Fermion localization in higher curvature spacetime

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Abstract

Fermion localization in a braneworld model in presence of dilaton coupled higher curvature Gauss–Bonnet bulk gravity is discussed. It is shown that the lowest mode of left handed fermions can be naturally localized on the visible brane due to the dilaton coupled higher curvature term without the necessity of any external localizing bulk field.

Keywords: Randall Sundrum model, braneworld gravity, modified gravity, higher dimensional gravity, Fermion localization, phenomenology from higher dimension

(Some figures may appear in colour only in the online journal)

1. Introduction

The Randall Sundrum (RS) warped geometry model \cite{RS1, RS2} is an eminently successful model in resolving the long standing gauge hierarchy/naturalness problem in an otherwise successful standard model (SM) of elementary particles. This resulted in extensive search for a signature of the RS model in large hadron collider (LHC) \cite{LHC1, LHC2, LHC3, LHC4, LHC5, LHC6, LHC7, LHC8} . When applied as a physics beyond the SM such a scenario is often based on an underlying assumption that the SM fermions in general can propagate in the bulk while their chiral states are appropriately localized in different regions of bulk spacetime producing the desired 4D fermion masses on the visible brane.

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Though the RS model itself can not provide any justification for this localization, there have been efforts to provide an explanation of this by introducing an ad-hoc scalar field in the anti-de Sitter bulk of the RS model [9–15]. Such a mechanism of localization however gives rise to the speculation about the possible back-reaction of the scalar to jeopardize the original RS solution for the warp factor. The origin of hierarchy of fermion masses in the SM is a problem yet to be resolved. In this work we show that a string inspired modification to Einstein gravity via dilaton and higher curvature effects can explain the hierarchy among the fermion masses, where the effective masses of fermions are determined by their couplings with the dilaton field. Our work is staged on a five-dimensional (5D) spacetime where our universe sits at one of the fixed points of the orbifolded extra spacetime dimension. This work thus is an attempt to look for an alternative pathway for this localization which resorts to the presence of higher curvature corrections to the classical Einsteinian gravity in the bulk as postulated in the RS model. Such a correction, though heavily suppressed in low energy world, assumes significance in an anti-de Sitter (AdS) bulk with curvature $\sim$ Planck scale as assumed in the RS model. At the leading order, such correction over Einstein gravity which is free from the appearance of any ghost field due to the higher derivative terms, is the Gauss–Bonnet (GB) gravity where the various quadratic curvature terms appear in suitable combination to make the theory stable. Such a correction is also inspired by string theory which in addition also predicts the presence of the scalar dilaton in the action.

Following the path adopted in the RS model, if the dilaton coupled GB gravity is compactified on $S^1/Z_2$ orbifold, it gives rise to two branches of warped solutions, one of which is stable and free of ghost. In this work we adopt this ghost free branch and study the effects of higher curvature couplings on the two chirality states of 5D SM fermions to study their localization on the standard model 3-brane.

As discussed earlier, if the standard model fermions also propagate in the bulk, just as graviton, then by appropriately choosing the interaction potential between the fermion and a localizing field one can try to obtain the appropriate overlap between the chiral states of the fermionic wave functions to localize the left chiral massless fermion states on the brane. However without resorting to any such external ad-hoc field can one produce this desired feature through the higher curvature GB terms in the bulk? This would then provide a possible explanation of observing only the left-handed neutrinos in our universe while the massive fermions appear with both the chiral states. We try to address this question in the present work in the framework of GB-dilaton induced warped geometry model [16].

In this work, we consider a bulk fermion and study the localization profile in the 5D warped geometry model in the backdrop of GB dilaton gravity. We will show that inclusion of higher curvature terms lead to the localization of left handed fermionic modes near the visible brane as their mass decreases while the right hand modes are localized within the bulk. In such localization scenario therefore one does not need to invoke an external bulk field as has been proposed earlier. Since the SM only includes the left handed modes of massless fermions, we can clearly see that this higher curvature setup will automatically allow the fermionic wave functions to localize themselves in the TeV brane. On the other hand, the delocalization of right handed modes inside the bulk can produce interesting phenomenological consequences like fermion mass generation as suggested in [15].

2. Fermion localization in Gauss–Bonnet dilaton gravity

Here we generalize the analysis in warped geometry in presence of GB coupling and gravidiiton coupling in a 5D bulk. The background warped geometry model is proposed by making use of the following sets of assumptions as a building block:
• The leading order Einstein’s gravity sector is modified by the GB [16–25] and dilaton coupling [16, 20, 21, 26] which originates from heterotic string theory.

