Power Grid Network Evolutions for Local Energy Trading

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Abstract

The shift towards an energy Grid dominated by prosumers (consumers and producers of energy) will inevitably have repercussions on the distribution infrastructure. Today it is a hierarchical one designed to deliver energy from large scale facilities to end-users. Tomorrow it will be a capillary infrastructure at the Medium and Low Voltage levels that will support local energy trading among prosumers. In [74], we analyzed the Dutch Power Grid and made an initial analysis of the economic impact topological properties have on decentralized energy trading. In this paper, we go one step further and investigate how different networks topologies and growth models facilitate the emergence of a decentralized market. In particular, we show how the connectivity plays an important role in improving the properties of reliability and path-cost reduction. From the economic point of view, we estimate how the topological evolutions facilitate local electricity distribution, taking into account the main cost ingredient required for increasing network connectivity, i.e., the price of cabling.

Keywords: Power Grid, Decentralized energy trading, Complex Network Analysis

1 Introduction

Something is changing both in the way energy is produced and distributed due to the combined effects of technological advancements and introduction of new policies. In the last decades a clear trend has invested the energy sector, that of unbundling. That is the process of dismantling monopolistic and oligarchic energy system, by allowing a greater number of parties to operate in a certain role of the energy sector and market. The goal of unbundling is that of reducing costs for the end-users and providing better services through competition (e.g., [28, 52]). At the same time from the technological perspective, new energy generation facilities (mainly based on renewable sources) are becoming more and more accessible. These are increasingly convenient and available at both the industrial and the residential scale [64, 62]. The term Smart Grid, which does not yet have a unique agreed definition [10, 69], is sometimes used to define the new scenario of a Grid with a high degree of delocalization in the production and exchange of energy. The new actors, who are both producers and consumers of energy, also known as prosumers, are increasing in number and will most likely demand a market with total freedom for energy trading [88]. In this coming scenario, the main role of the High Voltage Grid may change, while the Distribution Grid (i.e., Medium Voltage and Low Voltage end of the Power Grid) becomes more and more important, while requiring a major update. In fact, the energy interactions between prosumers will increase and most likely occur at a rather local level, therefore involving the Low and Medium Voltage Grids. This evolution of the energy sector will inevitably call for an upgrade of the enabling distribution infrastructure so to facilitate local energy exchanges. An infrastructure comparable more to a “peer-to-peer” system on the Internet, rather than the current strictly hierarchical system. But how will the infrastructure evolve to enable and follow this trend?

In [71] we laid the foundation for a statistical study of the Medium and Low Voltage Grid with the aim of identifying salient topological properties of the Power Grid that affect decentralized energy exchange. We based that study on real samples of the Dutch Grid and provided an initial economic analysis of the possible barriers to delocalized trades. In this follow up paper, we go one step beyond and consider growth models for network topologies providing an analysis of which models suit best the purpose of local energy exchange. The tool for our study is Complex Network Analysis (CNA) [6]. In particular, in the present case we use CNA as a synthesis
tool by synthesizing networks using topological models coming from the literature of modeling technological, infrastructural and social network evolutions.

In order to evaluate the adequacy of the generated networks, we develop a set of metrics that capture the various aspects that networks suited for small-scale energy exchange need to satisfy. It is then quite straightforward to compare the results of the synthetic models with the real samples analyzed in \cite{74} and on that ground propose network models that best suit a prosumer-based local energy exchange. Finally, a quantitative evaluation of how the improvement in the topology directly influences electricity prices is then possible.

We remark the novelty of this proposal with respect to previous CNA studies of the Power Grid. In fact, CNA has been used only on the High Voltage networks to get information on resilience to failures, and the Medium and Low Voltage Grids have been mostly ignored. Another novelty is the use of Complex Network Analysis not as a tool for pure analysis of the existing, but to exploit it as a design tool for an infrastructure. Using Graph Theory in the design of distribution systems is not completely new, several studies have incorporated Graph Theory elements in operation research techniques for Grid planning \cite{29, 88}, but never, to the best of our knowledge, has Graph Theory been combined with global statistical measures to design the Grid. In addition, we ground the design methods to investments by taking into account costs of Grid cabling based on the types of cables typically used in real Distribution Networks (i.e., Norther Netherlands Medium and Low Voltage network samples). In summary, the paper proposes which topologies according to CNA-based metrics are best suited in terms of performance and reliability of the infrastructure for a local energy exchange, give an estimation of the cabling cost for the realization of such topologies and assess the advantages from the electricity distribution point of view of the proposed topologies compared to the actual ones.

The paper is organized as follows. We open by analyzing the motivations for a new energy landscape and the required changes to the current Grid in Section 2. The background of Graph Theory necessary for the present study is presented in Section 3. Section 4 describes the main properties of the graph models; while the metrics exploited to compare the properties of the various generated graphs are described in Section 5. The analysis and discussion of the results is presented in Section 6. An overall discussion and illustration taking into account benefits and costs of evolution of topologies are considered in Sections 7. Section 8 reviews the main approaches to Electrical Grid and System design and evolution, while Section 9 provides a conclusion of the paper. A series of appendixes is included to provide extended coverage of topics related to the core of the paper, in particular, Appendix A describes statistical properties of power cables’ price and resistance; Appendix B provides an overview about the relationship between network topology a electricity price; appendix section concludes with Appendix C which describes an engineering process based on a Complex Network Analysis that can guide Grid and energy operators to shape their networks for the new local energy exchange paradigm.

2 The Need for Evolution of the Grid

In the XIX century electrical energy generation was considered a natural monopoly. The cheapest way to produce electricity was in big power plants and then transmitting it across a country through a pervasive network of cables operated by a monopolistic state owned company. The situation has changed and now more and more companies are present in the energy business from energy production, to energy transmission and distribution, to retail and service provided to the end-user. To enable and accelerate this process, governments in the western world have promoted policies to open the electricity business and facilitate competition with the final aim to both modernize the energy sector and provide a more convenient service for the end-user. Even more on this path of enabling everybody to be a producer of energy is the possibility (sometimes incentivized with governments’ policies) to have small-scale energy generation units such as photovoltaic panels, small-wind turbines and micro combined heat-and-power systems (micro-CHP) which are now all widely available and affordable for the end-user market.

Such small-scale approach is beneficial to the electricity system in many ways: from reduced losses since source and load are closer, to system modularity, to smaller investments compared to large-scale energy solutions \cite{52}. Local generation based on renewables is a boost for the transition towards a renewable-based energy supply. In fact, end-users generate their own energy and the additional supply is likely to be provided by other end-users in the neighborhood that have energy surplus generated by their renewable-based generating equipment. In such a context with many small-scale producers and still without an efficient and cheap energy storage technology a local energy exchange at the neighborhood or municipal level between end-users is foreseeable and desirable. Micro-grids increased performance in terms of reduced losses and power quality have been successfully tested \cite{57, 79}, but little attention has been devoted to the network topology of these type of Grids.

In the evolution of the actual electricity system to the Internet of Energy \cite{11, 88}, the end-users are producers in addition to the normal consumer role they have always had. Energy is then something that the user does no more need to negotiate through yearly or lifetime contracts, but can be traded between prosumers and consumers on a fully electronic and automated market. These energy exchanges are likely to be local inside a neighborhood, a village or a city where users that have small-scale renewable-based producing facilities can sell their energy surplus. This solution represents a “win-win” solution first for the environment envisioning more energy generated with renewable sources, and second for the prosumer who sells his surplus energy on the market obtaining some
profits out of it. This latter aspect helps accelerating the return on investment made in purchasing the generating
equipment. A benefit is also for the end-user that has more flexibility in choosing his energy provider and takes
advantage of the cheaper tariffs of the prosumers. The traditional energy providers and distributors still play
important roles even in this paradigm: the former provide traditional supply where or when prosumers are not
available, the latter has even a more critical role in monitoring and providing a Grid at Medium and Low Voltage
that is efficient, failure resistant and that satisfies the needs of this new energy exchange paradigm.

This future scenario might impact deeply on the actual Grid infrastructure especially the Medium and Low
Voltage section where the prosumer and consumer exchanges will happen. The Grid infrastructure with the
associated reliability, losses, quality, performances and related transmission costs might act as an enabler or
repression of the local energy exchange. The Medium and Low Voltage Grid is likely to face important changes
in its infrastructure to support Smart Grid [17] and even more in enabling a scenario where energy producers
are many and the interactions are at local scale. Usually, the lower end of the Grid have been considered of small
importance and less critical than the High Voltage infrastructures, however the tendency is likely to be reversed
in a prosumer-based energy paradigm.

In our study we resort to Complex Network Analysis, a branch of Graph Theory taking its root in the
eyear studies of Erdős and Rényi [30] on random graphs and considering statistical structural properties of very
large graphs. Although taking its root in the past, Complex Network Analysis (CNA) is a relatively young
field of research. The first systematic studies appeared in the late 1990s [94, 85, 8, 4] having the goal of
looking at the properties of large networks with a complex systems behavior. Afterwards, Complex Network
Analysis has been used in many different fields of knowledge, from biology [51] to chemistry [34], from linguistics
to social sciences [77], from telephone call patterns [1] to computer networks [39] and web [3, 33] to virus
spreading [55, 27, 43] and also inter-banking systems [13]. Men-made infrastructures are especially interesting to study under the Complex Network Analysis lenses, especially when they are large scale
and grow in a decentralized and independent fashion, thus not being the result of a global, but rather of many
local autonomous designs. The Power Grid is a prominent example. In this work we consider a novel approach
both in considering Complex Network Analysis tools as a design instrument (i.e., CNA-related metrics are used
in finding the most suited Medium and Low Voltage Grid for local energy exchange) and in focusing on the
Medium and Low Voltage layers of the Power Grid. In fact, traditionally, Complex Network Analysis studies
applied to the Power Grid only evaluate reliability issues and disruption behavior of the Grid when nodes or
edges of the High Voltage layer are compromised.

In summary, the requirements of the new Power Grid enabling decentralized trading are:

1. Realizing the small-scale network paradigm;
2. Improving local energy exchange;
3. Supporting renewable-based energy production;
4. Encouraging the end-user (technically, economically and politically) to buy/sell energy locally;
5. Realizing networks easy to repeat at different scales (i.e., neighborhood, small village, city, metropolis)
6. Reducing losses in the Medium and Low Voltage end of the Grid; and
7. Enabling smartness in the automation of energy exchanges and their accounting.

In the above general requirements several are tightly connected with the topology of the network, while others
are more related to the control and ICT-oriented aspects of the Smart Grid. For the former aspects we provided
a first investigation in our previous work [74] and in the present work; the latter aspects are out of the scope of
the present work and can be traced to other investigations such as [19, 18, 82, 78].

3 Graph Theory Background

The approach used in this work to model the Power Grid and its evolution is based on Graph Theory and
Complex Networks. Here we recall the basic definitions that we use throughout the paper and refer to standard
textbooks such as [11, 12] for a broader introduction. First we define a graph for the Power Grid [73].

**Definition 1** (Power Grid graph). A Power Grid graph is a graph $G(V, E)$ such that each element $v_i \in V$ is
either a substation, transformer, or consuming unit of a physical Power Grid. There is an edge $e_{i,j} = (v_i, v_j) \in E$
between two nodes if there is physical cable connecting directly the elements represented by $v_i$ and $v_j$.

One can also associate weights to the edges representing physical cable properties (e.g., resistance, voltage,
supported current flow).
Definition 2 (Weighted Power Grid graph). A Weighted Power Grid graph is a Power Grid graph $G_w(V,E)$ with an additional function $f: E \rightarrow \mathbb{R}$ associating a real number to an edge representing the physical property of the corresponding cable (e.g., the resistance, expressed in Ohm, of the physical cable).

A first classification of graphs is expressed in terms of their size.

Definition 3 (Order and size of a graph). Given the graph $G$ the order is given by $N = |V|$, while the size is given by $M = |E|$.

From order and size it is possible to have a global value for the connectivity of the vertexes of the graph, known as average node degree. That is $<k> = \frac{2M}{N}$. To characterize the relationship between a node and the others it is connected to, the following properties provide an indication of the bond between them.

Definition 4 (Adjacency, neighborhood and degree). If $e_{x,y} \in E$ is an edge in graph $G$, then $x$ and $y$ are adjacent, or neighboring, vertexes, and the vertexes $x$ and $y$ are incident with the edge $e_{x,y}$. The set of vertexes adjacent to a vertex $x \in V$, called the neighborhood of $x$, is denoted by $\Gamma_x$. The number $d(x) = |\Gamma_x|$ is the degree of $x$.

A measure of the average ‘density’ of the graph is given by the clustering coefficient, characterizing the extent to which vertexes adjacent to any vertex $v$ are adjacent to each other.

Definition 5 (Clustering coefficient (CC)). The clustering coefficient $\gamma_v$ of $\Gamma_v$ is

$$\gamma_v = \frac{|E(\Gamma_v)|}{\left(\frac{N_v}{2}\right)}$$

where $|E(\Gamma_v)|$ is the number of edges in the neighborhood of $v$ and $\left(\frac{N_v}{2}\right)$ is the total number of possible edges in $\Gamma_v$.

This local property of a node can be extended to an entire graph by averaging over all nodes.

Another important property is how much any two nodes are far apart from each other, in particular the minimal distance between them or shortest path. The concepts of path and path length are crucial to understand the way two vertexes are connected.

Definition 6 (Path and path length). A path of $G$ is a subgraph $P$ of the form:

$$V(P) = \{x_0, x_1, \ldots, x_l\}, \quad E(P) = \{(x_0, x_1), (x_1, x_2), \ldots, (x_{l-1}, x_l)\}.$$

such that $V(P) \subseteq V$ and $E(P) \subseteq E$. The vertexes $x_0$ and $x_l$ are end-vertexes of $P$ and $l = |E(P)|$ is the length of $P$. A graph is connected if for any two distinct vertexes $v_i, v_j \in V$ there is a finite path from $v_i$ to $v_j$.

Definition 7 (Distance). Given a graph $G$ and vertexes $v_i$ and $v_j$, their distance $d(v_i, v_j)$ is the minimal length of any $v_i - v_j$ path in the graph. If there is no $v_i - v_j$ path then it is conventionally set to $d(v_i, v_j) = \infty$.

Definition 8 (Shortest path). Given a graph $G$ and vertexes $v_i$ and $v_j$ the shortest path is the path corresponding to the minimum of to the set $\{|P_1|, |P_2|, \ldots, |P_k|\}$ containing the lengths of all paths for which $v_i$ and $v_j$ are the end-vertexes.

A global measure for a graph is given by its average distance among any two nodes.

Definition 9 (Average path length (APL)). Let $v_i \in V$ be a vertex in graph $G$. The average path length for $G$ $L_{av}$ is:

$$L_{av} = \frac{1}{N \cdot (N - 1)} \sum_{i \neq j} d(v_i, v_j)$$

where $d(v_i, v_j)$ is the finite distance between $v_i$ and $v_j$ and $N$ is the order of $G$.

Definition 10 (Characteristic path length (CPL)). Let $v_i \in V$ be a vertex in graph $G$, the characteristic path length for $G$, $L_{cp}$ is defined as the median of $d_{v_i}$, where:

$$d_{v_i} = \frac{1}{(N - 1)} \sum_{i \neq j} d(v_i, v_j)$$

is the mean of the distances connecting $v_i$ to any other vertex $v_j$ in $G$ and $N$ is the order of $G$.
To describe the importance of a node with respect to minimal paths in the graph, the concept of betweenness helps. Betweenness (sometimes also referred as load) for a given vertex is the number of shortest paths between any other nodes that traverse it.

**Definition 11 (Betweenness).** The betweenness \( b(v) \) of vertex \( v \in V \) is

\[
    b(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}
\]

where \( \sigma_{st}(v) \) is 1 if the shortest path between vertex \( s \) and vertex \( t \) goes through vertex \( v \), 0 otherwise and \( \sigma_{st} \) is the number of shortest paths between vertex \( s \) and vertex \( t \).

Looking at large graphs, one is usually interested in global statistical measures rather than the properties of a specific node. A typical example is the node degree, where one measures the node degree probability distribution.

**Definition 12 (Node degree distribution).** Consider the degree \( k \) of a node in a graph as a random variable. The function \( N_k = \{ v \in G : d(v) = k \} \) is called probability node degree distribution.

The shape of the distribution is a salient characteristic of the network. For the Power Grid, the shape is typically either exponential or a Power-law \([8, 5, 74, 80]\). More precisely, an exponential node degree \( (k) \) distribution has a fast decay in the probability of having nodes with relative high node degree. The relation:

\[
    P(k) = \alpha e^{\beta k}
\]

follows, where \( \alpha \) and \( \beta \) are parameters of the specific network considered. On the contrary, a Power-law distribution has a slower decay with higher probability of having nodes with high node degree. It is expressed by the relation:

\[
    P(k) = \alpha k^{-\gamma}
\]

where \( \alpha \) and \( \gamma \) are parameters of the specific network considered. We remark that the graphs considered in the Power Grid domain are usually large, although finite, in terms of order and size thus providing limited and finite probability distributions.

A Graph can also be represented as a matrix, typically an adjacency matrix.

**Definition 13 (Adjacency matrix).** The adjacency matrix \( A = A(G) = (a_{ij}) \) of a graph \( G \) of order \( N \) is the \( N \times N \) matrix given by

\[
    a_{ij} = \begin{cases} 
    1 & \text{if } (v_i, v_j) \in E, \\
    0 & \text{otherwise.}
    \end{cases}
\]

We have now provided the basic definitions needed to present the modeling tools for the Power Grid evolutions.

## 4 Modeling the Power Grid

To address the question of what are the best suited topologies to characterize the Medium and Low Voltage Grids, we study models for graph generation proposed for technological complex networks. For each model we evaluate the properties of the network for several values of the order of the graph. Following our analysis of the Northern Dutch Medium and Low Voltage \([74]\), we categorize networks as Small, Medium and Large, see Table 1.

We then analyze the properties of the networks coming from the generated models by applying relevant Complex Network Analysis metrics and combine them appropriately. In this way, Complex Network Analysis is not only a tool for analysis, but it becomes a design tool for the future electrical Grid.

Most studies using Complex Network Analysis focus on extracting properties of networks arising from natural phenomena (e.g., food webs \([35]\), protein interactions \([51]\), neural networks of microorganism \([93]\)), and human generated networks (computer networks \([39]\), the web \([3]\), transport systems \([47]\)) to try to understand which underlying rules characterize them. Here we look at network models that have proven successful in showing salient characteristics of technological networks (i.e., preferential attachment, Copying Model, power-law networks), social networks (i.e., small-world, Kronecker graph, recursive matrix) and natural phenomena as well (e.g., Random Graph, small-world, Forest Fire) to investigate which one is best suited for supporting local-scale energy exchange form a topological point of view. Next we provide a brief introduction to all the models used in the present study, while a more in-depth presentation is available for instance in \([20]\) or \([72]\).
### Random Graph

A Random Graph is a graph built by picking nodes under some random probability distribution and connecting them by edges. It is due to the pioneering studies of Erdős and Rényi [36, 37]. More precisely, there are two ways to build a Random Graph, (a) the $G_{N,p}$ model proposed by Erdős and Rényi considers a set of $N$ nodes and for each pair of nodes an edge is added with a certain probability $p$; (b) the $G_{N,M}$ model considers with equal probability all the graphs having $N$ vertexes and exactly $M$ edges randomly selected among all the possible pairs of edges. The models have the same asymptotic properties. We use the $G_{N,M}$ model since we are interested in setting both the number of nodes and edges for the networks to generate. A Random Graph with order 199 and size 400 is shown in Figure 1.

