Convective and absolute instability of horizontal flow in porous media

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Abstract. The onset of instability in flow systems has a dual nature depending on the dynamics of the growing normal modes. When the time evolution of a wave packet perturbation is tested, the growth of individual Fourier normal modes can be concealed to an observer in the laboratory reference frame. The reason is that the growing mode can be convected away by the basic flow, so that no effective growth is detected for the wave packet. The convective instability just focuses on the dynamics of each Fourier mode of perturbation, disregarding the actual amplitude growth of a wave packet, measured at a given position. When not only the Fourier modes, but all localised wave packets grow in time at a given position, then the instability becomes absolute. The two types of instability are generally distinct. This paper illustrates the transition from convective to absolute instability starting from a simple one-dimensional case. Then, this concept is employed for the stability analysis of a porous medium flow with heating from below. While the one-dimensional flow system is studied analytically, the porous medium flow stability is investigated numerically.

1. Introduction
It is well known that solutions of the governing equations of convection heat transfer may exist even if they cannot be observed in a laboratory experiment. The reason is the instability that can be developed when the dimensionless parameter or parameters describing the convection process belong to a specific domain. Such aspects are of paramount importance for the engineering community as the onset of convection cells may be undesirable when it comes to the optimisation and control of production processes. This, then, is what happens for the growth of crystals or the extrusion of plastics where the onset of convection cells should be inhibited. Likewise, the onset of thermal convection can be one of the causes of contamination for groundwater reservoirs. Conversely, many other practical situations exist where the thermal instability, and hence the emergence of cellular flow patterns, is a desirable condition meaning enhancement of heat transfer in a fluid system employed for a cooling process.

The initiation of instability is defined by the neutral stability threshold. Determining the neutral stability condition means testing the response of the flow system to quite peculiar perturbations: the normal modes. The normal modes are basically Fourier modes associated with a given wave number \( k \) and propagating either as plane waves, or as cylindrical waves, or as waves with a more complicated geometry depending on the system examined. Hence, the neutral stability threshold is a curve or a surface where the governing parameters of the flow vary as functions of \( k \).
The analysis of convective instability aims to establish the neutral stability threshold. The point is that a natural perturbation of a convection process is a wave packet that forms an envelope of normal modes, i.e. monochromatic waves, with infinitely many wave numbers $k$. The time evolution of perturbation wave packets depends in a non-trivial way on the growth rate of the normal modes included in the envelope. Testing the response of a convection heat transfer process to wave packet perturbations is the aim of an absolute instability analysis.

The interesting aspect of the distinction between convective and absolute instability is that there can be parametric ranges where the flow is convectively unstable, but not absolutely unstable. In these ranges, normal mode perturbations can destabilise the flow while a wave packet perturbation can not. The origin of the concept of absolute instability is usually dated back to studies in the field of plasma physics, as reported by Dysthe [1].

The fundamental idea is that the onset of instability to normal modes may be concealed to an observer in the laboratory reference frame. This happens when the basic flow is so intense as to convect downstream the growing normal modes of perturbation before they turn out to be amplified at any specific position.

The aim of the study presented here is to illustrate the criterion to establish quantitatively the emergence of absolute instability in a flow system. As a simplified arena amenable for the presentation of these technical aspects, we have chosen Burgers flow [2]. The illustration of this simple case serves to build-up the method, based on the Fourier transform, to work out the analysis of the transition from convective to absolute instability of a fluid flowing in a porous medium [3]. In particular, a numerical solution is presented for the transition from convective to absolute instability in the uniform flow driven by a horizontal pressure gradient along a horizontal porous layer bounded by impermeable planes with isoflux conditions. The adopted numerical technique is based on the shooting method coupled with a Runge-Kutta solver. This procedure allows one to manage the differential formulation of the stability dispersion relation.

### 2. Flow stability with Burgers equation

We examine the one-dimensional Burgers equation subject to a linear force,

$$\frac{\partial W}{\partial t} + W \frac{\partial W}{\partial x} = \frac{\partial^2 W}{\partial x^2} + R (W - W_0),$$

where $R \in \mathbb{R}$ and $W_0 \in \mathbb{R}$. The Burgers equation is just a toy model devised to study the one-dimensional flow of a fluid. In a paper by J. M. Burgers of 1939, entitled “Mathematical examples illustrating relations occurring in the theory of turbulent fluid motion”, an equation similar to (1) was studied in a simplified model of developing turbulence [2].

