Superconducting Dark Matter

Alexander Vikman

CEICO-Central European Institute for Cosmology and Fundamental Physics,
Institute of Physics of the Czech Academy of Sciences,
Na Slovance 2, 182 21 Prague 8, Czech Republic

E-mail: vikman@fzu.cz

Abstract. We extend recently proposed mimetic dark matter (DM) and dust of dark energy (DE) models to a U(1)-charged gauged scalar field. This extension yields a mass to the photon via the Higgs mechanism. The square of this photon mass is proportional to the DM energy density. Thus dense DM environments can screen the dark vector force. There is a substantial freedom in this gauged extension of the irrotational fluid-like DM. This freedom enables one to model the classical London equation of superconductivity or to obtain the photon mass which remains constant during the matter-dominated époque.
1 Introduction

Observations show\footnote{For review see e.g. [1][2]} that magnetic fields are present not only in galaxies, but also in intergalactic space on cosmological scales. The structure of these fields and the history of their evolution is rather complicated and heavily influenced by different astrophysical properties. The origin of these large-scale cosmological magnetic fields remains as unclear as the origin of dark matter. It would be great if inflation could produce proper seeds, as it was first suggested in [4], but it seems that it is not an easy task see e.g. [1][5].

Classical electrodynamics is conformally invariant, therefore photons cannot not be produced in evolving cosmological backgrounds, as the conformal vacuum is preserved in this case [6]. Thus the conformal invariance of the electromagnetism must be broken to produce long rage magnetic fields as a result of inflation. The simplest way to break conformal invariance is to introduce the mass to the photon. It is well known, that superconducting media forces the photon to become massive. This is very similar to the Higgs effect. The main difference is that Higgs effect yields the mass which is constant and does not depend on the environment, whereas the London equation introduces the mass proportional to the square root of the charge carrier density.

On the other hand, it is probably too naive and definitely premature to claim that the dark sector of the universe has a simple structure and cannot contain dark $U(1)$ field. Moreover, gauging the only anomaly-free global symmetry of the Standard Model - the difference between baryon number $B$ and lepton number $L$ could give rise to another $U(1)$ field. It is interesting whether these dark long range forces can be screened and avoid observations so far in this way. Making these dark photons to leave in a superconductor would enable the desired screening.

It seems that it is interesting to equip the dark sector with superconducting properties. In this paper we present a class of models which make a fluid-like DM to behave as a superconductor. In particular, we managed to obtain the London equation with the square of the photon mass proportional to the DM density.
2 Model

Standard irrotational dust can be described by the action
\[ S = \int d^4x \sqrt{-g} \frac{\rho}{2} \left( \frac{(\partial \varphi)^2}{M^4} - 1 \right), \]  
(2.1)

where \( \rho \) is a Lagrange multiplier playing the role of the energy density while the real scalar field \( \varphi \) is the velocity potential\(^2\). The mass scale \( M \) is introduced to keep the canonical dimensions of the noncanonical scalar field \( \varphi \). This action (2.1) also naturally appears [7–9] in the so-called mimetic gravity [10, 11]. For later it is useful to note that the mass scale \( M \) can be assumed to be a function of the field \( \varphi \), so that one can allow for a function
\[ M(\varphi) = \mu f(\varphi/\mu), \]  
(2.2)

where \( f \) is a free function and \( \mu \) is a constant mass scale. This extension does not change the dynamics, as the following field-redefinition\(^3\)
\[ d\phi = \frac{d\varphi}{f^2(\varphi/\mu)}, \]  
(2.3)

maps this theory back to (2.1) with \( \varphi \to \phi \) and \( M(\varphi) \to \mu \). We will later use this fact.

