The unconventional superconductor Sr$_2$RuO$_4$ has been studied in detail \cite{1, 2}, but the exact pairing mechanism remains still unclear. Inspired by the ferromagnetic order in the perovskite SrRuO$_3$ a p-wave triplet pairing mediated through ferromagnetic fluctuations was suggested shortly after the discovery of the superconductivity \cite{3}. In the meanwhile, there is experimental evidence that the order parameter indeed has a triplet character \cite{3}. In the meanwhile, there is pronounced nesting, as it has been first calculated in LDA \cite{5}. Still, there is a strong anisotropy with $\chi^{||}_{zz}$ close to three times larger than $\chi^{\perp\perp}_{ij}(q)$ \cite{6}. The question about the anisotropy of the incommensurate fluctuations is hence essential for the understanding of the superconducting pairing.

The anisotropy of the magnetic incommensurate fluctuations in Sr$_2$RuO$_4$ has been studied by inelastic neutron scattering with polarized neutrons. We find a sizeable enhancement of the out of plane component by a factor of two for intermediate energy transfer which appears to decrease for higher energies. Our results qualitatively confirm calculations of the spin-orbit coupling, but the experimental anisotropy and its energy dependence are weaker than predicted.

Several theoretical approaches explored the possible role of the different bands in the superconducting pairing \cite{7, 8, 9, 10, 11, 12}. From the integration over the Fermi-surface, Mazin and Singh \cite{8} conclude that the incommensurate fluctuations favor a d-wave singlet pairing in contrast to the experimental observations. However, such analysis completely neglects the anisotropy of the susceptibility $\chi^{||}_{\alpha\beta} \approx \chi^{\perp\perp}_{\alpha\beta}(q)$. More recent analyses conclude that the incommensurate fluctuations indeed may lead to triplet pairing provided that there is a strong anisotropy with $\chi^{||}_{zz}$ much larger than $\chi^{\perp\perp}_{ab}(q)$ \cite{13}. The question about the anisotropy of the incommensurate fluctuations is hence essential for the superconducting pairing.

Ng and Sigrist analyzed the Lindhard susceptibility taking the spin orbit coupling into account and indeed find an enhancement of the out of plane component close to the incommensurate position $q_i$ \cite{14}. This anisotropy appears to be rather modest in the bare susceptibility $\chi^0(q)$ but may become essential in the interaction enhanced susceptibility, which in the most simple RPA treatment is given by:

$$\chi^{\alpha\alpha}(q) = \frac{\chi^0_{\alpha\alpha}(q)}{1 - I(q) \cdot \chi^0_{\alpha\alpha}(q)},$$  

where $\alpha$ labels the components and $I(q)$ is the interaction parameter, which may be anisotropic. Since the enhancement parameter $S(q) = I(q)/\chi^0(q)$ is close to one in Sr$_2$RuO$_4$ near $q_i$, the enhanced susceptibility $\chi(q, \omega)$ is very sensitive to any, even small change in $\chi^0$. For the resulting $\chi^0(q_i, 6\text{meV})$, Eremin et al. obtain an anisotropy of at least one order of magnitude by full analysis of the spin orbit coupling \cite{14}. On the experimental side, evidence for an anisotropy was found in the NMR $\chi^{||}_{zz}/\chi^{\perp\perp}_{\alpha\beta}$ measurements by Ishida et al. who obtain a factor of three for the $\chi^{||}_{zz}/\chi^{\perp\perp}_{\alpha\beta}$ ratio at the NMR frequency \cite{15}. Qualitatively the anisotropy is also confirmed in Ti-doped sam-

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**Anisotropy of the incommensurate fluctuations in Sr$_2$RuO$_4$: a study with polarized neutrons**

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The anisotropy of the magnetic incommensurate fluctuations in Sr$_2$RuO$_4$ has been studied by inelastic neutron scattering with polarized neutrons. We find a sizeable enhancement of the out of plane component by a factor of two for intermediate energy transfer which appears to decrease for higher energies. Our results qualitatively confirm calculations of the spin-orbit coupling, but the experimental anisotropy and its energy dependence are weaker than predicted.
samples, where the incommensurate fluctuations condense to a static spin density wave ordering with the ordered moment being aligned perpendicular to the planes [17]. The direct analysis in Sr$_2$RuO$_4$ can be performed by INS. However, when using unpolarized neutrons the analysis is indirect and requires some knowledge or assumptions about the underlying form factor. Our previous rough estimation with INS pointed to some weak anisotropy [17], whereas the deeper study by Servant et al. [18] concluded that the incommensurate fluctuations would be isotropic. Another INS study with unpolarized neutrons concludes a high anisotropy of about a factor 4 [19].

In this work, we have used polarized neutrons in order to have direct access to the anisotropy of the quasi-antiferromagnetic fluctuations. In addition we have again looked for the existence of quasi-ferromagnetic fluctuations.

