Leptonic Flavor-changing $Z^0$ Decays
in $SU(2) \otimes U(1)$ Theories
with Right-handed Neutrinos

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ABSTRACT

We analyze possible lepton-flavor-violating decays of the $Z^0$ particle in a minimal extension of the Standard Model, in which one right-handed neutral field for each family has been introduced. Such rare leptonic decays are induced by Majorana neutrinos at the first electroweak loop level and are generally not suppressed by the ordinary "see-saw" mechanism. In particular, we find that experimental bounds on branching ratios of the order of $10^{-5} - 10^{-6}$ attainable at LEP may impose constraints on lepton-flavor-mixing parameters and the masses of the heavy Majorana neutrinos.

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Many theoretical scenarios have recently been proposed for the production and the lepton-number-violating decay of heavy Majorana neutrinos at accessible accelerator energies \[1,2,5\]. A low-energy realization of these theories could be the Standard Model (SM) with one right-handed neutrino field per family \[4\]. Such a three-generation "see-saw" model \[3\] can theoretically provide small Majorana masses for the ordinary neutrinos, relatively light masses for the heavy ones (e.g. of the order of 100 GeV), as well as large light-heavy neutrino mixings (i.e. \(\xi_{\nu N} \simeq 0.1\)) \[2\]. This possibility was systematically investigated in \[5\] and the notation given there will be used throughout this note. In this minimal class of models some interesting phenomenological applications to possible lepton-flavor-violating \(H^0\) decays have recently been presented \[6\]. In this note, however, we will calculate branching ratios of leptonic flavor-changing decays of the \(Z^0\) particle, i.e. \(Z^0 \to e\tau\) or \(\mu\tau\), in this minimal class of models. For realistic samples of \(10^6 - 10^7\) \(Z^0\) decays at LEP, one would be sensitive to branching ratios of the order of \(10^{-5} - 10^{-6}\).

We find that branching ratios of this order of magnitude could be easily understood within such minimal models. Of course, alternative theoretical models exhibiting lepton-flavor-nonconservation in \(Z^0\) decays, have been proposed over the years \[7,8\]. We briefly comment on the results of some of these calculations.

As has been extensively discussed in \[5\], in the Feynman-'t Hooft gauge the relevant couplings of Majorana neutrinos \(n_i\) (with \(i = 1, 2, \ldots, 2n_G\)) to the \(W^\pm\), \(Z^0\) and Goldstone bosons \(\chi^\pm\) are written down in terms of mass eigenstates as follows:

\[
\mathcal{L}^W_{\text{int}} = -\frac{g_W}{2\sqrt{2}} W^{-\mu} \bar{\tilde{l}}_i B_{i\mu}^\nu (1 - \gamma_5) n_j + \text{h.c.} \quad (1)
\]

\[
\mathcal{L}^Z_{\text{int}} = -\frac{g_W}{4 \cos \theta_W} Z^{0\mu} \bar{n}_i \gamma_\mu [i \text{Im}(C_{ij}) - \gamma_5 \text{Re}(C_{ij})] n_j \quad (2)
\]

\[
\mathcal{L}^\chi_{\text{int}} = -\frac{g_W}{2\sqrt{2} M_W} \chi^\mu \bar{l}_i \left[ m_l B_{i\mu}(1 - \gamma_5) - B_{i\mu}(1 + \gamma_5) m_{n_j} \right] n_j + \text{h.c.} \quad (3)
\]

where \(B_{i\mu}\) is a rectangular \(n_G \times 2n_G\) matrix given by

\[
B_{i\mu} = \sum_{k=1}^{n_G} V^I_{l_k} U^\nu_{k\mu} \quad (4)
\]

and \(C_{ij}\) is in general a nondiagonal \(2n_G \times 2n_G\) projection matrix defined by

\[
C_{ij} = \sum_{k=1}^{n_G} U^\nu_{k\mu} U^\nu_{k\mu} \quad (5)
\]
In eqs (4) and (5), $V_l$ and $U_\nu$ are the corresponding Cabbibo-Kobayashi-Maskawa (CKM) matrix for the leptonic sector and the unitary matrix which diagonalizes the symmetric "see-saw" neutrino mass matrix, respectively.

