The era for studying particle physics with the LHC at CERN is ongoing. Since 2010, the experiments have been collecting data from proton-proton collisions at a center-of-mass energy of 7 TeV. This has already enabled the exploration of new regimes of the current Standard Model (SM) as well as physics beyond the SM. One of the aims is to establish or exclude the presence of a SM Higgs boson. The latest public Higgs search results were presented by the ATLAS and CMS collaborations in March 2012 [1, 2]. These analyses exclude a SM Higgs in the range 127 – 600 GeV to the 95% confidence level (CL). It is however important to keep in mind that new particles can both contribute to the Higgs decay width and alter its production cross section. The exclusion limits on this range of Higgs boson masses might thus not be valid for a Higgs that is SM-like in many respects, but which also couples to states beyond the SM. This is of particular relevance to the present paper. Let us point out that while both the ATLAS and CMS experiments have started to see potential evidence for a particle signal at ∼ 125 GeV, the significance is not yet enough to claim discovery and establish this to be caused by the Higgs particle itself. Moreover, there have been other, perhaps interesting, excesses in the Higgs searches; e.g. at the 2σ level for a ∼ 320 GeV particle mass in CMS [3, 4], which was then not confirmed by the latest preliminary results from the ATLAS experiment [5].

Due to the nature of hadron colliders, the LHC has obvious advantages in probing beyond the SM scenarios that incorporate strong quantum chromodynamic (QCD) interactions, such as minimal low-energy supersymmetry models. So far the LHC searches have found no evidence for strongly interacting beyond the SM particles [6]. No-
toriously, scenarios without direct SM QCD interactions are expected to give lower signals – although many exceptions, such as resonances (e.g. $W^+$) or composite state effects (e.g. $\tilde{Q}$), may appear. From an empirical point of view, there is a priori no need for new QCD interacting sectors. Indeed, two of the major questions in particle physics and cosmology – the fine-tuning problem in the SM Higgs sector (commonly known as the “LEP paradox” or the hierarchy problem [10]), and the dark matter (DM) problem with a thermally produced weakly interacting massive particle (WIMP) as one of the long-standing candidate solutions [11–15] – are not directly connected to QCD properties.

Given the latter point of view, we study the inert doublet model (IDM): a minimal extension of the SM which contains one additional electro-weak scalar doublet and has the potential both to alleviate the mentioned fine-tuning in the SM and to provide a DM candidate. The IDM appeared already in the 1970s in [16], but received tuning in the SM and to provide a DM candidate. The model could provide both a DM candidate solutions [11–15] – are not directly connected to QCD properties.

In the second part of the paper, where we study a new potential discovery channel for the IDM in the form of multilepton events via heavy Higgs production at the LHC.

The prospects for detecting IDM signatures in the upcoming LHC data at 14 TeV has already been partly explored. In [17, 27] the authors studied how inert particles affect SM Higgs searches, by the opening of additional decay channels, as well as the discovery potential in the dilepton and missing energy channel. A more comprehensive study of this dilepton channel was done in [29], followed by a trilepton study [30]. None of these studies explore the possibility to detect the inert doublet model in the almost background-free multilepton (≥ 4 leptons) plus $E_T$ channel. Here we argue that it is natural to study the tetralepton channel in addition to the di- and trilepton channels. This has actually been done for many other popular models, e.g. in supersymmetry [31, 32] and extra dimension [33, 34] models.

The inert doublet contains four new particle states. The more massive states may be pair produced in proton collisions and subsequently cascade decay (in one or two steps) down to the lightest inert particle state, which remains stable due to the conserved $\mathbb{Z}_2$ parity. In each decay step, an electromagnetically gauge boson is produced and can decay into one or two charged leptons. If the lightest stable inert particle is electrically neutral, it will contribute to the missing transverse energy ($E_T$), and up to six charged leptons can be directly produced from the $W^\pm$ and $Z$ boson that participated in the cascade decay. We show that the (≥ 4l + $E_T$)-channel is an interesting test of the IDM and can provide an early discovery channel of the IDM when the LHC runs at 14 TeV.

In Sec. [II] and [II] we set up the IDM framework and the theoretical, experimental and observational constraints that will be imposed on the model. In Sec. [IV] we answer our first question, namely under what conditions a heavy SM-like Higgs can survive the recent and complementary constraints from the LHC and XENON. In Sec. [V] we turn to our second aim, to discuss the multilepton signal at the LHC in such scenarios. We perform detailed event simulations for a set of IDM benchmark models and the SM background and describe our analysis tools in Sec. [VI]. Our results and discovery prospects for IDM in the tetralepton+$E_T$ channel are presented in Sec. [VII] and in Sec. [VIII] we summarize and conclude.

II. THE INERT DOUBLET MODEL

The IDM consists of the SM, including the standard Higgs doublet $H_1$, and an additional Lorentz scalar in the form of an SU(2)$_L$ doublet $H_2$. An exact unbroken $\mathbb{Z}_2$ symmetry is introduced, under which $H_2$ is taken to be odd ($H_2 \rightarrow -H_2$) while $H_1$ and all other SM fields are even. This $\mathbb{Z}_2$ symmetry protects against the introduction of new flavor changing neutral currents and guarantees the absence of direct Yukawa couplings between the inert states and the SM fermions (hence the name inert doublet model). The symmetry also renders the lightest particle state of $H_2$ stable. If neutral, the latter can provide a good DM candidate. The new kinetic gauge term takes the usual form, $D^\mu H_2 D_\mu H_2$, and the most general renormalizable CP conserving potential for the model.
IDM scalar sector is

\[
V = \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 \\
+ \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1|^2 |H_2|^2 + \lambda_5 \Re[(H_1^* H_2)^2],
\]

where \( \mu_1^2 \) and \( \lambda_i \) are real parameters.

Four new physical particle states are obtained in this model: two charged states, \( H^\pm \), and two neutral states, \( H^0 \) and \( A^0 \). After standard electroweak symmetry breaking, the masses of the scalar particles (including the SM-like Higgs mass \( m_h \)) are given by:

\[
\begin{align*}
m_{H^0}^2 &= \mu_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) v^2 \equiv \mu_2^2 + \lambda_{H^0} v^2, \\
m_{A^0}^2 &= \mu_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) v^2 \equiv \mu_2^2 + \lambda_{A^0} v^2, \\
m_{H^\pm}^2 &= \mu_2^2 + \lambda_3 v^2, \\
m_h^2 &= -2\mu_1^2 = 4\lambda_1 v^2,
\end{align*}
\]

where \( v \approx 177 \text{ GeV} \) is the vacuum expectation value of the Higgs field \( H_1 \). In the following, we choose \( H^0 \) to be the lightest inert particle, and hence the potential DM candidate. Notice that the roles of \( A^0 \) and \( H^0 \) are equivalent in the IDM and our conclusions would remain unchanged if we had chosen \( A^0 \) to be the DM candidate. A convenient set of parameters to describe the full scalar sector are the four scalar masses \( \{m_{H^0}, m_{A^0}, m_{H^\pm}, m_h\} \), the self coupling \( \lambda_2 \) and \( \lambda_{H^0} \equiv \lambda_3 + \lambda_4 + \lambda_5 \).

### III. CONSTRAINTS ON IDM

There are several theoretical, experimental and observational constraints on the model that have to be considered. For all the models in this study we consistently impose:

- the requirements for vacuum stability \[38, 39\],
- that calculations should be within the perturbative regime (with \( \lambda_i < 4\pi \)) \[17, 32\],
- unitarity constraints (the absolute value of the eigenvalues of the S-matrix are required to be \( \leq 1/2 \) for scalar-to-scalar scatterings, including the longitudinal parts of the gauge bosons) \[40, 43\],
- consistency with electroweak precision tests (EWPT) (99% CL) \[47\],
- consistency with particle collider data from LEP (\( \sim 95\% \text{ CL} \)) \[17, 27, 28, 48\].

IDMs with large Higgs masses can potentially alleviate the fine-tuning present in the SM and thus address the LEP paradox \[17\]. While we choose not to impose any explicit naturalness constraints here, we will extensively comment on this in Appendix A.

For a review of many of the constraints on IDM we refer to \[39\]. We have implemented the above list of constraints, as described in the given references, into our own computer code. We stress the importance of combining all these bounds since, as we will see, their complementarity becomes a powerful tool in constraining the IDM.

We will present results of random scans over the whole viable IDM parameter space that is of interest for our study (from a few GeV to hundreds of GeV). More precisely, the free parameters were taken to be the three masses of the inert scalars, the Higgs mass and the coupling \( \lambda_L \). We scanned over the ranges:

\[
\begin{align*}
5 \text{ GeV} &\leq m_{H^0} \leq 170 \text{ GeV}, \\
5 \text{ GeV} &\leq m_{A^0} \leq 800 \text{ GeV}, \\
\max(m_{H^0}, 70 \text{ GeV}) &\leq m_{H^\pm} \leq 800 \text{ GeV}, \\
100 \text{ GeV} &\leq m_h \leq 900 \text{ GeV}, \\
10^{-5} &\leq \left| \lambda_L \right| \leq 4\pi.
\end{align*}
\]

Once these parameters were chosen randomly, the value of \( \lambda_2 \) was fixed to its minimal value satisfying the constraints from vacuum stability. The resulting IDMs were confronted with the constraints listed in this section and only models passing the full set of constraints were considered as viable.

A random scan is always incomplete in covering all possible models. By a combination of random scans, simple Markov chain Monte Carlo searches (following \[50, 61\]) and physical insight into where models could be expected to be found, we believe that we have been able to cover all relevant parts of the parameter space for our results with more than 100 000 models. For example, earlier studies \[50, 61\] have already shown that expanding the scan to larger \( H^0 \) masses is not relevant if \( H^0 \) should constitute a WIMP DM candidate. This is at least true for \( H^0 \) masses below 500 GeV, and higher masses are not relevant for the current LHC searches. It is worth noticing that this part of the IDM gives well isolated regions in all our presented quantities. In practice, no viable IDMs were found.

