Topological sound in two dimensions

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Abstract

Topology is the branch of mathematics studying the properties of an object that are preserved under continuous deformations. Quite remarkably, the powerful theoretical tools of topology have been applied over the past few years to study the electronic band structure of crystals. Topological band theory can explain and predict topological phase transitions in a material, and the unusual robustness of certain band structure shapes, such as Dirac cones, against small perturbations. These findings have also unveiled a new phase of matter—topological insulators—whose exotic transport properties at their boundaries are topologically protected against imperfections and disorder. The fascinating features of topological boundary states have triggered the search for their analogs in classical wave physics. Here, we focus on the peculiar features of two-dimensional topological insulators for sound and mechanical waves. Two-dimensional Dirac cones and phononic topological insulators can emerge under certain conditions in periodic acoustic metamaterials, demonstrating great potential for acoustic and mechanical systems to demonstrate, over a tabletop platform, complex fundamental phenomena driven by topological concepts. In addition, these discoveries offer a direct path toward new technologies for enhanced sound control and manipulation.

KEYWORDS

acoustics, elastodynamics, phononics, topological insulators, topological protection

INTRODUCTION

In recent years, condensed matter physics has experienced the breakthrough of two-dimensional (2D) quantum materials and of their topological states, whose robust electron transport is immune from disorder. Such discoveries have nurtured great excitement for the future of quantum devices and technologies.1–3 Topological physics concepts have naturally expanded into the classical realm, including photonics, acoustics, and mechanics, where exciting phenomena predicted by topological band theory can be exploited to control the dynamics of classical waves and are far less challenging to experimentally investigate in conventional platforms.4–10 Notably, systems that control the propagation of phonons—sound and mechanical waves—form a genuine platform to implement the analog of complex condensed matter Hamiltonians with nontrivial topological features in a laboratory setting.11–13 Although fundamental differences exist between the properties of electrons and phonons, a plethora of 2D phononic topological phases has been lately demonstrated, endowing classical acoustic platforms with topological protection for enhanced wave control and manipulation and opening the door to new technologies.

In this work, we review recent advances in the topological features of 2D phononic metamaterials. First, we revisit the underlying
TOPOLOGICAL BAND THEORY

Topology is a universal concept in mathematics dealing with the relation between components of space (geometry). As such, it does not deal with the details of a specific geometrical shape, but it rather introduces global features that can describe widely different geometries under the same principles. Topological invariants—powerful tools introduced by topology—are quantized values describing these global features, which do not change under continuous deformations of the underlying geometrical shape. One canonical example is the features of doughnuts, mugs, and pretzels: the first two belong to the same topological family since they are characterized by a single hole. A pretzel, however, has three holes, which puts it into another topological family. Objects belonging to the same topological class, like a mug and a doughnut, share profound similarities that are described by their topological invariant.

In physics, particularly in condensed matter, this concept has been extended to define interesting features of the electronic energy bands of quantum materials, resulting in the tantalizing discovery of new phases of matter known as topological insulators. The underlying theory, known as topological band theory, predicts the emergence of exotic electron conduction states at arbitrarily shaped boundaries of these topological insulators, associated with the topological features of the bulk properties, described by a topological invariant for their energy bands. Hence, studying the topological features of the infinite medium determines how the electrons behave when a finite volume is arbitrarily shaped.

Graphene—a 2D hexagonal lattice made out of carbon atoms—is a good example to unveil the unusual physics of topological insulators, as it allows to elegantly illustrate how topological band theory can be powerfully employed to describe its extraordinary electronic properties. The electronic π-orbital band structure of graphene, with its hexagonal unit cell shown in the inset of Figure 1A, is well known for hosting low-energy excitations associated with the two-fold degeneracy at a singular point in the energy bands known as the Dirac cone. Remarkably, this singular point mimics the properties of massless fermions in quantum electrodynamic systems. Symmetries ensure that two-fold Dirac cones cannot exist in odd numbers in a 2D system, due to the constraints associated with time-reversal symmetry (T symmetry), which flips the time flow from the future to the past, and inversion symmetry (P symmetry), which switches the signs of spatial coordinates. Therefore, Dirac cones must come in pairs and are commonly located at the border of the Brillouin zone (BZ), the primitive cell in the reciprocal space of the lattice. In the case of graphene, this BZ takes the shape of a hexagon and Dirac cones exist at its corners, namely the K and K′ points (Figure 1A), representing valley degrees of freedom (DOFs). In the vicinity of these valley points, the bands show a linear dispersion, associated with the massless features of Dirac fermions, hence providing a platform to demonstrate relativistic phenomena. For example, the Zitterbewegung phenomenon, related to the trembling motion of electrons, can be demonstrated near a Dirac point, as well as quantum Klein tunneling, based on which normally incident relativistic Dirac quasi-particles can tunnel through a potential barrier with unitary transmission independently of its width and height.

