Robust control of a supermarket refrigeration system using multi-stage NMPC

Sankaranarayanan Subramanian∗ Adeel Ahmad∗ Sebastian Engell∗

* Process Dynamics and Operations Group, TU Dortmund (e-mail: {sankaranarayanan.subramanian, adeel.ahmad, sebastian.engell}@bci.tu-dortmund.de).

Abstract: Using Nonlinear Model Predictive Control (NMPC), real world systems that are subject to stringent constraints can be controlled efficiently when accurate plant models are employed. Model uncertainties however may lead to poor performance of the controller and to constraint violations because of the wrong predictions. As in real applications, there often is a significant plant model mismatch, the controller must be robust to plant-model mismatch without deteriorating the performance of the controller significantly. Multi-stage NMPC is a robust scheme which has been proven to be less conservative than open loop worst case solutions because of the presence of feedback information at the future time stages is explicitly accounted in the problem formulation. In this paper, we study the application of this NMPC technique to a supermarket refrigeration system under uncertainty and show that the presence of uncertainties in the model lead to constraint violations when standard NMPC scheme is applied. The robust multi-stage NMPC scheme improves the controller performance by accounting for the uncertainties in the prediction and results in a reliable operation of the system.

Keywords: Multi-stage NMPC, Robust control, Hybrid system, Refrigeration system, Optimal control

1. INTRODUCTION

The performance of many real world systems can be significantly improved by using advance control techniques such as Nonlinear Model Predictive Control (NMPC). In NMPC, the model of the nonlinear system is directly employed to predict the future behavior of the system over a finite time period called prediction horizon. The sequence of future control inputs is obtained by solving a nonlinear programming problem for the required objective and given constraints recursively at every time step and the control input solved for the first time step in the sequence is applied to the system and the others in the sequence are discarded. Once the measurements are available at the next time step, the optimization problem is solved again by re-initialization of the problem with the current state of the system and this process continues in a receding horizon fashion. Since the constraints are handled explicitly in the optimization problem, the control inputs are calculated such that they satisfy the constraints at all times. In the objective, in addition to the conventional reference tracking, the economic goals can also be considered directly such that the control inputs are optimized for economically optimal operation of the system (see Engell (2007)).

The performance of NMPC depends on the accuracy of the prediction which in turn depends on the quality of the model. The uncertainties in the model will lead to predictions that are different from the reality and this may result in constraint violations and may even lead to an unstable closed loop behavior. Hence the uncertainties must be accounted for in the controller design. Multi-stage NMPC is a robust scheme where the evolution of the system for different realizations of the uncertainties are modeled by a scenario tree (Lucia et al., 2013). The key aspect of the approach is that it considers the fact that the measurement information is available at the future time instants in the prediction and hence the decisions at those instants can be different from each other depending on the realizations of the uncertainties observed thus far. Since the scheme allows the option of recourse in the optimization problem, it is less conservative compared to other robust NMPC schemes.

In this paper, we study a supermarket refrigeration system and focus our attention on the effects of uncertainties present in the model. There have been many previous studies on the application of (N)MPC algorithm for the supermarket refrigeration system. In Larsen et al. (2005), the authors show the advantages of MPC technique applied to a supermarket refrigeration system when compared to traditional control schemes. In Sonntag et al. (2007), the authors proposed a hierarchical scheme to solve the control problem and later improved it in Sonntag et al. (2009) with extensions to the scheme to handle a large system. Sarabia et al. (2007) proposed a scheme to parameterize the switching control inputs in terms of the events’ occur-
rence and reformulated the scheme into an NLP problem. We have also borrowed this idea for the implementation of both the standard and robust NMPC schemes in this paper. Recently Hovgaard et al. (2013) proposed a sequential convex programming scheme to solve the problem faster with economic objectives. However the influence of the presence of significant uncertainties in the system model have not been studied in detail. We show that the standard NMPC technique, which uses a nominal model of the system fails to satisfy the constraints and gives rise to unwanted oscillatory control movements because of the constraint violations whereas the robust multi-stage NMPC scheme not only satisfies the constraints for the system under uncertainty but also improves the overall performance of the system.