\footnote{The constraint in Eq. 17 of reference \[17\] that poses a sufficient condition not to affect their naturalness arguments for the IDM, is not included. Applying it does not change our conclusions, although it would reject the models in our scans which have \( m_{H^0} \geq 120 \text{ GeV} \) and correct relic density \( \Omega_{H^0} \approx \Omega_{\text{CDM}} \).

\footnote{See also \[44\], where the authors studied the constraints from unitarity on the IDM.}
with $m_h \gtrsim 700$ GeV, $m_{H^0} \gtrsim 150$ GeV, $m_A^0 \lesssim 50$ GeV and $|\lambda_{H^0}| \gtrsim 7$.

The DM relic density calculations have been performed by DarkSUSY \cite{57} interfaced with FormCalc \cite{58}. This code was originally developed in \cite{21}, but has now been updated to also include three-body final states (as in \cite{53}). Also, an upgrade of micrOMEGAs \cite{59} including annihilation into three-body final states \cite{55} has been used for the scans.

IV. IDM IN LIGHT OF XENON AND THE LHC HIGGS SEARCH

Dark matter direct detection and the LHC’s SM Higgs searches are known to be complementary in constraining Higgs portal DM models \cite{62–70}. Direct detection experiments pose upper limits on the DM coupling to the Higgs. This in turn restricts the Higgs decay rate into the invisible DM states, which makes it more difficult for such models to escape the bounds coming from LHC’s Higgs particle searches.

The constraints on singlet scalar DM from combining XENON-100 and the LHC SM Higgs searches were e.g. studied in \cite{63} for a wide range of Higgs masses. Let us emphasize that the latter analysis did not assume any explicit mechanism for evading EWPT constraints, which would otherwise constrain the SM Higgs mass to be below roughly 160 GeV. By contrast, the IDM provides such a mechanism, and can easily accommodate Higgs masses up to at least 600 GeV while still being in agreement with EWPT. Another difference to the singlet scalar DM model is that the IDM’s “dark” sector is composed of more than one particle state. The additional states potentially provide new contributions to the decay width of the SM-like Higgs boson, along with additional processes relevant for the determination of the DM relic density.

A. Constraints from direct detection DM searches

Figure \ref{fig:figure1} shows how XENON \cite{50,51} constrains the IDM models that have a relic density in agreement with WMAP. These constraints assume a local $H^0$ density of $\rho_0 = 0.3$ GeV/cm$^{-3}$ and a standard Maxwellian velocity distribution. The spin-independent cross section for IDMs is calculated as in \cite{17}:

$$\sigma_{SI|H^0-p} = \frac{m_n^2 \lambda_{H^0}^2 f^2}{16\pi(m_n + m_{H^0})^2 m_h^4},$$

where the form factor is taken to be $f = 0.3$ \cite{20,63,71}, and $m_n$ is the target nucleon mass. The loop induced contribution estimated in \cite{17} is also included, but it is very small.

This leaves a viable mass range roughly between 45 and 80 GeV for the DM candidate $H^0$. The range can be extended up to $\sim 150$ GeV with a few models marginally surviving the current XENON-100 bound.

The low mass region below $\lesssim 10$ GeV is excluded both by XENON-10 \cite{51} and by Fermi-LAT gamma-ray constraints \cite{63,72}. We will however include low $H^0$ masses in parts of the following discussion for illustrative purposes, although they are excluded once we impose all our constraints.

A viable large $H^0$ mass region above $\sim 500$ GeV also exists \cite{61}, but is not of interest for the present study. Such heavy IDM states would for kinematical reasons never alter the width of the Higgs boson (with a mass below 1 TeV) and therefore the LHC constraints apply exactly as in the SM. Such heavy IDM states will also

4 Concerning a low mass WIMP, there is a debate as to what extent the exclusion limits from direct detection results are reliable (see e.g. \cite{22}). In order to be conservative, we could therefore choose not to include the XENON-10 upper bounds. At the same time, we note that the WIMP signal constraints from the Fermi-LAT data on gamma-rays from e.g. dwarf galaxies \cite{63,72} also exclude this low $H^0$ mass region of the IDM. We therefore take the viewpoint that a light $H^0$ below 10 GeV is not a viable WIMP candidate within the current standard scenario \cite{53}.
be very difficult to probe directly at the LHC. On top of that, in order to get the correct relic density and to comply with EWPT, only small Higgs masses can be considered \[61\].

B. Constraints from Higgs boson searches

The latest results are based on analyses of \(\sim 5 \text{ fb}^{-1}\) of integrated luminosity. CMS set the strongest (preliminary) constraints on large Higgs masses until March 2012, excluding a SM Higgs over the mass range 127-600 GeV to 95\% CL, when all search channels are combined (4.6 - 4.7 \text{ fb}^{-1} of integrated luminosity) \[4\]. At that time, ATLAS presented their (preliminary) limits on large Higgs masses using up to 4.9 \text{ fb}^{-1} \[1\]. The CMS collaboration also updated their limits in some channels for 4.6 - 4.8 \text{ fb}^{-1} \[2\]. We will here use both the experiments’ current best exclusion limits on a Higgs signal \(\sigma/\sigma_{\text{SM}}\). Here \(\sigma/\sigma_{\text{SM}}\) denotes the signal rate in units of the expected SM Higgs production cross section \(\sigma_{\text{SM}}\). The 95\% CL upper limits, for all channels combined but for each experiment individually, will be used. In Figure 2 the excluded signal strength, as a function of the Higgs boson mass, is shown as the blue (gray) region. The exclusion region represents the strongest of the two limits from the CMS (dotted line) and the ATLAS (dashed line) experiments.

1. Reduction of the Higgs signal in the IDM

In the IDM, the new contributions to the SM-like Higgs width \(\Gamma_h\) can have a significant impact on the LHC Higgs searches by effectively reducing the Higgs production cross section into SM particles. Since \(H^0\) is neutral and stable, the Higgs decays into \(H^0\) pairs will necessarily contribute to an invisible width. However, let us emphasize that the Higgs can also decay into \(A^0\) and \(H^\pm\) pairs which would further increase the Higgs width. The latter processes give rise to the production of (off- or on-shell) \(Z\) and \(W\) bosons that can make them partly visible in the Higgs search channels.

Nevertheless, the exclusion limits on the Higgs mass range could very well be evaded within the IDM. The processes \(h \to H^0H^0\), \(h \to A^0A^0\) and \(h \to H^+H^-\) enhance the Higgs decay width by:

\[
\Delta \Gamma_{\text{IDM}} = \frac{v^2}{16\pi m_h} \left[ \lambda_{H^0}^2 \left( 1 - \frac{4m_{H^0}^2}{m_h^2} \right) \right]^{1/2} + \\
+ \lambda_{A^0}^2 \left( 1 - \frac{4m_{A^0}^2}{m_h^2} \right)^{1/2} + 2\lambda_{H^\pm}^2 \left( 1 - \frac{4m_{H^\pm}^2}{m_h^2} \right)^{1/2},
\]

where \(\lambda_{H^0, A^0, H^\pm}^2\) are given in Eq. \[1\] and \[2\].

In the narrow-width approximation, the signal strength \(\sigma/\sigma_{\text{SM}}\), or equivalently the reduction factor \(R\), for producing SM particles \(x \bar{x}\) is given by

\[
R = \frac{\sigma_{\text{IDM}}(pp \to h) \text{Br}(h \to x \bar{x})_{\text{IDM}}}{\sigma_{\text{SM}}(pp \to h) \text{Br}(h \to x \bar{x})_{\text{SM}}} = \frac{\Gamma_{h \to x \bar{x}}^{\text{IDM}}}{\Gamma_{h \to x \bar{x}}^{\text{SM}}} + \sum \epsilon \frac{\Gamma_{h \to \phi^+\phi^-}}{\Gamma_{h \to x \bar{x}}^{\text{SM}}},
\]

where the sum runs over \(\phi = A^0, H^\pm\) and \(\epsilon\) is the efficiency with which \(A^0, A^0\) and \(H^+H^-\) may contribute to the current \(x \bar{x}\) Higgs search. This efficiency may be expected to be low due to the fact that the final states will contain extra invisible \(H^0\) states and therefore in principle have different characteristics than the pure SM \(x \bar{x}\) final states.

This means that for the whole range of Higgs masses, even if excluded within the SM, the LHC limits could potentially be evaded within the IDM.

In the next subsection we will argue that, for the models of interest for our study, Higgs decay into all IDM particles will effectively be invisible. In that case the reduction factor in Eq. \[5\] reduces to

\[
R_{\text{cons}} = \frac{\Gamma_{\text{SM}}}{\Gamma_{\text{SM}} + \Delta \Gamma_{\text{IDM}}},
\]

In general, the IDM contribution could in principle also enhance certain SM Higgs signatures, depending on the specific model and search channel. However, the effect of such a contribution would only give stronger exclusion limits on large Higgs masses than in the SM. Taking \(R = R_{\text{cons}}\) is thus the most conservative choice when it comes to determining to what extent the IDM is excluded and therefore the one that we will adopt in the following.

2. Higgs searches and the IDM

The \(WW\) and \(ZZ\) search channels are the most effective ones in the search for heavy Higgs bosons, and below we list their most sensitive sub-channels. We quote the excluded SM Higgs masses, as this indicates where the searches could be sensitive enough to exclude Higgs masses in the IDM.

- \(h \to ZZ^{(*)} \to 4l\) with \(l = \{\text{electron, muon}\}\). By these lepton channels alone, the CMS experiment excluded at 95\% CL SM Higgs boson masses in the ranges 134-158, 180-305 and 340-465 GeV \[3\]. At the same confidence level, ATLAS excluded the ranges 134-156, 182-233, 256-265 and 268-415 GeV

\[5\] For an effect at the loop level, see e.g. the study in \[77\] of the \(\gamma\gamma\) channel.
Higgs decays into $H^0$ would be invisible, but might it also be the case that decays into $A^0$ and $H^\pm$ escape detection in the above search channels?

