Sensing electric and magnetic fields with Bose-Einstein Condensates

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We discuss the application of Bose-Einstein condensates (BECs) as sensors for magnetic and electric fields. In an experimental demonstration we have brought one-dimensional BECs close to micro-fabricated wires on an atom chip and thereby reached a sensitivity to potential variations of \( \sim 10^{-14} \) eV at 3\( \mu \)m spatial resolution. We demonstrate the versatility of this sensor by measuring a two-dimensional magnetic field map 10\( \mu \)m above a 100\( \mu \)m-wide wire. We show how the transverse current-density component inside the wire can be reconstructed from such maps. The field sensitivity in dependence on the spatial resolution is discussed and further improvements utilizing Feshbach-resonances are outlined.

The measurement of magnetic and electric fields in the immediate vicinity of surfaces allows to characterize micro-structured devices. These maps of the local fields provide an insight into details of the charge distribution and into the transport of the electron gas in various geometries. This is of great interest for fundamental studies as well as for technical matters of quality control of microchips [1]. Electric fields can for example be probed with high precision by means of single-electron transistors [2]. Conventionally available methods for measuring magnetic fields exist for either high field sensitivity at low spatial resolution (SQUID [3] and thermal atom magnetometers [4]) or high resolution at low sensitivity (MFM [5] and Hall probes [6]).

In this letter we study the performance of a novel field sensor [7] that is ideally suited for field measurements close (single microns) to micro-structures. It simultaneously features high spatial resolution and high field sensitivity. For sensing the fields, we use one-dimensional Bose-Einstein condensates (BEC) that we prepare close to the surface of atom chips [8]. High resolution imaging of these BECs enables us to measure the density profile of the cold atomic clouds. The variation of both magnetic and electric fields can be inferred as even slightest inhomogeneities in these fields measurably alter the trapping potentials. In demonstration experiments we have reached a sensitivity to potential variations of \( \sim 10^{-14} \) eV at a spatial resolution of 3\( \mu \)m. For magnetic field measurements this corresponds to a sensitivity of \( \sim 10^{-19} \)T; we could detect electric field modulations on the order of V/cm, corresponding to the field of \( \sim 10 \) elementary charges at a distance of \( \sim 10 \mu \)m.

We study in detail how various parameters determine the sensitivity of the sensor and outline a route to even enhanced performance. Under ideal conditions, the BECs outperform conventional devices over a wide spatial resolution range. Furthermore, we show how a magnetic field map measured near a conductor can be used to reconstruct the local current profile inside the conductor.

The basis of our sensor is a trapped highly elongated BEC which can be precisely positioned microns above a sample to be probed. We create these BECs in out atom chip apparatus. We start by collecting 87Rb atoms from a background vapor by an integrated mirror magneto-optical trap (MOT). The atoms are transferred to magnetic traps created by micro-fabricated gold wires mounted on a silicon surface (atom chip) [8, 9]. A quasi 1d BEC (\( > 1 \) mm long with aspect ratios up to several thousand) contains up to \( 10^5 \) atoms in the \( F = m_F = 2 \) state. The atom number and the desired chemical potential \( \mu \) can be adjusted during the final evaporative cooling stage [10] (See Fig. 1). The magnetic micro-trap is created by superimposing the magnetic field of a current carrying wire with an external homogeneous offset field \( B_{\text{offset}} \) perpendicular to the wire (Fig. 1, left). This trapping potential allows to arbitrarily position the minimum of the magnetic trap and thus the BEC by choosing the current in the wire and the offset field (magnitude and direction) in an appropriate way. The confinement in the direction along to the wire (z-direction) is generated by a varying magnetic field component \( B_z \) parallel to the wire [8].

