A bi-invariant Einstein-Hilbert action for the non-geometric string

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Abstract

Inspired by recent studies on string theory with non-geometric fluxes, we develop a differential geometry calculus combining usual diffeomorphisms with what we call $\beta$-diffeomorphisms. This allows us to construct a manifestly bi-invariant Einstein-Hilbert type action for the graviton, the dilaton and a dynamical (quasi-)symplectic structure. The equations of motion of this symplectic gravity theory, further generalizations and the relation to the usual form of the string effective action are discussed. The Seiberg-Witten limit, known for open strings to relate commutative with non-commutative theories, makes an interesting appearance.
1 Introduction

String theory is expected to be a consistent theory of quantum gravity. In this respect, it is interesting to note that a generic feature of all known string theories is that besides the graviton, there exist two additional massless excitations, the Kalb-Ramond field $B_{\mu\nu}$ and the dilaton $\phi$. At leading order, the dynamics of these fields is governed by the extension of the Einstein-Hilbert action

$$S = -\frac{1}{2\kappa^2} \int d^n x \sqrt{-G} e^{-2\phi} \left( R - \frac{1}{12} H^2 + 4(\partial \phi)^2 \right),$$

which has two types of local symmetries. Namely, it is invariant under diffeomorphisms of the space-time coordinates, and under gauge transformation of the Kalb-Ramond field. Note also, this action is a valid approximation for solutions with large radii.

Employing T-duality [1], methods of generalized geometry [2, 3, 4] and double field theory [5, 6, 7, 8], it has become clear during the last years that also a non-geometric frame exists, where the degrees of freedom are described by a metric on the co-tangent bundle, by a dilaton and by a (quasi-)symplectic structure $\beta^{ab}$. The latter gives rise to so-called non-geometric $Q$- and $R$-fluxes. In particular, the $R$-flux has been argued to be related to a non-associative structure [9, 10, 11, 12, 13]. However, in contrast to the well-established non-commutative behavior of open strings [14], the generalization to closed strings is more complex, as in a gravitational theory the non-commutativity parameter is expected to be dynamical.

Since in the non-geometric frame, apart from the dilaton, one deals with just a metric and a (quasi-)symplectic structure, it is natural to expect that both local symmetries of the string action can be given a description in terms of a (generalized) differential geometry. Starting from so-called double field theory, this question has already been approached in an interesting way in [15, 16] (see also [17]). However, the action derived in [15, 16] is not manifestly invariant under both local symmetries. It is the objective of this letter to construct such a manifestly bi-invariant action for the non-geometric string. The appropriate mathematical framework for this turns out to be the theory of Lie and Courant algebroids [18, 19], which we will mention only briefly. More details on the underlying mathematical structure of Lie algebroids and the details of the computations will appear in [20].

Here, we present the main steps of a construction of an Einstein-Hilbert type action, which is manifestly invariant under both usual diffeomorphisms and what we call $\beta$-diffeomorphisms. This bi-invariant action turns out to be closely related to the action derived for non-geometric fluxes using double field theory [15, 16]. Remarkably, relations familiar from the Seiberg-Witten limit for D-branes in a two-form background also appear in this closed-string framework.
2 \textbf{\(\beta\)-diffeomorphisms}

As mentioned in the introduction, in addition to the dilaton, we consider the cotangent bundle \(T^\ast M\) of a manifold with metric \(\hat{g} = \hat{g}^{ab} e_a \otimes e_b\) and an invertible anti-symmetric bi-vector \(\hat{\beta} = \frac{1}{2} \hat{\beta}^{ab} e_a \wedge e_b = \hat{\beta}^{ab} e_a \otimes e_b\), where our notation is \(e_a = \partial_a\) and \(e^a = dx^a\). Note that \(\hat{\beta}\) can be thought of as a (quasi-)symplectic structure giving rise to a (quasi-)Poisson structure \(\{f, g\} = \hat{\beta}^{ab} \partial_a f \partial_b g\), with Jacobi-identity \(\text{Jac}(f, g, h) = \hat{\Theta}^{abc} \partial_a f \partial_b g \partial_c h\). The \(R\)-flux is defined as \(\hat{\Theta}^{abc} = 3 \hat{\beta}^{[a[m} \partial_m \hat{\beta}^{bc]}\), where the (anti-)symmetrization of indices contains a factor of \((1/n!)\). Moreover, \(\hat{\beta}\) provides a natural (anchor) map \(\hat{\beta}^\sharp : T^\ast \! M \to TM\) via \(\hat{\beta}^\sharp e_a = \hat{\beta}^{am} e_m\). As we will see, it is essential that \(\hat{\beta}\) is invertible, which is however the generic situation. On the other hand, that means we can only describe backgrounds for which that requirement is satisfied.

