Strong Couplings of Heavy Mesons to A Light Vector Meson in QCD

Zuo-Hong Li\textsuperscript{a,b,c,d}, Tao Huang\textsuperscript{a,c} Jin-Zuo Sun\textsuperscript{a,b} and Zhen-Hong Dai\textsuperscript{b}

\textsuperscript{a} CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China
\textsuperscript{b} Department of Physics, Yantai University, Yantai 264005, China
\textsuperscript{c} Institute of High Energy Physics, P.O.Box 918(4), Beijing 100039, China
\textsuperscript{d} Department of Physics, Peking University, Beijing 100871, China

\textbf{ABSTRACT}

We make a detailed analysis of the $BB\rho(DD\rho)$ and $B^*B\rho(D^*D\rho)$ strong couplings $g_{BB\rho}(g_{DD\rho})$ and $g_{B^*B\rho}(g_{D^*D\rho})$ using QCD light cone sum rules(LCSR). The existing some negligence is pointed out in the previous LCSR calculation on $g_{B^*B\rho}(g_{D^*D\rho})$ and an updated estimate is presented. Our findings can be used to understand the behavior of the $B, D \to \rho$ semileptonic form factors at large momentum transitions.

PACS numbers: 11.55.Hx, 13.75.Lb
Keywords: Light Cone QCD Sum Rules, $BB\rho$ and $B^*B\rho$ Strong Couplings

\footnote{Email: lizh@ytu.edu.cn}
\footnote{Mailing address}
1. INTRODUCTION

At the present time, there is an increasing interest in exclusive $B$ decays in order to explore the sources of CP violation. However, the definite interpretations for the relevant experimental data demand that we have the ability to precisely compute the physical amplitudes. It is nonperturbative QCD dynamics not being dealt rigorously with that would hinder us from doing such a desired calculation. One is forced to use some approximate methods. Lattice QCD simulation is the most trustworthy approach to nonperturbative QCD effects, with no parameters or assumptions, but the precision of calculation is limited by the available computing resources and some certain restriction exists\[1\], for example, in describing $b \to u$ or $s$ transitions. The formulation of QCD factorization formula\[2\] is based on the first principle and is viewed as a great progress in phenomenology of heavy flavors; however, the underlying long distance effects included in a series of the hadronic matrix elements still confront us. Heavy quark symmetry can very well apply to describing the systems including one heavy quark, but is of less predictive power for heavy-to-light transitions. Although QCD sum rule method\[3\] builds its underlying physical assumptions on the field theory, and exhibits its decided superiority in dealing with some of nonperturbative quantities, such as decay constants, hadronic matrix elements and strong coupling constants, a problem with it is that the resulting form factors for heavy-to-light transitions can not behave very well in the heavy quark limit $m_Q \to \infty$. The successes of perturbative QCD in treating numerous hard exclusive processes have excited the occurrence of many excellent works. The most prominent of them is the development of QCD light-cone sum rules (LCSR)\[4, 5\]. It is the striking advantage of this approach to describe heavy-to-light transitions in a way consistent with the universally accepted physical picture that nonperturbative QCD dynamics occupies an dominant place and perturbative hard gluon exchanges contribute only a subleading effect in that case. LCSR approach takes the basic correlator, in which the proper current operators are sandwiched between the vacuum and an on shell light meson state, and adopts the operator product expansion (OPE) around the light cone $x^2 \approx 0$ instead of at the small distance $x \approx 0$. Nonlocal matrix elements occurring in this approach, which encode all the information on large distance dynamics, are parametrized in terms of a set of so-called light-cone wavefunctions classified by twist, which describe the momentum distributions of the quarks inside the relevant light mesons. This is identical with a summation over all the condensate terms in the short distance OPE. Consequently, the resulting form factors for heavy-to-light transitions exhibit the correct behavior with heavy quark mass, averting the problem with the traditional sum rules. This method has extensively been accepted and has found its successful applications, such as the investigations of the form factors for heavy-to-light transitions at small and intermediate momentum transfers\[5-11\] and
of the strong couplings between heavy and light mesons[8, 12, 13], since its presentation in[4]. Very recently, it has been generalized to study the nonfactorizable effects in $B \rightarrow \pi\pi$[14] and to probe heavy-to-light form factors in the whole kinematically accessible ranges[15]. The technical details of LCSR can be found, for instance, in [6], while for a detailed comparison with traditional sum rules, see [9].

An investigation of strong interactions between heavy mesons and a light vector meson is of important phenomenological interest, and especially the relevant strong couplings can be employed to understand the behavior of the corresponding heavy-to-light form factors at large momentum transfer in a pole dominance model, which has proven to be exact in an effective chiral Lagrangian approach[16]. The off-shell $B^* B\rho (D^* D\rho)$ strong coupling $g_{B^* B\rho}(g_{D^* D\rho})$, in fact, has been calculated in the standard LCSR approach[13]; however, it calls for a reexamination due to some negligence existing in calculations. Another nonperturbative quantity deserving of investigation is the $BB\rho (DD\rho)$ strong coupling $g_{BB\rho}(g_{DD\rho})$, which turns out to be equally important. Motivated by all these facts, in this paper we make a systematic study on them.

This presentation is organized as follows. The following Section is devoted to a detailed derivation of the sum rules for $g_{BB\rho}$ and $g_{B^* B\rho}$, using LCSR method. Then we give a numerical analysis of the resulting sum rules, including a discussion of error estimates, in Sec.3. The last Section give to a simple summary.

