1. Introduction

How close the strip profile, or the distribution of the thickness of the rolled product along the width direction in flat rolling, is in compliance with the desired profile is extremely important in the evaluation of the quality of the product. Naturally, engineers in modern hot and cold rolling mills are keenly interested in achieving the desired profile through precision process control. However, precision process control is a difficult task, since the strip profile is not only influenced by the distribution of the no-load clearance between the top and bottom work rolls (which may be termed no-load roll gap profile), but also by the complex nature of deformation of the strip, deformation of the work rolls, and deformation of backup rolls which are three-dimensional and interdependent.

In the past, numerous investigators have proposed various models for the prediction and control of the strip profile. Their models are mostly based on three constituent elementary models—an elementary model for the analysis of the deflection of the work rolls and backup rolls, such as the slit beam model, an elementary model for the analysis of roll flattening, such as a semi-infinite solid model, and an elementary model for the prediction of the roll force distributions along the strip width, such as a slab model. Then, the elementary models are integrated so as to satisfy all the mechanical contact conditions. However, the range of process conditions with which the desired prediction accuracy could be achieved is rather limited, due to many simplifying assumptions inherent to the elementary models. Recently, a remarkable deviation from the conventional approach is made, in which the three-dimensional finite element (FE) models in order to enhance the prediction accuracy.

As far as the prediction accuracy is concerned, a rigorously formulated FE-based process model would perhaps be the best choice. In this paper, we present a fully integrated finite element process model for the coupled analysis of the mechanical behavior of the strip, work roll, and backup roll in a four-high mill. A series of process simulation are conducted and the results are compared with the measurements made in hot and cold rolling experiments.

2. FE Model for Analysis of Rigid-viscoplastic Deformation of the Strip

Consider the strip domain $\Omega$ with the $h_t$ traction prescribed on a part $\Gamma_t$ of the surface $\Gamma$ and with the velocity $u_t$ prescribed on a part $\Gamma_u$. Let $\Gamma_r$ be the remainder of the surface and assume that $\Gamma_r$ represents the roll–strip interface. The boundary value problem associated with the steady-state plastic deformation of the strip may be given as:

Find the velocity field $\mathbf{u}$ and hydrostatic pressure $p$ satisfying...
Equation of motion (neglecting all the acceleration terms):

$$\sigma_{ij} + f_i = 0 \quad \text{(1)}$$

Constitutive relationship:

$$\sigma_y = -p\delta_y + \sigma_{0y} \quad \text{(2)}$$

$$\sigma_y = \frac{2\sigma}{3\varepsilon} \frac{1}{\varepsilon_y} \quad \text{(3)}$$

Where the flow stress $\sigma$ of the strip deforming at elevated temperatures may be represented by

$$\sigma = f(T) \quad \text{(4)}$$

Incompressibility condition:

$$u_i = 0 \quad \text{(5)}$$

Traction boundary conditions:

$$\sigma_{nj} = h_i \quad \text{on } \Gamma_k \quad \text{(6)}$$

Velocity prescribed boundary conditions:

$$u_i = \bar{u}_i \quad \text{on } \Gamma_u \quad \text{(7)}$$

- Boundary Conditions at the Roll–Strip Interface—I. Normal Stress:

A penalty algorithm was applied for the determination of the normal stress distributions at the roll–strip interface. For this purpose, the velocity prescribed boundary condition

$$u_i = \bar{u}_i \quad \text{on } \Gamma_u \quad \text{(8)}$$

was replaced by

$$\sigma_y = \frac{2\xi_i}{\varepsilon_y} (u_i - \bar{u}_i) \quad \text{on } \Gamma_u \quad \text{(9)}$$

where $\sigma_y$ denotes the normal stress, $u_i$ and $\bar{u}_i$ denote the normal component of the velocity vector of the strip and that of the roll, respectively. Considering that the magnitude of $\sigma_y$ should be finite in order to maintain stress equilibrium, it is evident that the contact condition represented by Eq. (8) is strictly enforced as $\xi_i$ becomes very large.

