Static Enforceability of XPath-Based Access Control Policies

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Abstract

We consider the problem of extending XML databases with fine-grained, high-level access control policies specified using XPath expressions. Most prior work checks individual updates dynamically, which is expensive (requiring worst-case execution time proportional to the size of the database). On the other hand, static enforcement can be performed without accessing the database but may be incomplete, in the sense that it may forbid accesses that dynamic enforcement would allow. We introduce topological characterizations of XPath fragments in order to study the problem of determining when an access control policy can be enforced statically without loss of precision. We introduce the notion of fair characterizations of XPath fragments in order to study the problem of dynamic enforcement itself.

1. Introduction

Access control policies for XML documents or databases have been studied extensively over the past 10 years [1, 6, 9, 12, 20, 21, 24, 26, 29, 32]. Most of this work focuses on high-level, declarative policies based on XPath expressions or annotated schemas; declarative policies are considered easier to maintain and analyze for vulnerabilities than the obvious alternative of storing ad hoc access control annotations directly in the database itself [13]. However, this convenience comes at a cost: enforcing fine-grained, rule-based policies can be expensive, especially for updates. In this paper we consider the problem of efficient enforcement of access control policies involving update operations, where permissions are specified using downward monotone XPath access control rules.

An example of such a policy, specifying the allowed and forbidden updates for nurses in a hospital database, is shown in Figure 1. The policy is parameterized by data values Swin (ward number) and Suid (user id); these values are available as part of the request so can be treated as constants.

The first three positive rules specify that nurses may insert data into any patient records, may update information about patients in their own ward, and may update their own phone number; the last two negative rules specify that nurses may not insert or update treatment elements. Some sample data is shown in Figure 2.

Most prior work on XML access control focuses on controlling read-access, and access control for read-only XML data is now well-understood. Some techniques, such as filtering [6, 21] and security views [12, 20], hide sensitive data by rewriting queries or providing sanitized views. Other access control techniques rely for efficiency on auxiliary data structures (such as access control annotations [19], or “compressed accessibility maps” [32]). Static analysis has been proposed to avoid dynamic checks [24] or speed reannotation [19].

However, access control for updates still poses challenges that previous work on read-only access does not fully address, and XML databases still typically lack support for fine-grained access control. Prior work [11, 19] suggests two obvious dynamic approaches to enforcement of write-access control policies: query-based enforcement, analogous to filtering, in which we use the policy rules and update request to generate Boolean queries that answer “true” if the update is allowed and “false” if not, and annotation-based enforcement, in which the rules are used to place annotations on the data indicating which updates are allowed on each node. In annotation-based enforcement, when an update is performed the annotations need to be updated to restore consistency with the policy; query-based enforcement has no such maintenance overhead.

To illustrate, consider the data tree in Fig. 2. Suppose nurse $n_{123}$ wishes to insert a new patient record represented by an XML tree $T$. A client-side program issues an XQuery Update expression $\text{insert } T \text{ into } /\text{hospital}/\text{patients}$. Executing this update yields an atomic update $\text{insert}(n_{2}, T)$ where $n_{2}$ is the node id of the /hospital/patients node. This update is allowed dynamically by the policy, and this can be checked by executing a query against the database to select those nodes where patient insertion is allowed, or by maintaining annotations that encode this information for all operations.

Since XPath evaluation is in polynomial time (in terms of data complexity) [16], both query-based and annotation-based approaches are tractable in theory, but can be expensive for large databases. Koromilas et al. [19] found that checking whether an update is allowed is much faster using annotations than using queries, but even with static optimizations, the overhead of maintaining the annotations can still be prohibitively expensive for large databases. Both approaches can in the worst case require a complete traversal of
of the database; in practice, Koromilas et al. [19] found that incremental maintenance of annotation-based enforcement requires a few seconds per update even for databases of modest size.

This strongly motivates an alternative approach that avoids any dependence on the actual data; static analysis of the rules and updates to check whether a proposed update is allowed [24]. This approach draws upon exact static analysis algorithms for intersection [17] and containment [22] of downward XPath. Intersection is decidable in polynomial time, but containment for expressive fragments of XPath can be intractable in the size of the path expressions involved; even so, for a fixed policy such tests could still be much faster than dynamic enforcement, because they depend only on the policy and update size, not that of the data.

To illustrate via our running example, instead of checking the actual atomic update against the actual data, we can consider a static approach, under the assumption that the database does not allow atomic updates directly but instead only accepts updates specified using a high-level update language such as XQuery Update [22]. For example, the user-provided update \( u \) could be

\begin{equation*}
\text{insert } T \text{ into } /\text{hospital/patients}
\end{equation*}

In prior work, we have introduced static analyses that provide a conservative static approximation of the possible effects of an update [2]. We call such representations update capabilities. In our approach, the system first approximates \( u \) via an update capability

\[ U = \text{insert}(/\text{hospital/patients}, \text{patient}) \]

Here, the second argument patient indicates the type of node being inserted, that is, the root label of \( T \). Again, in this case the access is allowed, since \( U \) is contained in the positive rule \( R_4 \) and does not overlap with any of the negative rules \( R_5 \).

However, purely static enforcement may not give the same results as dynamic enforcement: put another way, for some policies and updates, it may be impossible to statically determine whether the update is allowed. Static enforcement would either deny access in such a case or fall back on dynamic techniques. We call a policy fair when this is not the case: that is, when purely static and dynamic enforcement coincide.

For example, if we add a rule \(-\text{delete}(/\text{patient}[\text{treatment}])\) to the example policy in Figure 1 the resulting policy is unfair with respect to any monotone fragment of XPath, because there is no way to specify a static update request that guarantees the absence of a treatment child in the updated patient subtree. Fair policies are of interest because they can be enforced statically, avoiding any dependence on the size of the data.

In this paper we consider the fairness problem: given a policy language and a policy in that language, determine whether the policy is statically enforceable. We focus on subsets of downward, unordered, monotone XPath. In this context, downward and unordered refers to the fact that we consider only the self, child and descendant axes that navigate downward into the tree and are insensitive to order (though our results also apply to ordered trees), and monotone refers to the fact that we exclude features such as negative path tests or difference operations, so that all of the XPath expressions we consider have monotone semantics. We use notation \( XP(S) \), where \( S \) is a set of XPath features such as child (\( / \)), descendant (\( // \)), filter (\( [ ] \)) or wildcard (\( * \)) to denote different fragments of downward XPath.

Our key insight is based on a shift of perspective. A conventional view of the semantics of an XPath expression \( p \) over a given tree \( T \) is as a set of selected nodes \( n \) obtained by evaluating \( p \) from the root of \( T \). Instead, we consider the semantics of \( p \) to be the set of pairs \( (T, n) \). We consider the topological spaces generated by different fragments of XPath. A policy is fair (with respect to updates specified in a given fragment \( XP \)) if and only if its semantics denotes an open set in the topology generated by \( XP \). Intuitively, the reason for this is that a policy is fair if any update dynamically allowed by the policy is contained in a statically allowed update capability. The atomic updates are points of the topological space, the update capabilities denote basic open sets.

Based on this insight, we first prove that fairness is monotonic in the fragment \( XP \) used for updates: that is, making the XPath characterizations of updates more precise never damages fairness. Second, we show that all policies over \( XP(/,//,
\text{node}) \) are fair with respect to \( XP(/,//,
\text{node}) \) (or any larger fragment). We show that it is CONP-complete to decide whether a policy over \( XP(/,//,
\text{node}) \) is fair with respect to \( XP(/,//,
\text{node}) \); however, policies that only use filters in positive rules are always fair. We show that for update operations with a bounded number of descendant steps, static enforcement is decidable in polynomial time. We sketch how these results can be extended to handle policies with attributes and data value tests.

