Plastic rotation capacity of lightweight-aggregate concrete beams

Luis F. A. Bernardo, Miguel C. S. Nepomuceno & Hugo A. S. Pinto

To cite this article: Luis F. A. Bernardo, Miguel C. S. Nepomuceno & Hugo A. S. Pinto (2016): Plastic rotation capacity of lightweight-aggregate concrete beams, Journal of Civil Engineering and Management, DOI: 10.3846/13923730.2014.946442

To link to this article: http://dx.doi.org/10.3846/13923730.2014.946442

Published online: 28 Jan 2016.
PLASTIC ROTATION CAPACITY OF LIGHTWEIGHT-AGGREGATE CONCRETE BEAMS

Luis F. A. BERNARDO, Miguel C. S. NEPOMUCENO, Hugo A. S. PINTO

Department of Civil Engineering and Architecture, C-made: Centre of Materials and Building Technologies, University of Beira Interior, 6201-001 Covilhã, Portugal

Received 12 Mar 2014; accepted 23 May 2014

Abstract. This article describes a study on the plastic behaviour of lightweight-aggregate concrete beams. The experimental results of nineteen simply supported beams previously tested by the authors were used in this study. The experimental plastic rotation capacity of the tested beams was characterized by a parameter called Plastic Trend Parameter (PTP). The main variables studied were the concrete compressive strength and the longitudinal tensile reinforcement ratio. It was found that plastic rotation capacity slightly increases as the concrete compressive strength increases. An appropriate range for the longitudinal tensile reinforcement ratio to ensure plastic rotation capacity is proposed. The results of this study were also compared with the requirements from some codes of practice. From this analysis, it was shown that ACI Code requirements give more guarantees as far as plastic rotation capacity is concerned, when compared with European codes.

Keywords: beams, reinforced concrete, lightweight-aggregate concrete, flexure, plastic rotation capacity, codes of practice.

Introduction

The evolution of the chemical admixtures and minerals has evolved in the last decades. For this reason, lightweight aggregate concretes (LWAC) have seen successive improvements in terms of workability, durability and compressive strength. In several structural applications, advantages exist in using concretes with higher compressive strengths and, simultaneously, lighter weights. This is because self-weight can represent a very large percentage of the total load. These applications include, for instance, slabs in high rise buildings, bridge decks, pavement rehabilitation, precast elements, etc.

It is known that some characteristics of the concrete are deteriorated by increasing compressive strength and/or incorporating lightweight aggregates instead of normal-weight aggregates. For instance, it was shown that tensile/compressive strength ratio and fracture toughness are lower (Domagala 2011; Cui et al. 2012). This results in higher brittleness of the concrete (Jung et al. 2007).

However, as far as compressive strength is concerned, tests have shown that low deformability for high strength concrete does not necessarily result in brittle behaviour of structural elements made with this material. For beams under flexure using normal strength (NS) and high strength (HS) normal-weight aggregate concrete (NWAC), several studies have shown that such elements can positively combine the relative brittleness of the concrete with an adequate amount and detailing of ductile reinforcing bars (Shin et al. 1989; Shehata, I. A. E. M., Shehata, L. C. D 1996; Bernardo, Lopes 2003, 2004; Lopes, Bernardo 2003).

In general, this is also true for LWAC beams. However, LWAC and NWAC beams show marked differences in their flexural ultimate behaviour (Sin et al. 2010). For instance, the flexural ductility of HS LWAC beams (concrete compressive strength above 30 MPa (Ahmad et al. 1995)) is lower than NWAC beams (Liu et al. 2006; Jung et al. 2007; Bernardo et al. 2014). This is because LWAC is more brittle, both in tension and compression, when compared with NWAC. Moreover, it is expected that, for LWAC beams, the concern with ductility may arise for lower compressive strength of concrete when compared with NWAC beams (Bernardo et al. 2014).

Some techniques to improve ductility have been studied, such as the incorporation of steel fibres into LWAC and NWAC (Balendran et al. 2002; Domagala 2011). However, such techniques are expensive. For current structures, simple design detailing rules based on the control of key parameters to provide sufficient amount of flexural ductility are desirable.
1. Research significance and previous studies

Structural elements must not only provide adequate strength but should also ensure adequate ductility under overload condition. This important property is directly related to the capacity for redistributing internal forces with structural safety. Ductility is an important issue and still continues to be focused in recent studies (for instance, Lam et al. 2009a, 2009b; Kwan, Ho 2010).

The ductility of the structural elements depends directly on the plastic rotation capacity of the critical sections (CEB 1998; Pecce, Fabbrocino 1999; Ko et al. 2001). This property is related with the ductility and, therefore, the results obtained by studying ductility can be confirmed.

For NWAC, codes of practice provide detailed rules to ensure sufficient flexural ductility. In general, such rules attempt to control the relative depth of the compressed concrete in failure, which strongly depends on the characteristics of steel and the mechanical ratio of tensile longitudinal reinforcement. Other factors also influence flexural ductility, such as the concrete strength, the transverse and longitudinal compressive reinforcement ratios.

