Acoustic Source Localization in Shallow Water: A Probabilistic Focalization Approach

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Abstract—This paper presents a Bayesian estimation method for the passive localization of an acoustic source in shallow water. Our probabilistic focalization approach estimates the time-varying source location by associating direction of arrival (DOA) observations to DOAs predicted based on a statistical model. Embedded ray tracing makes it possible to incorporate environmental parameters and characterize the nonlinear acoustic waveguide. We demonstrate performance advantages of our approach compared to matched field processing using data collected during the SWellEx-96 experiment.

Index Terms—Array processing, data association, direction of arrival (DOA) estimation, source localization, ray tracing.

I. INTRODUCTION

In the shallow water environment, knowledge of the acoustic waveguide can be used to exploit multipath propagation for source localization. In particular, it has been demonstrated that a typical shallow water waveguide offers sufficient coherence times and ray-path diversity (or aperture) to determine the range and depth of an acoustic source from the measurements provided by a single vertical line array (VLA) [1].

A. State-of-the-Art

Matched field processing [2]–[4] (MFP) is a scientific approach to shallow water localization that compares modeled signal replicas with acoustic data in order to determine the location of one or multiple sources. A potential limitation of MFP is the fact that modeling the channel between a candidate source position and a VLA requires detailed knowledge of environmental parameters.

MFP is still an active research topic [5]–[10] because in real-world scenarios, knowledge of the acoustic environment, in particular an accurate model of the seabed, is typically unavailable. Recent approaches to shallow water localization explicitly consider environmental uncertainty to offer a degree of robustness to model mismatch [8], [11], [12].

B. Contribution and Paper Organization

This paper presents an innovative approach for the localization and tracking of an underwater source with a probabilistic model of the environment. Signals transmitted by an acoustic source are received by a VLA. Inspired by recently proposed graph-based methods for multitarget tracking and indoor localization and mapping [13]–[18], the proposed sequential probabilistic focalization [19] method associates direction of arrival (DOA) observations to modeled DOAs and jointly estimates the time-varying position of the source. Embedded ray tracing makes it possible to incorporate environmental parameters such as the SSP (sound speed profile) of the acoustic channel. Evaluation of the proposed method is performed based on acoustic data from the SWellEx-96 shallow water localization scenario shown in Fig. 1. The contributions of this paper are as follows:

- We introduce a new probabilistic model for the localization of an acoustic source in the shallow water waveguide by using a VLA.
- We establish a Bayesian estimation method based on the new model and evaluate its performance using data collected during the SWellEx-96 experiment.

An important aspect is the probabilistic data association of DOA observations to modeled propagation paths for a source and VLA geometry. The considered model calculates expected DOAs by means of ray tracing based on the SSP and, if available, a characterization of the seabed.

II. SYSTEM MODEL

We consider a mobile source with unknown time-varying position \( p_n \in \mathbb{R}^2 \) which, due to the azimuthal ambiguity of DOA information provided by the VLA, only consists of range and depth. There are \( K \) propagation paths that are used for localization. In each discrete time slot \( n \), the array acts as a receiver and provides DOA observations. A DOA estimation method [20], [21] processes the acoustic signals received by the VLA and returns \( M_n \) DOA estimates at time \( n \). \( M_n \) at time \( n \) is related to the number of propagation paths \( K \) as follows: It is possible that some propagation paths are not “detected” by the DOA estimator and thus do not generate a DOA estimate, and it is also possible that some DOA estimates do not correspond to a propagation path. Accordingly, \( M_n \) may be smaller than, equal to, or larger than \( K \). Note also that \( M_n \) depends on the source position \( p_n \) and on the environment. Selecting the number of propagation paths \( K \) requires some a priori understanding of the propagation environment.

