Coherent $\pi^0$ threshold production from the deuteron at $Q^2 = 0.1 \text{ GeV}^2/c^2$

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Abstract

First data on coherent threshold $\pi^0$ electroproduction from the deuteron taken by the A1 Collaboration at the Mainz Microtron MAMI are presented. At a four-momentum transfer of $q^2 = -0.1 \text{ GeV}^2/c^2$ the full solid angle was covered up to a center-of-mass energy of 4 MeV above threshold. By means of a Rosenbluth separation the longitudinal threshold $s$ wave multipole and an upper limit for the transverse threshold $s$ wave multipole could be extracted and compared to predictions of Heavy Baryon Chiral Perturbation Theory.

Keywords: Pion electroproduction; Threshold production; Deuteron
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1. Introduction

The electroproduction of neutral pions is one of the significant testing grounds of Chiral Perturbation Theory (ChPT, see [1,2] for an overview of the field).

The data of the production from the proton [3,4] are consistent with calculations of Heavy Baryon Chiral Perturbation Theory [5] and are in reasonable agreement with this approach for the free nucleon case. For this comparison still six low energy constants, introduced into ChPT by renormalizing counter terms, had to be adjusted to the data. However, for the free neutron a strong prediction without further freedom can be deduced from the proton production amplitude.
Unfortunately, no free neutron target exists and one has to cope as normal with the problem of a target bound in a complex nucleus. Therefore, a model is needed to separate the elementary production amplitudes and the nuclear effects introducing theoretical uncertainties. The most evident choice for investigating the neutron bound in a nucleus would be the quasi-free reaction. The momenta of the recoiling nucleons and the initial Fermi momenta are, however, of the same order and, therefore, the dynamics of the full reaction are not well under control. On the other hand, assuming that the proton production amplitude is known and that a reasonable model for the simplest nucleus is at hand today, the coherent production from the deuteron can be used. In this case, the initial and final Fermi motion can be included in the model description of the deuteron via structure form factors.

For the photoproduction case, a first coherent threshold measurement of $d(\gamma, \pi^0)d$ at SAL [6] extracted the transverse threshold $s$ wave multipole $|E_0|$ in good agreement with the predictions of a fourth order calculation in the framework of ChPTh [7]. In photoproduction the reaction is identified by the detection of the decay photons of the neutral pion, and the small incoherent break up channel has to be subtracted by model dependent assumptions. By contrast in an electroproduction experiment the recoil deuteron has to be detected due to large background of hard and soft photons, thus there is no incoherent contribution. The low kinetic energy of the heavy recoil deuteron (25 MeV < $T_d$ < 35 MeV) is the biggest challenge of such an experiment, the detection efficiencies and multiple scattering effects have to be measured at each state of the experiment.

On the other hand, the detection of the deuteron with a high resolution spectrometer allows for a good invariant mass resolution close to threshold. The focusing due to the Lorentz boost permits to cover the full solid angle in the center-of-mass system up to 4 MeV above threshold if one uses the large solid angle magnetic spectrometer A of of the A1 Collaboration with $\Delta \Omega = 21$ msr.

2. Kinematics

In Born approximation the virtual photon is defined by the four-vectors of incident electron $e = (E, \vec{k})$ and scattered electron $e' = (E', \vec{k}')$ as $q = e - e' = (q, \vec{q})$ (see figure 1). Denoting the variables in the photon-deuteron center-of-mass frame with an asterisk the cross section can be written as

$$\frac{d\sigma}{dE'd\Omega'_{\pi}} = \Gamma \frac{d\sigma}{d\Omega'_{\pi}}$$

with the virtual photon flux

$$\Gamma = \frac{\alpha}{2\pi^2} \frac{E'}{E} \frac{k_\gamma}{1 - \epsilon}$$

and the equivalent photon energy $k_\gamma = (W^2 - m_A^2)/2m_d$. The transverse and longitudinal degree of polarization of the virtual photon is given by

$$\epsilon = \left(1 - \frac{2(\omega^2 - q^2)}{q^2} \tan^2 \frac{\theta_e}{2}\right)^{-1}$$

$$\epsilon_L = \frac{q^2}{\omega^2}\epsilon.$$  \hspace{1cm} (3)

