Operatorial quantization of Born-Infeld Skyrmion model and hidden symmetries

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Abstract

The SU(2) collective coordinates expansion of the Born-Infeld Skyrmion Lagrangian is performed. The classical Hamiltonian is computed from this special Lagrangian in approximative way: it is derived from the expansion of this non-polynomial Lagrangian up to second-order variable in the collective coordinates. This second-class constrained model is quantized by Dirac Hamiltonian method and symplectic formalism. Although it is not expected to find symmetries on second-class systems, a hidden symmetry is disclosed by formulating the Born-Infeld Skyrmion model as a gauge theory. To this end we developed a new constraint conversion technique based on the symplectic formalism. Finally, a discussion on the role played by the hidden symmetry on the computation of the energy spectrum is presented.

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1 Introduction

The presence of symmetries in a constrained dynamical model reveals important physical contents of a given system. In particular, we can cite the energy spectrum that is an observable quantity invariant under gauge transformations. In views of this, some authors have proposed some processes to convert second-class systems to first-class ones. In this paper, we are interested in investigating this subject employing a new technique based on the symplectic procedure, called symplectic gauge-invariant method. To clarify our proposal we will quantize a Skyrme-like model, discussed in this work.

The Skyrme model is an effective field theory for baryons and their interactions. These hadronic particles are described from soliton solutions in the non-linear sigma model. Normally, in this Lagrangian, it is necessary to add the Skyrme term to stabilize the soliton solutions. In principle, the Skyrme term is arbitrary and there is not a concrete reason to fix it through a particular choice. Its importance resides in the fact that, maybe, it is the most simple possible quartic derivative term that we need insert in the Hamiltonian in order to obtain soliton solutions. However, it is possible avoid this ambiguity by adopting a nonconventional Lagrangian also based in a non-linear sigma model, given by

\[ L = -\frac{F_\pi}{16} \int d^3r \left[ Tr \partial_\mu U \partial^\mu U^+ \right]^2, \tag{1} \]

where \( F_\pi \) is the pion decay constant and \( U \) is a SU(2) matrix. This model, based on the ideas of Born-Infeld Electrodynamics, was proposed by Deser, Duff and Isham. The existence of soliton solution might be observed.

\footnote{According to Derrick scale theorem.}
by applying Derrick’s theorem in the static Hamiltonian derived from the
Lagrangian (1).

The main goal of this paper is to quantize the Born-Infeld Skyrmion
model through the symplectic gauge-invariant method and then to display
the role played by the hidden symmetry on the computation of the energy
spectrum. To this end, this model is expanded in terms of the collective rota-
tional coordinates and, subsequently, formulated as a gauge invariant theory.
Implementing the semi-classical approach after the usual collective canonical
quantization, the spin and isospin modes are obtained producing quantum
corrections to the baryons properties[8]. This process reduces the SU(2)
Skyrme model to a nonrelativistic particle constrained over a $S^3$ sphere, a
well known second-class problem[9][10]. Afterwards, the SU(2) Skyrme model
expanded in terms of the collective rotational coordinates[8] is quantized via
the Dirac Hamiltonian method[11] and symplectic formalism[12, 13], which
allows us to compute the field dependent Dirac’s brackets among the physi-
cal coordinates, assumed to be commutators at quantum level. We observe
that when we keep the non-causality sector of the soliton solution influencing
the physical values as minimum as possible, the commutators obtained are
the same of the Skyrme model. At this level, problems involving operator
ordering ambiguities[14, 15] arise, which can be avoided just formulating this
model as a gauge invariant theory. At this stage we unveil a hidden symme-
try of the model, that is an unexpected result since this nonlinear model is
originally a second-class model.

For the sake of self consistency, this paper was organized as follows. In
section 2, we propose the Born-Infeld Skyrmion model, obtaining the clas-
sical Hamiltonian by an approximative way from the expansion of the non-
polynomial Lagrangian up to second-order variable in the collective coordi-
nates since we take into account some causality arguments. In section 3, the second-class model will be quantized via Dirac and symplectic methods, where we demonstrate that the computed Dirac’s brackets are the same ones obtained for the Skyrme model\cite{16,17}. In section 4, the Born-Infeld Skyrme model will be reformulated as a gauge theory via symplectic gauge-invariant method. In this section we will also investigate the hidden symmetry lying on the original phase-space coordinates. In order to corroborate the previous results, we investigate in section 5 the hidden symmetry via the gauge unfixing Hamiltonian method\cite{18}. In section 6, the role played by the symmetry on the computation of the energy spectrum will be explored. The last section is dedicated to the discussion of the physical meaning of our findings together with our final comments and conclusions. In appendices A and B, we will present brief reviews about the new constraint conversion procedure, namely, the symplectic gauge-invariant formalism\cite{2}, and the gauge unfixing Hamiltonian formalism, respectively.

2 The Born-Infeld Skyrmion model

The dynamic system will be given performing the SU(2) collective semiclassical expansion\cite{8}. Substituting $U(\vec{r},t)$ by $A(t)U(\vec{r})A^+ (t)$ in (1), where $A$ is a SU(2) matrix, we obtain

$$L = -F_\pi \int d^3 r \left[ m - J Tr (\partial_0 A \partial_0 A^{-1}) \right]^{3/2}, \quad (2)$$

where $m$ and $J$, identified as being the soliton mass and the inertia moment\cite{8}, respectively, are functionals written in terms of the chiral angle $F(r)$, which satisfies the topological boundary conditions, $F(0) = \pi$ and $F(\infty) = 0$. Here, we use the Hedgehog ansatz for $U$, i.e., $U = \exp(i\tau \cdot \vec{r} F(r))$. The SU(2) matrix
A can be written as $A = a_0 + ia_i \tau_i$, where $\tau_i$ are the Pauli matrices, leading to the constraint

$$\sum_{i=0}^{i=3} a_ia_i = 1.$$  \hspace{1cm} (3)

