Reliability Life Estimation Considered fault Correlation of Aerial Bearing

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Abstract. The purpose of this paper is to study the reliability life calculation model of aerial rolling bearing considered the correlation of failure model. By mean of collecting relevant experiment data of vibration signal and temperature signal in the life test of the aerial rolling bearings, and analyse time domain and frequency domain, we can analyse the main failure modes and mechanism (e.g. wear, fatigue). The characteristic parameters that characterize the healthy state of aerial rolling bearings obviously are extracted further. Considering that the bearings expose multiple failure modes in the working process in most case, bearing fault diagnosis method is based on fuzzy theory. It is used to clarify the failure mechanism of different failure modes and the degradation rule of performance characteristic parameters. It’s very essential that establish the clear relationships between performance degradation and symptomatic set. According to the fault diagnosis results, each of failure modes is established the performance degradation model respectively. Considering the correlation between multiple different failure modes, Copula theory is introduced to analyse the correlation between multiple failure modes. Base on Copula theory, it’s obtained the life distribution model of aerial bearing based on multiple failure modes. Life distribution model are verified by subsequent experiments, which show that life distribution model of aerial rolling bearing in this paper, is feasible and necessary.

1. Introduction

Aerial rolling bearing is one of the most important parts in aeronautic equipment, on the other hand, Aerial rolling bearing is also vulnerable spot. Most of aircraft faults are closely related to bearings. Because of complex operating environment and diversified failure modes of aerial rolling bearing, it’s difficult to predict its reliability life accurately. In most instance that the bearing feed enough oil and kept away from salt mist, and the key failure modes of aerial rolling bearings are fatigue and wear.

Although the reliability life prediction of aerial rolling bearings under fatigue failure and wear failure have been studied by frequency and time analysis respectively⁴¹, there is few research on the reliability life prediction of aerial rolling bearing considering failure correlation between fatigue and wear⁴².

In order to achieve more accurate reliability life precision, the correlation theory of failure is introduced into bearing life evaluation in this paper creatively. Considered multi-failure modes of aerial rolling bearing, this paper has a research on key failure modes and mechanisms in
corresponding experimental condition, and by mean of Couple theory, predicts the reliability life of rolling bearings by performance degradation analysis method.

2. Math Method

2.1. Frequency and time analysis

It is assumed that the vibration acceleration amplitude of the bearing is the original signal in the time domain. Vibration signals of bearings are taken as key performance parameters. Common methods of feature extraction are time-domain analysis and frequency-domain analysis. The eigenvalue of time-domain analysis can be used as the basis for judging the severity and type of bearing fault, while frequency-domain analysis can be used as the basis for judging the location of bearing fault.

It’s obtained that the two main failure modes of bearing are fatigue and wear, and each of them can be revealed by research status and the time domain analysis results of vibration signals. By the time domain analysis of performance parameters, both of the bearing life prediction model for wear and fatigue, are able to be obtained respectively. Then, it is established that bearing life prediction model considered correlation of failure modes is based on single failure mode. The time domain analysis value which is chosen, is as follows:

- RMS (short for root-mean-square value) mainly reflects the energy of signals and is sensitive to wear faults. This is because in the process of bearing operation, impact faults have less obvious influence on RMS than wear faults. However, with the increase of impact energy, RMS will increase, so RMS is chosen as the performance parameter of wear failure mode.

- Maximum value reflects the impact force at a local fault point of bearing. The greater the impact force, the higher the peak value. Therefore, for impact vibration caused by cracks, spalling and pitting corrosion, the maximum value can more clearly reflect the health state of bearing. Maximum value is suitable for characterizing impact failure states such as surface pitting and spalling. Especially for initial surface peeling. Therefore, the maximum value is chosen as the performance parameter of fatigue failure mode.

At the beginning of frequency domain analysis, time domain single is transformed to frequency domain single by FFT transform. It’s clearly to judge the fault location by analyzing the relationship between peek frequency value and fault frequency value. The function of characteristic frequency is determined by the case of the inner ring is rotating and the outer is stationary.

