Abstract

Background/Objectives: Nowadays, most organizations acknowledge the role and significance of different energy resources in the supply of present and future needs. As a result, they make sizeable investments and undertake extensive research on strategy and policy making as well as definition of infrastructure projects to reduce energy consumption. According to the diversity of projects and their cost of implementation, ranking the projects with different criteria is currently one of the complex problems in organizations. This paper presents a new solution based on the specific architecture of Perceptual Computer (Per-C) for project selection. Methods/Statistical Analysis: In project selection problem, decision makers express their assessment in the form of the linguistic terms that they are vague and uncertain. Using Per-C method is a new experience in project selection problems that has not been mentioned in any literature so far. The Per-C has three components: an Encoder, which maps words into IT2 FS models; a CWW engine, which operates on the input words and whose outputs are FOU(s); and a Decoder, which maps these FOU(s) into a recommendation. We used the Interval Approach (IA) for Decoding, Linguistic Weighted Average (LWA) method is used in CWW engine and Centroid method is used for Decoding. Findings and Application/Improvements: A real case study from a power plant is used to illustrate the applicability of the approach. Using Per-C method is a new experience in project selection problems that has not been mentioned in any literature so far. The results show the applicability of the proposed methodology clearly.

Keywords: Computing with Word and Interval Type 2 Fuzzy Set, Fuel Consumption Reduction, Perceptual Computing Based, Project Selection

1. Introduction

Limited fossil resources, population growth and increased energy demand are matters with which most countries are facing. Various energy resources are in fact national wealth of every country and should be used in favor of sustainable development. Development of electric power plants is a fundamental factor for industrial expansion of every countries. Generation and distribution of electrical energy and sustainable development considerations are essential for future generations especially in developing countries. Fossil energy has an essential role in power generation. During the conversion of fossil energy to electrical energy and electricity generation, a significant loss of energy whose amount depends on the employed technology and design of power plants occurs. Nowadays, most countries acknowledge the role and significance of different energy resources in the supply of present and future needs. As a result, they make sizeable investments and undertake extensive research on strategy and policy making as well as defining infrastructure projects to reduce energy consumption. Regarding that fossil energies are used as fuel in power plants and that some of these plants in Iran have considered the concept of sustainable development in recent years, several projects have been defined in field of reducing fossil energy consumption for implementation.

Evaluating and selecting these projects has become an issue of a decision problem for mangers of the organizations. Multi Criteria Decision Making (MCDM)
has been broadly and effectively applied in Decision Problems. There are various methods based on Multi Criteria Decision Making (MCDM) methods for selecting project in the different fields, as some of them are mentioned in the following. Yu and Liu, introduced a new multi-criteria methodology for prioritizing highway safety improvement projects based on an extended Analytical Hierarchy Process (AHP) with fuzzy logic\(^1\). Wei and Chang, proposed a new approach which combines fuzzy set theory and multi-criteria group decision making method into a NPD project portfolio selection model. Their proposed approach consisted of two main phases, in the first phase, a new fuzzy multi-criteria group decision making approach was used to select project portfolio which considers project performance, project delivery and project risk, and in the second phase, a fuzzy linear programming model is formulated to analyze the best NPD portfolio that suits most with enterprises objective\(^1\). Ghapanchi et al.\(^3\), used Data Envelopment Analysis (DEA) to select the best portfolio of IS/IT projects while taking both project uncertainties (modeled as fuzzy variables) and project interactions into consideration simultaneously\(^3\). Bhattacharyya et al.\(^4\), presented a fuzzy multi-objective programming approach to facilitate decision making in the selection of R and D projects. They proposed a fuzzy tri-objective R and D portfolio selection problem and discussed how their methodology can be used to make decision support tools for optimal R and D project selection in a corporate environment\(^5\). Pakdin Amiri, formulated a project selection problem as a multi-criteria decision making (MCDM) problem and applied a utility function. They used AHP and Fuzzy TOPSIS techniques, the AHP is used to analyze the structure of a project selection problem and to determine weights of the criteria, and fuzzy TOPSIS method is used to obtain final ranking\(^6\).

