By insisting that the formal apparatus of quantum mechanics is a probability calculus, QBism opens the door to a deeper understanding of what quantum mechanics is trying to tell us. By insisting on a subjectivist Bayesian interpretation of probability in the context of quantum foundations, it closes this door again. To find the proper balance between subject and object, one must turn to Niels Bohr, the alleged “obscurity” of whose views casts a poor light on the current state of foundational research.

*Keywords*: QBism; Bohr; Kant; probability; nonlocality; manifestation.

### 1. Introduction

In a recent comment in *Nature*\textsuperscript{1} titled “QBism puts the scientist back into science,” David Mermin reports that something remarkable has happened at last: “it was only in the twenty-first century that US physicist Christopher Fuchs and British–German physicist Rüdiger Schack\textsuperscript{2,3,4} put forth an understanding of quantum mechanics that restored the balance between subject and object.” The new understanding of the theory is called “QBism,” where Q is for quantum and B is for the Bayesian theory of probability.

What Mermin seems to have momentarily forgotten is that therapies designed to cure quantum mechanics of its measurement problem by implicating the subject are almost as old as the theory itself. In 1932, none other than John von Neumann, using the simple example of a temperature measurement, argued that

> no matter how far we calculate—to the mercury vessel, to the scale of the thermometer, to the retina, or into the brain, at some time we must say: and this is perceived by the observer. That is, we must always divide the world into two parts, the one being the observed system, the other the observer.\textsuperscript{5}

What is also not new about QBism, nor should it be particularly noteworthy, is the view that the formal apparatus of quantum mechanics is a probability calculus. To make this as obvious as it ought to be, I formulate the theory’s standard axioms in Sect. \textsuperscript{2} and I show, in Sect. \textsuperscript{6} how these axioms can be derived (and thereby demystified) by *assuming* that the theory’s formal apparatus is nothing more (nor
What is new about QBism is its uncompromising personalist Bayesian stance. The central affirmations of QBism are listed in Sect. 4. That probability assignments can differ from person to person may be anathema to the objectivist, but where quantum mechanics is concerned, this follows from the fact that probabilities are assigned on the basis of measurement outcomes and thus depend on the outcomes constituting the assignment basis. As the assignment basis can differ from person to person, so can the probabilities assigned. The dependence of quantum-mechanical probabilities on data sets, however, does not by itself warrant a subjectivist stance. How far, then, and in what sense, is a subjectivist stance warranted? This question is discussed in the sections that follow, beginning with Sect. 5 in which the reader is reminded of the views of Niels Bohr, whose alleged “obscurity” casts a poor light on the current state of foundational research.

No one, to my mind, has elucidated our epistemological situation more clearly than Kant in the 18th Century and Bohr again in the 20th. Kant taught us to distinguish between two concepts of reality, a transcendental reality and an empirical one, and Bohr stressed the need for this distinction when he reminded us that “in our description of nature the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience” (pp. 17–18). In Sect. 6 the troubled state of quantum foundational research is traced to an endemic neglect of this crucial distinction, of which QBists are as guilty as realists about quantum states.

In his Nature comment, Mermin also expresses the hope that the QBist “conversation” can be broadened to provide a solution to “the problem of the Now.” How, he asks, can there be no place in physics for something as obvious as the experiential Now? The answer, as discussed in Sect. 7, ought to be obvious to anyone sufficiently acquainted with Bohr. Physics is not concerned with experiences qua experiences. Physics is concerned with the objectifiable aspects of our experience. Temporal relations are objectifiable. The experiential Now is not. There remains the question why a QBist should be worried about this. What does it matter if some aspects of our experience are not objectifiable? Are they therefore unreal? Only a greedy reductionist or a hard-core realist would think so.

Sections 8 and 9 examine how QBism meets the challenges posed by Wigner’s friend and by EPR-entangled states, Sect. 10 is a comment on the re-evaluation by Fuchs and Schack of Feynman’s claim that the double-slit experiment has in it the heart of quantum mechanics, and Sect. 11 examines the claim by Fuchs et al. that QBism eliminates the measurement problem.

As Mermin observed, today “the quantum measurement problem” has almost as many meanings as “the Copenhagen interpretation” (p. 156). While some versions of the problem can be solved without going to QBist extremes, others remain unaddressed by QBism, in particular the challenge to demonstrate the consistency of the quantum-mechanical probability calculus with the existence of the objective events it serves to correlate. Section 12 recapitulates a recent demonstration of this
consistency.\textsuperscript{10,11}

The QBist agenda was set out in a paper by Fuchs\textsuperscript{12} to find information theoretic reasons for the axioms of quantum mechanics, with a view to isolating that piece of the theory which lacks information theoretic significance—that piece which has “no need of an agent for its definition.” This, he reasoned, would be “our first unadorned glimpse of ‘quantum reality’.” What else can that piece be than the instruction to calculate the probabilities of events that can come about in alternative ways by adding the amplitudes associated with the alternatives, rather than their probabilities? The unadorned glimpse of quantum reality this instruction affords us is the subject of Sect.\textsuperscript{13} It provides a new way of understanding the necessary distinction between the macroscopic or classical domain, which contains the events the theory serves to correlate, and the quantum or non-classical domain, which “contains” the correlations between these events, as is explained in Sect.\textsuperscript{14}.

This new way of understanding Bohr’s distinction between those two domains makes it possible to conceive of a new kind of causality, in addition to that which links events across time and space, and which has meaningful application only within the macroscopic or classical domain. This new kind of causality, in turn, casts new light on the mystery of quantum nonlocality—the subject of Sect.\textsuperscript{15}—which Fuchs \textit{et al.}\textsuperscript{8} sweep under the rug by proclaiming that quantum correlations “refer only to time-like separated events” and “cannot assign correlations, spooky or otherwise, to space-like separated events.”

Section\textsuperscript{16} reflects on the meaning of probability, in general and in the particular context of quantum mechanics, and Sect.\textsuperscript{17} reviews the senses in which it is legitimate to consider probabilities objective, despite all QBist protestations to the contrary. Section\textsuperscript{18} finally, assesses the respective truth, falsity, or irrelevance of the QBist affirmations listed in Sect.\textsuperscript{4}.

2. The Standard Axioms

The first axiom typically tells us that the state of a system \(S\) is (or is represented by) a normalized element \(|v\rangle\) of a Hilbert space \(\mathcal{H}_S\).

The next axiom usually states that observables are (or are represented by) self-adjoint linear operators on \(\mathcal{H}_S\), and that the possible outcomes of a measurement of an observable \(\hat{O}\) are the eigenvalues of \(\hat{O}\).

Then comes an axiom concerning the time evolution of states. Between measurements, states evolve according to unitary transformations; at the time of a measurement they evolve as stipulated by the projection postulate. That is, if \(\hat{O}\) is measured, the subsequent state of \(S\) is the eigenvector \(|w\rangle\) corresponding to the outcome \(w\), regardless of the previous state of \(S\).

A further axiom stipulates that the states of composite systems are (or are represented by) vectors in the tensor product of the Hilbert spaces of the component systems.

Finally there is an axiom concerning probabilities. If \(S\) is in the state \(|v\rangle\) and we
do an experiment to see if $\hat{O}$ has the value $|w\rangle$, then the probability $p$ of a positive outcome is given by the Born Rule $p = |\langle w|v\rangle|^2$. Furthermore, the expectation value of $\hat{O}$ in the state $|v\rangle$ is $\langle v|\hat{O}|v\rangle$.

There is much here that is perplexing if not downright wrong. What is meant by saying that the state of a system is (represented by) a normalized vector in a Hilbert space? What does it mean to say that observables are (represented by) self-adjoint operators? And why is it that the states of composite systems are (represented by) vectors in the tensor product of the Hilbert spaces of the component systems? Axioms are supposed to be clear and compelling. The standard axioms of quantum theory lack both desiderata.

3. The Standard Axioms Demystified

*If quantum theory is so closely allied with probability theory, why is it not written in a language that starts with probability, rather than a language that ends with it? Why does quantum theory invoke the mathematical apparatus of complex amplitudes, Hilbert spaces, and linear operators?*

—Christopher A. Fuchs

The root cause of the perplexity induced by the standard axioms is that probabilities are introduced almost as an afterthought. The obvious way to demystify these axioms is to acknowledge that the irreducible empirical core of quantum theory is a probability calculus, and that the events to which it assigns probabilities are measurement outcomes (as are the data on the basis of which it assigns probabilities).

The irreducible empirical core of classical mechanics, too, is a probability calculus. However, because the classical probability calculus is trivial, in the sense that it only assigns the trivial probabilities 0 or 1, it allows us to think of a state in the classical sense of the word: a collection of possessed properties. We can represent the state of a classical system with $n$ degrees of freedom by a point $P$ in a $2^n$-dimensional phase space $S$, and we can represent the positive outcome of an elementary test by a subset $U$ of $S$. The probability of obtaining $U$ is 1 if $P \in U$ and 0 if $P \notin U$. Because in this calculus there are no probabilities that are both greater than 0 and less than 1, we are free to believe that the probability of finding the property represented by $U$ is 1 because the system has this property.

The irreducible empirical core of quantum theory is a nontrivial probability calculus. Arguably the most straightforward way to make room for nontrivial probabilities is to upgrade from a 0-dimensional point to a 1-dimensional line. The virtual inevitability of this “upgrade” was demonstrated by Jauch. Instead of representing the probability algorithm associated with a system by a point in a phase space, one represents it by a 1-dimensional subspace of a linear vector space $V$ equipped with a Hermitian inner product, and instead of representing measurement outcomes by subsets of a phase space, one represents them by (closed) subspaces of $V$. A 1-dimensional subspace $L$ can be contained in a subspace $U$, it can be
orthogonal to \( \mathcal{U} \), but now there is a third possibility, and this makes room for non-trivial probabilities. \( \mathcal{L} \) assigns probability 1 to outcomes represented by subspaces containing \( \mathcal{L} \), it assigns probability 0 to outcomes represented by subspaces orthogonal to \( \mathcal{L} \), and it assigns probabilities greater than 0 and less than 1 to measurement outcomes represented by subspaces that neither contain nor are orthogonal to \( \mathcal{L} \).

The one-to-one correspondence between subspaces and projectors allows us to formulate the following axiom:

**Axiom 1.** Measurement outcomes are represented by the projectors of a vector space.

The reason why the irreducible empirical core of quantum theory is a nontrivial probability calculus is the existence of mutually incompatible observables. The following relation between measurements of compatible observables is also readily established (see Sect. 6 of Ref. [14] and Sect. 8.5 of Ref. [16]):

**Axiom 2.** The outcomes of compatible elementary tests correspond to commuting projectors.

We now consider two measurements. The first, \( M_1 \), has three possible outcomes, represented by the subspaces \( A, B, \) and \( C \). The second, \( M_2 \), has two, represented by \( A \cup B \) and \( C \). Because the probabilities of all possible outcomes must add up to 1, we have

\[
p(A) + p(B) + p(C) = 1 \quad \text{as well as} \quad p(A \cup B) + p(C) = 1.
\]

If we require non-contextuality by demanding that the probability of an elementary test be independent of whichever other tests are simultaneously made, we can deduce from these equations that

\[
p(A \cup B) = p(A) + p(B),
\]

and this allows us to formulate the following axiom:

**Axiom 3.** If \( \hat{A} \) and \( \hat{B} \) are orthogonal projectors, then the probability of the outcome represented by \( \hat{A} + \hat{B} \) is the sum of the probabilities of the outcomes represented by \( \hat{A} \) and \( \hat{B} \), respectively.

What we are looking for is a probability calculus capable of accommodating nontrivial probabilities and incompatible tests. If this can be found while preserving common sense in the form of non-contextuality, we go for it. By hindsight we know that Nature concurs, but we also know that contextuality is an inescapable feature in situations where probabilities are assigned either on the basis of past and future outcomes [17] or to outcomes of measurements performed on entangled systems (Ref. [18] and Chap. 7 of Ref. [20]).

Armed with Axioms 1–3, one can prove Gleason’s theorem [19], which holds for vector spaces with at least three dimensions (Ref. [20], p. 190). The theorem states

\[\text{[14][22]}\]
that the probability of obtaining the outcome represented by $\hat{P}$ is given by the trace rule

$$p(\hat{P}) = \text{Tr}(\hat{W}\hat{P}),$$

where $\hat{W}$ is a unique operator with the following properties: it is linear, self-adjoint (so that $p(\hat{P})$ comes out real), and positive (so that $p(\hat{P})$ comes out non-negative); its trace is 1 (so that the probabilities of the possible outcomes of a measurement add up to 1), and it satisfies $\hat{W}^2 \leq \hat{W}$. If the density operator $\hat{W}$ is a pure state ($\hat{W}^2 = \hat{W}$), it is a 1-dimensional projector $|w\rangle\langle w|$, $|w\rangle$ is the corresponding state vector, and the trace rule takes the form $p(\hat{P}) = \langle w|\hat{P}|w\rangle$. If in addition $\hat{P} = |v\rangle\langle v|$, the trace rule boils down to the familiar Born rule $p(\hat{P}) = |\langle v|w\rangle|^2$.

