Mapping the dominant regions of the phase space associated with $c\bar{c}$ production relevant for the Prompt Atmospheric Neutrino Flux

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We present a detailed mapping of the dominant kinematical domains contributing to the prompt atmospheric neutrino flux at high neutrino energies by studying its sensitivity to the cuts on several kinematical variables crucial for charm production in cosmic ray scattering in the atmosphere. This includes the maximal center-of-mass energy for proton-proton scattering, the longitudinal momentum fractions of partons in the projectile (cosmic ray) and target (nucleus of the atmosphere), the Feynman $x_F$ variable and the transverse momentum of charm quark/antiquark. We find that the production of neutrinos with energies larger than $E_\nu > 10^7$ GeV is particularly sensitive to the center-of-mass energies larger than the ones at the LHC and to the longitudinal momentum fractions in the projectile $10^{-8} < x < 10^{-5}$. Clearly, these are regions where we do not control the parton, in particular gluon, densities. We also analyse the characteristic theoretical uncertainties in the charm production cross section coming from its QCD modelling. The precision data on the prompt atmospheric neutrino flux can efficiently constrain the mechanism of heavy quark production and underlying QCD dynamics in kinematical ranges beyond the reach of the current collider measurements.

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I. INTRODUCTION

The recent detection of ultra-high energy neutrino events with deposited energies up to a few PeV by the IceCube Observatory sets the beginning of neutrino astronomy [1–3] (for a review of IceCube potential for neutrino astronomy, see e.g. Ref. [4]). It is mandatory to know the flux of atmospheric neutrino produced in cosmic-ray interactions with nuclei in Earth’s atmosphere at different energies with high precision as an unavoidable background for cosmic neutrino studies. In recent years, the atmospheric high-energy neutrino flux became accessible to the experimental studies and, in particular, was constrained by several neutrino observatories [5–8].

The available data indicate that the atmospheric flux observed in the experiment is dominated at low energies ($E_\nu \lesssim 10^5$ GeV) by atmospheric neutrinos that arise from the decay of light mesons (pions and kaons), denoted as the conventional atmospheric neutrino flux [9–11], while the data for the higher energies ($E_\nu \gtrsim 10^7$ GeV) are most probably associated with cosmic neutrinos. In the intermediate energy range ($10^5$ GeV < $E_\nu < 10^7$ GeV), it is expected that the prompt atmospheric neutrino flux associated with the decay of heavy flavoured hadrons, composed of heavy quarks, become important [12–14]. In particular, it is typically considered that this contribution dominates the atmospheric neutrino flux for large neutrino energies ($E_\nu > 10^6$ GeV).

This expectation can be easily understood. The increasing competition between the interaction and decay lengths for pions and kaons at high energies, implies a reduction of the neutrino flux associated with decays of these particles.
This behaviour is related to the fact that the long-lived high-energy light mesons interact and lose their energy before decaying into neutrinos. In contrast, in the case of heavy hadrons, they have short lifetimes and decay into neutrinos almost immediately after their production. Consequently, at very high energies, the atmospheric neutrino flux is expected to arise from semi-leptonic decays of heavy, in particular charmed, hadrons.

Thus, the precise knowledge of the prompt atmospheric flux is crucial for the determination of the cosmic neutrino flux. This subject has been a theme of intense debate in the literature, mainly due to the fact that the calculation requires good knowledge of the heavy quark production cross section at high energies. In the last two years, results of many calculations of this flux were presented [14–22], focusing on the determination of the theoretical uncertainties present in the QCD calculations. These uncertainties are typically associated, for example, with the choice of the heavy quark masses, factorization and renormalization scales, as well as the contribution of higher order corrections, the choice of the parton distribution functions (PDFs) and the treatment of QCD dynamics at high energies (very small \(x\)). The overall theoretical uncertainty in QCD predictions of the prompt neutrino flux has been estimated to be a factor of three or a bit larger in Ref. [13]. The impact of nuclear effects, saturation and low-\(x\) resummation was studied in detail in Ref. [17]. The recent LHC data for the prompt heavy quark production cross sections (see e.g. Refs. [23, 24]) significantly reduced some of these uncertainties with direct impact on the predictions for the prompt neutrino flux. However, several questions still remain open.