The Lagrangian (2) can be written as a function of the $a_i$ as

$$L = -\frac{F_\pi}{16} \int d^3r [m - 2I \dot{a}_i \dot{a}_i]^2.$$ \hspace{1cm} (4)

From the Eq. (4) we can obtain the conjugate momenta, given by

$$\pi_i = \frac{\partial L}{\partial \dot{a}_i} = \frac{3F_\pi}{16} \dot{a}_i \int d^3r I [m - 2I \dot{a}_k \dot{a}_k]^{\frac{1}{2}}.$$ \hspace{1cm} (5)

The algebraic expression for the Hamiltonian is obtained applying the Legendre transformation, $H = \pi_i \dot{a}_i - L$. However, in some situations, due to the momenta expression given in Eq.(3), it is not possible to write the conjugate Hamiltonian corresponding to the Born-Infeld Skyrmion Lagrangian in terms of $\pi_i$ and $a_i$. An alternative procedure is to expand the original Lagrangian (3) in collective coordinates. Thus, considering the binomial expansion variable $\frac{1}{m} \dot{a}_i \dot{a}_i$, the Lagrangian sum is given by

$$L = -M + A(\dot{a}_i \dot{a}_i) + B(\dot{a}_i \dot{a}_i^2) + \ldots ,$$ \hspace{1cm} (6)

where $M = \frac{F_\pi}{16} \int d^3r \frac{m^2}{4}$, $A = \frac{3F_\pi}{16} \int d^3r I \sqrt{m}$, $B = \frac{3F_\pi}{32} \int d^3r \frac{I^2}{\sqrt{m}}$, and etc. In this step we would like to give physical argument that justifies this procedure. Even though not being a relativistic invariant model, we hope that

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2 In the context of semi-classical expansion, it is expected that the product of $\dot{a}_i \dot{a}_i$ by the expression $\frac{L}{m}$ given by the Euler-Lagrange equation, does not considerably modify this result.
the experimental results can be reproducible with a good accuracy when the soliton velocity is much smaller than the speed of light. From the relation given in ref.\[19\], $A^+ \partial_0 A = i/2 \sum_{k=1}^{3} \tau_k \omega_k$, where $\omega_k$ is the uniform soliton angular velocity, it is possible to show that $Tr[\partial_0 A \partial_0 A^+] = 2 \dot{a}_i \dot{a}_i = \omega^2/2$. If we require that the soliton rotates with velocity smaller than $c$, then $\omega r \ll 1$, leading to $\dot{a}_i \dot{a}_i = \frac{\omega^2}{4} \ll 1$, and consequently $\dot{a}_i \dot{a}_i \ll 1$ for all space. Thus, these results explain our procedure.

In this manner, the Hamiltonian is obtained by using the Legendre transformation

$$
H = \pi_i \dot{a}_i - L
= M + A(\dot{a}_i \dot{a}_i) - 3B(\dot{a}_i \dot{a}_i)^2 + \ldots.
$$

(7)

Obtaining the canonical momenta from Eq. (6), then writing the Lagrangian as $L = \pi_i \dot{a}_i - H$, and comparing with the expansion of the Lagrangian (6), it is possible to derive the expression of the Hamiltonian (7) as

$$
H = M + \alpha \pi_i \pi_i + \beta (\pi_i \pi_i)^2 + \ldots,
$$

(8)

with $\alpha = \frac{1}{4A}$ and $\beta = \frac{B}{16A^2}$. We will truncate the expression (8) in the second-order variable$^3$, and we will use this approximate Hamiltonian to perform the quantization.

$^3$ Due to the equation(5) together with the fact that $\dot{a}_i \dot{a}_i \ll 1$, we expect that terms like $(\pi_i \pi_i)^3$ or higher order degree do not alter our conclusion about the commutators of the quantum Born-Infeld Skyrmion.
3 Operatorial quantization at the second-class level

In this section the reduced Born-Infeld Skyrmion model will be quantized using the Dirac method [11] and the symplectic formalism [12, 13]. In order to apply the Dirac second-class Hamiltonian method, we need to look for secondary constraints, which can be calculated from the following Hamiltonian

$$H_T = M + \alpha \pi_i \pi_i + \beta (\pi_i \pi_i)^2 + \lambda_1 (a_i a_i - 1), \quad (9)$$

where $\lambda_1$ is a Lagrangian multiplier. Using the iterative Dirac formalism we get the following second-class constraints

$$\phi_1 = a_i a_i - 1 \approx 0, \quad (10)$$
$$\phi_2 = a_i \pi_i \approx 0. \quad (11)$$

After straightforward computations, the Dirac brackets among the phase spaces variables are obtained as

$$\{a_i, a_j\}^* = 0,$$
$$\{a_i, \pi_j\}^* = \delta_{ij} - a_i a_j, \quad (12)$$
$$\{\pi_i, \pi_j\}^* = a_j \pi_i - a_i \pi_j.$$

Through the well known canonical quantization rule $\{, \}^* \rightarrow -i [, ]$, we get the commutators

$$[a_i, a_j] = 0,$$
These results show that the quantum commutators of the reduced Born-Infeld Skyrmion model are equal to the Skyrme model\cite{16,17} when the Lagrangian is expanded up to the second-order term of the collective coordinates. This completes the Dirac’s quantization process.

