An oil sloshing study: adaptive fixed-mesh ALE analysis and comparison with experiments

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Abstract
We report in this work a numerical analysis of the sloshing of a squared tank partially filled with a domestic vegetable oil. The tank is subject to controlled motions with a shake table. The free-surface evolution is captured using ultrasonic sensors and an image capturing method. Only confirmed data within the error range is reported. Filling depth, imposed amplitude and frequency effects on the sloshing wave pattern are specifically evaluated. The experiments also reveal the nonlinear wave behavior. The numerical model is based on a stabilized finite element method of the variational multi-scale type. The free-surface is captured using a level set technique developed to be used with adaptive meshes in Arbitrary Lagrangian–Eulerian framework. The numerical results are compared with the experiments for different sloshing conditions near the first sloshing mode. The simulations satisfactorily match the experiments, providing a reliable tool for the analysis of this kind of problems.

Keywords Sloshing · Experimental validation · Arbitrary Lagrangian–Eulerian (ALE) · Stabilized finite element methods · Adaptive mesh

1 Introduction

Due to the importance of free-surface and two-fluid problems in many physical situations and engineering applications, such as, ship hydrodynamics, dam break, sloshing in tanks, shallow water, mold filling, or ink-jet analyses [1–4], among others, the development of efficient and accurate numerical schemes capable of representing these types of phenomena is of major importance.

Interface problems, like free-surface flows [5,6], phase-change thermally coupled analysis [7,8], or solid mechanics problems [9–11], are cases where the use of adaptivity could be relevant because thin layers of the full domain contain relevant information for the the global problem. In this work an adaptive fixed-mesh ALE method is used to numerically approximate the sloshing of a squared tank partially filled with a domestic oil subjected to a controlled movement generated by a shake table. Numerical results are compared with experimental data obtained via image processing and the use of ultrasonic sensors.

Several approaches to study moving interfaces have been proposed over the years, and therefore, devising a single classifications that covers all of them is not an easy task. However, one of the most general classification depends on the nature of the mesh used, which can be fixed or moving. A complete review of numerical methods used to solve free surface flow problems including experimental validation can be found in the literature (e.g., [4,12,13] and references therein).

In the computational fluid dynamics framework, the classical ALE approach has been proposed as a method where the computational mesh that covers the solved domain is deformed following the flow [14,15]. In any moving discretization method, the accuracy of the method depends on the mesh distortion, which can be large in the type of sloshing problems dealt with in this paper, particularly when the excitation frequency is close to the natural frequency. Several
methods have been devised to address this situation [16–19]. When an excessively distorted mesh is reached, also an effective approach consists in computing a new mesh that ensures good quality form, and later, project the results from the deformed mesh to the new mesh [20,21].

In [22], a fixed-mesh ALE approach for the numerical approximation of flows in moving domains was proposed. The key idea of this method consists in projecting the results of the ALE deformed mesh onto a fixed background mesh at each time step, prior to solving the flow equations. This procedure is known as a fixed mesh-ALE method, and in fact the ALE deformed mesh does not need to be explicitly solved. This fixed-mesh ALE approach was used satisfactorily both in free surface problems [23] as in the numerical simulation of floating solids [24], appearing as a powerful tool for the numerical approximation of moving domains. In [5], the fixed-mesh ALE method was coupled with an adaptive mesh algorithm giving place to a highly efficient and robust method. In that work, several numerical aspects were discussed in detail, such as, the global stabilization method used to ensure bounded pressure and to solve convective dominated flows for equal interpolation spaces between velocity and pressure, and additionally, the stability terms designed to stabilize the ill-conditioning introduced by the cuts on the background finite element mesh.

