Model of the discrete destruction process of a solid body

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Abstract. Destruction is considered as a discrete thermomechanical process, in which the deformation of a solid body is achieved by changing the boundary stresses acting on the part of the volume being destroyed with the external load unchanged. On the basis of the proposed concept, a model for adhesive stratification of a composite material is constructed. When adhesive stratification is used, the stress state of one or two boundaries of the adhesive layer changes to zero if the bonds with the joined body are broken. As a result of the stratification, the interaction between the part of the composite, which may include an adhesive layer and the rest of the body stops. When solving the elasto-plastic problem of cohesive stratification, the region in which the destruction criterion is achieved is identified. With the help of a repeated solution of the problem of subcritical deformation with the known law of motion of the boundary of the region, the distribution of the load (nodal forces) acting from the region to the body is located. The next step considers the change in the stress–strain state of the body in the process of destruction of the selected area. The elasto-plastic problem is solved with a simple unloading of the formed surface of the body and preservation of the external load corresponding to the beginning of the process of destruction.

1. Introduction
Modeling formation of new material surfaces involves the description of two stages of the process of loading the body: subcritical and postcritical. The subcritical stage can be described in the framework of the known equations of mechanics of a deformed solid body, both without taking into account nonlinear effects, and in a nonlinear (elastoplastic) formulation. The main problem is the formulation of criteria of the transition to the postcritical stage. The general formulation of the problem of postcritical behavior in elastoplastic deformation is far from being complete. This is due, firstly, to the choice of conditions for the termination of the interaction between surfaces, and secondly to the necessity of formulating boundary conditions on newly formed surfaces. When performing this or that criterion of destruction, the phase of formation of new material surfaces begins. The principal moment of this stage is the discreteness of the increment of the discontinuity surfaces [1] and the localization of the destruction process. In the modeling of destruction within the material surface of measure zero, cohesive elements are used [2,3]. The widely used bilinear law of the distribution of interaction in the cohesive zone from the opening of a crack with a falling section was considered in the works [4–5]. The main disadvantage of this approach is that the cohesive elements must be located on the trajectory of destruction, which, in general, depends on the new material surfaces. In addition, the various laws of cohesive interaction [6–8] and the material characteristics of cohesive elements significantly affect the distribution of stress–strain state [9] and require experimental confirmation.

Complexity of describing the destruction of the material volume when using the falling section of the tensile diagram is related to the construction of the determining relations of the unstable Drucker
deformation and their confirmation in experiments [10]. The main approach to finite element modeling of the destruction process is the "kill element" procedure, described in detail in [11]. In this case, when the criterion characteristic of destruction is reached in the final element, the isolated material volume is excluded from consideration by multiplying the local stiffness matrix of the element by a number close to zero. Note that this procedure is correct for elastic deformation, when the load and unloading are determined by the same modules in the determining ratios. In the case of elastoplastic deformation of a body with the element excluded in this way, it is necessary to take into account possible unloading.

The discretization of the problem by the finite element method implies the description of the interaction among the particles of the body by means of nodal forces. When the material volume, in the general case interacting with several finite elements, is destroyed, we assume that the node interaction forces will be unloaded to zero within a simple process, with the unchanged external load reached at the time of destruction. Thus, the unstable Drucker deformation is excluded from consideration. Note that the determination of the nodal forces of interaction with the excluded material volume, at the internal points of the deformed body, is a separate problem. In the article [12], a procedure is proposed for the repeated loading of a body without a destructible volume, but with a displacement field found along its boundary, to find the corresponding nodal forces. The finding of forces of interaction of material volumes, in such a way, does not require additional information on the characteristics of the connection of these volumes except the determining relationships of the material.

In this paper, when modeling the process of destruction of a composite along the adhesive layer, along with destruction along the adhesive array, we considered the procedure for stratification of the adhesive layer from the adjacent material. In this case, the determination of the interaction forces of the separated materials with the repeated loading method is considered on the surface of the measure zero, and the use of the averaged characteristics of the stress–strain state of the adhesive layer allows us to avoid the singularity at the blind point of the mathematical cut being formed.

2. Materials and methods

Figure 1 shows the excluded volume of the adhesive layer 3 with the size $\delta \times \delta$ in the state of pre–destruction that connects the bodies 1 and 2. Its effect on the body is compensated by the external nodal load $\mathbf{F}^+ (i)$, $\mathbf{F}^- (j)$ on the part of the element.