To implement the symplectic quantization procedure\cite{13}, let us consider the zeroth-iterative first-order Lagrangian

\[
L^{(0)} = \pi_i \dot{a}_i - V^{(0)},
\]

where the potential \(V^{(0)}\) is

\[
V^{(0)} = M + \alpha \pi_i \pi_i + \beta (\pi_i \pi_i)^2 + \lambda (a_i a_i - 1),
\]

with the enlarged symplectic variables given by \(\xi^{(0)}_\alpha = (a_j, \pi_j, \lambda)\). The symplectic tensor is

\[
f^{(0)} = \begin{pmatrix}
0 & -\delta_{ij} & 0 \\
\delta_{ij} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

where the elements of rows and columns follow the order: \(a_i, \pi_i, \lambda\). The matrix above is obviously singular, then it has a zero-mode that generates the following constraint

\[
\Omega^{(1)} = a_i a_i - 1 \approx 0,
\]

where the potential \(V^{(0)}\) is given by Eq.\,(13). Taking the time derivative of this constraint and introducing the result into the previous Lagrangian by means of a Lagrange multiplier \(\rho\), we get a new Lagrangian \(L^{(1)}\)
\[ L^{(1)} = (\pi_i + \rho a_i)\dot{a}_i - V^{(1)}, \quad (18) \]

where

\[ V^{(1)} = M + \alpha \pi_i \pi_i + \beta (\pi_i \pi_i)^2. \quad (19) \]

The matrix \( f^{(1)} \) is then

\[
f^{(1)} = \begin{pmatrix}
0 & -\delta_{ij} & -a_i \\
\delta_{ij} & 0 & 0 \\
a_i & 0 & 0
\end{pmatrix}, \quad (20)
\]

where rows and columns follow the order: \( a_i, \pi_i, \rho \). The matrix \( f^{(1)} \) is singular so it has a zero-mode, given by

\[
v^{(1)} = \begin{pmatrix}
0 \\
a_i \\
-1
\end{pmatrix}, \quad (21)
\]

that produces the constraint

\[ \Omega^{(2)} = a_i \pi_i \approx 0. \quad (22) \]

Here we must mention that these constraints, Eqs.(17) and (22), derived by the symplectic procedure are the same one obtained when the Dirac formalism is used. Following the iterative symplectic process, we get the new Lagrangian \( L^{(2)} \), given by

\[
L^{(2)} = (\pi_i + \rho a_i + \eta \pi_i)\dot{a}_i + \eta a_i \dot{\pi}_i - V^{(2)}, \quad (23)
\]

where \( V^{(2)} = V^{(1)} \). The new enlarged symplectic variables are \( \xi^{(2)}_\alpha = (a_j, \pi_j, \rho, \eta) \), where \( \rho \) and \( \eta \) are Lagrange multipliers. The corresponding symplectic matrix \( f^{(2)} \) is
The matrix \( f^{(2)} \) is not singular, then it is identified as the symplectic tensor of the constrained theory. The inverse of \( f^{(2)} \) gives the same Dirac brackets among the physical coordinates given in Eq.(13).

### 4 Gauging the Born-Infeld Skyrmion model

The Born-Infeld Skyrme model is a noninvariant model with field dependent Dirac’s brackets among the phase-space variables. Due to this, the quantization of the model is affected by operator ordering ambiguity. To overcome this problem at the commutator level, the model will be reformulated as a gauge invariant model. In this section, we will use the symplectic gauge-invariant method proposed in section 4. To implement this scheme, the second-order Lagrangian that governs the dynamics of the Born-Infeld Skyrmion model is reduced to its first-order form and an extra term \( G(a_i, \pi_i, \theta) \) is introduced into the first-iterative Lagrangian, namely,

\[
L^{(1)} = \pi_i \dot{a}_i + (a_i^2 - 1) \dot{\eta} - V^{(1)},
\]

where \(-\lambda \to \dot{\eta}\) and with \(V^{(1)}\) as

\[
V^{(1)} = V^{(0)}|_{(a^2 - 1 = 0)} = M + \frac{1}{4A} \pi_i^2 + \frac{B}{16A^4} \pi_i^4 - G(a_i, \pi_i, \theta),
\]

The symplectic variables are \(\xi_\alpha^{(1)} = (a_i, \pi_i, \eta, \theta)\) and the extra term, given by

\[
G(a_i, \pi_i, \theta) = \sum_n \mathcal{G}^{(n)}(a_i, \pi_i, \theta),
\]
satisfies the boundary condition,
\[ G(a_i, \pi_i, \theta = 0) = G^{(0)}(a_i, \pi_i, \theta = 0) = 0. \]  
(28)

The corresponding symplectic matrix, computed as
\[ f^{(1)} = \begin{pmatrix} 0 & -\delta_{ij} & 2a_i & 0 \\ -\delta_{ij} & 0 & 0 & 0 \\ -2a_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \]  
(29)

is singular and has a zero-mode,
\[ \nu^{(1)} = (0 \ a_i \ \frac{1}{2} \ 1). \]  
(30)

Following the prescription of the symplectic gauge-invariant formalism, giving in appendix A, the gauge invariant Lagrangian obtained after fourth iteration is
\[ L^{(1)} = \pi_i \dot{a}_i + (a_i^2 - 1)\dot{\theta} - V^{(1)}_{(4)}, \]  
(31)

where the symplectic potential \( V^{(1)}_{(4)} \) is identified as being the invariant Hamiltonian
\[ H = M + \frac{1}{4A}a_i^2 + \frac{B}{16A^4} \tilde{\pi}_i^4 - \left( \frac{1}{2A} + \frac{B}{4A^4} \pi_i^2 \right) (a_i \pi_i) \theta \\
+ \left( a_i^2 + a_i^2 \frac{B}{A^3} (a_i \pi_i)^2 + \frac{B}{2A^3} a_i^2 \pi_i^2 \right) \frac{\theta^2}{4A} - \frac{B}{4A^4} (a_i \pi_i) a_i \theta^3 \\
+ \frac{B}{16A^4} a_i^4 \theta^4. \]  
(32)

To complete our gauge invariant reformulation, the infinitesimal gauge transformation is also computed. In agreement with the symplectic method, the
zero-mode $\tilde{\nu}^{(1)}$ is the generator of the infinitesimal gauge transformation ($\delta O = \varepsilon \tilde{\nu}^{(1)}$),

\begin{align*}
\delta a_i &= 0, \\
\delta \pi_i &= \varepsilon a_i, \\
\delta \lambda &= \dot{\varepsilon}, \\
\delta \theta &= \varepsilon,
\end{align*}

where $\varepsilon$ is an infinitesimal time-dependent parameter.

