ONE-LOOP EFFECTIVE POTENTIAL FOR

SO(10) GUT THEORIES

IN DE SITTER SPACE

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Abstract. Zeta-function regularization is applied to evaluate the one-loop effective potential for $SO(10)$ grand-unified theories in de Sitter cosmologies. When the Higgs scalar field belongs to the 210-dimensional irreducible representation of $SO(10)$, attention is focused on the mass matrix relevant for the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry-breaking direction, to agree with low-energy phenomenology of the particle-physics standard model. The analysis is restricted to those values of the tree-level-potential parameters for which the absolute minima of the classical potential have been evaluated. As shown in the recent literature, such minima turn out to be $SO(6) \otimes SO(4)$- or $SU(3) \otimes SU(2) \otimes SU(2) \otimes U(1)$-invariant. Electroweak phenomenology is more naturally derived, however, from the former minima. Hence the values of the parameters leading to the alternative set of minima have been discarded. Within this framework, flat-space limit and general form of the one-loop effective potential are studied in detail by using analytic and numerical methods. It turns out that, as far as the absolute-minimum direction is concerned, the flat-space limit of the one-loop calculation about a de Sitter background does not change the results previously obtained in the literature, where the tree-level potential in flat space-time was studied. Moreover, when curvature effects are no longer negligible in the one-loop potential, it is found that the early universe remains bound to reach only the $SO(6) \otimes SO(4)$ absolute minimum.

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1. Introduction

Over the last ten years, the idea by Coleman and Weinberg (1973) on radiative corrections at the origin of spontaneous symmetry breaking has played a very important role also in cosmology. In particular, in Allen (1983) and Allen (1985) the one-loop approximation of path integrals in curved space was applied to study massless scalar electrodynamics and \( SU(5) \) non-Abelian gauge fields in de Sitter space. For this purpose, the author used zeta-function regularization (Hawking 1977, Esposito 1994), and was able to show that the inflationary universe can only slide into either the \( SU(3) \otimes SU(2) \otimes U(1) \) or the \( SU(4) \otimes U(1) \) extremum, in the case of \( SU(5) \) gauge models. In his analysis, Allen (1983, 1985) was dealing with Wick-rotated path integrals, leading to a Riemannian background 4-geometry with \( S^4 \) topology and constant scalar curvature, i.e. the Euclidean-time version of de Sitter space-time.

More recently, work by the authors (Buccella et al 1992, Esposito et al 1993) has led to a deeper understanding of the results in Allen (1985). However, since the technique described in Allen (1985) enables one to evaluate the one-loop effective potential for all non-Abelian gauge theories in de Sitter space, a naturally occurring question is whether one can repeat this analysis in the case of physically more relevant GUT theories in de Sitter cosmologies. For this purpose, our paper studies the one-loop effective potential of \( SO(10) \) GUT theories.

\( SO(10) \) gauge theories as unified models for strong, electromagnetic and weak interactions (Fritzsch and Minkowski 1975, Tuan 1992) have been studied over many years for their interesting physical properties. There are several motivations for this choice, and the
One-loop effective potential for SO(10) GUT theories in de Sitter space

strongest one can be found in the predictions for nucleon lifetimes. In this case in fact, SO(10) models enable one to obtain higher values for the masses of the lepto-quarks which mediate proton decay and which were predicted to be too low, with respect to the experimental lower limit, in the minimal SU(5) model. This property is essentially related to the presence of an intermediate symmetric phase between the SO(10) symmetry at GUT scale and the SU(3) ⊗ SU(2) ⊗ U(1) symmetry at weak scale.

A complete analysis of the possible symmetry-breaking patterns with Higgs particles in representations with dimension \( \leq 210 \) and with only an intermediate symmetry group \( G' \) between \( G \) and SO(10) has led to only four different possibilities for a physically relevant SO(10) unified model (Buccella 1988) [table I]. With the notation of table I, \( SU(3)_C \) is the colour group, \( SU(2)_{L,R} \) are the left and right \( SU(2) \) groups whose representations differ by their behaviour with respect to helicity. Moreover, \( B - L \) is the difference between baryonic and leptonic number, \( D \) is the discrete left-right interchanging symmetry (Kuzmin and Shaposhnikov 1980, Chang et al 1984), and \( SU(4)_{PS} \) denotes the \( SU(4) \) Pati-Salam group (Pati and Salam 1973). For these models, using the one-loop approximation for the renormalization-group equations, the upper limit for the values of the symmetry-breaking scales of SO(10) (\( M_X \)), and of \( G' \) (\( M_R \)), is reported for the different models in their minimal formulation [table II].

As one can see, both models without \( D \) symmetry yield sufficiently high values for the scale \( M_X \), and the model with \( G' \supset SU(4)_{PS} \) predicts \( M_R = 10^{11} \) GeV, while the one with \( G' \supset SU(3)_C \otimes U(1)_{B-L} \) gives rise to a value about two orders of magnitude smaller. Using these results and their implications for proton decay we can safely restrict
One-loop effective potential for $SO(10)$ GUT theories in de Sitter space

our analysis of the cosmological implications of $SO(10)$ GUT models, to the ones which appear physically more relevant and which contain the Higgs field in the 210-dimensional irreducible representation.

Our paper is organized as follows. Section 2, aimed at cosmologists who are not familiar with grand-unified theories, describes the basic elements of $SO(10)$ GUT models in particle physics, the tree-level potential for the 210-dimensional irreducible representation, and the mass matrix relevant for the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry-breaking direction. Section 3 derives the one-loop form of the effective potential for $SO(10)$ GUT theories studied about a de Sitter background. The numerical analysis of the corresponding flat-space limit is then carried out in section 4. Section 5 studies by numerical methods the one-loop effective potential in the region where no asymptotic expansion for infinite or vanishing curvature of the special function occurring in such a potential can be made. Results and concluding remarks are presented in section 6.

