Soft Set-Valued Mappings and their Application in Decision Making Problems

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Abstract. In this paper, we introduce the notion of a set-valued mapping on soft classes and study several properties of images and inverse images of soft sets supported by examples and counterexamples. Finally, these notions have been applied in decision making problems.

1. Introduction

It is known that classical mathematics methods are inadequate in modeling the problem in cases of uncertainty and ambiguity. In order to overcome such situations, researchers have begun new searches and introduced new theories such as theory of probability, fuzzy set theory [27], intuitionistic fuzzy sets [5], vague sets [13], theory of interval mathematics [14], rough set theory [24], etc. to model uncertainty situations.

One of the most important of these theories is the theory of fuzzy sets introduced by Zadeh [27]. This theory tries to digitize the uncertainties in human thoughts and perceptions and offers concepts and methods that bring certainty to uncertain situations and eliminate problems in solution. On the other hand, since the definition of membership function required for a fuzzy set depends on the person defining the function, fuzzy set operations can be far from reality. The difficulty of defining this membership function causes the fuzzy set theory to be insufficient in some cases. Molodtsov [23], who argued that the reason for similar problems existing in also other theories is that the elements of the sets cannot be adequately parameterized, put forward soft set theory, which is a novel theory alternative to these set theories to model uncertainties. The absence of any limitation in defining objects in soft set theory, that is, choosing any number, word or phrase can be selected as a parameter, enables much more suitable models for real-life problems by minimizing information loss.

Therefore, researchers has shown great interest to this new theory and studied its applications in different disciplines such as decision-makings [20], Perron integration, Riemann-integration, smoothness of functions, Theory of Probability, Theory of Measurement, the smoothness of functions [23], Game Theory, Optimization Theory, Operations Research [23], algebraic structures [1, 3, 11, 15, 18] and topological structures [9, 25, 26, 28, 29].
2. Preliminaries

Throughout the work, any universe of objects will be denoted by $U$, a set of parameters suitable for the elements in $U$ will be denoted by $E$, and the power set of $U$ will be also denoted by $P(U)$.

**Definition 2.1.** ([23]) The pair $(F, E)$ is called a soft set on $U$ where $F : E \rightarrow P(U)$ is a map.

Thus a soft set is a parameterized family of subsets of $U$ and for each $e \in E$, the set $F(e)$ can be considered as the set of $e$-elements or $e$-approximations of the soft set $(F, E)$.

According to Majumdar and Samanta [22], any $(F, A)$ soft set can be extended to a soft set $(F, E)$, where $F(e) \neq \emptyset$ when $e \in A$ and $F(e) = \emptyset$ when $e \in E \setminus A$. Based on this idea, Çağman and Enginoğlu [7] revised the algebraic operations of soft sets in [21] as follows. From now on, the soft set defined by a map $F$ with $F(e) \neq \emptyset$ when $e \in A \subseteq E$ and $F(e) = \emptyset$ when $e \in E - A$ be denoted by $F_A$ and this soft set will also be considered as the map $F_A : E \rightarrow P(U)$. Also, the family of all of soft sets on $U$ will be denoted by $S(U, E)$.

**Definition 2.2.** ([7]) Let $F_A, F_B \in S(U, E)$. Then:

1. If $F_A(e) \subseteq F_B(e)$ for all $e \in E$, then $F_A$ is a soft subset of $F_B$, denoted by $F_A \subseteq F_B$.
2. Union of $F_A$ and $F_B$, denoted by $F_A \cup F_B$, is a soft set defined by $(F_A \cup F_B)(e) = F_A(e) \cup F_B(e)$ for all $e \in E$.
3. Intersection of $F_A$ and $F_B$, denoted by $F_A \cap F_B$, is a soft set defined by $(F_A \cap F_B)(e) = F_A(e) \cap F_B(e)$ for all $e \in E$.
4. If $F_A(e) = \emptyset$ for all $e \in E$, then $F_A$ is called a empty soft set, denoted by $F_\emptyset$. $F_A(e) = \emptyset$ means that there is no element in $U$ related to the parameter $e \in E$.
5. If $F_A(e) = U$ for all $e \in E$, then $F_A$ is called a universal soft set, denoted by $F_U$.

**Definition 2.3.** ([2]) Let $F_A \in S(U, E)$. Then complement of $F_A$, denoted by $F^c_A$, is a soft set defined by $F^c_A(e) = U - F_A(e)$ for all $e \in E$.

