Double Phase Transition Model and the problem of entropy and baryon number conservation.

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Abstract

We continue to develop further the bag-type Double Phase Transition Model (DPTM) for transformation of Quark-Gluon Plasma (QGP) to normal hadronic matter (H-phase). The model is based on the assumed existence of an intermediate Q phase composed by massive constituent quarks and pions (as Goldstone bosons).

In the present paper the problem of entropy $S$ and baryon number $N_B$ conservation in phase transitions from deconfined phases (QGP and $Q$) to hadronic matter $H$ is considered. It is shown that standard construction of both first order phase transitions, $H \leftrightarrow Q$ as well as $Q \leftrightarrow QGP$ implies a discontinuous structure of entropy per baryon $S/N_B$ when crossing phase boundary; this results in impossibility of equilibrium transition from QGP to hadron gas.

We follow the way suggested recently by H.Satz et al. for the same problem concerning direct transition $H \leftrightarrow QGP$. They proposed a modification of bag pressure parameter $B_{QGP}$ by making it dependent on system temperature $T$ and baryon chemical potential $\mu$; this modification has been demonstrated to be sufficient to provide $S/N_B$ conservation.

Here we show that within DPTM such a modification turns out to be necessary and sufficient for bag pressure $B_Q$ in the $Q$ phase only. The DPTM modified in such a way is shown to satisfy equilibrium Gibbs criteria for phase transitions. Location of phase boundaries in $\mu - T$ plane has been demonstrated to be changed but slightly; the modification tells mainly on baryon number density within $Q$ phase. Two alternative descriptions of nucleon-nucleon interaction -the Hard Core Model and the Mean Field Approximation - have been tested; the results for both cases appeared to be similar. All the results are shown to be stable against rather broad variations of model parameters.
1 Introduction

According to the fundamental QCD predictions strongly interacting matter has to exist in (at least) two different phases:\footnote{Here the QCD lattice calculations confirm this concept indicating to the first order phase transition at $T \approx 200$ MeV. Well above this temperature matter behaviour is easily described by QCD perturbation theory. Close to the critical conditions perturbation theory fails. However critical behaviour of strongly interacting matter is very important to analyze the phase transition conditions and the QGP signals. To study this problem one has to use some phenomenological models. Thermodynamic approach with two-phase matter Equations of State (EOS) has been widely used recently\footnote{We consider, following the common way,\footnote{In our earlier works, this problem has been reconsidered following the same ideology, but with different physical approach to the choice of the bag parameters, since lattice approach (not securing pion description) does not seem to be a safe basis in the case of $H \leftrightarrow Q$ transformation where pions play the decisive role.} bag type EOS for $QGP$ and usual nonrelativistic EOS for $H$ phase (taking into account hadron interaction). Such type models\footnote{First attempt to investigate this problem within the thermodynamic models with bag type EOS belongs to the Bielefeld group, then this problem has been investigated in Refs.\footnote{In our earlier works, this problem has been reconsidered following the same ideology, but with different physical approach to the choice of the bag parameters, since lattice approach (not securing pion description) does not seem to be a safe basis in the case of $H \leftrightarrow Q$ transformation where pions play the decisive role.}.)\footnote{A possibility for existence of the intermediate phase $Q$ formed by deconfined constituent quarks and pions (as necessary Goldstone bosons). In the form proposed it was actually demonstrated that such a $Q$ phase could exist.}}:
- Quark-Gluon Plasma ($QGP$), i.e. gas of deconfined and massless quarks $q$, antiquarks $\bar{q}$ and gluons $g$, - at super high temperature $T$ and energy density $\epsilon$, and
- Hadron Gas, or $H$-phase at low $T$ and/or $\epsilon$.

QCD lattice calculations confirm this concept indicating to the first order phase transition $H \leftrightarrow QGP$ at some critical temperature $T_c$, \textit{if none intermediate phase is taken into account} (to be called later on Single Phase Transition Model, or SPTM). However, in distinction to the great majority of researchers considering only direct phase transition $H \leftrightarrow QGP$, and thus assuming that, both, quark deconfinement and chiral symmetry restoration phase could exist. We have worked out\footnote{First attempt to investigate this problem within the thermodynamic models with bag type EOS belongs to the Bielefeld group, then this problem has been investigated in Refs.\footnote{In our earlier works, this problem has been reconsidered following the same ideology, but with different physical approach to the choice of the bag parameters, since lattice approach (not securing pion description) does not seem to be a safe basis in the case of $H \leftrightarrow Q$ transformation where pions play the decisive role.}} the Double Phase Transition Model (DPTM) based on the assumption of an intermediate phase $Q$ formed by deconfined constituent quarks and pions (as necessary Goldstone bosons). In the form proposed it was actually demonstrated that such a $Q$ phase could exist.