In this paper we generalize the theory (2.1) to a complex scalar field \( \varphi \) interacting with a \( U(1) \) gauge field \( A_\mu \) in the standard covariant way
\[ S_\varphi = \int d^4x \sqrt{-g} \frac{\rho}{2} \left( \frac{|D_\mu \varphi|^2}{M^4} - 1 \right), \]  
(2.4)

where the \( U(1) \)-covariant derivative extends the usual Levi-Civita connection \( \nabla_\mu \) as
\[ D_\mu = \nabla_\mu + iqA_\mu, \]  
(2.5)

so that \( q \) is the coupling constant of \( \varphi \) to \( A_\mu \). This action can be obtained from the mimetic substitution into the standard action for gravity \( g_{\mu\nu} \) and matter
\[ g_{\mu\nu} = h^{\alpha\beta}D_\alpha \varphi D_\beta \bar{\varphi}, \]  
(2.6)

which is a particular Weyl (conformal) transformation of the auxiliary metric \( h_{\mu\nu} \). This substitution results in a degenerate higher derivative and Weyl-invariant and \( U(1) \) gauged-scalar (\( \varphi \))-tensor (\( h_{\alpha\beta} \))-vector (\( A_\mu \)) theory.

We assume that the dynamics of the free gauge field is given by the standard action
\[ S_a = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu}F^{\mu\nu}, \]  
(2.7)

\(^2\)We use: the standard notation \( \sqrt{-g} \equiv \sqrt{-\text{det}g_{\mu\nu}} \) where \( g_{\mu\nu} \) is the metric, the signature convention \((+, -, -, -)\), and the units \( c = \hbar = 1, M_{Pl} = (8\pi G_N)^{-1/2} = 1 \). However, sometimes for convenience we explicitly write \( M_{Pl} \).

\(^3\)Similar redefinition, along with an obvious redefinition of the Lagrange multiplier reduces the constraint in [12] to the form (2.1) with a constant mass-scale \( \mu \).
with the field strength \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). Gravity is minimally coupled to matter and to the scalar field and is described by the Einstein-Hilbert action. Both actions (2.4) and (2.7) are invariant under the gauge transformation

\[
\varphi \to e^{-iq\lambda} \varphi, \quad A_\mu \to A_\mu + \partial_\mu \lambda.
\]

It is important to stress that, contrary to the original irrotational dust described by the action (2.1), the shift-symmetry in field space \( \varphi \to \varphi + c \), is broken in the presence of the gauge field \( A_\mu \). Hence it is not forbidden by any symmetry to introduce a potential, \( V(|\varphi|) \), or to promote the scale \( M \) to be a function of the field: \( M(|\varphi|)^4 \). The shift-symmetry breaking makes the dynamics similar to the Dust of Dark Energy [12] and allow to unify DM and DE. Moreover, this construction can motivate other shift-symmetry breaking terms used in [10–13]. It is crucial to mention that some shift-symmetry breaking is needed for these models to survive inflation. Let us leave the case with potential for a future work, but allow for a nontrivial mass scale in (2.4)

\[
M(|\varphi|) = \mu f \left( |\varphi| / \mu \right).
\]  

(2.8)

The energy-momentum tensor (EMT) of the scalar sector is

\[
T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_\varphi}{\delta g^{\mu\nu}} = \frac{\rho}{2M^4} \left( D_\mu \varphi D_\nu \varphi + \overline{D_\mu \varphi} D_\nu \varphi \right).
\]  

(2.9)

Due to the interaction with the vector field \( A_\mu \) this EMT is not conserved. The \( U(1) \)-conserved current sourcing the Maxwell equations

\[
\nabla_\mu F^{\mu\nu} = J^\nu,
\]  

(2.10)

is given by

\[
J_\mu = -\frac{1}{\sqrt{-g}} \frac{\delta S_\varphi}{\delta A^\mu} = \frac{iq\rho}{2M^4} \left( \varphi D_\mu \varphi - \overline{\varphi} \overline{D_\mu \varphi} \right).
\]  

(2.11)

Other equations of motion are: the constraint

\[
|D_\mu \varphi|^2 = M^4,
\]  

(2.12)

and equation of motion for the scalar field

\[
D_\mu \left( \frac{\rho}{M^4} D^\mu \varphi \right) = 0,
\]  

(2.13)

along with its complex conjugated. Because of the constraint (2.12), either the field-derivatives \( \partial_\mu \varphi \) or the gauge field \( A_\mu \) (or both) are never vanishing. Moreover, either the absolute value of the field \( |\varphi| \) or its derivatives are always not vanishing as well.

