The generating function of correlators of dual operators on the boundary of (A)dS$_4$ space corresponding to the conformally coupled $\phi^4$-model is obtained up to first order in the coupling constant by using the conformal map between massless scalar fields in (Euclidean) Minkowski space and conformally coupled scalars on (Euclidean anti) de Sitter space. Some exact classical solutions of the nonlinear wave equation of massless (conformally coupled) $\phi^3$, $\phi^4$ and $\phi^6$-models in $D = 6, 4, 3$ Euclidean/Minkowski (AdS/dS) spaces are obtained.
1 Introduction

According to the generalized second law of thermodynamics [1] the total entropy of ordinary matter and black hole will never decrease in any physical process. This idea has led to the holographic principle [2, 3] which is a relation between information and geometry and/or the entropy and area of an event horizon, $S \sim A$. According to the holographic principle the physics of a generic space-time can be described in terms of a theory on the boundary (the holographic screen). A quantitative realization of the holographic principle is provided by the AdS/CFT correspondence [4]. The universality of the holographic principle and more specially the generalized second law of thermodynamics which includes de Sitter space [5] as well as AdS space has motivated many attempts to obtain a holographic duality between physics in de sitter space and some CFT on its boundary. In the absence of a well-posed stringy description of dS/CFT correspondence different approaches to this problem has been considered [6]. For example, holographic reduction of Minkowski space-time [8] provides a duality between Minkowski space-time and a CFT defined on the boundary of the light cone. The key-point in that method is the fact that Minkowski space time can be sliced in terms of Euclidian AdS and Lorentzian dS slices which correspond to the time-like and space-like regions respectively. Therefore known facts about AdS/CFT duality can be used to understand Minkowski/CFT and dS/CFT correspondence.

Recently in [9, 10], a holographic description of Euclidean/Minkowski is given for free massless scalar and Dirac fields. In that method, the action for free field theory on the boundary $t = 0$ is derived by solving the action of free scalars/Dirac fields in terms of the Cauchy data on the hypersurface $t = 0$. Using the conformal map between (Euclidean) Minkowski space and (Euclidean anti) de Sitter space which maps the hypersurface $t = 0$ to the boundary of (A)dS space, it is verified that the obtained action on the boundary $t = 0$ is exactly the generating function of CFT correlators of the dual operators recognized in AdS/CFT correspondence [11] and dS/CFT correspondence [6, 7]. Here, by a similar method, we derive the generating function of correlators of boundary operators dual to the $\phi^4$-model in (A)dS$_4$ using a holographic description of $\phi^4$-model in four dimensional (Euclidean) Minkowski space-time.

The paper is organized as follows. In section 2, we study free massless scalar field theory on Euclidean space $R^{d+1}$ and the conformal map to Euclidean AdS$_{d+1}$. We show that the CFT action on the boundary of AdS space can be obtained by solving the action of free scalars on $R^{d+1}$ in terms of the Cauchy data at $t = 0$. Finally we classify interacting scalar field theories that can be conformally mapped from $R^{d+1}$ to Euclidean AdS$_{d+1}$. In section 3, we study $D = 4 \phi^4$-model on Euclidean space. In section 4 some exact classical
solutions of non-linear Klein-Gordon equation of massless (conformally coupled) $\phi^3$, $\phi^4$ and $\phi^6$-models in $D = 6, 4, 3$ Euclidean/Minkowski (AdS/dS) spaces are obtained. Section 5 is devoted to scalar field theory on Minkowski space time and dS/CFT correspondence. We conclude in section 6. The free scalar field theory in curved spacetime is briefly reviewed in the appendix.

2 Massless Scalars on Euclidean Space

The equation of motion for free massive scalar fields, or free conformally coupled scalars on Euclidean AdS$_{d+1}$ with metric,

$$ds^2 = \frac{1}{t^2} \left( dt^2 + \sum_{i=1}^{d} dx_i^2 \right),$$

is

$$\left( t^2 \partial_t^2 + (1 - d)t \partial_t + t^2 \nabla^2 - m^2 \right) \Phi = 0,$$

where $\partial_t = \frac{\partial}{\partial t}$ and

$$\nabla^2 = \sum_{i=1}^{d} \frac{\partial^2}{\partial x_i^2}.$$  (3)

See the appendix for a brief review of scalar field theory in curved spacetimes. One can show that if $\phi$ is a massless scalar field in $d + 1$ dimensional Euclidean space $R^{d+1}$ with metric $ds^2 = dt^2 + dx_i^2$, satisfying the equation

$$(\partial_t^2 + \nabla^2)\phi = 0,$$

then

$$\Phi = t^{d-1} \phi,$$

is a solution of Eq.(2) with mass $m^2 = \frac{1-d^2}{4}$. Since $\frac{d^2}{4} < m^2 < 0$, this solution is stable in AdS$_{d+1}$. From AdS/CFT correspondence [11] it is known that the dual theory on the boundary $t = 0$ is a conformal theory with the following action,

$$I[\phi] = \int d^d y d^2 z \frac{\phi_0(y) \phi_0(z)}{|y-z|^{2(d+\lambda_+)}},$$

where $\lambda_+$ is the larger root of the equation $\lambda(\lambda + d) = m^2$.  $^1$ Here $\lambda_+ = \frac{(1-d)}{2}$ as far as $m^2 = \frac{(1-d^2)}{4}$. $\phi_0$ is a function on the boundary such that $\Phi(x, t) \sim t^{-\lambda_+} \phi_0$ as $t \to 0$. From the map (5), one can interpret $\phi_0(x)$ in Euclidean space $R^{d+1}$ as the initial data on the

$^1$I[\phi] is the generating function for CFT correlators i.e. $I[\phi] = \ln \langle \exp \int \phi \mathcal{O} \rangle$ where $\mathcal{O}$ is a dual operator.
hypersurface $t = 0$. Therefore one expects that the action (6) can be obtained from the action of scalar fields in $d + 1$ dimensional Euclidean space,

$$I[\phi] = \frac{1}{2} \int dt d^d x \left( (\partial_t \phi)^2 + (\nabla \phi)^2 \right),$$