The decay channel $h \to A^0A^0$ would give rise to two $Z$ bosons and could be visible in the above $ZZ$ search channels. It would however only give a visible contribution if $(m_{A^0} - m_{H^0})$ is large enough to produce on-shell $Z$ bosons via the decay $A^0 \to H^0 + Z$.\(^6\)

In the $WW \to 2l2\nu$ Higgs search channel, the final state is required to include two opposite-sign leptons and missing energy. The $h \to A^0A^0$ and $h \to H^+H^-$ production, with the subsequent decays $A^0 \to H^0 + Z$ and $H^\pm \to H^0 + W^\pm$, could pass these requirements and one can imagine that this could contribute to a signal in this search channel.

Let us therefore take a closer look at this possibility, to see if the contribution could be significant. So far this channel only excludes SM Higgs masses in the range 130-270 GeV, and we therefore expect that it is only within this same mass range that Higgs bosons can be excluded in the IDM. This statement is motivated by the use of

---

\(^6\) Even in the case of on-shell $Z$'s, the characteristics of the final states are altered by the presence of $H^0$'s giving rise to $E_T$. In the $ZZ \to 4l$ channel this would lead to a smearing of the $4l$ invariant mass spectrum, thereby evading a peak search, but could potentially contribute to the observation of a less constraining broad excess.
cuts on the transverse mass, that sets the SM Higgs mass for which the limit applies. This ‘transverse mass’ variable corresponds to the Higgs boson mass in the SM and should roughly do so also in the IDM. This entities the use of the same $\sigma / \sigma_{SM}$ limit for the Higgs in the IDM as in the SM.

In this specific mass range, Higgs decays into $A^0 A^0$ and $H^0 H^\pm$ will however never contribute to the $WW \rightarrow 2\ell 2\nu$ Higgs search channel. This is because for $m_h \lesssim 160$ GeV the LEP limits \cite{28,48} already exclude almost all inert particles $A^0$ and $H^\pm$ with masses less than 80 GeV, which are the only masses that could have been kinematically accessible for these Higgs decays. The exception, with lighter $m_{A^0,H^0}$, occurs only when the mass splitting $m_{A^0} - m_{H^0}$ is very small, and the final state fermions are then too soft to contribute. Moreover, in the region $160 \text{ GeV} < m_h < 270$ GeV it turns out that IDMs, which make up the DM, are excluded irrespective of whether the $A^0$ and $H^\pm$ states are invisible or not to the Higgs searches (see Figure 2).

This means that for many models, in particular those that have a mass difference $(m_{A^0} - m_{H^0})$ too small to produce $Z$ bosons on-shell, the IDM contributions to the Higgs width can be treated as invisible in the current LHC searches for heavy Higgs bosons. Our arguments for such a treatment were based on the channels important for the searches in the high $m_h$ region, while for low Higgs masses, other channels could be more important. Nevertheless, we will apply the same assumption to all our models as this will not alter our discussion.

### C. Constraints on IDM from LHC and XENON-100 combined

Figure 2 shows the LHC Higgs exclusion limit together with IDMs that have the largest invisible Higgs width possible and still pass XENON-100 direct detection constraints. As we can conclude from the above discussion, all the inert states resulting from Higgs decay can be regarded as effectively invisible when the mass difference $(m_{A^0} - m_{H^0})$ is less than $m_Z$, i.e. $\mathcal{R} = \mathcal{R}^{\text{cons}}$. In Figure 2 we present lines for when we take $\mathcal{R} = \mathcal{R}^{\text{cons}}$ for some representative $m_{H^0}$ masses.

Once $m_{H^0}$ and $m_A$ are fixed, Eq. (3) and the XENON-100 exclusion limit on $\sigma^{SI}$ determine the largest available value of $\lambda_{H^0}$, and consequently $\Delta \Gamma_{H^0 \rightarrow H^0 H^0}$. The largest values of $\lambda_{A^0}$ and $\lambda_3$, driving the two other contributions to $\Delta \Gamma_{\text{IDM}}$ in Eq. (4) can be found numerically under the imposition of all the other IDM constraints listed in Sec. IV. The only exception is that we do not yet impose that $H^0$ accounts for the total WMAP DM relic abundance. Instead, we immediately assume that the local $H^0$ density provides the observed DM density which is relevant for the constraints on DM direct and indirect detection. This is in order to keep the discussion more general at this stage, and not include constraints from the freeze-out process occurring in the early Universe. We notice that the LEP and EWPT bounds give the most crucial limits to constrain $\lambda_{A^0}$ and $\lambda_3$ after the XENON bound is imposed. Together with the XENON and LHC constraints, they are efficient in excluding IDMs with heavy Higgs masses.

We see that even without including the relic density calculations, the XENON and LHC Higgs searches, if taken at face value, exclude most of the IDM scenario with large Higgs masses. Only two exceptions appear – see the left plot in Figure 2.

First, we have the low mass WIMP, with e.g. $m_{H^0} = 8$ GeV, which could give rise to large Higgs decay branching ratio into $H^0$. As discussed in Sec. IV, this case is already excluded by XENON-10 and Fermi-LAT data and is presented for illustration only. The second exception arises in the large mass region for $m_{H^0} \sim 80$ GeV to 150 GeV, which might still be viable for the largest Higgs masses. However, if we take into account also the constraint from having the DM candidate, $H^0$, as a thermal relic, this region is no longer allowed. This is clearly seen in Figure 3 where the relic density calculation has been included. We are thus able to exclude the so-called, ‘new viable region’ of IDM found in \cite{54} even before direct detection experiments have fully probed this regime of the IDM. Therefore none of these exceptions provides good models.

Also the possibility to have models with Higgs masses...
The EWPT and unitarity constraints however limit the Higgs mass to be below \( \sim 700 \text{ GeV} \) (also the trivality/perturbativity bound would disfavor larger Higgs masses \( [84, 85] \)). As can be seen from Figure\[2\] when the thermal relic density calculation has been included, the DM mass range \( M_{H^0} \sim 40 \sim 80 \text{ GeV} \) with a very heavy Higgs in the range 600-700 GeV is still an allowed region.

In Figure\[3\] we present the result of a random scan in the \( m_{DM} \in [15-170] \text{ GeV} \) parameter space of the IDM giving rise to an \( H^0 \) relic abundance in agreement with WMAP \[40\] at the 3\( \sigma \) level. All the constraints from Sec.\[III\] are now included. The plot illustrates in the \( m_h - m_{H^0} \) plane the IDMs that pass the constraints set by XENON-100, the LHC Higgs searches and WMAP. We see that many models pass either the direct detection or the LHC Higgs bounds individually. In the heavy Higgs region there are no surviving models, except for the region \( m_h \gtrsim 600 \text{ GeV} \) and \( m_{H^0} \sim 50 \sim 80 \text{ GeV} \) (see also the plots in Figure\[2\]). We thus conclude that in order to have an IDM that makes up all the DM and has a SM-like Higgs in the 160 to 600 GeV mass range, at least one of our imposed constraints has to be relaxed.

D. Accommodating a heavy Higgs and DM in the IDM

One of the original motivations for studying the IDM was that it could alleviate the LEP paradox in the SM by allowing for a heavier Higgs particle while staying in agreement with EWPT. We have shown above that constraints from direct detection in combination with the SM Higgs search essentially rule out large Higgs masses up to \( \sim 600 \text{ GeV} \) in the IDM.

In this section, we investigate the assumptions that could be relaxed in order to allow for a large range of high Higgs masses \( (m_h > 160 \text{ GeV}) \) within the IDM. In particular, we will allow for larger values of \( \lambda_{H^0} \) by suppressing the bound that derives from direct detection searches. In that way, models with larger invisible Higgs branching ratios will become available, which consequently give lower signal strengths in the LHC Higgs searches. This is illustrated in the right panel of Figure\[2\].

The bound on \( \lambda_{H^0} \) can be suppressed in two ways:

1. by assuming that the DM from IDM does not account for the entire DM abundance: the green lines in the right panel of Figure\[1\] assumes that \( H^0 \) constitutes only 10\% of the local DM density \( \rho_0 \). This suppresses the constraint on \( \sigma_{SI}^{H^0,p} \) by the same factor.

2. by considering systematic uncertainties in direct detection: the dark blue lines in the right panel of Figure\[2\] takes into account a smaller form factor \( f = 0.26 \) \[63, 86\], a smaller local DM density \( \rho_0 = 0.2 \text{ GeV/cm}^3 \) \[87\], and, in addition, include a minor effect of 10\% weakening of the XENON-100 cross-section limits due to uncertainties in the local WIMP velocity distribution \[88\]. Concerning the local DM density, there have been recent improved measurements constraining it to the range \( \rho_0 = 0.3 \pm 0.1 \text{ GeV/cm}^3 \) \[88\] (see also \[89\]).

These reconsiderations weaken the constraints on \( \lambda_{H^0} \), and IDMs with \( m_h \gtrsim 500 \text{ GeV} \) could be allowed. Also \( m_h \) around 320 GeV could be allowed if only the LHC constraints from CMS are considered. However, the preliminary analysis recently presented by ATLAS \[1\] does not show any excess around \( m_h = 320 \text{ GeV} \) as CMS does, but instead puts very strong constraints in the 300 to 450 GeV mass range. There are also uncertainties related to the absolute calibration of cross-section limits at the LHC on \( \sigma/\sigma_{SM} \). We choose here not to take into account such potential additional uncertainties.

