

Interacting entropy-corrected holographic dark energy with apparent horizon as an infrared cutoff

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Abstract

In this work we consider the entropy-corrected version of interacting holographic dark energy (HDE), in the non-flat universe enclosed by apparent horizon. Two corrections of entropy so-called logarithmic 'LEC' and power-law 'PLEC' in HDE model with apparent horizon as an IR-cutoff are studied. The ratio of dark matter to dark energy densities $u$, equation of state parameter $w_D$ and deceleration parameter $q$ are obtained. We show that the cosmic coincidence is satisfied for both interacting models. By studying the effect of interaction in EoS parameter, we see that the phantom divide may be crossed and also find that the interacting models can drive an acceleration expansion at the present and future, while in non-interacting case, this expansion can happen only at the early time. The graphs of deceleration parameter for interacting models, show that the present acceleration expansion is preceded by a sufficiently long period deceleration at past. Moreover, the thermodynamical interpretation of interaction between LECHDE and dark matter is described. We obtain a relation between the interaction term of dark components and thermal fluctuation in a non-flat universe, bounded by the apparent horizon. In limiting case, for ordinary HDE, the relation of interaction term versus thermal fluctuation is also calculated.

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I. INTRODUCTION

The dark energy scenario has attracted a great deal of attention in the last decade. Many cosmological observations reveal that our universe evolves under an acceleration expansion\[1\]. This expansion may be driven by an unknown energy component with negative pressure, so called, dark energy (DE), which fills $\sim 70$ percent of energy content of our universe with an effective equation of state (EoS) parameter $-1.48 < w_{\text{eff}} < -0.72$\[2\]. Despite of many efforts in this subject, the nature of DE is the most mysterious problem in modern cosmology. The first and simplest candidate of dark energy is $\Lambda$CDM model, in which $w_\Lambda = -1$ is constant. Although this model is consistent very well with all observations, it faces with with the fine tuning and cosmic coincidence problem. After this, the dynamical DE models have been proposed to solve the DE problems. Among many dynamical models of DE, in which $w_D$ is not constant, the entropy-corrected dark energy models based on quantum field theory and gravitation have been widely extended by many authors in recent years\[3, 4\]. The motivation of these corrections has been based on black hole physics, where some gravitational fluctuations and field anomalies can affect the entropy-area law of black holes. The logarithmic and power-law corrections of entropy are two procedures in dealing with this fluctuations. First correction has been given by logarithmic fluctuations at the spacetime, in the context of loop quantum gravity (LQG)\[5\]. The entropy-area relationship leads to the curvature correction in the Einstein-Hilbert action and vice versa\[6, 7\]. In this case the corrected entropy is given by\[8\]

$$S_{\text{BH}} = \frac{A}{4G} + \tilde{\gamma} \ln \frac{A}{4G} + \tilde{\beta},$$

(1)

where $\tilde{\gamma}$ and $\tilde{\beta}$ are dimensionless constants of order unity. By considering the entropy correction, the energy density of logarithmic entropy-corrected holographic dark energy (LECHDE) can be given as\[9\]

$$\rho_D = 3n^2 M_p^2 L^{-2} + \gamma L^{-4} \ln(M_p^2 L^2) + \beta L^{-4}.$$  

(2)

Three parameters $n$, $\beta$ and $\gamma$ are parameters of model and $M_p$ is the reduced Planck mass. The correction terms (two last terms of (2)) are effective only at the early stage of the universe and they will be vanished when the universe becomes large, in which $\rho_D^{\text{EC}} \rightarrow \rho_D^O$, where $\rho_D^O = 3n^2 M_p^2 L^{-2}$ is the dark energy density of ordinary HDE model (more discussion
of HDE model is referred to [10]). In this model, the IR-cutoff \( L \) plays an essential role. If \( L \) is chosen as particle horizon, the HDE can not make an acceleration expansion [11], while for future event horizon, Hubble scale \( L = H^{-1} \), and apparent horizon (AH) as an IR-cutoff, the HDE can simultaneously drive accelerated expansion and solve the coincidence problem [12–14]. More recently, a model of interacting HDE (i.e. a non gravitational interaction between DE and dark matter (DM)) at Ricci scale, in which \( L = (\dot{H} + 2H^2)^{-1/2} \) has been proposed. The authors performed a detailed discussion on the cosmic coincidence problem, age problem and obtained some observational constraints on their’s model [15].