![Figure 1. Equivalent loads at the time of pre–destruction of a material volume.](image-url)

We assume that during the interval $\Delta t$ the $\delta$–element will be completely destroyed if the load acting on the part of the layer element on the body becomes zero. In this case, during the time interval $\Delta t$, with the external load unchanged $\mathbf{P}$, it is necessary to unload the new material surfaces. We assume that
the unloading process is simple, depending on one parameter $\Delta t$. This corresponds to the specification of the velocities of the external nodal loads:

$$\ddot{F}^{FF} = -\frac{F^{+}(i)}{\Delta t}, \quad \ddot{F}^{NN} = -\frac{F^{-}(j)}{\Delta t}. \quad (1)$$

Thus, the description of the destruction process reduces to solving the problem with boundary conditions (1) at zero external–load velocity $P = 0$. Note that the determination of the nodal forces of interaction with the excluded material volume, at the internal points of the deformed body, is a separate problem. To determine the nodal forces $F^{+}(i), F^{-}(j)$ it is proposed to use the procedure of repeated loading, which consists in replacing the action of the destructible element by setting the process of change with the "time" of nodal displacements of the element when the law of external action on the body is repeated, according to figure 2.

![Figure 2. The scheme of repeated loading.](image)

When the adhesive layer is delaminated the surface with the material adjacent to it, the state of pre–destruction is shown in figure 3. We will mentally divide the layer and the adjacent material along the length $\delta_0$, and apply oppositely directed nodal forces $F^{-(j)}$ corresponding to the stressed state at the boundary to the corresponding new surfaces. Without loss of generality, we assume that the separation occurs along the lower boundary of the layer. We assume that during the interval $\Delta t$ there will be a stratification in the $\delta$–part of the surface, if the load acting from the layer to the body becomes zero. In this case, during the time interval $\Delta t$, with the external load $P$ unchanged, the unloading of new material surfaces will correspond to the specification of the velocities of external nodal loads:

$$\ddot{F}^{NN} = \frac{F^{+}(i)}{\Delta t}, \quad \ddot{F}^{NN} = -\frac{F^{-}(j)}{\Delta t}. \quad (2)$$

As an example of the use of the proposed procedure, the problem of stratification of a composite material along the adhesive layer is considered. In this case, destruction is possible both over the adhesive array and along the boundary of the adhesive layer with the material adjacent to it. In the first case, we mean a cohesive character of the stratification, and in the second case, we mean an adhesional character of the stratification.
Figure 3. Stratification the surface.

The equilibrium flow conditions of the process with small deformation and rotations of material fibers was taken in the form [13]:

$$\int_S \mathbf{\dot{\sigma}} \cdot \mathbf{\dot{w}} \, ds = \int_L \mathbf{\dot{P}} \cdot \mathbf{\dot{v}} \, dl,$$

where \( \mathbf{v} \) is the velocity field; \( \mathbf{w} = 0.5(\mathbf{V} \mathbf{v} + \mathbf{v} \mathbf{V}) \) is the velocity strain tensor; \( \mathbf{V} = \mathbf{e}_i \frac{d}{dx_i} \); \( \mathbf{P} = \frac{d\mathbf{P}}{dt} \) is the velocity of external load on the circuit; \( \mathbf{\dot{\sigma}} = \frac{d\mathbf{\sigma}}{dt} \) is the velocity of the stress tensor; \( t \) is the time-like parameter; \( S \) is the inner region of the complex body; \( \delta \) is the variation.

In the adhesive layer, the stress–strain state is considered on the basis of the average stresses along the thickness \( \delta_0 \) and the associated boundary stress velocities [12]:

$$\bar{\sigma}_{21}(x_f) = \frac{\delta_0/2}{\delta_0 - \delta_0/2} \int_0^{\delta_0/2} \bar{\sigma}_{21}(x_f,x_2) \, dx_2 \, , \quad \bar{\sigma}_{12}(x_f) = \frac{\delta_0/2}{\delta_0 - \delta_0/2} \int_0^{\delta_0/2} \bar{\sigma}_{12}(x_f,x_2) \, dx_2 \, , \quad \bar{\sigma}_{22}(x_f) = \frac{\delta_0/2}{\delta_0 - \delta_0/2} \int_0^{\delta_0/2} \bar{\sigma}_{22}(x_f,x_2) \, dx_2 \, ,$$

