Superfluidity of fermionic pairs in a harmonic trap. Comparative studies: Local Density Approximation and Bogoliubov-de Gennes solutions

Agnieszka Cichy and Andrzej Ptok

1 Faculty of Physics, Adam Mickiewicz University, ul. Uniwersytetu Poznańskiego 2, PL-61-614 Poznań, Poland
2 Institut für Physik, Johannes Gutenberg-Universität Mainz, Staudingerweg 9, D-55099 Mainz, Germany
3 Institute of Nuclear Physics, Polish Academy of Sciences, ul. W. E. Radzikowskiego 152, PL-31342 Kraków, Poland
4 Author to whom any correspondence should be addressed. E-mail: agnieszka.cichy2311@gmail.com and aptok@mmj.pl

Keywords: superfluidity, harmonic trap, local density approximation, Bogoliubov-de Gennes equations

Abstract
Experiments with ultracold gases on the lattice give the opportunity to realize superfluid fermionic mixtures in a trapping potential. The external trap modifies the chemical potential locally. Moreover, this trap also introduces non-homogeneity in the superconducting order parameter. There are, among other approaches, two methods which can be used to describe the system of two-component mixtures loaded into an optical lattice: the Local Density Approximation (LDA) and the self-consistent Bogoliubov–de Gennes equations. Here, we compare results obtained within these two methods. We conclude that the results can be distinguishable only in the case of a small value of the pairing interaction.

1. Introduction
Recent progress in experimental techniques in ultracold quantum gases gives ground to believe that they are good candidates for realization of quantum simulators [1–5]. This progress gives the opportunity to study different types of quantum models, well established already in many-body physics [5–7].

Parameters such as the depth of the periodic trapping potential or the lattice geometry can be tuned with a high level of control in experiments. Moreover, the interaction strengths can also be fully controlled via so-called Feshbach resonances [8], giving the possibility of realization of repulsively and attractively interacting systems [9]. The latter allows for an experimental study of superfluid states [10–12].

Typically, a system of ultracold atoms is located in an external potential, for instance in a harmonic trap. This potential introduces some effective site-dependent chemical potential and non-homogeneous distribution of the particles in the system. As a consequence, non-uniform superconducting order parameter is expected. The system can be studied theoretically via several methods. In this paper, we compare the results obtained within the Local Density Approximation (LDA) and the self-consistent Bogoliubov–de Gennes (BdG) equations, for a system of an ultracold attractive fermionic gas loaded into a one-dimensional (1D) optical lattice, put in a harmonic trap.

The paper is organized as follows: first, we shortly introduce the model as well as the above mentioned methods (section 2). In section 3, we present and discuss numerical results. Finally, we summarize in section 4.

2. Model and techniques
We investigate a fermionic gas loaded onto a one-dimensional lattice, in a harmonic trap, which is schematically illustrated in figure 1. The system under consideration can be described by the Hamiltonian:

\[
\hat{H} = \sum_{\mathbf{r}, \sigma} \epsilon_{\mathbf{r}} \hat{c}_{\mathbf{r} \sigma}^\dagger \hat{c}_{\mathbf{r} \sigma} + \frac{1}{2} \sum_{\mathbf{r}, \mathbf{r}' \sigma, \sigma'} J_{\mathbf{r}, \mathbf{r}'} (n_{\mathbf{r}, \sigma} - n_{\mathbf{r}', \sigma}) (n_{\mathbf{r}', \sigma'} - n_{\mathbf{r}, \sigma'})
\]

where \(\epsilon_{\mathbf{r}}\) is the energy of the particle at site \(\mathbf{r}\), \(J_{\mathbf{r}, \mathbf{r}'}\) is the hopping integral between sites \(\mathbf{r}\) and \(\mathbf{r}'\), and \(n_{\mathbf{r}, \sigma}\) is the occupation number of the fermion at site \(\mathbf{r}\) and spin \(\sigma\). The Hamiltonian includes both the kinetic energy and the pairing interaction.
The LDA method is based on an assumption that, locally, the system behaves like a uniform gas [1, 2]. The parameters of the system depend solely on the value of the particle density \( n_i = n_{i1} + n_{i2} \) (i.e. the average number of particles), at each point \( i \) of space [13, 14]. Hence, the local SOP is defined only by \( n_i = n(\mu_i) \). In the presence of the trapping potential, local lattice potential is given by \( \mu_i = \mu_0(n_i) + V(\tau_i) \), called Thomas–Fermi equation [1, 2]. The chemical potential \( \mu_0 \) is fixed by the condition \( \sum_i n_i = N \), where \( N \) is the total number of particles in the system. In a general case, the functional \( \Delta_i \equiv \Delta(n_i) = \Delta(\mu_i) \) can be found starting from the solution for the homogeneous system, i.e., \( \Delta_i = \Delta(\mu = \mu_i) \).

