Research on main-lobe deceptive jamming against FDA-MIMO radar

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Abstract
A multiple-input multiple-output (MIMO) radar with frequency diverse array (FDA) produces a range-angle-dependent steering vector, which can suppress the deceptive jamming in joint range-angle domain. This in turn complicates the task of radar jamming. With the emergence and development of digital radio frequency memory (DRFM), radar electronic jamming technique has entered the age of coherent jamming. Based on the analysis of the superiority of the FDA-MIMO radar in cancelling the deceptive jammers, a novel range gate pull-off method via DRFM against FDA-MIMO radar is proposed. Compared with the traditional deceptive jamming methods, the proposed method cannot be suppressed due to the mismatch in range. In particular, even the anti-jamming methods based on power domain cease to be effective. The effectiveness of the proposed scheme in jamming the tracking performance of the FDA-MIMO radar is demonstrated via computer simulations.

1 INTRODUCTION

In modern electronic environment, electronic warfare with its silent and changeable technical means has become the fifth dimensional battlefield after land, sea, air and sky. Therefore, electronic countermeasures (ECMs) have become a research hotspot and undergone rapid developments [1–3]. Among ECMs techniques, deceptive jamming plays an important role. In particular, if the deceptive jamming exists in the same angle as the target, or in the angle covered by the main lobe of the radar, then such jamming called the main-lobe jamming can reduce the estimation precision and tracking accuracy of the victim radar seriously. Phased array (PA) can suppress the power of jamming into radar through a series of side lobe anti-jamming measures, but there is no effective means to reduce the main-lobe jamming power from the beam gain because if the gain reduces, the target power will reduce, respectively, and the detection performance of the target will be weakened. Specifically, the radar waveform is replicated by the false target generator (FTG) and its core technology is the digital radio frequency memory (DRFM) which can scale, delay, modulate and retransmit the intercepted radar waveform properly [4–6]. Moreover, based on appropriate DRFM timing, it is possible to produce the main-lobe coherent jamming, resulting in a big challenge to classify and suppress the intended false targets.

1.1 Related works

Since frequency diverse array (FDA) is first proposed in [7, 8], it draws great attention due to its range-dependent property [9, 10]. By using a tiny frequency offset across the array elements, FDA provides a time-range-angle-dependent beam pattern bringing the possibility of joint location and detection of range and angle parameters, which has promising broad potentials in electronic counter countermeasures (ECCMs). Furthermore, MIMO radar has received significant attention due to its various advantages over conventional radar system [11–13], such as enhanced detection performance, improved parameter identifiability and angular resolution, providing more degrees of freedom (DOF) and better spatial coverage [14]. By combining FDA and MIMO technology, FDA-MIMO system has the result of associating a...
waveform to each point range/angle of the space, with the possibility of recovering this information in the receiver after appropriate processing \cite{15}. Thus, the range-angle-dependent transmit steering vector is obtained. Several approaches have been suggested to suppress the deceptive jamming in FDA-MIMO radar schemes. In \cite{16}, a subarray-based FDA method was proposed to counteract the deceptive signals. In \cite{17}, the FDA-MIMO radar configuration with the generalized likelihood ratio test detector was considered to guarantee high rejection probability of deceptive jamming. In \cite{18}, the FDA-MIMO radar was investigated to suppress the deceptive jamming in the joint transmit–receive domain.

However, the modelling and analysis of the jamming signal of FDA are still insufficient. Based on the FDA-MIMO radar technology, the expression of target and deceptive jamming signal is derived in \cite{19}, and a method to distinguish true and false targets is proposed by taking advantage of the difference between the real target and the deceptive jamming false target in the space frequency domain of transmitting and receiving. In \cite{20}, a space-time adaptive processing method for enhanced 3-D joint domain localization is proposed. To solve the main-lobe jamming problem, a robust deception jamming suppression method based on covariance matrix reconstruction is proposed in \cite{21}, and a robust adaptive beamforming method based on direct data domain technology is proposed in \cite{22}. However, the jamming models in \cite{19–22} ignore the time delay modulation in the jammer part, hence the jamming models are incomplete. In \cite{23}, the neglected time delay problem is reduced and analysed in the deceptive jamming model of FDA. The authors in \cite{24, 25} proposed deceptive jamming suppression methods based on sample selection and nonuniform sample detection.

A further problem is that the main-lobe deceptive jamming methods studied in \cite{23–25} are based on the PA system, rather than designing a novel jamming method according to the characteristics of the FDA-MIMO radar. However, ECMs and ECCMs are coexisting, mutually promoted and developed. Hence with the deepening of the anti-jamming research based on FDA-MIMO radar, novel deceptive jamming against FDA-MIMO radar should be studied and proposed.

