Dynamical Casimir effect and the possibility of laser-like generation of gravitational radiation

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December 22, 2017

Abstract: In this paper, we address the question as to whether or not measurable sources for gravitational waves could possibly be made in the laboratory. Based on an analogy of the dynamical Casimir effect with the stimulated emission of radiation in the laser, our answer to this question is in the affirmative, provided that superconducting radio-frequency cavities in fact possess high quality factors for both electromagnetic and gravitational microwave radiation, as one would expect due to a quantum-mechanical gravitational Meissner-like effect. In order to characterize the response of matter to tensor gravitational fields, we introduce a prefactor to the source term of the gravitational wave equation, which we call the “relative gravitational permeativity” analogous to the “relative electric permittivity” and “relative magnetic permeability” that characterize the vector response of matter to applied fields in electromagnetism. This allows for a possibly large quantum mechanical enhancement of the response of a superconductor to an incident tensor gravitational wave field. Finally, we describe our experimental work with high-Q superconducting radio-frequency cavities, and propose a design for a coupled-cavity system with a flexible superconducting membrane in its middle as its amplifying element. This will then allow us to test for a Meissner-like expulsion, and therefore reflection, of incident tensor gravitational wave fields, and, above a certain threshold, to generate coherent gravitational radiation via the dynamical Casimir effect.

Text: The 2017 Nobel prize in physics was awarded for observations of gravitational waves arising from the inspiral of black hole pairs \cite{1,2}. Recently, the emission of gravitational waves was also observed due to the inspiral of a pair of neutron stars, along with the simultaneous observations of gamma ray and optical detections of the same event from the same source \cite{3}.

The question naturally arises: Is it possible to generate gravitational radiation in the laboratory? A common response to this question is the one given by Misner, Thorne, and Wheeler (MTW), in their classic text \cite{4}:
"The construction of a laboratory generator of gravitational radiation is a non-attractive enterprise in the absence of new engineering or a new idea or both."

This response was a result of Einstein’s calculation of the power emitted in gravitational radiation $P_{\text{Einstein}}$ by a rotating steel beam, which was based on his quadrupole formula $[4][5]$

$$P_{\text{Einstein}} = \frac{G}{45c^5} \left\langle \dot{Q}_{ij}^2 \right\rangle$$

(1)

where $G$ is Newton’s constant, $c$ is the speed of light, and where $[5]$

$$Q_{ij} = \int \rho \left( 3x_i x_j - \delta_{ij} x_k x_k \right) dV$$

(2)

is the mass quadrupole tensor (Einstein’s summation convention is being used here, with Latin indices denoting spatial dimensions).

Einstein $[6]$ calculated the gravitational radiation emitted from a massive steel beam with a length of 20 meters and a radius of 1 meter, whose mass is $4.9 \times 10^5$ kilograms, rotating end-over-end around its midpoint at its maximum possible angular velocity near its breaking point, which is determined by the tensile strength of steel, $3 \times 10^6$ J m$^{-2}$. Then the maximum possible angular velocity of the steel beam due to its tensile strength is 28 radians per second, and the gravitational radiation power predicted by the quadrupole formula (1) turns out to be

$$P_{\text{Einstein}} \simeq 2.2 \times 10^{-29} \text{ Watts}$$

(3)

which is a miniscule amount of power. The basic reason for this arises from the fact that the prefactor

$$G/c^5 \sim 10^{-53} \text{ (Watts)}^{-1}$$

(4)

in Einstein’s quadrupole formula (1), is an extremely small number. This is a consequence of the fact that Newton’s constant $G$, which is a tiny number to begin with, is combined with the inverse quintic power of the speed of light $c$, which is yet a much tinier number.

Put differently, there arises a characteristic power $P_{\text{GR}}$ in general relativity which is given by the fundamental constants $G$ and $c$ in the combination

$$P_{\text{GR}} \equiv \frac{c^5}{G} = 3.6 \times 10^{52} \text{ Watts}$$

(5)

As pointed out by MTW in the beginning of their classic text $[4]$, the only place where such enormous powers could occur naturally is in astrophysical sources, such as in supernova explosions. In fact, the first direct observation of gravitational waves by LIGO $[2]$, was in the merger of a pair of massive black holes orbiting each other at relativistic speeds, an extreme case of an astrophysical source. Thus it would seem impossible, for all practical purposes, for gravitational radiation power to ever be produced in laboratory sources.
However, note that Planck’s constant $\hbar$ is absent from Einstein’s quadrupole formula (1) for the emission of gravitational radiation. Could the “new engineering or a new idea or both,” as suggested in the above quotation from MTW, be “quantum engineering,” in which $\hbar$ somehow replaces $G$ and $c$, so that the necessity for the use of astrophysical sources for the generation of gravitational waves could somehow be avoided? Here we propose a possibly affirmative answer to this question that involves the laser-like generation of gravitational radiation via the process of the dynamical Casimir effect [7].

The starting point for this new “quantum engineering” approach to the generation of gravitational waves is the assumption that the uncertainty principle

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

(6)

leads to the existence of zero-point fluctuations with the zero-point energy

$$E_0 = \frac{1}{2} \hbar \omega$$

(7)

for any kind of wave which oscillates with a frequency $\omega$. In particular, this zero-point energy should apply to gravitational waves, as well as to electromagnetic waves. In the case of gravitational waves, note that the size of the zero-point energy (7) is independent of Newton’s constant $G$ and of the speed of light $c$. Rather, it depends solely on Planck’s constant $\hbar$ and the frequency $\omega$. Although $\hbar$ is a tiny number, its tininess can be compensated for by the exponential growth of the gravitational wave arising from the process of stimulated emission of radiation, just like in the case of the laser.