2. $SO(10)$ GUT theory in flat space-time

The group $SO(10)$ is defined as the set of $10 \times 10$ orthogonal matrices with unit determinant, and with the usual product rules. It has 45 generators here denoted by $T_{ij}$ ($i, j = 0, 1, \ldots, 9$) obeying the following commutation relations:

$$\left[ T_{jk}, T_{lm} \right] = i \left( \delta_{jl} T_{mk} + \delta_{jm} T_{kl} + \delta_{kl} T_{jm} + \delta_{km} T_{lj} \right). \tag{2.1}$$

Considering the vector irreducible representation $\phi_1$ of the group, which we indicate by $\varphi_1$, the action of the generators $T_{jk}$ on it is given by

$$T_{jk} \varphi_l \equiv i \left( \delta_{kl} \varphi_j - \delta_{jl} \varphi_k \right). \tag{2.2}$$
One-loop effective potential for SO(10) GUT theories in de Sitter space

To construct a satisfactory gauge theory based on the SO(10) local symmetry, which has the proper residual symmetry in the low-energy limit, we need a Higgs mechanism to break the symmetry spontaneously (O’Raifeartaigh 1986). This is based on the presence of a fundamental scalar particle (Higgs field), belonging to one or more irreducible representations (hereafter referred to as IRR’s) of the gauge group, whose dynamics is ruled by a Higgs potential. In the present case, we are going to study the most general, renormalizable and conformally invariant Higgs potential constructed by using only the IRR 210, which is obtained by the completely anti-symmetrized product of four different \( 10 \)’s as

\[
\Phi_{abcd} = N \mu_{[a} \otimes \nu_{b} \otimes \rho_{c} \otimes \sigma_{d]} \quad (2.3)
\]

where \( N \) is the normalization constant. The 210 IRR has four independent quartic invariants, i.e. \( \|\phi\|^4 \) and three non-trivial invariants (see (2.8)-(2.10)), hence the Higgs potential we are going to construct will be a function of these. Multiplying two 210 and symmetrizing one gets the Clebsch-Gordan decomposition

\[
(210 \otimes 210)_{\text{sym.}} = 1 \oplus 45 \oplus 54 \oplus 210 \oplus 770 \oplus (1050 \oplus \overline{1050}) \oplus 4125 \oplus 8910 \oplus 5940 \quad (2.4)
\]

where 45, 54, 770 and so on denote the IRR’s with dimension 45, 54, 770 respectively.

The IRR’s 45, 210 and \( (1050 \oplus \overline{1050}) \) give no contribution along the \( SO(6) \otimes SO(4) \)-invariant direction. This can be easily understood noticing that 45 and \( (1050 \oplus \overline{1050}) \) do not contain singlets along the above direction and that the only singlet contained in the 210 representation is such that \( C_{(1,1,1)}^{210} \otimes 210 \otimes 210 = 0 \). With our notation, \( (1,1,1) \) is the only singlet with respect to the \( SU(4) \otimes SU(2) \otimes SU(2) \) group contained in the
One-loop effective potential for SO(10) GUT theories in de Sitter space

The 210 representation. Moreover, we study the Clebsch-Gordan coefficient (Cornwell 1984b) corresponding to the decomposition \((1, 1, 1)_{210} \otimes (1, 1, 1)_{210} \rightarrow (1, 1, 1)_{210}\).

Hence the only quartic non-trivial invariants, apart from the fourth power of the 210 norm which is isotropic in the space of IRR’s and hence cannot discriminate among the invariant directions, are \(\|(\phi\phi)_{45}\|^2\), \(\|(\phi\phi)_{210}\|^2\) and \(\|(\phi\phi)_{1050}\|^2\) (where for example, the symbol \((\phi\phi)_{45}\) stands for the 45 IRR contained in the product of two 210).

By virtue of the above considerations, the most general renormalizable and conformally invariant Higgs potential, made out of the 210 representation only, turns out to be a linear combination of the above invariants, with arbitrary coefficients \(g_1, g_2, g_3\) and \(\lambda\)

\[
V(\phi) = g_1 \|(\phi\phi)_{45}\|^2 + g_2 \|(\phi\phi)_{210}\|^2 + g_3 \|(\phi\phi)_{1050}\|^2 + \lambda \|\phi\|^4. \quad (2.5)
\]

The IRR 1050 is quite complicated. We thus prefer to express the term \(\|(\phi\phi)_{1050}\|^2\) as a function of the representations 45, 54 and 210

\[
\|(\phi\phi)_{1050}\|^2 = -\frac{35}{6} \|(\phi\phi)_{45}\|^2 - \frac{7}{3} \|(\phi\phi)_{54}\|^2 + \frac{5}{4} \|(\phi\phi)_{210}\|^2 + \frac{1}{10} \|\phi\|^4. \quad (2.6)
\]

In other words, since the space of group invariants is a vector space, we can evaluate the components of \(\|(\phi\phi)_{1050}\|^2\) along the basis vectors. Thus, by inserting (2.6) in (2.5) we get the flat-space potential

\[
V(\phi) = \left( g_1 - \frac{35}{6} g_3 \right) \|(\phi\phi)_{45}\|^2 + \left( g_2 + \frac{5}{4} g_3 \right) \|(\phi\phi)_{210}\|^2
\]

\[
- \frac{7}{3} g_3 \|(\phi\phi)_{54}\|^2 + \left( \frac{1}{10} g_3 + \lambda \right) \|\phi\|^4. \quad (2.7)
\]
One-loop effective potential for SO(10) GUT theories in de Sitter space

