Optical model parallel description of elastic, fusion and breakup cross sections for systems with weakly bound projectiles

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Abstract. A brief description is presented of the results obtained in recent years for the simultaneous analysis of elastic and fusion cross section data of nuclear reactions for several nuclear systems with weakly bound and halo projectiles. The method used in this description, consists of simultaneously determine the parameters of fusion $U_F$ and direct reaction $U_{DR}$ polarization potentials of Woods-Saxon geometric shapes, that fit the elastic and fusion data. As a matter of fact, $U_F$ is an energy dependent potential, with real $V_F$ and imaginary $W_F$ components, that is responsible for fusion reactions. Similarly, $U_{DR}$ is also energy dependent with real $V_{DR}$ and imaginary $W_{DR}$ parts, that accounts for direct reactions. A general finding for all the systems presented is that, the real and imaginary parts of the fusion potential and direct reaction potentials, are related by a dispersion relation and their energy dependence around and below the Coulomb barrier, show the so-called Breakup Threshold Anomaly. The effect of breakup reactions on fusion cross sections is studied by analyzing the separate effect of the absorption potential $W_{DR}$ and the fusion barrier rising produced by $V_{DR}$.

1. Introduction
Nuclear reactions involving weakly bound and halo projectiles have been the source of extensive investigations in recent years[1]. Since this type of projectiles have low threshold energies, thus can be split into different fragments even at low collision energies, hence large breakup and transfer yields have been measured at energies around and below the Coulomb barrier. Similarly, due to the appreciable breakup cross sections, other nuclear reaction mechanisms like elastic scattering and fusion, are strongly affected by this process[1, 2, 3]. As a matter of fact, several nuclear processes are present in reactions with weakly bound and halo nuclei, besides elastic scattering and inelastic scattering, fusion can occur in several forms, i.e., a) Direct Complete Fusion (DCF), when the whole weakly bound projectile fuses to the target without splitting, b) Sequential Complete Fusion (SCF) when the projectile splits into different fragments but all of these fuse to the target, c) Incomplete Fusion (ICF), when after the splitting, only some of the projectile fragments are captured by the target and others feed the continuum d) Non-Capture Breakup NCBU when all the fragments feed the continuum, i.e., none is captured and e) Nucleon Transfer when some neutrons or protons are transferred to the
target. Experimentally, it is very difficult to distinguish between the CF and SCF processes, so these are normally reported as complete fusion. Total fusion is composed of all the fusion mechanisms, \( \sigma_{TF} = \sigma_{DCF} + \sigma_{SCF} + \sigma_{ICF} \). Very few experimental works have been performed that report exclusive CF and ICF cross section measurements\cite{4, 5, 6, 7}, in these cases complete fusion is considered as the fusion of the total charge of the projectile and the target. Basically, the small threshold energy associated to the breakup process influences fusion in two different manners. First, through the static effect connected to the \( l = 0 \) Coulomb barrier, that is, the height and curvature of the Coulomb barrier might enhance or hinder fusion. Secondly, through the dynamical effect, associated to the strong couplings to different reaction channels, that produce polarization potentials that can be attractive or repulsive. These polarization potentials, in turn, can lower or rise the fusion barrier and consequently enhance or hinder the fusion cross section. Normally, only the combined static and dynamical effects on fusion has been investigated, however, recently it has been possible to separate the particular effect for weakly bound systems\cite{8, 9}.

In several recent works, the effect of breakup on fusion has been studied by determining the dynamical polarization potential that arises from couplings between the incident elastic channel to different reaction channels. This polarization potential \( U(r, E) \) has been calculated by a simultaneous fitting to elastic and fusion cross section data for various weakly bound systems\cite{10, 11, 12, 13, 14, 15, 16, 17}. As a matter of fact, in order to do the simultaneous fitting, the energy dependent polarization potential \( U(r, E) \) is split into a fusion part \( U_F(r, E) \) responsible to fusion couplings and a direct reaction part \( U_{DR}(r, E) \) related to any other direct reaction couplings. In turn, the fusion and direct reaction polarization potentials have real and imaginary potentials, \( V_F(r, E), W_F(r, E) \) and \( V_{DR}(r, E), W_{DR}(r, E) \) respectively, that are correlated by the dispersion relation\cite{18, 19}, i. e.,