Compared to the standard differential geometry calculus, here, not only the tangent bundle but also the co-tangent bundle plays an important role. This suggests that the former principle of diffeomorphism covariance of gravity, the equivalence principle, should be extended by a second class of diffeomorphisms. Recall, that in the former case, infinitesimal diffeomorphisms \(x^a \to x^a + \xi^a(x)\) are given by the Lie derivative \(\delta_X = L_X\), which acts as the Lie bracket on vector fields and as the anti-commutator of the insertion map and the exterior differential on forms. For the second class, that is infinitesimal transformations parametrized by the components of a one-form \(\hat{\xi} = \hat{\xi}_a dx^a\), we note the following. The bracket, generalizing the commutator of vector fields to forms, is the so-called Koszul-bracket defined as

\[
[\hat{\xi}, \eta]_K = L_{\hat{\xi}} \eta - \iota_{\hat{\xi}} d\eta,
\]

where \(\iota\) denotes the insertion map. In addition, let us define the action of a one-form on a function \(\phi\) by the anchor map:

\[
dx^a(\phi) := \beta^a(dx^a)(\phi) = \hat{\beta}^{am} \partial_m \phi =: D^a \phi.
\]

Now, we can proceed as in ordinary differential geometry and define tensors by their infinitesimal transformation properties. In particular, a scalar field \(\phi\) is called a \(\beta\)-scalar if it transforms as

\[
\hat{\xi} \phi = L_{\hat{\xi}} \phi = \hat{\xi}(\phi) = \hat{\xi}_m D^m \phi,
\]

and a one-form \(\eta\) is a \(\beta\)-one-form if

\[
\hat{\xi} \eta = L_{\hat{\xi}} \eta = [\hat{\xi}, \eta]_K = \left(\hat{\xi}_m D^m \eta_a - \eta_m D^m \hat{\xi}_a + \hat{\xi}_m \eta_a Q_a^{mn}\right) e^a,
\]

\[
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\]
with $\hat{Q}_c^{ab} = \partial_c \hat{\beta}^{ab}$. The transformation properties of general $\beta$-tensors are then determined by requiring the Leibniz rule of $\delta_\xi$ for tensor products and contractions, which implies for instance that a $\beta$-vector field $X = X^a e_a$ transforms as

$$\delta_\xi X = \mathcal{L}_\xi X = \left( \hat{\xi}_m D^m X^a + X^m D^a \hat{\xi}_m - X^m \hat{\xi}_n \hat{Q}_m^{na} \right) e_a .$$

(6)

To continue, we have to fix the nature of the metric $\hat{g}^{ab}$ and the anti-symmetric bi-vector $\hat{\beta}^{ab}$. The former should be a tensor with respect to both diffeomorphisms and $\beta$-diffeomorphisms, while we require the latter only to be a tensor under diffeomorphisms. As will become clear below, it should transform under $\beta$-diffeomorphisms non-covariantly

$$\hat{\delta}_\xi \hat{\beta} := \mathcal{L}_\xi \hat{\beta} + \hat{\beta}^{am} \hat{\beta}^{bn} \left( \partial_m \hat{\xi}_n - \partial_n \hat{\xi}_m \right) e_a \otimes e_b$$

$$= \hat{\xi}_m \hat{\Theta}^{mab} e_a \otimes e_b .$$

(7)

