Kondo Effect in Quantum Dots Coupled to Luttinger Liquid Leads

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We consider the Kondo effect in quantum dots coupled to Luttinger liquid leads, focusing on the case of repulsive interactions and spin SU(2) symmetry in the leads. We find that the system can flow to the 1-channel or 2-channel Kondo fixed points, depending on the interactions in the system. We compute the conductance and find that the qualitative behavior is strongly dependent on the interactions. Finally, we point out a consequence of 2-channel Kondo physics, which should be observable in thermal conductance measurements.

The Kondo effect, which deals with a single magnetic impurity in a sea of conduction electrons, has received an enormous amount of attention over the years. Kondo’s original work over thirty-five years ago was intended to explain the anomalous resistivity observed in magnetic alloys. Since then, a variety of variations of the Kondo Hamiltonian have been introduced, and some interesting manifestations of Kondo physics have been suggested. One of the most recent manifestation of the Kondo effect has been provided by quantum dots, in quantum dots with an odd number of electrons, resonant transmission through the dot is observed due to the Kondo effect. The quantum dot provides a fascinating place to study Kondo physics because there are many parameters which can be controlled in an experiment. Therefore, many aspects of the Kondo effect can be probed.

The Kondo effect is the anomalous resistivity observed in magnetic alloys, and it is a consequence of the coupling of a magnetic impurity to a sea of conduction electrons. The coupling of the impurity to the leads leads to a ground state with a finite spin, which is called the Kondo state. The Kondo effect has been observed in many different systems, including magnetic impurities in metals, quantum dots, and quantum wires.

In most treatments of the Kondo effect in quantum dots, the leads were taken to be Fermi liquids and interactions in the leads were ignored. This is sufficient if the leads are two or three dimensional electron gases, where interactions affect the low energy properties only perturbatively. However, in one dimension, arbitrarily weak interactions completely modify the ground state; the low energy excitations are described by a Luttinger liquid rather than a Fermi liquid. In contrast to a Fermi liquid, the low energy excitations in a Luttinger liquid are spin and charge density fluctuations; there are no well defined single particle excitations. Moreover, the speeds of the spin and charge density fluctuations are different i.e. spin and charge separate. In this work, we consider the Kondo effect in quantum dots coupled to Luttinger liquid leads. The setup we consider is shown in Fig. 1. In what follows, we focus on the case of repulsive interactions in the leads and assume the system to have spin SU(2) symmetry.

To model the dot, we consider what has come to be the canonical model for small quantum dots — the Anderson impurity model. The Hamiltonian, including the coupling to the leads, is

\[ H_{\text{dot}} = \varepsilon_0 \sum_s d_s^\dagger d_s + U_0 \sum_s d_s^\dagger d_s^\dagger d_s d_s + t' \left[ \left( \psi_{1,s}^\dagger(0) + \psi_{2,s}^\dagger(0) \right) d_s + \text{h.c.} \right], \]

where \( \psi_{i,s}^\dagger \) creates an electron with spin-\( i \) in lead-\( i \); \( d_s^\dagger \) creates an electron with spin-\( s \) on the dot; \( \varepsilon_0 \) is the energy level of the dot, which can be controlled by a gate voltage; \( U_0 \) is the charging energy; \( t' \) is the tunneling matrix element between the leads and the dot. In this work, we will focus on the case where the tunneling matrix elements of both leads are equal. Being interested in the Kondo regime of the dot, we integrate out the charge fluctuations on the dot. Working to second order in perturbation theory, the following interactions are generated

\[ H_{\text{int}} = J \tau \cdot \sigma_{s,s'} \left( \psi_{1,s}^\dagger(0) \psi_{1,s'}(0) + \psi_{2,s}^\dagger(0) \psi_{2,s'}(0) \right) \]

\[ + J' \tau \cdot \sigma_{s,s'} \left( \psi_{1,s}^\dagger(0) \psi_{2,s'}(0) + \text{h.c.} \right) \]

\[ + V \left( \psi_{1,s}^\dagger(0) \psi_{2,s}(0) + \text{h.c.} \right) \]

\[ + V' \left( \psi_{1,s}^\dagger(0) \psi_{1,s}(0) + \psi_{2,s}^\dagger(0) \psi_{2,s}(0) \right). \]

The couplings can be computed for a given microscopic model of the leads (e.g. Hubbard model); we will take them as phenomenological parameters. However, it is important to note that \( J > 0 \) and \( J' > 0 \).

