Injection of a microsatellite in circular orbits using a three-stage launch vehicle

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Abstract. The injection of a satellite into orbit is usually done by a multi-stage launch vehicle. Nowadays, the space market demonstrates a strong tendency towards the use of smaller satellites, because the miniaturization of the systems improve the cost/benefit of a mission. A study to evaluate the capacity of the Brazilian Microsatellite Launch Vehicle (VLM) to inject payloads into Low Earth Orbits is presented in this paper. All launches are selected to be made to the east side of the Alcântara Launch Center (CLA). The dynamical model to calculate the trajectory consists of the three degrees of freedom (3DOF) associated with the translational movement of the rocket. Several simulations are performed according to a set of restrictions imposed to the flight. The altitude reached in the separation of the second stage, the altitude and velocity of injection, the flight path angle at the moment of the activation of the third stage and the duration of the ballistic flight are presented as a function of the payload carried.

1. Introduction
According to the Brazilian "National Space Activities Program" (PNAE), it is necessary the development of critical technologies in the Brazilian space industry [1]. During the design process and the launch of space rockets, it is required modelling and simulations. Recently, researchers at INPE and ITA presented a computational tool to simulate the motion of a launch vehicle in 3DOF and 6DOF [2,3]. Another studies explored various engine configurations for rocket liquid propellant and to calculate the performance of the Satellite Launch Vehicle – VLS [4]. The study to analyse the dynamics of the separation of the first stage of the VLM to quantify the displacements and rotations between the vehicle and the discarded stage was performed in 2015 [5]. The problem of the VLM trajectory optimization is performed by [2,6]. The main objective of this work is to make a preliminary analysis on the capacity of the VLM to inject payloads in circular orbits with inclination close to 2,3°.

2. Methodology
The Brazilian Microsatellite Launch Vehicle (VLM) is selected as the launch vehicle to perform the simulations. It is a vehicle with approximately 28 ton composed of three stages of solid propulsion configured in series (tandem) [6].

The launch site is Alcântara (CLA), with geodetic coordinates 2°22’39.52’’S, 44°23’57.71’’W at the sea medium level. The atmospheric model is the US ATM 1976, rotating with the planet and without winds [7]. The mathematical expansion of the Earth’s geopotential model up to the J₄ spherical harmonic was performed [8]. However, in this work it was considered up to the J₆ spherical harmonic due to the proximity of Alcântara with the terrestrial Equator. The rocket engine is ideal, the
thrust force is associated to the propellant specific impulse, without variations in thrust due to the atmospheric pressure. A Runge-Kutta-Fehlberg seventh order (RKF-7) numerical integrator with fixed stepsize ($10^{-3}$) solves the differential equations [9]. The VLM technical data are presented in Table 1.

Table 1. VLM’s data [6].

| PARAMETERS      | UNITS | STAGE I | STAGE II | STAGE III | FAIRING |
|-----------------|-------|---------|----------|-----------|---------|
| Diameter        | cm    | 135     | 135      | 101       | 120     |
| Liftoff weight  | kg    | 12900   | 13100    | 1004      | 110     |
| Propellant weight| kg   | 11500   | 11500    | 814       | -       |
| Specific impulse| s     | 286.4   | 286.4    | 289       | -       |
| Burn time       | s     | 85      | 85       | 71        | -       |

3. Formulation of the dynamical model

To determine the possible flight path of the VLM, it is necessary to analyze the initial conditions on the launch site and the forces applied in its center of gravity along the trajectory. Three forces are taking into account in the mass-point body or carrier rocket. The principal equation that describes the dynamical system is Equation 1, where $\ddot{r}$ is the inertial acceleration.

$$\ddot{r} = \frac{\vec{T}}{m} - \frac{\vec{D}}{m} - \vec{g}$$

where $\vec{T}$ is the force coming from the engine, $\vec{D}$ the aerodynamic drag, $\vec{g}$ the gravity and $m$ is the mass of the body, which changes as a function of time due to the mass flow consumption.

