Recurrent gas accretion by massive star clusters, multiple stellar populations and mass thresholds for spheroidal stellar systems

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ABSTRACT
We explore the gravitational influence of pressure-supported stellar systems on the internal density distribution of a gaseous environment. We conclude that compact massive star clusters with masses $\gtrsim 10^6 M_\odot$ act as cloud condensation nuclei and are able to accrete gas recurrently from a warm interstellar medium which may cause further star formation events and account for multiple stellar populations in the most massive globular and nuclear star clusters. The same analytical arguments can be used to decide whether an arbitrary spherical stellar system is able to keep warm or hot interstellar material or not. These mass thresholds coincide with transition masses between pressure supported galaxies of different morphological types.

Key words: ISM: general – ISM: kinematics and dynamics – globular clusters: general – globular clusters: individual: $\omega$ Cen – galaxies: fundamental parameters – galaxies: star clusters.

1 INTRODUCTION
Stars are believed to form during one single event in compact star clusters with a total mass ranging from a few solar masses to a few million solar masses. But, in recent years observations have revealed that stellar populations of compact star clusters are more complex than a single aged stellar ensemble.

Very young star clusters such as the Orion Nebula Cluster (ONC; Palla et al. 2005) and $\sigma$ Orionis (Sacco et al. 2007) may probably harbour a few low-mass stars which are more than 10 Myr older than the main bulk of their stars. This may be the result of extended star formation (Palla et al. 2005) or the capture of stars, which are born in surrounding former star formation events, through the deepening potential during cloud collapse (Pflamm-Altenburg & Kroupa 2007).

On the other hand, some of the old massive globular clusters (GCs) of both the Milky Way, Andromeda and M87 (Kaviraj et al. 2007) exhibit a spread in metallicity and/or have subpopulations with a helium overabundance suggesting the occurrence of multiple star formation events in these compact star clusters in the past (see Section 2 for a compilation of observational evidences).

All star clusters with evidences for multiple stellar populations have in common that their total mass is around $10^6 M_\odot$ or higher. Furthermore, star clusters with a mass larger than $\sim 10^6 M_\odot$ show a correlation between the cluster mass and the cluster size, whereas star clusters less massive than $\sim 10^6 M_\odot$ have a constant radius (Haségan et al. 2005; Walcher et al. 2005; Dabringhausen, Hilker & Kroupa 2008).

Mass thresholds of GCs have been considered by several authors (Morgan & Lake 1989; Shustov & Wiebe 2000; Recchi & Danziger 2005) addressing the question how massive a star cluster must be in order to keep ejected material by asymptotic giant branch (AGB) stars or supernovae. However, these thresholds can not explain the suggested growth of nuclear star clusters by repeated accretion (Walcher et al. 2005) and large internal spreads in metallicity.

Lin & Murray (2007) discussed the collective accretion of gas by GCs on to their member stars embedded in a $10^6 K$ hot background gas corresponding to today’s circumstances. But, GCs are assumed to have formed in denser environments, as for example nuclear star clusters are forming today. Additionally, the treatment by Lin & Murray (2007) does not reveal the existence of a mass threshold at $10^6 M_\odot$.

In this paper, we show that compact massive star clusters with masses $\gtrsim 10^6 M_\odot$ are able to accrete gas from an embedding warm interstellar medium (ISM). We first summarize the observational evidences for multiple stellar populations in massive star clusters in Section 2. The theoretical criterion for possible gas accretion by massive star clusters is derived in Section 3, and the $10^6 M_\odot$ accretion mass threshold is obtained in Section 4. Additional mass thresholds are found and identified with transitions in the radius–mass plane of spheroidal and pressure-supported stellar systems (Section 8).

2 EVIDENCE FOR MULTIPLE POPULATIONS
The most massive GC, $\omega$ Cen, with a mass of $\sim 2.5 \times 10^6 M_\odot$ (McLaughlin & van der Marel 2005) shows a wide spread in metallicity (Freeman & Rodgers 1975). Hilker & Richtler (2000) determined by Strömgren photometry three different stellar populations.

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being $\sim 1-3$ and $\sim 6$ Gyr younger than the oldest one. Bedin et al. (2004) confirmed the existence of multiple evolutionary sequences in $\omega$ Cen.

