The field of magnonics, which aims at using spin waves as carriers in data processing devices, has attracted increasing interest in recent years. We present and study micromagnetically a nonlinear nanoscale magnonic ring resonator device for enabling implementations of magnonic logic gates and neuromorphic magnonic circuits. In the linear regime, the transmission curve of this device exhibits the behavior of a notch filter, which filters out the spin waves of resonant frequencies. By increasing the spin-wave input power, this resonance frequency is shifted leading to transmission curves reminiscent to the activation functions of neurons. An analytical theory is developed to describe the transmission curve of the magnonic ring resonators in the linear and nonlinear regimes and it is validated by a comprehensive micromagnetic study. The proposed magnonic ring resonator provides a multi-functional nonlinear building block for unconventional magnonic circuits.

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I. Introduction

Spin waves are collective excitations of spin systems in magnetic materials, which can be considered as a potential data carrier in future low-energy data processing systems [1-4]. This is due to their ultrashort wavelengths, down to a few nanometers [5-6], high frequencies, up to few terahertz [7], ultralow losses [8-9], and abundance of associated nonlinear phenomena [10-12]. These features make spin waves highly attractive for wave-based and neuromorphic computing concepts. Several important milestones were achieved in the realization of magnonic data processing units, including logic gates [13-15], majority gates [16-18], a magnon transistor [19], building blocks for unconventional computing [20-22], auxiliary units for integrated circuits [12], magnonic directional couplers [23-26], and an integrated magnonic half-adder [27].

Here, we propose a nanoscale nonlinear magnonic ring resonator, the magnetic counterpart of the photonic ring resonator. The latter one is considered as a universal unit and widely used in integrated photonic circuits [28], photonic quantum computing [29], and photonic neuromorphic computing [30]. The concept of the magnonic ring resonator (see Fig. 1(a)) is similar to that of the photonic ring resonator [31] except that spin waves, instead of light, are used to carry information. A magnonic ring resonator of submillimeter sizes has been studied in linear regime using micromagnetic simulations in Ref. [32]. Although such “large” ring resonators demonstrate certain interesting features due to multimode coupling and external field sensitivity, their functionality and sizes are hardly compatible with the current state of the CMOS technology. Moreover, their multitude of width modes with different coupling strengths result in less effective energy transfer. Here, we study the single mode nanoscale magnonic ring resonator and demonstrate its functionality analytically and by simulation, including linear and nonlinear operation regimes as well as their anticipated applications. In spite of the fact that this is simulation, the recent progress in the realization of single-mode magnonic nano-conduits proves that everything written here can be realized on practice [33].

II. Theory and micromagnetic simulations of the linear regime

The basic configuration of the magnonic ring resonator under investigation, which consists of a ring of the mean radius $R$ and a straight waveguide of the width $w$, is shown in Fig. 1a. The static magnetization of the magnonic ring resonator obtained from micromagnetic simulation is shown in Fig. 1b (details of the micromagnetic
simulation are described in the Appendix). For a sufficiently narrow ring, as, e.g. the considered case of \( w = 100 \) nm, the static magnetization is in the vortex state with the magnetization lying along the ring. Such a vortex state in the ring structure is the ground state in the presence of zero external field (i.e. it corresponds to the global energy minimum), and can be easily achieved in an experiment \([34,35]\). The static magnetization of a straight waveguide is uniform and is along the waveguide, which is caused by the strong shape anisotropy and sufficiently small cross-section. We consider Yttrium-Iron-Garnet (YIG) as a material of both waveguide and ring, which is chosen for its low damping allowing for a long-range spin-wave propagation. The material parameters of YIG are described in the Appendix.

