Energy budget of the bifurcated component in the radio pulsar profile of PSR J1012+5307

J. Dyks* and B. Rudak

Nicolaus Copernicus Astronomical Center, Rabiańska 8, PL-87-100 Toruń, Poland

Accepted 2013 July 1. Received 2013 June 11; in original form 2013 February 25

ABSTRACT
The bifurcated emission component (BEC) in the radio profile of the millisecond pulsar J1012+5307 can be interpreted as the signature of the curvature radiation beam polarized orthogonally to the plane of electron trajectory. Since the beam is intrinsically narrow (\(\sim 1^\circ\)), the associated emission region must be small for the observed BEC to avoid smearing out by spatial convolution. We estimate whether the energy available in the stream is sufficient to produce such a bright feature in the averaged profile. The energy considerations become complicated by the angular constraints imposed by the width of the microbeam, and by the specific spectrum of the BEC which is found to have the spectral index \(\xi_{\text{BEC}} \approx -0.9\) in comparison to the index of \(\xi \approx -2\) for the total profile. For typical parameters, the luminosity of the BEC is determined to be \(4 \times 10^{25}\) erg s\(^{-1}\), whereas the maximum-possible beam-size-limited power of the stream is \(L_{\Delta\phi} \approx 2 \times 10^{29}\) erg s\(^{-1}\). This implies the minimum energy-conversion efficiency of \(\eta_{\Delta\phi} \approx 2 \times 10^{-4}\). The BEC’s luminosity does not exceed any absolute limits of energetics, in particular, it is smaller than the power of primary electron and/or secondary plasma stream. However, the implied efficiency of energy transfer into the radio band is extreme if the coherently emitting charge-separated plasma density is limited to the Goldreich–Julian value. This suggests that the bifurcated shape of the BEC has macroscopic origin; however, several uncertainties (e.g. the dipole inclination and spectral shape) make this conclusion not firm.

Key words: radiation mechanisms: non-thermal – pulsars: general – pulsars: individual: J1012+5307.

1 INTRODUCTION
Bifurcated emission components (BECs) have so far been observed in integrated radio profiles of J0437−4715 (Navarro et al. 1997) and J1012+5307 (Kramer et al. 1998; Dyks, Rudak & Demorest 2010, hereafter DRD10). It has been proposed in DRD10 that these features are produced when our line of sight crosses a split-fan beam emitted by a narrow plasma stream flowing along curved magnetic field lines (see fig. 1 in Dyks & Rudak 2012, hereafter DR12). The double-peaked shape of BECs has been attributed to the intrinsic bifurcation of the extraordinary mode of the curvature radiation beam in strongly magnetized plasma. The peaks in the observed BECs approach each other with increasing frequency at a rate that is roughly consistent with the curvature radiation origin (fig. 7 in DR12). In the case of J0437−4715, its BEC is considerably merged. The observed width of this BEC is consistent with the angular size of the curvature radiation beam (section 3.1.1 in DR12), and the energy contained in the BEC is a small fraction of that observed in the full profile.

In the case of J1012+5307, however, the feature is much more pronounced, wider and well resolved (Fig. 1). The BEC’s peaks are separated by a deep central minimum which reaches \(\sim 32\) per cent of the BEC’s peak flux at 820 MHz. If the shape of this feature is mostly determined by the intrinsic shape of the curvature radiation microbeam, the extent of the associated emission region must be small enough so that the spatial convolution of the curvature emission beams does not smear out the BEC.

At high frequencies (\(\nu \gtrsim 3\) GHz; see fig. 5 of Kramer et al. 1999), the BEC of J1012+5307 starts to have comparable flux to the main pulse (MP) in the averaged profile of this pulsar. Given the extreme narrowness of the curvature radiation beam (\(\sim 1^\circ\) for typically expected parameters), it is worth to verify if the energy supplied by the Goldreich–Julian density in such a narrow stream is sufficient to produce the observed flux of the BEC.

After introducing some energetics-related definitions in Section 2 we estimate the BEC’s luminosity (Section 3). In Section 4 we estimate the maximum energy flux that can be confined in the plasma stream, the width of which is limited by the resolved form of the
BEC. In Section 4.2.1 we compare our result to another published estimate, and reiterate our main conclusions in Section 5.

2 BASICS OF ENERGETICS

The radio luminosity of a pulsar beam cannot be accurately determined, because we do not know if our line of sight samples representative parts of the beam. Without the a priori knowledge of the emission beam and viewing geometry, we cannot tell how much the observed flux differs from the flux averaged over the full solid angle of pulsar emission. The missing information needs to be provided by some model of the beam and viewing geometry. The simplest model assumes a uniform emission beam of solid angle $\Delta \Omega(v)$, with the uniform emissivity determined by the observed flux $S_{\text{mean}}(v)$ averaged within the ‘on-pulse’ interval of pulse duration. This implies the pseudo-luminosity:

$$L = \int d^2 \Delta \Omega(v) S_{\text{mean}}(v) \, dv,$$

where the integration is within the frequency band of interest (between $v_{\text{min}}$ and $v_{\text{max}}$). Hereafter, the term ‘pseudo’, which expresses our assumption that the measured flux represents the beam-integrated flux, will not be used. For many pulsars the observed pulse width does not change with frequency or it changes slowly enough to consider the solid angle as $v$-independent, and to extract $\Delta \Omega$ from the integrand (Gold & Lyne 1998; Hankins & Rankin 2010). In our case $\Delta \Omega$ and $S_{\text{mean}}(v)$ must be determined for the BEC of J1012+5307. The BEC’s spectrum will be calculated further below for a $v$-independent pulse duration interval of 35°, marked in Fig. 1. The solid angle $\Delta \Omega$ will accordingly be considered fixed ($v$-independent). Equation (2) then becomes

$$L_{\text{BEC}} = d^2 \Delta \Omega \int S_{\text{BEC}}(v) \, dv,$$

where $S_{\text{BEC}}$ is the mean flux of the BEC.

The luminosity of equation (2) cannot exceed the maximum power which is theoretically available for the emitting stream:

$$L_{\Delta \phi} = e \Delta \Phi_{\text{pc}} c n_{\text{GJ}}(r) \, A(r),$$

where $e \Delta \Phi_{\text{pc}} = E_{\text{max}}$ is the energy corresponding to the potential drop above the polar cap, $n_{\text{GJ}}$ is the Goldreich–Julian density of the stream, and $A$ is the cross-sectional area of the stream, measured at the same radial distance $r$ as $n_{\text{GJ}}(r)$.

Note that equations (2) and (3) are not independent: the emitting area $A$ in $L_{\Delta \phi}$ refers to the same emission region as the solid angle $\Delta \Omega$ in $L$. The choice of the emission region simultaneously determines both $A$ and $\Delta \Omega$ in these equations.

The accelerating potential drop is approximated by the potential difference between the centre and the edge of the polar cap, as derived for a perfectly conducting neutron star with vacuum magnetosphere and no dipole inclination: $\Delta \Phi_{\text{pc}} = 6.6 \times 10^{12} \, V B_{\text{pc}, 12} r_6^3 / P^2$, where $B_{\text{pc}, 12}$ is the polar magnetic induction in TG, $r_6$ is the neutron star radius in units of 10 km and $P$ is the pulsar period (Goldreich & Julian 1969).

