Superrobust Geometric Control of a Superconducting Circuit

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Geometric phases accompanying adiabatic quantum evolutions can be used to construct robust quantum control for quantum information processing due to their noise-resilient feature. A significant development along this line is to construct geometric gates using nonadiabatic quantum evolutions to reduce errors due to decoherence. However, it has been shown that nonadiabatic geometric gates are not necessarily more robust than dynamical ones, in contrast to an intuitive expectation. Here we experimentally investigate this issue for the case of nonadiabatic holonomic quantum computation (NHQC) and show that conventional NHQC schemes cannot guarantee the expected robustness due to a cross-coupling to the states outside the computational space. We implement a different set of constraints for gate construction in order to suppress such cross coupling to achieve an enhanced robustness. Using a superconducting quantum circuit, we demonstrate high-fidelity holonomic gates whose infidelity against quasi-static transverse errors can be suppressed up to the fourth order, instead of the second order in conventional NHQC and dynamical gates. In addition, we explicitly measure the accumulated dynamical phase due to the above mentioned cross coupling and verify that it is indeed much reduced in our NHQC scheme. We further demonstrate a protocol for constructing two-qubit NHQC gates also with an enhanced robustness.

I. INTRODUCTION

Robust quantum operations are essential for noisy intermediate-scale quantum computation [1] with the existence of various error sources. Different strategies have been proposed for realizing such robust operations. One seminal example is quantum control based on geometric phases [2]. As a general and fundamental feature that accompanies quantum evolution, geometric phases are solely determined by global properties, rather than local details, of the evolution. Therefore, they are intrinsically robust against certain types of noise and control imperfections. Such a property has naturally been developed into the framework of geometric or holonomic quantum computation [3, 4], where quantum gates based on abelian or nonabelian geometric phases are realized through carefully engineering the involved quantum evolutions [5–13].

Early proposals of holonomic quantum computation utilize adiabatic evolution to suppress unwanted transitions among the instantaneous eigenstates of the Hamiltonians [14–16], which makes the resulting quantum gates have a long runtime and thus be sensitive to decoherence. To overcome such a problem, nonadiabatic holonomic quantum computation (NHQC) [17, 18] has been proposed to reduce the runtime of quantum gates [19–26]. Various NHQC schemes have been experimentally demonstrated on different physical platforms, including superconducting circuits [27–34], nuclear magnetic resonance [35–37], and nitrogen-vacancy centers in diamond [38–43]. However, it has been found that such NHQC gates are not significantly more robust than the standard dynamical ones. For example, infidelity of both types of gates exhibits a second-order dependence of control errors [44–49]. The missing of the “intrinsic” robustness theoretically expected for the NHQC gates (in fact, for other types of geometric gates as well) remains a puzzle in the community and needs to be resolved before such gates can ever become practically useful.

Here, by reexamining the design principles of NHQC gates following a recent theoretical work [50], we show that in conventional NHQC schemes, the phases used for gate construction may become a mixture of geometric and dynamical components due to a cross-coupling to the states outside the computational space, which compromises the robustness of geometric gates. This issue can be resolved by imposing a different set of constraints to the gate construction. Using a superconducting quantum circuit [51–53], we have experimentally demonstrated arbitrary single-qubit NHQC gates complying with such constraints, with an average fidelity of 0.9956 characterized by the standard randomized benchmark-
By defining a bright state as $|b⟩ = -\sin(\frac{\gamma}{2})e^{-i\theta}|g⟩ + \cos(\frac{\gamma}{2})|f⟩$ where $\phi = \phi_2(t) - \phi_1(t) - \pi$ and $\tan(\theta/2) = \Omega_{ge}(t)/\Omega_{ef}(t)$, one has $H(t) = \frac{1}{2}[\Omega(t)e^{i\phi(t)}|b⟩⟨e| + H.c.]$ with $\Omega(t) ≡ \sqrt{\Omega_{ge}(t)^2 + \Omega_{ef}(t)^2}$. We further define a dark state $|d⟩ = \cos(\theta/2)e^{-i\phi}|g⟩ + \sin(\theta/2)|f⟩$, which is decoupled from the system evolution since $H(t)|d⟩ = 0$. In the following discussion, we keep both $\theta$ and $\phi$ (thus $|b⟩$ and $|d⟩$) time independent and use $\Omega(t)$ and $\phi_1(t)$ as adjustable parameters for designing evolutions.

An arbitrary evolution of the system can be formulated as $|ψ_{0}(t)⟩, |ψ_{1}(t)⟩, |ψ_{2}(t)⟩ = U(t, 0) |d⟩, |b⟩, |e⟩$ with the evolution operator $U(t, 0) = τ e^{-i\int_{0}^{t}H(t')dt'} = \sum_{m=0}^{\infty} |ψ_{m}(t)⟩⟨ψ_{m}(0)|$, where $τ$ stands for time ordering. In NHQC schemes, a nonadiabatic cyclic evolution is engineered so that at the end moment $τ$, $U(τ, 0) |d⟩, |b⟩, |e⟩ = |d⟩, e^{iγ}|b⟩, e^{-iγ}|e⟩$, where $γ$ is a geometric phase determined by the path of evolution. When transformed and truncated into the computational subspace, $U(τ, 0)$ has a form of $e^{-i2π\frac{γ}{T}}σ$. Therefore, an arbitrary single-qubit gate can be realized by properly choosing $θ$, $ϕ$, and $γ$.

Conventional NHQC schemes impose the condition of parallel transport. (1) $⟨ψ_{m}(t)|H(t)⟩ψ_{n}(t)⟩ = 0$ ($m, n = 0, 1$), to ensure a geometric phase (i.e., the accumulated dynamical phase is zero) and the robustness of gates. However, it was proved that the infidelity of the resulting NHQC gates showed an identical dependence on control errors to the second order as a dynamical gate [45, 48]. In other words, the NHQC gates do not exhibit a better robustness against control errors as expected. This puzzle has recently been resolved by some authors of this work. Liu et al. [50] showed that the condition of parallel transport given above cannot alone guarantee that the resulting phases are pure geometric. Specifically, the “geometric” phases may become contaminated by a residual dynamical phase due to a nonzero cross-coupling of $⟨ψ_{1}(t)|H(t)⟩ψ_{2}(t)⟩$. This fact compromises the prerequisite for the robustness of geometric gates.

