Aharonov–Bohm effect in the presence of evanescent modes

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Abstract

It is known that differential magnetoconductance of a normal metal loop connected to reservoirs by ideal wires is always negative when an electron travels as an evanescent modes in the loop. This is in contrast to the fact that the magnetoconductance for propagating modes is very sensitive to small changes in geometric details and the Fermi energy and moreover it can be positive as well as negative. Here we explore the role of impurities in the leads in determining the magnetoconductance of the loop. We find that the change in magnetoconductance is negative and can be made large provided the impurities do not create resonant states in the systems. This theoretical finding may play an useful role in quantum switch operations.
Introduction

Recent advances in microfabrication technology [1–4] have made it possible to fabricate artificial structures with very fine control over design parameters and one has approached a technological level of solid state structures in which the energy and length scales are such that macroscopic quantum effects can be observed. Typical sizes of these systems vary from nanometer to micrometer. At very low temperatures (typically mK), inelastic scattering is significantly suppressed and the electron phase coherence length can become large compared to the system size. In this regime the electron maintains the single particle phase coherence across the entire sample and the system is called a mesoscopic system [1–4]. The idealized sample becomes an electron wave guide and transport properties are solely determined by the impurity configuration and geometry of the conductor and by the principles of quantum mechanics. Transport in these systems cannot be described by standard classical Boltzmannian theory, where self averaging over many macroscopic configurations is assumed. Instead, a mesoscopic system as a whole should be treated as a phase coherent elastic scatterer.

It has now become clear that transport behavior in these nanostructures (in the regime of quantum ballistic transport), resemble in many ways, properties of wave guides in electromagnetic wave propagation. This similarity with guided electromagnetic wave propagation has opened up the possibility of new quantum semiconductor devices [5]. These quantum devices rely on quantum effects for their operation, and are based on interferometric principles. The mechanism of switch operation by quantum interference is a new idea in electronic applications. Several switching devices have been proposed wherein one can control the relative phase difference between two interfering paths by applying electrostatic potential or magnetic fields [6–8]. The possibility of transistor action (quantum modulated transistor) in T shaped structures by varying effective length of an open ended lead have been explored [6–8]. Electron transmission in these devices is controlled by a remote gate voltage in a region where no classical current flows. The transmission across these devices can be varied between 0 and 1(100% modulation), for propagation in the fundamental transverse mode (single channel regime). This requires that the Fermi energy should lie between ground and
the first excited transverse mode. Only in this regime can one observe sharp and controllable variations in transmission. However, if more than single mode propagation is allowed, different modes with different wavelengths due to mode mixing will produce a more complex transmission pattern and oscillations in the total transmission are averaged out. In fact, quantum wires with a few modes have become a reality in the past few years. Devices operating in the fundamental mode promise to be much faster and will consume less power than the conventional devices. They can also drastically reduce the size of the electronic devices.

The conventional transistors operate in a classical diffusive regime, and are not very sensitive to variations in material parameters such as the dimensions or the presence of small impurity. These devices operate by controlling the carrier density of quasiparticles. Whereas the proposed quantum devices are not very robust in the sense that the operational characteristics depend very sensitively on material parameters \[9,10\]. Incorporation of a single impurity in the mesoscopic device may change nontrivially the interference of partial electron waves propagating through the sample and hence the electron transmission (operational characteristics) across the sample \[10\]. In such a device the actual problem of control and reproducibility of operating threshold becomes highly nontrivial. These devices can be exploited if we achieve the technology that can reduce or control the phase fluctuation to a small fraction of \(2\pi\) \[9\]. On the positive side, it should be noted that quantum devices can exhibit multifunctional property (e.g., single stage frequency multiplier) wherein the functions of an entire circuit can be performed within a single element \[11\].