$$\bar{\sigma}_{11}(x_f) = \frac{\delta_0/2}{\delta_0 - \delta_0/2} \int_0^{\delta_0/2} \bar{\sigma}_{11}(x_f,x_2) \, dx_2 \, ,$$

and the average velocities, and deformation velocities through their boundary values:

$$w_{22}(x_f) = \frac{v_2^+(x_f) - v_2^-(x_f)}{\delta_0} \, , \quad w_{11}(x_f) = 0.5 \left( \frac{dv_1^+(x_f)}{dx_f} + \frac{dv_1^-(x_f)}{dx_f} \right) \, ,$$

$$\frac{dv_{11}(x_f)}{dx_2} = \frac{v_{11}^+(x_f) - v_{11}^-(x_f)}{\delta_0} \, , \quad \frac{dv_{22}(x_f)}{dx_2} = 0.5 \left( \frac{dv_2^+(x_f)}{dx_f} + \frac{dv_2^-(x_f)}{dx_f} \right) \, ,$$

$$\bar{v}_1(x_f) = 0.5(\bar{v}_1^+(x_f) + \bar{v}_1^-(x_f)), \quad \bar{v}_2(x_f) = 0.5(\bar{v}_2^+(x_f) + \bar{v}_2^-(x_f)),$$

where \( \mathbf{v}^+ \), \( \mathbf{v}^- \) is the velocity vector of the upper and lower boundaries of the region 3.

From the expressions (5) we arrive at the representation of the average shear strain velocity along the considered region:
\[
\begin{align*}
\mathbf{w}_{21}(x_j) = 0.5 \left( \frac{d\mathbf{v}_2(x_j)}{dx_j} + \frac{d\mathbf{v}_1(x_j)}{dx_2} \right) = 0.5 \left( \frac{v^+_j(x_j) - v^-_j(x_j)}{\delta_0} + v^+_j(x_j) + \frac{d\mathbf{v}_2(x_j)}{dx_j} + \frac{d\mathbf{v}_2(x_j)}{dx_j} \right),
\end{align*}
\] (7)

We assume that the stress velocity vectors on the conjugate boundaries of the adhesive layer are equal and opposite to the stress velocity vectors of the conjugate boundaries of the body. In addition, rigid adhesion among the boundaries of the layer and regions 1, 2 is postulated (see figure 1). Considering the velocities of the boundary stress vectors of the layer as boundary conditions for regions adjacent to it [14], we arrive at a simultaneous solution of the variational equilibrium equations for the body 1:

\[
\begin{align*}
\int \sigma \cdot \delta \mathbf{w} ds + \int \sigma_{22} \delta v_2 dx_j + \int \sigma_{21} \delta v_1 dx_j + 0.5 \delta_0 \left( \int \sigma_{11} \frac{d\mathbf{v}_1}{dx_j} dx_j + \int \sigma_{21} \frac{d\mathbf{v}_2}{dx_j} dx_j \right) = \int \mathbf{P} \cdot \delta \mathbf{v} dl,
\end{align*}
\] (8)

and body 2:

\[
\begin{align*}
\int \sigma \cdot \delta \mathbf{w} ds - \int \sigma_{22} \delta v_2 dx_j - \int \sigma_{21} \delta v_1 dx_j + 0.5 \delta_0 \left( \int \sigma_{11} \frac{d\mathbf{v}_1}{dx_j} dx_j + \int \sigma_{21} \frac{d\mathbf{v}_2}{dx_j} dx_j \right) = \int \mathbf{P} \cdot \delta \mathbf{v} dl.
\end{align*}
\] (9)

The behavior of the material of bodies 1 and 2 under active loading (\( \sigma \cdot \delta > 0 \)) is determined by the following physical relationships:

\[
\hat{\sigma} = 2G^{(l)} \hat{\mathbf{w}}, \quad \hat{\rho} = 3K^{(l)} \hat{\theta},
\] (10)

where \( \hat{\sigma} \) is the velocity of the deviator of the stress tensor; \( i = 1,2; \hat{\mathbf{w}} \) is the deviator component of the strain velocity tensor; \( \hat{\rho} = \hat{\sigma} \cdot \mathbf{E} \); \( K^{(l)} \) is the volume compression module; \( G^{(l)} \) is the shear modulus \( G^{(l)} = G^{(l)}_e \) at \( T \leq T^{(l)}_p \), \( G^{(l)} = G^{(l)}_p \) at \( T > T^{(l)}_p \); \( T^{(l)}_p \) is the yield limit of the corresponding material; \( T \) is the intensity of tangential stresses.