To compare the BEC’s luminosity with the power given by equation (3) one can define the efficiency:

$$\eta_{\Delta \phi} \equiv \frac{L_{\text{BEC}}}{L_{\Delta \phi}},$$

which is expected to be much less than unity. Since the production of coherent radio emission is not understood in detail, it is also useful to compare the BEC’s luminosity to the power carried by the outflowing stream of particles (primary electrons and secondary $e_\pm$-pairs):

$$L_{\text{par}} = L_{\text{pr}} + L_\pm,$$

$$= (\gamma_{\text{pr}} + n_{\pm} \gamma_{\pm}) m c^3 n_{\text{GJ}}(r) \, A(r),$$

where $\gamma_{\text{pr}}$ denotes the Lorentz factor acquired by an accelerated primary electron, $\gamma_{\pm}$ is the initial Lorentz factor of $e_\pm$ pairs and $n_\pm$ is the number of pairs produced per one primary electron in the cascade which is responsible for the BEC. Equation (6) defines the power carried by the primary electrons ($L_{\text{pr}} = \gamma_{\text{pr}} m c^3 n_{\text{GJ}} A$) and the secondary $e_\pm$ plasma ($L_\pm = n_\pm \gamma_{\pm} m c^3 n_{\text{GJ}} A$). We will also use $L_{\pm,1}$, which is equal to $L_\pm$ taken for $n_\pm = 1$. Below we also discuss the radio-emission power $L_R$, determined by the minimum Lorentz factor $\gamma_R$ required for the curvature radio spectrum to extend at least up to the upper integration limit $v_s$ in equation (2). The stream’s radio power can be expressed by:

$$L_R = \gamma_R m c^3 n_{\text{GJ}}(r) \, A(r),$$

where a contribution due to $n_\pm$ is neglected, because it is electromagnetically difficult to separate plasma into charge density exceeding $n_{\text{GJ}}$ (Gil & Melikidze 2010, hereafter GM10). For all the afore-described power-related quantities, we define their corresponding efficiencies in the way analogous to equation (4), e.g.:

$$\eta_{\text{pr}} \equiv L_{\text{BEC}} / L_{\text{pr}}, \quad \eta_\pm \equiv L_{\text{BEC}} / L_\pm, \quad \eta_{\pm,1} \equiv L_{\text{BEC}} / L_{\pm,1}, \quad \eta_R \equiv L_{\text{BEC}} / L_R.$$
density for the total profile (Fig. 1). This is done by assuming that emission from J1012+5307 is negligibly low at the pulse longitude of the minimum flux (around 190° in Fig. 1). The result, illustrated in Fig. 1, is

\[ S_{\text{BEC}} = 0.73 \, S_{\text{mean}} \text{ at } 0.8 \text{ GHz} \] (8)

\[ S_{\text{BEC}} = 1.37 \, S_{\text{mean}} \text{ at } 1.4 \text{ GHz}, \] (9)

where \( S_{\text{mean}} = 14 \text{ mJy at } 0.8 \text{ GHz}, \) whereas \( S_{\text{mean}} = 3 \text{ mJy at } 1.4 \text{ GHz} \) (Kramer et al. 1998). The mean flux density within the BEC is then roughly the same as the mean flux density of full profile around 1 GHz: \( S_{\text{BEC}} \approx S_{\text{mean}} \). The case of the BEC of J1012+5307 is then considerably different from the standard case of luminosity estimate for normal pulsars with narrow beams. In the latter case, \( S_{\text{mean}} \) is an order of magnitude larger than \( S_{\text{mean}} \). This is because to determine the mean flux \( S_{\text{mean}} \), the energy contained within the narrow pulse is attributed to the full rotation period. Equation (3.41) in Lorimer & Kramer (2005, hereafter LK05) is sometimes used to quickly estimate radio pulsar luminosities. It is based on the exemplificative assumption that \( S_{\text{peak}} = 25 S_{\text{mean}} \) for narrow profiles of typical pulsars (with duty cycle 0.04 = 1/25). One should, therefore, resist from using this equation for the BEC of J1012+5307. In the case of this bifurcated component we have \( S_{\text{BEC}} \approx 0.45 S_{\text{peak}} \) (compare the level of bars at \( \theta_{\text{obs}} = 71° \) in Fig. 1 with the peaks of the BEC at the corresponding \( \nu \)). The narrow duty cycle expressed by the equation \( S_{\text{peak}} = 25 S_{\text{mean}} \) would then imply \( S_{\text{BEC}} \approx 10 S_{\text{mean}} \), to be compared with equations (8) and (9). Thus, if equation (3.41) from LK05 is directly used to calculate the luminosity of the BEC, the result becomes overestimated by one order of magnitude.

Kramer et al. (1999) find that between 0.4 and 5 GHz, the spectra of many MSPs can be well approximated by a power-law function of:

\[ S_{\text{mean}}(\nu) = S_{\text{mean},0} \left( \frac{\nu}{\nu_0} \right) ^{\xi}, \] (10)

where \( S_{\text{mean},0} \equiv S_{\text{mean}}(\nu_0) \) is the mean flux density at a frequency \( \nu_0 \). To perform the integration in equation (2) the integration limits are set to \( \nu_1 = 10 \text{ MHz} \) and \( \nu_2 = 100 \text{ GHz} \), which is much larger than has been observationally explored so far for J1012+5307. Data points in Fig. 2 present the \( vF_v \) spectrum of total profile of J1012+5307 based on available literature (Nicastro et al. 1995; Kramer et al. 1998, 1999; Stairs, Thorsett & Camilo 1999). Solid line presents the power law of equation (10) fitted to the six points in Fig. 2, with the index of \( \xi = -2.0 \pm 0.6 \) (\( vF_v \) slope of -1.0 in the figure). Using this value of \( \xi \), and the known flux-contributions of the BEC at 0.82 and 1.4 GHz (equations 8 and 9), we have determined the spectral index of the BEC \( \xi_{\text{BEC}} \approx -0.87 \pm 0.58 \). The BEC’s spectrum, shown in Fig. 2 with dashed line, is then completely different from the total one. At 5 GHz the flux density contained within the BEC (\( S_{\text{BEC}} \)) is \( \sim 6 \) times larger than the mean flux density of the total profile. However, the BEC’s flux, integrated between \( \nu_1 \) and \( \nu_2 \), is 14 times lower than the one calculated for the total profile with the ‘total’ spectral index of \(-2.0 \). This is caused by disparate levels of the BEC’s and total spectra at low frequencies (see Fig. 2).

### 3.1.1 Physical implications of the BEC’s spectral index

Interestingly, the BEC’s spectral index \( \xi_{\text{BEC}} \approx -0.87 \pm 0.58 \) is consistent with the value of \(-2/3 \) expected for distribution of charges that efficiently lose their energy in the form of the curvature radiation. For a narrow (delta-like) source function, the steady-state distribution of particles that undergo the curvature-radiation cooling has the power-law form \( N_e \propto \gamma^{-p} \) with \( p = 4 \) (see equation 3 in Rudak & Dyks 1999). The observed value of \( \xi_{\text{BEC}} \) implies \( p = 4.6 \pm 1.7 \).

However, the ‘curvature spectral index’ \( \xi_{\text{cr}} = -2/3 \) can extend down to the radio band (\( \nu \approx 400 \text{ MHz}, \) which is the lowest \( \nu \) at which the BEC has been detected so far), only when the curvature radiation can reduce the electron Lorentz factor down to \( \gamma \approx 40 \), as implied by equation (14) with \( \rho = 10^6 \text{ cm} \). This can occur provided the characteristic time-scale of particle escape from the emission region:

\[ t_{\text{esc}} \approx \rho/c \approx 3.3 \times 10^{-5} \text{ s} \rho_6, \] (11)

is longer than the time-scale of the curvature radiation cooling:

\[ t_{\text{cr}} \approx \frac{\gamma}{\Gamma_{\text{cr}}} \approx \frac{3 \, mc^2 \rho^2}{2 \, e^2 \kappa \gamma^3} = 1.2 \times 10^{14} \text{ s} \kappa^{-1} \rho_6^2 \gamma^{-7}, \] (12)

where \( \kappa \) has been introduced to take into account the increase of energy loss above the vacuum value as a result of unknown coherence mechanism. The condition \( t_{\text{esc}} \gtrsim t_{\text{cr}} \) with \( \gamma = 40 \) requires \( \kappa \gtrsim 5.5 \times 10^3 \), an apparently enormous value. Thus, the observed spectrum of the BEC can be understood as the curvature radiation from an initially narrow electron energy distribution, provided that the radiative energy loss rate \( \dot{\gamma}_{\text{cr}} \) is larger by the factor \( \kappa \) than the non-coherent value. However, the uncertainty of \( \rho \) is large, and one cannot exclude the possibility that other factors are responsible for the observed spectral slope.

### 3.2 The solid angle

The value of solid angle \( \Delta \Omega \) depends on the beam associated with the observed BEC. In the ‘stream-cut’ model, the BEC is observed when the line of sight is traversing through a narrow but elongated, fan-shaped emission beam (see figs 1, 2 and 4 in DR12). The transverse width of this beam corresponds to the angular size of the curvature radiation microbeam. In what follows the word ‘microbeam’ is used to mean the elementary pattern of radiation.
characteristic of the coherent emission process operating in pulsar magnetosphere. It should be discerned from the ‘pulsar beam’ which is observed at the Earth and results from spatial convolution of many microbeams. The BEC of J1012+5307, at least around $v \sim 1$ GHz, appears to be an intermediate case between the pure microbeam and the convolved case.

