Theoretical investigation and experimental support for the cavitation bubble dynamics near a spherical particle based on Weiss theorem and Kelvin impulse

Xiaoyu Wang\textsuperscript{a}, Guanhao Wu\textsuperscript{a}, Xiaoxiao Zheng\textsuperscript{a}, Xuan Du\textsuperscript{a}, Yuning Zhang\textsuperscript{a,\textdagger}, Yuning Zhang\textsuperscript{b,\textdagger}\textasteriskcentered

\textsuperscript{a} Key Laboratory of Power Station Energy Transfer Conversion and System (Ministry of Education), School of Energy Power and Mechanical Engineering, North China Electric Power University, Beijing 102206, China
\textsuperscript{b} College of Mechanical and Transportation Engineering, China University of Petroleum-Beijing, Beijing 102249, China
\textsuperscript{c} Beijing Key Laboratory of Process Fluid Filtration and Separation, China University of Petroleum-Beijing, Beijing 102249, China

\textbf{ARTICLE INFO}

\textbf{Keywords:}
Cavitation bubble dynamics  
Weiss theorem  
Kelvin impulse  
High-speed photography  
Particle-bubble interaction

\textbf{ABSTRACT}

In the present paper, the laser-induced cavitation bubble dynamics near a fixed spherical particle is comprehensively investigated based on the Weiss theorem, the Kelvin impulse theory and the high-speed photography experiment. Firstly, the applicability range of the theoretical model in the time and the space is statistically obtained based on sufficient experimental results. Then, the in-depth theoretical analysis is carried out in terms of the liquid flow field and the bubble Kelvin impulse with the corresponding experimental results as the reasonable support. In addition, the theoretical prediction model of the bubble movement is established and experimentally fitted from the analytic expression of the Kelvin impulse. Through our research, it is found that: (1) the applicability range of the Kelvin impulse theory for the bubble near the spherical particle is approximately the dimensionless distance between the bubble and particle ($\gamma$) greater than 0.50. (2) The effect of the particle on the liquid velocity between the bubble and the particle is mainly manifested in the form of the image bubble, which always causes the liquid velocity in this region to be significantly lower than other surrounding regions. (3) The average movement velocity of the bubble centroid can be reasonably predicted by establishing a directly proportional function between the Kelvin impulse and the velocity with the relationship constant ($\alpha$) equal to $3.57 \times 10^{-6} \pm 1.63 \times 10^{-7}$ kg.

\textbf{1. Introduction}

In the process of the hydroelectric power generation, the abrasive erosion caused by cavitation bubbles and sand particles is a serious threat to the safety of the power equipment and pipelines [1–3]. Generally speaking, the combined effect of the frequent particle impact and the cavitation collapsing jet would induce more severe damage to the wall compared with the cavitation erosion in the pure water [1]. However, the results could be reversed for the relatively close distance between particles and bubbles. For this situation, the interaction between particles and cavitation bubbles is dominant. And the presence of particles could significantly affect the bubble dynamic behaviors including the non-spherical deformation [4], the micro jet [5], the translational movement and so on. In addition, this situation is also widespread in the fields of ultrasonic cleaning [6,7], chemical processing [8–10] and biomedicine [11,12]. Therefore, in order to reveal the specific effect of particles on cavitation bubbles, the single bubble dynamic behaviors near a spherical particle and the flow field around the cavitation bubble are investigated in depth from both theoretical and experimental aspects in the present paper.

Firstly, in the theoretical research on the interaction between walls and bubbles, the Kelvin impulse theory is a set of relatively systematic theory, which can be employed to explain and predict the bubble dynamic behaviors (e.g. bubble movement, jet direction and jet strength) near various walls. Here, a brief review of the research progress on the Kelvin impulse theory will be presented. Initially, Benjamin and Ellis [13] applied the Kelvin impulse to cavitation bubble dynamics for the first time, and explained its rationality and possible application mathematically and physically. Subsequently, Blake et al. [14,15] established a strict theoretical system of the Kelvin impulse theory based on the first principle, and obtained the Kelvin impulse expressions for the cavitation dynamics near various walls. This is the so-called Kelvin impulse theory.
bubble near several simple walls (e.g. rigid boundary, free surface, two-fluid interface and membrane boundary). Specifically, in this theory, the effect of the cavitation bubble on its nearby liquid flow field is approximated as a point source with a time-varying intensity. Meanwhile, the effect of the walls on the liquid flow field is simplified by the corresponding method of images. And the influence of the wall on the cavitation bubble can be reflected by the velocity potential of the liquid around the bubble wall. Subsequently, Blake et al. [16–18] carried out the numerical analysis on the bubble dynamic behaviors near the rigid straight boundary and the free surface based on the boundary integration method. During the analysis, the bubble contour and the motion path of the cavitation bubble, as well as the pressure distribution of the surrounding liquid flow field were obtained numerically, which qualitatively verified the correctness of the Kelvin impulse prediction results near these walls. In addition, Blake [19] mentioned the possibility of using the Kelvin impulse theory to predict the non-spherical deformation of the bubble wall, and summarized the above research on the bubble behaviors near the walls. Based on the above research, Best and Blake [20] further obtained a simplified Kelvin impulse expression according to the unsteady Lagally theorem for deformable bodies [21]. Basically, the Lagally theorem allows to employ the velocity potential at the bubble centroid position to characterize that on the bubble wall. Therefore, the difficulty on the calculation of the Kelvin impulse is greatly reduced, and the Kelvin impulse for the cavitation bubble near the above walls can even be expressed analytically. Meanwhile, the application of the Lagally theorem also makes it possible to investigate the Kelvin impulse for the cavitation bubble near a spherical particle. And its numerical and analytical expressions are both obtained for the first time according to the Weiss theorem [22]. Recently, more scholars have combined the Kelvin impulse with the boundary integral method to predict the jet direction of the cavitation bubble at a corner based on the potential flow theory and the method of images. And Biju et al. [24] numerically studied the buoyancy-considering bubble located above the rigid boundary, proposed the “null final Kelvin impulse state”, and demonstrated two typical cases of jetting behavior near the state.