\[
V_{F,DR}(R_{sa}, E) = \frac{1}{\pi} \mathcal{P} \int_0^{\infty} \frac{W_{F,DR}(R_{sa}, E')}{E' - E} dE',
\]

where \( R_{sa} \) is the strong absorption radius. This relation establishes a constraint on the energy dependence of \( V_{F,DR} \) and \( W_{F,DR} \) leading to sharp variations of these functions around the Coulomb barrier \( V_B \). For tightly bound systems, the polarization potentials \( V_F(E) \) and \( V_{DR}(E) \), that arise from couplings to fusion and bound state excitation channels, become attractive. This is due to a decreasing behavior of \( W_F(E) \) and \( W_{DR}(E) \) as the energy decreases below the Coulomb barrier. That is, as the energy decreases, fusion and direct reaction channels become eventually closed, an effect manifested by decreasing functions \( W_F(E) \) and \( W_{DR}(E) \). Consequently, this is accompanied by an increase attraction represented by \( V_F(E) \) and \( V_{DR}(E) \). This conjugated energy behavior between the real and imaginary parts of the polarization potentials has been termed as Threshold Anomaly (TA)\cite{18, 19, 20} and is a direct consequence of the causality principle. An important immediate effect of the dynamical attractive potentials \( V_F(E) \) and \( V_{DR}(E) \), is to lower the nominal Coulomb barrier, that is, the barrier \( V_B \) resulting from the static nuclear bare \( V_{bare} \) and Coulomb \( V_{Coul} \) potentials. Hence, fusion is enhanced at energies around and below the barrier energy.

A different situation happens when weakly bound nuclei are considered. In these cases the breakup channel remains open well below the Coulomb barrier. The direct reaction polarization potential should account for appreciable couplings to channels in the continuum even at low energies. Then, the absorptive direct reaction \( W_{DR} \) can not decrease since important breakup reactions occur around and below \( V_B \). Hence, from the dispersion relation, the real polarization potential \( V_{DR} \) that arises from couplings to these continuum states results repulsive. On the other hand, fusion cross sections become hindered, a fact that is accounted for, by a decreasing \( W_F \) (fusion channel progressively closes as the energy decreases towards the barrier \( V_B \)), which is accompanied by an attractive \( V_F \). The effect of non-negligible breakup reactions at low energies
on fusion cross sections is twofold, namely, the incident flux that is absorbed into breakup reactions which considered by $W_{DR}$ and the rising of the fusion barrier due to $V_{DR}$. This particular energy dependent behavior, in contraposition to reactions with tightly bound systems, is known as Breakup Threshold Anomaly (BTA)[21, 22, 23]. The BTA has been studied in a variety of systems involving weakly bound and halo projectiles with targets of different charges and masses. For instance $^3$Be with $^{27}$Al[24], $^{64}$Zn[23, 25], $^{144}$Sm[26, 27], $^{208}$Pb and $^{209}$Bi[28, 29]. $^{6,7}$Li in reactions with $^{208}$Pb[21, 30], $^{27}$Al[31, 32], $^{144}$Sm[33], $^{58,64}$Ni[34, 35], $^{59}$Co[36], $^{90}$Zr[37], $^{138}$Ba[23, 38], $^{28}$Si[39]. Halo systems such as $^8$B with $^{58}$Ni[14], and $^6$He impinging on heavy targets $^{209}$Bi[40] and $^{208}$Pb[41].