Obviously, $W = W_0$ is a stationary solution of equation (1). In this simple model, one-dimensional flow occurs along the $x$-axis. To test the linear stability of this basic solution we carry out an analysis of small perturbations.

#### 2.1. Linear stability analysis

We superpose a small perturbation to the basic solution, $W = W_0$, namely

$$W = W_0 + \varepsilon w, \quad \varepsilon > 0,$$

where $\varepsilon$ is a perturbation parameter such that $\varepsilon \ll 1$. On substituting equation (2) into equation (1), we obtain

$$\varepsilon \frac{\partial w}{\partial t} + \varepsilon W_0 \frac{\partial w}{\partial x} + \varepsilon^2 w \frac{\partial w}{\partial x} = \varepsilon \frac{\partial^2 w}{\partial x^2} + \varepsilon R w.$$
Then, by neglecting terms $O(\varepsilon^2)$, equation (3) reads
\[
\frac{\partial w}{\partial t} + W_0 \frac{\partial w}{\partial x} = \frac{\partial^2 w}{\partial x^2} + R w.
\] (4)

Equation (4) can be solved by employing the Fourier transform. Thus, we define
\[
\tilde{w}(k,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} w(x,t) e^{-ikx} \, dx,
\]
\[
w(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{w}(k,t) e^{ikx} \, dk,
\] (5)

were $k$ is the wave number. We apply the Fourier transform to equation (4) so that we obtain
\[
\frac{\partial \tilde{w}}{\partial t} = \lambda(k) \tilde{w},
\] (6)

with $\lambda(k)$ defined as
\[
\lambda(k) = R - k^2 - ik W_0.
\] (7)

Equation (7) is usually called the dispersion relation. The solution of equation (6) is
\[
\tilde{w}(k,t) = \tilde{w}(k,0) e^{\lambda(k)t}.
\] (8)

By substituting equation (8) into equation (5), we express the perturbation as
\[
w(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{w}(k,0) e^{ikx} e^{\lambda(k)t} \, dk.
\] (9)

The solution $w(x,t)$ given by equation (9) depends on the initial condition, $w(x,0)$, through its Fourier transform $\tilde{w}(k,0)$. We note that $w(x,t)$ represents a wave packet where
\[
\omega(k) = -\Im[\lambda(k)] = k W_0
\] (10)
is the angular frequency, and
\[
b(k, t) = \frac{1}{\sqrt{2\pi}} \tilde{w}(k,0) e^{\Re[\lambda(k)] t}
\] (11)
is the time-dependent amplitude of the normal mode. The single normal mode, with a given wave number $k_s$, is the evolution of an initial plane wave perturbation, namely
\[
w(x,0) = \frac{1}{\sqrt{2\pi}} e^{ik_s x}.
\] (12)

**Convective Instability**

A single normal mode perturbation with a given wave number $k$ is called convectively stable if $\Re[\lambda(k)] < 0$. It is called convectively unstable if $\Re[\lambda(k)] > 0$. The marginal condition where $\Re[\lambda(k)] = 0$ is called neutral stability condition.

On account of equation (7), convective instability happens when
\[
R > k^2,
\] (13)
with the curve given by $R = k^2$ defining neutral stability.

A resume of the concept of convective instability and the marginal condition of neutral stability is displayed in Fig. 1(a). In this figure, only the domain of positive wave numbers is represented, as the condition of convective instability just involves $k^2$ and it is thus independent of the sign of $k$. Convective instability, to some wave numbers $k$, is possible only when $R$ is larger than its critical value, denoted as $R_c$, which corresponds to the absolute minimum of $R$ along the neutral stability curve. The corresponding value of $k$ is the critical wave number, $k_c$. Thus, we have

$$k_c = 0, \quad R_c = 0.$$  \hspace{1cm} (14)

Convective instability involves quite special initial perturbations of the basic solution, having the form of plane waves with a given wave number. Such perturbations are intrinsically non-local in character. More general perturbations are given by a superposition of infinite plane waves with all possible wave numbers, as represented by the Fourier integral, equation (9). These wave packets may describe perturbations with a local support as it could be, for instance, when the initial condition $w(x, 0)$ is a Gaussian signal.