2. MULTI-STAGE NMPC

2.1 NMPC problem formulation

We assume that a nonlinear model of the plant is given by:
\[ x_{k+1} = f(x_k, u_k, d_k), \]
\[ y_k = x_k, \]
where \( x_k \in \mathbb{R}^{n_x} \) is the state of the plant, \( u_k \in \mathbb{R}^{n_u} \) is the control input, \( d_k \in D \subset \mathbb{R}^{n_d} \) is the parametric uncertainty with the dimension \( n_d \), \( y_k \in \mathbb{R}^{n_y} \) is the measured output of the plant and it is assumed that all the states are measured and function \( f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_d} \to \mathbb{R}^{n_x} \) represents the system model.

The standard NMPC formulation of the optimization problem to be solved at each sampling time with full state information can be written as:
\[
\begin{align*}
\min_{x_{k+1}, u_k, \forall k \in I} & \sum_{k=0}^{N_p-1} \mathcal{L}(x_{k+1}, u_k) \quad (3a) \\
\text{subject to:} & \quad x_{k+1} = f(x_k, u_k), \quad (3b) \\
& \quad x_{k+1} \in X, \quad u_k \in U, \quad (3c)
\end{align*}
\]
where \( N_p \) denotes the prediction horizon. Equation (3b) represents the plant model and (3c) denotes all the state and input constraints. In the standard NMPC formulation, the effects of uncertainties are not explicitly considered and only the nominal model is used for the predictions. Since the problem is solved online at every time step, the feedback information gives a certain robustness against uncertainties. In reality, the uncertainties have to be explicitly considered in the problem formulation to have a stable closed loop and constraint satisfaction. In the next section a non-conservative robust NMPC formulation known as multi-stage NMPC is discussed (see Lucia et al. (2013), Lucia et al. (2014a)).

2.2 Robust Multi-stage NMPC scheme

In multi-stage NMPC, the future evolution of the plant is modeled as a scenario tree as shown in Fig. 1. Each node denotes a different possible future evolution of the plant for the different realizations of the uncertainty starting from a root node. The current state \( x_0 \) forms the root node of the tree. From the root node, the scenario tree branches into a number of nodes \( (x_1^i, x_2^i, x_3^i) \) according to the number of realizations of the uncertainties \( (d_{1,k}^r, d_{2,k}^r, d_{3,k}^r) \) in the first stage and each node in the first stage then give rise to as many nodes in the second stage. This process continues until the end of the prediction horizon. As it can be seen, the size of the problem grows exponentially with respect to the number of uncertainties and the prediction horizon. The branching of the tree can be limited to a stage before the end of the prediction horizon (known as robust horizon \( N_r \)) and the uncertainties can be assumed to be constant from then on. The controller performance even for the choice of robust horizon \( N_r = 1 \) is comparable to the full scenario tree while the size of the problem is significantly reduced as shown in Lucia et al. (2013). Depending on the choice of prediction and robust horizons, the tree branches from the root node to the leaf nodes (end nodes). A path from the root node to a leaf node is called a scenario.

The general formulation of the optimization problem that has to be solved at each sampling time can be written as:
\[
\begin{align*}
\min_{x_{k+1}^1, u_k^i, \forall (i,k) \in I} & \sum_{i=1}^{N} \sum_{k=0}^{N_p-1} \mathcal{L}_i(x_{k+1}^i, u_k^i) \quad (4a) \\
\text{subject to:} & \quad x_{k+1}^i = f(x_k^i, u_k^i, d_k^{r(i)}), \quad \forall (j,k+1) \in I, \quad (4b) \\
& \quad x_{k+1}^i \in X, \quad u_k^i \in U, \quad \forall (j,k) \in I, \quad (4c) \\
& \quad u_k^i = u_k^j \text{ if } x_k^j = x_k^i, \quad \forall (j,k), (l,k) \in I. \quad (4d)
\end{align*}
\]
stage cost can be either economical or a simple tracking term. The cumulative sum of the stage costs over the prediction horizon defines the cost of a scenario. The weighted sum of all the scenario costs gives the overall cost function of the optimization problem. The future control actions taken at different nodes can be different because the measurement information will be revealed at every time step and the decisions taken at those nodes can be adapted accordingly. However the decisions taken for all the branches having a common parent node has to be the same because the control inputs have to satisfy the constraints for all the possible realizations of the uncertainties and this is enforced by (4d). In Fig. 1, \( u_0^1 = u_0^2 = u_0^3 \) is enforced by non-anticipativity constraints given in (4d) but the decisions taken at different nodes at the next stage can be different (i.e. \( u_1^1 = u_1^2 = u_1^3 \), \( u_2^1 = u_2^2 = u_2^3 \) and \( u_3^1 = u_3^2 = u_3^3 \) but \( u_1^1, u_1^2 \) and \( u_1^3 \) can be different from each other). Since the feedback information is explicitly considered, the approach is less conservative when compared to open loop min-max approaches.

