Phenomenology of Space-time Imperfection II: Local Defects

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Abstract

We propose a phenomenological model for the scattering of particles on space-time defects in a treatment that maintains Lorentz-invariance on the average. The local defects considered here cause a stochastic violation of momentum conservation. The scattering probability is parameterized in the density of defects and the distribution of the momentum that a particle can obtain when scattering on the defect. We identify the most promising observable consequences and derive constraints from existing data.

1 Introduction

The phenomenology of quantum gravity proceeds by the development of models that parameterize properties which the, still unknown, theory of quantum gravity might have. These phenomenological models are constructed for the purpose of being testable by experiment and thereby guide the development of the theory. Among the best studied phenomenological consequences of quantum gravity are violations or deformations of Lorentz-invariance, additional spatial dimensions, and decoherence induced by quantum fluctuations of space-time. Research in the area of quantum gravity phenomenology today encompasses a large variety of models that have been reviewed in \[1,2,3\].

A possible observable consequence of quantum gravity that has so far gotten little attention is the existence of space-time defects. If the seemingly smooth space-time that we experience is not fundamental but merely emergent from an underlying, non-geometric theory then we expect it to be imperfect – it should have defects. We will here study which observational consequences can be expected from such space-time imperfections of non-geometric origin.
Space-time defects are localized both in space and in time and therefore, in contrast to defects in condensed-matter systems, do not have worldlines. There are two different types of defects: Local defects and nonlocal defects. Local defects respect the emergent locality of the space-time manifold. A particle that encounters a local defect will scatter and change direction, but continue its world-line continuously. Nonlocal defects on the other hand do not respect the emergent locality of the space-time manifold. A particle that encounters a nonlocal defect continues its path in space-time elsewhere, but with the same momentum. The nonlocal defect causes a translation in space-time, while the local defect causes a translation in momentum-space.

The present paper is the second part of a study of space-time defects and deals with local defects. Nonlocal defects have been subject of the first part [4]. In principle a space-time defect could cause both, a change of position and momentum. But before making the situation more complicated by combining these two effects, we will first study the two cases separately. In this paper, we develop a model for the local type of defects. Since Lorentz-invariance violation is the probably most extensively studied area of Planck-scale physics [5, 6], we will only consider the case where Lorentz-invariance is maintained on the average. A different model for local space-time defects has recently been put forward in [7]. We will briefly comment on the differences to this model in the discussion.

We use the unit convention $\hbar = c = 1$. The signature of the metric is $(+,-,-,-)$.

2 The distribution of defects

To develop our model for local defects, we will here start with the simplest case in which the emergent background space-time is flat Minkowski space, i.e., background curvature is not taken into account. This approximation will allow us to describe only systems in which gravitational effects are negligible, but it is good enough for Earth-based laboratories and interstellar propagation over intergalactic distances where curvature is weak and redshift can still be neglected. These will be the cases we consider in section 5.

The only presently known probability distribution for points in Minkowski space that preserves Lorentz-invariance on the average is the result of a Poisson process developed and studied in [8, 9]. With this distribution, the probability of finding $N$ points in a space-time volume $V$ is

$$P_N(V) = \frac{(\beta V)^N \exp(-\beta V)}{N!},$$  

where $\beta$ is a constant space-time density.

The average value of the number of points that one will find in some volume $V$ is then the expectation value of the above distribution and given by

$$\langle N(V) \rangle = \sum_{N=0}^{\infty} P_N(V)N = \beta V.$$
The variance that quantifies the typical fluctuations around the mean is $\Delta N \sim \sqrt{\beta V}$, and the corresponding fluctuations in the density of points are $\Delta(N/V) \sim \sqrt{\beta/V}$. In other words, the density fluctuations will be small for large volumes.

We will use the distribution $\mathcal{P}$ to seed the defects with an average density $\beta$. In the following, we will not be concerned with fluctuations in the density as our aim here is to first get a general understanding for the type and size of effects caused by local defects and using the average will suffice for the present purpose. The probability is a density over space-time $\beta = L^{d+1}$, where $L$ is a length scale and $d$ is the number of spatial dimensions. The ratio between the fundamental length scale, that we take to be the Planck length $l_P$, and $L$ is $\epsilon = l_p/L \ll 1$, ie the defects are sparse. Just exactly how sparse is a question of experimental constraints that we will address in section 5.

3 Kinematics of scattering on local defects

The idea which we want to parameterize here is that local defects are deviations from the smooth geometry of general relativity that cause a violation of energy-momentum conservation.

The requirement of Lorentz-invariance restricts what the particle can do when it encounters such a local defect. This restriction is more stringent than for normal scattering processes because we have fewer quantities as input. We only have the ingoing and outgoing momenta, whereas one normally has at least three momenta involved in scattering, leading to the three invariant Mandelstam variables.

Let us denote the momentum of the particle before it encounters the defect with $p$ and the momentum after it encountered the defect with $p'$, where boldfaced quantities denote four-vectors. Let us further formally assign the momentum $k = p' - p$ to the defect (see Figure 1 left). This assignment of momentum to the defect is a bookkeeping device that will allow us to think in terms of normal scattering processes. The space-time defect itself does however not actually have a momentum; it instead causes a violation of momentum conservation.