If \( H^0 \) particles constitute only a fraction of the DM density they would more easily pass direct detection constraints (now rescaled by \( \Omega_{CDM}/\Omega_{H^0} \)) while having a larger \( \lambda_{H^0} \) coupling, and then be able to evade the LHC Higgs limits. It then remains to be shown if such models exist that have such a low relic density while not exceeding the other constraints in Sec.\[III\]. The possible mechanisms for this in the IDM are 7:

- **Annihilation via \( h \) at the resonance** \( (m_{H^0} \sim m_h/2) \): In the case of a heavy Higgs, the resonance could only occur when \( m_{H^0} \gtrsim 80 \text{ GeV} \) and annihilations into gauge bosons already provide an efficient annihilation mechanism.

- **Coannihilations** \( (m_{H^0} \sim m_{A^0} \text{ or } m_{H^0} \sim m_{H^\pm}) \): This is relevant for small mass differences when \( m_{A^0, H^\pm}/m_{H^0} \lesssim 1.1 \). For large Higgs masses, the EWPT also requires that \( (m_{H^\pm} - m_{A^0}) \times (m_{A^0} - m_{H^0}) \) is positive \[17\]. This means that \( m_{H^\pm} > m_{A^0} \) and that the mass difference between the two neutral inert scalars has to be small. For the tetralepton search channel that we will investigate in the next section, this has the implication that the leptons from the decay \( A^0 \rightarrow H^0 \) are too soft to be detected at the LHC.

- **Annihilation to WW, ZZ and \( t\bar{t} \) (\( m_{H^0} \gtrsim m_{A^0} \)): Strong annihilation channels into gauge bosons be-

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7 Models with annihilation dominantly into fermions have \( \sigma v \propto \lambda^2_{H^0} \), and are already in the region excluded by direct detection searches. This can be seen in Figure\[2\] where \( m_{H^0} \lesssim 40 \text{ GeV} \) corresponds to models having annihilations into fermions only. In that framework, increasing \( \lambda_{H^0} \) would not alter the bounds from direct detection searches. Indeed, these bounds derives from the quantity \( \sigma_{SI} \times \Omega_{H^0} \propto \lambda^2_{H^0}/(\sigma v) \) which is unchanged under a rescaling of \( \lambda_{H^0} \).
come kinematically available already for $m_{H^0}$ just
below $m_W$, $m_Z$ or $m_t$.

Although all three of the above mechanisms could be
viable, we will in the next section only consider models
where the relic density is suppressed by the last type of
mechanism. This is because we want to investigate the
best prospects for detecting the IDM in the tetralepton
channel at the LHC, and the simplest scenario to consider
is then when the $WW$ annihilation channel regulates the
DM abundance.

We will also consider benchmark models that give a
relic density in agreement with 100 % of the observed
DM. However, for these models systematic uncertainties
for the direct detection searches have to be included, as
described above, to make them pass all constraints.

V. THE MULTILEPTON SIGNAL

The inert scalars can only be produced in pairs, since
each inert particle has negative $Z_2$ parity contrary to the
SM particles. At tree-level, the relevant hard processes
producing final states with four leptons or more, are via
the gauge bosons and the Higgs:

$$qar{q} \rightarrow W^\pm \rightarrow A^0 H^\pm \quad (7)$$

$$qar{q} \rightarrow Z/\gamma/h \rightarrow H^+ H^- \quad (8)$$

The tree-level contribution to $qar{q} \rightarrow h \rightarrow A^0 A^0$ is negligible
but at loop-level, gluon fusion into Higgs is important
for $A^0 A^0$ and $H^+ H^-$ production.

After the inert particles are produced, they will
cascade-decay through the processes:

$$H^\pm \rightarrow \begin{cases} H^0 W^\pm \\ A^0 W^\pm \end{cases} \quad \text{and} \quad A^0 \rightarrow H^0 Z \quad (9)$$

or

$$H^\pm \rightarrow H^0 W^\pm \quad \text{and} \quad A^0 \rightarrow \begin{cases} H^\pm W^\mp \\ H^0 Z \end{cases} \quad (10)$$

depending on whether $H^\pm$ or $A^0$ is the most massive
inert state. The gauge bosons will, with their respective
branching ratios, decay into fermions, $f$, according to

$$W^\pm \rightarrow f^\pm \nu \quad \text{and} \quad Z \rightarrow f^+ f^- \quad (11)$$

Figure 4 illustrates these production and decay chains.
Our focus will be on the production of four or more lep-
tons, $l$ (which in this context refers only to electrons and
muons), where the SM background is expected to be very
low.

The cross sections and decays widths will be calcu-
lated using MadGraph/3.8.1.10, and their stream-
lined interface with Pythia 6.4 and PGS 92 to simu-
late hadronization and detector response. To be able to
generate signal events in practice, we split the processes
into separable steps, in order to diminish the phase space
from the otherwise up to ten-body final-state processes.

A. Production of inert scalars via gauge fields

In this subsection, we discuss the general expectations
of the $\geq 4$ lepton signal strength from inert scalars pro-
duced via gauge bosons. Some of the contributing di-
agrams are shown in the first three panels of Figure 4.

As the gauge couplings are fixed, the production cross
sections of the heavier inert states are fully determined
by their masses, and their decay patterns by their mass

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This does however not make $A^0$ sufficiently long-lived to give rise
to displaced vertices.
small ∆m splitting would kinematically favor decay into m_A large mass splitting would kinematically favor decay into m_A important. For very small mass splittings ∆m ≲ 6.7 % which is the result for an on-shell A0 boson. A small ∆m also gives larger Br(H0 → AW) as a large mass splitting would kinematically favor decay into H0; especially if W becomes on-shell. Again a small mass shift becomes weighed against the ability to produce hard enough leptons for detection. For a fixed ∆m, increasing mH will typically increase Br(H± → A0), but at the cost of lowering the production cross section of heavier H±.

In Figure 5 we show the cross section for the gauge mediated contribution to the production of four or more leptons. We calculate the tree-level cross sections with MadGraph/MadEvent and apply a corrective factor, a so called K-factor, of 1.2 to achieve agreement with the NLO results in [93, 94].

As we will see, even for optimal parameter values, the gauge-mediated contribution to a four-lepton signal in the IDM will not be enough to render the model detectable. Here we merely study under what conditions the contribution from gauge mediated production can become non-negligible, and will turn to the more significant contribution from gluon fusion in the next section.

As our interest is in the detection of leptons, the branching ratios A0 → H0ℓ+ℓ− and H± → A0ℓν are important. For very small mass splittings ∆mA0,H0 the Br(A0 → H0ℓ+ℓ−) can be large, but give rise to leptons that are too soft to be isolated. For increased mass splitting, decay modes into the more massive quarks open up, and the branching ratio into leptons decreases, approaching 6.7 % which is the result for an on-shell Z boson. A small ∆m also gives larger Br(H0 → AW) as a large mass splitting would kinematically favor decay into H0; especially if W becomes on-shell. Again a small mass shift becomes weighed against the ability to produce hard enough leptons for detection. For a fixed ∆m, increasing mH will typically increase Br(H± → A0), but at the cost of lowering the production cross section of heavier H±.

In Figure 6 we show the cross section for the gauge mediated contribution to the production of four or more leptons. We calculate the tree-level cross sections with MadGraph/MadEvent and apply a corrective factor, a so called K-factor, of 1.2 to achieve agreement with the NLO results in [93, 94].

B. Production of inert scalars via SM Higgs

The SM Higgs production at LHC is dominated by gluon fusion – dominantly induced by the loop of a top-quark coupled to the Higgs [95]. The couplings of inert particles to the Higgs can then give a significant contribution to the production of four leptons through the processes

\[ gg \to h \to A^0 A^0, H^+ H^-. \]

In the A0A0 channel one obtains four leptons in the final states independently of the values of mH and Br(H± → A0). This process is shown in the last diagram of Figure 5. The signal strength will, apart from mH and mA, also depend on mh and λh. Unlike the processes considered in the previous section, the study of this process is strongly related to the SM Higgs search.
and to the search for DM in direct detection experiments. Given \( m_{H^0} \) and \( m_h \), direct detection data constrains the coupling \( \lambda_{H^0} \) between \( H^0 \) and the Higgs, which for a given mass \( m_{A^0} \) also limits the size of the Higgs coupling to \( A^0 \)

\[
\lambda_{A^0} = \lambda_{H^0} + \frac{m_{A^0}^2 - m_{H^0}^2}{4v^2}.
\]  

To generate \( gg \to A^0A^0, H^+H^- \) events we make use of MadGraph/MadEvent’s implementation of Higgs effective theory, where the Higgs boson couples directly to gluons. The effective coupling between the Higgs and the gluons depends on the Higgs mass and we match the cross sections obtained with MadGraph/MadEvent to the NNLO results for Higgs production via gluon fusion in the SM.

At the largest Higgs masses the vector boson fusion could also start to become relevant, but we are conservative in the sense that we do not include such, or other subdominant, Higgs production contributions to our IDM signal. In Figure 5 we show the IDM cross section to four or more leptons by the Higgs mediated interactions in Eq. 15.

### C. Background

The requirement of leptons in the final state enables a signal to be extracted from the otherwise huge QCD background at hadron colliders. In order to simulate the SM background in the \( \geq 4l + \slashed{E}_T \) channel, we include the following SM processes:

\[
VVV, ZZ, t\bar{t}Z, t\bar{t}, b\bar{b}Z \text{ and } t\bar{t}t\bar{t},
\]

where \( V = W, Z \) are allowed to be off-shell.

Out of the contributions to \( VVV \), \( WWZ \) is the most dominant contribution to our background and is the one we include in our analysis. We do not simulate \( VVVV \) processes, which are expected to be subdominant.

