LQR/Sliding Mode Controller Design Using Particle Swarm Optimization for Crane System

Hazem I. Ali¹, Azhar J. Abdulridha²*, Rawaa R. Khaleel³, Kareem A. Hussein⁴

Abstract

In this work, the design procedure of a hybrid robust controller for crane system is presented. The proposed hybrid controller combines the linear quadratic regulator (LQR) properties with the sliding mode control (SMC) to obtain an optimal and robust LQR/SMC controller. The controller is examined by considering a ± 50% variation in system parameters with applying an external disturbance input. Finally, the superiority of the proposed LQR/SMC controller over the SMC controller is shown in this work.

Keywords: Linear Quadratic Regulator (LQR), Sliding Mode Control (SMC), PSO, LQR/Sliding Mode Controller, Full State Feedback.

1. Introduction

A Crane system is used for educational purposes in the engineering of control. The system is suitable to prove a wide range of control theories [1]. This system has to meet stringent safety and efficiency requirements easily cause the vessel to move away from a desired position both horizontally and vertically. Therefore, the control of crane system is a challenging problem due to the presence of parameter variations and disturbances [2]. The gantry cranes are widely used, in modern industrial system, for the heavy loads transfer. The undesirable load swing causes by the crane acceleration and having negative consequences and safety performances on the system control. For the load position, a minimizing of load swinging is necessary for controlling on load swing angle. The derivations of swing angle and the cart position should be controlled. It is impossible to completely isolate the controlled object from undesired influences which are usually represented as external disturbances which may minimize by the controlled action [3].

* Corresponding author.

Authors affiliations:
1) Control and Systems Engineering Dept., University of Technology Baghdad–Iraq.
2) Control and Systems Engineering Dept., University of Technology Baghdad–Iraq.
3) Control and Systems Engineering Dept., University of Technology Baghdad–Iraq.
4) Control and Systems Engineering Dept., University of Technology Baghdad–Iraq.

Kareem_control1994@yahoo.com
rawaa521@yahoo.com
Kareem_control1994@yahoo.com

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On the other hand, the Linear Quadratic Regulator (LQR) is one of the optimal control methods and it is the most widely used in aerospace systems. This controller can achieve a desirable performance with control effort. However, the LQR controller cannot give the same performance in case of disturbance or in case of system parameters change [4, 5]. The Sliding Mode Control (SMC) is a robust technique which can be implemented by changing the structure of controller. In the SMC, state of the system trajectory is forced to move along a selected sliding surface. The sliding surface is always selected in such a way that the system is in a state depending on control law [6]. The SMC is low sensitivity to variations in system parameters and external disturbances [7].

There are several control schemes have been developed to improve the crane operation performance such as input shaping control, optimal control and fuzzy control [2]. Chang [8], proposed an adaptive fuzzy control to achieve precise position control and fast damping of load swing in spite of the load mass, flexible length of wire, and external disturbance of the crane. Hua et al [9], presented an adaptive nonlinear coupling control law based on Lyapunov stability for the motion control of the crane. The results showed better performance in comparison to the conventional PD control and sliding mode controller. Park et al [10], used an adaptive fuzzy sliding mode controller to achieve a better performance for the robust anti-sway trajectory tracking of the crane for both uncertainty and nonlinearity. Bruin [1], made a comparison among four control methods which are a parallel P-controller, cascade P-controller, fuzzy controller and an internal model controller. The controllers except the internal model controller have achieved the stabilization and tracking of the system to a desired position. Burul et al. [3] made a comparison between H∞ controller and pole placement control. H∞ controller has ensured a system response with minimum steady state error, while the pole placement method failed to achieve that. Fang et al. [11], achieved a superior performance using a δ-shape trajectory for the trolley and an adaptive controller to reduce payload swing. The adaptive controller has achieved an asymptotic tracking even in the presence of variations in system parameters and external disturbances. Jaafar et al [12], designed the optimal controller gains for PID and PD controllers by improved PSO algorithm using a priority-based fitness method. The gains was examined based on a control structure that utilized for position and oscillation control of the system. Ismail et al [2], designed a robust SMC for a crane system with disturbances and uncertain parameters. Wang et al [13], proposed a dynamic sliding mode variable structure control for controlling the anti-swing position of overhead traveling crane with external disturbance and parameters uncertainty.

In this work, the LQR/SMC controller is proposed to stabilize the crane system and achieve a desirable tracking. The PSO method is used to optimize the controller parameters subject to a proposed cost function.