3.1 Massless particles in 1+1 dimensions

We will first consider the case where the ingoing particle is right moving and massless, $m = 0$. Since $p^2 = 0$, we have two Lorentz-invariants left, $p \cdot k = M^2$ and $k^2 = a^2 M^2$ (thus $p'^2 = (a^2 + 2)M^2$), where $m$ and $M$ have dimension mass, and $a$ is a dimensionless parameter that we expect to be of order one. It is henceforth assumed that $a$ and $M$ are real-valued to avoid that the mass of the outgoing particle can become tachyonic.

We want to quantify now the probability $\mathcal{P}$ for what will happen when the particle encounters the defect. Lorentz-invariance requires $\mathcal{P}$ to be a function solely of $a$ and $M$, and our expectation is that it is actually a function of $a^2$ and $M^2$. The condition $p \cdot k = M^2$ selects a cut in the hyperboloid defined by $k^2 = a^2 M^2$. In 1+1 dimensions,
Figure 1: Assignment of momentum notation. Left: Simple vertex for scattering on local defect (dotted line). Right: Photon (wavy line) decays into a fermion pair (solid line) enabled by scattering on local defect (dotted line).

this specifies $k$ completely. The higher dimensional case brings additional complications that we will come to in section 3.3.

When we leave behind the classical particle and think about quantum particles, it does not seem to make much sense to have a particle scatter off a point to obtain a distinct momentum from that point. We should instead take into account that the point has a position uncertainty and the momentum it transfers will inevitably have an uncertainty too. That is the distribution $P(M^2, a^2)$ will not encode distinct values of $a$ and $M$ but these variables will have some spread to them. We will quantify their distribution only roughly by means of the average values $\langle a^2 \rangle, \langle M^2 \rangle$ and variances $\Delta(a^2), \Delta(M^2)$. For the sake of readability, we will in the following write the variances simply as $\Delta a^2, \Delta M^2$.

Let us assume that the massless particle moves into the positive $x_1$-direction, and denote $p^\nu = (E, E)$ and $k^\nu = (k_0, k_1)$, where $k_1 = \pm \sqrt{k_0^2 - M^2/a^2})$. Then the requirement $p \cdot k = M^2$ leads to

$$E = \frac{k_0}{a^2} \pm \sqrt{\frac{k_0^2}{a^4} - \frac{M^2}{a^2}}. \quad (3)$$

Or, solving for $k$ instead, one finds

$$k_0 = \frac{1}{2} \left( \frac{M^2}{E} + a^2 E \right), \quad k_1 = \frac{1}{2} \left( \frac{M^2}{E} - a^2 E \right). \quad (4)$$

This means in particular there is no threshold for the particle scattering; massless particles can scatter even if they have a very small energy.

Though the assignment of momentum to the defect is just a bookkeeping device it is instructive to make a Fourier transformation of the momentum distribution $P(M^2, a^2)$. To that end, we consider the distribution to be Gaussian

$$P(M^2, a^2) = \exp \left( -\frac{(M^2 - \langle M^2 \rangle)^2}{(2\Delta M^2)^2} - \frac{(a^2 - \langle a^2 \rangle)^2}{(2\Delta a^2)^2} \right) \left( 2\pi \sqrt{\Delta M^2 \Delta a^2} \right)^{-1}, \quad (5)$$
where we have placed the defect at the origin. We now want to compute the space-time distribution

\[ P(t, x) = \int d(M^2) d(a^2) P(M^2, a^2) e^{-i(k_0 x^0 + k_1 x^1)} . \]  

Since we are dealing with massless particles, it will be handy to introduce the light-cone momenta and

\[ k^\pm = k_\mp = (k^0 \pm k^1)/\sqrt{2} \]  

and \( p_+ = E, p_- = 0 \). We then rewrite the integral into lightcone coordinates and obtain

\[ P(x^+, x^-) = \int dk_+ dk_- P(k_+, k_-) e^{-i(k_+ x^+ + k_- x^-)} \]  

(6)

(7)

(8)

Here we have introduced the average values \( \langle k_+ \rangle = \langle M^2 \rangle / (\sqrt{2} E) \) and \( \langle k_- \rangle = \langle a^2 \rangle E / \sqrt{2} \).

The Fourier-transformation thus yields a Gaussian distribution in light-cone coordinates

\[ P(x^+, x^-) = \exp \left( \left( \frac{(x^+)^2}{2\sigma^+} + \frac{(x^-)^2}{2\sigma^-} \right) \right) \frac{e^{i\langle k_+ \rangle x^+ + \langle k_- \rangle x^-}}{2\pi \sqrt{\sigma^+ \sigma^-}} , \]  

with widths

\[ \sigma^+ = \frac{\sqrt{2} E}{\Delta M^2} \]  
\[ \sigma^- = \frac{\sqrt{2} \Delta a^2}{E \Delta a^2} . \]  

(9)

(10)

(11)

(12)

We see that the typical space-time patch covered in the direction of propagation

\[ \sigma^+ \sigma^- = 2 \left( \Delta M^2 \Delta a^2 \right)^{-1} \]  

has a Lorentz-invariant volume independent of \( E \), though it will deform under boosts that red- or blueshift \( E \) as one sees from Eqs. (11).