We expect to be able to efficiently reduce these backgrounds in order to discriminate the signal: \( ZZ \) production is the dominant source of hadronically quiet \( 4l \) events, but without invisible particles in the final states it can be efficiently removed by a cut on missing transverse energy. For IDs producing leptons from off-shell \( Z \) bosons, the SM backgrounds including on-shell \( Z \) can be further discriminated by reconstructing the invariant mass of same flavor, opposite sign lepton pairs. The \( t\bar{t}Z \) and \( t\bar{t}t\bar{t} \) backgrounds can also be reduced by vetoing b-tagged jets, which should leave most of the IDM signal events. For the low background levels in the four-lepton channel, a significant contribution could come from fake leptons. This is difficult to properly take into account in a study based on Monte Carlo simulation, and should be estimated from experimental data. We comment further on this in our discussion of systematic uncertainties in Sec. VId.

### VI. ANALYSIS

In order to study the signal expectations for IDM at the detector-level, we define a set of benchmark models in Table II. The models are divided into three subsets:

- The **A**-models are constructed to test how strong the signal can be when inert states are produced via gauge interactions, and the direct detection signal will be very weak. These models do however give invisible Higgs branching ratios that are too low to pass the current LHC constraints on a heavy Higgs, and are therefore ruled out but kept here for illustration of the strength of the gauge mediated production.

- The **B**-models represent IDM scenarios that explain all of the observed DM. They only pass all constraints if we add the systematic uncertainties to the XENON-100 limits, as discussed in Sec. IVd.

Models with a Higgs mass above 600 GeV (where no Higgs search limits have been presented) should more easily pass all constraints. Such models should be able to give similar \( 4l + \slashed{E}_T \) signal features as IDM-B2. However, a weaker signal is expected.
since the production cross section of inert states will be smaller once $\lambda_{H0}$ is adjusted to pass the direct detection constraint (unless we again allow for systematic uncertainties as for IDM-B2).

The model IDM-B1 has a Higgs boson mass of 320 GeV, motivated by the excess seen by the CMS experiment around this value. This possibility was however ruled out by the new ATLAS limits [1] that were presented during the preparation of this manuscript (see Figure 2).

- The C-models are illustrative examples of models that pass all constraints, but have a relic density that explains only a fraction of the observed total cold DM content. They are chosen such that IDM-C1 and IDM-C3 pass the XENON-100 constraint, but have some margin to the LHC Higgs bound. IDM-C2 and IDM-C4 instead just evade current Higgs searches at the LHC, but have larger margins to the XENON-100 limits.

The models IDM-C2, IDM-C3 and IDM-C4 give a relic DM contribution of 10% to $\Omega_{CDM}$, and IDM-C1 gives 1% of $\Omega_{CDM}$.

All the benchmark models pass all the other experimental and theoretical constraints listed in Sec. III.9 In our detector-level study, we take these as our representative IDM models for a tetralepton signature with heavy SM-like Higgs. In Table II and III we list the models’ properties relevant for the four-lepton signal.

These models may well show up in upcoming data from XENON-100 and LHC. The expected performance of LHC is an integrated luminosity of up to $\sim 15 \text{ fb}^{-1}$ collected by end of 2012 with an upgrade to 8 TeV for the rest of this year. The increase in sensitivity in the SM Higgs searches is about a factor 1.6 due to the integrated luminosity being 3 times larger and a factor about 1.2 due to the increased energy [98]. A factor up to about $\sqrt{2}$ could also come from combining the ATLAS and CMS data. This means that all our benchmark models, except possibly IDM-C3, should be reached by exclusion limits from LHC Higgs searches by the end of 2012. Detection of the Higgs bosons in any of our benchmark models, at the $5\sigma$ level, would however require more integrated luminosity, and these Higgs would most likely not be revealed before the LHC run at 14 TeV.

The cross-section sensitivity of XENON-100 will also improve by an order of magnitude by the end of 2012 [99, 100]. This is enough to start to probe all our benchmark models, except for the IDM-C1 and possibly IDM-C4 model (and of course the IDM-A models). The planned XENON-1T is expected to improve the sensitivity by more than an order of magnitude [99, 100].

9 The high Higgs mass in combination with large couplings actually renders the IDM-C3 model marginally in violation of the, somewhat arbitrary, choice for the tree-level unitarity limit given in Sec. III.
We therefore consider the complementary four-lepton plus missing energy channel as a potential step to further pin down or discover an IDM signal.

A. Event generation

We generate signal and background events with the MadGraph/MadEvent-4.4.32 package. From a user specified process, MadGraph creates Feynman tree-level amplitudes (including effective operators and using a HELAS \cite{101} implementation for the helicity amplitude calculations) for all relevant hard subprocesses. Once events are generated with MadGraph they are passed to Pythia \cite{91} for hadronization and decay. The events are then passed to the Pretty Good Simulator PGS \cite{92} to mimic the detector response.

For each background and signal process, we generate events corresponding to an integrated luminosity of at least 10 times the integrated luminosity that we make predictions for. In a few cases, however, we were limited by computer power, and for the IDM-A models and the SM backgrounds we have generated events corresponding to at least 3000 fb\(^{-1}\), except for \(bbZ\) and \(tt\) production for which we have generated 220 fb\(^{-1}\) and 160 fb\(^{-1}\) respectively.

B. Settings

We consider proton-proton collisions at 14 TeV, using the standard cteq6l1 for the parton distribution functions \cite{102}. In Pythia, we include initial and final state radiation but not multiple interactions. For our PGS settings we choose the options that mimic the ATLAS detector with a cluster finder cone size of \(\Delta R = 0.4\) for jet reconstruction, and keep the other parameters as described in our benchmark models together with the total SM background events, we perform cuts sequentially on the detector with a cluster finder cone size of \(\Delta R = 0.4\).

For the cases where we generate events including jet-matching (see Sec. \cite{VLD}), we use the so called MLM scheme \cite{103,104} with the minimum \(K_T\) jet measure for the phase space separation between partons set to 20 GeV.

The lepton isolation criteria are an important part of the lepton object definition in order to distinguish them from leptons that could have originated in jets. For electrons, PGS does this by default by requiring that the transverse calorimeter energy in a \((3 \times 3)\) cell grid around the electron, excluding the cell with the electron, has to be less than 10% of the electron’s transverse energy and that the summed \(p_T\) of tracks within a \(\Delta R = 0.4\) cone around the electron, excluding the electron, is less than 5 GeV. To mimic the ATLAS detector response, we also ignore electrons with a pseudorapidity \(\eta\) within \(1.37 \leq |\eta| \leq 1.52\) \cite{94}. For muons, that are not isolated by default in PGS (and we do not make use of the cleaning script that is default in MadGraph/MadEvent), we require the summed \(p_T\) in a \(\Delta R = 0.4\) cone around them, excluding the muon itself, to be less than 10 GeV to define them as isolated. For each lepton we also require a minimum distance of \(\Delta R = 0.4\) from the nearest lepton or jet (as reconstructed by PGS).

C. Cuts

In order to discriminate an IDM signal from SM background events, we perform cuts sequentially on the detector simulator’s reconstructed particle data.

To illustrate our cuts, we show in Figure \ref{fig:cuts} the event distributions after each cut. The plots include two of our benchmark models together with the total SM background and two of its main sub-process contributions in this tetralepton + \(E_T\) channel. These are the cuts specific to our IDM study:

- First, we require four or more isolated leptons. In order to make lepton isolation and event triggering in the four-lepton channel robust, we will require a leading lepton with \(p_T^l \geq 20\) GeV and that each of the additional leptons have \(p_T^{l,2,3,4} \geq 10\) GeV.

- In order to reduce the \(ZZ\) background efficiently, we require the missing transverse energy \((E_T)\) in each event to be larger than 25 GeV, as illustrated in the (upper left) panel of Figure \ref{fig:cuts}.

- We reject events with any pair of same flavor and opposite sign (SF-OS) leptons among the \(\geq 4\) leptons with an invariant mass that falls within the range of the \(Z\) resonance, 75 GeV < \(m_{\text{inv}}^{l,l'}\) < 105 GeV. We refer to this as our \(Z\) veto. The (upper right) panel in Figure \ref{fig:cuts} shows the distribution of events by the pair of SF-OS leptons giving an invariant mass closest to 91 GeV.

- The \(tt\bar{Z}\) background can be fairly efficiently discriminated against by requiring no \(b\)-tagged jets in the event, as illustrated in the (bottom left) panel of Figure \ref{fig:cuts}.

- In the (bottom right) panel of Figure \ref{fig:cuts} we show the distribution of events in the minimal SF-OS dilepton invariant mass (minimal since each event has at least four leptons, and may contain more than one pair of SF-OS leptons). This invariant mass is expected to be low for our benchmark models, as the \(Z\) decays off-shell, and we require the minimal invariant mass to be < 50 GeV.
FIG. 7: Top left: Missing transverse energy distributions in events with four isolated leptons. Top right: The invariant mass of SF-OS lepton pairs after the cut on $E_T$ has been applied. Bottom left: Distribution of b-tagged jets, after the cut on $E_T$ and $Z$ veto. Bottom right: Invariant mass distribution for the SF-OS lepton pair producing the minimal such value per event, after all other cuts have been performed. The shaded grey regions indicate the cuts on each quantity.

For the signal events, the position of the peak in the SF-OS dilepton invariant mass distributions is slightly below the mass difference $\Delta m_{A^0H^0}$ in a given model. The large fluctuations in the minimal invariant mass distribution of the total SM background (bottom right panel of Figure 7) come from the low statistics of our $t\bar{t}$ sample; only six $t\bar{t}$ events are left after the cuts, five of which lie in the 15-25 GeV bins. This makes it difficult to say something about the distribution of this specific background contribution.

What we can see is that if the $t\bar{t}$ events could be vetoed in some way, for example using the impact parameter for muons mentioned above, then the SF-OS dilepton invariant mass distribution can be used as a signature to clearly distinguish our models from the background.

A characteristic of our benchmark models is that the signal leptons originate in off-shell $Z$-bosons. Therefore our signal efficiency is sensitive to the isolation criteria and the minimum $p_T$ requirements on the leptons. Models with larger $\Delta m_{A^0H^0}$, which allows $A^0$ to decay to on-shell $Z$, would be more difficult to detect since in this case the signal cannot be distinguished from the background using the $Z$ veto.

