The Dark Side of a Patchwork Universe

Martin Bojowald
Institute for Gravitational Physics and Geometry, The Pennsylvania State University, 104 Davey Lab, University Park, PA 16802, USA

Abstract

While observational cosmology has recently progressed fast, it revealed a serious dilemma called dark energy: an unknown source of exotic energy with negative pressure driving a current accelerating phase of the universe. All attempts so far to find a convincing theoretical explanation have failed, so that one of the last hopes is the yet to be developed quantum theory of gravity. In this article, loop quantum gravity is considered as a candidate, with an emphasis on properties which might play a role for the dark energy problem. Its basic feature is the discrete structure of space, often associated with quantum theories of gravity on general grounds. This gives rise to well-defined matter Hamiltonian operators and thus sheds light on conceptual questions related to the cosmological constant problem. It also implies typical quantum geometry effects which, from a more phenomenological point of view, may result in dark energy. In particular the latter scenario allows several non-trivial tests which can be made more precise by detailed observations in combination with a quantitative study of numerical quantum gravity. If the speculative possibility of a loop quantum gravitational origin of dark energy turns out to be realized, a program as outlined here will help to hammer out our ideas for a quantum theory of gravity, and at the same time allow predictions for the distant future of our universe.

1 Introduction

A complete understanding of the universe currently faces several problems, most of which are occasionally expected to be solved by some version of quantum gravity. This also applies to the dark energy problem. Its first, older part is the cosmological constant problem, i.e. the divergence of vacuum energy density when it is naively computed in quantum field theory, and its being off by orders of magnitude if computed with reasonable-looking cut-offs. More precisely, the question is how vacuum energy couples to gravity, and which subtraction of the quantum field theoretical result has to be employed to derive the

\* Contribution to the special issue on Dark Energy by the journal General Relativity and Gravitation.
\[email address: bojowald@gravity.psu.edu]
gravitationally relevant value. Any fundamental theory must provide a solution to this part of the problem, since a fundamental theory must provide a well-defined answer to a well-defined question such as that of the energy density in a vacuum state. Whether or not quantum gravity can solve this part of the dark energy problem is thus part of the question whether quantum gravity will be a fundamental theory. This is sometimes expected, but there is an independent relation to quantum gravity: an ultraviolet cut-off would make the vacuum energy finite, which is often done by hand although it gives, at best, confusing results. Since one often associates an underlying space-time discreteness with quantum gravity, and thus an upper bound for wave numbers and frequencies, a natural cut-off may be provided to result in a finite vacuum energy. A precise implementation would provide means to determine the correct coupling of vacuum energy to gravity.

While the second part of the problem—nailing down a dynamical culprit for dark energy—is more specific, its relation to quantum gravity is only vague. This part of the problem became acute only after recent observational indications that the current evolution of the universe is best modeled by a Friedmann–Robertson–Walker space-time if a negative pressure component to its energy balance became dominant in the recent past. Giving a precise value for the corresponding dark energy contribution, combined with information on its equation of state, the problem becomes more specific, and also closer to providing explicit tests of falsifiable candidates for solutions to the dark energy problem or the quantum gravity theories on which they may be based. This is, for instance, possible in candidate theories leading to additional, so far unobserved, fields which could provide the dark energy component.

Loop quantum gravity (Rovelli, 2004; Ashtekar and Lewandowski, 2004; Thiemann, 2001), the topic of this article, is not of this kind of a theory with surplus fields, at least not in its presently realized form. There are no fields in addition to the gravitational one and whatever matter fields one assumed before setting out to quantize the theory. (It could, however, be possible that additional degrees of freedom emerge in a yet to be developed effective description; see e.g. (Bilson-Thompson et al., 2006) for a suggestion.) What is realized in loop quantum gravity and in fact presents one of its main features is an underlying discrete structure of space (Rovelli and Smolin, 1995a; Ashtekar and Lewandowski, 1997a,b), so that one may at least address the first part of the problem. This requires making precise the relation between spatial discreteness and quantum field theoretical cut-offs. As for the second part of the problem, what is provided by existing phenomenological descriptions of some regimes of loop quantum gravity are modifications to matter equations of state which, in an average effect, could resemble a dark energy contribution. These possibilities, which are currently all to be considered speculative, are described here.

The specific difficulty of addressing the dark energy problem from the viewpoint of loop quantum gravity is that it requires detailed information on the behavior of inhomogeneous configurations. While the dynamics of loop quantum gravity has been understood quite well in homogeneous models (Bojowald, 2005a), which already provides access to several

---

1Not all general relativistic situations allow a global energy concept, but at least an approximate notion of energy density must exist for nearly vacuum states of fields in a maximally symmetric space-time.
problems in cosmology, this cannot result in deviations from classical gravity resembling dark energy in large, very classical universes. Quantum corrections to homogeneous models only appear when the total spatial size is small, close to the Planck scale, or when anisotropies are so big that curvature components become large. This is certainly not realized in those recent phases of our universe which the dark energy problem refers to. Any dark energy contribution from loop quantum gravity must then be more indirect; it cannot be a single quantum correction term to a classical evolution equation but at best a cumulative effect, obtained by coherently adding up small corrections. A precedent for such an effect in loop cosmology has recently been found (Bojowald et al., 2007c): small quantum corrections can add up coherently during long cosmic evolution times to give more sizable and potentially observable effects. If something similar happens in space instead of time, a sizable dark energy component could result by adding coherently small quantum corrections occurring at separate spatial points in a nearly Friedmann–Robertson–Walker spacetime. As we will see below, this provides a well-defined setup which can be addressed, though not yet fully evaluated, in loop quantum gravity.

We will first introduce the basic features of loop quantum gravity and cosmology. This will then be used to review its present contributions to both parts of the dark energy problem. The first part is being addressed by an attempt to make contact between quantum gravity, in which all fields including the gravitational one are quantized, and quantum field theory on a classical curved manifold. The second part can be analyzed in the context of specific models which describe how a discrete quantum state of space-time evolves, including refinements of the discrete structure as the universe expands. This refers to detailed dynamical properties, which makes the problem technically complicated but also indicate that a dedicated analysis can provide stringent tests of the underlying formulation of loop quantum gravity. As we will see, some of the known properties of dark energy are successfully modeled in this way.

2 The universe according to loop quantum gravity

Universe dynamics is described in loop quantum gravity by an “evolving” graph as a superposition of states representing the spatial quantum geometry. Many of the typical properties of physical scenarios based on loop quantum gravity are directly related to this underlying discreteness, which will also be the main scheme used in this article. The emphasis on spatial geometry arises because of the canonical quantization used to set up the theory. As in classical canonical formulations, a covariant space-time will result only if certain constraint equations are satisfied. Solutions to the quantum constraints, taking the form of superpositions of spatial geometry states, encode the “evolution” of physical fields classically depicted as a space-time manifold.
2.1 Spatial geometry

In a canonical formulation of general relativity (Arnowitt et al., 1962) only the spatial metric components \(q_{ab}\), referring to a chosen slicing of space-time by a global time function, are dynamical fields and have conjugate momenta. The remaining components of the space-time metric tensor play the role of Lagrange multipliers of the constraints mentioned above, analogously to the time component of the vector potential in electrodynamics which becomes the Lagrange multiplier of the Gauss constraint. Accordingly, only spatial geometric objects such as areas of spatial surfaces or volumes of spatial regions are represented as operators in a canonical quantization. Loop quantum gravity proceeds by first reformulating the theory in terms of densitized triads \(E^a_i = \sqrt{\det(q_{bc})} e^a_i\) with the triad \(e^a_i\) canonically conjugate to a connection \(A^i_a\) (Ashtekar, 1987; Barbero G., 1995). This allows one to define holonomies

\[ h_e(A) = \mathcal{P} \exp \int_e dt \dot{e}^a_i A^i_a \tau_i \]

(1)

of the connection \(A^i_a\) along curves \(e\) in space with \(\mathcal{P}\) denoting path ordering of the non-Abelian SU(2) matrices (with generators \(\tau_i\)) along the path, and fluxes

\[ F_S(E) = \int_S d^2 y n_a E^a_i \tau^i \]

(2)

of the densitized triad through surfaces \(S\) with co-normal \(n_a\). These objects, unlike the local fields \((A^i_a, E^b_j)\) themselves, form a well-defined Poisson algebra without the delta functions one otherwise has in field theories. This algebra has a representation as operators on a Hilbert space, which is the basis of loop quantum gravity.