For the coherent production of pseudo scalar mesons from a spin 1 target the unpolarized differential cross section can be separated similarly to the production off a proton into four structure functions (see e.g. [8,9]):

$$\frac{d\sigma(\theta_\pi^*, \phi_\pi^*)}{d\Omega_\pi^*} = f_T(\theta_\pi^*) + \epsilon_L f_L(\theta_\pi^*)$$

$$+ \sqrt{2} \epsilon_L (1 - \epsilon) \cdot f_{TL}(\theta_\pi^*) \cdot \cos \phi_\pi^*$$

$$+ \epsilon \cdot f_{TT}(\theta_\pi^*) \cdot \cos 2\phi_\pi^*.$$  \hspace{1cm} (5)

In principle, the extraction of the transverse-longitudinal $f_{TL}$ and transverse-transverse interference structure functions $f_{TT}$ is possible by a measurement of the $\phi_\pi^*$ dependence of the differential cross section. In the presented experiment, however, the angular resolution of the detection of the
low energy deuterons was not sufficient to allow an extraction of these small structure functions.

The choice of the four-momentum transfer of $q^2 = -0.1 \text{GeV}^2/c^2$ as a lower limit was dictated by the detection efficiency of the deuteron as will be discussed in the next section. For this four-momentum transfer we chose three different values for the virtual photon polarization $\epsilon$ with the largest possible spread for a Rosenbluth separation. Table 1 summarizes the kinematical settings.

| $\epsilon$ | $E_0$ [MeV] | $E'$ [MeV] | $p_d$ [MeV/c] | $\theta_e$ [°] | $\theta_d$ [°] |
|------------|-------------|-------------|---------------|---------------|---------------|
| 0.854      | 720         | 554         | 339           | 29.00°        | 48.69°        |
| 0.590      | 435         | 269         | 339           | 55.10°        | 38.06°        |
| 0.364      | 345         | 178         | 339           | 79.22°        | 29.40°        |

3. Experimental Setup and Analysis

3.1. Particle Detection and Efficiencies

The experiment was performed at the three spectrometer setup of the A1 Collaboration at the MAMI accelerator (see ref. [10] for a detailed description of the setup). For the electron detection spectrometer B, a clamshell dipole spectrometer with an angular acceptance of 5.6 msr at a momentum resolution of $\Delta p/p = 10^{-4}$ was used. For the deuteron detection spectrometer A with a large solid angle of 21 msr was chosen.

A high power liquid deuteron target was used at luminosities of 15 MHz/µb, limited by the current in the drift chambers of the deuteron spectrometer. Special care had to be taken to minimize the pathlength of the deuterons in the target material. A long narrow cell of 4.8 cm length and 1 cm width was shifted towards the side of the scattered electrons. In this way an average pathlength of 3 mm for the deuterons in the target material could be achieved. In addition the beam had to be moved by a fast magnet across the target area in a time scale of several kHz to avoid boiling of the liquid deuterium. The wall of the target was built of a 10 µm Havar foil.

To further reduce multiple scattering and energy loss the vacuum of the scattering chamber was connected with the vacuum of the deuteron spectrometer. For the electron detection a focal plane detector consisting of 4 layers of vertical drift chambers for spatial resolution and two layers of scintillators for coincidence trigger and time of flight measurement was used. A halocarbon gas Čerenkov detector with an efficiency of 99.8% was used for the separation of electrons and charged pions dominantly produced off the target walls. For the deuteron detection, only one layer of scintillators after four layers of vertical drift chambers could be used, since the deuterons were already stopped after a pathlength of a few millimeters in the first scintillator layer.

The large energy loss and the deuteron loss due to nuclear reactions of the low energy deuterons made it necessary to monitor the efficiency of the deuteron detection very carefully. In order to estimate these effects we used the standard formulas for the energy loss [11] and the calculations of [12] for the contributions of nuclear reactions. Since these calculations are only valid with large restrictions in our energy range, we checked and calibrated in addition the deuteron detection efficiency by a coincidence measurement of the scattered electron and the deuteron in the elastic $d(e,e'd)$ reaction. We used several settings of the elastic line at different positions of the focal plane and compared the results with the known cross section in the parameterization of [13].

As expected from extensive simulations on the computer, we had to apply correction factors of the order of up to 1.5 for the lowest deuteron energies. Figure 2 shows this correction for the elastic measurements in comparison with the calculated elastic cross section.

3.2. Reaction Identification

The reaction $d(e, e'd)n^0$ was identified in two steps. First, the coincidence time was determined by measuring the time of flight of the deuteron and the electron and correcting it for the reconstructed path length inside the spectrometers and the measured momenta. The coincidence time resolution of 3.3 ns FWHM was limited by the uncertainty in the flight path reconstruction because of the large multiple scattering of the low energy deuterons.

