H-Infinity Model Following Control for Uncertain Systems by Using Digital Redesign Sliding Mode Control

JIUNN-SHIOU FANG1, (Member, IEEE), JASON S. H. TSAI1, (Senior Member, IEEE), JUN-JUH YAN2, P. L. CHEN1, AND SHU-MEI GUO3, (Member, IEEE)  

1Department of Electrical Engineering, National Cheng Kung University, Tainan 701, Taiwan  
2Department of Electronic Engineering, National Chin-Yi University of Technology, Taichung 41107, Taiwan  
3Department of Computer Science and Information Engineering, National Cheng Kung University, Tainan 701, Taiwan  
Corresponding author: Jun-Juh Yan (jjyan@ncut.edu.tw)  

This work was supported by the Ministry of Science and Technology, Taiwan, under Grant MOST 108-2221-E-006-213-MY3 and Grant MOST-109-2221-E-167-017.

ABSTRACT In this article, we discuss the $H_{\infty}$ model following control for uncertain continuous systems by using a discrete sliding mode control (SMC). In contrast to the reports in the literature, the digital redesign technology is newly integrated with SMC such that the traditional sliding mode controller can be discretized and easily implemented with digital signal processing technology. Under the proposed digital redesign SMC, the states of the controlled systems can be controlled and follow the trajectory of the model systems. Furthermore, by using the linear quadratic analog tracker (LQAT) design for model systems, the model system also can track the desired trajectory. Besides, to solve the chattering performance, a saturation function is introduced, and the upper-bound of the sliding mode function by utilizing the saturation function is well discussed. Finally, we use a numerical example to illustrate the robustness and effectiveness of the proposed digital redesign SMC in this article.

INDEX TERMS Sliding mode control, H-infinity control, digital redesign, linear quadratic analog tracker, model following control.

I. INTRODUCTION

In the control systems, because the system’s uncertainties and external disturbances can affect the control performance of the overall system, and even cause the failure of control systems, in the research literature, we can find many methods to ensure the stability of the controlled system, such as sliding mode control [1], [2], adaptive control [3], [4], neural network control [5], [6], etc. Especially due to its inherent robustness, the sliding mode control (SMC) has been applied in lots of actual applications to constrain the uncertainties and disturbances. We can see that SMC has widely developed in many papers, such as terminal SMC [7], neuro-fuzzy-SMC [8], and integral SMC [9], etc. However, because of the current advancement of digital signal processing technology and the reduction in the cost of related digital chips, the use of digital controllers to design and achieve control performance for a system has become the current trend. Thus, discussing the discrete control design is an important issue. The distinction between continuous-time SMC and discrete-time SMC (DSMC) is the sampling frequency [10]. There are many papers discussing DSMC [11]–[16] for the discrete systems. Before designing the DSMC, they have to transfer the continuous-time system to the discrete-time system. Then, they can design the digital controller. However, this design approaches increase the stability analysis difficulty when applied to continuous systems. As mentioned above, to solve this problem, we use the digital redesign method to transform the well-designed SMC for the continuous systems to a discrete SMC. By this proposed digital redesign method, we can not only efficiently analyze the controlled performance but also many well-developed designed approaches in the continuous-time domain can be directly applied.

The digital redesign is a technique to convert the continuous-time controller into the discrete-time controller [17], [18]. According to the digital redesign approach, we can easily obtain a discrete SMC from a well-designed continuous SMC. However, till now the digital redesign sliding mode
control (DRSMC) hasn’t been developed well in the literature, therefore the design of DRSMC is worthy to study. Many methods have been proposed for dealing with the tracking control problem while systems have external disturbances. For example, in [19], authors pointed out that the nonlinear perturbations do exist in systems, then the tracking controller design integrated with the adaptive method and fuzzy approach is discussed for processing nonlinear systems. In [20], while the complexity and the high accuracy are needed to be considered for system performances in practical systems, the proposed observer and controller are introduced to cope with these problems for the tracking control. As mentioned above, while systems are with non-linearity and the high accuracy performance is considered, the tracking controller needs to be well-developed to overcome the unexpected disturbances. Therefore, in this article, SMC will be introduced to deal with the unexpected disturbances. To complete the design of DRSMC, we first design a continue-time sliding mode controller to suppress the effect of the disturbances and uncertainties. Then, we discretize the continuous-time SMC by using the digital redesign approach based on LQAT. With the Euler approach, we also ensure that the hitting condition is satisfied, and the trajectories of the systems controlled by the proposed DRSMC can also converge to the sliding manifold.

However, the traditional SMC has the chattering problem, then we use the saturation function [12] to solve the problem. But, using the saturation function can’t let sliding mode function always be zero. Thus, we have to discuss the upper-bound of the sliding mode function and ensure the sliding mode function will converge within the predicted upper-bound. Thus, it’s worth exploring issues, and we analyze it in this article.

On the other hand, mismatched disturbances frequently occur in the control applications, thus it’s important to discuss the suppression for the effect of mismatched disturbances. Nowadays, many written works have discussed mismatched disturbances [1], [2], [12], [21]. $H_\infty$ control is concerned in lots of studies and this strategy can not only preserve the control performance but also suppress the mismatched disturbances. Besides, the authors indicate that the mismatch disturbances cannot be eliminated perfectly in the SMC; therefore, the effect of mismatched disturbances to the controlled system and the bound are discussed in [12]. When we design the $H_\infty$ control, we have to ensure the stability of the systems. Many control design methods were frequently adopted, such as ADP approach [21], Riccati approach [22], and linear matrix inequality (LMI) approach [23]. Besides, the $H_\infty$ problem can be applied to the filter design by using LMI [24]. Therefore, solving the $H_\infty$ problem is an important topic to deal with external disturbances and noises. However, we don’t have enough information about perturbations. Thus, we need to have a disturbances observer to estimate the external disturbances [25]–[28]. In [25], the authors considered a system with the external disturbance and the proposed sliding mode observer (SMO) for dealing with the system state, external disturbance, and fault-tolerant control by using the augmented method. In [26], it explores the disturbance is slowly time-varying in the continuous-time and design the disturbances estimator with SMC. In the works [27], [28], they discuss the disturbances observers in discrete time. In [29], the authors propose a one-step delay estimator to observe the external slow and smooth disturbance, and the estimator error belongs to $O(T_\epsilon)$ defined in Lemma 2. Due to the disturbance varies slowly, we can design the disturbance estimator and let the difference of compensation and disturbances converge to zero closely. In this article, the LMI-based approach will be introduced to solve the $H_\infty$ problem and the disturbance estimator will be designed to ensure the stability of the system.

Based on our knowledge, the digital redesign for the $H_\infty$ model following control by using DRSMC has not been well developed so far. For the proposed hybrid-controlled systems in this article, since the continuous-time SMC-based controller has been discussed well, the digital redesign approach can be utilized to discretize the controller directly such that the controller design process becomes easy. By applying the developed digital redesign method in this article, the reaching condition can be adopted to ensure the existence of the sliding surface on the sampling time. Hence, the continuous-time $H_\infty$ model following controller can be transferred to the discrete-time $H_\infty$ model following controller and the performance of the original continuous-time controller is remained as possible.

