Noise robust intuitionistic fuzzy c-means clustering algorithm incorporating local information

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Abstract
The human brain magnetic resonance image (MRI) is always contaminated by noise and has uncertainty on the boundary between different tissues. These characteristics bring challenges to the human brain image segmentation. To handle these limitations, many variants of standard fuzzy c-means (FCM) algorithm have been proposed. Some methods attempt to incorporate the local spatial information in the standard FCM algorithm. However, they can't solve the problem of data uncertainty very well. And some other methods can handle the problem of data uncertainty, but they are sensitive to noise since it doesn't incorporate any local spatial information. In this paper, we propose a noise robust intuitionistic fuzzy c-means (NR-IFCM) algorithm, which can handle noise and uncertainty problems simultaneously. In order to process the human brain MRI with noise better, we introduce a noise robust intuitionistic fuzzy set (NR-IFS) which is noise robust in this NR-IFCM algorithm. Meanwhile, in order to handle the data uncertainty, we also introduce a new intuitionistic fuzzy factor to this NR-IFCM algorithm which combine the local gray-level and the spatial information together. A large number of experimental results on human brain MRI validate the effectiveness and the superiority of our proposed NR-IFCM algorithm.

1 | INTRODUCTION

Image segmentation is a classical problem in the field of image processing. It has been widely used in various industries, such as human brain image segmentation. The magnetic resonance image (MRI) of human brain has three brain tissues [1]: the gray matter (GM), the white matter (WM) and the cerebrospinal fluid (CSF). Due to the complex structure of human brain, the boundaries between different tissues are not clear. In order to solve this problem, many researchers have proposed various segmentation algorithms for MRI, such as the threshold-based methods [2], the map-guided methods [3], the convolutional neural networks (CNN) [4–8] methods, the Markov random field models [9], the clustering algorithms [10, 11] and so on. Among them, the CNN-based method is a new image segmentation algorithm emerging in recent years. In the field of the CNN, the densely connected convolutional networks (DenseNet) proposed by Huang [3] can handle the noise and the data uncertainty, but it must have a lot of training data. Therefore, this paper does not focus on CNN-based image segmentation algorithms.

The clustering algorithm is considered to be extremely efficient for MRI brain image segmentation. However, traditional clustering methods cannot process the uncertain data which
is in the intersection of two clusters well. In the past few years, according to the fuzzy set (FS) theory [12], Bezdek had presented the fuzzy c-means (FCM) clustering algorithm for handling the problem of data uncertainty. However, there are two main drawbacks for FCM: (1) It is sensitive to noise since it does not consider any spatial information; (2) it still cannot handle the uncertain data on boundary between different tissues completely.

In the past decades, in order to overcome the influence of noise, some effective fuzzy clustering algorithms were presented in [13, 15]. Chen et al. [13] presented two algorithms named the FCM_S1 and the FCM_S2. The mean filtered and the median filtered of images are computed in the FCM_S1 and the FCM_S2 algorithms, respectively, but the two algorithms may change the pixel value of the original image and lose the continuity from the standard FCM. Tripathy et al. [14] proposed the spatial intuitionistic fuzzy c-means (SIFCM) clustering algorithm which uses the spatial function to update the membership function. However, the SIFCM algorithm only updates the membership function. The updating of the membership function does not depend on the objective function, so it may lead to this algorithm not converging. The fuzzy local information c-means (FLICM) algorithm [15] includes a fuzzy local neighbourhood factor in the objective function of standard FCM algorithm. The factor incorporates the local spatial information [16–18], such as gray-level and spatial relationship, and does not require to choose any parameters. However, this algorithm spends more time than any other algorithms. To solve the problem of data uncertainty, the intuitionistic fuzzy set (IFS) theory which is the generalisation of fuzzy set theory is proposed by Atanassov [19]. According to the IFS theory, Xu et al. [20] proposed the intuitionistic fuzzy c-means (IFCM) algorithm which use the intuitionistic fuzzy distance to replace fuzzy distance. However, the algorithm is greatly sensitive to noise and cannot handle the data certainty.

Based on the works mentioned above, the improved intuitionistic fuzzy c-means (IIFCM) algorithm which incorporates the local spatial information in IFCM was presented by Verma et al. [21].

### Table 1: The corresponding parameter setting

| Algorithm    | Input parameters | Proposed by          |
|--------------|------------------|----------------------|
| NR-IFCM      | \(m = 2, T = 300, \varepsilon = 10^{-5}, \alpha = 0.5, \gamma = 0.3, N_R = 9\) | this paper           |
| FCM_S1       | \(m = 2, T = 300, \varepsilon = 10^{-5}, \beta = 6, N_R = 9\) | Chen et al. [13]     |
| FCM_S2       | \(m = 2, T = 300, \varepsilon = 10^{-5}, \beta = 6, N_R = 9\) | Chen et al. [13]     |
| FLICM        | \(m = 2, T = 300, \varepsilon = 10^{-5}, N_R = 9\) | Krinidis et al. [15] |
| FCM          | \(m = 2, T = 300, \varepsilon = 10^{-5}, \lambda = 1.5\) | Xu et al. [20]       |
| IIFCM        | \(m = 2, T = 300, \varepsilon = 10^{-5}, \lambda = 1.5, N_R = 9\) | Verma et al. [21]    |
| DenseNet     | \(kernel = 5 \times 5, layer = 5\) | Huang et al. [4]     |
et al. [21]. In this algorithm, a new intuitionistic fuzzy factor has been introduced into fuzzy clustering to overcome the influence of noise and process the problem of data uncertainty. However, there are still two main drawbacks of the IIFCM algorithm: (1) The IFS theory is the theoretical basis for this algorithm while the IFS is also sensitive to noise. It is necessary to adopt more effective fuzzy theory in the IIFCM; (2) this algorithm still cannot solve the data uncertainty. Except that, Monalisa et al. [22] proposed a robust intuitionistic fuzzy c-means (RIFCM) clustering algorithm. Similar with the IIFCM, the RIFCM also has the disadvantages of noise sensitivity and data uncertainty.

