Semileptonic $B$ and $B_s$ decays into orbitally excited charmed mesons

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The BaBar Collaboration has recently reported products of branching fractions that include $B$ meson semileptonic decays into final states with charged and neutral $D_1(2420)$ and $D_2^*(2460)$, two narrow orbitally excited charmed mesons. We evaluate these branching fractions, together with those concerning $D_0^*(2400)$ and $D_1^*(2430)$ mesons, within the framework of a constituent quark model. The calculation is performed in two steps, one of which involves a semileptonic decay and the other is mediated by a strong process. Our results are in agreement with the experimental data. We also extend the study to semileptonic decays of $B_s$ into orbitally excited charmed-strange mesons, providing predictions to the possible measurements to be carried out at LHC.

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I. INTRODUCTION

Different collaborations have recently reported semileptonic $B$ decays into orbitally excited charmed mesons providing detailed results of branching fractions. The theoretical analysis of these data, which include both weak and strong decays, offers the possibility for a stringent test of meson models.

Moreover, an accurate determination of the $|V_{ub}|$ and $|V_{cb}|$ Cabbibo-Kobayashi-Maskawa matrix elements demands a detailed knowledge of semileptonic decays of $b$-hadrons. Decays including orbitally excited charmed meson in the final state provide a substantial contribution to the total semileptonic decay width. Furthermore, a better understanding of these processes is also necessary in the analysis of signals and backgrounds of inclusive and exclusive measurements of $b$-hadron decays.

The Belle Collaboration [1], using a full reconstruction tagging method to suppress the large combinatorial background, reported data on the product of branching fractions $B(B^+ \to D_{s(*)}^+ l^+ \nu_l)B(D_{s(*)}^+ \to D^{(*)} \pi)$, where, in the usual notation, $l$ stands for a light $e$ or $\mu$ lepton, the $D_0^*$, $D_1^*$, $D_1$, and $D_2^*$ mesons are denoted generically as $D^{(*)}$, and the $D^*$ and $D$ mesons as $D^{(*)}$.

$D^{(*)}$ decays are reconstructed in the decay chains $D^{(*)} \to D^* \pi^\pm$ and $D^{(*)} \to D^0 \pi$. In particular, the $D_0^*$ meson decays only through the $D^\pi$ channel, while the $D_1^*$ and $D_1$ mesons decay only via $D^* \pi$. Both $D^\pi$ and $D^* \pi$ channels are opened in the case of $D_2^*$.

In the case of BaBar data [2, 3] the branching fractions $B(D_0^* \to D^{(*)} \pi)$ include both the $D^*$ and $D$ contributions. As they also provide the ratio $B_{D_s(D)}$, we estimate the $D^*$ and $D$ contributions separately. The experimental results of both collaborations are given in Table I.

A similar analysis can be done in the strange sector for the $B_s$ meson semileptonic decays. Here the intermediate states are the orbitally charmed-strange mesons, $D_s^{**}$, and the available final channels are $DK$ and $D^*K$. The Particle Data Group (PDG) reports a value $B(B_s^0 \to D_{s(*)}^0(2536)^- \mu^+ \nu_\mu)B(D_{s(*)}^0(2536)^- \to D^{*-} K^0) = 2.4 \pm 0.7$ [4] based on their best value for $B(B \to B_s^0)$ and the experimental data for $B(B \to B_s^0)B(B_s^0 \to D_{s(*)}^0(2536)^- \mu^+ \nu_\mu)B(D_{s(*)}^0(2536)^- \to D^{*-} K^0)$ measured by the D0 Collaboration [5].

All these magnitudes can be consistently calculated in the framework of constituent quark models because they can simultaneously account for the hadronic part of the weak process and the strong meson decays. In this context, meson strong decay has been described successfully in phenomenological models, like the $3P_0$ model [6] or the flux-tube model [7], or in microscopic models (see Refs. [8, 9]). The difference between the two approaches lies on the description of the $q\bar{q}$ creation vertex. While the $3P_0$ model assumes that the $q\bar{q}$ pair is created from the vacuum with vacuum quantum numbers, in the microscopic model the $q\bar{q}$ pair is created from the interquark interactions already acting in the model. Both approaches will be used here to evaluate the strong decays. As for the weak process the matrix elements factorizes into a leptonic and a hadronic part. It is the hadronic part that contains the nonperturbative strong interaction effects and we shall evaluate it within a constituent quark model (CQM). We will work within the CQM of Ref. [10] which successfully describes hadron phenomenology and hadronic reactions [11–13] and has recently been applied to mesons containing heavy quarks in Refs. [14, 15].

The paper is organized as follows: In Sec. II we introduce the model we have used to get the masses and wave functions of the mesons involved in the reactions mentioned above. In Secs. III and IV we study the semileptonic and strong decay mechanisms, which constitute the two steps of the processes under study. Finally, we present our results in Sec. V and give some conclusions in Sec. VI.
lepton. The symbol ( indicates contributions, generated by Goldstone boson exchanges, should appear between constituent quarks.

A simple Lagrangian invariant under chiral transformations can be derived as

\[ \mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - MU^{\gamma_5}) \psi, \]

where \( U^{\gamma_5} = \exp(i\pi^a \lambda^a_5 / f_\pi) \), \( \pi^a \) denotes the pseudoscalar fields (\( \pi, K, K_0 \)) and \( M \) is the constituent quark mass. The momentum-dependent mass acts as a natural cutoff of the theory. The chiral quark-quark interaction can be written as

\[ V_{\pi q} (\vec{r}_{ij}) = V_{\pi q}^C (\vec{r}_{ij}) + V_{\pi q}^T (\vec{r}_{ij}) + V_{\pi q}^{SO} (\vec{r}_{ij}), \]

where \( C, T \) and \( SO \) stand for central, tensor and spin-orbit potentials. The central part presents four different contributions,

\[ V_{\pi q}^C (\vec{r}_{ij}) = V_{\pi q}^C (\vec{r}_{ij}) + V_{\pi q}^C (\vec{r}_{ij}) + V_{\pi q}^C (\vec{r}_{ij}) + V_{\pi q}^C (\vec{r}_{ij}), \]

given by
where \( Y(x) \) is the standard Yukawa function defined by \( Y(x) = e^{-x}/x \). We consider the physical \( \eta \) meson instead of the octet one and so we introduce the angle \( \theta_p \). The \( \lambda^a \) are the SU(3) flavor Gell-Mann matrices, \( m_i \) is the quark mass and \( m_\sigma, m_K \) and \( m_\eta \) are the masses of the SU(3) Goldstone bosons, taken from experimental values. \( m_\sigma \) is determined through the PCAC relation \( m_\sigma^2 \approx \lambda^2 + 4m_{ud}^2 \). Finally, the chiral coupling constant, \( g_{ch} \), is determined from the \( \pi N N \) coupling constant through

\[
g_{ch}^2 \frac{4\pi}{3} \approx \frac{g_{\pi NN}^2 m_{ud}^2}{4\pi m_N^2},
\]

which assumes that flavor SU(3) is an exact symmetry only broken by the different mass of the strange quark.

There are three different contributions to the tensor potential

\[
V_{qq}^T(\vec{r}_{ij}) = V_{\pi}^T(\vec{r}_{ij}) + V_K^T(\vec{r}_{ij}) + V_\eta^T(\vec{r}_{ij}),
\]

given by

\[
V_{\pi}^T(\vec{r}_{ij}) = g_{ch}^2 \frac{m_\pi^2}{4\pi} \frac{\lambda_\pi^2}{m_\pi^2 - m_\pi^2} \sum_{a=1}^3 \left( \Lambda_a^i \cdot \Lambda_a^j \right),
\]

\[
V_K^T(\vec{r}_{ij}) = g_{ch}^2 \frac{m_K^2}{4\pi} \frac{\lambda_K^2}{m_K^2 - m_K^2} \sum_{a=1}^3 \left( \Lambda_a^i \cdot \Lambda_a^j \right),
\]

\[
V_\eta^T(\vec{r}_{ij}) = g_{ch}^2 \frac{m_\eta^2}{4\pi} \frac{\lambda_\eta^2}{m_\eta^2 - m_\eta^2} \sum_{a=1}^3 \left( \Lambda_a^i \cdot \Lambda_a^j \right).
\]

The dynamics to be governed by QCD perturbative effects. In this way one-gluon fluctuations around the instanton vacuum are taken into account through the quark-gluon coupling

\[
\mathcal{L}_{qqg} = i\sqrt{4\pi} \bar{q}_x \gamma_\mu G^\mu \lambda^c \psi,
\]

with \( \lambda^c \) being the SU(3) color matrices and \( G^\mu \) the gluon field.