As mentioned above, the contributions of this article are highlighted below. Based on the model-following control strategy, we propose a digital-redesign SMC-based controller for suppressing the matched/mismatched disturbances. Besides, while the saturation function is adopted to reduce the undesirable chattering in traditional SMC, we discuss the effect and the upper-bound for the SMC by using saturation function; therefore, the upper-bound of the sliding mode function is well analyzed in this article.

The framework of the paper is given as follows. Section 2 is the system description. Section 3 is the method of DRSMC. Section 4 includes an illustrative example. Section 5 is the conclusion.

Notation. $w^T$ denotes the transport for a matrix $w$. $\|w\|$ is the Euclidean norm of the vector. $I_n \in R^{n \times n}$ denotes the identity matrix. $|w|$ represents the absolute value for $w$. $w^\dagger$ denotes to the pseudo inverse for a matrix $w$. $\text{sgn}(w) = [\text{sgn}(w_1), \text{sgn}(w_2), \cdots, \text{sgn}(w_m)]^T \in R^m$ and $\text{sgn}(w)$ is the sign function of $w$, if $w > 0$, $\text{sgn}(w) = 1$; if $w < 0$, $\text{sgn}(w) = -1$.

II. DESCRIPTION OF HYBRID MODEL-FOLLOWING CONTROL SYSTEM AND PROBLEM FORMULATION

The model following control system includes two subsystems, the controlled system, and the model system. In this article, we discuss the hybrid model following control design. The configuration of a hybrid model-following control system is given in Figure 1. A DRSMC is utilized to control...
a continue-time system and ensures the output signal of controlled systems can track the states and reference signals in the model system. In Figure 1, we can find the digital controller is designed to control the continue-time system and the one-step delay estimator is used to repress the mismatched disturbances. By the way, we also use the \( E_{md} \) and \( K_{ind} \) to make sure the model system output can track the reference signal. Since we use the digital controller to control the continue-time system, we apply Z.O.H. to transfer the reference signal. Since we use the digital controller to control the continue-time system, we apply Z.O.H. to transfer the reference signal.

As shown in Figure 1, the controlled system output including the matched/unmatched disturbances are described as follows

\[
\dot{x}(t) = Ax(t) + B_1 (g(x(t), t) + f(x(t), t) + u_c(t)) + B_2 d(t),
\]

\[
y(t) = Cx(t),
\]

where \( A \in \mathbb{R}^{n \times n} \), \( B_1 \in \mathbb{R}^{n \times m} \) and \( C \in \mathbb{R}^{p \times n} \) are the system matrices, \( B_2 \in \mathbb{R}^{n \times r} \) is the mismatched system matrix, \( x(t) \in \mathbb{R}^n \), \( u_c(t) \in \mathbb{R}^m \), \( y(t) \in \mathbb{R}^p \), \( g(x(t), t) \in \mathbb{R}^m \), \( f(x(t), t) \in \mathbb{R}^m \) and \( d(t) \in \mathbb{R}^r \) are the state vector, the input vector, the output vector, the known internal nonlinear uncertainties, the unknown external matched disturbance, and the unknown mismatched disturbance, respectively. Besides the unknown external disturbance satisfies

\[
f(x(t), t) = f_1(x(t), t) + f_2(t)
\]

and is assumed to be bounded by \( \|f_1(x(t), t)\| \leq \alpha_2 \|x(t)\| , \|f_2(t)\| \leq \alpha_3 \). The mismatched disturbance \( d(t) \) is assumed to vary slowly and \( \|d(t)\| \leq \alpha_1 \).

The model system in Figure 1 is described by

\[
\dot{x}_m(t) = Ax_m(t) + B_1 u_{mc}(t),
\]

\[
y_m(t) = Cx_m(t),
\]

where \( x_m(t) \in \mathbb{R}^n \), \( u_{mc}(t) \in \mathbb{R}^m \) and \( y_m(t) \in \mathbb{R}^p \) are the model system state vector, the model system input vector, and the model system output vector, respectively.

For the model following control considered in this article, we aim to design a hybrid \( H_\infty \) controller not only to force the controlled system output (1b) to track the model system output (2b) but also to track a desired reference trajectory. To achieve this control goal, we can first consider the tracker design with LQAT [30] for the model system (2), then the model system input can be designed as

\[
u_{mc}(t) = -K_{mc} x_m(t) + E_{mc} r(t),
\]

where \( K_{mc} \) is the feedback control gain, \( E_{mc} \) is the forward control gain, and \( r(t) \in \mathbb{R}^p \) is the desired reference trajectory. Similar to the design procedures in [30], to complete LQAT for the model system, the following cost function is considered

\[
J_m = \frac{1}{2} \int_0^{t_{end}} \left\{ e_{ym}^T(\tau) Q_m e_{ym}(\tau) + u_{mc}^T(\tau) R_m u_{mc}(\tau) \right\} d\tau,
\]

where \( e_{ym}(\tau) = y_m(\tau) - r(\tau), Q_m = 10^4 I_p, q \geq 0, \) is a positive definite or a positive semidefinite real symmetric matrix and \( R_m \) is a positive define matrix. As mentioned above, to calculate the gain of \( K_{mc} \), the Riccati equation is given as

\[
A^T P_m + P_m A - P_m B R_m^{-1} B^T P_m + C^T Q_m C = 0, \tag{5}
\]

where \( P_m \) is the positive symmetric define matrix satisfying the Riccati equation of (5). Hence, the feedback gain can be calculated as \( K_{mc} = R_m^{-1} B^T P_m \). Therefore, the forward gain can be designed as \( E_{mc} = -R_m^{-1} B^T \left[ (A - B K_{mc})^T \right]^{-1} C^T Q_m \) [30].

**Remark 1:** For the square system, if the control law is design as \( u_{mc}(t) = -K_{mc} x_m(t) + r(t) \) by forcing the forward gain \( E_{mc} = I_p \) in (3), then it reduces to the traditional model following control.

### III. DIGITAL REDESIGN SLIDING MODE CONTROLLERS

In this section, we will discuss the design of a hybrid \( H_\infty \) model following control for uncertain continuous systems by using a DRSMC. To complete the design, we first design continuous SMC and then use the digital redesign technology to discretize the continuous SMC to achieve a hybrid \( H_\infty \) model following control. The detailed design is described as follows.

#### A. CONTINUOUS \( H_\infty \) SLIDING MODE CONTROL DESIGN

Before we design the SMC, we calculate the following error dynamic equation from (1a) and (2a)

\[
\dot{e}(t) = Ae(t) + B_1 (g(x(t), t) + f(x(t), t) + u_c(t) - u_{mc}(t)) + B_2 d(t),
\]

where \( e(t) = x(t) - x_m(t) \).

