Application of intelligent algorithm in trajectory optimization of hypersonic vehicle

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Abstract. Aiming at the application problem of intelligent algorithm in trajectory optimization of hypersonic vehicle, the basic theoretical framework to solve this problem is discussed. Based on the flight characteristics of hypersonic flight vehicle, the demand for intelligent algorithm application is put forward. According to the principle of intelligent optimization, the key problems such as initial selection of trajectory optimization, intelligent algorithm and parallel computation are discussed. On this basis, the system application framework of intelligent algorithm is designed, which optimizes trajectory and guidance synchronously, and provides a new typical application mode for flight vehicle intelligent control technology.

1. Introduction

Hypersonic vehicle can be divided into two categories: unpowered and powered. They can carry out missions such as reconnaissance, delivery and launch load in the space. They mainly include reusable flight vehicle that can realize the transportation between space and earth, and hypersonic vehicle carrying rapid delivery and reconnaissance missions. At present, major aerospace countries in the world have invested a great deal of manpower, material resources and financial resources in the design and development of hypersonic vehicles [1]-[7]. Because hypersonic vehicles mainly fly in the near space, the dynamic model has many characteristics such as multi-constraints, strong coupling, fast time-varying, strong nonlinearity and strong uncertainty, which make the trajectory design of flight vehicle have many difficulties in theory, including small feasible region of trajectory optimization solution, large parameter uncertainty, stronger system coupling, multiple flight mission requirements, and it is urgent to break through the technologies such as online trajectory generation and control system reconfiguration. At present, trajectory design has become the core technology in flight vehicle design.

Bellman dynamic programming and Pontryagin maximum principle, i.e. indirect method [8], were mainly used in early trajectory optimization. This method based on variational principle is more effective for unconstrained and is suitable for solving simple optimal control problems. However, it is difficult for complex problems. With the development of computer technology, trajectory optimization method gradually changes to numerical method. At present, the method for solving optimal control problems is mainly based on direct method. The direct method is to transform the continuous trajectory optimal control problem into a Nonlinear Programming(NLP) problem, and then solve the parameterized NLP problem by numerical method to obtain the optimal trajectory. Since this method does not require the first-order necessity of the optimal solution, it has been widely used in trajectory
optimization of hypersonic vehicles with complex constraints in recent years. The commonly used methods include shooting method, collocation method, pseudospectral method, etc., such as trajectory optimization with waypoint constraints [9], optimal trajectory design of hypersonic vehicles [10] and boost-glide trajectory optimization [11], etc. At present, there are many kinds of software using pseudospectral method to realize optimization [12], which are widely used in offline trajectory optimization. As the environment for hypersonic flight vehicle becomes more and more complex and the tasks become more and more diverse, there are higher requirements for the efficiency of solving NLP problem. As a heuristic algorithm, intelligent optimization algorithms [13]-[17] are of low model dependency, strong robustness, insensitive to initial values, good global optimization ability and parallel computing ability, and have received considerable attention in trajectory optimization problem solving. This paper will discuss the application of intelligent algorithm in trajectory optimization of hypersonic vehicle.

2. Description of Trajectory Optimization Problem

2.1. Mathematical Representation of Trajectory Optimization Problem

Trajectory optimization problem can be expressed as a kind of optimal control problem in mathematics. For different types of hypersonic vehicle, the selection of differential equations and state parameters in optimal control problem will be different, and the constraints will vary greatly according to the task requirements. If different optimization indexes are considered, the trajectory optimization problem will be extremely complex. The following summarizes the main factors and characteristics of the current hypersonic vehicle trajectory optimization problem.

2.1.1. Kinetic Equations. Hypersonic vehicles have diverse trajectory characteristics. The flight phases include ascending phase, orbit changing phase, gliding phase, cruising phase, returning phase, landing phase, etc. Generally, its centroid dynamics equation can be established in the launch coordinate system or velocity coordinate system. In order to describe the flight characteristics of vehicles in different phases conveniently, normalized dynamics models are often established by pole transformation method. The dynamic equation will be appropriately simplified according to the engine performance, aerodynamic performance and structural performance of the flight vehicle.