(7)

if one solves equation (4) in terms of the initial data $\phi_0(\vec{x})$ given on the hypersurface $t = 0$. Proof is as follows:

The most general solution of the equation of motion that vanishes as $t$ tends to infinity is

$$\phi(\vec{x}, t) = \int d^d k \tilde{\phi}(\vec{k}) e^{i \vec{k} \cdot \vec{x}} e^{-\omega t},$$

(8)

where $\omega = |\vec{k}|$ and

$$\tilde{\phi}(\vec{k}) = \int d^d x \phi_0(\vec{x}) e^{-i \vec{k} \cdot \vec{x}}.$$  

(9)

Here, $\phi_0(\vec{x})$ is the initial value on the boundary $t = 0$. Inserting (9) into (8) one obtains

$$\phi(\vec{x}, t) = \int d^d y \mathcal{G}(\vec{x}, t; \vec{y}) \phi_0(\vec{y}),$$

(10)

where

$$\mathcal{G}(\vec{x}, t; \vec{y}) = \int d^d k e^{-\omega t} e^{i \vec{k} \cdot (\vec{x} - \vec{y})}.$$  

(11)

$\mathcal{G}(\vec{x}, t; \vec{y})$ is the solution of wave equation i.e. $\Box \mathcal{G} = 0$, with the initial condition $\mathcal{G}(\vec{x}, 0; \vec{y}) = \delta^d(\vec{x} - \vec{y})$. To obtain the action of the corresponding theory on the boundary $t = 0$ one should insert (10) into (7). But it is more suitable to rewrite the action (7) in the form,

$$I[\phi] = -\frac{1}{2} \int d^{d+1} x \phi \Box \phi - \frac{1}{2} \int d^d x \phi_0(\vec{x}) \partial_t \phi_0(\vec{x}),$$

(12)

which is obtained by an integration by part and under the assumption that $\phi(x)$ vanishes as $t$ tends to infinity and also at spatial infinity. Inserting (10) into (12), the first term vanishes and from the second term one obtains:

$$I[\phi] = \frac{1}{2} \int d^d y d^d z \phi_0(\vec{y}) \phi_0(\vec{z}) F(\vec{y} - \vec{z}),$$

(13)

in which

$$F(\vec{x}) = \int d^d k \omega e^{i \vec{k} \cdot \vec{x}}.$$  

(14)

As can be verified from the rotational invariance ($\vec{x} \rightarrow R \vec{x}$, $R \in SO(d)$), $F(\vec{x})$ depends only on the norm of $\vec{x}$. By scaling $\vec{x}$ by a factor $\lambda > 0$ one can also show that $F(\lambda \vec{x}) = \lambda^{-(d+1)} F(\vec{x})$ and consequently,

$$\vec{x} \cdot \nabla F(\vec{x}) = \lim_{\lambda \rightarrow 1} \frac{F(\lambda |\vec{x}|) - F(|\vec{x}|)}{\lambda - 1} = -(d + 1) F(|\vec{x}|).$$

(15)

Therefore, $F(\vec{x}) = \text{Const.} \ |\vec{x}|^{-(d+1)}$ and

$$I[\phi] = \text{Const.} \int d^d y d^d z \frac{\phi_0(\vec{y}) \phi_0(\vec{z})}{|\vec{y} - \vec{z}|^{d+1}},$$

(16)
which is equal to (6) derived in AdS/CFT correspondence.

One should note that in addition to the general solution (8), Eq. (4) has one further solution \( \phi(\vec{x}, t) = \alpha t \) where \( \alpha \) is some constant that can not be determined from the initial data \( \phi_0(\vec{x}) \). Although this solution is meaningless in Euclidean space, but \( \alpha \) is equal to the value of \( \Phi(\vec{x}, t) \) at \( t \to \infty \) which is an additional point on the boundary of AdS.

One can interpret the above result as a holographic description of Euclidean space. But our analysis considers only massless scalars. As can be easily verified, it is not possible to generalize the map (5) to include massive scalar fields in \( R^{d+1} \). A reason for this is the fact that the equation of motion of scalar fields \( (\Box + m^2)\phi = 0 \) is not conformally covariant unless \( m = 0 \).

By the same consideration, one can also show that the map between free scalar field theories can only be generalized to \( D = 6 \phi^3 \)-model, \( D = 4 \phi^4 \)-model and \( D = 3 \phi^6 \)-model in which no specific length (mass) is given in the model as far as \( g \), the coupling, is dimensionless. The proof is as follows: using the identity,

\[
\left( t^2 \partial_t^2 + (d - 1)t \partial_t - t^2 \nabla^2 + \frac{d^2 - 1}{4} \right) \Phi(\vec{x}, t) = t^{\frac{d+1}{2}} \left( \partial_t^2 - \nabla^2 \right) \phi, \tag{17}
\]

in which \( \Phi = t^{\frac{d+1}{2}} \phi \), and considering the potential \( V(\phi) = \frac{g}{n+1} \phi^{n+1} \), one can show that

\[
n = 1 + \frac{4}{d - 1}, \tag{18}
\]

if \( (\partial_t^2 - \nabla^2) \phi = -g\phi^n \), and

\[
\left( t^2 \partial_t^2 + (d - 1)t \partial_t - t^2 \nabla^2 + \frac{d^2 - 1}{4} \right) \Phi(\vec{x}, t) = -g\Phi^n. \tag{19}
\]

Since \( n \) is an integer, the identity (18) can be satisfied only for \( d = 2, 3, 5 \) and \( n = 5, 3, 2 \) respectively. All such theories are renormalizable.

### 3 \( D = 4 \) Massless \( \phi^4 \)-Model

In this section we study \( D = 4 \) massless \( \phi^4 \)-model and obtain the corresponding action on the boundary \( t = 0 \) up to first order in \( g \), the coupling constant. As is shown in section 2, this massless \( \phi^4 \) model can be directly mapped to conformally coupled scalar theory on Euclidean AdS. The same method can be used to obtained the action on the boundary corresponding to \( D = 6 \phi^3 \)-model and \( D = 3 \phi^6 \)-model.