In the unloading state (\( \sigma \cdot \delta \leq 0 \)), the defining relations are written as:

\[
\hat{\sigma} = 2G^{(l)}_e \hat{\mathbf{w}}, \quad \hat{\rho} = 3K^{(l)} \hat{\theta}.
\] (11)

In the material of the layer, the defining relationships are assumed to be valid for the medium–thick layer of the velocity characteristics of the stress–strain state:

\[
\hat{\sigma} = 2G^{(l)} \hat{\mathbf{w}}, \quad \hat{\rho} = 3K^{(l)} \hat{\theta}, \quad \sigma \cdot \hat{\sigma} > 0 ;
\] (12)

\[
\hat{\sigma} = 2G^{(l)}_e \hat{\mathbf{w}}, \quad \hat{\rho} = 3K^{(l)} \hat{\theta}, \quad \sigma \cdot \hat{\sigma} \leq 0;
\] (13)

where \( K^{(l)} \) is the volume compression module of the layer material; \( G^{(l)} \) is the shear modulus of the layer material \( G^{(l)} = G^{(l)}_e \) at \( T \leq T^{(l)}_p \), \( G^{(l)} = G^{(l)}_p \) at \( T > T^{(l)}_p \); \( T^{(l)}_p \) is the yield limit of the layer material.

As a result of substituting expressions in the defining relations (12) – (13) the component of the average velocity deformation, the average stress velocities are determined through the boundary velocities and their derivatives. Thus, the solution of the system (8) – (13) reduces to determining the velocity field \( \mathbf{v}(x_1, x_2) \) in bodies 1 and 2. In this case, boundary velocities of the body 2 adjacent to the layer (see) will be present in equation (8), and in equation (9) boundary velocities of the body 1, which adjacent to the layer. We note that the use of the average–thickness adhesive layer characteristics of the stress–strain state makes it possible to exclude the dependence on the form of the end of the adhesive layer in the formulation of problem (8) – (13). Therefore, in figure 1–3, the left and right boundaries of the layer are shown as a wavy dotted line. Fields of displacement and deformation are determined with the help of integration according to the parameter \( l \) of the found velocity fields.

In stratification along the surface (see figure 3), we will take into account that along the lower boundary of the layer and the body 2, on the exfoliated region, there is no rigid adherence condition.
\( \mathbf{v}_{-} = \mathbf{v}(x_{j}, -\delta_{0}/2) \), \( x_{j} \in [N, N_{f}] \) while along the upper boundary of the layer and body 1, there is a rigid adherence of the boundaries: \( \mathbf{v}_{+} = \mathbf{v}(x_{j}, \delta_{0}/2) \), \( x_{j} \in [F, F_{1}] \). In this case, due to the small thickness of the layer compared with the thicknesses of bodies 1 and 2, we consider the hypothesis of the membrane theory of the homogeneity of velocity fields over the layer thickness to be valid. Thus, in the exfoliated part of the layer, the deformation velocities (4) and (7) will be equal to:

\[
\begin{align*}
    w_{1j}(x_{j}) &= \frac{dv_{1j}(x_{j})}{dx_{j}}, \quad w_{2j}(x_{j}) = 0, \quad w_{2j}(x_{j}) = 0.5 \frac{dv_{3j}(x_{j})}{dx_{j}}.
\end{align*}
\]

3. Results and discussion

Calculations carried out for a composite material consisting of two elements in a state of a plane deformation, with material characteristics close to the aluminum alloy: \( G_{e} = 2.8 \cdot 10^{10} \text{Pa}; \)
\( G_{p} = 5.2 \cdot 10^{8} \text{Pa}; \) \( K = 6 \cdot 10^{10} \text{Pa}; \) \( T_{p} = 3 \cdot 10^{8} \text{Pa} \) is the elasticity limit; \( \sigma_{k} = 4.2 \cdot 10^{8} \) is the strength limit bonded epoxy resin with the following properties: \( G_{e} = 1.3 \cdot 10^{9} \text{Pa}; \) \( K = 1.7 \cdot 10^{9} \text{Pa}; \)
\( \sigma_{k} = 9 \cdot 10^{7} \text{Pa}. \) For adhesive, a variant of adhesion strength for peeling and shearing along the boundary between the material and the layer also considered. The geometric characteristics of the composite shown in figure 4 were taken as follows: \( AD = 5 \cdot 10^{-3} \text{m}; \) \( \delta_{0} = FN = 10^{-3} \text{m}; \) \( MQ = 2 \cdot 10^{-2} \text{m}; \) \( AB = 10^{-1} \text{m}; \) \( DF = 5 \cdot 10^{-2} \text{m}. \) The velocity of the external load \( \mathbf{P} = 1 \text{Pa/s} \) is directed at an angle \( \pi/4 \) to the axis \( OX_{1} \).