Transport properties arising from a fundamental mode propagation can be easily understood by a one dimensional (single channel) modeling of the system. In such a case potential felt by an electron can be related to the transverse width of the channel. Naturally, modulation of the width of the conducting channel gets related to spatial variation in the potential. In our earlier work \[12\] we have studied transport across a metallic loop in the presence of magnetic flux. In that case potential felt by an electron in the loop is \(V \neq 0\) and in the ideal connecting wires \(V = 0\). If the Fermi energy of the electron is less than \(V\), the electron on entering the loop propagates as evanescent modes. In such a situation, the contribution to
the conductance arises from two non-classical effects, namely, Aharonov–Bohm effect and quantum tunneling. For this case we have shown that, on application of a small magnetic field, the conductance always decreases, or small field differential magnetoconductance is always negative. This is in contrast to the behavior in the absence of barrier, where the small field differential magnetoconductance is negative or positive depending on the Fermi energy and other geometric details. The negative differential magnetoconductance for the case of evanescent modes can play an useful role in device operation. In our present treatment we have studied transport across a modified metallic loop in the presence of a magnetic flux. The propagation of electrons in the loop is via evanescent modes. We have obtained an analytical expression for the transmission coefficient and studied the effect of impurity on the magnetoconductance of the system. We show that in the presence of impurity, the conductance can still exhibit negative differential magnetoconductance on the application of magnetic field, provided the defects do not create resonant states in the system. The change in the magnetoconductance can be made large by intentionally incorporating impurities. This fact may play an useful role in the device operation.

**Theoretical treatment**

In our present work we have considered a system of metallic loop connected to ideal leads as shown in Fig.1. The geometry considered here is a modified form of geometry considered in an earlier work [12]. The upper arm of the ring is of length \( l_2 \) and the lower arm of length \( l_3 \) such that \( L = l_2 + l_3 \) is the circumference of the ring. The quantum mechanical potential \( V \) is positive and nonzero (barrier) in the regions drawn as thick lines whereas zero in the regions drawn as thin lines. The thick lines protrude into the leads for a length \( l_1 \) to the left and \( l_4 \) to the right. In our present problem we always take \( l_1 = l_4 \). The wave vector for the electron in the thin line region is \( k = \sqrt{E} \) where \( E \) is incident Fermi energy (we have set \( \hbar = 2m = 1 \)). Wave vector in the thick line region is \( q = \sqrt{E - V} \). The thick line region is considered as a system and the thin line regions are ideal leads, connecting the system to two reservoirs on two sides. We would like to emphasize that our one dimensional modeling of the system along with ideal wires corresponds to a situation where electrons propagate
only in the fundamental transverse mode in a quasi–2D system. If we have a situation in which the transverse width of the system is much less than the ideal wires, then due to the higher zero point energy arising from transverse confinement, fundamental subband minima in the system will be at higher energy than the value of a few subband minima in the ideal connecting wires. Then a situation can arise, where several propagating modes in the wire will have energy less than the minimum propagating subband energy in the system. Thus the electron propagating in a fundamental subband of the ideal wire feels a barrier to its motion (arising solely from the mismatch in the zero point energies) and electron tunnels across the system (due to evanescent mode propagation) experiencing an effective potential $V$. In our 1–D modeling in the presence of Aharonov–Bohm magnetic flux ($\phi$), if the incident energy is less than $V$, contribution to the transmission coefficient comes from two non-classical effects, namely, Aharonov–Bohm effect and quantum tunneling. We have also incorporated a delta function impurity of strength $V_0$ at a distance $l_5$ from the sample, marked as X in Fig.1. This will help us in understanding the role of impurities on the magnetoconductance behavior. Transport properties across a metallic loop for the case $V = 0$, and $V_0 = 0$ have been studied earlier by several workers [13–19]. Following the quantum wave guide theory on the network [20–22] one can readily calculate the transmission coefficient across the system and is given by

$$T = \frac{8k^4q^2[2 - \cos(2l_2q) - \cos(2l_3q) + 4\cos(\alpha)\sin(l_2q)\sin(l_3q)]}{\Omega}$$  \hspace{1cm} (1)$$

