Four-fermion production at LEP2 and NLC

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ABSTRACT

The present knowledge on four-fermion production in electron-positron collisions is reviewed, with emphasis on $W$ boson physics. Different methods to extract $M_W$ from the data are presented and the role of QCD loop corrections discussed.

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1 Introduction

Four-fermion processes represent the experimentally measured signal for $W$ boson physics at LEP2 and NLC. In fact, the produced $W$'s always decay, giving four-fermion final states [1]. One is primarily interested in measuring the $W$ mass [2], but also measurements of the trilinear gauge boson couplings [3] and the four-fermion cross sections [4] provide useful information. An accurate determination of $M_W$ can be combined with the precision data coming from LEP1 to further constrain the Standard Model of the Electroweak Interactions. In fact, a global fit to the LEP1 data [5] predicts the $W$ mass with an error of the same order as the expected final LEP2 error (35 MeV), while NLC will presumably reduce that error down to 15 MeV [6]. A strong discrepancy between predicted and measured $M_W$ would be a signal for new physics. Alternatively, an improvement on the measurement of $M_W$ can significantly tighten the present bounds on the Higgs mass through the relation

$$G_{\mu} = \frac{\alpha \pi}{\sqrt{2} M_W^2 (1 - M_W^2/M_Z^2)} \cdot \frac{1}{1 - \Delta_r(m_t, m_H)},$$

where $\Delta_r$ is a calculable contribution coming from radiative corrections.

Several tree-level four-fermion codes are available [4]. Electroweak radiative corrections are usually included at the leading log (LL) level, but very recently, non-factorizable QED corrections have been computed as well [7]. While tree level programs + LL corrections seem in general to be adequate to deal with LEP2 physics, further refinements, especially in the sector of the loop radiative corrections, are needed in view of the NLC precision physics.

In this contribution, I focus my attention on two particular aspects of four-fermion $W$ physics, namely $M_W$ measurement and QCD corrections.

In the next section, I describe two techniques to measure the $W$ mass, and present a new method to extract $M_W$ from the best measured variables.
When quarks are present in the final state, QCD loop contributions have to be included as well. Those corrections are discussed in the last section of the paper.

2 $M_W$ measurement

Two methods are mainly used to extract the $W$ mass: the threshold method and the direct reconstruction technique [2]. In the first case the total $W^+W^-$ cross section is measured near threshold (161 GeV), where the sensitivity to $M_W$ is stronger, and plotted as a function of the $W$ mass. At LEP2, that gives $M_W = 80.40 \pm 0.22$ GeV [5].

The direct reconstruction method is applied at higher energy, where the statistics increases. It requires three steps:

1. From the experimental data the invariant mass distribution $\frac{d\sigma}{dM}$ is reconstructed. To improve the mass resolution, a constrained fit is usually performed event by event, assuming no Initial State Radiation (ISR) and equality between the invariant masses coming from different $W$'s.
2. A theoretical distribution is taken for $\frac{d\sigma}{dM}$ (usually a convolution of a Breit-Wigner with a Gaussian) and a mass $M'_W$ fitted.
3. The value $M'_W$ is then corrected by Monte Carlo, from the bias introduced by the constrained fit, to get the reconstructed $W$ mass $M_R$ with an error $\Delta M_R$.

Such a measurement gave $M_W = 80.37 \pm 0.19$ GeV [5], in the LEP2 run at 172 GeV.

Recently, a new method has been proposed [8] (direct fit method), in which only the best measured quantities are used to extract the $W$ mass. The idea is simple. Given a set of well measured quantities $\{\Phi\}$ one computes, event by event, the theoretical probability $P_i$ of getting the observed set of values $\{\Phi_i\}$ for $\{\Phi\}$. This is a function of $M_W$ and is given by the ratio of the differential cross section in those variables, divided by the total cross section in the experimental fiducial volume

$$P_i(M_W) = \frac{\frac{d\sigma}{d\Phi}}{\sigma}.$$  \hspace{1cm} (2)

Given $N$ observed events, the logarithm of the likelihood function $L$

$$\log L(M_W) \equiv \log \prod_{i=1}^{N} P_i(M_W) = \sum_{i=1}^{N} \log \frac{d\sigma}{d\Phi_i}(M_W) - N \log \sigma(M_W)$$  \hspace{1cm} (3)

is distributed, for large $N$, as a quadratic function of $M_W$. The previous equation is then computed for different values of $M_W$ and a parabola fitted, from which the reconstructed $W$ mass $M_R$ is obtained with an error $\Delta M_R$. 

