Light vectors coupled to anomalous currents with harmless Wess-Zumino terms

Luca Di Luzio\textsuperscript{a,b}, Marco Nardecchia\textsuperscript{c}, Claudio Toni\textsuperscript{c}

\textsuperscript{a}Dipartimento di Fisica e Astronomia ‘G. Galilei’, Università di Padova, Italy
\textsuperscript{b}Istituto Nazionale Fisica Nucleare, Sezione di Padova, Italy
\textsuperscript{c}Physics Department and INFN Sezione di Roma La Sapienza, Piazzale Aldo Moro 5, 00185 Roma, Italy

We reconsider the case of light vectors coupled to anomalous fermionic currents, focussing on the interplay between UV and IR dynamics. Taking as a general framework the gauging of the Standard Model accidental symmetries, we show that it is possible to devise an anomaly-free UV completion with mostly chiral heavy fermions such that anomalous Wess-Zumino terms are suppressed in the IR, thus relaxing would-be strong bounds from the longitudinal emission of light vectors coupled to non-conserved currents. We classify such scenarios and show that they will be extensively probed at the high-luminosity phase of the LHC via the measurement of the $h \to Z\gamma$ rate and the direct search for non-decoupling charged leptons.

Contents

1 Introduction 2

2 Gauging the Standard Model accidental symmetries 3
  2.1 UV model .................................................. 3
    2.1.1 Anomaly cancellation .................................. 3
    2.1.2 Renormalizable operators .......................... 4
    2.1.3 Spectrum ........................................... 5
  2.2 EFT of a light vector and decoupling of WZ terms .......... 6
  2.3 Neutrino masses ......................................... 9

3 Electroweak anomalons phenomenology 10
  3.1 Electroweak precision tests ........................... 10
  3.2 Higgs physics ........................................ 10
  3.3 Direct searches ....................................... 11
    3.3.1 Stable charged leptons ............................ 11
    3.3.2 Unstable charged leptons .......................... 11

4 Conclusions 11

A Calculation of the Wess-Zumino terms 12
  A.1 Toy model .................................................. 12
  A.2 $\gamma_5$ in dimensional regularization ................. 14
  A.3 1-loop matching ....................................... 15
  A.4 Reproducing the chiral anomaly in the EFT ............ 17
  A.5 WZ terms for massive vector bosons and Goldstone bosons 18
  A.6 Properties of the WZ coefficients ................. 20
1 Introduction

The physics of light spin-1 dark bosons has witnessed a growing amount of interest in the recent years, from both a theoretical and phenomenological standpoint. A standard benchmark is that of a secluded U(1) gauge boson, kinetically mixed with the photon [1] and hence universally coupled to the Standard Model (SM) sector via the electromagnetic current. This framework, although elegant and predictive, can be too restrictive for phenomenological applications and hence more general forms of the light vector boson interactions with the SM fields can be envisaged. Going beyond the kinetic mixing framework, a theoretically motivated option is provided by the gauging of the accidental global symmetries of the SM, that is baryon number $U(1)_B$ and family lepton number $U(1)_{L_i}$ (with $i = e, \mu, \tau$) in the limit of massless neutrinos. Within the SM field content, only the $L_i - L_j$ combinations turn out to be anomaly free [2–4]. Hence, in order to consistently gauge a general linear combination

$$X = \alpha_B B + \sum_{i=e,\mu,\tau} \alpha_i L_i,$$  \hspace{1cm} (1.1)

one requires new fermions, also known as anomalons, which cancel the anomalies of the new $U(1)_X$ factor, also in combination with the electroweak gauge group. Note that Eq. (1.1) is the most general linear combination of abelian global symmetries of the SM that can be gauged, under the assumption that all SM Yukawa operators are allowed in the quark sector at the renormalizable level. Barring the cases of $B/3 - L_i$ and linear combinations thereof, the anomalons need to be charged under the electroweak gauge group (henceforth indicated more precisely as electroweak anomalons).\footnote{This is not the case if some of the quark Yukawa operators arise at the non-renormalizable level, as e.g. discussed recently in Ref. [5].} In the latter case, in order to evade detection at high-energy particle colliders, the new fermions need to be heavier than the electroweak scale. Consequently, their effects on the physics of the light vector boson associated with the $U(1)_X$ gauge symmetry, here denoted as $X$, can be described within an effective field theory (EFT) approach. In particular, after integrating out the new heavy fermions at one loop, one generates dimension-4 Wess-Zumino (WZ) terms, schematically of the form $X(W\partial W + WWW)$ and $XB\partial B$ (with $W$ and $B$ denoting $SU(2)_L$ and $U(1)_Y$ gauge bosons). These contact interactions have the role of compensating, in the EFT without electroweak anomalons, the anomalous shift of the effective action due to the anomalous SM fermion current coupled to $X$ (see e.g. [6–9]).

As it was emphasized more recently in Refs. [10, 11], WZ terms display an axion-like behaviour (as can be understood by applying the equivalence theorem to the longitudinal component of $X$) and lead to amplitudes that grow with the energy. The anomalous $XW\partial W$ vertex can be dressed with SM flavour-violating interactions leading to loop-induced flavour changing neutral current (FCNC) processes, while the anomalous $XB\partial B$ vertex is responsible for $Z \to \gamma X$ decays at the tree level (see also [12–15]). In both cases these processes are enhanced as $(\text{energy}/m_X)^2$, thus resulting into the typically most stringent bounds on light vectors with no direct couplings to electrons, as e.g. in the case of gauged baryon number.

It is known (see e.g. [11]) that in the limit where the mass of the anomalons stems from a SM-preserving vacuum expectation value (VEV), the low-energy coefficients of the WZ terms are entirely fixed by the requirement of cancelling the $SU(2)_L^2 U(1)_X$ and $U(1)_Y^2 U(1)_X$ anomalies of the SM sector. On the other hand, if the anomalons pick up a mass contribution from the electroweak VEV then the coefficients of the WZ terms become model-dependent. In particular, in the limit where the anomalons mass is completely due to electroweak symmetry breaking sources, the anomalous couplings of the longitudinal component of $X$ with SM electroweak gauge bosons goes to zero, thus relaxing the above mentioned strong bounds on light vectors.

In this paper, we revisit the argument why WZ terms become harmless in the limit where the electroweak anomalons obtain their mass solely from the Higgs, classify the structure of UV
completions that allow for such a pattern and discuss their electroweak-scale phenomenology. Due to its non-decoupling nature, the phenomenology of the electroweak anomalons is tightly constrained (but not yet ruled out) by Higgs couplings measurements and direct searches, thus making the whole setup testable at the high-luminosity phase of the LHC (HL-LHC). Chiral fermionic extensions of the SM, sharing some similarities with our setup, were previously discussed in a different context in Refs. [16, 17]. Here, the main phenomenological interest consists in the physics of light (i.e. sub-GeV) vector bosons coupled to anomalous SM currents and the possibility of re-opening a large portion of parameter space, which might be probed by several low-energy experiments or help in explaining current experimental anomalies, such as e.g. that of the muon $\mu - 2$ [18].

The paper is structured as follows. Sect. 2 is the core of the work, in which we provide the general setup for the gauging of the generic linear combination of $U(1)$ factors in Eq. (1.1). We discuss in particular the heavy anomalons sector leading to the cancellation of gauge anomalies and compute the resulting WZ terms in the EFT. In passing, we also deal with the issue of neutrino masses when lepton family generators are gauged. Sect. 3 is devoted instead to the phenomenology of the electroweak anomalons, in the limit where their mass dominantly stems from the Higgs VEV. We conclude in Sect. 4, while in App. A we collect a series of technical results about the calculation of WZ terms.

# 2 Gauging the Standard Model accidental symmetries

In this Section, we provide an explicit UV completion for the gauging of the most general combination of the SM global symmetries in Eq. (1.1), discuss the conditions for the cancellation of gauge anomalies, compute the spectrum and the EFT below the scale of the heavy fermions (anomalons) assuming that the only new physics light state is the vector boson $X$.

## 2.1 UV model

The field content of the model is displayed in Table 1, where the anomalon fields are highlighted in color and we also extended the scalar sector of the SM in order to spontaneously break the $U(1)_X$ symmetry. Similar setups for anomaly cancellation were considered e.g. in Refs. [19–24]. Here, the more general SM charges of the electroweak anomalon fields ($L, N, E$) are needed to evade LHC constraints on purely-chiral fermions for $\mathcal{Y} \approx 2, -1$ [16, 17], as it will be reviewed in Sect. 3. In fact, as already anticipated in the Introduction, we will be interested in exploring the limit in which the electroweak anomalon masses are dominantly due to the Higgs, so that the strong bounds stemming from the anomalous WZ couplings of the light vector with SM gauge bosons are relaxed. We have also included $N$ copies of chiral SM-singlet fermions $\nu_{R}^{\alpha}$ ($\alpha = 1, \ldots, N$) which allow to have more freedom for the cancellation of $U(1)_X$ and $U(1)_Y$ anomalies (as well as provide a seesaw setup for neutrino masses), but whose presence does not impact the calculation of the electroweak WZ terms.

### 2.1.1 Anomaly cancellation

The $U(1)_X$ charges are required to cancel all gauge anomalies. This corresponds to the following five conditions:

\[
\text{Gravity} \times U(1)_X : \quad 2(X_{L_L} - X_{L_R}) + (X_{e_L} - X_{e_R}) + (X_{N_L} - X_{N_R}) - \sum_{\alpha=1}^{N} X_{\nu_{R}^{\alpha}} + \alpha_\mu + \alpha_\tau = 0, \quad (2.1)
\]

\begin{footnote}{For other anomalon configurations leading to anomaly cancellation when baryon and/or lepton number generators are gauged see e.g. [25, 26].}
\end{footnote}
Further constraints on the $U(1)_X$ charges are obtained by the requirement that the electroweak anomalous fields pick up their mass from the VEV of $H$. Hence, the Yukawa Lagrangian involving the electroweak anomalon fields is (the discussion of neutrino masses is postponed to Sect. 2.3)

$$
\mathcal{L}_Y = y_1 \overline{\ell}_L \ell R H + y_2 \overline{\ell}_R \ell L H + y_3 \overline{\nu}_L N_R \tilde{H} + y_4 \overline{\nu}_R N_L \tilde{H} + \text{h.c.},
$$

(2.6)

with $\tilde{H} = i\sigma_2 H^\dagger$. The extra conditions on the $U(1)_X$ charges stemming from Eq. (2.6) read

$$
X_{\ell R} = X_{\ell L},
$$

(2.7)

$$
X_{\ell L} = X_{\ell R},
$$

(2.8)

$$
X_{N_R} = X_{\ell L},
$$

(2.9)

$$
X_{N_L} = X_{\ell R},
$$

(2.10)

thus reducing the number of independent charges to two. By substituting Eqs. (2.7)–(2.10) into Eqs. (2.1)–(2.5), we obtain the following non-trivial conditions

$$
X_{\ell R} - X_{\ell L} = 3\alpha_B + \alpha_e + \alpha_\mu + \alpha_\tau = 3\alpha_{B+L},
$$

(2.11)
\[
\sum_{\alpha=1}^{N} X_{\nu_{\alpha}} = \alpha_e + \alpha_\mu + \alpha_\tau, \tag{2.12}
\]
\[
\sum_{\alpha=1}^{N} (X_{\nu_{\alpha}})^3 = \alpha_e^3 + \alpha_\mu^3 + \alpha_\tau^3, \tag{2.13}
\]
where we have introduced the shorthand \(\alpha_{B+L}\) defined in Eq. (2.11). Note that the condition of cancellation of electroweak anomalies fixes only the difference \(X_{\nu_{R}} - X_{\nu_{L}}\), leaving us with one free charge that we choose to be \(X_{\nu_{L}}\). This redundancy is related to the electroweak anomalon number \(U(1)_A\), corresponding to a common re-phasing of the electroweak anomalon fields.

