Three and two-hadron correlations in $\sqrt{s_{NN}} = 200$ GeV proton-proton and nucleus-nucleus collisions

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We compare the azimuthal correlations arising from three and two hadron production in high energy proton-proton and nucleus-nucleus collisions at $\sqrt{s_{NN}} = 200$ GeV, using the leading order matrix elements for two-to-three and two-to-two parton-processes in perturbative QCD. We first compute the two and three hadron production cross sections in mid-rapidity proton-proton collisions. Then we consider Au + Au collisions including parton energy loss using the modified fragmentation function approach. By examining the geometrical paths the hard partons follow through the medium, we show that the two away-side partons produced in two-to-three processes have in average a smaller and a greater path length than the average path length of the away-side parton in two-to-two processes. Therefore there is a large probability that in the former processes one of the particles escapes while the other gets absorbed. This effect leads to an enhancement in the azimuthal correlations of the two-to-three with respect to the two-to-two parton-processes when comparing to the same processes in proton-proton collisions since in average the particle with the shortest path length loses less energy with respect to the away side particle in two-to-two processes. We argue that this phenomenon may be responsible for the shape of the away-side in azimuthal correlations observed in mid-rapidity Au + Au collisions at RHIC.

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Many interesting phenomena have been observed in high energy heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) since it began operation almost a decade ago. The suppression of the single hadron transverse momentum spectra in Au + Au collisions, as compared to normalized p + p collisions, is one of such phenomena. This is believed to result from the energy loss of fast partons traversing the medium and multiply scattering (both elastically and inelastically) from medium constituents. This picture of energy loss was further confirmed by azimuthal correlation studies where for certain combinations of leading and associated particles a disappearance of the away-side peak is observed. This is interpreted as the absorption of the away side parton by the medium. Combined with an enhancement of the single hadron production in d + Au collisions as compared to normalized p + p collisions, these observations established the presence of a final state medium produced in Au + Au collisions at RHIC. Furthermore, the strong collectivity of the medium, as measured by the flow parameter $v_2$, among other observations, have led to the consensus that the medium is partonic in origin as well as strongly interacting.

In spite of the success of the above interpretation of RHIC data, there still remain important aspects of energy loss dynamics which are poorly understood. The picture has been further complicated by the recent observation in azimuthal correlation studies in Au + Au collisions, where for the case when the magnitude of the momentum difference between leading and associated particles increases, either a double hump structure or a broadening of the away-side peak appears, while these structures are absent in p + p collisions at the same energy. This observation gave rise to a large number of theoretical explanations which are based on the assumption that collective phenomena are at work in A + A collisions in contrast to the case of p + p collisions. A hydrodynamic approach is used to attribute the double hump structure in the away side jet to the propagation of pressure gradients from dense zones in the plasma which can only be observed by averaging over a large number of random initial conditions and after subtracting the flow component of the azimuthal correlation function.

However, one can ask whether these features are already present at the level of p + p collisions, albeit obscured by the smallness of their intensity. For instance, two-to-three parton processes would lead to such a double hump structure even in p + p collisions, although these would be suppressed with respect to two-to-two parton processes. The question is whether in A + A collisions there exists a mechanism that amplifies hadron production from two-to-three processes with respect to that from two-to-two parton processes. This question has been recently partially investigated in Ref. using the event shape analysis in p + p collisions to distinguish events containing three jets from those containing two.

In this Letter, we study angular correlations between
three hadrons produced both in p + p and A + A collisions at RHIC using the full Leading Order (LO) matrix elements [13] for two-to-three parton processes and compare these to angular correlations between two hadrons using also LO matrix elements for two-to-two parton processes. To the best of our knowledge, this is the first study of its kind using the complete LO matrix elements to calculate three hadron production in A + A collisions. We should mention that the implementation of Next-to-Leading Order (NLO) matrix elements for two-to-three processes requires the corresponding nuclear parton distributions and medium modified fragmentation functions to matching accuracy. The former are available in the literature [12], the latter are not fully known. Therefore here we use the LO matrix elements, given that we focus on the nuclear effects. We use a medium modified fragmentation function in order to treat the medium-induced parton energy loss, following the approach of Ref. [16] where two-hadron production cross section in A + A collisions is studied. We argue that since in A + A collisions, the final state partons with the short (long) trajectory in the away side has a large probability to punch through (get absorbed), the effect produces, on the average, a double hump in the azimuthal correlation.

