Multi-color continuous-variable quantum entanglement in dissipative Kerr solitons

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In a traveling wave microresonator, the cascaded four-wave mixing between optical modes allows the generation of frequency combs, including the intriguing dissipative Kerr solitons (DKS). Here, we theoretically investigate the quantum fluctuations of the comb and reveal the quantum feature of the soliton. It is demonstrated that the fluctuations of Kerr frequency comb lines are correlated, leading to multi-color continuous-variable entanglement. In particular, in the DKS state, the coherent comb lines stimulate photon-pair generation and also coherent photon conversion between all optical modes, and exhibit all-to-all connection of quantum entanglement. The broadband multi-color entanglement is not only universal, but also is robust against practical imperfections, such as extra optical loss or extraordinary frequency shift of a few modes. Our work reveals the prominent quantum nature of DKSs, which is of fundamental interest in quantum optics and also holds potential for quantum network and distributed quantum sensing applications.

Introduction.- Over the past decades, nonlinear optics technologies become the backbone of modern optics, enabling frequency conversion, optical parametric oscillation, ultrafast modulation, and non-classical quantum sources. Especially, various nonlinear optics mechanisms enabled the generation of the optical frequency comb, which has attracted lots of research interests due to its revolutionary applications in precision spectroscopy, astronomy detection, optical clock, and optical communication [1–7]. Recently, by harnessing the enhanced nonlinear optical effect in a microresonator, the Kerr frequency comb, in particular, the dissipative Kerr soliton (DKS) as phase-locked frequency comb, has been extensively studied [8]. Such DKS have been realized on photonic chips with various χ⁽³⁾ materials, showing advances in compact size, scalability, low power consumption, great spectral range and large repetition rate[9–12].

Therefore, the DKS is appealing for photonic quantum information science. From one aspect, the DKS produces an array of stable and locked coherent laser sources with equally spaced frequencies, which provides a coherent laser source for driving nonlinear light-matter interaction as well as stable frequency reference of local oscillators for heterodyne measurements via frequency multiplexing. By taking advantage of the coherence over a large frequency band, the quantum key distribution utilizing DKS has been experimentally studied [13], which promises commercialized high-speed quantum communication. From another aspect, the Kerr nonlinear interaction is inherently a quantum parametric process that describes the annihilation of two photons and simultaneous generation of signal-idler photon pairs, which imply quantum correlations among comb lines. When working below the threshold, the DKS devices have been exploited to generate both discrete- and continuous-variable (CV) quantum entangled states[14–17], holding the potential for one-way quantum computing and high-dimensional entanglement between different colors that could be distributed over the quantum network [18]. For the case above the threshold, although previous studies of χ⁽²⁾ optical combs indicate the potential applications of comb in the multipartite cluster-state generation for quantum computing [7, 19, 20], the quantum nature of DKS have not been studied excepting one pioneer work by Chengbo [21], who pointed out the mode-pairs locating symmetrically in two sides of the pump mode exhibit significant squeezing.

In this Letter, we theoretically investigated the CV entanglement between comb lines in a microcavity and compare the quantum features of comb in different states, i.e. the state below the threshold, primary comb, and DKS. We showed that the DKS could stimulate pair-generation and coherent conversion interactions between optical modes and thus produces a complex network links all optical modes. Evaluated by the entanglement logarithm negativity, it is demonstrated that a large number of comb lines with different colors are entangled together when the cavity is prepared at the soliton state. In particular, a group of modes show all-to-all quantum entanglement, indicating that arbitrary two modes over a large bandwidth could be entangled. The all-to-all entanglement persists even some modes in this complex network are eliminated. This multi-color quantum-entangled state [22] holds great potential in application in multi-user quantum communication, quantum teleportation network [23, 24] and quantum-enhanced measurements [25–28].

Model and principle.- Figure 1(a) illustrates an optical microring cavity side-coupled to a waveguide for generating dissipative Kerr solitons [8]. By injecting a pump laser into a resonance of the cavity, the intracavity optical field builds up and the photons are converted between optical modes via cascaded four-wave mixing (FWM) due to the Kerr nonlinearity, leading to a broad comb spectrum at the output. For investigating the DKS in the microcavity, we consider a group of 2ᴺ + 1 modes belongs to the same mode family, with the spatial distributions described by Ψ(−→r, θ) = φ(−→r) eimθ [29].
interaction terms describe the simultaneous annihilation and the angular momentum conservation \[29, 31\]. The nonlinear mutations of indices the total photon numbers are conserved. By taking all permutation, the resonant frequencies could be expanded with respect to the index as

