The OPEs of spin-4 Casimir currents in the holographic \( SO(N) \) coset minimal models

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Abstract
The operator product expansion (OPE) between the spin-4 current and itself in the \( WD_4 \) coset minimal model was calculated using \( SO(8) \) current algebra. The right-hand side of this OPE contains the spin-6 Casimir current, which is also a generator of the \( WD_4 \) coset minimal model. Based on this \( N=8 \) result, the above OPE was generalized for the general \( N \) (in the \( WD_{2N} \) coset minimal model) using the two \( N \)-generalized coupling constants initiated by Hornfeck, which is the simplest OPE for the lowest higher spin currents. The similar OPE in the \( WB_3 \) (and \( WB_{N-1/2} \)) coset minimal model was also analyzed with the \( SO(7) \) current algebra. The large \( \Lambda 't \) Hooft limits are discussed. The results in two-dimensional conformal field theory provide asymptotic symmetry at the quantum level of the higher spin AdS3 gravity reported by Chen et al.

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1. Introduction
In the Gaberdiel and Gopakumar proposal [1, 2], the \( WAN_{N-1} \) minimal model conformal field theory is dual in the \( 't \) Hooft expansion to the higher spin theory of Vasiliev on the AdS3 coupled to one complex scalar. The higher spin gauge fields in bulk AdS3 couple to conserved higher spin currents (whose charges form an extended global symmetry of the conformal field theory) in the boundary theory. See recent review papers for more details [3, 4].

The \( SU(N) \) spin-3 Casimir construction in the \( WAN_{N-1} \) minimal model (described in terms of a coset) is reported in [5]. The four independent cubic terms are comprised of the spin-1 currents in the two factors in the numerator of the coset. The operator product expansion (OPE) between the spin-3 current and itself generates a spin-4 current [6], which consists of quartic and quadratic terms with derivatives in terms of the above spin-1 currents. Furthermore, the \( SO(N) \) spin-4 Casimir construction in the \( WD_{2N} \) and \( WB_{N-1/2} \) minimal models [7] is reported in detail in [8] with the observations of [9, 10]. The quartic terms, cubic terms with one derivative and quadratic terms with two derivatives in the spin-4 current (with two unknown coefficient.
functions), are given in terms of the spin-1 currents in the two factors in the numerator of the coset. Therefore, it is important to know what happens when the OPE between the spin-4 current and itself is calculated.

In this paper, the OPE between the spin-4 Casimir current and itself, the simplest OPE for the lowest higher spin currents, in the $WD_2$ coset minimal model with $N = 8$ is calculated. During this computation, the spin-6 Casimir current arises on the right-hand side of this OPE and the two unknown coefficients in the spin-4 current are fixed. The aim was to generalize the above OPE for the general $N$ using two $N$-generalization coupling constants \[11\]. The similar OPE in the $WB_{2\times 1}$ coset minimal model was also analyzed.

Reference \[11\] did not answer what types of field contents for $WD_4$ (or $WB_3$) algebra are present. This study considered the field contents for the higher spin currents in the specific coset model. In \[12\], they started with the most general ansatz for the OPEs between the spin-4, spin-6 and spin-8 currents, and determined the various structure constants using the Jacobi identities between these higher spin currents. This can be classified by approach 1 in the context of \[13\]. With the identification of the above algebra as a Drinfeld–Sokolov reduction (can be described as approach 2 in \[13\]) with a higher spin algebra, they applied to the coset model by comparing the central charge and self-coupling constant of the spin-4 current. On the other hand, it is unclear how one can see the explicit form for the higher spin currents on the coset model. Because approach 3 in \[13\] used in this study is based on the specific model and the higher spin currents are made of the fields in the coset model, the zero-mode eigenvalue equation can be analyzed, which is necessary to describe the three-point function with a real scalar, as reported in \[8, 14, 15\]. Note that the zero modes satisfy the commutation relations of the underlying finite-dimensional Lie algebra.

According to the results reported in \[13, 16\], the additional currents and $WA_{N-1}$ (or $WD_2^\perp$) currents with arbitrary levels are expected to appear. As one of the levels is equal to 1, this extra current in the OPE disappears completely. In the spirit of \[3, 17\], one can think of more general algebra rather than conventional $WA_{N-1}$ (or $WD_2^\perp$) algebra, i.e. the existence of additional higher spin currents. One of the levels in this general coset model is not equal to 1. Hence, the isomorphism between the coset construction and Drinfeld–Sokolov reduction cannot be used, as explained in \[12\] because this isomorphism restricts to one of the levels as 1 \[13\]. This suggests that for this general coset model with levels $(k, l)$, the procedure developed in \[12\] cannot be followed directly\(^1\). The presentation in this paper will provide some insights to describe the general coset model with levels $(k, l)$ in two-dimensional CFT.

Section 2 reviews the diagonal coset minimal models and describes the spin-2 current with the central charge.

In section 3, the spin-6 current is constructed in the $WD_2^\perp$ minimal model with $N = 8$ by taking the OPE between the spin-4 currents. By realizing the presence of two structure constants, which depend on the general $N$, the OPE between the spin-4 currents, which is valid for any $N$ can be given.

\(^1\) In other words, the direct construction using the Jacobi identity (without knowing the realization of the higher spin currents) is not connected directly to the Casimir (coset) construction. This suggests that although one has the self-coupling constant for the spin-4 current from the work reported in \[11, 12\], the coset construction itself (a realization in the specific coset model) can identify a complete set of generating currents that will have greater symmetry. Once the lower higher spin currents are determined using the coset construction explicitly, the next undetermined higher spin currents can be generated, in principle, by calculating the OPEs between the known higher spin currents repeatedly (i.e. by analyzing the various singular terms). In general, this procedure will be quite complicated. In contrast to approach 1 (which can be done only if one knows the number of currents with given spins), the resulting extended algebra via approach 3 is associative by construction. Therefore, there is no need to check the Jacobi identities separately. This more general coset model and its AdS3 gravity dual which will be beyond the scope of this paper is a good topic for future research.
Section 4 describes the similar OPE in the \( WB_{N-1} \) minimal model.

Section 5 summarizes the findings and comments on future directions.

The appendices, for convenience, present the relative coefficient functions in terms of the two undetermined ones for each minimal model. These were reported previously in [8].

During this preparation, it was found that the work of [12] overlaps somewhat with this work although they did not consider the explicit realizations.

This study used the package reported by Thielemans [18].

