Proximity effect in the asymmetrical incommensurable ferromagnet/superconductor/ferromagnet trilayer

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Abstract. In this report the original theory of a proximity effect is proposed for the ferromagnet/superconductor/ferromagnet (F\textsubscript{1}/S/F\textsubscript{2}) trilayer. The distinctions in ferromagnetic metals (F\textsubscript{1} and F\textsubscript{2}), in their thicknesses of \((d_{F1} \text{ and } d_{F2})\), in the transparencies of F\textsubscript{1}/S and S/F\textsubscript{2} interfaces, and so on are reviewed among the causes of incommensurability of system. The quasiclassical approach of Usadel equations for dirty limit case is used to find critical temperature \(T_c\). The peculiar \(T_c(d_{F1}, d_{F2})\) interference pattern is predicted for the F\textsubscript{1}/S/F\textsubscript{2} systems with disproportionate thicknesses \(d_{F1}\) and \(d_{F2}\). The possible applications to an observability of the spin-valve regime are discussed.

1. Introduction

In the last two decades after pioneer works [1, 2] the considerable success in the experimental and theoretical examination of proximity effect in the layered ferromagnetic metal/superconductor (F/S) systems which consist of the alternating layers of ferromagnetic (F) and superconducting (S) metals is reached (see reviews [3, 4] and references therein). As a rule, the multilayered systems with a high degree of symmetry (the materials, their interfaces, and boundary conditions and so on are considered with identical parameters) were explored. The finiteness of the F/S systems can lead to incommensurate F/S structures. It is linked with nonequivalence of layers which are made of identical material, but they have different local surrounding and, as a consequence, different boundary conditions. In turn the hierarchy of critical temperatures \(T_c\) may arise, as shown for the four-layered F/S/F'/S" structures [5].

In common case the transparencies of different F/S interface may be dissimilar. The different F metals or and different S metals may be used for fabrication of the layered F/S system. The layers may have different parameters: coherence lengths, thicknesses, free path length, Fermi velocities, zone parameters, exchange fields, and so on. Fauré \textit{et al.} [6] have shown in simple \textit{Cooper limit} for the F\textsubscript{1}/S/F\textsubscript{2} trilayer that the differences in the ferromagnetic layer thicknesses \((d_F)\), exchange fields \((I)\), interface transparencies, and spin-flip scattering times lead to significant modifications of phase diagrams and change (amplify or attenuate) the spin-valve effect [7]. The spin valve regime is possible for an F/S/F trilayer because the critical temperature \(T_c\) for the antiferromagnetic superconducting state (\(\pi\)-phase magnetic state, below \(\pi\)-phase),

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when the F layers magnetizations $\mathbf{M}$ are antiparallel, is higher than $T_c$ for the ferromagnetic superconducting state (0-phase magnetic state), i.e. $T_c(\pi) > T_c(0)$.

In this report the original theory of a proximity effect is proposed for asymmetrical incommensurable $F_1/S/F_2$ trilayer in the dirty limit. Unlike the work [6] we expand the theory beyond the simple Cooper limit. We also discuss possible spin-switch application.

2. Main results and discussion
To calculate $T_c$ we use our 1D theory [3] with the dirty limit conditions ($l_S \ll \xi_S \ll \xi_{S0}$, $l_F \ll a_F \ll \xi_F$) and usual relation between the energy parameters ($\varepsilon_F \gg 2I \gg T_{cS}$). $\varepsilon_F$ is the Fermi energy; $l_{S,F} = \nu_{S,F}\tau_{S,F}$ is the mean free path length for the S(F) layer; $\xi_{S,F}$ is the superconducting coherence length; $\xi_{S0}$ is the BCS coherence length; $T_{cS}$ is the critical temperature of the S material; $v_{S,F}$ is the Fermi velocity; $a_F = v_F/2I$ is the spin stiffness length.

For brevity we do not present here well known formulae [3, 4] of Usadel approach, but in common case all parameters for the $F_2$ layer may be different to the $F_1$ layer ones. They should be differ by the indexes 1 and 2. $\sigma_{S_i}(\varepsilon_{F_i})$ is the boundary transparency at the S/$F_i$ ($F_i$/S) side correspondingly ($0 \leq \sigma_{S,F} < \infty$). On each interface they satisfy the detailed balance condition: $\sigma_{F_i}/\sigma_{S_i} = v_{S_i}N_{S_i}/v_{F_i}N_{F_i} = n_i$ [3], where $N_{S(F)}$ is the Fermi level density of states. In common case we use complex diffusion coefficient $D_{F_i}(I\tau_F)$ in the F layer [3] according to discussion [8], but at $2I\tau_F \ll 1$ it is practically real and coincide with $D_F = v_FI/3$ [5, 8].

