Numerical methods and transition investigation of transient flows around a pitching hydrofoil

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Abstract. The numerical simulations for a NACA66 hydrofoil are performed by using the standard $k$-$\omega$ SST turbulence model and revised $\gamma$-$Re_{\theta}$ transition model respectively. The simulation results are compared with the experimental results, and the hydrodynamic property and the fluid structure during the pitching process is studied. It is revealed that, compared with the standard $k$-$\omega$ SST turbulence model, the revised $\gamma$-$Re_{\theta}$ transition model is able to present the hydrodynamic property and the fluid structure of the transient flow around a pitching hydrofoil more accurately, and better predict the separation and transition process in the boundary layer. The transient flow process around a pitching hydrofoil can be divided into 5 parts. At small angle of attack, transition is observed at the leading edge of the foil, resulting in the inflection of dynamic property curves. As the angle of attack increases, a clockwise trailing edge vortex expands toward the leading edge of the foil. At high angles of attack, large-scale load fluctuations are observed due to the stall caused by separation of the leading edge vortex. The flow transitions back to laminar during the downward pitching process.

1. Introduction
In order to avoid or to reduce undesirable effects such as dynamic stall, as well as structural instabilities such as flutter and resonance, a substantial number of experimental and numerical investigations on aerodynamic stall have been conducted [1,2]. In hydrodynamics-related fields such as hydraulic machinery and ship engineering, the pitching hydrofoil problem is relevant to the dynamic performance of turbomachineries, rudders, hydrofoils, and general control surfaces, where the effective angle of attack changes due to spatially and/or temporally varying inflow, and/or due to active/passive rigid/elastic body motions [3-5], while there is lack of the study of flow structure and dynamical property of pitching hydrofoil. Huang et al. [6] evaluate the predictive capability of popular transport equation-based cavitation models for the simulation of cavitating flows around a stationary NACA66 hydrofoil. Ducoin et al. [7] deal with an experimental and numerical investigation of the transition from laminar flow to turbulence flow on a NACA66 hydrofoil undergoing a transient up-and-down pitching motion at four pitching velocities. It is shown that increasing the pitching velocity tends to delay the laminar-to-turbulence transition and an increase of the hysteresis effect during pitch-down motion resulting to a significant increase of the hydrodynamic loading.

Transition is a complex process occurred in the boundary layer, which will result in rapid increase of wall friction, and have decisive influence on the separation state of boundary layer. Engineering transition predictions are based mainly on two modeling concepts [8]. The first is the use of low-Reynolds number turbulence models, where the wall damping functions of the underlying turbulence
model trigger the transition onset. However, experience has shown that this approach is not capable of reliably capturing the influence of the many different factors that affect transition. The second approach is the use of experimental correlations. The \( e^2 \) method [9] is based on the local, linear stability theory and the parallel flow assumption in order to calculate the growth of the disturbance amplitude from the boundary layer neutral point to the transition location. However, it cannot predict transition due to non-linear effects such as high free stream turbulence or surface roughness. The classical correlation-based transition models [10-12] typically correlate the transitional Reynolds number to local free-stream conditions, like turbulence intensity and pressure gradient. The difficulties associated with non-local formulations are exaggerated by modern CFD methods that are based on unstructured grids and massive parallel execution. A novel approach, the \( \gamma \)-Re\( \theta \) transition model [13,14] is built strictly on local variables and theory and thereby compatible with modern CFD methods. Dong et al. [15] performed numerical simulations for the flat plate boundary layer transition test at zero and non-zero pressure gradients. The results show that the \( \gamma \)-Re\( \theta \) transition model could predict accurately the transition and development process. Luo et al. [16] couple the \( \gamma \)-Re\( \theta \) transition model with SST turbulence model to simulate the turbine assembly under different conditions. The calculation method does have the ability to predict the effects of the Reynolds number on the dynamic performance.

