A Simple Description of Strange Dibaryons in the Skyrme Model

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Abstract

We study strange dibaryons based on the SU(2)-embedded $B = 2$ toroidal soliton. Treating the excursions of the soliton into strange directions as small rigid oscillations, we obtain a good approximation to the bound state approach. We calculate the dibaryon mass formula to order $1/N$ and find that the doubly strange $I = J = 0$ dibaryon is bound by about 90 MeV.

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The existence of stable multibaryon states with vanishing hypercharge, although not yet experimentally confirmed, remains an exciting possibility. The simplest of such states, the doubly strange dibaryon $H$, was first conjectured to exist [1]. On the basis of an MIT bag model calculation, its mass was predicted to be $m_H = 2150\text{ MeV}$, well below the $\Lambda\Lambda$ threshold. This means that all strong decays of $H$ are forbidden, and it is expected to have a long lifetime typical of weak decays. Experimental detection of doubly strange dibaryons is a subtle matter, and there are some remarkable ongoing efforts in this direction [2]. In the meantime it is important to sharpen the theoretical understanding of dibaryons by resorting to other available models of low-energy QCD.

One viable alternative to the quark models is the Skyrme model [3, 4, 5] where baryons are identified with solitons of the non-linear meson action, which is usually taken to be

$$S = N S_{WZ} + \int d^4x \left[ \frac{f_\pi^2}{16} \text{tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{tr}[\partial_\mu UU^\dagger, \partial_\nu UU^\dagger]^2 + \frac{f_\pi^2}{16} \text{tr} M(U + U^\dagger - 2) \right]$$

with $U(\vec{x}, t) \in SU(3)$. $N$ is the number of colors, and the Wess-Zumino term was first determined in [5]. $M = \begin{pmatrix} m_\pi^2 & m_\pi^2 & 2m_K^2 - m_\pi^2 \\ m_\pi^2 & m_\pi^2 & \frac{2m_K^2 - m_\pi^2}{2} \end{pmatrix}$ is proportional to the quark mass matrix. While $m_\pi = 138\text{ MeV}$ is small and is often neglected, the effects of $m_K = 495\text{ MeV}$ are significant. With the standard fit to the nucleon and $\Delta$ masses obtained with this Lagrangian [6], the parameters are assumed to be $f_\pi = 108\text{ MeV}$ and $e = 4.84$.

After a remarkable success in describing the properties of non-strange baryons, based on the SU(2) collective coordinate quantization of the Skyrme hedgehog [3, 4], there has been a number of attempts to model strange dibaryons. The first interesting idea appeared in [8] where an SO(3) imbedded soliton of baryon number $B = 2$ was constructed. In the limit of vanishing $m_\pi$ and $m_K$, the classical mass of the SO(3) soliton was found to be $1658\text{ MeV} = 1.92M_h$. The classical mass of the Skyrme hedgehog is $M_h = 863\text{ MeV}$. Thus, in the chiral limit, the SO(3) soliton is stable against decay into two $B = 1$ states. Its collective coordinate quantization leads to SU(3) multiplets of zero triality and $H$, the SU(3) singlet, is the lightest state [9]. An important feature of the SO(3) soliton, however, is that it extends significantly into the strange directions of SU(3). As $m_K$ is dialed to its physical value, which is appreciable, the SO(3) symmetry crucial to the soliton’s existence is destroyed. A perturbative estimate shows that inclusion of $m_K$ pushes $M_{SO(3)}$ well above $2M_h$, destroying its classical stability [10]. Thus, the dibaryon states constructed by the collective coordinate quantization of the SO(3) soliton, which were found to be stable in the chiral limit, may not survive the breaking of SU(3). This question requires further study.

