Induced On-shell Supersymmetry in Eikonal Scattering

Parthasarathi Majumdar

The Institute of Mathematical Sciences,
CIT Campus, Madras 600 113, India.

Abstract

Generically coupled neutral scalar bosons and chiral fermions are shown, in the eikonal kinematical limit, to be described by a reduced (free field) theory with $N = 1$ on-shell supersymmetry. Charged scalars and spinors turn out to be described in the eikonal limit by a reduced interacting theory with a modified and restricted on-shell $N = 1$ supersymmetry. Consequences of such a symmetry for the nontrivial scattering amplitudes in this latter case are discussed.
Spacetime supersymmetry has two major uses in high energy theory: as a means of resolving the problems of naturalness and the stability of the gauge hierarchy in the Standard Model \[1\]; as a device to eliminate spacetime tachyons from the spectrum of string theories \[2\]. Yet, there is so far no compelling experimental evidence at all of this symmetry, even as an approximate fundamental symmetry of nature.\[1]\ Nor are we any closer to finding a non-perturbative mechanism for supersymmetry breaking which would still lead to a naturally small cosmological constant for the real world. Recent advances in formal aspects of exactly supersymmetric field and string theories, related to electric-magnetic duality, have so far only a tenuous connection with reality. While prospects of a headway into strong coupling situations appear to be good, the dependence on unbroken supersymmetry seems crucial in the more interesting cases. Despite the unmistakeable beauty of the mathematical structures it embodies, spacetime supersymmetry remains an enigma.

Is it conceivable, however, that in nature supersymmetry is dynamically induced, rather than fundamental? In this letter we point out a possibility as to how a version of supersymmetry, realized as a transformation between physical scalar and spinor fields (with generic nonsupersymmetric Yukawa and scalar self couplings), must arise in a certain kinematical limit. Two major characteristics distinguish this version of supersymmetry from standard spacetime supersymmetry: first of all, it is an inevitable consequence of the kinematical restrictions imposed on a system whose parameters (masses and couplings) exhibit no intrinsic feature of spacetime supersymmetry; secondly, the transformation laws affect only matter fields leaving gauge bosons (including gravitons) inert by assumption. Consequently, this sort of supersymmetry has little to do with naturalness, the gauge hierarchy problem or tachyons. However, we shall argue that it will have non-trivial dynamical consequences, especially pertaining to small angle electromagnetic, and possibly gravitational, scattering

\[1\] The unification of gauge coupling constants inferred from LEP data can be neither uniquely attributed to, nor be taken as unambiguous evidence of, spacetime supersymmetry.
of bosons and fermions.

The kinematical limit in question is the so-called eikonal limit – the limit of arbitrarily large ratio of the squared center-of-mass energy \( (s) \) to the squared momentum transfer \( (t) \). Clearly, these are the kinematics of almost forward scattering with tiny scattering angles, well-studied in the context of electromagnetism \([3]\) and also including gravitation \([4] - [9]\), for scalar particles. In this rather singular limit, transverse photon/graviton exchange between scattering particles is severely suppressed; the amplitude is dominated by a semiclassical process induced by instantaneous classical shock wave gauge configurations, and becomes exactly computable. Corrections to the semiclassical approximation due to fluctuations around the shock wave become important only if one recedes from the eikonal limit. Now, within the kinematical restrictions of the eikonal, the large ratio of the transverse and longitudinal momentum scales allows a scaling of the longitudinal (lightcone) coordinates, relative to the transverse ones. In other words, one can transform to a very large momentum frame, and use the resulting scaling on fields to determine which of these participate in the scattering process \([7]\), and also the relevant interactions that survive such scaling. The resulting reduced theory describes particle scattering in the eikonal limit exactly, i.e., without further approximation. Other features of the reduced theory include the appearance of certain global symmetries absent in the original Lagrangian. These latter aspects will be our major concern in this paper.