![Figure 1: A Random Graph.](image)

### Small-world Graph

The small-world phenomenon became famous after the works of Milgram in the sociological context [87] who found short chains of acquaintances connecting random people in the USA. More recently, the small-world characterization of graphs has been investigated by Watts and Strogatz [93, 94] who showed the presence of the small-world property in many types of networks such as actor acquaintances, the Power Grid infrastructure and neural networks in worms. It is obtained from a regular lattice that connects the nodes followed by a process of rewiring the edges with a certain probability $p \in [0, 1]$. The resulting graph has intermediate properties between the extreme situations of a regular lattice ($p = 0$) and a random graph ($p = 1$). In particular, small-world networks hold interesting properties: the characteristic path length is comparable to the one of a corresponding random graph ($L_{sw} \gtrsim L_{random}$), while the clustering coefficient has a value bigger than a random graph and closer to the one of a regular lattice ($CC_{sw} \gg CC_{random}$). A small-world Graph with order 200 and size 399 is shown in Figure 2.

![Figure 2](image)
Preferential Attachment

The preferential attachment model represents the phenomenon happening in real networks, where a fraction of nodes has a high connectivity while the majority of nodes has small node degree. This model is built upon the observation by Barabási and Albert [8] of a typical pattern characterizing several type of natural and artificial networks. The basic idea is that whenever a node is added to the network and connects (through edges) to $m$ other nodes, those with higher degree are preferred for connection. In other words, the probability to establish an edge with an existing node $i$ is given by $P(k_i) = \frac{k_i}{\sum_j k_j}$ where $k_i$ is the node degree of node $i$. One can see then that the more connected nodes have higher chances to acquire more and more edges over time in a sort of “rich gets richer” fashion; a phenomenon studied by Pareto [77] in relation to land ownership. The preferential attachment model reaches a stationary solution for the node degree probability that follows a power-law with $P(k) = \frac{2m^2}{k^3}$. A graph based on preferential attachment with order 200 and size 397 is shown in Figure 3.
R-MAT

R-MAT (Recursive MATrix) is a model that exploits the representation of a graph through its adjacency matrix\cite{21}. In particular, it applies a recursive method to create the adjacency matrix of the graph, thus obtaining a self-similar graph structure. This model captures the community-based pattern appearing in some real networks. Moreover, the generated graph is characterized by a power-law node degree distribution while showing a small diameter. The idea is to start with an empty $N \times N$ matrix and then divide the square matrix into four partitions in which the nodes are present with a certain probability for each partition, specifically probabilities $a, b, c, d$ that sum to one. The procedure is then repeated dividing each partition again in four sub-partitions and associating the probabilities. The procedure stops when a $1 \times 1$ cell is reached in the iterative procedure. The $a, b, c, d$ partitions of the adjacency matrix have particular meaning: $a$ and $d$ represent the portions containing nodes belonging to different communities, while $b$ and $c$ represent the nodes that act as a link for the different communities (e.g., in a social network people with interests both in topics mostly popular in either $a$ or $d$ community). The recursive nature of this algorithm creates a sort of sub communities at each round. A graph based on R-MAT model with order 222 and size 499 is shown in Figure 4.

Models Independent from the Average Node Degree

When generating certain models there is no explicit dependence on the average node degree, these include Random Graph with power-law model, Copying Model, Forest Fire and Kronecker Graph which are presented next.

Random Graph with Power-law

A Random Graph with power-law model generates networks characterized by a power-law in the node degree distribution ($P(k) \sim k^{-\gamma}$) having the majority of nodes with a low degree and a small amount of nodes with a very high degree. Power-law distributions are very common in many real life networks both created by natural processes (e.g., food-webs, protein interactions) and by artificial ones (e.g., airline travel routes, Internet routing, telephone call graphs),\cite{6}. The types of networks that follow this property are also referred to as \textit{Scale-free networks} (\cite{9,7,4}). From the dynamic point of view, these networks are modeled by a preferential attachment model. In addition, reliability is a property of these graphs, that is, high degree of tolerance to random failures and high sensitivity to targeted attacks towards high degree nodes or hubs \cite{4,68,30}. This model is characterized by the exponent of the power-law (i.e., $\gamma$) which governs the degree of each node. The edges between the nodes are then wired in a random fashion. As we have shown earlier, the other way of constructing a graph that is compliant with a power-law based node degree distribution is through the growth of the network and preferential attachment based on node degree. A Random Graph with power-law with order 200 and size 399 is shown in Figure 4.

Copying Model

Replicating the structure underlying the links of WWW pages brought the development of the Copying Model \cite{56} capturing the tendency of members of communities with same interests to create pages
with a similar structure of links. The basic intuition is to select a node and a number \( k \) of edges to add to the node. Then with a certain probability \( \beta \), the edges are linked independently and uniformly at random to \( k \) other nodes, while with probability \( 1 - \beta \) the \( k \) edges are copied from a randomly selected node \( u \). If \( u \) has more than \( k \) edges, a subset is chosen, while if it has less than \( k \) edges they are anyway copied and the remaining are copied from another randomly chosen node. It leads to a distribution for the incoming degree that follows a power-law with a characteristic parameter \( \gamma_{in} = \frac{1}{1-\beta} \). A graph based on Copying Model with order 200 and size 199 is shown in Figure 6.

**Forest Fire** In order to capture dynamic aspects of the evolution of networks, Leskovec et al. [61] proposed the Forest Fire model. The intuition is that networks tend to densify in connectivity and shrink in diameter (i.e., the greatest shortest path in the network) during the growth process; technological, social and information networks show this phenomenon in their growth process. The model requires two parameters known as forward burning probability \( (p) \) and backward burning ratio \( (r) \). The graph grows over time and at each discrete time step a node \( v \) is added, then a node \( w \), known as ambassador, is chosen at random between the other nodes of the graph and a link between \( v \) and \( w \) is added. A random number \( x \) (obtained from a binormal distribution with mean \( (1 - p)^{-1} \)) is chosen and this is the number of out-links of node \( w \) that are selected. Then a fraction \( r \) times less than the out-links is chosen between the in-links and an edge is created with these as well. The process continues iterating choosing a new \( x \) number for each of the nodes \( v \) is now connected to. The idea, as the name of the model suggests, resembles the spreading of a fire in a forest that starts from the ambassador node to a fraction (based on the probability parameters) of nodes it is connected to and goes on in a sort of chain reaction. This model leads to heavy tails both in the distribution of in-degree and out-degree node degree. In addition, a power-law is shown in the densification process: a new coming node tends to have most of his links in the community of his ambassador and just few with other nodes. A graph based on Forest Fire model with order 200 and size 505 is shown in Figure 7.

**Kronecker Graph** A generating model with a recursive flavor similar to R-MAT uses the Kronecker product applied to the adjacency matrix of a graph [60]. The Kroneker product is a non conventional way of multiplying two matrices.

**Definition 14** (Kronecker product). Given two matrices \( A \) and \( B \) with dimension \((n \times m)\) and \((n' \times m')\) the Kroneker product between \( A \) and \( B \) is a matrix \( C \) with dimension \((n \times n') \times m \times m'\) with the following structure:

\[
C = A \otimes B = \begin{pmatrix}
    a_{1,1}B & a_{1,2}B & \cdots & a_{1,m}B \\
    a_{2,1}B & a_{2,2}B & \cdots & a_{2,m}B \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n,1}B & a_{n,2}B & \cdots & a_{n,m}B
\end{pmatrix}
\]
Figure 6: A Copying Model graph.

Figure 7: A Forest Fire graph.
**Definition 15** (Kronecker Graph). *Given two graphs $G$ and $H$ with adjacency matrices $A(G)$ and $A(H)$, a Kronecker graph is a graph whose adjacency matrix is obtained by the Kronecker product between the adjacency matrices of $G$ and $H*.

If the Kronecker product is applied to the same matrix, therefore multiplying the matrix with itself in the Kronecker product fashion, a self similar structure arises in the graph. This situation can be seen as the increase of a community in a network and the further differentiation in sub-communities while the network grows. This model creates networks that show a densification in the connectivity of its nodes, which provides a shrinking diameter over time. The idea is to apply the Kronecker product to the same matrix recursively. The procedure to create a graph based on the Kronecker product starts with a $N \times N$ matrix where each $x_{ij}$ element of the matrix represents a probability of having an edge between node $i$ and $j$. Thereafter, at each time step the network grows so that at step $k$ the network has $N^k$ nodes. By applying the Kronecker product to the same matrix leads to the emergence of self-similar fractal-like structures at different scales. This structure mimics a quite natural process that is the recursive growth of communities inside communities which are a miniature copy of a big community (i.e., the whole graph structure) [60]. A Kronecker Graph with order 167 and size 264 is shown in Figure 8.

![Figure 8: A Kronecker graph.](image)

### 5 Network metrics

In [74] we proposed a number of metrics useful for analyzing Power Grid topologies having in mind decentralized energy trading. We recall them here together with new ones, which we then apply to the evolution/growth models presented in Section 4. We set two main categories of requirements: qualitative and quantitative desiderata the network should satisfy.

#### Qualitative requirements

The main qualitative requirement we envision for the future Distribution Network relies on the modularity of the network topology. In the power system domain, the modularity is invoked as a solution that provides benefits reducing uncertainties in energy demand forecasting and costs for energy generation plants as well as risks of technological and regulatory obsolescence [52, 49]. Modularity is usually required not only in the energy sector, but more generally in the design and creation of product or organizations [44]. It is also a principle that is promoted in innovation of complex systems [38] for the benefits it provides in terms of reduced design and development time, adaptation and recombination. We assess the modularity of a network as the ability of building the network using a self-similar recurrent approach and having a repetition of a kind of pattern in its structure.
Quantitative requirements

As a global statistical tool, quantitative requirements are even more useful as they give a precise indication of network properties. Here are the relevant ones when considering efficiency, resilience and robustness of a power system.

- **Characteristic Path Length (CPL) lower or equal to the natural logarithm of order of graph**: \( \text{CPL} \leq \ln(N) \).
  This requirement represents having a general limited path when moving from one node to another. In the Grid this provides for a network with limited losses in the paths used to transfer energy from one node to another.

- **Clustering Coefficient (CC) which is 5 times higher than a corresponding random graph with same order and size**: \( \text{CC} \geq 5 \times \text{CC}_{RG} \).
  Watts and Strogatz [94] show that small-world networks have clustering coefficient such that \( \text{CC} \gg \text{CC}_{RG} \). Here we require a similar condition, although less strong by putting a constant value of 5. This requirement is proposed in order to guarantee a local clustering among nodes since it is more likely that energy exchanges occur at a very local scale (e.g., neighborhood) when small-scale distributed energy resources are highly implemented.

- **Betweenness-related requirements**:
  - A low value for average betweenness in terms of order of the graph \( \overline{\text{Betw}} = \frac{s}{N} \), where \( \overline{\text{Betw}} \) is the average betweenness of the graph and \( N \) is the order of the graph. For the Internet Vázquez et al. [89] have found for this metric \( \overline{\text{Betw}} \approx 2.5 \). Internet has proved successful to tolerate failures and attacks [25, 4], therefore we require a similar value for this metric for the future Grid.
  - A coefficient of variation for betweenness \( c_v = \frac{s}{x} < 1 \) where \( s \) is the sample standard deviation and \( x \) is the sample mean of betweenness. Usually distribution with \( c_v < 1 \) are known as low-variance ones.

The above two requirements are generally considered to provide network resilience by limiting the number of critical nodes that have a high number of minimal paths traversing them. These properties provide distributions of shortest paths which are more uniform among all nodes.

- **An index for robustness such that** \( \text{Rob}_N \geq 0.45 \). Robustness is evaluated with a random removal strategy and a node degree-based removal strategy by computing the average of the order of the maximal connected component (MCC) of the graph between the two situations when the 20% of the nodes of the original graph are removed [74]. It can be written as \( \text{Rob}_N = \frac{|\text{MCC}_{\text{Random 20%}}| + |\text{MCC}_{\text{Node Degree 20%}}|}{2} \). Such a requirement is about double the value observed for current Medium Voltage and 33% more for Low Voltage samples [74].

- **A measure of the cost related to the redundancy of paths available in the network**: \( \text{APL}_{10^{th}} \leq 2 \times \text{CPL} \).
  With this metric we consider the cost of having redundant paths available between nodes. In particular, we evaluate the 10\(^{th}\) shortest path (i.e., the shortest path when the nine best ones are not considered) by covering a random sample of the nodes in the network (40% of the nodes whose half represents source nodes and the other half represents destination nodes). The values for the paths considered are then averaged. In the case where there are less than ten paths available, the worst case path between the two nodes is considered. This last condition gives not completely significant values when applied to networks with small connectivity (i.e., absence of redundant paths).

| Metric       | Efficiency | Resilience | Robustness |
|--------------|------------|------------|------------|
| CPL          | ✓          |            |            |
| CC           |            | ✓          |            |
| Avg. Betweenness |            | ✓          |            |
| Betw. Coeff. of Variation |            |            | ✓          |
| Rob\(_N\)    |            |            | ✓          |
| \(\text{APL}_{10^{th}}\) | ✓          | ✓          |            |

Table 2: Metrics classification related to properties delivered to the network.

The above quantitative metrics can be categorized into three macro categories with respect to how they affect a Power Grid: efficiency in the transfer of energy, resilience in providing alternative path if part of the network is compromised/congested and robustness to failures for network connectivity. Table 2 summarized the property each metric assesses.
6 Generating Smart Grids

Having presented topological models and relevant Power Grid metrics to evaluate them, it is now time to perform the networks generation for the purpose of assessing their quality. The baseline network for metric analysis must be the real current Power Grid network. For this purpose, we use actual samples from the Medium and Low Voltage network of the Northern Netherlands (for a complete description of the data we refer to [74, 75]).

| Network sample | Model | Order | Size  | Avg. deg. | CPL     | CC | Removal robustness (RobN) | Redundancy cost (APL_{10th}) |
|----------------|-------|-------|-------|-----------|---------|----|--------------------------|------------------------------|
| LV-Small       | Real data | 21    | 22    | 2.095     | 4.250   | 0.00000 | 0.338                     | 12.364                       |
| LV-Medium      | Real data | 63    | 62    | 1.968     | 5.403   | 0.00000 | 0.245                     | 5.607                        |
| LV-Large       | Real data | 186   | 189   | 2.032     | 17.930  | 0.00000 | 0.154                     | 56.733                       |
| MV-Small       | Real data | 263   | 288   | 2.190     | 12.672  | 0.01117 | 0.134                     | 18.399                       |
| MV-Medium      | Real data | 464   | 499   | 2.151     | 13.107  | 0.00035 | 0.181                     | 20.905                       |
| MV-Large       | Real data | 884   | 1059  | 2.396     | 9.529   | 0.04024 | 0.208                     | 12.809                       |

Table 3: Metrics for Dutch Medium Voltage and Low Voltage samples.

Table [3] summarizes the values for the network metrics applied on the Dutch network samples. We notice that the average degree of the Medium and Low Voltage samples scores almost constantly around $\approx 2$ independently of the order of the network. In the Low Voltage networks we see a tendency towards the increase of characteristic path length, with a value about 18 when the order and size are about 200 nodes and edges, respectively. The same metric does not have the same clear tendency for the Medium Voltage samples. Considering the clustering coefficient there is a general tendency: a null value for the Low Voltage samples and small, but at least significant, values for the Medium Voltage samples. These differences in both characteristic path length and clustering coefficient come from the difference in topology of the two networks. Low Voltage is almost a non-mashed network which resembles for certain samples trees or closed chains with longer paths on average, especially when the network grows. On the other hand, the Medium Voltage network is more meshed (despite the same average node degree) with more connections that act as “shortcuts”. It also has to some extent some redundancy in the connections between the neighborhood of a node, which implies a more significant clustering coefficient compared to the Low Voltage network. The analysis of the robustness metric shows generally poor scores that decrease while the sample increase, at least for the Low Voltage networks, while the tendency is not clear for the Medium Voltage samples considered. A common behavior for the Medium Voltage samples is the problem they experience in the biggest component connectivity, when the 20% of the nodes with the highest degree are removed from the network: the robustness falls to 0.0456, 0.0366 and 0.0396 respectively for the Small, Medium and Large sample. Considering the additional effort required when the first nine shortest paths are not available, we see a general increase especially for the Low Voltage samples, where the 10th average path length (redundancy cost column in Table 3) increases three times for the Large sample analyzed; the increase is still present in Medium Voltage, but it is limited compared to the Low Voltage samples. This is again an indication that the Medium Voltage provides more efficient alternative paths to connect nodes. An exception in the results is the Low Voltage Medium size sample: here the 10th path average path length is really close to the traditional characteristic path length. This is basically due to the absence of alternative paths, therefore the only paths between nodes are at the same time the best and worst case too. This reinforces once again the idea of a Low Voltage network with a fixed structure (sort of chain or tree like) and a limited redundancy.

| Network sample | Model | Order | Size  | Avg. betweenness | Avg. betw/order | Coeff. variation |
|----------------|-------|-------|-------|------------------|-----------------|-----------------|
| LV-Small       | Real data | 21    | 22    | 70.286           | 3.347           | 0.643           |
| LV-Medium      | Real data | 63    | 62    | 255.016          | 4.048           | 0.733           |
| LV-Large       | Real data | 186   | 189   | 2928.227         | 15.743          | 1.207           |
| MV-Small       | Real data | 263   | 288   | 1237.711         | 4.706           | 1.517           |
| MV-Medium      | Real data | 464   | 499   | 3424.602         | 7.381           | 1.687           |
| MV-Large       | Real data | 884   | 1059  | 7755.542         | 8.773           | 2.875           |

Table 4: Betweenness for Dutch Medium Voltage and Low Voltage samples.

Considering the betweenness-related metrics shown in Table 3, one notices an increase in the average betweenness as the samples become more numerous in the two segments of the network (i.e., Medium Voltage and Low Voltage).
This same tendency is present also in the average betweenness to order ratio: the biggest samples in terms of order both of Low Voltage and Medium Voltage score highest. In particular, the Large sample belonging to the Low Voltage is almost twice the value of the biggest sample of the Medium Voltage. This again can be justified by the similar-to-tree structure of the Low Voltage sample for which nodes responsible for the paths that enable sub-trees or sub-chains to be connected are the most high scoring for betweenness. This highly increases the average betweenness (while the mode is usually null). The coefficient of variation is above one for all the big samples and reaches almost three for the biggest sample belonging to the Medium Voltage network. Such a high value implies a high standard deviation in the betweenness of the nodes, an indication for an heavy-tail distribution.