Smith-Perera et al.\(^6\) used Analytic Network Process (ANP) method to prioritize project portfolio. BuyukOzkan and Ozturkcan developed a novel approach based on a combined ANP and DEMATEL technique to help companies in order to determine critical Six Sigma projects and identify the priority of these projects especially in logistics companies\(^7\). Ebrahimnejad proposed a new two-phase Group Decision Making (GDM) approach. Their approach integrated a modified Analytic Network Process (ANP) and an improved compromise ranking method, known as VIKOR. In this approach, at First, a modified fuzzy ANP method is introduced to address the problem of dependence as well as a feedback among conflicting criteria to determine their relative importance. Then, a fuzzy VIKOR method is extended to rank potential projects on the basis of their overall performance\(^8\). Vetschera and Teixeira de Almeida used PROMETHEE outranking methods for portfolio selection problems\(^9\). N. Vandaeele et al.\(^10,11\) used a Data Envelopment Analysis (DEA) approach for selecting project. Tavana et al.\(^12\), proposed a Data Envelopment Analysis (DEA) model with ambiguity and vagueness. In their model, the vagueness of the objective functions is modeled by means of multi-objective fuzzy linear programming and the ambiguity of the input and output data modeled with fuzzy sets and a new a-cut based method.

In MCDM problems, the ratings and the weights of the criteria are known precisely\(^13,14\). In fact, decision makers’ judgments including preferences are often vague, so that a judge cannot estimate his preference with an exact numerical value. A more realistic approach may be to use linguistic assessments instead of pure numbers to describe the desired value and important weights of criteria because words can mean different things to different decision makers; it is very important to use a Fuzzy Set (FS) model for a word that lets us capture word uncertainty.

In this paper, we used a CWW approach based on the specific architecture of Perceptual Computer (Per-C) method for selecting projects. First, obtaining Interval Type 2 Fuzzy Set (IT2 FS) word models for the words in a pre-specified vocabulary we used the Interval Approach (IA). Second, we used Linguistic Weighted Average (LWA) method for aggregating all the words modeled by IT2 FSs. Then, a centroid-based ranking method is used to rank Projects.

The rest of this paper is organized as follows: in section 2, Interval Type 2 Fuzzy Sets theory is briefly reviewed. Section 3, some background and technical materials about CWW in decision-making are reviewed. Section 4 provides a numerical example and Section 5 draws conclusions.

### 2. Interval Type 2 Fuzzy Sets (IT2 FSs)

In this section, we briefly review basic concept of type 2 fuzzy sets and interval type-2 sets. A Type-2 fuzzy set (A T2 FS) in the universal of discourse X by a type-2 membership function \(\mu_A(x,u)\) are shown as follows\(^15\):

1. **2.1. Interval Type-2 Fuzzy Sets**

   An interval type-2 fuzzy set (IT2 FS) is a set whose membership function is an interval-valued function on the real line. The membership function is a function that assigns to each point in the universe of discourse a closed interval in the unit interval [0,1]. Formally, an IT2 FS \(\tilde{A}\) can be defined as a triplet \((\alpha, [\tilde{A}_L, \tilde{A}_U])\), where \(\alpha\) is a parameter called the degree of fuzziness, \([\tilde{A}_L, \tilde{A}_U]\) is the lower and upper membership functions, respectively, and \(\tilde{A}_L(x)\) and \(\tilde{A}_U(x)\) are interval-valued functions on \(X\). The membership function of an IT2 FS can be expressed as a function of the membership function of an interval type-1 fuzzy set, which is a function of the membership function of a crisp set.

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\[
\tilde{A} = \{(x,u), \mu_\tilde{A}(x,u)\} \ \forall x \in X, \forall u \in I_x \subseteq [0,1] \quad (1)
\]

In which \(0 \leq \mu_\tilde{A}(x,u) \leq 1\).

Where \(x\) is the primary variable, \(I_x \in [0,1]\) is the primary membership of \(x\), \(u\) is the secondary variable, and \(\mu_\tilde{A}(x,u)\), is the secondary membership function at \(x\). A Type 2 fuzzy set (A T2 FS), denoted as \(\tilde{A}\), can be characterized as:

\[
\tilde{A} = \int_{x \in X} \int_{u \in I_x} \frac{\mu_a(x,u)}{(x,u)} \quad (2)
\]

Where \(J_x \subseteq [0,1]\) and \(\int \int\) denotes union over all admissible \(x\) and \(u\). For discrete universes of discourse \(\int\) is replaced by \(\sum\). In a general T2 FS, when all \(\mu_\tilde{A}(x,u) = 1\) then \(\tilde{A}\) is an interval type-2 set (IT2 FS). Although the third dimension (i.e., \(\mu_\tilde{A}(x,u)\)) of the IT2 FSs is no longer needed because it conveys no new information about the IT2 FSs (i.e., \(\mu_\tilde{A}(x,u) = 1\) for all \(x\) and \(u\)), the IT2 FSs can still be expressed as a special case of the general T2 FSs as follows:

\[
\tilde{A} = \int_{x \in X} \int_{u \in I_x} \frac{\mu_a(x,u)}{(x,u)} \quad (3)
\]

