Implication of Classical Black Hole Evaporation Conjecture to Floating Black Holes

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In Randall-Sundrum single-brane (RS-II) model, it was conjectured that there is no static large black hole localized on the brane based on adS/CFT correspondence. Here we consider the phase diagram of black objects in the models extended from the RS-II model. We propose a scenario for the phase diagram consistent with the classical black hole evaporation conjecture. The proposed scenario indicates the existence of a rich variety of the families of black objects.

In the Randall-Sundrum single-brane (RS-II) model,\(^1\) no stable large black hole solution localized on the brane is known. We proposed a conjecture that such a large localized black hole solution does not exist,\(^2,3\) based on adS/CFT correspondence.\(^4,5\) Once gravitational collapse occurs on the brane, the collapsed object will form something like a black hole, but it should eventually evaporate within the classical dynamics. In the dual CFT picture this evaporation can be interpreted as back-reaction due to the Hawking radiation. Although there might be some possible objections to this conjecture,\(^7\) this naive correspondence works pretty well in all known examples.\(^5,8,12\)

In the previous studies,\(^13,14\) small black hole solutions localized on the brane have been constructed numerically, but the numerical construction of solutions becomes more and more difficult as the horizon size increases. There are a few analytical works on the existence of small localized black holes, but the results are not conclusive yet.\(^15,16\) Our interpretation of this numerical results is that localized black hole solutions exist if and only if their size is small compared with the bulk curvature scale, \(\ell\), although recently results which suggest the possibility that even a small localized black hole solution does not exist in a strict sense were given.\(^17\)

The phase diagram of the black objects in the RS-II model has not been clarified yet, but the diagram in the usual KK compactification (un-warped two-brane model) has been established.\(^18,19\) These two models are continuously connected with each other in the space of model-specifying parameters, the bulk curvature length \(\ell\) and the brane separation \(d\). As we vary these parameters, the phase diagram of the black objects will also change continuously. Then, there must be a consistent scenario for the phase diagram in which only small black hole solutions are allowed in the RS-II model.

In this letter we discuss the phase diagram of black objects, considering models continuously connected to the RS-II model. Our basic assumption is that, when the model-specifying parameters are continuously varied, a sequence of solutions should also change continuously. We propose a consistent scenario of the phase diagram, assuming that the classical black hole evaporation conjecture is correct.
Phase diagram in two-brane model: Here we begin with discussing the phase diagram of black objects in two-brane model with un-warped bulk. The phase diagram is summarized in Fig. [18–20] The horizontal axis represents the degree of deformation from the uniform black string. The non-uniform black string branch starts at the Gregory-Laframme instability point, where the horizon radius measured on the brane is as large as the brane separation $d$. At some point, the sequence of non-uniform black string solutions continues to that of localized black hole solutions. We refer to this branch as the primary branch. In the same plot, we have also shown another curve corresponding to the sequence starting at the second Gregory-Laframme instability point where the gradient of the zero mode has one node in the bulk. We refer to this branch as the secondary branch. There are infinitely many branches in a similar manner, but here we focus only on the primary and the secondary branches. In the un-warped case, when we trace a sequence of solutions, a non-uniform black string in the secondary branch is detached from both branes simultaneously. On the black hole branch, therefore the black hole is floating in the bulk.

The first question is how this diagram is modified once we introduce the warp in the bulk by adding a bulk negative cosmological constant and appropriate tensions on the branes. Notice that the primary branch has two solutions, depending on which brane has the larger rescaled cross-section with the horizon, where the rescaled cross-section means the cross-section divided by the squared background warp factor evaluated on the brane. When there is no warp, these two solutions are degenerate because two boundary branes are equivalent. Once the extra-dimension is warped, however, these two solutions are not equivalent any more. Here we focus on the solution whose rescaled horizon section is larger on the UV brane.

We point out that the floating black hole in the un-warped case cannot stay apart from the UV brane when $\ell$ gets smaller. This can be understood as follows. Here we assume the Randall-Sundrum condition, i.e. the brane tension is fine-tuned to accept Minkowski brane. The gravity between the UV brane, which has positive tension, and a particle floating in the bulk is repulsive. To see this, we evaluate the acceleration of a test particle fixed at a spatial point in the Poincare coordinates, in...
which adS metric is written as
\[ ds^2 = dy^2 + e^{-2y/\ell} (-dt^2 + dx^2) , \]

The acceleration is calculated as
\[ a = \frac{(\log g_{tt})_y}{\sqrt{g_{yy}}} = -\frac{1}{\ell} , \]

which means that the acceleration toward the UV brane needed to keep the position of a particle fixed in the bulk is \( y \)-independent. To have a static configuration, the only way to compensate this repulsive (attractive) force from the UV (IR) brane is the self-gravity caused by the mirror images of the particle itself on the other side of the branes.

