THE EFFICIENCY OF RF CURRENT DRIVE IN THE PRESENCE OF FAST PARTICLE LOSSES

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ABSTRACT. The effects of losses of the fast electron current carriers are included in a calculation of the efficiency of RF current drive. The analytical expressions obtained for the current drive efficiency \( J / P_A \) and bulk heating rate \( P_D / P_A \) are explicitly dependent on the fast electron confinement time, \( \tau_F \). In most present tokamak experiments, fast particle losses are predicted to reduce the current drive efficiency from 50% to more than an order of magnitude. However, in future fusion plasmas fast electron confinement appears to be sufficient to allow nearly ideal efficiencies to be achieved.

In tokamak experiments in which the plasma current is sustained by lower hybrid current drive, the current drive efficiency is generally found to be less than the efficiency predicted by Fisch and Boozer in Ref. [1]. The cause or causes for this apparent discrepancy are not as yet understood, and in order to provide credible extrapolation of present experiments to fusion plasma conditions it appears that refinements in the theory of Ref. [1] are necessary. Here the effects of losses of the fast electron current carriers are considered. The current drive efficiency, heating rate, and power loss rate are calculated, including the effect of a finite fast particle confinement time, \( \tau_F \). Fast particle confinement effects are also considered in the numerical code work reported in Ref. [2]. The efficiencies are calculated from response functions following the method of Ref. [1]. The response functions for the current, \( \chi_J \), collisionally dissipated power, \( \chi_D \), and power lost as a result of fast particle losses, \( \chi_L \), are given in Eqs (1) to (3):

\[
\chi_J = e \int_0^{\Delta t} \frac{dt}{\Delta t} v || \delta f
\]

(1)
\[ x_D = - \int_0^\Delta t \frac{dt}{\Delta t} \left[ \frac{dE}{dt} \right]_{\text{coll}} \delta f \]  
\[ x_L = - \int_0^\Delta t \frac{dt}{\Delta t} \left[ \frac{d\delta f}{dt} \right]_{\text{loss}} \]  

where \( E = (1/2) mv^2 \), \( v_\parallel \) is the velocity parallel to the magnetic field, \( (dE/dt)_{\text{coll}} \) is the collisional energy loss rate, \( (d\delta f/dt)_{\text{loss}} \) is the fast particle loss rate, and \( \Delta t \) is the averaging time which is taken long compared to the collisional and particle loss time-scales. The time integrals in Eqs (1) to (3) are taken along the velocity space trajectory \( \delta f \), which follows as a result of collisional slowing.

In treating fast particle losses the collisional power dissipated, \( P_D \), and the RF power absorbed, \( P_A \), must be distinguished. Absorption of a quantum of RF wave energy by the electron population \( \delta f \) causes \( \delta f \) to undergo a velocity space displacement in the direction \( s \). The time averaged power absorbed is given by:

\[ P_A = \mathcal{S} \cdot \nabla_V \left[ E \frac{\delta f_0}{\Delta t} \right] \]  

where \( \delta f_0 \) is the initial number of particles in the volume element \( d^3v \) affected by the wave absorption. In general, \( P_A \neq P_D \), where \( P_D \) is the power dissipated by collisions of the population \( \delta f \) with the bulk particles. The efficiency of interest gives the amount of current generated per unit of power absorbed; this efficiency is \( J/P_A \). The response functions in Eqs (1) to (3) yield the time averaged current \( J \), the collisionally dissipated power \( P_D \), and the power loss caused by fast particle losses:

\[ J = \mathcal{S} \cdot \nabla_V \chi_J \]  
\[ P_D = \mathcal{S} \cdot \nabla_V \chi_D \]  
\[ P_L = \mathcal{S} \cdot \nabla_V \chi_L \]

and the efficiencies of current drive, bulk heating, and power loss are given by ratios of the quantities in Eqs (5) to (7) to \( P_A \). Expressions (1) to (3) can be evaluated by writing the response function integrals in the velocity representation. Here, a dimensionless set of variables will be used with velocities normalized to \( v_\parallel = (T/m)^{1/2} \), and with times normalized to the bulk collision frequency,

\[ v_0 = \omega_{pe}^4 \ln \Lambda/(2\pi n_0 v^2) \]

the variables \( w \) and \( u \) are defined as usual,

\[ w = v_\parallel /v_\perp, \quad u^2 = (v_\parallel^2 + v_\perp^2)/v_\perp^2 \]

The change to the velocity representation is accomplished by using the collisional slowing, momentum loss, and particle confinement equations. The collisional energy loss of the population \( \delta f \) is described by the equation \( du/dt = -v_E u \), and in the high velocity limit of interest, \( v_E = 1/2 \cdot 1/u^3 \). The momentum loss rate is \( dw/dt = -\nu_m w \), where \( \nu_m = (2 + Z_i) v_E \) and \( Z_i \) is the ion charge state, and the fast particle loss rate \( (d\delta f/dt)_{\text{loss}} \) is given by:

\[ \delta f = (\delta f_0 \exp(-f_0 dt'/\tau_F)) \]  

with the solution \( \delta f = \delta f_0 \exp(-f_0 dt'/\tau_F) \).