The structure of the rest of this paper is as follows: In Section 2 we review the model of write-access control policies introduced in prior work. We define fairness and give its topological characterization in Section 3 and present the main results in Section 4. Section 5 discusses the implications of our results and generalizations. We conclude with discussions of related and future work in Sections 6 and 7.

2. Preliminaries

XML trees We model XML documents as unordered, unranked trees. Let \( \Sigma \) be an element name alphabet, \( \Gamma \) an attribute name alphabet, and \( D \) a data domain. We assume that \( \Sigma, \Gamma, \) and \( D \) are infinite and mutually disjoint. We consider an XML document to be a tree \( T = (V_T, E_T, R_T, \lambda_T) \), where \( \lambda_T : V_T \rightarrow \Sigma \cup (\Gamma \times D) \cup D \) is a function mapping each node to an appropriate label, \( E_T \subseteq \Sigma \cup (\Gamma \times D) \) is the edge relation, and \( R_T \) is a distinguished node in \( V_T \), called the root node. We distinguish between element nodes labeled with \( l \in \Sigma \), attribute nodes labeled with attribute-value pairs \( (\alpha, d) \in \Gamma \times D \), and data nodes labeled with elements of \( d \in D \); attribute and data nodes must be leaves. We do not assume that an XML DTD or schema is present.

XPath The fragment of downward XPath used in update operations and policies is defined as follows:

\begin{align*}
\text{Paths} & : p ::= \alpha \cdot \phi \mid p/p' \mid p[q] \\
\text{Filters} & : q ::= p \mid q \land q \mid \forall \alpha = d \mid \text{true} \\
\text{Axes} & : \alpha ::= \text{self} \mid \text{child} \mid \text{descendant} \mid \text{attribute} \\
\text{Node tests} & : \phi ::= l \mid * \mid f \mid \text{text()} \\
\end{align*}

Absolute paths are written \( /p \); we often omit the leading slash when this is obvious from context. Here, \( l \) is an element label from \( \Sigma, f \) is an attribute name from \( \Gamma, \) and \( d \) is a data value or parameter name. Wildcard \( * \) matches any element or text node. The expressions are built using only the child, descendant and attribute axes of XPath and conditions that test for the existence of paths or constant values of attributes. We use the standard abbreviated forms of XPath expressions in examples. For example, \( /\text{a}/b[@/\text{c}]/d \) abbreviates \( /\text{child}[:a/\text{child}]/b[\text{child}[@/\text{c}]/d] \). We write \( [p(T) \mid q(T)] \) for the set of nodes of a tree \( T \) obtained from evaluating XPath expression \( p \) on the root node of \( T \). We also write \( [\phi] \) for the subset of node labels \( \Sigma \cup (\Gamma \times D) \cup D \) matching \( \phi \). These semantics are defined in Figure 3, following standard treatments [4,15,31].

We write \( XP(S) \), for \( S \subseteq \{/,,//,[],\cdot,\alpha\} \), for the sublanguage of the above XPath expressions that includes the features in \( S \). For example, \( XP(/,//,\cdot,=) \) includes \( /a/b[@c = "foo"] \), but not \( /a// \).

We say that an XPath expression \( p \) is contained in another expression \( p' \) (written \( p \subseteq p' \)) if for every XML tree \( T \), \( p(T) \subseteq p'(T) \). We say that two XPath expressions are disjoint if their
intersection is empty: that is, for every $T$, $[p](T) \cap [p'](T) = \emptyset$. Otherwise, we say $p$ and $p'$ overlap.

As for relational queries, containment and satisfiability are closely related for XPath queries, and both problems have been studied for many different fragments of XPath. Containment has been studied for downward XPath expressions (XP($\langle ., ., . \rangle$)) by Miklau and Suciu [22] and for larger fragments by others [3, 25, 30]. Specifically, Miklau and Suciu showed that containment is coNP-complete for XP($\langle ., ., . \rangle$) and presented a complete, exponential algorithm and an incomplete, polynomial time algorithm, which is complete in restricted cases. Polynomial algorithms for testing overlap of XPath expressions in the fragment XP($\langle ., ., . \rangle$) have been studied in [17]; however, both satisfiability and containment for XPath with child axis, filters and negation is PSPACE-hard [3], and the complexity of containment increases to EXPTIME-hard when the descendant axis is added. Containment for XPath 2.0, which includes negation, equality, quantification, intersection, and difference operations, rapidly increases to EXPTIME or non-elementary complexity [30].

Atomic Updates We consider atomic updates of the form:

$$u ::= \text{insert}(n, T') \mid \text{update}(n, T') \mid \text{delete}(n)$$

where $n$ is a node expression, and $T'$ is an XML tree. An insert operation $\text{insert}(n, T')$ is applied to a tree $T$ by adding a copy of $T'$ as a child of node $n$ (recall that we consider unordered trees so the order does not matter). The operation $\text{delete}(n)$ deletes the subtree of $n$, and likewise the operation $\text{update}(n, T')$ replaces the selected node with $T'$. We write $U(T)$ for the set of all atomic updates applicable to the nodes of $T$. We omit a definition of the semantics of atomic updates on trees, since it is not necessary for the results of the paper.

Update Capabilities We consider update capabilities of the form

$$U ::= \text{insert}(p, \phi) \mid \text{update}(p, \phi) \mid \text{delete}(p)$$

where $p$ is an XPath expression, and $\phi$ is a node test constraining the tree that can be inserted. Intuitively, an update capability describes a set of atomic update operations that a user is allowed or forbidden to perform in the context of a given policy. An update capability is interpreted (with respect to a given tree) as defining a set of atomic updates:

$$[(\text{insert}(p, \phi))(T)] = \{\text{insert}(n, T') \mid n \in [p](T), \lambda_T(R_{T'}) \in [\phi]\}$$

$$[(\text{update}(p, \phi))(T)] = \{\text{update}(n, T') \mid n \in [p](T), \lambda_T(R_{T'}) \in [\phi]\}$$

$$[(\text{delete}(p))(T)] = \{\text{delete}(n) \mid n \in [p](T)\}$$

**Access Control Policies** Following prior work (e.g. [14, 19]), we define access control policies $\mathcal{P} = (\mathcal{D}, \mathcal{C}, \mathcal{A}, D)$ with four components: a default semantics $\mathcal{D} \subseteq \{+, -, \}$, a conflict resolution policy $\mathcal{C} \subseteq \{+, -, \}$, and sets $\mathcal{A}$ and $\mathcal{D}$ of allowed and denied capabilities, described by XPath expressions. The default semantics indicates whether an operation is allowed if no rules are applicable. The conflict resolution policy resolves conflicts when an operation matches both a positive rule and a negative rule. The semantics $[\mathcal{P}]$ of a policy $\mathcal{P}$ is defined in Figure 4 as a function from trees $T$ to sets of allowed atomic updates $[\mathcal{P}](T)$.

For example, in the deny–deny case, the accessible nodes are those for which there is a capability granting access and no capabilities denying access. Note that the allow–deny and deny–allow cases are degenerate cases of the other two when $\mathcal{A} = \emptyset$ or $\mathcal{D} = \emptyset$ respectively.

**Enforcement Models** We now define the two enforcement models: dynamic and static.

**Definition 1.** An update $u$ is (dynamically) allowed on tree $T$ if $[u](T) \subseteq [\mathcal{P}](T)$. An update capability $U$ is statically allowed provided that for all $T'$, we have $[U](T') \subseteq [\mathcal{P}](T')$.

For any policy, if $u \in [U](T) \subseteq [\mathcal{P}](T)$, then clearly $u$ is dynamically allowed on $T$. The reverse is not necessarily the case, depending on the policy and class $\mathcal{X}P$ of paths used in update capabilities.

**Definition 2.** A policy $\mathcal{P}$ is fair with respect to XPath fragment $\mathcal{X}P$ provided that whenever $\mathcal{P}$ allows $u$ on $T$, there exists $U$ expressible in $\mathcal{X}P$ such that $u \in [U](T)$ and $\mathcal{P}$ statically allows $U$.