The prompt neutrino flux is usually calculated using the semi-analytical \(Z\)-moment approach, proposed many years ago in Ref. [12] and discussed in detail e.g. in Refs. [16, 25]. One of the main inputs in this approach is the Feynman \(x_F\) distribution for the heavy quark production in hadronic collisions. As discussed e.g. in Refs. [23, 25], it is expected that the main contribution to the prompt neutrino flux comes from large values of \(x_F\), that are associated with the heavy quark production at forward rapidities. Moreover, the production of neutrinos at a given neutrino energy \(E_\nu\) is determined by collisions of cosmic rays with nuclei in the atmosphere at energies that are a factor of order \(100-1000\) larger. One also has that the prompt neutrino flux measured in the kinematical range that is probed by the IceCube Observatory and future neutrino telescopes is directly associated with the treatment of the heavy quark cross section at high energies. Currently, different experiments at the LHC probe a limited range in rapidity. In particular, they do not cover rapidities larger than 4.5, which corresponds to relatively small values of \(x_F \lesssim 0.1\). Therefore, the \(D\)-meson production in the kinematical range of large \(x_F\) values is not covered by the LHC detectors.

The main motivation of the current study is to clarify the kinematical range of energies and rapidities in the heavy quark production that determine the prompt atmospheric neutrino flux in the range probed by the IceCube Observatory. Such an aspect is fundamental if we would like to reduce the current theoretical uncertainties. Moreover, as the IceCube 2 program [27] is expected to measure neutrinos with energies that are three orders of magnitudes larger than the current coverage, it also will help to define what are the theoretical issues that should be resolved in order to obtain realistic predictions for the future neutrino telescopes.

In this paper, we concentrate on \(c\bar{c}\) production to understand to which extend the calculated prompt neutrino flux is reliable. We therefore neglect the \(b\bar{b}\) production as well as nuclear effects. The \(b\bar{b}\) component gives about 10% contribution to the corresponding \(x_F\)-distribution and, thus, to the neutrino flux [17].

The paper is organized as follows. In the next section we present a brief review of the \(Z\)-moment formalism for the calculation of the prompt atmospheric neutrino flux. Moreover, we describe the main assumptions of our analysis and present a comparison of our results and those obtained by the Prosa Collaboration [21]. In Section III we discuss the different cuts assumed in the calculations and analyse their impact on the neutrino flux, focussing on \(E_\nu > 10^6\) GeV. Finally, in Section IV we summarize our main conclusions.

II. PROMPT ATMOSPHERIC NEUTRINO FLUX

In order to determine the prompt atmospheric neutrino flux at the detector level we should describe the production and decay of the heavy hadrons as well as the propagation of the associated particles through the atmosphere. The evolution of the inclusive particle fluxes in the Earth’s atmosphere can be obtained using the \(Z\)-moment approach [12]. In this approach, a set of coupled cascade equations for the nucleons, heavy mesons and leptons (and their antiparticles) fluxes is solved, with the equations being expressed in terms of the nucleon-to-hadron (\(Z_{NH}\)), nucleon-to-nucleon (\(Z_{NN}\)), hadron-to-hadron (\(Z_{HH}\)) and hadron-to-neutrino (\(Z_{HV}\)) \(Z\)-moments. For a detailed discussion of the cascade equations, see e.g. Refs. [12, 16]. These moments are inputs in the calculation of the prompt neutrino flux associated with production of a heavy hadron \(H\) and its decay into a neutrino \(\nu\) in the low- and high-energy
fluxes, both aspects increase the uncertainty in the predictions for the flux in the high-energy regime. This point has

of the prompt atmospheric neutrino flux requires a folding of the heavy-quark cross section with the incoming cosmic

result is similar to the central prediction of the Prosa collaboration [21], as shown in Fig. 1. In this figure, we also

performed in Ref. [15]. Although we have done several approximations to compute the prompt neutrino flux, our

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assume that 

by a broken power-law spectrum [26], with the incident flux being represented by protons 

the effective interaction lengths \( \Lambda \) are given by \( \Lambda_i = \lambda_i/(1 - Z_{\nu i}) \), with \( \lambda_i \) being the associated interaction length 

\( i = N, H \). The expected prompt neutrino flux in the detector can be estimated using the geometric interpolation formula

\[
\phi_{\nu} = \sum_H \frac{\phi_{H, low} \cdot \phi_{H, high}}{\phi_{H, low} + \phi_{H, high}}.
\]