### 3.2.1 Curvature radiation microbeam

The beam of curvature radiation emitted in vacuum has a mostly filled-in, pencil-like shape. Deep in the magnetosphere, however, the ordinary-mode part of the beam can be damped and absorbed by plasma. The remaining part, which is the X mode polarized orthogonal to the plane of electron trajectory, has the two-lobed form which we associate with the BEC (DRD10).

The microbeam then consists of two lobes that point at a small angle $\psi$ with respect to the plane of electron trajectory, with no emission within the plane itself. The angle between the lobes is:

$$2\psi = \frac{0.8}{(\rho v_\psi)^{1/3}}$$

(13)

where $\rho = 10^3 \text{ cm} \times \gamma$ is the curvature radius of electron trajectory, and $v = 10^8 \text{ Hz} \times v_\psi$. Hence, for typical parameters ($\rho v \sim 1$, $v_\psi \sim 1$) the microbeam size is 10 times smaller than the observed separation of maxima in the BEC around 1 GHz: $\Delta_{\text{BEC}} = 7.9$. We assume that the large apparent width of the BEC results from the very small cut angle $\delta_{\text{cut}}$ between the beam and the trajectory of the line of sight. When we walk across a railway track at a decreasingly small angle, the distance between two points at which we cross each rail increases. Small $\delta_{\text{cut}}$ increases the apparent width of the BEC in a similar way (see fig. 2 in DR12, with the angle $\delta_{\text{cut}}$ marked on the right-hand side). A BEC produced by the beam of size $2\psi$ effectively has the observed width of $\Delta \sim 2\psi/\sin \zeta \sin \delta_{\text{cut}}$, where $\zeta$ is the viewing angle between the rotation axis and the line of sight.

There are several important reasons for why we use equation (13) instead of the popular result of $\psi \simeq 1/\gamma$, where $\gamma$ is the Lorentz factor of the radio-emitting electrons:

1. Equation (13) is valid for any frequency smaller than, or comparable to, the characteristic frequency of the curvature radiation spectrum:

$$v_{\text{crv}} = \frac{3c\gamma}{4\pi\rho} = 7 \text{ GHz} \left(\frac{\gamma^2}{\rho \text{[cm]}}\right).$$

(14)

Equation (13) does include the result of $\psi \sim 1/\gamma$ as a special case when $v = v_{\text{crv}}$, but it also holds true for any $v \leq v_{\text{crv}}$. This can be immediately verified by inserting $v_{\text{crv}}$ into equation (13), which gives $\psi = 0.78\gamma^{-1}$.

2. The approximation given by equation (13) is fairly accurate for frequencies extending all the way up to the peak of the curvature spectrum. The maximum of this spectrum occurs$^2$ at $v_{pk} = 0.28v_{\text{crv}}$.

At this spectral peak, equation (13) overestimates $2\psi$ by only a factor of 1.05. At $v = v_{\text{crv}}$, the angle is still overestimated only by a modest factor of 1.12.

3. Unlike equation (13), the formula $\psi \sim 1/\gamma$ is valid only in two cases: (i) for a frequency-integrated BEC; (ii) at the peak of the curvature spectrum: $v \approx v_{\text{crv}}$. Case (ii) does not have to apply for the actual, frequency-resolved BEC observed at a fixed $v$ by a real radio telescope.

4. The most important reason: within the validity range of equation (13), i.e. for $v \leq v_{\text{crv}}$, the angle $\psi$ does not depend on $\gamma$. Therefore, when the formula $\psi \sim 1/\gamma$ is used instead of equation (13), one may misleadingly invoke that for, e.g. $\gamma = 10^4$, the angle $\psi$ at $v = 1$ GHz is equal to 10$^{-4}$ rad = 0.006. This is in general wrong, because at $v = 1$ GHz (fixed by the properties of a radio receiver) the angle $\psi$ depends on the curvature radius only, and for $\rho = 10^3 \text{ cm}$ is of the order of 1 regardless of how high value of $\gamma$ is assumed. The curvature radiation has then this interesting property that as long as the curvature spectrum extends up to the receiver frequency $v$, the detected beam has the angular width which is fully determined by the curvature radius $\rho$ only. This angular width is with good accuracy the same for all values of $\gamma$ that ensure $v \leq v_{\text{crv}}$.

5. The separation of maxima in the BEC of J1012+5207 evolves with $v$ in a way expected in the limit of $v \ll v_{\text{crv}}$ (see fig. 7 in DR12). The use of equation (13), which is also valid in this limit, ensures consistency.

6. The formula $\psi \sim 1/\gamma$ is blind to the question of whether the curvature spectrum for a chosen $\gamma$ extends up to the observed frequency band. For example, for $\gamma \sim 10$, which neatly fits the observed $\Delta_{\text{BEC}}$ in the absence of any geometrical magnification, the curvature spectrum does not reach 1 GHz at all ($\rho \approx 10^3 \text{ cm}$). Whereas in the case of equation (13) it is immediately visible that an extremely small $\rho \sim 10^3 \text{ cm}$ is required to get $\Delta_{\text{BEC}} \approx 8^\circ$ at 1 GHz.

### 3.2.2 Value of solid angle

Since the BEC’s beam is emitted by the stream, the projection of the beam on the sky can be approximated by an elongated rectangle, described by two dimensions: one in the transverse direction orthogonal to plane of the stream (direction of the magnetic azimuth $\phi$), and the other parallel to the stream (direction of the magnetic colatitude $\theta$).

The flux contained in the BEC has been estimated in Section 3.1 through the integration over the pulse-longitude interval of 35°, which is 4.4 times larger than the separation of peaks in the BEC at 1 GHz ($\Delta_{\text{BEC}} \approx 8^\circ$). Since the peak separation itself is interpreted as the angle $2\psi$ given by equation (13), we assume that the transverse size of the solid angle is equal to $4.4 \times 2\psi = 0.06 \text{ rad}/(\rho v_{\psi})^{1/3}$. The line of sight may cut the beam at a small angle $\delta_{\text{cut}} \ll 1 \text{ rad}$, measured between the elongated projection of the beam on the sky and the path of sightline passage through the beam. If $\delta_{\text{cut}}$ is small whereas the viewing angle $\zeta$ is not, the beam needs to extend in the $\theta$ direction by an angle comparable to the BEC’s phase interval itself ($\sim 0.5 \text{ rad}$). The solid angle associated with the BEC can then be estimated as $\Delta \Omega \approx 0.03 \text{ sr}/(\rho v_{\psi})^{1/3}$. This value is similar to the solid angle of a typical polar beam of normal pulsar (LK05 assume 0.034 sr).

$^1$ Further below we will discuss wider beams with $\rho \gamma \ll 1$ that undergo less extreme geometrical magnification.

$^2$ The frequency $v_{\text{crv}}$ is sometimes defined to be twice larger than in equation (14). In such a case the curvature spectrum peaks at 0.14$v_{\text{crv}}$, i.e. at a frequency almost one order of magnitude lower than $v_{\text{crv}}$. The value of $v_{\text{crv}}$ itself is then located at the onset of the exponential high-frequency cut-off of the spectrum, where the flux has already dropped down to $\sim 30$ per cent of the peak flux. Note that the energy spectrum ($F_\nu$ convention) is assumed in this discussion of spectral peak location.

$^3$ It may be worth to note here that in the low frequency limit $v \ll v_{\text{crv}}$, the $\nu$-resolved intensity of the curvature radiation and the shape of the microbeam do not depend on the Lorentz factor either.
The BEC’s spectrum has been determined above by flux-integration within the same phase interval at different frequencies. For consistency, therefore, the solid angle is assumed to be \( v \)-independent by setting \( v_0 = 1, \) i.e. \( \Delta \Omega \approx 0.03 \text{ sr} / \rho_1^{1/3} \). For the specific spectral index of the BEC \((\xi_{\text{BEC}} \approx -0.87); \) see Fig. 2) the resulting luminosity changes only by a factor of 1.2 if the \( v \)-dependent solid angle is used in equation (2). Also note that a choice of wider longitude interval for the BEC does not change the result much, because the values of \( S_{\text{BEC}} \) given by equations (8) and (9) decrease for wider intervals. This compensates the increase of \( \Delta \Omega \).