2. The GKO coset construction: review

For the diagonal coset model

\[
\frac{G}{H} = \frac{\widehat{SO}(N)_k \oplus \widehat{SO}(N)}{SO(N)_{k+1}},
\]

the spin-1 fields, \( J^{ab}(z) \) with level 1 and \( K^{ab}(z) \) with level \( k \), generate the affine Lie algebra \( G \) where the indices of the finite-dimensional Lie algebra \( SO(N) \) are given by \( a, b = 1, 2, \ldots, N \). Their OPEs [19] are

\[
J^{ab}(z)J^{cd}(w) = \frac{-1}{(z-w)^2} \left( -\delta^{bc}\delta^{ad} + \delta^{ac}\delta^{bd} \right) + \frac{1}{(z-w)} \left[ \delta^{bc}J^{ad}(w) + \delta^{ad}J^{bc}(w) - \delta^{ac}J^{bd}(w) - \delta^{bd}J^{ac}(w) \right] + \cdots,
\]

and

\[
K^{ab}(z)K^{cd}(w) = \frac{-1}{(z-w)^2} \left( -\delta^{bc}\delta^{ad} + \delta^{ac}\delta^{bd} \right) + \frac{1}{(z-w)} \left[ \delta^{bc}K^{ad}(w) + \delta^{ad}K^{bc}(w) - \delta^{ac}K^{bd}(w) - \delta^{bd}K^{ac}(w) \right] + \cdots.
\]

The spin-1 field, \( J^{ab}(z) \) with level \( k+1 \), which generates the affine Lie subalgebra \( H \), can be expressed as

\[
J^{ab}(z) = J^{ab}(z) + K^{ab}(z).
\]

The Sugawara stress–energy tensor for the coset (2.1) with (2.4), can be written as

\[
T(z) = -\frac{1}{4(N-1)}(J^{ab}J^{ab})(z) - \frac{1}{4(k+N-2)}(K^{ab}K^{ab})(z) + \frac{1}{4(k+N-1)}(J^{ab}J^{ab})(z).
\]

The OPE between the spin-2 currents (2.5) can be expressed as

\[
T(z)T(w) = \frac{1}{(z-w)^2} \frac{c}{2} + \frac{1}{(z-w)^2} 2T(w) + \frac{1}{(z-w)} \partial T(w) + \cdots.
\]

The central charge in the highest singular term in (2.6) can be given by the following:

\[
c = \frac{N}{2} \left[ 1 - \frac{(N-2)(N-1)}{(N+k-2)(N+k-1)} \right] \leq \frac{N}{2}.
\]

The higher spin Casimir currents of spin 4 in the \( WD_{\frac{N}{2}} \) and \( WB_{N-1} \) minimal models were constructed in [8]. The subsequent sections will calculate the OPEs between these spin-4 currents in each minimal model.
3. The OPE between the spin-4 current and itself in the $WD_N^2$ minimal model with even $N$

In [8], the spin-4 current was determined with two unknown coefficient functions, as follows:

$$V(z) = c_3 f^{ef} J^{ef} K^{ef} (z) + c_3 f^{ef} J^{ef} K^{ef} (z) + c_9 f^{ef} K^{ef} K^{ef} (z) + c_9 f^{ef} K^{ef} K^{ef} (z) + c_{10} K^{ef} K^{ef} K^{ef} (z) + c_{10} K^{ef} K^{ef} K^{ef} (z) + c_{11} J^{ef} J^{ef} J^{ef} (z) + c_{11} J^{ef} J^{ef} J^{ef} (z) + c_{12} J^{ef} J^{ef} J^{ef} (z) + c_{12} J^{ef} J^{ef} J^{ef} (z) + c_{13} f^{ef} f^{ef} f^{ef} (z) + c_{13} f^{ef} f^{ef} f^{ef} (z) + c_{14} f^{ef} f^{ef} f^{ef} (z) + c_{14} f^{ef} f^{ef} f^{ef} (z) + c_{15} f^{ef} f^{ef} f^{ef} (z) + c_{15} f^{ef} f^{ef} f^{ef} (z) + c_{16} f^{ef} f^{ef} f^{ef} (z) + c_{16} f^{ef} f^{ef} f^{ef} (z) + c_{17} f^{ef} f^{ef} f^{ef} (z) + c_{17} f^{ef} f^{ef} f^{ef} (z) + d_1 J^{ef} J^{ef} J^{ef} (z) + d_1 J^{ef} J^{ef} J^{ef} (z) + d_2 J^{ef} J^{ef} J^{ef} (z) + d_2 J^{ef} J^{ef} J^{ef} (z) + d_3 J^{ef} J^{ef} J^{ef} (z) + d_3 J^{ef} J^{ef} J^{ef} (z) + d_4 J^{ef} J^{ef} J^{ef} (z) + d_4 J^{ef} J^{ef} J^{ef} (z),$$

(3.1)

where the coefficient functions are given in (A.1). First consider the particular $N = 8$ case in the $WD_N^2$ minimal model [11]. Calculating the OPE between this spin-4 current (3.1) and itself is straightforward for the given basic OPEs from (2.2) and (2.3). The final results are presented in detail as follows:

$$V(z) W(w) = \frac{1}{(z - w)^6} \left[ \frac{1}{4} + \frac{1}{2} \frac{2T(w)}{(z - w)^2} + \frac{1}{10} \frac{2\partial T(w)}{(z - w)} + \frac{1}{36} \frac{2\partial T(w)}{(z - w)^3} \right]$$

$$+ \frac{1}{3 \cdot 2 \cdot 5 \cdot 10} \frac{2\partial T(w)}{(z - w)^3} \left[ \frac{3}{20} \partial T + \frac{42}{(5c + 22)} \left( T^2 - \frac{3}{10} \partial T \right) + C_{44} V \right] (w)$$

$$+ \frac{1}{3 \cdot 2 \cdot 5 \cdot 10} \frac{2\partial T(w)}{(z - w)^3} \left[ \frac{1}{30} \partial T + \frac{1}{2} \frac{42}{(5c + 22)} \partial T + \frac{1}{36} C_{44} \partial T \right] (w)$$

$$+ \frac{1}{3 \cdot 2 \cdot 5 \cdot 10} \frac{2\partial T(w)}{(z - w)^3} \left[ \frac{1}{168} \partial T + \frac{5}{36} \frac{42}{(5c + 22)} \partial T + \frac{9}{36} C_{44} \partial T \right] (w)$$

