LEPTONIC CP VIOLATION AND LEPTOGENESIS

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The phenomenology of 3-neutrino mixing, the current status of our knowledge about the 3-neutrino mixing parameters, including the absolute neutrino mass scale, and of the Dirac and Majorana CP violation in the lepton sector, are reviewed. The problems of CP violation in neutrino oscillations and of determining the nature - Dirac or Majorana - of massive neutrinos, are discussed. The seesaw mechanism of neutrino mass generation and the related leptogenesis scenario of generation of the baryon asymmetry of the Universe, are considered. The results showing that the CP violation necessary for the generation of the baryon asymmetry of the Universe in leptogenesis can be due exclusively to the Dirac and/or Majorana CP-violating phase(s) in the neutrino mixing matrix $U$, are briefly reviewed.

Keywords: Neutrino mixing; Dirac and Majorana Leptonic CP violation; Neutrino Oscillations; Neutrinoless Double Beta Decay; Seesaw Mechanism; Leptogenesis.

1. Introduction: Neutrinos (Preliminary Remarks)

It is both an honor and a pleasure to speak at this Conference, organized in honor of Prof. Freeman Dyson’s 90th birthday. My talk will be devoted to aspects of neutrino physics, so I would like to start by recalling some basic facts about neutrinos. It is well established experimentally that the neutrinos and antineutrinos which take part in the standard charged current (CC) and neutral current (NC) weak interaction are of three varieties (types) or flavours: electron, $\nu_e$ and $\bar{\nu}_e$, muon, $\nu_\mu$ and $\bar{\nu}_\mu$, and tauon, $\nu_\tau$ and $\bar{\nu}_\tau$. The notion of neutrino type or flavour is dynamical: $\nu_e$ is the neutrino which is produced with $e^+$, or produces an $e^-$ in CC weak interaction processes; $\nu_\mu$ is the neutrino which is produced with $\mu^+$, or produces $\mu^-$, etc. The flavour of a given neutrino is Lorentz invariant. Among the three different flavour neutrinos and antineutrinos, no two are identical. Correspondingly, the states which describe different flavour neutrinos must be orthogonal (within the precision of the current data): $\langle \nu_1 | \nu_1 \rangle = \delta_{11}$, $\langle \bar{\nu}_1 | \bar{\nu}_1 \rangle = \delta_{11}$, $\langle \bar{\nu}_i | \nu_j \rangle = 0$.

It is also well-known from the existing data (all neutrino experiments were done so far with relativistic neutrinos or antineutrinos), that the flavour neutrinos $\nu_i$...
(antineutrinos $\bar{\nu}_l$), are always produced in weak interaction processes in a state that is predominantly left-handed (LH) (right-handed (RH)). To account for this fact, $\nu_l$ and $\bar{\nu}_l$ are described in the Standard Theory (ST) by a chiral LH flavour neutrino field $\nu_{LL}(x)$, $l = e, \mu, \tau$. For massless $\nu_l$, the state of $\nu_l$ ($\bar{\nu}_l$) which the field $\nu_{LL}(x)$ annihilates (creates) is with helicity (-1/2) (helicity +1/2). If $\nu_l$ has a non-zero mass $m(\nu_l)$, the state of $\nu_l$ ($\bar{\nu}_l$) is a linear superposition of the helicity (-1/2) and (+1/2) states, but the helicity +1/2 state (helicity -1/2 state) enters into the superposition with a coefficient $\propto m(\nu_l)/E$, $E$ being the neutrino energy, and thus is strongly suppressed. Together with the LH charged lepton field $l_{L}(x)$, $\nu_{LL}(x)$ forms an $SU(2)_L$ doublet. In the absence of neutrino mixing and zero neutrino masses, $\nu_{LL}(x)$ and $l_{L}(x)$ can be assigned one unit of the additive lepton charge $L_l$, and the three charges $L_l$, $l = e, \mu, \tau$, as well as the total lepton charge, $L = L_{e} + L_{\mu} + L_{\tau}$, are conserved by the weak interaction.

At present there is no compelling evidence for the existence of states of relativistic neutrinos (antineutrinos), which are predominantly right-handed, $\nu_R$ (left-handed, $\bar{\nu}_L$). If RH neutrinos and LH antineutrinos exist, their interaction with matter should be much weaker than the weak interaction of the flavour LH neutrinos $\nu_l$ and RH antineutrinos $\bar{\nu}_l$, i.e., $\nu_R$ ($\bar{\nu}_L$) should be “sterile” or “inert” neutrinos (antineutrinos). In the formalism of the Standard Theory, the sterile $\nu_R$ and $\bar{\nu}_L$ can be described by $SU(2)_L$ singlet RH neutrino fields $\nu_R(x)$. In this case, $\nu_R$ and $\bar{\nu}_L$ will have no gauge interactions, i.e., will not couple to the weak $W^\pm$ and $Z^0$ bosons. The simplest hypothesis (based on symmetry considerations) is that to each LH flavour neutrino field $\nu_{LL}(x)$ there corresponds a RH neutrino field $\nu_{LR}(x)$, $l = e, \mu, \tau$, although schemes with less (more) than three RH neutrinos are also being considered.

If present in an extension of the Standard Theory (even in the minimal one), the RH neutrinos can play a crucial role i) in the generation of neutrino masses and mixing, ii) in understanding the remarkable disparity between the magnitudes of neutrino masses and the masses of the charged leptons and quarks, and iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via the leptogenesis mechanism). In this scenario which is based on the see-saw theory, there is a link between the generation of neutrino masses and the generation of the matter-antimatter (or baryon) asymmetry of the Universe. In this talk we will review this remarkable connection. We will discuss also the interesting possibility that the CP violation necessary for the generation of the observed matter-antimatter asymmetry of the Universe in the leptogenesis scenario of the asymmetry generation can be provided exclusively by the Dirac and/or Majorana CP violation phases, present in the neutrino mixing matrix.

2. The Neutrino Mixing

There have been remarkable discoveries in the field of neutrino physics in the last 15 years or so. The experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidences for the existence of neutrino oscilla-
tions\textsuperscript{[10,11]}, transitions in flight between the different flavour neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$ (antineutrinos $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$), caused by nonzero neutrino masses and neutrino mixing (see, e.g., Ref.\textsuperscript{11} for review of the relevant data). The existence of flavour neutrino oscillations implies that if a neutrino of a given flavour, say $\nu_\mu$, with energy $E$ is produced in some weak interaction process, at a sufficiently large distance $L$ from the $\nu_\mu$ source the probability to find a neutrino of a different flavour, say $\nu_\tau$, $P(\nu_\mu \rightarrow \nu_\tau; E, L)$, is different from zero. $P(\nu_\mu \rightarrow \nu_\tau; E, L)$ is called the $\nu_\mu \rightarrow \nu_\tau$ oscillation or transition probability. If $P(\nu_\mu \rightarrow \nu_\tau; E, L) \neq 0$, the probability that $\nu_\mu$ will not change into a neutrino of a different flavour, i.e., the “$\nu_\mu$ survival probability” $P(\nu_\mu \rightarrow \nu_\mu; E, L)$, will be smaller than one. If only muon neutrinos $\nu_\mu$ are detected in a given experiment and they take part in oscillations, one would observe a “disappearance” of muon neutrinos on the way from the $\nu_\mu$ source to the detector.

The existing data, accumulated over more than 15 years allowed to firmly establish the existence of oscillations of the solar $\nu_\nu$ ($E \cong (0.23 - 14.4)$ MeV), atmospheric $\nu_\mu$ and $\nu_\tau$ ($E \cong (0.2 - 100)$ GeV) crossing the Earth, accelerator $\nu_\mu$ ($E \sim 1$ GeV) at $L = 250$; 295; 730 km and reactor $\nu_e$ ($E \cong (2.6 - 10.0)$ MeV) at $L \sim 1$; 180 km. The data imply the presence of mixing in the weak charged lepton current:

$$
\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \overline{\nu}_L(x) \gamma_\alpha \nu_l L (x) W^{\alpha\dagger}(x) + h.c., \quad \nu_l L (x) = \sum_{j=1}^{n} U_{ij} \nu_j L (x),
$$

(1)

where $\nu_l L (x)$ are the flavour neutrino fields, $\nu_j L (x)$ is the left-handed (LH) component of the field of the neutrino $\nu_j$ having a mass $m_j$, and $U$ is a unitary matrix - the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix\textsuperscript{[10,11]}, $U \equiv U_{PMNS}$. All compelling neutrino oscillation data can be described assuming 3-neutrino mixing in vacuum, $n = 3$. The number of massive neutrinos $n$ can, in general, be bigger than 3 if, e.g., there exist RH sterile neutrinos\textsuperscript{12} and they mix with the LH flavour neutrinos. It follows from the current data that at least 3 of the neutrinos $\nu_j$, say $\nu_1, \nu_2, \nu_3$, must be light, $m_{1,2,3} \lesssim 1$ eV, and must have different masses, $m_1 \neq m_2 \neq m_3$.\textsuperscript{13}