A solution of this issue is to impose the condition [50]

$$D_{mn} ≡ \int_{0}^{τ} d_{mn}(t)dt = 0, \quad m, n = 0, 1, 2, \quad (1)$$

where $d_{mn}(t) = ⟨ψ_{m}(t)|H(t)⟩ψ_{n}(t)⟩$. For $m, n = 0, 1$, this condition represents a relaxed version of the abovementioned condition of parallel transport. On the other hand, $D_{12} = 0$ ensures that the dynamical phase due to a nonzero $⟨ψ_{1}(t)|H(t)⟩ψ_{2}(t)⟩$ amounts to zero in a cyclic evolution, resulting in a pure geometric phase. SR-NHQC gates constructed following Eq.(1) exhibit an enhanced robustness against control errors or similar imperfections compared to conventional NHQC gates and the standard dynamical gates. In Appendix B 5, we show that for an error of the form $H′(t) = (1 + ε)H(t)$ [i.e., an error in the driving amplitude as $ε\Omega(t)$], the fidelity of the SR-NHQC gate is given by:

$$F(ε) = \sqrt{\cos^2 \frac{\gamma}{2} + \sin^2 \frac{\gamma}{2} \cos^4 \frac{πε}{2} (1 + \sin^2 \frac{πε}{2})^2}. \quad (2)$$

II. SR-NHQC SCHEME

We first present the theoretical framework of the SR-NHQC scheme for a single-qubit case. Consider a typical NHQC scheme comprising three states of $|g⟩, |f⟩, |e⟩$ driven by two resonant pulses with time-dependent amplitudes $Ω_{ge}(t)$ and $Ω_{ef}(t)$, and phases $ϕ_0(t)$ and $ϕ_1(t)$ (Fig. 1(a)). Here $|g⟩$ and $|f⟩$ form the computational basis and $|e⟩$ is an ancillary state. Under the rotating-wave approximation, the system Hamiltonian reads (with $\hbar ≡ 1$) [54]

$$H(t) = \frac{1}{2}[Ω_{ge}(t)e^{iϕ_0(t)}|g⟩⟨e| + Ω_{ef}(t)e^{iϕ_1(t)}|f⟩⟨e| + H.c.  \quad \text{(a)}$$

FIG. 1. Super robust holonomic gates. (a) The lowest three energy levels of a superconducting transmon qubit driven by two resonant microwave pulses on the transitions of $|g⟩ ↔ |e⟩$ and $|e⟩ ↔ |f⟩$, respectively. (b) Bloch sphere representation of quantum evolutions in the $|g⟩, |e⟩$ subspace for the single-qubit SR-NHQC gate, which comprises six sequential rotations $R_1(ϕ)$ in the equatorial plane, whose rotation angles and axes $(θ_i, ϕ_i)$ ($i = 1, ..., 6$) are specified alongside. (c)-(e) The measured populations of the resonant qubit for single-qubit SR-NHQC gates $U_{1}(θ, 0, π/2), U_{1}(π/2, 0, γ)$, and $U_{1}(π/2, ϕ, π/2)$. The initial states in the three cases are $|f⟩, |g⟩$, and $(|g⟩ + |f⟩)/\sqrt{2}$, respectively. Symbols and solid lines are experimental data and numerical simulations.
In the limit of \( |\varepsilon| \ll 1 \), \( F(\varepsilon) \approx 1 - \pi^4 \varepsilon^4 (1 - \cos \gamma)/32 \), exhibiting a fourth-order dependence on the error instead of second-order dependence.

When expressing the Hamiltonian \( H(t) \) as a function of \( \Omega(t) \) and \( \phi_1(t) \), we have many different choices of these adjustable parameters to fulfill the constraint of Eq. (1). Here for simplicity, we use a pulse sequence of six segments to construct the single-qubit SR-NHQC gates with a total gate time of 120 ns (details are given in Appendix B 1). Each segment contains a pair of microwave drives on resonance with the \(|g\rangle \leftrightarrow |e\rangle \) and \(|e\rangle \leftrightarrow |f\rangle \) transitions. The operation of each segment corresponds to a rotation in the subspace of \(|b\rangle, |e\rangle \) as shown in Fig. 1(b). In the computational subspace \(|g\rangle, |f\rangle \}, the evolution operator \( U(\tau, 0) \) has the form \( U_1(\theta, \phi, \gamma) = e^{-i\frac{\pi}{2} n \sigma} \) with \( n = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \), similar to the case of conventional NHQC discussed above.

III. SINGLE-QUBIT SR-NHQC GATE EXPERIMENT

The single-qubit SR-NHQC experiment is carried out using a superconducting circuit, where a superconducting transmon qubit [55] is dispersively coupled to two microwave cavities [56–59], one for readout of the qubit state and the other for storage of a microwave photonic qubit. The frequencies of the \(|g\rangle \leftrightarrow |e\rangle \) and \(|e\rangle \leftrightarrow |f\rangle \) transitions of the transmon qubit are \( \omega_{ge}/2\pi = 5.31 \text{ GHz} \) and \( \omega_{ef}/2\pi = 5.12 \text{ GHz} \), respectively. The readout cavity with a transition frequency of \( \omega_{R}/2\pi = 8.68 \text{ GHz} \) is used to perform high-fidelity and simultaneous readout of the \(|g\rangle, |e\rangle, \) and \(|f\rangle \) states. The storage cavity has a transition frequency of \( \omega_{S}/2\pi = 6.56 \text{ GHz} \) and allows for the implementation of two-qubit SR NHQC between the transmon states \(|g\rangle, |f\rangle \) and the Fock states \(|0\rangle, |1\rangle \) of the photonic qubit. More details of the device and measurement setup can be found in Appendix A.

We first demonstrate the tunability of \( \theta, \phi, \) and \( \gamma \) for realizing arbitrary single-qubit gates. Figures 1(c)-(e) show the measured population of qubit states as a function of \( \theta, \phi, \) and \( \gamma \) for the three representative gate sets \( U_1(\theta, 0, \pi/2), U_1(\pi/2, \phi, \pi/2), \) and \( U_1(\pi/2, 0, \gamma) \), respectively. The experimental results agree well with our numerical simulations.

We then characterize the single-qubit SR-NHQC gates using quantum process tomography (QPT) including all three states of \(|g\rangle, |e\rangle, \) and \(|f\rangle \) (details are given in Appendix B 2). Figure 2(a) shows the reduced quantum process matrix in the subspace of \(|g\rangle, |f\rangle \) for the four specific gates \( X = U_1(\pi/2, 0, \pi), Y = U_1(\pi/2, \pi/2, \pi), X/2 = U_1(\pi/2, 0, \pi/2), \) and \( Y/2 = U_1(\pi/2, \pi/2, \pi/2) \). The average process fidelity is 0.9858. We also use the Clifford-based RB [60–62] to characterize the single-qubit SR-NHQC gates. The reference RB experiment gives an average gate fidelity of 0.9956 for the single-qubit SR-NHQC gates in the Clifford group. The difference between the reference and interleaved RB experiments gives the gate fidelities 0.9957, 0.9960, 0.9958, and 0.9956 for the four SR-NHQC gates of \( X, Y, X/2, \) and \( Y/2 \), respectively. Infidelities of these gates mainly come from the decoherence of both \( |e\rangle \) and \( |f\rangle \) states of the qubit with a contribution of \( 4.3 \times 10^{-3} \) to gate errors (see Appendix D).