\[a_i a_j + a_j a_i\]

for all pairs in \(\xi\). Meanwhile, a single mode also connects to multiple \(\xi\). As an example, for mode \(m = -3\) in Fig. 1(b), it could be entangled with mode \(m = 3\) with \(\xi = 0\), and \(m = 2\) with \(\xi = -1\), as well as \(m = 7\) with \(\xi = 4\). In addition, for the phase-matching \(i + j = k + l\), there are also coherent photon conversion processes between mode \((i, l)\) simulated by mode \((j, k)\) and vice versa, which also distributes the quantum correlations between different modes. Consequently, the FWM in the microresonator results in a complex network, with optical modes serving as nodes, and the links corresponding to photon-pair generation and coherent photon conversion.

The complex network of FWM in a microcavity (Fig. 1(b)) implies complicated dynamics of optical fields, which can be numerically evaluated by solving the quantum dynamics of bosonic modes according to the Hamiltonian [Eq. (1)]. Instead of solving the intractable many-body nonlinear equations via full quantum theory, we adopt the mean-field treatment of the strongly pumped system, i.e. the mean and fluctuation of optical field in each mode are solved separately, due to the weak nonlinearity \(g_0/\kappa \ll 1\) in practice [32]. The operator of the cavity mode \(a_i\) is approximated by the sum of a classical field described by an amplitude \(\alpha_i\) and a fluctuation described by a bosonic operator \(\delta a_i\). According to the Heisenberg equation and discarding the high-order terms of the fluctuation operators, the dynamics of the classical fields and their fluctuations follow

\[
\frac{d}{dt} \alpha_i = \beta_i \alpha_i - ig_0 \sum_{jkl} \alpha_j^\dagger \alpha_k a_i + \epsilon_p \delta (i),
\]

\[
\frac{d}{dt} \delta a_i = \beta_i \delta a_i + \sqrt{2\kappa_0} a_i^m
+ ig_0 \sum_{jkl} \left( \alpha_k \alpha_l \delta a_j^\dagger + \alpha_j^\dagger \alpha_k \delta a_l + \alpha_l^\dagger \alpha_j \delta a_k \right),
\]

respectively. Here, the summation takes over all possible permutation \((j, k, l)\) with \(i = k + l - j\), \(\beta_i = -i\delta_i - \kappa_i\), \(\kappa_i\) is the total amplitude decay rate of the \(j\)-th mode, \(\epsilon_p\) is the pump strength, \(a_i^m\) is the input noise on mode \(i\) and fulfills \(\langle a_i^m(t) a_i^m(t') \rangle = \delta(t - t')\). From Eq. (3), the last three terms represent the classical-field-stimulated photon-pair generation in modes \((i, j)\), and also the coherent photon conversion between modes \((i, k)\) and \((i, l)\). Rather than merely produce down-conversion photons by drive lasers [14], these terms imply very rich quantum dynamics in the complex network, with the quantum correlations directly generated through photon-pair generation and also indirectly generated through coherently re-distributing the generated photons among the modes.
The classical fields have been extensively studied in previous works \cite{31, 33}. With the amplitudes of classical fields \(\alpha_i\) obtained, the CV quantum correlation \cite{34} between the comb lines can be evaluated by solving the dynamics of fluctuations \cite{35}. It is convenient to represent the fluctuations by the “amplitude” and “phase” field quadratures \(\hat{\delta X}_i = (\delta a_i + \delta a_i^*)/\sqrt{2}\), \(\delta Y_i = i(\delta a_i - \delta a_i^*)/\sqrt{2}\). The dynamics of the quadratures \[\hat{Q} = \{\delta X_{-N}, \delta Y_{-N}, \delta X_{-N+1}, \delta Y_{-N+1}, \ldots, \delta X_N, \delta Y_N\}^T\] follow \[\frac{d}{dt}\hat{Q} = \mathbf{M} \cdot \hat{Q} + \hat{\pi}(t)\], where \(\hat{\pi}^T = \{\sqrt{2\kappa N X_{-N}^2}, \sqrt{2\kappa N Y_{-N}^2}, \ldots\}\) is the input noise, and \(\mathbf{M}\) is a \((4N+2) \times (4N+2)\) matrix derived from Eq. (3). Then, the correlation matrix \(\mathbf{V}\) for all modes can be solved following the deviation in Ref. \cite{35} and we obtain

\[\mathbf{MV} + \mathbf{VM}^T = -\mathbf{D},\]

