An Elementary (Pseudo)Scalar at all scales.

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Abstract

We consider an extended linear $\sigma$ model in which the fermions are quarks and are coupled to gluons. Equivalently, this is QCD extended by coupling the quarks to a colour singlet chiral multiplet of $(\sigma, \vec{\pi})$ fields. This theory has a phase governed by a UV fixed point where all couplings are AF (asymptotically free). This implies that the scalars are elementary at high energies (UV) and, as they are colour singlets, they are not confined at low energy (IR). Thus, the scalar particles are elementary at all scales.

1 Introduction

The question of elementarity of a particle has been an abiding theme in Physics. From the atom to the nucleus, to the nucleon, all have been treated as elementary till proved otherwise. The higher the energies we explore, and this is a process that goes monotonically with time, the greater the resolution. It is, then, not surprising that elementarity has been a strongly time dependent attribute. However, we do have some long term 'elementary' survivors like the electron and the quark, to name a few. The electron has continued to be elementary from the time it was discovered to this day whereas the quark is a recent arrival. There is a rather important difference, the electron is manifest in a free state but the quark is not - it is confined.

From a field theory point of view this difference between the quark and the electron is perhaps even more interesting. We could consider the electron entirely within the framework of QED, which is not an asymptotically free theory whereas the quarks can be described by QCD which is an asymptotically free (AF) theory. In that case the electron couples to the photon more and more strongly as the energy goes up and so we cannot predict its fate.
(or its elementarity) at some very high energy when the coupling diverges at the so called Landau Singularity (LS). In QCD, the quark coupling to the gluons, however, decreases as we go to higher and higher energies and so it starts behaving like a free particle or parton and is in no danger of losing its elementarity as the energy goes higher.

So it seems that particles that are strictly governed by a non-abelian gauge AF theory can be elementary in the sense that they cannot resolve any further into constituents as the energy goes up. This is a new way to look at elementarity.

This, however, comes at a cost. Particles of a non-abelian gauge theory, like quarks and gluons, turn out to be confined and so cannot be seen in a free state. This means that these particles are absent from the spectrum of the theory. It is not meaningful, then, to talk about their elementarity at these scales.

This leaves us in a bit of a bind. Particles which belong to an AF theory like QCD are clearly elementary at asymptotic energies but are confined at low energy. At the other end particles that are seen only at low energies are generally composite. Can we find a particle that eludes this catch i.e one which is elementary at all scales?

In what follows, this problem is eluded by considering an extension of QCD by the addition of a colour singlet chiral multiplet, $(\sigma, \vec{\pi})$, that couples to quarks via a Yukawa coupling. This chiral multiplet should not be confused with the dynamically generated multiplet that consists of the Goldstone boson, the pion. This coupling, as it turns out, can be AF $^1$. The AF nature of the Yukawa coupling is possible only when the quarks are also coupled to an AF non-abelian gauge theory. Since it is the AF nature of the Yukawa coupling that makes the scalar elementary, the elementarity of the scalar can only be gained by riding piggy back on a nonabelian gauge theory.

Further, since the scalar is not confined like the quarks, it is elementary and visible at all energies! This is indeed a new situation in Particle Physics. The electron in QED has a problem at very high energy, when the coupling gets large and has a Landau Singularity and we have no way to handle the physics $^2$. The quark or gluons, on the other hand, are confined at low energy and so the question of their elementarity at low energy is not very meaningful.

$^1$ Of course, this is evaded when QED is embedded in a larger AF theory.
The issue of elementarity vs compositeness in local field theory has a long and interesting history. An important first step was the construction of a local field operator for composite particles [3, 4, 5]. A central idea in the field theoretic context was that compositeness is signalled by the vanishing of the wavefunction renormalisation. While this remained a conjecture for sometime which was supported in some specific models [6, 7, 8], Fried and Jin on the one hand [9], and independently Divakaran [10] on the other, were able to prove this on general grounds. Divakaran was also able to demonstrate the so called “indifference hypothesis” of Feynman according to which the Lagrangian can be written as a function of any one of all the irreducible sets of fields. In particular, sets including the composite fields can also be used. This immediately indicates that characterisations according to which elementary particles are those whose fields appear in the Lagrangian and composite the ones whose fields do not appear so [8] are rather naive. This fact is explicitly borne out in the works of Hasenfratz et al [11] and Zinn-Justin [12].

