DESCRIPTION OF THE LOW-ENERGY DOUBLET
NEUTRON-DEUTERON SCATTERING ON THE BASIS OF THE TRITON
BOUND AND VIRTUAL STATE PARAMETERS

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Low-energy doublet neutron-deuteron scattering is described on the basis of the triton bound and virtual state parameters — the energies and the nuclear vertex constants of these states. The van Oers-Seagrave formula is derived from the Bargmann representation of the $S$ matrix for a system having two states. The presence of a pole in this formula is shown to be a direct corollary of the existence of a low-energy triton virtual state. Simple explicit expressions for the $nd$ scattering length and for the pole of the function $k \cot \delta$ are obtained in terms of the triton bound and virtual state parameters. Numerical calculations of the $nd$ low-energy scattering parameters show their high sensitivity to variations in the asymptotic normalization constant of the virtual state $C_v^2$. The $C_v^2$ value fitted in our model to the experimental result for the $nd$ scattering length is $C_v^2 = 0.0592$.

1. INTRODUCTION

The low-energy characteristics of three-body systems were studied previously [1, 2] on the basis of the two-body model with the Hulthén potential; in particular, a correlation between the binding energy of three hadrons and the hadron-deuteron scattering length was analyzed over a wide region of three-body parameters. An important role of the virtual triton state was revealed in studying this dependence (Phillips line) in the region of the experimental values of the triton binding energy $E_T$ and the doublet $nd$ scattering length $a_{nd}$. The characteristics associated with the $T \rightarrow d + n$ decay vertex were also calculated for the bound ($T$) and virtual ($v$) triton states. The results of the calculations for the position of the virtual triton level, $B_v$, as reckoned from the threshold of the elastic $nd$ channel and for nuclear vertex constants $G_T^2$ and $G_v^2$ characterizing the ground and the virtual triton state, respectively, agree with the

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relevant results of three-body calculations and with experimental data. The results from [1, 2] evince a clear-cut correlation between the low-energy characteristics of two- and three-hadron systems.

It was established in [3–6] that the effective-range expansion of the doublet $S$-wave phase shift for $nd$ scattering involves a pole situated in the nonphysical region near the threshold. For the function $k \cot \delta$, the corresponding four-parameter representation involving a pole is referred to as the van Oers-Seagrave formula. It should be noted that this formula was found in a purely empirical way without any theoretical justification.

In order to analyze two- and three-hadron systems, we propose here an approach based on the $S$ matrix corresponding to so-called Bargmann potentials [7, 8]. We will consider the relation between the $S$-matrix representation introduced by Bargmann [7, 8] and the effective-range expansion and demonstrate that, on the basis of this representation, the effective-range approximation can be obtained for the case where the system in question has one state, bound or virtual. An example of such a situation is provided by neutron-proton scattering. We will then generalize this analysis to systems having two states, as in the case of doublet neutron-deuteron scattering. It will be shown that the van Oers-Seagrave formula for the function $k \cot \delta$ involving a pole follows directly from the Bargmann representation of the $S$ matrix for a system having two states. This analysis makes it possible to relate the parameters of the effective-range expansion to the characteristics of the bound and virtual states of the system.

We assume that any state of the system is specified by two parameters, the energy corresponding to the pole of the $S$ matrix on the imaginary axis in the complex plane of the wave number $k$ and the nuclear vertex constant, which is directly expressed in terms of the residue of the $S$ matrix at this pole [9]. Nuclear vertex constants are fundamental physical characteristics of nuclei like more conventional quantities, including mass, spin, and parity. Kinematical factors apart, nuclear vertex constants are related to the on-shell amplitude for the virtual or real decay (or fusion) of a nucleus into two fragments. The properties of vertex constants, their values for a number of nuclei, and methods for determining them experimentally and theoretically were surveyed elsewhere [9].
2. DERIVATION OF THE EFFECTIVE-RANGE APPROXIMATION FROM THE BARGMANN REPRESENTATION OF THE S MATRIX INVOLVING ONE STATE: NEUTRON-PROTON SYSTEM

