Dynamical Analysis of Fractional-order Permanent Magnet Synchronous Motor Based on Current Time-delayed Feedback

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Abstract: This paper investigates the nonlinear dynamics of a fractional-order PMSM depends on current time-delayed feedback. Firstly, model parameters of fractional-order PMSM are selected to display bifurcation and chaos in the case of no feedback. Secondly, the stability of equilibrium points and emergence of Hopf bifurcation in the system with feedback gain and time delay are derived. It is found that a smaller fractional-order can enhance the stability of fractional-order PMSM if all parameters are fixed in some cases. In addition, the research indicates that time delay can vary the stability interval, the properties of stability and Hopf bifurcation show chaos vanishes as the time delay reaches a certain value. Finally, numerical simulations are provided to illustrate the theoretical results and demonstrate the complex dynamic behaviors.

1. Introduction

Recently, fractional calculus has proved to be a useful tool for describing long memory, many systems with memory properties or complex materials can be described by fractional derivative [1], such as heat transfer process, diffusion process and the influence of frequency in induction motor. As fractional-order systems can be implemented by electrical circuits composed of resistors, fractional calculus has become a mathematical tool widely used by researchers in many fields of technology and theory [2], [3], it attracts the attention of many engineers.

Permanent Magnet Synchronous Motor (PMSM) is increasing used in aerospace, robotics, traction, technology and so on. The stability and secure operation of PMSM are the basic requirement of automatic control system and have been received considerable attention [4]. However, the PMSM has highly nonlinear dynamics phenomenon even under certain conditions, which is extremely sensitive to the change of parameters. Therefore, it is necessary to understand the bifurcation and chaotic behaviors of PMSM. Over the past few years, many large-scale complex dynamic behaviors have been found to have an impact on stability [5]. To study the dynamic behaviors of PMSM, a model without introducing any parameters artificially is proposed [6]. Many articles have shown that the PMSM may produce bifurcation, limit cycles, and chaos oscillation [7]. In recent articles, it also has been proved that fractional order system on modeling the PMSM can capture much better effect and robustness [8]. The study in this field is just starting.

However, the previous studies do not include current time-delayed feedback of fractional-order PMSM. In fact, time delay is a phenomenon of control system [9], and it exists widely and can not be avoided. Furthermore, time-delayed feedback has been widely used as a powerful tool for chaotic
systems [10]. Thus, it is meaningful to consider the effect of time-delayed feedback. For fractional-order PMSM with time-delay feedback, the objective of this article is to study how its dynamic behaviors. For dynamic behaviors of a nonlinear system, it is deemed to be an extremely efficient approach by using Hopf bifurcation analysis [11]. It has great significance to the study of many practical systems. Compared with other articles, the main contributions of out work are given as follows: (i) The fractional-order PMSM model with current time-delayed feedback is proposed. (ii) By employing the time delay of system as bifurcation parameter, the situations of Hopf bifurcation are derived. (iii) By taking several methods, we investigate nonlinear dynamics of fractional-order PMSM in the case of different feedback gains and time delay.

2. Preliminaries and model of fractional-order PMSM

2.1. The Caputo Fractional-order definition

**Definition 1.** The Caputo fractional derivative for \( f(t) \) is:

\[
D^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau,
\]

where \( \Gamma(\cdot) \) represents Gamma function, \( m \) satisfies \( m-1 \leq \alpha < m \), \( \alpha \) is the order.

2.2. Stability Theory of Fractional-order Systems

**Lemma 1.** [12] Consider the following fractional-order nonlinear system with time delay \( D^\alpha x(t) = f(x_1(t), \cdots, x_n(t), x_1(t-\tau), \cdots, x_n(t-\tau)) \) where \( 0 < \alpha \leq 1 \). The equilibrium point \( X^* = (x_1^*, \cdots, x_n^*) \) of system is asymptotically stable if all the roots \( \lambda \) of characteristic equation \(|J + e^{-\lambda\tau}J_r - \lambda J_f| = 0\) have negative real parts, where \( J = \frac{\partial f_i}{\partial x_j} \) and \( J_r = \frac{\partial f_i}{\partial x_j} \), \( i, j = 1, \cdots, n \).