**Example 1.** Fairness depends critically upon the class of paths that may be used to specify updates. If we consider updates with respect to $\mathcal{X}P$, an example of an unfair policy is $\mathcal{P} = (-, -, \{\text{delete}(a), \text{delete}(a[b])\})$. Static enforcement cannot ever allow a deletion at $a$ because there is no way (within $\mathcal{X}P$) to specify an update that only applies to nodes that have no $b$ child. Fairness could be recovered by increasing the expressive power of updates, for example to allow negation in filters; however, this makes checking containment considerably more difficult [3, 25, 30]. On the other hand, constraints such as attribute uniqueness mean that some policies with filters in negative rules are
fair: for example, ⟨⟨\{v\rangle⟩⟩ \subseteq ⟨⟨P⟩⟩ and suppose \langle⟨p\rangle⟩⟩ \subseteq ⟨⟨P⟩⟩ where \langle⟨p\rangle⟩⟩ = \{\{(T, v) | (T, R_Y, v) \in P \langle⟨p\rangle⟩\}\}. This implies that \langle⟨P⟩⟩ \subseteq P \langle⟨\alpha\rangle⟩ where \langle⟨\alpha\rangle⟩ is the basis for a topology \langle⟨α⟩⟩. Next, we consider fairness for \langle⟨XP\rangle⟩ policies with respect to \langle⟨\alpha⟩⟩ updates, and show that they can be unfair only if they involve filters in negative rules. We then show that deciding fairness for such policies is coNP-complete, and conclude by discussing how our results extend to the general case of \langle⟨XP\rangle⟩ = \{p | p \in XP\}.

Recall that a topological space is a structure \((X, \tau)\) where \(\tau \subseteq P(X)\) is a collection of open sets that contains \(\emptyset\) and \(X\), and is closed under finite intersections and arbitrary unions. The complement of an open set is called closed. A basis \(B\) for \(X\) is a collection of subsets of \(X\) such that \(\bigcup B = X\) and whenever \(x \in B_1 \cap B_2\) there exists \(B \subseteq B\) such that \(x \in B \subseteq B_1 \cap B_2\). A basis \(B\) for \(X\) gives rise to a topology \(\tau\) for \(X\), formed by closing \(B\) under arbitrary unions, which we call the topology generated by \(B\).

We consider topological spaces over the set MTree of marked trees, and the open sets are generated by the sets \{\langle⟨p\rangle⟩⟩ | p \in some fragment XP\}

**Theorem 1.** If \{\langle⟨p\rangle⟩⟩ | p \in XP\} is the basis for a topology \(\tau\) on MTree, then a policy \(P\) is fair with respect to \(XP\) if and only if \{\langle⟨P\rangle⟩⟩ \subseteq \langle⟨P\rangle⟩\} is open in \(\tau\).

Proof. If \(P\) is fair, then \{\langle⟨P\rangle⟩⟩ \subseteq \langle⟨P\rangle⟩\} is a (basic) open set, it is obvious that \{\langle⟨P\rangle⟩⟩ \subseteq \langle⟨P\rangle⟩\} is open. Conversely, if \{\langle⟨P\rangle⟩⟩ \subseteq \langle⟨P\rangle⟩\} is open, then \langle⟨P\rangle⟩⟩ = \{\{Y | Y \subseteq \langle⟨P\rangle⟩\}\}. Thus, it suffices to show that \{\{Y | Y \subseteq \langle⟨P\rangle⟩\}\} = \{\langle⟨P\rangle⟩⟩ \subseteq \langle⟨P\rangle⟩\}. The \(\emptyset\) direction is immediate since every \{\langle⟨P\rangle⟩⟩ \subseteq \langle⟨P\rangle⟩\} is a basic open set. For \(\emptyset\), suppose \(x \in \{\{Y | Y \subseteq \langle⟨P\rangle⟩\}\}\), that is, for some \(Y \in \tau\) with \(Y \subseteq \langle⟨P\rangle⟩\), we have \(x \in Y\). Any open set \(Y\) is the union of basic open sets, so \(x\) must be in some \{\langle⟨P\rangle⟩⟩ \subseteq \langle⟨P\rangle⟩\}. Hence \(x \in \langle⟨P\rangle⟩⟩ \subseteq \langle⟨P\rangle⟩\).

**4. Main results**

In this section we investigate fairness for different classes of policies. We first consider the simpler case of \langle⟨XP\rangle⟩ policies and show that they are always fair with respect to \langle⟨XP\rangle⟩. Next, we consider fairness for \langle⟨XP\rangle⟩ policies with respect to \langle⟨\alpha\rangle⟩ updates, and show that they can be unfair only if they involve filters in negative rules. We then show that deciding fairness for such policies is coNP-complete, and conclude by discussing how our results extend to the general case of \langle⟨XP\rangle⟩ = \{p | p \in XP\}.

**4.1 Fairness for \langle⟨XP\rangle⟩ policies**

We call elements of \langle⟨XP\rangle⟩ linear paths, and usually write them as \(\alpha, \beta\). For policies over \langle⟨XP\rangle⟩, we consider the basis given by linear path sets \{\{\langle⟨\alpha\rangle⟩\} | \alpha \in \langle⟨XP\rangle⟩\}.

**Proposition 2.** The linear path sets partition MTree (and hence also form a basis for a topology on MTree).

Proof. Every point \((T, n)\) in MTree is in a linear path set: take \(p\) to be the sequence of node labels along a path leading to \(n\) in \(T\). Moreover, two linear path sets are either equal or disjoint.

Consider the topology \(\tau_1 = \tau_{\langle⟨XP\rangle⟩}\) generated by the linear path sets. Clearly, as for any partition topology, we have:

**Proposition 3.** \(\tau_1\) is closed under set complement.

Next, we show that any path in \langle⟨XP\rangle⟩ denotes an open set in \(\tau_1\), \(vi\) an auxiliary definition.

**Definition 4.** We define the function LP mapping \(p \in \langle⟨XP\rangle⟩\) to a set of linear paths:

\[
\begin{align*}
LP(\text{self} :: \phi) &= \{\text{self} :: l | l \in \phi\} \\
LP(\text{child} :: \phi) &= \{\text{child} :: l | l \in \phi\} \\
LP(\text{descendant} :: \phi) &= LP(\text{child} :: \phi)^* \cdot LP(\text{child} :: \phi) \\
LP(p/p') &= LP(p) \cdot LP(p')
\end{align*}
\]

where \(S \cdot T\) stands for \(\{s/t | s \in S, t \in T\}\) and \(S^* = \bigcup_n S^n\).
Proposition 4. For every \( p \in X P^{(\L, \R, \L, \R)} \), we have \( P\{p\} = \bigcup \{ P\{\alpha\} \mid \alpha \in LP\{p\} \} \), and \( \{p\} = \bigcup \{ \{\alpha\} \mid \alpha \in LP\{p\} \} \), hence \( \{p\} \) is open in \( \tau_1 \).

Proof. The first part follows by induction on the structure of \( p \). The base cases for child :: φ and self :: φ are straightforward. For a path descendant :: φ, we reason as follows:

\[
P\{\text{descendant :: } \phi\} = \bigcup \{ (T,n,m) \mid (n,m) \in E^0_T, \lambda_T(m) \in [\phi]\} = \bigcup \{ (T,n,m) \mid (n,m) \in E^0_T, \lambda_T(m) = l, l \in [\phi]\} = \bigcup \{ (T,n,m) \mid (n_1) \in E_T, \lambda_T(n_1) = \alpha_1, \ldots, (n_k, m) \in E_T, \lambda_T(n_k) = \alpha_k, \lambda_T(m) = l, l \in [\phi]\} = \bigcup \{ P\{\alpha/l\} \mid \alpha \in \Sigma^*, l \in [\phi]\} = \bigcup \{ P\{\alpha\} \mid \alpha \in \text{LP}(\text{descendant :: } \phi)\}.
\]

For the fourth equation, observe that for any marked tree \( (T, n) \) there is a (possibly empty) path \( p \) formed of labels of nodes leading from the root of \( T \) to \( n \). Conversely, for any \( \alpha \) there is a (linear) tree \( T \) and node \( n \) such that \( \alpha \) is the list of labels of nodes from the root to \( n \).

