A novel spectral broadening from vector–axial-vector mixing in dense matter

Masayasu Harada\(^1\) and Chihiro Sasaki\(^2\)

\(^1\)Department of Physics, Nagoya University, Nagoya, 464-8602, Japan
\(^2\)Physik-Department, Technische Universität München, D-85747 Garching, Germany

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The presence of baryonic matter leads to the mixing between transverse \(\rho\) and \(a_1\) mesons through a set of \(\omega\rho a_1\)–type interactions, which results in the modification to the dispersion relation. We show that a clear enhancement of the vector spectral function appears below \(\sqrt{s} = m_\rho\) for small three-momenta of the \(\rho\) meson, and thus the vector spectrum exhibits broadening. We also discuss its relevance to dilepton measurements.

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In-medium modifications of hadrons have been extensively explored in the context of chiral dynamics of QCD \(\cite{1,2}\). Due to an interaction with pions in the heat bath, the vector and axial-vector current correlators are mixed. At low temperatures or densities a low-energy theorem based on chiral symmetry describes this mixing \(\text{(V-A mixing)} \cite{3}\). The effects to the thermal vector spectral function have been studied through the theorem \(\cite{3}\), or using chiral reduction formulas based on a virial expansion \(\cite{5}\), and near critical temperature in a chiral effective field theory involving the vector and axial-vector mesons \(\cite{6}\).

It has been derived, as a novel effect at finite baryon density, that a Chern-Simons term leads to mixing between the vector and axial-vector fields in a holographic QCD model \(\cite{7}\). This mixing modifies the dispersion relation of the transverse polarizations and will affect the in-medium current correlation functions independently of specific model dynamics. In this letter we study the effect of the vector–axial-vector mixing to the in-medium spectral functions which is the main input to the experimental observables. We show that the mixing produces a clear enhancement of the vector spectral function which appears below \(\sqrt{s} = m_\rho\), and that the vector spectral function is broadened due to the mixing. We will discuss its relevance to dilepton measurements.

At finite baryon density a system preserves parity but violates charge conjugation invariance. Chiral Lagrangians thus in general build in the term

\[
\mathcal{L}_{\rho a_1} = 2C' \epsilon^{\mu
u\lambda\sigma} \text{tr} \left[ \partial_\nu V_\lambda \cdot A_\sigma + \partial_\sigma A_\lambda \cdot V_\nu \right].
\]

This mixing results in the dispersion relation \(\text{(2)}\)

\[
p_0^2 - \vec{p}^2 = \frac{1}{2} \left[ m_\rho^2 + m_{a_1}^2 \pm \sqrt{(m_{a_1}^2 - m_\rho^2)^2 + 16C^2 \vec{p}^2} \right],
\]

which describes the propagation of a mixture of the transverse \(\rho\) and \(a_1\) mesons with non-vanishing three-momentum \(|\vec{p}| = \vec{p}\). The longitudinal polarizations, on the other hand, follow the standard dispersion relation, \(p_0^2 - \vec{p}^2 = m_\rho^2\). When the mixing vanishes as \(\vec{p} \to 0\), Eq. \(\text{(2)}\) with lower sign provides \(p_0 = m_\rho\) and it with upper sign does \(p_0 = m_{a_1}\). In the following, we call the mode following the dispersion relation with the lower sign in Eq. \(\text{(2)}\) “the \(\rho\) meson”, and it with the upper sign “the \(a_1\) meson”. In a holographic QCD approach the coupling \(C\) depends on the baryon density \(n_B\) and is found \(C = 1\, \text{GeV} \cdot (n_B / n_0)\) with normal nuclear matter density \(n_0 = 0.16 \, \text{fm}^{-3}\) \(\cite{7}\). Figure 1 shows the dispersion relation \(\text{(2)}\). For very large \(\vec{p}\) the longitudinal and transverse dispersions are in parallel with a finite gap, \(\pm C\). The dispersion relation \(\text{(2)}\) also indicates a possibility of vector condensation for a large \(C\) \(\cite{7}\).

The vector-current correlation function in matter is decomposed into the longitudinal and transverse parts as

\[
G_V^{\mu\nu}(p_0, \vec{p}) = P_L^{\mu\nu} G_V^L(p_0, \vec{p}) + P_T^{\mu\nu} G_V^T(p_0, \vec{p}),
\]

with the polarization tensors \(P_L^{\mu\nu}\) and momentum \(p^\mu = (p_0, \vec{p})\). Using the bare propagator inverse, \(D_{V,A} = s - m_{\rho,a_1} \pm i m_{\rho,a_1}(s)\), \(G_V^L\) and \(G_V^T\) are expressed as

