The multi-period multi-level capacitated lot-sizing and scheduling problem in the dairy soft-drink industry

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ABSTRACT
Lot-sizing and scheduling are a well-known problem. However, there are only few works addressing the problem in the food industry. We propose a mixed-integer-programming model for the lot-sizing and scheduling problem in a dairy production process. The latter is composed of standardisation tanks that can store different types of flavours and ultra-high temperature (UHT) lines. The model considers limited shelf life of liquid flavours in standardisation tanks and production bottleneck that can alternate between stages. Ten representative instances using real data are addressed with an exact method and a relax-and-fix heuristic. The results show that the assignment strategies increase the production costs. Moreover, these assignment strategies do not only reveal the total production capacity related to some industrial settings but also show the limit at which the bottleneck capacity of a given setting moves from the UHT line to the tanks.

Introduction
In the management of industrial systems, decisions related to production planning are closely linked to lot sizing and scheduling. The issues of lot sizing are mostly linked to the necessary configurations between batches and changeovers. Reducing the configurations by producing large quantities to satisfy the demand leads to the increase in storage costs. Therefore, production planning determines the periods in which production should take place and the quantities needed to satisfy the demand, while reducing production costs and stocks. In this regard, the production planning and scheduling problems can be represented and solved by lot-sizing and scheduling models (Toscano, Ferreira, and Morabito 2017). In this paper, we focus on lot sizing and scheduling in the food industry and, more specifically, on realistic industrial settings of a dairy soft-drink production company. The industrial problems are often characterised by a large number of constraints, limited production shelf life, variable demand and non-storable intermediate items (Ferreira, Morabito, and Rangel 2009; Ferreira et al. 2012; Toledo et al. 2015; Baldo et al. 2017). In contrast with earlier works, the lot-sizing and scheduling problem studied here considers the synchronisation between production stages and shelf life.

The remainder of the paper is organised as follows. Section 2 provides a literature review. Section 3 presents the studied problem in the dairy soft-drink production facility. Section 4 gives a mathematical model for this problem. Section 5 highlights the solution approaches for solving the model, including a relax-and-fix algorithm and an exact method. Section 6 presents some managerial insights and the numerical experiments. Section 7 concludes this study and proposes future research directions.

Literature review
Production management of short-life-cycle products, and particularly food products, faces specific challenges and risks (Voldrich, Fcplx, and Zufferey 2017; Berbain, Bourbonnais, and Vallin 2011). In this context, there has been an increasing interest in lot-sizing and scheduling problems (Copil et al. 2017; Toscano, Ferreira, and Morabito 2017) in different fields, such as wafer fabrication (Chen, Sarin, and Peake 2010), poultry industry (Boonme et al. 2016), paper industry (Furlan et al. 2015), glass container industry (Toledo et al. 2016), foundry companies (Camargo, Mattioli, and Toledo 2012), electro-fused grains (Luche, Morabito, and Pureza 2009), process industry (Tranchel et al. 2011), and animal feed production (Clark, Morabito, and Tos 2010). The multi-level lot-sizing problem consists of determining the production plan that supplies the demand for final items and/or their components without exceeding the available production capacity (Furlan et al. 2015). The objective is to minimise the setup, holding and changeover costs under the demand specified in each period of a finite
planning horizon. This problem has been extensively studied in the literature (Pochet and Wolsey 2006; Jans and Degraeve 2007). Diaz-Madronero, Mula, and Peidro (2014) highlighted several characteristics of tactical production planning models, whereas Copil et al. (2017) presented a review on simultaneous lot-sizing and scheduling problems. According to the same authors, models based on micro-periods (with at most one setup per period) and models based on macro-periods (with any number of setups) can be distinguished. There exist different models based on micro-periods such as the discrete lot-sizing and scheduling problem (DLSP) (Fleischmann 1990), the continuous setup lot-sizing problem (CSLP) (Karmarkar and Schrage 1985) and the proportional lot-sizing and scheduling problem (PLSP) (Drexel and Haase 1995). Macro-periods are used in the formulations of the general lot-sizing and scheduling problem (GLSP) (Fleischmann and Herbert 1997) and the capacitated lot-sizing problem with sequence-dependent setups (CLSD) (Haase 1996). All these models support lot-sizing as well as scheduling decisions with the objective to minimise the sum of setup and holding costs under consideration of due dates and finite capacities.

Different formulations are proposed to the lot-sizing and scheduling problem in the food industry, taking into consideration the product shelf-life issues (Alipour et al. 2020; Lütke Entrup et al. 2005; Amorim, Antunes, and Almada-Lobo 2011). Ferreira, Morabito, and Rangel (2009) offer a simplified MIP model integrating production, lot-sizing and scheduling decisions with sequence-dependent setup times and costs in two industrial process stages with the same number of elements, and where synchronisation is required along with the alternation of the production bottleneck. To solve the problem, they proposed different solution approaches, such as a relaxation approach and some strategies of the relax-and-fix heuristic.

