Quantum Hall effect in tunable 1-D lateral superlattice in graphene – role of crossed electric and magnetic fields

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(Dated: April 21, 2015)

We probe quantum Hall effect in a tunable 1-D lateral superlattice (SL) in graphene created using electrostatic gates. We create strong local electric field at the interface of regions of different charge densities. Crossed electric and magnetic fields modify the spectrum and wavefunction of the Landau Levels (LLs) which is reflected in magnetotransport. In the region of copropagating electrons and holes at the interface, the electric field is high enough to cause departure from full equilibration picture. This ability to modify LL wavefunctions with electric field is unique to graphene and cannot be seen in 2DEG.

Magnetotransport across one-dimensional superlattice (SL) had been studied in two-dimensional electron gas (2DEG) [1-6], reporting dissipationless transport across high potential barriers [1] and magnetic commensurability oscillations in longitudinal resistance [8]. The motivation was to study various competing length scales and energy scales between tunable SL potential and quantum Hall system. However, some regions of the parameter space were experimentally inaccessible due to technological challenges; these are now accessible due to new platforms, like graphene [7-9], where the energy scales of the Landau levels (large cyclotron gap) and the ability to create abrupt (\(\sim 10\) nm) tunable barriers allow new aspects to be explored. In addition, new physics, due to the role of crossed electric and magnetic field, that cannot be seen in conventional 2DEG can be studied in SL structures based on graphene.

SL in graphene modifies its bandstructure [10-15] and generates additional Dirac cones. A SL of periodicity (\(\lambda \sim 10\) nm) is induced in graphene due to its interaction with substrate like boron nitride [16, 17]. This led to the realization of a regime where magnetic length \(l_B \sim \lambda\) and observation of a quantum fractal spectrum [18-21]. The regime \(l_B \sim \lambda\) was difficult to realize in 2DEG systems due to experimental challenges of patterning SL (2DEG) [1-6], reporting dissipationless transport across high potential barriers [1] and magnetic commensurability oscillations in longitudinal resistance [8]. The motivation was to study various competing length scales and energy scales between tunable SL potential and quantum Hall system. However, some regions of the parameter space were experimentally inaccessible due to technological challenges; these are now accessible due to new platforms, like graphene [7-9], where the energy scales of the Landau levels (large cyclotron gap) and the ability to create abrupt (\(\sim 10\) nm) tunable barriers allow new aspects to be explored. In addition, new physics, due to the role of crossed electric and magnetic field, that cannot be seen in conventional 2DEG can be studied in SL structures based on graphene.

In this letter, we study magneto transport in an electrostatically defined 1D lateral SL in graphene [24]. In our device we apply a perpendicular magnetic field and periodically modulate the charge carrier density in adjacent “ribbons” of graphene, tuning from an array of p-p’ (or n-n’) to an array of p-n’ junctions. Changing the magnetic field allows us to vary \(l_B\) relative to \(\lambda\); and changing the gate voltage allows us to tune the SL potential strength relative to LL spacing. The relative abruptness, bipolarity of charge carriers, large modulation and unequally spaced LLs distinguishes the present work from the previous work on 1D SL using 2DEG systems [14, 25].

Apart from the length scales, we also study the energy scales involved. The competition between SL amplitude (\(V_0\)) and LL spacing (\(\hbar \omega_c\), where \(\hbar = h/2\pi\), \(h\) being the Planck’s constant, and \(\omega_c\) is the cyclotron frequency) gives rise to three regimes. When \(V_0 >> \hbar \omega_c\), SL effect dominates giving rise to extra Dirac points [15]. In the other extreme when \(V_0 << \hbar \omega_c\), quantum Hall effect in graphene is restored [15]. However, the situation is more complex and little explored when \(V_0\) and \(\hbar \omega_c\) have comparable contribution, and we have experimentally probed this regime in graphene.

We create a 1D tunable SL, of period \(\lambda\) by fabricating an array of thin finger gates on graphene. The schematic of a device is shown in Figure 1(a), and Figure 1(b) shows false colored scanning electron microscope image (details of fabrication in Supplemental Material). Electrostatically the region in graphene under the fingers is of width (\(w\) \(\sim 75\) nm and they have a period of \(\lambda = 150\) nm. Thus each region in this SL has the same width [24].

