Research Article

On the Unstable Solutions to Functional Vector Differential Equations of the Seventh Order

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This paper studies the instability of the zero solution for a certain nonlinear functional vector differential equation of the seventh order with multiple deviating arguments. Under sufficient conditions, we prove a result on the instability of the zero solution. This work contributes and complements to previously known results in the literature.

1. Introduction

More than 100 years ago, the world famous mathematician Lyapunov established the Lyapunov direct method to study stability problems. From then on, the Lyapunov’s direct method was also widely used to study the instability of solutions of ordinary differential equations and functional differential equations, see for example, Bereketoğlu [1], Sadek [2], Tejumola [3], Tunç [4–7], C. Tunç and E. Tunç [8], and the references therein. However, a review to date of the literature indicates that the instability of the solutions to the nonlinear functional vector differential equation of the seventh order has not been investigated. This paper is the first work on the subject. It is also known that the expressions of Lyapunov-Krasovskii functional are very complicated and hard to construct. In this paper, we define a Lyapunov-Krasovskii functional [9] and base on the Krasovskii criteria to prove a new theorem on the topic for the nonlinear functional vector differential equation of the seventh order. In this paper, intend to make a contribution to the subject since the functional differential equations have an important place in various fields of science and engineering.

Meanwhile, some respective contributions on the topic can be summarized as follows.

In 2000, Tejumola [3] discussed the instability of the zero solution of the seventh order scalar nonlinear differential equation without a deviating argument

\[ x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} + \psi_4 \left( x, x', . . . , x^{(6)} \right) x'' + \psi_5 \left( x' \right) x'' + \psi_6 \left( x, x', . . . , x^{(6)} \right) + \psi_7 \left( x \right) = 0. \]  

\[(1)\]

Later, in 2003, Sadek [2] proved an instability theorem for the seventh order scalar nonlinear differential equation without a deviating argument

\[ x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} + a_4 x''' + f \left( x' \right) x'' + g \left( x \right) x' + h \left( x \right) = 0. \] \[(2)\]

Recently, Tunç [6, 7] discussed the same problem for the seventh order scalar nonlinear delay differential equations of the form:

\[ x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} + a_4 x''' + f \left( x' \right) x'' + g \left( x \right) x' + h \left( x, x \left( t - r \right), x', . . . , x^{(6)} \left( t - r \right) \right) = 0, \] \[(3)\]
\[ x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} + \psi_4 \left( x, x(t - r), x', \ldots, x^{(6)}(t - r) \right) x'' + \psi_5 \left( x' \right) x'' + \psi_6 \left( x, x(t - r), x', \ldots, x^{(6)}(t - r) \right) + \psi_7 \left( x(t - r) \right) = 0. \]  

In this paper, instead of (4), we consider the seventh order nonlinear functional vector differential equation with \( n \)-deviating arguments, \( \tau_i, (i = 1, 2, \ldots, n) \):

\[ X^{(7)} + A_1 X^{(6)} + A_2 X^{(5)} + A_3 X^{(4)} + \psi_4 \left( X, X(t - \tau_1), \ldots, X(t - \tau_n), X^{(6)}(t - \tau_n) \right) X'' + \psi_5 \left( X' \right) X'' + \psi_6 \left( X, X(t - \tau_1), \ldots, X(t - \tau_n), X^{(6)}(t - \tau_n) \right) X' + \sum_{i=1}^n H_i \left( X(t - \tau_i) \right) = 0. \]  

