Phase transition between non-extremal and extremal Reissner-Nordström black holes

Yun Soo Myung

Institute of Mathematical Science and School of Computer Aided Science,
Inje University, Gimhae 621-749, Korea

Abstract

We discuss the phase transition between non-extremal and extremal Reissner-Nordström black holes. This transition is considered as the $T \to 0$ limit of the transition between the non-extremal and near-extremal black holes. We show that an evaporating process from non-extremal black hole to extremal one is possible to occur, but its reverse process is not possible to occur because of the presence of the maximum temperature. Furthermore, it is shown that the Hawking-Page phase transition between small and large black holes unlikely occurs in the AdS Reissner-Nordström black holes.

PACS numbers: 04.70.Dy, 04.60.Kz, 04.70.-s

\footnote{e-mail address: ysmyung@inje.ac.kr}
1 Introduction

Since the pioneering work of Hawking-Page on the phase transition between thermal AdS and AdS black hole in four dimensions [1], the research of the black hole thermodynamics has greatly improved. Especially, the phase transition of the AdS black hole in five dimensions inspired by string theory has generated renewed attention because it relates to confining-deconfining phase transition on the gauge theory side through the AdS/CFT duality [2]. It was extended to study the transition between AdS soliton and AdS black hole with Ricci flat horizon [3-4].

In addition to the usual transition between thermal AdS and AdS large black hole, it was suggested that there may exist a different phase transition between small and large black holes (HP1) in the AdS Reissner-Nordström (AdSRN) black holes for fixed charge \( Q < Q_c \) [5, 6] and AdS Gauss-Bonnet black holes [7].

In the conventional Hawking-Page phase transition (HP2), one generally starts with thermal radiation in AdS space. An unstable small black hole appears with negative heat capacity. The heat capacity changes from negative infinity to positive infinity at the minimum temperature \( T_0 \). Finally, the large black hole with positive heat capacity comes out as a stable object. There is a change of the dominance at the critical temperature \( T_1 \): from thermal radiation to black hole [1].

In the HP1, we start with a stable small stable black hole with positive heat capacity (SBH) because the thermal AdS is not an admissible phase for the AdSRN black holes [6]. The heat capacity changes from positive infinity to negative infinity at the maximum temperature \( T_m \). Then, an intermediate black hole with negative heat capacity (IBH) comes out. There is a change of the dominance at the critical temperature \( T = T_2 \): from SBH to large black hole with positive heat capacity (LBH). The HP1 can be described by two processes: SBH \( \rightarrow \) IBH \( \rightarrow \) LBH. However, it seems that the first process of SBH \( \rightarrow \) IBH is not permitted because there exists a temperature barrier between SBH and IBH.

In order to investigate the first process of SBH \( \rightarrow \) IBH, we have to study thermodynamics of the RN black hole [8]. In general, non-extremal RN black holes form two parameters of mass \( M \) and charge \( Q \) with the inequality of \( M > |Q| \). Here NOBH denotes non-extremal Reissner-Norström black hole with \( M > M_m = 2Q/\sqrt{3} \). On the other hand, extremal RN black holes (EBH) form one parameter with \( M = |Q| \). Their surface gravity vanishes and thus their Hawking temperature is zero. Therefore, for fixed-charge ensemble, the EBH can be the stable endpoint of the evaporation of NOBH. Recently, it was reported that there is no real discontinuity between EBH and NOBH, if one takes into account the flux due to the Hawking radiation properly [9].
In this work, we first check whether the first process of HP1 occurs or not by studying the phase transition between NOBH and near-extremal black hole (NEBH) with $Q < M < M_m$. In order to investigate this transition, we review thermodynamics of Reissner-Nordstr"{o}m black hole thoroughly. We study the off-shell (non-equilibrium) process of the growth of the RN black hole using the off-shell free energy and off-shell $\beta$-function. It turns out that the process of SBH $\rightarrow$ IBH is not allowed but its reverse process of SBH $\leftarrow$ IBH is permitted as an evaporating process. Hence, it is shown that the HP1 of SBH$\rightarrow$IBH$\rightarrow$LBH unlikely occurs in the AdSRN black holes.