The idea of possibility of two phase transitions goes back to E. Shuryak\footnote{In our earlier works, this problem has been reconsidered following the same ideology, but with different physical approach to the choice of the bag parameters, since lattice approach (not securing pion description) does not seem to be a safe basis in the case of $H \leftrightarrow Q$ transformation where pions play the decisive role.} who put forward various arguments to point out that temperatures of quark deconfinement $T_d$ and chiral symmetry restoration $T_{ch}$ may not coincide, and $T_d$ should be less than $T_{ch}$. Later this idea was supported by other works\footnote{First attempt to investigate this problem within the thermodynamic models with bag type EOS belongs to the Bielefeld group, then this problem has been investigated in Refs.\footnote{In our earlier works, this problem has been reconsidered following the same ideology, but with different physical approach to the choice of the bag parameters, since lattice approach (not securing pion description) does not seem to be a safe basis in the case of $H \leftrightarrow Q$ transformation where pions play the decisive role.}}. It leads to the conclusion that there may exist some temperature interval $T_d < T < T_{ch}$ where quarks are liberated off individual hadrons but still possess non-zero mass. Such objects are well known from Additive Quark Model (AQM) and called \textit{constituent quarks, or valons.} They are necessary entities for satisfactory description of moderate energy hadron phenomena. Thus an intermediate phase of deconfined massive constituent quarks, $Q$-phase, may exist.

First attempt to investigate this problem within the thermodynamic models with bag type EOS belongs to the Bielefeld group, then this problem has been investigated in Refs.\footnote{In our earlier works, this problem has been reconsidered following the same ideology, but with different physical approach to the choice of the bag parameters, since lattice approach (not securing pion description) does not seem to be a safe basis in the case of $H \leftrightarrow Q$ transformation where pions play the decisive role.}. A possibility for existence of the intermediate phase $Q$ formed by deconfined constituent quarks and pions was in fact demonstrated. However the choice of the key model parameters based on the lattice calculation data for baryonless matter resulted in the negative conclusion that $H \leftrightarrow QGP$ transition should proceed almost always directly, without any intermediate state, since in temperature $T$ - chemical potential $\mu$ plane $Q$ phase occupies only tiny petals and can hardly play any essential role in reality.

In our earlier works\footnote{In our earlier works, this problem has been reconsidered following the same ideology, but with different physical approach to the choice of the bag parameters, since lattice approach (not securing pion description) does not seem to be a safe basis in the case of $H \leftrightarrow Q$ transformation where pions play the decisive role.} the problem has been reconsidered following the same ideology, but with different physical approach to the choice of the bag parameters, since lattice approach (not securing pion description) does not seem to be a safe basis in the case of $H \leftrightarrow Q$ transformation where pions play the decisive role.

This resulted in conclusion that $Q$ phase seemingly exists almost always (at least, for $\mu \leq 1$). In $\mu - T$ plane it occupies a corridor between $H$ and $QGP$ phases having the width $\Delta T = T_{ch} - T_d \approx 50$ MeV. The value of $T_d$ was found to be equal to some 150 MeV and its physical meaning (the highest temperature allowing for existence of hadrons, above $T_d$ they have to decay into constituent quarks) enables to identify it with the Hagedorn temperature. Both phase transitions are of the first order.

The qualitative stability of these results for varying values of parameters (within reasonable limits) has been demonstrated. More details are given in the section 2.

The present paper is devoted to the problem of entropy and baryon number conservation when crossing phase transition boundaries. This problem appeared already in the common model...
with single (first order) phase transition, SPTM. It has been shown within SPTM that as a result of the transition \( H \leftrightarrow QGP \) the entropy per baryon \( S/N_B \) is discontinuous across the phase transition boundary:

\[
(S/N_B)_{QGP} > (S/N_B)_H.
\]

This means that transition \( H \leftrightarrow QGP \) is irreversible: it could not satisfy thermal and chemical equilibrium conditions, and, at the same time, fulfill baryon number and entropy conservation at the phase boundary. In particular, adiabatic transition \( QGP \rightarrow H \) is impossible as it should be accompanied by the entropy decrease (since conservation of the baryon number is secured).

Thus one has to choose between two possibilities:
- either the transition from \( QGP \) into \( H \)-phase can not proceed as an adiabatic one under any conditions,
- or EOS used are not fully correct.

The first possibility has been discussed recently in connection with large value of specific entropy \( (S/N_B)_H \) detected in experiments on high energy heavy ion collisions. It was used as an argument for assumption that \( QGP \) has been actually observed.

The later possibility seems rather realistic due to uncertainties presenting in phenomenological EOS. In particular, bag-type EOS used for \( QGP \) includes the key model parameter \( B_{QGP} \) (representing nonperturbative interactions of quarks and gluons with physical vacuum). In various works on SPTM its value was chosen rather arbitrary: \( B_{QGP} = 0.2 \pm 0.5 \) GeV/fm\(^3\). Usually it is treated as constant, but there are no reasons for \( B \) not to depend on \( T \) and/or \( \mu \).

Accordingly in there was suggested a certain modification of \( B_{QGP} \) making it \( \mu \) and \( T \) dependent, \( B(\mu, T) \), instead of commonly used \( B_{QGP} = \text{const} \). This enabled to restore the continuity of \( S/N_B \) and thus to solve the problem immediately. This modification of EOS was shown to change phase diagram of the system not considerably, while the transition \( QGP \leftrightarrow H \) becomes reversible.