Using the LO matrix elements for two-to-three parton processes, including the final state phase space factors and enforcing momentum conservation, the three hadron production cross section can be written as

\[
\frac{d\sigma_{pp\to h1h2h3X}}{dy_1dy_2dy_3h_{11}d h_{22}dh_{33}d \phi_2d \phi_3} = \frac{1}{2^8(2\pi)^4}S_{3t} \int |M|^2 d\phi_2 |\sin \phi_2| d y_1d y_2d y_3h_{11}d h_{22}dh_{33}d \phi_2d \phi_3 \times f_{j/p}(x_1, \mu^2)f_{j/p}(x_2, \mu^2)D_{h1/k}^0(z_1, \mu^2)D_{h2/m}^0(z_2, \mu^2)D_{h3/n}^0(z_3, \mu^2),
\]

where \( y_i, h_{it} \) are the rapidity and magnitude of transverse momentum of the \( i \)-th produced hadron, \( \phi_2 (\phi_3) \) is the azimuthal angle between the second (third) hadron and the first hadron, assumed to be the leading hadron and \( \sqrt{S} \) is the total center of mass energy. The parton distribution functions are denoted by \( f_{j/p}(x, \mu^2) \) and are given by the CTEQ6 parameterization [17]. \( D_{h1/k}^0(z, \mu^2) \) are the unmodified fragmentation functions given by the KKP parameterization [18]. All along this work, the scale \( \mu^2 \) is taken to be the same for all the distribution and fragmentation functions and given as the invariant mass squared of the final state hadrons, which we take as pions. We will focus on the mid-rapidity region and thus, hereafter, set all rapidities equal to zero. The momentum fractions \( x_1, x_2, z_1, z_2 \) are all given in terms of \( z_3 \) and the transverse momenta of the produced hadrons. Furthermore, requiring that all momentum fractions are between 0 and 1 introduces the lower and upper limits on the \( z_3 \) integration. Further details will be provided in Ref. [19].

We now use Eq. (1) to calculate the three hadron production cross section in p + p for mid-rapidity at \( \sqrt{S} = 200 \) GeV. The open circles on the left panel of Fig. 1 show the differential cross section as a function of the azimuthal angle \( \phi \) for the particular case where each of the three hadrons carry transverse momentum of \( h_i = 10 \) GeV/c and therefore, due to momentum conservation on partonic level, they are separated by an angle of \( \Delta \phi = 2\pi/3 \) radians. Only the angular distribution of the away side hadrons (with respect to the direction of one of the hadrons—the would be leading hadron—which in the symmetric case chosen here, can be any one of them) is shown.

In order to investigate the effects of a medium on three hadron production in A + A collisions, we use the modified fragmentation function proposed in [16] given by

\[
D_{h/k}(z, \mu^2) = (1 - e^{-\langle \hat{z}_i \rangle}) \left[ z_i' D_{h/k}^0(z, \mu^2) + \frac{L}{\lambda} \frac{\hat{z}_i'}{z_i} \right] \times D_{h/g}^0(z'_{g}, \mu^2) + e^{-\langle \hat{z}_i \rangle} D_{h/k}^0(z, \mu^2),
\]

where \( z_i' = \frac{h_i}{(b_{\perp} - \Delta E_i)} \) is the rescaled momentum fraction of the leading parton with flavor \( i \), \( z_i' = \langle \hat{z}_i \rangle \frac{1}{\langle \hat{z}_i \rangle} \) is the rescaled momentum fraction of the radiated gluon, \( \Delta E_i \) is the average radiative parton energy loss and \( \langle \hat{z}_i \rangle \) is the average number of scatterings. The energy loss \( \Delta E_i \) is related to the gluon density of the produced medium via

\[
\Delta E = \frac{dE}{dL} \int d\tau \frac{\tau - \tau_0}{\rho_0 \rho} \rho_g(\tau, \delta_{\perp}, \tau_t + n\tau),
\]

where \( \delta_{\perp} \) (note that \( b_0 \) is used to denote the transverse momentum of a parton) is the impact parameter of the collision. Since here we consider the most central collisions, we set \( \delta_{\perp} = 0 \) in the rest of the paper. \( \tau_t \) is the transverse plane location of the hard scattering where the partons are produced and \( \vec{n} \) is the direction in which the produced hard parton travels in the medium. The average number of scatterings \( \langle \hat{z}_i \rangle \), the one dimensional energy loss \( \langle \frac{dE}{dL} \rangle_{\perp} \) and the gluon density of the medium \( \rho_g \) which is related to the nuclear geometry of the produced medium are taken from Ref. [16] where we refer the reader also for details on the chosen parameters.