\section{Thermodynamics of RN black hole}

We consider the RN black hole whose metric is given by

$$ds^2_{RN} = -U(r)dt^2 + U^{-1}(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2$$  

with $U(r) = 1 - 2M/r + Q^2/r^2$. Here, $M$ and $Q$ are the mass and the electric charge of the RN black hole, respectively. Then, the inner ($r_-$) and the outer ($r_+$) horizons are obtained as $r_\pm = M \pm \sqrt{M^2 - Q^2}$, which satisfy $U(r_\pm) = 0$. For $M = Q$, we have an extremal RN black hole at $r_+ = Q$. In this work we consider the case of fixed charge $Q$ for simplicity [5]. The other case of the fixed potential $\Phi = Q/r_+$ will have parallel with the fixed charge case.

For the RN black hole, the relevant thermodynamic quantities are given by the ADM mass $M$, Bekenstein-Hawking entropy $S_{BH}$ and Hawking temperature $T_H$:

$$M(r_+, Q) = \frac{r_+}{2} \left( 1 + \frac{Q^2}{r_+^2} \right), \quad S_{BH}(r_+) = \pi r_+^2, \quad (2)$$

$$T_H(r_+, Q) = \frac{1}{4\pi} \left( \frac{1}{r_+} - \frac{Q^2}{r_+^3} \right). \quad (3)$$

Then, using the Eqs. (2) and (3), the heat capacity $C = (dM/dT_H)_Q$ for fixed charge and Helmholtz free energy $F$ are obtained to be

$$C(r_+, Q) = -2\pi r_+^2 \left( \frac{r_+^2 - Q^2}{r_+^2 - 3Q^2} \right), \quad (4)$$

$$F(r_+, Q) = E - T_H S_{BH} = \frac{r_+^2 + 3Q^2}{4r_+} - Q$$  

with $E = M - Q$. In this case, $E$ measures the energy above the ground state. We note that one has to use the extremal black hole as background for fixed charge ensemble [5].
As is shown in Fig. 1, the features of thermodynamic quantities are as follows: First of all, the whole region splits into the near-horizon phase of $Q < r_+ < r_m$ with $r_m = \sqrt{3}Q$ and Schwarzschild phase of $r_+ > r_m$. i) The temperature is zero ($T_H = 0$) at $r_+ = Q$, maximum $T_m = \frac{1}{\sqrt{3}Q}$ at $r_+ = r_m$, and for $r_+ > r_m$, it shows the Schwarzschild behavior. ii) The heat capacity $C$ determines the local stability of thermodynamic system. For $Q < r_+ < r_m$, it is locally stable because of $C > 0$, while for $r_+ > r_m$, it is locally unstable ($C < 0$) as is shown in the Schwarzschild case. Here, we observe that $C = 0$ at $r_+ = Q$ and importantly, it blows up at $r_+ = r_m$. iii) The free energy is an important quantity to determine where the presumed phase transition occurs. The free energy is negative for the near-horizon phase and it is increasing for the Schwarzschild phase. It takes the minimum value of $F_m = -\frac{Q(2-\sqrt{3})}{2}$ at $r_+ = r_m$ and $F = 0$ at $r_+ = Q$, $3Q$. We identify the thermal state of the EBH by the condition of $T_H = 0$, $C = 0$, $F = 0$ with the non-zero Bekenstein-Hawking entropy $S_{BH} = \pi Q^2$ [8]. Furthermore, the RN black hole is split into NEBH being in the region of $Q < r_+ < r_m$ and NOBH in the region of $r_+ > r_m$. In connection with HP1, we have a correspondence of NEBH ↔ SBH and NOBH ↔ IBH.

3 Off-shell (non-equilibrium) process

We use the off-shell free energy to study the growth of a black hole [10]. Also, the off-shell $\beta$-function is introduced to measure the mass of a conical singularity at the event horizon [11] [12] [13]. In order to explore a possible phase transition, we consider the off-shell free energy,

$$F_\text{off}(r_+, Q, T) = E - TS_{BH}$$

with the external temperature $T$. For fixed $Q$, it corresponds to the Landau function for describing the phase transition with the order parameter $r_+$ [6]. We note that $\partial F_\text{off}/\partial r_+ = 0 \rightarrow T = T_H$. Plugging this into Eq.(6) leads to the on-shell free energy $F(r_+, Q)$ in Eq.(5). In this sense, we call $F_\text{off}$ the off-shell free energy [14] [15]. In
other words, the first law of thermodynamics of $dE = TdS_{BH}$ holds for the on-shell but it does not hold for any off-shell process.