At this stage, some gauge fixing schemes will be implemented following the symplectic method that allows us to reveal a new and remarkable result. First, we require that $\chi_1 = \theta = 0$ (unitary gauge) that reduces the gauge invariant model to the original model with the same Dirac’s brackets among the phase-space variables $(a_i, \pi_i)$. The other one is

$$\chi_2 = \lambda = 0.$$  

(34)

With this gauge we have another noninvariant description for the nonlinear model with canonical Dirac’s brackets. In fact, the first-order Lagrangian becomes

$$L^{(1)} = \pi_i \dot{a}_i + \lambda \dot{\rho} - V^{(1)}_{(4)},$$

(35)

where the symplectic variables are $\xi^{(1)}_{\alpha} = (a_i, \pi_i, \lambda, \rho, \theta)$ and $V^{(1)}_{(4)}$ is

$$V^{(1)}_{(4)} = V^{(0)}_{(4)}|_{\lambda=0} = M + \frac{1}{4A} \pi_i^2 + \frac{B}{16A^4} \pi_i^4 - \left( \frac{1}{2A} + \frac{B}{4A^4 \pi_i^2} \right) (a_i \pi_i) \theta + \left( a^2 + \frac{B}{A^3} (a_i \pi_i)^2 + \frac{B}{2A^3} a^2 \pi_i^2 \right) \frac{\theta^2}{4A} - \frac{B}{4A^4} (a_i \pi_i) a^2 \theta^3,$$
The corresponding symplectic matrix, computed as

$$f^{(1)} = \begin{pmatrix} 0 & -\delta_{ij} & 0 & 0 & 0 \\ \delta_{ij} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

is singular and has a zero-mode that produces a new constraint,

$$\Omega_2 = \frac{1}{2A} (a_i \pi_i) + \frac{3B}{4A^4} (a_i \pi_i) a^2 \theta^2 - \frac{B}{4A^4} a^4 \theta^3 + \frac{B}{4A^4} \pi_i^2 (a_i \pi_i) - \frac{1}{2A} a_i^2 \theta - \frac{B}{2A^4} (a_i \pi_i)^2 \theta - \frac{B}{4A^4} \pi^2 a^2 \theta.$$  \hfill (38)

With the introduction of this constraint into the first-order Lagrangian \((L^{(2)})\), we have

$$L^{(2)} = \pi_i \dot{a}_i + \lambda \dot{\rho} + \Omega_2 \dot{\beta} - V^{(2)}_{(4)},$$

where \(V^{(2)}_{(4)} = V^{(1)}_{(4)} |_{\Omega_2=0}\). After that, a nonsingular symplectic matrix is set up and the Dirac’s brackets among the phase-space variables are identified as

$$\{a_i, a_j\}^* = 0, \quad \{a_i, \pi_j\}^* = \delta_{ij}, \quad \{\pi_i, \pi_j\}^* = 0.$$ \hfill (40)

From \(\Omega_2 = 0\), the \(\theta\) variable can be determined as
\[ \theta = \frac{1}{a^2} \left( a_i \pi_i + \frac{B}{2A^3} \pi_i^2 (a_i \pi_i) \right). \]  (41)

Bringing back this result into the symplectic potential \( V(2) = V(1) \big|_{\Omega_2=0} \) and collecting terms up to \( \pi_i^4 \), we obtain the following Hamiltonian,

\[ H = V(2) = M + \frac{1}{4A} \pi_i M_{ij} \pi_j + \frac{B}{16A^4} (\pi_i M_{ij} \pi_j)^2, \]  (42)

with the singular matrix \( M_{ij} \) defined as

\[ M_{ij} = \delta_{ij} - a_i a_j. \]  (43)

This Hamiltonian will be used to perform the computation of the energy spectrum. At this stage it is important to notice that the noninvariant model has a hidden symmetry that could not be detected by the symplectic method, due to the inexistence of a gauge generator. In spite of this, the Hamiltonian (42) is invariant under the gauge infinitesimal transformations (33), because the matrix \( M_{ij} \) has an eigenvector with eigenvalue null,

\[ a_i M_{ij} = 0. \]  (44)

In the next section, the hidden symmetry will be investigated using the gauge unfixing method.

5 The gauge invariant Born-Infeld Skyrmion model

In this section the hidden symmetry which underlies in the Born-Infeld Skyrmion model will be disclosed using the gauge unfixing Hamiltonian method [18], reviewed in the appendix. This model has a set of second-class
constraints, given in (10) and (11), that produces the nonvanishing Poisson bracket

\[ C = \{ \phi_1, \phi_2 \} = 2a_i a_i = 2. \]  

(45)

To obtain the first-class Hamiltonian in a systematic way we follow closely the procedure described in [20]. Initially, the set of constraints are redefined as

\[ \xi = C^{(-1)} \phi_1 = \frac{1}{2} a_i a_i - \frac{1}{2}, \]
\[ \psi = \phi_2. \]  

(46)

that generates the canonical Poisson bracket

\[ \{ \xi, \psi \} = 1. \]  

(47)

Subsequently, the Lagrange multipliers \( u_a \) with \( a = 1, 2 \) which appears in the total Hamiltonian

\[ H = H_c + u_1 \xi + u_2 \psi, \]  

