CRITICAL EXAMINATION OF THE ”FIELD-THEORETICAL APPROACH” TO THE NEUTRON-ANTINEUTRON OSCILLATIONS IN NUCLEI

Vladimir Kopeliovich\textsuperscript{a}\textsuperscript{*} and Irina Potashnikova\textsuperscript{b}\textsuperscript{†}

\textit{a) Institute for Nuclear Research of RAS, Moscow 117312, Russia}
\textit{b) Departamento de Física, Centro de Estudios Subatómicos, y Centro Científico - Tecnológico de Valparaíso, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile}

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Abstract

We demonstrate that so called ”infrared divergences” which have been discussed in some publications during several years, do not appear within the correct treatment of analytical properties of the transition amplitudes, in particular, of the second order pole structure of the amplitudes describing the $n \rightarrow \bar{n}$ transition in nuclei. Explicit calculation with the help of the Feynman diagram technique shows that the neutron-antineutron oscillations are strongly suppressed in the deuteron, as well as in heavier nuclei, in comparison with the oscillations in vacuum. General advantages and some difficulties of the field theoretical methods applied in nuclear theory are reminded for the particular example of the parity violating $np \rightarrow d\gamma$ capture amplitude.

\textsuperscript{*}\textit{e-mail: kopelio@inr.ru}
\textsuperscript{†}\textit{e-mail: irina.potashnikova@usm.cl}
1 Introduction

The neutron-antineutron transition induced by the baryon number violating interaction ($\Delta B = 2$) predicted within some variants of grand unified theories (GUT) has been discussed in many papers since 1970 [1], see [2] — [6]. Experimental results of searches for such transition are available, in vacuum (reactor experiments [7], and references therein), in nucleus $^{16}O$ [8] and in Fe nucleus [9], see also the PDG tables.

During the later time there have been many speculations that the neutron-antineutron oscillations in nuclei are not suppressed in comparison with the $n-\bar{n}$ transition in vacuum [10, 11]. The arguments were based on the ”true field-theoretical approach” to this problem. The result of [10] has been criticized in a number of papers [12, 13, 14, 15] which used somewhat different approaches (potential, S-matrix, diagram), and general physics arguments.

However, in view of continuing publications [11] containing same statement as in [10], it seems to be necessary to analyze this problem just within the quantum field theory based approach used in [10, 11]. Our consideration is close to the approach of paper [14] where the diagram technique has been applied to study neutron-antineutron transition in nuclei, although differs from [14] in some details. More recent realistic calculations of the neutron-antineutron transition in nuclei can be found in [16] (the diagram technique motivated consideration) and in [17] (potential approach).

In the next section the $n-\bar{n}$ oscillations in vacuum are considered and notations used in present paper are introduced. In section 3 we give some general arguments based on analytical properties of amplitudes in favour of suppression of the $n-\bar{n}$ transition in nuclei. The simplest example of the deuteron when the final result can be obtained in closed form, is considered in details in section 4, where the result of [14] for the case of the deuteron is reproduced. The analogy between analytical properties of the amplitude describing the $n-\bar{n}$ transition and the amplitude which corresponds to the nucleus formfactor at zero momentum transfer is noted in section 5. The specific difficulties of the field-theoretical methods applied to nuclear reactions are recollected for the case of the parity violating $np \rightarrow d\gamma$ amplitude in section 6. This concluding section contains also some explicit remarks on the approach of [10, 11] and on recent E-prints by V.Nazaruk.
2 The $n - \bar{n}$ transition in vacuum

To introduce notations, let us consider first the $n\bar{n}$ transition in vacuum which is described by the baryon number violating interaction (see, e.g. [2, 13, 14]) $V = \mu_{n\bar{n}}\sigma_1/2$, $\sigma_1$ being the Pauli matrix. $\mu_{n\bar{n}}$ is the parameter which has the dimension of mass, to be predicted by grand unified theories and to be defined experimentally \(^1\). As usually, a point-like $n - \bar{n}$ coupling is assumed here. The $n - \bar{n}$ state is described by the 2-component spinor $\Psi$, lower component being the starting neutron, the upper one - the appearing antineutron. The evolution equation is

$$i\frac{d\Psi}{dt} = (V_0 + V)\Psi$$

with $V_0 = m_N - i\gamma_n/2$ in the rest frame of the neutron ($m_N$ is the nucleon mass, $\gamma_n$ - the (anti)neutron normal weak interaction decay width, and we take $\gamma_{\bar{n}} = \gamma_n$, as it follows from $CP$-invariance of weak interactions). Eq. (1) has solution

$$\Psi(t) = \exp[-i(\mu_{n\bar{n}}t\sigma_1/2 + V_0t)]\Psi_0 = \left[\cos\frac{\mu_{n\bar{n}}t}{2} - i\sigma_1\sin\frac{\mu_{n\bar{n}}t}{2}\right]\exp(-iV_0t)\Psi_0,$$