For the later it is convenient to use the polar decomposition of the scalar field

\[
\varphi = \chi e^{iq\theta},
\]  

(2.14)

which is a nonsingular field-reparametrization provided \( \chi \neq 0 \). Using this parametrization we obtain

\[
D_\mu \varphi = e^{iq\theta} \left( \partial_\mu \chi + iq \chi (\partial_\mu \theta + A_\mu) \right),
\]  

(2.15)

\[\text{One could also promote } q \to q(|\varphi|), \text{ but we will not pursue this option in this paper.}\]
so that it is natural to introduce a \textit{gauge-invariant} vector field
\[ G_\mu \equiv A_\mu + \partial_\mu \theta. \] (2.16)

In terms of this new gauge-invariant dynamical variables \( \chi \) and \( G_\mu \) we have for the constraint equation (2.12)
\[ |D_\mu \varphi|^2 = (\partial_\mu \chi)^2 + q^2 \chi^2 G^\mu G_\mu = M^4. \] (2.17)

Hence by virtue of the polar decomposition (2.14) the new gauge-invariant vector field \( G_\mu \) becomes massive with the mass given by
\[ m^2_G = \frac{\rho}{M^4} q^2 \chi^2. \] (2.18)

The best way to detect the mass of the vector field \( G_\mu \) is to notice that the conserved current (2.11) becomes
\[ J_\mu = -m^2_G G_\mu, \] (2.19)

and introduces a mass term (2.18) in the Maxwell equations. One can expect this mass, \( m_G \), to be generically time-dependent. The other decisive difference from the standard canonical abelian Higgs mechanism is the prefactor \( \rho/M^4 \). Here \( \rho/M^4 \) is crucial. For example the mass scale
\[ M(\chi) = \sqrt{\mu \chi}, \] results in
\[ m^2_G = \frac{\rho}{\mu^2}, \] (2.20)

so that the mass of the vector field does not depend on the scalar field \( \chi \) at all. In this case the current is
\[ J = -q^2 \frac{\rho}{\mu^2} G, \] (2.21)

which gives the classical London equation in the superconductivity theory. Hence our dark matter is a \textit{superconductor}. In fact our system with mass scale \( M(\chi) \) above (2.20) exactly realizes the London equation in a manifestly covariant and gauge invariant manner. The mass scale \( \mu \) can be associated with the mass of the charged particle. It is important to note that this way of choosing the mass scale \( M(\chi) \) is rather nontrivial and forces the Lagrangian to be not an analytic function of fields around zero.

The EMT (2.9) reads in the new variables as
\[ T_{\mu\nu} = \frac{\rho}{M^4} \left( \chi_{,\mu} \chi_{,\nu} + q^2 \chi^2 G_\mu G_\nu \right). \] (2.22)

The current conservation \( \nabla_\mu J^\mu = 0 \) yields
\[ \nabla_\mu \left( \frac{\chi^2 \rho}{M^4} G^\mu \right) = 0. \] (2.23)

In particular, in cosmology this implies that the charge density redshifts as
\[ \frac{\chi^2 \rho}{M^4} G^0 = \frac{c}{a^3}. \] (2.24)

For timelike gradients of the radial field \( \chi \) one can introduce normalized vectors
\[ u_\mu = \frac{\chi_{,\mu}}{\sqrt{(\partial \chi)^2}}, \] (2.25)
which one can use as a local rest frame so that
\[ T_{\mu\nu} = E u_{\mu} u_{\nu} + m_G^2 G_{\mu} G_{\nu}, \]
(2.26)
where
\[ E = \rho (\partial_{\mu} \chi)^2 / M^4. \]
(2.27)

The total conserved EMT\(^5\) is
\[ T^{+}_{\mu\nu} = E u_{\mu} u_{\nu} + F^{\mu\lambda} F_{\nu}^{\lambda} + \frac{1}{4} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}^\mu G_{\mu} G_{\nu}, \]
(2.28)
with the field strength tensor \( F_{\mu\nu} = \partial_{\mu} G_{\nu} - \partial_{\nu} G_{\mu} \).