$$+ \frac{1}{3 \cdot 2 \cdot 5 \cdot 10} \frac{2\partial T(w)}{(z - w)^3} \left[ \frac{1}{28} \frac{2\partial T(w)}{(7c + 13)} \right]$$

$$+ \frac{1}{3 \cdot 2 \cdot 5 \cdot 10} \frac{2\partial T(w)}{(z - w)^3} \left[ \frac{1}{54} \frac{2\partial T(w)}{(7c + 13)} \right]$$

$$+ \frac{1}{3 \cdot 2 \cdot 5 \cdot 10} \frac{2\partial T(w)}{(z - w)^3} \left[ \frac{1}{2} \frac{2\partial T(w)}{(7c + 13)} \right]$$

$$+ \frac{1}{3 \cdot 2 \cdot 5 \cdot 10} \frac{2\partial T(w)}{(z - w)^3} \left[ \frac{1}{2} \frac{2\partial T(w)}{(7c + 13)} \right]$$

$$+ \frac{1}{3 \cdot 2 \cdot 5 \cdot 10} \frac{2\partial T(w)}{(z - w)^3} \left[ \frac{1}{2} \frac{2\partial T(w)}{(7c + 13)} \right]$$

(3.2)

In this case, the currents are given by the spin-4, spin-6 and the spin-2 currents. Moreover, there was one additional spin-4 current. For a general $N$, this last current has a spin-$(\frac{N}{2})$. This suggests that for large $N$ behavior, the OPEs between the lower higher spins in the $WD_N^2$ minimal model do not contain this spin-$(\frac{N}{2})$ field. On the other hand, for the $WD_4$ minimal model, the extra spin-4 current does not appear in the OPE between the other spin-4 currents. This can be explained by the existence of outer $Z_2$ automorphism [12, 20] under which the spin-2, spin-4 and spin-6 currents are even, whereas the extra spin-4 current is odd. For general $N$, the 'orbifold' subalgebra of $WD_N^2$ [21] is generated by the quadratic term in the extra spin-$(\frac{N}{2})$ (of spin $N$) as well as its higher derivative terms, in addition to the above spin-2, spin-4, . . . , spin-$(N - 2)$.
First consider the highest singular term. From the explicit structure of the eighth-order pole, one obtains the following expression:

\[ 192k(2 + k)(4 + k)(6 + k)(7 + k)(9 + k)(11 + k)(13 + k)c_{10}^2. \]  

By normalizing this expression (3.3) to be equal to \( \xi \) with

\[ c_{N=8} = \frac{4k(13 + k)}{(6 + k)(7 + k)} \]  \hspace{1cm} (3.4)

coming from the central charge (2.7), the unknown coefficient \( c_{10} \) can be determined as follows:

\[ c_{10}^2 = \frac{1}{192(2 + k)(4 + k)(6 + k)(7 + k)^2(9 + k)(11 + k)} \times \frac{(-4 + c)^2}{130,056 192(24 + c)(22 + 5c)}. \]  \hspace{1cm} (3.5)

The coefficient \( c_{10} \) is also expressed in terms of the central charge (3.4) for convenience. Therefore, the central term is given by \( \frac{3}{4} \), as shown in (3.2).

The next singular term, seventh pole, does not have any nonzero fields. In the sixth-order pole, there is a spin-2 current \( T(w) \) with coefficient 2, which is a structure constant between the spin-4, spin-4 and spin-2 currents. In the fifth-order pole, the descendant field \( \partial T(w) \) of \( T(w) \) can be obtained with the coefficient function \( \frac{1}{2} \times 2 = 1 \), where the relative coefficient function \( \frac{1}{2} \) is known from the spins of the current \( V(z) \), \( W(z) \), \( T(w) \) and the number of derivatives in the descendant field [13, 17, 22, 23].

In the fourth-order pole, a descendant field originating from the spin-2 current \( T(w) \) appears in the higher order singular term with a known coefficient, \( \frac{3}{20} \times 2 = \frac{3}{10} \). Furthermore, there is a spin-4 quasi-primary field as well as the spin-4 primary field \( V(w) \) by remembering the OPE of the spin-4 currents in the extended conformal algebra [23–27], which are denoted by \( \cal{W}(2,4) \) along the line of [13], where the higher spin current is of spin-4.3 The fourth-order pole is given by the one in (3.2), and the self-coupling constant for the spin-4 can be expressed as

\[ (C_{44}^4)^2 = \frac{12(4 + k)(9 + k)}{(2 + k)(11 + k)} = \frac{18(c + 24)}{(5c + 22)}, \]  \hspace{1cm} (3.6)

where \( C_{44}^4 \) can be expressed in terms of the central charge. The remaining unknown coefficient function \( c_k \) can also be determined as follows:

\[ c_{10} = \frac{(6 + k)}{105} (7 + k)^2(4 + k)(9 + k)(148 + k(13 + k) + (4 + k)(133 + k(16 + k)))c_{10} \]  \hspace{1cm} (3.7)

with (3.5). Of course, the structure constant (3.6) is different from the corresponding one in the \( \cal{W}(2,4) \) algebra as shown in [13]. For the \( \cal{W}_d \) algebra, the field contents are given by the spin-4, spin-4 and spin-6 currents, as stated in footnote 2, whereas the \( \cal{W}(2,4) \) algebra

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3 If the field contents in the fourth-order pole are not known, the method reported in [17] can be followed. The OPE between the spin-2 current \( T(z) \) and the fourth-order pole subtracted by \( \frac{3}{100} \partial^2 T(w) \) can be performed with focus on the fourth-order pole. One has the nonzero \( T(w) \) term on the right-hand side of the OPE. This suggests that one can consider the extra quasi-primary field containing the \( T^2(w) \) term with the derivative term because the OPE between \( T(z) \) and \( T^2(w) \) provides a term \( T(w) \) in the fourth-order pole. The coefficient \( -\frac{3}{100} \) in the derivative term can be checked by vanishing of the third-order pole in the OPE with \( T(z) \). Calculating the OPE between \( T(z) \) and the fourth-order pole subtracted by \( \frac{3}{100} \partial^2 T(w) \) is simple. Once again, the primary field condition fixes the constant as \( c_1 = \frac{3}{5c + 22} \). Of course, one should calculate the OPE between \( T(z) \) and \( \partial^2 T(w) \) explicitly to obtain this result.
contains only the spin-4 current. The above structure constant (3.6) coincides with the general N-dependent structure constant found by Hornfeck by substituting $N = 8$ with

\[
(C_{44}^4)^2 = \frac{n}{d},
\]

\[
n \equiv 18[2c^2(-18 + (-2 + N)N) + 2N(-28 + N(5 + 6N)) + 3c(-8 + N(80 + N(-49 + 6N)))],
\]

\[
d \equiv (22 + 5c)(c + (-5 + c)N + 4N^2)(c(-4 + N)(-3 + N) + N(-5 + 2N)) \times (2c(2 + N) + (-4 + N)(-2 + 3N)).
\]

(3.8)

Recently, this expression was reproduced in [12]. In the third-order pole, there are no additional spin-5 quasi-primary fields; there are one descendant field coming from $T(w)$ and two descendant fields coming from the spin-4 quasi-primary and primary fields.