Being interested in the low energy properties of the system, we expand the electron operators in the leads in terms of right and left movers

\[ \psi_{i,s}(x) = e^{ikFx} \psi_{R,i,s}(x) + e^{-ikFx} \psi_{L,i,s}(x), \]

where \( k_F \) is the Fermi wavevector, and \( \psi_{R,i,s} \) and \( \psi_{L,i,s} \) are the (slowly varying) right and left moving fermion operators. In bosonized form, the Hamiltonian describing low energy fluctuations in the leads is

\[ H = \frac{\nu_\sigma}{2} \sum_{i=1}^2 \int_{-\infty}^0 dx \ K \pi_{i,\rho}^2 + \frac{1}{K} (\partial_x \phi_{i,\rho})^2 \]

\[ + \frac{\nu_\sigma}{6\pi} \sum_{i=1}^2 \int_{-\infty}^0 dx \ J_{R,i}^2 + J_{L,i}^2, \]

where \( i = 1, 2 \) labels the leads; \( \phi_{i,\rho} \) and \( \pi_{i,\rho} \) are canonically conjugate fields describing charge fluctuations and
satisfy $[\phi_{i,\rho}, \pi_{j,\rho}] = i\delta_{ij}\delta(x_1 - x_2)$; $J_{R,i}$ and $J_{L,i}$ are currents satisfying the SU(2) Kac-Moody algebra at level-1 and describe spin fluctuations. The Luttinger parameter, $K$, is determined by the interactions in the system — $K < 1$ for repulsive interactions, $K > 1$ for attractive interactions, and $K = 1$ for a non-interacting system. At the ultraviolet fixed point, the leads are decoupled from the dot. Therefore the right and left moving fermion fields satisfy

$$\psi_{R,i,s}(x = 0) = \psi_{L,i,s}(x = 0).$$

Eq. 5 implies that $\psi_{L,i,s}(x) = \psi_{R,i,s}(-x)$. Hence, $\psi_{L,i,s}$ may be regarded as the analytic continuation of $\psi_{R,i,s}$ to positive $x$, and we can write our Hamiltonian solely in terms of right movers.

The Hamiltonian we consider (Eq. 2 and Eq. 4) is related to the problem of a magnetic impurity in a Luttinger liquid, which has been considered in several works. In that context, this Hamiltonian was first considered to describe a magnetic impurity in a Luttinger liquid with a strong scattering potential at the impurity site. There, they identified that the system could flow to the 1-channel or 2-channel Kondo fixed points, depending on the interactions in the system. In this work, our main goal is understanding transport through a quantum dot. More specifically, how do the various fixed points which arise influence the transport through the dot?

Before we proceed with the case of Luttinger liquid leads, it is useful to review the case of non-interacting leads. For non-interacting leads, one finds that $J = J'$ in Eq. 3. Also, the $V$ and $V'$ terms are usually ignored, since their effects are very small. Then, by working in the “even-odd” basis, $\psi_{e/o,s} = (1/\sqrt{2})(\psi_{1,s} \pm \psi_{2,s})$, the Hamiltonian becomes $H = H_0 + H_{\text{int}}$, where $H_0$ is the Hamiltonian for free electrons and

$$H_{\text{int}} = J \tau \cdot \sigma_{e,s'} \frac{1}{2} \psi_{e,s}^\dagger \psi_{e,s'}^\dagger.$$ 

We see that only the even channel couples to the impurity, giving us a 1-channel Kondo model; the odd channel remains free. The low energy (i.e. 1-channel Kondo) fixed point is given by an electron in the even channel bound in a singlet to the electron on the dot. Resonant tunneling occurs because electrons can tunnel freely through the odd channel. Essentially, the two semi-infinite leads have been joined to form one infinite lead, due to the Kondo effect.