Stimulated emission of gravitational-wave quanta, i.e., of gravitons, follows from a quantum treatment of the radiation oscillators [8] that result from a linearization of the theory of general relativity [9], in which the metric tensor $g_{\mu\nu}$ is decomposed into the Minkowski metric tensor components $\eta_{\mu\nu} = \text{diag} (-1, +1, +1, +1)$, which are large, and the metric deviation tensor components $h_{\mu\nu}$, which are small, viz.,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

(8)

The small, simple harmonic motion of the linearized gravitational radiation oscillators can be quantized by means of the standard quantization condition

$$[a_G, a_G^\dagger] = 1$$

(9)

where $a_G$ is the annihilation operator for a given gravitational radiation oscillator, and $a_G^\dagger$ is the creation operator for the same radiation oscillator (all other commutators for nonidentical radiation oscillators being set equal to zero). It follows from the canonical commutator (9) that

$$a_G^\dagger |n_G\rangle = \sqrt{n_G + 1} |n_G + 1\rangle$$

(10)

where $|n_G\rangle$ is the number state containing $n_G$ gravitons in a given mode of the radiation field (i.e., an excitation of a given radiation oscillator with $n_G$ quanta),
Figure 1: Sketch of the dynamical Casimir effect ("DCE"). A Fabry-Perot cavity consists of two parallel mirrors M1 and M2. Mirror M1 is moving back and forth sinusoidally with a time-dependent displacement $x(t)$ relative to the mirror M2, which is a stationary mirror. The piston-like pumping action of M1 upon the vacuum fluctuations contained inside the Fabry-Perot cavity amplifies them parametrically so that they will become macroscopically observable radiation (indicated in yellow) that fills up this resonator.

and $|n_G + 1\rangle$ is the number state containing $n_G + 1$ gravitons. As Feynman has pointed out in [10], the creation of an extra radiation quantum with the probability amplitude of $\sqrt{n_G + 1}$ in the recursion relationship [10] leads to the process of stimulated emission of radiation. Hence the recursion relationship [10] implies the possibility of the laser-like generation of gravitational radiation.
The action of the moving mirror is like that of a moving piston which pushes and pulls on a gas of photons or gravitons contained within the resonator. Thus the piston can impart energy into this gas. As a result, the action of the piston can parametrically amplify the radiation contained inside the resonator [13], so that it can become, via an exponential growth mechanism [7], a macroscopically observable beam of coherent radiation, just like in a laser.

Nation et al [14] have pointed out that the quantum amplification process in the dynamical Casimir effect (DCE) is equivalent to the amplification process in a parametric amplifier (paramp), such as the one in the “triple” cavity paramp configuration illustrated in Figure 2, in which a membrane is pumped into mechanical motion by the radiation pressure from pump microwaves in the leftmost “single” cavity. This membrane moves like the moving mirror M1 in Figure 1 with a sinusoidal displacement $x(t)$ that amplifies the signal and idler waves inside the “double” cavity on the right side of the membrane [15].

For this and similar paramps, Nation et al [14] give the following threshold:

$$v_{\text{max}} \geq \frac{c}{Q} \quad (11)$$

where $v_{\text{max}}$ is the threshold speed of the moving mirror in Figure 1, or of the
moving membrane in Figure 2, where \( c \) is the vacuum speed of light, and where \( Q \) is the quality factor of the cavity for producing the DCE. The maximum velocity amplitude of the moving membrane at threshold is given by

\[
v_{\text{max}} = \Omega \varepsilon_{\text{max}}
\]

where \( \Omega \) is the angular frequency of the sinusoidal mechanical motion of the moving mirror (i.e., of mirror M1 in Figure 1, or of the moving membrane in Figure 2), and where \( \varepsilon_{\text{max}} \) is the maximum displacement in the sinusoidal motion of this mirror at threshold.

For superconducting radio frequency ("SRF") cavities with \( Q \) on the order of \( 10^{10} \), we see the \( v_{\text{max}} \) will be on the order of centimeters per second, which is clearly a non-relativistic velocity scale that is readily achievable under laboratory conditions. Therefore although the generation of radiation is a relativistic effect, the motion of the mirror that generates the DCE at its threshold is non-relativistic due to the high quality factors of SRF cavities. One can understand the non-relativistic threshold condition (11) as arising from a multiple-imaging effect, along with its cumulative Doppler shifts, that occurs repetitively between the moving mirror M1 and the fixed mirror M2 of the Fabry-Perot cavity in Figure 1 [17].

Converting (11) into an expression for the kinetic energy in the motion of a mirror with a mass \( m \), we find

\[
U_{\text{kin}} = \frac{1}{2} m v_{\text{max}}^2 \geq \frac{1}{2} mc^2 \frac{1}{Q^2}
\]

If the mirror M1 in Figure 1 were to be driven on its left side by radiation pressure from a pump wave stored inside a separate, high-\( Q \) pump cavity on the left side of M1 (not shown in Figure 1, but shown in Figure 2), then by energy conservation, we find that the required amount of pump power that needs to be injected into the pump cavity for the DCE at threshold, would be

\[
P_{\text{thres}} \geq \frac{U_{\text{kin}}}{\tau_p} = \omega_p \frac{U_{\text{kin}}}{Q_p}
\]

where \( \tau_p \) is the "cavity ring-down time" for the energy stored inside the pump cavity. The last equality follows from the fact that the pump-cavity quality factor \( Q_p \) is related to the pump cavity ring-down time \( \tau_p \) by \( Q_p = \omega_p \tau_p \), where \( \omega_p \) is the angular frequency of the pump wave.

For the "triple-cavity" paramp pictured in Figure 2 whose membrane (red) is being pumped from the left by a radiation pressure force, the frequency of the mechanical motion of this moving membrane will be at the second harmonic \( 2\omega_p \) of the pump frequency. The meaning of the equality in (14) is that in steady-state equilibrium, the amount of pump power being injected into the "single" pump cavity through the left porthole of this cavity, must be balanced by the amount of mechanical power leaking away from the system due to the fact that pump waves which are driving the motion of the membrane, will also
be escaping from the “single” pump cavity through the same porthole, or will be lost into heat.