The properties of RR Lyrae stars observed in the GCs NGC 6388 and 6441 with masses of $\sim 1.1 \times 10^6$ and $\sim 1.6 \times 10^6 M_\odot$, respectively (McLaughlin & van der Marel 2005), can be reproduced by the composition of two distinct populations (Ree et al. 2002; Yoon et al. 2008).

Recently, Milone et al. (2008) showed that the split of the subgiant branch of the Galactic GC NGC 1851 with a mass of $\sim 3.1 \times 10^6 M_\odot$ (McLaughlin & van der Marel 2005) corresponds to the existence of two different stellar populations with an age spread of about 1 Gyr. Furthermore, Milone et al. (2008) concluded that the observed RR Lyrae gap in NGC 1851 requires an age difference of $\sim 2-3$ Gyr.

Lehnert, Bell & Cohen (1991) reported a possible metallicity spread in M22 with a mass of $4.4 \times 10^5 M_\odot$ (McLaughlin & van der Marel 2005), whereas Richter, Hilker & Richtler (1999) found no evidence for a metallicity spread using Strömgren photometry.

Metallicity spreads are also reported for G1 (Meylan et al. 2001), the most massive GC in M31 with a mass of $\sim 8 \times 10^6 M_\odot$ (Baumgardt et al. 2003) and in M54 (Sarajedini & Layden 1995) with a mass of $\sim 2.0 \times 10^6 M_\odot$ (McLaughlin & van der Marel 2005).

Evidence for metallicity spreads in the three massive GCs G78, G213 and G280 of M31 are reported by Fuentes-Carrera et al. (2008). These clusters have internal velocity dispersions as high as G1 (Djorgovski et al. 1997) and therefore must have masses larger than $10^6 M_\odot$.

Various explanations for the origin of multiple stellar populations in GCs and abundance anomalies such as ejecta from AGB stars (e.g. Ventura & D’Antona 2008) or rotating massive stars (e.g. Decressin et al. 2007) exist (see Renzini 2008 for a summary). Self-enrichment by rotating massive stars works for the first few Myr (Fig. 1) after star formation and might be attributed to observed anticorrelations of elements such as the Na–O anticorrelation for star clusters with masses $\gtrsim 10^7 M_\odot$.

Gas and metal return by supernovae occurs up to a few dozen Myr. As the ejecta by supernovae are much more energetic than winds by massive stars, star-cluster masses must be high in order to keep a significant fraction of the material supplied by supernovae. Using energy arguments, Baumgardt, Kroupa & Parmentier (2008) derived a lower mass threshold of $\sim 10^7 M_\odot$ for star clusters to retain their residual gas despite multiple supernova events. Wünsch et al. (2008) show using two-dimensional hydrodynamical simulations that also lower mass star clusters may be able to retain matter ejected by supernovae, as a substantial fraction of such material can thermalize its high kinetic energy before escape from the star cluster.

After all supernovae have exploded, the massive AGB stars begin to evolve and are able to continuously replenish the gas reservoir in the star cluster and in its vicinity (e.g. D’Antona & Caloi 2004).

After the epoch of massive energy feedback by the radiation and winds of massive ionizing stars and supernovae, which prevents gas of the surrounding ISM from being accreted, the gas of the ISM is able to react to the gravitational potential of the new star cluster. It is expected that the more massive the star cluster is the stronger is the gravitational influence on the ISM by the new star cluster. In this paper, we investigate this new possible scenario of gas accretion from the surrounding ISM by massive star clusters.

Additionally, Fellhauer, Kroupa & Evans (2006) showed that if $\omega$ Cen formed in a dwarf galaxy then it could have captured field stars during its formation from the underlying stellar field population contributing up to 40 per cent to the total final star-cluster mass and showing the complex stellar population composition of the host galaxy.

All existing explanations have in common that the material of further stellar generations has already assembled during cluster formation and thus their total mass does not increase with time.

In contrast, Walcher et al. (2005) determined the masses of nuclear star clusters near the photometric centre of bulge-less spiral galaxies to lie between $0.8 \times 10^6$ and $6 \times 10^6 M_\odot$. As these nuclear clusters have luminosities by up to two orders of magnitude larger than the most luminous Milky Way GCs, Walcher et al. (2005) suggested that these nuclear clusters grow by repeated accretion of gas and show subsequent star formation.

