Towards the identification of MARCOS models based on intuitionistic fuzzy score functions

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Abstract—We encounter uncertainty in many areas. In decision-making, it is an aspect that allows for better modeling of real-world problems. However, many methods rely on crisp numbers in their calculations. It makes it necessary to use techniques that perform this conversion. In this paper, we address the problem of score functions assessment regarding their effectiveness and usefulness in the decision-making field. The selected methods were used to convert the intuitionistic fuzzy set matrix into crisp data, then used in the multi-criteria assessment. Managing the theoretical problem showed that the used techniques provide high similarity values. Moreover, they proved to be helpful when dealing with intuitionistic fuzzy sets in the decision-making area.

I. INTRODUCTION

Many multi-criteria decision-making problems are considered in areas where data are represented using crisp numbers [1]. However, uncertainty problems are difficult to represent using this approach. Therefore, many tools based on classical arithmetic methods have been developed to model uncertainty in decision problems [2]. Such tools allow us to model real-world problems more accurately and reflect uncertain knowledge flexibly. Uncertainty modeling tools are often used in multi-criteria decision-making problems due to their high reliability [3].

Several popular tools can be used to represent uncertain knowledge. Among the classical approaches are fuzzy sets (FS), based on the idea related to partial membership [4]. Over the years, fuzzy sets have seen many developments: Hesitant Fuzzy Sets (HFS) [5], Fermatean Fuzzy Sets (FFS) [6], Picture Fuzzy Sets (PFS) [7], or Intuitionistic Fuzzy Sets (IFS) [8]. Indeed, the main advantage of the generalization of fuzzy sets is a new approach to uncertainty modeling that considers new degrees of membership, which gives the expert the ability to adapt to the characteristics of the problem [9].

One of the most popular tools based on the idea of fuzzy sets is Intuitionistic Fuzzy Sets. This tool introduces the possibility of determining the degree of membership and non-membership, thanks to which it is helpful in many areas such as decision-making and medical diagnosis [10], [11]. The wide use of Intuitionistic Fuzzy Sets has led to the development of this approach. A new similarity measure between intuitionistic fuzzy sets was proposed by Gohain et al. [12]. Szmidt et al. proposed a new proposal for attribute selection in models expressed by intuitionistic fuzzy sets [13]. Thao proposed new divergence measures of intuitionistic fuzzy sets from Archimedean t-conorm operators [14].

Using an extension of multi-criteria decision-making methods with fuzzy logic makes it possible to change the problem environment from crisp to uncertain. However, most Multi-Criteria Decision-Making (MCDM) related approaches operate in an environment based on crisp numbers [15]. To convert fuzzy data to crisp data, one can use point functions, whose idea in multi-criteria decision making was originally proposed by Chen and Tan [16]. However, the existence of multiple scoring functions means that their use within the same problem may be characterized by obtaining different results [17]. It creates a research gap that needs to be filled and determines which score function to select so that the results are satisfactory.

In this paper, we used five different score functions to convert Intuitionistic Fuzzy Sets to crisp values and assess the obtained decision matrix with the Measurement Alternatives and Ranking according to COmpromise Solution (MARCOS) method. The simulated data was used as the inputs to show the performance of the presented approach in the theoretical problem. Obtained results were then compared with selected correlation coefficients to point out the similarity of the used paths. The purpose of the study is to indicate the influence of the used score function regarding the differences obtained in multi-criteria ranking.

The rest of the paper is organized as follows. Section 2 presents the preliminaries of the IFS, the scores functions, the MARCOS method and selected similarity coefficients. In Section 3, the study case is shown, where the theoretical problem of the functioning of the different scores function is raised. Section 4 includes the description of the results obtained from the examined research. Finally, in Section 5, the summary is presented, and the conclusions are drawn.