In our device, graphene consists of two alternating regions - one without a top-gate (BG region) and the other under a top-gate (TG region). The difference in charge carrier density between BG and TG regions gives rise to a SL whose amplitude (\(V_0\)) is controlled by \(V_{bg}\) and \(V_{tg}\). \(V_0 = \sqrt{\pi \hbar v_F} \text{sgn}(C_{bg}V_{bg})\sqrt{|C_{bg}V_{bg}|} - \text{sgn}(C_{tg}V_{tg} + C_{bg}V_{bg})\sqrt{|C_{bg}V_{bg}|} + \text{sgn}(C_{bg}V_{bg})\sqrt{|C_{bg}V_{bg}|}\) [24], where \(V_{tg}\) (\(V_{bg}\)) is the top-gate (back-gate) voltage, \(C_{tg}\) (\(C_{bg}\)) is the capacitance per unit area of top-gate (back-gate), \(e\) is the electronic charge and \(v_F\) is the Fermi velocity. (Details of calculation and plot of \(V_0\) as a function of \(V_{bg}\) and \(V_{tg}\) is in Supplemental Material.)

We measure zero-bias four-probe longitudinal resistance (\(R_{xx}\)) while varying gate voltages at different magnetic fields (\(B\)) at a temperature of 2 K. The charge neutral point is at \(V_{tg} = -0.1\) V and \(V_{bg} = -2\) V (Figure 1(e)) suggesting low unintentional doping. The mean free path in our device is \(\sim 70\) nm and phase coherence length is \(\sim 600\) nm at 2 K [26]. As \(B\) increases, the magnetic length (\(l_B = \sqrt{\hbar/\pi B}\)) decreases and the charge carriers
encounter smaller periods of SL until they are confined at 0 T and 2 K when TG regions are n-doped. The oscillations at 0 T and 2 K when TG regions are n-doped. The oscillations correspond to band structure modification in graphene due to charge neutral region at 0 T and 2 K. (f) Line plot of resistance as a function of $V_{bg}$ in two regions. (d) Parameter space of $V_{bg}$ and $V_{tg}$ showing the different type of charge carriers in adjacent regions. (e) Line plot of resistance as function of $V_{bg}$ when $V_{bg}$ is biased in charge neutral region at 0 T and 2 K. (f) Line plot of resistance at 0 T and 2 K when TG regions are n-doped. The oscillations correspond to band structure modification in graphene due to superlattice (see [24] for details). (g) Schematic depicting edge state transport with three finger gates. The first two regimes are marked in blue and third in purple in Figure 1(d).

FIG. 1. Device geometry and three different regimes of edge state transport. (a) Schematic of a device. (b) False colored scanning electron microscope image of a device with a zoomed-in image of the finger like top gates. (c) Depiction of the periodic 1D potential and the influence of the two gates in two regions. (d) Parameter space of $V_{tg}$ and $V_{bg}$ showing the different type of charge carriers in adjacent regions. (e) Line plot of resistance as function of $V_{bg}$ when $V_{bg}$ is biased in charge neutral region at 0 T and 2 K. (f) Line plot of resistance at 0 T and 2 K when TG regions are n-doped. The oscillations correspond to band structure modification in graphene due to superlattice (see [24] for details). (g) Schematic depicting edge state transport with three finger gates. The first two regimes are marked in blue and third in purple in Figure 1(d).

