Research on the Dynamic Multisocial Networks Influence Maximization Problem Based on Common Users

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ABSTRACT The influence maximization problem of a single social network is to find a set of k seed nodes S so that the spread of information from the seed set to the single network has the largest influence. This problem has attracted the attention of many researchers worldwide. In recent years, with the rapid development of the internet and the popularity of social networks, a variety of social platforms have appeared, allowing people to have multiple social accounts simultaneously; that is, one person will participate in multiple social networks and spread information on the various social platforms simultaneously. Consequently, the problem of influence maximization has been extended from a single social network to multiple social networks. However, many studies are based on static networks, and the critical challenge is that social networks usually have dynamic characteristics. At present, there is almost no research on dynamic multiple social networks. Therefore, based on common users, this paper establishes a dynamic multisocial network communication model to study the dynamic multisocial network influence maximization problem (DMNIMP). In this model, multiple dynamic networks are merged into a dynamic network, in which the self-propagating edges of common users are added to the snapshots of each frame of the integrated network. Experimental analysis shows that the proposed model can not only accurately and vividly represent dynamic characteristics but also reflect the mutual influence of common users on multiple social networks. If common users are chosen as the nodes with greater influence in each network, the communication range of the integrated network is obviously larger than that of a single network, and the interaction of dynamic multisocial networks is more obvious.

INDEX TERMS Common users, dynamic multisocial networks, influence maximization.

I. INTRODUCTION

The rapid development of the internet has created massive data and diversified the communication channels between people. Studies have shown that current users usually participate in multiple social networks and enjoy more social network services simultaneously [1], [2]. Therefore, information dissemination in multiple social networks has become a trend. For a social network, one of the greatest advantages is that the network can promote a brand to quickly gain fame and fortune at a small cost [3] when an idea is generated, which is viral marketing based on word-of-mouth communication between users on the network [4]–[7]. Traditional marketing problems and methods are mainly concentrated in a single network [8], which has been known as the influence maximization problem.

In fact, the profiles of some online social network users are confidential, and only their close friends are allowed to send messages to them. The implementation of viral marketing on such social networks often fails to achieve the desired results [9]. For example, since 2012, the Centers for Disease Control and Prevention (CDC) has released the National

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Tobacco Education Campaign “Tips From Former Smokers” (http://www.cdc.gov/tobacco/campaign/tips/index.html). To expand the influence, the CDC also published the campaign on Facebook, Twitter and so on. The goal was to encourage former smokers to encourage smokers to give up smoking, but the advertising effect on Facebook was not as successful as that on other networks. HMC (https://www.healthmediacollaboratory.org/), which was supported to evaluate the advertising effects shows that Facebook allows users to choose their personal settings and choose who can see their personal information. Due to the privacy and security policies of the network, the advertising salesperson cannot obtain the user’s profile and cannot easily deliver the message to the users who need it most.

When the setting rules for user profiles in a social network are different, some user information cannot be disclosed, or some users intentionally set the information incorrectly or exclude it. The most common situation exists on WeChat or QQ, such as user’s age, gender and geography. This leads to a single network marketing strategy that cannot play a good role. However, the information in some networks is more open and transparent, and users are more easily accessible. This situation provides a good channel for communication between the advertising company and the audience and provides the possibility for spreading information on multiple social networks.

Some articles have performed data surveys. The data from Twitter and Foursquare show that information can not only spread in a single social network but also spread across the network through common users [2], [10]. When randomly extracting a common user set of 500 from the data, they found that 409 users will repost from Foursquare to Twitter, and this proportion accounts for 81.8% [10]. However, compared to the total number of posts made by these users in Foursquare, the number of reposts across the network is very small.

In general, the common (or shared) users across multiple social networks have been identified as anchor users [2] or “tipping users” [9]. Many social platforms provide users with a service to share posts so that they can effortlessly share pictures and videos from one social platform to another. It is the existence of these anchor users or “tipping users” as bridges between the two network platforms that enable information to be transmitted across the network. If the information dissemination in one social network does not achieve the desired effect, starting from another network in a roundabout way may produce unexpected results.

The traditional influence maximization problem method [11] considers only a single network. The goal is to find a set of $k$ seed nodes $S$, which makes the propagation of a single network from the seed set have the greatest impact. Studies have been involved not only in the static network environment [4], [5], [8], [12]–[18] but also in the dynamic network environment [19]–[29]. However, they mainly focused on a single social network.

The DMNIMP [9], [10], [30] is similar to a single social network. The seeds can be selected not only from a single dynamic social network but also from dynamic multiple social networks at the same time. The goal of the DMNIMP is to maximize the spread of information from the seed set to multiple dynamic social networks. However, in the current method of the multiple network influence maximization problem, every single social network is a static social network. In 2015, Zhan et al. [10] proposed a new model, the multialigned multirelational network influence maximizer (M&M), which is based on the metapath between and within the network. Then, they extended the model to a linear threshold model to characterize the information diffusion of multiple social networks. In 2016, this team proposed a new network information diffusion model, namely, cross-network information diffusion [9]. In this model, various heterogeneous network links were extracted and merged to the weight. The weight was used to calculate the probability of the user being activated, and then a new algorithm was proposed to identify the “tipping users” that can bring the greatest impact gain.

From the above discussion, we know that the traditional method of the influence maximization problem considers only a single network. However, in the current method of the influence maximization problem of multiple networks, every single social network is a static network. Previous scholars have hardly studied the influence maximization problem of dynamic multiple social networks. In practice, many forms of social interactions are transient in nature. Moreover, due to the emergence of multiple social network platforms, people will have multiple accounts and spread information on multiple social platforms at the same time. Exploring influence maximization for product publicity and advertisement marking in dynamic multiple social networks is more challenging. Therefore, research on dynamic multisocial networks is indispensable. This paper studies the dynamic multisocial network influence maximization problem based on common users.

Our contributions can be summarized as follows:

- In this paper, according to the self-propagation characteristics of common users, we propose a dynamic multisocial network information transmission model under the dynamic independent cascade model. This model, with the self-propagating edges of common users added to the snapshots of each frame of the integrated network, divides the information propagation into within- and between-dynamic multiple social networks.

- Moreover, we find that the multiple dynamic networks can be merged into one dynamic network for processing based on the dynamic multisocial network information transmission model. Thus, the DMNIMP can be regarded as a dynamic influence maximization problem in a signal dynamic social network. We can apply the existing method $T \times one$ hop approach [29] to dynamic multiple social networks.

- Finally, the experiment verifies that the model can well characterize the self-propagation characteristic of common users and the dynamic characteristics of the network.
The rest of this paper is organized as follows. The related work is reviewed in Section II. \( T \times \text{one-hop} \) approach in dynamic social networks is introduced in Section III. A dynamic multisocial network information transmission model is formed in Section IV. Influence maximization in dynamic multisocial networks is conducted in Section V. The simulation and experiments are achieved in Section VI. Section VII concludes this paper.