2. System Mathematical Model

The crane system can be represented as pendulum and cart. The mechanical model of the pendulum is presented as nonlinear system in Fig. 1 [14]:

![Figure 1: The mechanical model of the pendulum](image)

The summing of the forces and the moments acting on the pendulum and cart system can be expressed by the following nonlinear equations of motion [8, 14]:

\[
(m + M)\ddot{x} + b \dot{x} + m \dot{\theta} \cos \theta - m \dot{\theta}^2 \sin \theta = F \quad (1)
\]

\[
(l + m l^2) \ddot{\theta} - m g l \sin \theta + m l \dot{x} \cos \theta + d \dot{\theta} = 0 \quad (2)
\]

where \(m\) is the pole mass, \(M\) is the cart mass, \(b\) is the cart friction coefficient, \(l\) is the pole length, \(l\) is the moment of inertia of the pole, \(g\) is the earth gravity, \(d\) is the pendulum damping coefficient, \(x\) is the cart position, \(\dot{x}\) is the cart velocity, \(\dot{\theta}\) is the cart acceleration, \(\theta\) is the pendulum angular position, \(\ddot{\theta}\) is the pendulum angular velocity, \(\ddot{x}\) is the pendulum angular acceleration and \(F\) is the force applied on the crane system. The motion eq.1 and eq.2 can be linearized to be [8, 14]:

\[
(m + M)\ddot{x} + b \dot{x} - m \dot{\theta} = F \quad (3)
\]

\[
(l + m l^2) \ddot{\theta} + m g l \dot{\theta} - m \dot{x} + d \dot{\theta} = 0 \quad (4)
\]

To find \(\ddot{x}\) and \(\ddot{\theta}\), equations eq.3 and eq.4 are simplified to give:

\[
\dddot{x} = \frac{(l+m l^2)(F-b \ddot{x})-(m l g \dot{\theta}+d \dot{\theta})(m l)}{(m+m l^2)^2} \quad (5)
\]

\[
\dddot{\theta} = \frac{-(m l g \dot{\theta}+d \dot{\theta})(m+M)+(m l)(F-b \ddot{x})}{(m+m l^2)^2} \quad (6)
\]

Assuming \(x_1 = x, x_2 = \dot{x}, x_3 = \dot{x}_1, x_4 = \dot{x}_2\) and \(u = F\), the state equations of crane system are:

\[
\dot{x}_1 = x_3 \quad (7)
\]

\[
\dot{x}_2 = x_4 \quad (8)
\]

\[
\dot{x}_3 = (l+m l^2)(u-b x_2) - (m l g x_1 + d x_2)(m l) \quad (9)
\]

\[
\dot{x}_4 = \frac{-(m l g x_1 + d x_2)(m+M)+(m l)(u-b x_2)}{(m+m l^2)^2} \quad (10)
\]

The state space representation for linear model is:

\[
\dot{x} = Ax(t) + Bu(t) \quad (11)
\]

where \(x \in \mathbb{R}^n, u \in \mathbb{R}^m, n = 4, x = [x, \dot{x}, \dot{x}_1, \dot{x}_2]^T = [x_1, x_2, x_3, x_4]^T\) and the initial condition is: \(x_0 = [0.1 \ 190 \ 0 \ 0]^T\). The state space representation of Crane system will be:
The intersection of the system states are on the sliding phase in the SMC design \([6, 16, 17]\). Differentiating the sliding phase, when the states are on the sliding surface, an equivalent control law is applied to drive states along the sliding surface. In the reaching phase and the sliding phase, a reaching control law is applied to drive states to the sliding surface, an equivalent control law is applied to force the state of system along the sliding surface to the origin. Fig. 2 illustrates the reaching phase and the sliding phase in the SMC design \([6, 16, 17]\).

\[
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \dot{x}_3 \\
  \dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
  \frac{-g_3 g_2}{g_2 - g_3} & \frac{-g_3}{g_2 - g_3} & \frac{g_2}{g_2 - g_3} & \frac{-g_3}{g_2 - g_3} \\
  \frac{-g_3 g_2}{g_2 - g_3} & \frac{-g_3}{g_2 - g_3} & \frac{g_2}{g_2 - g_3} & \frac{-g_3}{g_2 - g_3}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  0 \\
  \frac{1}{g_3} \\
  \frac{1}{g_2 - g_3}
\end{bmatrix}
\begin{bmatrix}
  u
\end{bmatrix}
\]

where \(g_1 = (m + M)\), \(g_2 = (ml)\) and \(g_3 = (l + ml^2)\). Table 1 lists the Crane model parameters.

