Theoretical research of multifunctional tillage unit scheme

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Abstract. It is developed the multifunctional tillage unit includes replaceable sets of operation bodies in the form flat-cutting paws or in the form cultivation paws, as well as disk sections. To justify the rational layout of flat-cutting paws or of cultivation paws relative to the disk operating bodies theoretical studies have been conducted out to analysis the stability of movement of a multifunctional tillage unit in the longitudinal-vertical plane. The arrangement of flat-cutting paws or cultivator paws in front of the disc sections is more preferable.

1. Introduction
The making of new multifunctional units and the enhancement of existing machines that can execute the all complex of tillage operations and of preparing the soil for sowing is one of the most promising areas of modernization of the nomenclature of agricultural machinery [1-4].

The distinctive features of the region of the North-Eastern part of the European part of Russian Federation are the predominance of sod-podzolic soils with excessive accumulation of moisture, as well as the predominance of fields with an uneven terrain, small area and their length [5], which does not allow efficient use of energy-saturated tractors and of units with large grip width. Availability replaceable working bodies will allow quickly adapt the unit to seasonal production conditions.

At present in the region of the North-Eastern of the European part of Russia the agricultural technology has established the essence of which is to plowing during the one years and flat-cutting tillage during the one, two or three years [6-7].

2. Materials and methods
The construction scheme of a multifunctional unit that can perform basic soil tillage and of pre-sowing tillage operations has been developed.

Previous studies have shown that it is more expedient to install disk working bodies after the passage of flat-cutting paws, when the soil layer destroyed by the flat-cutting paw is processed more efficiently and with less energy consumption [8].

The mounted tillage unit is operated under a variety of changing conditions due to the different slope of the field, irregularities on its surface, heterogeneity of the physical and mechanical properties of the soil, etc. These changes give the tillage unit which can be considered as a mechanical system additional movements, speeds and accelerations [9].

The theoretical research were conducted the stability of the unit in the longitudinal-vertical plane when performing tillage. Considered are two options for placement of disc sections in the unit: in the
first embodiment the disc section located behind of flat-cutting hoes or cultivator hoes, in the second variant – placed front of flat-cutting paws or cultivator paws. Consider the force interaction of the machine-tractor aggregate with the supporting surface of soil (figure 1).

**Figure 1.** The scheme of the aggregate consisting of the tractor and of the mounted tillage unit taking into account possible displacements and active forces in the longitudinal-vertical plane where the coordinates of the centers of gravity is marked with: \( C_1 \) – the energy machine; \( C_2 \) – supporting wheel; \( C_3 \) – frame of the unit; \( C_4 \) and \( C_5 \) – the first and second rows of the flat-cutting hoes; \( C_6 \) – disc sections.

To do this we will use the results of research by A. B. Lurie and other scientists [10-12].

The system "tractor + mounted tillage unit" is considered as a two-mass system: the mass \( m_{TR} \) is located in the \( C_1 \) center mass of the tractor and a mass \( m_{UN} \) of the tillage unit consisting to the sum of the masses of frame, of supporting wheel, of flat-cutting hoes or of cultivator hoes and of disk sections is located in the center of mass \( C_{UN} \) or are applied to the axis of rotation of the unit.

We take two coordinate systems to describe the movement of a machine-tractor aggregate. One system is the mobile \( x_1y_1z_1 \). The second system is moving translational at the \( u_0 = \text{const} \) speed of the instantaneous center of rotation. The \( x_1 \) axis is going along the longitudinal axis of the tractor, the \( z_1 \) axis is vertical, the \( y_1 \) axis runs perpendicular to the \( x_1 \) and \( z_1 \) axes. The \( Oxyz \) system of reference is rigidly connected to the processed surface of the field. The \( Ox \) axis is directed along of movement of the aggregate, the \( Oz \) axis is vertically, the \( Oy \) axis runs perpendicular to the \( Ox \) and \( Oz \) axes.

As generalized coordinates for the accepted reference systems and accepted types of movement of the links of the mechanical system "tractor + mounted tillage unit" we will take the angle of rotation of the mounted tillage unit in relation to the instantaneous center of rotation \( \varphi_{UN} \), the angle of rotation of the tractor frame in relation to the horizontal \( \varepsilon_{TR} \) measured from the axis of the rear axle of the tractor and the path traveled by the machine-tractor aggregate \( x_{TR} \).