II. RELATED WORK

As an important issue of information dissemination, influence maximization has become a hot research topic due to its extensive applications, such as viral marking [31], [32], rumor control [33], personalized recommendation [34], and opinion maximization [35]–[40]. Finding the optimal influence maximization problem and proposed its mathematical formalizations [4], [5], [8], [12]–[18]. In fact, due to the social network structure formed by relations between individuals evolves constantly, some researchers not only are interested in static networks but also focus on dynamic networks.

In static networks, motivated by their extensive applications, many researchers have extensively studied the influence maximization problem and proposed its mathematical formalizations [4], [5], [8], [12]–[18]. Finding the optimal seed set was first regarded as an algorithmic problem to study [16], [17]. Under the independent cascade (IC) and linear threshold (LT) models, Kempe et al. [8] proved that the optimization of the influence maximization problem was NP-hard. A simple greedy hill-climbing algorithm was presented with a \((1 - 1/e)\)-approximation guarantee due to the submodularity and monotone properties [18]. Due to the inefficiency of the greedy hill-climbing algorithm, some scholars have concentrated on improving the efficiency. By utilizing the submodularity of the objective function to carry out a lazy evaluation, Leskovec et al. [41] proposed an optimization algorithm CELF to accelerate the greedy algorithm. Tang et al. [42] proposed hop-based algorithms that can provide certain theoretical guarantees.

Opinion maximization, as a new form and application of influence maximization, has emerged and developed [35]–[40] in static networks. Different from conventional influence maximization, the main goal of opinion maximization is to maximize the overall opinion instead of the number of influenced individuals, and the propagation models are the bounded confidence model, voter model, HK model and Axelrod model. He et al. [36], [37] described influence maximization for the opinion formation process mathematically and proposed different methods to determine the top-k influential nodes. Moreover, to ensure the efficiency and accuracy of the schemes, they also proposed a two-stage iterative framework [35]. In the real scenario, the influence power of one individual may be unknown, or the scale of the social network may be large. Faced with these two situations, the authors also proposed a novel influence power-based opinion framework [38] and community-based approach [39] to solve the opinion maximization problem.

From the former discussion, various recent works focusing on influence maximization in static networks have laid a good foundation for studying dynamic networks. Influence maximization in a dynamic network can be categorized into two types. The first type [23], focusing on finding a seed set at time \( t_0 \), maximized the influence during the interval \([t_0, t_0 + \Delta t]\). The second type [28], called the influential node tracking problem (INT), considers fast updates of the seed set across different snapshot graphs. Most scholars conducted the research based on the first type [19]–[27]. For the first type, they usually applied the centralities for dynamic networks [20]–[22] or used the heuristics method for approximate computation [19], [23]–[27], [29]. Scholars have proposed the \( T \times \text{one-hop} \) approach [29] to solve the dynamic influence maximization problem (DIMP) using a dynamic independent cascade model. Moreover, they prove that the function of influence spread is submodular and monotone and build the derivation of formulas.

III. T \( \times \) ONE- \text{HOP APPROACH IN DYNAMIC SOCIAL NETWORKS}

For any node \( v \in V \), let \( N_v \) denote the set of node \( v \)'s neighbors and \( \Gamma_v \) denote \( v \)'s inverse neighbors. In a dynamic social network \( G = (G_1, \cdots, G_T) \), when the process of information transfer is completed in the snapshot graph \( G_1 \), the nodes within one hop from \( S \) to be possibly activated are called pseudo-seed nodes \( S' \). \( S \) is the seed set. \( S' \) denotes the pseudo-seed nodes produced by \( S \) working on the dynamic network from \( G_1 \) to \( G_T \). The process of information transfer will repeat \( T \) hops, and we want to acquire the maximum expectation of influence spread \( \pi_v(\cdot) \). Since the seed set and pseudo-seed set always activate their neighbors within one hop in a snapshot graph whose total number is \( T \), the transfer process is equal to \( T \times \text{one-propagation hops in the static network} \). Therefore, it is the \( T \times \text{one-hop} \) approach [29].

For any node \( v \in V \), after the cascade effect is completed in the graph, its one-hop activation probability \( \pi^1_{\text{HOP}}(S_{t}^1 \cup S_{t-1}^2 \cup \cdots \cup S_{t-T}^2 \cup \{v\})(v) \) is as follows:

\[
\begin{cases}
1, & \text{if } v \in S, \\
1 - \left(1 - \pi^1_{\text{HOP}}(S_{t-1}^2 \cup \cdots \cup S_{t-T}^2 \cup \{v\})(v)\right) \Big(\prod_{w \in \Gamma_v \cap (S_{t-T}^2 \cup \cdots \cup S_{t-1}^2 \cup \{v\})} \left(1-\pi^1_{\text{HOP}}(S_{t-1}^2 \cup \cdots \cup S_{t-T}^2 \cup \{v\})(w)\right)^{p_w(v)}\Big), & \text{otherwise.}
\end{cases}
\]

\( \pi^1_{\text{HOP}}(S_{t}^1 \cup S_{t-1}^2 \cup \cdots \cup S_{t-T}^2 \cup \{v\})(v) \)

After adding a new seed node \( u \), the seed set is denoted as \( S^{+} = S \cup u \). Then, \( S'_u \) denotes the pseudo-seed set produced by \( S \cup u \) working on the dynamic network from \( G_1 \) to \( G_T \). Let \( \pi^1_{\text{HOP}}(S_{t}^1 \cup S_{t-1}^2 \cup \cdots \cup S_{t-T}^2 \cup \{v\})(v) \) denote the one-hop activated probability of node \( v \) after the cascade effect of \( S_{t} \) is completed in graph \( G_T \). Due to the cascade effect on \( G \), adding a node \( u \) to seed set \( S \) at the beginning of the network evolution, its cascade effect is reflected not only in its neighbors but also in the neighbors of its pseudo-seeds. Thus, the set whose activation probabilities are changed in graph \( G_{t-1} \) owing to adding \( u \) is denoted as Temp \( S \) (\( t - 1 \)). Thus, we only need to update the activation probabilities of partial nodes \( v \in N(t)_{\text{Temp } S(t - 1)} = \{\bigcup N_w^t, w \in \text{Temp } S(t - 1)\} \) on the basis
of $\pi^S_{t-1}(v)$, $\pi^S_t(v)$ and $\pi^S_{t-2}(v)$. Obviously, $\pi^S_{t-1}(u) = 1$. For any node $v \in V$, we construct a proportional expression $\Delta_t(v) = \frac{1 - \pi^S_{t-1}(v)}{1 - \pi^S_{t-2}(v)}$ in graph $G_t$ and the initial value of $\Delta_0(v) = 1$.