3. SUPERMARKET REFRIGERATION SYSTEM

In the supermarkets, cold storage of many of the goods is necessary in order to preserve them for a long period of time. Such goods are normally placed in open refrigeration systems that are present in supermarkets in the form of open display cases. These open display cases provide the required refrigeration to avoid the deterioration of the goods and allow an easy and self-access for the customers. In Larsen et al. (2007), the nonlinear hybrid model of the supermarket refrigeration system is discussed in detail along with the parametric values and only the important model dynamics are summarized here.

A typical supermarket refrigeration system consists of four major components: a compressor rack, a number of display cases, a suction manifold and a condenser unit. The compressor rack is the most important component of the system and consists of a number of compressors connected in parallel, which supply the flow of refrigerant to the system by compressing the low-pressure vapors from the suction manifold which are drained out from the display cases. This compressed refrigerant is passed on to a condenser unit which liquefies the compressed refrigerant and from there it is further passed on to the liquid manifold. From the liquid manifold, this liquid refrigerant is supplied to the evaporators inside the display cases through expansion valves where it evaporates by absorbing the heat from the goods placed in the open display cases (and thus cools the goods). From the display cases, these low-pressure vapors are drained out to the suction manifold and from where it is passed again to the compressors and thus closing the refrigeration cycle.

Supermarket refrigeration systems are of hybrid nature because the control inputs (expansion valves and compressors) can only switch discretely. The nonlinear hybrid model of the supermarket refrigeration system can be divided into four individual models: the display cases, the suction manifold, the compressor rack and the condensing unit. For controlling of the compressor(s), the dynamics of the display cases and the suction manifold are important and the models of these are given in the following sections.

3.1 The Display Cases

Each of the display case can be described by four states: the temperature of the goods \( (T_{\text{goods}}) \), the temperature of the evaporator wall \( (T_{\text{wall}}) \), the air temperature \( (T_{\text{air}}) \) and the mass of the liquefied refrigerant in the evaporator \( (M_{rfg}) \). The model is given as follows:

\[
\frac{dT_{\text{goods}}}{dt} = \frac{-\dot{Q}_{\text{goods-air}}}{M_{\text{goods}}C_p_{\text{goods}}},
\]

\[
\frac{dT_{\text{wall}}}{dt} = \frac{\dot{Q}_{\text{air-wall}} - \dot{Q}_e}{M_{\text{wall}}C_p_{\text{wall}}},
\]

\[
\frac{dT_{\text{air}}}{dt} = \frac{\dot{Q}_{\text{goods-air}} + \dot{Q}_{\text{airload}} - \dot{Q}_{\text{air-wall}}}{M_{\text{air}}C_p_{\text{air}}},
\]

\[
\frac{dM_{rfg}}{dt} = \begin{cases} 
\frac{M_{rfg\text{,max}} - M_{rfg}}{\tau_{\text{fill}}} & \text{if } u_{\text{disp}} = 1, \\
\frac{-\dot{Q}_e}{\Delta h_{\text{fg}}} & \text{if } u_{\text{disp}} = 0, M_{rfg > 0}, \\
0 & \text{if } u_{\text{disp}} = 0, M_{rfg = 0},
\end{cases}
\]

where \( \dot{Q}_{\text{airload}} \) is an external heat load (disturbance) on each of the display cases. \( M \) denotes the mass and \( C_p \) is the heat capacity with the subscript denoting the media (goods, wall and air). \( Q_e \) is the heat transferred in the evaporator which can also be called as cooling capacity.