![Figure 4. Scheme of composite loading.](image)

As a test, the problem of destruction of an element of a layer of the size \( \delta \times \delta \) in an elastic setting is considered. Let the critical state in the \( \delta \)-element be determined by the external load \( \mathbf{P} = 1 \text{ Pa}. \) The computational convergence of the solution establishes a partition of the boundary region conjugate to the \( \delta \)-element into four finite elements [15]. Taking into account the quadratic law of distribution of the velocity field on a finite element, nine nodal forces along the upper and lower boundaries of the destructible \( \delta \)-element will determine the equilibrium of the composite at the pre-destruction moment. Without loss of generality, the minimal module of projections of the found nodal forces can be considered as a local unloading parameter \( \Delta t \) for determining the boundary conditions (1). Graph 2 in figure 5 a) shows the distribution of dimensionless vertical displacement along the boundary of
body 1 and layer 3 toward the end of subcritical deformation, graph 3 shows the distribution of displacements due to the process of destruction of the structural element of the layer, graph 1 shows the superposition of displacement fields. Displacements related to the value of displacement \( u^b_x(0, \delta_0) / 2 \) at the beginning of the process of destruction. The dimensionless coordinate in the direction of the abscissa axis is defined as \( x^b_1 = x_1 / \delta_0 \). Graph 1 in figure 5 b) determines the corresponding vertical displacements when solving a problem without the first structural element of the layer at the end of the interval. This solution simulates the "kill element" approach used in finite element modeling [10] procedures, where the local stiffness matrix of the excluded element is multiplied by a number close to zero. Graph 2 in figure 5 b) repeats curve 1 in figure 5 a). As we can see from figure 5 b), the coincidence of graphs 1 and 2 shows the adequacy of the results of the proposed procedure of unloading new surfaces during the destruction process of the "kill element" approach within elastic deformation.

**Figure 5.** Distributions of vertical displacement in the process of destruction of the first structural element of the layer. In Fig. a) Graph 2 describes subcritical deformation, Graph 3 describes process of destruction of the structural element, Graph 1 describes a superposition of solutions. In Fig. b) Graph 1 repeats the corresponding line Fig. a), Graph 2 describes subcritical deformation without the first structural element.

Figure 6 shows a comparison of vertical displacement along the boundary of body 1 and layer 3 with total destruction of the layer material and when the layer is exfoliated along the boundary with body 2. Calculations made for the elastic deformation of the composite. Graph 1 in figure 6 a) shows the distribution of dimensionless vertical displacement \( u^b_2 \) along the boundary of body 1 and layer 3 with destruction of the first structural element of the layer with the size \( \delta_0 \times \delta_0 \). Graph 2 shows the distribution of displacements when the layer element is exfoliated along the length \( \delta_0 \). Displacements are referred to the value of the displacement \( u^b_2(0, \delta_0) / 2 \) toward the end of the process of destruction of the structural element of the layer. Graph 1 in figure 6 b) shows the distribution of dimensionless vertical displacements \( u^b_2 \) along the boundary of body 1 and layer 3 toward the end when the five structural elements of the layer are destroyed, and Graph 2 shows the distribution of displacements when the layer element is exfoliated along the length \( 5 \delta_0 \). From figure 6 we can see that the material of the layer creates additional rigidity due to tensile–compressive and shear stresses, and its account can play an important role in the stratification of the material along the length exceeding the thickness.
of the layer. For a stratification along the length equal to the size $\delta_0$, the results of calculation in comparison with the complete destruction of the structural element are practically identical.