In this paper, the commercial software ANSYS CFX is used. The numerical simulations for a pitching hydrofoil are performed by using the standard \( k-\omega \) SST turbulence model and revised \( \gamma \)-Re\( \theta \) transition model respectively. The simulation results are compared with the experimental results, and the hydrodynamic property and the fluid structure during the pitching process is studied.

2. Governing equations and numerical methods

2.1. Governing equations

The URANS equations, in their conservative form, for a Newtonian fluid without body forces and heat transfers are presented below:

\[
\frac{\partial (\rho u_j)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \mu + \mu_t \right) \frac{\partial u_j}{\partial x_i} - \frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_i} \left( \mu + \mu_t \right) \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} \right) \tag{1}
\]

\[
\frac{\partial (\rho u_i u_j)}{\partial t} + \frac{\partial (\rho u_i u_j u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu + \mu_t \right) \frac{\partial u_i}{\partial x_j} - \frac{\partial p}{\partial x_i} \left( \frac{\partial u_i}{\partial x_j} \right) \left( \frac{\partial u_j}{\partial x_j} \right) \tag{2}
\]

Where, \( \rho \) is the fluid density, \( u \) is the velocity, \( p \) is the pressure, \( \mu \) and \( \mu_t \) are respectively the laminar and turbulent viscosity. The subscripts \((i, j)\) denote the directions of the Cartesian coordinates.

2.2. Revised transition turbulence model based on standard \( k-\omega \) SST turbulence model

2.2.1. Standard \( k-\omega \) SST turbulence model. The \( k-\omega \) SST turbulence model proposed by Menter is shown as follows:

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_j k)}{\partial x_j} = P_k - D_k + \frac{\partial}{\partial x_j} \left[ \mu + \mu_t \rho \frac{\partial k}{\partial x_j} \right] \tag{3}
\]

\[
\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho u_j \omega)}{\partial x_j} = C_{\omega} P_{\omega} - \beta_{\omega} \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ \mu + \mu_t \rho \frac{\partial \omega}{\partial x_j} \right] + 2 \rho (1 - F_j) \sigma_{\omega} \frac{1}{\omega} \left[ \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \right] \tag{4}
\]

Where, \( P_k \) and \( P_{\omega} \) are production terms, \( D_k \) is the destruction term, \( F_j \) is blending functions.

2.2.2. \( \gamma \)-Re\( \theta \) transition model: \( \gamma \)-Re\( \theta \) transition model consists of intermittency transport equation and momentum thickness transport equation. The former associates transition property to the local variable, to turn on the production term of the turbulent kinetic energy downstream of the transition point and trigger the transition. The latter is solved to capture the nonlocal influence of the turbulence intensity, as it ties the empirical correlation to the onset criteria in the intermittency equation. It can be described in detail as follows:
\[
\frac{\partial (\rho y)}{\partial t} + \frac{\partial (\rho y U)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \mu_t \right) \frac{\partial y}{\partial x_j} \right] - \frac{\partial (\rho \delta y)}{\partial x_j} - \frac{\partial (\rho \delta y)}{\partial x_j} = P_t - \frac{\partial}{\partial x_j} \left[ \left( \mu + \mu_t \right) \frac{\partial y}{\partial x_j} \right] \]
\]

Where, \( \gamma \) is intermittency, \( P_t \) and \( P_\theta \) are transition sources, \( E_\gamma \) is the destruction/relaminarization term, \( Re_t \) is local transition Reynolds number.

It became apparent during the development of the present transition model that whenever a laminar boundary layer separation occurred, the model consistently predicted the turbulent reattachment location too far downstream, as the free stream turbulence intensity was lowered. Presumably this is because the turbulent kinetic energy, \( k \), in the separating shear layer is smaller at lower free stream turbulence intensities. To correct this deficiency, a modification to intermittency \( \gamma \) of the transition model was introduced that:

\[
\gamma_{\text{re}} = \min \left( \frac{k}{\gamma_{\infty}}, 1 \right) F_{\text{reattach}}^{\gamma^2} F_\theta \quad \gamma_{\text{eff}} = \max \left( \gamma, \gamma_{\infty} \right)
\]

Where, the size of the separation bubble can be controlled with the constant \( s_1 \), \( F_{\text{reattach}} \) is the function of the turbulence Reynolds number and disables the modification once the viscosity ratio is large enough to cause reattachment. In this way, the present intermittency will larger than 1 so that it allows \( k \) to grow rapidly once the laminar boundary layer separates.