\[
\dot{Q}_{\text{goods-air}} = UA_{\text{goods-air}} \cdot (T_{\text{goods}} - T_{\text{air}}),
\]

\[
\dot{Q}_{\text{air-wall}} = UA_{\text{air-wall}} \cdot (T_{\text{air}} - T_{\text{wall}}),
\]

\[
\dot{Q}_e = UA_{\text{wall-rfg}}(M_{rfg}) \cdot (T_{\text{wall}} - T_e(P_e)).
\]

In equations (9), (10) and (11) the term \( UA \) represents the heat transfer coefficient with the subscripts indicate the media between which the heat is transferred. \( T_e(P_e) \) in equation (10) represents the evaporation temperature and is a refrigerant-dependent function of the evaporation pressure \( P_e \). By assuming no pressure drop in the suction line, both the evaporation pressure \( P_e \) and the suction pressure \( P_{\text{suc}} \) are equal \( (P_e = P_{\text{suc}}) \). The term \( UA_{\text{wall-rfg}} \) in equation (11) is the heat transfer coefficient between the evaporator wall and the refrigerant and is a function of the mass of the refrigerant in the evaporator given by:

\[
UA_{\text{wall-rfg}}(M_{rfg}) = UA_{\text{wall-rfg\text{,max}}} \frac{M_{rfg}}{M_{rfg\text{,max}}},
\]

where \( M_{rfg} = M_{rfg\text{,max}} \) when the evaporator in a display case is completely filled with the refrigerant. The accumulation of the liquefied refrigerant in the evaporator is given by equation (8), in which the parameter \( \tau_{\text{fill}} \) is the filling time of the evaporator when a display valve is opened \( (u_{\text{disp}} = 1) \) and when the display valve is closed \( (u_{\text{disp}} = 0) \) and all the refrigerant has evaporated \( (M_{rfg} = 0) \), then the value of \( \Delta h_{\text{fg}} \) becomes zero. The term \( \Delta h_{\text{fg}} \) is the specific latent heat of the remaining refrigerant in the
evaporator of a display case which is a nonlinear function of the evaporation pressure \(P_e\).

3.2 The Suction Manifold

The dynamics of the suction manifold can be described by only one state, which is the suction pressure \(P_{suc}\) given by the following equation:

\[
\frac{dP_{suc}}{dt} = \frac{\dot{m}_{in-suc} + \dot{m}_{rfg, const} - \dot{V}_{comp} \cdot \rho_{suc}}{V_{suc} \cdot \frac{dP_{suc}}{dp_{suc}}}, \tag{13}
\]

where \(V_{comp}\) is the volume drained out of the suction manifold by the compressor, \(V_{suc}\) is the total volume of the suction manifold, \(\dot{m}_{rfg, const}\) is a constant mass flow into the manifold from other unmodelled refrigerated entities (e.g. cold storage rooms), \(\rho_{suc}\) is the density of the vapor refrigerant in the suction manifold and is a nonlinear refrigerant-dependent function of \(P_{suc}\) and \(T_{SH}\), which is the superheat in the suction manifold.

3.3 The Compressor

In a supermarket refrigeration system, the total compressor capacity is discrete-valued because compressors can only be switched on or off. The dynamics of the compressor bank can be modeled by a constant volumetric efficiency \(\eta_{vol}\) and by a total displacement volume \(V_d\). The volume flow \((\dot{V}_{comp})\) out of the suction manifold by a single compressor is given by

\[
\dot{V}_{comp} = u_{comp} \cdot C_{comp} \cdot \frac{1}{100} \cdot \eta_{vol} \cdot V_d \tag{14}
\]

where \(C_{comp}\) is the capacity of the compressor and \(u_{comp}\) is used for switching on and off the compressor and \(\dot{V}_{comp} = \sum_{i=1}^{n_c} \dot{V}_{comp}\) is the total volume flow.