The representation is most conveniently written in the connection representation. Then, being functionals of the connection as configuration variables, states are constructed from holonomies along paths in space and thus themselves refer to 1-dimensional objects. One can formulate this by using arbitrary graphs in space as a label of states, replacing the coordinate position of the classical fields \(Rovelli and Smolin, 1995b; Ashtekar et al., 1995\). Edges and vertices of the graphs carry further quantum labels, which for edges are half-integers \(j \in \frac{1}{2} \mathbb{N}\). (The resemblance to spin quantum numbers is due to the occurrence of the spatial rotation group, acting on the triad, in this context.) These quantum numbers assigned to a spatial graph determine the spatial geometry by combinatorial rules. The area of a surface, for instance, is obtained by summing contributions \(4\pi \gamma \ell_P^2 j(j+1)\) for all edges in a graph intersecting the given surface \(Rovelli and Smolin, 1995a; Ashtekar and Lewandowski, 1997a\), with a coefficient determined in terms of the Planck length \(\ell_P\) and the so-called Barbero–Immirzi parameter \(\gamma\) of loop quantum gravity \(Immirzi, 1997; Barbero G., 1995\). More precisely, this formula gives the eigenvalue of the area operator when it acts on a state associated with the given graph and edge labels. In particular, the area spectrum is discrete. This formula only applies to the generic case where the surface does not intersect any of the vertices of the graph. If this were the case, vertex labels would also be necessary to determine the action of the area operator, but the
spectrum remains discrete. For volume, only vertex labels play a role in a way more complicated to specify (Rovelli and Smolin, 1995a; Ashtekar and Lewandowski, 1997b). But the action is obtained in a similar combinatorial manner by summing contributions from vertices within a given region whose volume is to be determined. Moreover, the volume spectrum can also be shown to be discrete although it is not known explicitly.

2.2 Relational evolution

Such a state, associated with a single graph or as a superposition of states associated with different graphs, describes spatial quantum geometry at an instant, so far without an evolution or space-time picture. An arbitrary state does correspond to a physical quantum space-time only if it satisfies constraint equations, i.e. if it is annihilated by operators quantizing the classical constraints. This in general requires a physical state to be a superposition of different volume eigenstates. Evolution is encoded in such a superposition in a relational manner, for instance as the change of an area or of a matter field relative to a change in volume: For a positive real number $V$ we can project a given superposition to the volume eigenspace with eigenvalue $V$ (which may be the zero space) and compute the expectation value of an area $A$ or a matter field $\phi$ in the projection. Doing this for all values of $V$ results in a function $A(V)$ or $\phi(V)$ of volume which describes the relational evolution. This presents an implicit description of space-time not as a manifold but as the relational dynamics of the fields defined on it. We do not refer to $A(t)$ and $V(t)$ as functions of a coordinate time variable $t$ which, in a general relativistic situation, could be chosen arbitrarily. We rather eliminate $t$ in regions where $V(t)$ is invertible by inserting $t(V)$ in $A(t)$. The variable $A$ then evolves with respect to the “internal time” $V$. Just like coordinates, this description in general can be done only locally in patches corresponding to the image domain where $V(t)$ is invertible. To cover the whole dynamics relationally, one will have to select different internal times and derive transformation rules between them. (In a triad formulation one can even compute evolution for all real values of such an internal volume time including negative ones since the theory is sensitive also to the orientation and not just the size of space. This is relevant for the singularity issue, which is resolved in loop quantum cosmology by providing a well-defined relational evolution through zero volume or other degenerate configurations (Bojowald, 2001a; 2005b; 2007b).) Evolution in canonical quantum gravity thus does not happen in coordinate time since there is no time manifold, but in a relational manner. Observables have been defined directly in such a way (Bergmann, 1961; Rovelli, 1991b; Dittrich, 2006; 2005).

Just as not any 4-dimensional Lorentzian manifold is a physical space-time but only those are which satisfy Einstein’s equation, allowed superpositions of states in quantum gravity need to satisfy the constraint equations. These equations select physical superpositions of spatial geometry states giving rise to relational evolution. They are formulated by specifying the moves that can occur in the underlying graphs and labels of states when an internal time variable such as $V$ changes. Schematically, one has a picture where space is presented as a discrete structure building up from a small state at the big bang to a highly refined, nearly continuous fabric today. The evolution picture is thus that of an irregular
lattice structure which changes in internal time by elementary changes of geometry. Corresponding moves follow from the quantization procedure applied to the classical constraints. Although there is currently no uniquely specified version, several typical features of these moves are generic and can thus reveal properties of the fundamental quantum dynamics.

Unfortunately, a derivation of such rules from first principles, i.e. by quantizing the Hamiltonian constraint and reformulating the constraint equation in terms of an internal time, is highly complex. Fig. 1 illustrates the local action of a typical constraint operator as it follows from the usual constructions (Rovelli and Smolin, 1994; Thiemann, 1998a). The operator, when acting on a single graph state, changes labels as well as the graph itself, resulting in several new contributions to the total superposition. (However, also versions of Hamiltonian constraint operators based on fixed lattices have been constructed; see e.g. Giesel and Thiemann (2007).) Any single change of this form may increase or decrease the geometrical variable chosen as internal time, and thus represents elementary changes of the graph forward or backward in internal time. This is to be reformulated as global moves of the physical solution forward in time by rearranging the total solution as a superposition of internal time eigenstates. Since local moves acting on distant vertices can change the total internal time (e.g. volume) by equal amounts, the resulting forward moves, unlike the elementary ones in the quantized constraint, may no longer be local, adding to the technical complexity. We will now describe the form of the elementary constraint operator and later formulate basic assumptions on its implications for moves in internal time.

### 2.3 Properties of Hamiltonians

Properties of elementary moves are consequences of the main terms appearing in the classical Hamiltonian constraint of canonical general relativity in Ashtekar variables, quantized
by loop techniques. The main contribution to the constraint is

\[
H[N] = \frac{1}{16\pi G} \int_{\Sigma} d^3x N \left( \epsilon_{ijk} F^i_{ab} \frac{E_j^a E_k^b}{\sqrt{|\det E|}} \right)
\]

(3)

which classically must vanish for any spatial function \(N\). The constraint depends on the densitized triad as well as the connection through its Yang–Mills curvature \(F^i_{ab}\). Both dependencies require special care (Rovelli and Smolin, 1994; Thiemann, 1998a): Curvature terms can be quantized by expressing them as holonomies along closed loops which, by the non-Abelian Stokes theorem results in the exponentiated curvature. For a sufficiently small loop, the exponential becomes irrelevant and one obtains an expression of \(F^i_{ab}\) in terms of quantities acting in the loop representation. Such holonomies act as multiplication operators and either generate new edges in the graph of a state acted on (as in Fig. [1]) or, if the loop is only retracing edges already present in the graph, change the edge and vertex labels. In general, one thus expects from this action a change in the graph and a change in local geometrical values such as area and volume. Each state in the superposition on the right hand side of Fig. [1] in general has a volume expectation value different from the initial state. The action thus gives rise to the elementary moves in internal time mentioned above, which can in principle be derived once an explicit quantum operator for the constraint has been specified and physical solutions as superpositions of internal time eigenstates are analyzed.