If \( p \) is of the form \( p/p' \), then we reason as follows:

\[
P\{p/p'\} = \{ (T,n,m) \mid \exists k \in V_T, (T,n,k) \in P\{p\}, (T,k,m) \in P\{p'\}\} = \{ (T,n,m) \mid \exists k \in V_T, (T,n,k) \in \bigcup \{ P\{\alpha\} \mid \alpha \in \text{LP}(p) \}, (T,k,m) \in \bigcup \{ P\{\beta\} \mid \beta \in \text{LP}(p') \}\} = \{ (T,n,m) \mid \exists k \in V_T, (T,n,k) \in P\{\alpha\}, \alpha \in \text{LP}(p), (T,k,m) \in P\{\beta\}, \beta \in \text{LP}(p') \} = \{ (T,n,m) \mid \exists k \in V_T, (T,n,k) \in P\{\alpha\}, \alpha \in \text{LP}(p), (T,k,m) \in P\{\beta\}, \beta \in \text{LP}(p') \} = \bigcup \{ P\{\alpha\} \mid \alpha \in \text{LP}(p) \} = \bigcup \{ P\{\alpha\} \mid \alpha \in \text{LP}(p') \}.
\]

The second part is immediate since

\[
\{p\} = \{ (T,n) \mid (T,R_T,n) \in P\{p\}\} = \bigcup \{ P\{\alpha\} \mid \alpha \in \text{LP}(p) \} = \bigcup \{ P\{\alpha\} \mid \alpha \in \text{LP}(p') \}.
\]

which is a union of open sets in \( \tau_1 \).

Proposition 5. Every \( X P^{(\L, \R, \L, \R)} \)-policy \( P \) denotes an open set in \( \tau_1 \).

Proof. Clearly, the sets \( \{A\} \), \( \{D\} \) are open since they are unions of open sets. Since \( \tau_1 \) is closed under complement, the set \( \{D\} \) is closed so \( \{P\} \) is open.

Corollary 2. Every \( X P^{(\L, \R, \L, \R)} \)-policy is fair with respect to \( X P^{(\L)} \).

4.2 Fairness for \( X P^{(\L, \R, \L, \R)} \) policies

Linear path sets are not rich enough to make all expressions in \( X P^{(\L, \R, \L, \R)} \) denote open sets. For example, \( \{a[b]\} \) is not open in \( \tau_1 \); if it were, then it would be expressible as a (possibly infinite) union of basic open sets \( \{a\} \). However, clearly the only \( \alpha \) such that \( \langle\alpha\rangle \) overlaps with \( \langle a[b]\rangle \) is /a, and \( \langle a[b]\rangle \subseteq \langle a\rangle \). Thus, updates based on linear paths are not sufficiently expressive for policies involving filters.

Instead, we generalize to filter paths \( X P^{(\L, \R, \L, \R)} \). These paths correspond in a natural way to marked trees \( (T,n) \). We adopt a standard definition of a tree homomorphism \( h : T \rightarrow U \) as a function mapping \( V_T \) to \( V_U \) such that

1. \( R_U = h(R_T) \),
2. for each \( (v,w) \in E_T \) we have \( (h(v),h(w)) \in E_U \), and
3. for each \( v \in V_T \) we have \( \lambda_T(v) = \lambda_U(h(v)) \).

A marked tree \( (T,n) \) matches a tree \( U \) at node \( m \) (i.e., matches the marked tree \( (U,m) \)) if there is a tree homomorphism \( h : T \rightarrow U \) such that \( h(n) = m \). We refer to such a homomorphism as a marked tree homomorphism \( h : (T,n) \rightarrow (U,m) \), and write \( \langle T,n \rangle \) for the set of all homomorphic images of \( (T,n) \). If \( p \in X P^{(\L, \R, \L, \R)} \) corresponds to marked tree \( (T,n) \) then it is easy to show that \( \langle p \rangle = \langle T,n \rangle \).

Lemma 2. If \( \langle T,n \rangle \) and \( \langle U,m \rangle \) overlap, then there is a marked tree \( (V,k) \) such that \( \langle V,k \rangle = \langle T,n \rangle \cap \langle U,m \rangle \).

This proof is technical, but straightforward; the details are in an appendix.

Corollary 3. The sets \( \{\langle T,n \rangle \mid (T,n) \in MTree\} \) form a basis for a topology on \( MTree \).

Let \( \tau_2 \) be the topology generated by the sets \( \{T,n\} \).

Definition 5. The set \( FP(p) \) of filter paths of \( p \in X P^{(\L, \R, \L, \R)} \) is defined as

\[
FP(ax :: \phi) = LP(ax :: \phi) \\
FP(p/p') = FP(p) \setminus \text{FP}(p') \\
FP(p[q]) = \{p'[q'] \mid p' \in FP(p), q' \in FP^Q(q)\} \\
FP^Q(p) = \text{FP}(p) \\
FP^Q(q_1, q_2) = \{q'_1, q'_2 \mid q'_1 \in FP^Q(q), q'_2 \in FP^Q(q')\} \\
FP^Q(\text{true}) = \{\text{true}\}.
\]

Proposition 6. For every \( p \in X P^{(\L, \R, \L, \R)} \), we have \( P\{p\} = \bigcup \{ P\{p'\} \mid p' \in FP(p) \} \), and \( Q\{q\} = \bigcup \{ Q\{q'\} \mid q' \in FP^Q(q) \} \), hence \( \{p\} \) is open in \( \tau_2 \).

Proof. We show by induction that for every \( p \in X P^{(\L, \R, \L, \R)} \), we have \( P\{p\} = \bigcup \{ P\{p'\} \mid p' \in FP(p) \} \). The base cases are as in Prop. 4. The inductive step case for \( p/p' \) is straightforward, following the same idea as in Prop. 4. We give the inductive case for \( p[q] \) as follows.

\[
P\{p[q]\} = \{ (T,n,m) \mid (T,n,m) \in P\{p\}, (T,m) \in Q\{q\}\} = \{ (T,n,m) \mid (T,n,m) \in \bigcup \{ P\{p'\} \mid p' \in FP(p) \}, (T,m) \in Q\{q'\}\} = \{ (T,n,m) \mid (T,n,m) \in P\{p'[q']\}, (T,m) \in Q\{q'\}, p' \in FP(p), q' \in FP^Q(q)\} = \bigcup \{ \{ (T,n,m) \mid (T,n,m) \in P\{p'[q']\}, (T,m) \in Q\{q'\} | p' \in FP(p), q' \in FP^Q(q)\} \}.
\]
that by axes XPath query containment [22], which we first review.

If do have: policies (such as Example 1) exist for from paths in

is fair even though it involves negative filter paths, because the existence test =

The converse does not hold; for example, the policy is a conjunction

Note that = is a structure

The argument that ⟨⟨p⟩⟩ is open is similar to that for Prop. 4.

In other words, = is a homomorphic image of = ⟨⟨P⟩⟩, which implies ⟨⟨P⟩⟩ is not an open set so = is unfair.

Thus, = holds if and only if = is fair. Since containment of = paths is coNP-hard, fairness is also coNP-hard. ∎
For coNP-completeness we need the following lemma:

**Proposition 7.** A set $Y \subseteq \text{MTree}$ is open in $\tau_2$ if and only if $Y$ is closed under homomorphic images; that is, for all $(T, n) \in Y$ and $h : (T, n) \to (U, m)$ we have $(U, m) \in Y$.