In order to construct a tool for the evaluation of $P_i(M_W)$ one has to choose the set $\{\phi\}$ of accurately measured variables. Although one can always consider more sets $\{\phi\}$, the following choices seem reasonable in practice [9] for different four-fermion final states:

1. Semileptonic case: $q_1q_2\ell\nu$
   
   1a $\{\phi\} = \{E_\ell, \Omega_\ell, \Omega_{q_1}, \Omega_{q_2}\}$
   
   1b $\{\phi\} = \{E_\ell, \Omega_\ell, \Omega_{q_1}, \Omega_{q_2}, E_h\}$, where $E_h$ is the total energy of the jets.

2. Purely hadronic case: $q_1q_2q_3q_4$
   
   $\{\phi\} = \{\Omega_{q_1}, \Omega_{q_2}, \Omega_{q_3}, \Omega_{q_4}\}$.

3. Purely leptonic case: $\ell_1\nu_1\ell_2\nu_2$
   
   $\{\phi\} = \{E_{\ell_1}, \Omega_{\ell_1}, E_{\ell_2}, \Omega_{\ell_2}\}$

Since eight variables determine an event when no ISR is present, sets 1a and 3 would require one or two integrations, cases 1b and 2 none. Including ISR adds two integrations. When jets cannot be assigned to specific quarks a folding over the various possibilities should be included.

As an example of the direct fit method, I show, in table 1, the reconstructed masses obtained by fitting a sample of 1600 EXCALIBUR [1] CC3 events [4] including ISR, generated with an input mass of $M_W = 80.23$ GeV, at $\sqrt{s} = 190$ GeV. In all four cases, the $W$ mass is correctly reconstructed.

|       | 1a          | 1b          | 2            | 3            |
|-------|-------------|-------------|--------------|--------------|
| $M_W$ | 80.238 ± 0.049 | 80.238 ± 0.032 | 80.255 ± 0.036 | 80.209 ± 0.077 |

**Table 1.** Reconstructed $W$ mass (GeV) with four choices of the set $\{\phi\}$ (see text).

Cases 1b and 2 give better errors, because less information is integrated out. Conversely, in the leptonic case 3, the error is worse, since a large part of the kinematical information is actually missing.

In the direct reconstruction method, one wants to keep as much information as possible, also preferably photon momenta, in order to reconstruct the kinematics event by event. On the contrary, in the direct fit method, one has to integrate over all information that is not well determined, in particular ISR. Because of the fact that the integral over the $p_T$ distribution of ISR photons is theoretically better known than the distribution itself, one expects the details of the radiation to matter less in the direct fit method. That can be viewed as an advantage with respect to the direct reconstruction technique.

However, a last remark is in order. All numbers presented in table 1 refer to the partonic level, without inclusion of hadronization effects and detector
resolution. Therefore one still has to prove that the fitting procedure survives those effects. This question is currently under investigation [10].

It is also clear that the direct fit method is not only applicable to measure $M_W$, but can be used, in principle, to extract any parameter - as $\Gamma_W$ or a set of anomalous trilinear gauge couplings (TGCs) - from the data sample. Not surprisingly, the whole strategy for the direct fit method has been first discussed in the final report of the Workshop on Physics at LEP2, in the context of TGCs determination [3].

3 QCD corrections

QCD loop corrections to four-fermion production in $e^+e^-$ collisions can be divided in two classes, namely QCD corrections to $\mathcal{O}(\alpha^2\alpha_s^2)$ and $\mathcal{O}(\alpha^4)$ processes, respectively.

The first corrections appear as $\mathcal{O}(\alpha_s^2)$ contributions to four-jet production via QCD [11], while the second ones are relevant for studying $W$ boson physics, and, more in general, semileptonic four-fermion processes and fully hadronic final states mediated by electroweak bosons.

I shall concentrate here on the latter contributions, which can all be obtained by defining suitable combinations of loop diagrams plus real gluon radiation, as shown in ref. [12]. The calculation is simplified a lot by using the reduction procedure presented in ref. [13].

