Scalar FCNC and rare top decays in a two Higgs doublet model "for the top"

Itzhak Baum\textsuperscript{a,b,}\textsuperscript{*}, Gad Eilam\textsuperscript{b,}\textsuperscript{†} and Shaouly Bar-Shalom\textsuperscript{b,c,}\textsuperscript{††}

\textsuperscript{a} Dip. di Fisica, Università di Roma La Sapienza, Roma, Italy
\textsuperscript{b} Physics Department, Technion-Institute of Technology, Haifa 32000, Israel
\textsuperscript{c} Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA

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In the so called two Higgs doublet model for the top-quark (T2HDM), first suggested by Das and Kao, the top quark receives a special status, which endows it with a naturally large mass, and also potentially gives rise to large flavor changing neutral currents (FCNC) only in the up-quark sector. In this paper we calculate the branching ratio (BR) for the rare decays $t \rightarrow ch$ and $h \rightarrow tc$ ($h$ is a neutral Higgs) in the T2HDM, at tree level and at 1-loop when it exceeds the tree-level. We compare our results to predictions from other versions of 2HDM’s and find that the scalar FCNC in the T2HDM can play a significant role in these decays. In particular, the 1-loop mediated decays can be significantly enhanced in the T2HDM compared to the 2HDM of types I and II, in some instances reaching $BR \sim 10^{-4}$ which is within the detectable level at the LHC.

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I. INTRODUCTION

The Standard Model (SM) of elementary particles has been highly successful in describing the observed and measured phenomena. It contains, however, an unexplored sector, namely, the Higgs sector. The SM also has several problems, one of which is the fermion mass hierarchy problem, especially the top quark having a much larger mass than all other quarks.

In the T2HDM which was first suggested by Das and Kao [1] as an extension to the SM, and which can be viewed as a low-energy parametrization of a more fundamental theory, the top quark receives a special status by a particular Yukawa structure which endows the top quark with a naturally large mass, while at the same time giving rise to potentially large FCNC couplings in the up-quark sector. Such new FCNC interactions in the up-quark sector may drive FCNC decays such as $t \rightarrow ch$ and $h \rightarrow tc$ that we will consider in this paper.

Previous studies [2, 3] have shown that the $BR(t \rightarrow ch)$, where $h = H^0, h^0, A^0$, could reach up to $\sim 10^{-4}$ in the 2HDM type II and in the MSSM, and about $\sim 10^{-6}$ in the 2HDM type I. In addition, the FCNC top decays $t \rightarrow c\gamma$, $c g$, $c Z$ can range between $10^{-4} - 10^{-10}$ depending on the underlying Higgs sector. In order to give a feeling of the maximal values expected for the BR’s of FCNC top rare decays to gauge-bosons and scalars within different scalar models, we collect in table I some highlights of the results obtained in [2] and reported in the review [3]. Namely, the expected FCNC rates within the SM and the 2HDM of types I, II and III. Note that the values are given with: $m_t = 178$ GeV, $m_{H^0} = 2.74$ GeV and $m_{H^0} = 115$ GeV. The branching ratio depends strongly on these parameters, especially on $m_h$. Note also that the corresponding BR’s for the decays into a $u$ quark instead of a $c$ quark are a factor $|V_{ub}/V_{cb}|^2 \approx 0.0088$ smaller in the SM and in the 2HDM of types I and II.

The estimated LHC discovery limit for $t \rightarrow ch$ is $BR(t \rightarrow ch) \sim 5 \times 10^{-5}$ [3] for an integrated luminosity of 100 $fb^{-1}$. In the SM, this decay has a vanishingly small branching ratio $BR_{SM}(t \rightarrow cH^0) \sim 6 \times 10^{-15}$ [3–5], which is not accessible at the upcoming Large Hadron Collider (LHC). Thus the rare FCNC decays $t \rightarrow ch$ and $h \rightarrow tc$ are extremely sensitive probes of new physics in the scalar sector.

In this work we will explore these rare decay channels, $t \rightarrow ch$ and $h \rightarrow tc$, within the parameter space of the T2HDM, at the 1-loop level and, when allowed, also at the tree-level. We will focus on regions of the parameter space in which the $BR(t \rightarrow ch)$ can exceed the detection limit of the LHC, and also on regions where the $BR(t \rightarrow ch)$ and $BR(h \rightarrow tc)$ can be enhanced significantly compared to other 2HDM.

The paper is organized as follows: in sections II and III we describe the main features of the T2HDM relevant for our analysis and in section IV we shortly discuss the constraints on the parameter space of the model. In section V we outline our analytical derivation and in section VI we give our numerical results. In section VII we summarize. In Appendix A we list the required Feynman

|                | SM        | 2HDM-I,II | 2HDM-III |
|----------------|-----------|-----------|-----------|
| $BR(t \rightarrow c\gamma)$ | $\sim 5 \times 10^{-14}$ | $\lesssim 10^{-9}$ | $\lesssim 10^{-6}$ |
| $BR(t \rightarrow cg)$     | $\sim 5 \times 10^{-12}$ | $\lesssim 10^{-8}$ | $\lesssim 10^{-4}$ |
| $BR(t \rightarrow cZ)$     | $\sim 1 \times 10^{-14}$ | $\lesssim 10^{-10}$ | $\lesssim 10^{-7}$ |
| $BR(t \rightarrow ch)$     | $\sim 6 \times 10^{-15}$ | $\lesssim 10^{-5}$ | $\lesssim 10^{-3}$ |

TABLE I: Top quark rare decays into $c\gamma$, $cg$, $cZ$, $ch$, within various models: the SM, the 2HDM of type I, II and type III. Our results are consistent with [3–5].
rules, in Appendix B we give the 1-loop amplitudes, in Appendix C we define the 1-loop integrals and in Appendix D we derive the total Higgs width in the models considered.

II. THE TWO HIGGS DOUBLETS MODEL “FOR THE TOP”

In the T2HDM the second Higgs field couples only to the top-quark, while the first Higgs field couples to all other quarks [1]:

\[
L_Y = -\bar{Q}_L \Phi \gamma_i d_R - Q_L \bar{\Phi} \gamma_i G_{ij=1,2} \left( \begin{array}{c} u \\ c \end{array} \right)_R \\
-\bar{Q}_L \bar{\Phi} G_{ij} t_R + h.c.,
\]

where: \( i, j = 1, 2, 3 \) are flavor indices, \( L(R) \equiv (1 - (+)\gamma^5)/2 \) are the chiral left (right) projection operators, \( f_{L(R)} = L(R)f \) are left(right)-handed fermion fields, \( Q_L \) is the left-handed \( SU(2) \) quark doublet and \( F, G \) are general \( 3 \times 3 \) Yukawa matrices in flavor space. Also, \( \Phi_{1,2} \) are the Higgs doublets:

\[
\Phi = \left( \begin{array}{c} \Phi^+ \\ v + \Phi^0 \end{array} \right), \quad \bar{\Phi} = \left( \begin{array}{c} \Phi^+ \\ \sqrt{2}v - \Phi^0 \end{array} \right).
\]

The Yukawa texture of (1) can be realized in terms of a \( Z_2 \)-symmetry under which the fields transform as follows [1]:

\[
\Phi_1 \rightarrow -\Phi_1, \quad d_R \rightarrow -d_R, \quad u_R \rightarrow -u_R (u = u, c), \\
\Phi_2 \rightarrow +\Phi_2, \quad Q_L \rightarrow +Q_L, \quad t_R \rightarrow +t_R.
\]

The Higgs potential is a general 2HDM one [6]:

\[
\mathcal{L}_H = \lambda_1 \left( \Phi_1^+ \Phi_1 - v_1^2/2 \right)^2 + \lambda_2 \left( \Phi_2^+ \Phi_2 - v_2^2/2 \right)^2 \\
+ \lambda_3 \left[ \left( \Phi_1^+ \Phi_1 - v_1^2/2 \right)^2 + \left( \Phi_2^+ \Phi_2 - v_2^2/2 \right)^2 \right] \\
+ \lambda_4 \left[ \left( \Phi_1^+ \Phi_1 \right) \left( \Phi_2^+ \Phi_2 \right) - \left( \Phi_1^+ \Phi_2 \right) \left( \Phi_2^+ \Phi_1 \right) \right] \\
+ \lambda_5 \left[ \Phi_1^+ \Phi_2 - v_1/2 \right]^2.
\]

where we have included the term \( \propto \lambda_5 \) which softly breaks the \( Z_2 \)-symmetry in (2) and which can also give rise to CP-violation [7].