In this paper we study the problem of specific entropy conservation within DPTM. There appears the same discontinuity of \( S/N_B \) when crossing both transition boundaries, thus both phase transitions appear to be irreversible. It is shown that the reversibility of both phase transitions can be restored by modifying the \( Q \) phase EOS alone, without changing \( B_{QGP} \).

The paper is organized as follows. In section 2 we remind basic features of the DPTM. Specific entropy discontinuity and the method of its correction within SPTM is discussed in section 3; in section 4 the same problem is discussed within DPTM; results of numerical calculations are presented in subsection. Section 5 represents summary and discussion. Some details of hydrodynamical description of equilibrium system evolution are given in Appendix.

## 2 Basic features and main results of DPTM

We use, following common way[1][2][3], bag-type model EOS for \( QGP \) and \( Q \) phases: \( (p, g, m_i) \) are pressure, degeneracy factors, masses of \( i \)-th type particles respectively for each \( j \)-th phase; \( j \) means hadronic \( H \), valonic \( Q \), and \( QGP \) phases respectively.

\[
p_{QGP}(T, \mu, V) = \frac{\pi^2}{90}(g_g + g_q) \left[ T^4 + T^2 \mu^2_q + \frac{1}{2\pi^2} \mu^4_q \right] - B_{QGP}, \quad \text{(1)}
\]

\[
p_Q(T, \mu, V) = \int \frac{k^4dk}{\sqrt{k^2 + m_i^2}} \left[ \frac{1}{\exp\left(\frac{k^2 + m_i^2}{T}\right) - 1} + \frac{1}{\exp\left(\frac{\sqrt{k^2 + m_i^2} - \mu_{q_i}}{T}\right) + 1} \right] - B_Q, \quad \text{(2)}
\]

Massless gluons \( (g) \) and quarks \( (q) \) participate in \( QGP \); \( Q \) phase contains constituent quarks \( (u, d, s) \) with \( m_u \simeq m_d \simeq 320 \text{ MeV} \) and \( m_s \simeq 512 \text{ MeV} \), and pions; \( \mu_{q_i} \) being chemical potential of
constituent quarks, equal to that of corresponding current quarks \((\mu_q = \mu_u = \mu_d, \text{ and } \mu_s \text{ is taken to be zero})\).

The terms \(B_i\) in EOS of \(Q\) and \(QGP\) reflect effective interactions with vacuum. \(B_Q, B_{QGP}\) are free bag parameters chosen according to their physical meaning: \(B_{QGP}\) is QCD vacuum energy-density known \([8]\) to be \(\approx 0.5 \div 1 \text{ GeV/fm}^3\), and \(B_Q\) is estimated from low energy phenomena \([8]\) as \(50 \div 100 \text{ MeV/fm}^3\) (close to the B value of MIT-bag model). Note that this choice of \(B\) values (instead of that chosen in \([1,2]\)) is crucial for appearance of the intermediate \(Q\) phase.

For the hadronic phase \(H\):

\[
p_H(T, \mu, V) = \frac{g_π}{6\pi^2} \int \frac{k^4dk}{\sqrt{k^2 + m_π^2}} \frac{1}{e^{\frac{\sqrt{k^2 + m_π^2}}{T}} - 1} + \sum_i \frac{g_i}{6\pi^2} \int \frac{k^4dk}{\sqrt{k^2 + m_i^2}} \left[ \frac{1}{e^{\frac{\sqrt{k^2 + m_i^2 - \mu_i}}{T}} + 1} + \frac{1}{e^{\frac{\sqrt{k^2 + m_i^2 + \mu_i}}{T}} + 1} \right] + \phi(U(\nu))
\]

The first term in (3) represents pion contribution; the summation (in the second term) is over all \(i^{\text{th}}\) type stable hadrons dominating in the \(H\) phase \((\pi, N, \Lambda \text{ and } K\) were taken into account); \(m_i, \mu_i\) and \(g_i\) are the corresponding masses, chemical potentials and degeneracy factors. The last term stands for account of nucleon-nucleon interactions in the form of Mean Field Approximation (MFA) \([2]\); we have used also Hard Core Model (HCM) for nucleon interaction description \([3,4]\) (where the form of interaction account in the \(H\) phase is more complicated).

