Dynamic Topology Adaptation and Distributed Estimation for Smart Grids

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Abstract—This paper presents new dynamic topology adaptation strategies for distributed estimation in smart grids systems. We propose a dynamic exhaustive search–based topology adaptation algorithm and a dynamic sparsity–inspired topology adaptation algorithm, which can exploit the topology of smart grids with poor–quality links and obtain performance gains. We incorporate an optimized combining rule, named Hastings rule into our proposed dynamic topology adaptation algorithms. Compared with the existing works in the literature on distributed estimation, the proposed algorithms have a better convergence rate and significantly improve the system performance. The performance of the proposed algorithms is compared with that of existing algorithms in the IEEE 14–bus system.

Keywords—Dynamic topology adaptation, distributed estimation, smart grids.

I. INTRODUCTION

The electric power industry is likely to involve many more fast information gathering and processing devices (e.g., phasor measurement units) in the future, enabled by advanced control, communication, and computation technologies [1]. As a result, the need for more decentralized estimation and control in smart grids systems will experience a high priority. Several works in the literature have proposed strategies for distributed estimation [2, 3, 4]. With existing algorithms, the neighbors for each bus are fixed. When there are links that are more severely affected by noise or other disturbances, these approaches may not provide an optimized estimation performance for each specified bus. Moreover, with the number of neighbor buses increasing, each bus requires a large network bandwidth and transmit power. Therefore, a key problem with the strategies reported so far in the literature is that they do not exploit the topology of the smart grids system and the knowledge about the poor links to improve the performance of distributed estimation techniques.

The objective of this paper is to propose fully distributed dynamic topology adaptation algorithms for distributed estimation in smart grids system, in order to optimize the performance and minimize the mean-square error (MSE) associated with the estimates. We propose two dynamic topology adaptation strategies, the proposed algorithms exploit the knowledge about the poor links and the topology of the system to select a subset of links that results in an improved estimation performance. For the first approach, we consider a dynamic exhaustive search–based topology adaptation (DESTA) strategy. For the DESTA algorithm, we consider all possible combinations for each bus with its neighbors. Then we choose the combination associated with the smallest MSE value.

In the second approach, we introduce the dynamic sparsity–inspired topology adaptation (DSITA) algorithm. A reweighted zero attraction (RZA) strategy is incorporated into the dynamic topology adaptation algorithm. The RZA approach is usually employed in applications dealing with sparse systems in such a way that it shrinks the small values in the parameter vector to zero, which results in better convergence rate and steady–state performance. Different from prior work with sparsity–aware algorithms [5, 6, 7, 8], the proposed DSITA algorithm exploits the possible sparsity of the MSE associated with each of the links in a different way and employs the Hastings rule [9]. The DSITA shrinks to zero the links that have a poor performance. To implement DSITA, we introduce a convex penalty, i.e., an $\ell_1$–norm term to adjust the combination coefficients for each bus with its neighbors, in order to select the neighbor buses that yield the smallest MSE values.

The dynamic topology adaptation is achieved as follows:

• For a specified bus, we calculate the MSE at all its neighbor buses including the specified bus itself through the previous estimate.

• For the bus with the maximum MSE, we impose a penalty and give a reward to the bus with the smallest MSE.

The proposed DSITA algorithm performs this process automatically. By using the DSITA algorithm, some buses with unsatisfactory performance will be eliminated and some poor buses will be taken into account when their performance improves, which means the system topology will change automatically as well. To further improve the performance of distributed estimation techniques, we consider the Hastings rule [9] to construct the initial combination coefficients and incorporate it into the proposed algorithms.

This paper is organized as follows. Section II describes the system model and the problem statement. In section III, the proposed dynamic topology adaptation algorithms are introduced. The numerical simulation results are provide in section IV. Finally, we conclude the paper in section V.

Notation: We use boldface uppercase letters to denote matrices and boldface lowercase letters to denote vectors. We use $(\cdot)^{-1}$ to denote the inverse operator, and $(\cdot)^*$ for conjugate transposition.