(2)

Here $\Psi_0$ is the starting wave function, e.g. for the neutron in the initial state $\Psi_0 = (0, 1)^T$. In this case we have for an arbitrary time

$$\Psi(\bar{n}, t) = -i\sin\frac{\mu_{n\bar{n}}t}{2}\exp(-iV_0t), \quad \Psi(n, t) = \cos\frac{\mu_{n\bar{n}}t}{2}\exp(-iV_0t),$$

(3)

which describes oscillation $n - \bar{n}$. Evidently, for large enough observation times, $t_{obs}^\gamma \gg 1/\mu_{n\bar{n}}$, the average probabilities to observe neutron and antineutron are equal if we neglect the natural decay of the neutron (antineutron):

$$W(\bar{n}) = |\Psi(\bar{n})|^2 = |\Psi(n)|^2 = W(n) = 1/2.$$  

(4)

This case is, however, of academic interest, only, since $\gamma_n \gg \mu_{n\bar{n}}$ \(^2\). It should be stressed that in vacuum the neutron goes over into antineutron, also the

\(^1\)There is relation $\mu_{n\bar{n}} = 2\delta m$ with the parameter $\delta m$ introduced in [2]. The neutron-antineutron oscillation time in vacuum is $\tau_{n\bar{n}} = 1/\delta m = 2/\mu_{n\bar{n}}$, see also [17] and references in this paper.

\(^2\)It is a matter of simple algebra to calculate the integrals over time of the probabilities $|\Psi(n, t)|^2$ and $|\Psi(\bar{n}, t)|^2$:

$$\int_0^\infty |\Psi(n, t)|^2dt = \frac{2\gamma_n^2 + \mu_{n\bar{n}}^2}{2\gamma_n(\gamma_n^2 + \mu_{n\bar{n}}^2)}, \quad \int_0^\infty |\Psi(\bar{n}, t)|^2dt = \frac{\mu_{n\bar{n}}^2}{2\gamma_n(\gamma_n^2 + \mu_{n\bar{n}}^2)}$$

for neutron as initial state and for arbitrary, but different from zero $\gamma_n$. The difference between both quantities is obvious, and disappears when $\gamma_n \rightarrow 0$. 

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discrete localized in space state, which can go over again to the neutron, so the oscillation neutron to antineutron and back takes place.

Since the parameter $\mu_{n\bar{n}}$ is small, the expansion of $\sin$ and $\cos$ can be made in Eq. (3) at not too large times. In this case the average (over the time $t_{\text{obs}} \ll 1/\mu_{n\bar{n}}$) change of the probability of appearance of antineutron in vacuum is (for the sake of brevity we do not take into account the (anti)neutron natural instability which has obvious consequences)

$$W(\bar{n}; t_{\text{obs}})/t_{\text{obs}} = |\Psi(\bar{n}, t_{\text{obs}})|^2/t_{\text{obs}} \simeq \frac{\mu_{n\bar{n}}^2 t_{\text{obs}}}{4} \quad (5)$$

which has, obviously, dimension of the width $\Gamma$. So, in vacuum the transition $n \to \bar{n}$ is suppressed if the observation time is small, $t_{\text{obs}} \ll 1/\mu_{n\bar{n}}$. From existing data obtained with free neutrons from reactor the oscillation time is greater than $0.86 \cdot 10^8 \text{sec} \simeq 2.7 \text{ years}$ [7], therefore,

$$\mu_{n\bar{n}} < 1.5 \cdot 10^{-23} \text{ eV}, \quad (6)$$

very small quantity.