In thermodynamic equilibrium, according to Gibbs principle, the actually realized phase is that with the largest pressure at given \(\mu\) and \(T\). Then at given value of nucleon chemical potential, \(\mu\) there are 3 possible transition temperatures:

- Deconfinement transition \((H \leftrightarrow Q)\) at \(T_d\):
  \[
p_H(T_d, \mu) = p_Q(T_d, \mu/3)
\]

- Direct transition \((H \leftrightarrow QGP)\) at \(T_c\):
  \[
p_H(T_c, \mu) = p_{QGP}(T_c, \mu/3)
\]

- Chiral transition \((Q \leftrightarrow QGP)\) at \(T_{ch}\):
  \[
p_Q(T_{ch}, \mu/3) = p_{QGP}(T_{ch}, \mu/3)
\]

The case of coincidence of all three transitions corresponds to the triple point at \(T^*\):

\[
p_H(T^*, \mu) = p_Q(T^*, \mu/3) = p_{QGP}(T^*, \mu/3)
\]

\(Q\) phase actually exists if for rising temperature the deconfinement of valons occurs first, prior to direct formation of the \(QGP\) phase, i.e. \(T_d < T_c\); and, in the opposite direction (for decreasing temperature), if chiral transition from \(QGP\) to \(Q\) phase occurs prior to formation of \(H\) phase, i.e. \(T_{ch} > T_c\). Thus general condition for \(Q\) phase existence is:

\[
T_d(\mu) < T_c(\mu) < T_{ch}(\mu).
\]

In this case the direct transition does not occur. Otherwise DPTM reduces to a model with single phase transition and its results coincide with that of SPTM. This very case have been met in the papers \([6,7]\) due to specific choice of the \(B\) parameters.

The choice of model parameters based on its physical meaning resulted in the quite opposite conclusions, namely \([6,7]\):

i) \(H \leftrightarrow QGP\) transition proceeds almost exclusively via the \(Q\) phase, \(H \leftrightarrow Q \leftrightarrow QGP\).

ii) Deconfinement of valons \(H \rightarrow Q\) should occur at rather low energy density of nuclear matter \(\approx 0.3\div0.4 \text{ GeV/fm}^3\) (only three times larger than energy density in a normal nucleus, as it was roughly estimated in \([1]\)).

iii) Temperature interval for \(Q\) phase, \(\Delta T = T_{ch} - T_d\), amounts to \(\approx 50 \text{ MeV}\) (see Fig. 1). For baryonless matter, typically, \(T_d \approx 140 \text{ MeV} \text{ and } T_{ch} \approx 200 \text{ MeV}\). Thus \(T_d\) coincides with the well known Hagedorn temperature (as it should be since at \(T_d\) hadrons cease to exist and decay into constituent quarks), which had been treated earlier as some approximation to direct transition \(H \leftrightarrow QGP\) temperature \([8]\) and now is shown to have actually independent physical meaning.

These results turned out to be qualitatively stable against extended variations of model parameters and nucleon interaction description.
3 Restoration of specific entropy continuity in SPTM.

As it has been said above, within SPTM the specific entropy value turns out to be discontinuous when crossing the direct transition boundary (the ratio of specific entropy values above and below the direct transition are presented in Fig. 2a). However, it was shown\(^\text{18}\) that the value \((S/N_B)_{QGP}\) can be corrected by the modification: \(B_{QGP} \to B_{QGP}(\mu,T)\). Indeed, according to general thermodynamic relations entropy density \(s\) and baryon number density \(n\) in QGP are defined from:

\[
n_{QGP}(\mu,T) \equiv \frac{\partial p_{QGP}(\mu,T)}{\partial \mu} = n_{QGP}^0(\mu,T) - \frac{\partial B_{QGP}(\mu,T)}{\partial \mu},\tag{5}
\]

\[
s_{QGP}(\mu,T) \equiv \frac{\partial p_{QGP}(\mu,T)}{\partial T} = s_{QGP}^0(\mu,T) - \frac{\partial B_{QGP}(\mu,T)}{\partial T},\tag{6}
\]

where zero superscripts indicate corresponding values calculated for constant \(B\). Other thermodynamic functions in QGP (\(\epsilon, p\), enthalpy \(w\)) remain undependent on \(B\)'s derivatives.

Thus to restore conservation of specific entropy it is possible to determine \(B_{QGP}(\mu,T)\) from the differential equation:

\[
\frac{s_{QGP}^0(\mu,T) - \frac{\partial B_{QGP}(\mu,T)}{\partial T}}{n_{QGP}^0(\mu,T) - \frac{\partial B_{QGP}(\mu,T)}{\partial \mu}} = \frac{s_H(\mu,T)}{n_H(\mu,T)}.\tag{7}
\]

Since \(S/N_B\) value is not defined at the points \(\mu = 0\) and \(T = 0\) it seems natural to fix an integrating constant:

\[
B_{QGP}(0,T) = B_{QGP}(\mu,0) = B_{QGP}^0.
\]

Let us stress that the function \(B_{QGP}(\mu,T)\) satisfying eq. (7) provides equal behaviour of specific entropy functions in both phases everywhere in \(\mu-T\) plane, thus, in particular, conservation of the specific entropy at the phase transition boundary.

This equation has been solved in Ref.\(^\text{18}\); it was shown that the obtained \(B_{QGP}(\mu,T)\) does not change phase diagram considerably.

However, there still remain several questions concerning this procedure. In particular, \(B_{QGP}(\mu,T)\), as a solution of the eq. (7), is defined in the open region in \(\mu-T\) plane, i.e. for any temperature \(T > T_c(\mu)\). Since it depends on the value of specific entropy in \(H\) phase, it means that EOS of QGP (in particular, the bag constant) should store information on the \(H\)-phase EOS everywhere including far high-\(T\) limit. This looks suspicious since \(B_{QGP}\) represents pressure of the physical vacuum and has nothing in common with a particular hadronic system.

