Superconductor in static gravitational, electric and magnetic fields with vortex lattice

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Abstract

We estimate the conjectured interaction between the Earth’s gravitational field and a superconductor immersed in external, static electric and magnetic field. The latter is close to the sample upper critical field and generates the presence of a vortex lattice. The proposed interaction could lead to multiple, measurable effects. First of all, a local affection of the gravitational field inside the superconductor could take place. Second, a new component of a generalized electric field parallel to the superconductor surface is generated inside the sample.

The analysis is performed by using the time-dependent Ginzburg–Landau theory combined with the gravito-Maxwell formalism. This approach leads us to analytic solutions of the problem, also providing the average values of the generated fields and corrections inside the sample. We will also study which are the physical parameters to optimize and, in turn, the most suitable materials to maximize the effect.

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1 Introduction

The possible interaction between superconductors and gravitational field is an intriguing field of research, providing an interesting connection between condensed matter systems and gravitational interaction, with beneficial effects both in theoretical and applied physics. The seminal paper [1] set the stage for a deeper analysis of the phenomenon, while, in the following years, a certain amount of scientific literature on the subject was produced [2–11]. The underlying idea behind this line of research is that, under certain conditions, a gravity/supercondensate interplay should exist, resulting in a slight affection of the local gravitational field through the interaction with suitable condensate systems. Finally, in 1992 Podkletnov and Nieminen proposed a laboratory experimental configuration to detect the conjectured mutual interplay [12, 13]. Due to the complexity of the experimental setting and the high costs involved, the experiment was then repeated only in simplified versions [14], without achieving conclusive results.

After the above experiments, a series of theoretical explanations have been proposed in the literature [15–18], including the first and most convincing interpretation [19, 20] given in terms of a coupling of the gravitational field with an unconventional state of matter (superfluid) in a quantum gravity framework. In all these works, the contribution of the superfluid is included in the energy momentum tensor, exploiting the formalism of general relativity. Unfortunately, this approach does not allow to extrapolate quantitative predictions to be put in relation with possible laboratory experiments: one is then led to consider alternative, phenomenological evidences to better understand the proposed interplay.

Parallel to DeWitt (and subsequent) studies about the coupling between supercondensates and gravity, other theoretical [21–25] and experimental [26–28] researches were conducted about generalized electric-type fields induced in (super)conductors by the presence of the Earth’s weak gravitational field. The main result of those studies was the introduction of a generalized electric-type field, characterized by an electrical component and a gravitational one, leading to detectable corrections to the free fall of charged particles. These results then led to the definition of a set of generalized, fundamental fields featuring both gravitational and electromagnetic components [29–34]. This gravito-Maxwell formalism, valid in the weak gravity regime, is particularly suitable for treating the behaviour of a superfluid immersed in the Earth’s gravitational field [35–38].

In this paper we again use the time-dependent Ginzburg Landau theory combined with the gravito-Maxwell formulation. With respect to our previous analyses, this time we also consider the presence of external static electric and magnetic fields. In particular, the magnetic field value is very close to the critical magnetic field of the superconductor, and its presence also determines the presence of a vortex lattice. These new ingredients can lead to an enhancement of the interaction with the gravitational field, once chosen appropriate sample parameters and geometry.

2 Gravito-Maxwell formalism

Here we consider a weak gravitational background, where the (nearly-flat) spacetime metric $g_{\mu \nu}$ is expressed as

$$g_{\mu \nu} \approx \eta_{\mu \nu} + h_{\mu \nu},$$

(1)
with \( \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \) and where \( h_{\mu\nu} \) is a small perturbation of the flat Minkowski spacetime\(^1\). If we now introduce the symmetric traceless tensor

\[
\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h,
\]

with \( h = h^\sigma \sigma \), it can be shown that the Einstein equations in the harmonic De Donder gauge \( \partial^\mu \bar{h}_{\mu\nu} \approx 0 \) can be rewritten, in linear approximation, as [33, 35–38]