**Absolute Instability**

A perturbation $w(x, t)$ is defined as absolutely unstable if it is absolutely integrable over the real $x$-axis and if

$$\lim_{t \to +\infty} |w(x, t)| = +\infty,$$  \hspace{1cm} (15)

for every $x \in \mathbb{R}$.

One can decide whether a perturbation $w(x, t)$ expressed through equation (9) is absolutely unstable or not by checking the large-time behaviour of the Fourier integral on the right hand side of equation (9). This task can be accomplished by employing the steepest-descent approximation.
The first step is determining the saddle points of $\lambda(k)$. In fact, equation (7) yields

$$ \frac{d\lambda}{dk} = -2k - iW_0. \tag{16} $$

Equation (16) shows that there is just one, purely imaginary, saddle point,

$$ \frac{d\lambda}{dk} = 0 \implies k = k_0 = \frac{-iW_0}{2}. \tag{17} $$

One must check that the $\tilde{w}(k,t)$, expressed by equation (8), is holomorphic in the region of the complex plane $\mathbb{C}$ between the real axis and the saddle point $k_0$. We know that $\lambda(k)$ is holomorphic for every $k \in \mathbb{C}$. On the other hand, $\tilde{w}(k,0)$ is arbitrary, so that we must assume its holomorphy. If this hypothesis regarding the initial state $w(x,0)$ holds, we can approximately evaluate $|w(x,t)|$ for large times as $[4]$}

$$ |w(x,t)| \approx \frac{|\tilde{w}(k_0,0)|}{\sqrt{2\pi t}} e^{\Re[\lambda(k_0)]t}. \tag{18} $$

On account of equations (15) and (18), one can conclude that absolute instability is attained when $\Re[\lambda(k_0)] > 0$. On account of equations (7) and (17), this means

$$ R > R_a = \frac{W_0^2}{4}, \tag{19} $$

where $R_a$ denotes the threshold for the onset of absolute instability. The condition of absolute instability is independent on the details of the initial perturbation, $w(x,0)$, inasmuch as it is absolutely integrable over the real $x$-axis and its Fourier transform, $\tilde{w}(k,0)$, is holomorphic.

A qualitative sketch of the concepts of convective instability and absolute instability is displayed in Fig. 1(b). This figure highlights that absolute instability is not a modal condition, meaning that its validity does not depend on the behaviour of individual normal modes, but on the asymptotic behaviour of a general class of perturbations. Interestingly enough, the condition of absolute instability turns out to be a parametric condition, given by equation (19), mostly independent of the detailed characteristics of the initial perturbations superposed to the basic stationary solution through equation (2).

3. Mathematical model of porous flow

Let us consider a horizontal porous channel having a rectangular cross-section with height $H$. We adopt a two-dimensional description where the coordinates are chosen so that $x$ is the longitudinal horizontal axis and the $y$ axis is the transverse vertical axis (see Fig. 2). We are assuming a heating from below with a uniform heat flux at $y = 0$, while the upper boundary, $y = H$, is kept isothermal with temperature $T_0$.

The velocity field, $u = (u,v)$, and the temperature field, $T$, as well as the coordinates, $(x,y)$, and time, $t$, can be written in a dimensionless form by adopting the transformation

$$ (u,v) \frac{H}{\chi} = (u^*,v^*), \quad (T - T_0) \frac{\chi}{q_0 H} = T^*, \quad (x,y) \frac{1}{H} = (x^*,y^*), \quad t \frac{H^2}{\chi} = t^*, \tag{20} $$

where $\chi$ is the average thermal diffusivity and $\chi$ is the average thermal conductivity of the porous medium, while $q_0$ is the constant wall heat flux and $T_0$ is a constant reference temperature. In equation (20), the asterisks denote the dimensionless quantities. Hereafter, such asterisks will be omitted in order to avoid an uselessly complicated notation. This will not induce any ambiguity as all the forthcoming equations are dimensionless.
According to the Oberbeck-Boussinesq approximation and to Darcy’s law, the governing equations expressing the local balances of mass, momentum and energy can be written as