II. PRELIMINARIES

A. Intuitionistic Fuzzy Sets

Definition II.1. An IFS A in a universe X is defined as an object of the following form:
where \( \mu : X \rightarrow [0,1] \) and \( \nu : X \rightarrow [0,1] \) such that \( 0 \leq \mu_j + \nu_j \leq 1 \) for all \( x_j \in X \). The values of \( \mu_j \) and \( \nu_j \) represent the degrees of membership and non-membership of \( x_j \in X \) in \( A \) respectively [17].

For every \( A \in IFS(X) \) (the class of IFSs in the universe \( X \)), the value of

\[
\pi_j = 1 - \mu_j - \nu_j
\]

represents the degree of hesitation (or uncertainty) associated with the membership of element \( x_j \in X \) in IFS \( A \), where \( 0 \leq \pi_j \leq 1 \).

B. Score Functions

The purpose of the score function is to convert the uncertain data representation to a crisp value. Different approaches to performing such an action obtain diverse values as a final output. Selected score functions and the formulas for their calculations are presented below [17], [18], [19].

\[
S_I(X_{ij}) = \mu_{ij} - \nu_{ij}
\]

(3)

\[
S_{II}(X_{ij}) = \mu_{ij} - \nu_{ij} \cdot \pi_{ij}
\]

(4)

\[
S_{III}(X_{ij}) = \mu_{ij} - \left( \frac{\nu_{ij} + \pi_{ij}}{2} \right)
\]

(5)

\[
S_{IV}(X_{ij}) = \left( \frac{\mu_{ij} + \nu_{ij}}{2} \right) - \pi_{ij}
\]

(6)

\[
S_{V}(X_{ij}) = \gamma \cdot \mu_{ij} + (1 - \gamma) \cdot (1 - \nu_{ij}), \quad \gamma \in [0,1]
\]

(7)

where \( S_I(X_{ij}), S_{II}(X_{ij}), S_{III}(X_{ij}) \in [-1,1], S_{IV}(X_{ij}) \in [-0.5,1] \), and \( S_{V}(X_{ij}) \in [-1,0.5] \).

C. MARCOS method

The Measurement Alternatives and Ranking according to COMpromise Solution (MARCOS) method was introduced by Željko Stević in 2020 [20] as new multi-criteria decision making method, which was presented on study case of sustainable supplier selection in healthcare industries. This method provides new approach to solve decision problems, as it considers an anti-ideal and ideal solution at the initial steps of consideration of the problem. Moreover it proposes new way to determine utility functions and their further aggregation, while maintaining stability in the problems requiring large set of alternatives and criteria.

Step 1. The initial step requires to define set of \( n \) criteria and \( m \) alternatives to create decision matrix.

Step 2. Next, the extended initial matrix \( X \) should be formed by defining ideal (AI) and anti-ideal(AAI) solution.

\[
AI = \begin{bmatrix} x_{a1} & x_{a2} & \cdots & x_{aan} \\ x_{11} & x_{12} & \cdots & x_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \\ x_{a1} & x_{a2} & \cdots & x_{ain} \end{bmatrix}
\]

(8)

The anti-ideal solution (AAI) which is the worst alternative is defined by equation (9), whereas the ideal solution (AI) is the best alternative in the problem at hand defined by equation (10).

\[
AAI = \min_{i,j} x_{ij} \quad \text{if } j \in B \quad \text{and} \quad \max_{i,j} x_{ij} \quad \text{if } j \in C
\]

(9)

\[
AI = \max_{i,j} x_{ij} \quad \text{if } j \in B \quad \text{and} \quad \min_{i,j} x_{ij} \quad \text{if } j \in C
\]

(10)

where \( B \) is a benefit group of criteria and \( C \) is a group of cost criteria.

