Inelastic lifetimes of confined two-component electron systems in semiconductor quantum wire and quantum well structures

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Abstract

We calculate Coulomb scattering lifetimes of electrons in two-subband quantum wires and in double-layer quantum wells by obtaining the quasiparticle self-energy within the framework of the random-phase approximation for the dynamical dielectric function. We show that, in contrast to a single-subband quantum wire, the scattering rate in a two-subband quantum wire contains contributions from both particle-hole excitations and plasmon excitations. For double-layer quantum well structures, we examine individual contributions to the scattering rate from quasiparticle as well as acoustic and optical plasmon excitations at different electron densities and layer separations. We find that the acoustic plasmon contribution in the two-component electron system does not introduce any qualitatively new correction to the low energy inelastic lifetime, and, in particular, does not produce the linear energy dependence of carrier scattering rate as observed in the normal state of high-$T_c$ superconductors.

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I. INTRODUCTION

A central quantity in the theory of interacting electron systems is the quasiparticle inelastic lifetime, which is the inverse of the scattering rate due to electron-electron interaction and determines the many-body broadening of the electron spectral function due to Coulomb interaction. The quasiparticle inelastic lifetime is important in understanding many physical phenomena in electron systems, such as tunneling,\(^1\) transport,\(^2\) localization,\(^3\) etc. The concept of the inelastic lifetime is also important in electronic device operations because it controls the electron energy relaxation rate.\(^4\) Over the past several decades, confined electron systems in semiconductor heterostructures have been extensively studied for both fundamental and technological reasons. The electron systems in high mobility GaAs/AlGaAs heterostructures are especially suitable for studying electron-electron interaction effects because of reduced impurity scattering resulting from the modulation-doping technique. In this article, we present our calculations of Coulomb scattering lifetimes of electrons confined respectively in a two-subband quantum wire and in a double quantum well structure within the framework of the random-phase approximation (RPA). Our work is, therefore, a generalization of the existing work in the literature (which is mostly for the one-component electron systems) to the corresponding two-component one (quantum wire) and two (quantum well) dimensional electron systems. We find that the scattering rate of a two-subband quantum wire is quite different from that of a one-subband quantum wire. For a one-subband quantum wire, the scattering rate is identically zero at small wavevectors and only plasmon excitation contributes at large wavevectors. For a two-subband quantum wire, on the other hand, both quasiparticle and plasmon excitations contribute and the scattering rate is non-zero even at small wavevectors because of the quasiparticle contribution. For a double quantum well electron system, we compare individual contributions to the scattering rate from quasiparticle as well as from acoustic and optical plasmon excitations at different electron densities and interwell separations, and discuss implications of our results for other electronic systems with layered structures. In particular, we show that acoustic plasmon
excitations in a layered two-dimensional electron system do not provide the linear energy
dependence of carrier scattering rate which has been observed in the normal state of high-$T_c$
superconductors.

Technological progress has made it possible to fabricate high quality quantum wires
where only the lowest one or two subbands are populated by electrons. This has stimu-
lated an increasing interest in quantum wire structures. Much in the same way as
quantum well structures have generated intense activity in pure and applied research in
two-dimensional electron systems, quantum wire structures have created the potential for
new device applications and the opportunity to carry out experimental study on one-
dimensional Fermi systems, where many theoretical predictions can be tested. Because
of the low dimensionality, quantum wire systems have many unique characteristics. One of
them is that momentum and energy conservations severely suppress low energy scattering
processes and result in long lifetimes for electrons in quantum wires. The conservation laws
are much less restrictive on electron-electron scattering processes in two-subband quantum
wires than in one-subband quantum wires, and, as a result, the inelastic scattering rates in
a single subband quantum wire and in a two-subband quantum wire are quite different. In
a one-subband quantum wire, quasiparticle excitation does not contribute. The scattering
rate is identically zero until the electron momentum becomes larger than a threshold value,
where plasmon excitation begins to contribute. For a two-subband quantum wire, both
quasiparticle and plasmon excitations contribute to the scattering rate. The inelastic scat-
tering rate is non-zero even at small wavevectors because of the quasiparticle excitations.
The inelastic scattering rate in a two-subband quantum wire is, therefore, qualitatively dif-
ferent from the corresponding one component situation, and, in fact, bears some ingredient
reminiscent of a higher dimensional system. Our main motivation in studying the two-
component quantum wire systems is two-fold: (1) exploring its qualitative difference with
the purely one-subband situation; and (2) the fact that the narrowest existing doped semi-
conductor quantum wires have two occupied subbands. For simplicity, we only consider
two-subband quantum wires where electron-electron interaction potentials are independent
of the subband index. Our results are therefore strictly valid only in quantum wires where
the two subband occupations are due to Zeeman splitting. Qualitatively and even semi-
quantitatively, however, the results should also be applicable to other types of two-component
quantum wires, including two coupled-wire structures.