\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} + R \frac{\partial T}{\partial x} &= 0, \\
\xi \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2},
\end{align*}

where the local momentum balance equation (21b) has been expressed in its vorticity formulation. In fact, the dimensionless Darcy’s law comprehensive of the buoyancy force term can be written as

\begin{align*}
u = -\frac{\partial \Pi}{\partial x}, \quad v = RT - \frac{\partial \Pi}{\partial y},
\end{align*}

where \( \Pi \) is a suitably defined dimensionless pressure. By evaluating \( \partial u/\partial y \) and \( \partial v/\partial x \) from equation (22), one obtains equation (21b).

The parameter \( \xi \) in equation (21c) is the ratio between the average volumetric heat capacity of the saturated porous medium and the volumetric heat capacity of the fluid [5]. The Darcy-Rayleigh number \( R \) is defined as

\begin{align*}
R = \frac{\rho g \gamma_0 K H^2}{\mu^2 \chi},
\end{align*}

where \( \gamma \) is the thermal expansion coefficient of the fluid, \( g \) is the modulus of the gravitational acceleration, \( K \) is the permeability, \( \mu \) is the dynamic viscosity of the fluid, and \( \rho \) is the fluid density at the reference temperature \( T_0 \). The boundary conditions are given by

\begin{align*}
y = 0 : \quad v = 0, \quad \frac{\partial T}{\partial y} = -1, \\
y = 1 : \quad v = 0, \quad T = 0.
\end{align*}

4. Linear perturbation equations

A stationary solution of equations (21) satisfying the boundary conditions (24), which describes a parallel throughflow in the \( x \)-direction, is given by

\begin{align*}
(u_b, v_b) &= (P, 0), \quad T_b = 1 - y,
\end{align*}
where subscript “b” stands for “basic solution”, and the dimensionless parameter $P$ plays the role of the Péclet number associated with the basic horizontal throughflow. We can express small-amplitude perturbations to the basic solution as

$$(u, v) = (u_b, v_b) + \varepsilon (U, V), \quad T = T_b + \varepsilon \Theta,$$  \hspace{1cm} (26)$$

where $(U, V)$ and $\Theta$ are the velocity and temperature disturbances, respectively, while $\varepsilon$ is a perturbation amplitude such that $|\varepsilon| \ll 1$.

Taking into account that the basic solution (25) satisfies equations (21) and (24), the substitution of equation (26) into equations (21) and (24) upon neglecting terms $O(\varepsilon^2)$ yields the linearised governing equations for the perturbations. These equations can be expressed in a streamfunction formulation where

$$U = \frac{\partial \psi}{\partial y}, \quad V = -\frac{\partial \psi}{\partial x}.$$  \hspace{1cm} (27)$$

Thus, we can write

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + R \frac{\partial \Theta}{\partial x} = 0,$$  \hspace{1cm} (28a)$$

$$\xi \frac{\partial \Theta}{\partial t} + P \frac{\partial \Theta}{\partial x} + \frac{\partial \psi}{\partial x} = \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2},$$  \hspace{1cm} (28b)$$

$$y = 0 : \quad \frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \Theta}{\partial y} = 0,$$  \hspace{1cm} (28c)$$

$$y = 1 : \quad \frac{\partial \psi}{\partial x} = 0, \quad \Theta = 0.$$  \hspace{1cm} (28d)$$

Following equation (5), we apply the Fourier transform to equations (28), and express the Fourier transforms of $\psi$ and $\Theta$ as

$$\hat{\psi} = e^{\lambda(k)y} f(y), \quad \hat{\Theta} = -ik e^{\lambda(k)y} h(y).$$  \hspace{1cm} (29)$$