From the real part of the equation (2.13) we obtain\(^6\)
\[ \nabla_\mu (\rho \nabla^\mu \chi) = q^2 \rho \chi G^\mu G_\mu. \]
(2.29)

This equation is homogeneous in \( \rho \) and admits the trivial vacuum solution \( \rho = 0 \).

The field \( \rho \) moves almost as the dust energy density except of a small \( O(q^2) \) external force on right hand side of
\[ \dot{\rho} + \Theta \rho = q^2 \rho \left( \frac{2M^2 \chi G^2 + u^\lambda \nabla_\lambda (\chi^2 G^2)}{2M^4} + O(q^4) \right), \]
(2.30)
where \( \dot{\rho} = u^\lambda \nabla_\lambda \rho \) and \( \Theta = \nabla_\mu u^\mu \) is the expansion of the \( u^\mu \) congruence. In cosmology, the vector field can be only present on the level of perturbations. Hence
\[ \dot{\rho} + \Theta \rho \simeq 0, \]
(2.31)
up to small perturbations terms multiplied with a small \( q^2 \). The congruence \( u^\mu \) is almost geodesic as
\[ a_\mu = u^\lambda \nabla_\lambda u_\mu = \nabla_\mu \ln (\partial_{\mu} \chi)^2 \simeq - \frac{q^2}{M^4} \nabla_\mu \left( \chi^2 G^2 \right) + O(q^4), \]
(2.32)
where we used projector \( \perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu \), to introduce spatial derivative \( \nabla_\mu \perp = \perp_{\mu} \nabla_\nu \).

Hence in the limit
\[ q^2 \chi^2 |G^\mu G_\mu| \ll M^4, \]
(2.33)
the filed \( \chi \) plays the role of the velocity potential for dust while \( \rho \) is its energy density. In this case, on the background level in cosmology \( (\partial_{\mu} \chi)^2 \simeq M^4 \), so that \( \mathcal{E} \simeq \rho \). Hence this can be called “Higgs dust” or superconducting DM as because of the effective photon mass and the London equation (2.21).

For \( M = \text{const} \)
\[ \chi \simeq M^2 t, \]
(2.34)
while for more general mass-scales (2.8) one can use the redefinition (2.3) to obtain
\[ \int \frac{d\chi}{T^2 (\chi/\mu)} = \mu^2 t, \]
(2.35)

\(^5\)This total conserved EMT differs from the EMT for the Proca vector field and dust, by the absence of the \(-\frac{1}{2} g_{\mu\nu} m_G^2 G_\mu G^\alpha \) term. This term is absent due to the constraint equation (2.12) which we used.

\(^6\)Imaginary part of this equation gives the current conservation (2.23).
which gives an implicit function $\chi(t)$.

In this regime, when one can neglect the energy density stored in the massive term of the vector field, the energy density of the dust redshifts in the standard way

$$\rho \propto a^{-3},$$  \hspace{1cm} (2.36)

while the freedom in the choice of the mass-scale function $f(\chi/\mu)$ allows one to arrange for practically arbitrary evolution of $m_G(t)$. In particular, the mass redshifts slower than the dust energy density provided

$$\frac{d}{dt} \left( \frac{\chi}{M^2(\chi)} \right) > 0.$$ \hspace{1cm} (2.37)