In table 2, cross sections computed with the program in ref. [12] are presented, for the semileptonic process $e^+e^- \rightarrow \mu^-\bar{\nu}_\mu u \bar{d}$.

| $\sqrt{s}$ | Born       | NLO        | nQCD       |
|------------|------------|------------|------------|
| 161 GeV    | 24962 ± .00002 | 24760 ± .00002 | 24790 ± .00002 |
| 175 GeV    | .96006 ± .00007 | .94519 ± .00007 | .94613 ± .00007 |
| 190 GeV    | 1.184003 ± .00009 | 1.16681 ± .00009 | 1.16766 ± .00008 |
| 500 GeV    | .46970 ± .00006 | .47109 ± .00007 | .46131 ± .00006 |

Table 2. Cross sections in pb for $e^+e^- \rightarrow \mu^-\bar{\nu}_\mu u \bar{d}$ with canonical cuts [4].

The exact calculation (NLO) is compared with a “naive” approach to strong radiative corrections (nQCD), where the QCD contributions are simply included through the substitutions
\[ \Gamma_W \to \Gamma_W \left( 1 + \frac{2}{3} \frac{\alpha_s}{\pi} \right), \quad \sigma \to \sigma \left( 1 + \frac{\alpha_s}{\pi} \right). \]  

For the direct reconstruction of the \( W \) mass, the quantity
\[ (\Delta M) = \frac{1}{2\sigma} \int \left( \sqrt{s^+} + \sqrt{s^-} - 2M_W \right) d\sigma \]
is relevant [4], where \( M_W \) is the input mass in the program. One gets, with canonical cuts at \( \sqrt{s} = 175 \) GeV:
\[ (\Delta M)_{\text{NLO}} = -0.5585 \pm 0.0002 \text{ GeV} \]
\[ (\Delta M)_{\text{nQCD}} = -0.5583 \pm 0.0002 \text{ GeV}. \]  

Also, one can show that the angular distributions are distorted, in the NLO calculation, with respect to the nQCD prediction [12].

From the previous results it is clear that nQCD is adequate at LEP2 for semileptonic processes, but exact calculations are important at NLC and for anomalous couplings studies, where the angular distributions matter to constrain the anomalous contributions.

In table 3, I show results for the fully hadronic process \( e^+e^- \to u\bar{d}s\bar{c} \) at \( \sqrt{s} = 175 \) GeV. The numbers are obtained by using the program described in ref. [14]. In case (a) only canonical cuts are applied. In (b) two reconstructed masses \( M_{R1} \) and \( M_{R2} \) are determined by minimizing the quantity \( \Delta'_M = (M_{R1} - M_W)^2 + (M_{R2} - M_W)^2 \), and a cut \( |M_{Ri} - M_W| < 10 \) GeV is imposed. In (c) a smearing with a Gaussian with a 2 GeV width is introduced in addition, to mimic the experimental resolution.

| \( \sigma \) (pb) | (a)       | (b)       | (c)       |
|------------------|-----------|-----------|-----------|
| NLO              | 1.1493(4) | 0.7895(5) | 0.7758(9) |
| nQCD             | 1.1069(3) | 1.0545(3) | 1.0479(3) |

Table 3. Cross sections for \( e^+e^- \to u\bar{d}s\bar{c} \) at \( \sqrt{s} = 175 \) GeV.

For the process at hand one gets, for case (b),
\[ (\Delta M)_{\text{NLO}} = -0.2290 \pm 0.0010 \text{ GeV} \]
\[ (\Delta M)_{\text{nQCD}} = -0.0635 \pm 0.0004 \text{ GeV} , \]  

where now \( (\Delta M) = \frac{1}{2\sigma} \int \left( M_{R1} + M_{R2} - 2M_W \right) d\sigma \).

We are easily convinced, by the above results, that the naive QCD implementation fails in describing hadronic four-fermion final states at LEP2. In particular, the reduction in cross section (compare cases (a) and (b) in table 3) shows that many soft gluons are exchanged between decay products of different \( W \)'s that are not taken into account using nQCD. A failure of nQCD can also be proved at NLC energies [14].
4 Conclusions

Precise measurements of $M_W$ are performed at LEP2, using the threshold method and the direct reconstruction of $\frac{d\sigma}{dM_W}$.

An alternative technique is also available in which only well measured quantities are directly used to extract $M_W$ (and TGCs) from the data.

QCD loop corrections to semileptonic $\mathcal{O}(\alpha^4)$ processes are well approximated at LEP2, by nQCD, except for angular distributions, while the naive approach fails in describing hadronic four-fermion final states.

All existing calculations have to be refined and radiative corrections better understood in view of the NLC precision measurements.

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