First, let us shortly describe derivation of the equations which are useful in the LDA method [15, 16]. In the case of a homogeneous system (\( \mu_i \equiv \mu \) and \( \Delta_i \equiv \Delta \)), the Hamiltonian \( (3) \) takes the following form in momentum space:

\[
H^{\text{MF}} = \sum_{k} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + U \sum_k (\Delta^s_k c_{k1}^\dagger c_{-k1} + \text{h.c.}) - U \sum_i |\Delta_i|^2,
\]

where \( \Delta^s_k = -2t \cos(k) - \mu \) is the single-particle dispersion relation of the one-dimensional system. Here, it should be noticed that the presented derivations do not depend on the lattice dimensionality. Using anti-commutation relation, the first term can be rewritten as:

\[
\sum_k \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} = \sum_k [\varepsilon_k c_{k1}^\dagger c_{k1} + \varepsilon_k (1 - c_{k1}^\dagger c_{k1}^\dagger)].
\]

This allows us to write down the Hamiltonian \( H^{\text{MF}} \) in the Nambu notation:

\[
H^{\text{MF}} = \sum_k \Phi_k^\dagger \Xi_k \Phi_k + \sum_k (\varepsilon_k - U|\Delta|^2),
\]

where \( \Phi_k = (c_{k1}^\dagger, c_{-k1}) \) are the Nambu spinors. The system can be solved using the Bogoliubov transformation:

\[
\begin{pmatrix}
  c_{k1}^\dagger \\
  c_{-k1}
\end{pmatrix} =
\begin{pmatrix}
  u_k & v_k \\
  -v_k & u_k
\end{pmatrix}
\begin{pmatrix}
  \gamma_{k1}^\dagger \\
  \gamma_{k1}
\end{pmatrix},
\]

where \( \gamma^\dagger \) and \( \gamma \) are the quasiparticle operators. The spectrum of the Hamiltonian is given by \( \lambda_k = \pm \sqrt{\varepsilon_k^2 + \Delta^2} \), while
The above equation is called the gap equation. Here, \( f(\omega) = [1 + \exp(\omega/k_B T)]^{-1} \) is the Fermi–Dirac distribution at temperature \( T \), while \( N \) is the number of states. Similarly, the occupations \( n_{k\sigma} = \langle \epsilon_{k\sigma}^\uparrow, \epsilon_{k\sigma}^\downarrow \rangle \) can be found as:

\[
\frac{n^\uparrow_{k\sigma}}{n^\downarrow_{k\sigma}} = u^2_k f(\lambda^+_k) + v^2_k f(-\lambda^-_k),
\]

which is equivalent to \( \sum_{k\sigma} n_{k\sigma} = -\partial \Omega/\partial \mu \). As a result, one can find values of \( \Delta(\mu) \) and \( n(\mu) \) within the LDA method.

### 2.2. Bogoliubov–de Gennes solution

In the case of the non-homogeneous Hamiltonian (3), the system can be solved within the Bogoliubov–Valatin transformation:

\[
\epsilon_{in\sigma} = \sum_n \{E_{in\sigma} \gamma_{in\sigma} - \sigma v^*_{in\sigma} \gamma_{in\sigma} \},
\]

where \( \gamma_{in\sigma} \) and \( \gamma^*_{in\sigma} \) are the quasi-particle fermionic operators and \( \sigma = -\sigma \). The above transformation can be treated as an extended version of the transformation (7) in real space. If the Hamiltonian is diagonalized under the transformation (11), i.e., \( H^\text{MF} = \sum_{n\sigma} E_{in\sigma} \gamma^\dagger_{in\sigma} \gamma_{in\sigma} \), the relations [17]:

\[
[H^\text{MF}, \gamma_{in\sigma}] = -E_{in\sigma}, \quad [H^\text{MF}, \gamma^\dagger_{in\sigma}] = E_{in\sigma} \gamma^\dagger_{in\sigma}
\]

hold such that one straightforwardly finds Bogoliubov–de Gennes equations [18] (called also Blonder–Tinkham–Klapwijk equations):

\[
E_{in\sigma} \begin{pmatrix} u_{in\sigma} \\ v_{in\sigma} \end{pmatrix} = \sum_j \begin{pmatrix} H_{jj} & \Delta^\dagger_{ij} \\ \Delta_{ij} & H^\ast_{jj} \end{pmatrix} \begin{pmatrix} u_{in\sigma} \\ v_{in\sigma} \end{pmatrix},
\]

where \( H_{ij} = -t \delta_{ij} - \mu_i \delta_j \) is the single-particle Hamiltonian and \( \Delta_{ij} = U \Delta \delta_{ij} \) are the on-site SOPs. Due to the symmetry of the BdG equations with respect to spin inversion, we can find an identity relation between eigenpairs:

\[
E_{in\sigma} \begin{pmatrix} u_{in\sigma} \\ v_{in\sigma} \end{pmatrix} \rightarrow -E_{in\sigma} \begin{pmatrix} -v_{in\sigma} \\ u_{in\sigma} \end{pmatrix}.
\]

The SOPs can be found self-consistently as:

\[
\Delta_i = \langle \epsilon_{i\uparrow} \epsilon_{i\downarrow} \rangle = \sum_n [u^*_{in\uparrow} v_{in\downarrow} f(\epsilon_{n\uparrow}) - u_{in\downarrow} v^*_{in\uparrow} f(-\epsilon_{n\uparrow})].
\]

In a similar way, the site-dependent average number of particles, \( n_i = n_{i\uparrow} + n_{i\downarrow} \) can be determined, where:

\[
n_{in\sigma} = \langle \epsilon_{in\sigma}^\dagger \epsilon_{in\sigma} \rangle = \sum_n [u^*_{in\sigma} f(\epsilon_{n\sigma}) + |v_{in\sigma}|^2 f(-\epsilon_{n\sigma})].
\]

### 2.3. Numerical implementation

As has been pointed out in the previous paragraph, the chemical potential \( \mu_i \) of the system in the trapping potential is fixed by the total number of particles \( N \). Next, the solution of the system within the LDA method is defined by the effective potential \( \mu_i \) given by the Thomas–Fermi equation. Hence, the corresponding SOP \( \Delta \) can be found by minimization of \( \bar{\Omega}(\Delta) \) at fixed \( \mu_i \) [19–21]. In fact, this way is equivalent to the one in which equation (9) should be solved [22]. The huge advantage of this method is based on the fact that it is relatively simple to implement and quite quickly solvable even for a trapped system, due to an efficient procedure of finding the solution for a homogeneous system. For instance, in the two-dimensional (2D) case, the solution (the minimum of the grand canonical potential) can be found for the system with around a thousand sites [21].
In turn, the self-consistent solution $\Delta_i$ (15) and $n_{ii}$ (16) of the BdG equations (13), in the presence of the trapping potential, can be found only when all eigenpairs are known. Hence, the BdG equations are associated with the exact diagonalization of the Hamiltonian in matrix form, which can be found numerically using one of the available numerical libraries, e.g. LAPACK [23], IntelMKL [24] or MAGMA [25, 26]. However, this method is very computational time-consuming $\sim N^3$ and leads to a limitation in the system size $N$. This problem can be partially solved by using the sparse matrices storage implemented in some modern softwares, for instance ARPACK [27]. Moreover, there exist some iterative methods for solving the BdG equations [28–32], based on the kernel polynomial method [33]. For instance, in the Chebyshev–Bogoliubov–de Gennes method, the Chebyshev expansion of Green’s functions is implemented. This type of methods allows to study the large size systems, even with hundreds or thousands sites [32]. Moreover, in some physical problems, it is reasonable to limit the number of eigenpairs to only few nearest ones to ‘zero energy’ (i.e. the Fermi level). This issue is implemented, e.g., in FEAST [34] and it can be applicable to topological edge states investigations, for instance.