As the simplest example of the appearance of such symmetries, consider a real scalar field with the standard action in Minkowski 4-space

\[
S = \int d^4x \left[ \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right],
\]

where, the potential \( V(\phi) \) might have a mass term \( \frac{1}{2} \mu^2 \phi^2 \) and higher order self-coupling terms. Following \([7]\), we perform the following scaling of the lightcone coordinates: \( x^\pm \to \xi x^\pm \), with \( x^\pm \equiv x^0 \pm x^3 \), and \( \xi \sim t/s \). The transverse coordinates \( \vec{x}_\perp \) remained unchanged under the scaling. Under this scaling, of course, the scalar field undergoes no change, while the lightcone derivatives do scale by \( \xi^{-1} \). Taking account of the scaling of the
integration measure, the net effect of these on the action is

\[ S \rightarrow \int d^4x \partial_+ \phi \partial_- \phi + O(\xi^2) . \]  

(2)

In the eikonal limit \( \xi \rightarrow 0 \), the action reduces to a free field action with only lightcone derivatives; apart from the obvious symmetry associated with the conservation of the number of \( \phi \) particles, this reduced theory is also invariant under \( \phi \rightarrow \phi + \text{constant} \), a symmetry absent in the original formulation. Of course the price to pay is the loss of manifest Lorentz invariance. In any event, it is clear that \( \phi \)-particles scatter in the forward direction with unit amplitude because of the decoupling seen above. This result is consistent with earlier assertions [3] that for scalar exchanges, the eikonal approximation is never dominant; the theory is rendered trivial in this approximation.

Generalization to a theory of self-interacting complex scalars is straightforward: the reduced action in the eikonal limit becomes

\[ S_{\text{red}} = \int d^4x \partial_+ \phi^* \partial_- \phi \]  

(3)

which continues to be invariant under global rotations \( \phi \rightarrow e^{i\theta} \phi \), just as the original action (assuming that the potential is a function only of \(|\phi|\)). In addition, the number of positive and negative charges are separately conserved, and the reduced action is also invariant under the shift of \( \phi \) by a constant. Thus, in this limit, the theory has a two (real) dimensional space of vacua; one of the flat directions can be identified as a Goldstone direction if the potential \( V(|\phi|) \) of the theory before scaling has a minimum away from the origin, and the other (the Higgs) is a modulus. We shall return to this reduced theory later.

A similar symmetry enhancement takes place with the free massive Dirac theory in four dimensions as well;

\[ S = \int d^4x \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi . \]  

(4)

Under the above scaling of the lightcone coordinates, the spinor field \( \psi \rightarrow \xi^{-\frac{i}{2}} \psi \), so that

\[ S \rightarrow \int d^4x i\bar{\psi} \gamma_+ \partial_- \psi + O(\xi) , \]  

(5)
implying that in the eikonal limit $\xi \to 0$ one is left with a massless theory with only lightcone derivatives, and the (global) chiral symmetry under $\psi \to e^{i\gamma_5\theta}\psi$. Of course, for the free theory the symmetry simply ensures the separate conservation of left handed and right handed fermion number.

Now consider the most general theory of a Dirac spinor and a complex scalar field, with canonical kinetic energy terms,

$$S = \int d^4 x \left[ |\partial_{\mu}\phi|^2 - V(|\phi|) + \bar{\psi}(i\gamma^\mu\partial_{\mu} - H(\phi))\psi \right]. \quad (6)$$

The functions $V(\phi)$ and $H(\phi)$ are assumed to be completely arbitrary and mutually independent, unlike in an intrinsically supersymmetric theory where they are both derived from a holomorphic superpotential.

