Algorithmic Complexity in Noise Induced Transport Systems

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Abstract

Time correlated fluctuations interacting with a spatial asymmetry potential are sufficient conditions to give rise to transport of Brownian particles. The transfer of information coming from the nonequilibrium bath, viewed as a source of negentropy, give rise to the correlated noise. The algorithmic complexity of an object provides a means of quantitating its information contents. The Kolmogorov information entropy or algorithmic complexity is investigated in order to quantitate the transfer of information that occurs in computational models showing noise induced transport. The complexity is measured in terms of the average number of bits per time unit necessary to specify the sequence generated by the system.
Recently a lot of attention has been paid to simple stochastic models in which some of the energy in a nonequilibrium bath is used to obtain a net transfer of information. The best known examples of this process are thermal ratchets or correlation ratchets where a nonzero net drift speed may be obtained from time correlated fluctuations interacting with broken symmetry structures. Recently the study of a wide variety of ratchets has received much attention due to their general interest and possible technological applications in modeling molecular motors and other nanoscale and mesoscale systems. For a general review of thermal ratchets the reader is referred to Ref. [1].

As was demonstrated by M. Millonas [2], the net current is obtained by means of a source or sink of negentropy (physical information) that allows the engine to operate. Numerous existing studies have addressed the question of the efficiency in ratchets [3] but, despite this widespread interest none of them did it from the perspective of information theory and, up to our knowledge, a quantitative estimate of the amount of negentropy was never obtained. A possible reason that no effort has been done to study the entropy transfer on thermal ratchets may be that they constitute far from equilibrium, non-ergodic systems and usual entropy calculations rely on probabilistic ensemble concepts that require energy minimization procedures associated with equilibrium arguments. Algorithmic complexity was a concept introduced by Kolmogorov to avoid the probabilistic interpretation of Shannon that has been recently used to obtain the information content of far from equilibrium systems such as proteins and fractal growth processes [4]. This characteristic makes algorithmic complexity specially useful to obtain the variation of entropy or physical information of far from equilibrium systems such as thermal ratchets.

Following the idea developed by Crisanti et al [5] and Paladin et al [6] that applied algorithmic complexity to characterize the chaoticity of dynamical systems with noise, we study the algorithmic complexity of thermal ratchet motion.

The algorithmic complexity of an object is broadly defined as the length in bits of the shortest description for that object [4]. In other words, complexity can be characterized as the length of the shortest possible definition of the sequence itself.

In the framework of information theory, if one wants to transmit the sequence, through a noisy channel, until it deviates from the deterministic one a tolerance threshold $\Delta$, one can use the following strategy:

Specify the initial condition $x(1)$ with precision $\delta_0$ using a number of bits $n = \log_2(\Delta/\delta_0)$ which permits to arrive up to a time $\tau_1$ where the error equals $\Delta$. Then specify again a new initial condition $x(\tau_1 + 1)$ with a precision $\delta_0$ and arrive up to a time $\tau_2$, and so on $N$ times. The number of bits necessary to specify a sequence with a tolerance $\Delta$ up to $T_{\text{max}} = \sum_{i=1}^{N} \tau_i$ is $\simeq Nn$ and the mean information generated per time step is $\simeq Nn/T_{\text{max}} = K_\sigma/\ln 2$ bits. Following the idea of algorithmic complexity as the length in bits of the mean information, $K_\sigma$ is the complexity associated to stochastic motion. Thus, $K_\sigma$ can be obtained as [6]:

$$K_\sigma = \frac{1}{n} \ln \left( \frac{\Delta}{\delta_0} \right),$$

(1)
The model for ratchets that we have used is based on the model proposed by Astumian and Bier [7]. Two of the authors, recently used this model to study the approach to steady state in ratchets [8] and to study ratchets with finite inertia [9]. It consists of an asymmetric piecewise linear potential where the barrier height fluctuates between two states. Astumian and Bier showed that the fluctuations of the barrier height can produce a net flow even with the net macroscopic force that is zero at all times. The general Langevin equation describing the ratchets that we will consider is

$$\frac{dx}{dt} = -u'(x, t) + \sqrt{2k_B T} w(t),$$  \hspace{1cm} (3)

where $u'(x, t) = \frac{\partial u(x, t)}{\partial x}$, $T$ is the temperature and $k_B$ is the Boltzmann constant. The potential $u(x, t)$ switches on and off the $v(x)$ asymmetric potential, and $w(t)$ represents the Gaussian noise term with delta function correlation $\langle w(t)w(t') \rangle = \delta(t-t')$.