Now if we set \( Q_p = Q \) (i.e., that the pump and the DCE cavities to the left and to the right of the moving membrane in Figure 2, will have comparable \( Q \) values), then it follows from \([13]\) and \([14]\) that the injected pump power for achieving threshold for the DCE should be

\[
P_{\text{thres}} \geq \frac{1}{2} \frac{m \omega_p c^2}{Q^3}
\]

(15)

where \( m \) is the mass of the moving mirror, \( \omega_p \) is the pump frequency, and \( Q \) is the quality factor of cavities. Note that this DCE threshold power scales inversely as the cube of the \( Q \) value of the pump and the DCE cavities. This points out the importance of utilizing cavities with the highest possible \( Q \) values in order to be able to achieve the DCE with reasonable pump powers. Therefore SRF cavities with \( Q \approx 10^{10} \) \([16]\) would be good candidates for this purpose.

A more detailed derivation of the threshold power (15) is given in \([7]\), and yields the following result:

\[
P_{\text{thres}} \geq \frac{m \omega_p \omega_s \omega_i L^2}{4 Q_p Q_s Q_i}
\]

(16)

where \( m \) is the mass of the moving membrane, where \( \omega_p, \omega_s, \) and \( \omega_i \) are respectively, the pump, signal, and idler frequencies of the “triple” cavity depicted in Figure 2, where \( L \) is the length of the “double” cavity in Figure 2, and where \( Q_p, Q_s, \) and \( Q_i \) are respectively, the pump, signal, and idler quality factors of the three tandem SRF cavities that constitute the “triple” paramp cavity.

Numerically, if we assume that

\[
m = 3 \text{ milligrams} \quad (17)
\]
\[
\omega_p \approx \omega_i \approx \omega_s \approx 2\pi \times 10 \text{ GHz} \quad (18)
\]
\[
L \approx \lambda_i \approx \lambda_s \approx 3 \text{ cm} \quad (19)
\]
\[
Q_p \approx Q_i \approx Q_s \approx 10^{10} \quad (20)
\]

then we conclude that for observing the DCE, and thus the generation of gravitational microwave radiation, the threshold pump power at a frequency of 10 GHz to be injected through the left hole of the “triple” paramp cavity of Figure 2, needs to be at least

\[
P_{\text{thres}} \approx 0.17 \text{ microwatts} \quad (21)
\]

which is easily achievable experimentally.

A crucial question now arises: How can we construct a high-Q cavity for gravitational radiation, when we know that all known ordinary (that is, non-astrophysical) materials, are essentially completely transparent to this kind of radiation? To answer this question, consider a \((\times)\) polarized GR plane wave incident upon a square piece of SC (yellow square), as pictured in Figure 3(a).
Figure 3: (a) A quadrupolar pattern (blue) of an incident (×) polarized GR plane wave propagating into the page, impinges upon a square piece of superconductor (yellow). Tidal “forces” $F_G$ acting upon the square due to this wave exert a shear stress. (b) The strain of the ionic lattice of the square superconductor (SC) due to this stress causes a slight extrusion of positive (+) charge into corners of a rhombus (orange) produced by the wave, but the Cooper pairs (yellow) of the SC will not respond, since they are all Bose-condensed in a non-localizable, zero-momentum eigenstate. There results an extrusion of negative (−) charge into corners of an undistorted square (yellow), in a “Heisenberg-Coulomb” charge separation effect [18]. The Coulomb attraction of the (+) and (−) charges opposes the tidal “forces” $F_G$, leading to a huge GR-EM coupling.
The strain fields \( h_{\mu\nu} \) of the incident GR wave will interact with the stress-energy tensor \( T^{\mu\nu} \) of the SC via the interaction Hamiltonian density \( H' \) of the interaction
\[
H'_{\text{interaction}} = \frac{1}{2} h_{\mu\nu} T^{\mu\nu}
\] (22)

In particular, the instantaneous spatial components of the transverse-traceless metric deviation tensor \( h^{(x)}_{ij} \) for a \((\times)\) polarized plane wave, described in Cartesian \((x, y)\) coordinates in a plane \( z = \text{constant} \) perpendicular to the +z propagation direction of the wave, are given by the following \( 2 \times 2 \) matrix [1]:
\[
\begin{pmatrix}
  h_{xx} & h_{xy} \\
  h_{yx} & h_{yy}
\end{pmatrix} = h_0(z - ct) \begin{pmatrix}
  +\frac{1}{2} (x^2 - y^2) & xy \\
  xy & -\frac{1}{2} (x^2 - y^2)
\end{pmatrix}
\] (23)

where \((i, j) = (x, y)\) and where \( h_0(z - ct) \) is the dimensionless strain of space due to the passage of the plane wave. A snapshot of the tidal “force” fields that are produced by the metric deviation tensor \( h^{(x)}_{ij} \) (\(x, y, z, t\)) in (23) is represented by the hyperbolae (blue curves) in Figure 3(a). One can easily verify by direct substitution that the Ansatz given in (23) is a transverse-traceless vacuum solution to the wave equation that follows from linearized general relativity, viz.,
\[
\nabla_\perp^2 h^{(x)}_{ij} + \frac{\partial^2 h^{(x)}_{ij}}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 h^{(x)}_{ij}}{\partial t^2} = 0
\] (24)

where \( \nabla_\perp^2 \) is the transverse Laplacian in a Cartesian \((x, y)\) coordinate system, where +z is the direction of propagation of the plane wave \( h^{(x)}_{ij} \) (\(x, y, z, t\)) into the page that is depicted in Figure 3(a), and where \( c \) is the speed of light.

The highly unusual quantum response of the SC square (yellow) to this wave, which we called the “Heisenberg-Coulomb” effect in [18], is illustrated in Figure 3(b). Quantum mechanics on a macroscopic length scale inside the SC takes effect below the SC transition temperature, due to the fact that Cooper pairs are bosons that will all undergo Bose-Einstein condensation into the lowest possible energy state of the system, namely the unique ground state in which all the bosons occupy the same, single-particle zero-momentum eigenstate, relative to the center of mass of the SC (which is represented by the large black dot at the center of the yellow square in Figure 3(b)). Because their momenta will all be exactly known in the zero-momentum eigenstate (their momenta will all certainly be exactly zero), it follows from the Heisenberg uncertainty relations for momentum and position, that the locations of the Cooper pairs inside the SC square will be completely uncertain. Thus the Cooper pairs are all completely non-localizable within the SC square in Figure 3.