In this paper we focus on another $B = 2$ soliton solution, which is embedded entirely in
the light SU(2) subgroup of SU(3), and is based on the cylindrically symmetric ansatz [11],

\[ U_{B=2}(\vec{x}) = \begin{pmatrix} e^{i F(r, \theta) \hat{n} \cdot \vec{\tau}} & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{n} = \begin{pmatrix} \sin \Theta(r, \theta) \cos 2\phi \\ \sin \Theta(r, \theta) \sin 2\phi \\ \cos \Theta(r, \theta) \end{pmatrix}. \]  

The classical mass of this soliton has no dependence on \( m_K \). By numerically relaxing the two unknown functions with the boundary conditions

\[ F(r = 0) = \pi, \quad F(r = \infty) = 0, \quad \Theta(\theta = 0) = 0, \quad \Theta(\theta = \pi/2) = \pi/2, \]

the classical mass was found [11] to be \( M_{B=2} \approx 1660 \) MeV, which is considerably smaller than the mass of the SO(3) soliton evaluated with \( m_K = 495 \) MeV. Constructions of dibaryon states based on the soliton (2) may be found in the literature [12, 13, 14]. In [12], an SU(3) collective coordinate quantization of \( U_{B=2} \) was carried out, and certain stable dibaryon states were predicted. While these predictions are interesting, it should be noted that a similar approach to the octet and the decuplet of baryons has not been particularly successful because of problems with the breaking of SU(3) [15]. There exists another approach to strangeness [16], however, which makes no explicit mention of SU(3) multiplets but nevertheless appears to be more successful quantitatively [16, 17]. In this approach, hyperons are modeled by bound states of kaons and SU(2) solitons [16].

The bound state approach was first applied to strange dibaryons in [13]. To simplify calculations, the ansatz (2) was restricted to \( F(r, \theta) = F(r), \ \Theta(r, \theta) = \theta \). This restriction effectively forces the soliton into a spherically symmetric shape and pushes its classical energy above \( 2M_h \). While an interesting qualitative picture emerged, no definitive assessment of dibaryon stability could be made. More recently, an improved study of bound state dibaryons was made in [14]. In this paper, a better (although not minimum energy) \( B = 2 \) soliton ansatz was used, and kaon modes were studied in its background. The lightest dibaryon was predicted to be bound by about 35 MeV. This calculation, however, did not take full account of kaon-kaon interactions, which are difficult to include in the bound state approach, but are expected to be particularly important in this system.

In this letter, we propose a new description of the Skyrme model dibaryons which we call the rigid oscillator approach and which is intermediate between the collective coordinate and the bound state approaches. As in the collective coordinate quantization, we allow only the rigid motions of the soliton. As in the bound state approach, we expand in the deviations of the soliton into strange directions, which we denote by \( D \). Since \( D \) turns out to be of order \( 1/\sqrt{N} \), this generates a systematic \( 1/N \) expansion. The suppression of the strange deviations is related to the fact that, in the quark model language, we are dealing with states consisting of \( 2N - 2 \) light quarks and only 2 strange quarks. While increasing \( m_K \) further reduces the strange deviations, our methods work for any \( m_K \).  

An advantage of the rigid oscillator approach is a Skyrme model manifestation of the general fact that for large \( N \) baryons, flavor SU(3) breaking is large even for small \( m_K \) [18, 19].

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approach is its simplicity: it provides analytic formulae for various quantities in terms of \( m_K \), thus serving as a good physical guide to the bound state calculations. Furthermore, it is not hard to include all terms quartic in \( D \), which are necessary for complete order \( 1/N \) calculations. The rigid oscillator approach has been applied to the \( B = 1 \) sector in \([18, 20, 21]\), giving a reasonable approximation to the bound state approach. In \([22]\), it was further noted that the rigid oscillator approximation gets better with increasing baryon density and beyond some critical density becomes identical to the bound state approach. We may expect, therefore, that the rigid oscillator approach will work better for the denser \( B = 2 \) soliton than for the Skyrme hedgehog.