Moreover, the variation with respect to $\hat{\xi}$ should commute with partial derivatives, i.e. $[\delta_\xi, \partial_a] = 0$. The Lie brackets of infinitesimal diffeomorphisms and $\beta$-diffeomorphisms are

$$[\hat{\delta}_\xi, \delta_\eta] = \delta_\xi \delta_\eta (\hat{\xi}_2),$$

$$[\hat{\delta}_\xi, \hat{\delta}_\eta] = \delta_\xi \hat{\theta} + \delta_\eta \hat{\theta} (\hat{\xi}_2).$$

(8)

Ordinary differential geometry is based on the covariantization of the partial derivative of tensors, however, because of

$$\hat{\delta}_\xi (\partial_a \phi) = \mathcal{L}_\xi (\partial_a \phi) + (D^m \phi) (\partial_m \hat{\xi}_n - \partial_n \hat{\xi}_m),$$

(9)

under a $\beta$-diffeomorphism the partial derivative of a scalar does not transform as a $\beta$-vector. But, on the other hand, we have defined the transformation of $\hat{\beta}$ in eq. (7) such that the derivative $D^a \phi$ transforms precisely as a $\beta$-vector, i.e. $\hat{\delta}_\xi (D^a \phi) = \mathcal{L}_\xi (D^a \phi)$. Finally, using one of the Bianchi identities derived in [19, 15], we find that the $R$-flux is also a $\beta$-tensor, that is $\hat{\delta}_\xi \hat{\Theta}^{abc} = \mathcal{L}_\xi \hat{\Theta}^{abc}$.

3 Covariant derivative, torsion and curvature

As established in the last section, the role played by $\partial_a$ in usual gravity theories is now taken by the derivative $D^a$. Following the same steps as in standard differential geometry, we then define the covariantization of $D^a$ as

$$\hat{\nabla}^a X^b = D^a X^b - \hat{\Gamma}_c^{ab} X^c ,$$

(10)
and the action on forms reads $\hat{\nabla}^a \eta_b = D^a \eta_b + \hat{\Gamma}_b^{ac} \eta_c$. Demanding that the covariant derivative is a $\beta$-tensor requires that the $\beta$-connection cancels the anomalous transformation of the first term, leading to

$$\hat{\Delta}_\xi (\hat{\Gamma}^{ab}_c) = D^a \left( D^b \hat{\xi}_c - \hat{\xi}_m \hat{Q}^{mb}_c \right), \quad (11)$$

with $\hat{\Delta}_\xi = \delta_\xi - \mathcal{L}_\xi$. Under usual diffeomorphisms, $\hat{\Gamma}^{ab}_c$ needs to transform anomalously as

$$\Delta_\xi (\hat{\Gamma}^{ab}_c) = -D^a \left( \partial_c \xi^b \right). \quad (12)$$

Taking the commutator of two covariant derivatives defines the $\beta$-torsion

$$[\hat{\nabla}^a, \hat{\nabla}^b] \phi = -\hat{T}^{ab}_c D^c \phi, \quad (13)$$

which can be expressed as

$$\hat{T}^{ab}_c = \hat{\Gamma}^{ab}_c - \hat{\Gamma}^{ba}_c - \hat{Q}^{ab}_c, \quad (14)$$

with $\hat{Q}^{ab}_c = \hat{Q}^{ab}_c + \hat{\Theta}^{abm}_c \hat{\beta}^{mc}$. By construction, this is a usual tensor and a $\beta$-tensor. The curvature is defined by

$$[\hat{\nabla}^a, \hat{\nabla}^b] X^c = -\hat{R}^{cab}_m X^m - \hat{T}^{ab}_m \hat{\nabla}^m X^c, \quad (15)$$

leading to

$$\hat{R}^{cab}_m = D^a \hat{\Gamma}^{bc}_m - D^b \hat{\Gamma}^{ac}_m + \hat{\Gamma}^{bc}_n \hat{\Gamma}^{an}_m - \hat{\Gamma}^{ac}_n \hat{\Gamma}^{bn}_m - \hat{Q}^{ab}_c \hat{\Gamma}^{nc}_m. \quad (16)$$

The metric-compatible and torsion-free Levi-Civita connection takes the form

$$\hat{\Gamma}^{ab}_c = \hat{\Gamma}^{ab}_c - \hat{g}_{cq} \hat{g}^{ap} \hat{Q}^{bq}_p + \frac{1}{2} \hat{Q}^{ab}_c, \quad (17)$$

with

$$\hat{\Gamma}^{ab}_c = \frac{1}{2} \hat{g}_{cp} \left( D^a \hat{g}^{bp} + D^b \hat{g}^{ap} - D^p \hat{g}^{ab} \right). \quad (18)$$

Note that one can check explicitly that (17) has the right anomalous transformation behavior under diffeomorphisms (12) and $\beta$-diffeomorphisms (11).

