Dynamical solution to the $\mu$ problem at TeV scale

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Abstract

We introduce a new confining force ($\mu$-color) at TeV scale to dynamically generate a supersymmetry preserving mass scale which would replace the $\mu$ parameter in the minimal supersymmetric standard model (MSSM). We discuss the Higgs phenomenology and also the pattern of soft supersymmetry breaking parameters allowing the correct electroweak symmetry breaking within the $\mu$-color model, which have quite distinctive features from the MSSM and also from other generalizations of the MSSM.
I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) contains two different types of mass scales: (i) soft supersymmetry (SUSY) breaking parameters $m_{\text{soft}}$ including the soft scalar and gaugino masses and (ii) the $\mu$ parameter in the superpotential $W \ni \mu H_1 H_2$ where $H_1$ and $H_2$ are the MSSM Higgs doublets with opposite hypercharge. In the MSSM point of view, $\mu$ is entirely different from $m_{\text{soft}}$ since it has nothing to do with SUSY breaking [2]. In order to have correct electroweak symmetry breaking without severe fine tuning, both $m_{\text{soft}}$ and $\mu$ are required to be of order the electroweak scale. Although it is technically natural that both $m_{\text{soft}}$ and $\mu$ are much smaller than the cutoff scale of the model which may be as large as the Planck scale $M_{\text{Pl}}$, one still needs to understand the dynamical origin of these mass scales for deeper understanding of their smallness [1,2].

It is commonly assumed that $m_{\text{soft}}$ arises as a consequence of spontaneous SUSY breaking at high energy scales. The explicit relation between $m_{\text{soft}}$ and the scale of spontaneous SUSY breaking depends on how the SUSY breaking is transmitted to the MSSM sector: (i) $m_{\text{soft}} \sim F/M_{\text{Pl}}$ in the case of gravity mediation with SUSY breaking auxiliary component $F$ [1] and (ii) $m_{\text{soft}} \sim (\frac{\alpha}{4\pi}) F/M_X$ in the case of gauge mediation [3] by a messenger particle with mass $M_X$. In both cases $\sqrt{F}$ is significantly larger than 1 TeV, $\sqrt{F} \sim 10^8$ TeV for gravity-mediated case and $\sqrt{F} \gtrsim 20$ TeV for gauge-mediated case [4], so it is quite unlikely that SUSY breaking dynamics can be directly probed by future experiments.

About the dynamical origin of $\mu$, there have been many interesting suggestions in the literatures [5–10]. Perhaps the most attractive possibility would be that SUSY breaking dynamics provides a dynamical seed for both $\mu$ and $m_{\text{soft}}$ in a manner to yield $\mu \sim m_{\text{soft}}$. In most cases, these schemes are again based on high energy dynamics which is hard to be probed by future experiments. In this paper we wish to propose an alternative scheme replacing $\mu$ by a new confining force ($\mu$-color) at TeV scale which would lead to interesting phenomenologies in future experiments.

The $\mu$ term is essential in the MSSM for several phenomenological reasons. Its absence implies the absence of the associated $B$-term $(B\mu H_1 H_2)$ in the scalar potential, leading to $\langle H_1 \rangle = 0$ even when nonzero $\langle H_2 \rangle$ is radiatively induced by the large top quark Yukawa coupling and also to the phenomenologically unacceptable Weinberg-Wilczek axion [2]. The $\mu$ term is necessary also to render sufficiently large masses to the Higgsinos. In the $\mu$-color model, Yukawa couplings of $H_{1,2}$ with the $\mu$-colored matter fields generate effective $\mu$ terms involving the composite Higgs doublets. The unwanted axion is avoided due to the $U(1)_{PQ}$ breaking by the strong $\mu$-color anomaly, and also the correct electroweak symmetry breaking can be achieved by the combined effects of the $\mu$-color dynamics and soft SUSY breaking terms.