- Boundary Conditions at the Roll–Strip Interface—II. Tangential Stress:

The tangential stress distributions at the roll–strip interface due to friction may be modeled by the Coulomb friction model. It should be noted that the sticking zone is not known a priori before solving the boundary value problem. An approach that may be conceived is to develop and apply a special trial and error procedure to locate the sticking zone. Another possible approach, which is adopted in this investigation, is to replace the Coulomb friction model by a hypothetical model. The model closely resembles the Coulomb friction model but bears no difference between the sticking zone and the sliding zone as far as the mathematical expression of the frictional stress at each zone is concerned, so that the need for locating the sticking zone may be removed. A step-like function which varies continuously with the magnitude of $\Delta u$, the velocity vector of the strip relative to that of the work roll, is used for this purpose, as follows:

$$\sigma_t = -\mu |\Delta u| \quad \text{(10)}$$

where $\sigma_t$ denotes the tangential stress acting along the direction of $\Delta u$, and

$$g(|\Delta u|) = \frac{2}{\pi} \tan^{-1} \left( \frac{|\Delta u|}{a} \right) \quad \text{(11)}$$

$a$ a very small constant.

The inverse tangent function can closely approximate the Coulomb friction at the sliding zone (which is, $g(|\Delta u|) = 1$ when $|\Delta u| > 0$), as well as the Coulomb friction at the sticking zone (which is, $0 \leq g(|\Delta u|) \leq 1$ when $|\Delta u| = 0$), provided the constant $a$ is sufficiently small.

An integral statement equivalent to the above boundary value problem may be given as follows: Among all the velocity fields satisfying the velocity prescribed boundary conditions, find $\mathbf{u}$ satisfying the following variational equation for an arbitrary function field $\mathbf{w}$:

$$\int_I \sigma_{ij} \omega_i d\omega - \int_I -\xi_i \dot{\varepsilon}_{ij} \omega_i d\Omega - \int_I f_i \omega_i d\Omega$$

$$- \sum_{ij} \int_{\Gamma_i} h_i \omega_i d\Gamma - \int_{\Gamma} \left( -\xi_0 (u_i - \bar{u}_i) \omega_i d\Gamma \right)$$

$$- \int_{\Gamma} \mu \xi_0 (u_i - \bar{u}_i) g(|\Delta u|) \omega_i d\Gamma = 0 \quad \text{(12)}$$

where $\omega_i = (\omega_{i1} + \omega_{i2})/2$ and $\omega_{i} = \omega_{i1} - \omega_{i2}/3$. Note that the hydrostatic pressure $p$ was approximated by $p = -\xi_i \dot{\varepsilon}_{ij}$, which led to an implicit implementation of the incompressibility condition.

In the finite element approximation, $\mathbf{u}$ and $\mathbf{w}$ are approximated by

$$\mathbf{u} = N_i (\mathbf{x}) \mathbf{U}_i \quad \text{(13)}$$

$$\mathbf{w} = N_i (\mathbf{x}) \mathbf{W}_i \quad \text{(14)}$$

where $N_i$ are FEM basis functions, and $\mathbf{U}_i$ and $\mathbf{W}_i$ denote $\mathbf{u}$ and $\mathbf{w}$ evaluated at nodal point $L$, respectively. Substituting Eqs. (13) and (14) into Eq. (12) results in a set of non-linear algebraic equations that may be solved for $\mathbf{U}_i$, either by direct iteration method or by Newton–Rhapson method.

