Inertial Single Vehicle Trajectory Prediction Baselines and Applications with the NGSIM Dataset

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Abstract—In the recent vehicle trajectory prediction literature, the most common baselines are briefly introduced without the necessary information to reproduce it. In this article, we produce reproducible vehicle prediction results from simple models. For that purpose, the process is explicit, and the code is available. Those baseline models are a constant velocity model and a single-vehicle prediction model. They are applied on the NGSIM US-101 and I-80 datasets using only relative positions. Thus, the process can be reproduced with any database containing tracking of vehicle positions. Produced results on this database establish the three most used trajectory prediction performance indicators: Root Mean Squared Error (RMSE), Negative Log-Likelihood (NLL), and Mean Absolute Error (MAE). The NLL estimation needs attention because several formulations that differ from the mathematical definition are used in other works. This article is meant to be used along with the published code to establish baselines for further work.

I. INTRODUCTION

Automation of driving tasks aims for safety and comfort improvements. For that purpose, every Autonomous Driving (AD) system relies on an anticipation of the traffic scene movements. Freeway datasets NGSIM I-80 and US-101, have been extensively used for various applications, among which trajectory prediction. Many variations of trajectory prediction algorithms have been produced in the literature. Since they adopt different strategies, comparing them can be difficult. In those cases, performance evaluations of simple models are used as a reference. The most common reference model is kinematic prediction. The Kalman filter \cite{1} is used to compute the kinematic state of the vehicles and the associated uncertainty before producing predictions. Then its predictive step is repeatedly used to compute predictions and the associated error estimations. On freeway situations, the assumption of constant velocity is a good fit and leads to the simplest models. However, various choices of Kalman filter model and parameters may produce different results. Therefore, it is important to fit them to the dataset as cautiously as the parameters of the algorithm to be compared. In this article we present a process to fit a constant velocity Kalman filter to the NGSIM I-80 and US-101 dataset. We obtain better results on negative log-likelihood (NLL) than published results for the same model, and on the same data. Similarly to the conclusions made on pedestrian motion prediction in \cite{2}, comparing this new baseline with state of the art predictions using the NLL shows that error covariance prediction from state of the art models are to be improved.

As an application, we improve the covariance estimation of a machine learning model based on Long Short-Term Memory (LSTM \cite{3}) that predicts the future trajectory of a single vehicle without interaction. In our model, the LSTM function is used as a command prediction in the predictive step of a Kalman filter instead of the canonical encoder-decoder architecture. Comparing results from the two architectures shows an improvement on the NLL evaluation.

II. TRAINED KALMAN FILTER PARAMETERS FOR PREDICTION

The sanity check model for highway trajectory prediction is a constant velocity prediction. It seems simplistic and easy to implement but various results may be found in the literature with no explanation of the specific implementation and parameter choice. For instance \cite{4} and \cite{5} implemented two different Kalman filters that are both called constant velocity and produce predictions with associated error covariance matrices but no details are given to reproduce them. Consequently, the different results that they obtain cannot be interpreted.

This section writes the Kalman filter equations and describes the process to learn its parameters.

A. Definition of the constant velocity prediction

Constant velocity prediction is a short expression that efficiently describes what the model does. However, it is an abusive expression because the velocity is modeled as quasi-constant. In fact, an acceleration is represented by a zero-mean random noise.

Several expressions of constant velocity models for Kalman filtering with positions as observations can be made. We use the state vector $X = (x, v_x, y, v_y)^T$ because it is the simplest linear Kalman filter for a constant velocity prediction. However, other work might choose $X = (x, y, \theta, v)^T$, with the equality : $(v_x, v_y) = (\cos(\theta)v, \sin(\theta)v)$.

The bicycle model, with a state vector $(x, y, \theta, v, \omega, \alpha)$, respectively position, heading angle, velocity, wheel angle, and acceleration describes the actual vehicle dynamics with no slip approximation. The constant velocity model might have a different meaning with this last state vector because the wheel angle could be considered constant or set to 0 along with the acceleration. Many variations of these models may be used. Consequently, even if it is a simplistic model, the
The Kalman filter. For each sample from the training set, a predictive step is used for prediction with no observation. These observations are used sequentially to update a Kalman filter from its initial point, so it reaches a good state estimation at t₀. With our linear model, the evolution of the state X from step k to step k+1 is written as follow:

\[ X_{k+1} = AX_k + E\tilde{a}_k \]  

