Digital Modeling of Heat Transfer during the Baking Process

Heba Mosalam
Heliopolis University, Energy Engineering Program, Cairo, Egypt

Correspondence should be addressed to Heba Mosalam; heba.mosalam@hu.edu.eg

Received 25 July 2021; Revised 15 August 2021; Accepted 28 September 2021; Published 18 October 2021

Copyright © 2021 Heba Mosalam. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Numerical modeling and analysis of the baking process are challenging biochemical processes occurring in bread. These changes result from mass engineering tasks, usually characterized by the complex chain of chemical, physical, and heat transfer processes impacting the baking at the same time primarily caused by a variation of two dominating factors: (i) the heat and (ii) the internal moisture content at different temperatures and during the time process. This study presents an analysis of the 1-D computational fluid dynamics model for simultaneous heat transfer within a cylindrical bread sample. The numerical simulations were performed using the finite difference model (FDM) and the finite element model (FEM). In the first case, the proposed numerical model considered radiation and convection during sample heating and described the sample’s simultaneous heat, water, and vapor diffusion mechanisms. The calculations indicated that the FDM was susceptible to the time step; consequently, the range of 10 s and 100 s yielded the only relevant results. In the second case, the FEM was used to describe the phenomena of transportation during baking. Results obtained by the FEM showed a large temperature gradient near the surface. The study showed the presence of some critical cases that are considered the most influential on the stages of bread production. The first critical value is the time when the baking temperature reaches 100 °C. The second critical value is the time when the liquid water content in the baking medium reaches its peak. The boundary conditions were examined and illustrated by figures in the center and the surface of the bread.

1. Introduction

Baking is a way of loaf’s transformation during a variety of complicated chemical, biochemical, and physical processes occur within a product [1, 2]. The characteristics of items and ovens have evolved; the combining of fundamental physical and chemical transformations caused by heat transfer and the conversion of raw ingredients into finished products remain important to researchers and food producers. Although the separate transformations that occur during the baking process are well known, today, the development of processes for large-scale baking remains empiric.

Baking has historically been described as the combination of heat and mass transfer involving various hypotheses. Measuring airflow inside a baking oven numerically simulates oven efficiency. Simulation with computational fluid dynamics (CFD) also evaluates the thermal and mass transfer and keeps up the ideal moisture and temperature inside the oven [3, 4]. The nonuniform distribution of heat within the oven allows the moisture content and bread color to deviate. Using a correctly designed oven and ensuring specific baking conditions, including oven temperature (either uniform or nonuniform), heating capacity profile and the pattern of the air flow, loaf size, and baking time will reduce these bread quality variations [5]. Currently, many studies have emphasized these as the key components responsible for the bread quality using simulation tools such as CFD [6–8]. In 2006, a research has reported on the temperature profile of bread when effected by uncertainties with physical properties using 2-D geometry with sliding mesh. Additionally, in 2010 determined the temperature distribution inside an electric oven showing a significant difference between locations of trays using CFD with pilot scale electrical heating.

Additionally, Boulet in 2010 developed a baking oven pilot using CFD by incorporating a measuring device for heat flux into the geometry. Mondal demonstrated simultaneous mass and thermal transfer applying 2-D modeling during baking. Chhanwal et al. have developed an evaporation condensation model to predict the temperature and
browning profile for bread. In 2011 [9], they developed a CFD model to study bread’s heat and cooking properties. This model integrates evaporation phase changes and evaporation-condensation mechanisms during the baking process. However, this study assumes constant thermophysical factors that could result in incorrect predictions of these properties. Properties such as thermal conductivity and mass diffusivity have also been identified as functions of both temperature and moisture [10]. Zhang and Datta [11] developed a numerical model that completely joins equations for transport and deformation. Purlis and Salvadore [12] put forth a general relation between condensation/evaporation and the bread boundary generally applicable to baking bread. Ousegui et al. [13] modeled the baking and then following the approach of multiphase porous media. The author simulated conductive heat path using Fourier empirical. In contrast, Fick and Darcy laws were used to determine the mass through phenomena in the liquid and gas phases, respectively. Similarly, Purlis [14] studied the phenomenon of transportation and changes in bread quality during baking using mathematical simulation, including moving the boundary with changing phases. In [15], the authors used nanofluids shifting and two-dimensional, steady, and low speed flow solving method homotopy analysis (HAM). In [16], the authors used D3Q19 to solve the partial differential heat transfer equations for a vertical fin mounted on hot wall to study the effect of the vertical fin and nanoparticles on the flow. And the same solving tool was used in [17] for horizontal baffles.