The local density of trapped thermal clouds or BECs is imaged in situ or after ballistic expansion by high res-

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FIG. 1: (i) left: An elongated BEC held by a trapping wire can be arbitrarily positioned above the wire itself (i) or above an independent sample to be probed (ii) by freely adjusting the current in the wire and the offset-field. right: Using a BEC held above an independent wire (transverse trap frequency \( \omega_{\text{tr}} = 2 \pi \cdot 700 \) Hz) the potential variations along the z-direction have been measured. The blue (lower) curve has been measured with a current passing through the wire. The red (upper) curve has been measured at the same position but charging the wire to amplify the electric potentials. The dotted lines are a guide to the eye for comparing the different patterns. The red (upper) curve has been shifted by 40nK for visibility.
olution (3µm) absorption imaging. This density profile \( n_{1d}(z) \) can be converted into a map of the spatial profile of the longitudinal potential energy variations \( V(z) \). The sensitivity to potential variations of a thermal atomic cloud is given by their temperature \( T \): 
\[ n_{1d} \sim \exp \left( -V/k_B T \right) \]  
In the case of a BEC the relevant energy scale is given by the chemical potential \( \mu \) which can be orders of magnitude smaller than the temperature of a thermal cloud (\( \mu \ll k_B T \)). We were able to profit from this exceptional potential sensitivity because atom chips with very low disorder potentials have been used \[14\].

In elongated traps, the chemical potential \( \mu \) of the BEC is of the order of the energy level spacing in the transverse (strong confinement) direction (\( \mu \approx \hbar \omega_{1t} \)), but still much larger than the energy level spacing in the longitudinal (weak confinement) direction (\( \mu \gg \hbar \omega_{1t} \)). In this cross-over region the three-dimensional regime is smoothly connected to the one-dimensional mean-field regime by a local density approximation \[15\]. The longitudinal potential can be calculated from the 1d-density profile by

\[ V_0 + V(z) = -\hbar \omega_{1t} \sqrt{1 + 4a_{scat} n_{1d}(z)} \quad (1) \]

where \( a_{scat} \) is the s-wave scattering length (5.2nm for \(^{87}\text{Rb} \[16\]) and \( V_0 \) is an arbitrary offset. The sensitivity to potential variations is proportional to \( \hbar \omega_{1t} \). Weaker confinement allows to see smaller potential variations.

The optimal potential single shot sensitivity \( \Delta V \) of a BEC as field sensor is reached by detecting the density distribution atom shot-noise limited. The desired spatial resolution is not necessarily equal in the longitudinal (\( z_0 \)) and transverse (\( \rho_0 \)) directions. Ideally, the trap parameters are chosen such that the transverse ground state size matches \( \rho_0 \). In this optimal situation the sensitivity of a one-dimensional BEC in the mean-field regime is given by:

\[ \Delta V = \frac{\gamma \Delta N}{\rho_0^2 z_0}, \quad \gamma = \frac{2 \hbar^2}{m a_{scat}} \quad (2) \]

For our detection imaging noise of \( \Delta N \sim 4\text{atoms/pixel} \) a single-shot single-point sensitivity to potential variations of \( \sim 10^{-13} \text{eV} \) (\( \sim 10^{-14} \text{eV} \)) can be reached at \( \omega_{1t} = 2\pi \cdot 3\text{kHz} \) (\( \omega_{1t} = 2\pi \cdot 300 \text{Hz} \)). Variations in the longitudinal potential can originate from local magnetic as well as from electric fields (Fig. 4-right).

The sensitivity to electric field modulations can be enhanced by adding a homogeneous electric offset-field \( E_0 \) \[17\]. In this case the potential is related to the electric field by \( V(z) = -\alpha E_0 \) where \( \alpha \) is the polarizability. For our atom detection a field sensitivity of \( \Delta E = 0.4\text{V/cm} \) can be reached at \( \omega_{1t} = 2\pi \cdot 3\text{kHz} \) and \( E_0 = 1\text{V/cm} \). This equals the field strength produced by a fraction of an elementary charge (\( \sim 25\%) \) detected at a distance of 3µm.