### Table (1): Crane model parameters \([14]\).

| Parameter | Value/ Unit |
|-----------|-------------|
| \(g\)     | 9.81 m/sec^2 |
| \(M\)     | 2.4 kg      |
| \(m\)     | 0.23 kg     |
| \(l\)     | 0.099 kg.m² |
| \(b\)     | 0.005 N.m/sec/rad |

In a real-time, the cart position and the control signal are bounded when designing the controller. The range of the control signal should be around \([-2.51^\circ, +2.51^\circ]\) and the magnitude of generated force is \([-20.0N, +20.0N]\). By the rail length of the cart position which is bounded as \([-0.5m, +0.5m]\ \[14, 15\].

### 3. Controller Design

The SMC includes two phases which are the reaching phase and the sliding phase. In the reaching phase, a reaching control law is applied to drive states of the system to the sliding surface rapidly. In the sliding phase, when the states are on the sliding surface, an equivalent control law is applied to force the state of system along the sliding surface to the origin. Fig. 2 illustrates the reaching phase and the sliding phase in the SMC design \([6, 16, 17]\).

\[
S(t) = Gx(t)
\]

where \(G\) is an \(n \times n\) matrix which represents the gain to inform sliding surface. Differentiating eq.13 and substituting Eq.11 in the resulting equation, yields \[6\]:

\[
\dot{S}(t) = G(Ax(t) + Bu(t))
\]

The sliding mode control law is:

\[
u(t) = u_{eq}(t) + u_s(t)
\]

where \(u_{eq}(t)\) is an equivalent control input and \(u_s(t)\) is discontinuous control input. The vector \(S(t) = 0\) represents the intersection of the sliding surface passing through the origin. The equivalent control input \(u_{eq}(t)\), corresponds to \(\dot{S}(t)=0\). Therefore, from eq.14 \[6\], the equivalent control input \(u_{eq}(t)\) will be:

\[
u_{eq} = -(GB)^{-1}GAx\]

the matrix \(GB\) must be nonsingular. The reaching condition for the surface is \(S \dot{S} < 0\) so that

\[
\dot{S} = g_0 \text{sgn}(S)
\]

where \(g_0\) is a positive scalar gain.

The discontinuous control input \(u_s\) has the following form:

\[
u_s = -(GB)^{-1}g_0 \text{sgn}(S)
\]

Substituting eq.16 and eq.18 in eq.15, the control law becomes:

\[
u = -(GB)^{-1}(GAx + g_0 \text{tanh}(S))
\]

The \(G\) matrix is selected by trial and error as \([0.5, 0.5, 0.5, -0.1]\) and \(g_0 = 10\) for crane system. During sliding mode control implementation a chattering phenomenon is appeared. The chattering phenomenon happened because of unmolded dynamics of system and discretization chattering. To attenuate the undesired effects of the chattering phenomenon, Slotine \[16\] proposed an adoption of a thin boundary layer neighboring the switching surface, the Sign function replacing by a Hyperbolic Tangent function \((\tanh(S))\), and the control law becomes:

\[
u = -(GB)^{-1}(GAx + g_0 \tanh(S))
\]

To design an optimal SMC, the LQR will be added to the control law of the SMC. Consequently, the SMC law which is described in eq.15 will be:

\[
u = -Kx - g_0 \tanh(S)
\]

where \(K\) is derived from minimization of the cost function represented by:

\[
J = \int_0^\infty (x^TQx + u^TRu)\,dt
\]

where \(Q\) and \(R\) are positive semi-definite and positive definite Symmetric constant matrices, respectively. The weights \(Q\) and \(R\) can be time varying if needed. The pair \((A, B)\) must be controllable. Clearly, the controller cannot stabilize the system and perform as desired if the dynamics are not controllable. The LQR gain vector \(K\) is \[15\]:

\[
K = R^{-1}B^TP
\]

where \(P\) is a positive definite symmetric matrix and it obtained from the solution of the algebraic Riccati equation \[5\]:

\[
A^{-1}P + PA - PBR^{-1}B^TP + Q = 0
\]
In this work, the weights $Q$ and $R$ are selected using PSO which is an efficient method for global optimization problems. Assuming that particles are composed of a group in $n$ dimensional space, among them, the position and velocity of $i$ particle in the space are 

$$x_i = (x_{i1}, x_{i2}, ..., x_{id}), \quad v_i = (v_{i1}, v_{i2}, ..., v_{id})$$

where $i = 1, 2, ..., n$. The best position of the $i$ particle experiences is denoted by $p^{best}_{i}$, and the best position of all the particles in the group experience is denoted by $g^{best}$.