Let is accept the following assumptions to simplify because in general the equations of motion of the
mounted unit are not linear:  
In general case the equations of motion of the mounted unit are not linear, so to simplify the problem we assume the following assumptions:

- Deviations of the trajectories of definite points from their trajectories in stable move of aggregate are small and increments of variables of the second and higher levels cannot be consider;
- Modifications in outer forces and moments that related with deviations of the trajectory of the points of the machine-tractor aggregate from the stable trajectory are directly proportional to the digressions and the first derivatives (velocities) of these digressions;
- The tractor mass and the mounted tillage machine mass does not change;
- Air resistance during of the moving aggregate is neglected;
- All elements of the mechanical system are review as point masses, they do not have their own moments of inertia;
- The instantaneous center of rotation (i.c.r.) and instantaneous center of speed of the mounted machine are available, the angles of inclination of the hitch levers of the tractor less than 90°;
- The kinetic energy of the rotating mechanisms of the energy machine is not taken into account, movement of the machine is assumed to be rectilinear.

The distributed forces arising on the operating bodies of the mounted unit and at the locations of contact with the field surface of its support wheels as well as the tractor wheels are substituted by concentrated forces functioning in three projection planes.

If we neglect the inertia forces of unbalanced masses of the unit, then in addition to the weight \( G_k \) the reactions of the support surface \( N_i \) at the points of contact will act on it. When projected on the \( Ox, Oy, Oz \) axes these forces turn into the \( z \) vertical component, the \( y \) transverse constituent and the \( x \) longitudinal ingredient. This tillage unit has symmetrical operating bodies in relation to the trajectory of the moving unit, thus the horizontal constituents of the soil resistance will amount zero \( (R_y = 0) \).

It is input the certain restrictions on the values of these forces. The elementary normal forces from the bearing surface of the wheels with a pneumatic tire is replaced to the \( N_2 \) normal force. The \( N_2 \) force is focused to the center of contact passing vertically through the wheel axis. It is ignored also the moment of elementary forces relative to the transverse axis to the moving aggregate which passes through the center of contact.

We will form an equation of motion of a machine-tractor aggregate. We will take advantage of the Lagrange equation of the second kind for this.

\[
\frac{\partial}{\partial T} \left( \frac{\partial T}{\partial q} \right) - \frac{\partial T}{\partial q} + \frac{\partial \Phi}{\partial q} + \frac{\partial \Pi}{\partial q} = Q, \tag{1}
\]

where \( T \) – the kinetic energy of the power machine and of the mounted tillage unit; \( \Pi \) – the potential energy of the power machine and of the mounted tillage unit; \( \Phi \) – the resistance function; \( q \) – the generalized coordinate; \( Q \) – the generalized forces.

The kinetic energy of the machine-tractor aggregate is determined by the formula

\[
T = T_{TR} + T_{UN}, \tag{2}
\]

where \( T_{TR} \) – kinetic energy of the power machine (tractor); \( T_{UN} \) – kinetic energy of the tillage unit.

\[
T_{TR} = T_1; \quad T_{UN} = T_2 + T_3 + T_4 + T_5 + T_6, \tag{3}
\]

where \( T_2 \) – kinetic energy of the support wheel of tillage unit; \( T_3 \) – kinetic energy of the frame of unit; \( T_4, T_5 \) – kinetic energy of flat-cutting paws of unit; \( T_6 \) – kinetic energy of the disk sections of unit.

It is determined the kinetic energy of the power machine by the formula

\[
T_{TR} = \frac{1}{2} m_{TR} \cdot \ddot{x}_{TR}^2, \tag{4}
\]
where \( m_{TR} \) – the mass of the power machine; \( \dot{x}_{TR} \) – the linear velocity of the mass center of the power machine.

It is determined the kinetic energy of all components of the mounted tillage unit by the formula

\[
T_i = \frac{1}{2}(m_i \cdot \dot{x}_i^2 + J_i \cdot \dot{\varphi}_i^2),
\]

where \( m_i \) – the mass of the \( i \)-element of mounted tillage unit; \( \dot{x}_i \) – velocity of the mass center of the \( i \)-element of unit, at that \( \dot{x}_i = \dot{x}_{TR} \); \( J_i = m_i \cdot r_i^2 \) – axial moment of inertia of the construction elements of mounted tillage unit comparative to the i.c.r.; \( r_i \) – the spacing from the mass center of the \( i \)-element of the unit to the i.c.r.; \( \dot{\varphi}_i \) – angular velocity of rotation of components of the mounted tillage unit in the longitudinal-vertical plane, \( \dot{\varphi}_2 = \dot{\varphi}_3 = \ldots = \dot{\varphi}_6 = \dot{\varphi}_{UN} \).