Böker et al. (2004) found that nuclear star clusters have dimensions ($r_\text{adm} \approx 3.5$ pc) comparable to massive GCs, and some of these with masses $\gtrsim 10^8 M_\odot$ show evidence for multiple structural components and estimated periods between star formation episodes of about 0.5 Gyr (Seth et al. 2006).

3 MODEL

From the hydrodynamical point of view, the ISM as a self-gravitating gas is described by the three conservation equations for mass, momentum and energy, the Poisson equation and the equation of state (e.g. Martel, Evans & Shapiro 2006),

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi, \quad (2)$$

$$\frac{\partial \epsilon}{\partial t} + \mathbf{v} \cdot \nabla \epsilon = -P \nabla \cdot \mathbf{v} + \frac{\Gamma}{\rho} - \frac{\Lambda}{\rho}, \quad (3)$$

$$\nabla^2 \Phi = 4\pi G (\rho - \bar{\rho}), \quad (4)$$

$$P = f(\rho, \epsilon), \quad (5)$$

where $\rho$ is the mass density, $P$ is the pressure, $\epsilon$ is the specific energy, $\mathbf{v}$ is the velocity, $\Phi$ is the gravitational potential, $\Gamma$ is the radiative heating rate and $\Lambda$ is the cooling rate, $\bar{\rho}$ is the mean density. Following Martel et al. (2006), this term has to be included to prevent the overall collapse of the ISM and the term $-\bar{\rho}$ accounts for whichever process makes the ISM globally stable. If a compact massive star cluster is present within a homogeneous part of the ISM, then the potential in equation (2) has to be split into a self-gravitating part.
and the external potential of the compact star cluster acting on the ISM. If the density fluctuations \( \rho - \bar{\rho} \) are initially small then the total potential in the vicinity of the star cluster is mainly given by the star cluster itself. To further simplify the problem, we assume that the star cluster has no relative velocity with respect to the embedding ISM, i.e. \( \mathbf{v} = 0 \). With increasing relative velocity between the star cluster and the embedding ISM, the effect of gas accretion is expected to become less dominant. Therefore, the stationary case gives a lower limit of a resulting mass threshold.

As a final simplification, only the static case can be considered, i.e. all time derivatives vanish. This can be interpreted such that the out-of-equilibrium ISM, for which the density distribution does not follow the static solution, attempts to reach this state. Thus, the static solution can be used to explore the gravitational influence of a compact massive star cluster on the surrounding ISM and the full set of equations which are only numerically solvable reduces to the hydrostatic equation,

\[
\frac{1}{\rho} \nabla P = -\nabla \Phi_{\text{cl}},
\]

\[
P = f(\rho, \epsilon),
\]

where \( \Phi_{\text{cl}} \) is the potential of the massive star cluster.

The potential of the star cluster is assumed to be spherical. Then, for a simple isothermal ideal gas,

\[
P = \rho \frac{B}{\mu m_u} T,
\]

with a temperature \( T \) and a molecular mass \( \mu m_u \) of the isothermal ISM, the static solution is given by

\[
\rho(r) = \rho_0 e^{-\frac{\mu m_u}{B} [\Phi(r) - \Phi_{\text{cl}}]},
\]

where \( \rho_0 \) and \( \Phi_{\text{cl}} \) are the mass density and the star-cluster potential at a given reference radius, \( r_0 \), introduced by the integration. The integration constants in equation (9) are chosen such that the potential vanishes at infinity, i.e. for a large distance from the cluster centre, and the particle density is the particle density of the undisturbed ISM,

\[
\rho(\infty) = \mu m_u n_{\text{ISM}}, \quad \Phi_{\text{cl}}(\infty) = 0,
\]

where \( n_{\text{ISM}} \) is the particle density of the ISM.

The potential created by the compact star cluster is described analytically by a Plummer potential (Plummer 1911),

\[
\Phi_{\text{cl}}(r) = -G M_{\text{cl}} \left( r^2 + b_{\text{cl}}^2 \right)^{-1/2},
\]

where \( M_{\text{cl}} \) is the mass of the star cluster and \( b_{\text{cl}} \) is the Plummer parameter describing the compactness of the cluster potential and is equal to the projected half-mass radius (Heggie & Hut 2003).