Consider the next second-order singular term$^4$. A spin-6 primary field, which is a generator of the $W_{D_2}$ minimal model, is expected. The first line of the second-order pole in (3.2) is known and the remaining three terms are characterized by three spin-6 quasi-primary fields$^5$. Although the $V$-independent terms are characterized by $\delta^4 T(w)$, $\delta^2 T^{2}(w)$, $T^{3}(w)$ and $\delta T \delta T(w)$ (these are all possible spin-6 fields coming from the spin-2 current $T(w)$), it is very important to split the two descendant terms and two quasi-primary fields to determine the new quasi-primary fields for the given pole. These also arise in the $V(2, 4)$ extended conformal algebra. Finally, spin-6 primary field remains, where

\[
C_{44}^6 W(z) = \text{coeff}(k)J^{12}J^{12}J^{12}J^{12}(z) + \cdots,
\]

and coeff$(k)$ is a complicated function of $k$, which is not present here.

To determine the normalization of the spin-6 current, the highest singular term should be calculated from the OPE between the current (3.9) and itself. On the other hand, it is not possible to do this because the spin-6 current has too many terms. By demanding that this central term be equal to $\tilde{z}$, it is expected that the normalization factor would be obtained as follows:

\[
(C_{44}^6)^2 = \frac{6(-1 + k)(14 + k)(24 + 5k)(41 + 5k)}{(-6 + k)(13 + k)(119 + 4k(13 + k))} = \frac{12(c - 1)(11c + 656)}{(2c - 1)(7c + 68)}.
\]

(3.10)

This structure constant (3.10) was taken from the more general $N$-dependent expression reported by Hornfeck

$^4$ It took several months to calculate the complete pole structures (up to second-order pole) using several personal computers. Although the first-order pole was not checked explicitly, the first-order pole in (3.2) is expected to be correct. The point is whether there is an extra quasi-primary field of spin-7. Because $V(z) V(w) = V(w) V(z)$, one can reverse the arguments $z$ and $w$ in (3.2) with the insertion of some quasi-primary field of spin-7 and use the Taylor expansions about the coordinate $w$. An explicit expression can be obtained as expressed in the appendix of [16]. All the higher order terms greater than order 1 can appear as the derivative terms at the first-order pole. The first-order term in $V(w) V(z)$ appears as the first-order pole term in (3.2) and above spin-7 field with an opposite sign. Therefore, a comparison of both sides showed that the above quasi-primary field of spin-7 vanished.

$^5$ In [13], the expression for $\Omega_{23}$ in (5.11) is incorrect. The correct one is $\Omega_{23}(w) = T(T^2 - \frac{1}{2} \delta^2 T)(w) - \frac{1}{2} \delta^2 T T(w) + \frac{1}{4} \delta^3 T(w)$ as in (3.2). In addition, the notation for the normal ordering used here is different from that used in [23], as emphasized in [17]. Sometimes, there are several ways to express the quasi-primary field using the identities $T^2 = \delta^2 T$ and $\delta^3 T = (\delta \delta T + \frac{1}{2} \delta^2 T(z) + 2 \delta T \delta T(z) + 2 \delta^2 T(z) - \frac{1}{2} \delta^3 T(z))$. That is, $H_{23}(z)$ corresponds to $\Omega_{23}(z)$ of [23], $P_{23}(z)$ corresponds to $-\frac{1}{2} \delta T(z)$, and $Q_{23} - \frac{1}{4} P_{23}(z)$ corresponds to $\Delta(z)$.

Note that any linear combinations of quasi-primary fields for a given spin provides a different quasi-primary field. The convention for the quasi-primary fields in [26] is the same as that in [13], whereas the convention for the same quantity in [23] is the same as those in [12, 25].
\[
(C_{44}^{6})^2 = \frac{n}{d^2},
\]

\[
n \equiv 12(1 + c)(22 + 5c^2)(c(-6 + N)(-5 + N) + 2N(-7 + 2N))
\times (2c(4 + N) + (-8 + 3N)(-4 + 5N))(c(3 + N) + 2N(-7 + 6N)),
\]

\[
d \equiv (24 + c)(-1 + 2c)(68 + 7c)(c + (-5 + c)N + 4N^2)
\times (c(-4 + N)(-3 + N) + N(-5 + 2N))(2c(2 + N) + (-4 + N)(-2 + 3N)).
\]

(3.11)

As reported by Hornfeck [11], there is also an extended conformal algebra \(\mathcal{W}(2, 4, 6)\) in [27] which is unrelated to the present case but is related to the next example, the \(W_{B_1}\) coset minimal model. By substituting \(N = 3\) into the formulas, (3.8) and (3.11), the author could obtain the structure constants in [27] precisely. Note that the spin-4 and spin-6 currents are made of the stress–energy tensor and its structure constants in [27] precisely. Note that the spin-4 and spin-6 currents are made of the stress–energy tensor and its structure constants in [27] precisely. Note that the spin-4 and spin-6 currents are made of the structure constants in [27] precisely. Note that the spin-4 and spin-6 currents are made of the structure constant in [27]. The explicit form is given in [28]. Furthermore, a previous study [11] checked that the classical \(c \to \infty\) limit of (3.8) coincides with the one in [29], where the structure constant can be obtained from the one from the \(WA_{N-2}\) minimal model. Note that there is a factor \((c - 1)\) in the structure constant in (3.11). In other words, \(c = 1\) suggests that \(k = 1\) or \(k = 2(1 - N)\). Then, for \(k = 1\), the structure constant \(C_{44}^{6}\) vanishes. This is a type of ‘minimal’ extension of conformal algebra [17], where the only higher spin current is of spin 4, whereas the higher spin current of spin-6 vanishes.