### 3. Streamline Tracing and Free Surface Correction

The success of a steady-state approach for the precision analysis of the viscoplastic deformation of the strip in general heavily depends on the capability of predicting the streamlines and relocating the nodal points on the streamline. Streamline tracing may be efficiently performed if a domain mapping technique\textsuperscript{29} is used, in which a streamline is traced not in the actual element but in its parent domain. Also important is the capability of correcting the streamlines so as not to violate the fundamental requirement that a portion of the boundary of the strip should be in contact with the work roll and the streamline should not intersect the wok roll. Correction of a streamline may start from correcting the position at which the streamline would touch the roll. Details regarding the validity of the technique may be
found in the reference. On completing streamline tracing and correction, simulation is performed with the updated strip mesh, and the procedure is repeated until convergence.

4. FE Model for Analysis of Thermo-elastic Deformation of the Roll

Consider an undeformed roll \( \Omega \) with the traction \( h_i \) prescribed on a part \( \Gamma_i \) of the surface \( \Gamma \) and with the displacement \( u_i \) prescribed on a part \( \Gamma_i \). Let \( \Gamma_i \) be the remainder of the surface and assume that \( \Gamma_i \) represents the roll–roll interface where friction is negligible. The boundary value problem associated with the thermo-elastic deformation of the roll can be given as follows:

Find the displacement field \( u \) satisfying

Equation of motion (neglecting all the acceleration terms):

\[
\sigma_{ij,i} + f_i = 0 \quad \text{.................................(15)}
\]

Constitutive relationship:

\[
\sigma_{ij} = ku_{ij} + 2\mu \varepsilon_{ij} + 3k\alpha \Delta T \delta_{ij} \quad \text{.................................(16)}
\]

where \( k \) and \( \mu \) are bulk modulus and shear modulus, respectively, \( \alpha \) is the coefficient of linear thermal expansion, and \( \Delta T \) denotes the temperatures difference from the reference temperature.

Boundary conditions:

\[
s_{ij} = h_i \quad \text{on} \quad \Gamma_i \quad \text{.................(17)}
\]

\[
u_i = \bar{u}_i \quad \text{on} \quad \Gamma_i \quad \text{...............(18)}
\]

\[
u_i = -\xi(u_i - \bar{u}_i) \quad \text{on} \quad \Gamma_i \quad \text{...............(19)}
\]

where \( \bar{u}_i (=0) \) in Eq. (19) denotes the normal component of the displacement of the neighboring roll on \( \Gamma_i \).

A weak form equivalent to the above boundary value problem may be given as follows: Among all the displacement fields satisfying the displacement prescribed boundary conditions, find one which satisfies the following variational equation for an arbitrary function field \( \omega \),

\[
\int_\Omega \sigma_{ij} \omega_{ij} d\Omega - \int_\Omega \frac{\sigma_{kj}}{3} \omega_{ij} d\Omega - \int_\Omega f_i \omega_i d\Omega
\]

\[-\sum_{i} \int_{\Gamma_i} h_i \omega_i dI - \int_{\Gamma_i} \varepsilon_i(\bar{u}_i - \bar{u}_i) \omega_i dI = 0 \quad \text{.................................(20)}
\]

The finite element approximation of the variational equation may be conducted in the same manner as described in Sec. 2.

5. Treatment of Mechanical Contact

When an object is in contact with other one and both of them are deforming, the shape of the interface of the deformed state is not known a priori and consequently the mechanical boundary conditions at the interface can not be prescribed. Examples of such an interface are strip-work roll interface and roll-roll interface. An iterative solution scheme may be adopted to resolve this problem, as follows:

Step 1. Perform the mechanical analysis of object A, treating the interface as and assuming that object B, with its present configuration, is a rigid body (initially, the present configuration is an undeformed one). Predict the resulting stress distributions at the interface.

Step 2. Perform the mechanical analysis of object B, treating the interface as a traction prescribed boundary where the stress distributions at the interface predicted in step 1 are applied.

Step 3. Update the deformed configuration of object B, as follows:

\[
x_i = x_i + aU^n_i + (1-a)U^{-1}_i \quad \text{.....................(21)}
\]

where \( x_i \) and \( X_i \) denote the nodal point position vector at the deformed and at the undeformed configuration, respectively, \( U_i \) denotes the nodal point displacement vector, \( i \) denotes the iteration number, and \( a \) is a damping coefficient, which is introduced to facilitate solution convergence by preventing excessive deviation from the old configuration. \( A \) may be selected from the range \( 0 \leq a \leq 0.5 \).