3. Numerical results and discussion

3.1. Spin-balanced system

Here, we describe the balanced system case, which means the average number of particles with opposite spin in each site is equal ($n_{\uparrow \downarrow} = n_{\downarrow \uparrow}$). Due to the fact that the average number of particles $n_i$ in each site is determined by the effective chemical potential $\mu_i$, to perform the calculation within both methods, the chemical potential is discretized with the parametrization: $\mu_i = \mu + V(r_i)$.

Figure 2 shows $n$ versus $\mu$ and $\Delta$ versus $\mu$ dependences, obtained within LDA calculations. In the case of a one-dimensional system, the band edges are at $\pm 2t$. At $U/t = 0$, the chemical potential changes from $-2t$ to $2t$, which corresponds to the filling $n = 0$ and $n = 2$ at the band edges. The SOP is equal to zero for the non-interacting system. With increasing on-site interaction, the SOP increases. For intermediate couplings, the chemical potential drops below the lower band edge, which, according to the Leggett criterion [35], indicates the BCS–BEC crossover [36]. Found values of the $n(\mu)$ and $\Delta(\mu)$ can be used as a solution of the system for a given $U < 0$ from the LDA approach.

Figure 3 shows results in the case of the system in a harmonic trap, $V(r) = V_0(t_0 - r)^2$. We fix $V_0/t = 15 \times 10^{-6}$. The average number of particles $n_i$, obtained within these two methods, is comparable for the chosen parameters. A similar behavior is observed for the SOP $\Delta$ in the center of the trap. However, the situation looks differently at the edge of the harmonic trap (insets). Namely, LDA gives significantly different solutions for the SOP from those obtained within the BdG method at relatively small attractive interactions $U$.

![Figure 2. Average number of particles $n$ (blue) and superconducting order parameter $\Delta$ (green) as a function of the chemical potential $\mu$ for different values of the pairing potential $U < 0$, at fixed $k_B T / t = 10^{-3}$. The results were obtained within the LDA approach.](image-url)

3. J. Phys. Commun. 4 (2020) 055006 
A. Cichy and A. Ptok
A similar behavior has been observed in the case of a spin-imbalanced system in which unconventional pairing [37–39] can be realized at the edge of the system. It is worth emphasizing that the LDA is a reliable approach when the gradient corrections are relatively small. In turn, the BdG solutions in each site depend on the rest of the system via equations (13) and (15). Hence, the BdG method is a better approach to describe the system under consideration. Moreover, the LDA can also fail in the description of the system in a strong anisotropic trap [40].

### 3.2. Other applications and limitations

The theoretical predictions within the LDA method are in relatively good agreement with experimental results [41–44]. This method was already successfully implemented in the context of trapped ultracold quantum gases [37, 40, 45–47]. However, the LDA method has some limitations which can be lifted by applying the BdG technique. Below, we give two examples of the systems for which the LDA method fails.

#### 3.2.1. Spin-imbalanced systems

Both methods under consideration can be applied for a spin-imbalanced system, i.e., when $n_{i\uparrow} \neq n_{i\downarrow}$ [13, 48]. This corresponds to the experimental setup with a two-component Fermi mixture, with unequal number of atoms with opposite spins [49–51]. This leads effectively to the substitution $\mathcal{H} \rightarrow \mathcal{H} - \hbar \sum_i (c_{i\uparrow}^\dagger c_{i\uparrow} - c_{i\downarrow}^\dagger c_{i\downarrow})$, where $\hbar$ is the Zeeman magnetic field inducing the spin-imbalance. In the case of the LDA method, this is equivalent to the substitution $\mathcal{E}_k \rightarrow \mathcal{E}_k - \sigma \hbar$. Hence, an additional condition is needed in the numerical calculations, i.e., $\delta N = \sum (n_{i\uparrow} - n_{i\downarrow})$. Here, the $n_{i\uparrow} - n_{i\downarrow}$ corresponds to the magnetization at $i$-th site. It is worth mentioning, that the symmetry of the BdG equations, given by equation (14), is preserved.