We start with observing the temperature in Fig. 2. An important point is that there exists the maximum temperature $T = T_m = 0.3$ near the extremal black hole. The presence of this temperature makes the thermodynamic process different when comparing with the Hawking-Page phase transition (HP2) between thermal AdS and black hole [16]. Here we choose three external temperatures to study a possible phase transition between NEBH and NOBH. These are $T = T_m, 0.02, 0.005$. It is noted that in the limit of $T \to 0$, one may include the phase transition between EBH and NOBH. In this case, the starting
point approaches the extremal black hole \((r_i \to Q)\), while the ending point goes to infinity \((r_f \to \infty)\). For these temperatures, the corresponding graphs of off-shell free energy are shown in Fig. 2. Considering the condition of \(T = T_H\), we find their equilibrium points as the initial and final points. These are given by

\[
    r_i = \frac{1}{24\pi T} \left[ 2 + 4 \cos \left( \frac{\alpha}{3} - \frac{2\pi}{3} \right) \right], \quad r_f = \frac{1}{24\pi T} \left[ 2 + 4 \cos \frac{\alpha}{3} \right],
\]

(7)

where \(\cos \alpha = 1 - 2T^2 / T_m^2\) with \(2T_m^2 < \alpha \leq \pi\). In particular, for \(T \ll T_m\) \((\alpha \to 2T_m^2 \simeq 0)\), the initial point is located at \(r_i \simeq r_e\), while the final point is at \(r_f \simeq \frac{1}{4\pi T}\). In case of \(T = T_m(\alpha = \pi)\), one has a degenerate point at \(r_i = r_f = r_m\), which corresponds to the maximum point.

As is expected from the temperature graph, for \(T > T_m\), there is no equilibrium point where the on-shell curve of free energy meets the off-shell curve \((F = F_{off})\). For the maximum temperature \(T = T_m\), there is a degenerate equilibrium point at \(r_+ = r_m\), where the minimum on-shell free energy \(F_m = F_{off}\) appears and the heat capacity blows up. In case of \(T = 0.02\), we have two equilibrium points \(r_i = 1.2\) and \(r_f = 3.69\): \(r_i\) is a globally stable point because of \(C > 0, F < 0\) while \(r_f\) is a globally unstable point because of \(C < 0, F > 0\). For \(T = 0.005\), the situation is qualitatively similar to the case of \(T = 0.02\), but the difference is \(r_i = 1.03\) and \(r_f = 15.85\). Hence, the transition from NOBH to NEBH is likely allowed as an evaporating process \([17]\), while its reverse process is unlikely allowed because of the presence of the maximum temperature \(T = T_m\).

In order to investigate the off-shell process explicitly, we consider the off-shell parameter \(\alpha\) and the deficit angle \(\delta\) as

\[
    \alpha(r_+, Q, T) = \frac{T_H}{T}, \quad \delta(r_+, Q, T) = 2\pi(1 - \alpha).
\]

(8)

\(\alpha\) is zero at the extremal point of \(r_+ = Q\) and it is one at the equilibrium point \(T = T_H\). On the other hand, the range of deficit angle is given by \(0 \leq \delta \leq 2\pi\). \(\delta\) has the maximum value of \(2\pi\) at the extremal point and it is zero at \(T = T_H\). This implies that the extremal configuration at \(r_+ = Q\) has the narrowest cone of the shape \((\prec)\) near the horizon, while its geometry at \(T = T_H\) is a contractible manifold \((\subset)\) without conical singularity. For any off-shell process of the growth of black hole, we must have \(0 < \delta < 2\pi\) and a conical singularity of the shape \((\prec)\) near the horizon \([10, 11, 12, 13, 14, 15]\). However, as is shown in Fig. 3, there exist negative deficit angles for \(T < T_m\), while the deficit angle is always positive for \(T > T_m\). Explicitly, we find negative deficit angles for \(r_i < r_+ < r_f\). Hence we stress that these are not properly defined thermodynamic processes of growing black hole by absorbing radiation in the heat reservoir.
Furthermore, the off-shell process could be described by the off-shell $\beta$-function defined with the off-shell Euclidean action $I^{off} = F^{off}/T$ as

$$\beta(r_+, Q, T) \propto \frac{\partial I^{off}}{\partial r_+} = -r_+ \delta(r_+, Q, T)$$

which measures the deficit angle $\delta$ directly. The connection between mass of conical singularity and off-shell $\beta$-function is given by

$$M_{cs} = \frac{\beta}{4\pi} = -M_{pp},$$

where $M_{pp}$ is the mass of point particle at the event horizon. In general, the mass of a conical singularity is negative.