For the case of Luttinger liquid leads, the even-odd basis is not useful, because the Hamiltonian becomes non-local. Therefore, we will proceed along different lines and change variables in a different way. For the spin degrees of freedom, we work in terms of a total spin variable, $J_R = J_{R,1} + J_{R,2}$, and fields describing the relative spin fluctuations between the two leads. To deduce the fields describing the relative spin fluctuations, we note that $J_R$ are currents of the SU(2) WZW model at level-2; the WZW model at level-2 has a central charge of $3/2$. Our original description was in terms of the SU(2)$_1 \times$ SU(2)$_1$ WZW model with central charge 2. Therefore, the relative spin fluctuations must account for a missing central charge of $1/2$. This is precisely the central charge of the Ising model. Therefore, we can describe the spin degrees of freedom in terms of an SU(2) WZW model at level-2 for the total spin and an Ising model for the relative spin fluctuations between the two leads. The spectrum of the WZW model at level-2 is classified by its 3 primary fields — the spin-0 field $I$ of dimension 0 (the identity operator); the spin-1/2 field $g_s$ of dimension 3/16; and the spin-1 field $\phi$ of dimension 1/2. The spectrum of the Ising model is classified by its 3 primary fields — $I$ of dimension 0 (the identity operator); the order parameter field $\sigma$ of dimension 1/16; and the energy field $\epsilon$ of dimension 1/2. The WZW fields satisfy the operator product expansions (OPE’s)

$$[\sigma] \times [\sigma] \rightarrow [I] + [\phi] , \quad [\phi] \times [\phi] \rightarrow [I] ,$$

and the Ising fields satisfy

$$[\sigma] \times [\sigma] \rightarrow [I] + [\epsilon] , \quad [\epsilon] \times [\epsilon] \rightarrow [I].$$

In terms of the charge, WZW, and Ising fields, the electron operator is

$$\psi_{R,i,s} \sim e^{i\sqrt{2\pi}g_R/s}\phi \sigma.$$

It will also prove useful to form the combinations

$$\phi_{R,c/f} = \frac{1}{\sqrt{2}}(\phi_{R,1,\rho} \pm \phi_{R,2,\rho})$$

for the charge degrees of freedom. $\phi_{R,c}$ is a charge field describing total charge fluctuations of the system; $\phi_{R,f}$ is a flavor field describing the relative charge fluctuations between the two leads.

Let us first (formally) consider the case where the system has particle hole symmetry. In this case the $V$ and $V'$ terms are absent from Eq. 2. Using Eq. 3 and the OPE’s of Eq. 6 and 7 we obtain

$$H_{\text{int}} = v_\sigma \lambda_1 \tau \cdot J_R(0) + \sqrt{v_\sigma v_\rho} \lambda_2 \tau \cdot \phi \cos \sqrt{4\pi\phi_{R,f}}(0),$$

where $\lambda_1$ and $\lambda_2$ are dimensionless couplings, with $v_\sigma \lambda_1 \sim J$ and $\sqrt{v_\sigma v_\rho} \lambda_2 \sim J'$. We begin by considering the effects of Eq. 7 on the ultraviolet fixed point via a renormalization group (RG) analysis. To second order in the couplings, the RG equations for the parameters are

$$\frac{d\lambda_1}{dl} = \lambda_1^2 + \lambda_2^2 , \quad \frac{d\lambda_2}{dl} = \frac{1}{2} \left(1 - \frac{1}{K}\right) \lambda_2 + 2 \lambda_1 \lambda_2 .$$