Here we firstly design a controller \( u_c(t) \) in (6) to ensure that the states of controlled systems (1) can be forced to follow the states of model systems (2). To complete the design of the SMC \( u_c(t) \), it is necessary to design a proper sliding mode function \( s(t) \) and ensure \( e(t) \) can converge to the sliding mode (i.e. \( s(t) = 0 \)). Then, we design the SMC \( u_c(t) \) to guarantee the state trajectory of controlled states can be forced to the sliding manifold as expected. To complete the above design,
According to (6), we first select the sliding mode function as follows

\[ s(t) = C_s e(t) + \int (-C_s A e(t) + K_c e(t)) dt, \]  

(7)

where \( C_s = B_1^T \) and \( K_c \) is the controller gain to be designed. Based on the SMC theorem, if the model following error \( e(t) \) can converge to the sliding manifold (i.e., \( s(t) = 0 \) and \( \dot{s}(t) = 0 \)), we can obtain the equivalent control law by using the relation of \( s(t) = 0 \) and \( \dot{s}(t) = 0 \). Thus, one has

\[ \dot{s}(t) = C_s (A e(t) + B_1 (g(x(t), t) + f(x(t), t) + u_{ceq}(t)) - u_{mc}(t))) + C_s B_2 d(t) - C_s A e(t) + K_c e(t), \]

then, one has

\[ \dot{s}(t) = g(x(t), t) + f(x(t), t) + u_{ceq}(t) - u_{mc}(t) + C_s B_2 d(t) + K_c e(t) = 0 \]  

(8)

and the equivalent control law can be obtained as

\[ u_{ceq}(t) = -g(x(t), t) - f(x(t), t) - K_c e(t) + u_{mc}(t) - C_s B_2 d(t). \]  

(9)

Substituting (9) into (6), and we can have the equivalent dynamics in the sliding manifold as

\[ \dot{e}(t) = A e(t) + B_1 (-K_c e(t) - C_s B_2 d(t)) + B_2 d(t) = A e(t) + B_1 u_c^*(t) + B_w d(t), \]  

(10)

where \( u_c^*(t) = -K_c e(t) \) and \( B_w = (I_n - B_1 C_s) B_2 \). In (10), to efficiently suppress the mismatched disturbances, we have to design the controller gain \( K_c \) satisfying the \( H_\infty \) tracking performance in the SMC. Next, we discuss the design of controller gain \( K_c \) in Theorem 1.

**Theorem 1:** If the following LMIs are satisfied with specified matrices \( Y \) and \( Z \)

\[
\begin{bmatrix}
Y A^T + A Y - Z^T B_1^T - B_1 Z & B_w Y C^T \\
B^T w & -I_n & 0
\end{bmatrix} < 0,
\]

where \( Y \in \mathbb{R}^{m \times n} \) is a symmetric matrix and \( Z \in \mathbb{R}^{m \times n} \), then the controller gain can be calculated as \( K_c = Z Y^{-1} \) and the equivalent dynamics (10) in the sliding manifold is \( H_\infty \) stable with a pre-selected positive parameter \( \gamma \). One can see Appendix A for detailed proof.

In the following, it is still necessary to design the controller \( u_c(t) \) to guarantee the state trajectory of controlled states can be forced to the sliding manifold to obtain the equivalent dynamics (10) used in Theorem 1. Now, the proposed sliding mode control law can be designed as

\[ u_c(t) = u_c^*(t) + u_{mc}(t) + u_{\pm}(t), \]  

(11)

where \( u_{\pm}(t) = -\gamma_1 s(t) - (\gamma_2 + \gamma_3) \text{sgn}(s(t)), \gamma_1, \gamma_2 \) and \( \gamma_3 \) are positive parameters. Substitute (11) into (8), and then we can get

\[ \dot{s}(t) = \dot{g}(x(t), t) + f(x(t), t) + C_s B_2 d(t) - \gamma_1 s(t) - (\gamma_2 + \gamma_3) \text{sgn}(s(t)). \]  

(12)

To make the trajectories of the controlled system reach the sliding manifold, we consider a Lyapunov function as follows

\[ V(s(t)) = \frac{1}{2} s^T(t) s(t). \]

Taking the derivative of \( V(s(t)) \) yields \( \dot{V}(s(t)) = s^T(t) \dot{s}(t) \). Then, \( \dot{V}(s(t)) = s^T(t)(g(x(t), t) + f(x(t), t) + C_s B_2 d(t) - \gamma_1 s(t) - (\gamma_2 + \gamma_3) \text{sgn}(s(t))). \)

Also, one has

\[ \dot{V}(s(t)) \leq \|g(x(t), t)\| + \alpha_2 \|x(t)\| + \alpha_3 + \alpha_1 \|C_c\| \|B_2\| \|s(t)\| - \gamma_1 \|s(t)\|^2 - \gamma_2 \|s(t)\| - \gamma_3 \|s(t)\|, \]

where \( \gamma_3 = \|g(x(t), t)\| + \alpha_1 \|C_c\| \|B_2\| + \alpha_2 \|x(t)\| + \alpha_3. \) Thus, we have

\[ \dot{V}(s(t)) \leq -\gamma_1 \|s(t)\|^2 - \gamma_2 \|s(t)\| \leq 0, \]

where \( \gamma_1 > 0 \) and \( \gamma_2 > 0 \). According to the Lyapunov theory, we ensure \( \lim_{t \to \infty} V(s(t)) = 0 \) as well as \( \lim_{t \to \infty} s(t) = 0 \). Therefore, the sliding manifold is guaranteed.

**B. DESIGN DRSMC**

We have completely analyzed the sliding mode in the continuous-time domain. Now, we continue to discuss the sliding mode control in the discrete-time domain by using the digital redesign approach. To ensure the reaching condition in the discrete-time domain, the following Lemma 1 is introduced.

**Lemma 1** [12] The following reaching condition is considered.

For \( s(kT_s) > 0 \),

\[ \Delta s(kT_s) = s(kT_s + T_s) - s(kT_s) \leq -\gamma_1 T_s (s(kT_s) - \gamma_2 T_s \text{sgn}(s(kT_s))), \]

for \( s(kT_s) < 0 \),

\[ \Delta s(kT_s) = s(kT_s + T_s) - s(kT_s) \geq -\gamma_1 T_s (s(kT_s) - \gamma_2 T_s \text{sgn}(s(kT_s))), \]

where \( T_s \) is the sampling time. If the above reaching conditions of the sliding motion are satisfied, then the trajectories of the error dynamic system (6) converge to the sliding mode function \( s(kT_s) = 0 \) [12].

To prove the existence of sliding manifold in the discrete-time domain, discretizing (12) with Euler method [31], then one has

\[ s(kT_s + T_s) = s(kT_s) + T_s (g(x(kT_s), kT_s) + f(x(kT_s), kT_s)) \]
+ C_s B_2 d(kTs) - \gamma_1 s(kTs) - (\gamma_2 + \gamma_3)\text{sgn}(s(kTs))).