Other renormalizable operators, which are allowed by the SM gauge symmetry, may or may not be allowed by \(U(1)_X\) invariance. For instance, extra Yukawas of the type
\[
- \Delta L_Y = y_L \overline{e}_L \ell R S^* + y_e \overline{e}_L e R S + y_N \overline{N}_L N R S + \text{h.c.}, \tag{2.14}
\]
are only permitted for \(X_S = X_{\nu_{R}} - X_{\nu_{L}} = 3\alpha_{B+L}\). These terms would yield an additional vector-like mass to the anomalons after \(U(1)_X\) symmetry breaking. Finally, for specific values of \(U(1)_Y\) and \(U(1)_X\) charges, the electroweak anomalons can mix with the SM leptons at the renormalizable level. The classification of \(d = 4\) mixing operators is provided in Table 2, where we emphasized the phenomenologically relevant case \(Y = 2, -1\) (see Sect. 3.2). Note that in the presence of mixing operators the electroweak anomalon number is explicitly broken, and hence \(X_{\nu_{L}}\) gets fixed in terms of the coefficients of the \(X\) generator in Eq. (1.1).

### 2.1.3 Spectrum

By adding a proper term in the scalar potential, \(\Delta V(H, S)\), the following VEV configurations are generated
\[
\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle S \rangle = \frac{v_X}{\sqrt{2}}, \tag{2.15}
\]
with \(v \simeq 246\) GeV and \(v_X\) being the order parameter of \(U(1)_X\) breaking. The latter is responsible for the mass of the \(U(1)_X\) gauge boson, \(X^\mu\), that is
\[
m_X = X_S g_X v_X, \tag{2.16}
\]
where \(g_X\) is the \(U(1)_X\) gauge coupling entering the covariant derivative, i.e. \(D^\mu S = (\partial^\mu + ig_X X_S X^\mu) S\). The scalar field can be expanded around the vacuum as \(S = \frac{v_X e^{i\xi/v_X}}{\sqrt{2}} + \ldots\), where \(\xi\) is the Goldstone boson associated with the massive state \(X\) and we neglected the radial mode. After \(U(1)_X\) and electroweak symmetry breaking the Yukawa terms in \(\mathcal{L}_Y + \Delta \mathcal{L}_Y\) (see Eq. (2.6) and Eq. (2.14)) give mass to the electroweak anomalons (neglecting for simplicity possible mixings with the SM sector)
\[
- \mathcal{L}_{\text{mass}} = \overline{\Psi}_L^0 \mathcal{M}_L \Psi_R^0 + \overline{\Psi}_L^N \mathcal{M}_N \Psi_R^N + \text{h.c.}, \tag{2.17}
\]
which can be cast into 2-flavour Dirac fermions, \(\Psi_L^E, R = (\mathcal{E}_{\nu_{L,R}})\) and \(\Psi_L^N = (\mathcal{N}_{\nu_{L,R}}),\) with
\[
\mathcal{M}_L = \begin{pmatrix} m_L & m_1 \\ m_2 & m_{\overline{\epsilon}} \end{pmatrix}, \quad \mathcal{M}_N = \begin{pmatrix} m_L & m_3 \\ m_4 & m_{\overline{N}} \end{pmatrix}, \tag{2.18}
\]
and
\[
m_{\mathcal{E}, \mathcal{N}} = \frac{y_{\mathcal{E}, \mathcal{N}} v_X}{\sqrt{2}}, \quad m_{1,2,3,4} = \frac{y_{1,2,3,4}}{\sqrt{2}} v. \tag{2.19}
\]

\(^3\)The case \(S \rightarrow S^*\) is trivially obtained by replacing \(X_S \rightarrow -X_S\).
In the limit $y_X\ll 1$ this requires $\Psi_{\nu}$. For completeness, we also include mixings via RH neutrinos and/or $S$, whose $U(1)_X$ charges depend on the mechanism giving mass to neutrinos (see Sect. 2.3). Mixing operators via $S^*$ are trivially obtained by flipping the sign of $X_S$ in the third column.

Table 2: Renormalizable operators leading to a mixing between electroweak anomalons and SM leptons (first column) and required conditions on $U(1)_Y$ and $U(1)_X$ charges (second and third columns). For completeness, we also include mixings via RH neutrinos and/or $S$, whose $U(1)_X$ charges depend on the mechanism giving mass to neutrinos (see Sect. 2.3). Mixing operators via $S^*$ are trivially obtained by flipping the sign of $X_S$ in the third column.

| Mixing operator | $U(1)_Y$ | $U(1)_X$ |
|-----------------|----------|----------|
| $\bar{t}L\bar{e}_R H$ | $Y = 0$ | $X_{\ell_L} = \alpha_i$ |
| $\bar{t}L\bar{e}_R H$ | $Y = 1$ | $X_{\ell_L} = \alpha_i$ |
| $\bar{t}L(E_L)^c H$ | $Y = 2$ | $X_{\ell_L} = -\alpha_i - 3\alpha_{B+L}$ |
| $\bar{t}L(E_L)^c H$ | $Y = 1$ | $X_{\ell_L} = -\alpha_i - 3\alpha_{B+L}$ |
| $\bar{t}L(N_R)^c H$ | $Y = -1$ | $X_{\ell_L} = \alpha_i$ |
| $\bar{t}L(N_R)^c H$ | $Y = 0$ | $X_{\ell_L} = \alpha_i$ |
| $\bar{t}L(N_L)^c H$ | $Y = 1$ | $X_{\ell_L} = -\alpha_i - 3\alpha_{B+L}$ |
| $\bar{t}L(N_L)^c H$ | $Y = 0$ | $X_{\ell_L} = -\alpha_i - 3\alpha_{B+L}$ |
| $\bar{t}L\nu_{eR} H$ | $Y = 1$ | $X_{\ell_L} = \alpha_i$ |
| $\bar{t}L\nu_{eR} H$ | $Y = 0$ | $X_{\ell_L} = \alpha_i$ |
| $\bar{t}L(\nu_{eR})^c H$ | $Y = 1$ | $X_{\ell_L} = -\alpha_i - 3\alpha_{B+L}$ |
| $\bar{t}L(\nu_{eR})^c H$ | $Y = 0$ | $X_{\ell_L} = -\alpha_i - 3\alpha_{B+L}$ |
| $\bar{t}LS$ | $Y = 0$ | $X_{\ell_L} = \alpha_i + X_S - 3\alpha_{B+L}$ |
| $\bar{t}LS$ | $Y = 1$ | $X_{\ell_L} = \alpha_i + X_S - 3\alpha_{B+L}$ |
| $\bar{t}L\nu_{\ell R} S$ | $Y = 2$ | $X_{\ell_L} = -\alpha_i + X_S$ |
| $\bar{t}L\nu_{\ell R} S$ | $Y = 1$ | $X_{\ell_L} = -\alpha_i + X_S$ |
| $\bar{N}_L\nu_{\ell R} S$ | $Y = -1$ | $X_{\ell_L} = \alpha_i + X_S - 3\alpha_{B+L}$ |
| $\bar{N}_L\nu_{\ell R} S$ | $Y = 0$ | $X_{\ell_L} = \alpha_i + X_S - 3\alpha_{B+L}$ |
| $\bar{N}_L(\nu_{\ell R})^c S$ | $Y = 1$ | $X_{\ell_L} = -\alpha_i + X_S$ |
| $\bar{N}_L(\nu_{\ell R})^c S$ | $Y = 0$ | $X_{\ell_L} = -\alpha_i + X_S$ |

The mass matrices are diagonalized via the bi-unitary transformations $\Psi_{\ell,R}^{E,N} \rightarrow U_{\ell,N} \Psi_{\ell,R}^{E,N}$ and $\Psi_{L}^{E,N} \rightarrow V_{\ell,N} \Psi_{L}^{E,N}$, with the unitary matrices entering non-trivially into the gauge currents in the mass basis. In the limit $y_{E} = y_{N}$, $y_{1} = y_{3}$, $y_{2} = y_{4}$ (and hence $\mathcal{M}_{\ell} = \mathcal{M}_{N}$), the Yukawa Lagrangian features a custodial symmetry which helps in taming corrections to electroweak precision observables (see Sect. 3.1). In the following, we will stick to the custodial limit, while for the calculations in App. A we will consider the more general case.