To clarify the definitions of \((\phi \phi)_{45}\), \((\phi \phi)_{210}\) and \((\phi \phi)_{54}\), we point out that from the symmetrized product of two IRR’s \(210\) \((\phi_{abcd})\) it is possible, using the Levi-Civita symbol \(\epsilon_{i_{0}...i_{9}}\), to construct the IRR (hereafter we sum over repeated indices)

\[
(45)_{ab} = C_{cdef}^{210} g_{ghil}^{210} 45_{ab} \phi_{cdef} \phi_{ghil} = \frac{1}{\sqrt{70}} \epsilon_{abdefghil} \phi_{cdef} \phi_{ghil}.
\]

(2.8)

Analogously, using the SO(10) invariance of the Levi-Civita symbol, the \(210\) representation can be denoted by 4, or, equivalently, 6 indices of the completely antisymmetric tensor

\[
(210)_{abcd} = (210)_{efghil} = C_{abmn}^{210} mncd efghil \phi_{abmn} \phi_{cdmn}
\]

\[
= \frac{1}{\sqrt{90}} \epsilon_{abdefghil} \phi_{abmn} \phi_{cdmn}
\]

(2.9)

and

\[
(54)_{ab} = \frac{1}{\sqrt{112}} (\phi_{amno} \phi_{bmno} + \phi_{bmno} \phi_{amno}) \quad a \neq b.
\]

(2.10)

If \(a = b\) we have 9 more terms orthogonal to the trace, here omitted for the sake of brevity.

Starting from the general potential (2.7), a complete analysis of its absolute minima would require first of all, the computation of the above potential along every direction of possible residual symmetry and secondly the determination of the ranges for the parameters \(g_{i}\) corresponding to the different residual symmetries for the absolute minima. This is exactly what was done in the case of SU(5) (Allen 1985, Buccella et al 1992, Esposito et al 1993) with the Higgs scalar field in the adjoint representation.

Unfortunately, the technical difficulties due to the complexity of the group SO(10) with respect to the unitary ones and the size of the IRR used, make it impossible to extend the previous analysis to the present case. For this reason, at least at this stage,
we restrict our considerations to the study of the modifications, induced by one-loop and curvature effects (section 3), of the symmetry-breaking pattern, for choices of the parameters \(g_i\) corresponding to absolute minima of the potential at tree-level, invariant under the residual-symmetry group \(SU(3) \otimes SU(2) \otimes U(1)\). These are the only ones relevant for particle physics in flat space, because they predict the correct low-energy-limit phenomenology.

The most general singlet \(\phi_0\) with respect to the group \(SU(3) \otimes SU(2) \otimes U(1)\) contained in the 210 representation is

\[
\phi_0 = \frac{z_1}{\sqrt{3}} (\phi_{1234} + \phi_{3456} + \phi_{5612}) + \frac{z_2}{\sqrt{6}} (\phi_{1278} + \phi_{3478} + \phi_{5678} + \phi_{1290} + \phi_{3490} + \phi_{5690}) + z_3 \phi_{7890} \tag{2.11}
\]

where \((z_1^2 + z_2^2 + z_3^2) = 1\). Varying the \(z_i\) parameters in their ranges, we get the following residual-symmetry groups (see comments in section 1):

\[
z_2 = 0 \rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \tag{2.12a}
\]

\[
z_2 = z_3 = 0 \rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D \tag{2.12b}
\]

\[
z_1 = z_2 = 0 \rightarrow SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \tag{2.12c}
\]

\[
\frac{z_1}{\sqrt{3}} = \frac{z_2}{\sqrt{6}} = z_3 \rightarrow SU(5) \otimes U(1) \tag{2.12d}
\]

otherwise \(\rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_{T_{3R}} \otimes U(1)_{B-L} \tag{2.12e}\)

where \(T_{3R}\) is the \(z\)-component of the \(SU(2)_R\) group.
One-loop effective potential for SO(10) GUT theories in de Sitter space

Inserting (2.11) in (2.7) one gets in flat space-time

\[ \hat{V} \equiv V(\phi_0) = \left( \frac{\alpha}{8} f_{\alpha} + \frac{\gamma}{4} f_{\gamma} + \frac{\delta}{9} f_{\delta} + (\lambda - \delta) \right) \|\phi_0\|^4 \] (2.13)

where

\[ \alpha \equiv \frac{4}{945} \left( - 108 g_1 + 28 g_2 + 140 g_3 \right) \] (2.14)

\[ \gamma \equiv \frac{8}{35} g_1 \] (2.15)

\[ \delta \equiv -\frac{1}{10} g_3 \] (2.16)

\[ f_{\alpha} \equiv \left( z_1^2 + z_2^2 \right)^2 + z_2^2 \left( 2 z_1 + \sqrt{3} z_3 \right)^2 + \frac{3}{4} z_2^4 \] (2.17)

\[ f_{\gamma} \equiv \left( z_1 z_3 + \frac{z_2^2}{\sqrt{3}} \right)^2 + (z_1 z_2)^2 + f_{\alpha} \] (2.18)

\[ f_{\delta} \equiv 30 \left( z_1 z_3 + \frac{z_2^2}{\sqrt{3}} \right)^2 + 30 z_1^2 z_2^2 + \left( 2 z_1^2 - \frac{z_2^2}{2} - 3 z_3^2 \right)^2 \]
\[ + 5 \left( z_1^2 + z_2^2 \right)^2 + 5 z_2^2 \left( 2 z_1 + \sqrt{3} z_3 \right)^2 + \frac{15}{4} z_2^4. \] (2.19)

Since in the following analysis \( \delta \) is always negative and \( \alpha \) may take negative values, the tree-level potential (2.13) is unbounded from below, unless we impose the restriction

\[ \lambda \geq \left| \frac{\alpha}{8} \right| \left( f_{\alpha} \right)_{\text{max}} + \left| \frac{\delta}{9} \right| \left( f_{\delta} \right)_{\text{max}}. \] (2.20)

Note also that contributions proportional to a cubic term in the potential (denoted by \( \beta \) in Acampora et al (1994)) are set to zero, since we are assuming conformal invariance of
One-loop effective potential for $SO(10)$ GUT theories in de Sitter space

our model (Buccella et al 1992, Esposito et al 1993). This assumption enables one to be more predictive, because it leads to a smaller number of free parameters. In the models proposed in Acampora et al (1994) a complete study of the potential at tree-level for the case $z_2 = 0$ has been carried out, including the range of the bare-potential parameters such that the absolute minimum lies in the two-dimensional surface ($z_2 = 0$).