Under the scaling of the scalar and spinor fields induced by the scaling of the lightcone coordinates in the passage to the large momentum frame, this action reduces to

$$S_{\text{red}} = \int d^4 x \left[ \partial_{(+}\phi^*\partial_{(-)}\phi + \bar{\psi}\gamma_{(+}\partial_{(-)}\psi \right], \quad (7)$$

which is once again a free field theory action. However, $S_{\text{red}}$ has an additional symmetry property: the Lagrangian density changes by a total lightcone derivative

$$\delta L_{\text{red}} = \partial_{(+}\left(\bar{\epsilon}\partial_{(-)}\psi \phi \right) \quad (8)$$

under the transformations

$$\delta \phi = \bar{\epsilon}\psi \ , \ \delta \psi = -i\gamma_{(+}\partial_{(-)}\phi\epsilon \ , \quad (9)$$

where $\epsilon$ is a spacetime-independent spinorial parameter. While reminiscent of standard spacetime supersymmetry, the glaring absence of any auxiliary field reminds us of the on-shell nature of this version. Furthermore, irrespective of its symmetries, a free theory is intrinsically of extremely limited interest.

We now explore the possibility of realizing such a symmetry with less trivial consequences. To this end, we endow the fields with electric charge and couple them minimally
to electromagnetism. The behaviour of the Maxwell action under the scaling of lightcone coordinates is well-known \[7\], \[3\]

\[ S_{Max} = \int d^4x \left[ \frac{1}{\xi^2} F_{+-}^2 + F_{ai}^2 + O(\xi^2) \right], \quad (10) \]

where, \(\alpha = \pm\). Now, in the eikonal limit \(\xi \to 0\) the first term explodes, so that the dominant contribution to the partition function comes from gauge field configurations for which \(F_{+-} = 0\) which implies that \(A_\pm = \partial_\pm \Omega\). Thus, only the second term survives in the eikonal limit. The matter action, modified to include the coupling to electromagnetism, reduces in this limit to

\[ S_{red} = S_0 + S_{int}, \quad (11) \]

where, \(S_0\) is the reduced action given in eq. \(\[7\] and

\[ S_{int} = \int d^4x \left[ ieA_+ (\phi^* \partial_- \phi - c.c. - \bar{\psi} \gamma_- \psi) + e^2 A_+ A_- |\phi|^2 \right]. \quad (12) \]

All interactions of the transverse gauge potentials \(A_i, i = 1, 2\) drop out in the \(\xi \to 0\) limit, thereby rendering it a free field not warranting further consideration. We should point out a crucial assumption: both scalar and spinor fields have the same electric charge \(e\). This is imperative for the symmetry considerations to follow. Note, however, that, because of \(S_{int}\) the particles no longer forward scatter with unit amplitude in the eikonal domain.

Because we prefer not to introduce new fields like the gaugino, we must assume that whatever symmetry transformations the matter fields are subjected to must leave the gauge fields invariant. These assumptions suffice to demonstrate that, after some arithmetic, under the transformations

\[ \delta \phi = \bar{\epsilon} \psi; \quad \delta \psi = i\gamma_+(D_-)\phi \epsilon \quad (13) \]

the Lagrangian density in \(S_{red}\) transforms into

\[ \delta \mathcal{L}_{red} \sim \partial_+ \left[ \bar{\epsilon} (D_- \psi) \phi^* + c.c. \right], \quad (14) \]
where $D_\pm$ is the $U(1)$ covariant lightcone derivative

$$D_\pm \phi \equiv \partial_\pm \phi - ieA_\pm \phi.$$ 

In arriving at the result in eq. (14), we have made repeated use of the commutator $[D_+, D_-] \sim F_{+-} = 0$ on all fields. Observe that the transformation of the fermion fields is non-linear, involving both the gauge potential and the scalar. This is yet again a departure from standard super-transformation laws. Thus, it is not possible to interpret the transformations as being the ‘square root’ of spacetime translations. The question of interest is, however, whether this symmetry imposes non-trivial restrictions on boson and fermion scattering amplitudes, in analogy with restrictions on helicity-flip amplitudes obtained by Grisaru and Pendleton [11] more than two decades ago within standard globally supersymmetric 4d field theories. Such restrictions, if any, would be all the more predictive here given that the original theory (11) has no supersymmetry to begin with.

