A Theoretical Model for the $M_{bh}−\sigma$ Relation for Supermassive Black Holes in Galaxies

Fred C. Adams$^{1,2}$, David S. Graff$^2$, and Douglas O. Richstone$^2$

$^1$Physics Department, University of Michigan, Ann Arbor, MI 48109

$^2$Astronomy Department, University of Michigan, Ann Arbor, MI 48109

ABSTRACT

We construct a model for the formation of black holes within galactic bulges. The initial state is a slowly rotating isothermal sphere, characterized by effective transport speed $a_{\text{eff}}$ and rotation rate $\Omega$. The black hole mass is determined when the centrifugal radius of the collapse flow exceeds the capture radius of the central black hole. This model reproduces the observed correlation between black hole masses and galactic velocity dispersions, $M_{bh} \approx 10^8 M_\odot (\sigma/200 \text{ km s}^{-1})^4$, where $\sigma = \sqrt{2} a_{\text{eff}}$. This model also predicts the ratio $\mu_B$ of black hole mass to host mass: $\mu_B \approx 0.004 (\sigma/200 \text{ km s}^{-1})$.

Subject headings: black hole physics – galaxies: nuclei – galaxies: dynamics

1. INTRODUCTION

Two groups have recently reported an observed relationship between a galaxy’s velocity dispersion $\sigma$ and the mass $M_{bh}$ of its central (supermassive) black hole. This correlation can be written in the form

$$M_{bh} = M_0 (\sigma/200 \text{ km s}^{-1})^\gamma,$$

where the two groups find $\gamma = 3.75$ (Gebhardt et al. 2000) and $\gamma = 4.7$ (Farrarrese & Merritt 2000) and where the leading mass coefficient is $M_0 \approx 1.2 - 1.3 \times 10^8 M_\odot$. Both groups report little scatter about the relationship (the scatter in $M_{bh}$ at fixed $\sigma$ is bounded by 0.30 dex) and find no evidence for a subsidiary dependence on the Hubble type, profile type, or environment. This relationship supersedes a previous one claimed between the black hole mass and bulge luminosity (Richstone et al. 1998; Magorrian et al. 1998; van der Marel 2000; Kormendy & Richstone 1998). Furthermore, this measured scaling law poses a clear challenge for galaxy formation theories, which must ultimately account for this relationship.

Several theories imply a relationship between the central black hole mass and the galactic velocity dispersion. For example, a semi-analytic model of merger-driven starbursts with black hole accretion (Haehnelt & Kauffmann 2000; Kauffman & Haehnelt 2000) provides a correlation of the form [1]. Several models are based on black hole accretion influencing star formation and gas
dynamics in the host galaxy; this feedback can occur through ionization, mechanical work, and heating (e.g., Ciotti & Ostriker 1997, 2000; Blandford 1999; Silk & Rees 1998). The model of Blandford (1999) gives \( M_{\text{bh}} < \eta \frac{\sigma^5}{3} \) whereas Silk & Rees (1998) predict \( M_{\text{bh}} \propto \sigma^5 \). Finally, the accretion of collisional dark matter indicates the scaling relation \( M_{\text{bh}} \propto \sigma^{4-4.5} \) (Ostriker 2000).

In this letter, we present a new theory of the \( M_{\text{bh}} - \sigma \) relation, based on an idealized model for the collapse of the inner part of protogalaxies. The simplest variant of this theory is described in terms of particle dynamics in §2. The role of gas dynamics is explored in §3. We discuss the implications of this model and identify future issues in §4.

2. THE BASIC BALLISTIC MODEL

In this section, we examine the collapse of the inner part of a region that will form the bulge of a galaxy. The calculation starts at the time of maximum expansion for the main body of the bulge. For the sake of definiteness, we assume the following: [1] The dark matter and baryons are unsegregated. [2] The mass in this region is distributed like a singular isothermal sphere even though it is not in virial equilibrium. The initial density and mass distributions thus take the form

\[
\rho(r) = \frac{a_{\text{eff}}^2}{2\pi Gr^2} \quad \text{and} \quad M(r) = \frac{2a_{\text{eff}}^2}{G} r.
\]

The transport speed \( a_{\text{eff}} \) that specifies the initial conditions is related to the isotropic velocity dispersion \( \sigma \) according to \( \sigma = \sqrt{2a_{\text{eff}}} \) (see below). [3] This region is slowly rotating like a solid body (e.g., due to tidal torques) at a well-defined frequency \( \Omega \). Both dark matter particles and parcels of baryons that are initially located at radius \( r_\infty \) thus have initial angular momentum \( j = r_\infty^2 \Omega \sin^2 \theta \), where \( \theta \) is the polar angle in spherical coordinates. [4] The center of this region contains a black hole, which may have a tiny mass at the start. This initial “seed” black hole could form by the collapse of the densest (central) part of the perturbation or could be primordial.