In the weak coupling limit, at a large spin imbalance (i.e. polarization) [22, 38], the Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phase [52, 53] can occur. In this case, the SOP is characterized by oscillations in real space and at non-zero local polarization [13, 54]. If the polarization is equal to zero or low, the connection between the phase diagrams of the system without a trapping potential and in the presence of the harmonic trap [55] exists, in the LDA method [22]. Unfortunately, the LDA approach cannot account for the oscillating character of the SOP as well as the oscillations of the average number of particles in the FFLO phase [13, 56]. A similar problem has been observed for the spin-imbalanced gap [57] estimated within the LDA method and the Density Matrix Renormalization Group (DMRG). In this case, the oscillating character of $n_i$ at relatively low polarization has not been reproduced within the LDA method. Moreover, the LDA calculations underestimate critical parameters of the system, with respect to the results obtained within the DMRG [57] or the BdG approach [22].

![Figure 3. A comparison of the results obtained from the BdG method (solid lines) and the LDA approach (dashed lines). Black dashed line shows the effective chemical potential $\mu_i$, while red (yellow) and blue (light blue) solid (dashed) lines correspond to the BdG (LDA) results. Inset shows the comparison of results for BdG (red line) and LDA (blue squares) approaches around the edge of the system. The numerical simulations were performed at $k_B T / t = 10^{-4}$.](image-url)
The situation can be even more complicated in the two dimensional systems. In the case of the harmonic trap, the additional SOP oscillations along the radial direction have been reported [37, 58–60]. However, for the toroidal trap, the oscillations of the SOP can be function of angle, due to rotational symmetry of the system [61–63]. The combination of these two types of oscillations can be expected [64] as well. This case shows an advantage of the BdG technique in the comparison to LDA, i.e. the spin-imbalanced systems with unconventional superconducting phases.

3.2.2. Non-homogeneous systems

The strongly inhomogeneous systems, e.g. systems with long-range magnetic or charge orderings or disordered systems are examples for which the LDA method fails. Apart from the above systems, one should mention ultracold fermionic gases or the condensed matter systems [65]. Here, the impurities [66–68] or vortices [69–71] play the role of inhomogeneity. It is worth mentioning the superconducting systems with spin-density (SDW) or charge-density (CDW) waves coexistence [72–75]. The 'local' disorder in the system influences on the properties of the system. Because of the presence of inhomogeneity with 'long-range' impact on the system, the LDA method can not be used. In such cases, only the BdG technique is suitable [65].

4. Summary

In this paper, we studied an ultracold fermionic mixture loaded into a one-dimensional lattice, put in a harmonic trap. We briefly compared the Local Density Approximation and the self-consistent Bogoliubov–de Gennes equations—the main assumptions, implementations and limitations of both of these methods. We have shown that both approaches give comparable results in the case of relatively large pairing interactions \( U \). For smaller \( U \), the results for the superconducting order parameter \( \Delta \) can be significantly different, especially in the case of a small density of particles, i.e. small filling at the edge of the system. Concluding, the results obtained within the LDA method can be treated as a starting point for the more accurate BdG technique. One should be aware of the LDA method limitations. This method is strongly unrecommended to use for spin-imbalanced as well as strongly inhomogeneous systems.

Acknowledgments

We thank Krzysztof Cichy for careful reading of the manuscript, valuable comments and discussions. This work was supported by the National Science Centre (NCN, Poland) under grants UMO-2017/24/C/ST3/00357 (AC), and UMO-2017/25/B/ST3/02586 (AP).

ORCID iDs

Agnieszka Cichy  
https://orcid.org/0000-0001-5835-9807
Andrzej Ptok  
https://orcid.org/0000-0002-5566-2656

