SYNCHRONISED PS-SPS TRANSFER WITH BARRIER BUCKETS

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Abstract
For the future intensity increase of the fixed-target beams in the CERN accelerator complex, a barrier-bucket scheme has been developed to reduce the beam loss during the 5-turn extraction from the PS towards the SPS, the so-called Multi-Turn Extraction. The low-level RF system must synchronise the barrier phase with the PS extraction and SPS injection kickers to minimise the number of particles lost during the rise times of their fields. As the RF voltage of the wide-band cavity generating the barrier bucket would be too low for a conventional synchronisation, a combination of a feedforward cogging manipulation and the real-time control of the barrier phase has been developed and tested. A deterministic frequency bump has been added to compensate for the imperfect circumference ratio between PS and SPS. This contribution presents the concept and implementation of the synchronised barrier-bucket transfer. Measurements with high-intensity beam demonstrate the feasibility of the proposed transfer scheme.

INTRODUCTION

The Multi-Turn Extraction (MTE) scheme replaced the previous Continuous Transfer (CT) extraction method in the CERN Proton Synchrotron (PS) to deliver the high-intensity proton beams for the fixed-target physics at the CERN Super Proton Synchrotron (SPS) and the operational implementation of MTE allowed to significantly reduce extraction losses in the PS ring (see, e.g. [5][7]).

The circumference difference between the PS and the SPS, the latter eleven times longer than the first, suggests extracting the beam from the PS over five turns to maximise the duty factor for fixed-target experiments. This means two transfers from the PS which fills 10/11th of the SPS and leaves time for a gap for the injection kickers. MTE has been designed to split the beam into five beamlets in the horizontal plane by crossing adiabatically the fourth-order resonance [11], thus removing the losses due to the beam-foil interaction. Only those due to the longitudinal beam structure and the rise time of the extraction kickers remain.

Synchronisation is triggered by the SPS, which means that the extraction kickers of the PS are synchronous with the injection kickers of the SPS. In order to mitigate the losses coming from the rise time of the SPS extraction kickers, a longitudinal gap is made with the wide-band RF system of the PS. This new requirement means that RF systems that were previously designed to split the beam into five beamlets in the horizontal plane by crossing adiabatically the fourth-order resonance [11], thus removing the losses due to the beam-foil interaction. Only those due to the longitudinal beam structure and the rise time of the extraction kickers remain.

SYNCHRONISATION CONCEPT

Figure 1 shows the magnetic flux density during the acceleration cycle with and without the synchronisation process. After injection, the beam is accelerated in $h = 8$ buckets, then longitudinally blown up, split into $h = 16$ buckets and accelerated to the flat-top momentum of 14 GeV/c. The longitudinal blow-up is needed to stabilise the beam at transition crossing at high intensities. The bunches are then transversely split and debunched at the end of the cycle prior to extraction to the SPS.

The transfer of the coasting beam from the PS to the SPS is triggered by the SPS, which means that the extraction kickers of the PS are synchronous with the injection kickers of the SPS. In order to mitigate the losses coming from the rise time of the SPS extraction kickers, a longitudinal gap is made with the wide-band RF system of the PS. This new requirement means that RF systems that were previously designed to split the beam into five beamlets in the horizontal plane by crossing adiabatically the fourth-order resonance [11], thus removing the losses due to the beam-foil interaction. Only those due to the longitudinal beam structure and the rise time of the extraction kickers remain.

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Figure 1: A comparison of the magnetic field function of the operational SFTPRO cycle without the synchronisation and the modified cycle with the longer flat-top needed for synchronisation.
operated stand-alone must be synchronised with the SPS. The beam energy has to be changed in the PS slightly to adjust the position of the longitudinal gap with the rise time of the PS extraction kickers. This can only be done if there is enough voltage available in the main RF system, especially if the manipulation is to be performed on the order of ten milliseconds.