Particles in the main body of the initial distribution will fall toward the center. Because the dynamical time scales monotonically increase with radius, infalling shells do not cross. The mass contained inside a given spherical shell, which marks a particle’s location, does not change as the particle falls inward. As a result, orbital energy is conserved and is given (classically) by

\[
E = \frac{1}{2} v_r^2 + \frac{1}{2} \frac{j^2}{r^2} - \frac{GM}{r}.
\]

We consider orbits which fall a long way toward the center of the galaxy. Idealizing these trajectories as zero energy orbits permits the use of equation [3] to determine their pericenters \( p \). For particles falling within the equatorial plane (\( \theta = \pi/2 \)), the pericenter can be written in the form

\[
p = \frac{j^2}{2GM} = \frac{r_\infty^4 \Omega^2}{2GM} = \frac{(GM)^3 \Omega^2}{2^9 a_{\text{eff}}^8},
\]
where $r_\infty$ is the starting radius of an infalling particle and where we have used $M = M(r)$ as a label for $r_\infty$ in the final equality.

If this pericenter $p$ is sufficiently small, ballistic particles will pass inside the horizon of the black hole and be captured. As mass accumulates in the black hole, its horizon scale grows accordingly. The pericenter of particles in ballistic orbits, falling from our assumed mass distribution, increases as $p \propto r_\infty^3 \sim M^3$. In the earliest stages of the collapse, all of the falling material is thus captured by the black hole. Later, this growth mechanism cuts off sharply when the black hole mass reaches a critical point defined by equating the pericenter $p$ (for $\theta = \pi/2$ orbits) to the capture radius of the black hole. In Schwarzschild geometry, particles coming in from infinity on zero energy orbits are captured by the black hole if $p < 4R_S$ (Misner, Thorne, & Wheeler 1973), where $R_S = 2GM/c^2$ is the Schwarzschild radius. The condition $p = 4R_S$ thus defines the critical mass scale $M_C$ where direct accretion is compromised, i.e.,

$$M_C \equiv \frac{16a_{\text{eff}}^4}{Gc\Omega} = \frac{4\sigma^4}{Gc\Omega}. \tag{5}$$

In this scenario, the critical mass scale $M_C$ determines the observed black hole mass $M_{bh}$ (note that equation [5] displays the correct scaling with $\sigma$). Most of the baryonic material not captured by the black hole during this early collapse phase eventually forms stars in the galactic bulge. Dark matter with low angular momentum is captured into the black hole along with the baryons; dark matter with high angular momentum ($p > 4R_S$) passes right through the galactic plane and forms an extended structure.

We now use observations to specify the appropriate values of $\Omega$ and $a_{\text{eff}}$. In dissipationless collapse, the scale length of the mass distribution drops by 1/2 from maximum expansion, and the “flat rotation curve conspiracy” suggests that dissipation doesn’t alter $\sigma$ further. This argument implies that the observed velocity dispersion $\sigma$ is related to the initial transport speed $a_{\text{eff}}$ of the protogalactic material through the relation $\sigma^2 = 2a_{\text{eff}}^2$. To specify $\Omega$, we use the fundamental plane, which provides a well defined relationship (for normal ellipticals and bulges) between the half-light radii of galactic bulges and the corresponding velocity dispersions (see Binney & Merrifield 1998). For $\sigma = 200 \text{ km s}^{-1}$, the effective radius $R$ of a mean surface brightness elliptical on the fundamental plane is about 3.5 kpc. With this scale $R$ as the outer boundary of the forming bulge, the angular momentum of a ballistic particle cannot exceed the angular momentum of a circular orbit at this radius, i.e., $R^2\Omega = \sigma R$ or $\Omega = \sigma/R$. Evaluating this result for $\sigma = 200 \text{ km s}^{-1}$, we obtain a fiducial value of $\Omega = 5.8 \times 10^{-2}\text{Myr}^{-1} = 1.8 \times 10^{-15} \text{ rad s}^{-1}$. With the values of $\Omega$ and $\sigma$ now specified, we use equation [5] to find the desired $M_{bh} - \sigma$ relation

$$M_{bh} \approx 10^8M_\odot(\sigma/200 \text{ km s}^{-1})^4, \tag{6}$$

where we have written the result in terms of $\sigma$ rather than $a_{\text{eff}}$. This relation is in good agreement with the observed correlations (equation [1]), as shown in Figure 1.
3. GAS DYNAMICS AND OTHER EFFECTS