References

[1] Giorgini S, Pitaevskii L P and Stringari S 2008 Theory of ultracold atomic fermi gases Rev. Mod. Phys. 80 1215
[2] Sheehy D E and Radzihovsky L 2007 BCS–BCS crossover, phase transitions and phase separation in polarized resonantly-paired superfluids Ann. Phys. 322 1790–924
[3] Guan X-W, Batchelor M T and Lee C 2013 Fermi gases in one dimension: from Bethe ansatz to experiments Rev. Mod. Phys. 85 1633
[4] Georgescu I M, Ashhab S and Nori F 2014 Quantum simulation Rev. Mod. Phys. 86 153
[5] Dobrzyńiecki J and So wiński T 2020 Simulating artificial one-dimensional physics with ultra-cold fermionic atoms: three exemplary themes arXiv:2001.04223
[6] Bloch I, Dalibard J and Zwerger W 2008 Many-body physics with ultracold gases Rev. Mod. Phys. 80 885
[7] Dutta O, Gajda M, Hauke P, Lewenstein M, Lührmann D-S, Malomed B A, Sowiński T and Zakrzewski J 2015 Non-standard Hubbard models in optical lattices: a review Rep. Prog. Phys. 78 066001
[8] Chin C, Grimm R, Julienne P and Tiesinga E 2010 Feshbach resonances in ultracold gases Rev. Mod. Phys. 82 1225
[9] Koschorreck M, Pertot D, Vogt E, Frohlich B, Feld M and Kohl M 2012 Attractive and repulsive Fermi polarons in two dimensions Nature 485 619
[10] Partridge G B, Li W, Kamar R I, Liu Y -A and Hulet R G 2006 Pairing and phase separation in a polarized Fermi gas Science 311 503
[11] Zwierlein M W, Schunck C H, Schirotzek A and Ketterle W 2006 Direct observation of the superfluid phase transition in ultracold Fermi gases Nature 442 54
[12] Schunck C H, Shin Y, Schirotzek A, Zwierlein M W and Ketterle W 2007 Pairing without superfluidity: the ground state of an imbalanced fermi mixture Science 316 867
[13] Liu X -J, Hu H and Drummond P D 2007 Fulde–Ferrell–Larkin–Ovchinnikov states in one-dimensional spin-polarized ultracold atomic fermi gases Phys. Rev. A 76 043605
[14] Bulgac A 2007 Local-density-functional theory for superfluid fermionic systems: the unitary gas Phys. Rev. A 76 040502
[15] Bedaque P F, Caldas H and Rupak G 2003 Phase separation in asymmetrical fermion superfluids Phys. Rev. Lett. 91 247002
[16] Sedrakian A, Mur-Petit J, Polls A and Mütter H 2004 Deformed Fermi surfaces in ultracold Fermi gases arXiv:cond-mat/0405477
[17] Van Schaeybroeck B and Lazarides A 2009 Normal-superfluid interface for polarized fermion gases Phys. Rev. A 79 053612
[18] De Gennes P G 1999 Superconductivity Of Metals And Alloys (Advanced Books Classics Series) (Westview: Westview Press)
[19] Hu H and Liu J-X 2006 Mean-field phase diagrams of imbalanced Fermi gases near a feshbach resonance Phys. Rev. A 73 051603
[20] Januszevski M, Ptok A, Crivelli D and Garbacz B 2015 GPU-based acceleration of free energy calculations in solid state physics Comput. Phys. Commun. 192 220
[21] Ptok A and Crivelli D 2017 Influence of finite size effects on the Fulde–Ferrell–Larkin–Ovchinnikov state Commun. Comput. Phys. 21 748
[22] Ptok A, Cichy A, Rodríguez K and Kapcia K J 2017 Critical behavior in one dimension: Unconventional pairing, phase separation, BEC-BCS crossover, and magnetic Lifshitz transition Phys. Rev. A 95 033613
[23] Anderson E, Bai Z, Bischof C, Blackford S, Dongarra J, Du Croz J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 LAPACK Users’ Guide vol 9 (Philadelphia: Siam)
[24] Intel(R) Math Kernel Library (https://software.intel.com/en-us/mkl)
[25] Tomov S, Dongarra J and Baboulin M 2010 Towards dense linear algebra for hybrid GPU accelerated manycore systems Parallel Computing 36 322