In what follows, we will focus on vertical fluxes \( (\theta = 0) \) and assume that the cosmic ray flux \( \phi_N \) can be described by a broken power-law spectrum [20], with the incident flux being represented by protons \( (N = p) \). Moreover, we will assume that the charged hadron \( Z \)-moments can be expressed in terms of the charm \( Z \)-moment as follows:

\[
Z_{pH} = f_H \times Z_{pc}, \quad \text{where} \quad f_H \text{ is the fraction of charged particle which emerges as a hadron } H.
\]

As in Ref. [25], we will assume that \( f_{D^0} = 0.565, f_{D^+} = 0.246, f_{D^+} = 0.080 \) and \( f_A = 0.094 \).

It is important to emphasize that the composition of the particle content of the ultra high energy cosmic rays in the

region beyond the ankle \( (E \approx 5 \times 10^5 \text{ GeV}) \) still is an open question and no clear consensus exists. As the computation

of the prompt atmospheric neutrino flux requires a folding of the heavy-quark cross section with the incoming cosmic

both aspects increase the uncertainty in the predictions for the flux in the high-energy regime. This point has

been recently discussed in detail in Refs. [17, 21].

The charm \( Z \)-moment at high energies can be expressed by

\[
Z_{pc}(E) = \int_0^1 dx_F \phi_F(E/x_F) \frac{1}{\sigma_{pA}(E)} \frac{d\sigma_{pA \rightarrow \text{charm}}(E/x_F)}{dx_F},
\]

where \( E \) is the energy of the produced particle \( (\text{charm}) \), \( x_F \) is the Feynman variable, \( \sigma_{pA} \) is the inelastic proton-Air

cross section, which we assume to be given as in Ref. [13], and \( d\sigma/dx_F \) is the differential cross section for the charm

production, which we assume to be given by \( d\sigma_{pA \rightarrow \text{charm}}/dx_F = 2 \sigma_{pA \rightarrow \text{cc}}/dx_F \).

We compute the prompt neutrino flux associated with charged hadrons by evaluating all quantities entering the
different terms of Eq. [18]. In our analysis, we closely follow Refs. [17, 20]. In the analysis of Eq. [11] we will use the

standard QCD collinear factorization formalism allowing us to calculate the charm production cross section [28].

In the leading-order collinear factorization approach the differential cross section can be written as

\[
\frac{d\sigma}{dy_1 dy_2 dp_T} = \frac{1}{16\pi^2 s} \times [ \frac{[M_{gg \rightarrow cc}][x_1 g(x_1, \mu_F^2) x_2 g(x_2, \mu_F^2)]}{[M_{q\bar{q} \rightarrow cc}][x_1 \bar{q}(x_1, \mu_F^2) x_2 q(x_2, \mu_F^2)]} + \sum_f [M_{q\bar{q} \rightarrow cc}][x_1 q_f(x_1, \mu_F^2) x_2 \bar{q}_f(x_2, \mu_F^2)]
\]

where \( p_T \) is the heavy quark transverse momentum, and \( y_1 \) and \( y_2 \) are the charm and anticharm rapidities, respectively. The distribution in \( x_F \) is obtained by an appropriate binning. The PDFs will be assumed to be given by the CT14

parametrization [20] and the hard scattering will be estimated at the leading order taking into account both \( gg \rightarrow cc \)

and \( q\bar{q} \rightarrow cc \) subprocesses. The contribution of the next-to-leading order corrections for the \( x_F \)-distribution will be taken into account by multiplying our predictions by an effective \( K \)-factor that depends on \( x_F \), as proposed in Ref. [26]. We assume \( m_c = 1.5 \text{ GeV} \), the factorization and renormalization scales are taken as \( \mu_F^2 = \mu_R^2 = m_c^2 \equiv (p_T^2 + 4m_c^2) \).

We will disregard in the present analysis the nuclear effects, in particular, shadowing, i.e. we calculate the cross

section for collisions on nuclei as \( \sigma_{pA \rightarrow cc} = Z \times \sigma_{pp \rightarrow cc} + N \times \sigma_{nn \rightarrow cc} \approx A \times \sigma_{pp \rightarrow cc} \), where \( Z, N \) and \( A \) are the number of protons, neutrons and nucleons in the nucleus of the target, respectively. In practical calculations we take \( 14N \) nucleus as the most representative one. A more refine analysis is possible but would shadow our discussion of the selected issues.