Inelastic electron lifetime due to Coulomb interaction in two-dimensional systems has
been well studied\(^1\)–\(^4\) in the literature. There is a resurgent interest in the quasiparticle
properties of layered two-dimensional electron systems due to the discovery of high-\(T_c\)
superconductors which have layered structures. One of the many unexpected behaviors observed
in the high-\(T_c\) superconductors is a linear energy dependence of the normal state carrier
scattering rate. This anomalous linear energy dependence of the scattering rate is not ex-
pected from a single-layer uniform two-dimensional electron gas with Coulomb interaction,
where electron scattering rate is well known\(^1\)–\(^4\) to be \(\tau_{\mathbf{k}}^{-1} \sim \xi_{\mathbf{k}}^2 \ln(|\xi_{\mathbf{k}}|)\). The essential new
ingredient in a layered two-dimensional system is the presence of new low energy collective
modes, the acoustic plasmon excitations.\(^5\) There are recent suggestions\(^6\) that this anoma-
lous linear energy dependence of the carrier scattering rate in the normal state of high-\(T_c\)
superconductors may be explained by inelastic scattering processes due to acoustic plasmon
excitations. If this is true, \textit{i.e.} if acoustic plasmon scattering gives rise to a linear en-
ergy dependence of the inelastic Coulomb scattering rate, then the theoretical implications
are tremendous because it is generally assumed that the standard many-body Fermi liquid
theory is not capable of explaining this linear dependence ( without invoking very special
non-generic properties, for example, van Hove points or hot spots, of the Fermi surface). To
further investigate this important issue, we calculate the inelastic lifetime of two-dimensional
electrons in a double quantum well structure using the leading order many-body perturba-
tive GW approximation. We compare the contributions to the scattering rate from acoustic
plasmon excitations with contributions from quasiparticle and optical plasmon excitations
at different electron densities and interwell separations. We show that the scattering rate
due to the acoustic plasmon excitation is, in general, small, and does not provide either a
qualitative or a quantitative explanation for the linear energy dependence of carrier lifetimes
in the normal state of high-$T_c$ superconductors. We therefore conclude that, contrary to the claims of Ref. 17, layered structure by itself is not capable of producing a linear energy dependence in the Coulomb scattering rate of two dimensional electron systems.

In Sec. II, we briefly sketch the procedure we use for calculating the inelastic lifetime and then present and discuss our calculated results of electron lifetimes in a two-subband quantum wire and in double quantum wells. A short summary in Sec. III concludes this article.

II. INELASTIC LIFETIMES OF TWO-COMPONENT ELECTRON SYSTEMS

A. Formalism

The inelastic lifetime is obtained from a calculation of electron self-energy due to Coulomb interaction. In this work, we approximate the self-energy by the leading perturbative term in an expansion in the dynamically screened RPA exchange interaction. This approximation, commonly referred to as the GW approximation, is extensively employed in calculating electronic many-body effects and is exact in the high density limit. We restrict ourselves to the case where Coulomb interaction does not change the electron subband (layer) index. Under this condition, the electron self-energy is diagonal with respect to the subband (layer) indices, i.e. we assume that there is no Coulomb interaction induced intersubband (interlayer) scattering. In the Matsubara formalism, the electron self-energy within this GW approximation we employ is given by

$$\Sigma_{ii}(k, ip) = -\frac{1}{\nu} \sum_{q} \frac{1}{\beta} \sum_{i\omega} V_{ii}^{scr}(q, i\omega) G_0^{0}(q + k, i\omega + ip),$$