Given that the quantum-mechanical probability calculus assigns probabilities not only to measurement outcomes but also on the basis of measurement outcomes, we must now determine how the density operator depends on measurement outcomes. Suppose that $\hat{W}_1$ encapsulates a set of relevant outcomes obtained before the time $t_0$, that at $t_1$ a measurement is made, and that its outcome is $\hat{P}$. What density operator $\hat{W}_2$ should be used for assigning probabilities to the possible outcomes of whatever measurement is made next, at a time $t_2 > t_1$?

Let us call a measurement repeatable if, when performed twice in succession, the second measurement invariably yields the same outcome as the first. If (for the moment) we confine ourselves to repeatable measurements, $\hat{W}_2$ must satisfy the following conditions: (1) It is constructed out of $\hat{W}_1$ and $\hat{P}$. (2) It is self-adjoint. And it must satisfy (3) $\text{Tr}(\hat{W}_2\hat{P}) = 1$ and (4) $\text{Tr}(\hat{W}_2\hat{P}_\perp) = 0$, where $\hat{P}_\perp$ is any possible outcome of $M$ other than $\hat{P}$.

Conditions (1) and (4) are satisfied by $\hat{W}_1\hat{P}$, $\hat{P}\hat{W}_1$, and $\hat{P}\hat{W}_1\hat{P}$, but since $\hat{W}_1\hat{P}$ and $\hat{P}\hat{W}_1$ are not self-adjoint unless $\hat{W}_1$ and $\hat{P}$ commute, only $\hat{P}\hat{W}_1\hat{P}$ is certain to satisfy condition (2). To satisfy the third condition as well, all we have to do is divide by $\text{Tr}(\hat{W}_1\hat{P}) = \text{Tr}(\hat{P}\hat{W}_1) = \text{Tr}(\hat{P}\hat{W}_1\hat{P})$. The result is

$$\hat{W}_2 = \frac{\hat{P}\hat{W}_1\hat{P}}{\text{Tr}(\hat{W}_1\hat{P})}.$$ (4)

This is the projection postulate in its general form. If $\hat{P} = |w\rangle\langle w|$, then

$$\hat{W}_2 = \frac{|w\rangle\langle w|\hat{W}_1|w\rangle\langle w|}{\text{Tr}(\hat{W}_1|w\rangle\langle w|)} = |w\rangle\langle w|\hat{W}_1|w\rangle\langle w| = |w\rangle\langle w|.$$ (5)

What is noteworthy here is that the operator $|w\rangle\langle w|$ serves two purposes: for the purpose of updating the density operator, it represents the outcome obtained at $t_1$, and for the purpose of assigning probabilities to the possible outcomes of whatever measurement is made next, it represents the updated density operator.

Finally we need to find out how probabilities depend on the time of the measurement to the possible outcomes of which they are assigned. To this end we shall consider measurements that are verifiable. A measurement $M_1$, performed at a time $t_1$, is verifiable if it is possible to confirm its outcome by a measurement $M_2$.
performed at a time \( t_2 > t_1 \). If \( M_1 \) is repeatable, then it can be verified by simply repeating it. Otherwise the verification requires (i) the performance of a different measurement and (ii) the existence of a one-to-one correspondence, between the possible outcomes of \( M_1 \) and \( M_2 \), which makes it possible to infer the outcome of \( M_1 \) from the outcome of \( M_2 \). For complete measurements (a.k.a. maximal tests), there will be bases \( \{|a_i\}\) and \( \{|b_k\}\) such that the projectors \( \{|a_i\}\langle a_i|\) represent the possible outcomes of \( M_1 \) and the projectors \( \{|b_k\}\langle b_k|\) represent the possible outcomes of \( M_2 \), and the two bases will be related by a unitary transformation: \( |b_k\rangle = \hat{U}_i |a_k\rangle \).

So now we know why a (pure) state is represented by a normalized element \( |v\rangle \) of a vector space. The reason is that the projector \( |v\rangle \langle v| \) defined by \( |v\rangle \) represents the outcome \( v \) of a measurement in two kinds of calculations: when it comes to assigning a probability to \( v \) qua possible outcome, and again when it comes to updating the density operator if \( v \) is the actual outcome.

We also know now why states evolve according to unitary transformations between measurements, and why they “collapse” as stipulated by the projection postulate when a measurement is made. Except that they don’t, for the parameter \( t \) on which a state \( \hat{W} \) functionally depends (in the Schrödinger picture) represents the time of the measurement to the possible outcomes of which \( \hat{W} \) assigns probabilities. A probability algorithm is not the kind of thing that exists at every instant of time, waiting for a measurement to be made.

It is now also readily seen why observables are represented by self-adjoint operators, and why the expectation value of an observable \( O \) represented by \( \hat{O} \) “in” the state \( |v\rangle \) is \( \langle v|\hat{O}|v\rangle \). If we take the projectors \( |O_k\rangle \langle O_k| \) representing the possible outcomes of a measurement of \( O \), multiply them with the corresponding values \( O_k \), and sum the results, we obtain the self-adjoint operator \( \hat{O} = \sum_k O_k |O_k\rangle \langle O_k| \). Using this operator, the Born rule allows us to cast the expectation value of \( O \) associated with the state \( |v\rangle \), \( \sum_k O_k p(O_k|v) \), into the form \( \langle v|\hat{O}|v\rangle \).

It also ceases to be a mystery why the states of composite systems are represented by vectors in the tensor product of the vector spaces associated with the component systems. Consider two independent, distinguishable systems associated with the states \( |a\rangle \) and \( |b\rangle \). The joint probability of finding the two systems in possession of the respective properties \( c \) and \( d \), represented by the projectors \( |c\rangle \langle c| \) and \( |d\rangle \langle d| \), is \( p(c, d|a, b) = p(c|a) p(d|b) = |\langle c|a\rangle \langle d|b\rangle|^2 \). The inner product on the direct product space, defined by \( \langle c, d|a, b\rangle = \langle c|a\rangle \langle d|b\rangle \), allows us to cast this into the form \( p(c, d|a, b) = |\langle c, d|a, b\rangle|^2 \).

Finally there are the questions why the irreducible empirical core of quantum mechanics is a nontrivial probability calculus, why there are mutually incompatible observables, and why \( \mathcal{V} \) needs to be a complex vector space. These feature of the theory are all implied by the existence of stable objects—objects that occupy finite volumes of space without exploding or imploding, and whose probability to be found somewhere does not decrease with the passage of time \(^{13}\) (see also Ref. \(^{16}\) Sect. 5.7 and Chap. 8). \( \mathcal{V} \) becomes a Hilbert space if in addition we require that it
be complete, so that the limit of every strongly convergent sequence \( |v_k \rangle \) of elements in \( V \) is contained in \( V \).

The possibility of demystifying the standard axioms as outlined in this section demonstrates beyond reasonable doubt that the formal apparatus of quantum theory is a probability calculus, and that it serves to assign probabilities to possible measurement outcomes on the basis of actual measurement outcomes. It also demonstrates the absurdity of \( \psi \)-ontology\(^{b}\) which seeks to construe quantum states objectively or realistically. The transmogrification of a tool for assigning probabilities to possible measurement outcomes on the basis of actual measurement outcomes into an evolving instantaneous state (in the classical sense of “state”) cannot but lead to inconsistencies and absurdities.

4. The Central Affirmations of QBism

QBism makes the following affirmations\(^{2,3,8}\):

1. A probability is intrinsically and necessarily a degree of confidence or belief, a subject’s estimate. Probability assignments are therefore egocentric, single-user.
2. Probability theory is a calculus of consistency, a set of criteria for testing coherence between beliefs, such as beliefs relevant to decision making (for example, when placing a bet).
3. There are no external criteria for declaring a probability judgment right or wrong. The only criterion for adequacy is internal coherence between beliefs.
4. While probability judgments are necessarily subjective, they are not intrinsically measures of ignorance. What is responsible for uncertainty is not necessarily lack of knowledge of some true state of affairs. The subjectivity of probability assignments is therefore consistent with an objective indeterminism. It may be that “the world itself does not yet know what it will give”\(^{23}\)
5. Quantum theory is an addition to Bayesian probability theory. It provides each user with a calculus for gambling on his/her own experiences. Like probability theory, it is normative. It offers rules of consistency a user should strive to satisfy within his/her overall mesh of beliefs.
6. The Born Rule—a rule to guide the agent’s behaviour whenever he/she interacts with the physical world—is a method of transforming or relating probabilities, of getting new ones from old ones, not of setting probabilities by some external entity called “the quantum state.”
7. Making a quantum state assignment is equivalent to assigning probabilities to the outcomes of a well-selected set of measurements. It is as subjective and user-dependent as making a probability judgment. There can be no such thing as a true (or real, or objective) quantum state. Potentially there are as many quantum states for a given system as there are agents.

\(^{b}\)Not to be confused with Scientology. (See note 3 of Ref.\(^{3}\))
(8) Quantum correlations refer only to time-like separated events: the acquisition of experiences by any single agent.

(9) Subjective certainty—the assignment of probability 1 to some event—does not warrant objectivity or factuality. Certainty about what a measurement will reveal is not certainty about anything if no measurement is made. EPR’s reality criterion\[23\] and the so-called eigenvalue–eigenstate link\[5\] must therefore be rejected.

(10) Measurements are generative in a very real sense; they are “moments of creation”\[3\] “A quantum measurement finds nothing, but very much makes something”\[7\]

(11) The existence of an external reality, undisclosed in experience, is not denied; only that this reality can be represented by a quantum state.

(12) A measurement is any action an agent takes to elicit a set of possible experiences. It is an action on the quantum world that results in the creation of an outcome—a new experience for the agent.

(13) The formal apparatus of quantum mechanics does not account for the data on the basis of which probabilities are assigned.

(14) All external systems, whether they be atoms or other agents, are to be treated quantum-mechanically. The only exception is the subject’s “own direct internal awareness of her own private experience”\[8\] Thus Alice can assign to Bob a quantum state, which encodes her probabilities for the possible answers to any question she puts to him. Bob’s answer is created for Alice only when it enters her experience.

5. Bohr (and therefore Kant)

In my opinion, those who really want to understand contemporary physics—i.e., not only to apply physics in practice but also to make it transparent—will find it useful, even indispensable at a certain stage, to think through Kant’s theory of science. —Carl Friedrich von Weizsäcker\[25\]

Bohr’s claim was that the classical language is indispensable. This has remained valid up to the present day. At the individual level of clicks in particle detectors and particle tracks on photographs, all measurement results have to be expressed in classical terms. Indeed, the use of the familiar physical quantities of length, time, mass, and momentum–energy at a subatomic scale is due to an extrapolation of the language of classical physics to the non-classical domain. —Brigitte Falkenburg\[26\]

\[*\]Here is how this interpretive principle was formulated by Dirac\[24\] “The expression that an observable ‘has a particular value’ for a particular state is permissible in quantum mechanics in the special case when a measurement of the observable is certain to lead to the particular value, so that the state is an eigenstate of the observable.”
The demystification, in Sect. 3 of the axioms formulated in Sect. 2, leaves wide open a slew of philosophical issues, such as: What exactly is meant by “measurement”? Does a measurement necessarily involve an apparatus that can be described in classical terms? Does it necessarily involve a subject—an observer conscious of the outcome? Does it necessarily involve a free agent who designs, chooses, and performs the measurement? And what is probability? Can there be an objective probability or is “objective probability” an oxymoron? Is there a reality that is somehow described by the quantum-mechanical correlation laws? If so, how? More specifically, what does the existence of mutually incompatible observables teach us about that reality? And how is that reality related to measurements, ontologically or epistemologically, considering that the existence of measurement outcomes is presupposed (and thus cannot be accounted for) by the quantum-mechanical correlation laws?

What light (if any) does QBism throw on these issues? While I see no point in assessing the comparative merits of QBism and ψ-ontologist interpretations of quantum theory, I think it fruitful to use the views of Niels Bohr as a standard or foil. But what exactly are Bohr’s views? What, for instance, does he mean by “experience” in the following sentence which, for obvious reasons, is prominently displayed in Fuchs et al. 8:

In our description of nature the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience.

To ascertain the specific sense of an equivocal term like experience, one must pay attention to the context in which it is used. Here is the passage from which this quote has been harvested:

what gives to the quantum-theoretical description its peculiar characteristic is just this, that in order to evade the quantum of action we must use separate experimental arrangements to obtain accurate measurements of the different quantities, the simultaneous knowledge of which would be required for a complete description based upon the classical theories, and, further, that these experimental results cannot be supplemented by repeated measurements. . . . a subsequent measurement to a certain degree deprives the information given by a previous measurement of its significance for predicting the future course of the phenomena. Obviously, these facts not only set a limit to the extent of the information obtainable by measurements, but they also set a limit to the meaning which we may attribute to such information. We meet here in a new light the old truth that in our description of nature the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience. (pp. 17–18)

What Bohr concerned was “the task of bringing order into an entirely new field of experience” (Ref. 27, p. 228), a field of experience that places limits on “the extent
of the information obtainable by measurements” and on “the meaning which we may attribute to such information.”