2. LCSR’S FOR THE STRONG COUPLINGS

The $BB\rho$ and $B^* B\rho$ strong couplings $g_{BB\rho}$ and $g_{B^* B\rho}$ can be defined as

$$\langle \rho (q, e) B (p) | B (p + q) \rangle = g_{BB\rho} e^{*}_\lambda \cdot p, \quad (1)$$

$$\langle \rho (q, e) B^* (p, \eta) | B (p + q) \rangle = -g_{B^* B\rho} \epsilon_{\mu\alpha\beta\gamma} q^\alpha e^{*}_\beta p^\gamma. \quad (2)$$

The resulting findings can easily converted into the corresponding $c$-quark meson cases. According to the general strategy of QCD sum rules, it is needed to construct an adequate correlator in order to obtain an qualitative estimate for $g_{BB\rho}$ and $g_{B^* B\rho}$. As usual, we use two correlators of the following forms as the starting points of LCSR calculations on $g_{BB\rho}$ and $g_{B^* B\rho}$, respectively,

$$F (p, (p + q)) = i \int d^4xe^{ipx} \left\langle \rho (q, e) | T\pi(x) i\gamma_5 b(x), \bar{b}(0) i\gamma_5 d(0) | 0 \right\rangle$$

$$= \tilde{F} (p^2, (p + q)^2) e^{*} \cdot p, \quad (3)$$

$$G_\mu(p, p + q) = i \int d^4xe^{ipx} \left\langle \rho(q, e) | T\pi(x) \gamma_\mu b(x), \bar{b}(0) i\gamma_5 d(0) | 0 \right\rangle$$

$$= \tilde{G}_\mu (p^2, (p + q)^2) \epsilon_{\mu\alpha\beta\gamma} q^\alpha e^{*}_\beta p^\gamma. \quad (4)$$
For the correlator (3), isolating the pole contribution of the lowest $0^- B$ meson and parametrizing these from the higher $0^-$ states in a form of dispersion integral, the hadronic form of the invariant function $\tilde{F} \left( (p^2, (p + q)^2) \right)$ may be written as

\[
\tilde{F}^H \left( (p^2, (p + q)^2) \right) = \frac{m_b^4/m_b^2 \Gamma_B^2 g_{BB\rho}}{(m_B^2 - p^2) [(m_B^2 - p + q)^2]} + \int \int \frac{\rho_1^H (s_1, s_2)}{(s_1 - p^2) [s_2 - (p + q)^2]} ds_1 ds_2, \tag{5}
\]

with a double integral starting from the same threshold parameter $s_0$, which should be set in the neighborhood of the squared mass of the first excited $0^- B$ meson. Similarly we have for the invariant function $\tilde{G}^H \left( (p, (p + q)) \right)$,

\[
\tilde{G}^H \left( (p^2, (p + q)^2) \right) = \frac{m_B^4/m_B^2 \Gamma_B \Gamma_B' g_{BB'\rho}}{m_b (m_{B'}^2 - p^2) [m_B^2 - (p + q)^2]} + \int \int \frac{\rho_2^H (s_1, s_2) ds_1 ds_2}{(s_1 - p^2) [s_2 - (p + q)^2]} . \tag{6}
\]

QCD calculations of the underlying correlators may be allowed, on the other side, for the negative and large values of $p^2$ and $(p + q)^2$, in which case the $b$ quarks travel only a small distance $x$ and therefore the operator product expansion (OPE) goes effectively in powers of the deviation from the light cone $x^2 \approx 0$. We would like to work in the case where the interactions of the $b$ quarks with the background field gluons are omitted, since their influence on the sum rules is, as always, negligibly small. On contracting the $b$ quark operators into a free propagator,

\[
\langle 0 | T b(x) \overline{b} (0) | \rangle = \frac{1}{(2\pi)^4} \int d^4 k e^{-i k \cdot x} \frac{k^2 + m_b}{m_b^2 - k^2}, \tag{7}
\]

one gets,

\[
\tilde{F}^{QCD} \left( (p^2, (p + q)^2) \right) = \frac{1}{(2\pi)^4} \int \int d^4 x d^4 k \frac{1}{m_B^2 - k^2} e^{i(p-k) \cdot x} [k^\mu \langle \rho (q, e) | T \pi (x) \gamma_\mu d (0) | 0 \rangle - m_b \langle \rho (q, e) | T \pi (x) d (0) | 0 \rangle] . \tag{8}
\]

The nonlocal matrix elements $\langle \rho (q, e) | T \pi (x) \gamma_\mu d (0) | 0 \rangle$ and $\langle \rho (q, e) | T \pi (x) d (0) | 0 \rangle$ define the light cone wavefunctions of the $\rho$ meson[17-19] as,

\[
\langle \rho (q, e) | \pi (x) \gamma_\mu d (0) | 0 \rangle = f_\rho m_\rho \left\{ \frac{e^{(\lambda)^* \cdot x}}{q \cdot x} q_\mu \int_0^1 du e^{i u q \cdot x} \left[ \varphi_{||} (u, \mu) + \frac{m_\rho^2 u^2}{16} A (u, \mu) \right] \right.
\]

\[
+ \left( \frac{e^{(\lambda)^* \cdot x} - q_\mu e^{(\lambda)^* \cdot x}}{q \cdot x} \right) \int_0^1 du e^{i u q \cdot x} g_\perp^{(\nu)} (u, \mu) \]

\[
- \frac{1}{2} \frac{e^{(\lambda)^* \cdot x}}{(p \cdot x)^2} m_\rho^2 \int_0^1 du e^{i u q \cdot x} C (u, \mu) \right\} , \tag{9}
\]

\[
\langle \rho (q, e) | T \pi (x) d (0) | 0 \rangle = -i/2 f_\rho^T m_\rho^2 (\lambda) \cdot x \int_0^1 du e^{i u q \cdot x} h_\perp^{(\nu)} (u, \mu_b) . \tag{10}
\]