• The background warped metric has a RS like structure [1, 2] on a slice of AdS$_5$ geometry. For example, from 10D string model compactified on AdS$_5 \times S^5$, one typically obtains moduli from $S^5$ as scalar degrees of freedom. Such moduli can be stabilized by fluxes. In our model, which is similar to a 5D RS model, it is assumed that these degrees of freedom are frozen to their vacuum expectation value and are non-dynamical at the energy scale under consideration [27]. We therefore focus into the slice of AdS$_5$ as is done for the 5D RS model.

• The well known $S^1/Z_2$ orbifold compactification is considered.

• The dilaton degrees of freedom is assumed to be confined within the bulk.

• We allow the interaction between dilaton and the 5D bulk cosmological constant via dilaton coupling.

• The Higgs field is localized at the visible (TeV) brane and the hierarchy problem is resolved via Planck to TeV scale warping.

• The modulus can be stabilized by introducing scalar in the AdS$_5$ bulk without any fine tuning following Goldberger–Wise mechanism [28–31].

• Additionally while determining the values of the model parameters we require that the bulk curvature to be less than the 5D Planck scale $M_5$ so that the classical solution of the 5D gravitational equations can be trusted [8, 32].

2.1. The background setup

Before going to discuss the various features of fermion localization, we start with the 5D action for the two brane warped geometry model including higher curvature gravity as [16]:

$$S = \int d^5x \left[ \frac{M_5^4}{2} R^{(5)} + \frac{\alpha_5 M_5}{2} \left( R^{ABCD} R_{ABCD} - 4 R^{AB} R_{AB} + R_{(5)}^2 \right) - \frac{g^{AB}}{2} \partial_\alpha \chi(y) \partial_\beta \chi(y) - 2 \Lambda_5 e^{\chi(y)} \right] - \sum_{i=1}^2 \sqrt{-g^{(5)}(y)} T_i e^{\chi(y)} \delta(y - y_i) \right]$$

(2.1)

where $A, B, C, D = 0, 1, 2, 3, 4$. Here $i$ signifies the brane index, $i = 1$(hidden), 2(visible) and $T_i$ is the brane tension. Additionally $\alpha_5$ and $\chi(y)$ represent the GB coupling and dilaton. The background metric describing slice of the AdS$_5$ is given by,

$$ds_5^2 = g_{AB} dx^A dx^B = e^{-2\lambda(y) r_c} dy \delta(y - y_i) dx^\alpha dx^\beta + r_c^2 dy^2$$

(2.2)

where $r_c$ represents the compactification radius of extra dimension which has a unit of $M_{Pl}^{-1}$. Here the orbifold points are $y_i = [0, \pi]$ and periodic boundary condition is imposed in the closed interval $-\pi \leq y \leq \pi$. After orbifolding, the size of the extra dimensional interval is $\pi r_c$. Here it is important to note that the bulk extra dimension $y$ is dimensionless and plays a role of an angular coordinate in this context. Moreover in the above metric ansatz $e^{-2\lambda(y)}$ represents the warp factor while $\eta_{\alpha\beta} = (-1, +1, +1, +1)$ is the flat Minkowski metric. A more general brane metric for a purely Einsteinian bulk has been discussed in [33].
2.2. Warp factor from Gauss–Bonnet dilaton gravity

After varying the model action stated in equation (2.1) we get:

\[
\delta S = \int d^5x \left( \sqrt{-g^{(5)}} \left( M_5^2 G_{AB}^{(5)} + \alpha_5 M_5^2 H_{AB}^{(5)} + T_{AB} \right) + \sum_{i=1}^2 T_i \sqrt{-g^{(5)}} \delta_{\alpha_5} e^{\chi \delta (y - y_i)} \right) \delta g^{AB} \\
+ \int d^5x \left( \sqrt{-g^{(5)}} \left( -M_5^4 \Box^{(5)} \chi - 2 \Lambda^{(5)} e^{\chi \delta y} \right) + \sum_{i=1}^2 T_i \sqrt{-g^{(5)}} e^{\chi \delta (y - y_i)} \right) \delta \chi
\]

(2.3)

where the 5D Einstein’s tensor, the GB tensor are given by:

\[
G_{AB}^{(5)} = \left[ R_{AB}^{(5)} - \frac{1}{2} g_{AB} R^{(5)} \right],
\]

