Direct Observation of Topology from Single-photon Dynamics on a Photonic Chip

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Topology manifesting in many branches of physics deepens our understanding on state of matters. Topological photonics has recently become a rapidly growing field since artificial photonic structures can be well designed and constructed to support topological states, especially a promising large-scale implementation of these states using photonic chips. Meanwhile, due to the inapplicability of Hall conductance to photons, it is still an elusive problem to directly measure the integer topological invariants and topological phase transitions for photons. Here, we present a direct observation of topological winding numbers by using bulk-state photon dynamics on a chip. Furthermore, we for the first time experimentally observe the topological phase transition points via single-photon dynamics. The integrated topological structures, direct measurement in the single-photon regime and strong robustness against disorder add the key elements into the toolbox of ‘quantum topological photonics’ and may enable topologically protected quantum information processing in large scale.

The introduction of topology into condensed-matter and material sciences originates from the connection of integer quantum Hall conductances with topological Chern invariants [1], which greatly expands our knowledge on state of matters. With the birth of topological insulators, searching topological state of matters in solid state materials [2 3] and photonic systems [4 5] has recently become a leading research field. In contrast to the challenging experimental requirements for realizing topological states in solid state materials, photonic systems provide a convenient and versatile platform to design various topological lattice models and study different topological states, including topological insulator states [6 10] and topological Weyl points [11 12]. The found topological boundary states potentially can be utilized for developing inherently robust and efficient artificial photonic devices [13 16].

In the view of fundamental physics, the topological invariant is a crucial parameter to characterize the topological matter state. In fermion systems, the topological invariant can be revealed by conductance measurements, while the concept of Hall conductance is inapplicable in photonic systems. New methods for directly detecting the topological invariants in topological photonics remain to be developed. The pioneering proposals in theory [17 19] and experimental observations have been dedicated to be developed. The pioneering proposals in theory [17 19] and experimental observations have been dedicated in both integrated photonic lattices [20 22] and bulk optics [23 26].

Different from the efforts made to detect the topological invariant based on probing Berry curvature [17 20], non-Hermitian photon loss [18 21] or the dynamics of edge states [19 22], we propose a new approach to directly detect the topological invariant via the bulk-state photon dynamics in the real space, which beyonds the physical picture where topological invariant is defined on the equilibrium Bloch state in the momentum space.

To extend promised topological protection into the quantum regime, we have to find an appropriate system that is physically scalable and has inherently low loss when scaling up. Integrated photonics can meet the first requirement elegantly by constructing topological structures on a photonic chip in a physically scalable fashion, with which topological states can be generated, manipulated and detected in a very high complexity beyond that conventional bulk optics can do [23 26]. Meanwhile, realizing topological states in Hermitian systems can well meet the second requirement since the intrinsic loss in non-Hermitian systems [21 24 25] will induce an evolution of exponential decay for single photons and multiplicative inefficiency for multi-photons.

Here, we integrate topological waveguide lattices on a photonic chip and experimentally demonstrate a direct observation of the topological invariants in the...
constructed Hermitian system using bulk-state photon dynamics. Through initially injecting photons into the middle waveguide to excite the bulk state, the values of topological winding numbers can be extracted from the chiral photonic density centers associated with the final output distribution. We further extend the topological system and measurement into quantum regime by observing the topological phase transition point separating the topological trivial and nontrivial phases, even with artificially introduced disorder.

**Topological photonic lattice.** As is shown in Fig. 1(a), we fabricate waveguide lattices in borosilicate glass by using the femtosecond laser direct writing technique \([27,30]\) (see Methods for details). The constructed lattices are based on the Su-Schrieffer-Heeger model, which describes a one-dimensional lattice with alternating strong and weak couplings. The Hamiltonian of this model could be written as

\[
H = \sum_x (J_1 a_x^+ b_x + J_2 b_{x+1}^+ a_x) + h.c.,
\]

where each unit cell in the chain consists of two sites labeled as \(a\) and \(b\), the terms \(a_x^+ (a_x)\) and \(b_x^+ (b_x)\) are the creation (annihilation) operators for the two sites in the \(x\) unit cell, and the coefficients \(J_1\) and \(J_2\) represent the intra-cell and inter-cell coupling strengths, respectively.