Dirac cones carry topological charges, known as Berry phases, which can be evaluated as γ_n = ∮_{Γ} A_n(k).dk following a closed path Γ in the BZ, with A_n(k) being the Berry connection defined as A_n(k) = −i⟨u_n(k)|∇_k|u_n(k)⟩, where u_n(k) is the eigenstate for the nth energy band and k is the electron wave vector in the BZ. Remarkably, this quantity predicts the localization of the electron wave function at the edge of a finite size sample, depending on its orientation with regard to the lattice symmetry. For example, when the graphene boundary is cut in a zigzag shape, zero-energy edge states arise localized at this boundary and their bands connect the two valleys, as shown by gray lines in Figure 1A.

The features described above make Dirac cones the mother states of 2D topological materials, whose tantalizing topological properties emerge from lifting their degeneracy by either breaking T or P symmetry in the material. The first canonical model of a 2D topological material based on breaking T symmetry was introduced by Haldane in 1988. Here, the hoppings between second-neighbor sites in graphene are considered such that a periodic local magnetic flux within the honeycomb lattice is induced. In turn, this distribution of magnetic flux opens a bandgap around the Dirac point, changing graphene into an insulator (Figure 1B). For this structure, a relevant topological invariant can be introduced, known as the Chern number, describing the unusual topological features of the bands surrounding this bandgap. The Chern number of the nth energy band in 2D solids is obtained from the integral of the Berry curvature Ω_n(k) in the BZ, where the Berry curvature is the curl of the Berry connection: Ω_n(k) = ∇ × A_n(k). Chiral edge states emerge in the bandgap, sustained by the nontrivial topological features of the gapped Dirac cones in Figure 1B. The specific number of these edge modes is always equal to the difference in the Chern numbers of the insulators surrounding the interface. This powerful...
FIGURE 1  Schematics of Dirac cones, gapped Dirac cones in a 2D insulator, their topological invariants, and corresponding edge conduction states. (A) Two-fold Dirac cones in the Brillouin zone (BZ) and flat edge states between the two valleys. The inset shows the unit cell of a honeycomb lattice whose band structure supports two-fold Dirac cones. (B) Dirac cones are gapped by breaking time-reversal symmetry, for example, by applying an external magnetic field normal to the 2D insulator. According to the bulk-edge correspondence, chiral edge states (green lines) close the gap of Dirac cones and their propagation is confined to the boundary of the insulator and is unidirectional. (C) Dirac cones are gapped by breaking inversion symmetry, for example, by making the size of two atoms in each unit cell different. Valley edge states (red and blue lines) close the gaps of their respective valleys, and their propagation is confined to the interface between two topologically distinct domains, with valley-dependent propagation directions. (D) A four-fold Dirac cone can exist alone in BZ. (E) Four-fold Dirac cones are gapped by the incorporation of spin–orbital interactions in a 2D semiconductor/insulator; the subspace of each spin effectively experiences the synthetic magnetic fields with opposite directions and thus supports spin-polarized helical edge states with opposite directions. The edge states are confined to the boundaries obeying the spin-momentum locking rule due to the time-reversal symmetry.

