Generic Vertical Frame Model for Estimation of Floor Vertical Vibration

J B Sun$^{1,2}$ and P Pan$^2$

1 Institute of Engineering Mechanics, China Earthquake Administration; Key Laboratory of Earthquake Engineering and Engineering Vibration of China Earthquake Administration, China
2 Key Laboratory of Civil Engineering Safety and Durability of the China Education Ministry, Tsinghua University, Beijing 100084, China
E-mail: panpeng@tsinghua.edu.cn

Abstract. Seismic-isolation is an appealing design alternative for enhancing both the seismic safety and functionality of building structures against large earthquake motions. However, large floor vertical accelerations were observed in the conventionally designed base-isolated buildings. Previous researches indicate that column and beam vibrations should be considered when evaluating the floor vertical accelerations. The column and beam vibrations lead the representation of the Single Degree of Freedom system and corresponding spectra analysis ineffective. To evaluate the vertical floor vibration of building structures, the generic vertical frame (GVF) model was developed in this study. Comparing with the conventionally frame analysis model, the GVF model significantly reduced the DOFs. The GVF model is capable of estimating the vertical acceleration at interior spans for multiple span structures with errors not more than twenty percent, and becomes more accurate for structural systems with strong columns.

1. Introduction
Seismic-isolation has been accepted as an appealing design alternative for enhancing both the seismic safety and functionality of building structures against large earthquake motions. Since base-isolation techniques essentially isolate only against horizontal motions, effects of vertical motions on the base-isolated structure becomes a concern. Previous researches indicate that conventionally designed base-isolated buildings have a large margin of safety against vertical ground motions, but maximum floor vertical accelerations can reach as large as more than five times of the peak ground acceleration. Floor vertical accelerations are closely related to structural functionality, thus quantification of acceleration amplifications is critical [2], [3]. In the dynamic response analysis of structures subjected to vertical ground motion, detailed finite element analysis is often required to take into account floor vibration. It is rather time consuming for both modeling and computation. To generalize the behavior of floor vertical vibration, simplified models that can preserve the prototype’s vertical vibration characteristics are recommended [1]. Previous researches indicate that two types of vibrations should be considered when evaluating the floor vertical accelerations. One is the column vibration featured by column elongation and contraction, and the other is the beam vibration featured by beam deflection. The column and beam vibrations are coupled in many modes, leading the representation of the SDOF (Single Degree of Freedom) system and corresponding spectra analysis ineffective. To this end, a model, named the generic vertical frame (GVF) model, is developed. In this paper, development of the GVF model is detailed, and time history analyses are carried out to validate the effectiveness.
2. Development of Generic Vertical Frame Model

2.1. Vertical full structure, substructure, and generic frame models

A full vertical frame model (designated as the FVF model hereafter) is given in Fig. 1(a), and the corresponding substructure vertical frame models (designated as the SVF model hereafter) [1] are constructed as in Fig. 1(b). The horizontal movements are restricted for both FVF and SVF models. In the SVF model, all middle columns and associated half beams are included, and the boundary condition at the end of the half-beams is taken to slide without rotation. This is justified because the maximum deflection is located approximately at the middle of the beams where the shear force is nearly zero. The SVF model is further simplified to a GVF model. Specifically, the floor vibration is simplified to an equivalent SDOF (single degree of freedom) system. A hanging spring and a hanging mass are used to represent the equivalent SDOF system. A schematic illustration of the GVF model is given in Fig. 1(c). In the GVF model, only vertical DOFs are considered. The hanging spring with a stiffness of $K_H$ and a mass of $M_H$ are connected at the beam-to-column joint. The column is also represented by a spring with a stiffness of $K_C$, and a mass of $M_C$ is lumped at the beam-to-column joint. Values of $K_H$, $M_H$, $K_C$, and $M_C$ are derived in the following discussions.

![Figure 1](image1)

Figure 1. Full vertical frame (FVF), substructure vertical frame (SVF), and generic vertical frame (GVF) models: (a) FVF; (b) SVF; (c) GVF.