Then the kinetic energy of the machine-tractor aggregate in differential form will look like this:

\[
T = \frac{1}{2}(m_{TR} \dot{x}_{TR}^2 + m_2 \dot{x}_2^2 + J_2 \dot{\varphi}_2^2 + m_3 \dot{x}_3^2 + J_3 \dot{\varphi}_3^2 + m_4 \dot{x}_4^2 + J_4 \dot{\varphi}_4^2 + m_6 \dot{x}_6^2 + J_6 \dot{\varphi}_6^2).
\]

We take into account that the mass of the machine-tractor aggregate

\[
m_{AG} = m_{TR} + m_{UN},
\]

where \( m_{UN} \) – the mass components of the mounted tillage unit, \( m_{UN} = \sum m_i \).

\[
T = \frac{1}{2}(m_{AG} \dot{x}_{TR}^2 + (m_2(x_0 + x_3)^2 + m_3(x_0 + x_5)^2 + m_4(x_0 + x_6)^2 + m_6(x_0 + x_7)^2 + m_6(x_0 + x_g)^2) \cdot \dot{\varphi}_{UN}^2).
\]

The potential energy \( \Pi \) of the machine-tractor aggregate is more difficult to determine. The potential forces include the gravity of the power machine and of the elements of mounted tillage unit and the elastic forces of pneumatic tires.

In the scheme "tractor + mounted tillage unit" the tractor is view as a whole body based by four supports having the \( c_F \) stiffness of front wheels and the \( c_R \) stiffness of rear wheels. The elasticity of the supports in the longitudinal-transverse directions is not consider.

The potential energy of the machine-tractor aggregate fold by the potential energy of gravity and the potential energy of elastic forces.

Thus the potential energy of the system "tractor + mounted tillage unit" can be represented as:

\[
P = P_{TR} + P_{UN},
\]

where \( P_{TR} \) – the potential energy of the power machine; \( P_{UN} \) – potential energy of the mounted tillage unit.

\[
P_{TR} = P_{TR}^G + P_{TR}^E,
\]

where \( P_{TR}^G \) – the potential energy of the power machine from its gravity; \( P_{TR}^E \) – the potential energy of the power machine from the elastic force.

\[
P_{UN} = P_{2}^G + P_{3}^G + P_{4}^G + P_{5}^G + P_{6}^G,
\]

where \( P_i^G \) – potential energy of gravity of \( i \)-elements of the mounted tillage unit.

The potential energy of gravity of the aggregate determined by the formula:

\[
P_i^G = m_i \cdot g \cdot z_k,
\]

where \( z_k \) – the change in the coordinate when the unit moving lengthways on uneven field surface.

The potential energy of the aggregate from the elastic force

\[
P_i^E = \frac{1}{2} \sum c_k (\Delta z_k)^2,
\]

where \( c_k \) – the stiffness of the \( k \)-th elastic support of aggregate; \( \Delta z_k \) – value of deformation.
\[ \Delta z_k = z_k - \dot{z}_k - d_k. \]  

(14)

where \( \dot{z}_k \) – the coordinate of the wheel position on the field surface under the corresponding wheel; \( d_k \) – the static compressing in the equilibrium location.

The \( z_k \) values in the function of generalized coordinates for the front wheels \( z_F \) and rear wheels \( z_R \) of the tractor will have the following form:

\[ z_F = (x_1 + x_2)\varepsilon_{TR}. \]

(15)

\[ z_R = 0. \]

(16)

It is can write in this case for the tractor

\[ p_{TR}^F = \frac{1}{2} \sum_1^n c_k (z_k - \dot{z}_k - d_k)^2. \]