Ferreira et al. (2012) and Toscano, Ferreira, and Morabito (2020) consider a similar problem and introduce temporal clearings and sequence-dependent changeovers. Toledo et al. (2009) introduce an evolutionary algorithm for a more general problem with two interdependent levels, where decisions concern the different stages. As a solution, Toledo, de Oliveira, and França (2013) present an algorithm to solve the Multi-Level Capacitated Lot-sizing Problem (MLCLP) with backlogging. The proposed method combines a multi-population-based metaheuristic using the fix-and-optimise heuristic and mixed-integer-programming. Similar heuristics for lot sizing and scheduling are developed and discussed by Soler, Santos, and Akartunali (2019), who use an efficient relax-and-fix procedure, and by Sel and Bilgen (2014), who formulate the planning distribution problem in the dairy industry as a Mixed-Integer-Linear Program. They solve it with a fixed-and-optimise heuristic and simulation. In relation to the nature of our model, Copil et al. (2017) state that decomposition methods generate good solutions for the MLCLP. In this regard, and from a modelling point of view, the problem addressed in our paper is a MLCLP of a particular soft-drink production facility in a dairy company in Switzerland. It differs from the literature as:

- There are important differences between the stages in the production line. Thus, we consider, through several constraints, dynamic associations between the different stages.
- There is a major difference in the nature of the products considered, namely the soft drinks, which have different limited liquid-flavour conservation times. These set of constraints are formulated in our problem.

To solve the problem, we propose a relax-and-fix decomposition approach when the optimal solution cannot be reached by exact methods, since relax-and-fix techniques have proven their efficiencies for solving lot sizing problems (Absi and van Den Heuvel 2019). With respect to the literature, the contributions of this work are the following. First, we address a realistic and up-to-date multi-period multi-level Capacitated Lot-Sizing and Scheduling Problem in the soft-drink production process that considers constraints related to the product perishability. Second, we propose a relax-and-fix technique to solve the model using real data from the production facility and generate multi-period production plans to identify the impact of different industrial settings on the production plans. The results of the relax-and-fix technique are compared to the exact-method solutions.

Description of the production system

We consider a realistic MLCLP based on real settings of a dairy soft-drink production facility. The problem consists of determining a production plan for a planning horizon composed of T periods. This plan has a finite capacity since we dispose more than one production level and multiple resources (in each level) in which the capacity is limited. The production process, presented in Figure 1, is composed of streamlines of ingredients (i.e., cocoa powder, flavours, water, and milk powder), which are combined in different ways according to a total of eight recipes. Each streamline of ingredients is used to fill eight standardisation tanks (one at a time) in which ingredients are mixed. The standardisation tanks are then emptied by serving two UHT lines, where the product is pasteurised before the filling and packaging steps. All standardisation tanks in the production facility are similar in terms of capacity. They can store all types of recipes and are characterised by a minimum filling quantity to guarantee the homogeneity of the
standardised liquid flavour. Each tank cannot handle more than one liquid flavour at a time. Each tank requires cleaning each time there is a processing of the liquid flavour, thus generating more setup costs and times.

In contrast with the standardisation tanks, the two UHT lines have different capacities, but each one can process any type of liquid flavour. The production of a batch of soft drink on a UHT line requires a set of two operations before the production can start. The first operation is the warming-up and sterilising process (1 hour and 30 minutes in our case). The second one is the push-in operation (20 minutes in our case), where the UHT line is completely filled with the liquid flavour before being able to deliver product to the next part of the production line. Those two operation times, occurring at the beginning of a batch production on a UHT line, can be aggregated and thus considered as one setup with a corresponding time and cost. After sterilising a batch of soft drink, the UHT line must be emptied from the remaining liquid by injecting water or aseptic filling (cleaning sequence), and this operation is known as the push-out period (15 minutes in our case). UHT lines can process several batches of the same liquid flavour consecutively without any push-in/push-out periods, thus reducing the setup time and cost between batches. The setup time and cost of the UHT line is therefore sequence dependent.

The production process considered in this paper is made of two stages (I and II). In stage II, two UHT lines serve a downstream section of the production process that is beyond the scope of this work. Every UHT line must be filled from only one tank at a time, independently from the number of tanks containing products. In stage I (composed of eight standardisation tanks), a tank can feed simultaneously both UHT lines. Figure 1 shows an example where both UHT lines receive liquid flavour from the same tank (tank 2) at the same time. Therefore, the proposed model considers the dynamic assignment between the two stages. The packaging machines have a tight schedule that is defined in advance to satisfy the demand, which is defined at the packaging level. The objective for the UHT line is therefore to satisfy, at its best, the demand, which is defined daily for each type of soft-drink dairy product. Initially, each UHT line is configured to produce a specific flavour. A sequence-dependent setup time related to mechanical adjustments and/or cleaning is imposed, even if the same liquid flavour is being treated in consecutive processes. If the same dairy soft drink with a flavour is prepared during two successive periods, the cleaning time and cost will be smaller than those required for a changeover of two soft drinks using different flavours. Moreover, the storage time of liquid flavour in the tanks is limited due to the nature of the dairy products.

In the production planning, a UHT line must be prepared and configured to produce the soft drink before it can be fed with the necessary liquid flavour by a related tank. Moreover, a UHT line has to wait if the necessary liquid flavour is not ready to be delivered.

