Pauli Terms Must Be Absent In Dirac Equation

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ABSTRACT It should be of interest, whether Dirac’s equation involves all 16 basis elements of his Clifford algebra $Cl_D$. These include the 6 ‘tensorial’ $\sigma^{\mu\nu}$ with which the ‘Pauli terms’ are formed. We find that these violate a basic axiom of any *-algebra, when Dirac’s $\Psi$ is canonical. Then the Dirac operator is spanned only by the 10 elements $1, i\gamma_5, \gamma^\mu, \gamma^\mu\gamma_5$ (which don’t form a basis of $Cl_D$ because the $\sigma^{\mu\nu}$ are excluded).

Keywords: Quantum field theory, Dirac equation, Clifford algebra.

1 Motivation and conclusions

In Dirac’s equation

$$i\partial x^\mu \Psi = B \Psi \quad \text{with} \quad B := \gamma_\mu \frac{\partial}{\partial x^\mu}$$ (1.1)

the Bose field $B$ is a member of the Clifford algebra $Cl_D$. Hence it can be written as

$$B = S^+ + i\gamma_5 S^- + \gamma^\mu V_+^\mu + \gamma^\mu\gamma_5 V_-^\mu + \sigma^{\mu\nu} T_{\mu\nu}.$$ (1.2)

Here $S^\pm, V_\pm, T_{\mu\nu}$ are matrices which act on the flavors and colors of $\Psi$ (the Dirac field for leptons and quarks). In the excellently verified Standard Model, the matrices

$$S := S^+ + i\gamma_5 S^- \quad \text{and} \quad V_\mu := V_+^\mu + \gamma_5 V_-^\mu$$ (1.3)

contain all the fields of Higgs and Yang-Mills. Vice versa, the Standard Model requires (1.2) to contain the 10 basis elements $1, i\gamma_5, \gamma^\mu, \gamma^\mu\gamma_5 \in Cl_D$, but not the further $\sigma^{\mu\nu}$ or $\sigma^{\mu\nu}\gamma_5$ (6 of which are linearly independent).

Thus we encounter the question, to what extent (1.2) can involve $T_{\mu\nu} = -T_{\nu\mu}$. For this problem it is irrelevant whether $T_{\mu\nu}$ is a separate ‘tensor potential’ or a multiple of Maxwell’s $F_{\mu\nu} := A_{\mu,\nu} - A_{\nu,\mu}$ (as proposed by Pauli [1]) or of its dual $\varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$. Hence we always call $\sigma^{\mu\nu} T_{\mu\nu}$ the ‘Pauli

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term’ of (1.2). In rigorous, but not trivial ways, we find that $T_{\mu\nu}$ must be absent for a very basic reason: For the members $a, b, \ldots$ of any *-algebra and their conjugates $a^\dagger, b^\dagger, \ldots$, one postulates $(a^\dagger b)^\dagger = b^\dagger a$. This would be violated by any Pauli term. Hence we must demand

$$T_{\mu\nu} = 0 \quad \text{in order to keep} \quad (a^\dagger b)^\dagger = b^\dagger a. \quad (1.4)$$

In other words, our fields will not generate a *-algebra, unless (1.2) is restricted by (1.4). This clear result has not been found in the literature, because a familiar reciprocity [2] is generally misnamed a theorem, whereas we prove it to be a condition, which excludes (1.2) unless it satisfies (1.4).

Showing in Sections 2 and 3 the adopted foundations, we indicate the proof of (1.4) briefly in Section 4, and more elaborately in Appendix A. It rests on a reciprocity condition, which is discussed in Appendix B. Calling this a ‘relation’ [2], one generally suggests that it holds without restricting the Bose fields to be prescribed. That misnomer may have caused the absence of (1.4) in the literature.