Several studies on NWAC beams, including NS and HS concrete, confirmed the influence of the above-mentioned parameters (Shin et al. 1989; Bernardo, Lopes 2003, 2004). Previous studies also checked the influence of some of these parameters for LWAC beams (Ahmad, Barker 1991; Ahmad, Batts 1991; Ahmad et al. 1995). However, the flexural behaviour of LWAC beams, including NS and HS concrete, still continues to be studied because several requirements of codes of practice continue to be based on the experimental results of NWAC beams. Recent papers about this subject, including HS LWAC beams, can be found in the literature (Liu et al. 2006; Jung et al. 2007; Sin et al. 2010; Ho, Zhou 2011; Bernardo et al. 2014). Although some proposals for the extension of European codes of practice to include LWAC, including HS concrete, have been reported (Faust 2000; Fib 2000), they are still not fully incorporated. This is because several aspects about flexural performance of LWAC beams are not fully clarified. This shows that more studies need to be carried out. Moreover, only a limited number of studies specifically focused on the ductility of LWAC beams under flexure still exist in the literature (Ahmad, Barker 1991; Ahmad, Batts 1991; Liu et al. 2006; Jung et al. 2007; Bernardo et al. 2014). Also related with ductility, the rotation capacity of LWAC beams constitutes a very important issue, which should be fully studied in order to help to clarify some aspects about the flexural performance of LWAC beams.

In a previous article (Bernardo et al. 2014) the authors presented an experimental study on the flexural ductility of LWAC beams, by using ductility indexes. Some important findings about how concrete strength influences ductility and also on the appropriate range for the longitudinal tensile reinforcement ratio were presented. The results of this previous study were also compared with the requirements from some important codes of practice. The present article constitutes an extension of the previously referred study from the authors, and presents a study on the flexural plastic behaviour of the same set of tested beams, by using a parameter called Plastic Trend Parameter (PTP). Since ductility and plastic rotation capacity are directly related, the findings of this new study should confirm the previous findings of the authors. Moreover, no studies especially focused on the plastic rotation capacity of LWAC beams under flexure were found in the literature.

2. Description of the tested beams

A detailed description of the experimental work performed by the authors can be found in a previous article (Bernardo et al. 2014). However, a brief description of the tested beams and testing procedures is presented below.

Nineteen simply supported RC LWAC beams were tested until failure. The test setup, the geometry and the detail of the reinforcement are illustrated in Figure 1. Hot-rolled ribbed steel rebars (S400) were used for the reinforcement. The concrete cover was 2 cm.

Table 1 summarizes some relevant properties of each test beam, namely: the effective depth (d) of the cross-section, the average LWAC compressive strength (f_c) and dry density (δ_d), the average Young’s Modulus (E_{lc}), the area of longitudinal tensile reinforcement (A_s) and the reinforcement ratio ρ. The beams were classified into 3 series, depending on the range of concrete strength.

The mix proportions of the LWAC produced in the laboratory and the results from tensile tests carried out on samples of steel tensile reinforcement bars can be found in Bernardo et al. (2014). The average yielding stress (f_y) varied between 503 and 575 MPa, depending on the diameter of the bars. A Young’s Modulus (E_y) of 200 GPa was assumed to compute the yield strain values (ε_y).

Figure 2 illustrates the beam in test position, including the location of the external measuring instruments. An external grid of Demec targets placed on one face of the beam, between the load application points, was used to measure the strains along the height of the central sections. Resistance strain gauges were fixed to the longitudi-
3. Experimental rotation and deflection curves

The total load \( P \) – deflection \( \delta \) curves and the moment \( M \) – curvature \( \chi \) curves for each tested beam can be found in Bernardo et al. (2014). Figures 4(a) to 4(c) represent, for each series of beams, the experimental rotation \( \theta \) – deflection at mid-span \( \delta \) graphs.

Each graph shows experimental and theoretical curves. The theoretical curves were computed by using a theoretical elastic analysis (TEA) with homogenized section (considering the influence of the reinforcement) and a theoretical plastic analysis (TPA) of the tested beams (assuming a mechanism). The experimental and theoretical elastic rotations were obtained by multiplying the corresponding curvatures (theoretical and experimental) by a length of \( 1.2h \), being \( h \) the height of the cross-section (Fig. 5). This length, according to Eurocode 2 – EC2 (NP EN 1992-1-1 2010), corresponds to the length of the local plastic hinge of beams with ductile failure. The theoretical elastic curvatures were calculated from the elastic stress and strain diagrams, assuming a pure bending state acting on the homogenized section of the beams. The theoretical elastic deflections of the beams at mid-span were calculated using tables to compute elastic deflections. Since the slopes of the straight lines for the TEA are very similar, only an average straight line was drawn in the graphs of Figures 4(a) to 4(c). To compute the theoretical plastic rotation- deflection relationship (Eqn (1)), a global mechanism with a plastic hinge located at the mid-span section was assumed (Fig. 6):