To make sense of the scarcity of local defects, the typical size associated with the defect should be much smaller than the typical volume in which to find one defect, ie \( L^2 \Delta M^2 \Delta a^2 \gg 1 \). Strictly speaking, the above treatment is valid only for an incident plane wave with exactly defined momentum \( E \). If the incoming particle has an energy spread, \( \Delta p_+ \neq 0 \), this will contribute to \( \Delta k_\pm \) via the relations (7). The plane wave approximation is good only as long as \( \langle a^2 \rangle \Delta p_+ \ll \langle p_+ \rangle \Delta a^2 \) and \( \langle M^2 \rangle \Delta p_+ \ll p_+ \Delta M^2 \).

We will in the following assume that this approximation is valid.


3.2 Massive particles in 1+1 dimensions

If the incoming particle has a mass, \( m > 0 \), we have an additional dimensionful parameter to generalize our requirements on \( k^2 \) and \( p \cdot k \). To find a suitable way to treat massive particles, we first note that in the ultrarelativistic limit the relation for \( k^2 \) should reproduce the \( k^+ \) from the massless case. Thus, we will leave the requirement \( k^2 = a^2 M^2 \) the same for the massive case as it was for the massless case. We further note that in the limit of small energies \( k_0 \) and \( k_1 \) from the massless case tend towards the same value.

We therefore expect that for a massive particle in the restframe \( k_0 \) and \( k_1 \) have the same value.

We thus make the ansatz \( k \cdot p = M^2 + a^2 m^2 / 4 \), which in the restframe of the massive particle leads to

\[
k_0 = \frac{M^2}{m} + \frac{a^2 m}{4} \quad , \quad k_1 = \pm \left| \frac{M^2}{m} - \frac{a^2 m}{4} \right| .
\]

If the incoming particle has a mass, \( m \), then the defect will increase its mass to \( p'^2 = M^2 (a + 2) + m^2 (1 + a^2 / 2) \) and can thus result in a virtual particle as in the massless case.

This leads us to assign the following Gaussian distribution to the defect

\[
P(M^2, a^2) = \exp \left( -\frac{(M^2 - \langle M^2 \rangle)^2}{(2 \Delta_m M^2)^2} - \frac{(a^2 - \langle a^2 \rangle)^2}{(2 \Delta_m a^2)^2} \right) \left( 2\pi \Delta_m M^2 \Delta_m a^2 \right)^{-1}, \tag{14}
\]

where the index \( m \) on the variance stands for the mass of the particle and indicates that this distribution is a priori not the same as in the massless case. We proceed as previously and rewrite this into momentum space

\[
P(k_0, k_1) = \exp \left( -\frac{(k_0 - \langle k_0 \rangle + |k_1| - |\langle k_1 \rangle|)^2 m^2}{2 (\Delta_m M^2)^2} \right) \times
\]

\[
\exp \left( -\frac{(k_0 - \langle k_0 \rangle - |k_1| + |\langle k_1 \rangle|)^2}{2 m^2 (\Delta_m a^2)^2} \right) \left( 2\pi \Delta_m M^2 \Delta_m a^2 \right)^{-1}, \tag{15}
\]

where \( \langle k_0 \rangle \) and \( \langle k_1 \rangle \) are the mean values of \( k_1 \) and \( k_2 \) in (13). Since we expect the variances of \( k_0 \) and \( k_1 \) to be the same in the restframe we set \( \Delta_m a^2 m = \Delta_m M^2 / m = \Delta_m k \), which yields

\[
P(k) = \exp \left( -\frac{(k_0 - \langle k_0 \rangle)^2 + k_1^2}{(2 \Delta_m k)^2} \right) (2\pi \Delta_m k)^{-1} . \tag{16}
\]

In space-time the defect in the restframe of the incident particle is thus described by the Fourier-transformation

\[
P(x) = \exp \left( -\frac{(x_0^2 + x_1^2)}{(2 \Delta_m x)^2} \right) e^{i\langle k_0 \rangle x^0} (2\pi \Delta_m x)^{-1} , \tag{17}
\]
where $\Delta m x = m/\Delta_m M^2$.

Since the space-time volume is Lorentz-invariant, we know then that the typical volume occupied by the defect is $(m/\Delta_m M^2)^2$. If we want this volume to be the same as in the massless case, we can identify $(\Delta m M^2)^2 = m^2 M^2 \Delta a^2$ and $(\Delta_m a^2)^2 = (\Delta M^2 \Delta a^2)/m^2$.

### 3.3 Massive and massless particles in 3+1 dimensions

The reason we were able to construct the above Lorentz-invariant and normalizable distributions even though the Lorentz-group is not compact is that we have not treated the momentum distribution assigned to the defect as internal, but as dependent on the incident particle. Then, one can use the momentum vector of the particle that scatters as reference without affecting observer-independence of the outcome.

If we add two additional spatial dimensions, momentum space becomes larger and the two requirements on $k$ will no longer suffice to pick out a specific momentum that the defect transfers. The degeneracy in $k$ is due to the subgroup of the Lorentz-group that leaves the momentum vector $p$ invariant, known as the little group of the particle. If we have only $p$ as reference, then in each reference frame in which $p$ has the same components, the distribution for $k$ must be the same.

In 3+1 dimensions, the little group of a massive particle is $SO(3)$ and compact, so the treatment is straight-forward because we can use a uniform distribution over the additional degrees of freedom. We add the requirement that the restframe of the particle remains the same on the average, and the distribution is spherically symmetric for the spatial components of $k$ in the particle’s restframe.

