Critical spin transport in Bose gases

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Abstract. We consider spin transport in a two-component atomic Bose gas in three dimensions, at temperatures just above the critical temperature for Bose–Einstein condensation. In these systems, the spin conductivity is determined by spin drag, i.e. frictional drag between the two spin components due to interactions. We find that in the critical region, the temperature dependence of the spin conductivity deviates qualitatively from the Boltzmann result and is fully determined by the critical exponents of the phase transition. We discuss the size of the critical region where these results may be observed experimentally.

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1. Introduction

The research field of spin electronics or spintronics, concerned with practical applications of the electron spin, has recently shown renewed interest in spin currents [1]. In part as a result of these efforts, it is now understood that there are several fundamental differences between
charge currents and currents of spin angular momentum. For example, the latter are even under time reversal-symmetry operations and can thus in principle flow without dissipation in ordinary conductors \cite{2,3}, contrary to electric currents. Furthermore, the charge conductivity is infinitely large in Galilean invariant systems, whereas spin currents can then still decay due to spin-drag effects, i.e. the friction between two spin states due to interactions \cite{4,5}. Finally, spin and charge currents couple in a completely different way to other degrees of freedom in the system, most notably order parameters such as magnetization or a superconducting condensate. For example, a spin current can exert a so-called spin transfer torque on the magnetization of a ferromagnet \cite{6–9}, a phenomenon that is currently intensively studied in part because of its promise for magnetic-memory applications. On a more fundamental level, there have been several studies on the interplay between spin currents and the critical fluctuations in magnetization that occur close to the Curie temperature for the ferromagnetic phase transition \cite{10}. Such magnetic phase transitions form, together with superconducting phase transitions, the overwhelming majority of phase transitions occurring in electronic condensed-matter physics.

In this paper, we consider the effect of another phase transition, i.e. Bose–Einstein condensation, on spin transport. The system we consider is a spin mixture of trapped ultracold bosonic alkali atoms that differs in several important ways from electronic solid-state systems. Firstly, the particles are bosons rather than fermions (electrons). Secondly, cold-atom systems are disorder-free and hence the only contribution to spin conductivity is the above-mentioned spin-drag effect. The atomic interactions that are responsible for this drag are short-ranged as opposed to the Coulomb interactions between the electrons.

Spin drag in bosonic cold-atom mixtures was recently studied by two of us using an approach based on the Boltzmann equation. It was found that the bosonic nature of the particles lead to an enhancement of spin-drag effects at low temperatures \cite{11}. This should be contrasted with the Fermi-liquid behavior that, as a result of Pauli blocking, leads to the suppression of interaction effects at low temperatures so that the so-called spin-drag relaxation rate $1/\tau_{sd}(T)$, which is equal to the inverse of the spin-transport relaxation time $\tau_{sd}(T)$, vanishes quadratically with temperature $T$ \cite{12–14} except in the vicinity of a superconducting \cite{15} or ferromagnetic phase transition \cite{16}. For bosons close to the critical temperature for Bose–Einstein condensation, it was found that the Boltzmann approach incorporates the phase transition at the mean-field level and gives $1/\tau_{sd}(T) - 1/\tau_{sd}(T_C) \sim -1/\xi(T) \sim T_C - T$ \cite{17}, where $\xi(T)$ is the correlation length that diverges at the phase transition.

Our main findings are presented in figure 1, which shows the spin-drag relaxation rate as a function of the distance to the critical point, determined by the difference of the chemical potential $\mu$ from its critical value $\mu_C$. The dotted line shows the Boltzmann result (described above) and the solid lines are the results found using an approach based on the Kubo formula and approximating the atomic self-energy with the so-called sunset Feynman diagram shown in figure 2. The Hartree diagram should in principle be included in the self-energy, but since this can be achieved by a simple redefinition of the chemical potential we do not consider it here. Within the latter approximation we find that the spin-drag relaxation rate qualitatively agrees with the Boltzmann result for temperatures not too close to the critical temperature, but in the critical region deviates and goes to zero at the phase transition according to $1/\tau_{sd}(T) \sim 1/\xi(T)$. As discussed in detail below, an exact scaling ansatz confirms that the spin-drag relaxation rate
Figure 1. Spin-drag relaxation rate $1/\tau_{sd}$ at constant temperature as a function of distance from the critical point expressed in terms of the chemical potential difference $\mu - \mu_C$. Lines 1, 2 and 3 represent the relaxation rate for $a/\Lambda_{th} = 9 \times 10^{-3}$, $2(9 \times 10^{-3})$ and $3(9 \times 10^{-3})$, respectively, where $a$ is the scattering length and $\Lambda_{th} = \sqrt{2\pi\hbar^2/(mk_BT)}$ is the thermal de Broglie wavelength. Upon approaching the transition from above, the spin-drag relaxation rate shows an upturn due to Bose enhancement that is ultimately completely suppressed by fluctuations in the critical region. The dotted line 4 represents the Boltzmann result that does not include critical fluctuations [17]. The small quantitative difference between lines 1–3 and the Boltzmann results far from criticality arises because we neglect vertex corrections in the calculations that lead to curves 1–3.