FIG. 2. Longitudinal resistance ($R_{xx}$) at $B = 14$ T when LLs are well resolved in three different case of edge state transport. (a) $R_{xx}$ as function of $V_{tg}$ and $V_{bg}$ at 14 T at a temperature of 2 K. The white lines denote line of constant $\nu_{bg}$. $R_{xx}$ as function of $\nu_{bg}$ for (b) $\nu_{tg} = 2$ and (c) $\nu_{tg} = 6$. The green curves in (b), (c) denote experimental value with the black dashed lines representing the resistance plateau in case of equilibration of edge states. ($\nu_{bg}$, $\nu_{tg}$) values are denoted within the parenthesis. Equilibration of edge state occurs with observed resistance plateau coinciding with black dashed line when type of charge carrier is same in adjacent region (first two cases in the schematic of Figure 1(g)). However in p-n region (shaded region in (b), (c)) we do not observe plateaus and the resistance is higher than the value expected in case of full equilibration of edge states. The only exception occurs when $\nu$ is 2 and -2 in adjacent region giving rise to a resistance plateau at $h/e^2$ as expected.
have one edge state for both electrons and holes circulating in adjacent regions, the resistance plateau is seen at $h/e^2$ (25.8 kΩ) suggesting full equilibration in this state in p-n' region; this is also the result one expects for the case of single gate. However, when neighboring regions have other edge states, that is, $(\nu_{tg} \times \nu_{bg}) < 0$ and $|\nu_{tg}|$ or $|\nu_{bg}|$ is greater than 2, we find resistance significantly larger than $h/e^2$ (for example, (2,-6) in Figure 2(b), (6,-2) in Figure 2(c)); a feature not generally seen in single p-n'-p junction in the quantum Hall state. This large resistance with maximum $R_{xx} \sim 200$ kΩ at 14 T, is seen along the diagonal direction in Figure 2(a); it is precisely in this diagonal direction of the parameter space that $V_0$ increases. This feature is quite robust and is also observed at lower $B$ (details in Supplemental Material).

This absence of equilibration could be due to a) certain symmetry of the system that prevents scattering from electron edge state(s) to more than one hole edge states with equal probability [29,32]; or b) the strength of the SL potential which affects the edge state transport and incomplete equilibration results, this combined with the large number of gates contributes to this observation. We believe that the later mechanism is at play in our experiment.

We now try to understand the reason for the incomplete equilibration. Figure 3(a) shows a LL diagram along the length of the device when $(\nu_{tg}, \nu_{bg}) = (-2,6)$; where we have co-propagating electron and hole edge states at the junction. At the interface of these regions (shaded brown in Figure 3(a)) there is an electric field ($E$), due to the SL, and it has a significant effect on the LL spectrum and the wavefunctions. It has been shown by Lukose et al. [33] and later extended by Gu et al. [29] for the case of a top-gate geometry that the SL potential which affects the edge state transport and incomplete equilibration results, this combined with the LL index and effective magnetic length as function of $E$ varies periodically along the length of graphene. The gap between the two LLs (which also depends on the LL index) decreases with increasing $E$. (c) Numerical simulation of $E$ in our device in the bipolar region ($V_{bg} = -30$ V and $V_{tg} = 3$ V). The top-gates spaced 150 nm apart are along the length of graphene. $E$ varies periodically along the length of graphene with maxima and minima at the edge of the top-gate. (d) Spatial variation of potential and $E$ along the length of graphene in bipolar region ($V_{bg} = -30$ V and $V_{tg} = 3$ V). Yellow regions denote spatial position of top-gates. (e) Contour map of maximum $E$ at $1.2 \times 10^7$ V/m and $1.4 \times 10^7$ V/m as function of $V_{bg}$ and $V_{tg}$ overlaid on measured $R_{xx}$ in bipolar regime at 14 T. Around these contours $E$ is high enough to modify the Landau levels causing departure from full equilibration.

We note that in the bipolar regime, $V_0$ created is much larger than $\hbar \omega_C$. Large $V_0$ leads to large $E$ in the region between BG and TG region which modifies the LLs locally. Increasing $E$ modifies the gap between the LLs leading to their collapse when $E/(\nu F B) = 1$. The extent of the wavefunction of the $n^{th}$ LL is given by the Larmor radius $R_L = \sqrt{\mu_B}$. Effective magnetic length in the presence of $E$ can be written as $l_B' = l_B/(1 - (E/\nu F B)^2)^{1/4}$. $l_B'$ increases with increasing $E$ and rises rapidly when $E$ approaches $\nu F B$. Green curve in Figure 3(b) shows divergence of $l_B'$ at 14 T when $E = 1.4 \times 10^7$ V/m. In the presence of $E$, LL energy ($E_n$) gets modified as $E_n' = Sgn(n)\sqrt{2n} \times (h\nu F /l_B) \times (1 - (E/\nu F B)^2)^{3/4}$, where $n$ is the LL index. Line plot of $E_n'$ at 14 T as function of $E$ is shown in Figure 3(b). The gap between two LLs at 14 T decreases with increasing $E$ and becomes zero as $E$ approaches $1.4 \times 10^7$ V/m.