The second class of ECHDE, power-law correction of entropy (PLEC), is appeared in dealing with the entanglement of quantum fields in and out of the horizon [16]. In this model, the corrected-entropy is given by [3]

\[
S = \frac{A}{4G} [1 - K_\alpha A^{1-\alpha/2}],
\]

(3)

where \( \alpha \) is a dimensionless positive constant and

\[
K_\alpha = \frac{\alpha}{4 - \alpha} (4\pi r_c^2)^{\alpha/2 - 1}.
\]

(4)

Here \( r_c \) is the crossover scale. More detail is referred to [3, 16, 17]. It is worthwhile to mention that in the most acceptable range of \( 4 > \alpha > 2 \) [3, 16], the correction term (i.e. the second term of (3)), is effective only at small \( A \)'s and it falls off rapidly at large values of \( A \). Therefore, for large horizon area, the ordinary entropy-area law (first term of (3)) is recovered. However the thermodynamical considerations predict that the case \( \alpha \leq 2 \) may be acceptable, but as we will show in Sec. III, this range should be removed by cosmic coincidence consideration. Due to entropy corrections to the Bekenstein-Hawking entropy \( (S_{BH}) \), the Friedmann equation should be modified [3]. In comparison with ordinary Friedman equation, the energy density of PLECHDE, has been given by [18]

\[
\rho_D = 3n^2M_p^2L^{-2} - \delta M_p^2L^{-\alpha},
\]

(5)

where \( \delta \) and \( \alpha \) are the parameters of PLECHDE model. We must mention that the ordinary HDE is recovered for \( \delta = 0 \) or \( \alpha = 2 \).

In historical point of view, laws of black hole thermodynamics have made some relations between thermodynamics and a self gravitating system bounded by a horizon. In this theory, some thermodynamical quantities such as entropy and temperature are purely geometrical
quantities which have been obtained from area and surface gravity of horizon, respectively. In the Friedmann-Robertson-Walker (FRW) universe, with horizons, like future event horizon in black hole physics, by studying the thermodynamical quantities and generalized second law (GSL) [19], one can choose the best DE model or the best horizon. For example, it has been shown that in a non-flat FRW universe, enclosed by apparent horizon, the GSL is governed irrespective of any DE model [14]. The investigation of GSL for LECHDE and PLECHDE models has been performed in [3].

Recently, the HDE and agegraphic/new-agegraphic DE models have been extended regarding the entropy corrections (LECHDE, PLECHDE, PLECNADDE) and a thermodynamical description of the LECHDE model has been studied [4, 9, 18, 20]. Also at Ref. [14], thermodynamics interpretation of interacting holographic dark energy with AH-IR-cutoff, enclosed by apparent horizon, was studied. These papers give us a strong motivation to study the LECHDE and PLECHDE models with AH-IR-cutoff in a non-flat universe, enclosed by apparent horizon, which is a generalization of earlier works of Sheykhi et.al. [14, 18]. It should be mentioned that, the motivation of a closed universe has been also shown in a suite CMB experiments [21] and of the cubic correction to the luminosity-distance of supernova measurements [22].

The outline of our paper is as follows: In Sec. II the interacting LECHDE model with AH-IR-cutoff is studied and the evolution of dark energy, deceleration parameter and EoS parameter are calculated. Also these calculations are performed for PLECHDE model with AH-IR-cutoff in Sec. III. In Sec. IV the thermodynamical quantities such as entropy and Hawking temperature of apparent horizon are obtained only for LECHDE model and then the interaction term due to thermal fluctuation is obtained in Sec. V. We finish Our paper with some concluding remarks.

II. INTERACTING “LECHDE” MODEL WITH AH-IR-CUTOFF

The line element of a homogenous and isotropic FRW universe is given by

\[ ds^2 = h_{ab} dx^a dx^b + \tilde{r}^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]  

(6)

where \( \tilde{r} = a(t)r \), two non-angular metric \((x^0, x^1) = (t, r)\) and two dimensional metric is \( h_{ab} = diag[-1, a^2/(1-Kr^2)] \). Here \( K = 1, 0, -1 \) is the curvature parameter corresponding to
a closed, flat and open universe, respectively. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion is \( \tilde{r}_A = (H^2 + K/a^2)^{-1/2} \) which has been calculated by the relation \( h^{ab} \partial_a \tilde{r} \partial_b \tilde{r} = 0 \). This relation implies that the vector \( \nabla \tilde{r} \) is null on the apparent horizon surface. The apparent horizon may be considered as a causal horizon for a dynamical spacetime. Thus one can associate a gravitational entropy and surface gravity to it.