The word “experience” may be used with reference to either the experiencing subject or the experienced object. When Bohr spoke of “the great experience of meeting Einstein for the first time” (Ref. 27, p. 94), he was, in addition to referring to an objective event, indicating the subjective impression this event had made on him. But when he spoke of a “field of experience,” when he insisted that it “lies in the nature of physical observation, that all experience must ultimately be expressed in terms of classical concepts” 6 or when he pointed out that “the requirement of communicability of the circumstances and results of experiments implies that we can speak of well defined experiences only within the framework of ordinary concepts” 28 what he had in mind was the objectifiable content of experience—an objective reality conditioned by how we experience it and how we describe it but not dependent for its existence on the fact that we experience and describe it.

The reason why, for Bohr, the notion of disclosing the real essence of the phenomena was an oxymoron, was that it meant to describe a reality that is unconditioned by how we describe it. It did not mean, as it does for QBists, that the purpose of describing nature is to track down relations between the manifold aspects of an individual subject’s experience. On the contrary, it was to preserve the objectivity of classical descriptions that Bohr insisted on the indispensability of using such descriptions to communicate “what we have done and what we have learned” (Ref. 29, pp. 3, 24; Ref. 30, pp. 39, 72, 89). For him, the “description of atomic phenomena has . . . a perfectly objective character, in the sense that no explicit reference is made to any individual observer. . . . all subjectivity is avoided by proper attention to the circumstances required for the well-defined use of elementary physical concepts” (Ref. 29, pp. 3, 7). The reason he stressed the key role played by relations between the manifold aspects of our experience was that empirical knowledge owes its objectivity to such objectifiable relations as the relation between substance and property or the relation between cause and effect.

Nothing is more central to Bohr’s views than the dual necessity (i) of defining observables in terms of the experimental arrangements by which they are measured and (ii) of describing these arrangements in classical terms. Because the limits placed on simultaneous measurements of incompatible observables imply “a complementarity of the possibilities of definition” (Ref. 6, p. 78), observables can no longer be defined without reference to the “agency of observation” (pp. 54, 67) (i.e., the experimental apparatus), so that the “procedure of measurement has an essential influence on the conditions on which the very definition of the physical

\[ ^{d} \text{To support their claim that “[q]uantum correlations . . . refer only to . . . the acquisition of experiences by any single agent,” Fuchs et al. do not shy away from alluding to their favourite Bohr quote with a singular “her” substituted for Bohr’s plural “our”: “when any agent uses quantum mechanics to calculate [cor]relations between the manifold aspects of [her] experience”, those experiences cannot be space-like separated.”} \]
quantities in question rests" [31] The “distinction between object and agency of
measurement” is therefore “inherent in our very idea of observation” (Ref. [6], p. 68). And since “[o]nly with the help of classical ideas is it possible to ascribe an
unambiguous meaning to the results of observation” (p. 94), “it would be a mis-
conception to believe that the difficulties of the atomic theory may be evaded by
eventually replacing the concepts of classical physics by new conceptual forms” (p.
16). For summaries of Bohr’s mature views, which “remained more or less stable at
least over the latter thirty years of Bohr’s life” (Ref. [32] p. 138), see Kaiser [33] and
MacKinnon [34].

There are affinities between Bohr and Kant, as a number of commentators have
noted (Refs. [33–42], among them Cuffaro [43] who holds that any interpretation of
Bohr should start with Kant. Kant owes his fame in large part to his successful
navigation between the Scylla of transcendental realism (which eventually runs up
against the impossible task of explaining the subject in terms of the object) and
the Charybdis of a Berkeley-style idealism. What allowed him to steer clear of
both horns of the dilemma was a dramatic change of strategy. Instead of trying to
formulate a metaphysical picture of the world consistent with Newton’s theory, as
he had done during the pre-critical period of his philosophy, he inquired into the
cognitive conditions that are (i) necessary for the possibility of empirical knowledge
and (ii) sufficient for the objective status of Newtonian mechanics. He recognized, as
Bitbol [38] put it, “that the land sought by metaphysics can be foreign to us not due
to the excessive distance we have with it, but rather due to its excessive proximity
to us.”

Kant’s reading of classical mechanics dispelled many qualms shared by thinkers
at the end of the eighteenth century—qualms about the purely mathematical na-
ture of Newtonian mechanics, about its partial lack of intelligibility (e.g., action at
a distance), and about such unfamiliar features as Galileo’s principle of relativity.
The mathematical nature of Newton’s theory was justified not by the Neo-Platonic
belief that the book of nature was written in mathematical language but by being
a precondition of objective knowledge. What permits us to look upon the manifold
aspects of our experience as the constituents of an objective world is the mathe-
matical regularities that exist between them. They make it possible to synthesize
these aspects into a system of interacting re-identifiable objects—i.e., to think of
(subjectively) successive aspects of experience as (objectively) successive proper-
ties of objects and to think of (subjectively) coexisting aspects of experience as a
subject-independent system of coexisting objects.

Similarly, Newton’s refusal to explain gravitational action at a distance was due
not to any incompleteness of his theory but to the fact that the only causality
available to us consists in the same regular mathematical connections that permit
us to objectify the content of our experience. As for Galileo’s principle of relativity,
which asserts that motion is always relative, it is a direct consequence of the fact
that regular mathematical connections only exists between particular aspects of
experience, i.e., between successive events and between coexisting objects, but never between a particular aspect of experience and the general form of experience we call space.

In the course of the nineteenth century, most philosophers and apparently many of the scientists who cared to reflect on the nature of science came to adopt (or, in some cases, reinvent) Kant’s epistemology. It alleviated the loss of transcendental realism by justifying the use of a universal objectivist language, which made it possible to think and behave as if transcendental realism were true. The advent of quantum mechanics dealt a severe blow to this comfortable attitude. Atoms respected neither Kant’s apriorism, such as the assertion that the possibility of empirical knowledge requires the universal validity of the law of causality, nor his principle of thoroughgoing determination, which asserts that “among all possible predicates of things, insofar as they are compared with their opposites, one must apply to [each thing] as to its possibility” (Ref. 44, p. 553). Neither Kant’s apriorism nor his principle of thoroughgoing determination, however, though significant elements of his thinking, are central to his theory of science. Hence, there are more ways to respond to this blow than relapsing into transcendental realism, embracing a Berkeley-style idealism, or resigning oneself to a metaphysically sterile instrumentalism. In Bohr’s case, MacKinnon writes, the crisis precipitated something akin to a Gestalt shift, a shift in focal attention from the objects studied to the conceptual system used to study them. This effect was somewhat similar to the epoché advocated by Husserl. He stressed the need for philosophers to bracket the natural standpoint and consider it as a representational system, rather than the reality represented. Similarly, Bohr’s realization of the essential failure of the pictures in space and time on which the description of natural phenomena have hitherto been based shifted the focus of attention from the phenomena represented to the system that served as a vehicle of representation. (p. 97)

Bohr’s “epoché” mirrors the seminal change of strategy that had enabled Kant to navigate between the Scylla of realism and the Charybdis of idealism. Where Kant had famously stressed that “[t]houghts without content are empty, intuitions without concepts are blind” (p. 193), Bohr might have said that without measurements the formal apparatus of quantum mechanics is empty, while measurements without the formal apparatus of quantum mechanics are blind. What permits us to look beyond the directly experienced world, which is amenable to classical description, is the mutual exclusion of experimental procedures, but this also means that there is no quantum world that can be described independently of our experimental investigations: “it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the co-existence of which might at first sight appear irreconcilable with the basic principles of science” (Ref. 30, p. 61). To look beyond the classically describable world, one needs
the quantum-mechanical formalism, and one needs experimental arrangements situated in that world, “the specification of which is imperative for any well-defined application of the quantum-mechanical formalism” (p. 57).

For Bohr, as for Kant, there is an intimate connection between perception, which is conditioned by the spatiotemporal aspects of experience, and conception. For Kant, conception organizes the content of experience into an effectively subject-independent world of re-identifiable objects and causally connected events. For Bohr, conception provides the framework necessary for the formulation of unambiguous, communicable knowledge, which is required for a clear-cut separation of the subject from the object. Far from invalidating Kant’s epistemological stance, quantum theory reminds us of the roles perception and conception play in the construction and constitution of the objective world. Bohr’s departure from Kant is not a departure from Kant’s epistemological stance but a consequence of quantum theory’s departure from classical physics. By limiting the application of classical concepts, quantum theory extends the range of phenomena within our ken. It is an irony that Bohr, seeing Kant as arguing for the a priori necessity of classical concepts, regarded complementarity as an alternative to Kant’s theory of science, thus drawing the battle lines in a way which put Kant and himself on opposing sides.

If Bohr insisted on the necessary use of classical language in describing experiments and reporting results, it was because he considered the categorial structure implicit in that language a *sine qua non* of objective science. This categorial structure is in all significant respects the very system of categories that Kant had considered a precondition of the possibility of empirical knowledge.

I cannot but agree with Folse when he wrote: “It is often said that a work of genius resists categorization. If so, Bohr’s philosophical viewpoint easily passes this criterion of greatness. Surely this is one of the reasons for the commonplace complaints over Bohr’s ‘obscurity’.” Historically, Bohr’s reply to the EPR paper was taken as a definitive refutation by the physics community. By the time interpreting quantum mechanics became a growth industry, Bohr’s perspective was lost. His epistemological reflections generally came to be treated on a par with a proliferating multitude of ontological interpretations of a mathematically formulated theory, and as such they could not but seem amateurish, outdated, ad hoc, bizarre, or downright irrational. This is how Jeffrey Bub came to conclude that

The careful phraseology of complementarity . . . endows an unacceptable theory of measurement with mystery and apparent profundity, where clarity would reveal an unsolved problem,

how Abner Shimony came to confess that

after 25 years of attentive—and even reverent—reading of Bohr, I have not found a consistent and comprehensive framework for the interpretation of quantum mechanics,

and how Edwin Jaynes came to see in “our present QM formalism”
a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature—all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble.

It would therefore seem appropriate to close this section with the following words by Clifford Hooker:

it is a remarkable commentary on the state of confusion and misunderstanding now existing in the field that Bohr’s unique views are almost universally either overlooked completely or distorted beyond recognition—this by philosophers of science and scientists alike. Despite the fact that there are almost as many philosophies of quantum theory as there are major quantum theorists, the illusion somehow persists that they are all talking about the same thing and in essentially the same way... so many people can apparently read Bohr and not grasp the significance of what he was driving at.

6. A Tale of Two Realities

In the concluding section of their recent paper, Fuchs et al. hijack Schrödinger’s historical account of the failings of the modern scientific world view—essentially a critique of the classical mechanistic world view—to support a QBist interpretation of quantum mechanics, claiming that Schrödinger “takes a QBist view of science.” Schrödinger’s actual concerns can be gleaned from the following key passages of his essay, in which he explains why he believes “that I actually do cut out my mind when I construct the real world around me”:

I am not aware of this cutting out. And then I am very astonished that the scientific picture of the real world around me is very deficient. It gives a lot of factual information, puts all our experience in a magnificently consistent order, but it is ghastly silent about all and sundry that is really near to our heart, that really matters to us. It cannot tell us a word about red and blue, bitter and sweet, physical pain and physical delight; it knows nothing of beautiful and ugly, good or bad, God and eternity....

The scientific world-picture vouchsafes a very complete understanding of all that happens—it makes it just a little too understandable. It allows you to imagine the total display as that of a mechanical clockwork, which for all that science knows could go on just the same as it does, without there being consciousness, will, endeavour, pain and delight and responsibility connected with it—though they actually are. And the reason for this disconcerting situation is just this, that, for the purpose of constructing the picture of the external world, we have used the greatly simplifying device or cutting our own personality out, removing it; hence it is gone, it has evaporated, it is ostensibly not needed.
In particular, and most importantly, this is the reason why the scientific world-view contains of itself no ethical values, no aesthetical values, not a word about our own ultimate scope or destination. . . . Science cannot tell us a word about why music delights us, of why and how an old song can move us to tears. (pp. 95–97)

Depending on what one expects from science in general, one may deplore that there is no science of all those things that really matter to us, or that science cannot account for consciousness, qualia, emotions, or ethical and aesthetical values, not to mention God and eternity. But here we are concerned with physics, not science in general, and it just isn’t the business of physics to account for these wonderful things. We physicists and armchair philosophers of science ought to know our place in the scheme of things, and no one, to my mind, has elucidated our epistemological situation more clearly than Kant in the 18th Century and Bohr again in the 20th. Kant in particular taught us to distinguish between

(I) a transcendental reality external to the subject, undisclosed in experience, which he looked upon as the intrinsically unknowable cause of subjective experience,

(II) the product of a mental synthesis—a synthesis based on the spatiotemporal structure of experience, achieved with the help of spatiotemporal concepts, and resulting in an objective reality from which the objectifying subject can abstract itself.

