The $W\ell\nu$-vertex corrections to W-boson mass in the R-parity violating MSSM

Min-Di Zheng, Feng-Zhi Chen, and Hong-Hao Zhang

School of Physics, Sun Yat-Sen University, Guangzhou 510275, China

Abstract

Inspired by the astonished 7σ discrepancy between the recent CDF-II measurement and the Standard Model prediction on the mass of $W$-boson, we investigate its unique $\lambda'$-corrections to the vertex in $\mu \to \nu\mu\bar{e}_\nu$ decay in the context of the R-parity violating minimal supersymmetric standard model. This correction can raise the $W$-boson mass independently. Combined with recent $Z$-pole measurements, $m_W \lesssim 80.38$ GeV can be reached. However, this vertex correction cannot explain the CDF result entirely.

1 Introduction

In past decades, the observation of striking agreement between the Standard Model (SM) predictions and the experimental results in a vast of particle interactions has shown up the powerful predicted capacity of the SM. However, the SM is not the final answer to the particle physics, as it is unable to explain several phenomena, such as the matter-antimatter asymmetry, the origin of neutrino mass, the hierarchy problem, the candidate of dark matter, etc. These strongly call for some new physics (NP) beyond the SM. Up to date, although there is still no direct evidence that NP exist, there is still another indirect way, i.e., studying the loop-effects of NP on the low-energy process or electroweak observables, such as the persisting $B$-physics anomalies, the muon anomalous magnetic moment problem, and the very recent precision measurement of $W$-boson mass.
Recently, the Collider Detector at Fermilab (CDF) collaboration at Tevatron reported a high precision measurement on the mass of $W$ gauge boson with the CDF-II detector, the value is given by $m_{W}^{\text{CDF}} = 80.4335 \pm 0.0094$ GeV [1], with the precision which is better than all other previous measurements. The new measurement, $m_{W}^{\text{CDF}}$, is $7\sigma$ away from the SM prediction $m_{W}^{\text{SM}} = 80.357 \pm 0.006$ GeV [2]. Such an astonished tension, if is confirmed in the future, will undoubtedly be a strong challenge to the SM.

Before this profound changes on the situation of NP searches, there are already some anomalies indicating the clues of NP, e.g., the observable $R_{K}$, an ideal place for NP searching in $b \rightarrow s\ell^{+}\ell^{-}$ processes, reported by the LHCb Collaboration [3] that it is $3.1\sigma$ away from the SM prediction. As for the $R_{D^{(*)}}$, the average values [4] reported by the Heavy Flavor Averaging Group, are about $3.1\sigma$ away from the corresponding SM predictions [5–8]. Aimed at these anomalies, there are numbers of models to explain them. Among these models, the minimal supersymmetric standard model (MSSM) extended by the $R$-parity violation, can provide several explanations [9–11], thus, investigation on this model for the $m_{W}^{\text{CDF}}$ explanation is necessary.

Although it is found that MSSM can provide some parameter points which can raise $m_{W}^{\text{MSSM}}$ into the $2\sigma$ accordance region [12], through boson self-energy corrections, the solution requires the lightest squark masses at several hundreds of GeV, which is not suitable to more general scenarios in MSSM or the extended models under the Large Hadron Collider (LHC) searches. Above all, we will study the extra vertex correction from the $R$-parity violating interactions and get the independent enhancement to $m_{W}$, considering the general TeV scale bounds for colored sparticle masses as well as main constraints from $Z$-pole.

2 The contribution to $m_{W}$ from the R-parity violating MSSM

As we know, the $W$-boson mass can be determined from the muon decays with the relation

$$
\frac{m_{W}^{2}}{m_{Z}^{2}} = \frac{1}{2} + \frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_{\mu}m_{Z}^{2}}(1 + \Delta r),
$$

(2.1)
which comprises the three precise inputs, the $Z$-boson mass $m_Z$, the Fermi constant $G_\mu$, and the fine structure constant $\alpha$. Here the loop-corrections are contained in $\Delta r$,

$$\Delta r = \frac{\Sigma^W(0)}{m^2_W} + h^v + h^b + \cdots,$$

(2.2)

where $\Sigma^W(0)$ expresses the self-energy of the renormalized $W$-boson, and the vertex and box corrections to $\mu \to \nu_\mu e\bar{\nu}_e$ are denoted by $h^v$ and $h^b$, respectively. In MSSM, the pure squarks only engage the self-energy sector at one-loop level, and so are the pure slepton loops. The corrections to the vertex and box involve charginos and neutralinos. In MSSM, the dominant contribution to $m_W$ is the one-loop diagrams involving pure squarks. Thus, in this work we focus on the vertex corrections affected by the $\lambda'$ coupling in the $R$-parity violating MSSM (RPV-MSSM).