Currently, much effort is devoted to experimentally validate numerical models. To this end, controlled experiments are proposed to provide valuable data to be used for comparison with numerical results (e.g., [25–31]). In the present work, a novel sloshing experiment is reported and its numerical analysis is performed. The experiment consists of a square tank filled with a commercial vegetable oil and subjected to controlled imposed motion via a shake table. Sloshing experiments performed in water and their simulations were reported in [31,32] using a similar layout. Those papers report experiments in rectangular tanks for a water filling depth of 100mm and the numerical modelling performed with fixed-mesh stabilized finite element formulations using free surface tracking and capturing techniques. In the present work, oil is used and different filling depths are investigated. The aim of this study is to measure wave height evolution during sloshing of square-section tanks and evaluate its dependence on the filling depth. The measurements are made using two techniques to confirm and verify the experimental data. The modelling is performed with a VMS finite element adaptive level set formulation.

This work encompasses numerical and experimental analysis of an oil sloshing problem and its main contributions are:

- comprehensive analysis of an oil sloshing problem is made.

The remainder of this work is organized as follows. Section 2 presents the governing equations. The numerical strategy is presented in Sect. 3. Section 3.1, describes specific aspects of the stabilized finite element method used to solve free-surface problems. The time integration is shortly presented in Sect. 3.2. The level set method adopted and the adaptive mesh approach are reported in Sects. 3.3 and 3.4, respectively. The experimental study of an oil sloshing problem in square tanks is reported in Sect. 4. The modeling of the problem is presented in Sect. 5, comparison with experimental data validates the numerical model. Finally, conclusions are drawn in Sect. 6.

2 Problem statement

In fixed-domain free surface analyses, the computational domain represented by Ω can be split in two parts, the part of the domain effectively occupied by the fluid Ω1(t) and the remaining Ω\Ω1(t). The moving boundary of Ω1(t) is the free surface Γfree(t). Note that both Ω1(t) and Γfree(t) are time-dependent. This domain motion can be represented using an ALE domain velocity uΩ(x, t) ∈ Rd, where x ∈ Ω are the spacial coordinates. For an ALE approach, the domain velocity does not coincide in general with the velocity of the fluid in Ω1(t). The effective conditions between domain velocity and the flow velocity u are defined by n · uΩ = n · u in free ∪ @, where n represents the outward normal of Γfree(t). In the rest of the domain, a smooth velocity consistent with these boundary conditions is defined. Under these definitions, the conservation equations for momentum and mass in differential form for incompressible Newtonian fluids can be expressed as:

\[
\frac{\partial u}{\partial t} + \rho a \cdot \nabla u - \nabla \cdot (2\mu \nabla^2 u) + \nabla p = f, \quad (1)
\]

\[
\nabla \cdot u = 0, \quad (2)
\]

in a fixed computational domain Ω of Rd (d is the space dimensions) partially occupied by the fluid and, t ≡ [0, tf] is the time interval for which the problem is solved, where a = u – uΩ. In addition, ρ and μ denote the density and kinematic viscosity of the fluid, p : [0, tf] → R is the pressure field, f = ρg is the gravity force vector and, g is the gravity acceleration. As notation, \nabla^2 u represents the symmetrical gradient of the velocity \( \nabla u = \frac{1}{2} (\nabla u + (\nabla u)^T) \). The above equations need to be solved together with initial and appropriate boundary conditions. We define the fluid stresses as \[ \sigma = 2\mu \nabla^2 u - pI, \] where I is the identity tensor. As usual, boundary conditions can be subdivided into Dirichlet
and Neumann boundary conditions, i.e., \( \mathbf{u} = \mathbf{u}_D \) on \( \Gamma_D \) and, \( \sigma \cdot \mathbf{n} = \mathbf{h} \) on \( \Gamma_N \), where \( \mathbf{n} \) represents the outward unit normal vector, \( \Gamma_D \) is the Dirichlet boundary, and \( \Gamma_N \) is the Neumann boundary. The effect of any fluid outside the fluid domain is neglected. Another usual assumption consists in neglecting the surface tension effects, which is reasonable in most engineering applications. Under this assumption, the boundary condition we enforce in the free-surface interface, referred as \( \Gamma_{\text{free}}(t) \), is a traction-free condition \( \sigma \cdot \mathbf{n} = \mathbf{0} \) on \( \Gamma_{\text{free}}(t) \).