The final expression of the static solution is

\[
n(r) = n_{\text{ISM}} e^{-\frac{\mu m_u}{B} \left( r^2 + b_{\text{cl}}^2 \right)^{-1/2}}.
\]

The composition of the ISM is assumed to be primordial with \( X = 0.75 \) relative mass fraction of hydrogen and \( Y = 0.25 \) for helium. This corresponds to a mean molecular mass of \( \mu = 4/(4X + Y) = 1.23 \). In the following, the solutions for ISM-star-cluster systems are plotted as a function of the star-cluster mass and are parametrized by the ISM temperature and/or the Plummer parameter. If the ISM is enriched to for example \( Y = 0.3 \) then the mean molecular mass is \( \mu = 1.29 \), and the solutions correspond to the solution with the primordial composition but a star-cluster mass reduced by 4.8 per cent. In the \( n - M_{\text{cl}} \) plot, the curves then have to be shifted by 0.02 dex to the left. Thus, the choice of a primordial ISM composition is appropriate for this analysis.

4 THE \( 10^6 M_\odot \) MASS THRESHOLD

The Plummer parameter of a star cluster is equal to the projected half-mass radius (Heggie & Hut 2003). Assuming that the mass-to-light ratio of a massive star cluster is independent of the radius then the projected half-mass radius is equal to the half-light radius. For Galactic GCs, the median half-light radius is about 3.2 pc and ranges from 2 to 4 pc (Haegang et al. 2005; Dabringhausen et al. 2008). The ISM of the Galaxy consists of three main components (McKee 1995; Ferrière 2001; Cox 2005): a molecular and atomic cold component below 100 K but occupying only 1–2 per cent of the interstellar volume, a warm atomic and ionized component between \( 6 \times 10^3 \) and \( 10^5 \) K with a particle density of \( \sim 0.1–1.0 \) cm\(^{-3} \) corresponding to the ionization of hydrogen and a hot component with temperatures \( \gtrsim 10^6 \) K and a particle density less than \( 10^{-2} \) cm\(^{-3} \) fed by supernovae. The warm and the hot components have roughly the same volume filling factor of about 50 per cent. So, it is most likely that star clusters are embedded in the warm or hot component of the ISM.

The radial gas density profiles of three different star clusters \( (M_{\text{cl}} = 10^4, 10^5 \) and \( 10^6 M_\odot \) with a Plummer parameter \( b_{\text{cl}} = 3.2 \) pc embedded in a warm ISM with a temperature of 8000 K and a particle density of 0.5 cm\(^{-3} \) are plotted in Fig. 2. It can be seen that the hydrostatic gas density within the Plummer radius does not vary much. We therefore use the central gas density, \( n_c = n(r = 0) \), to characterize the static gas density in the inner part of a star cluster.

In order to explore the influence of the cluster potential on the embedding warm ISM, we calculate the central density of the static solution (equation 12) in dependence of the star-cluster mass with a Plummer radius of 3.2 pc shown in Fig. 3. To cover the full range of the observed warm ISM, four different models are calculated with temperatures of 6000 and \( 10^5 \) K and densities of 0.1 and 1 cm\(^{-3} \) corresponding to the boundary values of the observed warm ISM. For cluster masses smaller than \( 10^5 M_\odot \), the central particle density does not differ from the density of the undisturbed ISM. Between \( 10^5 \) and \( 10^6 M_\odot \), the central density starts to rise and becomes dramatically large above \( 10^6 M_\odot \).

The potential of massive star clusters with a mass larger than \( 10^6 M_\odot \) thus produces an instability in the warm ISM such that the ISM is expected to react with starting inflow towards the cluster centre. In other words, a self-gravitating ISM does not note the existence of star clusters with a total mass lower than \( 10^5 M_\odot \), but star clusters more massive than \( 10^6 M_\odot \) become immediately

![Figure 2](https://academic.oup.com/mnras/article-abstract/397/1/488/1007279)

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attracting holes in the warm ISM. Note, how rapidly the instability rises with increasing cluster mass only depends on the temperature but not on the density of the ISM. Keeping the properties of the ISM constant \( T = 8000 \) K and \( n = 0.5 \) cm\(^{-3}\), the instability region also appears at cluster masses of about \( 10^6 \) M\(_\odot\) when varying the Plummer parameter between 2 and 4 pc (Fig. 4).