The lowest OPE between the spin-4 current in the \(WD_2\) coset minimal model is characterized by (3.2), where the central charge is given by (2.7), the spin-4 current, spin-2 current, structure constant \(C_{44}^{4}\) and structure constant \(C_{44}^{6}\) are given by (3.1), (2.5), (3.8) and (3.11), respectively. We have found the spin-4 and spin-6 currents for the particular \(N = 8\) case in the \(WD_2\) minimal model (and \(N = 6\) case)\(^6\). For the spin-4 current at general \(N\), there are two unknown coefficient functions, as explained before. This spin-6 current, which holds for arbitrary \(N\), would be interesting to obtain. This can be achieved by calculating the OPEs between the spin-4 currents (3.1) by hand.

The large \((N, k)\) ’t Hooft limit provides the following limiting value for the central charge [1, 9, 10]:

\[
c \to \frac{N}{2} (1 - \lambda^2), \quad \lambda \equiv \frac{N}{N + k - 2}.
\]

(3.12)

Furthermore, the limiting values for structure constants can be obtained and by the following:

\[
(C_{44}^{4})^2 \to \frac{36(-19 + \lambda^2)^2}{5(-3 + \lambda)(-2 + \lambda)(2 + \lambda)(3 + \lambda)},
\]

\[
(C_{44}^{6})^2 \to \frac{150(-5 + \lambda)(-4 + \lambda)(4 + \lambda)(5 + \lambda)}{7(-3 + \lambda)(-2 + \lambda)(2 + \lambda)(3 + \lambda)}.
\]

(3.13)

The OPE (3.2) under the large \((N, k)\) limit can be obtained by substituting (3.12) and (3.13) into (3.2). See also related works [32, 33], where the large \((N, k)\) ’t Hooft limit on the OPE was described.

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\(^6\) The self-coupling structure constant between the spin-4 currents in the \(WA_{N-2}\) minimal model is given by [2, 30]

\[
(C_{44}^{4})^2 = \frac{(36(-19 + \lambda^2)^2}{N(N + k)(N - 2)(N - 4)}\text{, which is equal to the corresponding limit of (3.8) [29].}
\]

\(^7\) The OPE (3.2) was checked when \(N = 6\) and the result is the same as (3.2) by replacing the central charge \(c_{N=6} = \frac{2}{M_{N=6} + 4}\) and \(N = 6\) for the currents and structure constants. The field contents of the \(WD_2\) minimal model are given by the spins 2, 4, \ldots, \((N - 2), \frac{2}{N}\). The naive field contents for \(N = 6\) are given by spins 2, 3 and 4 in this formula. According to the observation of [31], the minimum value of \(N\) for the above field contents in the \(WD_2\) minimal model is equal to 8.
4. The OPE between the spin-4 current and itself in the $WB_{27}^{24}$ minimal model with odd $N$

In this case, the spin-1 field is realized by $N$ free fermions [19, 36, 37]

$$J^{ab}(z) = \psi^a\psi^b(z).$$

The OPE between the fermions is given by the following:

$$\psi^a(z)\psi^b(w) = \frac{1}{(z-w)}\delta^{ab} + \cdots.$$  \hspace{1cm} (4.2)

The OPE (2.2) can be observed using (4.1) and (4.2). One takes the other OPE (2.3). The Sugawara stress–energy tensor is given by (2.5) with a diagonal current (2.4). The OPE satisfies (2.6) with the central charge (2.7).

The spin-4 current in [8] can be, with two undetermined coefficient functions, expressed as

$$V(z) = c_5 J^{cd}J^{ef}K^{cd}K^{ef}(z) + c_9 J^{cd}J^{ef}K^{cd}K^{ef}(z) + c_{10} K^{cd}K^{cd}K^{ef}K^{ef}(z) + c_{11} J^{cd}J^{ef}J^{ef}(z) + c_{12} J^{cd}J^{ef}J^{ef}(z) + c_{13} J^{cd}J^{ef}K^{cd}K^{ef}(z) + \cdots$$

For the $WD_4$ algebra, the structure constant (3.10) vanishes at $c = 1$ or $c = -\frac{656}{7}$. In [23, 27], the $\mathcal{W}(2, 4, 4)$ algebra has been shown to be consistent with the values $c = 1$ and $c = -\frac{656}{7}$. As the $c \to \infty$ limit with fixed $N$ is taken, the structure constant behaves as $(cN^2)^2 \to \frac{250(-1+4N)(-3+4N)(13+4N)(1+4N)}{(-4+3N)(-5+3N)(-3+4N)(12+4N)}$, which is the ratio of each $c^2$ term in the denominator and numerator of $(C_3^{27})^2$. Therefore, for $N = 6$, this structure constant vanishes. The behavior of the structure constant $C_3^{27}$ can be observed in the classical limit. According to the observation of footnote 6, one has $(C_3^{27})^2 \to \frac{27}{7}$ by substituting $N = 6$ into the formula. Note that the numerical value $\frac{27}{7}$ is the same as the one in the classical $\mathcal{W}(2, 4)$ algebra because $\frac{54(6+24i)^2}{(72-128i)(12+24i)} \to \frac{54}{27} = \frac{27}{7}$. The $WD_4$ algebra reduces to the $\mathcal{W}(2, 4)$ algebra [34]. For $N = 5$, the above structure constant $C_3^{27}$ vanishes and from footnote 10, the structure constant $(C_3^{27})^2$ reduces to $\frac{27}{7}$. 

8
after calculating the OPE between the spin-6 current (4

\[ V(w) = \text{coeff}(k) \psi^a \bar{\psi}^b \bar{\phi}^c \phi^d + \cdots, \]

where the coefficient functions are given by (B.1).

The OPE \( V(z) \) \( V(w) \) can be reduced to equation (3.2). The highest singular term with \( N = 7 \) has

pole 8 \( = \frac{n}{d} \).

\[ n \equiv 21k(720c_7^{(-1+k)}(2+k)^2(3+2k)^2(10+3k)(21+4k)(285+31k)^2 + d_k^2(5+k)(1320+79k(11+k) \times (-898722+k(201615+k(578998+k(126529+92k(107+3k)))))) \]

\[ d \equiv (2+k)(5+k)(3+2k)(285+31k)^2(-250+23k(5+6k)), \]

where there are two unknown coefficients \( c_9 \) and \( d_k \). Normalizing (4.4) to \( \bar{\xi} \), where the central charge is

\[ c_{N=7}^7 = \frac{7k(11+k)}{2(5+k)(6+k)}, \] (4.5)

results in a single relation between the coefficients. The seventh-order pole vanishes as before. The sixth-order pole can be written in terms of \( 2T(w) \), where \( T(w) \) is given by (2.5) with \( N = 7 \).