(1)

A is the transition matrix, it represents the evolution model. In our case, \( A = \begin{pmatrix} A_x & 0 \\ 0 & A_y \end{pmatrix} \) and \( A_z = A_y \). With a timestep \( dt \), \( A_k = \begin{pmatrix} 1 & dt \\ 0 & 1 \end{pmatrix} \). E is the noise matrix and \( \tilde{a}_k = (\tilde{a}_{x_k}, \tilde{a}_{y_k})^T \) is the noise. We chose to represent the noise as an acceleration thus \( E = \begin{pmatrix} E_x & 0 \\ 0 & E_y \end{pmatrix} \) and \( E_x = E_y \).

With a timestep \( dt \), \( E_x = \frac{E_y^2}{2} dt^2 \).

The Kalman filter consists in three steps: prediction, innovation, update. The prediction uses the model to predict the future state estimation \( \hat{X}_{k+1|k} \) from the current state estimation \( \hat{X}_{k|k} \). The covariance matrix \( P \) is updated with the model matrix A and with a process noise Q. With the hypothesis that velocity variations are produced by a white noise centered Gaussian acceleration, the process noise \( Q \) can be written \( Q = EQ_d Q_d^T E_d^T \) with \( Q_d \) a square matrix of learned parameters acting as a factorized acceleration noise matrix. The observation at time \( t_k \) is \( Z_k = (x_k, y_k)^T \). It is matched with the positions from the state vector \( X \) with

\[ H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \]

in the innovation step.

Prediction:

\[ \dot{\hat{X}}_{k+1|k} = AX_{k|k} \]

\[ P_{k+1|k} = AP_{k|k}A^T + Q \]  

(2)

Innovation:

\[ \epsilon_{k+1} = Z_{k+1} - H\hat{X}_{k+1|k} \]

\[ S_{k+1} = HP_{k+1|k}H^T + R \]  

(3)

Update:

\[ K_{k+1} = P_{k+1|k}H^TS_{k+1}^{-1} \]

\[ \hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1}\epsilon_{k+1} \]

\[ P_{k+1|k+1} = P_{k+1|k} - K_{k+1}H_{k+1}P_{k+1|k} \]  

(4)

The algorithm 1 describes the kinematic prediction using the Kalman filter. For each sample from the training set, a state \( X_{-15} \) and the covariance matrix \( P_{-15} \) is initialized, then all three steps of the Kalman filter are computed on the 3 seconds history using the parameters \( \sigma_a \) and \( R \) and the observations \( Z_{k=-14|0} \). This allows it to reach the state estimation \( \hat{X}_0 \) at time \( t_0 \). From this time on, only the predictive step is used for prediction with no observation to update it. The predicted observation for \( k \geq 0 \) is \( \hat{Z}_k = H\hat{X}_k|0 \). The associated error covariance matrix estimation is \( P_k^Z = HP_k|0H^T \). Since there is no new observation after this step, the notation showing the conditioning to the last observation can be omitted because it is always 0. As many steps as necessary to fill the desired prediction horizon are made. We fixed this to 25 steps of 0.2 seconds producing a 5 seconds sequence as prediction.

Algorithm 1 Kalman prediction

Require: \( Q, R \)
for \( Z^i \) in training set do
Set: \( X^i_{-15}, P^i_{-15} \)
for \( k \) from -14 to 0 step 1 do
\( \hat{X}^i_k, P^i_k \leftarrow \text{KalmanFilter}(Z^i_k, \hat{X}^i_{k-1}, P^i_{k-1}, Q, R) \)
end for
for \( k \) from 1 to 25 step 1 do
\( \hat{X}^i_k, P^i_k \leftarrow \text{KalmanPrediction}(\hat{X}^i_{k-1}, P^i_{k-1}, Q) \)
end for
end for
return \( \hat{Z}^i_k, P^i_{Z^i_k}=1, k=1,25 \)

The predicted position sequences and the estimated error covariance are compared with the observed next 5 seconds from the dataset. The average over the predicted time sequence of the Negative Log-Likelihood loss (NLL) is to be minimized over a training set from the database. When averaged over time, it defines a scalar loss function to be minimized. Otherwise, at each prediction horizon, it used on the test set as performance indicators. Performance indicators are defined in the next section. The Kalman filter function arguments are the current observation and the trainable parameters written \( \text{args} = (\rho, \sigma_a, R, \text{init}) \in ([-1,1]^4, \mathbb{R}^2_+, \mathbb{Z}_n^2_+, (\mathbb{R}^4, \mathbb{R}^4_+)) \), \( \mathbb{S}^n_+ \) being the set of symmetric positive definite matrices of size \( n \times n \). The past observations written \( Z_h = \{Z_k\}_{k=-14,0} \). The predicted sequence computed using the algorithm 1 is written \( \text{KalmanPred}(Z_h, \text{args}) \). The future observations are written \( Z_f = \{Z_k\}_{k=1,25} \).

