Forty years of acting electron-positron colliders

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Abstract

Around forty years passed from the beginning of operation of the first electron-positron colliding beam facility VEPP-2 in Institute of Nuclear Physics (INP), Novosibirsk. Here I described development of electron-positron colliding beam project in INP, as well as advance of similar projects of the first generation at LAL, Orsay and at LNF, Frascati.

1 Introduction

In the mid of 50’s of last century electron accelerators of higher and higher energy were built, while their usefulness was limited by the fact the energy available for physics study, e.g. for analysis of interaction at small distances or for creating of new particles, is that measured in the center-of-mass system (c.m.s.) of target and incident electron. In the relativistic limit the energy in c.m.s. is \( \varepsilon_c = \sqrt{2\varepsilon_l mc^2}/2 \), where \( \varepsilon_l \) is the electron energy in the laboratory system, \( m \) is the mass of target. In electron-electron interaction for \( \varepsilon_l = 6 \) GeV (the highest electron energy planned at that time) one has only \( \varepsilon_c = 39 \) MeV. Because of this reason a new idea appeared: to create storage rings where two electron beams (or two proton beams) traveling in opposite directions could collide. The colliding beams idea was proposed nearly simultaneously in 1956 [1], [2] and actively discussed at 1956 CERN Accelerator Symposium. After several laboratories started serious R&D and design activity. SLAC proposal to study limits of quantum electrodynamics was issued in May 1958 [3].

After completing my post-graduated courses at Moscow Lebedev Institute at January 1959 I joined newly organized Institute of Nuclear Physics (INP) of Siberian Branch of Academy of Science of USSR[1]. Professor G.I.Budker was appointed di-
rector of the institute. This was 40-years ambitious physicist working on fairly non-standard problems of plasma and accelerator physics. The institute was formed on the basis of small Budker’s laboratory ”New acceleration methods” at Nuclear Energy Institute (now Kurchatov Institute) in Moscow. At the end of 1958 scientific staff of Institute was consisted of around 30 persons most of them were graduated students. The very new idea of electron-electron colliding beams, for realization of which many original development were needed, attracted Budkers’s attention and creation of electron-electron colliding beam facility became one of main purposes of newly created institute.

In 1959 the colliding beams team worked on technical design of facility including some specific issues such as fast injection kicker magnets, ultra-high vacuum system, detector of scattered electrons, etc. At the same time the program of physics research was under development. Budker seek for support of colliding beams project and invited known scientists to discuss the research program. In October 1959 I.Ya.Pomeranchuk visited institute. In Budker’s office it was quite long discussion of electron-electron colliding beams facility where study of quantum electrodynamics at small distances was mixed with technical details of installation, which was at very beginning of construction. Pomeranchuk was not in raptures concerning the discussion and no support to the project was expressed. Besides other participants of the meeting did not express any enthusiasm. After Pomeranchuk left institute, Budker came to my office, he was complaining that institute research program is not enough impressive and one has to think how improve it. I replied that the program becomes immeasurably more rich, if one creates electron-positron colliding beams. ”You are mad!” said Budker and left. But several minutes later he arrived back: ”Tell me once more!” In very intense discussion which followed we expressed many times the pros and cons realization of electron-positron colliding beams. In the end, late in the evening, Budker demanded that I had to lay aside all my business and concentrate on realization of new proposal. It was October 28, 1959.

The next day active work began. We considered the electron-positron colliding beams installation step by step from morning to late evening. Within a week the very rough draft of facility which later became VEPP-2 was prepared. The maximal energy per beam (700 MeV) was selected to produce K-mesons. Special attention was devoted to positron production by electrons in tungsten converter. The first design drawing of electron-positron facility were made in December 1959. It should be noted that at that time all electron-electron colliding beam projects were on the very preliminary stage.

The research program which was formulated at that time was [1]:

1. Study of elastic electron-positron scattering at large angle to test QED at small distances (similar to a goal of electron-electron colliding beams).

2. Study of annihilation electron-positron pair into two photons. This is an additional test of QED.
3. Study of conversion of $e^+e^-$ pair into pair of $\mu^+$ and $\mu^-$ mesons. This is a test if the $\mu$ meson is pointlike and if so this is an additional channel to test QED.

4. Study of conversion of $e^+e^-$ pair into pair of $\pi^+$ and $\pi^-$ mesons to study the electromagnetic form-factor of pion at time-like momentum transfer.

5. Study of conversion of $e^+e^-$ pair into pair of $K^+$ and $K^-$ mesons to study the electromagnetic form-factor of kaon at time-like momentum transfer.

It is reasonable to describe the scene in elementary physics research at that time. A few electron synchrotrons with energy slightly higher than 1 GeV were brought into operation in middle of 50’s, so the experience of work with such accelerators was quite limited. The table of elementary particles contained leptons: electron, positron, neutrino, antineutrino, $\mu^\pm$ meson; and strongly-interacting particles: proton, neutron, a few types of hyperons, as well as $\pi^\pm$ and $\pi^0$ mesons and different types of K mesons.

It is evident, that electron-positron colliding beams could give very new opportunities not only for test of QED at small distances but also for study of electromagnetic properties of mesons participating in strong interaction. At that time there was no information at all about these properties. Even the cross section of such simple process as transformation of electron-positron pair onto pair of $\mu^+$ and $\mu^-$ mesons was calculated by Berestetsky and Pomeranchuk only in 1955 [5].

How to realize electron-positron facility was the question. First of all it was necessary to develop positron production system. Positrons (antiparticles) were observed in cosmic rays and in reaction at accelerators. However considerable amount of positrons was never produced. In the 30’s-40’s the theory of electron-photon showers was developed. This was a basement. But for electron-positron facility one has to produce beam of positrons. The elaborated scheme of positron production is used now everywhere, naturally with many perfection: the electron beam with energy a few hundreds MeV is directed to heavy metal (e.g. tungsten) target with thickness 1-2 radiation length. In the target electron radiate a photon in collision with a nucleus, then this photon interacts with another nucleus and creates electron-positron pair. Created positrons should be collected and accelerated and then injected to storage ring. We started calculation of conversion of electrons into positrons nearly from the very beginning of project. The good piece of this work was done by Synakh [6] (at that time my post-graduated student). For example, the calculated conversion coefficient $\mu$ of electrons with energy $\varepsilon_0 = 500$ MeV into positrons with energy $\varepsilon = 250$ MeV into the energy interval $\Delta\varepsilon/\varepsilon \sim 5\%$ is $\mu \simeq 1/400$ for the converter with thickness around 1 radiation length. The created positrons are moving mostly ahead in the direction of electron momentum. In the mentioned example the angle of positron cone is $\sim 4^\circ$. So, other things being equal, for production of positron beam of some intensity one needs the electron beam
which is thousand more intense. If one accumulates positrons by small bunches, then the storage time will be thousand times longer.

A few graduated student were recruited to the team which started the development of electron-positron facility. They began more detail study of the project including electron injector, ejection from it using the fast kicker magnets, channels and conversion of electrons into positrons in one of channels, injection into sole storage ring where electrons and positrons are moving in opposite directions, ultra-high vacuum system in storage ring and stability of orbits in it.

At that time (end of 1959) the electron-electron colliding beam projects were only at very preliminary stage of development and some people doubted that the colliding beam technology could be used in high-energy physics research. In this situation the proposal to build the electron-positron facility with essentially higher requirements for intensity and quality of beams was coldly received by many famous members of Soviet Academy of Science. Only support of I.V.Kurchatov at that time very influential director of Nuclear Energy Institute permitted to start development of the project. Another factor which lowered level of opposition to the project was transfer of the all team from Moscow to the wild East: to Novosibirsk, where building of edifices of Nuclear Physics Institute began in 1959.

In 1960 when the staff was still very small all members of the team were working on various topics of both electron-electron and electron-positron installations. Alexander Skrinsky was appointed as a head of laboratory. I have found not long ago the internal report of INP-1960 with a title "Motion of particles in an accelerator with racetrack" by V.N.Baier, V.S.Synakh and I.B.Khriplovich devoted to study of particle dynamics in VEPP-2.

In 1961 the main part of staff (including Budker, Skrinsky and myself) was moved from Moscow to the new Academic town (Academgorodok) in 30 km south from Novosibirsk, where the first building of INP was constructed. Evidently, transfer of the team with equipment was an obstacle in the way of project development. However, it was quite good financial support of the new Institute in Novosibirsk and this permitted to create quite effective Institute’s workshop for manufacturing of parts of the facility, and to order most big parts in Novosibirsk industry from the one side, and to recruit many graduated students from Novosibirsk universities from the other side.

In the beginning of 1961 our library received Il Novo Cimento with announcement about Frascati storage ring [7]. This showed that we were not alone in the field. But from point of view of our experience at that time it was evident (we are worked hard at injection system and creation of positron beam) that at best the very limited amount of electrons and (or) positrons could be stored in storage ring without direct particle injection.

In 1962 some parts of VEPP-2 were ready and tested. The storage ring VEP-1 manufactured at big Novosibirsk factory was first moved in 1961 to the Kurchatov

\[^{2}\text{Contacts with foreign laboratories were so limited at that time, that the scientific information one drew only from journals.}\]
Institute in Moscow, then in summer of 1962 it was disassembled together with synchrotron B-2S which was its injector and moved to INP in Novosibirsk where both was assembled fast and immediately the test operations began.