(1)

where $\nu$ is the volume of the electron gas, $\beta$ is the inverse of temperature, $i$ is the subband (layer) index, $G^0$ is Green’s function of non-interacting electrons, and $V_{ii}^{scr}$ is the dynamically screened intrasubband (intralayer) electron-electron interaction potential for which an explicit expression will be given below. Inelastic lifetime of an electron with momentum $k$ in subband (layer) $i$ is obtained from the retarded self-energy by

$$\tau_{ki}^{-1} = -\frac{2}{\hbar} \text{Im} \Sigma_{ii}(k, \xi_{ki}),$$
where $\xi_{ki} = \frac{\hbar^2 k^2}{2m} - \epsilon_{Fi}$ is the bare energy of the electron measured with respect to the Fermi energy of subband (layer) $i$. The imaginary part of the retarded self-energy at zero temperature is given by Eq.(1) as

$$\text{Im} \Sigma_{ii}(k, \xi_{ki}) = \frac{1}{\nu} \sum_q \text{Im} \left[ V_{ii}^{scr}(q, \xi_{q-k} - \xi_{ki}) \right] \left[ \theta(\xi_{ki} - \xi_{q-k}) - \theta(-\xi_{q-k}) \right],$$

(2)

where $\theta$ is step function. The screened electron-electron interaction potential $V_{ii}^{scr}$ in the above expression is given by

$$V_{11}^{scr}(q, \omega) = \frac{v_a(q) + \chi^0_{11}(q, \omega) (v_a^2(q) - v_b^2(q))}{1 - v_a(q) (\chi^0_{11}(q, \omega) + \chi^0_{22}(q, \omega)) + \chi^0_{11}(q, \omega) \chi^0_{22}(q, \omega) (v_a^2(q) - v_b^2(q))},$$

(3)

where $v_a$ and $v_b$ are unscreened intra- and intersubband (layer) Coulomb interaction potential, and $\chi^0_{ii}$ is density-density response function of non-interacting electrons for which analytical expressions are known.\[11,21] Equations (2) and (3) form a complete description for the electron self-energy, from which the inelastic electron lifetime can be obtained. The self-energy in Eq.(2) contains two kinds of contributions. One is the quasiparticle contribution which occurs when either $\text{Im} \chi^0_{11}$ or $\text{Im} \chi^0_{22}$ are nonzero. The other is the plasmon contribution which occurs when both $\text{Im} \chi^0_{11}$ and $\text{Im} \chi^0_{22}$ are zero and the denominator of the expression in Eq.(3) vanishes. For a two-component electron gas, there are in general\[14] two branches of plasmon mode. In the following, we shall apply the above formalism to two-component quantum wire and quantum well structures and investigate the importance of individual scattering processes due to quasiparticle and plasmon excitations in these systems.

### B. Inelastic electron lifetime in a two-subband quantum wire

In the following, we evaluate electron inelastic lifetime due to Coulomb interaction in a two-subband quantum wire. Our purpose is to compare the result of the two-subband quantum wire with that of a one-subband quantum wire and illustrate the generic features of the inelastic lifetime in the two-subband case.

We consider the case where the two subbands are occupied by different numbers of electrons, so each subband $i$ has its own Fermi wavevector $k_{Fi}$ and Fermi energy $\epsilon_{Fi}$ ($i = 1, 2$).
As discussed before, we restrict ourselves to the case where the Coulomb potential is subband independent, \( v_a(q) = v_b(q) \) in Eq.(3). This is not a particularly severe approximation in view of the fact that the exact matrix elements of Coulomb interaction in a quantum wire depend on the details of subband confinement, which, in general, are not known for quantum wires. In our calculation, the bare potential \( v_a(q) \) is obtained by using the infinite-well confinement model. The input materials parameters in our calculation are for GaAs-based quantum wires.