The force opposite to the motion of the spacecraft, known as drag, is proportional to the projected area of the body ($A$), the drag coefficient ($C_D$), the atmospheric density ($\rho$) and the velocity of the spacecraft with respect to the atmosphere ($V$).

$$D = \frac{1}{2} \rho A C_D V^2$$

The description of the equations in the local reference system and the transformation between different reference systems are available in [10].

The dynamic equations in 3DOF are integrated in the Earth-Centered, Earth-Fixed (ECEF) system. The inertial reference system (ECI) is used to calculate the inclination of the injection orbit. The trajectory is guided through the local navigation system (ENU) whose origin is the center of mass of the rocket. These three coordinate systems are shown in Figure 1.

![Figure 1. ECEF, ECI and ENU coordinate systems.](image-url)
The equations that transform the position and velocity of the rocket from the rotating to the inertial coordinate system are, respectively,

\[
\begin{bmatrix}
P_X \\
P_Y \\
P_Z
\end{bmatrix}_{\text{ECI}} = \begin{bmatrix}
\cos(\omega t) & -\sin(\omega t) & 0 \\
\sin(\omega t) & \cos(\omega t) & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
P_X \\
P_Y \\
P_Z
\end{bmatrix}_{\text{ECEF}}
\]

(3)

\[
\begin{bmatrix}
V_X \\
V_Y \\
V_Z
\end{bmatrix}_{\text{ECI}} = \begin{bmatrix}
\cos(\omega t) & -\sin(\omega t) & 0 \\
\sin(\omega t) & \cos(\omega t) & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
V_X \\
V_Y \\
V_Z
\end{bmatrix}_{\text{ECEF}} + \begin{bmatrix}
-\omega \sin(\omega t) & -\omega \cos(\omega t) & 0 \\
\omega \cos(\omega t) & -\omega \sin(\omega t) & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
P_X \\
P_Y \\
P_Z
\end{bmatrix}_{\text{ECEF}}
\]

(4)

where \( \omega = 7,292,1159 \times 10^{-5} \) rad/s is the angular velocity of the Earth and \( t \) the flight time of the rocket.

3.1. Flight restrictions and simplifying assumptions
The constraints imposed in the algorithm that calculates the trajectory are as follows:
- the second stage is ignited immediately after the end of the firing of the first stage
- the mass of the fairing is subtracted at 105 km of altitude
- the third stage is limited to offering only horizontal velocity to the payload, i.e., the yaw correction maneuver (“dog leg”) is not performed
- the azimuth angle is maintained above 89° throughout the flight
- drag coefficient is constant and equal to 0.5
- the influence of the wind is not considered
- eccentricity of the injection orbit is maintained below 10^{-4}

4. Results
The results obtained for launching payloads from 150 to 225 kg mass, in the eastern direction of CLA, are shown in figures 2 to 5. According to Equation 1, the larger the mass of payload carries, the lower is the acceleration delivered by the rocket. This causes the rocket to reach lower altitudes which, consequently, reduces the radius of the injection orbit. However, the smaller the radius of the orbit, the larger the velocity required to keep the microsatellite in orbit, as shown in Figure 2. A linear ratio are inverse to the payload increase and the injection altitude, but the increment in payload mass requires larger injection velocity.

![Figure 2. Altitude and velocity injection vs. payload mass.](image-url)
The burning time of the three propellant stages of the VLM are well defined in Table 1. The duration of the ballistic flight phase is a parameter to be determined according to the restrictions imposed for the circular orbit. At this stage of flight there are no thrusters acting in the vehicle, it is a free or ballistic flight. The high altitudes and lower density atmosphere reduces the drag effect and the gravity is the main force acting along the trajectory. The increasing in the payload mass reduces the horizontal range of the final stage rocket, as well as the free flight time, because the mass increment generates a faster gravity turn, so reducing the horizontal velocity (Figure 3).

