A fuzzy multiple-attribute decision-making modelling for vulnerability analysis on the basis of population information for disaster management

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Research activity and published literature on the reliability and vulnerability analysis of urban areas for disaster management has grown tremendously in the recent past. Population information has played the most important role during the entire disaster management process. In this article, population information was used as the evaluation criterion, and a fuzzy multiple-attribute decision-making (MADM) approach was used to support a vulnerability analysis of the Helsinki area for disaster management. A kernel density map was produced as a result that showed the vulnerable spatial locations in the event of a disaster. Model results were first validated against the original population information kernel density maps. In the second step, the model was validated by using fuzzy set accuracy assessment and the actual domain knowledge of the rescue experts. This is a novel approach to validation, which makes it possible to see how and if computer decision-making models compare to a real decision-making process in disaster management. The validation results showed that the fuzzy model has produced a reasonably accurate result. By using fuzzy modelling, the number of vulnerable areas was reduced to a reasonable scale and compares to the actual human assessment of these areas, which allows resources to be optimised during the rescue planning and operation.

Keywords: fuzzy MADM; disaster management; population information modelling; GIS

1. Introduction

Natural hazards, disasters, and the increasing number of terrorist attacks are causing insecurity for society. In rescue operations, saving people’s lives is always the first priority. If the disaster occurs over a large area and the government has a limited amount of resources, rescue personnel needs to prioritise interventions at the most vulnerable locations ahead of other locations in order to save more lives. Therefore, there is a need to develop spatial vulnerability models to support rescue planning and optimise operations in the event of a disaster. In this article, we address this problem by developing a fuzzy multiple-attribute decision-making (MADM) spatial vulnerability model and evaluate it against the expert knowledge from the real rescue decision-making process.

Our model is based on population information, which plays an important role in the disaster management process (Figure 1). For example, it can be used for preparedness planning, i.e. to decide where to set up a rescue facility so that rescue personnel can save

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most lives with available resources. Another example is the evacuation process, where population information can help the rescue personnel to estimate the resources needed to transport the inhabitants from the endangered area.

Population information plays an important role in defining vulnerability for natural hazard and disaster management (Lewis 1999, Wisner et al. 2004). Various types of population information models have been developed to support vulnerability analysis. In this context and specifically for our study area in Finland, there are several studies that are of interest. Krisp and Karasová (2005) used population density maps to support fire and rescue operations in Helsinki area. Their approach is limited, as a population density map can only represent the location of the people and other relevant characteristics of population are not taken into account. Another similar study for Helsinki area (Špatenková and Stein 2010) applies statistical point pattern analysis to explore the causes of domestic fires based on population and building information. In this study the temporal aspect and individual characteristics of people were taken into consideration. However, the use of results is limited to domestic fires and the uncertainty of population information was not considered in the analysis. Another recent study (Tillander et al. 2010) compares statistical risk models to spatial risk models in order to predict accident rates in Finland. They found that inclusion of socio-economic information into spatial models improved prediction results, but the uncertainty in the risk modelling process and in the data needs to be taken into account.

There are two types of decision-making processes in fire and rescue operations: rapid decision-making and long-term decision-making (Castrén et al. 2007). Rapid decision-making occurs in emergency situations, and the decision will be made swiftly on the basis of how well the persons in charge of the situation know the area(s) at risk. Long-term decision-making is used in cases when the rescue mission is not extremely urgent and there is time to analyse the situation. In the long-term decision-making process, the rescue personnel will therefore not only consider the population density of the affected areas but also the individual characteristics of local population, such as age, disabilities and/or
health conditions, and how these variables affect the ability of individuals to deal with a disaster (Cutter et al. 2003, Castrén et al. 2007). Because of the difficulties in combining all these different types of population variables, the long-term decision-making process can be complex and time-consuming.