Moreover, we will calculate the effective hadronic interaction lengths \( \Lambda_i \) and the \( Z_{pp}, Z_{HH} \) and \( Z_{H\nu} \)-moments as performed in Ref. [15]. Although we have done several approximations to compute the prompt neutrino flux, our result is similar to the central prediction of the Prosa collaboration [21], as shown in Fig. 1. In this figure, we also
our calculation

Our calculation

show the current theoretical uncertainty band present in one of the most sophisticated calculations of the neutrino flux. Although the available data from collider experiments are used in Ref. [21] as an input to constrain the main uncertainties present in the treatment of heavy quark production, the associated predictions for the neutrino flux are still uncertain. The main sources of uncertainty here are associated to the modelling of the cosmic ray composition and renormalization/factorization scale variations.

III. RESULTS

In this Section, we wish to understand what is the range of several kinematical variables relevant for the production of the high energy neutrinos observed recently by IceCube or for higher energies than possible at present. To realize the goal we map the range of several kinematical variables such as: center-of-mass energies, charm transverse momentum ($p_T$), parton momentum fractions in the projectile ($x_1$) and target ($x_2$), and the Feynman-$x$ ($x_F$). All of them determine the size of the cross section and, as a consequence, the energy dependence of the prompt neutrino flux.

Let us analyze first how the flux of neutrinos from semileptonic decays of $D$ mesons depends on the maximal center-of-mass collision energy included in the calculation. In Fig. 2 we present our results obtained for different values of the maximal energies considered in the analysis of the differential cross section in Eq. (4). As $x_F$ is integrated and $d\sigma/dx_F$ is probed at the energy $E/x_F$, one have that $Z_{pc}(E)$ may be influenced by the behaviour of distribution at higher energies. In our calculation, we consider three different values for the maximum center-of-mass energy allowed in the $pp$ collision that generates the heavy quark pair. For comparison the full prediction for the flux, denoted as “no cuts” in the figure, is presented. Here, no energy limitations were imposed. Moreover, for illustration, the energy range probed by the recent IceCube data [3] is shown as well. The figure demonstrates that the flux depends on of the cross section for heavy quark production in the LHC energy range and at even larger energies. The latter unexplored region can also have a direct impact on the flux at high neutrino energies ($E_\nu \geq 10^6$ GeV).

Moreover, our results indicate that the prompt neutrino flux for $E_\nu \gtrsim 10^7$ GeV is determined by the behaviour of the differential cross section in the energy range beyond that probed in the Run 2 of the LHC. Consequently, the detection of prompt atmospheric neutrinos in this range by the IceCube experiment, its upgrade or by other future neutrino telescope, can significantly contribute to our understanding of several aspects associated with the heavy quark production at high energies. Whether we control at present the cross section for energies above those for the LHC is an open question, at least, in our opinion.

In Fig. 3 we present the sensitivity of the charm production cross section $d\sigma/dx_F$ (left panel) and the corresponding energy dependence of the prompt neutrino flux on $x_1$ cuts. The notation of the different curves indicate the range of $x_1$ values that is included in our calculations. The $x_1$ cut has a direct impact on the $x_F$ distribution, strongly suppressing the distribution at large $x_F$. Regarding the neutrino flux presented on the right panel, we observe that the main contribution comes from the intermediate $x_1$-range ($0.2 < x_1 < 0.6$). These results demonstrate that the significant portion of the neutrino flux comes from very forward (large $x_F$) charm production, with the incident parton
energy larger than 20% of the projectile nucleon energy at any probed neutrino energy.

Analogously, in Fig. 3 we show the corresponding sensitivity to the cuts on the target momentum fraction $x_2$. One finds that if the values of $x_2 < 10^{-5}$ are excluded, the $x_F$ distribution gets strongly suppressed at intermediate and large $x_F$. In particular, our results indicate that the main contribution to the distribution at proton energy $E_p = 10^9$ GeV comes from the $10^{-7} < x < 10^{-5}$ range of gluon longitudinal momentum fractions.

