Reflection and Transmission of Sound Waves through the Layer of Two-fractional Bubbly Liquid

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Abstract. The mathematical model that determines reflection and transmission of sound waves through a medium containing two-fractional bubbly liquid is presented. For the model of media with three layer the wave reflection and transmission coefficients are calculated. The influence of the volume content of bubbles on the investigated coefficients is shown.

1. Introduction

The investigations of wave dynamics and acoustics of disperse media are of significant interest. Currently, the basics of mechanics and thermophysics of bubbly liquids, as well as the most significant results in the study of wave processes in these environments are presented in monographs [1-5]. Hao and Prosperetti [6] obtained the results of study of the vapor bubble dynamics in liquid. Commander and Prosperetti [7] presented a model of the propagation of plane waves with small amplitude in a mixture of liquid and of gas bubbles. This model is adequate for the frequencies far from resonance at the volume content of a disperse phase of 1-2%. Nigmatulin et al. [8] showed the need of allowing for the effect of liquid compressibility for problems of acoustics of bubbly liquids. Shagapov [9] considered a problem of propagation of small disturbances in liquids with polydisperse bubbles. Nigmatulin et al. [10] studied the small radial oscillations of vapor-gas bubbles in liquid under the influence of an acoustic field. They showed that capillary effects and phase transitions lead to the new resonant frequency of small bubbles. This frequency is different from the Minnaert frequency.

The propagation of acoustic waves in a two-fractional mixture of liquid with bubbles is investigated in [11-13]. A mathematical model of the propagation of sound waves in the two-fractional mixture of liquid with polydisperse vapor-gas and gas bubbles with phase transitions is shown in [14-16]. The propagation of acoustic waves in multifractional mixtures of liquid with gas and vapor-gas bubbles of different sizes and different compositions is studied in [17-20].

In this study the sound wave reflection and transmission through a multilayer object containing a two-fractional bubbly fluid layer is investigated.

2. Dispersion relation

Linearized equations for one-dimensional disturbances in a two-fractional mixture of liquid with bubbles are obtained from the general equations of motion for bubbly mixtures [1]. The dispersed
phase consists of two fractions having various gases in bubbles and different in the bubbles radii. The total bubble volume concentration is small. Solving this system this dispersion relation is obtained:

\[
\left(\frac{K_s}{\omega}\right)^2 = \frac{a_{10}^2}{C_1^2} + \frac{a_{20}^2\rho_{10}}{p_0 N_k^2} \left(1 + \frac{H_1^n + H_2^n + H_3^n x_1}{M_1^2 x_1 / b_1^n}\right) + \frac{a_{20}^n\rho_{10}}{p_0 N_k^2} \left(1 + \frac{H_1^n + H_2^n + H_1^n x_1}{M_1^2 x_1 / b_1^n}\right)
\]

(1)

Dispersion relation (1) (i.e. the function of the complex wave number \(K_s\) on the frequency \(\omega\)) determines a propagation of sound waves in the two-fractional mixtures of liquid with vapor-gas bubbles of different gases (different initial radii, different initial volume contents and different thermal properties of fractions) with the interphase diffusion mass transfer.

3. Mathematical model

In analysing the interaction between an acoustic signal and a multilayer object (Fig. 1), the following method of calculations is used. According to [2], the result of the reflection and transmission of a plane monochromatic wave \(\exp(ikx + \omega t)\) from a multilayer object is the plane waves \(R \exp(ikx + \omega t)\) and \(T \exp(ikx - \omega t)\), where \(R\) and \(T\) are the wave reflection and transmission coefficients, respectively, determined in terms of the layer impedances \(Z_i\) and the entry impedances of the layer boundaries \(Z_{in}\). For a three-layer medium the coefficients \(R\) and \(T\) are as follows:

\[
R = \frac{Z_2(Z_2 - Z_3) - i(Z_3^2 - Z_2 Z_3)\tan(K_d d_2)}{Z_2(Z_2 + Z_3) - i(Z_3^2 + Z_2 Z_3)\tan(K_d d_2)}
\]

\[
T = \prod_{j=1}^{2} \left(\frac{Z_j^n + Z_j}{Z_j^n + Z_{j+1}}\right)^{\exp[iK_j d_j]}
\]

(2)

Here, \(d_2\) is the thickness of the layer with bubbles, \(K\) is the wave number, \(\omega\) is the perturbation frequency, and \(\rho\) is the layer density. For a homogeneous layer, the wave number is determined as \(K_j = \omega/C_j\), where \(C_j\) is the speed of sound in the \(j\)-th layer.