It is noted in [7] that $(F_A)^c = F_A$, $F_\emptyset = F_\emptyset$ and $F_U = F_U$.

Now let us express the uni – int decision making method of Çağman and Enginoğlu [7]. For this, we will first give the necessary definitions.

**Definition 2.4.** ([7]) If $F_A, F_B \in S(E, U)$, then $\land$-product of soft sets $F_A$ and $F_B$, denoted by $F_A \land F_B$, is a soft set defined by

$$F_A \land F_B : E \times E \rightarrow P(U), \quad (F_A \land F_B)(x, y) = F_A(x) \cap F_B(y)$$
Definition 2.5. ([7]) Let \( F_A, F_B \in S(E, U) \) and let \( \land(U) \) be the set of all \( \land \)-products of the soft sets over \( U \). Then \( \text{uni} – \text{int} \) operators for the \( \land \)-products, denoted by \( \text{uni}_s \text{int}_y \) and \( \text{uni}_s \text{int}_x \), are defined, respectively,

\[
\text{uni}_s \text{int}_y : \land(U) \to P(U), \quad \text{uni}_s \text{int}_y(F_A \land F_B) = \bigcup_{y \in \land(A)(F_A \land F_B)(x,y))}
\]

\[
\text{uni}_s \text{int}_x : \land(U) \to P(U), \quad \text{uni}_s \text{int}_x(F_A \land F_B) = \bigcup_{y \in \land(A)(F_A \land F_B)(x,y))}
\]

Each of them transforms the \( \land \)-product \( F_A \land F_B \) into a subset of the universe \( U \).

Definition 2.6. ([7]) Let \( F_A \land F_B \in \land(U) \). Then \( \text{uni} – \text{int} \) decision function for the \( \land \)-products, denoted by \( \text{uni} – \text{int} \), is defined by, \( \text{uni} – \text{int} : \land(U) \to P(U) \)

\[
\text{uni} – \text{int}(F_A \land F_B) = \text{uni}_s \text{int}_y(F_A \land F_B) \cup \text{uni}_s \text{int}_x(F_A \land F_B)
\]

that reduces the size of the universe \( U \). Hence, the values \( \text{uni} – \text{int}(F_A \land F_B) \) is a subset of \( U \) called \( \text{uni} – \text{int} \) decision set of \( F_A \land F_B \).

For details, reference [7] can be examined. Now, let us give the algorithm of the \( \text{uni} – \text{int} \) decision making method. According to the problem,

Step 1: Choose feasible subsets of the set of parameters,

Step 2: Construct the soft sets for each set of parameters,

Step 3: Find the \( \land \)-product of the soft sets,

Step 4: Compute the \( \text{uni} – \text{int} \) decision set of the product.

Note that obtained \( \text{uni} – \text{int} \) decision set is not small enough to work on it, subset of the decision set can be reached by the method.

Definition 2.7. Let \( X \) and \( Y \) be two sets. An \( F \) relation that corresponds to each element of \( X \) with a non-null subset of \( Y \), is called a set-valued mapping from \( X \) to \( Y \) and is denoted by \( F : X \rightsquigarrow Y \). The subset corresponding to \( x \in X \) is indicated by \( F(x) \).

Definition 2.8. For a set set-valued mapping \( F : X \rightsquigarrow Y \), the upper and lower inverse of any subset \( B \) of \( Y \), denoted by \( F^+(B) \) and \( F^-(B) \) respectively, are the subsets \( F^+(B) = \{ x \in X : F(x) \subseteq B \} \) and \( F^-(B) = \{ x \in X : F(x) \cap B \neq \emptyset \} \). In particular, \( F^+(y) = \{ x \in X : y \in F(x) \} \) for each \( y \in Y \), and the image of an \( A \subseteq X \) under \( F \) is \( F(A) = \bigcup \{ F(x) : x \in A \} \).

Theorem 2.9. Let \( X \) and \( Y \) be two sets and \( F : X \rightsquigarrow Y \) be a set-valued mapping. Then \( X – F^+(B) = F^-(Y – B) \) for each \( B \subseteq Y \).

3. Soft Set-Valued Mappings

In this section, a new mapping between two soft set families will be defined with help of set-valued mappings between classical sets. Then, an example of this mapping will be given and basic properties of it will be proven.