Step 3. After defining anti-ideal and ideal solutions, the extended initial matrix \( X \) needs to be normalized, by applying equations (11) and (12) creating normalized matrix \( N \).

\[
n_{ij} = \frac{x_{ai}}{x_{ij}} \quad \text{if } j \in C
\]

(11)

\[
n_{ij} = \frac{x_{ij}}{x_{ai}} \quad \text{if } j \in B
\]

(12)

Step 4. The weight for each criterion needs to be defined to present its importance in accordance to others. The weighted matrix \( V \) needs to be calculated by multiplying the normalized matrix \( N \) with the weight vector through equation (13).

\[
v_{ij} = n_{ij} \times w_j
\]

(13)

Step 5. Next, the utility degree \( K \) of alternatives in relation to the anti-ideal and ideal solutions needs to be calculated by using equations (14) and (15)

\[
K_i^- = \frac{\sum_{i=1}^n v_{ij}}{\sum_{i=1}^n v_{ai}}
\]

(14)

\[
K_i^+ = \frac{\sum_{i=1}^n v_{ij}}{\sum_{i=1}^n v_{ai}}
\]

(15)

Step 6. The utility function \( f \) of alternatives, which is the compromise of the observed alternative in relation to the ideal and anti-ideal solution, needs to be determined. Its done using equation (16).

\[
f(K_i) = \frac{K_i^+ + K_i^-}{1 + f(K_i^+)} + \frac{1 - f(K_i^-)}{f(K_i^-)}
\]

(16)

where \( f(K_i^-) \) represents the utility function in relation to the anti-ideal solution and \( f(K_i^+) \) represents the utility function in relation to the ideal solution.
Utility functions in relation to the ideal and anti-ideal solution are determined by applying equations (17) and (18).

\[
f(K^-_i) = \frac{K^+_i}{K^+_i + K^-_i} \quad (17)
\]

\[
f(K^+_i) = \frac{K^-_i}{K^+_i + K^-_i} \quad (18)
\]

**Step 7.** Finally, rank alternatives accordingly to the values of the utility functions. The higher the value the better is an alternative.

**D. Rank similarity coefficients**

In order to compare the performance of the score functions, it would be useful to compare the rankings obtained after evaluating the values calculated using these functions. For this purpose, one can use rank similarity coefficients, which are often used in the literature to compare the resulting rankings. In the case of our study, we decided to use weighted Spearman’s correlation coefficient, which allows comparing rankings considering alternatives rated the best as more significant, and the WS ranking similarity coefficient, which has as a main assumption that the positions of top of the rankings has a more significant influence on similarity. The formulas for calculation of both coefficients are presented below in equation (19) for weighted Spearman’s correlation and equation (20) for WS rank similarity coefficient.

\[
r_w = 1 - \frac{6 \cdot \sum_{i=1}^{n} (x_i - y_i)^2 ((N - x_i + 1) + (N - y_i + 1))}{n \cdot (n^3 + n^2 - n - 1)} \quad (19)
\]

\[
WS = 1 - \sum_{i=1}^{n} \left(2^{-x_i} \frac{|x_i - y_i|}{\max \{|x_i - 1|, |x_i - N|\}} \right) \quad (20)
\]

**III. Study case**

The use of fuzzy sets in multi-criteria problems is a popular approach to solving problems where uncertainty arises. It allows greater flexibility in modeling input data, thus ensuring that the actual values that determine the parameters can be represented. However, in many cases, the criteria are not considered in a binary way, or the corresponding values are not known precisely. Fuzzy sets are one of the possible ways to represent uncertainty [21]. In the following, we focus our attention on the problem of using Intuitionistic Fuzzy Sets and different score functions to point out differences and similarities in the results obtained by using these tools.

A randomly generated decision matrix of 6 alternatives and 4 criteria was used in the study. Each matrix element is represented in the form of an IFS, where the first value indicates the degree of membership, while the second determines the degree of indeterminacy. Then, based on the score functions described above, conversions of the uncertain matrix to a matrices represented in the form of sharp numbers were performed. The generated matrix is shown in Table I.

The purpose of this operation is the need to indicate how a given score function affects the process of converting the data to a crisp form. Furthermore, it is crucial to determine whether the obtained matrices influence the obtained result through a multi-criteria analysis.