However, since we are interested in the modification of the bare potential produced at one-loop by de Sitter curvature, we can use only part of the inequalities appearing in Acampora et al (1994). More precisely, the parameters are bound to lie in regions where the mass spectrum is positive and the first derivatives of the effective potential vanish.

Thus, the allowed ranges for the parameters become

(1) $z_1 = 0, \quad z_3 = 1 \Rightarrow SO(6) \otimes SO(4) \sim SU(4) \otimes SU(2)_L \otimes SU(2)_R$

$$\gamma > 0 \quad \delta < 0 \quad \beta = 0 \quad \alpha > -2\gamma$$

(2) $z_1^2 + z_3^2 = 1 \Rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

$$\alpha > 0 \quad \beta = 0 \quad -\frac{3}{5}\alpha < \gamma < -\frac{1}{2}\alpha$$

$$\frac{3\left(9\alpha^2 + 9\alpha\gamma - 18\gamma^2 + 4\left(3\alpha + 7\gamma\right)\sqrt{-3\gamma(\alpha + \gamma)}\right)}{320\left(\gamma - \sqrt{-3\gamma(\alpha + \gamma)}\right)} < \delta < \frac{3\gamma^2}{10(3\alpha + \gamma)}.$$ 

In the case $z_1 = 1, z_3 = 0$ one finds that it is impossible to get positive mass for both $(6,2,2,-2/3)$ and $(1,2,2,2)$. In fact this is a saddle point in the space representation. Indeed, since we are interested in the case when the intermediate symmetry group contains $SU(4)_PS$ for the reasons described in the introduction, we can restrict our analysis to case (1).
For our purposes we need to compute the mass matrix for the gauge bosons. This comes from the kinetic term for the Higgs field when we expand this scalar field around its vacuum expectation value $\phi_0$ to get

$$\left( D_\mu \phi_0, D^\mu \phi_0 \right) = \mathcal{G}^2 \left( T_{ab} \phi_0, T_{cd} \phi_0 \right) A^{ab}_\mu A^{cd}_\mu ,$$

where square brackets denote the scalar product in the 210-dimensional space and $\mathcal{G}$ is the gauge coupling constant of the $SO(10)$ group. Since the general form of $\phi_0$ is given, we can evaluate the action of the 45 generators $T_{ab}$ on it.

One now takes the decomposition of the adjoint representation 45 under the group $SU(3) \otimes SU(2) \otimes U(1)$. This makes it necessary to use a standard notation in particle physics, where $(l, r, x)$ denotes the tensor which behaves as an $l$-dimensional representation under $SU(3)$, $r$-dimensional under $SU(2)$ and takes a value $=x$ when acted upon by the $U(1)$ generator. By virtue of the Wigner-Eckart theorem (Cornell 1984a), defining $m^2 \equiv \mathcal{G}^2 \| \phi_0 \|^2$, and evaluating the Clebsch-Gordan coefficients, one finds that the non-vanishing eigenvalues $m^2_{(l,r,x)}$ are $m^2_{(1,1,1)} = m^2_{(1,1,-1)} = m^2 \left[ \frac{z_1^2}{2} \right]$ with degeneracy 1, $m^2_{(3,1,2/3)} = m^2 \left[ \frac{2}{3} \left( z_1^2 + z_2^2 \right) \right]$ with degeneracy 3, $m^2_{(3,2,1/6)} = m^2 \left[ \frac{2}{3} z_1^2 + \frac{z_2^2}{2} + z_3^2 - \sqrt{\frac{2}{3}} z_2 z_3 \right]$ with degeneracy 6, and $m^2_{(3,2,-5/6)} = m^2 \left[ \frac{2}{3} z_1^2 + \frac{z_2^2}{2} + z_3^2 + \frac{2\sqrt{2}}{3} z_1 z_2 + \sqrt{\frac{2}{3}} z_2 z_3 \right]$ with degeneracy 6 as well. Note that this is the mass matrix relevant for the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry-breaking direction. This choice is motivated by low-energy phenomenology of the particle-physics standard model, and all groups containing $SU(3) \otimes SU(2) \otimes U(1)$ lead to the same kind of mass matrix (of course, the $z_i$ parameters take different values for different groups). Hence we only rely on the $\phi_0$ singlet appearing in (2.11).
3. One-loop effective potential in de Sitter space

Within the framework of inflationary cosmology, the quantization of non-Abelian gauge fields has been recently studied in the case of \textit{SU}(5) GUT theories (Allen 1985, Buccella et al 1992, Esposito et al 1993). In this case one starts from a bare, Euclidean-time Lagrangian

\[
L = \frac{1}{4} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2} \text{Tr}(D_\mu \varphi)(D^\mu \varphi) + V_0(\varphi) \tag{3.1}
\]

where both the gauge-potential \(A^\mu\) and the Higgs scalar field \(\varphi\) are in the adjoint representation of \textit{SU}(5). Note that boldface characters are used to denote the curvature 2-form \(F\) in the non-Abelian case, to avoid confusion with the curvature 2-form \(F\) in the Abelian case. The background 4-geometry is de Sitter space with \(S^4\) topology. The background-field method is then used, jointly with the gauge-averaging term first proposed by 't Hooft

\[
L_g = \frac{\tilde{\alpha}}{2} \text{Tr}\left(\nabla_\mu A^\mu - i\tilde{\alpha}^{-1}[\varphi_0, \varphi]\right)^2. \tag{3.2}
\]