Let us start by reviewing the rigid oscillator approach to quantization of the \( B = 1 \) Skyrme hedgehog.

\[
U_h(\vec{x}) = \begin{pmatrix} e^{iF(\vec{r})} \hat{\vec{r}} \cdot \vec{\tau} \ 0 \ 1 \end{pmatrix}.
\]

While in \([18, 20, 21]\) the approximation with \( m_\pi = 0 \) was considered, here we extend it to include the pion mass. We consider only the rigid motions of the soliton,

\[
U(\vec{x}, t) = A(t)U_h(\vec{x})A(t)^\dagger.
\]

To separate the SU(2) rotations from the deviations into strange directions, we write \([20]\)

\[
A(t) = \begin{pmatrix} A(t) & 0 \\ 0^\dagger & 1 \end{pmatrix} S(t),
\]

where \( A(t) \in \text{SU}(2) \), and

\[
S(t) = \exp i \sum_{a=4}^7 d^a \lambda_a = \exp i D,
\]

where \( D = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 0 & \sqrt{2}D \\ \sqrt{2}D^\dagger & 0 \end{array} \right) \), \( D = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} d^4 - id^5 \\ d^5 - id^7 \end{array} \right) \).

In calculating to order \( N^0 \), we may neglect the dynamics of \( A(t) \) and treat the strange deviations in the harmonic approximation. This leads to the following effective Lagrangian,

\[
L = -M_h + 4\Phi_1 \dot{D}^\dagger \dot{D} + i \frac{N}{2} \left( D^{\dagger} \dot{D} - \dot{D}^{\dagger} D \right) - \Gamma_1 (m^2_K - m^2_\pi) D^{\dagger} D.
\]

The quantities \( \Phi_1 \) and \( \Gamma_1 \), whose integral expressions are given in \([21]\), may be evaluated numerically,

\[
\Phi_1 \approx 0.00186/\text{MeV}, \quad \Gamma_1 \approx 0.00398/\text{MeV}.
\]

Canonical quantization of \((6)\) leads to the following Hamiltonian,

\[
H = M_h + \frac{1}{4\Phi_1} \Pi^{\dagger} \Pi - i \frac{N}{8\Phi_1} \left( D^{\dagger} \Pi - \Pi^{\dagger} D \right) + \left( \Gamma_1 (m^2_K - m^2_\pi) + \frac{N^2}{16\Phi_1} \right) D^{\dagger} D.
\]
The order \( N \) piece of the Hamiltonian is the classical ground state energy \( M_h \). The order 1 piece includes the terms quadratic in \( D \) and \( \Pi \), and thus may be diagonalized exactly using creation and annihilation operators

\[
D^i = \frac{1}{\sqrt{N\mu_1}} (a^i + (b^i)^\dagger) \,, \quad \Pi^i = \frac{\sqrt{N\mu_1}}{2i} (a^i - (b^i)^\dagger) \,,
\]

where

\[
\mu_1 = \sqrt{1 + 16(m_K^2 - m_\pi^2)\Gamma_1\Phi_1/N^2} \,.
\]

The operators \( a^\dagger (b^\dagger) \) may be thought of as creation operators for constituent strange quarks (anti-quarks). The normal-ordered Hamiltonian to order 1 is given by

\[
H = M_h + \omega_1 a^\dagger a + \bar{\omega}_1 b^\dagger b ; \quad \omega_1 = \frac{N}{8\Phi_1}(\mu_1 - 1) \, , \quad \bar{\omega}_1 = \frac{N}{8\Phi_1}(\mu_1 + 1) \, .
\]

Thus, replacing a light quark with a strange quark (anti-quark) costs energy \( \omega_1 (\bar{\omega}_1) \). Note that for \( m_K = m_\pi \), \( \omega_1 \) vanishes thereby restoring the original SU(3) symmetry (it costs no energy to replace a \( u \) or \( d \) quark with an \( s \) quark) but that \( \bar{\omega}_1 \) tends to a rather large value, \( N/(4\Phi_1) \). The Wess-Zumino term, which acts as magnetic field in the \( D-D^\dagger \) plane, breaks the \( s \leftrightarrow \bar{s} \) symmetry. Using the calculated values of \( \Phi_1 \) and \( \Gamma_1 \), we find that \( \omega_1 \approx 200 \text{ MeV} \). This is a reasonable estimate of the difference between the strange and the light quark constituent masses. It is also a good upper bound on the similar quantity (the lowest mode energy) found in the bound state approach to strangeness. We conclude that the rigid oscillator approach is a sound approximation and proceed to apply it to the \( B = 2 \) soliton.