2 Gauge theory from Clifford algebra

For the vector field from (1.3) we must admit the gauge transformation

$$V_{\mu} \rightarrow e^{-i\omega} (V_{\mu} - i\partial_{\mu}) e^{i\omega} \approx V_{\mu} + \omega_{\mu} + i[V_{\mu}, \omega]. \quad (2.5)$$

In order to state that $V_{\mu}$ and $\omega$ are hermitian matrices, we write them

$$V_{\mu}(x) = t_Y V^Y_{\mu}(x) = V^\dagger_{\mu}(x) \quad \text{and} \quad \omega(x) = t_Y \omega^Y(x) = \omega(x)^\dagger. \quad (2.6)$$

While the constant matrices $t_Y$ act on flavors and colors, they contain all coupling constants and the $\gamma_5 := i\gamma^0\gamma^1\gamma^2\gamma^3$ from Dirac’s Clifford algebra $\text{Cl}_D$. We generate this $\text{Cl}_D$ by $\gamma^{(\mu, \rho)} = \eta^{\mu\rho}$ from

$$\gamma^\dagger_{\mu} = \gamma^{-1}_{\mu} = \gamma^\mu = \gamma^\mu \in \text{Cl}_D. \quad (2.7)$$

A transformation similar to the homogeneous part of (2.5) follows for the $S$ of (1.3). In order to prove (1.4), however, we must initially use (1.2) with (1.3) in the form

$$B = S + \gamma^\mu V_{\mu} + \sigma^{\mu\nu} T_{\mu\nu}, \quad \text{where} \quad T_{\mu\nu} \neq 0. \quad (2.8)$$

This together with $(a^\dagger b)^\dagger = b^\dagger a$ will in Section 3 yield a contradiction, which then proves (1.4). That proof holds in Quantum Induction [3] where the canonical relations

$$[\Psi(x), \Psi(0)^\dagger]_+ \delta(x^0) = \delta(x) \quad \text{and} \quad [\Psi(x), \Psi(0)^T]_+ \delta(x^0) = 0 \quad (2.9)$$

together with Dirac’s equation (1.1) are fundamental. Its possible validity under presumptions different from (1.1) through (1.3) is discussed in Appendix [3].
3 Short distance representation

For the bilocal, time ordered Dirac matrix

$$b(x, z) := (4\pi)^2 T \Psi(x + z) \overline{\Psi}(x - z),$$  \hfill (3.10)

(1.1) and (2.9) provide the differential equation

$$\{\slashed{D} + \slashed{D} + 2iB(x + z)\} b(x, z) = 2\pi^2 \delta(z) = i\partial_z \frac{z}{z - 3} - \frac{i}{z - 3},$$  \hfill (3.11)

where \( \frac{z}{z - 3} := (z^2 - i\epsilon)^{-2} \) with \( \epsilon \to +0 \). The representation used here for \( \delta(z) := \delta(z^0)\delta(z^1)\delta(z^2)\delta(z^3) \) follows directly from the familiar

$$\Box (z^2 - i\epsilon)^{-1} = (2\pi)^2 i\delta(z).$$

Writing (3.10) as

$$b(x, z) = i\frac{z}{z - 3} + (C^{-2} + C^{-1} + r^0)(x, z),$$  \hfill (3.12)

let us anticipate that the \( C^h \) can be made homogeneous in the sense that

$$C^h(x, \lambda z) = \lambda^h C^h(x, z) \quad \text{for} \quad h = -2, -1 \quad \text{and} \quad \lambda \in C.$$  \hfill (3.13)

Also using the ‘Taylor representation’

$$B(x + z) = B(x) + z^\mu B_{\mu}(x) + R(x, z)\frac{z}{z - 3}, \quad \text{where} \quad R(x, 0) = 0,$$  \hfill (3.14)

we can split (3.11) into

$$\slashed{D} C^{-2}(x, z) = 2B(x)\frac{z}{z - 3},$$  \hfill (3.15)

$$\slashed{D} C^{-1}(x, z) = 2z^\mu B_{\mu}(x)\frac{z}{z - 3} - \{2iB(x) + \slashed{D}\} C^{-2}(x, z),$$  \hfill (3.16)

$$\slashed{D} r^0(x, z) = \ldots - 2iR(x, z)z^{-2}.$$  \hfill (3.17)

Here the dots symbolize infinitely many unknown terms; they will be irrelevant because they are not more singular than \( \frac{z}{z - 3} \) (for \( z \to 0 \) at \( z^2 \neq 0 \)). As one easily verifies, (3.13) with (2.8) can be solved by

$$z^4 C^{-2}(x, z) = 2z^\mu \overline{T}_\mu(x) - z^2 S(x) + \frac{z^2}{z - 3},$$  \hfill (3.18)

This clearly satisfies (3.13) and due to Appendix A is the only solution of (3.15) which does so. Since it makes the right side of (3.16) homogeneous in \( z \) of the order \( h = -2 \), we can choose also \( C^{-1}(x, z) \) to obey (3.13). On the right side of (3.17), we have omitted terms which due to (3.18) are as homogeneous as \( \frac{z}{z - 3} \) (or less singular).