We find from Fig. 3 that the off-shell $\beta$-functions are negative for $T > T_m$, while these are positive for $T < T_m$. We have positive $\beta$-function between two equilibrium points, $r_i < r_+ < r_f$. Hence these are not regarded as a properly defined off-shell process of increasing black hole by absorbing radiation. However, its reverse process of decreasing black hole by emitting radiation as the Hawking radiation seems to be possible to occur.

4 AdSRN black hole

We consider the AdSRN black hole whose metric function is given by

$$U(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2},$$

where $l$ is the curvature radius of AdS space. Then, the inner ($r_-$) and the outer ($r_+$) horizons are obtained from the condition of $U(r_\pm) = 0$. In this work we consider the case of fixed-charge ensemble $Q < Q_c = l/6$ \cite{5}. The other case of the fixed potential $\Phi = Q/r$ will have parallel with the fixed charge case.

For the AdSRN black hole, the relevant thermodynamic quantities are given by the ADM mass $M$ and Hawking temperature $T_H$

$$M(r_+, Q) = \frac{r_+}{2} \left(1 + \frac{Q^2}{r_+^2} + \frac{r_+^2}{l^2}\right), \quad T_H(r_+, Q) = \frac{1}{4\pi} \left(\frac{1}{r_+} - \frac{Q^2}{r_+^3} + \frac{3r_+}{l^2}\right).$$

In the case that the horizon is degenerate, we have an extremal black hole with $M = M_e$. In general, one has an inequality of $M > M_e$. Then, using the Eqs. (12), the heat capacity $C = (dM/dT_H)_Q$ for fixed charge and Helmholtz free energy $F$ are obtained to be

$$C(r_+, Q) = 2\pi r_+^3 \left[\frac{3r_+^4 + l^2(r_+^2 - Q^2)}{3r_+^4 + l^2(-r_+^2 + 3Q^2)}\right],$$

$$F(r_+, Q) = E - T_H S_{BH} = \frac{1}{4r_+} \left(r_+^2 + 3Q^2 - \frac{r_+^4}{l^2}\right) - M_e.$$
Figure 4: Thermodynamic quantities of the AdSRN black hole as function of horizon radius \( r_+ \) with fixed \( Q = 1 \) and \( l = 10 \): temperature \( T_H \), heat capacity \( C \), and free energy \( F \).

with \( E = M - M_e \). In this case, \( E \) measures the energy above the ground state. Also the extremal black hole is chosen as background for fixed-charge ensemble [5].

The global features of thermodynamic quantities are shown in Fig. 4. It seems to be a combination of RN and AdS Schwarzschild black holes. Here we observe the local minimum \( T_H = T_0 \) at \( r_+ = r_0 \) (feature of AdS Schwarzschild black hole), in addition to the extremal temperature \( T_H = 0 \) at \( r_+ = r_e \) and the maximum value \( T_H = T_m \) at \( r_+ = r_m \) (feature of RN black hole).

For \( r_e < r_+ < r_m \), the black hole is locally stable because of \( C > 0 \) while for \( r_m < r_+ < r_0 \), it is locally unstable (\( C < 0 \)). For \( r_+ > r_0 \), the black hole becomes stable because of \( C > 0 \). Here, we observe that \( C = 0 \) at \( r_+ = r_e \), and it blows up at \( r_+ = r_m, r_0 \). Based on the local stability, the AdSRN black holes are split into SBH with \( C > 0 \) being in the region of \( r_e < r_+ < r_m \), IBH with \( C < 0 \) in the region of \( r_m < r_+ < r_0 \), and LBH with \( C > 0 \) in the region of \( r_+ > r_0 \).