For vanishing torsion, the Ricci tensor $\hat{R}^{ab} = \hat{R}^{amb}_m$ is symmetric and reads

$$\hat{R}^{ab} = D^m \hat{\Gamma}^{ba}_m - D^b \hat{\Gamma}^{ma}_m + \hat{\Gamma}^{ba}_n \hat{\Gamma}^{mn}_m - \hat{\Gamma}^{ma}_n \hat{\Gamma}^{nb}_m. \quad (19)$$
The Ricci scalar \( \hat{R} = \hat{g}_{ab} \hat{R}^{ab} \) can be expanded as
\[
\hat{R} = - \frac{1}{4} \hat{g}_{ab} \left( D^a \hat{g}_{mn} D^b \hat{g}^{mn} - 2 D^a \hat{g}_{mn} D^m \hat{g}^{nb} - \hat{g}_{mn} \hat{g}_{pq} D^a \hat{g}^{mn} D^b \hat{g}^{pq} \right) + \frac{1}{4} \hat{g}_{ab} \hat{g}^{pq} \hat{Q}^p_{mb} \hat{Q}^b_{np} + \frac{1}{2} \hat{g}_{ab} \hat{Q}^m_{ma} \hat{Q}^b_{nb} + \hat{g}_{ab} \hat{Q}_m^{ma} \hat{Q}_n^{nb} + 2 D^a \left( \hat{g}_{ab} \hat{Q}^{mb} - \hat{g}_{ab} \hat{g}_{mn} D^a \hat{g}^{mn} \hat{Q}_p^{bp} \right),
\]
which is the same expression as in [16] if one substitutes \( \hat{Q}^{abc} \leftrightarrow Q^{abc} \).

4 Bi-invariant action

After having defined a covariant curvature, we can now move forward and construct a bi-invariant action for the fields \((\hat{g}, \hat{\beta}, \phi)\), where the dilaton \(\phi\) is chosen to be a scalar under both transformations. Since by construction \(\Theta^{abc}\) is a tensor, the following combination
\[
\hat{L} = e^{-2\phi} \left( \hat{R} - \frac{1}{12} \hat{\Theta}^{abc} \hat{\Theta}_{abc} + 4 \hat{g}_{ab} D^a \phi D^b \phi \right)
\]
behaves as a scalar under both types of diffeomorphisms. Our aim is now to construct a bi-invariant action
\[
\hat{S} = - \frac{1}{2\kappa^2} \int d^n x \mu(\hat{g}, \hat{\beta}) \hat{L},
\]
where \(\mu\) denotes an appropriate measure.

An obvious first choice would be \(\mu = \sqrt{-\hat{g}}\), however, using that \(\hat{g}^{ab}\) is a \(\beta\)-tensor we find
\[
\delta \xi(\sqrt{-\hat{g}} \hat{L}) = \partial_k \left( \sqrt{-\hat{g}} \hat{L} \hat{\xi}_m \right) \hat{\beta}^{mk} - \sqrt{-\hat{g}} \hat{L} \hat{\xi}_m \partial_k \hat{\beta}^{mk},
\]
so that the sign in front of the last term does not complete the desired total derivative. Furthermore, one can show that the resulting action is not invariant under usual diffeomorphisms either. But because of the relation \(\delta \xi |\hat{\beta}^{-1}| = 2 |\hat{\beta}^{-1}| \hat{\xi}_m \partial_k \hat{\beta}^{mk} + \hat{\xi}_m \hat{\beta}^{mk} \partial_k |\hat{\beta}^{-1}| \) for the absolute value of \(\text{det}(\hat{\beta}^{-1})\), the missing terms can be accounted for by modifying the measure to \(\mu = \sqrt{-\hat{g}} |\hat{\beta}^{-1}|\). Analogously, this new measure also ensures the diffeomorphism invariance of the action. Note that \(|\beta^{-1}| = \text{Pf}(\beta^{-1})^2 \geq 0\). Thus, we have succeeded in constructing the bi-invariant action
\[
\hat{S} = - \frac{1}{2\kappa^2} \int d^n x \sqrt{-\hat{g}} |\beta^{-1}| e^{-2\phi} \left( \hat{R} - \frac{1}{12} \hat{\Theta}^{abc} \hat{\Theta}_{abc} + 4 \hat{g}_{ab} D^a \phi D^b \phi \right),
\]