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \partial^\rho g_{\mu\nu\rho} = 8\pi G T_{\mu\nu},
\]

having also defined the tensor

\[
g_{\mu\nu\rho} = \partial^\sigma [\bar{h}_{\rho\sigma}]_{[\mu} + \partial^\sigma \eta_{[\rho [\bar{h}_{\sigma]\sigma]}_{\mu]} \approx \partial^\sigma [\bar{h}_{\rho\sigma}]_{[\mu}.
\]

### 2.1 Gravitoe-Maxwell formulation

We then introduce the fields [29, 33]

\[
E_g = -\frac{1}{2} g_{00} = -\frac{1}{2} \partial_0 \bar{h}_{i0}, \quad A_g = \frac{1}{4} \bar{h}_{0i}, \quad B_g = \frac{1}{4} \varepsilon^{ijk} g_{0jk},
\]

for which we get, restoring physical units, the set of equations [29, 31–42]

\[
\nabla \cdot E_g = 4\pi G \rho_g, \quad \nabla \cdot B_g = 0,
\]

\[
\nabla \times E_g = -\frac{\partial B_g}{\partial t}, \quad \nabla \times B_g = 4\pi G \frac{1}{c^2} j_g + \frac{1}{c^2} \frac{\partial E_g}{\partial t},
\]

having defined the mass density \( \rho_g \equiv -T_{00} \) and the mass current density \( j_g \equiv T_{0i} \). The above equations have the same formal structure of the Maxwell equations, with \( E_g \) and \( B_g \) gravitoelectric and gravitomagnetic field, respectively.

### 2.2 Generalized fields and equations

Now we introduce generalized electric/magnetic fields, scalar and vector potentials, featuring both electromagnetic and gravitational contributions [29, 30, 33, 35]:

\[
E = E_e + \frac{m}{e} E_g, \quad B = B_e + \frac{m}{e} B_g, \quad \phi = \phi_e + \frac{m}{e} \phi_g, \quad A = A_e + \frac{m}{e} A_g,
\]

where \( m \) and \( e \) identify the mass and electronic charge, respectively. The generalized Maxwell equations for the new fields read [29, 33–38]:

\[
\nabla \cdot E = \left( \frac{1}{\varepsilon_0} + \frac{1}{\varepsilon_g} \right) \rho, \quad \nabla \cdot B = 0,
\]

\[
\nabla \times E = -\frac{\partial B}{\partial t}, \quad \nabla \times B = (\mu_0 + \mu_g) j + \frac{1}{c^2} \frac{\partial E}{\partial t},
\]

where \( \varepsilon_0 \) and \( \mu_0 \) are the vacuum electric permittivity and magnetic permeability. In the above equations, \( \rho \) and \( j \) identify the electric charge density and electric current density, respectively.

\(^1\) here we work in the ‘mostly plus’ convention and natural units \( c = \hbar = 1 \)
while the mass density and the mass current density vector can be expressed in terms of the latter as

$$\rho_g = \frac{m}{e} \rho, \quad j_g = \frac{m}{e} j,$$

(9)

while the vacuum gravitational permittivity $\varepsilon_g$ and permeability $\mu_g$ read

$$\varepsilon_g = \frac{1}{4\pi G m^2}, \quad \mu_g = \frac{4\pi G m^2}{e^2}.$$

(10)

3 The model

3.1 Time-dependent Ginzburg–Landau formulation

The time-dependent Ginzburg–Landau equations (TDGL) can be written as [43–50]:

$$\frac{\hbar^2}{2m^*} \left( i \nabla + \frac{2e}{\hbar} A \right)^2 \psi - a \psi + b |\psi|^2 \psi = - \frac{\hbar^2}{2m^*} D \left( \frac{\partial}{\partial t} + \frac{2i e}{\hbar} \phi \right) \psi,$$