**Proof.** If $Y$ is open, then suppose $(T, n)$ is a point in $Y$ and $h : (T, n) \to (U, m)$. Since $Y$ is the union of basic open sets, there must be some $(V, k)$ such that $(T, n) \in \langle\langle V, k \rangle\rangle \subseteq Y$. That is, there is a homomorphism $g : (V, k) \to (T, n)$, hence, $h \circ g : (V, k) \to (U, m)$ so $(U, m) \in \langle\langle V, k \rangle\rangle \subseteq Y$, as desired.

Conversely, if $Y$ is closed under homomorphic images, then we will show that $Y = \bigcup\langle\langle (T, n) \mid (T, n) \in Y \rangle\rangle$. The $\subseteq$ direction is immediate since $(T, n) \in \langle\langle T, n \rangle\rangle$; on the other hand, for each $(T, n) \in Y$, it follows that $\langle\langle T, n \rangle\rangle \subseteq Y$ since each element of $\langle\langle T, n \rangle\rangle$ is a homomorphic image of $(T, n) \in Y$. Hence, $Y$ is a union of basic open sets, so it is open.

The basic idea of the proof of the coNP upper bound is as follows. We need to show that for any policy $P$, it suffices to consider a finite set of trees (of size bounded by a polynomial in the policy size) in order to decide whether $P$ is closed under homomorphisms. To illustrate, let a counterexample consisting of trees $(T, n)$ and $(T', n)$ and homomorphism $h : (T, n) \to (T', n)$ be given, such that $(T, n) \in \langle\langle P \rangle\rangle$ and $(T', n) \notin \langle\langle P \rangle\rangle$.

First, consider a deny–deny policy, so that $(T, n) \in \langle\langle A \rangle\rangle \iff \langle\langle D \rangle\rangle$. Since $\langle\langle A \rangle\rangle$ is open and $(T, n) \in \langle\langle A \rangle\rangle$, it follows that $(T, n) \in \langle\langle D \rangle\rangle$. Moreover, there must exist paths $p \in A$ and $p' \in D$ such that $(T, n) \in \langle\langle p \rangle\rangle$ and $(T', n) \in \langle\langle p' \rangle\rangle$. It is easy to see that $(T', n) \in \langle\langle p' \rangle\rangle$ also, while $(T', n) \notin \langle\langle P \rangle\rangle$ means that $(T, n)$ does not satisfy any path in $\langle\langle P \rangle\rangle$. Observe that $(T, n)$ and $(T', n)$ could be much larger than $P$. It suffices to show that we can shrink $(T, n)$ and $(T', n)$ to a small counterexample by deleting nodes and edges that do not affect satisfiability of $p, p'$, using similar techniques to those used by Miklau and Suciu [22]. They considered how to shrink a counterexample to the containment problem $p \subseteq p'$, consisting of a single tree, whereas we need to shrink $(T, n)$, $(T', n)$ and $h$ while ensuring that $h$ is still a homomorphism, and also that the shrinking process does not cause the first tree to satisfy some other path in $D$. Thus, it suffices to search for small $O(|\langle\langle P \rangle\rangle|)$ counterexamples.

The reasoning for other kinds of policies (allow–allow, etc.) is similar. This in turn gives a coNP-time decision procedure to determine fairness: first we guess a pair of trees $(T, n)$, $(T', n)$ with $h : (T, n) \to (T', n)$ and $\langle\langle T \rangle\rangle, \langle\langle T' \rangle\rangle \subseteq O(|\langle\langle P \rangle\rangle|)$, then check whether $(T, n) \in \langle\langle P \rangle\rangle$ and $(T', n) \notin \langle\langle P \rangle\rangle$. If no such counterexamples exist, then $P$ is fair.

The proof makes use of the following facts which are immediate or proved by Miklau and Suciu [22].

**Lemma 3** [22].

1. If $h : (P, n) \to (T, n)$ is an embedding witnessing that $(T, n)$ matches some path $p$ with $\langle\langle p \rangle\rangle = \langle\langle P \rangle\rangle$, and $(T', n)$ is a subtree of $T$ such that $\overline{h}(T, n) \subseteq \overline{h}(T', n)$, then $h : (P, n) \to (T', n)$ witnesses that $(T', n)$ matches $p$.

2. If $(T, n) \notin \langle\langle P \rangle\rangle$ and $(T', n) \notin \langle\langle P \rangle\rangle$ is obtained by removing any subtree from $(T, n)$ then $(T', n) \notin \langle\langle P \rangle\rangle$.

3. If $(T, n)$ contains a path of child steps of length $w + 1$, where $w$ is the star length of $p$, $z$ is not present in $p$, and each node along the path is labeled, then we can form $(T', n)$ by removing one of the steps, such that $(T, n) \in \langle\langle P \rangle\rangle \iff (T', n) \in \langle\langle P \rangle\rangle$.

**Theorem 4.** Deciding whether a policy in $\text{XP}^{\langle\langle 1 \rangle\rangle, \langle\langle 1 \rangle\rangle}$ is fair with respect to $\text{XP}^{\langle\langle 1 \rangle\rangle, \langle\langle 1 \rangle\rangle}$ is coNP-complete.

**Proof.** For deny–deny policies, suppose $(T, n) \in \langle\langle P \rangle\rangle$ and $(T', n) \notin \langle\langle P \rangle\rangle$ where $h : (T, n) \to (T', n)$. This implies that there exists $p \in A, p' \in D$ with $(T, n) \in \langle\langle p \rangle\rangle \iff \langle\langle D \rangle\rangle$ and $(T', n) \in \langle\langle p' \rangle\rangle$. Construct expression $p''$ such that $(T, n) \in \langle\langle p'' \rangle\rangle \subseteq \langle\langle p \rangle\rangle \cap \langle\langle p' \rangle\rangle$ and $|p''| \leq |p| + |p'|$. Without loss of generality assume wherever not required by matching $p, p'$, the labels of $T, T'$ are some $z \in Z$ not appearing in $P$. (Relabeling $T, T'$ in this way cannot affect whether they satisfy $P$ since $z$ does not appear there). Then, using Lem. [2], we can shrink $T, T'$ and $h$ by removing subtrees that are not needed to ensure that $T, T'$ match $p, p'$ respectively; moreover, we can maintain $h$ so that it remains a homomorphism through this process. This yields trees where every leaf node is needed for matching $p, p'$, but where there may still exist long chains of $z$s that are only needed to match descendant steps in $p$ or $p'$. However, again using Lem. [2], we can remove $z$-labeled nodes from any chains longer than $W + 1$, where $W$ is the maximum star length of any path in $P$ and we can maintain $h$ so that it remains a homomorphism. Call the resulting trees $(U, n), (U', n)$. By the above lemma, $(U, n) \in \langle\langle p \rangle\rangle \iff \langle\langle D \rangle\rangle$ and $(U', n) \in \langle\langle p' \rangle\rangle$ still hold since $W$ is larger than the star height of any path in $D$. Moreover, $U, U'$ have at most $(|p| + |p'|)(W + 1)$ nodes because any two nodes needed for matching $p, p'$ can be separated by a chain of at most $W + 1$ $z$-nodes.

For allow–allow policies, the reasoning is slightly different. If $(T, n) \in \langle\langle P \rangle\rangle$ but $(T', n) \notin \langle\langle P \rangle\rangle$ then $(T, n)$ cannot match $\langle\langle A \rangle\rangle$ because if it did, then so would $(T', n)$. Thus, $(T, n)$ does not match $D$ either. Similarly, $(T', n)$ must match some negative rule $p \in D$ and no positive rules in $A$. The rest of the argument is similar; we obtain a small counterexample by replacing unimportant node labels with some fresh $z$, removing subtrees, and shortening long chains of $z$s.

