A GRAVITATIONAL POTENTIAL WITH ADJUSTABLE SLOPE: A HYDROSTATIC ALTERNATIVE TO CLUSTER COOLING FLOWS

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ABSTRACT

I discuss a new gravitational potential, $\Phi(r) \propto (r_0^n + r^n)^{-1/n}$, for modeling the mass distribution of spherical systems. This potential has a finite mass and generates a density profile with inner slope $2 - n$. A gas embedded in this potential has hydrostatic temperature and gas density distributions that are elementary functions of $n$, greatly simplifying the task of measuring the slope from X-ray data. I show that this model is successful in describing the rising temperature profile and steep gas density profile often seen in cooling flow clusters. An application to the Abell 478 cluster of galaxies yields an inner slope $2 - n = 1.0 \pm 0.2$ (90%), consistent with the inner regions of collisionless dark matter halos first simulated by Navarro, Frenk, and White. The potential is also useful for cluster dynamics: it is a generalization of the familiar Hernquist and Plummer potentials, and because it is invertible, it allows for easy analytic calculation of particle phase space distribution functions in terms of $n$.

1. INTRODUCTION

The X-ray emitting medium in clusters of galaxies is gravitationally heated, and therefore a tracer of dark matter. The current generation of X-ray satellites finally provides the spatial and spectral resolution necessary to constrain not only the total amount but also the distribution of matter in clusters. The shape of the mass distribution is important, because it can be usefully compared with N-body simulations of structure formation, allowing constraints on dark matter physics. For example, the inner slope of the dark matter density,

$$\nu \equiv - \lim_{r \to 0} \frac{d \ln \rho_{\text{dm}}}{d \ln r} \quad (1)$$

can distinguish between collisionless ($\nu \approx 1$) and self-interacting ($\nu \approx 0$) varieties of dark matter (Nakano & Makino 1999; Moore et al. 1999; Burkert 2000).

Direct measurements of $\nu$ from X-ray data are rare, however, for two reasons. The first is that most mass models in common use—e.g., the Navarro, Frenk, & White (1997; NFW) profile $\rho \propto (r_0^2 + r^2)^{-\frac{1}{2}}$ or the nonsingular isothermal sphere—do not have an adjustable inner slope $\nu$, and even if they do, the gas and temperature distributions they generate are not expressible in terms of elementary functions of $r$ and $\nu$ (Markovich & Vikhlinin 1997). This inconvenience makes fitting for $\nu$ an expensive and difficult task.

More importantly, our understanding gas physics in the cores of the galaxy clusters closest to equilibrium leaves something to be desired. In the ROSAT era, such clusters were observed to have cool cores and cooling times much shorter than the Hubble time; for these reasons they were thought to possess a quasihydrostatic cooling flow (Fabian 1994; Peres et al. 1998). However, the Chandra and XMM satellites have in general not detected the multiphase plasma required by cooling flow models (Böhringer et al. 2001; Johnstone et al. 2002; Lewis et al. 2002). It is therefore important at least to ask whether true hydrostatic models are capable of describing the properties of the gas—specifically the rising temperature profile—outside the central $\approx 10$ kpc, where the gas is often disrupted by a central radio source (Blanton et al. 2001).

In this Letter I consider a unified solution to the above two problems. First, I discuss a mass distribution with adjustable inner slope which does yield hydrostatic gas and temperature profiles in terms of elementary functions. Then, I show that together with an exponential pressure distribution, these hydrostatic models can reproduce the rising temperature profile and the steep density profile in the rich cluster Abell 478.

2. THE POTENTIAL

Consider a potential of the form

$$\Phi(r) = -\frac{G M_{\text{tot}}}{(r_0^n + r^n)^{1/n}} \quad (2)$$

Where $M_{\text{tot}}$ is the total mass of the system. The cumulative mass distribution and density profile that generates this potential are

$$M(r) = M_{\text{tot}} \left( \frac{r^n}{r_0^n + r^n} \right)^{1/n} \quad (3)$$

$$\rho(r) = \frac{M_{\text{tot}}(1 + n)}{4\pi r_0^n} \left( \frac{r}{r_0} \right)^{-n-2} \left( \frac{r_0^n}{r^n + r_0^n} + 1 \right)^{2+1/n} \quad (4)$$

The requirement that $M(0) = 0$ yields the lower bound $n > 0$, while the requirement that the density never increase with radius leads to an upper bound $n \leq 2$. This mass profile therefore corresponds to a broken power law with inner slope $\nu = 2 - n$ and outer slope $3 + n$. The $n = 1$ case corresponds to the Hernquist (1990) model, $n = 2$ is a Plummer sphere, and $n = 1/2$ yields a profile similar to that of collisionless dark matter halos in recent N-body simulations (Ghigna et al. 2000).