Formulating a complete operator also requires one to represent the triad dependent expression in the constraint. It appears as an inverse determinant, which classically can diverge and may be problematic in the quantum theory, too. As we have seen, the volume in loop quantum gravity acquires a discrete spectrum, and it does contain zero as an eigenvalue. Thus, there is no inverse operator and no obvious way to find a quantization of \(|\det(E_i^a)|^{-1/2}\) as required for the Hamiltonian constraint. The second characteristic aspect of a loop quantization, in addition to loop holonomies quantizing \(F^i_{ab}\), is a consequence of this behavior and is thus directly related to the discrete spectrum of the volume operator. Rather than taking a direct inverse, expressions such as that in the Hamiltonian constraint can be quantized by first rewriting them in a way involving Poisson brackets between holonomies and volume only requiring positive powers. The basic relation used is (Thiemann, 1998a)

\[
\left\{ A^i_a, \int \sqrt{|\det E|} d^3x \right\} = 2\pi \gamma G \epsilon^{ijk} \epsilon_{abc} \frac{E_j^b E_k^c}{\sqrt{|\det E|}}
\]

(4)

which follows from the fact that the connection \(A^i_a\) is canonically conjugate to the densitized triad. In such expressions, all ingredients can be turned into operators in a well-defined manner, and eigenvalues of the resulting operator do approach the classical behavior for large volume and small anisotropy. But at small volume or for large anisotropies there are deviations from the classical behavior giving rise to quantum corrections in any classical expression where an inverse of the densitized triad occurs.
The qualitative behavior of such inverse powers for small anisotropies can be illustrated well by explicit formulas from isotropic models (Bojowald, 2002b). Here, we have a single component \( p \) of the densitized triad \( E^a_i = p \delta^a_i \), and a single connection component \( c \) in \( A^i_a = c \delta^i_a \) satisfying \( \{ c, p \} = 8 \pi \gamma G / 3 \). Analogously to (4) we write

\[
\{ c, |p|^{1/2} \} = \frac{4}{3} \pi \gamma G |p|^{-1/2}
\]

showing the resulting inverse power of \( p \) even though no inverse is used on the left hand side.

Similarly to holonomies in the full theory, isotropic models of loop quantum cosmology only allow the quantization of exponentials \( e^{i c} \) on a Hilbert space \( L^2(\mathbb{R}) \) by \( (\psi_\nu)_\nu \mapsto (\psi_{\nu-\delta})_\nu \), not of \( c \) itself (Ashtekar et al., 2003). The Poisson bracket thus can be quantized only after \( c \) is expressed in terms of such exponentials, which in the isotropic case can be done exactly by

\[
\{ c, \sqrt{|p|} \} = i \delta^{-1} e^{i dc} \{ e^{-i dc}, \sqrt{|p|} \}
\]

\[
= \frac{i}{2\delta} \left( e^{i dc} \{ e^{-i dc}, \sqrt{|p|} \} - e^{-i dc} \{ e^{i dc}, \sqrt{|p|} \} \right).
\]

The parameter \( \delta \) is to be chosen as a quantization ambiguity or may follow from a symmetry reduction of the quantum operator; see (Bojowald and Kastrup, 2000; Engle, 2006; Koslowski, 2006; Engle, 2007; Koslowski, 2007) for these developments. In the full theory where SU(2) expression are used, values for \( \delta \) are restricted to be half-integers. In the second line of (6) we have ensured that the expression remains even under \( c \mapsto -c \) (including the derivative by \( c \) implicit in the Poisson bracket), which is required if the phase factors of an isotropic model are to be embedded as matrix elements of SU(2) holonomies in the full theory. (The map \( c \mapsto -c \) then appears as a consequence of the Weyl group.) Note, however, that the full situation with its non-Abelian holonomies taking values in SU(2) rather than just phase factors is more subtle (Bojowald, 2006a), giving rise to additional quantum corrections compared to models based on Abelian U(1) holonomies. In isotropic models we then proceed by turning elementary phase space functions into basic operators and the Poisson bracket into a commutator divided by \( i\hbar \),

\[
\frac{1}{\sqrt{|p|}} = \frac{3}{8 \pi \gamma \delta \ell_P^2} \left( e^{i dc} \{ e^{-i dc}, \sqrt{|p|} \} - e^{-i dc} \{ e^{i dc}, \sqrt{|p|} \} \right)
\]

(with the Planck length \( \ell_P = \sqrt{G \hbar} \) which is densely defined and even finite (Bojowald, 2001b). We only need to take the square root of the norm of \( \hat{p} \), which for a self-adjoint operator \( (\hat{p} \psi)_\nu = p_\nu \psi_\nu = \frac{8}{3} \pi \gamma \ell_P^2 \nu_\nu \psi_\nu \) can easily be done. Using the action of the basic operators one can directly compute the action and eigenvalues of (7) in explicit form:

\[
\left( \frac{1}{\sqrt{|p|}} \right)_\nu = \sqrt{\frac{3}{8 \pi \gamma \delta \ell_P}} \left( \sqrt{|\nu + \delta|} - \sqrt{|\nu - \delta|} \right).
\]
Figure 2: Eigenvalues cubed of the inverse volume operator compared to the classical expectation $a^{-3}$ (dashed). To the right of the peak deviations are due to perturbative corrections in $\ell_P/a$, while at and to the left of the peak stronger non-perturbative corrections occur. The functions are plotted with respect to a normalized scale factor $a/a_0$, $a_0^2 = \frac{8}{3}\pi\gamma\ell_P^2$, such that the peak occurs at $a/a_0 = 1$.

For $|\nu| \gg \delta$ the expression is close to the classical value,

$$\left(\frac{1}{\sqrt{|\nu|}}\right) = \frac{1}{\sqrt{|\nu|}} \left(1 + \frac{4\pi^2\delta^2\gamma^2\ell_P^4}{9p_\nu^2} + O(\ell_P^8/p_\nu^4)\right)$$

with only perturbative corrections in $\ell_P^2/p_\nu$, while there are strong, non-perturbative deviations for $|\nu|$ comparable to and smaller than $\delta$; see Fig. 2. The most drastic deviations occur at $\nu = 0$ where the classical expectation diverges. More general expressions including quantization ambiguities have been derived in Bojowald (2002c, 2004).

This illustrates why operators for inverse densitized triad components can be well-defined in loop quantum gravity and remove divergences in Hamiltonians which would arise when fields are quantized on a classical metric background.

In the isotropic example used here, the basic densitized triad component is related to the conventional scale factor by $|\nu| = a^2$. A distinguished value of $a$ defined in terms of the Planck length, such as the peak position of the quantized inverse, seems to be in conflict with the classical rescaling freedom of $a$ by an arbitrary positive factor present in a flat
isotropic model. This only poses an apparent problem which is resolved once operators are viewed in the proper inhomogeneous context (Bojowald, 2006b). Corrections to the classical behavior such as those embodied by the peak appear as functions of the local discrete scales, or local fluxes associated with individual edges in a graph. These local flux values can be viewed as elementary building blocks of a macroscopic region. Their size determines how many building blocks are present in a given region of a certain volume, and is thus relevant for physical implications: whether or not discreteness effects are significant depends on the ratios of elementary flux values to a macroscopic scale. In this way, the underlying discrete structure distinguishes scales even if the geometry is scale invariant macroscopically.

Inverse components of the densitized triad also appear in matter Hamiltonians, such as

\[ H_\phi[N] = \int_\Sigma d^3x \left( \frac{1}{2} \frac{p_\phi^2}{\sqrt{|\det E^a_i|}} + \sqrt{\det E^a_i} V(\phi) \right) \]

for a scalar field \( \phi \) with momentum \( p_\phi \). Again, the loop quantization proceeds by rewriting the classical inverse determinant in the kinetic term in a form accessible to quantization (Thiemann, 1998b), giving the classical behavior at large volume but quantum corrections when the total volume becomes smaller or anisotropies are large.