In order to overcome the two main problems mentioned above, a noise robust intuitionistic fuzzy c-means (NR-IFCM) clustering algorithm is proposed in this paper. In order to handle the noise of MRI better, we apply a noise robust intuitionistic fuzzy set (NR-IFS) [23] to the human brain MRI in the proposed NR-IFCM clustering algorithm. The NR-IFS representation is obtained by a majority dominated suppressed similarity measure which uses the neighbourhood statistics and the competitive learning. Compared with the IFS, the NR-IFS has favourable noise immunity. In addition, to overcome the problem of data uncertainty, we also present a new intuitionistic fuzzy factor that can combine the local gray-level and the spatial information together in the proposed NR-IFCM clustering algorithm. Due to the shorter the distance of an element from the cluster center, the more likely it is to belong to the cluster. The spatial information can more accurately distinguish different parts of the image. So, we add the spatial information into the novel intuitionistic fuzzy factor. Different from the intuitionistic fuzzy factor in [22], we integrate the spatial information into the similarity function and get a more compact intuitionistic fuzzy factor. So the running time of NR-IFCM can be reduced. After all, to evaluate the performance of our proposed NR-IFCM algorithm, we give some experiments on the brain images and compared the results with other six existing segmentation methods. A large number of experimental results
can validate the effectiveness and the superiority of our proposed NR-IFCM algorithm.

The rest of this paper is organized as follows: Section II presents a brief introduction of the IFCM. Section III shows the proposed NR-IFCM method. And then, the NR-IFCM method is verified by experiments on the simulated brain images in Section IV. Finally, Section V gives some concluding remarks and discussions.

2.1 Intuitionistic fuzzy set

The fuzzy set proposed by Zadeh [12] is defined on a finite set $X = \{x_1, x_2, ..., x_n\}$ as $S = \{ (x, \mu_S(x)) | x \in X \}$, where the function $\mu_S(x) : X \rightarrow [0, 1]$ denotes the membership degree of an element $x$ in the set $S$. However, fuzzy sets cannot deal with the uncertainty of the data well. In real situations, because the membership degree defined may has error with the real membership degree, another uncertainty will arise while defining the membership degree. To overcome this problem, Atanassov [19] proposed the IFS theory which introduces the non-membership degree and the hesitation degree on the basis of fuzzy sets. By adding the new attribute parameters, the fuzzy nature of the images can be described more delicately. The IFS $S$ in the finite set $X$ can be denoted as:

$$S = \{ (x, \mu_S(x), \nu_S(x), \pi_S(x)) | x \in X \},$$  (1)

where $\mu_S(x)$, $\nu_S(x)$, and $\pi_S(x)$ are the membership, the non-membership, and the hesitation degrees of the element $x$ in the set $S$, respectively, and it holds the condition $\mu_S(x) + \nu_S(x) + \pi_S(x) = 1$. The non-membership degree can be calculated by the fuzzy complement functions. There are two of the most common intuitionistic fuzzy complement functions [24–26]. The first intuitionistic fuzzy complement function [24] is shown as:

$$\nu_S(x) = N_1(\mu_S(x)) = (1 - \mu_S(x)) / (1 + \lambda \mu_S(x)),$$

$$-1 < \lambda < \infty.$$  (2)

The second intuitionistic fuzzy complement function [25, 26] is given by:

$$\nu_S(x) = N_2(\mu_S(x)) = (1 - \mu_S(x)^w)^w, \quad -1 < w < \infty.$$  (3)

As the lack of knowledge in defining the membership degree, the hesitation degree is defined as:

$$\pi_S(x) = 1 - \mu_S(x) - \nu_S(x), \quad 0 \leq \pi_S(x) \leq 1.$$  (4)

If $\pi_S(x) = 0$, we have $\mu_S(x) + \nu_S(x) = 1$, the IFS is decayed to the FS. As the existence of the hesitation degree, the membership degree lies in the interval $[\mu_S(x), \mu_S(x) + \pi_S(x)]$ or $[\mu_S(x), 1 - \nu_S(x)]$. 

2 INTUITIONISTIC FUZZY c-MEANS CLUSTERING ALGORITHM

In this section, we will review the IFS, and introduce the IFCM clustering algorithm.
### TABLE 2
Comparative performance ($V_{pe}, V_{pc}, DB$) of different segmentation algorithms on simulated brain images with different INU and noise levels