4. Results

We will consider the acoustic signal transmission through the three-layer medium, namely, water-water with bubbles-water. Disperse phase of bubbly liquid contain vapor-air bubbles and bubbles of helium with water vapor. Let the bubble layer thickness be \(d_2 = 5\) mm, the bubble radii – 2 mm (vapor-air bubbles), 1 mm (bubbles of helium with water vapor), \(f_0 = 1630\) Hz (resonance frequency of 2 mm bubbles) and the volume content \(a_{20} = 0.01\). The mixture pressure \(p_0 = 0.1\) MPa and the temperature \(T_0 = 288\) K. The calculations were performed according equations (1), (2). The dependences of the coefficients of the wave reflection and transmission through the given bubble layer on the
dimensionless disturbance frequency are presented in the figure 2. Clearly that a minimum of the transmission coefficient and a maximum of the reflection coefficient are observable in the region of the resonance frequency of the bubbles. This means that at the given frequency the bubble layer almost completely reflects the incident acoustic wave. In addition, two fractions with different initial radii of the disperse phase leads to the appearance of two local minima and maxima on this coefficients. This is due to the difference in the values of the resonance frequencies of the intrinsic vibrations of bubbles of each fraction.

On the figures 3, 4 the influence of the volume content of bubbles in bubble layer on the sound wave transmission and reflection is illustrated. Parameters of bubbly liquid are the same as on figure 2, curves 1 – \( \alpha_0 = 0.0025 \), curves 2 – \( \alpha_0 = 0.005 \), curves 3 – \( \alpha_0 = 0.01 \). An increasing the volume content of bubbles leads to a decrease in the transmission coefficient and, correspondingly, to an increase in the reflection coefficient on the entire frequency range. The increasing the volume content of bubbles, the range near the resonance frequency, where the wave transmission coefficient takes near-zero values (opposite effect on the reflection coefficient), also expands.
Figure 4. Coefficients of sound transmission through a bubbly screen at different volume content.

References

[1] Nigmatulin R I 1991 *Dynamics of Multiphase Media* (New York: Hemisphere)
[2] Temkin S 2005 *Suspension Acoustics: An Introduction to the Physics of Suspensions* (New York: Cambridge University Press)
[3] Leighton T G 1994 *The acoustic bubble* (London: Academic Press)
[4] Nakoryakov V E, Pokusaev B G and Shreiber I R 1993 *Wave Propagation in Gas-Liquid Media* (New York: CRC Press)
[5] Gubaidullin D A 1998 *Dynamics of two-phase gas-vapor-droplet media* (Kazan: Kazan mathematical society)
[6] Hao Y and Prosperetti A 1999 *Physics of Fluids* 11 2008
[7] Commander K W and Prosperetti A 1989 *Journal of the Acoustical Society of America* 85 732
[8] Nigmatulin R I, Shagapov V Sh and Vakhitova N K 1989 *USSR Academy of Sciences Doklady* 304 1077
[9] Shagapov V Sh 1977 *Journal of Applied Mechanics and Technical Physics* 18 77
[10] Nigmatulin R I, Khabeev N S and Nagiev F B 1981 *International Journal Heat and Mass Transfer* 24 1033
[11] Gubaidullin D A, Nikifirov A A and Gafiyatov R N 2012 *High Temperature* 50 250
[12] Gafiyatov R N, Gubaidullin D A and Nikiforov A A 2013 *Fluid Dynamics* 48 366
[13] Gubaidullin D A and Gafiyatov R N 2014 *J. Phys.: Conf. Ser.* 567 012020
[14] Gubaidullin D A, Gubaidullina D D and Fedorov Yu V 2014 *J. Phys.: Conf. Ser.* 567 012011
[15] Gubaidullin D A and Fedorov Yu V 2016 *Acoustical Physics* 62 179
[16] Gubaidullin D A, Gubaidullina D D and Fedorov Yu V 2016 *J. Phys.: Conf. Ser.* 669 012011
[17] Gubaidullin D A, Nikiforov A A and Gafiyatov R N 2015 *High Temperature* 53 240
[18] Gubaidullin D A, Nikiforov A A and Gafiyatov R N 2016 *J. Phys.: Conf. Ser.* 669 012019
[19] Gubaidullin D A and Gafiyatov R N 2017 *J. Phys.: Conf. Ser.* 789 012019
[20] Gubaidullin D A and Gafiyatov R N 2017 *Journal of Engineering Physics and Thermophysics* 90 1433
[21] Brekhovskikh L M and Godin O A 1989 *Acoustics of layered media* (Moscow: Nauka)