In this section, we include gas dynamics in our model for black hole formation during the collapse of galactic bulges. Because these structures become gravitationally bound, we expect the proto-bulge to collapse as a whole from an initial state (described here by equation [2]). To obtain a mathematical description of this collapse, we consider the flow that produces a galactic bulge to be a scaled up version of the collapse flows that have been studied previously for star formation (Shu 1977; Terebey, Shu, & Cassen 1984); this approach should thus capture the basic essence of the collapse problem. The collapse of the initial state (with density distribution [2]) proceeds from inside-out and the central portion of the flow approaches a ballistic (pressure-free) form: Dark matter always exhibits pressure-free behavior. Even for infalling gas, however, the inner limit of the collapse flow approaches pressure-free conditions. For collapse over sufficiently long time intervals, stars can form as parcels of gas fall inward and the resulting infalling stars are manifestly ballistic. The time scale for individual star formation events is $\tau_\ast \sim 10^5$ years (e.g., Adams & Fatuzzo 1996), whereas the time scale $\tau_{blg}$ for the entire bulge structure to form is much longer ($\tau_{blg} \sim 25 - 50$ Myr). We thus expect most of the stars to form while the overall collapse of the bulge is still taking place.

For a given gravitational potential, we find the orbital solutions for stars (or gas parcels or dark matter) falling towards the galactic center. In this initial calculation, the inner solution is derived using the gravitational potential of a point source. This form is only used in the innermost regime of the collapse flow where the potential is dominated by the forming black hole. As a result, this orbital solution is valid for the range of length scales $R_S \leq r \ll r_\infty$. \(^1\) Since this potential is spherically symmetric, angular momentum is conserved and the motion is confined to a plane described by the coordinates ($r, \phi$); the radius $r$ is the same in both the plane and the original spherical coordinates. The angular coordinate $\phi$ in the plane is related to the angle in spherical coordinates by the relation $\cos \phi = \cos \theta / \cos \theta_0$, where $\theta_0$ is the angle of the asymptotically radial streamline (see below). For zero energy orbits, the equations of motion imply a cubic orbit solution,

$$1 - \frac{\mu}{\mu_0} = (1 - \mu_0^2) \frac{j_\infty^2}{GMr},$$

where $j_\infty$ is the specific angular momentum of particles currently arriving at the galactic center along the equatorial plane. Here, the trajectory that is currently passing through the position ($r, \mu \equiv \cos \theta$) initially made the angle $\theta_0$ with respect to the rotation axis (where $\mu_0 = \cos \theta_0$). As in star formation theory, we define the centrifugal radius $R_C \equiv j_\infty^2 / GM$ ($= 2p$), which represents the radius of a circular orbit with angular momentum $j_\infty$ (for $\theta = \pi/2$). Our assumption of uniform initial rotation at rate $\Omega$ implies that $j_\infty = \Omega r_\infty^2$, where $r_\infty$ is the starting radius of the material that is arriving at the center at a given time. To evaluate the radii $R_C$ and $r_\infty$, we invert the mass distribution of the initial state (equation [2]) to find $r_\infty = GM / 2a_{\text{eff}}^2$ and $R_C = \Omega^2 G^3 M^3 / 16a_{\text{eff}}^8$.