In food engineering, baking’s mathematical simulation remains a challenge. So far, by numerical simulation, the experimental heating and drying curves were found as baking occurrences are difficult to reproduce, specifically, the typical sigmoid trend at the bread center of the temperature variance. Accurate expressions of bread’s thermal physical properties are still not available or only available during a small set of operating conditions, i.e., below 100°C.

Most published studies do not consider evaporation-condensation mechanisms on the baking process, and few published studies consider evaporation-condensation processes. Therefore, this study is aimed at developing two models for concurrent moisture and heat transfer in the interior of bread during cooking. The FDM model describes the instruments of concurrent heat, water, and vapor diffusion in a 1-D solid object warmed with radiation and convection from the exterior. We use the FEM model to evaluate the time changes in some characteristic parameters, mainly the moisture content of the bread.

2. Methodology

This study uses a cylindrical bread, 2 cm thick and 22 cm in diameter, which contains around 40 percent water (wet basis).

2.1. Finite Difference Method (FDM)

2.1.1. Applied Model. Here, in the presented paper’s modeling, applied is built with respect to a 1-D model, including a few modifications defined by Thorvaldsson and Janestad. Assuming a symmetric system, a bunch of three differential equations represent the model: (i) water vapor diffusion, (ii) thermal transfer, and (iii) liquid water diffusion. With a set of algebraic conditions, the three equations in the system are linked to each other, updating liquid water and water vapor using defined values determined for the saturated vapor pressure content. The power conservation equation is used to derive the heat transfer equation by adding a term representing the water evaporation’s latent heat. The temperature $T(x,t)$ can be defined as follows at the location $x$ during time $t$ [18]:

$$\frac{\partial T}{\partial t} = \frac{1}{\rho \cdot C_p} \frac{\partial}{\partial x} \left( k \cdot \frac{\partial T}{\partial x} \right) + \frac{\lambda}{C_p} \frac{\partial W}{\partial x} + \frac{\lambda}{C_p \cdot p} \frac{\partial T}{\partial t}, 0 < x < x_{L/2}. \quad (1)$$

With boundary conditions and initial conditions [19],

$$-k \frac{\partial T}{\partial x} \bigg|_{x=0} = \frac{1}{H_{\text{air}}}(T_n - T_{\text{sat}}) + h_{\text{conv}}(T_{\text{air}}, T_{\text{sat}}) - \lambda \cdot p \cdot D_{\text{H2OJ}} \frac{\partial W}{\partial x} \bigg|_{x=0}, \quad (2)$$

$$\frac{\partial T}{\partial x} \bigg|_{x=x_{L/2}} = 0, t > 0, T(x,0) = T_0(x), 0 < x < x_{L/2}. \quad (3)$$

Fick’s law can be used to derive equations for liquid water and vapor water diffusion. The equations are as follows [20]:

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\text{H2OJ}} \frac{\partial V}{\partial x} \right], 0 < x < x_{L/2}, t > 0, \quad (4)$$

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\text{H2OJ}} \frac{\partial W}{\partial x} \right], 0 < x < x_{L/2}, t > 0. \quad (5)$$

With boundary conditions and initial conditions,

$$\frac{\partial V}{\partial x} \bigg|_{x=0} = k_{d\text{H2OJ}} \cdot [V(0,t) - V_{\text{air}}] \frac{\partial V}{\partial x} \bigg|_{x=x_{L/2}} = 0, t > 0, \quad (6)$$

$$V(x,0) = V_0(x), 0 < x < x_{L/2}, \quad (7)$$

$$\frac{\partial W}{\partial x} \bigg|_{x=0} = k_{d\text{H2OJ}} \cdot [W(0,t) - W_{\text{air}}] \frac{\partial W}{\partial x} \bigg|_{x=x_{L/2}} = 0, t > 0, \quad (8)$$

$$W(x,0) = W_0(x), 0 < x < x_{L/2}. \quad (9)$$

The above equations (W) complement a mathematical expression that describes the connection between water vapor level ($V$) and water liquid level. Inside the bread’s gas cells, saturation status is assumed. For this purpose, we use a saturated steam table. Physical parameters are not constants in (1)–(9). These equations are reliant on $W(x,t)$ humidity level and $T(x,t)$ the local temperature (Table 1), which provides the MATLAB model’s material parameters.
and diffusion coefficient. In the vapor diffusion coefficient, coefficient of vapor mass has transferred; also, coefficient of water mass transfer and the rate of evaporation have been illustrated using different references.