Physics, being concerned with the latter, cannot be expected to account for the objectifying subject, its quality-rich experience, or its ethical and aesthetical values. It is unfortunate that not only ψ-ontologists but QBists as well fail to make that crucial distinction, as the following statement by Fuchs et al. illustrates:

when we attempted to understand phenomena at scales not directly accessible to our senses, our ingrained practice of divorcing the objects of our investigations from the subjective experiences they induce in us got us into trouble.

This statement is ambiguous in two respects: there is an ambiguity concerning the objects that induce subjective experiences in us, and there is an ambiguity concerning the objects of our investigations. One may think, as physicalists do, that our subjective experiences are induced by (if not identical with) physical processes in our brains, or one may think of subjective experiences as having their origin in the intrinsically unknown reality of type I. Likewise, one may think of the objects of our investigations as belonging to the epistemically inaccessible type-I reality, or one may think of them as objectifiable aspects of our experiences. No sooner than one learns to distinguish between those two concepts of reality, however, one realizes that only the epistemically inaccessible reality of type I can induce subjective experiences, and that the knowledge resulting from our investigations can only
refer to the type-II reality constructed and objectified by us. Hence, the objects of
our investigations cannot be equated with whatever it is that induces subjective
experiences in us, as the statement by Fuchs et al. implies.

What actually gets us into trouble is our failure to distinguish between these
two realities. Failing to make this distinction, Fuchs et al. attribute to the Copenhagen
interpretation the “view that measurement outcomes belong to an objective
(‘classical’) domain that is independent of agents and/or their experience.” Bohr
would have agreed that the “classical” domain is independent of any particular
subject’s experiences, but he would have insisted that we are able to know it as an
objective reality only because it conforms to the spatiotemporal structure of our
experience and therefore is describable in terms of concepts that owe their meanings
to this structure. Missing the objective “middle-ground” between the experiencing
subject and the transcendental object (Kant’s thing-in-itself), they blame Bohr and
Heisenberg for giving rise to the measurement problem.

The Founders of quantum mechanics were already aware that there was a
problem. Bohr and Heisenberg dealt with it by emphasizing the inseparability
of the phenomena from the instruments we devised to investigate them. Instruments
are the Copenhagen surrogate for experience. Being objective
and independent of the agent using them, instruments miss the central
point of QBism, giving rise to the notorious measurement problem, which
has vexed physicists to this day.

In point of fact, Bohr’s understanding of quantum mechanics is unique in that
it does not give rise to this notorious problem. The measurement problem is the
invention of von Neumann and the concern of those who uncritically accept his
particular codification of quantum mechanics. Clearly, Hooker’s observation that
“so many people can apparently read Bohr and not grasp the significance of what
he was driving at” applies to QBists as well.

If Bohr’s epistemological reflections were revolutionary, von Neumann’s ψ-
oniological thinking was counter-revolutionary. Bohr stands apart with his insistence
that what happens between a system preparation and a measurement is a
holistic phenomenon, which cannot be dissected into the unitary evolution of a
quantum state (called “pre-measurement” by measurement theorists) and a
subsequent “collapse” or “objectification.” It comes as no surprise that von Neumann
himself was led to conclude—like many of those who have looked on his
codification of quantum mechanics as a physical theory in need of an interpretation
(Refs. 53–58)—that the final termination of a measurement is in the conscious

*Here is how Bohr defines “phenomenon”: “all unambiguous interpretation of the quantum mechanical formalism involves the fixation of the external conditions, defining the initial state of the atomic system concerned and the character of the possible predictions as regards subsequent observable properties of that system. Any measurement in quantum theory can in fact only refer either to a fixation of the initial state or to the test of such predictions, and it is first the combination of measurements of both kinds which constitutes a well-defined phenomenon.”
observer.

The reason it is so hard to make sense of quantum mechanics is not our ingrained practice of divorcing the objects of our investigations from our subjective experiences. The possibility of drawing a line of separation between subject and object, in such a way as to be able to refer to objects and our surroundings without referring to our experiences, is the *sine qua non* of empirical science and the very reason for its spectacular success. That possibility is non-negotiable. What leads to the measurement problem is, as Mermin has put it elsewhere, \(^5^9\) “our habit of inappropriately reifying our successful abstractions.” The transmogrification of a probability algorithm into a physical reality in the transcendental (type-I) sense of “reality” calls for a “supraphysical” reality capable of turning possibilities into facts, and the obvious candidate for such a “supraphysical” reality is the consciousness of the observer.

Although von Neumann’s error does not lie in his conclusion but in his premise, the error cannot be rectified by a retreat to the respective private experiences of individual agents. Such a retreat would be a cop-out, not a solution. To rectify the error, one must not only recognize that objective reality is a mental construct from which the objectifying subject can abstract itself; one must also respect the conditions on which the possibility of this abstraction rests. Because these conditions are the same for classical as for quantum physics, “there is, strictly speaking, no new observational problem in atomic physics” (Ref. \(^3^0\) p. 89). There is no new observational problem because in quantum physics we are doing what we have always done: setting up experiments and reporting their results, in a language that is suitable for unambiguous communication. What makes this language suitable for unambiguous communication is that it refers to a system of interacting objects and causally connected events that makes no reference to the objectifying subject.

What is new, and radically so, is that the properties of quantum systems are defined by the (classically describable) experimental conditions in which they are measured, and that they only exist if and when they are measured. Phenomena are indeed inseparable from the experimental conditions in which they occur. Instead of being “the Copenhagen surrogate for experience,” instruments open up a new and wholly unanticipated field of experience—a field of experience that does not permit itself to be conceived as an autonomous system of objects with causally connected intrinsic properties but, instead, evinces itself through statistical correlations between (classically describable) preparations and (classically describable) outcomes. At the same time instruments ensure the necessary objectivity of science in this field. To say that “instruments miss the central point of QBism” because they are “objective and independent of the agent using them” is to put the cart before the horse. It is QBism that misses the central point of instruments, which is that they open up this new field of experience and ensure its requisite objectivity.
7. A Cause for Celebration

Classical physics “knows” about spatial relations between objects and temporal relations between events. It also knows about Lorentz invariant spatiotemporal relations and how to separate them into spatial and temporal relations given an inertial reference frame. But it knows nothing of absolute positions or times. It knows nothing of spatial relations between objects on the one hand and, on the other, either the general form of experience we call space or the experiencing subject’s “here.” It knows nothing of temporal relations between events and either the general form of experience we call time or the experiencing subject’s “now.” Nor does it know anything of the experiential quality—the respective qualia—of spatial and temporal relations. Hermann Weyl made this point with respect to space, stressing “with what little right mathematics may claim to expose the intuitional nature of space”:

Geometry contains no trace of that which makes the space of intuition what it is in virtue of its own entirely distinctive qualities which are not shared by “states of addition-machines” and “gas-mixtures” and “systems of solutions of linear equations”. It is left to metaphysics to make this “comprehensible” or indeed to show why and in what sense it is incomprehensible. We as mathematicians . . . must recognise with humility that our conceptual theories enable us to grasp only one aspect of the nature of space, that which, moreover, is most formal and superficial.

Mermin made the same point with respect to time, writing that physics “knows nothing of now and deals only with correlations between one time and another.” He appears to deplore that the “point on my world-line corresponding to now, obvious as it is to me, cannot be identified in any terms known to today’s physics,” comparing the ability of consciousness “to go beyond time differences and position itself absolutely along the world-line of the being that possesses it” to another “elementary constituent of consciousness which physics is silent on,” to wit, “the characteristic and absolutely unmistakable blue quality of the experience of blueness itself.”

Here I must dwell for a moment on the notion of a “point on my world-line corresponding to now,” a point where consciousness is positioned “absolutely along the world-line of the being that possesses it,” made famous by Weyl’s pronouncement that

The objective world simply is; it does not happen. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time.

I have commented on this incoherent notion in several papers. The following comment is from Ref.
If we conceive of temporal relations, we conceive of the corresponding relata simultaneously—they exist at the same time *in our minds*—even though they happen or obtain at different times in the objective world. Since we can’t help it, that has to be OK. But it is definitely not OK if we sneak into our simultaneous spatial mental picture of a spatiotemporal whole anything that advances across this spatiotemporal whole. We cannot mentally represent a spatiotemporal whole as a simultaneous spatial whole and then imagine this simultaneous spatial whole as persisting in time and the present as advancing through it. There is only one time, the fourth dimension of the spatiotemporal whole. There is not another time in which this spatiotemporal whole persists as a spatial whole and in which the present advances, or in which an objective instantaneous state *evolves*. If the present is anywhere in the spatiotemporal whole, it is trivially and vacuously everywhere—or, rather, everywhen.

Mermin’s new hope\(^1\) that QBism can assign to *now* a physical role corresponding to the central role it plays in our experience is misplaced. As my *here* cannot be objectified as a particular point in space from which I survey my surroundings—a fact about which nobody seems to complain—so my *now* cannot be objectified as a particular point in time at which I remember my past and anticipate my future. This, however, should be no cause for regret. It *only* becomes that if physical reality is mistaken for a transcendental (type-I) reality from which we—our internalities—are then missing.

Nothing is missing if physical reality is understood to result from the objectification of the objectifiable content of our experience. Our experiential relation to this reality remains intact. That we cannot objectify the qualitative aspects of our experience does not change the fact that this reality is *our* creation, extracted from *our* experience, supported and shared by *us* qua subjects. The reason we cannot project our subjectivities into it is that they encompass and exceed it. As we depend on an epistemically inaccessible type-I reality in which we are embedded, so the type-II reality created by us depends on us. Its spatial and temporal relations owe their qualitative character to the warp and woof of our experience, phenomenal space and phenomenal time.

The wonderful thing about our respective objectified worlds is that each of them contains all of us. It does not contain our internalities—you may thank God that *my* objective world does not contain *your* internalities—but it contains our externalities. This makes it a shared world, a common playground on which we can meet and communicate. It is a cause for celebration, not lamentation.\(^2\)

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4 Reporting on a conversation with Einstein, Rudolf Carnap\(^3\) wrote: “Einstein said that the problem of the Now worried him seriously. He explained that the experience of the Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics. That this experience cannot be grasped by science seemed to him a matter of painful but inevitable resignation. . . . Einstein
What do the quantum-mechanical correlation laws add to this picture? Since, in Bohr’s words (Ref. 29 p. 12), “the physical content of quantum mechanics is exhausted by its power to formulate statistical laws governing observations obtained under conditions specified in plain language,” the correlata remain the same. The classically describable world does not disappear, for it contains the correlata whose existence quantum mechanics presupposes and therefore has to take for granted. Nor do the quantum-mechanical correlations supplant the classical correlations between the manifold aspects of our experience, for without the classical correlations it would not be possible to synthesize an objective world from which the objectifying subject can abstract itself. What quantum mechanics adds to the picture is that quantum systems are known through (i) properties that only exist when they are indicated and (ii) statistical correlations between property-indicating events in the classically describable world—diachronic correlations between events in timelike relation as well as synchronic correlations between events in spacelike relation.

What allowed Bohr to go beyond Kant was his insight that empirical knowledge is not limited to what is accessible to our senses, and that it does not have to be a knowledge of interacting objects and causally connected events. It can reach beyond what is directly accessible to our senses. What it finds there, however, cannot be expected to be structured in the same way as what is directly accessible to our senses. We have no reason to expect that what is not directly accessible to our senses is subject to the spatiotemporal conditions of sensory experience, and we have no reason to expect that what is not subject to these conditions conforms to descriptions involving such spatiotemporal concepts as position and momentum, time and energy, causality and interaction. Since these are essentially all the descriptive concepts at our disposal, it should have come as no surprise that what is not directly accessible to our senses can only be described in terms of correlations between events that are directly accessible to our senses. Bohr still stands virtually alone with his insight that reality cannot be “squeezed” into Kant’s “pure forms of experience,” space and time. “It is my personal opinion,” he wrote, “that these difficulties are of such a nature that they hardly allow us to hope that we shall be able, within the world of the atom, to carry through a description in space and time that corresponds to our ordinary sensory perceptions.”

8. Wigner’s Friend

For QBists, who look on quantum theory as an addition to Bayesian probability theory, measurements are relative to individual “agents,” each of whom has to treat
everything he or she does not directly experience as subject to the superposition principle. Thus QBism directs Alice to treat all “external systems” on the same footing.

whether they be atoms, enormous molecules, macroscopic crystals, beam splitters, Stern-Gerlach magnets, or even agents other than Alice. In QBism the only phenomenon accessible to Alice which she does not model with quantum mechanics is her own direct internal awareness of her own private experience. When Alice elicits an answer from Bob, she treats this as she treats any other quantum measurement. Bob’s answer is created for Alice only when it enters her experience. A QBist does not treat Alice’s interaction with Bob any differently from, say, her interaction with a Stern-Gerlach apparatus, or with an atom entering that apparatus.