### 3.3 Luminosity of the BEC

Taking \( d = 520 \text{ pc}, \) \( v_0 = 1.4 \text{ GHz}, \) \( S_{\text{mean},0} = 3 \text{ mJy}, \)
\[ S_{\text{BEC}} = 1.37 S_{\text{mean},0} \]
\[ v_1 = 10 \text{ MHz}, \] \( v_2 = 100 \text{ GHz}, \)
\[ \xi_{\text{BEC}} = -0.87, \]
\[ \Delta \Omega = 0.03 \text{ sr} / \rho_1^{1/3} \]
we get
\[ L_{\text{BEC}} = 4 \times 10^{32} \rho_2^{-1/3} \text{ erg s}^{-1}. \]

(15)

The only previously known estimate of the luminosity of the BEC is the one by GM10, which has not yet been published in any astronomical journal, but is being widely broadcasted on most recent pulsar conferences. The value obtained by GM10 is 15 times larger than \( 4 \times 10^{35} \). The main reason for this is that GM10 used equation (3.41) from LK05, which assumes that because of the usually narrow duty cycle \( \delta \approx 0.04, \) the peak flux \( S_{\text{peak}} \) is 25 times larger than the mean flux \( S_{\text{mean}} \). In the case of the BEC of J1012+5307, we have \( S_{\text{peak}} \approx 2.5 S_{\text{mean}} \) at \( v \approx 1 \text{ GHz} \) (see Fig. 1). For the millisecond pulsars, it is worth to lower down the numerical coefficient in equation (3.41) of LK05 by a factor of 0.04/\( \delta \), where \( \delta \) is the MSP’s duty cycle (or to use their equation 3.40 instead of 3.41).

The other difference is that GM10 used the ‘global’ spectral index of the total pulsar population \((\xi = -1.8)\) instead of the index of the BEC, and assumed that 10 per cent of the flux calculated in such a way is contained in the BEC. Thus, they used the spectrum shown in Fig. 2 with dotted line, which is different from the BEC’s spectrum (dashed line in Fig. 2). For this reason their estimate of the integral in equation (2) is two times smaller than ours. Therefore, GM10 obtain the luminosity which is approximately larger by a factor of 25/2 than given by equation (15).

### 4 THE MAXIMUM POWER OF THE STREAM

#### 4.1 Transverse area of the stream

The transverse cross-section of the stream is assumed to extend laterally through the distance \( \Delta l_\perp \) (in the direction of magnetic azimuth \( \phi \)) and meridionally through \( \Delta l_\parallel \) (in the direction of magnetic colatitude \( \theta \)). Then the area of the cross-section \( A = \Delta l_\parallel \Delta l_\perp \). The size of \( \Delta l_\perp \) is limited by the spread of magnetic field lines within the emitting area \( A \), which must not be too large in comparison with \( 2 \psi \) (equation 13) to not blur the BEC. Let \( \theta_B \) denote the angle between the tangent to a dipolar B-field line and the magnetic dipole axis. For two points separated azimuthally by \( \Delta \phi \), and located at the same \( r \) and \( \theta \), the dipolar B-field lines diverge by the angle of \( \delta_B \) (the angle between the tangents to the field lines) given by
\[ \sin \frac{\delta_B}{2} = \sin \theta_B \sin \frac{\Delta \phi}{2}. \]

(16)

For inclination orthogonal to the dipole axis \((\theta_B = 90^\circ), \) equation (16) gives \( \delta_B = \Delta \phi \). For two points on the opposite sides of the polar cap \((\theta_B = 15^\circ \text{pc} \) and \( \Delta \phi = 180^\circ), \) it gives \( \delta_B = 30 \text{ pc} \ll \Delta \phi \). As can be seen, a specific difference \( \Delta \phi \) in the magnetic azimuth results in the B-field-line divergence that is smaller than \( \Delta \phi \) by the factor of \( \sin \theta_B \). This is because for small \( \theta_B, \) B-field lines become almost parallel to each other (and to the dipole axis) irrespective of \( \Delta \phi \).

The divergence \( \delta_B \) of the B-field lines within the emission region is allowed to comprise a fraction \( \epsilon_\phi \) of the microbeam’s width:
\[ \delta_B = \epsilon_\phi 2 \psi, \]

(17)

where \( \epsilon_\phi \) < 1 to avoid blurring. Assuming that the allowed angles \( \delta_B \) and \( \Delta \phi \) are small, from the last two equations we get:
\[ \epsilon_\phi 2 \psi = \sin \theta_B \Delta \phi \]

(18)

This gives the following limitation on the transverse size of the stream:
\[ \Delta \phi = r_\perp \Delta \phi = r_\perp \epsilon_\phi 2 \psi / \sin \theta_B, \]

(19)

where \( r_\perp \) is the distance of the stream from the dipole axis.\(^4\) For the rim of the polar cap of J1012+5307, we have: \( \theta_B = 17.3 \) which allows \( \Delta l_\perp \) to be 3.3 times larger than the value of \( r_\perp \epsilon_\phi 2 \psi \), expected for orthogonal viewing. The BEC of J1012+5307 is, however, observed 50° away from the phase of the interpulse (IP), which may suggest \( \theta_B > 50^\circ \), for which \((\sin \theta_B)^{-1} \approx 1.3 \). Since geometric effects can make the observed BEC–IP separation both smaller and larger than \( \theta_B \), below we assume that \((\sin \theta_B)^{-1} \approx 1.3 \).

The fraction \( \epsilon_\phi \) of the beam size that can be occupied by the stream can be estimated by making convolutions of various density profiles with the shape of the elementary microbeam, given by equation (11) in DR12.

Fig. 3 presents such results for the rectangular (top hat) density distribution (Fig. 3a) and the Gaussian distribution (Fig. 3b). The grey rectangle in the centre of both panels presents the observed level of the central minimum between 0.82 and 1.4 GHz: the minimum is at 0.32 and 0.5 of the peak flux of the BEC. In the top hat case this admits the range of 0.6 < \( \epsilon_\phi \) < 0.74. In the Gaussian case we have assumed that \( \epsilon_\phi = 1 \) corresponds to the width of the Gaussian function at the half power level (1.18\( \sigma \)). The observed depth of the central minimum then implies 0.44 < \( \epsilon_\phi \) < 0.57. Thus, the maximum allowed width of the stream depends on the sharpness of the density distribution. Below we assume \( \epsilon_\phi = 0.5 \) at 1 GHz, which approximately corresponds to the Gaussian case in Fig. 3.

If we now consider two points that have the same magnetic azimuth \( \phi \), but different colatitude \( \theta \), we have \( \delta_B = \Delta \theta_B \approx (3/2) \Delta \phi = (3/2) \Delta l_\parallel / r_\perp \), where \( r_\perp \) is the radial distance of the emission points from the neutron star centre.\(^5\) This implies \( \Delta l_\parallel \) which is larger than \( \Delta l_\perp \) by the factor of \((2/3)(r/r_\perp) \sin \theta_B \). However, the spread of B-field lines in the colatitude is not limited to a fraction of \( 2 \psi \), because the extent of the emission region within the plane of the B-field lines does not smear out the BEC. Effects of the colatitude extent are degenerate with the effect of motion of electrons along the B-field lines and do not (directly) smear out the BEC. This can make misleading impression that the colatitudeal extent is not limited at all by the unsmearing shape of the BEC. However, because the emission beam is instantaneously narrow, the parts of the emission region that are far from the line of sight do not contribute to the detectable flux. This may lead one to think that \( \Delta l_\parallel \) still needs to be constrained by \(~2 \psi \) so that \( \Delta \theta_B \) in the stream does not blur the BEC.

\(^4\) For consistency, \( r_\perp \) needs to refer to the same radial distance \( r \) from the star centre, as the Goldreich–Julian density does in equation (3).

\(^5\) Prompted by the referee we explain that dipolar B-field at locations with small magnetic colatitude \( \theta \) is inclined at the angle \((3/2)\theta \) with respect to the dipole axis. Hence the 3/2 factor.