Steps 1–3 are repeated until convergence is achieved for each object. For the convergence criterion, we may use:

\[
\left| \frac{U_i - U^n_i}{U_i} \right| \leq \varepsilon \quad \text{.................................(22)}
\]

where \( U_i \) denotes the nodal point displacement vector if the object is a roll, and the nodal point velocity vector if the object is a strip.

Details regarding the validity of the present scheme for a two dimensional deformation problem may be found in the reference.

6. Calculation of Normal Vector and Rotational Velocity Vector on the Surface of a Deformed Object

Proper evaluation of the normal vector at a nodal point on the surface of a deformed object B is important for the determination of the normal vector at a nodal point on the surface of the object A at the interface between the object A and the object B. (For example, B is the work roll and A is the strip in the coupled analysis of the strip and work roll). Also important is the evaluation of the velocity vector due to rotation at the surface of a deformed object, such as a deformed work roll. A deformation gradient-based approach may be taken for this purpose, as follows:

Let \( x_i \) and \( X_k \) denote the deformed and the initial configuration, respectively. Then,

\[
x_i = x_i(X_k) \quad \text{.................................(23)}
\]

\[
dx_i = x_i, k dX_k \quad \text{.................................(24)}
\]

\[
X_{ik} = x_{ik} \quad \text{.................................(25)}
\]

where \( x_{ik} \) is the deformation gradient tensor.

It may be shown that the normal vector \( n_i \) at a nodal point on the deformed surface \( ds \) is given by

\[
n_i ds = J dS \quad X_{ik} \quad \text{.................................(26)}
\]

where \( N_i \) is the normal vector at the nodal point on the undeformed surface \( dS \), and \( J = [X_{ik}] \). Consequently, \( n_i \) may be calculated from

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the velocity vector \( u_i \) at a nodal point on the deformed surface may be calculated from
\[ u_i = x_{i,j} V_j \]
where \( V_j \) is the velocity vector at the nodal point on the surface before deformation.

It is evident from Eqs. (27) and (28) that it is necessary to calculate \( x_{i,k} \) at a nodal point. However, \( x_{i,k} \) is discontinuous across the boundary of the two adjacent element surfaces. In order to resolve this problem, we may assume
\[ x_{i,k}^N = x_{i,k}^A (X_i^N - X_i^A) \]
where \( A \) denotes the nodal point at which the deformation gradient is to be calculated, and \( N \) denotes any nodal point on each element that includes the nodal point \( A \).

Applying the least square method, the average value of \( x_{i,k}^A \) may then be calculated from
\[ P_{id} = x_{i,k}^A Q_{id} \]
where
\[ P_{id} = \sum_N (x_i^N - x_i^A) (X_i^N - X_i^A) \]
\[ Q_{id} = \sum_N (X_i^N - X_i^A) (X_i^N - X_i^A) \]
Substituting Eq. (30) into Eqs. (27) and (28) leads to
\[ n_i = \text{normalize}(N_i X_{i,j}) \]
\[ u_i = P_{id} Q_{id}^{-1} V_M \]

7. Integrated FE Process Model

On the basis of the proposed iterative scheme for the treatment of the mechanical contact, the basic FE models may be integrated, in many different ways, to form a process model for the coupled analysis of the deformation of the rolls and strip in various types of mills. Figure 1 shows a computational sequence that can be taken for a four-high mill (see Fig. 2), if one can consider only a quarter system of a four-high mill because of the process symmetry. Note that the strip is regarded as an object A and the work roll is regarded as an object B for the treatment of contact between the strip and work roll, while the work roll is regarded as an object A and the backup roll is regarded as an object B for the treatment of roll-to-roll contact. Also, note that the work roll-to-backup roll iteration is completed before returning to the analysis of the strip deformation. The present computational sequence is believed to be optimal in terms of computational efficiency, since it is the analysis of the strip deformation that would consume most of the CPU time. Further, it can easily be extended to solve a full system, which may be required when dealing with a work roll-shift mill or a pair-cross mill.