Theorem 1: If we add a node $u$ to seed set $S$ at the beginning of the network evolution, based on $\pi^S_{t-1}(v)$, $\pi^S_t(v)$ and $\pi^S_{t-2}(v)$, we only need to update the activation probabilities of partial nodes $v \in N(t)_{\text{Temp} \_ S(t-1)}$ in $G_t$. Therefore, for any node $v \in N(t)_{\text{Temp} \_ S(t-1)}$, the formula is

$$\pi^S_{t-1}(v) = 1 - \Delta_t(v) \cdot \left(1 - \pi^S_{t-2}(v)\right),$$

(2)

If $\pi^S_{t-1,1}(w) \cdot p^t_{w,v} \neq 1$, $w \in \text{Temp} \_ S(t-1)$, then

$$\Delta_t(v) = \Delta_{t-1}(v) \cdot \prod_{w \in \text{Temp} \_ S(t-1)} \frac{1 - \pi^S_{t-1,1}(w) \cdot p^t_{w,v}}{1 - \pi^S_{t-1,1}(w) \cdot p^t_{w,v}}.$$  

(3)

If $\pi^S_{t-1,1}(w) \cdot p^t_{w,v} = 1$, or $\pi^S_{t-1,1}(w) = 1$, $p^t_{w,v} = 0$, then $\Delta_t(v) = 0$.

IV. DYNAMIC MULTISOCIAL NETWORK INFORMATION TRANSMISSION MODEL

A. DYNAMIC MULTISOCIAL NETWORKS

In recent years, the rapid development of the Internet and the popularity of social networks have increased the variety of social media platforms. It allows people to have multiple social accounts at the same time; that is, a person will participate in multiple social networks and the spread of information simultaneously. These users are called common users. This section redefines the model of dynamic multisocial networks and their related symbols. In dynamic multisocial networks, $G = \{G_{L_1}, \cdots, G_{L_N}\}$, $V = \{V_{L_1} \cup \cdots \cup V_{L_N}\}$ and $E = \{E_{L_1}, \cdots, E_{L_N}\}$ represent all the nodes and edges of network $G$, respectively.

In a single dynamic social network $G_{L_i}, i \in \{1, \cdots, N\}$, given the time period $t_0 \sim t_0 + \Delta t$, it is assumed that the dynamic structure partition time slot of the multiple social networks is the same, and it is denoted as $\omega$. The network dynamically changes with time according to a certain network evolution model and generates $T = \Delta t / \omega$ snapshots at a time. We rewrite the single dynamic social network as $G_{L_i} = \{G_{L_{i1}}, \cdots, G_{L_{iT}}\}, V_{L_i}$ and $E_{L_i} = \{E_{L_{i1}}, \cdots, E_{L_{iT}}\}$ represent all the nodes and edges of the single dynamic social network $G_{L_i}$, respectively. $G_{L_i}(V_{L_i}, E_{L_i})$ is the network structure at time $t_i$ which means that the nodes of a single dynamic social network remain unchanged during evolution, but the edges between each node of the network will increase or decrease with time. In a single dynamic social network, we assume that the information diffusion process is subject to a dynamic independent cascade model (DICM). However, the diffusion of information among multiple dynamic social networks is mainly based on the self-propagation characteristics of common users. In other words, a common user will forward information from one network to another with a certain forwarding probability.

We denote $C_{L_i} = C_{L_i} = \{V_{L_i} \cap V_{L_j}\}$ as common users of the dynamic networks $G_{L_i}$ and $G_{L_j}$. The sets $C_{L_i} \subset V_{L_i}$ and $C_{L_j} \subset V_{L_j}$, with the same number of elements $|C_{L_i}| = |C_{L_j}|$, represent different symbols of common users in the dynamic social networks $G_{L_i}$ and $G_{L_j}$. Namely, the corresponding elements indicate that the same user has different identities in different dynamic social networks $G_{L_i}$ and $G_{L_j}$.

To better introduce the structural model of the dynamic multinetwork, Fig. 1 shows the structure diagram of dynamic multiple social networks based on common users. Each layer of the network represents a single dynamic social network. There are $N$ dynamic social networks in Fig. 1. The white nodes represent ordinary users, and the blue nodes represent common users. The common user set of two dynamic networks $G_{L_i}$ and $G_{L_j}$ is $C_{L_i} \cap C_{L_j} = C_{L_{ij}} = \{V_{L_i} \cap V_{L_j}\} = \{a_1, b_1\} = \{a_2, b_2\}$, and $C_{L_i} = \{a_1, b_1\}$, $C_{L_j} = \{a_2, b_2\}$. So we obtain $|C_{L_i}| = |C_{L_j}|$. In other words, $a_1$ and $a_2$ are the corresponding elements of the two dynamic social networks $G_{L_i}$ and $G_{L_j}$, respectively. They represent different identity marks of the same user on different platforms. The dotted line between the two dynamic social networks indicates the self-propagation characteristics of the common user. How specific information is spread across the network is described in detail in part C.
B. DYNAMIC INDEPENDENT CASCADING MODEL IN DYNAMIC MULTISOCIAL NETWORKS

If we have a given time period \( t_0 \sim t_0 + \Delta t \), in a dynamic independent cascade model (DICM), a single dynamic social network \( G_{L,t}, i \in \{1, \cdots, N\} \) dynamically changes with time according to a certain network evolution model. Then, \( T \) network snapshots \( G_{L,1}, \cdots, G_{L,T} \) are generated. \( N_u \) represents the neighbor set of node \( u \) in the snapshot graph \( G_{L,t} \).

At time \( t \), an active node \( u \) attempts to activate the inactive node \( v \) within one hop in graph \( G_{L,t} \) with a certain probability \( p^L_t(u,v) \). If \( u \) successfully activates node \( v \) in the snapshot graph \( G_{L,t} \) at time \( t \), then node \( v \) remains active in any snapshot graph after \( G_{L,t} \). If \( u \) fails to activate \( v \) in graph \( G_{L,t} \) at time \( t \), then in graph \( G_{L,(t+1)} \) at time \( t+1 \), when nodes \( u \) and \( v \) are still connected, it attempts to activate node \( v \) again. This process is repeated until \( G_{L,T} \) at time \( T \). DICM is an extension of the independent cascade model (ICM) in a dynamic network. There are two differences between ICM and DICM. The former is applied to static networks, while the latter is applicable to dynamic networks. Therefore, ICM nodes can only be activated once, while DICM nodes can be activated more than once.

C. DISSEMINATION RULES IN DYNAMIC MULTISOCIAL NETWORKS

In multiple dynamic social networks, each dynamic network is subject to the dynamic independent cascade model (DICM); that is, the active node in each snapshot only attempts to activate the inactive neighbor nodes within one hop. The existence of common user self-propagation edges is also equivalent to a hop of information from one network snapshot to another network snapshot, which is also similar to the independent cascade model of a single network.

