction determination is much lower than suggested by equation (5), indicating that the model of the halo as a smooth ellipsoid is violated (10 halos with $\chi^2 < 0.1$), and those cases

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**Fig. 5.—Log-weighted projected density of four halos with a range of subhalo fractions $f_s$.** The subhalo fractions are 0.166 (top left), 0.065 (top right), 0.045 (bottom left), and 0.016 (bottom right). Axes are in units of $h^{-1}$ kpc from the halo center. All halos have masses in the range $(2-3) \times 10^{12}$ $h^{-1} M_\odot$.**
in which the phase does not evolve linearly with time (134 halos with $\chi^2_{\text{top}} > 10$). Finally, we eliminate halos where the axes do not lie on a common plane, i.e., the 32 halos where $\chi^2_{\text{top}} > 10$. Therefore, the final sample consists of 317 halos.

A sample halo is shown in the first five panels of Figure 6. It was chosen randomly from the halos with relatively low errors and typical pattern speeds. It has a mass of $1.9 \times 10^{12} \, h^{-1} M_\odot$ and a pattern that rotates at $0.32 \pm 0.01 \, h \, \text{km s}^{-1} \, \text{kpc}^{-1}$. It has a spin parameter $\lambda = 0.047$ and axis ratios of $b/a = 0.86$ and $c/a = 0.77$ at $z = 0$. The derived substructure fraction is $f_s = 0.045$, and the out-of-plane $\chi^2_{\text{top}} = 8.5$. The solid line shows the measured major axis in each snapshot, which rotates counterclockwise in this projection. The phase of its figure rotation as a function of time is shown in the bottom right panel.
of Figure 6. The zero point is arbitrary but is consistent from snapshot to snapshot. The linear fit is also shown, which has a reduced $\chi^2$ of 2.9.

3.4.2. Quaternion Method

For the quaternion method, we directly measure the rotation between the axes at two snapshot. If the major, intermediate, and minor axes for snapshot $j$ are $\hat{a}_j$, $\hat{b}_j$, and $\hat{c}_j$, respectively, we construct a rotation matrix $R_j$ that transforms vectors into the principal axis frame:

$$R_j = \begin{pmatrix} a_{ij} & a_{ij} & a_{ij} \\ b_{ij} & b_{ij} & b_{ij} \\ c_{ij} & c_{ij} & c_{ij} \end{pmatrix}.$$ \hspace{1cm} (11)

The matrix expressing the rotation from snapshot $j$ to snapshot $k$ is $dR = R_j R_k^T$. This rotation matrix can be represented as a quaternion,\textsuperscript{4} which then directly gives the figure rotation axis and the angle of rotation between the snapshots.

4. RESULTS

Figure 7 shows the measured figure rotation speeds of the major axes for all of the halos in the sample, as a function of their error using the plane method. Halos with measured pattern speeds less than twice as large as the estimated error (dashed line) are taken as nondetections; 278 of the 317 halos have detected figure rotation. A histogram of the pattern speeds is presented in Figure 8, expressed in log $\Omega_p$. The thin histogram contains all halos that have 2 $\sigma$ detections of figure rotation, i.e., those above the dashed line of Fig. 7, and is incomplete at $\Omega_p < 0.126$ km s$^{-1}$ kpc$^{-1}$ or equivalently log $\Omega_p < -0.9$. The thick histogram contains only those halos with errors less than 0.014 km s$^{-1}$ kpc$^{-1}$ and is incomplete at $\Omega_p < 0.015$ km s$^{-1}$ kpc$^{-1}$ or log $\Omega_p < -1.8$. The dashed curves are Gaussian fits to the histograms. The vertical dotted line shows the Nyquist limit.