This particular choice is necessary to eliminate in the total action cross-terms involving \(\text{Tr}(\nabla_\mu A^\mu)\) and the \textit{commutator} \([\varphi_0, \varphi]\), where \(\varphi_0\) is a constant background Higgs field. After sending \(\tilde{\alpha} \to \infty\) (Landau condition), and denoting by \(\Omega = \frac{8}{3}\pi^2 a^4\) the volume of a 4-sphere of radius \(a\), the resulting one-loop effective potential is (Allen 1985)

\[
V(\varphi_0) = V_0(\varphi_0) + \frac{1}{2\Omega} \log \det \mu^{-2} \left[ -g_{\mu\nu} \Box + R_{\mu\nu} + g_{\mu\nu} M_{ab}^2(\varphi_0) \right] \\
+ \frac{1}{2\Omega} \log \det \mu^{-2} \left[ -\delta_{ab} \Box + \frac{\partial^2 V_0}{\partial \varphi_a \partial \varphi_b} \varphi_0 \right] \tag{3.3}
\]

since the ghost determinant cancels the longitudinal one.
One-loop effective potential for $SO(10)$ GUT theories in de Sitter space

To understand how to generalize (3.3) to $SO(10)$ GUT theories, we have to bear in mind only the first line of (3.3), since, by virtue of the Coleman-Weinberg mechanism, only gauge-field loop diagrams contribute to the symmetry-breaking pattern in the early universe (Allen 1983, Allen 1985, Buccella et al 1992). Denoting by $\psi$ the logarithmic derivative of the $\Gamma$ function, and defining the functions $A$ and $P$ by means of

$$A(z) \equiv \frac{z^2}{4} + \frac{z}{3} - \int_{\frac{3}{2}}^{\frac{3}{2}+\sqrt{4-z}} y(\frac{y}{2} - 3)\psi(y) \, dy$$

$$- \int_{\frac{3}{2}}^{\frac{3}{2}-\sqrt{4-z}} y(\frac{y}{2} - 3)\psi(y) \, dy$$

$$P(z) \equiv \frac{z^2}{4} + z$$

one thus finds for the $SU(5)$ model (Allen 1985)

$$V(\varphi_0) = V_0(\varphi_0) - \frac{1}{2\Omega} \sum_{i=1}^{24} \left[ A(a^2m_i^2) + P(a^2m_i^2) \log(\mu^2a^2) \right]$$

(3.6)

where the $m_i^2$ are the 24 eigenvalues of the mass matrix $M_{ab}^2$.

In the case of the $SO(10)$ GUT model, the same method used for $SU(5)$ in Allen (1985) shows that the one-loop effective potential $V$ takes the form (see appendix)

$$V = \tilde{V}_c - \frac{1}{2\Omega} \sum_{i=1}^{45} \left[ A(a^2m_i^2) + P(a^2m_i^2) \log(\mu^2a^2) \right]$$

(3.7)

where (cf (2.13))

$$\tilde{V}_c = \left( \frac{\alpha}{8} f_\alpha + \frac{\gamma}{4} f_\gamma + \frac{\delta}{9} f_\delta + (\lambda - \delta) \right) \|\phi_0\|^4 + \frac{R}{12} \|\phi_0\|^2.$$

(3.8)
The corresponding one-loop effective potential in (3.7) is obtained by inserting the following formulae:

\[
\sum_{i=1}^{45} A(a^2 m_i^2) = 6A\left[ a^2 m^2 \left( \frac{2}{3} z_1^2 + \frac{2}{3} z_2^2 + \frac{2}{3} z_3 + \sqrt{\frac{2}{3}} z_2 z_3 \right) \right] \\
+ 6A\left[ a^2 m^2 \left( \frac{2}{3} z_1^2 + \frac{2}{3} z_2^2 - \sqrt{\frac{2}{3}} z_2 z_3 \right) \right] \\
+ A\left[ a^2 m^2 \frac{z_2^2}{2} \right] + 3A\left[ a^2 m^2 \frac{z_2^2}{3} (z_1^2 + z_2^2) \right] 
\]

(3.9)

\[
\sum_{i=1}^{45} P(a^2 m_i^2) = a^2 m^2 \left( 10z_1^2 + 4\sqrt{2} z_1 z_2 + \frac{17}{2} z_2^2 + 12z_3^2 \right) \\
+ a^4 m^4 \left( \frac{5}{3} z_1^4 + \frac{4}{3} \sqrt{2} z_1^3 z_2 + 4z_1^2 z_2^2 \right) \\
+ \sqrt{2} z_1 z_2^3 + \frac{55}{48} z_2^4 + \frac{4}{\sqrt{3}} z_1 z_2 z_3^2 + 4z_1^2 z_3^2 \\
+ 2\sqrt{2} z_1 z_2 z_3^2 + 5z_2^2 z_3^2 + 3z_3^4 \right) . 
\]

(3.10)

4. Flat-space limit

The one-loop effective potential (3.7)-(3.10) can hardly be used for an analytic or numerical study of the absolute minima, since it involves a large number of complicated contributions. We therefore begin by studying its flat-space limit, i.e. its asymptotic behaviour when the 4-sphere radius \( a \) tends to \( \infty \). The corresponding asymptotic form of \( A(z) \) is (Allen 1985)

\[
A(z) \sim - \left( \frac{z^2}{4} + z + \frac{19}{30} \right) \log(z) + \frac{3}{8} z^2 + z + \text{const.} + O(z^{-1}). 
\]

(4.1)
One-loop effective potential for $\text{SO}(10)$ GUT theories in de Sitter space