The minimization performed to learn the parameters is written:

\[ \arg\min_{\text{args}} \text{loss} (\text{KalmanPred}(Z_h, \text{args}), Z_f) \]

The model is implemented with the Pytorch library. The parameters args are fitted to the training set using a combination of the RAdam optimizer [6], and the look ahead optimization [7] called ranger and available on Github[3]. Our code for data preprocessing and model training is also accessible on Github[3].

B. Prediction performance indicators

1) Global indicators: Once the parameters of the model are computed, an evaluation of the predictive performance of

[3] https://github.com/jmercat/KalmanBaseline

Ranger-Deep-Learning-Optimizer
the model is made. In this section, the three most common performance indicators are defined and computed on a test set from the NGSIM database. This test set is built using the code from [4]. It is important to note that when used as performance evaluation indicators, the evolution of the predicted horizon is kept. The indicators are functions of the prediction step $k$ or as function of time $t$. The loss to be minimized is a scalar value computed as the average over time of the NLL.

The RMSE computation is made with equation (5) with $(x_k^i, y_k^i)$ the observed positions and $(\hat{x}_k^i, \hat{y}_k^i)$ the predicted positions of the $i^{th}$ sequence at time $t_k$. $N$ is the number of sequences in the subset of the database on which the computation is made. In our case, it is the number of test sequences which is more than 1.5 million sequences (with sequence overlap).

$$\text{RMSE}(k) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_k^i - \hat{x}_k^i)^2 + (y_k^i - \hat{y}_k^i)^2}$$

The MAE computation is made with equations (6).

$$\text{MAE}_x(k) = \frac{1}{N} \sum_{i=1}^{N} |x_k^i - \hat{x}_k^i|$$

$$\text{MAE}_y(k) = \frac{1}{N} \sum_{i=1}^{N} |y_k^i - \hat{y}_k^i|$$

Reported NLL values from articles [4], [8], [9] have chosen a variation of the NLL definition. However, to avoid confusion in our result comparison, the equation (9) using the NLL formulation from equation (8) is used.

By definition, with $f$ a probability density function, the NLL is written:

$$\text{NLL} = -\ln(f(Z_f|Z_h))$$

The global NLL over time is not used. Instead the NLL value for each timestep is computed. The time dependency is not explicitly written with index $k$ to simplify. At each timestep $k$, the prediction error is modeled as a bivariate Gaussian probability, thus $f(Z_f|Z_h)_k = \mathcal{N}((Z_f - \hat{Z}_f)_k, \Sigma_k)$ with

$$\Sigma_k = P_k = \begin{pmatrix} \sigma_x^2 & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{pmatrix}$$

The errors along $x$ and $y$ axis at time $t_k$ are written $d_x$ and $d_y$. The coefficients $\sigma_x$, $\sigma_y$, and $\rho$ are identified with the coefficients of $P_k$. Then the NLL of the prediction at a fixed timestep $k$ is given by equation (8).

$$\text{NLL}(dx, dy, \Sigma) = \frac{1}{2} \left( \frac{d_x^2}{\sigma_x^2} + \frac{d_y^2}{\sigma_y^2} - 2\rho \frac{d_x d_y}{\sigma_x \sigma_y} \right) + \ln\left(\frac{\sigma_x \sigma_y}{\sqrt{1-\rho^2}}\right) + \ln(2\pi)$$

Over the dataset, the mean NLL (MNLL) value at time $t_k$ is given by equation (9).

$$\text{MNLL}(k) = \frac{1}{N} \sum_{i=1}^{N} \text{NLL}((x_k^i - \hat{x}_k^i, y_k^i - \hat{y}_k^i, P_k))$$

Results from the computation of these performance indicators over the test set are reported in table I. All RMSE and Mean Absolute Error (MAE) values are similar to the same indicators from the literature. However, the NLL values vary. This means that all predictions share the same accuracy but have different standard deviation estimation accuracy.