**Mathematical formulation**

We model the problem as a mixed-integer-optimisation model that considers the synchronisation of the two capacitated production levels and the limited shelf life of liquid flavours. This model extends the one presented by Ferreira, Morabito, and Rangel (2009), where each element of the first stage has a dedicated packaging line, which makes it possible to eliminate the needs to dynamically match the lines with the tanks. The objective of the model is to
minimise the production cost, composed of inventory, backorder and changeover costs, related to both stages. The time-dependent problem is then discretised using the concept of macro-periods and micro-periods of time. Subdividing the macro-periods of the problem into several micro-periods leads to a discrete lot-sizing and scheduling problem (DLSP). The fundamental assumption of the DLSP is that only one item may be produced per period, and, if so, production uses the full capacity. The mathematical formulation of the problem is developed as follows.

### Indices

- \( i, j \in \{1, ..., J\} \) item number, i.e., dairy soft-drink product.
- \( e, u \in \{1, ..., U\} \) standardisation tank number.
- \( m \in \{1, ..., M\} \) UHT line number.
- \( k, l \in \{1, ..., L\} \) liquid flavour number.
- \( t \in \{1, ..., T\} \) macro-period number.
- \( s \in \{1, ..., N\} \) micro-period number.
- \( p_t \) first micro-period of period \( t \).

In this model, we also assume that the following data is known:

### Sets

- \( S_t \) micro-periods in macro-period \( t \).
- \( \lambda_j \) UHT line producing item \( j \).
- \( a_m \) products compatible with line \( m \).
- \( Y_m \) products compatible with line \( m \) and produced with liquid flavour \( l \).
- \( \mu_l \) tanks that can produce liquid flavour \( l \).

The superscript \( l \) (resp. \( ll \) ) is added to the variables and data used for the first (resp. second) stage of the production line.

### Parameters

- \( d_t\) demand of product \( j \) at period \( t \).
- \( h_t\) cost generated by inventory of product \( j \) (≥ 0).
- \( g_j\) cost generated by backorder of product \( j \) (≥ 0).
- \( s_{ij}\) cost generated by changeover from liquid flavour \( k \) to liquid flavour \( l \).
- \( c_{ij}\) cost generated by changeover from product \( i \) to product \( j \).
- \( b_{il}^{0}\) time generated by the changeover from liquid flavour \( k \) to liquid flavour \( l \).
- \( b_{il}^{1}\) product \( i \) to product \( j \) changeover time.
- \( a_{mij}\) production time of product \( j \) on line \( m \).
- \( k_u\) capacity of tank \( u \).
- \( \alpha_m\) capacity of line \( m \) at period \( t \).
- \( r_{ij}\) necessary quantity of liquid \( l \) to produce \( j \).
- \( q_{ij}\) minimum quantity of liquid \( l \) in tank \( u \) (to ensure homogeneity).
- \( \omega_t\) initial inventory of product \( j \).
- \( i_{uj}\) initial backorder of product \( j \).
- \( \gamma_{uj}\) 1 if the initial setup of \( u \) is made for \( j \), 0 otherwise.
- \( \delta_{uj}\) 1 if the initial setup of \( m \) is made for \( j \), 0 otherwise.
- \( w_{il}\) maximum storage time for liquid flavour \( l \) in tank \( u \) (in time units).

### Variables

- \( x_{ij}^{0, t}\) inventory of product \( j \) at the end of period \( t \).
- \( x_{ij}^{1, t}\) backorder of product \( j \) at the end of period \( t \).
- \( y_{mij}\) quantity of \( j \) produced on line \( m \) in micro-period \( s \).
- \( y_{ms}\) 1 if tank \( u \) is used for the liquid flavour \( l \) in micro-period \( s \); 0 otherwise.
- \( y_{mll}^{u}\) 1 if tank \( u \) is producing liquid flavour \( l \) in line \( m \) in micro-period \( s \); 0 otherwise.
- \( y_{mll}^{u}\) 1 if line \( m \) is used for product \( j \) in micro-period \( s \); 0 otherwise.
- \( z_{ms}\) 1 if there is changeover from liquid flavour \( k \) to liquid flavour \( l \) in tank \( u \) in (micro-period \( s \)); 0 otherwise.
- \( z_{mll}^{u}\) 1 if there is changeover in tank \( u \) for line \( m \) from liquid flavour \( k \) to liquid flavour \( l \) in (micro-period \( s \)); 0 otherwise.
- \( z_{mll}^{u}\) 1 if there is changeover from product \( i \) to product \( j \) in line \( m \) (in micro-period \( s \)); 0 otherwise.

In this model, we consider a production-planning horizon divided into \( T \) macro-periods, and each one is divided into \( S \) micro-periods. Figure 2 represents a macro-period, where the length is defined by the lot sizes of products (the liquid flavours in stage I).

The bottleneck of the production line may alternate between the two stages, and therefore the maximum/minimum capacity for the first stage and the maximum capacity for the second stage define the size of each micro-period considered. Since we have a different number of elements in both production stages, the set \( U \) represents the number of tanks, and the indices \( u \) and \( e \) have been added to represent the tanks numbers. Therefore, the set of tanks that can produce a flavour \( l \) is noted by \( \mu_{l} \), and \( w_{il} \) is the maximum time a flavour \( l \) can be stored in tank \( u \), expressed in time units.

The additional decision variables \( y_{ms}^{u} \) and \( z_{mll}^{u} \) are introduced to represent the attribution of tank \( u \) to machine \( m \) for liquid flavour \( l \) in micro-period \( s \). Augmenting the variables with index \( m \) could also solve the problem. However, the indices are required in only four constraints, and the introduction of these two variables only requires the addition of a few simpler constraints where they are used.