Since the \( B = 2 \) soliton is less symmetric (its flavor rotations are in general different from spatial rotations), we consider rigid rotations both in the flavor space and in the real space,

\[
U(\vec{x}, t) = A(t)U_{B=2}(R(t)\vec{x})A(t)^\dagger ,
\]

together with the ansatz of eqs. (3–5). To order 1 we obtain an effective Lagrangian very similar to that in the \( B = 1 \) sector,

\[
L = -M_{B=2} + 4\Phi_2 \hat{D}^\dagger \hat{D} + iN \left( \hat{D}^\dagger \dot{D} - \dot{\hat{D}}^\dagger \hat{D} \right) - \Gamma_2 (m_K^2 - m_\pi^2)D^\dagger D .
\]

(8)

The only modifications are the extra factor of 2 in the Wess-Zumino term, and the new integral expressions [12]

\[
\Gamma_2 = \frac{f_\pi^2}{2} \int d^3x \left( 1 - \cos F(r, \theta) \right) ,
\]

\[
\Phi_2 = \frac{f_\pi^2}{8} \int d^3x \left( 1 - \cos F \right) \left[ 1 + \frac{1}{e^2 f_\pi^2} \left\{ (FF) + (\Theta\Theta) \sin^2 F + 4\sin^2 F \frac{\sin^2 \Theta}{r^2 \sin^2 \theta} \right\} \right] ,
\]

where \( (FF) = (\partial F/\partial r)^2 + (\partial F/\partial \theta)^2/r^2 \). Numerical evaluation of the integrals in the \( B = 2 \) soliton background yields

\[
\Phi_2 \approx 0.0038/\text{MeV} \, , \quad \Gamma_2 \approx 0.0079/\text{MeV} \, .
\]
Canonical quantization of (8) leads to the Hamiltonian
\[ H = M_{B=2} + \frac{1}{4\Phi_2} \Pi^\dagger \Pi - i\frac{N}{4\Phi_2} (D^\dagger \Pi - \Pi^\dagger D) + \left( \Gamma_2(m_{K}^2 - m_{\pi}^2) + \frac{N^2}{4\Phi_2} \right) D^\dagger D. \] (9)
Diagonalizing it as before, we find
\[ H = M_{B=2} + \omega_2 a^\dagger a + \bar{\omega}_2 b^\dagger b; \quad \omega_2 = \frac{N}{4\Phi_2} (\mu_2 - 1) , \quad \bar{\omega}_2 = \frac{N}{4\Phi_2} (\mu_2 + 1) , \]
where
\[ \mu_2 = \sqrt{1 + 4(m_{K}^2 - m_{\pi}^2)\Gamma_2\Phi_2/N^2} . \]
Substituting the numbers we obtain \( \omega_2 \approx 198 \text{ MeV} \). This is only 2 MeV less than \( \omega_1 \), the corresponding quantity in the \( B = 1 \) calculation. In the bound state calculations [13, 14], the kaon mode energies in the \( B = 2 \) and \( B = 1 \) backgrounds were also close to each other.