(17)

\[ p_{TR}^R = \frac{1}{2} \sum_1^n c_F ((x_1 + x_2)\varepsilon_{TR} - \dot{z}_F)^2 - \sum_1^n c_F d_F((x_1 + x_2)\varepsilon_{TR} - \dot{z}_F) \]

\[ + \frac{1}{2} \sum_1^n c_R \ddot{z}_R^2 \sum_1^n c_R d_R \dot{z}_R + \frac{1}{2} \sum_1^n c_R \dot{d}_R^2. \]

(18)

For the energy machine the potential energy from its gravity can be determined by:

\[ P_{TR}^G = m_1 g dz_1 = m_1 g x_2 \varepsilon_{TR}. \]

(19)

For the mounted tillage unit the formula of determining the potential energy from the gravity of its elements will look like this:

\[ P_{UN} = m_2 g z_2 + m_3 g z_3 + m_4 g z_4 + m_5 g z_5 + m_6 g z_6. \]

(20)

Then the potential energy from the gravity for the unit using generalized coordinates is:

\[ P_{UN} = g \cdot (m_2 x_3 + m_3 x_4 + m_4 x_5 + m_5 x_6 + m_6 x_7) \cdot \varphi_{UN}. \]

(21)

According to expression (6) the total potential energy of the machine-tractor aggregate:

\[ P = \frac{1}{2} \sum_1^n c_F ((x_1 + x_2)\varepsilon_{TR} - \dot{z}_F)^2 - \sum_1^n c_d d_F((x_1 + x_2)\varepsilon_{TR} - \dot{z}_F) \]

\[ + \frac{1}{2} \sum_1^n c_d \dot{d}_F^2 + \frac{1}{2} \sum_1^n c_R \ddot{z}_R^2 \]

\[ + \frac{1}{2} \sum_1^n c_R d_R \dot{z}_R + \frac{1}{2} \sum_1^n c_R \dot{d}_R^2 + m_2 g x_2 \varepsilon_{TR} + g (m_2 x_3 + m_3 x_4 + m_4 x_5 + m_5 x_6 + m_6 x_7) \varphi_{OP}. \]

(22)

For the wheel machine-tractor aggregate the drag function can be written as follows:

\[ F_{AG} = \frac{1}{2} \sum_1^n d_k (\Delta \dot{z}_k)^2, \]

(23)

where \( \Delta \dot{z}_k \) – speed deformation \( k \)-th support, m/s; \( d_k \) – resistance coefficient.

\[ \Delta \dot{z}_k = z_R - \dot{z}_k, \]

(24)

therefore

\[ F_{AG} = \frac{1}{2} \sum_1^n d_F \dot{z}_F^2 - \sum_1^n d_R \dot{z}_R^2 - \sum_1^n d_R \dot{z}_R^2 \]

\[ + \frac{1}{2} \sum_1^n d_F \dot{z}_F^2 + \frac{1}{2} \sum_1^n d_R \dot{z}_R^2, \]

(25)

Or using generalized coordinates:

\[ F_{AG} = \frac{1}{2} \sum_1^n d_F ((x_1 + x_2)\varepsilon_{TR})^2 - \sum_1^n d_F ((x_1 + x_2)\varepsilon_{TR}) \dot{z}_F + \frac{1}{2} \sum_1^n d_R \dot{z}_R^2 + \frac{1}{2} \sum_1^n d_R \dot{z}_R^2. \]

(26)

By giving a mechanical system a virtual displacement we determine the generalized forces \( Q_i \) when all variations in the generalized coordinates except \( \partial q_i \) will be zero:

\[ \partial q_i \neq 0; \quad dq_1 = dq_2 = \cdots = dq_i = 0. \]

(27)
We determine the work of all active forces applied to the mechanical system due to this displacement:

\[ [\Sigma A(\vec{F}_i)]_{0q_1} = Q_l \cdot dq_l. \]  

(28)

We take into account that the multiplier for the variation \( dq_1 \) is equal to the generalized force \( Q_l \). We get the formulas:

\[ Q_x = \frac{\Sigma A(\bar{F}_i)}{\partial q_x} = \frac{-R_{4x}dx_{TR} - R_{5x}dx_{TR} - R_{6x}dx_{TR} + M_{FW}r_{FW} dx_{TR} + M_{RW}r_{RW} dx_{TR}}{dx_{TR}} = \]

\[ = \frac{M_{FW}}{r_{FW}} + \frac{M_{RW}}{r_{RW}} - (R_{4x} + R_{5x} + R_{6x}); \]