Where $\alpha = 2\pi\phi/\phi_0$, $\phi_0 = hc/e$, and $\phi$ is total flux piercing the loop. Expression for $\Omega$ is too long to reproduce here, we have given the expression for $\Omega$ separately in the appendix A. As expected the transmission coefficient is periodic in flux $\phi$ with period $\phi_0$ (Aharonov–Bohm oscillation) and, moreover, $T$ is symmetric in $\phi$. The transmission coefficient $T$ is related to two probe conductance ($G$) by the Landauer formula $G = (2e^2/h)T$ and to the dimensionless conductance $g$ by $g = G/2e^2/h = T$. Our expression (1) in the limit $V = 0$ and $V_0 = 0$ gives the expression obtained earlier in [18].

Results and discussions
We first show that design imperfections and Fermi energy can alter the nature of output characteristics or the conductance of the system in the presence of the propagating modes. In order to have propagating modes we set $V$ equal to 0. Also we set $V_0$ to 0 so that our system becomes the same as the earlier studied system [13–19] (this ensures that $q \to k$ in equation (1)). Design imperfections may lead to variations in the arm lengths of the loop. In fig.2 we have plotted the dimensionless conductance $g$ ($\equiv |T|^2$) versus dimensionless magnetic flux $\alpha$ for $\frac{l_2}{L} = 0.2$, $\frac{l_3}{L} = 0.8$, for different values of $kL$ (dimensionless Fermi wave vector). Although all the curves oscillate with a period of $2\pi$, as expected [13–19], these curves are, however, completely diverse in nature. The solid curve is plotted for $kL = 0.2$. It shows that the small field differential magnetoconductance is negative. The dashed curve and the dotted curve are plotted for $kL = 2.0$ and $kL = 3.5$, respectively. Both of them show more oscillations than the solid curve. The small field differential magnetoconductance is positive for both. However, for the dashed curve the absolute minima in the conductance occurs at $\alpha = \pi$ whereas for the dotted curve the absolute minima in the conductance occur at $\alpha = 0$. Similar diversities can be seen if we fix the Fermi energy and vary other parameters such as $\frac{l_2}{L}$ and $\frac{l_3}{L}$. One can readily check this from the fact that $kl_2$, $kl_3$, etc., always occur as a single variable in the expression for the dimensionless conductance $g$. A single impurity inside the ring or in the lead can also drastically alter the output characteristics [10]. In fig.3 we have shown the sensitivity in output characteristics in presence of propagating modes (i.e., $V = 0$ and $q \to k$ in equation (1)), due to a single defect in the lead and design imperfections. In this figure we have plotted the conductance versus $kL$ for different cases in the absence of magnetic field. In the case of the solid curve $\frac{l_2}{L} = 0.5$, $\frac{l_3}{L} = 0.5$ and $V_0L = 0$. In the case of the dashed curve $\frac{l_2}{L} = 0.75$, $\frac{l_3}{L} = 0.25$ and $V_0L = 0$ whereas the dotted curve is for $\frac{l_2}{L} = 0.5$, $\frac{l_3}{L} = 0.5$, $V_0L = 1.0$ and $\frac{l_5}{L} = 1.0$. In all these cases we have set $\alpha = 0$. From this figure one can readily notice the sensitive dependence of the output characteristic on material parameters. With such sensitivity of output characteristics with respect to the variation in the Fermi energy, the length parameters and the magnetic flux (fig.2 and fig.3) it is difficult to ensure stability in the device performance. This problem
does not arise in the case of evanescent modes in the system. In fig.4, we have plotted the conductance $g$ due to evanescent modes versus magnetic field $\alpha$ for three different values of Fermi energy. We have fixed other parameter values as $\frac{l_1}{L} = 0.1$, $\frac{l_2}{L} = 0.2$, $\frac{l_3}{L} = 0.8$, $VL = 16$ and $V_0 = 0$. The dotted curve, the dashed curve and the solid curve correspond to the $kL$ values 3.5, 3.0 and 2.0, respectively. To have evanescent modes in the ring it is necessary to take a sufficiently large $V$ such that the incident Fermi energy $E < V$. $V_0$ is still set to zero. Transmission occurs due to quantum mechanical tunneling and it is to be calculated by analytic continuation which means in the equation (1) we have to put $q \rightarrow iQ$, where $Q=\sqrt{V-k^2}$. We find that for all the three Fermi energies considered in fig.4 the conductance versus $\alpha$ plots are similar. All of them initially show negative differential magnetoconductance with the absolute minima occurring at $\alpha = \pi$. The physical reason for this is that for evanescent modes it is always the first harmonic in the Fourier expansion (in $\alpha$) of transmission that dominates over all the others in determining the conductance [12], which is not the case for the propagating modes. In the case of propagating modes for a particular configuration or for a particular Fermi energy, on the other hand, any one of the infinite number of harmonics in the Fourier expansion of transmission coefficient can give large contributions and hence the output characteristics changes drastically with change in the configuration or the Fermi energy. Such a systematic nature of output characteristics for evanescent modes will help devising robust switches. However the price we pay is loss in the sensitivity of conductance to change of $\alpha$. This is because evanescent modes are not so sensitive to changes in the boundary conditions induced by the change in the flux $\alpha$.

