Chaotic Analysis of Fractional-Order Permanent Magnet Synchronous Generator for Wind Turbine

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Abstract. Fractional-order wind turbine is a strongly coupled non-linear dynamic system. It mainly studies the significant chaos characteristics such as the complex chaotic motion with fractional order varying. According to the mathematical model of the system, the fractional order Lorenz chaotic equation is established by linear affine transformation and time scale transformation. The theory of Lyapunov stability analysis is adopted to deeply study the development process of the system from stable operation to chaotic motion. The correctness of the chaos characteristics of the system is verified.

Keywords: Fractional-order Wind Turbine, Chaos Characteristics, Lyapunov Stability, Nonlinearity

1 Introduction

Wind resource, as a clean and green energy, has attracted increasing attention from countries around the world, and wind power generation technology has also developed rapidly. Due to the elimination of gearboxes, Permanent Magnet Synchronous Generator (PMSG) has many advantages of good reliability, low maintenance and flexible control of the power grid. Then, PMSG has become the main development model of offshore wind farms [1]. However, under certain parameter ranges and working conditions, PMSG will have gap oscillations in speed, torque and output power, the control performance is unstable, and it will seriously affect the grid-connected operation of the wind turbine. Chaos control theory [2] has been widely used in many fields such as biology, economics, digital encryption, secure communication, and motor systems since its birth [3]. Chaos control is very sensitive to the factors such as initial conditions, parameter changes, and environmental noise, which makes it very suitable for studying the motion characteristics of Permanent Magnet Synchronous motor (PMSM). Hemati [4], Zhang Bo [5] et al have done a lot of research on the bifurcation and chaos characteristics of PMSM, but studies on the chaos characteristics of PMSG wind turbine system...
are relatively few [6, 7]. Literature [8] found the existence of chaos in direct-drive PMSG (D-PMSG) wind turbine, and designed the controller to suppress chaotic motion. Yu W and Zheng W J et Al. [9] obtained the fractional-order PMSM model by experimental data and identification algorithm, and established the fractional order model in time domain and frequency domain. The chaotic characteristics of the fractional-order PMSM are analyzed by means of three-phase diagram, Lyapunov exponent spectrum and Poincare map. But the analysis of the chaos characteristics for fractional order PMSG wind turbine are relatively few.

To get into study the chaotic motion of fractional order Direct drive PMSG(D-PMSG), firstly, this paper applies affine transformation and time transformation to convert its mathematical model into Lorenz chaotic model, then the fractional-order Lyapunov stability theory is adopted to analyze chaotic characteristics of fractional-order D-PMSG. Time domain diagrams, three-phase diagrams verify the chaotic characteristics of fractional-order D-PMSG.

2 Mathematical Model of Fractional-Order D-PMSG

2.1 Wind Turbine Model

As you can see from the aerodynamics mechanism of wind turbine, the power obtained by wind turbine blade is as follows [6]:

\[ T_r = P_r / \omega_r = 0.5 \rho AC (\lambda, \beta) v^3 / \omega_r \]  

(1)

Wherein, \( T_r \) is the output power of the wind wheel; \( \rho \) is the air density; \( A=\pi R^2 \) is the swept area of the wind turbine blade; \( \lambda = \omega_0 r / v \) is the blade tip speed ratio; \( \beta \) is the pitch angle; \( \omega_r \) is the wind wheel angular velocity; \( C_p (\lambda, \beta) \) is the non-linear function of the wind wheel power coefficients \( \lambda \) and \( \beta \); \( r \) is the wind wheel radius; \( v \) is the wind velocity; \( T_r \) is the aerodynamic torque of the wind turbine.

2.2 Mathematical Model of D-PMSG

Assuming that there is the motor core being unsaturated, each winding inductance being linear, eddy current and hysteresis losses being neglected and the counter electromotive force of the motor being sinusoidal, there is no damping winding action on the rotator, and the permanent magnet has no damping action. Taking \( i_d \), \( i_q \), \( \omega_b \) as the state variable, the model of PMSG with uniform air gap (\( L_d = L_q = L \)) is obtained:

\[
\begin{align*}
\dot{i}_d &= -R_s i_d / L + n_s i_q \omega_b + u_d / L \\
\dot{i}_q &= -R_s i_q / L - n_s i_d \omega_b - n_p u_d / L + u_q / L \\
\dot{\omega}_b &= (1.5 n_p \Psi d i_d - B_m i_q - T_0) / J_m 
\end{align*} \]  