Moreover, in the experimental research, the microscopic interaction between the single particle and the single cavitation bubble has also been investigated. And the previous research can be divided into the particle behaviors and the bubble behaviors according to the difference of the main research object. For the research on the particle behaviors, the particle motion characteristics (e.g. movement direction [25–30], spin of the particle [31], and accelerate movement [32–34]) are focused on primarily. And several degrees of freedom are often given to the particle in different ways in the research. For the research on the bubble behaviors, the bubble dynamic characteristics (e.g. nucleation [35,36], non-spherical collapse [37], and jets of the cavitation bubble [38]) are mainly concerned near a fixed particle. In most of the research, the main analysis methods are only to qualitatively describe the bubble behaviors, and to statistically classify several typical features based on the sufficient data from the high-speed photography experiment.

However, although Best and Blake [20] combined the Weiss theorem with the Kelvin impulse theory to provide a basic theoretical model for analyzing the dynamic behavior of the cavitation bubble near the spherical particle for the first time, there is still a lack of comprehensive and profound theoretical analysis methods and corresponding examples at present. Specifically, they are not clear enough about the circumferential inhomogeneity of the liquid radial velocity around the cavitation bubble obtained by the superposition of the potential flow, the quantitative relationship between the Kelvin impulse strength and the bubble movement characteristics, and even the applicability range of

| Nomenclature |
| --- |
| **Roman letters** |
| \( d_i \) | the displacement of the bubble centroid (m) |
| \( e \) | the circularity of the cavitation bubble in the high-speed photography |
| \( e_c \) | the critical circularity of the cavitation bubble in the high-speed photography |
| \( F \) | the Bjerkness force for the cavitation bubble (kg m/s²) |
| \( I \) | the Kelvin impulse for the cavitation bubble (kg m/s) |
| \( l \) | the distance between the particle centroid and the initial centroid position of the cavitation bubble (m) |
| \( m \) | the intensity of the bubble point source (m³/s) |
| \( n \) | the unit vector perpendicular to the bubble wall and pointing to the bubble interior |
| \( p_0 \) | the ambient pressure (Pa) |
| \( p_r \) | the first derivative of \( R \) with respect to the time (m/s) |
| \( R \) | the instantaneous radius of the cavitation bubble (m) |
| \( R_0 \) | the equilibrium bubble radius (m) |
| \( R_{max} \) | the maximum bubble radius during the first bubble oscillation period (m) |
| \( R_p \) | the radius of the spherical particle (m) |
| \( r \) | the position coordinate in the liquid flow field outside the walls and the cavitation bubble (m) |
| \( r_0 \) | the initial centroid coordinate of the cavitation bubble |
| \( r_i \) | the initial centroid coordinate of the image cavitation bubble |
| \( S \) | the bubble surface |
| \( t \) | the time (µs) |
| \( t_0 \) | the moment corresponding to the first frame when the cavitation bubble is captured by the high-speed camera (µs) |
| \( t_c \) | the typical moment at the bubble collapse stage (µs) |
| \( t_g \) | the typical moment at the bubble growth stage (µs) |
| \( \Delta t \) | the minimum time interval between adjacent frames in the high-speed photography experiment (µs) |
| \( T \) | the first oscillation period of the cavitation bubble (µs) |
| \( u \) | the instantaneous liquid velocity (m/s) |
| \( u_l \) | the liquid velocity at the left endpoints of the bubble wall (m/s) |
| \( u_r \) | the liquid velocity at the right endpoints of the bubble wall (m/s) |
| \( \Delta u \) | the dimensionless difference between \( u_l \) and \( u_r \) |
| \( v_c \) | the instantaneous velocity of the bubble centroid (m/s) |
| \( v_c, avg \) | the average movement velocity of the bubble centroid during the first bubble oscillation period (m/s) |
| **Greek letters** |
| \( a \) | the relationship constant between \( I \) and \( v_c, avg \) (kg) |
| \( \alpha \) | the dimensionless distance between the right end of the particle and the initial centroid position of the bubble |
| \( \gamma \) | the polynoptic exponent |
| \( \kappa \) | the density of the liquid (kg/m³) |
| \( \rho \) | the surface tension coefficient (kg/m²) |
| \( \phi \) | the velocity potential of the liquid caused by the cavitation bubble in the liquid flow field without walls (m²/s) |
| \( \phi' \) | the additional velocity potential of the liquid generated by the walls near the cavitation bubble (m²/s) |
| \( \Phi \) | the velocity potential of the liquid (m²/s) |

Among them, Wang et al. [23] investigate the jet of the cavitation bubble at a corner based on the potential flow theory and the method of images. And Biju et al. [24] numerically studied the buoyancy-considering bubble located above the rigid boundary, proposed the “null final Kelvin impulse state”, and demonstrated two typical cases of jetting behavior near the state.
2. Theoretical model

In this section, the Kelvin impulse theory and the method of images for the cavitation bubble near a spherical particle according to the Weiss theorem [22] are concisely introduced. Firstly, several assumptions in these theoretical models are employed as follows: (1) The liquid is incompressible. (2) The liquid is a potential flow. (3) The effect of the cavitation bubble on the surrounding liquid is regarded as a point source with the time-varying and isotropic source intensity and the fixed position. (4) The effect of the particle is only reflected in the velocity potential of the liquid flow field. (5) The effect of the buoyancy on the cavitation bubble is ignored.