In the year 1963 it was permitted the complete legalization of activity of INP.³ At the International Conference on high-energy accelerators in Dubna in August 1963 the collider activity of INP was presented for the first time [8]. The photo of VEPP-2 assembly process was contained in the report along with other information. The main contributors are the authors of the corresponding parts.

Before the Conference we got information that the Frascati storage ring AdA moved from Frascati to Orsay, where there was the high-energy linac which was used as an injector. The new interesting effect was observed: the loss of particles in storage ring due to electron-electron scattering inside the bunch [9]. Under some conditions the lifetime of a beam in a storage ring is determined just by this event which is now called the Touschek effect.

We have learned at the Conference that two more teams started the work in the same direction. These were the projects of Orsay electron-positron storage ring ACO with energy up to 450 MeV at Laboratoire de l’Accélérateur Linéaire (LAL) in Orsay, France [10] and electron-positron storage ring ADONE with energy up to 1.5 GeV at Laboratori Nazionali di Frascati (LNF) in Frascati, Italy [12]. Somewhat later we received description of ACO in detail [11].

This indicated that creation of electron-positron colliders became very respectable and perspective direction of development of high-energy accelerators. Some kind of race emerged for the physics meaningful results at these colliders.

The study of electron-electron scattering at VEP-1 storage ring began in 1964. The first small angle scattering event was registered on May 19.

A very important characteristics of colliding beam facility is the luminosity $L$. The number of events per unit time (usually 1 sec) $N$ of some process with the total cross section $\sigma$ is $N = L\sigma$. The luminosity is proportional to product of current in the beams and inverse proportional to the transverse section of beam $S$. To obtain acceptable luminosity one has to work with enough high currents and small size beams. A very high vacuum and damping of beam instabilities are the necessary conditions to have reasonable small beam size.

A new type of sophisticated detectors had to be created. Ben Sidorov was in charge of this direction in INP.

The data taking at VEPP-2 installation began in 1966 [13]. The results will be discussed below.

³Because of pathological secrecy adopted at that time in USSR, all activity in Kurchatov Institute was considered as "for restricted use only" and the special permission for publication in open journals or proceedings was necessary for each article.
2 Physics with electron-positron colliding beams

2.1 Radiative corrections

On the first stage of the electron-positron project development one of main goals was the check of the applicability of quantum electrodynamics at small distances. The qualitative estimate of measured distance\(^4\) is \(\lambda \sim 1/q = (\hbar/q)\), where \(q\) is the momentum transfer. For \(q \sim 1\) GeV the measured distance is \(\lambda \sim 0.2\) fm (1 fm=10\(^{-13}\) cm is a typical hadron scale).

The cross sections of electron-electron and electron-positron scattering in Born approximation (order \(\alpha^2\)) was calculated by Möller and Bhabha in 30’s. The fantastic development of Quantum Electrodynamics (QED) in 40’s permitted consideration of higher order corrections (the series with respect to powers \(\alpha = e^2\)) which are called radiative corrections (RC). In the late 50’s these corrections to the mentioned cross sections were the topic of QED textbook (e.g. [14, 15]). At high energy \(\varepsilon \gg m\) the actual parameter of decomposition is \((\alpha/\pi)\ln(\varepsilon/m)\). Calculation of RC includes obligatory (because of infrared divergence) contribution from radiation of real photons and because of this depends on the particular experimental set-up. In the specific conditions of binary (2 \(\rightarrow 2\)) reactions on colliding beams the photon emission from one of initial particles causes non-collinearity of the final particle momenta \(\Delta\theta\). Since for elastic cross section the events with minimal \(\Delta\theta\) are selected, this imposes substantial limitation on energy \(\Delta\varepsilon\) radiated from the initial particles. Accuracy of measurement of final particles energy in the 1st generation detector was quite poor. This means that hard photon emission from final particles are allowed and this complicates calculation. The cross section with the radiative corrections \(\delta_R\) taken into account are usually written in the form: 
\[
\frac{d\sigma}{d\omega} = d\sigma_0(1 - \delta_R),
\]
where \(d\sigma_0\) is the Möller (or Bhabha) cross section. The main term of RC of the lowest order \(\propto \alpha^3\) (so-called “double-logarithm term” containing the product of two large logarithm: logarithm of energy and logarithm of ratio \(\Delta\varepsilon/\varepsilon\), which arises from sum of contributions of soft virtual and real photons) is 
\[
\delta_R = (8\alpha/\pi)\ln(\varepsilon/m)\ln\varepsilon/\Delta\varepsilon,
\]
where \(\Delta\varepsilon\) is the total energy of emitted quanta. The different aspects of radiative effects in electron-electron (positron) collisions were analyzed with Sam Kheifets, Victor Fadin and Valery Khoze. The complete expressions for \(\delta_R\) in \(e - e\) scattering are given in [16, 17] and in \(e^+ - e^-\) scattering in [18].

For typical experimental conditions at \(\varepsilon \sim 1\) GeV one has the radiative correction \(\delta_R \sim 10\%\) and evidently it should be taken into account in comparison of theory and data.

At high energies and for \(\Delta\varepsilon/\varepsilon \ll 1\) the soft-photon corrections dominate, e.g. for \(\varepsilon = 7\) GeV and \(\Delta\varepsilon/\varepsilon = 10^{-2}\) one has \(\delta_R = 0.75\), and one can’t be restricted to the lowest order of perturbation theory. So the general analysis of RC in the all orders of perturbation theory is of significant interest. In the 50’s it was fashionable

\(^4\)Below the system of units where \(\hbar = c = 1\) is used.
to study structure of QED as a whole. The method of calculation of the cross sections in high energy QED proposed by Abrikosov [19] was used. Within framework of this method, only those contributions are retained which contain the maximum power of the large logarithms. For test of QED at small distances only processes with large momentum transfer are of interest. It’s remarkable that in this case in any order of the perturbation theory only contributions of diagrams with one photon exchange between charged particle lines survives, while all other contributions cancel each other, and in double-logarithm approximation the scattering cross section after inclusion of soft photon emission acquires the form

$$d\sigma(\vartheta) = d\sigma_0(\vartheta)e^{-\delta R}$$

It was shown that in double-logarithm approximation the cross sections of all processes of scattering and pair creation for large momentum transfers have soft-photon nature (see [20, 21, 22]). As it was mentioned, only diagrams with one photon exchange between charged particles contribute. An interesting application is the behavior of \(e^- + e^+ \rightarrow \mu^- + \mu^+\) cross section near threshold. In the case when there is no limitation on photon emission \(\delta R \rightarrow \delta \mu = (4 \alpha/\pi) \ln(\varepsilon/(\varepsilon - \mu)) \ln(\varepsilon/m)\), where \(\mu\) is the muon mass. For nonrelativistic muons one has [21]

$$\sigma(\vartheta) = \sigma_0(\vartheta) \left(\frac{q}{\sqrt{2\varepsilon \mu}}\right) \frac{\alpha^2 \ln \frac{\varepsilon}{m}}{16 \varepsilon^2} |\psi(0)|^2, \quad \sigma_0(\vartheta) = \frac{\alpha^2 q}{16 \varepsilon^3} \left[1 + \frac{\mu^2}{\varepsilon^2} + \frac{q^2}{\varepsilon^2 \cos^2 \vartheta}\right], \quad (1)$$

where \(q\) is the momentum of final muon and \(\vartheta\) is the angle between momenta of initial electron and final \(\mu^-\), factor \(|\psi(0)|^2\) takes into account the Coulomb interaction between final particles found by Sakharov [23]: \(|\psi(0)|^2 = (2\pi \alpha/v)/(1 - e^{-2\pi \alpha/v})\), here \(v\) is the muon velocity. In the region where the Coulomb interaction is insignificant \((2\pi \alpha/v \gg 1)\) one obtains \(\sigma \propto q^{15}\) in place of \(\sigma \propto q\); and for \(\tau\)-lepton \(\sigma \propto q^{1.15}\).

The test of the applicability of QED at small distances by electron-electron scattering was performed at VEP-1 storage ring in Novosibirsk [26] and at Princeton-Stanford storage ring [25, 27]. The data [25, 27] have been compared with the Møller formula modified by a form factor \(f(q^2)\) (in vertex \(\gamma \mu \rightarrow \gamma \mu f(q^2)\)) and \(1/q^2 \rightarrow f(q^2)/q^2\) and usually a simple parametrization is used \(f(q^2) = 1/(1 \pm q^2/\Lambda^2)\) and with radiative correction \(\delta R\), calculated by Tsai [28, 29], taken into account. The limit \(\Lambda = \infty\) corresponds to interaction of point particles. The results of experiment [27] are \(\Lambda^- > 4.4\) GeV and \(\Lambda^+ > 2.7\) GeV (95% confidence). The limit \(\Lambda^- > 4.4\) GeV means that QED was checked for distance \(l \geq 0.05\) fm. The distance \(l\) is much shorter than characteristic hadronic dimension.

Similar limitations where obtained for different processes at electron-positron colliders of the first generation (95% confidence):

1. Electron-positron elastic scattering measured in Orsay (ACO collider) with beam energy \(\varepsilon = 510\) MeV [30] with RC \(\delta R \sim 7%\) results \(\Lambda^- > 3.8\) GeV and \(\Lambda^+ > 2.8\) GeV and measured in Frascati (Adone collider) with beam energy \(\varepsilon = 700 - 1200\) MeV [31] gives \(\Lambda^+ > 6\) GeV.

