Joule heating generated by spin current through Josephson junctions

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We theoretically study the spin-polarized current flowing through a Josephson junction (JJ) in a spin injection device. When the spin-polarized current is injected from a ferromagnet (FM) in a superconductor (SC), the charge current is carried by the superconducting condensate (Cooper pairs), while the spin-up and spin-down currents flow in the equal magnitude but in the opposite direction in SC, because of no quasiparticle charge current in SC. This indicates that not only the Josephson current but also the spin current across JJ at zero bias voltage, thereby generating Joule heating by the spin current. The result provides a new method for detecting the spin current by measuring Joule heating at JJ.

Spin transport through a nonmagnetic metal has attracted much interest in magnetic nanostructures. In the tunnel junctions consisting of a ferromagnet (FM) and a normal metal (N) or superconductor (SC), the tunnel current driven from FM is spin-polarized and creates a nonequilibrium spin population in N or SC. Recently, there has been a number of experiments on suppression of superconductivity by injection of spin-polarized electrons using tunnel junctions of a high-Tc SC and a ferromagnetic manganite.

A double tunnel junction with a thin layer of SC sandwiched between two FM electrodes is a unique system in which is sandwiched between two FMs. The central part of the junction forms the Josephson junction, which is sandwiched between two FMs. The applied bias left and right SC (FM) are made of the same SC (FM). The magnetization of the left FM points up and that of the right FM is either up or down. The applied bias current $I_{\text{inj}}$ flows through the junctions of resistances $R_J$ and $R_T$ with the voltage drops $V_J$ and $V_T$, respectively.

We calculate the tunneling current using a phenomenological tunneling Hamiltonian that describes the transfer of electrons from one electrode to the other. If SC is in the superconducting state, it is convenient to rewrite the electron operators $\hat{a}_{k\sigma}$ in SC in terms of quasiparticle operators $\hat{\gamma}_{k\sigma}$ appropriate to the superconducting states, using the Bogoliubov transformation.

$$a_{k\sigma} = u_{k\sigma} \gamma_{k\sigma} + v_{k\sigma} \hat{S}_{k\sigma}, \quad a_{-k\sigma}^\dagger = -v_{k\sigma}^* \hat{S}_{-k\sigma}^\dagger + u_{-k\sigma}^* \gamma_{-k\sigma}^\dagger,$$

where $|u_{k\sigma}|^2 = \frac{1}{2} (1 + \xi_k / E_k)$, $|v_{k\sigma}|^2 = \frac{1}{2} (1 - \xi_k / E_k)$, $\hat{S}$ is an operator which annihilates a Cooper pair, while $\hat{S}^\dagger$ creates one, and $E_k = (\xi_k^2 + \Delta^2)^{1/2}$ is the quasiparticle dispersion of SC, $\xi_k$ being the one-electron energy relative to the chemical potential of the condensate and $\Delta$ being the isotropic superconducting gap.

From the Fermi’s golden rule result, the spin-dependent tunnel currents $I_{\sigma}$ across the ith junction are expressed as

$$I_{\sigma}^1(V_T) = (G_{1\text{t}}/e) [N - S_1], \quad I_{\sigma}^2(V_T) = (G_{1\text{r}}/e) [N + S_1], \quad I_{\sigma}^3(V_T) = (G_{2\text{t}}/e) [N + S_2], \quad I_{\sigma}^4(V_T) = (G_{2\text{r}}/e) [N - S_2].$$

Here, $G_{\sigma}$ ($i = 1, 2$) is the tunnel conductance for electrons with spin $\sigma$ when SC is in the normal state. The quantity $N$ is given by the usual expression

$$N(V_T) = \int_{-\infty}^{\infty} d\epsilon S_{\text{f}}(\epsilon) [f_0(\epsilon - eV_T) - f_0(\epsilon)] d\epsilon,$$

where $S_{\text{f}}(\epsilon) = \text{Re} [\epsilon / \sqrt{E^2 - \Delta^2}]$ is the normalized BCS density of states and $f_0(\epsilon)$ the Fermi function. The quantity $S_{\text{f}}$ is the quasiparticle spin density accumulated in SC.
The first term describes the phase coherent (Cooper pair) tunneling. The usual Josephson effect is associated with the sin \( \varphi \) term. Using the golden rule formula, we have the spin-dependent tunnel current of quasiparticles

\[
I_{\text{qp}}^\sigma(V_j) = \frac{1}{2eR_J} \int_{-\infty}^{\infty} D_S(E)D_S(E+eV_j) \times [f_{\uparrow\sigma}(E) - f_{\downarrow\sigma}(E+eV_j)] \, dE,
\]

with which the quasiparticle charge current \( I_{\text{qp}} \) and the spin current \( I_{\text{spin}} \) across JJ are written as

\[
I_{\text{qp}} = I_{\text{qp}}^\uparrow(V_j) + I_{\text{qp}}^\downarrow(V_j),
\]

\[
I_{\text{spin}} = I_{\text{qp}}^\uparrow(V_j) - I_{\text{qp}}^\downarrow(V_j).
\]

