A toy quantum model of psychology

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Abstract

We propose the ideal individual model and the behavior coordinate system. Based on the ideal individual model, the behavior coordinate system and the quantum probability, a toy quantum model of human psychology is presented here in a different way. It can give some enlightening viewpoints through which some phenomena can be discussed from a different perspective.

Keywords: Quantum probability, ideal individual, behavior coordinate system, behavior state function

1 Introduction

Quantum mechanics is a fundamental theory in physics and is widely applied to many areas, from natural sciences such as biology and chemistry (Brookes, 2017; Levine, 2000) to social sciences such as economics and psychology (Busemeyer & Bruza, 2012; Haven & Khrennikov, 2013; Mansfield & Spiegelman, 1989; Rossi, 1994).

Many phenomena in psychology which cannot be explained by classical theories have been investigated by employing the quantum concepts and methods. In Ref. (Valle, 1989), discussions on the duality of human existence, i.e., the “wave” side and “particle” side of human existence is given analogous to the wave-particle duality in quantum physics. In Refs. (Busemeyer & Bruza, 2012), a human is considered to be in an indefinite state (formally called a superposition state) at each moment in time before a decision is made, and uncertainty is also discussed. Khrennikov & Haven (2007) claim that quantum probability interference is present in cognition as well as in social phenomena. In Refs. (Barros & Suppes, 2009; Busemeyer, Wang & Townsend, 2006), the interference are discussed and the Schrödinger equation is used. In Ref. (Sozzo, 2014), a quantum probability model in Fock space is proposed. Quantum principles are introduced to the field of human judgement and decision making (Aerts & Aerts, 1995; Atmanspacher, Römer & Walach, 2002; Bordely, 1998; Busemeyer, Wang, Khrennikov & Basieva, 2014; Khrennikov, 1999). This new field is called quantum cognition in which puzzling behavioral phenomena are found and discussed by applying a probabilistic formulation with non-commutative algebraic principles (Aerts, 2009; Bruza, Wang & Busemeyer, 2015;
We present a toy quantum psychology model in a different way. The old things can be recognized from a new point of view. In addition, there is always the hope that the new point view will inspire an idea for the modification of present theories, a modification necessary to encompass present experiments. In Sec. 2, the toy quantum psychology model is presented. The conclusions are in Sec. 3.

2 Toy quantum psychology model

In this section, the toy quantum model of human psychology is presented. For simplicity, we borrow nomenclature in physics to denominate the quantities in the following part.

2.1 Ideal individual

Behavior refers to the observable actions of human beings, such as speech, body movement, emotional expression and so on. Human behavior can be observable, measurable and repeatable.

An ideal individual is an idealization of human beings. For an ideal individual, every possible behavior can occur with equal probability. The possible behaviors are the human behaviors that have occurred in the past, are occurring at present and will occur in the future.

Let $B$ be a behavior set which is the collection of the behaviors of the whole mankind. The elements in $B$ are the possible behaviors of an ideal individual. Let $B_1$, $B_2$, · · · be the subsets of $B$, and there is

$$B_1 \cup B_2 \cup \cdots = B,$$

where $B_i$ is the collection of the behaviors of the individual $i$. In general, there are the relations

$$B_i \neq B_j, \quad B_i \cap B_j \neq \emptyset, \quad i, j = 1, 2, \cdots,$$

where $\emptyset$ denotes the empty set. Different behaviors show different properties and make different individuals.

2.2 Behavior coordinate system

Except for the intrinsic properties such as age, gender, etc., generally speaking, other quantities, such as emotion, personality and so on, are assumed to be functions of behavior and the derivative of behavior with respect to time. In other words, these quantities can be revealed by behavior and its derivatives (Tolman, 1948).

Let the behavior coordinate system be a coordinate system that specifies each behavior point uniquely in a behavior space by a set of numerical behavior coordinates. Each reference line is called a coordinate axis or axis of the system. The point where they meet is its origin. The reference lines are assumed to be perpendicular to each other, which are the speech-axis ($Q_1$-axis), the movement-axis ($Q_2$-axis), the expression-axis ($Q_3$-axis) and so on.
For simplicity, we assume that the behavior coordinate system is a Cartesian coordinate system, and the behavior coordinate space is an n-dimensional Euclidean space $\mathbb{R}^n$. Of course, the behavior coordinate system is far from being well established, therefore, this quantum psychological model is only a toy model. However, it can give some enlightening results.

### 2.3 Behavior state function

The state of an ideal individual is assumed to be represented by a behavior state function $|\Psi(x, q, t)\rangle$ (Chen, 2019), where $t$ is time, $x$ denotes the spatial space vector $x = (x, y, z)$, $q$ denotes the behavior space vector, $q = (q_1, q_2, q_3, \cdots)$. $q_1$ is the speech coordinate, $q_2$ is the movement coordinate, $q_3$ is the expression coordinate, and $\cdots$ denotes other coordinates.