II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider an IEEE 14–bus system [11], where 14 is the number of substations. At every time instant $t$, each bus $k$, $k = 1, 2, \ldots, 14$, takes a scalar measurement $z_k(t)$ according to

$$z_k(t) = H_k(x(t)) + e_k(t), \quad k = 1, 2, \ldots, 14,$$  

(1)
where \( x(i) \) is the state vector of the entire interconnected system, \( H_k(x(i)) \) is a nonlinear measurement function for bus \( k \). The quantity \( e_k(i) \) is the measurement error with mean equal to zero and which corresponds to bus \( k \). Fig. 1 shows a standard IEEE–14 bus system with four nonoverlapping control areas.

\[
\begin{align*}
\text{Fig. 1. IEEE 14–bus system}
\end{align*}
\]

Initially, we focus on the linearized DC state estimation problem. The system is built with 1.0 per unit (p.u) voltage magnitudes at all buses and 1.0 p.u. branch impedance. Then, the state vector \( x(i) \) is taken as the voltage phase angle vector \( \theta \) for all buses. Therefore, the nonlinear measurement model for state estimation [1] is modified to

\[
z_k(i) = h_k(i)\theta + e_k(i), \quad k = 1, 2, \ldots, 14. \tag{2}
\]

where \( h_k(i) \) is the measurement Jacobian vector for bus \( k \). Then, the aim for the distributed estimation algorithm is to compute an estimate of \( \theta \), which can minimize the cost function

\[
J_x(x) = \mathbb{E}|z_k(i) - h_k(i)^*x_k(i)|^2, \tag{3}
\]

where \( \mathbb{E} \) denotes the expectation operator.

A LS–type distributed algorithm, named Modified–Coordinated State Estimation (M–CSE), has been reported in the literature [4]. In this strategy, the system is decomposed into \( N \) areas. Based on the current state vector \( x_n(i) \), where \( n = 1, 2, \ldots, N \), the exchanged data \( \{x_i(i)\}_{i \in \Omega_n} \) and the measurement vector \( z_n \), the estimate of the state at the \( n \)th control area can be updated via the following formula

\[
x_n(i + 1) = x_n(i) - \left[ \beta(i) \sum_{l \in \Omega_n} (x_l(i) - x_l(i)) \right] - \alpha(i)H_n^T(z_n - H_nx_n(i)), \tag{4}
\]

where \( \{\alpha(i)\}, \{\beta(i)\} \) are time–varying weight sequences.

For the existing strategies in the literature for smart grids, the system communication topology is fixed. This situation will cause a problem when some of the neighbor buses have a poor performance, or the links between buses experience a disturbance. Also, there is no chance for the bus to discard the poorly performing neighbors rather than continue to use their information. In order to solve these problems and optimize the distributed estimation process, we need to provide the system with the ability to adapt the topology dynamically.

### III. Proposed Dynamic Topology Adaptation Strategies

In this section, we introduce dynamic topology adaptation strategies for distributed estimation in smart grids. The aim of our proposed DESTA and DSITA algorithms is to optimize the distributed estimation process and improve the performance of the smart grids. These two algorithmic strategies give the buses the ability to choose their neighbors based on their MSE performance. Note that other performance criteria are possible.

#### A. Hastings Rule

We first describe a combination rule – Hastings rule that has an improved performance as compared to the Metropolis rule [9], and is incorporated into the proposed algorithms. The combination coefficient \( c_{kl} \) for a bus \( k \) and its neighbor bus \( l \), can be calculated under the Hastings rule as follows

\[
c_{kl} = \begin{cases} 
\frac{\sigma_{n,k}^2}{1 - \sigma_{n,k}^2} & \text{if } k \neq l \text{ are linked} \\
1 & \text{for } k = l 
\end{cases} \tag{5}
\]

where \( |N_k| \) denotes the cardinality of \( N_k \), and \( \sigma_{n,k}^2 \) stands for the noise variance on bus \( k \). All \( c_{kl} \) should satisfy

\[
\sum_l c_{kl} = 1, l \in N_k \forall k. \tag{6}
\]

The Hastings rule is a fully–distributed solution, as each bus only needs to obtain the degree–variance product \( (|N_k| - 1)\sigma_{l,k}^2 \) from its neighbour \( l \), to get the combination coefficient [10].

