The Cosmic X-ray Background – IRAS galaxy Correlation and the Local X-ray Volume Emissivity

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ABSTRACT

We have cross-correlated the galaxies from the IRAS 2 Jy redshift survey sample and the 0.7 Jy projected sample with the all-sky cosmic X-ray background (CXB) map obtained from the HEAO-1 A2 experiment. We have detected a significant correlation signal between surface density of IRAS galaxies and the X-ray background intensity, with $W_{xy} = \langle \delta I \delta N \rangle / \langle I \rangle \langle N \rangle$ of several times $10^{-3}$. While this correlation signal has a significant implication for the contribution of the local universe to the hard ($E > 2\text{keV}$) X-ray background, its interpretation is model dependent.

We have developed a formulation to model the cross-correlation between CXB surface brightness and galaxy counts. This includes the effects of source clustering and the X-ray – far infrared luminosity correlation. Using an X-ray flux limited sample of AGNs, which has IRAS 60 $\mu$m measurements, we have estimated the contribution of the AGN component to the observed CXB – IRAS galaxy count correlations in order to see whether there is an excess component, i.e. contribution from low X-ray luminosity sources. We have applied both the analytical approach and Monte-Carlo simulations for the estimations.

Our estimate of the local X-ray volume emissivity in the $2 - 10\text{keV}$ band is $\rho_x \approx (4.3 \pm 1.2) \times 10^{38} h_{50} \text{erg s}^{-1} \text{Mpc}^{-3}$, consistent with the value expected from the luminosity function of AGNs alone. This sets a limit to the local volume emissivity from lower-luminosity sources (e.g. star-forming galaxies, liners) to $\rho_x \lesssim 2 \times 10^{38} h_{50} \text{erg s}^{-1} \text{Mpc}^{-3}$.

Subject headings: Cosmology — galaxies: clustering — X-rays: sources

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1. Introduction

The origin of the Cosmic X-ray Background (CXB) is still an open question (see review by Fabian & Barcons 1992). The current popular idea of the origin of the CXB is that it is a superposition of unresolved sources. While much of the soft \( (E < 2\text{keV}) \) CXB is resolved into quasars by the ROSAT PSPC (Hasinger, Schmidt, & Trümper 1991; Shanks et al. 1991; Hasinger et al. 1993), the discrepancy between the \( \log N - \log S \) relations of hard \( (E \approx 2 - 10\text{keV}) \) and soft bands \( (E < 2\text{keV}) \) may indicate that those sources are not the only important contributor in the hard band.

One important quantity which can be determined observationally is the X-ray volume emissivity in the local universe. This would provide a constraint for models of the CXB with evolving populations of sources. A lower limit to the quantity can be set from the X-ray luminosity function (XLF) of resolved sources. The \( \text{2} - \text{10 keV} \) AGN XLF by Piccinotti et al. (1982) showed that it is well approximated by a power-law function. The uncertainty on the lower luminosity limit of the Piccinotti et al. XLF causes a large uncertainty of the local volume emissivity from AGNs. The AGN XLF is observed to flatten below \( L_{x,44} \approx 0.05 \) (where and hereafter \( L_{x,44} \) refers to the \( 2 - 10 \text{keV} \) X-ray luminosity in units of \( 10^{44} h^{-2}_{50} \text{erg s}^{-1} \)), which is the lower luminosity limit of the XLF given by Piccinotti et al. (1982), by using a flux-limited sample of AGNs (Grossan 1992, hereafter G92) extracted from the HEAO-1 MC-LASS Catalog of Identified, Hard X-ray Sources (Remillard, 1994), which exploits the data from HEAO 1 A1/A3 experiments. This gives \( \rho_{x,38} \approx 4 \) (where and hereafter, \( \rho_{x,38} \) refers to the volume emissivity in units of \( 10^{38} h_{50} \text{erg s}^{-1} \text{Mpc}^{-3} \) in the \( 2 - 10 \text{keV} \) band unless otherwise noted) for \( L_{x,44 \text{min}} \sim 0.01 \) (G92).

Also using the data from ROSAT and the Einstein Observatory, Boyle et al. (1993) have determined the local AGN luminosity function for \( L_{x,44}(0.3 - 3.5\text{keV}) > 0.01 \); this yields \( \rho_{x,38}(0.3 - 3.5 \text{keV}) = 1.7 \). The corresponding volume emissivity implied for the \( 2 - 10 \text{keV} \) band has been estimated by Leiter & Boldt (1994) based on the unified model in which the number of Seyfert 2’s is 2.3 times that of Seyfert 1’s (Huchra & Burg 1992), the Seyfert 1 X-ray spectrum is the relatively steep one assumed by Boyle et al. (1993), and the average Seyfert 2 spectrum exhibits absorption by \( N_H \sim 5 \times 10^{22} \text{cm}^{-2} \); this gives \( \rho_{x,38}(2 - 10 \text{keV}) \sim 2.3 \). For the G92 determined HEAO-1 based volume emissivity \( (2-10 \text{keV}) \) to be consistent with the ROSAT value obtained by Boyle et al. (1993) in the \( 0.3 - 3.5 \text{keV} \) band would require that the number of Seyfert 2’s be somewhat more than 4 times that of Seyfert 1’s.

Extrapolating the volume emissivity implied by these resolved sources to \( z \sim 5 \) explains \( 10 - 30\% \) of the \( 2 - 10 \text{keV} \) CXB intensity without evolution. If there are a large number of low luminosity sources \( (L_{x,44} \leq 0.01, \text{e.g. star-forming galaxies suggested by Griffiths & Padovani [1990] and/or liners}) \), these would not appear in the luminosity function, but would contribute to the CXB. Thus an observational constraint for the local volume emissivity from these sources gives an important constraint to the models of the CXB. Since galaxies are known to be clustered, one way to estimate the local volume emissivity from low-luminosity sources is to search for the
auto-correlation property of the CXB at a scale of degrees. An analysis using this method by Danese et al. (1993) set the upper limit to the local X-ray volume emissivity clustered like normal galaxies to $\rho_{x,38} \lesssim 6$.

Another method to probe the volume emissivity due to these low luminosity sources is to cross-correlate the CXB surface brightness with known catalogs of galaxies. The first such attempt was made by Turner & Geller (1980) who set an upper limit to the fraction of the CXB correlated with nearby galaxies. Persic et al. (1989) took a similar but slightly different approach. They looked for a surface brightness enhancement at positions of various classes of sources and set an upper limit to the contribution of low-luminosity sources to CXB. Jahoda et al. (1991; 1992, hereafter JLMB91, JLMB92 respectively or JLMB collectively) have used the HEAO-1 A2 all-sky X-ray map to cross-correlate with the galaxies in the UGC and ESO catalogs and found a correlation signal. Based on this correlation signal and effective depths of UGC and ESO catalogs, they have derived a local X-ray volume emissivity in the 2–10 keV band of $\rho_{x,38} = (12.5 \pm 7.0)^4$. Their calculation, however, was based on the assumptions that the effect of source clustering can be neglected and that the radial selection functions of cataloged galaxies and the associated X-ray sources are identical. Lahav et al. (1993) estimated the clustering effect and concluded that the effect is not at all negligible and that one thereby overestimates the volume emissivity by a large factor.

In this work, we have used two complete flux limited samples of galaxies from the IRAS point source catalog, i.e. the IRAS 2 Jy redshift survey (Strauss et al. 1990) and the 0.7 Jy (Meurs & Harmon 1988) projected samples to correlate with the all-sky CXB surface brightness map from the HEAO-1 A2 experiment. The 2 Jy sample is particularly useful because it has a redshift for each galaxy and thus we can calculate correlation coefficients for a few redshift-selected subsets of the sample. In § 2.1., we briefly summarize the data we have used and also explain the calculation of the correlation coefficients. In § 3.1. and 3.2., we develop analytical formulations of the correlation and the application of the formulations to this particular problem. This includes a full treatment of the effects of source clustering and the X-ray – 60 $\mu$m luminosity correlation. Some details of the derivations are shown in appendices A - B. The Monte-Carlo simulations are explained in § 3.3. and and the simulated models are compared with the observations as well as analytical calculations in § 3.4.. The results are discussed in § 4.