More details of this approach can be found in \([14,22,33]\), where different ALE methods have been proposed for moving domains. In this work a Fixed-Mesh ALE method is used. The particularity of this method is that, after each discrete time integration scheme, the results are projected from the deformed mesh to the initially undeformed mesh, from which the simulation is continued. Thus, the method falls in the fixed-mesh method family, but it allows to properly track the movement of the domain using an ALE approach.

### 3 Numerical model

This section summarizes the adaptive fixed-mesh ALE approach used to solve the free surface problem.

#### 3.1 Stabilized finite element formulation

It is well known that the standard Galerkin method fails when the nonlinear convective term dominates the viscous term. Another drawback is related to the discrete compatibility or inf-sup condition that must be satisfied by the \( \mathbf{V} \times \mathbf{Q} \) pair in order to have a well-posed problem with bounded pressure. As it is well known, equal order approximation for velocity and pressure does not yield a stable scheme even in a single fluid problem with fixed domain. A possible remedy to this situation consists of using stabilized formulations permitting any equal order interpolation of the unknowns \([34–40]\). The stabilized method used in this work is based on the variational multi-scale approach \([40–42]\).

The method consists on finding \( \mathbf{U}_h : [0, t_f] \rightarrow \mathbf{V}_h \) such that the weak form of the Navier–Stokes Eqs. (1)–(2) is satisfied:

\[
\rho \frac{\partial \mathbf{u}_h}{\partial t} + B(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h) + S_1(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h) + S_2(\mathbf{U}_h, \mathbf{V}_h) + S_{\text{ghost}}(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h) = \langle f, \mathbf{v}_h \rangle,
\]

for all \( \mathbf{V}_h \in \mathbf{V}_h \), where

\[
B(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h) = (2\mu \nabla \mathbf{u}_h, \nabla \mathbf{v}_h) + (\rho \mathbf{a}_h \cdot \nabla \mathbf{u}_h, \mathbf{v}_h) - (p_h, \nabla \cdot \mathbf{v}_h) + (\nabla \cdot \mathbf{u}, q),
\]

\[
S_1(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h) = \sum_{k} \alpha_1 \left( f - \rho \frac{\partial \mathbf{u}_h}{\partial t} - \rho \mathbf{a}_h \cdot \nabla \mathbf{u}_h + \mu \Delta \mathbf{u}_h - \nabla p_h, -\rho \mathbf{a}_h \cdot \nabla \mathbf{v}_h - \mu \Delta \mathbf{v}_h - \nabla q_h \right)_K.
\]

\[
S_2(\mathbf{U}_h, \mathbf{V}_h) = \sum_{k} \alpha_2 (\nabla \cdot \mathbf{u}_h, \nabla \cdot \mathbf{v}_h)_K.
\]

\[
S_{\text{ghost}}(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h) = \sum_{k} \left( c_4 h^2 \alpha_1^{-1} (\nabla \mathbf{v}_h, \mathbf{p}_f^+ (\nabla \mathbf{u}_h)) \right)_{\Omega_{\text{cut}}},
\]

\[
+ \sum_{k} (c_5 \alpha_2) (\nabla \mathbf{q}_h, \mathbf{p}_f^+ (\nabla \mathbf{p}_h - f))_{\Omega_{\text{cut}}},
\]

\[
S_1(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h), S_2(\mathbf{U}_h, \mathbf{V}_h) \text{ and, } S_{\text{ghost}}(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h) \text{ are the stabilization terms added to the Galerkin formulation, i.e, the first two terms of Eq. 3. The ghost penalty stabilization term } S_{\text{ghost}}(\mathbf{a}_h; \mathbf{U}_h, \mathbf{V}_h) \text{ is used to to overcome the ill-conditioning and local instability in the elements cut by the free surface and, consequently it is only active in the layers of elements hosting the free surface, see } [43] \text{ and } [44] \text{ for details. In such a term, } \Omega_{\text{cut}} \text{ represents the domain of the } K \text{ element cut by the free surface } \Gamma_{\text{free}}(t), c_4 \text{ and } c_5 \text{ are algorithmic constants, both taken as } 0.1 \text{ in this work. The orthogonal projections } P^\perp \text{ in the cut elements are defined as: } P^\perp_u (\nabla \mathbf{u}_h) = \nabla \mathbf{u}_h - P_u (\nabla \mathbf{u}_h) \text{ and, } P^\perp_p (\nabla p_h - f) = (\nabla p_h - f) - P_p (\nabla p_h - f), \text{ where } P_u \text{ represents the } L^2(\Omega) \text{ projection onto } \mathbf{V}_h, \text{ and } P_p \text{ the } L^2(\Omega) \text{ projection onto } H_0^1. \text{ Note that the } L^2(\Omega) \text{ projection is not the only possibility (see } [45]) \text{ for other options of interpolators used in the ghost penalty method). A more detailed description of this stabilization technique can be found in } [5].