Present-day Milky Way GCs do not belong to the disc and are not embedded in a comoving warm ISM. But, in early times when they formed they may have been embedded in a warm ISM, e.g. in a dwarf protogalactic building block. \( \omega \) Cen, for example, is believed to have been hosted in a dwarf galaxy disc before being accreted by the Milky Way. Because it was massive enough it could have accreted additional gas when the conditions were appropriate. Star formation events within the young \( \omega \) Cen or in its vicinity are expected to have caused varying ISM properties. Thus, it could have been placed alternately in the warm or hot ISM and the accretion history could have been fluctuating. Ionizing OB stars in the young \( \omega \) Cen would have prevented it from further accretion from the embedding ISM and star formation rested in \( \omega \) Cen. Meanwhile, star formation would have been ongoing in the ISM of the dwarf galaxy disc being continuously enriched with metals. After all ionizing sources in \( \omega \) Cen disappeared, accretion would have restarted if the comoving conditions were appropriate. When enough gas accumulated in \( \omega \) Cen, star formation would have restarted and the newly formed stars would have been more metal rich than the younger stellar populations. Thus, distinct populations with different metallicities are expected in such a scenario in \( \omega \) Cen as observed.

To underline the strength of this instability threshold for star-cluster masses \( \gtrsim 10^6 \) M\(_\odot\), the cooling time-scale, \( \tau_{\text{cool}} \), corresponding to the central gas density is calculated. We use the cooling function provided in Koeppen, Theis & Hensler (1995) in the temperature regime from 100 to \( 10^4 \) K giving an estimate of the cooling time-scale of

\[
\tau_{\text{cool}}/\text{yr} = 16400 \left( T / K \right)^{0.5} \left( n / \text{cm}^{-3} \right)^{-1}.
\]  

The resulting central cooling time-scale for the static solution of an ISM with a temperature of \( 8000 \) K and a density of \( 0.5 \) cm\(^{-3}\) is plotted in dependence of the star-cluster mass with a Plummer parameter of 3.2 pc (Fig. 5). If the gas of the ISM starts to flow into the massive star cluster due to the instability caused by the cluster potential, the increasing central gas density has a decreasing cooling time-scale supporting the gas accretion. For comparison, a thermal instability leading to an expected cooling flow of hot gas is known for galaxy clusters (Fabian 2003) and massive elliptical galaxies (Kroupa & Gilmore 1994). New star formation is inhibited by the ISM in massive elliptical galaxies being kept at a temperature of \( \approx 10^6 \) K due to the random stellar motions with velocities of a few hundred km s\(^{-1}\) (Mathews & Brighenti 2003; Parriott & Bregman 2008). The massive star-cluster instability, however, leads to a cooling instability and the onset of star formation until new OB stars reheat the cluster ISM.

The argument can also be turned around. If warm material is released inside the cluster then the gas attempts to reach the hydrostatic solution. The warm gas of a star cluster less massive than \( 10^6 \) M\(_\odot\) will be distributed nearly uniformly in space, i.e. it escapes from the cluster, whereas a star cluster more massive than \( 10^6 \) M\(_\odot\) is able to keep its warm gas.

It may be rashly argued that the reason for accretion or keeping of warm material is that the sound speed of the gas is smaller than the escape velocity from the star cluster. The sound velocity of an ideal gas is given by

\[
c_s = \sqrt{\frac{\gamma k_B T}{\mu m_u}} = 91.2 \mu^{-1/2} \left( \frac{T}{\text{K}} \right)^{1/2} \text{ km s}^{-1},
\]  

For a monatomic gas, \( \gamma = 5/3 \), the sound speed is about 10.6 km s\(^{-1}\) (\( T = 10^4 \) K) or 8.2 km s\(^{-1}\) (\( T = 6000 \) K). The escape velocity at a distance of \( r = h_{\text{cl}} \) from the centre of a Plummer
potential with mass $M_\odot$ and Plummer parameter $b_\odot$ is

$$v_{\text{esc}} = \sqrt{\frac{2G}{b_\odot}} \frac{M_\odot}{M_\odot + \frac{1}{2}} \approx 92.7 \text{ km s}^{-1} \ (1 + \frac{1}{2})^{-1/4} \sqrt{\frac{M_\odot}{M_\odot + \frac{1}{2}}} \ (15).$$

The lowest threshold mass of $2.5 \times 10^4 M_\odot$ is calculated in the case of the lowest temperature of the warm ISM (6000 K) and the central escape velocity ($l = 0$). The largest threshold mass of $6.4 \times 10^3 M_\odot$ is calculated in the case of the highest temperature of the warm ISM ($10^5 K$) and an escape velocity from the outer region of the cluster, e.g. the half-mass radius ($l = 1.305$). Nevertheless, by comparing these two characteristic velocities describing the potential of the star cluster and the internal energy content of the gas, threshold masses are obtained which are more than one order of magnitude smaller than the lower limit of masses of star clusters in which multiple stellar population are observed. In any case, the sound speed of a gas describes the velocity with which a pressure change propagates through space. No large-scale motion of matter is involved in the propagation of sound, whereas the accretion of material is such a process.