As we will see, the $\mu$-color model is distinguished from the MSSM (and also from many other generalizations of the MSSM) mainly by its Higgs sector. It is distinguished also by the pattern of soft parameters which would allow the correct electroweak symmetry breaking to take place. Some soft parameter values which would lead to a successful electroweak symmetry breaking within the MSSM can not work within the $\mu$-color model, while others which would not work within the MSSM do work in the $\mu$-color model. For instance, in the $\mu$-color case it is not necessary to have a negative mass squared of $H_1$ or $H_2$ for the electroweak symmetry breaking to take place. As another example of the difference, a large
portion of the \((\tan \beta, M_m)\) space in gauge-mediated SUSY breaking models appears to be incompatible with the \(\mu\)-color model where \(M_m\) is the messenger scale of SUSY breaking, though it can be compatible with the conventional \(\mu\)-term in the MSSM \([10]\). A potentially unattractive feature of the \(\mu\)-color model is that it requires that the \(\mu\)-color gaugino mass at the messenger scale is significantly smaller than the MSSM soft parameters (by the factor of \(1/16\pi^2\)). In gauge-mediated SUSY breaking models, such a small \(\mu\)-color gaugino mass can be achieved if the messenger particles are \(SU(2)_\mu\)-singlets. In gravity-mediated case, e.g. string effective supergravity in which SUSY breaking is mediated by string moduli, the \(\mu\)-color gaugino mass is small if the \(\mu\)-color gauge kinetic function does not depend on the messenger moduli at string tree level. Thus the small \(\mu\)-color gaugino mass may not be a serious drawback of the model. At any rate, we note that string effective supergravity models provide large varieties in the pattern of soft parameters \([11, 12]\), which are diverse enough to include those giving the correct Higgs phenomenology within the \(\mu\)-color model.

II. THE MODEL

The minimal \(\mu\)-color model includes, in addition to the MSSM gauge and matter multiplets, the \(\mu\)-color gauge group \(SU(2)_\mu\) which confines at \(\Lambda_\mu \sim 1\) TeV and also the \(\mu\)-colored matter superfields which transform under \(SU(2)_\mu \times SU(2)_L \times U(1)_Y\) as

\[
Y_{\alpha a} = (2, 2)_0, \quad X_{1a} = (2, 1)_{1/2}, \quad X_{2a} = (2, 1)_{-1/2},
\]

where \(a = 1, 2\) and \(\alpha = 1, 2\) denote the \(SU(2)_\mu\) and \(SU(2)_L\) doublet indices, respectively, and the subscripts of the brackets denote the \(U(1)_Y\) charge. Obviously these additional matters are free from (both perturbative and global) gauge and gravitational anomalies. The MSSM matter parity can be easily generalized to the \(\mu\)-color model such that the two MSSM Higgs doublets are even while all other matter multiplets are odd under the generalized matter parity. Then the most general scale-free tree-level superpotential with the generalized matter parity is given by

\[
W_{\text{tree}} = \lambda_1 H_1 Y X_1 + \lambda_2 H_2 Y X_2 \\
+ \lambda_d H_1 Q D^c + \lambda_u H_2 Q U^c + \lambda_l H_1 L E^c,
\]

where \(H_{1,2}, Q, U^c, D^c, L\) and \(E^c\) denote the MSSM fields in self-explanatory notation, and all the gauge and generation indices are omitted here.

For the \(\mu\)-colored matter contents of Eq. (1), the holomorphic \(\mu\)-color scale is given by

\[
\Lambda_\mu = M_{\text{GUT}} \exp\left(-\frac{2\pi^2}{g_\mu^2(M_{\text{GUT}})} + \frac{\theta_\mu}{4}\right),
\]

where \(g_\mu\) and \(\theta_\mu\) are the \(\mu\)-color gauge coupling and vacuum angle, respectively. Once the extra matter multiplets of (1) carrying \(SU(2)_L \times U(1)_Y\) charges are introduced, we lose the unification of gauge couplings at single energy scale. However this may not be a serious drawback of the model since there are many string theory models, e.g. heterotic string theory with a large threshold effects \([14]\) and/or Type I strings with different type of D-branes \([15]\), implying that the gauge couplings at the string or unification scale can take
different values. At any rate, we note that $\alpha_\mu(M_{GUT}) \sim 1/19$ and $M_{GUT} \sim 10^{16}$ GeV lead to $\Lambda_\mu \sim 1$ TeV, so having $\Lambda_\mu$ at TeV scale is a plausible possibility.

