Prospects for Measuring the Hubble Constant with Neutron-Star–Black-Hole Mergers

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Gravitational wave (GW) and electromagnetic (EM) observations of neutron-star-black-hole (NSBH) mergers can provide precise local measurements of the Hubble constant \(H_0\), ideal for resolving the current \(H_0\) tension. We perform end-to-end analyses of realistic populations of simulated NSBHs, incorporating both GW and EM selection for the first time. We show that NSBHs could achieve unbiased 1.5\%–2.4\% precision \(H_0\) estimates by 2030. The achievable precision is strongly affected by the details of spin precession and tidal disruption, highlighting the need for improved modeling of NSBH mergers.

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Introduction.—The current expansion rate of the Universe—the Hubble constant \(H_0\)—is at the heart of a significant cosmological controversy. Direct measurements in the local Universe by the SH0ES team’s Cepheid-supernova distance ladder [1] find \(H_0 = 74.03 \pm 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}\). This is discrepant at the 4.4-\(\sigma\) level from the 67.36 \(\pm\) 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1} value inferred from the Planck satellite’s observations of the cosmic microwave background (CMB) anisotropies, assuming the standard flat cosmological model [2].

There are two potential explanations for this discrepancy, the most exciting of which derives from the model-dependence of the CMB constraint: could the discrepancy be due to physics beyond the standard model? Despite extensive effort (e.g., Refs. [3,4]), consensus on a compelling theoretical explanation has not been reached. The more prosaic explanation posits undiagnosed systematic errors or underestimated uncertainties; however, despite multiple investigations of both the distance ladder [5] and CMB [6] datasets, no study has found incontrovertible evidence warranting a change of conclusions.

In the absence of conclusive evidence of systematic errors or consensus on an extended model, independent verifications of the two central measurements offer a promising route to resolving the tension. Independent verification of the CMB anisotropy constraints comes from recent inverse distance ladder datasets [7]. Local verification has, however, proven more challenging, with some alternative analyses supporting the SH0ES team’s findings [8] and others providing contradictory conclusions of varying significance [9], in some cases using the same data.

A direct, completely independent local measurement with percent-level precision is therefore needed to resolve the \(H_0\) tension. Combined gravitational-wave (GW) and electromagnetic (EM) observations of nearby compact-object mergers are ideal candidates to provide that measurement, yielding \(H_0\) estimates that depend on general relativity alone [10–33]. Thanks to their accompanying EM emission, the utility of binary neutron star (BNS) mergers is well established [12–25], but less attention has been paid to the potential contribution of as-yet undiscovered neutron-star-black-hole (NSBH) mergers with EM counterparts [13,16,34]. Using idealized, fixed-signal-to-noise simulations at indicative parameter values, Vitale and Chen [34] recently showed that catalogs of GW-selected NSBH observations may constrain \(H_0\) as well as BNSs, depending on the relative merger rates and BH spins. In particular, they showed explicitly that luminosity distance estimates could improve, as misaligned BH spins induce spin precession, helping break the degeneracy between the luminosity distance and inclination angle for some NSBH systems.

Here, we determine the \(H_0\) constraints realistic NSBH samples will achieve, by performing end-to-end analyses of simulated NSBH samples incorporating fully specified parent populations, combined GW and EM selection, and a complete noise treatment. We use state of the art GW waveforms [35,36] and EM outflow models [37], both calibrated to a suite of numerical relativity simulations, for
our GW and EM signals, and focus on the “A+” era of the mid-to-late 2020s, assuming an expanded GW network including LIGO India and KAGRA.