**Lemma 2.** Let \( A \in \mathbb{R}^{n \times n} \), the fractional-order system \( D^\alpha x(t) = Ax(t) \) is asymptotically stable if \(|\arg(\lambda_i(A))| > \pi\alpha/2\).

2.3. Model of Fractional-order PMSM

In Ref. [14], fractional-order PMSM is given as

\[
\begin{align*}
\frac{d^{\alpha_1} i_d}{dt^{\alpha_1}} &= -i_d + wi_q \\
\frac{d^{\alpha_2} i_q}{dt^{\alpha_2}} &= -i_q - wi_d + \gamma w \\
\frac{d^{\alpha_3} w}{dt^{\alpha_3}} &= \sigma(i_q - w)
\end{align*}
\]

(1)

where \( \alpha_i \in (0,1] \) (i = 1,2,3) is the fractional order, \( i_d, i_q \) and \( w \) are system states.

2.4. Model of Fractional-order PMSM with Current Time-Delayed Feedback

Now, consider the fractional-order PMSM system with current time-delayed feedback as follows

\[
\begin{align*}
\frac{d^{\alpha_1} i_d}{dt^{\alpha_1}} &= -i_d + wi_q + K(i_d(t-\tau) - i_d(t)) \\
\frac{d^{\alpha_2} i_q}{dt^{\alpha_2}} &= -i_q - wi_d + \gamma w + K(i_q(t-\tau) - i_q(t)) \\
\frac{d^{\alpha_3} w}{dt^{\alpha_3}} &= \sigma(i_q - w)
\end{align*}
\]

(2)

where \( \tau \) is the time delay, \( K \) is the feedback gain. System (2) reduces to the original PMSM model (1) when time delay parameter \( \tau = 0 \). When \( K \neq 0 \) and \( \tau \neq 0 \), \( K(i_d(t-\tau) - i_d(t)) \) and \( K(i_q(t-\tau) - i_q(t)) \) can be regarded as the control signal of the system (2). This paper mainly discusses the case of \( K \geq 0 \) and \( \tau \geq 0 \). The variation range of fractional-order is \( 0 < \alpha \leq 1 \) if there is no special instruction.
3. Nonlinear dynamics of fractional-order PMSM

The three balances of system (2) can be obtained: $X_1^* = (0,0,0), X_2^* = (\gamma - 1, \sqrt{\gamma - 1}, \sqrt{\gamma - 1}), X_3^* = (\gamma - 1, -\sqrt{\gamma - 1}, -\sqrt{\gamma - 1})$.

For system (8), assume that $X(t) = x(t) - x^*$, where $x = (i_{d}, i_{q}, w)$, setting a disturbance near the equilibrium point around $X^*$ by a time dependent function $e^{\omega t}$, then we have $D^a X = JX + J_\tau X_\tau$, where $X$ and $X_\tau$ are column vectors $(i_d, i_q, w)$ at $t$ and $t - \tau$, respectively, and

$$J = \begin{bmatrix} -1 - K & w^0 & 0 \\ -w^0 & -1 - K & \gamma - i_d^0 \\ 0 & -\sigma & -\sigma \end{bmatrix} \quad \text{and} \quad J_\tau = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Now using the theory of Lemma 1, the stability bounds can be got. The characteristic eigenvalue equation as follows

$$\lambda^3 + (\lambda^2 + \sigma)(\lambda + \omega)^2 + w_0^2) + \sigma(i_0 w_0^0 + (\lambda^2 + \omega)(i_0^0 - \gamma)) = 0 \quad (3)$$