$f_\rho$ stands for the usual vector decay constant of the $\rho$ meson and $f_\rho^T$ is defined as $\langle 0 | d_{\mu u} | \rho \rangle = i f_\rho^T \left( e^{(\lambda)}_\mu q_\nu - e^{(\lambda)}_\nu q_\mu \right)$; $\varphi_{||} (u, \mu)$ is the leading twist-2 wavefunction, $g_\perp^{(\nu)} (u, \mu)$ and $h_\perp^{(\nu)} (u, \mu_b)$ refer
to the twist-3 ones, and both \( A(u, \mu) \) and \( C(u, \mu) \) have twist-4, which parametrize the mass corrections. \( \varphi || (u, \mu) \), \( g_\perp^{(v)} (u, \mu) \) and \( h_\perp^{(s)} (u, \mu) \) are normalized as \( \int_0^1 du f(u) = 1 \), while \( C(u, \mu) \) satisfies \( \int_0^1 du C(u) = 0 \). A tedious but straightforward calculation yields

\[
\tilde{F}^{QCD} \left( p^2, (p + q)^2 \right) = f_{\rho} m_{\rho} \left[ \int_0^1 du \frac{\varphi || (u)}{m_b^2 - (p + u q)^2} - \frac{1}{m_b^2 - (p + q)^2} \right] + f_{\rho}^T m_{\rho} m_b \int_0^1 du \frac{h_\perp^{(s)} (u, \mu)}{[m_b^2 - (p + u q)^2]^2} \\
- \frac{1}{2} f_{\rho} m_{\rho}^3 \int_0^1 du \left\{ \frac{1}{2 \left[ m_b^2 - (p + u q)^2 \right]^2} + \frac{m_b^2}{[m_b^2 - (p + u q)^2]^3} \right\} \times [A(u) + 8 \tilde{C}(u)]
\]

(11)

In deriving Eq.(11), it proves to be convenient for a partial integration to introduce the auxiliary functions \( \tilde{f} (u) = \int_0^u f(v) dv \) for \( f(u) = \varphi || (u, \mu), g_\perp^{(v)} (u, \mu) \) and \( A(u, \mu) \), and \( \tilde{C}(u) = \int_0^u C(v) dv \), \( \tilde{C}(u) = \int_0^u \tilde{C}(v) dv \), which is equal to zero when \( u = 1 \). The twist-3 contribution from \( g_\perp^{(v)} (u, \mu) \) vanishes exactly due to cancellations in the partial integrations.

Furthermore, it is indispensable to convert Eq.(11) into a form of dispersion integral for upcoming continuum substitution. The relevant QCD spectral density can easily be obtained by virtue of the technique suggested in [20]. In the following we consider only the twist-2 and -3 terms and give a detailed deviation. First of all, we perform a double Borel transformation \( Q_1^2 = -p^2 \to M_1^2, Q_2^2 = -(p + q)^2 \to M_2^2 \), for the twist-2 term ( the term proportional to the inverse \( m_b^2 - (p + q)^2 \) disappears after doing that ), obtaining

\[
\hat{B} \left( M_1^2, Q_1^2 \right) \hat{B} \left( M_2^2, Q_2^2 \right) \int_0^1 du \frac{\varphi || (u)}{m_b^2 - (p + u q)^2} = \frac{M^2}{M_1^2 M_2^2} e^{-\frac{1}{M^2} \left[ m_b^2 + m_{\rho}^2 u_0 (1 - u_0) \right]} \varphi || (u_0),
\]

with \( u_0 = M_2^2 / (M_1^2 + M_2^2) \) and \( M^2 = M_1^2 M_2^2 / (M_1^2 + M_2^2) \). The symmetry of the correlator makes it natural to set \( M_1^2 = M_2^2 \) so that the wavefunctions \( \varphi || (u) \) may take its value at the symmetric point \( u_0 = 1/2 \). Further, making a replacement \( M_1^2 \to 1/\sigma_1, M_2^2 \to 1/\sigma_2 \) in Eq.(12) yields

\[
\hat{B} \left( \frac{1}{\sigma_1}, Q_1^2 \right) \hat{B} \left( \frac{1}{\sigma_2}, Q_2^2 \right) \int_0^1 du \frac{\varphi || (u)}{m_b^2 - (p + u q)^2} = \frac{\sigma_1 \sigma_2}{\sigma_1 + \sigma_2} e^{-(\sigma_1 + \sigma_2) (m_b^2 + \frac{1}{4} m_{\rho}^2)} = f \left( \sigma_1, \sigma_2 \right).
\]

(13)