A crucial feature of the $\mu$-color model is that there is no mass parameter in $W_{\text{tree}}$. Thus at scales above $\Lambda_\mu$, all the mass parameters are in the soft SUSY breaking terms which are presumed to be induced by SUSY breaking dynamics at scales far above $\Lambda_\mu$. For the scale-free tree level superpotential $W_{\text{tree}} = \lambda_{ijk}\Phi_i\Phi_j\Phi_k$, soft SUSY breaking terms can be written as

$$-\mathcal{L}_{\text{soft}} = m_i^2|\Phi_i|^2 + (A_{ijk}\lambda_{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}M_\mu\lambda^a\lambda^a + \text{h.c.})$$

$$= m_Y^2|Y|^2 + m_{X_1}^2|X_1|^2 + m_{X_2}^2|X_2|^2 + \left(\frac{1}{2}M_\mu\lambda^a\lambda^a\right) + A_1\lambda_1 H_1Y X_1 + A_2\lambda_2 H_2 Y X_2 + \ldots \text{h.c.},$$

(4)

where $\Phi_i$ in $\mathcal{L}_{\text{soft}}$ corresponds to the scalar component of the corresponding superfield, $\lambda^a$ are gauginos ($\lambda^\mu$ and $M_\mu$ are the $\mu$-color gaugino and its mass, respectively), and the ellipsis stands for the terms involving only the MSSM fields. In this paper, we will not address the origin of these soft parameters, but take an approach to allow generic forms of soft parameters as long as they are phenomenologically allowed. In this regard, we note that string theories with the SUSY breaking mediated by string moduli show enough varieties in the resulting soft parameters [11, 12].

Let us discuss some global symmetries and the associated selection rules which will be useful for the later discussion of the effective theory below $\Lambda_\mu$. In the limit that $W_{\text{tree}}, \mathcal{L}_{\text{soft}},$ and the standard model gauge couplings are all turned off, the model is invariant under the $SU(4)$ global rotation of the four $SU(2)_\mu$ doublets $X_{1a}, X_{2a}, Y = (Y_{1a}, Y_{2a})$. The model includes also several global $U(1)$ symmetries whose charge assignments are given by

$$U(1)_{PQ} : (Y, X_1, X_2, H_1, H_2, U^c, D^c, E^c, A_\mu)$$

$$= (-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 1, 1, -1, -1, -1, -\frac{1}{2}),$$

$$U(1)_R : (H_1, H_2, \lambda^a, A_{ijk}, M_a) = (2, 2, 1, -2, -2),$$

$$U(1)_\mu : (Y, X_1, X_2) = (1, -1, -1),$$

(5)

where the superfields that do not appear in this charge assignment are understood to have vanishing charge. Note that $U(1)_{PQ}$ is explicitly broken by the strong $SU(2)_\mu$ anomaly as indicated by that the holomorphic scale $\Lambda_\mu = M_{GUT} \exp(-\frac{2\pi^2}{g^2_{\mu}(M_{GUT})} + i\frac{\theta}{4})$ carries nonzero $U(1)_{PQ}$ charge. As a result, its spontaneous breaking at scales below $\Lambda_\mu \sim 1$ TeV does not lead to any phenomenologically harmful axion. $U(1)_R$ is free from the $SU(2)_\mu$ anomaly, however broken by the gaugino masses ($M_a$) and $A$-parameters ($A_{ijk}$) carrying $-2$ units of $U(1)_R$ charge. Finally $U(1)_\mu$ corresponds to the $\mu$-baryon number which is exactly conserved within our framework.

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1 This may be explained by the $U(1)_R$ symmetry of Eq. (5) which forbids the bilinear terms such as $YY, X_1X_2,$ and $H_1H_2$ in the superpotential.
III. EFFECTIVE THEORY BELOW $\Lambda_\mu$

In the limit that $m_{\text{soft}} \ll \Lambda_\mu$ and $\langle H_{1,2} \rangle \ll \Lambda_\mu$, light degrees of freedom at scales below $\Lambda_\mu$ correspond to $SU(2)_\mu$-invariant composite superfields describing $SU(2)_\mu$ $D$-flat directions \[13\]. In our case, the light composite fields are given by

$$Z_{AB} = \begin{pmatrix}
0 & T & Z_{11} & Z_{12} \\
- T & 0 & Z_{21} & Z_{22} \\
-Z_{11} & -Z_{21} & 0 & S \\
-Z_{12} & -Z_{22} & -S & 0
\end{pmatrix} \quad (6)$$

obeying the constraint \[13\]:

$$\text{Pf}(Z) = \frac{1}{2} \epsilon^{ABCD} Z_{AB} Z_{CD} = \epsilon^{\alpha\beta} Z_{1\alpha} Z_{2\beta} - ST = \hat{\Lambda}^2, \quad (7)$$

where

$$S \sim \frac{1}{\Lambda_\mu} \epsilon^{ab} Y_{1a} Y_{2b}, \quad T \sim \frac{1}{\Lambda_\mu} \epsilon^{ab} X_{1a} X_{2b},$$

$$Z_{1\alpha} \sim \frac{1}{\Lambda_\mu} \epsilon^{ab} X_{1a} Y_{ab}, \quad Z_{2\alpha} \sim \frac{1}{\Lambda_\mu} \epsilon^{ab} X_{2a} Y_{ab}. \quad (8)$$