The relation between a global property of the bulk material and the states existing at the boundaries of a finite sample is known as the bulk-edge correspondence principle.\textsuperscript{26,27} Interestingly, these edge states must exist in that defined number independent of the shape of the boundary, hence they are inherently robust to perturbations and disorder. The phase transition of graphene from a semimetal (Dirac cone) to an insulator (open bandgap) can be also understood phenomenologically: by applying a static magnetic bias on a 2D quantum material, a cyclotron motion of electrons is induced in the bulk, preventing conduction of currents. At the boundaries, however, the electron cyclotron orbits are bounded off, yielding one-way electron transport stuck to the boundaries. This phenomenon, known as the quantum Hall effect,\textsuperscript{25–27} is one of the earliest examples of a 2D topological phase of matter. The one-way nature of this edge conduction implies a strong robustness to defects, as predicted by the asymmetric orientation of the edge bands in Figure 1B: the associated chiral edge states propagate unidirectionally, circumventing any defect, and continue propagating in the same direction since no states exist that can carry electronic currents in the backscattering direction at a given boundary (lower panel in Figure 1B).

Conversely, a different form of topological phase can emerge when \( \mathcal{P} \) symmetry is broken in graphene, for example, when two atoms in each unit cell of the hexagonal lattice are no longer identical, which lifts the point degeneracy of the Dirac cone in Figure 1C. This broken symmetry results in two symmetric bands separated by a bandgap. Although being topologically trivial on average when their Berry curvature is integrated over the entire BZ, these bandgaps are characterized by local topological features—a nonzero Berry phase around the gapped Dirac cones, leading to what is known as the valley Hall effect.\textsuperscript{28} In this scenario, we can define a local topological invariant, the valley Chern number \( C_{K/K'} = \pm 1/2 \) for each valley, whose sign flips as we analyze one or the other bandgap. The bulk-edge correspondence predicts that an interface between topologically distinct domains with opposite valley Chern numbers, as shown in the lower panel of Figure 1C, supports edge modes whose propagation direction is locked to their valley DOF, as predicted by the dispersion of valley-polarized edge states in the upper panel of Figure 1C.

Since electrons are fermions, their time-reversal symmetry operator obeys \( T^2 = -1 \) and thus allows for Kramers degeneracy of the
energy bands. In this fermionic case, all eigenstates of a system with $T$ symmetry are at least two-fold degenerate, leading to the formation of two overlapping copies of the Dirac cone, each associated with a single electronic spin. This four-fold degeneracy relaxes the constraints dictated by symmetries on the necessity of an even number of Dirac cones, hence a four-fold Dirac cone can exist alone in the BZ, as demonstrated in a semiconductor quantum well (Figure 1D). Such four-fold Dirac cones open the door to another topological phase of matter: the QSHE, governed by spin–orbital interactions, a process in which the intrinsic spin and angular momentum of an electron are not independent any longer. We illustrate the QSHE with the model of a four-fold Dirac cone and study the corresponding spin–orbit coupling in the system. Under this scenario, a bandgap opens with topological features characterized by a $Z_2$ invariant taking either value 0 or 1, or an integer-quantized spin Chern number $C_{1,1} = \pm 1$, as seen in Figure 1E. Such simple model effectively behaves as two copies of a noninteracting Chern insulator, each linked to opposite electron spins experiencing an opposite magnetic bias, making this model superposition still obeying a global $T$ symmetry. According to the bulk-boundary correspondence, helical currents arise along the edges of the sample, demonstrating the defining feature of topological insulators—spin–momentum locking—observed in the fact that the spin polarization of an electron correlates directly with the direction of propagation. Spin-up states propagate in one direction, while spin-down states propagate in the opposite, as shown in the lower schematic of Figure 1E. These helical currents are inherently robust against defects along the interface, as long as the disorder asymmetry does not couple opposite electron spins.

Quite remarkably, topological band theory in quantum materials can be transposed to phononic systems, since the sound dynamics in periodic metamaterials can be described by band structures and the Bloch theorem. In this scenario, the operating frequency takes the role of Fermi energy. The phases of acoustic/mechanical Bloch waves in such materials are subject to periodic potential modulation, which is usually ignored in the study of periodic acoustic structures but becomes essential to define the emergence of nontrivial topological properties. In the following, we review the phononic counterparts of Dirac cones, gapped Dirac cones, and their topological properties, as well as their experimental realizations and emerging technological opportunities.