To estimate the order of magnitude of $E$ in our devices we performed numerical simulation of the electrostatics using finite element method. In our device geometry, the top-gates are spaced 150 nm apart along the length of graphene. At a given $V_{tg}$ and $V_{bg}$, the charge carrier density induced along the length of graphene is calculated, from which potential and $E$ is obtained. (Details of calculation in Supplementary Information.) $E$ is periodic along the length of graphene as seen in Figure 3(c). Spatial variation of potential and $E$ along length of graphene in bipolar region is shown in Figure 3(d). The maximum $E$ is higher in the bipolar region compared to the unipolar region (details in Supplementary Information). At a given $V_{tg}$ and $V_{bg}$, the maximum $E$ is obtained and contours of this maximum $E$ at $1.2 \times 10^7$ V/m. and $1.4 \times 10^7$ V/m. is overlaid on the measured data at 14 T as shown in Figure 3(e).
from full equilibration. Thus in our device geometry, $E$ created is high enough to modify LLs which is reflected in charge transport measurements. $E$ modifies the LLs and so the assumption of full equilibration that the electrons are scattered with equal probability in any of the available channels remains no longer valid. Full equilibration of the copropagating electron and hole edge states is disrupted due to combination of three possibilities discussed next. The spatial extent of the wavefunction ($v_B$) increases with increasing $E$ and approaches the width of TG (or BG) region. This leads to increased coupling of the wavefunction within TG (or BG) region of the superlattice resulting in enhanced scattering. Secondly, the LL energy gets modified with increasing $E$. The gap between two LLs decreases with increasing $E$ and coupled with the fact that LLs have finite broadening due to disorder, LLs can merge before $E$ reaches $v_F B$. Additionally, the higher LLs can merge at a lower $E$ compared to lower LLs since the gap between two LLs decreases with increasing LL index. Lastly, at high $E$, there could be LL mixing [33] (where electron in a particular LL has contribution from other LLs). Thus there is lack of full equilibration even though electrons and holes are in close proximity at the junction.

In the absence of equilibration, the resistance is not only high but also depends on $E$ and $B$ applied. For example, resistance values at $(v_{bg}, v_{tg}) = (6,-2)$ at 14 T and 8 T are different as the maximum $E$ results not only because the $E$ is low in this state compared to any other state in the bipolar region but also because this is a special state where there needs to be scattering between the same LLs ($n = 0$) for full equilibration and that the LL gap is maximum between $n = 0$ and $n = 1$ LL. The LL modification depends on the ratio of $E$ and $B$ and we observed this effect in the bipolar region at 14 T. In unipolar region where $E$ is lower compared to the bipolar region, we observed the LL modification at lower $B$ of 3.5 T (details in Supplementary Information).

Our experiments with tunable superlattices suggest that tuning of the LL wavefunctions in graphene can be done using $E$ at interfaces, which cannot be realized in 2DEG. In addition, the nature of the state that emerges after the collapse of the LLs is little understood and possibility of existence of correlations has been speculated [33]. The close proximity of co-propagating electron and hole edge states can be used to construct large class of topological states [35] and also offers an opportunity to study excitonic effects, this has been recently explored in bilayer quantum Hall systems [39]. There have been predictions of correlated states in $\nu = 0$ LL of graphene and LL mixing can enable exploration of such phases [37]. Tunable 1D lateral superlattice in magnetic field offers opportunity to explore diverse physical phenomena.

We thank Marcin Mucha-Krucyzinski, G. Baskaran, R. Shankar, Jainendra Jain, Vibhore Singh, Shamashis Sengupta and K. Sengupta for discussions and comments on the manuscript. We acknowledge Swarnajayanthi Fellowship of Department of Science and Technology and Department of Atomic Energy of Government of India for support.