Bargmann [7] proposed taking the Jost function \( f(k) \) entering into the well-known \( S \)-matrix expression [8]

\[
S(k) = \frac{f(-k)}{f(k)}
\]  

(1)

in the form of a rational function having some simple poles and zeros and exhibiting a correct asymptotic behavior at high energies — that is, approaching unity at infinity:

\[
\lim_{k \to \infty} f(k) = 1.
\]  

(2)

In the simplest case of only one state in the system, the Jost function has the form

\[
f(k) = \frac{k-i\alpha}{k+i\lambda},
\]  

(3)

where the parameter \( \lambda \) is always positive, whereas the parameter \( \alpha \) is positive for a bound state and negative for a virtual state. Upon the substitution of (3) into (1), the \( S \) matrix for the system having one state assumes the form

\[
S(k) = \frac{k+i\alpha}{k-i\alpha} \frac{k+i\lambda}{k-i\lambda}.
\]  

(4)

This state has the energy

\[
E_0 = -\frac{\hbar^2\alpha^2}{2m},
\]  

(5)

where \( m \) is the reduced mass of the system and \( \hbar \) is the Planck constant. The first pole factor in expression (4) for the \( S \) matrix corresponds to a physical bound or a virtual state of the system. The second factor in the \( S \) matrix (4) includes the well-known redundant pole [8, 10], which is associated with no bound state of the system. The redundant pole ensures the correct asymptotic behavior (2) of the Jost function. The nuclear vertex constant \( G^2 \) for the state being considered is directly expressed in terms of the residue of the \( S \) matrix at the pole \( k = i\alpha \) as

\[
G^2 = i\pi\lambda^2 \text{Res}_{k=i\alpha} S(k),
\]  

(6)
where $\lambda \equiv \hbar/mc$ is the reduced Compton wavelength of the system. Calculating the residue on the basis of (6) and (4), we express the vertex constant in terms of the parameters $\alpha$ and $\lambda$ as

$$G^2 = -2\pi\lambda^2 \frac{\alpha + \lambda}{\alpha - \lambda}.$$  \hfill (7)

Using (4) and considering that the $S$ matrix is expressed in terms of the phase shift $\delta(k)$ as

$$S(k) = e^{2i\delta(k)} = \frac{\cot \delta + i}{\cot \delta - i},$$  \hfill (8)

we represent the function $k \cot \delta$ in the form

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2}r_e k^2.$$  \hfill (9)

This is the expression of the effective-range approximation, with the scattering length $a$ and the effective range $r_e$ being given by

$$a = \frac{1}{\alpha} + \frac{1}{\lambda},$$  \hfill (10)

$$r_e = \frac{2}{\alpha + \lambda}.$$  \hfill (11)

Thus, the effective-range approximation (9) immediately follows from the Bargmann representation (4) of the $S$ matrix for a system having one state. Formulas (10) and (11) relate the parameters of the effective-range approximation to the $S$-matrix parameters and, together with (5) and (7), the low-energy scattering parameters to the binding energy $E_0$ and the nuclear vertex constant $G^2$ (parameters of the bound state of the system).

As a specific example, we will now consider neutron-proton scattering in the triplet spin state. In this case, the system features one bound state, the deuteron, with the binding energy being $\varepsilon_d = 2.225 \text{MeV}$. The nuclear vertex constant $G^2_d$ for the deuteron corresponds to the $d \to n + p$ vertex and takes the value of $G^2_d = 0.43 \text{fm}$ [9]. It should be recalled that the nuclear vertex constants are directly related to the asymptotic normalization factors for the bound-state wave functions. The latter factors are often introduced in the analysis along with the nuclear vertex constants. In the case under consideration, the constant $G^2_d$ is expressed in terms of the dimensionless asymptotic normalization factor $C_d$ for the deuteron wave function as

$$G^2_d = 2\pi\lambda^2 \alpha C^2_d,$$  \hfill (12)
where $\lambda = 2\lambda_N$, with $\lambda_N = \hbar/m_Nc$ being the nucleon Compton wavelength ($m_N$ is the nucleon mass). The relevant numerical value is $C_d^2 = 1.673$. With the aid of (7) and (12), we can easily express the $S$-matrix parameter $\lambda$ in terms of the deuteron normalization factor as

$$\lambda = a \frac{C_d^2 + 1}{C_d^2 - 1}.$$  

(13)