In Fig.1, the excitation spectrum of the two-subband quantum wire is shown, where the plasmon modes are represented by solid-lines, and the quasiparticle excitations are confined within the shaded area. The spectrum consists of two regions of quasiparticle excitations and two branches of plasmon excitations. Both the plasmons are approximately linear modes in the long wavelength limit. The high energy mode exists for all values of wavevectors, while the low energy mode exists only within a limited range of wavevectors. The most characteristic feature of the spectrum is that low energy quasiparticle excitations are prohibited, i.e. the phase space within \( 0 < q < 2k_F i \) and \( 0 < \omega < \frac{\hbar^2}{2m} (2qk_{Fi} - q^2) \) \( (i = 1, 2) \) is excluded from the quasiparticle excitation spectrum. Both quasiparticle and plasmon excitations contribute to the inelastic scattering rate in the two-subband quantum wires as we shall show below.

In Fig.2, the inelastic scattering rate of a two-subband quantum wire is shown as a function of electron momentum \( k \). The distinct feature of a one-subband quantum wire is the total suppression, due to restrictions from energy and momentum conservations, of the quasiparticle contribution to the inelastic scattering rate. Consequently, the scattering rate of a single-subband quantum wire is identically zero until the plasmon contribution is turned on at \( k > k_c > k_F \). Occupation of the second subband by electrons in a quantum wire opens up additional scattering channels and makes the energy-momentum conservation less restrictive on scattering processes. As a result, both quasiparticle and plasmon excitations contribute to the inelastic scattering rate. We see, from Fig.2, that the inelastic scattering rate has contributions from quasiparticle excitations at small wavevectors, from the second
(low-energy) plasmon excitation at larger wavevectors, and from the first (high energy) plasmon excitation at even larger wavevectors. Compared with that of a single-subband quantum wire, the inelastic scattering rate in a two-subband quantum wire contains some ingredient of the corresponding two-dimensional system. For quantum wires with increasing widths, one may expect an eventual crossover to higher-dimensional behavior as a large number of subbands become occupied by electrons.

In Fig. 3, we compare contributions to the inelastic scattering rate from the quasiparticle, the first (high-energy) plasmon, and the second (low-energy) plasmon excitations at different occupations of the second subband. We see that the quasiparticle excitation contributes to the inelastic scattering rate for \( k \leq k_{F2} \), and the second plasmon mode contributes for \( k \geq k_{F2} \) while the first plasmon mode contributes for \( k > k_c > k_{F1} \). The momentum thresholds for plasmon contributions increase as the density of electrons in the second subband increases.

The inelastic scattering rates in a two-subband quantum wire and in a single-subband quantum wire share qualitatively similar behaviors in some respects. For a single-subband quantum wire under the ideal condition of zero temperature and no disorder, \( \text{Im} \Sigma(k_F, \omega) \sim \omega[\ln(|\omega|)]^{1/2} \) for \( \omega \to 0 \), which comes entirely from the plasmon contributions. Following the same procedure, one can show that the same functional dependence, \( \text{Im} \Sigma_{11}(k_{F1}, \omega) \sim \omega[\ln(|\omega|)]^{1/2} \) for \( \omega \to 0 \), is valid for a two-subband quantum wire as well, which again arrives from the contribution of the first (high energy) plasmon excitation. We like to point out that this asymptotic dependence of \( \text{Im} \Sigma \), which could imply non-Fermi-liquid properties, may not apply in real situations where temperature is finite and disorder scattering is inevitably present. More detailed discussion in this respect can be found in Ref. 11. We should also mention that intersubband scattering (which is neglected in our model) leads to an off-diagonal self-energy contribution whose imaginary part (being quantitatively small for our two-subband model) has a different asymptotic \( \omega \)-dependence which eventually produces the two dimensional behavior in the limit of the occupancy of many one dimensional subbands.
C. Inelastic electron lifetime in double quantum wells

In the following, we investigate inelastic scattering lifetime of electrons in double quantum wells. We compare individual contributions to the scattering rate from quasiparticle and plasmon excitations at different electron densities and interwell separations. The emphasis is on understanding the acoustic plasmon contribution to the inelastic Coulomb scattering lifetime and its implications for other electronic systems with layered structures, such as high-$T_c$ superconductors. In particular, we want to correct some erroneous statements on this point in the literature.

