Gluon Sivers Function and Transverse Single Spin Asymmetries in $e + p^\uparrow \rightarrow \gamma + X$

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Abstract

We present estimates of transverse single-spin asymmetry (TSSA) in prompt photon production in the scattering of low virtuality photons off a polarized proton target and discuss the possibility of using this as a probe to get information about the gluon Sivers function (GSF). Using a generalized parton model (GPM) framework, we estimate the asymmetries at electron-ion collider (EIC) energy ($\sqrt{s} = 140$ GeV) taking into account both direct and resolved photon processes and find that the dominant contribution, upto 10%, comes from quark Sivers function (QSF) while the contribution from GSF is found to be upto 2%. However, upon taking account the effects of the process-dependent initial and final state interactions through the color gauge invariant generalized parton model (CGI-GPM) approach we find that the situation is significantly changed, with near zero contributions from the QSFs and upto a 1% level contribution from the f-type GSF. Our results indicate that this process may be useful for distinguishing between GPM and CGI-GPM models and can be used as a good probe of f-type GSF.

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I. INTRODUCTION

Transverse single-spin asymmetries (TSSAs) and the underlying physics is an area of hadron physics that has been explored with keen interest in the past years as these asymmetries provide useful tools for probing the three dimensional and spin structure of the nucleon. Ever since the first observation of large TSSAs in pion production in hadronic collisions involving protons and transversely polarized protons at Fermilab [1], a number of experimental and theoretical studies have been performed to measure and understand the TSSAs in various processes [2, 3]. At present, there are two approaches which are being used to explain these effects, among which one is based on collinear factorization at next-to-leading twist (twist-three) where SSAs are given by convolutions of hard scattering amplitudes with universal quark-gluon-quark and three-gluon correlation functions [4–8]. The second approach is based on a generalization of collinear factorization of QCD wherein the collinear parton distribution functions (PDFs) and fragmentation functions (FFs) are replaced by corresponding transverse momentum dependent (TMD) distribution functions which, on being integrated upon the transverse momentum variables, yield the collinear PDFs and FFs [9]. However, such a factorization (TMD factorization) has been established only for two scale processes like Semi-Inclusive Deep Inelastic Scattering (SIDIS) and Drell-Yan (DY) process and no such proof exists for single scale processes. A phenomenological approach which uses this for processes even where the TMD factorization has not been established is called the Generalised Parton Model (GPM). In GPM, TMDs such as Sivers function are assumed to be process independent. The GPM approach has been used by several groups for estimating asymmetries in processes like $p^+p \rightarrow \pi X$, $p^+p \rightarrow \gamma X$ and open and closed charm production under this assumption [10–16].

The transverse momentum dependent PDFs and FFs are collectively called TMDs and extensive experimental and theoretical work has been done and experiments proposed with the aim of determining these. One of the most important TMDs is Sivers function which quantifies the azimuthal asymmetry in the distribution of unpolarized quarks and gluons inside a proton which is transversely polarized with respect to the direction of motion [17, 18]. Sivers Effect has been used to explain the asymmetries in the process $p^+ + p \rightarrow \pi + X$ [19] and has since then been used in computation of asymmetries in processes involving a transversely polarized proton target. While the quark Sivers functions (QSFs) have been
studied extensively and a number of parametrizations have been proposed based on fits to data [20, 21], not much information is available on gluon Sivers function (GSF). The first indirect estimates of GSF were obtained based on GPM in Ref. [22] by fitting the gluon Sivers function to midrapidity data on SSA in $\pi^0$ production at RHIC and using the QSFs fitted earlier to SIDIS data. However, there is still need to have more direct and clean probes of GSF in which the contribution from QSFs and other TMDs is absent or negligible. Open and closed charm production in $p^+p$ and low virtuality $p^+e$ collisions as well as prompt photon production in $p^+p$ collisions have recently been proposed as probes of GSF within GPM framework [11, 15, 16, 23, 24]. As mentioned earlier, GPM assumes universality of Sivers function in addition to the assumption of TMD factorization. However, it is now well-established that universality of Sivers function does not hold in general and the Sivers function is process dependent. This process dependence of the Sivers function arises due to the process dependent initial state interactions (ISIs) and final state interactions (FSIs) between the active parton and the spectator partons of the polarized hadron. This observation led to a modification of GPM approach in which the process dependence of Sivers function is taken into account by a careful analysis of initial and final state interactions in one-gluon exchange approximation [23]. The modified GPM approach, now known as color-gauge invariant generalized parton model (CGI-GPM), amounts to absorbing the effects of ISIs and FSIs into the hard part of the process under consideration, thus restoring the universality of the Sivers function. The hadronic cross-sections are then obtained by convoluting the universal Sivers function with the modified hard part, which now contains the process dependence and which is obtained by combining the partonic cross section with the initial/final state interactions [26, 27].