In order to calculate the \( 1/N \) corrections we need to include the terms in the Lagrangian (8) which are quartic in \( D \) or involve the soliton angular velocities. For \( B = 1 \) this was done in [21], and the correction to the Hamiltonian was found to be
\[ \frac{1}{2\Omega_1} \left( (\vec{I}^b_f)^2 + 2c_1 \vec{I}^b_f \cdot \vec{T} + c_1 \vec{T}^2 \right) , \] (10)
\[ c = 1 - \frac{\Omega}{2\mu\Phi} (\mu - 1) , \quad \bar{c} = 1 - \frac{\Omega}{\mu^2\Phi} (\mu - 1) , \] (11)
where we omit the subscript 1 throughout the last equation. \( \vec{I}^b \), the isospin relative to the body fixed axes, is the momentum conjugate to \( \vec{\alpha} \) which is defined by
\[ A^\dagger \dot{A} = \frac{1}{2} i\vec{\alpha} \cdot \vec{T} . \]
For the non-exotic states containing no \( b \)-quanta, \( \vec{T} = \frac{1}{2} a^\dagger \vec{T} a \). The \( \Lambda \)-particle has \( \vec{I} = \vec{I}^b_f = 0 \) and \( T = 1/2 \), so that the order \( 1/N \) correction to its mass is
\[ \delta m_\Lambda = \frac{3c_1}{8\Omega_1} \approx 23 \text{ MeV} , \]
where we use \( \Omega_1 = 0.00514/\text{MeV} \). To order \( 1/N \), our methods give \( m_\Lambda \approx 1086 \text{ MeV} \), which is close to its physical value of 1115 MeV.

In extending the calculation to the \( B = 2 \) soliton, in addition to the SU(2) angular velocity \( \vec{\alpha} \) we include the spatial angular velocity \( \vec{\beta} \), which is also of order \( 1/N \). The complete order \( 1/N \) correction to the Lagrangian (8) is
\[ \delta L = \frac{1}{2} \Omega_2 \sum_{j=1}^{2} (\dot{\alpha}_j + i \dot{D}^\dagger \tau_j \dot{D} - i \dot{D}^\dagger \tau_j D)^2 + \frac{1}{2} \lambda (\dot{\alpha}_3 - 2 \dot{\beta}_3 + i \dot{D}^\dagger \tau_3 \dot{D} - i \dot{D}^\dagger \tau_3 D)^2 \\
+ \frac{1}{2} \lambda \sum_{j=1}^{2} (\dot{\beta}_j)^2 - ND^\dagger \vec{\alpha} \cdot \vec{T} D - \frac{2}{3} i N (D^\dagger \dot{D} - \dot{D}^\dagger D) D^\dagger D - 2i \Phi_2 (D^\dagger \vec{\alpha} \cdot \vec{T} \dot{D} - \dot{D}^\dagger \vec{\alpha} \cdot \vec{T} D) \\
- \frac{8}{3} \Phi_2 (D^\dagger D)(\dot{D}^\dagger \dot{D}) + \frac{2}{3} \Phi_2 (D^\dagger \dot{D} + \dot{D}^\dagger D)^2 + 2 \Phi_2 (D^\dagger \dot{D} - \dot{D}^\dagger D)^2 + \frac{2}{3} \Gamma_2 (m_{K}^2 - m_{\pi}^2) (D^\dagger D)^2 . \]
where the additional moments of inertia are numerically found to be
\[ \Omega_2 \approx 0.0106/\text{MeV}, \quad \lambda \approx 0.0072/\text{MeV}, \quad \tilde{\lambda} \approx 0.016/\text{MeV}. \]

The isospin and angular momentum relative to the body fixed axes are
\[ I_{bf}^i = \frac{\partial L}{\partial \dot{\alpha}_i}, \quad J_{bf}^i = \frac{\partial L}{\partial \dot{\beta}_i}, \]
and we find a constraint
\[ J_{bf}^3 = -2(I_{bf}^3 + T^3). \]

The calculation of the Hamiltonian is lengthy, but the result is quite simple,
\[ \delta H_{1/N} = \frac{1}{2\Omega_2} \left( (\vec{P}^{bf})^2 + 2c_2 \vec{P}^{bf} \cdot \vec{T} + \vec{c}_2 \vec{T}^2 \right) + \frac{1}{2\lambda}(\vec{J}_{bf})^2 + \left( \frac{1}{8\lambda} - \frac{1}{8\Omega_2} - \frac{1}{2\lambda} \right) (J_{bf}^3)^2, \tag{12} \]
where \( c_2 \) and \( \vec{c}_2 \) are given by (11) with subscripts 2 throughout. Note that, unlike in [13, 14], there is no explicit \( T^3 \) term in the Hamiltonian. This greatly simplifies the diagonalization of the Hamiltonian.