| INU     | Performance measure | Noise   | NR-IFCM | FCM_S1 | FCM_S2 | FLICM | IFCM | IIFCM | DenseNet |
|---------|---------------------|---------|---------|--------|--------|-------|------|-------|----------|
| INU=0   | $V_{pc}$            | 0%      | 0.9233  | 0.8794 | 0.8949 | 0.6219 | 0.8942| 0.9045| 0.8549   |
|         |                     | 1%      | 0.9230  | 0.8793 | 0.8945 | 0.6229 | 0.8943| 0.9052| 0.8545   |
|         |                     | 3%      | 0.9139  | 0.8676 | 0.8807 | 0.6204 | 0.8799| 0.8970| 0.8407   |
|         | $V_{pe}$            | 0%      | 0.1557  | 0.2380 | 0.2080 | 0.7330 | 0.2080| 0.1868| 0.2180   |
|         |                     | 1%      | 0.1564  | 0.2387 | 0.2093 | 0.7314 | 0.2084| 0.1857| 0.2193   |
|         |                     | 3%      | 0.1775  | 0.2651 | 0.2403 | 0.7356 | 0.2412| 0.2031| 0.2104   |
|         | $DB$                | 0%      | 0.3215  | 0.3307 | 0.3236 | 0.7312 | 0.3239| 0.3781| 0.3636   |
|         |                     | 1%      | 0.3255  | 0.3335 | 0.3270 | 0.7353 | 0.3273| 0.3786| 0.3271   |
|         |                     | 3%      | 0.3639  | 0.3712 | 0.3666 | 0.7277 | 0.3652| 0.4136| 0.3566   |
| INU=20  | $V_{pc}$            | 0%      | 0.9198  | 0.8745 | 0.8894 | 0.6208 | 0.8890| 0.9010| 0.8694   |
|         |                     | 1%      | 0.9199  | 0.8746 | 0.8892 | 0.6215 | 0.8894| 0.9020| 0.8492   |
|         |                     | 3%      | 0.9165  | 0.8716 | 0.8850 | 0.6213 | 0.8833| 0.8986| 0.8450   |
|         | $V_{pe}$            | 0%      | 0.1641  | 0.2492 | 0.2208 | 0.7348 | 0.2196| 0.1924| 0.2008   |
|         |                     | 1%      | 0.1641  | 0.2495 | 0.2215 | 0.7337 | 0.2193| 0.1908| 0.2027   |
|         |                     | 3%      | 0.1721  | 0.2570 | 0.2317 | 0.7341 | 0.2345| 0.2007| 0.2020   |
|         | $DB$                | 0%      | 0.3390  | 0.3457 | 0.3399 | 0.7230 | 0.3397| 0.3978| 0.3699   |
|         |                     | 1%      | 0.3409  | 0.3474 | 0.3420 | 0.7269 | 0.3412| 0.3987| 0.3920   |
|         |                     | 3%      | 0.3570  | 0.3643 | 0.3604 | 0.7394 | 0.3586| 0.3993| 0.3594   |

### TABLE 3
Comparative performance ($\rho, r_{fp}, r_{fn}$) of different segmentation algorithms on simulated brain images with different INU and noise levels for GM

| INU     | Performance measure | Noise   | NR-IFCM | FCM_S1 | FCM_S2 | FLICM | IFCM | IIFCM | DenseNet |
|---------|---------------------|---------|---------|--------|--------|-------|------|-------|----------|
| INU=0   | $\rho$              | 0%      | 0.9977  | 0.9836 | 0.9917 | 0.2580 | 0.9795| 0.9127| 0.9495   |
|         |                     | 1%      | 0.9882  | 0.9812 | 0.9841 | 0.2651 | 0.9781| 0.5221| 0.9494   |
|         |                     | 3%      | 0.9664  | 0.9669 | 0.9676 | 0.2971 | 0.9606| 0.9016| 0.9306   |
|         | $r_{fp}$            | 0%      | 0.0024  | 0.0147 | 0.0037 | 0.8455 | 0.0124| 0.1047| 0.1124   |
|         |                     | 1%      | 0.0085  | 0.0182 | 0.0109 | 0.8414 | 0.0111| 0.6351| 0.1117   |
|         |                     | 3%      | 0.0345  | 0.0340 | 0.0296 | 0.8200 | 0.0320| 0.1242| 0.0311   |
|         | $r_{fn}$            | 0%      | 0.0103  | 0.0181 | 0.0130 | 0.0433 | 0.0290| 0.0666| 0.0256   |
|         |                     | 1%      | 0.0152  | 0.0195 | 0.0210 | 0.0381 | 0.0332| 0.0329| 0.0242   |
|         |                     | 3%      | 0.0327  | 0.0321 | 0.0354 | 0.0313 | 0.0475| 0.0670| 0.0243   |
| INU=20  | $\rho$              | 0%      | 0.9732  | 0.4829 | 0.9699 | 0.2781 | 0.9678| 0.9043| 0.9255   |
|         |                     | 1%      | 0.9697  | 0.9632 | 0.9676 | 0.2871 | 0.9671| 0.9029| 0.9245   |
|         |                     | 3%      | 0.9560  | 0.9535 | 0.9573 | 0.3148 | 0.9538| 0.8921| 0.9333   |
|         | $r_{fp}$            | 0%      | 0.0243  | 0.0137 | 0.0238 | 0.8323 | 0.0176| 0.1199| 0.0171   |
|         |                     | 1%      | 0.0147  | 0.0322 | 0.0248 | 0.8269 | 0.0171| 0.1224| 0.0169   |
|         |                     | 3%      | 0.0453  | 0.0462 | 0.0411 | 0.8086 | 0.0403| 0.1423| 0.0314   |
|         | $r_{fn}$            | 0%      | 0.0295  | 2.0988 | 0.0368 | 0.0382 | 0.0479| 0.0664| 0.0352   |
|         |                     | 1%      | 0.0363  | 0.0418 | 0.0405 | 0.0399 | 0.0497| 0.0664| 0.0412   |
|         |                     | 3%      | 0.0426  | 0.0469 | 0.0444 | 0.0448 | 0.0527| 0.0652| 0.0447   |
### TABLE 4  Comparative performance ($\rho, r_{fp}, r_{fn}$) of different segmentation algorithms on simulated brain images with different INU and noise levels for WM