To study the topological feature, we rewrite Eq. (1) in momentum space as

\[
\hat{H} = \sum_k \hat{h}(k),
\]

where \(\hat{h}(k) = d_x \hat{\tau}_x + d_y \hat{\tau}_y,\) \(d_x = J_1 + J_2 \cos(k_x),\) \(d_y = J_2 \sin(k_x),\) and \(\hat{\tau}_x\) and \(\hat{\tau}_y\) are the Pauli spin operators defined in the momentum space. The energy bands of the Hamiltonian are nontrivial and nontrivial phases, respectively. The Hamiltonian is marked starting from the edge of the system with 1. Every unit cell consists of two sites labeled as \(a\) and \(b\), the terms \(a_x^+ (a_x)\) and \(b_x^+ (b_x)\) are the creation (annihilation) operators for the two sites in the \(x\) unit cell, and the coefficients \(J_1\) and \(J_2\) represent the intra-cell and inter-cell coupling strengths, respectively.

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Dynamical detection of topological winding number. To detect the winding number of the lattice on the topological photonic chip, we introduce a photon population difference center (PPDC) \(P_d = \sum_x x(a_x^+ a_x - b_x^+ b_x)\), where the unit cell index \(x\) is shown in Fig. 1(a). We inject photons into the middle waveguide to excite the bulk state. With the evolution of the photons over a distance \(z\) in the lattice, the corresponding PPDC can be expressed as

\[
\nu = \frac{1}{2\pi} \int dk_x \mathbf{n} \times \partial_{k_x} \mathbf{n}
\]

where \(\mathbf{n} = (n_x, n_y) = (d_x, d_y)/\sqrt{d_x^2 + d_y^2}\). We manipulate the coupling coefficients as \(J_1 = g + gt \cos(w\tau)\) and \(J_2 = g - gt \cos(w\tau)\), where \(g > 0, 0 < w < 1,\) and \(0 < t < 1\). The system is in the topological nontrivial phase with the winding number \(\nu = 1\) when \(J_1 < J_2\), i.e. \(w \in (0.5, 1)\). Otherwise, it is in the topological trivial phase with \(\nu = 0\) when \(J_1 > J_2\), i.e. \(w \in (0, 0.5)\). The topological phase transition point appears when \(J_1 = J_2\).
FIG. 2. Experimental results of PPDC. The measured values of PPDC for 10-sited (a, b) and 18-sited (c, d) lattices, which are found oscillating around 0 (a, c) and 0.5 (b, d) for the systems in topological trivial and nontrivial phase. The averaged values of PPDC $\bar{P}_d$ are $0.045 \pm 0.090$ (a) and 0.540 ± 0.070 (b) for the case of $t = 1.0$, and are 0.095 ± 0.16 (c) and 0.526 ± 0.014 (d) for the case of $t = 0.5$, respectively.

step size of 0.2mm. We excite one waveguide in the central unit cell ($x = 3$) with a narrowband coherent light at 852 nm, and measure the output density from each photonic lattice. The evolution-distance-dependent PPDC is extracted and shown in Fig.2(a–b). The result in Fig.2(a) shows that, when the system is in the topological trivial phase, the values of PPDC $\bar{P}_d$ keep oscillating centered at 0.045 ± 0.090. While the system is in the topological nontrivial phase, $\bar{P}_d$ keeps oscillating around 0.540 ± 0.070 as shown in Fig.2(b). According to Eq. (3), we obtain the topological winding numbers $\nu = 0.09 \pm 0.18$ and $\nu = 1.08 \pm 0.14$ for the two phases respectively. We can see that the oscillation of the measured $P_d$ values is more irregular than that of the simulated result, but the winding number can still be clearly extracted.

To further experimentally demonstrate the reliability and universality of our approach, we fabricate another set of photonic lattices consisting of 18 waveguides with $t = 0.5$. The evolution distance varies from 7 mm to 16 mm with a step size of 0.2 mm. As is shown in Fig.2(c–d), when the lattices are prepared in the topological trivial and nontrivial phases, the measured values of $\bar{P}_d$ are oscillating around 0.095 ± 0.16 and 0.526 ± 0.014, which lead to the topological winding numbers of $\nu = 0.19 \pm 0.32$ and $\nu = 1.052 \pm 0.28$, respectively. The results are well consisit with the simulated results shown in Fig.1(c) and suggest that our proposed dynamical approach of measuring topological invariants is insensitive to the detailed lattice configurations.