It therefore follows from Heisenberg’s uncertainty principle that the Cooper pairs cannot respond at all to the passage of the gravitational plane wave, in contrast to the response to this wave of the ions inside the ionic lattice, which are all completely localizable, since they will be located at the lattice sites of the ionic lattice inside the SC material. Since the microwave frequencies of...
Figure 4: Gravitational Meissner-like effect inside a SC square (yellow) subjected to tidal “forces” $F_G$ from the $(\times)$ polarized gravitational plane wave depicted in Figure 3. The stress-energy tensor $T_{xy}$ at a point along the main diagonal of the rhombus (orange) produced in response to the tidal “forces” $F_G$, is a tensor product of the two supercurrent vector components $v_x$ and $v_y$, both of which decay into the interior on the scale of the London penetration depth.

the incident gravitational plane wave in Figure 3 are typically orders of magnitude higher than the typical acoustical frequencies of the ionic lattice, it follows that the ions will move essentially as freely falling masses along the geodesics of general relativity, in their response to the passage of the plane wave. By contrast, the Cooper pairs are completely nonlocalizable due to the uncertainty principle, and therefore it is forbidden in quantum mechanics for them to follow any classical trajectory, including the geodesics of general relativity. This difference in response of the Cooper pairs and lattice ions has been demonstrated quantitatively in [20][21].

One can arrive at this same conclusion from another point of view. The quantum adiabatic theorem tells us that for any SC sample, the BCS ground state, which is separated from all possible excited states by the BCS energy gap $E_{\text{BCS}}$, cannot respond to any slowly-varying external perturbation whose characteristic frequency lies well below the BCS gap frequency of $f_{\text{BCS}} = E_{\text{BCS}}/2\pi\hbar$. For the case of niobium, $E_{\text{BCS}}$ is around 3 meV, corresponding to a BCS gap frequency $f_{\text{BCS}} \approx 730$ GHz. Therefore any perturbations arising from an incident GR wave whose typical frequency lies in the microwave range of around 10 GHz, which is much less than 730 GHz, cannot cause any transitions out of the BCS ground state. Hence the Cooper pairs inside the SC (niobium) square of Figure 3 will remain rigidly in the BCS ground state, and cannot respond to the incident GR microwaves at 10 GHz that we are using in our experiments.

However, the ions of the ionic lattice of the SC will undergo free fall in
response to the Newtonian tidal “forces” $F_G$ (i.e., the blue hyperbolae in Figure 3(a)). Thus the ionic lattice will undergo a shear strain that distorts the SC square (yellow) into a rhombus (orange), as shown in Figures 3 and 4. This rhomboidal distortion leads to an extrusion of positive ionic charges into the acute corners of the rhombus (orange corners labeled by (+) signs). The overall charge of the SC, however, must remain neutral. Hence the corners of the original square (yellow) (labeled by (−) signs) adjacent to obtuse corners of the rhombus must have compensating extrusions of negative charges arising from the negative charges of the Cooper pairs that remain in these corners during the rhomboidal distortion of the ionic lattice, because of the fact that these pairs must remain adiabatically in their zero-momentum ground state everywhere.

There results a “charge-separation effect” (or “Heisenberg-Coulomb effect”; see below) [18][20][22], in which positive charges appear near the acute corners of the rhombus of Figures 3 and 4, but negative charges appear near the obtuse corners of this rhombus. This leads to a huge Coulomb force of attraction between the separated positive and negative charges that strongly opposes the Newtonian tidal “forces” $F_G$ of the incoming gravitational wave that produced this charge separation in the first place. There arises an enormously stiff effective Hooke’s law, i.e., a strong restoring force inside the SC material, in that there will arise an enormous Coulomb-strength back-action that strongly resists the tidal action of the incoming gravitational plane wave, so much so that this wave is expelled, and therefore reflected, from the SC square, in what we shall call a “gravitational Meissner effect.” Since this effect results from a combination of the Heisenberg uncertainty principle with the huge Coulomb force of attraction that arises from the resulting separation of the ions from the Cooper pairs, we have dubbed this the “Heisenberg-Coulomb effect.” Therefore this effect differs from the usual “charge-separation effect” that occurs in electrically polarized dielectrics in response to the application of an electric field, because, firstly, it is a response to the tensor $h_{ij}$ field of gravitational radiation, and not to the vector electric field of electromagnetism, and, secondly, this response is purely quantum mechanical in nature, and possesses no classical explanation.

According to [18][22][23], the strength of the “Heisenberg-Coulomb” effect is characterized by the ratio of the strength of the Coulomb electrical force between two electrons to the strength of their Newtonian gravitational force

$$\left| \frac{F_{\text{Coulomb}}}{F_{\text{Newton}}} \right| = \frac{e^2}{4\pi \varepsilon_0 r^2 (Gm_e^2/r^2)} = \frac{e^2}{4\pi \varepsilon_0 Gm_e^2} \approx 4.2 \times 10^{42} \quad (25)$$

where $e$ is the electron charge, $\varepsilon_0$ is the electrical permittivity of free space, $G$ is Newton’s constant, and $m_e$ is the mass of the electron (note that the Coulomb and Newtonian forces both obey inverse-law force laws, so that this result is independent of distance $r$ between the two electrons). The ratio given by (25) is obviously a huge dimensionless number.

One surprising consequence of the enormous number [25] predicted in [18] is that it leads to such an enormous enhancement of the reflection process from the SC square, that the SC behaves like a material with hard-wall boundary
conditions with respect to the incident gravitational wave, and thus behaves like a highly reflective mirror. But for such a hard-wall reflection to occur, it is necessary that the incident gravitational wave would somehow generate sufficiently strong mass currents on the surface of the mirror, such that these currents would then re-radiate both a plane wave in the forwards direction that would cancel out the incident wave, and would simultaneously re-radiate a plane wave in the backwards direction that is 180 degrees out of phase with respect to the incident wave, in order to create the totally reflected wave.

