Dynamic model of the electrorheological fluid based on measurement results

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Abstract. To develop modern applications for vibration decoupling based on electrorheological fluids with suitable control strategies, an appropriate mathematical model of the ERF is necessary. The devices mostly used have annular-shape electrorheological valves. This requires the use of flow channels to measure the static and dynamic properties of the electrorheological fluids in similar flow conditions. Particularly for the identification of the dynamic behavior of the fluids, the influences of the non-electrorheological properties on the overall system must be taken into account. In this contribution three types of parameters with several nonlinear dependencies for the mapping of the static and dynamic properties of the ERF are considered: electro-rheological, hydraulic and electrical. The mathematical model introduced can precisely demonstrate the static and dynamic behavior of the electrorheological fluid and can be used for the future design of real systems for vibration decoupling or other systems with high dynamic requirements.

1. Introduction
To map the dynamic properties of the ERF in a simulation program, many approaches are in existence [1-7]. Here mainly phenomenological models are used, which describe in general the behavior of the overall electrorheological system quite well. But for designing highly dynamic systems these models cannot be used. On the one hand, unrealistic fluids and operating conditions are used. On the other hand, these models do not cover all outcomes which are necessary for the design and development of highly dynamic systems [8-10].

In the present work, a mathematical model to predict the static and dynamic behavior of the commercially available electrorheological fluid is presented [11]. The breakdown of the whole system into electrorheological, electrical and hydraulic areas allows the modeling of all effects which are relevant to develop systems for vibration decoupling or other systems with high requirements to the dynamics.

The full simulation model consists of the hydraulic and electric equations with field- and pressure-dependent parameters and can predict the following behaviors of the real system: the superharmonic oscillations of pressure, the amplitude- and phase-frequency characteristics, the shape of the pressure signals with all important nonlinear phenomena and the drift of the mean pressure in the overall

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Moreover, the representation of the dynamic properties of the electrorheological fluid itself without any significant influences is possible.

2. Measurement components and dynamic parameters

To measure the static and dynamic properties of the electrorheological fluids in conditions similar to the real applications, flow channels are required [12, 13]. The static (stationary) parameters of the electrorheological fluid, like pressure drop dependent on field strength and shear rate, can be measured with only one flow channel.

However, by measuring the dynamic properties of the electrorheological fluids, the influence of the hydraulic parameters as hydraulic capacity $C_H$ and inductance $L_H$ of the overall system need to be considered. These parameters are dependent, for the most part, on the design of the hydraulic components of the system. To reduce this influence, two reciprocally controlled flow channels are required (figure 1) [9]. In spite of this arrangement of the active components, it cannot be assumed that the true dynamic behavior of the ERF has been measured.

![Figure 1](image1.png)  
**Figure 1.** Schematic representation of hydraulic system for dynamic measurements and its dynamic parameters.

![Figure 2](image2.png)  
**Figure 2.** Connection of the high-voltage sources to the flow channels and required control signals.

For such kind of control of the electrorheological channels, two high-voltage sources are necessary. At least one HV-source must cover the dynamic of the measured system with its own dynamic characteristics. The second needs only the required electrical power which is necessary for the operation of the flow channel. The reciprocal dynamic control of the two valves is realized through the connection which couples the two channels electrically with each other. The connections and the required control signals are shown in figure 2. The required phase shift between the respective channels is realized by the phase shift in building up electric fields. Therefore, it makes use of the fact that for an electric field potential differences between the electrodes are required. Besides, the level of the voltage $U_{stat}$ satisfies the following relationship, wherein $U_0$ is the activation voltage of the electrorheological fluid:

$$U_{stat} = 2U_{dyn.mean} - U_0$$  

As well the HV-sources as the wires, the connectors and the flow channel itself are electrically less than ideal and have a transfer function, internal resistances and capacities. It follows that, to identify the dynamic behavior of electrorheological fluids, the dynamic behavior of the hydraulic and electrical parts of the overall system must be identified first.