8. Results and Discussion

Figures 3 and 4 show the typical finite element mesh for the backup roll, work roll, and the strip used in process sim-
elements are used for each object, especially near the contact surface. These problems may be resolved by performing a series of numerical test until the solution is not affected by further increase in the zone length as well as in the mesh density. The meshes shown in Figs. 3 and 4 reflect an outcome of such an effort. Also, it is revealed from the numerical test that a safe upstream (and downstream) zone length is at least 40 times larger than the bite zone length.

Regarding the criterion for solution convergence, \( e/H < 0.0005 \) is used for the rolls, while \( e/H < 0.00001 \) is used for the strip. The average number of iteration required to achieve solution convergence is 6, where 5–8 roll-to-roll iteration is performed in each iteration. The computation takes approximately 25 CPU minutes with an engineering workstation (Compaq XP1000).

As a preliminary step, the prediction accuracy of the constituent elementary models of the conventional approach is examined through comparison with the predictions from the finite element method. The process conditions and boundary conditions employed for the investigation are shown in Table 1.

Regarding the roll axis deflection, the difference between the predictions from the beam model and those from the finite element method is found to be less than 0.1 %, for \( L/D = 11.64 \), but greater than 5.5 % for \( L/D = 2.91 \), as shown in Fig. 5. It may be seen that predictions from the beam model tend to overestimate the deflection. Considering that \( L/D = 2.91 \) is close to the aspect ratios of the actual work rolls used in industrial hot rolling, and that the aspect ratios of the backup rolls are even smaller, it is evident that, for the precision analysis of roll axis deflection, the application of the beam model should be confined to some special cases with a sufficiently large aspect ratio.

For the prediction of the deformed roll profile at the bite zone via the finite element simulation, note that three different boundary conditions are tested, as shown in Table 1. B.C. 1 assumes that the end of the roll shaft is free while four nodal points on the cross-section at the roll center are clamped to prevent the rigid body motion, B.C. 2 allows the rotation of the end of the roll shaft in the roll chock, and

| Table 1. Process and boundary conditions for FE process simulation (I). |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| case | D (mm) | L (mm) | W (mm) | \( L/D \) | \( E/GPa \) | \( v \) | \( P_0 \) (KN) | \( P_s \) (KN/m²) | Id (mm) |
|------|--------|--------|--------|---------|--------|--------|-------------|-------------|--------|
| a    | 758    | 2200   | 1000   | 2.910   | 2.2    | 160    | 0.275       | 9.09        | 0.2578  | 77.582 |
| b    | 378    | 2200   | 1000   | 5.820   | 2.2    | 160    | 0.275       | 9.09        | 0.2578  | 77.582 |
| c    | 378    | 4000   | 2000   | 11.64   | 2.2    | 160    | 0.275       | 9.09        | 0.2578  | 77.582 |

Note. Roll crown is zero.
without the rotation of the roll chock, and B.C. 3 assumes that the end of the roll shaft is completely clamped. For the process conditions, case a in Table 1 is selected, since they reflect an actual situation occurring in an industrial hot strip mill. As may be seen from Figs. 6–8, the deformed roll profile predicted from B.C. 2 and B.C. 3 are similar, but prediction from B.C. 1 is quite different from the others, perhaps due to the unrealistic clamping of some of the nodal points on the central cross-section, indicating that either B.C. 2 or B.C. 3 may be adopted for the finite element simulation. Regarding the deformed roll profile at the bite zone, –250 μm is predicted at the strip edge with B.C. 2, as may be seen from Fig. 7, while –225 μm is predicted at the strip edge with B.C. 3, as may be seen from Fig. 8, indicating that the difference resulting from the different choice of the boundary condition is 25 μm. Assuming that the finite element model is precise and considering that the actual boundary condition in an industrial mill may be found between B.C. 2 and B.C. 3, it may be expected that the average difference between the prediction from the finite element model with either B.C. 2 or B.C. 3 and the measurement is approximately 12.5 μm.