In this work we further investigate the role of impurities in the connecting leads in modifying the above mentioned features. By definition electrons will always be in propagating modes in the connecting leads. Impurities inside the ring are not expected to produce any drastic changes if we have evanescent modes in the ring. This is due to the fact that, having evanescent modes in the ring the impurities cannot cause any additional resonance. Any additional phase change due to impurity scattering is almost equivalent to changing the effective arm lengths of the two loops in the ring. Thus to expect nontrivial effects due to
impurities in the presence of evanescent modes in the system, impurities must be located in
the connecting leads where electrons travel as propagating modes. To study this we have
incorporated a single delta function potential of strength $V_0$ at a distance $l_5$ from the loop at
the point $X$ shown in fig.1. We choose the typical values of parameters such that
$l_2 = 0.1, l_2 = 0.5, l_2 = 1.0, VL = 5.0$. Note that $l_2 = l_3$, i.e. we are considering a symmetric
loop. For such a system we have plotted in fig.5 the conductance $g$ versus incident Fermi
wave vector $kL$ in the energy interval where we have only evanescent modes in the ring.
Choosing $V_0L = 0$ we have plotted the conductance $g$ versus $kL$ for two values of $\alpha$ i.e.
$\alpha = 0$ (dotted curve) and $\alpha = 2$ (dot–dashed curve). Also for $V_0L = 1$ we have plotted the
conductance $g$ versus $kL$ for the same two values of $\alpha$ i.e. $\alpha = 0$ (solid curve) and $\alpha = 2$
(dashed curve). We find by comparing the dashed and solid curves that the magnetic field
decreases the conductance at all values of $kL$ signifying that the magnetoconductance of
the system is still negative inspite of the impurity in one of the leads. Qualitatively there
is no change compared to the situation when $V_0L = 0$. This feature remains unchanged
as one increases the impurity strength $V_0$. However, for large $V_0$ one can have a resonance
in the energy range 0 to $\sqrt{V}$ due to multiple scattering in the region of length $l_5$. This is
shown in fig.6 where we have plotted conductance versus $kL$ for the same two values of $\alpha$
and the same set of parameters as in fig.5 i.e. $\alpha = 0$ (solid curve) and $\alpha = 2$ (dashed curve)
for $l_2 = 0.1, l_2 = 0.5, l_2 = 0.5, l_2 = 1.0$ and $VL = 5.0$. Only the strength of the delta
potential has been increased to $V_0L = 3$ as compared to the situation in fig.5. One can
clearly see a well defined conductance peak or a resonance in the absence of magnetic flux
($\alpha = 0$). For nonzero $\alpha$ the scattering strength of the ring is much higher than that of the
impurity (differential magnetoconductance of isolated ring being negative definite, magnetic
field increases the scattering strength or decreases the transmission coefficient of the loop)
and this rules out the possibility of a sharp resonance. Superposed in this graph we have
also shown the $V_0L = 0$ situation for the same two values of $\alpha$ i.e. $\alpha = 0$ (dotted curve)
and $\alpha = 2$ (dot–dashed curve) for the same values of the other parameters. A comparison
of the two sets of graphs (each for $V_0L \neq 0$ and $V_0L = 0$) shows that the resonance not only
increases the conductance but also increases the sensitivity of the conductance to change in magnetic field or $\alpha$. At a Fermi energy marked as $k_1L$ in Fig.6, by increasing the magnetic field or $\alpha$ we can decrease the conductance much more when $V_0L = 3$ than when $V_0L = 0$. This is true for any $kL$ in general. To show this, in fig.7. we have fixed the incident energy such that $kL = 1.4$ (solid line), $kL = 1.6$ (dashed line) and $kL = 1.8$ (dotted line) for $\frac{k_1}{L} = 0.1$, $\frac{k_2}{L} = 0.5$, $\frac{k_3}{L} = 0.5$, $\frac{k_4}{L} = 1.0$, VL=5.0 and plotted $(g(\alpha = 0) - g(\alpha = 2))$ (this quantity can be taken as a measure of the magnitude of differential magnetoconductance or a measure of the flux sensitivity of the conductance) versus $V_0L$ i.e. the strength of the delta function potential. We find that initially the flux sensitivity of the system increases with the impurity strength, passes through a peak, and asymptotically decreases. For $V_0L = 0$ there can be no resonant transmission. Initially as $V_0$ increases we approach the resonance between the loop and the delta potential. This resonance not only increases the conductance but also increases the sensitivity of conductance to twisting of boundary condition by the magnetic field and hence the differential magnetoconductance increases in magnitude while being negative all the time. However, for very large value of $V_0L$ we move far away from resonance condition and then due to enhanced scattering by the delta potential the flux sensitivity of the conductance or the differential magnetoconductance decreases. In fig.7 as $(g(\alpha = 0) - g(\alpha = 2))$ is positive over the whole range of $V_0L$ the magnetoconductance is negative at these Fermi energies for the whole range of $V_0L$ shown.