Large single crystals of Sr$_2$RuO$_4$ were grown by a floating zone technique; the superconducting transition temperatures of all the samples used were above 1.3 K. Due to their stick like form, ten crystals of less than 30mm length were coaligned yielding a total mass of about 10g. INS experiments were performed on the IN20 triple axis spectrometer with polarization analysis on the monochromator and on the analyser sides as obtained by focusing Heusler crystals. Scans were performed with a final energy of 35 meV where a pyrolithic graphite filter allows efficient suppression of the second order contamination. The polarization ratio, $R$, was obtained by measuring phonon and Bragg scattering to $R \sim 15$. The monochromator and analyzer crystals polarize or analyze the neutron spin perpendicular to the scattering plane. In addition Helmholtz coils were used in order to rotate the polarization of the neutron at the sample to the desired orientation. A neutron spin flipper was put in front of the analyzer crystal. Since there is no incoherent magnetic signal from nuclear spins in Sr$_2$RuO$_4$, the analysis of the spin-flip signal directly gives the distinct components of the generalized susceptibility given by eqn. (1). We put the sample in the $[100]/[010]$ orientation and determined the scattering intensities in the three spin-flip channels with the neutron polarization (i) parallel to the scattering vector $Q$, (ii) perpendicular to $Q$ and within the $a$, $b$-plane, $I_{\perp a,b}$, and (iii) perpendicular to $Q$ and perpendicular to the planes, $I_{\perp c}$. Since we studied only $Q$-vectors within the $a$, $b$-plane, these three measured intensities sense the following components of the susceptibilities (neglecting corrections of the order of $1/R$) [20]:

$$I_{\parallel} = scale \cdot (\chi_{a,b}^{zz} + \chi_{zz}^{aa,b}) + I_{\text{BGR}} \quad (3)$$

$$I_{\perp a,b} = scale \cdot \chi_{zz}^{aa,b} + I_{\text{BGR}} \quad (4)$$

$$I_{\perp c} = scale \cdot \chi_{a,b}^{zz} + I_{\text{BGR}} \quad (5)$$

where $I_{\text{BGR}}$ is the spin-flip background scattering. In addition, for some cases, we have also determined the non-spin flip scattering to exclude a possible phonon contamination through the non-ideal polarization ratio. Typically, $I_{\parallel}$ was measured twice as long as the other components and the single contributions were obtained by subtracting the corresponding intensities assuming that $scale$ and $I_{\text{BGR}}$ are the same in all spin-flip channels.

Fig. 1 a) and b) show the raw scan data across the incommensurate position $Q=(0.7,0.3,0)$ for an energy transfer of 8meV and at $T=1.5K$ a-d) and at $T=300K$ e-f). Part a) shows the non-spin-flip scan data which does not exhibit a peak. Part b) shows the total sum of the spin-flip scattering and the corresponding spin-flip background. Part c) presents the spin-flip scattering for the different orientations of the neutron polarization including the spin-flip background. In part d) we show the out-of-plane and in-plane susceptibilities obtained from c) and equations (3-5). Parts e) and f) show the corresponding results obtained at 300K.

FIG. 1: Results of the scans across the incommensurate position at $Q=(0.7,0.3,0)$ for an energy transfer of 8meV and at $T=1.5K$ a-d) and at $T=300K$ e-f). Part a) shows the non-spin-flip scan data which does not exhibit a peak. Part b) shows the total sum of the spin-flip scattering and the corresponding spin-flip background. Part c) presents the spin-flip scattering for the different orientations of the neutron polarization including the spin-flip background. In part d) we show the out-of-plane and in-plane susceptibilities obtained from c) and equations (3-5). Parts e) and f) show the corresponding results obtained at 300K.
large in the integrated signal. This anisotropy seems to disappear upon heating to 300K, see Fig. 1 e) and f).

The sign and the temperature dependence of the magnetic anisotropy are in qualitative agreement with the calculations, but the experimental anisotropy at 1.5 K is much smaller than the calculation reported in reference [13] for comparable energy transfer. The experimentally determined anisotropy is, furthermore, slightly smaller than the factor of three obtained from the analysis of the NMR experiments by Ishida et al. [15]. The slight difference should reflect the much lower fluctuation energies by the NMR experiment in respect with the value of 8meV studied here. Eremin et al. indeed calculate a pronounced energy dependence for the anisotropy ratio at low temperature suggesting that only the out-of-plane component is approaching a SDW phase transition. In our previous work we have analyzed the energy dependence of the magnetic response at \( q_i \) with single relaxor behavior [23]:

\[
\chi''(q_i, \omega) = \chi'(q_i, \omega = 0) \frac{\Gamma \omega}{\omega^2 + \Gamma^2}
\]