Recalculation of the quantity $\mu_{n\bar{n}}$ or $\tau_{n\bar{n}}$ from existing data on nuclei stability [8, 9] is somewhat model dependent, and different authors obtained somewhat different results, within about 1 order of magnitude, see e.g. discussion in [14, 16, 17]. Most recent results for $\mu_{n\bar{n}}$ obtained from the nuclear stability data are close to (6) [16, 17], see also the next section.

3 Analyticity based arguments for the suppression of the $n - \bar{n}$ transition in a nucleus

In the case of nuclei the $n - \bar{n}$ line with the transition amplitude $\mu_{n\bar{n}}$ is the element of any amplitude describing the nucleus decay $A \to (A - 2) + \text{mesons}$, where $(A - 2)$ denotes a nucleus or some system of baryons with baryonic number $A - 2$, see Fig. 1. The decay probability is therefore proportional to $\mu_{n\bar{n}}^2$, and we can write by dimension arguments

$$\Gamma(A \to (A - 2) + \text{mesons}) \sim \frac{\mu_{n\bar{n}}^2}{m_0}, \quad (7)$$

where $m_0$ is some energy (mass) scale. For the result of [10, 11] to be correct, the mass $m_0$ should be very small, $m_0 \sim \mu_{n\bar{n}} \sim 10^{-23} \text{ eV}$, but we shall argue
that $m_0$ is of the order of normal hadronic or nuclear scale, $m_0 \sim m_{hadr} \sim (10 - 100)\,\text{MeV}$. We can obtain the same result from the above vacuum formula (5), if we take the observation time $t_{\text{obs}} \sim 1/m_{hadr}$.

![Feynman diagram](image)

Figure 1: The Feynman diagram describing the $n - \bar{n}$ oscillation in a nucleus $A$ with subsequent annihilation of antineutron to mesons. The final state has the baryon number $A - 2$.

Indeed, the matrix element of any Feynman diagram containing such transition

$$
T(A \to (A - 2) + \text{mesons}) \sim 
$$

$$
\sim \mu_{n\bar{n}}(A - Z) \int V(A; n, (A - 1)) \frac{\bar{T}(\bar{n} + (A - 1) \to (A - 2) + \text{mesons})}{(E_n - E_0^n + i\delta)^2} dE_n \simeq 
$$

$$
\simeq -2\pi i(A - Z) \frac{d(V \bar{T})}{dE_n}(E_n = E_0^n), \quad (8)
$$

according to the Cauchy theorem known from the theory of functions of complex variable. $E_n$ is the neutron (antineutron) energy - integration variable, $E_0^n$ is the (anti)neutron on-mass-shell energy $E_0^n \simeq m_N + \vec{p}^2/2m_N$. The energy-momentum conservation should be taken into account for the vertex $V(A \to n + (A - 1))$ which includes the propagator of the $(A - 1)$ system, and for the annihilation amplitude $\bar{T}$. The case of the deuteron considered below is quite transparent and illustrative.

The amplitude $\bar{T}$ which describes the annihilation of the antineutron, and the vertex function $V$ are of normal hadronic or nuclear scale and cannot, in principle, contain a very small factors in denominator (or very large factors, of the order of $10^{15}$, in the numerator). By this reason we come to the above Eq. (7), and the resulting decay width of the nucleus is very small,

$$
\Gamma(A \to (A - 2) + \text{mesons}) < 10^{-30} \mu_{n\bar{n}}, \quad (9)
$$
at least 30 orders of magnitude smaller than the inverse time of neutron-antineutron oscillation in vacuum $\mu_{n\bar{n}}$. From Eq. (7) or (9) we obtain

$$\mu_{n\bar{n}} \sim \sqrt{\Gamma(A \rightarrow (A - 2) + \text{mesons})m_0},$$

(10)

and when one tries to get the restriction on $\mu_{n\bar{n}}$ from the data on nuclei stability [8, 9] the result is close to that from the vacuum experiment [7], somewhat smaller, within one order of magnitude [5, 6, 14]. The result of [16] based on the intuitive physical picture of $n - \bar{n}$ transition in medium, differs from that of [14] for nuclei $^{16}O$ and $Fe$, and the authors [16] come to the conclusion, that experiments with free neutrons from reactor could provide stronger restriction on the neutron-antineutron transition parameter than experiments on stability of nuclear matter ³.