### 2.4 Summary

Writing the basic action of a Hamiltonian constraint operator through moves of a discrete graph while an internal time variable such as volume changes may be difficult. But there are rather robust general aspects of the picture which we are going to use in what follows. Hamiltonian operators are discrete, with countably many contributions. Each contribution receives characteristic corrections to the classical form, which includes discretization corrections but also quantization effects such as those seen in (9) for inverse powers of the spatial metric determinant. The size and number of all these contributions depends on the precise state which describes the universe. All ingredients are dynamical and in general change when an internal time variable changes. For instance, as the universe grows one expects the number of contributions to increase, corresponding to a refined lattice structure. A quantum correction from a single contribution may grow or shrink with this refinement, the precise behavior following from the operators.

Even in the absence of a direct line of derivations from a fundamental Hamiltonian to a refinement model, the form of operators indicates the general behavior of correction terms and how they change with a refined graph. The difficult part is to derive how an initial graph state evolves in internal time and how it is being refined. But once such a refinement prescription is known or assumed, the behavior of quantum corrections is rather robust. This makes it possible to construct phenomenological models based on an assumption for

---

This is a general expectation, not just based on basic properties of loop quantum gravity. It has been discussed in the context of string theory, too, for instance by Mathur (2003).

---
graph refinements occurring with a change of internal time, which then implies the internal time dependence of corrections. A phenomenological Hamiltonian will result which allows one to analyze potential physical implications of these effects. The robustness of the procedure is underlined by the fact that there are independent arguments for certain refinements even in symmetry reduced models where one can analyze the dynamical equations more easily. Good semiclassical behavior can then be used as a criterion to select some refinements and rule out others (Ashtekar et al., 2006b; Bojowald et al., 2007a).

What we will use therefore is a phenomenological Hamiltonian depending on a function $N(V)$ describing the average number of vertices of a state as a function of the spatial volume. Changes in $N(V)$ indicate refinements of the graph during evolution. We cannot compute this function at the current level of developments. It can be a complicated function, but for short evolution times, which is sufficient for the dark energy problem, one can assume a power law $N(V) \propto V^\alpha$. Theoretical considerations indicate that $\alpha$ should be in the range $0 < \alpha < 1$: a fundamental Hamiltonian constraint operator typically creates new vertices and increases the edge spins. If edge spins remained constant, the growth in volume could only come from the number of vertices at constant vertex contributions and thus $N \propto V$. If only edge spins change but the number of vertices remains constant, we obviously have $N = \text{const}$. Thus, since both vertex creation and edge spin increase happens at the same time, the actual power low must be somewhere in between. Restrictions could be found by detailed semiclassical analyses as in (Bojowald et al., 2007a), or phenomenologically as we will see below.

3 The dark energy problem: Possible contributions from loop quantum gravity

Although loop quantum gravity by its general formalism provides a well-defined formulation of possible quantum dynamics, including the gravitational as well as matter fields, the resulting equations for physical states annihilated by the constraint operators as sketched above are complicated. They do simplify in symmetric models, most notably in homogeneous ones as they are often used in cosmology, but this is unlikely to provide insights to the dark energy problem of a large, classical universe. All facets of the dark energy problem, from the viewpoint of loop quantum gravity, require properties of inhomogeneous states. This involves a more detailed analysis of the full constraint equations, which is still in progress. Nevertheless, despite this lack of understanding of details there are indications for possible contributions of loop quantum gravity to the dark energy problem.

\footnote{We only discuss here mechanisms by which dark energy could be provided through an ingredient of loop quantum gravity. Investigations in loop quantum cosmology where a fluid with negative equation of state parameter is assumed are not covered; see e.g. (Sami et al., 2006; Samart and Gumjudpai, 2007).}
3.1 Implications of connection variables

We start with results which do not, at least currently, use loop quantum gravity but which crucially rely on the same basic variables given by real Ashtekar variables. Since many general properties of loop quantum gravity—the use of holonomies and fluxes as basic operators of a well-defined background independent quantization with the resulting discrete flux spectrum—are consequences of the type of basic variables used, independent implications of these variables are likely to be realized in loop quantum gravity, too.

The Ashtekar connection $A^i_a = \Gamma^i_a + \gamma K^i_a$ is defined in terms of the spin connection $\Gamma^i_a$ compatible with the densitized triad $E^i_a$ and extrinsic curvature $K^i_a$ only up to a real parameter $\gamma > 0$. This constant, the Barbero–Immirzi parameter (Barbero G., 1995; Immirzi, 1997), can be changed arbitrarily by a canonical transformation for pure gravity or with bosonic fields. As pointed out recently by Perez and Rovelli (2006) and elaborated by Freidel et al. (2005) and Mercuri (2006), however, the value of $\gamma$ does have a physical effect on fermion fields due to their coupling to the connection. The fermion current provides a source for torsion. Solving for and eliminating torsion in the Dirac action, one obtains an effective current-current interaction whose strength depends on $\gamma$.

Alexander and Vaid (2007) observe that the form of this four-fermion interaction resembles what is used in the BCS theory of superconductivity. Non-perturbative effects imply the instability of the fermionic ground state and its decay into a condensate effectively described by a scalar. This condensate may provide a source of dark energy which is dynamical in the history of the universe. The corresponding scalar would then play the role of either an inflaton or quintessence field. Loop quantum gravity may help in formulating the specifics of this construction but has not been applied yet.

3.2 Finiteness of vacuum energy

The construction of Hamiltonian constraint operators and matter Hamiltonians sketched above results in operators which are well-defined mathematically. A normal ordering as used in quantum field theories on curved background space-times, where Hamiltonians would be diverging without proper subtractions, is not needed. This makes it possible, at least in principle, to determine the coupling of vacuum energy to the gravitational field within the theory. Additional assumptions on the form of subtraction need not be imposed, although there are quantization ambiguities which may influence the resulting value of gravitationally active vacuum energy. Despite of the presence of such ambiguities, a derivation would indicate whether or not a large amount of fine tuning would be necessary to be in agreement with current observations.

The finiteness of Hamiltonian operators in loop quantum gravity can be traced back to the underlying background independence (Thiemann, 1998a): by taking into account the full metric in the quantization, rather than treating it as a classical background field, regularization parameters which when sent to zero would give rise to divergences in a

---

4Related observations were made earlier by Ashtekar (1991) and Nicolai et al. (2005).
quantization on a background cancel out completely in the background independent quantization. Thus, Hamiltonians are well-defined operators with dense domains of definition, and any physical situation can be described by a quantum state with finite expectation values of Hamiltonians. (Since these operators are typically unbounded, not every state lies in their domain of definition. But any physical situation can be described arbitrarily closely by a state in the domain of definition due to its denseness, which is sufficient given the finite precision of any measurement.) In particular, the expectation value in the vacuum of a quantum field in a quantum universe must be finite, and the vacuum energy cannot diverge. The central question is whether or not this just reproduces the value obtained by choosing a Planck-size cut-off in a Fock quantization.

While these methods lead to such a very general statement about the finiteness of the vacuum energy, which could play the role of dark energy if its size has the correct magnitude, specific results are more difficult to derive. First, one would like to make contact between this fully background independent quantization and quantum field theories on a curved background so as to see how the common divergences are avoided precisely without an explicit normal ordering. Preliminary results have been derived by Sahlmann and Thiemann [2006a,b]. From the point of view of quantum field theory on curved space-times one can effectively view the finiteness of vacuum energy in loop quantum gravity as a cut-off provided by the underlying discrete structure of loop quantum gravity. On the grounds of dimensional arguments one would expect that the cut-off occurs at Planckian values of energy or length, which would certainly result in the well-known mismatch between the predicted and observed cosmological constants. To understand this issue one has to go beyond dimensional arguments and do a dedicated calculation of expectation values of Hamiltonians in semiclassical states which can be associated with a quantum field theoretical vacuum. Here, the full complexity of loop Hamiltonians strikes and makes the calculation very involved. Preliminary investigations have been done, but not yet resulted in any numerical estimates.