According to the definition of impulse, the Reynolds transmission theorem and the Bernoulli equation for unsteady flow in continuous medium, the Kelvin impulse for the cavitation bubble can be expressed as follows [14]:

\[ I = \int_{t_0}^{t_1} F \, dt \]  
(1)

where

\[ F = \rho \int \left( \frac{1}{2} \nabla \Phi^2 + (n \cdot \nabla \Phi) \nabla \right) dA \]  
(2)

\[ \Phi = \phi + \phi' \]  
(3)

Here, \( I \) means the Kelvin impulse for the cavitation bubble. \( T \) is the first oscillation period of the cavitation bubble. \( F \) is the Bjerkness force in Eq. (2). The effect of the particle is only reflected in the velocity potential of the liquid caused by the cavitation bubble in the liquid flow field without walls. And \( \phi' \) is the additional velocity potential of the liquid generated by the walls near the cavitation bubble.

In addition, according to the definition of the velocity potential, the liquid velocity can be expressed as follows [40]:

\[ u = \nabla \Phi \]  
(4)

Here, \( u \) represents the instantaneous liquid velocity.

Subsequently, according to the continuity equation and the unsteady Lagally theorem [21] for deformable bodies, the Bjerkness force in Eq. (2) can be simplified as follows [20]:

\[ F = -m \rho \nabla \Phi (r_0) \]  
(5)

where

\[ m = 4\pi R^2 \tilde{R} \]  
(6)

Here, \( m \) means the intensity of the bubble point source. \( R \) is the instantaneous radius of the cavitation bubble. And \( \tilde{R} \) is the first derivative of \( R \) with respect to \( t \).

Then, by combining Eqs. (1), (5) and (6), one can obtain the general expression of the Kelvin impulse for the cavitation bubble as follows [20]:

\[ I = \xi_0 A \int_{t_0}^{t_1} R^2 \tilde{R} \, dt \]  
(7)

where

\[ \Gamma = 4\nabla g(r_0) \]  
(8)

\[ g(r) = \frac{4\pi}{m} \]  
(9)

Here, \( r_0 \) is the initial centroid coordinates of the cavitation bubble. And \( r \) is the position coordinate in the liquid flow field outside the walls and the cavitation bubble.

Furthermore, for the fixed spherical particle model in the infinite liquid environment, the boundary condition of the liquid is that the

![Fig. 1. Schematic of the Weiss theorem [22] for the effect of a single spherical particle near the cavitation bubble on the surrounding liquid flow field. The solid circles on the left and right represent the particle and the cavitation bubble respectively. The blue dashed circle refers to the image cavitation bubble. And the red dashed line refers to the virtual sinks with a uniform linear distribution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)](image)
shows the schematic of the Weiss theorem [22] for the effect of a single axis of rotation obviously. For the convenience of understanding, Fig. 1 the Weiss theorem [22]. In general, the effect of the particle can be bubble near a spherical particle is introduced specifically according to the cavitation bubble are spherical particle near the cavitation bubble on the surrounding liquid characteristics of the system can be represented by any plane passing through the particle on the liquid field, the method of images for the cavitation normal velocity of the liquid on the particle surface is zero. In order to

Specific models and parameters of the main components in the experimental system.

| Components       | Models       | Details                                      |
|------------------|--------------|----------------------------------------------|
| Laser generator  | Nd: YAG      | Laser wavelength: 532 nm                     |
|                  |              | Laser energy range: 0 – 30 mJ                |
|                  |              | Laser diameter: 3 mm                         |
| High-speed camera| Phantom V1212| Shooting speed: 90,000 fps                    |
|                  |              | Scale: 256 x 256 pixels                      |
|                  |              | Image resolution: 0.02817 mm/pixel           |
| Water tank       | Customization| Material: Transparent acrylic sheet          |
|                  |              | Size: 100 mm × 100 mm × 100 mm               |

normal velocity of the liquid on the particle surface is zero. In order to satisfy the above boundary condition and obtain the explicit effect of the particle on the liquid field, the method of images for the cavitation bubble near a spherical particle is introduced specifically according to the Weiss theorem [22]. In general, the effect of the particle can be regarded as the combination of an image bubble and several virtual sinks with a uniform linear distribution, in order to satisfy the boundary conditions of the liquid on the surface of the particle surface. In addition, since the particle-bubble system is a body of revolution, the characteristics of the system can be represented by any plane passing through the axis of rotation obviously. For the convenience of understanding, Fig. 1 shows the schematic of the Weiss theorem [22] for the effect of a single spherical particle near the cavitation bubble on the surrounding liquid flow field. In Fig. 1, the two-dimensional Cartesian coordinate system is established with the particle centroid as the origin. The solid circles on the left and right represent the particle and the cavitation bubble respectively. The radius of the particle and the instantaneous radius of the cavitation bubble are \( R_p \) and \( R \) respectively. The blue dashed circle refers to the image cavitation bubble. The red dashed line refers to the virtual sinks with a uniform linear distribution. The initial centroid positions of the cavitation bubble and the image cavitation bubble are both located on the X-axis with \( r_0 \) and \( r_i \) as the position coordinates respectively.

