JOINT POWER AND ADMISSION CONTROL VIA $p$ NORM MINIMIZATION DEFLATION

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ABSTRACT
In an interference network, joint power and admission control aims to support a maximum number of links at their specified signal to interference plus noise ratio (SINR) targets while using a minimum total transmission power. In our previous work, we formulated the joint control problem as a sparse $\ell_0$-minimization problem and relaxed it to a $\ell_1$-minimization problem. In this work, we propose to approximate the $\ell_0$-optimization problem by a $p$ norm minimization problem where $0 \leq p \leq 1$, since intuitively $p$ norm will approximate $0$ norm better than $1$ norm. We first show that the $\ell_p$-minimization problem is strongly NP-hard and then derive a reformulation of it such that the well developed interior-point algorithms can be applied to solve it. The solution to the $\ell_p$-minimization problem can efficiently guide the link’s removals (deflation). Numerical simulations show the proposed heuristic outperforms the existing algorithms.

1. INTRODUCTION

Power control is an effective tool for interference management in cellular, ad-hoc, and cognitive underlay networks [1,2,3,4,5,6,7,8]. The prevailing formulation of power control aims to use a minimum total transmission power to support all links in an interference network at their desired SINR targets. A longstanding issue associated with power control is that the problem often becomes infeasible, i.e., it is not possible to simultaneously support all links in the network at their SINR targets. In this case, we must adopt a joint power and admission control approach to selectively remove some links from the network so that the remaining ones can be simultaneously supported at their desired SINR levels. Our goal is to maximize the number of simultaneously supportable links at their required SINR targets while using a minimum total transmission power.

Theoretically, the joint power and admission control problem is known to be NP-hard to solve to global optimality [1,3] and to approximate to constant ratio global optimality [6], so various heuristic algorithms [1,2,3,6,7,8] have been proposed for this problem. Among them, the reference [1] proposed a convex approximation-based algorithm for the joint power and admission control problem. Instead of directly solving the original NP-hard problem, the basic idea of the proposed linear programming deflation (LPD) algorithm in [1] is to approximate the problem by an appropriate convex problem. The solution to the approximation problem can be used to check the feasibility of the original problem and guide link’s removals. The removal procedure is terminated until all the remaining links in the network are simultaneously supportable. The recent work [6] developed another LP approximation-based new linear programming deflation (NLPD) algorithm for the joint power and admission control problem. In [6], the joint power and admission control problem is first equivalently reformulated as a sparse $\ell_0$-minimization problem and then its $\ell_1$-convex approximation is used to derive a LP, which is different from the one in [1]. Again, the solution to the derived LP can guide an iterative link removal procedure, and the removal procedure is terminated if all the remaining links in the network are simultaneously supportable.

Based on the sparse $\ell_0$-minimization reformulation in [6], this paper proposes a new deflation algorithm based on $p$ ($0 \leq p \leq 1$) norm minimization for the joint power and admission control problem. Compared to the $\ell_1$-minimization problem, the $p$ norm minimization problem is closer to the original $\ell_0$-optimization problem. The $\ell_p$-approximation problem is solved by applying the efficient interior-point algorithm in [9] to solve its equivalent reformulation. Numerical results show that the proposed algorithm compares favorably with the existing approaches [1,2,6] in terms of the number of supported links, the total transmission power, and the CPU time.

Notations: We adopt the following notations in this paper. We denote the index set $\{1,2,\cdots,K\}$ by $K$. Lowercase boldface and uppercase boldface are used for vectors and matrices, respectively. For a given vector $x$, the notations $\max\{x\}$, $|x|_k$ and $\|x\|_p := \sum_k |x_k|^p$ ($0 \leq p < 1$) stand for its maximum entry, its $k$-th entry, and its $p$ norm, respectively. In particular, when $p = 0$, $\|x\|_0$ stands for the number of nonzero entries in $x$. Finally, we use $e$ to represent the vector of an appropriate size with all components being one and $I$ to represent the identity matrix of an appropriate size, respectively.