Next, we demonstrate the enhanced robustness of the SR-NHQC gates by comparing them to the conventional NHQC gates and the standard dynamical gates. Using QPT measurements, we study the performance of the four specific gates \( X, Y, X/2, \) and \( Y/2 \) realized via the three different schemes in the presence of a Rabi error \( \varepsilon \). The measured process fidelities of these four gates as a function of \( \varepsilon \) are shown in Figs. 3(a)-(d). The SR-NHQC gates are clearly superior to the other two types of gate in terms of robustness.

As discussed above, the super robustness of the SR-NHQC gates is guaranteed by a suppression of accumulated dynamical phases via imposing Eq. (1). In order to examine whether this condition is satisfied, we directly measure the accumulation rates of dynamical phases given by \( d_{\alpha n}(t) \) for SR NHQC, conventional NHQC, and dynamical gates (details are given in Appendix B 4). For the case of an \( X \) gate, the results...
and are shown in Figs. 3(e)-(g). Integrating the measured $d_{mn}(t)$ gives the total dynamical phases $D_{11}$, $D_{22}$, and $D_{12}$, which are $-0.01\pi$, $0.02\pi$, $-0.01 - 0.05\pi$ for the SR-NHQC X gate, $-0.06\pi$, $0.08\pi$, $(0.97 + 0.038)i\pi$ for the conventional NHQC X gate, and $0.78\pi$, $-0.78\pi$, $0.47\pi$ for the dynamical X gate, respectively. Thus, we have indeed verified that Eq. (1) holds for the SR-NHQC gate. Specifically, the suppression of $D_{12}$ is critical for achieving the super robustness of the SR-NHQC scheme, which is not guaranteed in the conventional NHQC schemes.

IV. TWO-QUBIT SR-NHQC GATE

We also demonstrate nontrivial two-qubit SR-NHQC gates between a superconducting transmon qubit and a photonic qubit, using the scheme illustrated in Fig. 4(a). In our experiment, two microwave drives are applied on resonance with the $|0g\rangle \leftrightarrow |0e\rangle$ and $|0e\rangle \leftrightarrow |0f\rangle$ transitions. Here the numbers represent the Fock states of the photonic qubit. Similar to the single-qubit case, we thus realize an arbitrary holonomic gate $U_1(\theta, \phi, \gamma)$ with a total gate time of 2.76 $\mu$s in the $\{|0g\rangle, |0f\rangle\}$ subspace, while keeping $|2g\rangle$ and $|2f\rangle$ unaffected thanks to the strong dispersive $ZZ$ interaction (about 2 MHz). This operation thus creates a control gate in the two-qubit subspace of $\{|0g\rangle, |0f\rangle, |2g\rangle, |2f\rangle\}$ described by

$$U_2(\theta, \phi, \gamma) = \begin{pmatrix} U_1(\theta, \phi, \gamma) & 0 \\ 0 & I \end{pmatrix}.$$  

(3)

Of course, the strong $ZZ$ interaction between the two qubits also induces a conditional phase operation, which renders the overall evolution matrix to be different from Eq. (3). However, since this interaction is time independent, the resulting phase can always be zeroed by properly setting the overall evolution time. Therefore, we can safely neglect this phase in the following discussion.

In order to characterize the two-qubit SR-NHQC gates, we first prepare different initial Fock states of the photonic qubit by sequentially applying Raman drives on the transmon qubit corresponding to the three Fock states $|0\rangle$, $|1\rangle$, and $(|0\rangle + |2\rangle)/\sqrt{2}$ of the photonic qubit. The Fock states are generated by sequentially applying Raman drives on the transitions of $|0f\rangle \leftrightarrow |1g\rangle$ and $|1f\rangle \leftrightarrow |2g\rangle$ (solid gray arrow lines in (a)). (c) Populations of the transmon qubit as a function of $\gamma$ for the two-qubit SR-NHQC gate $U_2(\pi/2, 0, \gamma)$ with an initial state $(|0g\rangle + |2g\rangle)/\sqrt{2}$. Symbols and lines are experimental data and numerical simulations, respectively. (d) Population of the ground state of the transmon qubit as a function of the Rabi error for a two-qubit controlled-NOT (CNOT) gate realized by SR-NHQC and conventional NHQC means. The initial state is $|0f\rangle$ in both cases.
routinely achieved [65, 66]. We further demonstrate the super robustness of the two-qubit SR-NHQC CNOT gate by measuring the populations as a function of the Rabi error $\varepsilon$ and comparing to the conventional NHQC two-qubit CNOT gate, as shown in Fig. 4(d). The experimental results clearly show the superior robustness of the SR-NHQC CNOT gate. Again, the under performance of the SR-NHQC gate at small $\varepsilon$ is due to a longer gate operation time and can be greatly improved when implemented on a system of two transmon qubits and a tunable coupler.

V. CONCLUSION

In summary, we have experimentally demonstrated a universal gate set based on a super robust NHQC scheme in an architecture of circuit quantum electrodynamics. Compared to conventional NHQC schemes, the SR-NHQC scheme guarantees an enhanced robustness against quasistatic errors appearing in the transverse direction (e.g., Rabi error in the $xy$ control pulses) by imposing additional constraints that help suppress dynamical phases. The realized single- and two-qubit SR-NHQC gates achieve an average fidelities of 0.9956 and 0.944, respectively, and both show super robustness against Rabi errors, as predicted. In addition, for the single-qubit SR-NHQC gates, we have directly measured the residual dynamical phases and verified the suppression of such phases inherent in our scheme. Given its generality and simplicity, the SR-NHQC scheme can be implemented on other platforms such as trapped ions, quantum dots, Rydberg atoms, and nuclear magnetic resonance, etc. Our work thus paves a way to construct universal super robust holonomic quantum gates for future large-scale quantum computation.