#### B. Dynamic Exhaustive Search–Based Topology Adaptation (DESTA)

In the proposed DESTA algorithm, we divide the distributed estimation process into two steps. The first step is the adaptation step and the second step is the combination step. For the proposed DESTA algorithm, we employ the adaptation strategy given by

\[
\psi_k(i) = x_k(i - 1) + \mu_k h_k(i)[z_k(i) - h_k(i)^*x_k(i - 1)]^*. \tag{7}
\]

Following the adaptation step, we introduce the combination step for the DESTA algorithm, based on an exhaustive search strategy. At first, we introduce a tentative set \( \Omega_s \) using a combinatorial approach described by

\[
\Omega_s \triangleq C^T_T, \quad t = 1, 2, \ldots, T, \tag{8}
\]

where \( \{T\} \) is the total number of buses linked to bus \( k \) including bus \( k \) itself. This combinatorial strategy will cover all combination choices for each bus \( k \) with its neighbors. After the tentative set \( \Omega_s \) is defined, we redefine the cost function [3] for each bus as

\[
J_\psi(\psi) \triangleq \mathbb{E}|z_k(i) - h_k(i)^*\psi|^2, \tag{9}
\]

where

\[
\psi \triangleq \sum_{l \in \Omega_s} c_{kl} \psi_l(i) \tag{10}
\]

Then, we introduce the error pattern for each bus, which is defined as

\[
e_{\Omega_s}(i) \triangleq z_k(i) - h_k(i)^*[\sum_{l \in \Omega_s} c_{kl} \psi_l(i)]. \tag{11}
\]
For each bus $k$, the strategy that finds the best set $\Omega_s$ should solve the following optimization

$$\hat{\Omega}_s = \arg \min_{\Omega_s} J_{\psi}(\psi), \quad (12)$$

which is equivalent to minimizing the error $e_{\Omega_s}(i)$. After the adaptation steps have been completed, the combination step is performed as given by

$$x_k(i) = \sum_{l \in \Omega_s} c_{kl} \psi_l(i). \quad (13)$$

The DESTA algorithm corresponds to equations (7)-(13) and the combination weights are obtained from (5).

C. Dynamic Sparsity-Inspired Topology Adaptation (DSITA)

The DESTA algorithm previously described needs to examine all possible sets to find a solution, which might result in an unacceptable computational complexity for large systems such as the IEEE 118–bus system [1]. To solve this combinatorial problem with a low complexity, we propose the sparsity–inspired based DSITA algorithm, which bears the simplicity of a standard diffusion LMS algorithm and is suitable for adaptive implementations and scenarios where the parameters to be estimated are slowly time-varying.

The zero-attracting strategy (ZA), reweighted zero-attracting strategy (RZA) and zero-forcing (ZF) are reported in [5], [11] for sparsity aware technique. These approaches are usually employed in applications dealing with sparse systems in such a way that they shrink the small values in the parameter vector to zero, which results in better convergence and steady-state performances. Unlike existing methods that shrink the signal samples to zero, our proposed DSITA algorithm shrinks to zero the links that have a poor performance [7].

We follow the same processing in (4) for the adaptation step, then we redesign the combination step. First, we introduce the convex penalty term $\ell_1$–norm into the combination step. Different penalty terms have been considered for this task. We have adopted the heuristic approach [5], [12] called reweighted zero–attracting strategy, into the combination step, because this strategy has shown an excellent performance and is simple to use. Then, we consider the log-sum penalty function

$$f_1(e_l(i)) = \sum_{l \in \mathcal{N}_k} \log(1 + \varepsilon |e_l(i)|), \quad (14)$$

where the error pattern $e_l(i) (l \in \mathcal{N}_k)$ is defined as

$$e_l(i) \triangleq z_l(i) - \beta_l(i) \psi_l(i) \quad (15)$$

and $\varepsilon$ is the shrinkage magnitude. Then, the combination step can be defined as

$$x_k(i) = \sum_{l \in \mathcal{N}_k} c_{kl} - \rho \frac{\partial f_1(e_l(i))}{\partial e_l(i)} |\psi_l(i)|, \quad (16)$$

where $\rho$ is used to control the shrinkage intensity of the algorithm. After that, we calculate the partial derivative $e_l(i)$ of (14) by

$$\frac{\partial f_1(e_l(i))}{\partial e_l(i)} = \frac{\varepsilon \text{sign}(e_l(i))}{1 + \varepsilon |e_l(i)|^2}. \quad (17)$$

In (17), the parameter $\varepsilon_{\text{min}}$ stands for the minimum value of $e_l(i)$ in each group of buses including each bus $k$ and its neighbors. The function $\text{sign}(a)$ is defined as

$$\text{sign}(a) = \begin{cases} a/|a| & a \neq 0 \\ 0 & a = 0. \end{cases} \quad (18)$$

