Magnetic Morris-Thorne wormhole in 2+1-dimensions

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In the context of 2 + 1-dimensional gravity coupled to a particular nonlinear electrodynamics (NED), we obtain a class of traversable / Morris-Thorne type wormhole solutions. The problem is reduced to a single function dependence in which the shape function acts as generator to the wormholes. The field ansatz is pure magnetic and the nonlinear Lagrangian is $\sqrt{F_{\mu\nu}F^{\mu\nu}}$ i.e. the square root of the Maxwell Lagrangian. In 2 + 1-dimensions the source-free pure magnetic nonlinear Maxwell equation with square-root Lagrangian is trivially satisfied. The exotic energy density is found explicitly and the flare-out conditions are emphasized.

I. INTRODUCTION

Although the topic of spacetime wormholes was popularized in modern times by the seminal works of Morris and Thorne and Visser [1], the original idea traces back to the 'bridge' of Einstein and Rosen [2] and even to the embedding diagram of Flamm [3]. In brief, it is a hypothetical shortcut spacetime tunnel that connects vastly distant points belonging either to the same or different universes. Classical physics prohibits such travels due to instability unless an exotic matter source is taken for granted to support the tunnel. Similar to black holes wormholes are also exact solutions to Einstein’s field equations. The idea of time travel through a wormhole, however, transcends classical considerations. To draw a rough analogy we may refer to the Art of Escher [4], where in the same picture birds transmute into fishes etc. While this transmutation takes place in our minds, for the sake of Art, physical theory of wormholes demands far more than this kind of visualization. In brief an observer can travel from one universe into the other through a traversable wormhole can also connect distant parts of the same universe. Yet in this analogy we can say that in the realm of wormholes Einstein meets Escher. Wormholes demand physical transition between vastly separated points in warped spacetime in which curvature of spacetime plays the principal role in Einstein’s relativity and given the suitable energy-momentum such a travel becomes possible according to the laws of physics. Another interesting development took place recently in connection with wormholes: the Einstein-Rosen (ER) bridge and the spooky interaction of quantum particles known as Einstein-Podolski and Rosen (EPR) pair may be related. Symbolically this situation has been summarized by $ER = EPR$ [5], which may serve to connect wormholes with the realm of quantum theory.

For these reasons we took wormhole physics seriously and attempted to construct these objects on physical, i.e., non-exotic matter [6]. To certain extent we obtained results that employ non-circular / non-spherical throat topology in the wormholes [7]. In addition to the energy matters recently we have also revised the well-known flare-out conditions [8].

In this paper we resort to the non-linear electromagnetism to provide a possible source for our traversable wormhole in 2 + 1-dimensions. This is the square-root Lagrangian of the Maxwell invariant which breaks the scale invariance. Being a square-root expression our electromagnetic field is automatically pure magnetic, i.e., we have $F_{r\theta} \neq 0$, as the only non-zero electromagnetic field component. The energy density turns out to be exotic and under this condition we present exact wormhole solutions. In [1] the idea of a traversable wormhole is introduced and the flare-out conditions which every traversable wormhole must satisfy are also found. In accordance with [1] the general line element of a traversable, circularly symmetric wormhole in 2 + 1-dimensions is written as

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - b(r)} + r^2d\theta^2,$$

in which $\Phi (r)$ is called the redshift function and $b(r)$ stands for the shape function of the wormhole. If we consider the location of the throat which connects two distant spacetimes, at $r = b_0$, the flare-out conditions state that: i) $b (r_0) = r_0$ and ii) $b' (r) < \frac{b(r)}{r}$, where prime means $\frac{d}{dr}$. For $r \geq r_0$. Although, in [1] a throat is a gate between two asymptotically flat spacetimes (i.e., $\lim_{r \to \infty} \Phi = 0$ and $\lim_{r \to \infty} \frac{b(r)}{r} = 0$) this condition is not necessary due to the

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existence of non-asymptotically flat spacetimes as the solutions of the Einstein’s gravity coupled to different matter fields such as dilaton [9–12]. Therefore, the only constraint on $\Phi$ is to be finite in the domain of $r_0 \leq r$. Having the Einstein’s equations and flare-out conditions all satisfied yields a negative energy density $\rho < 0$. This can be seen from the $tt$ component of the Einstein’s equation where $G^{tt} = T^{tt}$ ($8\pi G = c = 1$). From the line element (1) and the fact that $T^{tt} = -\rho$, one finds

$$\frac{(b' - \frac{b}{r})}{2r^3} = \rho.$$  \hspace{1cm} (2)

From (2) one easily observes that with the flare-out condition fulfilled i.e., $b' - \frac{b}{r} < 0$, the energy density becomes negative. Hence, the traversable wormholes are supported by exotic matter which violates the null energy condition.