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whose form closely resembles the universal part of the low-energy effective action of string theory \(\text{(1)}\). We call this theory \textit{symplectic gravity} with a dilaton.

The equations of motion are derived by varying the action \(\text{(24)}\) with respect to the metric, the (quasi-)symplectic structure and the dilaton. Since \(\hat{\beta}_{ab}\) appears also through the \(\mathcal{D}a\) derivative, this is a non-trivial computation. Employing the relation

\[
\sqrt{-\hat{g}} |\hat{\beta}^{-1}| \hat{\nabla}^m X_m = \partial_k\left( \sqrt{-\hat{g}} |\hat{\beta}^{-1}| \hat{\beta}^{mk} X_m \right),
\]

we obtain the three independent equations

\[
\begin{align*}
0 &= \hat{R}^{ab} + 2 \hat{\nabla}^a \hat{\nabla}^b \phi - \frac{1}{4} \hat{\Theta}^{amn} \hat{\Theta}^{b}_{mn}, \\
0 &= -\frac{1}{2} \hat{\nabla}_m \hat{\nabla}^m \phi + (\hat{\nabla}_m \phi)(\hat{\nabla}^m \phi) - \frac{1}{24} \hat{\Theta}^{mnr} \hat{\Theta}_{mnr}, \\
0 &= -\frac{1}{2} \hat{\nabla}^m \hat{\Theta}^a_{m} \hat{\Theta}^{b}_{a} + (\hat{\nabla}^m \phi)\hat{\Theta}^a_{m} \hat{\Theta}^{b}_{a}.
\end{align*}
\]

These feature the same form as the usual string-frame equations of motion derived from the action \(\text{(1)}\). A more detailed derivation will be presented in \(\text{[20]}\).

Finally, a natural guess for the action of the massless bosonic states in the Ramond-Ramond sector is

\[
\hat{L}^{RR} = -\sum_n \frac{1}{2 n!} \hat{g}_{a_1 b_1} \cdots \hat{g}_{a_n b_n} \hat{F}^{a_1 \cdots a_n} \hat{F}^{b_1 \cdots b_n},
\]

where \(\hat{F}^{a_1 \cdots a_n} = n \hat{\nabla}^{[a_1} \hat{C}^{a_2 \cdots a_n]} + \mathcal{O}(\hat{\Theta})\) and \(n\) is even (odd) for type IIA (B) theories.

## 5 Relations to string theory

Generalized geometry and double field theory suggest that the relation between the geometric and non-geometric fields is given by

\[
\begin{align*}
\hat{g} &= (G + B)^{-1} G (G - B)^{-1}, \\
\hat{\beta} &= -(G + B)^{-1} B (G - B)^{-1}.
\end{align*}
\]

However, starting from the action \(\text{(1)}\) and inserting this transformation, the computation in \(\text{[15] [16]}\) shows that one does not find eq. \(\text{(24)}\). But, observing that the relation between the fields \((G, B)\) and \((\hat{g}, \hat{\beta})\) is formally the same as the one appearing in the study of D-branes in a two-form flux backgrounds, a second natural possibility arises. In particular, in the Seiberg-Witten limit \(\text{[14]}\), i.e. where a fluxed brane theory is effectively described by a non-commutative gauge theory, the relation between the two sets of fields reads

\[
B = \hat{\beta}^{-1}, \quad G = -\hat{\beta}^{-1} \hat{g} \hat{\beta}^{-1}.
\]
(Note that we are not taking a true limit $G \to 0$, that is we are not neglecting any terms from the action.) Now, a straightforward though tedious computation to be presented in detail in \[20\] shows that indeed the two actions \[11\] and \[24\] are related via this field redefinition, i.e.