Notably, in fig.6, inspite of the resonance the magnetoconductance is negative. However if the impurity strength is slightly higher than the scattering strength of the isolated ring then there can be sharp resonances in the absence of magnetic field ($\alpha = 0$) as well as in the presence of magnetic field ($\alpha = 2$). The reason is that for both $\alpha = 0$ and $\alpha \neq 0$ the scattering strength of the loop is comparable (in one case it is slightly smaller and in the other case it is slightly higher) to that of the delta potential. Then the resonant conductance at $\alpha = 2$ can exceed the nonresonant conductance at $\alpha = 0$ implying positive differential magnetoconductance in a small range of Fermi energy. This is shown in fig.8, where we have plotted dimensionless conductance $g$ versus $kL$ for $\alpha = 0$ (solid line) and
\( \alpha = 2 \) (dashed line) for \( L_1 = 0.1, L_2 = 0.5, L_3 = 0.5, L_5 = 1.0, VL = 5 \) and \( V_0L = 5.5 \). In a small energy range \( i.e. kL = 2.1 \) to \( 2.25 \) the \( \alpha = 2 \) (dashed) curve goes above the \( \alpha = 0 \) (solid) curve signifying positive magnetoconductance in this energy window although we have only evanescent modes in the ring. Hence the scattering strength of the impurity should lie above a critical value which is higher than the scattering strength of the loop. Otherwise the magnetic field will increase the scattering strength of the loop (the magnetoconductance of the isolated loop being negative definite) thus detuning the strength of the two scatterers and making sharp resonance for nonzero \( \alpha \) impossible, within the relevant energy window.