## 2. The Data and Correlation Analysis

### 2.1. The Data

4The value was initially given incorrectly in JLMB91 (Erratum: JLMB92). We show the corrected value here.
The X-ray data are from all-sky HEAO-1 A2 survey (Rothschild et al. 1979; Boldt 1987). The X-ray map has been constructed with the data from MED and HED # 3 of the A2 experiment (Jahoda & Mushotzky 1989; JLMB91) accumulated over the 180 day period beginning on day 322 of 1977. The period corresponds to one complete scan over the entire sky. The combination had sensitivity from 2 to 60 keV and quantum efficiency over 50% roughly between 3 and 17 keV. The all-sky X-ray map constructed from the data observed with the 1°.5 × 3°.0 FWHM collimators are used for the analysis. The point spread function (PSF) of the observation through these collimators can be well represented by

$$B_{psf} = \max(1 - \frac{|y|}{3}, 0) \max(1 - \frac{|x|}{1.5}, 0),$$

where $y$ is the coordinate along the scan path of the survey and $x$ is along the axis perpendicular to it (Shafer 1983). For this combination, one count per second corresponds to a 2 – 10 keV flux of $\sim 2.1 \times 10^{-11} \text{erg s}^{-1} \text{cm}^{-2}$ for the 40 keV thermal bremsstrahlung emission. The conversion differs only by a few percent between a power-law spectrum with the energy index of 0.65 and a 40 keV bremsstrahlung (Shafer 1983).

The map has been corrected for the Compton-Getting Effect (e.g. Boldt 1987) which is the all-sky dipole distortion due to the sun’s peculiar motion at 370 km s$^{-1}$ toward $(l, b) = (264.4, 48.4)$ inferred from the COBE DMR measurement of the cosmic microwave background dipole anisotropy (Kogut et al. 1993). A slight ($\sim 3\%$) sensitivity change over the 180 day period has been recognized. The map has been corrected for this effect by linearly modeling the sensitivity change.

The IRAS 2 Jy sample complete with redshift values (e.g. Strauss et al. 1990) and a deeper (0.7 Jy) sample of color selected IRAS galaxies (Meurs & Harmon, 1989) have been correlated with the CXB intensity. For the 2 Jy sample, we have limited our analysis to those galaxies with radial velocities between 500 and 8000 km s$^{-1}$, where the selection function is well defined (Strauss et al. 1992a). Although the 2 Jy sample is shallow, it has an advantage of having redshift on each object so the volume correlated with CXB is well defined. For the 0.7 Jy sample, we have used the selection function based on the luminosity function by Saunders et al. (1990). This sample is more sensitive to the low-luminosity behavior of the luminosity function.

**2.2. The Correlation Analysis**

To avoid confusion with galactic sources, we have limited our analysis to $|b| > 20^\circ$; regions around the Magellanic clouds are excluded. We have also excluded regions within one beam of sources in Piccinotti et al. (1982). But we have included nearby AGNs in that sample which may contribute to the correlation signal, i.e. AGNs with $v < 8000 km s^{-1}$ for the correlation with the 2 Jy sample and $z < 0.1$ for the correlation with the 0.7 Jy sample. These have been kept in the analysis because calculating the correlations including them is more convenient for the comparison with the formulations developed in the next section. In this work, we are not interested in the contributions from the X-ray emission of clusters of galaxies. They are rare in number density and known to evolve negatively with redshift (Edge et al. 1990); therefore their contribution to
the CXB is not very significant. Thus we excluded data from within one beam of clusters in Edge et al. (1990). Since the depths of the IRAS samples are shallow, the contribution from the X-ray clusters fainter than the flux limit of Edge et al. to the correlation signal is expected to be small. The regions of the sky which is not covered by the IRAS samples (see Strauss et al. [1990] for the 2Jy sample and Meurs & Harmon [1989] for the 0.7 Jy sample) are also excluded from the analysis.

The HEAO-1 A2 all-sky map is correlated with the galaxy surface density map created by smearing the position of the selected galaxies with the PSF of the A2 experiment. We have calculated the zero-lag cross-correlation in 3° × 3° square cells. The gridding has been made with a coordinate system which has one of its poles at the galactic center and the zero-longitude great circle on the galactic plane. Using this coordinate system, we have divided the sky into latitude strips of 3° width and subdivided each latitude strip every 3[cos(b_c)]^{-1} degrees in longitude, where b_c is the central latitude of the strip. With this division process, cells are made nearly 3° × 3° square except near the coordinate poles, which are in the galactic plane. We have accepted the cells for analysis only if the excluded region is no more than 20% of the cell. In this case, the small excluded region is assumed to have the mean X-ray surface brightness and galaxy surface density as the rest of the cell. This procedure leaves about 2300 cells for analysis, covering about a half of the whole sky.

As a statistical characterization of the zero-lag correlation, we have calculated the correlation coefficient:

\[
W_{xg} = \frac{N_{\text{cells}} \sum_i (I_i - \langle I \rangle)(N_i - \langle N \rangle)}{(\sum_i I_i)(\sum_i N_i)} \sim \frac{\langle \delta I \delta N \rangle}{\langle I \rangle \langle N \rangle},
\]

where \( I \) is the surface brightness of the X-ray background, \( N \) is the surface number density of the sample galaxies, and the summation is over \( N_{\text{cells}} \) cells. We have correlated the CXB with subsets of IRAS galaxies selected by the redshift range and the supergalactic latitude (\(|SGB|\)). The values of \( \langle I \rangle \) and \( \langle N \rangle \) are for the subsets and not the global averages. The results of the correlation are summarized in Table 1 with the 1σ errors estimated from the bootstrap resampling of the correlated data (JLMB and references therein). We measure stronger correlation signals inside the supergalactic plane (\(|SGB| < 20^\circ\)) than outside of it. The enhancement at low \(|SGB|\) is dominated by low redshift IRAS galaxies (\(V < 3500 \, km \, s^{-1}\)). We analyze the all-sky average behavior of the correlation signals in different redshift bins in the following section. For reference, we also show the case where all the AGNs in Piccinotti et al. (1982) are also excluded from analysis (fourth row in Table 1). This shows that there still is a significant correlation between IRAS galaxies and off-source X-ray sky. The exclusion procedure shown above, however, would make the analysis in the following section complicated and thus we model the correlation signal when resolved AGNs are included as described above.

3. Correlation and the X-ray Volume Emissivity

3.1. Analytical Formulations
In this section, we develop formulations relating the X-ray volume emissivity and the correlation signal. These are useful for investigating the dependence of the correlation signal on various parameters. In particular, we have included the effects of the source clustering and the X-ray – 60\,\mu m luminosity correlation (see also Lahav et al. 1993). The source clustering causes an enhancement of the correlation signal compared with the Poisson case for a fixed volume emissivity. The effect is partially due to the enhanced fluctuations associated with the angular clustering of the sample galaxies, some of which are X-ray sources. The effect is also due to the X-ray sources which themselves are not in the catalogued galaxies, but are clustered with the sample galaxies. If X-ray and infrared luminosities are correlated, the Poisson process should also be enhanced compared with the case with no such correlation.

We observe the galaxy counts and the X-ray intensities through effective cells, where each is the convolution the instrumental PSF (point spread function) and the square box profile of the cell. (Hereafter, the expression a ‘square cell’ refers to a cell with a square profile as well as a square projected shape.) We express the effective cell profile $B_{ec}(\hat{R} - \hat{R}_0)$ normalized at the center ($B_{ec}(0) = 1$), where $\hat{R}$ and $\hat{R}_0$ are the unit vectors of the current position and the cell center respectively. We also define the effective cell solid angle $\Omega_{ec} \equiv \int d\Omega B_{ec}(\hat{R} - \hat{R}_0)$. As Lahav et al. (1993), we express the expected correlation as a sum of the Poisson ($\hat{\eta}_p$) and the clustering ($\hat{\eta}_c$) terms:

$$W_{xg}(I\langle N \rangle) \equiv \langle (N - \langle N \rangle)(I - \langle I \rangle) \rangle = \hat{\eta}_p + \hat{\eta}_c$$  \hspace{1cm} (2)

Note that here $I$ and $N$ are the X-ray intensity and the IRAS galaxy number density per unit solid angle while the same symbols represent per beam values in Lahav et al. (1993).