In the conventional approach, the deformed roll profile at the bite zone is obtained by adding the roll axis deflection profile predicted from the beam model to the roll indentation profile at the bite zone, predicted from a semi-infinite solid model, such as one proposed in the reference. A substantial difference between the prediction from the finite element method and the prediction from the conventional approach may be seen from Figs. 7 and 8. When B.C. 2 is used, the difference is 50 μm at the strip edge, as may be seen from Fig. 7, while the difference is 75 μm when B.C. 3 is used, as may be seen from Fig. 8. Assuming that the finite element model is precise and considering that the actual boundary condition in an industrial mill may be found between B.C. 2 and B.C. 3, it may be expected that the average difference between the prediction from the conventional approach and the measurement is approximately 62.5 μm.

To investigate the validity of the integrated FE process model as applied to hot rolling, predictions from process simulation are compared with the measurements made by
The process conditions and boundary conditions are summarized in Tables 2 and 3, as cases 1–3. The predicted strip profiles are in excellent agreement with the measurements, as illustrated in Fig. 9.

The effect of roll crown on the strip profile can be observed from Fig. 10. Significant variation of the strip profile with roll crown is due to the fact that the distribution of roll force in the width direction at the roll–strip interface as well as at the roll–roll interface are dramatically affected by the roll crown, as shown in Figs. 11 and 12, while the total roll force is little affected. Regarding the indentation profile of the work roll, the indentation depth gradually increases with the distance from the roll entrance and then decreases, with the maximum indentation depth being found near the neutral point, as shown in Fig. 13. To investigate the validity of the integrated FE process model as applied to cold rolling, predictions from process simulation are compared to the measurements made in

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**Table 2.** Process and boundary conditions for FE process simulation (II).

| Case | H | W (mm) | H (mm) | b (mm) | V (m/min) | H | RG | WRC (µm) | BF (mm) | FT/BT | PC (°) |
|------|---|--------|--------|--------|-----------|---|----|-----------|--------|-------|--------|
| 1    | H | 251    | 3      | 2.1    | 55        | 0.25 | 1  | 100       | 0      | 0     | 0      |
| 2    | C | 320    | 5      | 0.792  | 12        | 0.13 | 2  | 0         | -12.3  | 0     | 0      |
| 3    | C | 300    | 1      | 0.792  | 12        | 0.13 | 2  | 0         | -12.3  | 0     | 0      |
| 4    | H | 320    | 5      | 0.792  | 12        | 0.13 | 2  | 0         | -12.3  | 0     | 0      |
| 5    | C | 400    | 2      | 1.4    | 10        | 0.03 | 4  | 0.5       | 10/5   | 0.5   | 1.5    |

Note: (1) H (hot/Cold rolling), FS (flow stress), W (strip width), H (strip inlet thickness), b (strip outlet thickness), V (rolling speed), RG (roll geometry). Refer Table 2, WRC (work roll crown), BF (work roll bender force), PC (pair cross angle), FT/BT (front/back tension), µ (coefficient of Coulomb friction).