Setting \( X = X_1, X' = X_2, X'' = X_3, X''' = X_4, X^{(4)} = X_5, X^{(5)} = X_6, X^{(6)} = X_7 \), we can write (5) in system form as follows:

\[
\begin{align*}
X_1' &= X_2, \\
X_2' &= X_3, \\
X_3' &= X_4, \\
X_4' &= X_5, \\
X_5' &= X_6, \\
X_6' &= X_7, \\
X_7' &= -A_1 X_7 - A_2 X_6 - A_3 X_5 - \psi_4 \left( X_1, \ldots, X_1(t - \tau_n), \ldots, X_7(t - \tau_n) \right) X_4 - \psi_5 \left( X_2 \right) X_3 - \psi_6 \left( X_1, \ldots, X_1(t - \tau_n), \ldots, X_7(t - \tau_n) \right) X_2 - \sum_{i=1}^n H_i \left( X(t - \tau_i) \right). 
\end{align*}
\]  

where \( \tau_i \) are certain positive constants, the fixed delays, \( t - \tau_i > 0 \), \( A_1, A_2, \) and \( A_3 \) are constant \( n \times n \)-symmetric matrices, the primes in (5) denote differentiation with respect to \( t \), \( \tau_i \in \mathbb{R}_+ \), \( \mathbb{R}_+ = [0, \infty) \); \( \psi_4, \psi_5, \) and \( \psi_6 \) are continuous \( n \times n \)-symmetric matrix functions for the arguments displayed explicitly, \( H_i : \mathbb{R}^n \to \mathbb{R}^n, H_i(0) = 0, H_i(X_1) \neq 0, (X_1 \neq 0) \), and \( H_i \) are continuous for all the respective arguments. The Jacobian matrices of \( H_i(X) \) are given by

\[ J_{H_i}(X) = \left( \frac{\partial h_{ij}}{\partial x_k} \right), \ldots, J_{H_n}(X) = \left( \frac{\partial h_{ni}}{\partial x_k} \right), \quad (i, j, k = 1, 2, \ldots, n), \]  

where \((x_1, \ldots, x_n)\) and \((h_{ij}), \ldots, (h_{ni})\) are the components of \( X \) and \( H_i \), respectively. It is also assumed that the Jacobian matrices \( J_H(X) \) exist and are continuous. The existence and uniqueness of the solutions of (5) are assumed (see El'sgol'ts [10, pages 14, 15]). Throughout what follows that \( X_1(t), \ldots, X_7(t) \) are abbreviated as \( X_1, \ldots, X_7 \), respectively.

Let \( r \geq 0 \) be given, and let \( C = C([-r, 0], \mathbb{R}^n) \) with

\[ \| \phi \| = \max_{-r \leq s \leq 0} | \phi(s) |, \quad \phi \in C. \]  

For \( H > 0 \) define \( C_H \subset C \) by

\[ C_H = \{ \phi \in C : \| \phi \| < H \}. \]  

If \( x : [-r, A) \to \mathbb{R}^n \) is continuous, \( 0 < A \leq \infty \), then, for each \( t \) in \([-r, A) \), \( x_i \) in \( C \) is defined by

\[ x_i(s) = x(t + s), \quad -r \leq s \leq 0, \quad t \geq 0. \]  

Let \( G \) be an open subset of \( C \) and consider the general autonomous delay differential system

\[ \dot{x} = F(x), \quad x(t) = x(t + \theta), \quad -r \leq \theta \leq 0, \quad t \geq 0, \]  

where \( F : G \to \mathbb{R}^n \) is continuous and maps closed and bounded sets into bounded sets. It follows from the conditions on \( F \) that each initial value problem

\[ \dot{x} = F(x), \quad x_0 = \phi \in G \]  

has a unique solution defined on some interval \([0, A), 0 < A \leq \infty \). This solution will be denoted by \( x(\phi)(t) \) so that \( x_0(\phi) = \phi \).

**Definition 1.** The zero solution, \( x = 0 \), of \( \dot{x} = F(x) \) is stable if for each \( \varepsilon > 0 \) there exists \( \delta = \delta(\varepsilon) > 0 \) such that \( \| \phi \| < \delta \) implies that \( |x(\phi)(t)| < \varepsilon \) for all \( t \geq 0 \). The zero solution is said to be unstable if it is not stable.