The fluctuations cause the potential $u(x, t)$ to switch between $u_+(x) = v(x)$ and $u_-(x) = 0$ with a dichotomous Markov process governed by the master equation

$$\frac{d}{dt} \begin{pmatrix} P_+(t) \\ P_-(t) \end{pmatrix} = \gamma \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} P_+ \\ P_- \end{pmatrix},$$  \hspace{1cm} (4)

where $\gamma$ represents the frequency of the flipping between $u_+(x)$ and $u_-(x)$.

In our numerical simulations we have followed the approach of Elston and Doering [10]. Details of the simulation procedure can be found in references [8] and [9].

In order to obtain the algorithmic complexity associated to ratchet motion, two different walkers are allowed to move under the ratchet potential with initial conditions separated by $\delta_0$, $x(1)$ and $x'(1) = x(1) + \delta_0$. Both motions continue until the difference equals $\Delta$ at time $\tau_1$. Then begin again with initial conditions $x(\tau_1 + 1)$ and $x'(\tau_1 + 1) = x(\tau_1 + 1) + \delta_0$, and so on $N$ times.

Simulations were carried out on ratchets with an asymmetric barrier [8] switched on and off with different flipping rates. The asymmetric parameter used was $\alpha = 10/11$ and the barrier height was $E_0 = 10$. This is a very important kind of ratchet because the experimental studies of ratchets done by Rousselet et al [11] and by Faucheux et al [12] were carried out with an asymmetric potential switched on and off periodically and Ajdari and Prost [13] used the on-off barrier fluctuation as a method for dielectrophoretic separation.

In the limits of low and high flipping rates the current tends to zero and the system behaves essentially as a random walk. The relative complexity for a given flipping rate is defined as:

$$\frac{K - K_{RW}}{K_{RW}},$$  \hspace{1cm} (5)
where $K$ is the complexity at the given flipping rate and $K_{RW}$ is the complexity for high flipping rate limit. The relative complexity and the net current as functions of the logarithm of the flipping rate $\gamma$ are shown in Fig. 1. In this figure it can be appreciated that the relative complexity tends to zero in the limits of low and high flipping rates, where the current tends to zero. On the other hand a dramatic increase of the relative complexity appears for flipping rates corresponding to the highest currents.

The proportionality between $K$ and $J$ can be explained using the well known statistical mechanical relationship derived by Zurek [14]

$$S = K + I,$$

where $S$ is the physical entropy, $K$ is the algorithmic complexity and $I$ is the Shannon information entropy. Zurek’s relationship says that the physical or thermodynamic entropy of a system is composed of two parts, that determined from the known information of the system $K$ and that determined from the unknown or probabilistic information $I$. When the microstates of a system are unknown $K = 0$ and the entire entropy is due to the Shannon information, i.e.,
Figure 2: Log-log plot of the relative complexity vs the error $\Delta$ for high and low current flipping rates.

$S = I$. As observations are made on the system the information content shifts from $I$ to $K$.

When there is a net current in the ratchet, the amount of known information about the system increases with respect to the case of no net current. The existence of a net current implies there is some more information known about the system. This amount of extra information comes from the negentropy supplied by the external nonequilibrium bath. As a consequence, the maximum amount of extra information or complexity is obtained when the current is maximum.

Another way to characterize the behavior of complexity in on-off ratchets is the scaling of the complexity with the error $\Delta$. We studied the scaling behavior of complexity for the random walk limit corresponding to a high flipping rate and for the highest current flipping rate. The scaling behavior for both cases is shown in Fig. 2 where $\log(K)$ vs $\log(\Delta)$ is plotted. In the random walk limit $K \sim \Delta^{-2}$ according to the standard diffusion law. For the highest current flipping rate, due to the existence of a net current, the complexity scales as $K \sim \Delta^{-1}$.

The simulations were carried out using a rather high value of $\Delta \simeq 10$ in order that the interaction between the ratchet potential and the on-off fluctuation may take effect. Results were averaged over 500 independent runs.
In conclusion, the net transfer of information from a source of negentropy that allows the net drift of particles in thermal ratchets may be obtained through the algorithmic complexity. The negentropy transference results in an increase of complexity observed when high ratchet currents take place (see Fig. 1). Additionally, a different scaling behavior of complexity at high currents compared with the no current case, i.e. random walk limit, is observed as can be seen in Fig. 2. As far as we know, the algorithmic complexity lets us obtain, for the first time, an estimate of the net transfer of information associated with ratchet motion.
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