Which gives

$$\frac{\dot{\chi}}{M^2} \left( 1 - \frac{2\chi M'}{M} \right) = 1 - \frac{2\chi M'}{M} > 0,$$ \hspace{1cm} (2.38)

or

$$\frac{d \ln M}{d \ln \chi} < \frac{1}{2}.\hspace{1cm} (2.39)$$

The equality in the last expression would yield the mass scale (2.20). For $M = const$ one can neglect the energy density stored in the massive term of the vector field provided

$$q^2 t^2 |G^\mu G_\mu| \ll 1.$$ \hspace{1cm} (2.40)

If we estimate the relevant time to be the cosmological time

$$t \simeq H^{-1},$$ \hspace{1cm} (2.41)

we obtain

$$q^2 |G^\mu G_\mu| \ll H^2.$$ \hspace{1cm} (2.42)

Let's first assume that there is only magnetic field $B_\ell$ on characteristic scale $\ell$. Then $G^\mu G_\mu = -G^2$ so that $(\partial \chi)^2 > 0$ and $\chi$ can be still used as a potential for some four-velocity. The condition that magnetic field corrections do not spoil the dust-like behavior of the in the Higgs phase implies that the charge is bounded by

$$q \ll \frac{H}{B_\ell \ell}.$$ \hspace{1cm} (2.43)

If the system violates this bound it i) either cannot be in the Higgs phase and form some magnetic tubes / strings or ii) the system does not behave like dust and has a strong influence of magnetic field on the motion.

In this paper we consider a general $U(1)$ field. However, we can try to apply this to the standard electromagnetic interactions in the Standard Model (SM). There are seem to be magnetic fields on large inter-cluster scales \cite{3} $B \sim 10^{-16} G \sim 10^{-35} \text{GeV}^2 \sim 10^{-73}$ on scales of $\ell \sim 1 \text{Mpc} \sim 3 \times 10^{24} \text{m} \sim 10^{57}$ so that

$$q \ll \frac{10^{-62}}{10^{-73}10^{57}} \simeq 10^{-46}$$ \hspace{1cm} (2.44)

Further for galactic magnetic field one can assume $B \sim \mu G \sim 10^{-25} \text{GeV}^2 \sim 10^{-63}$, $\ell \sim 1 \text{kpc} \sim 3 \times 10^{21} \text{m} \sim 10^{54}$ and $H \sim 10^{-61}$ so that

\[ \text{– 6 –} \]
\[ q \ll \frac{10^{-61}}{10^{-63} 10^{54}} \simeq 10^{-52}. \tag{2.45} \]

On the other hand for the case reproducing the London equation (2.20) we have
\[ q^2 \chi^2 |G^\mu G_\mu| \ll \mu^2 \chi^2, \tag{2.46} \]
so that
\[ G \ll \frac{\mu}{q}. \tag{2.47} \]

For a magnetic field on scale \( \ell \) this bound gives
\[ B_\ell \ell \ll \frac{\mu}{q}, \tag{2.48} \]
which for scales \( \ell \simeq \mu^{-1} \) for the electric field corresponds to the Schwinger limit. If we apply this boldly to the SM electrodynamics we get
\[ q \ll \frac{\mu}{B_\ell \ell} \simeq \left( \frac{\mu}{M_{Pl}} \right) \times 10^9. \tag{2.49} \]

3 “Photon” mass in cosmology, theoretical and observational constraints

In case of the Higgs dust with (2.20) reproducing the London equation, the mass of the photon always redshifts as \( a^{-3} \). On the other hand a naively more natural choice \( M = \text{const} \) gives a more interesting evolution \( m_G(t) \). Below we consider different cases.