vanishes, and in terms of the critical exponents $z$, $\eta$ and $\nu$ we find that $1/\tau_{sd}(T) \sim 1/\xi^{z-d+2-2\eta}$. Below, we also discuss the size of the critical region where these effects can be measured.

2. Spin-drag conductivity

We consider a three-dimensional (3D) homogeneous gas of bosonic atoms of mass $m$, with two spin states that couple with opposite sign to an external force $F$. As discussed in more detail below, this force can in a cold-atom experiment be implemented by a magnetic-field gradient. This force leads to a nonzero spin current $j_s$ according to $j_s = \sigma_s F$, where $\sigma_s = n\tau_{sd}/m$ is the spin conductivity in terms of the spin-drag relaxation time and the density $n$ per spin state (we consider here only the balanced case where the densities of the two spin states are equal). The Kubo formula for spin conductivity is given by $\sigma_s = \lim_{\omega \to 0} \Im \Pi^{(+)\mu\nu}(k = 0, \omega)]/\omega$ in terms of the Fourier transform of the retarded spin-current–spin-current correlation function $\Pi_{\mu\nu}(x, t; x', t') = i\theta(t-t')(\langle [\hat{j}_s^{(\mu}(x, t), \hat{j}_s^{(\nu}(x', t')) \rangle)/\hbar$, where the expectation value $\langle \cdots \rangle$ is taken in equilibrium and

$$
\hat{j}_s^{(\mu}(x, t) = \frac{\hbar}{2mi} \sum_{\alpha \in \{\uparrow, \downarrow\}} \alpha [\hat{\psi}_{\alpha}^+(x, t) \nabla^{(\mu}\hat{\psi}_{\alpha}(x, t) - \text{h.c.}].
$$
In the above expression, the bosonic Heisenberg annihilation operator is denoted by \( \hat{\psi}(x, t) \) with \( \alpha \in \{\uparrow, \downarrow\} \) labeling the spin states, and we note that \( \alpha \) takes the respective numerical value + or − if it is not used as a label.

To evaluate the correlation function in the Kubo formula, we neglect interactions between atoms of like spins that are of minor importance for spin-drag effects. The Hamiltonian then reads

\[
\hat{H} = \int \! dx \sum_{\alpha \in \{\uparrow, \downarrow\}} \hat{\psi}^{\dagger}(x, t) \left[ -\frac{h^2 \nabla^2}{2m} - \mu \right] \hat{\psi}(x, t) + T^{2B} \int \! dx \hat{\psi}^{\dagger}(x, t) \hat{\psi}^{\dagger}(x, t) \hat{\psi}(x, t) \hat{\psi}(x, t),
\]

with \( T^{2B} = 4\pi h^2 a/m \) being the two-body \( T \) matrix in terms of the interatomic scattering length \( a \) between the two spin states.

Upon ignoring vertex corrections to the correlation function \( \Pi^{(i)}_{\hat{\mu}(\mu)}(k, \omega) \), the Kubo formula for the spin conductivity is worked out to yield

\[
\sigma_s = -\frac{\pi h^3}{3m^2} \sum_{\alpha \in \{\uparrow, \downarrow\}} \int \! \frac{dk}{(2\pi)^3} k^2 \int \! d\omega \frac{dN_B(\hbar \omega)}{d\omega} \rho_\alpha^2(k, \omega),
\]