The powerful pair-breaking action of exchange field $I$ is the basic mechanism for the destruction of superconductivity in the F/S systems. We seek the solutions in the single-mode approximation [3, 9], that is enough for qualitative study of physical properties of the explored system. This permits the analytical solution of the complicated boundary value problem. The Abrikosov-Gor’kov type equation for the reduced temperature $t_c = T_c/T_{cS}$ looks like

$$\ln t_c = \Psi\left(\frac{1}{2}\right) - \text{Re} \Psi\left(\frac{1}{2} + \frac{D_S k_S^2}{4\pi T_{cS} t_c}\right),$$

where $\Psi(x)$ is the digamma function, and $k_S$ is the component of the wave vector describing spatial changes of the pair amplitudes across the layer S (along the z axis) independent of the frequency $\omega$. We have found that the pair-breaking parameter $D_S k_S^2$ can be determined from

$$\left[(D_S k_S)^2 - T_1 \times T_2\right] \tan (d_s k_S) = D_S k_S \left(T_1 + T_2\right),$$

$$D_S = \frac{v_S l_S}{3}, \quad T_i = \frac{\sigma_{S_i} v_S}{4 - \frac{\sigma_{S_i} v_S}{D_{F_i}(I\tau_F)k_{F_i}} \cot (d_{F_i} k_{F_i})}, \quad i = 1, 2. \tag{3}$$

The complex value of wave vectors $k_{F_i}$ for the 0-phase and for the $\pi$-phase are defined as

$$(k_{F_i}^0)^2 = (k_{F_i}^\pi)^2 = \mp \sqrt{\frac{2I_i}{D_{F_i}(\pm I_i \tau_{F_i})}}, \quad (k_{F_2}^0)^2 = \mp \sqrt{\frac{2I_2}{D_{F_2}(\pm I_2 \tau_{F_2})}}, \quad (k_{F_2}^\pi)^2 = \mp \sqrt{\frac{2I_2}{D_{F_2}(\mp I_2 \tau_{F_2})}}, \tag{4}$$

correspondingly. Here upper (lower) signs correspond to case when magnetization $\mathbf{M}_i$ is oriented up (down). After substitution $k_{F_i}^{(0,\pi)}$ from expressions (4) into (3) $k_S^{(0,\pi)}$ is found from (2). In turn $t_{c(0,\pi)}$ for the 0($\pi$)-phase magnetic alignment of magnetizations is obtained from the equation (4).

In the Cooper limit case ($k_S d_S \ll 1$), we obtain the main results of the paper [6]. If the layers $F_1$ and $F_2$, and interfaces $F_1/S$ and $S/F_2$ have identical parameters, the problem is reduced to the known expression for the symmetrical $F/S/F$ trilayer [3, 5, 6]. If we put $d_{F_2} = 0$ or $d_{F_1} = 0$ we also come to the known expressions for the bilayers $F_1/S$ or $S/F_2$ [3, 5]. A similar situation holds when one of the boundaries is nontransparent ($\sigma_{S2} = 0$ or $\sigma_{S1} = 0$).
 Firstly we consider the F$_1$/S/F$_2$ trilayer, in which only the thicknesses $d_{F1}$ and $d_{F2}$ may differ, other parameters of the metals and interfaces are identical. In the Figure 1 the phase diagram with the pronounced $T_c(d_{F1}, d_{F2})$ interference is presented for the F$_1$/S/F$_2$ system. The section $d_{F2} = 0$ corresponds to the weak oscillation $T_c(d_{F1})$ curve for the F$_1$/S bilayer. An addition of even thin F$_2$ layer drastically changes the phase picture. Initial weak minima of the F$_1$/S bilayer (section $d_{F2} = 0$) and the S/F$_2$ bilayer (section $d_{F1} = 0$) strongly reinforce each other in the $T_c(d_{F1}, d_{F2})$ dependence. Note, that the diagonal section $d_{F1} = d_{F2}$ corresponds to the known picture from review [3] with exotic case of periodically reentrant superconductivity. Note that only the 0-phase magnetic case is considered. Analogous $T_c(d_{F1}, d_{F2})$ phase diagram with reentrant superconductivity for the set of parameters found in [5] is presented in Figure 2. The reentrant superconductivity was theoretically predicted in [10] and recently observed [11].