| Beam series \((f_c - \sigma)\) | \(f_c\) MPa | \(\delta_l\) kg/m³ | \(E_c\) GPa | \(A_s\) cm² | \(d\) cm | \(\rho = A_s / bd\) % |
|-----------------------------|-------------|------------------|-------------|-------------|--------|-------------------------|
| 1(23.5-0.13)                | 23.5        | 1659             | 16.5        | 0.56 \((2\phi 6)\) | 27.7   | 0.13                    |
| 1(22.8-0.24)                | 22.8        | 1659             | 16.0        | 1.01 \((2\phi 8)\) | 27.6   | 0.24                    |
| 1(22.0-0.38)                | 22.0        | 1687             | 15.8        | 1.58 \((2\phi 10)\) | 27.5   | 0.38                    |
| 1(22.4-0.55)                | 22.4        | 1651             | 17.1        | 2.26 \((2\phi 12)\) | 27.4   | 0.55                    |
| 1(28.5-0.99)                | 28.5        | 1659             | 23.2        | 4.02 \((2\phi 16)\) | 27.2   | 0.99                    |
| 2(45.1-1.0-13)              | 45.1        | 1802             | 22.8        | 0.56 \((2\phi 6)\) | 27.7   | 0.13                    |
| 2(42.1-0.2-13)              | 42.1        | 1807             | 23.7        | 1.01 \((2\phi 8)\) | 27.6   | 0.24                    |
| 2(47.1-1.0-38)              | 47.1        | 1809             | 24.5        | 1.58 \((2\phi 10)\) | 27.5   | 0.38                    |
| 2(49.2-0.55)                | 49.2        | 1827             | 22.7        | 2.26 \((2\phi 12)\) | 27.4   | 0.55                    |
| 2(43.4-0.99)                | 43.9        | 1788             | 23.2        | 4.02 \((2\phi 16)\) | 27.2   | 0.99                    |
| 2(47.0-1.5-55)              | 47.0        | 1791             | 22.6        | 6.28 \((2\phi 20)\) | 27.0   | 1.55                    |
| 2(43.0-2.03)                | 43.0        | 1790             | 26.0        | 8.04 \((4\phi 16)\) | 26.4   | 2.03                    |
| 3(52.1-1.0-13)              | 52.1        | 1867             | 26.2        | 0.56 \((2\phi 6)\) | 27.7   | 0.13                    |
| 3(51.2-0.2-38)              | 51.2        | 1879             | 26.1        | 1.58 \((2\phi 10)\) | 27.5   | 0.38                    |
| 3(52.4-0.55)                | 52.4        | 1869             | 27.7        | 2.26 \((2\phi 12)\) | 27.4   | 0.55                    |
| 3(55.3-0.99)                | 55.3        | 1910             | 26.5        | 4.02 \((2\phi 16)\) | 27.2   | 0.99                    |
| 3(53.4-1.55)                | 53.4        | 1877             | 29.7        | 6.28 \((2\phi 20)\) | 27.0   | 1.55                    |
| 3(60.4-2.03)                | 60.4        | 1953             | 25.9        | 8.04 \((4\phi 16)\) | 26.4   | 2.03                    |
| 3(51.6-2.69)                | 51.6        | 1867             | 16.5        | 10.30 \((2\phi 16+2\phi 20)\) | 25.5 | 2.69                    |
\[ \theta = 2 \arctan \left( \frac{\delta}{1200} \right). \]  

(1)

The points of the experimental curves \( \theta - \delta \) were obtained directly from the experimental values recorded during testing. The experimental rotation \( \theta \) represents the rotation between the two surrounding sections of the plastic hinge. This rotation was obtained by multiplying the experimental curvatures by the length of the plastic hinge \((1.2h)\), as also done for the TEA. The experimental curvatures were obtained from the experimental strains along the height of the sections in the failure zone. These were measured from the external grid of Demec targets.

As previously showed by Bernardo et al. (2014), in the first beams of each series with the lowest tensile reinforcement ratio (Table 1), the reinforcement yielded suddenly after the first concrete crack occurred. Due to the limitation of the measuring instruments, it was not possible to record accurately the readings for the deformation state of these beams until the effective failure of the steel bars. For this reason, the results of these beams were not used by Bernardo et al. (2014) to study the flexural ductility. In this sense, these beams were also excluded in this study to evaluate the plastic rotation capacity of the tested LWAC beams under flexure.

In the next paragraphs, a summary of the calculation procedure to compute the PTP is presented.

From Figures 4(a) to 4(c) it can be seen that all beams show an almost perfect elastic behaviour during a relatively short interval of the deformations. The points of the curves from which the experimental values start to shift from the theoretical linear response correspond to the yielding of the tensile longitudinal reinforcement. When these points are reached, the experimental rotation \( (\theta) \) suddenly increases and the curve adopt new slopes that lie between the two straight lines of the theoretical analyses (TPA and TEA).

4. Plastic rotation capacity

4.1. Evaluation of the plastic trend parameter

In this section, the experimental and purely plastic behaviour of the tested beams is evaluated by analysing the experimental values for the plastic rotation. An experimental parameter, called Plastic Trend Parameter (PTP), was previously proposed and used in other studies to characterize the plastic rotation capacity of NWAC beams. These studies included NS and HC concrete beams under pure bending (Bernardo, Lopes 2009) as well as similar beams with the compressive concrete area confined with internal stirrups (Bernardo et al. 2008). Moreover, PTP has also been adapted and used to characterize the plastic twist capacity of hollow beams under torsion (Bernardo, Lopes 2013). From these earlier studies, it was found that the proposed PTP is very suitable to characterize and study the experimental plastic behaviour of RC beams. For this reason, PTP was considered sufficiently reliable to be also used in the present study to characterize the plastic rotation capacity of the tested LWAC beams under flexure.

In the next paragraphs, a summary of the calculation procedure to compute the PTP is presented.

As previously referred (Section 3), the experimental values of the rotations plotted in Figures 4(a) to 4(c) include both the elastic and plastic parts of the rotation. Therefore, they are referred to as total rotations.
to study only the plastic part of the rotation, the elastic part has to be eliminated from the experimental rotations. To achieve this goal, for each tested beam and for each deformation step, the elastic rotation computed from the experimental rotations is subtracted from the experimental plastic rotation \( \theta_p \). From this calculation, the experimental plastic rotation \( \theta_p \) – deflection at mid span \( \delta \) graphs are plotted for each series of beams in Figures 7(a) to 7(c).