Here $a, b$ and $\alpha, \beta$ are $SU(2)_\mu$ and $SU(2)_L$ doublet indices, respectively. For the composite fields normalized to have canonical kinetic terms, the supersymmetric naive dimensional argument (NDA) \[16,17\] leads to

$$\hat{\Lambda} \approx \Lambda_\mu / 4\pi. \quad (9)$$

The low energy effective action of the composite fields $Z_{AB}$ can be expanded in powers of $1/\Lambda_\mu$, more precisely in powers of $H_{1,2}/\Lambda_\mu$ and/or of $m_{\text{soft}}/\Lambda_\mu$, where each term in the expansion is consistent with the symmetries and selection rules discussed in the previous section. The NDA rule \[16,17\] then provides an order of magnitude estimate of the expansion coefficients at energy scales around $\Lambda_\mu$ at which the $SU(2)_\mu$ gauge coupling saturates the bound $g_\mu \lesssim 4\pi$. Let us normalize all superfields to have the canonical kinetic terms. Then applying the NDA rule together with the symmetries and selection rules of the underlying superpotential, we find the following form of the effective superpotential

$$W_{\text{eff}} = X(Z_1 Z_2 - ST - \hat{\Lambda}^2) + a_1 \hat{\Lambda}(\lambda_1 H_1 Z_1 + \lambda_2 H_2 Z_2) + W_{\text{MSSM}}, \quad (10)$$

where $W_{\text{MSSM}}$ stands for the Yukawa terms involving only the MSSM superfields, $a_1$ is a nonperturbative parameter of order unity, and the $SU(2)_L$ gauge indices are omitted. Here the Lagrange multiplier superfield $X$ is introduced to implement the constraint \[7\]. Note that $X$ is not a dynamical field and so does not appear in the Kähler potential. There may be additional terms in $W_{\text{eff}}$ which are higher order in $1/\Lambda_\mu$, but the NDA rule suggests that the effects of such higher order terms are suppressed by more powers of $\langle H_{1,2} \rangle / \Lambda_\mu$. As will be argued in the subsequent discussions, $m_{\text{soft}}$ and the Higgs VEVs are all comparable to $\hat{\Lambda} \approx \Lambda_\mu / 4\pi$ in our framework, and then the $1/\Lambda_\mu$ expansion whose coefficients obey the
NDA rule becomes essentially an expansion in powers of $1/4\pi$. Though not a terribly good approximation, we expect that this expansion is reasonably good and thus the leading order results are not significantly modified by higher order corrections.

In the $\mu$-color model, there are four doublet VEVs participating in the electroweak symmetry breaking:

$$\langle H_1 \rangle^2 + \langle H_2 \rangle^2 + \langle Z_1 \rangle^2 + \langle Z_2 \rangle^2 = (178 \text{ GeV})^2.$$ (11)

If any of $S$ and $T$ develops a nonzero VEV, $U(1)_{\mu}$ will be spontaneously broken, leading to a potentially dangerous Goldstone boson. To avoid this problem, we assume $\langle S \rangle = \langle T \rangle = 0$ which can be easily achieved by choosing appropriate values of $m^2_S$ and $m^2_T$. Then the constraint (7) gives $\langle Z_1 Z_2 \rangle = \hat{\Lambda}^2$, and so $\langle Z_1 \rangle^2 + \langle Z_2 \rangle^2 \gtrsim 2\hat{\Lambda}^2$. Furthermore, one would require $\langle H_2 \rangle$ not significantly smaller than 100 GeV in order to avoid a too large top quark Yukawa coupling. Combining these, one finds $\hat{\Lambda} < \sim 110$ GeV where the upper limit is saturated when $\langle Z_1 \rangle \approx \langle Z_2 \rangle \approx \hat{\Lambda}$. In most cases, it is phenomenologically desirable to have $\hat{\Lambda}$ close to its upper limit, and then we have

$$\Lambda_\mu = 4\pi \hat{\Lambda} \sim 1 \text{ TeV}.$$ (12)