Substituting (13) into (10) and (11), we obtain

$$a = 2 \alpha \frac{C_d^2}{C_d^2 + 1} ,$$  

(14)

$$r_e = \frac{1}{\alpha} \left( 1 - \frac{1}{C_d^2} \right) .$$  

(15)

Formulas (14) and (15) yield explicit expressions for the low-energy $np$ scattering parameters in terms of the bound-state parameters, the deuteron binding energy $\varepsilon_d$ and the deuteron nuclear vertex constant $G_d^2$. For the scattering length and the effective range, the substitution of the above experimental values of $\varepsilon_d$ and $G_d^2$ into expressions (14) and (15) yields

$$a = 5.41 \, fm ,$$  

(16)

$$r_e = 1.74 \, fm .$$  

(17)

These values are very close to the experimental values [11]

$$a^{\text{expt}} = 5.42 \, fm ,$$  

(18)

$$r_e^{\text{expt}} = 1.76 \, fm .$$  

(19)

This corresponds to the well-known fact that low-energy neutron-proton scattering can be very well interpreted by using the effective-range approximation (9). Thus, low-energy neutron-proton scattering in the triplet spin state can be accurately described on the basis of data on the deuteron bound state.
Let us now consider elastic neutron-deuteron scattering in the doublet spin state at energies below the threshold for deuteron breakup. In this case, there are two states in the system, the triton ground state and its virtual state. The existence of the latter has been firmly established \[12\]. For the case where the system has two states, we take the Jost function in the form of the rational function

\[
f(k) = \frac{k - i\alpha}{k + i\lambda} \frac{k - i\beta}{k + i\mu},
\]

which has the correct asymptotic behavior given by (2). The parameters \(\alpha, \lambda,\) and \(\mu,\) which appear on the right-hand side of (20), are positive, while the parameter \(\beta,\) which corresponds to the virtual state, is negative. Substituting (20) into (1), we obtain the \(S\) matrix for the system having two states in the form

\[
S(k) = \frac{k + i\alpha}{k - i\alpha} \frac{k + i\beta}{k - i\beta} \frac{k + i\lambda}{k - i\lambda} \frac{k + i\mu}{k - i\mu}.
\]

The energies of the bound and the virtual state are given by

\[
E_0 = -\frac{\hbar^2\alpha^2}{2m},
\]

\[
E_v = -\frac{\hbar^2\beta^2}{2m},
\]

where the reduced mass \(m\) is \((2/3)m_N\). The first and the second pole factor in expression (21) for the \(S\) matrix correspond to, respectively, the bound and the virtual triton state. The third and the fourth factor involve redundant poles of the \(S\) matrix. The nuclear vertex constants \(G^2_T\) and \(G^2_v\) for, respectively, the bound and the virtual state are directly expressed in terms of the residues of the \(S\) matrix at the respective poles \(k = i\alpha\) and \(k = i\beta\) as

\[
G^2_T = i\pi\alpha^2 \operatorname{Res}_{k=i\alpha} S(k),
\]

\[
G^2_v = i\pi\beta^2 \operatorname{Res}_{k=i\beta} S(k),
\]
where $\lambda = \frac{3}{2} \lambda_N$. Calculating the residues and using (24), (25), and (21), we find that the vertex constants are expressed in terms of the $S$-matrix parameters as

$$G_T^2 = -2\pi^2 \alpha \frac{\alpha + \beta + \lambda}{\alpha - \beta} \frac{\alpha + \mu}{\alpha - \lambda},$$

$$G_v^2 = 2\pi^2 \beta \frac{\alpha + \beta + \lambda}{\alpha - \beta} \frac{\beta + \mu}{\beta - \lambda}.$$

(26)

(27)