### 2.2 EFT of a light vector and decoupling of WZ terms

We are interested in the limit where the electroweak anomalons are heavier than the electroweak scale, while the vector $X$ is much lighter than the electroweak scale. Parametrically (see Eq. (2.16)), this can be obtained in two ways: i) $v_X \gtrsim v$ and $g_X \ll 1$ or ii) $v_X \ll v$ and $g_X \gtrsim 1$. In case ii) or if the $\Delta L_Y$ operators in Eq. (2.14) are absent due to charge assignment (i.e. $X_S \neq 3\alpha_{B+L}$), this requires $y_{1,2,3,4} \sim \sqrt{4\pi}$ in order for the anomalons to be heavier than the electroweak scale. Upon integrating out the electroweak anomalons at one loop one finds in the EFT given by the
where \(a, b = 1, 2, 3\) and we neglected non-abelian \(W\) terms scaling with an extra gauge coupling \(g\). In general, from the requirement that the electromagnetic group remains unbroken, one obtains

\[
C_{ab} = \begin{pmatrix} 11 & C_{12} & 0 \\ -C_{12} & 11 & 0 \\ 0 & 0 & 33 \end{pmatrix}, \quad C_{aB} = (0, 0, C_{3B}), \quad C_{Ba} = (0, 0, C_{B3}),
\]

and

\[
D_{ab} = \begin{pmatrix} D_{11} & 0 & 0 \\ 0 & D_{11} & 0 \\ 0 & 0 & D_{33} \end{pmatrix}, \quad D_{aB} = (0, 0, D_{3B}),
\]

together with the sum-rules

\[
\begin{align*}
C_{33} + C_{3B} + C_{B3} + C_{BB} &= 0, \\
D_{33} + 2D_{3B} + D_{BB} &= 0.
\end{align*}
\]

A relatively simple case is given in the limit where the masses of the anomalon fields stem completely from the VEV of \(S\), yielding

\[
\begin{align*}
C_{11} &= C_{33} = -C_{BB} = 3\alpha_{B+L}, \\
C_{B3} &= -C_{3B} = D_{3B} = C_{12} = 0, \\
D_{11} &= D_{33} = -D_{BB} = -9\alpha_{B+L},
\end{align*}
\]

where the effective coefficients are set by the anomalous trace of the SM current (see Eqs. (A.46)–(A.47)). Here, instead, we focus on the more general case where the anomalon masses have both a SM-singlet and an electroweak symmetry breaking source. Although we were not able to cast explicit expressions for the EFT coefficients into a simple analytical form (see Eqs. (A.28)–(A.29)), we will present them here under the simplified (but phenomenologically motivated) hypothesis in which the anomalon masses are degenerate, that is

\[
\mathcal{M}_\xi^\dagger \mathcal{M}_\xi = \mathcal{M}_N^\dagger \mathcal{M}_N = m_\psi^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\]

with \(m_\psi\) denoting the degenerate anomalon mass. Eq. (2.28) enforces the mass matrices to be

\[
\mathcal{M}_\xi = \mathcal{M}_N = \frac{1}{\sqrt{2}} \begin{pmatrix} y_S v_X & i y_H v \\ i y_H v & y_S v_X \end{pmatrix} = m_\psi \begin{pmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{pmatrix},
\]

with \(y_{S,H}\) real parameters,\(^4\) while

\[
m_\psi^2 = \frac{1}{2} \left( (y_H v)^2 + (y_S v_X)^2 \right),
\]

\(^4\)The accidental global \(U(1)_B\) symmetry corresponding to the re-phasing of the each electroweak anomalon field is broken by the \(\mathcal{L}_Y + \Delta \mathcal{L}_Y\), leaving an unbroken electroweak anomalon number \(U(1)_A\), that is the subgroup corresponding to the common re-phasing of all the anomalon fields. Hence, of the seven complex parameters introduced in \(\mathcal{L}_Y + \Delta \mathcal{L}_Y\), \(6 - 1 = 5\) phases are unphysical and can be rotated away. One possible choice is to set \(\text{Arg}(y_\xi) = \text{Arg}(y_E) = \text{Arg}(y_N) = 0\), \(\text{Arg}(y_1) = -\text{Arg}(y_2)\), and \(\text{Arg}(y_3) = -\text{Arg}(y_4)\).
and
\[ \tan \theta = \frac{y_{H^0}}{y_{S^0 X}} . \]  
(2.31)

Specializing the general expressions in Eqs. (A.28)–(A.29) to the degenerate case above, we find
\[ C_{11} = C_{33} = -C_{BB} = \frac{3}{4} \alpha_{B+L}(1 + 3 \cos 2 \theta) , \]  
(2.32)
\[ C_{B3} = -C_{3B} = \frac{9}{4} \alpha_{B+L}(1 - \cos 2 \theta) , \quad C_{12} = 0 , \]  
(2.33)
\[ D_{11} = D_{33} = -D_{BB} = -\frac{9}{2} \alpha_{B+L}(1 + \cos 2 \theta) , \quad D_{3B} = 0 . \]  
(2.34)

The important point to be noted is that when the anomalons pick up a mass from both electroweak preserving and breaking sources, the low-energy WZ coefficients acquire a model dependence through the angle $\theta$.

In order to understand the phenomenological implications of this model dependence, we briefly recall here the argument of Refs. [10, 11] regarding the energy-enhanced emission of the longitudinal modes of $X$ stemming from the WZ operators. Taking the limit $g_X \to 0$ and $m_X \to 0$, while keeping fixed the ratio $m_X/g_X \propto v_X$, the transverse modes of $X$ decouple, while the longitudinal mode is enhanced as $E/m_X$. In this regime, the equivalence theorem states that the longitudinally polarized vectors are equivalent to the corresponding scalar Goldstone bosons. This is readily seen by working in the so-called “Equivalent Gauge” of Ref. [27], where the longitudinally polarized state, $|X_L\rangle$, is represented as
\[ \langle 0 | X_\mu(x) X_L(p) \rangle = \epsilon_{L}^\mu(p) e^{-i p x} , \quad \langle 0 | \xi(x) X_L(p) \rangle = -i e^{-i p x} , \]  
(2.35)
with the polarization vector
\[ \epsilon_{L}^\mu(p) = -\frac{m_X}{E_p + |p|} \left\{ 1 , \frac{\vec{p}}{|p|} \right\} , \]  
(2.36)
vanishing in the $m_X \to 0$ limit. The advantage of this representation is that it makes the equivalence theorem explicit, since in the high-energy limit (or equivalently $m_X \to 0$) only the Goldstone mode survives. Hence, adopting the above prescription, only the diagrams with one external $\xi$ contribute to physical processes in the $m_X \to 0$ limit. For instance, upon integrating out the $W$ boson, the axion-like operator $\xi W^- \bar{W}^+$ proportional to $D_{11}$ in Eq. (2.20) yields the effective interaction
\[ g_{\xi_4 d_j} \bar{d}_j \gamma^\mu P_L d_4 \left( \partial_\mu \xi/m_X \right) + h.c. , \]  
(2.37)
in terms of the effective coupling [28]
\[ g_{\xi_4 d_j} = \frac{g_X g^4}{4\pi^4} D_{11} \sum_{\alpha = u,c,t} V_{\alpha \alpha} V_{\alpha j}^* F(m_{\alpha}^2/m_W^2) , \]  
(2.38)
with $D_{11} \propto (1 + \cos 2 \theta)$ given in Eq. (2.34), $V$ denoting the CKM matrix and the loop function
\[ F(x) = \frac{x(1 + x(\ln x - 1))}{(1 - x)^2} . \]  
(2.39)

This leads to FCNC processes, such as $K \to \pi X_L$, $B \to K X_L$, etc, whose rate is enhanced as $(E/m_X)^2$, where $E$ is the decay energy (cf. the derivative operator in Eq. (2.37)), thus implying strong bounds on light vector bosons coupled to anomalous currents [10, 11].

On the other hand, the above constraints from energy-enhanced $X_L$ emission disappear for $D_{11} = 0$, that is when the $U(1)_X$ Goldstone decouples from the electroweak anomalons. This corresponds to $\theta = \pi/2$, which implies that the anomalon masses are entirely due to the Higgs VEV (cf. Eq. (2.31)). From a top-down perspective this condition can be neatly obtained.
in terms of \( \text{U}(1)_X \) charges (\( X_S \neq 3 \alpha_{B+L} \)) which forbid the operators of \( \Delta \mathcal{L}_Y \) in Eq. (2.14). Alternatively, it can be parametrically obtained by taking \( v_X \ll v \) or \( y_S \approx 0 \). Note that the latter condition is radiatively stable, since it corresponds to an enhanced \( \text{U}(1)_1 \) global symmetry of the Lagrangian in which LH and RH anomalons fields are rotated with an opposite phase.

In conclusion, we have shown that the bounds of Refs. [10, 11] can be relaxed by assuming that the anomalons fields are mostly chiral (namely, their mass mostly stems from the Higgs VEV). This possibility, however, leads to non-decoupling signatures in Higgs observables and direct searches, to be discussed in Sect. 3.

2.3 Neutrino masses

If the \( X \) generator has a non-trivial projection on family lepton numbers, \( \alpha_i \neq 0 \) (\( i = e, \mu, \tau \)), then we need RH neutrinos, \( \nu^\alpha_R \) (\( \alpha = 1, \ldots, N \)), in order to cancel \( \text{U}(1)_X \) and \( \text{U}(1)_3^X \) anomalies (cf. conditions in Eqs. (2.12)-(2.13)). The simplest solution is to introduce one RH neutrino for each \( \alpha_i \neq 0 \) and set \( X^\alpha_{\nu_R} = \alpha_i \). Another possibility is to have universal charges \( X^\alpha_{\nu_R} = X_{\nu_R} \), so that the anomaly-free conditions are

\[
X_{\nu_R} = \left( \frac{\alpha_e^3 + \alpha_\mu^3 + \alpha_\tau^3}{\alpha_e + \alpha_\mu + \alpha_\tau} \right)^{1/2}, \quad N = \left( \frac{(\alpha_e + \alpha_\mu + \alpha_\tau)^3}{\alpha_e^3 + \alpha_\mu^3 + \alpha_\tau^3} \right)^{1/2}.
\]  

(2.40)

The SM-singlet states \( \nu^\alpha_R \) can be used to give mass to light neutrinos via the seesaw mechanism. In fact, SM gauge invariance would allow the operators

\[
-\mathcal{L}_Y^{\nu_R} = y_D^i j^i j^i \nu^\beta_R \tilde{H} + \frac{1}{2} b_{ij} \alpha_i \nu^\alpha_R \nu^\beta_S \times + \text{h.c.} \quad \rightarrow \quad m_D^i j^i j^i \nu^\beta_R + \frac{1}{2} M^\alpha_R \nu^\alpha_R \nu^\beta_R + \text{h.c.},
\]

(2.41)

with \( m_D = y_D v/\sqrt{2} \) and \( M_R = y_R v_X/\sqrt{2} \), leading to light neutrino masses via the seesaw mechanism

\[
m_\nu = m_D M_R^{-1} m_D^T.
\]

(2.42)

However, in order for the operators in Eq. (2.41) to be \( \text{U}(1)_X \) invariant, the following constraints on \( \text{U}(1)_X \) charges need to be satisfied

\[
-\alpha_i + X^\alpha_{\nu_R} = 0, \quad (2.43)
\]

\[
X^\alpha_{\nu_R} + X^\beta_{\nu_R} - X_S = 0. \quad (2.44)
\]

While the first condition can be easily fulfilled (since it also ensures the cancellation of \( \text{U}(1)_X \) and \( \text{U}(1)_3^X \) anomalies), the second one could imply texture zeros in \( M_R \) if some leptonic generators are non-universal \( \alpha_i \neq \alpha_j \). Consistency with light neutrino data might then require the introduction of extra scalars in order to reproduce realistic low-energy textures (see e.g. [29–31]).