The top-quark acquires a mass term primarily from the second Higgs vacuum expectation value (VEV), which we will choose to be much larger than the first Higgs VEV:

\[
\tan \beta \equiv \frac{v_2}{v_1} \gg 1.
\]

Eq. 4 above is the working assumption of the T2HDM.

The particular Yukawa structure of (1) gives rise to various other interesting features of the T2HDM:

- **Enhanced \( H^+cb \) coupling:** The \( H^+cb \) interaction term is naively enhanced by a factor of \( V_{tb}/V_{cb} \) compared to other 2HDM’s, where \( V \) is the CKM matrix. This property which motivated the analysis in [8, 9], also motivated the present work since, as we shall later see, the 1-loop FCNC decays \( t \rightarrow ch \) and \( h \rightarrow t\bar{c} \) are expected to be enhanced, due to this large \( H^+cb \) coupling, naively by this factor compared to e.g., the 2HDM of type II.

- **Tree-level FCNC couplings in the up-quark sector:** While there are no tree-level FCNC interactions in the down-quark sector (as for the example in the case of the type III 2HDM [10]), there are a-priori \( O(1) \) FCNC \( htc \) and \( htu \) couplings.

- **Enhanced couplings to up quarks:** The couplings of the three neutral scalars (\( h \equiv H^0, H^0, \) or \( A^0 \)) to all the quarks except for the top quark, increase with \( \tan \beta \). For example, the \( hcc \) coupling is \( \propto m_c/\tan \beta \) in the T2HDM as opposed to being \( \propto m_c/\tan \beta \) in e.g., the type II 2HDM which also underlies the MSSM. Since \( \tan \beta \gg 1 \) is the working assumption of the T2HDM, one expects a large enhancement of the \( hcc \) coupling in the T2HDM. This motivated the work in [11].

III. YUKAWA INTERACTIONS AND THE THEORETICAL SETUP

A detailed derivation of all Yukawa interactions in the physical mass basis can be found in [12] (see also [13]). We use the results of [12], which are also summarized as a list of scalar-quark-quark Feynman rules in Appendix A.

Below we highlight only the new Yukawa interactions that will be the focus of our analysis. In particular, as mentioned above, in the T2HDM the \( H^+cb \) interaction is different from the typical 2HDM scenario:

\[
\mathcal{L}_{H^+cb} = \frac{g}{\sqrt{2}m_W} H^+ \bar{c} \left[ \tan \beta V_{cb} M_d R + (-M_u \tan \beta + \sigma^f (\tan \beta + \cot \beta)) V L \right]_{cb} b \\
\sim \frac{g}{\sqrt{2}m_W} H^+ \bar{c} \left[ \tan \beta V_{cb} m_b R + m_c (-\tan \beta V_{cb} + \sigma^f (\tan \beta + \cot \beta) V_{tb} L \right] b,
\]

where \( M_d = diag(m_d, m_s, m_b) \), \( M_u = diag(m_u, m_c, m_t) \) and \( \Sigma \) is a new mixing matrix in the up-quark sector that
\[ \frac{\Sigma}{M_t} = \left( \begin{array}{c} \frac{m_t^2 \varepsilon^2}{m_t} |\xi|^2 \left( 1 - |\varepsilon| |\xi|^2 \right) \\ \frac{m_t^2 \varepsilon^2 \xi' \xi}{\sqrt{1 - |\varepsilon| |\xi|^2}} \\ \epsilon^3 \varepsilon \xi' \sqrt{1 - |\varepsilon| |\xi|^2} \\
- \epsilon^2 \xi' \sqrt{1 - |\varepsilon| |\xi|^2} \\
\end{array} \right) \]

where \( \varepsilon_{ct} \equiv m_c/m_t \) and \( \xi, \xi' \) are dimensionless parameters naturally of \( O(1) \).

As we can see from (5), the \( H^+cb \) vertex has terms proportional to \( V_{cb} \) which are common to other 2HDM's, but has an additional term which is not CKM suppressed and is proportional to \( (\tan \beta + \cot \beta) \times (\Sigma^I)_{cb} \sim m_c \xi^* \tan \beta V_{tb} \). As we shall see below, this apparent enhancement to \( H^+cb \) coupling will drive the main contribution to the 1-loop diagrams with internal \( H^+ \) and \( b \).

The \( H^+ \bar{b} \) vertex also receives an additional term within the T2HDM:

\[ L_{H^+b} \sim \frac{g}{\sqrt{2} m_W} H^+ \bar{b} \left\{ \tan \beta V_{tb} m_b R + \left[ m_t V_{tb} \cot \beta \right] L \right\} \bar{b}. \]

We see, however, that this new contribution to the \( H^+b \) coupling is sub-leading and vanishes in the limit \( \xi, \xi' \to 0 \), in which case the \( H^+b \) interaction in (7) converges to that of a 2HDM type I and II.

As for the neutral Higgs sector, there is no \textit{a priori} distinction between \( h^0 \) and \( H^0 \) other than the rotation angle \( \alpha \). In particular, the \( h^0tc \) and \( H^0tc \) Yukawa interactions are:

\[ L_{h^0tc} \sim h^0 \left[ - \frac{g}{2 m_W} \left( \frac{\cos \beta - \sin \beta}{\sin \beta} \right) m_c \xi (R + \varepsilon_{ct} L) \right] \bar{c} , \]

\[ L_{H^0tc} \sim H^0 \left[ \frac{g}{2 m_W} \left( \frac{\sin \beta - \cos \beta}{\cos \beta} \right) m_c \xi (R + \varepsilon_{ct} L) \right] \bar{c} , \]

where we have used the off-diagonal terms of \( \Sigma \) from Eq. 6, neglecting terms of order \( \varepsilon^2_{ct} \) (recall \( \varepsilon_{ct} = m_c/m_t \)) for which \( \Sigma_{tc} \sim m_c \xi \) and \( \Sigma^I_{tc} \sim m_c \varepsilon_{ct} \xi \).

For arbitrary \( \alpha \) and \( \beta \), the above FCNC interactions will lead to both \( t \to c H^0 \) and \( t \to c h^0 \) (or \( H^0 \to tc \) and \( h^0 \to tc \) if \( m_t < m_{H^0}, m_{h^0} \) decays at tree level). One can, however, eliminate one of the two tree-level \( h^0tc \) or \( H^0tc \) couplings by choosing a specific direction with respect to the mixing angles \( \alpha \) and \( \beta \) of the neutral Higgs sector. For example, the tree-level \( H^0tc \) coupling can be eliminated if one of the following two conditions is satisfied:

1. \( \xi = 0 \).
2. \(- \frac{\sin \alpha}{\sin \beta} + \frac{\cos \alpha}{\cos \beta} = 0 \) implying \( \alpha = \beta + n \pi \).