2) Covariance prediction assessment: Figure 1 shows a combined representation of the MAE as a parametric curve ($\text{MAE}_x(t), \text{MAE}_y(t)$), three of the mean error covariance estimations, and the matching computed error covariance estimations. The error covariance matrices are computed at 1s, 3s and 5s of the trajectory predictions over the test set. They are compared with the mean values of the estimated error covariance matrices. If the prediction error are unbiased, the average of the estimated error covariance matrices at time $t_k$ is an estimation of the global error covariance matrix at the same time:

$$\text{COV}_{Z_k \in \text{dataset}} (\hat{Z}_k - Z_k) \approx \frac{1}{N} \sum_{i=1}^{N} P_k$$

The error caused by averaging covariances with small bias grows linearly with the bias magnitude. In our case, the error bias is lower than 5% of the RMSE at all prediction time. Thus, the average of the error covariances is a good estimation of the predicted overall error covariance. Using this result, it can be compared with the overall error covariance estimated from the prediction error over the whole test set. In figure 1 the error covariances are represented as ellipses. There is a good match between predicted error covariance ellipses and the global error covariance ellipses. This is also assessed by the low MNLL values in table I.

The NGSIM dataset contains errors, as shown by the consistency analysis [10]. Thus, the distance between the predicted trajectories and the observed future is caused by prediction errors but also by measurements errors. This means that perfect predictions would produce positive error values. To appreciate this, the NLL value considers the estimated variance of the prediction. This accounts for both error
From [4] 3.72 5.37 6.40 7.16 7.76  
ours 0.11 0.21 0.30 0.39 0.47

estimated standard-deviation is written $\sigma_{\hat{u}_k}$. The command sources making it a preferable indicator for performance comparisons than RMSE or MAE. Moreover, the NLL is extendable for other distributions such as Gaussian Mixtures whereas MAE and RMSE are not good error indicators for multimodal predictions. For these reasons, having a well-defined baseline correctly estimating its error covariance and its NLL evaluation is important before producing more complex models.

### III. Model based RNN predictions

In this section, we extend the previous model with a prediction model that depends on the acceleration and a recurrent neural network for command prediction. The predicted command is a variation of the acceleration which can be interpreted as a jerk. This relies on a Kalman filter using a state with velocities and accelerations $X = (x, v_x, a_x, y, v_y, a_y)^T$.

The evolution of the state is written as follow:

$$X_{k+1} = AX_k + Bu_k + E\tilde{j}_k$$ (11)

with the transition matrix

$$A = \begin{pmatrix} A_x & 0 \\ 0 & A_y \end{pmatrix}$$

$$A_x = A_y = \begin{pmatrix} 1 & dt & dt^2/2 \\ 0 & 1 & dt \\ 0 & 0 & 1 \end{pmatrix}$$

We write $\tilde{j}_k = (\tilde{j}_{xk}, \tilde{j}_{yk})^T$ as the noise jerk vector. It is modeled as a zero-mean Gaussian random variable. Its estimated standard-deviation is written $\sigma_{\tilde{a}_k}$. The command matrix $B$ and the noise matrix $E$ are equal because the command and the noise are both jerk.

$$B = E = \begin{pmatrix} dt^3/6 & dt^2/2 & dt & 0 & 0 & 0 \\ 0 & 0 & dt^3/6 & dt^2/2 & dt \end{pmatrix}^T$$

Thus the evolution of the state simplifies to:

$$X_{k+1} = AX_k + B(\hat{u}_k + \tilde{j}_k)$$ (12)

The Kalman filter with recurrent command prediction consists of the three Kalman steps with an extra line for the command prediction in the prediction step written in equations (13). This new line defines a function with two outputs: $\hat{h}_k = (\hat{u}_k, \hat{q}_{\tilde{a}_k})$, and $c_k$, $\hat{h}_k$ contains the command and its variance, $c_k$ is a recurrent value that serves as memory from one timestep to the next. All three vectors are defined with a single equality in equation (13).