Finally, we take the function \( \tilde{f} (\sigma_1, \sigma_2) = \frac{1}{\sigma_1 \sigma_2} f (\sigma_1, \sigma_2) \) and perform one more Borel transformation in the variables \( \sigma_1 \) and \( \sigma_2: \sigma_1 \to 1/s_1 \) and \( \sigma_2 \to 1/s_2 \). The resulting QCD spectral density reads

\[
\rho^{(QCD)}_{tw2} (s_1, s_2) = f_{\rho} m_{\rho} \hat{B} \left( \frac{1}{s_1}, \sigma_1 \right) \hat{B} \left( \frac{1}{s_2}, \sigma_2 \right) \tilde{f} (\sigma_1, \sigma_2)
\]

\[
= f_{\rho} m_{\rho} \varphi || \left( \frac{1}{2} \right) \delta (s_1 - s_2) \Theta \left( s_1 - m_b^2 - \frac{1}{4} m_{\rho}^2 \right) \Theta \left( s_2 - m_b^2 - \frac{1}{4} m_{\rho}^2 \right).
\]

(14)
Applying all the same procedure to the twist-3 part, we have

\[ \rho_{\text{tw3}}^{QCD}(s_1, s_2) = f_\rho^T m_\rho^2 m_b \left( m_b^2 + \frac{1}{4} m_\rho^2 \right)^2 h^*_\parallel \left( \frac{1}{2} \right) \delta \left( s_1 - m_b^2 - \frac{1}{4} m_\rho^2 \right) \delta \left( s_2 - m_b^2 - \frac{1}{4} m_\rho^2 \right) \frac{1}{s_1 s_2}, \]

\[ \text{(15)} \]

which means that the twist-3 part receives no continuum substraction. With Eqs. (14) and (15), \( \tilde{F}^{QCD}(p^2, (p + q)^2) \) can be expressed as

\[ \tilde{F}^{QCD}(p^2, (p + q)^2) = f_\rho m_\rho \int \int ds_1 ds_2 \frac{\delta (s_1 - s_2) \Theta \left( s_1 - m_b^2 - \frac{1}{4} m_\rho^2 \right) \Theta \left( s_2 - m_b^2 - \frac{1}{4} m_\rho^2 \right)}{(s_1 - p^2) \left[ s_2 - (p + q)^2 \right]} \varphi \left( \frac{1}{2} \right) \]

\[ + \frac{f_\rho^T m_\rho^2 m_b h^*_\parallel \left( \frac{1}{2} \right)}{m_b^2 + \frac{1}{4} m_\rho^2 - p^2} \left( m_b^2 + \frac{1}{4} m_\rho^2 - (p + q)^2 \right) \]

\[ - \frac{1}{2} f_\rho m_\rho \int_0^1 du \left\{ \frac{1}{2 \left[ m_b^2 - (p + uq)^2 \right]^2} + \frac{m_b^2}{\left[ m_b^2 - (p + uq)^2 \right]^3} \right\} \left[ A(u) + 8 \tilde{C}(u) \right]. \]

\[ \text{(16)} \]

Making the Borel improvement \( p^2 \rightarrow M_1^2, \ (p + q)^2 \rightarrow M_2^2 \) for both the theoretical and hadronic expressions, which suppresses the higher state and twist-4 contributions, and then equating them by the use of the quark-hadron duality ansatz, the final sum rule for \( f_B^2 g_{BB_\rho} \) reads,

\[ f_B^2 g_{BB_\rho} = \frac{m_b^2}{m_B^4} \left\{ f_\rho m_\rho M^2 \left[ e^{-\frac{m_b^2}{M^2} (m_b^2 + \frac{1}{4} m_\rho^2)} - e^{-\frac{m_\rho^2}{M^2} (m_b^2 + \frac{1}{4} m_\rho^2)} \right] \varphi \left( \frac{1}{2} \right) + \frac{1}{4} m_\rho^2 e^{-\frac{m_b^2}{M^2} (m_b^2 + \frac{1}{4} m_\rho^2)} \right\} \]

\[ \times \left[ 4 m_b f_\rho^T h^*_\parallel \left( \frac{1}{2} \right) - f_\rho m_\rho \left[ A \left( \frac{1}{2} \right) + 8 \tilde{C} \left( \frac{1}{2} \right) \right] \left( 1 + \frac{m_b^2}{M^2} \right) \right]. \]

\[ \text{(17)} \]

Now let’s turn from this topic to a discussion of the sum rule for the \( B^* B_\rho \) strong coupling \( g_{B^* B_\rho} \) by expanding the relevant correlator (4) around the light cone \( x^2 = 0 \). Utilizing Eq.(7) it follows immediately that

\[ G^{QCD}_\mu (p^2, (p + q)^2) = \frac{i}{(2\pi)^4} \int \int d^4x d^4k \frac{1}{m_b^2 - k^2} e^{i(p-k)\cdot x} \left[ k_\nu \langle \rho | T \bar{\pi} (x) \gamma_\mu \gamma_\nu \gamma_5 d (0) | 0 \rangle \right] + m_b \langle \rho | T \bar{\pi} (x) \gamma_\mu \gamma_5 d (0) | 0 \rangle \].

\[ \text{(18)} \]