The allow–deny and deny–allow cases are special cases of the above. Hence, in any case, to decide whether $P$ is homomorphism-closed it suffices to check for counterexamples among trees of size bounded by $(|p| + |p'|)(W + 1)$.

**4.4 Polynomial-time static enforcement**

Fairness ensures static enforceability, but the problem of checking whether an update operation is statically allowed by a policy can still be expensive. Consider the common case of a deny–deny policy. An update $U$ is statically allowed if and only if it is contained in $A$, and does not overlap with $D$. Overlap testing is decidable in polynomial time [17], but containment of XPath expressions involving unions is coNP-complete. This high complexity is however dependent only on the policy and update size, not the size of the data, so may still be acceptable in practice; also, efficient-in-practice solvers are being developed for XPath containment and overlap tests [15].

We can take advantage of several observations to obtain efficient algorithms for special cases. First, we identify classes of XPath queries satisfying the following union decomposition property:

$p \subseteq p_1 \cdot \cdots \cdot p_n \iff p \subseteq p_1 \lor \cdots \lor p \subseteq p_n \tag{1}$

We need some auxiliary lemmas:

**Lemma 4.** Suppose $p \in \text{XP}^{\langle\langle 1 \rangle\rangle}$ and $Y_1, \ldots, Y_n$ are open sets in $\tau_2$. Then $\langle\langle p \rangle\rangle \subseteq Y_1 \cup \cdots \cup Y_n$ if and only if $\langle\langle p \rangle\rangle \subseteq Y_1 \lor \cdots \lor \langle\langle p \rangle\rangle \subseteq Y_n$.

**Proof.** Clearly $\langle\langle p \rangle\rangle$ is nonempty, and as discussed in Sec. [4] $\langle\langle p \rangle\rangle$ contains a tree $(T, n)$ such that $\langle\langle p \rangle\rangle = \langle\langle T, n \rangle\rangle$. Thus, $(T, n) \in Y_1 \cup \cdots \cup Y_n$, so for some $i$ we have $(T, n) \in Y_i$. By Prop. [7] we know that $Y_i$ is closed under homomorphic images of $(T, n)$, but the set of homomorphic images of $(T, n)$ is precisely $\langle\langle T, n \rangle\rangle = \langle\langle p \rangle\rangle$. □
Corollary 4. Suppose $p \in XP^{(\downarrow,1)}$ and $p_1, \ldots, p_n \in XP^{(\downarrow,1)}$. Then $p \subseteq p_1 \cdot \cdots \cdot p_n$ if and only if $p \subseteq p_1 \lor \cdots \lor p \subseteq p_n$.

Next, in order to prove union decomposition for containment problems whose left-hand side involves wildcards, we introduce relabeling functions $\rho : \Sigma \to \Sigma$. We define $\rho(T)$ in the obvious way: specifically,

$$\rho(T) = (V_T, E_T, R_T, \rho \circ \lambda_T).$$

Similarly, $\rho(T, n) = (\rho(T), n)$ and $\rho(T, n, m) = (\rho(T), n, m)$; furthermore if $Y$ is a set of (marked) trees then $\rho(Y) = \{\rho(y) \mid y \in Y\}$.

Definition 6. Suppose $C \subseteq \Sigma$ is finite. We say that $\rho$ fixes $C$ if $\rho(c) = c$ for each $c \in C$. A set $Y$ of trees, marked trees or doubly marked trees is called $C$-invariant if for all $\rho$ fixing $C$, we have $\rho(Y) \subseteq Y$. In other words, $Y$ is closed under relabelings that replace labels in $C$ with arbitrary labels.

We define the function labels mapping each path to the finite set of labels appearing in it, and likewise labels$^Q$ mapping each filter to its finite set of labels:

$$\text{labels}(ax :: a) = \{a\}, \quad \text{labels}(ax :: *) = \emptyset, \quad \text{labels}(p/p') = \text{labels}(p) \cup \text{labels}(p'), \quad \text{labels}(p[q]) = \text{labels}(p) \cup \text{labels}^Q(q),$$

$$\text{labels}^Q(q \circ q') = \text{labels}^Q(q) \cup \text{labels}^Q(q'), \quad \text{labels}^Q(p) = \text{labels}(p).$$

Thus, for example, labels$^Q([/a\times/*, b/c[)]\{a, b, c\}$ is the set of specific labels appearing in $/a\times/*, b/c[$. The semantics of a path $p$ is labels$^Q(p)$-invariant:

**Lemma 5.** For any $p \in XP^{(\downarrow,1)}$:

1. $P(\langle p \rangle)$ is labels$^Q(p)$-invariant.
2. $Q(\langle q \rangle)$ is labels$^Q(q)$-invariant.
3. $\langle p \rangle$ is labels$^Q(p)$-invariant.

**Proof.** Given $p$, define $C$ to be the set of all node labels from $\Sigma$ appearing in $p$. We proceed to prove parts (1,2) by simultaneous induction on the structure of path expressions and filters.

First, consider the case $ax :: a$. Observe that labels$^Q(ax :: a) = \{a\}$, so let $\rho$ be given such that $\rho(ax :: a) = a$. Then $\rho(ax :: a) = \rho(\langle T \rangle)$, and since the semantics of $\rho$ depends only on $\langle T \rangle$, we know that $(n, m) \in A[ax :: a]$.

Next, for the case $ax :: *$, observe that labels$(ax :: *) = \emptyset$, so we must consider arbitrary renamings $\rho$. Let $\rho$ be given and suppose $(T, n, m) \in P(ax :: *)$. Then $\rho(ax :: a) = \rho(\langle T \rangle)$, and since the semantics of $\rho$ depends only on $\langle T \rangle$, we can conclude that $(n, m) \in A[ax :: a]$. The cases for $p/p'$, $p[q]$, and filters are straightforward. For example, let $\rho$ fixing labels$(p/p') = \text{labels}(p) \cup \text{labels}(p')$ be given, and suppose $(T, n, m) \in P(p/p')$. Then there is some $k$ such that $(T, n, k) \in P(p)$ and $(T, k, m) \in P(p')$. Clearly, $\rho$ fixes labels$(p)$ and labels$(p')$ so $\rho(T, n, k) \in P(\langle p \rangle)$ by induction and similarly $\rho(T, k, m) \in P(\langle p' \rangle)$. So, we can conclude that $(\rho(T), n, m) \in P(\langle p/p' \rangle)$.

Finally, for part (3) if $(T, n) \in \langle p \rangle$ then $(T, R_T, n) \in P(\langle p \rangle)$ so $\rho(T, R_T, n) \in P(\langle \rho \rangle)$ and we can conclude that $(\rho(T), n) \in \langle \rho \rangle$.

**Lemma 6.** Suppose $p \in XP^{(\downarrow,1)}$ and $y_1, \ldots, y_n$ are open in $\tau_2$ and assume that each $y_i$ is $C$-invariant for some fixed $C$. Then $P(\langle \langle \rho \rangle \rangle) = \{ y_1 \cup \cdots \cup y_n \}$ if and only if $P(\langle \langle \rho \rangle \rangle) = \{ y_1 \cup \cdots \cup y_n \} \subseteq Y_n$.

**Proof.** Recall that we assume $\Sigma$ is infinite, so choose an infinite sequence $x_1, x_2, \ldots$ of elements of $\Sigma - C$. Form a new path expression $\rho'$ from $p$ by replacing each $\star$ occurring in $p$ with a distinct $x_i$. For example, if $p = /a/\times/[b]/x_2$ then $\rho' = /a/x_1[b]/x_2$.