The different terms of the potential derived from the Lagrangian contain central, tensor, and spin-orbit contributions and are given by

The operator and distribution coming from the scalar part of the interaction is determined through the PCAC relation
$V_{\text{CON}}^C(\vec{r}_{ij}) = \frac{1}{4} \alpha_s (\vec{\sigma}_i \cdot \vec{\sigma}_j) \left[ \frac{1}{r_{ij}} - \frac{1}{6m_i m_j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}^2} \right]$, \\
$V_{\text{CON}}^T(\vec{r}_{ij}) = -\frac{1}{16} \frac{\alpha_s}{m_i m_j} \left( \vec{S}_i \cdot \vec{S}_j \right) \left[ \frac{1}{r_{ij}} - \frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}} \left( \frac{1}{r_{ij}^2} + \frac{1}{3r_{ij}^2} + \frac{1}{r_{ij} r_0(\mu)} \right) \right] S_{ij}$, \\
$V_{\text{CON}}^{SO}(\vec{r}_{ij}) = -\frac{1}{16} \frac{\alpha_s}{m_i^2 m_j^2} \left( \vec{S}_i \cdot \vec{S}_j \right) \left[ \frac{1}{r_{ij}} - \frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}} \left( 1 + \frac{r_{ij}}{r_0(\mu)} \right) \right] \times$
\[ \times \left[ \left( (m_i + m_j)^2 + 2 m_i m_j \right) (\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2) (\vec{S}_- \cdot \vec{L}) \right], \] (10)

where $S_{ij}^\pm = \frac{1}{2} (\sigma_i \pm \sigma_j)$. Besides, $r_0(\mu) = r_0^{\mu_{ij}}$ and $r_\mu(\mu) = r_\mu^{\mu_{ij}}$ are regulators which depend on $\mu_{ij}$, the reduced mass of the $q\bar{q}$ pair. The contact term of the central potential has been regularized as

$$\delta(\vec{r}_{ij}) \sim \frac{1}{4\pi r_0^2} \frac{e^{-r_{ij}/r_0}}{r_{ij}} \quad (11)$$

The wide energy range needed to provide a consistent description of light, strange and heavy mesons requires an effective scale-dependent strong coupling constant. We use the frozen coupling constant of Ref. [10]

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln \left( \frac{\mu^2 + \mu_0^2}{\Lambda_0^2} \right)}, \quad (12)$$

in which $\mu$ is the reduced mass of the $q\bar{q}$ pair and $\alpha_0$, $\mu_0$ and $\Lambda_0$ are parameters of the model determined by a global fit to the meson spectra.

Confinement is one of the crucial aspects of QCD. Color charges are confined inside hadrons. It is well known that multigluon exchanges produce an attractive linearly rising potential proportional to the distance between quarks. This idea has been confirmed, but not rigorously proved, by quenched lattice gauge Wilson loop calculations for heavy valence quark systems. However, sea quarks are also important ingredients of the strong interaction dynamics. When included in the lattice calculations they contribute to the screening of the rising potential at low momenta and eventually to the breaking of the quark-antiquark binding string. This fact, which has been observed in $n_f = 2$ lattice QCD [18], has been taken into account in our model by including the terms

$$V_{\text{CON}}^C(\vec{r}_{ij}) = \left[ -a_c (1 - e^{-\mu_{ij} r_{ij}}) + \Delta \right] (\vec{X}_i \cdot \vec{X}_j),$$

$$V_{\text{CON}}^T(\vec{r}_{ij}) = -\left( \vec{S}_i \cdot \vec{S}_j \right) \frac{a_c e^{-\mu_{ij} r_{ij}}}{4m_i^2 m_j^2 r_{ij}} \left[ \left( (m_i^2 + m_j^2)(1 - 2a_s) + 4m_i m_j (1 - a_s) \right) (\vec{S}_+ \cdot \vec{L}) + \left( m_j^2 - m_i^2 \right) (1 - 2a_s) (\vec{S}_- \cdot \vec{L}) \right], \quad (13)$$

where $a_s$ controls the mixture between the scalar and vector Lorentz structures of the confinement. At short distances this potential presents a linear behavior with an effective confinement strength $\sigma = -a_c \mu_c (\vec{X}_i \cdot \vec{X}_j)$ and becomes constant at large distances with a threshold defined by

$$V_{\text{thr}} = \{-a_c + \Delta\} (\vec{X}_i \cdot \vec{X}_j). \quad (14)$$

No $q\bar{q}$ bound states can be found for energies higher than this threshold. The system suffers a transition from a color string configuration between two static color sources into a pair of static mesons due to the breaking of the color string and the most favored decay into hadrons.

Among the different methods to solve the Schrödinger equation in order to find the quark-antiquark bound states, we use the Gaussian Expansion Method [19] because it provides enough accuracy and it makes the subsequent evaluation of the decay amplitude matrix elements easier.

This procedure provides the radial wave function solution of the Schrödinger equation as an expansion in terms of basis functions

$$R_\alpha(r) = \sum_{n=1}^{n_{\text{max}}} c_n^\alpha \phi_n^\alpha(r), \quad (15)$$

where $\alpha$ refers to the channel quantum numbers. The coefficients, $c_n^\alpha$, and the eigenvalue, $E$, are determined
Quark masses

\begin{tabular}{lcc}
\hline
Parameter & Value (MeV) \\
\hline
$m_\alpha$ & 313 \\
$m_\beta$ & 555 \\
$m_\delta$ & 1763 \\
$m_\epsilon$ & 5110 \\
\hline

goldstone bosons

\begin{tabular}{lcc}
\hline
Parameter & Value (fm$^{-1}$) \\
\hline
$m_g$ & 0.70 \\
$m_k$ & 3.42 \\
$m_\delta$ & 2.51 \\
$m_\epsilon$ & 2.77 \\
$\Lambda_\alpha$ & 4.20 \\
$\Lambda_\beta$ & 4.20 \\
$\Lambda_\delta$ & 4.21 \\
$\Lambda_\epsilon$ & 5.20 \\
g_\alpha/4\pi & 0.54 \\
$\theta_\alpha$ (°) & -15 \\
\hline
\end{tabular}

\textbf{TABLE II.} Quark model parameters.

from the Rayleigh-Ritz variational principle

$$\sum_{n=1}^{n_{\text{max}}} \left[ (T_{\alpha_n} - E N_{\alpha_n}) c_{\alpha_n}^2 + \sum_{\alpha'} V_{\alpha_n}^{\alpha_\alpha'} c_{\alpha_n}^{\alpha'} = 0 \right],$$

where $T_{\alpha_n}$, $N_{\alpha_n}$, and $V_{\alpha_n}^{\alpha_\alpha'}$ are the matrix elements of the kinetic energy, the normalization and the potential, respectively. $T_{\alpha_n}$ and $N_{\alpha_n}$ are diagonal whereas the mixing between different channels is given by $V_{\alpha_n}^{\alpha_\alpha'}$.

Following Ref. [13], we employ Gaussian trial functions with ranges in geometric progression. This enables the optimization of ranges employing a small number of free parameters. Moreover, the geometric progression is dense at short distances, so that it allows the description of the dynamics mediated by short range potentials. The fast damping of the gaussian tail is not a problem, since we can choose the maximal range much longer than the hadronic size.

Table II shows the model parameters fitted over all meson spectra and taken from Refs. [10, 14].

**III. WEAK DECAYS**

In this section, we give an account of the semileptonic decays of the $B$ ($B$ or $B_s$) meson into orbitally excited charmed mesons. In the nonstrange sector, this has been studied before within heavy quark effective theory (HQET) in Refs. [20, 21]. There, only relative branching ratios could be predicted and their results depended on the approximation used and on two unknown functions, $\tau_1$, $\tau_2$, that describe corrections of order $\Lambda QC D/mQ$. Only the ratio $\Gamma_{D^*}/\Gamma_{D^0}$, semileptonic decay rate with a helicity 0 $D^*$ final meson over total semileptonic decay rate to that meson, seemed to be stable in the different approximations. We shall comment on this below.

In the context of nonrelativistic constituent quark models, the state of a meson is given by

$$|M, \lambda \vec{P} \rangle_{NR} = \int \frac{d^3 p}{(2\pi)^{3/2}} \sum_{\alpha_1, \alpha_2} \frac{(-1)^{1/2-s_1}}{\sqrt{2E_{\alpha_1} E_{\alpha_2}}} \times \phi_{\alpha_1, \alpha_2}(\vec{p}) | q, \alpha_1 \bar{p}_1 \rangle | q, \alpha_2 \bar{p}_2 \rangle,$$

where $\vec{P}$ is the three-momentum of the meson and $\lambda$ is the spin projection in the meson center of mass. The vector $\vec{p}$ is the relative momentum of the $q\bar{q}$ pair, $\vec{p}_1 = \frac{m_{q_1}}{m_{q_1} + m_{q_2}} \vec{P} - \bar{p}$ and $\vec{p}_2 = \frac{m_{q_2}}{m_{q_1} + m_{q_2}} \vec{P} + \bar{p}$ are the momenta of the antiquark and the quark, respectively, $\alpha_1$ and $\alpha_2$ are the spin, flavor and color quantum numbers. $(E(\bar{p}_i), \bar{p}_i)$ are the four-momenta and $m_i$ are the quark masses. The factor $(-1)^{1/2-s_1}$ is included in order that the antiquark spin states have the correct relative phase.

