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SCATTERING PROCESSES WITH THE EMISSION OF LONGITUDINAL OPTICAL PHONONS IN THE LANDAU LEVEL SYSTEM IN GaAs/AlGaAs QUANTUM WELL

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Abstract – The scattering processes of longitudinal optical phonons in GaAs/AlGaAs quantum wells in a quantizing magnetic field are considered. The time of intrasubband scattering between Landau levels is calculated by using Fermi’s golden rule. The dependence of the scattering rate on the magnitude of the magnetic field has been shown and the magnetic field can suppress scattering processes on longitudinal optical phonons. It is found that the scattering time depends linearly on the width of the quantum well.

Key word: quantum well; Landau level; optical phonons scattering

1. Introduction

Scattering by optical phonons is an important mechanism of electron energy relaxation in semiconductor quantum wells, which significantly determines their transport and optical properties. For this reason, much attention has been paid to the problem of scattering by phonons in two-dimensional subbands of quantum wells, and for A3B5 systems this problem has been thoroughly and comprehensively studied [1].

The situation is completely different in the case when a quantum well is superimposed with a quantizing magnetic field. In this case, a strong magnetic field changes the 2D parabolic energy dispersion of each size-quantized subband \( \varepsilon_v(k) \) into a set of discrete, equidistant, 0D-like, Landau levels, \( E_{(v,n)} = \varepsilon_v + \hbar \omega_c (n + \frac{1}{2}) \), separated by the cyclotron energy \( \hbar \omega_c \), where \( v \) is the subband index, \( n \) is the Landau level index [2, 3]. Moreover, the distance between adjacent levels in the subband – the Landau energy – significantly exceeds their thermal broadening and width due to intra-level scattering (determination of the quantizing magnetic field). As a result, the relative role and behavior of scattering processes, including the scattering process on optical phonons, change. In the system of Landau levels, scattering on acoustic phonons is significantly suppressed, and as a result, scattering on optical phonons becomes the only effective mechanism for relaxation of the energy of the electronic subsystem [11 – 13]. Thus, scattering with emission of optical phonons is an important scattering mechanism in the Landau level system of a quantum well.

The processes of scattering by optical phonons in quantum wells were considered in [4,5]. However, the calculation was carried out here only for the interstitial scattering processes in several structures. Also, the intersubband scattering processes were considered in [6 – 10], in relation to a very limited set of cascades of quantum cascade lasers. As for the intrasubband scattering by optical phonons, their calculation was performed in [11 – 13]. However, in these work, only one GaAs/Al_{0.3}Ga_{0.7}As quantum well with a width of 25 nm was calculated. In our work, we calculate the scattering time with the emission of an optical phonon between the Landau levels in the lower subband of the GaAs/AlGaAs quantum well, and the dependence of the scattering time on the width of the quantum well and the magnitude of the magnetic field.
is studied. As is known, in the oscillation spectrum of the GaAs crystal lattice, there is a sharp peak falling on the longitudinal optical phonon with a frequency of $\omega_{LO} = 36$ meV, so that the overwhelming contribution is made by scattering on the longitudinal optical phonon with a given frequency. Accordingly, the paper considers the scattering on this optical phonon.

2. Optical phonons scattering time between Landau levels in a quantum well

Consider electron in the quantum well structure in the magnetic field $B = B_x$, where $z$ is the growth axis. In Landau gauge $A = (B_x z - B_z y)e_x$ the single-electron spectrum is a set of confinement subbands each splitting into a series of Landau levels [2, 3]

$$E_{(\nu, n)} = \varepsilon_{\nu} + \hbar \omega_c (n + \frac{1}{2})$$  \hspace{1cm} (1)

with wave functions

$$\psi(x, y, z) = \frac{\exp(ikx)}{\sqrt{L}} \varphi_{\nu}(z) \Phi_n(y - k \ell^2)$$  \hspace{1cm} (2)

determined by three quantum numbers – subband index $\nu$, Landau level number $n$ and $x$-axis momentum component $k$. $\ell = \sqrt{\frac{\hbar e B}{m \omega_c}}$ is the magnetic length. $\Phi_n(y) = \frac{1}{\sqrt{2^{2n}n!}} \exp(-\frac{y^2}{2 \ell^2}) H_n\left(\frac{y}{\ell}\right)$ is the wave function of harmonic oscillator with effective mass $m$ and cyclotron frequency $\omega_c = \frac{eB}{mc}$. $\varepsilon_{\nu}$ and $\varphi_{\nu}(z)$ are the energy and wave function of $\nu$-th subband.