1.2 Contributions

The main-lobe deceptive jamming against FDA-MIMO is studied. The main contributions of this study are given as follows:

1. The accurate understanding about the effect of main-lobe deceptive jamming against FDA-MIMO radar is presented.
2. A novel understanding perspective is proposed to obtain the range information of the true target from false target jamming.
3. A novel range gate pull-off (RGPO) method against FDA-MIMO is proposed, and the special advantages compared with false target jamming are presented.

1.3 Study organization

The remainder of this study is organized as follows. Section 2 presents the signal model of the FDA-MIMO radar with deceptive jamming. Section 3 explores the main-lobe false target jamming methods against FDA-MIMO radar and corresponding anti-jamming measures. In Section 4, a novel RGPO method against FDA-MIMO radar is proposed. Section 5 shows the simulation results and performance analysis. Finally, conclusions are drawn in Section 6.

2 SIGNAL MODEL OF FDA-MIMO RADAR

We consider a colocated FDA-MIMO radar consisting of $N$ transmit elements and $M$ receive elements in an identical uniform linear array and the first element is recorded as the reference element. The configuration of FDA-MIMO is shown in Figure 1.

The radiation frequency fed to the $n$-th element is assigned as:

$$ f_n = f_0 + \Delta f_n, \quad n = 0, 1, ..., N - 1 $$

where $f_0$ is the carrier frequency, and $\Delta f_n$ is the frequency offset corresponding to the $n$-th element which is tiny enough when compared with the carrier frequency and can be expressed as:

$$ \Delta f_n = n\Delta f $$

where $\Delta f$ is the frequency increment cell. Under the narrowband assumption, the monochromatic continuous signal transmitted by the $n$-th element can be given as:

$$ s_n(t) = \text{rect}\left(\frac{t}{T_p}\right)\psi_n(t)\exp(j2\pi f_n t) $$

where $T_p$ denotes the pulse duration, $\psi_n$ denotes the baseband envelope of the $n$-th transmit element and the function $\text{rect}(x)$ is defined as:

$$ \text{rect}(x) = \begin{cases} 
1, & 0 \leq x \leq 1 \\
0, & \text{else}
\end{cases} $$

For better separation of waveforms at the receiver, without loss of generality, $\psi_n(t)$ are assumed to be orthogonal to each other, that is,

$$ \int \psi_{n1}^*(t) \cdot \psi_{n2}(t - \tau) dt = 0, \quad n1 \neq n2, \forall \tau $$
where $(\cdot)^*$ is the conjugate operation and $\tau$ denotes the time delay. Observed from a specific far-field location with an angle $\theta$ and a slant range $r$, the received signal corresponding to the $n$-th transmit and $m$-th receive elements can be expressed as:

$$s_{n,m}(t) = \text{rect} \left( \frac{t - \tau_{n,m}}{T_p} \right) \psi_n(t - \tau_{n,m}) e^{j2\pi f_0' \left( t - \tau_{n,m} \right)} \quad (6)$$

where $\tau_{n,m} = 2r/c - nd_T \sin \theta/c - md_R \sin \theta/c$ represents the transmit delay from the $n$-th transmit element to the $m$-th receive elements, $d_T$ and $d_R$ denote the interelement spacing of transmit and receive elements, respectively, $c$ denotes the light space.

Under the assumption of far-field narrow band, $\text{rect}[t - \tau_{n,m}] / T_p \approx \text{rect}[t - \tau_{0,0}] / T_p$, the phase difference between the signal of the first element and the reference element cannot be ignored. Hence, Equation (6) can be approximated to:

$$s_{n,m}(t) = \text{rect} \left( \frac{t - 2r/c}{T_p} \right) \psi_n(t - 2r/c) e^{j\varphi_n(t')} \quad (7)$$

where $t' = t - 2r/c \in [0, T_p]$ is the time index within pulse and $\varphi_n(t')$ is assigned as:

$$\varphi_n(t') = 2\pi \int_{0}^{t'} \frac{nd_T \sin \theta}{c} + f_0 + md_T \sin \theta}{c} + f_0 + md_T \sin \theta}{c} \, dx$$

$$= 2\pi \left[ f_0 t' + f_0 \frac{nd_T \sin \theta}{c} + f_0 \frac{md_T \sin \theta}{c} \right]$$

$$+ f_0 \frac{nd_T \sin \theta}{c} + f_0 \frac{md_T \sin \theta}{c} \Delta f_n \, dx$$

$$\approx 2\pi \left( f_0 t' + \Delta f_n t' + f_0 \frac{nd_T \sin \theta}{c} + f_0 \frac{md_T \sin \theta}{c} \right) \quad (8)$$