The \( \gamma \) algebraic relation \( \gamma_\mu \gamma_\nu \gamma_5 = - \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} \sigma^{\alpha \beta} + g_{\mu \nu} \gamma_5 \) can help us to write it down in a preferred form,

\[ G^{QCD}_\mu (p^2, (p + q)^2) = \frac{i}{(2\pi)^4} \int \int d^4x d^4k \frac{1}{m_b^2 - k^2} e^{i(p-k)\cdot x} \left[ k_\nu \langle \rho | T \bar{\pi} (x) \gamma_\mu \gamma_\nu \gamma_5 d (0) | 0 \rangle \right] - \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} k_\nu \langle \rho | T \bar{\pi} (x) \sigma^{\alpha \beta} d (0) | 0 \rangle + m_b \langle \rho | T \bar{\pi} (x) \gamma_\mu \gamma_5 d (0) | 0 \rangle \].

\[ \text{(19)} \]
The nonlocal matrix element $\langle \rho | \pi(x) \gamma_5 d(0) | 0 \rangle$ is exactly vanishing, as required by the parity conservation of strong interactions. The light cone expansions of the other two matrix elements reads[10, 19] respectively,

$$
\langle \rho | \pi(x) \sigma^{\alpha\beta} d(0) | 0 \rangle = -i f^T_{\rho} \left\{ \left( e^{*\lambda}_{(\rho)} q^\beta - e^{*\lambda}_{(\rho)} q^\alpha \right) \int_0^1 du e^{iuq \cdot x} \left[ \varphi_{\perp}(u) + \frac{1}{16} m^2_{\rho} x^2 A_T(u) \right] 
+ \left( q^\alpha x^\beta - q^\beta x^\alpha \right) \int_0^1 du e^{iuq \cdot B_T(u)} \right\},
$$

and

$$
\langle \rho | T \pi(x) \gamma_\mu \gamma_5 d(0) | 0 \rangle = \frac{1}{4} f_{\rho} m_\rho \epsilon_{\mu\alpha\beta\gamma} q^\alpha e^\beta x^\gamma \int_0^1 du e^{iuq \cdot x} g_{\perp}^{(a)}(u, \mu_b).
$$

$\varphi_{\perp}(u)$ and $g_{\perp}^{(a)}(u)$ stand for the twist-2 and -3 wavefunctions, respectively, and obey $f_0^1 f(u) du = 1$; the others are associated with twist-4 operators and parametrize the $\rho$ mass corrections, among which both $B_T(u)$ and $C_T(u)$ abide by $f_0^1 f(u) du = 0$ as it stands.

As with the $g_{BB\rho}$ case, in practical calculations we use the definitions $B_T(u) = f_0^u B_T(v) dv$, $\bar{B}_T(u) = f_0^u \bar{B}_T(v) dv$ and $G_T(u) = f_0^u G_T(v) dv$. Substituting Eqs. (20) and (21) into (19) we gain the theoretical expression for $\tilde{G}^{QCD}(p^2, (p + q)^2)$,

$$
\tilde{G}^{QCD}(p^2, (p + q)^2) = f^T_{\rho} \left\{ \int_0^1 du \frac{\varphi_{\perp}(u)}{m^2_b - (p + uq)^2} + \frac{1}{4} m^2_{\rho} \int_0^1 du A_T(u) \right\}
$$

$$
\times \left( \frac{3}{\left[ m^2_b - (p + uq)^2 \right]^2} - \frac{2 m^2_b}{\left[ m^2_b - (p + uq)^2 \right]^3} \right)
$$

$$
+ \frac{1}{2} m_b m_{\rho} f_{\rho} \int_0^1 du \frac{g_{\perp}^{(a)}(u)}{\left[ m^2_b - (p + uq)^2 \right]^2}.
$$

It is interesting to note that the $\rho$ mass effect is only offered by the twist-4 wavefunction $A_T(u)$. Obviously, the QCD spectral densities concerning the leading and next-to-leading terms are, except for a constant factor, the same as those in the $g_{BB\rho}$ case, respectively. Omitting details, we end up with the following sum rule for the product $f_B f_{BB^*B\rho}$,

$$
f_B f_{BB^*B\rho} = \frac{m_b}{m_B m_{B^*} m_{B\rho}} e^{\frac{m^2_b + m^2_{B\rho}}{2 M^2}} \left\{ f^T_{\rho} \left[ \left( e^{-\frac{1}{M^2} \left( m^2_b + \frac{1}{2} m^2_{B\rho} \right)} - e^{-\frac{m^2_b}{M^2}} \right) M^2 \varphi_{\perp} \left( \frac{1}{2} \right) \right] \right.
$$

$$
+ \frac{1}{4} m^2_{\rho} \left( 3 \frac{m^2_b}{M^2} e^{-\frac{1}{M^2} \left( m^2_b + \frac{1}{2} m^2_{B\rho} \right)} A_T \left( \frac{1}{2} \right) \right)
$$

$$
+ \frac{1}{2} f_{\rho} m_b m_{\rho} e^{-\frac{1}{2} \left( m^2_b + \frac{1}{2} m^2_{B\rho} \right)} g_{\perp}^{(a)} \left( \frac{1}{2} \right) \right\}.
$$