Wormholes in $2+1$–dimensions, relatively, received less attention than the $3+1$–dimensional counterparts [13–21]. It worths to mention that the first work on $2+1$–dimensional wormholes was studied by Perry and Mann in [14].

In this paper we consider traversable wormholes in $2+1$–dimensions supported by a nonlinear electrodynamic (NED) matter source. The nonlinear Maxwell’s Lagrangian which is employed in this study, namely the square root of the Maxwell Lagrangian is of the form given in [22] which was developed further in [23–32].

II. MORRIS-THORNE TYPE WORMHOLE IN $2+1$–DIMENSIONS

Let’s start with the line element

$$ds^2 = -dt^2 + dl^2 + (l^2 + b_0^2) \, d\theta^2$$  \hspace{1cm} (3)

which we wish to call the Morris-Thorne type wormhole (MTtW) in $2+1$–dimensions. Note that in $3+1$-dimensions such a wormhole was introduced by Ellis [33]. Herein, $b_0$ is a real parameter, $-\infty < t < \infty$, $-\infty < l < \infty$ and $0 \leq \theta \leq 2\pi$. The Ricci scalar of MTtW

$$R = -\frac{2b_0^2}{(l^2 + b_0^2)^2}$$  \hspace{1cm} (4)

FIG. 1: $z/b_0$ versus $l/b_0$, and $\theta$ in cylindrical coordinates (See Eq. (8)). We note that at $r = b_0$, $z = 0$ is where the throat lies. At the location of the throat the magnitude of the curvature scalar i.e. $|R|$ is maximum while at large $r$ it goes to zero. The negative energy density gets its maximum value also at the throat while at large $r$ it vanishes.
is clearly negative and the geometry is regular everywhere. The Ricci scalar admits an absolute / relative minimum located at \( l = 0 \) while for \( l \to \pm \infty \) it vanishes. Furthermore, the only nonzero component of the Einstein’s tensor is given by

\[ G^1_1 = \frac{b_0^2}{(l^2 + b_0^2)^2} \]  

which yields

\[ \rho = -\frac{b_0^2}{(l^2 + b_0^2)^2} = \frac{R}{2} \]  

where \( \rho \) is the energy density of the matter, supporting the MTtW. It can easily be seen that \( \rho \) behaves the same as \( R \) such that a minimum occurs at \( l = 0 \). Upon taking a time slice of the spacetime (3) and embedding the result in cylindrical coordinates as

\[ ds^2 = dt^2 + (l^2 + b_0^2) d\theta^2 = dr^2 + dz^2 + r^2 d\theta^2 \]  

yields \( l^2 + b_0^2 = r^2 \) and \( \left( \frac{dx}{dr} \right)^2 = \frac{b_0^2}{r^2 + b_0^2} \). These clearly imply that \( r^2 \geq b_0^2 \) and

\[ z = \pm \int_{b_0}^{r} \frac{dx}{\sqrt{\frac{r^2}{b_0^2} - 1}} = \pm b_0 \ln \left( \frac{r}{b_0} + \sqrt{\frac{r^2}{b_0^2} - 1} \right), \]  

which is the same paraboloid of revolution as in the 3 + 1–dimensional MTtW [1]. We note that \( r^2 = b_0^2 \) is equivalent to \( l^2 = 0 \). In Fig. 1 we plot \( \frac{x}{b_0} \) in terms of \( \frac{r}{b_0} \) and \( \theta \). This figure supports the idea of having a throat located at \( z = 0 \) corresponding to \( r = b_0 \) and therefore \( l = 0 \), where \( R_{\text{min}} = 2\rho_{\text{min}} = -\frac{2}{b_0} \). To complete this section we add that a transformation of the form we introduced above, i.e., \( l^2 + b_0^2 = r^2 \), helps us to find the more familiar form of the line element of the MTtW as

\[ ds^2 = -dt^2 + \frac{dr^2}{1 - \frac{b_0^2}{r^2}} + r^2 d\theta^2 \]  

which suggests that \( \Phi = 0 \) and \( b(r) = \frac{b_0^2}{r^2} \) in (1). Once more we stress that in (9), \( b_0 \leq r \) so that \( r = b_0 \) corresponds to \( l = 0 \) which defines the location of the throat. There is no need to state also that the flare-out conditions are perfectly satisfied.