The potential is invertible, and thus the density can be expressed as a function of the potential. If we define a dimensionless potential variable $\psi = -\Phi_0/GM$ then

$$\rho(\psi) = \frac{M_{\text{tot}}(n + 1)}{4\pi r_0^n} \frac{\psi^{n+3}}{(1 - \psi^n)^{2/n-1}} \quad (5)$$
This form of the density can be used to compute phase space distribution functions using the methods outlined by, e.g., Cuddeford (1991), and thus facilitate measurement of the mass profile through equilibrium particle dynamics.

3. X-ray Atmospheres of Dark Matter Halos

Here I investigate the luminosity and temperature distribution of a hot, X-ray emitting plasma embedded in the mass distribution given by (3) and governed by the equation of hydrostatic equilibrium:

$$\nabla P = -\rho_g \nabla \Phi,$$

where $P$ is the gas pressure and $\rho_g$ is the gas density, which in general is different from the total matter density traced by $\Phi$. With the assumption that the plasma is spherically symmetric, the equation becomes

$$\frac{1}{\rho_g} \frac{d\tilde{T} \rho_g}{dr} = -\frac{d\Phi}{dr} = -\frac{GM}{r^2}$$

where $\tilde{T} \equiv P/\rho_g$ is the thermal energy per unit mass. For an ideal gas in a cluster environment, $\tilde{T} = kT/\mu m_p$, where $T$ is the temperature, $m_p$ is the proton mass, $\mu = 0.61$ is the mean molecular weight, and $k$ is Boltzmann’s constant.

### 3.1. Polytopes and the $\beta$-model

Consider the possibility that the gas is a polytrope, $P \propto \rho^\gamma$, where $\gamma$ is the polytropic index. In this case, the potential yields

$$\tilde{T} = \frac{\tilde{T}_0}{(r/r_0^n + 1)^{1/n}}$$

$$\rho_g = \frac{\rho_0}{(r/r_0^n + 1)^{3\beta/n}}$$

$$\beta \equiv \frac{1}{3(\gamma - 1)} = \frac{GM_{\text{tot}}}{r_0 T_0} - 1.$$  

An important property of this model is that the $n = 2$ potential reproduces the familiar $\beta$-model, the gas density profile most widely used by X-ray astronomers. While most literature assumes that the $\beta$-model describes an isothermal gas, in the present context it corresponds to the temperature profile in equation (8).

Furthermore, the model suggests an important connection between the polytropic index $\gamma$ and the $\beta$ parameter. It is clear from equation (9) that, for the gas mass to be finite, we must have $\beta > 2/3$. On the other hand, for $\beta < 2/3$, the polytropic index $\gamma > 5/3$, and the plasma subject to convective instabilities (Nevalainen et al. 1999). Thus, two important physical criteria for the validity of the polytropic model above—convergence of the gas mass and lack of convective instabilities—are fulfilled by the very same constraints on $\beta$.

### 3.2. A Hydrostatic Alternative to Cooling Flows

The polytropic models have two marked disadvantages: they require that the X-ray emitting medium have the same characteristic radius, $r_0$, as the total matter distribution, which need not in general be true; they also do not reproduce the temperature structure of the clusters closest to equilibrium. In many of the most regular clusters

of galaxies, the temperature is observed to rise with distance from the center, leveling out or falling after a peak. However, it is clear from equation (8) that the temperature in the polytropic model decreases monotonically with radius, and is therefore unable to reproduce the rise in temperature observed in many clusters.

For this reason, I introduce a new hydrostatic pressure model that reproduces many of the properties observed in clusters with rising temperature profiles. Consider an exponential gas pressure profile:

$$P = P_0 e^{-((r/r_0)^a/b)}$$

where $P_0$ is the central pressure, and $b^{1/a} r_0$ is the scale height of the pressure distribution. Together with the equation of hydrostatic equilibrium and equation (2), this yields the following gas density and temperature profiles:

$$\rho_g = \frac{P_0 a r_0}{GM_{\text{tot}} b} \left( \frac{r}{r_0} \right)^{a-n} \left( 1 + \frac{r^n}{r_0^n} \right)^{1+1/n} e^{-((r/r_0)^a/b)}$$

$$\tilde{T} = \frac{GM_{\text{tot}} b}{a r_0} \left( \frac{r}{r_0} \right)^{-a-n} \left( 1 + \frac{r^n}{r_0^n} \right)^{-1-1/n}$$

In order for the gas density to decrease with the radius, we must have $n > a$. In this case, the temperature profile behaves as an increasing power law of index $n - a$, achieving a maximum at $r_{\text{max}} = r_0 [(n-a)/(1+a)]^{1/n}$ before falling to zero at infinity. When $n = a$, the temperature peak is at $r = 0$, giving us again a monotonically decreasing temperature profile.