It should be understood that the Borel variables $M^2$ and $M^2$ have been taken equal once again, for the $B$ and $B^*$ mesons are nearly degenerate in mass. This enables the relevant wavefunctions to take values at $u = 1/2$, to high accuracy.
We would like to emphasize that the previous LCSR results for \( g_{B^* B_\rho} \) and \( g_{D^* D_\rho} \)\cite{13} are questionable, because of a missing factor \( 1/2 \) in front of the term \(-\epsilon_{\mu\alpha\lambda\rho}\sigma^{\lambda\rho}\) in writing down the \( \gamma \) algebraic relation \( \gamma_\mu \gamma_\alpha \gamma_\beta = -1/2 \epsilon_{\mu\alpha\lambda\rho}\sigma^{\lambda\rho} + g_{\mu\alpha} \), and a misuse of the subtraction procedure \( e^{-\frac{m_\pi^2}{4\pi}(m_\rho^2 + \frac{1}{4}m_\rho^2)} \rightarrow e^{-\frac{m_\pi^2}{4\pi}(m_\rho^2 + \frac{1}{4}m_\rho^2)} - e^{-\frac{m_\pi^2}{4\pi}} \) imposed on the twist-3 parts.

### 3. Numerical Results

The parameters to need fixing for a numerical estimate are those concerning the \( B^* \) or \( B \) mesons and those describing the \( \rho \) meson. The former contain the decay constants \( f_B \) and \( f_{B^*} \), mass parameters \( m_B, m_{B^*} \) and \( m_\rho \), and threshold parameter \( s_0 \). We use \( m_\rho = 4.8 \pm 0.1 \) GeV, \( m_B = 5.279 \) GeV and \( m_{B^*} = 5.325 \) GeV. With the two point sum rules formulated in\cite{8}, the values of the decay constants \( f_B \) and \( f_{B^*} \) are fixed at \( f_B = 115 \) MeV and \( f_{B^*} = 125 \) MeV, corresponding to \( m_\rho = 4.8 \) GeV, \( s_0 = 33 \) GeV\(^2\) and the leading order in \( \alpha_s \). Also, all the parameters of the \( \rho \) meson involved become now numerically available. We take as inputs the experimental values \( m_\rho = 770 \) MeV, \( f_\rho = 198 \pm 7 \) MeV and the QCD sum rule result \( f_B^T = 152 \pm 9 \) MeV \cite{18} at the scale \( u_\rho = \sqrt{m_B^2 - m_\rho^2} \approx 2.5 \) GeV. Concerning the light cone wavefunctions appearing in our sum rules, there have been many discussions on them in the literature. It is in \cite{14} that QCD sum rule method is first applied to study the twist-2 distribution amplitudes of vector mesons. Later on, a more systematic discussion was given in \cite{18}, where results of \cite{17} were critically examined and updated. Very recently, the authors of \cite{19} took further the meson mass corrections into consideration by introducing some higher twist distributions in the light cone expansions of the relevant nonlocal matrix elements, extending the work \cite{18} by an additional use of the QCD equations of motion. The yielded findings, some of which will be used in our numerical analysis, have found applications\cite{14} in phenomenology of exclusive semileptonic and radiative \( B \) decays. The explicit forms of the light cone wavefunctions in relation to our sum rule calculations are

\[
\begin{align*}
\varphi_\perp (u, \mu_b) &= 6u(1-u) \left( 1 + a_2^\perp (\mu_b) \frac{3}{2} \left[ 5(2u-1)^2 - 1 \right] \right), \\
\varphi_\parallel (u, \mu_b) &= 6u(1-u) \left( 1 + a_2^\parallel (\mu_b) \frac{3}{2} \left[ 5(2u-1)^2 - 1 \right] \right), \\
h_\parallel^{(s)} (u, \mu_b) &= 6u(1-u)(1 + 0.15[5(2u-1)^2 - 1]), \\
g_\perp^{(a)} (u, \mu_b) &= 6u(1-u)(1 + [5(2u-1)^2 - 1]), \\
A_T (u, \mu_b) &= 24u^2 (1-u)^2, \\
A (u, \mu_b) &= \left[ \frac{4}{5} + \frac{20}{9} \xi_4 + \frac{8}{9} \xi_3 \right] 30u^2 (1-u)^2, \\
\bar{C} (u, \mu_b) &= - \left[ \frac{3}{2} + \frac{10}{3} \xi_4 + \frac{10}{3} \xi_3 \right] u^2 (1-u)^2,
\end{align*}
\]

with the coefficient \( a_2^\perp (\mu_b) = 0.17 \pm 0.09 \) and \( a_2^\parallel (\mu_b) = 0.16 \pm 0.09, \xi_3 = 0.023 \) and \( \xi_4 = 0.13 \).
Having all the input parameters at hand, we could carry out the numerical calculations. It is a critical step towards deriving a reliable sum rule prediction to look for a reasonable range of the Borel parameters. The standard procedure requires that the terms proportional to the highest inverse power of the Borel parameters stay reasonably small, which can fix the lower limit of the fiducial Borel interval, and that the higher resonance and continuum contribution should not become too large, which may determine the upper limit of the allowed range. For the two sum rules in consideration, we find that the Borel intervals to satisfy the above criteria are respectively \(8 \leq M^2 \leq 14 \text{ GeV}^2\) for the \(g_{BB\rho}\) case and \(8 \leq M^2 \leq 15 \text{ GeV}^2\) for the \(g_{B^*B\rho}\) case, where the twist-4 wavefunctions contribute less than 6% and 5% and the high states at the orders lower than 23% and 25%, respectively. The figure 1 shows the sensitivity of the sum rules for \(f^2_{BB\rho}\) and \(f_{B^*B\rho}\) to the Borel parameters. From the corresponding sum rule "windows", we arrive at \(f^2_{BB\rho} = 0.071 \pm 0.002 \text{ GeV}^2\) and \(f_{B^*B\rho} = 0.082 \pm 0.004 \text{ GeV}\), the uncertainties quoted being due to the variations of \(M^2\). By means of values of decay constants obtained previously it is immediate to get the desired sum rules for the strong couplings \(g_{BB\rho}\) and \(g_{B^*B\rho}\). If using the central values for all the relevant sum rule results, we have \(g_{BB\rho} = 5.37\) and \(g_{B^*B\rho} = 5.70 \text{ GeV}^{-1}\).