CGI-GPM approach has been applied to $J/\psi$ and D-meson production at RHIC [24] and has recently also been applied to the study of TSSA in prompt photon production at RHIC by us and also by D’Alesio et al. independently [28, 29]. Both of these studies indicate that prompt photon production in $p^+p$ collisions can be used to discriminate between GPM and CGI-GPM approaches.

In the present work, we have considered TSSAs in the low-virtuality electroproduction of prompt photons at EIC energy using both GPM and CGI-GPM approaches. We have taken into account along with the direct photon contribution, also the resolved photon contribution to prompt photon production where the partonic scattering takes place between a
parton originating from the quasi-virtual photon emitted by the incoming electron and the parton from the polarized proton \[30-32\]. Further in CGI-GPM computation, the major contribution comes from f-type GSF while contribution of QSF and d-type GSF to asymmetry are nearly zero. Our results indicate that this process can be used to discriminate between GPM and CGI-GPM and can also be used to extract information about the f-type GSF.

The plan of the paper is as follows: In section II, we give expressions for the differential cross section and TSSAs. In section III, we discuss the CGI-GPM approach and give expressions for the modified hard parts for the relevant processes. In section IV, parametrizations used for the gluon Sivers function and the quark Sivers function are given. Finally, in section V, we present numerical estimates of TSSAs in both GPM and CGI-GPM frameworks.

II. PROMPT PHOTON PRODUCTION IN THE GPM FORMALISM

Prompt photon production in electron proton collisions can take place through direct electromagnetic process \(\gamma + p^\uparrow \rightarrow \gamma + X\) or through a resolved process. An illustration of both mechanisms of prompt photon production is given in Fig.1. Direct process \(q\gamma \rightarrow \gamma q\) contributes to the hadronic process at \(\mathcal{O}(\alpha^2)\) to the hadronic cross section whereas resolved processes, \(q + \bar{q} \rightarrow \gamma + g\) and \(q + g \rightarrow \gamma + q\), contribute at \(\mathcal{O}(\alpha \alpha_s)\). Although resolved subprocesses are naively higher order in \(\alpha_s\) as compared to direct process, the parton distribution in photon has a leading behavior proportional \(\alpha/\alpha_s\) and therefore, these subprocesses contribute to the hadronic cross section effectively also at \(\mathcal{O}(\alpha^2)\). In addition, there can be photons originating from fragmentation of a quark or gluon in the final state. We would like to mention here that the fragmentation contribution also contribute effectively to \(\mathcal{O}(\alpha^2)\). These contribution can be separated by putting isolation cuts in the experiments and therefore, in this work, we have not included their contribution. We have also not taken into account \(\gamma g \rightarrow \gamma g\) process as it contributes at \(\mathcal{O}(\alpha_s^2 \alpha^2)\) and is therefore not relevant for our discussion at LO.