Soft SUSY breaking terms of the composite fields $Z_{AB}$ can be similarly expanded in powers of $m^2_{\text{soft}}/\Lambda_\mu$ (and also of $H_{1,2}/\Lambda_\mu$) where $m^2_{\text{soft}}$ denote the soft parameters of the $\mu$-colored elementary fields renormalized at the NDA scale. At the leading order, we find

$$-\mathcal{L}_{\text{soft eff}}^\text{eff} = m^2_S |S|^2 + m^2_T |T|^2 + m^2_{Z_1} |Z_1|^2 + m^2_{Z_2} |Z_2|^2 + (\hat{A}_1 \lambda_1 \hat{\Lambda} H_1 Z_1 + \hat{A}_2 \lambda_2 \hat{\Lambda} H_2 Z_2 + \hat{A}_3 \hat{\Lambda}^2 X + \text{h.c.}),$$ (13)

where

$$m^2_S = a_2 (m^2_{X_1} + m^2_{X_2}) + a_3 |M_\mu|^2,$$
$$m^2_T = 2a_2 m^2_Y + a_3 |M_\mu|^2,$$
$$m^2_{Z_1} = a_2 (m^2_{X_1} + m^2_Y) + a_3 |M_\mu|^2,$$
$$m^2_{Z_2} = a_2 (m^2_{X_2} + m^2_Y) + a_3 |M_\mu|^2,$$
$$\hat{A}_1 = a_1 A_1 + a_4 M_\mu,$$
$$\hat{A}_2 = a_1 A_2 + a_4 M_\mu,$$
$$\hat{A}_3 = a_5 M_\mu.$$ (14)

Here the nonperturbative parameters $a_i$ ($i = 2, 3, 4, 5$) are again of order unity when the soft parameters of the $\mu$-colored elementary fields are renormalized at the NDA scale $\Lambda_\mu$ at which $g_\mu(\Lambda_\mu) \sim 4\pi$.

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2 The soft SUSY breaking scalar potential includes also the additional $A$-term: $AX(Z_1 Z_2 - ST - \hat{\Lambda}^2)$, but this can be eliminated by the redefinition of the $F$-component of the Lagrange multiplier: $F_X \rightarrow F_X + AX$. 
When it is runned from the messenger scale $M_m$ of SUSY breaking to $\Lambda_\mu$, the $\mu$-color gaugino mass is enhanced by the nonperturbative factor $\sim 16\pi^2$:

$$M_\mu(\Lambda_\mu) \sim \frac{g_\mu^2(\Lambda_\mu)M_\mu(M_m)}{g_\mu^2(M_m)} \sim (4\pi)^2 M_\mu(M_m).$$

Furthermore if the soft SUSY breaking at $\Lambda_\mu$ is dominated by $M_\mu$, the renormalization group evolution makes the other soft parameters of the $\mu$-colored fields at $\Lambda_\mu$, i.e. $m_{\tilde{X}_{1,2}}, m_{\tilde{Y}}$, and $A_{1,2}$, to be comparable to $M_\mu(\Lambda_\mu)$ also. Thus if $M_\mu$ were comparable to the soft parameters of the MSSM fields at $M_m$, there will arise a $16\pi^2$-hierarchy between the MSSM soft parameters and the soft parameters of the $\mu$-colored fields at the NDA scale $\Lambda_\mu$, and thus the same hierarchy between the MSSM soft parameters and the soft parameters of the composite fields $Z_{AB} = \{S, T, Z_1, Z_2\}$. In order to provide a consistent framework, the soft parameters of both $Z_{AB}$ and the MSSM fields at the electroweak scale are required to be comparable to $\frac{\Lambda_\mu}{4\pi}$. This means that at $M_m$ the $\mu$-color gaugino mass must be smaller than the MSSM soft parameters by the factor of $\frac{1}{16\pi^2}$:

$$M_\mu(M_m) \lesssim \frac{1}{16\pi^2} m_{\text{soft}}(M_m). \quad (15)$$

In gauge-mediated SUSY breaking models [3], such a small $\mu$-color gaugino mass can be achieved if the messenger particles are $SU(2)_\mu$-singlets. In gravity-mediated case, e.g. string effective supergravity models in which SUSY breaking is mediated by string moduli, $M_\mu(M_m)$ is small if the $\mu$-color gauge kinetic function does not depend on the messenger moduli at string tree level [11,12].