In the above nonlinear hybrid model of a supermarket refrigeration system, a number of refrigerant-dependent functions are used, i.e. \(\rho_{suc}, T_e(P_e), \frac{d\rho_{suc}}{dT_{suc}}\) and \(\Delta h_{lg}\). More details about these functions and the values of all the parameters can be found in Larsen et al. (2007).

4. OPTIMIZATION PROBLEM

4.1 System states and inputs

In this work, we only consider one display case and one compressor and demonstrate the advantages of using multi-stage NMPC for the system under uncertainty. The states and inputs vectors for the overall model of the supermarket refrigeration system are given by

\[
x = [T_{goods}, T_{wall}, T_{air}, M_{rfg}, P_{suc}]^T,
\]

and

\[
u = [u_{disp}, u_{comp}]^T.
\]

The inputs \(u\) are integer variables and take either 0 or 1, i.e. \(u_{disp} \in \{0, 1\}\) and \(u_{comp} \in \{0, 1\}\). Because of the nonlinear dynamics and the presence of inputs as integer variables, the resulting optimization problem is a Mixed Integer Nonlinear Programming (MINLP) problem. In Sarabia et al. (2007), the authors proposed a scheme to convert the MINLP problem to a Nonlinear Programming (NLP) problem by parameterizing the input variables in terms of continuous variables which represent on/off times. The optimizer will choose the on and off times of display valves and the compressors instead of choosing the control inputs directly. With this scheme, the control inputs of the resulting NLP can be given as follows:

\[
u_p = [\dot{t}_{on}^{disp}, \dot{t}_{off}^{disp}, \dot{t}_{on}^{comp}, \dot{t}_{off}^{comp}]^T.
\]

At every sampling interval, the decision variables are \(t_{on}^{disp}, t_{off}^{disp}\) for the operation of the expansion valve for the display case and \(t_{on}^{comp}, t_{off}^{comp}\) are the decision variables to switch on and off the compressors. Based on the results of the optimization problem, the control is applied. If the decision is \(t_{on}^{disp} = t_{off}^{disp} = t_s/2\), the valve is opened for half the sampling time \(t_s\) and closed for the remaining time.

4.2 Objective function

In the supermarket refrigeration system, the control of the compressor is the most crucial part because a higher number of start/stops of the compressor may reduce its life. So, the main objective for the controller is to reduce the number of switches to prolong the lifetime of the compressor in the presence of uncertain parameters \(Q_{airload}\) (heat load on the display case) and \(\dot{m}_{rfg, const}\) (unmodeled mass flow of the refrigerant). The uncertainty \(Q_{airload}\) is assumed to be \(\pm 50\%\) of the nominal value and \(\dot{m}_{rfg, const} \in [0, 0.2]\). The air temperature \(T_{air}\) is used as an indirect measure of the temperature of the goods and hence the constraints on \(T_{air}\) are crucial in the presence of uncertainty because its violation will indirectly result in deterioration of the foods. In addition, there is also constraint on suction pressure \(P_{suc}\) which must be met at all times. The stage cost of the NMPC problem is given by

\[
\mathcal{L} = -K_{v1} \cdot (t_{on}^{disp} - t_{off}^{disp})^2
- K_{v2} \cdot (t_{on}^{comp} - t_{off}^{comp})^2 + \sum_{i=1}^{n_i} r_i \Delta u_{p,i}^2 \tag{15}
\]

where first term represents the control of the discrete expansion valve attached with the display case and the manipulated variables used for its control are \(t_{on}^{disp}\) and \(t_{off}^{disp}\) and the second term represents the control of the compressor with the manipulated variables \(t_{on}^{comp}\) and \(t_{off}^{comp}\). \(K_{v1} \) and \(K_{v2}\) are the weights associated with switching of each terms. \(r_i\) represents the weight of the penalty for change in the control movements for all \(n_i = 2 \cdot n_u\) parameterized inputs. By the choice of proper weights \(K_{v1}\) and \(K_{v2}\) the switching can be avoided within a given sampling time and \(r_i\) penalizes the change and thus tasks the optimizer to reduce the number of switches.