(2.4)

and

\[
H_{AB}^{(5)} = 2R_{ACDE}^{(5)} g^{DE} - 4R_{ACBED}^{(5)} g^{CD} - 4R_{AC}^{(5)} R_B^{(5)} + 2R^{(5)} R_{AB}^{(5)} \\
- \frac{1}{2} \delta_{AB} \left( R^{ACDE} g_{AC}^{(5)} R_{BCDE}^{(5)} - 4R_{AC}^{(5)} R_{B}^{(5)} - R_{AB}^{(5)} R^{(5)} \right).
\]

(2.5)

Also the 5D D’Alembertian operator is defined as, \( \Box^{(5)} \chi \delta y = \frac{\sqrt{-g^{(5)}}}{\delta y} \partial_{\delta y} \left( \sqrt{-g^{(5)}} \delta \chi \left( y \right) \right). \)

In this context the 5D dilaton stress tensor is defined as, \( T_{AB} = -\frac{2}{\sqrt{-g^{(5)}}} \frac{\delta S}{\delta g_{AB}} \left( \sqrt{-g^{(5)}} \chi \right). \)

where \( L_{\chi} \) is dilaton Lagrangian as given by, \( L_{\chi} = \left[ -M_5^3 \frac{\delta M_5^2}{\sqrt{-g^{(5)}}} \delta \chi \partial \chi - V(\chi) \right]. \) Here it is important to note that the 5D bulk dilaton potential is identified to be the following expression:

\( V(\chi) = 2 \Lambda_5 e^{\chi \delta y}, \)

where 5D cosmological constant \( \Lambda^{(5)} \) fix the scale of the dilaton potential.

By doing explicit computation one can show that the 5D dilaton stress tensor can be expressed as:

\[
T_{AB} = \left[ \partial_{\delta y} \chi \left( y \right) \partial \chi \left( y \right) + g_{AB} \left( \frac{g^{CD}}{2} \partial \chi \left( y \right) \partial \chi \left( y \right) + V(\chi) \right) \right].
\]

(2.6)

After varying the model action stated in equation (2.1) with respect to the 5D metric \( g_{AB} \) we get, \( \frac{\delta S}{\delta g_{AB}} = 0 \) using which the 5D bulk equation of motion turns out to be,

\[
\sqrt{-g^{(5)}} \left[ G_{AB}^{(5)} + \frac{\alpha_5^{(5)}}{M_5^{(5)}} H_{AB}^{(5)} \right] = -\frac{1}{M_5^{(5)}} \left[ \sqrt{-g^{(5)}} T_{AB} + \sum_{i=1}^2 T_i \sqrt{-g^{(5)}} \alpha_5^{(i)} \delta_{\alpha_5} e^{\chi \delta y} \left( y - y_i \right) \right].
\]

(2.7)

Now very far from the orbifold points the 5D field equation in the warped background can be simplified in the following form:

\[
\sqrt{-g^{(5)}} \left[ G_{AB}^{(5)} + \frac{\alpha_5^{(5)}}{M_5^{(5)}} H_{AB}^{(5)} \right] = -\frac{1}{M_5^{(5)}} \sqrt{-g^{(5)}} T_{AB}.
\]

(2.8)

Similarly varying equation (2.1) with respect to the dilaton field we get \( \frac{\delta S}{\delta \chi} = 0 \). Using this, the Klein–Gordon (KG) equation for the dilaton in the warped background turns out to be:

\[
\frac{1}{M_5^{(5)}} \sum_{i=1}^2 T_i \sqrt{-g^{(5)}} e^{\chi \delta y} \left( y - y_i \right) = \sqrt{-g^{(5)}} \left[ 2 \frac{\Lambda^{(5)}}{M_5^{(5)}} e^{\chi \delta y} + \Box^{(5)} \chi \right].
\]

(2.9)
Now very far from the orbifold points the KG equation for the dilaton in the warped background can be simplified in the following form:

$$\Box(\chi) = J(\chi). \quad (2.10)$$

Here $J(\chi)$ is identified to be the source function for the bulk localized dilaton field, which is given by the following expression:

$$J(\chi) = \frac{V(\chi)}{M_5} = 2 \frac{\Lambda(5)}{M_5} e^{\chi(y)}. \quad (2.11)$$

Before solving the equations (2.7) and (2.10), here it is important to note the following crucial facts which are sure to be helpful for us in understanding the nature of the field equations:

- First of all the derived two field equations are second order coupled differential equations. Here such complicated structures are appearing due to minimal interaction between gravity and dilaton in the bulk.
- On the other hand, the KG equation for the dilaton in the warped background is itself complicated as it contains an exponential source term, which is appearing as dilaton effective potential in the bulk.
- Both of the equations can be simplified as equations (2.8) and (2.10) if we go very far from the orbifold fixed points.