\(^1\) Notice that at late times, long after black hole formation is complete, the potential is no longer close to a point potential and this solution loses its validity; in addition, relativistic corrections become important as $r \to R_S$. 
With an isothermal profile as the initial state, the collapse solution indicates that the flow exhibits a well defined mass infall rate \( \dot{M} = m_0 a_{\text{eff}}^3 / G \), where \( m_0 \approx 0.975 \) (Shu 1977). This infall rate is constant in time and we can measure the time since the collapse began by the total mass \( M \) that has fallen to the galactic center. At early times, all of the mass falling to the center is incorporated into the central black hole. At later times, the mass supply is abruptly shut off by conservation of angular momentum. In this setting, the mass infall rate is quite large, \( \dot{M} \approx 650 M_\odot \, \text{yr}^{-1} \) (for \( \sigma = 200 \, \text{km s}^{-1} \) and \( \sigma^2 = 2 a_{\text{eff}}^2 \)). The time scale \( \tau_{\text{bh}} \) to form a typical supermassive black hole (with mass \( M_{\text{bh}} \sim 10^8 M_\odot \)) is thus about \( \tau_{\text{bh}} \sim 10^5 \, \text{yr} \), comparable to the time scale \( \tau_\ast \) for individual stars to form. The time scale to form the entire bulge is much longer, about \( \tau_{\text{blg}} \sim 25 - 50 \, \text{Myr} \), comparable to the crossing time \( t_{\text{cross}} = R/a_{\text{eff}} \).

In this collapse flow, streamlines entering the central region do not cross each other. As long as the (bulge) infall time is longer than the time scale for individual star formation events, the core regions that produce stars will not interact. However, this scenario has an initial transient phase \( \sim 10^5 \, \text{yr} \) in which the collapse of the bulge structure takes place faster than individual stars form. If stars are already condensing out of the collapse flow during this initial phase, they can interact and merge upon entering the central region. The resulting merger activity can lead to the initial production of the central black hole, which then gains additional mass as the collapse proceeds.

Given the orbital solution (equation [7]), we can find the velocity fields for the collapse flow. The density distribution \( \rho(r, \theta) \) of the infalling material can then be obtained by applying conservation of mass along a streamtube (Terebey, Shu, & Cassen 1984) and can be written in the form

\[
\rho(r, \theta) = \frac{\dot{M}}{4\pi v_r |r|^2} \frac{d\mu_0}{d\mu}.
\]  

The properties of the collapsing structure determine the orbit equation [7], which in turn determines the form of \( d\mu_0/d\mu \) and the radial velocity \( v_r \). The density field is thus completely specified (analytically, but implicitly).

This gas dynamical version of the model defines the same critical mass scale for central black holes as the ballistic model presented in §2. In the earliest stages of collapse, incoming material falls to small radii \( r \ll R_\text{S} \), where \( R_\text{S} \equiv 2GM/c^2 \) and \( M \) is the total mass \( M = \dot{M}t \) that has fallen thus far. In this early stage, the black hole mass \( M_{\text{bh}} = M \). As the collapse develops, incoming material originates from ever larger radii and carries a commensurate increase in specific angular momentum. The centrifugal barrier of the collapse flow grows with time. The black hole mass is determined when the centrifugal radius exceeds the capture radius of a black hole in Schwarzschild geometry. This condition takes the form \( R_\text{C} > \alpha R_\text{S} \), where \( \alpha = 8 \) and is determined by particle orbits in a Schwarzschild metric. This condition, \( R_\text{C} > \alpha R_\text{S} \), leads to the same critical mass scale \( M_C \) defined in equation [5]. Although \( M_C \) determines the black hole mass \( M_{\text{bh}} \), two additional effects conspire to make the final black hole mass somewhat larger:

[1] Even after the centrifugal barrier grows larger than the capture radius, the black hole continues to gain mass from infalling streamlines oriented along the rotational poles of the system.
The fraction of the infalling material that lands at such small radii is a rapidly decreasing function of time. As a result, this effect makes the black hole mass larger by only a modest factor $F_A$. The mass infall rate $\dot{M}_{\text{bh}}$ for material falling directly onto the black hole itself is given by

$$\dot{M}_{\text{bh}} = \int_{-1}^{1} d\mu \frac{2\pi (\alpha R_S)^2 |v_r| \rho(\alpha R_S, \mu)}{\mu},$$

where $\mu = \cos \theta$. Using equation [8] to specify the density, we evaluate the integral to obtain a differential equation for the time evolution of the black hole mass. Solving the resulting differential equation (Adams et al. 2001), we find that the black hole mass increases by a factor $F_A \approx 1.35$ due to direct infall.