2.2. Life distribution wear failure mode

2.2.1. Establishment of life degradation model for wear failure mode

For single failure mode of bearing wear, the RMS is selected as its performance parameter in this project. The trend of RMS characteristic parameters can be seen as an exponential form. Combining with the existing literature, the degradation process of RMS characteristic parameters $X_{rms}(t)$ can be described as follows:

$$Y'_1(t) = \ln(X_{rms}(t) - \phi) = \alpha_1 + \beta_1 t + \epsilon(t) = \alpha_1 + \beta_1 t + B_1(t)$$

Among them, $\phi$ is the RMS of the initial vibration signal, and it’s generally equal to zero. $\alpha_1$ is the RMS of the initial transformation, $\alpha_1 = \ln(\theta)$. $\beta_1$ is the drift coefficient, $\beta_1 = \beta - \sigma_1^2 / 2$. In order to characterize the randomness of products, both $\alpha_1$ and $\beta_1$ are assumed to be random variables, i.e., $\alpha_1 \sim N(\mu_{11}, \sigma_{11}^2)$, $\beta_1 \sim N(\mu_{12}, \sigma_{12}^2)$. $\sigma_1$ is the positive diffusion coefficient. $B_1(t)$ is the standard
Wiener stochastic process\textsuperscript{[12]}, which is used to describe the volatility and time-varying of the individual product, and it’s characterized by the followings:\textsuperscript{[13]}

- For any selected time series \( t_1 < t_2 < \ldots < t_n \), \( n \) is an arbitrary positive integer, so \( B_i(t) \) and the increments, i.e., \( B_i(t_n) - B_i(t_{n-1}) \) are independent of each other.
- For arbitrary \( i, j, t_i, t_j, i = 1, 2, \ldots, n \), incremental is shown as
  \[
  B_i(t_i) - B_i(t_j) \sim N\left(0, |t_i - t_j|\right),
  \]
  and \( \text{Cov}(B_i(t_i), B_i(t_j)) = \min\{t_i, t_j\} \).

2.2.2. Estimation of unknown parameters model

For the \( i \) th bearing, the performance characteristic values are obtained is \( y_{i1} < y_{i2} < \ldots < y_{in} \), at any selected \( n \) time \( t_{i1} < t_{i2} < \ldots < t_{in} \), that is to say, \( y_{ij} = Y_i(t_{ij}) \) performs characteristic values of the \( i \) th bearing measured at the \( j \) th test time point. It is described as follows:

\[
\sum_{i} = \sigma_{i1}^2 I_i I_i^T + \sigma_{i2}^2 t_i t_i^T + \sigma_i^2 Q_i
\]  

(2)

According to the basic properties of the Gauss process, \( y_i \) obeys normal distribution, i.e., \( y_i \sim N\left(\mu_i I_n + \mu_{2t} t_n\right) \). Among them, \( I_i \) is n dimension unit matrix, and \( I_i \) is n dimension vector whose elements are all ones. By introducing the estimated value \( \hat{\mu}_{i1}, \hat{\mu}_{i2}, \) and \( \hat{\sigma}_{i2}^2 \), the edge logarithmic likelihood function about \( \Theta = \left(\sigma_{i1}, \sigma_i\right) \) can be obtained as follows:

\[
\text{ln}(\Theta) = -\frac{N}{2} \left[\text{ln}(2\pi) + 1\right] - \frac{N}{2} \left[\text{ln}(\sigma_{i2}^2) + \frac{1}{2} \sum_{i=1}^{m} |y_i|\right]
\]  

(3)

Maximal likelihood estimators \( \hat{\sigma}_{i1} \) and \( \hat{\sigma}_{i2} \) can be obtained respectively by maximizing \( \text{ln}(\Theta) \).

In this paper, the RMS performance data are selected as input data. Using the above method of model parameter estimation, the estimated values of model parameters can be obtained.

2.2.3. Reliability life analysis of wear failure mode

\( T \) is respected the failure time of the test bearing, so the bearing reliability formula for any given failure threshold \( D_{11} \) is obtained as follows:\textsuperscript{[14]}

\[
R(t) = 1 - F_T(t/D_{11}) = \Phi \left( \left( D_{11} - \mu_{11} - \mu_{2t} t \right) \left( \sigma_{11}^2 + \sigma_{i2}^2 t + \sigma_i^2 t^2 \right)^{0.5} \right)
\]  

(4)

Among them, \( D_{11} = \mu_{11} + \mu_{2t} t - u_{pl} \sigma_i (t_{pl}) \), \( u_{pl} \) is the standard normal deviation with probability \( P \). Reliable life \( t_{pl} \) is obtained by solving equation (4).