In Generalized Parton Model (GPM) approach, the differential cross section for the direct photon production is,
FIG. 1. Representative diagrams for prompt photon production through direct subprocess (left) and through the resolved subprocess (right), in lepton-proton collisions. We consider a proton moving in the +Z direction, with a polarization along the +Y axis. The lepton is unpolarized moving along the Z direction.

\[
E_{\gamma} \frac{d\sigma^{ep\rightarrow\gamma X}}{d^3p_{\gamma}} = \int dx_{\gamma} dx_q d^2k_{\perp} f_{\gamma/e}(x_{\gamma}) \hat{f}_{q/p}(x_q, k_{\perp}, Q) \frac{\hat{s}}{x_{\gamma} x_q s} \frac{\hat{s}}{\pi} \frac{d\sigma^{q\gamma\rightarrow\gamma q}}{dt} \delta(\hat{s} + \hat{t} + \hat{u})
\]

\[
= \int dx_{\gamma} dx_q d^2k_{\perp} f_{\gamma/e}(x_{\gamma}) \hat{f}_{q/p}(x_q, k_{\perp}, Q) \alpha_{2} x_{\gamma} x_q s H^{U}_{q\gamma\rightarrow\gamma q} \delta(\hat{s} + \hat{t} + \hat{u}).
\]

\(x_{\gamma}\) and \(x_q\) are the light-cone momentum fractions of the incoming photon and incoming parton from the electron and proton respectively and \(p_{\gamma}\) is the 4-momentum of the outgoing photon. \(k_{\perp}\) is the intrinsic transverse momentum of the quark and \(\hat{s} = (p_q + p_{\gamma})^2\), \(\hat{t} = (p_q - p_{\gamma})^2\) and \(\hat{u} = (p_{\gamma} - p_{\gamma})^2\) are the Mandelstam variables for \(q\gamma\rightarrow\gamma q\) subprocess. \(Q\) is a factorization scale.

In above equation, \(f_{\gamma/e}(x_{\gamma})\) is the William-Weizacker distribution of quasi-real photons in electron given by,

\[
f_{\gamma/e}(x_{\gamma}) = \frac{\alpha}{2\pi} \left[ 2m_{e}^2 x_{\gamma} \left( \frac{1}{Q_{\min}^2} - \frac{1}{Q_{\max}^2} \right) + \frac{1 + (1 - x_{\gamma})^2}{x_{\gamma}} \ln \frac{Q_{\max}^2}{Q_{\min}^2} \right]
\]

where \(\alpha\) is the electromagnetic coupling and \(Q_{\min}^2 = m_{e}^2 \frac{x^2_{\gamma}}{1 - x_{\gamma}}\), \(m_{e}\) being the electron mass. We have used \(Q_{\max}^2 = 1\) in our analysis.

\(H^{U}_{q\gamma\rightarrow\gamma q}\) is the hard part for the direct process given by

\[
H^{U}_{q\gamma\rightarrow\gamma q} = -2e_{q}^4 \left( \frac{\hat{t}}{\hat{s}} + \frac{\hat{s}}{\hat{t}} \right)
\]
The differential cross section for the resolved process is

\[ E_\gamma \frac{d\sigma^{ep \rightarrow \gamma X}}{d^3p_\gamma} = \sum_{a,b=q,g,q'} \int dx_\gamma dx_a dx_b d^2k_{\perp a} f_{\gamma/e}(x_\gamma) \frac{\hat{f}_{b/\gamma}(x_b, Q) \hat{f}_{a/p}(x_a, k_{\perp a}, Q)}{x_a x_b x_\gamma s} \frac{\hat{s}}{\pi} \frac{d\sigma^{ab \rightarrow \gamma d}}{dt} \delta(\hat{s} + \hat{t} + \hat{u}) \]

(3)

\[ = \sum_{a,b=q,g,q'} \int dx_\gamma dx_a dx_b d^2k_{\perp a} f_{\gamma/e}(x_\gamma) \frac{\hat{f}_{b/\gamma}(x_b, Q) \hat{f}_{a/p}(x_a, k_{\perp a}, Q)}{x_a x_b x_\gamma s} \alpha \alpha_s \frac{\alpha^2}{N_c} \frac{\hat{s}}{\pi} \delta(\hat{s} + \hat{t} + \hat{u}) \]

where, \( x_\gamma \), \( x_a \), and \( x_b \) are the light-cone momentum fractions of the incoming photon, incoming parton in proton and incoming parton from the photon respectively. \( f_{b/\gamma}(x_b) \) is the distribution of parton \( b \) in the photon. The hard parts for the partonic processes involved in the resolved contribution are,

\[ H_{ab \rightarrow \gamma d}^U \delta(\hat{s} + \hat{t} + \hat{u}) \]