According to Lemma 1 and \( \gamma_3 = \|g(x(t), t)\| + \alpha_1 \|C_s\| \|B_2\| + \alpha_2 \|x(t)\| + \alpha_3 \),
when \( s(kTs) > 0 \),
\[
\Delta s(kTs) = -\gamma_1 T_s s(kTs) + T_s g(x(kTs), kTs) + f(x(kTs), kTs)
+ C_s B_2 d(kTs) - (\gamma_2 + \gamma_3)\text{sgn}(s(kTs)))
\leq -\gamma_1 T_s s(kTs) - \gamma_2 T_s \text{sgn}(s(kTs)).
\]
When \( s(kTs) < 0 \),
\[
\Delta s(kTs) = -\gamma_1 T_s s(kTs) + T_s g(x(kTs), kTs) + f(x(kTs), kTs)
+ C_s B_2 d(kTs) - (\gamma_2 + \gamma_3)\text{sgn}(s(kTs)))
\geq -\gamma_1 T_s s(kTs) - \gamma_2 T_s \text{sgn}(s(kTs)).
\]
Thus, by using Lemma 1, it ensures that \( s(kTs) \) also reaches the sliding manifold in the discrete-time domain. As discuss above, we have \( u_\epsilon(t) = u_{ceq}(t) \) in the sliding manifold. We discretize (10) by the sampling time \( T_s \), then one has
\[
e(kTs + T_s) = Ge(kTs) + H_1 u^*_d(kTs) + H_w d(kTs)
\]
where \( G = e^{AT_s}, H_1 = (G - I_n)A^{-1}B_1 \) and \( H_w = (G - I_n)A^{-1}B_w \). Then, using the digital redesign approach [17], the digital-redesign-based control law \( u^*_d(kTs) \) can be represented as
\[
u^*_d(t) = u^*_d(kTs) \equiv u^*_d(kTs + T_s) + u^*_d(kTs) - u^*_d(kTs + T_s), \quad \text{for } kTs \leq t < kTs + T_s.
\]
According to (14), we further have
\[
u^*_d(kTs) \equiv u^*_d(kTs + T_s) + u^*_d(kTs) - u^*_d(kTs + T_s).
\]
To implement the control input of DRSMC, the following Lemma 2 is introduced.

Lemma 2 [29]

We assume that the function \( f(t) \) is varying slowly. In [29], we can define that the \( f(kTs) \) is the order \( O(T_s) \), which is concerning the sampling time \( T_s \). Thus, the property is defined as follows

Property: \( f(kTs) = O(T_s), f(kTs) - f(kTs - T_s) = O(T_s^2), \) and \( f(kTs) - 2f(kTs - T_s) + f(kTs - 2T_s) = O(T_s^3) \).

In order to implement the proposed control law to the digital controller, we let
\[
O_1(T_s^2) = u^*_d(kTs) - u^*_d(kTs + T_s).
\]
The detailed proof of (16) will introduce later. Thus, we further rewrite (15) as follows
\[
u^*_d(kTs) = u^*_d(kTs + T_s) + O_1(T_s^2).
\]
In (17), by using the digital redesign approach to transfer \( u^*_d(t) \), one can get
\[
u^*_d(kTs + T_s) = -K_c e(kTs + T_s)
= -K_c(Ge(kTs) + H_1 u^*_d(kTs) + H_w d(kTs)).
\]
We utilize (18) to the first term of (17), and then we get
\[
u^*_d(kTs) = -K_c(Ge(kTs) + H_1 u^*_d(kTs) + H_w d(kTs))
+ O_1(T_s^2).
\]
We organize (19), it can be
\[
u^*_d(kTs) = -K_c Ge(kTs) - K_c H_w d(kTs)
+ O_1(T_s^2),
\]
then one has
\[
u^*_d(kTs) = -(I_m + K_c H_1)^{-1} K_c(Ge(kTs))
+ (I_m + K_c H_1)^{-1} \left[ -K_c H_w d(kTs) + O_1(T_s^2) \right].
\]
Therefore, one has
\[
u^*_d(kTs) = -(I_m + K_c H_1)^{-1} K_c(Ge(kTs))
+ (I_m + K_c H_1)^{-1} \left[ -K_c H_w d(kTs) + O_1(T_s^2) \right].
\]
(21)
where \( K_c = (I_m + K_c H_1)^{-1} K_c \) and \( d(kTs) = H_w d(kTs) \).

According to Lemma 2, \( d(kTs) \) belongs to \( O_2(T_s) \) and is unknown, but \( d(kTs) \) is bounded and grows slowly.

We design an estimator \( \hat{d}_d(kTs) \) to estimate \( d_d(kTs) \). Thus, (21) can be described as
\[
u^*_d(kTs) = -(I_m + K_c H_1)^{-1} K_c \left( d_d(kTs) \right)
+ \hat{d}_d(kTs) \quad \text{for } kTs \leq t < kTs + T_s.
\]
Then,
\[
u^*_d(kTs) = -(I_m + K_c H_1)^{-1} O_1(T_s^2)
+ (I_m + K_c H_1)^{-1} O_2(T_s^2)\].
(22)
where \( O_2(T_s) = d_d(kTs) - \hat{d}_d(kTs) \) and the estimation of \( d_d(kTs) \) is given as
\[
\hat{d}_d(kTs) = \hat{d}_d(kTs - T_s)
+ k_{\hat{d}} \left[ G(kTs) - Ge(kTs - T_s) - H_1 u^*_d(kTs - T_s) \right]
- k_{\hat{d}} d_d(kTs - T_s),
\]
(23)
where \( k_{\hat{d}} = \tilde{k}_{\hat{d}} I_n \) and \( \tilde{k}_{\hat{d}} \) is a positive scalar parameter that can be designed. The detailed proof of the proposed disturbance estimator (23) can seem in Appendix B. From Appendix B, we can know \( \hat{d}_d(kTs) \) is bounded. Therefore, the proposed digital controller can be designed as follows
\[
u^*_d(kTs) = -(I_m + K_c H_1)^{-1} K_c \hat{d}_d(kTs).
\]
The detailed proof of the proposed controller (24) and (16) can be found in Appendix C. As the results in Appendix C, the proposed controller in (24) belongs to \( O(T_s) \).

After the control law is discretized, to implement the discrete-time SMC, we discretize the sliding mode function (7) with Euler method [31] as follows
\[
s(kTs) = C_s e(kTs) + s_1(kTs),
\]
where \( \gamma_1(T_s^2)
(25)
where $s$ is an arbitrary and small positive constant. □

Let $\gamma_3 = \gamma_d$ and $\gamma_{\text{max}} = \max(\gamma_3)$, one has

$$
\sum_{i=1}^{m} s_i(t) \hat{s}_i(t) \leq \sum_{i=1}^{m} (\gamma_3 |s_i(t)| - \gamma_2 |s_i(t)|) - \gamma_2 s_i(t) \leq \gamma_2 \left( \sum_{i=1}^{m} (s_i(t) - |s_i(t)|) \right) + \gamma_2 \gamma_{\text{max}} |s_i(t)|
$$

(32)

(33)

Obviously, from (33), it implies $\dot{V}(s(t)) \leq 0$ when

$$
|s(t)| \leq \left( \frac{\gamma_2 + \gamma_{\text{max}}}{\gamma_2} \right) m \varepsilon.
$$

(34)

Hence, in (34), we can select a small enough parameter $\varepsilon$ and a large enough parameter $\gamma_2$ to achieve the desired performance in the SMC such that $s(t)$ approximates to zero closely.

Remark 2: Since $\gamma_3$ is a time-varying parameter, we use the maximum of $\gamma_3$ to calculate the upper-bound of the sliding mode trajectory.

Since the saturation function is utilized to eliminate chattering phenomenon by selecting a proper $\varepsilon$, the digital-redesign SMC-based control law can be rewritten as

$$
u_d(kT_s) = -K_d e(kT_s) - (I_m + K_c H_1)^{-1} K_c \hat{d}_d(kT_s)
$$

(35)

Also, the upper-bound of sliding mode trajectories can be found in (34). As discussed above, we have completed the digital controller and continue to verify the proposed digital controller. The flow chart of the hybrid model-following control is shown in Figure 2.