2.2.3. \( \gamma \)-Re\( _t \) transition turbulence model. \( \gamma \)-Re\( _t \) transition model is coupled with the standard k-\( \omega \) SST turbulence model, and the k-equation of the standard k-\( \omega \) SST turbulence model is revised as:

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k U)}{\partial x_j} = \bar{\rho}_t - \bar{D}_k + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_k \mu_t \right) \frac{\partial k}{\partial x_j} \right]
\]

Where, \( P_k \) and \( D_k \) are the original production and destruction terms from the turbulent kinetic energy equation.

One further modification to the standard k-\( \omega \) SST turbulence model is a change in the blending function \( F \) responsible for switching between the k-\( \omega \) and k-\( \varepsilon \) models:

\[
R_e = \frac{\rho \sqrt{k}}{\mu} \quad F_1 = e^{\frac{R_e}{120}} \quad F_2 = \max \left( F_{1,\text{orig}}, F_1 \right)
\]

2.3. Numerical description

Numerical predictions are compared with experimental measurements of a pitching NACA66 hydrofoil. The chord length \( c=150\text{mm} \). The computational domain and boundary conditions are given according to the experimental setup in Ref [17], which is shown in Fig. 1. The Reynolds number is \( Re=U_{\infty} c/\nu=7.5\times10^5 \).

![Figure 1](image1.png)

**Figure 1.** Computational domain and boundary conditions.

![Figure 2](image2.png)

**Figure 2.** Fluid mesh details.

![Figure 3](image3.png)

**Figure 3.** Variation of Angle of attack [17].

Using the dynamic mesh technique, the flow around a pitching hydrofoil is studied. The 2D fluid mesh (shown in Fig. 2) is composed of 120,000 elements with 50 structured elements across the foil boundary layer, to ensure \( y^+=y_{\tau}/\nu \approx 1 \), where \( y \) is the thickness of the first cell from the foil surface, and \( u_\tau \) is the wall frictional velocity. The regions outside the boundary layer have been discretized...
with unstructured triangular elements. Mesh refinements are performed at the foil leading edge, trailing edge, and in the wake region.

The transient pitching motion is defined as a single upward-downward motion from 0° to 15°, then back to 0°. The rotation axis is located at the midchord, i.e. $x/c=0.5$. The mean angular velocity is defined as $\dot{\alpha}=6^\circ/s$, which is accordance with the experiment [17]. The variation of the $\alpha$ with time is given in Fig.3. For clarity, the upward phase of the pitching cycle will be denoted with $\alpha^-$ and the downward phase will be denoted with $\alpha^+$.

3. Results and discussion

Figure 4. Comparison of the predicted hydraulic characteristic curve by different turbulence model.

Figure 4 shows the evolution of the lift($C_l=\frac{L}{(0.5\rho U_\infty^2 sc)}$), drag($C_D=\frac{D}{(0.5\rho U_\infty^2 sc)}$), suction side loading($C_{si}\sum C_s(x/c,t)M(x,c)$) and moment($C_m=\frac{M}{(0.5\rho U_\infty^2 sc^2)}$) coefficients with the geometric angle of attack ($\alpha$), where $L$, $D$ and $M$ are respectively the lift, drag, and moment about the quarter chord $x/c=0.25$, with the clockwise direction as positive, $C_s(x/c, t)$ is the suction side loading coefficient at $x/c$, $\tau$ is shear stress at the wall, $\rho_i$ is fluid density, $U_\infty$ is flow velocity, $s$ is the span of foil. Also shown in Figs. 4(a) and (b) are the measured lift coefficients for the same foil at static and measured suction side loading at dynamic [17], which compared well with the numerical predictions during the pitching process. Figure 5 shows the evolution of streamline distribution and $Q$ contours at eight representative times (from $t_1$ to $t_8$, as shown in Fig. 4(a)), where $Q=1/2(|\Omega|^2-|\Sigma|^2)$, $\Omega$ is vorticity tensor, $\Sigma$ is strain rate tensor. $Q>0$ means the rotational motion being dominant in the flow region.