In the receive chain, as shown in Figure 2, the received signal is mixed with $e^{j2\pi f_0' t}$ in the analogue device, mixed with $e^{j2\pi f_n' t}$ in the digital device and matched with $\psi_n(t')$ also in the digital device. Hence, the output signal with respect to the $n$-th transmit and $m$-th receive elements can be expressed as:

$$s_{\text{output}}^{n,m}(t') \approx \text{rect} \left( \frac{t'}{T_p} \right) \xi_n e^{j2\pi \left( -f_0 \frac{nd_T \sin \theta}{c} + f_0 \frac{md_T \sin \theta}{c} \right) \Delta f_n t'} \quad (9)$$

where $\xi_n$ is the complex coefficient after matched filtering. The array factor can be given by:
3 | PRINCIPLE AND PERFORMANCE OF DECEPTIVE FALSE TARGET JAMMING

In this section, the main-lobe deceptive false target jamming against FDA-MIMO radar is explored. First, the principle of the main-lobe deceptive false target jamming is studied. Second, a recently introduced false target deceptive jamming approach against FDA-MIMO is explored. Third, an approach to suppress the recently jamming approach is studied.

3.1 | Signal model of false target against FDA-MIMO radar

In the FTG, the emergence of DRFM makes the FTG capable of storing and reverting the radar’s transmit signal to deceive the radar systems. Particularly, the FTG can create a false target with an apparent negative range offset by delaying the stored pulse until a fixed time before the next incoming radar pulse.

Consider an FTG with an angle $\theta_i$ and a slant range $r_i$, which adopts the same frequency offset setting as FDA-MIMO. Let the transmitted signal be intercepted by the FTG and modulated by time delay $\tau = 2\Delta r/c$, where $\Delta r$ denotes the equivalent range corresponding to the modulated time delay. Thus, the generated false target can be expressed as:

$$s_j^n(t) = \text{rect} \left( \frac{t'}{T_p} \right) \sum_{n=0}^{M-1} c e^{j2\pi \Delta f_s n \Delta \phi + jf_0 n \Delta \phi} \sum_{m=0}^{N-1} e^{j2\pi \Delta f_p m \Delta \phi + jf_0 m \Delta \phi}$$

(10)

The steering vector can be expressed as:

$$v(r, \theta) = a(r, \theta) \otimes b(\theta)$$

(11)

where $\otimes$ is the Kronecker operator, $a(r, \theta)$ and $b(\theta)$ are the transmit and receive steering vectors, respectively, which can be expressed as:

$$a(r, \theta) = \left[ 1, \ldots, e^{j2\pi [-\Delta f_s \Delta r/c + f_0 (N-1) \Delta \phi \sin \theta / c]} \right]^T$$

(12)

$$b(\theta) = \left[ 1, \ldots, e^{j2\pi f_0 (M-1) \Delta \phi \sin \theta / c} \right]^T$$

(13)

As seen from Equation (11), transmit-receive beam pattern can be further divided into transmit beam pattern and receive beam pattern. Note that the transmit beam pattern herein is supposed to be the equivalent transmit beam pattern at receiver, which is different from the original transmit beam pattern.

$$x(l) = x_i(l) v(r_i, \theta_i) + \sum_{j=1}^{K} x_i(l) v(r_j, \theta_j) + n(l)$$

(16)
where \( x_i(l), x_j(l), n(l) \) are the desired signal, the \( j \)-th false target signal and additive white Gaussian noise vector, respectively.

To detect the target, but also to suppress the jamming, we can set the corresponding weight vector based on the adaptive beamforming technology at the receiver. Generally, minimum variance distortionless response (MVDR) algorithm is used to obtain adaptive weight vector \( w(r, \theta) \in \mathbb{C}^{MN \times 1} \), which can be expressed as:

\[
\begin{aligned}
\min_w \quad & w^H(r, \theta)R_{i+n}w(r, \theta) \\
\text{s.t.} \quad & w^H(r, \theta)^{\dagger}a(r_s, \theta_t) = 1
\end{aligned}
\]

where \( R_{i+n} \) is the jamming plus noise covariance matrix, which cannot be accurately estimated in the practice, and the covariance matrix is often replaced by the sample matrix, which can be expressed as:

\[
R_s = \frac{1}{L} \sum_{l=1}^{L} x(l)x^H(l)
\]

Hence, the adaptive weight vector can be written as:

\[
w_{opt} = \frac{R_{\theta}^{-1}a(r_s, \theta_t)}{a^H(r_s, \theta_t)R_{\theta}^{-1}a(r_s, \theta_t)} = uR_{\theta}^{-1}a(r_s, \theta_t)
\]

The output of the adaptive beamformer can be expressed as:

\[
y(l) = w_{opt}^Hx(l)
\]

Hence, the normalized antenna pattern can be expressed as:

\[
B(r, \theta) = \frac{w_{opt}^H \cdot v(r, \theta)}{\max(w_{opt}^H \cdot v(r, \theta))}
\]

Note that the MVDR beamforming method often suffers from severe performance degradation due to certain factors such as small number of training snapshots, corruption of training data by the desired signal and steering vector mismatches in many practical applications. However, as many robust adaptive beamforming methods [26–28] have been proposed to address the above problems, without loss of generality, we adopt MVDR algorithm to carry out adaptive beamforming under ideal conditions.

