Reduction of couplings in the MSSM

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based on

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Outline

• Introduction.

• General Method of Reduction.

• Application in the MSSM.

• Summary.
• To reduce the number of independent couplings of a theory one can impose a symmetry.

• Also we can adopt a more general approach.

• We reduce this number by imposing relations between the couplings.

• The relations between the dimensionless couplings are such that renormalizability is preserved and are independent of the renormalization point.

• The method is developed for the reduction from \( n+1 \) coupling parameters \( g_0, g_1, ..., g_n \) to a description in terms of \( g_0 \) only. (W. Zimmermann, 1985)
Our aim is to express $g_1, g_2, ..., g_n$ as functions of $g_0$ so that a model involving a single coupling parameter $\lambda_0$ is obtained which is again invariant under the renormalization group. They can be written as

$$g_j = g_j(g_0) \quad j = 1, ..., n$$

(1)

Invariance of the Green’s functions of the original system under renormalization group implies

$$\left( M \frac{\partial}{\partial M} + \sum_{j=0}^{n} \beta_j \frac{\partial}{\partial g_j} + \gamma \right) G(p_i, M, g_0, g_1, ..., g_n) = 0$$

where $M, \beta_j, \gamma$ are the renormalization mass, the beta functions and the anomalous dimension correspondingly.
And for the reduced system
\[
\left( M \frac{\partial}{\partial M} + \beta' \frac{\partial}{\partial g_0} + \gamma' \right) G'(p_i, M, g_0, g_1(g_0), ..., g_n(g_0)) = 0
\]

We can see that \( G' \) is obtained from \( G \) by substituting the functions (1)
\[
G' = G(g_0, g_1(g_0), ..., g_n(g_0))
\]

Differentiating with respect to \( g_0 \)
\[
\frac{dG'}{dg_0} = \frac{\partial G}{\partial g_0} + \sum_{j=1}^{n} \frac{\partial G}{\partial g_j} \frac{dg_j}{dg_0}
\]

From the above equations we have
\[
\beta' = \beta_0, \gamma' = \gamma, \beta' \frac{dg_j}{dg_0} = \beta_j
\]

So the functions (1) must satisfy the following differential equations, the Reduction Equations
\[
\beta_j = \beta_0 \frac{dg_j}{dg_0}
\]
For simplicity we assume that the original system has two coupling parameters, $g_0$ and $g_1$. The beta-functions can be written

$$\beta_0 = b_0 g_0^2 + \ldots$$

$$\beta_1 = c_1 g_1^2 + c_2 g_0 g_1 + c_3 g_0^2 \ldots$$

The reduction equation is

$$\beta_1 = \beta_0 \frac{dg_1}{dg_0}$$

Assuming power series solution:

$$g_1 = p_0^{(1)} g_0 + \sum_{n=1} \sum p_n^{(1)} g_0^{(n+1)}$$

at lowest order we end up with a quadratic equation:

$$c_1 p_0^2 + (c_2 - b_0) p_0 + c_3 = 0$$
Assuming that $\alpha_2$ gauge coupling can be related with the $\alpha_1$ gauge coupling ($\alpha_i = \frac{g_i^2}{4\pi}$) we have the following reduction equation

$$\beta_2 = \beta_1 \frac{d\alpha_2}{d\alpha_1}$$  \hspace{1cm} (2)$$

where

$$\beta_2 \equiv \frac{d\alpha_2}{dt} = \frac{b_2}{2\pi} \alpha_2^2, \quad \beta_1 \equiv \frac{d\alpha_1}{dt} = \frac{b_1}{2\pi} \alpha_1^2,$$

and

$$b_2 = 1, \; b_1 = 11$$
We can write $\alpha_2$ in lowest order in perturbation theory as

$$\alpha_2 = c_0 \alpha_1$$

Substituting this relation to the reduction equation (2)

$$c_0 = \frac{\beta_2}{\beta_1} = \frac{b_2 \alpha_2^2}{b_1 \alpha_1^2} = \frac{b_2 c_0^2 \alpha_1^2}{b_1 \alpha_1^2} \Rightarrow$$

$$c_0 = 11$$

Hence $\alpha_2$ can be written as

$$\alpha_2 = 11 \alpha_1$$
We can check now if this result is compatible with the experimental values.