This means

$$k_0 = \frac{M^2}{m} + \frac{a^2 m}{4}, \quad k = \left| \frac{M^2}{m} - \frac{a^2 m}{4} \right|, \quad (18)$$

where $k = \overline{k}$ and $\overline{k} = (k_1, k_2, k_3)$ and the 1+1 dimensional distribution generalizes to (compare to Eq. (16))

$$P(k) = \exp \left( - \frac{(k_0^2 + k^2)}{(2\Delta_m k)^2} \right) (2\pi \Delta_m k)^{-2}. \quad (19)$$

In space-time, the defect is thus described by the (four dimensional) Fourier-transformation

$$P(x) = \exp \left( - \frac{(x_0^2 + x^2)}{(2\Delta_m x)^2} \right) (2\pi \Delta_m x)^{-2}, \quad (20)$$

where $\Delta_m x = m/\Delta_m M^2$ as before and the typical volume occupied by the defect is $(m/\Delta_m M^2)^4$. This distribution is invariant under the little group of the incident massive particle that scatters on the defect and thus all observers who measure the particle must agree on the scattering result, as desired.
The little group for a massless particle it is $ISO(2)$, the symmetry group of the 2-dimensional Euclidean plane. It consists of rotations in the $x_2 - x_3$ plane and combinations of boosts and rotation.\footnote{The existence of these additional transformations goes back to the same property of the 3+1 dimensional Lorentz-group that gives rise to the Thomas-Wigner rotation.} For an explicit construction, see eg \cite{10}. $ISO(2)$ is not a semi-simple group and not compact. The rotational part into direction $x_2 - x_3$ is unproblematic, but the remaining transformations have a continuous set of two parameters corresponding to an infinite number of reference frames in which observers will all measure the same components of the incident massless particle’s four-momentum.

Since $k$ is then not uniquely defined but only up to the transformations in the little group of the incident particle, without further input, Lorentz-invariance would require that we have to treat all possible solutions equally. Because the parameter space is not compact, for a massless particle this would then entail a uniform distribution over two unbounded parameters which is not normalizable.

However, we deal with uniform distributions over non-compact spaces every time we use plane waves. It thus seems likely that the problem has the same origin and can be dealt with the same way, namely by taking into account that in reality the incoming particle is spread out only over a finite volume and has a non-zero momentum spread.

Concretely, the elements of the photon’s little group are besides the rotations in the $x_2 - x_3$ plane of the form

$$L^\mu_\nu = \begin{pmatrix} 1 + \xi & -\xi & \chi & \kappa \\ \xi & 1 - \xi & \chi & \kappa \\ \chi & -\chi & 1 & 0 \\ \kappa & -\kappa & 0 & 1 \end{pmatrix},$$  

where $2\xi = \chi^2 + \kappa^2$. One convinces oneself readily that indeed $L^\mu_\nu p^\nu = p^\mu$.

The solution for $k$ from the 1+1 dimensional case is still a solution to the requirements on $k \cdot p$ and $k^2$ (with two zero entries added), but under a transformation of the above photon’s little group element, one obtains

$$k^\nu = L^\mu_\nu k^\mu = k^\nu + \frac{M^2}{E} \nu^\nu,$$  

where $\nu^\nu = (\xi, \xi, \chi, \kappa)$. The probability distribution for $P(k)$ in four dimensions then should be independent of $\kappa$ and $\chi$, where $\kappa$ and $\chi$ could be expressed in terms of $k'_2$ and $k'_3$ as $\chi = k'_2 E/M^2$ and $\kappa = k'_3 E/M^2$.

To address the issue of normalizing the distribution, we now take into account that the incident particle’s wave-function has a finite spread in $p_2$ and $p_3$ direction, which we will label $\Delta p_2$ and $\Delta p_3$. The mean momentum vector we will denote $\langle p \rangle = \langle E \rangle, \langle E \rangle, 0, 0$. We would like to preserve the rotational symmetry, and so will assume $\Delta p_2 = \Delta p_3 = \vdots$\footnote{Note that this spread is not constant over time.}
These relevant point is that these extensions in direction \( x_2 \) and \( x_3 \) are perpendicular to the photon’s wave-vector and not invariant under the little group. Therefore, they provide us with additional information about the incident particle that we can use to put bounds on the integration over \( \kappa \) and \( \chi \), or (dropping the primes) on \( k_2 \) and \( k_3 \) respectively. We do this by assuming that the width of the defect in momentum space has \( \Delta k_2 = \Delta k_3 = \Delta \kappa_\perp = \Delta \kappa_\perp \) and so (compare to Eq. (9))

\[
P(k) = \exp \left( -\frac{(E)^2 k_\perp^2}{(\Delta M^2)\perp^2} - \frac{(k_- - \langle k_- \rangle)^2}{(\langle E \rangle \Delta a^2)^2} - \frac{k_\perp^2}{2 \Delta k_\perp^2} \right) \left( 4\pi^2 \Delta k_\perp \sqrt{\Delta M^2 \Delta a^2} \right)^{-1}
\]

where \( k_\perp^2 = k_2^2 + k_3^2 \).

In (23) we have set \( \langle M^2 \rangle = 0 \). In this case, the average of \( k \) is parallel to the average of \( p \). Any other choice will, in the plane wave limit (when \( \langle E \rangle = E \)), not be invariant under the little group as one sees from Eq (22). It is thus henceforth assumed that \( \langle M^2 \rangle = 0 \) and thus \( \langle k_\perp \rangle = 0 \). In the plane-wave limit where \( \Delta p_\perp \rightarrow 0 \), one then has \( \Delta k_\perp \rightarrow \infty \) and the distribution of \( k \) will go to the uniform distribution over the little group.