Without loss of generality, let us consider the case in which either the \( h^0tc \) or the \( H^0tc \) coupling vanishes at tree-level. For definiteness, we will adopt the second choice above, \( \alpha = \beta \), which sets:

\[ L_{H^0tc}^{\alpha=\beta} = 0 , \]

and gives:

\[ L_{h^0tc}^{\alpha=\beta} \sim h^0 i \left[ - \frac{g m_c \xi}{2 m_W} \left( \tan \beta - \cot \beta \right) (R + \varepsilon_{ct} L) \right] \bar{c} . \]

There are several reasons which motivate an analysis of the case \( \alpha = \beta \) rather than the case of generic mixing angles and for preferring this choice over the choice \( \xi = 0 \) which also eliminates the tree-level \( H^0tc \) coupling:

1. \( \xi = 0 \) is disfavored by the analysis in [13], as we will recapitulate in Sec. IV.
2. \( \xi = 0 \) diminishes the potentially enhanced term in the \( H^+b \) coupling, as is evident from Eq. 5.
3. The limit \( \alpha = \beta \) is a natural result of the MSSM, when the mass of the CP-odd neutral Higgs, \( A^0 \), is large (see e.g. [6] in the limit \( m_{A^0} \to \infty \)). The choice \( \alpha = \beta \) is widely used in the literature, partly for this reason. Thus, even though the T2HDM setup is not natural within the MSSM, this will help us compare our results with other existing results in different types of 2HDM’s.
4. The limit \( \alpha = \beta \) sets the scalar \( H^0 \) to be SM-like, in which case the direct bounds on the SM Higgs mass roughly apply to \( H^0 \). Also, \( H^0 \) will have SM-like Yukawa couplings to quarks:

\[ L_{Y_{H^0}}^{\alpha=\beta} \sim H^0 \left[ \frac{g M_t}{2 m_W} \right] d + H^0 \left[ \frac{g M_u}{2 m_W} \right] u . \]

Finally, the \( h^0it \) interaction, with \( \alpha = \beta \), reads:

\[ L \sim h^0 i \left\{ \frac{g m_t}{2 m_W} \left[ - \cot \beta + \varepsilon^2_{ct} \left( |\xi|^2 + |\xi'|^2 \right) (\tan \beta - \cot \beta) \right] R + \left( h.c. \right) L \right\} t , \]

where \( \Sigma_{tt} = \left( \Sigma^I \right)_{tt} \sim m_t - m_{ct} \left( |\xi|^2 + |\xi'|^2 \right) \). As in the case of \( H^+b \), the term \( \propto m_t \tan \beta \) cancels, and we are left with the usual \( m_t \tan \beta \) term plus terms that are suppressed either by \( \varepsilon^2_{ct} \) or \( \cot^2 \beta \).
IV. THE PARAMETER SPACE OF THE T2HDM

The parameter space of the T2HDM was recently analyzed in [13]. Here we recapitulate the bounds on the parameter space of T2HDM that were derived in [13] by performing a best fit to several experimentally measured observables mainly associated with B-decays. The processes that were selected were the ones that are potentially most sensitive to the charged sector of the T2HDM. The analysis in [13] is directly relevant to the present work, and so we list below the final results of [13] (recall that $\xi = |\xi| e^{i\varphi}$):

$$m_{H^\pm} = (660^{+390}_{-280}) \text{ GeV},$$

$$\tan \beta = 28^{+44}_{-8},$$

$$0.5 < |\xi| < 1,$$

$$\varphi_\xi = (110^{+30}_{-65})^\circ,$$

$$|\xi| \sim 0.21,$$

$$\varphi_\xi \sim 250^\circ.$$  \hfill (14)

Note that:

- These values for $\tan \beta$ and $m_{H^\pm}$ are allowed also within the framework of a type II 2HDM [14].
- The authors of [13] didn’t consider possible constraints on the FCNC $H^0$ – up-quark couplings of the T2HDM, coming from the recently measured $D^0$ oscillations. We note, however, that such contributions to the $D^0 – \bar{D}^0$ mass difference is suppressed by a factor of $(\frac{1}{\tan \beta} \frac{m_t}{m_t^R} \frac{m_t^L}{m_t^R})^2$ compared to the charged Higgs contribution that was considered in [13], and therefore does not impose further constraints on the FCNC parameter space of the neutral sector other than those found in [9, 10].

There are also direct constraints on the neutral Higgs masses from high-energy collider experiments [15]:

- The direct bound on the SM Higgs (which also applies to $H^0$ of the T2HDM when $\alpha = \beta$) is: $m_{H^0} > 114$ GeV.
- The bounds on the mass of lightest neutral scalar and the charged scalar in supersymmetry are: $m_h > 90$ GeV and $m_{H^+} > 80$ GeV.

V. ANALYTICAL RESULTS

A. The tree-level $t \rightarrow c h^0$ or $h^0 \rightarrow t c$

As stated above, when $\alpha = \beta$ the decays $t \rightarrow c h^0$ or $h^0 \rightarrow t c$ can proceed at tree-level (when kinematically allowed) while the corresponding decays involving $H^0$ are mediated at 1-loop. Using the tree-level coupling $h^0 t c$ in Eq. 11, the tree-level amplitude for the process $t \rightarrow c h^0$ is:

$$\mathcal{M} (t \rightarrow c h^0) = \bar{u}_c \left[ \frac{g}{2m_W} \left( (M_u)_{ct} \sin \alpha \cos \beta \right. \right.$$

$$- \Sigma_{ct} \left( \frac{\cos \alpha}{\sin \beta} + \frac{\sin \alpha}{\cos \beta} \right) R + (h.c.) L \bigg] u_t,$$

where from (6) we have:

$$\Sigma_{ct} = m_t \epsilon_{ct} \xi^* \sqrt{1 - |\epsilon_{ct}\xi|^2} \sqrt{1 - |\epsilon_{ct} \xi|^2} \sim m_t \epsilon_{ct} \xi^*,$$

$$\left( \Sigma_f \right)_{ct} = m_t \epsilon_{ct} \xi^* \sqrt{1 - |\epsilon_{ct}\xi|^2} \sqrt{1 - |\epsilon_{ct} \xi|^2} \sim m_t \xi^*.$$  \hfill (16)

Taking $\alpha = \beta$ we then obtain:

$$\mathcal{M} (t \rightarrow c h^0) = \bar{u}_c \frac{g}{2m_W} \left( \cot \beta + \tan \beta \right) \epsilon_{ct} R + L \bigg] u_t$$

$$\equiv \bar{u}_c \left[ m_R R + M_L L \bigg] u_t.$$  \hfill (17)

The squared amplitude, summed over initial and final state polarizations is then:

$$\sum_{pol} |\mathcal{M}|^2 = 2m_c m_t (M_L M_R^* + M_R M_L^*)$$

$$+ (m_t^2 + m_c^2 - m_h^2) (M_L M_R^* + M_R M_L^*)$$

$$= \frac{g^2 m_t^2}{4m_W^2} \left( \cot \beta + \tan \beta \right)^2 |\xi|^2$$

$$\times \left[ 2m_c m_t \epsilon_{ct} + \left( m_t^2 + m_c^2 - m_h^2 \right) \left( 1 + \epsilon_{ct}^2 \right) \right]$$

$$\sim \frac{g^2 m_t^2}{4m_W^2} \tan^2 \beta |\xi|^2 \left( m_t^2 - m_h^2 \right).$$  \hfill (18)

where we have neglected terms of $\mathcal{O}(m_c^2/m_t^2)$ and of $\mathcal{O}(\cot \beta)$ in accordance with the working assumption of the T2HDM, i.e., that $\tan \beta >> 1$. The width of $t \rightarrow c h^0$ then reads:

$$\Gamma (t \rightarrow c h^0) = 4\pi \cdot \lambda^\frac{1}{2} \left( \frac{1}{x} \frac{m_t^2}{m_t^L} \frac{m_t^2}{m_t^R} \right) \sum_{pol} |\mathcal{M}|^2$$

$$\sim \frac{g^2 |\xi|^2 m_t m_c}{128 \pi m_W^2} \tan^2 \beta \left( 1 - \frac{m_h^2}{m_t^2} \right)^2,$$  \hfill (19)

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ and $\sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} |\mathcal{M}|^2$ is the squared amplitude summed over final polarizations and averaged over the initial top polarizations.

We then obtain the following $BR (t \rightarrow c h^0)$ (for large $\tan \beta$):

$$BR (t \rightarrow c h^0) \sim \frac{|\xi|^2 m_t^2}{2m_W^2} \tan^2 \beta \left( 1 - \frac{m_h^2}{m_t^2} \right)^2 \left( 1 - \frac{m_h^2}{m_t^2} \right)^{-1}$$

$$\times \left( 1 - 2 \frac{m_W^2}{m_t^2} \frac{m_t^2}{m_W^2} \right)^{-1}.$$  \hfill (20)
where for the total top-quark decay width ($\Gamma_t$) we took (at tree-level and neglecting terms of order $m_H^2/m_t^2$ [15]):

$$\Gamma_t = \Gamma (t \to bW^+) \sim \frac{g^2 m_t}{64 \pi} \left(1 - \frac{m_H^2}{m_t^2}\right) \left(1 - 2 \frac{m_t^2}{m_H^2} + \frac{m_t^2}{m_W^2}\right). \quad (21)$$

For instance, taking $\tan \beta = 28$, $|\xi| = 0.8$ (compatible with the bounds in (14)) and $m_{h^0} = 91$ GeV, we get (when $\Gamma_t = \Gamma (t \to bW^+)$):

$$BR (t \to ch^0) \sim 0.0077. \quad (22)$$

Note that if $m_{H^+} < m_t$, then the top can also have an appreciable BR to $t \to H^+b$ which must be taken into account in $\Gamma_t$.