### III. MTTW IN NON-LINEAR ELECTRODYNAMICS (NED) COUPLED TO GRAVITY

Let’s start with the line element of a static and circularly symmetric spacetime given in (1) in which \( \Phi(r) \) and \( b(r) \) depend only on \( r \). We note that \(-\infty < t < \infty, 0 \leq r < \infty \) and \( \theta \in [0, 2\pi] \). The action for gravity coupled to NED in 2 + 1–dimensions is given by

\[ S = \frac{1}{2} \int d^3x \sqrt{-g} (R - 2\Lambda + \mathcal{L}) \]  

in which \( R \) is the Ricci scalar, \( \Lambda \) the cosmological constant, and \( \mathcal{L} = \alpha \sqrt{F} \) stands for the nonlinear Maxwell’s Lagrangian. Note that \( \alpha \) is a coupling constant and \( F = F_{\mu\nu} F^{\mu\nu} \) with \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the Maxwell’s invariant. Let us remind that historically it was Born and Infeld [34] who considered a non-linear version of electromagnetic Lagrangian that survived to present time. In their approach one could obtain the linear Maxwell Lagrangian as a limiting procedure. In our choice of square-root Maxwell Lagrangian, however, we shall have no such a limit. Once we let \( \alpha \to 0 \) with \( \Lambda \neq 0 \) we arrive at the BTZ [35] black hole solution. for the choice \( \Lambda = 0 \) (with \( \alpha = 0 \)) in 2+1–dimensions we recover nothing but the flat spacetime. It should also be added that the original motivation of NED was to eliminate the divergences in electromagnetic field due to the point charges. We comment that recently Einstein’s gravity coupled minimally to the nonlinear Maxwell’s Lagrangian of the form \( \mathcal{L}(F) \sim F^k \), received attentions from different aspects [22,32]. Here, we consider \( k = \frac{1}{2} \) i.e., \( \mathcal{L}(F) \sim \sqrt{F} \) with a pure magnetic field. Let us add that the particular power \( k = \frac{3}{4} \) corresponds to the scale invariant case, i.e. invariance under \( x_\mu \to \lambda x_\mu \) and \( A_\mu \to \frac{1}{\lambda} A_\mu \), for
\( \lambda = \text{constant, in } 2+1-\text{dimensions. Our choice } k = \frac{1}{2} \) therefore breaks the scale invariance with physical consequences. It should also be remarked that \( \mathcal{L}(F) \sim \sqrt{F} \) in flat spacetime had been studied long ago by Nielsen and Olesen \[36\] in string theory while 't Hooft \[37\] highlighted a linear potential term to be effective toward confinement. Our choice of the Maxwell’s 2-form is just a magnetic field of the form

\[ F = B(r) \, dr \wedge d\theta \]  

(11)

in which \( B(r) \) is a function of \( r \) to be found. Breaking the scale invariance the Lagrangian \( \sqrt{F} \) has the interesting property that it confines geodesics \[38\]. The source-free nonlinear-Maxwell’s equation for the specific Lagrangian chosen, is given by

\[ d \left( \frac{\ast F}{\sqrt{F}} \right) = 0 \]  

(12)

in which

\[ F = 2B^2 \left( 1 - \frac{b(r)}{r} \right) \]  

(13)

with its dual 1-form

\[ \ast F = \frac{Be^\Phi}{r} \sqrt{1 - \frac{b(r)}{r}} \, dt. \]  

(14)

Hence, (12) yields

\[ e^\Phi = \text{const.} \]  

(15)

and consequently

\[ \Phi = C \]  

(16)

in which \( C \) is an integration constant. We note that, \( e^C \) can be easily absorbed in time \( t \) and therefore without loss of generality we set \( C = 0 \). Next, the Einstein’s equations with a cosmological constant \( \Lambda \) are given by

\[ G_{\mu}^{\nu} + \frac{1}{3} \Lambda \delta_{\mu}^{\nu} = T_{\mu}^{\nu} \]  

(17)

in which

\[ T_{\mu}^{\nu} = \frac{\alpha}{2} \left( \mathcal{L}_{\mu}^{\nu} - 4 \mathcal{L}_F F_{\mu\lambda} F^{\nu\lambda} \right). \]  

(18)

Upon (11) and (18), one finds \( T_r^r = T_\theta^\theta = 0 \) and

\[ T_t^t = \frac{\alpha}{2} \sqrt{F} = \frac{\alpha}{\sqrt{2}} B \sqrt{f} = \frac{\alpha}{\sqrt{2}} B \sqrt{\frac{1 - \frac{b(r)}{r}}{r}}. \]  

(19)