In order to solve these coupled equations, we use some approximations as follows: first of all, it is important to note that we may neglect the derivative terms in the dilaton action with respect to its potential which is of the order of Planck scale. Due to this fact the stress energy tensor of the dilaton is approximately given by the expression:

$$T_{AB} \approx g_{AB} V(\chi) = 2 g_{AB} \Lambda(5) e^{\chi(y)}. \quad (2.12)$$

Consequently, the 5D field equation can be recast into the following simplified form:

$$\sqrt{-g(5)} \left[ G^{(5)}_{AB} + \frac{\alpha(5)}{M_5^2} H^{(5)}_{AB} \right] \approx - \frac{e^{\chi(y)}}{M_5^2} \Lambda(5) \sqrt{-g(5)} g_{AB}^{(5)} + \sum_{i=1}^{2} T_i \sqrt{-g(5)} g_{i,j}^{(5)} \delta^{\alpha\beta} \delta(\delta - y_i) \right], \quad (2.13)$$

which can be further simplified in the region far away from the orbifold fixed points as:

$$\sqrt{-g(5)} \left[ G^{(5)}_{AB} + \frac{\alpha(5)}{M_5^2} H^{(5)}_{AB} \right] \approx - \frac{e^{\chi(y)}}{M_5^2} \Lambda(5) \sqrt{-g(5)} g_{AB}^{(5)} = - \frac{1}{M_5^2} \sqrt{-g(5)} V(\chi) g_{AB}^{(5)} . \quad (2.14)$$

Moreover, considering the equation of motion for the dilaton, we further note that the source term in the right hand side is extremely suppressed due to the warping and therefore may be neglected. As a result the space variation of $\chi(y)$ can be derived from the equation:

$$\partial^2 \chi(y) \approx 0. \quad (2.15)$$

Now using the $Z_2$ orbifolding, we obtain at the leading order of $\alpha(5)$ [16]:

$$\chi(y) = (c_1 |y| + c_2) \quad (2.16)$$

where $c_1$ and $c_2$ are arbitrary integration constants in which $c_1$ characterizes the strength of the dilaton self interaction within the bulk. For our computation we fix $c_2 = 0$. As the nature of warping influences the localization profile of the bulk fermion, therefore it is expected that
the dilaton charge $c_1$ for a given fermionic field will determine the localization property and hence the effective fermion mass term on the brane.

The corresponding warp factor turns out to be [16]:

$$A(y) := A_{\pm}(y) = k_{\pm}(y) r_c |y|$$

(2.17)

where

$$k_{\pm}(y) = \frac{3M_5^2}{16\alpha_{(5)}} \left[ 1 \pm \sqrt{1 + \frac{4\alpha_{(5)}\Lambda_5 e^{\chi_5(y)}}{9M_5^2}} \right].$$

(2.18)

Also the localized brane tensions are given by:

$$T_2 = -T_1 = 24k_{\pm}(y) M_5^3 e^{-\chi_5} \left[ 1 - \frac{\alpha_{(5)} k_{\pm}^2(y) r_c^2}{3M_5^2} \right].$$

(2.19)

In the small $\alpha_{(5)}$, $c_1$ and $c_2$ limit we retrieve the results as in the case of RS model with:

$$k_-(y) \rightarrow k_{RS} = \sqrt{-\frac{\Lambda_5}{24M_5^3}},$$

(2.20)

and the corresponding brane tension is given by:

$$T_2^{RS} = -T_1^{RS} = 24k_{RS} M_5^3.$$

(2.21)

Here we have discarded the +ve branch of solution of $k_+$ which diverges in the small $\alpha_{(5)}$ limit, bringing in ghost fields [25, 34 – 38]. Now expanding equation (2.18) in the perturbation series order by order around $\alpha_{(5)} \rightarrow 0$, $c_1 \rightarrow 0$ and $c_2 \rightarrow 0$ we can write:

$$k_{\text{M}}(y) := k_-(y) = k_{RS} e^{\chi_5} \left[ 1 + \mathcal{O}(L^2) + \cdots \right].$$

(2.22)

Here $L$ is defined as:

$$L := \frac{4\alpha_{(5)} k_{RS}^2}{M_5^2}.$$ 

(2.23)

However, the results of this paper will be unchanged if we take the non zero value of the constant $c_2$. In the weak coupling regime of gravity and dilaton one can also consider non zero but small values of $c_2$ which finally appears as an overall factor $e^{c_2^2/2}$ in the expression for the warp factor in equation (2.22). More precisely the contribution in the warp factor can be written as:

$$e^{c_2^2/2} \approx 1 + \left( \frac{c_2}{2} + \cdots \right) \approx \left( 1 + \frac{c_2}{2} \right),$$