From Eq. (2), the energy density of LECHDE with apparent horizon, \( \tilde{r}_A \), as an IR-cutoff can be written as

\[
\rho_D = 3n^2 M_P^2 \tilde{r}_A^{-2} + \gamma \tilde{r}_A^{-4} \ln(\rho_D) + \beta \tilde{r}_A^{-4}.
\] (7)

The first Friedmann equation is

\[
\frac{1}{\tilde{r}_A^2} = H^2 + \frac{K}{a^2} = \frac{1}{3M_P^2} (\rho_m + \rho_D),
\] (8)

where \( H = \dot{a}/a \) is the Hubble parameter. In a FRW universe, the total energy density \( \rho = \rho_D + \rho_m \) is satisfied in a conservation equation as:

\[
\dot{\rho} + 3H(1 + w)\rho = 0
\] (9)

where \( w = p/\rho \) is the EoS parameter. Due to non-gravitational interaction between dark energy and pressureless cold dark matter (CDM) with subscript ‘m’, two energy densities \( \rho_D \) and \( \rho_m \) are not conserved separately and the conservation equation can be written as

\[
\dot{\rho}_D + 3H(1 + w_D)\rho_D = -Q,
\] (10)

\[
\dot{\rho}_m + 3H\rho_m = Q.
\] (11)

Here \( Q \) is the interaction term which has been usually considered in three forms as

\[
Q = \Gamma \rho_D = \begin{pmatrix} 3Hb^2 \rho_D \\ 3Hb^2 \rho_m \\ 3Hb^2(\rho_m + \rho_D) \end{pmatrix}.
\] (12)

In this equation, \( b^2 \) is coupling constant. Although a theoretical interpretation of this interaction has not been performed yet, as we see from Eqs. (10, 11), the interaction term \( Q \) should be as a function of \( H \) multiplied to energy density. Therefore in Eq. (12), the simplest form of \( Q \) is considered with a coupling constant \( b \). This term indicates the decay rate of DE to CDM as similar as standard ΛCDM model where vacuum fluctuations can
decay into matter. In many models the interaction term is necessary in order to solving the coincidence problem. It has been shown that this interaction can influence the perturbation dynamics, cosmic microwave background (CMB) spectrum and structure formation [26].

Differentiating Eq. (7) with respect to cosmic time and using the differentiation of apparent horizon with respect to cosmic time, we have

$$-\tilde{r}_A^{-3} = H(\dot{H} - \frac{K}{a^2}) = \frac{1}{6M_P^2}(\dot{\rho}_D + \dot{\rho}_m),$$

(13)

where from Eqs. (10, 11) we obtain

$$\tilde{r}_A = \frac{H}{2M_P^2} \tilde{r}_A^2 \rho_D (1 + u + w_D),$$

(14)

$$\dot{\rho}_D = -\frac{H\rho_D \tilde{r}_A^2}{M_P^2} (1 + u + w_D)[2\rho_D - \gamma \tilde{r}_A^{-4} - 3n^2 M_P^2 \tilde{r}_A^{-2}].$$

(15)

Here $u = \rho_m/\rho_D$ is the ratio of energy densities. Also from Eq. (8), we find that $3M_P^2 \tilde{r}_A^{-2} = (1 + u)\rho_D$ where $u$ is governed by

$$u = \frac{3M_P^2}{3n^2M_P^2 + \gamma \tilde{r}_A^{-2} \ln(M_P^2 \tilde{r}_A^2) + \beta \tilde{r}_A^{-2}} - 1.$$  

(16)

From Eq. (16), we see that at sufficient large $\tilde{r}_A$, where $\rho_D \approx 3n^2M_P^2\tilde{r}_A^{-2}$, the ratio of energy densities will tend to a constant value $u \to 1/n^2 - 1$. Also at present time, $u$ varies slowly up to reach a constant value, $u = 1/n^2 - 1$. In Fig. 11 the function $u$ is plotted in versus $\tilde{r}_A$ for fixed $\gamma$, $n$ and various $\beta$ in the Planck mass unit in which $M_P = 1/\sqrt{8\pi G} = 1$. From this figure, we conclude that the coincidence problem gets alleviated since for some values of model parameters, we get $u \sim O(1)$ for wide range of $\tilde{r}_A$ (including the present time), and it is growing so that finally reaches to a fixed value of order unity.

The deceleration parameter $q = -1 - \dot{H}/H^2$ may be obtained by using the Friedmann equation and continuity equation as follows [13, 14]

$$q = -(1 + \Omega_K) + \frac{3}{2} \Omega_D (1 + u + w_D),$$

(17)

where $\Omega_K = K/(a^2 H^2)$, $\Omega_D = \rho_D/(3M_P^2 H^2)$ and $\Omega_m = \rho_m/(3M_P^2 H^2)$ are the energy density parameters. From these dimensionless parameters, the first Friedmann equation can be
rewritten as: $1 + \Omega_K = \Omega_D + \Omega_m$. Using the third form of interacting term, in which $\Gamma/3H = b^2(1 + u)$ and combining Eq. (15) with (10), the EoS parameter $w_D$ is given by

$$w_D = -1 - \frac{u(2 \rho_D - 3 n^2 M_P^2 \tilde{T}_A^{-2} - \gamma \tilde{T}_A^{-4}) - b^2 (1 + u)^2 \rho_D}{(1 - u) \rho_D - 3 n^2 M_P^2 \tilde{T}_A^{-2} - \gamma \tilde{T}_A^{-4}}. \quad (18)$$