Uncertainty in the primary memberships of a general T2 FS, \(\tilde{A} = \int_{x \in X} \int_{u \in I_x} \frac{\mu_a(x,u)}{(x,u)}\) consists of a bounded region that we call the Footprint of Uncertainty (FOU). It is the union of all primary memberships, i.e.:

\[
FOU(\tilde{A}) = \bigcup_{x \in X} J_x, \quad (4)
\]

\[
\tilde{A} = \frac{1}{FOU(\tilde{A})} \quad (5)
\]

It is also observed that an IT2 FS is bounded from above and below by two T1 FSs, which are called Upper Membership Function (UMF) and Lower Membership Function (LMF), respectively.

In Figure 1 the upper Membership Function (MF) and lower MF of the T2FS \(\tilde{A}\) are T1 MFs. That bounded its Footprint of Uncertainty (FOU). An embedded T1 FS2 is any T1 FS within the FOU. The UMF denoted \(\bar{\mu}_a(x), \forall x \in X\) and the LMF denoted \(\underline{\mu}_a(x), \forall x \in X\) are type-1 membership functions, respectively:

\[
FOU(\tilde{A}) = \bigcup_{x \in X} \left[\mu_\tilde{A}(x), \bar{\mu}_a(x)\right] \quad (6)
\]

### 3. Perceptual Computer (Per-C)

This paper provides a new solution method for group multi-criteria decision making problems using Perceptual Computer (Per-C) theory. The Perceptual Computer (Per-C) is an instantiation of Zadeh’s CWW paradigm, as applied to assisting people in making subjective judgments. The Per-C consists of three components: an encoder, which maps words into IT2FS models; a CWW engine, which operates on the input words and whose outputs are FOU(s); and a decoder, which maps these FOU(s) into a recommendation. Conceptual structure of Per-C, which was proposed to make subjective judgments by CWW, is shown in Figure 2.

These three components are described as follows:

#### 3.1 Encoder

The Encoder converts linguistic terms or words from Decision Makers into IT2FS. Interval Approach (IA) is established to model a word using an IT2 FS. It is based on collecting interval end-point data from a group of subjects and does not require subjects to be knowledgeable about fuzzy sets; hence, it can be used by anyone. It is a very practical method for mapping subject’s data interval into a Footprint of Uncertainty (FOU) for a word. The resulting FOUs are either interior, left-shoulder, or
right-shoulder FOUs (Figure 3.) and it is the data that establishes which FOU models a word$^{16}$.

The IA consists of two parts, the Data Part and the Fuzzy Set (FS) Part$^{16}$. At first, encoding based on this approach, for each word in an application dependent encoding vocabulary, a group of subjects are asked the following question:

On a scale of 0-10, what are the endpoint of an interval that you associate with the word …..?

After some pre-processing, during which some intervals (e.g., outliers) are eliminated, each of the remaining intervals is classified as an interior, left-shoulder, or right-shoulder IT2 FS. Then, each of the word’s data intervals is individually mapped into its respective T1 interior, left-shoulder, or right-shoulder MF, after which the union of all of these T1 MFs is taken. The result is an FOU for an IT2 FS model of the word. The words and their FOUs constitute a codebook$^{16}$.

### 3.2 CWW

Lotfi Zadeh, the father of fuzzy logic, coined the phrase “computing with words” (CWWs), which is “a methodology in which the objects of computation are words and propositions drawn from a natural language”. Words in the CWW paradigm may be modeled by type-1 fuzzy sets (T1 FSs) or their extension, Interval Type-2 (IT2) FSs$^{17–20}$. There are two kinds of CWW engines. The CWW Engine maps the IT2 FSs into IT2FSs by means of Novel Weighted Average (NWA) method$^{21}$ for MCDM problems and Perceptual Reasoning (PR) method$^{22}$ for rule-based problems. As this paper is a decision problem, NWA approach is explained as follow:

#### 3.2.1 Novel Weighted Averages

An Novel Weighted Average (NWA) is a Weighted Average (WA) in which at least one sub criterion or weight is not a single real number; instead, it is an interval, T1 FS or an IT2 FS, in which cases such sub criteria, weights, and the WA are called novel models. There are different WAs, as summarized$^{16}$:

##### 3.2.1.1 Interval Weighted Averages (IWA)

When at least one sub criterion or weight is modeled as an interval, and all other sub criteria or weights are modeled by no more than such a model, the resulting WA is called an Interval Weighted Averages (IWA).