The decay width for the reverse $h^0 \to \bar{t}c$ process (corresponding to the case $m_{h^0} > m_t$) can be obtained by applying a crossing symmetry to the squared amplitude of the decay $t \to ch^0$ in Eq. 18 (see e.g., [16]):

$$\sum_{\text{pol}} |M|^2 (h^0 \to \bar{t}c) = \sum_{\text{pol}} |M|^2 (t \to ch^0) \sim \frac{g^2 m_t^2}{4m_W^2} \tan^2 \beta |\xi|^2 \left[m_{h^0}^2 - m_t^2\right], \quad (23)$$

from which we get:

$$\Gamma (h \to \bar{t}c + ct) = 2 \times \Gamma (h^0 \to \bar{t}c) = 2N_c \sum_{\text{pol}} |M|^2 (h^0 \to \bar{t}c) \sim N_c |\xi|^2 \frac{g^2 m_{h^0} m_t^2}{32 \pi m_W^2} \tan^2 \beta \left(1 - \frac{m_t^2}{m_{h^0}^2}\right)^2, \quad (24)$$

where $N_c = 3$ is a color factor, and $\sum_{\text{pol}} |M|^2 = \sum_{\text{pol}} |M|^2$, since the initial state is a scalar field.

To get an estimate of the $BR(h^0 \to \bar{t}c)$ we need the total width of $h^0$. For $\alpha = \beta$ and assuming also that $m_{h^0} < 2m_t, 2m_{H^+}$, the total decay width of $h^0$ is mainly comprised of fermion decays, since the couplings $W^+W^-h^0, Z^0Z^0h^0$ and $H^0H^0h^0$ are all $\propto \sin (\beta - \alpha)$ (see Table VII and App. D). Thus, below the $tt$ threshold (at about $m_{h^0} \approx 340$ GeV) the decay $h^0 \to bb$ dominates, with (see App. D):

$$\Gamma (h^0 \to bb) \sim \frac{N_c g^2 m_{h^0}^2}{32 \pi m_W^2} \tan^2 \beta. \quad (25)$$

In this case, we obtain:

$$BR (h^0 \to \bar{t}c + ct) \sim |\xi|^2 \frac{m_t^2}{m_{h^0}^2} \left(1 - \frac{m_t^2}{m_{h^0}^2}\right)^2, \quad (26)$$

which for e.g., $|\xi| = 0.8$ and $m_{h^0} = 300$ GeV, amounts to $BR (h^0 \to \bar{t}c + ct) \sim 0.023$.

### B. The 1-loop decays $t \to cH^0$ or $H^0 \to \bar{t}c$

The 1-loop $t \to cH^0$ decay amplitude is composed of 10 Feynman diagrams which are shown in Fig. 1. The individual amplitudes corresponding to each of the 10 diagrams are given in App. B. The calculation was performed in the t’Hooft Feynman gauge and was aimed to be as model-independent as possible, therefore assuming general vertices for the general fields $q_i, V_\alpha$ and $H_\alpha$ which stand for a quark (up or down type), vector (gauge) fields and scalar fields, respectively (see the calculation setup as defined by Fig. 13 in App. B). This allowed us to easily calculate the partial width for $t \to cH^0$ (or for $H^0 \to \bar{t}c$) in different multi-Higgs models, by inserting the appropriate vertices and fields.

The 1-loop integrals were evaluated numerically with FORTRAN (777) using the FF package [17]. The calculations were done using the Passarino-Veltman reduction scheme, which expresses the integrals in terms of basic scalar n-point functions. In particular, the vector and tensor integrals were computed using linear combinations of the scalar functions (for explicit formulae see e.g. App. A in [2]). In App. C we describe the reduction scheme used to calculate the 1-loop integrals.

As in the tree-level case, let us define the total amplitude as:

$$\mathcal{M} = \frac{i e \alpha}{16 \pi^2} (M_L L + M_R R) u_t, \quad (27)$$

where

$$M_{L,R} = \sum_{i=1-10} M_{L,R}^i, \quad (28)$$

and $M_{L,R}^i$ are parts of the amplitude corresponding to diagram $i$ which are given in App. B.

Using Eq. 27 we can write the squared amplitude summed over polarizations as:

$$\sum_{\text{pol}} |M|^2 = \frac{1}{2} \frac{1}{256 \pi^2} \left[2m_t m_i (M_L M_R^* + M_R M_L^*) \right], \quad (29)$$

from which we obtain:

$$\Gamma = 4\pi \cdot \lambda^2 \left(1, \frac{m_t^2}{m_{h^0}^2}, \frac{m_t^2}{m_{h^0}^2}\right) \frac{\sum_{\text{pol}} |M|^2}{64 \pi^2 m_t^4}. \quad (30)$$

where: $\sum_{\text{pol}} |M|^2 = \frac{4}{3} \sum_{\text{pol}} |M|^2$.

Finally, the BR for the decay $t \to cH^0$ is:

$$BR (t \to cH^0) \sim \frac{\Gamma (t \to cH^0)}{\Gamma (t \to bW^+)} \cdot \frac{\Gamma (t \to bW^+)}{\Gamma (t \to \bar{t}c + ct)}, \quad (31)$$

where $\Gamma (t \to bW^+)$ is given in Eq. 21. As before, if $m_{H^+} < m_t$, then the top can also have an appreciable
For the case $m_{H^0} > m_t$ we again consider the reversed 1-loop decay $H^0 \to \bar{t}c + \bar{c}t$, where

$$\Gamma(H^0 \to \bar{t}c + \bar{c}t) = 2 \times \Gamma(H^0 \to \bar{c}c)$$

$$= 2N_c\lambda^2 \left(1, \frac{m_c^2}{m_{H^0}^2}, \frac{m_{H^0}^2}{m_{H^0}^2}\right) \frac{\sum_{\text{pol}} |\mathcal{M}|^2(H^0 \to tc)}{16\pi m_{H^0}} \tag{32}$$

where, as in the tree-level case, $\sum_{\text{pol}} |\mathcal{M}|^2(H^0 \to \bar{t}c) = \sum_{\text{pol}} |\mathcal{M}|^2(H^0 \to \bar{c}c) = \sum_{\text{pol}} |\mathcal{M}|^2(t \to cH^0)$.

Here also, in order to obtain the $BR(H^0 \to \bar{t}c + \bar{c}t)$ we need to know the total width of $H^0$. We will include the leading-order contributions to the Higgs width from $H^0 \to \bar{q}q$, $H^0 \to VV$, $H^0 \to 2$ scalars and $H^0 \to \text{vector+scalar}$ [18]. The last channel to vector+scalar can be important in some regions of the parameter space such as low $\tan \beta$. The formulae used for the calculation of the total $H^0$-width are given in App. D, along with a plot (Fig. 14) of the SM Higgs width in the leading order approximation and compared with higher order predictions (recall that for our choice of $\alpha = \beta$, $H^0$ behaves as the SM-Higgs).

\section{VI. NUMERICAL RESULTS}

Before presenting our numerical results we note that we have performed several checks to validate our calculation:

1. We have successfully reproduced the results for $BR(t \to ch)$ obtained in [4] for the SM case and in [2] for the type II 2HDM case.

2. We have successfully reproduced the results for $BR(h \to \bar{t}c)$ obtained in [5] for the SM case and in [14] for the type II 2HDM case. However, we were not able to reproduce the values for $BR(h \to \bar{t}c)$ reported in [2], as was also stated in [14].

3. We have verified both analytically and numerically in the FORTRAN code, the cancellation of the UV divergences which appear in some of the individual 1-loop amplitudes.