Due to the enormity of the “Heisenberg-Coulomb effect” predicted in (25), there could indeed arise such enormous quantum-mechanical mass supercurrents, which are induced by the extremely tiny strains of space associated with the incident gravitational plane wave, so that even the thinness of Einstein’s coupling constant \(8\pi G/c^4\) that couples sources to fields in Einstein’s field equations, might somehow be overcome during reflection. But how could one possibly reconcile this with the Einstein’s field equations without somehow modifying its extremely tiny \(8\pi G/c^4\) coupling constant?

There already exists a hint as to how to handle this situation in magnetostatics, in which the field equation in the vacuum is given by Ampere’s law

\[
\nabla^2 \mathbf{A} = -\mu_0 \mathbf{j} \tag{26}
\]

where \(\mathbf{A}\) is the vector potential from which the magnetic field is derived, \(\mu_0\) is the magnetic permeability of free space (i.e., the vacuum without any medium), and \(\mathbf{j}\) is the total current density, which is the source of this field equation.

However, suppose that there exists a magnetic medium with a relative magnetic permeability \(\mu_r\), such as some ferromagnetic material that fills all of space. It is a well known empirical fact that the insertion of a high-permeability ferromagnetic material, such as iron, into the interior of an electromagnet will greatly enhance the strength of the magnetic field generated by this electromagnet. This empirical fact provides ample justification for a modification of the field equation (26), in which one inserts a prefactor \(\mu_r\) in front of the source-to-field coupling constant \(\mu_0\), so that this modified field equation now reads

\[
\nabla^2 \mathbf{A} = -\mu_r \mu_0 \mathbf{j} \tag{27}
\]

Thus in the presence of a homogeneous and isotropic magnetic medium, there exists a dimensionless number \(\mu_r\) (i.e., the “relative permeability” of the medium), which has a sign and a magnitude that must be determined by experiment, as the prefactor of the source term in the field equation (27).

Now for most “normal” materials, it turns out that the magnitude of \(\mu_r\) is very close to unity. Both signs of the permeability for magnetic materials (i.e., diamagnetic and paramagnetic signs) exist in nature, but all of these permeabilities are quantum mechanical, and not classical, in origin [24]. In both cases of diamagnetism and paramagnetism, quantum mechanical currents are required to explain the phenomena. Moreover, in the case of ferromagnetic materials, \(|\mu_r|\) has been observed to have very large values, such as \(10^6\) in iron-nickel alloys.
Note that one must carefully distinguish here between the “relative permeability” and the “relative permittivity” of material media, because the magnetic response of a given material is fundamentally different from its electric response, since the magnetic field is fundamentally different in nature from the electric field. Likewise, the question now arises: Does one need to make similar distinctions in the case of the different possible responses of various kinds of material media to the different kinds of gravitational fields in general relativity?

We argue here that the answer to this question is yes. One reason for this affirmative answer is that we know that in general relativity, there exists the Lense-Thirring field, which is a gravito-magnetic field, which is fundamentally different in nature from the usual Newtonian gravitational field, which is a gravito-electric field. However, in addition to these two kinds of fields, there exists in general relativity a third, fundamentally different kind of field, namely, the transverse-traceless $h_{ij}$ gravito-tensor field associated with gravitational radiation, which has no analog in electromagnetism. In general relativity, we know that the different components of the stress-energy tensor $T_{\mu\nu}$ can be sources for three different possible kinds of gravitational fields, and thus in principle can lead to three different possible kinds of responses of different material media to gravitational fields, namely, a scalar, a vector, and a tensor response, which correspond to the components $T_{00}$, $T_{0i}$, and $T_{ij}$ of the tensor $T_{\mu\nu}$, respectively.

The modification of Ampere’s law \[(27)\] in order to allow for the different possible responses of homogeneous and isotropic magnetic media due to an applied magnetic $\mathbf{H}$ field arising from a solenoid, for example, justifies a similar modification of Einstein’s field equations, after they have been reduced to a linearized wave equation for $h_{ij}$, in order to allow for the possibility of different responses of homogeneous and isotropic material media to a gravitational wave. In particular, there could arise enormous quantum-mechanical mass supercurrents induced in a superconductor due to even a tiny applied $T_{ij}$ stress field arising from the incident ($\times$) polarized plane wave depicted in Figures 3 and 4, which, in light of the above “Heisenberg-Coulomb” effect, would lead to internal electric fields inside the superconductor that would drive these enormous supercurrents.

Before modification, the wave equation for gravitational waves is \[(25)\]

$$\nabla^2 h_{ij} - \frac{1}{c^2} \frac{\partial^2 h_{ij}}{\partial t^2} = -2\kappa_0 T_{ij}$$

(28)

where the extremely tiny dimensionful constant

$$\kappa_0 = \frac{8\pi G}{c^4}$$

(29)

is Einstein’s coupling constant for the vacuum in the absence of any medium. The dimensionful constant $\kappa_0$ is analogous to the dimensionful constant $\mu_0$, the magnetic permeability of free space (i.e., for the vacuum in the absence of any magnetic medium) in Ampere’s law \[(26)\].

After making the proposed modification, in which one inserts a prefactor $\kappa_r$ in front of the source-to-field coupling constant $\kappa_0$, the wave equation for
gravitational waves now reads as follows:

$$\nabla^2 h_{ij} - \frac{1}{c^2} \frac{\partial^2 h_{ij}}{\partial t^2} = -2\kappa_r \kappa_0 T_{ij}$$  \hspace{1cm} (30)$$

where the dimensionless number $\kappa_r$ on the right hand side of this wave equation [25][27], is an empirically determined constant that we shall call the “relative gravitational permeativity” [28], in analogy with $\mu_r$, the “relative magnetic permeability,” that was introduced into Ampere’s law [27]. Although the typical sizes of the relative permeability observed in ferromagnetic media $|\mu_r| \sim 10^6$ may not be as large as the typical sizes of the relative gravitational permeativity $|\kappa_r|$ that may eventually be observed in future experiments in superconducting media, both the sign and the magnitude of $\kappa_r$ must ultimately be arrived at empirically; they cannot be ruled out on any a priori basis [29].