(11.i)

$$\nabla \times \nabla \times A - \nabla \times B_0 = \mu_0 (j_n + j_s),$$

(11.ii)

where $B_0$ is the external magnetic field, $D$ is the diffusion coefficient, $\sigma$ the conductivity in the normal phase and

$$a = a(T) = a_0 (T - T_c), \quad b = b(T_c),$$

(12)

$a_0, b$ being positive constants. The contributions related to the normal current and supercurrent densities can be explicitly written as

$$j_n = -\sigma \left( \frac{\partial A}{\partial t} + \nabla \phi \right),$$

$$j_s = -i \hbar \frac{e}{m^*} \left( \psi^* \nabla \psi - \psi \nabla \psi^* \right) - \frac{4e^2}{m^*} |\psi|^2 A.$$

(13)

Let us put ourselves in the London gauge $\nabla \cdot A = 0$ so that

$$\nabla^2 A = -\mu_0 (j_n + j_s).$$

(14)

We now consider a sample of thickness $L$ along the (vertical) $\vec{u}_z$ direction and very large dimensions along $\vec{u}_y, \vec{u}_z$, see Fig. 1. The sample features a square lattice of vortices, whose axes are directed along the direction of the static magnetic field $B_0$ that we choose to be

$$B_0 = B_0 \vec{u}_z,$$

(15)

together with a vector potential $A$ of the form

$$A = B_0 x \vec{u}_y.$$

(16)
We then consider the presence of a constant external (standard) electric field \( \mathbf{E}^{(e)}_0 \) along the \( \vec{u}_x \) direction, so that a *generalized* static field \( \mathbf{E}_0 \) can be expressed as

\[
\mathbf{E}_0 = \mathbf{E}^{(e)}_0 + \mathbf{E}^{(g)}_0 = \left( E^{(e)}_0 - E^{(g)}_0 \right) \vec{u}_x = \left( E^{(e)}_0 - \frac{m}{e} g \right) \vec{u}_x = E_0 \vec{u}_x ,
\]

with a related scalar potential \( \phi_0 = -E_0 x \).

The boundary and initial conditions are

\[
\begin{aligned}
\left( i \nabla \psi + \frac{2e}{\hbar} \mathbf{A} \psi \right) \cdot \mathbf{n} &= 0 \quad \text{on } \partial \Omega \times (0, t), \\
\nabla \times \mathbf{A} \cdot \mathbf{n} &= B_0 \cdot \mathbf{n} \\
\mathbf{A} \cdot \mathbf{n} &= 0 \\
\psi(x, 0) &= \psi_0(x) \\
\mathbf{A}(x, 0) &= \mathbf{A}_0(x)
\end{aligned}
\]

on \( \Omega \),

where \( \partial \Omega \) is the boundary of a smooth and simply connected domain in \( \mathbb{R}^N \). We denote by \( \mathbf{A}_0 \) the external vector potential, coinciding with the internal value for \( t \leq 0 \), i.e. when the sample is in the normal state and the material is very weakly diamagnetic. For \( t < 0 \) we also have \( T < T_c \) and \( B > B_c^2 \), while at \( t = 0 \) we still have \( T < T_c \) but \( B \approx B_c^2 \).

If we put ourselves very close to \( B_{c2} \), it is possible to find a solution of the linearized TDGL equations [51, 52] for the order parameter of the form

\[
\psi(x, y, t) = \sum_{n=-\infty}^{\infty} c_n \exp \left( i q n \left( y + \frac{E_0}{B_0} t \right) \right) \exp \left( -\frac{1}{2\xi(T)} \left( x - \frac{\hbar q n}{2e B_0} \right)^2 + i \frac{e E_0 \xi^2(T)}{\hbar D} \left( x - \frac{\hbar q n}{2e B_0} \right) \right).
\]

Since we are very close to \( B_{c2} \), this solution of the linearized TDGL equations describes the behaviour of an ordered vortex lattice, moving under the influence of the external electric field\(^2\,^3\). For a square lattice, \( q \) expresses the distance between adjacent vortices [54]

\[
q \simeq \frac{2\pi}{\xi(T)} ,
\]

and we can also replace the general \( c_n \) coefficient with the correspondent \( c_n \) expression for the square lattice:

\[
c_n \rightarrow c_n = \frac{2\sqrt{2\pi}}{\xi^2(T)} ,
\]

the \( c_n \) coefficients being then independent of \( n \).