Using the identities,
\[ \vec{T}^2 = \frac{1}{4}(a^\dagger a)^2 + \frac{1}{2}a^\dagger a = \frac{S}{2} \left( \frac{S}{2} - 1 \right), \]
\( (\vec{P}^{bf})^2 = J(J + 1), \) and \( (\vec{P}^{bf})^2 = I(I + 1), \) we have
\[ \delta M_{1/N} = \frac{1}{2\Omega_2} \left\{ c_2 K(K + 1) + (1 - c_2)I(I + 1) + \frac{\vec{c}_2 - c_2}{4}(S^2 - 2S) \right\} \]
\[ + \frac{1}{2\lambda} J(J + 1) + \left( \frac{1}{8\lambda} - \frac{1}{8\Omega_2} - \frac{1}{2\lambda} \right) (J_{bf}^3)^2, \]
where \( \vec{K} = \vec{P}^{bf} + \vec{T}. \) The significance of the quantum number \( K \) is that \( K(K + 1) \) is equal, up to an additive constant, to the quadratic Casimir of the SU(3) representation which emerges in the \( m_K = m_\pi \) limit. In such a limit, \( c_2 = \vec{c}_2 = 1 \) and we recover the mass formula of the SU(3) collective coordinate quantization. Furthermore, states with \( K = 0 \) merge into a \((0, N)\) SU(3) multiplet, states with \( K = 1 \) into \((2, N - 1)\), states with \( K = 2 \) into \((4, N - 2)\), etc.

As discussed in [14], not all possible quantum numbers are allowed. There are certain constraints which arise due to special symmetries of the soliton. They work, in effect, to insure the correct statistics of the overall wave function. Consider, for instance, dibaryons with \( S = -2 \) and \( J = 0 \) (which immediately implies \( T = 1 \) and \( J_{bf}^3 = 0 \)). Then one finds that the allowed \((I, K)\) quantum numbers are \((0, 1), (1, 1), (2, 1), (2, 3)\), etc., while the forbidden ones are \((1, 0), (1, 2), (2, 2)\), etc. In the SU(3) limit, the allowed \( J = 0 \) multiplets are \((2, N - 1), (6, N - 3)\), etc. Remarkably, these multiplets are also allowed in the quark model [1]. In general, the states constructed from the \( B = 2, \) SU(2) soliton together with the
states constructed from the SO(3) soliton appear to cover all the quantum numbers found in the quark model.

Among the $S = -2$ dibaryons that we constructed, the lightest one has quantum numbers $I = J = J^B_f = 0$ and $K = 1$. Its mass to order $1/N$ is

$$M = M_{B=2} + 2\omega_2 + \frac{\bar{\omega}_2}{\Omega_2} \approx 2084 \text{ MeV}. \quad (13)$$

The binding energy with respect to $\Lambda\Lambda$, calculated entirely within our approach, is $(66 + 4 + 18 = 88) \text{ MeV}$, where we separated the classical, order 1, and order $1/N$ contributions. The lightest $S = -2$ dibaryon we found is not the $H$ because it originates from the $(2, N - 1)$ SU(3) multiplet, not from the singlet. For arbitrary $N$ it has strangeness $S = -2$, while the $H$ has $S = -2N/3$. Thus, for large $N$ the breaking of SU(3) makes the state that we found lighter than the $H$, which appeared in the SO(3) dibaryon quantization. Whether this conclusion may be extrapolated to $N = 3$ is an open question. We believe, however, that the quantum numbers predicted by the Skyrme model are in accord with the quark model for any $N$, and that we have presented a good description of strange dibaryons for sufficiently large $N$.

How well can we trust the binding energy of 88 MeV? Indeed, no model of low energy QCD is perfect. In our approach to the dibaryon, quantization of other soliton modes, better treatment of the SU(3) breaking, etc., could affect the numbers appreciably. Nevertheless, our calculation is attractive for its simplicity, and it gives a good physical picture of both $B = 1$ and $B = 2$ states. Our results strongly suggest that a tightly bound doubly strange dibaryon is theoretically natural.

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