Cluster mass dependent truncation of the upper IMF: evidence from observations and simulations

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Abstract. We attempt to evaluate whether the integrated galactic IMF (IGIMF) is expected to be steeper than the IMF within individual clusters through direct evaluation of whether there is a systematic dependence of maximum stellar mass on cluster mass. We show that the result is sensitive to observational selection biases and requires an accurate knowledge of cluster ages, particularly in more populous clusters. At face value there is no compelling evidence for non-random selection of stellar masses in low mass clusters but there is arguably some evidence that the maximum stellar mass is anomalously low (compared with the expectations of random mass selection) in clusters containing more than several thousand stars. Whether or not this effect is then imprinted on the IGIMF then depends on the slope of the cluster mass function. We argue that a more economical approach to the problem would instead involve direct analysis of the upper IMF in clusters using statistical tests for truncation of the mass function. When such an approach is applied to data from hydrodynamic simulations we find evidence for truncated mass functions even in the case of simulations without feedback.

1. Introduction

Does the maximum stellar mass in a cluster depend systematically on cluster mass? Here we emphasise the word ‘systematically’ since we need to distinguish between the pure size of sample effect (that leads one to expect, on average, a lower maximum mass in a less populous cluster) and the systematic effect of cluster mass-dependent truncation of the IMF explored by Weidner & Kroupa (2006). A mere size of sample effect would still imply that the IMF assembled through combining all clusters (the integrated Galactic IMF, IGIMF) would have the same slope as the universal IMF from which the stellar content of each cluster was drawn. On the other hand, cluster mass-dependent truncation of the IMF within each cluster can result in an IGIMF with a different slope from the individual cluster IMF: in particular, if the distribution of cluster masses is strongly biased towards low mass clusters for which the IMF is truncated at a low value, then the IGIMF can be markedly steeper than the IMF within each cluster (Kroupa & Weidner 2003). This hypothesis would have important implications for how, for example, we normalise galaxy integrated quantities based on their high mass stellar content (e.g. supernova rate, blue light) to the total stellar mass/star formation rate in the galaxy. Weidner & Kroupa (2005) have furthermore postulated that if the cluster mass function is systematically truncated at a mass that depends on the galaxy mass, then the influence of small clusters is greater in dwarf galaxies and hence predict a steeper IGIMF in these systems. It is hard to test the IGIMF hypothesis directly because, for example, the observation of a low rate of massive star production (relative
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Figure 1. Probability density of the most massive star, \( p(m_{\text{max}}) \) (eq. 1) for a star cluster containing \( n = 30 \) stars. Characteristic quantities are the mean, \( \bar{m}_{\text{max}} \), and the median, \( m_{1/2} \). The 1/6 and 5/6 quantiles limit the shaded region containing 2/3rd of the most massive stars. (Figure taken from Maschberger & Clarke 2008).

to the available gas mass) may be interpreted either as indicating a steep IGIMF (so that the the total star formation rate is not anomalously low: Pilamm-Altenburg et al. 2007) or else a suppression of the star formation efficiency (star formation rate per unit gas mass) in that system (e.g. Kaufmann et al. 2007). In the absence of observations that probe the formation of stars on the lower IMF, both explanations are equally good.

One can in principle break this degeneracy by going directly to observations of young clusters in order to ascertain whether there is observational support for the effect that underpins the IGIMF hypothesis, i.e. the systematic dependence of maximum stellar mass on cluster mass as described above. This hypothesis was originally framed by inspecting the available cluster data and is thus an empirical hypothesis, without a quantitative theoretical basis (although one can argue plausibly about physical effects such as stellar feedback that may give rise to such behaviour). Therefore in order to establish the need for such a hypothesis it is first necessary to test the data against the readily quantifiable ‘null hypothesis’, i.e. that stars are assembled at random from a universal IMF. Only if the data is significantly discrepant with this hypothesis should we start to explore and constrain more complicated alternative hypotheses.