In this paper, a brief description of the results obtained in recent years, of the simultaneous analysis of fusion and elastic scattering angular distribution data for several nuclear systems with weakly bound projectiles. The basic idea is to determine the parameters of the real and imaginary parts of the fusion $U_F$ and direct reaction $U_{DR}$ polarization potentials by a $\chi^2$-analysis of the data. As will be described in the next sections, the calculated energy dependence of the real and imaginary parts of the fusion polarization potential $V_F$ and $W_F$ not only are linked by the dispersion relation, but also show a behavior consistent with the usual Threshold Anomaly. That is, the incident flux that is absorbed into the fusion channel, represented by $W_F(E)$ decreases as the energy also diminishes below the Coulomb barrier $V_B$. On the other hand, the associated virtual potential $V_F$, that arises from couplings between the incident elastic channel and the fusion ones, becomes attractive with a bell-shaped energy dependence centered around the barrier $V_B$. As a matter of fact, an important result of the present analysis is that, for all the systems studied, the calculated imaginary absorption potential $W_{DR}(E)$, which is the potential that accounts for the flux that is absorbed into direct reactions (basically breakup for weakly bound systems), does not decrease around and below the barrier, as does $W_F(E)$. Conversely, the determined dynamic direct reaction real polarization potential $V_{DR}(E)$ becomes repulsive around the barrier energy. Therefore, $V_{DR}(E)$ has the important effect of increasing the fusion barrier and thus hinders fusion. This particular energy dependent behavior of $V_{DR}(E)$ and $W_{DR}(E)$ is linked by the dispersion relation. Finally, the effect on fusion due to $V_{DR}(E)$, $W_{DR}(E)$ is investigated. First, the separate effect of either $V_{DR}(E)$ and $W_{DR}(E)$ is studied, then the combined effect is calculated. Since a detailed description of the methodology used in these calculations has been given in several papers, here only a very brief account is given. A complete description of the results of the calculations is given for some of the systems studied, but for other systems only the conclusions are mentioned.

2. Basic features of the model.

The total interaction between projectile and target is given by,

$$V(r, E) = V_{Coul}(r) - V_{bare}(r) - U(r, E),$$

(2)

where $V_{Coul}$ is the repulsive Coulomb interaction between the nuclei, $V_{bare}(r)$ is the energy independent average nuclear potential and $U(r, E)$ is the nuclear dynamic polarization potential that arises from couplings between the incident elastic channel to fusion and direct reaction ones. The polarization potential has real and imaginary parts, i.e.,

$$U(r, E) = V(r, E) + iW(r, E),$$

(3)

where, at the strong absorption radius $R_{abs}$, the energy dependent functions $V(E)$ and $W(E)$ are linked by the dispersion relation (Eq.(1)). The total absorption potential $W(r, E)$ can be split into fusion and direct reaction absorption parts,

$$W(r, E) = W_F(r, E) + W_{DR}(r, E).$$

(4)
A formal justification of why the total absorption potential $W$ can be divided into different absorption channels is given in Refs.[11, 42, 43]. Consequently, the total real polarization potential can also be split into fusion and direct reaction parts,

$$V(r, E) = V_F(r, E) + V_{DR}(r, E),$$

where $V_F(r, E)$ and $W_F(r, E)$ as well as $V_{DR}(r, E)$ and $W_{DR}(r, E)$ are related by a dispersion relation. Fusion and direct reaction cross sections are given by,

$$\sigma_{F,DR}(E) = \frac{2}{\hbar v} < \chi_\alpha^{(+)} | W_{F,DR}(E) | \chi_\alpha^{(+)} >,$$

where $\chi_\alpha^{(+)}$ represents the relative motion wave function between the nuclei and $v$ their relative velocity. Thus, the total reaction cross section is calculated by,

$$\sigma_R(E) = \sigma_F(E) + \sigma_{DR}(E).$$

The fusion potentials $V_F(r, E)$ and $W_F(r, E)$ are assumed of Woods-Saxon volume shapes, i.e,

$$V_F(r, E) = V_{0,F}(E)f_F(r) ; \quad W_F(r, E) = W_{0,F}(E)f_F(r),$$

where,

$$f_i(r) = \frac{1}{1 + \exp[(r - R_i)/a_i]}, \quad i = F, DR,$$

$V_{F,0}(E)$ and $W_{F,0}(E)$ are respectively the strengths of $V_F(r, E)$ and $W_F(r, E)$, $a_F$ the diffuseness and $R_F = r_F(A_p^{1/3} + A_F^{1/3})$, $r_F$ being the reduced radius parameter. Similarly, the direct reaction potentials have surface Woods-Saxon shapes,

$$V_{DR}(r, E) = -4a_{DR}V_{0,DR}(E)\frac{df_{DR}}{dr}, \quad W_{DR}(r, E) = -4a_{DR}W_{0,DR}(E)\frac{df_{DR}}{dr},$$

where $R_{DR} = r_{DR}(A_p^{1/3} + A_F^{1/3})$. $V_{0,DR}$ and $W_{0,DR}$ are the potential strengths, $a_{DR}$ the diffuseness and $r_{DR}$ the reduced radius parameters. The strengths, diffuseness and reduced radii of the potentials are fixed during the simultaneous fitting of elastic and fusion data. Clearly, the energy dependence of the potentials can be determined around the Coulomb barrier.