A. Source State and Association Vectors

The state of the source at time \( n \) is \( x_n \triangleq [p_n^T \ v_n]^T \), where \( v_n \) is the source speed in range. The source state \( x_n \) is assumed to evolve according to Markovian state dynamics,
The DOA observations $z_{m,n} \in [-90^\circ, 90^\circ]$, $m \in \{1, \ldots, M_n\}$ are subject to observation-origin uncertainty. That is, it is not known which observation $z_{m,n}$ is associated with which propagation path $\{1, \ldots, K\}$, or if an observation $z_{m,n}$ did not originate from any propagation path (this is known as a false alarm), or if a propagation path did not give rise to any observation (this is known as a missed detection). The probability that a propagation path is “detected” in the sense that it generates an observation $z_{m,n}$ in the DOA estimation stage is denoted by $d_k(p_n) \in [0, 1]$. For positions $p_n$ for which a propagation path $k$ is geometrically impossible, we set $d_k(p_n) = 0$. False alarms are independent and identically distributed as $f_{FA}(z_{m,n})$. The number of false alarms is assumed Poisson distributed with mean $\mu_{FA}$ [13], [14], [22]. $f_{FA}(z_{m,n})$, and $\mu_{FA}$ are known. The DOA observations resulting from propagation paths that are not modeled among the $K$ selected paths are treated as false alarms. Finally, we introduce the total observation vector $z_n \triangleq [z_{1,n} \cdots z_{M_n,n}]^T$ that is assumed sorted in descending order, i.e., $z_{m,n} \geq z_{m+1,n}$, $m \in \{1, \ldots, M_n - 1\}$.

The associations between DOA observation $m \in \{1, \ldots, M_n\}$ and propagation path $k \in \{1, \ldots, K\}$ at time $n$ can be described by the $K$-dimensional data association vector $a_n = [a_{1,n} \cdots a_{K,n}]^T$, whose $k$th entry is given by $a_{k,n} = m \in \{1, \ldots, M_n\}$ if propagation path $k$ generates observation $z_{m,n}$, and $a_{k,n} = 0$ if it does not generate any observation. Due to observation-origin uncertainty, $a_n$ is a random variable. However, since the DOAs of the propagation paths have a fixed order [23] and each propagation path can generate at most one DOA observation [13], [14], [22] at any time $n$, only certain data association vectors $a_n$ are valid. To facilitate identifying invalid vectors $a_n$, we order its elements $a_{1,n}, \ldots, a_{K,n}$ such that for the case where there are no false alarms and no missed detections, we have $z_n = [z_{a_{1,n},n} \cdots z_{a_{K,n},n}]^T$. With this order a general association vector $a_n$ is invalid if and only if there exist $k, k' \in \{1, \ldots, K\}$ with $k > k'$ such that $a_{k',n} \geq a_{k,n} \neq 0$.

For examples, let us assume that we use the $K = 4$ propagation paths shown in Fig. 1. According to the order defined above, we have $k = 1$ for the SB, $k = 2$ for the DP, $k = 3$ for the BB, and $k = 4$ for the SBB. It can easily be verified that for the case where there is a measurement for each propagation path and no false alarm, we obtain $a_{1,n} = 1$, $a_{2,n} = 2$, $a_{3,n} = 3$, and $a_{4,n} = 4$ as well as $z_{1,n} \geq z_{2,n} \geq z_{3,n} \geq z_{4,n}$ (cf. with the DOAs of the four propagation paths at the VLA shown in Fig. 1).
B. Observation Model

At each time step \( n \), a DOA estimation method [20], [21] provides DOAs \( z_{m,n}, m \in \{1, \ldots, M_n\} \). These DOAs are considered as the observations in our statistical model. Let \( z_{m,n} \) be the DOA observations related to propagation path \( k \in \{1, \ldots, K\} \). The observation model is then given by

\[
z_{m,n} = g_k(p_n) + w_{k,n}. \tag{1}
\]

Here, \( w_{k,n} \) is zero mean Gaussian distributed with variance \( \sigma_k^2 \) and \( g_k(p_n) \) is the DOA of propagation path \( k \). The observation noise \( w_{k,n} \) assumed statistical independent across time \( n \) and propagation path \( k \). From (1), we directly get the conditional PDF \( f_k(z_{m,n}|x_n) \). The functions \( g_k(p_n) \) are the DOAs of eigenrays used to the BELLHOP [24] ray-tracing software. Details on how the \( g_k(p_n); k \in \{1, \ldots, K\} \) can be precomputed offline will be discussed in Section IV.