Thus (3.17) can be solved by an \( r^0 \) which (due to \( R(x, 0) = 0 \)) satisfies

$$z^\mu r^0(x, z) \to 0 \quad \text{for} \quad z \to 0.$$  \hfill (3.19)
Therefore, we can include in $r^0$ those terms from $C^{-2}$ and $C^{-1}$ which are left arbitrary by (3.15) and (3.16) because they are independent of $z$. In order to learn much more about $r^0(x, z)$ than (3.19) shows, one would need ‘outer’ boundary conditions (at large $z$). These could be obtained from heat kernels (HK); but here the ‘inner’ condition (by (2.9) giving to (3.11) its right side) has been sufficient. No obstruction to solving (3.11) has been encountered in (3.18) or in Appendix A. Neither would any arise if we would extend our recursion for (3.12) or invoke HK[4]. Taking the Dirac adjoint of (3.10), however, we obtain the ‘reciprocity’ condition

$$z^4 b(x, z) = z^4 b(x, -z) \quad \text{due to} \quad (\Psi^\dagger \Psi^\dagger)^\dagger = \Psi^\dagger \Psi^\dagger.$$ (3.20)

The latter exemplifies a general rule for *-algebras.

4 Reciprocity as a condition

The $z^4$ in (3.20) removes the denominators of the terms in (3.12), so that the hermitian conjugation affects only their numerators. Hence these must (for $h = -2, -1$) satisfy

$$z^4 C^h(x, z) = z^4 C^h(x, -z) \quad \text{and} \quad z^4 r^0(x, z) = z^4 r^0(x, -z),$$ (4.21)

because all three are linearly independent. In (4.21), we did not mention the leading $i\xi^{-3}$ of (3.12), because it satisfies (3.20) trivially. Since (2.9) equals its adjoint $B = \mathcal{S} + \gamma^\mu V^\dagger_\mu + \sigma^\mu\nu T^\mu\nu$, that reciprocity is also verified for (3.18).

Instead of (4.21) with $h = -1$, however, in Appendix A we find

$$z^4 C^{-1}(x, z) \neq z^4 C^{-1}(x, -z) \quad \text{unless} \quad T^\mu\nu = 0.$$ (4.22)

Hence it is misleading when one calls (3.20) a reciprocity ‘theorem’ [2], as if it were fulfilled for arbitrarily prescribed Bose fields (2.8). Since (3.18) satisfies (4.21) even with $T^\mu\nu \neq 0$, we also see that (1.4) cannot be derived as long as one only examines that solution of (3.13). In other words, we do not know a way of reaching the conclusion (1.4) without noting that the solution of (3.10) provides (4.22).

By (1.4), however, the $C^{-1}(x, z)$ solving (3.16) is simplified greatly; and the further parts of (3.12) are shortened still more drastically. Hence superfluous work is done by those who pursue higher terms of (3.12) without inserting (1.4) quickly. They solve the differential equation (3.11) correctly but in excessive generality. Thus they miss the fact that (3.10) must also satisfy (3.20), which is a non-trivial condition not expressed by (3.11).
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A Appendix: Linear Differential Equations