Therefore, the transformed equations (28) read

$$f'' - k^2 f + R k^2 h = 0,$$  \hspace{1cm} (30a)$$

$$h'' - (k^2 + \xi \lambda + ik P) h + f = 0,$$  \hspace{1cm} (30b)$$

$$y = 0 : \quad f = 0, \quad h' = 0,$$  \hspace{1cm} (30c)$$

$$y = 1 : \quad f = 0, \quad h = 0.$$  \hspace{1cm} (30d)$$

where primes denote derivatives with respect to $y$. Equations (30) are linear and homogeneous. They define an eigenvalue problem whose solution provides us with the dispersion relation, i.e. an equation analogous to (7). However, things are significantly more complicated than in equation (7), as we are unable to find an explicit analytical expression for the complex valued function $\lambda(k)$. Then, a numerical solution of equations (30) is a sensible choice in this case. This approach can be implemented by employing the LSODE subroutine for the numerical solution of ordinary differential equations [6], and the shooting method (see, for instance, chapter 19 of Straughan [7]).

The first task to be accomplished is defining the neutral stability condition, i.e. the parametric threshold for convective instability. As illustrated for the sample case of Burgers equation, this means setting $\Re(\lambda(k)) = 0$ for $k \in \mathbb{R}$. It is readily proved that $\xi \Im(\lambda(k)) + k P = 0$ so that the principle of exchange of stabilities [8, 9] holds in a suitably defined moving reference frame.
Thus, the evaluation of the neutral stability data involves a simplified version of equations (30), namely

\[ f'' - k^2 f + Rk^2 h = 0 , \quad (31a) \]
\[ h'' - k^2 h + f = 0 , \quad (31b) \]
\[ y = 0 : \quad f = 0 , \quad h' = 0 , \quad (31c) \]
\[ y = 1 : \quad f = 0 , \quad h = 0 , \quad (31d) \]

where \( k \in \mathbb{R} \). The solution of equation (31) is employed to determine the numerical function \( R(k) \), i.e. the neutral stability function. In other words, \( R \) is computed as the eigenvalue of equations (31), for every prescribed value of \( k \in \mathbb{R} \). The result of this computation is provided in Fig. 3. This figure shows that the point of minimum \( R \) along the neutral stability curve, i.e. the critical point for convective instability, is given by

\[ k_c = 2.32621 , \quad R_c = 27.0976 . \quad (32) \]

4.1. From convective to absolute instability

Absolute instability in the horizontal flow through a porous medium occurs when localised wave packets of perturbation grow unboundedly at large times \( t \), for a given \( x \). As illustrated in Section 2, the study of absolute instability relies on the steepest-descent approximation of the perturbation wave packet. Hence, the first step is the determination of the saddle-point in the complex plane, \( k = k_0 + i k_1 \), such that \( d\lambda/dk = 0 \). The threshold of absolute instability occurs when the prescribed value of \( R \) matches the condition of zero asymptotic growth, \( \Re(\lambda(k_0)) = 0 \). This special threshold value of \( R \) is denoted as \( R_a \).

The basis for the evaluation of \( R_a \) is still the eigenvalue problem (30). However, its numerical solution must be approached with the specification that \( k = k_r + i k_i \) is a complex variable with real part \( k_r \) and imaginary part \( k_i \). The fulfilment of the saddle-point condition can be automatically implemented by forcing the constraint \( d\lambda/dk = 0 \). One can impose this constraint

\[ k_c = 2.32621 , \quad R_c = 27.0976 . \quad (32) \]
by doubling the order of the differential problem (30). To this end, we define

\[ \hat{f} = \frac{\partial f}{\partial k}, \quad \hat{h} = \frac{\partial h}{\partial k}. \] (33)

Then, we obtain the extended eigenvalue problem

\[ f'' - k^2 f + R k^2 h = 0, \] (34a)
\[ h'' - (k^2 + \xi \lambda + i k P) h + f = 0, \] (34b)
\[ \hat{f}'' - k^2 \hat{f} + R k^2 \hat{h} - 2 k f + 2 R k h = 0, \] (34c)
\[ \hat{h}'' - (k^2 + \xi \lambda + i k P) \hat{h} + \hat{f} - (2 k + i P) h = 0, \] (34d)
\[ y = 0 : \quad f = 0, \quad h' = 0, \quad \hat{f} = 0, \quad \hat{h}' = 0. \] (34e)
\[ y = 1 : \quad f = 0, \quad h = 0, \quad \hat{f} = 0, \quad \hat{h} = 0. \] (34f)

