Creep rupture as a non-homogeneous Poissonian process

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Creep rupture of heterogeneous materials occurring under constant sub-critical external loads is responsible for the collapse of engineering constructions and for natural catastrophes. Acoustic monitoring of crackling bursts provides microscopic insight into the failure process. Based on a fiber bundle model, we show that the accelerating bursting activity when approaching failure can be described by the Omori law. For long range load redistribution the time series of bursts proved to be a non-homogeneous Poissonian process with power law distributed burst sizes and waiting times. We demonstrate that limitations of experiments such as finite detection threshold and time resolution have striking effects on the characteristic exponents, which have to be taken into account when comparing model calculations with experiments. Recording events solely within the Omori time to failure the size distribution of bursts has a crossover to a lower exponent which is promising for forecasting the imminent catastrophic failure.

Materials subject to a constant external load below their fracture strength typically exhibit a time dependent response and fail in a finite time. Such creep rupture phenomena have an enormous technological importance and human impact since they are responsible for the collapse of engineering constructions and they lie at the core of natural catastrophes such as landslides, snow and stone avalanches and earthquakes. The acoustic waves generated by the nucleation and propagation of cracks allow for the monitoring of the failure process on the meso- and micro scales. Crackling noise is usually characterized by the integrated statistics accumulating all the events of the time series up to failure. Experiments revealed that the probability distribution of the energy of crackling bursts and of the interoccurrence times have power law functional form, which are considered to be the fingerprint of correlations in the microscopic breaking dynamics. The value of the exponents measured on different types of heterogenous materials show a surprisingly large scatter between 1 and 2 which has not been captured by theoretical studies. The approach to failure is usually characterized on the macroscale by the strain rate which proved to have a power law divergence as a function of time to failure.

Here we take a different strategy and investigate the details of the crackling time series in order to understand how the creeping system evolves towards catastrophic failure. We consider a generic fiber bundle model (FBM) of damage enhanced creep rupture which successfully reproduces measured creep behaviour of heterogeneous materials both on the micro and macro scales (Methods). In the model under a constant subcritical load the fibers break due to two physical mechanisms: immediate breaking occurs when the load of fibers exceeds the local failure strength. Time dependence emerges such that intact fibers accumulate damage which results in failure in a finite time. The separation of time scales of slow damaging and of immediate breaking together with the load redistribution following failure events lead to a highly complex time evolution where slowly proceeding damage sequences trigger bursts of immediate breakings. An example of the time series of bursts can be seen in Fig. 1 where the increasing burst size and the decreasing waiting time between consecutive events clearly mark the acceleration of the system towards failure. As a novel approach to creep we focus on the evolution of the rate of bursts and show that the temporal occurrence of crackling events and the power law statistics of waiting times can fully be described based on non-homogeneous Poissonian processes without assuming correlations of bursts. Our investigation unveils that limitations of measuring devices in experiments have astonishing effects on the outcomes of crackling noise analysis which can explain the strong scatter of measured critical exponents of crackling noise in creep, and the discrepancy between experimental findings and theoretical approaches. Studying how the time series evolves when approaching the catastrophe we address the possibility of forecasting the imminent failure. The importance of the results goes beyond fracture phenomena and catastrophic failures, recently the human activity has been found to exhibit similar bursty character where analogous problems of the evolution of time series and waiting time statistics occur.
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Results

Bursts of immediate breakings triggered by damage sequences are analogous to acoustic outbreaks in loaded specimens. However, damage breakings cannot be recorded by experimental means they determine the waiting time \( T \) between consecutive bursts. The competition of the two failure modes has the consequence that the system drives itself towards failure under a constant subcritical external load \( \sigma_0 < \sigma_c \). The global acceleration of the dynamics that can be observed in Fig. 1 is the consequence of the increasing load on the intact part of the system due to subsequent load redistributions, while the fluctuations of the burst size \( \Delta \) and waiting time \( T \) emerge due to the quenched heterogeneity of fibers' strength in qualitative agreement with experiments1-11.