\(^2\) we should also note that, although this is an exact solution just below \( B_{c2} \), this formula does not necessarily hold for different values of the magnetic field, such as the lower critical field \( B_{c1} \); indeed, close to \( B_{c2} \) the vortices are very close to each other, at distances of the order the coherence length \( \xi(T) \) (they are so densely packed that their cores are essentially touching [53]);

\(^3\) the motion of the vortices under the influence of the external electric field causes dissipative phenomena even in the superconducting state; one way to avoid the effect is to add defects to anchor the vortices (vortex pinning), in order to reduce or eliminate energy dissipation [51]
Dimensionless TDGL. In order to write eqs. (11) in a dimensionless form, the following expressions are introduced:

\[ \psi_0^2(T) = \frac{|a(T)|}{b}, \quad \xi(T) = \frac{\hbar}{\sqrt{2m_\star |a(T)|}}, \quad \lambda(T) = \frac{b m_\star}{4\mu_0 |a(T)| e^2}, \quad \kappa = \frac{\lambda(T)}{\xi(T)}, \]

\[ \tau(T) = \frac{\lambda^2(T)}{D}, \quad \eta = \mu_0 \sigma D, \quad B_c(T) = \frac{\mu_0 |a(T)|^2}{b} = \frac{\hbar}{2\sqrt{2} e \lambda(T) \xi(T)}, \]

where \( \lambda(T), \xi(T) \) and \( B_c(T) \) are the penetration depth, coherence length and thermodynamic critical field, respectively. We also define the dimensionless quantities

\[ \tau' = \frac{t}{\tau}, \quad x' = \frac{x}{\lambda}, \quad y' = \frac{y}{\lambda}, \quad x'_0 = \frac{x_0}{\lambda}, \quad \psi' = \frac{\psi}{\psi_0}, \]

and the dimensionless potentials, fields and currents can be expressed as:

\[ A' = \frac{A \kappa}{\sqrt{2} B_c \lambda}, \quad \phi' = \frac{\phi \kappa}{\sqrt{2} B_c D}, \quad E' = \frac{E \kappa}{\sqrt{2} B_c D}, \quad B' = \frac{B \kappa}{\sqrt{2} B_c}, \quad j' = \frac{1}{\mu_0} \frac{\lambda \kappa}{\sqrt{2} B_c}. \]

We now consider the dimensionless version of the London gauge, \( \nabla' \cdot A' = 0 \). Inserting eqs. (23) and (24) in eqs. (11), provides the dimensionless TDGL equations (we drop the primes for the sake of notational simplicity) in a bounded, smooth and simply connected domain in \( \mathbb{R}^N \) [43, 44]:

\[ \frac{\partial \psi}{\partial t} + i \phi \psi + \kappa^2 \left( |\psi|^2 - 1 \right) \psi + (i \nabla + A)^2 \psi = 0, \]

\[ \nabla \times \nabla \times A - \nabla \times B_0 = j_n + j_s = -\eta \left( \frac{\partial A}{\partial t} + \nabla \phi \right) - \frac{i}{2} (\psi^* \nabla \psi - \psi \nabla \psi^*) - |\psi|^2 A, \]

and the boundary and initial conditions (18) become, in the dimensionless form

\[ \begin{aligned}
(i \nabla \psi + A \psi) \cdot n &= 0 \quad \text{on } \partial \Omega \times (0, t); \\
\nabla \times A \cdot n &= B_0 \cdot n \\
A \cdot n &= 0 
\end{aligned} \]

\[ \left\{ \begin{array}{ll}
\psi(x, 0) &= \psi_0(x) \\
A(x, 0) &= A_0(x) 
\end{array} \right\} \quad \text{on } \Omega. \]

3.2 Solving dimensionless TDGL

We now discuss how to get an analytic approximate solution to the above dimensionless TDGL.