IV. HIGGS PHENOMENOLOGY

The key difference between the $\mu$-color model and the MSSM is in the Higgs sector. To see this, let us consider the neutral Higgs sector of the model in more detail. For notational simplicity, in this section let $Z_{1,2}$ and $H_{1,2}$ denote the neutral components of the corresponding composite and elementary Higgs doublets. Due to the exact $\mu$-baryon symmetry ($U(1)_\mu$), one can always adjust the parameters of the model, e.g. $m_{\tilde{S}}^2$ and $m_{\tilde{T}}^2$, to have $\langle S \rangle = \langle T \rangle = 0$. We then have five neutral complex scalar field fluctuations ($\delta \Phi = \Phi - \langle \Phi \rangle$) with masses of order the electroweak scale: the two composite singlet Higgs fluctuations $\delta S$ and $\delta T$, the two elementary doublet Higgs fluctuations $\delta H_1$ and $\delta H_2$, and finally one linear combination of the composite Higgs doublet fluctuations $\delta Z_1$ and $\delta Z_2$ obeying the constraint

$$\langle Z_1 \rangle \delta Z_1 + \langle Z_2 \rangle \delta Z_2 = 0.$$

In particular, we have three physical scalar and two pseudo-scalar particles arising from the neutral components of the doublet Higgs fluctuations.

To study the electroweak symmetry breaking and the Higgs mass spectrum, let us consider the scalar potential of the Higgs doublets while setting $S$ and $T$ to their vanishing VEVs. We first have the $F$-term potential arising from the superpotential:

$$V_F = |XZ_2 + \lambda_1 \hat{A}H_1|^2 + |XZ_1 + \lambda_2 \hat{A}H_2|^2 + |\lambda_1 \hat{A}^2Z_1|^2 + |\lambda_2 \hat{A}^2Z_2|^2 - (F_X(Z_1Z_2 - \hat{A}^2) + h.c.). \quad (16)$$
and also the contribution from soft SUSY breaking:

$$V_{\text{soft}} = (\hat{A}_1 \hat{\lambda}_1 H_1 Z_1 + \hat{A}_2 \hat{\lambda}_2 H_2 Z_2 + \hat{A}_3 \hat{\lambda}^2 X + h.c.)$$  \hspace{1cm} (17)

$$+ m^2_{H_1} |H_1|^2 + m^2_{H_2} |H_2|^2 + m^2_{Z_1} |Z_1|^2 + m^2_{Z_2} |Z_2|^2.$$  

Then the quation of motion for the auxiliary field $X$ yields

$$X = - \frac{(\lambda_1 H_1 Z_2^* + \lambda_2 H_2 Z_1^*) \hat{\lambda} + \hat{A}_3 \hat{\lambda}^2}{|Z_1|^2 + |Z_2|^2},$$  \hspace{1cm} (18)

leading to

$$V_F + V_{\text{soft}} = |\lambda_1 \hat{\lambda} Z_1|^2 + |\lambda_2 \hat{\lambda} Z_2|^2 + |\lambda_1 \hat{\lambda} H_1|^2 + |\lambda_2 \hat{\lambda} H_2|^2$$

$$- \frac{|(\lambda_1 H_1 Z_2^* + \lambda_2 H_2 Z_1^*) \hat{\lambda} + \hat{A}_3 \hat{\lambda}^2|^2}{|Z_1|^2 + |Z_2|^2} - (F_X (Z_1 Z_2 - \hat{\lambda}^2) + h.c.)$$

$$(\hat{A}_1 \lambda_1 \hat{\lambda} H_1 Z_1 + \hat{A}_2 \lambda_2 \hat{\lambda} H_2 Z_2 + h.c.)$$

$$(2 \lambda_2 \hat{\lambda}^2 |Z_1|^2 + 2 \lambda_1 \hat{\lambda}^2 |Z_2|^2 + m^2_{H_1} |H_1|^2 + m^2_{H_2} |H_2|^2 + m^2_{Z_1} |Z_1|^2 + m^2_{Z_2} |Z_2|^2).$$

There is also the $D$-term potential

$$V_D = \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2 - |Z_1|^2 + |Z_2|^2)^2.$$  \hspace{1cm} (20)

Putting these together,

$$V = V_F + V_D + V_{\text{soft}},$$  \hspace{1cm} (21)

we see that the Higgs potential takes a form very different from that of the MSSM or of other generalizations of the MSSM.