with \( \rho_\alpha(k, \omega) = -\frac{1}{\pi} \text{Im}[G_\alpha^{(r)}(k, \omega)]/\hbar \) being the spectral function for particles with spin \( \alpha \), obtained from the Fourier transform of their retarded Green’s function that is defined by \( G_\alpha^{(r)}(x, t; x', t') = i\theta(t - t')\langle [\hat{\psi}_\alpha(x, t), \hat{\psi}_\alpha^{\dagger}(x', t')] \rangle \). Furthermore, \( N_B(\hbar \omega) = [e^{\beta \hbar \omega} - 1]^{-1} \) is the Bose–Einstein distribution function, with \( \beta = 1/k_B T \) being the inverse thermal energy.
Figure 3. The spectral function at low momentum inside (left) and outside (right) the critical region. The dashed and dotted lines show the value of $\epsilon_k - \mu$ for the spectral function inside and outside the critical region, respectively.

The lowest-order (in interatomic interactions) diagram that gives a finite conductivity is the sunset diagram in figure 2 for the atomic self-energy

$$\Im \{\hbar \Sigma^{(+)} (k, \omega)\} = -\pi (T^{2B})^2 \int \frac{dk_2}{(2\pi)^3} \int \frac{dk_3}{(2\pi)^3} \int \frac{dk_4}{(2\pi)^3}$$

$$\times (2\pi)^3 \delta (k + k_2 - k_3 - k_4) \delta (\omega + \mu + \epsilon_k - \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4})$$

$$\times \{N_B(k_2)(1 + N_B(k_3))(1 + N_B(k_4)) - (1 + N_B(k_2))N_B(k_3)N_B(k_4)\},$$

with $\epsilon_k = \hbar^2 k^2/2m$. The imaginary part of the self-energy is calculated numerically from equation (3). Its real part is obtained by using a Kramers–Kronig relation. The spectral function then follows by using $G^{(+)}(k, \omega) = \hbar/(\hbar \omega - \epsilon_k + \mu - \hbar \Sigma^{(+)}(k, \omega)).$ This real part shifts the critical chemical potential from its noninteracting value of zero to the positive value $\mu_C = \Re \{\hbar \Sigma^{(+)}(0, 0)\}$.

The result for the spectral function is shown in figure 3 as a function of frequency. For a given momentum, the spectral function exhibits a rather sharp Lorentzian peak corresponding to a quasi-particle excitation. For such a Lorentzian spectral function, the frequency integral in equation (2) can be performed and the resulting conductivity is then found to be proportional to the lifetime $\tau(k) = -\hbar/(2\Im \{\hbar \Sigma^{(+)}(k, \omega)\})$ of the quasi-particle, where $\omega_k$ is the solution to $\hbar \omega_k = \epsilon_k - \mu + \Re \{\hbar \Sigma^{(+)}(k, \omega_k)\}$. The evaluation of the expression for spin conductivity in equation (2) with the above expression for self-energy leads to the results shown in figure 1 for various scattering lengths.

To gain more insight into the breakdown of the Boltzmann approach in the critical region, we consider the spectral function at low momentum inside (outside) the critical region, corresponding to the left (right) peak in figure 3. The vertical lines correspond to $\epsilon_k - \mu$. The peak in the spectral function is shifted considerably from its non-interacting value $\epsilon_k - \mu$ in the...
critical region. Since the Boltzmann approach does not take into account shifts in the quasi-particle energy beyond first order in the interaction, it does not correctly capture the low-momentum dependence of the lifetime of quasi-particles in the critical region. The latter is crucial for the divergence of the conductivity. The importance of the real part of atomic self-energy in the critical region is also demonstrated by its importance in determining the upward shift in the critical temperature due to interactions, which is correctly found to be of order $O(a/\Lambda_{th})$ [19].

3. Critical phenomena and scaling

From the calculation that is based on the sunset diagram for self-energy, we find numerically that the spin conductivity diverges as $\sigma_s \sim 1/\sqrt{\mu_C - \mu}$. This result is understood on a more general level by considering scale invariance of the system. Near criticality, we have as a result of the simple (linearized) renormalization-group flow near the fixed point that the spectral function scales as

$$\rho(\lambda k, \lambda^z \omega, \lambda^{1/\nu} (\mu - \mu_C)) = \frac{1}{\lambda^{2-\eta}} \rho(k, \omega, \mu - \mu_C),$$

(4)

where $\lambda$ is an arbitrary dimensionless scaling parameter and $z$, $\nu$ and $\eta$ are critical exponents. The relation between the correlation length $\xi$ and the chemical potential is $\xi \sim 1/|\mu - \mu_C|^\nu$. From this scaling ansatz and equation (2), we find that $\sigma_s \sim \xi^{z-d+2-2\eta} \sim |\mu - \mu_C|^{-\nu(z-d+2-2\eta)}$, with $d$ being the number of spatial dimensions.

