Robust quantum repeater with atomic ensembles against phase and polarization instability

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We propose an alternative scheme for quantum repeater without phase stabilization and polarization calibration of photons transmitted over long-distance channel. We introduce time-bin photonic states and use a new two-photon interference configuration to robustly generate entanglement between distant atomic-ensemble-based memory qubits. Our scheme can be performed with current experimental setups through making some simple adjustments.

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I. INTRODUCTION

The concept of quantum repeater\cite{1} was introduced for long-distance quantum communication in order to overcome the problems caused by inevitable photon loss in the transmission channel. Generating distant entanglement is a crucial ingredient of a quantum repeater protocol. In 2001, Duan, Lukin, Cirac and Zoller (DLCZ) proposed a original scheme\cite{2} to use atomic-ensembles and linear optics in which robustly generating entanglement over long distances can be achieved. Motivated by the DLCZ protocol, much experimental effort\cite{3,4,5} has been made in the last few years.

However, it has been shown that DLCZ protocol requires severe phase stability since entanglement generation and entanglement swapping in the protocol depend on single-photon Mach-Zehnder type interferences\cite{6}, and this problem make its experimental realization extremely difficult by the current technology. Hence a novel scheme\cite{6} based on DLCZ scheme was proposed to use phase-insensitive two-photon quantum interference which dramatically relax this phase stability requirements. A latest experiment\cite{7} has primarily demonstrated the scheme over a distance of 300 meters. Meanwhile, a fast and robust approach\cite{8} was proposed and also can solve the phase stability problem as long as the entanglement generation is performed locally, and they gave a detailed comparisons between the schemes. In addition, there are another novel schemes\cite{9,10} proposed to improve the efficiency of quantum repeaters.

These protocols greatly stimulate experimental implementations of quantum repeater, but there are still some problems need to be solved, such as reliable transmission of photon’s polarization states over noisy channel. Since the photon interferences rely on the polarization states, the ability to maintain photonic polarizations is indispensable in the process of distant entanglement generation or swapping. Most of the time, optical fibers are used as photon transmission channel. Due to the fiber birefringence, the photonic polarizations will be changed randomly\cite{11}. Experimentally, active feedback compensation could be applied to solve this problem\cite{12}, but it is efficient only when the thermal and mechanical fluctuations are rather slow. Furthermore, even though polarization compensations can be used efficiently, imperfect shared reference frame (SRF) for polarization orientation may cause some errors. It is difficult to correct this kind of errors since establishing a perfect SRF requires infinite communication\cite{13}. Due to these reasons, it would be better to have a quantum repeater scheme with inherent polarization insensitivity.

In this paper, we propose an alternative approach to create distant entanglement between atomic-ensembles. In our scheme neither the phase stabilizing nor the polarization calibrating is needed for photons transmitted over long distance. Through introducing time-bin photonic states and using a new two-photon interference configuration, we make only the unchanged part of initial polarization states contribute to the desired results. Combined with local entanglement swapping for entanglement connection, our scheme can be used to implement a robust quantum repeater.

II. ENTANGLEMENT GENERATION

Here optical thick atomic ensemble, which includes $N$ atoms with $\Lambda$ level structure (see inset of Fig.\ref{fig:1}), is used as quantum memory. Each atomic ensemble is illuminated by a short, off-resonant write pulse that induces a spontaneous Raman process. This process will produce a forward-scattered Stokes light and a collective atomic excitation state\cite{2}. The photon-atom system can be described as (neglecting the higher-order terms)

$$[1 + \sqrt{\chi^{\dagger} S^{\dagger} a^{\dagger} + \frac{1}{2}(S^{\dagger} a^{\dagger})^2}]|\text{vac}\rangle,$$