This question becomes serious at relatively large \(\mu \geq 0.9\) GeV/fm\(^3\) where hadron interactions become decisive in the \(H\) phase.\(^\text{6}\) One has to use again phenomenological models for description of those interactions (e.g., HCM and MFA) which give differing results for the \((S/N_B)_H\) value (see Fig. 2a). Correspondingly, \(B_{QGP}(\mu,T)\) modified according to the procedure described above should depend on description of nucleon interactions in the \(H\) phase.

Besides, there appears some uncertainty concerning fulfillment of Gibbs relation

\[
\epsilon + p - \mu n = sT,\tag{8}
\]

which is to be satisfied in an equilibrium system. Note that this imposes additional constraint on \(B\)'s derivatives in the case of modified \(B_{QGP}(\mu,T)\). Indeed, combining (8) with (5),(6) one gets the condition:

\[
\mu \frac{\partial B(\mu,T)}{\partial \mu} = -T \frac{\partial B(\mu,T)}{\partial T},
\]

which, seemingly, is not satisfied within the procedure used in\(^\text{18}\).

Another problem concerns dynamical evolution of the system in question. The analysis presented refers to the stationary systems. Being related to modern experiments on high energy heavy ion collisions, it is to be included in hydrodynamical model of the system evolution (for
qualitative analysis the simplest Bjorken scaling solution is used). However the condition (7) does not provide equilibrium character of first order phase transition in dynamically evolving systems. Such transitions are to proceed through the mixed phase state where not only entropy and baryon number are to be conserved, as for all equilibrium processes, but enthalpy of the system as well (the latent heat works on the system expansion).

To provide conservation of these three thermodynamic variables it is necessary and sufficient (see Appendix A, eq.(27)) to fulfill the condition:

$$\frac{w_{QGP}(T_c, \mu/3)}{w_H(T_c, \mu)} = \frac{s_{QGP}(T_c, \mu/3)}{s_H(T_c, \mu)} = \frac{n_{QGP}(T_c, \mu/3)}{n_H(T_c, \mu)}.$$  \hspace{1cm} (9)

Thus, both $B_{QGP}(\mu, T)$ derivatives are to be defined according to (9), at least, at the transition boundary $T_c(\mu)$. Then fixing integrating constant in both ending points, $\mu = 0$ and $T = 0$, as it was done in [4], makes the problem over-defined.

4 Entropy and baryon number conservation in DPTM

The same problem arises also for transitions considered within DPTM, i.e. for deconfinement and the chiral transition. The ratio $S/N_B$ turns out to be discontinuous when crossing both transition boundaries. It is illustrated in Fig.2b, where "jumps" of $S/N_B$ ratio at the corresponding transition boundaries are presented as calculated, both, in the HCM and the MFA phenomenological models describing nucleon interactions in the $H$ phase.

Following the same way as in [4] we try to reconsider EOS for deconfined phase. However, within DPTM it seems natural and reasonable to modify $Q$-phase EOS alone making the bag pressure parameter $B_Q$ $\mu$ and $T$ dependent, $B_Q(\mu, T)$, with QGP bag parameter $B_{QGP}$ remaining constant.

The reasons are as follows:

i). $Q$-phase is the intermediate phase between $H$ and QGP, thus $Q$ phase EOS can serve as a tool for compensation of defects of other phases EOS (remaining the same as earlier) which may occur invalid close to the transition boundaries.

ii). $B_Q$ is chosen even more arbitrarily than $B_{QGP}$ : the last one is to coincide with the QCD vacuum pressure estimated usually as 0.5 GeV/fm$^3$, while the $B_Q$ is to be closed to the bag pressure within the MIT-bag model, and thus varies within the interval: $B_Q \approx 50 \div 100$ MeV/fm$^3$.

iii). The discontinuity of $S/N_B$ is smaller when crossing the deconfinement and the chiral boundaries, or that for direct transition, (see Fig. 2), thus it is easier to modify $Q$-phase EOS only.

iv). $B_Q$ is defined for (and has a physical sense) within closed region of phase space where the $Q$ phase can exist (in $\mu - T$ plane: the region bounded by $T_d(\mu)$ and $T_{ch}(\mu)$ curves and $\mu=0$ axis).

Thus there arises no problem with securing proper $B_Q$ behaviour at high-$T$ limit, as it appeared for $B_{QGP}(\mu, T)$.

v). modified $B_Q(\mu, T)$ would also depend on hadron interaction description, and this dependence becomes considerable in high-$\mu$ limit. However it is quite natural for $B_Q$ to be model dependent: EOS of the intermediate phase has to vary in accordance with $H$-phase EOS variations (in distinction to model dependence of $B_{QGP}(\mu, T)$ in SPTM which seems to be unnatural).