Since the Higgs potential takes so different form, the soft parameter ranges for successful electroweak symmetry breaking can be different also. Some soft parameter ranges which would not lead to the correct electroweak symmetry breaking within the MSSM, e.g. positive $m^2_{H_1}$ and $m^2_{H_2}$ at the electroweak scale, can successfully generate the symmetry breaking in the $\mu$-color framework, while some others which would work in the MSSM do not work within the $\mu$-color framework. To see this more explicitly, let us consider the case that all Higgs doublet VEVs can be chosen to be real. Then the vacuum stability condition includes

$$\left\langle \frac{\partial^2 V}{\partial \text{Re}(H_2)^2} \right\rangle \geq 0,$$  \hspace{1cm} (22)

which corresponds to

$$2|\lambda_2 \hat{\lambda}|^2 - \frac{2|\lambda_2 \hat{\lambda}|^2 |Z_1|^2}{Z_1^2 + Z_2^2} + 2m^2_{H_2} + \frac{g^2 + g'^2}{2} (3H_2^2 - H_1^2 + Z_1^2 - Z_2^2) \geq 0,$$  \hspace{1cm} (23)

where all Higgs fields mean their VEVs which are assumed to be real. Combining this with Eqs. (7) and (11) which imply ($m_Z$ is the $Z$-boson mass)

$$2|\hat{\lambda}|^2 \lesssim Z_1^2 + Z_2^2 \lesssim 4m_Z^2,$$  \hspace{1cm} (24)
one easily finds (with $g^2 + g'^2 \approx 0.5$

\[ -m_{H_2}^2 \lesssim 3m_Z^2 + |\hat{\Lambda}|^2 (|\lambda_2|^2 - 1 - \frac{|\lambda_2|^2 Z_1^2}{2m_Z^2}) - \frac{1}{2} Z_2^2 \]

\[ \lesssim 3m_Z^2 + (2|\lambda_2|^2 - 1)|\hat{\Lambda}|^2 - \left(\frac{|\lambda_2\hat{\Lambda}|}{m_Z}\right)|\hat{\Lambda}|^2. \]

For $|\lambda_2| \lesssim 1.5$, the above limit gives

\[ m_{H_2}^2 \gtrsim -(174 \text{ GeV})^2, \]

which is in conflict with the large portion of the $(\tan \beta, M_m)$ space in gauge mediated SUSY breaking models \[^{11}\] where $M_m$ is the messenger scale of SUSY breaking. This shows that the $\mu$ color model can be incompatible with certain soft parameter ranges which would be fine with the conventional $\mu$ term in the MSSM.

Since the Higgs potential of the $\mu$-color model is too complex to get analytic vacuum solutions for generic parameter values, here we consider two cases one of which allows an analytic solution, while the other requires numerical analysis. The first case is when the parameters renormalized at the electroweak scale are all real and obey

\[ \lambda_1 \approx \lambda_2, \quad \hat{A}_1 \approx \hat{A}_2, \quad m_{Z_1}^2 \approx m_{Z_2}^2, \]

\[ m_{H_1}^2 \approx m_{H_2}^2 \approx \lambda_1\hat{\Lambda}\hat{A}_1 > 0. \]

In this case, it is straightforward to find that the Higgs potential has a (local) minimum at

\[ \langle H_1 \rangle \approx \langle H_2 \rangle \approx \langle Z_1 \rangle \approx \langle Z_2 \rangle \approx \hat{\Lambda}, \]

where all parameters are assumed to be real. The neutral components of the four Higgs doublets, $H_{1,2}$ and $Z_{1,2}$, constrained as $Z_1 Z_2 = \hat{\Lambda}^2$ contain three physical scalar and two pseudoscalar particles. After a tedious but still straightforward computation, we find the scalar mass eigenvalues are given by

\[ (\text{scalar mass})^2 \approx (m_{H_1}^2, m_1^2 + m_2^2, m_1^2 - m_2^2), \]

where

\[ m_1^2 = 2\lambda_1^2\hat{\Lambda}^2 + m_{Z_1}^2 + (g^2 + g'^2)\hat{\Lambda}^2 \]

\[ m_2^4 = 2\lambda_1^4\hat{\Lambda}^4 + 2m_{Z_1}^2\lambda_1^2\hat{\Lambda}^2 + 2\lambda_1^2(g^2 + g'^2)\hat{\Lambda}^2 + m_{Z_1}^4 \]

\[ - 4\lambda_1^2\hat{\Lambda}^2 m_{H_1}^2 - 2m_{Z_1}^2 m_{H_1}^2 + (g^2 + g'^2)^2\hat{\Lambda}^4, \]

and also the pseudoscalar mass eigenvalues

\[ (\text{pseudoscalar mass})^2 \approx (m_{H_1}^2, 2m_{H_1}^2 + 2\lambda_1^2\hat{\Lambda}^2). \]