As detailed in appendices A & B, the Poisson term can be expressed as:

$$\hat{\eta}_p = \frac{\int d\Omega B_{ec}^2(\hat{R} - \hat{R}_0)}{4\pi\Omega_{ec}}\rho_{xp}R_p \equiv \frac{s_p}{4\pi\Omega_{sq}}\rho_{xp}R_p, \hspace{1cm} (3)$$

where the R.H.S. of the equation expresses in terms of the square cell case with $\Omega_{sq} = a^2$ and a factor $s_p$ explaining the PSF smearing effect for convenience.

In the general case where $L_x$ may be correlated with $L_{60}$, the Poisson term effective depth $R_p$ can be expressed as:

$$R_p \equiv \int_{R_{\text{min}}}^{R_{\text{max}}} dRP_x(R), \hspace{1cm} (4a)$$

$$P_x(R) = \frac{\int_{4\pi R^2 f_{60,\text{lim}}}^\infty dL_{60} L_x(L_{60})\Phi_{60}(L_{60})}{\int_0^\infty dL_{60} L_x(L_{60})\Phi_{60}(L_{60})}, \hspace{1cm} (4b)$$

where $\Phi_{60}(L_{60})$ is the 60\,\mu m luminosity function and $L_x(L_{60})$ is the mean X-ray luminosity per IRAS source of a given 60\,\mu m luminosity. Note that if $\bar{L}_x$ is constant (no correlation between $L_x$ and $L_{60}$), then $P_x(R) = P(R)$ (the usual selection function).
Likewise, we express the clustering term. For the spatial correlation function between X-ray sources and IRAS galaxies $\xi_{xg}(r)$, the clustering term can be expressed as (Lahav et al. 1993):

$$\hat{\eta}_c = \frac{1}{\Omega_{ec}^2} \int d^3 R_1 \frac{\rho_{xc}}{4\pi R_2^2} \langle n \rangle P(R_1) \xi_{xg}(|R_2 - R_1|) B_{ec}(\hat{R}_1 - \hat{R}_0) B_{ec}(\hat{R}_2 - \hat{R}_0)$$

$$\equiv \frac{s_c}{4\pi\Omega_{sq}} \rho_{xc} R_c,$$

where $\langle n \rangle$ is the mean number density of the galaxies in the sample (above some minimum luminosity), $P(R)$ is the selection function of the sample, $\hat{R}_1$ & $\hat{R}_2$ are the unit vectors towards $R_1$ & $R_2$ respectively. Here again, the effect of the instrumental PSF is expressed by the factor $s_c$. Assuming a power-law spatial correlation function between IRAS galaxies and X-ray sources $\xi_{xg} = (r/r_0)^{-\gamma}$, and also assuming that the scale of the clustering is much smaller than the distances to the objects $(|R_1 - R_2| \ll R_1, R_2)$ (appendix C),

$$R_c \equiv \int_{R_{min}}^{R_{max}} dR P_c(R) \approx \langle n \rangle A_4 r_0^\gamma a^{3-\gamma} \int_{R_{min}}^{R_{max}} dR R^{3-\gamma} P(R),$$

where $A_4$ is the geometrical factor for the $\Omega_{sq} = a^2$ square cell depending on $\gamma$. While the volume emissivity in the Poisson term $\rho_{xp}$ is due to X-ray sources among the catalogued class of galaxies (i.e., those having IRAS luminosities above $L_{60,\text{min}}$), $\rho_{xc}$ is due to X-ray sources clustered with those galaxies, which may or may not be among those galaxies (if all the X-ray emitters responsible for the correlation are in the catalogued class of galaxies, then $\rho_{xp} = \rho_{xc}$).

### 3.2. Application to the HEAO 1 – IRAS Case

We now apply the general formalism developed above to our problem. In the case of the $3^\circ \times 3^\circ$ square cells ($\Omega_{sq} = 9 \, \text{deg}^2$) convolved with the A2 PSF we have used here, $\Omega_{ec} \approx 12 \, \text{deg}^2$, and $s_p \approx 0.42$. These values are insensitive (within a few percent) to the alignment between the square dividing cell and the scan path (i.e. orientation of the PSF) of the A2 experiment. For $\gamma = 1.8, s_c \approx 0.74$ and $A_{1.8} \approx 8.3$. (If the instrumental PSF could be considered a delta function, $\Omega_{sq} = \Omega_{ec}$ and $s_p = s_c = 1$.)

First, we consider the expected correlation signal from the population of X-ray sources whose X-ray luminosity function is reasonably known from the existing catalogs. The MC-LASS catalog of X-ray sources (Remillard 1994) include identifications of the X-ray sources using the data from HEAO 1 A1 (LASS) and A3 (Modulation Collimator) experiments. Grossan (G92) has made extensive studies of a complete flux limited sample of AGNs extracted from the MC-LASS catalog (flux limit: 0.95 $\mu$Jy at 5 keV, which is about a factor of two lower than that of Piccinotti et al. [1982]) (the LMA sample [LASS/MC identified sample of AGNs]). He constructed the luminosity function of the LMA AGNs giving $\rho_{x,38} = 4.1^{+1.3}_{-1.7}$, including AGNs down to $L_{x,44} \approx 0.01$. There are also extensive measurements over the electromagnetic spectrum for the LMA AGNs.
including redshifts and IRAS 60\(\mu\)m fluxes (G92). By comparing the observed correlations and the expected correlations from those AGNs, we can estimate or set a limit to the contribution of the lower-luminosity sources to the local volume emissivity.

As shown in appendix A, the numerator of the R.H.S. of Eq. 4b \((= \rho_{xp}P_x(R))\) can be estimated using a complete X-ray flux limited sample with red shifts and 60\(\mu\)m flux measurements \((L_{x,i}, L_{60,i})\). Neglecting the clustering of the sample sources, this is:

\[
\int_{L_{60,min}}^{L_{60,max}} dL_{60} L_x(L_{60}) \Phi_{60}(L_{60}) \approx \sum_{L_{60,i}\geq 4\pi r^2 f_{60,lim}} \frac{L_{x,i}}{V_{max}(L_x)},
\]

where \(f_{60,lim}\) is the limiting flux of the correlating IRAS sample and \(V_{max}(L_x)\) is the volume of space where a galaxy with an X-ray luminosity \(L_x\) would be in the X-ray flux limited sample.

As discussed in appendix A, \(P_x(R)\) estimated using Eq. 7 only represents the contribution to the Poisson term from the sources with the X-ray luminosities covered by the sample used to evaluate Eq. 7. Another Poisson term should be added for the contribution of lower luminosity objects. Thus it may be convenient to divide the Poisson term into two parts:

\[
W_{xg} = \frac{1}{4\pi \Omega_{sq}} [s_p(\rho_{xpA}R_{pA} + \rho_{xpB}R_{pB}) + s_c \rho_{xc} R_c],
\]

where \(\rho_{xpA,38} = 4.1^{+1.3}_{-1.7}\) (G92) is the volume emissivity of sources with \(L_{x,44} \geq 0.01\) (component [A]) and \(\rho_{xpB}\) is the X-ray volume emissivity of the sources which are in the IRAS catalog \((L_{x,44} < 0.01)\) (component [B]), for which we do not know the X-ray luminosity function. If most of the X-ray sources emit far infrared radiation also with \(L_{60} \geq L_{60, min}\), then \(\rho_{xc} \approx \rho_{xpA} + \rho_{xpB}\).

Applying Eq. 7 to the 61 AGNs from the LMA sample with radial velocities less than 20000 \(km \ s^{-1}\) (Fig. 1), we have evaluated \(P_{xA}(R)\) \((P_x(R)\) for component [A]) and \(R_{pA}\). Out of the 61 objects, 8 objects have only upper limits to the 60 \(\mu\)m flux. For these 8 objects, we have assigned 60 \(\mu\)m luminosities corresponding to a half of their 3\(\sigma\) upper limit fluxes. We also compared the resulting \(R_{pA}\) by excluding the 8 objects and observed that the effect of including/excluding these objects are much smaller than other sources of errors.

