Dual Actions for Chiral Bosons\footnote{Talk given at the International Workshop “Supersymmetry and Integrable Systems” (June 22–26, 1998, Dubna, Russia) and at the Xth School–Seminar on Recent Problems in Theoretical and Mathematical Physics “VOLGA 10’98” (June 22–July 2, 1998, Kazan, Russia).}

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Abstract

We study duality properties of actions for chiral boson fields in various space–time dimensions using $D = 2$ and $D = 6$ cases as examples. As a result we get dual covariant formulations of chiral bosons.

1 Introduction

During the last few years the study of duality–symmetric and self–dual fields (or chiral bosons) attracted much attention because of the role they play in theoretical models related to superstring theories revealing various types of dualities of these theories. The fields of this kind appear on the worldvolumes of heterotic strings, M–theory five–branes and in several field–theoretical limits of M–theory and superstrings.

Chiral bosons are associated with differential $p$–forms $A^{(p)}$ in the $D = 2(p + 1)$–dimensional space–time, whose external differential $F^{(p+1)}(A) = dA^{(p)}$ is restricted by a self–duality condition

$$\mathcal{F}^{(p+1)} \equiv F^{(p+1)}(A) - * F^{(p+1)}(A) = 0; \quad \mathcal{F}^{(p+1)} = - * \mathcal{F}^{(p+1)} . \quad (1)$$

When the dimension of a Lorentzian space–time is twice odd the chiral form is real, while if it is twice even the chiral field is complex (or described by two real $p$–forms). For

1In what follows we use a metric of mostly positive signature.
instance, in the $D = 4$ case ($p = 1$) a field theory with two vector potentials whose field strengths are connected by the duality relation describes duality–symmetric Maxwell electrodynamics.

The first–order differential equation is an equation of motion which defines the dynamics of the chiral boson. This feature distinguishes chiral bosons from other bosonic fields whose equations of motion are usually second–order differential equations. Thus it seems natural to try to construct an action for chiral bosons in a first order form. In such a formulation eq. appears as the equation of motion of an auxiliary Lagrange multiplier field. It turns out that for the Lagrange multiplier itself not to carry propagating degrees of freedom one has to introduce an infinite number of auxiliary fields “compensating” the dynamics of each other.

An action of another kind (which will be the subject of our discussion) is quadratic in field strengths and contains only one auxiliary scalar field ensuring manifest Lorentz invariance of the construction. This formulation is a covariant generalization of non manifestly space–time invariant actions for chiral bosons and it has been used for the construction of the effective action for the $D = 11$ super–five–brane coupled to a duality–symmetric $D = 11$ supergravity, Type IIB supergravity and for some other models.

In this paper we discuss duality properties of the covariant chiral boson action in diverse space–time dimensions considering the $D = 2$ and $D = 6$ cases as examples. The results obtained for the six–dimensional chiral boson represent basic duality features inherent to this formulation in any space–time dimension.

2 Doubly Self–Dual Action In $D = 2$

We begin with the simplest two–dimensional free chiral boson model. The literature devoted to studying $D = 2$ chiral bosons and their quantization is very extensive, and we are able to refer the reader only to some of the papers where different approaches were addressed.

In the $D = 2$ case the chiral boson field is a scalar whose “field strength” satisfies the following on–shell self–duality condition

$$F_m(\phi) = \partial_m \phi - \epsilon_{mn} \partial^n \phi = 0, \quad F_m(\phi) = -\epsilon_{mn} F^n(\phi)$$

This condition can be obtained from the action

$$S = \int d^2 x \left[ -\frac{1}{2} F_m(\phi) F^m(\phi) + \frac{1}{2(\partial a)^2 (\partial^2 a)} (\partial^m a F_m(\phi))^2 \right],$$

where $a(x)$ is the auxiliary scalar field mentioned above. It enters the action in a non–polynomial way, and to avoid singularities, we require $\neq 0$. This condition reflects a non–trivial topological structure of this theory and is present in the formulations of this kind in any space–time dimension.

The action possesses the following set of local symmetries

$$\delta a = \varphi(x), \quad \delta \phi = \frac{\varphi}{(\partial a)^2} F^m(\phi) \partial^m a,$$

$$\delta \phi = f(a(x)),$$
with the parameters $\varphi(x)$ and $f(a)$, respectively. Note that the latter depends on $x$ only through the field $a(x)$ and there is no any first class constraint associated with this symmetry. The only first class constraint, which one finds in the Hamiltonian formulation of this model, generates the symmetry \[ (4) \] \[ (5) \]. The self–duality condition \[ (2) \] appears as a general solution to the $\phi$–field equations of motion, obtained from the action \[ (3) \], upon gauge fixing the symmetries \[ (4) \] and \[ (5) \].