For the theoretical description of the power transmission in the ring resonator, we adopt the method of coupled modes, which is typically used in optics and microwave electronics \([31,36]\). Let us denote the complex amplitudes of input (output) spin waves in the waveguide and ring as \( a_{1,2} (b_{1,2}) \), as shown in Fig. 1a. As the reference plane (which is needed for non-point-like couplings) we use the position of the minimal distance between the ring and waveguide \((x=0)\), so that all \( a_i \) and \( b_i \) are the values, which would be at the point \( x = 0 \) if continuously extrapolated in the absence of coupling. If the coupling between the ring and waveguide is lossless, which is the case of dipolar coupling, it is described by the unitary scattering matrix \([31]\):

\[
\begin{pmatrix}
  b_1 \\
  b_2
\end{pmatrix} =
\begin{pmatrix}
  \tau & i\kappa \\
  i\kappa & \tau^* 
\end{pmatrix}
\begin{pmatrix}
  a_1 \\
  a_2
\end{pmatrix}
\]

(1)

The parameter \( \kappa \) is the coupling efficiency between the straight waveguide and the ring, which shows the fraction of the spin-wave amplitude coupled from the waveguide into the ring structure and vice versa. The parameter \( \tau \) is the transmission efficiency across the coupling region, which shows the fraction of the spin-wave amplitude passed through the coupling region. The transmission efficiency \( \tau \) is different from the transmittance \( T \), which shows the final transmitted power through all the structure and accounts for the interference in the ring. Naturally, \( |\kappa|^2 + |\tau|^2 = 1 \), which reflects the lossless nature of the coupling. The calculation of the coefficients \( \kappa \) and \( \tau \) for the dipolar coupled waveguide and ring is presented in Appendix.
Fig. 1 The modeling and characteristics of the magnonic ring resonator. a The generic model of the magnonic ring resonator. The widths of the waveguides are $w = 100 \text{ nm}$, and thicknesses are $h = 50 \text{ nm}$, radius (from the center of the ring to the center of the waveguide) is $R = 550 \text{ nm}$, the minimum gap between the ring and waveguide is $\delta = 20 \text{ nm}$. b The initial magnetization distribution of the investigated structure. The grey arrows and colors represent the in-plane orientations of the magnetization $M$.

The amplitudes $a_2$ and $b_2$ are connected by the circulation condition: $a_2 = b_2 \beta e^{i\theta}$. Here, $\beta = \exp\left(-2\pi R\Gamma / v_{gr}\right)$ is the loss coefficient describing what fraction of the spin-wave amplitude remains after one circulation in the ring, with $\Gamma$ and $v_{gr}$ being the spin-wave damping rate and group velocity in the ring, respectively. The parameter $\theta$ is the round-trip phase accumulation, which is equal to $\theta = 2\pi Rk$, where $k$ is the wavenumber of the spin wave in the ring. The wavenumber is determined by the input spin-wave frequency from the dispersion relation $\omega_k$ in the ring and, in a general case, can be different from the wavenumber in the straight waveguide.

Solving Eq. (1) together with the circulation condition one finds the transmission rate through the ring resonator structure:

$$T = \frac{|a_1|^2}{|a_2|^2} = \frac{\beta^2 + |\tau|^2 - 2\beta|\tau|\cos(\theta - \psi)}{1 + \beta^2|\tau|^2 - 2\beta|\tau|\cos(\theta - \psi)},$$

where $\psi = \text{Arg}[\tau]$. The optimal operation of the ring resonator is realized for so-called case of “critical coupling” with $|\tau| = \beta$, so that the output signal vanishes completely if $\theta - \psi = 2\pi n$, where $n = 0, 1, 2, 3, \ldots$ is the number of the resonance.
mode in the ring. The maximal transmission $T$ in the critical case, which is achieved if $\theta = (2n + 1)\pi$, is equal to $T = 4\beta^2 / (1 + 2\beta)^2$, and is increased with $\beta$. Therefore, it is desirable to work in the range of $\beta \approx |r| \to 1$ to achieve a large output power, i.e. have small losses in the ring and weak coupling between the ring and waveguide. However, the small loss and coupling increases the operational time of the resonator [37]. Thus, optimal values of the transmission efficiency and loss coefficient are in the range $\beta \approx |r| \sim 0.6 - 0.9$.

The coefficients $\kappa$, $\theta$ and $\beta$ depend on the spin-wave frequency. The coupling coefficient $\kappa$ significantly depends on the gap between waveguide and ring, which allows to fabricate a structure with $\kappa$ being optimal in a desirable frequency range. In our example simulations, the minimum gap is $\delta = 20$ nm. The ring radius determines the separation between the ring resonance frequencies. To set the loss coefficient to a value close to the optimum of $\beta \approx |r|$, in our simulations we increased the Gilbert damping in the ring structure to $\alpha_G = 2 \times 10^{-3}$. In experiment, increase of the YIG damping can be realized, e.g., by placing a normal metal on top of the ring to use the phenomenon of spin pumping [38]. Also, in our case of straight waveguide and ring of the same cross-section, the phase $\psi = 0$ (see Appendix).