A.1 Exactly homogeneous solutions

In (3.11) we have used

\[ i \partial / z z / - 3 = 2 \pi^2 \delta(z) \quad \text{for} \quad \delta = (z^{-2})^2 /, \]

where \( z_2 = (z^2 - i \epsilon)^{-1} \) with \( \epsilon \to +0 \). These obviously provide

\[ \partial / z z / - 3 = \gamma^\mu (z)^{-3}, \quad \partial / z z / - 2 = -2 \delta / (z^{-3}), \]
\[ \partial / z z / - 3 \sigma_{\mu \nu} = \gamma^\rho (z)^{-3} \sigma_{\mu \nu} \gamma_\rho = 2 \sigma_{\mu \nu} (z)^{-3}, \]  

which are derived most easily, when one uses (A.23) as often as possible. Thus (3.18) makes

\[ \partial / C_3 (x, z) = 2 \gamma^\mu (z) \nabla (z)^{-3} + 2 \delta / (z^{-3}) S(x) + 2 \sigma_{\mu \nu} T_{\mu \nu} (x), \]

so that (3.13) with (2.8) is fulfilled.

All these calculations of course do not make sense on the cone \( z^2 = 0 \). Hence throughout this paper we assume \( z^2 \neq 0 \), as we must clearly do in (3.12) through (3.17). This is also true for the limits with \( z \to 0 \), as needed in (3.19) and in the proof of (A.23). Hence such limit transitions can proceed on any path which ends at \( z = 0 \), except that it must not touch the cone \( z^2 = 0 \).

While (3.13) with (2.8) is due to (A.23) satisfied by (3.18), its most general solution follows when we add any matrix \( H \) which fulfills the homogeneous Dirac equation \( \partial / H = 0 \). For choosing this \( H \) we need the

**Singularity Theorem**: Every Poincaré covariant member \( H \) of Dirac’s Clifford algebra \( \mathcal{C}L_D \), which solves

\[ \partial / H(z) = 0 \quad \text{in a neighborhood of} \quad z = 0, \]

becomes for \( z \to 0 \) with \( z^2 \neq 0 \) either more singular than \( \delta / z^{-3} \) or less than \( \delta / z^{-1} \), hence yields either

\[ \lim_{z \to 0} \delta / H(z) = \infty \quad \text{or} \quad \lim_{z \to 0} \delta / z^\mu H(z) = 0. \]

One proves this easily when \( H \) depends only on \( z \). The general proof is lengthy and scarcely of interest to physicists; hence we shall show it whenever requested.
Obvious solutions of (A.26) and (A.27) are all \( H \) which do not depend on \( z \). Already the solution of (3.15) by (3.18) says that (3.13) with \( h = -2 \) can be satisfied. The theorem (A.27) proves that (3.18) yields the only \( C^{-2} (x, z) \) which does so (such that (3.13) and (3.14) make (3.18) necessary). It also shows that the leading term of (3.12) is determined uniquely. In the same way, (A.27) says that the solution of (3.16) and (3.13) will be unique when such a \( C^{-1} (x, z) \) can be found at all.

A.2 Relevant short distance terms

Inserting (2.8) and (3.18) in (3.16), we obtain

\[
\Psi C^{-1} (x, z) = 2 z^\mu \gamma^\rho \phi^{-3} (\nabla_{\rho, \mu} - \nabla_{\mu, \rho} - 2 i \nabla_{\rho} \nabla_{\mu}) \\
+ z^{-3} \gamma^{\rho} (S^\dagger_{\rho, \mu} + 2 i \nabla_{\mu} S^\dagger) + 2 \phi^{-3} z^\rho \phi (S^\dagger_{\rho, \mu} - 2 i \nabla_{\mu} \nabla_{\rho}) \\
+ 2 iz^{-2} S S^\dagger + 2 \phi^{-3} \gamma^{\rho \mu} \phi ST_{\rho \mu} \\
+ 2 \phi^{-3} \gamma^{\rho, \mu} \phi \phi^{\rho, \mu} T_{\rho \mu, \rho} + 2 \phi^{-3} \gamma^{\rho \mu} \phi \phi^{\rho, \mu} T_{\rho \sigma} \\
+ \gamma^{\rho \mu} \phi \phi^{\rho, \mu} \phi \phi^{\rho, \mu} (2 \phi \phi^{\rho, \mu} T_{\rho \mu, \rho} - 2 \phi^{-2} \gamma^{\rho \mu} \phi \phi^{\rho, \mu} T_{\rho \mu, \rho}) \\
(A.28)
\]