Consider the linear constant coefficient differential equation of the seventh order:

\[ x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} + a_4 \dot{x} + a_5 \ddot{x} + a_6 \dddot{x} + a_7 x = 0. \]  

It is known from the qualitative behavior of solutions of linear differential equations that the zero solution of (13) is unstable if and only if, the associated auxiliary equation:

\[ \psi(\lambda) \equiv \lambda^7 + a_1 \lambda^6 + a_2 \lambda^5 + a_3 \lambda^4 + a_4 \lambda^3 + a_5 \lambda^2 + a_6 \lambda + a_7 = 0 \]  

has at least one root with a positive real part. The existence of such a root naturally depends on (though not always all of) the coefficients \( a_1, a_2, \ldots, a_7 \) in (14). For example, if

\[ a_1 < 0 \]  

then it is clear from a consideration of the fact that the sum of the roots of (14) equals \( a_1 \), and that at the least one root of (14) has a positive real part for arbitrary values of \( a_2, a_3, a_4, a_5, a_6, \) and \( a_7 \). An analogue consideration, combined with the
fact that the product of the roots (14) equals \((-a_7)\) will verify that at least one root of (14) have a positive real part if
\[
a_1 = 0, \quad a_7 \neq 0
\]
for arbitrary \(a_2, a_3, a_4, a_5, a_6\). The condition \(a_1 = 0\) here in (16) is, however, superfluous when
\[
a_7 < 0;
\]
then for \(\psi(0) = a_7 < 0\) and \(\psi(R) > 0\) if \(R > 0\) is sufficiently large, thus showing that there is a positive real root of (14) subject to (17) and for arbitrary \(a_1, a_2, a_3, a_4, a_5\), and \(a_6\). Moreover, a necessary and sufficient condition for (14) to have a purely imaginary root \(\lambda = i\beta\) (\(\beta\) real) is that the two equations
\[
-a_1\beta^4 + a_3\beta^2 + a_5 = 0,
\]
\[
-a_2\beta^4 + a_4\beta^2 + a_6 = 0
\]
are satisfied at the same time. If
\[
a_1 \leq 0, \quad a_3 \geq 0, \quad a_5 \leq 0, \quad a_7 > 0
\]
or
\[
a_1 \geq 0, \quad a_3 \leq 0, \quad a_5 \geq 0, \quad a_7 < 0,
\]
then (14) cannot have any purely imaginary root whatever.

It should be noted that there are no restrictions on the constants \(a_2, a_4, \) and \(a_6\) in (13).

2. Main Result

First, we give the following lemma.

**Lemma 2.** Let \(A\) be a real symmetric \(n \times n\)-matrix and
\[
a^i \geq \lambda_i(A) \geq a > 0, \quad (i = 1, 2, \ldots, n),
\]
where \(a^i\) and \(a\) are constants, and \(\lambda_i(A)\) are the eigenvalues of the matrix \(A\).

Then
\[
a^i \langle X, X \rangle \geq \langle AX, X \rangle \geq a \langle X, X \rangle,
\]
\[
a^2 \langle X, X \rangle \geq \langle AX, AX \rangle \geq a^2 \langle X, X \rangle.
\]

(Bellman [11]).

Let \(\tau = \max \tau_i, \quad (i = 1, 2, \ldots, n)\).

The following theorem is our main result.

**Theorem 3.** In addition to all the assumptions imposed to \(A_1, A_2, A_3, \Psi_4, \Psi_5, \Psi_6, \) and \(H_1\) that appear in (5), we assume that there exist constants \(a_2 < 0, \alpha_i > 0, \beta_i > 0, \) and \(\delta > 0\) such that the conditions
\[
\lambda_i(A_2) \leq a_2, \quad \beta_i \leq \lambda_i(JH_1(X_1)) \leq a_i,
\]
\[
\frac{1}{4a_2^2} \left[\lambda_i(\Psi_4(\cdot))\right]^2 - \lambda_i(\Psi_6(\cdot)) \geq \delta > 0
\]
hold. If \(\tau < \delta/\sqrt{n}(\alpha_1 + \cdots + \alpha_n)\), then the zero solution of (5) is unstable.