[26] Tomov S, Nath R, Li Y and Dongarra J 2010 Dense linear algebra solvers for multicore with GPU accelerators Proc. of the IEEE IPDPS’10 (Atlanta, GA: IEEE Computer Society) p 1
[27] Lehoucq R B, Sorensen D C and Yang C 1998 ARPACK Users’ Guide: Solution of Large-Scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods vol 6 (University of Leeds: Siam) (https://caam.rice.edu/software/ARPACK/UG/ug.html)
[28] Cavcic I, Peeters F M and Berciu M 2010 Efficient numerical approach to inhomogeneous superconductivity: the Chebyshev–Bogoliubov–de Gennes method Phys. Rev. Lett. 105 167006
[29] Nagai Y, Ota Y and Machida M 2012 Efficient numerical self-consistent mean-field approach for fermionic many-body systems by polynomial expansion on spectral density J. Phys. Soc. Jpn. 81 024107
[30] Nagai Y, Shinohara Y, Futamura Y, Ota Y and Sakurai T 2013 Numerical construction of a low-energy effective hamiltonian in a self-consistent Bogoliubov–de Gennes approach of superconductivity J. Phys. Soc. Jpn. 82 094708
[31] Nagai Y, Shinohara Y, Futamura Y and Sakurai T 2017 Reduced-shifted conjugate–gradient method for a green’s function: efficient numerical approach in a nano-structured superconductor J. Phys. Soc. Jpn. 86 014708
[32] Nagai Y 2020 N-independent localized Krylov Bogoliubov–de Gennes method: Ultra-fast numerical approach to large-scale inhomogeneous superconductors arXiv:2001.02362
[33] Weile A, Wellein G, Alvermann A and Fehske H 2006 The kernel polynomial method Phys. Rev. Mod. Phys. 78 275–306
[34] Polzilli E 2009 Density-matrix-based algorithm for solving eigenvalue problems Phys. Rev. B 79 113112
[35] Leggett A J 1980 Cooper pairing in spin-polarized Fermi systems Rev. Mod. Phys. 52 339
[36] Machida K, Mizushima T and Ichioka M 2006 Generic phase diagram of fermion superfluids of unequal spin populations Phys. Rev. Lett. 97 120407
[37] Jinik S and Williams C J 2008 Polaron in a trapped Bose–Einstein condensate with unbalanced spin populations Phys. Rev. A 78 013613
[38] Imambekov A, Bolech C J, Lukin M and Demler E 2006 Breakdown of the local density approximation in interacting systems of cold fermions in strongly anisotropic traps Phys. Rev. A 74 053626
[39] Regal C A, Greiner M, Giorgini S, Holland M and Jin D S 2005 Momentum distribution of a Fermi gas of atoms in the BCS-BEC crossover Phys. Rev. Lett. 95 250404
[40] De Silva T N and Mueller E J 2006 Profiles of near-resonant population–imbalance trapped Fermi gases Phys. Rev. A 73 051602
[41] Haque M and Stoof H C T 2006 Pairing of a trapped resonantly interacting fermion mixture with unequal spin populations Phys. Rev. A 74 011602
[42] Bertaino G and Giorgini S 2009 Density profiles of polarized fermi gases confined in harmonic traps Phys. Rev. A 79 031616
[43] Chevy F 2006 Density profile of a trapped strongly interacting Fermi gas with unbalanced spin populations Phys. Rev. Lett. 96 130401
[44] Yi W and Duan L-M 2006 Trapped fermions across a Feshbach resonance with population imbalance Phys. Rev. A 77 033613
[45] Wang J, Guo H and Chen Q 2013 Exotic phase separation and phase diagrams of a Fermi-Fermi mixture in a trap at finite temperature Phys. Rev. A 87 041601
[46] Patton K R and Sheehy E 2020 Hartree–Fock–Bogoliubov theory of trapped one-dimensional imbalanced Fermi systems arXiv:2003.11659
[47] Bourdel T, Khaykovich L, Cubizolles J, Zhang J, Chevy F, Teichmann M, Tarruell L, Kokkelmans S J J M F and Salomon C 2004 Experimental study of the BEC–BCS crossover region in lithium 6 Phys. Rev. Lett. 93 090401