First of all, we write the following first-order expression for the dimensionless order parameter:

\[ \psi(x, y, t) = \sum_{n=-\infty}^{\infty} |c_n| \exp \left( i q n \left( y + \frac{E_0}{B_0} t \right) \right) \exp \left( -\frac{\kappa^2}{2} (x - n x_0)^2 + i \frac{E_0}{\kappa} (x - n x_0) \right), \]

with

\[ |\psi|^2 = \sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left( -\kappa^2 (x - n x_0)^2 \right). \]
The equations for the vector potential components can be written as

\[
\frac{\partial^2 A_x(x,t)}{\partial x^2} = \eta \left( \frac{\partial A_x(x,t)}{\partial t} - E_0 \right) + \left( A_x(x,t) - \frac{E_0}{\kappa} \right) \sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left( -\kappa^2(x-nx_0)^2 \right),
\]

\[
\frac{\partial^2 A_y(x,t)}{\partial x^2} = \eta \frac{\partial A_y(x,t)}{\partial t} + \sum_{n=-\infty}^{\infty} \left( A_y(x,t) - 2\pi \kappa n \right) |c_n|^2 \exp \left( -\kappa^2(x-nx_0)^2 \right),
\]

\[
\frac{\partial^2 A_z(x,t)}{\partial x^2} = \eta \frac{\partial A_z(x,t)}{\partial t} + \sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left( -\kappa^2(x-nx_0)^2 \right).
\]

(29)

We now want to obtain explicit solutions for the dimensionless order parameter at first order in \(E_0\). To this end, we have to estimate the summations

\[
\sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left( -\kappa^2(x-nx_0)^2 \right) \quad \text{and} \quad \sum_{n=-\infty}^{\infty} n |c_n|^2 \exp \left( -\kappa^2(x-nx_0)^2 \right).
\]

(30)

Since we are considering a square vortex lattice, we can replace the general coefficients \(c_n\) with \(c_0\) that, for the case under consideration, can be expressed as [54]

\[
c_0^2 = 2 \sqrt{2\pi} \kappa^2,
\]

(31)

being then a function of \(\kappa\) only. Moreover, we are interested in high-\(T_c\) superconductors, so that we also have large values for the \(\kappa\) parameter, \(\kappa^2 \gtrsim 10^4\). This gives us the following estimates for the above (30):

\[
\sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left( -\kappa^2(x-nx_0)^2 \right) 
\approx c_0^2 e^{-\kappa^2 x^2} \sum_{n=-\infty}^{\infty} e^{-\kappa^2 n^2 x_0^2} e^{2\pi i nx_0} \approx c_0^2 e^{-\kappa^2 x^2},
\]

(32)

\[
\sum_{n=-\infty}^{\infty} n |c_n|^2 \exp \left( -\kappa^2(x-nx_0)^2 \right) \approx 0,
\]

where only the \(n = 0\) term gives a non negligible contribution in the first summation. Equations (29) can be rewritten

\[
\frac{\partial^2 A_x(x,t)}{\partial x^2} = \eta \left( \frac{\partial A_x(x,t)}{\partial t} - E_0 \right) + \left( A_x(x,t) - \frac{E_0}{\kappa} \right) c_0^2 e^{-\kappa^2 x^2},
\]

\[
\frac{\partial^2 A_y(x,t)}{\partial x^2} = \eta \frac{\partial A_y(x,t)}{\partial t} + A_y(x,t) c_0^2 e^{-\kappa^2 x^2},
\]

(33)

\[
\frac{\partial^2 A_z(x,t)}{\partial x^2} = \eta \frac{\partial A_z(x,t)}{\partial t} + c_0^2 e^{-\kappa^2 x^2}.
\]