Let us now present our results for the 1-loop decays $BR(t \to cH^0)$ and $BR(H^0 \to \bar{t}c)$ in the T2HDM. We have taken the following set of assumptions/values on the relevant parameter space of the T2HDM:

- Set $\alpha = \beta$ for the reasons explained above.
- Set $|\xi| \sim 0.8$, as in [13].
• The other parameters of the T2HDM are set to their best-fit (central) value in (14) unless stated otherwise.

• For the analysis of the decay \( t \rightarrow cH^0 \) we set \( m_{H^0} = 91 \text{ GeV} \), which is the central (best fitted) value of the SM Higgs mass to EW precision data [15]. Recall that, in our setup, \( H^0 \) has couplings identical to the SM Higgs and we therefore expect the phenomenology of \( H^0 \) to roughly follow that of the SM’s Higgs.

• For the total top decay width we take \( \Gamma (t \rightarrow W^+b) = 1.55 \text{ GeV} \).

• For the process \( H^0 \rightarrow t\bar{c} \) we arbitrarily choose \( m_{H^0} = 300 \text{ GeV} \).

• We set \( m_{A^0} \approx 1 \text{ TeV} \) to enhance the triple-scalar coupling, which is roughly \( m_{A^0}^2 \) (see App. A).

• Other values used for the calculations were [15]: \( m_t = 172.5 \text{ GeV} \) (pole mass), \( m_c = m_{cW}(m_t) = 1.24 \text{ GeV} \), \( m_b = m_{bW}(m_t) = 4.20 \text{ GeV} \) (\( m_c \) and \( m_b \) are in the \( \overline{\text{MS}} \) renormalization scheme), \( m_W = 80.40 \text{ GeV} \), \( m_Z = 91.188 \text{ GeV} \), \( \cos \theta_W = m_W/m_Z \), \( \alpha(m_Z) = 1/128 \).

The mass values used are without running the energy scale, even though the BR’s were found to be sensitive to both \( m_c \) and \( m_b \) (our results are not sensitive to \( m_t \)). For example, the value \( BR(t \rightarrow cH^0) = 5.99 \times 10^{-5} \) quoted in the upper right corner of Table II would change to \( BR(t \rightarrow cH^0) = 1.28 \times 10^{-5} \) had we used \( m_b(m_Z) \sim 3 \text{ GeV} \) and \( m_c(m_Z) \sim 0.7 \text{ GeV} \) [19]. In addition see below for an example with explicit scale dependence.

A. The 1-loop decay \( t \rightarrow cH^0 \)

In Figs. 2a and 3a we give a 3D plot of \( BR(t \rightarrow cH^0) \) in the \( m_{H^+} - \tan \beta \) and \( m_{A^0} - \tan \beta \) planes, respectively. The flat grid in Figs. 2a and 3a represents the LHC detection limit which is \( \sim 5 \cdot 10^{-5} \), so that the colored surface above the grid is the region of the parameter space of the T2HDM which has a BR potentially within the sensitivity of the LHC.

The choice \( m_{h^0} = 1000 \text{ GeV} \) made in Fig. 2a suppresses the diagrams with \( h^0 \) in the loop and, thus, better explores the charged Higgs sector. As expected the BR rises with \( \tan \beta \) and is highest when \( m_{H^+} \) is lowest. The dominant Feynman diagram in this case is the one depicted in Fig. 2b with two \( H^+ \) and a \( b \)-quark in the loop. This diagram receives an enhancement from the 3-scalar vertex \( H^0 H^+ H^- \), as noted above.

The choice \( m_{H^+} = 1000 \text{ GeV} \) made in Fig. 3a suppresses the diagrams with \( H^\pm \) in the loop and, thus, is more sensitive to the neutral Higgs sector. In this case also, the BR rises with \( \tan \beta \) and drops as \( m_{h^0} \) is increased. The dominant diagram in this case is the one which has two \( h^0 \) scalars in the loop. This diagram receives an enhancement from the 3-scalar vertex \( H^0 h^0 h^0 \) when \( m_{A^0} \) is large, as mentioned above.

The two limits \( m_{h^0} = 1000 \text{ GeV} \) and \( m_{H^+} = 1000 \text{ GeV} \) have similar consequences, yet the \( BR(t \rightarrow cH^0) \) is higher when \( m_{H^+} < m_{h^0} \) in which case the charged
Higgs loop-exchange is the dominant source for the enhanced BR \( t \rightarrow cH^0 \). This is a distinctive property of the T2HDM, since, in this model, the charged Higgs coupling \( H^+bc \) is enhanced by \( V_{tb}/V_{cb} \) compared to other 2HDM's such as the type I and type II 2HDM.

In Fig. 4 we plot the BR \( t \rightarrow cH^0 \) in the \( m_{A^0} - m_{h^0} \) plane, where all other parameters are given the central values of Eq. (14). We see that the BR is highest for a large \( m_{A^0} \) and a small \( m_{h^0} \) where the diagram with the two \( h^0 \) in the loop dominates. The dip in the middle of the surface is due to cancellations in the \( H^0H^+H^+ \) vertex.

To better illustrate the dependence of the BR \( t \rightarrow cH^0 \) on tan \( \beta \) and \( m_{H^+} \) we give in Figs. 5 and 6 2D plots of the BR \( t \rightarrow cH^0 \) as a function of tan \( \beta \) and \( m_{H^+} \), respectively, using the same parameter set as in Fig. 2.

Finally, in order to demonstrate the difference in the BR \( t \rightarrow cH^0 \) expected within the T2HDM, the 2HDM-II, and the SM, we list in Table II the BR \( t \rightarrow cH^0 \) values within these 3 different models, for 4 different points in the relevant parameter space. Note that in the SM the 1-loop BR \( t \rightarrow cH^0 \) depends only on the SM's Higgs mass which we also set to \( m_{H^0} = 91 \) GeV. Also, recall that the type II 2HDM Yukawa potential is similar to that of the MSSM and that it has no tree-level FCNC.

The first two rows in Table II illustrate the impact of the charged sector, by setting a high \( m_{A^0} \) and a much smaller \( m_{H^+} \) (note that the value \( m_{H^+} = 200 \) GeV is outside the 1 \( \sigma \) bounds). In this case, the BR \( t \rightarrow cH^0 \) in the T2HDM is not as enhanced as expected relative to the 2HDM-II, where it is a bit higher in the T2HDM. Recall that we expected the diagram with the \( H^+b \) quark in the loop to be particularly enhanced in the T2HDM due to the enhanced \( H^+cb \) interaction in this model. The amplitude of this diagram is (see App. C):

\[
M_7 = \frac{-iU_{ec}^3}{16\pi^2} \frac{1}{\sin^2 \theta_W} \left[ m_b C_0 \left( A_{cb}^H B_{tb}^{H+*} L + B_{tb}^{H+} A_{cb}^{H+*} R \right) + m_c C_{12} \left( B_{cb}^{H+} B_{tb}^{H+*} L + A_{cb}^{H+} A_{tb}^{H+*} R \right) \right] m_t, \tag{33}
\]

where \( C_{ij} \) are the Passarino-Veltman scalar functions. The term \( m_b A_{cb}^H B_{tb}^{H+*} \) (multiplied by the left projection operator), which is sub-leading in the type II 2HDM, is enhanced in the T2HDM and dominates the other terms, being \( \propto \xi^* m_c m_b^2 \tan^2 \beta V_{cb} V_{tb} \). On the other hand, in the 2HDM of type II it is the term \( \propto m_b B_{cb}^{H+} B_{tb}^{H+*} \) which dominates for a large \( \tan \beta \gtrsim 10 \). Therefore, we see that the different leading terms in the T2HDM and the type II 2HDM are roughly of the same order of magnitude since \( m_t \cdot V_{cb} \sim m_c \), and therefore the enhancement in the T2HDM is not as significant as expected.

In the last two rows of Table II we set a high \( m_{H^+} \sim 1 \) TeV, thus exploring the impact of an EW-scale neutral.
We set $m_{H^0} = 300$, $\alpha = \beta$, and other parameters to their best-fit value of (14). In the Higgs sector. Evidently, in this case the $BR(t \to cH^0)$ is much larger in the T2HDM than in the 2HDM of type II. This is in fact expected since the type II 2HDM does not have any tree-level FCNC.