---

Energy budget of PSR J1012+5307 3065
not considerably exceed the beam size $2\psi$. This is not the case, because the emission from the poleward part of the stream (located closer to the dipole axis) becomes tangent to the line of sight at a slightly larger $r$ (as a result of the curvature of $B$-field lines). For an arbitrarily large colatitudinal extent $\Delta l_\theta$ there exists some radial distance $r$, at which the poleward extremes of the stream can become visible to the observer. Therefore, the extent $\Delta l_\theta$ can still appear to be constrained by the beam size. However, because of the $r$-dependent effects of aberration and retardation (Blaskiewicz, Cordes & Wasserman 1991; Kumar & Gangadhara 2012), the radial extent that is related to $\Delta l_\theta$ produces a pulse-longitude spread of $\Delta \phi_{\text{obs}} \simeq 2\Delta r/R_\phi$ (Dyks, Rudak & Harding 2004). This spread must be a small fraction of the size of the beam:

$$
\Delta \phi_{\text{obs}} \simeq 2\Delta r/R_\phi \leq \epsilon_\psi 2\psi,
$$

where $\epsilon_\psi < 1$. For $\epsilon_\psi = \epsilon_\phi = 0.5$ and $R_\phi = 25 \times 10^6$ cm, this constrains $\Delta r$ to $8.7 \times 10^4$ cm. The extent in colatitude $\Delta l_\theta$ is thereby also constrained to a value that can be determined as follows. Consider the aforesaid two points at the same $r$ and $\phi$, one of them located at $\theta$, whereas the other at a slightly smaller colatitude of $\theta(1 - \Delta s)$, where $\Delta s \ll 1$. Dipolar field lines are identically inclined to the dipole axis at all points which have the same magnetic colatitude $\theta$, irrespective of their radial distance $r$. Therefore, the $B$-field line that crosses the second (poleward) point becomes tangent to the line of sight at a slightly higher position ($r + \Delta r, \phi, \theta$) determined by the equation of dipolar field lines:

$$
\frac{\sin^2(\theta(1 - \Delta s))}{r} = \frac{\sin^2 \theta}{r + \Delta r},
$$

which in the small angle approximation gives

$$
\Delta r \simeq 2\Delta s \, r.
$$

The limit of equation (20) on $\Delta r$ then translates to

$$
\Delta s \leq \frac{\epsilon_\psi 2\psi R_\phi}{4} \frac{R_\phi}{r}.
$$

For $\epsilon_\psi = 0.5$, $2\psi = 0.8 = 0.014$ rad and $r = 10^6$ cm one obtains $\Delta s < 0.044$. Thus, the necessity to produce the sharply resolved BEC imposes indirect constraints on the stream’s extent in magnetic colatitude $\theta$:

$$
\Delta l_\theta = r \Delta \theta = r \Delta s \, \theta \simeq r \Delta s \, \theta \leq \frac{\epsilon_\psi 2\psi R_\phi}{4} \frac{R_\phi}{r}.
$$

The value of $\Delta l_\theta = (\sin \theta_\phi/4) (R_\phi/r) \Delta l_{0\phi}$ may then be a few times larger than $\Delta l_\phi$. Taking $r_\perp = r_{\text{pc}}$ (rim of the polar cap), one obtains $\Delta l_\theta = 0.044 r_{\text{pc}}$, which is twice as large as $\Delta l_{0\phi}$. The apparent bifurcation of the BEC does not therefore put equally tight constraints on the stream size in colatitude, as in the azimuth. Actually, it is possible to consider streams with elliptical cross-section, with the longer axis of the ellipse pointing towards the magnetic pole. Because of the curvature of magnetic field lines, pair production indeed tends to spread the pairs in the $\theta$ direction. Let $\Delta \theta_\pm$ denote the range of colatitudes over which $\epsilon_\psi$-pairs associated with a single primary electron were produced. Detailed numerical simulations of type such as in Daugherty & Harding (1982) suggest that $\Delta \theta_\pm$ does not exceed few hundredths of angular polar cap radius $\theta_{\text{pc}}$, i.e. $\Delta \theta_\pm$ is comparable to $\Delta\theta$ given by equation (24). For the sake of simplicity and minimalism, however, we will assume below that the stream has the same narrow size in both directions: $\Delta l_\theta = \Delta l_{0\phi}$, as given by equation (19).

Using $r_\perp = r_{\text{pc}} = 2 \times 10^6$ cm, $\epsilon_\phi(1$ GHz$) = 0.5$ and $(\sin \theta_\phi)^{-1} = 1.3$ in equation (19), we get $\Delta l_{0\phi} = 1.8 \times 10^3$ cm, and assuming $\Delta l_\theta = \Delta l_{0\phi}$, the stream’s cross-section has the area of $A = \Delta l_{0\phi} \Delta l_\theta = 3.3 \times 10^6$ cm$^2 \rho_{17}^{-2/3}$.

### 4.2 Kinetic luminosity of the stream

The electric potential difference between the centre and the edge of the polar cap of J1012+5307 is $\epsilon \Delta \Phi_{\text{pc}} = 1.45 \times 10^{14}$ eV = 232 erg which sets the upper limit to the Lorentz factor: $\gamma_{\text{max}} = 2.8 \times 10^9$

Using equation (3) with the surface value of the Goldreich–Julian density $n_{GJ} = 4.43 \times 10^{13}$ cm$^{-3} (P/P)^{1/2}$ this value of $\gamma_{\text{max}}$ corresponds to the following maximum kinetic luminosity of the stream:

$$
L_{\Delta \Phi} = 2 \times 10^{29} \text{ erg s}^{-1} \rho_{17}^{-2/3}.
$$

The corresponding value of minimum radio emission efficiency, calculated using $L_{\text{BEC}}$ from equation (15), is

$$
\eta_{\Delta \Phi} = 2 \times 10^{-4} \rho_{17}^{1/3},
$$

which fits the reasonable range expected for radio emission. Thus, even with the available energy limited by the narrowness of the beam, the stream has enough energy to power the bright BEC observed in the average pulse profile of J1012+5307.

We have retained dependence on $\rho$, because for $\rho_{17} = 1$ the stream must be observed at a very small angle (0.1 rad) so the intrinsic beam is enlarged to the apparent BEC. Non-dipolar values

---

**Figure 3.** Convolution of the curvature radiation microbeam with a rectangular distribution of emitting plasma density (a), and a Gaussian distribution (b). The lowermost curve in each panel presents the unconvolved microbeam with the peak separation of $2\psi$, marked by the vertical lines. Different curves correspond to different widths of a stream $\epsilon_\phi = 1.0, 0.9, 0.8, \ldots, 0.1, 0.0$ (top to bottom), a few values of $\epsilon_\phi$ are marked explicitly for the upper curves), where $\epsilon_\phi$ is a fraction of $2\psi$. The grey rectangle at the centre presents the level of flux at the centre of the BEC of J1012+5307, as measured relative to the BEC’s peaks, in the frequency range between 0.82 and 1.4 GHz (see Fig. 1).
of $\rho_0 \ll 1$ are, therefore, preferred to make the microbeam wider, and to place the stream-cut model in a more comfortable point of the parameter space. This makes the energy requirements even smaller: for $\rho = 10^6$ cm, $\eta_{\Delta \phi} = 10^{-4}$, whereas for $\rho = r_{\text{pc}} = 2 \times 10^4$ cm, $\eta_{\Delta \phi} = 6 \times 10^{-5}$.

Note that except for $E_{\text{max}}$, we have been conservative in our estimates, so some parameters may still be set to make the energy requirements even less demanding. For example, the spectrum of the BEC (Fig. 2) may be integrated only between 100 MHz and 10 GHz, which is already a wider interval than has ever been explored for J1012+5307. The stream may be assumed to have an elliptical shape with $\Delta \psi = 0.04 r_{\text{pc}}$, and the ‘multipolar’ $\rho_1 = 0.1$ may be taken. With all this optimism applied simultaneously, $\eta_{\Delta \phi} = 1.1 \times 10^{-5}$. The energy contained in the BEC is then a negligible fraction of the maximum energy that can possibly be attributed to the particle stream.