In this work, we choose the economical type of the $R$-parity violating MSSM which only takes the superpotential, $W = \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k \ (i, j, k = 1, 2, 3)$. Thus, the corresponding Lagrangian in the mass basis is,

$$L^{\text{LQD}} = \lambda'_{ijk} \left( \bar{\nu}_{Li} d_{Rk} d_{Lj} + \bar{d}_{Lj} d_{Rk} \nu_{Li} + \bar{d}^c_{Rk} \nu_{Li} d_{Lj} \right)$$

$$- \tilde{\lambda'}_{ijk} \left( \bar{\nu}_{Li} d_{Rk} u_{Lj} - \bar{u}_{Lj} d_{Rk} \nu_{Li} - \bar{d}^c_{Rk} \nu_{Li} u_{Lj} \right) + \text{h.c.},$$

(2.3)

where the generation indices $i, j, k = 1, 2, 3$, while the colour ones are suppressed and "c" indicates the charge conjugated fermions. The relation between $\lambda'$ and $\tilde{\lambda'}$ is $\tilde{\lambda'}_{ijk} = \lambda'_{ijk} K^*_{ij}$ and $K$ is the Cabibbo–Kobayashi–Maskawa matrix.

![Diagram](image)

Figure 1: The diagrams of $W\ell\nu$ and $Z\ell\ell$. Including the one-loop contribution from RPV-MSSM, the $W\ell_i\nu_{\ell_i}$ vertex then is described
by the following Lagrangian,
\[ \mathcal{L} = \frac{g}{\sqrt{2}} \bar{\ell}_i \gamma^\mu P_L (\delta_{ii} + h_{ii}) \nu_i W^-_\mu + \text{h.c.,} \tag{2.4} \]
where the correction part \( h_{ii} \) contributes to the \( h^r \) term exactly in Eq. (2.2) and the formula of the \( \lambda' \)-contributions is given by
\[ h'_{ii} = -\frac{3}{64\pi^2} x_t f_W(x_t) \tilde{\lambda}'_{a33} \tilde{\lambda}'_{a33}, \tag{2.5} \]
where \( x_t \equiv m_t^2/m_{\tilde{b}_R}^2 \) and the loop function \( f_W(x) \equiv \frac{1}{x-1} + \frac{(x-2) \log x}{(x-1)^2} \) and the non-dominant parts are eliminated. Here we consider the \( \lambda' \)-correction only to the \( W\mu\nu \)-vertex or to the \( We\nu \)-vertex at a time, which can be easily achieved by setting one of the couplings \( (\tilde{\lambda}'_{133}, \tilde{\lambda}'_{233}) \) dominant while neglecting the rest, that is inspired by the methods for explaining the \( B \)-physics anomalies in RPV-MSSM. Given this “single coefficient dominance” scenario, the \( \lambda' \)-corrections to the \( \mu \to \nu_{\mu}e\bar{\nu}_{e} \) box are also vanished, then the one-loop \( \lambda' \)-contribution to \( \Delta r \) only comes from \( h_{aa} \) (the index \( a \) here is restricted to 1 or 2).

Given the purpose of this work is to investigate that: to what degree, the pure RPV contribution\(^1\), \( h'_{aa} \), can accommodate the new \( W \)-boson mass data, we can define,
\[ m_W^{\text{New}} = 80.357 - 15.6387 h'_{aa} = 80.357 + 0.0743 x_t f_W(x_t) \left| \tilde{\lambda}'_{a33} \right|^2. \tag{2.6} \]
Thus, the RH sbottom mass \( m_{\tilde{b}_R} \) and the coupling \( \tilde{\lambda}'_{a33} \) are related to \( \lambda' \)-correction of \( m_W \).

\section{3 Numerical results and discussions}

If we let the pure RPV contribution explain the new \( W \)-boson mass at \( 2\sigma \) level, we need the \( h_{aa} \) fulfil, \(-6.34 < h_{aa} \times 10^3 < -3.47 \), which will induce the ratio,
\[ R_{\text{NP}/\text{SM}}^W = \frac{\Gamma(W\ell_a\nu_a)_{\text{NP}}}{\Gamma(W\ell_a\nu_a)_{\text{SM}}} = 1 + 2h_{aa}, \tag{3.1} \]
\(^1\)There are always contributions from the original MSSM framework, while we can set the sufficient heavy left-handed squarks and sleptons to screen these effects.
Observations | Experimental data
--- | ---
$R_{\mu/e}^W = \Gamma(W \rightarrow \mu\nu)/\Gamma(W \rightarrow e\nu)$ | $0.996 \pm 0.008$
$R_{\tau/\mu}^W = \Gamma(W \rightarrow \tau\nu)/\Gamma(W \rightarrow \mu\nu)$ | $1.070 \pm 0.026$
$R_{\tau/e}^W = \Gamma(W \rightarrow \tau\nu)/\Gamma(W \rightarrow e\nu)$ | $1.043 \pm 0.024$
$R_{\mu/e}^Z = \Gamma(Z \rightarrow \mu\mu)/\Gamma(Z \rightarrow e\nu)$ | $1.0001 \pm 0.0024$
$R_{\tau/\mu}^Z = \Gamma(Z \rightarrow \tau\tau)/\Gamma(Z \rightarrow \mu\mu)$ | $1.0010 \pm 0.0026$
$R_{\tau/e}^Z = \Gamma(Z \rightarrow \tau\tau)/\Gamma(Z \rightarrow e\nu)$ | $1.0020 \pm 0.0032$