Eq.(3) is transcendental equation when $\tau \neq 0$, rewrite the equation as

$$P_1(\lambda) + P_2(\lambda)e^{-\lambda \tau} + P_3(\lambda)e^{-2\lambda \tau} = 0 \quad (4)$$

where

$$P_1(\lambda) = (\lambda^2 + \sigma)((\lambda + 1 + K)^2 + w_0^2) + \sigma(i_0^0 w_0^0 + \sigma(i_0^0 \gamma)(\lambda + 1 + K)), P_2(\lambda) = 2K(\lambda^2 + \sigma)(\lambda + 1 + K) - \sigma K(i_0^0 \gamma), P_3(\lambda) = K^2(\lambda^2 + \sigma).$$

Denoting $P_n(\lambda) = A_n + iB_n (n = 1,2,3)$, where $A_n$ and $B_n$ denote the real and imaginary parts of $P_n(\lambda)$, then Eq.(4) is equivalent to the following equation

$$A_1 + iB_1 + (A_2 + iB_2)e^{\omega t} + (A_3 + iB_3)e^{-2\omega t} = 0 \quad (5)$$

Suppose that $\lambda = \omega i = \omega(cos \frac{\omega}{2} + isin \frac{\omega}{2}), \omega > 0$ is a root of Eq.(5). Separating real and imaginary parts gives

$$A_1 + A_2cos\omega t + B_2sin\omega t = -A_3cos2\omega t - B_3sin2\omega t \quad (6)$$

$$B_1 - A_2sin\omega t + B_3cos\omega t = A_3sin2\omega t - B_3cos2\omega t \quad (6)$$

It follows from Eq.(6) that

$$2A_1 A_2 + 2B_1 B_2)cos\omega t + (2A_1 B_1 - 2A_2 B_2)sin2\omega t = A_3^2 + B_3^2 - A_2^2 - B_2^2 - A_0^2 - B_0^2 - A_0^2 - B_0^2 \quad (7)$$

According to $sin2\omega t = \pm \sqrt{1 - \cos^2 \omega t}$, it is easy to obtain the expression of $cos\omega t$, the following equation hold

$$(m^2 + n^2)cos^2 \omega t - 2mncos \omega t + r^2 = 0 \quad (8)$$

where $m = 2A_1 A_2 + 2B_1 B_2, n = 2A_1 B_1 - 2A_2 B_2, r = A_3^2 + B_3^2 - A_2^2 - B_2^2 - A_0^2 - B_0^2$. Obviously, if $m^2 + n^2 > r^2$, then Eq.(8) has no roots. Assume that Eq.(8) has roots, suppose that $cos \omega t = f(\omega)$, the solution of $cos \omega t$ can be got from Eq.(7). Based on $cos \omega t = f(\omega)$, it can be derived that

$$\tau_j = \frac{1}{\omega} [arcos(f(\omega)) + 2j\pi], \quad j = 0, 1, 2 \cdots \quad (9)$$

To investigate more detailed dynamic behaviors, two cases should be considered, $\tau = 0$ and $\tau \neq 0$.

Now, the dynamics motion of system (2) is explored when $\tau = 0$.

**Lemma 3.** Assume that the fractional orders of system (2) are $\alpha_1$ and $\alpha_2$ separately, the stability regions of system with $\alpha_1$ are greater than that of $\alpha_2$ if $\alpha_1 < \alpha_2$ when $\tau = 0$.

**Proof.** According to Lemma 1, the stability of nonlinear system can be determined by Eq.(6). For Eq.(10), one has the following equation when $\tau = 0$

$$det \begin{bmatrix} \lambda^2 + 1 & -w_0^0 & -i_d^0 \\ w_0^0 & \lambda^2 + 1 & i_d^0 - \gamma \\ 0 & -\sigma & \lambda^2 + \sigma \end{bmatrix} = 0 \quad (10)$$

Apparently, it can be rewritten as $|J - \lambda^a I| = 0$, where $J$ is the linearization matrix. The condition of stability is that all the roots have negative real parts, which is equivalent to argument of $\lambda_{min}$ greater than $\pi \alpha / 2$. It can be deduced that if the orders are smaller, the stability regions are greater. This completes the proof.
Remark 1. In fact, with the fractional order decreasing, it doesn’t mean Re(λ) is decreasing too. It only changes the argument. And if \( τ \neq 0 \), characteristic equation is transcendental equation, the influence of the orders change on the system is more complex.