Now let us study the duality properties of this action. There are two fields which can be dualized, $\phi(x)$ and $a(x)$.

To get the $\phi$–dual formulation, we replace \[ (3) \] with

$$S = \int d^2x \left[ -\frac{1}{2} F_m F^m + \frac{1}{2(\partial_r a)(\partial^r a)} (\partial^m a F_m)^2 + G^m (F_m - \partial_m \phi) \right].$$

In this action $F_m$ and $G_m$ are regarded as independent vector fields and $F_m \equiv F_m - \epsilon_{mn} F^n$. The classical equivalence of the actions \[ (3) \] and \[ (6) \] is evident. Varying \[ (3) \] with respect to the Lagrange multiplier $G_m$ we obtain

$$F_m = \partial_m \phi,$$

which yields \[ (3) \] when substituted into \[ (6) \]. The variation of the action \[ (3) \] with respect to $F_m$ regarded as another auxiliary field produces an expression for $G_m$ in terms of $F_m$ which, when substituted into \[ (3) \], results in a dual action. If we perform this procedure, we shall realize that the dual action obtained this way coincides with \[ (3) \], so the action \[ (3) \] is self–dual with respect to the dualization of the chiral boson field. Note that this situation happens for the free chiral boson model in any even space–time dimension and reflects the basic (self–duality) property of the chiral bosons. The reader may find an explicit proof of this self–duality of $D = 2$ and $D = 4$ chiral boson actions in \[ 15 \].

Consider now the properties of \[ (3) \] with respect to the dualization of the auxiliary scalar $a(x)$. In order to do this we replace \[ (3) \] with the following classically equivalent action

$$S = \int d^2x \left[ -\frac{1}{2} F_m(\phi) F^m(\phi) + \frac{1}{2u ur} (u^m F_m(\phi))^2 + v^m (u_m - \partial_m a) \right]$$

which contains the independent auxiliary fields $v^m$ and $u^m$. The equation of motion for the field $v^m$ is

$$u_m = \partial_m a.$$  \[ (8) \]

It reduces the model to the one described by \[ (3) \].

The variation of \[ (3) \] with respect to $u^m$ produces the constraint

$$v^m = \frac{1}{(u ur)^2} \epsilon_{mn} u_n (u^p F_p(\phi))^2,$$

from which it follows, in particular, that the normalized vectors $u^m$ and $v^m$ are dual to each other \[ (9) \].

$$\frac{u^m}{\sqrt{(u)^2}} = \epsilon_{mn} \frac{v_n}{\sqrt{-(v)^2}}.$$  \[ (10) \]

\[^2\text{Note that this relation holds only off the mass shell, i.e. when } F_m \text{ is non–zero and the relation } (6) \text{ is non–degenerate.}\]
Substituting (9) into (7), we get
\[ S = \int d^2x \left[ -\frac{1}{2} F_m(\phi) F^m(\phi) - \frac{1}{2 v^r v_\rho} (v^m F_m(\phi))^2 + a(\partial_m v^m) \right]. \] (11)

The equation of motion of the field \(a(x)\) allows us to express \(v^m\) as
\[ v^m = \epsilon^{mn} \partial_n b, \] (12)
where \(b(x)\) is a scalar field. Inserting this expression back into the action (11) and taking into account the anti–self–duality of \(F^m(\phi)\), we recover the action (3) with the field \(a(x)\) replaced by \(b(x)\). Thus, the action (3) describing the free two–dimensional chiral scalar is self–dual with respect to the dualization of the auxiliary scalar field \(a(x)\) as well.

3 Dual Actions in \(D = 6\)

The situation changes when we consider such a dualization in a space–time of higher dimension. Double self–duality which we observed in \(D = 2\) is due to the duality relation between scalars (8), (10) and (12), which hold only in \(D = 2\). In higher space–time dimensions scalars are dual to tensors of a rank \(D–2 > 0\) and therefore the field contents of the dual actions will have no chance to coincide anymore. In this Section we demonstrate this fact with the example of a free chiral boson in \(D = 6\).