As an approximation of the ring dispersion relation, in principle, one can use the dispersion relation of a straight waveguide [39]. For our case it results in only a slight discrepancy of 80 MHz, and the discrepancy becomes more negligible for $R \gg w$ and $kr \gg 1$. In all the following calculations, we use a more accurate theory of the dispersion in the ring, as outlined in the Appendix. The spin-wave damping rate and group velocity, which determine the loss coefficient, are calculated from the dispersion relation.

The frequency dependences of the transmission and loss coefficient $\tau$ and $\beta$, together with the round-trip phase $\theta$ are shown in Fig. 2. In the chosen frequency range $\tau \sim \beta$, and the critical coupling condition is satisfied at a frequency around 2.55 GHz (not shown in Fig. 2). The frequency dependence of the transmission coefficient is more pronounced because of a significant wavenumber dependence of the dynamic dipolar fields, generated by a spin-wave propagating in the waveguide and the ring [23].
Fig. 2 The transmission efficiency ($\tau$), loss coefficient ($\beta$), and round-trip phase $\cos(\theta)$ as a function of frequency $f$.  

The theoretical transmission curve of the whole ring resonator calculated according to Eq. (2) and the results of micromagnetic simulations are shown in Fig. 3a. Two resonant frequencies of the magnonic ring resonator are observed in this frequency range, which corresponds to the 16th and 17th resonant modes. At these frequencies, the output signal is vanishing due to the destructive interference in the outgoing waveguide between the transmitted spin wave $a_1\tau$ and the coupled-back spin wave $ia_2\kappa$, which acquires the round trip phase of $\theta = 2\pi n$ and two $\pi/2$ phase shifts in the coupler (see Eq. (1)), being, in total, in antiphase to the first wave. At the resonance frequency, all the spin-wave power is concentrated in the ring (see Fig. 3b). In contrast, at the frequency of 2.724 GHz, which corresponds to $\theta = 2\pi \times 16.5$, the constructive interference conditions are satisfied, and a large part of input power is transmitted, while only a small amount is circulating in the ring (Fig. 3b). A strong frequency dependence of the output power is important as it allows one to realize notch filters with a magnonic ring.

In addition, Fig. 3 shows the transmission curves for the rings having different radii. As expected, the resonance frequencies change and the resonance curves are shifted, while preserving their shape. The resonant frequencies, calculated analytically, are 20 MHz greater than those found from the micromagnetic simulations. Also, in the simulations, the critical coupling condition is satisfied in the range 2.65 GHz-2.68 GHz, as it is evident from the vanishing output at the resonance frequency, while theory gives the critical coupling at a little bit different frequency of 2.55 GHz. Both
these weak discrepancies are mainly attributed to a slightly nonuniform width profile of the spin waves in the ring and the waveguide, and not ideal ring shape in the simulations due to the discretization effects, which both are not taken into account in the theory. Also, there is a certain difference of the maximum transmission energy between the simulations and theoretical calculations, which is attributed to the propagation losses in the straight waveguide and the coupling area.

Fig. 3 a The theoretical (top panel) and simulated (bottom panel) normalized transmission $T$ as a function of frequency $f$ for different diameters of the ring. b The snapshot of the out-of-plane component spin-wave amplitude $m_z$ in the ring resonator with radius of 550 nm for different excitation frequencies.

III. Nonlinearity of magnonic rings

Magnonic systems are known to involve rich nonlinear effects, which open a way for the development of various power-dependent devices. In general, all the parameters, which define the operation of the magnonic ring resonator, namely, $\theta$, $\beta$ and $\tau$ are power-dependent. However, it can be shown that the main impact of nonlinearities is caused by the nonlinear phase accumulation $\theta = \theta(b)$, where $b$ is the spin-wave amplitude, while the nonlinearities of the loss coefficient (due to the group velocity shift) and coupling lead only to a small (second-order) correction, and, therefore, they can be neglected in almost all experimentally achievable cases.