The axes of the graphs in Figures 7(a) to 7(c) are non-dimensional for better interpretation and comparison purposes. Parameter \( \theta_{p,th} \) represents the ultimate value of the theoretical plastic rotation computed from the TPA and corresponding to the ultimate experimental value of the deflection \( \delta_u \).

In the graphs of Figures 7(a) to 7(c), the last value for the experimental deflection \( \delta \) and plastic rotation \( \theta_p \) corresponds to a conventional ultimate value for the load which is defined from the \( P - \delta \) graphs. As assumed by Bernardo et al. (2014), to study the flexural ductility of the same tested beams, in this study it is also assumed that the ultimate point on the experimental \( P - \delta \) curves correspond to the point of intersection between the descending branch of the curve with a horizontal line that crosses at the point where the reinforcement starts to yield (in the ascending branch of the curve). If no intersection is found to occur between the aforementioned line and the experimental curve, the ultimate point is simply ascribed to the last point on the curve. This criterion considers that below the defined ultimate point the beam capacity to sustain substantial loads is no longer acceptable. A detailed illustration of this criterion and a discussion about its validity to analyse comparatively the flexural ductility among the tested beams can be found in Bernardo et al. (2014). Since the plastic rotation capacity is related with the flexural ductility, the referred criterion is also considered valid to perform a comparative analysis of the plastic rotation capacity between the tested beams.

From the graphs of Figures 7(a) to 7(c), two parameters, \( C_{p,exp} \) and \( C_{p,th} \), are computed. These parameters represent the area below the experimental curve \( C_{p,exp} \) and below the theoretical line corresponding to the TPA \( C_{p,th} \), respectively. From Figures 7(a) to 7(c), it can be seen that the value of \( C_{p,th} \) is constant and equal to 0.5 for all beams. The ratio \( C_{p,exp} / C_{p,th} \) is called Plastic Trend...
Parameter (PTP) and provides an indication of the experimental plastic rotation capacity level compared with the theoretical perfectly plastic behaviour (TPA). Therefore, the higher the value of the PTP the larger the experimental plastic rotation capacity for a given beam. The values obtained for the PTP are summarized in Table 2. Table 2 also presents the average compressive strength of the concrete ($f_{lc,m}$) for each series of beams, to be used latter (Section 4.3).

### 4.2. Influence of the concrete strength

In order to analyse the influence of the concrete compressive strength on the PTP, Table 3 groups the beams as a function of their reinforcement ratios ($\rho$) and presents the average longitudinal tensile reinforcement ratios values ($\rho_{m}$) for each group. Only groups with 3 beams at least are presented.

A global analysis of Table 3 shows that the plastic rotation capacity seems to increase slightly as the concrete compressive strength increases.

#### Table 2. Experimental values of PTP

| Beam     | $f_{lc,m}$ (MPa) | $\rho$ (%) | PTP (%) |
|----------|------------------|------------|---------|
| 1(22.8-0.24) | 23.9  | 0.24   | 9.8     |
| 1(22.0-0.38) | 22.0  | 0.38   | 5.8     |
| 1(22.4-0.55) | 22.4  | 0.55   | 3.2     |
| 1(28.5-0.99) | 28.5  | 0.99   | 1.8     |
| 2(42.1-0.24) | 42.1  | 0.24   | 11.2    |
| 2(47.1-0.38) | 47.1  | 0.38   | 6.9     |
| 2(49.2-0.55) | 49.2  | 0.55   | 4.4     |
| 2(43.9-0.99) | 43.9  | 0.99   | 2.7     |
| 2(47.0-1.55) | 47.0  | 1.55   | 1.5     |
| 2(43.0-2.03) | 43.0  | 2.03   | 0.9     |
| 3(51.2-0.38) | 51.2  | 0.38   | 6.5     |
| 3(52.4-0.55) | 52.4  | 0.55   | 3.8     |
| 3(55.3-0.99) | 55.3  | 0.99   | 2.6     |
| 3(53.4-1.55) | 53.4  | 1.55   | 1.9     |
| 3(60.4-2.03) | 60.4  | 2.03   | 0.9     |
| 3(51.6-2.69) | 51.6  | 2.69   | 0       |

#### Table 3. Beams groups with similar reinforcement ratio

| Group | Beam     | $\rho_{m}$ (%) | $f_{lc}$ (MPa) | PTP (%) |
|-------|----------|----------------|---------------|---------|
| I     | 1(22.0-0.38) | 0.38       | 22.0          | 5.8     |
|       | 2(47.1-0.38) |            | 47.1          | 6.9     |
|       | 3(51.2-0.38) |            | 51.2          | 6.5     |
| II    | 1(22.4-0.55) | 0.55       | 22.4          | 3.2     |
|       | 2(49.2-0.99) |            | 49.2          | 4.4     |
|       | 3(52.4-0.99) |            | 52.4          | 3.8     |
| III   | 1(28.5-0.99) | 0.99       | 28.5          | 1.8     |
|       | 2(43.9-0.99) |            | 43.9          | 2.7     |
|       | 3(55.3-0.99) |            | 55.3          | 2.6     |

Figure 8 shows the evolution of the PTP with the concrete compressive strength ($f_{lc}$) for Groups I, II and III (Table 3). The graphs also include a line calculated with linear regression. The tendencies observed graphically confirm that, for a constant longitudinal tensile reinforcement ratio, the plastic rotation capacity of the beams increases slightly with increasing concrete compressive strength.