From this equation and Eq. (16), we find

$$\tilde{T}_A' = \frac{3 M_P^2 \tilde{T}_A}{2} \left[ 3 n^2 M_P^2 \tilde{T}_A^2 + \gamma \ln(M_P^2 \tilde{T}_A^2) + \beta + 3 M_P^2 \tilde{T}_A^2 (b^2 - 1) \right] \left[ 3 M_P^2 \tilde{T}_A^2 (n^2 - 1) + 2 \gamma \ln(M_P^2 \tilde{T}_A^2) + 2 \beta - \gamma \right], \quad (19)$$

where “prime” denotes the differentiation with respect to $x = \ln a = -\ln(1 + z)$ in which $Hd/dx = d/dt$.

On the other hand, by using Eqs. (8) and (12), the evolution of dark energy density can be rewritten as

$$\rho_D' = - 3 \rho_D \left[ 1 + w_D + b^2 (1 + u) \right], \quad (20)$$

and then the evolution of $\Omega_D$ is calculated as:

$$\Omega_D' = - 3 \Omega_D \left[ (1 + w_D)(1 - \Omega_D) + b^2 (1 + u) - \Omega_D u + \frac{2}{3} \Omega_K \right]. \quad (21)$$

Using Eq. (17), the deceleration parameter is given by

$$q = - (1 + \Omega_K) - \frac{3}{2} \frac{\Omega_D (1 + u)[u - b^2 (1 + u)] \rho_D}{(1 - u) \rho_D - 3 n^2 M_P^2 \tilde{T}_A^{-2} - \gamma \tilde{T}_A^{-4}}. \quad (22)$$
It is worthwhile to mention that $\Omega_K$ and $\Omega_D$ is related by

$$\frac{\Omega_K}{\Omega_m} = a \frac{\Omega_{K0}}{\Omega_{m0}} \Rightarrow \Omega_K = \frac{e^\Gamma(1 - \Omega_D)}{1 - e^\Gamma},$$

where $\Gamma = \Omega_{K0}/\Omega_{m0}$ is a constant value, which from the recent data, is given by $\Gamma \approx 0.04$. Here the subscript '0', is used for the present time.

In the limiting case of ordinary HDE with $\gamma = \beta = 0$, Eqs. (16, 18, 22) reduce to the following simple forms

$$u = \frac{1}{n^2} - 1,$$

$$w_D = -(1 + \frac{1}{u}) \frac{\Gamma}{3H},$$

$$q = -(1 + \Omega_K) - \frac{3}{2} \Omega_D(1 + u)(\frac{\Gamma}{3Hu} - 1),$$

which have been also calculated by [14]. In this case, from Eq. (19), the radius of apparent horizon, $\tilde{r}_A$, can be obtained as

$$\tilde{r}_A = \tilde{r}_{A0} e^{\frac{3M_P^2}{2} \left(\frac{a^2 - 1}{n^2 - 1}\right)x} = \tilde{r}_{A0} (1 + z)^\frac{3M_P^2}{2} \left(\frac{a^2 - 1}{n^2 - 1}\right).$$

Here we can choose $\tilde{r}_{A0} = 1$ at present time: ($x = 0$ or vanishing redshift, $z = 0$). Therefore $\tilde{r}_A$ may be considered as a normalized horizon radius. From Eq. (27), we see that the radius of apparent horizon is increased by cosmic time provided that $|n| > 1$ or $|n| < \sqrt{1 - b^2}$.

From Eq. (25), we see that, in the absence of interaction, we have $w_D = 0$, but in LECHDE model, the EoS parameter may cross the phantom divide ($w_D < -1$) even in the absence of interaction. In Fig. 2, the evolution of the EoS parameter of LECHDE in versus of $\tilde{r}_A$ is studied, both in interacting and non-interacting modes for positive values of $\beta$, in the Planck mass unit. We consider specially the effect of coupling constant on behavior of $w_D$.

As it is shown in Fig. 2 by choosing the typical value of parameters of LECHDE model as: $\gamma = 0.1$, $\beta = 0.2$, $n = 0.8$, two distinct regions of $\tilde{r}_A$ are given as:

- **a**: $0.22 > \tilde{r}_A > 0$, Fig 2a. Neither of interacting and non-interacting cases can drive an expanding universe ($w_D > 0$).