In any real situation of interest, the gas will not be static and the ISM will be turbulent and clumpy. Thus, it might be questionable how much is learned by computing static gas densities. Indeed, the calculated static solutions are not reached by the gas. The static solution describes a stage of an equilibrium. In the case of the presence of a star cluster less massive than $10^6 M_\odot$, these equilibrium is described by a nearly uniform density distribution. That is, the warm ISM does not note the existence of the star cluster. In the case of a star cluster more massive than $10^6 M_\odot$, the static solution is characterized by a required central gas density many orders of magnitude larger than the surrounding gas density. At the position of the massive star cluster, a uniformly distributed ISM is far from equilibrium. Thus, the gas tries to asymptotically reach this equilibrium by increasing its density at the position of the massive star cluster, i.e. the star cluster accretes. It will never reach the exact static gas density distribution as the increase of the central gas density will result in a more efficient cooling and restarted star formation will stop further accretion by heating the infalling gas.

Turbulences and inhomogeneities of the warm ISM lead to the condensation of material and the formation of cold molecular clouds at random locations in the galaxies. In this context, compact star clusters with masses $\gtrsim 10^6 M_\odot$ stimulate this process at certain locations, i.e. the star clusters act as cloud condensation nuclei.

5 THE AMOUNT OF ACCRETED MASS BY A MASSIVE STAR CLUSTER

The threshold criterion derived here can be used to decide whether a star cluster is able to accrete gas from a warm medium or not. If a massive star cluster is able to accrete, the question arises how much material can be accreted and if the amount of accreted material can account for the observed multiple stellar populations. Finding an accurate answer to this question requires an extensive numerical simulation. However, a reasonable estimation can be done using the accretion rate expression for a star from its surrounding medium (Bondi 1952). The star has a potential of a point mass, whereas the potential of the star cluster is extended. But, due to the spherical symmetry of the star cluster the potential at a certain radius from the centre of the potential is given by the enclosed mass within this radius placed at the origin of the potential (Newton’s second theorem). Therefore, outside the cluster radius the cluster potential is the same as if the cluster is assumed to be a point mass and the application of the Bondi accretion is justified. The accretion rate, $A$, is given by

$$A = \frac{1}{\pi} \left(\frac{G M_\odot}{v_\text{esc}^3}\right)^2 c_s \rho_\infty, \quad (16)$$

where $c_s$ is the sound speed and $\rho_\infty$ is the mass density of the gas far from the cluster, i.e. the density of the surrounding ISM. With

$$A \left(\frac{M_\odot}{\text{Myr}^{-1}}\right) = 2.9 \times 10^{-6} \mu \left(\frac{M_\odot}{M_\odot}\right)^2 \left(\frac{c_s}{\text{km s}^{-1}}\right)^{-3} \left(\frac{\rho_\text{ISM}}{\text{cm}^{-3}}\right) \quad (17)$$

and for a metallicity of $\mu = 1.23$, a sound speed of $c_s = 10.2 \text{ km s}^{-1}$ and a particle density of $\rho_\text{ISM} = 1 \text{ cm}^{-3}$, the accretion rate is about $3400 M_\odot \text{ Myr}^{-1}$. Thus, on a time-scale of a few 100 Myr corresponding to derived age gaps between the multiple stellar populations the possible amount of accreted material is of the order of $3.4 \times 10^3 M_\odot$ for a $10^5 M_\odot$ star cluster. The accreted material can account for the observed multiple stellar populations.

We note in passing that the accretion may also be modulated by the cluster orbit about its host galaxy. For example, it may experience more significant accretion events when it passes through the ends of a galactic bar or through major spiral arms where the likelihood of it encountering cold gas is enhanced.