sample size but is incomplete at low $\Omega_p$, peaks at log $\Omega_p = -0.80$ and has a standard deviation of 0.29, while the thick curve, which contains fewer halos but is less biased toward large values of $\Omega_p$, peaks at log $\Omega_p = -0.84$ and has a standard deviation of 0.34. We give more weight to the thick histogram, whose points all have very small errors, and propose that the true distribution peaks at log $\Omega_p = -0.83$ with a standard deviation of 0.36. Expressed as a lognormal distribution, the probability is

$$P(\Omega_p) = \frac{1}{\Omega_p \sigma \sqrt{2\pi}} \exp\left[ -\frac{\ln^2(\Omega_p/\Omega_{p0})}{2\sigma^2} \right].$$ \hspace{1cm} (12)

where $\Omega_{p0} = 10^{-0.83} = 0.148$ km s$^{-1}$ kpc$^{-1}$ and the natural width $\sigma = 0.36 \ln 10 = 0.83$. This fit is shown in Figure 9, compared to the fractional distribution of halos in the full (thin line) and low error (thick line) samples, and encompasses both the large number of halos with low $\Omega_p$ seen when the errors are sufficiently small, and the tail at high $\Omega_p$ seen when the sample size is sufficiently large.

For comparison, the halo in BFP99 has a pattern speed of 2 km s$^{-1}$ kpc$^{-1}$. This lies slightly above the top end of our distribution; the maximum pattern speed in our sample is 1.01 km s$^{-1}$ kpc$^{-1}$. Based on the lognormal fit of equation (12), we estimate the fraction of halos with $\Omega_p \geq 2$ km s$^{-1}$ kpc$^{-1}$ to be $\sim 10^{-3}$. Therefore, this halo is unusual, but it is not unreasonable to find a halo with such a pattern speed in a large simulation. Given the size of the errors in P99, and that he found very few halos with figure rotation, it should not be surprising that P99 could only detect pattern speeds at the upper end of the overall distribution. The different adopted cosmologies may also influence the results (note, however, that this comparison is performed in $h$-independent units). Our results are also mostly consistent with D92, who finds pattern speeds of between 0.1 and 1.6 km s$^{-1}$ kpc$^{-1}$ in a sample of 14 halos. We have trouble reproducing the most rapidly rotating halos in D92, but this may be a product of the heuristic initial conditions in D92 compared to the cosmological initial conditions

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\textsuperscript{4} See, e.g., the Matrix and Quaternion FAQ at http://vamos.sourceforge.net/matrixfaq.htm.
conditions we use. Another useful reference value is $v_{200}/r_{200}$, the orbital frequency at $r_{200}$, which has a value of 1.0 $h$ km s$^{-1}$ kpc$^{-1}$. This is the same for all halos, because the only timescale in a purely gravitational system is $1/(\bar{\rho})^{1/2}$, and the mean density $\bar{\rho}$ inside $r_{200}$ is the same for all halos by definition.

In order to account for the spiral structure in NGC 2915, a triaxial figure would need to rotate at $7 \pm 1$ km s$^{-1}$ kpc$^{-1}$ (MB03). This is almost an order of magnitude faster than the fastest of the halos in our sample, and the lognormal fit from equation (12) suggests that the fraction of halos with $\Omega_p \geq 6$ km s$^{-1}$ kpc$^{-1}$ is $5 \times 10^{-7}$. Therefore, the figure rotation of undisturbed LCDM halos cannot explain the spiral structure of NGC 2915. SPH simulations of gas disks inside triaxial halos with pattern speeds of 0.77 km s$^{-1}$ kpc$^{-1}$, comparable to the fastest pattern speeds in our sample, show very weak, if any, enhancement of spiral structure compared to a static halo (Bekki & Freeman 2002, their Fig. 2). Therefore, it is unlikely that triaxial figure rotation can be detected in extended gas disks.

The dotted line in both Figures 7 and 8 shows the Nyquist frequency of 3.8 $h$ km s$^{-1}$ kpc$^{-1}$. If the measured distribution of pattern speeds extended up to the Nyquist frequency, the intrinsic distribution would likely extend above the Nyquist frequency, and the results would be affected by frequency aliasing. However, the measured distribution does not approach the Nyquist frequency. Therefore, any halo whose figure rotation is aliased would need to be wildly anomalous, with a pattern speed many times faster than any other halo in our sample. We consider this unlikely. Figure 10 shows the pattern speeds as a function of the error, as in Figure 7, except that it only uses snapshots b096–b102, so the maximum time between snapshots is 200 $h^{-1}$ Myr, and the corresponding Nyquist frequency is 7.6 $h$ km s$^{-1}$ kpc$^{-1}$, shown again as the dotted line. The top of the distribution does not change between Figures 7 and 10, demonstrating that the results are not affected by aliasing.