2. Testing the null hypothesis

If we pick \( n \) stars at random from a universal IMF \( f(m) \) (such that the fraction of stars with masses in the range \( m \) to \( m + dm \) is \( f(m)dm \)) then simple application of binomial statistics implies that the maximum stellar mass should be distributed according to the probability density function:

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p(m_{\text{max}}) = n \left( \int_{m_L}^{m_{\text{max}}} f(m')dm' \right)^{n-1} f(m_{\text{max}})
\]

(eq A5 in Maschberger & Clarke 2008) where \( m_L \) is lower limit of the stellar mass function, the (cluster mass independent) minimum stellar mass. (Note that \( m_{\text{max}} \) is not the upper limit, \( m_U \) of the IMF). This can be simply understood as representing the
Figure 2. Mass of the most massive star versus the number of stars in the cluster (for better visibility, a small random scatter was applied to the (discrete) masses). The data are collected from the literature, with the main sources Testi et al. (♦) and Weidner & Kroupa (■). The solid line is the mean value of $m_{\text{max}}$ depending on $n$. The dotted lines follow the $1/6$ and $5/6$ quantiles, and should confine $2/3$rd of the observed data. (Figure taken from Maschberger & Clarke 2008).

probability that one star has mass in the range $m_{\text{max}}$ to $m_{\text{max}} + dm_{\text{max}}$ and the remaining $n-1$ have masses less than this, with there being $n$ choices as to the identity of the most massive star. Figure 1 demonstrates (for the case $n=30$) that this probability density function is highly asymmetric, with the mean being significantly higher than the median. This immediately implies that with data drawn from the null hypothesis the maximum stellar mass is likely (for the majority of random realisations) to be less than the mean. It is therefore a mistake to compare the expectation value of the null hypothesis with sparsely sampled data, since one is then likely to conclude (erroneously) that the data is systematically low compared with the model prediction. The correct approach is to compare observational data with the quantiles of the distribution.

Figure 2 represents a recent attempt to collate all available data on the maximum stellar mass as a function of numbers of stars in the parent cluster and to compare with the null hypothesis of random drawing from a parent mass function (for details see Maschberger & Clarke 2008). There are several noteworthy features of the data. Firstly, one is struck by the large range in maximum mass at a given cluster $n$ (around a factor 5–10) so clearly one is not looking at a well defined ‘relation’; on the other hand, the dotted line shows the $1/6,5/6$ quantiles of the null hypothesis which show that such a range is actually expected in the case of random drawing. Secondly, the errorbars are large: we have not even attempted to assign errorbars on the ‘mass’ axis and our errors in cluster membership are a notional factor 2: we have corrected the number of stars in each cluster down to a common lower mass limit, assuming a universal (Kroupa) form for the lower IMF but we are obviously unable to accurately assign the numbers of stars which belong to a given cluster but whose surface density on the sky is less than the local background value. This is particularly acute in the case of small $n$ clusters which are liable to undergo significant dynamical evolution over the (few Myr) lifetimes of the clusters (Bonnell & Clarke 1999). This can result in a significant fraction of original
cluster members being dispersed into the neighbouring field and rendered undetectable, so that the associated errorbars are in reality larger for small \(n\) systems.

Thirdly, one cannot be struck by the fact that the provenance of the data determines which region of the diagram it populates. Specifically, the data of Testi et al.
(1997, 1998) lies significantly higher with respect to the model quantiles than does the other data. This is simply because the Testi et al. data is assembled by searching for clusters around known, apparently isolated, massive stars, whereas the other data is obtained by identifying the most massive member of known clusters. Clearly, the Testi data will tend to contain objects of relatively high mass compared with the cluster mass (see also Oey et al., this volume, for similar results obtained by imaging OB stars in the Magellanic Cloud). Since our test of the null hypothesis depends on how the data is distributed in relation to the model quantiles it follows that the types of objects that we include need to be representative of their incidence in the Galaxy. We are of course very far from this situation when we simply, as in Figure 2, assemble all measurements from the literature and this means that any conclusions from plots like Figures 2 and 3 must be highly provisional.

Laying these caveats aside, the analysis by Maschberger & Clarke (2008) of the data contained in Figure 2 implies that it is indeed consistent with the null hypothesis — we would therefore hesitate to argue for a cluster mass dependent maximum stellar mass based on this data.