The unknown two coefficients can be expressed as

\[ c_9^2 = \frac{1320+869k+79k^2}{24(2+k)(6+k)^2(9+k)(3+2k)(19+2k)(10+3k)(23+3k)^2}, \]

\[ d_k^2 = \frac{2(2+k)(3+2k)(10+3k)(285+31k)^2}{24(6+k)^2(9+k)(19+2k)(23+3k)(1320+869k+79k^2)}. \] (4.6)

Equation (4.6) can be expressed in terms of (4.5) but the expressions are rather complicated.

From the fourth-order pole, one obtains the self-coupling constant for the spin-4 current

\[ (C_{44}^4)^2 = \frac{150(7224+6677k+2180k^2+286k^3+13k^4)^2}{(2+k)(9+k)(3+2k)(19+2k)(10+3k)(23+3k)(1320+869k+79k^2)} = \frac{2(4214+627c+34c^2)^2}{(21+4c)(22+5c)(19+6c)(161+8c)}. \] (4.7)

This coincides with the results [38] from the quantum Miura transformation. (4.7) can be obtained from (3.8) by putting \( N = 7 \). The classical \( c \rightarrow \infty \) limit of (3.8) coincides with that in [29], where the structure constant can be obtained from that from the \( WA_{N-1} \) minimal model.

In addition, the spin-6 current can be expressed as

\[ C_{44}^6 W(z) = \text{coeff}(k) \psi^a \bar{\psi}^b \bar{\phi}^c \phi^d + \cdots, \] (4.8)

where coeff(\( k \)) is a complicated function of \( k \). The following structure constant can be obtained after calculating the OPE between the spin-6 current (4.8) and itself:}

\[^9\text{The self-coupling structure constant between the spin-4 currents in the } \mathcal{W}_{A_{N-1}} \text{ minimal model is given by [30],} \]

\[ (C_{44}^4)^2 = \frac{12(c_7^8(2c_7^8)_{8c_7^8}+3c_7^8(1-2)c_7^8(2-2c_7^8)_{8c_7^8}+3c_7^8(1-2)c_7^8(2-2c_7^8)_{8c_7^8}+3c_7^8(1-2)c_7^8(2-2c_7^8)_{8c_7^8})}{3c_7^8(1-2)c_7^8(2-2c_7^8)_{8c_7^8}} \]

\[ \times N \rightarrow \infty \text{ limit, this leads to} \]

\[ Nc_7^8(1-2)c_7^8(2-2c_7^8)_{8c_7^8}, \]

\[ \text{which is equal to the corresponding limit of (3.8) with } N \text{ replaced by } N+1 \text{ [29].} \]
This expression can be derived from (3.11) by taking $N = 7$. In addition, this structure constant (4.9) was reported in [38]. The lowest OPE between the spin-4 current and itself in the $WB_{5,1}$ coset minimal model is characterized by (3.2), where the central charge, spin-4 current, spin-2 current, structure constant $C_{44}$, and the structure constant $C_{5,44}$ are given by (2.7), (4.3), (2.5), (3.8) and (3.11), respectively. We have found the spin-4 and spin-6 currents for the particular $N = 7$ case (and $N = 5$ case$)$. As described before, when $N = 3$ (i.e., the $WB_{1}$ coset minimal model), the structure constants (3.8) and (3.11) produce the previous results reported in [27].

As before, the ‘minimal’ extension of conformal algebra arises for $k = 1$, where the only higher spin current is of spin-4, whereas the higher spin current of spin-6 vanishes.

5. Conclusions and outlook

The OPEs (3.2) between the spin-4 current and itself in the $WD_{2}$ and $WB_{5,1}$ coset minimal models were derived, by checking those in the $WD_{3}$ and $WD_{4}$ minimal models (and $WB_{2}$ and $WB_{3}$) explicitly. These are the simplest OPEs for the given minimal models. Using the holography between the above conformal field theory and higher spin $AdS_{3}$ gravity, it is expected that the bulk computation, at the quantum level, should possess the asymptotic symmetry corresponding to the OPE (3.2).

Finding the correct answers for the following interesting subjects is an open problem.

- The full expression of the spin-4 and spin-6 currents at general $N$. Thus far, the spin-4 current for general $N$ has been found up to two unknown coefficients. This can be achieved only after the OPE between the spin-4 current and itself is calculated by hand. After performing this complicated and lengthy computation, the spin-4 current and spin-6 current (up to an overall factor) can be extracted completely. The method in [8] can be followed to obtain the spin-6 current by imposing that the OPE with a diagonal current has no singular term and the spin-6 current is the primary field under the stress–energy tensor. Exhausting all the possible terms coming from sextic-, . . . , cubic- and quadratic-terms in the $WZ$ currents is nontrivial.

- The quantum Miura transformations and the corresponding higher spin currents. The higher spin currents can be found using the quantum Miura transformation. The OPE between the spin-4 and spin-6 and the OPE between the spin-6 and itself can be calculated in a straightforward manner. The nontrivial part in this direction is to obtain the primary fields under the stress–energy tensor by combining the nonprimary fields with fixed spins. In addition, it will be interesting to determine if the other structure constants in [11] are correct.

10 The OPEs in the $WB_{3}$ minimal model [37, 39], where the spin-6 current contains a term $\bar{U}U(w)$ and $U(w)$ is a spin-$\frac{1}{2}$ current [8], were also checked. The structure constants $(C_{44})^{2} = \frac{54c-490k^{4}+120k^{2}}{2800c+c^{2}}$ and $(C_{6,44})^{2} = \frac{120(-1+c)(15+4c)(22+5c^{2}+9c)}{12700c+c^{2}}$ in the $WB_{3}$ minimal model can be obtained from (3.8) and (3.11) using $N = 5$, respectively. There is a $(c-1)$ factor in $C_{6,44}$.