For a better understanding of the overall uncertainties in the coupling constants, it is highly advisable to employ the analytic forms instead of the numerical results in Eqs.(17) and (23) for the decay constants. When \(m_b\) keeps fixed while \(s_0\) changes between 32 – 34 \(\text{GeV}^2\), the resulting variations relative to the central values amount to \(\pm 6\%\) for \(g_{BB\rho}\) and to \(\pm 8\%\) for \(g_{B^*B\rho}\), if the uncertainties due to the Borel parameters are included. The influences on the sum rules can be investigated of the uncertainty in \(m_b\), by considering a correlated variation of \(m_b\) and \(s_0\) in the individually allowed ranges. Requiring the strong couplings \(g_{BB\rho}\) and \(g_{B^*B\rho}\) to take values only if the sum rules for the relevant decay constants show the best stability, we observe that the induced changes are typically of orders 7% and 6% respectively. It is of course important to investigate further the uncertainties from the wavefunctions. The simplest way to test the sensitivity of the sum rules to model wavefunctions is by putting all the corresponding nonasymptotic coefficients to zero, namely by using their asymptotic forms which are model-independent and completely dictated by perturbative QCD. The resulting sum rules deviate from their individual central values by about 7% in the \(g_{BB\rho}\) case and by about 8% in the \(g_{B^*B\rho}\) case. The effects observed in such a way come from an extreme treatment and therefore are anyway being overestimated. Taking it into account that the twist-4 wavefunctions bring only a correction of about 4% to both sum rules for \(g_{BB\rho}\) and \(g_{B^*B\rho}\), we can reasonably conjecture that the uncertainties due to neglected yet higher twists would be at best of the same orders as the twist-4 corrections. At present, the total uncertainties in the sum rules for \(g_{BB\rho}\) and \(g_{B^*B\rho}\)
can conservatively be estimated to be about 25% and 27%, respectively, by adding linearly up all the considered errors.

The same procedures may be used for a numerical discussion of LCSR for $g_{DD\rho}$ and $g_{D^*D\rho}$. The relevant parameters are taken as $m_c = 1.3$ GeV, $m_D = 1.87$ GeV, $m_{D^*} = 2.01$ GeV, $f_D = 170$ MeV, $f_{D^*} = 240$ MeV and $s_0 = 6$ GeV$^2$. In addition, we have to evolve the wavefunctions to a lower scale $\mu_c = \sqrt{m_D^2 - m_c^2}$. Using the standard criteria the fiducial intervals of $M^2$ turns out to be $4 < M^2 < 8$ GeV$^2$ in the $g_{DD\rho}$ case and $5 < M^2 < 8$ GeV$^2$ in the $g_{D^*D\rho}$ case. The stability of the sum rules for $f_D^2g_{DD\rho}$ and $f_{D^*}f_Dg_{D^*D\rho}$ is illustrated in Fig. 2. We have $f_D^2g_{DD\rho} = 0.11$ GeV$^2$ and $f_{D^*}f_Dg_{D^*D\rho} = 0.17$ GeV, with the negligibly small uncertainties due to $M^2$. As for the strong couplings $g_{DD\rho}$ and $g_{D^*D\rho}$, the resulting sum rules are predicted to be $g_{DD\rho} = 3.81$ and $g_{D^*D\rho} = 4.17$ GeV$^{-1}$, the total uncertainties being about 23% and 25%, respectively.

4. SUMMARY

We have made an intensive study on QCD interactions between heavy mesons and a light vector meson within the framework of LCSR. A detailed deviation of the sum rules is presented for the relevant strong coupling constants $g_{BB\rho}(g_{DD\rho})$ and $g_{B^*B\rho}(g_{D^*D\rho})$ and a systematic numerical analysis, including a painstaking investigation of the uncertainty arising from all the possible sources of error, is made. An existing negligence is pointed out in the previous LCSR calculation on $g_{B^*B\rho}$ and $g_{D^*D\rho}$, and an updated LCSR result is formulated.

The obtained predictions can be used to estimate the couplings for the other charge states using the relations from isospin symmetry. Also, it is straightforward to investigate the $B_sBK^*$, $B_s^*BK^*$ and $B^*B_sK^*$ strong couplings, and the corresponding those in $c$ quark meson case by making a corresponding parameter replacement in the relevant sum rules formulated.