According to [10, 11] the probability of the nucleus decay is proportional to

$$W(t_{\text{obs}}) \sim \mu_{n\bar{n}}^2 (t_{\text{obs}})^2$$

(the process proceeds similar to the vacuum case), where $t_{\text{obs}}$ is the large observation time, of the order of $\sim 1$ year or greater. By this reason the extracted value of $\mu_{n\bar{n}}$ is smaller than that given by Eq. (10), by about 15 orders of magnitude. Technical reason for strange result obtained in [10, 11] is the wrong interpretation of the second order pole structure of any amplitude containing the $n - \bar{n}$ transition. Instead of using the well developed Feynman diagram technique, the author [10, 11] tries to construct the space-time picture of the process by analogy with the vacuum case, which is misleading. Further discussion of papers [10, 11] and recent E-prints of this author can be found in concluding section 6.

4 The case of the deuteron

We continue our consideration with the case of the deuteron which is quite simple and instructive, and can be treated using the standard diagram technique. The point is that in this case there is no final state containing antineutron — it could be only the $p\bar{n}$ state, by the charge conservation. But this state is forbidden by energy conservation, since the deuteron mass is smaller than the

³There is, in fact some kind of competition between both methods, and final result will depend on the progress to be reached in both branches of experiments — with free neutrons and with neutrons bound in nuclei. Friedman and Gal [17] obtained the restriction $\tau_{n\bar{n}} > 3.3 \times 10^8$ sec from the latest datum on $^{16}O$ stability and using the potential approach. Experiments with ultracold neutrons in a trap have been proposed and discussed in [3, 18], but not performed till now.

⁴It has been considered in fact in [19] within the reasonable framework of the diagram technique. However, the author has drawn later wrong conclusions from this consideration.
sum of masses of the proton and antineutron. Therefore, if the \( n - \bar{n} \) transition took place within the deuteron, the final state could be only some amount of mesons. The amplitude of the process is described by the diagrams of the type

![Diagram](image)

Figure 2: The Feynman diagram describing \( n - \bar{n} \) oscillation in the deuteron with subsequent annihilation of antineutron and proton to mesons.

shown in Fig. 2 and is equal to

\[
T(d \to mesons) = i g_{dnp} m_N \mu_{n\bar{n}} \int \frac{T(\bar{n}p \to mesons)}{(p^2 - m_N^2)(\bar{d} - p)^2 - m_N^2} \frac{d^4p}{(2\pi)^4}. \tag{11}
\]

The constant \( g_{dnp} \) is normalized by the condition [20, 21, 22]

\[
g_{dnp}^2 = \frac{\kappa}{16\pi} \frac{m_N}{m_{\bar{n}}} = \sqrt{\frac{\epsilon_d}{m_N}} \simeq 0.049, \tag{12}
\]

which follows, e.g. from the deuteron charge formfactor normalization \( F_d(t = 0) = 1 \), see the next section. \( \kappa = \sqrt{\frac{m_N}{\epsilon_d}} \), \( \epsilon_d \simeq 2.22 \text{ MeV} \) being the binding energy of the deuteron. For the vertex \( d \to np \) we are writing \( 2m_N g_{dnp} \) to ensure the correct dimension of the whole amplitude, see also the next section.

The integration over internal 4-momentum \( d^4p \) in (11) can be made easily taking into account the nearest singularities in the energy \( p_0 = E \), in the non-relativistic approximation for nucleons. As we shall see right now, the integral over \( d^3p \) converges at small \( p \sim \kappa \) which corresponds to large distances, \( r \sim 1/\kappa \). By this reason the annihilation amplitude can be taken out of the integration in some average point, and we obtain the approximate equality

\[
T(d \to mesons) = g_{dnp} m_N \mu_{n\bar{n}} I_{dNN} T(\bar{n}p \to mesons) \tag{11a}
\]
with

\[ I_{dNN} = \frac{i}{(2\pi)^4} \int \frac{d^4p}{(p^2 - m_N^2)[(d - p)^2 - m_N^2]} \approx \]

\[ \approx \frac{i}{(2\pi)^4(2m)^3} \int \frac{d^4p}{[p_0 - m_N - \vec{p}^2/(2m_N) + i\delta][m_d - m_N - p_0 - \vec{p}^2/(2m_N) - i\delta]^2} = \]

\[ = \int \frac{d^3p}{(2\pi)^3 8m_N[\kappa^2 + \vec{p}^2]^2} = \frac{1}{64\pi m_N \kappa}, \quad (13) \]