2.2. Assignment of mass and stiffness

Rayleigh’s method [4], [5] is adopted to determine the equivalent stiffness and equivalent mass of the hanging part. For explanation, one story of a SVF model is examined in Fig. 2. Since the middle column deflection is negligible, only the beam deflection is considered here.

![Figure 2](image2)

Figure 2. One story of SVF model for interior span.

The defected shape function $\psi_s(x)$ resulting from a uniformly distributed vertical load $\bar{q}$ is given in Equation (1):
\[
\psi_b(x) = \frac{q}{24EI_b} (x^4 - 2L_b x^3 + L_b^2 x^2)
\]  

(1)

where, \( E \) is the modulus of elasticity, \( I_b \) is the second moment of area of the beam, \( L_b \) is the length of the beam, and \( x \) is the coordinate along the beam (Fig. 2).

The generalized mass is given in Equation (2):

\[
M_m = 2 \times \int_0^{L_b} m(x) (\psi_b(x))^2 \, dx
\]

(2)

Where, \( m(x) \) is mass per unit length of the beam. Since the model is symmetrical about the column, the generalized mass of the left half-beam equals to that of the right half-beam.

The generalized stiffness is given in Equation (3):

\[
K_m = 2 \times \int_0^{L_b} EI(x) (\psi_b^2(x))^2 \, dx
\]

(3)

The circular frequency \( \omega_m \) of the equivalent SDOF system is given in Equation (4):

\[
\omega_m = \frac{\sqrt{K_m}}{M_m}
\]

(4)

The participation factor \( \beta_m \) is expressed in Equation (5):

\[
\beta_m = \frac{L_m}{M_m}
\]

(5)

where \( L_m \) is defined in Equation (6):

\[
L_m = 2 \times \int_0^{L_b} m(x) \psi_b(x) \, dx
\]

(6)

The equivalent mass \( M_m^* \) and stiffness \( K_m^* \) are given Equation (7) and (8), respectively:

\[
M_m^* = \beta_m^2 M_m
\]

(7)

\[
K_m^* = M_m \omega_m^2
\]

(8)

Note that the subscript “\( \text{in} \)” in Equations (2) to (8) refers to interior spans. The equivalent SDOF system that represents the floor consists of the hanging spring and mass in the GVF model, thus for the interior span:

\[
M_H = M_m^*
\]

(9)

\[
K_H = K_m^*
\]

(10)

Since only the vertical direction is considered, the stiffness \( K_C \) of the spring that represents the column is as follows:

\[
K_C = \frac{E A_C}{L_C}
\]

(11)

where, \( A_C \) is the cross-section area of the column, and \( L_C \) is the length of the column.

To keep the total mass of the equivalent system equal to the original system, the mass \( M_C \) lumped at the beam-to-column joint is assigned as:
\[ M_c = 2 \times \int_0^{L_b} m(x)dx - M_H \]  

(12)

Note that in Equations (9) to (12), the subscripts “\( H \)” and “\( C \)” represent the parameters related to the hanging spring system and original column system, respectively.

2.3. Correlation on Floor Vertical Acceleration

The following equations are used to correlate the floor vertical accelerations of the GVF model with those of the FVF and SVF models:

\[ Acc(x) = Acc_C + \beta \times (Acc_H - Acc_C) \times \psi_b(x) \]  

(13)

In Equation (13), \( Acc_C \) is the acceleration at the beam-to-column joint in the GVF model, \( Acc_H \) is the acceleration of the mass attached to the hanging spring, \( \beta \) is the participation factor as defined in Equation (6), and \( \psi_b(x) \) is the assumed shape function. If the assumed shape function \( \psi_b(x) \) is normalized, i.e., \( \psi_b(L_b / 2) = 1 \), the floor acceleration at the middle span, is simplified in Equation (14):

\[ Acc(L_b / 2) = Acc_C + \beta \times (Acc_H - Acc_C) \]  

(14)

3. Representative Structures

As shown in Fig. 3, two steel moment frames AO-04 and AO-12 were chosen as the representative structures. AO-04 is four-story and four-span, and AO-12 is twelve-story and four-span. They are treated as planar structures, and the gravity load is taken to be 60 kN/m.