(48)

with

\[ H_c = M + \alpha \pi_i \pi_i + \beta (\pi_i \pi_i)^2, \]  

(49)

are determined as

\[ u_1 = -2\alpha \pi_i^2 - 4\beta (\pi_i^2)^2, \]
\[ u_2 = -2\alpha a_i \pi_i - 4\beta (a_i \pi_i) \pi_i^2, \]  

(50)
just imposing that the constraints has no time evolution. To obtain the first-class system, we maintain only $\xi$ as gauge generator. At first, $\{\xi, H\} \neq 0$, i.e., $\xi$ and $H$, in principle, do not satisfy a first-class algebra. Thus, the first-class Hamiltonian can be given by the following formula\textsuperscript{20}

$$
\tilde{H} = H - \psi\{\xi, H\} + \frac{1}{2!}\psi^2\{\xi, \{\xi, H\}\} - \frac{1}{3!}\psi^3\{\xi, \{\xi, \{\xi, H\}\}\} + \ldots,
$$

(51)

with

$$
H = M + \alpha\pi_i\pi_i + \beta(\pi_i\pi_i)^2 - 2\alpha(a_i\pi_i)^2 + 4\beta(a_i\pi_i)^2(\pi_j\pi_j),
$$

(52)

satisfying the first-class algebra

$$
\{\xi, \tilde{H}\} = 0.
$$

(53)

At this point, we start to compute each term of the Hamiltonian \textsuperscript{(51)}. The first one is

$$
\{\xi, H\} = -2\alpha(a_i\pi_i) + 12\beta(a_i\pi_i)(\pi_i\pi_i) + 8\beta(a_i\pi_i)^3,
$$

(54)

while the second and the remaining ones are given by

$$
\begin{align*}
\{\xi, \{\xi, H\}\} &= -2\alpha + 12\beta(\pi_i\pi_i) + 48\beta(a_i\pi_i)^2, \\
\{\xi, \{\xi, \{\xi, H\}\}\} &= 120\beta(a_i\pi_i), \\
\{\xi, \{\xi, \{\xi, \{\xi, H\}\}\}\} &= 120\beta.
\end{align*}
$$

(55)

The gauge invariant Hamiltonian is obtained as
\[ \tilde{H} = M + \alpha(\pi_i\pi_i) + \beta(\pi_i\pi_i)^2 - 2\beta(a_i\pi_i)^2(\pi_j\pi_j) - \alpha(a_i\pi_i)^2 + \beta(a_i\pi_i)^4 \]
\[ = M + \frac{1}{4A}\pi_i M_{ij}\pi_j + \frac{B}{16A_4}(\pi_i M_{ij}\pi_j)^2, \]

with the singular matrix \( M_{ij} \) given in Eq.(43).

It is easy to show that the gauge invariant Hamiltonian satisfies the non-involutive algebra

\[ \{\xi, \tilde{H}\} = 0. \]

Due to this, the constraint \( \xi \) is the gauge symmetry generator of the infinitesimal transformation

\[ \delta a_i = \{a_i, \xi\} = 0, \]
\[ \delta \pi_i = \{\pi_i, \xi\} = -\varepsilon a_i, \]

with \( \varepsilon \) as an infinitesimal time-dependent parameter. Note that the Hamiltonian (56) is invariant under this infinitesimal gauge transformation because \( a_i \) are eigenvectors of the phase-space metric \( M_{ij} \) with null eigenvalues \((a_i M_{ij} = 0)\).

6 The energy spectrum

In this section, we will derive the energy levels. Normally, these results were employed to obtain the baryons physical properties[8]. In this context, our perturbative approach plays an important role on the computation of the
energy spectrum since the quartic term presenting in the Hamiltonian leads to an extra term.

In the second-class formalism the energy spectrum is obtained calculating the mean value of the quantum Hamiltonian, which reads as

\[ E = \langle \psi | \tilde{H} | \psi \rangle, \]  

where \( \tilde{H} = M + \alpha \pi_i \pi_i + \beta (\pi_i \pi_i)^2 \). \( \tilde{H} \) is the quantum version of the second-class Hamiltonian, Eq. (8). The eigenvectors of the quantum Hamiltonian \( \tilde{H} \) are \( |\psi\rangle = \frac{1}{N(t)} (a_1 + ia_2)^l = |\text{polynomial}\rangle \). These wave functions are also eigenvectors of the spin and isospin operators, written in reference [8] as

\[ J_k = \frac{1}{2}(a_0 \pi_k - a_k \pi_0 - \epsilon_{klm} a_l \pi_m) \quad \text{and} \quad I_k = \frac{1}{2}(a_k \pi_0 - a_0 \pi_k - \epsilon_{klm} a_l \pi_m). \]

The expression for \( \pi_i \), satisfying the commutation relations, Eqs. (13), is given by

\[ \pi_i = \frac{1}{i} (\partial_i - a_i a_j \partial_j). \]  

The algebraic expression for \( \pi_i \) presents operator ordering problems. A possible choice, following the prescription of Weyl ordering [21] (symmetrization procedure) is given by

\[ [p_i]_{\text{sym}} = \frac{1}{6i} (6 \partial_i - a_i a_j \partial_j - a_i \partial_j a_j - a_j a_i \partial_i - a_j \partial_j a_i - \partial_j a_i a_j - \partial_j a_j a_i) \]

\[ = \frac{1}{i} \left( \partial_i - a_i a_j \partial_j - \frac{5}{2} a_i \right). \]  