where $|\text{vac}\rangle$ denotes that all the ensemble atoms are in the ground state $|g\rangle$ and the Stokes light in the vacuum state, $a^{\dagger}$ is the creation operator of the Stokes light, and the collective atomic excitation is defined by
There are two detection zones: one with the single photon pulses, and the other with PSBs. Due to channel noise or imperfect SRF, the polarization states of photons arrived at the middle point have been changed, and as long as the time intervals between the subsequent transmitted photon wave packets are small, it is reasonable to assume that the polarization changes are the same to them. After the PBS, there are also a long path and a short one whose difference is the same as the one at each node. The polarization transformation during this process are usually considered as a unitary transformation, so the evolution of the photonic polarization components can be described as:

$$A_{H,S}^{\dagger}\uparrow \downarrow \uparrow \downarrow H_{SS,LL}^{\dagger}(\phi_{SS,LL}^{\dagger})^{\dagger}$$

$$+ 2S_{a}^{\dagger}(S_{d}^{\dagger}a_{V,L}^{\dagger}a_{H,S}^{\dagger})^{2}\uparrow \downarrow\uparrow \downarrow H_{SS,LL}^{\dagger}$$

where the subscripts $u$ and $d$ are used to distinguish the two ensembles, and $a_{V,L}^{\dagger}(a_{H,S}^{\dagger})$ denotes the creation operator of the Stokes photon with vertical (horizontal) polarization passing through the long (short) path.

Assume two neighboring communication nodes, denoted by $A$ and $B$, are connected through certain transmission channel such as optical fiber with a distance of $L_0$. At the middle point between the two nodes, the Stokes photons are firstly directed to PBS. Due to channel noise or imperfect SRF, the polarization states of photons arrived at the middle point have been changed, and as long as the time intervals between the subsequent transmitted photon wave packets are small, it is reasonable to assume that the polarization changes are the same to them. After the PBS, there are also a long path and a short one whose difference is the same as the one at each node. The polarization transformation during this process are usually considered as a unitary transformation, so the evolution of the photonic polarization components can be described as:

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and states distinguished by the subscript $A$ and $B$. A coincidence count between the detectors at either detection zone, e.g., $D_1$ and $D_3$, will project the neighboring memory qubits into a complex state with contributions from second-order excitations. Note that in second order in $\chi$, the states $|\psi\rangle_{cross},_{A,B}$ can just trigger one detector at either detection zone at the time $SL(\text{LS})$ and consequently have no contribution to the coincidence. Thus a coincidence count between detectors, for instance $D_1$ and $D_3$, projects the two memory quits into

$$|\psi\rangle_{AB} = \frac{1}{2}e^{i(\phi+\phi')} \cos \theta \cos \theta' (S_{uA}^3S_{dA}^3 + S_{uA}^3S_{dA}^3) e^{2i\phi}\cos^2 \theta S_{uA}^S + e^{2i\phi'\cos^2 \theta} S_{uB}^S S_{dB}^S |\text{vac}\rangle$$

where $\theta(\theta')$ and $\phi(\phi')$ are the noise parameter and the polarization-independent phase that the photons acquires during the transmission from the node $A(B)$ to the middle point, respectively. The first part of the state is the desired maximally entangled state. The second part is unwanted two-excitation state and can be effectively eliminated by entanglement swapping. The success probability is on the order of $O(\chi^2R^2e^{-20L_{at}})$, where $\eta$ is the detection efficiency and $L_{at}$ is the channel attenuation length. Until now, we have generated the state similar to the one (Eq.(S3) in Supplementary information of Ref.[2]) which was created in the recent experimental demonstration of quantum repeater over a distance of 300 meters. It is obvious that the polarization noises only influence the success probability but have no effect on the fidelity of the desired state, and the phases $\phi$ and $\phi'$ only lead to a trivial global factor $e^{i(\phi+\phi')}$ on the desired state.