The simplest way to visualize the scattering processes we have in mind is to use the electromagnetic shock wave description [4], [7], [3]. Thus, for two-particle scattering, one chooses a Lorentz frame in which one of the particles is moving almost luminally, carrying with it a plane-fronted electromagnetic shock wave with the (infinitely extended) shock plane transverse to the direction of propagation. The fields due to this particle vanish everywhere except on the plane where they have a $\delta$ function singularity. Consequently, a test particle, slow-moving relative to our chosen frame, experiences no force except when the shock front passes it. The resulting phase factor induced instantaneously by the shock wave in the wave function of the test particle manifests in a non-trivial scattering amplitude (calculated as an overlap of the test particle wave functions before and after the impact [4]). In the center-of-mass frame of the particles, the amplitude can be calculated directly by a path integral approach [7], [3], and amounts to determining the amplitude for the elastic collision of two shock wave fronts in the eikonal kinematics. The issue of our concern here is whether the scattering amplitudes for elastic eikonal collisions of bosons are related to those of fermions.

To this end it suffices to restrict the matter field theory (11) to on-shell field configura-
tions, i.e., solutions to the free equations of motion obtained from varying the action $S_0$; it also makes sense to invoke the classical eikonal approximation, also referred to as the ‘geometrical optics’ approximation [12]. In this approximation, the fields, expressed as complex-valued functions, have phases which vary extremely rapidly relative to the variation of the moduli. E.g., the complex scalar field $\phi$ has the polar decomposition $\phi(x) = \rho(x) e^{i\theta(x)}$; in the eikonal approximation, we make the approximation $\partial_\pm \rho \ll \rho \partial_\pm \theta$, so that the modulus $\rho = \rho(\vec{x}_\perp)$. Thus,

$$S_0 \approx \int d^4x \left[ \rho^2 \partial_+ \theta \partial_- \theta + i\bar{\psi} \gamma_+ \partial_- \psi \right]$$

(15)

The corresponding equations of motions are

$$\rho^2 \partial_+ \partial_- \theta = 0$$

(16)

$$\rho \partial_+ \theta \partial_- \theta = 0$$

(17)

$$\partial_- \left[ \gamma_- \gamma_+ \psi \right] = 0 = \partial_+ \left[ \gamma_+ \gamma_- \psi \right] .$$

(18)

Since $\rho \neq 0$, eq. (14) has the solution

$$\theta(x^\pm, \vec{x}_\perp) = \theta^{(+)}(x^+, \vec{x}_\perp) + \theta^{(-)}(x^-, \vec{x}_\perp) .$$

(19)

Eq. (17) is simply the on-shell constraint for the (massless) scalar particles. As for eq. (18), defining $\chi^{(\pm)} \equiv \gamma^\pm \gamma^\mp \psi$, one has the solutions

$$\chi^{(+)} = \chi^{(+)}(x^+, \vec{x}_\perp) , \quad \chi^{(-)} = \chi^{(-)}(x^-, \vec{x}_\perp) .$$

(20)

Using these solutions, the interacting part of the action assumes the form

$$S_{\text{int}} = e \int \left[ A_- \left( i\rho^2 \partial_+ \theta^{(+)} - \bar{\chi}^{(+)} \gamma_+ \chi^{(+)} \right) \right]
+ e \int \left[ A_+ \left( - i\rho^2 \partial_- \theta^{(-)} - \bar{\chi}^{(-)} \gamma_- \chi^{(-)} \right) \right]
+ e^2 \int \rho^2 A_+ A_- ,$$

(21)

where $\int \equiv \int d^4x$. Thus, the first two lines represent the usual $j \cdot A$ type of gauge interaction, with
\[ j_{\pm} = j_{B,\pm} + j_{F,\pm} , \]  

where,

\[ j_{B,\pm} = \pm i \rho^2 \partial_\pm \theta^{(\pm)} \equiv \partial_\pm k_B^{(\pm)}(x^\pm, \vec{x}_\pm) \]
\[ j_{F,\pm} = - \bar{\chi}^{(\pm)} \gamma_\pm \chi^{(\pm)} \equiv \partial_\pm k_F^{(\pm)} . \]  

The first line of eq. (23) defines \( k_B \) in the same manner as in ref. [3]. For the spinor current, we have appealed to ‘bosonization’ of the fermionic field considered as functions only on the null plane; this does not entail any loss of generality since in the kinematical region of interest, derivatives with respect to the transverse coordinates do not appear in the interaction Lagrangian. Thus \( k_F^{(\pm)}(x^\pm, \vec{x}_\perp) \) is a Lorentz scalar. Both currents of course are conserved via eq.s (19) and (20).

