The Distinctive Feature of Weak Interactions and Some of Its Subsequences (Impossibility of Generation of Masses and Absence of the MSW Effect)

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Abstract

In the Quantum theory the wave functions form a full and orthonormalized functional space. For this reason we can use the equation on eigenfunctions and eigenstates and find the eigenenergies $E_n$ and eigenfunctions $\Psi_n$ to determine the physical characteristics of the considered systems (or models). In the Quantum theory the observed values are the average value of operators. Since the wave functions create a full and orthonormalized space, the average values of operators coincide with eigenvalues of operators. This situation takes place in the case of strong and electromagnetic interactions. However, the average values of the weak interaction operators are equal to zero since only the left-handed components of spinors participate in the weak interactions. It means that in these interactions the connected states cannot exist. Then the weak interaction of the particles (scatterings, decays) can be considered only in the framework of the standard perturbative approach. It is shown that the weak interactions cannot generate masses and the equation for Green’s function of the weak interacting fermions (neutrinos) in the matter coincides with the equation for Green’s function of fermions in vacuum. And in the result we come to a conclusion: the mechanism of resonance enhancement of neutrino oscillations in matter (i.e. MSW effect) cannot exist.

PACS: 12.15 Ff-Quark and lepton masses and mixings.
PACS: 96.40 Tv-Neutrinos and muons.
1 Introduction

In the strong and electromagnetic interactions the left-handed and right-handed components of spinors participate in a symmetric manner. In contrast to these interactions only the left-handed components of spinors participate in the weak interactions. This is a distinctive feature of the weak interactions.

In three different approaches: by using mass Lagrangian [1, 2], by using the Dirac equation [3, 2], and using the operator formalism [4], I discussed the problem of the mass generation in the standard weak interactions. The result was: the standard weak interaction cannot generate masses of fermions since the right-handed components of fermions do not participate in these interactions. Then using this result in works [4] it has been shown that the effect of resonance enhancement of neutrino oscillations in matter cannot exist.

At present there is a number of papers published (see [5] and references there) where by using the Green’s function method it is shown that the weak interactions can generate the resonance enhancement of neutrino oscillations in matter (it means that the weak interaction can generate masses). As we see below, this result is a consequence of using the weak interaction term $H_{\mu}^{int} = V_1 \gamma_2 (1 - \gamma_5)$ in an incorrect manner, and in the result they have obtained that the right-handed components of the fermions participate in the weak interactions.

Now let us come to a common consideration and then consider concrete examples and consequences of the distinctive feature of the weak interactions.

2 Common Consideration Consequences of Distinctive Feature of Weak Interactions

In the Quantum theory the wave functions form a full and orthonormalized functional space. For this reason we can use the equation on eigenfunctions and eigenvalues

$$\hat{E} \Psi(...) = F(...) \Psi(...) = 0,$$  \hspace{1cm} (1)
and find the eigenenergies $E_n$ and eigenfunctions $\Psi_n$ to determine the physical characteristics of the considered systems [6, 7] (or models). In the Quantum theory the observed values are the average value of operators

$$E_n = (\Psi_n, \hat{E}\Psi_n).$$

(2)

Indeed, since the wave functions create a full and orthonormalized space, the average values of operators coincide with the eigenvalues of operators. This situation takes place in the case of strong and electromagnetic interactions and in concrete objects which appear from these interactions. It takes place since the wave functions and their conjunction function exist in this case. An absolutely another situation takes place in the case of the weak interactions. In these interactions only the left-handed components of the wave function (wave vector) (i.e. the left-handed components of spinors) participate in these interactions. In the weak interaction the $P$-symmetry is violated. And what is more in the theory for his $\gamma_5$ invariance, it is more suitable to use the Dirac [8] or Veil [9] but not the Shrödinger type of equations. Naturally, the following question arises: To which consequences does this distinctive feature of the weak interactions lead?

Also in the strong and electromagnetic interactions in the weak interactions we can use the perturbative theory but in this case propagators must be propagators of free particles (without renormalization).