Simulations.—In this work we simulate the results of a circa-2025 GW detector network, consisting of LIGO A+, Virgo AdV+, KAGRA, and LIGO India [38,39] observing for $t_{\text{obs}} = 5 \text{ yr}$ with duty cycle of $\Delta_{\text{obs}} \approx 0.5$. We assume a constant rest-frame NSBH merger rate $\Gamma_{\text{fid}} = 610 \text{ yr}^{-1} \text{ Gpc}^{-3}$ (corresponding to the 90% upper limit of Ref. [40]), and cosmological parameters matching Ref. [2], with $H_0 = 67.36 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = -0.527$. Each simulation proceeds by drawing the total number of mergers from a Poisson distribution with mean $\lambda = \Delta_{\text{obs}} t_{\text{obs}} V_{\text{fid}}$, where $V$ is the redshifted volume calculated using a third-order cosmographic comoving volume element (see Supplemental Material [41] for details). To reduce computation time, the volume integral can be truncated at some redshift $z_{\text{max}}$ where there is negligible probability of even the loudest merger being detected: we find that $z_{\text{max}} = 0.44$ suffices for our setting. For our fiducial parameter set, the mean number of mergers $\lambda = 25160$; our particular Poisson draw yields a preselection total of 25 241.

For each merger, we draw a cosmological redshift $z$, assuming a constant source-frame rate [see, e.g., Eq. (28) of Ref. [24]], along with an isotropically distributed angular sky position, inclination angle phase, and polarization angle. We draw uniformly distributed BH masses from $P(m_{\text{BH}}) = U(2.5 M_\odot, 40 M_\odot)$, taking the upper limit from low-metallicity binary population synthesis simulations [42] (using solar metallicities would reduce this upper limit to $12 M_\odot$) and extending to low masses to reflect the detection of objects in the purported NS/BH mass gap [43]. NS masses are drawn from $P(m_{\text{NS}}) = U(1 M_\odot, 2.42 M_\odot)$, with an upper limit chosen to match that of the DD2 equation of state (EOS) [44]. Dimensionless BH and NS spin magnitudes are drawn from the uniform distributions $P(a_{\text{BH}}) = U(0,0.99)$ and $P(a_{\text{NS}}) = U(0,0.05)$ and are assumed to be isotropically oriented. Following Refs. [37,45], we use the component masses, NS compactness, and BH spins to calculate the baryonic mass ejected by each merger. This formula requires the assumption of a NS EOS (we again use DD2) and has been calibrated using simulations without precession due to misaligned BH spins. We use the same EOS to calculate tidal deformabilities for the NSs, and set the BH deformabilities to zero [46]. Finally, we generate a peculiar velocity $v$ for each merger from a zero-mean normal with a standard deviation of $500 \text{ km s}^{-1}$.

With the NSBH parameters in hand, we generate mock data for each merger and apply our selection criteria. To determine the impact of different physical effects on our results, we simulate two populations using different waveform approximants: the BNS-calibrated IMRPhenomPv2_NRTidal [35] and NSBH-specific SEOBNRv4_ROM_NRTidalv2_NSBH [36] (hereafter IMRPhenom and SEOBNR). We refer the reader to the Supplemental Material [41] (which includes Refs. [47–49]) for a complete discussion of the differences between the two waveforms. The SEOBNR waveform requires aligned or antialigned spins, so we set the transverse NS and BH spins to zero after sampling them isotropically (mimicking, in some sense, spins becoming aligned over time). For each merger, we generate a 32–sec segment of noisy (using spectra from Ref. [39]) GW data $\tilde{x}$ per waveform using the same random seed and a frequency range of 20–2048 Hz, considering it detected if the network signal-to-noise ratio (SNR) is at least $\rho_s = 12$. We assume that the GW detectors operate in concert with an EM follow-up program capable of detecting all mergers with ejecta mass greater than $m_{\text{ej}}^* = 0.01 M_\odot$, modeling for the first time a hybrid GW-EM selection function for NSBHs. This ejecta material is assumed to produce EM emission in the form of a gamma-ray burst, kilonova and/or afterglow, as opposed to the “baryonic” effect of Ref. [50]. Finally, we generate noisy measured redshifts and peculiar velocities by drawing from $P(z) = N(z,0.001)$ and $P(v) = N(v, 200 \text{ km s}^{-1})$, respectively. Of the 25 241 simulated mergers, 2477 (2954) are detected in GWs using the IMRPhenom (SEOBNR) waveform, 99 (75) of which have sufficient SNRs for $P(z) = N(z,0.001)$ and $P(v) = N(v, 200 \text{ km s}^{-1})$, respectively. The SNRs for SEOBNR waveforms are, on average, 5.9% larger than their IMRPhenom counterparts, resulting in the GW-detected SEOBNR sample containing ~500 more objects. (We hypothesise that this is due to the effects of generic spin precession on the IMRPhenom population. Differences in the lengths of GW signals within the detector frequency bands due to the two waveforms’ different merger frequencies would tend to boost the IMRPhenom SNRs.) Setting the transverse spins to zero for use with the SEOBNR waveform, however, has the side effect of reducing the typical ejecta mass [37] and hence the final GW + EM-detected sample.