4.3 Constraints on states and inputs

The constraints on the states and their initial conditions are given in Table 1 and the bounds on the manipulated variables are given in Table 2.
5. RESULTS

The nonlinear dynamics is discretized using the control vector parameterization method and the integration is performed using a fourth order Runge-Kutta integrator, and CasADi (see Andersson et al. (2012)) is used for the automatic generation of first and second order derivatives of the resulting NLP and it is solved using IPOPT (Wächter and Biegler, 2006). The efficient and modular implementation of multi-stage NMPC is available online and can be downloaded from: https://github.com/do-mpc/do-mpc (Lucia et al., 2014b).

The prediction horizon used for the controller is \( N_P = 10 \) time steps and its sampling time is \( t_s = 10 \) sec. The scenario tree for multi-stage NMPC is generated using the maximum and minimum values of the uncertainties \( Q_{	ext{airload}} \in [1500, 4500] \) and \( m_{rfg,\text{const}} \in [0, 0.2] \) with a robust horizon of \( N_r = 1 \). The possible combination of both the uncertainties gives rise to 4 scenarios \( \{(1500, 0), (1500, 0.2), (4500, 0), (4500, 0.2)\} \) in total and all the scenarios are assumed to have uniform probability. The objective function used is given by equation (15). The weights \( K_1 \) and \( K_2 \) are \( 7.5 \times 10^{-3} \) and \( 10^{-2} \) and the penalty for change in control movements is \( 10^{-3} \) for the control movements is \( 10^{-3} \) for the compressor. The path constraints are implemented as soft constraints as the standard NMPC solution became infeasible when the constraints are hard. In Figure 2, the simulation results for the case of true values of the uncertain parameters \( Q_{	ext{airload}} = 1500 \) J/sec and \( m_{rfg,\text{const}} = 0.2 \) kg/sec are shown. The control inputs for the standard NMPC and the multi-stage NMPC are shown separately, while the states are shown in the same subplots. It can be seen that the lower bound of the temperature constraint \( T_{	ext{air}} \) is violated when standard NMPC with nominal model is used. This is because the model used considers a high heat flow to the display case than the reality and because of this more cooling power is supplied. This results in violent oscillation of the input \( u_{\text{comp}} \) which is not desired. When multistage NMPC is used, the constraints are satisfied at all times and the plant is operated normally.

Figure 3 gives the simulation results of both the standard and the multi-stage NMPC for the true plant values of the uncertain parameters given as \( Q_{	ext{airload}} = 4500 \) J/sec and \( m_{rfg,\text{const}} = 0.2 \) kg/sec. The upper bound of the temperature constraint \( T_{	ext{air}} \) is violated when standard

Table 1. Initial conditions and constraints on states

| State      | Init.cond. | Min. | Max. | unit   |
|------------|------------|------|------|--------|
| \( T_{\text{goods}} \) | 1.0        | 0.0  | 5.0  | °C     |
| \( T_{\text{wall}} \) | 1.0        | -5.0 | 5.0  | °C     |
| \( T_{	ext{air}} \) | 3.5        | 2.0  | 5.0  | °C     |
| \( M_{rfg} \) | 0.0        | 0.0  | 1.0  | kg     |
| \( P_{\text{vac}} \) | 1.1        | 1.0  | 1.7  | bar    |

Table 2. Bounds on control inputs

| Input          | Min. | Max. | unit |
|----------------|------|------|------|
| \( u_{\text{comp}} \) | 0    | \( t_s \) | sec  |
| \( \text{on, off comp} \) | 0    | \( t_s \) | sec  |
NMPC is used and this happens because of the wrong predictions that result due to the wrong model. Multi-stage NMPC satisfies the constraints at all times but for both the discussed cases, the periodic switching frequency is higher in case of multi-stage NMPC compared to the standard NMPC controller. This is because multi-stage NMPC considers all different possible scenarios and includes the satisfaction of constraints for scenarios different to the reality as well. This is a small price to be paid for the robust scheme for the system under uncertainty. In return, we get constraint satisfaction and a better overall performance for all scenarios and violent oscillations are completely avoided.