In the case of 3 light neutrinos, the neutrino mixing matrix $U$ can be parametrised by 3 angles and, depending on whether the massive neutrinos $\nu_j$ are Dirac or Majorana particles, by one Dirac, or one Dirac and two Majorana, CP violation (CPV) phases\textsuperscript{16}:

$$
U = V P, \quad P = \text{diag}(1, e^{i \frac{\varphi_1}{2}}, e^{i \frac{\varphi_2}{2}}),
$$

(2)

\textsuperscript{b}At present there are several experimental inconclusive hints for existence of one or two light sterile neutrinos at the eV scale, which mix with the flavour neutrinos, implying the presence in the neutrino mixing of additional one or two neutrinos, $\nu_4$ or $\nu_{4,5}$, with masses $m_4$ ($m_{4,5}$) $\sim 1$ eV (see, e.g., Refs.\textsuperscript{12,13}). The discussion of these hints and of the related implications is out of the scope of the present article.
where $\alpha_{21,31}$ are two Majorana CPV phases and $V$ is a CKM-like matrix,

$$
V = 
\begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},
$$

In Eq. (3), $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$, and $\delta = [0, 2\pi)$ is the Dirac CPV phase. Thus, in the case of massive Dirac neutrinos, the neutrino mixing matrix $U$ is similar, in what concerns the number of mixing angles and CPV phases, to the CKM quark mixing matrix. The presence of two additional physical CPV differences $\Delta m_{i,j}^2 \equiv (m_i^2 - m_j^2)$, $i \neq j$ (see, e.g., Refs. 2, 7, 14). On the basis of the existing neutrino data it is impossible to determine whether the massive neutrinos are Dirac or Majorana fermions.

The neutrino oscillation probabilities depend on the neutrino energy, $E$, the source-detector distance $L$, on the elements of $U$ and, for relativistic neutrinos used in all neutrino experiments performed so far, on the neutrino mass squared differences $\Delta m_{i,j}^2 \equiv (m_i^2 - m_j^2)$, $i \neq j$ (see, e.g., Ref. [14]). In the case of 3-neutrino mixing there are only two independent $\Delta m_{i,j}^2$, say $\Delta m_{21}^2 \neq 0$ and $\Delta m_{31}^2 \neq 0$. The numbering of the neutrinos $\nu_j$ is arbitrary. We will employ the widely used convention which allows to associate $\theta_{13}$ with the smallest mixing angle in the PMNS matrix, and $\theta_{12}$, $\Delta m_{21}^2 > 0$, and $\theta_{23}$, $\Delta m_{31}^2$, with the parameters which drive the solar ($\nu_e$) and the dominant atmospheric $\nu_\mu$ and $\nu_\tau$ oscillations, respectively. In this convention $m_1 < m_2$, $0 < \Delta m_{21}^2 < |\Delta m_{31}^2|$, and, depending on $\text{sgn}(\Delta m_{31}^2)$, we have either $m_3 < m_1$ or $m_3 > m_2$. The existing data allow us to determine $\Delta m_{21}^2$, $\theta_{12}$, and $|\Delta m_{31}^2|$, $\theta_{23}$ and $\theta_{13}$, with a relatively good precision,[15,19]. The best fit values and the $3\sigma$ allowed ranges of $\Delta m_{21}^2$, $s_{12}^2$, $|\Delta m_{31}^2|$, $s_{23}$ and $s_{13}^2$ read[19]:

$$
(D\Delta m_{21})_{BF} = 7.54 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{21}^2 = (6.99 - 8.18) \times 10^{-5} \text{ eV}^2, \quad (\text{sin}^2 \theta_{12})_{BF} = 0.308, \quad 0.259 \leq \text{sin}^2 \theta_{12} \leq 0.359, \quad (|\Delta m_{31}^2|)_{BF} = 2.48 (2.44) \times 10^{-3} \text{ eV}^2, \quad (\text{sin}^2 \theta_{23})_{BF} = 0.425 (0.437), \quad 0.357 (0.363) \leq \text{sin}^2 \theta_{23} \leq 0.641 (0.659), \quad (\text{sin}^2 \theta_{13})_{BF} = 0.0234 (0.0239), \quad 0.0177 (0.0178) \leq \text{sin}^2 \theta_{13} \leq 0.0297 (0.300),
$$

where when there are two values one of which is in brackets, the value (the value in brackets) corresponds to $\Delta m_{31}^2 > 0$ ($\Delta m_{31}^2 < 0$). There are also hints from data about the value of the Dirac phase $\delta$. In both analyses[15,19] the authors find that the best fit value of $\delta \approx 3\pi/2$. The CP conserving values $\delta = 0$ and $\pi$ ($\delta = 0$) are disfavored at 1.6$\sigma$ to 2.0$\sigma$ (at 2.0$\sigma$) for $\Delta m_{31}^2 > 0$ ($\Delta m_{31}^2 < 0$). In the case of $\Delta m_{31}^2 < 0$, the value $\delta = \pi$ is statistically 1$\sigma$ away from the best fit value $\delta \approx 3\pi/2$ (see, e.g., Fig. 3 in Ref. [15].
Thus, we have $\Delta m^2_{21}/|\Delta m^2_{31(32)}| \cong 0.03$, and $|\Delta m^2_{31}| = |\Delta m^2_{32} - \Delta m^2_{21}| \cong |\Delta m^2_{12}|$. Maximal solar neutrino mixing, i.e. $\theta_{12} = \pi/4$, is ruled out at more than 6$\sigma$ by the data. Correspondingly, one has $\cos 2\theta_{12} \geq 0.28$ (at 99.73% C.L.). The angle $\theta_{13}$ was measured relatively recently - in the spring of 2012 - in the high precision Daya Bay experiment and RENO experiments.

The results quoted above imply also that $\theta_{23} \cong \pi/4$, $\theta_{12} \cong \pi/5.4$ and that $\theta_{13} < \pi/13$. Correspondingly, the pattern of neutrino mixing is drastically different from the pattern of quark mixing.

The existing data do not allow one to determine the sign of $\Delta m^2_{31(32)}$. In the case of 3-neutrino mixing, the two possible signs of $\Delta m^2_{31(32)}$ correspond to two types of neutrino mass spectrum. In the convention of numbering the neutrinos $\nu_j$ employed by us, the two spectra read:

i) *spectrum with normal ordering (NO):* $m_1 < m_2 < m_3$, $\Delta m^2_{31(32)} > 0$, $\Delta m^2_{21} > 0$, $m_2(3) = (m_1^2 + \Delta m^2_{21(31)})^{1/2}$;

ii) *spectrum with inverted ordering (IO):* $m_3 < m_1 < m_2$, $\Delta m^2_{32(31)} < 0$, $\Delta m^2_{21} > 0$, $m_2 = (m_3^2 + \Delta m^2_{23})^{1/2}$, $m_1 = (m_3^2 + \Delta m^2_{23} - \Delta m^2_{31})^{1/2}$.

Depending on the values of the lightest neutrino mass, $\min(m_j)$, the neutrino mass spectrum can also be:

a) *Normal Hierarchical (NH):* $m_1 \ll m_2 < m_3$, $m_2 \cong (\Delta m^2_{31})^{1/2} \cong 8.7 \times 10^{-3}$ eV, $m_3 \cong (\Delta m^2_{31})^{1/2} \cong 0.050$ eV; or

b) *Inverted Hierarchical (IH):* $m_3 \ll m_1 < m_2$, $m_1,2 \cong |\Delta m^2_{32}|^{1/2} \cong 0.049$ eV; or

c) *Quasi-Degenerate (QD):* $m_1 \cong m_2 \cong m_3 \cong m_0$, $m_0 \cong |\Delta m^2_{31(32)}|$, $m_0 \cong 0.10$ eV.

All three types of spectra are compatible with the existing constraints on the absolute scale of neutrino masses $m_j$. Determining the type of neutrino mass spectrum is one of the main goals of the future experiments in the field of neutrino physics (see, e.g., Refs. 11-13,21).

Information about the absolute neutrino mass scale (or about $\min(m_j)$) can be obtained, e.g., by measuring the spectrum of electrons near the end point in $^3$H $\beta$-decay experiments and from cosmological and astrophysical data. The most stringent upper bounds on the $\bar{\nu}_e$ mass were obtained in the Troitzk experiment:

$$m_{\bar{\nu}_e} < 2.05 \text{ eV} \quad \text{at 95\% C.L.}$$  \hspace{1cm} (10)

Similar result was obtained in the Mainz experiment: $m_{\bar{\nu}_e} < 2.3$ eV at 95\% CL.