This construction for massless particles in 3+1 dimensions is not invariant under the little group of the mean momentum of the incoming photon wave packet. But this is unnecessary because the incident particle, when it has a finite width, has the rotational symmetry in \( x_2 - x_3 \) as the only remaining symmetry. Thus, observer independence is preserved and the distribution is normalizable. In the plane-wave limit, the components of \( k \) can become arbitrarily large and then the defect can in principle transfer an arbitrarily large momentum to the particle. In reality, this momentum transfer will however be bounded because the incident particle has a finite width. We have essentially identified the non-compact part of \( ISO(2) \) with the non-compactness of the plane wave.

In summary, we have seen that Lorentz-invariance proves to be surprisingly restrictive on the possible scattering outcomes. We have dealt with the non-compactness of the Lorentz-group by using measurable properties of the incident particle to reduce the symmetry of the momentum non-conservation mediated by the defect while preserving observer-independence.

### 4 Dynamics of scattering on local defects

This now leads us to ask what we can say about scattering amplitudes. For concreteness, we assume that the defect makes itself noticeable in the covariant derivative since the local defect represents a non-geometric inhomogeneity that cannot be accounted for by the normal covariant derivative.

The local defect thus appears in the Lagrangian together with the derivative terms and gauge fields and can be implemented by replacing the normal gauge-invariant derivative \( D = \partial + eA \) with \( D = \partial + eA + \tilde{g} \partial P \), where \( P \) is the previously introduced Fourier-transform of the momentum distribution \( \mathcal{P} \), and the derivative \( \partial P \) is proportional to \( \mathbf{p}^\prime - 9 \).
\( \mathbf{p} = k \). \( e \) is some Standard Model coupling constant (not necessarily that of QED though this will be the case we consider later) and \( \tilde{g} \) is the coupling constant for the defects. Since the mass dimension of \( \lbrack P \rbrack = \lbrack E^2 \rbrack \), the dimension of \( \lbrack \tilde{g} \rbrack = \lbrack E^{-2} \rbrack \), and since we have only one scale of dimension mass at our hands, we use \( \tilde{g} = 1/\Delta M^2 \).

The defects are sprinkled over space-time according to the Poisson process described in section 2, but since we are considering the homogeneous case we will not treat the defects as individual events. We will instead replace the many single defects by a field \( P \) with which the standard model fields have a small interaction probability. That is, instead of interacting with defects in rare places, the particle interacts with the defect-field anywhere but with low probability.

In this limit, instead of having a sum over many defects, the probability to interact with the defect-field \( P \) is suppressed by the volume of the defect over the typical volume in which to find a defect, ie

\[
\tilde{g}^2 = \frac{1}{(\Delta M^2)^2} \frac{\sigma^+ \sigma^-}{\Delta k_{\perp}^2 L^4} .
\]

(24)

This is a similar approximation as we have made in [4] in that it is implicitly assumed here that scattering on defects happens often enough so that we can effectively describe it as a homogeneously occurring process. Concretely this means that this approximation is only good if the space-time volume swept out by a particle’s wave-function is large enough to contain a large number (\( \gg 1 \)) of defects. In this sense it is an effective description, not unlike in-medium propagation of photons, except that here we maintain Lorentz-invariance and thus the effects (have to) scale differently.

We note however that not only is the coupling constant dimensionful, but also goes to zero in the plane wave-limit, when \( \Delta p_{\perp} \to 0 \) and \( \Delta k_{\perp} \to \infty \). This behavior is an artifact of the normalization and dimension of the field \( P \) which contains a prefactor of \( \Delta k_{\perp} \). To avoid having to deal with quantities that diverge in the plane wave limit, we will thus shift this normalization factor from \( P \) into the coupling constant, so that both become dimensionless. We therefore define

\[
\hat{P}(x) := \frac{P(x)}{\Delta k_{\perp} \sqrt{\Delta M^2}} , \quad \hat{P}(k) := \frac{P(k)}{\Delta k_{\perp} \sqrt{\Delta M^2}} ,
\]

\[
g := \tilde{g} \Delta k_{\perp} \sqrt{\Delta M^2} = \frac{1}{\Delta M^2} \frac{1}{\sqrt{\Delta a^2 L^2}} .
\]

(25)

With this definition, the coupling to standard model fields is then \( D = \partial + eA + g\hat{P} \) and \( g \) is dimensionless and remains finite in the plane wave limit.

We will further in the following assume that the defects do not carry any standard model charges and do not change the type of the ingoing particle, ie the scattering is entirely elastic. In principle we could have a defect that changes not only the momentum but also the spin of the particle, but we will not consider this possibility here.

That the assignment of \( \hat{P}(x) \) to the defect is a bookkeeping device rather than the introduction of a new field is somewhat hidden in this notation, which looks like we have
introduced a new field. That $\hat{P}(x)$ is not actually an independent quantity can be seen most clearly from Eqs (6) and (4). The momentum $k$ that is assigned to the defect is a derived quantity from the particle’s incoming momentum $p$, and $\hat{P}$ is thus an operator acting on the same field that the gauge-covariant derivative acts on.