From the above observation, we expect that the equilibrium position of the floating black hole should move toward the UV brane as \( \ell \) decreases. However, the attractive force between two black holes will be, at most, of \( O(1/R) \) with \( R \) being the size of the black holes. If \( R \gg \ell \), the self-gravity of black holes will not be sufficient to compensate the repulsive force from the UV brane. Thus, a black hole with \( R \gg \ell \) cannot float in the bulk. In such cases, a black hole on the secondary branch necessarily touches the UV brane.

Now we are ready to discuss the phase diagram of black objects in the two-brane model with a warped extra dimension. If the topology of the phase diagram is preserved, there must be, at least, two black hole solutions localized on the UV brane for a large horizon radius (\( \gtrsim \ell \)), under the assumption that the brane separation is sufficiently large (\( d \gg \ell \)). We think that the co-existence of two branches of localized black holes looks quite unlikely.

Our basic assumption is that the classical black hole evaporation conjecture is correct. Then, most likely the phase diagram should be modified as shown in Fig. 2. The absence of a large black hole is explained by the reconnection between the primary and the secondary branches. If it were not for the secondary branch, the absence of a large localized black hole in the warped case would lead to sudden disappearance of a sequence of solutions, which would not be understood naturally.

When the brane separation is small compared with the bulk curvature length (\( d \ll \ell \)), the effect of the warp will not be significant. Hence, the phase diagram
Fig. 3. Effective potential for a small particle in the bulk in detuned-tension model with adS brane.

should be topologically identical to that given in Fig. 1. As the parameter $\ell$ decreases, the two branches will get closer. At a critical value of $\ell(\approx d)$, the two branches will touch and interchange. As a result, the phase diagram becomes topologically as given in Fig. 2.

*Extension to the detuned brane tension*: Our current discussion can be extended to more general cases by considering de-tuned brane tension, which is called the Karch-Randall model. First we consider the case that the deviation from the Randall-Sundrum condition is small. We introduce a parameter $\delta \sigma \equiv (6/\kappa_5 \ell) - \sigma > 0$, where $\sigma$ is the tension of the UV brane. Here we consider the large separation limit ($d \to \infty$) for simplicity. In order to describe the unperturbed solution with a detuned brane, it is convenient to use the coordinates

$$ds^2 = dy^2 + \ell^2 \cosh^2(y/\ell) ds^2_{adS^4},$$

where $ds^2_{adS^4}$ is the metric of four-dimensional anti-de Sitter (adS) space with unit curvature. The brane is on a $y =$ constant surface, and the value of $y$ on the brane, $y_b$, is determined by the condition

$$\kappa_5 \sigma = -\frac{6}{\ell} \tanh \frac{y_b}{\ell}.$$

The limit corresponding to the RS-II model is obtained by setting $\sigma = 6/\ell$ ($y_b \to -\infty$). When $\delta \sigma$ is small, we have $\delta \sigma \approx (12/\kappa_5 \ell) e^{2y_b/\ell}$. The very outstanding feature for $\delta \sigma \neq 0$ is that the warp factor $\ell^2 \cosh^2(y/\ell)$ is not monotonic but has a minimum at $y = 0$. When $\delta \sigma$ is sufficiently small, $-y_b$ is very large. Hence, significant deviation from the exact RS limit arises only in the region distant from the UV brane ($y \gtrsim 0$).

Let us consider a small black hole floating in the bulk. Since the acceleration of a static test particle is given by $a = (\log \sqrt{g_{00}})_y$, the effective gravitational potential (without self-gravity) becomes

$$U_{\text{eff}} = \log(g_{00}) = \log \left(\ell \cosh \frac{y}{\ell}\right).$$

The effective potential after taking into account the self-gravity will be modified as shown in Fig. 3. From this plot, we expect that there are two floating black hole
solutions when the size is small. The one close to the UV brane is unstable, while the other close to \( y = 0 \) is stable. When \( \delta \sigma \) is small, the stable floating black hole is very far from the UV brane. In the limit \( \delta \sigma \to 0 \), this black hole is infinitely far. As a result, this sequence of solutions disappears from the phase diagram of the RS-II model. The distance from the UV brane to the unstable equilibrium point will not be sensitive to the small change of \( \sigma \). Hence, this branch is smoothly connected to the diagram shown in Fig. 2.

In order to draw the phase diagram in the regime \( 0 < \delta \sigma \ll 1/\kappa_5 \ell \), it will be important to know how the branch of stable floating black holes extends to a larger size. It will be easy to imagine that this sequence also touches the UV brane when the area of the five dimensional horizon becomes sufficiently large. Again adS/CFT correspondence provides us with a method for estimating the critical size at which the floating black hole touches the UV brane.