For more general modification see [24]
2. Two-photon annihilation of electron-positron pair measured on VEPP-2 collider in Novosibirsk with beam energy $\varepsilon = 500$ MeV [32] results $\Lambda_\pm > 1.3$ GeV and measured in Frascati (Adone collider) with beam energy $\varepsilon = 700 - 1200$ MeV [31] gives $\Lambda_- > 2.0$ GeV and $\Lambda_+ > 2.6$ GeV. In this reaction both the modifications of vertex and of electron propagator in a consistent (gauge invariant) way were introduced.

3. Study of conversion of $e^+e^-$ pair into pair of $\mu^+$ and $\mu^-$ mesons on VEPP-2 collider in Novosibirsk with beam energy $\varepsilon = 500$ MeV [33] gives $\Lambda_\pm > 3.1$ GeV (95% confidence) and measured in Frascati (Adone collider) with beam energy $\varepsilon = 700 - 1200$ MeV [31] gives $\Lambda_- > 5$ GeV.

The tests of QED at small distances were continued at next generations of electron-positron colliders, which were built later in Novosibirsk, Stanford, Cornell, Orsay, Frascati, Tsukuba, Geneva. The described above limitations were substantially improved (by two orders of magnitude) due to higher energy, larger circulating current and better detectors. For example, the two(three)-photon annihilation of electron-positron pair measured on LEP collider in CERN (Geneva) with beam energy $\varepsilon = 45 - 101$ GeV [34] gives $\Lambda_- > 258$ GeV and $\Lambda_+ > 415$ GeV. The last limit means that QED is checked for distance $l \geq 5 \cdot 10^{-17}$ cm.

2.2 Inelastic processes

At low energy $\varepsilon \sim m$ the electromagnetic processes are sorted usually over powers of fine-structure constant $\alpha = e^2 = 1/137$ in frame of perturbation theory. The cross sections of simplest two-particle processes: electron(positron)-electron scattering, photon-electron scattering, annihilation of electron-positron pair into two photon or pair of charged particles are of order $\alpha^2/m^2 = r_0^2 \sim 10^{-25}$ cm$^2$. In many-particle processes each additional particle adds factor $\alpha$ to the cross sections. Such processes were considered only in the form of RC, as it was discussed above. At high energy $\varepsilon \gg m$ the situation changes; the magnitude of the cross sections is determined mainly by the dependence on energy. Understanding of importance of such classification arose along with development colliding beam program. The processes, diagram of which contains two blocks (each of which is attached to charged particle line) connected with photon(photons) line, have nondecreasing as a function of energy total cross section. Besides the power constancy in some cases there are the logarithmic growth with energy. The important example is the process of soft $n$-photon radiation in electron(positron)-electron scattering which was studied by Victor Galitsky and I [36].

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6To elastic $e^+e^-$ scattering and to the process $e^+e^- \rightarrow \mu^+\mu^-$ there is an additional (and quite significant for used energy) contribution from $Z$ boson. So these processes can’t be used for pure QED test.

7 The only process which is fallen out this scheme is the elastic electron(positron)-electron scattering where the total cross sections diverges at any energy.
of classical \((\omega \ll \varepsilon)\) photons occurs in an independent way so that the cross section of process with the emission of \(n\) photons may be represented as, see e.g. \[35\]:

\[
d\sigma_n = d\sigma_0 \prod_{i=1}^{n} dW(k_i)/n!, \]

where \(d\sigma_0\) is the cross section of elastic process, \(dW(k_i) = |j|^2 d^3k_i/2\omega_i\) here \(j_\mu\) is the "classical" current, for electron scattering off Coulomb center \(j_\mu = i\sqrt{2}\varepsilon(p_\mu/(kp) - p'_\mu/(kp'))/(2\pi)\), where \(p_\mu(p'_\mu)\) is the initial (final) electron momentum, generally each line of charged particle in the process diagram contributes to the current the combination \(\pm p_\mu(p)/kp\). For bremsstrahlung \(d\sigma_1 = d\sigma_0 dW(k)\), where \(d\sigma_0\) is the Rutherford formula. Integrating \(dW(k)\) over photon emission angles \(\Omega\) one obtains

\[
dI(\omega, x) = \int dW(k) = \frac{2\alpha d\omega}{\pi} \Phi(x), \quad \Phi(x) = \frac{2x^2 + 1}{x\sqrt{1 + x^2}} \ln(x + \sqrt{1 + x^2}) - 1, \quad (2)
\]

where \(4m^2 x^2 = -(p - p')^2 = 4p^2 \sin^2(\vartheta/2)\), \(\vartheta\) is the electron scattering angle. In the limiting cases one has: \(x \ll 1\), \(\Phi(x) = 4x^2/3\) and \(x \gg 1\), \(\Phi(x) = \ln 4x^2 - 1\). The universal function \(\Phi(x)\) defines the probability dependence on the momentum transfer in soft photon radiation.

To find the integral cross section one has to integrate \(d\sigma_0 dI(\omega, x)\) over the momentum transfer \(x\). Taking into account that the Rutherford cross section \(d\sigma_0 = (\pi Z^2\alpha^3/m^2)dx^2/x^4\) it is clear that the main contribution gives region \(x \ll 1\). The minimal value of \(x\) is attained when all the momenta are collinear: \(4m^2 x^2_{\text{min}} = \omega^2 m^4/4\varepsilon^4\). Within logarithmic accuracy one can put \(x_{\text{max}}^2 = 1\). Substituting the functions in \(d\sigma_0 dI(\omega, x)\) for \(x \ll 1\) and performing integration one find the spectrum of bremsstrahlung of the photon with energy \(\omega\):

\[
\sigma_1 = (16/3)(Z^2\alpha^3/m^2)(d\omega/\omega) \ln(4\varepsilon^2/m^2\omega). \quad \text{The region } x \gg 1 \text{ does not contribute because of fast decreasing of the Rutherford cross section with } x \text{ increase.}
\]

The result of similar analysis for bremsstrahlung at electron-electron(positron) scattering \((Z = 1)\) differs from this expression only by logarithm argument: \(4\varepsilon^2/m^2\omega \rightarrow 8\varepsilon^3/m^3\omega\) and radiation takes place in the direction of motion of both colliding particles.

In the case of many photons radiation the integration over photon emission angles can be performed independently, so that \(d\sigma_n = d\sigma_0 \prod_{i=1}^{n} dI(\omega_i, x)/n!\). Of course, for electron-electron(positron) scattering in c.m.s. one has to use the corresponding classical current. However the only region of small momentum transfer contributes and final results is expressed in terms of the function \(\Phi(x)\). Since at \(x \ll 1\), \(\Phi(x) \propto x^2\) starting from \(n \geq 2\) there is no divergence at \(x = 0\) and with high accuracy one can put \(x_{\text{min}} = 0\). So, 1) the cross section \(d\sigma_n\) does not contain large logarithm; 2) the value \(d\sigma_n\) can be calculated within power accuracy (discarded terms \(\sim m^2/\varepsilon^2\)); 3) the main contribution gives region \(x \sim 1\); 4) at \(x \gg 1\) the cross section looks like \(\ln^n x dx^2/x^4\) and because convergence of the integral one can put \(x_{\text{max}} = \infty\) within accuracy \(\sim m^2/\varepsilon^2\). Thus, the cross section of \(n\) photon radiation at electron-electron(positron) collision is

\[
d\sigma_n^e = 2\pi \frac{\alpha^2}{m^2} \left(\frac{4\alpha}{\pi}\right)^n \frac{\nu(n)}{n!} \prod_{i=1}^{n} \frac{d\omega_i}{\omega_i}, \quad \nu(n) = \int_0^{\infty} \Phi^n(x) \frac{dx}{x^2}, \quad (3)
\]
here \( \nu(2) = 5/4 + 7\zeta(3)/8, \ \nu(3) = 3[8\zeta(3) - 1]/5, \ \zeta(3) = 1.202.. \) is Riemann’s zeta function. The simple combinatorial analysis shows that when \( m \) photons are emitted in the direction of one particle and \( n - m \) photons are emitted in the direction of another particle the corresponding cross section is \( \sigma_\nu^m(n, n - m) = C_n^m d\sigma_\nu^m/2^n \).

For the double bremsstrahlung in the case when photons are emitted in opposite directions \( d\sigma_\nu^2(1, 1) = d\sigma_\nu^2/2 \) and in the case when both photons are emitted in the one direction \( d\sigma_\nu^2(2, 0) = d\sigma_\nu^2(0, 2) = d\sigma_\nu^2/4 \).

So considered cross section grows logarithmically with energy increase at \( n = 1 \), while at \( n \geq 2 \) it doesn’t dependent on energy.