In order to quantify how the accelerating dynamics appears on the microscale we determined the rate of bursts \( n(t) \) as a function of the distance from the critical point \( t_f - t \). Figure 2(a) shows that at the beginning of the creep process the event rate monotonically increases having a power law functional form. Approaching catastrophic failure \( n(t) \) saturates and converges to a constant. The most remarkable result is that the functional form of \( n(t) \) can be described by the modified Omori law26,27:

\[
 n(t) = \frac{A}{[1 + (t_f - t)/c]^p}, \tag{1}
\]

where \( A \) is the saturation rate or productivity at catastrophic failure, \( c \) denotes the characteristic time scale, and \( p \) is the Omori exponent. Perfect agreement can be observed in Fig. 2(a) between the simulated data and the analytic form of equation (1). In the case of earthquakes, the Omori law describes the decay rate of aftershocks following major earthquakes26,27. For creep rupture we observe the inverse process: considering the macroscopic failure as the main shock, the breaking bursts are foreshocks whose increasing rate is described by the (inverse) Omori law.

As a crucial point, our approach makes it possible to clarify how the characteristic time scale \( c \) of the Omori law emerges: Figure 2(b) illustrates that due to the increasing load on intact fibers shorter and shorter damage sequences are sufficient to trigger bursts. However, this acceleration is limited such that for \( t_f - t < c \) the average length of damage sequences \( \langle \Delta \rangle \) saturates between 1 and 2. The origin of this high susceptibility is that the load of intact fibers \( \sigma(t) \) gradually increases to its quasi-static critical value \( \sigma^s_c \) (see Fig. 2(b))16-22. Hence, the Omori time scale \( c \) is determined by the condition \( \sigma(t) - t = c = \sigma^s_c \), where \( \sigma^s_c \) can be obtained from the quasi-static constitutive equation of FBMs16-22. Note that the condition \( \langle \Delta \rangle = 1 \) marks the point of instability where the avalanche cannot stop anymore and it becomes catastrophic.

Figure 2(a) also demonstrates that the saturation rate \( A \) does not depend on the external load \( \sigma_0 \); however, the characteristic time scale \( c \) linearly increases \( c \sim \sigma_0 \) indicating that at higher load saturation sets on earlier. Our simulations revealed that the Omori exponent is \( p = 1 \), it does not depend on any details of the damage law21,22 such as the \( \gamma \) exponent and the disorder distributions until the load redistribution is long ranged.

The event rate \( n(t) \) is practically the inverse of the average waiting time between events occurring at time \( t \). More detailed characterization is provided by the probability distribution of waiting times \( P(T) \) which is presented in Fig. 3 corresponding to the system of Fig. 2. Along the distributions two characteristic time scales can be identified: for waiting times below a threshold \( T < T_b \) the distributions have constant values, while in the limit of large waiting times \( T > T_u \) a rapidly decreasing exponential form is obtained.

For the intermediate regime \( T_i < T < T_u \) the waiting time distributions exhibit a power law behavior

\[
P(T) \sim T^{-\gamma}, \tag{2}
\]

where the exponent proved to be universal \( \gamma = 1 \). Increasing the external load \( \sigma_0 \) the upper cutoff \( T_u \) decreases, however, the lower characteristic time \( T_i \) is independent of \( \sigma_0 \). Since the temporal occurrence of events is determined by the global increase of the breaking probability due to the increasing load on intact fibers, the above results suggest that the time evolution of crackling noise of heterogeneous materials undergoing creep rupture can be described as a
The comparison of theoretical results to the experimental findings and different types of measurements to each other can be problematic because in laboratory experiments the time series of bursts is never complete: small size bursts generate only low amplitude signals which may fall in the range of background noise\(^{1-11}\). Devices also have a finite time resolution resulting in a deadtime of detection, during which bursts generated at different positions in space cannot be distinguished from each other\(^{11,15,30}\). Incompleteness of the time series, especially in field observations, may also be caused by the fact that recording does not start exactly at the time when the load was set. Hence, the beginning of the time series is missing and the measurement is more focused on the vicinity of the failure point where intensive cracking occurs\(^{1-3,5,23}\).