6 ON THE COLD GAS THRESHOLD FOR STAR CLUSTERS AND THE STAR-CLUSTER BIRTH INSTABILITY

The criterion, how massive a star-cluster must be in order to cause a density instability of the ISM, depends only on the ratio, $n_\text{ISM}/n_\odot$, of the central and surrounding gas density in equation (12). The threshold expression is proportional to $e^{M_\odot/T_{\text{ISM}}}$. Thus, it should be easier to accrete cold cloud material than warm material. But, the volume filling factor of cold cloud material in galaxies is about 1–2 per cent and much smaller than the volume filling factor of the warm material ($\approx 50$ per cent). The possibility that a massive star cluster is long-term embedded in a cold molecular cloud is much smaller than the possibility that the star cluster is embedded in the warm ISM. If a star cluster is surrounded by cold dense material, it is expected that accretion occurs for even less massive star clusters. As shown above, the threshold is surpassed for typical values of $M_\odot/b_\odot = 10^5 M_\odot$, $b_\odot = 3 \text{ pc}$ and $T = 10^4 \text{ K}$. Star clusters should be able to accrete cold material with a temperature of about 100 K if the mass–size ratio is larger than $M_\odot/b_\odot = 10^4 M_\odot/3 \text{ pc}$. For young embedded star clusters with $b_\odot = 0.3 \text{ pc}$ (as the ONC for example), a total cluster mass of about $100 M_\odot$ is sufficient to accrete cold gas.

It may be possible that this characterizes the transition from distributed star formation within giant molecular clouds in the form of loose groups hosting a few low-mass stars to star formation in well-defined compact embedded star clusters: star formation in giant molecular clouds starts with distributed loose groups of a few low-mass stars. If regions in the low-mass star-forming giant molecular cloud surpass the threshold expression, runaway accretion starts and a well-defined star cluster appears. The ONC, for example, is a well-defined young embedded star cluster within a molecular cloud showing many loose groups of few low-mass stars distributed over the molecular cloud.

7 NUCLEAR STAR CLUSTERS

Nuclear star clusters, which are embedded long term in a warm comoving ISM and have masses less than $10^6 M_\odot$, may not accrete
additional material after their formation. But, nuclear star clusters which are born or end up through mergers of smaller clusters more massive than $10^6 \, M_\odot$ should be able to accrete additional gas after their formation and show further star formation events. Therefore, they are expected to grow as suggested by Walcher et al. (2005). The star-cluster mass region at $10^6 \, M_\odot$ should be underpopulated predominantly for star clusters located in gas-rich environments, i.e., the central regions of galaxies.

Indeed, the ensemble of nuclear clusters observed by Georgiev et al. (2009a) in gas-rich dwarf irregulars have absolute $V$-band luminosities of $-9.4$ mag and less corresponding to stellar masses of $\approx 10^6 \, M_\odot$ and more. Compared with the population of blue and red GCs hosted in dIrrs, dEs and dSphs, which populate the distribution of $V$-band luminosities uniformly down to $\approx -8.7$, these nuclear star clusters seem to form a distinct population as they are separated from the rest of the less massive GCs by a small gap in the luminosity sample (Georgiev et al. 2009a, fig. 1). These nuclear star clusters are candidates for repeated accretion events and so should contain complex stellar populations, as opposed to the clusters below the threshold which should not contain significant complex populations.

It may be argued that nuclear star clusters are just fed with additional gas because they are located at the origin of the potential of the galaxy. But, the location of nuclear clusters and the dynamical centres of disc galaxies do not always coincide (Matthews & Gallagher 2002; Walcher et al. 2005). Recently, Georgiev et al. (2009b) found that seven of 10 nuclear star clusters in dwarf irregular galaxies are off-centred by up to 480 pc. The overall process can be as follows.

The central regions of galaxies are fed with gas due to the deep gravitational potential of the galaxy. For a nuclear star cluster to be able to accrete gas from this continuously, refilled central gas reservoir then depends only on the mass of the massive star cluster. Due to differential rotation, the accreted material is expected to form a more rotationally than pressure-supported new stellar system in the star cluster. Furthermore, younger stellar populations in these compact massive star clusters should be more elliptical and aligned with the galactic gas disc, whereas the initial population should be more spheroidal if the formation of the star clusters occurred in a monolithic collapse. Indeed, it has been found that older stellar populations in nuclear star clusters have a more spheroidal morphology and nuclear star clusters appear flattened along the plane of the galaxy disc (Seth et al. 2008).