We investigate how the figure rotation axis relates to two other important axes. Both D92 and P99 claim that the major axis rotates around the minor axis. The direction cosine between the rotation axis and the minor axis is plotted both as a function of the pattern speed and as a histogram in Figure 11.

We confirm that the axis about which the major axis rotates aligns very well with the minor axis.

Measuring the rotation of the major axis using the plane rotation method precludes finding halos with figure rotation about the major axis, which is a theoretically stable configuration. Therefore, the first and last snapshots are compared using the quaternion method. A comparison between the derived rotation axes, for those halos with 2 $\sigma$ detections of figure rotation, is shown in Figure 12. The axes agree in the majority of the halos. There are three halos where the axes are antialigned. Examination of these halos reveals that in these cases, the figure rotation from snapshots b096–b102 is smooth, but snapshot b090 is anomalous (these halos have high values of $\chi^2$, although not high enough to have been excluded from the sample). More interesting are the halos where the alignment between the two determinations of the figure rotation axis lie between $-0.1$ and 0.5. In all of these cases, the quaternion figure rotation axis is aligned with the major axis. Some of the upper limits in Figure 7 may also be halos with rotation about the major axis. The quaternion figure rotation axes for most of the halos with upper limits are distributed randomly, but there is an excess of ~9 halos where the figure rotation axis aligns with the major axis. We have constructed a sample of potential major-axis rotators consisting of the 75 halos from Figure 12.
where the alignment lies between −0.1 and 0.5, plus the 17 “upper limit” halos where the quaternion figure rotation axis aligns with the major axis to better than 0.8 (of which we expect statistically that ~9 are real; examination of the results reveals that 10 are real). We use the plane method to calculate the figure rotation of these halos but investigate the evolution of the minor axis instead of the major axis. Out of the 92 halos, 37 have very large errors in the minor-axis determination in one of the snapshots and so are excluded. These are prolate halos where the quaternion method measures the fluctuations between the degenerate axes rather than true figure rotation. The pattern speeds for the remaining 55 halos are shown in Figure 13. We detect figure rotation about the major axis in 41 of the halos. Although the statistics are poorer, the range of pattern speeds for these halos is similar to the range of pattern speeds seen for the halos showing figure rotation about the minor axis. We conclude that 247 halos show minor-axis rotation, 41 show major-axis rotation, and 29 show no detectable figure rotation.

The rotation axis is compared to the angular momentum vector of the halo in Figure 14. Because the angular momentum is usually relatively well aligned with the minor axis of halos (Warren et al. 1992), it is no surprise that the rotation axis is also well aligned with the angular momentum vector. Because the alignment between the minor axis and angular momentum of halos is not perfect (Warren et al. 1992; J. Bailin & M. Steinmetz 2004, in preparation), some of the halos with perfectly aligned figure rotation and minor axes have less perfect alignment between the figure rotation axis and the angular momentum vector, seen as the bump in Figure 14 that extends down to a direction cosine of 0.6; the tail of the distribution extends all the way to antialignment. This indicates that the halos with retrograde figure rotation required for the polar ring and counterrotating disk mechanism of Tremaine & Yu (2000) are rare but do exist. The alignment between the rotation axis and the angular momentum vector is also plotted as a function of the pattern speed in the bottom panel of Figure 14. There is no trend for the halos with slow figure rotation, but all but two of the halos with \( \Omega_p > 0.4 \, h \, \text{km s}^{-1} \, \text{kpc}^{-1} \) have figure rotation axes and angular momentum vectors that are well aligned, with a direction cosine of 0.65 or higher.