The evolution of hydrofoil response with the angle of attacks can be divided into 5 phases:

(1) initial stage ($\alpha^-=0^\circ$~$4^\circ$): The lift and suction side loading coefficients calculated by the standard $k$-$\omega$ SST turbulence model and the $\gamma$-$Re_0$ transition turbulence model increase almost linearly with $\alpha$ because the flow is quasi-steady and laminar. Drag coefficient increases approximately as a quadratic function of $\alpha$. The moment coefficient is negative in this range because the center of pressure ($x_p/c$) is aft of the quarter chord. The slope of the hydraulic characteristic curves calculated by the $\gamma$-$Re_0$ transition turbulence model is larger and better agreement with the experimental results can be obtained.

(2) transition stage ($\alpha^+=4^\circ$~$6^\circ$): The results predicted by the standard $k$-$\omega$ SST turbulence model are increasing smoothly, while an inflection is observed on the hydraulic characteristic curves predicted by the $\gamma$-$Re_0$ transition turbulence model at time $t_1(\alpha^+=5.5^\circ)$, which is the same with the trend reflected by the experiment. This is due to the transition from laminar to turbulent in the leading edge of the foil, and the laminar separation bubble can be observed at time $t_1$ in the zoomed picture in Fig.5. However, the standard $k$-$\omega$ SST turbulence model cannot capture the transition.

(3) development stage ($\alpha^+=6^\circ$~$13.5^\circ$): In this range, the lift and drag coefficients predicted by the standard $k$-$\omega$ SST turbulence model and the $\gamma$-$Re_0$ transition turbulence model are basically identical and in accordance with the experimental results. As shown in Fig.5, a clockwise ("-") trailing edge vortex develops on the foil suction side. As $\alpha$ increases, the "-" trailing edge vortex expands toward the foil leading edge, which is responsible for the reduction in the slope of the lift and suction side loading coefficient curve, and increases in the drag coefficients. Simultaneously, the moment coefficient move toward zero. It should be noted that both the standard $k$-$\omega$ SST turbulence model and the $\gamma$-$Re_0$ transition
turbulence model can predict the expand process of trailing edge vortex well. In the development stage, compared with the trailing edge vortex, the leading edge vortex formed at the transition stage is weak and the hydraulic characteristic curves are mainly affected by the trailing edge vortex.