Importantly, the free energy plays a crucial role to test the phase transition. A black hole is globally stable when \( C > 0 \) and \( F < 0 \). We observe two extremal points for free energy: the local minimum \( F = F_{\text{min}} \) at \( r_+ = r_m \) and the maximum value \( F = F_{\text{max}} \) at \( r_+ = r_0 \). The free energy is negative for \( r_e < r_+ < r_m \) and it increases in the region of \( r_m < r_+ < r_0 \). For \( r_+ = r_1 > r_0 \), it is zero and remains negative for \( r_+ > r_1 \). The temperature \( T = T_1 \) determined from \( F = 0 \) at \( r_+ = r_1 \) plays a role of the critical temperature in HP2.

In order to investigate whether the HP1 is possible to occur in the AdSRN black holes, we study the off-shell process of the growth of a black hole. For this purpose, we introduce the off-shell free energy

\[
F^{\text{off}}(r_+, Q, T) = E - TS_{\text{BH}}
\]

with the temperature \( T \) of heat reservoir. The critical temperature \( T = T_2 \) is derived from the condition of \( F^{\text{off}}(r_s, 1, T_2) = F^{\text{off}}(r_l, 1, T_2) \), which means that the free energy
of SBH at $r_+ = r_s$ is equal to free energy of LBH at $r_+ = r_l$. This case is marked as the horizontal line in the right panel of Fig. 5. Then we expect to discuss a HP1 between SBH and LBH through IBH for $T_m = 0.0348 < T < T_2 = 0.0285$. As is shown in Fig. 5, we choose $T = 0.03$ for a transition temperature of the HP1. This transition consists of two processes: SBH with $C > 0 \rightarrow$ IBH with $C < 0 \rightarrow$ LBH with $C > 0$. We have global stabilities for SBH and LBH because of their free energies are positive. However, the IBH remains unstable and it may play a role of a mediator for HP1 as an unstable small black hole in HP2.

Let us study the HP1 further by introducing the deficit angle $\delta(r_+, Q, T)$ and the off-
shell $\beta$-function $\beta(r_+, Q, T)$ which are the same forms as in Eqs.(8) and (9) but different temperature $T_H$. From Fig. 6, we find that the HP1 is not possible to occur. The reason is that for the process of SBH → IBH, the off-shell (non-equilibrium) process is not well-defined because the negative deficit angle and positive off-shell $\beta$-function exist for $T = 0.03$. Hence, it seems that the HP1 is hard to occur in the AdSRN black holes.

### 5 Discussions

We discuss the phase transition between EBH and NOBH in the RN black hole. For this purpose, we may consider this transition as the $T \to 0$ limiting case of the assumed phase transition between the NEBH and NOBH. Our study was based on the on-shell observations of temperature, heat capacity and free energy as well as the off-shell observations of generalized (off-shell) free energy, deficit angle and $\beta$-function. In general, the on-shell thermodynamics describes relationships among thermal equilibria and not the transitions between equilibria. The off-shell thermodynamics is suitable for the description of transitions between thermal equilibria. As is summarized in Table 1, the NEBH is a globally stable object, whereas the NOBH is an unstable object. The $\leftarrow$ transition from NOBH to NEBH is likely allowed as an evaporating process [17], while the assumed phase transition

|                | APT ($\rightarrow$) | EP($\leftarrow$) |
|----------------|---------------------|------------------|
| starting point | $r_+ = r_i$(NEBH)  | $r_+ = r_f$(NOBH) | $r_+ = r_f$ | $r_+ = r_i$ |
| ending point   | $+$                 | $-$               | $-$          | $+$          |
| $C$            | $-$                 | $+$               | $+$          | $-$          |
| $F$            | stable              | unstable          | unstable     | stable       |

Table 1: Summary of the assumed phase transition (APT) and its reverse transition as Evaporating process (EP) for the RN black hole.

|                | starting point | intermediate point | ending point |
|----------------|----------------|--------------------|--------------|
| $r_+$          | $r_+ = r_s$(SBH) | $r_+ = r_i$(IBH)  | $r_+ = r_l$(LBH) |
| $C$            | $+$             | $-$                | $+$          |
| $F$            | $-$             | $+$                | $-$          |
| status         | stable          | unstable           | stable       |

Table 2: Summary of the assumed Hawking-Page transition (HP1:$\rightarrow$) for the AdSRN black hole.
(NEBH→NOBH) is unlikely allowed for $T < T_m$ \cite{16}. The main reason comes from the fact that for the latter, the off-shell (non-equilibrium) process is not well-defined because of the negative deficit angle and positive off-shell $\beta$-function. As is shown in Fig. 2, this arises because of the presence of temperature barrier. Furthermore, an evaporating process from NEBH to EBH is possible to occur as was discussed in Ref.\cite{18}.