To evaluate the clustering term, we note that the spatial correlation function \(\xi_{xg}(r)\) in Eq. 5 is the correlation function between IRAS galaxies and X-ray sources. An estimate of this correlation function from the available data may be found by finding the cross-correlation function between resolved X-ray sources and the IRAS galaxies. Since both the IRAS 2 Jy sample and the LMA sample have redshifts, it is easy to calculate the spatial correlation function between these. We estimate \(\xi_{xg}(r)\) by:

\[
\xi_{xg}(r) = \frac{N_{pair}(r)}{N_R} - 1,
\]

where \(N_{pair}(r)\) is the number of X-ray AGN - IRAS pairs separated by a distance \(r\) and \(N_{R}^R\) is the ensemble average of the number of pairs separated by the same distance between randomized
X-ray AGN and IRAS samples. The redshifts of the randomized samples are drawn from the real samples but the sky coordinates are randomized within the sampled region. This method of constructing randomized samples compensates for the effects of the radial selection functions and incomplete sky coverage of the LMA and IRAS 2 Jy samples. The result is shown in Fig. 2. Fig. 2 shows that the IRAS-LMA AGN correlation function is well represented by a power-law form \( \xi_{xg} = (r/r_0)^{-\gamma} \) with the \( r_0 \sim 400 \text{ km s}^{-1} \) and \( \gamma \sim 1.8 \) except about a factor of two deficit at \( r \approx 100 - 200 \text{ km s}^{-1} \). Here and in the next subsection, we use the power-law spatial correlation function with \( r_0 \sim 400 \text{ km s}^{-1} \) and \( \gamma \sim 1.8 \). This result is about a factor of two smaller than the IRAS QDOT - IRAS selected AGN correlation function (Georganopoulos & Shanks 1993). The origin of the discrepancy is unclear. Also it is somewhat steeper than the IRAS autocorrelation function \( (r_0 \sim 400 \text{ km s}^{-1}, \gamma \sim 1.6 ; \text{ Lahav, Nemiroff, Piran 1990; Strauss et al. 1992a; Saunders, Rowan-Robinson, & Lawrence 1992}) \). One source of systematic error may be that we have calculated \( \xi_{xg} \) using the redshift instead of the real space distance. Sensitivity of the results to the assumed correlation function is discussed in § 4.

To evaluate Eqs. 5 & 6, we need the radial selection function of the IRAS sample \( P(R) \). We used the Strauss et al. (1992a) selection function for the 2 Jy sample. We have constructed the radial selection function from the luminosity function by Saunders et al. (1990) for the 0.7 Jy. The 0.7 Jy luminosity selection function has been normalized to unity at \( v = 500 \text{ km s}^{-1} \) and assumed to be unity at nearer distances. Contribution of \( v < 500 \text{ km s}^{-1} \) to \( R_c \) for the 0.7 Jy sample is small (~2%). The lower and upper bounds of the integration to obtain \( R_c \) and \( R_{pA} \) for the 0.7 Jy sample were set at 0 and 20000 \( \text{ km s}^{-1} \) respectively. The upper bound was determined to give the observed surface number density for the given form of selection function. The calculated values of \( R_{pA} \) and \( R_c \) are summarized later along with the Monte-Carlo simulation results (§ 3.4.).

The selection functions multiplied by the smearing factors \( s_c P_c(R), s_p P_{xA}(R) \) evaluated above are plotted in Fig. 3 along with \( s_p P(R) \). Fig. 3 shows relative importance of each term as a function of redshift as compared to the purely Poisson no-luminosity correlation case \( (JLMB)(s_p P(R) \text{ curve}) \). Although Fig. 1 shows only a weak correlation between \( L_x \) and \( L_{60} \) for the X-ray selected sample, the increased fraction of the luminous X-ray sources with larger \( L_{60} \) makes \( P_{xA}(R) \) much larger than \( P(R) \) at large distances. (Remember that \( \bar{L}_x(L_{60}) \) in Eq. 4b is the mean X-ray luminosity per IRAS source with \( L_{60} \).) The peaked feature of \( s_c P_c(R) \) of the 0.7 Jy (Fig. 3(b)) can be understood as follows. The clustering selection function \( P_c(R) \) includes the product of \( P(R) \) and the volume integration of \( \xi_{xg} \). The former is a rapidly decreasing function of \( R \), while the latter increases with distance as an effective cell covers more of the clustered sources.

This analytical approach is useful in looking at the behavior of Poisson and clustering terms, investigating the sensitivity of the results to parameters, and understanding the principles of the surface brightness - number count correlations. However, the depth of the IRAS samples are such that the approximation used in Eq. 6 is not accurate and an elaborate Monte-Carlo integration is needed to evaluate the clustering term without this approximation. Also integrating the power-law correlation function to infinity may cause systematic errors. Also statistical uncertainties due
to the finite number of sources contributing to the correlation are hard to estimate with the analytical approach. Thus we use the Monte-Carlo simulations to compare the models with the data in the next section.

3.3. Monte-Carlo Simulations

We have made Monte-Carlo simulations in order to verify our analytical formulations and estimate the uncertainties of the correlations. The IRAS particles are drawn from N-body CDM simulation particles provided to us by White (1993), which is characterized by a spatial correlation function with a power-law index $\gamma \sim 1.8$ and a correlation length of 0.066 of the size of the box. The cubic box includes 9040 particles with a periodic boundary condition. We have scaled the volume to the correlation length of $r_0 = 400 \, km \, s^{-1}$ and assigned each CDM particle $60 \mu m$ and X-ray luminosities ($L_{60} \& L_x$). Fig. 2 shows the spatial correlation function of the scaled CDM particles as open triangles compared with the power-law form used for the analytical calculations and the LMA - IRAS correlation function. We have used multiple cubes of the provided CDM space making use of the periodic boundary conditions to simulate galaxies and X-ray sources up to $v = 9000$ and $20000 \, km \, s^{-1}$ for the correlations with the 2 Jy and 0.7 Jy samples respectively. The number density of the provided CDM particles ($5.1 \times 10^{-3} \, h_{50}^{-3} \, Mpc^{-3}$) is about the same as the number density of the IRAS galaxies. We assigned each CDM particle an IRAS luminosity drawn from the 2 Jy selection function (Strauss et al. 1992a) to simulate the 2 Jy sample and used the luminosity function by Saunders et al. (1990) to simulate the 0.7 Jy sample. We have also assigned a fraction of particles X-ray luminosities as described in the following paragraphs. Then all-sky X-ray surface brightness distributions contributed by these sources and flux limited samples of IRAS galaxies were simulated and correlation coefficients were calculated in $3^\circ \times 3^\circ$ square cells for a few hundred times each model. Running a few hundred Monte-Carlo simulations with the smearing process with the A2 instrumental PSF in each run is computationally impractical. Instead, we have multiplied the $W_{xg}$ distribution simulated with the non-smeared square cells by an attenuation factor. The factor should be the weighted mean of $s_p$ and $s_c$ ($\S$ 3.1.). The attenuation factor for each case was determined by the mean of five simulations where we evaluated the $W_{xg}$ values for both non-smeared and smeared cases.

We have included components [A] (AGNs with $L_{x,44} \geq 0.01$) and [B] lower X-ray luminosity sources defined in $\S$ 3.1. of X-ray populations in the simulations. In modeling component [A] in the simulation, we used the following information: (1) the 60 $\mu m$ luminosity distribution (Saunders et al. 1990; Strauss et al. 1992a ), (2) the X-ray luminosity distribution for AGNs with $L_{x,44} \geq 0.01$ (G92; Piccinotti et al. 1982), and (3) the $L_x$ vs $L_{60}$ correlation for these AGNs (G92). The luminosity correlation (3) was built into the simulation as follows. When an X-ray luminosity $L_{x,44}(\geq 0.01)$ is assigned to a particle, the corresponding IRAS luminosity was drawn from a galaxy randomly chosen from the LMA sample galaxies which have X-ray luminosities between half and twice that of the assigned one (see Fig. 1). Component [B] is added for some models by
randomly choosing a certain fraction of CDM particles (which are not a part of [A]) and assigning them a uniform X-ray luminosity with $L_{x,44} < 0.01$ (i.e., no luminosity correlation is assumed for component [B]). We have assumed the same clustering property for the component [B] sources. We have considered several models to search for the volume emissivity which fits the observed correlations well.