A few words are in order about the RG equations. At the ultraviolet fixed point, $\tau \cdot J_R$ is marginal. Since $\lambda_1 > 0$, quantum corrections push $\tau \cdot J_R$ to be marginally relevant. On the other hand, the interlead tunneling term
is irrelevant for repulsive interactions ($K < 1$). Therefore, $\lambda_2$ initially decreases under the RG. However, from Eq. [1] we see that the growth of $\lambda_1$ will also cause $\lambda_2$ to eventually grow. For $K \approx 1$, $\lambda_2$ will grow almost immediately. Similar to the case of non-interacting electrons, the system flows to the 1-channel Kondo fixed point. However, for $K$ sufficiently smaller than unity, we can have $\lambda_2 \ll 1$ while $\lambda_1$ has already grown to $O(1)$. If $\lambda_2 = 0$, we would have a 2-channel Kondo model, which is known to have a nontrivial $O(1)$ fixed point. For the case $\lambda_2 \ll 1$ with $\lambda_1 = O(1)$, the system flows close to the 2-channel Kondo fixed point. In that case, it is appropriate to consider the behavior near the 2-channel Kondo fixed point with $\lambda_2$ as a perturbation.

The 2-channel Kondo fixed point occurs for $\lambda_2^* = 1/2$. For this value of the coupling, we can define new spin currents, $J_R(x) = J_R(x) + 2\pi \tau \delta(x)$, which satisfy the same Kac-Moody algebra as the original spin currents. The impurity spin has now disappeared from the problem, leaving behind a new boundary condition. [14] The spectrum at the 2-channel Kondo fixed point is obtained by fusion with spin-1/2; the operator content is obtained by double fusion. [14] Upon double fusion,

$$[I] \to [I] + [\phi] , \ [g] \to [g] , \ [\phi] \to [I] + [\phi]. \quad (11)$$

To understand the properties near the 2-channel Kondo fixed point, we must consider the various operators which are allowed. Since the system has spin SU(2) symmetry, any operator from the spin sector must transform as a singlet. This forces $J_{-1} \cdot \phi(0)$, the first descendant of $\phi$ with dimension 3/2, as the leading irrelevant operator from the spin sector. In the ordinary 2-channel Kondo problem, only the spin sector enters; in this system, the other sectors contribute as well. Operators from the charge sector are not allowed, since the total charge of the system is conserved. Also, operators from the Ising sector are not allowed, since the Hamiltonian is invariant with respect to lead interchange — lead 1 $\leftrightarrow$ lead 2. [12]

However, operators from the flavor sector are allowed, due to the $\lambda_2$ term in Eq. [1]. The allowed operators can be deduced by double fusion. Using Eq. [11] we find that

$$\cos \sqrt{4\pi f} \phi_R(0) \quad (12)$$

is allowed. This operator has dimension $1/2K$; it is relevant for $K > 1/2$ and irrelevant for $K < 1/2$.

For $K < 1/2$, the operator in Eq. [12] is irrelevant, and the 2-channel Kondo fixed point is stable. However, for $K > 1/2$, this operator is relevant and induces a flow away from the 2-channel Kondo fixed point. The boundary conditions at the ultraviolet fixed point (Eq. [3]) imply that the flavor sector satisfies Dirichlet boundary conditions; near the 2-channel Kondo fixed point, the flavor sector continues to satisfy Dirichlet boundary conditions. The operator in Eq. [3] induces a flow from Dirichlet to Neumann boundary conditions. This operator is similar to what appears in the case of the anisotropic 2-channel Kondo model with a spin-1/2 impurity. [13] There, it is known that channel anisotropy is a relevant perturbation to the 2-channel Kondo fixed point and causes the system to ultimately flow to the 1-channel Kondo fixed point. Physically, this occurs because the channel with the larger exchange coupling screens the impurity. [16] Here, the physics is similar. The operator in Eq. [3] induces a flow from the 2-channel to the 1-channel Kondo fixed point (for $K > 1/2$), and the stronger channel screens the impurity. As in the case of non-interacting leads, the stronger channel is the “even” channel, $\psi_{e,s} = (1/\sqrt{2})(\psi_{1,s} + \psi_{2,s})$.