Furthermore, with \( \Phi = 0 \), the only nonzero component of the Einstein’s tensor is

\[ G_t^t = -\frac{(b' - \frac{b}{r})}{2r^2}, \]  

(20)

in which a prime stands for the derivative with respect to \( r \). We obtain as a result, \( \Lambda = 0 \) in order to have the \( rr \) and \( \theta\theta \) components of the Einstein-Maxwell’s equations satisfied. Next, we consider the \( tt \) component of the Einstein-Maxwell’s equation which reads

\[ -\frac{(b' (r) - \frac{b(r)}{r})}{2r^3} = \frac{\alpha}{\sqrt{2}} B (r) \sqrt{1 - \frac{b(r)}{r}}. \]  

(21)
This equation gives a relation between the magnetic field \( B(r) \) and the shape function \( b(r) \). In other words, a general class of solutions is determined by (21) such that the redshift function is zero while the shape function and the magnetic field satisfy the constraint

\[
B(r) = -\sqrt{2} \frac{(rb' - b)}{2\alpha r^3 \sqrt{1 - \frac{b}{r}}}.
\]  

(22)

From this expression one finds that a possible throat is located at \( r = b_0 \). We note that (22) may or may not result in a wormhole. For instance \( b(r) = 0 \) yields \( B(r) = 0 \) and the spacetime becomes flat. Hence, to have a traversable wormhole one should find specific function for \( b(r) \) such that the Morris-Thorne’s flare-out conditions are satisfied. In the following sections we give two specific wormhole solutions.

A. MTtW

In the first example we consider the shape function to be of the form \( b(r) = b_0^2 \) in which \( b_0^2 \) is the location of the throat. This shape function has been found above in MTtW in 2 + 1–dimensions (see Eq. (9)). Having \( b(r) \), one finds the form of the magnetic field which is determined as

\[
B(r) = \frac{b_0^2 \sqrt{2}}{\alpha r^3 \sqrt{r^2 - b_0^2}}.
\]  

(23)

This is a singular function of \( r \) such that at the location of the throat it diverges. The Maxwell invariant, however, \( F_\mu^\nu = \frac{2b_0^2}{r^8} \) is finite at \( r = b_0 \). In addition to that, at large \( r \) the magnetic field vanishes to give an asymptotically flat limit for the wormhole.

The Ricci and Kritchmann scalars, respectively, are

\[
R = -\frac{2b_0^2}{r^4}
\]  

(24)

and

\[
K = \frac{4b_0^4}{r^8}
\]  

(25)

which are clearly regular at the throat. We wish to proceed now with the investigation of geodesic completeness in order to verify that divergence of the magnetic field at the throat is of no significance. The geodesics Lagrangian is

\[
L = -\frac{1}{2}t^2 + \frac{1}{2} \left( 1 - \frac{b_0^2}{r^2} \right)^{-1} \dot{r}^2 + \frac{1}{2} r^2 \dot{\theta}^2
\]  

(26)

where a dot stands for derivative with respect to the proper time. The first integrals of \( t \) and \( \theta \) equations are

\[
\dot{t} = E = \text{cons.}
\]

\[
r^2 \dot{\theta} = \ell = \text{cons.}
\]  

(27)

The timelike geodesics (\( \ell = -\frac{1}{2} \)) give

\[
\frac{dr}{dt} = \frac{1}{E} \sqrt{ \left( \frac{E^2 - 1 - \ell^2}{r^2} \right) \left( 1 - \frac{b_0^2}{r^2} \right) }.
\]  

(28)

The radial geodesics (\( \ell = 0 \)) yields the hyperbolic curve

\[
r(t) = \sqrt{b_0^2 + \alpha_0 t^2}
\]  

(29)

in which \( \alpha_0^2 = 1 - \frac{1}{r_\infty} \). It is observed that for \(-\infty < t < \infty\), we have \( b_0 \leq r < \infty \) which reflects the completeness of geodesics in the wormhole spacetime.
Next, for \( \ell \neq 0 \) we arrive at the expression
\[
\int_{r_0}^{r} \frac{r \, dr}{\sqrt{((E^2 - 1) r^2 - \ell^2) \left(1 - \frac{b_0^2}{r^2}\right)}} = t - t_0 \tag{30}
\]
with the initial time constant \( t_0 \). This can be reduced to an elliptic integral form and naturally the geodesic completeness is valid here as well.