Let $\bar{\Psi}_L, \Psi_L, \bar{\Psi}_R, \Psi_R$-be wave functions (wave vectors) of spinor particles. Since we consider the weak interactions where the left-handed components of spinors participate, then

$$\bar{\Psi}_R = \Psi_R \equiv 0, \quad \bar{\Psi}_L = \frac{1}{2}(1 + \gamma_5), \quad \Psi_L = \frac{1}{2}(1 - \gamma_5)\Psi.$$  

(3)

If $\hat{B}$ is an operator of the weak interactions, then

$$\hat{B}\Psi_L = B\Psi_L,$$

(4)

and then the mean value of this operator (the observed value) is

$$\bar{B} = (\bar{\Psi}, \hat{B}\Psi_L) = B(\bar{\Psi}_R, \Psi_L) = 0.$$  

(5)

It is interesting to see: which values are zero in the weak interactions?
Now consider the problem of eigenstates and eigenvalues in the weak interactions. Let $\hat{F}$ be an operator and we divide it into two parts. The first part $\hat{A}$ characterizes the free particle, and the second part $\hat{B}$ is responsible for the weak interaction, then

$$\hat{F} = \hat{A} + \hat{B},$$

$$\hat{F}\Psi = \hat{A}\Psi + \hat{B}\Psi,$$  \hspace{1cm} (6)

and the mean value of $\hat{F}$ is

$$(\bar{\Psi}, \hat{F}\Psi) = (\bar{\Psi}, \hat{A}\Psi) + (\bar{\Psi}_R, \hat{B}\Psi_L) + (\bar{\Psi}_L, \hat{B}\Psi_R) =$$

$$(\bar{\Psi}, \hat{A}\Psi) + (\bar{\Psi}_R \equiv 0)(\bar{\Psi}_R, \hat{B}\Psi_L) + (\Psi_R \equiv 0)(\bar{\Psi}_L, \hat{B}\Psi_R) = (\bar{\Psi}, \hat{A}\Psi).$$  \hspace{1cm} (7)

The obtained result means that in the weak interactions there cannot arise the connected states in contrast to the strong and electromagnetic interactions. Besides, the average value of the polarization operators is equal to zero, i.e. the polarization of the matter is absent. In the same way we can show that the equation for renormcharge for the weak interaction is equivalent to the equation for the free charge, i.e. renormcharge $Q^2(t)$ in the weak interactions [10] (where $t$ is a transfer momentum squared) does not change and $Q^2(t) = const$ in contrast to renormcharges $e^2(t), g^2(t)$ of the electromagnetic and strong interactions [11] (it is necessary to remark that the neutral current of the weak interactions includes a left-right symmetrical part which is renormalized).

Let us consider the equation for Green’s function of fermions taking into account the standard weak interactions.

## 3 Equation for Green’s Function in Weak Interactions

The Green’s function method is frequently used for taking into account effects of electromagnetic interactions and strong interactions (chromodynamics) [12]. The equation for Green’s function has the following form:

$$[\gamma^\mu(i\partial_\mu - V_\mu) - M]G(x,y) = \delta^4(x-y),$$  \hspace{1cm} (8)
where $V_\mu$ characterizes the electromagnetic or strong interactions and

$$iG(x, y) = < T\Psi(x)\bar{\Psi}(y) >_o .$$

It is necessary to mention that the Green’s function method is a very convenient method for studying the electromagnetic and strong interaction effects since these interactions are left-right side symmetrical interactions.

At present a number of works has been published where Green’s function was used for taking into account the weak interaction. There was shown that the weak interaction can generate masses, i.e. masses of fermions are changed by the weak interactions and then the resonance enhancement of neutrino oscillations appears in the matter [5]. In this work we want to show that this result is a consequence that there the distinctive feature of the standard weak interactions is not used correctly, namely, that the right-handed components of fermions do not participate in these interactions (i.e. $\Psi_R = \bar{\Psi}_R \equiv 0$).