The action is

\[
I_g[\phi] = \int d^4x \left( \frac{1}{2} \left( (\partial_t \phi)^2 + (\nabla \phi)^2 \right) - \frac{g}{4} \phi^4 \right),
\]

\[
= -\int_{R^4} \left( \frac{1}{2} \phi \Box \phi + \frac{g}{4} \phi^4 \right) - \frac{1}{2} \int_{R^3} \phi_0(\vec{x}) \partial_t \phi_0(\vec{x}). \tag{20}
\]
The corresponding Eule-Lagrange equation of motion is the a non-linear Klein-Gordon equation on $R^4$ given as follows:

$$\Box \phi + g\phi^3 = 0.$$  \hfill (21)

As before, to obtain the second equality in Eq.(20), an integration by part is made. It is also assumed that $\phi(x)$ vanishes as $t \to \infty$ and at spatial infinity. $\phi_0(x)$ is the Cauchy data at $t = 0$. In the next section we obtain some exact solutions of Eq.(21), but here we are interested in solutions only correct up to $O(g^2)$. A formal solution of Eq.(21) is

$$\phi(x) = \eta(x) - g \int d^4y \, G_E(x; y)\phi^3(y),$$  \hfill (22)

where $\Box \eta = 0$ and $\Box G_E(x; y) = \delta^4_E(x - y)$. $G_E$ is the Euclidean Green function that is equal to the the Green function,

$$G(x, y) \sim \int d^3k \frac{e^{i\omega(x^0 - y^0)}}{\omega} e^{ik(\vec{x} - \vec{y})}, \quad \omega = |\vec{k}|,$$  \hfill (23)

after a Wick rotation $x^0 \to ix^0$. Using $G$ defined in Eq.(11), one can solve Eq.(22) in terms of $\phi_0(x) = \phi(x)|_{t=0}$ as follows:

$$\phi(x) = \eta_1(x) - g \int d^4y \, G(x; y)\eta^3_0(y) + O(g^2),$$  \hfill (24)

where $\eta_0(x) = \int d^3y \, G(\vec{x}, 0; \vec{y})\phi_0(\vec{y})$ and

$$\eta_1(x) = \eta_0(x) + g \int d^3y \, G(\vec{x}, t; \vec{y})G(\vec{y}, 0; y)\eta^3_0(y).$$  \hfill (25)

One can verify that $\Box \eta_1 = \Box \eta_0 = 0$ and $\eta_0(\vec{x}, 0) = \phi_0(\vec{x})$. Since,

$$\int d^3z \, G(\vec{x}, t; \vec{z})G(\vec{z}, 0; y) = G(x; y),$$  \hfill (26)

as can be verified from Eqs.(11) and (23) and using equations (24) and (25) one obtains,

$$\phi(x) = \eta_0(x) + O(g^2).$$  \hfill (27)

Consequently,

$$I_g[\phi] = \frac{1}{2} \int d^4x \partial_\mu \eta_0 \partial^\mu \eta_0 - \frac{g}{4} \int d^4x \, \eta^4_0 = \text{Const.} \int d^dyd^dz \frac{\phi_0(\vec{y})\phi_0(\vec{z})}{|\vec{y} - \vec{z}|^4} - \frac{g}{4} \int d^4x \, \eta^4_0,$$  \hfill (28)

where to obtain the second equality we have used Eq.(16). Consequently the only $g$-contribution in $I_g[\phi]$ is from the term $\int d^4x \, \eta^4_0(x)$ that can be written as follows:

$$\int_{R^4} \eta^4_0(x) = \int_{R^3} K_E(\vec{x}_1, \cdots, \vec{x}_4)\phi_0(\vec{x}_1) \cdots \phi_0(\vec{x}_4).$$  \hfill (29)
\[ K_E(\vec{x}_1, \cdots, \vec{x}_4) = \int d^4 x \, G(x; \vec{x}_1) \cdots G(x; \vec{x}_4) \]
\[ = \int d^3 k_1 \cdots d^3 k_4 \frac{e^{i \vec{k}_1 \cdot \vec{x}_1} \cdots e^{i \vec{k}_4 \cdot \vec{x}_4}}{\omega_1 + \cdots + \omega_4} \delta^3(\vec{k}_1 + \cdots + \vec{k}_4), \quad (30) \]

where to obtain the second equality we have used definition (11). To calculate \( K_E(\vec{x}_1, \cdots, \vec{x}_4) \) we note that

1. \( K_E \) is symmetric: \( K_E(\vec{x}_1, \cdots, \vec{x}_4) = K_E(\vec{x}_{p_1}, \cdots, \vec{x}_{p_4}) \) for any permutation \( p \),

2. \( K_E \) is invariant under translation: \( K_E(\vec{x}_1 + \vec{a}, \cdots, \vec{x}_4 + \vec{a}) = K_E(\vec{x}_1, \cdots, \vec{x}_4) \),

3. \( K_E(\lambda \vec{x}_1, \cdots, \lambda \vec{x}_4) = \lambda^{-8} K_E(\vec{x}_1, \cdots, \vec{x}_4) \), for any real-valued positive \( \lambda \),

4. \( \int d^3 x_4 \ K_E(\vec{x}_1, \cdots, \vec{x}_4) = K_E(\vec{x}_1, \vec{x}_2, \vec{x}_3) \),

where

\[ K_E(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \int d^3 k_1 d^3 k_2 d^3 k_3 \frac{e^{i \vec{k}_1 \cdot \vec{x}_1 + \vec{k}_2 \cdot \vec{x}_2 + \vec{k}_3 \cdot \vec{x}_3}}{\omega_1 + \omega_2 + \omega_3} \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3). \quad (31) \]

\( K_E(\vec{x}_1, \vec{x}_2, \vec{x}_3) \) is symmetric and invariant under translation, \( K_E(\lambda \vec{x}_1, \lambda \vec{x}_2, \lambda \vec{x}_3) = \lambda^{-5} K_E(\vec{x}_1, \vec{x}_2, \vec{x}_3) \) and

\[ \int d^3 x_3 \ K_E(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \int d^3 k_1 d^3 k_2 \frac{e^{i \vec{k}_1 \cdot \vec{x}_1 + \vec{k}_2 \cdot \vec{x}_2}}{\omega_1 + \omega_2} \delta^3(\vec{k}_1 + \vec{k}_2) \]
\[ = \text{Const.} \frac{1}{|\vec{x}_1 - \vec{x}_2|^2}. \quad (32) \]