A six–dimensional chiral boson field is a real antisymmetric field \(A_{mn}\), \((m, n = 0, \ldots, 5)\) whose field strength
\[ F_{mnp}(A) = \partial_m A_{np} + \partial_n A_{pm} + \partial_p A_{mn} \] (13)
is restricted (on the mass shell) by the self–duality condition
\[ F_{mnp}(A) = F_{mnp}(A) - \frac{1}{3!} \epsilon_{mnpqrs} F^{qrs}(A) = 0, \quad F_{mnp}(A) = -\frac{1}{3!} \epsilon_{mnpqrs} F^{qrs}(A). \] (14)

A \(D = 6\) analogue of the action (3) is [8]
\[ S = \int d^6x \left[ -\frac{1}{6} F_{mnp}(A) F^{mnp}(A) + \frac{1}{2(\partial_r a)(\partial^r a)} \partial^m a F_{mnp}(A) F^{npq}(A) \partial_q a \right], \] (15)

This action is invariant with respect to the following local transformations of the fields \(a\) and \(A^{mn}\) [8]:
\[ \delta a = \varphi(x), \quad \delta A_{mn} = \frac{\varphi}{2(\partial^r a)} F_{mnp}(A) \partial^p a; \] (16)
\[ \delta A_{mn} = \partial_{\langle m} \Phi_{n \rangle}; \] (17)
\[ \delta A_{mn} = \Psi_{\langle m} \partial_{n \rangle} a. \] (18)
Note that in contrast to the two–dimensional model (3) all the symmetries (16)–(18) are full–fledged local symmetries (i.e. in the Hamiltonian formulation of the model there is a first class constraint associated with each of these symmetries). Using these symmetries, we can reduce the general solution of the equations of motion of \(A_{mn}\) derived from (13) to the self–duality condition (14).
Consider now the duality properties of the action (15) with respect to the duality transform of the field \(a(x)\). In order to do this (as in the previous Section) we replace (15) by

\[
S = \int d^6x \left[ -\frac{1}{6} F_{mnp}(A) F^{mnp}(A) + \frac{1}{2u^r u_r} u^m F_{mnp}(A) F^{npq}(A) u_q + v^m (u_m - \partial_m a) \right],
\]

which is equivalent to (15) as a consequence of the equation of motion of the Lagrange multiplier \(v^m\). The variation of this action with respect to the field \(u^m\) gives an expression for \(v^m\) in terms of other fields

\[
v^m = -\frac{1}{u^2} F^{mnp}(A) F_{npq}(A) u^q + \frac{1}{(u^2)^2} u^m u^n F_{npq}(A) F^{pqr}(A) u_r,
\]

or, because of the anti–self–duality of \(F^{mnp}(A)\)

\[
v^m = \frac{1}{2(u^2)^2} \varepsilon^{mnpqr} u_t F_{nps} u^s F_{qrt} u^t
\]

from which it follows that, in particular, the vectors \(u^m\) and \(v^m\) are orthogonal

\[
u^m v_m = 0,
\]

and

\[
\frac{1}{2u^r u_r} u^m F_{mnp}(A) F^{npq}(A) u_q = -\frac{1}{2v^r v_r} v^m F_{mnp}(A) F^{npq}(A) v_q + 2 \tilde{F}^m \tilde{F}_m,
\]

where

\[
\tilde{F}_m \equiv \frac{1}{\sqrt{u^2 v^2}} F_{mnp} v^n u^p.
\]

Taking into account eq. (20) and the self–duality of \(F^{mnp}(A)\), one can show that the second term on the r.h.s. of (23) vanishes identically. To see this, replace one of \(v^n\) in \(\tilde{F}^m \tilde{F}_m\) with its expression (21), then

\[
\tilde{F}^m \tilde{F}_m = \frac{1}{2(u^2)^2 \sqrt{u^2 v^2}} \tilde{F}_m F^{mnp} u_p \varepsilon_{ntr1r2s1s2} u^r F^{r1r2s} u_s F^{s1s2} u_t.
\]

Now replace the second \(F^{mnp}\) on the right hand side of (25) with its antiselfdual and “eliminate” two epsilon–tensors. Eq. (25) takes the form

\[
\tilde{F}^m \tilde{F}_m = \frac{2}{(u^2)^2 \sqrt{u^2 v^2}} \tilde{F}_m F^{mnp} u_p F^{rst} u_s F_{nrs}.
\]

In virtue of (20) the last two \(F\) in (26) can be replaced with \(v^m\) which results in

\[
\tilde{F}^m \tilde{F}_m = -2 \tilde{F}^m \tilde{F}_m, \quad \Rightarrow \quad \tilde{F}^m \tilde{F}_m = 0.
\]