- **b**: $\tilde{r}_A > 0.23$, Fig 2b. Both of interacting and non-interacting cases may accelerate the expanding universe and cross the phantom divide. Interacting cases always remain under the quintessence wall, while in non-interacting mode, the EoS parameter grows from phantom regime, $w_D < -1$, to positive value of EoS parameter, ($w_D > -1/3$) at small values of
\( \tilde{r}_A < 1 \). Therefore the non-interacting case can not drive the late time acceleration in our universe.

By solving Eqs. (19, 21, 22, 23) numerically, the behavior of deceleration parameter, \( q \) with respect to \( x = \ln (a) \), in LECHDE model, is studied. In Fig. 3 as we can see, the present \((x \approx 0)\) accelerated stage \((q < 0\) is preceded by a sufficiently long period deceleration at the early time \((x < 0, \text{far from } x = 0)\). This is compatible with cosmic structure formation at matter dominated era and present accelerated expansion.

The typical values of \( \gamma, \beta, n \) are set, so that the function \( u \) becomes positive for all studied regions and gets \( u_0 \sim 0.4 \) at present time and rich to a constant value of order unity at the late time.

### III. INTERACTING “PLECHDE” MODEL WITH AH-IR-CUTOFF

From Eq. (31), the energy density of PLECHDE with apparent horizon, \( \tilde{r}_A \), as an IR-cutoff, is written as

\[
\rho_D = 3n^2 \tilde{M}_P^2 \tilde{r}_A^{-2} - \delta \tilde{M}_P^2 \tilde{r}_A^{-\alpha},
\]

where using (14, 28), the energy density evolution is given by

\[
\dot{\rho}_D = -3H \rho_D (1 + u + w_D) \left[ n^2 - \frac{\alpha \delta \tilde{r}_A^{-2-\alpha}}{6} \right].
\]

From Eqs. (8) and (28), the ratio of energy densities, \( u \), is given by

\[
u = \frac{1}{n^2 - \frac{\delta \tilde{r}_A^{-2-\alpha}}{3}} - 1.
\]

Also from Eqs. (28) and (30), as the same as Sec. II we see that at late time, for \( \alpha > 2 \), when \( \tilde{r}_A \) is large, we have \( \rho_D \approx 3n^2 \tilde{M}_P^2 \tilde{r}_A^{-2} \) and the ratio of energy densities \( u \), will tend to a constant value \( u \to 1/n^2 - 1 \), while this is not valid for \( \alpha < 2 \). In Fig. 4 we study the behavior of \( u \) in versus of \( \tilde{r}_A \), for various positive values of \( \delta \) and fixed value \( \alpha \). From this figure, we see that the function \( u \) is descending for \( \delta > 0 \) and the present value of \( u \) is satisfied for a typical set \((\alpha = 3, n = 0.89, \delta = 0.2)\) at \( \tilde{r}_A = 1 \) (present time). In this case \( u \sim O(1) \), only for \( \tilde{r}_A > 0.3 \). Also the coincidence problem can be solved, since for some
values of model parameters, we get $u_0 \sim \mathcal{O}(1)$, at present time, and it finally reaches to a fixed value of order unity.

FIG. 2: The evolution of EoS parameter, $w_D$, versus of $\bar{r}_A$ in LECHDE model, $a$: $0.22 > \bar{r}_A > 0.0$. $b$: $\bar{r}_A > 0.23$. 

FIG. 3: The evolution of $q$ in versus $x = \ln (a)$ in LECHDE model for ($n = 0.8, \gamma = 0.1, \beta = 0.2, b^2 = .1, \Gamma = 0.04$).

FIG. 4: The evolution of $u$ in versus of $\tilde{r}_A$ in PLECHDE model.
Similar to previous section, the EoS parameter $w_D$, $\tilde{r}_A'$, $\Omega_D'$ and deceleration parameter $q$ are calculated as

$$w_D = -\frac{1 - (1 + u)(n^2 - \frac{\alpha \delta \tilde{r}_A^{2-\alpha}}{6} - b^2)}{1 - (n^2 - \frac{\alpha \delta \tilde{r}_A^{2-\alpha}}{6})}, \quad (31)$$

$$\tilde{r}_A' = \frac{3\tilde{r}_A}{2} \left[ 1 + b^2 - (n^2 - \frac{\alpha \delta \tilde{r}_A^{2-\alpha}}{6}) \right], \quad (32)$$

$$\Omega_D' = -\Omega_D \left[ (1 + u + w_D)(3n^2 - \frac{\alpha \delta \tilde{r}_A^{2-\alpha}}{2} - 3\Omega_D) + 2\Omega_K \right], \quad (33)$$

$$q = -(1 + \Omega_K) + \frac{3\Omega_D}{2} \left[ \frac{u - b^2(1 + u)}{1 - (n^2 - \frac{\alpha \delta \tilde{r}_A^{2-\alpha}}{6})} \right]. \quad (34)$$