Observe also that the ‘transverse’ components \( A_i \) have decoupled from matter in our kinematic regime, and can therefore be set to zero without loss of generality. Recall now that \( A_\pm = \partial_\pm \Omega \) as a consequence of the constraint \( F_{+-} = 0 \). Thus, imposing the Lorentz-Landau gauge condition [3] on the gauge potential implies

\[ \partial_+ \partial_- \Omega = 0 \]  

so that, \( \Omega = \Omega^{(+)}(x^+, \vec{x}_\perp) + \Omega^{(-)}(x^-, \vec{x}_\perp) \). With these simplifications, both the Maxwell and the interaction Lagrangians can be expressed as total derivatives on the null plane:

\[ S_{Max} = \int \left[ \partial_- \left( \Omega^{(-)} \nabla^2_\perp \partial_+ \Omega^{(+)} \right) + \left( + \leftrightarrow - \right) \right] \]  

\[ S_{int} = e \int \left[ \partial_- \left\{ j_{B,+} + j_{F,+} \right\} \Omega^{(-)} \right] + \left( + \leftrightarrow - \right) \]  
\[ + \frac{1}{2} e^2 \int \left[ \partial_+ \left( \rho^2 \Omega^{(+)} \partial_- \Omega^{(-)} \right) + \left( + \leftrightarrow - \right) \right] . \]  

The action \( S = S_{Max} + S_{int} \) thus reduces to a field theory ‘living’ on the three dimensional space composed by the transverse plane and the boundary of the null plane. Parametrising the latter by \( \tau \) and indicating the \( \tau \)-derivative by an overdot, eq. (26) can be recast into [3].
\[ S_{\text{Max}} = \int d^2 \vec{x}_\perp \oint d\tau \left[ \Omega l(-) \nabla^2 \Omega^{(+)} \right], \quad (27) \]

where, the bar on \( \Omega \) indicates that it is evaluated on the boundary of the null plane. Similarly,

\[ S_{\text{int}} = \int d^2 \vec{x}_\perp \oint d\tau \left[ \left( k_B^{(+)} + k_F^{(+)} \right) \bar{\Omega}^{(-)} + \left( + \leftrightarrow - \right) \right] \]

\[ + e^2 \int d^2 \vec{x}_\perp \oint d\tau \partial_\tau \left( \rho^2 \bar{\Omega}^{(+)\bar{\Omega}^{(-)}} \right) \quad (28) \]

The seagull term drops out upon integration over \( \tau \), assuming that the gauge degrees of freedom are single-valued on the boundary of the null plane. Note that, in addition to the well-known interaction of the bosonic current \[3\], the action above includes the fermionic current. Thus, scattering amplitudes may be computed from the expectation values of the three vertex operators

\[ V_{BB} = \exp \left[ i \oint d\tau \int d^2 \vec{x}_\perp \left( \dot{k}_B^{(+)} \bar{\Omega}^{(-)} + \dot{k}_B^{(-)} \bar{\Omega}^{(+)} \right) \right] \]

\[ V_{FF} = \exp \left[ i \oint d\tau \int d^2 \vec{x}_\perp \left( \dot{k}_F^{(+)} \bar{\Omega}^{(-)} + \dot{k}_F^{(-)} \bar{\Omega}^{(+)} \right) \right] \]

\[ V_{BF} = \exp \left[ i \oint d\tau \int d^2 \vec{x}_\perp \left( \dot{k}_B^{(+)} \bar{\Omega}^{(-)} + \dot{k}_F^{(-)} \bar{\Omega}^{(+)} \right) \right]. \quad (29) \]