Remark 3: In order to implement the designed controller by using the digital controller, we propose a digital-redesign algorithm to discretize the continuous-time controller to discrete-time controller directly, and we discuss and prove the proposed digital-redesign-based controller belongs to $O(T_s)$. Also, in SMC, since the saturation function is often adopted to reduce the chattering phenomenon, we discuss the effect of the proposed SMC in this article.

IV. NUMERICAL SIMULATION

In this example, by using the proposed DRSMC, we design the digital controller to control the $n$-scroll Chua’s circuit [32]. The original continuous $n$-scroll Chua’s circuit can
FIGURE 2. The flow chart of the hybrid model-following control.

Step 1.
- Continuous-time controller design
- Select the sliding mode function in (7)
- Obtain the continuous-time SMC in (11)

Step 2.
- Digital redesign approach
- Obtain the discrete-time sliding mode function in (25)
- Design the disturbance estimator in (23)
- Obtain the digital-redesign SMC-based control in (35)

According to Theorem 1, by solving LMIs with γ = 0.3, we obtain \( K_c = \left[ \begin{array}{cccc} -3.9616 & -1.1215 & -0.005 \end{array} \right] \).

For the model system (2a) and (2b), by selecting the weighting matrices \( Q_m = 10^7 \) and \( R_m = 1 \), we obtain \( K_{mc} = \left[ \begin{array}{c} -3.1623 \times 10^3 & -1.0001 \times 2.9246 \times 10^{-5} \end{array} \right] \) and \( E_{mc} = \left[ -3.1623 \times 10^3 \right] \). The sampling time is selected as \( T_s = 10^{-3} \) (s). With (13), (21), and (28), we disretize \( A, B, K_c, K_{mc}, \) and \( E_{mc} \) to have

\[
G = \left[ \begin{array}{ccc} 9.995 \times 10^{-4} & 0.999 \times 10^{-4} \end{array} \right], \quad H_1 = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \end{array} \right], \quad K_d = \left[ \begin{array}{cccc} -0.0108 \times 10^{-6} \end{array} \right], \quad K_{md} = \left[ \begin{array}{cccc} -0.4052 \times 10^{-6} \end{array} \right], \quad E_{md} = \left[ \begin{array}{cccc} -3.8000 \times 10^{-6} \end{array} \right], \quad E_{mc} = \left[ \begin{array}{cccc} -3.1623 \times 10^{-6} \end{array} \right] \].

For simulation analysis, the initial conditions of the controlled system and the model system are given by \( x(0) = [3 -1 -2]^T \) and \( x_m(0) = [0 0 0]^T \). The controller parameters are given by \( \gamma_1 = 30, \gamma_2 = 0.1, \alpha_1 = 0.35, \alpha_2 = 0.1, \alpha_3 = 0.2, k_g = 0.8 \) and \( \epsilon = 0.05 \).

The reference trajectory is given by \( \tau(t) = 0.5 \cos(t) \). The disturbances are given by

\[
g(x(t), t) = \left[ \begin{array}{cc} 0.11 \pi / 2.6 & (x_1(t) - 18.2), & x_1(t) \geq 18.2 \\ -0.11 \sin(\pi x_1(t) / 2.6), & -18.2 \leq x_1(t) \leq 18.2 \\ 0.11 \pi / 2.6 & (x_1(t) + 18.2), & x_1(t) \leq -18.2 \end{array} \right],
\]

\[
f(x(t), t) = 0.1 x_1(t) \sin(x_3(t)) + 0.2 \sin(2\pi / 15),
\]

\[
d(t) = \left[ \begin{array}{cccc} 0.2 \cos(2\pi / 15) & 0.2 \sin(2\pi / 15) & 0.2 \cos(2\pi / 15) \end{array} \right]^T.
\]

The simulation results are shown in Figures 3-9. Fig. 3 shows the chaotic attractor of the original n-scroll Chua’s circuit [32]. Fig. 4 shows the state responses of \( x(t) \) and \( x_m(t) \). It reveals the proposed digital-redesign SMC-based controller makes the system state \( x(t) \) track the model system state \( x_m(t) \) closely. The control input is shown in Figure 5.
Fig. 6 demonstrates the tracking performance of $y(t)$ and $r(t)$. Fig. 7 illustrates the sliding mode trajectory. Besides, we can see the disturbance estimator $d_0(kT_s)$ estimate $d_a(kT_s)$ precisely in Figure 8 and the error between $\hat{d}_a(kT_s)$ and $d_a(kT_s)$, which is very close to zero in Figure 9. After calculation, we have

$$\gamma^2 = 0.09 > \int_0^{t_{end}} (e^T(\tau)e(\tau))d\tau \int_0^{\infty} (d^T(\tau)d(\tau))d\tau = 0.0193,$$
and
\[ \gamma^2 = 0.09 \]
\[ > \int_{\|s(t)\|=0}^{t_{\text{end}}} (e^T_s(\tau)e_s(\tau))d\tau / \int_{\|s(t)\|=0}^{t_{\text{end}}} (d^T(\tau)d(\tau))d\tau \]
\[ = 5.4859 \times 10^{-4}, \]
where \( t_{\text{end}} \) is the final simulation time. Therefore, the \( H_\infty \) tracking performance can be guaranteed by using the proposed digital redesign based tracking controller.

V. CONCLUSION
In this article, a digital redesign technology is newly proposed by integrating with sliding mode control. The problem of the hybrid \( H_\infty \) model following control for uncertain continuous systems can be well solved by using the proposed digital redesign based discrete sliding mode control. It has been verified that the states of the controlled systems can perfectly track the trajectory of the model systems. Also, by using the linear quadratic analog tracker, we force the model system to track the desired trajectory. In contrast to the reports in the literature, the matched/mismatched disturbances are also considered and efficiently suppressed by the proposed \( H_\infty \) discrete sliding mode controller. While the saturation function is adopted to replace the sign function for reaching the chattering phenomena, the effect and the upper-bound to the sliding mode function are analyzed in this article. A numerical example is included to verify the proposed digital redesign SMC in this article.