Definition 3.1. Let \( u : U \rightsquigarrow V \) and \( p : E \rightsquigarrow K \) be two set-valued mappings. Then a soft set-valued mapping \( \psi_p : S(U, E) \rightsquigarrow S(V, K) \) is defined as below:

(1) Let \( F_A \in S(U, E) \). The upper and lower images of \( F_A \) under \( \psi_p \), denoted by \( \psi^+(F_A) \) and \( \psi^-(F_A) \) respectively, are defined as

\[
\psi^+(F_A)(k) = \begin{cases} 
\bigcup_{e \in p^+(k)} u(F_A(e)) & ; p^+(k) \neq \emptyset \\
\emptyset & ; p^+(k) = \emptyset 
\end{cases}
\]

\[
\psi^-(F_A)(k) = \begin{cases} 
\bigcap_{e \in p^-(k)} u(F_A(e)) & ; p^-(k) \neq \emptyset \\
\emptyset & ; p^-(k) = \emptyset 
\end{cases}
\]
Then we have

$u_p^-(F_A)(k) = \begin{cases} \cup_{e \in p^-(k)} u(F_A(e)) & ; p^-(k) \neq \emptyset \\ \emptyset & ; p^-(k) = \emptyset \end{cases}$

for all $k \in K$.

(2) Let $G_B \in S(V,K)$. The upper and lower invers images of $G_B$ under $u_p$, written as $u_p^+(G_B)$ and $u_p^-(G_B)$ respectively, are defined as

$u_p^+(G_B)(e) = u^+(\cup_{k \in p(e)} G_B(k))$

and

$u_p^-(G_B)(e) = u^-(\cup_{k \in p(e)} G_B(k))$

for all $e \in E$.

**Example 3.2.** Let $E = \{e_1, e_2, e_3, e_4\}$, $K = \{k_1, k_2, k_3\}$, $U = \{u_1, u_2, u_3, u_4\}$ and $V = \{v_1, v_2, v_3, v_4\}$. Let $u : U \rightarrow V$ and $p : E \rightarrow K$ be set-valued mappings defined as $u(1) = \{v_1\}$, $u(2) = \{v_2, v_3\}$, $u(3) = \{v_4\}$, $u(4) = \{v_1, v_2\}$, $p(e_1) = \{k_1, k_2, k_3\}$, $p(e_2) = \{k_1, k_3\}$, $p(e_3) = \{k_2\}$, $p(e_4) = \{k_3\}$. Choose the soft set in $S(U,E)$ and $S(V,K)$ respectively, $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_3\}), (e_3, \{u_4\}), (e_4, \{u_2\})\}$ and $G_B = \{(k_1, \{v_1, v_3\}), (k_2, \{v_3, v_4\}), (k_3, \{v_2, v_4\})\}$. Then we have

$u_p^-(F_A)(k_1) = \cup_{e \in p^-(k_1)} u(F_A(e)) = \emptyset$, $u_p^+(F_A)(k_1) = \cup_{e \in p^+(k_1)} u(F_A(e)) = u(1)$, $u_p^-(G_B)(e_1) = u^-(\cup_{k \in p(e_1)} G_B(k))$, $u_p^+(G_B)(e_1) = u^+(\cup_{k \in p(e_1)} G_B(k))$, $u_p^+(G_B)(e_2) = [u_1, u_3, u_4]$, $u_p^-(G_B)(e_2) = [u_1, u_3, u_4]$, $u_p^+(G_B)(e_3) = [u_2, u_3, u_4]$, $u_p^-(G_B)(e_3) = [u_2, u_3, u_4]$, $u_p^+(G_B)(e_4) = [u_1]$ and $u_p^-(G_B)(e_4) = [u_1]$, $u_p^+(G_B)(e_4) = [u_2, u_3, u_4]$ and $u_p^-(G_B)(e_4) = [u_2, u_3, u_4]$ and so we obtain that $u_p^+(G_B) = \{(e_1, U), (e_2, \{u_1, u_3, u_4\}), (e_3, \{u_3\}), (e_4, \{u_3\})\}$ and $u_p^-(G_B) = \{(e_1, U), (e_2, U), (e_3, \{u_2, u_3, u_4\}), (e_4, \{u_2, u_3, u_4\})\}$.