Finally, the tidal forces at the throat can be analyzed through the geodesics deviation equation
\[
\frac{D^2 \xi^i}{d\tau^2} = -R^i_{\, jk\ell} \frac{\partial \xi^j}{\partial \tau} \frac{dx^k}{d\tau} \frac{dx^\ell}{d\tau} \tag{31}
\]
where \( \xi^i, (i = 1, 2) \) are displacements along the radial and angular directions. One obtains
\[
\frac{D^2 \xi^1}{d\tau^2} = \frac{b_0^2 \ell^2}{r^8} \xi^1 \tag{32}
\]
and
\[
\frac{D^2 \xi^2}{d\tau^2} = \frac{b_0^2}{4} \xi^2 \left( E^2 - 1 - \frac{\ell^2}{r^2} \right) \left(1 - \frac{b_0^2}{r^2}\right) \tag{33}
\]
which indicate the finiteness of tidal forces in the vicinity of the wormhole throat.

**B. Generalized MTtW**

As a second example we consider the shape function to be of the form \( b(r) = \frac{b_0^{\mu+1}}{r^{\mu+1}} \) in which \( \mu \) is a free, real parameter. Note that in order to have the flare-out conditions satisfied we must impose \( \mu > -1 \). The case \( \mu = 1 \) has already been considered in our example A. Among other possibilities, we consider \( \mu = 0 \) which yields \( b(r) = b_0 \). Consequently, the magnetic field and the energy density become
\[
B(r) = \frac{b_0 \sqrt{2}}{2 \alpha r^3 \sqrt{1 - \frac{b_0}{r}}} \tag{34}
\]
and
\[
\rho = -\frac{b_0}{2 r^3}. \tag{35}
\]
It should be stressed here also that the Maxwell invariant in the present case is \( F_{\mu \nu} F^{\mu \nu} = \frac{b_0}{\alpha^2 r^2} \), which is regular at the throat. One finds that the proper distance takes the form
\[
l(r) = \pm \left[ r^2 \left(1 - \frac{b_0}{r}\right) + \frac{b_0}{2} \ln \left( \frac{2r}{b_0} \left(1 + \sqrt{1 - \frac{b_0}{r}}\right) - 1 \right) \right] \tag{36}
\]
and the shape function is
\[
z(r) = \pm 2 \sqrt{b_0 (r - b_0)}. \tag{37}
\]
The Ricci and Kretchmann scalars become now
\[
R = -\frac{(\mu + 1) \frac{b_0^{\mu+1}}{r^{\mu+3}}}{r^{\mu+3}} \tag{38}
\]
and
\[
K = \frac{(\mu + 1)^2 \frac{b_0^{2(\mu+1)}}{r^{2(\mu+3)}}}{r^{2(\mu+3)}} \tag{39}
\]
which imply that the throat is a regular hypersurfacet. Given the analysis of the previous section it is not difficult to anticipate that the tidal forces / accelerations are finite in this generalized MTtW model as well. Due to the power \( \mu \) however the integrals of geodesics will not be any simpler.
We constructed a class of traversable wormhole solutions in the theory of gravity coupled to nonlinear electrodynamics in 2+1−dimensions. A similar model of wormhole with an anisotropic fluid source was considered in [39]. For specific choice of the shape function the solution is the Morris-Thorne type wormhole in 2+1−dimensions which shares most of its properties with its 3+1−dimensional version. The matter source which supports our wormhole solution is a pure magnetic field of the form given in (22). The square-root of pure magnetic Maxwell Lagrangian provides automatic satisfaction of the non-linear Maxwell equation in 2+1−dimensions. Confining of geodesics is another interesting property of such a square-root Lagrangian [38]. We comment that the magnetic field diverges at the throat and vanishes fast with \( r \to \infty \). A particularly simple example with \( b(r) = b_0 = \text{const.} \) is considered. In this example also the magnetic field diverges at the throat while the Maxwell invariant \( F_{\mu
u}F^{\mu
u} \) is finite at the throat. Next, we consider a more general ansatz which involves an arbitrary parameter \( \mu \). We note that divergence of the magnetic field at the throat was used in [40] as a counter argument against existence of such 2+1-dimensional wormholes. The only singularity of the problem lies at \( r = 0 \) which is a naked spacetime singularity but since the wormhole condition stipulates that \( r \geq b_0 \) the singularity at \( r = 0 \) where the scalar curvature invariants diverge remains ineffective for particle geodesics. Finally, we should add that the class of solutions found in this paper consists of a large number of solutions which only depends on the form of \( b(r) \). Any choice of \( b(r) \) satisfying the flare-out conditions acts as a generator and gives rise to a new Morris-Thorne type wormhole. The fact that we work in the reduced 2+1−dimensions simplifies the problem to a great extend. In 3+1−dimensions obviously wormholes can’t be generated from a single throat function.

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