Usually the equation for Green’s function for fermion (neutrino) with weak interactions is taken in the following form:

$$[\gamma^\mu(i\partial_\mu - V_\mu) - M]G(x, y) = \delta^4(x - y), \quad (9)$$

where $V_\mu$ is

$$V_\mu = V_\mu \frac{1}{2}(1 - \gamma_5) = V_\mu P_L. \quad (10)$$

It is supposed that the term (10) in Eq.(9) reproduces the distinctive feature of the weak interactions

$$V_\mu G(x, y) \to V_\mu \frac{1}{4}(1 - \gamma_5)^2T(\Psi(x)\bar{\Psi}(y)) = V_\mu T(\Psi_L(x)\bar{\Psi}_R(y)).$$

However, this operation is not correct since it does not reproduce the standard weak interaction. We see that, if we directly use the distinctive feature of these interactions, then the equation for Green’s function must be rewritten in the form

$$[\gamma^\mu(i\partial_\mu - V_\mu \left[ \begin{array}{c} \Psi_R = 0 \\ \bar{\Psi}_R = 0 \end{array} \right]) - M]G(x, y) = \delta^4(x - y). \quad (11)$$
Then the interaction term in Eq. (11) is

\[ V_\mu \left[ \begin{array}{c} \Psi_R = 0 \\ \bar{\Psi}_R = 0 \end{array} \right] T(\Psi_L \bar{\Psi}_R) = \]

\[ = T(\Psi_L \bar{\Psi}_R(\bar{\Psi}_R = 0) + (\Psi_R = 0)\Psi_R \bar{\Psi}_L) = V_\mu 0 \equiv 0, \quad (12) \]

and then Eq. (11) is transformed in the following equation:

\[ [\gamma^\mu (i\partial_\mu) - M] G(x, y) = \delta^4(x - y), \quad (13) \]

which coincides with the equation for free Green's function (i.e. equation without interactions). So, we see that the equation for Green's function with weak interactions (in matter) coincides with the equation for Green's function in vacuum.

4 Impossibility to Realize the Mechanism of Resonance Enhancement of Neutrino Oscillations in Matter

In the previous part we have obtained that the equation for Green’s function of fermions with weak interactions has the form (13). It is a consequence of the fact that the right-handed components of fermions (neutrinos) do not participate in the weak interactions. It means that the weak interaction cannot generate masses (see also works [1-4]) and, correspondingly, the weak interactions do not give a deposit to effective masses of fermions (neutrinos) therefore, the mixing angle cannot be changed in weak interactions (in matter) and it coincides with the mixing angle in vacuum.

The two neutrino \((a, b)\) mixing angle in vacuum is given by the expression [13]

\[ \sin^2 2\theta = \frac{(2m_{\nu_a\nu_b})^2}{(m_{\nu_a} - m_{\nu_b})^2 + (2m_{\nu_a\nu_b})^2}, \quad (14) \]

and this mixing angle \(\theta_m\) in the matter is

\[ \sin^2 2\theta_m = \frac{(2m'_{\nu_a\nu_b})^2}{(m'_{\nu_a} - m'_{\nu_b})^2 + (2m_{\nu_a\nu_b})^2}, \quad (15) \]
where \( m_{\nu_a}, m_{\nu_b}, m_{\nu_a\nu_b} \) are masses of neutrinos \( a, b \), nondiagonal mass term, and \( m'_{\nu_a}, m'_{\nu_b} \)-effective masses of the same neutrinos in matter. Since the masses of neutrinos \( a, b \) in vacuum and in the matter

\[
m'_{\nu_a} = m_{\nu_a} \quad m'_{\nu_b} = m_{\nu_b}
\]

are equal for the distinctive feature of the weak interactions, then the mixing angles in vacuum \( \sin^2 2\theta \) and in the matter \( \sin^2 2\theta_m \) coincides. Hence, the mechanism of the resonance enhancement of neutrino oscillations in the matter (MSW effect) cannot exist.