For a further analysis, it is worthwhile to introduce the following combinations of the $S$-matrix parameters:

$$p \equiv \alpha + \beta,$$

$$q \equiv \alpha \beta,$$

$$u \equiv \lambda + \mu,$$

$$v \equiv \lambda \mu.$$

(28)

(29)

(30)

(31)

Multiplying the corresponding factors in the numerator and the denominator of (21), we can represent the $S$ matrix in the form

$$S(k) = \frac{k^4 - (q + v + pu) k^2 + qv + \frac{i}{k} [(p + u) k^2 - pv - qu]}{k^4 - (q + v + pu) k^2 + qv - \frac{i}{k} [(p + u) k^2 - pv - qu]}.$$

(32)

Comparing (32) with the representation in (8), we recast the expression for $k \cot \delta$ into the form

$$k \cot \delta = \frac{qv - (q + v + pu) k^2 + k^4}{-pv - qu + (p + u) k^2}.$$

(33)

Upon dividing the polynomial in the numerator by the polynomial in the denominator, we arrive at

$$k \cot \delta = -A + Bk^2 - \frac{C}{1 + Dk^2}.$$

(34)

This expression is nothing but the well-known empirical van Oers-Seagrave formula [4], which describes well low-energy neutron-deuteron scattering in the doublet spin state. The parameters in expansion (34) were obtained in [4] by fitting low-energy experimental data. Comparing (34) and (33), we express the van Oers-Seagrave parameters in terms of the $S$-matrix parameters (28)–(31) as

$$A = \frac{pq + uv + pu(p + u)}{(p + u)^2},$$

$$B = \frac{1}{p + u},$$

(35)

(36)
Thus, the van Oers-Seagrave formula (34) immediately follows from the Bargmann representation (21) of the $S$ matrix for the system having two states. The same two states in the system are responsible for the pole in the expression for the function $k \cot \delta$. We note that, in the $S$ matrix, it is necessary to take into account, along with two physical poles corresponding to the bound and the virtual state, two redundant poles, which ensure the correct asymptotic behavior of the Jost function. For the case of potentials leading to a finite number of poles in the $S$ matrix, it follows from the Levinson theorem that the number of redundant poles is determined by the total number of bound, virtual, and quasistationary states. If we use an interaction leading to two states in the system, a ground and a virtual one, the $S$ matrix will therefore have two redundant poles.

Since the neutron-proton system considered above has a single state, a bound state in the triplet channel or a virtual one in the singlet channel, the $S$ matrix for a system having one state and the corresponding effective-range approximation for the function $k \cot \delta$ provide a good approximation for the $np$ interaction. The limiting transition from the van Oers-Seagrave formula to the effective-range approximation can easily be obtained by recasting formula (33) into the form

$$k \cot \delta = \frac{-1/a + c_2 k^2 + c_4 k^4}{1 + D k^2}.$$  

(39)

The two-state $S$ matrix (21) reduces to the one-state $S$ matrix (4) if the second state goes to infinity; that is, $\beta \to \infty$ and $\mu \to \infty$. It can easily be seen that, in this case, the coefficients $c_4$ and $D$ in (39) vanish, so that expression (39) reduces to the effective-range approximation (9). Thus, we can see that the van Oers-Seagrave formula is a direct generalization of the effective-range approximation to the case of a system having two states.

The nuclear vertex constants $G_T^2$ and $G_v^2$ for, respectively, the bound and the virtual triton state are expressed in terms of the corresponding dimensionless asymptotic constants $C_T^2$ and $C_v^2$ as

$$G_T^2 = 3\pi \chi^2 \alpha C_T^2,$$  

(40)

$$G_v^2 = 3\pi \chi^2 \beta C_v^2.$$  

(41)
Using (26), (27), (40), and (41) and taking into account (30) and (31), we obtain

\[ u = - (\alpha + \beta) \frac{4 (\alpha + \beta)^2 + 6 (\alpha^2 - \beta^2) (C_T^2 - C_v^2) - 9 (\alpha - \beta)^2 C_T^2 C_v^2}{4 (\alpha + \beta)^2 - 6 (\alpha + \beta) (C_T^2 + C_v^2) + 9 (\alpha - \beta)^2 C_T^2 C_v^2}, \]