D. Sources of systematic uncertainties

Systematic uncertainties in our $4l + E_T$ signal study are due to limited statistics in some of our background samples, sensitivity to lepton efficiencies and fake-lepton contributions to the background.

In our statistical analysis we fix the signal and background cross-section expectations to our average results but, as mentioned in Sec. VLA, generation of enough $bbZ$ and $t\bar{t}$ events were limited by computer power. We trust
that our cuts remove any contribution from $b\bar{b}Z$, but the $t\bar{t}$ contribution gives uncertainty to our background estimation. Our $t\bar{t}$ sample consists of only 5 events after all cuts, and for a Poisson distribution the upper expectation value is 9.3 events at 90% CL. Taking this upper value as the average $t\bar{t}$ result instead would increase our total background cross section with only 30% (and a similar relative increase in the expected needed luminosities to discover the signals).

The lepton efficiency is low for our models compared to the SM background. This is because the leptons in the model events originate in off-shell $Z$-boson decay, and our signal predictions are thus sensitive to the lepton isolation and $p_T$ requirements. For comparison, if we use our $p_T$ requirements and decrease the lepton efficiencies, as in [112], both the signal and the SM background cross sections are reduced by about 50%. Due to the increase of pile-up effects as the experiment reaches design luminosity, the isolation criteria might have to be loosened and the $p_T$ requirement raised in compensation. For our study, a raise to $p_T^{i,j} > 15$ GeV would leave only 32% of the total background (completely remove the contribution from $t\bar{t}$), while still leaving 20-60% of the signal in our benchmark models.

Since the background in this channel is low, we could be very sensitive to the contribution from fake leptons. In order to make use of PGS’s ability to generate fake electrons, we show in Table IV the results of our cuts applied on some SM processes that naively give three lepton final states (such as $WZ$) and include explicit jets. The table shows that these type of fake-lepton contributions seem not to be very important. Likewise, we find that including jet-matching would not alter the result in our final analysis that is presented in Table IV (where jet-matching, for consistency, is not included for neither the backgrounds nor the models).

A proper inclusion of backgrounds involving fake leptons has to be based on experimental data. In a recent ATLAS analysis [113] of the $4l + E_T$ channel, the systematic uncertainty due to differences in fake rate between simulation and data was estimated to be around 10% for the background processes $t\bar{t}$ and $t\bar{t}Z$. They also find that the $Z$+jets give a significant contribution to the background, potentially dominated by electron Bremsstrahlung in the detector material that subsequently pair produce leptons. However, these events are found to contain $E_T$ of 20-60 GeV and hard jets, as can be seen in Figure 2 in [113]. Requiring $<3$ jets with $p_T > 40$ GeV and optimizing the $E_T$ cut could potentially reject this background effectively without loss of more than $\sim 10\%$ of the signal events in our benchmark models. The uncertainties in the estimation of the $Z$+jets contribution are however large and an inclusion of this background is beyond the scope of our phenomenological study.

Sources of systematic uncertainties will not be included in the following statistical analysis.

### E. Results

In Table V, we show the results after the signal and background events have been passed through the PGS detector simulation as we successively perform the cuts described in Sec. VIIC.

To obtain a statistical measure for when our signal could be observed or excluded, we assume the number of events to be Poisson distributed. The probability of

| Process | $n_l \geq 4$ | $E_T \text{ cut}$ | $Z \text{ veto}$ | $n_b = 0$ | $m_{\min}^{t\bar{t}} \text{ cut}$ |
|---------|-------------|-----------------|----------------|-----------|----------------|
| WWW     | 0.0049      | 0.0025          | 0.0025         | 0.0025    | 0 (< 0.0025)   |
| WZ($j$) | 3.4         | 2.2             | 0.24           | 0.24      | 0 (< 0.059)    |
| ZZ($j$) | 2900        | 23              | 0.59           | 0.53      | 0.46           |
| $t\bar{t}W$($j$) | 1.1 | 1.1 | 0.80 | 0.47 | 0.19 |
| $t\bar{t}Z$($j$) | 150 | 140 | 13 | 6.3 | 3.5 |
| $t\bar{t}\bar{t}$($j$) | 0.62 | 0.61 | 0.41 | 0.14 | 0.038 |

| Proc./Model | $n_l \geq 4$ | $E_T \text{ cut}$ | $Z \text{ veto}$ | $n_b = 0$ | $m_{\min}^{t\bar{t}} \text{ cut}$ |
|-------------|-------------|-----------------|----------------|-----------|----------------|
| ZWW | 15 | 13 | 0.92 | 0.92 | 0.42 |
| ZZ   | 2700       | 16              | 0.62           | 0.62      | 0.47           |
| $t\bar{t}Z$ | 130         | 120             | 13             | 6.7       | 4.1            |
| $b\bar{b}Z$ | 7.2       | 0.89            | 0 (< 0.45)     | 0         | 0              |
| $t\bar{t}$ | 7.6       | 6.9             | 5.0            | 4.4       | 3.2            |
| $t\bar{t}\bar{t}$ | 0.56 | 0.56 | 0.46 | 0.093 | 0.031 |

| Total bkg | 2900 | 160 | 20 | 13 | 8.2 |

| IDM-A1 | 4.6 | 3.5 | 3.3 | 3.2 | 3.2 |
| IDM-A2 | 7.8 | 7.1 | 5.7 | 5.5 | 5.5 |
| IDM-B1 | 17  | 14  | 13  | 13  | 13  |
| IDM-B2 | 20  | 18  | 14  | 14  | 13  |
| IDM-C1 | 41  | 31  | 29  | 28  | 27  |
| IDM-C2 | 110 | 90  | 88  | 88  |     |
| IDM-C3 | 34  | 30  | 26  | 26  | 26  |
| IDM-C4 | 22  | 19  | 15  | 15  | 15  |

Table IV: Cross sections for backgrounds that require fake leptons to produce 4 final state leptons (the WWW, WZ processes). We also test if our backgrounds are sensitive to including jet-matching. None of these effects seem important if compared to the values used in our final analysis presented in Table V (where jet-matching is not included). We have required four isolated leptons and add the respective cuts for each column successively. Results are presented in units of $10^{-2}$ fb. We have included K-factors of 1.6 for $ZZ(j)$, 1.9 for the $WZ(j)$ [105] and 1.4 for $t\bar{t}Z$ [107].
observing $N$ or fewer events is then

$$P(N; B) = \sum_{n=0}^{N} \frac{B^n e^{-B}}{n!},$$

(17)

given that the background expectation value $B$ is the true mean. For a one-sided 3(5)$\sigma$ detection, we take the probability $(1 - P(N; B))$ of having this number of events or more due to a statistical background fluctuation to be less than 0.13% ($2.9 \times 10^{-5}$%). With a signal expectation $S$, the probability to observe such an excess signal is $1 - P(N; S + B)$, and we request this probability $P_{\text{obs}}$ to be 50%.

In Table VI we show the prospects for when detection or exclusion of our benchmark models at the LHC will occur. The quoted integrated luminosities are for a 50% or 95% CL exclusion of our benchmark models.

Because of the sometimes low statistics needed to detect these models, the use of Poisson statistics should be more correct than e.g. the commonly used rule of thumb of a 5$\sigma$ discovery when $S > \text{max}(5, 5\sqrt{B})$. In Appendix B this and other commonly used statistical measures are compared. For the benchmark models with the strongest signal, and thus the lowest number of expected events at the time of a discovery, Poisson statistics lead to about a factor of two larger required integrated luminosity than a naive Gaussian approximation. In Appendix B we also show that increasing the prospect from 50% to 90% probability to find evidence for a signal can require a factor two in increase integrated luminosity.

| Model | IDM-A1 | IDM-A2 | IDM-B1 | IDM-B2 | IDM-C1 | IDM-C2 | IDM-C3 | IDM-C4 |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| 3$\sigma$ evidence, $P_{\text{obs}} = 50\%$ (fb$^{-1}$) | 810 | 300 | 64 | 64 | 19 | 3.8 | 20 | 50 |
| 5$\sigma$ detection, $P_{\text{obs}} = 50\%$ (fb$^{-1}$) | 2300 | 820 | 180 | 180 | 53 | 9.0 | 55 | 140 |
| 95% CL exclusion, $P_{\text{obs}} = 50\%$ (fb$^{-1}$) | 280 | 110 | 30 | 30 | 13 | 3.1 | 14 | 20 |

TABLE VI: The expected integrated luminosities needed at 14 TeV for a 3$\sigma$ and 5$\sigma$ detection in the inert doublet benchmark models. Alternatively, the expected luminosity needed for a 95% CL exclusion of these benchmark models.

In particular, the combination of constraints utilized in this work completely rules out the, so-called, ‘new viable region’ found in [50] where $H^0$ masses are in range 80 to 150 GeV. Moreover, we conclude that the ensemble of constraints are in conflict with the IDM for its whole viable cold DM mass range if the models shall also incorporate the Higgs boson in the mass range 160-600 GeV. This conclusion can be avoided if either 1) the canonical experimental bounds can be relaxed or 2) the IDM does not account for all the DM.

We investigate the prospects of detection/exclusion in the near future of models belonging to these types of ‘escape’ scenarios. Adding the systematic uncertainties to the observational constraints, and at the prize of some fine-tuning, we found that we can still obtain IDMs that contain both a heavy Higgs ($\gtrsim 500$ GeV) and a good DM candidate. We also looked into the possibility that IDM explains only a fraction of the universe’s DM content, and thereby more easily evades current constraints from both LHC and DM direct detection experiments. Some of these models can be efficiently probed by the foreseen data from XENON and LHC before the end of 2012.