### B. The 1-loop decay $H^0 \to \bar{t}c$

In the results that follow, we assume that the Higgs decays that enter its total width (when kinematically allowed) are: $H^0 \to q\bar{q}$, $H^0 \to VV$, $H^0 \to h_i h_j$ and $H^0 \to V_i h_j$. The partial widths for these decay channels are given in App. D.

We first plot in Fig. 7 the SM value for the $BR(H^0 \to \bar{t}c + \bar{c}t)$, as a function of the Higgs mass, for two b-quark masses: $m_b(m_b) = 4.2$ GeV and $m_t(m_t) = 3$ GeV. Our results are in agreement with the results reported in [5].

![Graph showing the SM value for the BR(H^0 \to \bar{t}c + \bar{c}t) as a function of the Higgs mass, for two b-quark masses: mb(mb) = 4.2 GeV and mt(mt) = 3 GeV.](image)

FIG. 7: The SM value for the $BR(H^0 \to \bar{t}c + \bar{c}t)$ as a function of the Higgs mass, for $m_b(m_b) = 4.2$ GeV and for $m_t(m_t) = 3$ GeV [15]. The BR is not sensitive to $m_c$.

Next we turn to our results in the T2HDM. In Fig. 8 we give a 3D plot of $BR(H^0 \to \bar{t}c + \bar{c}t)$ in the $m_{H^0} - \tan \beta$ plane in the T2HDM, and (b) the dominant diagram. We set $m_{H^0} = 1000$ GeV and $m_{A^0} = 1000$ GeV.

![Graph showing the 3D plot of BR(H^0 \to \bar{t}c + \bar{c}t) in the m_{H^0} - tan beta plane.](image)

FIG. 8: (a) 3D plot of $BR(H^0 \to \bar{t}c + \bar{c}t)$ in the $m_{H^0} - \tan \beta$ plane in the T2HDM, and (b) the dominant diagram. We set $m_{H^0} = 1000$ GeV and $m_{A^0} = 1000$ GeV.

Finally, in table III we give the $BR(H^0 \to \bar{t}c + \bar{c}t)$ in the three different models (SM, type II 2HDM and the T2HDM) for 4 points of the relevant parameter space. As can be seen, the behavior is similar to the reversed top
Yukawa interactions and the scalar self interactions of transitions mediated by the charged Higgs. These enhanced FCNC in the up-quark sector and large flavor mass than all other quarks in a natural manner.

\[ \beta \]

with the top-quark, while the lighter Higgs doublet (i.e., \( m_H^0 \)) couples to all other quarks. In particular, the working assumption of the T2HDM is that \( \tan \beta \equiv v_2/v_1 \gg 1 \), so that the top quark receives a much larger mass than all other quarks in a natural manner.

The Yukawa sector of the T2HDM exhibits potentially enhanced FCNC in the up-quark sector and large flavor transitions mediated by the charged Higgs. These Yukawa interactions and the scalar self interactions of the model were explicitly (and independently) derived. For example, it was shown that the \( H^+ bc \) Yukawa coupling, which (in this model) is enhanced by a factor of \( V_{cb}/V_{ub} \) compared to the corresponding 2HDM type II coupling, enhances the 1-loop \( t \rightarrow ch \) and \( h \rightarrow tc \) (where \( h = h^0 \) or \( H^0 \) are the two CP-even neutral Higgs) within the T2HDM. In this model the Higgs doublet with the heavier VEV (\( v_2 \)) couples only to the top-quark, while the lighter Higgs doublet (i.e., with \( v_1 \ll v_2 \)) couples to all other quarks. In particular, the working assumption of the T2HDM is that \( \tan \beta \equiv v_2/v_1 \gg 1 \), so that the top quark receives a much larger mass than all other quarks in a natural manner.

The Yukawa sector of the T2HDM exhibits potentially enhanced FCNC in the up-quark sector and large flavor transitions mediated by the charged Higgs. These Yukawa interactions and the scalar self interactions of

VII. SUMMARY

We have studied the top and neutral Higgs FCNC rare decays \( t \rightarrow ch \) and \( h \rightarrow tc \) (where \( h = h^0 \) or \( H^0 \) are the two CP-even neutral Higgs) within the T2HDM. In this model the Higgs doublet with the heavier VEV (\( v_2 \)) couples only to the top-quark, while the lighter Higgs doublet (i.e., with \( v_1 \ll v_2 \)) couples to all other quarks. In particular, the working assumption of the T2HDM is that \( \tan \beta \equiv v_2/v_1 \gg 1 \), so that the top quark receives a much larger mass than all other quarks in a natural manner.

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The Yukawa sector of the T2HDM exhibits potentially enhanced FCNC in the up-quark sector and large flavor transitions mediated by the charged Higgs. These Yukawa interactions and the scalar self interactions of
and $H^0 \rightarrow \bar{c}c$ can reach $10^{-5} - 10^{-4}$ in a favorable scenario - a value higher than the LHC detection threshold for the top-decay and above their expected value within the SM and the type I and II 2HDM. Thus, even if $h^0$ decouples (i.e., too heavy), the 1-loop FCNC top-decay $t \rightarrow cH^0$ may still be accessible to the LHC.

### APPENDIX A: FEYNMAN RULES FOR TWO HIGGS DOUBLET MODELS

![Feynman diagram]

**FIG. 13:** Feynman rules.

**TABLE IV:** Feynman rules for vector-scalar-scalar interactions as in [6]. The second particle is outgoing.

The relevant Feynman rules for the 2HDM's that were used in this work are summarized in Fig. 13 and in Tables VI, VII, IV and V. The notation is given in Fig. 13 and the various couplings in the T2HDM and in the 2HDM of type II are collected in the Tables.

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**TABLE V:** Feynman rules for triple-scalar interactions [2, 6].

In particular, in Table VI we list the Yukawa couplings, in Table VII we give the vector-vector-scalar couplings, in Table IV we give the vector-scalar-scalar couplings and the triple-scalar couplings, which are common to any 2HDM [6], are given in Table V. Note that the vertices $Z^0G^0H^0$ and $Z^0G^0h^0$ do not participate in the calculations since the corresponding Yukawa vertex $qqG^0$ does not generate FCNC.

### APPENDIX B: 1-LOOP AMPHITUDES

Here we give the 1-loop amplitudes corresponding to the 10 diagrams shown in Fig. 1. The calculation was done in the ’t Hooft Feynman gauge and the following notation was used:

**definitions:**

- $M_n$ – the amplitude corresponding to diagram $n$.
- $h$ – the external neutral scalar.
- $i = (= t)$ when used as index, the incoming fermion - the top.
- $j = (= c)$ when used as index, the outgoing fermion - the charm.
- $\alpha, \beta$ – when used as indices, internal bosons (vectors or scalars) in the loop.
- $l, k, q$ – when used as indices, internal fermions.
- $L, R$ – the Left,Right projection operators.
- $\bar{u}_j = (= \bar{u}(P_1))$ the outgoing spinor of the charm.
- $u_i = (= u(P_1))$ the incoming spinor of the top.
- $B_0, B_1, C_0, C_{ij}$ – the n-point integral functions, defined in App. C.
- $A^0_{ab}, B^0_{ab}$ – the left,right-handed parts of the fermion-fermion-scalar vertex.
- $c_{ab}, b_{ab}$ – the left,right-handed parts of the fermion-fermion-vector vertex, for both charged and neutral gauge bosons.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
T2HDM & 2HDM-II \cite{6} \\
\hline
$H^0 \bar{u}_j u_i$ & $\frac{g}{2m_W} \left( -M_a \cos \alpha \cos \beta + \Sigma (\sin \alpha \cos \beta + \cos \alpha \sin \beta) \right) R + (h.c.) L$ \\
& $-\frac{g_\mu}{2m_W} \sin \alpha \sin \beta L$ \\
\hline
$h^0 \bar{u}_j u_i$ & $\frac{g}{2m_W} \left( -M_a \cos \alpha \cos \beta - \Sigma (\cos \alpha \sin \beta + \sin \alpha \cos \beta) \right) R + (h.c.) L$ \\
& $-\frac{g_\mu}{2m_W} \sin \alpha \sin \beta L$ \\
\hline
$A^0 \bar{u}_j u_i$ & $i \frac{g_{\mu\alpha}}{2m_W} (\bar{A}_R \alpha \tan \beta + \Sigma (\tan \beta + \cot \beta)) R + (h.c.) L$ \\
& $i \frac{g_{\mu\alpha}}{2m_W} \cot \beta (R - L)$ \\
\hline
$G^0 \bar{u}_j u_i$ & $i \frac{g_{\mu\alpha}}{2m_W} (R - L)$ \\
& $i \frac{g_{\mu\alpha}}{2m_W} (R - L)$ \\
\hline
$H^+ \bar{d}_d d_i$ & $\frac{\sqrt{2} m_W}{2m_W} \tan \beta (\Sigma \delta_{CKM} \delta_{M_4} R + \Sigma (\delta_{CKM} \delta_{M_4} R + \cot \beta \delta_{M_6} \delta_{CKM} L$ \\
& $\frac{\sqrt{2} m_W}{2m_W} \Sigma \delta_{CKM} \delta_{M_4} R + \cot \beta \delta_{M_6} \delta_{CKM} L$ \\
\hline
$G^+ \bar{d}_d d_i$ & $\frac{\sqrt{2} m_W}{2m_W} (\Sigma \delta_{CKM} \delta_{M_4} R)$ \\
& $\frac{\sqrt{2} m_W}{2m_W} (\Sigma \delta_{CKM} \delta_{M_4} R)$ \\
\hline
\end{tabular}
\caption{Feynman rules for Yukawa interactions in the T2HDM and in the 2HDM-II.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
$W^+ W^- H^0$ & $\frac{g_{\mu hhh} e^a}{g_{\mu hhh} e^b}$ \tabularnewline
$W^+ W^- h^0$ & $\frac{g_{\mu hhv}}{g_{\mu hhv}}$ \tabularnewline
$Z^0 Z^0 H^0$ & $\frac{g_{\mu hhv}}{g_{\mu hhv}}$ \tabularnewline
$Z^0 Z^0 h^0$ & $\frac{g_{\mu hhv}}{g_{\mu hhv}}$ \\
\hline
\end{tabular}
\caption{Feynman rules for vector-vector-interactions \cite{6}.}
\end{table}

\begin{align}
M_1 &= \frac{i\bar{u}_j}{16\pi^2} \frac{-1}{m_i^2 - m_i} \left[ m_{m_k} B_0 \left( B_{iL} h^0 \gamma^\ast$ \right) + A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right] - m_{m_i} B_1 \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) \\
&+ m_{m_j} B_0 \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) - m_{m_i} B_1 \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) \frac{u_i}{(B)} \\
\text{where } B &= \left( m_i^2, m_j^2, m_k^2 \right). \\
M_2 &= \frac{i\bar{u}_j}{16\pi^2} \frac{-1}{m_i^2 - m_i} \left[ m_{m_k} B_0 \left( A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) + m_{m_i} B_1 \left( A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) \\
&- m_{m_j} B_0 \left( A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) - m_{m_j} B_1 \left( A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) \frac{u_i}{(B)} \\
\text{where } B &= \left( m_i^2, m_j^2, m_k^2 \right). \\
M_3 &= \frac{i\bar{u}_j}{16\pi^2} \frac{-1}{m_i^2 - m_i} \left[ 4 m_{m_k} B_0 \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + 2 m_{m_i} B_1 \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) \\
&+ 4 m_{m_j} B_0 \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + 2 m_{m_j} B_1 \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) \frac{u_i}{(B)} \\
\text{where } B &= \left( m_i^2, m_j^2, m_k^2 \right). \\
M_4 &= \frac{i\bar{u}_j}{16\pi^2} \frac{-1}{m_i^2 - m_i} \left[ 4 m_{m_i} B_0 \left( B_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) + 4 m_{m_j} B_0 \left( A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) + 2 m_{m_j} B_0 \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) \frac{u_i}{(B)} \\
\text{where } B &= \left( m_i^2, m_j^2, m_k^2 \right). \\
M_5 &= \frac{-i\bar{u}_j}{16\pi^2} \left( A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) \left[ \hat{C}_0 + \hat{C}_1 \right] \left( A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) \\
&- m_{m_i} C_{10} \left( A_{iL} h^0 \gamma^\ast + B_{iL} h^0 \gamma^\ast \right) + m_{m_j} C_{10} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + m_{m_j} C_{10} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + m_{m_j} C_{10} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) \\
&+ m_{m_j} \left( C_{12} - C_{11} \right) \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + m_{m_j} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + m_{m_j} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + m_{m_j} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) \\
&- m_{m_j} \left( C_{12} + C_{11} \right) \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + m_{m_j} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + m_{m_j} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) + m_{m_j} \left( B_{iL} h^0 \gamma^\ast + A_{iL} h^0 \gamma^\ast \right) \frac{u_i}{(B)} \\
\end{align}
where \( C = C \left( m^2_k, m^2_a, m^2_q, m^2_l, m^2_h, m^2_\mu \right) \).

\[
M_6 = \frac{i \bar{u}_i}{16 \pi^2} \left[ \left( 4 \hat{C}_0 + 2 \left( m^2_l - m^2_h \right) C_{11} + 2 \left( -m^2_l + m^2_h \right) C_{12} \right) \left( \bar{a}^0_{jk} B_{kq} A^a_{ik} L + \bar{a}^0_{jk} A_{kj} B^{a*}_{ik} R \right) 
+ 2 m_q m_C 11 \left( \bar{a}^0_{jk} A_{kj}^{a*} b^a_{ik} L + \bar{a}^0_{jk} B^{a*}_{kq} a^a_{ik} R \right) - 2 m_q m_j C_{12} \left( a^0_{jk} B_{kq} a^a_{ik} L + \bar{a}^0_{jk} A_{kj} b^a_{ik} R \right) 
+ 4 m_q m_h C_0 \left( b^a_{jk} A_{kj}^{a*} a^a_{ik} L + \bar{a}^0_{jk} B^{a*}_{kj} b^a_{ik} R \right) + 2 m_q m_k (C_{11} + C_0) \left( b^a_{jk} B_{kj}^{a*} b^a_{ik} L + \bar{a}^0_{jk} A_{kj} a^a_{ik} R \right) 
- 2 m_j m_C (C_{12} + C_0) \left( \bar{a}^0_{jk} A_{kj}^{a*} b^a_{ik} L + \bar{a}^0_{jk} B^{a*}_{kj} b^a_{ik} R \right) \right] u_i. 
\]

\[
M_7 = \frac{-i \bar{u}_j}{16 \pi^2} g^{\alpha h} f_{\alpha h} \left[ m_k C_0 \left( \bar{A}_{j}^{\alpha} B_{ik}^{a*} L + B_{jk}^{\beta} A_{ik}^{a*} R \right) - m_j C_{12} \left( B_{jk}^{\beta} B_{ik}^{a*} L + A_{jk}^{\beta} A_{ik}^{a*} R \right) 
+ m_i (-C_{11} + C_{12}) \left( \bar{A}_{j}^{\alpha} A_{ik}^{a*} L + B_{jk}^{\beta} B_{ik}^{a*} R \right) \right] u_i, 
\]

where \( C = C \left( m^2_k, m^2_a, m^2_q, m^2_l, m^2_h, m^2_\mu \right) \).

\[
M_8 = \frac{-i \bar{u}_j}{16 \pi^2} g^{\alpha h} f_{\alpha h} \left[ 4 m_k C_0 \left( \bar{A}_{j}^{\alpha} B_{ik}^{a*} L + a_{jk}^{a} b_{ik}^{a*} R \right) + 2 m_i (C_{11} - C_{12}) \left( \bar{a}^{\beta}_{jk} b_{ik}^{a*} L + a_{jk}^{a} b_{ik}^{a*} R \right) 
+ 2 m_j C_{12} \left( a_{jk}^{a} b_{ik}^{a*} L + \bar{a}^{\beta}_{jk} b_{ik}^{a*} R \right) \right] u_i, 
\]

where \( C = C \left( m^2_k, m^2_a, m^2_q, m^2_l, m^2_h, m^2_\mu \right) \).