Model Parameters

To model the future Power Grid we compare network topologies that quantitatively evolve in size and order. In particular, we consider the increase of average node degree (< k >)≈ 2, < k >≈ 4 and < k >≈ 6. This evolution implies new cables and costs. For the Random Graph, small-world, preferential attachment and R-MAT models, we consider an evolution in the magnitude of average node degree of ≈ 2 then ≈ 4 and ≈ 6. The idea behind these values for average node degree is to study how the properties and metrics of networks change when increasing the connectivity of the network.

Each of the models introduced in Section 4 is defined by a set of parameters. Let us now consider some meaningful values for each one of them.

- **Random Graph.** For the \( G_{N,M} \) model, the only parameters needed are the order and size of the graph to be generated. We use the values shown in Table 1 for the order, and the size is chosen accordingly to obtain an average node degree of two, four and six, respectively.

- **Small-world Graph.** In addition to order, the small-world model requires the specification of the average out-degree and the edge rewiring probability. For the first parameter, we simply provide a value to obtain the desired average node degree (i.e., < k >≈ 2, < k >≈ 4 and < k >≈ 6). The latter parameter represents the probability of rewiring an edge connecting a source node to a different destination node chosen at random. We choose an intermediate approach between the regular lattice (i.e., rewiring probability \( p = 0 \)) and random graph extremes (i.e., rewiring probability \( p = 1 \)). In fact, we choose a rewiring probability \( p = 0.4 \). This is to give slightly more emphasis to the regular structure of lattice than to the rewiring, since we expect the future Grid to have more emphasis on a regular structure than random cabling. This last aspect also helps to satisfy the qualitative requirement of modularity.

- **Preferential Attachment.** For the creation of a graph based on growth and preferential attachment model of Barabási-Albert [8], the only parameters needed are the order and size of the graph to be generated. We use the values shown in Table 1 for order parameter, while size parameter is chosen accordingly to obtain an average node degree of two, four and six, respectively.

- **R-MAT.** The R-MAT model requires several parameters. First of all, order and size of the network, then the \( a, b, c, d \) parameters which represent the probabilities of the presence of an edge in a certain partition of the adjacency matrix. The order of the graph is chosen so that the nodes are a power of two, in particular, \( 2^n \) where usually \( n = \lceil \log_2 N \rceil \). Therefore, we consider for this model the following values for the order: \{32,128,256\} for comparison with the Low Voltage, and \{256,512,1024\} for comparison with the Medium Voltage Grids. For the probability parameters, since we have an undirected graph, we have \( b = c \), in addition the ratio found between \( a \) and \( b \) as in many real scenarios according to [21], is about 3:1. We assume a more highly connected community (\( a = 0.46 \)) and a less connected community (\( d = 0.22 \)) and a relative smaller connectivity between the two communities (\( b = c = 0.16 \)).

- **Copying Model.** The copying model requires, in addition to the order of the graph, a value for the probability of copying (or not) edges from existing nodes. \((1 - \beta)\) is the probability of copying nodes from another node. In the present study, we fix \( \beta = 0.2 \) so as to have a high probability of having a direct (just one-hop since with probability 0.8 each new node copies the connections of another node and attaches directly to them) connection to what might be considered the most reliable energy sources present in the city or villages (at Medium Voltage level), while it represents single users or small aggregation of users with high energy capacity at Low Voltage level.

- **Forest Fire.** The Forest Fire model requires, in addition to the order of the graph, two values representing the probability of forward and backward spread of the “burning fire”. We choose the same value for both probabilities since our graph is not directed. To avoid a flooding of edges, we choose few small values to assign to forward and backward probability (\( p_{fwd} = p_{bwd} = 0.2 \); \( p_{fwd} = p_{bwd} = 0.3 \); \( p_{fwd} = p_{bwd} = 0.35 \)) that give realistic amounts of edges incident to a node on average and that can be compared with the models for which one is able to directly set order and size.
**Random Graph with Power-law.** For the model representing Random Graph with power-law in node degree distribution, the parameters required are essentially the order of the network and the characteristic parameter of the power-law (known as the $\gamma$ coefficient). For the first parameter, we use the usual dimensions (see Table 1), while for the latter some additional considerations are necessary. We test different types of power-law coefficients characterizing real technological networks. For the non-electrical technological networks (i.e., technological networks which are not Power Grids) we average the values of the power-law characteristic parameter described in [24]; the details of the parameters are shown in Table 5. For the Power Grid networks the $\gamma$ values represent:

- the findings for the Western and Eastern High Voltage U.S. Power Grid in [22]; the values are averaged to have a single $\gamma$, the details are shown in Table 6;
- the findings for the High Voltage U.S. Western Power Grid in [8] which reports a value $\gamma = 4$;
- the findings for the Medium and Low Voltage Dutch Grid that follow a power-law in [74]; the values are averaged to have a single $\gamma$, the details are shown in Table 7.

| Type of network       | $\gamma$ |
|-----------------------|----------|
| Internet degree       | 2.12     |
| Telephone calls received | 2.09    |
| Blackouts             | 2.3      |
| Email address book size | 3.5     |
| Hits to web-sites     | 1.81     |
| Links to web sites    | 2.336    |
| Average               | 2.359    |

Table 5: Power-law $\gamma$ parameters for technological networks [24].

| Type of network     | $\gamma$ |
|---------------------|----------|
| Eastern Interconnection | 3.04   |
| Western System      | 3.09     |
| Average             | 3.065    |

Table 6: Power-law $\gamma$ parameters for High Voltage U.S. Power Grid [22].

**Kronecker Graph.** For the Kronecker model, the required parameter is the initial dimension of the square matrix to apply the Kronecker product: a $2 \times 2$ initiation matrix is a good starting model [60]. Once the structure of the matrix is defined the initial parameters for the generation matrix need to be evaluated. With a $2 \times 2$ adjacency matrix for the initial graph $G$:

$$A(G) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

the parameters can be interpreted in a similar fashion as in R-MAT:

- $a$ models the “core” part of the network and the tightness of its connectivity.
- $d$ models the “periphery” part of the network and the connectivity inside it.
- $b, c$ model the relationships and interconnections between the core and the periphery.

The findings of Leskovec et al. [60] applying the Kronecker modeling to many different networks report a common recurrent structure for the parameters of the $2 \times 2$ Kronecker matrix initiator. In particular, the

| Type of network | $\gamma$ |
|-----------------|----------|
| LV#5            | 2.402    |
| LV#10           | 1.494    |
| MV#2            | 1.977    |
| MV#3            | 2.282    |
| Average         | 2.039    |

Table 7: Power-law $\gamma$ parameters for Dutch Medium and Low Voltage Power Grid [74].
Table 8: Probability parameters for the $2 \times 2$ Kronecker matrix.

| Type of network          | $2 \times 2$ Kronecker generator parameters |
|--------------------------|---------------------------------------------|
|                          | $a$  | $c$  | $b$  | $d$  |
| Social-technological     | 0.9578 | 0.4617 | 0.4623 | 0.3162 |
| Power Grid               | 0.4547 | 0.8276 | 0.8504 | 0.0186 |

parameters tend to follow the empirical rule $a \gg b \geq c \gg d$ and are usually $a \approx 1$, $b \approx c \approx 0.6$ and $d \approx 0.2$. In this work, we consider two sets of parameters characterizing the Kronecker initiator matrix. The first set is extracted and averaged from the technological and social networks parameters extracted from real sample data [60]. The second set of parameters is obtained applying the fitting procedure to a Kronecker graph to the UCTE High Voltage Power Grid data set used in [80, 84], the High Voltage U.S. Western Power Grid data set used in [94], and the Medium and Low Voltage samples data set used in [74]. All these values have been averaged to obtain just one $2 \times 2$ Kronecker generation matrix. A summary of the values for the parameters of the Kronecker matrix used is given in Table 8. One notices a very different structure in the matrix parameters between the social and other diverse technological networks and the Power Grid.

Table 9: Metrics for small-world, preferential attachment, Random Graph and R-MAT models with average node degree $\approx 2$.

| Network type    | Model         | Order | Size | Avg. deg. | CPL  | CC        | Removal robustness (RobN) | Redundancy cost (APL$_{10th}$) |
|-----------------|---------------|-------|------|-----------|------|-----------|---------------------------|-------------------------------|
| LV-Small        | Small-world   | 20    | 20   | 2.000     | 4.053| 0.00000   | 0.330                     | 7.580                         |
| LV-Medium       | Small-world   | 90    | 90   | 2.000     | 11.820| 0.01593   | 0.167                     | 12.932                        |
| LV-Large        | Small-world   | 200   | 201  | 2.000     | 17.397| 0.01083   | 0.109                     | 21.544                        |
| MV-Small        | Small-world   | 250   | 250  | 2.000     | 24.237| 0.00000   | 0.087                     | 24.534                        |
| MV-Medium       | Small-world   | 500   | 501  | 2.004     | 28.084| 0.00000   | 0.057                     | 35.413                        |
| MV-Large        | Small-world   | 1000  | 1001 | 2.002     | 47.077| 0.00000   | 0.040                     | 60.074                        |
| LV-Small        | Preferential attachment | 20    | 19   | 1.900     | 2.579 | 0.00000   | 0.349                     | 2.800                         |
| LV-Medium       | Preferential attachment | 90    | 89   | 1.978     | 4.315 | 0.00000   | 0.263                     | 4.471                         |
| LV-Large        | Preferential attachment | 200   | 199  | 1.990     | 6.523 | 0.00000   | 0.206                     | 6.375                         |
| MV-Small        | Preferential attachment | 250   | 249  | 1.992     | 5.426 | 0.00000   | 0.245                     | 5.570                         |
| MV-Medium       | Preferential attachment | 500   | 499  | 1.996     | 5.705 | 0.00000   | 0.231                     | 5.745                         |
| MV-Large        | Preferential attachment | 1000  | 999  | 1.998     | 6.976 | 0.00000   | 0.187                     | 6.908                         |
| LV-Small        | Random Graph  | 17    | 21   | 2.471     | 2.938 | 0.07451   | 0.390                     | 7.472                         |
| LV-Medium       | Random Graph  | 78    | 92   | 2.359     | 5.987 | 0.03547   | 0.418                     | 10.974                        |
| LV-Large        | Random Graph  | 172   | 207  | 2.407     | 6.254 | 0.00736   | 0.354                     | 10.796                        |
| MV-Small        | Random Graph  | 224   | 259  | 2.313     | 7.269 | 0.00000   | 0.322                     | 12.002                        |
| MV-Medium       | Random Graph  | 435   | 516  | 2.372     | 8.380 | 0.00138   | 0.321                     | 12.818                        |
| MV-Large        | Random Graph  | 863   | 1026 | 2.378     | 9.061 | 0.00070   | 0.328                     | 13.446                        |
| LV-Small        | R-MAT         | 27    | 31   | 2.296     | 3.615 | 0.00000   | 0.356                     | 7.830                         |
| LV-Medium       | R-MAT         | 88    | 125  | 2.841     | 4.115 | 0.05688   | 0.369                     | 6.418                         |
| LV-Large        | R-MAT         | 199   | 261  | 2.623     | 5.495 | 0.00737   | 0.364                     | 8.774                         |
| MV-Small        | R-MAT         | 195   | 263  | 2.697     | 5.629 | 0.00885   | 0.378                     | 8.642                         |
| MV-Medium       | R-MAT         | 365   | 523  | 2.866     | 5.740 | 0.01090   | 0.386                     | 7.646                         |
| MV-Large        | R-MAT         | 728   | 1056 | 2.901     | 5.940 | 0.00589   | 0.363                     | 7.887                         |

Model Generation

Given the parameters presented above, we generate the graphs with respect to the different models and analyze them according to the significant Power Grid metrics described in Section 5. We begin with the models for which
| Network type | Model                     | Order | Size | Avg. betweenness | Avg. betw/order | Coeff. variation |
|--------------|---------------------------|-------|------|-----------------|----------------|-----------------|
| LV-Small     | Small-world               | 20    | 20   | 62.300          | 3.115          | 0.804           |
| LV-Medium    | Small-world               | 90    | 90   | 385.956         | 10.955         | 1.307           |
| LV-Large     | Small-world               | 200   | 201  | 3429.720        | 17.149         | 1.260           |
| MV-Small     | Small-world               | 250   | 250  | 5881.296        | 23.525         | 1.598           |
| MV-Medium    | Small-world               | 500   | 501  | 13980.228       | 27.960         | 1.745           |
| MV-Large     | Small-world               | 1000  | 1001 | 47919.616       | 47.920         | 2.279           |
| LV-Small     | Preferential attachment   | 20    | 19   | 31.400          | 1.570          | 2.344           |
| LV-Medium    | Preferential attachment   | 90    | 89   | 293.400         | 3.260          | 3.068           |
| LV-Large     | Preferential attachment   | 200   | 199  | 1089.260        | 5.446          | 3.288           |
| MV-Small     | Preferential attachment   | 250   | 249  | 1096.144        | 4.385          | 3.972           |
| MV-Medium    | Preferential attachment   | 500   | 499  | 2401.680        | 4.803          | 5.049           |
| MV-Large     | Preferential attachment   | 1000  | 999  | 6061.288        | 6.064          | 6.240           |
| LV-Small     | Random Graph              | 17    | 21   | 31.059          | 1.827          | 1.157           |
| LV-Medium    | Random Graph              | 78    | 92   | 408.308         | 5.235          | 1.126           |
| LV-Large     | Random Graph              | 172   | 207  | 938.512         | 5.456          | 1.276           |
| MV-Small     | Random Graph              | 224   | 259  | 1474.143        | 6.581          | 1.265           |
| MV-Medium    | Random Graph              | 435   | 516  | 3415.890        | 7.853          | 1.204           |
| MV-Large     | Random Graph              | 863   | 1026 | 7081.119        | 8.205          | 1.264           |
| LV-Small     | R-MAT                     | 27    | 31   | 70.593          | 2.615          | 1.320           |
| LV-Medium    | R-MAT                     | 88    | 125  | 282.500         | 3.210          | 1.540           |
| LV-Large     | R-MAT                     | 199   | 261  | 957.578         | 4.711          | 1.297           |
| MV-Small     | R-MAT                     | 195   | 263  | 959.118         | 4.919          | 1.395           |
| MV-Medium    | R-MAT                     | 365   | 523  | 1692.910        | 4.638          | 1.581           |
| MV-Large     | R-MAT                     | 728   | 1056 | 3633.473        | 4.991          | 2.004           |

Table 10: Betweenness metrics for small-world, preferential attachment, Random Graph and R-MAT models with average node degree $\approx 2$. 
| Network type | Model            | Order | Size | Avg. deg. | CPL    | CC     | Removal robustness $\left(R_{rob}\right)$ | Redundancy cost $\left(APL_{10^{th}}\right)$ |
|--------------|------------------|-------|------|-----------|--------|--------|------------------------------------------|-------------------------------------------|
| LV-Small     | Small-world      | 20    | 39   | 3.900     | 2.289  | 0.26000 | 0.721                                    | 4.720                                     |
| LV-Medium    | Small-world      | 90    | 177  | 3.933     | 3.652  | 0.14646 | 0.780                                    | 6.032                                     |
| LV-Large     | Small-world      | 200   | 399  | 3.990     | 4.407  | 0.15967 | 0.767                                    | 6.631                                     |
| MV-Small     | Small-world      | 250   | 498  | 3.984     | 4.566  | 0.12581 | 0.779                                    | 6.836                                     |
| MV-Medium    | Small-world      | 500   | 1000 | 4.000     | 5.067  | 0.19681 | 0.764                                    | 7.231                                     |
| MV-Large     | Small-world      | 1000  | 1998 | 3.996     | 5.479  | 0.10879 | 0.781                                    | 7.910                                     |
| LV-Small     | Preferential attachment | 20 | 37   | 3.700     | 2.263  | 0.47341 | 0.554                                    | 4.380                                     |
| LV-Medium    | Preferential attachment | 90 | 177  | 3.933     | 2.910  | 0.11216 | 0.426                                    | 4.788                                     |
| LV-Large     | Preferential attachment | 200 | 397  | 3.970     | 3.322  | 0.09566 | 0.448                                    | 5.047                                     |
| MV-Small     | Preferential attachment | 250 | 497  | 3.976     | 3.504  | 0.08400 | 0.419                                    | 4.998                                     |
| MV-Medium    | Preferential attachment | 500 | 997  | 3.988     | 3.687  | 0.03929 | 0.401                                    | 5.232                                     |
| MV-Large     | Preferential attachment | 1000 | 1997 | 3.994     | 4.211  | 0.01536 | 0.401                                    | 5.678                                     |
| LV-Small     | Random Graph     | 20    | 40   | 4.000     | 2.079  | 0.17667 | 0.733                                    | 4.350                                     |
| LV-Medium    | Random Graph     | 87    | 180  | 4.138     | 3.174  | 0.03418 | 0.735                                    | 5.368                                     |
| LV-Large     | Random Graph     | 199   | 400  | 4.020     | 3.869  | 0.03964 | 0.734                                    | 6.107                                     |
| MV-Small     | Random Graph     | 247   | 500  | 4.049     | 4.057  | 0.01681 | 0.740                                    | 6.432                                     |
| MV-Medium    | Random Graph     | 494   | 1000 | 4.049     | 4.495  | 0.00823 | 0.749                                    | 6.670                                     |
| MV-Large     | Random Graph     | 987   | 2001 | 4.055     | 5.082  | 0.00539 | 0.738                                    | 7.150                                     |
| LV-Small     | R-MAT            | 30    | 59   | 3.933     | 2.517  | 0.27360 | 0.579                                    | 4.511                                     |
| LV-Medium    | R-MAT            | 105   | 250  | 4.762     | 3.019  | 0.13039 | 0.581                                    | 4.490                                     |
| LV-Large     | R-MAT            | 227   | 504  | 4.441     | 3.619  | 0.04683 | 0.601                                    | 5.392                                     |
| MV-Small     | R-MAT            | 230   | 496  | 4.313     | 3.736  | 0.02940 | 0.626                                    | 5.381                                     |
| MV-Medium    | R-MAT            | 420   | 1004 | 4.781     | 3.915  | 0.00450 | 0.591                                    | 5.249                                     |
| MV-Large     | R-MAT            | 932   | 2039 | 4.376     | 4.562  | 0.00875 | 0.600                                    | 6.251                                     |