| INU   | Performance measure | Noise | NR-IFCM  | FCM_S1  | FCM_S2  | FLICM  | IFCM  | IIFCM  | DenseNet |
|-------|---------------------|-------|----------|----------|----------|--------|-------|--------|-----------|
|       |                     | 0%    | 0.9998   | 0.9932   | 0.9962   | 0.7380 | 0.9948 | 0.9578 | 0.9248    |
|       |                     | 1%    | 0.9964   | 0.9942   | 0.5303   | 0.7393 | 0.9932 | 0.5221 | 0.9211    |
|       |                     | 3%    | 0.9897   | 0.5252   | 0.5248   | 0.7455 | 0.9878 | 0.5201 | 0.9221    |
|       | $r_{fp}$            | 0%    | 0.0000   | 0.0079   | 0.0059   | 0.0000 | 0.0000 | 0.0000 | 0.0059    |
|       |                     | 1%    | 0.0047   | 0.0060   | 0.6385   | 0.0000 | 0.0010 | 0.6351 | 0.0316    |
|       |                     | 3%    | 0.0107   | 0.6417   | 0.6425   | 0.0101 | 0.0146 | 0.6322 | 0.0415    |
|       | $r_{fp}$            | 0%    | 0.0016   | 0.0057   | 0.0017   | 0.7102 | 0.0104 | 0.0881 | 0.1077    |
|       |                     | 1%    | 0.0026   | 0.0056   | 0.0017   | 0.7054 | 0.0034 | 0.0329 | 0.1057    |
|       |                     | 3%    | 0.0106   | 0.0062   | 0.0057   | 0.6828 | 0.0098 | 0.0381 | 0.1260    |
| INU   |                     | 0%    | 0.9917   | 0.5227   | 0.5237   | 0.7416 | 0.9899 | 0.5204 | 0.9217    |
|       |                     | 1%    | 0.9966   | 0.9866   | 0.9899   | 0.7432 | 0.9897 | 0.5200 | 0.9129    |
|       |                     | 3%    | 0.9865   | 0.5195   | 0.5204   | 0.7497 | 0.9857 | 0.5170 | 0.9214    |
|       | $r_{fp}$            | 0%    | 0.0191   | 0.6447   | 0.6447   | 0.0546 | 0.0147 | 0.6353 | 0.0367    |
|       |                     | 1%    | 0.0111   | 0.0128   | 0.0124   | 0.0177 | 0.0153 | 0.6354 | 0.0474    |
|       |                     | 3%    | 0.0131   | 0.6467   | 0.6459   | 0.0257 | 0.0162 | 0.6362 | 0.0471    |
|       | $r_{fp}$            | 0%    | 0.0075   | 0.0042   | 0.0042   | 0.6969 | 0.0054 | 0.0368 | 0.0032    |
|       |                     | 1%    | 0.0076   | 0.0099   | 0.0076   | 0.6911 | 0.0083 | 0.0376 | 0.0026    |
|       |                     | 3%    | 0.0039   | 0.0069   | 0.0068   | 0.6676 | 0.0124 | 0.0435 | 0.0058    |

### TABLE 5  Comparative performance ($\rho, r_{fp}, r_{fn}$) of different segmentation algorithms on simulated brain images with different INU and noise levels for CSF

| INU   | Performance measure | Noise | NR-IFCM  | FCM_S1  | FCM_S2  | FLICM  | IFCM  | IIFCM  | DenseNet |
|-------|---------------------|-------|----------|----------|----------|--------|-------|--------|-----------|
|       |                     | 0%    | 0.9910   | 0.9767   | 0.9862   | 0.5586 | 0.9468 | 0.9109 | 0.9248    |
|       |                     | 1%    | 0.9838   | 0.9726   | 0.3172   | 0.2898 | 0.9516 | 0.9173 | 0.9146    |
|       |                     | 3%    | 0.9540   | 0.3377   | 0.3374   | 0.3099 | 0.9468 | 0.9174 | 0.9146    |
|       | $r_{fp}$            | 0%    | 0.0026   | 0.0189   | 0.0118   | 0.6111 | 0.0466 | 0.1060 | 0.0486    |
|       |                     | 1%    | 0.0209   | 0.0288   | 0.0160   | 0.4381 | 0.0487 | 0.1062 | 0.0478    |
|       |                     | 3%    | 0.0587   | 0.0000   | 0.0000   | 0.3920 | 0.0622 | 0.1176 | 0.0682    |
|       | $r_{fp}$            | 0%    | 0.0006   | 0.0278   | 0.0158   | 0.0035 | 0.0606 | 0.0678 | 0.0606    |
|       |                     | 1%    | 0.0115   | 0.0259   | 4.2209   | 2.3161 | 0.0481 | 0.0549 | 0.0441    |
|       |                     | 3%    | 0.0320   | 3.9718   | 3.9272   | 2.3161 | 0.0431 | 0.0414 | 0.0445    |
| INU   |                     | 0%    | 0.9830   | 0.9676   | 0.9762   | 0.2851 | 0.9509 | 0.9132 | 0.9069    |
|       |                     | 1%    | 0.9745   | 0.9651   | 0.9701   | 0.2951 | 0.9502 | 0.9219 | 0.9056    |
|       |                     | 3%    | 0.9524   | 0.9494   | 0.9530   | 0.3156 | 0.9506 | 0.9243 | 0.9146    |
|       | $r_{fp}$            | 0%    | 0.0092   | 0.0281   | 0.0195   | 0.4487 | 0.0436 | 0.1056 | 0.0486    |
|       |                     | 1%    | 0.0231   | 0.0356   | 0.0309   | 0.4261 | 0.0410 | 0.1051 | 0.0470    |
|       |                     | 3%    | 0.0563   | 0.0566   | 0.0561   | 0.3786 | 0.0558 | 0.1105 | 0.0501    |
|       | $r_{fp}$            | 0%    | 0.0250   | 0.0368   | 0.0283   | 2.3161 | 0.0553 | 0.0645 | 0.0535    |
|       |                     | 1%    | 0.0280   | 0.0341   | 0.0288   | 2.3161 | 0.0405 | 0.0466 | 0.0450    |
|       |                     | 3%    | 0.0311   | 0.0440   | 0.0370   | 2.3161 | 0.0424 | 0.0351 | 0.0442    |
2.2 | IFCM clustering algorithm