The tendencies observed before are very similar to the ones observed by Bernardo et al. (2014) for the same tested beams and from the study of the flexural ductility by using ductility indexes. The above tendencies also agree with the ones observed by Bernardo and Lopes (2003, 2009) based on the plastic rotation capacity analysis of NWAC beams.

As previously explained by Bernardo et al. (2014), the flexural behaviour of RC beams is governed by the mechanical percentage of steel reinforcement ($\rho_{m}/f_{c}$). Therefore, the flexural ductility increases as the mechanical percentage of steel decreases. Hence, beams with a lower longitudinal reinforcement ratio and higher concrete compressive strength can have higher ductility than beams with a higher longitudinal reinforcement ratio and lower concrete compressive strength. Since the plastic rotation capacity is related with the flexural ductility, the above explanation is also valid to explain the observed tendencies from Figure 8.

### 4.3. Influence of the tensile reinforcement ratio

The analysis of the influence of the longitudinal tensile reinforcement ratio on the PTP requires grouping the beams with similar or equal concrete compressive strength (Table 2).

A global analysis of Table 2 shows that, for each group of beams, the plastic rotation capacity tends to decrease with increasing longitudinal reinforcement ratio. This tendency seems to be enhanced for the beams with low longitudinal tensile reinforcement ratio. This behaviour, already expected, may be explained by a fall in the ultimate deformation of the beam as the longitudinal tensile reinforcement ratio increases, thereby causing a fall of the plastic rotation capacity.
The above tendencies agree with the ones observed by Bernardo et al. (2014) for the same tested beams and based on the study of flexural ductility. The above tendencies also agree with the ones observed by Bernardo and Lopes (2003, 2009) based on the plastic rotation capacity analysis of NWAC beams.

Figure 9 presents the graph showing the evolution of the PTP as a function of the longitudinal tensile reinforcement ratio \( r \). Beam 3(51.6-2.69) was not included because no plastic rotation capacity was observed (PTP = 0, see Table 2). The graph in Figure 9 also includes a potential tendency curve, which fit well the tendency of the results. In Section 4.2, it was observed that the Influence of reinforcement ratio on the PTP

\[
y = 2.2217x^{1.08}
\]

The plastic rotation capacity of the LWAC beams, from the two studied, is the longitudinal tensile reinforcement ratio.

5. Comparative analyses with codes requirements

As previously performed by the authors (Bernardo et al. 2014), this study also aimed to analyse some simplest design rules intended for the assurance of enough ductility in LWAC beams, in the light of the test results described in this paper and related with the plastic rotation capacity. The analysis presented in this section is only focused in the limitation of the amount of longitudinal tensile reinforcement (both maximum and minimum). This simplest approach was previously justified by Bernardo et al. (2014) and also successfully used in previous studies focused on the flexural ductility and plastic rotation capacity of RC beams (Bernardo, Lopes 2003, 2004, 2009).

The following codes of practice were analysed: Model Code 1990 – MC90 (CEB 1990), Model Code 2010 – MC2010 (Fib 2010), Eurocode 2 – EC2 (NP EN1992-1-1 2010) and ACI 318-11 (2011). A detailed discussion on the code rules to compute the limits for the amount of longitudinal tensile reinforcement for LWAC beams under flexure can be found in Bernardo et al. (2014). The equations from each code of practice, as well as the values obtained for the beams, for the maximum and minimum longitudinal tensile reinforcement ratios \( (\rho_{\text{min}} \text{ and } \rho_{\text{max}}) \) are summarized in Table 4.

Table 4 includes the first beams of each series (beams with the lowest longitudinal tensile reinforcement ratio) as well as Beam 3(51.6-2.69) with PTP = 0 (Table 2). The incorporation of the first beams of each series is important for the comparison because they suffered a premature failure due to insufficient reinforcement (bar’s failure), so the codes should not allow such beams. Codes should also not allow Beam 3(51.6-2.69) which suffers a purely brittle failure due to concrete crushing.

From Table 4, it can be seen that ACI 318-11 (2011) is clearly more restrictive as far as the maximum and minimum amount of longitudinal tensile steel reinforcement is concerned. With one exception, all codes do not allow Beams 1(23.5-0.13), 2(45.1-0.13) and 3(52.1-0.13). The exception is Beam 1(23.5-0.13) that is allowed by MC2010 (Fib 2010). As previously referred, those beams have not a sufficient tensile reinforcement ratio. Then, it can be stated that almost all the studied codes provide adequate minimum limits for the longitudinal tensile reinforcement ratio, when applied to the tested LWAC beams. In addition, ACI 318-11 (2011) do not allow Beams 1(22.8-0.24) and 2(42.1-0.24). Such beams have low longitudinal tensile reinforcement ratios but they showed to have enough plastic rotation capacity. From this point of view, the minimum limits for the longitudinal tensile reinforcement ratio specified by ACI 318-11 (2011) seems to be somewhat excessive when applied to the tested LWAC beams.