The $\langle \delta I \delta N \rangle$ values have been calculated for 300 times each model, where $I$ and $N$ are the X-ray surface brightness and the IRAS galaxy surface number density of the simulated particles. These values are then divided by $\langle I_{CXB} \rangle N$, where $\langle I_{CXB} \rangle$ is the real mean CXB intensity, and corrected for the smearing effect as discussed above. The distribution of the corrected values can now be directly compared with the observed $W_{xg}$ and its bootstrap histogram. The simulation results are compared with the observations and analytical predictions in § 3.4.

3.4. Comparison with the Observations

Because the result from the 2 Jy 3500 - 8000 km s$^{-1}$ bin is least sensitive to the local large scale structures (see Table 1 for the supergalactic latitude divisions) and also because the IRAS luminosity functions from various works match very well with one another at higher luminosities (e.g. Saunders et al. 1990; Strauss et al. 1992a), the most reliable quantitative estimates for the local volume emissivity should be drawn from the 2 Jy sample 3500-8000 km s$^{-1}$ bin. This subsample has an advantage over the 0.7 Jy sample also because it has a well-defined edges in the redshift space, although statistical errors are somewhat larger.

A model showing a good fit to the observed correlation for the 2 Jy 3500 - 8000 km s$^{-1}$ bin consists of only component [A] with $\rho_{xA,38} = 4.3$ (model I). This value is within the error of the volume emissivity inferred from the luminosity function by G92. Fig. 4 compares the $W_{xg}$ histograms of model I simulation runs (300 for each; Thick Solid Lines) along with the observed values and their bootstrap histograms (Hatched). Also shown for reference is the simulated histogram of another model with both components with $\rho_{xA,38} = 4.3$ and $\rho_{xB,38} = 3.0$, where 15% of the CDM particles have $L_{x,44} = 0.0039$ as component [B] (model II; 100 runs each: Dashed lines). Table 2 compares the observed correlation (with the standard deviation of the bootstrap histogram $\sigma_{bs}$) with the median, mean, and the standard deviation ($\sigma_{mc}$) of the Monte-Carlo runs for model I. The results of the analytical calculations for model I is also shown. In any case, the analytical values are somewhat larger than the simulated mean values. The discrepancy is probably caused by the approximation in Eq. 6 ($|R_1 - R_2| \ll R_1, R_2$).

Fig. 4 and Table 2 shows that model I predicts somewhat larger correlations than observed for the 2 Jy 500-3500 km s$^{-1}$ division and the 0.7 Jy bin, but within 1 sigma errors. These two have larger weight in the nearby universe and thus are more subject to the local over/under densities and behavior of the IRAS luminosity function. In any case, Fig. 4 shows that model II is certainly rejected.
It is noteworthy that the spread of the $W_{xg}$ values from the Monte-Carlo simulations for the model I are similar to the bootstrap histogram for corresponding observations. In particular, the 500 - 3500 km s$^{-1}$ division of the 2 Jy sample shows the wider spreads of $W_{xg}$ in both the bootstrap and the Monte-Carlo histograms compared with those of the 3500 - 8000 km s$^{-1}$ division, although these two divisions contain roughly the same number of galaxies. This means that the bootstrap method is a good estimator of the main source of the spread, i.e., the shot noise due to the finite number of X-ray sources contributing to the correlation signal. The tendency that the simulation histogram has a tail at higher $W_{xg}$ values can be understood in terms of the rare occasions of a few extremely high flux sources contributing to the correlation signal.

Using the 2 Jy 3500-8000 sample vs CXB correlation as the optimum estimator, the comparison of the simulation and observed correlation leads to an estimated volume emissivity of $\rho_{x,38} = 4.3 \pm 1.2$ assuming that the IRAS galaxy vs X-ray emitter correlation can well be represented by a power law form of $\gamma = 1.8$ and $r_0 = 400$ km s$^{-1}$. This is appropriate for the X-ray AGN - IRAS galaxy correlation function (Fig. 2). The error corresponds to the 1$\sigma$ of the bootstrap runs of the observed correlation, which is approximately equal to the 1$\sigma$ of the simulated correlations. Considering that the AGN X-ray luminosity function of G92 (for $L_{x,44} \approx 0.01$) gives $\rho_{x,38} = 4.1^{+1.3}_{-1.7}$, the total volume emissivity derived from the IRAS - CXB correlation allows the local volume emissivity of the low luminosity objects of $\rho_{xB,38} \lesssim 2$.

4. Discussion

Fig. 3 shows that the clustering term contribution dominates the Poisson term (§ 3.) contribution to the correlation in our experiment. It means that our results are insensitive to the details of the $L_x - L_{60}$ correlation but sensitive to the clustering property between IRAS galaxies and contributing X-ray sources. This is complementary to the work by Carrera et al. (1994) of the similar nature using the GINGA scan data and the IRAS 0.7 Jy sample, where the Poisson term contribution is dominant. The factor of $\sim 2$ deficit compared with the power-law used in the X-ray AGN vs IRAS galaxy spatial correlation function (Fig. 2) near $r \sim 150$ km s$^{-1}$ causes $R_c$ to be reduced by 5 – 9 %, allowing only 4 – 7 % more volume emissivity for the given correlation signal.

The X-ray AGN vs IRAS galaxy correlation function may not be a good description of the clustering property between lower-luminosity X-ray sources and IRAS galaxies. If there are X-ray sources which are clustered with IRAS sources only weakly, one may squeeze more volume emissivity from these sources without violating the observed correlation signal. However, we can limit the contributions from lower X-ray luminosity galaxies because there is a reasonable range of spatial cross-correlation functions among various galaxy catalogs (e.g. Lahav, Nemiroff, & Piran 1990). For example, using the IRAS auto-correlation parameters ($\gamma \approx 1.65$, $r_0 \approx 400$ km s$^{-1}$), $R_c$ typically reduces by $\sim 15$ % allowing slightly (by about 10 %) more volume emissivity. If we use the optical correlation parameters, ($\gamma \approx 1.8$, $r_0 \approx 500$ km s$^{-1}$), $R_c$ is typically about 50 % larger, allowing less volume emissivity. Thus our result gives a strong constraint on the volume emissivity.
from the low X-ray luminosity sources such as starburst galaxies and liners under the reasonable assumption that their clustering property is not very different from that of known galaxies. The CDM model we have used in our Monte-Carlo simulations well represents $\xi_{g}(r)$ (Fig. 2) at smaller scales ($r \lesssim 1000 \, km \, s^{-1}$). However, it is known that the observed large scale power of the galaxy distribution exceeds that of the CDM model. If such large scale structures affect the correlation signal, the estimated volume emissivity should be smaller, giving a more strict upper limit to the contribution of the low luminosity objects to the local X-ray volume emissivity. A quantitative estimation of the effect may be obtained by comparing the spherical harmonic powers of the CXB surface brightness and the IRAS galaxies (c.f. Scharf et al. 1992); this may be addressed further in the future.

If the case of no evolution and the energy spectral index of 0.7 appropriate for nearby AGNs in the $\Omega_0 = 1$ universe, the effective look-back factor (Boldt 1987; Lahav et al. 1993) is $f = 0.46$. The low local volume emissivity derived with our experiment explains about $20\pm 5\%$ of the total 2 – 10 keV CXB intensity under these assumptions. If these sources with the same spectrum have undergone a luminosity evolution in 2 – 10 keV similar to that in the soft X-ray band (Boyle et al. 1993), i.e. $L_x \propto (1 + z)^{2.7}$ up to $z_{\text{max}} \sim 2$ for the $\Omega_0 = 1$ universe, the corresponding effective look-back factor is $f = 1.46$ and explains about 65% of the 2 – 10 keV CXB. If we include sources with $z > 2$, the fraction becomes larger. Although our results suggest that the contributions of low luminosity sources such as star-forming galaxies and liners to the 2 – 10 keV volume emissivity at present is small, our results do not exclude the possibility that such sources have undergone stronger evolution (luminosity and/or number) contributing significantly to the total CXB intensity in the past. Therefore it is important to investigate the correlation property between the CXB surface brightness and number counts from catalogs of higher redshift objects.