Coming back to the question of wavefunction renormalisation, if $Z_\phi$ vanishes at some scale, it implies that the particle loses its kinetic term and that it is composite from this scale onwards. However, often it is not possible to compute $Z_\phi$ except in perturbation theory - making it difficult to extend the calculation all the way to the region where $Z_\phi = 0$. The criterion is therefore more useful as a rough guide to the compositeness scale. As we point out, $Z_\phi = 0$ often occurs simultaneously with the coupling diverging at a LS. In the case of confinement, we expect $Z_\phi = 0$ at the confinement scale, which is inherently non perturbative and at present not calculable.

Even within the framework of S-matrix theory it is interesting to raise the distinction between elementarity and compositeness. At first sight it would appear that since both of them manifest as poles no clear distinction would be possible. Within the Lee model [13] Vaughn et al [8] indeed found a distinction that would be reflected in the properties of the S-matrix alone. This was in the high energy behaviour of the scattering phase shifts similar to the case in potential scattering as indicated by the Levinson [14] theorem. Rajasekaran [15] has proposed another way of distinguishing elementary and composite particles within S-matrix theory; according to him absence of a pole in the K-matrix signals compositeness.

We have scenarios of dynamical symmetry breaking (see for example, Refs. [16], [17]) where a sufficiently strong effective four fermion interaction
at the compositeness scale can generate kinetic terms for a new particle mode below the compositeness scale and also spontaneously break an explicit symmetry which will convert some particle modes to goldstone bosons (for example the QCD ‘pion’). In these models the low energy theory is a renormalizable sigma model. Using perturbative renormalization group (RG) it is found that as we approach the scale of the LS at which the yukawa coupling diverges, $Z_\phi$ (for the scalar /pseudoscalar (PS)) simultaneously goes as an inverse power of the yukawa coupling. As we approach this scale we may use the approximation of dropping the kinetic term for the scalar. In this circumstance, the four fermi interaction is recovered using the field equation. This scale may be loosely identified with the compositeness scale for the scalar (PS). Evidently, this procedure is far from rigorous, since the analysis breaks down for large coupling long before the LS for the yukawa coupling is reached. The above considerations are then only a pointer to compositeness and not a proof.

1.1 Elementarity and fixed points.

The question of fixed points is of fundamental importance in understanding the behaviour of a field theory; in particular of the scaling properties of a quantum field theory.

In four dimensions Yang Mills gauge theories are the only theories that have a non trivial ultra violet (UV) fixed point \cite{18}. The presence of a UV free fixed point for this theory is equivalent to the property of asymptotic freedom (AF) which, in turn, translates into the scaling behaviour of scattering amplitudes.

On the other hand, the $\lambda\phi^4$ field theory (nonperturbatively) is governed by only a gaussian IR fixed point and is therefore trivial. Perturbatively, as we know, the coupling diverges at LS.

The conventional wisdom on scalar field theories is that they are generally not AF. The introduction of fermions that interact with the scalars, e.g. the linear $\sigma$ model of Gell Mann and Levy, makes these theories more interesting but nevertheless it has been shown that they cannot be asymptotically free \cite{18}. This implies that they must be theories defined by a finite cut off. However, this does not clear up the issue of their triviality; in the sense that as the UV cut off is removed it is not clear that such theories go to free field theories. It is conceivable that these theories may be governed by a non
gaussian IR fixed point.

Hasenfratz et al \[11\] and Zinn-Justin \[12\] consider a large N yukawa field theory in which the fermions (quarks) are \(N\) component colour fields that couple to scalar fields. The colour index is a free index as there are no colour interactions. Effectively, the \(N\) components perform a counting operation to permit a large N expansion. They find that these theories are governed by an IR gaussian fixed point in analogy to the \(\lambda \phi^4\) theory. They also find that the Nambu-Jona-Lasinio model which has the same symmetry but is perturbatively non-renormalizable, is also governed by the gaussian IR fixed point in the large N limit.