After the timing cut, the missing mass was calculated from the measured four-vectors of incident electron $e$, scattered electron $e'$, and initial and final
Differential Cross Section $d\sigma/d\Omega [\mu\text{barn/sr}]

Fig. 2. The elastic cross section of the d(e,e'd) reaction at an incident energy of $E_0 = 420$ MeV. The solid line shows the known cross section calculated with the parameterization of [13]. The open circles show the coincidence measurement without efficiency correction, the solid circles show the measured cross section after full efficiency correction.

deuteron $d$ and $d'$ by $m^2_{\text{miss}} = (e + d - e' - d')^2$. Figure 3 shows the distribution of the missing mass before and after background subtraction. As can be seen, the $\pi^0$ can be identified with a missing mass resolution of 2.28 MeV/$c^2$ FWHM, which is again determined by the multiple scattering of the low energy deuterons in the wire chambers.

The effect of the radiative tail at higher missing masses was corrected by a simulation using the standard formulas of [14] in the peaking approximation.

Since the recoil deuteron is measured directly, no model dependent correction for the deuteron break up is necessary.

3.3. Systematic errors

The error of this experiment is dominated by the systematic errors. As known for threshold measurements the calibration of the measured electron momentum which is almost proportional to the center-of-mass energy causes the largest contribution.

Table 2

| Setting | $\epsilon = 0.854$ | $\epsilon = 0.590$ | $\epsilon = 0.364$ |
|---------|-------------------|-------------------|-------------------|
| $\Delta W$ | stat. | [\%] | sys. | [\%] | stat. | [\%] | sys. | [\%] | stat. | [\%] | sys. | [\%] |
| [MeV] | | | | | | | | | | | | |
| 0.5 | 7.5 | 22.7 | 6.9 | 20.3 | 7.1 | 22.9 |
| 1.5 | 3.7 | 12.7 | 3.2 | 9.1 | 3.8 | 10.4 |
| 2.5 | 2.9 | 6.0 | 2.8 | 3.3 | 3.0 | 4.1 |
| 3.5 | 2.8 | 3.6 | 3.0 | 3.7 | 3.2 | 4.8 |

The systematic error caused by the efficiency correction of the deuteron detection could be checked by our elastic calibration measurements and was determined to be 1.7%. Further the detector efficiencies, contributions of cuts and phase space integration, errors of the luminosity summation, and condensation on the cold target walls were taken into account.

The total systematical errors, together with the statistical errors, are compiled in table 2.
The measured differential cross section is compiled in table 3. For one setting ($\epsilon = 0.590$), figure 4 shows the data points, including the combined statistical and systematical errors. As expected, only angular structures up to $O(\cos^{2}\theta_{\pi}^*)$ appear and justify a fit with the assumption of only $s$ and $p$ waves contributing to the cross section. Figure 5 shows the total cross section and the result of a $\chi^2$ fit to the data with the assumption of a constant $s$ wave amplitude and $p$ wave amplitudes rising linearly with $p^*_n$. At the precision of these data, no cusp effects of the opening deuteron break up threshold at 2.2 MeV above threshold can be observed.

At present, only threshold calculations are available in the framework of ChPTh [15] which provide a prediction for the $s$ wave multipoles. From the fit we can extract the $s$ wave multipoles at threshold through the reduced threshold $s$ wave cross section

$$a_0 = \frac{3}{8} \frac{k^*_n}{p_n^*} \frac{d\sigma}{d\Omega^*_T} \bigg|_{\Delta W \to 0} = |E_d|^2 + \epsilon_L |L_d|^2$$

3. In this energy range the difference between a linear behavior as found empirically [6] and the theoretically predicted behavior $\sim p_n^*/\omega$ can be neglected.

Since $\epsilon_L \approx 9\epsilon$ the longitudinal amplitude $L_d$ contributes with a much higher weight to the $s$ wave cross section than $E_d$ and only an upper limit of one standard deviation can be extracted for $E_d$. From this analysis one gets

$$|E_d| \leq 0.42 \cdot 10^{-3}/m_{\pi}$$
$$|L_d| = (0.50 \pm 0.11) \cdot 10^{-3}/m_{\pi}$$

The classical Rosenbluth method, i.e. the total cross section plotted against the photon polarization $\epsilon$, represents a consistency check (Figure 6). For each energy bin a straight line fit was performed to separate $f_T$ by its offset and $f_L$ by its slope. At threshold, only $|L_d|^2$ contributes to the longitudinal cross section and can be determined by an extrapolation of $f_L$ to the threshold point. By this technique $|L_d| = (0.47 \pm 0.18) \cdot 10^{-3}/m_{\pi}$ is extracted in good agreement with the previous analysis. Again, the kinematically suppressed multipole $E_d$ cannot be determined.