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Supplemental Material: Quantum Hall effect in tunable 1-D lateral superlattice in graphene – role of crossed electric and magnetic fields

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I. DEVICE FABRICATION

Graphene is mechanically exfoliated on degenerately doped silicon (Si++) substrate having 300 nm of silicon dioxide. Si++ acts as global back-gate, that is, it tunes the charge carrier density of the entire graphene flake. Standard e-beam lithography is used to pattern Cr/Au contacts in Hall bar geometry. The device is then thermally annealed to remove residual PMMA from graphene. 3 nm of Al₂O₃ is then deposited using e-beam lithography which acts as seed layer for subsequent deposition of 20 nm Al₂O₃ using atomic layer deposition. Al₂O₃ acts as top-gate dielectric above which narrow (∼27 nm wide), periodic fingers of Pd are patterned using e-beam lithography. All the fingers of top-gate are connected to one electrode outside the graphene flake so that all the fingers are at the same potential. The period of these fingers is 150 nm.

II. VARIATION OF AMPLITUDE OF SUPERLATTICE POTENTIAL WITH BACK-GATE AND TOP-GATE VOLTAGE

In our device there are two alternating regions in graphene - one without a top-gate (BG region) and the other under a top-gate (TG region). The amplitude of superlattice potential ($V₀$) created is given by the difference of doping in the BG and TG region. In case of no unintentional doping, the number density of charge carriers in the BG region is given as $n_{bg} = \frac{C_{bg}V_{bg}}{e}$; and in the TG region, the net number density of charge carriers is the algebraic sum of the density of carriers induced by the two gates $n_{tg} = \frac{C_{tg}V_{tg} + C_{bg}V_{bg}}{e}$; where $C_{bg}$ ($C_{tg}$) is the capacitance per unit area between graphene and back-gate (top-gate) and $V_{bg}$ ($V_{tg}$) is the back-gate (top-gate) voltage. Using parallel plate capacitor geometry $C_{bg}$ is given by $\epsilon_0 \epsilon_r / d$, where $\epsilon_0 = 8.85 \times 10^{12}$ F/m, $\epsilon_r = 3.9$ and $d = 300$ nm. $C_{tg}$ is obtained from the slope of charge neutral line in the plot of resistance as function of $V_{bg}$ and $V_{tg}$ at zero magnetic field (Figure S1(a)). From the slope, we get $C_{tg} = 18 \times C_{bg}$. In case of graphene, due to its linear dispersion, $E_F = \sqrt{\pi \hbar v_F \sqrt{n}}$, where $v_F$ is the Fermi velocity and $E_F$ is the Fermi energy, with all energies measured relative to the charge neutrality point. Thus $V₀$ is given by

$$V₀ = \sqrt{\pi \hbar v_F} \left( \frac{C_{bg}V_{bg}}{e} - sgn(C_{tg}V_{tg} + C_{bg}V_{bg}) \sqrt{\frac{|C_{bg}V_{bg}|}{e}} \right) (1)$$
Magnitude of $V_0$ is referred as $V_0$ since it represents the amplitude of superlattice potential. Figure S1(b) shows the variation of $V_0$ with $V_{bg}$ and $V_{tg}$.

**FIG. S1.** Amplitude of superlattice potential (a) Resistance as function of $V_{tg}$ and $V_{bg}$ at zero magnetic field at temperature of 2 K. White arrow points to the charge neutral line. (b) Contourplot of $V_0$ as a function of $V_{bg}$ and $V_{tg}$. $V_0$ is higher in the bipolar region where there is a series of $p$-$n'$ junctions compare to the unipolar region.

**III. QUANTUM HALL IN THE ABSENCE OF SUPERLATTICE POTENTIAL**

Figure S2(a) shows the magnitude of Hall resistance as function of $V_{tg}$ and magnetic field (B) when $V_{tg}$ is biased at charge neutral point ($V_{tg}$ = 0.1). The fan diagram corresponds to monolayer graphene. Figure S2(b) is a line plot at 10 T showing plateau at $\nu = 2$ ($\equiv 12.9 \text{ k}\Omega$), $\nu = 6$ ($\equiv 4.3 \text{ k}\Omega$) and $\nu = 10$ ($\equiv 2.5 \text{ k}\Omega$), where $\nu$ is filling factor. We can observe the Landau levels from 2 T.