In connection with HP1, we have a correspondence of NEBH ↔ SBH and NOBH ↔ IBH. The process of SBH→IBH represents the feature of RN black hole, while the process of IBH→LBH denotes the feature of AdS Schwarzschild black hole. Thus, one expects that for $T < T_{2}$, the SBH is more favorable than the LBH, while for $T > T_{2}$, the LBH is more favorable than the SBH. However, the HP1 of SBH→IBH→LBH is hard to occur because SBH→IBH (NEBH→NOBH) is included as the first process in the AdSRN black holes. The presence of temperature barrier in Fig. 4 supports this argument. Further, its reverse transition is not allowed because both starting and ending points are globally stable (see Table 2).

Finally, we propose that NEBH→NOBH and NEBH←NOBH for the RN black holes, while SBH→LBH and SBH←LBH for the AdSRN black holes.

We conclude that an evaporating process from non-extremal black hole (NOBH) to extremal one (EBH) is possible to occur, but its reverse process is not possible to occur. However, considering the quantum tunnelling process through temperature barrier, one may expect to have NEBH→NOBH for the RN black holes and SBH→LBH for the AdSRN black holes.

\section*{Acknowledgement}

The author thanks Young-Jai Park and Yong-Wan Kim for helpful discussions. This work was supported by the Korea Research Foundation (KRF-2006-311-C00249) funded by the Korea Government (MOEHRD).

\section*{References}

[1] S. W. Hawking and D. N. Page, Commun. Math. Phys. 87 (1983) 577.

[2] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 505 \textcolor{blue}{[arXiv:hep-th/9803131]}.

[3] G. T. Horowitz and R. C. Myers, Phys. Rev. D 59 (1999) 026005 \textcolor{blue}{[arXiv:hep-th/9808079]}. 

11
[4] S. Surya, K. Schleich and D. M. Witt, Phys. Rev. Lett. 86 (2001) 5231 [arXiv:hep-th/0101134].

[5] A. Chamblin, R. Emparan, C. V. Johnson and R. C. Myers, Phys. Rev. D 60 (1999) 064018 [arXiv:hep-th/9902170].

[6] T. K. Dey, S. Mukherji, S. Mukhopadhyay and S. Sarkar, JHEP 0709 (2007) 026 [arXiv:0706.3996 [hep-th]].

[7] T. K. Dey, S. Mukherji, S. Mukhopadhyay and S. Sarkar, JHEP 0704 (2007) 014 [arXiv:hep-th/0609038].

[8] Y. S. Myung, Y. W. Kim and Y. J. Park, Mod. Phys. Lett. A 23 (2008) 91 [arXiv:0707.3314 [gr-qc]].

[9] R. Balbinot, A. Fabbri, S. Farese and R. Parentani, Phys. Rev. D 76 (2007) 124010 [arXiv:0710.0388 [hep-th]].

[10] V. P. Frolov, D. V. Fursaev and A. I. Zelnikov, Phys. Rev. D 54 (1996) 2711 [arXiv:hep-th/9512184].

[11] J. L. F. Barbon and E. Rabinovici, JHEP 0203 (2002) 057 [arXiv:hep-th/0112173].

[12] J. L. F. Barbon and E. Rabinovici, Found. Phys. 33 (2003) 145 [arXiv:hep-th/0211212].

[13] J. L. F. Barbon and E. Rabinovici, arXiv:hep-th/0407236.

[14] Y. S. Myung, Phys. Lett. B 638 (2006) 515 [arXiv:gr-qc/0603051].

[15] Y. S. Myung, Int. J. Mod. Phys. A 22 (2007) 41 [arXiv:hep-th/0604057].

[16] Y. S. Myung, Y. W. Kim and Y. J. Park, Phys. Lett. B 656, 221 (2007) [arXiv:gr-qc/0702145].

[17] D. Pavon, Phys. Rev. D 43, 2495 (1991).

[18] A. Fabbri, D. J. Navarro and J. Navarro-Salas, Nucl. Phys. B 595 (2001) 381 [arXiv:hep-th/0006035].