Dynamical detection of topological phase transition. We further extend the topological system and measurement into quantum regime by observing the topological phase transition point via single-photon dynamics. The topological phase transition in our photonic lattices can also be directly measured from the output density distribution. The transition point can be revealed by the generalized photon population center $\bar{P}_c = \sum_x x^2 (a_x^+ a_x + b_x^+ b_x)$, where the label $x$ is marked as shown in Fig.3(a) for a concise expression. With the bulk state excited from the central unit cell by single photons, the value of the generalized photon population center can be derived as $\bar{P}_c(z)$ for an evolution distance $z$ (see Methods). We can further obtain the topological phase transition signal (TPTS) $S_t = \bar{P}_c(z)/z^2$, and we find that (see Methods)

$$S_t = \begin{cases} \frac{\nu^2}{2}, & J_1 < J_2 \\ \frac{\nu^2}{2}, & J_1 > J_2 \end{cases}$$ (4)

The simulated results are illustrated in Fig.3(b). For a continuously transitive system from topological non-
FIG. 4. Experimental results of TPTS. a, The relation between the coupling strength and the separation between adjacent waveguides. The blue dash lines mark the experimentally accessible range of $d_1$ and $d_2$. b, The coupling strength of $J_1$ and $J_2$ of the 11 lattices used in experiment (red squares). c, The evolution probability distribution of single photons in topological nontrivial phase, transition point and trivial phase. The blank arrows mark the excited sites in experiment. d, The measured results of TPTS. The topological transition point appears when the system undergoes the phase transition from the topological nontrivial to trivial phase (red circles), even with artificially introduced disorder (black and blue circles).

The demonstrated key elements, including integrated topological structures, direct measurement in single-
photon regime and strong robustness against disorder, can enrich the emerging field of ‘quantum topological photonics’. With the primary attempt to combine topology with quantum integrated photonics, it is promising to explore scalable topologically protected quantum information processing on topological photonic chips beyond classical topological photonics. The prompt questions, but remain open, will be whether we can directly observe topology with multi-photon dynamics and how qubit and entanglement behave.

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Methods

Fabrication and measurement of the integrated topological photonic lattices: According to the characteristics coupling coefficients modulated by the separation between two adjacent waveguides, the lattices are designed and written in borosilicate glass (refractive index $n_0 = 1.514$) with femtosecond laser with repetition rate 1MHz, pulse duration 290fs and working wavelength 513nm. Before the laser writing beam is focused inside the borosilicate substrate with a 50X objective lens (numerical aperture of 0.55), we control the shape and size of the focal volume of the beam with a beam-shaping cylindrical lens. A high-precision three-axis motion stage is used to move the photonic chip during fabrication with a constant velocity of 5 mm/s.

Experiments are performed by injecting the photons (herald single photon) into the lattices using a 20X objective lens. The evolution output is observed using a 10X microscope objective lens and the CCD (ICCD) camera after a total evolution distance within the lattice.

The relationship between topological winding number and PPDC: Suppose one waveguide in the middle unit cell is initially excited, then the initial state of the photonic chip can be written as $|\psi(0)\rangle$. To find the relationship between the photon dynamics in the waveguide lattice and the winding number, we introduce population difference in each unit cell and define a photon population difference center (PPDC), i.e.,

$$P_d = \sum_{x=1}^{N} x(P_{a_x}^c - P_{b_x}^c),$$

(5)

where $P_{a_x}^c = |e\rangle \langle e| (m = a_x, b_x)$ is the photon population probability. The PPDC associated with the photonic evolution in the waveguide lattice can be described as

$$\bar{P}_d(z) = \langle \psi(0) \rangle e^{iHZ} P_d e^{-iHZ} |\psi(0)\rangle.$$  

(6)