The normalization of the quark-antiquark states is

$$\langle \alpha' \bar{p}'| \alpha \bar{P} \rangle = \delta_{\alpha', \alpha} (2\pi)^3 2 E_{\alpha}(\bar{p}) \delta(\bar{p}' - \bar{p}),$$

and the momentum space wave function $\phi_{\alpha_1, \alpha_2}(\vec{p})$ normalization is given by

$$\int d^3 p \sum_{\alpha_1, \alpha_2} (\phi_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p}))^* \phi_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p}) = \delta_{\lambda, \lambda}.$$
\[ \langle D(0^+) , \lambda \bar{P}_D | J_{t\mu}^{BC}(0) | B(0^-) , \bar{P}_B \rangle = P_\mu F_+(q^2) + q_\mu F_-(q^2), \]
\[ \langle D(1^+) , \lambda \bar{P}_D | J_{t\mu}^{BC}(0) | B(0^-) , \bar{P}_B \rangle = \frac{-1}{m_B + m_D} \epsilon_{\mu \nu \alpha \beta} \epsilon^{*}_{\lambda}(\bar{P}_D) P_\alpha \sigma^\beta A(q^2) \]
\[ -i \left\{ (m_B - m_D) \epsilon^{*}_{\lambda \mu}(\bar{P}_D) V_0(q^2) - \frac{P_\mu \epsilon^{*}_{\lambda}(\bar{P}_D)}{m_B + m_D} [P_\nu V_+(q^2) + q_\nu V_-(q^2)] \right\}, \]  
(22)
\[ \langle D(2^+) , \lambda \bar{P}_D | J_{t\mu}^{BC}(0) | B(0^-) , \bar{P}_B \rangle = \epsilon_{\mu \nu \alpha \beta} \epsilon^{*\lambda}_{\lambda}(\bar{P}_D) P_3 P_\alpha \sigma^\beta T_4(q^2) \]
\[ -i \left\{ \epsilon^{*}_{\lambda \mu \delta}(\bar{P}_D) P_\delta T_1(q^2) + P_\nu P_\delta \epsilon^{*}_{\lambda \nu \delta}(\bar{P}_D) [P_\mu T_2(q^2) + q_\mu T_3(q^2)] \right\}. \]

In the expressions above, \( P = P_B + P_D \) and \( q = P_B - P_D \), \( P_B \) and \( P_D \) being the meson four-momenta. \( m_B \) and \( m_D \) are the meson masses, \( \epsilon^{\mu \nu \alpha \beta} \) is the fully antisymmetric tensor, for which the convention \( \epsilon^{0123} = +1 \) is taken, and \( \epsilon^{*}_{\lambda}(\bar{P}_D) \) and \( \epsilon^{*\lambda}_{\lambda}(\bar{P}_D) \) are the polarization vector and tensor of vector and tensor mesons, respectively. The meson states in the Lorentz decompositions of Eq. (22) are normalized such that
\[ \langle M, \lambda' \bar{P}' | M, \lambda \bar{P} \rangle = \delta_{\lambda, \lambda'}(2\pi)^3 2E_M(\bar{P}) \delta(\bar{P'} - \bar{P}). \]
(23)
where \( E_M(\bar{P}) \) is the energy of the \( M \) meson with three-momentum \( \bar{P} \). Note the factor 2\( E_M \) difference with respect to Eq. (20).

The form factors will be evaluated in the center of mass of the 0\(^{-} \) meson, taking \( \vec{q} \) in the \( \hat{z} \) direction, so that \( \bar{P}_D = 0 \) and \( \bar{P}_D = -\vec{q} = -|\vec{q}|\hat{k} \), with \( \hat{k} \) representing the unit vector in the \( \hat{z} \) direction. We have taken the phases of the states such that all form factors are real. \( F_+ \), \( F_- \), \( A_\nu \), \( V_\lambda \), \( V_\mu \), \( V_- \) and \( T_1 \) are dimensionless, whereas \( T_2 \), \( T_3 \) and \( T_4 \) have dimension of \( E^{-2} \). Defining vector \( V_\lambda^\mu(|\vec{q}|) \) and axial \( A_\lambda^\mu(|\vec{q}|) \) matrix elements such that
\[ V_\lambda^\mu(|\vec{q}|) = \langle M_F , \lambda - |\vec{q}|\hat{k} , J_{t\mu}^{BC}(0) | M_I , \vec{0} \rangle , \]
\[ A_\lambda^\mu(|\vec{q}|) = \langle M_F , \lambda - |\vec{q}|\hat{k} , J_{t\mu}^{BC}(0) | M_I , \vec{0} \rangle , \]
(24)
we have for a 0\(^{-} \) \( \rightarrow \) 0\(^{+} \) decay, that the form factors are given in terms of vector and axial matrix elements as
\[ F_+(q^2) = -\frac{1}{2m_B} \left[ A^0(|\vec{q}|) + \frac{A^3(|\vec{q}|)}{|\vec{q}|} (E_D(-\vec{q}) - m_B) \right], \]
\[ F_-(q^2) = -\frac{1}{2m_B} \left[ A^0(|\vec{q}|) + \frac{A^3(|\vec{q}|)}{|\vec{q}|} (E_D(-\vec{q}) + m_B) \right]. \]
(25)

In the case of a 0\(^{-} \) \( \rightarrow \) 1\(^{+} \) transition, the corresponding expressions for the form factors are
\[ A(q^2) = -\frac{i}{\sqrt{2}} \frac{m_B + m_D}{m_B|\vec{q}|} A_{\lambda = -1}^1(|\vec{q}|), \]
\[ V_+(q^2) = -\frac{m_B + m_D}{2m_B} m_B \left\{ V_{\lambda = 0}^0(|\vec{q}|) - \frac{m_B - E_D(-\vec{q})}{|\vec{q}|} V_{\lambda = 0}^3(|\vec{q}|) + \sqrt{2} \frac{m_B E_D(-\vec{q}) - m_D^2}{|\vec{q}| m_D} V_{\lambda = -1}^1(|\vec{q}|) \right\}, \]
\[ V_-(q^2) = -i \frac{m_B + m_D}{2m_B} m_B \left\{ -V_{\lambda = 0}^0(|\vec{q}|) - \frac{m_B + E_D(-\vec{q})}{|\vec{q}|} V_{\lambda = 0}^3(|\vec{q}|) + \sqrt{2} \frac{m_B E_D(-\vec{q}) + m_D^2}{|\vec{q}| m_D} V_{\lambda = -1}^1(|\vec{q}|) \right\}, \]
\[ V_0(q^2) = +i \sqrt{2} \frac{1}{m_B - m_D} V_{\lambda = -1}^1(|\vec{q}|). \]
(26)

Finally, the form factors for a 0\(^{-} \) \( \rightarrow \) 2\(^{+} \) transition are given by the relations
The CQM evaluation of the vector and axial matrix elements $V^a_\lambda(|\vec q|)$ and $A^a_\lambda(|\vec q|)$ can be found in the Appendix. For a $B$ meson at rest and neglecting the neutrino mass, we have the double differential decay width

$$
\frac{d^2\Gamma}{dq^2dx_t} = \frac{G_F^2}{64\pi^3} \frac{|V_{bc}|^2}{8\pi^3} \frac{\lambda^{1/2}(q^2,m_{B}^2,m_{D}^2) \lambda^{3/2}(q^2,m_{B}^2,m_{D}^2)}{2m_B} \frac{q^2 - m_I^2}{q^2} \times H_{\alpha\beta}(P_B,P_D)\mathcal{C}^{\alpha\beta}(p_t,p_v),
$$

where $x_t$ is the cosine of the angle between the final meson momentum and the momentum of the final charged lepton measured in the lepton-neutrino center of mass frame. $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant [4], $m_I$ is the charged lepton mass, $\lambda(a,b,c) = (a + b - c)^2 - 4ab$ and $V_{bc}$ is the $bc$ element of the Cabibbo-Kobayashi-Maskawa matrix for which we shall use $V_{bc} = 0.0413$. $H_{\alpha\beta}$ and $\mathcal{C}^{\alpha\beta}$ represent the hadron and lepton tensors. $P_B, P_D, p_t$ and $p_v$ are the meson and lepton momenta.

Working in the helicity formalism of Ref. [22] and after integration on $x_t$ we have

$$
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{bc}|^2}{8\pi^3} \frac{2(q^2 - m_I^2)^{\lambda^{1/2}(q^2,m_{B}^2,m_{D}^2)}}{12m_B^2q^2} \frac{2m_B}{m_I} \times (H_U + H_L + \tilde H_U + \tilde H_L + \tilde H_S),
$$

where the suffixes $U, L, S$ stand for the unpolarized-transverse, longitudinal and scalar components of the hadronic tensor, and $\tilde H = \frac{m_I^2}{m_B^2} H$. Integrating over $q^2$ we obtain the total decay width that can be written as

$$
\Gamma = \Gamma_U + \Gamma_L + \tilde \Gamma_U + \tilde \Gamma_L + \tilde \Gamma_S,
$$

with $\Gamma_J$ and $\tilde \Gamma_J$ partial helicity widths defined as

$$
\Gamma_J = \int dq^2 \frac{G_F^2 |V_{bc}|^2}{8\pi^3} \frac{2(q^2 - m_I^2)^{\lambda^{1/2}(q^2,m_{B}^2,m_{D}^2)}}{12m_B^2q^2} \frac{2m_B}{m_I} H_J
$$

and similarly for $\tilde \Gamma_J$ in terms of $\tilde H_J$. The evaluation of the different form factors, and thus of the different helicity amplitudes of the hadronic tensor, has been done following Ref. [23].

**IV. STRONG DECAYS**

Meson strong decay is a complex nonperturbative process that has not yet been described from QCD first principles. Instead, several phenomenological models have been developed to deal with this topic, the $^3P_0$ [8], the flux-tube [9], and the Cornell $^3P_0$ [8] models being the most popular.

Some models describe the decay process assuming that the extra quark-antiquark pair is created from the vacuum. This is the case of the $^3P_0$ model, which borrows its name from the quantum numbers of the created pair, or the flux-tube model, which in addition to the creation vertex incorporates the overlaps between the color flux tubes of the initial and final states.