In the case of local magnetic field variations \( \mu \) the longitudinal potential is related to the magnetic field by \( V(z) = m_F g_F \mu_B B(z) \) where \( \mu_B \) is Bohr’s magneton, \( m_F \) is the quantum number associated with the Zeeman state of the atom and \( g_F \) is the Lande-factor. To compare the performance of the BEC-sensor to commonly used magnetic field detectors (Fig. 2) the dependence of the field sensitivity on the spatial resolution of the measurement has to be taken into account.

Two regimes can be distinguished which are depicted in Figure 2. The field sensitivity scales most strongly with the spatial resolution of the measurement if \( z_0 = \rho_0 \). This can be achieved if the transverse ground-state size matches the desired resolution and if the resolution of the imaging system is better than \( z_0 \). By using a transition in the blue (around \( \lambda = 421.67 \text{nm} \) for the Rubidium \( 5\text{S}_1/2 \rightarrow 6\text{P}_1/2 \) resonance \[18\]) an imaging resolution of \( \sim 500\text{nm} \) can be achieved. Consequently, the spatial resolution of the magnetic field measurement in this range \( \Delta B = 0.5 \sim 10\text{µm} \) can be assumed to be limited by the imaging system only. Since generation of BECs in traps which are shallower than \( \omega = 2\pi \cdot 1\text{Hz} \) (more than 10µm ground-state size) seems to be challenging \[19\], sensitivity can further be enhanced by decreasing \( z_0 \) or a fixed transverse confinement of \( 2\pi \cdot 1\text{Hz} \). This leads to an anisotropic spatial resolution of the sensor.

The sensitivity is given by the chemical potential for a specific longitudinal 1d-density and its shot-noise. Decreasing the strength of the interactions between the atoms leads to a lower chemical potential at the same \( n_{1d} \) and consequently higher sensitivity (Eq. 2). The sensor can be operated at an arbitrary homogeneous field \( B_z \); the s-wave scattering length \( a_{scat} \) can be adjusted using Feshbach resonances. In the case of Cesium, \( a_{scat} \) becomes zero around an easily manageable magnetic field value of 17.0G and the linearized slope around this zero-crossing of \( a_{scat} \) can be estimated to be \( \sim 6\text{nm/G} \) \[20\]. Additionally the higher mass of Cesium compared to Ru-

![FIG. 2: The potential sensitivity versus spatial resolution for the BEC-sensor has been plotted according to Eq. (2). The solid curve shows the sensitivity of the demonstrated sensor using a Rubidium BEC. Tuning the scattering length of the atoms by means of a Feshbach resonance leads to even higher sensitivity, e.g. the estimated sensitivity of a Cesium BEC has been plotted as dashed curve. In comparison the field sensitivity versus spatial resolution for state-of-the-art magnetic microscopes is shown (Scanning Hall Probe Microscopy \[21\], \[22\], \[23\], \[24\], \[25\], \[26\], Superconducting Quantum Interference Device \[27\], \[28\], \[29\], \[30\], \[31\], and thermal atom magnetometer \[32\]). The dark grey shaded region indicates the sensitivity-resolution range currently accessible only to the demonstrated BEC sensor.](image)
bium further increases the sensitivity. Decreasing the scattering length to 0.1nm which requires an easily manageable control of the magnetic field of 15mG would gain a factor of \(\sim 100\) in sensitivity (dashed line in Fig. 2).

As an application of our potential imaging we have measured a map of the local longitudinal magnetic field variation 10\(\mu\)m above a flat (cross-section 100 \(\times\) 3.1\(\mu\)m\(^2\)) current-carrying wire (Fig. 3a) consisting of 28 equally spaced positions along the transverse direction of the wire. From this map we have reconstructed the local transverse current flow in the conductor.