Gas that falls to the midplane of the system can collect into a disk structure surrounding the nascent black hole. The presence of the disk is consistent with the current theoretical ideas about AGNs and the jets they produce. In order to retain the desired scaling law $M_{\text{bh}} \sim \sigma^4$, however, the total mass added to the black hole through disk accretion must be less than (or comparable to) the original mass scale $M_C$. We can estimate the maximum amount of mass that can be added to the black hole through the disk by using the constraint that disk accretion cannot operate faster than the orbit time at the outer disk edge (which is determined by $R_C$). For reasonable assumptions, this maximum mass scale in the limit of efficient disk accretion is about $10 M_C$ (Adams et al. 2001).

This model also predicts a mass scale $M_B$ for the bulge itself. If the initial protobulge structure is rotating at angular velocity $\Omega$, then only material within a length scale $R = a_{\text{eff}}/\Omega$ can collapse to form the bulge. Material at larger radii, $r > R$, is already rotationally supported and will not fall inwards. The length scale $R$ thus defines an effective outer boundary to the collapsing region that forms the bulge. This boundary $R$, in turn, defines a mass scale for the bulge, i.e.,

$$M_B = \frac{2a_{\text{eff}}^3}{G\Omega} \approx 2.4 \times 10^{10} M_\odot (\sigma/200 \text{ km s}^{-1})^3. \quad (10)$$

Both the bulge mass $M_B$ and the black hole mass $M_{\text{bh}} \approx M_C$ have the same dependence on the rotation rate $\Omega$. We can divide out the rotation rate and find a robust estimate for the mass fraction $\mu_B$, i.e.,

$$\mu_B \equiv \frac{M_{\text{bh}}}{M_B} = \sqrt{\frac{32}{c}} \frac{\sigma}{c} \approx 0.0038 (\sigma/200 \text{ km s}^{-1}). \quad (11)$$

This mass fraction $\mu_B$ is roughly comparable to the observed ratio of black hole masses to bulge masses in host galaxies (e.g., Richstone et al. 1998; Magorrian et al. 1998), although the data show appreciable scatter (see Figure 2).

4. CONCLUSION

In this paper, we have presented a simple model to describe the collapse flow that produces galactic bulges and the supermassive black holes living at their centers. The initial (pre-collapse)
state is a slowly rotating isothermal sphere, characterized by an effective sound speed $a_{\text{eff}}$ and an angular velocity $\Omega$. These parameters $(a_{\text{eff}}, \Omega)$ thus represent the specification of the initial conditions. The velocity dispersion of the final stellar system is comparable to the initial sound speed and we make the identification $\sigma \approx \sqrt{2} a_{\text{eff}}$. In developing this basic picture, we find the following results:

[1] The black hole mass $M_{\text{bh}}$ is determined by the condition that the centrifugal radius exceeds the capture radius of a Schwarzschild black hole. This requirement leads to the scaling law $M_{\text{bh}} = M_0 (\sigma/200 \text{ km s}^{-1})^4$, which is consistent with observations both in its dependence on velocity dispersion $\sigma$ and for the mass scale ($M_0 \approx 10^8 M_\odot$) of the leading coefficient (see equation [6] and Figure 1).

[2] The bulge mass is determined by the outer boundary of the collapsing region – material at initial radii $r > R$ is rotationally supported and cannot collapse. This condition leads to the scaling law $M_B \propto \sigma^3$ (equation [10]) and predicts the ratio $\mu_B$ of black hole mass to bulge mass (equation [11]). This mass fraction scales weakly with velocity dispersion and has a typical value $\mu_B \approx 0.004$, similar to observed mass ratios (Figure 2).

Elliptical galaxies (and bulges) are described by four parameters: effective radius $R_e$, luminosity $L$, velocity dispersion $\sigma$, and central black hole mass $M_{\text{bh}}$. For this discussion, we substitute the total mass $M_B$ for the luminosity (for a given dark matter fraction, we thus assume that the transformation $M_B \to L$ is determined by known, but perhaps complicated, stellar physics). We need to make the connection between these four basic quantities $(\sigma, R_e, M_B, M_{\text{bh}})$ and our model, which is described by only two variables: the transport speed $a_{\text{eff}}$ and the rotation rate $\Omega$. Our model implies the following transformation between initial conditions and final system properties:

1. $\sigma = \sqrt{2} a_{\text{eff}}$,
2. $R_e = a_{\text{eff}}/(2\Omega)$,
3. $M_B = 2a_{\text{eff}}^3/(G\Omega)$,
4. $M_{\text{bh}} = 16F_A a_{\text{eff}}^4/(cG\Omega)$, and
5. $\mu_B = 8F_A a_{\text{eff}}/c$, where $F_A \approx 1.35$. The fifth relation is not independent, but because the rotation rate cancels out, it represents a robust prediction. The implications of these five scaling laws must be tested further against observations.