C. Measurement Model

The conditional PDFs \( f_k(z_{m,n}|x_n) \) obtained from (1) characterize the statistical relation between the observations \( z_{m,n} \) and the states \( x_n \). This PDF is a central element in the conditional PDF of the total observation vector \( z \) given \( x_n \), \( a_n \), and \( M_n \). Let us introduce the set of detected paths at time \( n \) as \( D_{a_n} \triangleq \{ k \in \{1, \ldots, K\} \mid a_{k,n} \neq 0 \} \). Conditioned on \( x_n \), we assume that DOAs generated by the source are statistical independent of false alarms, i.e., we can write

\[
f(z_n|x_n, a_n, M_n) = \left( \prod_{m=1}^{M_n} f_k(z_{m,n}|x_n) \right) \prod_{k \in D_{a_n}} f_{FA}(z_{k,n}) \tag{2}
\]

if the number of elements in the vector \( z_n \) is equal to \( M_n \) and \( f(z_n|x_n, a_n, M_n) = 0 \), otherwise. For example, let us again assume \( K = 4 \) and discuss three example realizations of \( M_n \) and \( a_n \). These three example realizations are shown in Fig. 2. In the ideal case where there is no missed detection and no false alarm, i.e., \( M_n = 4 \) and \( a_n = [1 2 3 4]^T \), the conditional PDF in (2) reads \( f(z_n|x_n, a_n, M_n) = \prod_{k=1}^4 f_k(z_{k,n}|x_n) \). Similarly, in case there is no detected path and two false alarms, i.e., \( M_n = 2 \) and \( a_n = [0 0 0 0]^T \), we have \( f(z_n|x_n, a_n, M_n) = \prod_{m=1}^2 f_{FA}(z_{m,n}) \). Finally, if we only detect the DP and the SB and there is also one missed detection, e.g., \( M_n = 3 \) and \( a_n = [2 3 0 0]^T \), we have \( f(z_n|x_n, a_n, M_n) = \prod_{k=1}^3 f_k(z_{k,n}|x_n) f_{FA}(z_{4,n}) \).

Let us consider \( f(z_n|x_n, a_n, M_n) \) as a likelihood function, i.e., a function of \( x_n \), \( a_n \), and \( M_n \), for observed \( z_n \). If \( z_n \) is observed and therefore fixed, also \( M_n \) is fixed, and we can rewrite (2), up to a constant normalization factor, as

\[
f(z_n|x_n, a_n, M_n) \propto \prod_{k=1}^K h_k(x_n, a_{k,n}; z_n).
\]

Here, the factors \( h_k(x_n, a_{k,n}; z_n) \) are defined as

\[
h_k(x_n, a_{k,n}; z_n) = \frac{f_k(z_{k,n}|x_n)}{f_{FA}(z_{k,n})}, \quad a_{k,n} \in \{1, \ldots, M_n\}
\]

Finally, the likelihood function for \( z_{1:n} \), involving the observations \( z_{m,n} \) of all time steps \( m = 1, \ldots, n \), is obtained as

\[
f(z_{1:n}|x_{1:n}, a_{1:n}, m_{1:n}) = \prod_{n=1}^n f(z_n|x_n, a_n, M_n) \tag{3}
\]

where we introduced \( m_n = [M_1 \cdots M_n]^T \). (Recall that \( M_n \) is the number of detected DOAs at time \( n \).

D. Prior Information

Now, one can derive [14], [22] the following expression of the joint conditional prior probability mass function (pmf) of the data association vector \( a_n \), and the number of observations \( M_n \), conditioned on the state \( x_n \) of the mobile source,

\[
p(a_n, M_n|x_n) = \frac{e^{-\mu_a} M_n^{-|D_{a_n}|}}{M_n!} \prod_{k \in D_{a_n}} d_k(p_n) \times \psi(a_n) \prod_{k' \notin D_{a_n}} \left( 1 - d_k(p_n) \right). \tag{4}
\]

Here, \( \psi(a_n) \) checks the validity of association vector \( a_n \) as discussed in Section II-A. In particular, it is defined to be 0 if there exist \( k, k' \in \{1, \ldots, K\} \) with \( k > k' \) such that \( a_{k,n} \neq a_{k',n} \), and to be 1 otherwise. The function \( \psi(a_n) \) enforces \( p(a_n, M_n|x_n) = 0 \) if any observation is associated with more than one propagation path. For future reference, we can also express (4) as

\[
p(a_n, M_n|x_n) \propto C(M_n) \psi(a_n) \prod_{k=1}^K v(x_n, a_{k,n}; M_n)
\]

where the factors \( v(x_n, a_{k,n}; M_n) \) are defined as

\[
v(x_n, a_{k,n}; M_n) \triangleq \frac{\mu_{FA}}{d_k(p_n)} a_{k,n} \in \{1, \ldots, M_n\}
\]

and \( C(M_n) = e^{-\mu_a} \mu_{FA}^{M_n} |D_{a_n}|! / M_n! \).