**IV. Results**

**A. Small example**

Each type of previously presented score function was used to calculate crisp values for the matrix, which were shown in Tables respective to the used function. Table II presents values obtained by use of score function \(S_f\). In the case of this score function, the spread of values in the range \([-1, 1]\) is around 1.69, which might mean that this specific score function differentiates well between alternative values.

| \(A_1\) | \(C_1\) | \(C_2\) | \(C_3\) | \(C_4\) |
|--------|--------|--------|--------|--------|
| 0.041174 | 0.510925 | 0.243452 | 0.726737 |
| 0.205952 | 0.075299 | 0.021391 | 0.089091 |
| 0.041371 | 0.060761 | 0.568287 | 0.587950 |
| -0.141754 | 0.930204 | 0.001438 | 0.324662 |
| 0.506242 | -0.056156 | 0.849442 | 0.389249 |
| 0.383334 | 0.368876 | -0.004356 | -0.014798 |

In Table III values calculated through execution of score function \(S_{II}\) are presented. This function is defined as the degree of membership minus the product of the non-membership and hesitation degrees, and even though it provides values from the same range as \(S_f\), it can be seen that there are less negative values. Moreover, it is clear that in this example, the spread of calculated values is significantly smaller, as in this case, it’s around 0.99.

| \(A_1\) | \(C_1\) | \(C_2\) | \(C_3\) | \(C_4\) |
|--------|--------|--------|--------|--------|
| 0.041174 | 0.510925 | 0.243452 | 0.726737 |
| 0.205952 | 0.075299 | 0.021391 | 0.089091 |
| 0.041371 | 0.060761 | 0.568287 | 0.587950 |
| -0.141754 | 0.930204 | 0.001438 | 0.324662 |
| 0.506242 | -0.056156 | 0.849442 | 0.389249 |
| 0.383334 | 0.368876 | -0.004356 | -0.014798 |

The values obtained through the equation of score function \(S_{III}\) are shown in Table IV. This specific function operates in the range \([-0.5, 1]\) and is similar to the previous one but subtracts the arithmetic mean of the non-membership and hesitation degrees. As a result, provided values spread around 1.24, which translates into a high differentiation of the individual IFS values from the initial decision matrix.
Table I presents preference values calculated by execution of MARCOS method. The values of preference for respective alternatives show the differences between considered score functions. The function $S_I$ has irregular distribution, where only one value is significantly higher than the rest. But in the case of this function, the difference between the highest and lowest value is almost 0.3, which shows that the values do not have a high spread. On the contrary, the score function $S_{III}$ provides a higher spread of 0.426, which might be preferable as it better distinguishes the differences between alternatives.

Score function $S_{III}$ provides the smallest spread of values of all functions, namely 0.047. In such a case, it may be perceived as the difference between alternatives is insignificant, which is rarely preferable in case of decision problems. Evaluated values from score function $S_{IV}$ yielded values that spread around 0.23, which is not the highest of presented score functions but might be useful in some cases. The last score function $S_V$ provided the highest values in this Table, which might be visually better perceived by some decision-makers, as the differences between the alternatives are more readily apparent. The spread is around 0.33, which is the second-highest. Considering those values, functions $S_{II}$ and $S_V$ are the most representative and might be preferred by numerous decision-makers.

Values for last score function, namely $S_V$ are presented in Table VI. This function represents a mixed result of positive and negative outcome expectations and operates in the same range as $S_I$ and $S_{II}$. In this case, no negative values were received even though the range in which operates this function includes negative values. The spread of values received from this function is around 0.85, which is the lowest of presented score functions, considering its range.
Figure 1 presents alternatives ranked by preference obtained through considered score functions. On the graph, the differences in evaluation are clearly visible as, for example, the score function $S_{III}$ and $S_I$ ranked alternative $A_1$ as the worst. In contrast, score function $S_V$ ranked this alternative as the worst. On the other hand, almost all functions placed alternative $A_6$ fourth.