Then, the velocity potential of the liquid flow field in the particle-bubble system can be obtained. According to the introduction of the Weiss theorem in Fig. 1, \( \phi \) and \( \phi' \) can be expressed as follows [20]:

\[
\phi = -\frac{m}{4\pi} \frac{1}{|r - r_0|} \tag{10}
\]

\[
\phi' = \frac{mR_p}{4\pi l} \frac{1}{|r - r_i|} + \frac{m}{4\pi R_p} \int_0^{\phi^{(r)}} \frac{ds}{|r - (s, 0)|} \tag{11}
\]

Here, \( R_p \) is the radius of the spherical particle. \( l \) is the distance between the particle centroid and the initial centroid position of the cavitation bubble. \( r_0 \) and \( r_i \) represent the initial centroid coordinates of the cavitation bubble and the image cavitation bubble with \( r_0 = (l, 0) \) and \( r_i = (R_p^2/l, 0) \) respectively. \( s \) represents the length integral variable. From Eq. (11), the intensity of the image cavitation bubble and the intensity density of the linear sinks are \( mR_p/l \) and \( -m/R_p \) respectively. For the specific proof process of \( \phi' \) based on the Weiss theorem, readers can refer to pages 496-497 of ref. [22].

Moreover, in order to close the theoretical model, the Rayleigh-Plesset equation [41] is adopted to calculate the instantaneous value of the point source strength in Eq. (6). Although this typical isotropic equation is best for describing the radial motion of the spherical bubble far from the wall, it is acceptable to employ the Rayleigh-Plesset equation to approximate the quasi-spherical bubble behaviors with slight anisotropy. Subsequently, the simplified Rayleigh-Plesset equation can be expressed as follows [41]:

\[
R\ddot{R} \frac{3}{2} R^2 = \frac{p_{in}(R,t) - p_0}{\rho} \tag{12}
\]

where

\[
p_{in}(R,t) = \left( p_0 + \frac{2\sigma}{R_p} \right) \frac{R_p^3}{R} - \frac{2\sigma}{R} \tag{13}
\]

Here, \( \ddot{R} \) is the second derivative of \( R \) with respect to \( t \). \( p_0 \) is the ambient pressure. \( \sigma \) is the surface tension coefficient. \( R_p \) is the equilibrium bubble radius. \( k \) is the polytropic exponent. By employing the ode45 with the adaptive step size for Eq. (12), \( R \) and \( \dot{R} \) in Eqs. (6) and (7) can be obtained numerically. Meanwhile, several important parameters for the numerical simulation are adopted as follows: \( p_0 = 1 \times 10^5 \) Pa, \( R_p = 1 \times 10^{-3} \) m, \( k = 1.4 \). \( \rho = 1 \times 10^3 \) kg/m³. \( \sigma = 7.25 \times 10^{-2} \) kg/s². Moreover, the time corresponding to the minimum \( R \) is defined as \( t = 0 \) in the analysis of the present paper.

Furthermore, for the framework of the theoretical model in the present paper, the dynamic behaviors of the non-spherical bubble can actually be employed. This can be achieved by employing the bubble wall motion equations with anisotropy to close the model. And even if the isotropic equation is employed, the superposition of the liquid velocity potential by the image bubble and the linear sinks based on the Weiss theorem can affect the distribution of the liquid equipotential surface around the bubble surface, which also can reflect the non-spherical properties of the bubble to some extent.

3. Experimental setup

In this section, the experimental system based on the high-speed photography and related parameters for investigating the interaction between the single particle and the single bubble are described in detail. In addition, the typical high-speed photographs are also completely exhibited and introduced.

Fig. 2 shows the brief description of the experimental system for the bubble-particle interaction. In Fig. 2, it can be seen that the experimental system mainly consists of a high-speed camera, a laser generator, a digital delay generator, a computer, an experimental water tank, a light source and a focusing lens. As shown in Fig. 2, the experimental principle and process can be described specifically as follows: Initially, the water tank is filled with the deionized water. And the spherical particle is fixed by a slender needle which can be moved and controlled precisely. In addition, the laser generator outside the tank excites the cavitation bubble through the focus lens. Furthermore, the digital delay generator is controlled by the computer, which is employed to synchronize the high-speed camera, the laser generator and the light source.
And the whole behaviors of the bubble are recorded by the high-speed camera and the images are preserved in the computer.

Subsequently, Table 1 shows the specific models and parameters of the main components in the experimental system corresponding to Fig. 2.

Then, Fig. 3 shows the definitions of the main parameters in the experiment. In Fig. 3, \( R_{\text{max}} \) represents the maximum bubble radius during the first bubble oscillation period. Moreover, for the convenience of the subsequent analysis, a dimensionless parameter is defined as follows:

\[
\gamma = \frac{l - R_p}{R_{\text{max}}} 
\]

Here, \( \gamma \) means the dimensionless distance between the right end of the particle and the initial centroid position of the bubble. In addition, the experiment, the value of the particle radius is consistent with that in the theoretical analysis. And the maximum bubble radius is selected as \( R_{\text{max}} = 1.15 \times 10^{-3} \text{ m} \).