Figure 2 also shows that, in micro-period \( s \), it is decided to produce the liquid flavour \( k \) in the tank \( u \) (i.e., \( y_{ms}^{u} = 1 \)). The tank \( u \) is matched to line \( m \) (i.e., \( y_{ms}^{u} = 1 \)), and it is necessary to obtain the liquid flavour that the related tank \( u \) must prepare by the changeover decision \( z_{ms}^{u} = 1 \) (the tank’s setup of the liquid flavour \( k \), which takes \( b_{il}^{1} \) periods).

For the second stage and on the matched line \( m \), it is decided to produce the product \( i \) by defining \( y_{ms}^{u} = 1 \). The necessary time for this production is given by the unitary production time \( a_{mij} \) multiplied by the produced quantity \( y_{ms}^{u} \) and therefore the line \( m \) must be tuned for product \( i \) by the changeover decision \( z_{ms}^{u} = 1 \). The changeover process time is indicated by \( b_{il}^{1} \) and \( y_{ms}^{u} \) refers to the waiting time that line \( m \) must wait until the liquid flavour in tank \( u \) is ready.
Figure 2. Schematisation of a macro-period.

The two-level multi-period lot-sizing and scheduling model is then:

\[
\text{MinZ} = \sum_{j=1}^{J} \sum_{t=1}^{T} \left( h_{j} r_{j} + g_{j} r_{j} \right) + \sum_{s=1}^{U} \sum_{u=1}^{M} \sum_{m=1}^{M} \sum_{k=1}^{K} s_{k} x_{j}^{k} y_{j}^{k} + \sum_{s=1}^{U} \sum_{m=1}^{M} \sum_{k=1}^{K} s_{k} x_{j}^{k} y_{j}^{k}
\]

\[
\text{subject to:}
\]

**Stage I (Standardisation tanks)**

\[
\sum_{m=1}^{M} \sum_{k \in \beta_{u}} y_{emb}^{k} \leq 1 \quad u = 1, \ldots, U; s = 1, \ldots, N
\]

Constraint (2) ensures that each tank with only one attributed flavour can feed only one line at a time.

\[
\sum_{l=1}^{L} \sum_{k \in \beta_{u}} y_{emb}^{k} \leq 1 \quad m = 1, \ldots, M; s = 1, \ldots, N
\]

Constraint (3) defines that a tank can only feed a line with only one attributed flavour at a time. Those two constraints ensure the dynamic match between the two production stages due to the difference between the numbers of elements in each stage.

\[
\sum_{m=1}^{M} y_{emb}^{k} \leq 1 \quad u = 1, \ldots, U; s = 1, \ldots, N; l \in \beta_{u}
\]

Constraint (4) ensures that a tank can only feed a line if it is full (setup state is one), and one at a time (note that a tank can be full and does not feed any line).

\[
\sum_{m=1}^{M} z_{emb}^{l} = z_{emb}^{l} u = 1, \ldots, U; s = 1, \ldots, N; k \in \beta_{u}
\]

Constraint (5) ensures that the changeover of a tank takes place only on the tank.

\[
\sum_{l} y_{uls}^{l} \leq 1 \quad u = 1, \ldots, U; s = 1, \ldots, N
\]

Constraint (6) ensures that only one fabricated product is produced at a time in each tank.

\[
\sum_{j \in \Phi_{l}} x_{emb}^{j} u = 1, \ldots, U; m = 1, \ldots, M; s = 1, \ldots, N; l \in \beta_{u}
\]

The demand in each tank for liquid flavour \( l \) in each micro-period \( u \) is \( \sum_{j \in \Phi_{l}} x_{emb}^{j} \). Therefore, constraints (7) and (8) ensure that the amount of product produced on the UHT line is defined between the minimum quantity of tank filling and the maximum capacity of the tank. For the first stage, there is no production variable; the quantity of liquid flavours to be filled in the tanks is defined by the quantity of products that need to be produced in the second stage.

\[
\sum_{j \in \Phi_{l}} x_{emb}^{j} \geq \sum_{j \in \Phi_{l}} x_{emb}^{j} u = 1, \ldots, U; s = 1, \ldots, N; m = 1, \ldots, M; l \in \beta_{u}
\]
The idle micro-period, which can occur at the end of a macro-period, is presented by constraint (9). Figure 3 shows the implications of this constraint: it forces all the production on a given tank-line to occur as soon as possible in the macro-period. All idle micro-periods are placed at the end of the macro-period.

\[
\sum_{j \in \text{prod}} a_{m}^j x_{mij}^j + \sum_{m \in \text{types}} \sum_{i \in \text{areas}} b_{m}^{ij} y_{mij}^j + v_{ms}^j - \sum_{i \in \text{areas}} \sum_{k \in \text{beta}} b_{k}^j x_{amks}^j \\
\leq \sum_{m \in \text{types}} w_{j}^m y_{amms}^j, m = 1, \ldots, M; l = 1, \ldots, F; s = 1, \ldots, N
\]

(10)

Constraint (10) refers to the dairy products shelf life, which is the maximum time that a liquid flavour can be stored in stage \(l\). The left part of the equation corresponds to the storage duration of a liquid flavour \(l\), expressed as the total duration of the micro-period, which, as shown in Figure 2, is the sum of the setup, waiting and production times of the UHT line \(m\) on which the flavour is processed, reduced by the setup time of the associated tank. The right part of the constraint represents the storage time limit of the tank associated with the UHT line \(u\).