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**Fig. 3.** Factor graph representing the factorization of the joint posterior PDF \( f(x_n, a_{1:n}|z_{1:n}) \) in (7) depicted for two time steps \( n' \in \{n-1, n\} \). The time index is omitted and the following short notations are used: \( f \triangleq f(x_n'|z_{1:n-1}) \), \( r_k \triangleq r_k(x_n', a_{k,n}; z_{1:n}) \), \( x \triangleq x_n \), \( a \triangleq a_n \), \( x^- \triangleq x_{n-1} \), and \( a^- \triangleq a_{n-1} \).
Using common assumptions [13], [14], [22], the joint prior distribution for all source states, association variables, and number of observations up to time \( n \), is given by

\[
f(x_{1:n}, a_{1:n}, m_{1:n}) = f(x_0) \prod_{n'=1}^{n} p(a_{n'}, M_{n'} | x_{n'}) f(x_{n'} | x_{n'-1}).
\]  

(5)

III. PROBLEM FORMULATION AND ESTIMATION

At time \( n \), our goal is to estimate the source state \( x_n \) from the total observation vector \( z_{1:n} \). For estimating \( x_n \), we will develop an approximate calculation of the minimum mean-square error (MMSE) estimator [25]

\[
\hat{x}_n^{\text{MMSE}} \triangleq \int x_n f(x_n | z_{1:n}) \, dx_n.
\]

(6)

This estimator involves the posterior PDF \( f(x_n | z_{1:n}) \) which is a marginal of the joint posterior PDF \( f(x_{1:n}, a_{1:n} | z_{1:n}) \) and involves all the source states, all the association variables, and all the observations, at all times up to the current time \( n \).

In the following derivation of the factorization of \( f(x_{1:n}, a_{1:n} | z_{1:n}) \), the observations \( z_{1:n} \) are considered observed and thus fixed, and consequently the numbers of observations \( m_{1:n} \) are fixed as well. Then, using Bayes’ rule and the fact that \( z_{1:n} \) implies \( m_{1:n} \), we obtain

\[
f(x_{1:n}, a_{1:n} | z_{1:n}) = f(x_{1:n}, a_{1:n}, m_{1:n} | z_{1:n}) \propto f(z_{1:n} | x_{1:n}, a_{1:n}, m_{1:n}) f(x_{1:n}, a_{1:n}, m_{1:n}).
\]

Inserting (5) for \( f(x_{1:n}, a_{1:n}, m_{1:n}) \) and (3) for \( f(z_{1:n} | x_{1:n}, a_{1:n}, m_{1:n}) \), then yields the final factorization

\[
f(x_{1:n}, a_{1:n} | z_{1:n}) \propto f(x_0) \prod_{n'=1}^{n} \Psi(a_{n'}) f(x_{n'} | x_{n'-1}) \prod_{k=1}^{K} r_k(x_{n'}, a_{k,n'}; z_{n'})
\]

(7)

with

\[
r_k(x_{n'}, a_{k,n'}; z_{n'}) \triangleq v(x_{n'}, a_{k,n'}; M_n) h_k(x_{n'}, a_{k,n}; z_{n'}) \quad \text{and} \quad \Psi(a_n) \triangleq |D_{a_n}| ! \psi(a_n).
\]

The factor graph representing this factorization of the joint posterior PDF is depicted for one time step in Fig. 3. By applying the sum-product algorithm (SPA) [26] on this factor graph, accurate approximations (“beliefs”) \( \tilde{f}(x_{n} | z_{1:n}) \) of the marginal posterior PDFs \( f(x_{n} | z_{1:n}) \), used for estimation in (6), are obtained in an efficient way.

In order to obtain accurate nonlinear estimation by means of the SPA, we represent messages related to continuous state variables by random samples or particles [13], [27]. Thus, our algorithm produces a particle representation \( \{ (x_{n,j}^m, \omega_{n,j}^m) \} \) of the posterior PDF \( f(x_{n} | z_{1:n}) \) at each time step \( n \), i.e., a particle-based approximation of the posterior \( f(x_{n} | z_{1:n}) \) is given by

\[
\tilde{f}(x_{n} | z_{1:n}) = \frac{1}{N_p} \sum_{j=1}^{N_p} \delta(x_{n} - x_{n,j}^m).
\]

For the considered estimation problem, the particle-based SPA is asymptotically optimum. This means that it can provide an approximation of the MMSE estimate in (6) that can be made arbitrarily good by choosing \( J \) sufficiently large [27]. For the processing of the SWellEx-96 data we set \( J = 10^4 \).