When a sharp resonance appears the output characteristics become very sensitive to the material parameters. For small field, the change in the magnetoconductance can become either negative or positive with the absolute minima at \( \alpha = 0 \) as well as at \( \alpha = \pi \) depending on the exact choice of Fermi energy. This behavior is similar to that in the presence of propagating modes along the ring, \( e.g. \) see fig.2.

These features of the output characteristics in the presence of defect, remain unchanged even if there are design imperfections like unequal arm lengths. Again, positive differential magnetoconductance occurs only when the resonant conductance at \( \alpha \neq 0 \) exceeds the nonresonant conductance at \( \alpha = 0 \). In this case as one of the arms is much shorter than the other it shunts most of the current and reduces the scattering strength of the loop. Hence resonances can occur for smaller values of the delta potential strength than those in the case of fig.8. If \( L_2 = 0.2 \) and \( L_3 = 0.8 \) then \( V_0L = 5 \) can give rise to appreciable positive magnetoconductance in a small energy range for the same values of the other parameters as in Fig.8.

It is evident from fig.8 that even if the delta potential strength is strong enough to produce resonance, it reduces the energy window where we have negative differential magnetoconductance, by a small amount. The range of Fermi wave vector in which we have only evanescent modes in the sample is \( k = 0 \) to \( k = \sqrt{V} \). The resonance condition being \( kl_5 = 2\pi \), we cannot have a resonance in the relevant energy range unless \( l_5 \geq \frac{2\pi}{\sqrt{V}} \). So
for $l_5 \leq \frac{2\pi}{\sqrt{V}}$ we can never have positive magnetoconductance in the presence of evanescent modes.

However if $l_5 >> \frac{2\pi}{\sqrt{V}}$ then one can have many resonances in the energy interval where we can have only evanescent modes in the loop. And whenever there is a resonance, there is a possibility of obtaining positive differential magnetoconductance. This is shown in fig.9, where we have plotted conductance for two values of $\alpha$ i.e. $\alpha = 0$ (solid line) and $\alpha = 2$ (dotted line) versus incident energy $kL$ for $\frac{l_1}{L} = 0.1$, $\frac{l_2}{L} = 0.2$, $\frac{l_3}{L} = 0.8$, $\frac{l_5}{L} = 5.0$, $VL = 5.0$ and $V_0L = 5.0$. We find that in the energy window where we have only evanescent modes in the loop there can be three resonances. Near each of these resonances in a very narrow energy window the dotted curve exceeds the solid curve signifying that differential magnetoconductance is positive in these regions. At other energies change in magnetoconductance is always negative.

In conclusion, we have investigated transmission across a loop in the presence of magnetic field and an impurity. In our case electronic wave travels as evanescent waves throughout the circumference of the loop. In such a situation we have shown that initial change in the magnetoconductance is negative even in the presence of impurities, provided impurities do not create resonant states in the system. For small fields the change in magnetoconductance can be made large by intentionally incorporating impurity. This fact can be used for an operation of a quantum switch. Where the on and the off states can correspond to transmission in the absence or in the presence of magnetic field, respectively i.e., the on state has always a larger conductance than the off state. The magnitude of negative differential magnetoconductance may also be enhanced in a multichannel situation. In this case, one can populate electrons in connecting leads in many lower subband channels (multichannels) corresponding to different transverse quantum numbers, such that the Fermi energy lies below the lowest subband of the loop. This can be achieved by making the width of the loop much smaller than the width of the connecting leads, i.e. the quantum zero point energy in the loop will be much higher than several subband energies in the connecting leads. Then all these subbands in the leads will contribute to the conductance through evanescent modes.
Appendix A