It is to be expected that vacuum energy in this formalism does not only depend on the matter state but also on quantum geometry, i.e. the precise discrete structure emerging from a physical state as described in Sec. [2]. Dimensional arguments can then easily fail due to the presence of several length scales: the Planck length, a macroscopic scale such as the Hubble length in cosmology, and the discreteness scale somewhere in between the former two scales. Loop quantum gravity in general gives only a lower bound for the discreteness scale close to the Planck length, but the precise value can well be larger depending on the actual quantum state. It has to be determined from properties of a physical solution of quantum gravity which can describe our universe. For a discreteness scale different from the Planck length, dimensional arguments are not sufficient for simple estimates of results and detailed calculations are required. This is similar to effective theories in a cosmological context as pointed out in Bojowald [2007a]. Effective arguments also apply to the problem at hand here, as described next.
3.3 Effective negative pressure

Rather than understanding dark energy as the vacuum energy of quantum fields, it could be a quantum effect which, when expressed in a Friedmann–Robertson–Walker solution, resembles an effective matter contribution giving rise to negative pressure. In loop quantum gravity, an explanation of dark energy could be provided in this manner. In isotropic models, loop quantum cosmology has revealed a source for negative pressure from quantum corrections (Bojowald, 2002a). This happens on small scales where quantum geometry modifies the behavior of functions such as the inverse volume which classically diverge at zero spatial extension. As seen in Fig. 2, the isotropic quantum version of the inverse cuts off the divergence and bends the curve down to zero (Bojowald, 2001b). Effective matter Hamiltonians, which always contain the inverse determinant of the densitized triad, are thus increasing as functions of volume at early times which replaces the classical decreasing behavior. It is then easy to see why negative pressure arises: by thermodynamics pressure is defined as the negative derivative of energy by volume. If energy starts to increase with volume when small scales are reached, negative pressure automatically results.

3.3.1 The scenario

While these effects can be used in inflationary scenarios which require negative pressure at early times (Bojowald, 2002a; Tsujikawa et al., 2004; Bojowald et al., 2004a; Date and Hossain, 2005), the effects quickly subside once the universe has expanded to some size very small compared to its current one. In fact, such a quantum geometry epoch of inflation typically does not last long enough to provide all 60 e-foldings required for successful structure formation. Moreover, such an isotropic model with only inverse volume corrections is not very accurate at large volume because it does not fully take into account the dynamical discreteness of space manifesting itself in lattice refinements determined by the elementary moves of a Hamiltonian constraint. Rather, during expansion the discrete structure of space subdivides as described in Sec. 2 which can be modeled by adding new small, discrete patches resulting from new vertices of graphs. The number of patches is expected to grow with the volume, \( N \sim V^\alpha \) for some constant \( 0 < \alpha < 1 \), which indeed has recently been recognized to resolve some technical problems related to isotropic models on large scales (Ashtekar et al., 2006a; Bojowald, 2006b; Bojowald et al., 2007a). The parameter \( \alpha \) is to be determined from the precise behavior as depicted in Fig. 1. When the number of patches increases with volume, their size stays nearly constant or could even decrease. Thus, some of the small-scale corrections active in the very early phase can remain or re-emerge later if they had subsided. How precisely this happens is a complicated dynamical process of a system with many independent but dynamically coupled ingredients. While a

---

5This is no longer the case for the eigenvalues of analogous operators in anisotropic situations. However, in that case it is not the full spectrum which is relevant but the part of the spectrum realized along a dynamical state (i.e. evaluated in volume eigenstates in its superposition if volume is used as internal time). In isotropic models, both concepts coincide since the minisuperspace is 1-dimensional and thus any geometrical trajectory reaching zero volume must fill out the whole small volume region. This is different in the absence of isotropy where dynamical information is much more crucial (Bojowald, 2006a).
derivation from first principles is currently out of reach, one can arrive at estimates on the overall behavior based on the history of the universe. After the first, quantum geometry powered phase of inflation the strong effects causing this phase must subside due to growing edge labels. It follows a long stretch of evolution with only minor corrections, during which most of the standard model of cosmology takes place (including a second phase of inflation). While there are no strong quantum gravity effects in this regime, small perturbative corrections are always present and may change some of the observable properties due to their effect on perturbative inhomogeneities (Bojowald et al., 2007c). Such effects of perturbations around Friedmann–Robertson–Walker space-times do not contribute negative pressure to the energy balance. But when inhomogeneous properties of non-perturbative states are considered, a negative pressure contribution results as we will show below.

This scenario is in principle testable by direct calculations in the full theory which would have to demonstrate the sketched behavior of edge spins: they first have to grow on average when the total volume is still rather small leading to a semiclassical regime, but then remain nearly constant (or even start to decrease) while the universe as a whole expands. Thus, there should be a point where an overall growth in both edge spins and the number of vertices turns over to growth dominated by the number of vertices at nearly constant edge spins. Independently of whether or not this can be shown from a fundamental Hamiltonian, one can already test the scenario phenomenologically. The question is if this effect will be sufficient for the amount of dark energy needed, or maybe even too much?

3.3.2 A phenomenological model

To determine this more quantitatively one needs to use the relevant equations as they are corrected by quantum gravity effects. Rather than dealing directly with Hamiltonian operators and semiclassical states we write a classical Hamiltonian which includes some of the effects described previously. This can be seen to arise as an expectation value of the fundamental matter Hamiltonian in a semiclassical state, along the lines of a general scheme to derive effective equations (Bojowald and Skirzewski, 2006; 2007). In particular, we incorporate the discreteness of space, building the total matter energy by summing over discrete patches. The size of each patch is taken to be dynamical to take into account the occurrence of lattice refinements. Finally, whenever there is an inverse of densitized triad components in the matter Hamiltonian we include correction functions for effects seen in quantum inverse operators. Then, matter energy with a classical form (10) for a scalar field $\phi$ as before is given by a Hamiltonian

$$ H_{\text{matter}} = \sum_v \left( \frac{1}{2} \frac{p_\phi(v)^2}{R(v)^3} + V(\phi) R(v)^3 \right) = \sum_v R(v)^3 \rho_{\text{matter}}(v). $$

6We just use this as a typical example but do not assume the presence of any fundamental scalar field; expressions will be similar for realistic matter fields. In fact, a strength of the loop quantum cosmological mechanism to provide negative pressure is that it would work for any matter field through its kinetic term in the Hamiltonian, rather than a fine-tuned potential of a postulated scalar. See (Bojowald and Das, 2007; Bojowald et al., 2007b) for analogous quantum corrections to Maxwell and Dirac Hamiltonians.
In this expression, we use the patch position \( v \), its radial size \( R(v) \) and local energy density \( \rho(v) \). The inverse patch volume \( R(v)^{-3} \) present in the kinetic term is the main term receiving quantum corrections in its effective expression

\[
d(R) = R^{-3}(1 + CR^{-2}\ell_P^2 + \cdots)
\]

with a constant \( C > 0 \) which is determined by the specific quantization. Here, we are using only perturbative corrections to the inverse, excluding non-perturbative ones which are realized at much smaller values of the patch size where the universe would not be semiclassical at all. The value of \( C \) in inverse volume operators is subject to quantization ambiguities (Bojowald, 2002c, 2004) also affecting the Hamiltonian operator itself from which an effective Hamiltonian (11) is derived. The eigenvalue expression (9), for instance, used for an expression \( (|p|^{-1/2})_v^{(a)} \) resulting in \( a^{-3} \) for large \( a \), gives a coefficient \( C = 4\pi^2\gamma^2\delta^2/3 \approx 0.742\delta^2 \) which, using the value \( \gamma \approx 0.2375 \) as derived from black hole entropy calculations (Ashtekar et al., 1998, 2000; Domagala and Lewandowski, 2004; Meissner, 2004), is of order one if \( \delta \) is of order one as expected; see (Bojowald, 2006b). However, in this expansion the leading correction term is of the order \( \ell_P^4/p^2 = \ell_P^4/a^4 \), and there is no term of order \( \ell_P^2/p \) since the quantization is invariant under orientation reversal \( p \mapsto -p \). Terms of second order can nevertheless result when one uses expectation values in semiclassical states, as they are relevant for effective equations (Bojowald and Skirzewski, 2006; Skirzewski, 2006; Bojowald and Skirzewski, 2007), rather than eigenvalues. The computation of the corresponding \( C \) then requires more detailed knowledge of the state but can in principle be performed.