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Appendix A: EXPERIMENTAL DETAILS

1. Device and setup

The experimental device with a circuit quantum electrodynamics (cQED) architecture [67, 68] contains a three-dimensional (3D) coaxial stub cavity as the storage cavity ($S$), a superconducting transmon qubit ($Q$), and a stripline readout resonator ($R$). The schematic diagram of the device is shown in Fig. 5. The 3D coaxial stub cavity is first machined from a single block of high purity aluminum and then chemically etched in order to improve the quality factor of the microwave
TABLE I. Hamiltonian parameters.

| Frequencies (GHz) | Couplings (MHz) |
|-------------------|-----------------|
| $\omega_R/2\pi$   | 8.68            |
| $\omega_S/2\pi$   | 6.56            |
| $\omega_{ge}/2\pi$ | 5.31            |
| $\omega_{ef}/2\pi$ | 5.12            |
| $\chi_{gR}/2\pi$  | 2.52            |
| $\chi_{gS}/2\pi$  | 2.39            |
| $\chi_{SQ}/2\pi$  | 2.87            |
| $\chi_{ef}/2\pi$  | 2.08            |

2. System Hamiltonian and coherence properties

In our device, a superconducting transmon qubit is dispersively coupled to two cavity modes: a storage cavity mode and a readout cavity mode. The transmon qubit has a large anharmonicity and is considered as a three-level artificial atom, while each cavity mode is considered as a harmonic oscillator. Thus, the Hamiltonian of the whole system can be described as

$$\mathcal{H} = \omega_R \left( a_R^\dagger a_R + 1/2 \right) + \omega_S \left( a_S^\dagger a_S + 1/2 \right) + \omega_{ge} \langle e | e \rangle + \omega_{ef} \left( | f \rangle \langle f | + | g \rangle \langle g | \right)$$

where $\omega_{R,S}$ are the resonant frequencies of the readout and the storage cavities, respectively; $\omega_{ge}$ and $\omega_{ef}$ are transition frequencies among the lowest three energy levels $\{|g\rangle, |e\rangle, |f\rangle\}$; and the $\chi$ are corresponding dispersive couplings between the qubit and the cavity modes.

All the parameters in the previous Hamiltonian are calibrated with the standard cQED technique. The dispersive couplings $\chi_{ge}^{SQ}$ and $\chi_{ef}^{SQ}$ between the qubit and the storage cavity mode are calibrated with a number splitting experiment through qubit $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transition frequency spectroscopy with a coherent state in the storage cavity. The experiment results are shown in Fig. 7. The dispersive couplings $\chi_{ge}^{RQ}$ and $\chi_{ef}^{RQ}$ between the qubit and

FIG. 7. Calibration of dispersive couplings between the qubit and the storage cavity mode with number splitting experiments. (a) Qubit $|g\rangle \leftrightarrow |e\rangle$ transition frequency spectroscopy with a coherent state in the storage cavity mode. (b) The resonant qubit $|g\rangle \leftrightarrow |e\rangle$ transition frequencies as a function of the photon numbers in the storage cavity. Solid line is a linear fit. (c) Qubit $|e\rangle \leftrightarrow |f\rangle$ transition frequency spectroscopy with a coherent state in the storage cavity mode. (d) The resonant qubit $|e\rangle \leftrightarrow |f\rangle$ transition frequencies as a function of the photon numbers in the storage cavity. Solid line is a linear fit.

TABLE II. Coherence properties of the system.

| Modes          | $T_1$  | $T_2$  |
|----------------|--------|--------|
| Readout cavity | 66 ns  | -      |
| Storage cavity | 334 $\mu$s | 243 $\mu$s |
| Qubit $|g\rangle \leftrightarrow |e\rangle$ | 18.9 $\mu$s | 25.9 $\mu$s |
| Qubit $|e\rangle \leftrightarrow |f\rangle$ | 12.7 $\mu$s | 12.9 $\mu$s |
the readout cavity are calibrated through a readout frequency spectroscopy experiment with the qubit prepared in different initial states $|g\rangle$, $|e\rangle$, and $|f\rangle$, respectively. The experimental results are shown in Fig. 9(a). All the parameters in Eq. A1 are measured and listed in Table I.

In addition, the coherence properties of the whole system are also experimentally measured and listed in Table II. The relaxation time of the readout cavity is extracted from the linewidth of the readout frequency spectroscopy. The relaxation times of the $|e\rangle$ and $|f\rangle$ states of the transmon qubit are obtained by measuring the free evolutions of the populations $P_{ge}$, $P_{ef}$, and $P_{gf}$ with an initial $|f\rangle$ state, following the technique described in Ref. [75]. The populations $P_{ge}$ and $P_{ef}$ are measured by mapping them onto the population of ground state $|g\rangle$ through a $\pi$ pulse and two sequential $\pi$ pulses, respectively. The experimental results are shown in Fig. 8(a).

The decay curves are globally fitted with the rate equation $d\vec{p}/dt = \Gamma \cdot \vec{p}$, where $\vec{p} = (P_{ge}, P_{ef}, P_{gf})^T$, and the decay rate matrix $\Gamma$ is

$$\Gamma = \begin{pmatrix} 0 & \Gamma_{gf} & 0 \\ -\Gamma_{ge} & 0 & \Gamma_{ef} \\ 0 & -\Gamma_{gf} & -\Gamma_{ef} \end{pmatrix}$$

where the negligible upward transition rates are ignored and only the downward transition rates $\Gamma_{ge}$, $\Gamma_{ef}$, and $\Gamma_{gf}$ are considered. We note that the transition $f \rightarrow g$ is forbidden from parity considerations for a single-junction transmon qubit. However, in a realistic device, there are small nonsequential decay rates, which are mainly dominated by some nonquasiparticle processes, such as dielectric loss or coupling to other cavity modes [75]. Since the nonsequential decay rate $\Gamma_{gf}$ is much slower than the sequential decay rates $\Gamma_{ge}$ and $\Gamma_{ef}$, the corresponding relaxation times $1/\Gamma_{ge}$ and $1/\Gamma_{ef}$ of qubit are listed as $T_1$ in Table II.

Then, we measure the dephasing rates between $|g\rangle$ and $|e\rangle$ states, and between $|e\rangle$ and $|f\rangle$ states of the qubit with Ramsey interference experiments; the results are shown in Figs. 8(b,c).

The Ramsey fringes are fitted with an exponentially damped sinusoidal function $y = y_0 + e^{-t/T_2^*} A \cos(2\pi f t + \phi)$ and the extracted $T_2^*$ are also listed in Table II.

The coherence times $T_1$ and $T_2$ of the storage cavity are measured through the relaxation of Fock state $|1\rangle$ and the dephasing of $(|0\rangle + |1\rangle)/\sqrt{2}$, respectively [71]. Both initial states are generated with selective number-dependent arbitrary phase gates [76].