The limiting case of Eqs. (30, 31, 34), with $\delta = 0$ or large $\tilde{r}_A$, has been given by Eqs. (24, 25, 26). Also in this case the eq. (32) reaches to Eq. (27) in the previous section. In PLECHDE model, the EoS parameter may cross the phantom divide ($w_D < -1$) even in the absence of interaction. In Fig. 5 the EoS parameter of PLECHDE is studied both in various interacting and non-interacting modes. As it is shown in Fig. 5 by choosing the typical value of parameters of PLECHDE as: ($\alpha = 3$, $\delta = +0.2$, $n = 0.89$), we encounter with two distinct regions of $\tilde{r}_A$ in behavior of EoS parameter as below:

**a:** ($0.08 > \tilde{r}_A > 0$), Fig 5a. We find: ($w_D > 0$). So the model can not drive an acceleration expansion irrespective of interaction.

**b:** ($\tilde{r}_A > 0.09$), Fig 5b. Both of interacting and non-interacting cases may accelerate the expansion of the universe and the phantom divide is crossed. Interacting cases always remains under the quintessence regime ($w_D < -1/3$), while in non-interacting mode, the EoS parameter grows from phantom regime, $w_D < -1$, to above the quintessence regime ($w_D > -1/3$) very soon. Therefore, same as previous section, the non-interacting case can not drive the late time acceleration.

Now we want to study the deceleration parameter of PLECHDE model. By solving Eqs. (32, 33, 34, 23), numerically, the behavior of $q$ with respect to $x$ can be studied. In Fig. 6 similar to previous case, the present ($x \approx 0$) acceleration has been supported by a long period deceleration phase at past ($x < 0$).

It must be mention that, similar to previous model, the typical values of $\alpha$, $\delta$, $n$ are set, so that the function $u$ become positive for all studied regions and gets $u_0 \sim 0.4$ at present time.
FIG. 5: The evolution of EoS parameter, $w_D$, in versus of $\bar{r}_A$ in PLECHDE model. 

**a:** $0.08 > \bar{r}_A > 0.0$ and $\delta = 0.2$. **b:** $\bar{r}_A > 0.09$ and $\delta = 0.2$. “Q” the Quintessence barrier ($w_D = -1/3$).

### IV. THERMODYNAMICS OF NON-INTERACTING LECHDE WITH AH-IR-CUTOFF

In this section we want to associate a thermodynamical description to cosmological horizons, similar to black hole physics. In a FRW universe enclosed by an apparent horizon, one can associate the Hawking temperature to the horizon, which is inversely proportional to size of the apparent horizon. We know that the FRW universe may consist several cosmic ingredients including dark energy, dark matter, radiation and baryonic matter. However
many cosmological evident reveal that the dark energy and matter are two dominant components in our universe. At following, we will consider only LECHDE and CDM components in a non-flat FRW universe enclosed by apparent horizon. In a local thermal equilibrium, where there is not any heat flow from the apparent horizon, the temperature of the energy content of the universe \( T \) should be equal to the temperature which is associated with apparent horizon \( T_h \). In non equilibrium case, the heat will flow outside (inside) the apparent horizon if the temperature of cosmic fluid is hotter (colder) than the apparent horizon, respectively. The thermal equilibrium state can be accessed at a finite time and therefore we can consider a unit temperature for whole spacetime (contain DE, CDM and AH). The equilibrium entropy of the LECHDE is connected with its energy and pressure, \( p_D \), through the Gibbs law of thermodynamics

\[
TdS_D = dE_D + p_D dV,
\]

where \( V = (4\pi/3)r_A^3 \) is the volume of whole space up to horizon surface and \( S_D \) is the entropy of DE component. The equilibrium temperature \( T \), can be obtained from the surface gravity \( (\kappa_H) \) of horizon as follows

\[
T = \frac{|\kappa_H|}{2\pi} = \frac{1}{4\pi \sqrt{-h}} \left| \partial_a (\sqrt{-h} h^{ab} \partial_b \tilde{r}) \right|.
\]
From this equation, the temperature of apparent horizon is calculated as

\[ T = \frac{1}{2\pi \tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right). \]  

(37)

Following Cai and Kim [23], the apparent horizon radius \( \tilde{r}_A \) should be regarded to have a fixed value in thermal equilibrium. It means that \( \dot{\tilde{r}}_A \approx 0 \). Thus the temperature is given by

\[ T = \frac{1}{2\pi \tilde{r}_A^{(0)}}. \]  

(38)

Now from Eq. (35), we have

\[ TdS = \rho_D (1 + w_D) dV + V d\rho_D, \]  

(39)

and by using Eq. (17), we can obtain

\[ \frac{dS_D^0}{d\tilde{r}_A^{(0)}} = \frac{8}{3} \pi^2 (\tilde{r}_A^{(0)})^3 \left[ 6n^2 M_p^2 (\tilde{r}_A^{(0)})^{-2} ight. \]

\[ + 2\gamma (\tilde{r}_A^{(0)})^{-4} - \rho_D^0 (1 - 3w_D^0) \], \]  

(40)

where superscript \((0)\) denotes that the universe is in a stable thermodynamical equilibrium state.