##### 3.2.1.2 Fuzzy Weighted Averages (FWA):

When at least one sub criterion or weight is modeled as a T1 FS, and all other sub criteria or weights are modeled by no more than such a model, the resulting WA is called a Fuzzy Weighted Averages (FWA).

##### 3.2.1.3 Linguistic Weighted Averages (LWA):

When at least one sub criterion or weight is modeled as an IT2 FS, the resulting WA is called a Linguistic Weighted Averages (LWA).

In the following, each of the methods is described:

- **Interval Weighted Average**

The IWA is defined as:

$$Y_{IWA} = \frac{\sum_{i=1}^{n} W_i \times X_i}{\sum_{i=1}^{n} W_i} = [l, r]$$

Where

$$X_i = [a_i, b_i], \quad i = 1, \ldots, n$$

$$W_i = [c_i, d_i], \quad i = 1, \ldots, n$$

And $Y_{IWA}$ is also an interval completely determined by its two endpoints l and r, with:

$$l = \min_{w_i \in W_i} \frac{\sum_{i=1}^{n} w_i \times X_i}{\sum_{i=1}^{n} w_i}$$

$$r = \max_{w_i \in W_i} \frac{\sum_{i=1}^{n} w_i \times X_i}{\sum_{i=1}^{n} w_i}$$

The variables l and r can easily be computed by the Karnik-Mendel (KM) or enhanced Karnik-Mendel (EKM) algorithms$^{23–25}$.

- **Fuzzy Weighted Average**

The FWA$^{26,27}$ is defined as:

$$Y_{FWA} = \frac{\sum_{i=1}^{n} W_i \times X_i}{\sum_{i=1}^{n} W_i}$$

![Figure 3. FOUs for CWWs$^{17}$.](image-url)
Where $X_i$ and $W_i$ are T1 FSs, and $Y_{FWA}$ is also a T1 FS.

Note that Equation (12) is an expressive way to represent the $Y_{FWA}$ because it is not computed using multiplications, additions, and divisions, as expressed by it. Instead, it has been shown\(^\text{22}\) that the $FWA$ can be computed by using the $\alpha$-cut decomposition theorem\(^\text{17}\), where each $\alpha$-cut on $Y_{FWA}$ is an $IWA$ of the corresponding $\alpha$-cuts on $X_i$ and $W_i$, as described by the following algorithm.

1) For each $\alpha \in [0, 1]$, the corresponding $\alpha$-cuts of the T1 FSs $X_i$ and $W_i$ are first computed, i.e., compute

$$X_i(a) = \left[ a_i(\alpha), b_i(\alpha) \right], \quad i = 1, \ldots, n$$

$$W_i(a) = \left[ c_i(\alpha), d_i(\alpha) \right], \quad I = 1, \ldots, n$$

(13) (14)

2) For each $\alpha \in [0, 1]$, compute the $\alpha$-cut of the $FWA$ by recognizing that it is an $IWA$, i.e., $Y_{FWA}(\alpha) = Y_{IWA}(\alpha)$, where $Y_{IWA}(\alpha) = [l(\alpha), r(\alpha)]$

In which:

$$l(a) = \min_{\forall w_i(a) \in [c_i(a), d_i(a)]} \frac{\sum_{i=1}^{n} a_i(a) w_i(a)}{\sum_{i=1}^{n} w_i(a)}$$

(15)

$$r(a) = \min_{\forall w_i(a) \in [c_i(a), d_i(a)]} \frac{\sum_{i=1}^{n} b_i(a) w_i(a)}{\sum_{i=1}^{n} w_i(a)}$$

(16)

And the KM or EKM algorithms\(^\text{23,28}\) are used to compute $l(\alpha)$ and $r(\alpha)$.

3) Connect all left coordinates $[l(\alpha), \alpha]$ and all right coordinates $[r(\alpha), \alpha]$ to form the T1 FS $YFWA$.

### C. Linguistic Weighted Average (LWA)

The LWA is defined as\(^\text{22}\)

$$\tilde{Y}_{LWA} = \frac{\sum_{i=1}^{n} \tilde{W}_i \times \tilde{X}_i}{\sum_{i=1}^{n} \tilde{W}_i}$$

(17)

where $\tilde{X}_i$ and $\tilde{W}_i$ are IT2 FSs, and $\tilde{Y}_{LWA}$ is also an IT2 FS. Again Equation (17) is an expressive way to describe the LWA. To compute $\tilde{Y}_{LWA}$, one only needs to compute its $LMF \quad Y_{LWA}$ and $UMF \quad \tilde{Y}_{LWA}$.