2.1.2. Constraint Conditions. There are many constraints in the flight process of hypersonic flight vehicle, including flight process constraints, mission constraints, control variable constraints and terminal condition constraints. The mathematical models established by these constraints can usually be described by equality constraints and inequality constraint equations. The main constraint conditions shown in Table 1.

| Constraint type | Constraints | Description                                                                 | Mathematical form |
|-----------------|-------------|------------------------------------------------------------------------------|-------------------|
| Process constraint | Heat flux | Aerodynamic heating effect in high speed flight in atmosphere               | Inequality        |
|                  | Dynamic pressure | The hinge moment of the aerodynamic control surface of the flight vehicle is ensured to be within a controllable range | inequality        |
|                  | Overload | Ensure the structural integrity of the flight vehicle and avoid structural damage | inequality        |
|                  | Equilibrium glide | Realize smooth gliding flight and effectively avoid jumping of flight tracks | inequality        |
| Task constraint | Way points | Flight trajectory constraints for arrival observation or others             | Inequality or equality |
No-fly zones
Aerodynamic control constraint
Engine thrust control
Terminal position constraint
Terminal flight-path angle constraint
Terminal velocity constraint
Avoidance geography constraints or others
Constraints on the range and rate of change of control variables (such as angle of attack and pitch angle, etc.)
Control of engine fuel
Constraints on longitude, latitude and altitude of endpoints
It is usually required that the terminal track angle is not greater than a certain threshold.
Constraint endpoint speed is not less than a certain threshold value

2.1.3. Performance Indices. According to different missions, the index requirements of hypersonic flight vehicle vary greatly, and the common performance index function shown in the following table.

| Index                      | Description                                                                 |
|----------------------------|-----------------------------------------------------------------------------|
| Maximum range              | Realize the maximum longitudinal flight range, which reflects the maximum range capability |
| Minimum range              | Realize the minimum longitudinal flight range, which reflects the minimum range capability |
| Maximum cross range        | The maximum cross range reflects the capability of lateral maneuver         |
| Minimum heat absorption    | To reduce the requirement on the thermal performance of materials            |
| Minimum heat flux          | To reduce the requirement on the thermal performance of materials            |
| Trajectory smoothness      | It can reduce the jump of trajectory and increase the smoothness             |
| Optimization of energy     | Improves the efficiency-cost ratio                                          |
| Maximum overload           | Improving maneuver capability                                               |
| Comprehensive performance  | Corresponding weighting factors are defined for the above different indexes to achieve comprehensive performance index |

2.2. Characteristics of Intelligent Computing in trajectory Optimization
As can be seen from the previous section, the dynamic equations of hypersonic vehicle trajectory optimization problem are extremely complex. The constraint conditions include a variety of equality and inequality constraints, and the index functions also include Lagrangian type, Mayer type and Polza type. It is difficult to solve the problem directly by indirect method. The above problem can be transformed into a NLP problem by direct method, and the difficulty of solving the problem will be different according to the complexity of flight vehicle dynamics problem. It is usually represented as an ultra-high dimensional Large Scale Global Optimization (LSGO) problem, which will lead to dimensional disaster in solving the problem. There are two reasons for the dimension disaster: first, the complexity of the problem becomes larger and the local extreme point increases rapidly; second, the solution space expands exponential with the increase of the dimension of the problem scale variable.

At present, there are two ideas to solve the dimension disaster: (1) decomposing or simplifying the high-dimensional problem into low-dimensional problem; (2) using parallel computing method to solve
multi-thread solution, which can also be combined to improve the efficiency of the algorithm. The core idea of dimension reduction is to split the high-dimensional optimization problem by dividing the high-dimensional space into several low-dimensional subspaces, and to realize the information exchange and spatial reconstruction of different subspaces by defining the reference vector. These algorithms can be divided into accurate algorithms based on gradient and intelligent algorithms, which mainly rely on gradient and other information. Intelligent algorithms include genetic algorithm, evolutionary strategy, simulated annealing method, neural network method, the particle swarm optimization algorithm, ant colony algorithm and so on.

In recent years, intelligent optimization algorithm has attracted much attention in LSGO problem solving of trajectory optimization. Compared with accurate algorithm, intelligent optimization algorithm as a heuristic algorithm has low model dependence, strong robustness and insensitive initial value. Good global optimization ability and parallel computing ability. The problem of intelligent optimization algorithm is that when the objective function of the optimization problem has too many local extreme points, it is easy to fall into local optimization in the process of evolution, resulting in premature phenomenon. The emergence of this phenomenon makes the algorithm stop evolution prematurely, which greatly affects the solving effect of the problem. The solution of premature is mainly to enhance the memory space of the individual, so that the individual can cover the solution space as much as possible. And concentrated around individuals with good qualities. That is to say, the cognition obtained by each generation through learning and communication is preserved and utilized, and when false cognition occurs, the wrong cognition can be weakened by making full use of the correct cognition in the past, thus the probability of an individual getting rid of the shackles of a local extreme point is increased.