Since we are dealing with materials featuring a very large value for the \(\kappa\) parameter, \(\kappa^2 \gtrsim 10^4\), we can also use the approximation

\[
e^{-\kappa^2 x^2} \approx \frac{\sqrt{\pi}}{\kappa} \delta(x),
\]

(34)
and our equation for the vector potential components become

\[
\frac{\partial A_x(x, t)}{\partial t} \simeq \frac{1}{\eta} \frac{\partial^2 A_x(x, t)}{\partial x^2} - \left( A_x(x, t) - \frac{E_0}{\kappa} \right) c^2 \frac{\sqrt{\pi}}{\eta \kappa} \delta(x) + E_0, \tag{35.i}
\]

\[
\frac{\partial A_y(x, t)}{\partial t} \simeq \frac{1}{\eta} \frac{\partial^2 A_y(x, t)}{\partial x^2} - A_y(x, t) c^2 \frac{\sqrt{\pi}}{\eta \kappa} \delta(x), \tag{35.ii}
\]

\[
\frac{\partial A_z(x, t)}{\partial t} \simeq \frac{1}{\eta} \frac{\partial^2 A_z(x, t)}{\partial x^2} - c^2 \frac{\sqrt{\pi}}{\eta \kappa} \delta(x). \tag{35.iii}
\]

The initial conditions for the vector potential components are:

\[ A_x(x, 0) = 0, \quad A_y(x, 0) = B_0 x, \quad A_z(x, 0) = 0, \tag{36} \]

and the generalized electric field \(E\) inside the superconductor can be obtained from

\[
E = -\frac{\partial A}{\partial t} - \nabla \phi. \tag{37}
\]

**Averaging over space.** Let us now study the average effects of the generalized electric field inside the superconductor. To this end, we integrate the vector potential components (35) over the \(x\)-variable [55].

First, we integrate over \(x\) eq. (35.iii), for an interval \(x \in [-L/2, L/2]\). This gives

\[
\frac{\partial \bar{A}_z(t)}{\partial t} = -c^2 \frac{\sqrt{\pi}}{\eta \kappa L}, \tag{38}
\]

having defined the averaged component \(\bar{A}_z(t) = \frac{1}{L} \int_{-L/2}^{L/2} dx A_z(x, t)\) and having used symmetric conditions for the first derivatives with respect to \(x\). The above equation is easily solved as

\[
\bar{A}_z(t) = -c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t + \bar{A}_z(0) = -c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t, \tag{39}
\]

having used initial conditions (36) to set \(\bar{A}_z(0) = 0\). This in turn gives for the averaged, generalized electric field \(\bar{E}_z\) component

\[
\bar{E}_z = c^2 \frac{\sqrt{\pi}}{\eta \kappa L} = \frac{2\sqrt{2} \pi \kappa}{\eta L}. \tag{40}
\]

The averaged differential equation for the \(\bar{A}_y(t)\) component is obtained from (35.ii) and reads

\[
\frac{\partial \bar{A}_y(t)}{\partial t} = -\bar{A}_y(t) c^2 \frac{\sqrt{\pi}}{\eta \kappa L}, \tag{41}
\]

having used the approximation \(A_y(0, t) \approx \bar{A}_y(t)\). The above equation would give for the averaged

\[^{4}\text{we emphasize that we have dropped the primes for the sake of notational simplicity, so here both } L \text{ and } x \text{ correspond to the dimensionless } L' \text{ and } x' \text{ of (23); in particular, here one would explicitly have for the dimensionless thickness } L' = L/\lambda, \text{ } L \text{ being the physical thickness and } \lambda \text{ the penetration depth.}\]
component
\[ A_y(t) = A_y(0) \exp \left( -c_0^2 \frac{\sqrt{\pi}}{\eta \kappa L} t \right) = 0, \]
(42)

using again initial condition (36), the resulting field \( E_y(t) \) being then zero.