An expression for $\Omega$

$$\Omega = (C^2 + D^2)$$

$$C = \cos(kl_5) \cos(\alpha) \left( 2k^2V_0 \cos(2l_1q) - 2k^2V_0 - 4k^2q \sin(2l_1q) \right) + \cos(\alpha) \sin(kl_5) \left( 2k^3 - 2kq^2 - 2k^3 \cos(2l_1q) - 2kq^2 \cos(2l_1q) - 2kqV_0 \sin(2l_1q) \right) + \cos(l_2q) \cos(l_3q) \left( 2k^2V_0 \cos(kl_5) - 2k^2V_0 \cos(kl_5) \cos(2l_1q) - 2k^3 \sin(kl_5) + 2kq^2 \sin(kl_5) + 2k^3 \cos(2l_1q) \sin(kl_5) + 2kq^2 \cos(2l_1q) \sin(kl_5) + 4k^2q \cos(kl_5) \sin(2l_1q) + 2kqV_0 \sin(kl_5) \sin(2l_1q) \right) + \cos(l_3q) \sin(l_2q) + \cos(l_2q) \sin(l_3q) - \sin(l_2q) \sin(l_3q) \left( k^2V_0 \cos(kl_5) + 3k^2V_0 \cos(kl_5) \cos(2l_1q) + k^3 \sin(kl_5) - kq^2 \sin(kl_5) - 3k^3 \cos(2l_1q) \sin(kl_5) - 3k^2q \cos(2l_1q) \sin(kl_5) - 6k^2q \cos(kl_5) \sin(2l_1q) - 3kqV_0 \sin(kl_5) \sin(2l_1q) \right)$$

$$D = \cos(kl_5) \cos(\alpha) \left( 2kq^2 - 2k^3 + 2k^3 \cos(2l_1q) + 2kq^2 \cos(2l_1q) + 2kqV_0 \sin(2l_1q) \right) + \cos(l_2q) \cos(l_3q) \left( 2k^3 \cos(kl_5) - 2kq^2 \cos(kl_5) - 2k^3 \cos(kl_5) \cos(2l_1q) - 2kq^2 \cos(kl_5) \cos(2l_1q) - 2q^2V_0 \sin(kl_5) - 2q^2V_0 \cos(2l_1q) \sin(kl_5) - 2kqV_0 \cos(kl_5) \sin(2l_1q) + 4k^2q \sin(kl_5) \sin(2l_1q) \right) + \cos(\alpha) \sin(kl_5) \left( 2q^2V_0 + 2q^2V_0 \cos(2l_1q) - 4k^2q \sin(2l_1q) \right) + \cos(l_3q) \sin(l_2q) + \cos(l_2q) \sin(l_3q) - \sin(l_2q) \sin(l_3q) \left( k^3 \cos(kl_5) + kq^2 \cos(kl_5) + 3k^3 \cos(kl_5) \cos(2l_1q) + 3k^2q \cos(kl_5) \cos(2l_1q) + q^2V_0 \sin(kl_5) + 3q^2V_0 \cos(2l_1q) \sin(kl_5) + 3kqV_0 \cos(kl_5) \sin(2l_1q) - 6k^2q \sin(kl_5) \sin(2l_1q) \right)$$
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Figure Captions

Fig. 1. A normal metal ring connected to two electron reservoirs by ideal leads. Quantum mechanical potential in the sample is $V$. At site X there is a delta function impurity potential of strength $V_0$. The magnetic flux $\phi$ pierces through the ring. The different lengths $l$ are marked in the figure.