From the above observations we conclude that

\[ K_E(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \text{Const.} \left( |\vec{x}_1 - \vec{x}_2| |\vec{x}_2 - \vec{x}_3| |\vec{x}_3 - \vec{x}_1| \right)^{-\frac{5}{4}}, \quad (33) \]

as far as

\[ M(\vec{x}_1, \vec{x}_2) = \int d^3 x_3 \left( |\vec{x}_2 - \vec{x}_3| |\vec{x}_3 - \vec{x}_1| \right)^{\frac{5}{8}} \sim |\vec{x}_1 - \vec{x}_2|^\frac{1}{8}, \quad (34) \]

and consequently Eq.(33) is satisfied. The validity of Eq.(34) can be verified by noting that \( M(\vec{x}_1, \vec{x}_2) = M(|\vec{x}_1 - \vec{x}_2|) \) and \( M(\lambda \vec{x}_1, \lambda \vec{x}_2) = \lambda^{-1/4} M(\vec{x}_1, \vec{x}_2) \). Using this result we claim that,

\[ K_E(\vec{x}_1, \cdots, \vec{x}_4) = \text{Const.} (|\vec{x}_1 - \vec{x}_2| |\vec{x}_1 - \vec{x}_3| |\vec{x}_1 - \vec{x}_4| |\vec{x}_2 - \vec{x}_3| |\vec{x}_2 - \vec{x}_4| |\vec{x}_3 - \vec{x}_4|)^{-\frac{5}{8}}. \quad (35) \]

Summarizing our results, we have found that

\[ I_g[\phi] = \int_{R^4} \left( \frac{1}{2} \left( (\partial_i \phi)^2 + (\nabla \phi)^2 \right) - \frac{g}{4} \phi^4 \right) \]
\[ \sim \text{Const.} \int_{R^3} \frac{\phi_0(\vec{y}) \phi_0(\vec{z})}{|\vec{y} - \vec{z}|^4} \]
\[ + g \int_{R^3} \frac{\phi_0(\vec{x}_1) \phi_0(\vec{x}_2) \phi_0(\vec{x}_3) \phi_0(\vec{x}_4)}{|\vec{x}_1 - \vec{x}_2| |\vec{x}_1 - \vec{x}_3| |\vec{x}_1 - \vec{x}_4| |\vec{x}_2 - \vec{x}_3| |\vec{x}_2 - \vec{x}_4| |\vec{x}_3 - \vec{x}_4|}^{\frac{5}{8}}. \quad (36) \]
4 Exact Solutions of Non-Linear Wave Equation

In sections 2 and 3 we inserted the solutions of the Klein-Gordon equation into the action in order to find an action for the fields on the boundary \( t = 0 \). The solutions of Euler-Lagrange equations obtained from the action on the boundary are the only configurations that can be considered as initial condition (Cauchy-data) which solve the non-linear wave equation (Klein-Gordon equation) with solutions that vanish as \( t \) tends to infinity, since they are *on-shell* configurations by construction. In section 2 we verified that the action on the boundary \((\mathbb{R}^d)\) that was obtained by inserting the exact solutions of the mass-less free field equation into the action of fields in the bulk \( \mathbb{R}^{d+1} \) is exactly the CFT action recognized in AdS\(_{d+1}\)/CFT\(_d\) correspondence. In section 3 we considered \( \phi^4 \)-model in four dimension \((\mathbb{R}^4)\) and derived the action on the boundary \( \mathbb{R}^3 \) (at \( t = 0 \)) by solving the action in terms of solutions of the non-linear wave equation obtained by perturbation. As far as the massless \( \phi^4 \)-model in \( \mathbb{R}^4 \) is in one-to-one correspondence to conformally coupled \( \phi^4 \) scalar theory on Euclidean AdS space, one may conjecture that the corresponding (CFT) action on the boundary of AdS space can be exactly obtained if one repeats the steps of sections 2 and 3 using the exact solutions of the non-linear Klein-Gordon equation. In what follows we consider \( \phi^n \)-model in \( d + 1 \) dimension and derive the exact plane wave solutions. Furthermore, in the case of conformally coupled scalar theories we also obtain some \( SO(d+1) \)-invariant solutions. As we will see for all such solutions, the scalar field is determined at all space-time points even on the boundary and no initial condition in the sense of free field solutions or solutions obtained by perturbation exist. The initial condition in the case of non-linear Klein-Gordon equation can at most clarify which of the solutions among the others should be considered and determine some constant coefficients that may appear in such solutions. By inserting any of the exact solutions into the action one does not obtain an action governing the fields on the boundary. This incommodity is caused by the lack of any superposition rule for solutions of non-linear differential equations. As far as the exact classical solutions can not be quantized by the same reason and one should use perturbation to obtain the \( \phi^4 \) quantum field theory, perturbation can also be considered as a reasonable method to obtain the boundary action.

**Plane Wave Solutions**

The non-linear Klein-Gordon equation in \( d + 1 \) dimension for plane wave solutions is,

\[
\frac{d^2}{du^2}\phi(u) + \frac{1}{k^2} \frac{dV(\phi)}{d\phi} = 0, \tag{37}
\]

where \( u = k_\mu x^\mu \) and \( k^2 = k_\mu k^\mu \) for some wave vector \( k = (k_1, \cdots, k_{d+1}) \). To obtain Eq.(37) we have inserted \( \Box\phi(u) = k^2 \frac{d^2}{du^2}\phi(u) \) into the Klein-Gordon equation \( \Box\phi + \frac{dV(\phi)}{d\phi} = 0 \).
From Eq.(37) one verifies that plane wave solutions are not sensitive to the dimension of space-time. The most general solutions of Eq.(37) are a one-parameter family that can be obtained by solving the following integral equation:

$$\int \frac{d\phi}{\sqrt{-k^2 V(\phi) + c}} = u,$$  \hspace{1cm} (38)

where $c$ is some real-valued constant. For example if $V(\phi) = \frac{4}{3} \phi^4$, $(g < 0)$ then,

$$\phi_{c=0}(x) = \frac{1}{k \cdot x}, \quad \tilde{k}^2 = -\frac{g}{2} \left( \tilde{k}^\mu = \sqrt{\frac{-g}{2} k^\mu} \right).$$  \hspace{1cm} (39)