The potential detection of a heavy Higgs and/or a signal in direct DM detection experiments in the viable IDM DM mass range, although striking features in favor of an IDM-like scenario, would not exclusively point to the IDM. A way to pin down the identity of the new physics further would be to compare different complementary channels. The prospects for detection of the IDM in the 14 TeV LHC data have been studied previously for channels with two or three leptons, together with missing energy [29, 30]. In this work, we have investigated the possibility of a four-lepton plus missing energy signature at the LHC coming from IDM. The models with a heavy Higgs that evade the current constraints typically have large couplings between the inert states and the SM-like Higgs. As a result, the production of four-lepton final states via gluon fusion Higgs production becomes a particularly promising channel to track, and even discover, the IDM during the early runs at LHC’s design center-of-mass collision energy.

We find that in the four-lepton plus missing energy channel our benchmark points, where the inert particles are mainly produced via Higgs, should show up early in the 14 TeV LHC run. Our models IDM-B1, IDM-B2 and IDM-C1 to IDM-C4) should be seen at integrated luminosities of 3.8-64 fb$^{-1}$ (9-180 fb$^{-1}$) at the 3$\sigma$ (5$\sigma$) CL. We can note that the IDM benchmark points that were stud-
ied in the previous works \cite{29,30} for the di- and trilepton channels, only one survives the current direct DM detection and SM Higgs searches. Nevertheless, according to these references, our benchmark points satisfy properties, such as favorable $\Delta m_{\chi}\mu_{\chi}$, that should also render them detectable in the di- and trilepton channels at integrated luminosities of 100-300 $f^{-1}$. We thus conclude that, compiling recent experimental constraints, the IDM with a SM-like Higgs heavier than about 160 GeV could very well first show up in the tetralepton channel.

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Appendix A: Naturalness

The IDM serves as an explicit framework where a heavy Higgs, up to around 700 GeV, can be incorporated and still be in agreement with EWPT. While this possibility is interesting in itself, it has also served as an additional motivation for the model. Indeed, a larger Higgs boson mass could alleviate the fine-tuning in the SM and make the model more natural by pushing the need for new divergence-canceling physics to higher energy scales \cite{17}.

Raising the Higgs mass within the IDM does however not necessarily lead to improved naturalness as compared to the SM \cite{19}. The new inert scalars contribute with additional corrections to the SM-like Higgs mass, as well as exhibit quadratic divergences of their own. This can lead to increased over-all fine-tuning although a larger Higgs mass naively renders it less sensitive to corrections from new physics at high energies.

Let $F^2(p_i)$ be a quantity that depends on some independent input parameters $p_i$. The amount of fine-tuning in $F^2$ associated with $p_i$ can then be taken to be $\Delta F^2$, defined by \cite{114}

$$ \frac{\delta F^2}{F^2} \equiv \Delta F^2 \frac{\delta p_i}{p_i}. \quad (A1) $$

A model is said to be natural, up to an energy scale $\Lambda$, if the total amount of fine-tuning is sufficiently small. The exact upper limit on $\Delta F^2$ is in order for the quantity not to be considered to be fine-tuned is somewhat arbitrary.

The scalar masses are the parameters that receive the dangerous quadratically ultraviolet-divergent contributions. Using momentum cut-off regularization, the one-loop corrections to the scalar mass parameters $\mu_i^2 = \mu_i^2 + \delta \mu_i^2$ can be written (as in \cite{19})

$$ \delta \mu_i^2 = \frac{3}{64\pi^2} \left[ -8\lambda_i^2 \Lambda_i^2 + (3g_i^2 + g^2)\Lambda_i^2 + 8\lambda_i^2 \Lambda_i^2 \right] \quad (A2) $$

and

$$ \delta \mu_2^2 = \frac{3}{64\pi^2} \left[ (3g_2^2 + g^2)\Lambda_2^2 + 8\lambda_2^2 \Lambda_2^2 \right] \quad (A3) $$

where $\lambda_i$ is the top Yukawa coupling, $g'$ and $g$ are the U(1) and SU(2) gauge couplings and we have assumed independent cut-offs $\Lambda_i$. The loop contribution from internal gauge-fields are sufficiently small that $\Lambda_i$ will be irrelevant compared to $\Lambda_i$. For large scalar couplings the most relevant ones will be $\Lambda_{11,12}$ and $\Lambda_{22,21}$ - the momentum cut-offs of the loop contributions from fields associated with the SM doublet and the inert doublet, respectively. In our case the relevant fundamental parameters are $\lambda_i^2, \lambda_i \in p_i$. We will start by focussing on the $\Lambda_i$ to assess the models sensitivity to physics at higher energy scales.

Taking $p_i = \Lambda_i^2$ for $F^2 = \mu_1^2, \mu_2^2$, Eq. (A1) implies

$$ \Delta F^2 = \frac{\partial \ln \mu_i^2}{\partial \ln \Lambda_i^2}. \quad (A4) $$

For each model, we take the fine-tuning to be $\Delta = \max(|\Delta F^2|)$. Specifying an acceptable level of fine-tuning thus determines the cut-off scale up to which the theory is natural without introducing any new physics.

In Figure 8 we plot the fine-tuning $\Delta$ for a given cut-off scale of $\Lambda_i = 1.5$ TeV. This cut-off scale corresponds to the perturbativity scale (in the SM) at which the one-loop RG corrections to the Higgs self-coupling grow to the same level as its tree-level value for a $m_h \sim 700$ GeV. In \cite{17} it was also used as the upper naturalness scale\cite{10}, and it was argued that with such a high scale one can no longer be certain that any new physics canceling the divergences will be observable at the LHC.

The plot includes all the IDMs from our scans that give a relic density in accordance with WMAP and pass

\footnotetext{10 In \cite{17} the no-fine-tuning scale associated to the Higgs mass quadratic corrections was derived to be $\Lambda = 1.3$ TeV and to be independent of the Higgs mass.}
Here we also impose the constraint in Eq. 17 of reference [17]. All our renormalization conditions were set at $Q = m_h$. The blue dashed line is the SM result with RG running of the couplings included. The solid blue line shows the results within the SM, and the mass range 115 to 129 GeV (the only span left for the SM Higgs given the current LHC limits) is marked as a thicker part of the solid blue line. This gives that $\Delta \approx 10$ for the SM. The kink on the blue curve around 350 GeV is when the fine-tuning goes from being dominated by $\Delta_{\Lambda_i}$ to more sensitive to the Higgs cut-off $\Lambda_i$. We see that this measure $\Delta$ gives a large fraction of the IDMs (green circles) that are less fine-tuned than the SM ($\Delta \approx 10$), but also many models that are not.

A similar measure to Eq. [A4] was used in [19], but with the running of the parameters up to the cut-off scale also taken into account. With $\Delta = \max(|\Delta_{\Lambda_i}|^2)$ and the RG equations deduced from [108–110], we find that with the fine-tuning condition $\Delta \leq 5$ on $\mu_{1,2}^2$ our benchmark models are natural up to cut-off scales $\Lambda = 1.0–2.4$ TeV. Figure [B] shows the running of the IDM parameters in the case of our benchmark model IDM-B1. For comparison, the SM is now left natural up to $\Lambda = 1.2$ TeV, with the SM Higgs mass bound to be $m_h < 129$ GeV [3].

This measure leaves half of our benchmark models less fine-tuned than the SM. In Figure [S] we also added the SM result, for $\Lambda = 1.5$ TeV, when the RG running of couplings is included.

In [19] they used $\Delta = \max \left( \sqrt{\sum_i \Delta_{\Lambda_i}^2} \right)$, where the contributions to $\Delta_{\Lambda_i}$ associated with $\lambda_i$ are also included. Here, the tuning with respect to $\lambda_i$ has no significant impact (on our benchmark models), but we comment further on this below. In Figure [B] we show this measure together with the individual contributions to the fine-tuning for our benchmark model IDM-B1.

We note, however, that in the case of our benchmark models, the large quartic couplings are compen-

\[\sum \Delta_{\Lambda_i}^2 \]

\[\max(\Delta_{\Lambda_i}^2) \]

**FIG. 8:** Fine-tuning $\Delta$, without RG effects, as a function of the SM Higgs mass for IDMs (red marks for the benchmark models and green for models in the scan) and the SM (blue solid line) given a cut-off scale of 1.5 TeV. The circles show the result for IDMs using Eq. [A3] and the crosses the result using Eq. [A5]. The thick part of the blue line corresponds to the remaining Higgs mass window allowed within the SM. The blue dashed line is the SM result with RG running of the couplings included.

**FIG. 9:** The running of IDM parameters with energy scale $Q$ for one of our benchmark models (IDM-B1). Where the curves flatten out at around 150 TeV is when we terminate the calculation because the perturbativity limit of at least one $\lambda_i > 4\pi$ is reached. The $m_{h_{1,0}}^2$ and $\mu_{1,0}^2$ curves are normalized into units of their values $m_{h_{1,0}}^2, \mu_{1,0}^2$ at the scale $Q = m_h$.

**FIG. 10:** Fine-tuning measures, with RG effects included, for one of our benchmark models (IDM-B1). The notation for the plot legend is that $\Delta = \max(\Delta_{\Lambda_i})$, and $\sqrt{\sum_i \Delta_{\Lambda_i}} = \Delta$ (as defined in the text).

\[\Delta = \max(\Delta_{\Lambda_i}^2) \]

\[\sqrt{\sum_i \Delta_{\Lambda_i}^2} \]

\[\Delta = \max(\Delta_{\Lambda_i}^2) \]

\[\sqrt{\sum_i \Delta_{\Lambda_i}^2} \]

\[\Delta = \max(\Delta_{\Lambda_i}^2) \]

\[\sqrt{\sum_i \Delta_{\Lambda_i}^2} \]

[11] Here we also impose the constraint in Eq. 17 of reference [17] even though we note that this does not qualitatively change the result.

[12] All our renormalization conditions were set at $Q = m_h$. 