The numerical results presented here should be updated, once our understanding of the meson wavefunctions, $b$-quark mass and decay constants became more clear, and the QCD radiative corrections are included in sum rule calculation.
References

[1] L. Lellouch, Nucl. Phys. B 497 (1996) 353; J. M. Flynn, hep-lat/9611016; J. M. Flynn, C. T. Sachrajda, hep-lat/9710057; K. C. Bowler et al. (UKQCD Coll.), hep-lat/9910011; A. Abada et al. (APE Coll.), hep-lat/9910021; S. Hashimoto et al. (JLQCD Coll.), Phys. Rev. D 58 (1998)014502; D. R. Burford et al. (UKQCD Coll.), Nucl. Phys. B 447 (1995) 425; J. M. Flynn et al. (UKQCD Coll.), Nucl. Phys. B 461 (1996) 327; L. DeL Debbio et al. (UKQCD Coll.), Phys. Lett. B 416 (1998) 392.

[2] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; Nucl. Phys. B 59 (2000) 313.

[3] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147 (1979) 385; B 147 (1979) 448.

[4] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B 312 (1989) 509.

[5] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B 345 (1990) 137.

[6] V. M. Braun, hep-ph/9801222, R. Rückl, hep-ph/9810338, A. Khodjamirian and R. Rückl, hep-ph/9801443.

[7] V. M. Belyaev, A. Khodjamirian and R. Rückl, Z. Phys. C 60 (1993) 349; A. Ali, V. M. Braun and H. Simma, Z. Phys. C 63 (1994) 437; A. Khodjamirian, R. Rückl, S. Weinzierl and O. Yakovlev, Phys. Lett. B 410 (1997) 275; E. Bagan, P. Ball and V. M. Braun, Phys. Lett. B 417 (1998) 154; P. Ball, JHEP 9809 (1998) 005; A. Khodjamirian, R. Rückl, and C. Winhart, Phys. Rev. D 58 (1998) 054013; O. Yakovlev, R. Rückl, S. Weinzierl, hep-ph/0007344, T. M. Aliev, A. Ozpineci and M. Savci, Phys. Rev. D 56 (1997) 4260; A. Khodjamirian, R. Rückl, S. Weinzierl, C. W. Winhart and O. Yakovlev, Phys. Rev. D 62 (2000) 114002.

[8] V. M. Belyaev, V. M. Braun, A. Khodjamirian, and R. Rückl, Phys. Rev. D 51 (1995) 6177.

[9] P. Ball and V. M. Braun, Phys. Rev. D 55 (1997) 5561.

[10] P. Ball and V. M. Braun, Phys. Rev. D 58 (1998) 094016.

[11] T. Huang, Z. H. Li and X. Y. Wu, Phys. Rev. D 63 (2001) 094001; Z. H. Li, F. Y. Liang, X. Y. Wu and T. Huang, Phys. Rev. D 64 (2001) 057901; T. Huang and Z. H. Li, Phys. Rev. D 57 (1998) 1993; T. Huang, Z. H. Li, and H. D. Zhang, J. Phys. G 25 (1999) 1179.
[12] A. Khodjamirian, R. Rückl, S. Weinzierl and O. Yakovlev, Phys. Lett. B 457 (1999) 245; P. Colangelo, F. De Fazio, N. Di Bartolomeo, R. Gatto and G. Nardulli, Phys. Rev. D 52 (1995) 6422; P. Colangelo and F. De Fazio, Eur. Phys. J. C 4 (1998) 503.

[13] T. M. Aliev, D. A. Demir, E. Iltan and N. K. Pak, Phys. Rev. D 53 (1996) 355.

[14] A. Khodjamirian, [hep-ph/0012271].

[15] S. Weinzierl and O. Yakovlev, JHEP 0101 (2001) 005; R. Rückl, S. Weinzierl and O. Yakovlev, [hep-ph/0105161].

[16] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B 292 (1992) 371; J. Schechter and A. Subbaraman, Phys. Rev. D 48 (1993) 332; P. Ko, Phys. Rev. D 47 (1993) 1964.

[17] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. 112 (1984) 173.

[18] P. Ball and V. M. Braun, Phys. Rev. D 54 (1996) 2182.

[19] P. Ball, V. M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. B 529 (1998) 323; P. Ball, V. M. Braun and G. Stoll, Nucl. Phys. B 543 (1999) 201.

[20] V. A. Nesterenko and A. V. Radyushkin, Sov. J. Nucl. Phys. 39 (1984) 811; V. A. Beylin and A. V. Radyushkin, Nucl. Phys. B 260 (1985) 61.
FIGURE CAPTIONS

Fig.1: The stability of LCSR for the products $f_{B}^{2}g_{BB\rho}$(Fig.1 (a)) and $f_{B^{*}}f_{B}g_{B^{*}B\rho}$(Fig.1 (b)), with $m_{b} = 4.8 \text{ GeV}$ and $s_{0} = 33 \text{ GeV}^{2}$.

Fig.2: The stability of LCSR for the products $f_{D^{2}}g_{DD\rho}$(Fig.2(a)) and $f_{D^{*}}f_{D}g_{D^{*}D\rho}$(Fig.2(b)), with $m_{c} = 1.3 \text{ GeV}$ and $s_{0} = 6 \text{ GeV}^{2}$.
Fig. 1

(a) \( f_B^2 g_{BB\rho} (GeV^2) \)

(b) \( f_B f_{B^*} g_{B^*B\rho} (GeV) \)
\begin{align*}
\int_{D^2} g_{D\rho}(GeV^2) \\
M^2(\text{GeV})
\end{align*}

\begin{align*}
\int_{D^*} f_{D^*g_{D^*\rho}}(GeV) \\
M^2(\text{GeV})
\end{align*}

Fig. 2