The three produced partons will in general travel through the medium at different angles with respect to the direction of \( \vec{r}_1 \) until they leave the medium after which they hadronize. Therefore, the fragmentation function
for each produced parton will have a different dependence on the path length traveled through the medium. Based on simple geometry one can show that out of the three angles involved, only one is independent which we take to be the angle that the direction of motion of particle 1 makes with the position vector of the scattering center. The normalized cross section is obtained from Eq. (1) using the modified fragmentation functions given by Eq. (2), integrating over the location of the hard scatterings and dividing by the nuclear overlap area. Due to the kinematics involved in mid-rapidity collisions, we expect the nuclear modification of parton distribution functions to be small \cite{20,21} and therefore ignore them.

The open circles on the right panel of Fig. 1 show the results of the above analysis for the three hadron production differential cross section as a function of azimuthal angle $\phi$ for mid-rapidity Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. As in the p + p case, we also consider that each of the three hadrons carry momentum with magnitude $h_t = 10$ GeV/c. Since in our picture fragmentation occurs collinear to the direction of the original parton, the final hadrons are separated by an angle $\Delta \phi = 2\pi/3$ radians. One can check that the ratio of the distributions in the Au + Au to the p + p case is as a function of $\phi$ approximately constant $\sim 0.7$.

In order to compare the three-hadron to the two-hadron production differential cross section in the p + p case, we compute the latter as a function of the azimuthal angle $\phi$. To this end we use the LO DIPHOS algorithm \cite{22}. We use CTEQ6 distribution functions and in the p + p case, unmodified fragmentation functions given by the KPP parametrization. For the A + A case, we use the modified fragmentation functions in Eq. (2) integrating over the location of the hard scatterings and dividing by the nuclear overlap area. To have a fair comparison to the already calculated cross section for two-to-three processes, and similar to the experimental situation, we take the leading and away side particles also having $h_t = 10$ GeV/c. The full circles in Fig. 1 show this cross section. On the left panel we show the results for p + p collisions and on the right panel, the results of Au + Au, both for $\sqrt{s_{NN}} = 200$ GeV collisions at mid-rapidity. Notice that in both cases the two-to-three hadron production cross section is suppressed with respect to the two-to-two result. However, and this is the main result of our work, this suppression is smaller in A + A collisions than in p + p collisions. Dividing this ratio in A + A to that in p + p, we get as a function of $\phi$ approximately a constant $\sim 2.26$.

Since in the present analysis, the sole ingredient that distinguishes between the p + p and the A + A cases is the energy loss of partons that hadronize collinearly, then the only reason for the cross sections in two-to-three processes to be less suppressed in A + A than in p + p, when compared to the two-to-two processes, is the different geometry for the trajectories of three as opposed to two particles in the final state.

To test this idea, we compute the distribution of path lengths with two and three hadrons in the final state. We take a nuclear overlap area with a distribution of scattering centers denser in the middle and decreasing toward the edge. In each case we disregard the shortest path length (the one that would correspond to the trigger particle) and compute the distribution of the other path lengths (the ones corresponding to the away side particles). This is shown in Fig. 2. The upper panel shows the distribution of scattering centers and the lower
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In conclusion, we have presented a LO calculation in p + p collisions for the cross section of two-to-three processes and compared to the corresponding cross sections for two-to-two processes, using the same approximations. We have also computed the same processes in A + A collisions taking into account a detailed Glauber approach, similar to the one employed in Ref. [23]. The results show that the cross section in two-to-three processes in A + A is less suppressed —with respect to two-to-two processes— than in the case of p + p. We attribute this result to a purely geometric effect associated with the differences in the path lengths of three as opposed to two particles in the final state in the case of A + A collisions. These results raise several interesting possibilities that need to be addressed, among them we can point out the following:
1) Two-to-three processes are naturally expected and the strength of their signal is not beyond the capabilities of current experiments. 2) Considering that two-to-three processes exist in p + p collisions, as suggested also in Ref. [14], our work shows that their observation in A + A should be enhanced with respect to the strongly energy-loss suppressed two-to-two processes and thus, that this effect may have a bearing on the shape of the away side for different kinematical cuts in azimuthal correlations in A + A. A detailed study of this effect is currently under way and will be reported elsewhere.

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