In this low-energy model, Majorana neutrinos can give rise to flavor-changing neutral current effects in the leptonic sector at the first electroweak loop level. The Feynman diagrams responsible for the decay process $Z^0 \rightarrow l_1 \bar{l}_2 \ (l_1 \neq l_2)$ are shown in figs (1a)--(1k). Before we proceed to list their individual contributions to the above leptonic rare decays, we first introduce some useful abbreviations:

$$C_{UV} = \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi - \ln \frac{M_W^2}{\mu^2}$$  \hspace{1cm} (6)

$$\Delta_{ij} = \frac{ig_W\alpha_W}{8\pi\cos\theta_W} B_{l_1 l_2} \bar{u}_i \gamma_\mu (1 - \gamma_5) v_{l_2} \epsilon_\mu^\nu$$ \hspace{1cm} (7)

$$\lambda_i = \frac{m_i^2}{M_W^2}, \quad \lambda_Z = \frac{M_Z^2}{M_W^2}$$ \hspace{1cm} (8)

We also work in the massless limit for the external leptons. Then, the results for the different contributing graphs to the decay $Z^0 \rightarrow l_1 \bar{l}_2$ are obtained in the on-shell renormalization scheme [9] by the following expressions:

$$i\Gamma_a = \frac{1}{2} \Delta_{ij} \left\{ C_{ij} \left[ \int_0^1 dx dy \ln B_2(\lambda_i, \lambda_j) - \lambda_i \int_0^1 dx dy \frac{y}{B_2(\lambda_i, \lambda_j)} \right] - \lambda_Z \int_0^1 dx dy \frac{1}{B_2(\lambda_i, \lambda_j)} \left[ 1 - y + y^2 x(1 - x) \right] \right\}$$

$$+ C_{ij}^* \sqrt{\lambda_i \lambda_j} \int_0^1 dx dy \frac{y}{B_2(\lambda_i, \lambda_j)} \right\}$$ \hspace{1cm} (9)

$$i\Gamma_b = - \frac{1}{4} \Delta_{ij} \left\{ C_{ij} \lambda_i \lambda_j \int_0^1 dx dy \frac{y}{B_2(\lambda_i, \lambda_j)} + C_{ij}^* \sqrt{\lambda_i \lambda_j} \left[ \frac{1}{2} C_{UV} - \frac{1}{2} \right] \right\}$$

$$+ \lambda_Z \int_0^1 dx dy \frac{y^2 x(1 - x)}{B_2(\lambda_i, \lambda_j)} - \int_0^1 dx dy \frac{y}{B_2(\lambda_i, \lambda_j)} \right\}$$ \hspace{1cm} (10)

$$i\Gamma_c = - \Delta_{ii} \left[ \lambda_Z \int_0^1 dx dy \frac{y}{B_1(\lambda_i)} y^2 [1 - y x(1 - x)] + 3 \cos^2 \theta_W \int_0^1 dx dy \frac{y}{B_2(\lambda_i, \lambda_j)} \right]$$ \hspace{1cm} (11)

$$i\Gamma_d = \frac{1}{8} \Delta_{ii} (1 - 2 \sin^2 \theta_W) \lambda_i \left[ C_{UV} - 2 \int_0^1 dx dy \frac{y}{B_2(\lambda_i)} \right]$$ \hspace{1cm} (12)
\[ i \Gamma_e + i \Gamma_f = - \Delta_{ii} \frac{\sin^2 \theta_W}{\cos \theta_W} \lambda_i \int_0^1 \frac{dxdy \ y}{B_1(\lambda_i)} \] (13)