We have $m_{\bar{\nu}_e} \cong m_{1,2,3}$ in the case of QD spectrum. The KATRIN experiment is planned to reach sensitivity of $m_{\bar{\nu}_e} \sim 0.20$ eV, i.e., it will probe the region of the QD spectrum.

The Cosmic Microwave Background (CMB) data of the WMAP experiment, combined with supernovae data and data on galaxy clustering can be used to obtain an upper limit on the sum of neutrinos masses (see and, e.g., Ref. 27). Depending on

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*For a brief discussion of experiments which can provide data on the type of neutrino mass spectrum see, e.g., Ref. 19 for some specific proposals see, e.g., Ref. 20.*
the model complexity and the input data used one obtains\cite{27}: \[ \sum_j m_j \lesssim (0.3 - 1.3) \text{ eV}, \quad 95\% \text{ CL.} \]

In March of 2013 the Planck Collaboration published their first constraints on \[ \sum_j m_j \] 27. Assuming the existence of three massive neutrinos and the validity of the \( \Lambda \) CDM (Cold Dark Matter) model, and combining their data on the CMB temperature power spectrum with the WMAP polarisation low-multiple (\( \ell \leq 23 \)) and ACT high-multiple (\( \ell \geq 2500 \)) CMB data,\cite{28,29} the Planck Collaboration reported the following upper limit on the sum of the neutrino masses,\cite{27}:

\[
\sum_j m_j < 0.66 \text{ eV}, \quad 95\% \text{ CL.}
\]

Adding the data on the Baryon Acoustic Oscillations (BAO) lowers significantly the limit\cite{27}:

\[ \sum_j m_j < (0.23 \text{ eV}), \quad 95\% \text{ CL.} \]

It follows from these data that neutrino masses are much smaller than the masses of charged leptons and quarks. If we take as an indicative upper limit \( m_j \lesssim 0.5 \text{ eV} \), we have \( m_j/m_{l,q} \lesssim 10^{-6} \), where \( m_l \) and \( m_q \) are the charged lepton and quark masses, \( l = e, \mu, \tau \), \( q = d, s, b, u, c, t \). It is natural to suppose that the remarkable smallness of neutrino masses is related to the existence of a new fundamental mass scale in particle physics, and thus to new physics beyond that predicted by the Standard Theory.

3. CP Violation in the Lepton Sector

3.1. Dirac CP Violation

The relatively large value of \( \sin \theta_{13} \approx 0.15 \) measured with a high precision in the Daya Bay\cite{17} and RENO\cite{18} experiments has far-reaching implications for the program of research in neutrino physics, and more specifically,

i) for the determination of the type of neutrino mass spectrum (or of \( \text{sgn}(\Delta m^2_{31(32)}) \)) in neutrino oscillation experiments (see, e.g., Refs.\cite{20,21});

ii) for understanding the pattern of the neutrino mixing and its origins (see, e.g., Ref.\cite{31} and the references quoted therein);

iii) for the predictions for the (\( \beta\beta \))\text{$_{0v}$}-decay effective Majorana mass in the case of NH light neutrino mass spectrum (see, e.g., Ref.\cite{19}).

The relatively large value of \( \sin \theta_{13} \approx 0.15 \) combined with the value of \( \delta = 3\pi/2 \) has far-reaching implications for the searches for CP violation in neutrino oscillations (see further). It has also important implications for the “flavoured” leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU). As we will discuss in greater detail in Section 5, if all CP violation necessary for the generation of BAU is due to the Dirac phase \( \delta \), a necessary condition for reproducing the observed BAU is\cite{15} \[ |\sin \theta_{13} \sin \delta| \gtrsim 0.09, \]  which is comfortably compatible with the measured value of \( \sin \theta_{13} \) and with best fit value of \( \delta \approx 3\pi/2 \).

A CP nonconserving value of the Dirac phase \( \delta \) will cause CP violation in flavour neutrino oscillations, \( \nu_l \rightarrow \nu_q, \bar{\nu}_l \rightarrow \bar{\nu}_q \), \( l \neq l' = e, \mu, \tau \). Indeed, CP-, T- and CPT-
invariance imply for $\nu_l \to \nu_{l'}$ oscillation probabilities:

\[ P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_l \to \bar{\nu}_{l'}), \quad \text{CP - invariance}, \quad (12) \]

\[ P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l), \quad \text{T - invariance}, \quad (13) \]

\[ P(\bar{\nu}_l \to \bar{\nu}_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l), \quad \text{T - invariance}, \quad (14) \]

\[ P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l), \quad \text{CPT - invariance}, \quad (15) \]

where $l, l' = e, \mu, \tau$. It follows from CPT-invariance that for $l = l' = e, \mu, \tau$ we have:

\[ P(\nu_l \to \nu_l) = P(\bar{\nu}_l \to \bar{\nu}_l). \quad (16) \]

From the comparison of Eqs. (12) and (16) it is clear that if CPT invariance holds, which we will assume to be the case, the “disappearance” neutrino oscillation experiments in which one gets information about the probabilities $P(\nu_l \to \nu_l)$ and $P(\bar{\nu}_l \to \bar{\nu}_l)$, $l = e, \mu, \tau$, are not sensitive to CP-violation. Therefore, a measure of $CP$- and $T$- violation is provided by the asymmetries:

\[ A_{\text{CP}}^{(l,l')} = P(\nu_l \to \nu_{l'}) - P(\bar{\nu}_l \to \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau, \quad (17) \]

\[ A_{\text{T}}^{(l,l')} = P(\nu_l \to \nu_{l'}) - P(\nu_{l'} \to \nu_l), \quad l \neq l' = e, \mu, \tau. \quad (18) \]

For 3-$\nu$ oscillations in vacuum one has:

\[ A_{\text{CP}}^{(e,\mu)} = A_{\text{CP}}^{(\mu,e)} = -A_{\text{CP}}^{(e,\tau)} = A_{\text{CP}}^{(\mu,\tau)} = A_{\text{T}}^{(e,\mu)} = A_{\text{T}}^{(e,\tau)} = -A_{\text{T}}^{(\mu,\tau)} = J_{\text{CP}} F_{\text{osc}}^{\text{vac}}, \quad (19) \]

\[ J_{\text{CP}} = \text{Im} \left\{ U_{e1} U_{\mu2} U_{e1}^* U_{\mu2}^* \right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta, \quad (20) \]

\[ F_{\text{osc}}^{\text{vac}} = \sin(\frac{\Delta m_{12}^2}{2E} L) + \sin(\frac{\Delta m_{32}^2}{2E} L) + \sin(\frac{\Delta m_{13}^2}{2E} L). \quad (21) \]

Thus, the magnitude of CP violation effects in neutrino oscillations is controlled by the rephasing invariant associated with the Dirac phase $\delta$, $J_{\text{CP}}$. The latter is analogous to the rephasing invariant associated with the Dirac phase in the Cabibbo-Kobayashi-Maskawa quark mixing matrix, introduced in Ref. [3]. The existence of Dirac CPV in the lepton sector would be established if, e.g., some of the vacuum oscillation asymmetries $A_{\text{CP}}^{(e,\mu)}$, $A_{\text{CP}}^{(e,\tau)}$, etc. are proven experimentally to be nonzero. This would imply that $J_{\text{CP}} \neq 0$, and, consequently, that $\sin \theta_{13} \sin \delta \neq 0$, which in turn would mean that $\sin \delta \neq 0$ since $\sin \theta_{13} \neq 0$.

Given the fact that $\sin 2\theta_{12}$, $\sin 2\theta_{23}$ and $\sin 2\theta_{13}$ have been determined experimentally with a relatively good precision, the size of CP violation effects in neutrino oscillations depends essentially only on the magnitude of the currently not well determined value of the Dirac phase $\delta$. The current data implies $|J_{\text{CP}}| \lesssim 0.038 |\sin \delta|$, where we have used the $3\sigma$ ranges of $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ given in Eqs. (41) - (44). For the best fit values of $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ and $\delta$ we find in the case
of $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$): $J_{CP} \cong -0.032 (-0.031)$. Thus, if the indication that $\delta \cong 3\pi/2$ is confirmed by future more precise data, the CP violation effects in neutrino oscillations would be relatively large if the factor $F_{osc}^{\nu}$ is not suppressing the CPV asymmetries. We would have $F_{osc}^{\nu} \cong 0$ and the CPV asymmetries will be strongly suppressed, as it follows from Eqs. (19) and (21), if under the conditions of a given experiment one of the two neutrino mass squared differences, say $\Delta m^2_{31}$, is not operative, i.e., $\sin(\Delta m^2_{31} L/(2E)) \cong 0$. In this case the CP violation effects in neutrino oscillations will be hardly observable.