In order to capture the effect of the detection threshold of the measuring equipment in the data evaluation, we introduced a threshold value \(\Delta_{th}\) for the size of bursts \(\Delta\), i.e. bursts with size \(\Delta \leq \Delta_{th}\) are ignored in the time series. Since the size of bursts increases when approaching global rupture (see Fig. 1), the detection threshold removes events typically at the beginning of the time series decreasing the rate of events in this regime. It can be observed in Fig. 4(a) that as \(\Delta_{th}\) increases the functional form of the event rate \(n(t, \Delta_{th})\) remains nearly the same described by the Omori law equation (1), however, the exponent \(p\) monotonically increases with the threshold value \(\Delta_{th}\). For the corresponding waiting time distributions \(P(T, \Delta_{th})\) in Fig. 4(b), the exponent \(z\) of the power law regime also increases with increasing \(\Delta_{th}\), however, the NHPP nature of the event series is preserved at any values of \(\Delta_{th}\). For NHPPs the two exponents \(z\) and \(p\) have the simple relation\(^{28}\)

\[
z(\Delta_{th}) = 2 - 1/p(\Delta_{th}),
\]

which holds with a high accuracy in our system for the numerically determined exponents at any values of \(\Delta_{th}\) (see Fig. 4(c)). It is important to emphasize that the exponents increase due to the non-stationarity of creep rupture: the simultaneous increase of the rate and size of events towards failure has the consequence that the finite detection threshold mainly affects the beginning of the time series resulting in a few long waiting times up to the first bigger bursts. Their statistics is characterized by a peak or small bump in Fig. 4(b), while the rest of waiting times have a steeper power law distribution. The results demonstrate that the detection threshold has a dramatic effect on the outcomes of the analysis of cracking time series, just varying \(\Delta_{th}\) practically any values can be obtained for the waiting time exponent \(z\) between 1 and 2.

The finite time resolution \(t_d\) of the detectors has the consequence that bursts pile up, i.e. since bursts cannot be distinguished within the duration \(t_d\) the size of bursts sums up giving rise to larger event sizes in the time series. The effect of the deadtime is captured in the data evaluation such that if an avalanche of size \(\Delta\) occurred at time \(t\), all those avalanches which appeared in the interval \(t < t < t + t_d\) are added to \(\Delta\). In Fig. 5(a) at zero deadtime \(t_d = 0\) where all avalanches

\[
\begin{align*}
\sigma_0/\sigma_c &= 10^{-1} \quad \text{or} \quad 10^{-2} \\
\sigma_0/\sigma_c &= 10^{-3} \\
\sigma_0/\sigma_c &= 10^{-4}
\end{align*}
\]

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\]
are distinguished the size distribution $P(\Delta, t_d = 0)$ has a power law form

$$P(\Delta, t_d=0) \sim \Delta^{-\gamma} \quad (5)$$

followed by an exponential cutoff. The value of the exponent $\gamma = 2.5$ is equal to the usual mean field burst exponent of FBMs\textsuperscript{11-12}. As $t_d$ increases the pile up of bursts promotes large events while the smaller ones get suppressed. Due to the acceleration of the failure process pile-up gets dominating in the vicinity of macroscopic failure, hence, in Fig. 5(a) the value of $t_d$ is compared to $T_1$ of the waiting time distribution. As a consequence, the waiting time distributions hardly change, however, the burst size distribution $P(\Delta, t_d)$ has a crossover to a power law of a significantly lower exponent $\gamma = 2.0$ showing the higher frequency of large events in the statistics. In Fig. 5(a) pile up becomes dominating already at $t_d/T_1 = 0.001$, which shows the importance of the results for real experiments.

Recently, laboratory experiments on earth materials have revealed that the b-value, i.e. the exponent of the probability distribution of the energy of the time series of acoustic events of rupture cascades decreases in the vicinity of failure\textsuperscript{13-14}. To investigate the possibility of an analogous phenomenon in creep rupture, we constrained the data evaluation to events occurring in a time interval of duration $t^*$ preceding macroscopic failure and determined the probability distribution $P(\Delta, \tau_f = t < t^*)$. It can be observed in Fig. 5(b) that approaching macroscopic rupture $t^* < c$, where the largest avalanche are triggered, the burst size distribution $P(\Delta, \tau_f = t < t^*)$ exhibits a crossover: at a characteristic burst size $\Delta_c$, the exponent of $P(\Delta, \tau_f = t < t^*)$ has a striking change from $\gamma = 2.5$ to a surprisingly low value $\gamma = 1.4$. The value of $\Delta_c$ extends to the largest avalanche as $t^* < c$ decreases. The crossover is accompanied by the change of the waiting time distribution, as well: Since in the regime $t^* < c$ the event rate is constant, the power law regime of $P(\Delta, \tau_f = t < t^*)$ disappears and the distribution turns to a pure exponential as it is expected for homogeneous (constant event rate) Poissonian processes\textsuperscript{39}.