The low volume emissivity implied by this work is hardly consistent with the independent estimate of the local volume emissivity using the all-sky X-ray dipole moment and the Local Group’s peculiar motion (JLMB92; Boldt 1990), i.e. $\rho_{x,38} \approx 30(b_x \Omega_0^{-0.6})^{-1}$ predicting $b_x \Omega_0^{-0.6} \sim 7$. Using the apparent dipole saturation at $v \approx 4500 \, km \, s^{-1}$ (the dipole moment of IRAS galaxies also seems to saturate at about the same distance), Miyaji & Boldt (1990); Miyaji, Jahoda, & Boldt (1991) derived $b_x \Omega_0^{-0.6} \approx 2.6 \pm 0.5$ for X-ray selected AGNs. This kind of estimation using flux-limited catalogs, however, could be subject to an misesimation by a factor of 2 or more (e.g Strauss et al. 1992b; Peacock 1992; Lahav, Kaiser, & Hoffman 1990; Juszkiewics, Vittorio, & Wyse 1990) considering that the mass within the apparent dipole saturation does not necessarily account for all the gravitational acceleration at the local group, even though the flux dipole within that depth appears to align with the peculiar velocity. There still are uncertainties on the all-sky extragalactic X-ray dipole in the subtraction of the galactic component and structures behind the galactic plane. Apparently more study is needed to pursue this comparison further.

5. Conclusion
The zero-lag cross correlation between the Cosmic X-ray Background and the IRAS galaxy surface number density has been investigated. Two flux-limited IRAS samples, i.e. the 2 Jy sample with redshift information and the 0.7 Jy projected sample, are used to cross-correlate with the all-sky hard X-ray map from the HEAO 1 A2 experiment. The cross-correlation study gives an statistical estimation of the local volume emissivity from the faint X-ray sources which are not resolved as point sources.

We have detected the zero-lag correlation signals between the X-ray surface brightness and the IRAS galaxy counts in the 9 deg$^2$ cells of $W_{xg} \sim (3 - 11) \times 10^{-3}$ for selected IRAS samples. Both Poisson and clustering effects contribute to the correlation signal. The correlation between far infrared and X-ray luminosities of galaxies affects the Poisson contribution of the correlation. We have developed an analytical formulation relating $W_{xg}$ and the X-ray volume emissivity including these effects. We have also made Monte-Carlo simulations using the particles from a CDM simulation, which fairly represent the clustering properties of galaxies at the scales affecting our correlation signal, by assigning these particles X-ray and infrared luminosities and observing through the same cells as the real observations.

In the case of our observation, the clustering term is the dominant term of the correlation and thus the result is insensitive to the detail of the far infrared - X-ray luminosity correlation. The volume emissivity estimated from the correlation strength between the X-ray surface brightness and IRAS 2 Jy 3500 - 8000 km s$^{-1}$ sample, which is least subject to systematic errors in the IRAS luminosity function and local large scale structures, is $\rho_{x,38} = 4.3 \pm 1.2$. This can be explained by AGNs with $L_{x,44} > 0.01$ alone, whose volume emissivity can be evaluated from the X-ray luminosity function. Thus our correlation study implies that the contribution of lower luminosity sources (e.g. star forming galaxies and liners) to the local volume emissivity is not larger than that of AGNs and could be substantially smaller. The derived local volume emissivity (2 - 10 keV) explains about 20% and 65% of the CXB intensity with no evolution and a luminosity evolution (up to $z \sim 2$) similar to that of AGNs in soft X-rays (Boyle et al. 1993) respectively.

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Appendix
A. The Poisson Term

We here develop the formulation of the cross-correlation between the intensity at a wavelength and a number density of objects selected by the flux at another wavelength due to the Poisson process. Here we use the example of X-ray intensity and IRAS galaxies. For simplicity, we consider here observations through cells with a square profile and express fluxes and number counts per cell by script characters.

Suppose we have two populations of objects with $N_a$ and $N_b$ per cell. The cross correlation of counts at zero-lag square-profile cells of solid angle $\Omega$ is separated into Poisson and clustering terms:

$$\langle \delta N_a \delta N_b \rangle = \langle N_o \rangle + \text{clustering term}, \quad (A1)$$

where $N_o$ is the number of objects per cell which overlap in both populations $a$ and $b$ (Lahav 1992). Hereafter in this appendix, we only consider the Poisson term of the correlation. Suppose we are to correlate the flux from population $a$ per cell, considering only the Poisson term ($I_a$) with the number counts $N_b$, then:

$$\langle \delta I_a \delta N_b \rangle = \bar{f}_o \langle N_o \rangle, \quad (A2)$$

where $\bar{f}_o$ is the mean flux of the overlapped objects, which is, in general, different from the mean flux of the population $a$ objects.

As a simple illustration of the effect of the luminosity correlation, let us first consider the case where we have an X-ray emitting population with a luminosity function $\Phi_x(L_x)$ (normalized to the spatial number density). Let us cross-correlate the X-ray fluxes $I_x$ and number counts $N_x$ of an X-ray flux limited (at $f_{x,lim}$) sample:

First, let us consider the contribution of the objects in a thin shell at a distance $R$ from us [$R, R + \Delta R$]. The number count from the shell ($\Delta R N_x$) is:

$$\Delta R N_x = \int_{4\pi R^2 f_{x,lim}}^{\infty} dL_x \Phi_x(L_x) R^2 \Omega \Delta R, \quad (A3)$$

and the mean flux of the sources in the flux limited sample from the thin shell (corresponding to the overlapped objects in Eq. A2):

$$\bar{f}_x(R) = (\Delta R N_x)^{-1} \int_{4\pi R^2 f_{x,lim}}^{\infty} dL_x \frac{L_x}{4\pi R^2} \Phi_x(L_x) R^2 \Omega \Delta R. \quad (A4)$$

Note that this is a function of $R$. Thus the correlation $\langle \delta I_x \delta N_x \rangle$ from the sources between distances $[R_{min}, R_{max}]$ can be found by radially integrating the contributions from radial shells:

$$\sum_{\text{shells}} \bar{f}_x(R) \Delta R N_x = \int_{R_{min}}^{R_{max}} dR \left[ \frac{1}{4\pi} \int_{4\pi R^2 f_{x,lim}}^{\infty} dL_x L_x \Phi_x(L_x) \Omega \right]. \quad (A5)$$
From the Bayes theorem, $\Psi(L)$ where $P$ of $L$ bivariate function (c.f. Sodrè & Lahav 1993) in the ($L_x$, $L_\delta$) space Ψ($L_x$, $L_\delta$) is the mean X-ray luminosity weighted radial selection function, we get,

$$\langle \delta L_x \delta N_x \rangle = \frac{\rho_x \Omega}{4\pi} \int_{R_{\min}}^{R_{\max}} dR P_x(R).$$  \hfill (A7)

When we have a galaxy sample which is flux limited at a different wavelength (60$\mu$m in this case, with the luminosity function $\Phi_{60}(L_{60})$ and the limiting flux of $f_{60,\lim}$) to correlate with the X-ray surface brightness, we have to modify the expression of $P_x(R)$ accordingly. In this case, the contribution of the thin shell to $\langle \delta L_x \delta N_{60} \rangle$ is $\bar{f}_x(R)\Delta R N_{60}$, where $\bar{f}_x(R)$ is now the mean X-ray flux of the objects which are in the 60$\mu$m flux limited sample at the distance $R$, i.e. the objects with $L_{60} \geq 4\pi R^2 f_{60,\lim}$. Then, this can be expressed as:

$$\bar{f}_x(R)\Delta R N_{60} = \int_0^\infty dL_{60} \Phi_{60}(L_{60}) \int_0^\infty dL_x \frac{L_x}{4\pi R^2} p(L_x|L_{60}) R^2 \Omega \Delta R$$

$$\equiv \frac{\Omega}{4\pi} \int_0^\infty dL_{60} L_x(L_{60}) \Phi_{60}(L_{60}) \Delta R,$$  \hfill (A8)

where $p(L_x|L_{60})dL_x$ is the normalized probability that an object has an X-ray luminosity between $L_x$ and $L_x + dL_x$ given that its 60$\mu$m luminosity is $L_{60}$ and $\bar{L}_x(L_{60})$ is the mean X-ray luminosity of objects with 60$\mu$m luminosity of $L_{60}$, i.e. $\bar{L}_x(L_{60}) = \int_0^\infty dL_x L_x p(L_x|L_{60})$. As before, we can radially integrate this to find $N_{60}$ (and $N_{60}$ defined as quantities per solid angle):

$$W_{60} \langle I_x \rangle_{N_{60}} = \frac{\langle \delta I_x \delta N_{60} \rangle}{\Omega^2} = \frac{\rho_x \Omega}{4\pi} \int_{R_{\min}}^{R_{\max}} dR P_x(R)$$  \hfill (A9)

$$P_x(R) = \frac{\int_0^{\infty} dL_{60} L_x(L_{60}) \Phi_{60}(L_{60})}{\int_0^{\infty} dL_{60} \bar{L}_x(L_{60}) \Phi_{60}(L_{60})},$$  \hfill (A10)

with a note that the denominator of $P_x(R)$ is equal to $\rho_x$. This is the square profile cell case of Eqs. 3 - 4b.