The phase coherent tunneling terms are obtained as

\[
I_{J1} = \frac{\Delta^2}{eR_J} \int_{-\infty}^{\infty} dE \frac{D_S(E)D_S(E+eV_j)}{|E+E-V_3|} \times \sum_{j=1}^{2} \left[ 1 - f_{j\sigma}(|E|) - f_{j\sigma}(|E|) \right],
\]

\[
I_{J2} = \frac{\Delta^2}{eR_J} \int_{-\infty}^{\infty} dE \frac{D_S(E)D_S(E+eV_j)}{E+eV_3} \times \sum_{\sigma} \left[ f_{\downarrow\sigma}(E+eV_j) - f_{\uparrow\sigma}(E) \right].
\]

When the thickness of SC is much smaller than the spin diffusion length, the distribution of quasiparticles is spatially uniform in SC. Then, the distribution function \( f_{i\sigma} \) is described by \( f_0 \), but the chemical potentials of the spin-up and spin-down quasiparticles are shifted oppositely by \( \delta \mu_i \) from the equilibrium one to create the spin density;

\[
f_{i\uparrow}(E) = f_0(E-\delta \mu_i), \quad f_{i\downarrow}(E) = f_0(E+\delta \mu_i).
\]

The gap parameter \( \Delta \) in SCs is determined by \( f_{i\sigma} \) through the BCS gap equation

\[
\ln \left[ \frac{\Delta}{\Delta_0} \right] + \int_{\Delta_0}^{\infty} \frac{f_{i\uparrow}(E) + f_{i\downarrow}(E)}{\sqrt{E^2 - \Delta^2}} \, dE = 0,
\]

where \( \Delta_0 \) is the gap at \( T = 0 \) in the equilibrium state \( (\delta \mu_i = 0) \).

In the following we calculate the tunnel current at zero bias voltage \( (V_j = 0) \). In this case, a DC Josephson current flows across JJ unless the bias current exceeds the Josephson critical current \( J_c = I_c(0) \). From Eq. (12), we notice that \( I_{\text{spin}} \) in the parallel alignment becomes finite even at \( V_j = 0 \) if \( \delta \mu_1 \) and \( \delta \mu_2 \) take nonzero values of different sign \( (\delta \mu_1 = -\delta \mu_2) \), while \( I_{\text{spin}} \) in the antiparallel alignment is zero because of \( \delta \mu_1 = \delta \mu_1 = \delta \mu_2 \). The currents \( I_{J2} \) and \( I_{\text{qp}} \) vanish at \( V_j = 0 \) irrespective of the value of \( \delta \mu_i \). These facts indicate that the Josephson current as well as the spin current flow across JJ at \( V_j = 0 \) in the parallel alignment, while only the Josephson current flows in the antiparallel alignment.
In other words, the half of Joule heating generated by spin current has strong spin-
Josephson current flow through the JJ at drop across JJ. Therefore, the spin current as well as the
for the spin-down current, resulting in no net voltage
is positive for the spin-up current, while it is negative
\( \delta \mu \) (see the inset) in the current bias
mode. A slight decrease of the critical temperature and
a depression of
caused by the pair breaking effect by spin accumulation
in SC. The power of Joule heating generated by spin
current is estimated by evaluating the value of \( W_0 \), since
\( W \) is comparable to \( W_0 \). If one uses the values of an area
resistance \( R_4 A = 100 \Omega \mu m^2 \) and \( \Delta_0 = 0.3 \) meV (Al), we
have \( W_0 / A = 90 \) mW/cm\(^2\) per area of JJ. Therefore, \( W \)
was fairly large for observing the Joule heating experimentally.

In conclusion, we propose a new method for detecting
the spin current by measuring the Joule heating generated
at JJ. This is attributed to the fact that the spin-
up and spin-down currents flow in the opposite direction
in SC. For measuring the Joule heating, it may be
important for the tunnel junctions to satisfy the condition
\( R_J \gg R_T \), in order to make the Joule heating \( W \) dominant
compared with the injection power \( (I_{inj}^2 R_T) \).

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Figure 2 shows the Joule heating \( W \) normalized to
\( W_0 = \Delta_0^2 / e^2 R_4 \) as functions of temperatures for the
injection current \( I_{inj} / J_0 = 0.1 \) and 0.2 in the parallel
alignment [10], where \( J_0 = \pi \Delta_0 / (2e R_4) \) is the Josephson
critical current. As the temperature is lowered, \( W \)
increases monotonically. This is due to the increase of the
spin-splitting of \( \delta \mu \) in the current bias
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[10] To avoid a spurious logarithmic divergence arising from
convolving the BCS density of states with an isotropic
gap in Eq. (11), we introduce a small broadening in the
BCS density of states using Dynes’s formula
\[ \Sigma(E) = Re[(E - i\gamma) / \sqrt{(E - i\gamma)^2 - \Delta^2}] \]
with broadening parameter \( \gamma \) much less than \( \Delta \).