11 That is, $(C_{44})^{2} = \frac{54c-82+7c^{2}+10c}{2800c+c^{2}}$ and $(C_{5,44})^{2} = \frac{144c-1+c^{2}}{2800c+c^{2}}$. There is a $(c-1)$ factor in the second structure constant.
Any supersymmetric extensions? Thus far, the supersymmetric versions of minimal model holography have been described [32, 33, 40–50]. In particular, it would be interesting to determine if the $WB_{\Delta=1}$ minimal model can be generalized to the supersymmetric extension. In the coset model (2.1), one of the levels was fixed by $1$. What happens if this level is given by $N$ along the line of [17]? As pointed out in [12], determining the other supersymmetric coset minimal models is an open problem.

Further studies should examine whether the next higher spin-5 Casimir current in the context of [6] can be obtained from the OPE between the spin-3 current and spin-4 current, and construct the spin-5 current explicitly.

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Appendix A. Coefficients in the spin-4 current of the $WD^{\frac{T}{T}}$ minimal model

The explicit coefficient functions [8] in (3.1), in terms of $c_8$ and $c_{10}$, are given by

\[
c_3 = (c_8(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2))
- 2c_{10}(42k^4 + k^3(-338 + 163N) + 2(-2 + N)^2(76 - 67N + 12N^2)
+ k^2(1000 - 953N + 219N^2) + k(-1288 + 1826N - 835N^2 + 122N^3)))/
(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2)),
\]
\[
c_9 = -4c_{10}(-2 + k + N),
\]
\[
c_{11} = -k(-c_8(-1 + k)(5 - 3N + N^2)(k^2(44 + 5N) + 44(2 - 3N + N^2)
+ k(-132 + 73N + 10N^2)) + 2c_{10}(44(-2 + N)^3(4 - 5N + N^2)
+ 6k^2(29 - 15N + 7N^2) + k^2(-1438 + 1433N - 683N^2 + 163N^3)
+ 2k(-2 + N)^2(648 - 876N + 371N^2 - 92N^3 + 12N^4)
+ 3k^3(1536 - 2249N + 1377N^2 - 471N^3 + 73N^4)
+ k^2(-7096 + 13610N - 10495N^2 + 4424N^3 - 1090N^4 + 122N^5)))/
(2(-1 + N)^2(2 - 5N + 2N^2)(k^2(44 + 5N) + 44(2 - 3N + N^2)
+ k(-132 + 73N + 10N^2))),
\]
\[
c_{12} = (2(-c_8(-1 + k)(5 - 3N + N^2)(k^2(44 + 5N) + 44(2 - 3N + N^2)
+ k(-132 + 73N + 10N^2)) + 2c_{10}(44(-2 + N)^3(4 - 5N + N^2)
+ 6k^2(29 - 15N + 7N^2) + k^2(-1438 + 1433N - 683N^2 + 163N^3)
+ 2k(-2 + N)^2(648 - 876N + 371N^2 - 92N^3 + 12N^4)
+ 3k^3(1536 - 2249N + 1377N^2 - 471N^3 + 73N^4)
+ k^2(-7096 + 13610N - 10495N^2 + 4424N^3 - 1090N^4 + 122N^5)))/
((-2 + 7N - 7N^2 + 2N^3)(k^2(44 + 5N) + 44(2 - 3N + N^2)
+ k(-132 + 73N + 10N^2))).
\]
\[ c_{13} = (-3c_{6}(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2)) \\
+ 2c_{10}(44(-2 + N)^3(-1 + N) + 6k^4(5 + 2N) + 2k(-2 + N)^2(-101 + 73N + 7N^2) \\
+ k^3(-238 + 64N + 41N^2) + k^2(672 - 656N + 74N^2 + 43N^3))/((2 - 3N + N^2)(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2)) \\
+ 12c_{10}(-2 + k + N)(18 + 7k^2 - 15N + 4N^2 + 7k(-3 + 2N))), \]
\[ c_{14} = k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2)), \]
\[ c_{15} = \frac{-k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2)}{3c_{10}(18 + 7k^2 - 15N + 4N^2 + 7k(-3 + 2N))}, \]
\[ c_{18} = -\frac{6c_8 + 4c_{10}(3k^2 + 5k(-2 + N) + 2(-2 + N)^2)}{2 + N}, \]
\[ c_{21} = (k(c_6(-1 + k)(-8 + N) + 2c_{10}(6k^3 + 2(-4 + N)(-2 + N)^2) \\
+ k^2(-32 + 13N) + k(56 - 46N + 9N^2)))/(2(-2 + 7N - 7N^2 + 2N^3)), \]
\[ c_{22} = -\frac{1}{2 - 5N + 2N^2} 2(c_{6}(-1 + k)(-8 + N) \\
+ 2c_{10}(6k^3 + 2(-4 + N)(-2 + N)^2 + k^2(-32 + 13N) + k(56 - 46N + 9N^2))), \]
\[ d_{1} = (k(-c_{6}(-1 + k)(-2 + N)(k^2(44 + 5N) + 44(2 - 3N + N^2) \\
+ k(-132 + 73N + 10N^2)) + 2c_{10}(6k^3(-8 + N) + k^4(472 - 322N + 25N^2) \\
- 8(-2 + N)^2(-28 + 55N - 32N^2 + 5N^3) + k^3(-1768 + 2172N - 705N^2 + 35N^3) \\
+ k^2(3168 - 5644N + 3294N^2 - 663N^3 + 20N^4) \\
+ k(-2720 + 6408N - 5460N^2 + 2000N^3 - 274N^4 + 4N^5)))/(4(-1 + N)(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2))), \]
\[ d_{2} = (k(c_{6}(-1 + k)(-4 + N^2)(k^2(44 + 5N) + 44(2 - 3N + N^2) \\
+ k(-132 + 73N + 10N^2)) - 2c_{10}(18k^2(-8 + N^2) \\
+ 3k^4(384 - 152N - 82N^2 + 25N^3) \\
- 8(-2 + N)^2(-40 + 58N - 8N^2 - 13N^3 + 3N^4) \\
+ k^3(-3632 + 2992N + 464N^2 - 727N^3 + 105N^4) \\
+ k^2(5632 - 7304N + 1064N^2 + 1778N^3 - 705N^4 + 60N^5) \\
+ 2k(-2144 + 3928N - 1764N^2 - 552N^3 + 590N^4 - 127N^5 + 6N^6)))/(4(1 - 3N + 2N^2)(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2))), \]
\[ d_{3} = (3c_{10}(2k^2(-8 + N) + k^4(96 - 76N + 8N^2) + k^2(-104 + 216N - 103N^2 + 12N^3) \\
+ 4(36 - 84N + 73N^2 - 27N^3 + 4N^4) + k(-120 + 62N + 57N^2 - 42N^3 + 8N^4))/(2(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2))), \]
\[ d_{4} = -c_{10}(2k^2(-8 + N) + k^3(96 - 76N + 8N^2) + k^2(-104 + 216N - 103N^2 + 12N^3) \\
+ 4(36 - 84N + 73N^2 - 27N^3 + 4N^4) + k(-120 + 62N + 57N^2 - 42N^3 + 8N^4)))/(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2)), \]
\[ d_{5} = (6c_8(-1 + k)(-3 + N^2)(k^2(44 + 5N) + 44(2 - 3N + N^2) \\
+ k(-132 + 73N + 10N^2)) + c_{10}(6k^3N(-197 + 15N + 14N^2) \\
+ k^4(96 + 9502N - 5303N^2 - 445N^3 + 350N^4) \\
+ 4(-2 + N^2)(-64 + 1060N - 1945N^2 + 1237N^3 - 316N^4 + 28N^5) \\
+ k^3(-320 - 31012N + 33947N^2 - 8004N^3 - 1485N^4 + 490N^5)). \]
\[ d_6 = (5c_9(-2 + N)(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2))) + 4c_{10}(3k^2(-8 + N) + k^4(368 - 212N + 5N^2) - k^3(1720 - 1893N + 492N^2 + 10N^3)) - 4(-2 + N)^2(-72 + 143N - 87N^2 + 16N^3) + k^2(3520 - 5888N + 3141N^2 - 494N^3 - 20N^4) - 2k(1648 - 3786N + 3149N^2 - 1110N^3 + 130N^4 + 4N^5))/((1 - 3N + 2N^2)(k^2(44 + 5N) + 44(2 - 3N + N^2) + k(-132 + 73N + 10N^2))) \]