Table 11: Metrics for small-world, preferential attachment, Random Graph and R-MAT models with average node degree $\approx 4$. 
| Network type | Model                    | Order | Size  | Avg. betweenness | Avg. betw/order | Coeff. variation |
|--------------|--------------------------|-------|-------|------------------|-----------------|------------------|
| LV-Small     | Small-world              | 20    | 39    | 24.900           | 1.245           | 0.554            |
| LV-Medium    | Small-world              | 90    | 177   | 235.244          | 2.014           | 0.654            |
| LV-Large     | Small-world              | 200   | 399   | 683.780          | 3.419           | 0.783            |
| MV-Small     | Small-world              | 250   | 498   | 897.568          | 3.590           | 0.653            |
| MV-Medium    | Small-world              | 500   | 1000  | 2043.600         | 4.087           | 0.706            |
| MV-Large     | Small-world              | 1000  | 1998  | 4762.808         | 4.763           | 0.677            |
| LV-Small     | Preferential attachment  | 20    | 37    | 23.100           | 1.155           | 1.505            |
| LV-Medium    | Preferential attachment  | 90    | 177   | 170.644          | 1.896           | 2.219            |
| LV-Large     | Preferential attachment  | 200   | 397   | 463.060          | 2.315           | 2.733            |
| MV-Small     | Preferential attachment  | 250   | 497   | 611.520          | 2.446           | 3.017            |
| MV-Medium    | Preferential attachment  | 500   | 997   | 1342.864         | 2.686           | 3.484            |
| MV-Large     | Preferential attachment  | 1000  | 1997  | 3179.750         | 3.180           | 3.450            |
| LV-Small     | Random Graph             | 20    | 40    | 23.600           | 1.180           | 0.807            |
| LV-Medium    | Random Graph             | 87    | 180   | 196.345          | 2.257           | 0.850            |
| LV-Large     | Random Graph             | 199   | 400   | 589.849          | 2.964           | 0.889            |
| MV-Small     | Random Graph             | 247   | 500   | 766.389          | 3.103           | 0.857            |
| MV-Medium    | Random Graph             | 494   | 1000  | 1768.757         | 3.580           | 0.972            |
| MV-Large     | Random Graph             | 987   | 2001  | 4068.393         | 4.122           | 0.942            |
| LV-Small     | R-MAT                    | 30    | 59    | 44.000           | 1.467           | 1.342            |
| LV-Medium    | R-MAT                    | 105   | 250   | 223.733          | 2.131           | 1.695            |
| LV-Large     | R-MAT                    | 227   | 504   | 609.419          | 2.685           | 1.493            |
| MV-Small     | R-MAT                    | 230   | 496   | 650.574          | 2.828           | 1.468            |
| MV-Medium    | R-MAT                    | 420   | 1004  | 1285.786         | 3.061           | 1.652            |
| MV-Large     | R-MAT                    | 932   | 2039  | 3422.348         | 3.672           | 1.506            |

Table 12: Betweenness metrics for small-world, preferential attachment, Random Graph and R-MAT models with average node degree $\approx 4$. 
| Network type | Model          | Order | Size | Avg. deg. | CPL  | CC        | Removal robustness ($Rob_N$) | Redundancy cost ($APL_{10th}$) |
|--------------|----------------|-------|------|-----------|------|-----------|------------------------------|---------------------------------|
| LV-Small     | Small-world    | 20    | 59   | 5.900     | 1.816 | 0.32350   | 0.775                        | 3.470                           |
| LV-Medium    | Small-world    | 90    | 266  | 5.911     | 2.809 | 0.20131   | 0.794                        | 4.508                           |
| LV-Large     | Small-world    | 200   | 598  | 5.980     | 3.324 | 0.15596   | 0.797                        | 4.895                           |
| MV-Small     | Small-world    | 250   | 747  | 5.976     | 3.486 | 0.14477   | 0.799                        | 5.039                           |
| MV-Medium    | Small-world    | 500   | 1494 | 5.976     | 3.968 | 0.14477   | 0.799                        | 5.518                           |
| MV-Large     | Small-world    | 1000  | 2996 | 5.992     | 4.429 | 0.14854   | 0.797                        | 5.905                           |
| LV-Small     | Preferential attachment | 20    | 54   | 5.400     | 1.868 | 0.34839   | 0.749                        | 3.460                           |
| LV-Medium    | Preferential attachment | 90    | 264  | 5.867     | 2.466 | 0.16601   | 0.742                        | 3.933                           |
| LV-Large     | Preferential attachment | 200   | 594  | 5.940     | 2.854 | 0.08772   | 0.671                        | 4.130                           |
| MV-Small     | Preferential attachment | 250   | 744  | 5.952     | 2.926 | 0.08676   | 0.705                        | 4.257                           |
| MV-Medium    | Preferential attachment | 500   | 1495 | 5.980     | 3.185 | 0.05017   | 0.667                        | 4.481                           |
| MV-Large     | Preferential attachment | 1000  | 2994 | 5.988     | 3.487 | 0.03335   | 0.679                        | 4.664                           |
| LV-Small     | Random Graph   | 20    | 60   | 6.000     | 1.684 | 0.29599   | 0.775                        | 3.370                           |
| LV-Medium    | Random Graph   | 90    | 270  | 6.000     | 2.640 | 0.06987   | 0.791                        | 4.298                           |
| LV-Large     | Random Graph   | 200   | 600  | 6.000     | 3.141 | 0.03991   | 0.777                        | 4.693                           |
| MV-Small     | Random Graph   | 249   | 750  | 6.024     | 3.230 | 0.01934   | 0.793                        | 4.884                           |
| MV-Medium    | Random Graph   | 499   | 1500 | 6.012     | 3.620 | 0.00976   | 0.792                        | 5.284                           |
| MV-Large     | Random Graph   | 998   | 3000 | 6.012     | 4.022 | 0.00544   | 0.791                        | 5.662                           |
| LV-Small     | R-MAT          | 32    | 87   | 5.338     | 2.194 | 0.21179   | 0.760                        | 3.945                           |
| LV-Medium    | R-MAT          | 123   | 374  | 6.081     | 2.926 | 0.08173   | 0.717                        | 4.377                           |
| LV-Large     | R-MAT          | 249   | 759  | 6.096     | 3.165 | 0.04444   | 0.786                        | 4.622                           |
| MV-Small     | R-MAT          | 236   | 747  | 6.331     | 3.143 | 0.04982   | 0.746                        | 4.389                           |
| MV-Medium    | R-MAT          | 466   | 1512 | 6.489     | 3.427 | 0.04365   | 0.743                        | 4.885                           |
| MV-Large     | R-MAT          | 925   | 3035 | 6.562     | 3.742 | 0.02960   | 0.723                        | 4.925                           |

Table 13: Metrics for small-world, preferential attachment, Random Graph and R-MAT models with average node degree $\approx 6$. 
| Network type | Model               | Order | Size  | Avg. betweenness | Avg. betw/order | Coeff. variation |
|--------------|---------------------|-------|-------|------------------|----------------|-----------------|
| LV-Small     | Small-world         | 20    | 39    | 15.800           | 0.790          | 0.584           |
| LV-Medium    | Small-world         | 90    | 177   | 163.778          | 1.820          | 0.555           |
| LV-Large     | Small-world         | 200   | 399   | 464.330          | 2.222          | 0.617           |
| MV-Small     | Small-world         | 250   | 498   | 621.488          | 2.486          | 0.609           |
| MV-Medium    | Small-world         | 500   | 1000  | 1479.404         | 2.959          | 0.565           |
| MV-Large     | Small-world         | 1000  | 1998  | 3441.742         | 3.442          | 0.564           |
| LV-Small     | Preferential attachment | 20    | 37    | 15.900           | 0.795          | 1.292           |
| LV-Medium    | Preferential attachment | 90    | 177   | 133.378          | 1.482          | 2.640           |
| LV-Large     | Preferential attachment | 200   | 397   | 374.970          | 1.875          | 2.401           |
| MV-Small     | Preferential attachment | 250   | 497   | 485.352          | 1.941          | 2.514           |
| MV-Medium    | Preferential attachment | 500   | 997   | 1095.116         | 2.190          | 2.894           |
| MV-Large     | Preferential attachment | 1000  | 1997  | 2447.594         | 2.448          | 3.283           |
| LV-Small     | Random Graph        | 20    | 40    | 14.700           | 0.735          | 0.662           |
| LV-Medium    | Random Graph        | 87    | 180   | 151.489          | 1.683          | 0.809           |
| LV-Large     | Random Graph        | 199   | 400   | 431.090          | 2.155          | 0.835           |
| MV-Small     | Random Graph        | 247   | 500   | 563.839          | 2.264          | 0.710           |
| MV-Medium    | Random Graph        | 494   | 1000  | 1328.405         | 2.662          | 0.745           |
| MV-Large     | Random Graph        | 987   | 2001  | 3051.922         | 3.058          | 0.774           |
| LV-Small     | R-MAT               | 30    | 59    | 38.000           | 1.188          | 0.989           |
| LV-Medium    | R-MAT               | 105   | 250   | 247.008          | 2.008          | 1.351           |
| LV-Large     | R-MAT               | 227   | 504   | 550.538          | 2.211          | 1.352           |
| MV-Small     | R-MAT               | 230   | 496   | 530.093          | 2.246          | 1.357           |
| MV-Medium    | R-MAT               | 420   | 1004  | 1169.382         | 2.509          | 1.506           |
| MV-Large     | R-MAT               | 932   | 2039  | 2599.496         | 2.810          | 1.731           |

Table 14: Betweenness-related metrics for small-world, preferential attachment, Random Graph and R-MAT models with average node degree $\approx 6$. 
| Network type | Model                                      | Order | Size | Avg. deg. | CPL   | CC    | Removal robustness (Rob_N) | Redundancy cost (APL_10n) |
|-------------|-------------------------------------------|-------|------|-----------|-------|-------|----------------------------|---------------------------|
| LV-Small    | Copying Model                             | 26    | 19   | 1.990     | 2.053 | 0.00000 | 0.312                      | 2.060                     |
| LV-Medium   | Copying Model                             | 99    | 89   | 1.978     | 3.337 | 0.00000 | 0.287                      | 3.996                     |
| LV-Large    | Copying Model                             | 200   | 199  | 1.990     | 3.588 | 0.00000 | 0.271                      | 3.875                     |
| MV-Small    | Copying Model                             | 250   | 249  | 1.992     | 3.253 | 0.00000 | 0.287                      | 2.958                     |
| MV-Medium   | Copying Model                             | 500   | 499  | 1.996     | 3.762 | 0.00000 | 0.273                      | 3.775                     |
| MV-Large    | Copying Model                             | 1000  | 999  | 1.998     | 3.898 | 0.00000 | 0.280                      | 3.782                     |
| LV-Small    | Forest Fire (fwdPb=bckPb=0.2)             | 26    | 22   | 2.200     | 3.263 | 0.11476 | 0.352                      | 6.760                     |
| LV-Medium   | Forest Fire (fwdPb=bckPb=0.2)             | 99    | 131  | 2.911     | 5.691 | 0.25303 | 0.279                      | 7.681                     |
| LV-Large    | Forest Fire (fwdPb=bckPb=0.2)             | 200   | 265  | 2.930     | 5.475 | 0.27328 | 0.278                      | 7.323                     |
| MV-Small    | Forest Fire (fwdPb=bckPb=0.2)             | 250   | 336  | 2.688     | 6.725 | 0.26020 | 0.212                      | 8.551                     |
| MV-Large    | Forest Fire (fwdPb=bckPb=0.2)             | 500   | 792  | 3.168     | 7.084 | 0.34939 | 0.236                      | 8.425                     |
| MV-Large    | Forest Fire (fwdPb=bckPb=0.2)             | 1000  | 1468 | 2.936     | 10.341| 0.28623 | 0.170                      | 11.393                    |
| LV-Small    | Forest Fire (fwdPb=bckPb=0.3)             | 26    | 34   | 3.400     | 2.711 | 0.44833 | 0.535                      | 4.750                     |
| LV-Medium   | Forest Fire (fwdPb=bckPb=0.3)             | 99    | 163  | 3.622     | 5.264 | 0.51427 | 0.317                      | 6.648                     |
| LV-Large    | Forest Fire (fwdPb=bckPb=0.3)             | 200   | 505  | 5.050     | 4.224 | 0.43460 | 0.370                      | 5.410                     |
| MV-Small    | Forest Fire (fwdPb=bckPb=0.3)             | 250   | 527  | 4.216     | 5.231 | 0.39254 | 0.323                      | 6.508                     |
| MV-Medium   | Forest Fire (fwdPb=bckPb=0.3)             | 500   | 1185 | 4.470     | 5.260 | 0.39249 | 0.350                      | 6.264                     |
| MV-Large    | Forest Fire (fwdPb=bckPb=0.3)             | 1000  | 2461 | 4.922     | 5.606 | 0.35872 | 0.346                      | 6.416                     |
| LV-Small    | Forest Fire (fwdPb=bckPb=0.35)            | 26    | 38   | 3.800     | 2.421 | 0.64278 | 0.421                      | 4.880                     |
| LV-Medium   | Forest Fire (fwdPb=bckPb=0.35)            | 99    | 212  | 4.711     | 3.697 | 0.49510 | 0.431                      | 5.359                     |
| LV-Large    | Forest Fire (fwdPb=bckPb=0.35)            | 200   | 702  | 4.707     | 3.472 | 0.41380 | 0.401                      | 4.738                     |
| MV-Small    | Forest Fire (fwdPb=bckPb=0.35)            | 250   | 1013 | 8.104     | 3.671 | 0.44863 | 0.411                      | 4.870                     |
| MV-Medium   | Forest Fire (fwdPb=bckPb=0.35)            | 500   | 2369 | 9.476     | 3.587 | 0.49452 | 0.409                      | 4.402                     |
| MV-Large    | Forest Fire (fwdPb=bckPb=0.35)            | 1000  | 7777 | 15.554    | 3.311 | 0.48042 | 0.408                      | 4.382                     |
| LV-Small    | Kronecker (Power Grid parameters)         | 25    | 25   | 2.000     | 4.500 | 0.00000 | 0.332                      | 7.269                     |
| LV-Medium   | Kronecker (Power Grid parameters)         | 100   | 110  | 2.200     | 6.465 | 0.00000 | 0.318                      | 12.358                    |
| LV-Large    | Kronecker (Power Grid parameters)         | 202   | 233  | 2.307     | 6.973 | 0.00000 | 0.325                      | 11.928                    |
| MV-Small    | Kronecker (Power Grid parameters)         | 202   | 233  | 2.307     | 7.012 | 0.00000 | 0.314                      | 11.747                    |
| MV-Medium   | Kronecker (Power Grid parameters)         | 397   | 504  | 2.539     | 6.290 | 0.00000 | 0.359                      | 9.554                     |
| MV-Large    | Kronecker (Power Grid parameters)         | 809   | 1047 | 2.588     | 6.741 | 0.00185 | 0.352                      | 9.750                     |
| LV-Small    | Kronecker (Social and technological networks parameters) | 25    | 27   | 2.160     | 3.958 | 0.04800 | 0.401                      | 7.577                     |
| LV-Medium   | Kronecker (Social and technological networks parameters) | 90    | 124  | 2.756     | 4.393 | 0.01638 | 0.392                      | 7.222                     |
| LV-Large    | Kronecker (Social and technological networks parameters) | 176   | 273  | 3.102     | 4.206 | 0.01559 | 0.375                      | 6.709                     |
| MV-Small    | Kronecker (Social and technological networks parameters) | 176   | 273  | 3.102     | 4.206 | 0.01933 | 0.374                      | 6.701                     |
| MV-Medium   | Kronecker (Social and technological networks parameters) | 315   | 603  | 3.216     | 4.628 | 0.00457 | 0.393                      | 6.653                     |
| MV-Large    | Kronecker (Social and technological networks parameters) | 763   | 1326 | 3.476     | 4.799 | 0.00787 | 0.391                      | 6.478                     |

Table 15: Metrics for Copying, Forest Fire and Kronecker models.
| Network type | Model | Order | Size | Avg. betweenness | Avg. betw/order | Coeff. variation |
|--------------|-------|-------|------|------------------|-----------------|-----------------|
| LV-Small     | Copying Model | 20    | 19   | 20.300           | 1.015           | 3.646           |
| LV-Medium    | Copying Model | 90    | 89   | 254.978          | 2.833           | 3.524           |
| LV-Large     | Copying Model | 200   | 199  | 555.330          | 2.777           | 5.369           |
| MV-Small     | Copying Model | 250   | 249  | 523.216          | 2.093           | 3.524           |
| MV-Medium    | Copying Model | 500   | 499  | 1303.884         | 2.788           | 8.300           |
| MV-Large     | Copying Model | 1000  | 999  | 2857.788         | 2.858           | 11.396          |
| LV-Small     | Forest Fire (fwdPb=bckPb=0.2) | 20    | 22   | 46.300           | 2.315           | 1.315           |
| LV-Medium    | Forest Fire (fwdPb=bckPb=0.2) | 90    | 131  | 410.556          | 4.562           | 2.280           |
| LV-Large     | Forest Fire (fwdPb=bckPb=0.2) | 200   | 295  | 943.690          | 4.718           | 2.683           |
| MV-Small     | Forest Fire (fwdPb=bckPb=0.2) | 250   | 336  | 1460.200         | 5.841           | 3.049           |
| MV-Medium    | Forest Fire (fwdPb=bckPb=0.2) | 500   | 792  | 3147.128         | 6.294           | 3.569           |
| MV-Large     | Forest Fire (fwdPb=bckPb=0.2) | 1000  | 1468 | 9416.916         | 9.417           | 4.324           |
| LV-Small     | Forest Fire (fwdPb=bckPb=0.3) | 20    | 34   | 35.500           | 1.775           | 1.363           |
| LV-Medium    | Forest Fire (fwdPb=bckPb=0.3) | 90    | 163  | 371.200          | 4.341           | 2.245           |
| LV-Large     | Forest Fire (fwdPb=bckPb=0.3) | 200   | 505  | 688.270          | 4.212           | 3.018           |
| MV-Small     | Forest Fire (fwdPb=bckPb=0.3) | 250   | 527  | 1060.408         | 4.411           | 2.773           |
| MV-Medium    | Forest Fire (fwdPb=bckPb=0.3) | 500   | 1185 | 2205.412         | 4.411           | 3.000           |
| MV-Large     | Forest Fire (fwdPb=bckPb=0.3) | 1000  | 2461 | 4728.648         | 4.729           | 3.000           |
| LV-Small     | Forest Fire (fwdPb=bckPb=0.35) | 20    | 38   | 29.700           | 1.485           | 1.798           |
| LV-Medium    | Forest Fire (fwdPb=bckPb=0.35) | 90    | 212  | 269.644          | 2.996           | 2.025           |
| LV-Large     | Forest Fire (fwdPb=bckPb=0.35) | 200   | 707  | 565.740          | 2.829           | 1.673           |
| MV-Small     | Forest Fire (fwdPb=bckPb=0.35) | 250   | 1013 | 755.272          | 3.021           | 2.302           |
| MV-Medium    | Forest Fire (fwdPb=bckPb=0.35) | 500   | 2369 | 1446.308         | 2.893           | 2.507           |
| MV-Large     | Forest Fire (fwdPb=bckPb=0.35) | 1000  | 7777 | 2598.518         | 2.599           | 3.020           |
| LV-Small     | Kronecker (Power Grid parameters) | 25    | 25   | 86.360           | 3.462           | 1.198           |
| LV-Medium    | Kronecker (Power Grid parameters) | 100   | 110  | 555.640          | 5.556           | 1.357           |
| LV-Large     | Kronecker (Power Grid parameters) | 200   | 233  | 1224.347         | 6.061           | 1.499           |
| MV-Small     | Kronecker (Power Grid parameters) | 202   | 233  | 1231.416         | 6.066           | 1.483           |
| MV-Medium    | Kronecker (Power Grid parameters) | 202   | 233  | 1231.416         | 6.066           | 1.483           |
| MV-Large     | Kronecker (Power Grid parameters) | 397   | 504  | 2166.096         | 5.456           | 1.448           |
| LV-Small     | Kronecker (Social and technological networks parameters) | 809   | 1047 | 4782.022         | 5.911           | 1.418           |
| LV-Medium    | Kronecker (Social and technological networks parameters) | 25    | 27   | 68.400           | 2.736           | 1.381           |
| LV-Large     | Kronecker (Social and technological networks parameters) | 90    | 124  | 316.844          | 3.520           | 1.340           |
| MV-Small     | Kronecker (Social and technological networks parameters) | 176   | 273  | 584.193          | 3.519           | 1.601           |
| MV-Medium    | Kronecker (Social and technological networks parameters) | 375   | 603  | 1386.560         | 3.697           | 1.816           |
| MV-Large     | Kronecker (Social and technological networks parameters) | 763   | 1326 | 2992.993         | 3.923           | 1.894           |