The storage time limit in the industrial setting is close to the production time of one full tank of liquid flavour. Therefore, consecutive batches of liquid flavour are not allowed without cleaning the tank in between. Consecutive batches of the same product with reduced changeover time are only allowed on UHT lines. Therefore, the time storage limitation is set on the size of the micro-periods in constraint (10), instead of limiting the storage duration of consecutive batches without any cleaning process in between.

\[
z_{alks}^j \geq y_{alk(s-1)}^j + y_{alk}^j - 1, u = 1, \ldots, U; s = 1, \ldots, N; k, l \in \beta_u
\]

(11)

Constraint (11) controls the liquid flavours changeover, except when the changeover corresponds to the changeover between macro-periods (as the setup state in the last micro-periods might be 0). As the changeover variable is sequence dependent, it is possible to represent both the push-out period of the last micro-period and the setup period of the current micro-period (with the changeover time and cost of the current micro-period), thus having only one non-productive period instead of two.

\[
z_{amks}^j \geq \sum_{j \in \text{prod}} y_{m(i-1)}^j + y_{amks}^j - 1
\]

(12)

\[
u = 1, \ldots, U; t = 2, \ldots, T; k, l \in \beta_u; m = 1, \ldots, M; s = P_t
\]

In stage II, the liquid flavour prepared in the last non-idle micro-period can be indicated by setup variables. Thus, the changeover between each macro-period can be ensured by constraint (12), which considers the setup state of the UHT line in the last micro-period of the previous macro-period as the setup state of the associated tank in the same last micro-period.

\[
\sum_{k \in \beta_u, l := (1, \ldots, 1)} z_{alks}^j \leq 1, u = 1, \ldots, U; t = 1, \ldots, T; s \in S_t
\]

(13)

Constraint (13) guarantees the existence of at most one changeover in tank \(u\) at micro-period \(s\).

\[
\sum_{k = 1}^j y_{alk}^j \geq y_{alk}^j, u = 1, \ldots, U; t = 1, \ldots, T; s \in (S_t - P_t); l = 1, \ldots, F
\]

(14)

For computational reasons, and due to the nature of the experiments where optimal solutions are required, constraint (14) is added to some instances only. It forces the different flavours, produced in the same tank during a macro-period, to be filled in a specific order. For example, a tank that needs to provide flavours 1 to 3 during a macro-period will be forced to store flavours 1 to 3 in that particular order. This eliminates symmetrical sequences and allows for a faster computing of the optimal solution.

**Figure 3.** UHT line capacity defined by the macro period.
Stage II (UHT lines)

\[
I_{ji(t-1)}^+ + f_{ji}^+ + \sum_{m \in A_n} \sum_{s \in S_i} x_{mjs}^i = I_{ji}^+ + f_{ji}^+ + d_f j
\]

\[
= 1, \ldots, J; t = 1, \ldots, T
\]  

(15)

Constraint (15) ensures that the inventory balancing constraints for each product in each macro-period \( s \) is fulfilled. For a given micro-period, the production variable is defined. Therefore, calculating the production variables related to \( j \) over all lines where it can be produced allows the determination of the total production quantity.

\[
\sum_{m \in A_n} \sum_{s \in S_i} d_m^s x_{mjs}^i + \sum_{m \in A_n} \sum_{s \in S_i} b_{mjs}^i + \sum_{s \in S_i} v_{ms}^i \leq k_{mt}^l
\]  

(16)

Constraint (16) represents the UHT line capacity in each macro-period. This capacity is in units and defines the size of a macro-period (see Figure 3). This constraint ensures that the time occupied by the sum of its micro-periods does not exceed the UHT line capacity.

\[
v_{ms}^i \geq \sum_{k \in I_s^i} \sum_{l \in I_s^i} b_{klms}^i - \sum_{m \in A_n} \sum_{s \in S_i} b_{mjs}^i m
\]

\[
= 1, \ldots, M; s = 1, \ldots, N
\]  

(17)

Constraint (17) controls the waiting time that each line \( m \) must wait from the beginning of each micro-period \( s \), until the liquid flavour in the tank is ready. This waiting time exists only when the setup time of the attributed tank is larger than the setup time of the UHT line (see Figure 3), thus establishing the synchronisation between the two production stages.

\[
x_{mjs}^i \leq \frac{k_{mt}^l}{d_f} v_{ms}^i t = 1, \ldots, T; m = 1, \ldots, M; j \in a_m; s \in S_i
\]  

(18)

Constraint (18) ensures that the production of \( j \) in line \( m \) at micro-period \( s \) takes place only if the correct setup variable is equal to one. It also ensures that the production quantity can fit in the macro-period.

\[
\sum_{j \in A_m} y_{mjs}^i = 1 s = 1, \ldots, N; m = 1, \ldots, M
\]  

(19)

Constraint (19) imposes that each line \( m \) is tuned for one product at each micro-period \( s \), and that only one mode of production is allowed.

\[
x_{mjs}^i \geq y_{mjs}^i + y_{mjs}^i - 1 s = 1, \ldots, N; m
\]

\[
= 1, \ldots, M; i, j \in a_m
\]  

(20)