(2) Flow stress equation:

\[\sigma = \begin{cases} 
0.157\varepsilon^{0.25} & (I) \\
0.53(0.001 + \varepsilon)^{0.25} & (II) \\
0.157\varepsilon^{0.25} - 0.212\varepsilon^{0.25} & (III) \\
9.81 \times 10^3 \sigma - f(f) & (IV) \\
0.657(0.003 + \varepsilon)^{0.25} & (V) 
\end{cases}\]

---

**Table 3.** Process and boundary conditions for FE process simulation (III).

| Case | RG | A100 (mm) | B100 (mm) | C100 (mm) | D100 (mm) | E100 (GPa) | A50 (mm) | B50 (mm) | C50 (mm) | D50 (mm) | E50 (GPa) |
|------|----|-----------|-----------|-----------|-----------|-------------|-----------|-----------|-----------|-----------|-------------|
| 1    | 1  | 370        | 300       | 300       | 300       | 300         | 300       | 300       | 300       | 300       | 300         |
| 2    | 2  | 490.5      | 249       | 200       | 200       | 200         | 490.5     | 249       | 200       | 200       | 200         |
| 3    | 3  | 1277.6     | 1025      | 500       | 376       | 220         | 1521      | 1012      | 990       | 1600      | 210         |
| 4    | 4  | 500        | 301       | 130       | 226       | 206         | 550       | 305       | 320       | 540       | 206         |

Note: (1) E, Eb denote Young’s modulus, and Poisson ratio of work roll and backing roll are 0.3.
POSCO. The process conditions and boundary conditions are summarized in Tables 2 and 3, as cases 4–6. The predicted strip profiles are in excellent agreement with the measurements, as illustrated in Fig. 14. The strip profiles reveal a relatively uniform thickness distribution and a sharp decrease in the thickness at the strip edge, which is known as the edge drop phenomenon. It is found that if the flows stress of the strip is dependent on the strain rate, the edge drop may disappear, as illustrated in Fig. 15(a). On the other hand, if \( \frac{w}{l} \) (\( w= \)strip width, \( l= \)roll contact arc length) is not sufficiently large (or the deformation mode of the strip is not sufficiently close to the plane strain mode), the edge drop may also disappear, as illustrated in Fig. 15(b). It may be explained from these results that the edge drop phenomenon occurs mostly in cold rolling while it is relatively rare in hot rolling.

The integrated FE process model is also applied to a pair-cross mill. Solved is a full system including the strip, two work rolls, and two backup rolls, as shown in Fig. 2. It is clearly seen from Fig. 16 that an effect of applying the pair-cross angle is to twist the rolled strip. Process simulation of a full system, however, may be impractical in terms of the required CPU time (process simulation of a full system takes 5.5 CPU hours, approximately). Therefore, explored is an equivalent quarter system that can lead to a similar strip profile. As shown in Fig. 17, the system that is tested for this purpose consists of a work roll with a convex roll crown and a backup roll with a concave roll crown,
such that there is no gap between the work roll and the backup roll before deformation. The magnitude of the roll crown, which may be calculated from the change in the no-load roll gap profile due to the presence of a non-zero pair-
cross angle, is given by

$$\delta = \frac{w^2 \tan^2 \theta}{2D_w} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (35)$$

where $\theta$ is the pair cross angle, $D_w$ is the diameter of the work roll, $w$ is barrel length of the work roll.

As shown in Fig. 18, the predicted strip profiles are in excellent agreement with the measurements made by Kajihara for various pair-cross angles, except when the angle is very large, indicating that the proposed quarter system can effectively replace the full system in a practical sense. However, further investigation may be necessary to find an equivalent quarter system that can deal with the cases where, the pair-cross angle being non-zero, either the work roll or the backup roll has its own roll crown.

9. Concluding Remarks

Developing a model for the precision analysis of the mechanical behavior occurring in flat rolling in general requires rigorous prediction and implementation of the boundary conditions at the roll–strip interface as well as at the roll–roll interface. It is demonstrated through this investigation that such a precision process model can be constructed in a relatively straight manner by combining the basic FE models, with the aid of an iterative strategy for the treatment of mechanical contact. Then, it is shown that the proposed model may reflect the detailed aspects of the effect of various process variables on the strip profile. Coupled with precision modeling of the changes in the no-load roll gap profile due to roll thermal expansion and roll wear, the proposed model is expected to serve as an effective tool for enhancing the dimensional accuracy of the product in flat rolling via precision process control.

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