**Theorem 3.3.** Let $u_p : S(U,E) \rightarrow S(V,K)$ be a soft set-valued mapping and $F_A, G_B \in S(V,K)$. Then the following are true:

1. $u_p^+(F_A) = F_A$ and $u_p^-(F_A) = F_A$
2. $u_p^+(F_K) = F_K$ and $u_p^-(F_K) = F_K$
3. $u_p^+(F_A \sqcup G_B) \subseteq u_p^+(F_A) \cup u_p^+(G_B)$
4. $u_p^-(F_A \sqcup G_B) = u_p^-(F_A) \cup u_p^-(G_B)$
5. $u_p^+(F_A \sqcap G_B) \subseteq u_p^+(F_A) \cap u_p^+(G_B)$
6. $u_p^-(F_A \sqcap G_B) \subseteq u_p^-(F_A) \cap u_p^-(G_B)$
7. If $F_A \sqsubseteq G_B$, then $u_p^+(F_A) \subseteq u_p^+(G_B)$ and $u_p^-(F_A) \subseteq u_p^-(G_B)$.

**Proof.** (1) Let us prove $u_p^+(F_A) = F_A$. The other can be done in a similar way. For all $e \in E$, we have that

$u_p^+(F_A)(e) = u^+(\cup_{k \in p(e)} F_A(k)) = u^+(\cup_{k \in p(e)} \emptyset) = \emptyset$

This shows that $u_p^+(F_A) = F_A$.

(2) Let us prove $u_p^-(F_K) = F_K$. For all $e \in E$, we have that

$u_p^-(F_K)(e) = u^-(\cup_{k \in p(e)} F_K(k)) = u^-(\cup_{k \in p(e)} V) = U$
This shows that $u_p^+(F_E) = F_E$.

Let us prove (3) and (4). (5) and (6) can be proved similarly.

(3) For all $e \in E$, we have that

$$u_p^+(F_A \cup G_B)(e) = u^+(\bigcup_{k \in p(e)} (F_A \cup G_B)(k))$$

$$= u^+(\bigcup_{k \in p(e)} F_A(k) \cup G_B(k))$$

$$= u^+(\bigcup_{k \in p(e)} (F_A(k) \cup G_B(k)))$$

$$\supseteq u^+(\bigcup_{k \in p(e)} F_A(k)) \cup u^+(\bigcup_{k \in p(e)} G_B(k))$$

$$= u_p^+(F_A)(e) \cup u_p^+(G_B)(e)$$

$$= (u_p^+(F_A) \cup u_p^+(G_B))(e)$$

This shows that $u_p^+(F_A \cup G_B) \supseteq u_p^+(F_A) \cup u_p^+(G_B)$.

(4) For all $e \in \hat{E}$, we have that

$$u_p^+(F_A \cap G_B)(e) = u^+(\bigcap_{k \in p(e)} (F_A \cap G_B)(k))$$

$$= u^+(\bigcap_{k \in p(e)} F_A(k) \cap G_B(k))$$

$$= u^+(\bigcap_{k \in p(e)} (F_A(k) \cap G_B(k)))$$

$$\subseteq u^+(\bigcap_{k \in p(e)} F_A(k)) \cap u^+(\bigcap_{k \in p(e)} G_B(k))$$

$$= u_p^+(F_A)(e) \cap u_p^+(G_B)(e)$$

$$= (u_p^+(F_A) \cap u_p^+(G_B))(e)$$

This shows that $u_p^+(F_A \cap G_B) \subseteq u_p^+(F_A) \cap u_p^+(G_B)$.

(7) Let $F_A \subseteq G_B$. Then for all $k \in K$, we have that

$$u_p^+(F_A)(k) = u^+(\bigcup_{k \in p(e)} F_A(k)) \subseteq u^+(\bigcup_{k \in p(e)} G_B(k)) = u_p^+(G_B)(k)$$

This shows that $u_p^+(F_A) \subseteq u_p^+(G_B)$. The other is similar. $\square$

**Theorem 3.4.** Let $u_p : S(U, E) \rightarrow S(V, K)$ be a soft set-valued mapping and $F_A, G_B \in S(U, E)$. Then the following are true:

1. $u_p(F_\emptyset) = F_\emptyset$ and $u_p(F_A) = F_\emptyset$.
2. If $p$ and $u$ are surjective, then $u_p(F_E) = F_K$.
3. $u_p(F_A \cup G_B) = u_p(F_A) \cup u_p(G_B)$.
4. $u_p(F_A \cap G_B) = u_p(F_A) \cap u_p(G_B)$.
5. $u_p(F_A \cap G_B) \subseteq u_p(F_A) \cap u_p(G_B)$.
6. $u_p(F_A \cap G_B) \subseteq u_p(F_A) \cap u_p(G_B)$.
7. If $F_A \subseteq G_B$, then $u_p(F_A) \subseteq u_p(G_B)$ and $u_p(F_A) \subseteq u_p(G_B)$.