While a measurement, according to QBism, is an “action an agent takes to elicit a set of possible experiences,” a “measurement outcome” is “the particular experience of that agent elicited in this way.” One is left to wonder how QBists would describe the action taken by the agent, including beam splitters and Stern-Gerlach magnets, and how they would describe the particular experience elicited. I surmise they would describe both the action and the experience in the classical language we all use to communicate what we have done and what we have learned. In doing so they would situate both what the agent has learned—the outcome indicated by the measurement apparatus—and what the agent has done to elicit the outcome in one and the same objective world. My conjecture seems confirmed by the following passage:

The world is filled with all the same things it was before quantum theory came along, like each of our experiences, that rock and that tree, and all the other things under the sun; it is just that quantum theory provides a calculus for gambling on each agent’s own experiences—it doesn’t give anything else than that. . . . quantum theory, from this light, takes nothing away from the usual world of common experience we already know. It only adds.

If the world is filled with all the same things it was before quantum theory came along, then it contains measurement outcomes, and it contains them not because they are or have been present in an agent’s subjective experience but because they are situated in a conceived world that we share, a world that is modelled after the objectifiable content of our [plural!] experience. Apparently, though, QBists will have none of this, insisting as they do that

An outcome is created for the agent who takes the measurement action only when it enters the experience of that agent. The outcome of the measurement is that experience. Experiences do not exist prior to being experienced.
This is confusing if not confused. What was the outcome before it became an outcome by entering the agent’s experience? Whence did it enter, and in what sense? And what about the measurement action taken by the agent? Did that also become a measurement action only when it entered her experience? And what if several agents collaboratively perform a measurement? Could it be that what enters their respective experiences becomes a different outcome in each agent’s experience? And if not, why not?

To find out what QBists actually mean to say, it may help to turn to two time-honoured testing grounds for quantum philosophies. First, Wigner’s friend, who, secluded in her laboratory, performs a measurement and obtains an outcome that Wigner, outside her laboratory, does not know. If it makes sense for Wigner to assign a quantum state to everything he does not directly experience, and

—if he believes what his friend has told him about her plans in her laboratory, he will assign an entangled state to her, her apparatus, and the system on which she is making her intervention. Wigner’s state superposes all the possible reports from his friend about her own experience, correlated with the corresponding readings of her apparatus.

“Intervention” is another QBist term for “measurement.” According to Fuchs et al., the difference between Wigner’s state assignment and his friend’s “is paradoxical only if you take a measurement outcome to be an objective feature of the world, rather than the contents of an agent’s experience.” How so? If the world is filled with all the same things it was before quantum theory came along, then it is also filled with measurement outcomes. Outcome-indicating events occur in the objective (type-II) world, which is independent of any particular agent’s experience in his or her here and now. The disparity is paradoxical if a quantum state (rather than a measurement outcome) is taken to be an objective feature of the world. The paradox vanishes with the recognition that probabilities are assigned on the basis of outcome-indicating events. State assignments can differ because they can be based on different sets of such events.

Needless to say, this does not justify Wigner’s assigning a superposition to his friend. Because the outcome obtained by her is part of the objective (type-II) world, Wigner must represent his expectations concerning her outcome by a proper (ignorance-interpretable) mixture. In fact, since a measurement is something that by definition has an outcome, he ought to assign a proper mixture to every measurement that is actually made. There is no earthly reason for Wigner to assign a state that “superposes all the possible reports from his friend about her own experience, correlated with the corresponding readings of her apparatus.”

We are informed by Fuchs et al. that “the disagreement is resolved the moment he [Wigner] receives her report, i.e. when it enters his own experience.” Anyone

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*This is the reason why the probabilities of the possible outcomes of a measurement add up to unity.*
but a QBist (or a Berkeleyan idealist) would infer from this statement that saying “something enters someone’s experience” is tantamount to saying that “someone becomes aware of something that happens or is the case in the objective world, something that would also happen or be the case if this particular someone did not become aware of it.” For a QBist, on the other hand, it means that the Moon exists only for those who have seen it or have received trustworthy reports of its existence, and that a trustworthy report exists only for someone into whose experience it has entered (and has thereby become a trustworthy report).

Much of what we know or believe to be true of the objective world is based not on direct sensory evidence but on information we trust. And therefore much of what we think we know may not be as trustworthy as we believe it is. If I receive a report of a measurement outcome, I tend to trust it, knowing it to be based on an honest person’s direct sensory experience of an outcome-indicating event. If, on the other hand, I receive a report of a superposition of alternative measurement outcomes, I tend to distrust it, to say the least, not simply because no one has ever experienced a superposition of measurement outcomes but because such a superposition is a contradiction in terms.

9. EPR

That the world should violate Bell’s theorem remains, even for QBism, the deepest statement ever drawn from quantum theory.

—Christopher A. Fuchs

Another—and surely the most popular—testing ground for quantum philosophies is the experiment discussed by Einstein et al. in their seminal paper of 1935. Bohm’s version of EPR’s setup features a spinless diatomic molecule dissociating into two spin-1/2 particles “in” the singlet state.

To say that a system is “in” a quantum state is to affirm the occurrence of an event (or set of events) that makes it possible to assign probabilities to the possible outcomes of any measurement to which the system may subsequently be subjected. The quantum state thereby “prepared” is the algorithm to be used for the assignment. In Bohm’s setup, the event is the dissociation of a molecule into two spin-1/2 particles, the algorithm for assigning probabilities is the singlet state, and the measurement contemplated is a joint measurement of one spin component of each particle. The outcomes of the two measurements are correlated: if the two spins are measured with respect to axes that differ by an angle $\alpha$, the probability of obtaining opposite outcomes (one “up” and one “down”) is $\cos^2(\alpha/2)$.

The implication of Bell’s theorem is that there can be no common-cause explanation for these correlations, whereas Lorentz invariance rules out explanations involving actions at a distance. What gives? The conclusion to be drawn is simply that Bohr was right all along. What gives is that the quantum-mechanical correlations—the synchronic ones between outcomes of measurements performed on entangled systems as well as the diachronic ones between outcomes of mea-
measurements performed on the same system at different times—describe indivisible quantum phenomena in the only way they can be described, i.e., in terms of correlations between state preparations and measurement outcomes, both expressed in classical terms. Neither can we reify our calculational tools in the manner of von Neumann, by dissecting a quantum phenomenon into a deterministic state evolution followed by an indeterministic “collapse,” nor is it possible to decompose a holistic quantum phenomenon into interacting objects and causally connected events. Unlike the classical world, which is conceived as if it were directly accessible to our senses on all scales, what is not directly accessible to our senses is not subject to the spatiotemporal conditions of sensory experience and therefore cannot be analyzed into substances with intrinsic properties and causal relations holding among them.

QBists deal with the mystery of EPR correlations by claiming that “quantum correlations are necessarily between time-like separated events, and therefore cannot be associated with faster-than light influences.”

Quantum correlations, by their very nature, refer only to time-like separated events: the acquisition of experiences by any single agent. Quantum mechanics, in the QBist interpretation, cannot assign correlations, spooky or otherwise, to space-like separated events, since they cannot be experienced by any single agent. Quantum mechanics is thus explicitly local in the QBist interpretation.

It is true that quantum correlations cannot be associated with faster-than light influences, which indeed “removes any tension between quantum mechanics and relativity” but the reason is not that quantum mechanics only gives us diachronic correlations. The reason EPR correlations cannot be associated with faster-than light influences is that quantum correlations cannot be analyzed into substances with intrinsic properties and causal relations holding among them. And the reason there is no tension between quantum mechanics and relativity is not that quantum correlations only exist between time-like separated events but that the correlations depend on the objective locations and times of the correlata (property-indicating events), rather than on the inertial coordinate frame we use to label locations and times.

According to Fuchs, it is by giving “each quantum state a home . . . localized in space and time—namely, the physical site of the agent who assigns it” that QBism “expels once and for all the fear that quantum mechanics leads to ‘spooky action at a distance.’” It is true that each quantum state—in fact, each calculational tool of every physical theory—has a “home” in someone’s mind, but if the spatiotemporal structure of the objective (type-II) world has its roots in the spatiotemporal structure of sensory experience, then giving someone’s mind (including the quantum state residing in it) a home in space and time is problematic, to say the least. However, the potential circularity of placing an agent’s body inside a world created by his mind does not concern us here.
When Alice and Bob, having performed a large number of spin measurements on particles prepared "in" the singlet state, get together to compare notes, they have clear evidence of the EPR correlations predicted by the singlet state. The claim that quantum correlations refer only to time-like separated events is therefore simply wrong. Contrary to what is claimed by Fuchs, the "protection" provided by "truly personal quantum-state assignments" does nothing to prevent the spectre of action at a distance from being there "as doggedly as it ever was." The reason local explanations—whether in terms of a common cause or by action at a distance—do not work needs to be understood, not swept under the rug by declaring that quantum correlations "by their very nature" (!) refer only to time-like separated events.

When Alice and Bob examine how the frequencies of their outcomes depend on the angle between their respective measurement axes, they view their past experiences as a spatiotemporal array of outcome-indicating events, from a vantage point that is not localized anywhere inside the past they are surveying, in the same way that physics in general "views" the spatiotemporal whole from no particular vantage point in space and time. What this spatiotemporal whole contains is actual events, regardless of whether they have happened or are yet to happen (as seen from the vantage point of a particular agent at a particular time). It does not contain possible events, and therefore it does not contain probabilities. De Finetti was right: probability does not exist. (He stressed this by writing it in all capital letters and centred on a separate line.) Probabilities—as well as the possible events to which they are assigned—do not exist in the objective world "known" to physics. Possible events only exist as non-objectifiable ideas in our minds, situated in a subject’s non-objectifiable future relative to his or her non-objectifiable now, and so do the probabilities we assign to them.

Perhaps, then, the formal apparatus of quantum mechanics should be characterized as a calculus of correlations rather than as a calculus of probabilities. While we use the quantum-mechanical correlation laws to predict probabilities on the basis of accessible events, what is at the heart of quantum mechanics is not probabilities but correlations. When we use ensembles of identically prepared systems to make approximate probability measurements, what we are actually measuring is correlations, and what we are actually testing is the quantum-mechanical correlation laws.

10. Quantum Interference

Although a distinction has to be made between formulations and interpretations of quantum mechanics, the choice of a formulation cannot but bias the available interpretations. (Among the better known formulations are Heisenberg’s matrix formulation, Schrödinger’s wave-function formulation, Feynman’s path-integral formulation, the density-matrix formulation, and Wigner’s phase-space formulation.) To my mind, one of the reasons it is so hard to see the light is the manner in
which quantum mechanics is generally taught. While junior-level classical mechanics courses devote a considerable amount of time to different formulations of classical mechanics (such as Newtonian, Lagrangian, Hamiltonian, least action), even graduate-level quantum mechanics courses emphasize the wave-function formulation almost to the exclusion of all variants. This is how \( \psi \)-ontologists come to think of quantum states as evolving physical states, and how QBists come to think of them as evolving states of belief.

Schrödinger’s formulation exposes us not only to the temptation of reifying quantum states but also to the allure of the eigenvalue–eigenstate link (see Dirac’s statement of the same in note \( \text{c} \)). Fuchs et al.\(^8\) illustrate just how powerful is this allure when they write: “That probability-1 (or probability-0) judgments are still judgments, like any other probability assignments, may be the hardest principle of QBism for physicists to accept.”

That this should be hard to accept is something I find hard to comprehend. How can an increase from probability 0.999999 to probability 1 make the difference between a probability assignment and a statement of fact? What we learn from an eigenvalue equation \( \hat{Q} \mid \psi(t) \rangle = q \mid \psi(t) \rangle \) is that the value \( q \) is certain to be found at the time \( t \) if and only if the observable \( Q \) is measured at the time \( t \). As I wrote in one of my first papers,\(^7\) “[e]ven the step from probability 1 to factuality crosses a gulf that quantum mechanics cannot bridge.”

The formulation of quantum mechanics arguably best suited to resist these temptations is Feynman’s,\(^7\)\(^3\), which has the following principle at its core:\(^7\)\(^4\)

**Core Principle (Feynman)** Any determination of the alternative taken by a process capable of following more than one alternative destroys the interference between alternatives.

Here is a more detailed formulation of Feynman’s principle (Ref.\(^1\)\(^3\) Sect. 11; Ref.\(^1\)\(^6\) Sect. 5.1):

**Premise A.** Quantum mechanics provides us with algorithms for assigning probabilities to possible measurement outcomes on the basis of actual outcomes. Probabilities are calculated by summing over alternatives. Alternatives are possible sequences of measurement outcomes.\(^8\) Associated with each alternative is a complex number called “amplitude.”

**Premise B.** To calculate the probability of a particular outcome of a measurement \( M_2 \), given the actual outcome of a measurement \( M_1 \), choose a sequence of intermediate measurements, and apply the appropriate rule.\(^8\)

**Rule A.** If the intermediate measurements are made (or if it the setup makes it possible to infer from other measurements what their outcomes would have

\(^8\)It deserves to be stressed that alternatives are defined in terms of measurement outcomes. The only referents needed to formulate the laws of quantum mechanics are property-indicating events.