The impact of our selection function is illustrated in Fig. 1, in which we plot histograms of our full population (dotted lines), GW-selected events (dashed lines), and GW + EM-selected mergers (colored bars) for a subset of our parameters. The prior curves are identical in both cases apart from the BH spin magnitudes, where zeroing the transverse spins has made the SEOBNR population’s distribution nonuniform. The primary impact of the GW SNR threshold is, as expected, to select nearby mergers; it also imparts very slight preferences for low mass ratios and prograde BH $z$ spins [51]. It is interesting to note that the GW-selected SEOBNR distance distribution is broader than that of IMRPhenom and peaked at slightly higher distances: this is a direct consequence of the SEOBNR injections’ systematically higher SNRs. Further, the presence of spin precession permits the detection of more edge-on IMRPhenom waveforms.
The ejecta-mass threshold (i.e., EM selection) strongly impacts the observed distributions. The GW + EM-detected distributions are shifted to even smaller distances, particularly for the SEOBNR waveform, as the low-mass-ratio systems which produce significant ejecta mass can only be detected nearby. There is a very strong preference for mass ratios under 10 (again, especially so for SEOBNR) and large spins [37,45], and the preference for positive $z$ spins is much more pronounced. (Populations with $\sim$ solar metallicities produce NSBHs with lower BH masses [42] and hence more GW + EM-detectable mergers.) As expected from Refs. [37,45,50], the bulk of detected systems have BH masses below $10 \, M_\odot$. The differences between the two waveforms’ GW + EM-selected distributions are slightly obfuscated by the small sample sizes, but the SEOBNR sample is shifted towards lower distances and mass ratios. As the SEOBNR mergers’ BH spin magnitudes are smaller than those of their IMRPhenom counterparts, they require smaller mass ratios to produce significant ejecta [37], and the resulting systems are harder to detect at distance. Our implementation of EM selection captures the current best understanding of the dependence on ejecta mass, but we note that a fully self-consistent model of EM selection does not yet exist. This selection does not, for example, incorporate any viewing-angle dependence (see Ref. [52] for a treatment in BNS mergers) or EM survey selection effects [53–56].

**Methods.**—The probabilistic inference of the Hubble constant from catalogs of compact object mergers has been described in detail in the literature [10,12–14,16,19,21–25,57,58]. In the following, we adopt a slight variant of the formalism set out in Ref. [24], whose Fig. 9 depicts a network diagram for the model we use to describe the data. (The only addition required to the network diagram of Ref. [24] is the dependence of the selection $S$ on an intrinsic parameter: the merger’s ejecta mass.) The precise posterior we evaluate is defined in the Supplemental Material [41]. We infer the parameters of this model in two parts, using two sampling methods. First, we process each merger individually in order to obtain the GW likelihoods marginalized over all parameters $\theta_i$ other than the luminosity distance $d$ to the merger ($\theta_i$ here comprising the $i$th merger’s component masses, spin magnitudes and orientations where used, inclination, polarization angle, NS tidal deformability, and time and phase at coalescence). We adopt priors identical to the distributions used in the generative model for all parameters other than the masses. Convergence is greatly improved by sampling chirp masses and mass ratios instead of component masses, and we therefore sample using interim priors that are uniform in these parameters (over the ranges permitted by our component-mass extrema), before importance sampling the outputs to reinstate our desired component-mass priors. The marginal GW likelihoods are sampled with the pypolychord nested sampler [59], wrapped by bilby [60], using 1000 live points and bilby’s marginalize_phase, time and_distance settings. Each 15 (11)-dimensional IMRPhenom (SEOBNR) sampling run takes 6–14 (4–6) days to complete on one Intel Xeon 2.7 GHz CPU.

