Strings with a confining core in a Quark-Gluon Plasma

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Abstract. We consider the intersection of N different interfaces interpolating between different $Z_N$ vacua of an SU(N) gauge theory using the Polyakov loop order parameter. Topological arguments show that at such a string-like junction, the order parameter should vanish, implying that the core of this string is in the confining phase. Using the effective potential for the Polyakov loop proposed by Pisarski for QCD, we use numerical minimization technique and estimate the energy per unit length of the core of this string to be about 2.7 GeV/fm at a temperature about twice the critical temperature. For the parameters used, the interface tension is obtained to be about 7 GeV/fm$^2$. Lattice simulation of pure gauge theories should be able to investigate properties of these strings. With the interpretation that quark contributions lead to explicit breaking of this $Z_N$ symmetry, such QGP strings may play important role in the evolution of the quark-gluon plasma phase and in the dynamics of quark-hadron transition.

1. The Polyakov loop model
For pure SU(N) gauge theory, an order parameter for the confinement-deconfinement phase transition is the Polyakov loop $l(x)$ which is defined as [1],

$$l(x) = \frac{1}{N} tr \left( P exp \left( ig \int_0^\beta A_0(x, \tau) d\tau \right) \right).$$

(1)

For temperatures above $T_c$, non-zero value of the expectation value of $l(x)$ ($\equiv l_0$ (say)) breaks the $Z(N)$ symmetry spontaneously and gives rise to N degenerate vacua. For the case of QCD with $N = 3$, effective Lagrangian density is written as [2],

$$L = \frac{N}{g^2} \left| \partial_\mu l \right|^2 T^2 - V(l).$$

(2)

$$V(l) = \left( - \frac{b_2}{2} |l|^2 - \frac{b_3}{6} (l^3 + (l^*)^3) + \frac{1}{4} |l|^2 \right) b_4 T^4.$$  

(3)

Values of various parameters [3, 4] in Eq.(2) and Eq.(3) are fixed by making correspondence to lattice results. The potential in Eq.(3) gives rise to $Z(3)$ degenerate vacua for $T > T_c$. The $Z(3)$ interface solution will correspond to a planar solution (say, in x-y plane) where $l$ starts at one of the minimum of $V(l)$ at $z = -\infty$ and ends up at the other minimum of $V(l)$ at $z = +\infty$. 

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Consider now the junction of all three different Z(3) interfaces, say with the line like junction being along the z axis. The interfaces then will be perpendicular to the x-y plane. It is immediately clear that as one considers a closed loop in the physical space encircling the z axis, then one encircles \( l = 0 \) point in the complex \( l \) plane. It is then obvious from the continuity of \( l \) that \( l \) must vanish along the z axis. These are topological strings (We call it QGP string to distinguish from the QCD string.). Potential \( V(l) \) in Eq.(3) is shown below. An important thing
to note from Fig.1,2, is that the barrier height between different Z(3) vacua is much higher than the barrier height between the metastable vacuum at \( l = 0 \) and the true vacuum. This situation is in complete contrast to the standard axionic models where the central bump is higher than the barrier between different Z(N) vacua. The solution for kink solitons are well known in such axionic models. Such a solution cannot be found here, as the height of barrier between the Z(3) vacua is much higher than the central bump. Fig.3 shows the surface plot of the potential \( V(l) \) in Eq.(3) in the complex \( l \) plane for \( T = 400 MeV \). From Fig.3, it may give the impression that the Z(3) interface will have \( l = 0 \) in the middle of the interface. However, as we discuss below, the actual domain wall cannot have \( l = 0 \) inside it.

2. Numerical techniques and the Z(3) interface profile
For a planar wall in the x-y plane we can write down the field equation as,

\[
\frac{d^2 l_i}{dz^2} = \frac{g^2}{N T^2} \frac{\partial V(l_1, l_2)}{\partial l_i}, \quad (l = l_1 + i l_2).
\] (4)