The $\mu$ color confining scale $\hat{\Lambda} \approx 90 \text{ GeV}$ is fixed by Eq.(11), and the Higgs spectrums are distributed in hundred GeV range if the soft parameters are also in few hundred GeV range. The lightest Higgs mass can be large enough to satisfy the current experimental lower bound,
particularly when the one-loop corrections involving the large top Yukawa coupling are taken into account.

Different types of VEVs and spectrums are obtained by alleviating the relations among the parameters given in (27). Note that the conditions in (27), especially \( m^2_{H_1} \approx m^2_{H_2} \) at the electroweak scale, are difficult to be achieved in the popular minimal supergravity model or gauge mediated models, though possible in string theory models with moduli-mediated SUSY breaking [11,12]. As another example of the successful Higgs phenomenology, we consider the parameter values at the electroweak scale:

\[
\lambda_1 \approx 0.5, \quad \lambda_2 \approx 0.9, \quad \hat{A}_1 \approx \hat{A}_2 \approx 70 \text{ GeV},
\]
\[
m^2_{Z_1} \approx m^2_{Z_2} \approx m^2_{H_1} \approx (270 \text{ GeV})^2, \quad m^2_{H_2} \approx -(30 \text{ GeV})^2.
\]

We then find the Higgs VEVs given by

\[
\langle H_1 \rangle \approx 7 \text{ GeV}, \quad \langle H_2 \rangle \approx 120 \text{ GeV},
\]
\[
\langle Z_1 \rangle \approx \langle Z_2 \rangle \approx \hat{\Lambda} \approx 90 \text{ GeV},
\]

and also the masses of the three scalar and two pseudoscalar neutral Higgs particles

scalar mass = (90, 270, 390) GeV
pseudoscalar mass = (92, 270) GeV.

If we include the loop corrections involving the top Yukawa coupling, the scalar mass can be increased by 10 \sim 30 \text{ GeV} depending on the top-stop mass ratio.

V. CONCLUSION

In this paper, we introduced a new confining force, the \( \mu \)-color, at TeV scale to replace the \( \mu \) parameter in the MSSM superpotential. Below the \( \mu \)-color scale, the model predict composite Higgs doublets and singlets whose mass spectrum has been analyzed for certain parameter range. The \( \mu \) color model has very distinctive electroweak symmetry breaking mechanism which differs entirely from the conventional radiatively generated one. Electroweak symmetry is broken by the \( \mu \) color dynamics together with soft SUSY breaking terms. The soft parameter ranges for successful electroweak symmetry breaking can be quite different from the MSSM and other generalizations of the MSSM. Some soft parameter ranges which would not lead to the correct electroweak symmetry breaking within the MSSM, e.g. positive \( m^2_{H_1} \) and \( m^2_{H_2} \) at the electroweak scale, can successfully generate the symmetry breaking in the \( \mu \)-color framework, while some others which would work in the MSSM do not work within the \( \mu \)-color framework.

It would be fair to finally summarize the potentially unattractive features of the \( \mu \)-color model which have been noticed in sections III and IV. First, we lose the unification of gauge couplings at single scale due to the extra \( \mu \)-colored matter multiplets carrying \( SU(2)_L \times U(1)_Y \) charges. However this may not be so serious in view of that many string theory scenarios imply that generically gauge couplings at the string or unification scale can take different values. Second, in order to implement the electroweak symmetry breaking
without fine tuning, it is required that the $\mu$-color gaugino mass at the SUSY breaking messenger scale $M_m$ is smaller than the MSSM soft parameters by the factor of $1/16\pi^2$. Such a small $\mu$-color gaugino mass can be easily achieved within gauge-mediated and/or gravity-mediated SUSY breaking models. In particular, string effective supergravity models would give a small $\mu$-color gaugino mass if the $\mu$-color gauge kinetic function does not depend on the messenger moduli at string tree level [11,12]. Finally the model does not provide a rationale for $\mu \sim m_{\text{soft}}$ since these two mass scales have different dynamical origin. Even with these features, it appears to be worthwhile to study the phenomenological aspects of the $\mu$-color model in view of its very rich phenomenologies at TeV scale.

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