2. PROBLEM FORMULATION

Consider a $K$-link (a link corresponds to a transmitter-receiver pair) interference channel with channel gains $g_{kj} \geq 0$ (from transmitter $j$ to receiver $k$), noise power $n_k > 0$, SINR target $\gamma_k > 0$, and power

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1Strictly speaking, $\|x\|_p$ with $0 \leq p < 1$ is not a norm, since it does not satisfy the triangle inequality. However, we still call it a norm for convenience in this paper.
budget $\bar{p}_k > 0$ for $k, j \in \mathcal{K} := \{1, 2, \cdots, K\}$. Denote the power allocation vector by $p = (p_1, p_2, \cdots, p_K)^T$ and the power budget vector by $\bar{p} = (\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_K)^T$. Treating interference as noise, we can write the SINR at the $k$-th receiver as

$$\text{SINR}_k = \frac{g_{kk}p_k}{\eta_k + \sum_{j \neq k} g_{kj}p_j}, \quad \forall k \in \mathcal{K}.\]$$

The joint power and admission control problem can be mathematically formulated as a two-stage optimization problem. Specifically, the first stage maximizes the number of admitted links:

$$\begin{aligned}
\max_{p, S} & \quad |S| \\
\text{s.t.} & \quad \text{SINR}_k \geq \gamma_k, k \in S \subseteq \mathcal{K}, \\
& \quad 0 \leq p_k \leq \bar{p}_k, \quad k \in S.
\end{aligned} \tag{1}$$

We use $S_0$ to denote the optimal solution for problem (1) and call it the maximum admissible set. Notice that the solution for (1) might not be unique. The second stage minimizes the total transmission power required to support the admitted links in $S_0$:

$$\begin{aligned}
\min_{(p_k)_{k \in S_0}} & \quad \sum_{k \in S_0} p_k \\
\text{s.t.} & \quad \text{SINR}_k \geq \gamma_k, k \in S_0, \\
& \quad 0 \leq p_k \leq \bar{p}_k, \quad k \in S_0.
\end{aligned} \tag{2}$$

Due to the special choice of $S_0$, power control problem (2) is feasible and can be efficiently and distributively solved by the Foschini-Miljanic algorithm.

3. REVIEW OF THE NLPD ALGORITHM

Since the developed algorithm in this paper follows the similar idea as the NLPD algorithm in [6], we first briefly review the NLPD algorithm in this section. The basic idea of the NLPD algorithm is to update the power and check whether all links can be supported or not. If the answer is yes, then terminate the algorithm; else drop one link from the network and update the power again. The above process is repeated until all the remaining links are supported.

We begin with the introduction of an equivalent normalized channel on which the NLPD algorithm is based. In particular, we use $q = (q_1, q_2, \cdots, q_K)^T$ with $q_k = p_k/\bar{p}_k$ to denote the normalized power allocation vector, and use $c = (c_1, c_2, \cdots, c_K)^T$ with $c_k = (\gamma_k/\eta_k)/(g_{kk}/\bar{p}_k) > 0$ to denote the normalized noise vector. We denote the normalized channel matrix by $A \in \mathbb{R}^{K \times K}$ with the $(k, j)$-th entry

$$a_{kj} = \begin{cases} 1, & \text{if } k = j; \\ -\frac{\gamma_k g_{kj} \bar{p}_j}{g_{kk} \bar{p}_k}, & \text{if } k \neq j. \end{cases}$$

In fact, $|a_{kj}|$ is the normalized channel gain. It is simple to check that $\text{SINR}_k \geq \gamma_k$ if and only if $|Aq - c| \geq 0$.

In [6], we reformulate the two-stage joint power and admission control problem (1) and (2) as a single-stage optimization problem

$$\begin{aligned}
\min_{q_c, q} & \quad \|q_c\|_0 + \alpha_\ell p^T q \\
\text{s.t.} & \quad q_c = c - Aq, \\
& \quad 0 \leq q \leq e.
\end{aligned} \tag{3}$$

where $0 < \alpha < \alpha_\ell := 1/e^T p$, and $[q_c]_k$ measures the excess transmission power [1] that the transmitter of link $k$ needs to maintain for the normalized channel in order to serve its desired SINR target (assuming all other links keep their transmission powers unchanged).