where the element of the correlation matrix \(V_{ij} = \langle Q_i Q_j + Q_j Q_i \rangle/2\), the noise term \(D_{ij} = \langle n_i n_j + n_j n_i \rangle/2\) can be derived from \(\langle X_m^\dagger(t)X_m(t')\rangle = \delta(i-j)\delta(t-t')\) and \(\langle X_m^\dagger(t)Y_m(t')\rangle = 0\). The bipartite CV entanglement between comb lines could be evaluated by the logarithmic negativity

\[E_{nm}^{\mathbf{V}} = \max\{0, -\ln \sqrt{2\eta}\},\]

where \(\eta = \sqrt{\Theta - \sqrt{\Theta^2 - 4\det V}}, \Theta = \det A + \det B - 2\det C\), with \(A, B\) and \(C\) are the elements of \(V_{nm} = \{\{A, C\}, \{C^T, B\}\}\), which is a sub-matrix of \(V\) representing the bipartite correlation matrix between \(\delta X_m, \delta Y_m, \delta X_n, \delta Y_n\). \(E_{nm}^{\mathbf{V}}\) is a measure of the CV entanglement \cite{35-37}, and the mode-pair is entangled only if \(E_{nm}^{\mathbf{V}} > 0\). By calculating all combinations of the modes, one obtains the entanglement matrix \(E_{nm}^{\mathbf{V}}\) of the cavity field.

**Multi-color entanglement.-** Figure 2 depicts the typical results of Kerr combs in a microring resonator at different states \cite{31, 38}, with the upper and bottom rows show the classical intracavity fields and the entanglement \(E_{nm}^{\mathbf{V}}\) between mode-pairs. Here, we consider a monochromatic field driving on the 0-th mode to initially excite the mode-pairs with \(\xi = 0\), and the system parameters are chosen from a typical AIN microring in the experimental work \cite{31}.

(i) Below the threshold [Fig. 2(a) and (e)]. For weak pump power, the parametric gain on mode-pair of \(\xi = 0\) can not compensate the dissipation in these modes, thus the system stays below the optical parametric oscillation threshold and generates thermal photon-pairs by the spontaneous parametric down-conversion (SPDC) \cite{14}. From the comb spectrum of classical field [Fig.2(a)], only the 0-th mode is sufficiently excited, and the intracavity fields in all the other modes are negligible. Figure 2(e) shows the matrix \(E_{nm}^{\mathbf{V}}\), which only has positive value at its diagonal elements, indicating that only the photon-pair generation between mode-pair \((+l, -l)\) for \(\xi = 0\) are initiated. Due to the dispersion of the cavity resonance, modes with indices away from 0 experience poorer phase-matching condition and thus the \(E_{nm}^{\mathbf{V}}\) decays with \(l\).

(ii) Primary comb [Figs. 2(b) and (f)]. As the pump power increases above the OPO threshold and the cavity field can be prepared into a stable Turing pattern (Fig. 2(b)). In this state, several equally-spaced comb lines are efficiently excited, with the space approximately be \(m \times \text{FSR}\) determined by the dissipation rate \(\kappa\) and the dispersion \(D_2\) \cite{38}. Even though only \(\xi = 0\) is initially excited, the generated comb lines can stimulate the cascaded FWM and produce entanglement for mode-pairs belongs to other \(\xi\). Consequently, \(E_{nm}^{\mathbf{V}}\) in Fig. 2(f) shows non-zero values at elements on several diagonal lines, with each diagonal line corresponding to \(\xi = 2nm\) with \(n \in \mathbb{Z}\).

In the primary comb, the power of the comb lines decreases with the mode index, the degree of entanglement decreases from the main-diagonal line (\(\xi = 0\)) to the high-order diagonal (\(\xi = 2nm \neq 0\)).

(iii) Soliton state [Figs. 2(c), (d), (g) and (h)]. With appropriate laser power and frequency detuning, the intracavity field can be driven to the soliton states, which are ultra-short pulses circulating inside the cavity \cite{8, 39}. As shown by the spectra in Fig.2(c)-(d), when tens of modes are efficiently excited in both the two and single soliton states, the envelopes show a profile of sech\(^4\)-function. The strong comb line with index number \(l\) generated by the the pump mode can further drive the mode-pairs with \(\xi = 2l\), leading to the positive \(E_{nm}^{\mathbf{V}}\) on the corresponding diagonals of the entanglement matrix in Fig. 2(g) and 2(h). Due to the intensity distribution of the spectra, the entanglement matrix \(E_{nm}^{\mathbf{V}}\) of the two-soliton state has positive values only on diagonal lines corresponding to even mode index \((m/2 \in \mathbb{Z})\), while all diagonals near the center have positive values for the single-soliton state. Since multi-soliton state has higher energy than the single-soliton state \cite{39}, the FWMs are excited more efficiently, resulting in a higher \(E_{nm}^{\mathbf{V}}\). For the case in Fig. 2, the two-soliton state has a maximum \(E_{nm}^{\mathbf{V}}\) of 0.259, in comparison with 0.117 for the single-soliton state.