Another issue in the long-term decision-making process is the uncertainty or vagueness of population information. For instance, it is difficult to say exactly what number of people living in a particular area constitutes a high-risk area in disaster management. Moreover, this evaluation often depends on the implicit knowledge used in the experts’ decision-making process which is difficult to quantify. Therefore it is reasonable to use a fuzzy approach in the definition of what constitutes a high-risk area or a low-risk area in vulnerability models rather than setting crisp boundaries.

The objective of this study is to develop a model which can identify vulnerable locations in Helsinki area by combining various population information variables in order to support the long-term decision process in rescue operations. The model uses fuzzy MADM modelling and integrates the knowledge of the rescue experts into the process, which is an important part of the spatial decision-making process (Yuan 1997). We use fuzzy logic to address the uncertainty in population information and experts’ knowledge together with MADM procedure that allows us to combine population information variables into a measure of vulnerability that can be used to optimise the efficiency of the long-term decision-making process. We evaluate our model using an innovative methodology to validate the model results versus the actual knowledge of the experts.

The rest of the article is organised as follows: Section 2 gives an overview of the theoretical background relevant to development of our model. The methodology is presented in Section 3, followed by the evaluation of the methods in Section 4. Sections 5 and 6 present conclusions and discussion, respectively.

2. Theoretical background
In this section we introduce the mathematical concepts relevant to our methodology: the fuzzy MADM model and the fuzzy accuracy assessment.

2.1. Fuzzy MADM model
Multi-criteria decision-making (MCDM) analysis is a mathematical framework that provides tools for analysis of choice alternatives in planning process. It is a method for decision support where a number of different criteria are combined to meet one or several objectives and help to make a decision about a problem. The decision rule in the MCDM is a procedure that combines criteria, often representing conflicting and complementary objectives, into a single composite index. The value of this index is then used as the basis for a decision. This method has been frequently combined with GIS to solve spatial decision-making problems (Carver 1991, Malczewski 2006).

In a review (Malczewski 2006), the GIS-based multiple-criteria decision-making (GIS-MCDM) analyses are classified into decisions under conditions of certainty and uncertainty. Of interest to our study are 49 articles listed in this review that develop a fuzzy version of a GIS-MCDM that takes into account the uncertainty in the information and process. Fuzzy theory has been frequently applied to GIS-MCDM analysis in the fields of land allocation, land evaluation, and water management (Jiang and Eastman 2000, Joerin et al. 2001, Makropoulos et al. 2003). However, in the field of disaster management, only a few studies were found. For instance, Rashed and Weeks (2003)
combined an MCDM analysis approach with fuzzy modelling to assess the vulnerability analysis of an urban area in the modelling of an earthquake hazard. They see a spatial MCDM approach to urban vulnerability as a process that combines and transforms spatially referenced data into a resultant vulnerability score.

The MADM analysis is a subfield of MCDM. In MADM the preference decisions are made over the available alternatives that are characterised by multiple, often conflicting attributes (Hwang and Yoon 1981). The original MADM uses crisp logic and does not allow for uncertainty. To address this, we incorporate fuzzy logic and fuzzy set theory into the MADM analysis.

Fuzzy set theory was first introduced by Zadeh (1965) as a mathematical theory of vagueness. A fuzzy set $A$ in $X$ is characterised by a membership function (MF). If $X$ is the universe of discourse, and its elements are denoted by $x$, then the fuzzy set $A$ in $X$ is defined as a set of ordered pairs: $A = \{x, \mu_A(x)|x \in X\}$, where $\mu_A(x)$ is called the MF of $x$ in $A$. It maps each element of $X$ to a membership value between 0 and 1.