Hence, when MVDR algorithm is effective, the FDA-MIMO radar obtains the highest gain at the target position and the lowest gain at the jamming position, which can be expressed as:

\[
B(r, \theta)_{|r=r, \theta=\theta_t} = \frac{w_{opt}^H \cdot v(r_s, \theta_t)}{\max(w_{opt}^H \cdot v(r_s, \theta_t))} = 1
\]

\[
B(r, \theta)_{|r=\theta, \theta=\theta_t} = \frac{w_{opt}^H \cdot v(r_s, \theta_t)}{\max(w_{opt}^H \cdot v(r_s, \theta_t))} \approx 0
\]

As seen from Equations (22) and (23), at the receiver of FDA-MIMO radar system, after digital weighting processing, the gain of signals with different range and angle parameters is different. The former deceptive jamming generated by direct time delay cannot obtain high gain at beam gain level, and it appears in the side lobe position of range dimension, so its power into radar system will be greatly reduced, which directly determines the energy of signal entering radar and affects the performance of deceptive jamming.

Then, the principal range parameter information of the target is determined by estimating the range gate, which is mainly determined by time delay \( \tau \) and pulse repetition \( T_p \), for example,

\[
r = \text{mod}(c \times \tau, R_{\max})
\]

where \( \text{mod}(a, b) \) denotes the remainder of \( a/b \), \( R_{\max} \) denotes the maximum unambiguous range related to \( T_p \). The signal processing flow related to range parameters in FDA-MIMO radar receiver is shown in Figure 3.

Many methods have been studied to increase \( R_{\max} \) and resolve range ambiguity. For the sake of model simplification, we suppose that the true and false targets don't exceed the unambiguous range in this article. Hence, after range gate analysis, the range information of target can be obtained. Note that for the false target jamming caused by time delay, the energy entering the FDA-MIMO radar receiver is greatly degraded due to the mismatch of range in the beam level. Therefore, the range information of the true target can be effectively obtained by range gate estimation.

### 3.2 New false target jamming method against FDA-MIMO radar

The author in [29] analysed a new jamming pattern which could effectively utilise the range dimension beam gain and then form effective deceptive jamming at the range gate estimation level. Assume that the FDA-MIMO waveform can be separated at the jammer, and the receiving processing flow of the new false target jamming model is shown in Figure 4.

The intercepted signal is first amplified by a low noise amplifier. Then, the down conversion is carried out by local oscillator \( f_{fgc} \). Then, the signal is divided into \( P \) components. Suppose \( f_{fgc} = f_0 \), the time delay of signal processing is \( T_{fgc} \) and \( P \) bandpass filter bank with a bandwidth of \( \Delta f \) and a centre frequency of \( p\Delta f, p = 0, \ldots, P-1 \) are used for matched filtering. Hence, the output signal of the \( n \)-th filter can be expressed as:
**FIGURE 3** Signal processing flow related to range parameters in FDA-MIMO radar receiver

**FIGURE 4** The receiving signal processing flow of new false target jamming model
\[ x_n(t) = \text{rect} \left( \frac{t - \tau_i/2 - \tau_{jn}}{T_p} \right) \psi_n(t - \frac{\tau_i}{2} - \tau_{jn}) \]
\[ \times e^{j2\pi \left[ f_a \left( -\frac{\tau_i}{2} - \tau_{jn} \right) \right]} e^{-j2\pi f_{FTG}(t-\tau_{FTG})} \]
\[ = \varphi_r \text{rect} \left( \frac{t - \tau_i/2 - \tau_{jn}}{T_p} \right) \psi_n(t - \frac{\tau_i}{2} - \tau_{jn}) \]
\[ \times e^{j2\pi \left[ f_a \left( -\frac{\tau_i}{2} - \tau_{jn} \right) - f_0 \left( \frac{\tau_i}{2} + \tau_{jn} \right) \right]} \] (25)

where \( \varphi_r = e^{j2\pi f_{FTG}/T_p} \) denotes the common phase change, \( \tau_i = \tau_i/c \) denotes the reference transmit delay and \( \tau_{jn} = -d_n \sin \theta \) denotes the transmit delay difference between the \( n \)-th transmit element and reference elements. Note that, \( n \leq N \leq P \). Then, after time delay modulation \( \tau_{jam} = 2\Delta r/c \) and phase modulation \( \varphi_{jam} = 2\pi \Delta f_n \tau_{jam} \) in \( n \)-th DRFM the obtained false target can be expressed as:

\[ x_n' (t) = A_{jam} \varphi_r \text{rect} \left( \frac{t - \tau_i/2 - \tau_{jn} - \tau_{jam}}{T_p} \right) \]
\[ \psi_n(t - \frac{\tau_i}{2} - \tau_{jn} - \tau_{jam}) \]
\[ \times e^{j2\pi \left[ f_a \left( -\frac{\tau_i}{2} - \tau_{jn} - \tau_{jam} \right) - f_0 \left( \frac{\tau_i}{2} + \tau_{jn} \right) + \Delta f_n \tau_{jam} \right]} \]
\[ = A_{jam} \varphi_r \text{rect} \left( \frac{t - \tau_i/2 - \tau_{jn} - \tau_{jam}}{T_p} \right) \]
\[ \psi_n(t - \frac{\tau_i}{2} - \tau_{jn} - \tau_{jam}) \]
\[ \times e^{j2\pi \left[ f_a \left( -\frac{\tau_i}{2} - \tau_{jn} - \tau_{jam} \right) - f_0 \left( \frac{\tau_i}{2} + \tau_{jn} \right) \right]} \] (26)

where \( A_{jam} \) denotes the amplitude modulation, after up-conversion processing, the signal turns to:

\[ x_n'' (t) = A_{jam} \varphi_r \text{rect} \left( \frac{t - \tau_i/2 - \tau_{jn} - \tau_{jam}}{T_p} \right) \]
\[ \psi_n(t - \frac{\tau_i}{2} - \tau_{jn} - \tau_{jam}) \]
\[ e^{j2\pi \left[ f_a \left( -\frac{\tau_i}{2} - \tau_{jn} \right) \right]} \] (27)

Hence, the received signal of the \( m \)-th receive element can be expressed as:

\[ y_{jm} = A_{jam} \varphi_r \text{rect} \left( \frac{t - \tau_j - \tau_{jn} - \tau_{jm} - \tau_{jam}}{T_p} \right) \]
\[ \cdot \sum_{n=1}^{N} \psi_n(t - \frac{\tau_j}{2} - \frac{\tau_{jn}}{2} - \frac{\tau_{jm}}{2} - \tau_{jam}) e^{j2\pi \left[ f_a \left( -\frac{\tau_j}{2} - \frac{\tau_{jm}}{2} - \frac{\tau_{jn}}{2} \right) \right]} \] (28)

After the digital signal processing flow shown in Figure 2, the output signal can be expressed as:

\[ y_{m,n} = \xi_{jm} e^{j2\pi \left[ -f_j \sin \theta - \frac{\tau_j}{2} - f_0 \sin \theta - \frac{\tau_i}{2} \right]} \]
\[ = \xi_{jm} e^{j2\pi \left[ -n\Delta f + f_0 \sin \theta - \frac{\tau_i}{2} \right]} \] (29)

where \( \xi_{jm} \) denotes the complex coefficient after matched filtering and \( \xi_{jm} = \xi_{jm} e^{j2\pi f_0 \sin \theta / c} \). Consider a self-defence jammer, that is, \( \tau_j = \tau_i \) and \( \theta_j = \theta_i \).

Hence, the beam gain of FDA-MIMO radar at the false target position is:

\[ G = B(r, \theta) \big|_{r=r_i, \theta=\theta_i} = \left| \frac{w_{opt}^H \cdot a(r_j, \theta_j)}{\max (w_{opt}^H \cdot a(r_j, \theta_j))} \right| = 1 \] (30)

It can be seen from Equation (30) that the deceptive jamming proposed in [29] can obtain the same maximum beam gain as the expected target. Therefore, its power will not be significantly reduced in the beam level, and then it can enter the next stage of radar signal processing which provides power guarantee for the formation of effective countermeasures against FDA-MIMO radar. Note that the time delay of the false target is \( \tau = \tau_i + \tau_{jam} \), which means that its equivalent distance parameter is \( r_f = r_0 + \Delta r \). Besides, by adjusting the time delay, the false target can appear in different range gates, forming range dimension deceptive jamming, which can affect the normal target detection and positioning of radar.

### 3.3 Countermeasures against new jamming methods

Although the new deceptive jamming can appear in any range gate, its total power is affected by the beam gain just like the target. The higher the beam gain, the greater the total power. Note that unlike PA radar, the guidance vector of FDA-MIMO radar is range-angle dependent which means that FDA-MIMO radar can utilise DOF in the range-angle domains to jointly determine the range and angle parameters of the target [30]. Hence, the spatial spectrum searching based on FDA-MIMO radar has been widely suggested for range and angle estimation of targets. [31–33].

Suppose there are \( K \) new false targets and one true target in the airspace, and the Equation (16) can be rewritten as:

\[ \mathbf{x}(l) = \mathbf{x}_a(l) \mathbf{v}(r_i, \theta_i) + \sum_{j=1}^{K} \mathbf{x}_j(l) \mathbf{v}(r_j, \theta_j) + \mathbf{n}(l) \] (31)
Note that $r_i = r_j$ and $\theta_i = \theta_j$ ensure the false targets will not experience energy attenuation due to the mismatch of range or angle while $x_r$ makes the radar not accurately judge the target range through the range gate analysis.