APPENDIX A
The detailed proof of Theorem 1:
If the controlled system can converge to the sliding manifold (i.e. \( s(t) = 0 \) and \( \dot{s}(t) = 0 \) \( s(t) = 0 \) \( \dot{\bar{s}}(t) = 0 \) \( \text{and} \)), the equivalent error dynamics in the sliding manifold can be obtained as (10). The following Lyapunov function is introduced
\[ V_e(t) = e^T(\tau)Pe(\tau), \]
where \( P \) a symmetric and positive definite matrix. By introducing (10) into \( \dot{V}_e(t) \), we have
\[ \dot{V}_e(t) = e^T(\tau)Pe(\tau) + e^T(\tau) Pe(\tau) \]
and
\[ \dot{V}_e(t) = e^T(\tau)Pe(\tau) + e^T(\tau)(A-B_1K_c)e(\tau) \]
\[ + e^T(\tau)PBw(\tau) + d^T(\tau)Bw^T(\tau)Pe(\tau). \]
\[ (A.2) \]
We consider the following \( H_\infty \) tracking performance
\[ J_\infty = \int_0^\infty \left\{ \gamma^{-2}e^T_y(\tau)e_y(\tau) - d^T(\tau)d(\tau) \right\}d\tau, \]
\[ \text{where } e_y(\tau) = y(\tau) - y_m(\tau) \text{ is the tracking error and } \gamma \text{ is a pre-selected positive parameter. Then,} \]
\[ J_\infty = \int_0^\infty \left\{ \gamma^{-2}e^T_y(\tau)e_y(\tau) - d^T(\tau)d(\tau) + \dot{V}_e(\tau) \right\}d\tau \]
\[ \text{therefore,} \]
\[ J_\infty = \int_0^\infty \left\{ \gamma^{-2}e^T_y(\tau)e_y(\tau) - d^T(\tau)d(\tau) + \dot{V}_e(\tau) \right\}d\tau \]
\[ - \lim_{t \to \infty} V_e(\tau) + V_e(0). \]
\[ (A.4) \]
For zero initial condition, we can rewrite (A.4) as
\[ J_\infty \leq \int_0^\infty \left\{ \gamma^{-2}e^T_y(\tau)e_y(\tau) - d^T(\tau)d(\tau) + \dot{V}_e(\tau) \right\}d\tau \]
\[ - \lim_{t \to \infty} V_e(\tau). \]
\[ (A.5) \]
Hence, we can have the \( H_\infty \) tracking performance as below
\[ J_\infty \leq \int_0^\infty \left\{ \gamma^{-2}e^T_y(\tau)e_y(\tau) - d^T(\tau)d(\tau) + \dot{V}_e(\tau) \right\}d\tau \leq 0. \]
\[ (A.6) \]
As long as we can ensure that (A.6) always holds, we can guarantee the \( H_\infty \) tracking performance with a pre-selected positive parameter \( \gamma \); therefore, one has \( \int_0^\infty ||e_y(\tau)||_2^2d\tau \leq \int_0^\infty ||\dot{e}_y(\tau)||_2^2d\tau + \int_0^\infty ||d(\tau)||^2d\tau \). Now, introduce (A.2) into (A.6) and use Schur complement [23], then the LMI is obtained as
\[ \left[ \begin{array}{ccc} P(A-B_1K_c) + (A-B_1K_c)^T P & PBw & CT \\
PB^T & -I_n & 0 \\
CY & 0 & -\gamma^2I_n \end{array} \right] < 0. \]
\[ (A.7) \]
By using \( \text{diag} \{Y, I_n, I_n\} \), where \( Y = P^{-1} \), presenting a congruence transformation to (A.7), we can rewrite (A.7) as follows
\[ \left[ \begin{array}{ccc} YA^T + AY - Z^TB_1^T - B_1Z & Bw & YC^T \\
Bw^T & -I_n & 0 \\
CY & 0 & -\gamma^2I_n \end{array} \right] < 0. \]
\[ (A.8) \]
From (A.8), it reveals that when having proper \( Y \) and \( Z \) with the pre-specified positive parameter \( \gamma \) and we can ensure \( H_\infty \) tracking performance in the sliding manifold.

APPENDIX B
The detailed proof of the estimator (23):
According to (21), we define \( d_a(kT_s) = H_\infty d(kT_s) \), then rewrite (13) as follows
\[ e(kT_s + T_s) = Ge(kT_s) + H_1u^g(kT_s) + d_a(kT_s), \]
\[ (B.1) \]
where \( d_a(kT_s) \in O(T_s) \). Based on (B.1), the disturbance estimator (23) can be described as
\[ \hat{d}_a(kT_s + T_s) = \hat{d}_a(kT_s) + k_g \left[ d_a(kT_s) - \hat{d}_a(kT_s) \right], \]
\[ (B.2) \]
where \( k_g = \tilde{k}_g I_n, \tilde{k}_g \) is a positive parameter, and \( \tilde{k}_g \) is designed to satisfy \( \text{eig}(1 - \tilde{k}_g I_n) < 1 \). To calculate the error between \( d_a(kT_s + T_s) \) and \( \hat{d}_a(kT_s + T_s) \), (B.2) can be rewritten as
\[ d_a(kT_s + T_s) - \hat{d}_a(kT_s + T_s) \]
\[ = d_a(kT_s + T_s) + d_a(kT_s) - d_a(kT_s) \]
\[ \text{where we use} \]
\[ 147207 \]
where \( \hat{\alpha}_d(kT_s + T_s) = (I_n - k_g)\hat{\alpha}_d(kT_s) + O_d(T_s) \). (B.3)

Now, let \( \|O_d(T_s^2)\| < \beta \) which is concerning the sampling time \( T_s \) to have

\[
\left\| \hat{\alpha}_d(kT_s + T_s) \right\| < (1 - \tilde{k}_g) \left\| \hat{\alpha}_d(kT_s) \right\| + \beta,
\]

then when \( k \to \infty \) one has

\[
\lim_{k \to \infty} \left\| \hat{\alpha}_d(kT_s + T_s) \right\| = \lim_{k \to \infty} \left[ (1 - \tilde{k}_g)^k \left\| \hat{\alpha}_d(kT_s) \right\| + \sum_{j=0}^{k-1} (1 - \tilde{k}_g)^j \beta \right]. \quad (B.4)
\]

Therefore, we have the following equation

\[
\lim_{k \to \infty} \|\hat{\alpha}_d(kT_s + T_s)\| < \lim_{k \to \infty} \left[ (1 - \tilde{k}_g)^k \|\hat{\alpha}_d(kT_s)\| + \sum_{j=0}^{k-1} (1 - \tilde{k}_g)^j \beta \right] = \left[ (1 - \tilde{k}_g)^0 + (1 - \tilde{k}_g)^1 + \cdots + (1 - \tilde{k}_g)^\infty \right] \beta = \left[ 1 - (1 - \tilde{k}_g)^{-1} \right] \beta = \tilde{k}_g^{-1} \beta. \quad (B.5)
\]

As the discussion above, \( \hat{\alpha}_d(kT_s) \) is bounded and \( \beta \) is a very small value with respect to the small enough sampling period. Thus, \( \hat{\alpha}_d(kT_s) \) converges to zero closely. Besides, the proposed estimator \( \hat{\alpha}_d(kT_s) \) is designed to estimate the \( \alpha_d(kT_s) \) which changes slowly.