The all particle swarm updates the position and velocity through tracking the individual extreme value and the optimal value. The particle optimization process is expressed in [18]:

$$\begin{align*}
  x_{id}^{k+1} &= x_{id}^{k} + v_{id}^{k} \\
  v_{id}^{k+1} &= w v_{id}^{k} + c_1 a(p_{id}^{k} - x_{id}^{k}) + c_2 \beta(p_{gd}^{k} - x_{id}^{k})
\end{align*}$$

where $w$ is the inertia weight; $c_1$ is the optimal value of the weight coefficient that the particle tracks its history; $c_2$ is the weight coefficients that particle track the optimal value; and $\alpha$ and $\beta$ are the random numbers changing in $[0, 1]$. $p_{id}^{k}$ is the individual optimal solution of the particle after the $k$ iterations; $p_{gd}^{k}$ is the global optimal solution of the group in the $k$ iterations [18, 19]. The linear quadratic performance index eq.22 is adopted as a fitness function. $Q$ and $R$ are symmetric positive definite matrix [20]. To simplify the problem, the parameters of $Q$ is selected as a diagonal matrix of $4 \times 4$ and $R$ is matrix of $1 \times 1$ as:

$$Q = diag[999.93, 913.14, 511.14, 999.56],$$

$$R = 0.94.$$  

The resulting optimal feedback matrix is:

$$K = \begin{bmatrix} 28.037 & 61.22 & 32.31 & 1.57 \end{bmatrix}$$

The PSO method parameters after $k = 10000$ iterations are set as $c_1 = c_2 = 1.5$ and $w = 2$.

Fig.3 represents the block diagram for proposed LQR/SMC controller.

Figure (3): A full state feedback LQR/SMC controller.

4. Results and Discussion

Fig.4 shows the behavior of a Crane system before applying the proposed controller with zeros’ initial condition. It is shown that the system cannot be tracking the desired position and it is out-off control and also the pendulum angle deviates with large values.

Figure (4): Time response of a Crane System.

Fig.5 represents of the nonlinear Crane system for SMC controller and LQR/SMC controller by the Simulink Matlab program.

Figure (5): Simulink diagram of the nonlinear Crane system for SMC controller and LQR/SMC controller.

Fig.6 represents the time response of Crane system for cart position and pendulum angle using the proposed controller with initial conditions $\{0.1 \text{ m}, 190^\circ, (0, 0) \text{ deg/sec} \}$. The state trajectories using LQR/SMC controller approaches the equilibrium point faster than the state trajectories obtained using SMC controller.

(a) cart position.
Figure (6): The states trajectories of the nonlinear Crane system for SMC controller and LQR/SMC controller.

(b) pendulum angle.

(c) control signal.

Figure (7): Tracking properties of the nonlinear Crane system using SMC controller and LQR/SMC controller.

(a) cart position.
(b) pendulum angle.
(c) control signal.

To show the robustness of the proposed LQR/SMC controller, an external disturbance is applied to the system at time $t_s = 4$ sec as shown in Fig.8. It can be noticed that the proposed LQR/SMC controller can effectively reject the external disturbance and achieve the required robustness in comparison to SMC for the system.

Figure (8): Disturbance properties of the nonlinear Crane system using SMC controller and LQR/SMC controller.

(a) cart position.
(b) pendulum angle.

Fig.9 shows the state trajectories of the system with LQR/SMC controller when a variation in parameters of the system about $\pm 50\%$ is considered. It is apparent that the LQR/SMC controller can robustly compensate a variation in parameters and maintain the desired performance of the system.
The efficiency of the proposed LQR/SMC controller was examined with ±50% variation in the system parameters and external disturbance. The results showed the superiority of the LQR/SMC controller over SMC controller.

5. Conclusion:

In this paper, the design of the hybrid LQR/SMC controller was presented. This controller combined the properties of LQR and SMC to obtain a new controller. This controller achieved a better performance than if SMC controller is used. The Crane system which is nonlinear and uncertain system was used to verify the effectiveness of the proposed LQR/SMC controller. The efficiency of the proposed LQR/SMC controller was examined with ±50% variation in the system parameters and external disturbance. The results showed the superiority of the LQR/SMC controller over SMC controller.

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