\[
M_9 = \frac{i \bar{u}_j}{16 \pi^2} g^{\alpha h} f_{\alpha h} \left[ \left( \bar{C}_0 + 2 m^2_l C_{11} + m^2_h C_{12} - 2 m^2_h C_{12} \right) \left( \bar{A}_{j}^{\alpha} B_{ik}^{a*} L + a_{jk}^{a} A_{ik}^{a*} R \right) 
- m_i m_j (C_{12} + C_{11}) \left( a_{jk}^{a} A_{ik}^{a*} L + \bar{A}_{j}^{\alpha} A_{ik}^{a*} R \right) + m_j m_k (C_{12} + C_{12}) \left( a_{jk}^{a} A_{ik}^{a*} L + \bar{A}_{j}^{\alpha} A_{ik}^{a*} R \right) \right] u_i, 
\]

where \( C = C \left( m^2_k, m^2_a, m^2_q, m^2_l, m^2_h, m^2_\mu \right) \).

\[
M_{10} = \frac{i \bar{u}_j}{16 \pi^2} g^{\alpha h} f_{\alpha h} \left[ \left( \bar{C}_0 + 2 m_l (C_{12} - C_{11}) - 2 m^2_l C_{11} - 2 m^2_h (C_{12} - C_{11}) \right) \left( \bar{A}_{j}^{\alpha} A_{ik}^{a*} L + B_{jk}^{\beta} A_{ik}^{a*} R \right) 
+ m_i m_j (2 C_{11} - C_{12}) \left( B_{jk}^{\beta} A_{ik}^{a*} L + A_{jk}^{\beta} A_{ik}^{a*} R \right) + m_j m_k (C_{11} + 2 C_0) \left( B_{jk}^{\beta} A_{ik}^{a*} L + A_{jk}^{\beta} A_{ik}^{a*} R \right) 
+ m_k (C_{11} - C_{12} - C_0) \left( \bar{A}_{j}^{\alpha} A_{ik}^{a*} L + B_{jk}^{\beta} A_{ik}^{a*} R \right) \right] u_i, 
\]

where \( C = C \left( m^2_k, m^2_a, m^2_q, m^2_l, m^2_h, m^2_\mu \right) \).

**APPENDIX C: 1-LOOP INTEGRALS**

The 1-loop scalar, vector and tensor integrals are defined as:

\[
B_0; B_\mu \left( m^2_l, m^2_h, p_1^2, p_2^2 \right) = \int \frac{d^4 k}{i \pi^2} \frac{1; k_\mu}{\left[ k^2 - m_l^2 \right] \left[ (k + p_1)^2 - m_l^2 \right] \left[ (k + p_2)^2 - m_l^2 \right]}, \quad (C1)
\]

\[
C_0; C_{\mu \nu}; \bar{C}_0 \left( m^2_l, m^2_h, m^2_3, m^2_2, p_1^2, p_2^2, p_3^2 \right) = \int \frac{d^4 k}{i \pi^2} \frac{1; k_\mu; k_\nu; k^2}{\left[ k^2 - m_l^2 \right] \left[ (k + p_1)^2 - m_l^2 \right] \left[ (k + p_2)^2 - m_l^2 \right] \left[ (k + p_3)^2 - m_l^2 \right]}, \quad (C2)
\]

where \( \sum_i p_i = 0 \) and the reduction to the 1-loop scalar functions is:

\[
B_\mu = p_\mu B_1,
C_\mu = p_1 \mu C_{11} + p_2 \mu C_{12},
C_{\mu \nu} = p_1 \mu p_1 \nu C_{21} + p_2 \mu p_2 \nu C_{22} + \{ p_1 p_2 \}_\nu C_{23} + g_{\mu \nu} C_{24}, \quad (C3)
\]
with \( \{ab\}_{\mu\nu} = a_\mu b_\nu + a_\nu b_\mu \).

**APPENDIX D: THE HIGGS WIDTH**

The total Higgs width, \( \Gamma^{tot} \), is derived from:

\[
\Gamma^{tot} = \Gamma(h \to \bar{q}q) + \Gamma(h \to VV) + \Gamma(h \to H_i H_j) + \Gamma(h \to VH),
\]

(D1)

when kinematically allowed (i.e., the decay products are assumed to be on-shell). All the above partial widths were calculated at tree-level. The relevant couplings follow the definition in Fig. 13.

The decay width for \( h \to \bar{q}q \) is [6]:

\[
\Gamma(h \to \bar{q}q) = \frac{N_c A_{hqq}^2}{8\pi} m_h \left( 1 - \frac{4m_q^2}{m_h^2} \right)^\frac{3}{2}, \tag{D2}
\]

where \( A_{hqq} = -\frac{g_{hqq} m_q}{2m_W \cos \beta}, \frac{g_{hqq} m_q}{2m_W \cos \beta} \) for \( h = H^0, h^0 \), respectively, and \( N_c = 3 \). For example, the width for the decay \( h^0 \to b\bar{b} \) in the T2HDM with \( \alpha = \beta \) is:

\[
\Gamma(h^0 \to b\bar{b}) = \frac{3g^2 m_b^2}{32\pi m_W^2} m_h \tan^2 \beta \left( 1 - \frac{4m_b^2}{m_h^2} \right)^\frac{3}{2}. \tag{D3}
\]

The decay width for \( h \to W^+W^- \) is [6]:

\[
\Gamma(h \to W^+W^-) = \frac{g_{hWW}^2 m_h^3}{64\pi m_W^2} (1 - x)^\frac{3}{2} \left( 1 - x + \frac{3}{4} x^2 \right), \tag{D4}
\]

where \( g_{hWW} = g m_W \cos (\beta - \alpha); g m_W \sin (\beta - \alpha) \) for \( h = H^0, h^0 \), respectively, and \( x = \frac{4m_W^2}{m_h^2} \).

The decay width for \( h \to Z^0Z^0 \) is [6]:

\[
\Gamma(h \to Z^0Z^0) = \frac{g_{hZZ}^2 m_h^3 \cos^4 \theta_W}{32\pi m_W^2} (1 - x)^\frac{1}{2} \left( 1 - x + \frac{3}{4} x^2 \right), \tag{D5}
\]

where \( g_{hZZ} = \frac{gm_Z}{\cos \theta_W} \cos (\beta - \alpha); \frac{gm_Z}{\cos \theta_W} \sin (\beta - \alpha) \) for \( h = H^0, h^0 \), respectively, and \( x = \frac{4m_Z^2}{m_h^2} \). Note that by choosing \( \alpha = \beta \) one sets the couplings \( W^+W^- h^0 \) and \( Z^0Z^0 h^0 \) to zero in which case \( \Gamma(h^0 \to W^+W^-), \Gamma(h^0 \to Z^0Z^0) = 0 \) while \( \Gamma(H^0 \to W^+W^-) \) and \( \Gamma(h^0 \to Z^0Z^0) \) are maximal.

The decay width for \( h \to H_i H_j \) (where \( H_i, H_j \neq h \)) is:

\[
\Gamma(h \to H_i H_j) = \frac{g_{hH_iH_j}^2}{16\pi m_h} \lambda^\frac{3}{2} \left( 1, \frac{m_H^2}{m_h^2}, \frac{m_H^2}{m_h^2} \right), \tag{D6}
\]

where \( g_{hH_iH_j} \) are the triple scalar couplings, and we recall that:

\[
\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.
\]

In order to demonstrate the role of radiative corrections to the leading order tree-level total width, we plot in Fig. 14 the total SM Higgs width at the tree-level (i.e., as calculated in this work) compared to the width calculated including higher-order corrections [20]. As can be seen, the discrepancy between the lowest order and the higher order calculations is significant only below the WW threshold (at about 160 GeV). In this mass range the \( b\bar{b} \) decay channel dominates for which radiative corrections can have an appreciable impact. This mass range is, however, below the \( h \to \ell\ell \) threshold and therefore irrelevant to the present work, and so the use of the lowest order widths is justified.
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