The follow procedure is used to implement the model:

(a) Under the conditions in (2) and (3), the thermal transfer Equation (1) gets the temperature

(b) The content of the water vapor is measured according to the saturation condition using the new temperature; this step determines the amount of water evaporation

(c) Under the conditions set out in (6) and (7), the diffusion Equation (4) determines the vapor content

(d) Again, adjust the water vapor content V using the state of saturation. This step determines the volume of water condensation after the diffusion of water vapor

(e) Then, under the conditions set out in (8) and (9), the diffusion Equation (5) determines the water content

(f) For each time step, the whole process is replicated

2.1.2. The Finite Difference Method. Finite difference method with unconditional stability was the numerical method used for the differentiation, implicit Euler and the partial water vapor pressure related to the equations. We used MATLAB to write the source code. The phase sizes with 1.25 mm space and with time 15 s in seconds.

When the time period is >15 s, for example, at \( \Delta t = 10 s \) and \( \Delta t = 5 s \), a mathematical equation provides temperature and moisture with divergent outcomes. If there is a wide time interval, these higher moisture content values do not greatly impact the governing set of formulas, and in the subsequent sequence, an increase in moisture values corresponds to a rise in temperature concerning the higher time interval. Meaning the temperature in each step is getting higher as the time increment increases, and hence, the moisture values must be higher.

### Table 1: The MATLAB model’s material parameters and diffusion coefficients.

| Parameters       | Dependence                        |
|------------------|-----------------------------------|
| \( k = 0.1133 \) W/m K | -                                |
| \( \rho = 380 \) kg/m^3     | 170 + 380 \times W               |
| \( c_p = 1941 \) J/kg K   | -                                |
| \( h_{\text{conv}} = 8.687 \) W/m^2 K | -                      |
| \( h_{\text{rad}} = 15.42 \) W/m^2 K | -                          |
| \( \varepsilon = 0.9 \) | -                                |
| \( D_{(\text{H}_2\text{O})} = 9.58 \times 10^{-9} \) m^2 s^{-1} | -                        |
| \( W_{\text{sat}}/kg(\text{H}_2\text{O}) kg^{-1} \) | -                          |
| \( D_{(\text{H}_2\text{O})} = 0.0354 \) m^2 s^{-1} | -                        |
| \( k_{d(\text{H}_2\text{O})} = 120 m^{-1} \) | -                          |
| \( k_{d(\text{H}_2\text{O})} = 0.01 m^{-1} \) | -                          |

### Table 2: The dough/bread properties, values, and descriptions for moisture loss in the COMSOL model.

| Parameters       | Value                |
|------------------|----------------------|
| \( T_{\text{air}} \) (°C) | 220                  |
| \( T_0 \) (°C) | 20                   |
| \( \rho \) (kg/m^3) | 380                  |
| \( h \) (W/m^2 K^{-1}) | 24.1              |
| \( M_{\text{H}_2\text{O}} \) (g mole^{-1}) | 18                 |
| \( C_{\text{n}} \) (kg/m^3) | 0.42 \times \rho |
| \( \varepsilon_1 \) (kg/m^3) | 8.95 \times 10^{-5} \times \rho |
| \( C_m ((kg_{\text{mol}}) (kg_{\text{b/d}})^{-1}) \) | 0.42             |
| \( k_m \) (kg m^{-1} s^{-1}) | 1.53 \times 10^{-6}   |
| \( h_m \) (kg m^{-2} s^{-1}) | 1.8 \times 10^{-3}  |
| \( D \) (m^2 s^{-1}) | \( k_m \times \rho^{-1} \times C_m^{-1} \) |
| \( k_s \) (m s^{-1}) | \( h_m \times \rho^{-1} \times C_m^{-1} \) |
| \( D_m \) (m^2 s^{-1}) | 5 \times 10^{-10}  |
| \( \lambda \) (I kg^{-1}) | 2.33 \times 10^6  |