The matrix of stabilization parameters \( \alpha \), has been study by different authors e.g., [36,46–49]. In the present work, the definition given in [50,51] is used:

\[
\alpha = \text{diag}(\alpha_1 I_d, \alpha_2),
\]

with \( I_d \) the identity on vectors of \( \mathbb{R}^d \) and the parameters \( \alpha_i, i = 1, 2, \) are computed as:

\[
\alpha_1 = \left[ \frac{c_1}{h_1^2} + \frac{c_2 \rho}{h_2} \right]^{-1},
\]

\[
\alpha_2 = \frac{h_2^2}{c_3 \alpha_1},
\]

In these expressions, \( h_1 \) corresponds to a characteristic element length calculated as the square root of the element area in a two-dimensional case and the cubic root of the element volume in 3D, and \( h_2 \) corresponds to another characteristic length calculated as the element length in the streamline direction. The constants \( c_i, i = 1, 2, 3 \) are algorithmic parameters in the formulation. The values used in this work.
are \( c_1 = 12 \), \( c_2 = 2 \), and \( c_3 = 4 \), which can be derived from the numerical analysis of the one-dimensional convection-diffusion-reaction problem. These values have proven to be robust in different problems and for different applications.

### 3.2 Time discretization

To discretize in time we use the second order backward difference scheme, defined as

\[
\frac{\partial u^{j+1}_h}{\partial t} = \frac{3u^{j+1}_h - 4u^j_h + u^{j-1}_h}{2\delta t} + O(\delta t^2),
\]

where \( \delta t \) corresponds to the size of a uniform partition of the time interval \([0, T]\), while \( O(\cdot) \) represents the approximation order of the scheme. The superscript indicates the time step where the variable is being approximated, so that \( u^j \) is an approximation to \( u \) at time \( t^j = j\delta t \). Note that this time marching scheme is independent of the method for spatial discretization.

### 3.3 Interface capturing

The common option to solve free surface problems using fixed mesh approaches is to solve a two-fluid flow problem. However, in a liquid-air interaction problem, the effect of air on water could be neglected, and thus air does not need to be modelled. This allows one to solve a single fluid flow problem. The key point to do this is to disconnect both domains, which is achieved by imposing a traction-free condition over the interface. Using this approach, the Navier–Stokes problem is solved only for the liquid domain. The air region is not simulated, but for numerical smoothness of the solution, in the air domain an extrapolation function is used for the velocity and the pressure fields. In this work, a Stokes problem is used in the air domain.

For the tracking of the two fluids interface, the level set method is used [52,53]. The pure advection of a smooth marked function \( \phi(x,t) \) can be defined in an Arbitrary-Lagrangian–Eulerian (ALE) frame by the following equation:

\[
\frac{\partial \phi}{\partial t} + \mathbf{a} \cdot \nabla \phi = 0,
\]

in the domain \( \Omega \) for \( t \in [0, t_f] \) with the corresponding initial and boundary conditions. This equation allows to define the interface position defined by a isovalue \( \phi(x,t) = \phi_c \), usually taken as \( \phi_c = 0 \). For the numerical solution of the level set equation, standard numerical techniques are used. In this work, the level set Eq. (8) is solved by using the classical SUPG method [54] to stabilize the convective nature of the equation. The time derivative is discretized in the same way as in the momentum equation described in Sect. 3.2.