Afterwards, Fig. 4 shows the high-speed photographs of the bubble dynamics near a spherical particle for the typical distance between the particle and the cavitation bubble with \( \gamma = 1.10 \). In Fig. 4, several representative frames are selected and arranged in chronological order to demonstrate the bubble behaviors during the first two periods of the bubble oscillations. To be specific, the serial numbers and the moments corresponding to each frame are placed on the left and the right side of the frames respectively. Among them, \( t_0 \) refers to the moment corresponding to the first frame when the cavitation bubble is captured by the high-speed camera, which can be obtained based on the numerical method by taking the measured bubble radius in the first frame into the \( R-t \) results from Eq. (12). And \( t_0 = 15.28 \mu \text{s} \) in this figure. In addition, \( \Delta t \) represents the minimum time interval between adjacent frames. And \( \Delta t = 11.11 \mu \text{s} \) which is approximately calculated from the camera shooting speed. Moreover, the red dashed lines represent the initial centroid position of the cavitation bubble. The scale bar is shown at the upper-right corner of the first frame. And the moment when the bubble radius reaches to \( R_{\text{max}} \) refers to the frame (4).

As shown in Fig. 4, during the first period oscillation (referring to the...
frames (1)-(10)), the considerable non-spherical deformation and the movement of the cavitation bubble is exhibited. Specifically, at the growth stage of the first period, the curvature of the bubble wall on the left side is significantly larger than that on other sides (referring to the frames (2)-(4)). And at the collapse stage of the first period, a bulge gradually forms and develops on the left side of the bubble wall (referring to the frames (6)-(10)). Moreover, an obvious leftward movement can be seen according to the red dashed line in the figure. Subsequently, during the second period oscillation (referring to the frames (11)-(15)), the non-spherical deformation and the movement of the cavitation bubble still remains with more violent and chaotic phenomena. Hence, in the subsequent analysis, only the first oscillation period of the cavitation bubble is concerned to analyze. In addition, for more high-speed photographs of the bubble dynamics with differently, readers can refer to Appendix I.

4. Analysis of bubble sphericity and applicability range of theoretical model based on experimental results

In this section, the sphericity of the bubble near the spherical particle is analyzed in detail in both the time and the space, based on sufficient results of high-speed photography experiments. And the approximate range of the time and the dimensionless distance \( \gamma \) which can make the bubble conform to the quasi-spherical shape is given qualitatively.

Firstly, since just one single shooting angle is adopted in the experiment, the bubble can only be represented in two-dimensional form. Therefore, the bubble circularity \( (e) \) in this two-dimensional plane is chosen to characterize the bubble sphericity and calculated by \textit{imageJ} software. According to the definition of the circularity, the closer the circularity value is to 1, the closer the cavitation is to a spherical shape. During analysis, the value of critical circularity \( (e_c) \) is set to 0.80, and the bubble shape with circularity not less than \( e_c \) is regarded as the quasi-spherical shape. And the bubble satisfying the above condition is considered to satisfy the theoretical model in Section 2.

Then, Fig. 5 shows the categorization of the bubble circularity in terms of the time and the space according to the high-speed photography during the first period oscillation. In Fig. 5, the green square and the red triangle refer to the bubble circularity not less than and less than the critical circularity \( e_c \) respectively. The black dash line is the dividing line between above two situations. And the numbers in parentheses indicate the specific circularity values corresponding to the adjacent data points \( e_c = 0.80 \). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

![Fig. 5. Categorization of the bubble circularity in terms of the time and the space according to the high-speed photography during the first period oscillation. The green square and the red triangle refer to the bubble circularity not less than and less than the critical circularity \( e_c \) respectively. The black dash line is the dividing line between above two situations. And the numbers in parentheses indicate the specific circularity values corresponding to the adjacent data points \( e_c = 0.80 \).](image)

In the following analysis, the bubbles that can be regarded as quasi-spherical will also be shown in Appendix II to illustrate the limitations of the theoretical model in Section 2.

5. Analysis and experimental support for liquid flow field

In this section, the liquid velocity fields around the particle and the cavitation bubble are shown and discussed in detail based on the point source assumption and the Weiss theorem [22] introduced in Section 2.
Moreover, the corresponding experimental results are also displayed and described in the form of the bubble contours to directly support the theoretical velocity field results.

5.1. Theoretical results

Firstly, two typical moments with distinctive characteristics at both the bubble growth and collapse stages are selected for the analysis in this section according to the description of Fig. 4. Among them, \( t_g \) refers to the typical moment at the bubble growth stage with \( t_g = t_0 + 2\Delta t \). And \( t_c \) refers to the typical moment at the bubble collapse stage with \( t_c = t_0 + 16\Delta t \). In addition, \( t_0 = 15.28 \mu s \) is employed which is consistent with Fig. 4. In addition, the dimensionless distance between the bubble and the particle is chosen as \( \gamma = 0.74, 1.10 \) and \( 1.95 \) respectively. According to the analysis of Fig. 5, the cavitation can be regarded as quasi-spherical for the above combination of the time and \( \gamma \), and the theoretical model can be safely applied.

Subsequently, Figs. 6-8 show the velocity nephograms and vectors of the liquid according to the theoretical results with \( \gamma = 0.74, 1.10 \) and \( 1.95 \) respectively. In Figs. 6-8, the subgraphs (a) and (b) refer to \( t = t_g \) and \( t = t_c \) respectively. In addition, the white semicircle on the left side of each subgraph represents the particle, and the white circle on the right represents the cavitation bubble with the instantaneous radius at the corresponding moment. And the arrows in the subgraphs represent the magnitude and direction of the liquid velocity at each point.