This integral converges at small \( |\vec{p}| \sim \kappa \), more details can be found in the next section: the integral \( I_{dNN} \) enters also the deuteron charge formfactor at zero momentum transfer. The decay width (probability) is, by standard technique,

\[ \Gamma(d \to mesons) \simeq \mu_{\bar{n}n} \sigma_{\bar{n}n}^{ann} \int |T(\bar{n}p \to mesons)|^2 d\Phi(mesons), \quad (14) \]

\( \Phi(mesons) \) is the final states phase space. Our final result for the width of the deuteron decay into mesons is

\[ \Gamma_{d \to mesons} \simeq \frac{\mu_{\bar{n}n}^2}{16\pi \kappa} m_N^2 [v_0 \sigma_{\bar{n}n}^{ann}(\bar{n}p)]_{v_0 \to 0} \simeq \frac{\mu_{\bar{n}n}^2}{8\pi \kappa} m_N [p_{c.m.} \sigma_{\bar{n}n}^{ann}]_{p_{c.m. \to 0}}, \quad (15) \]

where \( p_{c.m.} \) is the (anti)nucleon momentum in the center of mass system. This result is very close to that obtained by L.Kondratyuk (Eq. (17) in [14]) in somewhat different way, using the induced \( \bar{n}p \) wave function.

The annihilation cross section of the antineutron with velocity \( v_0 \) on the proton at rest equals

\[ \sigma(\bar{n}p \to mesons) = \frac{1}{4m_N^2 v_0} \int |T(\bar{n}p \to mesons)|^2 d\Phi(mesons). \quad (16) \]

According to PDG at small \( v_0 \), roughly, \( [v_0 \sigma_{\bar{n}n}^{ann}]_{v_0 \to 0} \approx (50 - 55)mb \simeq (130 - 140) GeV^{-2} \). So, we obtain from (15) \( \mu_{\bar{n}n} \leq 2.5 \times 10^{-24} eV \), or \( \tau_{\bar{n}n} > 5 \times 10^8 \ sec \) if we take optimistically the same restriction for the deuteron stability as it was obtained for the \( Fe \) nucleus, \( \tau_d \approx \tau_{Fe} > 6.5 \times 10^{31} \ yr \) [9]. Our result (15) is valid up to numerical factor of the order \( \sim 1 \), since we did not consider explicitly the spin dependence of the annihilation cross section and the spin structure of

\[ \text{The result Eq. (17) in [14] can be rewritten in our notations as} \]

\[ \Gamma_{d \to mesons} \simeq 0.01 \mu_{\bar{n}n}^2 m_N^2 \kappa [v_0 \sigma_{\bar{n}n}^{ann} \bar{n}p]_{v_0 \to 0}, \quad (17') \]

which differs from our result by some numerical factor, close to 1 and not essential for our conclusions.
the incident nucleus. Same holds in fact for the results obtained in preceding papers, see e.g. [2, 14].

Additional suppression factor in comparison with the case of a free neutron is of the order of

$$\frac{\mu_{\bar{n}n}}{\kappa} \sim 10^{-31}$$

in agreement with our former rough estimate (9), and disappears, indeed, when the binding energy becomes zero. The binding energy of the deuteron should be very small, to provide the value $\kappa \sim \mu_{\bar{n}n}$, to avoid such suppression. At such vanishing binding energy the nucleons inside the deuteron are mostly outside of the range of nuclear forces, similar to the vacuum case.

Results similar to (15) can be obtained for heavier nuclei, see [5, 6, 14, 16, 17]. The physical reason of such suppression is quite transparent and has been discussed in the literature long ago (see e.g. [2, 13, 15]): it is the localization of the neutron inside the nucleus, whereas no localization takes place in the vacuum case. In the case of the deuteron or heavier nucleus the annihilation of antineutron takes place, and final state is some continuum state containing mesons. By this reason the transition of the final state back to the incident nucleus is not possible in principle, and there cannot be oscillation of the type, e.g. $d \rightarrow \text{mesons} \rightarrow d$. This is important difference from the case of the free neutron.