As the level set interface evolves in time, it will intersect the elements of the finite element mesh in an arbitrary manner. To properly integrate the physical properties in the computational domain, a modified integration rule is used in the elements cut by the interface [5,6]. The use of the enhanced integration allows to impose the zero traction boundary condition correctly at the exact position of the interface.

The advection of the level set function does not guarantee global mass conservation. Depending on the space-time discretization employed, the amount of volume loss of the solved fluid can be important. Mass correction algorithms were proposed in several works (e.g., see [28,55,56] and references therein). In the present work, we use a very simple method to ensure the global mass conservation, which consists in measuring the total mass at the end of each time step and compute the amount of mass lost, and then accordingly displace the level set function in a uniform global manner in the direction orthogonal to the free-surface in order to recover the lost mass (see [5] for more details).

### 3.4 Adaptive approach

A key ingredient in the numerical simulations presented in this work is an adaptive mesh refinement strategy that tracks the position of the free surface, and allows one to achieve accurate solutions by using reasonably small number of elements. For this, the Fixed-Mesh ALE method is coupled with the parallel adaptive mesh refinement library [57]. The particularity of applying adaptive refinement approaches to free surface problems is that in many cases, the precision of the numerical simulation depends mostly on the accuracy with which the interface between the two materials is tracked [5]. Thus, an adaptive mesh refinement criterion based on layers of elements around the fluids interface is used in this work.

In the present work, the mesh refinement algorithm proposed in [5,57] is used to validate the numerical results with the experimental data also proposed in this work. The adaptive mesh refinement algorithm used in the work is based on layers of elements of different size. In general, a free surface problem could be divided into four zones: the most important one is the region (a layer) close to the free surface that contains elements occupied by the fluid as well as elements outside of the fluid. A second layer could be a region located neither so far nor so close to the free surface and adjacent to the first layer. A third zone is represented by the region far away from the free surface, but inside of the fluid, and finally, a fourth zone is represented by the region far away from the free surface but outside the fluid.

The adaptive mesh refinement algorithm for free surface problems proposed in this work is based on three positive integers parameters that refer to the level of refinement in each of these layers. Calling the parameters as \( a \), \( b \) and \( c \),
we show as the algorithm work for a sloshing case. Figure 1 shows the first eight time steps of a sloshing case, starting from an extremely coarse mesh (left top) called $M_0$ composed of elements of size $h_0$.

For the case shown in Fig. 1, the integer parameters were set as $a = 2$, $b = 2$ and $c = 8$. The first value refers to the level of refinement taking as reference the original mesh $M_0$. The value $a = 2$ means that the elements far away from the free surface, but inside of the region occupied by the fluid have two levels of refinement, and therefore, the element size is a quart of the originals ($h_e = \frac{1}{4} h_0$). The value $b = 2$ refers to the level of refinement of the elements cut by the fluid interface and very close to the free-surface both out as in of the fluid, giving place to elements of size equal to $1/16$ of the originals. The $c$ parameter allows us to define the number of element layers that conform the zone where the finest mesh is used ($h_e = 1/16$). For $c = 8$ layers of, we have 8 elements of size $h_e = 1/16 h_0$ of the original one associated to $M_0$.

In Fig. 2, an example of the adaptive mesh refinement algorithm is presented for a sloshing case. In this figure the capability of the algorithm for refining and coarsening the mesh in a time dependent problem is shown.
Fig. 3  Experimental setup of the shake table loaded with the tank partially filled with vegetable oil, and the ultrasonic sensors installed at the specified control points overall view

In this work, we make a mesh convergence study using a fixed mesh size, but changing the thickness of the region of smallest elements. In other words, we leave the values of $a$ and $b$ fixed, and change the value of the parameter $c$, as will be described in Sect. 5.