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**FIG. 8:** Fine-tuning $\Delta$, without RG effects, as a function of the SM Higgs mass for IDMs (red marks for the benchmark models and green for models in the scan) and the SM (blue solid line) given a cut-off scale of 1.5 TeV. The circles show the result for IDMs using Eq. [A3] and the crosses the result using Eq. [A5]. The thick part of the blue line corresponds to the remaining Higgs mass window allowed within the SM. The blue dashed line is the SM result with RG running of the couplings included.

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sated by large negative values of $\mu_2^2$ to give small masses to the inert particles. This introduces an additional source of fine-tuning, even if each scale $\mu_2^2$ and $\mu_3^2$, associated with the two Higgs doublets, is individually not severely tuned. This we can incorporate by introducing a fine-tuning measure on, e.g., the mass of the lightest inert particle

$$\Delta_{\Lambda_i}^{m_H^0} = \frac{\partial \ln m_{H_i}^0}{\partial \ln \Lambda_i} \equiv \frac{1}{2} \left( \frac{\partial (\mu_2^2 - \lambda_H^0 \mu_2^2 / 2 \Lambda_i)}{m_{H_i}^0 \partial \ln \Lambda_i} \right) \quad (A5)$$

This reflects better the increased fine-tuning in models with high $\lambda_H^0$ and low $m_{H_i}^0$ values. It also cures the artificial large fine-tuning that arises when $\mu_2^2$ goes through zero and drives $\Delta_{\Lambda_i}^{m_H^0}$ to infinity (any tuning around $\mu_2^2 = 0$ is irrelevant as it then does not contribute to any of the inert particle masses). The resulting $\Delta = \max(|\Delta_{\Lambda_i}^{m_H^0}|)$ are represented by crosses in Figure 5 (without RG improvement), as well as the solid black curve in Figure 10 for IDM-B1 (including RG improvement). As our benchmark models come with rather large $\Lambda_i$ this measure typically leaves them less natural. $\Delta$ is less than 5 up to cut-off scales $\Lambda = 0.4 - 1.4$ TeV when including the RG evolution. With this fine-tuning measure, our benchmark models can thus hardly be considered to be less fine-tuned than the SM.

We here also note that the sensitivity to variations in $p_i = \lambda_i$ could be significant already at tree-level. The tree-level contribution to

$$\Delta_{\lambda_i}^{m_H^0} = \frac{\partial \ln m_{H_i}^0}{\partial \ln \lambda_i} \quad (A6)$$

already gives $\Delta = \max(|\Delta_{\lambda_i}^{m_H^0}|) \sim 6 - 25$ for our benchmark models, and is independent of $\Lambda_i$. This type of fine-tuning is however not directly related to the unknown contributions beyond the cut-off scale, and would be absent if we take our $\lambda_i$ to be fixed and known parameters for each model.

**Appendix B: Statistical measures**

It is desirable to have a statistical measure of the integrated luminosity $\mathcal{L}$ expected to be needed to detect a signal with cross section $\sigma_S$ above a background with cross section $\sigma_B$.

We denote the probability of observing $N$ or fewer events from a distribution with expectation value $X$ by

$$P_0(N; X), \quad (B1)$$

where the index D distinguishes between different distributions. In the following, D = G and P to denote Gaussian and Poisson statistics, respectively. $B \equiv B(\mathcal{L}) = \sigma_B \mathcal{L}$ and $S \equiv S(\mathcal{L}) = \sigma_S \mathcal{L}$ denote the expectation values of the number of background and signal events, respectively.

To claim that an observation of $N_{\text{obs}}$ events is an excess, i.e. to reject the null hypothesis of a background expectation $B$, it has to lie outside the interval specified by the background model’s $P_1$ confidence level (CL). For a one-sided bound, this requires $N_{\text{obs}} \geq N_{\min}(\mathcal{L})$, where $N_{\min}$ is the minimum integer number satisfying

$$P_0(N_{\min}; B) \geq P_1. \quad (B2)$$

For such a future observation to occur with a probability $P_2$, when the underlying true scenario has an expectation value $S + B$, it is required that $N_{\min}$ also fulfills $N_{\min} \leq N_{\max}(\mathcal{L})$, where $N_{\max}$ is the maximum number satisfying

$$1 - P_0(N_{\max}; S + B) \geq P_2. \quad (B3)$$

For a given distribution function $P_0(N; X)$, the system of equations $B2$–$B3$ can then be solved to find the smallest required integrated luminosity $\mathcal{L}$ that has an integer solution $N$:

$$N_{\min}(\mathcal{L}, P_1) \leq N \leq N_{\max}(\mathcal{L}, P_2) \quad (B4)$$

Note that $P_0(N; X)$ are distribution functions, whereas $P_{1,2}$ are requested probabilities.

It can be convenient to phrase the probabilities $P_{1,2}$ in terms of a corresponding number $n_{1,2}$ of standard deviations ($n$-$\sigma$) for a one-sided normal distribution. We define such a correspondence by

$$P_{1,2} = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{n_{1,2}}{\sqrt{2}} \right) \right], \quad (B5)$$

where erf is the Gaussian error function

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dt \, e^{-t^2}. \quad (B6)$$

Eq. B5 thus defines what we refer to as an $n$-$\sigma$ observation, independently of the type of distribution function $P_0$. For example $3(5)\sigma$ correspond to $1 - P_{1,2} = 1.35 \times 10^{-3} (2.87 \times 10^{-7})$.

If the number $N$ of events is Poisson distributed, then the one-sided cumulative distribution function $P_0 = P$, can be expressed as

$$P_0(N; X) = \frac{\gamma(N + 1; X)}{\Gamma(N)}, \quad (B7)$$

where $\Gamma$ and $\gamma$ are the ordinary and the lower incomplete gamma function respectively,

$$\frac{\gamma(N + 1; X)}{\Gamma(N)} \text{ for integer } N \geq 0 \sum_{i=0}^{N} e^{-X} \frac{X^i}{i!}. \quad (B8)$$

Strictly speaking, $N$ can only take integer values – as it represents the number of observed events – and in
TABLE VII: Integrated luminosities $\mathcal{L}$, in units of fb$^{-1}$, required to detect our benchmark models under different statistical measures. The $3(5)\sigma$ columns give the required integrated luminosity in order to observe a $3(5)\sigma$ evidence (discovery) with a probability $P_2 = 90\%$, when the number of event counts is assumed to be Gaussian, $\mathcal{G}$, or Poisson, $\mathcal{P}$, distributed. In the columns with no $P_2$ value quoted, the commonly used criterion $S \geq 3(5)\sqrt{B}$ has been used (i.e. the Gaussian approximation in Eq. B12). In the last column we give the required luminosity to have a 95% probability to exclude the models with at least 95% confidence. (See the text for further information.)

| Model   | $3\sigma, \mathcal{G}$ | $3\sigma P_2=90\%, \mathcal{G}$ | $3\sigma P_2=90\%, \mathcal{P}$ | $5\sigma, \mathcal{G}$ | $5\sigma P_2=90\%, \mathcal{G}$ | $5\sigma P_2=90\%, \mathcal{P}$ | $P_1 = P_2 = 95\%$ |
|---------|------------------------|-----------------------------------|-----------------------------------|------------------------|-----------------------------------|-----------------------------------|------------------------|
| IDM-A1  | 720                    | 1600                              | 1700                              | 2000                   | 3400                              | 3600                              | 1000                   |
| IDM-A2  | 240                    | 500                               | 630                               | 680                    | 1200                              | 1300                              | 350                    |
| IDM-B1  | 44                     | 120                               | 140                               | 120                    | 240                               | 290                               | 88                     |
| IDM-B2  | 44                     | 120                               | 140                               | 120                    | 240                               | 290                               | 88                     |
| IDM-C1  | 10$^a$ (19)            | 36                                | 44                                | 28                     | 66                                | 90                                | 30                     |
| IDM-C2  | 0.95$^a$ (5.7)         | 5.8$^a$                           | 8.3                               | 2.6$^a$                | 9.3                               | 16                                | 5.2                    |
| IDM-C3  | 11$^a$ (19)            | 38                                | 49                                | 30                     | 70                                | 96                                | 30                     |
| IDM-C4  | 33                     | 97                                | 110                               | 91                     | 190                               | 230                               | 70                     |

$^a$Requiring a signal expectation of at least 5 events gives the value quoted in parentheses in the first column.

Although $N$ should be an integer, we will follow common practice and leave out this additional requirement when we present results for Gaussian distributions in Table VII. For $n_1 = n$ and $n_2 = 0$, this gives the commonly used criterion for expecting an $n$-sigma detection

$$S \geq n\sqrt{B}, \quad \text{(B12)}$$

which corresponds to a probability $P_2 = 50\%$ to observe the required $N_{\text{obs}}$ from a Gaussian distribution that, in fact, also spans over negative $N_{\text{obs}}$. For $n_1 = n_2 = n$, Eq. (B11) gives the sometimes seen criterion

$$S \geq n^2 + 2n\sqrt{B}. \quad \text{(B13)}$$

From these equations the minimum $\mathcal{L}$ is easily derived by substituting $B = \sigma_B \mathcal{L}$ and $S = \sigma_S \mathcal{L}$.

This defines our statistical measures to determine the expected integrated luminosity needed to observe a $n_1$-sigma detection with a probability $P_2$. Equivalently, this formalism also gives the expected integrated luminosity needed to exclude the signal expectation $S + B$ at the $P_2$ CL with a probability $P_1$.

In Table VII we present integrated luminosities required to detect our benchmark models with different probabilities $P_{1,2}$ under different assumed distribution functions $P_2$ for the number of event counts.

[1] The ATLAS Collaboration, ATLAS-CONF-2012-019 (2012)
[2] The CMS Collaboration, CMS-PAS-HIG-12-008 (2012)
[3] S. Chatrchyan et al. [CMS Collaboration], arXiv:1202.1997 [hep-ex].
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