**SO($d+1$)-Invariant Solutions**

The Klein-Gordon equation for SO($d+1$)-invariant solutions $\phi = \phi(s)$, where $s = \sqrt{x_\mu x^\mu}$ is

$$\left( \frac{d^2}{ds^2} + \frac{d}{s} \frac{d}{ds} \right) \phi + g \phi^n = 0.$$  \hspace{1cm} (40)

One solution of this equation is

$$\phi_0(s) = \frac{2(d - 1 - \frac{2}{n-1})}{(n - 1)} \left( \frac{1}{gs^2} \right)^{\frac{n-1}{2}}$$  \hspace{1cm} (41)

These solutions can be obtained by considering the ansatz $\phi(s) = \alpha s^\beta$ and solving the wave equation to determine $\alpha$ and $\beta$. The above solutions become singular as $g \to 0$. One can show that Eq.(40) has also solutions like,

$$\phi(s) = \frac{\alpha}{(\beta + s^2)^\gamma},$$  \hspace{1cm} (42)

only for conformally coupled theories, i.e. for $d = 2, 3, 5$ and $n = 5, 3, 2$ respectively. The solutions are:

$$\frac{\alpha}{\left( \frac{\alpha^2}{3} g + s^2 \right)^{\frac{n-1}{2}}}, \quad n = 2, \quad d = 5,$$

$$\frac{\alpha}{\left( \frac{\alpha^2}{8} g + s^2 \right)^{\frac{n-1}{2}}}, \quad n = 3, \quad d = 3,$$

$$\frac{\alpha}{\sqrt{\frac{\alpha^2}{3} g + s^2}}, \quad n = 5, \quad d = 2,$$  \hspace{1cm} (43)

where $\alpha$ is some arbitrary real-valued constant. By Wick rotation $t \to it$ one obtains the solutions of wave equation in Minkowski space-time. Using the map $\phi \to \Phi = t^{d+1} \phi$, defined in section 2 one can also obtain the corresponding SO($d$)-invariant solutions in Euclidean AdS$_{d+1}$ space and dS$_{d+1}$ space. For example, a solution for the Klein-Gordon equation for $\phi^4$ model in the $\mathcal{O}^-$ region of dS$_4$ space is

$$\phi_{dS_4}(t, \vec{x}) = \frac{\alpha t}{\left( \frac{\alpha^2}{5} g - t^2 + |\vec{x}|^2 \right)^{\frac{d+1}{2}}}, \quad \alpha \in \mathbb{R}.$$  \hspace{1cm} (44)
A method to obtain solutions given in Eq.(43) and more such solutions for conformally coupled models is as follows. Using the solutions \( \phi_0(s) \) given in Eq.(41) one can try to solve the Klein-Gordon equation for \( \phi(s) = \phi_0(s) \eta(s) \). The resulting equation for \( \eta(s) \) is

\[
\left( s \frac{d}{ds} \right) \left( s \frac{d}{ds} \right) \eta + \left\{ \begin{array}{ll}
4\eta(\eta - 1) = 0, & n = 2, \ d = 5, \\
\eta(\eta^2 - 1) = 0, & n = 3, \ d = 3, \\
\frac{1}{4}\eta(\eta^4 - 1) = 0, & n = 5, \ d = 2. 
\end{array} \right. 
\] (45)

The solutions given in Eq.(43) are obtained by solving Eq.(45) with vanishing constant of integration.

## 5 Scalars on Minkowski Space

In this section we consider \( \phi^4 \) scalar field theory on Minkowski space time. We solve the Klein-Gordon equation in terms of the initial conditions \( \phi_+(\vec{x}) = \phi(\vec{x}, 0) \) and \( \phi_-(\vec{x}) = i\partial_t \phi(\vec{x}, 0) \) up to first order in \( g \) and obtain the action on the boundary by solving the \( D = 4 \phi^4 \)-action in terms of these solutions. In the case of free field theory, we show that this action is similar to the boundary action found in dS/CFT for conformally coupled scalars.

### 5.1 Free-Scalar Theory and dS/CFT Correspondence

Similar to section 2, one can show that massless scalar fields in \( d+1 \) dimensional Minkowski space-time \( M_{d+1} \) can be mapped by (5) to scalars with mass \( m^2 = \frac{d^2-1}{4} \) or the conformally coupled scalars on \( dS_{d+1} \). To verify this, one can use the following metrics for \( dS_{d+1} \) and \( M_{d+1} \) respectively:

\[
d s_{dS}^2 = \frac{1}{t^2} \left( -dt^2 + \sum_{i=1}^{d} dx_i^2 \right), \quad (46)
\]

\[
d s_{M}^2 = \left( -dt^2 + \sum_{i=1}^{d} dx_i^2 \right) \quad (47)
\]

The metric (46) covers only half of dS space. This region called \( \mathcal{O}^- \) is the region observed by an observer on the south pole \( \mathcal{I}^- \) but is behind the horizon of the observer on the north pole \( \mathcal{I}^+ \). By construction \( t > 0 \). Following the Strominger proposal [6], dual operators living on the boundary \( \mathcal{I}^- \) obey,

\[
\langle \mathcal{O}_\phi(z, \bar{z}), \mathcal{O}_\phi(v, \bar{v}) \rangle = \text{const.} \frac{1}{|z - v|^{2h_+}}, \quad (48)
\]

where

\[
h_+ = \frac{1}{2} \left( d \pm \sqrt{d^2 - 4m^2} \right) \quad (49)
\]
Again, the existence of the map (5) suggests that Eq.(48) can be obtained by solving the equations of motion of massless scalar fields on $M_{d+1}$ in terms of the initial data at $t = 0$. If yes then the final result can be interpreted as a holographic description of massless scalars in Minkowski space time. Such a description can be made covariant by considering a covariant boundary [3] instead of the hypersurface $t = 0$, which here corresponds to the $\mathcal{I}^-$ by (5).