2.3. Life distribution fatigue failure mode

2.3.1. Establishment of life degradation model for fatigue failure mode

For single failure mode of bearing fatigue, the maximum value \( X_{\text{max}} \) is selected as its performance parameter in this paper. From the variation trend of peak characteristic parameter, the degradation process can be described by equation (5) in combination with existing literature.
2.3.2. reliability life analysis of fatigue failure mode

\( T \) is respected the failure time of the test bearing, so the bearing reliability formula for any given failure threshold \( D_f \) is obtained as follows:

\[
R(t) = 1 - F(t/D_f) = \Phi\left((D_{f2} - \mu_{21} - \mu_{22}t)\left(\sigma_{21}^2 + \sigma_{22}^2t + \sigma_{22}^2t^2\right)^{-0.5}\right)
\]

Among them, \( D_{f2} = \mu_{21} + \mu_{22}t - u_{p2}\sigma_y(t_{p2}) \), \( u_{p2} \) is the standard normal deviation with probability \( P \). Reliable life \( u_{p2} \) is obtained by solving equation (6).

2.4. Life distribution mode considering failure correlation

This paper intends to use copula theory to predict the life of rolling bearing considering failure correlation between fatigue and wear. Compared with other theories for relationship representation, the Copulas theory provides a convenient to build the reliability model for multiple performance degradation characteristics. The advantages of Copulas theory are that it’s used to construct reliability function of multiple failure modes through the edge reliable function of the single failure modes directly. Multiple performance degradation function can be characterized the correlation feasibly, no matter they are the linear functions or the nonlinear functions, and no matter the forms of the function are different or not.

2.4.1. Representation of correlation

In view of the correlation between bearing fatigue and wear failure modes, in order to characterize its correlation, it can be converted to correlation between \( Y_1(t) \) and \( Y_2(t) \), that are both the characteristic parameters RMS \( X_{rms} \) and maximum value \( X_{max} \). According to Copula theory, the correlation between \( Y_1(t) \) and \( Y_2(t) \) can be expressed by the following functions:

\[
C(Y_1(t), Y_2(t), \kappa) = \kappa^{-1} \ln\left(\frac{1 - \left(\exp(\kappa Y_1(t))\right)\left(1 - \left(\exp(\kappa Y_2(t))\right)\right)}{1 - \left(\exp(-\kappa)\right)^{-1}}\right)
\]

Among them, \( C(Y_1(t), Y_2(t), \kappa) \) is the Frank Copula function, and \( \kappa \) is the correlation coefficient of multiple failure modes. According to Copula theory, correlation between failure modes can be deduced as follows:

\[
\rho = 1 + 4 \left(D(\kappa) - 1\right) \kappa^{-1}
\]

\[
D(\kappa) = \left(\kappa \int_0^{\kappa} u \left(e^u - 1\right)^{-1} du\right)^{-1}
\]

2.4.2. Estimation of correlation

In order to calculate the correlation between the two failure modes, it is necessary to estimate the correlation coefficients \( \kappa \) between the characteristic parameter \( Y_1(t) \) and \( Y_2(t) \) in the correlation function. In order to improve the estimation efficiency, this project uses the maximum likelihood estimation theory to estimate the correlation coefficient \( \kappa \), based on the independent incremental
characteristics of performance characteristic parameters. The logarithmic likelihood function for correlation coefficient $\kappa$ can be defined as follows:

$$l(\kappa) = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \ln C \left( F_{Y_1}(\Delta y_{ij}), F_{Y_2}(\Delta y_{2ij}), \kappa \right) + \sum_{k=1}^{2} \sum_{i=1}^{m} \sum_{j=1}^{n_i} f_{X_{ij}}(\Delta x_{ij})$$

(10)

Order $\partial l(\kappa)/\partial \kappa = 0$, according to the multi-dimensional search algorithms, the parameter estimation can be obtained by solving the equation.

2.4.3. Reliability analysis considering failure correlation

According to Copula theory, the reliability function of bearing considering failure correlation between fatigue and wear can be deduced as follows:

$$R(t) = R_1(t) + R_2(t) - 1 + C \left( R_1(t), R_2(t), \kappa \right)$$

(11)

3. Experiment and result

3.1. Test specimens

In order to avoid interference of other fault models, the life experimental bearing is used for aeroplane motor. The bearing type for test is 16004/P6. Every two test bearings and two assistant bearings are in a group, which are worn together on a test shaft. There are twenty sets of test bearings in life experiment, which are used for failure mode and mechanism analysis, and they are divided into ten groups equally.