We see that for $K > 1/2$, the system ultimately flows to the 1-channel Kondo fixed point. We must now consider the physics near this fixed point. In the spin sector, the leading irrelevant operator is now the dimension-2 operator $J_R \cdot J_R(0)$. Performing an instanton gas expansion, [17] we find the leading irrelevant operator $\cos \sqrt{16\pi f} \phi_R(0)$ in the flavor sector. Due to the Neumann boundary conditions, this operator has dimension $2K$ and is irrelevant for $K > 1/2$. Hence, we see that the 1-channel Kondo fixed point is stable for $K > 1/2$.

Now we consider the more general (and probably more realistic) case where there is no particle-hole symmetry, and the $V$ and $V'$ terms in Eq. [2] are present. Using Eq. [8] and the OPE’s of Eqs. [1] and [2] we obtain

$$H_{\text{int}} = v_\sigma \lambda_1 \tau \cdot J_R(0) + \sqrt{v_\sigma v_R} \lambda_2 \tau \cdot \phi \cos \sqrt{4\pi f} \phi_R(0)$$

$$+ \sqrt{v_\sigma v_R} \lambda_3 \epsilon \sin \sqrt{4\pi f} \phi_R(0) + v_\rho \lambda_4 \partial_\tau \phi_R(0), \quad (13)$$

where $\{\lambda_i\}$ are dimensionless couplings with $v_\sigma \lambda_1 \sim J$, $\sqrt{v_\sigma v_R} \lambda_2 \sim J'$, $\sqrt{v_\sigma v_R} \lambda_3 \sim V$ and $v_\rho \lambda_4 \sim V'$. To begin with, we can take $\lambda_4$ term into account exactly. Its effect is to produce a small phase shift. At the ultraviolet fixed point, $\lambda_1$ and $\lambda_2$ satisfy the same RG equations as Eq. [11]; $\lambda_3$ satisfies

$$\frac{d\lambda_3}{dt} = \frac{1}{2} \left( 1 - \frac{1}{K} \right) \lambda_3, \quad (14)$$

and hence is irrelevant. Therefore, the flows near the ultraviolet fixed point are the same as for the particle-hole symmetric case. For $K \approx 1$, the system flows to the 1-channel Kondo fixed point, and for $K$ sufficiently less than unity, the system flows toward the 2-channel Kondo fixed point. Near the 2-channel Kondo fixed point, we still have the operators perturbing this fixed point as we did for the particle-hole symmetric case. However, now the $V$ and $V'$ terms in Eq. [3] allow terms which break particle-hole symmetry. Similar to the ultraviolet fixed point, the most relevant particle-hole symmetry breaking terms are $\partial_\tau \phi_R(0)$ of dimension 1 and $\epsilon \sin \sqrt{4\pi f} \phi_R(0)$ of dimension $(1/2)(1 + 1/K)$. Therefore, as in the particle-hole symmetric case, the 2-channel Kondo fixed point is stable for $K < 1/2$; for $K > 1/2$, the 2-channel Kondo fixed point is unstable and the system flows to the 1-channel Kondo fixed point.

For $K > 1/2$, the system flows to the 1-channel Kondo fixed point. Now, we must consider the properties of the system near this fixed point. As we saw for the case of
non-interacting electrons, at the 1-channel Kondo fixed point, the two semi-infinite leads are joined into one infinite lead. For an infinite Luttinger liquid, if we allow terms which break particle-hole symmetry, it is well known that the most relevant perturbation is potential scattering; \[15\] in bosonized form, it is \(\epsilon \sin \sqrt{4\pi \phi_{R.f}}(0)\).