General properties of the quantization imply that \( C \) must be positive, a property which will play an important role. Moreover, generically only even powers of \( \ell_P/R \) occur in the correction terms because the expression for inverse powers is a function of basic, area-like fluxes, used in the underlying quantum theory, whose norm is the square of \( R \). With this correction, there will be a new term

\[
H_{\text{quantum}} = \frac{1}{2} \sum_v \left( d(R(v)) - R(v)^{-3} \right) p_\phi(v)^2 = \frac{C}{2} \sum_v p_\phi(v)^2 \frac{\ell_P^3}{R(v)}
\]

in any matter Hamiltonian \( H_{\text{matter}} = H_{\text{classical}} + H_{\text{quantum}} \). This can be estimated using an approximately homogeneous matter distribution and the virial theorem which states that energy contributed by the kinetic term should be half the total energy. Thus,

\[
\frac{1}{2} \frac{p_\phi(v)^2}{R(v)^3} \sim \frac{1}{2} R(v)^3 \rho_{\text{matter}}(v)
\]

and

\[
H_{\text{quantum}} = \frac{C}{2} \sum_v \rho_{\text{matter}}(v) \ell_P^3 R(v).
\]

\[\text{Although (9) was derived for an isotropic model with } a^{-3} \text{ being the total inverse volume, a very similar expression results for the local patch sizes (Bojowald et al., 2007d).}\]
Finally, the patch size is the total size of the universe divided by the number $\mathcal{N}$ of patches, $R(v)^3 = a^3/\mathcal{N}$, resulting in

$$H_{\text{quantum}} = \frac{C}{2} \sum_v \rho_{\text{matter}}(v) \ell_p^2 a / \mathcal{N}^{1/3} \sim \frac{C}{2} \mathcal{N}^{2/3} \rho_{\text{matter}} \ell_p^2 a$$

for a nearly homogeneous matter distribution among the $\mathcal{N}$ patches.

This is the contribution obtained by adding up quantum corrections for all $\mathcal{N}$ patches. Since $\mathcal{N}$ is increasing with $a$, $H_{\text{quantum}}$ indeed contributes negative pressure resulting from an energy contribution increasing with volume. More precisely, for the case $\mathcal{N}(a) \sim \mathcal{N}_0 (a/\ell_P)^3$ with some constant $\mathcal{N}_0$ of order one, we obtain

$$H_{\text{quantum}} \sim \Lambda a^3$$

with a (cosmological) constant $\Lambda$. This implies an equation of state where pressure is the negative energy density, the currently preferred value by observations. Moreover, the size of $\Lambda = \frac{1}{2} C \mathcal{N}_0^{2/3} \rho_{\text{matter}}$ is the same as that of matter up to constant factors of order one. This case corresponds to $\alpha = 1$ in the parameterization $\mathcal{N}(V) \propto V^\alpha$ as a function of volume motivated in Sec. 2. As explained there, the balance between changing edge spins and the creation of new vertices provides this value as a limiting case of a general range $0 < \alpha < 1$. For $\alpha \neq 1$, we have $\mathcal{N}(a) \sim \mathcal{N}_0 (a/\ell_P)^{3\alpha}$, resulting in

$$H_{\text{quantum}} \sim \frac{C}{2} \mathcal{N}^{2/3} \rho_{\text{matter}} \ell_p^2 a = \lambda a^{1+2\alpha}$$

(13)

with a constant $\lambda = \frac{1}{2} C \mathcal{N}_0^{2/3} \rho_{\text{matter}} \ell_p^2 (1-\alpha)$. In this case, the energy density of the quantum correction “fluid” is

$$\rho_{\text{quantum}} = a^{-3} H_{\text{quantum}} \sim \lambda a^{2(\alpha-1)}$$

and its pressure (defined as the negative derivative of energy by volume)

$$P_{\text{quantum}} = -\frac{dH_{\text{quantum}}}{d(a^3)} = -\frac{1+2\alpha}{3} \lambda a^{2(\alpha-1)} = -\frac{1+2\alpha}{3} \rho_{\text{quantum}}.$$ 

The equation of state parameter is thus

$$w = -\frac{1+2\alpha}{3}$$

(14)

which is between $-1/3$ and $-1$ for the range $0 < \alpha < 1$.

Qualitatively, we rather naturally obtain all the observational properties without additional input. Although the precise form of lattice refinements or a precise estimate for $\alpha$ is not yet available, falsifiable predictions are possible. In particular, one can relate the

---

8We normalize the scale factor such that $\mathcal{N} = \mathcal{N}_0$ for $a = \ell_P$ is the number of vertices of a Planck size universe, which one typically expects to be of order one. This implies that the current scale factor in this normalization is huge.
equation of state parameter to the size of the current contribution by dark energy. If $\alpha = 1$, as we have seen, we obtain $w = -1$ and a value of the cosmological constant of the same order as the matter energy density. The Planck length dropped out of the final expression, and thus there is no remaining factor of $a/\ell_P$ in relating $\rho_{\text{quantum}}$ to $\rho_{\text{matter}}$, which could give large or small factors compared to the matter density. If such factors were present, they would result in a cosmological constant off by orders of magnitude unless they appear in a form taken only to small powers. If $\alpha \neq 1$, the Planck length does not cancel out completely and one has to worry about large factors in the final value for $\rho_{\text{quantum}}$ not of the same order as the matter density. Ratios of $a/\ell_P$ appear in

$$\frac{\rho_{\text{quantum}}}{\rho_{\text{matter}}} = \frac{1}{2} C N_0^{2/3} \left( \frac{a/\ell_P}{a} \right)^{2(1-\alpha)}$$

(15)

with small powers only if $\alpha$ is near one. For $\alpha \approx 1$ one still obtains a dark energy contribution of the order of the matter density, giving a phenomenological constraint on the value for $\alpha$. Even if $\alpha$ cannot be computed easily from the fundamental gravitational Hamiltonian to confirm that a value near $\alpha = 1$ is indeed realized, there is an interesting phenomenological cross-check: As we have seen, the value of $\alpha$ also influences the equation of state of dark energy. Near $\alpha = 1$ the equation of state is close to that of a cosmological constant, $w = -1$. For smaller values of $\alpha$ one approaches the equation of state parameter $w = -1/3$. Thus, the scenario described here, giving a relation between the equation of state parameter and the order of magnitude of dark energy density in terms of a single parameter $\alpha$, is testable by precise observations of the dark energy parameters, including its equation of state.