Figure 3 however tells a different story, being a similar compilation that extends to richer clusters (i.e. to those containing more than the few thousand stars contained in Figure 2, from Weidner et al. 2010). The dashed lines represent the 1/6, 5/6 quantiles of the null hypothesis and show that the data in the region that overlaps in mass with Figure 2 is still broadly consistent with this (random drawing) null hypothesis. At higher masses, however, one sees that the data lies progressively lower with respect to the model quantiles — i.e. the maximum stellar mass would indeed appear to be suppressed, in these massive clusters, compared with the expectations of random drawing.

In the following section we explore whether we expect the results contained in Figure 3 to have significant implications for the IGIMF, but first we need to enquire whether there are any systematics that might explain trends in the data. We draw attention to the fact that plots like Figures 2 and 3 have to be constructed from clusters that are young enough that their most massive stars have not yet expired as supernovae. For low mass clusters, the main sequence lifetime of stars with the rather low maximum masses expected is fairly long, but as one goes to more massive systems it follows that one should include only the youngest clusters. Although Weidner & Kroupa have been

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1It is sometimes stated that one can reduce the problem of errorbars on cluster membership by plotting this diagram as a function of cluster mass rather than \(n\); this approach is favoured by Weidner & Kroupa, even though it then has the mild disadvantage that the quantiles have to be computed through Monte-Carlo simulation rather than using the analytic form, equation (1). The argument is that since lower mass stars contribute less to the total mass than to the total number, then uncertainties associated with extrapolation to a given lower stellar mass limit are smaller when expressed in terms of mass rather than number. This however does not translate into a greater power to distinguish between rival hypotheses since, just as the total mass is less sensitive to errors in membership at the lower mass end, so, correspondingly, are the predicted quantiles of the distribution more sensitive to the total mass. We have established through Monte-Carlo simulations that, whether one expresses the independent variable in terms of stellar mass or number, the uncertainty associated with where a datapoint is placed with respect to the quantiles of the null hypothesis is about the same — in other words, it does not greatly matter whether one constructs plots as a function of mass or membership number.
conscientious in applying these age constraints, one nevertheless has to bear in mind that the ages of young clusters are not necessarily very accurately known and that age uncertainties could—in principle—explain the trend seen in Figure 3.

3. IGIMF Implications

We now lay aside all the caveats mentioned above in relation to Figures 2 and 3 and ask whether—if Figure 3 were taken at face value—it would have ramifications for the IGIMF. Since we have no theoretical guidance as to how the maximum mass should depend on cluster mass, we just adopt a ‘toy’ relationship which brings the data into acceptable agreement with the quantiles of the ‘toy model’ (see Figure 4). Assuming this form, we then consider that the Galactic field is composed of clusters whose spectrum, by mass, $M_c$, scales as $\propto M_c^{-\beta}$ and then compute the expected IGIMF.

Unsurprisingly, the result of this exercise depends on $\beta$. If $\beta$ is 2 or below, the relative contribution of large and small clusters as a source of stars of given mass is weighted towards large clusters, wherein the maximum stellar mass ($m_U$) is expected to tend to its global maximum value (here set to $150 M_\odot$: see Weidner & Kroupa 2004; Oey & Clarke 2005). Consequently, the IGIMF is indistinguishable from the IMF in each cluster (here assumed to follow a Salpeter slope for its massive stars); this only breaks down if we limit the maximum cluster mass to a scale ($< 10^4 M_\odot$) where the maximum stellar mass becomes less than $\sim 150 M_\odot$: in this case the IGIMF has a Salpeter slope but is truncated at the maximum stellar mass that we have assigned to the maximum cluster mass.

On the other hand, a rather modest change in $\beta$ changes the result quite markedly. Figure 4 shows the expected IGIMF in the case that the slope of the cluster mass func-
Figure 4. Integrated galactic IMFs for different values of the upper limit of the cluster mass function (solid lines) in the range $10^3$ to $10^7 M_\odot$. The cluster mass function follows $M_c^{-2}$ (left) and $M_c^{-2.35}$ The dotted line shows a Salpeter stellar mass function $\propto m^{-2.35}$.

4. Discussion

In some ways Figures 2 and 3 represent an exercise that is rather wasteful of observational data — i.e. for each cluster we use only one piece of data (apart from the cluster mass or $n$) namely the mass of its most massive star. Inevitably this means that we need a large number of clusters in order to populate this plane and then we should worry (as discussed above) whether we have in fact populated this plane in an unbiased way.