One of the major goals of the future experimental studies in neutrino physics is the searches for CPV effects due to the Dirac phase in the PMNS mixing matrix (see, e.g., Refs. [31,32]). It follows from the preceding discussion that in order for the CPV effects in neutrino oscillations to be observable, both $\sin(\Delta m^2_{31} L/(2E))$ and $\sin(\Delta m^2_{21} L/(2E))$ should be sufficiently large. In the case of $\sin(\Delta m^2_{31} L/(2E))$, for instance, this requires that, say, $\Delta m^2_{31} L/(2E) \sim 1$. The future experiments on CP violation in neutrino oscillations are planned to be performed with accelerator $\nu_{\mu}$ and $\bar{\nu}_{\mu}$ beams with energies of a few GeV. Taking as an instructive example $E = 1$ GeV and using the best fit value of $\Delta m^2_{31} = 2.48 \times 10^{-3}$ $eV^2$, it is easy to check that $\Delta m^2_{31} L/(2E) \sim 1$ for $L \sim 10^3$ km. Thus, the study of neutrino oscillations requires experiments to have relatively long baselines. The MINOS, T2K and OPERA experiments (see, e.g., Ref. [1] and references quoted therein), which have provided and continue to provide data on $\nu_{\mu}$ oscillations, have baselines of approximately 735 km, 295 km and 730 km, respectively. The NO$\nu$A experiment, which is under preparation and is planned to start taking data in 2014, has a baseline of 810 km.

Thus, in the MINOS, OPERA, NO$\nu$A and in the future planned experiments (see, e.g., Ref. [39] the baselines are such that the neutrinos travel relatively long distances in the matter of the Earth mantle. As is well known, the presence of matter can modify drastically the pattern of neutrino oscillations [37]. When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrons, which generates an effective potential $V_{eff}$ in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$. This modifies the neutrino mixing since the eigenstates and the eigenvalues of $H_{vac}$ and of $H = H_{vac} + V_{eff}$ are different, leading to different oscillation probabilities with respect to those of oscillations in vacuum. Typically, the matter background is not charge conjugation (C-) symmetric: the Earth and the Sun, for instance, contain only electrons, protons and neutrons, but do not contain their antiparticles. As a consequence, the oscillations taking place in the Earth, are neither CP- nor CPT- invariant [35]. This complicates the studies of CP violation due to the Dirac phase $\delta$ in long baseline neutrino oscillation experiments since neutrinos have relatively long paths in the Earth (see, e.g., Refs. [38,39]). The matter effects in neutrino oscillations in the Earth to a good precision are not T-violating [33] since the Earth matter density distribution is to a good approximation spherically symmetric. In matter with constant density, e.g., the Earth mantle, one has [33]: $A_{T(\mu)}^{(\nu)} = J_{CP}^{\nu} F_{osc}^{\nu}$, $J_{CP}^{\nu} = J_{CP}^{\nu} R_{CP}$, where the dimensionless function $R_{CP}$
by studying the energy dependence of $P$ formation about the Dirac CP violation phase in $U$

ever, we have $A$
effects of the Earth matter and of $J$
effects of the Earth matter. It will be important to experimentally disentangle the

$\nu$ mixing matrix $U$
term $P$
mally from that for $J$

The $N$
density approximation $N$
mummary rate in what concerns the calculation of $\nu$ oscillation probabilities along the given $\nu$ path in the Earth, was shown to be sufficiently accurate in what concerns the calculation of $\nu$ oscillation probabilities.

$N_e^{\text{man}}$ being the electron number density of the Earth mantle. Thus, the quantity $A$ accounts for the Earth matter effects in $\nu$ oscillations. The mean electron number density in the Earth mantle is $N_e^{\text{man}} \approx 2.2$ cm$^{-3}$ $N_A$, $N_A$ being Avogadro’s number. In the case of the experiments under discussion, the electron number density $N_e$ changes relatively little around the indicated mean value along the trajectories of neutrinos in the Earth mantle and the constant density approximation $N_e^{\text{man}} = \text{const.} = N_e^{\text{man}}$, $N_e^{\text{man}}$ being the mean density along the given $\nu$ path in the Earth, was shown to be sufficiently accurate in what concerns the calculation of $\nu$ oscillation probabilities.

The expression for the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probability can be obtained formally from that for $P_{\mu e}^{\nu \text{man}}(\nu_\mu \rightarrow \nu_e)$ by making the changes $A \rightarrow -A$ and $J_{CP} \rightarrow -J_{CP}$, with $J_{CP} \cot \delta \equiv \text{Re}(U_{\mu 3} U_{e 3}^\ast U_{e 2} U_{\mu 2}^\ast)$ remaining unchanged. The term $P_{\sin \delta}$ in $P_{\mu e}^{\nu \text{man}}(\nu_\mu \rightarrow \nu_e)$ would be equal to zero if the Dirac phase in the $\nu$ mixing matrix $U$ possesses a CP-conserving value. Even in this case, however, we have $A_{CP}^{(ep) \text{man}} \equiv (P_{\mu e}^{\nu \text{man}}(\nu_\mu \rightarrow \nu_e) - P_{\mu e}^{\nu \text{man}}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)) \neq 0$ due to the effects of the Earth matter. It will be important to experimentally disentangle the effects of the Earth matter and of $J_{CP}$ in $A_{CP}^{(ep) \text{man}}$; this will allow to get direct information about the Dirac CP violation phase in $U$. This can be done, in principle, by studying the energy dependence of $P_{\mu e}^{\nu \text{man}}(\nu_\mu \rightarrow \nu_e)$ and $P_{\mu e}^{\nu \text{man}}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$.

$\dagger$The conditions of validity of the analytic expression for $P_{\mu e}^{\nu \text{man}}(\nu_\mu \rightarrow \nu_e)$ given above are discussed in detail in Ref. 40.
the vacuum limit of $N_{man}^{\nu\nu} = 0$ ($A = 0$) we have $A_{CP}^{(\nu\mu)} = A_{CP}^{(\nu\mu)}$ (see Eq. (19)) and only the term $P_{\sin \delta}$ contributes to the asymmetry $A_{CP}^{(\nu\mu)}$.

The preceding remarks apply also to the probabilities $P_{\nu\nu}^{\nu\nu} (\nu_e \rightarrow \nu_\mu)$ and $P_{\nu\nu}^{\nu\nu} (\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$. The probability $P_{\nu\nu}^{\nu\nu} (\nu_e \rightarrow \nu_\mu)$, for example, can formally be obtained from the expression for the probability $P_{\nu\nu}^{\nu\nu} (\nu_\mu \rightarrow \nu_e)$ by changing the sign of the term $P_{\sin \delta}$.

3.2. Majorana CP Violation Phases and $(\beta\beta)_{0\nu}$-Decay

The massive neutrinos $\nu_j$ can be Majorana fermions. Many theories of neutrino mass generation predict massive neutrinos to be Majorana fermions (see, e.g., Refs. [6][4][5]. If $\nu_j$ are proven to be Majorana particles, the neutrino mixing matrix $U$, as we have already emphasised, will contain two additional CP violation “Majorana” phases, $\alpha_{21}$ and $\alpha_{31}$. Getting experimental information about the Majorana CPV phases $\alpha_{21}$ and $\alpha_{31}$ in $U$ will be remarkably difficult [10][21]. The oscillations of flavour neutrinos, $\nu_l \rightarrow \nu_l$ and $\bar{\nu}_l \rightarrow \bar{\nu}_l$, $l = e, \mu, \tau$, are insensitive to the phases $\alpha_{21,31}$ [7][3]. The phases $\alpha_{21,31}$ cannot affect significantly the predictions for the rates of the (LFV) decays $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. in a large class of supersymmetric theories incorporating the see-saw mechanism [22]. As we will discuss further, the Majorana phase(s) in the PMNS matrix can play the role of the leptogenesis CPV parameter(s) at the origin of the baryon asymmetry of the Universe [8].