with \( \gamma^{\mu \nu} := \gamma^{\mu | \nu} = -i \sigma^{\mu \nu} = \gamma^{\nu | \mu} - \eta^{\mu \nu} \). Here all the fields \( S, \nabla_{\mu}, T_{\rho \mu} \) and their partial derivatives are localized at \( x \); hence this parameter has been suppressed in the notation. Although (A.28) is for \( C^{-1} \) a linear differential equation in \( z \) of the first order, deriving a solution was tedious; but after such a \( C^{-1} \) has been found, only differentiations are needed to verify its validity. Only a few readers, however, would perform this extremely easy but time-consuming task; thus let us merely say that the \( C^{-1} (x, z) \) solving the differential equation (A.28) is twice as lengthy as this.

It implies (4.22) and thus contradicts (4.21), unless all 5 local field polynomials

\[
Z^{1}_{\rho \sigma, \tau} := \{ T_{\rho \sigma, \tau}, T_{\rho, \mu, \mu} \} + , \\
Z^{2}_{\rho \sigma, \tau} := T_{\rho \sigma} \nabla_{\tau} + \nabla_{\tau} T_{\rho \sigma}, \\
Z^{3}_{\rho} := \{ \nabla_{\mu}, T_{\mu \rho} \} \\
Z^{4}_{\rho \sigma} := T_{\rho \sigma} S^\dagger + S T_{\rho \sigma}, \\
Z^{5}_{\rho \sigma, \tau} := T_{\rho \sigma, \tau} + i \nabla_{\tau} T_{\rho \sigma} - i T_{\rho \sigma} \nabla_{\tau} \\
(A.29)
\]

satisfy

\[
Z^{n}_{\rho \sigma, \tau} (x) = 0. \\
(A.30)
\]

These derivations have been rigorous, because we did not admit any approximation (rarely possible in physics). Our solution (1.4) is clearly the only which holds irrespective of \( S \) and \( \nabla_{\mu} \). If (A.30) were mathematically
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solvable by any $T_{\mu\nu} \neq 0$ (a case we can’t examine exactly), it would restrict $S$ and $V_\mu$ in complicated and extremely unphysical ways.

Within the permitted size of this paper, we can’t show the complete proof for the necessity of (A.30); the known extension to quantum fields [3] would enlarge it enormously. A result as simple as (1.4), however, should find a much shorter proof. Hence we would prefer to delay the publication until that simplicity is achieved. If we would not show the result (1.4) and indicate our lengthy derivation from (3.20), however, it would be hard to get anyone interested in such a problem.

B The Reciprocity Violation

Let us finally collect further remarks about our result (1.4) and its absence from the literature:

(a) The reciprocity condition (3.20) has been called [2] a relation, as if it had been proved with (1.1) containing the most general Bose field (1.2).

(b) We had to perform extensive computer algebra for the derivation of $C^{-1}(x, z)$ from its differential equation (A.28). Still more would be required if one were to make the compact result from HK explicit. Neither is needed any longer, because it is easy to verify a known solution of any differential equation, no matter how hard its integration had been. Because of (a), however, nobody found this tedious search worthwhile.

(c) In the mathematics of HK, the boundary conditions [4] at large $z$ are presently more interesting than the behavior of (3.12) at small $z$.

(d) It is fashionable, instead of the differential equation (3.11) to solve a related integral equation, with an ‘inner’ boundary condition given by the right side of (3.11) and a purely mathematical condition at some outer boundary (where $z$ is large or infinite). For our problem, no choice of the latter makes sense because the result (A.30) depends only on (3.11) and its boundary condition at $z = 0$. Why should we use a physically irrelevant integral equation for a conclusion which is completely determined by a differential equation together with a single, well justified boundary condition?

(e) Methods of HK have been initiated [5] for classical field theories, where the reciprocity arises from a symmetry of their Green functions.
(f) In many treatments by HK, not only the Bose field $B$ but also Dirac’s $\Psi$ is non-quantized (or not even mentioned). Then (3.11) is regarded as an equation for a classical Green function $b(x, z)$, not related to any quantum field such as (3.10). In that approach, one hardly sees whether a reciprocity (3.20) should be desired.