Ten vertical ground motions recorded in the 1995 Hyogoken-Nanbu (Kobe) earthquake designated as the Kobe set [6], were chosen as representative ground motions. The pseudo acceleration spectra corresponding to 5\% critical damping and normalized by the corresponding peak ground acceleration (by dashed line), and median of these values (by the solid line) are plotted in Fig. 4. The normalized spectra of the representative ground motions are anchored to unity and having peaks in the periods of 0.1 to 0.2 seconds. If the period is not greater than 0.05 seconds, the normalized spectrum is nearly unity, indicating that the amplification is very small [2].

Figure 3. Sample structures: (a) AO-04; (b) AO-12.
Figure 4. Pseudo acceleration spectra (5% critical damping) normalized by PGA.

4. Validation by Comparative Study

4.1. Analysis of FVF, SVF, GVF models

A finite element program code named “CLAP” was used to analyse the FVF and SVF models. In order to take into account floor vertical vibration, each beam was divided into six elements in the FVF model, and each half beam was divided into 3 elements in the SFV model. In both FVF and SVF models, the associated lumped mass was assigned at each node.

In the GVF model, a deflection shape function should be specified. According to Equation (1), it is a four order polynomial function with the coefficients depending on the span length $L_b$. In the example frames, $L_b$ is 6 m for both AO-04 and AO-12, and the shape function is:

$$\psi_b(x) = x^4 - 12x^3 + 36x^2$$

(15)

Substituting the shape function to the associated equations in the previous section, $K_H$, $M_H$, $K_C$, and $M_C$ of the GVF model are determined.

The number of DOFs of FVF, SVF, and GVF models are compared in Table 1 for AO-04 and AO-12. It is clear that the number of DOFs decreases significantly from the FVF to GVF models, with a ratio of 35 for these specific cases.

|        | FVF  | SVF  | GVF  |
|--------|------|------|------|
| AO-04  | 280  | 72   | 8    |
| AO-12  | 840  | 216  | 24   |

4.2. Comparison between FVF, SVF, and GVF models

In order to investigate whether the SVF model is capable of representing the FVF model, a series of time history analyses were carried out. Here the focused are the accelerations at the interior middle span of the FVF model and the beam end of the SVF model. To qualify the closeness of the responses obtained from the FVF and SVF models, statistical evaluation was conducted, and the ratio of the maximum acceleration obtained from SVF to that from FVF was adopted as an index. Figure 5 shows the medians and SD’s [7] for the ratios. They are presented for all stories of each example structure. The figure also include the SD’s of the maximum accelerations obtained from the FVF model, which serves as an index to examine the degree of dispersion associated with the ground motions. The
The medians of the ratios ranged from 0.8 to 1.0, and 0.9 to 1.0 for AO-04 and AO-12, respectively, and the SD’s of the ratios are smaller than 0.15 and 0.10 for AO-04 and AO-12, respectively. Also, the SD’s for the maximum acceleration obtained from FVF model are three to four times larger than the SD’s of the ratios. These observations indicate that the responses of the SVF model are reasonably close to that of the FVF model. The SVF model gives better estimation for the twelve-story structure (AO-12) than the four-stories structure (AO-04). This is because the column vibrations, which are faithfully modeled in the SVF model, are more significant in taller frames.

Figure 6 shows detailed comparisons between the acceleration obtained from the FVF and SVF models. The responses subjected to JMA record, one of the most popular motions in the Kobe set, is shown in the figure. All the accelerations obtained from the time history analyses are included. For each time step, plotted is a pair of data, the floor maximum acceleration at top floor obtained from the FVF and SVF models. In the figure, all the data scatter near the 45-degree line. This indicates that the response obtained from the FVF and SVF models are similar not only in the absolute value but also in the vibration phase. This is a further confirmation of the conclusion drawn from Fig. 5.