Fig. 2. Plot of conductance $g$ versus $\alpha$ for $kL = 0.2$ (solid curve), $kL = 2.0$ (dashed curve) and $kL = 3.5$ (dotted curve). $\frac{l_1}{L} = \frac{l_4}{L} = 0$, $\frac{l_2}{L} = 0.2$, $\frac{l_3}{L} = 0.8$, $VL = 0$ and $V_0L = 0$ for all the cases.

Fig. 3. Plot of conductance $g$ versus $kL$ with and without impurity when electrons travel in propagating modes. $\frac{l_1}{L} = \frac{l_4}{L} = 0.5$, $\frac{l_2}{L} = 0$, $V_0L = 0$ (solid curve), $\frac{l_1}{L} = \frac{l_4}{L} = 0.5$, $\frac{l_2}{L} = 1.0$, $V_0L = 1.0$ (dotted curve), $\frac{l_1}{L} = 0.75$, $\frac{l_2}{L} = 0.25$, $\frac{l_3}{L} = 0$, $V_0 = 0$ (dashed curve) and $\frac{l_1}{L} = \frac{l_4}{L} = 0$, $VL = 0$ for all the cases.

Fig. 4. Plot of conductance versus $\alpha$ for different values of $kL$ when electron in evanescent modes. $kL = 2.0$ (solid curve), $kL = 3.0$ (dashed curve), $kL = 3.5$ (dotted curve). $\frac{l_1}{L} = \frac{l_4}{L} = 0.1$, $\frac{l_2}{L} = 0.2$, $\frac{l_3}{L} = 0.8$, $\frac{l_5}{L} = 1.0$, $VL = 16$, $V_0L = 0$ for all the curves.

Fig. 5. Plot of conductance $g$ versus $kL$ in presence and absence of both impurity ($V_0$) and magnetic flux ($\alpha$). $V_0L = 0$ and $\alpha = 0$ (dotted curve), $V_0L = 0$ and $\alpha = 2$ (dot-dashed curve), $V_0L = 1.0$ and $\alpha = 0$ (solid curve), $V_0L = 1.0$ and $\alpha = 2$ (dashed curve). $\frac{l_1}{L} = \frac{l_4}{L} = 0.1$, $\frac{l_2}{L} = \frac{l_5}{L} = 0.5$, $\frac{l_3}{L} = 1.0$, $VL = 5.0$ for all the curves.

Fig. 6. Plot of conductance $g$ versus $kL$ in presence and absence of both impurity and magnetic flux when electrons travel in evanescent modes. $V_0L = 0$ and $\alpha = 0$ (dotted curve), $V_0L = 0$ and $\alpha = 2$ (dot-dashed curve), $V_0L = 3.0$ and $\alpha = 0$ (solid curve), $V_0L = 3.0$ and $\alpha = 2$ (dashed curve). All other parameters are same as in fig.5.

Fig. 7. Plot of the difference between the conductances $(g(0)–g(2))$ in the absence ($\alpha = 0$) and in the presence ($\alpha = 2$) of magnetic flux versus impurity ($V_0L$) when electrons travel in evanescent modes. $kL = 1.4$ (solid curve), $kL = 1.6$ (dashed curve) and $kL = 1.8$ (dotted curve). All other parameters are same as in fig.5.
**Fig. 8.** Plot of conductance $g$ versus $kL$ in the presence and in the absence of magnetic flux with impurity. $V_0L = 5.5$ and $\alpha = 0$ (solid curve), $V_0L = 5.5$ and $\alpha = 2$ (dashed curve). All other parameters are same as in fig.5.

**Fig. 9.** Plot of conductance $g$ versus $kL$ in presence or absence of magnetic flux with impurity. $\frac{L_1}{L} = \frac{L_4}{L} = 0.1$, $\frac{L_2}{L} = 0.2$, $\frac{L_3}{L} = 0.8$, $\frac{L_5}{L} = 5.0$, $VL = 5.0$, $V_0L = 5.0$, $\alpha = 0$ (solid curve) and $\alpha = 2$ (dotted curve).