Transverse single-spin asymmetry in the electroproduction of prompt photons is defined as

\[ A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \]

(4)

where \( d\sigma^{\uparrow(\downarrow)} \) is the invariant differential cross-section for the process \( ep^{\uparrow(\downarrow)} \rightarrow \gamma + X \) with the spin of the transversely polarized proton being aligned in the \( \uparrow(\downarrow) \) direction with respect to the production plane. Here, \( \uparrow \) would be the +\( Y \) direction in a frame where the polarized proton is moving along the +\( Z \) direction and the photon is produced in the \( XZ \) plane.

For the direct process, expressions for the denominator and numerator of Eq. (4) are

\[ (d\sigma^\uparrow + d\sigma^\downarrow)_{\text{Direct}} = E_\gamma \frac{d\sigma^{ep \rightarrow \gamma X}}{d^3p_\gamma} + E_\gamma \frac{d\sigma^{ep \rightarrow \gamma X}}{d^3p_\gamma} \]

(5)

\[ = 2 \int dx_\gamma dx_q d^2k_{\perp q} f_{\gamma/e}(x_\gamma) \frac{\hat{f}_{q/p}(x_q, k_{\perp q}, Q)}{x_\gamma x_q s} \Delta \frac{\alpha^2}{N_c} H_{q\gamma \rightarrow q\gamma}^U \delta(\hat{s} + \hat{t} + \hat{u}). \]

and

\[ (d\sigma^\uparrow - d\sigma^\downarrow)_{\text{Direct}} = E_\gamma \frac{d\sigma^{ep \rightarrow \gamma X}}{d^3p_\gamma} - E_\gamma \frac{d\sigma^{ep \rightarrow \gamma X}}{d^3p_\gamma} \]

(6)

\[ = \int dx_\gamma dx_q d^2k_{\perp q} f_{\gamma/e}(x_\gamma) \Delta N \frac{\hat{f}_{q/p}(x_q, k_{\perp q}, Q)}{x_\gamma x_q s} \frac{\alpha^2}{N_c} H_{q\gamma \rightarrow q\gamma}^U \delta(\hat{s} + \hat{t} + \hat{u}). \]
For the resolved process, expressions for the denominator and numerator of Eq. (4) are

\begin{equation}
(d\sigma^\uparrow + d\sigma^\downarrow)_{\text{Resolved}} = E_\gamma \frac{d\sigma^{e\gamma^\uparrow \rightarrow \gamma X}}{d^3p_\gamma} + E_\gamma \frac{d\sigma^{e\gamma^\downarrow \rightarrow \gamma X}}{d^3p_\gamma}
\end{equation}

\begin{equation}
= 2 \sum_{a,b=g,q,\bar{q}} \int dx_\gamma dx_a dx_b d^2k_{\perp a} f_{\gamma/e}(x_\gamma) f_{b/\gamma}(x_b, Q) \hat{f}_{a/p}(x_a, k_{\perp a}, Q)
\end{equation}

\begin{equation}
\frac{\alpha_s}{x_a x_b x_\gamma} H_{ab \rightarrow \gamma}^U \delta(\hat{s} + \hat{t} + \hat{u}).
\end{equation}

and

\begin{equation}
(d\sigma^\uparrow - d\sigma^\downarrow)_{\text{Resolved}} = E_\gamma \frac{d\sigma^{e\gamma^\uparrow \rightarrow \gamma X}}{d^3p_\gamma} - E_\gamma \frac{d\sigma^{e\gamma^\downarrow \rightarrow \gamma X}}{d^3p_\gamma}
\end{equation}

\begin{equation}
= \sum_{a,b=g,q,\bar{q}} \int dx_\gamma dx_a dx_b d^2k_{\perp a} f_{\gamma/e}(x_\gamma) f_{b/\gamma}(x_b, Q) \Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}, Q)
\end{equation}

\begin{equation}
\frac{\alpha_s}{x_a x_b x_\gamma} H_{ab \rightarrow \gamma}^U \delta(\hat{s} + \hat{t} + \hat{u})
\end{equation}

where, \(\Delta^N f_{a/p^\uparrow}\) is the Sivers function the functional form for which is given in Sec. [IV].