2.2.1 Single bremsstrahlung in electron-(electron)positron collision

This simplest inelastic process is represented by 8 Feynman diagrams and the differential cross section is very cumbersome. In high-energy region which is of main interest, one can decompose cross section over powers of \( m/\varepsilon \). Moreover the calculation simplified essentially if one integrates contributions of the radiation block of diagram in tensor form taking into account invariance properties of QED [37]. In the center of mass system (c.m.s.) of initial particles the emitted photons are concentrated mainly in the narrow cones along momenta of each of initial particles. The integral spectral cross section in the each direction [38, 39] with power accuracy (to within terms \( \sim m^2/\varepsilon^2 \)) is

\[
\begin{align*}
\frac{d\sigma}{d\varepsilon}(1) = \frac{d\sigma}{d\varepsilon}(2) &= \frac{4\alpha^3}{m^2} \frac{d\omega}{\omega} \frac{\varepsilon'}{\varepsilon} \left( \frac{\varepsilon'}{\varepsilon} + \frac{\varepsilon}{\varepsilon'} - \frac{2}{3} \right) \ln \left( \frac{4\varepsilon^2\varepsilon'}{m^2\omega} \right) - \frac{1}{2},
\end{align*}
\]

where \( \varepsilon' = \varepsilon - \omega \). This cross section is the largest which can be observed in colliding beam experiment and grows logarithmically with energy, e.g. for \( \varepsilon = 1 \) GeV and in the interval \( 0.1 \leq \omega/\varepsilon \leq 1 \) it attains \( \sigma \sim 10^{-25}\text{cm}^2 \). The main contribution to the cross section gives the interval of low momentum transfer \( q = \sqrt{-q'^2} : (m^2\omega/(4\varepsilon^2\varepsilon') \leq q \leq m) \) so that deviation angle of radiating particle is less than \( m/\varepsilon \). When the scattering angle of an electron(positron) \( \vartheta \gg m/\varepsilon \) the radiation (within a logarithmic accuracy) is directed along momenta of charged particles and photon emission cross section from initial(i) and final(f) particle is (see e.g. [40])

\[
\begin{align*}
\frac{d\sigma}{d\vartheta}(1) &= \frac{\alpha}{\pi} \frac{d\omega}{\omega} \left( 1 + \frac{\vartheta^2}{\varepsilon'^2} \right) \ln \left( \frac{\vartheta}{m} \right) d\sigma_{\epsilon^+\epsilon^-}^e, \ d\sigma_f(1) &= \alpha \frac{d\omega}{\omega} \left( 1 + \frac{\vartheta^2}{\varepsilon'^2} \right) \ln \left( \frac{\vartheta}{m} \right) d\sigma_{\epsilon^+\epsilon^-}^e, \\
\end{align*}
\]

where \( d\sigma_{\epsilon^+\epsilon^-}^e \) is the electron-positron scattering cross section in the c.m.s. of final particles and \( d\sigma_{\epsilon^+\epsilon^-}^e \) is the electron-positron scattering cross section in the c.m.s. of initial particles.

2.2.2 Double bremsstrahlung in electron-(electron)positron collision

Radiation of two photons at \( e^- - e^- (e^+) \) collision is of evident interest for colliding beam experiments. The most interesting is the case when photons are emitted in...
opposite directions along the momenta of colliding particles, because the coincidence of two photon registration permits to separate the effect from background. This process was used as a monitor of beam collisions and for cross sections normalization. The use of method of invariant integration of tensors representing the radiation blocks mentioned in previous subsection, simplifies essentially the calculation of integral spectrum (the process is represented by 40 diagrams). The qualitative properties of dependence of the process cross section on momentum transfer given above for soft photons emission are valid for any energy of photons, but the radiation blocks should be found for hard photons. The spectrum of double bremsstrahlung in c.m.s. has the form

\[
d\sigma_{\omega_1\omega_2} = \frac{8\alpha^4}{\pi m^2} \left\{ (1 + \frac{\omega_1}{\varepsilon}) (1 + \frac{\omega_2}{\varepsilon}) \eta_1 + \left[ (1 + \frac{\omega_1}{\varepsilon}) \frac{\omega_2^2}{\varepsilon^2} + (1 + \frac{\omega_2}{\varepsilon}) \frac{\omega_1^2}{\varepsilon^2} \right] \eta_2 + \frac{\omega_1^2 \omega_2^2}{\varepsilon^2} \right\} d\omega_1 \frac{d\omega_2}{\omega_2}, \quad \eta_1 = \nu(2) = \frac{5}{4} + \frac{7\zeta(3)}{8}, \quad \eta_2 = \frac{1}{2} + \frac{7\zeta(3)}{8}, \quad \eta_3 = \frac{7\zeta(3)}{8}.
\]

Within a good numerical accuracy (better than 1%) the expression in curly brackets can be represented in the multiplicative form: \{\ldots\} = R(\omega_1)R(\omega_2), where \(R(\omega) = \sqrt{\eta_1(1 - \omega/\varepsilon)} + \sqrt{\eta_3 \omega^2/\varepsilon^2}\). This form is very convenient for comparison with experimental data. For soft photon the spectrum coincides with Eq.(3).

The first observation of double bremsstrahlung was done in Novosibirsk [42]. A special study of the double bremsstrahlung process as monitoring device was performed at ACO in Orsay [43]. Achieved accuracy (~3%) was record for high energy QED. It is striking that it was in measurement of 4-th order process. Bearing in mind that the double bremsstrahlung can be observed in quite clean conditions and has enough large cross section which is known within very good accuracy, this process was used as standard method for luminosity measurement in Novosibirsk, Orsay and Frascati.

### 2.2.3 Pair creation in electron-(electron)positron collision

Another 4-th order process, which cross section doesn’t decrease with energy, is the electroproduction process \(e^+e^- \rightarrow e^+e^- + N\). There are two types of diagrams presenting this process: 1) one-photon, where the final particles are created by a photon radiated from one of lines of the initial electron or positron; 2) two-photon, where the final particles are created at collision of two photons, radiated from each of initial particles (photon-photon colliding beams). The last mechanism is especially important since the final states, including hadrons, which are even at charge conjugation \((C = 1)\), can be produced with cross section which doesn’t decrease with energy, while in the one-photon channel \(C = -1\) and cross section of annihilation into hadron is decreasing as \(1/\varepsilon^2\).

The properties as well as values of contributions of one-photon and two-photons diagrams differ significantly. The main contributions is given by the two-photon
diagrams. For creation of $e^+e^-$ pair in electron-positron collision this contribution to the total cross section is (with an accuracy up to terms $\sim m^2/\varepsilon^2$)

$$
\sigma_2 = \frac{\alpha^4}{27\pi m^2} \left[ 28L^3 - 178L^2 + (490 - 82\pi^2)L + 1203\zeta(3) + \pi^2 \left( 78\ln 2 + \frac{458}{3} \right) \right] - 676 = \frac{\alpha^4}{\pi m^2} \left[ 1.04L^3 - 6.59L^2 - 11.8L + 104 \right]
$$

(7)

where $L = \ln 4\varepsilon^2/m^2$. The main term ($\propto L^3$) was found in 1934 by Landau and Lifshitz [44], the rest of logarithmic terms were calculated in [45], the constant was calculated in [46], see also review [48].

Let us discuss this result.

1. In the limit $\varepsilon \gg m$ the cross section increases as a cub of logarithm of energy.
2. Two of these logarithms originate from integration over the transverse momenta of photons emitted from the initial particles, the third one from integration over the longitudinal momenta of the created pair.
3. At moderate energy the main term ($\propto L^3$) is compensated essentially by the rest logarithmic terms and constant, e.g. for $\varepsilon = 5$ GeV the compensation diminishes the cross section $\sigma_2$ Eq.(7) which is about 2/3 of the main term.

The contribution to the total cross section of each set of one-photon diagrams (connected with one line of initial particles) is [45]

$$
\sigma_1 = \frac{\alpha^4}{162\pi m^2} (231\pi^2 - 2198)L = \frac{\alpha^4}{\pi m^2} 0.51L.
$$

(8)

It is significantly smaller than $\sigma_2$.

If detectors measure outgoing particles at large polar angles only, another kinematic region than in the main term of the cross section $\sigma_2$ Eq.(7) contributes in the corresponding cross section. In the case when the both polar angle of created particles are $\vartheta_+ = \vartheta_- = \pi/2$ the differential over the angles of created pair cross section of pair electroproduction has the form [47]

$$
\frac{d\sigma}{dc_+dc_-d\varphi} = \frac{\alpha^4 \ln (2\varepsilon^2(1 - \cos \varphi)/m^2)}{2\pi\varepsilon^2 \sqrt{m^2/\varepsilon^2 + 2(1 - \cos \varphi)}},
$$

(9)

where $c_\pm = \cos \vartheta_\pm$, $\varphi = \varphi_+ - \varphi_- + \pi$ is the non-coplanarity angle, $\varepsilon_0$ is the lowest energy of particles of created pair (registration threshold). This cross section has very sharp peak at $\varphi = 0$. This important peculiarity was used for observation. Large angle electroproduction of electron-positron pair was first observed at VEPP-2 [49, 50]. Data support the distribution Eq.(9).

2.3 Hadron production

2.3.1 Vector mesons

One of the main goals of electron-positron colliders of the first generation was production of pions and kaons to study electromagnetic form factors of pions and kaons
at the positive (time-like) momentum transfers. In 1960, when the INP project was in progress, Sakurai [51] proposed the non-Abelian gauge theory of strong interactions constructed upon the QED pattern. The gauge invariance in QED means that the invariance under local phase transformation \( \psi \rightarrow \exp(i\Lambda(x))\psi \) forces one to introduce a new field, which is to be identified with the electromagnetic field \( A_\mu \) coupled universally (with the constant \( e \)) to the conserved current constructed out of electrically charged fields. To maintain the invariance under the mentioned transformation it’s necessary also to perform the transformation \( A_\mu \rightarrow A_\mu + \partial \Lambda / \partial x^\mu \).