Suppose that the additive noise is independent and identically distributed Gaussian process with zero mean and variance $\sigma_n^2$. The noise-plus-jamming covariance matrix can be expressed as:

$$R_x = E\{xx^H\} = \sum_{i=1}^{K+1} \alpha_i^2 \mathbf{a}(r_i, \theta_i)\mathbf{a}^H(r_i, \theta_i) + \sigma_n^2 \mathbf{I}$$  \hspace{1cm} (32)

where $\mathbf{I}$ is the identity matrix, and $\alpha_i^2$ denotes the averaged signal power. According to [31], for independent target signal and noise, the covariance matrix $R_x$ can be reformulated as:

$$R_x = (\mathbf{U}_A, \mathbf{U}_n) \begin{pmatrix} \Lambda + \sigma_n^2 \mathbf{I} & 0 \\ 0 & \sigma_n^2 \mathbf{I} \end{pmatrix} (\mathbf{U}_A, \mathbf{U}_n)^H$$  \hspace{1cm} (33)

where $\Lambda$ denotes the diagonal matrix with $\alpha_i, i = 1, 2, ..., K + 1$ as diagonal elements, $\mathbf{U}_A$ denotes the signal subspace composed of feature vectors of target and jamming signal and $\mathbf{U}_n$ denotes the noise subspace composed of feature vectors of noise. Once the number of targets is obtained, the size of $\mathbf{U}_A$ and $\mathbf{U}_n$ is known accordingly. According to the MUSIC algorithm, the targets can then be localized by searching the following peaks:

$$P(\theta, r) = \frac{1}{\mathbf{a}^H(\theta, r)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta, r)}$$  \hspace{1cm} (34)

As seen from Figure 3, when the range gate analysis of FDA-MIMO radar cannot work properly, it can still obtain the range information of the true target by spatial spectrum searching. Hence, by combining adaptive beamforming technology and two ranging methods, the traditional deceptive jamming and the new deceptive jamming proposed in [29] can be both countered effectively.

4 | A NOVEL RANGE GATE PULL-OFF METHOD

4.1 | The principle of range gate pull-off jamming

For pulse radar, the process of RGPO is that after receiving the radar transmitting pulse, the jammer sends back a coherent pulse signal with a slight delay after modulation, and the jamming power is slightly higher than that of the real target, and then it gradually increases the delay. Thus, the radar range tracking gate is moved backwards, which affects the accurate tracking of the real target. A typical range pull is to repeat the following three stages called lingering phase, towing phase and closing phase, respectively.

Stage1: The jammer receives the radar signal and forwards a jamming echo with the minimum delay time and the jamming amplitude is larger than the target signal. The purpose is to make the jamming signal and the target echo signal act on the range gate at the same time.

Stage2: When the range gate accurately tracks the jamming pulse, the jammer will gradually leave the target echo with the jamming movement by gradually changing the time delay of the transmitting radar signal.

Stage3: When the jamming pulse drags the range gate away from the target enough distance, the jammer will shut down and the radar will turn to the search phase again.

Under the condition of self-defence jamming, the modulated time delay transmitted by jammer can be expressed as:

$$\tau_{jam}(t) = \begin{cases} 
0, & 0 \leq t \leq t_1, \text{lingering phase} \\
\frac{2\nu_j}{c}(t_o - t_1), & t_1 \leq t \leq t_2, \text{towing phase} \\
\text{off}, & t_2 \leq t \leq T, \text{closing phase}
\end{cases}$$  \hspace{1cm} (35)

where $\nu_j$ denotes the towing speed, $T$ denotes the jamming period, $t_o, t_1, t_2$ denote the time after jammer starts working, the end time of lingering phase and the end time of towing phase, respectively.

According to the analysis in the previous section, FDA-MIMO radar can not only obtain range information through range gate, but also judge range by spatial spectrum, which means traditional false target jamming cannot affect FDA-MIMO radar effectively.

4.2 | The principle of novel range gate pull-off method

The receiving processing flow of the proposed deceptive jamming model is shown in Figure 5.

As seen from Figure 5 that unlike the new false target jamming model, the proposed model adopts time varying delay modulation and phase modulation to ensure that FDA-MIMO radar cannot accurately obtain target information through range gate analysis and spatial spectrum searching. Take one jamming period as an example, and after time delay modulation $\tau_{jam}(t_o)$ and phase modulation $\varphi_{jam}(t_o)$ in DRFM, similar to
Equation (25), the obtained false target in $n$-th DRFM can be rewritten as:

$$x_n(t, t_0) = A_{jam}\phi_i \text{rect} \left( \frac{t - \tau_j / 2 - d_j - r_{jam}(t_0)}{T_p} \right)$$

$$\cdot \psi_a(t - \tau_j / 2 - d_j - r_{jam}(t_0))$$

$$e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j / 2 - d_j - r_{jam}(t_0) \right) \right]}$$