The IFCM clustering algorithm [17] has the advantages of intuitionistic fuzzy set, and can handle the problem of data uncertainty. It describes the relationship of the center of the cluster and the elements in the cluster in more details. So the IFCM can get more precise segmented results than the FCM. The objective function of the IFCM is shown as:

\[ J_{IFCM} = \sum_{i=1}^{K} \sum_{j=1}^{N} (\eta_{ij})^m d^2(c_{IFS}^i, x_{IFS}^j). \]  

The optimisation problem of the IFCM is given as:

\[ \min \sum_{i=1}^{K} \sum_{j=1}^{N} (\eta_{ij})^m d^2(c_{IFS}^i, x_{IFS}^j) \]

s.t. \( \sum_{i=1}^{K} \eta_{ij} = 1, \quad 1 \leq j \leq N \),

where \( X_{IFS} = (x_{IFS}^1, x_{IFS}^2, \ldots, x_{IFS}^N) \) denotes a data set with \( N \) intuitionistic fuzzy data, \( m (1 < m < \infty) \) is the fuzzifier coefficient, \( K (1 < K < N) \) is the number of clusters of the data, \( C_{IFS} = (c_{IFS}^1, c_{IFS}^2, \ldots, c_{IFS}^K) \) are the centroids of cluster, and \( c_{IFS}^i = (\mu(c_i), v(c_i), \pi(c_i)) \) and \( x_{IFS}^j = (\mu(x_j), v(x_j), \pi(x_j)) \) are the intuitionistic fuzzy representations of the center of cluster \( c_{IFS}^i \) and the element \( x_{IFS}^j \), respectively. The \( \eta_{ij} (0 \leq \eta_{ij} \leq 1) \) denotes the membership degree value of \( x_j \) belonging to \( i \)-th center of cluster. The term \( d^2(c_{IFS}^i, x_{IFS}^j) \) is the Euclidean intuitionistic fuzzy distance [27] between \( C_{IFS} \) and \( X_{IFS} \), and it is defined as follows:

\[ d^2(c_{IFS}^i, x_{IFS}^j) = \left\| \mu(c_i) - \mu(x_j) \right\|^2 + \left\| v(c_i) - v(x_j) \right\|^2 \]

\[ + \left\| \pi(c_i) - \pi(x_j) \right\|^2. \]

By using the Lagrangian multiplier method, the membership degree function \( \eta_{ij} \) [19] and the cluster center \( c_{IFS}^i \) [19] are updated as follows:

\[ \eta_{ij} = \frac{1}{\sum_{i=1}^{K} \left( d^2(c_{IFS}^i, x_{IFS}^j) \right), \quad 1 \leq i \leq K, 1 \leq j \leq N \]
Comparative performance ($\rho, r_{fp}, r_{fn}$) of different segmentation algorithms on simulated brain images with different INU and noise levels for GM

\[ i_{i}^{\text{IFS}} = (\mu(c_i), u(c_i), \pi(c_i)) \]

\[ i_{i}^{\text{IFS}} = \left( \sum_{j=1}^{N} \mu_{ij} \mu(x_j), \sum_{j=1}^{N} \mu_{ij} u(x_j), \sum_{j=1}^{N} \mu_{ij} \pi(x_j) \right) \]

\[ 1 \leq i \leq K. \]

3 | THE PROPOSED ALGORITHM

The IFCM algorithm introduced above has two main drawbacks: (1) The membership degree of the IFCM is just about the distance between the gray value and the cluster center. Pixels which close to the centroid have a high degree of membership and pixels which far from the centroid have a low degree of membership. Therefore, the degree of membership is sensitive to the noise; (2) in the MRI, the boundaries between different brain tissues are always ambiguous. However, the objective function of the IFCM algorithm does not incorporates any local spatial information, and it deals each pixel as a separate point. So it is easy to bring about data uncertainty.

Many improved algorithms take only one problem into account, but our proposed algorithm can solve both problems simultaneously. In the following two subsections, we will describe our proposed the NR-IFCM clustering algorithm in detail.

3.1 | NR-IFS based intuitionistic fuzzification of images

The NR-IFS, which uses the neighbourhood statistics and the competitive learning, was proposed by Zhao et al. [23]. In NR-IFS theory, a NR-IFS on a finite $X = \{x_1, x_2, \ldots, x_n\}$ can be defined as:

\[ A = \left\{ (x, \mu_A(x), u_A(x), \pi_A(x)) \mid x \in X \right\}, \]

where $\mu_A(x)$, $u_A(x)$, and $\pi_A(x)$ denote the membership degree, non-membership degree, and hesitation of a grey value $x$ in the set $X$. The NR-IFS theory also proposed novel functions of the membership and the non-membership degrees.

Firstly, the membership degree $\mu_A(x_j)$ [23] of the $j$-th pixel is written as:

\[ \mu_A(x_j) = \left[ \sum_{k \in N_j} w_{jk} \cdot x_k \right] / 255, \]

where $w_{jk} = \delta y_{jk} / \sum_{r \in N_j} \delta y_{jr}$, $N_j$ is a set of the pixels which are in a neighbouring window centered at the $j$-th pixel, and $\delta y_{jk}$ is a suppressed similarity measure between the $j$-th pixel and the $k$-th pixel in $N_j$. The gate symbol means rounding. The suppressed similarity measure uses the advantages of competitive learning and golden ratio [28–30] to calculate the relationship
among pixels in the neighbouring window. The detail is given in Algorithm 1.

Then, the non-membership degree \( \nu_A(x_j) \) [23] is calculated as follows:

\[
\nu_A(x_j) = \begin{cases} 
N_3(\mu_A(x_j)) \\
1 - \frac{1-\gamma}{\gamma} \mu_A(x_j), & 0 \leq \mu_A(x_j) \leq \gamma \\
\frac{\gamma}{1-\gamma} \mu_A(x_j), & \gamma \leq \mu_A(x_j) \leq 1,
\end{cases}
\]

where \( \gamma \) (0 \leq \gamma \leq 0.5) is the fixed point of the fuzzy complement function. Compared with the classical complement functions in Equations (2) and (3), it is piecewise linear and the parameter \( \gamma \) can be easily determined.