The coefficients \( c_8 \) and \( c_{10} \) for \( N = 8 \) were determined by (3.7) and (3.5). The coefficients for \( N = 6 \) were also obtained. For general \( N \), they are not known.

**Appendix B. Coefficients in the spin-4 current of the WB_{42} minimal model**

The explicit coefficient functions [8] in (4.3), in terms of \( c_9 \) and \( d_8 \), are given by

\[
\begin{align*}
c_3 &= -\frac{d_8(-8 + N)(-19 + 7k + 12N)}{2(2 + k)(68 - 39N + 10N^2 + k(-4 + 5N))}, \\
c_{10} &= -\frac{d_8k(6k + 11(-2 + N))}{4(-2 + k + N)}, \\
c_{11} &= -\frac{d_8k(6k + 11(-2 + N))}{4(-1 + N)(68 - 39N + 10N^2 + k(-4 + 5N))}, \\
c_{12} &= \frac{d_8k(6k + 11(-2 + N))}{68 - 39N + 10N^2 + k(-4 + 5N)}, \\
c_{13} &= -\frac{3d_8(-8 + N)(2k^2(5 + 2N) + 22(-2 + 3N + N^2) + k(-46 + 18N + 7N^2))}{2(2 + k)(-4 + 2k + N)(2 - 3N + N^2)(68 - 39N + 10N^2 + k(-4 + 5N))}. 
\end{align*}
\]
\[ c_{14} = (d_k(8 + N))^2(-2 + k + N)(-16 + 11N) \\
+ c_9(2992 + 4164N - 2456N^2 + 779N^3 - 129N^4 + 10N^6) \\
- 6k^3(4 - 13N + 10N^2) + k^2(416 - 1080N + 561N^2 - 130N^3) \\
+ k(40 + 54N - 135N^2 + 79N^3 - 15N^4)) / ((68 - 39N + 10N^2 + k(-4 + 5N)) \\
- 20(-2 + N)^2 + 6k(4 - N + N^2) + k(-8 + 6N - 3N^2 + N^3)), \\
\]
\[ c_{15} = (-d_k(-8 + N)^2(-2 + k + N)(-16 + 11N) + c_9(2992 - 4164N + 2456N^2) \\
- 779N^3 + 129N^4 - 10N^5 + 6k^3(4 - 13N + 10N^2) + k^2(-416 + 1080N \\
- 561N^2 + 130N^3) + k(-40 - 54N + 135N^2 - 79N^3 + 15N^4)) / \\
(4(-2 + k + N)(68 - 39N + 10N^2 + k(-4 + 5N))(-20(-2 + N)^2 \\
+ 6k^3(4 - N + N^2) + k(-8 + 6N - 3N^2 + N^3)), \\
\]
\[ c_{18} = (2 + k)(-2 + N)(-4 + 2k + N)(68 - 39N + 10N^2 + k(-4 + 5N)) \\
3d_kk(-4 + N + 2N^2 + k(4 + N))^3, \\
\]
\[ d_1 = \frac{-4(68 - 39N + 10N^2 + k(-4 + 5N))^3}{d_kk(-4 + N + 2N^2 + k(4 + N))}, \\
\]
\[ d_2 = \frac{-4(68 - 39N + 10N^2 + k(-4 + 5N))^3}{d_kk(-4 + N + 2N^2 + k(4 + N))}, \\
\]
\[ d_3 = \frac{-4(68 - 39N + 10N^2 + k(-4 + 5N))^3}{d_kk(-4 + N + 2N^2 + k(4 + N))}, \\
\]
\[ d_4 = \frac{-4(68 - 39N + 10N^2 + k(-4 + 5N))^3}{d_kk(-4 + N + 2N^2 + k(4 + N))}, \\
\]
\[ d_5 = \frac{-4(68 - 39N + 10N^2 + k(-4 + 5N))^3}{d_kk(-4 + N + 2N^2 + k(4 + N))}, \\
\]
\[ d_6 = \frac{-4(68 - 39N + 10N^2 + k(-4 + 5N))^3}{d_kk(-4 + N + 2N^2 + k(4 + N))}, \\
\]
\[ d_7 = \frac{-4(68 - 39N + 10N^2 + k(-4 + 5N))^3}{d_kk(-4 + N + 2N^2 + k(4 + N))}. \\
\]
The coefficients, $c_0$ and $d_s$, for $N = 7$ are determined by (4.6). For general $N$, they are not known. The coefficients for $N = 5$ were also obtained.

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