**Proof.** (1) Let us prove $u_p(F_\emptyset) = F_\emptyset$. The other can be done in a similar way. For all $k \in K$, we have that

$$u_p(F_\emptyset)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k)} u(F_\emptyset(e)) & p^{-1}(k) \neq \emptyset \\
\emptyset & p^{-1}(k) = \emptyset \end{cases}$$

This shows that $u_p^{-1}(F_\emptyset) = F_\emptyset$.

(2) Let $p$ and $u$ be surjective. Then for all $k \in K$, we have that

$$u_p(F_E)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k)} u(F_E(e)) & p^{-1}(k) \neq \emptyset \\
\emptyset & p^{-1}(k) = \emptyset \end{cases} = \bigcup_{e \in p^{-1}(k)} u(U) = V$$
This shows that $u_p(F_E) = F_K$.

Here we will provide proofs of (3) and (4). (5) and (6) can be proved similarly.

(3) For all $k \in K$, we have that

$$u_p(F_A \cup G_B)(k) = \begin{cases} \bigcup_{e \in p^+(k)} u(F_A(e) \cup G_B(e)) & \text{if } p^+(k) \neq \emptyset \\ \emptyset & \text{if } p^+(k) = \emptyset \end{cases}$$

$$= \begin{cases} \bigcup_{e \in p^+(k)} u(F_A(e)) & \text{if } p^+(k) \neq \emptyset \\ \emptyset & \text{if } p^+(k) = \emptyset \end{cases} \bigcup \begin{cases} \bigcup_{e \in p^+(k)} u(G_B(e)) & \text{if } p^+(k) \neq \emptyset \\ \emptyset & \text{if } p^+(k) = \emptyset \end{cases}$$

$$= u_p(F_A)(k) \cup u_p(G_B)(k)$$

This shows that $u_p(F_A \cup G_B) = u_p(F_A) \cup u_p(G_B)$.

(4) For all $k \in K$, we have that

$$u_p(F_A \cup G_B)(k) = \begin{cases} \bigcup_{e \in p^+(k)} u(F_A(e) \cup G_B(e)) & \text{if } p^+(k) \neq \emptyset \\ \emptyset & \text{if } p^+(k) = \emptyset \end{cases}$$

$$= \begin{cases} \bigcup_{e \in p^+(k)} u(F_A(e)) & \text{if } p^+(k) \neq \emptyset \\ \emptyset & \text{if } p^+(k) = \emptyset \end{cases} \bigcup \begin{cases} \bigcup_{e \in p^+(k)} u(G_B(e)) & \text{if } p^+(k) \neq \emptyset \\ \emptyset & \text{if } p^+(k) = \emptyset \end{cases}$$

$$= u_p(F_A)(k) \cup u_p(G_B)(k)$$

This shows that $u_p(F_A \cup G_B) = u_p(F_A) \cup u_p(G_B)$.

(7) Let $F_A \subseteq G_B$ Then for all $k \in K$, we have that

$$u_p(F_A)(k) = \begin{cases} \bigcup_{e \in p^+(k)} u(F_A(e)) & \text{if } p^+(k) \neq \emptyset \\ \emptyset & \text{if } p^+(k) = \emptyset \end{cases}$$

$$= \begin{cases} \bigcup_{e \in p^+(k)} u(G_B(e)) & \text{if } p^+(k) \neq \emptyset \\ \emptyset & \text{if } p^+(k) = \emptyset \end{cases}$$

$$= u_p(G_B)(k)$$

This shows that $u_p(F_A) \subseteq u_p(G_B)$. The proof that $F_A \subseteq G_B$ requires $u_p(F_A) \subseteq u_p(G_B)$ is similar.

\[ \square \]

**Remark 3.5.** Even if $p : E \sim K$ and $u : U \sim V$ are surjective, $u_p(F_E) = F_K$ may not for $u_p : S(U,E) \sim S(V,K)$. Consider Example 3.2. Then since

$$u_p(F_E)(k_1) = \bigcup_{e \in p^+(k_1)} u(F_E(e)) = \emptyset$$

$$u_p(F_E)(k_2) = \bigcup_{e \in p^+(k_2)} u(F_E(e)) = u(F_E(e_1)) = u(U) = V$$

$$u_p(F_E)(k_3) = \bigcup_{e \in p^+(k_3)} u(F_E(e)) = u(F_E(e_4)) = u(U) = V$$

we have $u_p(F_E) = \{(k_2, V), (k_3, V)\} \neq F_K = \{(k_1, V), (k_2, V), (k_3, V)\}$.

