Microscopic cluster model of $\alpha + \alpha$ bremsstrahlung following a Siegert approach

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Abstract. We have recently developed a microscopic cluster model of nucleus-nucleus bremsstrahlung, which takes implicitly account of a part of the meson-exchange current effects, by applying an extension of the Siegert theorem valid for arbitrary photon energy. This model is applied to the $\alpha + \alpha$ bremsstrahlung. The differences between the bremsstrahlung cross sections obtained in the non-Siegert and Siegert approaches are displayed and discussed. The theoretical bremsstrahlung cross sections are compared with experimental measurements. Whereas the agreement with quite old experimental data is fair, a rather important discrepancy between the theoretical results and the more recent experimental data is noticed. Some paths to investigate this disagreement are proposed.

1. Introduction

The $\alpha + \alpha$ bremsstrahlung is an $\alpha + \alpha$ collision during which a part of the kinetic energy between the $\alpha$ particles is converted to a photon. In other words, this is a radiative transition between $\alpha + \alpha$ scattering states corresponding to different energies. This specific nucleus-nucleus bremsstrahlung is particularly interesting to be studied because on the one hand, the $\alpha + \alpha$ scattering states are well described by relatively simple cluster models and on the other hand, some experimental $\alpha + \alpha$ bremsstrahlung cross sections are available [1–4] making possible a comparison between theory and experiment.

While most of previous nucleus-nucleus bremsstrahlung studies neglect the effects of meson-exchange currents, we have recently proposed [5,6] to include them partially in an implicit way by using an extended version of the Siegert theorem [7]. Following this approach, a microscopic cluster model of bremsstrahlung has been developed and applied to the $\alpha + \alpha$ [5] and $\alpha + N$ [6] bremsstrahlung reactions. Recently, new measurements of $\alpha + \alpha$ bremsstrahlung cross sections have been performed for four beam energies [4]. We apply the model developed in [5] at these colliding energies and compare the theoretical and experimental results each other.

2. The $\alpha + \alpha$ bremsstrahlung cross sections

In the center-of-mass (c.m.) frame, two $\alpha$ particles collide with initial relative wave vector in the $z$ direction and relative energy $E_i$. After emission of a photon with energy $E_\gamma = \hbar c k_\gamma$, the nuclear system has relative wave vector $k_f$ in direction $\Omega_f = (\theta_f, \phi_f)$ and energy $E_f$ given by

$$E_f = E_i - E_\gamma,$$

(1)
up to small recoil corrections.

The multipole matrix elements are defined by

$$u_{\lambda\mu}^{\sigma}(\Omega_f) = 2\pi^{1/2}\alpha_{\lambda}^{\sigma} \sum_{l_i f_j} (i_\mu \lambda \mu | l_f \mu) (2l_f + 1)^{-1/2} Y_{l_f}^\mu(\Omega_f) e^{2i(\sigma l_f + \delta l_f)} \langle \psi_f^l \| M_{\lambda}^{\sigma} \| \psi_i^l \rangle$$  \hspace{1cm} (2)

where \( \sigma l_f \) and \( \delta l_f \) are the Coulomb and quasinuclear phase shifts, \( M_{\lambda}^{\sigma} \) are the electromagnetic transition multipole operators, \( \lambda \) is the order of the multipole, \( \mu \) is its component, \( \sigma = 0 \) or \( E \) corresponds to an electric multipole and \( \sigma = 1 \) or \( M \) corresponds to a magnetic multipole, \( \alpha_{\lambda}^{\sigma} \) is given by

$$\alpha_{\lambda}^{\sigma} = -\sqrt{\frac{2\pi(\lambda + 1)\lambda^{\sigma} \kappa_0^\lambda}{\lambda(2\lambda + 1)(2\lambda - 1)!!}}$$  \hspace{1cm} (3)

the reduced matrix elements are defined by the following convention

$$\langle \psi_f^{l_{m_f}} | M_{\lambda\mu}^{\sigma} | \psi_i^{l_{m_i}} \rangle = (i_\mu \lambda m_i \mu | l_f m_f) \langle \psi_f^l \| M_{\lambda}^{\sigma} \| \psi_i^l \rangle,$$  \hspace{1cm} (4)

and \( \psi_f^{l_{m_f}} \) are the partial waves of the unit-flux \( \alpha + \alpha \) scattering wave function at colliding energy \( E_c \) with \( c = i \) or \( f \). The form of the partial waves \( \psi_c^{l_{m_c}} \) in the microscopic cluster approach is discussed in section 4.