In Figure 1, I show that this model fits the temperature and the emissivity profile of Abell 478 as observed by the Chandra satellite. The deprojected data are from Sun et al. (2002); all analysis and model fitting is mine. The minimized six-dimensional merit function is

$$\chi^2 = \sum_i \left[ \frac{n_i - \rho_g(r_i)/m_p}{\Delta n_i} \right]^2 + \sum_i \left[ \frac{T_i - \tilde{T}(r_i)}{\Delta T_i} \right]^2,$$

where $N_n$ is the total number of density data points $n_i$, and $N_T$ is the total number of temperature data points $T_i$. The minimized value is 39.0 for 49 degrees of freedom, indicating that the fit is of a high quality.

There are 6 total parameters, and therefore $6 \times 5/2 = 15$ unique pairs of parameters. I derive marginalized joint probability distributions for all these pairs via a four-dimensional integration of the Bayesian probability distribution $\exp(-\chi^2/2)$, with uniform prior. Six of these marginalized distributions are shown in Figure 2. To obtain closed confidence contours, it was necessary to substitute, without loss of generality, the parameters $r_e \equiv b^{1/a} r_0$ and $M_{0.5} \equiv M(< 0.5 \text{ Mpc})$. These substitutions are sensible, as $r_e$ is just the exponential scale height of the gas, and $M_{500}$ is the mass within a spherical radius of 500 kpc—about the extent of the Chandra field of view.

Finally, in order for the previous discussion to be physically self-consistent, three criteria must apply. First, the gas density must never exceed the total matter density. This corresponds to the requirement that the baryon fraction $\rho_g/\rho < 1$ everywhere throughout the cluster. Second, the gas must not experience convective instabilities $(d \ln \rho_g/\ln r \lesssim 5/3)$. Third, the inner slope of the dark matter density $\rho_{dm} = \rho - \rho_g$ must be checked against the
inner slope of the total density $\rho$. Figure 1 shows that all these criteria are abundantly fulfilled. The average baryon fraction within 0.5 Mpc is $0.08 \pm 0.07$ (at 68\% confidence), and the radial variation of it within that range is equally well-constrained. The effective polytropic index is always $< 5/3$, and the slope of the dark matter density profile matches that of the total profile. Hence the model as applied to Abell 478 is self-consistent.

Of course, any hydrostatic model has to contend with the fact that the cooling time remains quite short. Figure 1 shows that the cooling time on the average is smaller than the Hubble time in the region observed by *Chandra*, in agreement with Sun et al. (2002); the gas should radiate away all its energy in under ten billion years. However, the failure of *Chandra* and *XMM* observations to detect the temperature distribution predicted by cooling flow models (Lewis et al. 2002; Peterson et al. 2001) has lead some to infer that an unknown heating mechanism must balance the cooling (Loeb 2002; Brüggen & Kaiser 2002). While the details of this balancing system are beyond the scope of this Letter, its net effect may be to make the hydrostatic solutions empirically valid in at least some cases.

4. CONCLUSION

I suggest a new potential, $\Phi \propto (r^n + r_0^n)^{-1/n}$, for modeling the matter distribution of galaxy clusters. This potential has an inner density cusp of slope $2 - n$, and is particularly useful because (1) it is a generalization of potentials in common use; (2) it is invertible; and (3) a gas embedded in it can have density and temperature distributions that are elementary functions of $n$. As an example, I successfully apply a hydrostatic model to published *Chandra* observations of the galaxy cluster Abell 478, which is generally believed to contain a cooling flow. The resulting inner slope $2 - n = 1.0 \pm 0.2$ (90\% confidence interval) is well constrained, and consistent with N-body simulations in which massive halos are composed of collisionless matter.

This work demonstrates that cold cores and steep density gradients are compatible with hydrostatic models in all respects except for the cooling time.

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Fig. 1.— Fits of the present model to the temperature and electron density distribution of Abell 478. The data are taken from Sun et al. (2002); the model fits are mine. (a) Deprojected temperature profile; (b) deprojected electron density. Shown also are 68% confidence limits on the following quantities as a function of radius: (c) the cooling time; (d) the slope of the dark matter density $\nu = d \ln \rho_{dm} / d \ln r$; (e) the effective polytropic index $d \ln P / d \ln \rho_g$; and (f) the baryon fraction (solid lines) as well as on the average baryon fraction with 0.5 Mpc (dashed horizontal lines).
Fig. 2.— Marginalized Bayesian probability distributions for 6 of the 15 unique parameter pairs. Shown are the 68%, 95%, and 99.7% confidence contours.