Constraint (20) sets the changeover variables for the second stage in each line \( m \) in each micro-period \( s \). As the setup state for stage II, \( y_{mjs}^i \) is always set to one for one of the products \( j \), the changeover variable is also set to one for one of the changeovers in every micro-period, unlike in stage I, where both the setup and changeover variables can be zero for every product/changeover.

\[
\sum_{i \in A_m} \sum_{j \in A_m} x_{mjs}^i \leq 1 s = 1, \ldots, N; m = 1, \ldots, M
\]  

(21)

Constraint (21) imposes a maximum of one changeover in each line.

\[
f_{ji}^+ \geq 0
\]  

(22)

\[
x_{mjs}^i \geq 0
\]  

(23)

\[
y_{mjs}^i, y_{mjs}^i, y_{mjs}^i, y_{mjs}^i, y_{mjs}^i, y_{mjs}^i = 0 / 1
\]  

(24)

Finally, constraints (22–24) define the variable domains.

The model presented differs from the formulation developed by Ferreira, Morabito, and Rangel (2009). Indeed, in our case, the number of tanks does not correspond to the number of pasteurisation lines. In fact, the elements of the two stages are coupled with each other by Constraints (2) and (3). The storage time of raw dairy products in tanks before pasteurisation is limited and developed in Constraint (10). Constraint (6) represents the dynamic allocation of lines to tanks.

Solving the problem

The model was coded and run under CPLEX 12.5, using an Intel Core i5-4210 U 1.70 GHz processor (4GB memory) computer.

Presentation of the instances

Table 1 presents the experiments with the following representative industrial settings:

- **Tank/flavour assignment:** every tank can process only one liquid flavour (with the assumption that the number of tanks is equal to the number of liquid flavours).
- **Product/UHT line assignment:** each line can produce only a particular type of product.
- **Demand type:** excessive demand, either surpassing the capacity of production at each macro-period, or close to the maximum threshold that can be satisfied by the production line (feasible demand).
- **Same numbers of Tanks and UHT lines:** the number of tanks is reduced to be equal to the number of UHT lines.
- **Bottleneck location:** in the proposed model, the bottleneck may alternate between the two production stages at each macro-period. Therefore, the
size of a micro-period is defined by the minimum/maximum tank capacity if the bottleneck is located in stage I, and limited by the maximum UHT line capacity if the bottleneck is located in stage II.

10 experiments are designed according to 20 criteria. Each one is selected in a way that responds to a particular production strategy. Those criteria are listed below:

- Number of lines
- Number of tanks
- Number of products
- Customer demand depending on the period
- Unit storage cost
- Unit backorder cost
- Cost of the first-level changeover
- Cost of the second-level changeover
- First-level changeover time
- Second-level changeover time
- Unit production time
- Total capacity of each tank
- Total capacity of each line
- Quantity of liquid water for each unit production
- Minimum production quantity for each tank
- Initial storage quantity of the product
- Initial product backorder
- Initial configuration of the tank
- Initial configuration of the UHT line
- Maximum shelf life of each liquid flavour in each tank

**Exact Method**

Table 2 presents the results obtained with the exact branch-and-cut method. For each experience, the following information is reported: the best obtained objective-function value; the computation time required to find such value; the percentage gap between the obtained value and the best bound (identified by CPLEX). An optimal solution is found anytime the gap is zero. Two main observations can be made:

- An optimal solution is always found quickly (i.e., within 10 seconds) if each line can produce only a particular type of product (i.e., for Exp1, Exp2, Exp3, Exp5, Exp7, Exp8, and Exp9). This can be explained as this restriction is able to significantly reduce the search tree explored by the exact method.
- Optimality is not proven, and the computation times are large for the other instances (i.e., for which the product/UHT line assignment restrictions are not true). Indeed, the gaps are 45.5%, 18.45%, and 2% for Exp4, Exp6 and Exp10, respectively. Moreover, the associated computation times are 48 hours, 48 hours, and 72 hours, respectively. For these three instances, the excessive demand assumption is true, which also contributes to their complexity. Moreover, the gaps of Exp6 and Exp10 are smaller than the gap of Exp4, because the tank/flavour restriction is not true for the latter instance, which means that the search tree is much larger.

Note that for Exp10, regarding CPLEX, we have considered a gap tolerance of 2% (instead of 0% for the other experiments). The best solution value is reached after 72 hours. The choice of this gap tolerance is made after reaching a search tree size of 189 Go. We have here a smaller gap performance (2%) when compared to Exp4 and Exp6 because the bottleneck is located at stage I instead of stage II.

Table 1. Settings used in each experiment.

| Experience | Tank / Flavour assignment | Product / UHT line assignment | Excessive demand | Feasible demand | Same numbers of Tanks / UHT lines | Bottleneck location |
|------------|--------------------------|-------------------------------|------------------|----------------|----------------------------------|-------------------|
| Exp1       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage II          |
| Exp2       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage II          |
| Exp3       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage I           |
| Exp4       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage II          |
| Exp5       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage II          |
| Exp6       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage I           |
| Exp7       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage II          |
| Exp8       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage II          |
| Exp9       | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage I           |
| Exp10      | ✓                        | ✓                             | ✓                | ✓              | ✓                                | stage I           |

Table 2. Results obtained with the exact method.