4 Experimental work

An acrylic tank filled with commercial vegetable oil is mounted on a shake table and subject to controlled vibrations. The experimental setup is shown in Fig. 3. The dimensions of the square section tank are shown in Fig. 4, where $H$ is the oil depth. The shake table is actuated by an engine attached to a rotating screw, which produces a one-dimension time-varying motion. The amplitude and frequency of the imposed motion can be set by a user interface tool [58]. An experimental study of water sloshing in a rectangular tank has been also presented in [32]. In such a work, experiments using only one water depth were reported, using a unique procedure to measure wave heights. The present experimental work is focused on the wave height evolution for different oil filling depths, and encompass experimental sweep analyses. In particular, four oil depths $H$ have been used: 100 mm, 150 mm, 200 mm and 250 mm. The imposed motions have amplitudes $A$ of 5 mm, 7.5 mm and 10 mm, while the imposed frequency $f$ has been defined in terms of the analytic first natural frequency for non-viscous fluids ($f_n$) [59], the geometry of the tank, and the liquid’s depth (see Table 1). In this work, the values of the imposed frequency goes from 0.55 $\times$ $f_n$, up to 1.45 $\times$ $f_n$. Moreover, two different techniques for measuring wave heights are used with the aim of confirming and to completing the experimental data.

The wave height is measured at four control points (CP in Fig. 4) using ultrasonic sensors with resolution of 1 mm [60]. The ultrasonic interface detection can be distorted due to: the shape of the free surface, ultrasonic beam reflection at wall’s container or distance from the emitting point and the free surface. These facts are taken into account to define the sensors’ location in the experiment. Because of these aspects, under certain experimental conditions, the water level can
where $f_{n,a} = (g/(4\pi) \tanh(\pi H/l))^{1/2}$ with $l = 288$ mm.

Table 1  Analytic first natural frequencies

| $H$ (mm) | 100  | 150  | 200  | 250  |
|----------|------|------|------|------|
| $f_{n,a}$ (Hz) | 1.46998 | 1.58511 | 1.62962 | 1.62962 |

not be measured using the ultrasonic sensors, and the quality of the obtained data can not be assured. It could typically happen when sharp waves or strong 3D effects evolve and the free surface is highly distorted (even when no separation is obtained). However, the ultrasonic sensors provide information for long term analyses. The experiments are also recorded using a high speed camera [61]. This procedure is limited to the total time that can be registered. In the present study 10s of the steady state time-periodic regime were recorded with a resolution of 800 × 800 pixels at 120 frames per second. To process the videos a Python homemade code has been developed using the OpenCV library [62]. This code handles a video file and treats it frame-by-frame, from where the wave height is measured. The error bound in these measurements is 1.1 mm and they help to verify and complete the experimental data.

Moreover, sensors located at CP3 and CP4 help to detect the evolution of 3D effects when signals at CP1 and CP3 (or CP2 and CP4) are different, while for 2D motions CP1 and CP3 (or CP2 and CP4) practically coincide.

To illustrate the typical free surface responses during the time-periodic regime, the wave height evolution at CP1 for the entire analysis using $H = 100$ mm, $A = 7.5$ mm and $f = 1.69$ Hz ($1.15 \times fn$, $fn = 1.469$ Hz) is presented in Fig. 5. The ultrasonic sensors and image capturing measurements practically coincide. Only confirmed measurements are used to evaluate maximum and minimum wave amplitudes during the time-periodic regime for all studied cases.

The first natural frequency of the oil sloshing has also been determined experimentally by examination of the damped decaying regime via a Fast Fourier Transform (FFT). From these analyses, the natural frequencies coincide with the val-
Fig. 7 Experimental sweep analysis: ratio between maximum and minimum wave amplitudes at CP1 for each oil depth $H$, image data reported in Table 1, showing independence from the fluid’s viscosity for the studied cases.

Figure 6 presents the maximum and minimum wave amplitudes $\eta^+/-$ during the time-periodic regime for the
Fig. 9 Frequency sweep analysis: experimental data versus numerical results for $H = 100$ mm and $A = 7.5$ mm. Maximum and minimum wave amplitudes at CP1. Mesh (top) and time step (bottom) convergence analyses.