**APPENDIX C**

From (10), we have the equivalent dynamics in the sliding manifold as

\[
\dot{e}(t) = A e(t) + B_1 u^*_{c}(t) + B_w d(t), \quad (C.1)
\]

where \( u^*_{c}(t) = -K_c e(t) \). Discretizing (C.1) yields

\[
e(kT_s + T_s) = G e(kT_s) + H_1 u^*_{c}(kT_s) + H_w d(kT_s), \quad (C.2)
\]

where \( e(kT_s) = x(kT_s) - x_m(kT_s) \), \( d(kT_s) = H_w d(kT_s) \), and \( A - B_1 K_c \) is a stable matrix. By using the digital-redesign approach, one has

\[
u^*_d(kT_s) \geq u^*_c(kT_s + T_s) + u^*_c(kT_s + T_s) = -K_c g e(kT_s + T_s) + O_1(T_s^2), \quad (C.3)
\]

where \( O_1(T_s^2) = u^*_c(kT_s) - u^*_c(kT_s + T_s) \). Then, one can get

\[
u^*_c(kT_s) = -K_c (G e(kT_s) + H_1 u^*_c(kT_s) + d(kT_s)) + O_1(T_s^2). \quad (C.4)
\]

Now, we rewrite (C.4) as

\[
u^*_c(kT_s) = (I_m + K_c H_1)^{-1} \left\{ -K_c G e(kT_s) - K_c d(kT_s) + O_1(T_s^2) \right\}
\]

\[
= -(I_m + K_c H_1)^{-1} K_c G e(kT_s) - (I_m + K_c H_1)^{-1} K_c d(kT_s) + (I_m + K_c H_1)^{-1} O_1(T_s^2), \quad (C.5)
\]

where \( K_d = (I_m + K_c H_1)^{-1} K_c G \). Substituting the disturbance estimator \( \hat{\alpha}_d(kT_s) \) into (C.5) yields

\[
u^*_d(kT_s) = -K_d d(kT_s) - (I_m + K_c H_1)^{-1} K_c \hat{\alpha}_d(kT_s) + O_2(T_s) + (I_m + K_c H_1)^{-1} O_1(T_s^2), \quad (C.6)
\]

Next, based on (C.6), we design the proposed control law as follows

\[
u^*_c(kT_s) = -K_d e(kT_s) - (I_m + K_c H_1)^{-1} K_c \hat{\alpha}_d(kT_s). \quad (C.7)
\]

Substitute (C.7) into (C.2) to have

\[
 e(kT_s + T_s) = G e(kT_s) + H_1 (I_m + K_c H_1)^{-1} K_c \hat{\alpha}_d(kT_s) + d(kT_s), \quad (C.8)
\]

where \( G_c = (G - H_1 K_d) \) is a stable matrix. According to Appendix B, we have \( \hat{\alpha}_d(kT_s) = d(kT_s) - \hat{\alpha}_d(kT_s) \), then one has

\[
\hat{\alpha}_d(kT_s + T_s) = (I_n - k_g)^{k+1} \hat{\alpha}_d(0) + \sum_{j=0}^{k} (I_n - k_g)^j (d(kT_s + T_s) - d(kT_s)) \quad (C.9)
\]

and

\[
\lim_{k \to \infty} \| \hat{\alpha}_d(kT_s + T_s) \| = k^{-1} O_d(T_s^2). \quad (C.10)
\]

where \( O_d(T_s^2) = d(kT_s + T_s) - d(kT_s) \). Thus, \( \hat{\alpha}_d(kT_s + T_s) \in O(T_s^2) \) and \( \hat{\alpha}_d(kT_s) \in O(T_s) \). Now, from (C.8), the following result can be calculated

\[
\lim_{k \to \infty} \| e(kT_s + T_s) \| \leq \frac{O(1)}{(I_n - \| G_c \|^{-1}) O(T_s)} \times \left\| d(kT_s) - H_1 (I_m + K_c H_1)^{-1} K_c \hat{\alpha}_d(kT_s) \right\|. \quad (C.11)
\]

Finally, we conclude that \( e(kT_s) \in O(T_s) \), then we have

\[
u^*_d(kT_s) = -K_d e(kT_s) - (I_m + K_c H_1)^{-1} K_c \hat{\alpha}_d(kT_s). \quad (C.12)
\]

Therefore, we can further prove \( \nu^*_c(kT_s) \) belongs to \( O(T_s) \) and \( \nu^*_c(kT_s) - \nu^*_c(kT_s + T_s) \) belongs to \( O(T_s^2) \).
[1] D. Ginoya, P. D. Shendge, and S. B. Phadke, “Disturbance observer based sliding mode control of nonlinear mismatched uncertain systems,” *Commun. Nonlinear Sci. Numer. Simul.*, vol. 26, nos. 1–3, pp. 98–107, Sep. 2015.

[2] J. Yang, S. Li, J. Su, and X. Yu, “Continuous nonsingular terminal sliding mode control for systems with mismatched disturbances,” *Automatica*, vol. 7, no. 7, pp. 2287–2291, Jul. 2013.

[3] B. Niu, P. Zhao, J.-D. Liu, H.-J. Ma, and Y.-J. Liu, “Global adaptive control of switched uncertain nonlinear systems: An improved MDADT method,” *Automatica*, vol. 115, May 2020, Art. no. 108872.

[4] M. Zhang, M.-G. Gan, J. Chen, and Z.-P. Jiang, “Data-driven adaptive optimal control of linear uncertain systems with unknown jumping dynamics,” *J. Franklin Inst.*, vol. 356, no. 12, pp. 6087–6105, Nov. 2019.

[5] C. Xi and J. Dong, “Adaptive neural network-based control of uncertain nonlinear systems with time-varying full-state constraints and input constraint,” *Neurocomputing*, vol. 357, pp. 108–115, Sep. 2019.

[6] Q. Yin, M. Wang, X. Li, and G. Sun, “Neural network adaptive tracking control for a class of uncertain switched nonlinear systems.” *Neurocomputing*, vol. 301, pp. 1–10, Aug. 2018.

[7] W. Jie, H.-H. Kim, K. Dad, and M.-C. Lee, “Terminal sliding mode control with sliding perturbation observer for a hydraulic robot manipulator,” *IFAC-PapersOnLine*, vol. 51, no. 22, pp. 7–12, 2018.

[8] I. F. Bouguenna, A. Azaiz, A. Tahour, and A. Larbaoui, “Robust neuro-fuzzy sliding mode control with extended state observer for an electric drive system,” *Energy*, vol. 169, pp. 1054–1063, Feb. 2019.

[9] S. Das, M. Salim Qureshi, and P. Swarnkar, “Design of integral sliding mode control for DC-DC converters,” *Matex Today, Proc.*, vol. 5, no. 2, pp. 4290–4298, 2018.

[10] H. Du, X. Yu, M. Z. Q. Chen, and S. Li, “Chattering-free discrete-time sliding mode control,” *Automatica*, vol. 68, pp. 87–91, Jun. 2016.

[11] J.-J. Yan, C.-Y. Chen, and J.-S.-H. Tsai, “Hybrid chaos control of continuous unified chaotic systems using discrete rippling sliding mode control,” *Nonlinear Anal.*, *Hybrid Syst.*, vol. 22, pp. 276–283, Apr. 2020.

[12] J.-S.-H. Tsai, J.-S. Fang, J.-J. Yan, M.-C. Dai, S.-M. Guo, and L.-S. Shieh, “Hybrid robust discrete sliding mode control for generalized continuous chaotic systems subject to external disturbances,” *Nonlinear Anal.*, *Hybrid Syst.*, vol. 29, pp. 74–84, Aug. 2018.

[13] S. Chakraborty and A. Bartoszewicz, “Improved robustness and performance of discrete time sliding mode control systems,” *ISA Trans.*, vol. 65, pp. 143–149, Nov. 2016.

[14] B. Rahmani, “Robust output feedback sliding mode control for uncertain discrete time systems,” *Nonlinear Anal.*, *Hybrid Syst.*, vol. 24, pp. 83–99, May 2017.