An interesting question then arises as to whether the introduction of non-abelian gauge fields that couple to the fermions can change the fixed point structure of the theory.

In \[1, 2\] we considered a chirally invariant Yukawa-gauge theory in which a colour singlet, flavour chiral multiplet, was coupled to quarks in QCD, for the case \(N_C = 3\) and \(N_F = 6\). Another such theory in which the degrees of freedom are quarks, gluons, Higgs bosons, weak vector bosons and leptons is the standard EW model (the GSW model) \[19, 20\]. We have shown for such theories that: i) if the initial data on the couplings falls in a certain region in the parameter space of the couplings, namely, \(\rho = g_y^2/g_3^2 < \rho_c\) where \(g_y\) and \(g_3\) are the yukawa and QCD couplings respectively (\(\rho_c\) is determined by \(N_C\) and \(N_F\)), and, ii) only on a specific trajectory (the Invariant Line, IL) in the \([R, \rho]\) parameter space (where \(R = \lambda/g_y^2\), and \(\lambda\) the scalar self coupling), the theory can be asymptotically free. For couplings outside of this specific region the theory is not asymptotically free.

The AF branch above gives us a chiral invariant and completely asymptotically free theory (that is AF for all couplings), which can be a candidate theory for the strong interactions, similar to QCD. In \[2\], the difference between this theory and QCD is made explicit. Due to AF the entire spectrum of this theory, including the colour singlet scalar, is parton like or elementary at high energy.

This result runs counter to the popular belief that a generic non-abelian gauge theory with scalars may not be AF. This demonstrates that when we couple non-abelian gauge fields, gluons, to a chiral theory of quarks and scalars the theory can have an entirely new fixed point - an UV fixed point, as in QCD. This is in contrast to the case in the absence of the gauge fields, when the theory is governed by the IR free fixed point alone.
2 The theory

We begin with a spontaneously broken Yukawa gauge theory given by the Lagrangian below [1, 2, 21]:

\[
\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^{a} G^{a\mu\nu} \big| \text{colour} - \sum \bar{\psi} (D + g_{y}(\sigma + i\gamma_{5} \vec{\tau} \cdot \vec{\pi})) \psi - \frac{1}{2} (\partial_{\mu}\sigma)^{2} - \frac{1}{2} (\partial_{\mu}\vec{\pi})^{2} - \frac{1}{2} \mu^{2} (\sigma^{2} + \vec{\pi}^{2}) - \frac{\lambda}{4} (\sigma^{2} + \vec{\pi}^{2})^{2} + \text{const}
\]  

(1)

We consider here a particular chiral theory. The fermions are quarks. The scalars \((\sigma, \vec{\pi})\) are a colour singlet chiral multiplet of fields. The quarks belong to the fundamental representation of the colour group \(SU(N_{C})\). All quark generations couple to the chiral multiplet identically with an \(SU_{2}(L) \times SU_{2}(R)\) symmetry.

We find that when \(N_{C}\) and \(N_{G}\) are mutually constrained and so also the ratio of the gauge and Yukawa couplings, this theory can be completely asymptotically free (AF) for all couplings. It should however be noted that the various couplings need not maintain fixed ratios with respect to each other.

We give below the 1 loop beta functions of this theory:

\[
\frac{\partial g_{3}(t)^{2}}{\partial t} = -g_{3}^{4} / 8\pi^{2} P
\]  

(2)

where

\[ P = 11N_{C}/3 - 4N_{G}/3. \]

Likewise

\[
\frac{\partial g_{y}(t)^{2}}{\partial t} = \frac{1}{8\pi^{2}} g_{y}^{2} (4N_{G}N_{C}g_{y}^{2} - 3g_{3}^{2}(N_{C}^{2} - 1)/N_{C})
\]  

(4)

Note, that in our case \(N_{F} = 2N_{G}\). These equations can be brought to the form

\[
\frac{\partial \rho}{\partial g_{3}^{2}} = -\frac{\rho}{2g_{3}^{2} P} L
\]  