3.2. Result

3.2.1. The time to fault in the life experiment

The results and failure reasons of life experiment are shown in Table 1.

| No. of group | No. of test bearing | Total running time | Termination condition | Failure form               |
|--------------|---------------------|--------------------|----------------------|----------------------------|
| 1            | 1-2                 | $L_1$              | exceeds the threshold| Fatigue inner ring of No.1 |
| 2            | 3-4                 | $L_4$              | exceeds the threshold| Fatigue inner ring of No.4 |
| 3            | 5-6                 | $L_5$              | exceeds the threshold| Fatigue inner ring of No.5 |
| 4            | 7-8                 | $L_7$              | exceeds the threshold| Fatigue inner ring of No.7 |
| 5            | 9-10                | $L_{10}$           | exceeds the threshold| Fatigue inner ring of No.10|
| 6            | 11-12               | $L_{11}$           | exceeds the threshold| Fatigue inner ring of No.11|
| 7            | 13-14               | $L_{14}$           | exceeds the threshold| Fatigue inner ring of No.14|
| 8            | 15-16               | $L_{15}$           | exceeds the threshold| Fatigue inner ring of No.15|
| 9            | 17-18               | $L_{18}$           | exceeds the threshold| Fatigue inner ring of No.18|
| 10           | 19-20               | $L_{19}$           | exceeds the threshold| Sliding inner ring of No.20|

3.2.2. Result of radial internal clearance

Before the life experiment, the radial clearance of the test bearing is ranged from 15 to 20 μm. However, the minimum value of radial clearance is 17 μm, and the maximum value change to 26 μm after the life experiment. It’s identifiable that wear is included in the fault model of test bearing during the life experiment.
3.2.3. Interface morphology

Disassemble the bearing and check the main interfaces of the ten failed bearings were examined, and the result shows that fatigue is included in the fault model of test bearing. The results were obtained as following:

- There are peeling pits on the inner ring of No.1, 4, 5, 7, 10, 11, 14 and 15 bearing obviously, as it’s shown in figure 1;
- There are several peeling pits on the steel balls of No. 18 bearing, as it’s shown in figure 2;
- There are several peeling pits on the outer ring of No. 19 bearing, and there are skid marks on the inner ring of No. 20 bearing.

3.2.4. Time domain analysis of vibration signal

The sampling frequency of the test bearing is 20 kHz, and life experiment keeps carrying on until one of in bearings group reaches the failure threshold. The data of life experiment for rolling bearings are intercepted to calculate RMS and maximum value every one hour, which are both dimensionless. The time domain analysis results, that is to say, RMS and maximum value of No.1 bearing are shown in figure 3 and figure 4 respectively.

3.2.5. Frequency domain analysis of vibration signal
In this paper, wavelet transform is adopted to carry out frequency domain analysis of vibration signals\cite{17}. The results of frequency domain analysis for No. 2 and No. 19 bearings are shown respectively:

- There are peaks near the vibration amplitude at 989 Hz, 1829 Hz and 2946 Hz, in the figure 5, and the three points correspond to the inner ring fault characteristic frequency 981.1Hz, the double frequency 1962.2Hz and triple frequency 2943.3Hz, which is consistent with the results of disassembly analysis for test bearings;
- There are peaks near the vibration amplitude at 680 Hz and 1372 Hz the figure 6, and it’s consistent with disassembly analysis, that the characteristic frequency of outer ring fault is 685.5 Hz and the double frequency is 1371 Hz.

![Figure 5. Frequency domain analysis for No. 2 bearing.](image1)

![Figure 6. Frequency domain analysis for No. 19 bearing.](image2)

4. Analysis

4.1. Life distribution for Single failure mode

4.1.1. Life distribution for wear failure mode

The RMS of performance data is selected as input, which is shown in figure 7. According to the parameter estimation method, it’s obtained that the parameter estimation values of the performance degradation model for wear failure mode as follow:

\[
\mu_1 = -21.65, \quad \sigma_1 = 1.74 \times 10^{-5}, \quad \mu_2 = 0.0875, \quad \sigma_2 = 8.92 \times 10^{-4}, \quad \sigma_1 = 0.4461.
\]