IV. RAY TRACING

At each time step \( n \), the proposed particle-based algorithm evaluates the nonlinear DOA function \( g_k(p_n) \) for each propagation path \( k \in \{1, \ldots, K\} \) of a total of \( J \) times. DOAs are obtained by using BELLHOP [24] to calculate the eigenrays for the \( J \) source positions based on a SSP along the water column and the bathymetry information.

To reduce the runtime of the proposed algorithm, we precompute a 3-D matrix \( Z \in \mathbb{R}^{N_r \times N_d \times K} \) of DOAs that is used to interpolate the DOAs for each propagation path \( k \in \{1, \ldots, K\} \) and each source position \( p_n \), \( j \in \{1, \ldots, J\} \) during runtime.

The matrix \( Z \) is obtained by performing the following steps:

1) we define a regular \( N_r \times N_d \) grid of source positions \( p_n \), \( i_r \in \{1, \ldots, N_r\} \), \( i_d \in \{1, \ldots, N_d\} \) that covers the entire region of interest;

2) we use BELLHOP to calculate the eigenrays related to each source position on the grid \( p_{n,i_r,i_d} \), \( i_r \in \{1, \ldots, N_r\} \), \( i_d \in \{1, \ldots, N_d\} \) and select the \( K \) eigenrays that are expected to result in propagation paths for acoustic signals; and

3) we compute a 3-D matrix \( Z \in \mathbb{R}^{N_r \times N_d \times K} \) that consists of the DOAs of the \( K \) eigenrays at each of the \( N_r \times N_d \) grid points; if eigenray \( k \in \{1, \ldots, K\} \) is geometrically impossible at a specific grid point, we set the corresponding element of matrix \( Z \) to \(-\infty\) and the corresponding detection probability \( d_k(p_n) \) to zero.
For the processing of the SWellEx-96 data we choose $N_r = 2400$ and $N_d = 165$, which correspond to a resolution of 1 m.

V. Results

We validate the proposed method and compare it with matched field processing by using experimental data from a complex multi-path shallow-water environment [28], [29].

A. Setup of Experiment and DOA Estimation

Acoustic data sampled at 1500 Hz was recorded by a 64-element vertical linear array with a uniform inter-sensor spacing of 1.875 m that spanned water depths 94.125-212.25 m. The surface ship R/V Sproul traveled with a radial speed of 2.5 m/s towards the VLA with closest point of approach (CPA) at approximately 1 km. The ship towed an acoustic source at a depth of roughly 60 m that projects an acoustic signal that includes 13 tones at frequencies \{49, 64, 79, 94, 112, 130, 148, 166, 201, 235, 283, 338, 388\} Hz. The source track is shown in Fig. 1(a).

We process all 13 tones for the 20 minute interval indicated in Fig. 1(a). In particular, we perform beamforming by means of multifrequency sparse Bayesian learning (SBL) [20] as follows. First, the data are split into 1758 snapshots that consist of 2048 samples (1.37 s) and have 50% overlap. For each snapshot a rectangular window is applied and a fast Fourier transform (FFT) is performed. The FFT length of 2048 samples corresponds to a bin width of 0.73 Hz. To accommodate Doppler shifts, for each of the 13 tones, we of 2048 samples corresponds to a bin width of 0.73 Hz. To accommodate Doppler shifts, for each of the 13 tones, we search the ±1 FFT bins adjacent to the expected FFT bin and extract the FFT value with the maximum power. Next we perform multifrequency SBL for all tones and $L = 3$ consecutive snapshots. This results in 586 SBL solutions. Finally, we extract the DOA estimates from each SBL solution by finding (i) the 4 highest peaks and (ii) any additional peak that is within 35% of the highest peak. These DOA estimates are used as observations for the proposed method.

The SBL 586 solutions are shown in Fig. 4. In Fig. 4(a) and Fig. 4(b) expected DOAs related to the four dominate propagation paths are overlaid. These DOAs are obtained by using a source range calculate from the GPS system of the R/V Sproul, a reference depth of 60 m, and by applying the considered propagation model. In Fig. 4(b), the DOA estimates used as observations by the proposed method are highlighted in red. Note that around time 7 min and 11 min for an interval of approximately 74 s, the transmission of the 13 tones was interrupted. The 72 data segments corresponding to these time intervals were discarded, i.e., no DOA estimates were extracted. The remaining 514 data segments define the number of time steps of the proposed method.