The problem of the resonance enhancement of neutrino oscillations in the matter can be solved by another approach. Namely, while neutrino passing through the matter there can arise a polarization of the matter. Then the problem is in computation of the energy of polarization of the matter. In Wolfenstien’s equation for neutrinos [14] the term responsible for the (hypothetical) weak interaction is exactly the energy of polarization [15] (it is worth reminding that the energy and momentum transfer at elastic scattering in the forward direction is zero). As it is stressed above, we can use the previous scheme for computation of the polarization energy of matter by neutrinos in the weak interactions. If \( \epsilon \) is the operator for polarization energy of matter by neutrinos, then the average value is

\[
\bar{\epsilon} = (\bar{\psi}_R, \hat{\epsilon}\Psi_L) = \epsilon(\bar{\psi}_R \equiv 0)(\Psi_R, \Psi_L) = 0.
\]

We see that the Wolfenstein’s equation for (real) neutrino in the matter coincides with the equation for free neutrinos, then no resonance enhancement of neutrino oscillations in the matter appears.

5 Conclusion

In the Quantum theory the wave functions form a full and orthonormalized functional space. For this reason we can use the equation on eigenfunctions and eigenstates and find the eigenenergies \( E_n \) and eigenfunctions \( \Psi_n \) to determine the physical characteristics of the considered systems (or models). In the Quantum theory the observed values are the average value of operators. Since the wave functions create a full and
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In conclusion I would like to stress that in the experimental data from [16] there is no visible change in the spectrum of the $B^8$ Sun neutrinos. The measured spectrum of neutrinos lies lower than the computed spectrum of the $B^8$ neutrinos [17]. In the case of realization of the resonance enhancement mechanism this spectrum must be distorted. Also, the day-night effect on the neutrinos regeneration in bulk of the Earth is preserved within the mistakes [16], i.e. it is not observed.

References
1. Kh.M. Beshtoev, JINR Communication P2-93-44, Dubna, 1993.
2. Kh.M. Beshtoev, Fiz. Elem. Chastitz At. Yadra (Phys. Part. Nucl.), 27, 53(1996).
3. Kh.M. Beshtoev, JINR Communication E2-93-167, Dubna, 1993.
4. Kh. M. Beshtoev, [hep-ph/9912532], 1999;
   Hadronic Journal, 22, 235(1999).
5. C.Y. Cardall, D.J. Chung, Phys. Rev. D60, 073012(1999).
6. V. Pauli, Principles of Wave Mechanics, OGIS, Moscow, 1947;
   P. Dirac, Principles of Quantum Mechanics, Oxford,
   Clarendon Press, 1958.
7. S. S. Schweber. An Introduction to Relat. Quantum Field Theory,
Row, Peterson and Co., N.Y., 1961.

8. Particles and Fields, Phys. Rev. **D45**, N11(1998);
   L.B. Okun, Leptons and Quarks, M., Nauka, 1990.

9. S.S. Schweber, An Introduction to Relativistic Quantum
   Field Theory, Row, Peterson and Co., N.Y., 1961, p.114.

10. Kh.M. Beshtoev, JINR Communication E2-94-31, Dubna, 1994.

11. T. Izikson and K. Suber, Quant. Field Theory, M., 1984;
   N.N. Bogolubov, D.V. Schirkov, Intr. to Theory of Quant.
   Field, M., 1994, p.469.

12. J. Schwinger, Phys. Rev. **76**, 790(1949);
   F.J. Dyson, Phys. Rev. **85**, 631(1952);
   N.N. Bogolubov, D.V. Schirkov, Intr. to Theory of Quant.
   Field, M., 1994, p.372, 437;
   Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345(1961);
   M.K. Volkov, Fiz. Elem. Chastits At. Yadra, **17**, 433(1986);
   A.A. Abrikosov et al., Methods of Quant. Theory Field in
   Statistic Phys. M., 1962.

13. Kh.M. Beshtoev, JINR Communication E2-99-307, Dubna, 1999;
   \[\text{hep-ph/9911513}\]

14. L. Wolfenstein, Phys. Rev. **D14**, 2369(1978).

15. Kh.M. Beshtoev, Hadronic Journ., **22**, 477(1999).

16. K.S. Harita, et al., Phys. Rev. Lett. **65**, 1297(1991);
   Phys. Rev., **D44**, 2341(1991);
   Y. Totsuka, Rep. Prog. Physics 377, (1992).
   Y. Suzuki, Proceed. of the Intern. Conf. Neutrino-98, Japan, 1998.

17. J.N. Bahcall, Neutrino Astrophysics, Cambridge U.P.
   Cambridge, 1989.