To address a more fundamental description of the decay mechanism, one has to describe hadron strong decays in terms of quark and gluon degrees of freedom. However, there has been little previous work in this area. Two different examples are the study of open-charm decays of $c\bar c$ resonances by Eichten et al. [8], who assumed that the decays are due to pair production from the static part of a Lorentz vector confining interaction, and the study of a few strong decays in the light sector by Ackleh et al. [9], where the $q\bar q$ pair production comes from the one-gluon exchange and a scalar confining interaction.

As we mentioned in the introduction, we shall use both the $^3P_0$ model and a microscopic one, resembling those of Refs. [8] and [9], that originates from the different interaction pieces present in our interquark potential. These two approaches to meson production are introduced in the following subsections.
A. The $^3P_0$ model

It was first proposed by Micu \cite{Micu} and further developed by Le Yaouanc et al. \cite{LeYaouanc}. To describe the meson decay process $A \rightarrow B + C$, the $^3P_0$ model assumes that a quark-antiquark pair is created with vacuum tails of the resulting matrix elements for different cases. Definitions are the solutions of the Schrödinger equation using quantum numbers. The created quark-antiquark pair together with the $q\bar{q}$ pair present in the original meson regroup in the two outgoing mesons via a quark rearrangement process.

The interaction Hamiltonian which describes the production process is given by \cite{Micu}

$$H_I = g \int d^3x \bar{\psi}(\vec{x}) \psi(\vec{x})$$

where $g$ is related to the dimensionless constant giving the strength of the $q\bar{q}$ pair creation from the vacuum as $\gamma = \frac{g}{2m_q}$, $m_q$ being the mass of the created quark. Note that the operator $g\bar{\psi}\psi$ leads to the decay $(q\bar{q})_A \rightarrow (q\bar{q})_B + (q\bar{q})_C$ through the $a^0b^1$ term.

B. The microscopic model

In microscopic decay models one attempts to describe hadron strong decays in terms of quark and gluon degrees of freedom. The quark-gluon decay mechanism should give similar predictions to the reasonably accurate $^3P_0$ model and should determine the strength of the $q\bar{q}$ pair creation, $\gamma$, of the $^3P_0$ model in terms of more fundamental parameters.

Following Ref. \cite{Micu}, the strong decays should be driven by the same interquark Hamiltonian which determines the spectrum, the one-gluon exchange, and the confining interaction appearing as the kernels. These interactions and their associated decay amplitudes are undoubtedly all present and should be added coherently. We already mentioned that our constituent quark model for the heavy quark sector has a one-gluon exchange term and a mixture of Lorentz scalar and vector confining interactions. This completely defines our microscopic model for strong decays. Unlike previous works we use a screening confinement interaction and also a mixture between scalar and vector Lorentz structures, which is already fixed.

The Hamiltonian of the interaction can be written as

$$H_I = \frac{1}{2} \int d^3xdy J^a(\vec{x})K(|\vec{x} - \vec{y}|)J^a(\vec{y}).$$

The current $J^a$ in Eq. (33) is assumed to be a color octet. The currents, $J$, with the color dependence $\lambda^a/2$ factored out and the kernels, $K(r)$, for the interactions are

\begin{itemize}
  \item Currents
  \begin{equation}
  J(\vec{x}) = \bar{\psi}(\vec{x})\Gamma(\vec{x}) = \begin{cases}
    \bar{\psi}(\vec{x}) \mathbb{I} \psi(\vec{x}) & \text{Scalar Lorentz current,} \vspace{0.5em} \\
    \bar{\psi}(\vec{x}) \gamma^0 \psi(\vec{x}) & \text{Static part of vector Lorentz current,} \\
    \bar{\psi}(\vec{x}) \gamma^\mu \psi(\vec{x}) & \text{Spatial part of vector Lorentz current,}
  \end{cases}
  \end{equation}

  \item Kernels
  \begin{equation}
  K(r) = \begin{cases}
    -4a_s \left[-a_s(1 - e^{-\mu r}) + \Delta \right] & \text{Confining interaction,} \\
    +\alpha_s & \text{Color Coulomb OGE,} \\
    -\frac{\alpha_s}{r} & \text{Transverse OGE.}
  \end{cases}
  \end{equation}
\end{itemize}

For the Lorentz vector structure of the confinement we use $K(r) = \pm(1 - a_s)4[-a_s(1 - e^{-\mu r}) + \Delta]$, where \pm refers to static and transverse terms, respectively. We refer, following Ref. \cite{Micu}, to this general type of interaction as a $JKJ$ decay model, and to the specific cases considered here as $sKS$, $j^0Kj^0$ and $j^JKj^J$ interactions.

The wave functions for the mesons involved in the reactions are the solutions of the Schrödinger equation using the Gaussian Expansion Method mentioned above. Details of the resulting matrix elements for different cases are given in Ref. \cite{LeYaouanc}.

C. Strong decay width

The total width is the sum over the partial widths characterized by the quantum numbers $J_{BC}$ and $l$

$$\Gamma_{A \rightarrow BC} = \sum_{J_{BC},l} \Gamma_{A \rightarrow BC}(J_{BC},l)$$

where

$$\Gamma_{A \rightarrow BC}(J_{BC},l) = 2\pi \int dk_0 \delta(E_A - E_{BC})|\mathcal{M}_{A \rightarrow BC}(k_0)|^2$$

and $\mathcal{M}_{A \rightarrow BC}(k_0)$ is calculated according to Refs. \cite{LeYaouanc}. \cite{LeYaouanc, LeYaouanc}.\"
Using relativistic phase space, we arrive at

$$\Gamma_{A \rightarrow BC}(J_{BC}, l) = \frac{2\pi E_BE_C}{m_A k_0} |M_{A \rightarrow BC}(k_0)|^2, \quad (38)$$

where

$$k_0 = \frac{\lambda^{1/2}(m_A^2, m_B^2, m_C^2)}{2m_A} \quad (39)$$

is the on shell relative momentum of mesons $B$ and $C$.

V. RESULTS

For the low-lying positive parity excitations, any quark model predicts four states that in the $^{2S+1}L_J$ basis correspond to $^1P_1$, $^3P_0$, $^3P_1$ and $^3P_2$. As charge conjugation is not well defined in the heavy-light sector, $^1P_1$ and $^3P_1$ states can mix under the interaction.

In the infinite heavy quark mass limit, heavy quark symmetry (HQS) predicts two degenerated $P$-wave meson doublet, labeled by $j_q = 1/2$ with $J^P = 0^+, 1^+ ((1/2, 0^+), (1/2, 1^+))$ and $j_q = 3/2$ with $J^P = 1^+, 2^+ ((3/2, 1^+), (3/2, 2^+))$. In this limit, the meson properties are governed by the dynamics of the light quark, which is characterized by its total angular momentum $j_q = s_q + L$, where $s_q$ is the light quark spin and $L$ the orbital angular momentum. The total angular momentum of the meson $J$ is obtained coupling $j_q$ to the heavy quark spin, $S$.

Moreover, in the infinite heavy quark mass limit the strong decays of the $D_J (j_q = 3/2)$ proceed only through $D$-waves, while the $D_J (j_q = 1/2)$ decays happen only through $S$-waves [27]. The $D$-wave decay is suppressed by the barrier factor which behaves as $q^{2L+1}$ where $q$ is the relative momentum of the two decaying mesons. Therefore, the states decaying through $D$-waves are expected to be narrower than those decaying via $S$-waves.

A change of basis allows to express the above states in terms of the $^{2S+1}L_J$ basis, by recoupling angular momenta, as

\begin{align}
|1/2, 0^+\rangle &= +|^3P_0\rangle \\
|1/2, 1^+\rangle &= +\sqrt{\frac{1}{3}}|^1P_1\rangle + \sqrt{\frac{2}{3}}|^3P_1\rangle \\
|3/2, 1^+\rangle &= -\sqrt{\frac{2}{3}}|^1P_1\rangle + \sqrt{\frac{1}{3}}|^3P_1\rangle \\
|3/2, 2^+\rangle &= +|^3P_2\rangle
\end{align}

(40)

where in the $^{2S+1}L_J$ wave functions we couple heavy and light quark spins, in this order, to total spin $S$.

In the actual calculation the ideal mixing in Eq. (40) between $^1P_1$ and $^3P_1$ states changes due to finite charm quark mass effects. Our CQM model predicts the mixed states shown in Table III which are very similar to the HQS states. This is expected since the $c$-quark is much heavier ($m_c = 1763$ MeV) than the light ($m_u = 313$ MeV) or strange ($m_s = 555$ MeV) quarks. Note that now we have mixing, even if small, between the $^3P_2$ and $^3F_2$ partial waves in $2^+$ mesons. This is due to the OGE tensor term.