Due to the low RF voltage requirements of the transverse beam splitting, $V_{RF}$ must be lowered well before the end of the cycle. The Finemet system can only generate a lot RF voltage, insufficient to synchronise the beam alone. Hence the synchronisation is proposed to take place at the beginning of the magnetic flat top, where the RF voltage is still sufficient to re-phase the beam quickly. Due to the periodicity it only requires at maximum a half of a $h = 16$ bucket phase change in either direction. This is also required because the barrier-bucket transfer from $h = 16$ is non-adiabatic, which means that the barrier is most effective once the voltage is already raised between two existing $h = 16$ buckets, as shown in Fig. 8.

The beam phase measurements for $h = 16$ and then $h = 1$ have to take place at the same frequency as the SPS; with a fixed B field in the PS this means that the mean radial position at the time of the measurements must be the same as the one at extraction. This poses a problem for the transverse splitting process as its ideal radial position is centred. Hence, in open loop, a constant frequency excursion is programmed to bring the beam centred for the transverse splitting and then after the process steer it to the extraction orbit with the RF systems. This only causes a fixed phase offset that does not affect the synchronisation.

Since the bunches after the cogging are $h = 16$ synchronous with the SPS, the final $h = 1$ synchronisation is only a bucket selection, which can be calculated. Therefore, no change in beam energy is needed for the $h = 1$ part.

The steps of the proposed synchronisation sequence are summarised in Table 1.

| Cycle time [ms] | Action                        |
|-----------------|-------------------------------|
| 590-600         | loops off, $h = 16$ phase measurement |
| 600             | $h = 1$ phase measurement     |
| 600-620         | cogging                      |
| 620-650         | radial steering to central orbit |
| 650-770         | constant offset for MTE       |
| 770-800         | radial steering to extraction orbit |
| 800-815         | raise barrier-bucket voltage |
| 800-815         | debunching                   |
| 835             | extraction                   |

Table 1: Synchronisation sequence in PS

Cogging

The principle of phase correction in a fixed magnetic field $B$ is to change the frequency of the beam such that the phase aligns with the SPS, i.e. the integral of the frequency change equals the desired phase difference. This requires a phase measurement at the reference frequency, a phase correction, and then returning to the original frequency, thus implying that the beam has to be accelerated and then decelerated.

Since the $B$-field is constant, the beam is also radially offset according to the following relation \[ \frac{df}{f} = \frac{\gamma^2}{\gamma_\text{tr}^2 - \gamma^2} \frac{dR}{R}. \] (1)

where $f$ is the beam frequency, $R$ the PS radius, $\gamma$ the relativistic factor, and $\gamma_\text{tr}$ the gamma-transition. This frequency offset needs to be small enough such that the beam does not get too close to the beam pipe. Longitudinal macro-particle tracking simulations have been performed to validate the following phase curve \[ \phi(\phi_{\text{set}}, T, t) = \phi_{\text{set}} \left[ t - \frac{T}{2\pi} \sin \left( \frac{2\pi t}{T} \right) \right] \] (2)

\[ t \in [0, T], \quad \phi_{\text{set}} \in [-2\pi, 2\pi], \] (3)

which corresponds to the programmed frequency curve of:

\[ f(\phi_{\text{set}}, T, t) = \frac{\phi_{\text{set}}}{2\pi T} \left[ 1 - \cos \left( \frac{2\pi t}{T} \right) \right] \] (4)

that defines all the frequency curves for the $h = 16$ correction, which can be seen in Fig. 2 (middle).

**IMPLEMENTATION**

Synchronisation can be tried with current PS beam control if one modifies the master direct digital synthesizer (DDS) frequency as depicted in Fig. 3. Phase slips and frequency steering can be implemented for all RF cavities and all RF systems simultaneously if the master clock frequency that drives the clock signal for the low-level RF (LLRF) boards is changed.

Since this is an open-loop manipulation, the handover from closed-loop to open-loop has to be implemented. Furthermore, the frequency precision required is higher than what the current control system can perform in an open loop. A diagram of the $h = 16$ phase correction is shown in Fig. 2.

Since the higher precision is needed on the frequency word, to implement the synchronisation new prototype hardware was designed and manufactured alongside. Also new firmware had to be designed.