(2.24)

where we have neglected all the higher powers of $c_2$ as in the weak coupling regime of the gravity and dilaton always $c_2 \ll 1$ approximation holds good. On the other hand it is important to note that in this context strictly one cannot consider very large values of $c_2$, as in that case the perturbative solution in the weak coupling approximation itself is not valid for dilaton and consequently in the solution for the warp factor. Additionally in the weak coupling regime of gravity and dilaton the dilatonic charge $c_1$ is small compared to unity at the orbifold point...
y = π where the visible brane is placed. Here, to visualize the effect of dilaton in the phenomenon of localization of fermions we further use an approximation that the contribution from the dilatonic charge $c_1$ is larger than the contribution from $c_2$ at the orbifold point $y = π$. For a similar reason here also large values of $c_1$ are not strictly allowed. Using these set of approximations one can finally write down the warp function as:

$$k_M(y) ≈ k_{RS} e^{\frac{\pi y}{L_5}} \left(1 + \frac{c_2}{2}\right) \left[1 + L + O(L^2) + \cdots\right].$$

(2.25)

Further equation (2.25) can be used to solve the naturalness or gauge hierarchy problem and for this purpose one need to consider the following modified constraint at the orbifold point $y = π$ as given by:

$$k_M(π)r_c ≈ k_{RS} r_c e^{\frac{π y}{L_5}} \left(1 + \frac{c_2}{2}\right) \left[1 + L + O(L^2) + \cdots\right] ≈ 12.$$  

(2.26)

As in the weak coupling regime the constant $c_2$ is just playing the role of a correction term or an overall normalization factor, one can neglect the contribution from the coupling $c_2$ completely without taking care of any small contributions from the small corrections. This is exactly equivalent to the similar effect if we set $c_2 = 0$ from the starting point of our computation. In this case equation (2.25) can be written as:

$$k_M(y) ≈ k_{RS} e^{\frac{\pi y}{L_5}} \left[1 + L + O(L^2) + \cdots\right],$$

(2.27)

and consequently the modified constraint condition to solve the naturalness or gauge hierarchy problem can be recast at the orbifold point $y = π$ as:

$$k_M(π)r_c ≈ k_{RS} r_c e^{\frac{π y}{L_5}} \left[1 + L + O(L^2) + \cdots\right] ≈ 12.$$  

(2.28)

Now if we set the limit $α_3 → 0$ and $c_1 → 0$ then one can get back the RS result, which is $k_{RS}r_c ≈ 12$. Here it is important to note that, both the solution of the warp factor and the dilaton is consistent with equations (2.14) and (2.15) in the weak coupling regime of gravity (graviton) and dilaton in the bulk.

### 2.3. Localization scenario for fermions

We will now start our discussion regarding the localization scenario of fermionic modes. The 5D action for the massive fermionic field can be written as:

$$S_f = \int d^5x [\text{Det}(V)] \left\{ i\bar{Ψ}(x,y)γ^α\mathcal{D}_α^\mu \bar{Ψ}(x,y)δ^y_{\mu} - sgn(y)m_fΨ(x,y)Ψ(x,y) + \text{h.c.} \right\}$$

$$= \int d^5x e^{-4\text{det}(y)r_c} \left\{ \bar{Ψ}(x,y) \left[ ie^{4\text{det}(y)r_c}γ^α\hat{D}_α^\mu \right] Ψ(x,y) - sgn(y)m_B \right\} Ψ(x,y) + \text{h.c.}$$

(2.29)

where the differential operator $\hat{D}_\mu^\gamma$ is defined as $\hat{D}_\mu^\gamma := \left(\hat{\partial}_\mu^\gamma + \Omega_\mu^\gamma\right)$, which represents the covariant derivative in presence of fermionic spin connection:

$$\Omega_\mu^\gamma = \frac{1}{8} \omega^{\Lambda^\mu}_\nu \left[ \Gamma^\Lambda_{\alpha\beta}, \Gamma_\nu^\gamma \right].$$

(2.30)