For the purpose of numerical analysis at $a \to \infty$, the expansion (4.1) can be further approximated as

$$A(z) \sim \frac{z^2}{8} \left(3 - \log(z^2)\right). \quad (4.2)$$

Thus, defining (cf end of section 2)

$$h_1 \equiv \frac{2}{3} z_1^2 + \frac{z_2^2}{2} + z_3^2 + \frac{2\sqrt{2}}{3} z_1 z_2 + \sqrt{\frac{2}{3}} z_2 z_3 \quad (4.3)$$

$$h_2 \equiv \frac{2}{3} z_1^2 + \frac{z_2^2}{2} + z_3^2 - \sqrt{\frac{2}{3}} z_2 z_3 \quad (4.4)$$

$$h_3 \equiv \frac{z_2^2}{2} \quad (4.5)$$

$$h_4 \equiv \frac{2}{3} \left(z_1^2 + z_2^2\right) \quad (4.6)$$

$$h_5^2 \equiv \frac{3}{2} h_1^2 + \frac{3}{2} h_2^2 + \frac{h_3^2}{2} + \frac{3}{4} h_4^2 \quad (4.7)$$

$$y \equiv \frac{m}{\mu} \quad (4.8)$$

equations (3.7)-(3.10) and (4.2) lead to

$$\frac{V}{\mu^4} \sim \frac{\hat{V}}{\mu^4} - \frac{3}{8\pi^2} y^4 \left[ \frac{3}{4} h_1^2 \left(3 - \log(h_1^2)\right) + \frac{3}{4} h_2^2 \left(3 - \log(h_2^2)\right) \right. \left. + \frac{h_3^2}{8} \left(3 - \log(h_3^2)\right) + \frac{3}{8} h_4^2 \left(3 - \log(h_4^2)\right) - h_5^2 \log\left(y^2\right)\right]. \quad (4.9)$$

The problem now arises to find the absolute minima of the potential (4.9) by numerical methods. Since $z_1, z_2, z_3$ lie on a unit 2-sphere, they can be expressed as $z_1 =$
One-loop effective potential for SO(10) GUT theories in de Sitter space

\[ \sin(\theta) \cos(\varphi), z_2 = \sin(\theta) \sin(\varphi), z_3 = \cos(\theta). \]

For given values of the parameters \( \alpha, \gamma, \lambda, \delta \) appearing in (3.8), we have thus to minimize with respect to \( \theta, \varphi, y \). For this purpose, we point out that \( y_{\min} \) should be \( \leq 1 \), since it is the ratio of the gauge-boson mass to the cut-off value. Hence one gets a further restriction on \( \lambda \) which, combined with the inequality (2.20), yields the sufficient condition

\[
\lambda \geq \lambda_0 + \frac{|\alpha|}{8} (f_\alpha)_{\max} + \frac{|\delta|}{9} (f_\delta)_{\max}. \tag{4.10}
\]

With our notation, \( \lambda_0 \) is given by

\[
\lambda_0 \equiv \frac{3G^4}{8\pi^2} \left[ \frac{3}{4} h_1^2 \left( 3 - \log(h_1^2) \right) + \frac{3}{4} h_2^2 \left( 3 - \log(h_2^2) \right) \right. \\
+ \frac{h_3^2}{8} \left( 3 - \log(h_3^2) \right) + \frac{3}{8} h_4^2 \left( 3 - \log(h_4^2) \right) - \frac{1}{2} h_5^2 \log(y^2) \right]_{\min(\theta, \varphi)} - \tilde{f}_{\min} \tag{4.11}
\]

where \( \tilde{f} \) is the function such that \( (\tilde{f} + \lambda) y^4 = \hat{V} \). The corresponding numerical analysis, carried out by using the MINUIT minimization program available in the CERN libraries, shows that the absolute minimum always lies in the \( \theta = 0 \) direction. This is the \( SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \) symmetry-breaking direction (see (2.12c)). Thus, as far as the absolute-minimum direction is concerned, the flat-space limit of the one-loop calculation about a de Sitter background does not change the results found in Buccella et al (1986), where the tree-level potential in flat space-time was studied. Remarkably, since the value of \( y \) leading to the absolute minimum of \( V \) in the presence of symmetry breaking has been found to be \( y_{\min} \in [0.4, 0.8] \) in the regions where the inequality (4.10) is satisfied, one can evaluate \( \mu \) from (4.8) as

\[
\mu = \frac{M_X}{y_{\min}}. \tag{4.12}
\]
One-loop effective potential for SO(10) GUT theories in de Sitter space

This formula for $\mu$ can be used to derive the values of the 4-sphere radius corresponding to given values of the dimensionless parameter $\mu a$ (see below).

To complete this section, we find it helpful for the reader to evaluate the behaviour of the flat-space-limit one-loop effective potential as $\theta \to 0$, since $\theta = 0$ yields the absolute-minimum direction as we just said. The analytic calculation shows that the potential $V$ in (4.9) obeys the relation

$$\lim_{\theta \to 0} V(\lambda, y, \theta) \equiv V_{\text{lim}} = \frac{y^4}{\pi^2} \left[ \frac{625}{4} \lambda - \frac{27}{16} + \frac{9}{4} \log(y) \right].$$

(4.13)

The corresponding behaviours of $V_{\text{lim}}$ for various values of $\lambda$ ($\lambda = 0.03, 0.02, 0.015, 0.012$) are plotted in figure 1, when $y \in [0, 1]$.

5. Numerical evaluation of the absolute minima

As one can see from equations (3.7)-(3.10), the one-loop effective potential for our cosmological model takes a complicated form, and it is not clear whether curvature can modify the results of section 4, once the same values for $\alpha, \gamma, \delta, \lambda$ have been chosen. The corresponding absolute minima have been evaluated using again the MINUIT minimization program and choosing different values for the dimensionless parameter $\mu a$, since it is convenient to work with the dimensionless form of the one-loop potential, obtained dividing (3.7)-(3.10) by $\mu^4$. Of course, the parameters in the effective potential are $\alpha, \gamma, \delta, \lambda, \mu a$, whereas the arguments are $y, \theta, \varphi$.