Table 1: Current status of $W/Z$-boson partial width ratios.

in the region,

$$-12.7 < (R_{NP/SM}^W - 1) \times 10^3 < -6.9.$$  \hspace{1cm} (3.2)

Comparing this with the experimental results [14] of the $W$-related partial width ratio shown in table 1, one can see this bound, Eq. (3.2) makes stronger constraints than these recent measurements at $2\sigma$ level, whenever the NP is in the $\mu$ or $e$ channel. Then Eq. (3.2) induces the following bound,

$$0.7313 < x_t f_W(x_t) \left| \tilde{\lambda}^l_{a33} \right|^2 < 1.3357.$$  \hspace{1cm} (3.3)

The effects on the $W\ell_R\ell_L$-vertex will also affect $Z$-pole measurements, also shown in table 1, which constrain stringently the parameters $m_{b_R}, \tilde{\lambda}^l_{a33}$ contained in the coupling $g_{\ell_L}$, within the effective Lagrangian,

$$\mathcal{L}_\text{eff} = g_{\ell_L} \bar{\ell}_i \gamma^\mu \left[ g^{ij}_{\ell_L} P_L + g^{ij}_{\ell_R} P_R \right] \ell_j Z^\mu,$$  \hspace{1cm} (3.4)

where the exact formula of $g^{ij}_{\ell_L}$ is

$$(32\pi^2) g^{ij}_{\ell_L} = 3 \lambda^l_{j33} \lambda^r_{333} \{ -x_{b_R} (1 + \log x_{b_R}) + \frac{m_Z^2}{18 m_{b_R}^2} \left[ (11 - 10 \sin^2 \theta_W) + (6 - 8 \sin^2 \theta_W) \log x_{b_R} + \frac{1}{10} (-9 + 16 \sin^2 \theta_W) \frac{m_Z^2}{m_t^2} \right] \}.  \hspace{1cm} (3.5)$$

Here we can define $B^{ij} \equiv (32\pi^2) g^{ij}_{\ell_L}$ and get the bound $|B^{aa}| < 0.35$ at $2\sigma$. Then the region combining the new $W$-boson mass explanations and this bound is shown in figure 2.

One can see that in the allowed region of partial width ratio at $2\sigma$ level, the $m_W^{\text{New}}$ can
Figure 2: The regions of $(m_{bR}, \lambda'_{233})$ for the pure RPV contributions explaining the $m_W^{\text{CDF}}$ at $2(3)\sigma$ are denoted by dark (light) blue area, and the bound from $R_{Z\ell/\ell'}^Z$ at $2(3)\sigma$ level is denoted by the green area with dark (light) opacity. The dashed lines express the $m_W^{\text{New}}$ value (GeV) enhanced by vertex corrections.

be raised to around 80.38 GeV at most, while it cannot reach the value to explain the $m_W^{\text{CDF}}$ sufficiently. Even at $3\sigma$ level for both the observables, there are still none common areas besides the extremely narrow ones when $m_{bR} \approx 600$ GeV, however, this mass scale is already excluded by LHC searches in the vast majority of scenarios. So we find that, the pure RPV effects on the vertex cannot solve this $m_W$ problem, unless along with other contributions, like the oblique corrections from squarks [12, 15].

4 Conclusions

In this paper, inspired by the astonished $7\sigma$ discrepancy between the recent CDF-II measurement and the SM prediction on the mass of $W$-boson, we performed a phenomenological analysis on the muon decays which are relevant to the $W$ mass under the framework of RPV-MSSM, and to access whether such a deviation can be accommodated by this NP model. We focused on the one-loop corrections to the the vertex of $\mu \rightarrow \nu_e e\nu_e$ decay with the assumption that the vertex correction only affected by the $\lambda'$ coupling in the RPV-MSSM. Our constraint result from the muon decays, i.e. Eq. (3.3) is stringent than the recent measurement at $2\sigma$ level. As the $W_{\mu\nu}/We\nu$-vertex also affect the $Z$-pole measurement, which would lead to a situation shown...
in figure 2. This implies that the RPV-MSSM is hard to accommodate the CDF measurement.

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