**Remark 4.** For the proof of the theorem, under the conditions stated in the theorem, it suffices to find that there exists a continuous Lyapunov-Krasovskii functional \(V(\cdot) = V(X_1, \ldots, X_7)\), which has the Krasovskii properties [9], say \((K_1), (K_2), \) and \((K_3)\).

\((K_1)\) In every neighborhood of \((0, 0, 0, 0, 0, 0, 0)\) there exists a point \((\xi_1, \ldots, \xi_7)\) such that \(V(\xi_1, \ldots, \xi_7) > 0\).

\((K_2)\) The time derivative \(\dot{V}(\cdot) = (d/dt)V(X_1, \ldots, X_7)\) along solution paths of (6) is positive semidefinite.

\((K_3)\) The only solution \((X_1, \ldots, X_7) = (X_1(t), \ldots, X_7(t))\) of (6) which satisfies \((d/dt)V(X_1, \ldots, X_7) = 0, (t \geq 0),\) is the trivial solution \((0, 0, 0, 0, 0, 0, 0)\).

**Proof.** Consider the Lyapunov-Krasovskii functional \(V(\cdot) = V(X_1, \ldots, X_7)\) defined by
\[
V(\cdot) = \langle X_7, X_2 \rangle + \langle A_1X_2, X_6 \rangle
\]
\[
+ \langle A_2X_2, X_5 \rangle + \langle A_3X_2, X_4 \rangle
\]
\[
- \langle X_3, X_6 \rangle - \langle A_1X_3, X_5 \rangle
\]
\[
- \langle X_2X_3, X_4 \rangle + \langle X_4, X_5 \rangle
\]
\[
- \frac{1}{2} \langle X_3X_3, X_5 \rangle + \frac{1}{2} \langle A_1X_4, X_4 \rangle
\]
\[
+ \int_0^1 \langle H_1(\sigma X_1_1), X_1 \rangle d\sigma
\]
\[
+ \cdots + \int_0^1 \langle H_n(\sigma X_1_1), X_1 \rangle d\sigma
\]
\[
+ \int_0^1 \langle \Psi_5(\sigma X_2), X_2 \rangle d\sigma
\]
\[
- \sum_{i=1}^n \lambda_i \int_{\tau_i}^{\tau_i+s} \|X_2(\theta)\|^2 d\theta d\sigma,
\]
where \(s\) is a real variable such that the integrals \(\int_{\tau_i}^{\tau_i+s} \|X_2(\theta)\|^2 d\theta d\sigma\) are nonnegative, and \(\lambda_i\) are certain positive constants to be determined later in the proof.

We see that
\[
V(0, 0, 0, 0, 0, 0, 0) = 0.
\]
Since \((\partial/\partial s)H_j(\sigma X_1) = J_{H_j}(\sigma X_1)X_1, H_j(0) = 0\), then

\[
\begin{align*}
\int_0^1 \langle H_1 (\sigma X_1), X_1 \rangle \, d\sigma &= \int_0^1 \langle \sigma_1 J_{H_1} (\sigma_1 \sigma_2 X_1), X_1 \rangle \, d\sigma_2 \, d\sigma_1 \\
&\geq \beta_1 \|X_1\|^2, \\
\int_0^1 \langle H_2 (\sigma X_1), X_1 \rangle \, d\sigma &= \int_0^1 \langle \sigma_1 J_{H_2} (\sigma_1 \sigma_2 X_1), X_1 \rangle \, d\sigma_2 \, d\sigma_1 \\
&\geq \beta_2 \|X_1\|^2, \\
\vdots \\
\int_0^1 \langle H_n (\sigma X_1), X_1 \rangle \, d\sigma &= \int_0^1 \langle \sigma_1 J_{H_n} (\sigma_1 \sigma_2 X_1), X_1 \rangle \, d\sigma_2 \, d\sigma_1 \\
&\geq \beta_n \|X_1\|^2.
\end{align*}
\]

Let

\[
\bar{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_n).
\]

Then, we get

\[
\dot{V}(\bar{\varepsilon}, 0, 0, 0, 0, 0, 0) = \left( \sum_{i=1}^{\infty} \beta_i \right) \|\bar{\varepsilon}\|^2 > 0
\]

for all \(\bar{\varepsilon}, \varepsilon \in \mathbb{R}\), so that every neighborhood of the origin in the \((X_1, \ldots, X_7)\)-space contains points \((\xi_1, \ldots, \xi_7)\) such that \(V(\xi_1, \ldots, \xi_7) > 0\).