We have attempted to see if the pattern speed is correlated with other halo properties, in particular its mass and angular momentum. Figure 15 shows the pattern speed of the figure rotation versus the halo mass. Error bars are 1 \( \sigma \) errors, with 2 \( \sigma \) upper limits plotted for halos that lie below the dashed line of Figure 7. There is no apparent correlation between the halo mass and its pattern speed.

Figure 16 shows the pattern speed versus the spin parameter \( \lambda \), where

\[
\lambda \equiv \frac{J|E|^{1/2}}{GM^{3/2}}
\]  

(Peebles 1969). We use the computationally simpler \( \lambda' \) as an estimate for \( \lambda \), where

\[
\lambda' \equiv \frac{J}{\sqrt{2MFV}}
\]  

(Bullock et al. 2001). There is a tendency for halos with fast figure rotation to have large spin parameters; in particular, all but one of the halos with \( \Omega_p > 0.4 \, h \, \text{km s}^{-1} \, \text{kpc}^{-1} \) have \( \lambda > 0.024 \). These are the same halos that are shown to have
particularly well-aligned rotation axes and angular momentum vectors in Figure 14. We have calculated the median value of $\Omega_p$ including the upper limits for bins of width $\Delta\lambda = 0.01$. The median rises steadily from $0.12 \, h \, \text{km s}^{-1} \, \text{kpc}^{-1}$ for $\lambda < 0.02$ to $0.44 \, h \, \text{km s}^{-1} \, \text{kpc}^{-1}$ for $\lambda > 0.06$. Note that P99 only detected figure rotation in halos with $\lambda > 0.05$ (see his Fig. 5.24).

The degree of alignment between the figure rotation axis and the angular momentum vector may depend on the angular momentum content of the halo. Figure 17 shows how this alignment depends on the spin parameter $\lambda$. There is no trend for $\lambda < 0.05$, but the halos with $\lambda > 0.05$ show particularly good alignment. This is a natural consequence of the tendency for halos with rapid figure rotation to have well-aligned figure rotation and angular momentum axes (Fig. 14), and the correlation between figure pattern speed and spin parameter $\lambda$ (Fig. 16).

Figures 18 and 19 show how the figure rotation changes with radius. Figure 18 shows how the pattern speeds at different radii are related, while Figure 19 shows the alignment of the figure rotation axes between radii. Each panel includes only the halos that have at least 4000 particles within the inner radius, pass all of the tests of $\S$ 3.4.1 for both radii, and have 2 $\sigma$

measurements of figure rotation at both radii. Because of the smaller number of particles in spheres of smaller radii, there are progressively fewer halos with good measurements at smaller radii. The top panels show that the figure rotation in the outer regions of the halo is very coherent. To some degree, this is by construction; the test for substructure is equivalent to a cut in $\Delta\Omega_p$ between adjacent radii. However, gradual drifts of $\Omega_p$ and changes in the figure rotation axis with radius are still possible. The bottom panels of Figures 18 and 19 show that this indeed happens. In particular, while the figure rotation within $0.12r_{\text{vir}}$ and $0.6r_{\text{vir}}$ are strongly correlated, the pattern speeds within $0.12r_{\text{vir}}$ are slightly smaller and the alignment of the rotation

Fig. 15.—Pattern speed of the figure rotation vs. the mass of the halo. Error bars are 1 $\sigma$ errors, for halos with at least 2 $\sigma$ detections of figure rotation. The upper limits are the halos with a measured $\Omega_p < 2 \sigma$ and are plotted at the 2 $\sigma$ limit.

Fig. 16.—Pattern speed of the figure rotation vs. the spin parameter of the halo. Error bars and upper limits are as in Fig. 15.

Fig. 17.—Direction cosine between the angular momentum vector of the halo and the figure rotation axis vs. the spin parameter $\lambda$.