Now for most “normal” materials, the relative gravitational permeativity $\kappa_r$ will most likely be very close to unity, so that these media will be essentially completely transparent to gravitational waves. Note, however, that the wave equation (30) is still linear, even after the inclusion of the prefactor $\kappa_r$. This linearity leads to the applicability of the superposition principle for the solutions of this wave equation, and also permits the resulting classical waves to be quantized using the canonical quantization procedure outlined above.

For a superconductor, however, $\kappa_r$ may turn out to be huge. Although an estimate based on an incorrect vector coupling theory yields $|\kappa_r| \sim 10^{42}$ (as in [25] based on [18][23]), both the sign and the magnitude of this empirical constant must ultimately be determined by measurements, such as via the Fresnel reflection coefficient $|\rho(\omega)|^2$ off of the surface of a square plate, which is given by

$$|\rho(\omega)|^2 = \left| \frac{n(\omega) - 1}{n(\omega) + 1} \right|^2$$  \hspace{1cm} (31)$$

where $n(\omega)$ is given by a plasma-like formula for the index of refraction, as shown in Appendix B. This measurement of $|\rho(\omega)|^2$, however, has never been performed, since there exist at the present time no laboratory sources for gravitational waves.

But perhaps the strongest reason for introducing the “relative gravitational permeativity” $\kappa_r$ into the wave equation (30), would be the existence of a “gravitational Meissner-like effect.” To this end, let us consider evaluating the stress-energy tensor $T_{xy}$ evaluated at a point along the major diagonal of the rhombus sketched in Figures 3 and 4. Since any second-rank tensor can be written as a tensor product of two vectors, one can always express $T_{xy}$ as the direct product

$$T_{xy} \propto v_x v_y$$  \hspace{1cm} (32)$$

where $v_x$ and $v_y$ are the $x$ and $y$ components of some vector field inside the SC. But the only physically relevant vector field in the problem at hand is the quantum-mechanical supercurrent velocity vector field that is induced by the incident ($\times$)-polarized gravitational plane wave sketched in Figure 3(a).
Now the supercurrent velocity field \( \mathbf{v} \) is directly proportional to the supercurrent density \( \mathbf{j} \), which in turn is directly proportional to vector potential \( \mathbf{A} \) via London’s constitutive relationship. This leads to the following proportionalities:

\[
\mathbf{A} \propto \mathbf{j} \propto \mathbf{v} \tag{33}
\]

But Ampere’s law leads to the following equalities:

\[
\nabla \times \mathbf{B} = \nabla \times \nabla \times \mathbf{A} = \mu_0 \mathbf{j} \tag{34}
\]

Using the vector identity

\[
\nabla \times \nabla \times \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \tag{35}
\]

and using the London gauge \( \nabla \cdot \mathbf{A} = 0 \), one then arrives at London’s equation, i.e., the following Yukawa-like equation with an empirical constant \( \kappa_L \):

\[
\nabla^2 \mathbf{A} - \kappa_L^2 \mathbf{A} = 0 \tag{36}
\]

which is a linear PDE. Using London’s constitutive relations \( \tag{33} \), we also arrive at the following Yukawa-like, linear PDE for the supercurrent velocity field:

\[
\nabla^2 \mathbf{v} - \kappa_L^2 \mathbf{v} = 0 \tag{37}
\]

For the transverse supercurrent velocities flowing in the SC square configurations of Figures 3 and 4, we find the following two PDE’s:

\[
\frac{\partial^2 v_x}{\partial z^2} - \kappa_L^2 v_x = 0 \tag{38}
\]

\[
\frac{\partial^2 v_y}{\partial z^2} - \kappa_L^2 v_y = 0
\]

These equations possess the following exponentially decaying solutions:

\[
v_x (z) = v_x (0) \exp (-\kappa_L z) = v_x (0) \exp (-z/\lambda_L) \tag{39}
\]

\[
v_y (z) = v_y (0) \exp (-\kappa_L z) = v_y (0) \exp (-z/\lambda_L) \tag{40}
\]

where the London penetration depth \( \lambda_L \) is given by

\[
\lambda_L = \frac{1}{\kappa_L} \tag{41}
\]

For superconducting niobium, \( \lambda_L \) is measured to be around 40 nm.

From the tensor product relationship \( \tag{32} \) and from the solutions for the supercurrent velocity field components \( \tag{39} \) and \( \tag{40} \), we conclude that the stress-energy tensor has the following \( z \) dependence

\[
T_{xy} (z) \propto v_x (z) v_y (z) = (v_x (0) \exp (-z/\lambda_L)) \cdot (v_y (0) \exp (-z/\lambda_L)) \tag{42}
\]
Therefore it follows that the exponential decay solution for the stress-energy tensor in the $z$ direction is given by

$$T_{xy}(z) = T_{xy}(0) \exp\left(-\frac{2z}{\lambda_L}\right) \propto \exp\left(-\frac{2z}{\lambda_L}\right)$$

so that $T_{xy}(z)$ decays twice as fast as the supercurrent velocity field into the depth of the SC. Therefore the exponential decay length scale of $T_{xy}$, i.e., its gravitational penetration depth, is half that of the electromagnetic London penetration depth \((41)\).

Now from the linearity of the wave equation \((30)\) and from the solution \((43)\), we conclude that the solution for the gravitational wave field $h_{xy}$ penetrating into the SC square must also obey the proportionality relations

$$h_{xy}(z) \propto T_{xy}(z) \propto \exp\left(-\frac{2z}{\lambda_L}\right)$$

Therefore we conclude that the gravitational wave amplitude $h_{xy}$, like $T_{xy}$, decays twice as fast as the supercurrent velocity field into the depth of the SC. Hence the exponential decay length scale of gravitational plane amplitude $h_{xy}(z)$ deep inside the SC is also half that of the usual electromagnetic London penetration depth \((41)\), i.e., around 20 nm for the case of niobium. This is a “gravitational Meissner-like effect” that will lead to the expulsion of the incident gravitational plane wave from the interior of the SC square in Figures 3 and 4, and therefore will lead to a mirror-like, total reflection of this wave.

Now we present a progress report concerning our experiments towards achieving the goal of observations of the DCE and of the laser-like generation of gravitational waves. Figure 5 is a photograph of a silicon nitride membrane sample, which is coated with niobium on its back side. [33] This membrane will be the active amplifying element in our paramps. We are planning to place the sample shown in Figure 5 at the center of a degenerate paramp as a vibrating membrane (red) driven by pump microwaves, as sketched in Figure 6.