5 The deuteron charge formfactor

As we noted previously, the presence of the second order pole in intermediate energy variable is characteristic for the processes with the neutron - antineutron transition, but it is in fact not a new peculiarity, it takes place also for the case of the nucleus formfactor with zero momentum transfer, $F_A(q = 0)$. Let us consider as an example the deuteron charge formfactor. In the zero range approximation it can be written as

$$F_d(q) = \frac{i(2mg_{\text{dnp}})^2}{(2\pi)^4} \int \frac{d^4p}{(p^2 - m_N^2)((d - p)^2 - m_N^2)[(d - p + q)^2 - m_N^2]}.$$  \hspace{1cm} (17)

Behind the zero range approximation $g_{\text{dnp}}$ should be considered as a function of the relative $n - p$ momentum, not as a constant. For $q = 0$ second order pole

\footnote{There is no final formula. for $\Gamma_{d\rightarrow\text{mesons}}$ in [19] to be compared with our result (14), (15). Numerically, however, the result of [19] is in rough agreement with our and [14] estimates.}
appears, and we come to the expression for \( F(q = 0) \) containing the integral \( I_{dNN} \) introduced above in Eq. (13):

\[
F_d(0) = (2m_N g_{dnp})^2 I_{dNN}.
\] (18)

\[
\begin{aligned}
\gamma \\
p \\
d & g_{dnp} \\
n \\
d & g_{dnp}
\end{aligned}
\]

Figure 3: The Feynman diagram describing the deuteron charge formfactor.

In the nonrelativistic approximation, when only the nearest in energy \( E = p_0 \) singularities are taken into account, the integral over the energy has the structure

\[
I_{dNN} \sim \int \frac{dE}{(E - a + i\delta)(E - b - i\delta)^2} = \frac{-2\pi i}{(a - b)^2},
\] (19)

\( a = m_N + \vec{p}^2/2m_N, b = m_d - m_N - \vec{p}^2/2m_N, a - b = \epsilon_d + \vec{p}^2/m_N, \) and can be calculated using the lower contour which includes the pole at \( E = a - i\delta \), or the upper contour, including the second order pole at \( E = b + i\delta \), with the help of formulas known from the theory of functions of complex variables. After this we obtain

\[
F_d(q = 0) = \frac{g_{dnp}^2 m_N}{16\pi^3} \int \frac{d^3p}{(\kappa^2 + \vec{p}^2)^2} = \frac{g_{dnp}^2 m_N}{16\pi \kappa}.
\] (20)

Since \( F_d(0) = 1 \), this leads to the above mentioned normalization condition

\( g_{dnp}^2/(16\pi) = \sqrt{\epsilon_d/m_N} \).  

This relation between the constant \( g_{dnp} \) and the binding energy of the weakly bound system (deuteron in our case) is known for a long time [20, 21, 22]. It was obtained in [20, 21, 22] using different methods, dispersion relation, for

\[\text{As it is known from the nonrelativistic diagram technique, the wave function of the deuteron in momentum representation is } \Psi_d(\vec{p}) = \frac{g_{dnp}}{[4\pi^{3/2}(\kappa^2 + \vec{p}^2)]}, \text{ therefore, the normalization of the charge formfactor } F_d(0) = 1 \text{ follows from the normalization of the deuteron wave function, which is also well known from quantum mechanics.} \]
example. We shall demonstrate here for completeness, following to Landau [22], that relation (12) between the constant $g_{dnp}$ and binding energy appears from the consideration of the pole contribution to the two-particle scattering amplitude, the $np$-scattering in our case, see Fig. 4.

\[ T_{\text{pole}}^{np \rightarrow np} = \frac{(2m_N g_{dnp})^2}{s - m_d^2}, \]

(21)

where the Mandelstam variable $s = (p_n + p_p)^2$. At the threshold, $s = 4m_N^2$, we have

\[ T_{\text{pole}}^{np \rightarrow np}(s = 4m_N^2) = \frac{m_N^2 g_{dnp}^2}{\kappa^2}, \]

(22)

since at the threshold $s - m_d^2 = 4m_N \epsilon_d = 4\kappa^2$ and we assume for simplicity that both the proton and neutron masses are equal to $m_N$.