To discuss the stability of system (2) when \( τ = 0 \), the following hypothesis is made

\[
\begin{align*}
(H_1) & \quad \sigma > -2 \\
(H_2) & \quad 2 + 2w^0 + 4\sigma + \sigma_i^0 - \sigma_\gamma + 2\sigma^2 + \sigma^2_i - \sigma^2_\gamma > 0 \\
(H_3) & \quad \sigma(1 + w^0 + \iota_i^0 w_0 + \iota_i^0 - \gamma) > 0
\end{align*}
\]

Theorem 1. For fractional-order system (2), if \((H_1), (H_2)\) and \((H_3)\) hold, system is asymptotically stable when \( τ = 0 \).

Proof. When \( τ = 0 \), it follows from Eq.(3) that

\[
f(\lambda) = \lambda^{3\sigma} + (2 + \sigma)\lambda^{2\sigma} + (1 + w^0 + 2\sigma + \sigma_i^0 - \gamma)\lambda^{\sigma} + \sigma(1 + w^0 + \iota_i^0 + \iota_i^0 - \gamma) = 0 \quad (11)
\]

If \( \alpha = 1 \), then Eq.(11) can be rewritten as

\[
f(\lambda) = \lambda^2 + (2 + \sigma)\lambda^2 + (1 + w^0 + 2\sigma + \sigma_i^0 - \gamma)\lambda + \sigma(1 + w^0 + \iota_i^0 + \iota_i^0 - \gamma) = 0 \quad (12)
\]

By using Routh-Hurwitz, if \( \sigma > 2 \), \( 2 + 2w^0 + 4\sigma + \sigma_i^0 - \gamma > 2\sigma^2 + \sigma^2_i - \sigma^2_\gamma + \sigma_i^0 w_0 > 0 \), and \( \sigma(1 + w^0 + \iota_i^0 + \iota_i^0 - \gamma) > 0 \), then the real parts of the roots in Eq.(11) are negative. Thus, the system (2) is asymptotically stable when \( \alpha = 1 \).

If \( \alpha < 1 \), according to Lemma 3, it is easy to obtain that Re(λ) < 0 if all the roots of Eq.(12) have negative real parts. This completes the proof.

For the sake of simplicity in investigating the of roots of Eq.(4), the following assumption is given

\[
(H_4) \quad \frac{d\ln(\lambda)}{dt} = \frac{d\ln(\eta)}{dt} \neq 0
\]

Lemma 4. Suppose that \( \lambda(\tau) \) is a root of Eq.(12), \( \lambda(\tau) = \eta(\tau) + i\omega(\tau) \) near \( \tau = \tau_j \) satisfying \( \eta(\tau_j) = 0 \), \( \omega(\tau_j) = \omega_0 \), the transversality condition for Hopf bifurcation holds \( \text{Re}\left(\frac{d\lambda}{d\tau}\right)_{\tau=\tau_0,\omega=\omega_0} \neq 0 \).

Proof. According to Eq.(4), one can get the following equation

\[
\frac{d\lambda}{d\tau} = \frac{P_0(\lambda)e^{-\lambda\tau} + 2P_2(\lambda)e^{-2\lambda\tau}}{P_1(\lambda) + f(P_2, \lambda) + f(P_3, \lambda)} \quad (13)
\]

where

\[
f(P_2, \lambda) = P_0(\lambda)e^{-\lambda\tau} - P_2(\lambda)e^{-\lambda\tau}, \quad f(P_3, \lambda) = P_0(\lambda)e^{-2\lambda\tau} - 2P_2(\lambda)e^{-2\lambda\tau}, \quad P_n'(\lambda)(n = 1, 2, 3)
\]