The Majorana nature of massive neutrinos manifests itself in the existence of processes in which the total lepton charge changes by two units, $|\Delta L| = 2$: $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$, $e^- + (A, Z) \rightarrow e^+ + (A, Z - 2)$, etc. The only feasible experiments which at present have the potential of establishing the Majorana nature of light neutrinos $\nu_j$ and of providing information on the Majorana CPV phases in PMNS matrix are the experiments searching for neutrinoless double beta $(\beta\beta)_{0\nu}$-decay, $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$, of even-even nuclei $^{48}\text{Ca}$, $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{100}\text{Mo}$, $^{116}\text{Cd}$, $^{130}\text{Te}$, $^{136}\text{Xe}$, $^{150}\text{Nd}$, etc. (see, e.g., Refs. [2][5]). In $(\beta\beta)_{0\nu}$-decay, two neutrinos of the initial nucleus $(A, Z)$ transform by exchanging the virtual light massive Majorana neutrino(s) $\nu_j$ into two protons of the final state nucleus $(A, Z + 2)$ and two free electrons. The corresponding $(\beta\beta)_{0\nu}$-decay amplitude has the form (see, e.g., Refs. [1][5]):

$$A((\beta\beta)_{0\nu}) = G_F^2 <m> M(A, Z),$$

where $G_F$ is the Fermi constant, $<m>$ is the $(\beta\beta)_{0\nu}$-decay effective Majorana mass and $M(A, Z)$ is the nuclear matrix element (NME) of the process. The $(\beta\beta)_{0\nu}$-decay effective Majorana mass $<m>$ contains all the dependence of the $(\beta\beta)_{0\nu}$-decay amplitude on the neutrino mixing parameters. We have (see, e.g., Ref. [5]):

$$|<m>| = |m_1| |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i(\alpha_{31} - 2\delta)}|,$$

(28)

$|U_{e1}| = c_{12}c_{13}$, $|U_{e2}| = s_{12}c_{13}$, $|U_{e3}| = s_{13}$. For the normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) neutrino mass spectra $|<m>|$ is given by (see, e.g., Ref. [55]).
Fig. 1. The effective Majorana mass $|\langle m \rangle|$ (including a 2σ uncertainty), as a function of $m_{\text{min}} = \min(m_j)$ for $\sin^2 \theta_{13} = 0.0236 \pm 0.0042^{+0.001}_{-0.001}$, $\delta = 0$ and using the 95% C.L. allowed ranges of $\Delta m^2_{21}$, $|\Delta m^2_{31(32)}|$, $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ found in Ref. [63]. The phases $\alpha_{21,31}$ are varied in the interval $[0, \pi]$. The predictions for the NH, IH and QD spectra are indicated. The red regions correspond to at least one of the phases $\alpha_{21,31}$ and $(\alpha_{31} - \alpha_{21})$ having a CP violating value, while the blue and green areas correspond to $\alpha_{21,31}$ possessing CP conserving values. (From Ref. [1].)

$$|\langle m \rangle| \approx |\sqrt{\Delta m^2_{21}} s^2_{12} + \sqrt{\Delta m^2_{31}} s^2_{13} e^{i(\alpha_{32} - 2\delta)}|,$$ NH,

$$|\langle m \rangle| \approx \sqrt{|\Delta m^2_{32}|} |c^2_{12} + s^2_{12} e^{i\alpha_{21}}|,$$ IH,

$$|\langle m \rangle| \approx m_0 |c^2_{12} + s^2_{12} e^{i\alpha_{21}}|,$$ QD,

where $\alpha_{32} = \alpha_{31} - \alpha_{21}$. Obviously, $|\langle m \rangle|$ depends strongly on the Majorana phase(s): the CP-conserving values of $\alpha_{21} = 0, \pm \frac{\pi}{2}$, for instance, determine the range of possible values of $|\langle m \rangle|$ in the cases of IH and QD spectrum. As is well-known, if CP-invariance holds, the phase factor

$$\eta_{jk} = e^{i\alpha_{jk}} = \pm 1, j > k, j, k = 1, 2, 3,$$

represents the relative CP-parity of Majorana neutrinos $\nu_j$ and $\nu_k$,

$$\eta_{jk} = \eta^\nu_{jk} (\eta^\nu_{k}^{-1})^* = \eta^\nu_{j(k)} = \pm i$$ being the CP-parity of $\nu_{j(k)}$.

Using the 3σ ranges of the allowed values of the neutrino oscillation parameters quoted in Eqs. (9) - (11), one finds that:

1) $0.70 \times 10^{-3} \text{ eV} \lesssim |\langle m \rangle| \lesssim 4.51 \times 10^{-3} \text{ eV}$ in the case of NH spectrum;

2) $1.44 \times 10^{-2} \text{ eV} \lesssim |\langle m \rangle| \lesssim 4.80 \times 10^{-2} \text{ eV}$ in the case of IH spectrum;

3) $2.80 \times 10^{-2} \text{ eV} \lesssim |\langle m \rangle| \lesssim m_0 \text{ eV}$, $m_0 \gtrsim 0.10 \text{ eV}$, in the case of QD spectrum.

The difference in the ranges of $|\langle m \rangle|$ in the cases of NH, IH and QD spectrum opens up the possibility to get information about the type of neutrino mass spectrum.
from a measurement of $|<m>|$\textsuperscript{62}. The main features of the predictions for $|<m>|$ are illustrated in Fig. 1, where $|<m>|$ is shown as a function of the lightest neutrino mass $m_{\text{min}} \equiv \min(m_j)$.

The experimental searches for $(\beta\beta)_{0\nu}$-decay have a long history (see, e.g., Ref. \textsuperscript{68}). A positive $(\beta\beta)_{0\nu}$-decay signal at $> 3\sigma$, corresponding to $T_{1/2}^{0\nu} = (6.9 - 4.18) \times 10^{25}$ yr (99.73% C.L.) and implying $|<m>| = (0.1 - 0.9)$ eV, is claimed to have been observed in\textsuperscript{60}, while a later analysis\textsuperscript{63} reports evidence for $(\beta\beta)_{0\nu}$-decay at $6\sigma$ with $T_{1/2}^{0\nu}(76\text{Ge}) = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yr, corresponding to $|<m>| = 0.32 \pm 0.03$ eV. The best lower limit on the half-life of $76\text{Ge}$, $T_{1/2}^{0\nu}(76\text{Ge}) > 2.1 \times 10^{25}$ yr (90% C.L.), was found in the GERDA $76\text{Ge}$ experiment\textsuperscript{61}. By combining the limits obtained in the Heidelberg-Moscow\textsuperscript{64}, IGEX\textsuperscript{65} and GERDA experiments one gets\textsuperscript{66} $T_{1/2}^{0\nu}(76\text{Ge}) > 3.0 \times 10^{25}$ yr (90% C.L.).

Two experiments, NEMO\textsuperscript{3} with $^{100}\text{Mo}$ and CUORICINO\textsuperscript{67} with $^{130}\text{Te}$, obtained the limits: $|<m>| < (0.61 - 1.26)$ eV\textsuperscript{63} and $|<m>| < (0.16 - 0.68)$ eV\textsuperscript{65} (90% C.L.), where estimated uncertainties in the NME are accounted for. The best lower limits on the $(\beta\beta)_{0\nu}$-decay half-life of $^{136}\text{Xe}$ were reported by the EXO\textsuperscript{68} and KamLAND-Zen\textsuperscript{66} collaborations: $T_{1/2}^{0\nu}(136\text{Xe}) > 1.6 \times 10^{25}$ yr\textsuperscript{69} and $T_{1/2}^{0\nu}(136\text{Xe}) > 1.9 \times 10^{25}$ yr\textsuperscript{67} (90% C.L.).

Most importantly, a large number of experiments of a new generation aim at sensitivity to $|<m>| \sim (0.01 - 0.05)$ eV (see, e.g., Ref.\textsuperscript{63}): CUORE ($^{130}\text{Te}$), GERDA ($^{76}\text{Ge}$), SuperNEMO, EXO ($^{136}\text{Xe}$), MAJORANA ($^{76}\text{Ge}$), AMoRE ($^{100}\text{Mo}$), MOON ($^{100}\text{Mo}$), COBRA ($^{116}\text{Cd}$), CANDLES ($^{48}\text{Ca}$), KamLAND-Zen ($^{136}\text{Xe}$), SNO+ ($^{130}\text{Te}$), etc. GERDA, EXO and KamLAND-Zen have provided already the best lower limits on the $(\beta\beta)_{0\nu}$-decay half-lives of $^{76}\text{Ge}$ and $^{136}\text{Xe}$. The experiments listed above are aiming to probe the QD and IH ranges of $|<m>|$; they will test the positive result claimed in Ref.\textsuperscript{60}. If the $(\beta\beta)_{0\nu}$-decay will be observed in these experiments, the measurement of the $(\beta\beta)_{0\nu}$-decay half-life might allow to obtain constraints on the Majorana phase $\alpha_{21}$\textsuperscript{60,65} (see also Ref.\textsuperscript{69}).