Regarding the neutrino flux, one can see that in the kinematical range probed by the recent IceCube data [8], one observes a strong sensitivity to the region of $x_2 < 10^{-5}$. For neutrino energies $E_\nu > 10^7$ GeV, even the region of $x_2 < 10^{-7}$ becomes important. These values of $x$ are beyond those probed by the $pp$ and $ep$ colliders, currently and in the past. For instance, the charm production at the LHC (LHCb detector) is sensitive to $x_2 > 10^{-5}$, while the HERA data lead to constraints on the gluon distributions for $x_2 > 10^{-4}$. The smallest values of $x_2 \sim 10^{-6}$ can be obtained from the inclusive production of $\chi_c$ mesons [30, 31] in $pp$ collisions and in the exclusive $J/\Psi$ photoproduction in hadronic collisions [36]. However, these possible constraints were not used so far to extract the gluon distributions. The models of gluon distributions in proton for $x_2 < 10^{-5}$ are therefore rather uncertain (see e.g. Ref. [33]). Consequently, the future neutrino telescopes will probe the prompt neutrino flux in a weakly explored small-$x$ range of the QCD dynamics.

Very recently, however, in Ref. [32] the combined set of the LHCb data on $D$-meson production at $\sqrt{s} = 5, 7$ and 13 TeV has been shown to constrain the gluon PDF reasonably well down to $x \sim 10^{-6}$. Namely, the combined analysis has resulted in an order-of-magnitude reduction of uncertainties in the gluon PDF compared to such well-known parameterization as the NNPDF3.0 [33] (for a more recent alternative analysis of the low-$x$ gluon PDF driven by the charm LHCb data, see e.g. Ref. [34]). Implications of such a reduction in the gluon PDF uncertainties for the prompt neutrino flux have been discussed in Ref. [33].

In Fig. 3 (left panel) we present the results for the prompt neutrino flux for different cuts on the target momentum fraction $x_2$. We find that the dominant contribution to the neutrino flux comes typically from $x_F$ in the region $0.2 < x_F < 0.5$, which is consistent with our previous results for the impact of the $x_1$ and $x_2$ cuts.

In Fig. 4 (right panel) we show a two-dimensional plot in $(x_1, \log_{10} x_2)$ for this $x_F$ range. For simplicity, in this calculation only the gluon-gluon fusion was taken into account, which is dominant mechanism at large energies (see below). In particular, one can see that the dominant contribution comes from the region of $x_1 \in (0.2-0.6)$ and $x_2 \in (10^{-8} - 10^{-5})$. We wish to stress that in both these regions of longitudinal momentum fractions gluon distribution is poorly constrained (see e.g. Ref. [29]). The behaviour of the $x_F$ distribution at intermediate $x_F$ is directly associated with the charm production at large rapidities, beyond those probed currently by the LHC detectors.

For completeness, in Fig. 5 we analyze the effect of cuts on the quark transverse momentum $p_T$ on the prompt neutrino flux. Our results indicate that the prompt neutrino flux is strongly affected by the charm production with transverse momentum in the $2 < p_T < 5$ GeV range. As the description of the transverse momentum spectra for the $D$-meson production at the LHC in this $p_T$ range has a larger theoretical uncertainty (see e.g. Ref. [21]), it also implies a large uncertainty in the neutrino flux predictions in the kinematical range probed by the IceCube.

In order to estimate the sensitive of our predictions on the PDF choice, in Fig. 7 we show the distributions in
Feynman $x_F$ for two different parton distribution sets and for two different energies of the incident cosmic rays, assumed to be protons. In particular, we will compare our previous estimates obtained using the CT14LL parametrization [29], with those derived using the MMHT2014LO one [37]. For completeness, in Fig. 7 we show also the contribution of the quark-antiquark annihilation process. Here, both PDF sets give quite similar cross sections for both energies. Regarding to the $gg \rightarrow c\bar{c}$ contribution, one finds that at lower energies (left panel) both PDF sets give the same $x_F$-distributions while at higher energies (right panel) they lead to quite different results. Clearly the present experimental data obtained at the LHC cannot constrain the gluon distributions at $x < 10^{-5}$. Since variations in the $x_F$ distribution at intermediate values of $x$, have a direct impact on the neutrino flux, we are forced to conclude that the current predictions for the prompt neutrino flux at very high neutrino energies are still not reliable.