(g) Under infinite renormalizations, (1.1) and (2.9) and therefore (3.12) break down. Hence we can’t prove (1.4) in familiar settings (although it might be true even there).

(h) Any significant $\mathcal{T}_{\mu\nu} \neq 0$ would damage the excellent verification of the Standard Model [7] by the magnetic moment of the electron. This agreement [8] had formerly been regarded as a brilliant confirmation of renormalized QED. Under the present philosophy [9] of ‘effective’ actions, however, it is an unimportant result of imprecise measurements.

(i) Instead of (3.10), one often uses $\beta(e, t) := (4\pi)^2 T \Psi(e+t) \overline{\Psi}(e) = b(e + \frac{1}{2}t, \frac{1}{2}e)$ with the eccentric coordinates $e = x - z$ and $t = 2z$. These simplify (especially under gravity) the derivation of (3.11), but make the analysis of (3.20) complicated.

(j) The singularity at $z^2 = 0$ makes (3.12) dependent on the time ordering of (3.10). Hence (3.20) can’t simply be written $b(x, z) = b(x, -z)$ because an hermitian conjugation reverses the time order.

(k) Wherever basic ‘tensor potentials’ $\mathcal{T}_{\mu\nu}$ have found any attention, one has coupled them to each other or further Bose fields [10], leaving their interaction with Dirac’s $\Psi$ open.

(l) Whenever ‘tensor couplings’ are mentioned in phenomenology [11], it is unclear whether they are fundamental or caused by bound states or by ‘radiative’ corrections.

(m) The differential or integral equation for (3.10) can ‘mathematically’ be solved [4] without any concern about the reciprocity (3.20) needed in physics. However, (3.11) without (3.20) does not exhaust the contents of (3.10).

(n) The leading terms $i\xi^{-3}$ and $C^{-2}(x, z)$ of (3.12) satisfy (3.20) even when (2.8) has $\mathcal{T}_{\mu\nu} \neq 0$. Hence the contradiction between (4.21) and (4.22) is not recognized until one also determines $C^{-1}(x, z)$.

(o) A further reason for the usual rejection of (1.4) may be that such a clear result deserves a simple derivation. Instead our
proof has required the lengthy (but straightforward) deduction of the differential equation (A.28) and its explicit solution. That \( C^{-1}(x, z) \) is twice as long as (A.28) and therefore not shown here; but we hope that others can simplify our arguments.

(p) In order to examine (3.20) completely, the reciprocity (4.21) should also be checked for the ‘remainder’ \( r^0(x, z) \). Analyzing those parts of it which in \( z \) are homogenous of the orders \( h = 0 \) and \( h = 1 \), we have not found any restriction beyond \( T_{\mu \nu} = 0 \) (which simplifies those parts enormously). Our local approach (without outer boundary conditions) cannot extend that result to all orders. This should be taken as an incentive to treat the condition (3.20) globally, but not as excuse for discarding our result (4.22). Doing so would correspond to ignoring the singular part of a Laurent series until all its orders are known.

(q) For many authors, the Higgs field does not contribute to \( E \), because they attach it in isospinors to \( \Psi \).

(r) Many authors prefer two-component spinors instead of Dirac’s \( \Psi \).

(s) Some authors use other notations for Dirac matrices, for instance \( \alpha, \beta \) instead of \( \gamma^\mu \) or explicit \( 4 \times 4 \) squares.

(t) For Dirac’s \( \gamma^\mu \), one sometimes uses representations in which \( \gamma^\mu \) is not its own adjoint or the \( \beta \) in \( \Psi = \Psi^\dagger \beta \) differs from \( \gamma^0 \).

(u) From (2.8) we derived (1.4) by ‘reductio ad absurdum’, which not every mathematician appreciates.

(v) Readers may dislike (1.4), because beautiful theories are no longer expected to be simple but to offer rich mathematical structures.

(w) Instead of the Dirac equation for physics (which is of first order in Minkowski space), mathematicians prefer the elliptic equation given by its iteration in Euclidean space.

(x) One often uses Dirac’s Clifford algebra without any basis, hence not separating the scalar, vectorial and tensorial parts of \( B \).
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