$$= A_{jam}\phi_i \text{rect} \left( \frac{t - \tau_j / 2 - d_j - r_{jam}(t_0)}{T_p} \right)$$

$$\cdot \psi_a(t - \tau_j / 2 - d_j - r_{jam}(t_0))$$

$$e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j / 2 - d_j - r_{jam}(t_0) \right) \right]}$$

$$= A_{jam}\phi_i \text{rect} \left( \frac{t - \tau_j / 2 - d_j - r_{jam}(t_0)}{T_p} \right)$$

$$\cdot \psi_a(t - \tau_j / 2 - d_j - r_{jam}(t_0))$$

$$e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j / 2 - d_j - r_{jam}(t_0) \right) \right]}$$

$$= \sum_{n=1}^{N} \psi_a(t - \tau_j / 2 - d_j - r_{jam}(t_0))$$

$$e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j / 2 - d_j - r_{jam}(t_0) \right) \right]}$$

After up-conversion processing, the signal turns to:

$$x_n(t, t_0) = A_{jam}\phi_i \text{rect} \left( \frac{t - \tau_j / 2 - d_j - r_{jam}(t_0)}{T_p} \right)$$

$$\cdot \psi_a(t - \tau_j / 2 - d_j - r_{jam}(t_0)) e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j / 2 - d_j - r_{jam}(t_0) \right) \right]}$$

Hence, the received signal of the $m$-th receive element can be rewritten as:

$$y_{m,n} = A_{jam}\phi_i \text{rect} \left( \frac{t - \tau_j - d_j - r_{jam}(t_0)}{T_p} \right)$$

$$\cdot \sum_{n=1}^{N} \psi_a(t - \tau_j - d_j - r_{jam}(t_0))$$

$$e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j - d_j - r_{jam}(t_0) \right) \right]}$$

After the digital signal processing flow shown in Figure 2, the output signal can be expressed as:

$$y_{m,n} = \xi_{m,n} \cdot \sum_{n=1}^{N} \psi_a(t - \tau_j - d_j - r_{jam}(t_0))$$

$$e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j - d_j - r_{jam}(t_0) \right) \right]}$$

where $R(t_0)$ denotes the towing range. For simplicity, we generally choose $t_0$ as a constant to study the effect of three stages of the proposed jamming. Hence, Equation (39) can be rewritten as:

$$y_{m,n} = \begin{cases} \xi_{m,n} e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j / 2 - d_j - r_{jam}(t_0) \right) \right]}, & 0 \leq t_0 \leq t_1 \\ \xi_{m,n} e^{j2\pi \left[ \frac{A_{jam}}{T_p} \left( t - \tau_j / 2 - d_j - r_{jam}(t_0) \right) \right]}, & t_1 \leq t_0 \leq t_2 \\ \text{off}, & t_2 \leq t_0 \leq T \end{cases}$$
5 | SIMULATION RESULTS

In this section, simulations are performed to verify the effectiveness of the proposed method. The transmit array and receive array consisting of $M = 10$ and $N = 10$ elements with half a wavelength spacing is considered. For easy identification, we name the false targets produced by traditional deceptive jammer as existing deceptive jamming (EDJ), EDJ1 and EDJ2, and those produced by new deceptive jammer presented in [29] as new deceptive jamming (NDJ), NDJ1 and NDJ2. Because there is no change except the magnitude to the jamming model with changing jamming to noise ratio (JNR), we choose the fixed JNR and signal to noise ratio (SNR) for simplicity. The pulse repetition period of radar is 0.2 s, and thus the maximum unambiguous distance is $R_{\text{max}} = 30$ km. Simulation parameters are shown in Table 1.

### Table 1 Simulation parameters of FDA-MIMO radar

| Parameter                  | Value          |
|----------------------------|----------------|
| Elements number            | (10, 10)       |
| Carrier frequency          | 3 GHz          |
| Frequency offset           | 1 kHz          |
| Wavelength                 | 0.1 m          |
| Array element spacing      | 0.05 m         |
| Number of range bins       | 300            |
| Input SNR                  | 10 dB          |
| Input JNR                  | 20 dB          |
| Snapshot                   | 2000           |
| Pulse repetition period    | 0.2 s          |
| $R_{\text{max}}$           | 30 km          |
| Target location            | (0°, 15 km)    |
| EDJ1 time delay            | 0.0333 ms      |
| EDJ2 time delay            | 0.0667 ms      |
| NDJ1 time delay            | 0.0333 ms      |
| NDJ2 time delay            | 0.0667 ms      |

Note: The range information of EDJ1, EDJ2, NDJ1 and NDJ2 is 20, 25, 20 and 25 km, respectively, which means the false target signal and true target signal appear in the same pulse period.