The geographic coordinates which correspond to the position where the payload separation occurred is shown in Figure 4. The small variation in latitude with respect to the position of the CLA is due to the constraint imposed on the azimuth angle of the VLM ($\beta = 90^\circ$).

The second stage burnout altitude and FPA reduces due to the increment in the payload mass (Figure 5). These variations generate changes in the trajectory and in the final orbital injection.
To determine the final ballistic time and the payload injection, the selected parameter was the FPA. A value around 0° is critical for the third stage ignition to guarantee the impulsive force in the same direction of the velocity and the increase of the final velocity. This relation is quasi-linear and presented in Figure 6.

5. Conclusions and recommendations
The implementation of the mathematical model and the VLM performance in a computational code to simulate the flight trajectory are useful to determine the payload orbital conditions and the variations in the trajectory caused by the possible payload masses. In this case, the results show an increment in the orbital velocity proportional to the payload increment and a reduction in the injection altitude. The horizontal range and the ballistic time are inversely proportional to the payload mass increment. The payload mass increment generates altitude losses in the second stage because the gravity attraction and the lower acceleration generates a higher FPA changes due to the gravity turn.

The small satellite industry requires a new configuration in the Launch Vehicles. The paper presents in 3DOF the trajectory and the orbital injection changes due to the small payload variations. The Brazilian VLM was selected for the study. The simulated data are useful to compare the future flight data in the VLM’s missions. It is recommended an implementation of the mathematical model in 6DOF for better approximations.
Acknowledgements
The authors wish to express their appreciation for the support provided by the National Council for the Improvement of Higher Education (CAPES), the National Institute for Space Research (INPE), the grants # 406841/2016-0 and 301338/2016-7 from the National Council for Scientific and Technological Development (CNPq), and the grants # 2011/08171-3, 2016/14665-2, from Sao Paulo Research Foundation (FAPESP).

References
[1] AEB, National Program of Space Activities PNAE: 2012-2021, Brasilia, 2012.
[2] Souza C H M Development of a computational tool for use in the design of launch vehicles and application to the Vehicle Microsatellite Launcher. 65p. Undergraduate thesis – Aeronautics Institute of Technology (ITA), São José dos Campos – Brazil, 2012.
[3] Silveira G Development of a computational tool for flight simulation of launch vehicles. 151p. Master Thesis – National Institute for Space Research (INPE), São José dos Campos – Brazil, 2014.
[4] Mota F A S Modeling and simulation of launch vehicles using object-oriented programming. 181p. Doctoral thesis – National Institute for Space Research (INPE), São José dos Campos – Brazil, 2015.
[5] Souza C H M Analysis of the VLM-I first stage separation dynamics. Master Thesis – National Institute for Space Research (INPE), São José dos Campos – Brazil, 2015.
[6] Miranda D J F Trajectory optimization of a satellite launch vehicle using pseudo-spectral methods. 170p. Undergraduate thesis – Aeronautics Institute of Technology (ITA), São José dos Campos – Brazil, 2012.
[7] NOAA; NASA; US Air Force. US Standard Atmosphere, 1976. Washington, D. C.: Government Printing Office, 1976.
[8] Mooij, I. E. The motion of a vehicle in a planetary atmosphere, Delft University of Technology, Faculty of Aerospace Engineering, Delft – Netherlands, pages 29 - 31, 1994.
[9] Fehlberg, E. Classical Fifth-, Sixth-, Seventh-, and Eighth-Order Runge Kutta Formulas with Stepsize Control – NASA Technical Report-287. George C. Marshall Space Flight Center, Huntsville, Alabama – USA, 1968.
[10] Benavoli A, Chisci L, and Farina A 2007 Tracking of a Ballistic Missile with a-priori information IEEE Transactions on Aerospace and Eletronic Systems vol 43 issue 3 pp. 1000-1016. DOI: 10.1109/TAES.2007.4383589