In\(^\text{27}\), Wu and Mendel proved that computing LWA could be substituted with computing two FWAs of $\tilde{Y}_{LWA}$ and $\tilde{Y}_{LWA}$, and therefore, $\tilde{Y}_{LWA}$ can be computed as follows:

$$\tilde{Y}_{LWA} = \frac{1}{\frac{Y_{LWA}}{\tilde{Y}_{LWA}}}$$

(18)

Where $X_i(\tilde{X}_i)$ and $W_i(\tilde{W}_i)$, $i = 1, \ldots, n$, are the LMF (UMF) of $\tilde{X}_i$ and $\tilde{W}_i$, correspondingly. The $\alpha$-cut based approach\(^\text{22}\) is also used to compute $Y_{LWA}$ and $\tilde{Y}_{LWA}$.

In this paper, the Linguistic Weighted Average (LWA) is used for the CWW Engine because all the weights and ratings are modeled by IT2 FSs.

### 3.3 Decoder

The decoder maps the CWW Engine output FOUs into a recommendation. The recommendation from the decoder can have three forms: Word, Rank and Class. In the following, each of the forms is described:

#### 3.3.1 Word

This is the most typical case. In this case, the output of CWW (IT2) must map into a word; therefore, it must be possible to compare the similarity between two IT2FSs. Then, the word with maximum similarity is chosen as the decoder’s output. There are only six similarity (compatibility) measures for IT2 FSs\(^\text{29-33}\).

#### 3.3.2 Rank

In some decision-making situations, several alternatives are compared so that the best one (s) can be chosen. In these situations, each alternative is represented by an IT2FS; hence, the decoder must rank them to find the best alternative (s). There are different Ranking methods such as Reasonable Ordering Properties method, Mitchell’s method and Centroid-Based Ranking Method\(^\text{17,33}\).

#### 3.3.3 Class

In some decision-making applications, the output of the CWW engine must be mapped into a class. These IT2 FSs must be mapped into one of three decision classes: accept, rewrite, or reject\(^\text{16}\).

In this paper, we used Centroid-based ranking method for decoding, and then in this section we described this method.
3.3.3.1 Centroid-Based Ranking Method

The centroid \( C(\tilde{A}) \) of IT2 FSs \( \tilde{A} \) is the union of the centroids of all its embedded T1 FSs \( A \), i.e.

\[
C(\tilde{A}) = \bigcup_{A} C(A) = \{C_l(\tilde{A}), C_r(\tilde{A})\}
\]  

(21)

Where \( \bigcup \) is the union, and

\[
C_l(\tilde{A}) = \min C(A)
\]  

(22)

\[
C_r(\tilde{A}) = \max C(A)
\]  

(23)

\[
C(A) = \frac{\sum_{i=1}^{N} x_i \times \mu_\tilde{A}(x_i)}{\sum_{i=1}^{N} \mu_\tilde{A}(x_i)}
\]  

(24)

It has been shown\(^{32-34} \) that \( C_l(\tilde{A}) \) and \( C_r(\tilde{A}) \) can be expressed as:

\[
C_l(\tilde{A}) = \frac{\sum_{i=1}^{L} x_i \times \mu_\tilde{A}(x_i) + \sum_{i=L+1}^{N} x_i \times \mu_\tilde{A}(x_i)}{\sum_{i=1}^{L} \mu_\tilde{A}(x_i) + \sum_{i=L+1}^{N} \mu_\tilde{A}(x_i)}
\]  

(25)

\[
C_r(\tilde{A}) = \frac{\sum_{i=1}^{R} x_i \times \mu_\tilde{A}(x_i) + \sum_{i=R+1}^{N} x_i \times \mu_\tilde{A}(x_i)}{\sum_{i=1}^{R} \mu_\tilde{A}(x_i) + \sum_{i=R+1}^{N} \mu_\tilde{A}(x_i)}
\]  

(26)

Switch points L and R, as well as \( C_l(\tilde{A}) \) and \( C_r(\tilde{A}) \), are computed by iterative KM Algorithms\(^{33,34} \).

Centroid-based ranking method: First compute the average centroid for each IT2FS \( \tilde{A}_i \),

\[
C(\tilde{A}_i) = \frac{C_l(\tilde{A}_i) + C_r(\tilde{A}_i)}{2}, i = 1, ..., N
\]  

(27)

And then sort \( C(\tilde{A}_i) \) to obtain the rank of \( \tilde{A}_i \).