Clearly, by construction $P(\langle \langle \rho' \rangle \rangle) = \langle P(\langle \rho' \rangle) \rangle$ and $\rho(U) = T$ and $\rho$ fixes $C$. Let $(T, n) \in \rho$ be given, and let $\{m_1, \ldots, m_n\} = V_T$ be some enumeration of the $k$ vertices of $T$. Let $P(v)$ be a tree pattern corresponding to $p$, and let $U : (P, v) \rightarrow (T, n)$ be an embedding witnessing the fact that $(T, n) \triangleq p$. Define $U$ and $p$ as follows:

$$U = (V_T, E_T, R_T, \lambda')$$

$$\lambda'(m_i) = \begin{cases} \lambda_T(m_i), & m_i = x_i \\ a & \text{otherwise} \end{cases}$$

Clearly, $\rho(a) = \lambda_T(m_i) = a$.

That is, $U$ has the same nodes and edges as $T$, and its node labels are equal to those of $T$ for nodes that match a fixed label $a$ in $p$ (i.e., when $h(m_i) = m'$ and $\lambda_T(m_i) = a$), and the labels of nodes $m_i$ matching occurrences of $\star$ are reassigned to the corresponding $x_i$. Also, $\rho$ maps each $x_i$ to the corresponding label $a$ in $T$, so that by construction $p(U) = T$. Since the $x_i$ are chosen from outside $C$, it follows that $\rho$ fixes $C$ by construction.

Next, we show that $P(\langle \langle \rho \rangle \rangle) \subseteq Y_i$. Let $(T, n) \in \langle p \rangle$. Then $(T, n) \in \langle p \rangle$ and let $U, \rho$ be constructed as above, so that $\rho(U) = T$ and $(T, n) \in \langle \langle \rho \rangle \rangle$. Clearly, $(T, n) \in \langle \langle \rho \rangle \rangle \subseteq Y_i$, and $\lambda_T(m_i) = a$.

Therefore, by the assumption that each $y_i$ is $C$-invariant, we have that $(T, n) = (\rho(U), n) \in P(\langle \langle \rho \rangle \rangle) \subseteq P(\langle \langle \rho \rangle \rangle)$.

**Lemma 7.** The containment problem $p \not\subseteq p_1 \cdots \cdot p_n$ satisfies union decomposition provided that $p \in XP^{(\downarrow,1)}$ and $p_i \in XP^{(\downarrow,1)}$.

**Proof.** From Lem.5 we have that all of the sets $\langle p_i \rangle$ are labels$(p_i)$-invariant, so they are all $\cup$ labels$(p_i)$-invariant. Thus, by Lem.6 we must have $\langle p \rangle \subseteq \langle p_1 \rangle \cup \cdots \cup \langle p_n \rangle$ if and only if $\langle p \rangle \subseteq p_i$ for some $i$. This is equivalent to union decomposition for the problem $p \subseteq p_1 \cdots \cdot p_n$.

Note that for this proof, the assumption that $\Sigma$ is infinite was necessary: otherwise, if $\Sigma = \{a_1, \ldots, a_n\}$, then $\star \not\subseteq /a_1\times/\star\times/\star/a_1$ holds but does not satisfy union decomposition.

**Theorem 5.** Static enforcement of update capabilities in $XP^{(\downarrow,1)}$ is checkable in $PTime$ for any fixed policy $P$ over $XP^{(\downarrow,1)}$.

**Proof.** We consider the deny–deny case where $P = (-, -, A, D)$. Consider an update capability $U$ characterized by a path $p \in XP^{(\downarrow,1)}$. We need to ensure that $P(p) \subseteq \langle A \rangle$ and $P(p) \subseteq \langle D \rangle$. By Lem.7 the first part can be checked by testing whether $p \subseteq p_i$ for each $p_i \in A$. Each such test can be done in polynomial time since $p \in XP^{(\downarrow,1)}$. The second part amounts to checking that $p$ does not overlap with any element of $D$, which also takes polynomial time.

The allow–allow case is similar, but more involved. Given $p$, we first check whether it overlaps with any elements of $D$. For each $p_i \in D$ such that $p$ overlaps with $p_i$, we need to check whether $P(p_i) \subseteq \langle A \rangle$. The intersection of two paths in $XP^{(\downarrow,1)}$
can be expressed by another path in $XP(\lnot)$ and this can be computed in PTIME; the required containment checks are also in PTIME as per Lemma 7. Hence, allow-allow policies can also be statically enforced in PTIME. Other policies are special cases.

Unfortunately, union decomposition does not hold for problems where $p \in XP(\lnot)$. For example, $(a/b) \subseteq (a/b) \cup (a/b)$ holds, but neither $(a/b) \subseteq (a/b) \lor (a/b) \subseteq (a/b)$ nor $(a/b) \subseteq (a/b)$ hold. In any case, even without union, containment of $XP(\lnot)$ paths is coNP-complete. As noted earlier, Miklau and Suciu showed that containment is decidable in polynomial time if the number of descendant steps in $p$ is bounded. Using an adaptation of this result, together with Theorem 5 we can extend this result to handle problems of the form $p \subseteq p_1 \ldots p_n$ where all paths are in $XP(\lnot)$ and $p$ has at most $d$ descendant steps:

**Corollary 5.** Static enforcement of update capabilities in $XP(\lnot)$ having at most $d$ descendant steps is checkable in PTIME for any policy $P$ over $XP(\lnot)$.

**Proof.** Given a problem $p \subseteq p_1 \ldots p_n$, consider all expansions $p[\bar{u}]$ where $u_i \leq W + 1$, where $W$ is the maximum star length of the paths $p_1, \ldots, p_n$. There are at most $m = (W + 2)^d$ such expansions where $d$ is the number of descendant steps in $p$. Each of these paths is in $XP(\lnot)$ so by Lem. 7 we can check in PTIME whether all are contained in $p_1 \ldots p_n$. If not, then clearly $p$ itself is not contained in $p_1 \ldots p_n$. Conversely, if $p$ is not contained in $p_1 \ldots p_n$, then (using Lem. 3) we can find a small counterexample that matches one of the $p[\bar{u}]$, which implies that the algorithm will detect non-containment for this $p[\bar{u}]$.

5. Discussion

5.1 Generalizations

In the previous section we have simplified matters by considering only deletion capabilities; we also have not discussed attribute equality tests. Our framework extends to policies over the full language $XP(\lnot)$ and to policies consisting of multiple different kinds of operations (insert, delete, rename, replace). To handle multiple kinds of operations, we need to consider the topologies over the set of paths $T_u$ of trees and atomic operations $u \in T(U)$. Attribute steps and value tests complicate matters: because attribute values must be unique, the policy $(-,-,\{a[\bar{a}] = c\},\{a[\bar{a}] = d\})$ is fair. Verifying this requires taking the uniqueness constraint into account, or more generally, testing containment or overlap modulo key constraints. This can be done using more expressive logics for XPath over data trees [10] [11] or general-purpose solvers [12]. However, the coNP-hardness proof given earlier is not applicable if only attribute-based filters are allowed, so it may be possible to check fairness in the presence of negative attribute tests in PTIME.

Fairness can also be affected by the presence of a DTD or schema that constrains the possible trees. It is easy to see that a policy that is fair in the absence of a schema remains fair if we consider only valid documents. On the other hand, an unfair policy may become fair in the presence of a schema or other constraints (as illustrated above using attributes). For example, the unfair policy from Ex. 1 becomes fair if the schema eliminates uncertainty as to whether $a$ has a $b$ child. This can happen even if $a$ cannot have any $b$ children or always has at least one. However, checking containment and satisfiability often become more difficult when a DTD is present.

The fairness picture changes if we consider extensions to the XPath operations allowed in the update requests. For example, for $XP(\lnot,\lnot)$ policies, it appears possible to recover fairness by allowing negation in filter expressions (e.g. $a[\not(b)]$). However, XPath static analysis problems involving negation are typically not in PTIME [3] [24] [35]. Thus, there is a tradeoff between the complexity of determining that a policy is fair and the complexity of statically enforcing fairness, governed by the expressiveness of the set $XP$ of XPath expressions allowed in update capabilities. The more expressive $XP$ is, the easier it is to check fairness and the harder it is to enforce the policy.