We assume that the superposition principle holds for $\Psi$. If $\Psi_i$ describe possible states of an ideal individual, $\Psi = \sum_{i=1}^{n} c_i \Psi_i$ \hspace{1cm} (3) is also a possible state. The superposition principle demands that the dynamics equation for $\Psi$ should be a linear equation.

Adopting the Born hypothesis which gives the probability interpretation of the wave function in quantum mechanics, the behavior state function $|\Psi(x, q, t)\rangle$ is a probability amplitude and $||\Psi(x, q, t)||^2$ is interpreted as the probability of finding an ideal individual at a behavior point $q$ at time $t$ and spatial space $x$. There is the normalization condition

$$\int ||\Psi(x, q, t)||^2 dq dx = 1.$$ \hspace{1cm} (4)

The integration is over whole behavior space and whole spatial space.

In physics, the position of a particle and its derivative respective to time are used to discuss mechanical problems. Similarly, in psychology, the behavior of an ideal individual and its derivative respective to time are concentrated. In many cases, the spatial coordinates have small or even no effects on an individual’s behavior and then the spatial coordinates $x$ can be neglected. In some cases, the spatial space can be taken as part of the environment. It is not spatial space but the environment that affects the behaviors greatly. In consequence, the behavior coordinates are highlighted in these cases and the spatial coordinates can be integrated out

$$|\psi(q, t)\rangle = c \int |\Psi(x, q, t)\rangle dx, \quad \int ||\psi(q, t)||^2 dq = 1,$$ \hspace{1cm} (5)

where $c$ is a normalization factor. In the following part, the behavior state function is of $q$ and $t$. In many references (Busemeyer & Bruza, 2012; Haven & Khrennikov, 2013), the behavior state function $|\psi(q, t)\rangle$ has been used actually to discuss problems in different forms although the behavior variable $q$ is not given explicitly.

### 2.4 Operators

Similar to quantum mechanics, we assume that each measurable quantity $F$ has a corresponding operator $\hat{F}$. Any quantity that can be measured is an observable. It is obvious that the result of
a measurement of a dynamical variable must always be a real number (Dirac, 1958), therefore, the operator should be Hermitian, \( \hat{F} = \hat{F}^\dagger \). The superposition principle leads to the linearity of the operators. Thus the operators should be linear and Hermitian. The eigenvalue equation for the operator \( \hat{F} \) reads

\[
\hat{F}\phi = f\phi,
\]

where \( \phi \) is an eigenfunction of \( \hat{F} \) with eigenvalue \( f \).

The behavior position operator is the behavior space vector itself

\[
\hat{q} = q.
\]

Its components are

\[
\hat{q}_1 = q_1, \quad \hat{q}_2 = q_2, \quad \hat{q}_3 = q_3, \quad \cdots.
\]

The behavior momentum operator is assumed to be expressed as

\[
\hat{p} = -ib\nabla
\]

and its components are

\[
\hat{p}_1 = -ib\frac{\partial}{\partial q_1}, \quad \hat{p}_2 = -ib\frac{\partial}{\partial q_2}, \quad \hat{p}_3 = -ib\frac{\partial}{\partial q_3},
\]

where \( b \) is a new constant which plays the same role in quantum psychology as \( \hbar \) plays in quantum physics (Busemeyer, Pothos, Franco & Trueblood, 2011; Khrennikov, 1999). Using Eqs. (8) and (10), the commutators are obtained

\[
[\hat{q}_i, \hat{p}_j] = \delta_{ij}b, \quad [\hat{q}_i, \hat{q}_j] = 0, \quad [\hat{p}_i, \hat{p}_j] = 0,
\]

where \( \delta_{ij} \) is the Kronecker delta function.

### 2.5 Time evolution

Suppose the time evolution equation of the behavior state function can be written in the form of the Schrödinger equation as (Chen, 2019; Khrennikov, 1999; Pothos & Busemeyer, 2013; Triftet & Green, 1996)

\[
ib\frac{\partial|\psi(q, t)\rangle}{\partial t} = \hat{H}|\psi(q, t)\rangle,
\]

where \( \hat{H} \) is the Hamiltonian operator. The concrete form of \( \hat{H} \) is unknown to us now due to its complexity and the unwell-established behavior coordinate system. The differential equation (12) and the Schrödinger equation in quantum physics take the same form, however, they are fundamentally different. In Ref. (Busemeyer, Wang & Townsend, 2006; Pothos & Busemeyer, 2009), the Schrödinger equation is used to discuss quantum dynamics of human decision-making. But the new key constant \( b \) is not given.