The modification in question, $B_Q(\mu, T)$, results in change of entropy and baryon number density within $Q$ phase:

$$n_Q(\mu, T) = n_Q^0(\mu, T) - \frac{\partial B_Q(\mu, T)}{\partial \mu},$$ \hspace{1cm} (10)

$$s_Q(\mu, T) = s_Q^0(\mu, T) - \frac{\partial B_Q(\mu, T)}{\partial T},$$ \hspace{1cm} (11)

where zero superscripts indicate corresponding values calculated for constant $B_Q$. Other thermodynamic variables in $Q$-phase ($\epsilon_Q, p_Q, w_Q$) remain unchanged, i.e. do not depend on $B_Q$’s derivatives.
To provide conservation of specific entropy when crossing transition boundaries and proper behavior during the equilibrium mixed phase evolution one needs to fulfill the following conditions (see Appendix, eq.(28)):

\[
\frac{w_Q(T_d, \mu/3)}{w_H(T_d, \mu)} = \frac{s_Q(T_d, \mu/3)}{s_H(T_d, \mu)} = \frac{n_Q(T_d, \mu/3)}{n_H(T_d, \mu)}.
\]

\[
\frac{w_{QGP}(T_{ch}, \mu/3)}{w_Q(T_{ch}, \mu/3)} = \frac{s_{QGP}(T_{ch}, \mu/3)}{s_Q(T_{ch}, \mu/3)} = \frac{n_{QGP}(T_{ch}, \mu/3)}{n_Q(T_{ch}, \mu/3)},
\]

valid at \(Q\)-phase boundaries \(T_d(\mu)\) and \(T_{ch}(\mu)\). Combining together (10)-(13) we get certain constraints on \(B_Q(\mu, T)\) function instead of differential equation similar to (7). Namely:

- at the deconfinement boundary, \(T(\mu) = T_d(\mu)\):
  \[
  \left(\frac{\partial B_Q}{\partial \mu}\right)_{T=T_d(\mu)} = (n_Q^0 - n_H \frac{w_Q^0}{w_H})_{T=T_d(\mu)};
  \]
  \[
  \left(\frac{\partial B_Q}{\partial T}\right)_{T=T_d(\mu)} = (s_Q^0 - s_H \frac{w_Q^0}{w_H})_{T=T_d(\mu)};
  \]

- at the chiral restoration boundary, \(T(\mu) = T_{ch}(\mu)\)
  \[
  \left(\frac{\partial B_Q}{\partial \mu}\right)_{T=T_{ch}(\mu)} = (n_{QGP}^0 - n_Q \frac{w_Q^0}{w_{QGP}})_{T=T_{ch}(\mu)};
  \]
  \[
  \left(\frac{\partial B_Q}{\partial T}\right)_{T=T_{ch}(\mu)} = (s_{QGP}^0 - s_Q \frac{w_Q^0}{w_{QGP}})_{T=T_{ch}(\mu)};
  \]

- at \(\mu = 0\):
  \[
  B_Q(0, T) = B_Q^0 = \text{const}
  \]

- inside the \(Q\) phase region there is no special constraints on \(B_Q\) (besides the Gibbs relation (8) which is valid for any \(T\) and \(\mu\), and solution of the problem is not unique, thus the function \(B_Q(\mu, T)\) can be chosen rather arbitrarily.

In classical physics similar problems arise, eg., when simulating soap films stretched on some hard contour (so called two-dimensional Plateau problem). Variation methods for such problems are well elaborated. We have used a numerical procedure providing function \(B_Q(\mu, T)\) which belongs to the so called minimal surface class, securing local minimum of the surface functional:

\[
\int \int <Q> d\mu dT \sqrt{1 + \partial_\mu B_Q(\mu, T)^2 + \partial_T B_Q(\mu, T)^2}
\]

with given boundary conditions (14)-(18). Fulfillment of the Gibbs relation (8) has been tested.

Note that this procedure fails near \(T = 0\) region where the accuracy of calculation becomes worse; this case needs special investigation. However we are interested mainly in high-\(T\) region because this very case can be related to modern experiments on heavy ion ultrarelativistic collisions where formation of deconfined phase(s) seems to be rather probable.

### 4.1 Results of numerical calculations

The procedure described has been fulfilled within HCM and MFA models for the following parameter values:

\[
B_{QGP} = 0.5 \text{ GeV/fm}^3, \quad B_Q^0 = 70 \text{ MeV/fm}^3.
\]
Fig. 3a represents the crossection of the obtained $B_Q(\mu, T)$ surface for HCM by the deconfinement transition boundary $T = T_d(\mu)$ (solid line) and the chiral transition boundary $T = T_{ch}(\mu)$ (dashed line). The same curves for MFA model are presented in Fig. 3b. It is seen that $B_Q(\mu, T)$ remains practically unchanged for small $\mu$ values $\mu \leq 0.6 GeV$. For larger $\mu$, $B_Q(\mu, T)$ value varies within rather broad interval (relatively to its average value): $50 \text{ MeV/fm}^3 \leq B_Q(\mu, T) \leq 90 \text{ MeV/fm}^3$. However the interval of $B_Q$ variation is small as compared to difference between $B_{QGP}$ and $B_Q^0$. Moreover, phase diagrams before and after $B_Q$ modification (see Fig. 4) do not differ significantly. The region of $Q$ phase existence remains rather broad with varying $B_Q$ as well.