\(^8\)The parenthetical phrases are needed for dealing with “quantum eraser” setups like that discussed by Englert, Scully, and Walther.\(^7\)\(^5\),\(^7\)\(^6\)
been if they had been made), first square the absolute values of the amplitudes associated with the alternatives and then add the results.

**Rule B.** If the intermediate measurements are not made (and if the setup does not make it possible to infer from other measurements what their outcomes would have been), first add the amplitudes associated with the alternatives and then square the absolute value of the result.

From the Schrödinger/von Neumann point of view, Rule B seems uncontroversial. Superpositions are “normal,” and what is normal does not call for explanation. What calls for explanation is the existence of mixtures that admit of an ignorance interpretation. From Feynman’s point of view, the uncontroversial rule is Rule A, inasmuch as classical probability theory leads us to expect it. What calls for explanation is why we have to add amplitudes, rather than probabilities, whenever the conditions stipulated by Rule B are met. It is this change of viewpoint that makes it possible to obtain the insight Fuchs appears to call for when he exclaims 3: “The thing that needs insight is not the quantum-to-classical transition, but the classical-to-quantum!”

The core principle of Feynman’s formulation covers not only the two-slit experiment with electrons, which according to Feynman 77 “has in it the heart of quantum mechanics,” but other salient peculiarities of the theory as well: the “miraculous identity of particles of the same type,” which according to Misner et al. 78 “must be regarded, not as a triviality, but as a central mystery of physics,” and entanglement, which for Schrödinger 79 was “not . . . one but rather the characteristic trait of quantum mechanics.” And this is as it should be, since empirically Feynman’s formulation is equivalent to Schrödinger’s (Ref. 16, Chapters 5 and 7).

In the two-slit experiment with electrons, the outcome of $M_1$ is the launch of an electron in front of the slit plate, which can be inferred from the detection of the electron behind the slit plate; the outcome of $M_2$ is the detection of the electron by a specified detector; and the intermediate measurement (which may or may not be made) determines the slit taken by the electron.

Misner et al.’s central mystery can be illustrated by an elastic scattering event involving two particles of the same type. In this case the outcome of $M_1$ indicates the preparation of two particles moving in specific directions (one northward and one southward, say), and the outcome of $M_2$ indicates the particles’ subsequent directions of motion (one eastward and one westward, say). The intermediate measurement, if it were made, would tell us which incoming particle is identical with which outgoing particle, and the probability sought is (in obvious notation)

$$|\langle EW|NS\rangle \pm \langle WE|N\rangle^2|,$$

where the sign depends on whether the particles are bosons or fermions. One obtains the same result using the Born rule with these initial and final states:

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}\left(|NS\rangle \pm |SN\rangle\right), \quad |\psi_f\rangle = \frac{1}{\sqrt{2}}\left(|EW\rangle \pm |WE\rangle\right).$$

(6)
EPR–Bohm correlations can be calculated in exactly the same way if we take into account that Born probabilities are time-symmetric: we can assign probabilities to the possible outcomes of an earlier measurement on the basis of the actual outcome of a later measurement as well as vice versa. (This is one more reason why quantum states should not be thought of as evolving states.) Thus $M_1$ can be a spin measurement on particle 1 with respect to axis $a$, $M_2$ can be a spin measurement on particle 2 with respect to axis $b$, and the (counterfactual) intermediate measurement can be a spin measurement on particle 1 with respect to any axis, made right after the molecule’s dissociation into two particles “in” the singlet state. Adding the two amplitudes and taking the absolute square of the result gives us the conditional probability $p(b|a)$:

$$\left| \langle b|u \rangle \langle d|a \rangle - \langle b|d \rangle \langle u|a \rangle \right|^2.$$

The left-hand side reflects the logical order (as usual, from right to left): $|a\rangle$ (“up” with respect to axis $a$) represents the outcome on the basis of which $p(b|a)$ is assigned, $|b\rangle$ (“up” with respect to axis $b$) represents the outcome to which $p(b|a)$ is assigned, and $|u\rangle$ and $|d\rangle$ represent the possible outcomes of the unperformed intermediate measurement. If this measurement, made on particle 1, were to yield $u$, then particle 2 would start out “in” the state $|d\rangle$, and if it were to yield $d$, then particle 2 would start out “in” the state $|u\rangle$. The negative sign appears because the two amplitudes differ by an exchange of fermions. The right-hand side restores the temporal order. If we simplify it to

$$\left| \langle ba|ud \rangle - \langle ba|du \rangle \right|^2,$$

the analogy with Eq. 6 jumps out at us. One obtains the same conditional probability by calculating the joint probability $|\langle ba|S \rangle|^2$, where $|S\rangle$ is the singlet state $(|ud\rangle - |du\rangle)/\sqrt{2}$, and dividing it by the marginal probability of finding “up” with respect to axis $a$. It is worth noting, though, that in order to do the Feynmanesque calculation, we don’t need to know how to write the singlet state. All we need to know is that the two spins are anti-correlated.

Fuchs and Schack recently observed that “[i]n the history of foundational thinking, the double-slit experiment has fallen by the wayside.” It is no longer considered to have in it “the heart of quantum mechanics.” If so, the reason can only be that it has not been understood as an instance or illustration of Feynman’s core principle, which also covers Misner et al.’s central mystery and what Schrödinger regarded as the characteristic trait of quantum mechanics.

What Feynman means by “interference between alternatives” can be made obvious (if it isn’t already) by comparing Rules A and B. The difference between $|A_1 + A_2|^2$ and $|A_1|^2 + |A_2|^2$ is the so-called interference term $A_1 A_2^* + A_2 A_1^*$. If it is positive, one speaks of “constructive interference,” and if it is negative, one speaks of “destructive interference.” As I wrote in my textbook (Ref. 16, Sect. 5.3),

This terminology is prone to cause a great deal of confusion. Quantum-mechanical interference is not a physical mechanism or process. Whenever
we say that “constructive interference occurs,” we simply mean that a probability calculated according to Rule B is greater than the same probability calculated according to Rule A. And whenever we say that “destructive interference occurs,” we simply mean that a probability calculated according to Rule B is less than the same probability calculated according to Rule A.

Fuchs and Schack, too, appear to have hit on this insight:

we have recently started to appreciate that there may be something of substance in Feynman’s argument [that the two-slit experiment has in it the heart of quantum mechanics] nonetheless. It is just not something so easily seen without the proper mindset. The key point is that the so-called “interference” in the example is not in a material field—of course it was never purported to be—but in something so ethereal as probability itself (a logical, not a physical, construct). Most particularly, Feynman makes use of a beautiful and novel move: He analyzes the probabilities in an experiment that will be done in terms of the probabilities from experiments that won’t be done.

To be precise, he uses Rule B to calculate the probabilities of the possible outcomes of a measurement that will be done on the basis of the outcome of a measurement that has been done using the amplitudes associated with the possible outcomes of an unperformed intermediate measurement (rather then their probabilities). Essentially, that “something of substance” is quantum mechanics itself, as formulated by Feynman.

11. The Measurement Problem

Fuchs et al\cite{Fuchs8} claim that by restoring the “subject-object balance,” QBism inter alia “eliminates the measurement problem.” As Mermin, “a QBist in the making”\cite{Mermin80} points out (Ref.\cite{Mermin80} p. 156), “the quantum measurement problem” today has almost as many meanings as “the Copenhagen interpretation.” Of these he mentions two.

The first is how to account for an objective physical process called the collapse of the wave function, which supersedes the normal unitary time evolution of the quantum state in special physical processes known as measurements.

It is true that QBism eliminates this problem, but one doesn’t need to go to QBist extremes to see that (in Mermin’s words) “this version of the problem is based on an inappropriate reification of the quantum state.” It ought to be obvious from the fact that Feynman’s formulation of the theory is doing just fine—thank you very much—without evolving quantum states.

The second meaning of “quantum measurement problem” mentioned by Mermin concerns the question “whether there can be quantum interference between
quantum states that describe macroscopically distinct physical conditions,” including the challenge of appropriately defining “macroscopically distinct.” Here it is far from obvious that QBists have solved the problem, as against having swept it under the rug or having buried their heads in the sand.

It is true that “the basis for one’s particular quantum-state assignment is always outside the formal apparatus of quantum mechanics,” inasmuch as the theory’s formal apparatus presupposes the events it correlates and thus cannot account for their existence. This, however, leaves us with a consistency problem. If “[t]here is nothing in quantum theory making it applicable to three atoms and inapplicable to \(10^{23}\),” as Peres and Zurek observed, and as the unparalleled range of the theory’s predictions and explanations suggests, then quantum mechanics has to be consistent with the existence of two distinct kinds of property: properties of the first kind, which only exist if and when their existence is indicated by property-indicating events, and properties of the second kind, which exist whether or not they are measured and therefore are capable of indicating the existence of properties of the first kind. QBists are of course free to take “the real world . . . with its objects and events . . . for granted” and not to worry about the consistency of quantum mechanics with the existence of the two kinds of property, but then they have no right to claim they have solved this version of the measurement problem. There is a difference between not worrying about a problem and solving it.

12. Macroscopic Objects

The objective world is not the epistemically inaccessible (type-I) world—the world as it is in itself, unperceived and unconceived by us. Nor is it the world as we perceive it. The objective world is the world as we conceive it, and if we want our conception of it to be consistent with the quantum-mechanical correlations laws, then we must not conceive of it as if it were directly accessible to our senses on all scales, as is the imaginary world of classical physics. We must not conceive of it as being spatially differentiated “all the way down.” What renders quantum theory’s consistency problem insoluble is precisely the assumption that the spatial differentiation of the objective world is complete, and what makes it possible to demonstrate the internal consistency of quantum mechanics—i.e., its consistency with the existence of properties of both kinds—is quantum mechanics itself, for it implies, via the indeterminacy principle, that the spatial differentiation of the objective world cannot be complete.

Here is how. One point on which I am in full agreement with QBists is that probability 1 is not sufficient for “is” or “has”; a property of the first kind is
possessed only if (and only when) it is measured. Measured values do not pre-exist the act of measurement. A measurement does not merely ‘read off’ the values, but enacts or creates them by the process itself. A so-called region of space, therefore, cannot become the property of a particle unless it is realized by being monitored by a detector. A detector is needed not only to indicate the presence of a particle in a region of space but also—and in the first place—to realize a region of space, so as to make it possible to attribute to the particle the property of being inside. Speaking more generally, a measurement apparatus is needed not only to indicate the possession of a property by a quantum system but also—and in the first place—to realize a set of properties so as to make them available for attribution to the system.

If a detector is needed to realize a region of objective space, objective space cannot be conceived as intrinsically differentiated or partitioned. It can be conceived as partitioned only as far as regions of space can be realized, i.e., to the extent that the requisite detectors are physically realizable. This extent is limited by the indeterminacy principle, which forbids the realization of sharply localized and sharply bounded regions. Objective space therefore cannot be modelled as an actually existing transfinite manifold of triplets of real numbers or of the spatial points they designate. Any attempt to partition it into progressively smaller regions will come to a halt when the detectors needed for the purpose cease to exist. It will reach a point beyond which the distinctions we make between regions of space can no longer be objectified. The spatial differentiation of the objective world is therefore incomplete; it does not go “all the way down.”

In an incompletely differentiated objective world, there will be objects whose position distributions are and remain so narrow that there are no detectors with narrower position distributions. If anything truly deserves the label “macroscopic,” it is these objects. While arguments based on environmental decoherence cannot solve the objectification problem, or can solve it only “for all practical purposes,” they quantitatively support the existence of macroscopic positions—positions whose indefiniteness is never revealed in the only way it could be revealed—through a manifest departure from what the classical laws predict. Macroscopic objects follow trajectories that are only counterfactually indefinite. Their positions are “smeared out” only relative to an imaginary spatiotemporal background that is more differentiated than the objective world. The testable correlations between the outcomes of measurements of macroscopic positions are therefore consistent with both the classical and the quantum laws. It is because these correlations are consistent with the classical laws that we can attribute to the values of macroscopic positions a

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k This way of putting it is potentially misleading in that it suggests that a measured value “post-exists” the act of measurement. What post-exists is the effects of the outcome-indicating event—the traces it leaves in the objective world.

l The verb is used here in the epistemological sense of “representing a concept concretely, or presenting it as an object,” not in the peculiar sense in which it is used by measurement theorists.
measurement-independent reality, and it is this that allows us to attribute to these positions the measurement-independent reality that is needed to avoid “von Neumann’s catastrophe of infinite regression.”

13. Why Rule B?

\textit{I received a telephone call one day at the graduate college at Princeton from Professor Wheeler, in which he said, “Feynman, I know why all electrons have the same charge and the same mass.” “Why?” “Because, they are all the same electron!”}

—Richard P. Feynman

\textit{Omnibus ex nihilo ducendis sufficit unum. —Gottfried Wilhelm Leibniz}

As Bernard d’Espagnat has argued eloquently and convincingly (Refs. \[\text{S1} - \text{S2}\]), quantum mechanics requires us to make a clear-cut distinction between two kinds of reality, a “veiled” reality and an empirical one. This is the same distinction that was introduced in Sect. \[\text{S1}\]. Without interposing an empirical (type-II) reality between the empirically inaccessible (type-I) reality and the perceiving, conceiving, and objectifying subject, we may be left to choose between two extremes of which it is hard to say which is worse, the objectivist extreme of \(\psi\)-ontology and the subjectivist extreme of QBism.