In general, \( \tau_F \) may be a function of \( u \) and \( w \) and represents an arbitrary fast particle transport process which causes current carriers to be lost from the plasma. The response functions obtained by changing to velocity variables are:

\[ \chi_J = \frac{2w}{\Delta t} \left[ u \right] \left[ u \right] \left[ u \right] \left[ u \right] \left[ u \right] \exp \left[ 2 \left[ u \right] \left[ u \right] \left[ u \right] \left[ u \right] \right] \]  
\[ \chi_D = \frac{1}{\Delta t} \left[ u \right] \left[ u \right] \left[ u \right] \left[ u \right] \left[ u \right] \exp \left[ 2 \left[ u \right] \left[ u \right] \left[ u \right] \right] \]  
\[ \chi_L = \frac{1}{\Delta t} \left[ u \right] \left[ u \right] \left[ u \right] \left[ u \right] \left[ u \right] \exp \left[ 2 \left[ u \right] \left[ u \right] \right] \]
Using Eqs (9) to (11) in expressions (5) to (7) yields the efficiencies of current generation $J/P_A$, bulk heating $P_D/P_A$, and power loss $P_L/P_A$:

$$\frac{J}{P_A} = \frac{u}{s + v^2} \left( \frac{w}{u^2 + Z_1} \right) \int_0^u u^4 \exp \left( -\frac{u}{\tau_F} \right) du$$

$$\times \exp \left[ 2 \int_0^u \frac{u^{n-2}}{\tau_F} \frac{du}{u} \right]$$

$$= \frac{2}{s + v^2} \left( \frac{w}{u^2 + Z_1} \right) \int_0^u u^4 \exp \left( -\frac{u}{\tau_F} \right) \frac{du}{u}$$

$$\times \exp \left[ 2 \int_0^u \frac{u^{n-2}}{\tau_F} \frac{du}{u} \right]$$

$$= \frac{2}{s + v^2} \left( \frac{w}{u^2 + Z_1} \right) \int_0^u u^4 \exp \left( -\frac{u}{\tau_F} \right) \frac{du}{u}$$

$$\times \exp \left[ 2 \int_0^u \frac{u^{n-2}}{\tau_F} \frac{du}{u} \right]$$

(12)

(13)

(14)

Neglecting $v_2^2/v^2$ compared to $v^2/v^2$, Eq. (15) reduces to:

$$\frac{J}{P_A} = \frac{\tau_F}{w} \left[ 1 + \frac{3\tau_F}{w^3} \left[ 1 - \left(1 + \frac{w^3}{\tau_F} \right) \exp \left( -\frac{2w^3}{3\tau_F} \right) \right] \right]$$

(16)

FIG. 1. Current drive efficiency $J/P_A$ versus normalized parallel velocity $w$ for various values of $\tau_F$, the fast electron confinement time (in units of the bulk collisional slowing time, $\nu_0^2$).

In the limit $\tau_F \rightarrow \infty$, the Fisch-Boozer result $J/P_A = 4/3 w^2$ is recovered (note that $O(w^6)$ terms must be retained in the exponential in order to obtain the correct limit). The current drive efficiency as a function of $w$ is shown in Fig. 1 for various values of $\tau_F$, where $\tau_F$ is the fast particle confinement time in units of the bulk collisional slowing time $\nu_0^2$. As is shown in Fig. 1, at low velocities, $J/P_A$ approaches the ideal Fisch-Boozer efficiency. When $w$ is small the collisional slowing time is much faster than the particle loss time, so the ideal efficiency is achieved. At higher velocities, $J/P_A$ first increases then reaches a maximum and begins to decrease slowly with $J/P_A \sim \tau_F/w$ for large $w$.

The heating efficiency, $P_D/P_A$, and the power loss fraction, $P_L/P_A$, are obtained in this model in terms of the functions $\phi_n(x) = e^{-x^3} f_n^0 z^n e^{z^3} dz$.