**PHONONIC DIRAC POINTS**

The Dirac model outlined in the previous section offers a straightforward translation to the macroscopic scale of topological concepts and their application to sound propagation. The atomic orbitals can be replaced by resonant acoustic cavities, and electron hopping can be mapped into the coupling of these cavities through tubes with a small cross-section. Arranging such components as a honeycomb lattice reproduces the Hamiltonian of graphene, yielding acoustic Dirac cones at the corner of the BZ (Figure 2A). Like their condensed matter counterparts, these Dirac cones are protected by both $P$ and $T$ symmetry due to the high symmetry of the honeycomb lattice. Sonic crystals, as well as locally resonant metamaterials supporting airborne surface acoustic waves, can also host such Dirac degeneracies, as long as they are designed to obey the same high symmetries (Figure 2B). In particular, metamaterials can be realized as arrays of small acoustic resonators, such as Helmholtz resonators as simple as soda cans, offering the advantage of being open to the surrounding space and hence allowing for the straightforward measurement of the band structure and interaction with sound at the surface of the metamaterial. Dirac points have been also observed in elastic systems, both at the macroscale and microscale (Figure 2C). Similar to graphene, for frequencies close to the singularity, wave propagation can be described using the low-energy Dirac Hamiltonian, leading to dramatic consequences for sound propagation. For instance, the transmission of acoustic waves through a metamaterial slab excited around the Dirac point is inversely proportional to its thickness. This is related to the unusual beating that arises in the temporal propagation of pulses, a phenomenon which can be interpreted as the classical analog of Zitterbewegung of quasi-relativistic electrons in graphene. Another surprising feature of acoustic Dirac cones is Klein tunneling. The acoustic version of this intriguing phenomenon has been demonstrated using three sonic crystal slabs with carefully designed Dirac cone frequencies, as shown in Figure 2D. Remarkably, at frequencies close below the Dirac point, normally incident sound waves are fully transmitted through the barrier.

Furthermore, like their electronic counterparts, phononic Dirac cones possess a nontrivial Berry phase, which, albeit not providing topological protection, is still associated with peculiar topological phenomena. For instance, in the case of a finite honeycomb lattice zigzag-shape or beard-shape edge, the edge mode dispersion connects Dirac points with opposite topological charges, which have been studied as a function of the edge termination of a sonic crystal. Moreover, although being protected by $P$ and $T$ symmetries, long-range deformations of the lattice are able to move the cones within the first BZ. In the extreme case where Dirac cones with opposite topological charges, typically time-reversed partners, get closer to each other, they successively merge into a parabolic touching point before opening a bandgap. Interestingly, these specifically engineered global deformations, such as strain fields, can be described as an effective magnetic field bias (Figure 2E), leading to the acoustic analog of the topological insulator in Figure 1B, along with the emergence of helical edge states. These intriguing phenomena have been demonstrated not only in tightly coupled cavities and sonic crystals but also for mechanical waves at the macroscopic scale and for on-chip systems. Finally, Dirac cones found in honeycomb or Kagome lattices usually involve two bands, mimicking two-fold Dirac cones in graphene. Three- and four-band Dirac cones have been implemented in acoustic and mechanical lattices, unveiling new properties, such as spin-one Dirac particles, zero-index media, or non-Abelian properties. Interestingly, these conical degeneracies also exist in continuous media, such as in soft elastic strips, opening the door to the possible implementation of these concepts in biological tissues.
BREAKING T: CHERN AND FLOQUET PHONONIC TOPOLOGICAL INSULATORS