The essential difference between the vertex operators in eq. (29) stem from the difference between \( k_B \) and \( k_F \). Indeed, as functions of the basic fields they are quite distinct. However, the distinction blurs when two point-particle scattering in the eikonal limit is considered. The main reason for this, in the shock wave picture, has to do with the fact that, the restriction of the electric and magnetic fields to the transverse shock plane implies that helicity-flip effects are absent in the eikonal regime. The shock wave impinging upon the test particle does not ‘see’ its spin. Another way of seeing this is to appeal to the so-called Gordon decomposition of the spinor current and observe that the Pauli term cannot contribute in the eikonal approximation. The Dirac term, on the other hand, for almost luminal particles will reduce to a form which is almost identical to the current for an ultrarelativistic point boson. We should point out that these somewhat heuristic considerations hold only for point particles; their validity beyond that is not claimed. In any event, the net upshot is that the amplitudes
where,

\[ k^{(\pm)}(x^{(\pm)}(\tau), \vec{x}_\perp) \equiv e \Theta \left( x^{(\pm)}(\tau) - x^{(A)\pm} \right) \delta^{(2)}(\vec{x}_\perp - \vec{x}^{(A)}) \] (31)

with \( A = 1, 2 \) for two-particle scattering, and \( \Theta \) is the unit step function. This result is the same as in ref. [3] for scalar particles.

The identity of the bosonic and fermionic amplitudes in eq. (30) has rather remarkable ramifications. First of all, helicity-flip amplitudes vanish because of the behaviour of electromagnetism in the eikonal domain, consistent with the results of [11], as anticipated on the basis of the on-shell supersymmetry of the reduced action. Secondly, the fact that the Yukawa and scalar self-couplings drop out in the eikonal kinematics might have implications for pion-nucleon dynamics. Consider, e.g., the Gell Mann-Levy \( \sigma \) model [13], proposed decades ago as a model for PCAC and low energy theorems in pion-nucleon interactions. It is amusing to examine this model in the eikonal regime for pion-nucleon, pion-pion and proton-proton elastic scattering processes. The action has a chiral \( SU(2) \times SU(2) \) symmetry in the absence of nucleon masses; it is given by

\[
S_\sigma = \int \left( \bar{N} i \gamma^\mu \partial_\mu N + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 \right) \\
- \int \left( \lambda \left( \sigma^2 + \vec{\pi}^2 - a^2 \right)^2 - \hbar \bar{N} \left( \sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi} \right) N \right),
\] (32)

where, \( N \) is the nucleon isodoublet, \( \vec{\pi} \) is the isotriplet of pions. The isospin symmetry is, of course, broken by the electromagnetic interactions which must be there between protons and charged pions, but this breaking is a tiny perturbation on the strong interactions that usually dominate pion-nucleon dynamics. However, our considerations above would imply that, in the eikonal regime, strong interactions, as depicted in the \( \sigma \) model, would effectively be suppressed, so that, in this kinematical region, electromagnetism should take over. Furthermore, small-angle elastic scattering of charged pions should have identical amplitudes as
those of protons. In other words, their behaviour in these kinematics should be extremely similar, to the extent that it is describable in terms of the $\sigma$ model. Perhaps an analysis of the data for elastic $p-p$, $\pi-\pi$ and $\pi-p$ scattering at $\sqrt{s} \gg 1\text{Gev}$ and $t \to 0$ (almost-forward scattering) is in order to test the validity of these ‘predictions’.

Finally, the induced supersymmetry discerned above in eikonal scattering through electromagnetism might reappear for gravitational scattering of light point particles in Minkowski space. If so, it ought to find application in the analysis of Hawking radiation from black holes, taking into account the interaction of the outgoing radiation with collapsing matter, recalling that \cite{14}, \cite{15} such interactions typically involve large centre-of-mass momenta and small momentum transfers. The intriguing question to probe in this problem is how the supersymmetry disappears from the Hawking spectrum.

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