3 Electroweak anomalons phenomenology

In the previous Section we have seen that mostly chiral electroweak anomalons (i.e. which take their mass mostly from the Higgs VEV) allow to decouple dangerous WZ terms, which would otherwise lead to the energy-enhanced longitudinal emission of light vectors coupled to anomalon currents. We are hence interested in studying the electroweak phenomenology of the exotic leptons \( \mathcal{L} + \mathcal{N} + \mathcal{E} \), whose quantum numbers are displayed in Table 1. In particular, following the analysis of Refs. [16, 17], we will argue that phenomenology requires \( Y \approx 2, -1 \).

---

\(^{5}\) We neglect here bare Majorana mass terms, since in that case RH neutrinos would not contribute to the cancellation of \( \text{U}(1)_X \) and \( \text{U}(1)_3^X \) anomalies. Instead, possible mixings between RH neutrinos and electroweak anomalons have been classified in Table 2.
3.1 Electroweak precision tests

The contribution of the electroweak anomalous in terms of mass eigenstates (cf. Table 1 and Eq. (2.17)) to the $S$ and $T$ parameters is [16]

$$S = \frac{1}{6\pi} \left[ \left( 1 - 2\langle Y \rangle - \frac{1}{2} \log \frac{m_N^2}{m_{\xi_1}^2} \right) + \left( 1 + 2\langle Y \rangle - \frac{1}{2} \log \frac{m_{N_2}^2}{m_{\xi_2}^2} \right) \right] + \mathcal{O} \left( \frac{m_Z^2}{m_{\xi_1,\xi_2}^2} \right) \approx \frac{1}{3\pi}, \quad (3.1)$$

$$T = \frac{1}{16\pi c_W s_W m_Z^2} \left( m_{N_1}^2 + m_{\xi_1}^2 - 2 \frac{m_{N_2}^2 m_{\xi_1}^2}{2 m_{N_1}^2 - m_{\xi_1}^2} \log \frac{m_{N_1}^2}{m_{\xi_1}^2} \right) + \frac{1}{16\pi c_W s_W m_Z^2} \left( m_{N_2}^2 + m_{\xi_2}^2 - 2 \frac{m_{N_2}^2 m_{\xi_2}^2}{2 m_{N_2}^2 - m_{\xi_2}^2} \log \frac{m_{N_2}^2}{m_{\xi_2}^2} \right) \approx 0, \quad (3.2)$$

where the approximation in the last steps holds in the custodial limit $m_{N_{1,2}} \approx m_{\xi_{1,2}}$. Recent fits for oblique parameters, e.g. from Gfitter [32], yield

$$S = 0.05 \pm 0.11, \quad T = 0.09 \pm 0.13, \quad (3.3)$$

which are easily satisfied in the custodial limit, although a mass splitting might play a role to explain the recent $M_W$ anomaly [33].

3.2 Higgs physics

We now consider the constraints from Higgs couplings measurements. In particular, we assess the impact of the new heavy fermions on the decay rate of the Higgs boson to two photons, or to a photon and a $Z$ boson. Taking a fermion $\psi$ of mass $m_\psi$ the interaction Lagrangian is given by

$$\mathcal{L}_{\psi}^{\text{int}} = -\frac{m_\psi}{v} h \bar{\psi} \psi + e Q_\psi \bar{\psi} \gamma^\mu \psi A_\mu + \frac{e}{c_W s_W} \bar{\psi} \gamma^\mu \left( \frac{T^3_\psi}{2} - Q_\psi \frac{s_W^2}{2} - \frac{T^3_\psi}{2} \gamma_5 \right) \psi Z_\mu, \quad (3.4)$$

where $h$ is the 125 GeV Higgs, $A_\mu$ and $Z_\mu$ the photon and Z boson fields, $T^3_\psi$ is the eigenvalue of the third generator of SU(2)$_L$ when it acts on the left-handed component of $\psi$, so that $T^3_\psi = \pm \frac{1}{2}$ when $\psi_L$ arises from a doublet in the fundamental of SU(2)$_L$. Its one-loop contributions to the amplitudes $h \rightarrow \gamma \gamma$ and $h \rightarrow \gamma Z$ are [34]

$$A_{\gamma\gamma}^{\psi} \approx -\frac{4}{3} Q_\psi, \quad A_{Z\gamma}^{\psi} \approx -\frac{1}{3} Q_\psi \frac{T^3_\psi}{c_W} - 2 Q_\psi \frac{s_W^2}{c_W}, \quad (3.5)$$

where we assumed that $\psi$ is much heavier than the Higgs and the Z boson, which holds for the heavy fermions we consider here. In the SM, these amplitudes are dominated by the loop of the $W$ gauge boson interfering negatively with the loop of the top quark and they amount to $A_{\gamma\gamma}^{\SM} \approx -6.5$ and $A_{Z\gamma}^{\SM} \approx 5.7$ at leading order. In the presence of a single Higgs doublet, the new physics contribution yields $A_{\gamma\gamma}^{\NP} \approx \frac{2}{3}(1 - 2Y + 2Y^2)$. Writing the modified Higgs width to photons as

$$R_{\gamma\gamma} = \frac{|A_{\gamma\gamma}^{\SM} + A_{\gamma\gamma}^{\NP}|^2}{|A_{\gamma\gamma}^{\SM}|^2}, \quad (3.6)$$

a recent ATLAS analysis found $R_{\gamma\gamma} = 1.00 \pm 0.12$ [35]. There is in fact the possibility that the new physics contribution interferes negatively with the SM amplitude, namely $A_{\gamma\gamma}^{\NP} \approx -2 A_{\gamma\gamma}^{\SM} \approx 13.0$. This is obtained either for $Y \approx 2$ (1.93 $\lesssim Y \lesssim 2.03$ [2$\sigma$ range]) or $Y \approx -1$ ($-1.03 \lesssim Y \lesssim -0.93$ [2$\sigma$ range]), both yielding $A_{\gamma\gamma}^{\NP}(Y = 2) = A_{\gamma\gamma}^{\NP}(Y = -1) \approx 13.3$. A
correlated signal in the $\gamma Z$ channel yields $A_{NP}^{\gamma Z} \approx -\frac{2}{3} c_W [1 - (3 - 8Y + 8Y^2)t_W^2]$, leading to a large deviation $A_{NP}^{\gamma Z}(Y = 2) = A_{NP}^{\gamma Z}(Y = -1) \approx 2.33$ in the region where the value of $Y$ is compatible with the di-photon channel. The $\gamma Z$ decay channel of the Higgs has not been observed yet and HL-LHC is expected to measure $\kappa_{\gamma Z}$ within 10% precision [36]. Thus the model with a single Higgs doublet predicts a strong departure of $R_{Z\gamma}$ from its SM value, although extended Higgs sectors can help to tame modifications of Higgs signals (see e.g. [17]).

3.3 Direct searches

Direct searches at high-energy particle colliders depend on whether the exotic leptons mix with the SM leptons. In fact, this is possible only for the values $Y = 0, \pm 1, 2$ (see Table 2), including the phenomenologically favoured case $Y = 2, -1$. We discuss in turn the two different scenarios corresponding to $Y \neq 2, -1$ (stable charged leptons) and $Y = 2, -1$ (unstable charged leptons).

3.3.1 Stable charged leptons

For $Y \approx 2, -1$ (but $Y \neq 2, -1$) the exotic leptons do not mix the SM ones and the lightest state of the spectrum is electrically charged and stable due to exotic lepton number. Charged relics are cosmologically dangerous and largely excluded. To avoid cosmological problems one has to invoke low-scale inflation, such that charged relics are either diluted by inflation or never thermally produced. On the other hand, stable charged particles yield striking signatures at colliders in the form of charged track, anomalous energy loss in calorimeters, longer times of flight, etc. Applying the experimental limits of [37] at 13 TeV LHC with the leading-order Drell-Yann cross-sections rescaled for $|Q| = 2$ (see also [38]), Ref. [17] obtained $m_{N,E} \gtrsim 800$ GeV. Since $m_{N,E} = y_{N,E} v/\sqrt{2}$, direct searches imply Yukawa couplings, $y_{N,E} \approx 4.6$, at the boundary of perturbative unitarity (see e.g. [39, 40]).

3.3.2 Unstable charged leptons

For $Y = 2, -1$ the electroweak anomalons have electric charge $Q = 2, -1$ ($N$ components) and $Q = 1, -2$ ($E$ components). The $|Q| = 2$ states can decay into a $W$ and a $|Q| = 1$ fermion, while the latter can mix with SM leptons and decay into $Z\ell$ or $h\ell$. Signatures of this type were previously studied in Ref. [41], which estimated a mass reach at the LHC 14 up to $m_{N,E} \sim 800$ GeV (depending on the integrated luminosity). To our knowledge, however, such an analysis has never been performed by the experimental collaborations. Anyhow, the bounds appear to be of the same order of those obtained in the case of stable charged leptons.

4 Conclusions

In this work we have revisited the case of light vector bosons coupled to anomalous currents which are UV completed by new anomaly-cancelling heavy fermions (anomalons). After the latter have been integrated out, WZ terms of the type in Eq. (2.20) are generated. On the one hand, they take care of anomaly cancellation in the IR and, on the other, they source the energy-enhanced emission of longitudinally polarized vectors, $X$, which typically results in very strong bounds on $g_X/m_X \propto 1/v_X$ whenever the decay channels $Z \rightarrow \gamma X$, $B \rightarrow KX$, $K \rightarrow \pi X$, etc, are kinematically open [10, 11]. Here, we have studied the model-dependence of such bounds, considering as a paradigmatic framework the gauging of the most general (anomalous) linear combination of SM global symmetries, $U(1)_X$, with the generator $X$ given in Eq. (1.1). To this end, we provided a UV completion including electroweak anomalons $\mathcal{L} + \mathcal{E} + \mathcal{N}$ (cf. Table 1) to

\footnote{Large higher-order corrections (starting at two loops) are expected for Higgs decays and they might slightly change the solutions $Y \approx 2, -1$.}
cancel $U(1)_X$ anomalies in combination with electroweak gauge factors and RH neutrinos to take
care of $U(1)_X$ anomalies in isolation when the lepton number generators are gauged. An extra
scalar $S$ provides the spontaneous breaking of the $U(1)_X$ factor and gives mass to the vector
$\mathcal{X}$. Then, we have computed the EFT of a light $\mathcal{X}$ when the heavy anomalons are integrated,
keeping in general both electroweak symmetry breaking and preserving sources for the mass of
the anomalons (see App. A for details). This allowed us to conclude (cf. e.g. Eq. (2.38)) that the
bounds mentioned above on light $\mathcal{X}$ can be evaded in the limit where the mass of the electroweak
anomalons comes mostly from the Higgs VEV. This condition can be neatly imposed in terms
of $U(1)_X$ gauge charges (so that the operators in Eq. (2.6) are allowed while those in Eq. (2.14)
are forbidden) or parametrically by decoupling the vector-like masses of the exotic leptons by
taking a small yukawa and/or a small VEV for $S$. On the other hand, mostly chiral exotic
leptons (receiving their mass mostly from the Higgs VEV) are strongly constrained due to their
non-decoupling nature by electroweak-scale phenomenology, in particular Higgs couplings and
direct searches. We have reviewed in Sect. 3 those constraints, based on the previous analyses
in [16, 17], and argued that it is possible to evade $h \rightarrow \gamma\gamma$ bounds for $\mathcal{Y} \approx 2, -1$ (including the
exact cases $\mathcal{Y} = 2, -1$ allowing for mixings between anomalons and SM leptons, cf. Table 2).
For $\mathcal{Y} \approx 2, -1$, however, the $h \rightarrow \gamma Z$ channel differs $\mathcal{O}(1)$ from the SM and it will be possible
to test this scenario at the HL-LHC. Direct searches, whose signatures depend on whether
the electroweak anomalons mix or not with the SM leptons, are also very stringent and they
practically push the Yukawas of the exotic leptons to the boundary of perturbativity.