In [51] the Yang-Mills theory was used: if one requires that the non-Abelian gauge transformation associated with the isospin \( I \) conservation is local in character then one is forced to introduce the vector field with the isospin \( I = 1 \) (\( \rho^\pm, \rho^0 \) mesons in modern notation) coupled universally (with the constant \( f_\rho \)) to the isospin current constructed out of all fields having nonvanishing isospins. In [51] this result was generalized by adding the baryon and hypercharge conservation. This means appearance of two vector fields (\( \omega, \phi \) neutral mesons in modern notation) coupled universally (with the constants \( f_B, f_Y \)) to the baryon \( B \) and hypercharge \( Y \) currents constructed out of all fields having nonvanishing baryon number (hypercharge). This development indicated that one can hope for a first class physics at electron-positron colliders.

Side by side with outlined above theory, the indications that strong-interacting vector mesons play important role followed from analysis of nucleon electromagnetic form factors and some inelastic \( \pi^-p \) reactions. Connection between these approaches was established by Gell-Mann and Zachariasen [52], where it was stressed that in isovector electromagnetic form factors of hadrons the diagrams dominate, where photon interacts with hadrons via \( \rho^0 \) meson. The model which takes into account only such diagrams

\[
\begin{array}{c}
\gamma \\
q \\
g_{\gamma\rho} \\
\rho^0 \\
f_{\rho\pi\pi} \\
k_1 \\
k_2 \\
\end{array}
\]

was called the vector dominance model (VDM). In this model the pion electromagnetic form factor is \( eF_\pi(t) = g_{\gamma\rho}f_{\rho\pi\pi}/(m^2 - t) \), where \( g_{\gamma\rho} \) is the amplitude of \( \gamma \rightarrow \rho^0 \) transition, \( t = q^2 \), \( q \) is the photon momentum, \( m \) is the mass of \( \rho^0 \) meson. Similarly, the isovector electromagnetic form factor of nucleon is \( eF_{1N}(t)/2 = g_{\gamma\rho}f_{\rho\NN}/2(m^2 - t) \). At zero momentum transfer \( F_\pi(0) = F_{1N}(0) = 1 \) because the electric charge is universal. From \( g_{\gamma\rho}f_{\rho\pi\pi}/m^2 = g_{\gamma\rho}f_{\rho\NN}/m^2 = e \) it follows for all particles with isospin 1:

\[
1) f_{\rho\pi\pi} = f_{\rho\NN} = \ldots = f_\phi; \quad 2) g_{\gamma\rho} = em^2/f_\phi.
\]

(10)

This is the consequence of \( \rho^0 \) meson dominance in isovector form-factor. What was
done above can be expressed in the form current-field identity:

\[ j_{\mu}^{\text{had}}(x) = m_{\varrho}^2 \phi_{\mu}(x)/f_{\varrho}, \]  

(11)

where \( \phi_{\mu}(x) \) is the vector field describing \( \varrho \) mesons \((\alpha = 1, 2, 3)\).

The general form of the hadron electromagnetic current in the vector dominance model, which is expressed in terms of \( \varrho, \omega \) and \( \phi \) mesons fields, reads [53]:

\[ j_{\mu}^{\text{had}}(x) = e(m_{\varrho}^2 \phi_{\mu}(x)/f_{\varrho} + m_{\omega}^2 \phi_{\mu}(x)/f_{\omega} + m_{\phi}^2 \phi_{\mu}(x)/f_{\phi}). \]  

(12)

The field \( \phi_{\mu}^3 \) is connected with isovector states (e.g. \( \pi^+ \pi^- \)), while the fields \( \omega \) and \( \phi \) are connected with isoscalar states (e.g. \( \pi^+ \pi^- \pi^0, K^+ K^- \)) and can be mixed up.

The corresponding currents are [54]

\[ j_{\mu}^{Y} = \left[ m_{\varrho}^2 \cos \vartheta_Y \phi_{\mu}(x) - m_{\omega}^2 \sin \vartheta_Y \phi_{\mu}(x) \right]/f_Y, \]

\[ j_{\mu}^{B} = \left[ m_{\varrho}^2 \sin \vartheta_B \phi_{\mu}(x) + m_{\omega}^2 \cos \vartheta_B \phi_{\mu}(x) \right]/f_B, \]  

(13)

where \( \vartheta_Y \) and \( \vartheta_B \) are the mixing angles. The hadron electromagnetic current is \( j_{\mu}^{\text{had}} = j_{\mu}^{3} + j_{\mu}^{Y}/2 \). In the limit of exact \( SU(3) \) symmetry \( \vartheta_B = \vartheta_Y = 0, f_Y = \sqrt{3}f_{\varrho}/2 \). In the broken \( SU(3) \) symmetry \( \vartheta_B \neq 0, \vartheta_Y \neq 0 \), in mass mixing model \( \vartheta_B = \vartheta_Y = 39^\circ \), in current mixing model \( \vartheta_B = 21^\circ, \vartheta_Y = 33^\circ \).

Taking into account that \( \varrho^0 \) meson is highly unstable, so that \( m = m_{\varrho} - i\Gamma_{\varrho}/2 \), one has for \( \varrho^0 \) contribution to pion electromagnetic form factor \( F_{\pi}(t) = m_{\varrho}^2/(m_{\varrho}^2 - t - i\Gamma_{\varrho}m_{\varrho}) \). This means that

\[ |F_{\pi}(t)|^2 = \frac{m_{\varrho}^4}{(m_{\varrho}^2 - t)^2 + \Gamma_{\varrho}^2 m_{\varrho}^2} \]  

(14)

has sharp resonance peak at \( m_{\varrho}^2 = t = 4\varepsilon^2 \) with the enhancement \( \sim m_{\varrho}^2/\Gamma_{\varrho}^2 \).

Similarly, the sharp peaks at \( t = m_{\omega}^2 \) and \( t = m_{\phi}^2 \) should be observed in the cross sections of production of isoscalar states (e.g. \( \pi^+ \pi^- \pi^0, K^+ K^- \)).

Let us consider decay of vector meson into the electron-positron pair and decay of \( \varrho \) meson into the pion pair.

The partial width of decay \( V \to e^+e^- \) is

\[ \Gamma_{V\rightarrow e^+e^-} = \frac{4\pi \alpha^2 m_V}{3 f_V^2} \left( 1 + \frac{2m_e^2}{m_V^2} \right) \left( 1 - \frac{4m_e^2}{m_V^2} \right)^{1/2} = \frac{4\pi \alpha^2 m_V}{3 f_V^2} \left[ 1 + O \left( \frac{m_e}{m_V} \right)^4 \right], \]  

(15)
where $m_e$ is the electron mass, the equality $g_{V\gamma} = em^2_\gamma/f_V$ is used. Since the value $\Gamma_{V\gamma \gamma}$ is measured quite accurately, this expression can be used for determination of constants $f_V$. The constant $f_{\rho\pi\pi}$ is determined from the width of decay $\rho \to \pi\pi$

$$\Gamma_{\rho\pi\pi} = \frac{f_{\rho\pi\pi}^2 m_\rho}{4\pi} v_R^3,$$

(16)

where $v_R = \sqrt{1 - 4\mu^2/m_\rho^2}$ is the velocity of the created pion, $\mu$ is the pion mass.

The total cross section of $e^+e^-$ annihilation into pair of pseudoscalar ($\pi, K$) particles is

$$\sigma = \frac{\pi\alpha^2}{3t} v^3 |F(t)|^2,$$

(17)

where $v = \sqrt{1 - 4\mu^2/t}$ and $\mu$ are the velocity and mass of produced particle, $F(t)$ is the electromagnetic form factor of corresponding particle, $t = 4\varepsilon^2$. So at the resonance energy $m_\rho^2 = t$ one has the cross section $\sigma_R = (12\pi/m_\rho^2)(\Gamma_{\rho\pi\pi}\Gamma_{\rho\pi\pi}/4\pi^2)$. This formula has a transparent meaning, since in the quantum theory the resonance cross section in the channel with angular momentum $J$ is $\sigma_R = \pi\lambda^2(2J + 1)\Gamma_J/\Gamma_V^2 = 4\pi(2J + 1)/m_\rho^2(\Gamma_J/\Gamma_V^2)$, where $\lambda = 1/\varepsilon = 2/m_\rho$, $\Gamma_J(f)$ is the width of the resonance into channel $i(f)$, $\Gamma_V$ is the total width.

The transition of photon into the vector meson (which is the contribution to the hadronic polarization of vacuum) can appear in the purely QED processes such as $e^-e^+$ elastic scattering or the conversion process $e^-e^+ \to \mu^-\mu^+$. The last reaction is more appropriate since only one (annihilation) diagram contributes. The most pronounced effect will be near resonance $2\varepsilon \simeq m_\rho$. The cross section of the $e^-e^+ \to \mu^-\mu^+$ process (see Eq.(11)) with the transition $\gamma V$ taken into account acquires an additional factor [55]

$$|1 + \frac{g^2}{\alpha} \frac{m_\rho}{2(2\varepsilon - m_\rho)} + i\Gamma_V|^2,$$

(18)

where $g$ is the effective coupling constant $V e^-e^+$ or $V \mu^-\mu^+$. It can be expressed in terms of branching ratio $B_{V\gamma \gamma} = \Gamma_{V\gamma \gamma}/\Gamma_V$ and $B_{V\mu^-\mu^+}$, see Eq.(15). The prediction [55] was made for only known in 1963 narrow $\omega$ meson. The factor Eq.(18) results in oscillation of the process cross section with respect to the QED prediction in the narrow energy interval (the width $\sim \Gamma_V$) near $\varepsilon \simeq m_\rho/2$: first the cross section is going down, than it turns up and crosses the prediction very close to $\varepsilon = m_\rho/2$, attains some maximal value and than returns to the prediction.