![Fig. 1. Radar chart of MARCOS rankings.](image)

To better visualize differences between presented score functions, rankings obtained by execution of the MARCOS method were compared using similarity coefficients. The first coefficient, namely Weighted Spearman’s correlation coefficient, is presented in Figure 2 as a correlation matrix in the form of a heatmap. This coefficient shows which pair of compared rankings are not symmetrical, meaning that rankings are not identical neither the change in position is between exactly the same alternatives. As it can be seen once again, the pair $S_I$ and $S_{III}$ and pair $S_{II}$ and $S_V$ are characterized by a high degree of similarity. Moreover, comparing $S_{II}$ and $S_{IV}$ where $S_{II}$ is treated as yields high similarity.

![Fig. 2. Weighted Spearman’s correlation heatmap of MARCOS rankings for small decision matrix.](image)

Additionally, the rankings were compared using the WS rank similarity coefficient, which as well is presented as a correlation matrix in the form of a heatmap as Figure 3. This coefficient shows which pair of compared rankings are not symmetrical, meaning that rankings are not identical neither the change in position is between exactly the same alternatives. As it can be seen once again, the pair $S_I$ and $S_{III}$ and pair $S_{II}$ and $S_V$ are characterized by a high degree of similarity.

![Fig. 3. WS correlation heatmap of MARCOS rankings for small decision matrix.](image)

**B. Big example**

The next example that was taken into consideration consists of twenty alternatives and six criteria, which created the decision matrix presented in Table VIII. This approach provides a view of how specific score functions behave in larger multi-criteria decision problems.

Similarly to the smaller example, for a decision matrix consisting of IFS, crisp matrix was calculated using score
functions. The resultant matrix with crisp values is presented in Table IX.

**TABLE IX**
PREFERENCES FOR BIG DECISION MATRIX COMPUTED WITH MARCOS METHOD FOR $S_{II}$-SIV SCORE FUNCTIONS.

| $A_{i}$ | $S_{I}$ | $S_{II}$ | $S_{III}$ | $S_{IV}$ | $S_{V}$ |
|-------|--------|---------|---------|---------|--------|
| $A_{1}$ | -0.036549 | 0.190882 | -0.08514 | -0.041990 | 0.451390 |
| $A_{2}$ | -0.030858 | 0.162654 | -0.10952 | 0.059656 | 0.465973 |
| $A_{3}$ | -0.08761 | 0.126735 | -0.129638 | 0.086150 | 0.495595 |
| $A_{4}$ | 0.033628 | 0.355654 | 0.068392 | 0.049577 | 0.658271 |
| $A_{5}$ | 0.067533 | 0.500764 | 0.122254 | 0.119319 | 0.763359 |
| $A_{6}$ | -0.05921 | 0.174922 | 0.097213 | 0.057124 | 0.488577 |
| $A_{7}$ | -0.019398 | 0.278088 | -0.00707 | -0.101019 | 0.487457 |
| $A_{8}$ | 0.029490 | 0.462233 | 0.143382 | -0.127860 | 0.643773 |
| $A_{9}$ | -0.050486 | 0.13036 | -0.126984 | 0.006457 | 0.411298 |
| $A_{10}$ | 0.030223 | 0.483942 | -0.014051 | -0.105493 | 0.646673 |
| $A_{11}$ | 0.017131 | 0.371025 | 0.056299 | -0.051530 | 0.590749 |
| $A_{12}$ | 0.014285 | 0.170247 | -0.07856 | 0.216041 | 0.596157 |
| $A_{13}$ | 0.036399 | 0.465783 | 0.134848 | -0.048441 | 0.671756 |
| $A_{14}$ | -0.020070 | 0.282331 | -0.011157 | -0.077203 | 0.501384 |
| $A_{15}$ | 0.04216 | 0.482037 | 0.126290 | -0.027080 | 0.674939 |
| $A_{16}$ | -0.037072 | 0.129525 | -0.105932 | 0.042165 | 0.452381 |
| $A_{17}$ | 0.003562 | 0.395817 | 0.049615 | -0.063564 | 0.574704 |
| $A_{18}$ | -0.033528 | 0.026961 | -0.155368 | 0.105713 | 0.449290 |
| $A_{19}$ | 0.034387 | 0.394112 | 0.073114 | 0.025449 | 0.572488 |
| $A_{20}$ | 0.008353 | 0.327543 | 0.020393 | -0.003544 | 0.576575 |