3. Qubit readout

Here, we probe the qubit states $|g\rangle$, $|e\rangle$, and $|f\rangle$ via the dispersive readout technique [68]. We first perform the readout cavity frequency spectroscopy experiment with different qubit initial states $|g\rangle$, $|e\rangle$, and $|f\rangle$. The experimental results are shown in Fig. 9(a). We optimize the readout pulse amplitude, duration, frequency, record pulse delay, and integration length to maximize the readout discrimination of all the $|g\rangle$, $|e\rangle$, and $|f\rangle$ states simultaneously. We perform the single-shot experiments with 20 000 repetitions for each initial qubit state, and record the I and Q quadratures of the readout signals with the experimental results shown in Fig. 9(b). The I-Q plane is divided into three regions corresponding to the assignments of the $|g\rangle$, $|e\rangle$, and $|f\rangle$ states. By counting the number of I-Q data points in the three regions, we could obtain the assignment probability $\tilde{P} = (P_0, P_1, P_2)^T$ corresponding to that initial basis state. After repeating the experiments for the three initial qubit states, we could obtain an assignment probability matrix $\mathcal{M}$ in span $\{|g\rangle, |e\rangle, |f\rangle\}$ with

$$\mathcal{M} = \begin{pmatrix} 0.942 & 0.080 & 0.076 \\ 0.040 & 0.908 & 0.077 \\ 0.018 & 0.012 & 0.847 \end{pmatrix},$$

where each column represents the qubit assignment probabilities after preparing the qubit in the corresponding basis state. Then the readout errors can be corrected by multiplying the inverse of the assignment matrix $\mathcal{M}$ with the measured probability $\tilde{P}$. Therefore, $\tilde{P}_{corr} = \mathcal{M}^{-1} \cdot \tilde{P}$ represents the actual occupation probabilities of the $|g\rangle$, $|e\rangle$, and $|f\rangle$ states of the transmon qubit. Notably, due to the parameter fluctuations in experimental device, this assignment matrix may be a little
different from that in Eq. A3, thus making the final probabilities slightly over or inadequately corrected.

Appendix B: SINGLE-QUBIT SR-NHQC GATES

1. State population evolution of single-qubit gates

Single-qubit SR-NHQC gates are constructed by a pulse sequence of six segments. Each segment contains a pair of microwave drives on resonance with the $|g⟩ - |e⟩$ and $|e⟩ - |f⟩$ transitions, thus corresponding to a Hamiltonian of $H(t) = 1/2 \{Ω(t)e^{iφ(t)}|b⟩⟨e| + H.c.\}$. The operation of each segment corresponds to a rotation $R_φ(θ)$ in the equatorial plane of $\{b⟩, |e⟩\}$ subspace, with rotation angle $θ = ∫_0^δ Ω(t)dt$ and phase $φ = φ₁$. The total evolution time ($τ = 120 ns$) is divided into six intervals, with the rotation angle and phase in each segment satisfying

$$
\begin{align*}
\theta_1 &= π/2, \quad φ_1 = γ - π, \quad t \in [0, τ/8], \\
θ_2 &= π, \quad φ_2 = γ - π/2, \quad t \in [τ/8, 3τ/8], \\
θ_3 &= π/2, \quad φ_3 = γ - π, \quad t \in [3τ/8, τ/2], \\
θ_4 &= π/2, \quad φ_4 = 0, \quad t \in [τ/2, 5τ/8], \\
θ_5 &= π, \quad φ_5 = π/2, \quad t \in [5τ/8, 7τ/8], \\
θ_6 &= π/2, \quad φ_6 = 0, \quad t \in [7τ/8, τ].
\end{align*}
$$

(B1)

In the computational subspace $\{|g⟩, |f⟩\}$, the evolution operator $U(τ, 0)$ has form $U_1(θ, φ, γ) = e^{-iτ/2}n$, corresponding to a rotation operation around the axis $n = (sin θ cos φ, sin θ sin φ, cos θ)$ by an angle of $γ$. The total evolution of these six segments satisfies condition (1) in the main text, thus corresponding to super robust nonadiabatic holonomic gates with enhanced robustness. We note that this construction of holonomic gates is similar to composite pulses [77, 78], whose robustness also originates from satisfying condition (1) in the main text.

In our experiment, each resonant microwave drive of the six segments is implemented with the cosine-shape envelope pulse shown in Fig. 10(a), with the pulse amplitudes and phases satisfying Eq. (B1) to achieve better robustness. We first measure the state dynamics of the single-qubit SR-NHQC gates. Firstly, we initialize the qubit with $|g⟩$ and $(|g⟩ - i|f⟩)/\sqrt{2}$ states, respectively. Then, six pairs of resonant pulses are applied on the qubit to realize single-qubit SR-NHQC gates $X$ and $Y/2$, respectively. We measure the qubit state populations as a function of the gate duration, with the experimental results given in Figs. 10(b-e), which agree well with our numerical simulations. In addition, we demonstrate the arbitrary tunability of the parameters $θ$, $φ$, and $γ$ for the single-qubit SR-NHQC gates $U_1(θ, φ, γ)$. Here we present extended data to that in Fig. 1(a) in the main text. Figs. 11(a-c) are the measured qubit state populations as a function of $θ$, $γ$, and $φ$ for the realized single-qubit SR-NHQC gates $U_1(θ, φ, γ)$ and $U_1(π/2, φ, π/2)$, respectively. The experimental results agree well with our numerical simulations, indicating arbitrary control of the super robust single-qubit holonomic gates.

2. Quantum process tomography

The single-qubit SR-NHQC gates are first characterized by a full quantum process tomography with all the three-level system. We first initialize the three-level transmon qubit with the nine states $\{|g⟩, |e⟩, |f⟩, (|g⟩ + |e⟩)/\sqrt{2}, (|e⟩ + |f⟩)/\sqrt{2}, (|g⟩ + |f⟩)/\sqrt{2}, (|g⟩ - i|e⟩)/\sqrt{2}, (|e⟩ - i|f⟩)/\sqrt{2}, (|g⟩ - i|f⟩)/\sqrt{2}\}$, then apply the SR-NHQC gates, and finally perform state tomography measurements of the final states. The state tomography measurement requires nine prerotations to reconstruct the density matrix of the three-level qubit state:
\{ I, X^{\pi/2}, X^{\pi/2}, X^{\pi/2}, X^y \} \text{ with the rotation operators read from right to left. The measurements give the result } \langle M_k \rangle = \text{Tr}(\rho U_k^\dag M_1 U_k) \text{ for each prerotation } U_k \text{ with } k = 0, 1, 2, \ldots, 8, \text{ where } M_1 = \ket{g}\bra{g}. \text{ The density matrix of the three-level qubit state can then be reconstructed by the maximum likelihood estimation method [80]. With the nine initial states } \rho_i, \text{ the experimental process matrix } \chi_{\text{exp}} \text{ can be extracted from the nine corresponding final states } \rho_f \text{ through } \rho_f = \sum_{m,n} \chi_{mn} E_m \rho_i E_n^\dag [81], \text{ where the full set of nine orthogonal basis operators is chosen as \{I_{gf}, \sigma_y^{gf}, -i\sigma_x^{gf}, \sigma_y^{ef}, -i\sigma_x^{ef}, \sigma_z, -i\sigma_y^{ge}, \sigma_x^{ge}, -i\sigma_z^{ge}, \} \text{ [27]}, \text{ where the } \sigma^{mn} \text{ are the Pauli operators acting on the } m \text{ and } n \text{ energy levels, } I_{gf} = \ket{g}\bra{g} + \ket{f}\bra{f}, \text{ and } I_e = \ket{e}\bra{e}. \text{ For the single-qubit SR-NHQC gates on the transmon qubit, the state } \ket{e} \text{ serves as an auxiliary state. Therefore, we have calculated the reduced process matrix } \chi_R \text{ that describes the process only involving } \ket{g} \text{ and } \ket{f}, \text{ and ignore any operators acting on the auxiliary state. In order to compare with the process acting on a two-level system, the reduced process matrix } \chi_R \text{ is obtained by a normalization factor of } 3/2. \text{ The basis operators of the reduced process matrix are } \{I_{gf}, \sigma_y^{gf}, -i\sigma_x^{gf}, \sigma_y^{eg}, \sigma_x^{eg}, -i\sigma_z^{eg}, \} \text{ or simply } \{I, \sigma_y, -i\sigma_x, \sigma_z\} \text{ as in the main text. The quantum process fidelity of the corresponding gate is defined as } F = \frac{\text{Tr}(\chi_R \chi_{\text{ideal}})}{\text{Tr}(\chi_{\text{ideal}})}, \text{ where } \chi_{\text{ideal}} \text{ is the ideal process matrix for the corresponding gate.}