Table 16: Betweenness metrics for Copying, Forest Fire and Kronecker models.
| Network type | Model | Order | Size | Avg. deg. | CPL | CC | Removal robustness (RobN) | Redundancy cost (APL10%) |
|--------------|-------|-------|------|-----------|-----|----|--------------------------|--------------------------|
| LV-Small     | Power Law (Social and technological networks parameters) | 20    | 21   | 2.100     | 3.526| 0.00000 | 0.469 | 7.460                   |
| LV-Small     | Power Law (East-West U.S. HV Power Grid networks parameters) | 20    | 19   | 1.900     | 4.105| 0.00000 | 0.300 | 4.120                   |
| LV-Small     | Power Law (Western U.S. HV Power Grid networks parameters) | 20    | 25   | 2.500     | 4.632| 0.00000 | 0.329 | 4.800                   |
| LV-Medium    | Power Law (Social and technological networks parameters) | 89    | 118  | 2.652     | 4.105| 0.04839 | 0.357 | 6.626                   |
| LV-Medium    | Power Law (Eastern U.S. HV Power Grid networks parameters) | 90    | 95   | 2.111     | 5.169| 0.02386 | 0.259 | 10.241                  |
| LV-Medium    | Power Law (Western U.S. HV Power Grid networks parameters) | 90    | 90   | 2.000     | 7.416| 0.00000 | 0.329 | 10.286                  |
| LV-Medium    | Power Law (Dutch MV-LV Power Grid networks parameters) | 90    | 136  | 3.022     | 3.371| 0.14367 | 0.354 | 5.362                   |
| LV-Large     | Power Law (Social and technological networks parameters) | 200   | 280  | 2.800     | 4.497| 0.01370 | 0.351 | 6.688                   |
| LV-Large     | Power Law (East-West U.S. HV Power Grid networks parameters) | 200   | 210  | 2.100     | 6.975| 0.00000 | 0.227 | 12.646                  |
| LV-Large     | Power Law (Western U.S. HV Power Grid networks parameters) | 200   | 199  | 1.990     | 11.256| 0.00000 | 0.139 | 12.774                  |
| LV-Large     | Power Law (Dutch MV-LV Power Grid networks parameters) | 200   | 257  | 2.056     | 9.127| 0.00269 | 0.210 | 15.052                  |
| MV-Small     | Power Law (Social and technological networks parameters) | 250   | 383  | 3.064     | 3.831| 0.05247 | 0.357 | 5.498                   |
| MV-Small     | Power Law (Eastern U.S. HV Power Grid networks parameters) | 250   | 257  | 2.056     | 9.127| 0.00269 | 0.210 | 15.052                  |
| MV-Small     | Power Law (Western U.S. HV Power Grid networks parameters) | 250   | 249  | 1.992     | 13.277| 0.00000 | 0.129 | 14.047                  |
| MV-Small     | Power Law (Dutch MV-LV Power Grid networks parameters) | 250   | 456  | 3.648     | 3.424| 0.08564 | 0.354 | 4.893                   |
| MV-Medium    | Power Law (Social and technological networks parameters) | 499   | 721  | 2.890     | 4.026| 0.04681 | 0.436 | 5.530                   |
| MV-Medium    | Power Law (Eastern U.S. HV Power Grid networks parameters) | 500   | 533  | 2.132     | 8.179| 0.00202 | 0.246 | 11.922                  |
| MV-Medium    | Power Law (Western U.S. HV Power Grid networks parameters) | 500   | 500  | 2.000     | 17.385| 0.00271 | 0.090 | 18.189                  |
| MV-Medium    | Power Law (Dutch MV-LV Power Grid networks parameters) | 500   | 1122 | 4.488     | 3.253| 0.13604 | 0.364 | 4.474                   |
| MV-Large     | Power Law (Social and technological networks parameters) | 1000  | 1717 | 3.434     | 4.178| 0.04216 | 0.345 | 5.567                   |
| MV-Large     | Power Law (Eastern U.S. HV Power Grid networks parameters) | 1000  | 1086 | 2.172     | 8.194| 0.00232 | 0.254 | 11.699                  |
| MV-Large     | Power Law (Western U.S. HV Power Grid networks parameters) | 1000  | 999  | 1.998     | 16.409| 0.00000 | 0.085 | 16.009                  |
| MV-Large     | Power Law (Dutch MV-LV Power Grid networks parameters) | 1000  | 2404 | 4.808     | 3.224| 0.15003 | 0.364 | 4.446                   |

Table 17: Metrics for random connected graphs showing a power-law in node degree distribution.
| Network type | Model                                                                 | Order | Size  | Avg. betweenness | Avg. betw/order | Coeff. variation |
|--------------|-----------------------------------------------------------------------|-------|-------|------------------|-----------------|-----------------|
| LV-Small     | Power Law (Social and technological networks parameters)              | 20    | 21    | 52.600           | 2.630           | 1.112           |
| LV-Medium    | Power Law (Social and technological networks parameters)              | 89    | 118   | 278.382          | 3.128           | 1.755           |
| LV-Large     | Power Law (Social and technological networks parameters)              | 200   | 280   | 715.980          | 3.580           | 2.021           |
| MV-Small     | Power Law (Social and technological networks parameters)              | 250   | 383   | 760.968          | 3.044           | 2.946           |
| MV-Medium    | Power Law (Social and technological networks parameters)              | 499   | 721   | 1577.323         | 3.161           | 4.760           |
| MV-Large     | Power Law (Social and technological networks parameters)              | 1000  | 1717  | 3323.160         | 3.323           | 3.883           |
| LV-Small     | Power Law (East-West U.S. HV Power Grid networks parameters)          | 20    | 19    | 59.200           | 2.960           | 1.213           |
| LV-Medium    | Power Law (East-West U.S. HV Power Grid networks parameters)          | 90    | 95    | 415.600          | 4.618           | 2.082           |
| LV-Large     | Power Law (East-West U.S. HV Power Grid networks parameters)          | 200   | 210   | 1230.020         | 6.150           | 2.271           |
| MV-Small     | Power Law (East-West U.S. HV Power Grid networks parameters)          | 250   | 257   | 2098.936         | 8.396           | 2.061           |
| MV-Medium    | Power Law (East-West U.S. HV Power Grid networks parameters)          | 500   | 533   | 3792.020         | 7.584           | 2.293           |
| MV-Large     | Power Law (East-West U.S. HV Power Grid networks parameters)          | 1000  | 1086  | 7716.384         | 7.716           | 2.864           |
| LV-Small     | Power Law (Western U.S. HV Power Grid networks parameters)            | 20    | 19    | 71.200           | 3.560           | 0.906           |
| LV-Medium    | Power Law (Western U.S. HV Power Grid networks parameters)            | 90    | 90    | 584.756          | 6.497           | 1.521           |
| LV-Large     | Power Law (Western U.S. HV Power Grid networks parameters)            | 200   | 199   | 2122.370         | 10.612          | 1.983           |
| MV-Small     | Power Law (Western U.S. HV Power Grid networks parameters)            | 250   | 249   | 3136.032         | 12.544          | 2.307           |
| MV-Medium    | Power Law (Western U.S. HV Power Grid networks parameters)            | 500   | 500   | 8524.172         | 17.048          | 2.553           |
| MV-Large     | Power Law (Western U.S. HV Power Grid networks parameters)            | 1000  | 969   | 15956.820        | 15.957          | 3.103           |
| LV-Small     | Power Law (Dutch MV-LV Power Grid networks parameters)                | 20    | 25    | 31.800           | 1.590           | 1.400           |
| LV-Medium    | Power Law (Dutch MV-LV Power Grid networks parameters)                | 90    | 136   | 221.172          | 2.457           | 2.284           |
| LV-Large     | Power Law (Dutch MV-LV Power Grid networks parameters)                | 200   | 399   | 455.410          | 2.277           | 3.007           |
| MV-Small     | Power Law (Dutch MV-LV Power Grid networks parameters)                | 250   | 456   | 647.864          | 2.591           | 2.836           |
| MV-Medium    | Power Law (Dutch MV-LV Power Grid networks parameters)                | 500   | 1122  | 1209.380         | 2.419           | 4.086           |
| MV-Large     | Power Law (Dutch MV-LV Power Grid networks parameters)                | 1000  | 2404  | 2356.038         | 2.356           | 5.758           |

Table 18: Betweenness metrics for random connected graphs showing a power-law in node degree distribution.
it is possible to explicitly assign order and size (or one of these quantities and the average node degree); we then proceed analyzing the other models that do not explicitly allow to set the average node degree parameter.

Model generation implementation and metrics computation

The values and graphs of generated topologies are obtained using software applications for network generation and analysis. In particular, for the model generation we developed C++ programs using the Stanford Network Analysis Project (SNAP) [http://snap.stanford.edu/] library that enables the generation of the network topologies described in Section 4 and the assignment of the parameters described earlier in this section. The analysis of the generated graphs according to the metrics described in Section 5 is performed with had-loc created software based on the JAVA graph library JGraphT (http://www.jgrapht.org/). The only metric computed with the SNAP software is ‘betweenness’ whose computation is based on the algorithm developed by Brandes [14]. To perform the generation and computation of the metrics we used a PC with Intel Core2 Quad CPU Q9400 2.66GHz with 4GB RAM. The Operating system is based on the Linux kernel 2.6.32 with a 4.4.3 GCC compiler and JAVA framework 1.6. The versions of SNAP and JGraphT software libraries used are respectively v10.10.01 and v0.8.1.

Comparison of models with average node degree $< k > \approx 2$

The results for the metrics with average degree $< k > \approx 2$ for the small-world, preferential attachment, Random Graph and R-MAT models score quite poorly, cf. Table 9. These low values are due to the small connectivity the networks show. Especially, we highlight the poor results of the small-world model under these conditions: with such a small average degree, the characteristic path length tends to be very high particularly as the network grows, with a value that for the biggest network generated is higher than 45 and about 60 for the $10^6$ path measure. In such a graph with small amount of edges, the clustering coefficient is also affected: the neighbors of a node are not organized in tight clusters because the numbers of links available are limited, only the Random Graph and R-MAT have non zero values for some samples, although few. The robustness to failure is limited under these conditions, with the worst case corresponding to the small-world samples, while better results are shown by preferential attachment, Random Graph and R-MAT models. The last two score higher than 0.33 for this metric. Considering the cost of redundancy, we generally see an increase in the characteristic path length as the order of the graph grows; the best results are shown by the networks generated with the preferential attachment model that presents values close to the best ones of the average path length. This might be due to the absence of many redundant paths in such a loosely connected network (less than 10 shortest paths without cycles) between any two nodes. A graphical comparison for the results of the Large sample for the Medium Voltage type considering characteristic path length, clustering coefficient and robustness metrics are given in Figure 9.

The betweenness analysis, whose results are presented in Table 10 shows an average for each node that increases with the size of the graph. The difference is in the value of average betweenness for the small-world model compared to other models: for the largest networks (500 and 1000 nodes) the value is almost one order of magnitude higher. This is due to the lattice structure of small-world that with a $< k > \approx 2$ degenerates in a long “closed-chain” topology which involves many nodes. The amount of edges that provide a “shortcut” in the graph is limited. This is in line with the high characteristic path length just described. The R-MAT model scores well considering the desiderata we imposed for average betweenness order ratio and coefficient of variation; the former is below 5 even for the biggest sample and the latter stays below 2. For the small-world sample, we experience a small coefficient of variation which reinforces the result indicating that almost all nodes have the same high betweenness close to the average. A graphical comparison for the results of the Large sample for the Medium Voltage type considering average betweenness order ratio and coefficient of variation metrics are shown in Figure 10.

Comparison of models with average node degree $< k > \approx 4$

Table 11 shows the results for small-world, preferential attachment, Random Graph and R-MAT models with an average degree $< k > \approx 4$. One notices the scores for the metrics improve compared to the $< k > \approx 2$ case. The average over the characteristic path length of all the samples reduces from around 10 to a value that is slightly less than 5. The clustering coefficient has values that are significant and all positive. The small-world model scores best in this specific metric since it relies on the lattice topology that with an average degree of 4 connects each node with four neighbors. In particular 3 triangle structures emerge in each neighborhood of a node. This provides a substantial contribution to the quite high clustering coefficient. Generally, all models score higher than the random graph with respect to the clustering coefficient (one of our desired properties). The addition of links provides enhanced robustness for the network too. Generally the order of the biggest connected component is about 63% of the initial order of the network (averaging all the result for the models) while with a $< k > \approx 2$ networks the value is just 27%. Not surprisingly, the best scores for robustness are obtained by the Random
(a) Characteristic path length.  

(b) Clustering coefficient.  

(c) Removal robustness.  

Figure 9: Metrics for the Large sample of Medium Voltage network type with average node degree $\approx 2$.  

(a) Betweenness to order ratio.  

(b) Betweenness coefficient of variation.  

Figure 10: Metrics for the Large sample of Medium Voltage network type with average node degree $\approx 2$.  

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Figure 11: Results for metrics for the Large sample of Medium Voltage network type with average node degree ≈ 4.

Graph model since in this type of graph the nodes tend to have the same characteristics, and hubs are not present in the network. Quantitatively, quite similar results to Random Graph for robustness are shown by small-world graphs (for some samples the metric scores even higher) with a robustness that is close to 0.8. Preferential attachment and R-MAT models score lower than random and small-world models with values around 0.45 for the former and 0.6 for the latter (in both cases these values are almost double than those for the $<k> ≈ 2$ case). An explanation for this lower score compared to other models for preferential attachment and R-MAT models resides in their building properties: they admit the presence of hubs (the node degree distribution is characterized by a power-law) that are highly sensitive for network robustness when targeted for removal. Considering the cost for the redundancy related to alternative paths, lower values appear for the preferential attachment model followed by the R-MAT and slightly higher for Random Graph and small-world. The worst case for this last model is a little smaller than 8 which is anyway only increased by 2 compared to the characteristic path length for the same sample. A graphical comparison for the results of the Large sample for Medium Voltage type considering characteristic path length, clustering coefficient and robustness metrics are shown in Figure 11.

Analyzing the betweenness we see a general improvement in the metrics compared to the $<k> ≈ 2$ case, cf. Table 12. The most important improvement is for the small-world model which, with approximately 4 connections per node, substantially reduces the average betweenness by a factor of 10 compared to the $<k> ≈ 2$ case. Although the small-world model performs worse than other models for the average betweenness order ratio, the coefficient of variation performs the best. It reinforces the idea that is in the model itself: nodes that do not differ much in their properties (the underlying lattice structure) have a small variation in the betweenness of nodes. The preferential attachment and R-MAT models, which generate networks with a fraction of nodes that have a very high connectivity due to the power-law in the node degree distribution, reach a higher coefficient of variation for betweenness. A graphical comparison for the results of the Large sample for Medium Voltage type considering average betweenness order ratio and coefficient of variation metrics are shown in Figure 12.

Comparison of models with average node degree $<k> ≈ 6$

Table 13 shows the results for small-world, preferential attachment, Random Graph and R-MAT models with an average degree $<k> ≈ 6$. The scores for the metrics considered improve even more with respect to those of Tables 9 and 11. The characteristic path length of all the samples has reduced to a value that, considering the average over all the samples with $<k> ≈ 6$, is about 3; yet 2 hops lower than the situation with $<k> ≈ 4$. The same tendency for clustering coefficient found for samples in Table 11 applies to this situation too. The small-world model scores highest since the neighbors of a node have nine connections with each other, thus substantially contributing to a high coefficient. For the R-MAT and preferential attachment models the clustering coefficient decreases as the order of the graph increases, but still for the biggest sample generated (1000 nodes) it is about one order of magnitude higher than a corresponding random graph. It is interesting to highlight how the clustering
coefficient for the small-world model tends to stabilize over the 0.14 value for the biggest samples (250, 500 and 1000 nodes, respectively). Regarding robustness, on average it increases to a value higher than 0.75. However it worths to notice how the increment mainly involves the preferential attachment and the R-MAT models which improve respectively from 0.44 and 0.61 to 0.70 and 0.73, on average. Therefore, the additional connectivity is more beneficial to power-law distributions than the others which seem to have already hit the upper bound for this metric with the \(< k > \approx 4\) situation. The cost of the redundant paths with this enhanced connectivity is reduced even more and on average the 10\(^{th}\) shortest path is just 1.5 hops higher than the characteristic path length for the same network. A graphical comparison for the results of the Large sample for Medium Voltage type considering characteristic path length, clustering coefficient and robustness metrics are shown in Figure 13.