3. The Breakup Threshold Anomaly Investigation.

The energy dependence of the fusion and direct reaction dynamic polarization potentials has been studied for several systems with weakly bound projectiles[23-41]. Here, only a general discussion is given for two cases. Figures 1 and 2, show the results of the fusion and direct reaction dynamic polarization potentials for the systems $^8\text{B}+^{58}\text{Ni}$ and $^6\text{Li}+^{28}\text{Si}$. The values of $W_F(E)$ at $R_{sat} = 12.1$ fm, as obtained by the $\chi^2$-analysis of elastic and fusion data for $^8\text{B}+^{58}\text{Ni}$, shown by the full circles in Fig.1b, decrease as the energy is lowered below the Coulomb barrier $V_B = 20.8$ MeV. This means that the fusion channel becomes progressively closed. On the other side, the real fusion polarization potential $V_F(E)$ as obtained by the fitting calculation and shown by the full circles of Fig.1a, has an increasing-decreasing behavior as the energy is lowered just around $V_B$ and is attractive. The line that follows the behavior of $V_F(E)$ is the integration according to the dispersion relation of the linear functions given for $W_F(E)$(see details in Ref.[44]). So, $W_F(E)$ and $V_F(E)$ have an energy dependence consistent with the usual Threshold Anomaly as in the case of tightly bound systems. The obtained values for the direct reaction imaginary $W_{DR}(E)$ (squares in Fig.1b) are very stable at high energies but shows an
Figure 1. (a) Energy dependence of the real $V_{\text{DR}}$ and imaginary $W_{\text{DR}}$ fusion and direct reaction polarization potentials at the strong absorption radius $R_{sa}$ for the system $^{8}\text{B}+^{58}\text{Ni}$. Appreciable increase at the lowest energy. This is related to the fact that flux into breakup reactions continues to be important at low energies. The real potential $V_{\text{DR}}(E)$, that accounts for coupling effects to breakup channels, becomes repulsive and with increasing strength, as seen by the squares of Fig.1a. The corresponding line is that obtained by the dispersion relation using the parametrization assumed for $W_{\text{DR}}(E)$. Figure 2 shows the results for the system $^{6}\text{Li}+^{28}\text{Si}[15]$, where as before, $W_{\text{DR}}(E)$ increases just as the energy approaches the barrier, while $V_{\text{DR}}(E)$ is repulsive. Also, the fusion potentials show the usual threshold anomaly around the barrier energy. The energy dependence of the polarization potentials for these two systems, show that instead of the TA, the BTA is present. As is the case of many other weakly bound and halo systems investigated in recent years.

4. Effect of the direct reaction real polarization potential on the Coulomb barrier. The effect on the nominal Coulomb barrier $V_B$, due to the calculated dynamical fusion $V_F$ and direct reaction $V_{\text{DR}}$, real potentials, can be calculated from Eq.(2),

$$Re[V(r, E)] = V_{\text{Coul}}(r) - V_{\text{bare}}(r) - V_F(r, E) - V_{\text{DR}}(r, E),$$

(11)

where $Re[V(r, E)]$ is the real part of the potential $V(r, E)$ of Eq.(2). So, the attractive or repulsive effects of $V_F(r, E)$ and $V_{\text{DR}}(r, E)$ on the nominal Coulomb barrier can be studied. In figure 3, the effects on the nominal Coulomb barrier $(l=0)$ by the fusion $V_F$ and direct reaction $V_{\text{DR}}$ polarization potentials for the system $^{8}\text{B}+^{58}\text{Ni}$ at the lowest measured energy $E_{\text{c.m.}} = 18.17$ MeV is presented. The nominal Coulomb barrier (full line) is given by the Coulomb potential $V_{\text{Coul}}$ and the attractive average nuclear potential $V_{\text{bare}}$, which is determined at the highest energy where polarization effects are negligible. The barrier lowering effect of the attractive fusion potential $V_F(r)$ of Eq.(8) is shown by the dashed-line. However, when $V_{\text{DR}}$ is considered (dotted-line), the total effect is a net barrier increase. Thus, the incident flux faces a higher barrier for fusion, hence it is suppressed. This fusion suppression is dynamic, i.e., varies with the energy as seen in Fig.4, where the barrier rising effect decreases as the energy increases.
5. Effect of the real and imaginary direct reaction polarization potential on the fusion cross section.