The reality that is not directly accessible to our senses, however, may not be altogether inaccessible or veiled. Though all the testable statements we can make about it concern correlations between system preparations and measurement outcomes, there is something we can learn about it from these correlations. Why, we need to ask ourselves, do we have to add amplitudes, rather than probabilities, whenever Rule B requires us to do so? This question has a straightforward answer:

\textbf{Interpretation} Whenever quantum mechanics instructs us to use Rule B, the distinctions we make between the alternatives cannot be objectified. They correspond to nothing in the objective world.

In the context of the two-slit experiment discussed by Feynman, what cannot be objectified (under the conditions stipulated by Rule B) is the distinction we make between “the electron went through the left slit” and “the electron went through the right slit.” It is because this distinction cannot be objectified that objective space cannot be an intrinsically differentiated expanse.

In the context of the EPR–Bohm experiment, the (logically rather than temporally) intermediate measurement determines whether the initial spins of the two particles are, respectively, “up and down” or “down and up” with regard to some specified axis. In this case what cannot be objectified is the distinction we make between these two possibilities.

In the context of the scattering experiment illustrating the “miraculous identity of particles of the same type,” what has no objective answer is the question: which outgoing particle is identical with which incoming particle? There are no
transtemporal individuators, whether they be individuating properties or individuating substances (carriers of properties). Nothing therefore stands in the way of the claim that the incoming particles (and therefore the outgoing ones as well) are one and the same entity. What’s more, there is no compelling reason to believe that this identity ceases when it ceases to have observable consequences owing to the presence of individuating properties. We are free (though of course not forced) to take the view that, intrinsically, each particle is numerically identical with every other particle; what presents itself here and now with these properties and what presents itself there and then with those properties is one and the same entity. In what follows I shall simply call it “Being.” If you prefer any other name, be my guest.

14. Manifestation

We should be relentless in asking ourselves: From what deep physical principles might we derive this exquisite mathematical structure? Those principles should be crisp; they should be compelling. They should stir the soul.

—Christopher A. Fuchs

Felix qui potuit rerum cognoscere causas.

—Virgil, Georgics, Book 2

One reason it is so hard to beat sense into quantum mechanics is that the theory answers a question we are not in the habit of asking. Instead of asking what the ultimate constituents of matter are and how they interact and combine, what we should ask is: how are forms manifested? This question too has a straightforward answer. The shapes of things are manifested with the help of reflexive spatial relations. By entering into (or entertaining) reflexive spatial relations, Being gives rise to what looks like a multiplicity of relata if the reflexive quality of the relations is ignored.

While the non-relativistic theory allows us conceive of a physical system as being composed of a definite number of parts, and to conceive of its form as being composed of a definite number of spatial relations (to which values can be attributed only if and when they are measured), the relativistic theory requires us to treat the number of a system’s parts as just another quantum observable, which has a definite value only if and when it is measured. There is therefore a “deeper” level of reality at which a quantum system is always one, the number of its parts having a definite value only if and when it is measured.

To my mind, the most fruitful way to understand Bohr’s distinction between the classical domain and the non-classical or quantum domain is that it is essentially a distinction between the manifested world and its manifestation. The manifested world consists of the relative positions that exist between macroscopic objects, as defined in Sect. 12, and of whatever supervenes on them. While it allows itself to be conceived as a world of interacting objects and causally connected events, its manifestation cannot be understood in these terms. There is, however, something we
can learn from the quantum-mechanical correlations between system preparations and measurement outcomes, and this is that the number of the world’s “ultimate constituents” is one. An intrinsically undifferentiated and therefore intrinsically unqualifiable Being manifests the world by entering into reflexive spatial relations.

The manifestation of the world consists in a transition from a condition of complete indefiniteness and indistinguishability to a condition of complete definiteness and distinguishability, and what occurs in the course of this transition—what is not completely definite or distinguishable—can only be described in terms of what is completely definite and distinguishable. This is why the manifestation of the world can only be known through the quantum-mechanical correlations between system preparations and measurement outcomes. What is instrumental in the manifestation of the world can only be described in terms of its result, the manifested world.

The least definite stage of this transition, probed by high-energy physics, is known to us through correlations between the counterfactual clicks of non-existent detectors, i.e., in terms of transition probabilities between in-states and out-states. At energies low enough for atoms to be stable, it becomes possible to conceive of objects with fixed numbers of components, and these we describe in terms of correlations between the possible outcomes of unperformed measurements. Molecules, arising at the next stage, are the first objects with forms that can be visualized, to wit, their atomic configurations. But it is only the finished product—the manifested world—that gives us the actual detector clicks and the actual measurement outcomes which allow us to study the correlations in terms of which quantum mechanics describes the various stages of the process of manifestation.

This comes to saying that quantum mechanics presents us with a so far unrecognized kind of causality—unrecognized, I believe, within the scientific literature albeit well-known to metaphysics, for the general philosophical pattern of a single world-essence manifesting itself as a multiplicity of physical individuals is found throughout the world. The causality that effects the transition from the complete indefiniteness of Being to the definiteness of the manifested world, in which a detector either clicks or does not click, is the only kind of causality that is applicable to the quantum domain, inasmuch as the causality that links states or events across time or spacetime has meaningful application only within the manifested world.

When I speak of an intrinsically undifferentiated (and therefore intrinsically unqualifiable) Being that manifests the world by “entering into reflexive relations,” am I not speaking of a transcendental (type-I) reality—a reality that we do not

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80As always, we should keep in mind that saying that an atom is “in” a certain quantum state is the same as saying that certain measurements have been made, and that their outcomes define an algorithm—the so-called quantum state—which serves to assign probabilities to the possible outcomes of any subsequent measurement. Here it should also be mentioned that unperformed measurements need not be capable of being performed, and that, consequently, “possible” outcomes need not be capable of being obtained.

81Some of its representatives in the Western hemisphere are the Neoplatonists, John Scottus Eriugena, and the German idealists. The quintessential Eastern example is the original (pre-illusionist) Vedanta of the Upanishads.
perceive and therefore should not presume to be able to conceive? After all, as the philosopher–poet Xenophanes already observed some twenty-five centuries ago, even if my conceptions represented the world exactly as it is, I could never know that this is the case.

To say that our models of reality are mental constructs is to state the obvious, but it does not explain why most of our theoretical constructs turn out to be non-objectifiable. There is something about the (type-I) reality at the origin of our perceptions that permits the objectification of only a very specific set of mental constructs. From this “license” we cannot conclude that the “world as we know it” faithfully represents the “world in itself”—we cannot conclude that the former matches the latter. We may, however, conclude that the former fits the latter.

The distinction between a match and a fit was introduced by the epistemologist Ernst von Glasersfeld. He illustrated it by imagining a skipper who, in the dark of a stormy night, without navigational aids, passes a narrow strait whose contour he does not know. Epistemologically, we are in the skipper’s position. If he reaches the open sea without mishap, he has found a course that fits the strait; if next time he takes the same course, he will again pass safely. What he has not obtained is a map that matches the coastline. To precisely locate at least one point of the coastline, he must come into contact with it—at the risk of wrecking his ship. My contention is that the ideas put forward here and in greater detail in Refs. 10 and 11 fit the reality at the origin of our perceptions. No interpretation of quantum mechanics can achieve more than that.

15. The Question of Nonlocality

Another celebrated part of the muddle produced by the exclusion of the perceiving subject is ‘quantum non-locality’, the belief of some quantum physicists and many mystics, parapsychologists and journalists that an action in one region of space can instantly alter the real state of affairs in a faraway region.

—N. David Mermin

Fuchs et al. are surely right in calling quantum nonlocality “an artifact of inappropriate interpretations of quantum mechanics”—if what is meant by “quantum nonlocality” is that an action in one region of space can instantly alter the real state of affairs in a faraway region. But nonlocality is much more than that: it is first of all a question, and disposing of the wrong answer does not amount to giving the correct answer, nor does it dispose of the question. As was pointed out in Sect. 9 we need to understand why local explanations fail, not sweep the question under the rug by declaring that quantum correlations refer only to timelike separated events, as is done by Fuchs et al.

In point of fact, quantum correlations between events in timelike relation are as inexplicable as quantum correlations between events in spacelike relation. While we know how to calculate either kind of correlation, and therefore know how to calculate the probabilities of possible events on the basis of actual events, we know as
little of a physical process by which an event here and now contributes to determine
the probability of a later event happening here as we know of a physical process by
which an event here and now contributes to determine the probability of a distant
event happening now. “Quantum nonlocality” refers not only to the mystery posed
by the reality of synchronic correlations but also to the mystery posed by the reality
of diachronic ones.

The so far unrecognized kind of causality associated with the process of mani-
festation casts new light on these mysteries. The reason why local explanations
do not work may be the same as the reason why the manifestation of the world
including its spatiotemporal aspects cannot be explained by processes that connect
events within space and time. The causality connecting events within space and
time being part of the manifested world, it cannot be invoked to explain the mani-
festation of the world, including the correlations that exist between events in space
and time. Being part of the world drama, this causality does not take part in setting
the stage for it. Attempts to use it to explain the quantum-mechanical correlation
laws are putting the cart before the horse; it is the quantum-mechanical correlation
laws that determine the extent to which traditional (spatiotemporal) concepts of
causality are of use.

Fuchs et al. attribute the claim that quantum mechanics is nonlocal to the
denial of “at least one of three fundamental precepts of QBism”:

(1) A measurement outcome does not pre-exist the measurement. An out-
come is created for the agent who takes the measurement action only when
it enters the experience of that agent. The outcome of the measurement is
that experience. Experiences do not exist prior to being experienced.
(2) An agent’s assignment of probability 1 to an event expresses that agent’s
personal belief that the event is certain to happen. It does not imply the
existence of an objective mechanism that brings about the event. Even
probability-1 judgments are judgments. They are judgments in which the
judging agent is supremely confident.
(3) Parameters that do not appear in the quantum theory and correspond
to nothing in the experience of any potential agent can play no role in the
interpretation of quantum mechanics.

Fuchs et al. are right about three things: (i) A measurement outcome does not pre-
exist the measurement but rather is created by the measurement. (ii) An assignment
of probability 1 to a possible outcome does not imply the existence of the property
or value that would be indicated by the outcome if the measurement were made.
(iii) The reason why hidden variables are hidden is that they do not exist. The
rest—the song and dance about outcomes being experiences—is nothing short of a
cop-out. Since the celebrated part of the muddle is not produced by the exclusion
of the perceiving subject, it cannot be cleared up by the inclusion of it.
16. Probability

What do we gain for our theoretical conceptions by saying that along with each actual event there is a ghostly spirit (its “objective probability,” its “propensity,” its “objective chance”) gently nudging it to happen just as it did? Objects and events are enough by themselves. —Christopher A. Fuchs

In what seems like a previous incarnation, Mermin noted that quantum mechanics “is the first example in human experience where probabilities play an essential role even when there is nothing to be ignorant about.” This led him to ask:

How is probability to be understood as an intrinsic objective feature of the physical world, rather than merely as a tactical device for coping with our ignorance? How is one to make sense of fundamental, irreducible correlation?

The idea that probability is an intrinsic objective feature of the physical world is anathema to the QBists, for whom a probability is intrinsically a subjective degree of confidence or belief. Today, Mermin shares the belief that “probabilities in quantum mechanics, like all probabilities, express the willingness of the agent using them to take or place bets. That willingness is based on personal judgment, informed by the agent’s beliefs.”

The argument about whether probabilities are subjective or objective appears to me rather pointless. Just as there are valid reasons to think of the pure forms of our experience—phenomenal space and phenomenal time—as aspects of an objective world, and just as there are valid reasons to think of the redness of a ripe tomato as an objective property of ripe tomatoes (though this would be anathema to the physicalist), so there are valid reasons to think of quantum-mechanical probability assignments as objective. That the meaning of “space” or the meaning of “red” is rooted in experience prevents neither the expanse of space nor the colour red to be a property of the objective world or of things therein. Why then should the fact that the meaning of probability is rooted in the “feeling” of an individual prevent probabilities to be features of the objective world?

According to de Finetti, as formulated by Hampton et al., probability “expresses

Mermin’s next sentence presents something of a puzzle: “The correlations quantum mechanics describes prevail among quantities whose individual values are not just unknown: they have no physical reality.” If the correlata lack physical reality, what exactly do their correlations refer to? This question has been answered in the previous section: The least definite stage of the transition from a condition of complete indefiniteness and indistinguishability to a condition of complete definiteness and distinguishability is known to us through correlations between the counterfactual clicks of non-existent detectors. At energies low enough for atoms to be stable, it becomes possible to conceive of objects with fixed numbers of components, and these we describe in terms of correlations between the possible outcomes of unperformed measurements.