Interestingly, if $\mu a \leq 1$, the term $\frac{R}{12} \|\phi\|^2$ in the potential (3.7)-(3.10) dominates over all other contributions, and hence does not lead to any symmetry breaking. Thus, only
One-loop effective potential for $SO(10)$ GUT theories in de Sitter space

intermediate values of $\mu a$ are relevant for the symmetry-breaking pattern. In this case the absolute minima are still found to be $SO(6) \otimes SO(4)$-invariant (for them $\theta = 0$), providing the inequality (4.10) is satisfied. In figures 2-3, obtained setting $\mu a = 30, 300$ respectively, the one-loop effective potential is plotted as a function of $y$ when $\alpha = \gamma = \delta = 0$. Note that these particular values are chosen since the flat-space effective potential (4.13) is independent of $\alpha, \gamma, \delta$. Hence $\alpha, \gamma, \delta$ can be set to zero for simplicity when curvature vanishes, whereas in the presence of curvature they are set to zero to compare the flat-space analysis with the de Sitter case.

A naturally occurring question is what can be learned from the comparison of figure 1 with figures 2-3. Indeed, the values of the independent variable $y$ for which the absolute minima are attained are modified in the presence of curvature. The smaller $\mu a$ (stronger curvature), the more substantial the change of the shape of our plots. In particular, figure 2 shows that for $\mu a = 30$ and $\alpha = \gamma = \delta = 0$ no symmetry breaking occurs even though for other choices of values for $\alpha, \gamma, \delta$, non-trivial absolute minima are present. By contrast, from figure 3, corresponding to $\mu a = 300$, the effects of curvature on the absolute minima of the potential can be easily seen (cf figure 1).

Remarkably, our numerical investigation shows that, providing the mass matrix is positive-definite, the potential is bounded from below, and the gauge-boson mass remains smaller than the cut-off value, the absolute-minimum direction remains $SO(6) \otimes SO(4)$-invariant in flat or de Sitter space, if the tree-level potential has this invariance property. To help the reader, table III shows, for the same values of the parameters used in figure 3, the values taken by $y_{\min}$ and the corresponding values of the dimensionless one-loop
effective potential. The $\theta$ and $\varphi$ entries are omitted since $\theta = 0$ and $\varphi$ is undetermined in the presence of spontaneous symmetry breaking along the $SO(6) \otimes SO(4)$ direction.

6. Results and concluding remarks

The main results of our investigation are as follows.

First, the one-loop effective potential of $SO(10)$ GUT theories in de Sitter space has been obtained for the first time. This analytic result represents the continuation of the program initiated in Allen (1985), where the tools necessary for any non-Abelian gauge theory in de Sitter space were described in detail. Note that, while (3.7) holds for any irreducible representation of $SO(10)$, (3.8) relies on the $210$ representation, and (3.9)-(3.10) lead to a particular form of such potential, once $SU(3) \otimes SU(2) \otimes U(1)$ invariance for the mass matrix is required to agree with electroweak symmetry.

Second, the flat-space limit of the corresponding Coleman-Weinberg effective potential has been evaluated for the $210$ representation.

Third, the numerical analysis of absolute minima has been carried out in the case of the mass matrix relevant for low-energy-limit phenomenology. Interestingly, de Sitter curvature does not affect the flat-space symmetry-breaking pattern, leading only to the $SO(6) \otimes SO(4)$ symmetry-breaking direction.

A naturally occurring question is whether the analytic study of absolute minima can be performed, to check the results of our numerical investigation. In principle, this research appears possible, although it goes beyond the author’s computational skills, due to the many parameters appearing in the $SO(10)$ effective potential. For the time being, we
One-loop effective potential for SO(10) GUT theories in de Sitter space

should emphasize that our results, although obtained after a time-consuming numerical analysis, remain preliminary.

It has been our task to work under the restrictive conditions summarized at the end of section 5, while other forms of the mass matrix remain unknown in the literature. Thus, a complete mathematical treatment similar to what was done in Allen (1985) for SU(5) theories is lacking, and appears to be a topic for further research. Moreover, since the Higgs field (if it exists) is actually varying in time, it appears necessary to evaluate the one-loop effective potential of non-Abelian gauge theories in closed FRW cosmologies, de Sitter being just a particular case. This more complicated analysis would supersede the approximations made in Allen (1983) and Esposito et al (1993) to study the evolution of the early universe.

Appendix

To obtain the one-loop effective potential (3.7)-(3.8) one starts from the bare, Euclidean-time Lagrangian (cf (3.1))

$$L = \frac{1}{4} \text{Tr} \left( F_{\mu\nu} F^{\mu\nu} \right) + \frac{1}{2} \text{Tr} \left( D_\mu \phi \right) \left( D^\mu \phi \right) + V_0(\phi)$$

(A.1)

where $D_\mu \equiv \partial_\mu - i G A_{\mu}^{ab} T^{ab} \forall a, b = 0, ..., 9$. According to the background-field method, one expands the field $\phi$ as

$$\phi = \phi_0 + \tilde{\phi}$$

(A.2)

where $\phi_0$ is the background value, and $\tilde{\phi}$ is a perturbation. The 4-metric $g$ is also expanded as in Allen (1983, 1985). The resulting one-loop form of $L$, i.e. the Lagrangian quadratic
One-loop effective potential for SO(10) GUT theories in de Sitter space

in the perturbations, is

\[
L^{(1)} = \frac{1}{4} \text{Tr} \left( F_{\mu\nu} F^{\mu\nu} \right) + \frac{1}{2} \text{Tr} \left( D_\mu \tilde{\phi} \right) \left( D^\mu \tilde{\phi} \right) \\
+ \frac{iG}{2} \left[ \left( \nabla_\mu A_\mu \right)^{lm} \right] \left[ < \tilde{\phi} \mid T^{lm} \mid \phi_0 > - < \phi_0 \mid T^{lm} \mid \tilde{\phi} > \right] \\
+ \frac{G^2}{2} A^{lm}_\mu < \phi_0 \mid T^{lm} T^{pq} \mid \phi_0 > A^{pq}_\mu + V_0 + \frac{1}{2} \left( \frac{\partial^2 V_0}{\partial \phi^2} \right) \bigg|_{\phi=\phi_0}. \quad (A.3)
\]