(42)

\[ v = \alpha \beta \frac{4 (\alpha + \beta)^2 + 6 (\alpha + \beta)^2 (C_T^2 + C_v^2) + 9 (\alpha - \beta)^2 C_T^2 C_v^2}{4 (\alpha + \beta)^2 - 6 (\alpha + \beta)^2 (C_T^2 + C_v^2) + 9 (\alpha - \beta)^2 C_T^2 C_v^2}. \]

(43)

Formulas (35)–(38), together with (28), (29), (42), and (43), provide explicit expressions for the van Oers-Seagrave parameters in terms of the parameters of the bound and the virtual triton state (their energies and nuclear vertex constants). Since these general expressions are very cumbersome, we consider here only the expressions for the scattering length and for the pole of the function \( k \cot \delta \).

4. EXPRESSIONS FOR THE nd SCATTERING LENGTH AND FOR THE POLE OF THE FUNCTION \( k \cot \delta \) IN TERMS OF THE PARAMETERS OF THE BOUND AND THE VIRTUAL TRITON STATE

As can be seen from (34), the doublet nd scattering length \( a \equiv a_{nd} \) is expressed in terms of the van Oers-Seagrave parameters as

\[ a = \frac{1}{A + C}, \]

(44)

or in terms of the S-matrix parameters as

\[ a = \frac{p}{q} + \frac{u}{v} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\lambda} + \frac{1}{\mu}. \]

(45)

Substituting (28), (29), (42), and (43) into (45), we obtain

\[ a = 6 \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) - 2 (\alpha + \beta) (\beta C_T^2 + \alpha C_v^2) + 3 (\alpha - \beta)^2 C_T^2 C_v^2 \]

\[ \frac{2 (\alpha + \beta)^2 [2 + 3 (C_T^2 + C_v^2)] + 9 (\alpha - \beta)^2 C_T^2 C_v^2}{2 (\alpha + \beta)^2 [2 + 3 (C_T^2 + C_v^2)] + 9 (\alpha - \beta)^2 C_T^2 C_v^2}. \]

(46)

This formula expresses the doublet nd scattering length in terms of the parameters of the bound and the virtual triton state. From (38), we similarly find that the pole \( k_0^2 \equiv 1/D \) of the function \( k \cot \delta \) is given by

\[ k_0^2 = \alpha \beta \frac{2 (\alpha + \beta) (\beta C_T^2 + \alpha C_v^2) + 3 (\alpha - \beta)^2 C_T^2 C_v^2}{2 (\alpha + \beta) (\alpha C_T^2 + \beta C_v^2) - 3 (\alpha - \beta)^2 C_T^2 C_v^2}. \]

(47)

We note that the effective range for nd scattering can be expressed in terms of the van Oers-Seagrave parameters as

\[ r_e = 2 (B + C D). \]

(48)
Along with the effective range, which is anomalously large, \( r_e \sim 500 \text{ fm} \), for doublet \( nd \) scattering, the so-called amplitude slope parameter [6], given by the dimensionless quantity \( \frac{1}{2} \alpha_d^3 a^2 r_e \) with wave number \( \alpha_d = 0.2316 \text{ fm}^{-1} \) corresponding to the deuteron, is often used in the literature.