**Figure 6.** Distributions of vertical displacements in the process of destruction and stratification of the adhesive layer. In Fig. a) the solid line (Graph 1) shows the distribution of displacements during the cohesive destruction of the layer along the length $\delta_0$, and the dotted line (Graph 2) shows the distribution of displacements during adhesive delamination along the length $\delta_0$. In Fig. b) the solid line (Graph 1) shows the distribution of displacements during the cohesive destruction of the layer along the length $5\delta_0$, and the dotted line (Graph 2) shows the distribution of displacements during adhesive delamination along the length $5\delta_0$.

Let us consider the case of cohesive destruction of the adhesive layer of a composite, taking into account the elastoplastic properties of the materials. Figure 7 shows the area of plastic deformation in the state of pre–destruction. The value of the external load is denoted as $P_k$. The maximum principal stress in the adhesive layer on the characteristic element is equal to the strength limit, and the strength limit is not reached outside the layer. Consequently, destruction in the layer will occur faster than in the materials conjugate with it. It was assumed that the adhesive bond – resin–alloy is sufficiently strong, and the destruction will take place along the array of the adhesive component.

**Figure 7.** Zone of plasticity in the state of pre–destruction. The area is highlighted
in color.

Destruction of the first element leads to redistribution of the plasticity zone and unloading of a number of elements of the composite. In this case, the result of consideration of destruction as a thermomechanical process is different from the result received with "kill element" procedure. In figure 8, the zone of plastic loading is marked with a dark filling, and the lighter are the elements where elastic unloading from the plastic region occurred, after the destruction of the first $\delta$-element.

Figure 8. Evolution of the plasticity zone and unloading in the process of destruction of the first element. The unloading region is highlighted in gray, and the elastoplastic region is black.

Solving the problem of loading a composite with a critical load without the first structural element of the layer, we arrive at the distribution of the plastic region shown in figure 9. Comparing figure 9 and figure 8 we see that unloading from the plastic region is not taken into account in the "kill element" approach, the plasticity region is slightly larger than the combination of the unloading regions from the plastic region and the additional plastic load shown in figure 8.

Figure 9. Zone of plasticity in the composite without the first structural element of the adhesive layer. The area is highlighted in color.

Destruction of the first element leads to an excess of the strength limit on the second element of the layer, which means its destruction at a fixed external load.

4. Conclusion
The results of the calculations according to the proposed model within the elastic behavior of the material do not contradict the known method of calculation – the "kill element". In the case with an elastoplastic material, the stress state obtained by simulating the destruction process with the local unloading method may differ significantly from the state determined by the kill element method. The proposed approach allows taking into account the redistribution of plastic zones and the possibility of forming new zones of destruction as a result of local unloading.

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References
[1] Petrov Y V 1996 Physics of the Solid State 38 pp 1846–50
[2] Xiangting Su, Zhenjun Yang and Guohua Liu 2010 Acta Mechanica Solida Sinica 23 pp 271–282
[3] Sua X T, Yang Z J and Liu G H 2010 Int. J. Solids Struct. 47 pp 2336–45
[4] Dávila C G, Camanho P P and Turon A 2008 Journal of Aircraft 42 pp 663–672
[5] De Moura MFSF and Gonçalves JPM 2008 Int. J. Solids Struct. 51 pp 1123–31
[6] Jain S, Na S R, Liechti K M and Bonnecaze R T 2017 Int. J. Solids Struct. 129 pp 167–176
[7] Olsson E and Larsson P–L 2013 J. Mech. Phys. Solids 61 pp 1185–1201
[8] Mesarovic S D and Johnson K L 2000 J. Mech. Phys. Solids 48 2009–33
[9] Panettieri E, Fanteria D and Danzi F 2016 Composite Structures 137 pp 140–7
[10] Lebedev A A and Chausov N G 1983 Strength of Materials 15 pp 155–160
[11] ANSYS 2006 User's Guide, Release 11.0 (Pennsylvania USA: ANSYS Inc.)
[12] Glagolev V V, Markin A A and Fursaev A A 2017 PNRPU Mechanics Bulletin pp 45–59
[13] Markin A A and Sokolova M Y 2014 Thermomechanics of Elastoplastic Deformation (Cambridge International Science Publishing) p 380
[14] Glagolev V V, Glagolev L V and Markin A A 2015 Acta Mechanica Solida Sinica 28 pp 375–383
[15] Glagolev V V, Markin A A and Fursaev A A 2016 PNRP Mechanic Bulletin pp 34–44