As a preparation to practically evaluate $P_x(R)$ from available information, let us consider the bivariate function (c.f. Sodrè & Lahav 1993) in the ($L_x$, $L_{60}$) space $\Psi(L_x, L_{60})dL_x dL_{60}$, defined as the mean space density of galaxies within the luminosity-luminosity space element $dL_x dL_{60}$. From the Bayes theorem, $\Psi(L_x, L_{60}) = p(L_x|L_{60})\Phi_{60}(L_{60})$. Then the numerator of the expression of $P_x(R)$ in Eq. A10 is:

$$\int_0^{\infty} dL_{60} L_x(L_{60}) \Phi_{60}(L_{60}) = \int_0^{\infty} dL_{60} \int_0^{\infty} dL_x L_x \Psi(L_x, L_{60}).$$  \hfill (A11)
Using the available sample of X-ray flux selected galaxies with measured redshifts and 60\(\mu\)m fluxes (the LMA sample \[G92\] in this work), the bivariate function \(\Psi(L_x, L_{60})\) can be constructed above some minimum X-ray luminosity \(L_{x,\text{min}}\) (defined by the sample). Neglecting the clustering effect, the bivariate function can be estimated by plotting the objects in the sample \((L_{x,i}, L_{60,i})\) on the luminosity- luminosity plane weighted by \(V_{\text{max}}(L_{x,i})^{-1}\), where \(V_{\text{max}}(L_x)\) is the maximum volume of space where an object with \(L_x\) would be in the sample. Dividing by \(V_{\text{max}}(L_x)\) compensates for the effect of using the X-ray flux-limited sample and also gives the proper normalization. Then the double integral (lower integration limit for \(L_x\) is now \(L_{x,\text{min}}\) instead of zero) in Eq. A11 can then be expressed by the sum over the sample with \(L_{60} \geq 4\pi R^2 f_{60,\text{lim}}\):

\[
\int_{L_{x,\text{min}}}^{\infty} dL_x \int_{L_{x,\text{min}}}^{\infty} dL_{60} L_x \Psi(L_x, L_{60}) \sim \sum_{L_{60,i} \geq 4\pi R^2 f_{60,\text{lim}}} \frac{L_{x,i}}{V_{\text{max}}(L_{x,i})}.
\]

(A12)

This is Eq. 7 in § 3.1. Because of the lower X-ray luminosity limit from the available sample, we emphasize that the above expression actually represents the portion of \(\rho_x P_x(R)\) contributed by the sources with \(L_x \geq L_{x,\text{min}}\), expressed in terms of \(\rho_{xpA}, P_{xA}(R)\), and \(R_{pA}\) in the main text.

B. Effect of the Non-Square Profile of the Effective Cell on the Poisson Term

We use the same notations as in § 3.1. in the main text and appendix A unless otherwise noted. In many cases (including this work), we measure the X-ray intensities in observing cells with a size comparable to the instrumental point spread function (PSF). In our case, the IRAS galaxy distributions have also been smeared with the same PSF to perform the correlation. In that case, we have to consider the effective cell profile \(B_{ec}(\hat{R} - \hat{R}_0)\), which is the convolution of the PSF \(B_{psf}(\hat{R} - \hat{R}_0)\) with the square cell profile \(B_{sq}(\hat{R} - \hat{R}_0)\)(=1 in the cell; =0 outside of the cell),

\[
B_{ec}(\hat{R} - \hat{R}_0) \propto \int d\Omega_1 B_{psf}(\hat{R} - \hat{R}_1) B_{sq}(\hat{R}_1 - \hat{R}_0),
\]

(B13)
normalized to unity at the center.

Now let us consider the quantities observed through this effective cell (expressing by script characters as before). Then, noting that \(N\) and \(I\) (surface number density and surface brightness) are the functions of the sky position,

\[
\langle N \rangle = \int d\Omega N(\hat{R}) B_{ec}(\hat{R} - \hat{R}_0) = \langle N \rangle \Omega_{ec},
\]

(B14a)

\[
\langle I \rangle = \int d\Omega I(\hat{R}) B_{ec}(\hat{R} - \hat{R}_0) = \langle I \rangle \Omega_{ec}
\]

(B14b)

In the Poisson term of the correlation, since both flux and surface number density of the overlapped objects (see Eq. A2) should be weighted by the profile, noting that \(N_o(\hat{R})\) is the sum of randomly placed \(\langle N_o \rangle\) delta functions per solid angle:

\[
\langle \delta I \delta N \rangle = \int d\Omega \delta_o N_o(\hat{R}) B_{ec}^2(\hat{R} - \hat{R}_0) = f_x \langle N \rangle \int d\Omega B_{ec}^2(\hat{R} - \hat{R}_0).
\]

(B15)
Therefore, reading \( f_o \) as \( f_x \) and \( N_o \) as \( N \),

\[
W_{xg}(I)(N) = \frac{\int d\Omega B_{cc}^2(\hat{R} - \hat{R}_0)}{\Omega_{cc}^2} f_x(N).
\] (B16)

Thus the PSF smearing effect, or more in general, the case of non-square profile of effective cells, can be taken into account by replacing \( \frac{1}{\Omega} \) in Eq. A9 by \( \int d\Omega B_{cc}^2(\hat{R} - \hat{R}_0)\Omega_{cc}^{-2} \) (Eq. 3).

C. The Clustering Term with a Power-Law Correlation Function

In the case of a power law spatial correlation function \( \xi_{xg} = (r/r_0)^{-\gamma} \), the clustering term \( \hat{\eta}_c \) (Eq. 5) can be expressed in a more convenient form under the approximation that the clustering scale length is much smaller than the distance to the objects (\( |\mathbf{R}_1 - \mathbf{R}_2| \ll R_1, R_2 \)). Under this approximation, changing the variables to \( u = R_1 - R_2, x = \frac{R_1 + R_2}{2} \), then \( R_1 \approx x \) and \( r \equiv |\mathbf{R}_1 - \mathbf{R}_2| \approx (u^2 + x^2\theta^2)^{\frac{1}{2}} \), where \( \theta \) is the angle between \( \mathbf{R}_1 \) and \( \mathbf{R}_2 \). Then Eq. 5 can be rewritten as:

\[
\hat{\eta}_c \approx \langle n \rangle \rho \eta^{1-\gamma} \int dx x^2 P(x) \int d\Omega_1 d\Omega_2 B_{cc}(\hat{R}_1 - \hat{R}_0) B_{cc}(\hat{R}_2 - \hat{R}_0) \int_{-\infty}^{\infty} du (u^2 + x^2\theta^2)^{-\frac{\gamma}{2}}. \] (C17)

The integration in \( u \) can be calculated (Peebles 1980, Eq. 52.9):

\[
\int_{-\infty}^{\infty} du (u^2 + x^2\theta^2)^{-\frac{\gamma}{2}} = H_\gamma (x\theta)^{1-\gamma}, \quad H_\gamma = \frac{\Gamma\left(\frac{\gamma}{2}\right) \Gamma\left(\frac{\gamma-1}{2}\right)}{\Gamma\left(\frac{\gamma}{2}\right)}.
\] (C18)

For example, \( H_{1.8} = 3.68 \) and \( H_{1.65} = 4.29 \).