Therefore, the spread of information in dynamic multisocial networks based on the existence of common users is mainly divided into the spread of information in a single dynamic social network and the spread of information between multiple dynamic social networks. Taking the dissemination of information between two social networks as shown in Fig. 2, the following rules are followed:

1) The spread of single dynamic social network information is based on a dynamic independent cascade model, as shown in the horizontal network of Fig. 2. (2) Information dissemination among multiple social networks is based on the common user self-propagation characteristic. In this paper, we assume that each snapshot has self-propagating edges based on common users. We take the information dissemination between two social networks as an example, as shown in the dotted line of Fig. 2. Considering the self-propagation characteristics of the common user as two newly added information propagation edges between the corresponding snapshots of the two dynamic networks, the two dynamic social networks can be regarded as one dynamic network for information processing. In this case, the influence maximization problem of dynamic multisocial networks is similar to that of single dynamic social networks.

Fig. 2 shows the propagation model of two dynamic social networks \( G_{L,a} \) and \( G_{L_j} \) in the time period \((t_0, t_0 + \Delta t)\). The red nodes represent seed nodes, and the white nodes represent ordinary users. The blue nodes represent the common users of the two dynamic networks. Nodes \( a \) and \( a' \) are the same entity in different social networks \( G_{L,a} \) and \( G_{L_j} \), respectively, which is called a common user. In the time period \( \Delta t \), \( p^L_{0,a} \) represents the probability that the information is successfully forwarded from node \( a \) in network \( G_{L,a} \) to node \( a' \) of the network \( G_{L_j} \), and \( p^L_{a,a'} \) represents the probability that the information is successfully forwarded from node \( a' \) of network \( G_{L_j} \) to node \( a \) of network \( G_{L,a} \). In theory, due to different statistical time ranges, \( p^L_{a,a'} \), \( p^L_{a,a} \) or \( p^L_{a',a} \) and \( p^L_{a,a'} \) are different. This can be determined according to the specific dataset. For simplicity, it can be assumed that the two have the same probability.

To simplify the symbols, in this article, the probability symbol of a single social network node does not reflect the effect of pseudoseed nodes, but it can reflect the effect of pseudoseed nodes for the single dynamic social network after the integration of multisocial networks. \( \pi^S_{L,t}(a) \) represents the activation probability of node \( a \) in the snapshot graph \( G_{L,t} \) at time \( t \), and \( \pi^S_{L,t}(a') \) represents the activation probability of node \( a' \) in the snapshot graph \( G_{L,t} \) at time \( t \).

Horizontal single dynamic social network information dissemination and vertical multiple dynamic social network information dissemination are carried out simultaneously, as shown in Fig. 2. When the cascading effect starts at the snapshot graph at time \( t \), there are two edges that affect node \( a \) in graph \( G_{L,t} \), which are \( 3 \rightarrow a \) and \( a' \rightarrow a \). When the cascading effect is completed on the snapshot graph \( G_{L,t} \) at time \( t \), the activation probability of node \( a \) is \( \pi^S_{L,t}(a) = 1 - (1 - \pi^S_{L,t(\cdot-1)}(a))(1 - \pi^S_{L,t(\cdot-1)}(3))(1 - \pi^S_{L,t(\cdot-1)}(a'))p^L_{a,a} \). In this formula, it can be seen that \( 1 - \pi^S_{L,t(\cdot-1)}(a')p^L_{a,a} \leq 1 \) and \( 1 - \pi^S_{L,t(\cdot-1)}(a) \leq 1 \). When the self-propagating edge of \( a' \rightarrow a \) is added, the activation probability of node \( a \) becomes larger. This is the obvious effect of the common
node interaction, and this effect cascades through the cascade effect. Similarly, for node $a'$, there are three edges in graph $G_{Lt}$ that affect node $a'$: $7 \rightarrow a'$, $5 \rightarrow a'$ and $a \rightarrow a'$. When the cascading effect is completed on the snapshot graph $G_{Lt}$ at time $t$, the activation probability of node $a'$ is $P_{s_t}^L(a') = (1 - \pi_s^{L(t-1)}(a') \cdot (1 - \pi_s^{L(t-1)}(7) \cdot p_{7u}^L \cdot (1 - \pi_s^{L(t-1)}(5) \cdot p_{5u}^L \cdot (1 - \pi_s^{L(t-1)}(a) \cdot p_{au}^L)).$ It can also be seen that the activation probability of node $a'$ increases after adding the self-propagating edge $a \rightarrow a'$.

V. INFLUENCE MAXIMIZATION IN DYNAMIC MULTISOCIAL NETWORKS

A. DYNAMIC MULTISOCIAL NETWORKS INFLUENCE THE MAXIMIZATION PROBLEM

The goal of DMNIMP, which is similar to a single dynamic social network impact maximization problem, is to find a seed set $S$ with $k$ nodes at a time $t_0$. In the time period $(t_0, t_0 + \Delta t)$, the expected number of active nodes for information dissemination in the multisocial network starting from the seed set $S$ is the largest. The differences between DMNIMP and a single dynamic social network impact maximization problem are as follows. (1) The selection range of seed nodes is different. The latter can only select seed nodes from a single network, and the former can be selected from a single network, and the latter can only select seed nodes from a single network. (2) The spread of information is different. In multiple dynamic social networks, the spread of information from the seed set to multiple social networks is the largest, and a single network can only affect one dynamic social network.

Given a positive integer $k$ and the multiple dynamic social networks $G = \{G_{L_1}, \cdots, G_{L_N}\}$. DMNIMP aims to find a seed set $S$ with $k$ nodes at time $t_0$ to maximize the $\sigma_{s_t}^N(T(S^*))$ of any node set $S \in V$. We denote $\sigma_{s_t}^N(T(S^*))$ as the expected activation nodes after the impact of the information diffusion process across $N$ dynamic multisocial networks. According to the dynamic multisocial network propagation model, we add the edges and self-propagation probability between common users to the snapshot graph of each frame of the merged network. Thus, dynamic multisocial networks can be merged into dynamic social networks. The synthetic dynamic network still follows a dynamic independent cascade model. The single dynamic network influence maximization problem is an NP-hard problem [8], [29], and the influence propagation function has submodularity and monotone properties [18], [29]. Therefore, the DMNIMP in this paper is also an NP-hard problem, and the influence propagation function is submodular and monotone. The proof is referenced in the paper [29].

Under the dynamic independent cascade model, $S^*$ represents the optimal seed set that maximizes the impact of the multidynamic social networks $G = \{G_{L_1}, \cdots, G_{L_N}\}$, that is, $\sigma_{s_t}^N(T(S^*)) = \max_{S \in V} \sigma_{s_t}^N(T(S)).$ If the seed set $S$ is obtained according to algorithm 1 in paper [29], there are inequalities $\sigma_{s_t}^N(T(S) \geq (1 - 1/e) \cdot \sigma_{s_t}^N(T(S^*))$, which means the seed set $S$ provides a $(1 - 1/e)$ - approximate guarantee.