New Prediction:

$$(\hat{u}_k, \hat{q}_{\tilde{a}_k}, c_k) = \text{RNNCell}(\hat{X}_{k|k}, (\hat{u}_{k-1}, \hat{q}_{\tilde{a}_{k-1}}, c_{k-1}))$$

$$\hat{X}_{k+1|k} = AX_k + B\hat{u}_k$$

$$P_{k+1|k} = AP_{k|k}A^T + Bq_{\tilde{a}_{k}}q_{\tilde{a}_{k}}^TB$$ (13)

Innovation and update steps are written the same way as equations (3) and (4). For the RNNCell function we chose an LSTM cell. Its computation is written in equation (14), with the output $h_k = (\hat{u}_k, \hat{q}_{\tilde{a}_k})$. The symbol "$\ast$" denotes element-wise multiplication. The functions $\sigma : x \rightarrow \frac{1}{\sqrt{\pi \sigma^2}}$ and tanh are applied elementwise. The LSTM cell depends on parameters stored in matrices $W^i, W^f, W^o, W^g, U^i, U^f, U^o, U^g$. These parameters are fitted in the loss minimization process.

$$i_k = \sigma(\hat{X}_kU^i + h_{k-1}W^i)$$

$$f_k = \sigma(\hat{X}_kU^f + h_{k-1}W^f)$$

$$o_k = \sigma(\hat{X}_kU^o + h_{k-1}W^o)$$

$$c_k = \tanh(\hat{X}_kU^g + h_{k-1}W^g)$$

$$h_k = \tanh(c_k) * c_k$$

$$h_k = \tanh(c_k) * o_k$$ (14)

Exactly as before, predictions are produced with the algorithm [4] however they are computed with this new Kalman filter. Another modification is that the predicted commands are computed with the RNNCell but not used during the filtering phase. This allows the initialization of
TABLE II: Comparison of RMSE and NLL results for single learned dynamic models observing the trajectory of a single vehicle without context nor interaction. The NGSIM test set preprocessed with the code published by [4] is used. *Results from our training of their model.

| Time horizon | 1s   | 2s   | 3s   | 4s   | 5s   |
|--------------|------|------|------|------|------|
| RMSE         |      |      |      |      |      |
| V-LSTM [4]*  | 0.64 | 1.57 | 2.79 | 4.30 | 6.06 |
| ours         | 0.68 | 1.54 | 2.72 | 4.20 | 5.95 |
| MNLL         |      |      |      |      |      |
| V-LSTM [4]*  | 2.24 | 3.94 | 4.90 | 5.58 | 6.11 |
| ours         | 0.57 | 1.55 | 2.50 | 3.19 | 3.73 |

the state estimation and recurrent parameters of the RNNCell without demanding to the command predictor to be robust to the initially bad estimations of the state. Then, during the prediction phase, the commands are computed and used. The model parameters are learned on the training set with the minimization of the NLL loss.

We obtain the results shown in table II compared with results from the V-LSTM prediction from [4] that is made with an LSTM encoder and LSTM decoder (we recomputed the results to harmonize the NLL definition). This shows that forcing this structure is not detrimental for RMSE performances and the Kalman filter makes a better estimation of the covariance. Using this, maximum acceleration and velocity or other physical constraints are easy to guarantee. Moreover, the method is usable with any Kalman model in the generic form of equations (2), (3), (4). In [8], a Kalman filter using predicted accelerations from a neural network is also used. However, acceleration predictions and Kalman filtering are separated. The neural network takes observations as input and produces the acceleration prediction sequence. This sequence is then fed to the Kalman filter. In contrast, our method takes advantage of the whole kinematic state evaluation to predict the commands at each step. In [11], another combination of LSTM and Kalman filter is made. They replaced the state update with the LSTM cell (intuitively, instead of $X_{k+1} = AX_k$ they use $X_{k+1} = \text{LSTM}(X_k)$). In our case, the kinematic model forces the trajectory to have inertia which is known to play an important part. It keeps a kinematic interpretation of the state $X$ and does not require the LSTM Jacobian matrix computation.

IV. CONCLUSIONS

We have established reproducible baselines results for simple prediction methods on the NGSIM dataset. Surprisingly, NLL results were much lower than comparable published baselines and should push further work toward better estimation of prediction error covariance. A global covariance estimation visual assessment has been presented with superposition of ellipses from computed prediction error covariance knowing the future estimations and mean estimated error covariance at prediction time. Finally, an easy to implement extension of the constant velocity model allowing command prediction was produced. It gives a baseline for machine learning trajectory prediction models with a good error covariance estimation.

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