![Figure 2: h = 16 cogging configuration.](image)

To reduce development time and cost, we implemented a rapid prototyping solution, also taking into account the...
challenge of part availability. A custom NIM-sized board was made to interface an ARM®Cortex®M7 controller (STM32H723ZG) on a development board with the current beam control (see Fig. 3). The controller’s digital interface is compatible with the LVTTL/TTL signals of the frequency distribution [33]. Hence, adding only a thin layer of power management and IO interface was needed. The analogue part of the $h = 16$ board contains operational amplifier (ADA4891) based circuits suitable for interfacing RF signals from DC to 22 MHz or DC to $h = 46$ with the 16 bit ADCs of the controller.

![Figure 3: Ultra low-cost prototype module to evaluate the $h = 16$ synchronisation feature with beam using the current NIM based control system in the PS (designed with the open source KiCAD).](image)

At the hardware architecture level (see Fig. 4), the CPU-intensive low-latency processing uses the Instruction and Data Tightly Coupled Memories (ITCM/DTCM). The peripheral connections to the advanced high performance bus (AHB) that do not require immediate CPU action are handled by the Direct Memory Access (DMA) controller. The corresponding data is stored in the slower RAM without interrupting the CPU. The Advanced Peripheral Bus (APB) connects the architecture’s advanced timers used for the phase measurement. Therefore, the CPU is not required for the measurement itself. The job of the custom bootloader and of the initialisation routines when parameters change is to transfer the instructions and data used for the real-time frequency bump calculations from the Flash or RAM to the tightly coupled memories dedicated to instructions and data. Therefore, flash and RAM access is not needed to compute these parameters in real-time.

The software architecture (see Fig. 5) was kept very simple in order to implement only the strictly necessary components. No operating system is needed for the synchronisation-related functions, since a reliable and predictable scheduling can be achieved by ARM’s Nested Vector Interrupt Controller hardware.

![Figure 4: The parts of the STM H7 memory and bus architecture used in the implementation.](image)

![Figure 5: The structure of the firmware with the programming languages used (ASM for ARM Assembler and C).](image)

**Frequency word generation**

The implementation of Eq. (4) in discrete time taking the architecture of the controller into account takes the following form:

$$T = N \tau_s \quad ; \quad t = n \tau_s$$  \hspace{1cm} (5)

$$f(\phi_{\text{set}}, N, \tau_s, n) = \frac{h_{\text{CLK}}}{h_{\text{RF}}} \frac{\phi_{\text{set}}}{2 \pi N \tau_s} \left[ 1 - \cos \left( \frac{2 \pi n}{N} \right) \right]$$  \hspace{1cm} (6)

where $h_{\text{CLK}} = 256$, $h_{\text{RF}} = 16$, $N$ is the number of points stored in memory, $n$ is the running sample index, $\tau_s$ is the adapting sampling period. It can be seen that the cosine term on the right-hand side of Eq. (6) does not depend on time or the chosen sampling frequency and therefore does not need to be recalculated even if the frequency-bump parameters, such as the length or the desired phase offset change. The parameter that typically changes from cycle to cycle is $\phi_{\text{set}}$, which is the result of the phase measurement. This means that only one multiplication per output sample must be performed in real time, in addition to the encoding, and that multiplication takes only one clock cycle at 544 MHz.

Furthermore, to test the limit of adiabaticity of beam manipulations, the sampling time $\tau_s$ can be automatically adjusted if a very fast ($< 5$ ms) frequency bump has to be performed. If the computing power is not sufficient to output a high-resolution waveform in time, then the sampling rate is automatically dropped, while still keeping the integral, i.e. the desired phase offset, the same. Formally, this change does not affect the cosine term in Eq. (6), since the samples are simply skipped and only the scaling factor is adjusted. This means in practise that the expensive non-linear calculation does not have to be performed, and therefore the controller can act quickly if the load conditions change. The trade-off is that the precision at which the curve approxi-
mates the desired phase change is lower when more samples are skipped.

**Phase measurements and barrier-bucket selection**

The $h = 16$ phase measurement is performed reading the output of a $h = 16$ phase discriminator in PS as shown in Fig. [3]. This phase discriminator has a dead zone, but it is sufficiently small that it only causes a very rare poor measurement, approximately one every hour or so. However, if one were to implement a $h = 1$ measurement in the same way, then the available equipment would have a much larger dead zone.