The ratio between maximum and minimum wave height is plotted in Fig. 7. The error bars are also included, showing the influence of the low wave amplitudes measured with the declared error in the computation of the ratio. The nonlinear behaviour, represented by ratios not equal to one, is apparent in the figure. High nonlinearities are found near the first resonant frequency and at low filling depth. The ratios increase when the imposed amplitude increases. It can also be seen that near the resonant frequency the ratios remain practically constant. The ratios decrease when the filling depth increases, and they tend to be similar when the filling depth increases, denoting that the wave pattern is practically invariant at large filling depths.

Numerical and experimental results are presented in Sect. 5 to validate the proposed methodology for analyzing free surface flow problems. Numerical solutions confirm image measurements.
Fig. 12  Free surface evolution in a period for $f = 1.25 \text{ Hz } (f_r = 0.85)$

Fig. 13  Free surface evolution in a period for $f = 1.32 \text{ Hz } (f_r = 0.9)$
5 Modelling the experiments

The present Section reports the numerical analyses made using the proposed fully formulation described in Sect. 2. The numerically studied cases are those with \( H = 100 \text{ mm} \) and \( A = 7.5 \text{ mm} \) at different imposed frequencies. In all cases the properties of the fluid were set as \( \rho = 912.4 \text{ kg/m}^3 \) and \( \mu = 71 \text{ m Pa s} \).

Figure 9 summarizes the numerical results computed using different meshes and time step sizes in comparison with experimental data. The three meshes used (named as \( M1, M2 \) and \( M3 \)), are shown in Fig. 8. Note that the element size is the same in all the meshes, but the number of elements in the layer of finest elements changes between them. In this work we study the effect of this parameter on the convergence of the results using the experimental data. In mesh \( M1 \) we set the \( c \) parameter described in Sect. 3.4 as \( c = 8 \). For mesh \( M2 \) the value was set at \( c = 10 \) and for \( M3 c = 12 \).

Simulations using \( M1 \) and time steps of 0.005 s, 0.0025 s and 0.00125 s reveal practically independent time step size results (see Fig. 9, right). The mesh convergence analysis, made using a time step of 0.00125 s, presents improved results when finer meshes are used (see Fig. 9, left). Note that for \( M3 \) the numerical results fit well with the experimental data in the zone near the first mode. These results corroborate that the most efficient way to solve a problem of free surface is with adaptability. Additionally, it is evident that the fixed mesh ALE approach is a robust method to solve cases even close to resonance.

Figures 10 and 11 report experimental data and numerical results during time-periodic and transient regimes respectively. Figure 10 illustrates that wave height evolution at CP1 is in phase with shake table motion for frequencies lower than the first natural frequency, while the phase is \( 2\pi \) at frequencies greater than the first frequency. Moreover, as expected, the wave height evolution at CP2 has a \( 2\pi \) phase with respect to wave height evolution at CP1.

Snapshots from the videos and simulations are also plotted in Fig. 12, 13, 14 and 15 for imposed frequencies of 1.25 Hz, 1.32 Hz, 1.62 Hz, and 1.69 Hz. In all these analyses, the simulations satisfactorily match the experiments.

6 Conclusions

An exhaustive numerical and experimental analysis of the sloshing of a squared tank partially filled with a domestic vegetable oil has been presented. A variational multi-scale stabilized finite element method with a level set technique developed for adaptive meshes using a Fixed-Mesh Arbitrary Lagrangian–Eulerian method, has been presented.

The main results of the work can be summarized as:
– mesh and time step size refinement reveal practically independent numerical behaviour, but better results are computed with small sizes near resonant conditions.
– the adaptivity procedure warrants good results also when relatively coarse meshes are used, benefitting computational time.
– maximum and minimum wave amplitudes increase with the amplitude of the imposed motion for a given frequency of the imposed motion and, with the filling depth for a given imposed motion, and they tend to reach similar values for higher filling depth, i.e., the wave pattern becomes independent of the filling depth.
– the wave nonlinearities, expressed by the ratio between maximum and minimum wave amplitudes, are strong at the studied low filling depths.
– the results obtained in the simulation satisfactorily adjust to experimental data, validating the proposed numerical model.

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