Consequently, \( \pi_j \pi_j \) symmetrized can be written as
\[ [\pi_j \pi_j]_{\text{sym}} = -\partial_j \partial_j + \frac{1}{2} \left( OpOp + 2Op + \frac{5}{4} \right), \quad (62) \]

where \( Op \) is defined as \( Op \equiv a_i \partial_i \). The symmetrized second-class Hamiltonian operator is

\[ [\tilde{H}]_{\text{sym}} = M + \alpha \left( -\partial_j \partial_j + \frac{1}{2} \left( OpOp + 2Op + \frac{5}{4} \right) \right) + \beta \left( -\partial_j \partial_j + \frac{1}{2} \left( OpOp + 2Op + \frac{5}{4} \right) \right)^2. \quad (63) \]

Substitution of the expression \((63)\) in the mean value, \((59)\), leads to the energy levels

\[ E_l =_{\text{phys}} \langle \psi | \tilde{H} | \psi \rangle_{\text{phys}} = M + \alpha \left( l(l+2) + \frac{5}{4} \right) + \beta \left( l(l+2) + \frac{5}{4} \right)^2. \quad (64) \]

Notice that these energy levels have a quartic extra term, indicating some modifications on the calculation of the physical parameters, previously obtained in the context of the SU(2) Skyrme model\cite{8}. Furthermore, we remark that the adopted ordering scheme produces a constant value on the energy levels formula. It is an important subject since different ordering schemes can lead to distinct physical results, as pointed out in Refs.\cite{15, 17}.

In the first-class scenario the quantum Hamiltonian is

\[ \tilde{H} = M + \alpha(\pi_i \pi_i) + \beta(\pi_i \pi_i)^2 - 2\beta(a_i \pi_i)^2(\pi_j \pi_j) - \alpha(a_i \pi_i)^2 + \beta(a_i \pi_i)^4. \quad (65) \]

\footnote{Note that the eigenvalues of the operator Op are defined by the following equation: \( Op|\text{polynomial} \rangle = l|\text{polynomial} \rangle \).}
$\tilde{H}$ is the quantum version of the first-class Hamiltonian, Eqs. (12). The quantization is performed, following the prescription of the Dirac method [11], imposing that the physical wave functions are annihilated by the first-class operator constraint

$$\phi_1 |\psi\rangle_{phys} = 0,$$ (66)

where $\phi_1$ is

$$\phi_1 = a_ia_i - 1.$$ (67)

The physical states that satisfy (66) are

$$|\psi\rangle_{phys} = \frac{1}{V} \delta(a_ia_i - 1) |\text{polynomial}\rangle,$$ (68)

where $V$ is the normalization factor. Thus, in order to obtain the spectrum of the theory, we take the scalar product, $\langle \text{phys} | \tilde{H} | \psi\rangle_{phys}$, that is the mean value of the first-class Hamiltonian. We begin by calculating the scalar product

$$\text{phys} \langle \psi | \tilde{H} | \psi\rangle_{phys} = \langle \text{polynomial} | \frac{1}{V^2} \int da_i \delta(a_ia_i - 1) \tilde{H} \delta(a_ia_i - 1) |\text{polynomial}\rangle,$$ (69)

where $\tilde{H}$ is defined in Eqs. (65). Note that due to $\delta(a_ia_i - 1)$ in (69) the scalar product can be simplified. Then, integrating over $a_i$, we obtain
\[ \langle \text{polynomial} | M + \alpha (\pi_i \pi_i) + \beta (\pi_i \pi_i)^2 - 2\beta (a_i \pi_i)^2 (\pi_j \pi_j) - \alpha (a_i \pi_i)^2 \\
+ \beta (a_i \pi_i)^4 | \text{polynomial} \rangle. \quad (70) \]

Here we would like to comment that the regularization delta function squared \( \delta (a_i a_i - 1)^2 \) is performed using the delta relation, \((2\pi)^2 \delta (0) = \lim_{k \to 0} \int d^2 x e^{ik \cdot x} = \int d^2 x = V. \) In this manner, the parameter \( V \) is used as the normalization factor. The Hamiltonian operator inside the kets, Eq.(70), can be rewritten as

\[ \langle \text{polynomial} | M + \alpha [p_k \cdot p_k] + \beta [p_k \cdot p_k]^2 | \text{polynomial} \rangle, \quad (71) \]

where \( p_k = \pi_k - a_k (a_j \pi_j) \). The operators \( \pi_k \) describe a free particle. Then, the \( p_k \) operators are identical to the canonical momenta obtained for the second-class theory. Consequently, the algebraic expression for the quantum Hamiltonian inside the scalar product(71) is the same obtained in a second-class theory, Eq.(63), naturally leading to the same energy levels, Eq.(64).

This important result shows the equivalence between the second-class collective coordinates Skyrme model and its first-class version.

7 Conclusions

The Born-Infeld Skyrmion Lagrangian has a nonconventional structure that allows to stabilize the soliton solutions without adding higher derivative order terms. However, due to the nonanalytical structure of Born-Infeld Skyrmion
Lagrangian, a perturbative treatment becomes necessary. The expansion of the non-polynomial Born-Infeld Lagrangian in terms of dynamic variables is possible if we pay attention to the problem of breaking the relativistic invariance in the collective coordinates expansion. The contributions to the physical parameters due to the non-causal soliton solution have no physical relevance if the chiral angle $F(r)$ satisfies the relation, $\lim_{r \to \infty} F(r) = 0$, together with the fact that we impose that the soliton angular velocity be small, i.e., $\omega \ll 1$. In views of this, the perturbative expression for the Hamiltonian could be truncated at the quartic-order term in the canonical momenta.

To obtain the quantum structure for the Born-Infeld Skyrmion model, the Dirac Hamiltonian method and the Lagrangian symplectic formalism were used. In these contexts, we verified that all constraints are second-class and the symplectic matrix is nonsingular, showing that is not expected to find symmetries. Afterward, the quantum commutators of the model were computed. These results are the same ones obtained for the conventional SU(2) Skyrme model. In spite of this, the energy levels with a quartic correction term together with an operator ordering scheme can change the baryons static properties, previously obtained for the usual SU(2) Skyrme model.