[48] Partridge G B, Li W, Liao Y A, Hulet R G, Haque M and Mueller E J 2010 Spin-imbalance in a one-dimensional Fermi gas Nature 467 567
[49] Fulde P and Ferrell R A 1964 Superconductivity in a strong–spin–exchange field Phys. Rev. 135 A530
[50] Larkin A I and Ovchinnikov Y N 1964 Nonuniform state of superconductors Zh. Eksp. Teor. Fiz. 47 1136
[51] Liu X-J and Liu X-J 2006 Mean-field phase diagrams of imbalanced Fermi gases near a feshbach resonance Phys. Rev. A 78 023601
[52] Cichy A, Kapcia K J and Ptok A 2019 Phase separations induced by a trapping potential in one-dimensional fermionic systems as a source of core–shell structures Sci. Rep. 9 6719
[53] Silotri S A 2010 Trap- and population-imbalance two-component Fermi gas in the Bose–Einstein-condensate limit Phys. Rev. A 81 013623
[54] Heidrich-Meisner F, Orso G and Feiguin A E 2010 Phase separation of trapped spin-imbalanced Fermi gases in one-dimensional optical lattices Phys. Rev. A 81 053602
[55] Kinunnen J, Jensen L M and Törmä P 2006 Strongly interacting Fermi gases with density imbalance Phys. Rev. Lett. 96 110403
[59] Jensen L M, Kinnunen J and Törmä P 2007 Non-BCS superfluidity in trapped ultracold Fermi gases Phys. Rev. A 76 033620
[60] Liu X-J, Hu H and Drummond P D 2007 Mean-field thermodynamics of a spin-polarized spherically trapped Fermi gas at unitarity Phys. Rev. A 75 023614
[61] Yanase Y 2009 Angular Fulde–Ferrell–Larkin-Ovchinnikov state in cold fermion gases in a toroidal trap Phys. Rev. B 80 220510
[62] Chen Y, Wang Z D, Zhang F C and Ting C S 2009 Exploring exotic superfluidity of polarized ultracold fermions in optical lattices Phys. Rev. B 79 054512
[63] Yoshida T and Yanase Y 2011 Rotating Fulde–Ferrell–Larkin-Ovchinnikov state in cold Fermi gases Phys. Rev. A 84 063605
[64] Ptok A 2012 The Fulde–Ferrell–Larkin-Ovchinnikov state in quantum rings J. Supercond. Nov. Magn. 25 1843
[65] Balatsky A V, Vekhter I and Zhu J X 2006 Impurity-induced states in conventional and unconventional superconductors Rev. Mod. Phys. 78 373
[66] Wang Q, Hu C-R and Ting C-S 2007 Impurity-induced configuration-transition in the Fulde–Ferrell–Larkin-Ovchinnikov state of a d-wave superconductor Phys. Rev. B 75 184515
[67] Ptok A 2010 The Fulde–Ferrell–Larkin-Ovchinnikov superconductivity in disordered systems Acta Phys. Pol. A 118 420
[68] Ptok A, Głowdzik S and Domasiński T 2017 Yu-Shiba-Rusinov states of impurities in a triangular lattice of NbSe2 with spin–orbit coupling Phys. Rev. B 96 184425
[69] Han Q 2009 A method of studying the Bogoliubov–de Gennes equations for the superconducting vortex lattice state J. Phys.: Condens. Matter 22 035702
[70] Smith E D B, Tanaka K and Nagai Y 2016 Manifestation of chirality in the vortex lattice in a two-dimensional topological superconductor Phys. Rev. B 94 064515
[71] Pérez L A, Galván C G and Wang C 2016 Vortices in Hubbard superconductors: A Bogoliubov–de Gennes approach J. Supercond. Nov. Magn. 29 285
[72] Chen Y and Ting C S 2002 Magnetic-field-induced spin–density wave in high-temperature superconductors Phys. Rev. B 65 180513
[73] Schmid M, Andersen B M, Kampf A P and Hirschfeld P J 2010 d-wave superconductivity as a catalyst for antiferromagnetism in underdoped cuprates New J. Phys. 12 053043
[74] Ptok A, Matka M M and Mierzejewski M 2011 Coexistence of superconductivity and incommensurate magnetic order Phys. Rev. B 84 094526
[75] Lin S-Z, Kim D Y, Bauer E D, Ronning F, Thompson J D and Movshovich R 2019 Interplay of the spin density wave and a possible Fulde–Ferrell-Larkin-Ovchinnikov state in CeCoIn5 in rotating magnetic field arXiv:1902.04797