Having increased the average degree to 6 brings benefits to the betweenness statistics too, cf. Table 14. The benefits on the average betweenness order ratio are about 25% higher compared to the \(< k > \approx 4\) situation; this ratio therefore is now very close to the experimental values that have been found for the Internet (i.e., \(\approx 2.5\)) which is one of our desiderata. The preferential attachment model, especially scores lower than the Internet threshold value for all the categories of samples considered. As already mentioned for the samples with \(< k > \approx 4\), the coefficient of variation for betweenness, even in this \(< k > \approx 6\) situation, scores best for the non power-law topologies (i.e., small-world and Random Graph) that show a value below the unit for all the dimensions of samples considered. The improvement for this metric for preferential attachment and R-MAT models are present but limited, in fact, they score higher than 3 and 1.7, respectively, in the worst case. A graphical comparison for the results of the Large sample for Medium Voltage type considering average betweenness order ratio and coefficient of variation metrics are shown in Figure 14.
Models Independent from the Average Node Degree

The Copying, Forest Fire, and Kronecker models are not generated using explicitly the average node degree, cf. Section 6. Therefore, we consider the Power Grid metrics on them separately. We remark however that, though not explicitly used as input parameter, the average node degree of the generated graphs has similar values to those of Random Graphs, small-world and preferential attachment models generated with the same order. Tables 15 and 17 contain the results for the metrics analyzed, while Tables 16 and 18 contain the results for betweenness.

The Copying Model results are comparable with the values for the metrics analyzed in Table 9, since the way the model is created provides a constant average node degree <k> ≈ 2. One can see that the Copying Model, which leads to a power-law in the node degree distribution, scores better than the small-world and preferential attachment ones in characteristic path length and robustness. The small cost in the average 10<sup>th</sup> shortest path is due to the computation of the worst case path (in fact in these conditions of network connectivity there are not ten paths between two nodes) which due to the very small meshed structure that is created it admits in the majority of cases just one path. With the way the model is implemented in the simulation environment used (Stanford Network Analysis Platform - SNAP), a node just copies a link from another chosen node and therefore there is no possibility to generate “triangle” structures between nodes which are essential to have a non-null clustering coefficient; that is why this metrics has such score. Considering the results for betweenness and comparing the values for Copying Model with the results obtained for models with imposing <k> >≈ 2, the Copying Model scores with an average smaller betweenness, this translates into a betweenness to order ratio that is better than other samples. On the other hand, the coefficient of variation is quite high given the difference in betweenness: extremely high only for few nodes in the network that sustain the majority of the shortest paths, and comparing the values for Copying Model with the results obtained for models with imposing <k> >≈ 2, the Copying Model scores with an average smaller betweenness, this translates into a betweenness to order ratio that is better than other samples. On the other hand, the coefficient of variation is quite high given the difference in betweenness: extremely high only for few nodes in the network that sustain the majority of the shortest paths, while the majority of the nodes participates only in the shortest paths for which they are end nodes for the path. The statistical mode for the betweenness values of each category for Copying Model is in fact null.

For the Forest Fire model, we assign different forward and backward burning probabilities to obtain values for the average degree to some extent comparable with the other models. The model with p<sub>_fwd</sub> = p<sub>_bwd</sub> = 0.2 can be compared to models with <k> >≈ 2. The Forest Fire scores definitely better than all the others in clustering coefficient. This is not surprising, if one recalls the algorithm behind the model: an ambassador node is chosen and with a certain probability a certain number of ambassador’s neighbors nodes are chosen to establish link to. One can see how many triangle-like structures tend to appear from such a generating method. The same observations can be done for the Forest Fire with p<sub>_fwd</sub> = p<sub>_bwd</sub> = 0.3 when compared to models with <k> >≈ 4: the characteristic path length scores almost like the other models, while this model suffers deeply in the robustness metric which for the biggest samples obtain a score which is half compared to the other generating models with <k> >≈ 4. This is due to the very high damages imposed to network connectivity when high degree nodes are removed: for the biggest sample (order of about 1000 nodes), when the 20% of nodes with highest degree are removed, the biggest connected component is just 2% of the original graph order. This is typical of heavy-tailed distributions which Forest Fire models empirically [61]. The metric that scores best is again the clustering coefficient that is three times higher (for the biggest sample) than the already quite high value of the small-world model. Even when we consider denser Forest Fire networks (i.e., p<sub>_fwd</sub> = p<sub>_bwd</sub> = 0.35) the comparison with the model with <k> >≈ 6 brings to the same conclusions: far better clustering coefficient, but an important weakness to node removal. Betweenness for the Forest Fire Model shows a known trend when varying the average node degree, the more the networks becomes connected the better the metrics related to betweenness become. For the samples with a burning probability of p<sub>_fwd</sub> = p<sub>_bwd</sub> = 0.35, the betweenness to order ratio stays below 3. The same behavior applies to the coefficient of variation, although it generally scores worse than the samples already analyzed with similar average degree.

The results shown by the networks generated with the Kronecker model using the parameters extracted from the Power Grid networks show metrics values similar to the ones computed from the physical samples.

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<http://snap.stanford.edu/>
with almost equal order. Especially the parameters for the Power Grid create networks with an average node degree, characteristic path length and robustness that mimic what we found for the current Dutch Medium and Low Voltage samples. Even the very low clustering coefficient (very often down to zero) is something we already recorded in real Power Grid samples [74]. When the networks generated with parameters extracted from Power Grid are compared with the networks generated from social and technological networks, one sees a general improvement in all the metrics under analysis: a reduction of a couple of hops in the characteristic path length, a higher clustering coefficient which is similar to the values obtained for random graphs. Generally the social-technological based Kronecker networks score more than 30% better than the corresponding based on Power Grid parameters for characteristic path length. We also see how the networks based on the Kronecker model show an almost constant, or decreasing value, for the average 10th path and a characteristic path length that very slowly increases with the order of the network. To some extent this tendency is something that the Kronecker model aims to achieve: densification of the network over time, i.e., when more nodes become part of the network the effective diameter of the networks becomes smaller. Considering betweenness, one sees a smaller average betweenness for the networks based on technological and social parameters than the ones generated with Power Grid parameters. Despite quite high values for both average betweenness and standard deviation, the samples produced with Power Grid parameters have a smaller coefficient of variation compared to the technosocial networks. The comparison of Kronecker models with networks generated with $<k> \approx 2$ models shows better values of the former compared to small-world and preferential attachment models while the results are quite similar for Random Graph and R-MAT models.

Considering the results of Random Graph with Power-law models, there is a difference for the networks generated with smaller $\gamma$ parameters (i.e., Medium and Low Voltage Dutch Grid $\gamma \approx 2$ and social and technological networks $\gamma \approx 2.3$) which score better than the ones with higher $\gamma$ (i.e., U.S. Eastern Interconnect and Western Grid $\gamma \approx 3$ and U.S. Western Power Grid $\gamma \approx 4$). The first two sets of samples show a denser network with higher average node degree, almost double compared to the other two sets, this results in a beneficial behavior for the metrics computed which present a smaller characteristic path length. This set of networks with small $\gamma$ is comparable for the characteristic path length property to the values obtained for networks generated with $<k> \approx 4$. The second set of samples (i.e., higher $\gamma$ parameter) shows results that are similar to the ones obtained for samples generated with $<k> \approx 2$. A general property that applies to all these power-law based samples is the problem they suffer, as already mentioned, from targeted attack involving the nodes with high degree, which justifies very poor scores for robustness metric. The betweenness analysis for the power-law based models shows an average betweenness value that is smaller for the networks with a lower value for the $\gamma$ coefficient so that they score best in the betweenness to order ratio. On the other hand, a lower $\gamma$ implies a higher probability in the presence of nodes that have higher node degree; usually there is quite a good positive correlation between the node degree and the betweenness the nodes have to sustain (high degree implies high betweenness for that node). It is therefore understandable why the coefficient of variation is higher for the networks characterized by a low $\gamma$ than the ones with higher power-law characteristic parameter.

A graphical comparison of the results for networks without explicit dependence to average node degree for the Large sample for Medium Voltage type considering characteristic path length, clustering coefficient and robustness metrics are shown in Figure 15 while a summary of the results for betweenness for the same sample are illustrated in Figure 16.

Comparing The Generated Topologies with the Physical Ones

The analysis of the Northern Netherlands Grid shows an average degree almost constant of about $<k> \approx 2$. Thus, it is fair to compare the generated models with similar average degree, the Copying Model ones and the Random Graphs with power-law in node degree distribution based on the data of Eastern and Western High Voltage U.S. Power Grid and the U.S. Western High Voltage Power Grid since all generate networks with average node degree $<k> \approx 2$. Generated models, except the model based on Random Graph with power-law, score better than the physical topologies for all the metrics considered; the characteristic path length scores half for the R-MAT and Copying Model cases in comparison to the real data. Also synthetic networks are more robust than the real data samples: R-MAT and Random Graph score constantly above 0.3 for robustness metric while real data hardly obtain this value. Clustering coefficients are quite similar since in this configuration with limited connectivity having triangle structures in the network is rare, however we see that R-MAT model has almost always significant clustering coefficient values. An exception is the small-world model which scores almost always worse than the real data samples, in fact, under this situation of limited average node degree it is actually not fully correct to consider this synthetic topology a “small-world”. The same sort of considerations can be done considering betweenness values: except the small-world model all the other synthetic ones score better for the average betweenness to order ratio metric, while for the coefficient of variation the situation is similar. If one considers the satisfaction of the desiderata for the actual samples of the Dutch Medium and Low Voltage Grid, summarized in Table 19 we notice that all parameters are not satisfied. However, networks generated according to the models with almost the same average node degree (networks with $<k> \approx 2$ in Table 20) and networks
Figure 15: Metrics for the Large sample of Medium Voltage network type for models independent from node degree.

Figure 16: Metrics for the Large sample of Medium Voltage network type for models independent from node degree.
based on Random Graph with power-law based on data from Eastern and Western High Voltage U.S. Power Grid and the U.S. Western High Voltage Power Grid in Table 22 do not satisfy all the desiderata as well. Therefore, this highlights that the first ingredient for the next generation of Grids to enable local energy exchange is an increase connectivity.

| Desiderata          | Northern Netherlands Medium and Low Voltage samples |
|---------------------|----------------------------------------------------|
| Modularity          | X                                                  |
| CPL \(\leq \ln(N)\) | X                                                  |
| \(CC \geq 5 \times CC_{RG}\) | X                                                  |
| \(\sigma = \frac{k}{N} \approx 2.5\) | X                                                  |
| \(c_0 \leq 1\)      | X                                                  |
| Rob\(N \geq 0.45\)  | X                                                  |
| \(APL_{100\%} \leq 2 \times CPL\) | \(\approx\) |

Table 19: Desiderata parameter compliance of real Medium and Low Voltage network samples of Northern Netherlands Grid.

Increasing the average node degree naturally provides for better values for the metrics, as shown in Table 20. The case of the small-world model is emblematic. While the \(< k > \approx 2\) case scores extremely poor as there are not enough “shortcuts” in the network so that they can not improve much the characteristic path length. Actually under such small average degree the condition Watts and Strogatz impose for their model is not completely satisfied (i.e., \(n > k > \ln(n) > 1\), where \(k\) is the average node degree and \(n\) is the order of the graph). When we move closer to satisfying the small-world condition by increasing the average node degree, the value of the metrics suddenly changes and the models score extremely high. The small-world scores best for the clustering property and resilience to failures in \(< k > \approx 4\) situations. Under these conditions also the betweenness values are quite concentrated around the mean with a coefficient of variation that does not exceed the unit.

Comparing the average values of the generated models for increasing node degree, one notices a natural improvement of the metrics, cf. Table 21. In fact, we have a reduction in characteristic path length of about 60% and an increase in the clustering coefficient of one order of magnitude, at the same time the robustness doubles. With \(< k > \approx 6\) the improvement compared to the metrics is less prominent, being between 10% and 20%. From the comparison of the metric results in Table 20 one sees that the small-world model almost always satisfies the desiderata requirement from a quantitative point of view when the average node degree is at least 4. From a qualitative point of view, the small-world model shows to some extent certain characters of modularity being generated starting from a regular lattice and then rewiring a certain fraction of the edges.

Comparing the metrics of the model and the results from Table 22 one sees that the general prevalence of requirement dissatisfaction, especially parameters involving betweenness are never satisfied by these generated samples.

From the topological analysis one can see that between the models analyzed when there is a minimal connectivity \((< k > \approx 4\) or \(< k > \approx 6\)) the small-world stands out, cf. Table 20. In Table 23 the models with explicit dependence on node degree are once again compared by assigning a “tick” sign (✓) for the fulfillment of each of the following properties: qualitative topological parameters (i.e., modularity), quantitative topological parameters (Table 20) and the thrift in network realization (e.g., addition of cables which represent a cost). This last parameter given is just a rough estimation, a more detailed analysis of cost in realizing a network belonging to Medium or Low Voltage with a certain size (i.e., Small, Medium or Large) and the economic benefits in electricity distribution arising from the enhanced connectivity is provided in Section 7. From Table 23 we conclude that networks generated with small-world model with average degree \(< k > \approx 4\) provide the the best balance to satisfy the desiderata of the future Power Grid.

7 Economic Considerations

Traditionally the problem of evaluating the expansion of an electrical system is a complex task that involves both the use of modeling, usually based on operation research optimization techniques and linear programming [12, 59, 10], and the experience and vision of experts in the field supported by computer systems. In this latter case computers acquire knowledge based on previous experts’ decision and, based on the electrical physical constraints of the domain, are then able to support Power Grid evolution decision [58] finding the most suitable technical and economical solution. With more distributed generating facilities at local scale, traditional methods have limits and need to be modified or updated to take into account the new scenario the Smart Grid framework brings into play. The models that we have so far analyzed as being candidates for the vision of the future Smart Grid need also to be evaluated from the economic point of view. How much will it cost to generate electrical infrastructures according to these models? What is the actual cost of adding a physical edge to the topology?
| Desiderata                  | Average node degree < $k$ $\approx$ 2 | Average node degree < $k$ $\approx$ 4 | Average node degree < $k$ $\approx$ 6 |
|----------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| Modularity                 | SW          | Pref. Attach. | RND. Graph | R-MAT | SW          | Pref. Attach. | RND. Graph | R-MAT | SW          | Pref. Attach. | RND. Graph | R-MAT |
| CPL $\leq \ln(N)$          | $\times$    | $\times$     | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $CC$ $\geq 5 \times CC_{RG}$ | $\times$ | $\times$ | N/A | $\checkmark$ | $\checkmark$ | N/A | $\checkmark$ | $\checkmark$ | N/A | $\checkmark$ | $\checkmark$ | N/A | $\checkmark$ |
| $\omega$ $\approx 2.5$    | $\times$ | $\times$ | $\times$ | $\times$ | $\approx$ | $\times$ | $\times$ | $\approx$ | $\times$ | $\approx$ | $\approx$ | $\times$ | $\checkmark$ |
| $c_v$ $\leq 1$             | $\times$ | $\times$ | $\times$ | $\times$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $RobN$ $\geq 0.45$        | $\times$ | $\times$ | $\times$ | $\times$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $APL_{10th}$ $\leq 2 \times CPL$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |

Table 20: Desiderata parameter compliance of the generated models with node degree < $k$ $\approx$ 2, 4, 6.
The Cost of Adding Edges

One important difference that a physical infrastructure such as the Power Grid has compared to the WWW or social networks is the physical presence of cables that have to connect the Medium Voltage substations or Low Voltage end-users generating units. If establishing a link from a Web page to another one is free, on the other hand, each increase in connectivity in the Power Grid implies costs in order to adequate the substation or end-user premise involved and the cables required for the connection. To assess these cost in the Medium and Low Voltage infrastructure, we consider a simple relation where the cost of cabling and cost of substations are added:

\[ C_{imp} = \sum_{j=1}^{N} Ssc_j + \sum_{i=1}^{M} Cc_i \]  

(1)

where \( C_{imp} \) stands for cost for implementation, \( Ssc_j \) is the adaptation cost for the substation \( j \) and \( Cc_i \) is the cost for the cable \( i \). The cost of the cable can be expressed as a linear function of the distance the cable \( i \) covers: \( Cc_i = C_{uc} \cdot l \) where \( C_{uc} \) is the cable cost per unit of length and \( l \) is the length of the cable. Several types of cables exist which are used for power transmission and distribution with varying physical characteristics and costs, in addition also the cost for installation can vary significantly [71]. In the present work, though, we simply consider cabling costs and ignore substation ones. While the former are directly tied to the topology and length of the links, the latter pricing is too dependent on other factors. As a source of data for cable type and pricing, we have been provided (courtesy of Enexis B.V.) with cables characteristics and prices together with topological information for 11 network samples belonging to the Low Voltage network and 13 samples belonging to the Medium Voltage of the Northern Netherlands.

Statistical consideration over cables’ price

Extracting probability distributions of physical and price data out of North Netherlands data samples, shows interesting correlations. The length of the cables plays an important role for both total resistance and price. If one considers the correlation between the price and resistance, high values are found. Using Spearman’s rank correlation coefficient [54], shown in Table 24, one can evaluate to what extent the variation tendency characterizing two variables can be described by a monotonic relationship. In other words, one has an indication of the correlation between price and resistance. Especially, for generating synthetic networks it is important to obtain values for both the properties of the cables that are similar to the ones actually used in practice. Plotting the correlation between price and resistance. Especially, for generating synthetic networks it is important to obtain values for both the properties of the cables that are similar to the ones actually used in practice. Plotting the two variables characterizing each cable one notices that the majority of the samples concentrates in the lower tails for the joint distribution. Figures 17 and 18 show the relation between the price and resistance where the values concentrate in the lower corner of the price x resistance space.

In the chart in Figure 18 one notices the two distinct lines that deviate from the low-left corner. They represent the two main types of cables that are used in that sample of the Low Voltage network to cover different distances and that result in increasing in price and resistance when longer lines are realized. This opens a new perspective: evaluate for each type of cable used in a certain sample (Small, Medium and Large) how the length of the cables used are distributed. In fact, given a certain type of cable and its length all other interesting properties for our analysis are then available (i.e., cable total resistance, cable total cost and cable current supported).