In order to disclose the hidden symmetry, two different approaches were used: the symplectic gauge-invariant scheme and the unfixing Hamiltonian formalism. The usual directions[22, 23, 24] point out the enlargement of the phase space with WZ variables, and consequently the symmetries arise. In our work, those symmetries are revealed only on the original phase space, where the quantum structure was obtained using the Dirac first-class procedure. In this scenario, the energy levels were computed, reproducing the spectrum of the original second-class system. Our findings point out the
consistency of those first-class conversion procedures and propose the gauge invariant version for the Born-Infeld Skyrmion model, dynamically equivalent to the usual SU(2) Skyrme model.

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A The symplectic gauge-invariant method

There are several schemes to reformulate noninvariant models as gauge invariant theories in the literature. However, recently, some constraint conversion formalisms, based on the Dirac’s method [11], were developed using Faddeev’s idea of phase-space extension with the introduction of auxiliary variables [22]. Among them, the Batalin-Fradkin-Fradkina-Tyutin(BFFT) [23] and the iterative [24] methods were powerful enough to be successfully applied to a great number of important physical models. Although these techniques share the same conceptual basis [22] and treat constrained systems following the Dirac process [11], the implementation of the constraint conversion methods are different. Historically, both BFFT and the iterative methods were applied in linear systems, such as chiral gauge theories [24, 25], in order to eliminate the gauge anomaly that hampers the quantization process. In spite of the great success achieved by these methods, some ambiguities which appear in the constraint conversion process make these iterative methods a hard task to implement [26]. It happens because these formalisms are based on the Dirac’s
method. In this section, we reformulate noninvariant systems as gauge invariant theories using a new technique which is not affected by those ambiguities problems. This technique follows the Faddeev suggestion\[22\] and is set up on a contemporary framework to handle noninvariant model, namely, the symplectic formalism[12, 13].

In order to systematize the symplectic gauge-invariant formalism, a general noninvariant mechanical model whose dynamics is governed by a Lagrangian $\mathcal{L}(a_i, \dot{a}_i, t)(\text{with } i = 1, 2, \ldots, N)$ is considered, where $a_i$ and $\dot{a}_i$ are the space and velocities variables respectively. Notice that this consideration does not lead to the loss of generality or of physical content. Following the symplectic method, the first-order Lagrangian, written in terms of the symplectic variables $\xi^{(0)}_\alpha(a_i, p_i)$ (with $\alpha = 1, 2, \ldots, 2N$), reads as

$$\mathcal{L}^{(0)} = A^{(0)}_\alpha \dot{\xi}^{(0)}_\alpha - V^{(0)},$$

(72)

where $A^{(0)}_\alpha$ is the one-form canonical momenta, $(0)$ indicates that it is the zeroth-iterative Lagrangian and, $V^{(0)}$, the symplectic potential. After that, the symplectic tensor, defined as

$$f^{(0)}_{\alpha\beta} = \frac{\partial A^{(0)}_\beta}{\partial \xi^{(0)}_\alpha} - \frac{\partial A^{(0)}_\alpha}{\partial \xi^{(0)}_\beta},$$

(73)

is computed. Since this symplectic matrix is singular, it has a zero-mode ($\nu^{(0)}$) that generates a new constraint when contracted with the gradient of potential, namely,

$$\Omega^{(0)} = \nu^{(0)}_\alpha \frac{\partial V^{(0)}}{\partial \xi^{(0)}_\alpha}.$$

(74)
Through a Lagrange multiplier $\eta$, this constraint is introduced into the zeroth-iterative Lagrangian (72), generating the next one

$$\mathcal{L}^{(1)} = A^{(0)}_\alpha \dot{\xi}^{(0)}_\alpha - V^{(0)} + \eta \Omega^{(0)},$$

$$= A^{(1)}_\alpha \dot{\xi}^{(1)}_\alpha - V^{(1)}, \quad (75)$$

where

$$V^{(1)} = V^{(0)}|_{\Omega^{(0)}=0},$$

$$\xi^{(1)}_\alpha = (\xi^{(0)}_\alpha, \eta),$$

$$A^{(1)}_\alpha = A^{(0)}_\alpha + \eta \frac{\partial \Omega^{(0)}}{\partial \xi^{(0)}_\alpha}. \quad (76)$$

The first-iterative symplectic tensor is computed and since this tensor is nonsingular, the iterative process stops and the Dirac’s brackets among the phase-space variables are obtained from the inverse matrix. On the contrary, if the tensor is singular, a new constraint arises and the iterative process goes on.

After this brief review, the symplectic gauge formalism will be systematized. It starts after the first iteration with the introduction of an extra term dependent on the original and Wess-Zumino(WZ) variable, $G(a_i, p_i, \theta)$, into the first-order Lagrangian. This extra term, expanded as

$$G(a_i, p_i, \theta) = \sum_{n=0}^{\infty} G^{(n)}(a_i, p_i, \theta), \quad (77)$$

where $G^{(n)}(a_i, p_i, \theta)$ is a term of order $n$ in $\theta$, satisfies the following boundary condition,

$$G(a_i, p_i, \theta = 0) = G^{(n=0)}(a_i, p_i, \theta = 0) = 0. \quad (78)$$
The symplectic variables are extended to contain also the WZ variable $\tilde{\xi}_a^{(1)} = (\tilde{\xi}_a^{(0)}, \eta, \theta)$ (with $\tilde{a} = 1, 2, \ldots, 2N + 2$) and the first-iterative symplectic potential becomes

$$
\tilde{V}^{(1)}_n(a, p, \theta) = V^{(1)}(a, p) - \sum_{n=0}^{\infty} \mathcal{G}^{(n)}(a, p, \theta).
$$