Due to the Neumann boundary conditions in the flavor sector, this operator has dimension \((1/2)(1 + K)\), and hence is relevant. This operator causes a flow to an insulating fixed point. Physically, this insulating fixed point corresponds to an electron in the “even” channel bound in a singlet to the electron on the dot, with an infinite potential in the “odd” channel. \[20\] The physics near this fixed point is basically that of two semi-infinite Luttinger liquids coupled by weak tunneling. The operator for weak tunneling is the most relevant operator near this fixed point, and has dimension \((1/2)(1 + 1/K)\); \[15\] hence, it is irrelevant. Therefore, this insulating fixed point is stable, and is the ultimate fixed point to which the system flows for \(K > 1/2\).

Experimentally, the quantity of interest is the conductance through the dot, \(G\). From the above discussion, there are three distinct cases to consider — case 1: \(K \approx 1\); case 2: \(K\) sufficiently less than unity, but \(K > 1/2\); case 3: \(K < 1/2\). Following Ref. \[13\] we compute the conductance perturbatively near the various fixed points, focussing on the case where the system does not have particle-hole symmetry. The conductance is plotted schematically in Fig. 2 for the three cases.

However, potential scattering is a relevant perturbation to the 1-channel Kondo fixed point and causes \(G\) to decrease as \(T^{K-1}\). Finally, as \(T \to 0\), \(G\) goes to zero as \(T^{1/K-1}\).

case 2: Near the ultraviolet fixed point, \(G\) decreases as \(T^{1/K-1}\) as the system flows toward the 2-channel Kondo fixed point. However, the 2-channel Kondo fixed point is unstable, and \(G\) increases as \(T^{1/K-2}\) as we flow away from the 2-channel Kondo fixed point to the 1-channel Kondo fixed point. Near the 1-channel Kondo fixed point, \(G\) comes close to its maximum value \(2e^2K/h\).

As \(T \to 0\), \(G\) goes to zero as \(T^{1-1/K}\).

case 3: \(K_\rho < 1/2\) — Near the ultraviolet fixed point, \(G\) decreases as \(T^{1/K-1}\) as the system flows toward the 2-channel Kondo fixed point. Near the 2-channel Kondo fixed point, \(G \sim T^{1/K-2}\). However, in this case the 2-channel Kondo fixed point is stable. As \(T \to 0\), we remain near the 2-channel Kondo fixed point and \(G\) goes to zero as \(T^{1/K-2}\).

In all three cases, \(G \to 0\) as \(T \to 0\). However, it is important to remember that the fixed points to which the system flows are different. For \(K > 1/2\), the system ultimately flows to an insulating fixed point of two semi-infinite Luttinger liquids coupled by weak tunneling; for \(K < 1/2\), the system flows to the 2-channel Kondo fixed point. This difference has observable consequences. In particular, if we consider the spin conductance, \[13\] we find that the 2-channel Kondo fixed point has perfect spin conductance, whereas the insulating fixed point has vanishing spin conductance. Though the spin conductance is difficult to measure, this effect should be observable in thermal conductance measurements. In particular, we expect a thermal conductance, \(\kappa\), such that \[13\]

\[
\frac{\kappa}{T} \to \left(\frac{\pi^2}{3}\right) \frac{k_B^2}{h} \text{ as } T \to 0 \tag{15}
\]

if the system flows to the 2-channel Kondo fixed point, whereas we expect a thermal conductance such that \(\kappa/T\) goes to zero as \(T^{1/K-1}\) if the system flows to the insulating fixed point.

To summarize, we considered the Kondo effect in quantum dots coupled to Luttinger liquid leads, focussing on the case of repulsive interactions \((K < 1)\) and spin SU(2) symmetry in the leads. We found that the system flows to the 2-channel Kondo fixed point for \(K < 1/2\) and flows to the 1-channel Kondo fixed point for \(K > 1/2\). For the particle-hole symmetric case, the 1-channel Kondo fixed point is stable; for the particle-hole asymmetric case, the system ultimately flows to an insulating fixed point. Furthermore, we computed the conductance and found that the qualitative behavior is strongly dependent on the value of \(K\). Finally, we pointed out that the 2-channel Kondo fixed point has perfect spin conductance, which should be observable in thermal conductance measurements.

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