Using the Fermi golden rule approximation, the total optical phonons scattering time from the Landau level $i = (\nu_i, n_i)$ to the Landau level $f = (\nu_f, n_f)$ is given by [12, 13]

$$\frac{1}{\tau_{i \rightarrow f}} = \frac{2 \pi}{S \alpha} \sum_{k_i, k_f, q} \left| \left\langle \hat{H}_{e-ph}(q) \right| \psi_{f,i} \right\rangle \left\langle \psi_{i,f} \right| \delta(E_{\nu_i, n_i} - E_{\nu_f, n_f} - \hbar \omega_{LO})$$  \hspace{1cm} (3)

Here $S$ is the transverse area of the sample (the area of the plane of the layers).

$$\hat{H}_{e-ph}(q) = \frac{i g}{q} \exp(-i q r) \sqrt{1 + N_B(\hbar \omega_{LO})}$$

is the Frelich Hamiltonian for the interaction of an electron with a polar longitudinal optical phonon with a wave vector $q$. $g = 2 \pi \hbar \omega_{LO} \frac{\varepsilon_s}{\varepsilon_x}$ - the force of the interaction of an electron with a phonon (Fleurich multiplier); $\omega_{LO}$ - optical phonon frequency; $\frac{1}{\varepsilon_x} = \frac{1}{\varepsilon_y} - \frac{1}{\varepsilon_z}$, $\varepsilon_s$ is high frequency permittivity and $\varepsilon_x$ - static permittivity; $V = L_z S$ is sample volume; $N_B(x) = \frac{1}{\exp(x/k_B T) - 1}$ - Bose Einstein distribution; $T$ - lattice temperature.

Substituting the expressions for the wave function (1) and energy levels (2) into the expression for the scattering time (3), and performing the integration by the coordinates in the plane of the layers $x$ and $y$, by the projections on the $x$-axis of the wave vector of the initial $k_i$ and final $k_f$ states, as well as by the projection $q_z$ on the $z$-axis of the wave vector $q$ and the direction of its component in the plane of the layers $q_z = q_x e_x + q_y e_y$, we obtain the following
expression for the scattering time from the Landau level \( n_i \) of the subband \( \nu_i \) to the Landau level \( n_f \) of the subband \( \nu_f \)

\[
\frac{1}{\tau_{i \rightarrow f}} = W_{i \rightarrow f} \delta \left( e_{\nu_i} - e_{\nu_f} + \hbar \omega_c (n_i - n_f) - \hbar \omega_{LO} \right). \tag{4}
\]

Where the probability of a transition

\[
W_{i \rightarrow f} = \frac{\sqrt{2} \pi \omega_{LO}}{n_i! n_f!} \left[ 1 + N_g (\hbar \omega_{LO}) \right] \frac{e^2}{\varepsilon_p} \int_0^\infty d\beta f \left( \frac{\sqrt{2} \beta}{\ell} \right) \beta^{2(n_i - n_f)} P_n^2 (\beta) \exp(-\beta^2). \tag{5}
\]

Here \( \beta = q_\perp \ell \) is the dimensionless module of the phonon wave vector component \( q_\perp = q_x + q_y \) in the plane of the layers. The polynomial \( P \) is given by the expression

\[
P_n (\beta) = \sum_{j=0}^{n_f} (-1)^j C_n^j C_{n_f}^j \beta^{2(n_f - j)}. \tag{6}
\]