As shown in Fig. 6(a), the liquid moves away from the cavitation bubble. And the velocity of the liquid between the particle and the cavitation bubble is significantly lower than that in other liquid positions around the cavitation bubble. In addition, further focusing on the liquid velocity on the bubble wall, the liquid velocity on the left side of the bubble wall is significantly lower than that on other sides, and the minimum velocity occurs at the left endpoint. According to the continuity equation, the movement velocity of the bubble wall is basically the same as the liquid velocity on the bubble wall. Therefore, it is foreseeable that the cavitation bubble cannot continue to maintain the spherical shape. Moreover, the above analysis also suggests that the particle can still significantly affect the bubble dynamics although the bubble does not touch the particle. Then, as shown in Fig. 6(b), the liquid around the cavitation bubble flows towards the cavitation bubble at the collapse stage. And a low velocity region still exists between the particle and the cavitation bubble. In addition, as for the liquid velocity on the bubble wall, the liquid velocity on the left side of the bubble wall is still significantly lower than that on other sides, which is relatively similar to the situation at the growth stage. Hence, predictably, the circumferential inhomogeneity of the bubble wall will further develop.

Furthermore, by comparing Figs. 6-8, it can be found that a low velocity region always appears and is located around the right endpoint of the particle due to the presence of the particle. And the difference in liquid velocities around the left and right sides of the bubble wall

![Fig. 7. Velocity nephograms and vectors of the liquid according to the theoretical results. (a) The bubble growth stage with \( t_g = t_0 + 2\Delta t \). (b) The bubble collapse stage with \( t_c = t_0 + 16\Delta t \). \( \gamma = 1.10 \). \( t_0 = 15.28 \mu s \).](image1)

![Fig. 8. Velocity nephograms and vectors of the liquid according to the theoretical results. (a) The bubble growth stage with \( t_g = t_0 + 2\Delta t \). (b) The bubble collapse stage with \( t_c = t_0 + 16\Delta t \). \( \gamma = 1.95 \). \( t_0 = 15.28 \mu s \).](image2)
gradually decreases with the increase of $\gamma$. That is to say, the influences of the particle on the liquid velocity field and the bubble wall still exist, and these influences gradually decrease with the increase of $\gamma$.

Next, in order to further explore and explain the low velocity region between the bubble and the particle, the liquid velocity components between the bubble and the particle caused by the cavitation bubble, the image bubble and the linear sinks will be respectively demonstrated. Fig. 9 shows the variations of the liquid velocity and its components between the bubble and the particle with the position of $X$. In Fig. 9, the subgraphs (a) and (b) refer to the bubble growth stage and collapse stage respectively. The black, red, blue and green lines refer to the liquid velocity and its components caused by the bubble, the image bubble and the linear sinks respectively. And the points A, B and C correspond to those in Fig. 7. $\gamma = 1.10$. $t_0 = 15.28 \mu s$. $Y = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
The liquid velocities and these dimensionless differences at the left and right endpoints of the bubble wall with the different $\gamma$. $u_l$ and $u_r$ represent liquid velocities at the left and right endpoints of the bubble wall respectively. And $\Delta u'$ means the dimensionless difference between $u_l$ and $u_r$. $t_0 = 15.28 \mu s$.

| Stages       | $\gamma$ | $u_l$(m/s) | $u_r$(m/s) | $\Delta u'$ |
|--------------|----------|------------|------------|------------|
| Growth ($t_g = t_0 + 2\Delta t$) | 0.74     | 1.27       | 8.80       | 85.57 %    |
|              | 1.10     | 7.76       | 8.73       | 11.69 %    |
|              | 1.95     | 8.60       | 8.68       | 0.92 %     |
| Collapse ($t_c = t_0 + 16\Delta t$) | 0.74     | 4.28       | 10.29      | 58.41 %    |
|              | 1.10     | 9.32       | 10.20      | 8.63 %     |
|              | 1.95     | 10.08      | 10.14      | 0.59 %     |

Fig. 10. Contours and centroid positions of the cavitation bubble according to the experimental results. (a) The bubble growth stage. (b) The bubble collapse stage. $\gamma = 0.74$. $t_0 = 15.28 \mu s$. $t_0 = 15.28 \mu s$. $\gamma = 1.10$. $t_0 = 15.28 \mu s$. $Y = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
and green dashed lines refer to the liquid velocity components caused by the bubble, the image bubble and the linear sinks respectively. The points $A$, $B$ and $C$ mean the right endpoint of the particle, the left endpoint of the bubble at $t_g$ and $t_c$ respectively. And other parameters are all consistent with those in Fig. 7. As shown in Fig. 9, the absolute value of the liquid velocity between the bubble and the particle always increases from zero as $X$ increases, and it is always less than the velocity component caused by the cavitation bubble. In addition, both velocity components caused by the image bubble and the linear sinks tend to zero with the opposite direction as $X$ increases. And the absolute value of the velocity component caused by the image bubble is always larger than that caused by the linear sinks. That is to say, the effect of the particle on the liquid velocity is mainly reflected in the form of the image bubble with a deceleration effect on the liquid, which can always induce a low velocity region between the bubble and the particle.