However, the efficiency is larger when the BEC’s luminosity is compared to the energy of primary electrons or secondary pairs. Let us define the efficiency: $\eta(\gamma) = L_{\text{BEC}}/(\gamma mc^2 n_{\text{GJ}}(A))$. By setting the upper limit of $\eta(\gamma) = 1$ one can calculate the minimum Lorentz factor that the radio-emitting particles need to have, to supply the energy observed in the BEC:

$$\gamma_{\text{min}} \simeq 6 \times 10^4 (\nu_{100} \rho_1^{1/3}),$$

(27)

where $\nu_{100} = v_{\text{max}}/(100 \text{ GHz})$ is the upper limit of the frequency-integration in equation (15). The energy transformed into the radio BEC needs to outflow at least at a rate exceeding $L_{\text{min}} \approx \gamma_{\text{min}} mc^2 n_{\text{GJ}}(A)$.

In the strongly curved $B$-field lines of MSPs, a balance between the radiative cooling and acceleration will constrain the Lorentz factor of primaries to $9.4 \times 10^3 B_{12}^{1/2} P_{-1}^{-1}$ (e.g. Rudak & Ritter 1994), hence $\gamma_{\text{pr}} = 2.8 \times 10^7$. This is clearly larger than $\gamma_{\text{min}}$, and implies $\eta_{\gamma} \simeq 2.2 \times 10^{-3}$, i.e. only 0.2 per cent of the primary electron energy is needed to explain the energy of the BEC. The value obtained in GM10 is $\eta_{\gamma} \simeq 1$ (100 per cent).

In the case of secondary pairs, their initial Lorentz factor can be estimated from Sturrock’s pair condition: $\gamma_{\text{pr}} \simeq 9.6 \times 10^4 B_{12}^{1/2} P_{-1}^{-3/2}$. For J1012+5307, $\rho_{\pm} \approx 3.6 \times 10^8$ which is six times larger than $\gamma_{\text{min}}$. Thus, it is enough that only one secondary particle (out of $n_{\pm}$ pairs produced per each primary electron) transfers 17 per cent of its initial energy into the BEC.

However, the secondary electron with so high Lorentz factor will lose almost all its energy in the form of synchrotron X-rays, not the radio waves. As shown in the Appendix, the remaining energy of parallel motion is $\gamma_0 \approx 25 P_{12}^{1/2}$, thus $\gamma_0 \approx 57$ for J1012+5307. This would have implied $\eta_{\gamma} \approx L_{\text{BEC}}/(\gamma_0 mc^2 n_{\text{GJ}}(A)) \sim 10^{-4}$; however, such a value of $\gamma_0$ is too low for the curvature spectrum to reach the upper limit $v_{\text{max}}$ of our integration range (if $\rho \sim 10^7$ cm). This means that either the secondaries are accelerated or the curvature radius $\rho$ is much smaller than dipolar. Therefore, to estimate the upper limit for radio emission efficiency, it is necessary to calculate the minimum Lorentz factor $\gamma_R$ for which the peak of CR spectrum reaches the radio band. For $v_{\text{max}} = 100 \text{ GHz}$ and $\rho_0 = 1$, equation (14) gives $\gamma_R \approx 522$. Hence $\eta_{\gamma} \approx 118$. We emphasize that $\eta_{\gamma}$ is independent of curvature radius of electron trajectory $\rho$, because both $\gamma_{\text{pr}}$ and $\gamma_0$ are proportional to $\rho^{1/3}$. GM10 obtain $\eta_{\gamma} \approx 2 \times 10^{-4}$ in their optimistic case, or $\eta_{\gamma} \sim 10^6$ for parameters that they call more realistic.

Thus, although no absolute energy limit is exceeded (there is initially $\delta n_{\pm}$ times more energy in the pair plasma than $L_{\text{min}}$), to explain the observed flux of the BEC, the available energy would have to be transformed into radio waves at an extreme rate. If the charge-separated (bunched) secondaries lose most of their energy in the form of X-rays, some process is required to draw the energy from another source, e.g. from the primaries or the charge-unseparated plasma, which outnumbers the Goldreich–Julian energy flux by the factor of $n_{\pm}$. Alternatively, super-Goldreich–Julian charge densities would have to emit coherently, i.e. $n_{\GJ}$ in equation (7) would have to be replaced by $n > n_{\GJ}$.

### 4.2.1 Comparison with the result of GM10

Contrary to GM10, we find that the energy content of the BEC does not break any strict upper limits. For example, we find $\eta_{\Delta \phi} \simeq 2 \times 10^{-4}$, $\eta_{\rho} \simeq 2 \times 10^{-3}$ (in GM10 $\eta_{\rho} \approx 1$), $\eta_{\gamma} \approx 0.17$. For the radio emission efficiency we get $\eta_{\gamma} \approx 10^{-2}$, i.e. we confirm the need for extremely efficient energy transport into the radio band. However, GM10 using a similar method estimate $\eta_{\rho} \approx 10^{-10}$–$10^{-8}$, which is in a notable disagreement with our result. There are several reasons for this difference.

(1) GM10 have overestimated $L_{\text{BEC}}$ by a factor of 15, because the duty cycle of J1012+5307 is much larger than 0.04, which is the typical duty cycle of normal pulsars assumed in LK05.

(2) GM10 assume $\gamma_{\rho} = 5 \times 10^4$, i.e. for unspecified reason they assume that only 1.8 per cent of available potential drop can be used up for powering the stream. We use the maximum Lorentz factor that the primary electrons can reach in the radiation-reaction-limited acceleration. We emphasize, however, that the energy available for radio-emitting $e_\pm$ pairs may actually be larger than $\gamma_{\rho}$, because when primary electrons are moving up with a fixed Lorentz factor (balanced by the energy losses to the curvature radiation), the energy is anyway being produced in the form of curvature photons that can produce the radio-emitting electron–positron plasma. It is then possible to produce the energy in the form of the electron–positron plasma without any change of electron energy (or even while the electron energy is increasing). For this reason the energy available for the stream may have more to do with the maximum potential drop rather than with the maximum achievable Lorentz factor.

(3) GM10 neglect the factor $(\sin \theta_B)^{-1}$ in equation (19), which at the polar cap’s rim of J1012 can increase the maximum allowed width of the stream 3.3 times. We assume $\theta_B = 50^\circ$ and $(\sin \theta_B)^{-1} = 1.3$.

(4) Furthermore, GM10 suggest that $L_{\Delta \phi}$ should be decreased by a factor of 7 because the double-lobed, orthogonally polarized part of the curvature microbeam comprises only 1/7 part of the total energy contained in the vacuum curvature beam. However, the form in which the energy of the parallel mode leaves the stream is not obvious.

(5) GM10 assume the Lorentz factor of radio emitting plasma $\gamma_{\pm} = 400$, the stream size of $e_\pm \times (1/\gamma_{\pm})$, and $e_\pm = 0.1$ to obtain $L_{\Phi}$ which is several orders of magnitude lower than $L_{\text{BEC}}$. However, their set of parameters ($\gamma_{\pm} = 400$, $e_\pm = 0.1$) is self-inconsistent because the BEC has the well-resolved double form at $v \simeq 1 \text{ GHz}$, whereas for typical $\rho_1 = 1$, the value of $\gamma_{\pm} = 400$ corresponds to $v_{\text{crv}} = 45 \text{ GHz}$, which is in the range where the BEC is unresolved and $e_\pm$ can considerably exceed 1. Around 1 GHz the flux observed at the minimum between the peaks increases quickly up and it is reasonable to expect that the BEC is fully merged at $v \gg 1 \text{ GHz}$. Thus, the condition $e_\pm = 0.1$ may apply only for $v \lesssim 1 \text{ GHz}$, whereas at high $v$ the stream may well be wider than the microbeam ($e_\pm > 1$).

The BEC is well-resolved around 1 GHz, and the formula for the beam size used by GM10 ($\psi \sim (1/\gamma_{\pm})$) is only valid at $v \simeq v_{\text{crv}}$. Therefore, both $e_\pm$ and the beam size (hence, the value of $\gamma_{\pm}$) must
refer to $\nu \simeq 1 \text{GHz}$. However, the large value of $\gamma_\pm = 400$ can only be made consistent with $v_{cr} = 1 \text{GHz}$ if $\rho = 448 \times 10^6 \text{cm}$ (as implied by equation 14 with $\gamma_\pm = 400$ and $v_{cr} = 1 \text{GHz}$). The value of $\rho$ implicitly present in their beam-size calculation is $\rho = 18R_\nu$, where $R_\nu = 25 \times 10^6 \text{cm}$ is the light cylinder radius. Thus, to justify their pessimistic values of $L_\rho$, GM10 assume parameters that imply the curvature radius several times larger than the light cylinder radius. This should not be practiced to find the maximum available kinetic luminosity.