For $n = 0$, we have

$$
\tilde{V}^{(1)}_{(n=0)}(a, p, \theta) = V^{(1)}(a, p).
$$

Subsequently, we impose that the symplectic tensor $(f^{(1)})$ be a singular matrix with the corresponding zero-mode

$$
\tilde{\nu}_{\tilde{a}}^{(1)} = \begin{pmatrix} \nu_{\tilde{a}}^{(1)} & 1 \end{pmatrix},
$$

as the generator of gauge symmetry. Due to this, all correction terms $\mathcal{G}^{(n)}(a, p, \theta)$ in order of $\theta$ can be explicitly computed. Contracting the zero-mode $(\tilde{\nu}_{\tilde{a}}^{(1)})$ with the gradient of potential $\tilde{V}^{(1)}_{(n)}(a, p, \eta, \theta)$ and imposing that no more constraints are generated, the following differential equation is obtained

$$
\tilde{\nu}_{\tilde{a}}^{(1)} \frac{\partial \tilde{V}^{(1)}_{(n)}(a, p, \theta)}{\partial \tilde{\xi}_{\tilde{a}}^{(1)}} = 0, \quad \nu_{\tilde{a}}^{(1)} \frac{\partial V^{(1)}(a, p)}{\partial \xi_{\tilde{a}}^{(1)}} - \sum_{n=0}^{\infty} \frac{\partial \mathcal{G}^{(n)}(a, p, \theta)}{\partial \theta} = 0,
$$

that allows us to compute all correction terms in order of $\theta$. For linear correction term, we have

$$
\nu_{\tilde{a}}^{(1)} \frac{\partial V^{(1)}_{(n=0)}(a, p)}{\partial \xi_{\tilde{a}}^{(1)}} - \frac{\partial \mathcal{G}^{(n=1)}(a, p, \theta)}{\partial \theta} = 0.
$$
while for quadratic one,

\[ \nu^{(1)}_\alpha \frac{\partial V^{(1)}_{(n=1)}(a_i, p_i, \theta)}{\partial \xi^{(1)}_\alpha} - \frac{\partial G^{(n=2)}(a_i, p_i, \theta)}{\partial \theta} = 0. \]  \hspace{1cm} (84)

From these equations, a recursive equation for \( n \geq 1 \) is proposed as

\[ \nu^{(1)}_\alpha \frac{\partial V^{(1)}_{(n=1)}(a_i, p_i, \theta)}{\partial \xi^{(1)}_\alpha} - \frac{\partial G^{(n)}(a_i, p_i, \theta)}{\partial \theta} = 0, \]  \hspace{1cm} (85)

that allows us to compute each correction term in order of \( \theta \). This iterative process is repeated successively until the equation (82) becomes identically null, consequently, the term \( G(a_i, p_i, \theta) \) is obtained explicitly. At this stage, the gauge invariant Hamiltonian, identified as being the symplectic potential, is obtained as

\[ \tilde{H}(a_i, p_i, \theta) = V^{(1)}_{(n)}(a_i, p_i, \theta) = V^{(1)}(a_i, p_i) + G(a_i, p_i, \theta), \]  \hspace{1cm} (86)

and the zero-mode \( \tilde{\nu}^{(1)}_\alpha \) is identified as being the generator of an infinitesimal gauge transformation

\[ \delta \tilde{\xi}_\alpha = \varepsilon \tilde{\nu}^{(1)}_\alpha, \]  \hspace{1cm} (87)

where \( \varepsilon \) is an infinitesimal time-dependent parameter.

**B The gauge unfixing Hamiltonian formalism**

The main idea of the unfixing gauge procedure is to consider half of the total second-class constraints as gauge fixing terms \([20, 27]\) and the remaining as gauge generators of symmetry. To obtain a first-class Hamiltonian in a
systematic way we follow closely the procedure described by Vytheeswaran in [20]. To start we consider a system with two second-class constraints, \( \phi_1 \) and \( \phi_2 \), where the Poisson bracket is

\[
C = \{ \phi_1, \phi_2 \}.
\]  

(88)

Using this relation and redefining the second-class constraints as

\[
\xi \equiv C^{-1} \phi_1,
\]

\[
\psi \equiv \phi_2,
\]

(89)

we have

\[
\{ \xi, \psi \} = 1 + \{ C^{-1}, \psi \} C \xi,
\]

(90)

so that \( \xi \) and \( \psi \) are canonically conjugate on the surface defined by \( \xi = 0 \).

The total Hamiltonian is

\[
H = H_c + u_1 \xi + u_2 \psi.
\]

(91)

To obtain the first-class system, we maintain only \( \xi \) as a constraint relation. At first, \( \{ \xi, H \} \neq 0 \), i.e., \( \xi \) and \( H \), in principle, do not satisfy a first-class algebra. Thus, the first-class Hamiltonian can be expressed by the formula given in [20], reads as

\[
\tilde{H} = H - \psi \{ \xi, H \} + \frac{1}{2!} \psi^2 \{ \xi, \{ \xi, H \} \} - \frac{1}{3!} \psi^3 \{ \xi, \{ \xi, \{ \xi, H \} \} \} + \ldots,
\]

(92)

which satisfies the first-class condition.
\{\xi, \tilde{H}\} = 0. \quad (93)

The first-class Hamiltonian $\tilde{H}$ can be elegantly rewritten in a projection equation form, given by

$$\tilde{H} = PH \equiv \exp^{-\psi} : H,$$

(94)

with $\psi$ respecting the ordering rule, that is, it should come before the Poisson bracket. The procedure described above is an outline of a formalism that converts a second-class system into first-class one without enlargement of the phase space.

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