**IV. EDGE STATE TRANSPORT FOR DIFFERENT FILLING FACTORS IN TG AND BG REGION**

The filling factor is given by $\nu = nh/Be$, where $n$ is the charge carrier density, $h$ is the Planck’s constant, $B$ is the magnetic field and $e$ is the electronic charge. For constant $B$, filling factor in BG region ($\nu_{bg}$) is controlled by $V_{bg}$, and filling factor in TG region ($\nu_{tg}$) is controlled by both $V_{tg}$ and $V_{bg}$. Changing $V_{bg}$ and $V_{tg}$ at a given magnetic field, there are three scenarios (I) series of $p$-$p'$ (or $n$-$n'$) region and $\nu_{tg}$ is more than $\nu_{bg}$, (II) series of $p$-$p'$
FIG. S2. (a) Magnitude of Hall resistance as a function of $V_{bg}$ and $B$ (b) Line plot of magnitude of Hall resistance as function of $V_{bg}$ at 10 T showing plateaus at $\nu = \pm 2, \pm 6, +10$.

(or n-n'') region and $\nu_{tg}$ is less than $\nu_{bg}$, and (III) series of p-n' region. Depending on these three cases, longitudinal resistance ($R_{xx}$) for a four-probe p-n-p device with a single gate is given by Equation I, II or III:

$$R_{xx} = \begin{cases} \frac{\hbar}{e^2} \frac{|\nu_{tg}| - |\nu_{bg}|}{|\nu_{bg}|} & \nu_{tg}\nu_{bg} > 0 \quad |\nu_{tg}| > |\nu_{bg}| \quad \text{(I)} \\
\frac{\hbar}{e^2} \frac{|\nu_{bg}| - |\nu_{tg}|}{|\nu_{bg}|} & \nu_{tg}\nu_{bg} > 0 \quad |\nu_{bg}| > |\nu_{tg}| \quad \text{(II)} \\
\frac{\hbar}{e^2} \frac{|\nu_{bg}| + |\nu_{tg}|}{|\nu_{bg}|} & \nu_{tg}\nu_{bg} < 0 \quad \text{(III)} 
\end{cases}$$

Thus $R_{xx}$ is proportional to the difference in filling factor in adjacent region in case (I) and (II), and is proportional to their sum in case (III). (The schematic of the three cases are shown in Figure 1(g) of the main text.)

FIG. S3. (a) Longitudinal resistance ($R_{xx}$) as function of $V_{tg}$ and $V_{bg}$ at 8 T. $R_{xx}$ as function of $\nu_{bg}$ for (b) $\nu_{tg} = 10$ and (c) $\nu_{tg} = 14$. The green curves in (b), (c) denote experimental value with the black dashed lines representing the resistance plateau in case of equilibration of edge states. (($\nu_{tg}, \nu_{bg}$) values are denoted within the parenthesis.)

Line slices showing longitudinal resistance ($R_{xx}$) as function of $\nu_{bg}$ for $\nu_{tg} = 2$ and 6 are shown in Figure 3 in main text. For $\nu_{tg} = 10$ and 14, we measured $R_{xx}$ as function of $V_{tg}$ and
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Vbg (Figure S3(a)) at a lower field of 8 T. Figure S3(b,c) shows $R_{xx}$ as function of $\nu_{bg}$ at $\nu_{tg}$ = 10 and 14. In the line slices (Figure S3(b, c)), the green curve is the experimental data with the black dashed line marking the calculated plateau in case of edge state equilibration. We observe edge state transmission and partial equilibration in the unipolar regime where there are same type of charge carrier in the adjacent TG and BG region. Incomplete equilibration is observed in bipolar regime where there is co-propagating electron and hole edge states in the junction between TG and BG region.

V. NUMERICAL SIMULATION OF ELECTROSTATICS USING COMSOL

We have modeled our device geometry in COMSOL software and obtained the electric field from numerical simulation of the electrostatics. In the simulation, we used 15 top-gates along the length of graphene and obtained the induced charge carrier density in graphene for a given back-gate and top-gate voltage. This induced charge carrier density is periodic along the length of graphene from which the potential is obtained. The electric field in the plane of graphene is obtained by taking the numerical derivative of the potential. The potential and the electric field variation along length of graphene for $V_{bg} = -30$ V and $V_{tg} = 3$ V in the bipolar region are shown in Figure S4(a,b). The top-gates are 150 nm apart similar to our device geometry. For clarity, magnified images with 3 top-gates along length of graphene are shown in unipolar region (Figure S4(c)) and bipolar region (Figure S4(d)). We observe the magnitude of electric field to be larger in bipolar region compared to the unipolar region. Another interesting thing to note is that not only the magnitude but also the shape of the potential is different in the two cases.