represent the derivative of \( P_n(\lambda) \), when \( \lambda = i\omega_0, \tau = \tau_0 \), it is easy to deduce that

\[
\frac{d\lambda}{d\tau} = \frac{d_1 + i d_2}{d_3 + i d_4} \quad (14)
\]

where

\[
\begin{align*}
d_1 &= A_2\cos\omega_0\tau_0 + B_2\sin\omega_0\tau_0 - 2\omega_0B_2\cos2\omega_0\tau_0 + 2\omega_0A_2\sin2\omega_0\tau_0, d_2 = -A_2\sin\omega_0\tau_0 + B_2\cos\omega_0\tau_0 + 2\omega_0A_2\cos2\omega_0\tau_0 + 2\omega_0B_2\sin2\omega_0\tau_0, d_3 = A_2'\cos\omega_0\tau_0 + B_2'\sin\omega_0\tau_0 + 2\omega_0A_2\cos2\omega_0\tau_0 + 2\omega_0B_2\sin2\omega_0\tau_0, d_4 = -A_2'\sin\omega_0\tau_0 + B_2'\cos\omega_0\tau_0 + 2\omega_0A_2\sin2\omega_0\tau_0 + 2\omega_0B_2\cos2\omega_0\tau_0 - A_2\sin2\omega_0\tau_0 + B_2'\cos2\omega_0\tau_0 - 2\tau_0A_2\sin2\omega_0\tau_0 + 2\tau_0B_2\cos2\omega_0\tau_0. \quad (14)
\end{align*}
\]

In the above equation, \( A_n'(n = 1, 2, 3) \) are the derivative of real parts of \( P_n(\lambda) \) with respect to \( \lambda \), and \( B_n' \) are the imaginary parts.

By some computation, one can obtain from Eq.(11)

\[
\text{Re}\left(\frac{d\lambda}{d\tau}\right) = \frac{d_1 d_3 + d_2 d_4}{d_3^2 - d_4^2} \quad (15)
\]

It is easy to obtain that transversality condition is verified by hypothesis \((H_4)\).

Theorem 2. Suppose that the condition \((H_4)\) is satisfied for the system (2), one has

(i) A periodic orbit is produced for \( \tau \in (\tau_j, \tau_{j+1}) \), if the system is asymptotically stable for \( \tau \in (\tau_j, \tau_{j+1}) \).

(ii) The system with \( \tau = \tau_j(j = 1, 2, \ldots) \) undergoes a Hopf bifurcation.

(iii) The system is asymptotically stable if \((H_1) - (H_3)\) hold when \( \tau \in [0, \tau_0) \).
4. Numerical results and discussion

In this section, the dynamical behaviors of system (2) are analyzed by numerical simulation. When $\sigma = 4$ and $\gamma = 65$, one of the non-zero equilibrium points turns out to be $(64, 8, 8)$. For feedback gain $K = 3$ as the time delay varies, whose diagrams are displayed in Figure 1.

![Diagram](image1)

(a) (b)

Fig. 1. The largest real part of the complex eigenvalues $\text{Re}(\lambda)$ versus time delay $\tau$ when $\alpha$ takes different values. (a) Feedback gain $k = 3$; (b) Feedback gain $k = 0.5$.

According to Lemma 1, one can obtain that when $\text{Re}(\lambda) < 0$, the system (2) is stable. From Figure 1, it’s clearly to observe that, the curve moves down as the fractional order $\alpha$ going down when $\tau = 0$, the stable region of the system is also getting bigger, this can be verified the result of Lemma 3. As $\alpha$ decreases to cross a critical value, the system is stable, no matter what the value of time delay is. For a fixed fractional order $\alpha$, if the state of the system is instability when $\tau = 0$, with time delay increases to some extent ranges, system becomes stable. However, the system comes back to an unstable state when $\tau$ continues to increase in a small range. This also proves that the system is extremely sensitive to time delay $\tau$. For different gains, the change area of $\tau$ has different influence on the system. Figure 2 are diagrams of the relationship between the largest real part of eigenvalues and feedback gain for a given $\tau$.