In Ref. [15] we have studied the $J^P = 1^+$ charmed-strange mesons, finding that the $J^P = 1^+ D_{s1}(2460)$ has an important non-qq contribution whereas the $J^P = 1^+ D_{s1}(2536)$ is almost a pure $q\bar{q}$ state. The presence of non-qq degrees of freedom in the $J^P = 1^+$ charmed-strange meson sector enhances the $j_q = 3/2$ component of the $D_{s1}(2536)$. This wave function explains most of the experimental data, as shown in Ref. [15], and it is the one we shall use here. For this sector only the $q\bar{q}$ probabilities are given in Table III.

| $D_0^-$ | $D_1$ | $D_1'$ | $D_2$ |
|--------|-------|--------|-------|
| $^3P_0$ | +, 1.0000 | - | - |
| $^1P_1$ | - | −, 0.5903, −, 0.4097 | - |
| $^3P_1$ | - | +, 0.4097, −, 0.5903 | - |
| $^3P_2$ | - | - | +, 0.99993 |
| $^1/2, 0^+$ | +, 1.0000 | - | - |
| $^1/2, 1^+$ | - | +, 0.0063, −, 0.9937 | - |
| $3/2, 1^+$ | - | +, 0.9937, +, 0.0063 | - |
| $3/2, 2^+$ | - | - | +, 0.99993 |

TABLE III. Probability distributions and their relative phases for the four states predicted by CQM in the two basis described in the text. In the $1^+$ strange sector the effects of non-qq components are included, see text for details.

A. $B$ semileptonic decays into $D^{\ast\ast}$ mesons

1. Semileptonic $B \rightarrow D_0^-(2400)\ell\nu_\ell$ decay

The measured branching fractions are $B(B^+ \rightarrow D^0 l^+\nu_\ell)B(D^0 l^- \rightarrow D^- \pi^+) = B(B^0 \rightarrow D_0^- l^+\nu_\ell)B(D_0^- \rightarrow D^0 \pi^-) = B(D_0^- \rightarrow D^- \pi^-) = 2/3$ coming from isospin symmetry.

Table [IV] shows the different helicity contributions to the semileptonic width. In both cases the dominant contribution is given by $\Gamma_1$, while the rest are negligible. The difference between the semileptonic width of the charged
and neutral $B$ meson is due to the large mass difference between the $D^*_0$ and $D^*_0$ mesons for which we take the masses reported in Ref. [4].

Figure 6 shows the $q^2$ dependence in the form factors and in the differential decay width for $\mathcal{B}(B^+ \to D^*_0 l^+ \nu_l)$, panels (a) and (b), respectively. Similar results (not shown) are obtained for the $\mathcal{B}(B^0 \to D^*_0 l^+ \nu_l)$ case.

The final results for the product of branching fractions are

$$\begin{align*}
\mathcal{B}(B^+ &\to D^*_0 l^+ \nu_l) \mathcal{B}(D^*_0 \to D^- \pi^+) = 2.15 \times 10^{-3}, \\
\mathcal{B}(B^0 &\to D^*_0^- l^+ \nu_l) \mathcal{B}(D^*_0^- \to D^0 \pi^-) = 1.80 \times 10^{-3},
\end{align*}$$

(41)

which compare very well with Belle data [1].

2. **Semileptonic $B \to D^*_1(2420)l \nu_l$ decay**

The only Okubo-Zweig-Iizuka (OZI)-allowed decay channel for the $D^*_1$ meson is $D^*_1 \to D^*+\pi$ so that isospin symmetry predicts a branching fraction $\mathcal{B}(D^*_1 \to D^*+\pi) = 2/3$.

Table V shows the different helicity contributions to the semileptonic width of $B^+ \to D^*_1 l^+ \nu_l$ and $B^0 \to D^*_1^- l^+ \nu_l$ calculated in the framework of the CQM. In this case, $\Gamma_U$ and $\Gamma_L$ are of the same order of magnitude and give the total semileptonic decay rate.

Panels (a) and (b) of Fig. 7 show the $q^2$ dependence of the form factors and the differential decay width for the neutral $D^*_1$ channel. A very similar result is obtained for the $D^*_0$ case.

![Image](image_url)

**TABLE IV.** Helicity contributions and total decay widths, in units of $10^{-15}$ GeV, for the $D^*_0$ meson.

| $B^+ \to D^*_0 l^+ \nu_l$ | $B^0 \to D^*_0 l^+ \nu_l$ |
|--------------------------|--------------------------|
| $\Gamma_U$              | 0.00                     |
| $\tilde{\Gamma}_U$      | 0.00                     |
| $\Gamma_L$              | 1.30                     |
| $\tilde{\Gamma}_L$      | 1.16                     |
| $\Gamma_S$              | 6.83 x 10^{-7}           |
| $\tilde{\Gamma}_S$      | 6.45 x 10^{-7}           |
| $\Gamma$                | 1.30                     |

**TABLE V.** Helicity contributions and total decay widths, in units of $10^{-15}$ GeV, for the $D^*_1$ meson.

$\mathcal{B}(B^+ \to D^*_1 l^+ \nu_l) \mathcal{B}(D^*_1 \to D^*+\pi^+) = 2.57 \times 10^{-3},$

(42)

As in the previous case, the branching fraction $\mathcal{B}(D^*_1 \to D^*+\pi)$ is again 2/3 in our model because $D^*_1 \to D^*\pi$ is the only OZI-allowed decay channel.

Table VI shows the different helicity contributions to the semileptonic width of the reactions $B^+ \to D^*_1 l^+ \nu_l$ and $B^0 \to D^*_1^- l^+ \nu_l$. The most important contribution is given by $\Gamma_L$. The ratio $\Gamma_L/\Gamma = 0.75$ gives the probability for the final $D^*_1$ meson to have helicity 0. This result is in agreement with the values 0.72 – 0.81 obtained in the HQET calculation of Ref. [21].

Figure 8 shows the $q^2$ dependence of the form factors and the differential decay width for neutral $D^*_1$ channel, in panels (a) and (b), respectively. Again, a very similar result is obtained for the charged case.

The product of branching fractions are

$$\begin{align*}
\mathcal{B}(B^+ &\to D^*_1 l^+ \nu_l) \mathcal{B}(D^*_1 \to D^*+\pi^+) = 2.57 \times 10^{-3}, \\
\mathcal{B}(B^0 &\to D^*_1^- l^+ \nu_l) \mathcal{B}(D^*_1^- \to D^0 \pi^-) = 2.39 \times 10^{-3},
\end{align*}$$

(43)

which in this case compare very well with the latest BaBar data [3].

4. **Semileptonic $B \to D^*_2 l \nu_l$ decay**

The semileptonic decay is studied by reconstructing the decay channel $D^*_2 \to D^*(\pi^-\nu)$, using the decay chain $D^* \to D^0\pi$ for $D^*$ meson and $D^0 \to K^-\pi^+$ or $D^+ \to K^-\pi^+\pi^+$ for $D$ meson. What is actually measured is the product of branching fractions $\mathcal{B}(B^+ \to D^*_2 l^+ \nu_l) \mathcal{B}(D^*_2 \to D^*+\pi) + \mathcal{B}(B^+ \to D^*_2 l^+ \nu_l) \mathcal{B}(D^*_2 \to D^-+\pi)$. 
The first step of this decay involves a semileptonic process which can be calculated using Eq. (30). In Table VII we show the different helicity contributions to the total width. The main contribution is $\Gamma_L$ in both neutral and charged $D_2^*$ channels, providing almost $2/3$ of the total width. The following one is $\Gamma_U$, the rest of the contributions being negligible. Again our ratio $\Gamma_L/\Gamma = 0.67$ is in agreement with the values $0.63 - 0.64$ obtained in Ref. [21] using HQET.

Figure 4 shows the $q^2$ dependence in the form factors and in the differential decay width, panels (a) and (b) respectively, for the $B^+ \to D_2^{*0}l^+\nu_l$ decay. Very similar results (not shown) are obtained for the $B^0 \to D_2^{-}l^+\nu_l$

| $B^+ \to D_2^{*0}l^+\nu_l$ | $B^0 \to D_2^{-}l^+\nu_l$ |
|-----------------------------|-----------------------------|
| $\Gamma_U$                 | 0.44                        |
| $\tilde{\Gamma}_U$        | $2.56 \times 10^{-8}$      |
| $\Gamma_L$                 | 0.90                        |
| $\tilde{\Gamma}_L$        | $5.27 \times 10^{-7}$      |
| $\tilde{\Gamma}_S$        | $1.54 \times 10^{-6}$      |
| $\Gamma$                  | $1.34$                      |
|                            | $1.35$                      |

TABLE VII. Helicity contributions and total decay widths, in units of $10^{-15}$ GeV, for the $D_2^*$ meson.
TABLE VIII. Branching ratios for $D^+_s$ decays collected by the PDG [4] and our theoretical results calculated through the two strong decay models.

| Branching ratio | Exp. | $3P_0$ Microscopic |
|-----------------|------|--------------------|
| $\Gamma(D^0\pi^+)/\Gamma(D^{*0}\pi^+)$ | $1.9 \pm 1.1 \pm 0.3$ | $1.80 \pm 0.19$ |
| $\Gamma(D^{*+}\pi^-)/\Gamma(D^{*+}\pi^-)$ | $1.56 \pm 0.16$ | $1.82 \pm 0.19$ |
| $\Gamma(D^{*+}\pi^-)/\Gamma(D^{*+}\pi^-)$ | $0.62 \pm 0.03 \pm 0.02$ | $0.65 \pm 0.06$ |

The subsequent strong decays which appear are $D^*_0 \rightarrow D^*\pi^- \rightarrow D\pi^-$ and $D^*_2 \rightarrow D\pi^-$. In Table VIII we show the strong decay branching ratios obtained with the $3P_0$ and microscopic models. They are in good agreement with experimental data [4].