**Theorem 3.6.** Let $u_p : S(U,E) \sim S(V,K)$ be a soft set-valued mapping and $G_B \in S(V,K)$. Then the following are true:

1. $u_p^+(G_B) = (u_p^+(G_B))^\sim$
2. $u_p^-(G_B) = (u_p^-(G_B))^\sim$
Proof. (1) For all $e \in E$, we have that

$$u_p^*(G^c_B)(e) = u^*(\cup_{k \in p(e)} u_G(k))$$

$$= u^*(\cup_{k \in p(e)} (V \setminus G_B(k)))$$

$$= u^*(V \setminus (\cap_{k \in p(e)} G_B(k)))$$

$$= U \setminus u^*(\cup_{k \in p(e)} G_B(k))$$

$$= U \setminus u^*_p(G_B)(e)$$

This shows that $u_p^*(G_B^c) = (u^*_p(G_B))^\circ$.

(2) It is similar to that of (1). \(\Box\)

4. An Application

Decision making is the process of choosing the best one among some alternatives based on some criteria. However, in the developing world, these processes in many fields such as Engineering, Economy, Management, Medicine and Social Sciences are often encountered as very complex systems due to uncertain and inaccurate data. In fact, the human mind has the ability to make decisions in many of such situations. However, if the selection criteria are too large to be held in human memory, and complex relationships exist, mathematical methods are needed.

For example, if we want to decide which sector is advantageous for the investor who wants to open a branch of one of the retail industry, in this decision process, consumers’ needs and the features of the product that the consumers are interested such as quality, well-known brand, cheapness etc. affect the decision. Therefore, there are many parameters that will affect the decision, but are not directly related to the objects in the universe to be selected, and in a different universe. So it does not seem possible to implement uni – int decision making method, and therefore decision making appears to be a difficult process. The following example shows the solution to the above problem using soft set-valued mappings.

Let us assume that an investor wants to open one of the stores $U = \{u_1, u_2, u_3, u_4\}$ which sells from the sectors $E = \{e_1 = \text{clothing}, e_2 = \text{sports}, e_3 = \text{cosmetics}, e_4 = \text{toys}\}$. Let us assume that, $u_1$ shop sells clothing and sports equipment, $u_2$ shop sells clothing and toys, $u_3$ shop sells sports equipment and $u_4$ shop sells cosmetics. Then we can write the soft set

$$F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_3\}), (e_3, \{u_4\}), (e_4, \{u_2\})\}$$

that shows these relationships.

On the other hand, the investor wants to use the tendency of consumers of different sex and age groups

$K = \{k_1 = \text{male}, k_2 = \text{female}, k_3 = \text{children}\}$ to the qualifications of the stores $V = \{v_1 = \text{reasonable price}, v_2 = \text{well-known brand}, v_3 = \text{quality}, v_4 = \text{plentiful}\}$ in decision making. Let us assume that the above soft set $G_B$ gives these trends.

$$G_B = \{(k_1, \{v_1, v_3\}), (k_2, \{v_3, v_4\}), (k_3, \{v_2, v_4\})\}$$

Assume that the relationship of sectors with gender and age groups gives a set-valued mapping $p : E \sim K$ defined by $p(e_1) = \{k_1, k_2, k_3\}$, $p(e_2) = \{k_1, k_2\}$, $p(e_3) = \{k_2\}$, $p(e_4) = \{k_3\}$. Again relationships between stores and qualifications of their gives a set-valued mapping $u : U \sim V$ defined by $u(u_1) = \{v_1\}$, $u(u_2) = \{v_2, v_3\}$, $u(u_3) = \{v_4\}$, $u(u_4) = \{v_1, v_4\}$. Then we have that

$$u^*_p(G_B) = \{(e_1, U), (e_2, \{u_1, u_3, u_4\}), (e_3, \{u_4\}), (e_4, \{u_2\})\}$$

and

$$u_p^*(G_B) = \{(e_1, U), (e_2, U), (e_3, \{u_2, u_3, u_4\}), (e_4, \{u_2, u_3, u_4\})\}$$
Then we have
and then we calculate that the best result for investment Therefore we obtain that