The double integral over the solid angles:

\[
X = \int d\Omega_1 d\Omega_2 \theta^{1-\gamma} B_{cc}(\hat{R}_1 - \hat{R}_0) B_{cc}(\hat{R}_2 - \hat{R}_0)
\] (C19)

is now separated from the radial integrations. This can be integrated numerically by the Monte-Carlo method. For the \( a \times a \) square cell case instead of the real effective cell, this double integral is expressed as \( C_\gamma a^{5-\gamma} \) with \( C_{1.8} = 2.25 \) and \( C_{1.65} = 1.87 \) (Totsuji & Kihara 1969). Then defining \( A_\gamma = H_\gamma C_\gamma \) immediately gives Eq.6. For our convolved effective cell, \( X \) is \( \approx 99 \ deg^3 \) and \( \approx 102 \ deg^3 \) corresponding to \( s_c = 0.74 \) and \( 0.77 \) (second raw of Eq. 5) for \( \gamma = 1.8 \) and 1.65 respectively.
Table 1: The IRAS Galaxy – CXB Correlation: Results

| IRAS Sample | $v_{\text{min}}$ [km s$^{-1}$] | $v_{\text{max}}$ [deg] | $SGB$ | $N_{\text{cells}}$ | $\langle I \rangle$ | $\langle N \rangle$ [deg$^{-2}$] | $W_{\text{eg}}$ [deg$^{-2}$] |
|-------------|-------------------------------|-------------------------|-------|-------------------|----------------|--------------------------|-----------------|
| 2 Jy        | 500                           | 8000                    | ...   | 2328              | 0.71           | $5.4 \times 10^{-2}$     | $(7.6 \pm 1.6) \times 10^{-3}$ |
| 2 Jy        | 500                           | 8000                    | $\geq 20^\circ$ | 1413              | 0.71           | $4.5 \times 10^{-2}$     | $(5.6 \pm 1.9) \times 10^{-3}$ |
| 2 Jy        | 500                           | 8000                    | $< 20^\circ$ | 915               | 0.71           | $6.7 \times 10^{-2}$     | $(9.3 \pm 2.6) \times 10^{-3}$ |
| 2 Jy$^a$    | 500                           | 8000                    | ...   | 2275              | 0.71           | $5.2 \times 10^{-2}$     | $(4.1 \pm 1.2) \times 10^{-3}$ |
| 2 Jy        | 500                           | 3500                    | ...   | 2328              | 0.71           | $2.8 \times 10^{-2}$     | $(8.6 \pm 2.8) \times 10^{-3}$ |
| 2 Jy        | 500                           | 3500                    | $\geq 20^\circ$ | 1413              | 0.71           | $2.2 \times 10^{-2}$     | $(4.3 \pm 2.8) \times 10^{-3}$ |
| 2 Jy        | 500                           | 3500                    | $< 20^\circ$ | 915               | 0.71           | $3.8 \times 10^{-2}$     | $(11.3 \pm 4.3) \times 10^{-3}$ |
| 2 Jy        | 3500                          | 8000                    | ...   | 2328              | 0.71           | $2.5 \times 10^{-2}$     | $(6.3 \pm 1.7) \times 10^{-3}$ |
| 2 Jy        | 3500                          | 8000                    | $\geq 20^\circ$ | 1413              | 0.71           | $2.3 \times 10^{-2}$     | $(5.3 \pm 2.0) \times 10^{-3}$ |
| 2 Jy        | 3500                          | 8000                    | $< 20^\circ$ | 915               | 0.71           | $2.9 \times 10^{-2}$     | $(6.9 \pm 2.8) \times 10^{-3}$ |
| 0.7 Jy      | ...                           | ...                     | 2379  | 0.71              | $2.7 \times 10^{-1}$ | $(3.6 \pm 0.7) \times 10^{-3}$ |
| 0.7 Jy      | ...                           | $\geq 20^\circ$         | 1453  | 0.71              | $2.6 \times 10^{-1}$ | $(2.8 \pm 0.8) \times 10^{-3}$ |
| 0.7 Jy      | ...                           | $< 20^\circ$            | 926   | 0.71              | $2.9 \times 10^{-1}$ | $(4.5 \pm 1.0) \times 10^{-3}$ |

$^a$All AGNs in Piccinotti et al. (1982) are excluded.
Table 2: Comparison of Observations with Model I ($\rho_{x,38} = 4.3$)

| IRAS Sample | $R_{p,A}^a$ [km s$^{-1}$] | $R_{c}^a$ [km s$^{-1}$] | observed | $W_{xg} \times 10^3$ | nominal | $\sigma_{bs}^b$ | median | mean | $\sigma_{mc}^c$ | analytical |
|-------------|--------------------------|--------------------------|----------|----------------------|---------|----------------|---------|------|----------------|------------|
| 2 Jy        | 500-3500                 | 2390                     | 560      | 8.6                  | 2.8      | 10.7           | 13.0    | 6.1  | 17.0           |            |
| 2 Jy        | 3500-8000                | 980                      | 1100     | 6.3                  | 1.7      | 6.2            | 6.4     | 1.9  | 7.9            |            |
| 0.7 Jy      | ...                      | 6700                     | 11000    | 3.6                  | 0.7      | 4.7            | 5.0     | 1.5  | 6.5            |            |

$^a$Effective depths defined in §§ 3.1. & 3.2.

$^b$The standard deviation of $W_{xg}$ for the bootstrap runs.

$^c$The standard deviation of $W_{xg}$ for the Monte-Carlo runs.
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Fig. 1.— The \( L_x \) vs \( L_{60} \) Plot of X-ray AGNs
The IRAS 60 \( \mu m \) luminosity is plotted against the 2 – 10 keV X-ray luminosity for the LMA sample AGNs (G92) with \( v \leq 20000 \ km\ s^{-1} \). The 60 \( \mu m \) luminosity is in the solar unit and the X-ray luminosity is in \( 10^{44} h_{50}^{-2} \ erg\ s^{-1} \).

Fig. 2.— The X-ray AGN - IRAS galaxy Correlation Function
The spatial correlation function \( \xi_{xg}(r) \) between LMA AGNs (G92) and IRAS 2 Jy galaxies is shown by filled hexagons with error bars. The power-law function with \( \gamma = 1.8, \ r_0 = 400 \ km\ s^{-1} \) is shown by a dashed line. Open triangles show the spatial correlation of the CDM particles (§ 3.3.) rescaled to the correlation length of \( r_0 = 400 \ km\ s^{-1} \).

Fig. 3.— Comparison of Radial Selection Functions.
The selection functions multiplied by the PSF smearing factors are drawn for (a) the 2 Jy sample, and (b) the 0.7 Jy sample. These curves show the relative contributions of the clustering and Poisson terms as functions of \( R \). Solid: \( s_p P(R) \), the IRAS 2 Jy and 0.7 Jy selection functions multiplied by the Poisson smearing factor for reference; Dashed: \( s_c P_c(R) \), the effective radial selection function for the clustering term defined in Eq. 6 multiplied by the clustering smearing factor, and Dot-dashed: \( s_p P_x(R) \), the X-ray selection function of the 2 Jy and 0.7 Jy samples evaluated using Eqs. 4b & 7 multiplied by the Poisson smearing factor. The wiggle in the \( P_x(R) \) curve is caused by the discreetness of the summation in Eq. 7.

Fig. 4.— The Observed and Simulated Correlation Coefficients
The observed correlation coefficients (arrows with the bootstrap histograms) are compared with the Monte-Carlo simulations for two models (model I: component [A] only \( \rho_{x,38} = 4.3 \) and model II: including components [A] and [B] \( \rho_{x,38} = 7.3 \). The IRAS samples used are: (a) the 2Jy sample with \( 500 \leq V[km\ s^{-1}] < 3500 \), (b) the 2 Jy sample with \( 500 \leq V[km\ s^{-1}] < 3500 \), and (c) the 0.7 Jy sample. The histograms in each figure are normalized to have the same area.