Notice that the formulation (3) is capable of finding the maximum admissible set with minimum total transmission power. Since problem (3) is still NP-hard, we further consider its $\ell_1$-convex approximation (equivalent to a LP)

$$\begin{aligned}
\min_{q_c, q} & \quad \|q_c\|_1 + \alpha_\ell p^T q \\
\text{s.t.} & \quad q_c = c - Aq, \\
& \quad 0 \leq q \leq e.
\end{aligned} \tag{4}$$

By solving (4), we know whether all links in the network can be simultaneously supported or not. If not, we drop the link

$$k_0 = \arg \max_{k \in K} \left\{ \sum_{j \neq k} \left( |a_{kj}| \left| [q_c]_j + |a_{jk}| \left| [q_c]_k \right| \right) \right\}. \tag{5}$$

An easy-to-check necessary condition

$$\left( \mu^+ \right)^T e - \left( \mu^- + e \right)^T c \geq 0 \tag{6}$$

for all links in the network to be simultaneously supported is also derived in [6], where $\mu^+ = \max \{ \mu, 0 \}$, $\mu^- = \max \{ -\mu, 0 \}$, and $\mu = A^T e$. The necessary condition allows us to iteratively remove strong interfering links from the network. In particular, we remove the link $k_0$ according to the scheme

$$k_0 = \arg \max_{k \in K} \left\{ \sum_{j \neq k} |a_{kj}| + \sum_{j \neq k} |a_{jk}| + c_k \right\} \tag{7}$$

until (6) becomes true.

The NLPD algorithm can be described as follows.

The NLPD Algorithm

Step 1. Initialization: Input data $(A, c, \bar{p})$.

Step 2. Preprocessing: Remove link $k_0$ iteratively according to (7) until condition (6) holds true.

Step 3. Power control: Solve problem (4); check whether all links are supported: if yes, go to Step 5; else go to Step 4.

Step 4. Admission control: Remove link $k_0$ according to (5), set $K = K/\{k_0\}$, and go to Step 3.

Step 5. Postprocessing: Check the removed links for possible admission.

4. A $p$ NORM MINIMIZATION DEFLATION ALGORITHM

In this section, we develop a new deflation algorithm based on $\ell_p$-minimization for the joint control problem (1) and (2). As seen in Section 3 the original $\ell_0$-minimization problem (1) is successively approximated by the $\ell_1$-minimization problem (3) in the NLPD algorithm. Intuitively, the $p$ ($0 < p < 1$) norm minimization problem

$$\begin{aligned}
\min_{q_c, q} & \quad \|q_c\|_p + \alpha_\ell p^T q \\
\text{s.t.} & \quad q_c = c - Aq, \\
& \quad 0 \leq q \leq e.
\end{aligned} \tag{8}$$


Theorem 4.2 The $\ell_p$-minimization problem (8) is strongly NP-hard if $0 \leq p < 1$.

The complexity result in Theorem 4.1 motivates us to approximately solve problem (8). Next, we first give a reformulation of problem (8), and then propose to use the interior-point algorithm developed in [7] to solve it.

Theorem 4.2 The $\ell_p$-minimization problem (8) can be equivalently reformulated as

$$\min_{\mathbf{q} \in \mathbb{R}^m} \sum_k |\mathbf{q}_k|^p + \alpha \mathbf{p}^T \mathbf{q}$$

s.t. $\mathbf{q} = \mathbf{c} - \mathbf{A} \mathbf{q}$,

$$0 \leq \mathbf{q} \leq \mathbf{e}, \quad \mathbf{q}_e \geq 0.$$  

Having obtained the solution $(\mathbf{q}_e, \mathbf{q}, \mathbf{s})$ of (10), we use the removal strategy called SMART rule in [3] to drop the link $k_0$ according to

$$k_0 = \arg \max_{k \in K} \left\{ \sum_{j \neq k} |a_{kj}| q_j + \sum_{j \neq k} |a_{jk}| q_k + c_k \right\}.$$  

The above operation can be interpreted as removing the link with the largest interference plus noise footprint in the normalized network.