Finally, we obtain the products of branching fractions for both decay chains considering that the total width of the $D^*_2$ meson is the sum of the partial widths of $D^*\pi$ and $D\pi$ channels since these are the only OZI-allowed processes

\[
\begin{align*}
B(B^+ \rightarrow D^*_2 l^+\nu_l)B(D^*_2 \rightarrow D^+\pi^-) &= \left\{ \begin{array}{c}
1.44 \times 10^{-3} \\
1.48 \times 10^{-3}
\end{array} \right. \\
B(B^+ \rightarrow D^*_2 l^+\nu_l)B(D^*_2 \rightarrow D^{*+}\pi^-) &= \left\{ \begin{array}{c}
0.79 \times 10^{-3} \\
0.75 \times 10^{-3}
\end{array} \right. \\
B(B^0 \rightarrow D^*_2 l^-\nu_l)B(D^*_2 \rightarrow D^0\pi^-) &= \left\{ \begin{array}{c}
1.34 \times 10^{-3} \\
1.38 \times 10^{-3}
\end{array} \right. \\
B(B^0 \rightarrow D^*_2 l^-\nu_l)B(D^*_2 \rightarrow D^{*0}\pi^-) &= \left\{ \begin{array}{c}
0.74 \times 10^{-3} \\
0.70 \times 10^{-3}
\end{array} \right.
\end{align*}
\]

where the first one refers to the calculation using the $3P_0$ model and the second one comes from the microscopic model. These results are in very good agreement with BaBar data [3].

5. Summary of the results

Final results and their comparisons with the experimental data are given in Table IX. Except for the $D'_1(2430)$, the predictions are in very good agreement with the latest experimental measurements, Belle for $D_0(2000)$ and BaBar for $D_1(2420)$ and $D'_2(2460)$. For the $D'_1(2430)$ there is also a strong disagreement between experimental data in the neutral case.

B. $B_s$ semileptonic decays into $D^{*+}_s$ mesons

The semileptonic decays of $B_s$ meson into orbitally excited $P$-wave charmed-strange mesons ($D^{*+}_s$) provides an extra opportunity to get more insight into this system.

The $J_q = 1/2$ doublet, $D^{*0}_s(2318)$ and $D^{*+}_s(2321)$, shows surprisingly light masses which are below the $DK$ and $D^*K$ thresholds, respectively. These unexpected properties have triggered many theoretical interpretations, including four quark states, molecules, and the coupling of the $q\bar{q}$ components with different structures. As mentioned before, the $D^{*+}_s(2321)$ meson has an important non-$q\bar{q}$ contribution.

We have calculated the semileptonic $B_s$ decays assuming that the $D^{*+}_s$ mesons are pure $q\bar{q}$ systems. For the $D^{*0}_s(2318)$ and $D^{*+}_s(2321)$, which are below the corresponding $D^{(*)}K$ thresholds, we only quote the weak decay branching fractions. Concerning the $D^{*+}_s(2321)$, and as shown in Ref. [15], the $1P_1$ and $3P_1$ probabilities
change with the coupling to non-$q\bar{q}$ degrees of freedom. What we do here is to vary these probabilities (including the phase) in order to obtain the limits of the decay width in the case of the $D_{s1}(2460)$ being a pure $q\bar{q}$ state, see Fig. 3. Assuming that non-$q\bar{q}$ components will give a small contribution to the weak decay, experimental results lower than these limits will be an indication of a more complex structure for this meson.

For the decay into $D_{s1}(2536)$, our model predicts the weak decay branching fraction $\mathcal{B}(B_s^0 \to D_{s1}(2536)\mu^+\nu_\mu) = 4.77 \times 10^{-3}$ and the strong branching fractions $\mathcal{B}(D_{s1}(2536)^- \to D^{*-}\bar{K}^0) = 0.43(0.47)$ for the $^3P_0$ (microscopic) models. The final result appears in Table IX. It is in good agreement with the existing

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & Belle & BaBar & $^3P_0$ & Mic. \\
 & ($10^{-3}$) & ($10^{-3}$) & ($10^{-3}$) & ($10^{-3}$) \\
\hline
$D_0^*(2400)$ & & & & \\
$\mathcal{B}(B^+ \to \bar{D}_0^{*0}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_0^{*0} \to D^-\pi^+)$ & 2.4 & 2.6 & 2.15 & 2.15 \\
$\mathcal{B}(B^0 \to \bar{D}_0^{*0}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_0^{*0} \to D^0\pi^-)$ & 2.0 & 4.4 & 1.80 & 1.80 \\
\hline
$D_1^*(2430)$ & & & & \\
$\mathcal{B}(B^+ \to \bar{D}_1^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_1^{*+} \to D^{*-}\pi^+)$ & < 0.7 & 2.7 & 1.32 & 1.32 \\
$\mathcal{B}(B^0 \to \bar{D}_1^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_1^{*+} \to D^{*0}\pi^-)$ & < 5 & 3.1 & 1.23 & 1.23 \\
\hline
$D_1(2420)$ & & & & \\
$\mathcal{B}(B^+ \to \bar{D}_1^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_1^{*+} \to D^{*-}\pi^+)$ & 4.2 & 2.97 & 2.57 & 2.57 \\
$\mathcal{B}(B^0 \to \bar{D}_1^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_1^{*+} \to D^{*0}\pi^-)$ & 5.4 & 2.78 & 2.39 & 2.39 \\
\hline
$D_2^*(2460)$ & & & & \\
$\mathcal{B}(B^+ \to \bar{D}_2^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_2^{*+} \to D^{*-}\pi^+)$ & 2.2 & 1.4 & 1.43 & 1.47 \\
$\mathcal{B}(B^+ \to \bar{D}_2^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_2^{*+} \to D^{*0}\pi^-)$ & 1.8 & 0.9 & 0.79 & 0.75 \\
$\mathcal{B}(B^+ \to \bar{D}_2^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_2^{*+} \to D^{(*)-}\pi^+)$ & 4.0 & 2.3 & 2.22 & 2.22 \\
$\mathcal{B}(B^0 \to \bar{D}_2^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_2^{*+} \to D^{*-}\pi^+)$ & 2.2 & 1.1 & 1.34 & 1.38 \\
$\mathcal{B}(B^0 \to \bar{D}_2^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_2^{*+} \to D^{*0}\pi^-)$ & < 3 & 0.7 & 0.74 & 0.70 \\
$\mathcal{B}(B^0 \to \bar{D}_2^{*+}(1^{+})\nu_\mu)\mathcal{B}(\bar{D}_2^{*+} \to D^{(*)0}\pi^-)$ & < 5.2 & 1.8 & 2.08 & 2.08 \\
\hline
$B_{D/D^*}$ & 0.55 & 0.62 & 0.65 & 0.66 \\
\hline
\end{tabular}
\caption{Most recent experimental measurements reported by Belle and BaBar Collaborations and their comparison with our results. The symbol (*) indicates the estimated results from the original data using $B_{D/D^*}$.}
\end{table}
TABLE X. Our predictions and their comparison with the available experimental data for semileptonic $B_s$ decays into orbitally excited charmed-strange mesons.  

| Decay mode | Experiment ($\times 10^{-3}$) | Theory ($\times 10^{-3}$) |
|------------|-------------------------------|--------------------------|
| $B(B^0_s \rightarrow D_{s1}^*(2318)^- \mu^+\nu_\mu)$ | 0.43 | 4.43 |
| $B(D_{s1}^0 \rightarrow D_{s1}(2460)^- \mu^+\nu_\mu)$ | - | 1.74 - 5.70 |
| $B(D_{s2}^0(2573)^- \mu^+\nu_\mu)$ | 3$P_0$ Mic. | 2.4 ± 0.7 [4, 5] 2.05 2.24 |
| $B(B^0_s \rightarrow D_{s2}^*(2573)^- \mu^+\nu_\mu)$ | 1.70 1.77 |
| $B(D_{s2}^0(2573)^- \mu^+\nu_\mu)$ | 0.18 0.11 |
| $B(B^0_s \rightarrow D_{s2}^*(2573)^- \mu^+\nu_\mu)$ | 1.88 1.88 |

FIG. 5. Decay width for the $B^0_s \rightarrow D_{s1}(2460)^- \mu^+\nu_\mu$ decay as a function of the $^3P_1$ component probability. The sign reflects the relative phase between $^3P_1$ and $^3P_1$ components: -1 opposite phase and +1 same phase.

We have performed a calculation of the branching fractions for the semileptonic decays of $B$ and $B_s$ mesons into final states containing orbitally excited charmed and charmed-strange mesons, respectively. 

We worked in the framework of the constituent quark model of Ref. [10]. The model parameters were fitted to the meson spectra in Refs. [10, 14]. Our meson states are close to the ones predicted by HQS as expected. 

We have calculated the semileptonic decay rates within the helicity formalism of Ref. [22] and following the work in Ref. [23]. The strong decay widths have been calculated using two models, the $^3P_0$ model and a microscopic model based on the quark-antiquark interactions present in the CQM model of Ref. [10]. 

From the experimental point of view, Belle and BaBar Collaborations provide their most recent measurements for the $B$ meson in Refs. [1] and [2, 3] respectively. For the $B_s$ meson only the product of branching fractions decays into orbitally excited charmed-strange mesons.  

decl part of the reaction, we obtain in our model 

$$B(D_{s2}^* \rightarrow D^+K^-) = \begin{pmatrix} 0.45 \\ 0.47 \end{pmatrix}$$

$$B(D_{s2}^* \rightarrow D^{*-}K^0) = \begin{pmatrix} 0.047 \\ 0.030 \end{pmatrix},$$

where the first one refers to the calculation using the $^3P_0$ model and the second one comes from the microscopic model. Our final results can be seen in Table X.