For the sunset diagram in three dimensions, we have $\nu = 1/2$, $z = 2$ and $\eta = 0$, in agreement with the numerical results. It is interesting to note that the behavior of the spin conductivity and the spin-drag relaxation time depends not only on the static critical exponents $\nu$ and $\eta$, but also on the dynamical exponent $z$. We also note that, even though we ignored interactions between atoms with the same spin in our perturbative calculation based on the Feynman diagram in figure 2, the results based on the scaling ansatz are exact close to the critical temperature and do include these interactions.

Following the reasoning of Hohenberg and Halperin [20], the factor $\xi^{2-d}$ in these results is understood as follows. A spin-dependent force acting on a region with (fluctuating) spin density $n_s$ is balanced by viscous forces so that $n_s \xi^3 F \sim \xi^3 \eta_v v_s / \xi^2$, where $v_s$ is the spin velocity and $\eta_v$ the viscosity. Using that $J_s = n_s v_s = \sigma_s F$, this yields $\sigma_s \sim \xi^2 \langle n_s^2 \rangle / \eta_v$. We have that $\langle n_s^2 \rangle \sim \chi_s / \xi^d$ [20], with $\chi_s$ being the spin susceptibility. In order to obtain full agreement with our result found from the scaling ansatz, we thus need to have that the ratio $\chi_s / \eta_v \sim \xi^{z-2\eta}$. In future work we intend to investigate this conjecture in more detail.

4. Discussion and conclusions

We have incorporated the effect of critical fluctuations on the behavior of the spin-drag relaxation rate near the critical temperature for Bose–Einstein condensation. We found that the enhancement of the spin-drag relaxation rate due to Bose enhancement of interatomic interactions, predicted by the Boltzmann equation, is suppressed by critical fluctuations sufficiently close to the critical temperature. Numerically, we found the critical region to be proportional to the square of scattering length $|\Delta \mu| \approx 60(a/\Lambda_{th})^2$. An estimate based on the Ginzburg criterion [22] confirms this result. Hence, the size of the critical region may be
enlarged by increasing interatomic interactions near a Feshbach resonance. Furthermore, the Ginzburg criterion in $d$ dimensions leads to $|\Delta \mu| \sim \left( a / \Lambda_{\text{th}} \right)^{2/(4-d)}$. Reducing the dimensionality of the system therefore also increases the critical region. With respect to these remarks it is important to note that recent experiments with ultracold bosonic atoms have succeeded in accessing the critical region and measuring the exponent $\nu$ [21].

The spin conductivity and spin-drag relaxation rate can be measured directly in a drag measurement in which the two clouds of different spins feel a different force due to a magnetic-field gradient. Another method is to study the damping of the spin-dipole mode that is fully determined by the spin-drag relaxation rate.

The main approximation leading to our results is to neglect vertex corrections in the evaluation of the spin-current–spin-current response function. The Boltzmann equation is known to include vertex corrections that essentially lead to a replacement of the single-particle relaxation time by the appropriate transport relaxation time. In the absence of exact cancellations, which we do not expect to occur for the spin-drag conductivity, there is only a quantitative difference between these two time scales, and we attribute the difference between our Kubo approach and the Boltzmann approach sufficiently far away from the critical region to this difference in time scales and hence to neglecting vertex corrections. This assumption is strengthened by noting that far away from the critical region the Kubo and the Boltzmann approach have qualitatively the same temperature dependence. Unfortunately, an explicit inclusion of the effects of vertex corrections is required to fully corroborate our expectations, which is a highly challenging problem that is beyond the scope of this paper.

In this work, we have considered the effect of thermal critical fluctuations since the phase transition to the Bose–Einstein-condensed state takes place at nonzero temperature. An interesting direction for future work is to investigate also the influence of the vicinity of a quantum critical point [23] on the spin transport in Bose gases, for example by considering the system in an optical lattice where the Mott-insulator–superfluid quantum phase transition occurs [24]. An additional interesting feature of this system is that due to the presence of the optical lattice, which breaks Galilean invariance, now also charge (mass) transport can be considered.

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