(29)

\[ Q_{\phi} = \frac{\Sigma A(\bar{F}_i)}{\partial q_{\phi}} = ((R_{4x} + R_{5x} + R_{6x})z_0 + (R_{4x} + R_{5x})h_{FCH} + R_{6x}h_D) - \]

\[ -g((m_2x_3 + m_3x_4 + m_4x_5 + m_5x_6 + m_6x_7) + x_0(m_2 + m_3 + m_4 + m_5 + m_6)) + \]

\[ + (N_{2x}\bar{I}_2 + N_{4z}\bar{I}_4 + N_{5z}\bar{I}_5 + N_{6z}\bar{I}_6) + (F_{EL2}\bar{I}_2 + F_{EL4}\bar{I}_4 + F_{EL5}\bar{I}_5 + F_{EL6}\bar{I}_6); \]

(30)

\[ Q_{\varepsilon} = \frac{\Sigma A(\bar{F}_i)}{\partial q_{\varepsilon}} = N_{1FW}x_1 - N_{2x}\bar{I}_2 - N_{4z}\bar{I}_4 - N_{5z}\bar{I}_5 - N_{6z}\bar{I}_6 - R_{6x}h_D + x_2(N_{1FW} - m_1g) + \]

\[ + x_0(N_{2z} + N_{4z} + N_{5z} + N_{6z}) - r_{RW}(R_{4x} + R_{5x} + R_{6x}) - r_{RW}(R_{4x} + R_{5x}), \]

where \( M_{FW}, M_{RW} \) – the torques accordingly on the axis of the front and rear wheels; \( r_{FW}, r_{RW} \) – the radii respectively of the front and rear wheels; \( h_D, h_{FCH} \) – the depth of processing correspondingly with the disc sections and with the flat-cutting hoes or cultivator paws; \( \bar{I}_2, \bar{I}_4, \bar{I}_5, \bar{I}_6 \) – respectively the distance from the i.c.r. accordingly to the points of application of forces \( R_{4x}, R_{4x}, R_{5x} \) and \( R_{6x} \) horizontally.

We use expression (1) to determine the partial derivatives of formulas (8), (22) and (26) in order to find all their components:

\[ \sum_{i=1}^{2} c_F(x_1 + x_2)^2 \varepsilon_{TR} - \sum_{i=1}^{2} c_F(x_1 + x_2)\bar{z}_F - \sum_{i=1}^{2} c_F\partial_F(x_1 + x_2) + m_1g x_2 + \varepsilon_{TR} - \]

\[ - \sum_{i=1}^{2} d_F(x_1 + x_2)\bar{z}_F = N_{1FW}x_1 - N_{2x}\bar{I}_2 - N_{4z}\bar{I}_4 - N_{5z}\bar{I}_5 - N_{6z}\bar{I}_6 - R_{6x}h_D + \]

\[ + x_2(N_{1FW} - m_1g) + x_0(N_{2z} + N_{4z} + N_{5z} + N_{6z}) - r_{RW}(R_{4x} + R_{5x} + R_{6x}) - \]

\[ - r_{RW}(R_{4x} + R_{5x}); \]

(32)

\[ m_{AG}\ddot{x}_{TR} = \frac{M_{FW}}{r_{FW}} + \frac{M_{RW}}{r_{RW}} - (R_{4x} + R_{5x} + R_{6x}); \]

(33)

\[ (m_2(x_0 + x_3)^2 + m_3(x_0 + x_4)^2 + m_4(x_0 + x_5)^2 + m_5(x_0 + x_6)^2 + m_6(x_0 + x_7)^2)\bar{\psi}_{OP} = \]

\[ = (m_2(x_0 + x_3)^2 + m_3(x_0 + x_4)^2 + m_4(x_0 + x_5)^2 + m_5(x_0 + x_6)^2 + m_6(x_0 + x_7)^2)\bar{\psi}_{OP} = \]

\[ = N_{2x}\bar{I}_2 + N_{3z}\bar{I}_4 + N_{5z}\bar{I}_5 + N_{6z}\bar{I}_6 - m_{AG}g x_0 - (R_{4x} + R_{5x} + R_{6x})z_0 - (R_{4x} + R_{5x})h_{FCH} \]

\[ - (R_{4x} + R_{5x})h_{FCH} - R_{6x}h_D - 2g(m_2x_3 + m_3x_4 + m_4x_5 + m_5x_6 + m_6x_7). \]