B. THE ALGORITHM OF DYNAMIC MULTISOCIAL NETWORKS INFLUENCES MAXIMIZATION

According to the dynamic multisocial network propagation model, the nodes of $N$ dynamic social networks $G_{L_1}, \cdots, G_{L_N}$ are relabeled in order and integrated into one dynamic network $G = \{G_1, \cdots, G_T\}$. Then, we can regard the DMNIMP as a signal dynamic social network influence maximization problem. We use the existing method $T \times onehop$ to solve the DMNIMP. Thus, the algorithm of dynamic multisocial networks influence maximization is divided into three steps.

(1) Relabel all nodes in order in dynamic multisocial networks.

Assume that $G_{L_1}$ has $m_1$ nodes, $G_{L_2}$ has $m_2$ nodes and $G_{L_N}$ has $m_N$ nodes; we relabel all nodes of $N$ dynamic social networks in order. The new number of all nodes is as follows. $\{V_1, \cdots, V_{m_1}\} \in G_{L_1}, \{V_{m_1+1}, \cdots, V_{m_1+m_2}\} \in G_{L_2}, \{V_{m_1+\cdots+m_{N-1}+1}, \cdots, V_{m_1+\cdots+m_{N-1}+m_N}\} \in G_{L_N}.$ It can be seen that the integrated dynamic network $G$ has a total of $m_1 + \cdots + m_N$ nodes, and the corresponding edges of each snapshot are relabeled according to the newly edited node number. As a consequence, the common users in different social networks must be marked by different numbers.

(2) Add the self-propagation characteristics of common users to the integrated dynamic network $G$.

Assuming that the edge between every two common users of a dynamic social network is a two-way edge, the probability of self-propagation is calculated. All common users are integrated to form a snapshot structure diagram between common users $C = \{C_1, \cdots, C_T\}$. Common user snapshot $C_i, t \in 1, \cdots, T$ corresponds to the integrated dynamic network snapshot $G_i, t \in 1, \cdots, T$, and we merge $C_i$ into the integrated dynamic network snapshot $G_i$. Thanks to the different numbers of common users in different social networks, although multiple dynamic social networks are integrated, each dynamic social network does not interfere with the others except for common users.

In the integrated dynamic social network $G = \{G_1, \cdots, G_T\}, V = \{V_1, \cdots, V_{m_1+\cdots+m_N}\}$ denotes all nodes of the integrated dynamic network.

(3) Applying the $T \times onehop$ approach to dynamic multiple social networks.

After completing the information cascading effect of $T$ snapshots, the probability of one-hop activation of any node $v \in V$ refers to formula (1). When a new seed node $u$ is added, the activation probability formula for any node is given by Theorem 1 in $T \times onehop$.

After adding the seed $u$, algorithm 1 gives the calculation process of marginal gain for the integrated dynamic network $G_i$ as shown in Table 1. All the symbols in algorithm 1 are related to the integrated dynamic network $G_i, t \in 1, \cdots, T$. In algorithm 1, we must first find the corresponding number of the node (that is, $num$), and then calculate the marginal gain according to the idea of the $T \times onehop$ algorithm. Therefore, the range of the pseudoseed sets $S'$ and $S''$, involved in algorithm 1 is the entire integrated network. Similarly,
TABLE 1. The calculation process of marginal gain after adding new seed nodes for the integrated dynamic network G.

| Algorithm 1: Dynamic multisocial network increment (G₁, G₂, . . . , Gₚ, S, u) |
| --- |
| 1. For network Gᵢ, i ∈ {1, 2, . . . , p} |
| 2. Determine which node u belongs to node u ∈ Gᵢ |
| 3. \[ p_{Δₛ(u)}^{Gᵢ}(t) = \frac{1}{\sum_{u \in Gᵢ} p_{Δₛ(u)}^{Gᵢ}(u)} \] |
| 4. For each node u ∈ N(u) ∪ N(Δₛ(u)) stop |
| 5. \[ Δₛ(u) = \Deltaₛ(u) - \Deltaₛ(u) \] |
| 6. If there exist two special cases |
| 7. \[ Δₛ(u) = 0 \] |
| 8. else Δₛ(u) = Δₛ(u) − 1 |
| 9. For each node u ∈ N(u) ∪ N(Δₛ(u)) stop |
| 10. \[ Δₛ(u) = \frac{1}{\sum_{u \in Gᵢ} p_{Δₛ(u)}^{Gᵢ}(u)} \] |
| 11. \[ Temp_S(t - 1) = Temp_S(t - 1) \cup N(Δₛ(u)) \] |
| 12. return \[ p_{Δₛ(u)}^{Gᵢ}(t) = \frac{1}{\sum_{u \in Gᵢ} p_{Δₛ(u)}^{Gᵢ}(u)} \] |

VI. EXPERIMENTS

In this paper, we establish a dynamic multisocial network information transmission model, which merges multiple dynamic networks into one dynamic network for processing. It easily extends the dynamic influence maximization problem of one network to dynamic multisocial networks. We conduct extensive experiments to verify whether the dynamic multisocial network information transmission model conforms to the theoretical analysis and evaluate the influence of the different common users on dynamic multisocial networks.

A. EXPERIMENT SETTING

Assuming there are common users between the datasets of two different network platforms, we conduct extensive experiments on the combination of two real-world dynamic networks. The real-world dynamic networks are Email [43] and CollegeMsg [44]. Emails were generated using e-mail data from a large European research institution. The emails only represent communication between institution members. A direct edge (u, v, t) denotes that person u sent an e-mail to person v at time t. CollegeMsg is comprised of private messages sent on an online social network at the University of California, Irvine. Similarly, an edge (u, v, t) denotes that user u sent a private message to user v at time t. The basic statistics of the datasets are summarized in Tables 2 and 3. In addition, the extracted time span is the actual time span used from the whole dataset.

There are two widely adopted models for the propagation probability on each edge. The first is uniform activation (UA), which assigns probability uniformly. We set all the propagation probabilities to 0.05 in our experiments. The second is degree weighted activation (DWA), which assigns the probability of each edge (u, v, t) as \[ p_{Δₛ(u)}^{Gᵢ}(t) = 1/\gamma_v \] .

We assume that each dynamic social network is evenly divided and that the self-propagation probability of common users is different. To judge the advantages of our model, we compare the \( T \times \) one hop algorithm with the weight degree and forward influence algorithm (FIA) [23]. The weight degree represents the sum of all degrees, and we simply select the seed nodes whose weight degree ranks top \( k \). The forward influence algorithm works by maintaining a current set \( S \) with \( k \) nodes and continually improving it over time with the use of successive replacement. All the experiments are carried out on a computer with 8 cores (3.6 GHz) and 32 GB of memory.