In the asymptotically flat case we cannot construct a static quantum black hole solution due to the quantum back-reaction.\(^{24}\) In this case the back-reaction is too strong to keep the asymptotic spacetime structure unchanged. However, it is not the case in asymptotically adS spacetime.\(^{25,26}\) One possible choice of static quantum state in which the energy density on the event horizon is regular is the thermal Hartle-Hawking (HH) state. In the asymptotically flat case, the HH state leads to a constant energy density in the asymptotic region. Hence, the total mass diverges. In contrast, in the case of asymptotically adS spacetime vacuum energy density exists from the beginning. As a result, the lapse function does not converge to a constant value at a large \( r \). In fact, the lapse function of the background four-dimensional adS space is given by \( \sqrt{f} \) with

\[
 f = 1 - \left( 2\kappa_4 M/r \right) + \left( r^2 / L^2 \right),
\]

where \( L(\ll \ell) \) is the four-dimensional adS curvature scale. Since the temperature red-shifts in proportion to \( 1/\sqrt{f} \), the energy density decreases very rapidly for \( r \gg L \).

When the size of the black hole is large, the black hole temperature is low. Therefore the energy density of CFT becomes important only at a large distance. If this scale is larger than the adS curvature scale \( L \), the back-reaction effect is cut off due to the red-shift factor. Therefore there is a possibility of having a static large black hole configuration consistent with the back-reaction due to CFT. To the contrary, if the size of the black hole is not large, the back-reaction becomes important below the adS curvature scale. Then, a static quantum black hole solution will not exist as in the asymptotically flat case. This means that there is a critical size beyond which a static quantum black hole solution exists.

Following the above picture, we can estimate the minimum size of a large static black hole in adS space. We quote the results from Refs. 25). Substituting the effective number of species \( \approx \ell^2 / \kappa_4 \) that adS/CFT correspondence tells, we find that the back reaction becomes important when \( M < \sqrt{\ell L} / \kappa_4 \). This result suggests that the sequence of four dimensional large localized black holes with a small value of \( \delta \sigma \) starts with \( M \approx \sqrt{\ell L} / \kappa_4 \). In the five dimensional picture this means that the sequence of stable floating black holes should touch the UV brane when the size of the black hole is as large as \( \sqrt{\ell L} \). Thus, the phase diagram in this regime will be
There must be a sequence of solutions on the four dimensional CFT side which corresponds to the sequence of stable floating BHs that is expected to exist on the five dimensional bulk gravity side. The four dimensional counterpart should be a sequence of star solutions composed of CFT matter. Such CFT stars will be well approximated by stars made of radiation fluid, which was already studied long time ago by Page and Phillips.\(^{28}\) If we substitute the effective number of species \(\approx \ell^2/\kappa_4\) into their results, we find that the sequence of star solutions terminates where the mass of star is \(O(\sqrt{\ell L}/\kappa_4)\). The end point of the sequence is a singular solution which has diverging central energy density. Diverging density means diverging temperature, which indicates that the lapse function also vanishes at the center in this limiting case. The corresponding five dimensional metric should also have vanishing lapse function at the center on the brane. For a static black hole spacetime, the lapse function will vanish only on the event horizon. Hence, we can imagine that the moment when the bulk floating black hole just touches the brane in the five dimensional bulk gravity picture corresponds to the end point of the sequence of CFT stars in the four dimensional CFT picture. At this moment the size of the five dimensional black hole is estimated from the entropy of the corresponding CFT star to be \(O((\ell L)^{3/2})\). How the sequence continues after formation of black hole in the four dimensional CFT picture will be discussed in the forthcoming paper.\(^{29}\)

Let us further reduce the brane tension. When \(\sigma \approx 0\), \(y_b\) is close to 0. In the most of region in the bulk, the repulsive force from the UV brane is screened by the attractive nature of the bulk negative cosmological constant. As a result, a test particle feels net repulsive force from the brane only in the limited small region near the brane. Thus, only a small black hole which can fit within this tiny region can float in the bulk.

In the limit \(\sigma \rightarrow 0\) the floating branches are not allowed at all. The size of the horizon measured on the brane at the transition point becomes zero in this limit. As a result, one sequence of black hole solutions localized on the brane remains. In this limit, this sequence of solution is nothing but adS-Schwarzschild solution cut by a tensionless brane placed on the equatorial plane. It is already proven that there is
no branching point (= solution with a zero mode) along this sequence.\(^{30}\) This fact is completely in harmony with our phase diagram.

To summarize, we have shown it possible to describe a scenario of the phase diagram evolution of black objects in models connected to RS-II brane world model, with a small number of assumptions. The obtained phase diagram is perfectly consistent with adS/CFT correspondence. An interesting point to stress is that in the Karch-Randall detuned tension brane world the existence of large static black holes localized on the brane is naturally expected in contrast to RS-II model. We will apply the numerical methods used to investigate black hole solutions in the RS-II model\(^{13\text{),}31}\) to the Karch-Randall case in the future work.

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