V. THERMODYNAMICS OF INTERACTING LECHDE WITH AH-IR-CUTOFF

In the presence of interaction, \( Q \neq 0 \), the thermal equilibrium is no further maintain due to thermal fluctuation which has been arose from decaying of dark energy to dark matter. The conservation equations for \( \rho_m \) and \( \rho_D \), have been given by Eqs. (10) [11]. In this case, however the Gibbs law of thermodynamics may hold only approximately for dynamical apparent horizon, the entropy affected under a first order logarithmic correction \( (S_D^{(1)}) \) involving temperature \( T \) and the heat capacity \( C \), as bellow [27]

\[ S_D^{(1)} = -\frac{1}{2} \ln(CT^2). \]  

(41)

Hence, the entropy should be modified as: \( S_D = S_D^{(0)} + S_D^{(1)} \). The heat capacity in thermal equilibrium has been defined as: \( C = T \partial S_D^{(0)}/\partial T \). Using (38), the heat capacity can be rewritten as: \( C = -(\tilde{r}_A^{(0)}) \partial S_D^{(0)}/\partial \tilde{r}_A^{(0)} \). Using Eq. (40) in thermal equilibrium, the corrected term \( S_D^{(1)} \) is calculated as

\[ S_D^{(1)} = -\frac{1}{2} \ln \left[ \rho_D^0 (\tilde{r}_A^{(0)})^2 (1 - 3w_D^0) - 6n^2 M_p^2 - 2\gamma (\tilde{r}_A^{(0)})^{-2} \right] \]

\[ - \frac{1}{2} \ln \left( \frac{2}{3} \right). \]  

(42)
similar to Eq. (40) with interaction, one obtains

$$dS_D = \frac{8}{3} \pi^2 \tilde{r}_A^2 \left[ 6n^2 M^2 P^2 \tilde{r}_A^{-2} + 2 \gamma \tilde{r}_A^{-4} - \rho_D (1 - 3w_D) \right] d\tilde{r}_A,$$

where from $dS_D = dS_D^{(0)} + dS_D^{(1)}$, we can find

$$1 - 3w_D = \left[ 6n^2 M^2 P^2 \tilde{r}_A^{-2} + 2 \gamma \tilde{r}_A^{-4} \right. \left. - \frac{3}{8\pi^2 \tilde{r}_A^2} \left( \frac{dS_D^{(0)}}{d\tilde{r}_A} + \frac{dS_D^{(1)}}{d\tilde{r}_A} \right) \right] \rho_D^{-1}. \quad (44)$$

From Eqs. (40) (42), it is obtained

$$\frac{dS_D^{(0)}}{d\tilde{r}_A} = \frac{dS_D^{(0)}}{d\tilde{r}_A} = \frac{8}{3} \pi^2 (\tilde{r}_A)^3 \left[ 6n^2 M^2 P (\tilde{r}_A)^{-2} \right. \left. + 2 \gamma (\tilde{r}_A)^{-4} - \rho_D^0 (1 - 3w_D^0) \right] \frac{d\tilde{r}_A^0}{d\tilde{r}_A}, \quad (45)$$

$$\frac{dS_D^{(1)}}{d\tilde{r}_A} = \frac{dS_D^{(1)}}{d\tilde{r}_A} = -\frac{1}{2} \left[ 2 \rho_D^0 (\tilde{r}_A^0)^3 (1 - 3w_D^0) + 4 \gamma (\tilde{r}_A^0)^{-3} \right. \left. + (\tilde{r}_A^0)^2 \frac{d}{d\tilde{r}_A^0} [\rho_D^0 (1 - 3w_D^0)] / \left[ \rho_D^0 (\tilde{r}_A^0)^2 (1 - 3w_D^0) \right] \right] \frac{d\tilde{r}_A^0}{d\tilde{r}_A}, \quad (46)$$

where from (18) and (16), we have

$$1 - 3w_D = \quad (47)$$

$$4 + 3u (2 \rho_D - 3n^2 M^2 P^2 \gamma r_A^{-2} - \gamma r_A^{-4}) - \frac{\Gamma}{\Delta H} (1 + u) \rho_D$$

$$1 - 3w_D^0 = \quad (48)$$

$$4 + 3u^0 \frac{2 \rho_D^0 - 3n^2 M^2 P^0 (\tilde{r}_A^0)^{-2} - \gamma (\tilde{r}_A^0)^{-4}}{(1 - u^0) \rho_D^0 - 3n^2 M^2 P^0 (\tilde{r}_A^0)^{-2} - \gamma (\tilde{r}_A^0)^{-4}},$$

$$\frac{du^0}{d\tilde{r}_A^0} = -(1 + u^0) \left[ \frac{2}{\tilde{r}_A^0} + \frac{d}{d\tilde{r}_A^0} \ln(\rho_D^0) \right]. \quad (49)$$