(2)

Wherein, \( u_d, u_q \) are the motor d-axis and q-axis terminal voltage respectively; \( i_d, i_q \) are the motor d-axis and q-axis stator current; \( R_s \) is the stator winding resistance; \( n_p \) is the number of pole pairs of the motor; \( \omega_b \) is the angular velocity of the generator rotor; \( \omega_b = \omega_r \); \( J_m \) is the equivalent moment of inertia of wind turbine; \( \Psi d \) is the flux linkage in the permanent magnet; \( B_m \) is the rotational viscosity coefficient of the generator; \( T_L \) is the load torque and \( T_r = T_L \).

Applying linear affine transformation and time scale transformation, i.e. \( x = \xi \tilde{x}, t = \tau \tilde{t} \), the Lorenz chaotic model of D-PMSG can be constructed as follows:

\[
\begin{align*}
\dot{i}_d &= -\tau^{-1} i_d + \tau \tilde{\omega}_b + \tilde{u}_d \\
\dot{i}_q &= -\tau^{-1} i_q - \gamma \tilde{\omega}_b + \tilde{u}_q \\
\dot{\tilde{\omega}}_b &= \sigma (\tilde{i}_q - \tilde{i}_d) - \tilde{\omega}_b 
\end{align*} \]  

(3)
Wherein, $\xi_1 = \xi_2 = B_n/(\tau_1 \nu_1 \nu_2^2)$, $\xi_3 = 1/(\tau_2 \nu_3)$, $\tau = L/R$, $\gamma = -(3 \tau_2 \nu_3 \nu_4^2)/(2 B_n L)$, $\sigma = (\tau B_n)/J_m$, $\sigma$ and $\gamma$ are determined by system parameters.

\[ \ddot{u}_x = (\tau u_x)/(\xi_3 L), \quad \dddot{u}_y = (\tau u_y)/(\xi_3 L), \quad \dddot{r}_e = (\tau L)/(\xi_3 J_m). \]

3 Chaos Characteristics of D-PMSG

Definition 1 [10]: If there is any positive real number $\alpha > 0$, the lower limit of the differential is assumed to be $t_0$, the upper limit is assumed to be $t$, if the integral time step is $h \to 0$, then the Caputo fractional differential for the function $f(t)$ is as follows:

\[ D^\alpha_t f(t) = \frac{d^n}{dt^n} \int_0^t \frac{1}{\Gamma(n-\alpha)} (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau \]

(4)

Wherein, $n-1 \leq \alpha \leq n$, $\alpha \in \mathbb{R}$, $n \in \mathbb{N}$, $\Gamma(\cdot)$ is Gamma function.

Definition 2 [11]: Considering fractional order non-linear dynamical system $Dt \chi(t) = f(t, x)$, if $x$ satisfies $f(t, x) = Dt \chi x$, then $x$ is referred to the equilibrium point for fractional order systems.

In general, $x = 0$ is assumed to be the equilibrium point of the system.

Definition 3 [11]: For an autonomous fractional differential system

\[ D^\alpha_t x(t) = f(t, x) \]

(5)

Wherein, $a_i \in (0,1)$, $D^\alpha_t = [D^{a_1}, D^{a_2}, \ldots, D^{a_n}]$, and $f(\cdot) \in \mathbb{R}^n$. Supposing $a_i = p_i/q_i$ and $N$ being the least common multiple of all $q_i$, if $x$ satisfies $f(t, x) = D^\alpha_t x$, then $x$ is the equilibrium point for the Equation (5).

Lemma 1 [12]: The necessary conditions for the existence of chaos in fractional order systems are follows: For a fractional order system of the same magnitude with $a_1 = a_2 = \ldots = a_n = a$, then Jacobian matrix of the eigenvalue for $J = \partial f/\partial x|x = \lambda i(i = 1, 2, \ldots, n)$, and meeting $\min\{|\arg(\lambda_i)|\} \leq \alpha \pi/2$.

If the equilibrium point of fractional order system does not satisfy the phase relation of Lemma 1, then the equilibrium point is asymptotically stable.