To further simplify the expression in (16), we introduce the vector and matrix quantities required to describe the combination step. We first define a vector $e$ that contains the combination coefficients for each group of buses including bus $k$ and its neighbors as described by

$$e \triangleq [c_{kl}] \quad l \in \mathcal{N}_k. \quad (19)$$

Then, we introduce a matrix $\Psi$ that includes all the estimated vectors, which are generated after the adaptation step in (7), for each group as given by

$$\Psi \triangleq [\psi_l(i)] \quad l \in \mathcal{N}_k. \quad (20)$$

An error vector $e$ that contains all the error values calculated through (15) for each group is expressed by

$$e \triangleq [e_l(i)] \quad l \in \mathcal{N}_k. \quad (21)$$

To devise the sparsity–inspired approach, we have modified the vector $e$ in the following way: the maximum value $e_1(i)$ in $e$ will be set to $|e_1(i)|$; the minimum value $e_1(i)$ will be set to $-|e_1(i)|$, while the remaining entries will be set to zero. Finally, by inserting (12)-(21) into (16), the combination step will be changed to

$$x_k(i) = \sum_{j=1}^{N_k} [c_j - \rho \frac{\partial f_1(e_j)}{\partial e_j}] \psi_j \quad (22)$$

The proposed DSITA algorithm performs dynamic topology adaptation by the adjustment of the combination coefficients through $e$ in (22). For the neighbor bus with the largest MSE value, after our modifications for $e$, its $e_1(i)$ value in $e$ will be a positive number which will lead to the term $\rho \frac{\text{sign}(e_1)}{1 + \varepsilon |e_1|^2}$ in (22) being positive too. This means that the combining coefficient for this bus will be reduced and the weight for this bus will be reduced as well. In contrast, for the neighbor bus with the minimum MSE, as its $e_1(i)$ value in $e$ will be a negative number, the term $\rho \frac{\text{sign}(e_1)}{1 + \varepsilon |e_1|^2}$ in (22) will be negative too. As a result, the weight for this node associated with the minimum MSE to build the $x_k(i)$ is increased. For the remaining neighbor buses, the $e_1(i)$ value in $e$ is zero, which means the term $\rho \frac{\text{sign}(e_1)}{1 + \varepsilon |e_1|^2}$ in (22) is zero and there is no change for their weights to build the $x_k(i)$. The constraint on the combination of the coefficients in (6) is still satisfied. In conclusion, each bus $k$ will first obtain an local estimate through (7). Then, each bus will employ (15)-(22) to perform the dynamic topology adaptation.

IV. SIMULATIONS

In this section, we compare our proposed dynamic topology adaptation algorithms, DESTA and DSITA, with the $M$–$\mathcal{CS}E$ and traditional diffusion LMS algorithm based on the MSE performance and the Phase Angle Gap. The MSE comparison is used to determine the accuracy of the algorithms, and the Phase Angle Gap is used to compare the convergence rate. In our scenario, ‘Phase Angle Gap’ stands for the phase angle difference between the target $\Theta$ and the estimate $\theta$ for all buses. We define the IEEE–14 bus system as in Fig. 2.

All buses are corrupted by additive white Gaussian noise with equal variance $\sigma^2 = 0.001$. The step size for the proposed
It can be seen that our proposed DESTA algorithm has the best performance, and significantly outperforms the standard diffusion ATC algorithm and $M$–CSE algorithm. DSITA is slightly worse than DESTA, which outperforms the remaining techniques.

To compare the convergence rate, we use the term – ‘Phase Angle Gap’ to describe the results. We pick bus 5 and the first 90 iterations as an example to show our results. In Fig. 4, the DESTA algorithm still has the fastest convergence rate, while the DSITA algorithm is the second fastest. The estimates $x$ made from our proposed dynamic topology adaptation algorithms can quickly reach the target $\theta$, which means the Phase Angle Gap will converge to zero.

V. Conclusion

In this paper, two dynamic topology adaptation strategies have been proposed for distributed estimation in smart grids. The DESTA algorithm uses an exhaustive search to perform the dynamic topology adaptation, and DSITA employs a sparsity–inspired approach with the $\ell_1$–norm penalization. Numerical results have shown that the two proposed algorithms achieve a better convergence rate and lower MSE values than the existing distributed state estimation algorithms. These results hold also when employing other algorithms including RLS and distributed CG techniques.

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