From this function of $\hat{P}(x)$ it is also clear that the scattering on the local defect, in the form that we have introduced it here, will break gauge invariance. This might not come as a surprise since the simplest Lorentz-invariant change to the dynamics of a massless gauge field is adding a mass term. Here, the underlying reason for the breaking of gauge invariance is that the defect itself is not gauged, i.e., the $k$ that is derived from the incoming momentum $p$ does not respect the gauge of the incoming particle.

One could fix gauge invariance by appropriately adding the gauge field to $k$ from the beginning on, but then one would obtain a nasty integral with gauge fields in exponents. Because of this complication together with the breaking of gauge invariance being not unexpected, we will accept that gauge invariance is broken. In the abelian case the additional terms for coupling a fermion field and its gauge-field to the defect are then of the form $g\bar{\Psi}(\partial\hat{P})\Psi$, $e^2g\partial A(\partial\hat{P})A$, $e^2g^2(\partial\hat{P})A(\partial\hat{P})A$.

With this prescription one can now calculate the scattering amplitudes for processes of interest. For each amplitude where one of the incoming particles previously scattered on the defect, one gets an additional integral over $d(a^2)d(M^2)dk_2dk_3$, or $dk_+dk_-d^2k_\perp = d^4k$ respectively. From this one obtains a cross-section or decay rate as usual.

Before we turn towards some examples, let us make a general observation. Since $\langle M^2 \rangle = 0$ and $\mathcal{P}$ does not vanish for $M^2 = 0$ if we assume a Gaussian distribution, it seems that a massless on-shell particle could remain on-shell and the momentum of the outgoing particle $p'$ would then be a multiple of the momentum of the ingoing particle $p$. However, unless the incident particle is virtual, the vertex factors all vanish because there is then no non-zero contraction of the momentum $k$ and the massless particle’s momentum or its transverse polarization tensor. Thus, if the incoming massless particle is real, the outgoing massless particle will necessarily be virtual.

For the massive particle, when $M^2 = 0$ the outgoing momentum must fulfill $p'^2 = (1 + a^2)m^2$ and thus the particle is virtual unless also $a^2 = 0$ in which case the particle only receives a transverse momentum from the defect that is equally distributed in its restframe. Alas, if $M^2 = a^2 = 0$, the particle did not acquire kinetic energy and thus a non-zero transverse momentum would violate the mass-shell condition. In other words, the massive particle too must be off-shell after changing its momentum by scattering on the defect.

Since the defects are sparse or the coupling constant is small respectively, we expect effects to be small and noticeable primarily for long-lived particles.
5 Constraints from observables

We will in the following make the assumption that $\langle a^2 \rangle \sim \Delta a^2 \sim 1$. Then we are left with two parameters, $\Delta M^2$ and $L$. (For some discussion on this and the other assumptions, see next section.)

Let us start with some general remarks. Since we are looking for constraints on (normally) stable particles that propagate over long times or distances respectively, we will focus on the QED sector of the standard model and on particles that we observe coming from distant astrophysical sources. This makes in particular the photon a sensitive probe for the presence of local defects.

If a photon scatters on the defect and becomes a virtual photon with mass $\sim \sqrt{3} M$, it can only decay into a fermion pair if $\sqrt{3} M$ is larger than twice the mass of the fermion. If $\sqrt{3} M$ is smaller than twice the electron mass, this leaves decay into a neutrino and antineutrino as the only option, which would necessitate a higher-order electroweak process and dramatically lower the cross-section. If $M$ is smaller than even the lightest neutrinos, the only option left for the virtual photon is to scatter on another defect.

The phenomenology thus depends significantly on the value of $\Delta M^2$ because it determines the relevance of possible decay channels through the typical range in the probability distribution $P$. There is always some possibility for $M^2$ or $a^2$ to take on very small or very large values, but the probability for this to happen is highly suppressed if $M^2$ is many orders of magnitude beyond $\Delta M^2$. For the rest of this section, we will only estimate the orders of magnitude for typical values of $M$, and will therefore from now on just write $M^2$ for $\Delta M^2$ and omit factors of order one.

In order to get a grip on the phenomenology, let us thus identify and focus on the parameter range that seems most interesting.

As mentioned previously, to make sense of the defects, we expect their typical volume to be small in comparison to the typical distance between the defects which is given by the density $\beta$. Or, in other words, we expect the effective coupling constant in the homogeneous limit, $g$, to be much smaller than one. Since we have already introduced one small parameter $\epsilon = l_p/L$ that is the (fourth root of) the density of the defects over a Planck density, the range $M^2 L^2 \sim 1/\epsilon \gg 1$ is of particular interest. If furthermore $L$ is in the same range as the length scale associated with the cosmological constant (a tenth of a millimeter or so), then $M$ is approximately in the TeV range. This is the parameter range that we will focus on in the following. In this parameter range, there is then no problem for the virtual photon to decay into fermions, and there are three processes of main interest:

1. **Photon decay**: The photon can make a vacuum-decay into a pair of electrons via a diagram as shown in Fig. right. This results in a finite photon lifetime, and leads
to electron-positron pair production. This process is similar to pair production in the presence of an atomic nucleus in standard QED.

2. **Photon mass**: The photon acquires an effective mass by scattering into a virtual photon on a first defect, and then converting back into a real photon by interacting with the defect a second time. This process is depicted in Fig. 2 left.

3. **Vacuum Cherenkov radiation**: An electron can emit a (real) photon after scattering on a defect, shown in Fig 2 right. This process is similar to Bremsstrahlung in standard QED.