2.2. Finite Element Method (FEM). For the Chemical Engineering Model (CEM), we used a preapplied time-dependent iterative nonlinear solver, and the COMSOL Multiphysics’ finite element method forms this model. We use the following hypotheses to replicate the coinciding thermal transfer during baking: (i) there was no formation of crust or shrinkage; (ii) only a single component (i.e., only water) was considered for mass transfer; (iii) moisture was diffused to the surface of the substance and evaporated only on the surface.

Table 2 illustrates the parameters, its values, and explanations of the precooked or dough and cooked bread for the initial COMSOL model of water content loss. The dough is initially at room temperature; the specific capacity of moisture, the conductivity of moisture, coefficient of mass transfer in mass units, coefficient of surface water content, and the latent heat needed for vaporization are adopted from the previous studies.
2.2.1. Governing Equations. The solid’s energy balance, which is based on Fourier’s law [18], results in
\[ \rho \cdot c_p \cdot \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = 0. \] (10)

Flick’s law-based mass equilibrium results in [21]
\[ \frac{\partial c}{\partial t} + \nabla \cdot (-D \nabla c) = 0. \] (11)

Assuming that, in the most general case, the above components (\( \rho, c_p, k, \) and \( D \)), depending on the local water concentration and temperature of food, (10) and (11) create a nonlinear differential equation partial system. We based the bread’s thermal physical properties on Hines and Mad- dox [20].

\[ \frac{1 + c}{\rho} = \frac{1}{\rho_d} + \frac{c}{\rho_w}, \]
\[ D = \frac{K}{\rho c}. \] (12)

\[ \epsilon_p = 3017.2 + 2.05xT + 0.24xT^2 + 0.002xT^3, \]
\[ K = 0.194 + 0.436 \times cK = 0.194 + 0.436 \times c. \]

2.2.2. Initial and Boundary Conditions. Uniform distribution of heat and water content is given as the initial conditions:
\[ (c)_{t_0} = c_0, \quad (T)_{t_0} = T_0. \] (13)

When applying (10), boundary conditions to outside food surfaces where no build up occurs show that convection heat transfer from air to food is partly to allow the free evaporation of water and to raise dough temperature by conduction:
\[ -n \cdot (-k \nabla T) = Q_a + h_{con} (T_{ds} - T_{air}). \] (14)

The boundary conditions relating to (11) used on the outside of the bread describe the relationship of the prolonged flow of water moisture moving through the middle of the bread and the vapor exiting the surface of the food that has been transitioned to the drying air.
\[ -n \cdot (-D \nabla c) = k_c (c - c_{env}). \] (15)

In COMSOL Multiphysics, we solved the nonlinear partial differential Equations (13) and (14) using FEM. We obtained the solution using the time-dependent iterative nonlinear solver already implemented in the CEM.
3. Results and Discussion

The results of a mathematical solution for the 1-D case of this parallel transfer model by using finite difference techniques (FDM) and finite element techniques (FEM) are discussed under the following headlines as bellow.

3.1. Finite Difference Method (FDM). Figure 1 shows that as temperature increases, simulated water content on the outside decreases quickly. The water content part way to the middle increases slightly, continuing to decrease, while the temperature plateaus as moisture begins to decline. The water content initially rises in the center, and then, it begins to decline steadily.

When we place the sample object in the oven, a rapid increase in surface temperature occurs, with a slower increase in the center. Additionally, within the pores, the partial water vapor pressure increases as temperature increases. This implies that near the surface, the partial water vapor pressure becomes greater than in the core. The water vapor content then travels in the direction of the middle and exterior to reduce the difference in pressure. Where middle temperature is lower, however, the water vapor condenses. Due to high temperatures, partial water vapor pressure is not saturated in the oven. Movement of the water vapor is from the surface to the air, resulting in the beginning of drying on the surface. Water that still exists under the surface evaporates to saturate the partial water vapor pressure. This spreads to the outside or middle, and additional water leaves to the level zero of its content internally. An increase slowly happens in size due to a drying zone has found in some researches studied the modeling of heat and water diffusion in food and also in some researches studied the energy management in baking industry. One should remember that liquid water content rises towards and in the middle where it condenses, demonstrating the setup of liquid water gradient. Therefore, the liquid water continues to travel in an opposing direction from water vapor, i.e., moving to the exterior. However, the transport of liquid water takes more time than the transport of vapor.