Finally, for the vertical component we have from (35.1)
\[ \frac{\partial \bar{A}_x(t)}{\partial t} = -\left( \frac{\bar{A}_x(t)}{L} - \frac{E_0}{\kappa} \right) c_0^2 \frac{\sqrt{\pi}}{\eta \kappa} + E_0, \]
(43)

that is solved, always using the approximation \( A_x(0, t) \approx \bar{A}_x(t) \), for
\[ \bar{A}_x(t) = \bar{A}_x(0) \exp \left( -c_0^2 \frac{\sqrt{\pi}}{\eta \kappa L} t \right) + E_0 \left( \frac{L}{\kappa} + \frac{\eta \kappa L}{c_0^2 \sqrt{\pi}} \right) \left( 1 - \exp \left( -c_0^2 \frac{\sqrt{\pi}}{\eta \kappa L} t \right) \right) = \]
(44)

having again used conditions (36). We finally get for the averaged \( E_x(t) \) component of the generalized electric field
\[ \bar{E}_x(t) = E_0 - E_0 \left( \frac{L}{\kappa} + \frac{\eta L}{2 \sqrt{2 \pi \kappa}} \right) \frac{2 \sqrt{2 \pi \kappa}}{\eta L} \exp \left( -\frac{2 \sqrt{2 \pi \kappa}}{\eta L} t \right). \]
(45)

4 Discussion

The solution of the differential equations for the vector potential gives rise to two very interesting situations. The first one is connected with a non-zero value of the generalized \( E_z \) component, determining the emergence of a new, detectable contribution along the direction of the magnetic field. The value of this new electric field is expressed, in dimensional units, as
\[ E_z = \frac{4 \pi B_c(T) D}{\eta L}. \]
(46)

For an YBCO sample of thickness \( L = 10 \text{ cm} \) at a temperature \( T = 85 \text{ K} \), this would correspond to a value \( E_z \approx 10 \text{ V/m} \).

The second remarkable effect is related to the expected variation of the gravitational field along the \( x \) direction inside the superconductor. In fact, it is possible to see from eq. (45) and from Fig. 2 that the field is reduced with respect to its external value. Moreover, in analogy to what we found in [35], for very short time scales (~ \( 10^{-9} \text{s} \)), the gravitational field seems to change sign: this could only happen in absence of suitable physical cutoffs preventing arbitrary growth of instabilities, giving rise to negative values [19]. It can also be noted that it is possible to find observable effects of gravitational field affection in different time scales; in principle, when dealing with samples of larger thickness and at temperatures very close to \( T_c \), larger observation times could be achieved.

In order to measure the effect it is necessary to determine suitable sample dimensions and chemical composition. In fact, large dimensions of the sample give rise to an increase for the time scales in which the effect occurs, combined with a decreasing intensity of the phenomenon. At the
same time, large values of the $\eta$ parameter (related to the sample characteristics) determines similar effect, that is, a reduced intensity of the correction together with an increase of the observation times. In this regard, the $L$ and $\eta$ parameters act in the same way: disordered materials (small $\eta$ values, i.e. bad conductors in the normal state) with negligible thickness dimension (small $L$ values) give rise to larger effects for a shorter time\textsuperscript{5}. On the contrary, large dimensions materials with good conductivity in the normal state (large $\eta$), would exhibit a weaker effect for a longer time scale. All this observations are in agreement with what we found in [35].

In Table 1 we report typical values for the physical parameters of two common high-$T_c$ cuprates, YBCO and BSCCO [33, 56–58]. We also point out that the presence of disorder also determines an increase of the $\lambda$ penetration depth and, being the square of the latter proportional to the time scale ($\tau \propto \lambda^2$), the duration of the phenomenon increases with beneficial effects for direct measurements. Finally, if we put ourselves at temperatures very close to $T_c$, we again find an increase in the $\lambda$ values together with larger time scales; in the latter case, however, the effects of thermal fluctuations should also be considered [59].