3. Randomized benchmarking

In the single-qubit RB experiment, we perform both the reference RB and interleaved RB experiments with the experimental sequences shown in the inset of Fig. 2(b) in the main text. In the reference RB experiment, we first apply a random sequence of } m \text{ quantum gates chosen from the single-qubit Clifford group, then append a recovery gate (} C_R \text{) to invert the whole sequence, and finally measure the ground-state probability as the sequence fidelity. The whole experiment is repeated for } k = 50 \text{ different sequences to get the average sequence fidelity. In the interleaved RB experiment, a specific gate } G \text{ is interleaved into the } m \text{ random Clifford gates, and a similar recovery gate is applied to invert the whole sequence. The experimentally measured sequence fidelity decay curves as a function of the number of Clifford gates } m \text{ for both the reference RB and interleaved RB experiments are fitted to } F = A p^m + B \text{ with different sequence decays } p = p_{\text{ref}} \text{ and } p = p_{\text{gate}}. \text{ The average gate fidelity is given by } F_{\text{gate}} = 1 - (1 - p_{\text{gate}})(d - 1)/d.1.875 \text{ with } d = 2^N \text{ for } N \text{ qubits. Here the number } 1.875 \text{ accounts for a total } 45 \text{ physical gates to construct the } 24 \text{ Clifford gates in the single-qubit Clifford group [82]. The difference between the reference and interleaved RB experiments gives the specific gate fidelity } F_{\text{gate}} = 1 - (1 - p_{\text{gate}}/p_{\text{ref}})(d - 1)/d.

4. Measurement of dynamical phases

In the main text, we have explicitly measured the accumulated dynamical phase for our SR-NHQC gates, as well as conventional NHQC gates and dynamical gates. In Sec.B 1 above, we introduced the construction of SR-NHQC gates in our scheme with a total evolution time } \tau = 120 \text{ ns. Conventional NHQC gates are easily realized according to Refs. [20, 23] with a total evolution time } \tau = 60 \text{ ns. Dynamical gates are constructed following Ref. [25]. A quantum system driven by the Hamiltonian } H(t) = \frac{1}{2} \{ \Omega(t) e^{i\phi(t)} |b\rangle\langle e| + h.c. \} \text{ can evolve along state } |\psi_1(t)\rangle = e^{-i\frac{\tau}{2}} [\cos(\chi/2) e^{-i\phi/2} |b\rangle + \sin(\chi/2) e^{i\phi/2} |e\rangle], \text{ where } f, \phi \text{ and } \chi \text{ are time-dependent auxiliary parameters, which satisfy the equations}

\begin{align}
\dot{\phi} &= -f \cos \chi, \\
\phi &= \text{atan} (\chi \cot \chi/\phi) - \varphi, \\
\Omega &= -\dot{\chi} \sin (\phi_1 + \varphi), \tag{B2}
\end{align}

where the dot represents the time differential. To construct arbitrary dynamical gates, we divide the evolution path into two segments. At the first segment } [0, \tau/2], \text{ we set } \chi = \pi \sin^2(\pi/\tau), f = \chi - \frac{1}{2} \sin(2\chi), \text{ and } \varphi = -\frac{1}{2} \sin \chi. \text{ The resulting evolution operator is } U_D(\tau/2,0) = |d\rangle\langle d| + e^{-i\pi/2} |e\rangle\langle e|. \text{ At the second segment } [\tau/2, \tau], \text{ we set } \chi = \pi \sin^2(\pi/\tau), f = -\chi + \frac{1}{2} \sin(2\chi), \text{ and } \varphi = \frac{1}{2} \sin \chi - \chi'. \text{ The resulting evolution operator is } U_D(\tau,\tau/2) = |d\rangle\langle d| + e^{-i\pi/2} |e\rangle\langle e|. \text{ Then, the total dynamical evolution operator can be expressed as } U_D(\tau,0) = |d\rangle\langle d| + e^{-i\pi/2} |b\rangle\langle e| + e^{i\pi/2} |e\rangle\langle b|. \text{ Thus, arbitrary dynamical gates are constructed. In our experiment, the total evolution time } \tau = 105 \text{ ns is chosen to ensure the same maximum coupling strength as SR-NHQC and NHQC gates. Note that, in general, the construction scheme of dynamical gates does not satisfy the condition (1) in the main text, and so does not exhibit robustness.}

Now, we show how to measure dynamical phases } D_{mn} = \int_0^\tau d\tau m(t)dt \text{ in our experiment. Here, we first measure the dynamical phase accumulated rate } d_m(t) = \langle \psi_m(t)| H(t) |\psi_n(t)\rangle \text{ with } |\psi_0(t)\rangle = U(t,0)|d\rangle = |d\rangle, |\psi_1(t)\rangle = U(t,0)|b\rangle, \text{ and } |\psi_2(t)\rangle = U(t,0)|e\rangle. \text{ Since the dark state is always decoupled, } d_{mn}(0) \text{ and } d_{on}(t) \text{ are always zero at each time. Thus, we only need to measure } d_{mn}(t) \text{ for } m, n = 1, 2 \text{ at each evolution time, and then } D_{mn} \text{ can be obtained by an integration.}