In relation to time, the time dependence of temperature, liquid water, and water vapor is presented by their profiles. Several critical values are taken as the process’s characteristic values in the temperature and moisture profiles. When the bread reaches 100°C, its temperature is taken for the first time. This is the temperature at which all liquid water evaporates; the crumb is converted to crust during baking (see Figure 2(a)). The second critical value occurs in the middle of the bread during peak liquid water content (see Figure 2(b)). Figure 2(c) demonstrates the third important aspect, the central bread level of maximum liquid water content. Although the maximum liquid water content is higher than the initial moisture content of the dough, it should be fairly restricted due to the low movement of water vapor in the bread.

Figure 2: (a) Time when 100°C reaches to the baking. (b) Time of peak liquid water content. (c) Peak liquid water content level.
The algebraic approach was sentient to the time step value $\Delta t$. Both large and small $\Delta t$ resulted in unworkable results. Figures 2(a)–2(c) demonstrate that at 10 s–1000 s, the numerical solution is sensitive to $\Delta t$, producing relevant outcomes.

3.2. Finite Element Method (FEM). This simulation uses one-quarter of a plant intersection because of the bread’s axisymmetric as shown in Figure 3, representing moisture concentration at 1800 seconds.

Figure 3 shows a 2-D axisymmetric at 1800 seconds for the bread’s water concentration, demonstrating a drier surface in comparison to the middle. We used the following to estimate the water loss per hour: $8859.28 \, \text{mol} \, \text{m}^{-3}$ dough moisture concentration at time 0 and $4991.3 \, \text{mol} / \text{m}^3$ dough moisture concentration at 30 minutes. The dough experienced $867.98 \, \text{mol} / \text{m}^3$ of water loss. The dough’s properties with its initial conditions are listed in Table 3.

As shown in Table 4, the moisture loss decreased when the dough/bread ($c_0$) initial moisture content was changed from $0.42 / \rho / M_{\text{H}_2\text{O}}$ to $0.40 / \rho / M_{\text{H}_2\text{O}}$. This is predicted because there has been a decrease in the gap between the initial bread water content and the humidity content. There was an increase in water loss of the product when the water concentration of the air ($c_1$) was reduced; this is predicted because of an increase in the variation amount the original moisture content of the product and the air’s water content.

Table 5 shows moisture loss without heat transfer and convection COMSOL model. As expected, a lower diffusion coefficient resulted in a lower moisture loss from the dough/bread.

The moisture loss reduces with an increase in the availability of the product to hold water ($c_0$), we predicted this because the specific moisture capacity is equivalent to a substance’s specific heat capacity. The specific heat of the product correlates directly to the quantity of energy needed to raise its temperature, equivalent to the energy needed to minimize a substance’s moisture content, which is dependent on the specific moisture capacity of the product. Lastly, the moisture loss of the dough/bread increased due to an increase in the mass transfer coefficient in mass units ($h_m$); this is predicted as the mass transfer coefficient, which the water level loss of the dough/bread is correlated to.

4. Conclusion

We developed a mathematical model for the thermal transfer that occurs during the baking process. The primary findings of the study are summarized below. The FDM model describes the method of concurrent heat, water, and vapor diffusion in a 1-D solid material using radiation and convection to heat it on the exterior. The techniques used in this study include a dynamic fluid design using COMSOL Multiphysics and a mathematical design using MATLAB. The model shows the temperature goes higher with increasing time, where it increases from $20^\circ\text{C}$ to $132.4^\circ\text{C}$ when the time increases from 0 min to 30 min at an oven temperature of 180°C. In contrast, the temperature ranges from $20^\circ\text{C}$ to $179.5^\circ\text{C}$ to $260^\circ\text{C}$ in the oven.