5. NUMERICAL CALCULATION OF THE LOW-ENERGY \( nd \) SCATTERING PARAMETERS AND THE CONSTANT \( C_v^2 \)

With the parameters specifying the bound and the virtual triton state, we have calculated numerically the low-energy \( nd \) scattering parameters \( a, \varepsilon_0 = -\frac{\hbar^2 k_0^2}{2m} \), and \( \frac{1}{2} \alpha_d^3 a^2 r_e \) by formulas \((46)-(48)\). The calculation has revealed that these parameters of \( nd \) scattering are weakly sensitive to variations in the energies \( E_0 \) and \( E_v \) of the ground and the virtual state, respectively, and to the asymptotic normalization factor \( C_T^2 \) for the bound state. At the same time, these parameters greatly depend on the asymptotic normalization factor \( C_v^2 \) for the virtual state. The results of the calculations for the low-energy \( nd \) scattering parameters are quoted in the table for various values of the constant \( C_v^2 \). In the calculations, the parameters of the triton were set to the values of \( E_T = 8.48 \text{ MeV} \) [13] \((E_T = |E_0| + \varepsilon_d)\), \( B_v = |E_v| = 0.482 \text{ MeV} \) [12], and \( C_T^2 = 3.5 \) \([1, 2, 6]\), which follow from a direct analysis of experimental data.

From the results quoted in the table, it can be seen that the parameters of \( nd \) scattering are highly sensitive to variations in the constant \( C_v^2 \). The experimental value of \( C_v^2 = 0.0504 \) [12] adopted at present leads to the \( nd \) scattering length \( a = 1.03 \text{ fm} \), which differs considerably from its experimental value now established to a high precision. The experimental value of the \( nd \) scattering length [14],

\[
a^{\text{expt}} = 0.65 \text{ fm}
\]

(49)
can be fitted with the asymptotic-constant value

\[
C_v^2 = 0.0592,
\]

(50)
which agrees well with the value of \( C_v^2 = 0.06 \) calculated in the two-body model of \( nd \) interaction simulated by the Hulthén potential [2]. Thus, the numerical value in (50) for the asymptotic normalization factor \( C_v^2 \) for the virtual state must be treated as the result of the theoretical calculation on the basis of our model. The high sensitivity of the scattering parameters to this quantity and a deviation of the theoretical results from its available experimental
value indicate that a refinement of the experimental value of $C^2_v$ is necessary. The above highlights the crucial importance of the factor $C^2_v$ for exploring the properties of $nd$ scattering in the doublet spin state. The experimental value in (49) for the $nd$ scattering length and, accordingly, the value in (50) for $C^2_v$ lead to the following values for the pole of the function $k \cot \delta$ and the amplitude slope parameter:

$$\varepsilon_0 = -0.1313 \text{ MeV}, \quad (51)$$

$$\frac{1}{2} \alpha_d^3 a^2 r_e = 1.52. \quad (52)$$

The value calculated for the pole position $\varepsilon_0$ and presented in (51) agrees well with $\varepsilon_0 = -0.15 \text{ MeV}$ quoted in [15] as an experimental value. The slope parameter (52) complies fairly well with the value of $\frac{1}{2} \alpha_d^3 a^2 r_e = 1.35$ [6] calculated theoretically from the Faddeev equations with separable nucleon-nucleon potentials.

For the van Oers-Seagrave parameters, a numerical calculation with $C^2_v$ from (50) yields

$$A = 0.3198 \text{ fm}^{-1}, \quad (53)$$

$$B = 0.7698 \text{ fm}, \quad (54)$$

$$C = 1.2190 \text{ fm}^{-1}, \quad (55)$$

$$D = 236.927 \text{ fm}^2. \quad (56)$$

The function $k \cot \delta$ calculated in the van Oers-Seagrave approximation with the parameter values (53)–(56) is shown in the figure versus the energy $k^2$. It can be seen that the theoretical curve faithfully reproduces experimental data from [4]. In summary, we have obtained a complete description of low-energy doublet $nd$ scattering in terms of the parameters of the bound and the virtual triton state and demonstrated that this description is consistent with experimental data.

6. CONCLUSION

It has been shown that the van Oers-Seagrave formula for doublet $nd$ scattering — it was originally deduced from a purely empirical consideration — immediately follows from the Bargmann $S$-matrix representation corresponding to the presence of two triton states, a
bound and a virtual one, in the system. That the pole term proves to be necessary in the function \( k\cot \delta \) is also due to the presence of two states in the system. Our analysis has given simple expressions relating the van Oers-Seagrave parameters to the \( S \)-matrix parameters. It has been shown that the van Oers-Seagrave formula is a direct generalization of the effective-range approximation to the case where the system being considered has two states; when one of these states goes to infinity, the former reduces to the latter.