Using generalization of standard techniques [5], we interpret \( z \) as time, and \( l_1 \) and \( l_2 \) as the two dimensional position space for a particle which is moving under the influence of potential
\[ V(l_1, l_2) \left( g^2 \right) \]. Domain wall solution will then correspond to the particle trajectory which for \( z \to -\infty \) approaches one of the minima of \( V(l) \), while for \( z \to +\infty \) it approaches a different minima of \( V(l) \). Fig.4 shows the plot of the inverted potential, i.e. \( -V(l_1, l_2) \). Minima of \( V \) now become maxima of \( -V \). As we mentioned above, the domain wall solution will correspond to the particle trajectory starting at the top of one of the hills, say, at \( P \), and ending at the top of another hill, say at \( Q \). With this picture it becomes immediately obvious that the domain wall solution cannot go through \( l = 0 \). The particle starting at \( P \), and rolling down to \( l = 0 \) cannot turn back to end up at \( Q \). It will rather go to the other side and roll away downward, as shown by the dashed curve in Fig.4. To end up at \( Q \), the trajectory must loop back before reaching \( l = 0 \), as shown by the solid curve in Fig.4. Thus, \( l \) must remain non-zero inside the domain wall. However, in contrast to the real scalar field case, it is very difficult to numerically solve Eq.(4) to get this "generalized" bounce solution. In the absence of such a technique for complex \( l(x) \), we resort to numerical minimization [3] of the energy to determine the appropriate solutions. For time independent case, the energy density from Eq.(2),(3) is,

\[ \mathcal{E} = \frac{NT^2}{g^2} |\nabla l|^2 + V(l). \]  

(5)

For the energy minimization (see ref.[3] for detailed discussion), we have used a code as was used in ref.[6]. To determine the domain wall solution we only need to consider the profile of \( l \) in one dimension (along \( z \)). We fix the values of \( l \) at the two boundaries of the one dimensional lattice as \( l = l_{01} \) and \( l = l_{02} \) where \( l_{01} \) and \( l_{02} \) are the values of \( l \) corresponding to the two distinct minima of \( V(l) \). For the intermediate lattice points we use an interpolating configuration to get the final wall solution (Fig.5). The surface tension of the wall in this case is found out to be about 7 GeV/fm\(^2\). Note, \( l \) remains non-zero in the profile of the wall, as we argued above.

![Figure 4](image1.png)  
**Figure 4.** Plot of the inverted potential, i.e. \(-V(l_1, l_2)\). Plot range is restricted for negative values to show the shape of the potential clearly.

![Figure 5](image2.png)  
**Figure 5.** The profile for the domain wall solution (centered near \( z = 10 \) fm) for \( T = 400MeV \). Note that \( l \) remains non-zero inside the wall, with the lowest value of \( l \) being about 0.3. Wall thickness (where \( l \approx 0.9 \)) is seen to be about 0.5 fm.

3. Junctions of Z(3) interfaces, the string profile
To determine the profile of the QGP string, we use the numerical minimization with a two dimensional lattice. The string is taken to be perpendicular to the lattice. To get the string
configuration (see, ref. [3] for detailed discussion), we fix the center of the string [6] at the middle of an elementary lattice square. We then fix $l$ at the four lattice points forming this particular lattice square. Everywhere else $l$ is fluctuated, and the energy is minimized to get lowest energy string profile. Fig.6 shows the surface plot of $-l$ showing clearly the string, connected to three interfaces. To separate out the string core energy contribution from the contribution of the energy of the interfaces, we use following parametrization of $E(r)$,

$$E(r) = E_0(r), \quad 0 < r < r_0$$  \hspace{1cm} (6)

$$E(r) = E_0(r_0) + 3\sigma(r - r_0), \quad r > r_0$$  \hspace{1cm} (7)

Here, $E_0(r)$ denotes the core energy contribution which should be the dominant contribution up to some distance $r_0$. Beyond $r_0$, the linear contribution of interfaces becomes significant. By plotting $E(r)$ vs. $r$, we can get $\sigma$ as well as the core energy $E_0(r_0)$. Fig.7 shows this plot. We have fitted the large $r$ part of the plot with a straight line (dashed line). Its slope is found to be about 23 GeV/fm$^2$, giving the value of $\sigma \simeq 7.7$ GeV/fm$^2$ in reasonably good agreement with our numerical estimate for the wall tension given in the previous section. The core energy $E_0(r_0)$ is found to be about 2.7 GeV/fm.

![Figure 6](image-url)  \hspace{1cm} ![Figure 7](image-url)

**Figure 6.** Surface plot of $-l$ for a small portion of the two dimensional lattice.  \hspace{1cm} **Figure 7.** Plot of $E(r)$ in GeV/fm vs. $r$.

4. **Conclusions**

We have discussed special configurations of junctions of $Z(N)$ interfaces for an SU(N) gauge theory and have shown that these correspond to topological strings which have confining phase in the core. The energy of the string is found out to be about 2.7 GeV/fm at $T \simeq 2T_c$. Lattice simulation of pure gauge theories should be able to investigate properties of these strings. With the interpretation that quark contributions lead to explicit breaking of this $Z_N$ symmetry [2, 4], such QGP strings may play important role in the evolution of the quark-gluon plasma phase and in the dynamics of quark-hadron transition.

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