This paper found 20 pairs of common users from the two network datasets Email and CollegeMsg and conducts experiments under the DWA model and UA model. The self-propagation probability of 20 pairs of users is randomly generated between zero and one. The selection of common users can be divided into two situations: 20 pairs of common users are randomly selected; 5 pairs are the nodes that rank top five in their network and the rest are randomly generated. A supplementary explanation for the second situation is that the nodes ranking in the top five nodes in CollegeMsg also rank in the top five in Email.

To better analyze the simulation results, we start with the concept of an isolated node and list the statistics of the number of isolated and nonisolated nodes in the different networks in Table 4. An isolated node indicates a node without neighbors and inverse neighbors in every \( T \) network snapshots. As seen in Table 4, approximately half of the nodes are isolated nodes in the CollegeMsg network, and 1/3 of the nodes are isolated nodes in the Email network.

B. RESULTS ANALYSIS UNDER THE DWA MODEL

We first compare the \( T \times \) one hop algorithm with the weight degree, degree and forward influence algorithm (FIA). Because the running time was analyzed in the paper [29], we only analyze the influence range of different algorithms under the DWA and UA models in Fig. 3 and Fig. 4.

In Fig. 3(a) and 3(b), we can clearly observe the following results. Regardless of which method we use to choose the common users, the \( T \times \) one hop algorithm achieves the best performance, and the forward algorithm ranks second. However, they are all much better than the weight degree and
degree. Under the UA model, Fig. 4(a) and 4(b) show similar results to the DWA model. However, the only difference is that the performance of the $T \times \text{one hop}$ algorithm and forward influence algorithm is much closer because the propagation probability of the UA model is too small to transform the information.

1) THE COMMON USERS SELECTED RANDOMLY
When 20 pairs of common users are randomly selected, Fig. 5 describes the influence range of different datasets under the DWA model.

'S from the whole dynamic multisocial' represents the information dissemination of dynamic multisocial networks. In other words, seed nodes are selected in the entire integrated network, and information is disseminated in the integrated network. 'S from CollegeMsg' or 'S from Email' indicates that the seed nodes are only selected in the CollegeMsg or Email network, and the information is spread in the integrated network. 'CollegeMsg' or 'Email' indicates that the seed nodes are only selected in the CollegeMsg or Email network, and the information is only spread in the corresponding single dynamic social network. We introduce the three graphs in Fig. 5. We set their abscissa to represent the number of seed budgets, and the ordinate to represent the number of activated nodes. In this paper, the meanings of curves and coordinates in similar diagrams refer to Fig. 5.

Fig. 5(a) describes the different influences of the existence of common users on dynamic multisocial networks and single networks. Fig. 5(b) and Fig. 5(c) depict the influence of the selection range of seeds on information dissemination. To visually compare the figure, we assign the three curves in Fig. 5(a) to Fig. 5(b) and Fig. 5(c). Therefore, the red star curves in the three figures are the same. The blue dotted curves are assigned to Fig. 5(b), and the cyan triangle curve...
is assigned to Fig. 5(c). The newly added purple triangle (or dotted) curves indicate that the seed nodes are only selected in the CollegeMsg or Email network, and the information is spread in the integrated network. We must note that the vertical starts at a different point in Fig. 5(b). In this paper, the assignments of the curves in Fig. 6, Fig. 7 and Fig. 8 are similar to those in Fig. 5.

Compared to a single dynamic social network, Fig. 5(a) shows the influence of the existence of common users on the information dissemination on the multiple dynamic social networks. The information dissemination range curve of dynamic multisocial networks (shown by the red star curve) is significantly higher than that of a single dynamic social network (CollegeMsg or Email). This result indicates that the existence of common users makes the influence of the two dynamic social networks’ information dissemination obviously mutually strengthen each other. Fig. 5 (b) and Fig. 5(c) show the influence of the selection range of seeds on information dissemination. In the two figures, a comparison of the red star curves and the purple triangle (or dotted) curves shows that the red curves are significantly higher than the purple curves. In other words, when the seeds are selected from the entire integrated network, the spread of information in the integrated network is significantly greater than selecting in a single social network.

However, when the seeds are selected from a single social network, the spread of information in the integrated network is also greater than the spread of information in the single social network. This situation can be clearly shown between the purple dotted curve and cyan triangle curve in Fig. 5(c). This result also indicates that the existence of common users makes the influence of the two dynamic social networks’ information dissemination obviously mutually strengthen each other. Similarly, we also find such a case from the purple triangle curve and blue dotted curve in Fig. 5(b), but it is not obvious because two-thirds of CollegeMsg nodes are in an isolated state and cannot be connected to others. The role of common users is relatively small. If we only select seed nodes in the CollegeMsg network and spread the information in the whole integrated network, the spread of information will not change much more than that in a single social network. Although we find this phenomenon in the Email network, the interaction of the common users selected from the email on the two networks is significantly greater than that of the CollegeMsg network, which means that information can be effectively transmitted from the Email network to the CollegeMsg network through the common users.

2) 1/4 COMMON USERS WITH GREATER INFLUENCE ARE SELECTED

Fig. 6 shows the influence range of different datasets under the DWA model when 1/4 common users with greater influence are selected. Since 5 of the 20 pairs of common users are the top five influential nodes in their respective networks,
the possibility of information dissemination between the two social networks is greatly enhanced. Whether the seed nodes are selected from the whole integrated network or from a single dynamic social network, the starting point of the information dissemination curves of the entire dynamic multisocial network is significantly higher than that in Fig. 5.

This also indicates that information has already been transferred from one network to another at the beginning of its transmission; that is, when common users have high influence in the two networks, it is easier for the information to be transferred from one network to the other. Such examples are very common in reality. For example, a celebrity has many fans on Weibo and blogs. Once the celebrity forwards a message from Weibo to the blog, the speed of this news in the blog is significantly faster than that of an unknown person. Due to the existence of 1/4 of the most influential common users, it compensates for the influence of many isolated nodes in the CollegeMsg network, making the propagation range of the seed selection by CollegeMsg significantly higher than that of a single CollegeMsg network. This result can be verified in Fig. 6(b).

C. RESULTS ANALYSIS UNDER THE UA MODEL

1) THE COMMON USERS SELECTED RANDOMLY

Since the propagation probability of each node under the UA model is relatively small, this obviously leads to the propagation range of Fig. 7 and Fig. 8 being smaller than that of Fig. 5 and Fig. 6. Under the UA model, although the propagation range is smaller than that under the DWA model, the trend of the spread range of the curve and the relative position of each curve is consistent with the DWA model. That is, regardless of whether under the UA model or the DWA model, the meaning of the existence of courses is the same, and it can strengthen the influence of the information dissemination of two dynamic social networks.