\begin{equation}
\Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}, Q) = \hat{f}_{a/p^\uparrow}(x_a, k_{\perp a}, Q) - \hat{f}_{a/p^\downarrow}(x_a, k_{\perp a}, Q)
\end{equation}

\begin{equation}
= \Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}, Q) \cos \phi_a
\end{equation}

\begin{equation}
= -2 \frac{k_{\perp a}}{M_p} f_{1T}(x_a, k_{\perp a}, Q) \cos \phi_a
\end{equation}

III. THE CGI-GPM FORMALISM

In the generalized parton model, it is assumed that all TMDs are universal and therefore, Sivers function extracted in SIDIS can be used to make predictions for inclusive particle production in hadron hadron scattering. However, we know that operator definition of Sivers function contains Wilson lines (gauge links) which are required for the gauge invariance of Sivers function. This presence of Wilson lines makes Sivers function process dependent. In the Color Gauge Invariant Generalized Parton Model, this process dependence is taken into account by considering the initial state interactions (ISI) between active parton from incoming unpolarized hadron and the spectators and the final state interactions (FSI) between active parton from the final state and the spectators of incoming polarized hadron. The ISIs and FSIs are then approximated by single eikonal gluon exchange, which is equivalent to the leading order expansion of Wilson line in the strong coupling constant \(g_s\). These ISI/FSIs provide an imaginary part to the amplitude that is required for the existence of
a SSA. The only effect of these ISI/FSI is to modify the color factor of the participating processes. In CGI-GPM, this modified color factor is absorbed into the hard part leading to process dependent hard parts in GPM expressions. These modified hard parts now can be used alongwith the universal Sivers function in the calculation of the asymmetry.

To illustrate this, we have shown an example in Fig. 2. Fig. 2(a) is the resolved subprocess $qg \rightarrow \gamma q$ in GPM formalism and Fig. 2(b) is the same process with FSI in CGI-GPM formalism. This approach was first proposed in Ref. [25] for the process $p^+p \rightarrow \pi^+X$ and has been extended to the case of GSF in Ref. [24] for $p^+p \rightarrow J/\psi + X$ and $p^+p \rightarrow D + X$. To obtain the asymmetry in CGI-GPM approach, one has to to make the following replacement for the QSF in Eq. 6 and 8,

$$f_{1T}^U g_{b\rightarrow cd} \rightarrow f_{1T}^U g_{b\rightarrow cd}^{mod} = \frac{C_I + C_{Fc}^d}{C_U} f_{1T}^U g_{b\rightarrow cd}$$

(10)

Here, $C_U$ is the color factor corresponding to unpolarized cross section, $C_I$ and $C_{Fc}$ are color factors for the case of ISI and FSI respectively and $f_{1T}$ is the Sivers function fitted to SIDIS data.

In the CGI-GPM framework, the process-dependent gluon Sivers function can be written as a linear combination of two independent universal gluon distributions $f_{1T}^U g(f)$ and $f_{1T}^U g(d)$, which correspond to the two different ways in which color can be neutralized. Therefore, for the case of GSF the substitution required in Eq. 8 is,

$$f_{1T}^U g_{b\rightarrow cd} \rightarrow f_{1T}^U g_{b\rightarrow cd}^{mod} = \frac{C_I^f + C_{Fc}^d}{C_U} f_{1T}^U g_{b\rightarrow cd} + \frac{C_I^d + C_{Fc}^d}{C_U} f_{1T}^U g_{b\rightarrow cd}$$

(11)