An increase of the spin-wave power results in a nonlinear shift of the spin-wave resonant frequency, $\omega_k(b) = \omega_k^{(lin)} + W_k |b|^2$, where $W_k$ is the nonlinear shift coefficient.
Consequently, a wave of a constant frequency possesses a power-dependent wavenumber \( k \approx k_0 - (W_k / v_{sw})|b_1|^2 \), which directly affects the phase accumulation during the spin-wave propagation. The integration over the ring yields the round-trip phase \( \theta(b_2) = 2\pi R (k_0 - K|b_2|^2) \), where \( K = W_e (1 - e^{-4\pi R/v_{sw}}) / (4\pi R) \approx W_e \beta / v_{sw} \) is the averaged coefficient of the nonlinear shift of the spin-wave wavenumber. Then, Eq. (1) together with the circulation condition yields the following relation:

\[
|b_2|^2 = |a_1|^2 \frac{\kappa^2}{1 + \beta^2 \tau^2 - 2\beta \tau \cos[\theta(b_2) - \psi]},
\]

which implicitly determines the amplitude in the ring. The transmission \( T \) is given by the same Eq. (2), in which one should use the nonlinear phase accumulation \( \theta(b_2) \) with the amplitude of \( b_2 \), found from Eq. (3).

Fig. 4 The transmission as a function of spin-wave frequencies for different excitation field \( b_e \) (top panel: theory, bottom panel: simulation). Dashed magenta lines represent the second solution in the bistability range for \( b_e = 12 \) mT.

The simulated transmission curves of the magnonic ring resonator for different excitation field \( b_e \) are shown in Fig. 4. A pronounced shift of the resonance frequency, at which transmission is minimal, is observed. A similar power-dependent transmission curve shifts was observed in optical ring resonators [28,40,41]. To plot theoretical curves, we use the nonlinear frequency shift value of \( W = -2\pi \times 2.6 \) GHz, which is calculated for a straight waveguide [27,42]. As one can see, this approximation is reasonable and gives similar shift of the transmission minimum position.
For a large enough input spin-wave power, the transmission curve becomes bistable (see magenta line in Fig. 4). The appearance of bistability is clear from Eq. (3), which, by expanding the denominator near the resonance frequency, becomes of the same structure as the nonlinear ferromagnetic resonance curve, demonstrating the foldover effect [43, 44]. In the bistability range, the realization of one or another curve depends on the experiment (simulation) conditions. If the input spin-wave amplitude is varied, the solid curve is realized. To access the dashed curve in the simulations, we gradually decrease the excitation frequency starting outside of the bistability range with a constant spin-wave amplitude. The small discrepancy between theory and simulation is mainly attributed to nonlinear shift coefficient which is calculated from a straight waveguide not from the ring structure and the previous mentioned nonuniform spin-wave profile in the ring structure.

In neuromorphic systems, the so-called ”activation function” plays an important role. It describes how strong an incoming stimulus (here, the incoming spin-wave amplitude) needs to be to lead to the ”firing” of an artificial neuron, i.e. to the emission of an output signal. In order to characterize the activation function of the magnonic ring resonator, the spin-wave excitation frequency is fixed at \( f = 2.662 \text{ GHz} \), which coincides with the 16\(^{th}\) resonance in the linear regime, as depicted in Fig. 4 by the vertical dashed line. The output spin-wave intensity depends on the input power because this frequency does not correspond anymore to a resonance in the nonlinear regime. Figure 5 shows the transmission power for \( f = 2.662 \text{ GHz} \) as a function of the excitation field amplitude \( b_e \) and out-of-plane component of magnetization \( m_z \) (top
axis). The transmission \( T \) is almost constant and below 1% in the excitation field range from 0.6 mT to 4 mT and then dramatically increases from \( T = 0.78 \% \) at \( b_e = 4 \) mT to \( T = 51.5 \% \) at \( b_e = 13 \) mT. A high contrast of around 18 dB between the output states is observed. The contrast, threshold and maximum transmission level can be tuned by adjusting the radius of the ring \( R \), the transmission efficiency \( \tau \), and the loss coefficient \( \beta \). It is worth to note that the precession angle is around 5 degrees even for the high excitation field of 13 mT which reveals the fact that the energy consumption is very low in the magnon domain. The power-dependent transmission coefficient of the magnonic ring resonator can be used to build elements of magnonic logic and signal processing circuits. For example, the low input spin-wave power is stored in the ring structure and the ring only generates an output if the input power exceeds a threshold.