This reconstruction involves the deconvolution of Biot Savart’s law which can be performed if the current density is assumed to be confined to a 2d-plane and if boundary conditions on the geometry of the wire are assumed. In this case only \(j_x\) contributes to the magnetic field \(B_z\) and can be calculated according to

\[
j_x(x, z) = \mathcal{F}^{-1}\{\tilde{B}(k_x, k_z) e^{i(k_x x + k_z z)}\}(x, z)
\]

where \(\tilde{B}(k_x, k_z) = \mathcal{F}\{B_z(x, z)\}(k_x, k_z)\) and \(\mathcal{F}\) indicates a two-dimensional Fourier transform.

In the example presented here two effects have to be taken into account when estimating the spatial resolution \(\Delta s_B\) of the reconstructed current-density: (i) The magnetic field has been measured at a finite distance of \(y = 10\mu\)m above the surface resulting in a smoothing of the magnetic field variations. (ii) The field has been mapped on a grid given by the spatial resolution of the imaging system in the z-direction and by the transverse positioning of the BEC in the x-direction. Here, two regimes have to be considered: If the spatial resolution \(\Delta s_B\) of the magnetic field measurement is coarser than the distance \(y\) to the wire, one finds \(\Delta s_j \approx \Delta s_B\). In the opposite limit \((y > \Delta s_B)\) the resolution of the current-density can be found as follows: Two point-like current-density components positioned at a distance \(\Delta s_j\) can be linked by Biot-Savart’s law to the length scale \(\Delta s_B\) of the modulation of the resulting magnetic field, yielding \(\Delta s_j \approx y + \frac{\Delta s_B^2}{y}\). In our measurement of the 100\(\mu\)m-wide wire, the spatial resolution of \(j_x\) is limited by the distance to the wire. To reveal more details the atom-surface distance in the field measurement has to be reduced. It has been shown that a surface approach of single microns is possible \([14]\). Constraints are the limited trap depth in the presence of attractive surface potentials \([21]\) and the finite trap lifetime \([22]\). Trap lifetimes have to be sufficiently long for the condensate to reach thermodynamic equilibrium. This is typically not problematic as lifetimes on the order of seconds can be achieved down to single microns from insulators \([23]\) or thin conducting layers \([24]\) that are prone to be probed by BEC sensors.

If the current density is reconstructed using Eq. (3), high frequency noise in the magnetic field map resulting from the imaging process has to be taken into account. This noise causes artificial structures which do not represent the actual current density distribution. Using the estimation of the expected current density resolution discussed above, a filter function can be designed:

\[
F^{-1}\left(\frac{k_x}{k_z}\right) = \left[1 + \exp\left(\frac{k_x}{c_x}\right)\right] \left[1 + \exp\left(\frac{k_z}{c_z}\right)\right]
\]

This has been applied to \(\tilde{B}(k_x, k_z)\) before computing \(j_x\) (Fig. 3b) using Eq. (3). A reasonable choice of the filter parameters is \(c_x = 0.32\mu m^{-1}\), \(c_z = 0.22\mu m^{-1}\), and \(s_x = s_z = 0.05\mu m^{-1}\). As a test of the applied deconvolution methodology, \(B_z\) has been calculated from the reconstructed \(j_x\) using Biot-Savart’s law directly (Fig. 3c). Good agreement between this calculated magnetic field and the measured magnetic field map is found.

In conclusion, we have shown that high resolution potential images can be derived from quasi 1d BECs used as sensor. Imaging the condensate 1d density one obtains a high resolution (~\(\mu m\)) potential map along a line on a mm scale. Our experiments demonstrate the unique sensitivity of the 1d BEC when applied to magnetic fields, surpassing conventional measurement methods by orders of magnitude for spatial resolutions in the \(\mu m\)-range. As an application of this field sensor we have demonstrated that BECs can be used to reconstruct the current-density in a micro-fabricated wire. In future experiments a BEC sensor could be used to obtain a deeper understanding of the local current flow for example in superconductors and two-dimensional electron gases.

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