Since the ratio $\Gamma_e/m_\rho$ turns out to be not very small the corrections $\propto \Gamma_e/m_\rho$ and dependence of term with $\Gamma_e$ in the resonance denominator $m_\rho^2 - t - i\Gamma_e m_\rho$ on the pion momentum becomes significant [75]: $\Gamma_\rho m_\rho \to \Gamma_\rho (p/p_0)^3 m_\rho^2/2\varepsilon$, where $p$ is the momentum of created pion and $p_0$ is the momentum at $m_\rho = 2\varepsilon$. With regard for final width of $\rho$ meson Eq.(11) is modified: $f_{\rho\pi\pi}^2 = 1.15 f_\rho^2$. Besides, since the masses of $\eta^0$ and $\omega^0$ mesons appears to be very close, the contribution of process
\(e^+e^- \rightarrow \omega^0 \rightarrow \pi^+\pi^-\) (the \(g - \omega\) interference) should be taken into account, in spite of the fact that in the mentioned channel the isospin invariance is violated, because at resonance the cross section \(\propto (m_V/\Gamma_V)^2\) and \(\Gamma_\omega \ll \Gamma_g\). As a result the \(g\)-meson excitation curve becomes asymmetric.

The first indications on existence of the vector mesons was obtained in the hadronic reactions in 1961. The particle which is now called \(g\) meson was observed in inelastic \(\pi p\) collisions \[56\] with mass \(m_g\) in interval 700-770 MeV and width \(\Gamma_g \approx 90\) MeV. The \(\omega\) was seen in reaction \(\bar{p}p \rightarrow 2\pi^+\pi^-\pi^0\) \[57\] with mass \(m_\omega = 787\) MeV and \(\Gamma_\omega < 30\) MeV. Extraction of properties of the vector mesons in the hadronic reaction is quite ambiguous due to involvement of strong interactions and only electron-positron colliders permitted to perform full scale study of the vector mesons.

The neutral vector mesons at colliding \(e^-e^+\) beams were observed first at VEPP-2 storage ring in INP, Novosibirsk in 1967 \[58\], \[59\], where the cross section of production of the \(\pi^-\pi^+\) pair was measured in the region of \(g\)-resonance and the excitation curve was obtained. Later the same measurement was performed at ACO storage ring in LAL, Orsay \[60\], \[61\], \[62\].

In isoscalar channel production of \(\pi^-\pi^+\pi^0\) was observed first at ACO storage ring in LAL, Orsay in the region of \(\omega\)-resonance \[63\] and the region of \(\phi\)-resonance \[64\]. The last channel was observed also in INP, Novosibirsk \[66\]. The reactions \(\phi \rightarrow K^0_LK^0_S, K^+K^-\) were observed at ACO storage ring in LAL, Orsay \[64\], \[65\] and at VEPP-2 storage ring in INP, Novosibirsk \[66\]. In these experiments the excitation curves were measured and resonance parameters were obtained.

Radiative modes of decay of \(\omega\) and \(\phi\) mesons into \(\eta\gamma, \pi^0\gamma, \pi^+\pi^-\gamma\) were investigated in LAL, Orsay \[69\]. The multi-hadron production in electron-positron annihilation was discovered at VEPP-2 storage ring in INP, Novosibirsk \[67\]. The production of \(\pi^+\pi^-\), \(K^+K^-\) pairs in electron-positron annihilation at energy higher than \(\phi\) resonance mass was observed at VEPP-2 storage ring in INP, Novosibirsk \[68\]. The vacuum polarization in the process \(e^-e^+ \rightarrow \mu^-\mu^+\) due \(\phi\) meson contribution according to Eq.(18) (where \(g^2 = 3B, B = \sqrt{B_{\phi e^-e^+}B_{\phi\mu^-\mu^+}}\)) was observed in LAL, Orsay \[70\], the magnitude of oscillation was \(\sim 10\%\). The calibration of storage ring energy was performed using the angular distribution of pions in reaction \(e^+e^- \rightarrow \phi \rightarrow K^0_LK^0_S, K^0_SK^0_S \rightarrow \pi^+\pi^-\) \[71\].

For review see e.g. \[73\], \[74\].

The results obtained at the electron-positron colliders confirmed the basic predictions of the vector dominance model, which appears to be remarkably successful,
and become outstanding achievement of the new method.

The recent parameters of vector mesons are given in the Table 1 below. These parameters differ from measured in cited above experiments 1967-1972 on the level of one standard deviation but here the accuracy is improved significantly.

**Table 1** Parameters of vector mesons (PDG 2004)

| meson | \( m_V \) (MeV) | \( \Gamma_V \) (MeV) | \( \Gamma_{Ve^+e^-} \) (keV) | \( f_V^2/4\pi \) | \( g_{V\gamma} \) (GeV²) |
|-------|-----------------|-----------------|-----------------|----------------|------------------|
| \( \rho \) | 775.8 ± 0.5 | 146.4 ± 1.5 | 7.02 ± 0.11 | 1.96 ± 0.03 | 0.121 ± 0.001 |
| \( \omega \) | 782.59 ± 0.11 | 8.49 ± 0.08 | 0.60 ± 0.02 | 23.2 ± 0.8 | 0.036 ± 0.001 |
| \( \phi \) | 1019.456 ± 0.020 | 4.26 ± 0.05 | 1.27 ± 0.04 | 14.2 ± 0.4 | 0.078 ± 0.001 |

For these parameters one has using Eq. (16) \( f_{\rho\pi\pi}^2/4\pi = 2.79 \pm 0.03 \).

Just by that time when the main results obtained at the first generation of electron-positron colliders were published (1972-1973), the quantum chromodynamics (QCD), which is the non-Abelian gauge theory, emerged \[76, \] and in a short time was accepted as a strong interaction theory. In QCD the basic components are quarks and gluons. In this theory the vector mesons discussed above are the composite systems each consisting of light (u,d,s) quark and antiquark with parallel spins coupled by the gluon field (e.g. the state of \( \rho \) meson is \( \rho = (u\bar{u} + d\bar{d})/\sqrt{2} \)). For this picture the VDM is an effective theory valid for energies up to \( \sim 1 \) GeV. Since the parameters of vector mesons are now measured within percent accuracy the deviations from exact VDM are seen (e.g. from parameters given above the ratio \( f_{\rho\pi\pi}^2/f_{\rho}^2 = 1.42 \) and not 1.15). Description of vector mesons in QCD frame for mentioned parameters lies indeed in region of strong coupling and should be done in scope of non-perturbative methods. Such analysis should not only explain the origin of the VDM but also clarify deviations from exact VDM. In lattice QCD the recent progress is on the level of \( \rho \) meson mass calculation \[78\]. Since there is no other reliable methods, the vector dominance is still a challenge for QCD.

### 2.3.2 Radiative return

The cross section of the process passing through the vector meson contains the resonant factor of the type of Eq. (14). Because of this the radiative corrections to the cross section of such process, considered first by Victor Fadin and I \[79\], depended strongly on energy. This is a consequence of photon emission from initial particles, which leads to decrease of produced particle energy.

If the initial particle energy is higher than the resonance one \( \varepsilon > \varepsilon_R = m_V/2 \), "then the initial particle radiation can "turn" the cross section back to the resonance (when final particles energy in their c.m.s. is equal to the resonant one).
Since the resonance cross section essentially exceeds the cross section far away from resonance, this leads to a fast increase of the radiative corrections at $\varepsilon > \varepsilon_R$.

Within the logarithmic accuracy in the case of soft photons the radiative corrections, given only by the initial electron and positron, calculated in [79] are written as

$$
\frac{2\tau(0)}{\Gamma_V} \arctan \left( \frac{2\omega\Gamma_V}{\tau(\omega)\tau(0) + \Gamma_V^2} \right) + \frac{13}{3} \ln 2\gamma
$$

where $d\sigma_0$ is the process cross section without radiative corrections depending on final particles momenta, $\varepsilon$ is the initial energy of electron/positron in their c.m.s., $\gamma = \varepsilon/m_e$, $\omega$ is the maximal permissible by the event selection energy of photon emitted in the direction of initial particle, $\tau(\omega) = 2(2\varepsilon - m_V - \omega)$, note that the square of invariant mass of final system is $\Delta^2 = 4\varepsilon(\varepsilon - \omega)$. This formula can be applied not far from resonance \(^8\). The above effect is determined mainly by the second term in the square brackets Eq.(19). It may turn out that the term $\tau(0)/\Gamma_V \gg 1$ and its factor is of the order of 1. Then it is possible that $4\alpha\tau(0)\ln 2\gamma/\pi\Gamma_V \geq 1$ or $\delta(\varepsilon) \geq 1$! Such paradoxical situation arose due to the fact that cross section of the process with photon emission by the initial particle turns out to be larger than the cross section of elastic process (without inclusion of radiative corrections) at a given energy of the initial particles. The increase of $\delta(\varepsilon)$ stops when the condition of event selection forbids emission of photon with an energy sufficient for the shift to the resonance.

It should be noted that Eq.(19) can be applied for an arbitrary process of particle production passing through the resonant state.

The radiative return method basing on equations of the type Eq.(19) is widely used now at meson factories (BELLE, BABAR, CLEO-C, DAPHNE) for study of particular hadronic reactions from production energy threshold up to the energy close to the machine energy $2\varepsilon$ (for recent review see e.g. [81]). The behavior of reaction $e^+e^- \rightarrow p\bar{p}$ ($p$ is a proton) near threshold, process $e^+e^- \rightarrow 3\pi$ for energy $\varepsilon > 0.7$ GeV (higher than operational energy of VEPP-2) are among the results obtained.