Since, somewhat surprisingly, mostly chiral exotic leptons are not yet ruled-out, it would be
interesting to see the potential implications of the setup discussed in this paper on the physics
of light vectors coupled to anomalous currents. The most relevant application turns out to be
for the case of electro-phobic light vectors ($\alpha_e = 0$ in Eq. (1.1)), for which the bounds stemming
from WZ terms are typically the most important ones. These include, for instance, baryonic
forces ($\alpha_B \neq 0$) which have a rich accelerator and collider phenomenology (see e.g. [42, 43]),
while a case motivated by the $g$-2 anomaly [18] is that of purely muonic forces ($\alpha_\mu \neq 0$) which
are in principle distinguishable from more standard scenarios based on $U(1)_{\mu-\tau}$ (see e.g. [44]).
The general framework discussed in Sect. 2 provides a consistent UV completion for light vectors
coupled to anomalon SM fermionic currents, allowing to free large portions of parameter space
from the bounds of Ref. [10, 11], until the HL-LHC will give the final word on the existence of
mostly chiral exotic leptons.

Acknowledgments

The work of LDL is partially supported by the European Union’s Horizon 2020 research and in-
novation programme under the Marie Skłodowska-Curie grant agreement No 860881 - HIDDEN.
The work of MN and CT was supported in part by MIUR under contract PRIN 2017L5W2PT

A Calculation of the Wess-Zumino terms

In this Appendix we present the calculation of the effective WZ terms involving gauge and Gold-
stone bosons that arise after integrating out heavy fermionic degrees of freedom. In particular,
we focus on the 3-point vertices involving the epsilon tensor $\epsilon^{\alpha \beta \mu \nu}$ (with $\epsilon^{0123} = 1$) which are
related to anomaly cancellation in the EFT when the heavy fermions are integrated out.

A.1 Toy model

We assume a toy model with a set of gauge bosons $G_{\mu}^A$ related to the generators $Q^A$ of the
gauge symmetry group $\mathcal{G}$ (that can be in general semi-simple). The model contains a fermionic
sector, whose fields are labeled as $\psi_i$, that acquire a mass term $M_{ij}$ after a spontaneously
symmetry breaking (SSB) mechanism. The \( \left( \frac{1}{2}, 0 \right) \) and \( \left( 0, \frac{1}{2} \right) \) Lorentz components of the \( \psi \) field are separately (reducible) representations of \( \mathcal{G} \) and the generators act on them as

\[
Q^A \psi_i = \sum_j (Q^A_{L})_{ij} \psi_{jL} + \sum_j (Q^A_{R})_{ij} \psi_{jR},
\]

where \( (Q^A_{L,R})_{ij} \) are the matrix representation of the gauge multiplets \( \psi_{L,R} \equiv P_{L,R} \psi \). We restrict ourselves to models with a U(1) symmetry corresponding to the fermionic number of the \( \psi \) fields \( (\psi_i \to e^{i\phi} \psi_i) \). The real scalar Higgs fields, responsible for the SSB mechanism, are labeled as \( H_a = (H_a)^* \) and belong to \( (1, 0) \) (reducible) representation of the gauge group \( \mathcal{G} \). By performing an infinitesimal transformation of angle \( \alpha_A \) along the \( Q^A \) generator, the \( H_a \) fields transform like

\[
\delta H_a = \sum_b g_A \alpha_A (iQ^A_H)_{ab} H_b,
\]

where \( (iQ^A_H)_{ab} \) is a real and antisymmetric matrix. Hence,

\[
\mathcal{L}_{\text{toy model}} \supset \sum_i \overline{\psi}_i i \partial \psi_i - \sum_{a,i,j} H_a (\overline{\psi}_{iL} Y^a_{ij} \psi_{jL} + \text{h.c.}) - \sum_A g_A G^A \mu J^\mu_A,
\]

with

\[
J^\mu_A = \sum_{i,j} \left[ \overline{\psi}_{iL} \gamma^\mu (Q^A_{L})_{ij} \psi_{jL} + \overline{\psi}_{iR} \gamma^\mu (Q^A_{R})_{ij} \psi_{jR} \right].
\]

The Yukawa couplings must preserve gauge invariance and hence they satisfy

\[
\sum_k \mathcal{Y}^a_{ik} (Q^A_{R})_{kj} - \sum_k (Q^A_{L})_{ik} \mathcal{Y}^a_{kj} + \sum_b \mathcal{Y}^b_{ij} (Q^A_H)_{ba} = 0.
\]

The Higgs fields acquire the VEVs \( \langle H_a \rangle = v_a \) which break the gauge group, leaving an unbroken subgroup \( \mathcal{G}_0 \). Then, the mass matrix of the \( \psi \) fields is given by

\[
\mathcal{M}_{ij} = \sum_a \mathcal{Y}^a_{ij} v_a,
\]

leading to

\[
\mathcal{L}_{\text{toy model}} \supset \sum_i \overline{\psi}_i i \partial \psi_i - \sum_{i,j} (\overline{\psi}_{iL} \mathcal{M}_{ij} \psi_{jL} + \text{h.c.}) - \sum_{a,i,j} \tilde{H}_a (\overline{\psi}_{iL} \mathcal{Y}^a_{ij} \psi_{jL} + \text{h.c.}) - \sum_A g_A G^A \mu J^\mu_A,
\]

where \( \tilde{H}_a = H_a - v_a \) are the Higgs fluctuations around the vacuum.

In order to go in the mass basis, the mass matrix \( \mathcal{M} \) is diagonalized via the bi-unitary transformations \( \psi_R \to U_R \psi_R \) and \( \psi_L \to U_L \psi_L \), which by construction satisfy \( U_R^\dagger \mathcal{M} U_L = \text{diag}(m_1, m_2, ...) \). This yields

\[
\mathcal{L}_{\text{toy model}} \supset \sum_i \overline{\psi}_i (i \partial - m_i) \psi_i - \sum_A g_A G^A \mu J^\mu_A - \sum_{a,i,j} \tilde{H}_a (\overline{\psi}_{iL} \mathcal{Y}^a_{ij} \psi_{jL} + \text{h.c.}),
\]

where \( \mathcal{Y}^a_R = U_L^\dagger \mathcal{Y}^a U_R = (\hat{\mathcal{Y}}^a_L)^\dagger \), while the gauge currents in the mass basis are equal to

\[
J^\mu_U = \sum_{i,j} \left[ \overline{\psi}_{iL} \gamma^\mu (U_L^A Q^A_{L})_{ij} \psi_{jL} + \overline{\psi}_{iR} \gamma^\mu (U_R^A Q^A_{R})_{ij} \psi_{jL} \right].
\]
After integrating out the heavy fermion fields, we get EFT operators of the type

\[ \mathcal{L}_{\text{toy model}} \supset \sum_{A,B,C} \frac{g_A g_B g_C}{48\pi^2} \epsilon^{\alpha\mu\nu\beta} G_\alpha^A G_\mu B G_\nu C \]

\[ - \sum_{a,b,c} \frac{g_B g_C}{48\pi^2} D^a_{\alpha\mu} \epsilon_{\alpha\mu\nu\beta} \tilde{H}_a \partial_\nu G^B B \partial_\beta G^C C, \]

in terms of the EFT coefficients \( C^{ABC} = -C^{BAC} \) and \( D^a_{BC} = D^{aCB} \) that we want to compute. Moreover, integrating by parts the term on the first line of Eq. (A.10), one also obtains

\[ C^{ABC} + C^{CAB} + C^{BCA} = 0. \]  

(A.11)

### A.2 \( \gamma_5 \) in dimensional regularization

Dimensional regularization allows to regularize the divergences arising from loop calculations in 4 dimensions, while explicitly preserving Lorentz covariance and gauge invariance. In the \( d \)-dimensional spacetime, the mass dimensions of the quantum fields are equal to

\[ [\psi] = \frac{d-1}{2}, \quad [H] = [G_\mu] = \frac{d-2}{2}. \]  

(A.12)

Hence, in order to keep the gauge couplings dimensionless, one introduces the renormalization scale \( \mu \) by the substitution

\[ g \rightarrow \mu^\frac{4-d}{2} g \]  

(A.13)

in the Lagrangian. The use of dimensional regularization poses some potential problems in calculations where the \( \gamma^5 \) matrix is involved. In fact, \( \gamma^5 \) (or equivalently the antisymmetric tensor \( \epsilon^{\alpha\beta\mu\nu} \)) is a quantity whose definition is strictly connected to the fact that space-time is four dimensional, and a definition in \( d \) dimensions requires special care. Here, we adopt the Breitenlohner-Maison-'t Hooft-Veltman (BMHV) scheme, which is able to reproduce the chiral anomaly (see [45] for a recent review).