Moreover, the gauge-averaging term we are looking for is (cf Allen 1985)

\[
L_{\text{gauge}} = \frac{\tilde{\alpha}}{2} \text{Tr} \left( \left( \nabla_\mu A_\mu \right)^{lm} \right) + \tilde{\beta} \left( < \tilde{\phi} \mid T^{lm} \mid \phi_0 > - < \phi_0 \mid T^{lm} \mid \tilde{\phi} > \right) \right)^2. \quad (A.4)
\]

By virtue of equations (A.3)-(A.4), cross-terms disappear in \( L^{(1)} + L_{\text{gauge}} \) if and only if

\( \tilde{\beta} = -\frac{iG}{2} \tilde{\alpha}^{-1} \). This leads to

\[
L^{(1)} + L_{\text{gauge}} = \frac{1}{4} \text{Tr} \left( F_{\mu\nu} F^{\mu\nu} \right) + \frac{1}{2} \text{Tr} \left( D_\mu \tilde{\phi} \right) \left( D^\mu \tilde{\phi} \right) \\
+ \frac{\tilde{\alpha}}{2} \text{Tr} \left( \left( \nabla_\mu A_\mu \right)^{lm} \right)^2 - \frac{G^2}{8\tilde{\alpha}} \text{Tr} \left[ < \tilde{\phi} \mid T^{lm} \mid \phi_0 > - < \phi_0 \mid T^{lm} \mid \tilde{\phi} > \right]^2 \\
+ \frac{G^2}{2} A^{lm}_\mu < \phi_0 \mid T^{lm} T^{pq} \mid \phi_0 > A^{pq}_\mu + V_0 + \frac{1}{2} \left( \frac{\partial^2 V_0}{\partial \phi^2} \right) \bigg|_{\phi=\phi_0}. \quad (A.5)
\]

By splitting the gauge potential into transverse and longitudinal part on the \( S^4 \) background, and following Allen (1985), one obtains an equation similar to (3.3), where the mass matrix has 45 eigenvalues rather than 24. Hence (3.7) is proved.
One-loop effective potential for $SO(10)$ GUT theories in de Sitter space

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Figure captions:

**Figure 1.** Flat-space limit of the dimensionless one-loop effective potential at $\theta = 0$ (4.13) versus $y$ (4.8) is here shown. The curves correspond to the $\lambda$-values 0.03, 0.02, 0.015, 0.012 respectively.

**Figure 2.** The dimensionless form of the one-loop effective potential in de Sitter space at $\theta = 0$, and $\mu a = 30$ versus $y$ is here shown. The curves correspond to $\alpha, \gamma, \delta = 0$ and the same values of $\lambda$ of figure 1.

**Figure 3.** The one-loop potential of figure 2 is evaluated for $\mu a = 300$.

Table captions:
One-loop effective potential for SO(10) GUT theories in de Sitter space

Table I. The intermediate symmetries, the Higgs directions and the IRR’s of SO(10) used for the Higgs scalar fields are here reported for the most physically relevant SO(10) GUT models (Acampora et al 1994). With our notation, $\omega_{ab}$ denotes the 54-dimensional irreducible representation of SO(10).

Table II. For the same models of table I, the masses of gauge bosons are shown, following Acampora et al (1994).

Table III. For the same values of the parameters used in figure 3, the values taken by $y_{\text{min}}$ and by the dimensionless one-loop effective potential are shown.
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This figure "fig1-3.png" is available in "png" format from:

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### TABLE I

| $G'$                                      | Higgs direction                                                                 | Representation |
|-------------------------------------------|--------------------------------------------------------------------------------|----------------|
| $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R \times D$ | $2(\omega_{11} + \ldots + \omega_{66}) - 3(\omega_{77} + \ldots + \omega_{00})$ | $54$           |
| $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R$         | $\Phi_T = \Phi_{7890}$                                                       | $210$          |
| $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D$ | $\Phi_L = \frac{(\Phi_{1234} + \Phi_{1256} + \Phi_{3456})}{\sqrt{3}}$   | $210$          |
| $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ | $\Phi(\theta) = \cos(\theta) \Phi_L + \sin(\theta) \Phi_T$          | $210$          |

### TABLE II

| $G'$                                      | $M_X/10^{15}$ GeV | $M_R/10^{11}$ GeV |
|-------------------------------------------|------------------|------------------|
| $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R \times D$ | $0.55 \cdot 1.64^{0\pm1}$ | $343.70 \cdot 1.25^{0\pm1}$ |
| $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R$         | $5.30 \cdot 1.87^{0\pm1}$ | $1.45 \cdot 2.09^{0\pm1}$ |
| $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D$ | $1.64 \cdot 2.83^{0\pm1}$ | $0.32 \cdot 1.81^{0\pm1}$ |
| $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ | $11.26 \cdot 2.06^{0\pm1}$ | $0.03 \cdot 3.34^{0\pm1}$ |
| \( \lambda \) | \( y_{\text{min}} \) | \( V(y_{\text{min}})/\mu^4 \) |
|---|---|---|
| 0.030 | 0 | no symmetry breaking |
| 0.020 | 0 | no symmetry breaking |
| 0.015 | 0.22 | \(-0.6 \cdot 10^{-4}\) |
| 0.012 | 0.32 | \(-0.26 \cdot 10^{-3}\) |