The fuzzy set theory in the field of MADM is justified when the intended goals or their attainment cannot be defined crisply but only as a fuzzy set. Bellman and Zadeh (1970) introduced decision-making in a fuzzy environment. This is defined as:

$$D = G \cap C$$

(1)

where $G$ is the fuzzy goal, $C$ is the fuzzy constraint, and $D$ is the fuzzy decision. $G$ and $C$ are fuzzy sets represented by their respective MFs. The MF of the fuzzy set $D$ representing the decision is then defined as:

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x))$$

(2)

The maximising decision is then defined as follows:

$$\max_{x \in X} \mu_D(x) = \max_{x \in X} \min(\mu_G(x), \mu_C(x))$$

(3)

To generalise this to more than one fuzzy goal and fuzzy constraint, we take $k$ fuzzy goals and $m$ fuzzy constraints, and define the fuzzy decision as:

$$D = G_1 \cap G_2 \cap \ldots \cap G_k \cap C_1 \cap C_2 \cap \ldots \cap C_m$$

(4)

Then the corresponding maximising decision is:

$$\max_{x \in X} \mu_D(x) = \max \min(\mu_{G_1}(x), \ldots, \mu_{G_k}, \mu_{C_1}, \ldots, \mu_{C_m}(x))$$

(5)

In our MADM model we use triangular and trapezoidal MFs, which are shown in Figure 2 together with corresponding fuzzy rules. The mathematical form of a triangular fuzzy MF curve is shown by Equation (6) (Kaufmann and Gupta 1985). The parameters $a$, $b$, $c$ (with $a < b < c$) determine the $x$ coordinate of the three corners of the triangular MF.
A trapezoidal curve indicates the MF of a vector $x$, defined by the scalar parameters $a$, $b$, $c$, and $d$ is shown by Equation (7) (Ai et al. 2013). The parameter $a$, $b$, $c$, $d$ (with $a < b < c < d$) determine the $x$ coordinates of the four corners of the trapezoidal MF. Trapezoidal MF reduces to a triangular MF when $b$ is equal to $c$.

$$
\text{trapezoid}(x; a, b, c, d) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a < x \leq b \\
1, & b < x \leq c \\
\frac{d-x}{d-c}, & c < x \leq d \\
0, & d < x 
\end{cases}
$$

(7)

In this study, we use fuzzy logic to capture the experts’ knowledge, and the experts’ decisions about the vulnerability value of each building. These decisions are represented by using fuzzy rules, as explained earlier. In this process, the input attributes and output values were described in terms of linguistic variables, such as high, middle, and low and these linguistic variables were then represented by a fuzzy MF. Fuzzy rules such as ‘If $X$ is high and $Y$ is high then $Z$ is high’ were then used to obtain the relationship between input and output. These rules work as follows (Zimmermann 2001): the if part (antecedent part) of the rule partitions the input space into a number of fuzzy regions (MFs), and the then part (consequent part) describes the behaviour of the system in these fuzzy regions. The word and in the fuzzy rule is a fuzzy operator and used to describe a fuzzy intersection or conjunction of the input MF. In this study, the experts do not make decisions about choosing the most vulnerable building in a disaster. Instead, we use a fuzzy logic to...
capture the experts’ knowledge about the vulnerability value of each building that are based on input population information are represented by using fuzzy rules.

2.2. Fuzzy set accuracy assessment

The performance of spatial models is often assessed using evaluation methods stemming from remote sensing, such as confusion matrices and accuracy indices derived from these matrices (Campbell 2008). These methods compare results of a model versus the so-called ground truth, which represent the real values of the modelled index at certain locations. The performance of the model is then described in terms of the frequency of correct versus incorrect model results at each location. Such evaluation methods work well for models where resulting categories are defined using crisp logic, but are not directly applicable to fuzzy models. In our work we therefore use the fuzzy set accuracy assessment (Gopal and Woodcock 1994) as an evaluation alternative.

Let \( X \) be a finite universe of discourse, which in this article is the set of buildings in the study area. Let \( \zeta \) denote the finite set of attribute MF classes (or categories) assigned to the data in \( X \); let \( m \) be the number of categories \(|\zeta|=m\). For each \( x \in X \), we define \( \chi(x) \) as the MF classes assigned to \( x \). The set

\[
M = \{(x, \chi(x))|x \in X\}
\]

defines the data. The subset \( S \subset X \) of \( n \) data is used. A fuzzy set

\[
A_c = \{(x, \mu_c(x))|x \in S\}
\]

is associated with each class \( C \in \zeta \) where \( \mu_c(x) \) is the characteristic or MF of \( C \).