As regards the beams with the highest longitudinal tensile reinforcement ratios, they are all allowed by the European codes. ACI 318-11 (2011) do not allow Beams
Table 4. Limits for the tensile reinforcement ratios

| Beam          | PTP | $\rho$ % | $\rho_{\text{min}}$ % | $\rho_{\text{max}}$ % | $\rho_{\text{min}}$ % | $\rho_{\text{max}}$ % | $\rho_{\text{min}}$ % | $\rho_{\text{max}}$ % | $\rho_{\text{min}}$ % | $\rho_{\text{max}}$ % |
|---------------|-----|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 1(23.5-0.13)  | –   | 0.13     | 0.15                  | 4.33                  | 0.09                  | 4.33                  | 0.15                  | 4.33                  | 0.35                  | 1.38                  |
| 1(22.8-0.24)  | 9.8 | 0.24     | 0.15                  | 4.35                  | 0.10                  | 4.35                  | 0.15                  | 4.35                  | 0.35                  | 1.17                  |
| 1(22.0-0.38)  | 5.8 | 0.38     | 0.15                  | 4.36                  | 0.09                  | 4.36                  | 0.15                  | 4.36                  | 0.35                  | 1.09                  |
| 1(22.4-0.55)  | 3.2 | 0.55     | 0.15                  | 4.38                  | 0.09                  | 4.38                  | 0.15                  | 4.38                  | 0.35                  | 1.20                  |
| 1(28.5-0.99)  | 1.8 | 0.99     | 0.15                  | 4.41                  | 0.12                  | 4.41                  | 0.15                  | 4.41                  | 0.35                  | 1.37                  |
| 2(45.1-0.13)  | –   | 0.13     | 0.15                  | 4.33                  | 0.18                  | 4.33                  | 0.18                  | 4.33                  | 0.35                  | 2.31                  |
| 2(42.1-0.24)  | 11.2| 0.24     | 0.15                  | 4.35                  | 0.17                  | 4.35                  | 0.17                  | 4.35                  | 0.35                  | 1.94                  |
| 2(47.1-0.38)  | 6.9 | 0.38     | 0.15                  | 4.36                  | 0.19                  | 4.36                  | 0.19                  | 4.36                  | 0.35                  | 2.00                  |
| 2(49.2-0.55)  | 4.4 | 0.55     | 0.15                  | 4.38                  | 0.20                  | 4.38                  | 0.20                  | 4.38                  | 0.35                  | 2.23                  |
| 2(43.9-0.99)  | 2.7 | 0.99     | 0.15                  | 4.41                  | 0.18                  | 4.41                  | 0.18                  | 4.41                  | 0.35                  | 1.86                  |
| 2(47.0-1.55)  | 1.5 | 1.55     | 0.15                  | 4.44                  | 0.19                  | 4.44                  | 0.19                  | 4.44                  | 0.35                  | 1.96                  |
| 2(43.0-2.03)  | 0.9 | 2.03     | 0.15                  | 4.55                  | 0.17                  | 4.55                  | 0.17                  | 4.55                  | 0.35                  | 1.84                  |
| 3(51.0-1.13)  | –   | 0.13     | 0.15                  | 4.33                  | 0.21                  | 4.33                  | 0.21                  | 4.33                  | 0.35                  | 2.51                  |
| 3(51.2-0.38)  | 6.5 | 0.38     | 0.15                  | 4.36                  | 0.20                  | 4.36                  | 0.20                  | 4.36                  | 0.35                  | 2.10                  |
| 3(52.4-0.55)  | 3.8 | 0.55     | 0.15                  | 4.38                  | 0.21                  | 4.38                  | 0.21                  | 4.38                  | 0.35                  | 2.31                  |
| 3(55.3-0.99)  | 2.6 | 0.99     | 0.15                  | 4.41                  | 0.22                  | 4.41                  | 0.22                  | 4.41                  | 0.35                  | 2.13                  |
| 3(53.4-1.55)  | 1.9 | 1.55     | 0.15                  | 4.44                  | 0.21                  | 4.44                  | 0.21                  | 4.44                  | 0.34                  | 2.11                  |
| 3(60.4-2.03)  | 0.9 | 2.03     | 0.15                  | 4.55                  | 0.24                  | 4.55                  | 0.24                  | 4.55                  | 0.37                  | 2.22                  |
| 3(51.6-2.69)  | 0   | 2.69     | 4.71                  | 4.71                  | 4.71                  | 4.71                  |                      |                      |                      |                      |

$d, h$ = effective depth and height of the cross section;
$f_c^r$ = concrete strength in compression;
$f_{ck}$ = characteristic compressive strength ($f_{ck} = 8$ MPa);
$f_{c}$ = mean value of compressive strength of LWAC;
$f_{c\text{cm}}$ = mean tensile strength for LWAC (Fib 2010: $f_{c\text{cm}} = 0.3 f_{ck}$ for $f_{ck} \leq 50$ MPa and $f_{c\text{cm}} = 0.4 f_{ck}$ for $f_{ck} > 50$ MPa);
$f_{\rho, f_y}$ = mean and characteristic value of the steel yielding stress;
$\lambda$ = modification factor to account for LWAC (in this study $\lambda = 0.85$, Section 8.6.1 from ACI318 2011);
$\rho_b$ = longitudinal steel reinforcement ratio that leads to the so-called balanced strain conditions:

$$\rho_b = (0.85 b_1 f_{ck}/f_y) \times [600 / (600 + f_y)].$$

$2(43.0-2.03)$ and $3(51.6-2.69)$. Beam $3(51.6-2.69)$, with $\text{PTP} = 0$, suffered a brittle failure with no inelastic deformation and should not be allowed by the codes of practice. From this point of view, the maximum limit for the longitudinal tensile reinforcement ratio specified by ACI318 seems to be adequate when applied to the tested LWAC beams.