General arguments show that a massive scalar field behaves as $t^{h_+} \phi_+$ near $\mathcal{I}^-$ [6]. Since $h_- = \frac{d-1}{2}$ for $m^2 = \frac{d^2 - 1}{4}$, using Eq.(5) one verifies that, $\phi_-(\vec{x}) = \phi(\vec{x}, t)|_{t=0}$. As will be exactly shown, $\phi_+(\vec{x}) = i \partial_t \phi(\vec{x}, t)|_{t=0}$. Two evidences for this claim are:

1. Since $h_+ = h_- + 1$ for $m^2 = \frac{d^2 - 1}{4}$, using Eq.(5) one can verify that $\partial_t \phi$ mapped to $dS_{d+1}$ behaves as $t^{h_+}$ near $\mathcal{I}^-$ as demanded.

2. A general solution of equation of motion for massless scalars in $M_{d+1}$, contains oscillating terms with both positive and negative frequencies. Therefore $\phi(\vec{x}, t)$ can only be given in terms of both $\phi_0(\vec{x}, 0)$ and $\partial_t \phi_0(\vec{x}, 0)$.

In other words the main purpose of this subsection is to derive the CFT action of the dual theory on the boundary of dS space,

$$I[\phi] = \int_{\mathcal{I}^-} d^d y d^d z \left( \frac{\phi_-(\vec{y}) \phi_-(\vec{z})}{|\vec{y} - \vec{z}|^{2h_+}} + \frac{\phi_+(\vec{y}) \phi_+(\vec{z})}{|\vec{y} - \vec{z}|^{2h_-}} \right)$$

(50)

obtained in ds/CFT correspondence, by inserting the solution of Klein-Gordon equation for massless scalar fields in $M_{d+1}$,

$$\left( \partial_t^2 - \nabla^2 \right) \phi = 0,$$

(51)

satisfying the initial conditions

$$\phi(\vec{x}, 0) = \phi_-(\vec{x}), \quad \partial_t \phi(\vec{x}, 0) = i \phi_+(\vec{x}),$$

(52)

into the action of massless scalar fields in $M_{d+1}$,

$$I[\phi] = \frac{1}{2} \int dt d^d x \left( (\partial_t \phi)^2 - (\nabla \phi)^2 \right).$$

(53)

The most general solution of Eq.(51) satisfying the initial conditions (52) is

$$\phi(\vec{x}, t) = \frac{1}{2} \int d^d y d^d k \left( \phi_-(\vec{y}) - \frac{\phi_+(\vec{y})}{\omega} \right) e^{i \vec{k}.(\vec{x} - \vec{y})} e^{-i \omega t}$$

$$+ \frac{1}{2} \int d^d y d^d k \left( \phi_-(\vec{y}) + \frac{\phi_+(\vec{y})}{\omega} \right) e^{i \vec{k}.(\vec{x} - \vec{y})} e^{i \omega t}$$

$$= \int d^d y \left( \mathcal{G}_-(x; \vec{y}) \phi_-(\vec{y}) + \mathcal{G}_+(x; \vec{y}) \phi_+(\vec{y}) \right),$$

(54)
where $\omega = |\vec{k}|$ and

$$
\mathcal{G}_-(\vec{x}, t; \vec{y}) = \int d^d k e^{i\vec{k} \cdot (\vec{x} - \vec{y})} \cos(\omega t),
$$

$$
\mathcal{G}_+(\vec{x}, t; \vec{y}) = i \int d^d k e^{i\vec{k} \cdot (\vec{x} - \vec{y})} \frac{\sin(\omega t)}{\omega}.
$$

(55)

To obtain the action (53) in terms of the solution (54), one should do calculations like:

$$
\int dtd^4 x \partial_t \mathcal{G}_-(\vec{x}, t; \vec{y}) \partial_t \mathcal{G}_-(\vec{x}, t; \vec{z}) = \int d^d k \omega^2 e^{i\vec{k} \cdot (\vec{y} - \vec{z})} \int dt \sin^2(\omega t)
$$

$$
= \int d^d k \omega^2 e^{i\vec{k} \cdot (\vec{y} - \vec{z})} \left( \frac{\pi}{\omega} \right)
$$

$$
\sim \frac{1}{|\vec{y} - \vec{z}|^{d+1}}.
$$

(56)

and

$$
\int dtd^4 x \partial_t \mathcal{G}_-(\vec{x}, t; \vec{y}) \partial_t \mathcal{G}_+(\vec{x}, t; \vec{z}) = i \int d^d k \omega e^{i\vec{k} \cdot (\vec{y} - \vec{z})} \int dt \sin(\omega t) \cos(\omega t) = 0
$$

(57)

Finally one obtains,

$$
I[\phi] \sim \int d^d y d^4 z \left( \frac{\phi_-(\vec{y})\phi_-^*(\vec{z})}{|\vec{z} - \vec{y}|^{d+1}} + \frac{\phi_+(\vec{y})\phi_+^*(\vec{z})}{|\vec{z} - \vec{y}|^{d-1}} \right).
$$

(58)

5.2 Massless $\phi^4$-Model on $M_4$

In this subsection we repeat the calculations of section 3 in the case of Minkowski space-time. As is explained before, the boundary action that we obtain here is related to the action on the boundary of dS$_4$ space corresponding to the conformally coupled $\phi^4$-model.