The possibility of establishing CP violation in the lepton sector due to Majorana CPV phases has been studied in Refs.\textsuperscript{47,48} and in much greater detail in Refs.\textsuperscript{49,50}. It was found that it is very challenging: it requires quite accurate measurements of $|<m>|$ (and of $m_0$ for QD spectrum), and holds only for a limited range of values of the relevant parameters. More specifically\textsuperscript{49,50}, establishing at $2\sigma$ CP-violation associated with Majorana neutrinos in the case of QD spectrum requires for $\sin^2 \theta_{12} = 0.31$, in particular, a relative experimental error on the measured value of $|<m>|$ and $m_0$ smaller than 15%, a “theoretical uncertainty” $F \lesssim 1.5$ in the value of $|<m>|$ due to an imprecise knowledge of the corresponding NME, and value of the relevant Majorana CPV phase $\alpha_{21}$ typically within the ranges of $\sim (\pi/4 - 3\pi/4)$ and $\sim (5\pi/4 - 7\pi/4)$.

The knowledge of NME with sufficiently small uncertainty\textsuperscript{6} is crucial for obtain-

\textsuperscript{6}A possible test of the NME calculations is suggested in Ref.\textsuperscript{47} and is discussed in greater detail.
ing quantitative information on the neutrino mixing parameters from a measurement of $(\beta\beta)_{0\nu}$-decay half-life\textsuperscript{[4].} The observation of a $(\beta\beta)_{0\nu}$-decay of one nucleus is likely to lead to the searches and eventually to observation of the decay of other nuclei. One can expect that such a progress, in particular, will help to solve completely the problem of the sufficiently precise calculation of the nuclear matrix elements for the $(\beta\beta)_{0\nu}$-decay\textsuperscript{[57].}

If the future $(\beta\beta)_{0\nu}$-decay experiments show that $|<m>| < 0.01$ eV, both the IH and the QD spectrum will be ruled out for massive Majorana neutrinos. If in addition it is established in neutrino oscillation experiments that the neutrino mass spectrum is with inverted ordering, i.e. that $\Delta m^2_{31(32)} < 0$, one would be led to conclude that either the massive neutrinos $\nu_j$ are Dirac fermions, or that $\nu_j$ are Majorana particles but there are additional contributions to the $(\beta\beta)_{0\nu}$-decay amplitude which interfere destructively with that due to the exchange of light massive Majorana neutrinos. The case of more than one mechanism generating the $(\beta\beta)_{0\nu}$-decay was discussed recently in, e.g., Refs. \textsuperscript{[72]}, where the possibility to identify the mechanisms inducing the decay was also analysed. If, however, $\Delta m^2_{31(32)}$ is determined to be positive in neutrino oscillation experiments, the upper limit $|<m>| < 0.01$ eV would be perfectly compatible with massive Majorana neutrinos possessing NH mass spectrum, or mass spectrum with normal ordering but partial hierarchy, and the quest for $|<m>|$ would still be open.

Let us emphasise that determining the nature of massive neutrinos is one of the fundamental, most challenging and pressing problems in today’s neutrino physics (see, e.g. Refs. \textsuperscript{[1,53]}). Establishing whether the neutrinos with definite mass $\nu_j$ are Dirac fermions possessing distinct antiparticles, or Majorana fermions, i.e., spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for understanding the origin of neutrino masses and mixing and the underlying symmetries of particle interactions (see, e.g., Ref. \textsuperscript{[44]}. We recall that the neutrinos $\nu_j$ will be Dirac fermions if the particle interactions conserve some additive lepton number, e.g., the total lepton charge $L = L_e + L_\mu + L_\tau$. If no lepton charge is conserved, the neutrinos $\nu_j$ will be Majorana fermions. As we have seen, the massive neutrinos $\nu_j$ are predicted to be of Majorana nature by the see-saw mechanism\textsuperscript{[6]}. The observed patterns of neutrino mixing and of neutrino mass squared differences can be related to Majorana massive neutrinos and the existence of an approximate flavour symmetry in the lepton sector (see, e.g., Ref. \textsuperscript{[15]}. Determining the nature (Dirac or Majorana) of massive neutrinos $\nu_j$ is one of the major goals of the program of research in neutrino physics.

\textsuperscript{[4]}For discussions of the current status of the calculations of the NMEs for the $(\beta\beta)_{0\nu}$-decay see, e.g., the third article quoted in Ref. \textsuperscript{[53]} and Ref. \textsuperscript{[71]}.
4. The See-Saw Mechanism and Leptogenesis

A natural explanation of the smallness of neutrino masses is provided by the see-saw mechanism of neutrino mass generation[10]. An integral part of the simplest version of this mechanism - the so-called “type I see-saw”, are the $SU(2)_L$ singlet RH neutrinos $\nu_{R, l} = e, \mu, \tau$. The latter are assumed to possess a Majorana mass term as well as Yukawa type coupling with the Standard Theory lepton and Higgs doublets $\psi_{L}(x)$ and $\Phi(x)$, respectively, $(\psi_{L}(x))^T = (\nu_{L}^T(x) \quad I^T_{L}(x))$, $l = e, \mu, \tau$, $(\Phi(x))^T = (\Phi(0) \quad \Phi(-))$. The Standard Theory admits such a minimal extension which does not modify any of the basic attractive features of the Theory (unitarity, renormalisability, etc.). In the basis in which the Majorana mass matrix of RH neutrinos is diagonal we have:

$$\mathcal{L}_{V,M}(x) = - (\lambda_{kl} \overline{N}_{kR}(x)\Phi^\dagger(x) \psi_{L}(x) + \text{h.c.}) - \frac{1}{2} M_{k} \overline{N}_{k}(x) N_{k}(x), \quad (29)$$

where $\lambda_{kl}$ is the matrix of neutrino Yukawa couplings and $N_{k}(x)$ is the heavy (RH) Majorana neutrino field possessing a mass $M_{k} > 0$, $M_{1} < M_{2} < M_{3}$. The fields $N_{k}(x)$ satisfy the Majorana condition $CN_{k}^T(x) = \xi_{k}N_{k}(x)$, where $C$ is the charge conjugation matrix and $\xi_{k}$ is a phase. When the electroweak symmetry is broken spontaneously, the neutral component of the Higgs doublet field develops non-zero vacuum expectation value $v = 174$ GeV and the neutrino Yukawa coupling generates a neutrino Dirac mass term: $m_{kl}^{D} \overline{N}_{kR}(x) \nu_{lL}(x) + \text{h.c.}$, with $m_{kl}^{D} = v \lambda$. In the case when the elements of $m_{kl}^{D}$ are much smaller than $M_{k}$, $|m_{ij}^{D}| \ll M_{k}$, $j, k = 1, 2, 3$, $l = e, \mu, \tau$, the interplay between the Dirac mass term and the Majorana mass term of the heavy singlets $N_{k}$ generates an effective Majorana mass (term) for the LH flavour neutrino fields $\nu_{lL}(x)$[30].

$$\langle m_{\nu} \rangle_{\nu_{lL}} \cong v^2 (\lambda^T M^{-1} \lambda)_{lL} = ((m_{kl}^{D})^T M^{-1} m_{kl}^{D})_{lL} = (U^* m U^\dagger)_{lL}, \quad (30)$$

where $M = \text{Diag}(M_{1}, M_{2}, M_{3})$ ($M_{1,2,3} > 0$), $m = \text{Diag}(m_{1}, m_{2}, m_{3})$, $m_{j} \geq 0$ being the mass of the light Majorana neutrino $\nu_{j}$, and $U$ is the PMNS matrix The diagonalisation of the mass matrix $m_{\nu}$ leads to the appearance of the PMNS neutrino mixing matrix in the charged current weak interaction Lagrangian $\mathcal{L}_{CC}(x)$, Eq. (1).

In grand unified theories, $m_{D}$ is typically of the order of the charged fermion masses. In SO(10) theories[30], for instance, $m_{D}$ coincides with the up-quark mass matrix. Taking indicatively $m_{\nu} \sim 0.05$ eV, $m_{D} \sim 100$ GeV, one finds $M_{k} \sim 2 \times 10^{14}$ GeV, which is close to the scale of unification of electroweak and strong interactions, $M_{\text{GUT}} \cong 2 \times 10^{16}$ GeV. In GUT theories with RH neutrinos one finds that indeed the heavy singlets $N_{k}$ naturally obtain masses which are by few to several orders of magnitude smaller than $M_{\text{GUT}}$ (see, e.g., Ref. [44]).

One of the characteristic predictions of the see-saw mechanism is that both the light and heavy neutrinos $\nu_{j}$ and $N_{k}$ are Majorana particles. As we have discussed, the Majorana nature of the light neutrinos can be revealed in the $\beta\beta$-decay experiments.