In Fig. 7(a) and 7(b), we can observe another obvious phenomenon: the first half of the propagation range curve selected by the seed from the integrated network and the propagation range curve selected by the seed from a single CollegeMsg network almost coincide. This shows that the seeds selected by the two methods are almost the same in the first half and that the spread range is the same. In other words, when seeds are selected from the integrated network, almost all the initial seeds belong to the CollegeMsg network because the number of non-isolated nodes in the CollegeMsg network is greater than that in the Email network. The influence of selecting seeds from the CollegeMsg network is definitely higher than that of selecting seeds from the Email network. We conclude that when selecting seeds in the beginning, the nodes in the CollegeMsg network must be selected first. However, when the seeds are selected from the Email network alone and information is disseminated in the entire integrated network, this phenomenon does not occur. We can observe the evidence from the red star curve and purple dotted curve in Fig. 7(c).
From the above analysis, if the number of nodes in the two dynamic social networks is quite different, when the seeds are selected from the integrated network, the initial seed node is likely to be selected from the social network with more nodes because social networks with more nodes have more users and that users possess greater influence. This phenomenon is also obvious in the actual network. For example, if we want to promote a thing, we must choose social media with more network users.

There is a special phenomenon in Fig. 7(b) and 7(c). The purple triangle curve and the blue dotted curve in Fig. 7(b) almost overlap. Similarly, this situation emerges between the purple dotted curve and the cyan triangle curve in Fig. 7(c). This special phenomenon is not only due to the existence of isolated nodes in the networks but also the most important thing is that the propagation probability selected under the UA model is too small to cause the failure of information propagation.

2) 1/4 OF THE COMMON USERS WITH GREATER INFLUENCE ARE SELECTED

Similar to the DWA model, the role of 1/4 of the common users with greater influence greatly increases the possibility of communication between multiple dynamic social networks. As the propagation probability of nodes is small under the UA model, this effect is significantly smaller than that under the DWA model. However, because of the existence of these 1/4 of the common users with greater influence, the gap between the two curves increases. The first curve shows when the seeds are selected from a single social network and the information spreads in the integrated network. The second is that when the seed is selected from a single network and the information spreads in a single network. The result can be seen in the comparison of the purple triangle curve and the blue dotted curve in Fig. 7(b) and Fig. 8(b). Similarly, it can also be seen in the comparison of the purple triangle curve and the cyan dotted curve in Fig. 7(c) and Fig. 8(c).

D. SIMULATION TIME ANALYSIS UNDER DIFFERENT PROBABILITY MODELS

The simulation times under the DWA model and UA model are shown in Tables 5 and 6, respectively. In Tables 5 and 6, under different probability models, time1 represents the simulation time when 20 pairs of common users are randomly selected, and time2 represents the simulation time when 1/4 of the common users with greater influence are selected.

In the two tables, it can be seen that regardless of the probability model, the simulation time is the longest when the seed is selected from the entire integrated network and the information propagates in the integrated network. Moreover, it is longer than the sum of the simulation time of the Email network and the CollegeMsg network alone, which also shows that the common user makes the two networks interact. Therefore, the time in ‘S from whole’ is the longest. It is longer than the sum of the time in ‘S from Email’ and ‘S from CollegeMsg’.

Theoretically speaking, the simulation time of seeds selected in a single network and information spreads in the integrated network is longer than that of seeds selected in a single network and the information spread in a single network. This is also proved in the two tables. Both times of ‘S from Email’ are longer than ‘Email’. When 1/4 of the common users with greater influence are selected, both times of ‘S from CollegeMsg’ are longer than ‘CollegeMsg’.

However, when the seeds are selected randomly, the simulation time of ‘S from CollegeMsg’ is slightly longer than that of ‘CollegeMsg’ because randomly selected common users are not conducive to the dissemination of information from the CollegeMsg network to the Email network. This indirectly proves that if the seeds are only selected in the CollegeMsg network and the information spreads in the entire integrated network, the dissemination range does not change much compared with that of the seeds selected and the information spreads in the CollegeMsg network. As described in Fig. 5(b), there is little difference between the purple triangle and the blue dotted curve.

VII. CONCLUSION

To meet different social services, people have multiple social accounts simultaneously; that is, one person will participate in multiple social networks and spread information simultaneously. We discover that many social networks in reality have dynamic characteristics, and there is almost no research on the dynamic multisocial network influence maximization problem. Therefore, this paper studied the dynamic multisocial network influence maximization problem (DMNIMP) based on common users. According to the self-propagation characteristics of common users and the dynamic characteristics of the network, we established a dynamic multisocial network propagation model based on the dynamic independent cascade model. Through this model, the multiple dynamic networks can be merged into one dynamic network for processing; that is, the mutual self-propagation edges of common users are added to the snapshot of each frame of the integrated network. Thus, the $T \times one$ hop approach can be extended to dynamic multiple social networks. Above all, the experiments show that the proposed dynamic multisocial network propagation model can not only accurately and vividly characterize dynamic characteristics but also reflect the mutual influence of common users on multiple social networks. If common users are nodes with greater influence in each network, this

| TABLE 5. Simulation time analysis under the DWA model. |
|---------|----------------|----------------|-----------------|-----------------|-----------------|
| Dataset | S from Whole  | S from Email   | S from CollegeMsg | Email           | CollegeMsg      |
| time1   | 117.8s       | 92.8s         | 41.1s           | 20.1s          | 39.7s          |
| time2   | 133.7s       | 99.0s         | 50.4s           | 20.1s          | 39.7s          |

| TABLE 6. Simulation time analysis under the UA model. |
|---------|----------------|----------------|-----------------|-----------------|-----------------|
| Dataset | S from Whole  | S from Email   | S from CollegeMsg | Email           | CollegeMsg      |
| time1   | 58.1s         | 22.8s         | 42.7s           | 14.7s          | 41.8s          |
| time2   | 95.2s         | 49.8s         | 48.2s           | 14.7s          | 41.8s          |
interaction will be more obvious. This can be reflected not only in the simulation figures but also in the comparison of simulation times.

In future work, we can not only use the study in this paper to find the influence of the common users of dynamic multisocial networks, that is, the influence of the ‘bridge’ connecting two networks but also discover the influence of nodes that connect multiple communities with each other. Although this method is very similar to the measure of betweenness centrality, it is a method for finding important bridges from the perspective of propagation.

Although our dynamic multisocial network propagation model has many advantages, the $T \times one$ hop approach is essentially a heuristic algorithm. Facing the future of massive users, our method can hardly guarantee efficiency in a dynamic environment. We need to develop more efficient methods based on deep learning.

REFERENCES

[1] J. Zhang, P. S. Yu, and Z.-H. Zhou, “Meta-path based multi-network collective link prediction,” in Proc. 20th ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining (KDD), Aug. 2014, pp. 1286–1295.