The first function, namely $S_I$ yielded results presented in Table X. In this case, the standard deviation is 0.411, which is pretty high considering the range of this function, and it tells us that this specific function provided differentiated results. Moreover, the spread of those values is 1.76, which once again, as in the smaller numerical example, shows that this function makes use of a significant part of the range it operates in.

**TABLE X**
CRISP BIG DECISION MATRIX CALCULATED WITH $S_I$ SCORE FUNCTION.

| $A_{I}$ | $C_{1}$ | $C_{2}$ | $C_{3}$ | $C_{4}$ | $C_{5}$ | $C_{6}$ |
|-------|--------|--------|--------|--------|--------|--------|
| $A_{1}$ | -0.6276 | -0.3539 | 0.2009 | -0.8417 | 0.9213 | -0.3310 |
| $A_{2}$ | -0.5725 | -0.6886 | -0.0828 | 0.2502 | -0.1487 | 0.3191 |
| $A_{3}$ | -0.0708 | -0.0688 | -0.1500 | -0.4354 | -0.6519 | 0.2811 |
| $A_{4}$ | 0.7605 | -0.0669 | 0.6912 | 0.1899 | -0.1135 | -0.5556 |
| $A_{5}$ | 0.5340 | 0.2855 | 0.2184 | 0.7794 | 0.1150 | 0.3181 |
| $A_{6}$ | 0.1626 | 0.1802 | -0.4781 | 0.4516 | -0.5629 | -0.4935 |
| $A_{7}$ | -0.0215 | 0.5710 | -0.2478 | -0.7878 | 0.0027 | -0.2930 |
| $A_{8}$ | 0.4644 | 0.6029 | -0.3617 | 0.3153 | 0.2572 | 0.5909 |
| $A_{9}$ | 0.9083 | -0.7432 | -0.3246 | -0.7262 | 0.4246 | -0.1831 |
| $A_{10}$ | 0.1436 | 0.2437 | 0.0738 | 0.1083 | 0.1212 | 0.2895 |
| $A_{11}$ | -0.1362 | 0.4315 | 0.0555 | 0.0331 | 0.1276 | 0.0750 |
| $A_{12}$ | 0.2727 | 0.1301 | 0.0710 | 0.4297 | -0.1792 | 0.2395 |
| $A_{13}$ | 0.1720 | -0.5045 | 0.4656 | 0.5900 | -0.1670 | 0.7214 |
| $A_{14}$ | -0.2417 | -0.5374 | 0.6060 | -0.6640 | 0.6439 | -0.3144 |
| $A_{15}$ | -0.1812 | 0.6133 | -0.2880 | 0.9599 | 0.7743 | 0.0662 |
| $A_{16}$ | -0.0260 | 0.2676 | -0.5009 | -0.2770 | -0.5746 | -0.0635 |
| $A_{17}$ | 0.1189 | 0.1396 | 0.0863 | -0.6625 | 0.6823 | -0.1134 |
| $A_{18}$ | -0.4305 | 0.1306 | -0.2260 | 0.0619 | -0.1626 | -0.5317 |
| $A_{19}$ | 0.3906 | -0.0399 | -0.2211 | 0.6039 | 0.7927 | -0.0755 |
| $A_{20}$ | -0.3248 | 0.0300 | -0.1724 | 0.5492 | 0.239 | -0.0408 |