The model used shows that the concentration in the water declines with time. It decreases from $8866.6$ moles/m$^3$ to $4991.3$ moles/m$^3$ when the time increases from 0 min to 30 min at an oven temperature of $220^\circ\text{C}$. The model shows that the moisture content % decreases with increasing time. It decreases from 42% to 23.6% as the time goes from 0 min to 30 min at $220^\circ\text{C}$ oven temperature. The mathematical model well describes the mechanisms of simultaneous diffusion of heat, water, and vapor into a one-dimensional solid material that is heated by radiation and convection from...
the outside. The simulated water content at the surface decreases rapidly when the temperature increases. The simulated water content halfway to the center at first increases a little and then starts to decrease, and the simulated temperature remains on a plateau while the water content starts to decrease.

The mathematical model was very sensitive to the time stage, and satisfactory results were obtained only on a range of time stages, between 15 s and 100 s. Smaller temporal stages produced an erroneous and divergent result, while a larger temporal stage rendered a result useless.

**Nomenclature**

**Abbreviations**

- FEM: Finite element method
- CFD: Computational fluid dynamics
- FDM: Finite difference method.

**Latin Symbols**

- $c$: Specific heat capacity (J/kg K)
- $c_p$: Concentration amount (mol/m$^3$)
- $c_0$: Specific heat with constant pressure (J/kg K)
- $c_{i0}$: Initial dough moisture concentration (kg m$^{-3}$)
- $c_i$: Air moisture concentration (kg m$^{-3}$)
- $C_{m0}$: Specific moisture capacity ((kg$_{mos}$)/(kg$_{b/d}$))
- $D$: Diffusion coefficient (m$^2$ s$^{-1}$)
- $D_m$: Surface moisture diffusivity (m$^2$ s$^{-1}$)
- $h$: Coefficient of heat transfer (W m$^{-2}$ K$^{-1}$)
- $h_m$: Mass transfer coefficient in mass units (kg m$^{-2}$ s$^{-1}$)
- $k$: Thermal conductivity (W m$^{-1}$ K$^{-1}$)
- $k_m$: Mass transfer coefficient (m s$^{-1}$)
- $k_d$: Mass transfer coefficient of water at the bread surface (m$^{-1}$)
- $k_{m0}$: Moisture conductivity (kg m$^{-1}$ s$^{-1}$)
- $M$: Molar mass (kg mol$^{-1}$)
- $Q_{fl}^s$: Heat flow rate per unit area (W m$^{-2}$)
- $T$: Temperature (K and °C)
- $t$: Time, s
- $V$: Water vapor content (kg (H2O,g) kg$^{-1}$(product))
- $W$: Liquid water content (kg (H2O) kg$^{-1}$(product))
- $x$, $y$, $z$: Cartesian space coordinates (m).

**Greek Symbols**

- $\rho$: Mass density (kg m$^{-3}$)
- $\varepsilon$: Emissivity (–)
- $\lambda$: The latent heat of vaporization (kJ/kg).

**Subscripts**

- $b/d$: Bread/dough
- $cnv$: Convection
- $dr$: Dry
- $env$: Environment
- $ff$: Fluid flowing stream
- $g$: Gas
- $l$: Liquid
- $m$: Moisture
- $rad$: Radiation
- $rs$: Radiating source
- $sr$: Surface
- $w$: Wet
- $0$: Conditions of the thermodynamic environment.

**Data Availability**

All the data that were listed in the research are the result of using simulation programs and are limited to the purpose of the research, and no data from an external source was based in the research.

**Disclosure**

The abovementioned author, “Heba Ahmed Mosalam,” declares that this manuscript contains original, unpublished results which are not currently being considered for publication elsewhere while for all reused data, the agreement of the publisher is attached. I confirm that the manuscript has been read and ready to be published. I further confirm that the author listed in the manuscript is the corresponding author and the sole contact for the Editorial process. The author is also responsible for communicating about progress, submissions of revisions, and final approval of proofs.

**Conflicts of Interest**

The author declares no conflicts of interest.

**References**

[1] M. Al-Nasser, I. Fayssal, and F. Moukalled, “Numerical simulation of baking in a convection oven,” *Applied Thermal Engineering*, vol. 184, p. 116252, 2021.

[2] R. S. Reddy, D. Arepally, and A. K. Datta, “Estimation of heat flux in baking by inverse problem,” *Journal of Food Engineering*, vol. 271, p. 109774, 2020.