To implement a decision, the model uses a Boolean function \( \sigma \) that returns a result of 0 or 1 based on whether \( x \) belongs to the class \( C \) with respect to the matrix \( A \). We define \( MAX \) function which can be used as \( \sigma \) and is illustrated below:

\[
MAX(x, C) = \begin{cases} 
1 & \text{if } \mu_c(x) \geq \mu_{c'}(x) \text{ for all } \forall C' \in \zeta \\
0 & \text{otherwise}
\end{cases}
\]

That is, \( MAX(x, C) \) is 1 if the numeric scale of the MF for \( x \) in category \( C(\mu_c(x)) \) is the maximum among all the categories \( \mu_c(x) \).

In our model the MFs are derived from linguistic scales and further converted into numeric scale provided by the experts. This approach is used to evaluate the accuracy of the labels which are assigned to the observation objects, in our case buildings. The data set used for the analysis is an \( n \times m \) matrix. The \( n \) rows in the matrix represent buildings and the \( m \) columns represent the MF categories which are derived from experts. Expert-derived categories are compared versus the model-derived categories and the performance of the model is described in terms of the frequency of match and miss-match between the two category types.

3. Methodology

In this article, a fuzzy MADM model was used to create vulnerability maps for disaster management. The evaluation criteria/attributes of the alternatives considered in this fuzzy
ADAM model consisted of the total number of people (TP) and the total number of children (TC) (aged 6 years old or less) inside a building. We set the cut-off for the children category to 6 years, since in Finland, primary school starts at age 7 and safety education is provided from the first year of school. In the safety education, children learn basic skill of safety and are considered able to independently respond to accidents. Therefore, the variable ‘children’ was defined as children under age 7, as the population who cannot respond to the crisis and who needs to be prioritised by the rescue personnel.

In our model, the number of people in a building (TP) was used as a proxy of how densely a building is inhabited. The number of children (TC) was considered as a proxy of necessity of rescue intervention. The vulnerability of a building was therefore defined by these two variables: the higher the TP and TC, the more vulnerable the building, where the exact criteria to derive the level of vulnerability were based on fuzzy logic and experts’ knowledge.

In order to validate the model, we used two separate methodologies. First, the fuzzy model result was compared to maps of the original population information in order to evaluate the spatial performance of the model. In the second step, we used the fuzzy set accuracy assessment approach to compare the results of the method with the actual domain knowledge of the experts who are frequently in situations where such decision-making has to be performed on the spot in the real decision-making process.

3.1. Data

Data used in this experiment come from the residential building register of the city of Helsinki in Finland. This register contains information on the number of persons who registered a certain building as their home address. This number was our TP variable. The register also contains age information on the building residents, which allowed us to extract the TC variable for each building. There are 31,988 buildings in total in the register and the attribute ranges are 1–338 persons for TP variable and 0–63 for TC variable.

3.2. Capturing the domain knowledge in the disaster management process in Helsinki area – the questionnaire

In Finland, the rescue operations are provided by the fire and rescue organisation, police stations, and military services (Castrén et al. 2007). These organisations often plan the rescue activities together in order to make the evacuation process more efficient. The Helsinki area is divided into eight rescue zones that are based on the locations of the rescue stations (see the figures in Section 3.5) (City of Helsinki 2013). In the event of a disaster, the rescue personnel will perform a rescue operation in their own zone first and may only later help other zones if there are resources available.

We were interested in how actual rescue experts would evaluate vulnerability of buildings in a real crisis. We captured this knowledge through a questionnaire that asked the experts – experienced crisis decision-makers – to estimate various levels of vulnerability based on the values of TP and TC variables. This questionnaire is provided in an appendix to this article. Our group of experts included seven participants: three rescue officers from the Helsinki Rescue Department, a lieutenant-colonel from the Military Science Security Committee, and three senior researchers from the Emergency Service College. The questionnaire consisted of two questions and had a dynamic design, where the answers to the first question were used to create the second question, to be able
to capture the variability in their knowledge as closely as possible. In order to achieve this, the decision-makers were asked to answer the first question before seeing the second part. The results from this questionnaire were used in the development of the model and further in evaluation, as described in the next section.