$$S(G(\hat{g}, \hat{\beta}), B(\hat{g}, \hat{\beta}), \phi) = \hat{S}(\hat{g}, \hat{\beta}, \phi).$$ \hfill (30)

As an immediate consequence, the action which appeared in \[15, 16\] and eq. \[24\] are related via the field redefinition $\hat{\beta} = \tilde{\beta} - \tilde{\beta}^{-1} \tilde{g}$ and $\hat{g} = \tilde{g} - \tilde{g} \tilde{\beta}^{-1} \tilde{g}$.

Let us provide more arguments for the relation among the actions. Instead of the infinitesimal variations $\delta \xi$ and $\hat{\delta} \hat{\xi}$, consider $\hat{\delta} \hat{\xi}$ and the linear combination $\hat{\xi} X = L(\beta, \hat{\xi}) X - \hat{\delta} \hat{\xi} X$, where $X$ is assumed to be tensor with respect to diffeomorphisms but not necessarily with respect to $\beta$-diffeomorphisms. We then find

$$\hat{\delta} \hat{\xi} \hat{g}^{ab} = -2 \hat{\beta}(a|m) (\partial_m \hat{\xi}_n - \partial_n \hat{\xi}_m) \hat{g}^{n|b},$$
$$\hat{\delta} \hat{\xi} \hat{\beta}^{ab} = -\hat{\beta}^a m (\partial_m \hat{\xi}_n - \partial_n \hat{\xi}_m) \hat{\beta}^{m|b}. \hfill (31)$$

Using then the transformation \[29\], we can compute the resulting infinitesimal transformations $\hat{\delta} \hat{\xi} G_{ab} = 0$ and $\hat{\delta} \hat{\xi} B_{ab} = (\partial_m \hat{\xi}_n - \partial_n \hat{\xi}_m)$, which is precisely the gauge transformation of the Kalb-Ramond field $B$. Thus, also the local symmetries map correctly under \[29\].

Finally, employing \[29\] we can translate the $\alpha'$-corrections to \[11\] into the non-geometric frame. This provides an expansion in the derivative $D^a$, and thus \[24\] is a valid approximation for solutions with large radii $\hat{g}^{ab} \sim \hat{r} \gg 1$. At second order, this expansion reads

$$\hat{S}^{(1)} = \frac{1}{2\kappa^2} \frac{\alpha'}{4} \int d^{26} x \sqrt{-|\hat{g}|} |\hat{\beta}^{-1}| e^{-2\phi} \left( \hat{R}^{abcd} \hat{R}_{abcd} - \frac{1}{2} \hat{R}^{abcd} \hat{\Theta}_{abm} \hat{\Theta}_{cd} \right. \left. + \frac{1}{24} \hat{\Theta}_{abc} \hat{\Theta}^{mn}_{ab} \hat{\Theta}^{mp} \hat{\Theta}^{nm} - \frac{1}{8} (\hat{\Theta}^2)_{ab} (\hat{\Theta}^2)^{ab} \right). \hfill (32)$$

### 6 Conclusions

We close with some comments on open questions and future directions. It would be interesting to study solutions to the equations of motion \[20\] of the novel symplectic gravity action. In particular, we expect analogues of the elementary string and the solitonic five-brane solution. It would also be interesting to compute next to leading order terms in the action and to include space-time fermions, as well as to study the up-lift of symplectic gravity to M-theory.

The presence of a dynamical (quasi-)Poisson structure and the appearance of the Seiberg-Witten limit in relating the non-geometric frame to the geometric one suggests that it might be possible to perform a deformation quantization of the classical symplectic gravity action. If that is feasible, we expect the non-associative structures observed in \[10, 11\] to play an essential role.
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