Besides we predict the ratio 

$$\frac{\Gamma(D_{s2}^* \rightarrow DK)}{\Gamma(D_{s2}^* \rightarrow DK) + \Gamma(D_{s2}^* \rightarrow D^*K)} = \begin{pmatrix} 0.91 \\ 0.94 \end{pmatrix},$$

VI. CONCLUSIONS
\[ B(B^0 \rightarrow D_{s1}(2536)^- \mu^+ \nu_\mu)B(D_{s1}(2536)^- \rightarrow D^{*-} \bar{K}^0) \]

has been determined [4] using the experimental data on
\[ B(b \rightarrow B^0_s)B(B^0 \rightarrow D_{s1}(2536)^- \mu^+ \nu_\mu)B(D_{s1}(2536)^- \rightarrow D^{*-} \bar{K}^0) \]
measured by the D0 Collaboration [3] and the PDG’s best value for \( B(b \rightarrow B^0_s) \) [4].

Our results for \( B \) semileptonic decays into \( D_s^0(2400), D_s(2420) \) and \( D_s(2460) \) are in good agreement with the experimental data. Note however the disagreement between BaBar and Belle data for the neutral case.

In the case of \( B_s \) semileptonic decays, our prediction for the \( B(B^0_s \rightarrow D_{s1}(2536)^- \mu^+ \nu_\mu)B(D_{s1}(2536)^- \rightarrow D^{*-} \bar{K}^0) \) product of branching fractions is in agreement with the experimental data. This, together with the properties calculated in Ref. [12], is to us evidence of a dominant \( q\bar{q} \) structure for the \( D_{s1}(2536) \) meson. We also give predictions for decays into other \( D_{s*}^0 \) mesons which can be useful to test the \( q\bar{q} \) nature of these states.

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Appendix A: Form factor decomposition of hadronic matrix elements

Here we give general expressions valid for transitions between a pseudoscalar meson \( M_f \) at rest with quark content \( \bar{q}_f \), a final \( M_F \) meson with total angular momentum and parity \( J^P = 0^+, 1^+, 2^+ \), three-momentum \(-|\bar{q}|\hat{k} \), and quark content \( \bar{q}_f \). The transition changes the antiquark flavor. Following Ref. [22] we evaluate \( V^\alpha_\lambda(|\bar{q}|) \) and \( A^\alpha_\lambda(|\bar{q}|) \) in the CQM through the relations

\[ V^\alpha_\lambda(|\bar{q}|) = \sqrt{2m_f}E_F(-\bar{q})\int d^3p\frac{1}{4\pi|p|}\left(\tilde{\phi}^{(M(0^+))}_{f_1 f_2}(|p|)\right)\left(\tilde{\phi}^{(M(0^-))}_{f_1 f_2}(|p|)\right)\left(\frac{m_f}{m_f + m_{f_2}}\bar{q}\bar{k} - \bar{p}\right) \]
\[ \textrm{and} \]
\[ A^\alpha_\lambda(|\bar{q}|) = \sqrt{2m_f}E_F(-\bar{q})\int d^3p\frac{1}{4\pi|p|}\left(\tilde{\phi}^{(M(0^+))}_{f_1 f_2}(|p|)\right)\left(\tilde{\phi}^{(M(0^-))}_{f_1 f_2}(|p|)\right)\left(\frac{m_f}{m_f + m_{f_2}}\bar{q}\bar{k} - \bar{p}\right) \]

For the different cases under study we will have the following.

1. Case \( 0^- \rightarrow 0^+ \)

\[ A^0(|\bar{q}|) = \sqrt{2m_f}E_F(-\bar{q})\int d^3p\frac{1}{4\pi|p|}\left(\tilde{\phi}^{(M(0^+))}_{f_1 f_2}(|p|)\right)\left(\tilde{\phi}^{(M(0^-))}_{f_1 f_2}(|p|)\right)\left(\frac{m_f}{m_f + m_{f_2}}\bar{q}\bar{k} - \bar{p}\right) \]
\[ \times \int d^3p\frac{1}{4\pi|p|}\left(\tilde{\phi}^{(M(0^+))}_{f_1 f_2}(|p|)\right)\left(\tilde{\phi}^{(M(0^-))}_{f_1 f_2}(|p|)\right)\left(\frac{m_f}{m_f + m_{f_2}}\bar{q}\bar{k} - \bar{p}\right) \]

\[ \times \left\{ \frac{1}{E_{f_1}E_{f_2}} \left[ \left( -\frac{m_{f_1}}{m_{f_1} + m_{f_2}}\bar{q}\bar{k} - \bar{p} \right) \left( -\frac{m_{f_2}}{m_{f_1} + m_{f_2}}\bar{q}\bar{k} - \bar{p} \right) \right] + \frac{1}{E_{f_1}E_{f_2}} \left[ \left( -\frac{m_{f_1}}{m_{f_1} + m_{f_2}}\bar{q}\bar{k} - \bar{p} \right) \right] \right\}. \]
$E_{f_1}$ and $E_{f_1}$ are shorthand notations for $E_{f_1}(-\frac{m_{f_1}'}{m_{f_1}+m_{f_2}}|\vec{q}|\vec{k}-\vec{p})$ and $E_{f_1}(\frac{m_{f_2}'}{m_{f_1}+m_{f_2}}|\vec{q}|\vec{k}-\vec{p})$ respectively and $E_f = E_f + m_f$.

2. Case $0^- \rightarrow 1^+$

Here we have to distinguish two different cases that depend on the total spin $S$ of the quark-antiquark system.

i) Case $S = 0$

\[
V_{\lambda=0}^{(1^+,S=0)}(|q|) = -i\sqrt{3}\sqrt{2m_1E_F(-q)} \int d^3p \sqrt{\frac{E_{f_1'}E_{f_1}}{4E_{f_1'}E_{f_1}}} \frac{1}{4\pi p} \left( \phi_{f_1',f_2}^{(M_F(1^+,S=0))}(p) \right)^* \times \phi_{f_1,f_2}^{(M_f(0^-))}(|\vec{p} - \frac{m_{f_2}}{m_{f_1}+m_{f_2}}|\vec{q}|\vec{k}|) p_z \left[ 1 + \frac{\left( -\frac{m_{f_1}'}{m_{f_1}+m_{f_2}}|\vec{q}|\vec{k} - \vec{p} \right) \cdot \left( \frac{m_{f_2}'}{m_{f_1}+m_{f_2}}|\vec{q}|\vec{k} - \vec{p} \right)}{E_{f_1'}E_{f_1}} \right],
\]

\[
V_{\lambda=-1}^{(1^+,S=0)}(|q|) = i\sqrt{3}\sqrt{2m_1E_F(-q)} \int d^3p \sqrt{\frac{E_{f_1'}E_{f_1}}{4E_{f_1'}E_{f_1}}} \frac{1}{4\pi p} \left( \phi_{f_1',f_2}^{(M_F(1^+,S=0))}(p) \right)^* \times \phi_{f_1,f_2}^{(M_f(0^-))}(|\vec{p} - \frac{m_{f_2}}{m_{f_1}+m_{f_2}}|\vec{q}|\vec{k}|) p_z \left( \frac{1}{E_{f_1}} + \frac{1}{E_{f_1'}} \right),
\]

\[
V_{\lambda=0}^{(1^+,S=0)}(3)|q|) = -i\sqrt{3}\sqrt{2m_1E_F(-q)} \int d^3p \sqrt{\frac{E_{f_1'}E_{f_1}}{4E_{f_1'}E_{f_1}}} \frac{1}{4\pi p} \left( \phi_{f_1',f_2}^{(M_F(1^+,S=0))}(p) \right)^* \times \phi_{f_1,f_2}^{(M_f(0^-))}(|\vec{p} - \frac{m_{f_2}}{m_{f_1}+m_{f_2}}|\vec{q}|\vec{k}|) p_z \left( \frac{\frac{m_{f_2}'}{m_{f_1}+m_{f_2}}|\vec{q}| - p_z}{E_{f_1}} - \frac{\frac{m_{f_1}'}{m_{f_1}+m_{f_2}}|\vec{q}| + p_z}{E_{f_1'}} \right),
\]

\[
A_{\lambda=-1}^{(1^+,S=0)}(|q|) = -i\sqrt{3}\sqrt{2m_1E_F(-q)} \int d^3p \sqrt{\frac{E_{f_1'}E_{f_1}}{4E_{f_1'}E_{f_1}}} \frac{1}{4\pi p} \left( \phi_{f_1',f_2}^{(M_F(1^+,S=0))}(p) \right)^* \times \phi_{f_1,f_2}^{(M_f(0^-))}(|\vec{p} - \frac{m_{f_2}}{m_{f_1}+m_{f_2}}|\vec{q}|\vec{k}|) \frac{p_z^2|\vec{q}|}{E_{f_1}E_{f_1}'},
\]