In the resulting expressions we will replace the variables:
\[ A = \sum_1^2 c_F(x_1 + x_2)^2 + 1; \quad B = \sum_1^2 c_F(x_1 + x_2)\dot{z}_F - \sum_1^2 c_F(x_1 + x_2) \ddot{z}_F. \] (28)

The force of resistance to the moving of the tillage unit is calculated by the formula
\[ R_{UN} = R_{4x} + R_{5x} + R_{6x}. \] (36)

The formulas for determining the transmitted traction forces relative to the front wheel and to the rear wheel of tractor are as follows:
\[ T_{FW} = \frac{M_{FW}}{r_{FW}}; \quad T_{RW} = \frac{M_{RW}}{r_{RW}}. \] (37)

We also denote the moment from the resistance forces of the processing bodies of the multifunctional tillage unit in relation to the rear wheel axis:
\[ M_{UN,RW} = r_{RW}(R_{4x} + R_{5x} + R_{6x}) + (R_{4x} + R_{5x})h_{FC}\frac{h}{R_{6x}h_D}. \] (38)

The reduced moment of inertia of the mounted tillage unit in relation to the instantaneous center of inertia is calculated by the expression
\[ J_{AG} = m_2(x_2 + x_3)^2 + m_3(x_0 + x_4)^2 + m_4(x_0 + x_5)^2 + m_5(x_0 + x_6)^2 + m_6(x_0 + x_7)^2. \] (39)

The moment of gravity force of the mounted tillage unit in relation to the rear wheel axis of the tractor determined by the formula
\[ M_G = g(m_2x_3 + m_3x_4 + m_4x_5 + m_5x_6 + m_6x_7). \] (40)

The moment of gravity force of the unit relative to the instantaneous center of inertia
\[ M_{UN,G} = m_{AG}g\dot{x}_0 + g(m_2\dot{x}_3 + m_3\dot{x}_4 + m_4\dot{x}_5 + m_5\dot{x}_6 + m_6\dot{x}_7). \] (41)

The moment from the resistance forces of the operating bodies of the mounted tillage unit relative to the instantaneous center of inertia is as follows
\[ M_{UN,ICR} = (R_{4x} + R_{5x} + R_{6x})\dot{x}_0 + (R_{4x} + R_{5x})h_{FC}\frac{h}{R_{6x}h_D}. \] (42)

The moment from the reaction of the surface soil relative to the instantaneous center of inertia and equal in the case of stable moving \( M_{UN,G} \) :
\[ M_{SOIL,ICR} = N_{22e}\dot{l}_2 + N_{32e}\dot{l}_4 + N_{52e}\dot{l}_5 + N_{62e}\dot{l}_6. \] (43)

Then expressions (25), (26) and (27) will take the following form:
\[ A\dot{\phi}_{TP} - B + m_4x_2 = -N_{1FW}\dot{x}_1 + M_{SOIL,ICR} + x_2(m_4g - N_{1FW}) + M_{OP,RA}; \] (44)
\[ M_{UN,T}\dot{\phi}_{TP} = T_{FW} + T_{RW} - R_{OP}; \] (45)
\[ J_{np,\phi}_{OP} = M_{SOIL,ICR} + M_{OP,ICR} - M_G - M_{OP,G} + (F_{EL2}\dot{l}_2 + F_{EL4}\dot{l}_4 + F_{EL5}\dot{l}_5 + F_{EL6}\dot{l}_6). \] (46)

The resulting equations (27)...(29) enough fully describe the movement of the power machine with the mounted tillage unit taking into account their construction parameters \( m_1 ... m_6 \) masses; dimensions \( x_1 ... x_7, \dot{l}_2 ... \dot{l}_6, r_{FW}, r_{RW}; \) traction force on the front and rear axes of power machine \( (M_{FW}, M_{RW}), \) the locations of the operating bodies \( (x_3 ... x_7), \) the setting of the power machine hitch elements \( (x_0), \) disturbances from the field surface \( (z_{SOIL}) \) and disturbances from the resulting unevenness of the traction force \( R_x, N_x. \)

The obtained formulas indicate that the stability of the moving of the mounted tillage unit is most affected by the value of its reduced moment of inertia, also the locate of the mass center of the mounted tillage unit. Gravity force and traction resistance on the moving stability of the mounted tillage unit does not have such a significant impact.
3. Results and discussions

We believe that to compare the moving stability of the mounted tillage unit in the longitudinal-vertical plane with various placement of flat-cutting paws or of cultivator paws and of disk sections it is quite sufficient to take into score the free vibrations of the system at ignoring the viscosity of the environment, not taking into account the fading nature of vibrations and using equation (46).