3.3. Creating the fuzzy MADM model by using expert knowledge in the domain

The first question in the questionnaire was used to create the MF for the fuzzy model. Each attribute had three MFs which were expressed as linguistic variables: a high number of people/children (Hₚ/Hₖ), a medium number of people/children (Mₚ/Mₖ), and a low number of people/children (Lₚ/Lₖ). Figure 3 illustrates the MFs of the two attributes. The corresponding attribute values of each MF were defined by using the range values of the seven decision-makers’ answers and the results are illustrated in Table 1. The range values of the experts’ answers were used to create the boundary line of the MFs. The boundary line for Lₚ MF is 1 and 100. For the low and high MFs we used the trapezoidal functions, because the peak value of the MFs should stay the same over a certain range. In order to calculate the peak value of each MF, we use the average value of the experts’ answers to Question 1. The average value of Lₚ MF is 61, therefore the peak value for the Lₚ MF should reach the middle point between the values 0 and 61, which is equal to 31. After the value 31 the Lₚ function starts to decrease. The peak value for the Hₚ MF is the maximum

![Figure 3. Fuzzy membership function curves for the TP and TC attributes.](image-url)
value of the experts’ answers for the MF value, which is 250. All the experts thought after the value 250, the MF should stay as high MF. We used a triangular function for the \( M^P \) MF, because the peak value of the function should only be reached at one single point and the degree of the membership should start to decrease immediately after that point. The peak value of the \( M^P \) was calculated by using Equation (11). The MF for the TC attribute was created in the same way.

\[
\text{Peak} \left( M^p \right) = \frac{\text{avg}(M^p) - \text{avg}(L^p)}{2} + \text{avg}(L^p) = \frac{(178 - 61)}{2} + 61 = 120 \quad (11)
\]

In the next step, the answers to Question 2 were used to create fuzzy rules as follows. Nine unique combinations of MFs can be formed by selecting one MF from each attribute. These are: \( H^P H^C \), \( H^P M^C \), \( M^P H^C \), \( H^P L^C \), \( L^P H^C \), \( M^P M^C \), \( M^P L^C \), \( L^P M^C \), and \( L^P L^C \). For instance, the combination ‘\( H^P M^C \)’ represents a building which has a high number of people and a middle number of children living inside it. Question 2 asked the decision-makers to rank the nine combinations of MF versus each other. We expected that the combination of MFs ‘\( H^P H^C \)’ should receive the highest rank (the most vulnerable combination) and the combination of MFs ‘\( L^P L^C \)’ should receive the lowest rank (the least vulnerable combination). However, some combinations of MFs can be difficult to rank. For instance, it is difficult to choose the more vulnerable combination between ‘\( M^P L^C \)’ and ‘\( L^P M^C \)’. Therefore, to make it easier for the participants, each MF was represented by its peak values instead of using exclusively linguistic terms. We provided several different scenarios in Question 2. In each scenario, there were two buildings (building A and building B) with different numbers of people and children, and the experts had to select the building which they thought was more vulnerable in the event of an emergency. The answers to Question 2 are summarised in Table 2, where the final choice is based on the majority decision of the experts. The answer B \(_{(6)}\) means 6 out of 7 experts think building B is more vulnerable than building A.