In the expression (6) \( C_n^j = \frac{n!}{(n-j)!j!} \) is binomial coefficient. The function \( f \) is a double integral along the coordinate along the growth axis of the structure from the product of the wave functions of the initial \( \varphi_{\nu_i} (z) \) and final \( \varphi_{\nu_f} (z) \) subbands of dimensional quantization and such an exponent

\[
f \left( \frac{\sqrt{2} \beta}{\ell} \right) = \int dz_1 dz_2 \varphi_{\nu_i}^{*} (z_1) \varphi_{\nu_i} (z_1) \varphi_{\nu_f}^{*} (z_2) \varphi_{\nu_f} (z_2) \exp \left( -\frac{\sqrt{2} \beta |z_1 - z_2|}{\ell} \right). \tag{7}
\]

As a rule, the Landau levels lying deep in the quantum well are of interest, so that the penetration of their wave functions into the barrier can be neglected. Therefore, in further calculations, we use expressions for wave functions of dimensional quantization along the structure axis for an infinitely deep quantum well

\[
\varphi_{\nu} (z) = \sqrt{\frac{2}{a}} \sin \left( \frac{\nu \pi}{a} z \right). \tag{8}
\]

Here \( a \) is the width of the quantum well. In this case, the integral \( f \) is calculated analytically and we get the following expression for the probability of a transition \( W_{i \rightarrow f} \)

\[
W_{i \rightarrow f} = \frac{\sqrt{2} \pi \omega_{LO}}{n_i! n_f!} \left[ 1 + N_g (\hbar \omega_{LO}) \right] \frac{e^2}{\varepsilon_p} \frac{1}{a} F \left( \frac{\ell}{a} \right). \tag{9}
\]

Where

\[
F \left( \frac{\ell}{a} \right) = \frac{\sqrt{2}}{2} \int_0^\infty d\beta \left( \frac{\delta_{\nu_i \nu_f}}{\beta^2 + 2 \omega_c (\nu_i - \nu_f) \beta + \frac{1}{2 \omega_c^2 (\nu_i - \nu_f)^2}} \right) \beta^{2(n_i - n_f) + 1} P_n^2 (\beta) \exp(-\beta^2) +
\]

\[
+ \frac{\ell}{a} \int_0^\infty d\beta \left( \frac{\delta_{\nu_i \nu_f}}{\beta^2 + 2 \omega_c (\nu_i - \nu_f) \beta + \frac{1}{2 \omega_c^2 (\nu_i - \nu_f)^2}} \right) \left[ (-1)^{\nu_i + \nu_f} \exp \left( \frac{\sqrt{2} \beta a}{\ell} \right) - 1 \right] \beta^{2(n_i - n_f) + 2} P_n^2 (\beta) \exp(-\beta^2)
\]
Thus, the expression for the scattering time is significantly simplified by analytical integration: the initial problem of calculating a six-dimensional integral is reduced to the problem of calculating two one-dimensional integrals.

3. Results of the numerical calculation and discussion

The expression for the scattering time (4) includes the Dirac delta function, which expresses the law of conservation of energy. The zero of its argument corresponds to the situation when the difference between the electron energy at the initial and final levels is equal to the energy of the optical phonon, i.e. when there is a scattering resonance. However, Landau levels always have a finite width. Therefore, in reality, instead of the delta function, there is a peak of finite width. The problem of the exact shape and width of the peak is quite complex, and its solution depends on many factors. First of all, the width and shape of the peak are determined by the density of states, and strongly depend on the concentration of carriers, the ratio between the magnetic length and the average distance between the scatterers, for example, impurities and the radius action of their forces. Moreover, the width and shape depend very much on technological factors, in particular on the structure of the interfaces [14]. Thus, the width, and in particular, the shape of the peak depends on both the physical parameters and the technological parameters of the structures. The question of the shape and width of the peak is beyond the scope of this work. Here, in fact, we study the optical phonon scattering time in resonance, when the distance between the initial and final Landau levels is equal to the energy of the optical phonon, i.e., in fact, we study the behavior of the multiplier $W$ (probability of a transition) before the delta function. In resonance, the value of this peak is usually inversely proportional to the width of the levels. For clarity, in order to set the scale, in further calculations, the delta function is replaced by a Gaussian

$$
\delta(E_{v_i,n_i} - E_{v_f,n_f} - \hbar \omega_{LO}) = \frac{1}{\sqrt{2\pi}\Gamma} \exp\left(-\frac{(E_{v_i,n_i} - E_{v_f,n_f} - \hbar \omega_{LO})^2}{2\Gamma^2}\right)
$$

(10)

with a typical width of $\Gamma = 1$ meV.