The results obtained using $S_{II}$ function are presented in Table XI. In this case, values are characterized by a standard deviation of 0.29 and a spread of 1.16. Because this function operates in the same interval as $S_I$, namely $[-1, 1]$, they can be easily compared. And just as in the small numerical example, here too, the function $S_{II}$ achieves smaller values of spread and standard deviation.
The calculated spread of values, being around 1.32, similar deviation value of about 0.36 is close to the value obtained by alternatives. While the obtained spread value is 0.88. This function operates in the same range as functions $S_I$ and $S_{II}$, which makes it the worst in diversifying values in comparison to those two. Even though a small standard deviation and spread characterize those values, this function might be useful when such values are expected.

Table XIV presents results obtained using function $S_{IV}$. The calculated spread of values, being around 1.32, similar to the functions $S_I$ and $S_{III}$ shows significant use of the range in which this function operates. Moreover, the standard deviation value of about 0.36 is close to the value obtained by the function $S_{III}$, which might indicate that those functions might yield similar results. The rankings obtained using the MARCOS method are grouped in the barplot shown in Figure 4. As can be seen, the obtained rankings differ significantly from each other, highlighting how important it is to choose an appropriate score.
function. Additionally, it can be seen that on the podium of the ranking, the functions $S_{Ii}$, $S_{III}$, and $S_{V}$ behave similarly. Still, in the further positions, significant discrepancies appear.

![Fig. 5](image1.png)  
**Fig. 5.** Weighted Spearman’s correlation heatmap of MARCOS rankings for big decision matrix.

The correlations of the rankings obtained from the big decision matrix data are shown in Figures 5 and 6 using heatmaps. The former, describing values for the weighted Spearman’s correlation coefficient, shows high correlation values for all scoring functions, excluding the $S_{IV}$ function. When it was used, the rankings calculated using the MARCOS method were significantly different.

![Fig. 6](image2.png)  
**Fig. 6.** WS correlation heatmap of MARCOS rankings for big decision matrix.

In contrast, the similarity of the rankings calculated with WS coefficient, shown in Figure 6 also indicated that the $S_{IV}$ function showed the least consistent results with the other techniques used. The strongest similarity of rankings could be observed for the pair of methods $S_{I}$ and $S_{V}$, which is 0.99. In contrast, the lowest consistency of rankings is 0.14 for the pair of methods $S_{III}$ and $S_{IV}$. It indicates a significant discrepancy, which confirms the importance of the influence of the used scoring function on the obtained results.

C. **WS comparison**

To generalize the results and examine the similarities between the scoring functions used, 1000 simulations were performed for randomly generated decision matrices. Each of the generated matrices was subjected to the techniques described earlier, and the resulting crisp matrices were used in a multi-criteria analysis using the MARCOS method. The figures and tables below show the values calculated for the similarities of the obtained rankings. The WS rank similarity coefficient determined their consistency.

Visualizations for selected scoring functions are presented below, together with tables describing selected statistics of the obtained data. Figure 7 shows the distribution of ranking similarity values for the simulations performed. The rankings obtained using the $S_{II}$ function were compared with the other methods. It is worth noting that for the functions $S_{III}$ and $S_{V}$, the similarity of the rankings was high and concentrated in a narrow area. It shows a high consistency in how IFS conversions to crisp values are performed, which translates into high reproducibility in evaluating alternatives.

![Fig. 7](image3.png)  
**Fig. 7.** Distribution of rankings similarity values using the $S_{II}$ score function.