Figure 7 shows the extracted $s$ wave multipoles in comparison with the prediction of Chiral Perturbation Theory [15]. These calculations are performed.
to third order in the chiral expansion and are shifted to reproduce the result of the fourth order calculation at the photon point [7]. The solid line shows the full calculation, for the dashed lines the calculated free neutron amplitude was varied by $\pm 10^{-3}/m_\pi$ to indicate the sensitivity of the calculation to this amplitude. The dash-dotted line shows the calculation without two body currents, i.e. only the amplitudes of the free nucleons folded with the pertinent deuteron form factors are included and no pion exchange between the two nucleons was taken into account. For this picture, we assumed the sign for $L_d$ to be the same as calculated in ChPTh.

As stated in [6], their transverse multipole $E_d$ is already at the photon point $\Delta E_d = 0.35 \cdot 10^{-3}/m_\pi$ above the ChPTh prediction [7]. A shift by this amount would make our result for $E_d$ consistent with the calculations of [15].

5. Summary

A first measurement of the coherent threshold electroproduction of neutral pions off the deuteron was performed and the differential cross section could be determined up to 4 MeV above threshold for three different values of the photon polarization. The absolute values of the $s$ wave multipoles were extracted and compared to the predictions of ChPTh. Although smaller than expected, our extracted upper limit for $|E_d|$ is consistent with ChPTh calculations. $|L_d|$ is overestimated by the theory by a factor of 2. More dramatic is this discrepancy in terms of cross sections: The reduced threshold $s$ wave cross section $a_0$ is one order of magnitude smaller than expected.

Both, in photo- and in electroproduction the measured differential cross sections allow tests of predictions for the contributing $p$ waves which are not yet calculated in the framework of ChPTh.

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The differential cross section. The cross section is integrated over the complete accepted out-of-plane angle $\phi_n$. 