Given the marginal GW likelihoods, we use No-U-Turn Sampling as implemented by the pystan package [61] to infer the cosmological and population parameters. To connect to the cosmological parameters, we adopt a third-order distance-redshift relation matching our volume element [62], with $H_0$ and $q_0$ allowed to vary but the jerk set to one. We assume a broad Gaussian prior on $H_0$, 

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**FIG. 1.** Distributions of a subset of parameters from our SEOBNR (top) and IMRPhenom (bottom) samples, as drawn from the prior (dotted), selected by GW SNR (dashed) and selected by GW and EM emission (colored histograms). The bins are colored by the fractional $H_0$ uncertainty the mergers within the bin achieve: the yellowest and lightest bins are most informative.
We therefore perform a Gaussian mixture model fit to each merger’s marginal distance likelihood using pomegranate [63]. We fit each likelihood with an integer grid of 2–10 mixture components, repeating 10 times at each grid point and selecting the best fit using the Akaike information criterion [64].

Finally, we must evaluate the expected number of detected mergers \( N \) at each sampled value of the cosmological and population parameters. We do so by resimulating the catalogs 100 times at each point of a \( 5 \times 5 \) grid in \( \{ H_0, q_0 \} \) assuming our fiducial rate, \( \Gamma_{\text{fid}} \), and interpolating the results using a 2D fourth-order interpolation. The dependence on the sampled rate is captured by multiplying the interpolation coefficients by \( \Gamma/\Gamma_{\text{fid}} \). The resulting 153 (193)-dimensional pystan inference runs take less than a minute to generate 20 000 well converged samples on a 3.1 GHz Intel Core i7 CPU. The set of true redshifts and peculiar velocities are uninteresting for the purposes of cosmology inference, and we marginalize over these parameters when quoting the results below.

**Results.**—Processing the simulated SEOBNR and IMRPhenom catalogs through our two-stage inference pipeline produces the cosmology and population parameter posteriors shown in the Supplemental Material [41]. In both cases, the recovered \( H_0 \), \( q_0 \), and rate posteriors are completely consistent with the input values, indicating, as expected, that the selection effects are correctly accounted for [24]. The 68% credible intervals on the near-Gaussian \( H_0 \) marginal posteriors are \( 68.8 \pm 1.6 \) km s\(^{-1}\) Mpc\(^{-1}\) for the SEOBNR sample and \( 66.5 \pm 1.0 \) km s\(^{-1}\) Mpc\(^{-1}\) for IMRPhenom. As the IMRPhenom sample contains 99 objects to the SEOBNR sample’s 75, we should therefore expect that the IMRPhenom sample’s \( H_0 \) posterior be roughly 13% narrower than that of the SEOBNR sample. The remaining reduction, therefore, reflects the ability for precessing spins to break the distance-inclination degeneracy [21,65–67]. This additional constraining power is equivalent to an approximate doubling of the catalog size.

The \( H_0 \) uncertainties we find for both waveforms are comparable to the current Cepheid-SN distance ladder precision [1]. NSBH populations—should they produce EM counterparts and occur at rates roughly matching our assumptions—will therefore strongly inform the outcome of the current \( H_0 \) tension, particularly when combined with accompanying BNS populations, likely of comparable size [21,23,24,34]. (We note that comparable \( H_0 \) precision had been hoped for by 2023 from catalogs of BNS mergers [21]. That timescale, however, now appears optimistic due to the lack of EM counterpart detections following GW170817.) The mergers are also informative about the deceleration parameter, \( q_0 \), shrinking its uncertainty from 0.5 to 0.32 or 0.27, depending on the waveform. This further implies that NSBH catalogs will be able to begin constraining parameters such as the matter density and dark energy EOS (in the context of \( \Lambda \)CDM and extended models), complementary to BBH results from higher redshifts [31,33,68]. The merger rates are recovered with roughly 10% precision [40,43,69,70].