Subsequently, in order to further reveal the influence of $\gamma$ on the bubble wall, the liquid velocities at two characteristic positions on the bubble wall are quantitatively compared and analyzed. Table 2 shows the liquid velocities and these dimensionless differences at the left and right endpoints of the bubble wall with the different $\gamma$. In Table 2, the
analyzed moments of the bubble growth and collapse stages and the values of $\gamma$ and $t_0$ are all consistent with those in Figs. 6-8 respectively. $u_l$ and $u_r$ represent the liquid velocities at the left and right endpoints of the bubble wall respectively. And $\Delta u'$ means the dimensionless difference between $u_l$ and $u_r$, which is defined as follows:

$$\Delta u' = \frac{u_r - u_l}{u_r}$$ (15)

As shown in Table 2, the results and the changing trends of $u_l$ and $u_r$ are considerably different respectively. Specifically, at both growth and collapse stages, the change of $u_r$ is basically inconsiderable, while $u_l$ increases significantly and keeps getting closer to $u_r$ with the increase of $\gamma$. As a result, the dimensionless difference $\Delta u'$ is significant when $\gamma$ is small and critically decreases with the increase of $\gamma$. Through the above analysis, it indicates that the effect of particle on the circumferential inhomogeneity of the liquid radial velocity around the bubble decreases rapidly and eventually disappears as $\gamma$ increases.

In addition, the circumferential inhomogeneity of the liquid radial velocity around the bubble at the bubble collapse stage can characterize the initial evolution of the jet formation. In general, liquid jets are most likely to develop at the location with the largest liquid radial velocity around the bubble, which can be supported by the adequate and detailed experimental research on the jet by Zhang et al. [39].

5.2. Experimental support

In order to properly support the above theoretical results for the...
the theoretical results of the liquid velocity fields in Fig. 6.

Moreover, for the different $\gamma$, the phenomena of the bubble contours can still properly correspond to the theoretical results of the liquid velocity fields. As $\gamma$ increases (referring to Fig. 11), the difference between the left and right sides of the bubble contours is still evident at both growth and collapse stages. By comparing the contours intervals at different moments, the contour movement velocity on the left side of the bubble wall is relatively lower than that on the right side at the two stages. However, as $\gamma$ further increases (referring to Fig. 12), the influence of the particle on the bubble contours is small enough to be ignored, and the bubble contours always keep round. In addition, by comparing Figs. 10-12, the difference of the movement velocities between the left and right sides of the bubble contours becomes smaller and smaller with the increase of $\gamma$, which is in particular agreement with the theoretical results in Table 2.

6. Analysis and experimental support for Kelvin impulse

In this section, the Kelvin impulse for the cavitation bubble near the spherical particle is investigated in detail based on the theoretical model in Section 2. Meanwhile, the corresponding experimental results are also exhibited in the forms of the bubble centroid displacements and velocities as reasonable supports for the theoretical results of the Kelvin impulse.

6.1. Theoretical results

Although the values of the Kelvin impulse at any point where $\gamma > 0$
can be calculated according to Eq. (7), it is undeniable that the calculation has limitation in space. According to the analysis of Fig. 5, the cavitation bubble and the particle is always in contact with each other in the first oscillation period, and the bubble circularity is less than the critical circularity in most of the time for the situations with too small $\gamma$ ($\gamma < 0.5$). Therefore, $\gamma > 0.5$ is chosen for the analysis of the Kelvin impulse in this Section.

Subsequently, Fig. 13 shows the variation of the Kelvin impulse for the cavitation bubble with the dimensionless distance between the particle and cavitation bubble based on Eq. (7). In Fig. 13, the positive direction of the Kelvin impulse corresponds to the positive direction of the X-axis in Fig. 1. As shown in Fig. 13, all values of $I$ is negative. And with the increase of $\gamma$, $I$ critically decreases and gradually tends to zero. That is to say, on the time scale of the first bubble oscillation period, the effect of the particle on the cavitation bubble is attraction, and its intensity can be severely weakened by increasing the dimensionless distance between the particle and cavitation bubble.

6.2. Experimental support

According to the definition of impulse, the Kelvin impulse can be used to describe the bubble movement characteristics. Therefore, the instantaneous displacement and velocity of the bubble centroid are selected and analyzed as reasonable supports for the Kelvin impulse results in Section 6.1.
Theoretical prediction model of bubble movement

According to the analysis in Section 6, it is obtained that the variation trend of the Kelvin impulse with \( \gamma \) is highly consistent with that of the instantaneous velocity of the bubble centroid. Notably, the Kelvin impulse corresponds to the entire process during the first period oscillation according to Eq. (1). Therefore, it is a reasonable choice to select the average movement velocity in the first period oscillation trend of the Kelvin impulse with \( \gamma \). Theoretical prediction results for the average movement velocity of the bubble centroid in the first bubble oscillation period with \( \gamma \) fitted by the experimental results. As shown in Fig. 16, the scatter points refer to the experimental results. The red line refers to the experimental fitting results based on Eq. (18). Moreover, according to the above fitting result, Fig. 16 shows the variation of the theoretical prediction results for the average movement velocity of the bubble centroid in the first bubble oscillation period with \( \gamma \) fitted by the experimental results. In Fig. 16, the scatter points refer to the experimental results. The red line refers to the experimental fitting results based on Eq. (18).

Notably, the fitting of the average velocity based on the Kelvin impulse also has limitations in the space. And the bubble movement behavior cannot be predicted correctly for the situations where \( \gamma \) is too small (\( \gamma < 0.50 \)). For more details for the limitations, readers can refer to Fig. 20 in Appendix II.