Let

\[
(X_1, \ldots, X_7) = (X_1(t), \ldots, X_7(t))
\]

be an arbitrary solution of (6).

Calculating the time derivative of the Lyapunov-Krasovskii functional \(V\) along this solution, we obtain

\[
\dot{V}(\cdot) = \langle X_5, X_5 \rangle - \langle A_2 X_4, X_4 \rangle - \langle \Psi_4(\cdot) X_2, X_4 \rangle \\
- \langle \Psi_6(\cdot) X_2, X_2 \rangle \\
+ \left\langle X_2, \int_{t-\tau_1}^{t} J_{H_1}(X_1(s)) X_2(s) \, ds \right\rangle \\
+ \cdots + \left\langle X_2, \int_{t-\tau_n}^{t} J_{H_n}(X_1(s)) X_2(s) \, ds \right\rangle \\
- \langle \lambda_1 \tau_1 X_2, X_2 \rangle - \cdots - \langle \lambda_n \tau_n X_2, X_2 \rangle \\
+ \lambda_1 \int_{t-\tau_1}^{t} \|X_2(\theta)\|^2 \, d\theta \\
+ \cdots + \lambda_n \int_{t-\tau_n}^{t} \|X_2(\theta)\|^2 \, d\theta.
\]

Using the assumption \(0 < \lambda_i (J_{H_i}(X_1)) \leq \alpha_i\) and the Schwarz inequality, we have

\[
\begin{align*}
\left\langle X_2, \int_{t-\tau_1}^{t} J_{H_i}(X_1(s)) X_2(s) \, ds \right\rangle &\geq -\|X_2\| \left\| \int_{t-\tau_1}^{t} J_{H_i}(X_1(s)) X_2(s) \, ds \right\| \\
&\geq -\sqrt{n}\alpha_1 \|X_2\| \left\| \int_{t-\tau_1}^{t} \|X_2(s)\|^2 \, ds \right\| \\
&\geq -\frac{1}{2}\sqrt{n}\alpha_1 \tau_1 \|X_2\|^2 - \frac{1}{2}\sqrt{n}\alpha_1 \int_{t-\tau_1}^{t} \|X_2(s)\|^2 \, ds, \\
\left\langle X_2, \int_{t-\tau_2}^{t} J_{H_2}(X_1(s)) X_2(s) \, ds \right\rangle &\geq -\|X_2\| \left\| \int_{t-\tau_2}^{t} J_{H_2}(X_1(s)) X_2(s) \, ds \right\| \\
&\geq -\sqrt{n}\alpha_2 \|X_2\| \left\| \int_{t-\tau_2}^{t} \|X_2(s)\|^2 \, ds \right\| \\
&\geq -\frac{1}{2}\sqrt{n}\alpha_2 \tau_2 \|X_2\|^2 - \frac{1}{2}\sqrt{n}\alpha_2 \int_{t-\tau_2}^{t} \|X_2(s)\|^2 \, ds, \\
\vdots \\
\left\langle X_2, \int_{t-\tau_n}^{t} J_{H_n}(X_1(s)) X_2(s) \, ds \right\rangle &\geq -\|X_2\| \left\| \int_{t-\tau_n}^{t} J_{H_n}(X_1(s)) X_2(s) \, ds \right\| \\
&\geq -\sqrt{n}\alpha_n \|X_2\| \left\| \int_{t-\tau_n}^{t} \|X_2(s)\|^2 \, ds \right\| \\
&\geq -\frac{1}{2}\sqrt{n}\alpha_n \tau_n \|X_2\|^2 - \frac{1}{2}\sqrt{n}\alpha_n \int_{t-\tau_n}^{t} \|X_2(s)\|^2 \, ds.
\end{align*}
\]