Substituting (23) with \(\tilde{F}^m \tilde{F}_m = 0\) into the action (19), we get

\[
S = \int d^6x \left[ -\frac{1}{6} F_{mnp}(A) F^{mnp}(A) - \frac{1}{2v^r v_r} v^m F_{mnp}(A) F^{npq}(A) v_q + a \partial_m v^m \right].
\]
Then, solving the dynamical equation of the field \( a(x) \) we express \( v^m(x) \) in terms of a 4-form field \( B_{mnpq} \):

\[
v^m = \epsilon^{mnpqrs} \partial_n B_{pqrs},
\]

and substituting this expression into (27) we obtain the dual action for the \( D = 6 \) chiral boson in the following form

\[
S = \int d^6x \left[ -\frac{1}{6} F_{mnp}(A) F^{mnp}(A) - \frac{1}{2 v^r v_r} v^m F_{mnp}(A) F^{npq}(A) v_q \right],
\]

(28)

\[
v^m = \epsilon^{mnpqrs} \partial_n B_{pqrs}.
\]

We see that, as in the \( D = 2 \) case, the vector fields \( u^m \) and \( v^m \) are dual to each other in a sense that \( u^m \) is the “field strength” of the scalar field \( a(x) \) and \( v^m \) is the dual “field strength” of the 4-form field \( B_{mnpq} \). Therefore, as we expected, the dual action (28) does not coincide with the initial one. Moreover it has a different symmetry structure. Eq. (28) is invariant under the following local transformations

\[
\delta B_{mnpq} = \partial_m C_{npq};
\]

\[
\delta B_{mnpq} = \epsilon_{mnpqrs} v^r \Lambda^s, \quad \delta A_{mn} = 2 \epsilon_{mnpqrs} F^{pqrs} v^r \Lambda^s,
\]

(29)

\[
\delta v^m = \Lambda^m \partial_n v^m - v^n \partial_n \Lambda^m + v^m \partial_n \Lambda^n,
\]

\((C_{mnp} \text{ and } \Lambda_m \text{ are the parameters})\), and under

\[
\delta A_{mn} = \frac{1}{\sqrt{v^2}} \epsilon_{mnpqrs} v^p \Phi^{qrs},
\]

(30)

where the parameter \( \Phi^{mnp} \) is restricted by the differential condition

\[
v^t \partial_t \left[ \frac{1}{\sqrt{v^2}} \epsilon_{mnpqrs} v^p \Phi^{qrs} \right] - (\partial_{[m} v^t) \frac{1}{\sqrt{v^2}} \epsilon_{n]} p q r s v^p \Phi^{qrs} = 0.
\]

(31)

Because of this restriction eq. (31) is not a conventional local symmetry and is an analogue of the symmetry (3) of the chiral scalar action (1). In order to see this consider (31) in the gauge

\[
\frac{v^m}{\sqrt{v^2}} = n^m, \quad n^m = \text{const},
\]

(32)

which we can impose using the symmetry (29). Now eq. (31) takes the following form

\[
n^t \partial_t [\epsilon_{mnpqrs} v^p \Phi^{qrs}(x)] = 0,
\]

which is solved in terms of an arbitrary 3–form depending on five independent arguments

\[
\Phi_{mnp} = \Phi_{mnp}(y), \quad y^m = x^m - n^m (n_p x^p),
\]

which are coordinates transversal to the vector \( n^m \). If we consider the symmetry (3) in the non–covariant gauge \( a = n^m \epsilon_{mn} x^n \), we will see that its parameter also depends on the coordinates transversal to the vector \( n^m \).

Using the symmetries (29) and (30) one can reduce the equations of motion of \( A_{mn} \)

\[
\partial_m \left( \frac{1}{v^2} v^m F^{npq}(A) v_q \right) = 0,
\]

(33)
which follow from (28), to the self–duality condition (14). Thus, the dual actions (13) and (28) both give a consistent description of the six–dimensional free chiral boson field.

It is instructive to note that in the gauge (32) the equations of motion (33) can be reduced to Maxwell–type equations for a self–dual 3–form field strength. To see this notice that the following identity holds for any \( F^{(3)} = dA^{(2)} \)

\[
\partial^m \epsilon_{mnpsqt} n^q F^{rst}(A)n_t = -\frac{1}{3} (n^m \partial_m) \epsilon_{npqrst} n^q F^{rst}(A).
\]

Using this identity one can rewrite eqs. (33) (with \( n^m \) satisfying (32)) in the form

\[
\partial_m T^{mnp} = 0,
\]

where

\[
T^{mnp} = 3n^{[m} F^{np]q}(A)n_q + \frac{1}{2} \epsilon^{mnpqs} n_q F^{rst}(A)n_t \equiv \frac{1}{6} \epsilon^{mnpqs} T_{qrs}
\]

is a self–dual combination of \( F^{mnp} \). We observe that the corresponding anti–self–dual combination of \( F^{mnp} \) does not enter the \( A_{mn} \)–field equations of motion and, hence, decouples from the classical degrees of freedom. This implies from a somewhat different point of view that the model under consideration indeed has the required properties to reproduce the self–duality condition (14) and describes the dynamics of a single chiral boson.