IV. Conclusion

In conclusion, a nanoscale nonlinear magnonic ring resonator is proposed, and its functionality is demonstrated using micromagnetic simulations. The transmission curve of the ring resonator is of a notch filter type. Spin waves at resonant frequencies are stored in the ring and cannot pass through it, while spin waves of a frequency in between the resonances pass the ring resonator with only a small loss. Nonlinear shift of the spin-wave resonant frequency and, consequently, of the spin-wave phase accumulation, leads to a power dependence of the magnonic ring transmission curves. In the nonlinear regime, the resonance frequencies are shifted, the transmission curves become asymmetric and, at large enough input power, exhibit a bistability. The transmission rate at the linear resonant frequency shows a threshold-like behavior: low input spin-wave power is stored in the ring structure and the ring only generates an output if the input power exceeds a threshold, which can be useful for magnonic logic applications. The obtained results are supported by the developed analytical theory, which allows to calculate the ring resonator characteristics in both the linear and nonlinear regimes.

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Appendix

Spin-wave dispersion in a ring. Dispersion and structure of spin waves in a magnonic rings can be calculated similarly to those of a vortex-state magnetic disk [45,46]. In our case, the width of the ring is sufficiently small [39] leading to an almost uniform (unpinned) spin-wave profile across the ring width, which greatly simplifies the calculations. In this approximation, the dispersion relation is given by

$$\omega_k^2 = \omega_M^2 (\lambda^2 k^2 + F_k^{(rr)})(\lambda^2 (k^2 - R^2) + F_k^{(zz)}),$$  \hspace{1cm} (A1)

where $\omega_M = \gamma \mu_0 M_s$, $M_s$ is the saturation magnetization, $\gamma$ is the gyromagnetic ratio, $\lambda$ is the exchange length, and $\hat{F}_k = \iint \hat{G}(r,r')\exp[ikR(\phi - \phi')]drdr'/ (2\pi Rw)$ is the effective dynamic demagnetization tensor with $\hat{G}(r,r')$ being magnetostatic Green’s function in the polar coordinate system [47] and the integration going over the ring surface. For an arbitrary wavenumber, the calculation of $\hat{F}_k$ is complicated. However, for $k_n = n/R$, which are the wavenumbers corresponding the ring resonant modes, it is greatly simplified, and yields

$$F_k^{(rr)} = \frac{1}{4Rw} \int_0^\infty f(kh)[I_{n+1}(kh) - I_{n-1}(kh)]^2 dk,$$  \hspace{1cm} (A2)

$$F_k^{(zz)} = \frac{1}{hRw} \int_0^\infty [1 - e^{-ikh}]I_n^2(kh) dk,$$  \hspace{1cm} (A3)

where $f(kh) = 1 - (1 - \exp(-kh)) / (kh)$, and we use the notation

$$I_n(k) = \int_{R-w/2}^{R+w/2} J_n(kr)rdr,$$  \hspace{1cm} (A4)

with the Bessel functions $J_n$. The function $I_n(k)$ can be expressed via a combination of hypergeometric functions or calculated numerically. The complete spin-wave dispersion $\omega_k$ can be numerically found by interpolation of the dispersion relations of the ring resonant frequencies $\omega_n$. The spin-wave group velocity is found via $v_{gr} = d\omega_k / dk$. The spin-wave damping rate is calculated using the general formalism [48]:
\[ \Gamma_k = \alpha_G \omega_h \left( \lambda^2 (2k^2 - R^2) + F^{(r)}_k + F^{(z)}_k \right) / 2. \] 

(A5)

**Coupling between waveguide and ring.**

The dynamics of spin-wave amplitudes \( a_1(x) \) and \( a_2(x) \) in coupled waveguides is described by the following system of equations [23]:

\[
\begin{aligned}
\nu \frac{da_1(x)}{dx} &= i \omega_c a_2(x), \\
\nu \frac{da_2(x)}{dx} &= i \omega_c a_1(x),
\end{aligned}
\]