We decompose all matrices into a four-dimensional (denoted by bars) and an extra-dimensional (also called “evanescent”, denoted by hats) component:

\[ \gamma^\mu = \bar{\gamma}^\mu + \hat{\gamma}^\mu, \]  

(A.14)

where \( \bar{\gamma}^\mu \) is non-zero only when \( \mu \) takes the ordinary values 0, 1, 2, 3 and \( \hat{\gamma}^\mu \) vanishes for \( \mu = 0, 1, 2, 3 \). Correspondingly, the matrix tensor \( g_{\mu\nu} \) has a four-dimensional and an extra-dimensional part,

\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + \hat{g}_{\mu\nu}, \]  

(A.15)

while mixed components vanish. The gamma matrices satisfy

\[ \{\bar{\gamma}^\mu, \bar{\gamma}^\nu\} = 2\bar{g}^{\mu\nu}, \quad \{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} = 2\hat{g}^{\mu\nu}, \quad \{\bar{\gamma}^\mu, \hat{\gamma}^\nu\} = 0. \]  

(A.16)

Then, we simply define \( \gamma^5 \) as in four dimensions, that is

\[ \gamma^5 = \bar{\gamma}^0 \bar{\gamma}^1 \bar{\gamma}^2 \bar{\gamma}^3. \]  

(A.17)

It is easy to check that the definition in Eq. (A.17) implies

\[ \{\gamma^5, \bar{\gamma}^\mu\} = 0, \quad [\gamma^5, \hat{\gamma}^\mu] = 0, \]  

\[ \text{Tr} \gamma^5 \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu = \text{Tr} \gamma^5 \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu = 4i\epsilon^{\alpha\beta\mu\nu}, \]  

(A.18)

which is the correct four-dimensional result.
In a general chiral gauge theory, the fermion fields are introduced as Weyl spinors whose formalism is intrinsically tied to 4-dimensional space. In $d$ dimensions, we replace the Weyl spinors by projections of Dirac spinors, which can be generalized to arbitrary dimensions. The right and left projections are $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$, as in the 4-dimensional space. Then, there are three possible inequivalent choices for the $d$-dimensional extension of the right-handed chiral current $\bar{\psi}_i R \gamma^\mu \psi_j$ coupled to gauge bosons, which are

$$\bar{\psi}_i P_L \gamma^\mu \psi_j, \quad \bar{\psi}_i P_R \gamma^\mu \psi_j, \quad \bar{\psi}_i P_L \gamma^\mu P_R \psi_j.$$  

They are different because $P_L \gamma^\mu \neq \gamma^\mu P_R$ in $d$ dimensions. Each of these does lead to valid $d$-dimensional extensions of the model that are perfectly renormalizable using dimensional regularization and the BMHV scheme. However, the intermediate calculations and the final $d$-dimensional results will differ, depending on the choice for this interaction term. Our choice for this work is to use the third option, that is

$$\bar{\psi}_i P_L \gamma^\mu P_R \psi_j = \bar{\psi}_i \gamma^\mu \psi_j,$$  

is the most symmetric one, and leads to the simplest expressions. Similar considerations hold for the left-handed chiral current $\bar{\psi}_i L \gamma^\mu \psi_j$. A different choice has to be taken instead for the kinetic terms $\bar{\psi}_i R \gamma^\mu \psi_i$ and $\bar{\psi}_i L \gamma^\mu \psi_i$. Indeed, in order to properly regularize the theory, we need to consider the full Dirac fermion kinetic term $\bar{\psi}_i i \gamma^\mu \psi_i$, including the evanescent terms.

Once the regulated amplitude is well-defined, we can perform all the necessary subtractions of the divergences of its sub-diagrams and the resulting finite expression is interpreted in the physical 4-dimensional space by setting all quantities to their 4-dimensional values, i.e. first taking the $d \rightarrow 4$ limit and then, setting all remaining evanescent terms to zero.

### A.3 1-loop matching

The epsilon tensor structure occurs in the 3-point functions $\Gamma_{ABC}^{\mu\nu}(x,y,z)$ and $\Gamma_{aBC}^{\mu\nu}(x,y,z)$ at 1-loop through fermionic triangle diagrams (see Fig. 1). The amplitudes in momentum space are defined via

$$\int d^4x \, d^4y \, d^4z \, e^{i(x_{q_1}+y_{q_2}+z_{q_3})} \Gamma_{ABC}^{\mu\nu}(x,y,z) \big|_{1\text{-loop}} = (2\pi)^4 \delta^{(4)}(q_1 + q_2 + q_3) \mu^{\frac{4-d}{2}} i M_{ABC}^{\mu\nu}(q_1, q_2, q_3),$$  

and

$$\int d^4x \, d^4y \, d^4z \, e^{i(x_{q_1}+y_{q_2}+z_{q_3})} \Gamma_{aBC}^{\mu\nu}(x,y,z) \big|_{1\text{-loop}} = (2\pi)^4 \delta^{(4)}(q_1 + q_2 + q_3) \mu^{\frac{4-d}{2}} i M_{aBC}^{\mu\nu}(q_1, q_2, q_3),$$

which yield

$$M_{ABC}^{\mu\nu} = \sum_{i,j,k} g_{A_iB_jC_k}(U_{x_1}^i Q_{x_1} A_{x_1})_{jk}(U_{x_2}^i Q_{x_2} B_{x_2})_{ki}(U_{x_3}^i Q_{x_3} C_{x_3})_{ij}$$

$$\times \frac{i \mu^{4-d}}{(2\pi)^d} \int \frac{dk^d}{(k^2 - m_i^2)(k + q_3)^2 - m_j^2}$$

$$+ \sum_{i,j,k} g_{A_iB_jC_k}(U_{x_1}^i Q_{x_1} A_{x_1})_{kj}(U_{x_3}^i Q_{x_3} C_{x_3})_{ji}(U_{x_2}^i Q_{x_2} B_{x_2})_{ik}$$

$$\times \frac{i \mu^{4-d}}{(2\pi)^d} \int \frac{dk^d}{(k^2 - m_i^2)(k + q_3)^2 - m_j^2}.$$

(A.24)
Figure 1: Feynman diagrams relative to the 3-point functions in Eqs. (A.22)–(A.23).

and

\[
M^\mu_{\alpha\beta\gamma} = \sum_{i,j,k} \frac{g_{B} g_{C}}{\chi_{1}^{*}} \langle \hat{H}_{A} \rangle_{ijk} (U_{i}^\dagger Q_{B} U_{j}^{\dagger} Q_{C} U_{k}^{\dagger} Q_{B} U_{j}^{\dagger} Q_{C} U_{k}^{\dagger} Q_{B} U_{j})
\]

\[
\times \langle \hat{H}_{A} \rangle_{ijk} (U_{i}^\dagger Q_{B} U_{j}^{\dagger} Q_{C} U_{k}^{\dagger} Q_{B} U_{j})
\]

\[
\times \int \frac{d^4k}{(2\pi)^4} \frac{Tr[D_{\mu} P_{\chi_{1}} (k + m_{i})]}{[(k^2 - m_{i}^2)(k^2 - q_2^2 - m_{j}^2) - (k^2 - q_3^2 - m_{k}^2)]}
\]

\[
\times \int \frac{d^4k}{(2\pi)^4} \frac{Tr[D_{\mu} P_{\chi_{1}} (k + m_{i})]}{[(k^2 - m_{i}^2)(k^2 - q_2^2 - m_{j}^2) - (k^2 - q_3^2 - m_{k}^2)]}
\]

\[
\times \frac{g_{B} g_{C}}{24\pi^2} \epsilon_{\alpha\mu\nu\beta}(C^{ABC} i q_3 + C^{CAB} i q_2 + C^{BCA} i q_1)
\]

\[
(A.25)
\]

Since we have regularized the theory, the loop integrals over momentum \(k\) are convergent and can be evaluated with the usual well-known techniques. Next, we perform the traces over the Dirac indices to extract the terms involving the epsilon tensor structure we are interested on. One finds that such terms are finite, i.e. they do not contain \(1/(d-4)\) poles, and are independent from the renormalization scale \(\mu\). Hence, we can send \(d \to 4\) and set the evanescent components to zero.

In order to obtain the EFT coefficients in Eq. (A.10), we have to match the expressions that we have calculated above to the EFT matrix elements in the limit of heavy fermion masses, i.e.

\[
\lim_{m_{i,j,k}^2 \gg q_2^2 q_3^2 q_4^2} M_{\alpha\beta\gamma}^{\mu\nu} |_{\epsilon-\text{tensor}} = \frac{g_{A} g_{B} g_{C}}{24\pi^2} \epsilon_{\alpha\mu\nu\beta} (C^{ABC} i q_3 + C^{CAB} i q_2 + C^{BCA} i q_1)
\]

\[
(A.26)
\]

and

\[
\lim_{m_{i,j,k}^2 \gg q_2^2 q_3^2 q_4^2} M_{\alpha\beta\gamma}^{\mu\nu} |_{\epsilon-\text{tensor}} = \frac{g_{B} g_{C}}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} q_{2\alpha} q_{3\beta}
\]

\[
(A.27)
\]
Thus we get

\[ C^{ABC} = \int_0^1 ds \int_0^1 dx \int_0^1 dy \int_0^1 dz \ 2 \delta(1-x-y-z) \times \]

\[ \times \text{Re} \left\{ 3y \text{Tr} \left[ e^{-s\gamma_5 M^A_{L\gamma} M^B_{L\gamma}} Q^B_{R\gamma} M^A_{L\gamma} Q^C_{R\gamma} \right] \right. \]

\[ -3y \text{Tr} \left[ e^{-s\gamma_5 M^A_{L\gamma} M^B_{L\gamma}} Q^B_{R\gamma} M^A_{L\gamma} Q^C_{R\gamma} \right] \]

\[ +3y \text{Tr} \left[ e^{-s\gamma_5 M^A_{R\gamma} M^B_{R\gamma}} Q^B_{L\gamma} M^A_{R\gamma} Q^C_{L\gamma} \right] \]

\[ -3y \text{Tr} \left[ e^{-s\gamma_5 M^A_{R\gamma} M^B_{R\gamma}} Q^B_{L\gamma} M^A_{R\gamma} Q^C_{L\gamma} \right] \]

\[ +y \text{Tr} \left[ e^{-s\gamma_5 M^A_{R\gamma} M^B_{L\gamma}} Q^B_{L\gamma} M^A_{R\gamma} Q^C_{L\gamma} \right] \]

\[ -x \text{Tr} \left[ e^{-s\gamma_5 M^A_{L\gamma} M^B_{R\gamma}} Q^B_{R\gamma} M^A_{L\gamma} Q^C_{R\gamma} \right] \]

\[ +x \text{Tr} \left[ e^{-s\gamma_5 M^A_{L\gamma} M^B_{R\gamma}} Q^B_{L\gamma} M^A_{L\gamma} Q^C_{L\gamma} \right] \]

\[ -y \text{Tr} \left[ e^{-s\gamma_5 M^A_{R\gamma} M^B_{L\gamma}} Q^B_{L\gamma} M^A_{R\gamma} Q^C_{L\gamma} \right] \}