4. Case Study

In this section, the proposed approach has been applied to a power plant which wants to prioritize projects in field of fuel consumption reduction in Iran. Details are as follows:

In this study, in order to rank alternatives, a committee composed of three Decision Makers (DMs) has been formed. In this case, assume the weights from DMs are \( \tilde{W}^1 = \text{Medium}, \tilde{W}^2 = \text{High}, \) and \( \tilde{W}^3 = \text{Medium High} \). They were asked to identify the desired criteria for evaluation of the identified projects by taking the concept of sustainability into consideration. These criteria are presented in Table 1.

The identification of fuel consumption reduction projects was conducted in four meetings with the DMs and 7 projects were identified as follow and hierarchical evaluating structure for this study is shown Figure 4:

- Using the equipment-wasted heat for heating of environment.
- Using IBMS.
- Optimal design of lighting systems.
- Using fibre optic cables to transmit sunlight.
- Using motorized dampers to control the input airflow.
- Using heating output of Heat Recovery Steam Generator (HRSG).
- Using solar cells.

Recall that the Per-C methodology has three steps: encoder, the CWW engine and decoder. Each of these three steps is described for the case study as follows:

![Figure 4. Hierarchical evaluating structure for projects selection.](image-url)

**Table 1. Criteria for evaluation Alternatives**

| Dimension of Sustainability | Criteria | Description |
|-----------------------------|----------|-------------|
| Economic                    | Time     | The cost of the project implementation. |
|                             | Cost     | The duration of project implementation. |
|                             | Risk     | The Risk of project implementation |
| Social                      | Organizational Readiness | Previous and common experiences of the organization in implementation of the project. |
| Environment                 | Resource consumption | The effect of the implementation of the project on Resource consumption. |

**Table 1** Criteria for evaluation Alternatives
4.1 Encoder
In this case, two codebooks are required to determine weights of criteria and evaluate rating of projects’ performance. For obtaining these codebooks, we used the Interval Approach (IA). Seven words were used in two codebooks, namely Very Low, Low, Medium Low, Medium, Medium High, High and Very High. These words should be obtained by collecting data from decision makers and their data intervals should be mapped into word FOUs using Interval Approach (IA) method that details can be found in 17. For this purpose, at first, the decision makers were asked to determine interval numbers for assigning to these words that are presented in Figures 5 and 6. After the answering of decision makers, we used IA method and finally the codebooks are calculated, these are and their FOUs are presented in Tables 2 and 3.

4.2 CWW
The importance of criteria represented as words were shown in Table 4. Every criterion has three weights that assigned by the three decision makers. Words are used for determining the weights of criteria are given in Table 2.

Three decision makers used the words presented in table 3 to evaluate the ratings of alternatives with respect to each criterion. The ratings of the seven alternatives

Table 2. Words and FOUs for determining weights

| Linguistic Terms | FOUs |
|------------------|------|
| Very Low (VL)    | [0.0,0.0,5.63;1] |
| Low (L)          | [0.9,1.46,1.60;0.41] |
| Medium Low (ML)  | [1.38,3.47,6.62;0.63] |
| Medium (M)       | [4.79,5.30,5.71;0.42] |
| Medium High (MH) | [6.19,6.76,7.21;0.37] |
| High (H)         | [7.69,8.21,8.91;0.49] |
| Very High (VH)   | [8.48,9.82,10.10;1] |

Table 3. Words and FOUs for evaluating alternatives

| Linguistic Terms | FOUs |
|------------------|------|
| Very Low (VL)    | [0.0,0.0,5.63;1] |
| Low (L)          | [2.15,2.40,2.60;0.42] |
| Medium Low (ML)  | [1.38,3.50,4.75;0.53] |
| Medium (M)       | [4.79,5.30,5.71;0.42] |
| Medium High (MH) | [6.19,6.76,7.21;0.37] |
| High (H)         | [7.69,8.21,8.91;0.49] |
| Very High (VH)   | [8.48,9.82,10.10;1] |

Table 4. Importance weight of Criteria from DMs

| Dimension of Sustainability | Criteria         | D₁ | D₂ | D₃ |
|-----------------------------|------------------|----|----|----|
| Economic                    | Time (C₁₁)       | H  | VH | VH |
|                             | Cost (C₁₂)       | VH | VH | H  |
|                             | Risk (C₁₃)       | H  | H  | H  |
| Social                      | Organizational Readiness (C₂₁) | MH | H  | MH |
|                             | Resource consumption (C₃₁) | VH | VH | VH |
by the decision makers under the various criteria are presented in Table 5.