Figure 1. Time (a) and rate (b) of optical phonon scattering from the Landau levels $n = 6; 5; 4$ and $3$ of the lowest subband ($\nu = 1$) to the underlying Landau levels ($1; 0$) in $\text{GaAs/Al}_{0.3}\text{Ga}_{0.7}\text{As}$ quantum well with a width of $a = 25$ nm as a function of the magnetic field.
We numerically calculate the intrasubband optical phonon scattering times and rate. The calculations were carried out for GaAs/Al$_{0.3}$Ga$_{0.7}$As quantum wells of 25 nm width. Figure 1 shows the dependence on the magnetic field of the scattering time (a) and rate (b) from the excited Landau levels to the zero Landau of the lowest subband. As expected, in a magnetic field, when the distance between the corresponding levels is equal to the energy of the optical phonon, time minima are observed, and, accordingly, the scattering rate maxima between these levels are observed. For example, the leftmost minimum corresponds to the situation when there is a scattering resonance from the sixth to the zero Landau level. With a further increase in the magnetic field, the Landau energy increases. The distance between the sixth and zero levels is growing, and these levels are out of resonance. Accordingly, the scattering time between these levels increases. However, at the same time, the fifth and zero levels begin to approach the resonance. Accordingly, the scattering time between levels decreases. Starting from a certain magnetic field, this transition becomes predominant, and its time reaches a minimum in a magnetic field when the distance between the levels is compared with the energy of an optical phonon. Then this distance becomes greater than the energy of the optical phonon, and this transition begins to turn off. But the transition from the fourth level begins to turn on, etc.

Figure 2. The dependence of the optical phonon scattering time from Landau levels a) $n=6$; b) $n=5$; c) $n=4$ and d) $n=3$ of the first subband to the underlying Landau levels, calculated with a magnitude of the magnetic field in resonance (eq. 11).

For optical phonon scattering between Landau levels of the same subband, the magnitude of the magnetic field when the scattering time is minimal, and, accordingly, the
maximum scattering speed does not depend on the width of the quantum well. This magnitude of the magnetic field is given by the following expression

$$B_{n_i,n_j}^{res} = \frac{mc}{e} \frac{\hbar \omega_{LO}}{n_i - n_f}. \quad (11)$$

Therefore, it is natural to pose the problem of the dependence of this minimum time on the width of quantum well. Figure 2 shows the dependence of the minimum scattering time calculated for a magnetic field in resonance (eq. 11) on the width of the quantum well for transitions between different Landau levels. It can be seen that the minimum time for all transitions depends linearly on the width of the quantum well in a wide range of values of the width of the quantum well from 5 to 30 nm. This is exactly the range of widths of GaAs/Al$_{0.3}$Ga$_{0.7}$As quantum wells that are usually used in practice. Pits of smaller widths are technologically difficult to grow. In wells with width above 30 nm, as a rule, the quantum-dimensional effect becomes insignificant. Apparently, this linear dependence is associated with a change in the scattering intensity when the ratio of the magnetic length and width of the quantum well changes. The magnetic field in the resonance of this transition does not depend on the width of the quantum well. Therefore, when the width of the quantum well changes, the ratio changes inversely proportional to the width of the quantum well.

This behavior of the optical phonon scattering time between Landau levels differs significantly from its behavior in the continuous subband of quantum wells. In the latter case, this dependence is practically absent. The result obtained is new and rather unexpected.

4. Conclusion

In this work, the intrasubband scattering processes in GaAs quantum wells in a quantizing magnetic field are studied. A numerical calculation of the scattering time has been performed. It is shown that the scattering time increases linearly with an increase in the width of the quantum well, which is qualitatively different from the behavior of the scattering time in the continuous subband of the quantum well, where this dependence is practically absent.

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