[2] X. Kong, J. Zhang, and P. S. Yu, “Inferring anchor links across multiple heterogeneous social networks,” in Proc. 22nd ACM Int. Conf. Conf. Inf. Knowl. Manage. (CIKM), 2013.

[3] D. Scott, The New Rules of Marketing and PR: How to Use News Releases, Blogs, Podcasting, Viral Marketing, & Online Media to Reach Buyers Directly. Hoboken, NJ, USA: Wiley, 2010.

[4] W. Chen, C. Wang, and Y. Wang, “Scalable influence maximization for prevalent viral marketing in large-scale social networks,” in Proc. 16th ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining (KDD), 2010, pp. 1029–1038.

[5] N. Du, L. Song, M. G. Rodriguez, and H. Zha, “Scalable influence estimation in continuous-time diffusion networks,” in Proc. Adv. Neural Inf. Process. Syst., 2013, pp. 3147–3155.

[6] J. Goldenberg, B. Libai, and E. Muller, “Talk of the network: A complex systems look at the underlying process of word-of-mouth,” Marketing Lett., vol. 12, no. 3, pp. 211–223, 2001.

[7] X. He, G. Song, W. Chen, and Q. Jiang, “Influence blocking maximization in social networks under the competitive linear threshold model,” in Proc. SIAM Int. Conf. Data Mining, Philadelphia, PA, USA: SIAM, 2012, pp. 463–474.

[8] D. Kempe, J. Kleinberg, and É. Tardos, “Maximizing the spread of influence through a social network,” in Proc. 9th ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining (KDD), 2003, pp. 137–146.

[9] Q. Zhan, J. Zhang, P. S. Yu, S. Emery, and J. Xie, “Discovering influential users for cross network influence spreading (invited paper),” in Proc. IEEE 17th Int. Conf. Inf. Reuse Integrate (IRI), Jul. 2016, pp. 67–76.

[10] Q. Zhan, J. Zhang, S. Wang, P. S. Yu, and J. Xie, “Influence maximization across partially aligned heterogeneous social networks,” in Proc. PacificAsia Conf. Knowl. Discovery Data Mining, 2015, pp. 58–69.

[11] S. Banerjee, M. Jenamani, and D. K. Pratihar, “A survey on influence maximization in continuous-time diffusion networks,” in Proc. Adv. Neural Inf. Process. Syst., 2013, pp. 112–125, Feb. 2017.

[12] H. Zhuang, Y. Sun, J. Tang, J. Zhang, and X. Sun, “Influence maximization in social networks,” in Proc. 13th ACM Int. Conf. Data Mining, Dec. 2013, pp. 1313–1318.

[13] D. Kim, D. Hyeon, J. Oh, W.-S. Han, and H. Yu, “Influence maximization based on reachability sketches in dynamic graphs,” Inf. Sci., vol. 394–395, pp. 217–231, Jul. 2017.

[14] G. Song, Y. Li, X. Chen, X. He, and J. Tang, “Influential node tracking on dynamic social network: An interference greedy approach,” IEEE Trans. Knowl. Data Eng., vol. 29, no. 2, pp. 359–372, Feb. 2017.

[15] Y. Meng, Y. Yi, F. Xiong, and C. Pei, “$T \times one$ hop approach for dynamic influence maximization problem,” Phys. A, Stat. Mech. Appl., vol. 515, pp. 575–586, Feb. 2019.

[16] J. Zhang and P. S. Yu, Cross-Platform Social Network Analysis. New York, NY, USA: Springer, 2017.

[17] H. T. Nguyen, M. T. Thi, and T. N. Dinh, “A billion-scale approximation algorithm for maximizing benefit in viral marketing,” IEEE/ACM Trans. Netw., vol. 25, no. 4, pp. 2419–2429, Aug. 2017.

[18] S. Tang, “When social advertising meets viral marketing: Sequencing social advertisements for influence maximization,” in Proc. AAAI Conf. Artif. Intell., vol. 32, 2018, pp. 1–8.

[19] M. Apte, G. K. Palschikar, and S. Baskaran, “Frauds in online social networks: A review,” Social Netw. Survell. Soc., pp. 1–18, 2019.

[20] X. Meng, S. Wang, K. Shu, J. Li, B. Chen, H. Liu, and Y. Zhang, “Personalized privacy-preserving social recommendation,” in Proc. AAAI Conf. Artif. Intell., vol. 32, 2018, pp. 3796–3803.

[21] Q. He, X. Wang, Z. Lei, M. Huang, Y. Cai, and L. Ma, “TIFIM: A two-stage iterative framework for influence maximization in social networks,” Appl. Math. Comput., vol. 354, pp. 338–352, Aug. 2019.

[22] Q. He, X. Wang, M. Huang, J. Lv, and L. Ma, “Heuristics-based influence maximization for opinion formation in social networks,” Appl. Soft Comput., vol. 66, pp. 360–369, May 2018.

[23] Q. He, X. Wang, C. Zhang, M. Huang, and Y. Zhao, “IMOF: An iterative framework to settle influence maximization for opinion formation in social networks,” IEEE Access, vol. 6, pp. 49654–49663, 2018.

[24] Q. He, X. Wang, B. Yi, F. Mao, Y. Cai, and M. Huang, “Opinion maximization through unknown influence power in social networks under weighted voter model,” IEEE Syst. J., vol. 14, no. 2, pp. 1874–1885, Jun. 2020.

[25] Q. He, X. Wang, F. Mao, J. Lv, Y. Cai, M. Huang, and Q. Xu, “CAOM: A community-based approach to tackle opinion maximization for social networks,” Inf. Sci., vol. 513, pp. 252–269, Mar. 2020.

[26] F. Mao, L. Ma, Q. He, and G. Xiao, “Match making in complex social networks,” Appl. Math. Comput., vol. 371, Apr. 2020, Art. no. 124928.

[27] J. Leskovec, A. Krause, C. Guestrin, C. Faloutsos, J. VanBriesen, and N. Glance, “Cost-effective outbreak detection in networks,” in Proc. 13th ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining (KDD), 2007, pp. 420–429.
[42] J. Tang, X. Tang, and J. Yuan, “Influence maximization meets efficiency and effectiveness: A hop-based approach,” in Proc. IEEE/ACM Int. Conf. Adv. Social Netw. Anal. Mining, Jul. 2017, pp. 64–71.

[43] J.-G. Liu, R.-D. Li, Q. Guo, and Y.-C. Zhang, “Collective iteration behavior for online social networks,” Phys. A, Stat. Mech. Appl., vol. 499, pp. 490–497, Jun. 2018.

[44] P. Panzarasa, T. Opsahl, and K. M. Carley, “Patterns and dynamics of users’ behavior and interaction: Network analysis of an online community,” J. Am. Soc. Inf. Sci. Technol., vol. 60, no. 5, pp. 911–932, May 2009.

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