The Linguistic Weighted Average (LWA) was used for the CWW Engine. According to Equation (28), after the application of this method for each project, $\tilde{Y}_{LWA}^k$ is obtained by decision makers. The results are presented in Table 6 to 8.

$$\tilde{Y}_{LWA}^k = \frac{\sum_{i=1}^{n} W_i \times \tilde{X}_i^k}{\sum_{i=1}^{n} W_i} \quad k = 1, 2, 3 \quad i = 1, ..., n$$ (28)

Where, $\tilde{Y}_{LWA}^k$ is the value of the alternative $A_i$ with respect to the major criterion of $C_{ij}$ provided by $D_k$ and $\tilde{W}_i$ is linguistic weight assigned by $D_k$ for criterion $C_{ij}$.

For computing Aggregation of the overall performance $Y_{LWA}^k$, According to Equation (29), $Y_{LWA}^k$ is obtained for every projects, that are presented in Table 9.

$$\tilde{Y}_{LWA} = \frac{\sum_{i=1}^{n} W_k \times \tilde{Y}_{LWA}^k}{\sum_{i=1}^{n} W_k} \quad i = 1, ..., k = 1, 2, 3$$ (29)

Where $W_k^k$ is the weight of $k$th decision maker and $\tilde{Y}_{LWA}^k$ is aggregation of the performance of each project.

4.3 Decoding

After computing Aggregation of the overall performance ($\tilde{Y}_{LWA}$), the projects should be ranked by using one of the methods mentioned in the Decoder. Among the methods mentioned in Decoding, we used Centroid-based ranking method by Equations (21–27) and the results are presented in Table 10 and Figure 6. Accordingly, priority projects are as follows:

$$P_5 > P_4 > P_6 > P_7 > P_3 > P_1 > P_2$$

5. Conclusion

Project selection is a decision making process with multiple criteria and often conflicting objectives. This paper presented a new solution using a CWW method based on the specific architecture of Perceptual Computer (Per-C) for selecting project. The proposed approach for project selection is the unique feature of the present study, which has not been reported in literature. The main findings are summarized as follows:

- In the classic methods for evaluating and selecting projects, only the economic criterion is used. According to the development of new concepts such as sustainability and its role in the development of enterprises, in this paper, the concept is used for determining the criteria in project selection. The result shows that the main criteria for selecting a project are Time, Cost, Risk, Organizational Readiness and Resource consumption.

- In MCDM problems, decision makers tend to use linguistic terms for expressing their opinions instead of pure numbers because of their different backgrounds and preferences, one of the main advantages of the approach is all the inputs to the project selection model which are words and all the uncertainties about the words are aggregated all through the calculations and reflected in the overall results; in consequence, any information is not missed. Even according to this method, the outputs can be words.
### Table 6. $\tilde{Y}_{LWA}^k$ for each projects

| Alternative | $D_1$ | $D_2$ | $D_3$ |
|-------------|-------|-------|-------|
| $P_1$       | ![Graph](image1) | ![Graph](image2) | ![Graph](image3) |
| $\tilde{Y}_{LWA}^k$ | $[(4.93,5.23,5.54,5.93;0.27)$ | $[(5.59,5.91,6.22,6.66;0.27)$ | $[(5.08,5.90,5.42,6.35;0.27)$ |
|             | $(2.40,4.50,6.18,8.07;1)]$   | $(2.94,5.04,6.82,8.65;1)]$   | $(2.41,4.67,6.39,8.24;1)]$   |
| $P_2$       | ![Graph](image4) | ![Graph](image5) | ![Graph](image6) |
| $\tilde{Y}_{LWA}^k$ | $[(5.52,5.81,6.18,6.58;0.27)$ | $[(4.89,5.24,5.75,6.25;0.27)$ | $[(4.44,4.75,4.95,5.31;0.27)$ |
|             | $(3.23,5.15,6.68,8.51;1)]$   | $(2.06,4.40,6.24,8.33;1)]$   | $(2.07,3.99,5.58,7.61;1)]$   |
| $P_3$       | ![Graph](image7) | ![Graph](image8) | ![Graph](image9) |
| $\tilde{Y}_{LWA}^k$ | $[(4.95,5.29,5.75,6.17;0.27)$ | $[(6.30,6.57,7.03,7.41;0.27)$ | $[(4.11,4.42,4.87,5.31;0.27)$ |
|             | $(2.37,4.58,6.25,8.16;1)]$   | $(4.22,6.94,7.38,8.92;1)]$   | $(1.71,3.65,5.37,7.55;1)]$   |