[15] M.-C. Pai, “Discrete-time output feedback quasi-sliding mode control for robust tracking and model following of uncertain systems,” *J. Franklin Inst.*, vol. 351, no. 5, pp. 2623–2639, May 2014.

[16] A. Norouzi, K. Ebrahimzadeh, and C. R. Koch, “Integral discrete-time sliding mode control of homogeneous charge compression ignition (HCCI) engine load and combustion timing,” *IFAC-PapersOnLine*, vol. 52, no. 5, pp. 153–158, 2019.

[17] S. M. Guo, L. S. Shieh, G. Chen, and C. F. Lin, “Effective chaotic orbit tracker: A prediction-based digital redesign approach,” *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 47, pp. 1557–1569, Sep. 2000.

[18] X. Li, C. Wang, Q. Wen, X. Yu, and B. Wang, “Digital redesign of sliding mode control of LTI systems,” in *Proc. 6th World Congr. Intell. Control Autom.*, Dalian, China, Jun. 2006, pp. 2431–2435.

[19] Y. Chang, Y. Yang, F. E. Alsaadi, and G. Zong, “Adaptive fuzzy output-feedback tracking control for switched stochastic pure-feedback nonlinear systems,” *Int. J. Adapt. Control, Vol. 33*, no. 10, pp. 1567–1582, 2019.

[20] L. Ma, G. Zong, X. Zhao, and X. Huo, “Observed-based adaptive finite-time sliding control for a class of nonstrict-feedback nonlinear systems with input saturation,” *J. Franklin Inst.*, Sep. 2019, doi: 10.1016/j.jfranklin.2019.07.021.

[21] Q. Qu, H. Zhang, R. Yu, and Y. Liu, “Neural network-based H∞ sliding mode control for nonlinear systems with actuator faults and unmatched disturbances,” *Neurocomputing*, vol. 275, pp. 2009–2018, Jun. 2018.

[22] A. A. Stoorvogel, “Stabilizing solutions of the algebraic Riccati equation,” *Automatica*, vol. 240, pp. 153–172, Jun. 1996.

[23] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory* (The Society for Industrial and Applied Mathematics), Philadelphia, PA, USA: SIAM, 1994.

[24] X.-H. Chang and G.-H. Yang, “Nonfragile H∞ filtering of continuous-time fuzzy systems,” *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1528–1538, Apr. 2011.

[25] H. Yang, Y. Jiang, and S. Yin, “Fault-tolerant control of time-delay Markov jump systems with Itô stochastic process and output disturbance based on sliding mode observer,” *IEEE Trans. Ind. Informat.*, vol. 14, no. 12, pp. 5299–5307, Dec. 2018.

[26] J.-S. Fang, J.-S.-H. Tsai, J.-J. Yan, C.-H. Tsou, and S.-M. Guo, “Design of robust trackers and unknown nonlinear perturbation estimators for a class of nonlinear systems: HTRDNA algorithm for tracker optimization.” *Mathematics*, vol. 7, no. 12, p. 1141, Nov. 2019.

[27] T. H. Yan, B. He, X. D. Chen, and X. S. Xu, “The discrete-time sliding mode control with computation time delay for repeatable run-out compensation of hard disk drives,” *Math. Problems Eng.*, vol. 2013, pp. 1–13, Mar. 2013, doi: 10.1155/2013/505846.

[28] K.-S. Kim and K.-H. Rew, “Reduced order disturbance observer for discrete-time linear systems,” *Automatica*, vol. 49, no. 4, pp. 968–975, Apr. 2013.

[29] Q. Xu and Y. Li, “Micro-/nanopositioning using model predictive output integral discrete sliding mode control,” *IEEE Trans. Ind. Electron.*, vol. 59, no. 2, pp. 1161–1170, Feb. 2012.

[30] E. Fehribahizadeh, J. S.-H. Tsai, Y. T. Liao, M.-C. Chung, S.-M. Guo, L.-S. Shieh, and L. Wang, “A generalised optimal linear quadratic tracker with universal applications—Part 1: Continuous-time systems,” *Int. J. Syst. Sci.*, vol. 48, no. 2, pp. 376–396, Mar. 2016.

[31] S. Li, H. Du, and X. Yu, “Discrete-time terminal sliding mode control systems based on Euler’s discretization,” *IEEE Trans. Autom. Control*, vol. 59, no. 2, pp. 546–552, Feb. 2014.

[32] A. Senouci and A. Boukabou, “Fuzzy modeling, stabilization and synchronization of multi-scroll chaotic systems,” *Optik*, vol. 127, no. 13, pp. 5351–5358, Jul. 2016.

**REFERENCES**

**JUINN-SHIOU FANG** (Member, IEEE) received the B.S. and M.S. degrees from the National Chianghua University of Education, Taiwan, and the Ph.D. degree in electrical engineering from National Cheng Kung University, Taiwan. His research interests include nonlinear control and intelligent algorithms.

**JASON S. H. TSAI** (Senior Member, IEEE) received the M.S. and Ph.D. degrees in electrical engineering from the University of Houston, Houston, TX, USA, in 1985 and 1988, respectively. Since August 1988, he has been an Associate Professor with the Department of Electrical Engineering, National Cheng Kung University, Taiwan. He has been a Full Professor, since August 1992, and a Distinguished Professor, since August 2002. His research interests include state-space self-tuning control, chaotic system control, partial differential system control, numerical analysis, and robotics. He was an (Executive) Editor of *Science Development*, published by the National Science Council, China, an Editor of the *Journal of the Chinese Institute of Electrical Engineering*, and an Associate Editor of the *International Journal of Systems Science* and the *Journal of The Franklin Institute*. 

**VOLUME 8, 2020**

**147209**
JUN-JUH YAN received the B.S. degree in electrical engineering from National Cheng Kung University, Taiwan, in 1987, the M.S. degree in electrical engineering from National Central University, Taiwan, in 1992, and the Ph.D. degree in electrical engineering from National Cheng Kung University, in 1998. He is currently a Professor with the Department of Electronic Engineering, National Chin-Yi University of Technology, Taichung, Taiwan. His main research interests include multi-robot dynamic systems, chaotic systems, neural networks, variable-structure control systems, and adaptive control.

P. L. CHEN received the B.S. degree from the National Changhua University of Education, Taiwan, and the M.S. degree in electrical engineering from National Cheng Kung University, Taiwan. His research interests include nonlinear control and sliding mode control.

SHU-MEI GUO (Member, IEEE) received the M.S. degree from the Department of Computer and Information Science, New Jersey Institute of Technology, Newark, NJ, USA, in 1987, and the Ph.D. degree in computer and systems engineering from the University of Houston, Houston, TX, USA, in May 2000. Since June 2000, she has been an Assistant Professor with the Department of Computer Science and Information Engineering, National Cheng Kung University, Tainan, Taiwan, where she has been a Full Professor, since August 2010. Her research interests include various applications on evolutionary programming, chaos systems, Kalman filtering, fuzzy methodology, neural networks, sampled-data systems, and computer and systems engineering.

* * *

**IEEE Access**

J.-S. Fang et al.: H-Infinity Model Following Control for Uncertain Systems by Using Digital Redesign SMC

VOLUME 8, 2020

147210