The description of the QCD dynamics at small-$x$ and the heavy-quark production at large energies and forward rapidities are currently the subjects of intense debate. Basically, different formalisms based on different assumptions are able to describe the current experimental data. As the behaviour of the prompt neutrino flux at high energies is determined by the $x_F$-distribution at intermediate values of $x_F$, it is interesting to compare the predictions of these formalisms for energies probed in neutrino physics. In Fig. 8 we compare the charm production cross section obtained in different underlying QCD approaches – the collinear factorization approach (solid and dotted lines), the $k_T$-factorization approach [38-41] (dashed line) and the dipole model accounting for the saturation phenomena [42-45] (dash-dotted line). These distinct approaches for the heavy quark production in hadronic collisions differ in their
FIG. 5: The effect of cuts on the Feynman variable $x_F$ on the prompt neutrino flux (left), and the two-dimensional differential cross section for charm production in $pp$-collisions as a function of $x_1$ and $\log_{10}(x_2)$ (right).

FIG. 6: The effect of cuts on the quark transverse momentum $p_T$ on the prompt neutrino flux.

basic assumptions and partonic pictures.

While in the collinear framework, all particles involved are assumed to be on mass shell, carrying only longitudinal momenta, and the cross section is averaged over two transverse polarizations of the incident gluons, in the $k_T$-factorization approach the Feynman diagrams are calculated taking into account the virtualities and all possible polarizations of the incident partons. Moreover, in the $k_T$-factorization approach the unintegrated gluon distributions are employed instead of the usual collinear distributions.

In contrast, in the color dipole formalism the basic partonic picture of heavy quark production in gluon-gluon interactions is such that, before interacting with the hadron target, a gluon is emitted by a projectile and fluctuates into a color octet pair $Q\bar{Q}$, its lowest-order Fock component. The dipole approach does not rely on QCD factorisation and is based upon the universal ingredients such as the dipole cross section and the light-cone wave function for a given Fock component of the projectile that undergoes scattering off the target nucleon. One of the main motivations to use this approach is that it allows us to take into account the non-linear effects in the QCD dynamics,
FIG. 7: The charm production cross section $d\sigma/dx_F$ obtained with the leading-order collinear factorization for two different energies (left and right panel) and for two different PDF sets. Here, the $gg \rightarrow c\bar{c}$ and $q\bar{q} \rightarrow c\bar{c}$ components are shown separately.

FIG. 8: The charm production cross section $d\sigma/dx_F$ obtained with three different QCD approaches: collinear factorization (solid and dotted lines), $k_T$-factorization with the KMR UGDF (dashed line) and the dipole model presented in Refs. [51, 58] (dash-dotted line).

expected to be important at large energies, the QCD factorisation breaking effects at large Feynman $x_F$, as well as the higher-order QCD corrections and coherence phenomena (for more details, see e.g. Refs. [47–51] and references therein).

The framework of $k_T$-factorization used here was successfully applied by two of us for single [52] and double [53] open charm meson production at the LHC, as well as for leptons from semileptonic decays of heavy mesons at RHIC [54]. As a default choice, we use the Kimber-Martin-Ryskin (KMR) [55, 56] unintegrated gluon distribution functions (uGDFs) that were shown recently to effectively include a part of real higher-order corrections in charm production [57]. In the case of the dipole approach, we will consider the predictions obtained recently in Refs. [51, 58] which describe the current LHCb data, at least, in the high-$p_T$ domain. As discussed above, here the Feynman $x_F$ distributions are very sensitive to the very small transverse momenta.

We observe in Fig. 8 a significant order-of-magnitude difference between the predictions of the dipole and collinear
QCD approaches, with the $k_T$-factorization result being in between. We wish to point out here that the contributions of the gluon bremsstrahlung off light $q \rightarrow q + (G \rightarrow Q\bar{Q})$ and heavy $Q(\bar{Q}) \rightarrow Q(\bar{Q}) + G$ (anti)quarks are not included in the current dipole-model analysis and that are worth further exploration. The large cross section in the dipole model is somewhat unexpected as this approach includes saturation effects that should lead rather to a reduction of the cross section compared to the traditional collinear factorization approach. One the other hand, a similar effect has been observed at low heavy quark (heavy meson) transverse momenta $p_T < m_Q$ where the dipole results overshoot the LHC data on open heavy flavor production. Can this effect be caused by an approximate treatment of the kinematics and the dipole cross section or due to the missing higher-Fock (higher-twist) contributions in the current dipole-model analysis? Recently, the Drell-Yan process in the dipole picture and the associated kinematic constraints has been thoroughly discussed by some of us in Refs. \cite{49,50,59} while the higher-twist corrections remain uncertain. A proper analysis of this issue for heavy flavor production in the dipole picture is left for a future work.