A general tendency appears when fitting the distribution of lengths to cable types belonging to Low Voltage and Medium Voltage: a fast decay in lengths’ probability distribution with the majority of lengths for the Low Voltage cables types in the order of tens of meters, and Medium Voltage cables hundreds of meters. Fitting the length to a statistical probability distribution gives a good approximation for the Low Voltage cable lengths as exponential distributions (\( y = f_X(x; \mu) = \frac{1}{\mu} e^{-\frac{x}{\mu}} \)). Figures 19 and 21 show respectively the cumulative distribution probability and the probability density functions for a certain type of cable belonging to the Low Voltage network. The use of the Kolmogorov-Smirnov test [65] lets us accept the hypothesis in favor of this distribution. The situation is slightly different for the Medium Voltage cables where the distribution that generally fits best the data is the generalized extreme value distribution (\( y = f_X(x; k, \mu, \sigma) = \frac{1}{\sigma} \left( 1 + \left( \frac{x - \mu}{\sigma} \right)^k \right)^{-\frac{1}{k}} \exp\left\{ -\left( 1 + \left( \frac{x - \mu}{\sigma} \right)^k \right)^{-\frac{1}{k}} \right\} \)); even in this case the Kolmogorov-Smirnov test supports this hypothesis. A graphical representation of the probability cumulative distribution function and the probability density function per cable type of to the Medium Voltage network are shown in Figures 20 and 22 respectively.

| Avg. node degree transition | Average metric improvement (%) | CPL | CC | Robustness |
|-----------------------------|--------------------------------|-----|----|-----------|
| \(< k > \approx 2 \rightarrow < k > \approx 4\) | 61.7 | 941.6 | 128.5 |
| \(< k > \approx 4 \rightarrow < k > \approx 6\) | 18.0 | 11.8 | 19.6 |

Table 21: Comparison of generated topologies for varying average node degree.
| Desiderata       | Copycat Model | Forest Fire (pf=0.2) | Forest Fire (pf=0.3) | Kronecker (PG params) | Kronecker (social net params) | HG with power-law (NL MLV RG params) | HG with power-law (West US HV PG params) | HG with power-law (East-West US HV PG params) | HG with power-law (social net params) |
|------------------|---------------|----------------------|----------------------|-----------------------|-------------------------------|--------------------------------------|------------------------------------------|--------------------------------------------|--------------------------------------|
| Modularity       | ≈             |                      |                      |                       |                               |                                      |                                           |                                           |                                      |
| $CPL \leq \ln(N)$ | $\times$      | $\times$             | $\times$             | $\times$              |                               |                                      |                                           |                                           |                                      |
| $CC \geq 5 \times CC_{RG}$ | $\times$      | $\times$             | $\times$             | $\times$              |                               |                                      |                                           |                                           |                                      |
| $\nu \approx 2.5$ | $\times$      | $\times$             | $\times$             | $\times$              |                               |                                      |                                           |                                           |                                      |
| $Rob \geq 0.45$  | $\times$      | $\times$             | $\times$             | $\times$              |                               |                                      |                                           |                                           |                                      |
| $A_{L_{max}} \leq 2 \times CPL$ | N/A           | $\times$             | $\times$             | $\times$              |                               |                                      |                                           |                                           |                                      |

Table 22: Desiderata parameter compliance of the generated models.
Figure 17: Price-Resistance pairs joint plot for the Medium Voltage Small size sample.

Figure 18: Price-Resistance pairs joint plot for the Low Voltage large size sample.
Network Model & Avg. node degree $< k \approx 2$ & Avg. node degree $< k \approx 4$ & Avg. node degree $< k \approx 6$
Small-world & // & // & //
Preferential Attachment & \checkmark & \checkmark & \checkmark
Random Graph & \checkmark & \checkmark & \checkmark
R-MAT & // & // & //

Table 23: Summary table considering satisfaction of modularity, performance and cabling cost for generated models with node degree $< k \approx 2, 4, 6$.

| Sample type          | Spearman’s rank correlation Price-Resistance |
|----------------------|----------------------------------------------|
| Low Voltage - Small  | 0.962                                        |
| Low Voltage - Medium | 0.974                                        |
| Low Voltage - Large  | 0.937                                        |
| Medium Voltage - Small | 0.787                                    |
| Medium Voltage - Medium | 0.634                                |
| Medium Voltage - Large | 0.946                                    |

Table 24: Spearman’s rank correlation for Low Voltage and Medium Voltage representative samples.

Assume that, statistically speaking, the distribution of the lengths for each type of cable in the synthetic networks are the same as in the real samples. Therefore, knowing the probability of using a certain type of cable $i$ ($p_{\text{cable}_i} = \frac{\#\text{cable}_i}{\sum_k \#\text{cable}_k}$ where $\#\text{cable}_i$ is the number of occurrences of cable type $i$ in a certain network sample) that has a certain cost and resistance per meter and a specific current supported, it is then possible to estimate the cables that are used in the synthetic samples together with their properties.

**Economic Benefits of Highly Connected Topologies**

Once the information about cable prices is available, it is possible to estimate the cost for realizing a network with a certain connectivity and if such networks are able to lower the (economic) barrier towards decentralized trading. The results for Low Voltage networks with an average node degree $< k \approx 2$ are shown in Table 25.

The results for $< k \approx 4$ and $< k \approx 6$ are about two and three times more expensive since there is an increase in the number of edges by the same quantity; for completeness the results are summarized in Tables 26 and 27.

For Medium Voltage networks, it is important to clarify that the information available for cables’ prices in this study are only partial and limited to some technologies (only few cross sections of aluminum and copper cables). Anyway, in order to have a glimpse of costs for this type of the network, we fitted to the best interpolating curve the available prices as a function of the cross section. The relation between price and cross section for aluminum cables fits best to a cubic polynomial, while for the copper ones is linear; in this way we can have an estimation for the prices for all the types of cables involved knowing their cross section. The results for the networks with an average node degree $< k \approx 2$ are shown in Table 28. The results for $< k \approx 4$ and $< k \approx 6$ are just two and three times more expensive since there is an increase in the number of edges by these same factors; for completeness, the results are shown in Tables 29 and 30. The small difference in costs between the Medium and Large types of networks is related mainly to the different technologies (i.e., cable types) in the cables that are used for these types of networks.

Price alone is not enough to describe future scenarios. It is important to investigate how an enhanced connectivity is beneficial to the electricity distribution costs. We have provided the benefits for more connected networks at the beginning of this section, however those results consider only the topology without any parameter related to the properties of the cables (e.g., resistance and supported current). In order to consider the effects of topology in electricity distribution costs, we have developed a set of metrics that associate topological properties of Power Grid networks to costs in electricity distribution. We have applied these metrics in the analysis of the Medium and Low Voltage Grid of the Northern Netherlands in [74]. In order to apply these metrics to Power Grid networks weights are essential, representing physical quantities such as resistance of the cable and maximal

| Sample type          | Size | Cost (thousand euro) |
|----------------------|------|----------------------|
| Low Voltage - Small  | \approx 20 | \approx 30          |
| Low Voltage - Medium | \approx 90  | \approx 78           |
| Low Voltage - Large  | \approx 200 | \approx 449         |

Table 25: Cabling cost for $< k \approx 2$ synthetic samples for Low Voltage networks.
Figure 19: Cumulative distribution function for cable length (meters) for cable type “VMvK(h)as 4x150 al” in Northern Netherlands sample Low Voltage size Large.

Figure 20: Cumulative distribution function for cable length (meters) for cable type “3x1x70al” in Northern Netherlands Medium Voltage sample size Medium.

| Sample type          | Size | Cost (thousand euro) |
|----------------------|------|----------------------|
| Low Voltage - Small  | ≈ 40 | ≈ 51                 |
| Low Voltage - Medium | ≈ 180| ≈ 174                |
| Low Voltage - Large  | ≈ 400| ≈ 827                |

Table 26: Cabling cost for $k \approx 4$ synthetic samples for Low Voltage networks.
Figure 21: Probability density function for cable length (meters) for cable type “VMvK(h)as 4x150 al” in Northern Netherlands sample Low Voltage size Large.

Figure 22: Probability density function for cable length (meters) for cable type “3x1x70al” in Northern Netherlands Medium Voltage sample size Medium.

| Sample type          | Size | Cost (thousand euro) |
|----------------------|------|----------------------|
| Low Voltage - Small  | ≈ 60 | ≈ 76                 |
| Low Voltage - Medium | ≈ 270| ≈ 254                |
| Low Voltage - Large  | ≈ 600| ≈ 1239               |

Table 27: Cabling cost for $< k > \approx 6$ synthetic samples for Low Voltage networks.
Table 28: Cabling cost for \(< k > \approx 2\) synthetic samples for Medium Voltage networks.

| Sample type          | Size  | Cost (millions euro) |
|----------------------|-------|----------------------|
| Low Voltage - Small  | \(\approx 250\) | \(\approx 32\) |
| Low Voltage - Medium | \(\approx 500\)  | \(\approx 42\)  |
| Low Voltage - Large  | \(\approx 1000\) | \(\approx 43\)  |

Table 29: Cabling cost for \(< k > \approx 4\) synthetic samples for Medium Voltage networks.

| Sample type          | Size   | Cost (millions euro) |
|----------------------|--------|----------------------|
| Low Voltage - Small  | \(\approx 500\)  | \(\approx 55\)  |
| Low Voltage - Medium | \(\approx 1000\) | \(\approx 88\)  |
| Low Voltage - Large  | \(\approx 2000\) | \(\approx 86\)  |

operating current supported by the cable. Once we have the statistical information about the types and the length of the cables used in a specific type of physical network, (i.e., Medium or Low Voltage and its Small, Medium or Large size) it is possible to assign weights to the edges of the generated graphs. This is done under the assumption that the same type of cables are used and that the distances covered in general (i.e., statistically) remain the same. In [74] we proposed to consider two types of measures that influence electricity price: one related to the dissipation and losses aspects on the Grid called \(\alpha\), and a second one which takes into account the aspects of reliability in the network called \(\beta\). Formally, these measures have the following expressions:

\[
\alpha = f(L_{\text{line}_N}, L_{\text{substation}_N});
\]

\[
\beta = f(\text{Rob}_N, \text{Red}_N, \text{Cap}_N).
\]

In equation (2) the factors influencing \(\alpha\) are the losses happening in the electrical lines \((L_{\text{line}_N})\) and the losses arising at substations \((L_{\text{substation}_N})\). On the other hand, the parameters influencing \(\beta\) consider the robustness of the network to failures \((\text{Rob}_N)\), the loss experienced in following redundant paths between nodes \((\text{Red}_N)\) and the available capacity in the lines connecting nodes of the network \((\text{Cap}_N)\). The relationship between \(\alpha\) and \(\beta\) and the price of electricity is considered quadratic as other components (e.g., fuel price) influencing electricity price hold this relationship [48]. Considering how \(\alpha\) and \(\beta\) parameters are computed [74], the smaller the values for each, the smaller the impact topology has on electricity prices (see Appendix B). For completeness the essential information about topology and electricity cost-related metrics are more thoroughly explained in Appendix B.

Figure 23 shows the values for the \(\alpha\) and \(\beta\) metrics for the synthetic networks generated following the small-world model with an increasing average node degree \((< k > \approx 2, < k > \approx 4\) and \(< k > \approx 6\)). It is not surprising to see the samples with \(< k > \approx 2\) score poorer than the other networks. The networks with higher average node degree are better visualized in Figure 24. One sees how the network with Medium size scores best and the difference between the network with \(< k > \approx 6\) and the network with \(< k > \approx 4\) is limited. Robustness (i.e., \(\beta\) parameter) for the Medium and Large size networks reaches a high value just with a sufficient connectivity (i.e., \(< k > \approx 4\) and more connectivity (i.e., \(< k > \approx 6\)) does not improve much this metric. The samples with Small size score better in the \(\alpha\) metric and this is quite reasonable since the paths, especially in terms of the number of substations traveled in the shortest path, are limited, of course due to the reduced order of the network.

The \(\alpha\) and \(\beta\) metrics for the networks generated for Medium Voltage purposes are shown in Figure 25. The same tendency appears: once the network is sufficiently connected (i.e., \(< k > \approx 4\)) the metrics score definitely better than the \(< k > \approx 2\) situation. If we dig into the most connected samples (Figure 26), we see how the values are quite concentrated with the exception of the Large sample with \(< k > \approx 4\). It is interesting to see the change in the \(\alpha\) value once there are more links: the value of the metric almost halves with an increase of connectivity i.e., \(< k > \approx 6\) situation.

Let us compare the \(\alpha\) and \(\beta\) metrics of the synthetic networks with the values of the real Power Grid samples of the Northern Netherlands. Considering the Low Voltage samples and the synthetic networks designed for

Table 30: Cabling cost for \(< k > \approx 6\) synthetic samples for Medium Voltage networks.
Figure 23: Transport cost of electricity based on the topological properties for synthetic networks based on small-world model for Low Voltage Grid.

Figure 24: Transport cost of electricity based on the topological properties for synthetic networks based on small-world model for Low Voltage Grid (detail of $<k> \approx 4$ and $<k> \approx 6$ samples).
Figure 25: Transport cost of electricity based on the topological properties for synthetic networks based on small-world model for Medium Voltage Grid.

Figure 26: Transport cost of electricity based on the topological properties for synthetic networks based on small-world model for Medium Voltage Grid (detail of $<k> \approx 4$ and $<k> \approx 6$ samples).
this purpose, we generally see an improvement in the metrics especially in the $\alpha$ values for the $< k > \approx 4$ and $< k > \approx 6$ networks. In fact, if we do not consider the synthetic networks with $< k > \approx 2$, because of the problems of small-world topology with such small connectivity, there is an improvement on average in the $\alpha$ metric of more than 50% comparing the Northern Netherlands samples with the $< k > \approx 4$ synthetic ones. In fact, for the $\alpha$ metric from an average of about 13 for the physical samples, the $< k > \approx 4$ synthetic ones score about 6. The improvement is more than 60% when considering the $< k > \approx 6$ ones where the average for these synthetic networks scores just below 5. There are improvements also in the $\beta$ metric, although limited. From an average around 4 for the physical samples the $< k > \approx 4$ on average score just below 2.75; while a better result is obtained by $< k > \approx 6$ which on average score 2.30 (about 40% improvement). The graphical comparison is shown in Figure 27.

Taking into account the Medium Voltage Netherlands samples and the small-world synthetic networks for this purpose, we see an important improvement in the metrics both in the $\alpha$ and $\beta$ values for the $< k > \approx 4$ and $< k > \approx 6$ networks. As already mentioned, synthetic networks with $< k > \approx 2$ should not be considered. The improvement on average in $\alpha$ metric is more than 65% comparing to the $< k > \approx 4$ synthetic samples (from an average of $\alpha$ about 33 for the physical samples, the $< k > \approx 4$ synthetic ones score about 11), and an improvement of more than 75% when comparing to the $< k > \approx 6$ ones (the average $\alpha$ for $< k > \approx 6$ synthetic networks scores around 7.3). There are improvements also in the $\beta$ metric. In particular, from an average around 3.55 for the physical samples the $< k > \approx 4$ score on average just below 1.15; a similar result is obtained by $< k > \approx 6$ which on average score about 1.2 (more than 65% improvement). The graphical comparison is shown in Figure 28.

Discussion

Watts and Strogatz’s small-world model, as shown in Tables 20, 22 and 23, is the model that captures best the requirements for the new Grid compared to the other ones analyzed being these dependent on the average node degree (preferential attachment, R-MAT and Random Graph) or not (Copying Model, Forest Fire, Kronecker and Power Laws). The tighten clustering that this models exhibits provides efficient local distribution with paths that are locally short; at the same time the shortcuts between the local clusters are the elements that keep the average path extremely limited. These two aspects influence the $\alpha$ parameter which then stays relatively limited. At the same time, the small-world model benefits from a general robustness against failures: the absence of big hubs that keep the network together (which are present on the other hand in the power-law-based topologies, for instance) improves the reliability against attacks which help obtaining good scores for the $\beta$ parameter. More quantitatively, one sees the general improvement in the metrics characterizing both the parameters influencing the losses (i.e., $\alpha$ parameter) and the reliability of the Grid (i.e., $\beta$ parameter) while the network becomes more dense, i.e., more edges are added. On average, we see an improvement of at least 50% when comparing the
Figure 28: Comparison for transport cost between synthetic small-world networks (black dots) and Northern Netherlands Medium Voltage samples (red dots).

physical samples of Northern Netherlands with the small-world networks with an average degree $< k > \approx 4$, while better results are obtained with more density (i.e., $< k > \approx 6$) where the improvement are 60% compared to the physical samples. This is indeed beneficial to the Power Grid and, according to the relationship with the topology, it should translate into a reduction in the costs for electricity distribution.

These benefits come literally at a cost. The network needs more connectivity therefore costs for extra cabling need to be considered in addition to the cost for upgrading the substations and end-users electricity gateways. A return on investment analysis on this aspect is beyond the scope of the present study. Nevertheless, it is interesting to see how with the $\alpha$ and $\beta$ metrics it is possible to consider how a certain physical sample belonging to a certain size category (Small, Medium and Large) would improve in its performance if its topology is arranged according to the principles of a synthetic model and more connections are added accordingly.

The benefits reached for $\alpha$ and $\beta$ should translate into a reduction in the cost for electricity transport and distribution since the parameters that influence these metrics are directly connected to aspects related to costs. However the significant investment required to add more connectivity in the network might not immediately enable cheaper electricity costs, but on the contrary make it more expensive.

8 Related Work

Complex Network Analysis works take into account the Power Grid at the High Voltage level usually to analyze the structure of the network without considering in detail the physical properties of the power lines. In our previous work [73], we have analyzed several works that investigate Power Grid properties using Complex Network Analysis approach. There are two main categories: 1) understand the intrinsic property of Grid topologies and compare them to other types of networks assessing the existence of properties such as small-world or scale-free [5, 53, 94, 8]; 2) better understand the behavior of the network when failures occur (i.e., edge or node removal) and analyze the topological causes that bring to black-out spread and cascading failures of power lines [51, 2, 31]. Few studies in the Complex Network Analysis landscape consider the possibility of using the insight gained through the analysis to help the design. These few cases consider the addition of lines in the network to assess the increase in the reliability of the entire Power Grid. Examples are the study of the Italian High Voltage Grid [32] and the study of improvement by line addition in Italian, French and Spanish Grids [81]. Also Holmgren [50] uses the Complex Network Analysis to understand which Grid improvement strategies are most beneficial showing the different improvement of typical Complex Network Analysis metrics (e.g., path length, average degree, clustering coefficient, network connectivity) although in a very simple small graph (less than 10 nodes) when different edges and nodes are added to the network. Wider is the work of Mei et al. [66] where a self-evolution process of the High Voltage Grid is studied with Complex Network Analysis methodologies. The model for Power Grid
expansion considers an evolution of the network where power plants and substations are connected in a “local-world” topology through new transmission lines; overall the Power Grid reaches in its evolution the small-world topology after few-steps of the expansion process. Wang et al. [91, 92] study the Power Grid to understand the kind of communication system needed to support the decentralized control required by the new Power Grid applying Complex Network Analysis techniques. The analyses aim at generating samples using random topologies based on uniform and Poisson probability distributions and a random topology with small-world network features. The simulation results are compared to the real samples of U.S. Power Grid and synthetic reference models belonging to the IEEE literature. These works also investigate the property of the physical impedance to assign to the generated Grid samples. Complex Network Analysis is not generally used as a design tool to propose new topologies for the future Smart Grid as we use in this paper where we also assess the benefits in terms of economical improvement.