In order to measure } d_{11}(t) \text{ for a specific gate } U_1(\theta, \phi, \gamma) \text{ we first prepare the initial state } |b\rangle = \sin(\frac{\theta}{2}) e^{i\phi}|g\rangle + \cos(\frac{\theta}{2})|f\rangle. \text{ Then, we apply a quantum gate } U_1(\theta, \phi, \gamma) \text{ with confirmed Hamiltonian } H(t) \text{ to drive the qubit as } |\psi_1(t)\rangle = |\psi_1(t)\rangle|\psi_1(t)\rangle. \text{ At each evolution time, we use the quantum state tomography technique to record the density matrix } \rho_1(t) = \langle \psi_1(t)| H(t) |\psi_1(t)\rangle \text{ and record the density matrix } \rho_2(t) = \langle \psi_2(t)| H(t) |\psi_2(t)\rangle \text{ at each evolution time. Thus, } d_{22}(t) = \langle \psi_2(t)| H(t) |\psi_2(t)\rangle = Tr[\rho_2(t) H(t)] \text{ can be obtained.}
However, \( d_{12}(t) = d_2^{12}(t) \) cannot be directly measured. Here, we measure both its real and imaginary parts. In order to measure the real part \( \operatorname{Re}[d_{12}(t)] \), we first prepare the initial state \(|\psi(0)\rangle = (|b\rangle + |e\rangle)/\sqrt{2} \) and apply a quantum gate \( U_1(\theta, \phi, \gamma) \) with confirmed Hamiltonian \( H(t) \) to drive the qubit. Then, we use the quantum state tomography technique at each evolution time to obtain the density matrix \( \rho_{12}(t) = \langle \psi(t)|\langle \psi(t)|\psi(t)\rangle + \langle \psi(t)|\langle \psi(t)| + |\psi(t)\rangle\langle \psi(t)| + [\psi(t)\rangle\langle \psi(t)|/2 = \rho_{12}(t) + \rho_{21}(t) + \rho_{11}(t) + \rho_{22}(t)/2 \). Thus, the real part of \( d_{12}(t) \) can be obtained as

\[
\operatorname{Re}[d_{12}(t)] = \operatorname{Re}[\langle \psi(t)|H(t)|\psi(t)\rangle] = \frac{1}{2} \left( \langle \psi(t)|H(t)|\psi(t)\rangle + \langle \psi(t)|H(t)|\psi(t)\rangle \right) = \frac{1}{2} \operatorname{Tr}[\rho_{12}(t)H(t) + \rho_{21}(t)H(t)] = \operatorname{Tr}[\rho_{12}(t)H(t)] - \frac{d_{11}(t)}{2} - \frac{d_{22}(t)}{2}.
\]

Similarly, we can measure the imaginary part \( \operatorname{Im}[d_{12}(t)] \) for a specific gate \( U_1(\theta, \phi, \gamma) \) by preparing the initial state \(|\psi(0)\rangle = (|b\rangle + i|e\rangle)/\sqrt{2} \) and obtaining the density matrix \( \rho_{12}(t) = \langle i\psi(t)|\langle \psi(t)|\psi(t)\rangle - i\langle \psi(t)|\langle \psi(t)| + \langle \psi(t)|\langle \psi(t)| + [\psi(t)\rangle\langle \psi(t)|/2 = [\rho_{12}(t) - i\rho_{21}(t) + \rho_{11}(t) + \rho_{22}(t)]/2 \). Thus, the imaginary part can be obtained as

\[
\operatorname{Im}[d_{12}(t)] = \operatorname{Im}[\langle \psi(t)|H(t)|\psi(t)\rangle] = \frac{1}{2} \left( -i\langle \psi(t)|H(t)|\psi(t)\rangle + i\langle \psi(t)|H(t)|\psi(t)\rangle \right) = \frac{1}{2} \operatorname{Tr}[-i\rho_{21}(t)H(t) + i\rho_{12}(t)H(t)] = \operatorname{Tr}[\rho_{21}(t)H(t)] - \frac{d_{11}(t)}{2} - \frac{d_{22}(t)}{2}.
\]

We have therefore measured the dynamical phase accumulated rate \( d_{mn}(t) \) \((m, n = 1, 2) \) at each evolution time. Thus, the total dynamical phases \( D_{mn} \) can be obtained by a time integration of the corresponding rate \( d_{mn}(t) \). We present the measured dynamical phase accumulated rate \( d_{mn}(t) \) \((m, n = 1, 2) \) for both the conventional NHQC and dynamical gates, as well as the SR-NHQC gate, in the main text.

5. Analytical calculation of the robustness of SR NHQC

Here, we theoretically calculate the gate fidelity as a function of the Rabi error \( \varepsilon \) of the control field. Specifically, we consider the driving amplitude \( \Omega(t) \) with an additional small error fraction of \( \varepsilon \Omega(t) \). In other words, the Hamiltonian \( H(t) \) becomes \( H'(t) = (1 + \varepsilon)H(t) \). The corresponding evolution operator in the basis \(|g\rangle, |e\rangle, |f\rangle \) is given by

\[
U_e(\tau) = \mathcal{T} e^{-i \int_0^\tau (1+\varepsilon)H(t)dt}.
\]

Applying perturbation theory [83] to Eq. (B3), we have

\[
U_e(\tau, 0) = I + \sum_{n=1}^\infty R_n(\tau, 0).
\]

The time-ordered products \( R_n(\tau, 0) \) are given by

\[
R_1(\tau, 0) = (1 + \varepsilon) \int_0^\tau dt_1 d(t_1)

R_2(\tau, 0) = (1 + \varepsilon)^2 \int_0^\tau dt_1 \int_0^{t_1} dt_2 d(t_1) d(t_2),
\]

where \( d(t) = \sum_{m,n} d_{mn}(t) |\psi_m(0)\rangle\langle \psi_n(0)| \) is the instantaneous dynamical part. Applying the settings \( d_{mn} = |\psi_m(t)\rangle\langle \psi_n(t)|, |\psi_1(0)| = |U(0)| b(\varepsilon) \), and \( D(\tau) = \int_0^\tau d(t)dt = 0 \) in the main text, the evolution operator of Eq. (B3) in the basis \(|\xi_0\rangle, |\xi_1\rangle \rangle \) can be expressed as

\[
U_e(\tau, 0) = X |\xi_0\rangle \langle \xi_1(0)| + |\xi_0\rangle \langle \xi_0|,
\]

where \( X \equiv \{1 - (1 - e^{i\gamma}) \cos^2(\pi\varepsilon/2) + \sin^2(\pi\varepsilon/2)\} \). Hence, a SR-NHQC gate under the Rabi control error in the basis \(|g\rangle, |f\rangle \) is given by

\[
U'_1(\theta, \phi, \gamma) = \left[ c_{q/2}^2 + c_{s q/2}^2 X \frac{(1-X)}{2} s_{q} e^{i\phi} \right],
\]

where \( c_q \equiv \cos(q/2) \), and \( s_q \equiv \sin(q/2) \). Using Eq. (B7), the gate fidelity is

\[
F(\varepsilon) = \frac{1}{2} \left| \operatorname{Tr} \left[ U'_1(\theta, \phi, \gamma) U'_1^\dagger(\theta, \phi, \gamma) \right] \right| = \sqrt{\cos^2 \gamma/2 + \sin^2 \gamma/2 \cos \pi \varepsilon/2 \left( 1 + \sin^2 \pi \varepsilon/2 \right)}.
\]

where \( U_1(\theta, \phi, \gamma) \) is the ideal evolution operator with \( \varepsilon = 0 \). When the error fraction \( |\varepsilon| \ll 1 \), \( F(\varepsilon) \approx 1 - \pi^4 \varepsilon^4 (1 - \cos \gamma)/32 \), exhibiting a fourth-order dependence on the Rabi error.