5.2 Implications

Having established some technical results concerning policy fairness and the complexity of determining fairness and of static enforcement with respect to fair policies, what are the implications of these results? We believe that there are three main messages:

- Policies without filters are always fair. However, such policies may not be sufficiently expressive for realistic situations; for example, the policy in Figure 1 would become much too coarse if we removed the filters. Policies with filters only in positive rules are also always fair, and are more expressive; for example, the policy in Figure 2 is in this fragment. Therefore, policy authors can easily ensure fairness by staying within this fragment.

- Checking policy fairness for policies with filters in negative rules may be computationally intensive; it may be worthwhile investigating additional heuristics or static analyses that can detect fairness for common cases more efficiently. Also, as discussed in the previous section, it may be possible to check fairness for policies with negative attribute tests in PTIME.

- We established that for relatively tame update capabilities (with limited numbers of descendant axis steps), static enforcement remains in PTIME. Static enforcement depends directly on the complexity of checking containment and overlap problems. Containment checking is not symmetric in $p$ and $p'$, so it may be profitable to investigate ways to make policies richer while retaining fairness with respect to less expressive classes of updates.

6. Related Work

Most prior work on enforcing fine-grained XML access control policies has focused on dynamic enforcement strategies. As discussed in the introduction, previous work on filtering, secure query evaluation and security views has not addressed the problems that arise in update access control, where it is important to decide whether an operation is allowed before performing expensive updates.

Murata et al. [24] previously considered static analysis techniques for rule-based policies, using regular expressions to test inclusion in positive rules or possible overlap with negative rules, but their approach provides no guarantee that static enforcement is fair; their static analysis was used only as an optimization to avoid dynamic checks. Similarly, Koromilas et al. [19] employed static analysis techniques to speed annotation maintenance in the presence of updates. In contrast, our approach entirely obviates dynamic checks.

The consistency problem for XML update access control policies involves determining that the policy cannot be circumvented by simulating a forbidden operation through a sequence of allowed operations. Fundulaki and Maneth [14] introduced this problem and showed that it is undecidable for full XPath. Moore [25] further investigated the complexity of special cases of this problem. Bravo et al. [17] studied schema-based policies for which consistency is in PTIME and also investigated repair algorithms for inconsistent policies. Jacquemard and Rusinowitch [13] studied complexity and
algorithms for consistency of policies with respect to richer classes of schemas. Fairness and consistency are orthogonal concerns. Language-based security, particularly analysis of information flow, is another security problem that has been studied extensively [28], including for XML transformations [5]. This paper considers only classical access control (deciding whether to allow or deny actions specified by a policy), a largely separate concern. Thus, while our approach draws on ideas familiar from language-based security such as static analysis, the key problems for us are different. Typically, language-based information flow security aims to provide a conservative upper bound on possible run-time behaviors of programs, for example to provide a non-interference guarantee. Thus, sound over-approximation is tolerable for information-flow security. In contrast, we wish to exactly enforce fine-grained access control policies, so we need to consider exact static analyses and related properties such as fairness.

7. Conclusion

Fine-grained, rule-based access control policies for XML data are expensive to enforce by dynamically checking whether the update complies with the rules. In this paper, we advocate enforcement based on static analysis, which is equivalent to dynamic enforcement when the policy is fair. We gave a novel topological characterization of fairness, and used this characterization to prove that for policies over $XP(//,/*,[])$, all policies without filters in negative rules are fair (with respect to $XP(//,[])$, and fairness is decidable in $\text{compNP}$-time.

There are natural next steps for future work, including investigating fairness for larger fragments of XPath or in the presence of schema or constraints on the data, and generalizing the approach to ordered trees and the full complement of XPath axes. Implementing and evaluating the practicality of fair policy enforcement or fairness checking is also of interest. Finally, our approach places a desired update in a conservative upper bound on possible run-time behaviors of programs, for example to provide a non-interference guarantee. Thus, sound over-approximation is tolerable for information-flow security. In contrast, we wish to exactly enforce fine-grained access control policies, so we need to consider exact static analyses and related properties such as fairness.

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A. Proofs

Proof of Lem. 2. Suppose \((V', k) \in \langle\langle T, n\rangle\rangle \cap \langle\langle U, m\rangle\rangle\). Then there must exist witnessing homomorphisms \(h_1 : (T, n) \to (V, k)\) and \(h_2 : (U, m) \to (V, k)\). Without loss of generality, assume that \(h_1\) and \(h_2\) are injective and \(\text{rng}(h_1) \cap \text{rng}(h_2)\) consists only of the vertices between \(R_V\) and \(k\). Observe that \(h_1\) and \(h_2\) are invertible when restricted to \(\text{rng}(h_1) \cap \text{rng}(h_2)\).

Construct \((V, k)\) from \((V', k)\) by deleting all subtrees that do not contain a node from \(\text{rng}(h_1) \cup \text{rng}(h_2)\). Observe that this implies that \(V' = \text{rng}(h_1) \cup \text{rng}(h_2)\). To see that \(\langle\langle V, k\rangle\rangle = \langle\langle T, n\rangle\rangle \cap \langle\langle U, m\rangle\rangle\), the forward inclusion \(\langle\langle V, k\rangle\rangle \subset \langle\langle T, n\rangle\rangle \cap \langle\langle U, m\rangle\rangle\) is immediate. Suppose \((W, l) \in \langle\langle T, n\rangle\rangle \cup \langle\langle U, m\rangle\rangle\), and suppose \(h_1' : (T, n) \to (W, l)\) and \(h_2' : (U, m) \to (W, l)\) are homomorphisms witnessing this. Choose a function \(g : (V, k) \to (W, l)\) such that:

\[
g(x) = \begin{cases} 
    h_1'(y) & x \in \text{rng}(h_1), h_1(y) = x \ 
    h_2'(z) & x \in \text{rng}(h_2) - \text{rng}(h_1), h_2(z) = x
\end{cases}
\]

We first show that \(h_1' = g \circ h_1\) and \(h_2' = g \circ h_2\). The first equation is immediate; for the second, clearly \(g(x) = h_2'(x)\) when \(x \in \text{rng}(h_2) - \text{rng}(h_1)\). If \(x \in \text{rng}(h_1) \cap \text{rng}(h_1)\) then \(x\) is between \(R_V\) and \(k\), so \(h_1^{-1}(x) = \{y\}\) and \(h_2^{-1}(x) = \{z\}\) where \(y\) and \(z\) are in the corresponding position on the paths between \(R_V\) and \(n\) and \(R_U\) and \(m\) respectively. Thus, we must have that \(g(z) = h_1'(y) = h_2'(z)\) because both \(h_1'\) and \(h_2'\) are homomorphisms.

To show that \(g\) is a homomorphism, first \(g(R_V) = h_1'(R_T) = R_W\). Second, for any edge \((v, w) \in E_V\), there are several cases to show that \((g(v), g(w)) \in E_W\). If \(w \in \text{rng}(h_1)\) then clearly \(v \in \text{rng}(h_1)\) also, and \(v = h_1(v'), w = h_1(w')\) where \((v, w) \in E_T\) by the injectivity of \(h_1\), so then \((g(v), g(w)) = (g(h_1(v')), g(h_1(w')))) = (h_1'(v'), h_1'(w')) \in E_W\). Similarly, if \(w \in \text{rng}(h_2)\) we are done. Finally, for any \(v \in V_V\), there are several cases to consider in showing \(\lambda_W(g(v)) = \lambda_V(v)\). If \(v \in \text{rng}(h_1)\) then suppose \(v = h_1(v')\) for some \(v' \in V_T\).

Then \(\lambda_W(g(v)) = \lambda_W(g(h_1(v'))) = \lambda_W(h_1'(v')) = \lambda_V(v') = \lambda_V(h_1(v')) = \lambda_V(v)\). The case for \(v \in \text{rng}(h_2)\) is similar. □