Traditionally power system engineers adopt techniques which are different from Complex Network Analysis although sometimes exploiting graph theory principles [93, 29]. The traditional techniques applied by Power Engineers involve the individuation of an objective function representing the cost of the power flow along a certain line which is then subject to physical and energy balance. This problem translates in an operation research problem. These models are applied both for the High Voltage planning [72, 29] and the Medium and Low Voltage [90, 29] since long time. Not only operation research, but also expert systems [63] are developed to help in the process of designing grounding stations based on physical requirements as well as heuristic approaches from engineering experience. The substation grounding issue is approached as an optimization problem of construction and conductor costs subject to the constraints of technical and safety parameters, its solution is investigated through a random walk search algorithm [15]. In [30] a pragmatic approach using sensitivity analysis is applied to a linear model of load flow related to various overloading situations and a contingency analysis (N-1 and N-2 redundancy conditions) is performed with different grades of uncertainty in medium and long term scenarios. In the practice the planning and expansion problem is even more complex since it implies power plants, transmission lines, substations and Distribution Grid. In [46] all these aspects are assessed separately and several challenges appear. For instance in the planning of a High Voltage overhead transmission line specific clearance code must be followed and not only load is a key element, but also topography and weather/climatic (above all wind and ice) conditions play an import role in the planning of the infrastructure. For substation planning the authors of [46] emphasize, in addition to the need for upgrading the Grid (e.g., load growth, system stability) and budgeting aspects, the multidisciplinary aspects which involve from environmental and civil to electrical and communications engineering. A more general approach proposed in [18] to deal with power system planning might be regarded as a multi-objective (e.g., economics, environment, feasibility, safety) decision problem thus requiring the tools typical of decision analysis [52].

The works mentioned so far take into account mainly the High Voltage end of the Grid while not least important is the Distribution Grid especially in the vision of the future electrical system as proposed in this work where the end-user plays a vital role. The integrated planning of Medium and Low Voltage networks is tackled by Paiva et al. [79] who emphasize the need of considering the two networks together to obtain a sensible optimal planning. The problem is modeled as a mixed integer-linear programming one considering an objective function for investment, maintenance, operation and losses costs that need to be minimized satisfying the constraints of energy balance and equipment physical limits.

Even more challenges to Electrical system planning is posed by the change in the energy landscape with several companies running different aspects of the business (generation, transmission, distribution). In addition, accommodating more players in the wholesale market transmission expansion should follow (as it is already for generation) a market based approach i.e., the demand forces of the market and its forecast should trigger the expansion of the Grid [15]. The same consideration regarding the need of a different approach in planning in a deregulated market are expressed in [83] where an optimization of an objective function in the market environment is applied. Another method to evaluate transmission expansion plan takes into account the probability reliability criteria of Loss Of Load Expectation (LOLE); in particular, in [23] an objective function is proposed that takes into account the cost in constructing a transmission line between all buses involved in the line which is then subject to constrains in peak load demand satisfaction and a certain level of LOLE that the line should not outrun.

In the Smart Grid framework the planning techniques might be revised especially for the Distribution Grid which is the segment that is likely to face the greatest changes due to the presence of Advanced Metering Infrastructure (i.e., bidirectional intelligent digital meters at customer location) and Distribution Automation (i.e., feeders can be monitored, controlled in automated way through two-way communication). In addition, the Medium and Low Voltage Grid is no longer a layer where only energy is consumed, but Distributed Energy Generation facilities (small-scale photovoltaic systems and small-wind turbines) will be attached to this segment of the Grid; altogether these elements are likely to reshape the way planning for Medium and Low Voltage is realized [17].
Conclusions

In an evolving electricity sector with end-users able to produce their own energy and sell it on a local-scale market, the Grid plays the essential enabling role of supporting infrastructure. Local scale energy exchange is in fact beneficial for several aspects such as the increase in renewable-based energy production, the possibility for the end-user to have an economic contentment by selling surplus energy and, not less important, a step forward to the unbundling of the electricity sector. We studied how different topologies inspired from technological and social network studies have varying properties and can be (or not) adequate for the future Smart Grid networks. We showed that between the various models analyzed, the small-world model appears to have many supporting characteristics, according to a set of topological metrics defined for Power Grids. We also showed how these topological benefits can be related to economical aspects of electricity distribution through an improvement in the $\alpha$ and $\beta$ parameters. We also performed a statistical investigation related to cables' properties used in Medium and Low Voltage samples to evaluate the cost of cables to be used to realize synthetic networks in order to estimate the investment required for such networks. The benefits reached through topological properties are significant and beneficial to enable a local energy exchange, however the quantification from and economic point of view is not easy due to the high investment in realizing a more connected Medium and Low Voltage Grid.

The underlying motivation for the present work, is to develop decision support techniques based on Complex Network Analysis metrics to upgrade the Power Grid to a Smart Grid and to assess the current infrastructures. In addition, it enables to predict how a change in the topology, according to a certain network model, can be beneficial for the network from an efficiency, resilience and robustness perspective. Finally, the approach enables to quantify how the topology can help in reducing the parameters influencing electricity costs while considering the evolution of the Medium and Low Voltage Power Grid network into an infrastructure to support Smart Grid.
Appendix A  Price and Resistance Distribution

In Section 4 we illustrate how physical properties of cables and their prices have a correlation. A proper analysis must then follow a bivariate approach. However, one might be interested in studying only one characteristic of cables (e.g., price) separated from others (e.g., resistance). Here we investigate what is the statistical distribution of these separate properties of cables. In particular we look for the presence of power-laws since they appear in several natural phenomena and man-created infrastructures [24].

Fitting the data regarding prices to the most likely distribution obtained from the three different sample sizes of the Dutch Low Voltage Grid gives usually distributions very concentrated towards small price values and only very few cables have very high prices (i.e., more than 10000 euros) which are particularly long, or their technology is extremely expensive. The distribution that best fits the data for the Low Voltage samples is the Log-Normal distribution \( y = f_X(x; \mu, \sigma) = \frac{1}{x\sigma \sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \). Comparing the fitted distribution with the original empirical cumulative distribution function of the data provides significant p-values with the Kolmogorov-Smirnov test. The analytical parameters for the fitted distribution obtained through log-likelihood estimation are shown in Table 31.

| Sample type    | Log-Normal distribution parameters |
|----------------|-----------------------------------|
|                | \( \mu \) | \( \sigma \) |
| Low Voltage - Small | 6.104     | 1.513     |
| Low Voltage - Medium | 6.075     | 1.397     |
| Low Voltage - Large  | 6.939     | 1.258     |

Table 31: Log-Normal distribution \( \mu \) and \( \sigma \) parameters for cable price distribution for Low Voltage samples.

Information related to prices for Medium Voltage cables are only partially available for this study and limited to some technologies and cross-sections of aluminum and copper cables. In order to have an estimate of costs, we fit the prices available to the best interpolating curve. For the aluminum cables we used a cubic polynomial, while for the copper ones a linear relation between price and cross-section. We performed the same probability distribution fitting procedure with the Medium Voltage samples. Also in this case, the distributions that best approximate the sample data show a “fat-tail” behavior. For the three representative classes of samples, we consider that the best approximation is given by the theoretical distribution of generalized extreme value \( y = f_X(x; k, \mu, \sigma) = \frac{1}{\sigma} (1 + k \frac{x-\mu}{\sigma})^{-1 \frac{1}{k}} \exp\left(-\left(1 + k \frac{x-\mu}{\sigma}\right)^{-\frac{1}{k}}\right) \). The sample distributions have significant p-values with the Kolmogorov-Smirnov test indicating to accept the hypothesis of this underlined probability law for the Small and Medium classes of samples. The Large class sample poses more problems since the p-value resulting form the test is under the 5% acceptance threshold. Although the test suggests to reject the hypothesis of the underlying distribution, we consider it anyway a good distribution approximation since this type of distribution is the one that has a p-value closer to significance compared to others distribution tested (e.g., log-normal, exponential, Gaussian). The analytical parameters for the fitted distribution obtained through log-likelihood estimation are shown in Table 32.

Similar statistical considerations can be applied for fitting the resistance characterizing the cables. The obtained distributions both for Low Voltage and Medium Voltage reference networks present once again a “fat-tail” characteristic since, although the most of the cables have a small resistance properties, there are some cables with far higher resistance properties. The distributions that best fit the data are either generalized extreme values \( y = f_X(x; k, \mu, \sigma) = \frac{1}{\sigma} (1 + k \frac{x-\mu}{\sigma})^{\frac{1}{k}} \exp\left(-\left(1 + k \frac{x-\mu}{\sigma}\right)^{-\frac{1}{k}}\right) \) or log-normal \( y = f_X(x; \mu, \sigma) = \frac{1}{x\sigma \sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \). The parameters are shown in Tables 33 and 34 for Low Voltage and Medium Voltage respectively.

Both price and resistance distributions present usually a high probability that is concentrated in the lower values, however there are overall small, but highly significant in terms of their values, contributions in the tail of the distribution. We perform also an investigation considering if this “heavy-tail” contribution have power-law properties. We apply the fitting techniques proposed by Clauset et al. [24] to understand the presence of significant power-law contributions in these distributions. From this analysis it appears that there are marks of

| Sample type       | Extreme value distribution parameters |
|-------------------|---------------------------------------|
|                   | \( k \) | \( \sigma \) | \( \mu \) |
| Medium Voltage - Small | 0.547     | 33082.4    | 31988.8    |
| Medium Voltage - Medium | 0.419     | 32569.4    | 35880.8    |
| Medium Voltage - Large  | 0.490     | 16925.2    | 16766.9    |

Table 32: Extreme values distribution \( k \), \( \mu \) and \( \sigma \) parameters to fit cable price distribution for Medium Voltage samples.
| Sample type          | Distribution type | Distribution parameters | $k$     | $\mu$     | $\sigma$ |
|---------------------|-------------------|-------------------------|---------|-----------|----------|
| Low Voltage - Small | Log-normal        |                         | -2.27846 | 1.97188   |          |
| Low Voltage - Medium| Generalized       | extreme values          | 0.994657 | 0.054877  | 0.058296 |
| Low Voltage - Large | Log-normal        |                         | -0.881168 | 1.25617   |          |

Table 33: Distribution parameters to fit cable resistance for Low Voltage samples.

| Sample type          | Extreme value distribution parameters | $k$     | $\sigma$ | $\mu$     |
|---------------------|---------------------------------------|---------|----------|-----------|
| Medium Voltage - Small|                                       | 1.09862 | 4.1366   | 2.96819   |
| Medium Voltage - Medium|                                      | 0.613803 | 3.35594  | 3.41663   |
| Medium Voltage - Large|                                       | 0.619069 | 3.59693  | 3.35337   |

Table 34: Extreme values distribution $k$, $\mu$ and $\sigma$ parameters to fit cable resistance for Medium Voltage samples.

power-law distribution in both the probability of cable prices and cable resistance. These power-law contributions are generally significant in the middle part of the distribution, while the very initial part of the distribution and the final part of the tail tend to deviate from the power-law rule. In fact, the $p$-value that characterizes the Kolmogorov-Smirnov test is generally higher than the 5% null hypothesis rejection for the power-law hypothesis in the central part of the distribution. Two examples for the Low Voltage and Medium Voltage samples are given in Figures 29 and 30 related to cable resistance, while Figures 31 and 32 are related to cable prices. Each figure represents the cumulative probability distribution (complementary) on double logarithmic scale where the blue circles represent the samples data, the red line is the best fitting probability distribution over the whole sample (described above) and the black dashed line represents the best fitting power-law distribution in the interval of the sample closer to power-law.

### Appendix B Relating Topological Properties to Economical Distribution Benefits

In Section 7 we introduce the concept to associate Grid topology and cost of electricity. Here we give a thorough explanation of these concepts based on the findings in our previous work [74], where we developed a set of metrics to relate topological aspects and electricity cost and applied it to existing Dutch Medium and Low Voltage infrastructure. As described in Section 7, we take advantage of that proposal and apply the same metrics to the generated topologies suitable for the Smart Grid. The goal is to consider from a topological perspective those measures that are critical in contributing to the cost of electricity as elements in the Transmission and Distribution Networks as described in economic studies such as the one of Harris and Munasinghe [48, 70]:

- losses both in line and at transformer stations,
- security and capacity factors,
- line redundancy, and
- power transfer limits.

The topological aspects that we consider provide two sorts of measures, the first one $\alpha$ gives an average of the dissipation in the transmission between two nodes

$$\alpha = f(L_{\text{line}}, L_{\text{substation}});$$

(4)

the second one $\beta$ is a measure of reliability/redundancy in the paths among any two nodes

$$\beta = f(\text{Rob}, \text{Red}, \text{Cap}).$$

(5)

The functions to explicitly compute $\alpha$ and $\beta$ parameters can be expressed as follows:

- Losses on the transmission/distribution line can be expressed by the quotient of the weighted characteristic path length and the average weight of a line (a weighted edge in the graph):

$$L_{\text{line}} = \frac{WCPL_N}{W}$$

(6)
Figure 29: Cumulative probability distribution (complementary) for cable resistance M-size sample Low Voltage network (double logarithmic scale).

Figure 30: Cumulative probability distribution (complementary) for cable resistance M-size sample Medium Voltage network (double logarithmic scale).
Figure 31: Cumulative probability distribution (complementary) for cable price M-size sample Low Voltage network (double logarithmic scale).

Figure 32: Cumulative probability distribution (complementary) for cable price M-size sample Medium Voltage network (double logarithmic scale).
• Losses at substation level are expressed as the number of nodes (on average) that are traversed when computing the weighted shortest path between all the nodes in the network:

\[ L_{\text{substation}} = \frac{\text{Nodes}_{\text{WCPL}}}{N} \]  

(7)

• Robustness is evaluated with random removal strategy and the weighted-node-degree-based removal by computing the average of the order of maximal connected component between the two situations when the 20% of the nodes of the original graph are removed. It can be written as:

\[ \text{Rob}_N = \frac{|\text{MCC}_{\text{Random20\%}}| + |\text{MCC}_{\text{NodeDegree20\%}}|}{2} \]  

(8)

• Redundancy is evaluated by covering a random sample of the nodes in the network (40% of the nodes whose half represents source nodes and the other half represents destination nodes) and computing for each source and destination pair the first ten shortest paths of increasing length. If there are less than ten paths available, the worst case path between the two nodes is considered. To have a measure of how these resilient paths have an increment in transportation cost, a normalization with the weighted characteristic path length is performed. We formalized it as:

\[ \text{Red}_N = \sum_{i \in \text{Sources}, j \in \text{Sinks}} \frac{SP_{w_{ij}}}{\text{WCPL}} \]  

(9)

• Network capacity is considered as the value of the weighted characteristic path length, whose weights are the maximal operating current supported, normalized by the average weight of the edges in the network (average current supported by a line). That is:

\[ \text{Cap}_N = \frac{\text{WCPL}_{\text{current}}}{\text{w}_{\text{current}}} \]  

(10)

With these instantiations, equations (4) and (5) become:

\[ \alpha = f(L_{\text{line}}N, L_{\text{substation}}N) = L_{\text{line}}N + L_{\text{substation}}N \]  

(11)

\[ \beta = f(\text{Red}_N, \text{Rob}_N, \text{Cap}_N) = \frac{\text{Red}_N}{\text{Rob}_N \cdot \ln(\text{Cap}_N)} \]  

(12)

The aspects here considered are just some of the factors (the ones closely coupled to topology) that influence the overall price of electricity. Naturally, there are other factors that influence the final price, e.g., fuel prices, government policies and taxation, etc., as illustrated for instance in the economic studies of Harris and Munasinghe [48, 70].

Appendix C  The Grid Engineering Process based on Complex Network Analysis

In our previous analysis work [71] we considered a topological analysis of the Dutch Medium and Low Voltage Power Grid, while in this work we generate synthetic networks to assess which ones are better to support a Smart Grid where prosumers exchange energy at local scale. Based on both these studies we can define an engineering process to upgrade the existing infrastructures towards a Smart Grid of prosumers. The engineering process is thus based on Complex Network Analysis metrics and techniques.

This process is intended for energy distribution companies to assess what is the current state of their infrastructures considering the influence of the topology on the electricity transport prices. In a totally unbundled scenario for the electricity market the distribution company might be incentivized in providing a better infrastructure closer to prosumer and consumer needs. Distribution company might charge them based on indicators that not only take into account downtime periods, but also topological efficiency which is based on the influence of topology on electricity prices. Figure 33 presents the flow of this process. Each big rectangular box represents a phase of this process and each contains a number of operation in a flow represented by small rectangles. The initial phase is basically an analysis of the existing infrastructure and computation of its topological properties (Phase 1 light orange box) extracted from Grid sample data input (trapezoidal block in the figure). Economic factors (Phase 2 violet box) play a role too which are accounted in considering the desired costs in electricity distribution and the investment available by the electricity distribution company (trapezoidal blocks in Phase 2 box). The match to the actual infrastructure to the desired one is realized (Phase 3 light yellow box) and reports for cost benefits and cost for the investment are provided (trapezoidal blocks in Phase 3 box). In particular, each phase has more articulated processes internally which are detailed below.

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Figure 33: Engineering process for Medium and Low Voltage Grid optimized for prosumer-based energy exchange.

The **first phase** of the process starts with the acquisition of the (complete) network topological information, that is, information about the nodes of the Grid (substations, transformers, end-users) and the lines connecting those nodes (cables and links). Also physical parameters characterizing the cables are necessary such as resistance per unit of length, length of cables and capacity of cables (supported current). Once the information is available it is possible to build a Power Grid graph. The following step is performing a Complex Network Analysis extracting the metrics that are essential to assess the influence of topology in electricity prices as shown in Appendix B (e.g., weighted characteristic path length, average number of nodes involved in the shortest path...). From these metrics the summary indicators that relate electricity prices to topology are computed, namely, the $\alpha$ and $\beta$ values are extracted as shown in Section 7 and Appendix B.

The **second phase** represents the input of the requirements for the evolution of the Grid. These translate into constrains for generation of a network topology which satisfies the desiderata parameters of the electricity distributor provider, or any other actor (e.g., the municipality, a cooperative of users in the neighborhood, a venture capitalist), interested in realizing a Distribution Network that is more prone to the small-scale energy exchange paradigm. The stakeholder in the network defines a cost for the electricity distribution (most likely a range in the cost). This target cost is translated into topological measures ($\alpha$ and $\beta$ parameters) that the target network should satisfy. In addition to the constraint regarding $\alpha$ and $\beta$ parameters, the stakeholder provides additional constraints such as the number of nodes (or transmission lines) the network without improvements should have (it could be the same as the original sample or be different in case of planned increase/decrease in the network assets) and the available budget. The budget quantifies the investment in realizing/upgrading the network to make it more prone to prosumer-based energy exchange (this influences the possibility of increasing the number of substations and power lines).

The **third phase** consists of adapting the physical sample network to the synthetic one, once the two sets of topological measures coming from phase 1 and 2 have been compared. This phase therefore provides a new network that is optimized for the local-scale energy exchange considering the constrains given in Phase 2. Once the network is available it is then possible to compare it with the physical Power Grid sample in order to evaluate the presumed benefits in terms of topology and its advantages in electricity distribution costs. On the other hand it is possible to evaluate the foreseen cost for the investment to achieve this kind of network.

We leave as future work to define in details all the steps of the engineering process. Though most of pieces of the puzzles are there: Phase one is covered by [74]; while the current work covers Phase 2 of the picture by offering ways to compare synthetic and physical samples in terms of $\alpha$ and $\beta$. In addition, an assessment of the costs required to build synthetic networks is also given as an aspect of evaluation in the current work.

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