The modification procedure described above influences mainly the baryon number density in the $Q$ phase (see Fig. 5), corresponding entropy density corrections satisfy Gibbs relation and are relatively small (everywhere except low $T$ region).

It deserves stressing that $B_Q(\mu, T)$ differs for HCM and MFA models reflecting intrinsic incorrectness of the models themselves: $Q$ phase does play its role of intermediate state compensating entire defects of $H$ phase EOS. The boundary of chiral transition does not depend practically on peculiarities of nucleon interaction in $H$ phase; this means that EOS of $QGP$ does not remember the $H$-phase interactions. This seems to be quite reasonable.

5 Summary and discussion

It has been shown that within DPTM the modification $B_Q \rightarrow B_Q(\mu, T)$ enables to provide proper behaviour of thermodynamic functions for reversible equilibrium phase transition, in particular, the mixed phase scenario. Phase diagram of three-phases matter is not practically changed. The correction concerns mainly baryon number density inside the intermediate $Q$-phase (its changes are not essential). The main result of DPTM - existence of a broad corridor of $Q$ phase in $\mu - T$ plane - remains entirely valid.

Note that within DPTM modifications are necessary and considerable only for sufficiently large chemical potential, $\mu \geq 0.6-0.8$ GeV. For small $\mu$ (most interesting for experimental data analysis) the equilibrium character of deconfinement phase transition is almost automatically saved, and the change of the chiral transition boundary is negligible.

Let us stress that EOS of $QGP$ remains unchanged within the method used, thus the problem of the direct transition $QGP \leftrightarrow H$ irreversibility remains as well. But within DPTM the transition is to proceed through the intermediate $Q$ phase so that direct transition $H \leftrightarrow QGP$ should not occur normally. However there still remains the possibility for $QGP$ to overcool (too fast) below the critical temperature $T_{ch}$, then the nonequilibrium and thus irreversible phase transition should occur.

It deserves mentioning that the interest to the problem of specific entropy discontinuity has been inspired mainly by recent experimental data on heavy ion high energy collision reporting a large value of $S/N_B$ for hadrons resulting from the collision. In this very connection it has been pointed out that experimental data does agree with the value typical for $QGP$ and could appear in experiment due to abrupt nonequilibrium phase transition $QGP \rightarrow H$. However, it should be stressed that the ratio $S/N_B$ calculated within DPTM for the resulting hadron gas (see Fig. 6) is much higher than that for direct transition (and almost the same as in intermediate $Q$ phase). It is connected with relatively low temperature of $H$-phase formation, $T_d \simeq 140 \text{ MeV}$. Thus experimental data could be as well described by equilibrium transition $Q \rightarrow H$ instead of abrupt direct transition.

In conclusion let us point out another problem concerning EOS uncertainties. There were put out arguments based on theoretical analysis of effective Lagrangian that hadron (and valon as well) masses are to decrease for temperature increasing (since the mass of any hadron is believed to be proportional to the quark condensate to the order $1/3$), with the fastest decrease (down to zero) being close to critical temperature of direct transition in SPTM. Actually this decrease means that the deconfined phase with broken chiral symmetry transforms smoothly into $QGP$ without the (at least, first order) phase transition. In accordance with such approach DPTM is to be modified in such a way that change of valon masses and corresponding change of bag pressure parameters (which have to be connected with quark condensate and gluon condensate) are taken into account. It seems natural that in this case there occurs the only phase transition, the deconfinement one, $H \leftrightarrow Q$, while transformation of the $Q$ phase into $QGP$ proceeds smoothly, with constituent
quark mass approaching zero. Then the problem of the deconfinement transition irreversibility remains actual and needs special analysis.

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Appendix

Hydrodynamical equations describing the system evolution are to be written for energy-momentum tensor; they express the local conservation laws. Neglecting dissipative effects, they are:

\[ \partial_{\mu} T^{\mu\nu}(x) = 0, \quad T^{\mu\nu}(x) = [\epsilon(x) + p(x)]u^\mu(x)u^\nu(x) + p(x)g^{\mu\nu}, \]  

(20)

where \( \epsilon \) is the energy density, \( p \) the pressure in the proper system of fluid element, \( g^{\mu\nu} \) the metric tensor (with \( g^{00} = -1 \)) and \( u^\mu \) is the four velocity of the local flow, \( x \) being the coordinate of the fluid element in the 4-dimension space. Conservation of the entropy and baryon number requires for:

\[ \partial_\nu (u^\nu s) = 0, \]  

(21)

\[ \partial_\nu (u^\nu n) = 0, \]  

(22)

where \( s \) and \( n \) being respectively the entropy density and baryon number density in the proper system of fluid element. In the case of one-dimensional scaling solution commonly used for qualitative description of evolution of the matter created in heavy ion collisions, local conservation of energy-momentum tensor takes the form (\( \tau \) being the proper time of fluid element):

\[ \frac{d\epsilon}{\epsilon + p} = -\frac{d\tau}{\tau}. \]  