Fig. 18.—Pattern speed of figure rotation $\Omega_p$ at 0.4, 0.25, 0.12, and 0.06 of the virial radius $r_{\text{vir}}$ (top to bottom) as a function of the pattern speed at $0.6r_{\text{vir}}$. Only halos where both radii in the comparison contain at least 4000 particles, pass all of the tests of $\S$ 3.4.1, and have 2 $\sigma$ detections of figure rotation are included. All units are $h \, \text{km s}^{-1} \, \text{kpc}^{-1}$. The solid line corresponds to equal pattern speeds.
axes is not quite as strong. The bottom panels show that in the innermost regions, within 0.06 \( r_{\text{vir}} \), the pattern speeds are significantly smaller than for the halo as a whole, particularly for those halos with high pattern speeds, and more than half of the halos show no alignment between the figure rotation axes.

We examine three possible explanations for these trends with radius. First, it may be that the halos with high pattern speeds are still affected by residual substructure in the outer regions. However, the gradual decline for all halos seen as the radius shrinks suggests that the mechanism responsible for the difference affects all halos equally and gradually, rather than affecting a few halos at a specific radius. Another piece of evidence that argues against this explanation is that the halos with the highest measured pattern speeds do not have preferentially high values of \( f_s \); they have values evenly spread between 0 and the cutoff of 0.05. A second possibility is that the figure rotation, although steady on timescales of 1 Gyr, may be fundamentally a transitory feature caused by a tidal encounter or the most recent major merger. The inner region of the halo has a shorter dynamical time, and therefore the effects of such a disturbance will be erased faster in the inner regions than in the outer regions. This is consistent with the gradual decrease in pattern speed with radius and the decrease in alignment. However, the halo of BFPM99 has fast figure rotation (faster than any of our halos) and yet shows steady figure rotation at all radii for 5 Gyr. We propose instead that the effects of force softening are becoming important at the smaller radii. The radius of the innermost sphere for the halos plotted in the bottom panels of Figures 18 and 19 range from three to five force softening lengths, where the effects of the gravitational softening can still be important (Power et al. 2003). The weaker gravitational force results in a more spherical potential, consistent with the weaker figure rotation and lack of alignment.

We have calculated the rate of change of the \( b/a \) and \( c/a \) axis ratios over the five snapshots using linear regression. The evolution of the axis ratios with time is linear for almost all halos. Figure 20 shows the fractional rate of change, \((b/a)/(b/a)\) and \((c/a)/(c/a)\), of the axis ratios as a function of the value of the axis ratio in the final snapshot. The median and standard deviation of the distribution of \((b/a)/(b/a)\) are 0.0089 and 0.0349 h Gyr\(^{-1}\), respectively. For \(c/a\) they are 0.0093 and 0.0297 h Gyr\(^{-1}\), respectively. Therefore, there is a weak tendency for undisturbed halos to become more spherical with time. Most halos require several Gyr before their flattening changes significantly; there are, however, a few outliers with quite significant changes in their axis ratios. Figure 20 demonstrates that there is no trend of \((b/a)/(b/a)\) or \((c/a)/(c/a)\) with the value of the axis ratio except for the outliers with very high (low) values of \(b/a\), which could not have such high (low) rates of change if the values of \(b/a\) were not very high (low) in the final snapshot. We find no trend with any other halo property such as mass, spin parameter, pattern speed, substructure fraction, or alignment of the figure rotation axis with the angular momentum vector or minor axis.

5. CONCLUSIONS

We have detected rotation of the orientation the principal axes in most undisturbed halos of a \( \Lambda \)CDM cosmological
FIGURE ROTATION OF DARK MATTER HALOS

The axis around which the figure rotates is very well aligned with the minor axis in ~85% of the cases, and well aligned with the major axis in the remaining halos. It is also usually well aligned with the angular momentum vector. The distribution of pattern speeds is well fitted by a lognormal distribution,

\[ P(\Omega_p) = \frac{1}{\Omega_p \sigma \sqrt{2\pi}} \exp \left[ -\frac{\ln^2(\Omega_p/\Omega_{0p})}{2\sigma^2} \right], \tag{15} \]

with \( \Omega_{0p} = 0.148 \text{ km s}^{-1} \text{ kpc}^{-1} \) and \( \sigma = 0.83 \).