\[
G_V^L = \left( \frac{g_\rho}{m_\rho} \right)^2 \frac{-s}{D_V}, \quad G_V^T = \left( \frac{g_\rho}{m_\rho} \right)^2 \frac{-sD_A + 4C^2 \vec{p}^2}{D_V D_A - 4C^2 \vec{p}^2},
\]
with $s = p^2$ being the squared four-momentum and $g_\rho$ the coupling strength of the $\rho$ meson to the vector current. We have imposed gauge invariance on the vector current to get the form $\Gamma_1$. The spin-averaged correlator is given by $G_V = \frac{1}{3} [G_V^I + 2G_V^T]$. The vector spectral function is defined as the imaginary part of the vector correlator $\Im G_V$. We define the integrated spectrum over three momentum by

$$\Im G_V(s) = \int \frac{d^3\vec{p}}{2p_0} \Im G_V(p_0, \vec{p}).$$

Equation (4) indicates that the mixing at finite three momentum $\vec{p}$ affects the real part of the transverse $\rho$ self-energy. We use the vacuum decay widths $\Gamma$ to illustrate its influence over the spectrum although $\Gamma$ in dense matter are considered to be broadened $\Gamma_2$. We take the following experimental values for further calculations: $m_\pi = 0.14$ GeV, $m_\rho = 0.77$ GeV, $m_{a_1} = 1.26$ GeV, $g_\rho(s = m_\pi^2) = 0.15$ GeV $\Gamma_3$. For the $a_1$ meson we use $\Gamma_{a_1}(s = m_{a_1}^2) = 0.3$ GeV as a typical example.

We show the vector spectral function in Fig. 2. The transverse spectrum presents two bumps due to the mixing: the lower one corresponds to the $\rho$ whose mass is shifted downward, and the upper one to the $a_1$ whose mass is shifted upward in compared with the longitudinal polarizations (see Fig. 1). Since two pion annihilation is assumed to be dominant in the $\rho$ meson decay, the contribution at low $\sqrt{s}$ is cut off at threshold $\sqrt{s} = 2m_\pi$. Figure 2 (left) shows a clear enhancement of the spectrum below $\sqrt{s} = m_\rho$ due to the mixing. This enhancement becomes much suppressed when the $\rho$ meson is moving with a large three-momentum as shown in Fig. 2 (right). The upper bump now emerges more remarkably and becomes a clear indication of the in-medium effect from the $a_1$ via the mixing. The presence of the two bumps in the transverse part leads to some broadening of the spin-averaged spectrum.

For more realistic evaluations one needs to include nuclear many-body dynamics into meson decay widths. This will be another source of in-medium broadening and eventually the vector correlator may not exhibit a clear maximum. Besides the iso-vector $\rho-a_1$ mesons, the mixing between the $\omega$ and $f_1(1285)$ mesons as well as that between the $\phi$ and $f_1(1420)$ mesons in isoo-scalar channel also exists and changes the dispersion relations. This is controlled by the same mixing strength $C$ which can be smaller in three-color QCD than the value predicted in holographic QCD models. In such a case the spectrum enhancement in low $\sqrt{s}$ region becomes more moderate but the effect is still relevant to the vector spectrum of the $\rho$ mesons carrying large $\vec{p}$. As a result, the averaged spectrum might have a broad bump with its maximum slightly shifted downward due to the mixing. Thus, it is expected that those mixing have some relevance to explain in-medium “mass shift” of the $\rho$, $\omega$ and $\phi$ mesons observed by CBELSA/TAPS and KEK-PS-E325 $\Gamma_4$.

As an application of the above in-medium spectrum, we calculate the production rate of a lepton pair emitted from dense matter through a decaying virtual photon. The differential production rate in a medium for fixed temperature $T$ and baryon density $n_B$ is expressed in terms of the imaginary part of the vector current correlator as $\Gamma_5$

$$\frac{dN}{d\vec{p}}(p_0, \vec{p}; T, n_B) = \frac{\alpha^2}{\pi^3} \frac{1}{e^{p_0/T} - 1} \Im G_V(p_0, \vec{p}; T, n_B),$$

where $\alpha = e^2/4\pi$ is the electromagnetic coupling constant. The three-momentum integrated rate is given by

$$\frac{dN}{ds}(s; T, n_B) = \int \frac{d^3\vec{p}}{2p_0} \frac{dN}{d\vec{p}}(p_0, \vec{p}; T, n_B).$$

FIG. 2: The vector spectral function for $C = 1$ GeV. The curves of the left figure are calculated integrating over $0 < \vec{p} < 0.5$ GeV, and those of the right figure over $0.5 < \vec{p} < 1$ GeV.
Figure 3 presents the integrated rate at $T = 0.1$ GeV for $C = 1$ GeV. One clearly observes a strong three-momentum dependence and an enhancement below $\sqrt{s} = m_\rho$ due to the Boltzmann distribution function which result in a strong spectral broadening. The total rate is mostly governed by the spectrum with low momenta $\bar{p} < 0.5$ GeV due to the large mixing parameter $C$. When density is increased, the mixing effect gets irrelevant and consequently in-medium effect in low $\sqrt{s}$ region is reduced in compared with that at higher density. The calculation performed in hadronic many-body theory in fact shows that the $\rho$ spectral function with a low momentum carries details of medium modifications \[11\]. One may have a chance to observe it in heavy-ion collisions with certain low-momentum binning at J-PARC, GSI/FAIR and RHIC low-energy running.