Here $\omega^{\Lambda^\mu}_\nu$ represents the gauge field respecting $SO(3,1)$ transformation on the vierbein coordinate. Here we assume that the bulk fermion mass $m_B$ originates through an underlying
spontaneous symmetry breaking in bulk via 5D Higgs mechanism \[11, 39–42\]. The 5D Gamma matrices:
\[\Gamma^\hat{A} = (\gamma^\mu, \gamma_5) = i\varepsilon^{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta = i\gamma_4\] (2.31)
satisfy the Clifford algebra anti-commutation relation \[\{\Gamma^\hat{A},\Gamma^\hat{B}\} = 2\eta^{\hat{A}\hat{B}}\] with \[\eta^{\hat{A}\hat{B}} = \text{diag} (-1, +1, +1, +1, +1)\]. In this context
\[g_{MN} := (\gamma^4 \otimes \gamma^\mu) \eta_{\hat{A}\hat{B}},\] (2.32)
where \(V_{\hat{A}M}\) are characterized by the usual conditions:
\[V_{44} = 1,\] (2.33)
\[V_{\hat{A}\mu} = e^{-\kappa_{\hat{A}(y)r}\gamma^r} \eta_{\hat{A}\hat{B}},\] (2.34)
\[\text{Det}(V) = e^{-4\kappa_{\hat{A}(y)r}\gamma^r},\] (2.35)
and \(\hat{A}, \hat{B}\) are tangent space indices. For our set up \(SO(3,1)\) spin connection can be written as:
\[\Omega_4 = 0,\] (2.36)
\[\Omega_\mu = -\frac{1}{2} e^{-\kappa_{\hat{A}(y)r}\gamma^r} k_M(y)r_c\gamma_5\gamma_\mu.\] (2.37)
Here we focus our attention only to the lowest fermionic mode in the brane. To serve this purpose we decompose the 5D spinor as
\[\Psi(x, y) = \psi(x)\xi(y).\] (2.38)
In the massless case the definite chiral states \(\psi_L(x)\) and \(\psi_R(x)\) correspond to left and right chiral states in four dimensions. The \(\psi_L\) and \(\psi_R\) are constructed by,
\[\psi_{L,R} = \frac{1}{2} (1 \pm \gamma^5)\psi.\] (2.39)
Here \(\xi\) denotes the extra dimensional component of the fermion wave function. We then can decompose the 5D spinor in the following way:
\[\Psi(x, y) = \psi_L(x)\xi_L(y) + \psi_R(x)\xi_R(y).\] (2.40)
Substituting the above decomposition in equation (2.29) we obtain the following equations for the fermions,
\[e^{-\kappa_{\hat{A}(y)r}\gamma^r} \left[ \pm (\partial_y - 2r_c\partial_y (k_M(y)|y|) + \text{sgn}(y)m_B) \xi_{L,R}(y) = -m \xi_{L,R}(y)\right]\] (2.41)
where \(m_B\) and \(m\) represent the 5D bulk mass and lowest effective mass of 4D fermion respectively. The 4D fermions obey the canonical equation of motion, \(i\gamma^\mu\partial_\mu\psi_{L,R} = m\psi_{L,R}\). Also it is important note that the left and right handed part of the extra dimensional wave function satisfies the usual ortho-normality condition. Finally, the solution of equation (2.41) turns out to be:
\[\xi_{L,R}(y) = \mathcal{N} \exp\left[ 2e^{-\kappa_{\hat{A}(y)}\gamma^r} k_M(y)|y| r_c \pm m \int dy e^{\kappa_{\hat{A}(y)r}\gamma^r} \pm \text{sgn}(y)m_B|y|} \right]\] (2.42)
where $\mathcal{N}$ represents the normalization constant$^5$.

Figures 1(a)–(f) describe the localization profiles of the fermion wave function inside the bulk. They clearly depict that for both massive as well as massless fermions, the left handed mode is localized on the brane while the right handed fermions are localized inside the bulk. Additionally, from the prescribed analysis we can observe that the gradual increment in the dilaton coupling $c_1$ for a fixed value of GB parameter $L$ within the window$^6$:

$$10^{-3} < L < 10^{-7},$$  (2.43)

will shift the peak position of the right handed fermionic wave function towards the left side of the visible brane towards the bulk. In such a situation the height of the right fermionic mode increases, the amount of localization increases and the localization position of the left fermionic mode slightly shift from the visible brane towards the bulk. Here it is clear from the figures 1(a)–(f), the mass of the different generation fermions increases from MeV to TeV, the wave function gets more and more sharply peaked towards the visible brane, implying more localization. We also observe that the the effective 4D mass $m$ term decreases as the peak position of the left handed fermionic mode shifts towards visible brane. Additionally it is important to note that left handed mode always touches the visible brane at $y = \pi$ with a finite height of the wave function for the different values of the effective 4D mass $m$ lying within MeV to TeV window. Additionally it is important to note that, from various experiments and observations following constraints are available for the GB parameter $L \sim \alpha_5$ (where $k_{RS} \sim M_5$), which are perfectly consistent with the present window of the the GB parameter considered in this paper:

- Astrophysical constraints from the perihelion precession of planetary orbits and the bending angle of null geodesics suggests that the bound on the GB parameter $L \sim \alpha_5$ lie within the following window $[43]$:  
  $$0 < L < 10^{-7}.$$  (2.44)

- Collider constraints from the Higgs mass of the resonance discovered near 125 GeV and the constraints from the $\mu$ parameter for Higgs diphoton and dilepton decays using ATLAS $[44]$ and CMS $[45]$ data within the $5\sigma$ statistical C.L. suggests that the bound on the GB parameter $L \sim \alpha_5$ lie within the following window $[20]$:  
  $$10^{-7} < L < 0.2.$$  (2.45)

- Another phenomenological constraint from the the lower bound on the lightest Kaluza–Klein (KK) graviton mass as obtained from the ATLAS $[44]$ dilepton search in 7 TeV proton–proton collision suggests that the bound on the GB parameter $L \sim \alpha_5$ lie within the following window $[20]$:  

$^5$ Applying the normalization of the extra dimensional wave function for left and right chiral fermionic modes the normalization constant can be expressed as:

$$\mathcal{N} = \frac{1}{\int_0^\pi dy \exp \left[ \left( 4e^{\frac{2\pi}{L}} - 3 \right) k_{BB}(y) r_c |y| \right]}$$

$^6$ Here we choose this specific window for the GB parameter $L$ to confront with other bounds on $L$ obtained from various astrophysical observations and collider search, which we have mentioned later in table 1. Also there is another motivation for choosing the small values of the GB parameter $L$ for our present setup is to justify the validity of the perturbation theory in the weak coupling regime of the gravity sector.
Figure 1. Localization of the left and right handed fermionic profile $\xi_{L,R}(y)$. For all situations we have taken $\alpha(5) = 10^{-3}$, $c_1 = 0.17$, $m_B = (+1(\text{left}), -1(\text{right}))$ which fixes, $k_1 = 12$, is necessarily required to solve the hierarchy problem. Here all the masses are given in the Planckian unit. The black colored vertical lines represent the hidden and visible branes which are placed at the orbifold points $y = 0$ and $y = \pi$ respectively. Also the purple colored dashed line represent that left handed mode always touches the visible brane at $y = \pi$ with a finite height of the wave function. (a) Left fermionic mode with $m \sim \mathcal{O}(\text{TeV})$. (b) Right fermionic mode with $m \sim \mathcal{O}(\text{TeV})$. (c) Left fermionic mode with $m \sim \mathcal{O}(\text{GeV})$. (d) Right fermionic mode with $m \sim \mathcal{O}(\text{GeV})$. (e) Left fermionic mode with $m \sim \mathcal{O}(\text{MeV})$. (f) Right fermionic mode with $m \sim \mathcal{O}(\text{MeV})$. 

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In the context of AdS/CFT the viscosity entropy ratio can be computed in presence of GB parameter $L \sim \alpha_{(5)}$ using the well known Kubo formula as \[21, 46\]:

$$\frac{\eta}{S} = \frac{1}{4\pi}(1 - 4L) + \mathcal{O}(L^2).$$

To satisfy the constraint on the positivity of the viscosity entropy ratio the upper bound on the GB parameter $L \sim \alpha_{(5)}$ is given by \[20, 46\]:

$$L < 0.25.$$

In Table 1 for the comparison of different sources of contraints on GB coupling we have mentioned all of these results, which shows that the result obtained in this paper is perfectly consistent with the astrophysical and particle collider data.

| Serial number | Constraint on GB coupling ($L$) | Different sources for the constraint on GB coupling | Final remarks |
|---------------|--------------------------------|---------------------------------------------------|--------------|
| I.            | $0 - 10^{-7}$                  | Perihelion precession of planetary orbits and the bending angle of null geodesics | Astrophysical constraint |
| II.           | $10^{-7} - 0.2$                | The discovered Higgs mass and Higgs diphoton and dilepton decays using ATLAS and CMS data | LHC constraint |
| III.          | $(4.8 - 5.1) \times 10^{-7}$  | Lower bound on the lightest KK graviton mass from ATLAS dilepton search | Search for extra dimensions at LHC |
| IV.           | $< 0.25$                      | Positivity of viscosity entropy ratio              | AdS/CFT correspondense |
| V.            | $10^{-3} - 10^{-7}$           | Localization of lowest mode of left handed fermions without using any bulk field | Constraint from extra dimensions and consistent with I, II, III, IV. |

\[ 4.8 \times 10^{-7} < L < 5.1 \times 10^{-7}. \]  (2.46)