3.1 Matter domination

In the universe completely dominated by this Higgs dust we have
\[ \rho = 3M_{Pl}^2 H^2 = \frac{4}{3} M_{Pl}^2 \cdot \frac{1}{l^2}, \tag{3.1} \]
so that the mass of the \( U(1) \)-vector field (2.18) is time-independent for the constant mass scale \( M \)
\[ m_G = \frac{2}{\sqrt{3}} q M_{Pl}. \tag{3.2} \]

In the DM-dominated and spatially-flat universe where only \( \Omega_H \) fraction of energy is in the Higgs dust we have
\[ \rho = 3M_{Pl}^2 H^2 \Omega_H, \tag{3.3} \]
so that the photon mass is again time-independent
\[ m_G = q \sqrt{\frac{4\Omega_H}{3}} M_{Pl}, \tag{3.4} \]
where \( \Omega_H \) remains constant. To develop intuition we can boldly apply this construction to the electrodynamics from the SM. Then we can use the astrophysical constraint, (see discussion in [14]) on the photon mass
\[ m_\gamma^a \lesssim 10^{-26} \text{eV}, \tag{3.5} \]
and obtain that

$$ q \sqrt{\frac{\Omega_H}{6\pi}} \lesssim \frac{10^{-26} \text{eV}}{1.22 \times 10^{28} \text{eV}} \simeq 10^{-54}. \tag{3.6} $$

On the other hand, if our system works as a type II superconductor where the coherence length is smaller than the penetration depth $m_G^{-1}$ this astrophysical bound is invalid [14]. Then we need to use the much weaker laboratory Coulomb law bound

$$ m^l_\gamma \lesssim 10^{-14} \text{eV}, \tag{3.7} $$

which still yields a tide constraint

$$ q \sqrt{\frac{\Omega_H}{6\pi}} \lesssim \frac{10^{-14} \text{eV}}{1.22 \times 10^{28} \text{eV}} \simeq 10^{-42}. \tag{3.8} $$

In any case, either the fraction of the Higgs dust (with constant mass scale $M(\chi) = \text{const}$) in the energy budget is vanishingly small or the coupling constant is extremely tiny.

It is interesting to mention that there is the so-called *Gravity as the Weakest Force* (GWF) conjecture [15] which states that the cutoff in gauge theories is

$$ \Lambda \simeq q M_{\text{Pl}}. \tag{3.9} $$

If we assume that this condition is also applicable to the environmental masses which are not fundamental we have $m_G \lesssim q M_{\text{Pl}}$ and therefore

$$ \Omega_H \lesssim \frac{3}{4}. \tag{3.10} $$

In this case the GWF conjecture (if applicable) forces the Higgs dust with the $M = \text{const}$ to be a heavily subdominant component in DM budget.

On the other hand, for the choice (2.20) reproducing the London equation we have a time dependent mass

$$ m_G(t) = \frac{q}{\mu} H M_{\text{Pl}} \sqrt{3 \Omega_H} \lesssim q M_{\text{Pl}}. \tag{3.11} $$

Hence the GWF provides a very weak bound from below on the mass of the charge carrier

$$ \mu \gtrsim H \sqrt{3 \Omega_H}. \tag{3.12} $$

If we again boldly apply this construction for the usual electrodynamics from the SM and use the constraint (3.5) we obtain the bound on the mass of the electric charge carrier building the superconductor

$$ \mu \gtrsim q \sqrt{\frac{3\Omega_H}{8\pi}} \left( \frac{1.22 \times 10^{28} \text{eV} 10^{-33} \text{eV}}{10^{-26} \text{eV}} \right) \simeq q \sqrt{\frac{3\Omega_H}{8\pi}} \times 10^{12} \text{GeV}, \tag{3.13} $$

which is stronger than (2.49) and can be clearly satisfied by some heavy particle. If again our system works as a type II superconductor, then we reserve to the weaker laboratory bound (3.7) which yields

$$ \mu \gtrsim q \sqrt{\frac{3\Omega_H}{8\pi}} \text{GeV}, \tag{3.14} $$

which is weaker than (2.49).
3.2 Radiation domination

During the radiation dominated èpoque $a \propto t^{1/2}$ so that
\[ \rho \propto a^{-3} \propto t^{-3/2}. \]  
so that for $M = const$ the mass of the gauge boson $m_G^2 = \rho q^2 t^4$, grows with time to the maximal constant value (3.4) as
\[ m_G \propto t^{1/4}. \]