Finally, the hesitation degree \( \pi_A(x_j) \) [23] of the \( j \)-th can be obtained as follows:

\[
\pi_A(x_j) = 1 - \mu_A(x_j) - \nu_A(x_j).
\]

### 3.2 The NR-IFCM clustering algorithm

In this subsection, we propose a novel NR-IFCM clustering algorithm for image segmentation. This algorithm integrates the superiority of the IFCM, the FLICM and the NR-IFS. In addition, it also introduces a novel intuitionistic fuzzy factor \( H_j \).

**Algorithm 1** Majority dominated suppressed similarity measure

1. Let \( X = x_1, x_2, x_3, \ldots, x_n \) denote an image, \( N_j \) denote the neighbouring window centered at \( x_j \), and it is defined as:

\[
1N_j = \{x'_k | k = 1, 2, \ldots, N_R - 1\},
\]

where \( N_R \) denotes the cardinality of the neighbouring window \( N_j \).

2. Sort the pixels in \( N_j \) in ascending order based on gray level values. Then, select the middle \([0.618 \times (N_R - 1)]\) pixels with the minimum variance in \( N_j \) to constitute the reward subset \( N_R^p \). Finally, use the other pixels in \( N_j \) to compose the punishment subset \( N_R^p \).

3. Compute the majority dominated suppressed similarity \( SS_{jk} \), its formula is written as follows:

\[
SS_{jk} = \begin{cases} 
(1 + \alpha)S_{jk}, & \text{if } k \in N_R^p \\
(1 - \alpha)S_{jk}, & \text{if } k \in N_R^p,
\end{cases}
\]

where \( \alpha \) is the suppressed factor. \( S_{jk} \) is the local gray-level similarity between the \( j \)-th pixel and the \( k \)-th pixel in \( N_j \). It can be written as follows:

\[
S_{jk} = \exp(-\|x_j - x'_k\|^2/\sigma^2),
\]

where \( \sigma_j = \sqrt{\sum_{p \in N_j} \|x_j - x'_p\|^2/N_R} \) is the scale factor of the spread of the gray-level similarity in the local window \( N_j \).
The proposed algorithm solves the problem of noise by utilizing the advantages of intuitionistic fuzzy set theory. It can also handle the uncertainty caused by the complex boundaries in the medical images by introducing an intuitionistic fuzzy factor incorporating the local spatial information. The objective function of the NR-IFCM is written as:

\[ J_m(C_{NIS}, X_{NIS}) = \frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{N} (u_{ij})^m d^2(c_{NIS}^i, x_{NIS}^j)H_{ij} \]  \hspace{1cm} (14)

The non-linear optimisation problem of the NR-IFCM is defined as:

\[ \min \sum_{i=1}^{K} \sum_{j=1}^{N} (u_{ij})^m d^2(c_{NIS}^i, x_{NIS}^j)H_{ij} \]

s.t. \[ \sum_{i=1}^{K} u_{ij} = 1, \quad 1 \leq j \leq N \]  \hspace{1cm} (15)

where \( X_{NIS} = (x_{NIS}^1, x_{NIS}^2, ..., x_{NIS}^N) \) is a dataset with noise robust intuitionistic fuzzy data, \( m (m \geq 1) \) is the fuzzifier constant, \( K (1 < K < N) \) is the number of clusters of the data, \( C_{NIS} = (c_{NIS}^1, c_{NIS}^2, ..., c_{NIS}^K) \) is the centroids of cluster, and \( c_{NIS}^i = (\mu(c_i), v(c_i), \pi(c_i)) \) and \( x_{NIS}^j = (\mu(x_j), v(x_j), \pi(x_j)) \) are the noise robust intuitionistic fuzzy representations of \( K \)-th center of cluster and the element \( x_j \) in \( X \), respectively. The \( u_{ij} (0 \leq u_{ij} \leq 1) \) denotes the membership degree value of \( x_j \) belonging to \( i \)-th center of cluster. The term \( d^2(c_{NIS}^i, x_{NIS}^j) \) is the Euclidean intuitionistic fuzzy distance between \( C_{NIS}^i \) and \( X_{NIS}^j \), and it is defined as:

\[ d^2(c_{NIS}^i, x_{NIS}^j) = \left\| \left( \mu(c_i) - \mu(x_j) \right)^2 + \left( v(c_i) - v(x_j) \right)^2 + \left( \pi(c_i) - \pi(x_j) \right)^2 \right\| \]  \hspace{1cm} (16)

The intuitionistic fuzzy factor \( H_{ij} \) is determined by sliding a fixed size window across every level in the image. This intuitionistic fuzzy factor includes the local gray-level information and spatial information by considering the similarity measures and membership degree, whose function is to control the balance between the membership degree and similarity measure and to maximize the membership degree and similarity measure into a specified local window, and it is written as follows:

\[ H_{ij} = \frac{1}{N_B} \sum_{l \in N_j} [(1 - u_{il})^m + (U_{il})^m] \]

s.t. \[ 1 \leq i \leq K, 1 \leq j \leq N \]  \hspace{1cm} (17)
where $N_j$ represents the set of neighbouring pixels around the centered pixel $x_j$ and does not contain $x_j$, the $N_R$ denotes the cardinality of $N_j$, $u_{ij}$ represents the membership degree values of $i$-th pixel in $N_j$ to the $i$-th cluster center, and $m$ ($m \geq 1$) is a fuzzifier constant, and $US_i$ which is the similarity function between $i$-th pixel in $N_j$ to the $i$-th cluster can be defined as:

$$US_i = 1 - \frac{1}{4} \left( \sqrt{\left\| \mu(c_i) - \mu(x_j) \right\|^2 + \left\| \nu(c_i) - \nu(x_j) \right\|^2} + \frac{m^2 d^2(c_{i}^{N_S}, x_{j}^{N_S}) H_{ij}}{2} \right), \quad 1 \leq i \leq K, 1 \leq j \leq N_R. \quad (18)$$