where \( \Gamma \) is the characteristic energy and \( \chi'(q_i, 0) \) the amplitude which corresponds to the real part of the generalized susceptibility at \( \omega = 0 \) according to the Kramers-Kronig relation. In strength, one would need to allow for two different characteristic energies and amplitudes for the in-plane and out-of-plane spectra which become superposed in the unpolarized experiment. Such analysis, however, cannot be made with the statistics achievable. Fig. 2 shows scans across the incommensurate position for different energies in the spin-flip channel with polarization parallel to \( Q \). The incommensurate scattering can be followed up to 40meV whereas the unpolarized analyzes were restricted to the energy range below 12meV due to contaminations with phonon scattering. Besides the conclusion that the nesting signal extends to higher energies than previously studied, we obtain the anisotropy ratios at higher energies. The incommensurate signal appears to become more isotropic with increasing energy. However, the statistics of these data remains rather poor due to the larger Q-value required for the scattering geometry and due to the limitation of the complete polarization analysis to single points. Fig. 3) resumes all results together with the NMR-anisotropy at low frequency. Taking account of the average between in- and out-of-plane susceptibility determined in the unpolarized INS experiments [5, 17], one may describe such energy dependence with \( \Gamma_{zz} \sim 8 \text{meV}, \chi_{zz}(q_i, 0) \sim 220 \mu_B^2/eV \) and \( \Gamma_{ab} \sim 13 \text{meV}, \chi_{ab}(q_i, 0) \sim 140 \mu_B^2/eV \), see the line in Fig. 3. Qualitatively, this behavior agrees with the analysis by Eremin et al., but the experimentally observed differences in \( \Gamma \) and \( \chi'(q_i, \omega = 0) \) are much smaller (this disagreement is far beyond the statistical errors).

In the theory of Kuwabara and Ogata [2] the ability of the incommensurate fluctuations to stabilize a triplet pairing is studied quantitatively. Following reference [4] we take \( S(q_i = 0) \) to 0.8 and use the q-dependence of the interaction \( I(q) \) given in [5]. With the experimental spin-susceptibility of \( \chi'(0, 0) = 9 \times 10^{-5} \text{emu/mol} \sim 28 \mu_B^2/eV \) and the absolute values of \( \chi_{zz}(q_i, 0) \) determined above [5] one obtains the enhancement parameter at the incommensurate \( q \)-value : \( S(q_i) = 0.97 \) [24]. The resulting anisotropy in the enhanced susceptibility -- taking equation (2) for the two channels [5] -- is then driven only by a small anisotropy in \( \chi^0 : \) for an anisotropy in the enhanced part of 2.5 one gets \( \frac{\chi_{zz}^0(q_i, 0)}{\chi_{ab}^0(q_i, 0)} = 1.045 \), whereas triplet pairing is stabilized only for \( \frac{\chi_{zz}^0(q_i, 0)}{\chi_{ab}^0(q_i, 0)} > 1.14 \) [24]. Comparing the ratios in the enhanced parts one finds that for \( S(q_i) = 0.97 \) the theory requires an effective anisotropy larger than 5.5 compared to the experimental value of 2.5. The energy dependence of the anisotropy discussed above will even enhance the discrepancy since it reduces the anisotropy in the real part of the susceptibility at zero energy; the rough estimate amounts only to a fac-
tor of 1.6, see above. Therefore, the sizeable anisotropy reported here does not still allow to explain triplet superconductivity within the theory of \[ \mathbb{R} \]; a more precise theoretical treatment is highly desirable.

Fig. 4 shows the results of a scan across \( Q=(0.7,0.3,0) \) in diagonal direction connecting to the two-dimensional zone-center \((1,0,0)\) where scattering related to a ferromagnetic instability should be observable. In agreement with all unpolarized previous studies \cite{17} one may conclude that there is no strong quasi-ferromagnetic scattering for the energy transfer studied, 8 meV. The statistics is not sufficient for complete polarization analysis, but the spin-flip analysis still indicates some broad magnetic scattering around \( Q=(1,0,0) \). Correcting for the form-factor, this signal is about a factor of five weaker than that of the peak in the incommensurate signal in rough agreement with the mapping of scattering with unpolarized neutrons \cite{17}. Furthermore, nearly the same factor of five can be seen in the scans in Fig. 1b) and d) at \( k \sim 0.1 \) and at low temperature, and this signal seems to increase with temperature, see Fig. e) and f). Such weakly \( q \)- and temperature-dependent scattering perfectly agrees with the interpretation of the NMR experiments \cite{13}.

In conclusion INS with polarized neutrons has shown that there is an anisotropy in the dynamic susceptibility at the incommensurate position. The enhancement of the out-of-plane component by a factor of 2-2.5 for an energy transfer of 8 meV qualitatively confirms calculations based on spin-orbit coupling. But the anisotropy as well as its temperature dependence is much weaker than the theoretical prediction. Comparing the anisotropy ratio at 8 meV with results at higher energy and with the NMR-analysis referring to very low energy, one has to conclude some energy dependence of the anisotropy. Again there is qualitative agreement with theory, but the experimentally observed energy dependence is much smaller. Close to the quantum critical point of the related SDW order-