The solution of the Klein-Gordon equation $\Box \phi + g\phi^3 = 0$ is

$$
\phi(x) = \eta_{(1)}(x) - g \int d^4 y \ G(x; y) \eta_{(0)}^3(y) + \mathcal{O}(g^2),
$$

(59)

where

$$
\eta_{(0)}(x) = \int d^4 y \ (\mathcal{G}_-(x; \vec{y})\phi_-(\vec{y}) + \mathcal{G}_+(x; \vec{y})\phi_+(\vec{y})),
$$

(60)

and

$$
\eta_{(1)}(x) = \eta_{(0)}(x) + g \int d^4 y \left( \int d^3 z \mathcal{G}_-(x; \vec{z})G(\vec{z}, 0; y) + i\mathcal{G}_2(\vec{z}, 0; y)\partial_t G(\vec{z}, 0; y) \right) \eta_{(0)}^3(y).
$$

(61)

Similar to Eq.(27), it is easy to verify that $\phi(x) = \eta_{(0)}(x) + \mathcal{O}(g^2)$ and consequently the action is,

$$
I_g[\phi] = \int_{M_4} \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{g}{4} \phi^4 \right)
$$

$$
= \text{Const.} \int_{R^3} \left( \frac{\phi_-(\vec{y})\phi_-^*(\vec{z})}{|\vec{z} - \vec{y}|^{d+1}} + \frac{\phi_+(\vec{y})\phi_+^*(\vec{z})}{|\vec{z} - \vec{y}|^{d-1}} \right)
$$

$$
- \frac{g}{4} \int_{R^3} K_-(-\vec{x}_1, \ldots, \vec{x}_4)\phi_-(-\vec{x}_1) \cdots \phi_-(-\vec{x}_4) + (- \rightarrow +)
$$

$$
- \frac{g}{4} \int_{R^3} \tilde{K}(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4)\phi_-(-\vec{x}_1) \phi_-(\vec{x}_2) \phi_+(\vec{x}_3)\phi_+(\vec{x}_4).
$$

(62)
where Eq.(58) is used and $K_{\pm}$ and $\tilde{K}$ are defined as follows:

\[
K_{\pm}(\vec{x}_1, \cdots, \vec{x}_4) = \int d^4x \, G_{\pm}(x; \vec{x}_1) \cdots G_{\pm}(x; \vec{x}_4)
\]

\[
\tilde{K}(\vec{x}_1, \cdots, \vec{x}_4) = \frac{1}{4} \sum_p \int d^4x \, G_-(x; \vec{x}_{p_1}) G_-(x; \vec{x}_{p_2}) G_+(x; \vec{x}_{p_3}) G_+(x; \vec{x}_{p_4}).
\] (63)

## 6 Conclusion

Considering the hypersurface $t=0$ as the boundary (the holographic screen) of Minkowski (Euclidean) space, and using the conformal map between massless scalar fields in Euclidean (Minkowski) space and Euclidean AdS (the past/future region of dS) we obtained the generating function of CFT correlators of operators on the boundary of (A)dS space, recognized in (A)dS/CFT dual to free field theories, by solving the corresponding action in terms of the Cauchy data given at the boundary. In the case of four dimensional $\phi^4$-model, we derived the generating function up to first order in $g$, the coupling constant. We also obtained some exact solutions of the non-linear Klein-Gordon equation for $\phi^3$, $\phi^4$ and $\phi^6$-models in $D = 6, 4, 3$ Euclidean/Minkowski (AdS/dS) spaces.

The conformal map between (Euclidean) Minkowski and (Euclidean anti) de Sitter spaces is a useful method to examine (A)dS/CFT for various interacting theories, at least at tree-level (quantum corrections may introduce a mass scale to the theory). For example, in the case of free spinors, the same method affirmed the surface term recognized in AdS/CFT [12] to be added to the standard Dirac action in the case of dS/CFT [10]. Another attempt could be studying the $D = 4$ minimal-coupled spinor-scalar theory ($\bar{\psi}\phi\psi$-model) to obtain and verify the role of the surface term considered in [13] to study AdS/CFT for interacting spinor-scalars action in the case of de Sitter space.

## Acknowledgement

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7 Appendix A

In this appendix we briefly review free scalar field theory in \( D = d + 1 \) dimensional curved spacetime. The action for the scalar field \( \phi \) is

\[
S = \int d^D x \sqrt{|g|^{\frac{1}{2}}} \left( g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - (m^2 + \xi R) \phi^2 \right),
\]

for which the equation of motion is

\[
\left( \Box + m^2 + \xi R \right) \phi = 0, \quad \Box = |g|^{-1/2} g^{\mu \nu} \partial_\mu \partial_\nu.
\]

(With \( \hbar \) explicit, the mass \( m \) should be replaced by \( m/\hbar \).) The case with \( m = 0 \) and \( \xi = \frac{d-1}{4d} \) is referred to as conformal coupling [14].

Using Eq.(20) it is easy to show that the Ricci scalar \( R \) for \( dS_{d+1} \) space is

\[
R = \frac{d(d+1)}{2},
\]

where we have set the dS radius \( \ell = 1 \). Therefore, the action for conformally coupled scalars in \( dS_{d+1} \) is

\[
S = \int d^D x \sqrt{|g|^{\frac{1}{2}}} \left( g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{d^2 - 1}{4} \phi^2 \right),
\]

Similar result can be obtained for AdS space using Eq.(1).

8 Appendix B

In this appendix, we modify the equation of motion of the phi-fourth model in the presence of the brane located at \( t = 0 \), and give a method that can be used to calculate the solution of equation of motion to arbitrary order of perturbation in \( g \), the coupling constant. At the first order in \( g \), we show that the generating function for the correlators of dual operators on the brane is the one given in Eq.(28).

Considering a free field theory in the presence of the brane at \( t = 0 \), one can simply verify that Eq.(10) gives the correct solution to the equation of motion for \( t > 0 \). But \( \mathcal{G} \) defined in Eq.(11) not only is a solution of the equation of motion for \( t > 0 \), but also indicates the brane located at \( t = 0 \). To see this, we make an analytic continuation to complex \( k^0 \)-plane to rewrite \( \mathcal{G} \) as follows,

\[
\mathcal{G}(\vec{x}, t; \vec{y}) = \int \frac{d^d k}{(2\pi)^d} e^{-\omega_k t} e^{i\vec{k}.(\vec{x} - \vec{y})}
\]

\[
= -2 \int \frac{d^{d+1} k}{(2\pi)^{d+1}} \frac{\omega_k}{k^2} e^{i\vec{k}.(x - y)},
\]

where \( \omega_k = |\vec{k}|, k^2 = (k^0)^2 + \omega_k^2, x^0 = t \) and \( y^0 = 0 \). From Eq.(67) one verifies that

\[
\Box \mathcal{G}(x; \vec{y}) = 2\delta(t) \int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}.(\vec{x} - \vec{y})}
\]

\[
= 2\delta(t) F(|\vec{x} - \vec{y}|),
\]

(68)
in which $\Box = \partial_t^2 + \partial_x^2$ and $F(|z|)$ has been defined in Eq.(14). Furthermore,

$$\mathcal{G}(\vec{x}, 0; \vec{y}) = \delta^d(\vec{x} - \vec{y}). \quad (69)$$

Thus $\phi$ is Eq.(10) satisfies the modified equation of motion given as follows,

$$\Box \phi = 2\delta(t) \int d^d y F(|\vec{x} - \vec{y}|) \phi_0(\vec{y}). \quad (70)$$

This is the true equation of motion as it indicates the source on the barne at $t = 0$.