\( \text{(A.28)} \)

and

\[ D^{aBC} = \int_0^1 ds \int_0^1 dx \int_0^1 dy \int_0^1 dz \ 6 \delta(1-x-y-z) \times \]

\[ \times \text{Im} \left\{ x \text{Tr} \left[ e^{-s\gamma_5 M^A_{L\gamma} M^B_{L\gamma}} Q^B_{R\gamma} M^A_{L\gamma} Q^C_{L\gamma} \gamma^\mu \right] \right. \]

\[ +x \text{Tr} \left[ e^{-s\gamma_5 M^A_{R\gamma} M^B_{L\gamma}} Q^B_{L\gamma} M^A_{R\gamma} Q^C_{L\gamma} \gamma^\mu \right] \]

\[ +y \text{Tr} \left[ e^{-s\gamma_5 M^A_{L\gamma} M^B_{R\gamma}} Q^B_{R\gamma} M^A_{L\gamma} Q^C_{R\gamma} \gamma^\mu \right] \]

\[ +y \text{Tr} \left[ e^{-s\gamma_5 M^A_{R\gamma} M^B_{L\gamma}} Q^B_{L\gamma} M^A_{R\gamma} Q^C_{L\gamma} \gamma^\mu \right] \]

\[ +y \text{Tr} \left[ e^{-s\gamma_5 M^A_{L\gamma} M^B_{R\gamma}} Q^B_{R\gamma} M^A_{L\gamma} Q^C_{R\gamma} \gamma^\mu \right] \]

\[ +y \text{Tr} \left[ e^{-s\gamma_5 M^A_{R\gamma} M^B_{L\gamma}} Q^B_{L\gamma} M^A_{R\gamma} Q^C_{L\gamma} \gamma^\mu \right] \}

\( \text{(A.29)} \)

### A.4 Reproducing the chiral anomaly in the EFT

A consistent gauge theory must be anomaly free and hence the chiral anomaly needs to cancel when we sum over all the fermion fields of the theory. If we integrate out a heavy fermionic sector of the complete UV model, the corresponding chiral anomaly is reproduced in the EFT action \( S_{\text{eff}} \) thanks to the WZ effective operators in Eq. (A.10). To show this, we make an infinitesimal transformation of angle \( \alpha_A \) along the \( Q^A \) generator. The gauge fields \( G^B_\mu \) and the Higgs fields \( \tilde{H}_a \) transform like

\[ \delta \tilde{H}_a = \sum_b \alpha_A (iQ^A_{H})_{ab} v_b + \text{linear terms} , \]

\( \text{(A.30)} \)
\[ \delta G^B_\mu = -\delta_{AB} (\partial_\mu \alpha_B)/g_B + \text{linear terms}, \] (A.31)

which, from the variation of the effective Lagrangian in Eq. (A.10), yields

\[ \delta S_{\text{eff}} = \sum_{BC} \frac{g_B g_C}{48\pi^2} \left[ C_{ABC} + C^{ACB} + D^{\alpha\beta} (iQ_H^A)_{ab} v_b \right] \int d^4 x \partial_\alpha G_\mu^B \partial_\beta G_\nu^C \epsilon^{\mu\nu\alpha\beta} \]

\[ = \sum_{BC} \frac{g_B g_C}{48\pi^2} \left[ \text{Tr} Q_R^A (Q_R^B, Q_R^C) - \text{Tr} Q_L^A (Q_L^B, Q_L^C) \right] \int d^4 x \partial_\alpha G_\mu^B \partial_\beta G_\nu^C \epsilon^{\mu\nu\alpha\beta}. \] (A.32)

### A.5 WZ terms for massive vector bosons and Goldstone bosons

The VEVs of the Higgs fields contribute to the mass matrix \( M^2_{\text{gauge}} \) for the gauge bosons \( G^A_\mu \) with elements

\[ (M^2_{\text{gauge}})_{AB} = \sum_{a,b,c} g_A (iQ_H^A)_{ca} v_a g_B (iQ_H^B)_{cb} v_b. \] (A.33)

Since the matrix is real and symmetric, we can diagonalize it through an orthogonal matrix \( O_{AB} \) such that

\[ \sum_{A,B} O_{DB} O_{CA} (M^2_{\text{gauge}})_{AB} = m^2 C \delta_{CD}. \] (A.34)

The massive eigenstates are then defined by

\[ Z^A_\mu = \sum_{B} O_{AB} G^B_\mu, \] (A.35)

with corresponding symmetry generator

\[ \tilde{g}_A T^A = \sum_{B} O_{AB} g_B Q^B. \] (A.36)

There are two scenarios for each generator \( T^A \):

- \( (iT_H^A)_{ab} v_b = 0 \), if \( T^A \) belongs to the unbroken subgroup \( G_0 \), such that the corresponding vector boson \( Z^A_\mu \) is then massless, i.e. \( m^A = 0 \);

- \( (iT_H^A)_{ab} v_b \neq 0 \), if \( T^A \) is spontaneously broken by the Higgs VEVs. The corresponding Nambu-Goldstone boson \( \eta^A \) is given by

\[ \eta^A = \sum_{a} t^A_a \tilde{H}_a, \] (A.37)

where \( t^A_a = \tilde{g}_A (iT_H^A)_{ab} v_b / m^A \) are a (incomplete) set of orthogonal vectors in the Higgs space, i.e. \( \sum_a t^A_a t^A_a = \delta_{AB} \). The Goldstone field is then eaten by the vector boson \( Z^A_\mu \) which acquires a mass \( m^A \).

The \( \tilde{H}_a \) contains the Goldstone modes along the \( t^A_a \) directions while the remaining modes, orthogonal to the Goldstones, are all physical, i.e.

\[ \tilde{H}_a = \sum_{\text{NG modes}} t^A_a \eta^A + \ldots. \] (A.38)

The interaction terms between the \( \psi \) fields and the Goldstone bosons are given by

\[ \sum_a \gamma^a_{ij} t^A_a = \frac{ig_A}{m^A} \sum_k \left[ (T^A_L)_{ik} M_{kj} - M_{ik} (T^A_R)_{kj} \right], \] (A.39)
because of the gauge invariance of the Yukawa couplings. Upon an infinitesimal transformation of angle $\alpha_A$ along a broken $T^A$ generator, the Goldstone field $\eta^A$ transform like

$$\delta \eta^A = \alpha_A m_A + \text{linear terms}.$$  \hspace{1cm} (A.40)

Finally, the effective operators in Eq. (A.10) written in terms of the $Z^A_\mu$ and $\eta^A$ fields are

$$\sum_{A,B,C} \frac{g_A g_B g_C}{48\pi^2} C_\mu^{ABC} \epsilon_{\alpha\mu\nu} Z^A_\alpha \partial_\mu Z^B_\nu - \sum_{A,B,C} \frac{g_A g_B g_C}{48\pi^2} D_\eta^{ABC} \epsilon_{\mu\alpha\beta} \frac{\eta^A}{m_A} \partial_\mu Z^B_\alpha \partial_\beta Z^C_\nu + \ldots, \hspace{1cm} (A.41)$$

where the dots contain the interaction terms with the Higgs physical modes. The rotated EFT coefficients are equal to

$$C_\mu^{ABC} = \int_0^{+\infty} ds \int_0^1 dx \int_0^1 dy \int_0^1 dz \; 2 \delta(1-x-y-z) \times \Re\left\{ 3y \text{Tr} \left[ e^{syM^A_T^T M^A_R M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} T^C_L M} \right] 
-3y \text{Tr} \left[ e^{syM^A_T^T M^B_T M^A_R e^{-szM^A_M M^B_L e^C_T T^C_L M} T^C_L M} \right] 
+3y \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} T^A_R M^C_R} \right] 
-3y \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} T^C_R e^{-szM^A_M M^B_L e^C_T T^C_L M} T^A_R M^C_R} \right] 
+y \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} T^A_R M^C_R} \right] 
-x \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} T^A_R M^C_R} \right] 
+x \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} T^A_R M^C_R} \right] 
-y \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} T^A_R M^C_R} \right] \right\}, \hspace{1cm} (A.42)$$

and

$$D_\eta^{ABC} = \int_0^{+\infty} ds \int_0^1 dx \int_0^1 dy \int_0^1 d\zeta \; 6 \delta(1-x-y-z) \times \Re\left\{ x \text{Tr} \left[ e^{szM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} e^{syM^A_M M^B_L e^C_T T^C_L M} (T^A_R M - MT^A_R)} \right] 
+y \text{Tr} \left[ e^{szM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} e^{syM^A_M M^B_L e^C_T T^C_L M} (T^A_R M - MT^A_R)} \right] 
+y \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} e^{syM^A_M M^B_L e^C_T T^C_L M} (T^A_R M - MT^A_R)} \right] 
+y \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} e^{syM^A_M M^B_L e^C_T T^C_L M} (T^A_R M - MT^A_R)} \right] 
+y \text{Tr} \left[ e^{syM^A_T^T M^B_T e^{-szM^A_M M^B_L e^C_T T^C_L M} e^{syM^A_M M^B_L e^C_T T^C_L M} (T^A_R M - MT^A_R)} \right] \right\}. \hspace{1cm} (A.43)$$
A.6 Properties of the WZ coefficients

In general, the expressions (A.42) and (A.43) involve non-trivial integrations which are difficult to compute. Special simplifications occur if the fermion mass term $\bar{\psi}_{Li} M_{ij} \psi_{Rj}$ is invariant under any of the symmetry generators $T^A$. If so, the mass matrix $M$ satisfies

$$\sum_k M_{ik} (T^A_R)_{kj} - \sum_k (T^A_L)_{ik} M_{kj} = 0.$$  \hspace{1cm} (A.44)

Alternatively, the invariance of the mass term reads

$$\sum_{a,b} Y_{ij} (T^A_H)^{ab} v_b = 0,$$  \hspace{1cm} (A.45)

which could occur if $T^A$ belongs to $G_0$ or some Yukawa coupling vanishes. Then, one finds

$$C_{ABC}^Z = \begin{cases} 
\text{Tr} T^A_R \{ T^B_R, T^C_R \} - \text{Tr} T^A_L \{ T^B_L, T^C_L \} & \text{if } \sum_{a,b} Y_{ij} (T^B_H)^{ab} v_b = 0 , \\
\text{Tr} T^A_L \{ T^B_L, T^C_L \} - \text{Tr} T^A_R \{ T^B_R, T^C_R \} & \text{if } \sum_{a,b} Y_{ij} (T^C_H)^{ab} v_b = 0 , \\
0 & \text{if } \sum_{a,b} Y_{ij} (T^B_H)^{ab} v_b = 0 . 
\end{cases}$$  \hspace{1cm} (A.46)

and

$$D_{\eta}^{ABC} = 3 \left[ \text{Tr} T^A_L \{ T^B_L, T^C_L \} - \text{Tr} T^A_R \{ T^B_R, T^C_R \} \right] \text{ if } \sum_{a,b} Y_{ij} (T^B_H)^{ab} v_b = 0 .$$  \hspace{1cm} (A.47)

Note that $D_{\eta}^{ABC}$ vanishes if

$$\sum_{a,b} Y_{ij} (T^A_H)^{ab} v_b = 0,$$  \hspace{1cm} (A.48)

i.e. if the fermion mass term is invariant under symmetry generator $T^A$.