The spin-valve effect was observed recently in various F/S/F systems [12]. However, the measured difference between $T_c(\pi)$ and $T_c(0)$ was small, and it did not exceed 0.05 K. It was due to not quite optimal choice of parameters of the trilayers [5, 6]. In the paper [5] one of possible optimal sets of parameters suitable to superconducting spin-switch realization was found. It is just the appearance of reentrant superconductivity in the 0-phase magnetic state can lead to prominent difference between the $\pi$- and 0-phase magnetic states of the F$_1$/S/F$_2$ and F/S/F'/S' systems that is sufficiently large to be observed [5]. Here we use real diffusion coefficient $D_{F1}$ in the F layers for this parameter set to compare present results with ones [5].

Let us consider how the possible difference and asymmetry affect the phase diagram closed to the “optimal” regime. In the Figure 3 results of corresponding calculation is shown. Only one parameter for the right F$_2$ layer differs in each panel (a–c) from symmetrical case shown in the Figure 2. The bold black lines in the Figure 3 correspond to completely symmetrical case [5]. In panel (a) the parameter $2I_1\tau_{F1}$ is changed. So, the F metals should be sufficiently dirty or (and) weak enough in regard to its magnetic properties. In the panel (b) the parameter $n_2 = v_SN_S/v_{F2}N_{F2}$ is changed. In the panel (c) the transparency parameter $\sigma_{S2}$ of the S/F$_2$
boundary is changed, whereas the transparency parameter σS1 for the S/F1 interface is retained. So, the boundaries should be sufficiently transparent. We note that not great deviations of parameters may lead to essential change of the $T_c(d_F)$ shape and significant decrease (increase) of the difference $[T_c(\pi) - T_c(0)]$! Even one can find the curves with more pronounced difference than in [5]: curves (2-2a) in panel a, and curves (3-3a) in panels b and c.

In conclusion it may be said that the theory of a proximity effect is proposed for the F1/S/F2 trilayer in the dirty limit. Critical temperature $T_c$ as function different parameters of materials (F1 and F2), layer thicknesses ($d_{F1}$ and $d_{F2}$), and parameters of interfaces (F1/S and S/F2) is found beyond the simple Cooper limit. Obtained equations go to the expressions of work [6] and the known bi- and trilayer formulae in corresponding limit cases. The peculiar $T_c(d_{F1},d_{F2})$ interference is predicted for the F1/S/F2 systems in which thicknesses $d_{F1}$ and $d_{F2}$ may be different. In reality even little excursion from normal technological regime in the fabricating procedure of layered systems may lead to appearance of nonsymmetry or incommensurability. In turn these deviation may essentially change the phase diagrams of layered system. This is especially marked in the reentrant superconductivity case. The last is very important for finding the better parameters of the spin-valve on base of the F1/S/F2 trilayer.

References
[1] Radović Z, Ledvij M, Dobrosavljević-Grujić L, A.I. Buzdin A I and Clem J R 1991 Phys. Rev. B 44 759
[2] Buzdin A I, Vujčić B and Kupriyanov M Yu 1992 Sov. Phys. JETP 74 124
[3] Izyumov Yu A, Proshin Yu N and Khusainov M G 2002 Physics - Uspekhi 45 109
[4] Buzdin A I 2005 Rev. Mod. Phys. 77 935
[5] Proshin Yu N, Zimin A, Fazleev N G and Khusainov M G 2006 Phys. Rev. B 73 184514
[6] Fauré M, Buzdin A I and Gusakova D 2007 Physica C 454 61
[7] de Gennes P G 1966 Phys. Lett. 23 10; Buzdin A I, Vedyayev A V and Ryzhanova N V 1999 Europhys. Lett. 48 686; Tagirov L R 1999 Phys. Rev. Lett. 83 2058
[8] Khusainov M G and Proshin Yu N 2003 Physics - Uspekhi 46 1311
[9] Fominov Ya V, Chchelkatchev N M and Golubov A A 2002 Phys. Rev. B 66 014507
[10] Proshin Yu N and Khusainov M G 1997 Phys. Rev. B 56 15746
[11] Zdralkov V, et al. 2006 Phys. Rev. Lett. 97 057004;
[12] Gu J Y, et al. 2002 Phys. Rev. Lett. 89 267001; Moraru I C, Pratt W P and Birge N.O. 2006 Phys. Rev. B 74 220507(R); Rusanov A Yu, Habraken S and Aarts J 2006 Phys. Rev. B 73 060505(R)