(4) dynamic stall stage ($\alpha^*=13.5^\circ$~$\alpha^*=12.5^\circ$): The hydraulic characteristic curves calculated by the standard $k$-$\omega$ SST turbulence model develop slowly, with the lift, suction side loading and moment coefficients increasing with $\alpha$ during the upward process and decreasing with $\alpha$ during the downward process. However, obvious fluctuations can be observed on the hydraulic characteristic curves predicted by the $\gamma$-Re$_0$ transition turbulence model. Even though there exists some discrepancy between the numerical and experimental results, which is probably attributed to the inability of URANS models to accurately capture the flow dynamics when the response is dominated by large-scale transient vortices because of the temporal averaging of the turbulent fluctuations, the general response is well captured by the $\gamma$-Re$_0$ transition turbulence model. As shown in Fig. 5, between $t_3$ and $t_4$ ($\alpha^*=13.5^\circ$~$14.2^\circ$), the lift and suction side loading coefficients increase rapidly while the moment coefficient drops to a high negative value because of the formation and expansion of the leading edge vortex toward the foil trailing edge. At $t_4$ ($\alpha^*=14.2^\circ$), the leading edge vortex develops to the maximum degree and attach to the suction side completely, resulting that the lift and drag coefficients reach a local maxima while the moment coefficient reaches a local minima due to moment stall. The lift and drag coefficients drop rapidly and the moment coefficient increases between $t_4$ and $t_6$ ($\alpha^*=14.2^\circ$~$14.5^\circ$). With the formation and growth of counterclockwise ("+" )trailing edge vortex, the interaction between the leading and trailing edge vortex forces the vortices to shed downstream. The local minima for $C_L$ and maximum for $C_m$ at $t_6$ ($\alpha^*=14.5^\circ$) corresponds to the moment just before the leading edge vortex completely sheds downstream of the foil TE, after which the lift increases again as the "+" trailing edge vortex also sheds downstream followed by the growth of the "-" trailing and leading edge vortex. This behavior is repeated. When the angle of attack is close to the maximum (15°), the leading edge vortex develops rapidly and obvious fluctuations of the hydraulic characteristic curves can be observed due to the dynamic stall caused by the leading edge vortex separation. The numerical results predicted by the standard $k$-$\omega$ SST turbulence model cannot capture the leading edge vortex caused by the transition and the interaction between the leading and trailing edge vortex, so that no fluctuations appear on the hydraulic characteristic curves and the prediction precision of the standard $k$-$\omega$ SST turbulence model is greatly reduced.

(5) recovery stage ($\alpha^*=12.5^\circ$~0°): The evolution of hydraulic characteristic curves in this range are almost the same with that in the first three stage ($\alpha^*=0^\circ$~13°). During the downward pitching process of the foil, the trailing edge vortex gradually shrinks with the angle of attack decreases, and the flow transitions back to laminar.

Figure 5. Comparison of the predicted $Q$ contours and flow streamline predicted by different turbulence models.
In order to clarify the effects of the transition model on the prediction of the flow characteristics, Figure 6 shows the wall friction coefficients predicted by the standard $k-\omega$ SST turbulence model and the $\gamma$-Re$_\theta$ transition turbulence model. It is commonly known that transition appears when the wall friction coefficient reaches the minimum and disappears when it reaches the maximum. According to the Prandtl separation criterion, it defines the laminar separation point at $(\partial u_x/\partial y)_{y=0}=0$, that is $r=\mu(\partial u_x/\partial y)_{y=0}=0$. As shown in Fig. 6, the wall friction coefficient predicted by the standard $k-\omega$ SST turbulence model is not less than 0 during the whole pitching process and it cannot exhibit the flow separation and laminar transition phenomenon. Nevertheless, when the angle of attack is small ($\alpha^+=5.5^\circ$), the wall friction coefficient predicted by the $\gamma$-Re$_\theta$ transition turbulence model drops clearly at leading edge ($x/c=0.05$) of the foil, which is correspond to the flow separation and transition from laminar to turbulent. When the angle of attack reaches $14.2^\circ$, the wall friction coefficient predicted by the $\gamma$-Re$_\theta$ transition turbulence model keeps at a low level, when the transition phenomenon disappear and the flow separates completely, which is due to the vortex developing to the maximum degree and attaching to the whole suction side of foil and dynamic stall occurs.

$\tau=\mu(\partial u_x/\partial y)_{y=0}=0$.

To explain the reason for the $\gamma$-Re$_\theta$ transition turbulence model of predicting the transition exactly, Figure 7 shows the distribution of blending function in the standard $k-\omega$ SST turbulence model and the $\gamma$-Re$_\theta$ transition turbulence model. It is known that the $k-\varepsilon$ model is adopted when the blending function $F_1$ is equal to zero and the $k-\omega$ model is adopted when $F_1$ is equal to one during the calculation process. From the Eqn. 12, one of the modifications to the SST model is a change in the blending function $F_1$ responsible for switching between the $k-\varepsilon$ and $k-\omega$ model. From Fig. 7, we can see that the blending function $F_1$ predicted by the standard $k-\omega$ SST turbulence model is equal to one near the surface and decreases to a value of zero outside the boundary layer. It was found that $F_1$ could potentially switch from one to zero in the center of the laminar boundary layer, as the equations used to define $F_1$ were intended solely for use in turbulent boundary layers.