If we denote by \( \lambda_r \) and \( \lambda_i \) the real part and the imaginary part of \( \lambda \), then the solution of equations (34) must be sought by keeping \( \lambda_r = 0 \). This means that we have four real values to get as an output of the solution process, namely

\[ k_r, \quad k_i, \quad \xi \lambda_i, \quad R. \] (35)

In fact, the number of boundary conditions (34e) and (34f) matches exactly the order of the ordinary differential equations (34a)–(34d). However, the homogeneous nature of the differential problem (34) implies that we are allowed to introduce scale fixing extra conditions,

\[ f'(0) = 1, \quad \hat{f}'(0) = 0. \] (36)

The first equation (36) introduces a normalisation, or scale gauge, for the eigenfunctions. The second equation (36) is just a consequence of the first one, obtained by employing equation (33).

Since the eigenfunctions \( (f, h, \hat{f}, \hat{h}) \) are complex valued, equation (36) is effectively a system of four real constraints that increments the number of boundary conditions given by equations (34e) and (34f). Thus, the extra conditions are necessary and sufficient to compute the eigenvalue tuple listed in equation (35). The numerical method to achieve this task is just the same as that described for the eigenvalue problem (31). In this case, it is more demanding in terms of coding and computational time, but nothing changes on the algorithmic side. One obtains a threshold value for absolute instability, \( R_a \), for every prescribed Péclet number, \( P \).

4.2. Discussion of the results

In order to study the onset of absolute instability, the eigenvalue search process must be initialised in a convenient way. There is, in fact, a case where the value of \( R_a \) for a given \( P \) can be easily guessed. It is the case \( P = 0 \) where one expects \( R_a = R_c = 27.0976 \). Consistently, in this case, one expects also

\[ k_r = k_c = 2.32621, \quad k_i = 0, \quad \xi \lambda_i = 0. \] (37)

These expectations are grounded on the idea that a rest state \( (P = 0) \) is one where there is no parametric gap between convective and absolute instability, namely \( R_a = R_c \). In fact, when there is no basic flow driving the perturbation downstream, the Fourier normal modes are non-travelling, so that they amplify or damp in place. This is a consequence of the principle of exchange of stabilities. In such situations, even a single Fourier mode which undergoes an
Figure 4. Migration of the saddle point in the complex $k$--plane for different values of $P$ (a); plot of $\xi \lambda_i$ versus $P$ (b)

Figure 5. Plots of $R_c$ and $R_a$ versus $P$

exponential amplification is sufficient to cause an amplification of the whole perturbation wave packet, for large times.

One can track the continuous displacement of the saddle point starting from the real axis $k_0 = 2.32621$, when $P = 0$, to the complex $k$--plane when $P > 0$. Figure 4(a) displays the migration of the saddle point in the $k$--plane as $P$ increases from 0 to 50. This figure reveals that the imaginary part of $k$ continuously decreases from 0, while the real part undergoes a non-monotonic trend. Figure 4(b) shows that the imaginary part of the growth rate $\lambda$ continuously decreases from 0 as $P$ increases.

Figure 5 illustrates the change of the threshold values, $R_c$ and $R_a$, for the onset of convective and absolute instability, respectively. The dashed horizontal line, relative to the critical value $R_c$, reflects the independence of $P$ for the threshold to convective instability. On the other hand the threshold to absolute instability depends significantly on $P$, so that the parametric gap
between convective and absolute instability gradually expands as $P$ becomes larger and larger. We mention that qualitatively similar behaviour is observed for the case where the porous layer is bounded by isothermal boundaries [10, 11].

5. Conclusions
The transition from convective to absolute instability in a flow system has been illustrated relative to a simple example involving the one-dimensional Burgers equation. Then, this concept has been employed for the stability analysis of a porous layer with horizontal through flow. A parametric gap has been found between the threshold of convective instability and that of absolute instability. The parametric gap becomes larger and larger as the Péclet number, $P$, of the basic flow increases. When $P$ tends to zero, the onset of instability has an absolute nature.

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