6. Robustness of SR-NHQC gates against decoherence and Rabi errors

The gate time of the demonstrated single-qubit SR-NHQC gates is generally longer than conventional DRAG-based gates, thus meaning that the gate infidelity mainly arises from
a finite qubit coherence time. The ac Stark shift due to off-resonant levels in the weakly anharmonic qubits prevents us from achieving faster gate operations. The current results represent a balance between the efforts of reducing errors due to leakage and decoherence. With further improved qubit coherence time, the drawback associated with long gate operation times will be more and more compensated by the gained robustness. Here, we perform a comparison between SR-NHQC gates and conventional DRAG-based gates by numerically simulating the gate fidelity as a function of the Rabi error and qubit decoherence error rate, with the results shown in Fig. 12. Obviously, with improved qubit coherence times, coherence-limited errors for both methods are negligible. Since the robustness against the Rabi error for the SR-NHQC gates (fourth order) is obviously superior to the conventional DRAG scheme (only second order), our SR-NHQC approach is more preferable in the near future.

Appendix C: TWO-QUBIT SR-NHQC GATES

1. Initialization of two-qubit states

Before implementing the two-qubit SR-NHQC gates, initialization of two-qubit states in the two-qubit subspace \{0g⟩, 0f⟩, 2g⟩, 2f⟩} is needed. Qubit initial states can be easily prepared by sequential π pulses on |g⟩ ↔ |e⟩ and |e⟩ ↔ |f⟩ transitions, while the initial Fock state of the cavity is more difficult. Here, we present the details of initialization of the cavity states, which is generated through sequential Raman transition drives on |0⟩ ↔ |1g⟩ and |1⟩ ↔ |2g⟩, respectively. The Raman transition drive of |0⟩ ↔ |1⟩ is calibrated by first preparing initial state |0⟩ through qubit sequential π pulses on |g⟩ ↔ |e⟩ and |e⟩ ↔ |f⟩ transitions, then applying a cosine-shaped microwave pulse with a variable driving frequency to achieve the |0⟩ ↔ |1⟩ coupling, and finally measuring the qubit populations. The corresponding pulse sequence and experimental results are shown in Figs. 13(a,c). Thus, we could extract the resonant frequency of the Raman transition drive to achieve the |0⟩ ↔ |1⟩ coupling. Besides, we further measure the Rabi oscillation between |0⟩ and |1⟩ states, with the corresponding pulse sequence and measurement results shown in Figs. 13(b,d).

2. State population evolution of two-qubit SR-NHQC gates

The two-qubit SR-NHQC gates are implemented by applying two microwave drives resonantly with the |0⟩ ↔ |0⟩ and |0⟩ ↔ |0⟩ transitions, while keeping |2⟩ and |3⟩ unaffected thanks to the strong dispersive ZZ interaction. The total evolution time \(\tau = 2760\) ns is chosen in our experiment such that the two drives are weaker enough to avoid driving undesired transitions. As for the single-qubit SR-NHQC gate, the total evolution time is divided into six segments with the driving parameters also satisfying Eq. (B1). In the main text, we have demonstrated the arbitrary tunability of the parameter γ for the two-qubit SR-NHQC gates \(U_{2}(\pi/2,0,\gamma)\) for an initial state \(|0g⟩ + |2g⟩⟩\). Here, we present extended data in Fig. 14, where we have measured the qubit state populations as a function of γ for the two-qubit SR-NHQC gate \(U_{2}(\pi/2,0,\gamma)\) with initial states |0⟩ and |2⟩, respectively. For the two-qubit SR-NHQC CNOT gate defined by \(U_{2}(\pi/2,0,\pi)\), a gate fidelity of 0.944 is obtained from the average of the correct state populations of the CNOT gate operation on initial states |0⟩ and |2⟩. The infidelity mainly comes from the decoherence of the qubit due to a long gate operation time and can be reduced with a short gate time when implemented on a system of two transmon qubits and a tunable coupler.
The infidelity of the single- and two-qubit SR-NHQC gates is given in Table III.

TABLE III. Infidelities of the single- and two-qubit SR-NHQC gates.

| Error sources                  | Single-qubit SR-NHQC gates | Two-qubit SR-NHQC gates |
|--------------------------------|----------------------------|-------------------------|
| Decoherence error of transmon qubit | 0.0043                     | 0.050                   |
| Decoherence error of photonic qubit | -                           | 0.008                   |
| Driving-induced leakage error   | 0.0006                      | < 0.001                 |
| Total                          | 0.0049                      | 0.058                   |

Time $T_{2E} = 38 \mu s$ is directly measured from the corresponding experiment. Because of the lack of direct measurements of $T_{2E,\gamma}$ and $T_{2E,f}$, we estimate these two dephasing times as $T_{2E,\gamma} \approx 26 \mu s$ and $T_{2E,f} = 2/(\Gamma_1 + \Gamma_2) \approx 31 \mu s$, respectively. For the single-qubit SR-NHQC gates with a gate time of 120 ns, the estimated coherence-limited error is about $4.3 \times 10^{-3}$. For the two-qubit SR-NHQC gates with a gate duration of 2760 ns, the decoherence error is estimated by the average errors from the decoherence of both the transmon qubit and photonic qubit, resulting in a decoherence error of $5.8 \times 10^{-2}$.

2. There are also small contributions from the driving-induced leakage errors due to the weakly anharmonic qubits. We estimate these leakage errors from numerical simulations without considering any decoherence of the qubits. The simulation results give leakage errors of $6 \times 10^{-4}$ and $< 1 \times 10^{-3}$ for the single- and two-qubit SR-NHQC gates, respectively.

We summarized these results in Table III. As a result, the totally estimated infidelities are consistent with our experimental results for both the single- and two-qubit SR-NHQC gates and the dominant error source is the qubits decoherence for both gates.

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