Also shown in Fig. 7, the $\gamma$-Re$_\theta$ transition turbulence model redefines $F_1$ in terms of a blending function that will always be equal to 1.0 in a laminar boundary layer. At the time of $\alpha^+=5.5^\circ$, transition occurs at the leading edge of the foil and the leading edge vortex is formed. The value of $F_1$ decreases with the turbulence intensity increasing in the downstream of the separation point. With the development of the leading edge vortex and the trailing edge vortex, at the time of $\alpha^+=14.2^\circ$, the flows around the suction side of the foil separate completely and $F_1$ is equal to 1.0 in the suction side region, so that assureing the accurate prediction of the flow details in the laminar boundary layer at leading edge of the foil.

Figure 6. Comparison of the predicted drag coefficients distributions.

Figure 7. Comparison of the predicted values of blending function $F_1$. 
Figures 8 and 9 show the evolution of intermittency and transition onset Reynolds to analyze the transition property of the flow around the pitching foil. Turbulence intermittency is a characteristics presented during the interaction process of the turbulent flow and non-turbulent flow. Assuming a point in the system, the turbulent flow and non-turbulent flow can appear by each time-ratio. It defines the intermittency as the time-ratio of the turbulent flow appearance. \( \gamma = 1 \) respond to turbulent flow and \( \gamma = 0 \) to non-turbulent flow. The intermittency is now set to be equal to one in the free stream and turbulent boundary layer, and zero in the laminar boundary layer. As shown in Fig. 8, at initial time, near the wall in the viscous sublayer, the intermittency is always small because the viscosity ratio is small and as a result the destruction terms (seen in Eqn. 6) of the intermittency equation are active. At time of \( \alpha = 5.5^\circ \), the laminar boundary layer thickness decreases and the local intermittency at leading edge of the foil is larger than 1, which is correspond to the flow separation. For this case, there is also a region at the downstream of the separation point where the intermittency is approximately equal to 0.5. This is caused by the large laminar separation upstream of the reattachment point. Moreover, the intermittency is also larger than 1 at the trailing edge of the foil due to the flow separation caused by the reverse pressure gradient. As the angle of attack increases, the region with the intermittency above 1 moved to the stagnant point at the leading edge of the foil, and the angle between the region and the foil increases. The flow separates completely and the intermittency is equal to 1, with the local turbulent performance being significant.

4. Conclusions

Numerical results are presented for a pitching NACA66 hydrofoil by the standard \( k-\omega \) SST turbulence model and the \( \gamma - Re_0 \) transition turbulence model respectively. The transition property is investigated. The primary findings include:

(1) Compared with the numerical results by the standard \( k-\omega \) SST turbulence model, the \( \gamma - Re_0 \) transition turbulence model can predict the flow structure and hydrodynamic property effectively and capture the flow separation and transition in the boundary layer.
(2) The flow around the foil during the pitching process can be divided into 5 phases: initial stage, transition stage, development stage, dynamic stall stage and recovery stage. The hydrodynamic coefficients are affected by the laminar to turbulent transition at low angles of attack. At high angles of attack, large-scale, low-frequency load fluctuations are observed due to strong interactions between the leading edge vortex and the trailing edge vortex caused by stall. During the downward pitching process, the flow transitions back to laminar.

(3) The transition result in the inflexion on the hydrodynamic characteristic curves, and decrease of the wall friction coefficients. The main reason for the dynamic stall is separation of the leading edge vortex, resulting in large-scale fluctuations on the dynamic characteristic curves.

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