First, let us apply the uni – int decision method for sets $F_A$ and $u^*_p(G_B)$. For ease of operation, assume that $u^*_p(G_B) = F_C$. In this case, we have

$$F_A \land F_C = \{(e_1, e_1), (u_1, u_2), (e_1, e_2), (u_1), (e_1, e_3), (u_2), (e_1, e_4), (u_2), (e_2, e_1), (u_1, u_3), (e_2, e_2), (u_1), (e_2, e_3), (u_3), (e_2, e_4), (u_3), (e_3, e_1), (u_1), (e_3, e_2), (u_1), (e_3, e_3), (u_3), (e_3, e_4), (u_3), (e_4, e_1), (u_1), (e_4, e_2), (u_1), (e_4, e_3), (u_3), (e_4, e_4), (u_3), (e_4, e_4), (u_3), \}$$

and then we calculate that

$$\text{uni}_x \text{int}_y(F_A \land F_C) = \bigcup_{x \in A} \bigcap_{y \in C} ((F_A \land F_C)(x, y))$$

$$= \bigcup \left\{ \bigcap \right\}$$

$$= \{u_3\}$$

and

$$\text{uni}_y \text{int}_x(F_A \land F_C) = \bigcup_{y \in C} \bigcap_{x \in A} ((F_A \land F_C)(x, y))$$

$$= \bigcup \left\{ \bigcap \right\}$$

$$= \emptyset$$

Therefore we obtain that

$$\text{uni} – \text{int}(F_A \land u^*_p(G_B)) = \text{uni} – \text{int}(F_A \land F_C) = \{u_3\} \cup \emptyset = \{u_3\}$$

This means that when using soft sets $F_A$ and $u^*_p(G_B)$ in the uni – int decision making method, the store $u_3$ is the best result for investment.

Now let us apply the uni – int decision method for sets $F_A$ and $u^*_p(G_B)$ and assume that $u^*_p(G_B) = F_D$. Then we have

$$F_A \land F_D = \{(e_1, e_1), (u_1, u_2), (e_1, e_2), (u_1, u_2), (e_1, e_3), (u_2), (e_1, e_4), (u_2), (e_2, e_1), (u_1, u_3), (e_2, e_2), (u_1), (e_2, e_3), (u_3), (e_2, e_4), (u_3), (e_3, e_1), (u_1), (e_3, e_2), (u_1), (e_3, e_3), (u_3), (e_3, e_4), (u_3), (e_4, e_1), (u_1), (e_4, e_2), (u_1), (e_4, e_3), (u_3), (e_4, e_4), (u_3), (e_4, e_4), (u_3), \}$$

$$\text{uni}_x \text{int}_y(F_A \land F_D) = \bigcup_{x \in A} \bigcap_{y \in D} ((F_A \land F_D)(x, y))$$

$$= \bigcup \left\{ \bigcap \right\}$$

$$= \{u_2, u_3, u_4\}$$

$$\text{uni}_y \text{int}_x(F_A \land F_D) = \bigcup_{y \in D} \bigcap_{x \in A} ((F_A \land F_D)(x, y))$$

$$= \bigcup \left\{ \bigcap \right\}$$

$$= \emptyset$$
and we conclude that

\[ \text{uni} - \text{int}(F_A \land u_p^c(G_B)) = \text{uni} - \text{int}(F_A \land F_D) = \{u_2, u_3, u_4\} \cup \emptyset = \{u_2, u_3, u_4\} \]

Accordingly, the investor can choose any of the stores \( u_2, u_3 \text{ or } u_4 \) when using soft sets \( F_A \) and \( u_p^c(G_B) \) in the \( \text{uni} - \text{int} \) decision making method.

Consequently, \( u_3 \) store is the most suitable store for the investor. However, the investor may also consider \( u_2 \text{ or } u_4 \) stores besides the \( u_3 \) store. Another result of the decision-making method is that the shop \( u_1 \) is not suitable for investment.

5. Conclusion

The main aim of this paper is to define soft set-valued mappings, investigate basic properties of them and to expand the application areas of soft set decision making methods. Mappings introduced in this study can be used not only for the uni-int decision making but also all decision making methods created with soft sets. Therefore, I hope that this study will be a useful guide for new studies.

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