We consider the case where the two quantum wells have equal electron densities. The unscreened intra- and interwell Coulomb potentials in Eq. (3) are $v_a(q) = \frac{2\pi e^2}{q}$ and $v_b(q) = v_a(q) \exp(-qd)$, where $d$ is the interwell separation. The effect of finite well-thickness is ignored, as it does not change any of the results qualitatively, although incorporating the effect into our calculations is straightforward. We neglect interwell tunneling effect and interwell Coulomb scattering (i.e. the off-diagonal element of the Coulomb interaction).

In Fig. 4, we show the excitation spectrum of a double quantum well system, where plasmon modes are represented by solid-lines, and quasiparticle excitations are within the shaded area. The quasiparticle excitation of a double quantum well system remains the same as that of a single quantum well system. The essential new feature in a double quantum well system is that there are two branches of plasmon excitations. In the present case of equal electron densities in the two layers, the plasmon dispersions $\omega_{\pm}(q)$ are solutions to

$$1 = [v_a(q) \pm v_b(q)] \chi^0(q, \omega_{\pm}),$$

where $\chi^0(q, \omega)$ is density-density response function of a single-layer non-interacting two-dimensional electron gas. In the long wavelength limit, $\omega_-(q)$ is a linear acoustic plasmon mode whose contribution to the scattering rate is the major concern of our calculation.

In Fig. 5, we show the calculated inelastic scattering rate of electrons in double quantum wells, along with each individual contribution from quasiparticle as well as acoustic and optical plasmon excitations. Quasiparticle excitations contribute essentially at all values
of wavevectors, and it is the only non-zero contribution at small wavevectors. Acoustic and optical plasmon excitations begin to contribute when the wavevector is larger than the respective thresholds \( k_{ac} \) and \( k_{op} \), where \( k_F < k_{ac} < k_{op} \). The contributions from acoustic and optical plasmon excitations are peaked within narrow windows of wavevectors just above the respective thresholds, and diminish as the wavevector is further increased. The threshold wavevectors, which depend on the parameters (such as electron densities, layer separation, etc.,) are \( k_{ac} \sim 1.5k_F \) and \( k_{op} \sim 2k_F \) for the particular choice of sample parameters of Fig.5.

It is clear that neither plasmon excitation contributes to the electron scattering close to the Fermi surface as they both become operative only above some threshold wavevectors. Therefore, the inelastic electron lifetimes close to the Fermi surface are determined entirely by quasiparticle excitations.

In a single quantum well system, inelastic scattering rate of electrons close to the Fermi surface has an energy dependence \( \text{Im}\Sigma(k, \xi_k) \sim \xi_k^2 \ln(|\xi_k|) \) for \( \xi_k \to 0 \), which comes from the contribution by quasiparticle excitations. In a double quantum well system, the scattering rate of electrons close to the Fermi surface is also determined by quasiparticle excitations. Following a similar procedure, the scattering rate in double quantum wells is found to have the same energy dependence \( \text{Im}\Sigma(k, \xi_k) \sim \xi_k^2 \ln(|\xi_k|) \) for \( \xi_k \to 0 \). The fact that the inelastic Coulomb scattering lifetime in both a double quantum well system and a single quantum well system has the same asymptotic energy dependence is very easy to understand physically. Scattering processes for electrons close to the Fermi surface involve only long wavelength excitations, where the double quantum well system can be viewed as having zero interwell separation because the interwell separation \( d \) is finite whereas the effective excitation wavelength is infinite—equivalent to a single-well two-dimensional electron system with an additional degeneracy of 2 (similar to the valley degeneracy in Si-MOS system.)

In Fig.6, we compare individual contributions to the scattering rate from quasiparticle as well as acoustic and optical plasmon excitations at different electron densities. As the density decreases, the effective strength of interaction increases, all the contributions to the
scattering rate increase, while their relative strengths do not change significantly.