Finally, in Fig. 9 we show the six-year experimental data collected by the IceCube Observatory \cite{3} together with our predictions for the neutrino flux calculated with two different current gluon PDFs. Both theoretical fluxes are below the IceCube data but unfortunately we cannot draw at present too strong conclusions. For comparison, we show a result of a simple fit (unbroken power-law isotropic distribution) proposed in Ref. \cite{3}, which is consistent with the yet low-statistics data.

Considering the several aspects discussed above one finds that in order to disentangle the magnitude of the astrophysical contribution to the neutrino flux, it is mandatory to get a better theoretical control of the prompt neutrino flux. Although the new experimental data from the LHC will be useful, they will not well constrain the charm production and the QCD dynamics in the kinematical ranges that determine the prompt neutrino flux at IceCube and future neutrino telescopes. Therefore, the experimental measurement of the neutrino flux and the separation of the prompt contribution are important challenges that should be surpassed in order to improve our understanding of strong interactions at high energies as well as of neutrino physics in astrophysical events.

### IV. SUMMARY

One of the current challenges in neutrino physics is to disentangle the signals of astrophysical origin from those associated with atmospheric interactions. The precise determination of the conventional and prompt atmospheric neutrino fluxes is fundamental for the interpretation of the results from neutrino observatories, such as the IceCube. In the last years, several groups estimated the prompt neutrino flux using different theoretical approaches e.g. for the calculation of the charm production cross section, charm fragmentation, cosmic ray flux etc. These studies demonstrated that the theoretical uncertainties are large, although they were reduced by the recent collider data and theoretical developments for the heavy quark production. Consequently, it is important to map the kinematical range that is probed by high-energy atmospheric neutrinos in order to clearly define the next steps that should be performed to obtain precise predictions for the atmospheric neutrino flux. This has been one of the main goals of our current
study.

In this paper, we have presented a detailed analysis of the kinematical domains that dominate the charm and prompt atmospheric neutrino production in cosmic rays relevant for the IceCube experiment by exploring the sensitivity of the corresponding neutrino flux and the charm cross section to the cuts on the maximal $pp$ c.m. energy, the longitudinal momentum fraction in the target and projectile, the Feynman $x_F$ and $p_T$ variables included in the calculation. We have found that in order to address production of high-energy neutrinos ($E_\nu > 10^7$ GeV) one needs to know the charm production cross section for energies larger than those available at the LHC as well as the parton/gluon distributions for the longitudinal momentum fractions in the region $10^{-8} < x < 10^{-5}$. Since this region of $x$ is not available at the collider measurements in the moment, the predictions in the collinear factorization approach and the $k_T$-factorization approach are not very reliable. If it was possible to disentangle the prompt atmospheric contribution from the cosmogenic one, it could perhaps become possible to put some contraints on the gluon distributions for extremely small longitudinal momentum fractions. This option requires a more dedicated study in the future.

We have also indicated the characteristic theoretical uncertainties in the charm production cross section obtained within different QCD approaches typically used by different groups in the analysis of prompt neutrino fluxes such as the leading-order collinear factorization approach, $k_T$-factorization and the dipole model accounting for the saturation phenomena.

Our results demonstrate that in order to predict the prompt neutrino flux for typical neutrino energies at the IceCube Observatory and future neutrino telescopes, we should extrapolate the behaviour of the heavy quark cross sections and energy distributions beyond the range accessible experimentally by current collider measurements. These results indicate that theoretical and experimental studies of the prompt atmospheric neutrino flux can provide an important information about the mechanism of heavy quark production as well as the description of the QCD dynamics in a kinematical range beyond that reached by the current colliders. At the current stage of research, it is premature to decide whether the measurement at the IceCube Observatory can provide a new information on the gluon distribution at very low longitudinal fractions $x \sim 10^{-7}$.

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