**Discussion**

Acoustic outbreaks generated by nucleating and propagating cracks provide the main source of information on the microscopic temporal dynamics of creep rupture. For the understanding of acoustic monitoring data of engineering constructions and of field measurements on steep slopes or rock walls in mountains requires the application of statistical physics. Our analysis showed that time-to-failure power laws of macroscopic quantities such as creep rate commonly observed in experiments are accompanied by the emergence of Omori type acceleration of the bursting activity on the microscale. The origin of the Omori time scale is that the aging system drives itself to a critical state where a few breakages are sufficient to trigger extended bursts. The Omori law is known to describe the relaxation of the rate of aftershocks following major earthquakes\textsuperscript{26-27}, and it has also been confirmed for foreshocks when observed\textsuperscript{32}. Our results suggest the interpretation that acoustic bursts in creep behavie like foreshocks of the imminent catastrophe.

Our investigations unveiled that the evolving time series of cracking events is the result of an underlying non-homogeneous Poissonian process. It has the striking consequence that observing power law distributed waiting times in fracture may not imply the presence of dynamic correlations, up to a large extent it can be caused by the global acceleration of the system. We showed that special care should be taken when comparing results of model calculations to measurements on cracking noise, since the deadline of devices and the finite background noise to signal ratio can even affect the measured value of critical exponents. Varying solely the detection threshold of events, for the distribution of waiting times any exponents can be obtained between 1 and 2 covering the range of experimental results\textsuperscript{15-17}. Due to the finite deadline of electronics bursts pile up which gives rise to a crossover to a lower exponent of the size distribution of bursts. Recently, avalanches have been identified with a high spacial resolution along a propagating crack front using optical imaging techniques\textsuperscript{19-20}. Considering global avalanches in the same experiment integrates bursts along the front, giving rise to a significantly lower exponent in agreement with our predictions\textsuperscript{15-30}.

Components of engineering constructions are mainly subject to creep loads\textsuperscript{9}, and creep rupture often lies at the core of natural catastrophes such as landslides, snow and stone avalanches, as well\textsuperscript{13-14}. We demonstrated that restricting the measurement to the close vicinity of ultimate failure, the size distribution of bursts exhibits a crossover to a significantly lower exponent, which is accompanied by the change of the functional form of the waiting time distribution. Since the crossover is controlled by the time scale of the Omori law these results can be exploited for forecasting the imminent catastrophic failure event.

Here we focused on the case of long range load redistribution following failure events. When the load sharing is localized the spatial correlation of failure events leads to the emergence of a propagating crack front. The load accumulated along the crack front gives rise to an overall acceleration of the failure process again with a non-homogeneous Poissonian character. However, at short time scales correlated clusters of events may arise inside the time series. The results imply the interesting question to clarify when studying burst time series under creep whether there is anything in the dynamics beyond non-homogeneous Poissonian processes.

**Methods**

We use a generic fiber bundle model\textsuperscript{28-29} of the creep rupture of heterogeneous materials which has successfully reproduced measured creep behavior\textsuperscript{9-10}. The sample is discretized in terms of a bundle of N parallel fibers having a brittle response with identical Young modulus $E$. The bundle is subject to a constant external load $\sigma_{ext}$ below the fracture strength $\sigma_f$ of the system. It is a crucial element of the model that the fibers break due to two physical mechanisms: immediate breaking occurs when the
local load $\sigma$ on fibers exceeds their fracture strength $\sigma_{fr}$. Time dependence is introduced such that those fibers, which remained intact, undergo an aging process accumulating damage $d(t)$ of $\gamma$. The rate of damage accumulation $\Delta \sigma(t)$ is assumed to have a power law dependence on the local load $\Delta \sigma(t) = \sigma(t)^\gamma$, where $\alpha$ is a constant and the exponent $\gamma$ controls the time scale of the accumulation process. Fibers can tolerate only a finite amount of damage so that the lifetime $t_f$ decreases as a power law of the external load $\Delta \sigma(t)$.

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Author contributions
Zs. D. performed computer simulations and evaluated the numerical data. F. K. designed the research, evaluated numerical data and wrote the manuscript.

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