5.1 | False target jamming and corresponding anti-jamming method

Figure 6 demonstrates the adaptive range-angle-dependent beam pattern results. As shown in Figure 6(a) and (b), using the range-angle 2-D beamforming, only the target with the specific range and angle can be detected. We can find that the EDJ1 and EDJ2 are suppressed due to mismatch in the range, while the NDJ1 and NDJ2 can obtain the beam gain as same as the real target, which means the energy from NDJ can fully enter into the FDA-MIMO receiver.

The output power through the transmit–receive 2-D matched filter is presented in Figure 7 which further indicates that the EDJ can be suppressed and the NDJ can affect the range gate of FDA-MIMO radar effectively. Note that the NDJ can be arbitrarily settled in any range bin to confuse the radar by adjust time delay.

Figure 8 demonstrates a method based on FDA-MIMO to find the location information of the true target in the presence of NDJ. Because NDJ aims to affect radar range gate without losing jamming energy, leading to the mismatch of the equivalent range between airspace and the range displayed on the range gate. At the same time, FDA-MIMO radar can carry out range-angle 2-D spatial spectrum searching. Thus, we can distinguish the NDJ from the real target by comparing the range information obtained by

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**Figure 6** The adaptive beam pattern results of the FDA-MIMO radar: (a) the 3-D beam pattern and (b) the 2-D beam pattern at the main lobe.
FIGURE 7  Output power of range-angle adaptive matched filter with the FDA-MIMO radar: (a) the 3-D normalized energy distribution in the presence of EDJ only, (b) the 2-D normalized energy distribution in the presence of EDJ only, (c) the 3-D normalized energy distribution in the presence of NDJ only and (d) the 2-D normalized energy distribution in the presence of NDJ only

FIGURE 8  Music spectrum distributions of the true target and false targets with FDA-MIMO radar: (a) the Music spectrum of the true target and EDJ and (b) the Music spectrum of the true target and NDJ
spatial spectrum searching and the range gate. We can find that although EDJ1 and EDJ2 can appear in the 200th range gate (equivalent 20 km), and the 250th range gate (equivalent 25 km), respectively, the range information displayed by 2-D Music algorithm are both 10 km which is the same as the true target. Note that even if the range information of NDJ obtained through spatial spectrum searching is not as same as the true target because of the error, it must be near the true target, otherwise the jamming energy will similarly be attenuated due to the range mismatch.

5.2 The novel range gate pull-off method

Simulation parameter details of the proposed method are shown in Table 2.

Table 2 Simulation parameters of novel range gate pull-off method

| Parameters                          | Value   |
|-------------------------------------|---------|
| Linger phase                        | 1 s     |
| Towing phase                        | 20 s    |
| Closing phase                       | 2 s     |
| Towing speed                        | 100 m/s |
| Input JNR                           | 20 dB   |
| Jamming pulse repetition period     | 0.2 s   |
| Maximum towing distance             | 2 km    |

Figure 9 demonstrates the dragging process of the novel RGPO method in range gate. It is obvious that the proposed

![Figure 9](image-url)

**Figure 9** Display of dragging process in range gate: (a) initial stage of dragging process ($t_0 = 1$), (b) false target overlaps with target partially ($t_0 = 5$), (c) the false target drags 10 range bins from the true target ($t_0 = 10$) and (d) the false target drags 20 range bins from the true target ($t_0 = 20$)
method can affect the tracking to real target in range gate effectively.

Figure 10 demonstrates the results of range-angle 2-D spatial spectrum searching under the proposed jamming. By comparing Figure 9(a) and (d), it indicates that the range information of the proposed jamming matched in the range gate and beam space.

Figure 11 demonstrates the adaptive range-angle-dependent beam pattern results under the proposed jamming. As shown in Figure 11(a), when the false target coincides with the true target, the false target cannot be suppressed by adaptive beamforming algorithm. Figure 11(b) indicates that when the false target drags a certain distance from the true target, the novel RGPO method can cause the distortion of adaptive beamforming pattern in the FDA-MIMO radar.

6 | CONCLUSION

We present the accurate understanding about the effect of main-lobe deceptive jamming against the FDA-MIMO radar. The joint ranging method based on range gate and spectrum estimation is explored, which can counter the range false target jamming effectively. Furthermore, a novel RGPO method against FDA-MIMO is proposed, and the proposed method has more advantages when compared with false target jamming. Three advantages of the proposed method which should be highlighted are given as follows: (1) As the modulated time delay in DRFM is time-varying, it is difficult for the FDA-MIMO radar to obtain the range information through spectrum estimation. (2) As the range information is matched in the range gate and beam space, it is difficult to judge whether the target is true or false through range
information comparison. (3) As the false target is towed from the true target position, it can lead to the failure of the adaptive beamforming algorithm. This work provides a distinct understanding of the range parameters of FDA. It is not only helpful to the extended development of deceptive jamming against the FDA-MIMO radar, but also helpful to the anti-jamming of FDA-MIMO radar.

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CONFLICT OF INTEREST
The authors have no conflict of interest to declare.

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