4 Conclusions

Loop quantum gravity becomes better and better understood and its crucial properties are studied in more and more realistic models. Since recently, inhomogeneous situations are being looked at in detail and the basic mathematical framework has been derived. This has led to several possible ingredients to solving the dark energy problem, although no complete proposal for a solution exists so far. Well-defined matter Hamiltonians make it possible at least in principle to determine what the size of vacuum energy in a semiclassical state is, and how it couples to gravity to play the role of dark energy. While explicit calculations at a fundamental level are still out of reach, a framework for effective equations applicable to this situation exists. In this way, several technical and conceptual difficulties can be circumvented. For instance, one would not need completely specified semiclassical states but can systematically determine their properties order by order in a semiclassical expansion. Also this is still being worked out, and so no clear result concerning dark energy exists. What does exist are phenomenological models as described here which allow one to test if typical effects of quantum gravity can have implications even in a large, classical universe and contribute to the understanding of dark energy.
The same basic ingredients which in isotropic models lead to inflationary behavior can be seen to imply properties of dark energy even in a large universe. Although these applications come from the same basic expression, different corrections are being probed. In isotropic models of the early universe main effects arise from non-perturbative corrections, while the effects used here for dark energy rely on perturbative ones. Moreover, dark energy effects are much more sensitive to properties of the full quantum dynamics, especially the occurrence of lattice refinements which does not play a large role in the early universe. Some of the effects present in isotropic models have been shown to be dual to versions of string or brane-world cosmology (Lidsey, 2004; Copeland et al., 2006; Singh, 2006). An extension to inhomogeneous situations might provide interesting comparisons between the different frameworks.

A successful implementation of a scenario describing dark energy depends on several different aspects which allow qualitative evaluations of the viability of effects: First and foremost, the main correction arises in the kinetic term. Matter components in this term always appear in positive combinations such as $p^2$ for a scalar, and quantum corrections generally contribute a positive quantity since $C > 0$ in Eq. (12). This implies that small corrections in patches can indeed add up coherently in such a way that they produce positive dark energy. Dark energy could thus provide an example for magnification effects of tiny quantum corrections by adding them up over many contributions all over space. A similar effect, where corrections add up during long evolution times, may leave observable imprints in the CMB (Bojowald et al., 2007c). Other crucial ingredients are the kinds of basic variables used, which affects the $a$-dependence of quantum corrections, and the discrete structure. All this results in corrections which are not yet uniquely determined from the theory but whose qualitative properties can be estimated up to constants of order one.

Qualitative explanations which are realized non-trivially are: (i) the subdivision into patches as it follows from technical expectations agrees with the observed range for the equation of state of dark energy; (ii) the size of $\Lambda$ is automatically related to the energy density of matter because it results from quantum corrections to its kinetic term; (iii) there is a tight relation between the size and equation of state of dark energy, Eqs. (15) and (14). These observations are not independent but closely related to each other: if functions $N(a)$ not close to cubic behavior had occurred, powers of $a/\ell_P$ had appeared in $\rho_{\text{quantum}}/\rho_{\text{matter}}$ which are far from being of order one. While still related to $\rho$, $\Lambda$ would then have been too big ($\alpha > 1$) or too small ($\alpha < 1$) by many orders of magnitude. That the technical expectation agrees with the observation in this way is a non-trivial feature.

The scenario is thus constrainable and testable by more detailed observations of cosmic acceleration, as well as structure formation which probe different parts of the theoretical curve $d(a)$. This has to be combined with detailed calculations and a dedicated effort in numerical quantum gravity to simulate the subdivision better and to obtain precise predictions. While some numerical aspects have occurred in loop quantum gravity, this was so far only in homogeneous models (Bojowald et al., 2004b; Ashtekar et al., 2006a). Inhomogeneous ones as needed for dark energy require much more refined methods. Interestingly, though, current investigations of the semiclassical behavior of homogeneous models in loop
quantum cosmology indicate that a value near $\alpha = 1$ is preferred, while values near $\alpha = 0$ would not provide the correct semiclassical limit (Ashtekar et al., 2006b; Bojowald et al., 2007a). This can be seen as circumstantial evidence for the dark energy scenario proposed here, which is to be corroborated by dedicated studies of inhomogeneous models. It may be worth the effort because a detailed knowledge of $N(a)$ would allow one to predict the far future of the universe from such models: If an analysis confirms that $N(a) \sim a^3$, the patch size remains nearly constant during expansion and there will be no dramatic changes in the future evolution. But if $N$ increases by a different power, the equation of state will change. Moreover, if the power becomes smaller than three, the patch size increases with expansion and dark energy will die off. If the power exceeds three, the patch size decreases and slowly approaches the strongly modified non-perturbative part of $d(a)$. At this point, the behavior of the universe would change dramatically through quantum effects on large scales. This case would require the edge spins to decrease systematically while the total volume grows which is, for better or worse, not supported by current constructions of loop quantum gravity.

Acknowledgements

This work was supported in part by NSF grant PHY 05-54771.

References

S. Alexander and D. Vaid. A fine tuning free resolution of the cosmological constant problem. hep-th/0702064, 2007.

R. Arnowitt, S. Deser, and C. W. Misner. The Dynamics of General Relativity. Wiley, New York, 1962.

A. Ashtekar. New hamiltonian formulation of general relativity. Phys. Rev. D, 36(6):1587–1602, 1987.

A. Ashtekar. Lectures on non-perturbative canonical gravity. World Scientific, Singapore, 1991.

A. Ashtekar, J. C. Baez, A. Corichi, and K. Krasnov. Quantum geometry and black hole entropy. Phys. Rev. Lett., 80:904–907, 1998.

A. Ashtekar, J. C. Baez, and K. Krasnov. Quantum geometry of isolated horizons and black hole entropy. Adv. Theor. Math. Phys., 4:1–94, 2000.

A. Ashtekar, M. Bojowald, and J. Lewandowski. Mathematical structure of loop quantum cosmology. Adv. Theor. Math. Phys., 7:233–268, 2003.
A. Ashtekar and J. Lewandowski. Quantum theory of geometry I: Area operators. *Class. Quantum Grav.*, 14:A55–A82, 1997a.

A. Ashtekar and J. Lewandowski. Quantum theory of geometry II: Volume operators. *Adv. Theor. Math. Phys.*, 1:388–429, 1997b.

A. Ashtekar and J. Lewandowski. Background independent quantum gravity: A status report. *Class. Quantum Grav.*, 21:R53–R152, 2004.

A. Ashtekar, J. Lewandowski, D. Marolf, J. Mourão, and T. Thiemann. Quantization of diffeomorphism invariant theories of connections with local degrees of freedom. *J. Math. Phys.*, 36(11):6456–6493, 1995.

A. Ashtekar, T. Pawlowski, and P. Singh. Quantum nature of the big bang: An analytical and numerical investigation. *Phys. Rev. D*, 73:124038, 2006a.

A. Ashtekar, T. Pawlowski, and P. Singh. Quantum nature of the big bang: Improved dynamics. *Phys. Rev. D*, 74:084003, 2006b.

J. F. Barbero G. Real ashtekar variables for lorentzian signature space-times. *Phys. Rev. D*, 51(10):5507–5510, 1995.

P. G. Bergmann. Observables in general relativity. *Rev. Mod. Phys.*, 33:510–514, 1961.

S. O. Bilson-Thompson, F. Markopoulou, and L. Smolin. Quantum gravity and the standard model. [hep-th/0603022], 2006.

M. Bojowald. Absence of a singularity in loop quantum cosmology. *Phys. Rev. Lett.*, 86:5227–5230, 2001a.

M. Bojowald. Inverse scale factor in isotropic quantum geometry. *Phys. Rev. D*, 64:084018, 2001b.

M. Bojowald. Inflation from quantum geometry. *Phys. Rev. Lett.*, 89:261301, 2002a.

M. Bojowald. Isotropic loop quantum cosmology. *Class. Quantum Grav.*, 19:2717–2741, 2002b.

M. Bojowald. Quantization ambiguities in isotropic quantum geometry. *Class. Quantum Grav.*, 19:5113–5130, 2002c.

M. Bojowald. Loop quantum cosmology: Recent progress. In *Proceedings of the International Conference on Gravitation and Cosmology (ICGC 2004)*, Cochin, India. *Pramana*, 63:765–776, 2004.