The intriguing features of 2D topological insulators, in particular the robust nature of their boundary states, have stimulated an intense search for classical analogs of topological states in phononic platforms in recent years. In electronics, time-reversal symmetry $T$ breaking is obtained with the addition of a DC magnetic bias. However, phonons are neutral charges and, therefore, are magnetically inert. However, $T$ symmetry can be broken in acoustics systems in other ways, for instance, by applying an angular momentum bias, that is, rotating the medium in which the wave propagates. For example, a vortex within irrotational fluid mimics the Aharonov–Bohm effect for sound waves\(^\text{56}\) and large acoustic nonreciprocity, that is, broken $T$ symmetry in transmission, can be achieved in a circular cavity with rotating air flow.\(^\text{57}\) Following the analogy of electron cyclotron orbits under a magnetic bias, Chern insulators for sound have been demonstrated both numerically and experimentally in tight-binding lattices formed by connected circulators\(^\text{58-61}\) (Figure 3A) or in a viscous fluid with embedded rotating rods.\(^\text{62}\) As soon as the medium rotation starts, the Dirac point degeneracy endowed by the lattice symmetry is lifted and the resulting bandgap hosts robust one-way topological edge states whose presence is governed by the nonzero Chern number of the corresponding bandgap. The phononic platform allows investigating phenomena that would be very challenging to observe in condensed matter systems, such as the emergence of antichiral states in honeycomb lattices whose elements have alternating opposite directions of rotation.\(^\text{63}\) Following these principles, Ding et al. have managed to build an experimental version of a topological acoustic Chern insulator with high-quality cavities\(^\text{61}\) (Figure 3B), but the impact of loss, the required rotation speed, and the size of such devices make it difficult to extend them to practical devices. One promising solution consists of the use of active liquids, in which a spontaneous flow can be generated using self-propelling particles,\(^\text{64-67}\) opening the door to active matter and chemically designed topological phononic devices (Figure 3C).

These concepts have been also transposed to mechanical waves, for which rotating devices are readily available. For example, tight-binding lattices made of springs and masses coupled to individual gyroscopes support topological bandgaps whose nonzero Chern numbers predict one-way edge propagation of mechanical vibrations\(^\text{68-70}\) (Figure 3D). The corresponding experiment has been demonstrated with high-speed cameras, allowing a very clear understanding of the physics at play\(^\text{70}\) (Figure 3E), leading to its extension to amorphous lattices.\(^\text{71}\) Finally, delocalizing the rotation to the entire spring mass lattice allows to exploit the Coriolis force as a uniform effective

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**Figure 2** Phononic Dirac cones. (A) A unit cell of tight-binding acoustic honeycomb lattice (left) and corresponding Dirac cone (right) in the band structure. Image reproduced from Ref. 33 with permission. (B) Experimental study of a Dirac cone medium for sonic surface waves. Image reproduced from Ref. 37 with permission. (C) On-chip phononic Dirac material. Image reproduced from Ref. 40 with permission. (D) Acoustic analog of Klein tunneling. Image reproduced from Ref. 41 with permission. (E) Strain field acts as a gauge field for phononic waves. Image reproduced from Ref. 44 with permission. (F) Phononic helical edge states are induced by gauge fields. Image reproduced from Ref. 48 with permission.
magnetic bias, which also produces topological bandgaps with nonzero Chern numbers.\textsuperscript{72}

The challenges linked to the practical implementations of these proposals have led to the search for other ways to break $T$ symmetry in phononic systems. One of the successful approaches consists of modulating the medium properties in time with precisely designed phase patterns. For example, Fleury et al. demonstrated an acoustic nonreciprocal device made of a trimer of cavities whose density is modulated in time with a rotating phase pattern.\textsuperscript{73} Arranging these circulators in a honeycomb pattern generates a so-called Floquet topological phase, whose bandgaps host one-way topological edge modes not only robust to defects but also to phase distribution disorder at the scale of the bulk (Figure 3F). If the time modulation of acoustic properties is achievable using mechanical modulation\textsuperscript{74} or feedback electric circuitry,\textsuperscript{75} it is still challenging to implement for large-scale lattices. Fortunately, it is simpler to temporally modulate mechanical waves, since commercially available piezo-electric patches allow to change locally the Young’s modulus of the medium. Using these devices, Darabi et al. carried out an experiment reproducing the model of Ref. 73 for mechanical waves\textsuperscript{76} (Figure 3G). In addition, taking advantage of the opto-mechanical interaction at the nanoscale, several authors have studied nonreciprocal topological phases for phonons mediated by light.\textsuperscript{77–81}