(A3)

ii) Case $S = 1$
\begin{align}
V^{(1+,S=1)}_{\lambda=0}(|\vec{q}|) &= i\sqrt{\frac{3}{2}} \sqrt{2m_12E_F(-\vec{q})} \int d^3p \sqrt{\frac{E_{f'_1}E_{f_1}}{4E_{f'_1}E_{f_1}}} \frac{1}{4\pi p} \left( \phi^{(M_F(1+,S=1))}_{f'_1f_2}(p) \right)^* \\
&\times \phi^{(M_f(0^-))}_{f'_1f_2}(|\vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}}|\vec{q}||\vec{k}|) |\vec{q}||p_x^2 + p_y^2 + p_z^2| |\vec{q}||m_{f'_1} + m_{f_2}| E_{f'_1}, \\
V^{(1+,S=1)}_{\lambda=-1}(|\vec{q}|) &= -i\sqrt{\frac{3}{2}} \sqrt{2m_12E_F(-\vec{q})} \int d^3p \sqrt{\frac{E_{f'_1}E_{f_1}}{4E_{f'_1}E_{f_1}}} \frac{1}{4\pi p} \left( \phi^{(M_F(1+,S=1))}_{f'_1f_2}(p) \right)^* \\
&\times \phi^{(M_f(0^-))}_{f'_1f_2}(|\vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}}|\vec{q}||\vec{k}|) (p_x^2 + p_y^2) \left( \frac{1}{E_{f'_1}} - \frac{1}{E_{f_1}} \right), \\
V^{(1+,S=1)}_{\lambda=0}(3)(|\vec{q}|) &= i\sqrt{\frac{3}{2}} \sqrt{2m_12E_F(-\vec{q})} \int d^3p \sqrt{\frac{E_{f'_1}E_{f_1}}{4E_{f'_1}E_{f_1}}} \frac{1}{4\pi p} \left( \phi^{(M_F(1+,S=1))}_{f'_1f_2}(p) \right)^* \\
&\times \phi^{(M_f(0^-))}_{f'_1f_2}(|\vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}}|\vec{q}||\vec{k}|) |\vec{q}||\vec{k}| (|\vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}}|\vec{q}|) \\
&\times \left\{ p_z \left[ 1 - \frac{\left( -\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} \frac{1}{m_{f'_1} + m_{f_2}} |\vec{q}||\vec{k} - \vec{p}| \right)}{E_{f'_1}E_{f_1}} \right] + \frac{m_{f_2} - m_{f'_1}}{m_{f'_1} + m_{f_2}} \frac{p_z^2|\vec{q}|}{E_{f'_1}E_{f_1}} \right\}. \quad (A4)
\end{align}
3. Case $0^+ \rightarrow 2^+$

Here we have to distinguish between $L = 1$ and $L = 3$.

i) Case $L = 1$

\[
V_{\lambda=1}^{(2^+,L=1)}(|\vec{q}|) = -i \frac{\sqrt{3}}{2} \sqrt{2m_f 2E_F} \left( \frac{1}{4E_f' E_f 4\pi p} \left( \phi_{M_1(0^-)}^{(M_1(2^+,L=1))}(p) \right)^* \right.
\]
\[
\times \phi_{f_1f_2}^{(M_1(0^-))}(|\vec{p}| - \frac{m_f}{m_f + m_f} |\vec{q}| |\vec{p}|) \left( \frac{p^2 - p^2 - p_2 |\vec{q}| m_f}{E_f'} - \frac{p_y^2 + p_y^2 + p_2 |\vec{q}| m_f}{E_f} \right),
\]

\[
A_{\lambda=0}^{(2^+,L=1)}(|\vec{q}|) = -i \frac{\sqrt{3}}{2} \sqrt{2m_f 2E_F} \left( \frac{1}{4E_f' E_f 4\pi p} \left( \phi_{M_1(0^-)}^{(M_1(2^+,L=1))}(p) \right)^* \right.
\]
\[
\times \phi_{f_1f_2}^{(M_1(0^-))}(|\vec{p}| - \frac{m_f}{m_f + m_f} |\vec{q}| |\vec{p}|) \left( p^2 + p^2 - 2p_2 |\vec{q}| m_f \right) \left( \frac{m_f}{E_f'} \right),
\]

\[
A_{\lambda=1}^{(2^+,L=1)}(|\vec{q}|) = i \frac{\sqrt{3}}{2} \sqrt{2m_f 2E_F} \left( \frac{1}{4E_f' E_f 4\pi p} \left( \phi_{M_1(0^-)}^{(M_1(2^+,L=1))}(p) \right)^* \right.
\]
\[
\times \phi_{f_1f_2}^{(M_1(0^-))}(|\vec{p}| - \frac{m_f}{m_f + m_f} |\vec{q}| |\vec{p}|) \left( 1 - \left( \frac{m_f}{m_f + m_f} \right) \right) \left( \frac{m_f}{E_f'} \right),
\]

\[
A_{\lambda=0}^{(2^+,L=1)}(|\vec{q}|) = -i \sqrt{2} \sqrt{2m_f 2E_F} \left( \frac{1}{4E_f' E_f 4\pi p} \left( \phi_{M_1(0^-)}^{(M_1(2^+,L=1))}(p) \right)^* \right.
\]
\[
\times \phi_{f_1f_2}^{(M_1(0^-))}(|\vec{p}| - \frac{m_f}{m_f + m_f} |\vec{q}| |\vec{p}|) \left( 1 - \left( \frac{m_f}{m_f + m_f} \right) \right) \left( \frac{m_f}{E_f'} \right),
\]

ii) Case $L = 3$

\[
(p^2 + p_y^2 \left( -p^2 + \frac{m_f}{2(m_f + m_f)} \right) \right). \quad (A5)
\]
\[ V_{\lambda=+1}^{(2^+, L=3)}(\{q\}) = \frac{i}{\sqrt{8}} \sqrt{2m_1 E_F(-q)} \int d^3p \sqrt{E_{f_1} E_{f_2}} \frac{1}{4E_{f_1} E_{f_2} 4\pi p^3} (\phi^{(M(2^+, L=3))}_{f_1, f_2}(p))^* \]
\[ \times \phi^{(M(0^-))}_{f_1, f_2}(\mid p - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \mid q \mid \vec{k} \mid) \]
\[ \times \left[ \frac{1}{E_{f_1}} \left( p^2 \left( 2p_y - 3p_z \left( \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \mid q \mid - p_z \right) \right) \right) + 5p_z \left( -2p_y^2 p_z + \left( \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \mid q \mid - p_z \right) (p_x^2 - p_y^2 + p_z^2) \right) \right] \]
\[ + \frac{1}{E_{f_1}} \left( p^2 \left( -2p_y^2 + 3p_z \left( -\frac{m_{f_1}}{m_{f_1} + m_{f_2}} \mid q \mid - p_z \right) \right) \right) \]
\[ - 5p_z \left( -2p_y^2 p_z + \left( -\frac{m_{f_1}}{m_{f_1} + m_{f_2}} \mid q \mid - p_z \right) (p_x^2 - p_y^2 + p_z^2) \right) \right] \]
\[ A^{(2^+, L=3)}_{\lambda=0}(\{q\}) = -i \sqrt{\frac{3}{4}} \sqrt{2m_1 E_F(-q)} \int d^3p \sqrt{E_{f_1} E_{f_2}} \frac{1}{4E_{f_1} E_{f_2} 4\pi p^3} (\phi^{(M(2^+, L=3))}_{f_1, f_2}(p))^* \]
\[ \times \phi^{(M(0^-))}_{f_1, f_2}(\mid p - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \mid q \mid \vec{k} \mid) \]
\[ \times \left[ \frac{5p_y^2}{p^2} - 3 \left( \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \mid q \mid - p_z \right) - \frac{m_{f_1}}{m_{f_1} + m_{f_2}} \mid q \mid + p_z \right] \]
\[ \frac{1}{E_{f_1}} \left( p^2 \left( p_x^2 + p_y^2 - \left( \frac{m_{f_1}}{m_{f_1} + m_{f_2}} \mid q \mid - p_z \right) \right) \right) \]
\[ - p_z \left( \frac{5p_y^2}{p^2} - 3 \right) \left( 1 - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \mid q \mid - p_z \right) \]
\[ A^{(2^+, L=3)}_{\lambda=1}(\{q\}) = -i \sqrt{\frac{3}{8}} \sqrt{2m_1 E_F(-q)} \int d^3p \sqrt{E_{f_1} E_{f_2}} \frac{1}{4E_{f_1} E_{f_2} 4\pi p^3} (\phi^{(M(2^+, L=3))}_{f_1, f_2}(p))^* \]
\[ \times \phi^{(M(0^-))}_{f_1, f_2}(\mid p - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \mid q \mid \vec{k} \mid) \]
\[ \times \left[ \frac{m_{f_2} - m_{f_1}}{m_{f_1} + m_{f_2}} \mid q \mid - 2p_z \right) \]
\[ \left( \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \mid q \mid - p_z \right) \]
\[ \frac{2p_z^2}{p^2} - 1 \left( \frac{m_{f_2} - m_{f_1}}{m_{f_1} + m_{f_2}} \mid q \mid - 2p_z \right) \]
\[ + 2p_z \left( \frac{5p_y^2}{p^2} - 1 \right) \left( \frac{m_{f_2} - m_{f_1}}{m_{f_1} + m_{f_2}} \mid q \mid - 2p_z \right) \]
\[ + 20 \frac{p_z p_y^2}{E_{f_1} E_{f_2} p^2} \right] \]
\[ (A6) \]
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