![Diagram](image2)

(a) (b)

Fig. 2. The largest real part of the complex eigenvalues $\text{Re}(\lambda)$ versus feedback gain $k$ when $\alpha$ takes different values. (a) Time delay $\tau = 0.8$; (b) Time delay $\tau = 1.0$.

As it’s shown in Figure 2, a stability area appears when K increases to cross a critical value. The system is unstable when K is on the left side of the critical point, and the system is stable with K on the right side. When system parameters are selected as $\sigma = 5$ and $\gamma = 10$, it’s easy to verify that the conditions of Theorem 1 are satisfied, the system of (2) is stable when $\tau = 0$, as shown in Figure 3.
Fig. 3. Fractional-order $\alpha = 1$. (a) Phase portrait of system; (b) Time domain waveform in the case of $\alpha = 1$; (c) Time domain waveform in the case of $\alpha = 0.95$.

To obtain more detailed dynamics motion of the system (2), system parameter is selected to instability the system when $\tau = 0$. A fixed feedback gain is chosen to analyze the effect of time delay on the system, where $K = 0.5$, it’s easy to obtain that the transversality condition $Re\left(\frac{d\lambda}{dt}\right)|_{\tau = 0, \omega = \omega_0} \neq 0$ holds. Table 1 lists some representative bifurcation points in a certain interval $\tau \in [0, 1.5]$.

Table 1. Bifurcation points of fractional-order PMSM with different fractional orders $\alpha$ or different time delay $\tau$.

| Fractional-order $\alpha$ | $\tau_i$ | $\tau_{i+1}$ | $\tau_j$ | $\tau_{j+1}$ |
|---------------------------|---------|-------------|---------|-------------|
| 0.99                      | 0.1572  | 0.5614      | 0.9083  | 1.2430      |
| 0.98                      | 0.1114  | 0.5929      | 0.8407  | 1.2670      |
| 0.97                      | 0.0415  | 0.6589      | 0.7372  | 1.3329      |

Asymptotic stability intervals are $(\tau_i, \tau_{i+1})$ and $(\tau_j, \tau_{j+1})$. Figure 4 shows the detailed dynamical behaviors of system with different $\tau$ when $\alpha = 0.99$. System (2) shows two chaotic attractors when $\tau \in (0, \tau_0)$, the motion is shown in Figure 4a. As $\tau$ increases to cross a critical value $\tau_i$, system become stable, the phase portrait is presented in Figure 4b. On the further increase of $\tau$ near bifurcation point $\tau_i + 1$, a periodic oscillation appears, the diagram can be seen in Figure 4c. When $\tau$ keeps increasing, the dynamical behaviors will repeat, which can be seen in Figure 4d-f.

As illustrated in Figure 4, it is clear to see that how the system evolves from a stable state to an unstable state with a change of time delay, where the initial values of the system are selected to be near the non-zero equilibrium point.

5. Conclusion

In this paper, the model of fractional-order PMSM with current time-delayed feedback is studied, the stability of system and emergence of Hopf bifurcation are analyzed. Dynamic behaviors are investigated analytically and numerically by various methods such as the characteristic eigenvalue, phase trajectories and time-domain diagram. It is found that the stability of system can be changed as the time delay increases if all the system parameters are fixed. The obtained results are easy and effective to be checked. To more efficient analysis of fractional-order PMSM, it is necessary to discuss more complicated system involving dynamic time delay, which will be considered in the future.
Fig. 4. Dynamical behaviors of fractional-order PMSM with different $\tau$. (a) $\tau = 0.08$; (b) $\tau = 0.25$; (c) $\tau = 0.55$; (d) $\tau = 0.85$; (e) $\tau = 1.10$; (f) $\tau = 1.24$.

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