B. System Parameters

We consider a region of interest of [100m, 2500m] × [10m, 175m] in range and depth. Source motion is modeled by assuming a nearly constant-velocity model [30] in range and a nearly constant-location model in depth, i.e.,

\[
x_n = \begin{pmatrix} 1 & 0 & T_n \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x_{n-1} + \begin{pmatrix} T_n^2 & 0 \\ 0 & T_n \\ T_n & 0 \end{pmatrix} u_n
\]

where $u_n$ is the driving noise and $T_n$ is the length of the time step. If there is no data dropout, we have $T_n = 2.048s$. If data segments are missing, $T_n$ is increased accordingly.

The driving noise $u_n \sim N(0, \Sigma_u)$ with $\Sigma_u = \text{diag}\{0.05 m^2/s^4, 0.1 m^2/s^2\}$ is an independent and identically distributed (iid) sequence of 2D Gaussian random vectors. Note that (8) fully defines the state transition function $f(x_n|x_{n-1})$ discussed in Section II-A.

We process the DOA observations with the proposed method by considering the $K = 4$ propagation paths shown it Fig. 1. Furthermore, we also process the DOA observations by only considering the $K = 2$ propagation paths that are not affected by the seabed (DP and SB). Our model uses the SSP measured on the day of the event and shown in Fig. 1.

The observation noise standard deviations (cf. (1)) are set to $\sigma_k = 0.5^\circ$ for $k \in \{1,2\}$ and $\sigma_k = 2^\circ$ for $k \in \{3,4\}$. The larger standard deviation for the propagation paths that involve bottom bounces is motivated by the fact that constant bathymetry used by the model is inaccurate. The detection probabilities are set to $d_k(p_n) = 0.9$ if propagation path $k$ is geometrically possible and set to zero otherwise. The false alarm PDF $f_{\text{FA}}(z_m,n)$ is uniform on $[-90^\circ, 90^\circ]$. The mean number of false alarms is $\mu_{\text{FA}} = 2$ for $K = 4$ and $\mu_{\text{FA}} = 4$ for $K = 2$. At time $n$, the prior distribution has the form $f(x_0) = f(p_0)f(v_0)$, where $f(p_0)$ is uniform on $[100m, 2500m] \times [10m, 175m]$ and $f(v_0)$ is zero-mean Gaussian with standard deviation 5 m/s. Our implementation of the proposed method used $J = 10^4$ particles.
C. Performance Comparison

As a reference method, we consider MFP by means of the Bartlett processor. Both methods use the SSP, a constant bathymetry of 216.5 m, as well as the geo-acoustic parameters discussed in Ref. [8]. As a reference for the true range of the source, we use the heading and GPS position of the RV Sproul and assume that the source is towed 100 m behind the vessel. As a reference for true source depth, we use the MFP solution compensated for bathymetry mismatch based on Eq. (10) in Ref. [28] and true seabed depths at the source locations.

Fig. 5 shows the shallow water source localization results for the considered 20 min of data. Range and depth estimation results for probabilistic fociation are shown in Fig. 5 (a) & (b). While only $K = 2$ propagation paths are needed for localization, using $K = 4$ instead of $K = 2$ propagation paths can slightly increase the range and depth estimation accuracy of the proposed method.

MFP results are shown in Fig. 5 (c) & (d). The range and depth bias of the MFP solution, also known as the mirage effect [28], is related to the fact that the assumed constant bathymetry is not accurate. In the considered scenario, the proposed method can outperform MFP despite relying on less environmental information. In particular, compared to MFP it is less sensitive to inaccurate bathymetry information. This is enabled by the proposed statistical model which makes it possible to assign different observation noise uncertainties to different propagation paths.

VI. CONCLUSION

We introduced a probabilistic fociation approach for the localization and tracking of an acoustic source in shallow water. Our method probabilistically associates observed DOAs to modeled DOAs and jointly estimates the time-varying source location. We demonstrated performance advantages compared to MFP using data collected during the SWellEx-96 experiment. Notably, despite relying on less environmental information, the proposed method can outperform MFP in terms of range and depth estimation accuracy.

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