For the photonic SSH model, we can connect the above dynamical center to the winding number. We rewrite the PPDC in the momentum space as

$$\bar{P}_d(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_x \langle \chi(0) | e^{i(k_x)z} \partial_{k_x} \tau_z e^{-i(k_x)z} | \chi(0) \rangle.$$  

(7)

By substituting $h(k_x) = d_x \sigma_x + d_y \sigma_y$ into (7), we find that $\bar{P}_d(Z)$ can be connected with the topological winding number $\nu$ defined, i.e.,

$$\bar{P}_d(z) = \frac{\nu}{2} - \frac{1}{4\pi} \int dk_x \cos(2Ez) n \times \partial_{k_x} n,$$  

(8)

where $n = (n_x, n_y) = (d_x, d_y)/E$ and $E = \sqrt{J_1^2 + J_2^2 + 2J_1J_2 \cos(k_x)}$. In the long evolution distance limit, the second term in the above equation will vanish. Then we can obtain a relationship between the winding number and the evolution-distance-averaged PPDC, i.e.,

$$\nu = 2 \bar{P}_d,$$  

(9)

where the evolution-distance-averaged PPDC is

$$\bar{P}_d = \lim_{Z \to \infty} \frac{1}{Z} \int_0^Z dz \bar{P}_d(z).$$  

(10)

where $Z$ is the total evolution distance for the photons in the waveguide lattice. Note that $\bar{P}_d$ is just the oscillation center of $\bar{P}_d(z)$ varying with $z$. The topological winding number is twice this oscillation center.

The relationship between TPTS and photon population center: The initial state of the photonic chip is the same as the one in the winding number detection. To find the relation between the photon dynamics in the waveguide lattice and the winding number, we introduce a generalize photon population center operator, i.e.,

$$P_c = \sum_{x=1}^{N} x^2 (P_{a_x}^c + P_{b_x}^c),$$  

(11)
where $P_m^e = |e\rangle_m \langle e|$ $(m = a_x, b_z)$ is the photon population operator. Then the generalized photon population center associated with the evolution of photons in the waveguide lattice can be described as

$$\hat{P}_z(z) = \langle \psi(0) | e^{iHz} P e^{-iHz} | \psi(0) \rangle.$$  \hspace{1cm} (12)

Based on the above equation, we define a new quantity called as topological phase transition signal (TPTS), which is expressed as

$$S_t = \frac{\hat{P}_z(z)}{z^2}. \hspace{1cm} (13)$$

By transferring the above equation into the momentum space, we can further get

$$S_t = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_x \langle \psi(0) | e^{iH_k} | \psi(0) \rangle^2.$$ \hspace{1cm} (14)

In the long evolution distance limit, the terms proportional to $1/z$ can be omitted and the above identity can be simplified into

$$S_t = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_x \langle \partial_{k_x} E \rangle^2$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_x \frac{J_1^2 J_2^2 \sin^2(k_x)}{J_1^2 + J_2^2 + 2J_1^2 J_2^2 \cos(k_x)}. \hspace{1cm} (15)$$

Based on residue theorem, we can analytically solve the above integral and get

$$S_t = \begin{cases} \frac{J_1^2}{J_2^2}, & J_1 < J_2 \\ \frac{J_2^2}{J_1^2}, & J_1 > J_2 \end{cases} \hspace{1cm} (16)$$

This equation shows that the topological phase transition in the photonic Su-Schrieffer-Heeger model can be directly observed from the single photon dynamics in bulk state.

The generation and imaging of the heralded single photons: We obtain the single-photon source with the wavelength of 810 nm generated from periodically-poled KTP (PPKTP) crystal via spontaneous parametric down conversion (SPDC). After a long-pass filter and a polarized beam splitter (PBS), the photon pairs are separated to two components, horizontal and vertical polarization. The measured evolution patterns would come from the thermal-state light rather than single-photons if we inject only one polarized photon into the lattices without external trigger. Therefore, we inject the horizontally polarized photon into the lattices, while the vertically polarized photon acts the trigger for heralding the horizontally polarized photons out from the lattices with a time slot of 10ns. We capture each evolution result using ICCD camera after accumulating in the ‘external’ triggering mode for 2000s.

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