(5)

with

\[ L = 8N_{C}N_{G}\rho + 2P - 6(N_{C}^{2} - 1)/N_{C} \]

(6)

where \(\rho\) is as defined before. The zero of \(L\) occurs at \(\rho = \rho_{c}\) where \(\rho_{c}\) is given by

\[
\rho_{c} = \frac{4(2N_{G} - N_{C})/3 - 6/N_{C}}{8N_{C}N_{G}}
\]  

(7)
Note that the derivative of $\rho$ above is positive when $L$ becomes negative, i.e. for $\rho < \rho_c$. In this case $\rho$ increases as $g_3^2$ increases. Since $g_3$ decreases with increasing momentum scale (since it is AF), this means that $\rho$ will also decrease with increasing momentum scale. The implication then is that if $\rho < \rho_c$ then $g_y^2$ is also AF and decreases faster than $g_3^2$ in the UV. It is then clear that the the behaviour of $g_y^2$ is now controlled not by the IR free fixed point [11, 12] but by the UV free fixed point that controls the AF QCD coupling. This is possible only if $N_G > (N_C/2 + 9/(2N_C))$.

On the other hand AF of $g_3$ requires P to be positive, i.e. $N_C > 4N_G/11$. The constraint required to have both an AF $g_3^2$ as well as an AF $g_y^2$ is, for large $N_C$, $2N_G > N_C > 4N_G/11$.

Moving on to the scalar self coupling, the RG flow equation is

$$\frac{\partial \lambda(t)}{\partial t} = 2N_G(4\lambda N_C g_y^2 + 12\lambda^2/(2N_G) - 4N_C g_y^4)/8\pi^2$$

(8)

After a little algebra this can be brought to the form

$$\frac{\partial R}{\partial \rho} = \frac{4N_G}{L} \left( 2R - 4 + R \frac{3}{2\rho} \right)$$

(9)

with $R$ as defined before. As observed in [13, 20, 21], for the constraints given above it has three quasi-fixed points which are also the zero slope points in the above equation. These quasi-fixed points are given by: i) $\rho = 0, R = 0$; ii) $\rho = \rho_c, R(\rho_c)$; iii) $\rho = \infty, R = 2$.

The above equation is characterised by an invariant line (IL) [22, 23, 24] that goes through all the above points. It divides the parameter space into two regions (see [13, 21] for details):

i) **The AF phase : $\rho < \rho_c$.** In this region the IL is the only trajectory on which an AF theory in all couplings can be defined. Wherever we are on the IL, we move to the origin as we go to the UV. All other trajectories diverge. There is a reduction in coupling constants on the IL [23, 24]; $\lambda$ is fixed once $g_y$ is fixed. Around the origin, it is easy to see that $R$ is proportional to $\rho$ on the IL. It is then clear that as $R$ goes to zero in the UV, that $\lambda$ goes to 0 even faster than $g_y^2$ in the UV.

This completes the demonstration that the coupling, $\lambda$, is also AF on one trajectory (IL) and governed by a UV free fixed point. Since all couplings are AF, the entire spectrum, including the scalars, is elementary at high energy.
ii) The non AF phase: $\rho > \rho_c$. In this region the theory is not AF in the yukawa and scalar self couplings. The fixed point structure is rather complicated and the scalars are not elementary. This phase will be considered separately.

2.1 Remarks

1. Till now we have considered the RNG evolution equations in the absence of the electroweak gauge coupling. The effect of the weak gauge coupling is to increase the value of $\rho_c$, permitting us to find a completely AF theory for smaller, $N_F > 3$, whereas, in the absence of the gauge coupling we had an AF solution only for $N_F > 4$, where $N_F$ is the number of flavours [21].

2. Is the Standard Model Higgs Composite? It is a simple exercise to read off the compositeness condition from the value of $\rho_c$. If $\rho < \rho_c$ then the theory is AF and the higgs elementary. If on the other hand $\rho > \rho_c$, the higgs is composite. It follows, from substituting the known top quark yukawa coupling (the other yukawa couplings may be neglected as they are much smaller in comparison) and the QCD and weak gauge couplings at the weak scale, that, even on including the weak coupling contribution to $\rho_c$, we are still left with a composite higgs. A full analysis of this will be presented elsewhere.