![Image of photon mass and vacuum Cherenkov radiation](image)

Figure 2: Left: Contribution to photon mass from twice scattering on a defect. Right: Vacuum Cherenkov radiation, enabled by scattering on a defect.

### 5.1 Constraints on photon mass

The contribution to the photon mass come from terms of the form $e^2 g^2 (\partial \hat{P}) A (\partial \hat{P}) A$. Since the derivative in $\partial \nu \hat{P}$ by definition just creates the defect’s momentum $k$ and $k^2 = M^2$, this term produces a photon mass of approximately $m_\gamma \approx g M$. The best current constraint on the photon mass is $m_\gamma \leq 10^{-18}$ eV [11]. This leads to the bound

$$ML^2 \geq 10^{-12} \text{m}.$$  \hspace{1cm} (27)

### 5.2 Photon lifetime

If photons of the Cosmic Microwave Background (CMB) decay before reaching us, this would lead to deviations from the thermality of the CMB spectrum [12]. To obtain a constraint, let us therefore estimate the decay rate of photons from scattering on defects, induced by the process depicted in Fig 1 right. The amplitude for this process has the form

$$\mathcal{M} \sim -i \left( \frac{eg}{\Delta k_+ \Delta k_- \Delta k_{\perp}^2} \right) \left( \frac{e_\mu}{\sqrt{E}} \right) u(q) u(q') \times$$

$$\int d^4p' \int d^4k \frac{\eta_{\mu\nu}}{p'^2} \delta(p' - q - q') p'^2 \delta(p' - p - k) \hat{P}(k),$$  \hspace{1cm} (28)
where we have omitted some factors of $2\pi$ and $\sqrt{2}$.

In the order displayed, the amplitude (28) is composed of the coupling constant for the first and second vertex, the normalization of $\hat{P}(k)$, the polarization tensor of the incoming photon $e_\mu$ with (dimensionful) normalization, the spinor wave-functions of the outgoing electron and positron $u(q)$ and $u(q')$ with normalization omitted, the photon propagator, the first vertex and the second vertex, multiplied with the probability distribution and integrated over the momentum of the virtual photon and that of the defect.

Omitting the polarization and spinor structure, we can perform the integral over $q$ and estimate the integral over $\hat{P}(k)$ by evaluating the integrant at one standard deviation (times the variances, which cancel with the prefactor). This gives

$$M \sim -i\frac{eg}{\sqrt{E}} \delta(p' + k - q - q') \ ,$$

where $k$ is given by Eqs. 4 (recall that we replaced $\Delta M^2$ with $M^2$). From this we obtain an estimate for the decay rate

$$\Gamma(\gamma \rightarrow e^+e^-) \approx E\alpha g^2 \ .$$

The photon half-life time $\tau_\gamma$ is thus

$$\tau_\gamma \approx \frac{L^4 M^4}{\alpha E_0 z_0} \int_0^{z_0} da(z) \ ,$$

where $E_0$ is the photon energy at the present time, $z_0 \approx 1100$ the redshift at the time of production of the photon, and $a(z)$ is the redshift-dependent scale factor. With $E_0 \sim 10^{-2}\text{eV}$ for a typical CMB photon, the requirement that no more than about $10^{-4}$ CMB photons should have decayed at the present time leads to $\tau_\gamma \geq 10^{21}\text{s}$ and

$$LM \geq 10^8 \ .$$

This constraint however assumes that the density of defects remains constant in time, the case that was also considered in [13]. If the density of defects dilutes, then it would have been higher in the past, thereby decreasing the average decay time and tightening the constraint. It would need a more sophisticated model for the generation of defects to know how the density evolves in time. However, if one makes the ad-hoc assumption that $L(z) \sim L_0 a(z)$, $M = M_0$, then the constraint (on $L_0 M_0$) is by a factor of about $10^3$ stronger.

### 5.3 Cosmological vacuum opacity

Besides affecting the CMB spectrum, decaying photons will furthermore generally diminish the luminosity of faraway sources while at the same time not changing the redshift. Constraints on such a cosmological vacuum opacity have recently been summarized in [14]. However, the constraints from the CMB are stronger than the constraints.
from emission of distant astrophysical light sources, owing to the long travel-time of CMB photons and the excellent precision by which their spectrum has been measured.

Because the photon sources in this case are localized however, the astrophysical constraints on vacuum opacity would be interesting to look for effects of inhomogeneities that might be difficult to extract from the CMB data. Since here we do not consider inhomogeneities we will not quantify this constraint, but just mention that it could prove interesting in the more general case.

### 5.4 Heating of the CMB

As one expects from our previous discussion, the total decay rate of photons is finite due to the normalization procedure with a finite width of the $k$ distribution. If we take the plane wave limit, the total cross-section remains unmodified by construction but the differential cross section now includes arbitrarily high momenta. The typical momentum of the outgoing electron is then of the order $\Delta M^2 \Delta x$, where $\Delta x$ is the width of the incident particle's wave-packet. If we assume $\Delta x \sim 1/E_0$ (note that this is not an observer-independent statement), the momentum will be of the order of the Planck mass in the restframe where $\Delta x$ takes on this value (that we identify with the CMB or Earth restframe, the distinction does not matter for our estimate).