The previous observations show that, when compared to European codes, ACI 318-11 (2011) provides higher guaranty for plastic rotation capacity for the tested LWAC beams. This conclusion was also observed by Bernardo et al. (2014) by analysing the same beams and by studying the flexural ductility using ductility indexes. As already stated by these authors, the higher guaranty from ACI 318-11 (2011) to ensure plastic rotation capacity can be explained by the inclusion, into the equations, of the amount of reinforcement relative to concrete strength. In fact, this parameter governs the flexural behaviour of the beams. In European codes, the maximum longitudinal tensile reinforcement ratio is only fixed by mean of a constant percentage of the cross section area.

Figures 10(a) to 10(e) present graphically the PTP as a function of the longitudinal tensile reinforcement ratio, regardless of the concrete strength. The graphs also show on the background the range of normative limit values $\rho_{\text{min}}$ and $\rho_{\text{max}}$ from the codes of practice. Conventional points corresponding to the first beams of each series were also highlighted (with symbol ×). Since their plastic rotation capacity was not computed, a conventional
minimum value was assumed. The arrow drawn over the points mean that the value of the PTP is actually higher, despite these beams should not be allowed.

Figures 10(d) and 10(e) correspond to ACI 318-11 (2011). Since the maximum tensile reinforcement ratio is related to the concrete strength, the corresponding range of values will be too large if all the beams were included in the same graph. Then, two graphs were drawn. Figure 10(d) presents the graph for the beams from Series 1, while Figure 10(e) includes beams from Series 2 and 3 (with similar concrete strengths).

Observing Figures 10(a) to 10(e), it can be confirmed that ACI 318-11 (2011) requirements provides more guarantees as far as the plastic rotation capacity is concerned.

Conclusions

In this study, a comparative analysis on the plastic rotation capacity of LWAC beams was presented. The Plastic Trend Parameter (PTP) used in this study was found to be very suitable to characterize and study the experimental plastic rotation capacity of the tested LWAC beams.

It was observed that, for approximately constant values of concrete compressive strength, the plastic rotation capacity of the beams decreases as the longitudinal reinforcement ratio increases. The results also shown that, for similar longitudinal tensile reinforcement ratios, an increase in the concrete compressive strength causes a slight increase in the plastic rotation capacity.

The results also shown a high reduction on the plastic rotation capacity as the longitudinal tensile reinforcement ratio increases until approximately $r \approx 1.5–2.0\%$. After these values, plastic rotation capacity of the tested LWAC beams is very low or null (failure tends to be brittle), regardless of the compressive concrete strength. This limit for $r$ is somewhat smaller when compared with the same one previously reported by other authors for nWAC beams. This seems to show that the range of longitudinal tensile reinforcement ratio compatible with plastic rotation capacity are somewhat lower for LWAC beams, when compared to nWAC beams. More studies are needed in order to clarify this important aspect and, eventually, to propose limit values for the reinforcement ratio for LWAC beams.

When comparing the limit values for the amount of longitudinal tensile steel reinforcement from different codes of practice, it was observed that ACI Code (ACI 318-11 2011) is more restrictive than the European codes (MC90 (Fib 1990), MC2010 (Fib 2010) and EC2 (NP EN1992-1-1 2010). The difference is more noticeable as far as the upper limit is concerned. When compared with the European codes, it was observed that ACI Code ensures higher plastic rotation capacity for the tested LWAC beams.

The experimental results obtained in this article by using the Plastic Trend Parameter (PTP) to study the plastic rotation capacity shown very good agreement with previous results obtained by Bernardo et al. (2014) for the same beams and based on the study of flexural ductility by using ductility indexes.

![Fig. 10. Graphs $p$–PTP: (a) MC90 (Fib 1990); (b) MC2010 (Fib 2010); (c) NP EN1992-1-1 (2010); (d, e) ACI 318-11 (2011)](fig10.png)
References