(A6)

where \( \omega_c \) is the coupling frequency, which has the meaning of splitting of the symmetric and antisymmetric collective modes in the coupled waveguides. The difference in dispersion relations (and, consequently, in \( \nu_{gr} \)) in the waveguide and ring leads to only a small (second order) correction and is neglected here. The coupling frequency is given by

\[ \omega_c = \frac{\Omega^\gamma F_{\nu_1}^{\gamma}(d) + \Omega^\nu F_{\nu_2}^{\nu}(d)}{\omega_h}, \]

(A7)

and it depends on the coordinate \( x \) via the dependence of the distance between centers of straight and ring waveguides \( d(x) = d_0 + (R + \sqrt{R^2 - x^2}) \) with \( d_0 = \delta + w \). The position-dependent angle between the waveguide and ring and, consequently, between their static magnetizations, is not accounted for, since in the region which contributes most to the overall coupling this angle is negligible. Here and below the tensors \( \hat{\Omega}, \hat{F} \) and \( \hat{N}_k \) are defined as in Ref. [23].

From the solution of (A6) one finds the coupling and transmission coefficients, mentioned in Eq. (1):

\[ \tau = \cos(2\bar{\omega}_c R / \nu_{gr}), \quad \kappa = \sin(2\bar{\omega}_c R / \nu_{gr}), \] 

(A8)

where \( \bar{\omega}_c = (1/2R) \int_{-R}^{R} \omega_c(x)dx \) is the “averaged” coupling frequency. This equation can be used for any shape of the coupling area, for example, if the ring is changed to a polygon. For the ring structure, the calculation of \( \bar{\omega}_c \) is greatly simplified (note, that \( \hat{F}(d) \) is an integral itself) noting that the coupling frequency decays fast with the separation \( d \), so we can use the approximation \( d(x) \approx d_0 + x^2 / (2R) \) and change the integration limits to \((-\infty, \infty)\). Then,

\[ \bar{\omega}_c = \frac{\omega_0}{\omega_k} (\Omega^\gamma \Phi_{\gamma}^{\gamma} + \Omega^\nu \Phi_{\nu}^{\nu}), \]

(A9)

where
\[ \Phi = \frac{1}{\sqrt{2\pi R}} \int_{0}^{\infty} \tilde{N}_k \cos \left( k_y d_0 + \frac{\pi}{4} \right) \frac{dk_y}{\sqrt{k_y}}. \] (A10)

**Micromagnetic simulations.** The micromagnetic simulations were performed by FastMag developed at the University of California, San Diego. This software uses a finite element method to solve the LLG equation and can use the power of modern Graphics Processing Units (GPUs), which leads to the capability to handle ultra-complex geometries at a high speed [49]. The finite element method is especially useful if non-rectangular systems like the presented ring are simulated. The simulated structure of magnonic ring resonator is shown in Fig. 1a. The parameters of the nanometer-thick YIG are obtained from experiment as following: saturation magnetization \( M_s = 1.4 \times 10^5 \) A/m, exchange constant \( A = 3.5 \) pJ/m, and Gilbert damping for most of the structure \( \alpha = 2 \times 10^{-4} \), except for the ring structure. The Gilbert damping in the ring structure is increased to \( 2 \times 10^{-3} \) to meet the critical coupling condition and the damping at the ends of the simulated structure is set to exponentially increase to 0.2 to prevent spin wave reflection. The high damping region can be realized in the experiment by putting another magnetic material or metal on top of YIG. The averaged cell size was set to \( 10 \times 10 \times 10 \) nm\(^3\). To excite a propagation spin wave, a sinusoidal magnetic field \( b = b_e \sin(2\pi f t) \) was applied over an area of 40 nm in length, with a varying oscillation amplitude \( b_e \) and microwave frequency \( f \). The magnetization \( M_z(x,y,t) \) was obtained over a period of 250 ns which is long enough to reach a stable dynamic equilibrium. The spin-wave intensity of the transmitted waves is calculated by performing Fourier transform from 200 ns to 250 ns, which corresponding to the dynamic equilibrium.

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