On the basis of the results shown in Table 2, the nine MF combinations were ranked to produce the order of the antecedent parts of the fuzzy rules used in our model. Table 3 shows these rules: each column in this table represents one rule, which consists of an antecedent and a consequent part. The antecedent parts are the MF combinations of nine attributes. The consequent part of the fuzzy rule contains five MFs (vulnerability values), and they are extremely high vulnerability (EH), high vulnerability (H), medium vulnerability (M), low vulnerability (L), and extremely low vulnerability (EL). For instance, the fuzzy rule for the first column is: If TP is High and TC is High, then the vulnerability of the building is extremely High.

| Attributes | Range value (number of people) | Average value (number of people) | Linguistic variable |
|------------|--------------------------------|---------------------------------|--------------------|
| Total number of people (TP) | 1–100 | 61 | \( L_P \) |
| | 30–250 | 178 | \( M_P \) |
| | 100–338 | \( H_P \) |
| Total number of children (TC) | 1–10 | 9 | \( L_C \) |
| | 3–50 | 33 | \( M_C \) |
| | 13–63 | \( H_C \) |
The rules were derived from the experts’ answers (Table 2) as follows. The MF combinations which received the highest (H) and lowest (L) ranks received the ‘EH’ and ‘EL’ MF, respectively, in the consequent part. The MF combinations which have the ‘M’ and ‘H’ combination received an ‘H’ vulnerability value. The ‘L’, ‘LH’, and ‘ML’ MF combinations received the ‘M’ vulnerability value. The ‘L’ and ‘ML’ membership combinations receive the ‘L’ vulnerability value.

### 3.4. Running the model

The fuzzy model takes the building data set with TP and TC as input attributes and calculates the vulnerability value for each building as output. For this, the model uses fuzzy rules, derived from experts’ knowledge, as explained earlier. Figure 4 illustrates the surface view of using fuzzy model to map input attribute data into output vulnerability.
value. The X- and Y-axes represent the input values of the TP and TC attributes. The Z-axis refers to the vulnerability output that is based on the fuzzy rules. When the degree of the MFs of the TP and TC attributes increases, the corresponding vulnerability value also increases.

3.5. Visualising the results of the fuzzy model with kernel density estimation

In the next step, we used the kernel density estimation on normalised results of the fuzzy model to produce a vulnerability map of the Helsinki area. Kernel density estimation creates a smooth curved surface with a certain search radius (bandwidth) in which to calculate the magnitude per unit area from a point using the kernel function. The volume under the surface equals the field value for the point (O’Sullivan and Unwin 2002). We created the kernel density surface using our building data as the spatial point data set with vulnerability from our model as the field value.

In Finland, fire and rescue services use population density maps based on a 250-metre-by-250-metre grid. Therefore, in the first step the kernel density search radius was defined as 250 metres. Later, search radius values of 500 and 750 metres were applied in order to see how the vulnerability map changes according to different search radii. Areas which have a high kernel density value in the resulting vulnerability map refer to places which have a high density of buildings or/and high output values in the model. The Helsinki area is divided into eight rescue zones, zones were then visualised together to produce a model result map for the entire Helsinki area. In this way, the kernel density at a specific location is created on the basis of a certain zone’s data value. This avoids visual overpowering of a low kernel value for a rescue zone which has low vulnerability values or only a few vulnerable buildings compared to another rescue zone with high vulnerability values.

Figure 5 shows kernel density maps based on results from the fuzzy models. The hot spots in blue represent the most vulnerable locations. In the maps with 500- and 750-meter search radii, the most vulnerable regions are regions Nos. 0, 8, 30, 44, 61, and 77 and the intersection part between regions 1, 2, 3, and 4, the intersection parts between regions 15 and 18, the intersection parts between regions 33, 34, and 36, and the intersection parts between regions 51 and 52. In Figure 5, the vulnerable locations in the event of a disaster are clearly pointed out. However, the accuracy of the results depends on the results of the validation of the fuzzy model which we describe in the next section.

4. Model validation

In this section we present the validation of our model in two ways: we first visually compare the vulnerability maps created from the results of the fuzzy model to actual population density maps. Then we validate the model against the real knowledge of the rescue experts using the fuzzy accuracy assessment.

4.1. Model validation against original population information maps

The model results were validated against the original population density maps (Figures 6 and 7) in order to see if the area’s vulnerability value matches the original population information. For example, the model is not performing correctly if the model produces an area with high vulnerability where the original population information maps show that
there are only a few people in that area. We used the same normalisation and cartographic methods for model maps and original density maps to enable this comparison.