| Experiment | Resolution time (s) | Objective value | GAP (%) |
|------------|---------------------|-----------------|----------|
| Exp1       | 3.7                 | 84,449.4        | 0        |
| Exp2       | 3.5                 | 20.6            | 0        |
| Exp3       | 5.1                 | 18.59           | 0        |
| Exp4       | 172,800             | 80,849.23       | 45.5     |
| Exp5       | 6                   | 84,446.4        | 0        |
| Exp6       | 172,800             | 80,522.26       | 18.45    |
| Exp7       | 8                   | 84,452          | 0        |
| Exp8       | 7                   | 16,522.59       | 0        |
| Exp9       | 7                   | 22.6            | 0        |
| Exp10      | 259,200             | 789,167.22      | 2        |
Relaxation approach

We consider a relax-and-fix approach to better tackle Exp4 and Exp6, for which the optimal solutions are not found. In fact, these experiments are characterised by large solution spaces due to the presence of the bottleneck of the production line in stage II, and to the absence of restrictions related to tank/flavour and UHT line/product. It consists of assuming that the capacity of the standardisation tanks is large enough to deliver a specific liquid flavour when it is requested by the UHT lines to produce a batch of a specific product. Indeed, the relax-and-fix methods have shown their benefit for different lot sizing problems (Ferreira, Morabito, and Rangel 2009). In all the experiments, the preparation of liquids in the tanks does not present a bottleneck in the production process except in Exp8 and Exp9, for which the optimal solutions are obtained in a reasonable time. Therefore, it is assumed that the capacities of the tanks are large enough to deliver a liquid of a given flavour. This approach is based on the idea that once the production decision at the UHT line level is made, the tank-level decisions can be easily determined. The method relies on a 3-step algorithm:

(1) Step 1: Solve the one-stage model.
(2) Step 2: Depending on the production plan generated by the previous step, fix the second-level configuration variable to 1 ($y_m^{il} = 1$) in each micro-period where a batch is produced ($x_{mij} > 0$).
(3) Step 3: Solve the two-stage model, considering the fixed configurations in stage II.

The assumption allows to remove the variables $x_{mij}$ and $y_m^{il}$ (i.e., no need to control synchronisation and changeovers). The two-stage model is then transformed into a one-level model, while keeping the constraints related to the batches upper/lower bounds and the dynamic assignment between those two stages through constraints (2–3).

The objective function of the one-stage model is redefined as follows:

$$\text{Min} Z = \sum_{j=1}^{J} \sum_{t=1}^{T} \left( h_{ij}^x + g_{ij}^x \right) + \sum_{s=1}^{N} \sum_{m=1}^{M} \sum_{i<j} \sum_{l<j} s_{ij}^y x_{mij}$$

(25)

Constraint (16) is replaced by:

$$\sum_{j<i_{loc}} a_{ij} x_{mij} + \sum_{j<i_{loc}} b_{ij} y_{mij} \leq k_{mt}$$

(26)

Table 3. Results obtained with the relaxation approach.

| Experiment | Computation time (s) | Objective value | GAP (%) |
|------------|-----------------------|-----------------|---------|
| Exp1       | Step1 = 3.1; Step3 = 1.3 | 84,450.3        | 0       |
| Exp2       | Step1 = 2; Step3 = 1.6 | 21.8            | 5.83    |
| Exp3       | Step1 = 3.9; Step3 = 5.0 | 21.3            | 14.58   |
| Exp4       | Step1 = 172,800; Step3 = 90 | 80,132.9        | 0.01    |
| Exp5       | Step1 = 2.5; Step3 = 2  | 84,452.4        | 0.01    |
| Exp6       | Step1 = 172,800; Step3 = 60 | 80,129.1        | 0.01    |
| Exp7       | Step1 = 3.3; Step3 = 2.1 | 84,457          | 0.01    |
| Exp8       | Step1 = 6.03; Step3 = 3.1 | 16,527.6        | 0.03    |
| Exp9       | Step1 = 3.6; Step3 = 1.6 | 23.1            | 2.21    |
| Exp10      | Step1 = 259,200; Step3 = 2700 | 789,573.6       | 0.05    |

The objective function (25) and the set of constraints {2}, {3}, {7}, {8}, {12}, {15}, {18}, {19}, {20}, {21}, {22}, {24} and {26} represent the one-stage multi-machine model.

Table 3 presents the results of the relaxation approach. For each experiment, we indicate: the best obtained objective-function value, the computation time required to find such value (only for Steps 1 and 3, as Step 2 is not time-consuming) and the percentage improvement gap with respect to the solution found by the exact approach (i.e., a negative gap means an improvement).

The following observations and comments can be made:

- For the instances having the Product/UHT line assignment restriction, the relaxation approach is often able to generate results that are comparable to the ones obtained by the exact method (i.e., close-to-optimal solutions within 10 seconds).
- For the other (more complex) instances (i.e., Exp4, Exp6 and Exp10), the relaxation approach provides improvements for Exp4 and Exp6, but a minor degradation for Exp10.
- In Exp4, the computation time for the first step is too high to achieve optimality and the generated production plan is not satisfactory. Therefore, the best solution after a computation time of 48 hours (i.e., the same time considered as for the exact method) is taken as a result. According to this solution, the third step of the relaxation approach gives a result within 90 seconds with an improvement of the objective function amounting to 716.33. The same process is employed for Exp6 and Exp10. For Exp6, it results in an improvement equal to 393.16.