Finally, there is a direct mapping between Floquet Hamiltonians and scattering network systems,\textsuperscript{82,83} leading to straightforward and practical implementations in passive media. For instance, a so-called anomalous Floquet topological insulator has been demonstrated in a square lattice of circular acoustic cavities whose counter-rotating modes are linked through adequate intercell forward couplings\textsuperscript{84–86} (Figure 3H). By mapping the time dimension to a third spatial dimension, it is also possible to mimic anomalous Floquet topological phases in passive media, such that the couplings’ chirality along the third dimension effectively breaks time-reversal symmetry,\textsuperscript{87} in which the role of time is here taken by the third dimension. In these passive anomalous Floquet insulators, the topological properties are crucially linked to the couplers’ design and strength, in sharp contrast with genuine Floquet topological insulators whose topological order is realized by actively modulating the resonant elements or couplings in time.
BREAKING $P$: PHONONIC VALLEY-HALL INSULATORS

The design flexibility of phononic metamaterials allows us to exploit the effects of inversion symmetry breaking to create phononic analogs of the valley-Hall effect. Breaking $P$ symmetry is significantly easier than breaking $T$ symmetry. For instance, it can be obtained by changing the orientation of triangular scatterers within a triangular lattice\cite{88-90} (Figure 4A). The resulting modes below and above the bandgap display acoustic vortices whose sense of rotation is linked to the bands’ valley DOFs, which is directly related to the corresponding local topological invariants. Topological valley edge modes propagate along the interface between domains with different invariants, taking sharp turns with very little backscattering. Such a straightforward protocol has paved the way to the transposition of phononic valley Hall insulators to a large number of platforms, for sound,\cite{91-101} water waves, and mechanical waves\cite{102-112} (Figure 4B), even at the scale of a phononic chip.\cite{113-115} The robustness of edge modes is sustained by the symmetries of the $P$-broken lattice, hence they are protected only against defects that do not mix chiralities, allowing for topological beam splitters\cite{113} (Figure 4E), as well as directional antennas\cite{116,117} (Figure 4D) and acoustic delay lines\cite{118} (Figure 4D) that can benefit from the high degree of tunability offered by phononic platforms.\cite{119,120} Bilayer metamaterials offer valley topological phases with more DOFs, such as valley or layer polarization in the same device.\cite{121-123} (Figure 4F).
PSEUDO-SPINS: PHONONIC SPIN-HALL INSULATORS

If phononic Chern insulators show promising transport with topologically robust properties, the fact that they rely on a strictly controlled angular momentum bias or modulation can become an issue for practical devices. Alternatively, as illustrated in Figure 1D,E, we can seek topological modes in spin systems obeying $T$ symmetry. Unfortunately, phononic waves are bosons, meaning that in this case the time reversal operator follows $T^2 = 1$, thus Kramers degeneracy does not hold, preventing the existence of a natural quantum spin Hall effect for phonons and the associated topological phenomena.

Nevertheless, this challenge can be circumvented by introducing external DOFs within a tight-binding structure, a sort of pseudo-spin, by engineering couplings that mimic a spin–orbit interaction. A first example was proposed in lattices of pendula dimers coupled with designed multilayer spring arrangements.$^{124,125}$ (Figure 5A). Here, the polarizations of each dimer act as a pseudo-spin degree of freedom, and the experimental demonstration using high-speed cameras demonstrates quite remarkably the robust helical edge propagation.$^{124,126}$

A different demonstration has relied on the addition of a second layer, along with chiral interlayer couplings.$^{126}$ as shown in Figure 5B. Remarkably, Deng et al. demonstrated a similar effect for sound waves and proved that, even if the topological protection of these helical edge modes remains, it is possible to flip the corresponding pseudo-spin by cutting the crystal in the middle of the unit cell.$^{127}$ This property is used to implement a so-called spin-flipper (Figure 5C).

A careful design of the lattice internal or external spatial symmetries $\Sigma$ can create pseudo-spins and the related pseudo-time reversal operator $\Sigma T$ such as $(\Sigma T)^2 = -1$. The idea is to find the equivalent of a pseudo-spin–orbital coupling for the particular symmetry $\Sigma$ that causes bandgap opening with a band inversion. A powerful example consists of a triangular lattice of pillars in the air whose band structure has two overlapping Dirac cones at the center of the BZ.$^{128}$ Subtle tuning of the scatterers’ radius lifts the four-band degeneracy, resulting in two pairs of bands separated by a bandgap. These pairs of bands are related to dipolar and quadrupolar symmetries, which are associated with pseudo-spins. Depending on the increase or decrease of the radius size, the quadrupolar and dipolar bands are inverted, leading to two distinct topological behaviors. An interface between the two topologically distinct media carries helical waves whose direction of propagation is locked to the sense of rotation of local acoustic vortices along the interface (Figure 5D). However, the topological protection of these modes directly relies on the spatial symmetries of the lattice that construct the pseudo-spin states of the modes. As the interface inevitably breaks these symmetries, the topological protection is hampered, leading to a small bandgap opening between the helical edge bands, hence making these interface modes inherently less robust than their electronic counterparts or even the phononic bilayer designs mentioned above.$^{124,126,127}$ which are independent of the lattice symmetries. Nevertheless, their peculiar features allow designing topological beam splitters ruled by pseudo-spin conservation.$^{128}$