**All-to-all entanglement.-** For single-soliton state, modes are excited efficiently so that the multi-color entanglement is distinguishable from other cases. In particular, we can find a group of modes \(|i| < 12\) in Fig. 2(h)) where all-pairs of modes are entangled. The all-to-all entanglement indicates a fully connected complex network. This phenomenon manifests the distinct physical mechanisms of entanglement generation in the soliton state: two-mode and single-mode \((l \approx (a_i^2 + a_i^2))\) due to the comb lines in \(\xi = 2l\) squeezing generation, and the linear conversion between two modes \((i, j)\) stimulated by comb lines \((\xi = i, \xi = j) \approx (\alpha_i^2 + \alpha_i^2)\) for all \(\xi\), compared to the SPDC in conventional studies [Fig. 2(a)]. Since the modes have relatively uniform intensities around the 0-th mode, i.e. \(|\alpha_i|\) decays with \(j\) as indicated by the sech\(^4\)-function, the soliton state has higher degree of entanglement at the center compared with modes away from the pump in Figs. 2(g) and (h).

According to Eq. 3, the entanglement is stimulated by classical fields and thus the all-to-all entanglement region could be further spread by flatten the comb spectrum. Since the spec-
trum bandwidth of DKS could be efficiently controlled by the
dispersion $D_2$ [39], the multi-color entanglement for various
$D_2$ is investigated. Figures 3(a)-(b) show the the entangle-
ment matrix $E_{ij}$ for $D_2/\kappa = 1.645$ and $D_2/\kappa = 0.329$, re-
spectively. The $D_2/\kappa = 0.329$ case supports an all-connected
group involving more modes than that of the $D_2/\kappa = 1.645$
case but has less degree of entanglement, as marked by the
black square. Defining the edge length of the square as the
size of the fully connected group, the size of the quantum state
decreases with the dispersion $D_2$, as summarized in Fig. 3(c).
All-to-all entanglement among more than 50 colors is pre-
dicted for $D_2/\kappa = 0.329$. Thus, by designing a microring with
smaller dispersion and larger cavity size, potentially hundreds
of modes can be all-to-all entangled. By distributing the pho-
ton to different users via wavelength multiplexing, this all-to-
all entangled state is promising to build a multi-party quantum
telegation network [40].

**Entanglement and comb percolation.** For a complex net-
work, it is always curious about how the system performs if
some nodes are removed. The network with defects corre-
sponds to the optical cavities in practice, where some modes
show extraordinary low quality factor or large frequency shift-
ing, due to the perturbation of environments or avoid-mode
crossing induced by other mode families. The absorption and
shifting of the resonance can significantly suppress the intensi-
ties in these modes and modify the soliton spectrum. There-
fore, we further investigate the multi-color entanglement of
the soliton state in a defective mode family. The defects are
simulated by magnifying the dissipation rate of certain modes
to $1000 \kappa$ to eliminate the mode density of state and thus sup-
presses the corresponding comb lines. As shown by the nu-
merical results in Figs. 4(a)-(b), the single-soliton state can
still exist even if 5 modes are eliminated, and the spectra re-
main a envelope of sech$^2$-function except the lines of the lossy
modes being suppressed. Due to the nature of the complex
network, each mode can participate many different frequency
mixing processes with different $\xi$, thus the network is still
fully connected even in the absence of a few nodes, and show-

![FIG. 2. Quantum entanglement in different comb states. (a)-(d) Optical spectra of below-threshold comb state (a), primary comb (b), two-
soliton state (c), and single-soliton state (d). (e)-(h) The corresponding entanglement measure $E_{ij}$ between all mode-pairs. The $E_{ij} > 0$ is painted with colors whereas $E_{ij} = 0$ is painted white.](image1)

![FIG. 3. All-to-all quantum entanglement via dispersion engineering. (a)-(b) The matrices $E_{ij}$ of single-soliton state with $D_2/\kappa = 1.645$
(a) and $D_2/\kappa = 0.329$ (b), respectively. (c) The relationship between
the size of the group and the cavity dispersion. The frequency and
strength of the pump field are fixed.](image2)
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