In Fig. 7, we compare the individual contributions to the scattering rate from quasiparticle and plasmon excitations at different interwell separations. The influence of the well separation on the quasiparticle (plasmon) contribution is relatively weak (strong). The two plasmon excitations are determined by the effective interaction $v_\pm(q)$, respectively, where $v_\pm(q) = v_a(q) \pm v_b(q)$, depending strongly on the well separation through the $e^{-qd}$ factor. As the separation decreases, $v_+(q)$ increases, so the optical plasmon contribution increases, and the threshold momentum $k_{op}$ also increases. The situation for the acoustic plasmon is the opposite. As the well separation decreases, $v_-(q)$ decreases, so the acoustic plasmon contribution decreases, and the threshold $k_{ac}$ decreases as well. It should be noted that the acoustic plasmon contribution diminishes quickly as the well separation decreases, so the electron lifetimes for energies close to the Fermi surface are not affected by the acoustic plasmon excitations at any well separations. The acoustic plasmons contribute significantly to the inelastic scattering rate only at large layer separations and at higher energies.

We now briefly discuss implications of our results for high-$T_c$ superconductors. In fact, our results essentially have no implications for the observed linear energy dependence of carrier scattering rates in the normal state of high-$T_c$ superconductors except to correct some recent erroneous and misleading claims in the literature. Many experiments suggest linear energy and temperature dependence of lifetimes for carriers close to the Fermi energy in the normal state of high-$T_c$ superconductors. It is well known that the anomalous linear energy and temperature dependence of the lifetime is not expected from a Fermi-liquid type many-body theory for a single-layer uniform two-dimensional electron gas. In fact, the standard Fermi-liquid result for the scattering rate of a two-dimensional electron gas is an $(\epsilon - \epsilon_F)^2 \ln(\epsilon - \epsilon_F)$ behavior for $\epsilon$ close to $\epsilon_F$, which has been extremely well-verified experimentally. From the discussion above, it is clear that the acoustic plasmon contributions in a layered two-dimensional electron gas do not produce a linear $(\epsilon - \epsilon_F)$ behavior for the scattering rate either. It is well-established that both two- and three-dimensional electron systems, at least within the standard perturbative many-body picture,
have scattering rates for electrons close to the Fermi surface which are essentially quadratic in energy, \( i.e. (\epsilon - \epsilon_F)^2 \) within logarithmic corrections. We show that the same is also true for a layered two-dimensional electron system.

III. SUMMARY

In this work, we have studied inelastic scattering rates due to electron-electron Coulomb interaction for two-component electron systems confined in semiconductor quantum wire and quantum well structures. We showed that the scattering rate of a two-subband quantum wire is quite different from that of a one-subband quantum wire because momentum and energy conservations are less restrictive on scattering processes in two-subband systems where additional scattering channels are available. For double quantum well systems, we compared individual contributions to the scattering rate from quasiparticle as well as acoustic and optical plasmon excitations. We emphasized, in particular, that the acoustic plasmon contributions in a layered electron system do not produce the linear energy dependence of carrier scattering rate observed in the normal state of high-\( T_c \) superconductors.

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FIGURES

FIG. 1. The excitation spectrum of a two-subband quantum wire electron system. The plasmon modes are represented by solid-lines. The quasiparticle excitations are confined within the shaded area. The electron densities in the two subbands \( n_1 = 2.0 \times 10^5 \text{cm}^{-1} \) and \( n_2 = 0.4n_1 \). The wire widths \( L_y = L_z = 100\text{Å} \).

FIG. 2. The inelastic scattering rate of electrons in the first subband of a two-subband quantum wire. The electron densities in the two subbands \( n_1 = 2.0 \times 10^5 \text{cm}^{-1} \) and \( n_2 = 0.4n_1 \). The wire widths \( L_y = L_z = 100\text{Å} \).

FIG. 3. The inelastic scattering rate of electrons in the first subband of a two-subband quantum wire due to the first plasmon excitations (a), the second plasmon excitations (b), and the quasiparticle excitations (c) at different occupations of the second subband. The electron density in the first subband \( n_1 = 2.0 \times 10^5 \text{cm}^{-1} \). The wire widths \( L_y = L_z = 100\text{Å} \).