The concept of opening a nontrivial topological bandgap from two overlapping Dirac cones in the center of the BZ can also be applied within triangular lattices consisting of six “meta-atoms,” which can be either shrunk or expanded.$^{129}$ In this case, the initial four-band degeneracy of the unperturbed lattice is purely due to artificial band folding. The relative simplicity of this protocol has motivated a plethora of studies of various platforms, ranging from soda can metamaterials$^{130}$ and macroscopic phononic crystals$^{131–133}$ (Figure 5E) to on-chip systems,$^{144–148}$ whose reconfigurability opens the door to practical devices.$^{141,142}$ Moreover, Lamb waves in thin plates and their intrinsic polarization provide another handle to realize a pseudo-spin Hall effect and implement helical edge states.$^{149,150}$ By breaking the out-of-plane symmetry of precisely patterned plates, a topological bandgap of the mechanical waves is opened, and it is populated by edge states protected from backscattering against defects (Figure 5F). The rotational degree of freedom of beads within granular media can also induce a pseudo-spin Hall topological phase.$^{151}$ In addition, the great flexibility inherent to phononic systems makes it straightforward to combine different topological phases, such as pseudo-spin and valley Hall topological insulators, to enhance wave manipulation and control through valley splitting of helical edge states.$^{152}$

BEYOND PHONONIC 2D TOPOLOGICAL INSULATORS

If phononic 2D topological insulators find their roots in the transposition of solid-state physics concepts, the transition to macroscopic scales, and the enhanced tunability that comes along with that, allows to envision new topological phases for waves that can go far beyond what can be expected in condensed matter. A first example relies on the control of the coupling sign in the phononic tight-binding model. Indeed, cavities with multipolar modes can be linked with positive or negative interaction, depending on the position where the coupling tubes are connected. This opens the door to the direct implementation of more elaborate tight-binding Hamiltonians relevant to new topological phases. The most relevant example is the realization of phononic higher-order topological insulators (HOTIs),$^{153–155}$ which not only carry wave propagation along their edges but also support field localization at their corners, referred to as second-order topological states. These lower dimension modes are predicted by bulk topological invariants, ensuring their protection against perturbations. A first family of these HOTIs relies on quantized multipole moments that are induced by the adequate distribution of positive and negative couplings within the lattice, leading to a so-called $\pi$-flux per unit-cell.$^{153}$ One of the first experimental characterizations of this topological phase for sound was performed using an elastic plate consisting of a square lattice of quadrupolar resonators$^{156}$ (Figure 6A), before being transposed to sound in a tight-binding medium of dipolar cavities.$^{157}$ Interestingly, these quantized HOTIs also exist in nonsymmorphic phononic crystals$^{158,159}$ without $\pi$-flux, pushing the implementation of topological corner states beyond the
FIGURE 5  Phononic spin-Hall insulator analogs. (A) Mechanical implementation with pendula and springs. Image reproduced from Ref. 124 with permission. (B) Mechanical implementation with bilayer media. Image reproduced from Ref. 126 with permission. (C) Acoustic implementation with bilayer tight-binding media and corresponding spin-flipper. Image reproduced from Ref. 127 with permission. (D) Acoustic implementation using in-plane lattice symmetries and beam splitter based on pseudo-spin degree-of-freedom. Image reproduced from Ref. 128 with permission. (E) Elastic implementation using in-plane lattice symmetries. Image reproduced from Ref. 136 with permission. (F) Thin plate implementation using Lamb wave polarizations. Image reproduced from Ref. 149 with permission.