Results show that the model maps display a similar spatial pattern as the original TP and TC attribute kernel density maps. In the model map, the hot spots refer to the places which have both high TP and TC values. For example, the intersection region between regions 48 and 50 did not receive a high kernel density value on the fuzzy map even though this region has high values on the TC kernel density map. This is because the region has a low value on the TP kernel density map.

Of interest to the rescue personnel in these maps is the number and location of hot spots, as these define the number and locations of areas where intervention are necessary. Results showed that the number of hot spots in the model map when compared to the original population information map was reduced to a reasonable number. This enables the rescue personnel to focus on the places which have large numbers of people and children living in them in order to optimise the efficiency of the rescue operation.
4.2. Model validation by using fuzzy set accuracy assessment and real domain knowledge

In the fuzzy set accuracy assessment, a numeric scale of MFs ranging from 0 to 7 was developed, based on the number of questionnaire participants. A value of 7 on this scale means that if all seven experts thought a building should belong to a certain attribute MF then this conclusion is absolutely right. This was used as an input to the fuzzy accuracy assessment described in Section 2.2.

Table 4 illustrates an example of using $3 \times 6$ matrices (see Section 2.2). This table also includes four additional columns representing sample building numbers ($x = 1, \ldots, 3$), numeric scales of $\mu(X)$ which are assigned to each buildings on the basis of the $\text{MAX}$ function, labels assigned to the building according to experts’ opinion with the $\text{MAX}$ function, and labels assigned to the buildings on the basis of the model results $\chi(x)$.
Table 4. Example of using fuzzy set accuracy assessment to validate the fuzzy model’s result. 
\( n = 3, m = 6, \text{AND } C = \{H_R, M_R, L_R, H_C, M_C, L_C\}. \) The columns H_R, M_R, L_R, H_C, M_C, L_C form the \( 3 \times 6 \) matrix \( A. \)

| Building | \( \mu(X) \) | MAX function | Experts suggested labels | Models labels |
|----------|----------------|---------------|-------------------------|---------------|
|          | TP | TC | TP | TC | TP | TC | TP | TC |
| 1        | 0  | 1  | 6  | 0  | 0  | 7  |     |     |
| 2        | 0  | 5  | 2  | 2  | 4  | 1  |     |     |
| 3        | 1  | 5  | 1  | 0  | 1  | 6  |     |     |
In order to perform the matching process of the model results against the experts’ knowledge, the original data (i.e. the building locations) were first divided into nine categories by using labels suggested by the experts (MAX function). Each category represents one attribute’s MF combinations and contains a set of buildings. We refer to these as the expert-derived categories. The original data were then sorted according to the vulnerability results of the fuzzy model from the highest to lowest order, and then divided into nine categories which represent the MF combinations of the attributes. This is called the model-derived category. Both the expert- and model-derived categories were then arranged according to the experts’ rankings (Table 3), from the highest to the lowest. The performance of the model was then assessed by comparing the number of buildings in each model-derived category with their corresponding value in the expert-derived category. For each category, we calculated the frequency of the matching and mismatching of buildings between the expert-derived category and the model-derived category (Table 5). According to these results, the fuzzy model produced high accuracy in the \( HP_HC \), \( MP_HC \), \( HP_MC \), \( HP_LC \), \( LP_HC \), \( MP_MC \), \( LP_MC \), \( MP_LC \), and \( LP_LC \) categories and low accuracy in the \( MP_HC \), \( HP_MC \), \( HP_LC \), and \( LP_MC \) categories. The low accuracy of the results mainly occurs in the categories which contain less than 40 buildings, except for the most vulnerable category.