FIG. 2. LO diagrams for the resolved subprocess $qg \rightarrow \gamma q$ in GPM formalism (a) and in CGI-GPM formalism(b). Here only ISI contributes through resolved channel.
Now, for the case of direct subprocess $q\gamma \rightarrow \gamma q$, both initial particle and the final observed particles are photons which can not emit eikonal gluon. Hence, there are no ISI/FSI diagrams to provide an imaginary part in any of the amplitudes that are interfering (which is the case with the standard unpolarised amplitude), and hence there will not be any SSA. In other words, there is a modified hard-part for direct subprocess $q\gamma \rightarrow \gamma q$, but it vanishes. It should be noted that this argument is valid only when considering the CGI-GPM framework wherein the ISI/FSI is explicitly taken to be a prerequisite for a non-zero SSA. In the context of standard GPM framework where the Sivers function is considered to be universal, the standard hard part does contribute to the numerator of the asymmetry. Finally, we would like to stress the point that although there are no ISI and FSI in direct subprocess $q\gamma \rightarrow \gamma q$, there is FSI from the unobserved particle which is quark. However, contribution from this FSI vanishes, when we sum the different cut diagrams which are shown in Fig. 3.

![LO diagrams of FSI of an unobserved particle for the direct subprocess $q\gamma \rightarrow \gamma q$ for the unobserved particle](image)

FIG. 3. LO diagrams of FSI of an unobserved particle for the direct subprocess $q\gamma \rightarrow \gamma q$ for the unobserved particle

The modified hard parts for the processes under consideration have been calculated in \cite{28, 29} and are given below for the sake of completeness:

\[
H_{q\bar{q} \rightarrow \gamma g}^{\text{mod}} = -H_{\bar{q}q \rightarrow \gamma g}^{\text{mod}} = \frac{e_q^2}{N_c^2} \left( \frac{\hat{t}}{t} + \frac{\hat{t}}{\bar{u}} \right)
\]

\[
H_{gq \rightarrow \gamma q}^{\text{mod}} = -H_{\bar{g}q \rightarrow \gamma q}^{\text{mod}} = -\frac{N_c}{N_c^2 - 1} e_q^2 \left( \frac{\hat{t}}{s} + \frac{\hat{s}}{\bar{t}} \right)
\]

\[
H^{(f)}_{gq \rightarrow \gamma q} = H^{(f)}_{\bar{g}q \rightarrow \gamma \bar{q}} = -\frac{1}{2} H_{gq \rightarrow \gamma q}^U
\]

\[
H^{(d)}_{gq \rightarrow \gamma q} = -H^{(d)}_{\bar{g}q \rightarrow \gamma \bar{q}} = \frac{1}{2} H_{gq \rightarrow \gamma q}^U
\]
IV. PARAMETRIZATION OF THE TMDS

For the unpolarized TMDs, we adopt the commonly used form with the collinear PDF multiplied by a Gaussian transverse momentum dependence,

\[ f_{i/p}(x, k_\perp; Q) = f_{i/p}(x, Q) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle} \]  \hspace{1cm} (12)

with \( \langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2 \).

Since we give predictions using the GSF fits of Ref. [22], we adopt the functional form of the Sivers functions used therein:

\[ \Delta^N f_{i/p}(x, k_\perp; Q) = 2 N_i(x) f_{i/p}(x, Q) \frac{\sqrt{2} e}{\pi} \sqrt{1 - \frac{\rho}{\rho} k_\perp} \frac{e^{-k_\perp^2 / \rho \langle k_\perp^2 \rangle}}{\langle k_\perp^2 \rangle^{3/2}} \]  \hspace{1cm} (13)

with \( 0 < \rho < 1 \). Here \( N_i(x) \) parametrises the \( x \)-dependence of the Sivers function given by

\[ N_i(x) = N_i (1 - x)^{\beta_i} \frac{1}{\alpha_i^{\alpha_i + \beta_i}} \]  \hspace{1cm} (14)

For the Sivers function to satisfy the positivity bound,

\[ \frac{|\Delta^N f_{i/p}(x, k_\perp)|}{2 f_{i/p}(x, k_\perp)} \leq 1 \quad \forall x, k_\perp, \]  \hspace{1cm} (15)

it is necessary to have \( |N_i(x)| < 1 \).