| $\cos \theta^*_n$ | $\epsilon = 0.854$ | $\epsilon = 0.590$ | $\epsilon = 0.364$ |
|-----------------|-----------------|-----------------|-----------------|
| $0 \text{ MeV} < \Delta W < 1 \text{ MeV}$ | | | |
| $-0.917$ | $16.79 \pm 7.65$ | $10.76 \pm 4.17$ | $3.94 \pm 2.52$ |
| $-0.750$ | $20.14 \pm 8.22$ | $10.47 \pm 4.03$ | $4.37 \pm 2.58$ |
| $-0.583$ | $21.73 \pm 8.36$ | $5.17 \pm 2.81$ | $2.48 \pm 2.00$ |
| $-0.417$ | $6.45 \pm 4.46$ | $9.96 \pm 3.76$ | $8.07 \pm 3.30$ |
| $-0.250$ | $10.54 \pm 5.19$ | $8.91 \pm 3.51$ | $6.25 \pm 2.78$ |
| $-0.083$ | $15.04 \pm 6.42$ | $6.69 \pm 2.92$ | $7.38 \pm 3.00$ |
| $0.083$ | $6.22 \pm 3.98$ | $4.55 \pm 2.51$ | $4.52 \pm 2.28$ |
| $0.250$ | $9.97 \pm 4.98$ | $8.00 \pm 3.33$ | $8.47 \pm 3.35$ |
| $0.417$ | $7.40 \pm 4.50$ | $6.12 \pm 3.04$ | $5.00 \pm 2.45$ |
| $0.583$ | $13.89 \pm 5.95$ | $4.27 \pm 2.66$ | $8.08 \pm 3.31$ |
| $0.750$ | $8.43 \pm 4.66$ | $8.96 \pm 3.72$ | $5.16 \pm 2.51$ |
| $0.917$ | $4.31 \pm 3.18$ | $4.65 \pm 2.69$ | $4.24 \pm 2.02$ |
| $1 \text{ MeV} < \Delta W \leq 2 \text{ MeV}$ | | | |
| $-0.917$ | $28.14 \pm 6.43$ | $21.28 \pm 3.87$ | $16.13 \pm 3.31$ |
| $-0.750$ | $29.94 \pm 5.96$ | $23.95 \pm 4.16$ | $17.51 \pm 3.46$ |
| $-0.583$ | $29.05 \pm 6.55$ | $19.55 \pm 3.76$ | $11.71 \pm 2.69$ |
| $-0.417$ | $22.11 \pm 5.50$ | $20.41 \pm 3.86$ | $11.97 \pm 2.70$ |
| $-0.250$ | $30.73 \pm 6.75$ | $17.56 \pm 3.55$ | $10.05 \pm 2.45$ |
| $-0.083$ | $25.77 \pm 6.12$ | $14.82 \pm 3.26$ | $11.29 \pm 2.69$ |
| $0.083$ | $16.52 \pm 4.85$ | $16.41 \pm 3.52$ | $11.60 \pm 2.71$ |
| $0.250$ | $15.79 \pm 4.73$ | $18.96 \pm 3.80$ | $12.74 \pm 2.90$ |
| $0.417$ | $17.36 \pm 5.18$ | $15.89 \pm 3.40$ | $10.54 \pm 2.59$ |
| $0.583$ | $15.60 \pm 4.92$ | $16.11 \pm 3.57$ | $10.16 \pm 2.62$ |
| $0.750$ | $15.55 \pm 5.17$ | $18.59 \pm 4.07$ | $7.38 \pm 2.27$ |
| $0.917$ | $19.54 \pm 5.97$ | $11.53 \pm 3.33$ | $5.59 \pm 1.99$ |
| $2 \text{ MeV} < \Delta W \leq 3 \text{ MeV}$ | | | |
| $-0.917$ | $50.15 \pm 6.42$ | $41.17 \pm 3.93$ | $27.56 \pm 3.33$ |
| $-0.750$ | $40.63 \pm 5.67$ | $35.07 \pm 3.60$ | $26.64 \pm 3.22$ |
| $-0.583$ | $44.79 \pm 6.01$ | $33.97 \pm 3.66$ | $24.83 \pm 3.04$ |
| $-0.417$ | $37.20 \pm 5.55$ | $29.93 \pm 3.55$ | $22.36 \pm 2.90$ |
| $-0.250$ | $43.22 \pm 6.31$ | $28.80 \pm 3.85$ | $24.67 \pm 3.13$ |
| $-0.083$ | $36.12 \pm 5.79$ | $27.28 \pm 3.90$ | $17.91 \pm 2.90$ |
| $0.083$ | $34.45 \pm 5.96$ | $24.34 \pm 3.70$ | $17.80 \pm 2.98$ |
| $0.250$ | $28.86 \pm 5.46$ | $28.86 \pm 4.07$ | $22.75 \pm 3.33$ |
| $0.417$ | $31.57 \pm 5.97$ | $27.80 \pm 3.97$ | $17.19 \pm 2.94$ |
| $0.583$ | $31.43 \pm 6.61$ | $24.81 \pm 3.89$ | $13.91 \pm 2.72$ |
| $0.750$ | $19.19 \pm 5.80$ | $22.81 \pm 4.13$ | $15.64 \pm 2.89$ |
| $0.917$ | $20.34 \pm 7.45$ | $22.58 \pm 4.99$ | $10.57 \pm 2.66$ |
| $3 \text{ MeV} < \Delta W \leq 4 \text{ MeV}$ | | | |
| $-0.917$ | $70.78 \pm 6.66$ | $56.58 \pm 5.57$ | $49.62 \pm 5.51$ |
| $-0.750$ | $63.13 \pm 6.18$ | $46.77 \pm 4.78$ | $36.83 \pm 4.41$ |
| $-0.583$ | $56.53 \pm 6.06$ | $46.71 \pm 5.12$ | $33.87 \pm 4.33$ |
| $-0.417$ | $47.84 \pm 6.02$ | $45.64 \pm 5.64$ | $35.88 \pm 4.82$ |
| $-0.250$ | $57.50 \pm 7.04$ | $41.79 \pm 5.78$ | $34.65 \pm 5.08$ |
| $-0.083$ | $55.06 \pm 7.23$ | $22.19 \pm 4.95$ | $22.52 \pm 4.40$ |
| $0.083$ | $41.72 \pm 6.70$ | $31.30 \pm 5.77$ | $32.23 \pm 5.26$ |
| $0.250$ | $41.41 \pm 7.06$ | $45.12 \pm 6.52$ | $25.87 \pm 4.89$ |
| $0.417$ | $26.78 \pm 6.49$ | $30.65 \pm 5.92$ | $23.35 \pm 4.76$ |
| $0.583$ | $24.05 \pm 6.84$ | $32.06 \pm 6.23$ | $27.31 \pm 4.74$ |
| $0.750$ | $29.06 \pm 8.52$ | $29.70 \pm 6.98$ | $28.11 \pm 4.99$ |
| $0.917$ | $16.78 \pm 10.93$ | $23.53 \pm 8.55$ | $23.70 \pm 5.06$ |