For the doublet \( nd \) scattering length and the pole of the function \( k\cot \delta \), we have obtained simple explicit expressions in terms of the energies of the bound and the virtual triton state and the nuclear vertex constants for these states. The resulting formulas make it possible to perform numerical calculations and to analyze the parameters of low-energy \( nd \) scattering versus the parameters of the triton. In particular, the calculations have revealed that the parameters of \( nd \) scattering are highly sensitive to variations in the asymptotic normalization factor \( C_v^2 \) for the virtual triton state. The experimental value of the \( nd \) scattering length is fitted at the value of \( C_v^2 = 0.0592 \), which differs from the experimental value presently accepted for this constant. Therefore, the constant \( C_v^2 \) is of paramount importance for studying the properties of the \( nd \) system and needs further experimental refinement.

REFERENCES

1. N. M. Petrov, Yad. Fiz. 48, 50 (1988) [Sov. J. Nucl. Phys. 48, 31(1988)].

2. Yu. V. Orlov, N. M. Petrov, and G. N. Teneva, Yad. Fiz. 55, 38 (1992) [Sov. J. Nucl. Phys. 55, 23 (1992)].

3. L. M. Delves, Phys. Rev. 118, 1318 (1960).

4. W. T. H. van Oers and J. D. Seagrave, Phys. Lett. 24B, 562 (1967).

5. J. S. Whiting and M. G. Fuda, Phys. Rev. C 14, 18 (1976).

6. I. V. Simenog, A. I. Sitnichenko, and D. V. Shapoval, Yad. Fiz. 45, 60 (1987) [Sov. J. Nucl. Phys. 45, 37 (1987)].

7. V. Bargmann, Rev. Mod. Phys. 21, 488 (1949).
8. R. G. Newton, *Scattering Theory of Waves and Particles*, 2nd ed. (Springer-Verlag, New York, 1982).

9. L. D. Blokhintsev, I. Borbeli, and E. I. Dolinskiï, Fiz. Elem. Chastits At. Yadra **8**, 1189 (1977) [Sov. J. Part. Nucl. **8**, 485 (1977)].

10. A. G. Sitenko, *Scattering Theory* (Springer-Verlag, Berlin, 1991).

11. O. Dumbrajs, R. Koch, H. Pilkuhn, *et al.*, Nucl. Phys. **B 216**, 277 (1983).

12. B. A. Girard and M. G. Fuda, Phys. Rev. **C 19**, 579 (1979).

13. D. R. Tilley, H. R. Weller, and H. H. Hasan, Nucl. Phys. **A 474**, 1 (1987).

14. W. Dilg, L. Koester, and W. Nistler, Phys. Lett. **36B**, 208 (1971).

15. Yu. V. Orlov, Yu. P. Orevkov, and L. I. Nikitina, Izv. Akad. Nauk, Ser. Fiz. **60** (11), 152 (1996).
Low-energy \( nd \) scattering parameters versus \( C_v^2 \)

| \( C_v^2 \) |  \( a \text{, } fm \) | \( \varepsilon_0 \text{, } MeV \) | \( \frac{1}{2}(\alpha)^3a^2r_v \) |
|--------|--------|--------|---------|
| 0.01   | 3.12   | −0.43  | 0.51    |
| 0.02   | 2.54   | −0.38  | 0.80    |
| 0.03   | 2.01   | −0.32  | 1.04    |
| 0.04   | 1.51   | −0.26  | 1.24    |
| 0.05   | 1.05   | −0.20  | 1.40    |
| 0.06   | 0.62   | −0.13  | 1.53    |
| 0.07   | 0.21   | −0.05  | 1.63    |
| 0.08   | −0.17  | 0.04   | 1.72    |
| 0.09   | −0.52  | 0.13   | 1.79    |
Function $k \cot \delta$ calculated for doublet $nd$ scattering in the van Oers-Seagrave approximation with the parameter values (53)-(56) versus the energy $k^2$. The experimental data were taken from [4].
This figure "figure.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/0402055v1