Discussion and managerial insights

Products with shelf-life constraints are always important in terms of storing strategies and production planning. In fact, results, generated from a dairy soft-drink
production facility, is used to develop production plans, allowing the decision makers to study the impact of the different production-process strategies. In addition, the production plans of the real industrial settings show the limit at which the bottleneck capacity of a particular setting moves from the UHT line to the tanks.

In Exp1, the production plan for four types of products on five macro-periods is generated with a total cost equal to 84,449.4, whereas the cost of the production plan generated in Exp2 is equal to 20.6. Those two experiences use the same settings except the nature of demand, which exceeds the UHT lines capacity in Exp1. The significant cost difference is due to the existence of stocks and backorders in Exp1. The generated production schedule also shows that each lot is produced on a single micro-period, except a batch of product 1 corresponding to a size of 221.55 tons, and divided into two micro-periods surpassing the capacity of tank 1 (i.e., 128 tons). Therefore, the proposed model defines the minimum/maximum lot sizes based on the assigned tank capacity.

The results generated from Exp2 and Exp9 (20.6 vs 22.6) show that shifting the bottleneck to stage I increases the production costs. In fact, the generated production plan in Exp9 shows the absence of any idle micro-periods and a size admitting a lower bound of 41 tons for the tanks feeding the UHT line 1 (characterised by the shortest treatment time) and 27 tons for the tanks feeding the line 2.

A comparison of the objective-function values of Exp4 and Exp5 (80,123 vs 84,446) shows that the assignment of some products to each UHT line limits the objective-function minimisation. This is explained by the differences in product processing times from a line to another. Therefore, this assignment limits the total quantity of goods that can be processed within the time horizon and generates a high objective-function value for the instances where the demand is excessive. The difference between results of Exp8 and Exp1 (16,522.59 vs 84,449.4) is caused by the shift of the bottleneck from UHT lines to the tanks where an excessive demand is considered. Additionally, even if the objective-function values in Exp2 and Exp3 show a slight difference (20.6 vs 18.59), this implies that the assignment of a unique flavour to each tank increases the total costs. In fact, the generated production plans show the reduction of the lot sizes in Exp2, which increases the number of changeovers in the UHT lines. This observation is reproduced in Exp1 and Exp5 (84,449.4 vs 84,446.4). We can therefore conclude that the strategy of assigning flavours to tanks, and products to lines, leads to the increase of the total production costs.

The results generated from Exp7 and Exp5 (84,452 vs 84,446.4) show that the reduction of the number of tanks (if there are as many tanks as lines, and that every tank can process any flavour type) does not affect the lot sizes and the production planning for products 1 and 2 produced in line 1 (characterised by the shortest processing time). However, it slightly affects the production planning of products 3 and 4 produced in line 2, where more backorders are generated given the nature of demand.

In Exp10, which is considered without any simplification (considering eight tanks, 2 UHT lines and eight products), the generated production plan indicates a total production of 2282 tons for the total time horizon, which is not sufficient to satisfy the demand used in this experience. In fact, the production plan shows the absence of any idle micro-periods in UHT lines where products 1, 3 and 4 are processed in line 1, and products 2 and 7 are processed in line 2. According to the results, the best strategy to minimise the costs is to entirely backorder products 5, 6 and 8.

Finally, it is important to notice that the large computing times associated with Exp4, Exp6 and Exp10 are likely to be prohibitive from a practical standpoint, as several hours are requested to obtain efficient solutions. For such configurations, two options are possible from a managerial point of view:

(a) Develop dedicated and fine-tuned metaheuristics (for example, based on the works of Respen, Zufferey, and Amaldi (2016) and Thevenin and Zufferey (2019)).

(b) Employ the proposed relaxation approach, but only after initially reducing the search space by restricting the number of liquid flavours that can be assigned to some tanks (which is less restrictive than assigning a single liquid flavour to each tank), and/or by restricting the number of products that can be assigned to some lines (which is less restrictive than assigning a single product to each line). The design of such restriction and filtering techniques can be derived from the works of Coindreau et al. (2021) and Hertz, Schindl, and Zufferey (2005).

**Conclusion**

This paper addresses the multi-period multi-level capacitated lot-sizing and scheduling problem. A mixed-integer-programming formulation for a real-case dairy soft-drink production line is proposed. The model represents two production levels with different resources in each level and where the capacity is limited. It dynamically reassigns the stages to each other according to specific constraints, such as the storage-time limitations of dairy products (because of the nature of the products). The model aims at minimising the production costs (i.e., backorders, inventory, and changeovers).
Ten different data configurations are used for the experiments to cover the various situations that the company is likely to face. Generating optimal solutions with the proposed exact method is not always possible, especially when the solution space is too large because there is no flavour-to-tank restriction and/or no product-to-line restriction. For such complex cases, a relax-and-fix approach is developed to generate improved production plans (with respect to the costs), and managerial insights are proposed to favour its use in practice.

Future research directions could be the introduction of improvement steps for the solution method. In addition, the designed model can be applied to evaluate different strategies related to other lot-sizing settings, such as lot-sizing with uncertain demand/setup times and lot sizing in different machine configurations. Finally, in line with the works of Argilaguet Montarelo, Gardon, and Zufferey (2017) and Damand et al. (2019), the considered production planning problem could be integrated within the global supply chain planning, for instance by adding upstream decisions such as request management and capacity planning.

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