(6) GM10 increase $\gamma_\pm$ to decrease the stream cross-section $A \propto (1/\gamma_\pm)^2$, while keeping the BEC’s luminosity fixed, which again implies $\eta_R \gg 1$. One should remember, however, that the beam size also determines the size of the solid angle $\Delta \Omega$ in equation (2) for the BEC’s luminosity. For the narrowing split-fan beam, our line of sight must cut it at a smaller angle $\delta_{cut}$ so that the 8°-wide BEC is observed. The value of the solid angle $\Delta \Omega$, which is proportional to the beam’s width $2\psi$ (or, in the case of GM10, to $1/\gamma_\pm$), should therefore be decreased accordingly. For $\Delta l_0 \propto \Delta l_0 \propto \psi$ the ratio $L_{\text{BEC}}/L_\rho$ is then proportional to $\psi^{-2}$ instead of being proportional to $\psi^{-1}$. It is then necessary to treat equations (2) and (3) as related to the same beam opening angle to avoid the overestimate of $\eta_R$.

5 CONCLUSIONS

We have calculated the radio luminosity of a single component selected from an average radio pulsar profile: the bifurcated component of J1012+5307. This luminosity, equal to $4 \times 10^{25} \text{erg s}^{-1}$, has been attributed to a narrow stream of radio-emitting plasma. The width of this stream is limited by the opening angle of the curvature radiation microbeam at 1 GHz, as determined by the well-resolved double-peak form of the BEC at this frequency. It has been shown that the efficiency of converting the stream’s energy into the BEC’s radio luminosity is of the order of $\eta_{\text{crv},\Delta} \approx 2.2 \times 10^{-3} \rho_1^{1/3}$, $\eta_{\text{crv}} \approx 2 \times 10^{-3}$, $\eta_{\Delta,1} \approx 0.17$, when the cross-cap potential drop, maximum energy of primary electrons, or initial energy of pairs is taken as a reference, respectively. Thus, no absolute energy limits are violated, and there is no energy deficit that can definitely be considered ‘fatal’ (as phrased by GM10) for the microbeam model of the BEC.

However, this result implies that large fraction of initial energy of a single secondary electron (per one primary) needs to be transferred to the radio BEC. This is unlikely, because such pairs lose most of their energy in the form of synchrotron X-ray photons. To power the radio emission, therefore, the energy would have to be drawn from the primary electrons or from the remaining charge-unseparated plasma (there is an extra energy of $n_b - 1$ uncharged secondary particles per each primary). Instead of assuming that such processes occur, it may be more natural to conclude that the BEC of J1012+5307 has macroscopic origin.

There are indeed some aspects that make the BEC of J1012 different from the rest of double features: (1) it is much wider, see Fig. 4, and (2) the outer wings of the BEC are much less steep than in the double notches of B1929+10 (see section 3.3 in DR12).

Nevertheless, the locally bidirectional emission (whether of either micro- or macroscopic origin) remains a valid and successful model for the BEC of J1012+5307. Note that it was not the BEC of J1012, but the absolute depth of double notches, which has decisively supported the model of bidirectional curvature radiation in section 4 of DRD10 (for physical details on the beam see Gil, Lyubarsky & Melikidze 2004). The curvature microbeam model continues to remain a valid and successful explanation for all the other pulsar double features (DR12).

We have assumed throughout this paper that J1012+5307 is a highly inclined rotator, with a large dipole inclination $\alpha \sim 90^\circ$, and a large viewing angle $\xi$, measured between the star’s rotation axis and the line of sight. The orthogonal geometry is supported by the presence of the IP separated by half of the rotation period from the MP, as well as by the width of the MP itself (about 40°), which is close to the opening angle of the surface polar beam (35°). For small $\alpha$ and $\xi$, which we consider unlikely, both the luminosity of the BEC, and the radio efficiency $\eta_R$ become smaller than quoted above. This is because the observed width of the BEC (a few times larger than $\Delta_{\text{BEC}}$) corresponds to the intrinsic solid angle $\Delta \Omega$ that becomes smaller by a factor of $\sin^2 \xi$.

Our results can additionally be affected by the uncertainty in the spectral index, bulk shape of spectral energy distribution (spectral breaks or cut-offs in the yet-unexplored frequency range), distance and scintillation-affected mean flux. However, since most of these factors can bias the result in both directions, it is unlikely that they can considerably decrease the energy requirements.

The value of the polar cap radius $r_{\text{cap}}$ that enters equations (19) and (24) is known at best with the accuracy of 16 per cent, which is the difference between the vacuum and force-free case (fig. 4 in Bai & Spitkovsky 2010). This implies a 36 per cent uncertainty in the stream area $A$. If a more narrow spectral band is assumed (0.1–10 GHz instead of 0.01–100 GHz) the BEC’s luminosity estimate decreases by a factor of 2.1. However, widening the band up to the range (1 MHz–1000 GHz) increases $L_{\text{BEC}}$ by only a factor of 1.6. If the spectral index of the BEC is increased or decreased by 0.5 the BEC’s luminosity increases by a factor of ~2. This is because in both these cases the spectrum becomes steep in comparison to the present slope of +0.12 in the $vFv$ convention.

The possibility of spectral breaks beyond the GHz band can reduce the energy requirements. Most millisecond pulsars do not
exhibit any spectral breaks above 100 MHz (Kuzmin & Losovsky 2001, however, the Pushchino measurements at 102 MHz (KL01; Malooffe, Malov & Shchegoleva 2000) suggest a break at ~600 MHz. This break is not included in our analysis (and not shown in Fig. 2), because the BEC is not discernible in the low-frequency profiles (see fig. 2 in Kondratiev et al. 2012). It is therefore not possible to determine the fractional energy content of the BEC below the spectral break. However, for the BEC spectrum shown in Fig. 2 with the dashed line, the mean flux of the BEC becomes comparable to the mean flux of total profile near 100 MHz (30 mJy; KL01). The lack of strong BEC in the 100 MHz profiles implies that the actual spectrum of the BEC at low frequencies is steeper than in the GHz range.

The Goldreich–Julian density in the emission region, which is proportional to $\mathbf{B} \cdot \mathbf{B}$, can be much smaller than we assume, if the local $\mathbf{B}$ is orthogonal to $\mathbf{B}$. This can happen because we assume rather large viewing angle. However, although we fix $\zeta$ to 90° for practical reasons, any values in the broad range of $\zeta = 90° \pm 45°$ are possible. Moreover, local non-dipolar enhancements of $\mathbf{B}$ are capable of increasing the density to a level considerably higher than the dipolar one. If the dipole inclination is not orthogonal, the magnetic field derived from the dipolar radiation energy loss should furthermore be increased by the factor $(\sin \alpha)^{-1}$. Other breaking mechanisms introduce additional uncertainty. For example, the magnetospheric currents in PSR B1931+24 change $\dot{P}$ by a factor of 1.5 (Kramer et al. 2006), which implies a 22 per cent error in $\dot{P}$.