An alternative observational route is however to seek evidence of truncation of the IMF within individual clusters, through analysis of the distribution of stellar masses in the high mass tail of the mass function. Maschberger & Kroupa (2009) present a ready tool for testing for truncation in power law data which consists of first fitting the data with a truncated power law and then assessing the significance of the best fit model, i.e. through testing how discrepant is the data with other hypotheses. This latter is achieved by means of a stabilised probability-probability (SPP) plot, which is closely allied to the Kolmogorov-Smirnov (KS) test in that it compares the (stabilised) cumulative distribution of the model with that predicted by other hypotheses. Here ‘stabilised’ refers to a transformation of the cumulative distribution in a way designed to achieve uniform variance at all quantiles in the case of randomly sampled data. This transformation therefore corrects a well known drawback of the KS test, i.e. that it is relatively insensitive to differences occurring near the extremes of the distribution. Obviously, this is a particular drawback if the feature of interest in the distribution is a truncation at the high mass end!

The SPP methodology can also be applied to simulation data and has revealed, in the analysis of stellar mass distributions generated by the turbulent fragmentation calculations of Bonnell et al. (2003, 2008) that there is some evidence for truncated mass distributions within individual clusters (Maschberger et al. 2010). This indeed...
Cluster mass dependent truncation of the upper IMF gives rise to an (extremely mild) IGIMF effect in the data — i.e. the IMF of the stars contained within all the clusters in the simulation volume is slightly steeper (at a level of about 0.2 in the mass function index) than the IMF within individual clusters. What is interesting here is not the magnitude of the effect (which is far smaller than what results from the ‘sorted sampling’ algorithms with which [Weidner & Kroupa (2006)] illustrate their IGIMF concept) but rather the remarkable fact that there should be any such effect in simulations which omit all forms of feedback and in which, therefore, one cannot argue that any abrupt switch prevents stars from growing to beyond a certain (cluster mass dependent) mass. Instead it would seem that in the calculations, the result is simply due to the fact that at any point in the simulation the stars have only had a finite time in which to acquire mass: the most massive stars are those that form first and, statistically, are those that are involved in more successive cluster mergers and which end up in the rich accretion environment of massive cluster cores. Since stars are often formed in small n groupings, these groups of massive stars tend to ‘travel together’ up the cluster merger tree and share similar accretion histories. Thus the most massive stars are relatively closely bunched in mass and the resulting IMF is best fit by a truncated form.

These calculations are from representing a ‘realistic’ picture of mass acquisition on the upper IMF given the omission of feedback processes; it is interesting that the history of cluster and massive star formation in these simulations is nevertheless such that it gives rise to these first hints of the reality of an IGIMF effect.

5. Conclusions

We have shown that current observational data on the relationship between maximum stellar mass and cluster mass does not strongly argue for a systematic (non-random) relationship between these quantities in the case of clusters that number thousands of stars or less. At higher cluster masses there is an apparent suppression of maximum stellar mass compared with the expectations of ‘random drawing’. We highlight the fact that the conclusions drawn from such plots are sensitive to the selection criteria involved in acquiring observational data. We also note that the inference of relatively low maximum stellar masses at high cluster mass (Figure 3) could in principle be explained by errors in cluster ages.

We briefly examine whether the available data on maximum stellar mass versus cluster mass could give rise to a significant ‘IGIMF effect’. We emphasise that the answer to this question is extremely sensitive to the power law slope of the cluster mass function, with the predicted results changing markedly for small variations in this index around the value of 2.

We then argue that it may be more profitable to look for direct evidence for IMF truncation in massive clusters rather than relying on one piece of observational information (the mass only of the most massive star) per cluster. Methods for detecting truncated mass functions and assessing their significance are available in the astronomical literature. We show that when such methods are applied to the output of hydrodynamic simulations of star cluster formation, we indeed find evidence for truncated mass functions and a (very mild) IGIMF effect. This truncation can be explained in terms of the finite time available for stars to grow by accretion and would probably be more marked in simulations that additionally included some form of dynamical or thermal feedback.
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