From the multipole matrix elements, the bremsstrahlung cross sections \( d\sigma/dE_\gamma \) can be evaluated,

$$\frac{d\sigma}{dE_\gamma} = \frac{E_\gamma k_f^2}{2\pi^2 \hbar c} \sum_{\lambda\mu} \int_0^\pi (2\lambda + 1)^{-1} |u_{\lambda\mu}(0) |^2 \sin \theta_f d\theta_f.$$  \hspace{1cm} (5)

Explicit formulas giving the angular differential bremsstrahlung cross sections from the multipole matrix elements \( u_{\lambda\mu}^{\sigma} \) can be found in [8]. Since the \( E2 \) transitions are dominant for the \( \alpha + \alpha \) bremsstrahlung, the sums in (5) are restricted to \( \sigma = E \) and \( \lambda = 2 \), here. The Siegert and non-Siegert forms of the electric transition multipole operators are defined in the next section.

In the experiments from [3,4], some \( \alpha + \alpha \) bremsstrahlung cross sections, integrated over the angles and some range of photon energies, are measured. They are defined from \( d\sigma/dE_\gamma \) by

$$\sigma = \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{d\sigma}{dE_\gamma} dE_\gamma,$$  \hspace{1cm} (6)

where \( [E_{\text{min}}, E_{\text{max}}] \) is the energy range of the detected photons.

3. Electric transition multipole operators

In the non-Siegert approach, the electric transition multipole operators are defined from the intrinsic nuclear current density \( J \) by

$$M_{\lambda\mu}^E = \sqrt{\frac{\lambda}{\lambda + 1}} \frac{(2\lambda + 1)!!}{k_\gamma^2 c} \int J \cdot A_{\lambda\mu}^E \, dr,$$  \hspace{1cm} (7)

where \( A_{\lambda\mu}^E \) is the electric multipole defined, in the Coulomb gauge, as [9]

$$A_{\lambda\mu}^E(r) = \frac{i}{k_\gamma \sqrt{\lambda(\lambda + 1)}} \left( k_\gamma^2 r + \nabla \frac{\partial}{\partial r} r \right) \phi_{\lambda\mu}(k_\gamma r)$$  \hspace{1cm} (8)

with

$$\phi_{\lambda\mu}(kr) = j_\lambda(kr)Y_{\lambda\mu}^0(\Omega)$$  \hspace{1cm} (9)
and \((r, \Omega)\) are the spherical coordinates of \(r\).

In the Siegert approach, the electric multipole is divided into a gradient term and a rest,

\[
A^{E}_{\lambda \mu}(r) = \frac{i}{k_c} \sqrt{\frac{\lambda}{\lambda + 1}} \nabla \phi_{\lambda \mu}(k_c r) + A^{'E}_{\lambda \mu}(r). \tag{10}
\]

The gradient term dominates at low photon energy. The Siegert electric transition multipole operators are defined by \([5]\)

\[
M^{E(S)}_{\lambda \mu} = \frac{(2\lambda + 1)!!}{k_c^\lambda} \int \left[ \phi_{\lambda \mu}(k_c r) \rho(r) + \frac{1}{c} \sqrt{\frac{\lambda}{\lambda + 1}} \mathbf{J} \cdot A^{'E}_{\lambda \mu} \right] d\mathbf{r}, \tag{11}
\]

where \(\rho(r)\) is the charge density. The label \((S)\) is added to recall that the extended Siegert theorem is used. At low photon energy, since the density-dependent term is dominant, the main effects of meson-exchange currents are implicitly included in the Siegert approach. The explicit forms of the electric transition multipole operators, which are considered here, can be found in \([5]\).

If consistent current and charge densities are considered and the exact scattering wave functions are used, the non-Siegert operators \((7)\) and the Siegert operators \((11)\) lead exactly to the same results. However, in microscopic cluster approaches, simplified versions of the current and charge densities and of the wave functions are considered to keep the complexity of the models reasonable. In this case, differences between Siegert and non-Siegert results occur and give some hint about the accuracy of these approximations.

4. Microscopic cluster approach

The microscopic description of the \(\alpha + \alpha\) system relies on the internal 8-body Schrödinger equation

\[
H \Psi = E_T \Psi, \tag{12}
\]

where \(H\) is the internal Hamiltonian, \(\Psi\) is the internal wave function, and \(E_T\) is the total energy of the system in the c.m. frame. The internal Hamiltonian \(H\) is given by

\[
H = \sum_{i=1}^{8} \frac{\mathbf{p}_i^2}{2m_N} + \sum_{i>j=1}^{8} v_{ij} - T_{c.m.}, \tag{13}
\]

where \(\mathbf{p}_i\) is the momentum of nucleon \(i\), \(m_N\) is the nucleon mass, \(v_{ij}\) is the two-body potential describing the interaction between nucleons \(i\) and \(j\), and \(T_{c.m.}\) is the center-of-mass (c.m.) kinetic energy. The nucleon-nucleon potential \(v_{ij}\) is divided into two parts: a nuclear part \(v^N_{ij}\) and a Coulomb part \(v^C_{ij}\). For the nuclear part, a central potential with spin-isospin dependence, namely the Minnesota potential \([10]\), is considered. This potential leads to a good description of the \(\alpha + \alpha\) elastic phase shifts by the microscopic cluster approach.