\[ i \Gamma_g + i \Gamma_j = \frac{1}{4} \Delta_{ii}(1 - 2 \sin^2 \theta_W) \left[ \frac{\lambda_i}{1 - \lambda_i} + \frac{\lambda_i^2 \ln \lambda_i}{(1 - \lambda_i)^2} \right] \] (14)

\[ i \Gamma_h + i \Gamma_k = - \frac{1}{8} \Delta_{ii}(1 - 2 \sin^2 \theta_W) \lambda_i \left[ C_{UV} + \frac{3}{2} - \frac{1}{1 - \lambda_i} - \frac{\lambda_i^2 \ln \lambda_i}{(1 - \lambda_i)^2} \right] \] (15)

Above, the summation convention is assumed for repeated indices, which run over all the Majorana neutrino mass-eigenstates \( n_i \). Also, the functions \( B_1(\lambda_i) \) and \( B_2(\lambda_i, \lambda_j) \) appearing in eqs (9)–(13) are defined as

\[ B_1(\lambda_i) = (1 - y)\lambda_i + y[1 - \lambda_Z \ yx(1 - x)] \] (16)

\[ B_2(\lambda_i, \lambda_j) = 1 - y + y[\lambda_i + (1 - x)\lambda_j - \lambda_Z \ yx(1 - x)] \] (17)

Notice that the \( UV \) divergences are mass-dependent expressions and vanish when all the diagrams are added together. To make this explicit, we list the following useful identities [5,6]:

\[ \sum_{i=1}^{2n_G} B_{li} B_{l^*_i} = \delta_{l_1l_2} \] (18)

\[ \sum_{i=1}^{2n_G} B_{li} C_{ij} = B_{lj} \] (19)

\[ \sum_{i=1}^{2n_G} m_{ni} B_{li} C^*_{ij} = 0 \] (20)

\[ \sum_{k=1}^{n_G} B^*_{l^*_k} B_{l^*_k} = C_{ij} \] (21)

\[ \sum_{i=1}^{2n_G} m_{ni} B_{l^*_i} B_{l^*_i} = 0 \] (22)

Analytically, eq. (18) represents the generalized version of the \( GIM \) mechanism which results in the cancellation of the mass-independent \( UV \) divergences in the graphs (1a), (1c), (1g) and (1j). The diagram (1d) cancels the \( UV \) pole of (1h) and (1k), while the \( UV \) constant in eq. (10) vanishes due to eq. (20). As a result of these cancellations, the transition amplitude for \( Z^0 \rightarrow l_1 \bar{l}_2 \) will be finite. Another consequence of the identities (18)–(22) is...
that they enormously reduce the number of independent mixing parameters $B_{ij}$ and $C_{ij}$. Thus, in our numerical analysis we make use of the fact that

$$B_{l_1 l_2} B_{l_2 l_1}^* F(\lambda_i) = B_{l_1 N_i} B_{l_2 N_i}^* [F(\lambda_{N_i}) - F(0)] \quad (23)$$

$$B_{l_1 l_2} C_{l_2 l_1} G(\lambda_i, \lambda_j) = B_{l_1 N_i} B_{l_2 N_j}^* \left\{ C_{N_i N_j} \left[ G(\lambda_{N_i}, \lambda_{N_j}) - G(\lambda_{N_i}, 0) - G(0, \lambda_{N_j}) \right. \right.$$

$$\left. + G(0, 0) \right\} + \delta_{N_i N_j} \left[ G(\lambda_{N_i}, 0) + G(0, \lambda_{N_i}) - G(0, 0) \right] \quad (24)$$