Contrary to GM10 we find that the BEC’s flux comprises a tiny part of the maximum limit for the stream energy ($\eta_B \approx 2 \times 10^{-3}$, as compared to $\eta_B \approx 1$ in GM10). The radio emission efficiency is indeed extreme (we find $\eta_B \approx 10^{2}$), yet it is 2–4 orders of magnitude smaller than in GM10. The determination of luminosity and efficiency for isolated components in pulsar profiles is more complicated than standard energy considerations. Special care is required because: (1) such an isolated component may have considerably different spectrum than the total pulsar spectrum and it may have the flux density which can be at any ratio with the mean flux density of the total profile. (2) Fraction of the polar cap outflow that is responsible for the observed component needs to be carefully estimated, with the constraint of not blurring the component which is observed sharp and resolved at a frequency $\nu$. In the case of the split-fan beam, this ‘sharp view’ condition is different in the transverse (ψ-⊥) direction than in the meridional direction of $\theta$. (3) For $v \leq \nu_{\text{cvm}}$, the size of the curvature microbeam at a fixed frequency $\nu$ is independent of the Lorentz factor $\gamma'$ of the radio emitting plasma. The popular formula $\nu \sim \gamma'^{-1}$ can only be used when $\nu_{\text{cvm}}(\gamma', \rho) = \nu$, where $\nu$ is the central frequency of the observed bandwidth. If the ratio $\nu/\nu_{\text{cvm}}$ is not known, and the spectrum extends up to the observation band, it may be more safe to use equation (13). (4) Both the radio luminosity of the BEC, through $\Delta \Omega$, and the maximum power of the stream, through $A$, depend on the width of the microbeam. Any decrease of available power imposed by the decreasing width of the beam is alleviated by the simultaneous decrease of BEC’s luminosity.

ACKNOWLEDGEMENTS

We thank Paul Demorest for providing us with GBT data on J1012+5307. This work was supported by the National Science Centre grant DEC-2011/02/A/ST9/00256 and the grant N203 387737 of the Polish Ministry of Science and Higher Education.

REFERENCES

Bai X.-N., Spitkovsky A., 2010, ApJ, 715, 1282
Blaszkiewicz M., Cordes J. M., Wasserman I., 1991, ApJ, 370, 643
Daugherty J. K., Harding A. K., 1982, ApJ, 252, 337
Daugherty J. K., Harding A. K., 1983, ApJ, 273, 761
Dyks J., Rudak B., 2012, MNRAS, 420, 3403 (DR12)
Dyks J., Rudak B., Harding A. K., 2004, ApJ, 607, 939
Dyks J., Rudak B., Demorest P., 2010, MNRAS, 401, 1781 (DRD10)
Gil J. A., Melikidze G. I., 2010, preprint (arXiv:1005.0678) (GM10)
Gil J., Lyubarsky Y., Melikidze G. I., 2004, ApJ, 600, 872
Goldreich P., Julian W. H., 1969, ApJ, 157, 869
Gould D. M., Lyne A. G., 1998, MNRAS, 301, 235
Hankins T. H., Rankin J. M., 2010, ApJ, 139, 168
Kondratiev V., Stappers B., the LOFAR Pulsar Working Group, 2012, in van Leeuwen J., ed., Proc. IAU Symp. 291, New Results from LOFAR. Cambridge Univ. Press, Cambridge, p. 47
Kramer M., Dulkisur M. K., Lorimer D., Doroshenko O., Jessner A., Wielebinski R., Wolszczan A., Camilo F., 1998, ApJ, 501, 270
Kramer M., Lange C., Lorimer D., Backer D. C., Dulkisur M. K., Jessner A., Wielebinski R., 1999, ApJ, 526, 957
Kramer M., Lyne A. G., O’Brien J. T., Jordan C. A., Lorimer D. R., 2006, Sci, 312, 549
Kumar D., Gangadharra R. T., 2012, ApJ, 746, 157
Kuzmin A., Losovsky B. Ya., 2001, A&A, 368, 230 (KL01)
Lorimer D. R., Kramer M., 2005, Handbook of Pulsar Astronomy. Cambridge Univ. Press, Cambridge (LK05)
Malooffe V. M., Malov O. I., Shchegoleva N. V., 2000, Astron. Rep., 44, 436
Navarro J., Manchester R. N., Sandhu J. S., Kulkarni S. R., Bailes M., 1997, ApJ, 486, 1019
Nicastro L., Lyne A. G., Lorimer D. R., Harrison P. A., Bailes M., Skidmore B. D., 1995, MNRAS, 273, L68
Rudak B., Dyks J., 1999, MNRAS, 303, 477
Rudak B., Ritter H., 1994, MNRAS, 267, 513
Stairs I. H., Thoret S. E., Camilo F., 1999, ApJS, 123, 627
Sturrock P. A., 1971, ApJ, 164, 529
Zhang B., Harding A. K., 2000, ApJ, 532, 1150

APPENDIX A

The energy of parallel motion of $e_\parallel$ pairs can be estimated in the following way. Let us consider a secondary electron with initial Lorentz factor $\gamma'_\parallel$. Let $v'_\parallel$ be the component of this electron’s velocity parallel to the magnetic field, and $\gamma'_\parallel = (1 - (v'_\parallel/c)^2)^{-1/2}$ is the corresponding Lorentz factor. Now consider a primed Lorentz frame which moves along $\mathbf{B}$ with the velocity $v'_\parallel$. In this frame our electron has purely transverse velocity $v'_\perp$ and a Lorentz factor $\gamma'_\perp = (1 - (v'_\perp/c)^2)^{-1/2}$. The Lorentz transformation of velocities implies:

$$\gamma'_{\parallel} = \frac{\gamma}{\gamma'_{\perp}}.$$  \hfill (A1)

In the case of millisecond pulsars, Sturrock’s condition for pair creation is

$$\chi = \frac{1}{2} \frac{\epsilon}{mc^2} \frac{B}{B_0} \sin \psi \approx \frac{1}{11.5}.$$  \hfill (A2)

(Sturrock 1971), where $\epsilon$ is the energy of the pair-producing photon, $mc^2$ is the electron rest energy, $B_0 \approx 44$ TG is the critical magnetic field value, and $\psi$ is the angle at which the photon crosses the magnetic field. For classical pulsars the number on the right-hand side is closer to 1/15. In the case of $\chi \ll 1$, each component of the created pair takes up half energy of the parent photon:

$$\gamma'_\perp mc^2 \approx \frac{\epsilon}{2}.$$  \hfill (A3)
(Daugherty & Harding 1983) and follows the photon’s propagation direction. In the relativistic limit of $\gamma_{\pm} \gg 1$, it holds that $\cos \psi = v_\parallel / v_{\pm} \approx v_\parallel / c$, hence: $1 / \gamma_{\parallel} \approx \sin \psi$. Equations (A2) and (A3) then give:

$$y_{\perp}' = 3.8 \times 10^3 \left( \frac{r}{R_{\text{ns}}} \right)^3 B_{\text{pc}}^{-1},$$  \hspace{1cm} (A4)

where the local $B$ field is $B = B_{\text{pc}} (r/R_{\text{ns}})^{-3}$.

The Lorentz factor $\gamma_{\pm}$ can also be estimated from (A3) and (A2) by noting that a photon emitted in dipolar field at $(r, \theta)$ encounters the largest value of $B \sin \psi = 0.085 \theta B(r)$ (Rudak & Ritter 1994). This is approximately the place where the one-photon absorption coefficient is maximum, and the pair production is most likely and efficient. By inserting the last formula into (A2) one can derive the so-called ‘escape energy’, which is the minimum photon energy required to produce pairs in pulsar magnetosphere:

$$\epsilon_{\text{esc}} \approx 1.0 \times 10^5 \text{ MeV} \ R_{\text{ns}}^{-1/2} (r/R_{\text{ns}})^{5/2} B_{\text{pc}}^{-1} P_3^{1/2},$$  \hspace{1cm} (A5)

where $R_{\text{ns},6} = R_{\text{ns}}/(10^6 \text{ cm})$, $P_3 = P/(10^{-3} \text{ s})$ and $\theta \approx (r/R_{\text{Ns}})^{1/2}$ was assumed to correspond to the polar cap rim. By inserting (A5) into (A3) we get

$$\gamma_{\pm} = 9.4 \times 10^3 (r/R_{\text{ns}})^{5/2} P_3^{1/2} B_{\text{pc}}^{-1}.$$  \hspace{1cm} (A6)

From (A1), (A4) and (A6) we obtain

$$\gamma_{\parallel} = 25 (r/R_{\text{ns}})^{-1/2} P_3^{1/2},$$  \hspace{1cm} (A7)

which, in the limit of near-surface emission and pair production ($r \sim R_{\text{ns}}$), is used in the main text. The derived estimates well reproduce the results of exact numerical simulations (see the distributions of $\gamma_{\parallel}$ and $\gamma_{\perp}'$ for a normal and millisecond pulsar in Fig. 1 of Rudak & Dyks 1999). They are also useful in semi-analytical modelling of pair cascades (Zhang & Harding 2000).