[3] H. W. Park and W. B. Yoon, “Computational fluid dynamics (CFD) modelling and application for sterilisation of foods: a review,” *Energies*, vol. 6, no. 6, p. 62, 2018.

[4] A. Tank, N. Chhanwal, D. Indrani, and C. Anandharamakrishnan, “Computational fluid dynamics modeling of bun baking process under different oven load conditions,” *Journal of Food Science and Technology*, vol. 51, no. 9, pp. 2030–2037, 2014.

[5] N. Therdthai, W. Zhou, and T. Adamczak, “Three-dimensional CFD modelling and simulation of the temperature profiles and airflow patterns during a continuous industrial baking process,” *Journal of Food Engineering*, vol. 65, no. 4, pp. 599–608, 2004.

[6] H. Alfar, A. Kianifar, and H. Zamani, “Investigation of the effect of variable heat flux on energy consumption and bread quality in the flat baking process by experimental and numerical methods,” *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, pp. 1–16, 2020.

[7] P. Jayapragasam, P. BideauLe, and T. Loulou, “Approximation of heat and mass transport properties for one sided cake baking,” *Journal of Food Engineering*, vol. 290, p. 110211, 2021.
[8] M. Khodeir, O. Rouaud, A. Ogé, V. Jury, P. Le-Bail, and A. Le-Bail, "Study of continuous cake pre-baking in a rectangular channel using ohmic heating," Innovative Food Science & Emerging Technologies, vol. 67, p. 102580, 2021.

[9] N. Chhanwal, D. Indrani, K. S. Raghavarao, and C. Anandharamakrishnan, "Compututional fluid dynamics modeling of baking process," Food Research International, vol. 44, no. 4, pp. 978–983, 2011.

[10] I. Zheleva and V. Kambourova, "Identification of heat and mass transfer processes in bread during baking," Thermal science, vol. 9, no. 2, pp. 73–86, 2005.

[11] J. Zhang and A. K. Datta, "Mathematical modeling of bread baking process," Journal of Food Engineering, vol. 75, no. 1, pp. 78–89, 2006.

[12] E. Purlis and V. O. Salvadori, "Bread baking as a moving boundary problem. Part 1: mathematical modelling," Journal of Food Engineering, vol. 91, no. 3, pp. 428–433, 2009.

[13] A. Ousegui, C. Moresoli, M. Dostie, and B. Marcos, "Porous multiphase approach for baking process – explicit formulation of evaporation rate," Journal of Food Engineering, vol. 100, no. 3, pp. 535–544, 2010.

[14] E. Purlis, "Bread baking: technological considerations based on process modelling and simulation," Journal of Food Engineering, vol. 103, no. 1, pp. 92–102, 2011.

[15] D. D. O. M. A. R. I. GANJI, M. PEIRAVI, and M. ABBASI, "Evaluation of the heat transfer rate increases in retention pools nuclear waste," International Journal of Nano Dimension, vol. 6, no. 4, pp. 385–398, 2015.

[16] M. M. Peiravi, J. Alinejad, D. Ganji, and S. Maddah, "Numerical study of fins arrangement and nanofluids effects on three-dimensional natural convection in the cubical enclosure," Challenges in Nano and Micro Scale Science and Technology, vol. 7, no. 2, pp. 97–112, 2019.

[17] M. M. Peiravi, J. Alinejad, D. D. Ganji, and S. Maddah, "3D optimization of baffle arrangement in a multi-phase nanofluid natural convection based on numerical simulation," International Journal of Numerical Methods for Heat & Fluid Flow, vol. 30, no. 5, 2020.

[18] P. F. Dickson, "Heat transfer, 4th Ed., J. P. Holman, McGraw Hill Book Company. 530 pages, price:$17.00," AIChE Journal, vol. 23, no. 4, 1977.

[19] M. BALABAN and G. M. PIGOTT, "Mathematical model of simultaneous heat and mass transfer in food with dimensional changes and variable transport parameters," Journal of Food Science, vol. 53, no. 3, pp. 935–939, 1988.

[20] A. L. Hines and R. N. Maddox, Mass Transfer, Fundamentals and Applications, vol. 7, no. 63, 1985Prentice Hall Inc, 1985.

[21] K. Kuttler, "Boundary Value Problems, Fourier Series," Elementary Differential Equations, 2018.