Now, we want to find a relation between the interaction term and the thermal fluctuation. For this purpose, by comparing Eqs. (44) (47), the interaction term can be calculated with
respect to thermal fluctuation as

\[
\frac{\Gamma}{3H} = \frac{2}{3(1 + u) \rho_D^2} \left\{ (2\rho_D - 3n^2M_p^2\tilde{r}_A^{-2} - \gamma\tilde{r}_A^{-4}) \right. \\
\left. \left( (1 + \frac{u}{2})\rho_D - 3n^2M_p^2\tilde{r}_A^{-2} - \gamma\tilde{r}_A^{-4} \right) + \left( \frac{d\tilde{r}_A}{d\tilde{r}_A^0} \right) \times \\
\frac{3\tilde{r}_A^{-1}}{32\pi^2\tilde{r}_A^3} \left[ (1 - u)\rho_D - 3n^2M_p^2\tilde{r}_A^{-2} - \gamma\tilde{r}_A^{-4} \right] \\
\left( \frac{16}{3} \pi^2 \left( 6n^2M_p^2 + 2\gamma(\tilde{r}_A^0)^{-2} - \rho_D^0(\tilde{r}_A^0)^2(1 - 3w_D^0) \right) \right)^2 \\
\left. + 4\gamma(\tilde{r}_A^0)^{-4} + 2\rho_D^0(1 - 3w_D^0) + \tilde{r}_A^0 \frac{d}{d\tilde{r}_A^0} [\rho_D^0(1 - 3w_D^0)] \right\}. \tag{50}
\]

In limiting case, for ordinary HDE ($\gamma = \beta = 0$), where $w_D^0 = 0$ and $\rho_D = 3n^2M_p^2\tilde{r}_A^{-2}$, from Eqs. (25, 50), we can obtain

\[
\frac{\Gamma}{3H} = \frac{1 - n^2}{3} \left[ 1 - \tilde{r}_A^0 \frac{d}{d\tilde{r}_A^0} \ln(\tilde{r}_A) \right]. \tag{51}
\]

VI. CONCLUSION

In this paper the logarithmic and power-law entropy-corrected version of interacting HDE with AH-IR-cutoff in a non-flat universe enclosed by apparent horizon have been studied. In fact we generalized the ordinary HDE model by considering the entropy correction due to fluctuation of spacetime and AH-IR-cutoff. In LECHDE model, corrections are restricted to the leading order correction which contains the logarithmic of area. In PLECHDE model, the correction is based on the gravitational fluctuations which affect the area law of entropy to a fractional power of area, which is arisen by entanglement of quantum field theory.

The ratio of dark matter to dark energy densities $u$, EoS parameter $w_D$ and deceleration parameter $q$ have been obtained. We showed that the cosmic coincidence is satisfied for appropriate model parameters. In dealing with cosmic coincidence problem, we found an appropriate set of values for LECHDE model as: ($\gamma = 0.1$, $\beta = 0.2$, $n = 0.8$) and for PLECHDE model as: ($n = 0.89$ $\alpha = 3$, $\delta = 0.2$). These parameters have been chosen in order to get $u_0 \sim 0.4$ and finally, reaches slowly to a constant value of order unity. By studying the effect of interaction in EoS parameter, we saw that the phantom divide may be crossed and also find that the interacting models can drive an acceleration expansion at the present and future, while in non-interacting case, this expansion can happen only at the
early time. The graphs of deceleration parameter for interacting models, showed that the present acceleration expansion is preceded by a sufficiently long period deceleration at past.

Moreover, the thermodynamical interpretation of interaction between LECHDE and dark matter was described. Based on the Gibbs law of thermodynamics, for dark energy sector of the universe in non-interacting case, we calculated a differentiation of entropy of DE with respect to $\tilde{r}_A$. Although in the absence of interaction between dark energy and dark matter, these two dark components conserved separately, while by imposing an interaction term, a stable fluctuation around equilibrium is expectable. Therefore, in the interacting case, where the entropy affected under a first order logarithmic correction, we obtained a relation between the interaction term and thermal fluctuation in a non-flat universe enclosed by the apparent horizon. Also in limiting case for ordinary HDE, the relation of interaction term versus thermal fluctuation was calculated.

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