An identity useful to calculate the action $I[\phi]$ given in Eq.(12) is

$$\partial_t \mathcal{G}(\vec{x}, t; \vec{y})|_{t=0} = -F(|\vec{x} - \vec{y}|), \quad (71)$$

which can be used to show that,

$$I[\phi] = -\frac{1}{2} \int d^d x d^d y \phi_0(\vec{x}) F(|\vec{x} - \vec{y}|) \phi_0(\vec{y}). \quad (72)$$

The minus sign in the above equation is crucial as it makes the action $I[\phi]$ positive valued since for example for $d = 3$,

$$F(|z|) = -\frac{1}{\pi^2} |z|^{-4}. \quad (73)$$

In the case of $\phi^4$ model, the equation of motion for $t > 0$ is

$$\Box \phi + g\phi^3 = 0. \quad (74)$$

A solution of this equation with the boundary condition $\phi_0(\vec{x})$ given at $t = 0$ is Eq.(22),

$$\phi(x) = \int d^d y \mathcal{G}(x; \vec{y}) \phi_0(y) - g \int_{0\leq y^0 \leq t} d^{d+1} y G_E(x, y) \phi^3(y), \quad (75)$$

where

$$G_E(x, y) = -\int \frac{d^{d+1} k}{(2\pi)^{d+1}} \frac{e^{ik.x-y}}{k^2}. \quad (76)$$

is the Euclidean Green function given in Eq.(23). In fact from the definition (76) one can show that

$$G_E(x, y) = -\int \frac{d^d k}{(2\pi)^d} e^{ik.(\vec{x}-\vec{y})} \int \frac{dk^0}{2\pi} \frac{e^{ik^0(x^0-y^0)} - \omega_k}{(k^0)^2 + \omega_k^2}, \quad (77)$$

and

$$\int \frac{dk^0}{2\pi} \frac{e^{ik^0(x^0-y^0)}}{(k^0)^2 + \omega_k^2} = \begin{cases} 
-\frac{1}{2\omega_k} e^{-\omega_k(x^0-y^0)} & x^0 > y^0, \\
\frac{1}{2\omega_k} e^{\omega_k(x^0-y^0)} & x^0 < y^0.
\end{cases} \quad (78)$$

Eq.(69) can be used to show that $\phi$ given in Eq.(75) satisfies the boundary condition $\phi(\vec{x}, 0) = \phi_0(\vec{x})$. Furthermore, using the identity $\Box G_E = \delta^{d+1}(x - y)$, one obtains,

$$\Box \phi + g\phi^3 = 2\delta(t) \int d^d y F(|\vec{x} - \vec{y}|) \phi_0(\vec{y}), \quad (79)$$
which is the equation of motion modified by the brane at $t = 0$.

To obtain the action $I[\phi]$ we use Eq.(20). Defining

$$\eta_0(x) = \int d^d y G(x; \vec{y}) \phi_0(\vec{y}), \quad (80)$$

It is not difficult to verify that $I[\phi]$ given in Eq.(20) is equivalent to

$$I[\phi] = \frac{1}{2} \int_{R^3} \eta_0 \partial_\tau \eta_0 + \frac{g}{4} \int_{R^4} \phi^4. \quad (81)$$

In fact to obtain the above equation one should only be careful about $\delta(t)$ appearing in $\Box \phi$ and the condition $0 \leq y^0 \leq t$ in the second term in Eq.(75). Thus to the lowest order in $g$ the action functional $I[\phi]$ is given by Eq.(28). Unfortunately Eq.(27) is not correct as we will show in the following. In fact there is a first order correction term in $g$ which makes contribution to the higher order terms in $I[\phi]$ and will not change the result given in Eq.(28).

Before going to the second part of this appendix where a method to calculate $\phi$ in terms of a perturbation series in $g$ is given I do like to comment on the claim (35). It is now clear to me that the conditions on $K_E$ enumerated in section 3, can not uniquely determine $K_E$ as there are some other functions which satisfy all these conditions.

In general $\eta_k$ is given by the diagram given in Fig.(1), where $\eta'$ is defined by the identity,

$$\eta(x) = -\int d^{d+1} y G_E(x, y) \eta'(y). \quad (85)$$

I am grateful to Karl-Henning Rehren who notified me of this fact on Jan 2005.

2I am grateful to Karl-Henning Rehren who notified me of this fact on Jan 2005.
\[ \eta'_k = \eta'_m \times C_{lmn}^k. \]

Figure 1: Diagrammatic representation of Eq.(83).

Each line in Fig.(1) stands for the operation

\[ -\int' G_E, \tag{86} \]

and the combinatoric factor is

\[ C_{lmn}^k = \begin{cases} 1, & l = m = n, \\ 3, & l = m \neq n, \\ 6, & \text{otherwise.} \end{cases} \tag{87} \]

More explicitly,

\[ C_{lmn}^k = \frac{3!}{S}, \tag{88} \]

in which \( S \) is the symmetry factor. For example,

\[ C_{000}^1 = \frac{3!}{3!}, \]
\[ C_{100}^2 = \frac{(3!)^2}{2!3!} = 3. \tag{89} \]

In Fig.(2) we represent \( \eta'_k \) for \( k = 1, 2, 3 \) diagrammatically. A dashed-line stands for \( \eta_0 \).
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