Table XV contains the statistics calculated from the simulations, including a comparison of the performance of the function $S_{I}$ with the others. The variance and standard deviation were most negligible for the functions $S_{III}$ and $S_{V}$, as confirmed by the data shown in Figure 7. On the other hand, the most significant standard deviation (0.384707) was seen when comparing the results obtained using the $S_{I}$ function.

| **TABLE XV**
| **STATISTICS FOR RESULTS OBTAINED USING THE S_{II} SCORE FUNCTION.** |
| $S_i$ | Standard deviation | Variance | Mean |
|---|---|---|---|
| $S_{II}$ | 0.384707 | 0.147999 | 0.446851 |
| $S_{III}$ | 0.098412 | 0.009685 | 0.967222 |
| $S_{IV}$ | 0.118155 | 0.013961 | 0.274657 |
| $S_{V}$ | 0.060845 | 0.003702 | 0.910355 |
Figure 8 shows the similarity distribution obtained for the comparison of results using the $S_{III}$ function together with the other functions. As in the previous case, the highest similarity of rankings was observed for the functions $S_{II}$ and $S_{V}$. In addition, the lowest consistency of results was noted when comparing with the ranking obtained using $S_{IV}$.

Table XVI describes the statistical values for the data obtained when comparing the rankings of the functions $S_{III}$ with the others. The highest average ranking similarity value is 0.968083 for the method pair $S_{III}$ and $S_{II}$. It demonstrates the high consistency of the results and shows that the two functions can be used interchangeably without much effect on the rankings in most cases.

A visualization of the similarity distribution of the rankings obtained using the scoring function $S_{IV}$ compared to the other functions is shown in Figure 9. It can be seen that none of the techniques used gives a strong rankings correlation. Instead, it causes the results obtained to vary, making it essential to bear in mind that the choice of scoring function directly impacts the results obtained.

The determined statistical features for the comparisons of the function $S_{IV}$ with the others are listed in Table XVII. The average correlation value oscillates between a value of...
The similarity results for the other functions used in the study, i.e., $S_I$ and $S_V$, show that the first function gives a similar similarity of rankings to the other techniques. Still, it oscillates within a value of 0.4, indicating low consistency of the results. On the other hand, the second function shows a high similarity of performance together with the functions $S_{II}$ and $S_{III}$. It confirms the trend of possible interchangeable use of these functions in converting IFS to crisp values in multi-criteria problems.

V. Conclusion

Decision-making appears in many parts of life, so developing this particular branch of technology is crucial. However, often in decision-making problems, the problem of uncertainty and fuzzy values arise, which makes standard methods inapplicable. For this reason, it is worth taking a closer look at the possibilities of defuzzification of such problems.

In the study carried out, five score functions that allow achieving crisp values from intuitionistic fuzzy sets were compared. Each of the functions allows obtaining completely different values, which ultimately will significantly influence the results of the rankings. The study showed that in the smaller problem, the functions $S_I$ and $S_{II}$ should be preferred in decision-making problems because of the high distinction of individual values between them. However, the more extensive problem and simulations for 1000 decision matrices showed that functions $S_{II}$, $S_{III}$ and $S_V$ proved to be the most coherent techniques. Moreover, those functions presented high similarity in resulting rankings rendering them equally capable.

In future studies, it would be meaningful to address this issue regarding the reference ranking to compare the performance of the used score functions to indicate their reliability in practical problems. In addition, it would verify the usefulness and effectiveness of presented score functions in the decision-making process, which is obligatory to obtain credible results. In addition, future research would need to consider real decision-making tasks.

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0.284001 \text{ for feature } S_{II} \text{ and } 0.522578 \text{ for feature } S_V. \text{ It shows that a low consistency of results is obtained regardless of the technique used. On the other hand, the standard deviation for the similarity of the rankings is similar across all functions. It shows that the quality of the correlation is also affected by the input data, which can improve or worsen the consistency of the rankings.}
\]

| $S_i$ | Standard deviation | Variance | Mean |
|-------|--------------------|----------|------|
| $S_I$ | 0.145812           | 0.021261 | 0.491630 |
| $S_{II}$ | 0.124837 | 0.015584 | 0.289953 |
| $S_{III}$ | 0.133994 | 0.017794 | 0.284001 |
| $S_V$ | 0.147851 | 0.021860 | 0.522578 |

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