We will discuss next briefly the interesting possibility[44] that the CP violation necessary for the generation of the baryon asymmetry of the Universe, $Y_{B}$, in the
leptogenesis scenario can be due exclusively to the Dirac and/or Majorana CPV phases in the PMNS matrix, and thus can be directly related to the low energy leptonic CP violation (e.g., in neutrino oscillations, etc.). We recall that leptogenesis is a simple mechanism which allows to explain the observed baryon asymmetry of the Universe, namely the observed difference in the present epoch of the evolution of the Universe of the number densities of baryons and anti-baryons, \( n_B \) and \( n_{\bar{B}} \):

\[
Y_B = \frac{n_B - n_{\bar{B}}}{s_0} = (8.67 \pm 0.15) \times 10^{-11},
\]

where \( s_0 \) is the entropy density in the current epoch. The simplest scheme in which the leptogenesis mechanism can be implemented is the type I seesaw model. In its minimal version it includes the Standard Theory plus two or three heavy (RH) Majorana neutrinos, \( N_k \). Thermal leptogenesis (see, e.g., Ref. [74]) can take place, e.g., in the case of hierarchical spectrum of the heavy neutrino masses, \( M_1 \ll M_2 \ll M_3 \), which we consider in what follows. The lepton asymmetry is produced in the Early Universe in out-of-equilibrium lepton number and CP nonconserving decays of the lightest heavy Majorana neutrino, \( N_1 \), mediated by the neutrino Yukawa couplings, \( \lambda \). The lepton asymmetry is converted into a baryon asymmetry by \((B-L)\)-conserving but \((B+L)\)-violating sphaleron interactions which exist within the Standard Theory and are efficient at temperatures \( T \sim 100 \text{ GeV} \). In grand unified theories the heavy neutrino masses fall typically in the range of \( \sim (10^8 - 10^{14}) \text{ GeV} \) (see, e.g., Ref. [44]). This range coincides with the range of values of \( M_k \), required for a successful thermal leptogenesis [74]. For hierarchical spectrum of the heavy neutrino masses \( M_1 \ll M_2 \ll M_3 \) we consider, leptogenesis takes place in the Early Universe typically at temperatures somewhat smaller than the mass of \( N_1 \), but not smaller than roughly \( 10^9 \text{ GeV} \), \( 10^9 \text{ GeV} \ll T < M_1 \).

In our further discussion it is convenient to use the “orthogonal parametrisation” of the matrix of neutrino Yukawa couplings [35]:

\[
\lambda = v^{-1} \sqrt{m} R \sqrt{m} U^\dagger, \quad R R^T = R^T R = 1,
\]

where \( R \) is, in general, a complex matrix. It is parametrised, in general, by six real parameters (e.g., three complex angles), of which three parameters can have CP violating values.

In the setting we are considering the only source of CP violation in the lepton sector is the matrix of neutrino Yukawa couplings \( \lambda \). It is clear from Eq. (32) that the CP violating parameters in the matrix \( \lambda \) can have their origin from the CP violating phases in the PMNS matrix \( U \), or from the CP violating parameters present in the matrix \( R \), or else from both the CP violating parameters in \( U \) and in \( R \).

For determining the conditions under which the CP-violation responsible for leptogenesis is due exclusively to the Dirac and/or Majorana CPV phases in the

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\( ^6 \)The entropy density \( s \) at temperature \( T \) is given by \( s = g_*(2\pi^2/45)T^3 \), where \( g_* \) is the number of (thermalised) degree of freedom at temperature \( T \). In the present epoch of the evolution of the Universe we have \( s_0 = 7.04 n_{\gamma 0} \), \( n_{\gamma 0} \) being the number density of photons.
PMNS matrix, it is useful to analyze the constraints which the requirement of CP-invariance imposes on the Yukawa couplings $\lambda_{jl}$, on the PMNS matrix $U$ and on the matrix $R$. These constraints read (in a certain well specified and rather widely used convention)\[3\]:

$$\lambda_{jl}^* = \lambda_{jl} \rho_j^N, \quad \rho_j^N = \pm 1, \quad j = 1, 2, 3, \quad l = e, \mu, \tau, \quad (33)$$

$$U_{ij}^* = U_{ij} \rho_j^\nu, \quad \rho_j^\nu = \pm 1, \quad j = 1, 2, 3, \quad l = e, \mu, \tau, \quad (34)$$

$$R_{jk}^* = R_{jk} \rho_j^N \rho_k^\nu, \quad j, k = 1, 2, 3, \quad (35)$$

where $i \rho_j^N = \pm i$ and $i \rho_k^\nu = \pm i$ are the CP-parities of the heavy and light Majorana neutrinos $N_j$ and $\nu_k$ (see, e.g., Refs. [2][14]). Obviously, the last would be a condition of reality of the matrix $R$ only if $\rho_j^N \rho_k^\nu = 1$ for any $j, k=1,2,3$. However, we can also have $\rho_j^N \rho_k^\nu = -1$ for some $j$ and $k$ and in that case $R_{jk}$ will be purely imaginary. Of interest for our further analysis is, in particular, the product

$$P_{jkml} = R_{jk} R_{jm} U_{ik}^* U_{lm}, \quad k \neq m. \quad (36)$$

If CP-invariance holds, we find from the conditions given above that $P_{jkml}$ has to be real\[3\]:

$$P_{jkml}^* = P_{jkml} (\rho_j^N)^2 (\rho_k^\nu)^2 (\rho_m^\nu)^2 = P_{jkml}. \quad (37)$$

Consider the case when CP-invariance conditions for the PMNS matrix are satisfied and $U_{\tau k} U_{\tau m}$ for given $k$ and $m$, $k < m$, $k = 1, 2, m = 2, 3$, is purely imaginary, i.e., $\text{Re}(U_{\tau k} U_{\tau m})=0$. This can be realised for $\delta = \pi q$, $q=0,1,2$, and $\rho_k^\nu \rho_m^\nu = -1$, i.e., if the relative CP-parity of the light Majorana neutrinos $\nu_k$ and $\nu_m$ is equal to $(-1)$, or, correspondingly, if $\alpha_{mk} = \pi(2q'+1)$, $q'=0,1,...$. In this case CP-invariance holds in the lepton sector at “low” energies. In order for CP-invariance to hold at “high” energy, i.e., for $P_{jkml}$ to be real, the product $R_{jk} R_{jm}$ also has to be purely imaginary, $\text{Re}(R_{jk} R_{jm}) = 0$. Thus, in the case considered, purely imaginary $U_{\tau k} U_{\tau m} \neq 0$ and real $R_{jk} R_{jm} \neq 0$, i.e., $\text{Re}(U_{\tau k} U_{\tau m}) = 0$, $\text{Im}(R_{jk} R_{jm}) = 0$, in particular, imply violation of CP-symmetry at “high” energy by the interplay of the matrices $U$ and $R$.

The realization that the CP violation necessary for the generation of the baryon asymmetry of the Universe can be due exclusively to the CPV phases in the PMNS matrix, is related to the progress in the understanding of the importance of lepton flavour effects in leptogenesis\[20,27\] (for earlier discussion see Ref. [78]. In the case of hierarchical heavy neutrinos $N_k$, $M_1 \ll M_2 \ll M_3$, the flavour effects in leptogenesis can be significant for $\text{matter} 10^9$ GeV $\lesssim M_1 \lesssim (0.5 - 1.0) \times 10^{12}$ GeV. If the requisite lepton asymmetry is produced in this regime, the CP violation necessary for successful leptogenesis can be provided entirely by the CPV phases in the neutrino mixing matrix\[13\].

Indeed, suppose that the mass of $N_1$ lies in the interval of interest, $10^9$ GeV $\lesssim M_1 \lesssim 10^{12}$ GeV. The CP violation necessary for the generation of
Fig. 2. The baryon asymmetry $|Y_B|$ as a function of the Dirac phase $\delta$ varying in the interval $\delta = [0, 2\pi]$ in the case of Dirac CP-violation, $\alpha_{32} = 0; 2\pi$, hierarchical heavy neutrinos and NH light neutrino mass spectrum, for $M_1 = 5 \times 10^{11}$ GeV, real $R_{12}$ and $R_{13}$ satisfying $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1$, and for i) $\alpha_{32} = 0, s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line), ii) $\alpha_{32} = 2\pi, s_{13} = 0.2$ (light blue line). (From Ref. [8].)

the baryon asymmetry $Y_B$ in “flavoured” leptogenesis can arise, as we have already noted, both from the “low energy” neutrino mixing matrix $U$ and/or from the “high energy” part of the matrix of neutrino Yukawa couplings $\lambda$ - the matrix $R$, which can mediate CP violating phenomena only at some high energy scale determined by the masses $M_k$ of the heavy Majorana neutrinos $N_k$. The matrix $R$ does not affect the “low” energy neutrino mixing phenomenology. Suppose further that the matrix $R$ has real and/or purely imaginary CP-conserving elements: we are interested in the case when the CP violation necessary for leptogenesis is due exclusively to the CPV phases in $U$. Under these assumptions, $Y_B$ generated via leptogenesis can be written as:

$$|Y_B| \cong 3 \times 10^{-3} |\epsilon_\tau \eta|,$$

where $\epsilon_\tau$ is the CPV asymmetry in the $\tau$ flavour (lepton charge) produced in $N_1$-decays$^4$.