To obtain a picture of the parameter combinations that are most important for the \( H_0 \) constraints, we return to Fig. 1. Here, the colors of the histogram bins indicate the fractional \( H_0 \) uncertainty the mergers within each bin attain, with the most-constraining bins colored yellow and the least-constraining blue. For both waveforms, the bulk of the \( H_0 \) constraining power comes from mergers out to roughly 700 Mpc, not just the very nearest (\( \approx 200 \) Mpc), loudest events. For the IMRPhenom mergers, all mass ratio bins less than \( \approx 8 M_\odot \) contribute equally, despite the frequency dropping rapidly with mass ratio. For the SEOBNR mergers, the constraints are instead driven by the lowest-mass ratio events [65,66]. From the IMRPhenom spin panels, it is clear that highest-spin events constrain \( H_0 \) most strongly, with the full \( H_0 \) constraint coming almost entirely from the highest-spin (and most populated) bin. The SEOBNR constraint, on the other hand, is sourced by events with a broader range of spins, though this is likely driven by the prior.

We further highlight the importance of precession in breaking the distance-inclination degeneracy in Fig. 2. In the first three panels we plot distance and inclination constraints for a selection of mergers when using the SEOBNR (red, filled) and IMRPhenom (grey, filled) waveforms. For the first two mergers, detected with high SNR, the long degeneracies present in their SEOBNR posteriors are almost completely broken when using the IMRPhenom waveform, in which the spins precess. We illustrate this point further by re-running the IMRPhenom case assuming aligned spins, having set the transverse spins to zero. These results are overlaid as dashed dark red contours. Both distance-inclination degeneracies blow up, increasing the distance uncertainties by a factor of over three, with commensurate consequences for the mergers’ ability to constrain \( H_0 \). In the third panel, we show equivalent posteriors for the IMRPhenom merger whose BH spin is closest to being aligned: the effect of switching waveforms here is markedly reduced. The impact on the population level is clear from the right-hand panel of Fig. 2, in which we plot the distributions of individual mergers’ fractional errors on distance (dashed) and \( H_0 \) (solid) when using the SEOBNR (red) and IMRPhenom (gray) waveforms. The SEOBNR distributions are shifted to significantly higher errors than their IMRPhenom counterparts, despite the SEOBNR mergers typically having higher SNRs. The smallest percentage error for any individual merger is 2.8% for the IMRPhenom case and 6.1% for...
SEOBNR; the medians are 13.2% and 17.3%, respectively. The $H_0$ constraints imparted by both “golden” and normal events are therefore stronger when spins precess significantly. Finally, we note from the lower limits of the dashed curves that peculiar velocity and redshift uncertainties strongly suppress the constraining power of the nearest and loudest events.

Conclusions.—In this Letter, we present the results of the first end-to-end inference of $H_0$ from realistic simulated catalogs of NSBH mergers incorporating GW and EM selection effects. The precision we should expect from such catalogs is very promising for resolving the current $H_0$ tension, with five years of A+ era observations yielding $H_0$ uncertainties of 1.5%–2.4%. We find, however, that the detailed physics of the NSBH waveforms strongly impacts the achievable precision. Using the SEOBNRv4-ROM_NRTidalv2_NSBBH waveform with nonprecessing BH spins results in boosted SNRs, and an increase of $\sim500$ GW-detected NSBHs. However, including precessing spins using the IMRPhenomPv2_NRTidal waveform markedly increases the typical ejecta mass and hence the number of combined GW + EM detections. Critically, precessing spins also break the distance-inclination degeneracy in the resulting GW parameter posteriors, yielding a significant improvement ($\sim40\%$ after accounting for differing catalog sizes) in the resulting $H_0$ constraint. Our results strongly highlight the need for improved modeling of NSBH signals in both gravitational waves (see, e.g., Ref. [71]) and the electromagnetic spectrum.

The Python simulation and inference software developed for this analysis are publicly available [72].

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