3. Modelling of the hydraulic system

The differential equations which describe the dynamic behavior of the hydraulic system with two reciprocally controlled flow channels form the basis of the model. The pressure $p_{1,2}$ upstream of each flow channel can be calculated from the different pressures as a result of the electrorheological effect
The volume flow rates in each valve are different and are dependent on the compressibility of the fluid and the pressure lines. They can be determined from the continuity equation as follows:

\[ Q_{1,2} = Q_{\text{pump},1} - C_{H1,2} \frac{dp_{1,2}}{dt} \] (4)

To fill these equations electrorheological and hydraulic parameters are needed. The static electrorheological properties of the ERF, hydraulic resistance \( R_{H1,2} = \frac{dp}{dQ_{1,2}} \) and linearization factor \( k_e1,2 = \frac{dp}{dE_{1,2}} \), can be implemented in the model as nonlinear functions of the field strength and the volume flow rate (figure 3 (a), (b)). For this purpose, a modified Carreau-Yasuda equation can be used [14]:

\[
\frac{\eta_s}{\eta_B} = 1 + \frac{(k_e \lambda^n - \eta_B)}{\eta_B} \left[1 + (\lambda \lambda')^\alpha \right]^{\beta-1}
\] (5)

Table 1: Numerical values for equation (5)

| Parameter | Value |
|-----------|-------|
| \( \eta_s \) (Pa·s) | 0.027 |
| \( k_e \) (Pa·s\(^{-1}\)·kV\(^{-n}\)·mm\(^{-m}\)) | 314.1 |
| \( m_e \) (-) | -0.893 |
| \( \lambda_s \) (s·kV\(^{-n}\)·mm\(^{-m}\)) | 1.745e-4 |
| \( \beta \) (-) | 0.677 |
| \( n_e \) (-) | 3.831e-8 |
| \( \alpha \) (-) | 1.5 |

Figure 3. Static parameters of a hydraulic system. Hydraulic resistance \( R_{H1} \) (a), linearization factor \( k_e \) (b) at 60°C and compression modulus \( K_{ERF} \) (c).
The details of the geometric parameters of the flow channels, operating temperature, density and
the basic viscosity of the fluid complete the hydraulic model, thus allowing calculation of the pressure
drop in the channels as a function of the applied field strength, shear rate, frequency and operating
temperature.

4. Modelling of the electrical system
Because of the strong coupling of the hydraulic and electrical parameters of the system, the derived
hydraulic model with implemented nonlinear hydraulic and electrorheological parameters must be
extended with equations to map the electrical properties of the overall system.

To describe the electrical properties, the system can be further subdivided into static and dynamic.
The static properties describe the well-known, almost quadratic, dependence of the electrical current
on the applied electric field strength [15]. This nonlinear static behavior of current-voltage
characteristics is independent from the shear rate in the flow channel. Hence, the only differential
electrical resistance $R_{el1,2} = dU_{1,2}/dI_{1,2}$, which is dependent only on the applied electrical field, can be
derived. In this case Ohm’s law has no validity.

To describe the dynamics of the electrical system the well-known phenomenological model of a
dielectric under the influence of electric fields is used (figure 4). This model allows the calculation of
important electric parameters like the capacity or loss factor of the system and can be mathematically
defined as follows:

$$Z_{1,2} = \frac{U_{1,2}}{I_{1,2}} = \frac{R_{el1,2}}{j\omega C_{el1,2} R_{el1,2} + 1}$$

(6)

Moreover, the frequency response of the introduced static electrical parameters at the operating
frequency can be derived. For this purpose the impedance, which is dependent on the working
frequency of the system, is determined from the measured current-voltage relationship (figure 5).
However, this measurement requires a separate control of the flow channels.

![Figure 4. Phenomenological model of a dielectric with parallel connection of the electrical capacitance $C_{el}$ and resistance $R_{el}$](image)

![Figure 5. Measured frequency response of the impedance of the overall electrical system (HV-source – connections – flow channel - ERF)](image)
to the first order system with lumped parameters. Therefore, the dynamic behavior of the electrical system with electrorheological fluid can be defined with a nonlinear Debye-like model.