In this paper, we adopt the Lagrangian multiplier method to solve the optimisation problem Equation (15). Its Lagrangian function is defined as:

$$J^*_m(C^{N_S}, X^{N_S}, \lambda) = \sum_{i=1}^{K} \sum_{j=1}^{N} (u_{ij})^m d^2(c_{i}^{N_S}, x_{j}^{N_S}) H_{ij} - \sum_{j=1}^{N} \lambda_j \left( \sum_{i=1}^{K} u_{ij} - 1 \right) \quad (19),$$

where $\lambda_j$ denotes the Lagrangian constants. We calculate the partial derivatives of $J^*_m$ with respect to $u_{ij}$ and $\lambda_j$. Then we equate them zero. Finally we get the following formulations:

$$\frac{\partial J^*_m}{\partial u_{ij}} = mn_{ij} (\mu(c_i) - \mu(x_j))^2 H_{ij} = 0 \quad (20)$$

$$\frac{\partial J^*_m}{\partial \lambda_j} = \sum_{i=1}^{K} u_{ij} - 1 = 0.$$

After simplifying Equation (20), we can get

$$n_{ij} = \frac{1}{\sum_{i=1}^{K} \left( \frac{d^2(c_{i}^{N_S}, x_{j}^{N_S}) H_{ij}}{\sqrt[2]{d^2(c_{i}^{N_S}, x_{j}^{N_S}) H_{ij}}} \right)}, \quad 1 \leq i \leq K, 1 \leq j \leq N. \quad (21)$$

Similarly, we calculate the partial derivatives of $J^*_m$ with respect to $\mu(c_i), \nu(c_i)$ and $\pi(c_i)$, and equate them zero. Then we have:

$$\frac{\partial J^*_m}{\partial \mu(c_i)} = \sum_{j=1}^{N} n_{ij} (\mu(c_i) - \mu(x_j))^2 H_{ij} = 0 \quad (22)$$

$$\frac{\partial J^*_m}{\partial \nu(c_i)} = \sum_{j=1}^{N} n_{ij} (\nu(c_i) - \nu(x_j))^2 H_{ij} = 0 \quad (22)$$

$$\frac{\partial J^*_m}{\partial \pi(c_i)} = \sum_{j=1}^{N} n_{ij} (\pi(c_i) - \pi(x_j))^2 H_{ij} = 0.$$

With simplifying Equation (22), we have:

$$c_{ij}^{N_S} = (\mu(c_i), \nu(c_i), \pi(c_i))$$

$$= \begin{pmatrix} \frac{\sum_{j=1}^{N} u_{ij} \mu(x_j) H_{ij}}{\sum_{j=1}^{N} u_{ij} H_{ij}} & \frac{\sum_{j=1}^{N} u_{ij} \nu(x_j) H_{ij}}{\sum_{j=1}^{N} u_{ij} H_{ij}} & \frac{\sum_{j=1}^{N} u_{ij} \pi(x_j) H_{ij}}{\sum_{j=1}^{N} u_{ij} H_{ij}} \end{pmatrix}, \quad 1 \leq i \leq K. \quad (23)$$

## 4 EXPERIMENTS

In this section, some experiment were carried out to validate the performance of the proposed algorithm NR-IFCM. Firstly, we introduce the experimental dataset and evaluation index, and then, we show the experiential data and analyse the experimential results.

### 4.1 Preparation of the experiments

To validate the effectiveness of the proposed NR-IFCM algorithm, the experiment is carried out on some brain images. In this experiment, we choose other five existing methods as compared algorithm, for example, FCM_S1, FCM_S2, FLICM, IFCM, IIFCM and DenseNet. All of them are implemented in Python and performed on a computer with Intel Core i7-7700HQ CPU, 8G RAM and Windows 10.

The brain images can be obtained from Brain Web [31]. There are only 180 images with different intensities and proportions in this dataset. The brain images are slices of 3D brain data with size $217 \times 181$ and its segmented image consisting of CSF, GM and WM. The brain images are 2D axial view of T1-weighted and slice thickness 1mm with different intensity non-uniformity (INU=0 and INU=20) and noise level (0%, 1%, 3%). For a 0% level (INU=0), the multiplicative INU field has a value of 1.0 over the brain area. For a 20% level (INU=20), the multiplicative INU field has a range of values of [0.90,1.10] over the brain area. The INU fields were estimated from real MRI scans, so they are realistic. These fields are non-linear and slowly-varying fields of a complex shape. The Gaussian noise is native to the dataset. In this dataset, the challenge is that the dataset has few images and the boundaries of the brain tissue are complex.

During the experiment, the brain image is segmented into four classes: the gray matter (GM), the white matter (WM), the cerebrospinal fluid (CSF) and the background. The background part is not considered in the performance measures. The segmentation results of the images are compared in terms of partition coefficient $V_p$ [32], partition entropy $V_e$ [33], Davies–Bouldin index $DB$ [34], similarity index $\rho$ [35], false negative ratio $r_{fn}$ [35] and false positive ratio $r_{fp}$ [35]. These performance
measures are respectively calculated as:

\[ V_{pc} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{K} \rho_{ij}^2}{N}, \]  
\[ V_{pe} = \frac{-\sum_{i=1}^{N} \sum_{j=1}^{K} \rho_{ij} \log(\rho_{ij})}{N}, \]

where \( V_{pc} \) and \( V_{pe} \) are the greatly representative functions for fuzzy partition. The algorithm with maximising \( V_{pc} \) or minimising \( V_{pe} \) is considered as better than others.