We expect that experimental issues would reside in the very short observation time. Since even detecting a reduced effect would be a remarkable result, it would be better to privilege long time scales setup, rather than configurations leading to larger effects of affection of the gravitational field. The latter would in fact imply strong difficulties in the measurements, since they would manifest themselves only for very short time scales. In light of these considerations, we suggest to consider big samples featuring large $\eta$ values.

5 Concluding remarks

A deeper interweaving between condensed matter and gravitational theories has already proven to be a powerful tool for inspecting many aspects of fundamental physics [60–68], while also providing new insights into related unresolved issues\textsuperscript{6}.

In this paper we have exploited a multidisciplinary approach to describe a gravity/superfluid interplay, in a specific physical setup involving the presence of external electric and magnetic fields, which in turn determine the presence of a vortex lattice. This situation could induce enhanced effects in the proposed interplay, leading to a non-negligible local affection of the gravitational field within the condensate, together with a predicted electric field generated inside the sample, parallel to the superconductor plane. The experimental verification of the emergence of this new component would result in a great step forward in the study of the interaction of the gravitational field with quantum condensates, opening new and unsuspected horizons both in the theoretical and applicative fields.

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\textsuperscript{5} this would be the case, for example, of superconducting films with high disorder

\textsuperscript{6} see for example ‘analogue gravity’ techniques exploiting a bottom-up formulation for condensed matter systems featuring analogues of gravitational effects [69–80], or top-down holographic approaches, where a substrate description comes from the geometric formulation of a suitable gravitational model [81, 82].
Fig. 1: Physical setup.

Fig. 2: Variation of gravitational field as a function of time inside superconductive samples of YBCO (black, red and green solid line, $T = 85\,\text{K}$) and BSCCO (dark blue, blue and orange solid line, $T = 102\,\text{K}$) for different thickness values ($L = 0.1\,\text{cm}$, $L = 1\,\text{cm}$, $L = 10\,\text{cm}$). In the inset, the same results are plotted in a different scale (same axes labels), to better appreciate the variations at smaller times.
|                | YBCO       | BSCCO      |
|----------------|------------|------------|
| $T_c$          | 89 K       | 107 K      |
| $T_\star$      | 85 K       | 102 K      |
| $\xi(T_\star)$ | $8.49 \cdot 10^{-9}$ m | $4.63 \cdot 10^{-9}$ m |
| $\lambda(T_\star)$ | $8.02 \cdot 10^{-7}$ m | $1.11 \cdot 10^{-6}$ m |
| $\sigma^{-1}$  | $4.0 \cdot 10^{-7}$ $\Omega$ m $^{(*)}$ | $3.6 \cdot 10^{-6}$ $\Omega$ m $^{(**)}$ |
| $B_{c2}(0)$    | 61 Tesla   | 113 Tesla  |
| $B_{c2}(T_\star)$ | 6 Tesla    | 11 Tesla   |
| $B_c(T_\star)$ | 0.25 Tesla | 0.32 Tesla |
| $\lambda(0)$   | $1.7 \cdot 10^{-7}$ m | $2.4 \cdot 10^{-7}$ m |
| $\xi(0)$       | $1.8 \cdot 10^{-9}$ m | $10^{-9}$ m |
| $\kappa$       | 94.4       | 240.0      |
| $\tau(T_\star)$ | $2.01 \cdot 10^{-9}$ s | $1.23 \cdot 10^{-9}$ s |
| $\eta$         | $1.0 \cdot 10^{-3}$     | $3.5 \cdot 10^{-4}$     |
| $D$            | $3.2 \cdot 10^{-4}$ $m^2/s$ | $10^{-3}$ $m^2/s$ |

$^{(*)}$ $T = 90$ K  
$^{(**)}$ $T = 108$ K

Table 1: YBCO vs. BSCCO [33, 56–58]
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