5. Coupling of hydraulic and electrical model

In the presented models the mapping of the nonlinear static and the dynamic behavior of each system is performed by the addition of linear differential equations with required nonlinear parameters. The subsystems contain the electrorheological, hydraulic and electrical parameters with corresponding dependencies on the strength of the applied electric field, the volume flow rate (shear rate), the operating frequency and temperature. To obtain an overall dynamic model, the individual models will be coupled to each other.

Theoretical and experimental studies have shown that the overall system can be represented as a series connection of the electrical and hydraulic subsystems. It was found that the negative effects at higher frequencies are caused by non-measurable electrical interactions within the electrorheological fluid. For this reason a fictive voltage is introduced, which is responsible for the effective viscosity change in the fluid.

This voltage is also a connecting link between the two introduced subsystems. The electrical system is excited with a voltage from the high voltage source. In the existing electrical model the electric currents flowing through the electrorheological fluid can be determined. These currents are used for the excitation of the fictive auxiliary system to calculate a fictively effective voltage. The fictive auxiliary system consists of a parallel arrangement of a capacitor and electrical resistance. The values of these parameters are identical to the calculated quantities of the electrical subsystem. The fictively effective voltage is used for the excitation of the hydraulic subsystem. The resulting overall dynamic model is shown in figure 6.

![Schematic representation of the derived overall dynamic model.](image)

The dynamic model of the high voltage source describes the behavior of the input voltage at the electrodes. In using high-voltage sources, the dynamics of the voltage build-up in the electrically active gap of the flow channels is load dependent. The higher the amplitude of high voltage, the slower is its rise and fall times. Consequently, this decreases the frequency bandwidth wherein the full amplitude of the voltage can be generated.

In addition, the entire system was provided with the characteristics of the pump and the associated control algorithm for keeping the flow rate constant at each operating point of the flow channels.
6. Results and discussion

The experimental data must now be compared with the simulation results of the new dynamic model. Firstly, for the design of highly dynamic systems the amplitude and phase response of the system is important. Figure 7 shows the measured and simulated dynamic behavior of electrorheological fluid in a frequency bandwidth of 10...1000 Hz. The excitation of the system is performed by a dynamic voltage of $U_{II} = 2.5\pm2.3$ kV/mm and static voltage of $U_I = 4.8$ kV/mm. The two flow channels are geometrically identical and are 100 mm long and 30 mm wide. The height of the electrically active gap is 1 mm. The flow rate is 5 L/min and kept constant by an integrated control algorithm. This corresponds to a shear rate of $16.7\times10^3$ s$^{-1}$.

![Figure 7. Measured and simulated frequency response of the electrorheological fluid.](image1)

![Figure 8. Time plot of the measured and simulated pressure.](image2)

The relatively large deviation between the measured and simulated frequency response occurs only at very high frequencies. In bandwidth, which can be used effectively, the mapping quality is about 90%. These deviations are due to the strong pulsations of the pump, which are relatively difficult to reproduce in the model.

Secondly, the time response of the pressure increase needs to be considered. As shown in figure 8, the pressure in the system increases about three milliseconds earlier than the voltage, which should theoretically be responsible for the increase in pressure. This effect is caused by the characteristic phase shift between the electrical voltage and current. It follows that not only the voltage, but also the electric current, which is passed through the fluid, is responsible for the increase in pressure. The simulation can reproduce with high accuracy the effect due to the presented auxiliary model and fictively effective voltage.

Furthermore, the model can represent with high accuracy the super harmonic vibrations of pressure in the flow channels. The problem of mapping the descending mean pressure at constant mean values of the voltage at higher frequencies is achieved with the new model.

7. Conclusions

The presented full-simulation model consists of hydraulic and electric equations with the most important process variables-dependent parameters and can predict the static and dynamic behavior of the electrorheological system. The effects, which are important for the design of highly dynamic systems and corresponding control algorithms, can be accurately predicted with the introduced model. These include the superharmonic oscillations of the pressure, the amplitude- and phase-frequency characteristics, the negative phase shift between pressure and voltage and the drift of the mean pressure in the system. Moreover, the representation of the dynamic properties of the electrorheological fluid itself is possible without almost any influences.
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