4 Dual Actions With External Sources

In this Section we discuss the problem of coupling chiral boson fields to external field sources. As in the previous Section we will use the \( D = 6 \) case as an example [21], [22].

If the \( D = 6 \) gauge field \( A_{mn} \) were not self–dual the action (15) would not contain the second term, and its minimal coupling to external field sources would be described solely by the standard term

\[
S_{int} = -\int d^6x j^{mn} A_{mn},
\]

where \( j^{mn} \) is a conserved charged current \( (\partial_m j^{mn} = 0) \). The conservation of the current ensures the gauge symmetry of the action under \( A_{mn} \rightarrow A_{mn} + \partial_m \varphi_n \).

In the case of the chiral field the situation with coupling becomes much more complicated. The interaction terms must be now compatible with the symmetry structure (16)–(18) of the free self–dual theory. To find this term notice that, because of self–duality, the equation of motion of the chiral field coincides with the Bianchi identities for its field strength. Therefore, in order to introduce sources while maintaining the self–duality condition (14), one should require them to possess equal “electric” and “magnetic” charges, that is to be dyons. This leads to a modification of the Bianchi identities which acquire non–zero right hand side. This means that the field strength is not simply the curl of the gauge field potential anymore and includes a so called “Dirac membrane” (a two–dimensional analogue of the Dirac string [23]) which accumulates on its surface singularities of the chiral gauge fields associated with the charged sources. The Dirac membrane is described by an antisymmetric tensor \( S_{mnp} \) which, by definition, satisfies the equation

\[
\partial_m S^{mnp} = j^{np}.
\]

Thus, in general, \( S_{mnp} \) is a nonlocal solution of (35) in terms of the current.
The chiral gauge field strength now gets modified as follows
\[
F_{mnp}(A) \rightarrow \hat{F}_{mnp} \equiv F_{mnp}(A) + \frac{1}{6} \epsilon_{mnpqrs} S^{qr}, \quad (36)
\]
and the action describing the coupling of the chiral boson to charged sources has the form
\[
S = \int d^6 x \left[ -\frac{1}{6} \hat{F}_{mnp}(A) \hat{F}^{mnp}(A) + \frac{1}{2} (\partial r O A) (\partial r A) \hat{F}_{mnp}(A) \hat{F}^{mpq}(A) \partial q a - j^{mn} A_{mn} \right]. \quad (37)
\]
It is invariant under (16)–(18) with the transformation laws modified according to (36). The equations of motion derived from this action are reduced to the modified self–duality condition [21, 22]
\[
\hat{F}_{mnp}(A) = 0; \quad (\hat{F}_{mnp}(A) = \hat{F}_{mnp}(A) - \frac{1}{3!} \epsilon_{mnpqrs} \hat{F}^{qr}(A)). \quad (38)
\]
Taking an exterior derivative of (38) and taking into account (35) we obtain the chiral field equations with sources
\[
\partial_m \hat{F}^{mnp} = j^{np}.
\]
Coupling of matter sources in the dual formulation based on the action (28) is carried out the same way as discussed above, or can be obtained from (37) by the duality transform of the scalar field.

5 Conclusion

We considered duality properties of the Lorentz–covariant actions for chiral bosons and showed that the duality transform of the auxiliary scalar field \(a(x)\) produces a dual covariant action which consistently describes self–dual gauge fields. In the \(D = 2\) case the formulation considered turns out to be self–dual with respect to this dualization [15]. In higher dimensions, however, it is not the case and the dual actions differ from each other. In \(D = 4\), for example, as was demonstrated in [15], such a duality transform allows one to connect two different non–covariant versions of the duality–symmetric Maxwell action [1]–[3].

We also discussed the problem of coupling the dual actions to external sources and showed that due to the dyonic nature of the latter the consistent coupling preserving the symmetry structure of the model should be non–local (as in formulations considered earlier [23, 21, 22]).

Better understanding the origin of the local symmetries of the duality–symmetric actions and of their topological structure can provide new information about the quantum properties of chiral boson fields and dualities which they are related to.

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