Now we should compare this result with the known quantum-mechanical expression for the scattering amplitude in the zero range approximation

\[ f(k) = \frac{1}{\kappa + ik}, \]

(23)

$k$ being the value of the nucleon 3-momentum in the center of mass frame. Using the known relation between the relativistic invariant and quantum-mechanical expressions.
scattering amplitudes, \( T(s) = 8\pi \sqrt{s} f(k) \), at the threshold (\( k = 0 \)) we obtain

\[
\frac{m_N^2 g_{dpn}^2}{\kappa^2} = \frac{16\pi m_N}{\kappa},
\]

and relation (12) follows from (24) immediately.

If the infrared divergence discussed in [10, 11] took place for the process of \( n - \bar{n} \) transition in nucleus, it would take place also for the nucleus formfactor at zero momentum transfer. But it is well known not to be the case, as we also illustrated in this section.

6 Concluding discussion and remarks

The field-theoretical description of nuclear reactions and processes is potentially useful, it allows to study some effects which is not possible, in principle, to study in other way, e.g. relativistic corrections to different observables. One should be, however, very careful to treat adequately analytical properties of contributing amplitudes.

In the case of the parity violating amplitude of the radiative capture of the low energy neutrons by protons relativistic contributions change the nonrelativistic weak interaction isospin selection rules for the parity violating observables: photon circular polarization (neutrons unpolarized) and photon asymmetry in the capture of polarized neutrons. This has been a motivation to study such relativistic contributions to parity violating observables in the \( np \rightarrow d\gamma \) - reaction [23]. In this case it was necessary to take into account contributions of all singularities (poles) of the amplitude in the complex energy plane of the virtual nucleon, not only contributions of the nearest poles in the energy variable, as it is made usually in the nonrelativistic calculations. Besides, and it is the specifics of the processes with photon emission, the contact terms should be reconstructed to ensure the gauge invariance of the whole amplitude of the photon radiation [23]. The nonrelativistic diagram technique developed up to that time turned out to be misleading for the case of physics problem considered in [23]. The cancellation between contributions of different poles has not been noted in first publications on this subject [24]. As a result of this cancellation the relativistic contributions to the observables turned out to be not greater than nonrelativistic values, in spite of the change of the isospin selection rules.
This particular example is only one of many possible illustrations of the difficulties of field-theoretical methods applied to various nuclear physics problems. It is often not so easily and straightforwardly to resolve appearing contradictions with widely known methods and results, as it happened in the case of papers [10, 11]. The author of [10, 11] tries to reconstruct the space-time picture of the process, but the correspondence of this picture to the well justified amplitude, as it appears from the Feynman diagrams, is questionable. The infrared divergence discussed in [10, 11] is an artefact of this inadequate space-time picture of the whole process of $n - \bar{n}$ transition with subsequent antineutron annihilation. A reasonable and logically consistent way would be to rewrite the amplitude which corresponds to Feynman diagrams with second order pole in energy-momentum variables (Fig.1 and 2) in space-time variables which will provide the correct space-time picture of the process, instead of writing ad hoc the amplitude in space-time variables similar to that of the process in vacuum. Another quite unrealistic consequence of this space-time picture [10, 11] is the nonexponential law of the nucleus decay. There is no ”new limit on neutron - antineutron transition” [10]; instead, one should treat correctly singularities of the transition amplitudes in the complex energy plane.

In his comment [25] Nazaruk makes the statement: ”For the propagator in the loop the infrared divergence (for $n\bar{n}$ transition, nucleus formfactor and so on) cannot be in principle. In order to obtain the infrared divergence the neutron line entering the $n\bar{n}$ transition vertex should be the wave function.” It means that the author of [25] agrees that within the Feynman diagram technique the ”infrared divergence” does not appear, but new rules seem to be proposed in [10, 11, 25] instead of well known Feynman rules. These ”new rules” should be, at least, clearly formulated, and, second, these rules should allow to reproduce all well known results of nuclear theory. In [25] there is also the statement concerning section 3 of [26] (preliminary version of present paper): ”The main statement of this section is completely wrong”. However, there are neither proof, nor scientific arguments that our results are invalid.

In recent E-prints [27] the upper bound for the free-space $n - \bar{n}$ oscillation time is extracted from existing nuclear data to be $\sim 10^{16}$ years. This is the result of the so called ”model with bare propagator” and repeats the previous statements of [10, 11]. We have just shown in present paper that calculations made in [10, 11] according to this ”model with bare propagator” are wrong. At the same time, the questions put in [26] and here, are not answered in [27].
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