An electron of that high an energy however has a very short lifetime because it will undergo inverse Compton scattering on CMB photons. It has a huge $\gamma$-factor of about $10^{22}$ and thus an average mean free path $l$ of about \[ l \sim \frac{10^{-12}}{\gamma} \text{lightyears} \sim 10^{-4} \text{fm} \quad , \] which means we'll never see it; it will just deposit its energy into the CMB. At such high energies, even the outgoing photon will have a short mean free path because it scatters on other CMB photons via box diagrams.

Effectively, the two processes of photon decay and vacuum Cherenkov radiation therefore just heat up the CMB. Or rather, they prevent it from cooling. Since the universe contains more free photons than electrons, photon decay is the more relevant of these processes. This then allows us to make the following rough estimate. The energy that is deposited into the CMB by the photons’ scattering on defects should not significantly raise the CMB temperature. This means that the typical probability for the photon decay to happen, $g^2 \alpha$, should be less than the ratio of the initial photon’s energy over the outgoing photon’s energy $\Delta M^2/E_0$. This leads to the bound

\[ L^2 M \geq 10^{-2} m \quad , \] which is considerably stronger than the bound from photon masses.
5.5 Summary of constraints

The constraints on the parameter space from the previous sections are summarized in Figure 3.

![Figure 3: Summary of constraints. The coral (dark) shaded region is excluded. The peachpuff (light) shaded region indicates the stronger constraint from photon decay with the ad-hoc assumption that the typical distance between defects increases with the cosmological scale factor. The dotted (black) line indicates the length scale associated with the cosmological constant. The dashed (blue) line is the case $LM = 1$ and the solid (green) line is the case $LM = 1/\epsilon$.](image)

In hindsight, it is not entirely surprising that the bound from CMB heating is the strongest. The specific property, in fact the defining property, of the defects is that they violate momentum conservation and, due to Lorentz-invariance, we cannot put a hard cutoff on the magnitude of this momentum non-conservation. It is only bounded by the properties of the incident particle and the larger the spatial spread of the incident particle’s probability distribution, the more focused the defect and the larger the spread of the outgoing particle’s momentum.

As we also found in [4], the present bounds are about 10 orders of magnitude away from fully exploring the parameter range where the density of defects is comparable to the cosmological constant, which in the cosmological setting represents a natural range of parameters. However, the constraints that we have considered here are only estimates to gauge the promise of exploring space-time defects as a signature for quantum gravitational effects. With a more sophisticated model that takes into account background

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4It does not make sense to use values of $1/M$ smaller than shown in Figure 3 because then we come into the Planckian regime and the defects would be so dense they become the norm rather than the exception.
curvature, more of the existing cosmological data could be analyzed. This would open the possibility of finding evidence for space-time defects or at least deriving better constraints on their density.

6 Discussion

Let us first summarize the assumptions we have made that can in principle be relaxed. We have assumed that the defects don’t carry quantum numbers, no spin or gauge charges. We have restricted the study of phenomenological consequences to the case where $M$ is larger than the electron mass. We looked at the parameter range $\langle a^2 \rangle \sim 1$. The latter assumption in particular could be modified. One could use $\Delta a^2$ to normalize the extension of the wave-packet into the third spatial direction in a similar way as the perpendicular directions. We also remind ourselves that we have worked in the plane-wave approximation, where $p_+ \gg \Delta p_+$. If this approximation is not good, then the width of the defects can have a different dependence on the momentum of the incident particle and the scaling of effects might change.

As previously mentioned in [4], a certain case of nonlocal defects effectively makes itself noticeable as a local defect. That will be the case when the nonlocal translation can occur in both directions between the same locations. In this case, a particle that makes a nonlocal jump to another location will be replaced at its point of departure with a different particle, making it appear like a non-elastic scattering on a local defect. The problem with this kind of scenario is that the probability for a particle to appear at a certain location would depend on the total volume of spacetime, past and future, where it could have originated from. In this case, it is then impossible to say anything about the interaction rates without first developing a model for the generation of defects in a time-dependent background.

Finally, let us investigate the difference between the approach discussed here and the one in [7]. In the model [7], interaction with the local defects is mediated exclusively by a scalar field. The probability distribution of the momentum that is assigned to the defect is not constrained by a requirement similar to our requirements $p \cdot k = M^2$ and $k^2 = a^2 M^2$. As a consequence, Lorentz-invariance necessitates that the defect be able to inject momenta from the full Lorentz-group, which is no longer normalizable. Thus there arises the need to introduce a cutoff on the momentum integration. While the model in [7] offers an concrete realization of coupling quantum fields to space-time defects, the need to eventually introduce a Lorentz-invariance violating cutoff defeats the point of requiring a Lorentz-invariant distribution and coupling to begin with. The more relevant difference between the two models is however that we have here assumed the coupling to appear as a contribution to the covariant derivative and not as an independent interaction vertex.

These approaches are presently the only existing models to describe space-time defects and the study of the effects is in its infancy. It is possible, in fact likely, that elements
of both approaches will turn out to be necessary for the development of more sophisticated models.

7 Summary

We have proposed a model for the scattering of particles on space-time defects that induce a violation of energy-momentum conservation. In the plane wave-limit, the energy-momentum non-conservation can become arbitrarily large due to Lorentz-invariance, but it remains bounded if one takes into account the finite widths of the incident particle’s wave-function. We have looked at various phenomenological consequences and estimated that the best constraints come from energy deposited by decaying photons into the cosmic microwave background.

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