(23)

Until \( p \) (as well as \( T \)) is constant during the mixed phase the last equation presents enthalpy \( w = \epsilon + p \) conservation law. Local conservation of entropy and baryon number is expressed in a similar form:

\[ \frac{ds}{s} = -\frac{d\tau}{\tau}, \quad \frac{dn}{n} = -\frac{d\tau}{\tau}. \]  

(24)

Thus to provide a possibility for an equilibrium transition process through the mixed phase state the following conditions are required:

\[ \frac{dw}{w} = \frac{ds}{s} = \frac{dn}{n} = -\frac{d\tau}{\tau}. \]  

(25)

In the case of direct transition the mixed phase state is described as a mixture of \( H \) and \( QGP \) fractions taken at constant temperature \( T_c \) and given chemical potential \( \mu \):

\[ w = w_H(T_c, \mu)\lambda(\tau) + w_{QGP}(T_c, \mu/3)(1 - \lambda(\tau)), \]  

(26)

\( \lambda(\tau) \) being the share of the \( H \)-phase admixture which changes during the mixed phase expansion from zero (for pure \( QGP \) phase) to the unity (for pure hadronic matter). The same representation is taken for all additive thermodynamic variables (\( s, n, \epsilon \)). Thus, (25) and (26) combining together lead to the following requirement:

\[ \frac{w_{QGP}(T_c, \mu/3)}{w_H(T_c, \mu)} = \frac{s_{QGP}(T_c, \mu/3)}{s_H(T_c, \mu)} = \frac{n_{QGP}(T_c, \mu/3)}{n_H(T_c, \mu)}. \]  

(27)
In general case of any first order equilibrium phase transition at some transition temperature $T_{tr}$ corresponding requirement takes the form:

$$\frac{w^+ (T_{tr}, \mu)}{w^- (T_{tr}, \mu)} = \frac{s^+ (T_{tr}, \mu)}{s^- (T_{tr}, \mu)} = \frac{n^+ (T_{tr}, \mu)}{n^- (T_{tr}, \mu)},$$

(28)

where (+) and (-) refer to thermodynamic variables above and below the transition boundary $T_{tr}(\mu)$.

Let us stress that for equilibrium mixed phase scenario it is necessary and sufficient to fulfill the conditions (28) only at transition boundary, but not for any $T$ and $\mu$. Besides, for $\mu = 0$ (when $n=0$) and $T=0$ (when $s=0$) cases these conditions are satisfied automatically due to common thermodynamic Gibbs relation (8).

It deserves stressing (and can be easily shown) that fulfillment of the conditions (28) provides automatically securing of the Gibbs relation (8) in one of the neighboring phases if it is valid for the another one.
Figure Captions

**Figure 1.** Phase diagram in $\mu - T$ plane for nucleon interaction description within HCM (solid) and MFA (solid with symbols). Dashed lines correspond to the direct transition in SPTM.

**Figure 2a.** Ratio of specific entropy values above (+) and below (-) the *direct* transition, $\frac{S}{N_B}^+ : \frac{S}{N_B}^-$, as a function of baryonic chemical potential $\mu$ along the transition boundary. Calculated for HCM (solid) and MFA (solid-symbols).

**Figure 2b.** The same ratio as in Fig. 2a at the deconfinement (solid) and chiral (dashed) transition boundaries; curves for MFA model are indicate by symbols. Note the different ordinate scales.

**Figure 3a.** Crossection of the obtained $B_Q(\mu, T)$ surface by the deconfinement transition boundary $T = T_d(\mu)$ (solid line) and the chiral transition boundary $T = T_{ch}(\mu)$ (dashed line) as a function of baryonic chemical potential $\mu$. Calculated within HCM.

**Figure 3b.** The same as in Fig. 3a calculated within MFA.

**Figure 4a.** Phase diagram within HCM for $B_Q=70$ MeV/fm$^3$ =Const (dashed) and modified $B_Q(\mu, T)$ (solid). Short-dashed lines correspond to the limiting values of modified $B_Q(\mu, T)$: $B_Q=50$ MeV/fm$^3$ and $B_Q=90$ MeV/fm$^3$ for deconfinement and chiral transitions respectively.

**Figure 4b.** The same as in Fig. 4a for MFA.

**Figure 5a.** Baryon number density in $Q$ phase $n_Q$ as a function of baryonic chemical potential $\mu$ at deconfinement (d) and chiral (ch) transition boundaries calculated for $B_Q=$Const (dashed) and modified $B_Q(\mu, T)$ (solid) for HCM.

**Figure 5b.** The same as in Fig. 5a for MFA.

**Figure 6.** Entropy per baryon in $H$ phase near the deconfinement in DPTM (solid) and direct in SPTM (dashed) transition boundaries as function of quark fugacity $\lambda = \exp(\mu_q/T)$. Short-dashed line corresponds to the same ratio in $QGP$ phase. Calculated within HCM.
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Figure 1
Figure 4a

Figure 4b
Figure 5a

Figure 5b
Figure 6