Hence

\[
\dot{V}(\cdot) \geq \|X_5\|^2 - \alpha_2 \int_{t-\tau_1}^{t} \|X_2\|^2 \, ds \\
+ \frac{1}{4\alpha_2} \left\langle \Psi_4(\cdot) X_2, \Psi_4(\cdot) X_2 \right\rangle \\
- \left( \lambda_1 + \frac{1}{2}\sqrt{n}\alpha_1 \right) \int_{t-\tau_1}^{t} \|X_2\|^2 \\
+ \cdots + \left( \lambda_n + \frac{1}{2}\sqrt{n}\alpha_n \right) \int_{t-\tau_n}^{t} \|X_2\|^2.
\]
\begin{equation}
- \cdots - \left( \lambda_n + \frac{1}{2} \sqrt{n} \alpha_n \right) r_n \|X_2\|^2 \\
+ \left( \lambda_1 - \frac{1}{2} \sqrt{n} \alpha_1 \right) \int_{t-\tau_1}^{t} \|X_2(s)\|^2 ds \\
+ \cdots + \left( \lambda_n - \frac{1}{2} \sqrt{n} \alpha_n \right) \int_{t-\tau_n}^{t} \|X_2(s)\|^2 ds.
\end{equation}

(32)

Let \( \lambda_i = (1/2) \sqrt{n} \alpha_i \). Then, using the assumptions of the theorem and the estimate \( \tau = \tau_i \), we have

\begin{equation}
\dot{V}(\cdot) \geq \|X_2\|^2 - a_2 \|X_4 + 2^{-1} a_1^{-1} \Psi_i(\cdot) X_2 \|^2 \\
+ \frac{1}{4a_2} \left( \left\langle \Psi_i(\cdot) X_2, \Psi_i(\cdot) X_2 \right\rangle - \left\langle \Psi_6(\cdot) X_2, X_2 \right\rangle \right) \\
- \sqrt{n} \alpha_i \tau_i \|X_2\|^2 - \cdots - \sqrt{n} \alpha_n \tau_n \|X_2\|^2 \\
\geq \{ \delta - \sqrt{n}(\alpha_1 + \cdots + \alpha_n) \} \|X_2\|^2.
\end{equation}

(33)

If \( r < \delta / \sqrt{n}(\alpha_1 + \cdots + \alpha_n) \), then, for some positive constant \( k \), we have

\begin{equation}
\dot{V}(\cdot) \geq k \|X_2\|^2 > 0.
\end{equation}

(34)

Finally, \( V(\cdot) = 0 \) for all \( t \geq 0 \) necessarily implies that \( X_2 = 0 \). Hence, it follows that

\begin{align*}
X_1 &= \xi_1 \text{ (constant vector),} \\
X_2 &= X' = 0, \\
X_3 &= X'' = 0, \\
X_4 &= X''' = 0, \\
X_5 &= X^{(4)} = 0, \\
X_6 &= X^{(5)} = 0, \\
X_7 &= X^{(6)} = 0,
\end{align*}

(35)

for all \( t \geq 0 \) so that

\begin{equation}
X_1 = \xi_1 \text{ (constant vector),} \\
X_2 = X_3 = X_4 = X_5 = X_6 = X_7 = 0, \quad (t \geq 0).
\end{equation}

(36)

Therefore, the estimates \( (d/dt)V(\cdot) = 0 \) and (6) imply \( X_1 = X_2 = X_3 = X_4 = X_5 = X_6 = X_7 = 0 \) since \( H_1(\xi_1) = 0 \) if and only if \( \xi_1 = 0 \). It now follows that functional \( V \) thus has all the requisite Krasovskii [9] properties subject to the conditions of the theorem. Hence, we can conclude that the zero solution of (5) is unstable.

The proof of the theorem is completed. \( \square \)

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