The PNMD Algorithm

Step 1. Initialization: Input data $(\mathbf{A}, \mathbf{c}, \mathbf{p})$.

Step 2. Preprocessing: Remove link $k_0$ iteratively according to (7) until condition (6) holds true.

Step 3. Power control: Compute the parameter $\alpha$ and solve problem (10); check whether all links are supported: if yes, go to Step 5; else go to Step 4.

Step 4. Admission control: Remove link $k_0$ according to some removal strategy, set $K = K \setminus \{k_0\}$, and go to Step 3.

Step 5. Postprocessing: Check the removed links for possible admission.
Fig. 1. Average number of supported links versus the number of total links.

Fig. 2. Average CPU time versus the number of total links.

Fig. 3. Average transmission power versus the number of total links.

5. NUMERICAL SIMULATIONS

We generate the same channel parameters as in [11] in our numerical simulations, i.e., each transmitter’s location obeys the uniform distribution over a 2 Km × 2 Km square and the location of its corresponding receiver is uniformly generated in a disc with radius 400 m; channel gains are given by $g_{kj} = 1/d_{kj}^2$ ($\forall k, j \in K$), where $d_{kj}$ is the Euclidean distance from the link of transmitter $j$ to the link of receiver $k$. Each link’s SINR target is set to be $\gamma_k = 2$ dB ($\forall k \in K$) and the noise power is set to be $\eta_k = -90$ dBm ($\forall k \in K$). The power budget of the link of transmitter $k$ is $p_k = 2p_{k,\min}$ ($\forall k \in K$), where $p_{k,\min}$ is the minimum power needed for link $k$ to meet its SINR requirement in the absence of any interference from other links.

The parameter $p$ in problem (6) is set to be 0.5 and the ones in (12) are set to be $c_1 = c_2 = 0.2$ and $c_3 = 4$. The number of supported links, the total transmission power, and the CPU time are the metrics we employ to compare the performance of the proposed PNMD algorithm with that of the LPD algorithm in [1], the Algorithm II-B in [2], and the NLPD algorithm in [6]. All figures are obtained by averaging over 200 Monte-Carlo runs.

Figs. 1, 2, and 3 indicate that the PNMD algorithm can take less CPU time to support more links while with less total transmission power than the existing algorithms (except the Algorithm II-B). As shown in Fig. 3, the Algorithm II-B transmits the least power among the tested algorithms. This is because the Algorithm II-B supports the least number of links; see Fig. 1. In particular, compared to the NLPD algorithm, the proposed PNMD algorithm can support (slightly) more links with much less total transmission power, and at the same time takes less CPU time.

The performance improvement of the proposed PNMD algorithm over the NLPD algorithm is mainly attributed to the $\ell_p$-approximation problem (6). The simulation results in Fig. 1 and Fig. 3 show that the admissible set $S_1$ obtained by the proposed PNMD algorithm based on the $\ell_p$-approximation problem (6) is “better” than the admissible set $S_2$ obtained by the NLPD algorithm based on the $\ell_1$-approximation problem (3), i.e., although the cardinality of the two admissible sets $S_1$ and $S_2$ is nearly equal to each other, it takes much less total transmission power to support the links in $S_1$ than to support the links in $S_2$. This is consistent with our intuition that the $p$ ($0 < p < 1$) norm minimization problem (8) is capable of approximating the $\ell_0$-minimization problem (3) better than the $\ell_1$-minimization problem (9) and the fact that the maximum admissible set for the joint power and admission control problem may not be unique.

6. CONCLUSIONS

In this paper, we have developed a $p$ ($0 < p < 1$) norm minimization deflation algorithm for the joint power and admission control problem. Numerical simulations show the proposed algorithm outperforms state-of-the-arts in [126] in terms of the number of supported links, the total transmission power, and the CPU time.

\footnote{To the best of our knowledge, the NLPD algorithm is so far the best removal-based algorithm for the joint power and admission control problem. It is shown in [6] that the NLPD algorithm can achieve more than 98% of global optimality in terms of the number of supported links when $K \leq 18$.}
7. REFERENCES

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