In this dual-SRF cavity setup, the pump injected into the left chamber is identical in frequency to the signal and idler frequencies that will be produced in the DCE in the right chamber (yellow in Figure 6) above a certain threshold. Due to our prediction that the London penetration depth for GR waves will be half that for EM waves, the modal volume for the right chamber at resonance will be slightly smaller for the case of GR wave generation as compared to the case of EM wave generation. Hence there should be a well-resolved difference in the position of the tuner (green) for EM wave production relative to that for GR wave production inside the right chamber. This difference will be a convenient signature that we can use to distinguish between the two cases.

However, since the detection of GR waves will be difficult, we will first try to indirectly infer that these invisible waves are in fact being generated by looking for a “pump depletion effect” in which there will arise a dip the reflected pump signal from the left chamber at the threshold for GR wave generation. This dip will arise from the “missing energy” that will be escaping in the form of these invisible waves from the right chamber. Thus we can infer from energy
Figure 5: A flexible silicon nitride membrane (green; 500 nm thick) is stretched over a circular window frame of an etched silicon wafer (gray; 50 mm diameter). A niobium coating (not shown) is sputtered onto the other side of the membrane.
Figure 6: Degenerate parametric amplifier/oscillator design for generating gravitational radiation (GR) in the signal/idler cavity (yellow) via the mechanical motions (double-headed black arrow) of a silicon nitride membrane coated with SC niobium (red) driven by microwaves in the pump cavity with tuner (green).

Figure 7: Exponential decay curve of a TEM microwave mode excitation of an SRF stub cavity with a resonant frequency around 10 GHz. The exponential ring-down time is 7.3 ms, implying a $Q$ of $1.7 \times 10^9$ at a temperature of 55 mK.
conservation that GR waves are in fact being generated, although they will not be directly detectable.

In a follow-up experiment, we plan to make a copy of the degenerate paramp apparatus pictured in Figure 6 as a “receiver,” and place it side-by-side with respect to the “transmitter,” in a Hertz-like “transmitter-receiver” configuration. The amplification of GR waves in the “receiver” paramp can serve as a low-noise preamp, i.e., a first-stage amplifier, of a GR-wave detection system, whose final stage could consist of a membrane-displacement measurement of a final-stage SC membrane, whose displacement arises from the radiation pressure being exerted on the membrane from the received GR waves.

In Figure 7, we show the progress that we have been making concerning the Q problem. It turns out that gaps and other imperfections in the joints between the cylindrical body of the SRF cavities and their endcaps can degrade the Q of the cavity by orders of magnitude. However, by fabricating a seamless resonator using a coaxial stub cavity, one can evade these kinds of degradations of the Q. Figure 7 is a plot of data from a ring-down measurement of a SC niobium stub resonator that demonstrates that we can achieve a Q on the order of a billion at the typical temperature of 55 millikelvin that we have been using in our dilution refrigerators. If we can achieve such a high Q in the dual-SRF cavity sketched in Figure 6, we will be well on our way towards demonstrating the DCE and, possibly, the laser-like generation of gravitational waves.

It should be emphasized at this point that we are not trying to detect the received GR waves by measuring the dimensionless strain of space produced by these waves, which would be exceedingly tiny, (see Appendix A), but rather we shall try to detect the radiation pressure, and hence the received power, associated with these waves.

Appendix A: The strain of space produced by one milliwatt of gravitational microwave power

Since spacetime can be thought of as an extremely stiff medium, the question naturally arises: How could one possibly produce any measurable amount of strain of space, even if one were to succeed in a laser-like scheme for generating gravitational (GR) waves? The short answer is this: One does not need to be able to directly measure the strain of space; one only needs to be able to directly measure the power in a laser-like beam of GR waves. Nevertheless, it will be instructive to put in some numbers in order to answer this question.

Suppose that one were able to generate one milliwatt of power in a laser-like beam of a GR wave. The gravitational analog of the time-averaged Poynting vector, which is the flux of energy, is given by

\[
\langle S \rangle = \frac{\omega^2 c^3}{32\pi G} h_x^2
\]  

(45)

where \( h_x \) is the strain of space for a \((\times)\) polarized plane wave. For one milliwatt of power in such a plane wave at 30 GHz, say, focused by a Newtonian SC
telescope to a 1 cm$^2$ Gaussian beam waist, one obtains a strain of space of
\[ h_x \approx 0.8 \times 10^{-28} \] (46)

within the focal area. Such a tiny strain of space would be exceedingly difficult
to directly detect, even using advanced LIGO. However, it is unnecessary to
directly measure the strain of space in order to detect the GR wave, just as it
is unnecessary to directly detect the optical electric field amplitude of a laser
beam in order to detect the light wave. Rather, one can directly measure the
power carried by the laser-like GR beam, for example by measuring the back-
conversion of one milliwatt of the incident GR wave power into one milliwatt
of EM wave power via a measurement of the radiation pressure exerted by the
received GR wave upon a SC membrane in a time-reversed parametric process
inside a replica of the dual-SRF cavity of Figure 6. It would then be easy to
detect one milliwatt of the back-converted EM microwave power.

