**ρ**-mass Modification in $He^3$ - a Signal of Restoration of Chiral Symmetry or Test for Nuclear Matter Models?

Abhijit Bhattacharyya$^{1,a}$, Sanjay K. Ghosh$^{2,b}$ and Sibaji Raha$^{2,c}$

$^{1}$) Variable Energy Cyclotron Centre, 1/AF, Bidhannagar, Calcutta 700 064, INDIA
$^{2}$) Department of Physics, Bose Institute, 93/1 A.P.C.Road, Calcutta 700 009, INDIA

Two recent experiments have demonstrated that the effective $\rho$-mass in nuclear medium, as extracted from the $^3He(\gamma, \pi^+\pi^-)$ reaction, is substantially reduced. This has been advocated as an indication of partial restoration of chiral symmetry in nuclear matter. We show that even in the absence of chiral symmetry, effective mean field nuclear matter models can explain these findings quantitatively.

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In-medium properties of hadrons is a field of high current interest. In high energy heavy ion collisions, where an environment of hot and dense hadronic matter is expected to be formed, the modification of hadronic properties can indeed have important physical consequences. The recent observation of enhanced dilepton production in the low invariant mass domain in heavy ion collider experiments has triggered speculation that the effective $\rho$-meson mass in the nuclear medium is decreased. Simultaneously, theoretical studies based on chiral perturbation theory ($\chi$PT) have led to the expectation that even at finite densities (at or above nuclear density) there may be a partial restoration of chiral symmetry, leading to the decrease of vector meson masses from their free values. These questions assume great importance also in the context of quark-gluon plasma searches in heavy ion collisions, where the hadronic matter constitute the background to the sought-for signals and thus must be controlled to a high degree of accuracy before conclusive evidence for the quark-gluon plasma can be extracted from the experimental data.

The evidence (or more appropriately, indication) of the $\rho$ mass renormalization, referred to above, is indirect. Very recently, however, a direct measurement of the invariant $\rho$ mass in photoproduction of $\rho^0$ on $He^3$ has been reported in the literature. The decrease in the $\rho$ mass found by these authors is quite substantial ($\delta m_{\rho} \sim 280 \pm 40$ MeV), so much so that it has been argued that such large decrease cannot be explained by the mean field picture of nuclear matter. These authors suggest that this should be taken as a signature of (partial) restoration of chiral symmetry in ground state nuclei. In this work, we show that such a conclusion is premature since a proper inclusion of the relevant interactions in a mean field description has the effect of reducing the $\rho$-mass to the desired level.

The most popular mean field model for nuclear matter is the Walecka model, which was proposed first in 1974 and has been greatly modified over the years by a number of authors; for a recent comparative study among the various versions, see [1]. This serves as the prototype of most effective mean field theories of nuclear matter and for the present purpose, we concentrate our attention only to the Walecka model, the basic Lagrangian for which is given by [12]:

\[
\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m_n)\psi - \left[ g_\sigma \bar{\psi}\gamma_\mu \sigma \partial^\mu \psi + \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \right] \\
+ \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) + g_\rho \bar{\psi}\sigma \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
+ \frac{1}{2} m_\rho^2 \rho^2 + g_\rho \bar{\psi}\gamma_\mu \tau \psi \rho_\mu + \frac{1}{2} \bar{\psi}\gamma_\mu \tau \psi \rho_\mu \rho_\mu \\
\]  

(1)

In the above equation $\psi$, $\sigma$, $\omega$, $\rho$ and $\rho^0$ are, respectively, the nucleon, the sigma, the omega and the $\rho$ meson fields; $m_n$, $m_\sigma$, $m_\omega$ and $m_\rho$ are the corresponding masses; $g_\sigma$ and $g_\omega$ are the couplings of the nucleon to $\sigma$ and $\omega$ mesons, respectively; $g_\rho$ and $f_\rho$ are the vector and tensor couplings of rho meson; $G_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $F_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + i g_\rho [\rho_\mu, \rho_\nu]$.