8. Conclusion

In the present paper, the cavitation bubble dynamics near a spherical particle is investigated based on the Weiss theorem the Kelvin impulse theory and the high-speed photography experiment. And the in-depth theoretical analysis and the corresponding experimental support are carried out in terms of the liquid flow field and the bubble Kelvin impulse. Through our investigation, the correctness of the theoretical models for the liquid velocity field and the Kelvin impulse results has been demonstrated comprehensively by the corresponding experimental results. And it is found that these theoretical models can be employed to effectively reveal and predict the dynamic behaviors of the cavitation bubble near the spherical particle. Specifically, several important conclusions are obtained as follows:

1. the applicability range of the Kelvin impulse theory for the bubble near the spherical particle is approximately the \( \gamma > 0.50 \). Only if \( \gamma \) is large enough, the cavitation bubble can be regarded as quasi-spherical.
2. The effect of the particle on the liquid velocity between the bubble and the particle is mainly manifested in the form of the image bubble, which always causes the liquid velocity in this region to be significantly lower than other surrounding regions. And the liquid low velocity region would become more obvious as \( \gamma \) decreases.
3. The Kelvin impulse for the cavitation bubble is always towards the particle centroid, and gradually decreases and approaches zero as \( \gamma \) increases. Based on the analytical expression of the Kelvin impulse, the directly proportional function can be developed for predicting the average movement velocity of the bubble centroid. And the relationship constant \( (\alpha) \) between the Kelvin impulse and the average movement velocity is obtained by experimental fitting with \( \alpha = 3.57 \times 10^{-6} \) kg.

CRediT authorship contribution statement

Xiaoyu Wang: Conceptualization, Methodology, Investigation, Software, Writing – original draft. Guanhao Wu: Validation, Formal analysis, Visualization. Xiaoxiao Zheng: Resources, Software,
Appendix I. High-speed photographs of bubble dynamics for different $\gamma$

Fig. 17 shows the high-speed photographs of the bubble dynamics near a particle with different $\gamma$. In Fig. 17, subgraphs (a) and (b) refer to $\gamma = 0.74$ and $\gamma = 1.95$ respectively. And the $t_0$ in the two subgraphs refer to $t_0 = 15.28$ $\mu$s and $t_0 = 10.85$ $\mu$s respectively. As shown in Fig. 17, for a small $\gamma$ (referring to Fig. 17(a)), the cavitation bubble contacts the particle at $t = t_0 + 9\Delta t$ and maintains contacting until the end with a progressively violent non-spherical collapse. While for a large $\gamma$ (referring to Fig. 17(b)), the cavitation bubble is almost unaffected by the particle and remains spherical at both the growth and collapse stages. Comparing Fig. 17 and Fig. 4, it can be found that the influence of the particle on the bubble dynamics is gradually weakened with the increase of $\gamma$.

Appendix II. Typical examples exhibition for theoretical model limitations in time and space

For the theoretical model in Section 2, the effect of the bubble oscillation on the liquid velocity distribution near the particle is approximated as a point source with an isotropic source intensity in order to simplify the model. However, such approximation does have certain limitations in both the time and the space. Specifically, the anisotropic property of the cavitation bubble would be very significant for the situations where the distance between the cavitation bubble and the particle is too small, or when the cavitation bubble is at the late stage of the collapse. In these situations, the sphericity of the bubble would be much low and theoretical results would lose reference value. In this Appendix, the limitations in both the time and the space will be elaborated separately based on the analysis of Fig. 5.

Firstly, Fig. 18 shows the comparison of the experimental and theoretical results for the situation that the bubble is at the last stage of the collapse. In Fig. 18, the subgraphs (a) and (b) refer to the high-speed photography result and the theoretical liquid velocity nephogram result respectively. And the bubble is at $t = t_0 + 18\Delta t$ with a circularity of 0.75. In addition, the value of $\gamma$ is consistent with that in Fig. 6. As shown in Fig. 18, there is a significant difference between the experimental and theoretical results, and the theoretical result is no longer valid. According to the experimental results, the bubble is always in contact with the particle during the collapse stage, and deforms into a slender shape. And such adsorption of the particle to the bubble and the subsequent bubble dynamic behaviors cannot be predicted and analyzed by the theoretical model provided in the present paper.

Subsequently, Fig. 19 shows the comparison of the experimental and theoretical results for the situation that $\gamma$ is too small. In Fig. 19, the subgraphs (a) and (b) refer to the experimental result and the theoretical result respectively. And the bubble is at $t = t_0 + 16\Delta t$ which is consistent with the time in subgraphs (b) of Figs. 6-8. However, the bubble circularity is only 0.76 which is much smaller than them in Figs. 6-8 because of the too small $\gamma$. As shown in Fig. 19, the difference between the experimental and theoretical results is also significant. From the experimental results, it can be seen that the bubble is in the shape of “mushroom” [37] with an obvious sunken at the arrow in the subgraph (a). While it can only be found that the liquid velocity is slightly larger at the contact between the bubble and the particle, and the generation and development process of the bubble sunken is hard to predict and analyze according to the theoretical results in the subgraph (b).

In addition, Fig. 20 shows the variation of the average movement velocity of the bubble centroid in the first bubble oscillation period with $\gamma$ based on Fig. 16. In Fig. 20, the black scatter points and the red line have the same meaning as them in Fig. 16. The blue triangles are newly added experimental data points with smaller $\gamma$. And the black dashed line indicates $\gamma = 0.50$. As shown in Fig. 20, the three new experimental data points on the left side of the dashed line obviously cannot satisfy the fitting results obtained from the Kelvin impulse. Therefore, it can be qualitatively obtained that the applicable range of the Kelvin impulse to predict the cavitation movement is $\gamma > 0.50$ by further combining the circularity analysis in Fig. 5.