FIG. 4. The excitation spectrum of a double quantum well electron system. The plasmon modes are represented by solid-lines. The quasiparticle excitations are confined within the shaded area. The electron densities in dimensionless unit \( r_{s1} = r_{s2} = 2 \). The interwell distance \( d = 2.3a_B \). (In GaAs-based materials, \( a_B \sim 100\text{Å} \), \( r_{s1} = r_{s2} = 2 \) corresponds to \( n_1 = n_2 \sim 8 \times 10^{10}\text{cm}^{-2} \).)

FIG. 5. The inelastic scattering rate of electrons in a double quantum well system. The electron densities \( r_{s1} = r_{s2} = 1.4 \). The interwell distance \( d = 1.5a_B \).

FIG. 6. The inelastic scattering rate of electrons in a double quantum well system due to the optical plasmon excitation (a), the acoustic plasmon excitation (b), and the quasiparticle excitation (c) at different electron densities. The solid-line is for \( r_{s1} = r_{s2} = 1 \) and the dotted-line is for \( r_{s1} = r_{s2} = 2 \). The interwell distance \( d = 1.1a_B \).
FIG. 7. The inelastic scattering rate of electrons in a double quantum well system due to the optical plasmon excitation (a), the acoustic plasmon excitation (b), and the quasiparticle excitation (c) at different well separations. The solid-line is for $d = 1.1a_B$ and the dotted-line is for $d = 0.6a_B$. The electron densities $r_{s1} = r_{s2} = 1$. 
Fig 1 Zheng & Das Sarma
Fig 3(a) Zheng & Das Sarma

1st plasmon contribution

\[ \frac{-\text{Im}[\Sigma_{\text{II}}(k, \xi_k)]}{\varepsilon_{F1}} \]

- \[ n_2/n_1 = 0.2 \]
- \[ n_2/n_1 = 0.4 \]
- \[ n_2/n_1 = 0.6 \]
- \[ n_2/n_1 = 0.8 \]

\[ k/k_{F1} \]
Fig 3(b) Zheng & Das Sarma

2nd plasmon contribution

\[-\text{Im} \left[ \Sigma_{11}(k, \xi_k) \right] / \epsilon_{F1} \]

\begin{align*}
n_2/n_1 &= 0.2 \\
n_2/n_1 &= 0.4 \\
n_2/n_1 &= 0.6 \\
n_2/n_1 &= 0.8
\end{align*}

\( \frac{k}{k_{F1}} \)
Fig 5 Zheng & Das Sarma

\[ -\text{Im}[\Sigma(k, \xi_k)]/\epsilon_F \]

- total
- quasiparticle
- acoustic plasm.
- optical plasm.
Fig 6(a) Zheng & Das Sarma

![Graph showing optical plasmon contribution with k/k_F on the x-axis and $-\text{Im} \Sigma(k, \xi_k)/\epsilon_F$ on the y-axis.]

- For $r_s = 2$, there is a significant increase in the graph around $k/k_F = 2$. 
- For $r_s = 1$, there is a smaller increase at a slightly lower $k/k_F$ value.
Fig 6(b) Zheng & Das Sarma

\[ -\text{Im}[\Sigma(k, \xi_k)] \big/ \epsilon_F \]

- \text{Acoustic plasmon contribution}

\[ r_s = 2 \]
\[ r_s = 1 \]

\[ k/k_F \]
Fig6(c) Zheng & Das Sarma

\[ -\text{Im}[\Sigma(k, \xi_k)]/\epsilon_F \]

quasiparticle contribution

\[ r_s=2 \]

\[ r_s=1 \]

\[ k/k_F \]
optical plasmon contribution

\[ \text{Im}[\Sigma(k, \xi_k)]/\epsilon_F \]

\[ k/k_F \]

\[ d = 0.6a_B \]

\[ d = 1.1a_B \]
Fig 7(b) Zheng & Das Sarma

\[-\text{Im}[\Sigma(k, \xi_k)]/\epsilon_F\]

- Acoustic plasmon contribution

- $d = 1.1 a_B$
- $d = 0.6 a_B$

$k/k_F$

1.0 1.5 2.0 2.5 3.0
Fig7(c) Zheng & Das Sarma

\[ -\text{Im}[\Sigma(k, \xi_k)]/\epsilon_F \]

\[ \text{quasiparticle contribution} \]

\[ d=1.1a_b \]

\[ d=0.6a_b \]