It is straightforward to introduce other V-A mixing between $\omega-f_1(1285)$ and $\phi-f_1(1420)$. We use the constant widths of narrow peaked mesons above threshold: $\Gamma_\omega = 8.49$ MeV, $\Gamma_\phi = 4.26$ MeV, $\Gamma_{f_1(1285)} = 24.3$ MeV and $\Gamma_{f_1(1420)} = 54.9$ MeV \[8\]. The coupling constants of $\omega$ and $\phi$ mesons to the vector current are given by

$$g_\omega = \frac{1}{3} \frac{m_\omega^2}{m_\rho^2} g_\rho, \quad g_\phi = \frac{\sqrt{2}}{3} \frac{m_\phi^2}{m_\rho^2} g_\rho.$$  \hspace{1cm} (8)

Figure 4 shows the integrated rate at $T = 0.1$ GeV with several mixing strength $C$ which are phenomenological option. One observes that the enhancement below $m_\rho$ is suppressed with decreasing mixing strength. This forms into a broad bump in low $\sqrt{s}$ region and its maximum moves toward $m_\rho$. Similarly, some contributions are seen just below $m_\phi$. This effect starts at threshold $\sqrt{s} = 2m_K$ in the present analysis because of $\Gamma_\phi(s) = \Theta(s - 4m_K^2)\Gamma_\phi(m_\phi)$. Self-consistent calculations of the spectrum in dense medium will provide a smooth change and this eventually makes the $\phi$ meson peak somewhat broadened.

The relevance of this mixing in dense matter essentially relies on how the strength $C$ is precisely determined. Holographic QCD approach predicts a strong mixing. However, the models based on the gravity-gauge correspondence are formulated in large $N_c$ limit. Their prediction of observables may have a non-negligible $1/N_c$ correction \[12\]. This suggests a possibility that $C$ is smaller in realistic QCD. One might consider to replace the mixing term \[11\] with the $\omega-\rho-\pi_1$ term which has been shown to arise from the gauged Wess-Zumino-Witten term in chiral Lagrangians \[13\] or alternatively from the reduction of five-dimensional Chern-Simons term to four dimensions \[14\],

$$\mathcal{L}_{\omega\rho\pi_1} = g_{\omega\rho\pi_1}(\omega_0) \epsilon^{\nu\omega\lambda\sigma} \text{tr} \left[ \partial_\nu V_\lambda \cdot A_\sigma + \partial_\nu A_\lambda \cdot V_\sigma \right],$$  \hspace{1cm} (9)

where the $\omega$ field is replaced with its expectation value given by $\langle \omega_0 \rangle = g_{\omega NN} \cdot n_B/m_\omega^2$. One finds with empirical numbers $C = g_{\omega\rho\pi_1}(\omega_0) \approx 0.1$ GeV at normal nuclear matter density. This relatively much weaker mixing has little importance in the correlation functions. It is plausible to assume an actual value of $C$ in QCD in the range $0.1 < C < 1$ GeV since the strong mixing in holographic QCD models contains higher members of Kaluza-Klein (KK) modes other than the lowest $\omega$ meson and those higher excitations are embedded in $C$. Some importance of the higher KK modes even in vacuum in the context of holographic QCD can be seen in the pion electromagnetic form factor at the photon on-shell: This is saturated by the lowest four vector mesons in a top-down holographic QCD model \[15\]. In hot and dense environment those higher members get modified and the masses might be somewhat decreasing evidenced in an in-medium holographic model \[17\]. This might provide
a strong V-A mixing $C > 0.1$ GeV in three-color QCD and the dilepton measurements may be a good testing ground.

It is also an interesting issue to address a change of the vector correlator with the V-A mixing toward chiral symmetry restoration. The mixing is chirally symmetric and thus does not approach zero toward the chiral restoration in contrast to the vanishing V-A mixing near the critical temperature $T_c$ without baryon density [6]. A spontaneous breaking of Lorentz invariance via the omega condensation could increase the mixing strength $C$ near chiral restoration [18]. Furthermore, if meson masses drop due to partial restoration of chiral symmetry assuming a second- or weak first-order transition in high baryon density but low temperature region, the ground state near the critical point may favor vector condensation even for a moderate mixing strength. This will be reported elsewhere.

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