![Figure 5. Comparison of vibration magnitude between SVF and FVF models.](image)

![Figure 6. Comparison of vibration phase between SVF and FVF models.](image)
It is also notable from Fig. 5 that although small, the SVF model tends to give a slightly smaller estimate. This is primarily because the constraints associated with the SVF model. The maximum deflection point is assumed to be located at the middle span when developing the SVF model. This is based on the assumption that all interior columns do not sustain any deflection. However, this is not entirely true since the columns next to the exterior columns sustain some deflection. The situation is less relevant if the columns are stiffer. Two further structures, designated as BO-04 and CO-04, were chosen as variation of AO-04. BO-04 and CO-04 are identical with AO-04 except that bending stiffness of columns are increased two and ten times, respectively. Following the same way in Fig. 6, the responses of the FVF and SVF models are compared for BO-04 and CO-04 in Fig. 7. It is apparent that compared with AO-04, the responses of the FVF and SVF models become closer for BO-04, and become almost identical for CO-04. This observation justified the interpretation of underestimation, and also indicated that the SVF model is more precise for structures with strong columns.

The responses are compared between the SVF and GVF models. Specifically, comparison is focused on the acceleration at the end of the half-beams for the SVF model and the acceleration of the hanging masses [corrected by Equation (14)] for the GVF model. Here the ratio of the maximum acceleration obtained from the GVF model to that obtained from the SVF model is adopted as an index to quantify the closeness of the responses. They are presented for all stories, and the ratios are estimated for each story. As shown in Fig. 8, the medians of the ratios are almost unity with a difference less than 0.02 for both AO-04 and AO-12, and the SD’s of the ratios are almost zero. These observations indicate that the response obtained from the GVF model is almost identical with that obtained from the SVF model.

Using the JMA ground motion, the accelerations at the top floor of AO-04 and AO-12 are investigated in detail to examine the closeness of responses between the GVF and SVF models. All the accelerations obtained from the time history analyses are included in Fig. 9. In the figure, all the data scatter very close to the 45-degree line. This indicates that responses obtained from the GVF and SVF model are very close in both vibration magnitude and phase, further confirming the conclusion drawn from Fig. 8.
5. Conclusions
To evaluate the vertical floor vibration of building structures, a generic vertical frame (GVF) model was developed in this study. Major conclusions obtained are summarized as follows:

(1) Compared with the FVF and SVF model, the GVF model significantly reduces the DOFs treated in the analysis.

(2) The GVF model is capable of estimating the vertical acceleration at interior spans for multiple span structures with errors not more than twenty percent.

(3) The GVF model becomes more accurate for structural systems with strong columns.

6. References
[1] Inoue K, Higashi K, Ogawa K, Tada M, and Hasegawa T 1995 Earthquake response of structural members of rigid frames with RHS columns (Part 1, design of model frames), *Summaries of Technical Papers of Annual Meeting C-1, AIJ*, pp 269-70.

[2] Elnashai AS 1997 Seismic design with vertical earthquake motion, *Seismic Design Methodologies for the next generation of codes*, Faifa & Krawinkler (eds), Balkema: Rotterdam

[3] Chopra A K 2001 *Dynamics of Structures: Theory and Applications to Earthquake Engineering*.
2nd edn, Prentice Hall: New Jersey

[4] Clough R W and Penzien J 1993 *Dynamics of Structures; 2nd edn*, McGraw Hill: New York

[5] Leung A Y T 1993 *Dynamic Stiffness and Substructures*; Springer-Verlag: London

[6] Nakashima M, Matsumiya T, and Asano K 2000 Comparison in Earthquake Responses of Steel Moment Frames Subjected to Near-Fault Strong Motions Recorded in Japan, Taiwan, and the U.S., *International Workshop on Annual Commemoration of Chi-Chi Earthquake*, Taiepi, Taiwan, pp 112-23.

[7] Luco N and Cornell C A 2000 Effect of connection fractures on SMRF seismic drift demands. *J Struct Eng-ASCE* **126**(1):127-36

**Acknowledgments**

Financial support from the Scientific Research Fund of the Institute of Engineering Mechanics, China Earthquake Administration (Grant No. 2017D07); and that from the Opening Funds of the State Key Laboratory of Building Safety and Built Environment and the National Engineering Research Center of Building Technology (Grant No. BSBE2017-01), are gratefully acknowledged.