### Table 7. $\tilde{Y}_{LWA}^k$ for each projects

| Alternative | $D_1$ | $D_2$ | $D_3$ |
|-------------|-------|-------|-------|
| $P_4$       | ![Graph](image10) | ![Graph](image11) | ![Graph](image12) |
| $\tilde{Y}_{LWA}^k$ | $[(5.56,5.86,6.20,6.60;0.27)$ | $[(6.02,6.28,6.72,7.07;0.27)$ | $[(6.89,7.18,7.66,8.09;0.27)$ |
|             | $(3.13,5.12,6.75,8.54;1)]$   | $(4.11,5.82,7.04,8.58;1)]$   | $(4.86,6.52,7.98,9.35;1)]$   |
| $P_5$       | ![Graph](image13) | ![Graph](image14) | ![Graph](image15) |
| $\tilde{Y}_{LWA}^k$ | $[(6.59,6.84,7.42,7.73;0.27)$ | $[(6.50,6.97,7.44,7.90;0.37)$ | $[(6.21,6.60,7.13,7.68;0.3)$ |
|             | $(4.72,6.49,7.66,9.01;1)]$   | $(4.03,6.29,7.90,9.09;1)]$   | $(3.48,5.82,7.52,9.09;1)]$   |
| $P_6$       | ![Graph](image16) | ![Graph](image17) | ![Graph](image18) |
| $\tilde{Y}_{LWA}^k$ | $[(6.26,6.55,6.83,7.21;0.27)$ | $[(5.57,5.87,6.30,6.69;0.27)$ | $[(6.85,7.31,7.55,8.25;0.37)$ |
|             | $(3.97,5.80,7.37,8.92;1)]$   | $(3.32,5.29,6.73,8.56;1)]$   | $(3.79,6.31,8.07,9.53;1)]$   |
Table 8. $\tilde{Y}_{LWA}^k$ for each projects

| Alternative $i$ | $D_1$ | $D_2$ | $D_3$ |
|-----------------|-------|-------|-------|
| $P_7$           | ![Graph](image1) | ![Graph](image2) | ![Graph](image3) |
| $\tilde{Y}_{LWA}^k$ | $[(5.62,5.92,6.34,6.73;0.27)\ldots]$ | $[(4.80,5.15,5.55,6.03;0.27)\ldots]$ | $[(5.70,6.01,6.47,6.89;0.27)\ldots]$ |

Table 9. Aggregation of the overall performance $\tilde{Y}_{LWA}$

| $A_i$ | $\tilde{Y}_{LWA}$ | $A_i$ | $\tilde{Y}_{LWA}$ |
|-------|------------------|-------|------------------|
| $P_1$ | ![Graph](image4) | $P_2$ | ![Graph](image5) |
| $\tilde{Y}_{LWA}$ | $[(5.22,5.72,5.78,6.38;0.27)\ldots]$ | $\tilde{Y}_{LWA}$ | $[(4.81,5.21,5.60,6.06;0.27)\ldots]$ |
| $P_3$ | ![Graph](image6) | $P_4$ | ![Graph](image7) |
| $\tilde{Y}_{LWA}$ | $[(5.15,5.49,6.00,6.45;0.27)\ldots]$ | $\tilde{Y}_{LWA}$ | $[(6.15,6.45,6.91,7.32;0.27)\ldots]$ |
| $P_5$ | ![Graph](image8) | $P_6$ | ![Graph](image9) |
| $\tilde{Y}_{LWA}$ | $[(6.41,6.79,7.35,7.79;0.27)\ldots]$ | $\tilde{Y}_{LWA}$ | $[(6.13,6.47,6.93,7.38;0.27)\ldots]$ |
| $P_7$ | ![Graph](image10) | | |
| $\tilde{Y}_{LWA}$ | $[(5.27,5.62,6.05,6.52;0.27)\ldots]$ | | |
Results from using the proposed method show that the method can help in the decision-making process where uncertainty is strongly presented. It is recommended to use other decision making methods, including combined techniques, instead of the proposed method, and compare the obtained results with the findings of the proposed technique. The application of this approach on other decision making problems is also suggested.

We used MATLAB software to solve the problem. All basic codes are available in this website (http://sipi.usc.edu/~mendel/software).

6. Author Contributions

All the authors contributed equally to this work.

7. Conflicts of Interest

The authors declare no conflict of interest.

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