\(^8\)The process $e^+e^- \rightarrow \pi^+\pi^-\gamma$ was analyzed in [80], where the exact in Born approximation and very compact expression for the integral spectrum in terms of $\Delta^2$ was calculated.
2.4 Polarization

2.4.1 Radiative polarization of electrons in storage rings

During extended motion in a magnetic field electrons and positrons can be polarized as a result of photon emission. The polarization arises because the probability of radiative transition with spin flip depends on the orientation of the initial spin. Existence of this mechanism was pointed out by Ternov, Sokolov et al. [82], [83]. The solution of Dirac equation in the uniform magnetic field was used in these paper. However, it is known that typical conditions accelerators correspond to very high quantum numbers, e.g. for \( H \sim 10^4 \) Oe and energy \( \sim 1 \) GeV the main quantum number \( \sim 10^{15} \). This means that the motion of particle in accelerator and storage ring is almost classical. We with Valery Katkov developed an operator method for investigation of spin phenomena\(^9\) in a quasiclassical approximation\(^10\).

The total probability of spin-flip radiative transition per unit time valid in an arbitrary magnetic field is \(^85\)

\[
W^\zeta = \frac{1}{2T} \left[ 1 - \frac{2}{9} (\zeta \mathbf{v})^2 - \frac{8\sqrt{3}}{15|\mathbf{v}|} (\zeta (\dot{\mathbf{v}} \times \mathbf{v})) \right], \quad \frac{1}{T} = \frac{5\sqrt{3}}{8} \frac{\alpha \gamma^5 |\mathbf{v}|^3}{m^2}, \tag{20}
\]

where \( \zeta = \zeta(t) \) is the unit spin vector of an electron, \( \mathbf{v} \) and \( \dot{\mathbf{v}} \) are the velocity and acceleration of an electron, \( T \) is the characteristic time of polarization\(^11\). For the longitudinal polarization \((\mathbf{v} \times \mathbf{v}) = 0\) the remaining terms \( 1 - (2/9)(\zeta \mathbf{v})^2 \) do not depend on whether the spin is directed along or opposite to the velocity, so that the radiation does not change the spin states with longitudinal polarization. A different situation arises in the case of transverse polarization \((\zeta \mathbf{v}) = 0\). In this case the transition probability depends on the spin orientation. For electrons \((e < 0)\) the probability of a transition from a state with spin along the field to a state with spin opposite to the field is higher than the probability of the inverse transition. For positrons \((e > 0)\) the opposite situation occurs. Thus, the resulting polarization (radiative polarization) is transverse and for electrons is directed opposite to the field and for positrons along it.

It is very important that the probability Eq.\(^\text{20}\) is given in the same terms as used in the quasiclassical equation for spin motion of Bargmann-Michel-Telegdi (BMT) in an external field [88]. The point is that the radiative polarization is rather slow process which evolves at background of rather complicate spin motion (described by BMT equation) in a storage ring. The kinetic equation which takes into account both factors was derived by Valery Katkov, Volodya Strakhovenko

\(^9\)Later the general quasiclassical operator method was developed by Katkov and I, which is actually the formulation of QED in an arbitrary electromagnetic field at high energy [86], [87].
\(^10\)Similar procedure was used by Schwinger[84] to find the first quantum correction to the intensity of electron radiation in a magnetic field.
\(^11\)For magnetic radius \( r = 150 \) cm (VEPP-2 facility) and \( \varepsilon = 700 \) MeV one has \( T = 38 \) minutes.

19
and I [80], [81]:

\[
\frac{d\zeta}{dt} = \frac{e}{c} (\zeta \times (\mu H_R + H_E)) - \frac{1}{T} \left[ \zeta - \frac{2}{9} (vH) \frac{8\sqrt{3}}{15|v|} (v \times v) \right],
\]

\[
H_R = \gamma \left[ H - \frac{v(vH)}{1 + 1/\gamma} - (v \times E) \right], \quad H_E = H - \frac{(v \times E)}{1 + 1/\gamma}, \tag{21}
\]

where \(\mu = \alpha/2\pi\) is the anomalous magnetic moment of an electron, \(E\) and \(H\) are the fields in the laboratory system, \(H_R\) is the magnetic field in the rest system of the electron. The first term in this equation is just BMT equation, while the second term appearing due to the spin-flip transitions leads to variation of \(|\zeta|\).

In the simplest case of circular motion in a homogeneous magnetic field decomposing the vector \(\zeta\) over the unit vectors \(e_1 = v/|v|, e_2 = \dot{v}/|\dot{v}|\) and \(e_3 = (e_1 \times e_2)\) one has from the above equation

\[
\zeta_1 = -\frac{7}{9} \frac{\zeta_1}{T}, \quad \zeta_2 = \Omega \zeta_1 - \frac{\zeta_2}{T}, \quad \zeta_3 = -\frac{1}{T} \left( \zeta_3 + \frac{8}{5\sqrt{3}} \right), \tag{22}
\]

where \(\Omega = \mu \gamma |\dot{v}|\). The solution of this set is

\[
\zeta_\perp(t) = \zeta_\perp(0) \exp \left( -\frac{8t}{9T} \right), \quad \zeta_3(t) = -\frac{8}{5\sqrt{3}} + \left( \zeta_3(0) + \frac{8}{5\sqrt{3}} \right) \exp \left( -\frac{t}{T} \right), \tag{23}
\]

where \(\zeta_\perp(t) = \sqrt{\zeta_1^2(t) + \zeta_2^2(t)}\), it was taken into account that \(\Omega \gg 1/T\). So the spin rotates around the \(e_3\) axis, the transverse component decays during a time \(\sim T\), while the nondecaying term \(-8/5\sqrt{3}\) in \(\zeta_3\) gives a finite polarization \((\sim 0.924)\) which does not depend on the initial value of the vector \(\zeta\). The polarization is oriented along the vector \((\dot{v} \times v)\).

Side by side with outlined development the very important result concerning behavior of spin vector was obtained by Derbenev, Kondratenko and Skrinsky [91]. It was shown that the stable direction of polarization exists for solution of BMT equation \((n(t) = n(t + \tau), \tau\) is the period of revolution\) for closed orbits in storage ring with arbitrary field.

Basing on mentioned above results and analysis of depolarization effects [92] Derbenev and Kondratenko obtained the following equation for the equilibrium degree of polarization for the time essentially larger than \(T\) [93]

\[
\zeta n \equiv \zeta_n = -\frac{8}{5\sqrt{3}} < |\dot{v}|^2 (v \times \dot{v}) (n - \gamma \frac{\partial n}{\partial \gamma}) >
\]

\[
< v^2 |1 - \frac{4}{3} (nv)^2| >
\]

where \(< \ldots >\) means averaging over azimuth and particle ensemble in storage ring. This formula summarize many contributions: the external factor \(8/5\sqrt{3}\) was found by Sokolov and Ternov [83], the terms \(< |\dot{v}|^2 (v \times \dot{v}) n >\) and \(< |\dot{v}|^2 |1 - \frac{4}{3} (nv)^2| >\) follow directly from Eq. (21), the term \(\gamma \frac{\partial n}{\partial \gamma}\) reflecting perturbation of quantization.
axis \( n \) due to influence of spin-dependent part of the magnetic bremsstrahlung is the invention of Derbenev and Kondratenko [93], the term \( \frac{41}{18} < \left( \frac{\partial n}{\partial \gamma} \right)^2 > \) describes electron beam depolarization due to chaotic jumps of the trajectory because of quantum nature of radiation process discovered by me and Yurii Orlov during his short stay in Novosibirsk [94]. Emerging, conservation, manipulation and measurement of radiative polarization are discussed in detail in [95], see also Sec.14 in [96].

2.4.2 Measurement of electron polarization

I. **High energy processes**

We have shown with Victor Fadin that the cross sections of two-particle production at electron-positron annihilation are extremely sensitive to electron and positron polarizations [97], so these reactions can be used for polarization measurement.

The cross section for production of a pair of pseudoscalar particles \( (\pi^+\pi^- , K^+K^- , K^0S_K^0K^0_L) \) in annihilation of transversely (and antiparallel) polarized electrons and positrons has the form

\[
\sigma_{2p}(\vartheta, \varphi) = \sigma_{2p}^0(\vartheta) \left[ 1 - |\zeta_1||\zeta_2| \cos 2\varphi \right],
\]

where \(|\zeta_1|\) and \(|\zeta_2|\) are the degrees of polarization of the positrons and electrons, \( \varphi \) is the angle between the plane of production (the plane passing through the momenta of the initial particle \( p \) and the final particle \( q \)) and the plane perpendicular to the spin direction (the plane of the orbit, \( \sigma_{2p}^0(\vartheta) \) is the cross section for unpolarized particles: \( \sigma_{2p}^0(\vartheta) = \alpha^2 v^3 \sin^2 \vartheta |F(t)|^2 / 8t \) (cf with Eq.(17)), \( \vartheta \) is the angle between \( p \) and \( q \). If the initial particles are completely polarized \(|\zeta_1| = |\zeta_2| = 1\), then \( \sigma_{2p}(\vartheta, \varphi = 0) = 0 \) (the production plane coincides with the orbit plane) and \( \sigma_{2p}(\vartheta, \varphi = \pi/2) = 2\sigma_{2p}^0(\vartheta) \) (the production plane is perpendicular to the orbit plane, so that the spin vector lies in the production plane).