The particular effect on the fusion cross section due to the lowering and rising effects of $V_F(r, E)$ and $V_{DR}(r, E)$ respectively, can be calculated by,

$$ R_{V_F, V_{DR}}(E) = \frac{\sigma_F(V_F(E), V_{DR}(E))}{\sigma_F(V_F = 0, V_{DR} = 0)} . \tag{12} $$

In the numerator, $\sigma_F(V_F(E), V_{DR}(E))$, two cases are considered, that when $V_F(E) \neq 0$ but $V_{DR}(E) = 0$, and both $V_F(E) \neq 0, V_{DR}(E) \neq 0$. The denominator, $\sigma_F(V_F = 0, V_{DR} = 0)$ is the
fusion cross section when both potentials are turned-off, that is, it represents the fusion cross section corresponding to the nominal barrier. \( \sigma_F(V_F(E) \neq 0, V_{DR}(E) \neq 0) \) corresponds to the actual calculation for fusion cross section resulting from the simultaneous analysis. Fig.5a shows the results for the \(^{8}\text{B}+^{58}\text{Ni}\) system. As seen, \( R_{V_F, V_{DR}} > 1 \) (full circles), i.e., fusion is enhanced by \( V_F \). When \( V_{DR} \) is considered, \( R_{V_F, V_{DR}} < 1 \), thus the net effect is of fusion suppression.

Similarly, the effect of breakup reactions on fusion due to barrier rising produced by \( V_{DR} \) and loss of incident flux into direct reactions accounted for by \( W_{DR} \), can be studied by,

\[
R_{V_{DR}, W_{DR}}(E) = \frac{\sigma_F(V_{DR}(E), W_{DR}(E))}{\sigma_F(V_{DR} = 0, W_{DR} = 0)}. \tag{13}
\]

Fig.5b shows the results, where three cases are calculated, \( V_{DR}(E) = 0 \) and \( W_{DR}(E) \neq 0 \) (circles) \( V_{DR}(E) \neq 0 \) and \( W_{DR}(E) = 0 \) (squares), and finally \( V_{DR}(E) \neq 0 \) and \( W_{DR}(E) \neq 0 \) (stars). Both, \( V_{DR} \) and \( W_{DR} \) suppress fusion, but the effect is stronger for \( V_{DR} \) and for low energies.

6. Summary.

Important results obtained in recent works, regarding the simultaneous \( \chi^2 \)-analysis of elastic scattering angular distributions and fusion cross sections for weakly bound systems have been discussed. In particular, the cases \(^{8}\text{B}+^{58}\text{Ni}\) and \(^{6}\text{Li}+^{58}\text{Ni}\) have been presented in detail. However, similar conclusions are valid for other systems, for instance, the weakly bound \(^{6}\text{Li}\) and \(^{7}\text{Li}\) on targets \(^{28}\text{Si}\) and \(^{58}\text{Ni}\), \(^{9}\text{Be}\) on \(^{208}\text{Pb}\) or the neutron halo \(^{6}\text{He}\) on \(^{209}\text{Bi}\). It has been shown that the Breakup Threshold Anomaly appears for these systems. This is a direct consequence of a non-decreasing direct reaction imaginary potential \( W_{DR}(E) \) (open channels) and the repulsive character of \( V_{DR}(E) \) at energies around and below the barrier \( V_B \). On the contrary, the fusion potentials \( V_F(E) \) and \( W_F(E) \) show the usual Threshold Anomaly. The effect on the fusion barrier due to the real parts \( V_F(E) \) and \( V_{DR}(E) \) has been studied, where a net fusion suppression is found, particularly at the lowest energies. Also, the effect on fusion by
Figure 5. (a) Effect on fusion due to barrier lowering and rising effects produced by the attractive $V_F$ and repulsive $V_{DR}$ real polarization potentials. (b) Effect due to direct reactions (basically breakup), on fusion cross sections from barrier rising ($V_{DR}$) and loss of flux ($W_{DR}$).

breakup reactions due to loss of flux and barrier rising, accounted for by $W_{DR}(E)$ and $V_{DR}(E)$ respectively, has been calculated. It has been found that both of these potentials contribute to fusion suppression.

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