Evaluating the integrals in Eqs (13) and (14) gives:

$$\frac{P_D}{P_A} = 1 - 3au \phi_1(au)$$

(17)

$$\frac{P_L}{P_A} = \frac{3}{2} au^3 - \frac{9}{2} au \phi_4(au)$$

(18)
where

\[ \alpha = \left( \frac{2}{3 \tau_F^2} \right)^{1/3} \]

\( P_D/P_A \) and \( P_L/P_A \) obey the conservation law \( P_D/P_A + P_L/P_A = 1 \); \( P_D/P_A \) is plotted in Fig. 2. Surprisingly, \( P_D/P_A \) reached negative values at sufficiently large \( w \), that is, an incremental increase in power absorption at large \( w \) can remove more energy because of increased particle losses than is supplied by the RF power absorbed. That the heating efficiency can be negative can be seen by considering a simple example. Suppose there is a velocity space loss process for which \( \tau_F = \infty \) for \( w \leq w_0 \) and \( \tau_F = 0 \) for \( w > w_0 \). A population of electrons \( \delta f \) located at \( w = w_0 \) will transfer all of its energy \( E_0 \) to the bulk by collisions in the absence of RF power absorption since \( \tau_F = \infty \) at \( w = w_0 \). If a quantum \( \Delta E_0 \) of RF energy is absorbed by these particles, the energy \( (E_0 + \Delta E_0) \delta f_0 \) is lost from the plasma since \( \tau_F = 0 \) for \( w > w_0 \), where the RF power absorption has pushed the population \( rf \) beyond \( w_0 \). The heating efficiency \( P_D/P_A \) of this process is therefore

\[ \left( E_{\text{coll}}^0 - E_{\text{coll}}^1 \right)/\Delta E_0 = -E_0/\Delta E_0 < 0 \]

where \( E_{\text{coll}}^0 \) and \( E_{\text{coll}}^1 \) are the energies dissipated via collisions, respectively, before and after absorption of the RF energy. If the velocity at which the power is absorbed is increased, the result will be increased power flow to the bulk and higher current drive efficiency only if fast particle loss processes are negligible, but if \( \tau_F \) is finite, then, at sufficiently high velocities, the collisional slowing time becomes long and the energy loss due to fast particle losses can dominate.

Before Eq. (16) can be applied to experiments, a value for the fast particle confinement time \( \tau_F \) is needed. Although \( \tau_F \) has been measured in only a few cases it appears that for the electron energies of present experiments, \( \tau_F \leq (1/2) \) MeV, \( \tau_F \) can be taken as approximately equal to the bulk electron energy confinement time, \( \tau_e \) [4]. In the case of a medium sized tokamak plasma with \( T_e = 1 \) keV, \( n_e = 1 \times 10^{13} \) cm\(^{-3} \), \( \tau_e = 5 \) ms, \( \tau_F \equiv \nu_F \tau_e = 10^3 \), and \( w = 10 \) for a typical fast electron velocity, Eq. (16) gives \( J/P_A = 67 \) as compared to \( J/P_A = 133 \) if the electron confinement were ideal. In this example, the energetic electron loss reduces the efficiency by approximately 50%. In smaller tokamaks, \( \tau_e \) is smaller, and typical parameters are \( T_e = 200 \) eV, \( n_e = 1 \times 10^{13} \) cm\(^{-3} \), \( \tau_e = 0.5 \) ms, \( \tau_F \equiv \nu_F \tau_e = 580 \), and \( w = 20 \). Equation (16) gives \( J/P_A = 35 \) as compared to the ideal confinement prediction of \( J/P_A = 532 \). In this case, the current drive efficiency is reduced by a factor of \( \sim 15 \). In most small to medium sized tokamak experiments, fast particle losses are therefore predicted to significantly degrade the current drive and bulk heating efficiencies (Figs 1 and 2).

In future fusion plasmas, the normalized fast particle confinement time is expected to be much improved over the above examples, and \( w \) values will tend to be smaller because of the increased bulk temperatures. These effects will tend to improve the current drive efficiency. For example, with \( T_e = 10 \) keV, \( n_e = 5 \times 10^{14} \) cm\(^{-3} \), \( \tau_e = 1 \) s, and \( w = 6 \), Eq. (16) predicts essentially the ideal efficiency. It appears that in moving to reactor plasma conditions significant improvements in the current drive efficiency can be anticipated in comparison to those achievable in most present experiments.

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