\[
\frac{1}{\alpha_{em}} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \Rightarrow \\
\alpha_{em} = \frac{11}{12}\alpha_1
\]

We know that

\[
\sin^2 \theta_w = \frac{\alpha_{em}}{\alpha_2} \Rightarrow \\
\sin^2 \theta_w = \frac{11}{12 \cdot 11\alpha_1} = \frac{1}{12} = 0.08333
\]

which is unacceptable because

\[
\sin^2 \theta_w^{\exp} = 0.23146 \pm 0.00017
\]
• Following the same procedure we assume that \( \alpha_{top} \) Yukawa coupling can be related with the \( \alpha_{bottom} \) Yukawa coupling, so they must satisfy the reduction equation

\[
\beta_{top} = \beta_{bottom} \frac{d\alpha_{top}}{d\alpha_{bottom}} \Rightarrow
\]

\[
\frac{d\alpha_{top}}{d\alpha_{bottom}} = \frac{\beta_t}{\beta_b} = \frac{\alpha_t(6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}{\alpha_b(6\alpha_b + \alpha_t + \alpha_\tau - \frac{7}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}
\]

We can for simplicity neglect the contribution from the \( \tau \) and the small difference between \( \frac{13}{15} \) and \( \frac{7}{15} \), so

\[
\frac{\beta_t}{\beta_b} = \frac{\alpha_t(6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}{\alpha_b(6\alpha_b + \alpha_t - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)} \tag{3}
\]
Assuming again power series solution of the reduction equation we can write

\[ \alpha_t = d_0 \alpha_b \]

The derivative of the ratio of the two Yukawa couplings must be zero

\[
\frac{d}{dt} \left( \frac{\alpha_t}{\alpha_b} \right) = 0 \Rightarrow
\]

\[
\frac{1}{\alpha_t^2} (\alpha_b \beta_t - \alpha_t \beta_b) = 0 \Rightarrow
\]

\[
\frac{\alpha_t}{\alpha_b} = \frac{\beta_t}{\beta_b}
\]
Eqn. (3) becomes

\[
\frac{\alpha_t}{\alpha_b} = \frac{\alpha_t \left(6\alpha_t + \alpha_b - \frac{13}{15} \alpha_1 - 3\alpha_2 + \frac{16}{3} \alpha_3\right)}{\alpha_b \left(6\alpha_b + \alpha_t - \frac{13}{15} \alpha_1 - 3\alpha_2 + \frac{16}{3} \alpha_3\right)} \Rightarrow \\
6\alpha_t + \alpha_b - \frac{13}{15} \alpha_1 - 3\alpha_2 + \frac{16}{3} \alpha_3 = 6\alpha_b + \alpha_t - \frac{13}{15} \alpha_1 - 3\alpha_2 + \frac{16}{3} \alpha_3 \Rightarrow \\
\alpha_t = \alpha_b
\]
The next thing to do is to solve numerically the one-loop coupled differential equations of top and bottom Yukawa couplings taken account the $\tau$ contribution and the difference between the numerical factors, to see if such a relation like the previous one can exist.

First, we solve the differential equations for the gauge and Yukawa couplings in the SM. And then at $M_{SUSY}$ we impose the next boundary conditions for some values of $\tan \beta$

\begin{align*}
\alpha_t(SM) &= \alpha_t(MSSM) \sin^2 \beta \\
\alpha_b(SM) &= \alpha_b(MSSM) \cos^2 \beta \\
\alpha_\tau(SM) &= \alpha_\tau(MSSM) \cos^2 \beta
\end{align*}
Application in the MSSM

Graphs showing the relationship between $h_t / h_b$ and Log$_{10} (E/\text{GeV})$ for different values of $\tan \beta$.

Graphs showing the derivative $d(\frac{h_t}{h_b})/dt$ as a function of Log$_{10} (E/\text{GeV})$ for different values of $\tan \beta$. 

- $\tan \beta = 10$
- $\tan \beta = 52.25$
- $\tan \beta = 56$
- $\tan \beta = 58.5$

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Summary

- No possible reduction between the gauge couplings.
- Assuming $\tan \beta = 56$ we can relate top and bottom Yukawa couplings.
- The idea of reduction of couplings in a field theory is very appealing, since it increases its predictive power. This method has led to Finite Unified Theories with successful calculation of top quark mass.

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