Appendix B: Plasma-like gravitational-wave refractive index of a
superconductor
The modified gravitational wave equation in a medium (such as a supercon-
ductor (SC)) is
\[ \nabla^2 h_{ij} - \frac{1}{c^2} \frac{\partial^2 h_{ij}}{\partial t^2} = -2\kappa T_{ij} \] (47)
where we define \( \kappa \equiv \kappa_r \kappa_0 \) (48)
where \( \kappa_0 = 8\pi G/c^4 \) is Einstein’s coupling constant in vacuum, and where \( \kappa_r \) is
the “relative gravitational permeativity” of the medium (to be determined by
experiment). We shall call \( \kappa \) the “gravitational permeativity” of a SC medium,
in analogy with the “magnetic permeability” of a magnetic medium
\[ \mu \equiv \mu_r \mu_0 \] (49)
where \( \mu_r \) is the relative magnetic permeability that appears as the prefactor of
the source term for Ampere’s law in a medium
\[ \nabla^2 \mathbf{A} = -\mu \mathbf{j} \] (50)

Let us define the constitutive relation of a SC medium as follows:
\[ T_{ij} \equiv -\mu_G h_{ij} \] (51)
where \( \mu_G \) is the “gravitational shear modulus” of the material to an applied \( h_{ij} \)
field. Substituting this constitutive relation into the modified wave equation in
a medium (47), we find
\[ \nabla^2 h_{ij} - \frac{1}{c^2} \frac{\partial^2 h_{ij}}{\partial t^2} = -2\kappa \mu_G h_{ij} \] (52)
Upon substitution of the monochromatic plane wave Ansatz,

\[ h_{ij}(x, y, z, t) = A \exp(ikz - i\omega t) \]  

(53)

into this equation, we obtain the implicit dispersion relation

\[ k^2 - \frac{\omega^2}{c^2} = -2\kappa \mu_G \]  

(54)

Let us now define the “gravitational plasma frequency” as

\[ \omega_G \equiv \sqrt{2\kappa c^2 \mu_G} \]  

(55)

This agrees with [20] since

\[ \kappa = \kappa_r \kappa_0 \]  

(56)

Solving for \( k(\omega) \) from (54), one finds the explicit dispersion relation

\[ k(\omega) = \frac{\omega}{c} \sqrt{1 - \frac{\omega_G^2}{\omega^2}} \]  

(57)

from which we see that the meaning of the plasma frequency is that

\[ k(\omega_G) = 0 \]  

(58)

i.e., that the plasma frequency is a cutoff frequency below which a gravitational wave cannot propagate inside the SC medium, because the propagation wavenumber \( k(\omega) \) becomes a pure imaginary quantity.

Alternatively, let us introduce the index of refraction \( n(\omega) \) as follows:

\[ k(\omega) = \frac{n(\omega)\omega}{c} \]  

(59)

Comparing this with (57), we see that

\[ n(\omega) = \sqrt{1 - \frac{\omega_G^2}{\omega^2}} \]  

(60)

which is a plasma-like index of refraction. Note that for \( \omega < \omega_G \), the refractive index becomes a pure imaginary quantity, which implies total reflection, just like the reflection from a plasma of an EM wave whose frequency is below cutoff.

Thus the Fresnel reflection formula [31] is

\[ |\rho(\omega)|^2 = \left| \frac{n(\omega) - 1}{n(\omega) + 1} \right|^2 \]  

(61)

where \( n(\omega) \) is given by the plasma-like formula for the index of refraction (60).

**Acknowledgments:** This work was supported in part by DARPA. We thank Professors Douglas Singleton and Gerardo Muñoz for their help on the theory, and Jacob Pate for his help on the experiments. This paper, based on a talk given on November 3, 2017 by RYC at the Aerospace Advanced Propulsion Workshop, will appear in the Journal of the British Interplanetary Society.
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However, the enormous enhancement found in [18] of the reflectivity of a SC mirror by huge dimensionless ratio $|F_{\text{Coulomb}}/F_{\text{Newton}}| \sim 10^{42}$ was derived on the basis of the incorrect vector interaction Hamiltonian of the form $\mathbf{p \cdot A}$ between the Cooper pairs and vector gravitational fields, and not on the basis of the correct tensor interaction Hamiltonian of the form $h_{\mu\nu}T^{\mu\nu}$ between the Cooper pairs and tensor gravitational fields used in [20].

Niels Bohr, PhD thesis.
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Note that the introduction of the relative gravitational permeativity $\kappa_r$ into the source of the wave equation (30) affects solely the source of the tensor $h_{ij}$ field associated with gravitational waves. This is not equivalent to the introduction of an effective Newton’s gravitational constant $G_{\text{eff}}$ to replace Newton’s constant $G$ in front of the source term in the full Einstein’s field equations so that they would now read as follows:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G_{\text{eff}}}{c^4}T_{\mu\nu}$$

since this replacement $G \rightarrow G_{\text{eff}}$ would affect all the components of $T_{\mu\nu}$ equally. Such a replacement makes no distinction at all between the scalar, vector, or tensor sources $T_{00}, T_{0i},$ or $T_{ij}$ of the three different kinds of gravitational fields. However, the Newtonian gravitational field of the Earth, whose source is $T_{00},$ will not be affected by the introduction of the relative gravitational permeativity $\kappa_r$ in (30). Hence there cannot be any “anti-gravity” arising from the presence of superconductors.

Since $\kappa_r$ affects solely the metric deviation tensor $h_{ij}$ field associated with gravitational waves, but not the other components of $h_{\mu\nu}$ whose sources are the other components of the full stress-energy tensor $T_{\mu\nu},$ the question naturally arises as to how the introduction of a non-unity relative gravitational permeativity $\kappa_r$ into the wave equation (30), would affect the conservation law $T_{\mu\nu}^{;\nu} = 0.$ The same question, however, arises also in Maxwell’s equations modified by the permittivity and permeability of electrical and magnetic media (for example, in the case of the large measured relative magnetic permeability $\mu_r \sim 10^6$ for a Metglas alloy). As the ongoing Abraham-Minkowski controversy shows, the answer to this question in the case of electromagnetic fields that pervade the space inside dielectric and magnetic media, requires a careful rethinking of the physical meaning of the concept of momentum of these fields inside a medium, as well as a rethinking of the physical meaning of the concept of inertia (see https://en.wikipedia.org/wiki/Abraham–Minkowski_controversy). One must also confront this same rethinking in the case of media for the various kinds of gravitational fields in general relativity. Ultimately, experiments must settle this controversy.

We have coined a new word “permeativity” to supplement the words “permeability” and “permittivity.”

Note that Szekeres [30], Press [31], and Flanagan and Hughes [32] do not distinguish between values of $\kappa_r$ for the scalar, vector, and tensor parts of Einstein’s equations. However, following the analogy with electromagnetism, where $\varepsilon$ and $\mu$ can vary independently for different kinds of macro-
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