For the sake of illustration, we shall restrict our discussion to a two generation model in the following. Employing eqs (18)–(22), we find the following useful relationships among the different CKM-mixing combinations:

$$B_{l_1 N_2} B_{l_2 N_1}^* = \frac{m_{N_2}}{m_{N_1}} B_{l_1 N_1} B_{l_2 N_1}^* \quad B_{l_1 N_2} C_{N_2 N_2} B_{l_2 N_2}^* = \frac{m_{N_2}^2}{m_{N_1}^2} B_{l_1 N_1} C_{N_1 N_1} B_{l_2 N_1}^* \quad (25)$$

$$B_{l_1 N_1} C_{N_1 N_2} B_{l_2 N_1}^* = \frac{m_{N_1}}{m_{N_2}} B_{l_1 N_1} C_{N_1 N_1} B_{l_2 N_1}^* \quad (26)$$

$$B_{l_1 N_1} C_{N_1 N_2} B_{l_2 N_1}^* = \frac{m_{N_1}}{m_{N_2}} B_{l_1 N_1} C_{N_1 N_1} B_{l_2 N_1}^* \quad (27)$$

In order to pin down numerical predictions, we can estimate $C_{N_1 N_1}$ by using Schwartz’s inequality in eq. (21), namely

$$C_{N_1 N_1} \geq 2 |B_{l_1 N_1} B_{l_2 N_1}^*| \quad (28)$$

Then, in this illustrative two generation model we are left with only one free mixing parameter; the CKM-mixing combination $\zeta = B_{l_1 N_1} B_{l_2 N_1}^*$. In the $e - \mu$ system, this quantity is strongly constrained to be $\zeta \leq 3 \times 10^{-3}$ from the non-observation of the decay mode $\mu \rightarrow e\gamma$ [10]. Constraints referring to $e - \tau$ or $\mu - \tau$ system are, however, much weaker and allow for mixing values of $\zeta \leq 10^{-1}$ [11].

In the heavy neutrino limit (e.g. $m_{N_1} \simeq m_{N_2} \simeq m_N \gg M_Z$) the amplitude $\mathcal{A}$ for the lepton-flavor-violating decay of the $Z^0$ behaves like

$$\mathcal{A}(Z^0 \rightarrow l_1 l_2) \simeq - \frac{g_W v_w}{16\pi \cos \theta_W} \frac{\zeta^2 m_N^2}{M_Z} \bar{u}_l \gamma^\mu(1 - \gamma_5) v_{l_1} \epsilon_Z^\mu \quad (29)$$
From this we can get a first estimate for the branching ratio of the rare decays,
\[ Br(Z^0 \rightarrow l_1 \bar{l}_2 + \bar{l}_1 l_2) \simeq \frac{\alpha_W^3}{192\pi^2 \cos^2 \theta_W} \frac{M_Z}{\Gamma_Z} \zeta^4 \frac{m_N^4}{M_W^4} \]  
(30)

where \( \Gamma_Z \) denotes the total decay width of the \( Z^0 \) boson, given by the central value \( \Gamma_Z = 2.534 \text{ GeV} \) [12]. For a given fixed value of \( \zeta \), eq. (30) shows a dramatic fourth power dependence of the heavy neutrino mass \( m_N \). In table 1 we present the numerical results for an exact numerical computation of the branching ratios \( Br(Z^0 \rightarrow l_1 l_2) \), using a value of \( \zeta = 0.05 \). We can readily see that experimental bounds on \( Br(Z^0 \rightarrow e\tau, \mu\tau) \) of the order of \( 10^{-5} - 10^{-6} \) [12] lead to combined constraints on the lepton-flavor-mixing parameter \( \zeta \) and the heavy neutrino mass \( m_N \). In fig. (2) we show the results of such an analysis for two nearly degenerate neutrinos. However, we have also checked that the numerical results do not show any essential change when one considers large mass differences between the two heavy Majorana neutrinos. In that case, \( m_N \) in fig. (2) indicates the mass of the lightest heavy Majorana neutrino.