In order to study the efficacy of the probe, we explore the following choices for the Sivers functions:

- Quark and gluon Sivers functions with the positivity bound saturated, viz. \( N_i(x) = 1 \) and \( \rho = 2/3 \).

- The SIDIS1 [20] and SIDIS2 [21] fits of the QSFs, along with the associated GSF fits from Ref. [22].

We will refer the first choice as ‘saturated’ Sivers function which provides the maximum value of Sivers function allowed by the positivity bound for a fixed width \( \langle k_\perp^2 \rangle \) and \( \rho \), and with a particular choice of unpolarized collinear parton distribution function. The parameter \( \rho \) is set to 2/3 in order to maximize the first \( k_\perp \)-moment of the Sivers function, following Ref. [33].
V. NUMERICAL ESTIMATES

First, we consider the unpolarized Lorentz-invariant cross-section for the production of prompt photons at EIC energy, $\sqrt{s} = 140 \text{ GeV}$. In Fig. 4 we show $x_F$ distribution of cross section in the left panel at fixed $p_T = 3 \text{ GeV}$ and $p_T$ distribution of cross section in the right panel at fixed rapidity $\eta = -2$. Fragmentation contributions are not considered here since, as mentioned earlier, these can be eliminated by applying isolation cuts in the experiment. We have used CTEQ6L [34] PDFs for the collinear parton distribution functions and the AFG04 [35] parton distribution in photon. A flavour independent gaussian width $\langle k^2 \rangle = 0.25 \text{ GeV}^2$ has been used for TMD-PDFs. Factorization scale $Q$ for evaluating the PDFs and $\alpha_s$ in the cross-section expressions is chosen to be the transverse momentum of produced photon $p_T$.

As evident from Fig. 4, the contribution to the unpolarised cross-section from direct subprocess $q\gamma \rightarrow \gamma q$ is dominant in comparison to the resolved contribution. As far as resolved contributions are concerned, the gluon initiated process i.e. $gq \rightarrow \gamma q$, where gluon is coming from proton and quark is coming from the photon, is the dominant one amongst
the resolved subprocesses since the region that we have considered here, which is $x_F < 0$, where low values of $x_a$ are being probed. At these low values of momentum fraction the distribution of gluon inside the proton is dominant over the distribution of quarks. The fact that cross section for direct subprocess is of the same order as the cross section from total resolved contribution will be crucial for discriminating between GPM and CGI-GPM formalism. This is due to the fact that the direct subprocess has no initial state or final state interaction and therefore, it does not contribute to the numerator of the asymmetry in the CGI-GPM formalism.

In the following subsections, we present the estimates of asymmetry obtained using GPM and CGI-GPM frameworks.

1. Estimates of asymmetry using saturated Sivers functions

Here, we calculate asymmetry using saturated Sivers functions, which satisfy the positivity bound of Eq.\((15)\). In Fig. 5, we show asymmetry estimates using saturated quark and saturated gluon Sivers functions.

As shown in left panel of Fig.5, asymmetry in GPM is dominated by quark Sivers function. At fixed $p_T$ value of 3 GeV, the saturated QSF contribution to asymmetry in $x_F$-distribution is found to be upto 13% at midrapidity and upto 3% to 7% at $x_F < -0.2$, whereas saturated gluon Sivers function contribution is upto 3% to 4% in this region. In the right panel of Fig. 5, we show the asymmetry estimates for $p_T$ distribution at fixed rapidity $\eta = -2$. In this case, we find that saturated QSF contributes 8.5% to the SSA, whereas saturated GSF contributes upto 5% at $p_T = 2$ GeV.

Next, we consider the estimates of SSA in color gauge invariant generalized parton model. In this model, we relax the assumption of universality of Sivers function taking into account the initial and final state interactions. We do not need to consider the direct subprocess in the numerator of SSA for the reason mentioned earlier. However, it does contribute to the denominator which is twice the unpolarized cross section.