Consider now an unbroken generator $Q \in G_0$, hence satisfying

$$\sum_k M_{ik} (Q_R)_{kj} - \sum_k (Q_L)_{ik} M_{kj} = 0,$$  \hspace{1cm} (A.49)

with the commutation rules

$$[Q, T^A] = \sum_B q_{AB} T^B .$$  \hspace{1cm} (A.50)

Thanks to (A.49) and the cyclic property of the trace, the expression

$$C_{Z}^{ABC} |_{T^A \to [Q, T^A]} + C_{Z}^{ABC} |_{T^B \to [Q, T^B]} + C_{Z}^{ABC} |_{T^C \to [Q, T^C]} = 0$$  \hspace{1cm} (A.51)

is identically zero. Then, we find that the EFT coefficients satisfy

$$\sum_D (q_{AD} C_{Z}^{DBC} + q_{BD} C_{Z}^{ADC} + q_{CD} C_{Z}^{ABD}) = 0 .$$  \hspace{1cm} (A.52)

The same argument yields

$$\sum_D (q_{AD} D_{\eta}^{DBC} + q_{BD} D_{\eta}^{ADC} + q_{CD} D_{\eta}^{ABD}) = 0 .$$  \hspace{1cm} (A.53)
References

[1] B. Holdom, “Two U(1)’s and Epsilon Charge Shifts,” *Phys. Lett. B* **166** (1986) 196–198.

[2] R. Foot, “New Physics From Electric Charge Quantization?,” *Mod. Phys. Lett. A* **6** (1991) 527–530.

[3] X. G. He, G. C. Joshi, H. Lew, and R. R. Volkas, “New Z-prime Phenomenology,” *Phys. Rev. D* **43** (1991) 22–24.

[4] X.-G. He, G. C. Joshi, H. Lew, and R. R. Volkas, “Simplest Z-prime model,” *Phys. Rev. D* **44** (1991) 2118–2132.

[5] A. Greljo, Y. Soreq, P. Stangl, A. E. Thomesen, and J. Zupan, “Muon Force Behind Flavor Anomalies,” arXiv:2107.07518 [hep-ph].

[6] E. D’Hoker and E. Farhi, “Decoupling a Fermion Whose Mass Is Generated by a Yukawa Coupling: The General Case,” *Nucl. Phys. B* **248** (1984) 59–76.

[7] E. D’Hoker and E. Farhi, “Decoupling a Fermion in the Standard Electroweak Theory,” *Nucl. Phys. B* **248** (1984) 77.

[8] J. Preskill, “Gauge anomalies in an effective field theory,” *Annals Phys.* **210** (1991) 323–379.

[9] F. Feruglio, A. Masiero, and L. Maiani, “Low-energy effects of heavy chiral fermions,” *Nucl. Phys. B* **387** (1992) 523–561.

[10] J. A. Dror, R. Lasenby, and M. Pospelov, “New constraints on light vectors coupled to anomalous currents,” *Phys. Rev. Lett.* **119** no. 14, (2017) 141803, arXiv:1705.06726 [hep-ph].

[11] J. A. Dror, R. Lasenby, and M. Pospelov, “Dark forces coupled to nonconserved currents,” *Phys. Rev. D* **96** no. 7, (2017) 075036, arXiv:1707.01503 [hep-ph].

[12] A. Ismail and A. Katz, “Anomalous $Z'$ and diboson resonances at the LHC,” *JHEP* **04** (2018) 122, arXiv:1712.01840 [hep-ph].

[13] L. Michaels and F. Yu, “Probing new $U(1)$ gauge symmetries via exotic $Z \rightarrow Z'\gamma$ decays,” *JHEP* **03** (2021) 120, arXiv:2010.00021 [hep-ph].

[14] J. Davighi, “Anomalous $Z'$ bosons for anomalous B decays,” *JHEP* **08** (2021) 101, arXiv:2105.06918 [hep-ph].

[15] G. D. Kribs, G. Lee, and A. Martin, “Effective Field Theory of Stückelberg Vector Bosons,” arXiv:2204.01755 [hep-ph].

[16] N. Bizot and M. Frigerio, “Fermionic extensions of the Standard Model in light of the Higgs couplings,” *JHEP* **01** (2016) 036, arXiv:1508.01645 [hep-ph].

[17] Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul, and A. N. Rossia, “The anomalous case of axion EFTs and massive chiral gauge fields,” *JHEP* **07** (2021) 189, arXiv:2011.10025 [hep-ph].

[18] Muon $g-2$ Collaboration, B. Abi et al., “Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm,” *Phys. Rev. Lett.* **126** no. 14, (2021) 141801, arXiv:2104.11225 [hep-ex].

[19] M. Duerr, P. Fileviez Perez, and M. B. Wise, “Gauge Theory for Baryon and Lepton Numbers with Leptoquarks,” *Phys. Rev. Lett.* **110** (2013) 231801, arXiv:1304.0576 [hep-ph].

[20] M. Duerr and P. Fileviez Perez, “Baryonic Dark Matter,” *Phys. Lett. B* **732** (2014) 101–104, arXiv:1309.3970 [hep-ph].

[21] B. A. Dobrescu and C. Frugiuele, “Hidden GeV-scale interactions of quarks,” *Phys. Rev. Lett.* **113** (2014) 061801, arXiv:1404.3947 [hep-ph].
[22] B. A. Dobrescu, “Leptophobic Boson Signals with Leptons, Jets and Missing Energy,” arXiv:1506.04435 [hep-ph].

[23] P. Fileviez Pérez, E. Golias, R.-H. Li, C. Murgui, and A. D. Plascencia, “Anomaly-free dark matter models,” Phys. Rev. D 100 no. 1, (2019) 015017, arXiv:1904.01017 [hep-ph].

[24] P. Fileviez Pérez, C. Murgui, and A. D. Plascencia, “Neutrino-Dark Matter Connections in Gauge Theories,” Phys. Rev. D 100 no. 3, (2019) 035041, arXiv:1905.06344 [hep-ph].

[25] P. Fileviez Perez and M. B. Wise, “Breaking Local Baryon and Lepton Number at the TeV Scale,” JHEP 08 (2011) 068, arXiv:1106.0343 [hep-ph].

[26] P. Fileviez Perez, S. Ohmer, and H. H. Patel, “Minimal Theory for Lepto-Baryons,” Phys. Lett. B 735 (2014) 283–287, arXiv:1403.8029 [hep-ph].

[27] A. Wulzer, “An Equivalent Gauge and the Equivalence Theorem,” Nucl. Phys. B 885 (2014) 97–126, arXiv:1309.6055 [hep-ph].

[28] E. Izaguirre, T. Lin, and B. Shuve, “Searching for Axionlike Particles in Flavor-Changing Neutral Current Processes,” Phys. Rev. Lett. 118 no. 11, (2017) 111802, arXiv:1611.09355 [hep-ph].

[29] K. Asai, “Predictions for the neutrino parameters in the minimal model extended by linear combination of U(1)$_{L_\mu - L_\tau}$, U(1)$_{L_\mu - L_\tau}$ and U(1)$_{B-L}$ gauge symmetries,” Eur. Phys. J. C 80 no. 2, (2020) 76, arXiv:1907.04042 [hep-ph].

[30] T. Araki, K. Asai, J. Sato, and T. Shimomura, “Low scale seesaw models for low scale U(1)$_{L_\mu - L_\tau}$ symmetry,” Phys. Rev. D 100 no. 9, (2019) 095012, arXiv:1909.08827 [hep-ph].

[31] Gfitter, “Constraints on the oblique parameters and related theories.” http://project-gfitter.web.cern.ch/project-gfitter/Oblique_Parameters/.

[32] CDF Collaboration, T. Aaltonen et al., “High-precision measurement of the W boson mass with the CDF II detector,” Science 376 no. 6589, (2022) 170–176.

[33] A. Djouadi, “The Anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model,” Phys. Rept. 457 (2008) 1–216, arXiv:hep-ph/0503172.

[34] ATLAS Collaboration, G. Aad et al., “Combined measurements of Higgs boson production and decay using up to 80 fb$^{-1}$ of proton-proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment,” Phys. Rev. D 101 no. 1, (2020) 012002, arXiv:1909.02845 [hep-ex].

[35] M. Cepeda et al., “Report from Working Group 2: Higgs Physics at the HL-LHC and HE-LHC,” CERN Yellow Rep. Monogr. 7 (2019) 221–584, arXiv:1902.00134 [hep-ph].

[36] L. Di Luzio, J. F. Kamenik, and M. Nardecchia, “Implications of perturbative unitarity for scalar di-boson resonance searches at LHC,” Eur. Phys. J. C 77 no. 11, (2017) 122004, arXiv:1609.08382 [hep-ex].

[37] L. Di Luzio, R. Gröber, J. F. Kamenik, and M. Nardecchia, “Accidental matter at the LHC,” JHEP 07 (2015) 074, arXiv:1504.00359 [hep-ph].

[38] L. Di Luzio, J. F. Kamenik, and M. Nardecchia, “Implications of perturbative unitarity for scalar di-boson resonance searches at LHC,” Eur. Phys. J. C 77 no. 1, (2017) 30, arXiv:1604.05746 [hep-ph].

[39] L. Allwicher, P. Arnan, D. Barducci, and M. Nardecchia, “Perturbative unitarity constraints on generic Yukawa interactions,” JHEP 10 (2021) 129, arXiv:2108.00013 [hep-ph].

[40] T. Ma, B. Zhang, and G. Cacciapaglia, “Doubly Charged Lepton from an Exotic Doublet at the LHC,” Phys. Rev. D 89 no. 9, (2014) 093022, arXiv:1404.2375 [hep-ph].
[42] P. Ilten, Y. Soreq, M. Williams, and W. Xue, “Serendipity in dark photon searches,” *JHEP* **06** (2018) 004, arXiv:1801.04847 [hep-ph].

[43] B. Batell, J. L. Feng, M. Fieg, A. Ismail, F. Kling, R. M. Abraham, and S. Trojanowski, “Hadrophilic Dark Sectors at the Forward Physics Facility,” arXiv:2111.10343 [hep-ph].

[44] D. W. P. Amaral, D. G. Cerdeño, A. Cheek, and P. Foldenauer, “Distinguishing $U(1)_{L_{\mu}} - L_{\tau}$ from $U(1)_{L_{\mu}}$ as a solution for $(g-2)_{\mu}$ with neutrinos,” arXiv:2104.03297 [hep-ph].

[45] H. Bélusca-Maïto, A. Ilakovac, M. Madjor-Božinović, and D. Stöckinger, “Dimensional regularization and Breitenlohner-Maison/’t Hooft-Veltman scheme for $\gamma_5$ applied to chiral YM theories: full one-loop counterterm and RGE structure,” *JHEP* **08** no. 08, (2020) 024, arXiv:2004.14398 [hep-ph].