\( \hat{H} \) will be in a complicated form and is expected to take the following form

\[
\hat{H} = \hat{T} + \mathcal{V}(V),
\]
where $\hat{T}$ is related to the properties of an individual, $V$ is the external environment which is independent on the individual and $\mathcal{V}(V)$ is the mapped environment by the individual from the external environment $V$ and his mind. In some cases, $\mathcal{V}(V)\approx V$. In some cases, $\mathcal{V}(V) = V + V_c$, where $V_c$ is a correction term. Sometimes, $\mathcal{V}(V)$ will be in a complicated form.

Let the Hamiltonian $\hat{H}$ be not explicitly time-dependent. The variables $q$ and $t$ of the time-dependent differential equation can be separated. With $|\psi(q,t)\rangle = |\Phi(q)\rangle f(t)$ we have two differential equations,

$$i\hbar \frac{\partial f}{\partial t} = \lambda f(t), \quad \hat{H}|\Phi(q)\rangle = \lambda |\Phi(q)\rangle. \quad (14)$$

Solving the first equation in the above equation gives the time factor $f(t) = \exp\left[-i\lambda t/\hbar\right]$. The second equation in Eq. (14) is a stationary differential equation.

### 2.6 Uncertainty principle

Let two observables be described by operators $\hat{A}$ and $\hat{B}$. The commutator of these two operators is written as

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} = i\hat{C}. \quad (15)$$

There is the Heisenberg’s uncertainty principle (Greiner, 2001)

$$\overline{(\Delta A)^2 (\Delta B)^2} \geq \frac{(\overline{C})^2}{4}, \quad (16)$$

where

$$\overline{\Delta A} = \int \psi^* \hat{A} \psi dq, \quad \overline{\Delta B} = \int \psi^* \hat{B} \psi dq, \quad \overline{\Delta C} = \int \psi^* \hat{C} \psi dq,$$

$$\overline{(\Delta A)^2} = \overline{A^2} - \overline{A}^2, \quad \overline{(\Delta B)^2} = \overline{B^2} - \overline{B}^2. \quad (17)$$

Eq. (16) can be written in a short form as

$$\Delta A \Delta B \geq \frac{1}{2} |\overline{C}|, \quad (18)$$

where $\Delta A = \sqrt{\overline{A^2} - \overline{A}^2}$, $\Delta B = \sqrt{\overline{B^2} - \overline{B}^2}$. From Eqs. (11) and (18), we have

$$\Delta q_i \Delta p_i \geq \frac{\hbar}{2}, \quad i = 1, 2, \ldots. \quad (19)$$

### 2.7 The simplest toy model

Let

$$\mathbf{p} = m \frac{d\mathbf{q}}{dt} = m\mathbf{v}, \quad (20)$$
where \( p \) is the behavior momentum, \( m = \frac{p}{v} \) is referred to as the behavior inertial mass, \( v \) is the behavior velocity. Due to the complexity of the discussed problems, we assume temporarily that the behavior kinetic energy takes the simple form

\[
\mathcal{T} = \frac{p^2}{2m}.
\]  

(21)

In case of one dimension, the stationary differential equation \((14)\) is written as

\[
\lambda |\Phi(q)\rangle = \hat{\mathcal{H}} |\Phi(q)\rangle, \quad \hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + \mathcal{V}(V), \quad \hat{p} = -i\hbar \frac{\partial}{\partial q},
\]  

(22)

where \( \lambda \) is the behavior energy.

According to Eqs. \((11)\) and \((18)\), we have the Heisenberg’s uncertainty in one dimension

\[
\Delta q \Delta p \sim \hbar.
\]  

(23)

In general, it is impossible to determine a human’s motivation according to one action. The motivation is often confined by observing enough behaviors. Suppose the motivation can be expressed in a function of behavior and behavior momentum, \( F(q, p) \). If \( \hat{F}(q, \hat{p}) = \sum_{m,n} c_{mn} q^m \hat{p}^n \) where \( c_{mn} \) are coefficients, there is

\[
[q, \hat{F}] = i\hbar \frac{\partial \hat{F}}{\partial \hat{p}}, \quad [\hat{p}, \hat{F}] = -i\hbar \frac{\partial \hat{F}}{\partial q}.
\]  

(24)

Then, we can obtain the uncertainty relation from Eqs. \((18)\) and \((24)\)

\[
\Delta q \Delta F \geq \frac{1}{2} |\hat{C}|, \quad \hat{C} = \hbar \frac{\partial \hat{F}}{\partial \hat{p}}.
\]  

(25)

3 Conclusions

We have proposed a toy quantum model of human psychology. This model is based on the ideal individual model, the behavior coordinate system and the quantum probability. The behavior coordinate system is far from being well established, therefore, this quantum psychological model is temporarily a toy model. However, it can give some enlightening viewpoints through which some phenomena can be discussed from a different perspective. In fact, many problems have been discussed qualitatively or even quantitatively by employing the behavior state function.

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