In general, for medium and heavy nuclei one uses the mean field approximation (MFA) and then a set of coupled differential equations is solved self-consistently to get the field values as a function of $r$. But for a light nucleus like $He^3$, the MFA may not be reliable. Therefore, we have used a simple approach, a la Saito et al., to calculate the effective $\rho$ mass in helium.

In this paper we have used a simple Gaussian form for the density distribution of $He^3$, in which the width parameter $\beta_3$ is fitted to reproduce the rms charge radius of $He^3$ i.e. 1.88 fm [4]. The density profile is given in figure 1. So once we know the density distribution of $He^3$, one can easily calculate the effective $\rho$ mass, $m_{\rho^*}$, as a function of radius, all the fields being known as a function of baryon density.

The $\rho$ mass has been calculated from eq. (1), in the usual manner, at the one loop level. The expression for the $\rho$-mass is given by
\[ m^*_2 = m^*_2 + \Pi_{\text{vac}} + \Pi_{\text{med}} \]  

where

\[ \Pi_{\text{med}} = \sum_{B=n,p} \frac{8g^2}{\pi^2} \int_0^{k_{FN}} \frac{p^2 dp}{E_p (m^*_2 - 4E_p^2)} \times \left[ \frac{2}{3} (2p^2 + 3m_n^2) + m^*_2 \left\{ 2m_n \left( \frac{c}{2m_n} \right) \right. \right. \\
\left. \left. \quad \quad - \frac{2}{3} \left( \frac{c}{2m_n} \right)^2 (p^2 + 3m_n^2) \right\} \right] \]

\[ \Pi_{\text{vac}} = \frac{g^2}{\pi^2} m^*_2 \left[ I_1 + m_n^* \left( \frac{c}{2m_n} \right) I_2 \right. \right. \\
\left. \left. \quad \quad \quad \quad + \frac{1}{2} \left( \frac{c}{2m_n} \right)^2 (m^*_2 I_1 + m_n^2 I_2) \right] \]

\[ I_1 = \int_0^1 dx (1-x) \ln \left[ \frac{m^*_2 - m^*_2 x(1-x)}{m_n^2 - m^*_2 x(1-x)} \right] \]

\[ I_2 = \int_0^1 \ln \left[ \frac{m^*_2 - m^*_2 x(1-x)}{m_n^2 - m^*_2 x(1-x)} \right] \]

In the above set of equations \( c_p = \frac{f_p}{g_p} \).

There are two coupling constants involved in the above set of equations. One is the vector coupling of the \( \rho \)-meson \( g_\rho \) and the other is the tensor coupling \( f_\rho \) (or equivalently \( c_\rho \)). This is where the difference between our approach and the earlier works arises; previous authors \cite{9} neglected the tensor coupling of the \( \rho \) to the nucleon. We have used three sets of coupling constants, shown in table 1. The density dependence of the \( \rho \)-meson mass, for these three sets of parameters, has been shown in figure 2.

| model                  | \( g_\rho \) | \( c_\rho \) |
|------------------------|-------------|-------------|
| Bonn Potential \cite{15} | 2.63        | 6.1         |
| QCD Sum rule \cite{16}  | 2.5 ± 0.2   | 8.0 ± 2.0   |
| Walecka Model          | 8.912       | 6.1         |

**TABLE I.** Parametr values for different models.
In order to compare the results for the effective $\rho$ mass with the experimental values, we calculate the average mass of the $\rho$-meson in the $^3\text{He}$ nucleus. The average mass is defined as

$$\langle m^*_\rho \rangle = \frac{\int d^3r m^*_\rho(r)\rho_B(r)}{\int d^3r \rho_B(r)}$$

(7)

In table 2, we show the average $\rho$-mass for the different sets of parameters.

| Model                | Average Mass $(\text{MeV})$ |
|----------------------|-----------------------------|
| Bonn Potential [15]  | 536                         |
| QCD Sum rule [16]    | $449 - 565$                 |
| Walecka Model        | 304                         |

TABLE II. Average mass of $\rho$ meson for different models.