3.2.1 $\Lambda$CDM

In the universe with a cosmological constant and DM the scale factor evolves as, (see e.g. [16])
\[ a(t) = a_m \left( \sinh \frac{3}{2} H_\Lambda t \right)^{2/3}, \]  
where $H_\Lambda = H_0 \sqrt{1 - \Omega_m}$, and $\Omega_m = \Omega_{DM} + \Omega_H$. After some straightforward manipulations one finds
\[ m_G(t) = q M_{Pl} \sqrt{\frac{4 \Omega_H}{3 (\Omega_H + \Omega_{DM})}}. \]  
Hence the mass is decreasing in the late times when $\Lambda$ starts to dominate. The maximal mass is
\[ m_G(t) = q M_{Pl} \sqrt{\frac{3 \Omega_H}{\Omega_m}} \left( \frac{H_0 \sqrt{1 - \Omega_m} t}{\sinh \left( \frac{3}{2} (H_0 \sqrt{1 - \Omega_m}) t \right)} \right). \]

3.3 Decay of perturbations

The usual excitations of the Higgs field are unstable and decay. This is also the case in our construction. On the perturbative level the mass scale defines the coupling constants and decay rates for fluctuations $\delta \rho$ and $\delta \chi$. Indeed, the cubic vertices in the presence of a background $\chi$ are
\[ \left( \frac{q^2 \chi^2}{M^4} \right) \delta \rho G^a G_\mu, \]  
and
\[ q^2 \rho \left( \frac{\chi^2}{M^4} \right)' \delta \chi G^a G_\mu, \]  
where $(\cdot)' = d(\cdot)/d\chi$. It is interesting to note that for (2.20) the last interaction vertex (3.21) is vanishing. These couplings are time dependent. In particular, for a constant $M$ both coupling constants are growing with time.

However, both fields $\delta \chi$ and $\delta \rho$ are not independent degrees of freedom and are not canonical wave-like fields with usual propagators. Hence the standard field theory methods cannot be directly applied here. Yet, it is clear that we are interested in rather small coupling constants $q$ so that the decay time-scales can be longer than the age of the universe. A proper estimation of the possible instability rate due to the production of two massive “photons” lies beyond the scope of this work and will be addressed somewhere else.
4 Discussion and Further Directions

We presented a model of superconducting DM. In particular we managed to derive the London equation. However, inclusion of a potential can promote this to the model of superconducting dark energy. Or superconducting unification of DM and DE.

It is very interesting to work out perturbation theory in this models and consider how this construction can influence the structure formation. Of course it is important to work out observation constraints in more details. Another issue is the quantization. Indeed, these models behave like fluid-like dust and do not have propagating DM waves. In particular it is important to see the limitations coming from potential strong couplings. It is well known that this systems are plagued by caustics. In the usual Higgs mechanism the theory restores the symmetry on short scales and the mass disappears. Maybe this softer behavior on short scales can help to solve these issues.

It is also interesting to generalize this conformal gauged mimetic substitution (2.6) to promote the disformal transformations [17] in a similar way

\[
g_{\mu\nu} = \omega \cdot h_{\mu\nu} + F \cdot D_{(\mu} \phi D_{\nu)} \phi, \tag{4.1}
\]

where both functions \(\omega\) and \(F\) can depend on standard kinetic term \(h^{\alpha\beta} D_\alpha \phi D_\beta \phi\) and on \(\phi \bar{\phi}\). It was demonstrated before [18] that singular disformal transformations do introduce new degrees of freedom.

Other possible extensions include direct couplings to curvature or / and incorporation of higher derivative operators which can give a small sound speed to small fluctuations of \(\phi\), e.g. one can add a term \(\left| D_\lambda D^\lambda \phi \right|^2\) to the Lagrangian. Further one can try to gauge other beyond Horndeski theories.