\[ DB = \frac{1}{K} \sum_{i=1}^{K} \max_{j \neq i} \left( \frac{\sigma_i + \sigma_j}{d(c_i, c_j)} \right), \]

where \( \sigma_\chi \) is the mean distance between the cluster centroid \( c_i \) and all the elements in the cluster with cluster centroid \( c_j \). The \( d(c_i, c_j) \) denotes the distance between the cluster centroid \( c_i \) and \( c_j \). Algorithm with the lower DB index leads to better clustering.

\[ \rho = \frac{|X_i \cap Y_j|}{|X_i| + |Y_j|}, \]
\[ r_{fn} = \frac{|X_i| - |X_i \cap Y_j|}{|X_i|}, \]
\[ r_{fp} = \frac{|Y_j| - |X_i \cap Y_j|}{|Y_j|}, \]

where \( X_i \) denotes the pixels set of the original image and \( Y_j \) denotes the pixels set of experimental segmented image. \(|X_i|\) denotes the cardinality of \( X_i \). It is worth mentioning that the better the segmentation results have the higher similarity index \( \rho \), the lower false negative ratio \( r_{fn} \) and the lower false positive ratio \( r_{fp} \).

### 4.2 Results on the brain images corrupted by different levels of noise

To compare the denoising performance of the proposed NR-IFCM algorithm with FCM_S1, FCM_S2, FLICM, IFCM, IIFCM and DenseNet algorithms, we apply these algorithms on the brain images with different intensity non-uniformity (INU = 0 and INU = 20) and noise levels (0%, 1%, 3%). In the experimental setup, the parameters are from [23]. We fixed the fuzzifier constant \( m = 2 \), the maximum number of iterations \( T = 300 \), the minimum deviation \( \varepsilon = 10^{-5} \) and the window size \( 3 \times 3 \) (i.e. \( N_{rl} = 9 \)). We set the parameter \( \beta = 6 \) in FCM_S1 and FCM_S2 algorithms after searching optimal value in interval [0, 10] with increment 0.2. We also conduct an experiment with different values of \( \lambda \), and find that segmentation result is better for \( \lambda = 1.5 \) in IFCM and IIFCM. By many experiments, we observe that it is the best segmentation result in NR-IFCM when the parameter \( \alpha = 0.5 \) and \( \gamma = 0.3 \), and the parameters of the DenseNet are consistent with [4]. The respective parameters of all the methods in our experiments are presented in Table 1.

The brain images are slices of 3D brain data with different intensity non-uniformity (INU = 0 and INU = 20) and noise levels (0%, 1%, 3%). Figures 1–9 shows the segmentation results. Figure 1 shows the original images and the correctly segmented GM, WM and CSF images. Figures 2–9 visually show the segmented GM, WM and CSF images of the brain image by FCM_S1, FCM_S2, FLICM, IFCM, IIFCM, DenseNet and NR-IFCM algorithms, respectively. As can be seen from Figures 2–9, the WM images of our proposed NR-IFCM are more complete than FCM_S1, FCM_S2, FLICM, IFCM, IIFCM and DenseNet algorithms. In summary, compared with other six algorithms, it can be seen from these figures that our proposed NR-IFCM algorithm has clearer boundaries.

To evaluate the performance of the proposed NR-IFCM algorithm quantitatively, we give their performances in terms of six evaluation indexes. The comparison results in terms of partition coefficient \( V_{pc} \), partition entropy \( V_{pe} \), Davies–Bouldin index \( DB \), similarity index \( \rho \), false negative ratio \( r_{fn} \) and false positive ratio \( r_{fp} \) are shown in Tables 2–5. The algorithm with maximising \( V_{pc} \), minimising \( V_{pe} \) and minimising DB is considered as better than others. The better segmentation results have higher similarity index \( \rho \), lower false negative ratio \( r_{fn} \) and lower false positive ratio \( r_{fp} \). We also draw line charts corresponding to the tables 2–5, which are shown in Figures 10–13, respectively. From Table 2 and Figure 10, it can be seen that the partition coefficient \( V_{pc} \) of NR-IFCM is bigger than that of other existing algorithms, and the partition entropy \( V_{pe} \) and the Davies–Bouldin index \( DB \) of NR-IFCM are smaller than those of other existing algorithms. The \( V_{pc} \) and \( DB \) can achieve average improvement of 7%, 6% and 2%. From Tables 3–5 and Figures 11–13, it can be seen that the similarity index \( \rho \) of NR-IFCM is bigger than that of other algorithms for GM, WM and CSF, and the false negative ratio \( r_{fn} \) and false positive ratio \( r_{fp} \) of NR-IFCM are minimum for most of the images. \( \rho \), \( r_{fn} \) and \( r_{fp} \) can achieve average improvement of 2%, 3% and 1%. Above all, our proposed NR-IFCM has better performances in terms of the six evaluation indexes and can handle the noise and data certainty at the same time. Therefore, our proposed NR-IFCM can get better performance than FCM_S1, FCM_S2, FLICM, IFCM, IIFCM and DenseNet algorithms.

### 5 Conclusion

This paper proposes an NR-IFCM algorithm, which can process the human brain MRI with noise better and solve the problem of boundary uncertainty. In this proposed algorithm, we apply the NR-IFS to get better noise immunity. In addition, we also propose a new intuitionistic fuzzy factor to handle boundary uncertainty. We perform experiments on MRI brain images using our proposed algorithm and some compared algorithms, and compare their segmentation performances in terms of partition coefficient, partition entropy, Davies–Bouldin index,
similarity index, false negative ratio and false positive ratio. The experimental results show that the performances of our proposed NR-IFCM algorithm are significantly better than the compared algorithms. In our future work, we will try to use the deep learning, which involves the U-Net neural network to study the image segmentation algorithm. We will design a new network architecture based on the U-Net.

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