### III. LOCAL ENTANGLEMENT SWAPPING FOR ENTANGLEMENT CONNECTION

The entanglement swapping setup is depicted in Fig.2, which is the same as the one in the schemes[6]. Note that there is no path difference at this situation. Consider three communication nodes $A$, $B$, and $C$, and assume that we have created the complex entangled states (given by Eq.(3)) $|\psi\rangle_{ABL}$ and $|\psi\rangle_{BRC}$ both in $(A,B_L)$ and in $(B_R,C)$, respectively. The memory qubits $B_L$ and $B_R$ at node $B$ are illuminated simultaneously by retrieval laser pulses. The retrieved anti-Stokes photons are subject to Bell-state measurement (BSM) which is used to eliminate the two-excitation terms since the arrangement of the $\text{PBS}S$ is to identify $|\phi^\pm\rangle$ at $|\pm\rangle$ polarization basis, where $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|\pm\rangle \pm |\mp\rangle)$. Therefore the two-photon states generated from the unwanted two-excitation terms are directed into the same detection zone and will not induce any coincidences. In addition, the capability of distinguishing photon numbers is technically demanding, and the retrieve efficiency is determined by the optical depth of the atomic ensembles[15].

Taking into account of these imperfections, the coincidence counts in BSM actually prepare the memory qubits into a mixed entangled state of the form $\rho_{AC} = p_2p_2 + p_1p_1 + p_0p_0$, where the coefficients $p_2$, $p_1$, and $p_0$ are determined by the retrieval efficiency and detection efficiency[6]. Here $p_2 = |\langle \phi|AC\rangle|^2$ is the maximal entangled state, where $|\phi\rangle_{AC} = \frac{1}{\sqrt{2}}(S_{uA}^3S_{uC}^3 + S_{uA}^3S_{dC}^3)|\text{vac}\rangle$, $p_0$ is the vacuum state that all the atomic ensembles are in the ground states, and $p_1$ is a maximally mixed state in which only one of the four atomic ensembles has one excitation.

Although $\rho_{AC}$ still includes unwanted terms, it can be projected automatically to maximally entangled states in the entanglement-based quantum cryptography schemes. When implementing quantum cryptography via Ekert protocol[10], only the coincidence counts between the detectors at two remote nodes are registered and used for quantum cryptography. Therefore only the maximally entangled state in $\rho_{AC}$ will contribute to the experimental results. In this sense, $\rho_{AC}$ is equivalent to the Bell state $|\phi\rangle_{AC}$.

To implement a quantum repeater protocol, further entanglement swapping is required. Since the noise parameters only change the coefficients of the obtained states, the analysis of the entanglement connection in Ref[6] can be directly used here, that is, as long as the excitation probability $\chi$ is small enough, the contributions from the higher-order excitations can be safely neglected. The probability to find an desired entangled pair in the remaining memory qubits is almost a constant and will not decrease significantly during the entanglement connection process.

### IV. CONCLUSION

In the existing quantum repeater protocols, the ability to reliably transfer photon’s polarization is indispensable, but it is not easy to meet the prerequisite in practice. For this reason, we have proposed an alternative approach with inherent polarization insensitivity to
generate entanglement between distant communication nodes. In our scheme, neither the phase stabilizing nor the polarization calibrating is needed for photons transmitted over long distance. Through introducing path difference, we make photon wave packets with different polarizations correspond to different time-bins, and use the two-photon interference with different configuration to generate entanglement between remote communication nodes. Hence only the unchanged part of initial polarization states can induce the coincidences between detectors, so that the polarization noise only have influence on the success probability but the fidelity of the desired states. Consider the function of atom ensembles as quantum memory for entanglement of a storage time up to milliseconds[17], the capacity of creating high-fidelity entangled states may be preferred even if at the cost of some efficiencies. Combined with local entanglement swapping for entanglement connection, our scheme can be used to implement a robust quantum repeater. Comparing with the DLCZ scheme and the scheme of Ref.[6], the generated entangled states in our scheme are of higher fidelity.

Finally, it is pointed out that our scheme can be used to perform remote entanglement swapping after locally generating entanglement, which would make the quantum repeater scheme more efficient[10]. For instance, the local entangled state (eq.(10) in Ref.[6]) generated via the setup in Fig[2] may be robustly connected to longer communication distance by use of our scheme.

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