| Category  | Total | Match  | Mismatch |
|-----------|-------|--------|----------|
| \( HP_HC \) | 12    | 11 (92%) | 1 (8%)   |
| \( MP_HC \) | 11    | 3 (27%)  | 8 (72%)  |
| \( HP_MC \) | 21    | 6 (29%)  | 15 (71%) |
| \( HP_LC \) | 22    | 0       | 22 (100%)|
| \( LP_HC \) | 0     | 0       | 0        |
| \( MP_MC \) | 431   | 322 (75%) | 109 (25%)|
| \( LP_MC \) | 36    | 0       | 36 (100%)|
| \( MP_LC \) | 2779  | 2344 (84%) | 435 (16%)|
| \( LP_LC \) | 28676 | 28344 (99%) | 322 (1%) |

5. Conclusion
In this article, a fuzzy MADM approach was used to estimate the vulnerability of buildings in the Helsinki area based on demographic information about their inhabitants. The results show that fuzzy model was highly accurate in the most vulnerable category and the categories which contain more than 40 buildings. On the other hand, the accuracy of the fuzzy model was low for the categories which contain less than 40 buildings. Further, by using fuzzy modelling, the numbers of hot spots from the original population information maps (i.e. the areas where rescue intervention is most needed) were reduced to a reasonable scale and the locations which have large numbers of people and children living in them were clearly identifiable on the fuzzy vulnerability map.

Experts’ knowledge acts as an important element in many spatial decision problems, but it is often not easy to obtain or represent. We demonstrate that a fuzzy model can systematically and mathematically emulate human reasoning, and allows for experts’ knowledge to be incorporated into the modelling process. To achieve this, we implicitly
captured this knowledge through a dynamic questionnaire and based our fuzzy model on the empirical results.

Another challenge in the spatial modelling problem is how to validate the mathematical models against experts’ knowledge and real-world phenomena. In this article, a fuzzy set accuracy assessment approach was used to validate the fuzzy models against empirical results from the questionnaire.

6. Discussion

While introducing the fuzzy concepts into the MADM modelling of uncertainty has its advantages, the model is dependent on several choices that can affect its results. One of these choices is the number of MFs. We decided to keep this number low (three MFs) in order to keep the number of scenarios in Question 2 of the questionnaire manageable for seven experts. Our decision was based on classification in similar work, where three proved to be a good number of categories to evaluate the level of risk and vulnerability (Špatenková and Stein 2010).

The second source of uncertainty in the model is conflicting questionnaire answers as given by the experts. Since our seven experts came from different rescue organisations and working environments, we expected that their answers may in some cases differ and vary to significant extent. The fuzzy MFs were therefore created on the basis of the range and average values of the seven experts’ answers to Question 1 in the questionnaire. Further, the model validation procedure used the majority results from Question 2. This meant that the fuzzy decision rules in our model reflected the average procedure used in the disaster response across all zones and all types of expertise. To improve the local accuracy of the fuzzy model in each rescue zone, it would be necessary to build local fuzzy decision rules, where only rescue personnel from that particular zone should participate in the knowledge acquisition process.

The method developed in this work takes into account only two population variables, the total number of inhabitants of a building and the number of children in a building. To expand the model, other demographic variables relevant to disaster management could be taken into account, such as the number of elderly people and/or people with disabilities, who also have a limited capability to independently respond to a disaster due to health issues or issues with mobility. We have not considered this due to lack of data, but it is a possibility that could be further explored in collaboration with the rescue services.

In our model we used data on inhabitants of the residential buildings, where habitation was defined as a static property of the building, i.e. we knew how many people were registered in a particular building, but we did not know when they were using this building. The vulnerability measure resulting from our model was therefore also static, as it did not take into consideration the temporal differences in the occupation of buildings. This limits the use of the results because the model assumes that people are at home all of the time and is therefore only realistic at particular times (mostly during the night or perhaps during weekends when people stay at home). However, the vulnerability for disaster management is dynamic, as has been reported in the literature (Ahola et al. 2007, Špatenková and Stein 2010, Zhang et al. 2010) and this should be taken into consideration if at all possible. A dynamic vulnerability model could be therefore developed by including information on other building types (not only residential) and on people’s daily migration behaviour, which could be derived from various sources, such as commuting information derived from volunteered GPS trajectories (Shen et al. 2013).
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