We assume that the gravity of the aggregate is balanced by the preliminary deformation of the soil at the zero level for the z coordinate taking \(M_{SOIL,ICR} = M_{UN,G}\). Then the soil reaction \(N_z\) can be substitute by the elastic force \(F_{ELz}\) of the soil

\[
F_{ELz} = c \cdot z, \tag{47}
\]

where \(c\) – the coefficient of rigidity; \(z\) – the elementary moving which movement that will be equal to \(z_2 = \varphi_{UN} \cdot (x_0 + x_3); \ z_4 = \varphi_{UN} \cdot (x_0 + x_5); \ z_5 = \varphi_{UN} \cdot (x_0 + x_6); \ z_6 = \varphi_{UN} \cdot (x_0 + x_7). \tag{48}\)

Then the system of equations for determining the elastic forces for the support wheel, the first and second rows of flat-cutting paws or of cultivator paws and the disk sections will look like this:

\[
\begin{align*}
F_{ELz2} &= c_2 \cdot z_2 = c_2 \cdot \varphi_{UN} \cdot (x_0 + x_3), \\
F_{ELz4} &= c_4 \cdot z_4 = c_4 \cdot \varphi_{UN} \cdot (x_0 + x_5), \\
F_{ELz5} &= c_3 \cdot z_5 = c_3 \cdot \varphi_{UN} \cdot (x_0 + x_6), \\
F_{ELz6} &= c_4 \cdot z_6 = c_4 \cdot \varphi_{UN} \cdot (x_0 + x_7),
\end{align*}
\tag{49}
\]

where \(c_2, c_3, c_4\) – generalized coefficients of rigidity of the support wheels, of flat-cutting paws or cultivator paws and the of disk sections.

It is determined experimentally and analytically the generalized stiffness coefficients for all elements of the mounted tillage unit [4]. The values of coefficients for support wheels, disc sections, cultivator paws and flat-cutting paws respectively are present 6.5, 8.1, 4.2 and 5.1 kN/m.

We substitute the variables from the system of equations (49) into the formula (46) and transform it, so we get:

\[
\ddot{\varphi}_{UN} - \varphi_{UN} \cdot \frac{c_2(x_0+x_3)l_2+c_3(x_0+x_5)l_3+c_4(x_0+x_6)l_4+c_4(x_0+x_7)l_5}{J_{AG}} = \frac{M_{UN,ICR}-M_G}{J_{AG}}. \tag{50}
\]

Denoting a number of expressions in terms of variables we get:

\[
k^2 = \frac{c_2(x_0+x_3)l_2+c_3(x_0+x_5)l_3+c_4(x_0+x_6)l_4+c_4(x_0+x_7)l_5}{J_{AG}}, \quad H = \frac{M_{UN,ICR}-M_G}{J_{AG}}. \tag{51}
\]

Then the equation of vibrations of the aggregate will have the form:

\[
\ddot{\varphi}_{UN} - k^2 \varphi_{UN} = H. \tag{52}
\]

This expression refers to linear differential equations. It is necessary to determine the constant coefficients for the general solution of homogeneous differential equation \(\varphi_{UN}^*\) and the particular solution of non-homogeneous equation \(\varphi_{UN}^{**}\):

\[
\varphi_{UN} = \varphi_{UN}^* + \varphi_{UN}^{**}. \tag{53}
\]

The general solution of a homogeneous differential equation \(\varphi_{OP}^*\) has the form:

\[
\dot{\varphi}_{UN} - k^2 \varphi_{UN} = 0. \tag{54}
\]

We solve the expression (54) using the characteristic equation

\[
\lambda^2 - k^2 = 0, \tag{55}
\]

the roots of the equation of which is \(\lambda_{1,2} = k\).

\[
\varphi_{UN}^* = C_1 \sin kt + C_2 \cos kt. \tag{56}
\]
where $C_1$, $C_2$ – the coefficients of integration.