tight-binding systems to the realm of phononic crystals described by Bragg interference, hence expanding the range of platforms supporting higher-order topological phases. In a related context, a second family of HOTIs is characterized by bulk polarization and it does not require negative coupling to achieve topological corner modes, since it only relies on the difference between inter- and intra-cell couplings.155 This phase has also been widely investigated in phononics,160–173 as shown in Figure 6B for the case of breathing Kagome lattices, along with lattices made of rotated scatterers174–176 and combined with Floquet physics.177 New exciting phenomena based on projective symmetry, such as Möbius-twisted topological phases, have been implemented in coupled acoustic resonators using couplings of both positive and negative signs.178,179 Besides, macroscopic phononic media allow to finely tune the couplings’ orientation and range, leading to remarkable phenomena, such as non-Abelian phenomena54,180,181 and nonlinearly induced topological interface modes.182

While precisely controlling loss and gain within a solid-state material can be very challenging, phononic systems at the macroscopic scale allow easy control of loss and gain channels in the medium, offering new avenues to the experimental realization of topological phenomena of non-Hermitian nature. For example, the combination of the valley-Hall effect with gain and loss generates topological edge states with enhanced amplitude.183 Notably, fine control of phase distribution within the unit cell of the lattice induces topological “audio lasing” of whispering gallery modes with a specific handedness,184 as shown in Figure 6C. Other intriguing phenomena, such as the non-Hermitian phononic skin-effect185–188 (Figure 6D) or non-Hermitian higher-order acoustic phases188–190 (Figure 6E), have also been investigated experimentally based on this platform.
Finally, phononic metamaterials offer a straightforward platform to go beyond purely periodic topological phases and investigate the impact of the disorder on wave propagation. One example is the demonstration of phononic topological defects, which can be used as robust waveguides but also to realize topological cavities induced by disclination, as respectively shown in Figure 6F and G. Topological cavity modes can also be induced through a Kekulé distortion conferring on them strong robustness against spatial perturbations (Figure 6H). Nonperiodic phononic media have also been studied in the context of elastic Chern insulators and higher-order phases.

CONCLUSION AND FUTURE DIRECTIONS

In this article, we have discussed how Dirac cones in 2D electronic materials and the associated topological phases can be transposed to sound waves. Starting from reproducing graphene’s Dirac cones in phononic crystals and metamaterials, several research groups have spent tremendous efforts in the last decade trying to lift the Dirac cone degeneracy and thus induce topological features for sound based on various approaches. These efforts have led to the demonstration of phononic Chern insulators and time-Floquet media supporting nonreciprocal edge propagation of sound with broken symmetry, valley edge states for broken lattices, and helical boundary modes in the context of pseudo-spin-Hall insulators. Novel topological phases have been tested for sound and elastic waves thanks to the genuine tunability of phononic crystals and metamaterials. This substantial advantage, along with the diversity of platforms presented in this review, ranging from macroscopic pendula to on-chip nano-resonators, has enabled the demonstration of a wide range of topological phases for sound and elastic waves, offering new possibilities for practical devices achieving enhanced wave control and steering, unusually robust acoustic devices, and providing platforms for exploring exciting fundamental phenomena.

Two-dimensional topological phononics research opens several exciting research directions, in particular when considering the possibility of including non-Hermitian as well as non-Abelian features in...
the phononic platforms, which are absent in conventional models for the topological matter. These features show great potential to implement extreme phenomena for sounds, such as topological lasing,\textsuperscript{184} or non-Abelian operations,\textsuperscript{54} Moreover, the emergence of disordered topological phases, such as topological Anderson insulators,\textsuperscript{199} and the study of nonlinearities\textsuperscript{182} in the context of topological phononics opens numerous new avenues. From a practical perspective, the translation of 2D topological phononic phenomena to on-chip systems may pave the way to a plethora of stimulating applications, not only in the domain of communications but also for biological sensing in the context of lab-on-a-chip,\textsuperscript{200} among other opportunities. Such miniaturization processes will strongly benefit from recent advances in topological phononics since the described functionalities, translated to chip scale, may enable a higher degree of robustness to fabrication imperfections due to topological protection.

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