There have been two recent papers on the density variation of $\rho$-mass inside the $^3\text{He}$ nucleus. Both of them are the results from the $\rho^0$ photoproduction experiment of $^3\text{He}$. The first one is in the energy range $E_\gamma = 800 - 1120\text{MeV}$ and the second for $E_\gamma = 380 - 700\text{MeV}$. The first paper finds a drop in the $\rho$-mass of $160 \pm 35\text{MeV}$, i.e. $m^*_\rho$ is in the range $575 - 645\text{MeV}$. The other study finds an effective $\rho$-mass in the range $450 - 530\text{MeV}$.

In almost all the previous studies of the $\rho$-meson inside a light nucleus from the mean field approach, the tensor coupling of the $\rho$-meson to the nucleon [14,9] was not included, as already mentioned. As a result, the variation of $\rho$-mass was rather soft in all the previous cases. Here, the incorporation of the tensor coupling leads to a change in the $\rho$-meson mass which is much larger and we get results which are very close to the experimental findings. For example, the Bonn potential parameter set [15] yields $\langle m^*_\rho \rangle = 536\text{MeV}$. On the other hand, for the QCD sum rule case [16], we get $\langle m^*_\rho \rangle = 449 - 565\text{MeV}$. For the Walecka model parameter set, the value of $\langle m^*_\rho \rangle$ is somewhat lower.

On the basis of above observations we argue that the reduction of the effective mass of $\rho$-meson in $^3\text{He}$ need not be an unambiguous signal for the restoration of chiral symmetry, as suggested by the authors of ref. [8]. In particular, even the mean field model of nuclear matter is capable of accommodating such substantial changes in the effective $\rho$ mass, if all the interactions are properly taken into account.

We would like to mention here that the present calculation is really an estimate of the behaviour of $\rho$ meson mass inside a nucleus. To get a quantitative estimate and compare with the experiments mentioned above, one should do a full calculation of photoproduction processes. Such a non-trivial calculation is presently under study.

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FIG. 1. Density profile of $He^3$

FIG. 2. Density dependence of rho mass, Q1 and Q2 are the upper and lower mass limits for the QCD sum rule parameter set, B for Bonn potential parameters and W is for Walecka model parameters.
Electronic Mail : abhijit@veccal.ernet.in
Electronic Mail : phys@boseinst.ernet.in
Electronic Mail : sibaji@boseinst.ernet.in

[1] CERES collaboration, Th. Ulrich et al., Nucl. Phys. A610, 313c (1996).
[2] HELIOS Collaboration, M. Masera et al., Nucl. Phys. A590, 93c (1995).
[3] NA50 collaboration, E. Scomparin et al., Nucl. Phys. A610, 331c (1996).
[4] G. Q. Li, C. M. Ko and G. E. Brown, Phys. Rev. Lett. 75, 4007 (1995).
[5] G. E. Brown et al., Nucl. Phys. A343, 295 (1995).
[6] J. Alam, S. Raha and B. Sinha, Phys. Rep. 273, 243 (1996) and references therein.
[7] G. J. Lolos et. al., Phys. Rev. Lett. 80, 241 (1998).
[8] G. M. Huber et. al., Phys. Rev. Lett. 80, 5285 (1998).
[9] K. Saito, A. W. Thomas and K. Tsushima, Phys. Rev. C56, 566 (1997).
[10] G. E. Brown, M. Buballa and M. Rho, Nucl. Phys. A609, 519 (1996).
[11] J.D.Walecka, Ann. Phys. (N.Y.) 83, 491 (1974).
[12] A. Bhattacharyya and S. K. Ghosh, Int. J. Mod. Phys. 7, 495 (1998).
[13] J. Piekarewicz and A. G. Williams, Phys. Rev. C47, 2462 (1993).
[14] K. Saito, K. Tsushima and A. W. Thomas, Phys. Rev. C55, 2637 (1997).
[15] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. 149, 1 (1987).
[16] S-L. Zhu, Phys. Rev. C59, 435 (1999).