- In the context of AdS/CFT the viscosity entropy ratio can be computed in presence of GB parameter $L \sim \alpha_{(5)}$ using the well known Kubo formula as \[21, 46\]:

$$\frac{\eta}{S} = \frac{1}{4\pi}(1 - 4L) + \mathcal{O}(L^2).$$

To satisfy the constraint on the positivity of the viscosity entropy ratio the upper bound on the GB parameter $L \sim \alpha_{(5)}$ is given by \[20, 46\]:

$$L < 0.25.$$  (2.48)

The overlap wave function of the left and right handed mode on the visible brane determines the effective mass of the fermion on the 3 brane. The effective 4D mass can be computed from the overlap integral as:

$$I_{\text{overlap}} = m_B \int d^5x \left| \text{Det}(V) \right| \text{sgn}(y) \bar{\psi}(x, y) \psi(x, y)$$

$$= \int d^4x mL \left[ \bar{\psi}_L(x) \psi_R(x) + \bar{\psi}_R(x) \psi_L(x) \right],$$

where the 4D effective mass $m_{L,R}$ is given by:

$$m_{L,R} = m_B \int_0^\pi \text{e}^{-4kM(y)\xi}\text{sgn}(y) \xi^L(y) \xi_R(y).$$

The equation above clearly indicates that if the bulk mass $m_B = 0$, then the mass of the lowest mode of fermions will also be zero. Further substituting equations (2.42) in (2.50) the effective mass $m = m_{L,R}$ can be recast as:
where we fix $k_{RS,c} = 12$, which is a necessary condition to resolve the gauge hierarchy or naturalness problem. In equation (2.51), we have introduced imaginary error function, which is defined as:

$$\text{Erfi}(x) = -i \text{Erfi}(ix) = \frac{2}{\sqrt{\pi}} e^{-x^2} \mathcal{D}(x)$$

where $\mathcal{D}(x)$ is the Dawson function, is given by:

$$\mathcal{D}(x) = e^{-x^2} \int_{x=0}^x e^t \, dt. \quad (2.53)$$

In table 2 we have shown the total parameter space for the GB coupling $L$, dilaton coupling $c_1$ and the 5D bulk mass $m_B$ required to generate the overlap of left and right handed fermion wave functions at the visible brane via arying the 4D effective mass within the window $10^{-9}$ GeV < $m$ < $10^3$ GeV.

### Table 2. Parameter space required to generate overlap of left and right handed fermion wave functions at the visible brane via 4D effective mass.

| 4D mass $m$ (in GeV) | 5D bulk mass $m_B$ (in $M_{Pl}$) | GB coupling $L$ | Dilaton coupling $c_1$ |
|----------------------|-------------------------------|-----------------|-----------------------|
| $10^3$               | 1                             | $10^{-3} - 10^{-7}$ | 0.007 |
| $1$                  | 1                             | $10^{-3} - 10^{-7}$ | 0.033 |
| $10^{-3}$            | 1                             | $10^{-3} - 10^{-7}$ | 0.057 |
| $10^{-6}$            | 1                             | $10^{-3} - 10^{-7}$ | 0.078 |
| $10^{-9}$            | 1                             | $10^{-3} - 10^{-7}$ | 0.098 |

where $m_B = 2m_B \sqrt{5[1 + L + \mathcal{O}(L^2)]} \exp \left[ \left\{ 48\pi e^{\frac{c_1}{4\pi}} \left( e^{\frac{c_1}{4\pi}} - 1 \right) + \frac{5}{s_{51}} \right\} [1 + L + \mathcal{O}(L^2)] \right] \sqrt{5} \pi c_1 \left[ \text{Erfi}(\sqrt{\frac{5}{s_{51}}}) - \text{Erfi}\left( \sqrt{\frac{5}{s_{51}}}[1 + L + \mathcal{O}(L^2)](1 + 5c_1\pi) \right) \right]$. 

(2.51)

3. Conclusion

The localization of left handed standard model fermions requires an external 5D bulk field. In this work, we have shown that it is possible to localize the SM fermions in the bulk using the higher curvature dilaton coupled gravity set-up without invoking any external scalar field in the bulk. This, then naturally explains the origin of localization of left handed fermions in the visible brane whereas the right handed fermionic modes get delocalized and obtain their peak inside the bulk. We have also obtained the effective 4D mass term in the brane which depends on the GB coupling parameters and dilaton coupling. Thus a string inspired background with higher curvature Gauss–Bonnet term and dilaton field in the bulk offers a possible explanation for the fermion localization on our brane.

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