The right side of the equation with the constant coefficient determine the partial solution:

$$\varphi_{UN}(t) = a.$$  \hspace{1cm} (57)

It is found the general solution of equation (52) by double differentiating of the formula (57):

$$\varphi_{UN} = C_1 \cdot \sin kt + C_2 \cdot \cos kt + \frac{H}{k^2},$$  \hspace{1cm} (58)

$$\dot{\varphi}_{UN} = C_1 \cdot k \cdot \cos kt - C_2 \cdot k \cdot \sin kt.$$  \hspace{1cm} (59)

For initial conditions $t = 0$; $\varphi_{UN}(0) = \varphi_{UN0}$; $\dot{\varphi}_{UN}(0) = 0$ we get:

$$C_1 = 0; \quad C_2 = \varphi_{UN0} - \frac{H}{k^2}.$$  \hspace{1cm} (60)

Then the equation of movement of the tillage unit when working with flat-cutting paws or cultivator paws and a disk working bodies will take the form:

$$\varphi_{UN} = \left(\varphi_{UN0} - \frac{H}{k^2}\right) \cos kt + \frac{H}{k^2}.$$  \hspace{1cm} (61)

It are calculated the periods of free vibrations of the mechanical system for four variants of the placement of operating bodies on the frame of the tillage unit with the purpose to determine the moving stability of the of the machine-tractor aggregate: 1 – flat-cutting paws are located in front of the disc sections; 2 – disc section are located in front of the flat-cutting paws; 3 – the cultivation paws are located in front of the disk sections; 4 – disc sections are located in front of cultivator paws.

The damped nature of vibrations due to the viscosity of the soil did not take into the calculations. For all variants of the arrangement of the operating bodies of the mounted tillage unit the values of the free vibration period of this system are calculated by the formula

$$\tau = \frac{2\pi}{k}.$$  \hspace{1cm} (62)

The graphics of variation in the turning angle $\varphi_{OP}$ (when the step of change $\varphi_{OP} = 1^\circ$) of the tillage unit are shown in figure 2, when the machine-tractor aggregate is moving during tillage obtained as a result of calculations.

Figure 2. Change in the period of free vibrations of the mounted tillage unit into subjection on the disposition of the operating bodies: 1 – the flat-cutting paws are located in front of the disc sections; 2 – the disc section are located in front of the flat-cutting paws; 3 – the cultivation paws are located in front of the disk sections; 4 – the disc sections are located in front of cultivator paws.

We consider that the more shortly the time to the first crossing of the $0t$-axis, the quicker the comeback of the operating bodies of the mounted tillage unit to the tolerance zone of the depth of tillage will occur and consequently the more steady moving of the unit.

The time of the vibrations period for the first variant (the flat-cutting paws place in front the disk sections) was 1.213 s which is less than for the second variant (the disk sections locate in front the flat-
cutting paws), when time of the vibrations period was 1.385 s, i.e. less by 13.3%.

When using a replacement set of cultivator paws the indicators of stability of the moving unit have the following values: the free oscillation period for the third variant (the cultivator paws locate in front the disk sections) was 1.204 s; the period for the fourth variant (the disk sections set in front the cultivator paws) was 1.253 s; the difference between the variants is 4.22%.

4. Conclusion
A design and technological scheme of a multifunctional mounted tillage unit with replaceable operating bodies is proposed for execution of flat-cutting paws or cultivator paws tillage and of surface tillage at applying a single unit.

In intention to substantiation the rational disposition of flat-cutting paws or cultivator paws relatively to the disk operating bodies theoretic research were conducted to the moving stability of the mounted multifunctional tillage unit in a longitudinally-vertical plane, when tillage of flat-cutting paws or cultivator paws and of disk sections is carried out. The stability of the movement of the tillage unit in the longitudinal-vertical plane was evaluated using the period of free vibrations of the mechanical system by different schemes of placement of operating bodies.

The outcome of the computation of the time of free vibrations of the mounted multifunctional tillage unit is testify that the location of flat-cutting paws in front of disk sections reduces the period of vibrations of the unit by 13.3% in comparison with the location disk sections in front of flat-cutting paws. At using in the unit the cultivator paws, when the cultivation paws are located in front of the disc sections, reduces the period of oscillation of the unit by 4.22%.

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