Then, Combine the estimates of parameter into the wear degeneration failure modes, and the reliability of bearing wear failure modes can be obtained by introducing equation (4), which is shown in figure 8. According to equation (4), the life prediction results of bearing wear failure modes can be obtained as follows: \( t_{0.9} = 226.1 \text{h}, \quad t_{0.5} = 229.1 \text{h}. \)
4.1.2. Life distribution for fatigue failure mode

The maximum value of performance data is selected as input, which is shown in figure 9. According to the parameter estimation method, it is obtained that the parameter estimation values of the bearing performance degradation model for fatigue failure mode as follows:

\[ \mu_{21} = -35.47 \, \mu \]
\[ \sigma_{21} = 16.06 \times 10^3 \, \mu \]
\[ \mu_{22} = 0.1545 \, \mu \]
\[ \sigma_{22} = 5.25 \times 10^3 \, \sigma \]
\[ \sigma_2 = 0.9454 \, \sigma \]

Then, combine the estimates of parameter into the fatigue degeneration failure modes, and the reliability of bearing degradation model for fatigue failure mode can be obtained by introducing equation (6), which is shown in figure 10. The life prediction results of bearing fatigue failure modes can be obtained as follows: \( t_{0.9} = 222.4 \, \text{h}, t_{0.5} = 232.1 \, \text{h} \).

4.2. Life distribution mode considering failure correlation

The RMS and Max performance data are selected as input. According to the above parameter estimation method, the estimated values of correlation coefficients of performance parameters of two failure modes of bearings can be obtained: \( \kappa = 38.43 \). According to equation (8), the correlation between the two failure modes can be obtained: \( \rho = 0.9 \).
The reliability of bearing considering failure correlation (fatigue and wear) can be obtained by introducing the parameter estimation and correlation coefficient estimation $\kappa=38.43$ of single failure mode degradation model of wear and fatigue into equation (11), and compared with the reliability of single failure mode, as shown in figure 11. The life prediction results of bearing wear failure modes can be obtained as follows: $t_{0.9}=222.4\,\text{h}$, $t_{0.5}=229\,\text{h}$.

It’s clearly shown in figure 11 that the reliability life curve of considering failure correlation is the same as the curve of single wear failure mode before 227 hours, however, the reliability life curve of considering failure correlation is the same as the curve of single fatigue failure mode after 227 hours. It’s clearly shown that, at the front part of life experiment, wear is more important than fatigue between the two main fault mode; and at the second part of life experiment, fatigue is more important than wear between the two main fault mode. It is consistent with the results of disassembly analysis for test bearings, that there are several failure spalling on the interface of test bearings.

![Figure 11. Reliability of bearing considering failure mode dependence.](image)

There is some interference in the signal acquisition process, at the same time, it’s difficult to distinguish between wear and fatigue, when life experiment runs uninterruptedly. It’s lack of correlation data for verifying the accuracy of the model accurately, which is mainly shortcoming of this paper. Maybe it’s possible to verify the life calculation model considering failure correlation by carrying out estimate the remaining life.

5. Conclusions

In this paper, based on acquisition and processing analysis for vibration signal acquisition and processing in the life experiment, the characteristic signal for characterizing the fault time and fault modes is obtained by time domain analysis and frequency domain analysis for the vibration signal. Based on the common fault mechanism theory of rolling bearing, it’s analysed the variation rules of performance degradation for wear and fatigue respectively. The Couple theory method was introduced to characterize the relationship between wear failure and fatigue failure. Under the condition of considering the failure correlation, it’s obtained that life distribution mode of the rolling bearing considering failure correlation further. The main conclusions that can be drawn from the present work are followed:

- According to life calculation wear failure mode for bearing, the life prediction results are described as follows: $t_{0.9}=226.1\,\text{h}$, $t_{0.5}=229.1\,\text{h}$;
- According to life calculation fatigue failure mode for bearing, the life prediction results are described as follows: $t_{0.9}=222.4\,\text{h}$, $t_{0.5}=232.1\,\text{h}$;
- According to life calculation mode considering failure correlation for bearing, the life prediction results are described as follows: $t_{0.9}=222.4\,\text{h}$, $t_{0.5}=229\,\text{h}$, and correlation coefficients $\rho=0.9$;
• At the front part of life experiment, wear is more important than fatigue between the two main fault mode. At the second part of life experiment, fatigue is more important than wear between the two main fault mode.

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