The pattern speed \( \Omega_p \) is correlated with spin parameter \( \lambda \). The median pattern speed rises from 0.12 km s\(^{-1}\) kpc\(^{-1}\) for halos with \( \lambda < 0.02 \) to 0.44 km s\(^{-1}\) kpc\(^{-1}\) for halos with \( \lambda > 0.06 \), with a spread of almost an order of magnitude around this median at a given value of \( \lambda \).

The 11% of halos in the sample with the highest pattern speeds, \( \Omega_p > 0.4 \text{ km s}^{-1} \text{ kpc}^{-1} \), not only have large spin parameters, but also show particularly strong alignment between their figure rotation axes and their angular momentum vectors. There is no obvious correlation of the figure rotation properties with mass. The pattern speed and figure rotation axis is coherent in the outer regions of the halo. Within 0.12\( r_{\text{vir}} \), the pattern speed drops, particularly for those halos with fast figure rotation, and the internal alignment of the figure rotation axis deteriorates. This is probably an artifact of the numerical force softening.

BFPM99 hypothesized that the spiral structure in NGC 2915 is due to figure rotation of a triaxial halo. The required pattern speed of 7 ± 1 km s\(^{-1}\) kpc\(^{-1}\) (MB03) is much higher than the pattern speeds seen in the simulated halos and is estimated to have a probability of \( 5 \times 10^{-7} \). We therefore conclude that the figure rotation of undisturbed \( \Lambda \)CDM halos is not able to produce this spiral structure. Halos with large values of \( \lambda \) tend to have more substructure (Barnes & Efstathiou 1987), so there is a deficiency of halos with very high \( \lambda \) in our sample. Because \( \Omega_p \) correlates with \( \lambda \), we cannot exclude the possibility that there exist halos with very high \( \lambda \) whose figures rotate sufficiently quickly. However, halos with such high \( \lambda \) are themselves very rare (MB03), and if such halos fall out of our sample because of the presence of strong substructure, the effects of the substructure on the gas disk of NGC 2915 would be of more concern than the slow rotation of the halo figure, a possibility BFPM99 rule out because of the lack of any plausible companion in the vicinity.

More generally, Bekki & Freeman (2002) found very weak if any enhancement of spiral structure in disk simulations with triaxial figures rotating at 0.77 km s\(^{-1}\) kpc\(^{-1}\), a value similar to the highest pattern speed seen in our sample. Therefore, it is unlikely that triaxial figure rotation can be detected by looking for spiral structure in extended gas disks.

The mechanism of Tremaine & Yu (2000) for creating polar rings and counterrotating disks requires halos with retrograde figure rotation whose pattern speed slowly drops to zero or smoothly reverses direction. While our temporal sampling is not sufficient to detect slow changes in the pattern speed, we note that halos with figure rotation retrograde to their angular momentum do exist.

We have found that the axis ratios of undisturbed halos tend to become more spherical with time, with median fractional decreases in the \( b/a \) and \( c/a \) axis ratios of \( \approx 0.009 \text{ h Gyr}^{-1} \). The distributions of \( b(a)/b(a) \) and \( c(a)/c(a) \) are relatively wide, with standard deviations of \( \approx 0.03 \text{ h Gyr}^{-1} \). A few outliers have axis ratios that change quite significantly over the span of 1 Gyr. The rate of change of the axis ratios is not correlated with any other halo property.

This work has been supported by grants from the US National Aeronautics and Space Administration (NAG 5-10827), by the David and Lucile Packard Foundation, and by the Bundesministerium für Bildung und Forschung (FKZ 05EA2BA1/8). We would like to thank the anonymous referee for valuable comments and suggestions, in particular regarding the quaternion method. We thank Volker Springel for providing us with an advance version of GADGET2. J. B. thanks Chris Power and Brad Gibson for useful discussions. In addition, we thank Ken Freeman and Martin Bureau for their repeated exhortations to pursue these calculations.

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