ACI 318-11 Building code requirements for reinforced concrete and commentary. American Concrete Institute, USA, 2011. 503 p.
Ahmad, S. H.; Barker, R. 1991. Flexural behavior of reinforced high-strength lightweight concrete beams, ACI Structural Journal 88(1): 69–77.
Ahmad, S. H.; Battis, J. 1991. Flexural behavior of reinforced high-strength lightweight concrete beams with web reinforcement, ACI Structural Journal 88(3): 251–258.
Ahmad, S. H.; Xie, Y.; Yu, T. 1995. Shear ductility of reinforced lightweight concrete beams of normal strength and high strength concrete, Cement and Concrete Composites 17(2): 147–159. http://dx.doi.org/10.1016/0958-9465(94)00029-X
Balendran, R. V.; Zhou, F. P.; Nadeem, A.; Leung, A. Y. T. 2002. Influence of steel fibres on strength and ductility of normal and lightweight high strength concrete, Building and Environment 37(12): 1361–1367. http://dx.doi.org/10.1016/S0360-1234(01)00109-3
Bernardo, L. F. A.; Lopes, S. M. R. 2003. Flexural ductility of high-strength beams, Structural Concrete 4(3): 135–154. http://dx.doi.org/10.1680/stcou.2003.4.3.135
Bernardo, L. F. A.; Lopes, S. M. R. 2004. Neutral axis depth versus flexural ductility in high-strength concrete beams, Journal of Structural Engineering 130(3): 452–459. http://dx.doi.org/10.1061/(ASCE)0733-9437(2004)130:3(452)
Bernardo, L. F. A.; Lopes, S. M. R. 2009. Plastic analysis of HSC beams in flexure, Materials and Structures 42(1): 51–69. http://dx.doi.org/10.1617/s11527-008-9366-x
Bernardo, L. F. A.; Lopes, S. M. R. 2013. Plastic analysis and twist capacity of high-strength concrete hollow beams under pure torsion, Engineering Structures 49: 190–201. http://dx.doi.org/10.1016/j.enstruct.2012.10.030
Bernardo, L. F. A.; Nepomuceno, M. C. S.; Pinto, H. A. S. 2014. Flexural ductility of lightweight-aggregate concrete beams, Journal of Civil Engineering and Management (in Press).
Bernardo, L. F. A.; Oliveira, L. A. P.; Pinto, D. G. 2008. Influence of transversal reinforcement on plastic rotation capacity of high strength beams, in Proc. of the V International ACI/CANMET – Conference on High Performance Concrete Structures and Materials, 2008, Manaus, Brazil, 305–322.
CEB. 1990. CEB-FIP model code. Comité Euro-International du Béton, 1990. 437 p.
CEB. 1998. Ductility of reinforced concrete structures, Bulletin d’Information, No 242, 1 May, 1998.
Cui, H. Z.; Lo, T. Y.; Memon, S. A.; Xu, W. 2012. Effect of lightweight aggregate on the mechanical properties and brittleness of lightweight aggregate concrete, Construction and Building Materials 35: 149–158. http://dx.doi.org/10.1016/j.conbuildmat.2012.02.053
Domagala, L. 2011. Modification of properties of structural lightweight concrete with steel fibres, Journal of Civil Engineering and Management 17(1): 36–44. http://dx.doi.org/10.3846/13923730.2011.553923
Faust, T. 2000. Supplements to MC-90 for lightweight aggregate concrete, LACER 5: 75–84.
Fib. 2000. Lightweight aggregate concrete. Recommended extension to the Model Code 90. Fib Bulletin no 8. Lausanne: The International Federation for Structural Concrete, 118 p.
Fib. 2010. Model Code 2010. First complete draft. Bulletin no 55. Lausanne: The International Federation for Structural Concrete, 111 p.
Ho, J. C. M.; Zhou, K. J.-H. 2011. Limited deformability design of high-strength concrete beams in low to moderate seismicity regions, Journal of Civil Engineering and Management 17(3): 409–423. http://dx.doi.org/10.3846/13923730.2011.594219
Jung, I. H.; Yang, W. J.; Yi, W. H.; Jee, S. W.; Lee, S. Y. 2007. Flexural behavior of lightweight concrete beam with eco lightweight aggregates, in Proc. of the 6th International Conference on Fracture and Damage Mechanics, 17–19 July 2007, Madeira, Portugal, 733–736.
Ko, M. Y.; Kim, S. W.; Kim, J. K. 2001. Experimental study on the plastic rotation capacity of reinforced high strength concrete beams, Materials and Structures 34: 302–311. http://dx.doi.org/10.1007/PL02482210
Kwan, A. K. H.; Ho, J. C. M. 2010. Ductility design of high-strength concrete beams and columns, Advances in Structural Engineering 13(4): 651–664. http://dx.doi.org/10.1260/1369-4332.13.4.651
Lam, J. Y. K.; Ho, J. C. M.; Kwan, A. K. H. 2009a. Flexural ductility of high-strength concrete columns with minimal confinement, Materials and Structures 42(7): 909–921. http://dx.doi.org/10.1617/s11527-008-9431-5
Lam, Y. K.; Ho, J. C. M.; Kwan, A. K. H. 2009b. Minimum flexural ductility design of high-strength concrete columns, HKIE Transactions 16(4): 2–9.
Liu, M. Y.; Lin, C.; Li, O.; Ding, Q. J.; Hu, S. G. 2006. Flexural performance of reinforced high-strength lightweight concrete beams, in Proc. of the 6th International Symposium on Structural Engineering for Young Experts, 18–21 August 2006, Fuzhou, China, 264–270.
Lopes, S. M. R.; Bernardo, L. F. A. 2003. Plastic rotation capacity of high-strength concrete beams, Materials and Structures 36: 22–31. http://dx.doi.org/10.1007/PL02481567
NP EN 1992-1-1 Eurocode 2: Design of Concrete Structures – Part 1: General Rules and Rules for Buildings. European Committee of Standardization, Brussels, 2010. 225 p.
Pecece, M.; Fabbricino, G. 1999. Plastic rotation capacity of beams in normal and high-performance concrete, ACI Structural Journal 96(2): 290–296.
Shehata, I. A. E. M.; Shehata, L. C. D. 1996. Ductility of high-strength concrete beams in flexure, in Proc. of the 4th International Symposium on Utilization of High-Strength / High-Performance Concrete, 29 May – 3 June 1996, Paris, France, 945–953.
Shin, S.-W.; Ghosh, S. K.; Moreno, J. 1989. Flexural ductility of ultra-high-strength-concrete members, ACI Structural Journal 86(4): 394–400.
Sin, L. H.; Huan, W. T.; Islam, M. R.; Mansur, M. A. 2010. Reinforced lightweight concrete beams in flexure, ACI Structural Journal 108(1): 3–12.