At this point an additional remark on the heavy neutrino masses is in order. For a fixed value of the mixing parameter \( \zeta \), the heavy neutrino mass cannot be arbitrarily large, since \( m_N \) is constrained by the requirement of pertubative unitarity. We can safely estimate this unitarity bound by imposing the inequality condition \( \Gamma_N/m_N \leq 1/2 \), where \( \Gamma_N \) is the width of the heavy neutrino \( N \). As calculated in [5], we obtain the inequality
\[ m_N \leq 2M_W \sqrt{\frac{1}{\alpha_W \zeta}} \]  
(31)

It should be also emphasized that, unlike the leptonic flavor-changing \( Z^0 \) decays, the rare decays \( \mu \rightarrow e\gamma \) or \( \tau \rightarrow \mu\gamma \) show a constant behavior with increasing heavy neutrino mass in most theories. This can be attributed to the fact that the Feynman graph (1b), which gives the crucial \( m_N^2 \)-dependence in eq. (29), is absent when the \( Z^0 \) boson is replaced by a photon.

It is also important to notice that similar strong mass dependence of the transition amplitude (i.e. eq. (29)) has previously been observed in the quark sector in \( Z^0 \) decays [13], e.g. in \( Z^0 \rightarrow b\bar{b}, Z^0 \rightarrow b\bar{s} \), where the top quark plays the role of the heavy neutrinos. This
strong mass dependence will be a common feature for theories based on the spontaneous symmetry breaking mechanism. In ref. [8], the possibility of lepton-flavor-changing $Z^0$ decays was studied in a superstring inspired SM. The enhancement effect due to intermediate heavy neutrinos should be also present in that model, if terms proportional to $\zeta^2 \frac{m_N^2}{M_W^2}$ are not neglected and the whole perturbatively allowed parameter space is considered.

In conclusion, we have explicitly demonstrated that $SU(2) \otimes U(1)$ models with more than one right-handed neutrino can theoretically account for sizeable lepton-flavor-nonconservation effects at the $Z^0$ peak. Another attractive theoretical feature of these models is that such rare leptonic $Z^0$ decays are generally not suppressed by the usual "see-saw" mechanism. Obviously, an improvement of the current experimental limits on possible non-universality effects in the tau lifetime [14] and on $Br(Z^0 \rightarrow \tau e$ or $\tau \mu)$ provides a possibility of imposing upper bounds on the masses of the heavy Majorana neutrinos.

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Figure and Table Captions

Fig. 1: Feynman graphs responsible for the effective $Z^0 - l_1 - l_2$ coupling ($l_1 \neq l_2$) in the Feynman–’t Hooft gauge.

Fig. 2: Theoretical bounds on the mass of the heavy Majorana neutrino $m_N$ and the $CKM$ mixing combination $\zeta = B_{l_1N}B_{l_2N}^*$ in a two generation model. These bounds result from the following requirements: (i) $Br(Z^0 \rightarrow \bar{l}_1 l_2 + l_1 \bar{l}_2) \leq 10^{-5}$ (solid line), (ii) $Br(Z^0 \rightarrow \bar{l}_1 l_2 + l_1 \bar{l}_2) \leq 10^{-6}$ (dashed line), (iii) the validity of pertubative unitarity (i.e. $\Gamma_N/m_N \leq 1/2$) (dash-dotted line). The area lying to the right of the curves is not allowed due to the conditions mentioned above.

Tab. 1: Numerical results of leptonic flavor-changing $Z^0$ decays in a two generation model. We have further assumed that $m_{N_1} \simeq m_{N_2}$ and $\zeta = B_{l_1N_1}B_{l_2N_1}^* = 0.05$. 

Table 1

| $m_N$ [TeV] | $Br(Z^0 \rightarrow \bar{l}_1 l_2 + l_1 \bar{l}_2)$ |
|------------|---------------------------------|
| 0.5        | $3.4 \times 10^{-8}$            |
| 1.         | $3.4 \times 10^{-7}$            |
| 2.         | $3.6 \times 10^{-6}$            |
| 3.         | $1.6 \times 10^{-5}$            |
| 4.         | $4.7 \times 10^{-5}$            |
| 5.         | $1.1 \times 10^{-4}$            |
| 6.         | $2.3 \times 10^{-4}$            |