Hence, a dedicated measurement for the $h = 1$ bucket selection (see Fig. [6]) was developed on the controller, directly requiring only a minimal hardware interface in the form of a development board hat. Using a combinations of advanced timers on the controller, the gap selection can be achieved with about a degree precision. In theory, this can still cause the wrong bucket to be selected occasionally. However, this has not been observed so far, but is expected to happen. Nevertheless, this implementation does not have a dead zone, which would result in the barrier being at a completely different azimuth location compared to the target.

Figure 6: $h = 1$ configuration.

**VALIDATION WITH BEAM**

As described above, the synchronisation occurs at fixed bending field, as indicated by Fig. [7](top). The plot was generated using measured data with beam during setup, hence it also shows potential pitfalls with their mitigation as described below.

First, the frequency at the flat top is fixed, and the loops are opened. The radial position has to be carefully matched such that there is no sudden jump in the programmed fixed frequency corresponding to the SPS reference frequency, and the closed loop frequency of the PS. If there is a considerable difference, oscillations visible on Fig. [7](bottom) before 610 ms happen. If the radial positions are matched to about 0.1 mm using radial loop corrections before switching to the open loop, the smooth switch in the firmware can bridge the small remaining difference. However, these oscillations must be corrected to keep the longitudinal emittance lower and the radial position at extraction within acceptable limits, especially with increasing intensity.

The phase is measured with respect to the SPS in $h = 16$ at fixed frequency, and a correction with the main RF system is performed (see Fig. [7](middle)). This correction varies from cycle to cycle as this is the main action to synchronise the beams. Therefore, the overlapping graphs (in the middle and bottom plots) from different cycles show the envelope of all the corrections between 600 ms–620 ms in the cycle. Then the $h = 1$ phase is measured with respect to the SPS and this information is sent to the barrier-bucket front end over the network during transverse beam splitting. The beam is brought to a favourable radial position for transverse splitting, with a constant phase difference between PS and SPS. Oscillations of the mean radial position (MRP) at the beginning of the transverse beam splitting decay during the process. Note that the frequency of these is different from the ones during non-smooth switching. Their amplitude during a slower switching (purple curve) is lower, therefore these are related to the too quick, controlled $f_{rev}$ correction to bring the beam centred for the transverse splitting. Since the splitting itself occurs towards the end of the process, the effect of these is not significant. Once transversely split, the beam is brought to back to the SPS reference frequency. At
this point, the barrier front end received the correct barrier phase over the network and selected the correct gap between the $h = 16$ buckets to raise the barrier.

Two barrier-bucket voltage ramping schemes were tried. The traces marked in purple in Fig. 7 (middle) and (bottom) show a case when the barrier voltage is increased as the final frequency excursion in the PS is performed. This effectively results in a decelerating barrier-bucket configuration. The advantage of the decelerating barrier is that the frequency change can be spread out in time, but has to be tuned to the longitudinal dynamics during debunching to keep the depth of the gap. This scheme is not stable for trial runs yet, it is currently under study. The barrier at fixed position was used for further tests because of its simplicity.

The detailed quantitative consequences of the longitudinal emittance on the losses at extraction and the limits of the scheme in terms of beam intensity are subject to active studies. Current tests were carried out to about 15% less than nominal intensity or $2.1 \times 10^{13}$ ppp, which is considered a high intensity in PS.

In order to evaluate the reduction of PS beam losses, similarly to the tests reported in [28], the dummy septum has to be retracted. Since the present tests were performed in parallel to the operational beams where the septum must be present for radiation protection reasons, these aspects can not be reported on in this paper.

### CONCLUSIONS

Barrier bucket PS-SPS synchronisation scheme was devised and tried with a technical demonstrator hardware connected to the present PS beam control. All elements of the synchronization have been validated with beam. The high intensity beam in barrier buckets was extracted to SPS and delivered to North Area experiments. Further studies are needed to investigate intensity limits of the scheme. The implementation in C is a good starting point on the road to operation, as hardware can be defined on various platforms based on the prototype implementation.

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