$$\epsilon_\tau = -\frac{3M_1}{16\pi^2} \frac{\text{Im}(\sum_{j<k} m_j^{1/2} m_k^{3/2} U_{\tau j}^* U_{\tau k} R_{1j} R_{1k})}{\sum_i m_i |R_{1i}|^2},$$

$^4$We have given the expression for $Y_B$ normalised to the entropy density, see, e.g., Ref. [8].
$\eta$ is the efficiency factor \cite{76},

$$|\eta| \equiv |\eta(0.71\tilde{m}_2) - \eta(0.66\tilde{m}_\tau)|,$$  \hspace{1cm} (40)

$\tilde{m}_{2,\tau}$ being the wash-out mass parameters which determine the rate of the processes in the Early Universe that tend to “erase”, or “wash-out”, the asymmetry,

$$\tilde{m}_2 = \tilde{m}_e + \tilde{m}_\mu, \quad \tilde{m}_l = |\sum_j m_j R_{1j} U_{lj}^*|^2, \quad l = e, \mu.$$  \hspace{1cm} (41)

Approximate analytic expression for $\eta(\tilde{m})$ is given in \cite{76,77}. We shall consider next a few specific examples.

A. NH Spectrum, $m_1 \ll m_2 \ll m_3 \approx \sqrt{\Delta m_{21}^2}$.

Assume for simplicity that $m_1 \equiv 0$ and $R_{11} \equiv 0$ ($N_3$ decoupling). If $R_{12}R_{13}$ is real and $\alpha_{32}=0$, the only source of CP-violation is the Dirac phase $\delta$ in $U$, and $\epsilon_\tau \propto \sin \theta_{13} \sin \delta$. For $R_{12}R_{13} > 0$, $s_{13} = 0.15$, $\delta = 3\pi/2$, and $R_{12} \equiv 0.86$ (which maximises $|Y_B|$), we have \cite{8}: $|Y_B| \approx 2.7 \times 10^{-13} (\sqrt{\Delta m_{21}^2}/0.05 \text{ eV}) (M_1/10^9 \text{ GeV})$, where we have used the best fit values of $\Delta m_{21}^2$, $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ (see Fig. 2).

For the values of $M_1 \lesssim 5 \times 10^{11}$ GeV for which the flavour effects in leptogenesis can be significant, the observed value of the baryon asymmetry, taken conservatively to lie in the interval $|Y_B| \approx (8.1 - 9.3) \times 10^{-11}$, can be reproduced if

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09, \quad \text{or} \quad |J_{CP}| \gtrsim 2.0 \times 10^{-2}.$$  \hspace{1cm} (42)

The ranges of values of $\sin \theta_{13}$ and of $|J_{CP}|$ we find in the case being studied are comfortably compatible with the measured value of $\sin \theta_{13}$ and with the hints that $\delta \equiv 3\pi/2$. Since both $Y_B$ and $J_{CP}$ depend on $s_{13}$ and $\delta$, for given values of the other
The asymmetry $|Y_B|$ as a function of the Dirac phase $\delta$ in the case of hierarchical heavy neutrinos, IH light neutrino mass spectrum, Dirac CP-violation, $\alpha_{21} = \pi R_{11} R_{12} = i \kappa |R_{11} R_{12}|$ ($|R_{11}|^2 - |R_{12}|^2 = 1$), $\kappa = -1$ (red and dark blue lines), $\kappa = +1$ (light blue and green lines), for $M_1 = 2 \times 10^{11}$ GeV, and $s_{13} = 0.1$ (red and green lines) and $s_{13} = 0.2$ (dark blue and light blue lines). Values of $|R_{11}|$, which maximise $|Y_B|$ have been used: $|R_{11}| = 1.05$ in the case of $\kappa = -1$, and $|R_{11}| = 1.3$ (1.6) for $\kappa = +1$ and $s_{13} = 0.2$ (0.1). (From Ref. [8].)

relevant parameters there exists a correlation between the values of $|Y_B|$ and $J_{CP}$. This correlation is illustrated in Fig. 3.

As was shown in [8], we can have successful leptogenesis also if the sole source of CP-violation is the difference of the Majorana phases $\alpha_{32} = \alpha_{31} - \alpha_{21}$ of $U_{PMNS}$. In this case values of $M_1 \approx 4 \times 10^{10}$ GeV are required.

B. IH Spectrum, $m_3 \ll m_{1,2} \approx \sqrt{\Delta m^2_{32}}$.

Under the simplifying conditions of $m_3 \approx 0$ and $R_{13} \approx 0$ ($N_3$ decoupling), leptogenesis can be successful for $M_1 \approx 10^{12}$ GeV only if $R_{11} R_{12}$ is not real [8] so we consider the case of purely imaginary $R_{11} R_{12} = i\kappa R_{11} R_{12}$, $\kappa = \pm 1$. The requisite CP-violation can be due to the i) Dirac phase $\delta$ (Fig. 4, and/or ii) Majorana phase $\alpha_{21}$ (Fig. 5), in the neutrino mixing matrix $U$. If, e.g., in the second case we set $\sin \delta = 0$ (say, $\delta = \pi$), the maximum of $|Y_B|$ for, e.g., $\kappa = -1$, is reached for $|R_{11}|^2 \approx 1.4$ ($|R_{12}|^2 = |R_{11}|^2 - 1 = 0.4$), and $\alpha_{21} \approx 2\pi/3; 4\pi/3$, and at the maximum $|Y_B| \approx 1.5 \times 10^{-12}(\sqrt{\Delta m^2_{32}}/(0.05 \text{ eV})/M_1/10^9 \text{ GeV})$. The observed $|Y_B|$ can be reproduced for $M_1 \approx 5.4 \times 10^{10}$ GeV. Since both $|Y_B|$ and the effective Majorana mass in $(\beta \beta)_{0\nu}$-decay, $|<m>|$, depend on the Majorana phase $\alpha_{21}$, there exists a correlation between the values of $|Y_B|$ and $|<m>|$.

Similar results can be obtained [8] in the case of quasi-degenerate in mass heavy Majorana neutrinos.

The interplay in “flavoured” leptogenesis between contributions in $Y_B$ due to the “low energy” and “high energy” CP violation, originating from the PMNS matrix $U$ and the $R$-matrix, respectively, was investigated in Ref. [9]. It was found, in
Fig. 5. The asymmetry $|Y_B|$ versus the Majorana phase $\alpha_{21} = [0, 2\pi]$, for IH spectrum, purely imaginary $R_{11} R_{12} = i \kappa |R_{11} R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $M_1 = 2 \times 10^{11}$ GeV, $\delta = 0$ and $\sin \theta_{13} = 0.2$ - blue (red) line. (From Ref. [8])

particular, that under certain conditions which can be tested in low energy neutrino experiments (IH spectrum, $(-\sin \theta_{13} \cos \delta) \approx 0.1$), the “high energy” contribution in $Y_B$ due to the $R$-matrix, can be so strongly suppressed that it would play practically no role in the generation of baryon asymmetry compatible with the observations. One would have successful leptogenesis in this case only if the requisite CP violation is provided by the Majorana phases in the PMNS matrix $U$.

5. Conclusions

The program of research in neutrino physics aims at shedding light on some of the fundamental aspects of neutrino mixing:

i) the nature of massive neutrinos $\nu_i$, which can be Dirac fermions possessing distinct antiparticles, or Majorana fermions, i.e., spin 1/2 particles that are identical with their antiparticles;

ii) the type of spectrum the neutrino masses obey;

iii) the status of CP symmetry in the lepton sector;

iv) the absolute scale of neutrino masses.

The program extends beyond the year 2025 (see, e.g., Refs. [13,36]. Our ultimate goal is to understand at a fundamental level the mechanism giving rise to neutrino masses and mixing and to non-conservation of the lepton charges $L_l$, $l = e, \mu, \tau$. This includes understanding the origin of the patterns of neutrino mixing and of neutrino masses suggested by the data. The remarkable experimental program of
research in neutrino physics (the cost of which is expected to exceed altogether 1.3 billion US dollars) and the related theoretical efforts are stimulated by the fact that the existence of nonzero neutrino masses and the smallness of the neutrino masses suggest the existence of new fundamental mass scale in particle physics, i.e., the existence of New Physics beyond that predicted by the Standard Theory. It is hoped that progress in the theory of neutrino mixing will also lead, in particular, to progress in the theory of flavour and to a better understanding of the mechanism of generation of the baryon asymmetry of the Universe.

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