In Fig. 6, we have given estimates of SSA using saturated Sivers functions in CGI-GPM formalism. As we can see from the plots, saturated QSF contribution is negligible in this case. This is due to following two reasons: First, the quark initiated direct subprocess, which contributes predominantly in the kinematic regions under consideration, does not
FIG. 5. Estimates for single spin asymmetry in prompt photon production using saturated quark and gluon Sivers function in GPM. Dashed blue line indicates contribution from quark Sivers function and dashed red line indicates contribution from gluon Sivers function. Results are obtained using CTEQ6L [34] parametrization for collinear PDFs for partons in the proton and AFG04 [35] parametrizations for parton distribution in photon.

contribute to the numerator of SSA in CGI-GPM and secondly, in the resolved channel, the quark initiated processes contribute negligibly even to the unpolarised cross section. In case of saturated GSF, f-type GSF contribution is negative and is around 1.5% to 2.5% in magnitude, which is 50% of the GPM estimates. The change in sign is due to the fact that \( H_{gq(q)\rightarrow\gamma q(q)}^{(f)} = -\frac{1}{2} H_{gq\rightarrow\gamma q}^{U} \). The reason for vanishing contribution from d-type GSF is that the modified hard parts have opposite signs for quarks and antiquarks \( H_{gq\rightarrow\gamma q}^{(d)} = -H_{g\bar{q}\rightarrow\gamma\bar{q}}^{(d)} \). This combined with the fact that the distribution of quarks and antiquarks is same in the photon leads to zero contribution from d-type GSF in this process. The absence of d-type GSF contribution to the SSA makes this probe especially useful for extracting information on f-type GSF, which has the dominant contribution to the SSA in CGI-GPM formalism.

2. Estimates of asymmetry using SIDISI1 and SIDIS2 parametrization of Sivers functions

In this section, we consider estimates of SSA using presently available parametrization for Sivers function mentioned in Section [IV] which will be referred to as SIDIS1 and
FIG. 6. Results for single spin asymmetry in prompt photon production using saturated quark and gluon Sivers function in CGI-GPM. Estimates are given as a function of $x_F$ (at $p_T = 3$ GeV, left panel) and $p_T$ (at rapidity $\eta = -2$, right panel). Dashed blue line corresponds to the contribution from QSF, red line corresponds to contribution from f-type GSF and green line shows d-type GSF contribution.

SIDIS2. SIDIS1 set has been obtained from fitting pion production data at HERMES and COMPASS[20]. In obtaining this fit, only valence quarks contributions were taken into account and therefore, only $u$ and $d$ quark Sivers functions are provided in this set. SIDIS2 set is obtained in Ref [21] by fitting both pion as well as Kaon production data and contains valence as well as sea quark contributions. In Ref. [22], D’Alesio et al. fitted gluon Sivers function using mid-rapidity data on pion production at RHIC [36] using SIDIS1 and SIDIS2 quark Sivers functions. We will also refer to the corresponding sets of GSF as SIDIS1 and SIDIS2. These fits have been performed using Generalized Parton Model expressions.

In Fig. 7, we present $x_F$ distribution of rapidity at fixed $p_T = 3$ GeV in GPM formalism. In case of GPM, for both SIDIS1 and SIDIS2 parameter sets, we find negligible gluon Sivers asymmetries and around 0.1% to 0.2% quark Sivers asymmetries.
VI. CONCLUSIONS

In this work, we have studied the Sivers asymmetry in prompt photon production in electron proton collisions. We have taken into account both direct as well as resolved contributions to the production of photons. We find that this probe can be useful for discriminating between GPM and CGI-GPM as the predictions of asymmetry given by these models are entirely different. Using GPM, we find, at $x_F < -0.2$, positive asymmetry with contribution from saturated QSF being up to 3% to 7% while it is found to be 3% to 4% from GSF. On the other hand, in case of CGI-GPM, we find negligible asymmetry due to saturated QSF and up to 2.5% asymmetry due to f-type GSF. We find that the d-type GSF do not contribute to SSA in CGI-GPM and hence, we conclude that this probe can be useful for exploring f-type GSF at future EIC.

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