The internal wave function \(\Psi\) is expanded in partial waves \(\psi_{lm}\) which in turn expanded as a sum of Generator Coordinate Method (GCM) basis functions \(\Phi_{lm}(R_n)\),

\[
\psi_{lm} = \sum_n f_n^l \Phi_{lm}(R_n), \tag{14}
\]

where \(R_n\) are discrete values of the so-called generator coordinate. The GCM functions are given by

\[
\Phi_{lm}(R) = A \phi_{\alpha \phi} \int Y_l^m(\Omega_R) \Gamma(\rho - R) d\Omega_R, \tag{15}
\]
where \( \phi_{\alpha} \) is the internal \( \alpha \) wave function, \( \rho \) is the relative coordinate between the centers of mass of the two \( \alpha \) particles, \((R, \Omega, \rho)\) are the spherical coordinates of \( R \), and \( \Gamma(\rho - R) \) is a Gaussian centered at \( R \). The internal \( \alpha \) wave function \( \phi_{\alpha} \) is obtained by removing a c.m. factor from the Slater determinant describing the ground state of the \( \alpha \) cluster within the harmonic oscillator shell model. The same oscillator parameter is considered for the cluster wave function \( \phi_{\alpha} \) and the relative wave function \( \Gamma \). The GCM functions, defined by (15), have the advantage that they can be written, after multiplication by a c.m. factor, as integrals of Slater determinants, which makes rather easy the calculation of the matrix elements between these functions. However, the GCM functions are not able to reproduce the correct asymptotic behavior of a scattering wave function. This problem is solved by the Microscopic \( R \)-matrix Method (MRM) [11,12]: the configuration space is divided in two regions at the channel radius \( a \): an internal region \((\rho < a)\) where the partial wave functions are described by the GCM, i.e. by (14), and an external region \((\rho > a)\) where the antisymmetrization between nucleons is reduced to the symmetrization between the \( \alpha \) particles and the \( \alpha \alpha \) interaction is considered purely Coulombic. The wave functions are evaluated by solving the Bloch-Schrödinger equation, derived from (12), on the internal region and by requiring the continuity of the wave function at the channel radius \( a \). They are insensitive to the value of \( a \) if \( a \) is chosen large enough.

5. Results

In [5], some differential \( \alpha + \alpha \) bremsstrahlung cross sections as well as some integrated ones are calculated within the microscopic cluster model and compared with experimental data [1–3]. All partial waves are included up to \( l_i, l_f = 8 \). Coulomb effects from higher partial waves are approximately included following the approach developed in [8]. The theoretical and experimental bremsstrahlung cross sections are noted to be in good agreement. However, these experimental data are few and quite inaccurate. Recently, new measurements of integrated \( \alpha + \alpha \) bremsstrahlung cross sections have been performed for colliding energy \( E_i = 9.22, 10.90, 12.04, \) and 14.20 MeV [4]. The experimental error bars associated with these measurements are significantly smaller than the ones associated to the previous experimental data from [4]. In figure 1, the integrated bremsstrahlung cross sections \( \sigma \), defined by (6), are calculated in the microscopic cluster approach for these four colliding energies and compared with these recent experimental data. Needless to say, the photon-energy ranges that are considered in the calculations are the same as the experimental ones [13]. The number and the values of the generator coordinates, the value of the channel radius, and other computational details can be found in [5].

Except at \( E_i = 12.04 \) MeV, a large discrepancy is noted between the calculated and measured bremsstrahlung cross sections. There are several possible explanations to this discrepancy but further investigations are required to verify or reject them. On the theoretical side, the use of cluster basis functions (15) could be responsible for a part of the discrepancy between theory and experiment. However, the gap between the Siegert and non-Siegert bremsstrahlung cross sections, which is an indicator of the validity of the cluster approximation, is significantly smaller than the gap between the calculated and measured bremsstrahlung cross sections. It is possible that the Minnesota potential, which is designed to reproduce the elastic \( \alpha + \alpha \) phase shifts, is not able to reproduce the \( \alpha + \alpha \) bremsstrahlung cross sections. The development of \textit{ab initio} methods of nucleus-nucleus bremsstrahlung would be useful to determine if more realistic potentials lead to \( \alpha + \alpha \) bremsstrahlung cross sections in better agreement with the experimental data from [4]. However, even the description of the \( \alpha + \alpha \) elastic scattering is not yet reachable by \textit{ab initio} approaches. Finally, more experimental data for a larger range of colliding energies and for smaller photon-energy ranges would be interesting to understand the gap between experiment and theory at some colliding energies and the good agreement at some other ones.
Figure 1. The $\alpha + \alpha$ integrated bremsstrahlung cross sections $\sigma$ in the Siegert (full lines) and non-Siegert (dashed lines) approaches for different ranges of the photon energy $E_\gamma$. Experimental data from [4].

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