VI. ABSENCE OF FULL EQUILIBRATION DUE TO ELECTRIC FIELD

As seen in the previous section, the potential and the electric field is high in bipolar region where we have a series of $p-n'$ junctions compared to unipolar regime where there is series of $n-n'$ (or $p-p'$) junctions. The electric field causes modification of Landau level wavefunction. With increasing electric field the spread of the wavefunction increases and when it is of the order of the width of TG or BG region, there is enhanced scattering across Landau levels, giving rise to departure from full equilibration and large resistance. This effect is manifested
FIG. S4. Numerical simulation with 15 top-gates along length of graphene using COMSOL. (a) Variation of potential along length of graphene when $V_{tg} = 3$ V and $V_{bg} = -30$ V. (b) Spatial variation of electric field when $V_{tg} = 3$ V and $V_{bg} = -30$ V. Magnified image of spatial variation of potential and electric field in (c) unipolar region when $V_{tg} = 3$ V and $V_{bg} = 30$ V and (d) bipolar region when $V_{tg} = 3$ V and $V_{bg} = -30$ V. Yellow regions denote spatial position of top-gates.
FIG. S5. Electric field and magnetotransport in the bipolar region (a) Maximum electric field as function of $V_{tg}$ and $V_{bg}$. (b) Contour of electric field of $8 \times 10^6$ V/m is overlaid on $R_{xx}$ as function of $V_{tg}$ and $V_{bg}$ at 8 T. $R_{xx}$ as function of $\nu_{bg}$ for $\nu_{tg} = 6$ at (c) 14 T and (d) 8 T. In the absence of full equilibration resistance value for $(\nu_{tg}, \nu_{bg}) = (6, -2)$ is different at different magnetic field.

VII. EFFECT OF ELECTRIC FIELD ON MAGNETOTRANSPORT IN UNIPOLAR REGION

In the unipolar region, the electric field is low compared to the bipolar region. So, to observe the modification of Landau levels by electric field in the unipolar region, we measured $R_{xx}$ as a function of gate voltages from 2 T to 3.5 T. At a constant $V_{bg}$ of 20 V, $R_{xx}$ as a function of $V_{tg}$ shows SdH oscillations as magnetic field is increased above 3 T as seen in Figure S6(a).

We measured $R_{xx}$ as a function of $V_{tg}$ and $V_{bg}$ at 3.5 T (Figure S6(b)) in the unipolar region. Line plot of $R_{xx}$ vs $V_{bg}$ at $V_{tg} = 1$ V is shown by the green curve in Figure S6(c). At a constant $B$ of 3.5 T, we observe $R_{xx}$ to oscillate as function of $V_{bg}$ for a given $V_{tg}$. The oscillations arise due to periodic decrease of back-scattering as the Fermi energy moves through different Landau levels in BG region.

At $V_{tg} = 2.8$ V, the electric field is higher compared to $V_{tg} = 1$ V. So we took line slice of $R_{xx}$ vs $V_{bg}$ at $V_{tg} = 2.8$ V (orange curve in Figure S6(c)) and observed the fading of SdH oscillations. So with increasing electric field, the Landau levels come close to each other resulting in a decrease in the modulation in resistance. Thus the decrease in amplitude of
oscillation is consistent with the modification of Landau levels with increasing electric field.

FIG. S6. Effect of electric field on resistance oscillations in unipolar region (a) $R_{xx}$ as a function of $V_{tg}$ at $V_{bg} = 20$ V at different magnetic field. SdH oscillations are observed when magnetic field is increased above 3 T. The plots at different magnetic fields are offset for clarity. (b) $R_{xx}$ as a function of $V_{tg}$ and $V_{bg}$ at 3.5 T in the unipolar region. (c) Line plot of $R_{xx}$ as a function of $V_{bg}$ at $V_{tg} = 1$ V (green curve) and 2.8 V (orange curve). Amplitude of oscillations decreases with increasing $V_{tg}$ and hence increasing electric field.

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