Acknowledgments

It is a pleasure to thank Gia Dvali, Andrei Gruzinov, Slava Mukhanov and Iggy Sawicki for useful discussions, encouragement and criticism. This work was supported by the J. E. Purkyňe Fellowship of the Czech Academy of Sciences and by the funds from Project CoGraDS - CZ.02.1.01/0.0/0.0/15_003/0000437 from the European Structural and Investment Fund and the Czech Ministry of Education, Youth and Sports (MŠMT).

References

[1] R. Durrer and A. Neronov, “Cosmological Magnetic Fields: Their Generation, Evolution and Observation,” *Astron. Astrophys. Rev.* 21 (2013) 62, arXiv:1303.7121 [astro-ph.CO].

[2] D. Grasso and H. R. Rubinstein, “Magnetic fields in the early universe,” *Phys. Rept.* 348 (2001) 163–266, arXiv:astro-ph/0009061 [astro-ph].

[3] A. Neronov and I. Vovk, “Evidence for strong extragalactic magnetic fields from Fermi observations of TeV blazars,” *Science* 328 (2010) 73–75, arXiv:1006.3504 [astro-ph.HE].

[4] M. S. Turner and L. M. Widrow, “Inflation Produced, Large Scale Magnetic Fields,” *Phys. Rev.* D37 (1988) 2743.

[5] V. Demozzi, V. Mukhanov, and H. Rubinstein, “Magnetic fields from inflation?,” *JCAP* 0908 (2009) 025, arXiv:0907.1030 [astro-ph.CO].

[6] L. Parker, “Particle creation in expanding universes,” *Phys. Rev. Lett.* 21 (1968) 562-564.
[7] A. Golovnev, “On the recently proposed Mimetic Dark Matter,” *Phys. Lett.* B728 (2014) 39–40, arXiv:1310.2790 [gr-qc].

[8] A. O. Barvinsky, “Dark matter as a ghost free conformal extension of Einstein theory,” *JCAP* 1401 (2014) 014, arXiv:1311.3111 [hep-th].

[9] K. Hammer and A. Vikman, “Many Faces of Mimetic Gravity,” arXiv:1512.09118 [gr-qc].

[10] A. H. Chamseddine and V. Mukhanov, “Mimetic Dark Matter,” *JHEP* 11 (2013) 135, arXiv:1308.5410 [astro-ph.CO].

[11] A. H. Chamseddine, V. Mukhanov, and A. Vikman, “Cosmology with Mimetic Matter,” *JCAP* 1406 (2014) 017, arXiv:1403.3961 [astro-ph.CO].

[12] E. A. Lim, I. Sawicki, and A. Vikman, “Dust of Dark Energy,” *JCAP* 1005 (2010) 012, arXiv:1003.5751 [astro-ph.CO].

[13] L. Mirzagholi and A. Vikman, “Imperfect Dark Matter,” *JCAP* 1506 (2015) 028, arXiv:1412.7136 [gr-qc].

[14] E. Adelberger, G. Dvali, and A. Gruzinov, “Photon mass bound destroyed by vortices,” *Phys. Rev. Lett.* 98 (2007) 010402, arXiv:hep-ph/0306245 [hep-ph].

[15] N. Arkani-Hamed, L. Motl, A. Nicolis, and C. Vafa, “The String landscape, black holes and gravity as the weakest force,” *JHEP* 06 (2007) 060, arXiv:hep-th/0601001 [hep-th].

[16] V. Mukhanov, *Physical Foundations of Cosmology*. Cambridge University Press, 2005. http://www-spires.fnal.gov/spires/find/books/www?cl=QB981.M99::2005.

[17] J. D. Bekenstein, “The Relation between physical and gravitational geometry,” *Phys. Rev.* D48 (1993) 3641–3647, arXiv:gr-qc/9211017 [gr-qc].

[18] N. Deruelle and J. Rua, “Disformal Transformations, Veiled General Relativity and Mimetic Gravity,” *JCAP* 1409 (2014) 002, arXiv:1407.0825 [gr-qc].