Milosavljević (2004) calculated the time-scale required for nuclear star clusters to have migrated from distant eccentric orbits in the disc towards the central regions of their host galaxies. Such migration time-scales are too long and nuclear star clusters must have formed in situ in the centres. However, if massive star clusters accrete gas from the long-term embedding ISM their masses grew. If the angular momentum of the star cluster is conserved their orbit must shrink and the inspiral may be accelerated. Places of the formation of such massive star cluster could have been for example the massive clumps observed in chain galaxies at higher redshift (Elmegreen & Elmegreen 2006).

8 OTHER MASS THRESHOLDS

In the previous sections, the influence of a few pc sized star cluster on the warm ISM has been explored. But, such globular/nuclear star cluster type objects constitute only a subset of pressure-supported stellar systems, which do not have arbitrary dynamical proper-

![Figure 6](https://academic.oup.com/mnras/article-abstract/397/1/488/1007279)

*Figure 6.* Illustration of the half-light radius dynamical mass relation of pressure supported stellar systems from Dabringhausen et al. (2008). See the text for details.

ties but show distinct size–mass relations (Haşegan et al. 2005; Dabringhausen et al. 2008; Forbes et al. 2008).

Fig. 6 is a sketch of fig. 2 of Dabringhausen et al. (2008) showing the half-light–radius–dynamical-mass relations of pressure-supported stellar systems. Here, the dynamical mass, $M$, is the total mass within the optical extend of the object as obtained by solving the Jeans equations. Globular clusters do not show a mass-dependence of their half-light radius, whereas above about $10^6 \, M_\odot$ the half-light radius becomes mass dependent. The ultra compact dwarfs (UCDs), bulges of spiral galaxies and high-luminous ellipticals constitute a steep branch (SB) in the half-light–dynamical-mass plane. The radius–mass relation of these objects lying on the SB presented in Dabringhausen et al. (2008) can be converted into a $b_{cl} - M$ relation,

$$b_{cl} = 2.95 \left( \frac{M}{10^6 \, M_\odot} \right)^{0.60}.$$  \hfill (18)

The flat branch (FB) is build by dwarf spheroidals and low-luminous ellipticals. The $b_{cl} - M$ of the FB is

$$b_{cl} = 92.8 \left( \frac{M}{10^6 \, M_\odot} \right)^{0.33},$$  \hfill (19)

corresponding to a constant mass density.

These two branches can be combined with the warm and the hot ISM. The ratio of the central density and the density of the gas at infinity of the static solution (equation 12) is plotted in Fig. 7 as a function of the total mass of the system. Systems with $M \geq 10^6 \, M_\odot$ on the SB are able to keep their warm gas, whereas

![Figure 7](https://academic.oup.com/mnras/article-abstract/397/1/488/1007279)

*Figure 7.* Density ratio of the static gas as a function of the total system mass of stellar systems lying on the FB and SB (Fig. 6) for the cases of the warm and hot ISM.
systems on the FB are able only to do so if their mass is $10^9 \, M_\odot$ or larger. This mass threshold coincides with the mass separating the dwarf spheroidals and low-luminous ellipticals from each other. The hot ISM can only be kept by systems more massive than about $10^{11} \, M_\odot$. This threshold mass corresponds to the transition region from low-luminous ellipticals on the FB to high-luminous spirals and bulges of disc galaxies on the SB.

One may speculate if these coincidences of the mass thresholds obtained from the instability argument and the mass transitions observed in the mass–radius plane of pressure-supported stellar systems has a direct physical origin.

9 CONCLUSION

We have shown that an ISM instability occurs for massive compact star clusters with masses of $M_\text{cl} \gtrsim 10^6 \, M_\odot$. This instability may relate to episodic gas accretion from the embedding ISM and subsequent star formation. Such an extended star formation history leads naturally to a spread in metallicity as observed in globular star clusters more massive than $\approx 10^6 \, M_\odot$.

This type of gas accretion can also account for the ongoing star formation in massive nuclear star clusters which are still embedded in a comoving dense ISM.

The ‘massive star-cluster’ instability adds to the well-known Rayleigh–Taylor instability.

Furthermore, for the combination of observed size–mass relations of pressure-supported systems and different phases of the ISM the same analysis reveals other mass thresholds which coincide with transitions in the size–mass diagram of pressure-supported stellar systems composed of GCs, UCD galaxies, bulges of disc galaxies, dwarf spheroidals and dwarf and large ellipticals.

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