While it used to be said that qualities such as colours are “nothing but” quantities, it may be closer to the truth to say that quantities are “nothing but” means of manifesting qualities, just as reflexive spatial relations may be said to be “nothing but” means of manifesting forms.
the feeling of an individual and cannot have meaning except in relation to him.”
What would be the alternative? According to Hampton et al., it would be “to assume that the probability of some event has a uniquely determinable value.”
And what would be the alternative to claiming, as Fuchs does, that a probability is “a feeling, an estimate inside” an agent? It would be to claim that a probability is “a solid object, like a rock or a tree that the agent might bump into.”

Inasmuch as the probability of obtaining a possible measurement outcome depends on the particular set of actual outcomes on the basis of which it is assigned, it lacks a uniquely determinable value. I don’t see how this eliminates all possible alternatives to the Bayesian way of thinking about probability. Quantum-mechanical probabilities can be objective without implying ghostly spirits that gently nudge events to happen as they do. They can be objective in the sense that they are determined by combinations of objective events and by correlation laws that owe nothing to the discretion of agents. That different sets of actual outcomes are accessible to Alice and Bob in their respective spatiotemporal locations does not mean that quantum-mechanical probabilities exist only in their respective minds—any more than space and time do. Nor does the commonplace that a probability is not a solid object—“like a rock or a tree that the agent might bump into”—leave us with no choice but to conclude that a probability is a feeling or estimate “inside” an agent.

False dilemmas also underlie the following statements or their invocations:

The inability of the earlier, classical and frequentist, definitions to deal with statements referring to unique events indicates why the probability required in a decision analysis is necessarily one that has been subjectively assessed.

The troubled history of quantum foundations is a telling indictment of the proposition that probabilities, even including probabilities 1 and 0, are not personal judgments but facts, backed up by objective features of the world.

Since in quantum mechanics we are not dealing with unique events but with events whose probabilities can be measured using ensembles of identically prepared systems (like any other quantity, with limited precision), it does not follow that

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8The passage in full:

“Bruno de Finetti (1964) believes there is no need to assume that the probability of some event has a uniquely determinable value. His philosophical view of probability is that it expresses the feeling of an individual and cannot have meaning except in relation to him.”

9The passage in full:

“A probability is not a solid object, like a rock or a tree that the agent might bump into, but a feeling, an estimate inside himself.”

10See Refs. 63 (Sect. 7) and 96. Fuchs and Schack cite the latter among a “selection of misunderstandings” of their position, comprising eleven papers, without giving any evidence of what the misunderstandings in this particular paper might be. (There is a difference between misunderstanding the QBist position and not agreeing with it.)
quantum-mechanical probabilities are necessarily ones that have been subjectively assessed. On the contrary, no decision analysis and therefore no subjective assessment is involved. Again, while it is true that probabilities—including probabilities 1 and 0—are neither facts nor backed up by a uniquely determinable quantum state, the troubled history of quantum foundations is a telling indictment, not of the denial that probabilities are intrinsically and necessarily ego-centric degrees of belief but of the claim that the formal apparatus of quantum mechanics is anything other than a probability calculus (or a calculus of correlations). This leaves the meaning of probability up for grabs. (As Appleby [97] remarked in 2005, “[w]hereas the interpretation of quantum mechanics has only been puzzling us for about 75 years, the interpretation of probability has been doing so for more than 300 years.”)

I cannot agree more with Fuchs and Schack [7] when they write:

As the foundations of probability theory dismisses the questions of where data comes from and what constitutes an agent—these questions never even come to its attention—so can the foundations of quantum theory dismiss them too.

All that needs to be established is what von Weizsäcker has called “semantic consistency”: “Semantic consistency of a theory will mean that its preconceptions, how we interpret the mathematical structure physically, will themselves obey the laws of the theory” [98]. To demonstrate the semantic consistency of quantum mechanics means to establish the consistency of the theory’s formal apparatus with the outcome-indicating events whose existence it presupposes and which it serves to correlate—in other words, to establish the consistency of the correlations with their correlata. This consistency is made problematic by the all but universally accepted view that the points and instants on which a wave function depends correspond one-to-one to the elements of an intrinsically and completely differentiated spacetime. As was demonstrated in Sect. [12], it can be established by relinquishing this view. (To establish the consistency of the theory’s formal apparatus with the existence of autonomous agents, on the other hand, one has to give up the physicalist reductionism that aspires to explain life in general and living organisms in particular in terms of “ultimate constituents,” laws of physics, chance occurrences, and natural selection. The question how this might be done lies well outside the foundations of quantum theory [1].) But if the foundations of quantum theory can and should dismiss the questions of where data comes from and what constitutes an agent, then they can and should also be indifferent with regard to the meaning of probability.

Fuchs [3] appears to think that the freedom of the experimenter, which the engineering sciences tacitly assume, has its roots in the mutual autonomy of quantum systems: “if one is willing to throw away one’s belief in systems’ autonomy from each other, why would one ever believe in one’s own autonomy?” This prompts me to ask: if the notion of an agent cannot and should not be derivable from the quantum formalism itself, as Fuchs maintains, why should the autonomy of an agent depend on the autonomy of quantum systems from each other?
What makes “objective probability” problematic is the circularity it shares with attempts to define consciousness, qualia, or even space and time, in terms that owe nothing at all to experience. Consciousness, Stuart Sutherland wrote in the *The International Dictionary of Psychology* [99] “is impossible to define except in terms that are unintelligible without a grasp of what consciousness means.” The *Cambridge Dictionary of Philosophy* [100] similarly notes, after quoting definitions of time by Plato, Aristotle, Plotinus, and Augustine, that these definitions, “like all attempts to encapsulate the essence of time in some neat formula, are unhelpfully circular because they employ temporal notions.” Much the same holds for attempts to define the expanse of space or the blueness of blue.

The circularity of frequentism has been captured by Fuchs [3] in a dialogue between a Bayesian and a frequentist. Invoking the Law of Averages, the frequentist claims that if one repeats a random experiment over and over, independently and under identical conditions, the fraction of trials that result in a given outcome converges to a limit as the number of trials grows without bound. While he admits that “incorrect” frequencies can be obtained as long as the number of trials is finite, he insists that they will be less and less likely. The Bayesian then asks: “Tell me once again, what does ‘likely’ mean?”

By saying that A will occur with probability $2/3$, a Bayesian means that he believes twice as strongly that A will happen as he believes that A will not happen. This could prompt a frequentist to ask: “What do you mean by saying that you believe something twice as strongly as something else? Do you not mean, rather, that you believe that something is twice as likely as something else?”

For the general meaning of probability, we may have to settle for a paraphrase of Augustine’s famous statement about time [101]: “What, then, is time? If no one ask of me, I know; if I wish to explain to him who asks, I know not.” When it comes to the meaning of probability in quantum mechanics, however, where we are dealing with events whose probabilities can be measured using ensembles of identically prepared systems, the frequentist approach appears to me to be the more rational, for I find it irrational to deny that $|N_H - N_T|$ increases more slowly than $N = N_H + N_T$, the sum of the number of heads and the number of tails obtained in $N$ tosses of a coin, or (what comes to the same) to deny that the fractions of heads and tails converge. But then, at the end of the day, when all is said and done, rationality itself may be but a feeling.

17. **Indeterminism or Indeterminacy?**

*QBism finds its happiest spot in an unflinching combination of ‘subjective probability’ with ‘objective indeterminism.’* —Christopher A. Fuchs [3]

Old habits die hard. In a deterministic world, physics separates neatly into kinematics, describing the state of a system at a given time or on a given spacelike hypersurface, and dynamics, describing the state’s evolution in time. In the indeterministic world of quantum physics, the division of labour into states and their
evolution is an anachronism. Quantum states are probability algorithms, not evolving instantaneous states. The remedy does not consist in relocating evolving states in the minds of agents but in getting rid of them. As stressed not least by Asher Peres,\textsuperscript{102} “there is no interpolating wave function giving the ‘state of the system’ between measurements”—not even in the minds of agents.

In physics, what is usually meant by “determinism” is the notion that the state of the world at any given time is fully determined by the state of the world at any other (usually but not necessarily earlier) time, and what is usually meant by “indeterminism” is the notion that this is not the case. If one nevertheless believes in an evolving instantaneous state of the world, one is led to believe in von Neumann’s alternation between unitary evolution and “collapse.” This fantasy relinquished, “objective indeterminism” can only refer to the irreducibly statistical character of the quantum-mechanical calculus of correlations. Whence this irreducibly statistical character? Since this is, in fact, a consequence of the existence of mutually incompatible observables, the obvious next question is: who ordered that? What is the “little more”\textsuperscript{12} that requires the existence of mutually incompatible observables?

As said, a measurement apparatus is needed not only to indicate the possession of a property by a quantum system but also—and in the first place—to realize a set of properties so as to make them available for attribution to the system. If it is to perform its dual function, it must have the following properties. It must be in possession of a form and, like every form, its form has to be manifested through spatial self-relations of an intrinsically formless Being. If the reflexive quality of the relations is ignored, it will therefore appear to be “made” of a finite number of formless relata—particles that do not “occupy” space. Yet it itself must “occupy” a finite volume, and it must be stable for a significant amount of time. The question we thus need to ask is: what does the existence of objects with these properties entail?

It entails, \textit{inter alia}, the existence of mutually incompatible observables (Ref. \textsuperscript{14} and Ref. \textsuperscript{16} Chap. 8). The argument in brief: the fact that “ordinary” objects (including measuring devices) occupy as much space as they do depends, \textit{inter alia}, on the fact that atoms occupy as much space as \textit{they} do, and the fact that the simplest atom occupies as much space as it does depends, \textit{inter alia}, on the existence of an inequality of the form

\[ \Delta r \Delta p \geq k, \]

where \( \Delta r \) quantifies the indeterminacy of the radial component of the electron’s position relative to the proton, \( \Delta p \) quantifies the indeterminacy of the corresponding momentum, and \( k \) is a positive quantity whose value depends on the units in which \( r \) and \( p \) are measured. The existence of measuring devices—and thus the very consistency of the quantum-mechanical correlation laws with the existence of their correlata—requires that positions and the corresponding momenta be incompatible.

While relations of the form (10) are essential for the stability of atoms, what makes atoms occupy as much space as they do is the indeterminacy of their internal
relative positions. What, then, is the meaning of calling a measurable quantity or its value “indefinite” or “indeterminate”? Think of the internal relative position (or momentum) of a hydrogen atom that has been prepared by a simultaneous measurement of its energy, its total angular momentum, and one component of its angular momentum—a hydrogen atom that, in common parlance, is “in” one of its stationary states. As long as no measurement is made, all we have to go by is probability distributions over possible outcomes of unperformed measurements. But that is enough. All we are saying by saying that a measurable quantity is indefinite is that it is prepared by earlier measurements without being subsequently measured, and that the prepared probability distribution has positive dispersion.

But if atoms (and hence “ordinary objects”) occupy as much space as they do because of the indeterminacy of their internal relative positions, then indeterminacy is closer to the foundations of quantum mechanics than indeterminism—the irreducibly statistical character of the quantum-mechanical calculus of correlations. Quantum indeterminism is an observable consequence of quantum indeterminacy in the same way that relative frequencies are observable consequences of probabilities. This ought to be reason enough to consider indeterminacy to be as objective as indeterminism, and to consider the probabilities in terms of which indeterminacy is defined to be objective as well. This is a far cry from saying that objective probabilities are relative frequencies, or that they are determined by a single objective quantum state, or that events happen as they do because of some gentle nudging by ghostly spirits. If quantum-mechanical probabilities are also subjective, it is because they have this in common with consciousness, qualia, and the qualitative aspects of space and time, that our knowledge of them is “knowledge by acquaintance” or “tacit knowledge,” rather than “knowledge by description” or “explicit knowledge.”

18. Concluding Assessment

While QBism must be lauded for insisting that the formal apparatus of quantum mechanics is a probability calculus, which opens the door to a deeper understanding of what quantum mechanics is trying to tell us, it again closes this door by its insistence on the Bayesian interpretation of probability in the context of quantum foundations.

Of the fourteen QBist affirmations listed in Sect. 4, some were found to be true or partly true, some false or partly false, and some irrelevant or partly irrelevant. The first two affirmations are irrelevant if not false. Points 9, 10, 11, and 13 are

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true, and points 4, 6, and 7 are partly true. Specifically, it is true

- that probabilities are not intrinsically measures of ignorance, and that what is responsible for uncertainty is not necessarily lack of knowledge of some true state of affairs (point 4),
- that the Born rule can be used iteratively to obtain new probabilities from old, and that it is not a method of setting probabilities by some external entity called “the quantum state” (point 6),
- that making a quantum state assignment may be user-dependent in the sense of being dependent on the measurement outcomes accessible to a particular user, that it is equivalent to assigning probabilities to the outcomes of a well-selected set of measurements, and that there can be no such thing as a true (or real, or objective) quantum state (point 7).

Points 3, 5, 8, 12, and 14, on the other hand, are false. Specifically, there are external criteria for declaring a probability judgment right or wrong: they are the objective data on the basis of which probabilities are assigned (point 3).

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