In this letter, we have presented a working collapse model for black hole and bulge formation. In future work, a number of issues should be addressed, including relativistic corrections to the infall solutions, additional black hole mass contributions from disk accretion, and feedback from the central black hole on the collapse. In order to successfully reproduce the observed correlations, our model uses initial conditions with a particular profile of specific angular momentum (determined by the initial $\rho(r)$ and $\Omega$); we thus need to explore more general initial conditions and make the connection between the required initial conditions and earlier stages of galaxy formation. In addition, galactic bulges are likely to experience merger events (White & Rees 1978) which lead to modest increases in the overall velocity dispersion (White 1979); we must determine how the black holes produced through this model evolve in the context of such galactic mergers. In any case, the properties of these supermassive black holes – and their connection to their host galaxies – will ultimately provide a vital diagnostic for the galaxy formation process.
We would like to thank Gus Evrard, Luis Ho, Greg Laughlin, Manasse Mbonye, and Scott Tremaine for useful discussions. This work was supported by funding from the University of Michigan, bridging support from NASA Grant No. 5-2869, the NASA Long Term Space Astrophysics program, and Space Telescope Science Institute.

REFERENCES

Adams, F. C., & Fatuzzo, M. 1996, ApJ, 464, 256

Adams, F. C., Graff, D. S., Mbonye, M., & Richstone, D. O. 2001, ApJ, in preparation

Binney, J., & Merrifield, M. 1998, Galactic Astronomy (Princeton: Princeton Univ. Press)

Blandford, R. D. 1999, in “Origin and Evolution of Massive Black Holes in Galactic Nuclei”, ed. Merritt, Valluri & Sellwood, in press, astro-ph/9906025

Ciotti, L., & Ostriker, J. P. 1997, ApJ, 487, L105,

Ciotti, L., & Ostriker, J. P. 2000, submitted to ApJ, astro-ph/9912064

Farrarese, L., & Merritt, D. 2000, ApJ, 539, L9

Gebhardt, K., et al. 2000, ApJ, 539, L13

Haehnelt, M., & Kauffmann, G. 2000, MNRAS, in press, astro-ph/0007369

Kauffmann, G., & Haehnelt, M. G. 2000, MNRAS, 311, 576

Kormendy, J., & Richstone, D. 1995, ARA & A, 33, 581

Magorrian, J. et al. 1998, AJ, 115, 2285, astro-ph/9708072

van der Marel, R. P. 2000, in “Galaxy Interactions at Low and High Redshift”, eds. Sanders & Barnes, in press

Misner, C. W., Thorne, K. S., & Wheeler, J. A. 1973, Gravitation (New York: Freeman)

Ostriker, J. P. 2000, Phys. Rev., in press, astro-ph/9912548

Richstone, D. et al. 1998, Nature, 395, A14

Shu, F. H. 1977, ApJ, 214, 488

Silk, J., & Rees, M. J. 1998, A & A, 331, L1

Terebey, S., Shu, F. H., & Cassen, P. 1984, ApJ, 286, 529

White, S.D.M. 1979, MNRAS, 189, 831
White, S.D.M., & Rees, M. J. 1978, MNRAS, 183, 341

FIGURE CAPTIONS

Figure 1. The correlation between black hole mass $M_{bh}$ and velocity dispersion $\sigma$ of the host galaxy. The data points (adapted from Gebhardt et al. 2000) represent the observed correlation for ellipticals (circles), S0 galaxies (squares), and spirals (triangles). The solid curve shows the theoretical result of this paper (using equation [6] and the 35% correction from equation [9]). The dashed and dotted curves show the observational fits advocated by Gebhardt et al. (2000) and Farrarese & Merritt (2000), respectively.

Figure 2. The ratio $\mu_B$ of black hole mass to host mass plotted as a function of the velocity dispersion $\sigma$ of the host galaxy. The solid curve shows the prediction of this paper. The data points (adapted from Gebhardt et al. 2000) exhibit considerable scatter, but their mean value is in reasonable agreement with theoretical expectations; the various symbols represent ellipticals (circles), S0 galaxies (squares), and spirals (triangles).