The simulation results of the standard NMPC and multi-stage NMPC for the true value of the uncertain parameters \(Q_{\text{airload}} = 4500 \text{ J/sec}\) and \(\dot{m}_{\text{rfg, const}} = 0 \text{ kg/sec}\) are shown in Fig. 4. Both the temperature constraint \(T_{\text{air}}\) and lower constraint on the suction pressure \(P_{\text{suc}}\) are violated when standard NMPC with the nominal model is used and the control input starts to oscillate as seen earlier. Since the predictions are always significantly wrong, the controller is not able to recover and this case represents the worst case behavior of the standard NMPC controller. In the case of multi-stage NMPC, the constraints are satisfied and the control input of the compressor switches only when necessary and the switching of the compressor is significantly reduced. The average computation time per step for the standard NMPC is 0.27 sec and for the multi-stage NMPC scheme, it is 1.94 sec.

6. CONCLUSION

Standard NMPC applied without regard to the presence of uncertainty lead to constraint violations and an unstable behavior for the example of a supermarket refrigeration system. Multi-stage NMPC showed a stable performance of the controller by satisfaction of the constraints at all times and the unwanted switching behavior because of the presence of the uncertainty is mitigated. In future works, the results will be compared with the original MINLP solution and the approximation of the MINLP with regularization functions will be studied.

REFERENCES

Andersson, J., Åkesson, J., and Diehl, M. (2012). CasADi – A symbolic package for automatic differentiation and optimal control. In S. Forth, P. Hovland, E. Phipps, J. Utke, and A. Walther (eds.), Recent Advances in Algorithmic Differentiation, Lecture Notes in Computational Science and Engineering, 297–307. Springer, Berlin.

Engell, S. (2007). Feedback control for optimal process operation. Journal of Process Control, 17, 203–219.

Hovgaard, T.G., Boyd, S., Larsen, L.F., and Jørgensen, J.B. (2013). Nonconvex model predictive control for commercial refrigeration. International Journal of Control, 86(8), 1349–1366.

Larsen, L.F., Geyer, T., and Morari, M. (2005). Hybrid model predictive control in supermarket refrigeration systems. In World Congress, volume 16, 335–335.

Lucia, S., Izadi-Zananaabadi, R., Wisniewski, R., and Sonntag, C. (2007). Supermarket refrigeration systems—a benchmark for the optimal control of hybrid systems. Hycon, http://astwww. bci.uni-dortmund.de/hycon4b/wpcontent/sr.pdf.

Lucia, S., Andersson, J., Brandt, H., Diehl, M., and Engell, S. (2014a). Handling uncertainty in economic nonlinear model predictive control: a comparative case-study. Journal of Process Control, 24, 1247–1259.

Lucia, S., Finkler, T., and Engell, S. (2013). Multi-stage nonlinear model predictive control applied to a semi-batch polymerization reactor under uncertainty. Journal of Process Control, 23, 1306–1319.

Lucia, S., Tatulea-Codrean, A., Schoppmeyer, C., and Engell, S. (2014b). An environment for the efficient testing and implementation of robust NMPC. In Proc. of the 2014 IEEE Multi-Conference on Systems and Control, 1843–1848.

Sarabia, D., Capraro, F., Larsen, L.F.S., and de Prada, C. (2007). Hybrid Control of a Supermarket Refrigeration System. In American Control Conference, 4178–4185.

Sonntag, C., Devanathan, A., Engell, S., and Stursberg, O. (2007). Hybrid nonlinear model-predictive control of a supermarket refrigeration system. In IEEE International Conference on Control Applications, 1432–1437.

Sonntag, C., Kölling, M., and Engell, S. (2009). Sensitivity-based predictive control of a large-scale supermarket refrigeration system. In Internaional Symposium on Advanced Control of Chemical Processes (ADCHEM), 354–359.

Wächter, A. and Biegler, L. (2006). On the implementation of a primal-dual interior point filter line search algorithm for large-scale nonlinear programming. Mathematical Programming, 106, 25–57.