For production of a pair of muons one has

\[
\sigma_{2\mu}(\vartheta, \varphi) = \frac{\alpha^2}{4t} v[2 - v^2 \sin^2 \vartheta] [1 - |\zeta_1||\zeta_2| \cos 2\varphi].
\]  

(26)

For relativistic muons \( v \simeq 1 \), and we have for completely polarized particles \( \sigma_{2\mu}(\vartheta = \pi/2, \varphi = \pi/2) = 0 \) (muon momentum directed along the spin) and \( \sigma_{2\mu}(\vartheta = \pi/2, \varphi = 0) = 2\sigma_{2\mu}^0(\vartheta) \) (muon momentum perpendicular to the spin).

II. **Internal scattering effects and polarization measurement** [98]

It is well known that an important cause of the loss of particles in storage ring is the electron-electron scattering inside the bunch [9]. If this scattering occurs into a sufficiently large angle and is such that particles with a large transverse momentum and small longitudinal momentum (in the rest system of the beam) acquire a large longitudinal moment, then in conversion to the laboratory system the longitudinal
momentum is subject to the relativistic transformation and can turn out to be larger than the permissible value. As a result the particles are lost. Under some conditions the lifetime of a beam in a storage ring is determined just by the Touschek effect. Internal scattering effects depend on the particle polarization, since the electron-electron scattering cross section at the large angles which determine the internal scattering effect depends substantially on electron polarization. The beam lifetime \( \tau \) (\( \tau \) is the time in which the number of particles decreases by a factor of two) is determined by the coefficient \( \alpha_b \): 
\[
\frac{1}{\tau} = \alpha_b N_0, \quad N_0 \text{ is the initial number of particles in the beam.}
\]
For example, for a Gaussian distribution of radial momenta of the electrons in the beam one has
\[
\alpha_b = \frac{2\sqrt{\pi} \alpha^2 m}{V(\Delta p)^2 \delta q} \left[ \ln \frac{2\varepsilon}{\Delta p} - \frac{7}{4} - \frac{\xi_1 \xi_2}{4} + 2\sqrt{\pi} \frac{\delta q}{m} \exp \left( \frac{m^2}{\delta q^2} \right) \left( 1 + \frac{m^2}{2\delta q^2} \right) \right] 
\times \left( 1 - \Phi \left( \frac{m}{\delta q} \right) \right) - \sqrt{\pi} \int_0^m e^{x^2} (1 - \Phi(x)) dx,
\]
where \( V \) is the volume of the beam in the laboratory system, \( \Delta p \) is the maximum permissible deviation of momentum from the equilibrium value in the laboratory system, \( \delta q \) is the mean-square value of the momentum distribution, \( \varepsilon \) is the electron energy in the laboratory system, \( \xi_{1,2} \) are the polarization vectors of electrons in the bunch, \( \Phi(x) \) is the probability integral. This dependence of the internal scattering effect on polarization is used to measure the polarization of electrons in a storage ring.

### III. Measurement of polarization by means of Compton scattering [99]

In Compton scattering of circularly polarized photons by transversely polarized high-energy electrons, terms in the cross section arise which depend on the electron polarization vector. In head-on collisions of laser photons (with energy \( \omega_1 \)) with high-energy electrons, the final photons are emitted mainly in a narrow cone with an angle \( \sim 1/\gamma \) relative to the initial electron direction. The cross section can be written in a form
\[
d\sigma = d\sigma_0 + d\sigma_1 |\xi_1| |\xi_2| \sin \varphi, \tag{28}
\]
where \( d\sigma_0 \) is the cross section for unpolarized particles, \( \xi_2 \) is the degree of circular polarization of the photons, and \( \varphi \) is the angle between the plane perpendicular to the vector \( \xi_1 \) and the scattering plane. The azimuthal asymmetry coefficient has the form
\[
P = \frac{d\sigma_1}{d\sigma_0} = -\frac{2\lambda n(1 + n^2)}{2\lambda^2(1 + n^2) + (1 + n^2 + 2\lambda)(1 + n^4)}, \tag{29}
\]
where \( \lambda = 2\omega_1 \varepsilon/m^2 \), the photon scattering angle measured from the direction of electron momentum is written as \( \vartheta = n/\gamma \ll 1 \).
IV. The first experiment

The first experimental study of the radiative polarization of electrons has been carried out in the storage ring VEPP-2 in INP, Novosibirsk (see [35]). The polarization measurement was accomplished by the method described above in the paragraph II, which utilizes the dependence of internal scattering effects on the polarization of the electrons in the bunch (see Eq. (27)). For the energy chosen ($\varepsilon = 650$ MeV) the polarization time is $T \simeq 50$ min and the theoretical degree of polarization during the experiment is $|\zeta_3(2T)| \simeq 0.80$ (see Eq. (23)). In this experiment it was extremely important to exclude the effect of depolarizing factors. For this purpose it is necessary first of all to be sufficiently far from spin resonances. If the depolarizing effects are taken into account, then the expected degree of radiative polarization is $|\zeta_3^{th}(2T)| \simeq 0.66$.

The measurements were made in the following way. The electron beam in the storage ring was polarized for a time $t \simeq 2T$, and the particles leaving the beam as a consequence of internal scattering effects were recorded by two counters. Then the beam was depolarized by application of an external longitudinal field. In this case the rate of departure of particles from the beam increases (i.e., the number of counts in the counters increases). The experimental results was obtained for an energy $\varepsilon = 638.8 \pm 0.8$ MeV. A jump was seen in the counting rate, occurring at the turning on of the depolarizing field. From the size of the jump one can deduce the following value of the degree of polarization of the electron beam:

$$|\zeta_3^{exp}(2T)| \simeq 0.52 \pm 0.13,$$

which is consistent with the expected value of the degree of polarization given above with inclusion of depolarizing effects $|\zeta_3^{th}(2T)| \simeq 0.66$, although it is somewhat smaller. This was the first experimental proof of the existence of radiative polarization.

3 Conclusion

Let us list the main results obtained at the electron-positron colliders of the first generation.

At the electron-positron colliding beam facility VEPP-2 in INP, Novosibirsk (the maximal observed luminosity $L = 3 \times 10^{28}$ cm$^{-2}$ s$^{-1}$):

1. The first observation of hadron production at electron-positron collider (1967), study of $\phi$ meson.
2. The first observation of two-photon annihilation ($e^+e^- \rightarrow 2\gamma$).
3. The first observation and study of the radiative polarization of beam in storage ring.
4. The first observation and study of the two-photon process (production of additional electron-positron pair).
5. Check of QED at $e^+e^-$ collision.
6. Check of QED in reaction $e^+e^- \rightarrow \mu^+\mu^-$.  
7. Systematic study of $\varrho, \omega, \phi$ mesons.  
8. Discovery of the multi-hadron production in electron-positron annihilation.  
9. Study of production $\pi^+\pi^-$, $K^+K^-$ pairs in electron-positron annihilation at energy higher than $\phi$ resonance mass.

At the electron-positron colliding beam facility ACO in LAL, Orsay (the maximal observed luminosity $L = 10^{29} \text{cm}^{-2}\text{s}^{-1}$):

1. The first observation and study of $\omega$ meson.  
2. The first observation and study of $\phi$ meson.  
3. Study of $\varrho$ meson.  
4. Study of $\phi - \omega$ and $\varrho - \omega$ interference.  
5. Study of radiative modes of decay of $\omega$ and $\phi$ mesons into $\eta\gamma$, $\pi^0\gamma$, $\pi^+\pi^-\gamma$.  
6. Study of $\mu$ meson pair creation ($e^+e^- \rightarrow \mu^+\mu^-$).  
7. Check of QED at $e^+e^-$ collision.  
8. Study of vector dominance model.  
9. Observation of $\phi$ meson contribution to vacuum polarization.

At AdA storage ring constructed in LNF, Frascati and brought to LAL, Orsay (the maximal observed luminosity $L \sim 10^{25} \text{cm}^{-2}\text{s}^{-1}$):

1. Discovery of Touschek effect [9].  
2. The first observation of $e^+e^-$ collision (1964) in the bremsstrahlung reaction $e^+e^- \rightarrow e^+e^-\gamma$ [72].

So, during quite short time new type of accelerator was developed. This included fast ejection of beam from accelerators which were used as injectors (where it was necessary), development of channels, convertors of electron beam into positron one, fast injection of beams into storage ring, prolong operation of storage ring with enough small beam dimensions (to have an acceptable luminosity), which required high vacuum and damping of many instabilities evolved during operation.

Both first generation detectors at VEPP-2 and ACO had some specific features.  
1) A good solid angle.  
2) Ability to identify the particles in an observed event.  
3) Reasonable track position accuracy.  
4) Momentum analysis.  
5) Background rejection.  

The first colliding beam experiments tested QED up to distances more than 100 times smaller than characteristic hadron dimension. Described above results completely changed understanding of electromagnetic structure of hadrons supporting from one side the basic idea of vector dominance model, but from other side showing shortages of this model. $SU(3)$ symmetry was tested as well as $SU(3)$ breaking effects.
Thus, the electron-positron colliding beam project started in INP in 1959 as exotic venture, within a decade became one of the main roads of high energy accelerator development. New discoveries were ahead including November revolution of 1974.

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