The dynamic effect caused by the internal damping of direct drive wind turbine is described by using fractional calculus. Assuming that the state variable is $[\ddot{\bar{s}}_u, \ddot{\bar{s}}_v, \dot{\bar{s}}_3]^T = [\bar{s}_u, \bar{s}_v, \bar{s}_3]^T$ and the external input is $[\ddot{\bar{s}}_u, \ddot{\bar{s}}_v, \dot{\bar{s}}_3]^T = [u_1, u_2, u_3]^T$, the chaos model of fractional direct drive wind turbine can be established by combining equation (3).

\[ \begin{align*}
D^\alpha_t \bar{s}_u &= -\bar{s}_u + \bar{s}_v + u_1 \\
D^\alpha_t \bar{s}_v &= -\bar{s}_v - \bar{s}_3 + \gamma \bar{s}_3 + u_2 \\
D^\alpha_t \bar{s}_3 &= \sigma(\bar{s}_3 - \bar{s}_v) - u_3
\end{align*} \]

(6)

The parameter conditions can be taken as $\sigma = 16$, $\gamma = 20$, $u_1 = -0.542$, $u_2 = 0.824$, $u_3 = 1.16$. The three equilibrium points of the system are $E01 = (45.0319, 6.7872, 6.7147), E02 = (44.8090, -6.6982, -6.7707), E03 = (-0.5429, 0.0559, -0.0165)$. Then the eigenvalues of the Jacobian matrix at the three equilibrium points are $(-18.7297, 0.3648 + 8.7983i), (-18.854, 0.4297 + 8.78i), (-36.7782, -1.0001, 19.7782)$. Since each equilibrium point has a positive real part eigenvalue, the system (18) will produce attractors around the equilibrium point, showing chaos.

4 Simulation Analysis

An important characteristic of chaotic motion is the emergence of chaotic limit cycles. Based on the above theoretical research, the time domain diagram of state operation and the three-phase diagram of $\ddot{i}_q, \ddot{i}_q, \dot{\omega}_q$ are drawn, showing the development process of the system from stability to limit cycle to chaos, which has important theoretical significance for the actual motion analysis of D-PMSG.
From the definition 1 and lemma 1, we can get that there are \( \min \{|\text{arg}(\lambda_i)|\} = 0.9736, 0.9689 \) for the equilibrium point E01 and E02. When the following inequality (19) holds, the system (18) will show chaotic motion.

\[
\alpha \geq \max \left\{ \frac{2}{\pi} \min \left[ \left| \text{arg} \left( \lambda_i \right) \right| \right] \right\} = 0.9736 \approx 0.97
\]  

(7)

When \( \alpha < 0.9689 \), the system is stable. Fig. 1-3 show that under the initial value being (20, -5, 1), the state time domain diagram and three-phase diagram of fractional order D-PMSG when \( \alpha = 1, \alpha = 0.97, \) and \( \alpha = 0.96 \) respectively. It can be seen from Fig. 3, when \( \alpha = 0.96 \), the system is in quasi periodic stable state. It can be seen from Fig. 2 that when \( \alpha = 0.97 \), the system has chaotic attractor and presents chaotic motion. Fig. 1-3 are the motion states of the system when \( \alpha = 1, \alpha = 0.97 \) and \( \alpha = 0.96 \) respectively. It is concluded that when \( \alpha \in (0.96, 1] \), the system will present chaos. According to the analysis of chaotic characteristics in Fig. 1-3, not only the system parameters and external interference will affect the operation characteristics of D-PMSG, but also the fractional order varying will influence the operation state of D-PMSG. Compared with the integer-order model, the fractional-order D-PMSG is more suitable for practical engineering application.

![Fig 1](image1.png)

**Fig 1.** When \( \alpha = 1 \), system state time domain diagram and three-phase diagram

![Fig 2](image2.png)

**Fig 2.** When \( \alpha = 0.97 \), system state time domain diagram and three-phase diagram
Fig 3. When $\alpha = 0.96$, system state time domain diagram and three-phase diagram

5 Conclusions
Based on the chaotic model of fractional-order D-PMSG, applying fractional Lyapunov stability theory, the chaotic characteristics of fractional order D-PMSG with fractional order change are studied. The simulation results show that when fractional order change, the system is from stable operation state to chaotic operation state, which confirms the existence of D-PMSG chaotic attractor under certain conditions. For the actual wind power system, its safe and stable operation, parameter uncertainty and external interference, as well as fractional order characteristics are more complex, which makes preparations for the next study of chaotic motion and chaos control of fractional order direct drive wind power system.

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