M. Bojowald. Loop quantum cosmology. *Living Rev. Relativity*, 8:11, 2005a. [http://relativity.livingreviews.org/Articles/lrr-2005-11/](http://relativity.livingreviews.org/Articles/lrr-2005-11/).
M. Bojowald. Non-singular black holes and degrees of freedom in quantum gravity. *Phys. Rev. Lett.*, 95:061301, 2005b.

M. Bojowald. Degenerate configurations, singularities and the non-abelian nature of loop quantum gravity. *Class. Quantum Grav.*, 23:987–1008, 2006a.

M. Bojowald. Loop quantum cosmology and inhomogeneities. *Gen. Rel. Grav.*, 38:1771–1795, 2006b.

M. Bojowald. Quantum gravity and cosmological observations. In *Proceedings of the VIth Latin American Symposium on High Energy Physics (Puerto Vallarta, Mexico)*. [gr-qc/0701142](http://arxiv.org/abs/gr-qc/0701142), 2007a.

M. Bojowald. Singularities and quantum gravity. In *Proceedings of the XIIth Brazilian School on Cosmology and Gravitation*. [gr-qc/0702144](http://arxiv.org/abs/gr-qc/0702144), 2007b.

M. Bojowald, D. Cartin, and G. Khanna. Lattice refining loop quantum cosmology, anisotropic models and stability. [arXiv:0704.1137](http://arxiv.org/abs/0704.1137), 2007a.

M. Bojowald and R. Das. The radiation equation of state and loop quantum gravity corrections. *Phys. Rev. D*, to appear, 2007.

M. Bojowald, R. Das, and R. Scherrer. In preparation, 2007b.

M. Bojowald, H. Hernández, M. Kagan, P. Singh, and A. Skirzewski. Formation and evolution of structure in loop cosmology. *Phys. Rev. Lett.*, 98:031301, 2007c.

M. Bojowald, H. Hernández, M. Kagan, and A. Skirzewski. Effective constraints of loop quantum gravity. *Phys. Rev. D*, 75:064022, 2007d.

M. Bojowald and H. A. Kastrup. Symmetry reduction for quantized diffeomorphism invariant theories of connections. *Class. Quantum Grav.*, 17:3009–3043, 2000.

M. Bojowald, J. E. Lidsey, D. J. Mulryne, P. Singh, and R. Tavakol. Inflationary cosmology and quantization ambiguities in semi-classical loop quantum gravity. *Phys. Rev. D*, 70:043530, 2004a.

M. Bojowald, P. Singh, and A. Skirzewski. Coordinate time dependence in quantum gravity. *Phys. Rev. D*, 70:124022, 2004b.

M. Bojowald and A. Skirzewski. Effective equations of motion for quantum systems. *Rev. Math. Phys.*, 18:713–745, 2006.

M. Bojowald and A. Skirzewski. Quantum gravity and higher curvature actions. In *Current Mathematical Topics in Gravitation and Cosmology (42nd Karpacz Winter School of Theoretical Physics)*. *Int. J. Geom. Meth. Mod. Phys.*, 4:25–52, 2007.
E. J. Copeland, J. E. Lidsey, and S. Mizuno. Correspondence between loop-inspired and braneworld cosmology. *Phys. Rev. D*, 73:043503, 2006.

G. Date and G. M. Hossain. Genericity of inflation in isotropic loop quantum cosmology. *Phys. Rev. Lett.*, 94:011301, 2005.

B. Dittrich. *Aspects of Classical and Quantum Dynamics of Canonical General Relativity*. PhD thesis, University of Potsdam, 2005.

B. Dittrich. Partial and complete observables for hamiltonian constrained systems. *Class. Quantum Grav.*, 23:6155–6184, 2006.

M. Domagala and J. Lewandowski. Black hole entropy from quantum geometry. *Class. Quantum Grav.*, 21:5233–5243, 2004.

J. Engle. Quantum field theory and its symmetry reduction. *Class. Quant. Grav.*, 23:2861–2893, 2006.

J. Engle. On the physical interpretation of states in loop quantum cosmology. gr-qc/0701132, 2007.

L. Freidel, D. Minic, and T. Takeuchi. Quantum gravity, torsion, parity violation and all that. *Phys. Rev. D*, 72:104002, 2005.

K. Giesel and T. Thiemann. Algebraic quantum gravity (AQG) I. Conceptual setup. *Class. Quantum Grav.*, 24:2465–2497, 2007.

G. Immirzi. Real and complex connections for canonical gravity. *Class. Quantum Grav.*, 14:L177–L181, 1997.

T. Koslowski. Reduction of a quantum theory. gr-qc/0612138, 2006.

T. Koslowski. A cosmological sector in loop quantum gravity. 2007.

J. E. Lidsey. Early universe dynamics in semi-classical loop quantum cosmology. *JCAP*, 0412:007, 2004.

S. Mathur. How does the universe expand? *Int. J. Mod. Phys. D*, 12:1681–1686, 2003.

K. A. Meissner. Black hole entropy in loop quantum gravity. *Class. Quantum Grav.*, 21:5245–5251, 2004.

S. Mercuri. Fermions in Ashtekar–Barbero connections formalism for arbitrary values of the immirzi parameter. *Phys. Rev. D*, 73:084016, 2006.

H. Nicolai, K. Peeters and M. Zamaklar. Loop quantum gravity: an outside view. *Class. Quantum Grav.*, 22:R193–R247, 2005.
A. Perez and C. Rovelli. Physical effects of the immirzi parameter. *Phys. Rev. D*, 73: 044013, 2006.

C. Rovelli. Quantum reference systems. *Class. Quantum Grav.*, 8:317–332, 1991a.

C. Rovelli. What is observable in classical and quantum gravity? *Class. Quantum Grav.*, 8:297–316, 1991b.

C. Rovelli. *Quantum Gravity*. Cambridge University Press, Cambridge, UK, 2004.

C. Rovelli and L. Smolin. The physical hamiltonian in nonperturbative quantum gravity. *Phys. Rev. Lett.*, 72:446–449, 1994.

C. Rovelli and L. Smolin. Discreteness of area and volume in quantum gravity. *Nucl. Phys. B*, 442:593–619, 1995a. Erratum: *Nucl. Phys. B*, 456:753, 1995.

C. Rovelli and L. Smolin. Spin networks and quantum gravity. *Phys. Rev. D*, 52(10): 5743–5759, 1995b.

H. Sahlmann and T. Thiemann. Towards the QFT on curved spacetime limit of QGR. I: A general scheme. *Class. Quantum Grav.*, 23:867–908, 2006a.

H. Sahlmann and T. Thiemann. Towards the QFT on curved spacetime limit of QGR. II: A concrete implementation. *Class. Quantum Grav.*, 23:909–954, 2006b.

D. Samart and B. Gumjudpai. Phantom field dynamics in loop quantum cosmology. *arXiv:0704.3414*, 2007.

M. Sami, P. Singh, and S. Tsujikawa. Avoidance of future singularities in loop quantum cosmology. *Phys. Rev. D*, 74:043514, 2006.

P. Singh. Loop cosmological dynamics and dualities with randall-sundrum braneworlds. *Phys. Rev. D*, 73:063508, 2006.

A. Skirzewski. *Effective Equations of Motion for Quantum Systems*. PhD thesis, Humboldt-Universität Berlin, 2006.

T. Thiemann. Quantum spin dynamics (QSD). *Class. Quantum Grav.*, 15:839–873, 1998a.

T. Thiemann. QSD V: Quantum gravity as the natural regulator of matter quantum field theories. *Class. Quantum Grav.*, 15:1281–1314, 1998b.

T. Thiemann. Introduction to modern canonical quantum general relativity. *gr-qc/0110034*, 2001.

S. Tsujikawa, P. Singh, and R. Maartens. Loop quantum gravity effects on inflation and the cmb. *Class. Quantum Grav.*, 21:5767–5775, 2004.