3 Conclusions.

We have found interesting structure in field theories with scalars and gauge fields. Whereas the AF of the gauge coupling is gained for large $N_C$ and minimal $N_G$, the AF for the yukawa coupling can happen only when $N_G > (N_C/2 + 9/(2N_C))$, for only then can we have a positive $\rho_c$ (inclusion of the weak gauge coupling modifies this condition). Therefore, there is a window in which AF for all couplings is possible.

To sum up we have found such a window in the above theory in which all couplings are AF. This guarantees that all particles in the spectrum of the theory are partonic/ elementary at arbitrarily high energies and their wave function renormalization remains non-zero. As we go to low energies, however, the coloured part of the spectrum is expected to get confined into
hadrons. Since the scalar/pseudoscalars are colour singlets, they are not confined, but remain part of the low energy spectrum. Thus, we have demonstrated that in a renormalizable field theory the scalars/pseudoscalars can be elementary particles and visible at all scales.

Harada et al. [20] have also looked at the nontriviality of the gauge higgs yukawa system and the gauged NJL model. Their emphasis is, however, different and not on the issue of elementarity. Also, they have not considered the effect of the weak gauge coupling.

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References

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[1] V. Soni, Mod. Phys. Lett. A, Vol.11, 331 (1996).

[2] N. D. Hari Dass and V. Soni, Mod. Phys. Lett. A, Vol.14, 589 (1999).

[3] R. Haag, Phys. Rev. 112, 669(1958)

[4] K. Nishijima, Progr. Theoret. Phys. (Kyoto) 111, 995(1958).

[5] W. Zimmermann, Nuovo Cimento 10, 597(1958).

[6] B. Jouvet, Nuovo Cimento 5, 1(1957).

[7] J.C. Howard and B. Jouvet, Nuovo Cimento 18, 466(1960).

[8] M.J. Vaughn, R. Aaron and R.D. Amado, Phys. Rev. 124, 1258(1961).

[9] H.M. Fried and Y.S. Jin, Phys. Rev. Letts. 17,1152(1966).

[10] P.P. Divakaran, Phys. Rev. Vol 160, No 5, 1468(1967).
[11] A. Hasenfratz, P. Hasenfratz, K. Jansen, J. Kuti and Y. Shen, Nuc. Phy. B 365, 79 (1991).
[12] J. Zinn-Justin, Nuc. Phy. B 367, 105 (1991).
[13] T.D. Lee, Phys. Rev. 95, 1329 (1954).
[14] N. Levinson, Kgl. Danske. Videnskab. Selskab. Mat.-fys. Medd. 25, No 9 (1949).
[15] G. Rajasekaran, Phys. Rev. D 5, 610 (1972).
[16] W.A. Bardeen, C.T. Hill and M. Lindener, Phys. Rev. D 41, 1847 (1990).
[17] V.A. Miransky, M. Tanabashi and K. Yamawaki, Phys. Lett. B 221, 177 (1989).
[18] S. Coleman and D.G. Gross, Phy. Rev. Lett. 31, 851 (1973); D. Gross, in Les Houches Lectures 1975, North Holland 341 (1976).
[19] B. Schrempp and F. Schrempp, Phys. Lett. B 299, 3221 (1993).
[20] M. Harada, Y. Kikukawa, T. Kugo and H. Nakano, Prog. Theor. Phys. 92, Vol. 6 (1994) 1161.
[21] N. D. Hari Dass and V. Soni, hep-th/9911074 (1999) to appear in Phys. Rev.D.
[22] B. Pendelton and G. G. Ross, Phys. Lett. 98B, 291 (1981).
[23] W. Zimmermann, Comm. Math. Physics 97 (1985) 211
[24] J. Kubo, K. Sibold and W. Zimmermann, Nucl. Phys. B 259 (1985) 331