According to the characteristics of intelligent algorithm, several theories should be pay attention: simplification of the original problem, the construction of the initial value space of the optimization problem, intelligent algorithm design and parallel computing architecture design with pattern classification ability. In terms of intelligent optimal modeling theory, emphasis is placed on the latter three.

3. Intelligent Optimal Modeling Theory

3.1. Construction of Initial Value Space

The rapid optimization of intelligent optimization algorithm depends largely on the selection of initial values, so the construction of initial value space is the first problem to be discussed in intelligent optimization theory. This problem can be solved by analysing the dynamic characteristics of the flight vehicle. The flight envelope can be established for different flight phases by establishing different complexity flight vehicle dynamic models. Vehicle flight envelope calculation is a set of three-dimensional space location domains that a flight vehicle can reach when given initial terminal constraints and process constraints. The set boundary determines the constraint boundary of the flight trajectory. Because the flight process of hypersonic vehicle is extremely complicated, the flight envelope can be calculated in sections based on the flight profile research. Based on the profile-based calculation method, the structure and expression method of envelope state parameters are given. Considering the uncertainty of parameters, the outward extension method of flight envelope is studied. The research results can provide the constraint range of state variables for trajectory optimization and narrow the space search domain of optimization algorithm.

3.2. Intelligent Algorithm Design Based on Pattern Classification Characteristics

The intelligent algorithm is sensitive to the initial value. A good initial value can greatly improve the convergence speed of the solution. In order to avoid premature phenomenon, the initial value selection is required to be distributed as evenly as possible in the solution space, and the initial value selection method needs to be designed. Under the condition that the initial population is determined, the intelligent algorithm can generate a new population by selecting, crossing, mutating, repairing and other
optimization operators. The operators need to be redesigned according to actual problems and models. On the one hand, the design of the operators should improve the generalization ability of the algorithm, on the other hand, it should avoid falling into local minima, and at the same time, it should improve the convergence speed of the algorithm.

If individuals in each generation of population are directly calculated and evaluated without classification, the amount of calculation and storage will be greatly increased. Non-linear classification methods can be used to classify the population, such as support vector machine technology, etc. Intelligent algorithms with better convergence and adaptability are designed in the initial value space. Through this method, cognitive results are retained through classification, thus greatly reducing the solution range of performance function and further improving the operation efficiency.

3.3. Parallel computing architecture
No matter which kind of intelligent algorithm is adopted, due to the parallel computing characteristics of the intelligent algorithm, the parallel computing framework can be further adopted to improve the computing efficiency of the optimization method. For different algorithms, the parallel computing framework should basically follow the framework shown in figure 1 to design parallel computing for computing tasks, and the performance evaluation function is used in the calculation to evaluate the balance and timeliness of the calculation.

The framework can also be used to propose the comprehensive evaluation conditions of the optimal trajectory and the non-inferior solution trajectory under the population classification model, at last give the optimal and suboptimal solutions with different properties.

4. Application of Intelligent Algorithm
The modeling of hypersonic vehicle based on intelligent algorithm needs to comprehensively implement the dynamic model, experimental design, intelligent algorithm, parallel computation, distributed simulation, computer system, etc. under the existing flight vehicle system. Trajectory generation of flight vehicle needs to be realized by trajectory design and guidance technology under deviation conditions. From the realization approach, the two parts can be organically combined to construct a unified system with coordinated trajectory and guidance. The unified structure is conducive to the online generation of trajectory and finally realize the systematic application of intelligent algorithm.
Online trajectory optimization and robust adaptive guidance are both optimal control problems in essence, but depend on different optimization models, and the two are closely related to each other in implementation and can be implemented under a unified control strategy, as shown in figure 2. \( x \) is the state variable, \( u \) is the control variable, \( J \) is the performance index, \( t \) is the time and \( * \) stand for the optimal value. Using the control strategy to call the trajectory optimization model, the optimal control solution \( u^* \) with the index function \( J_1 \) can be obtained. The trajectory optimization model takes the optimal control and the state variables as input to the tracking guidance module. The guidance model can output the control compensation \( \delta u \) with the index function \( J_2 \), and the guidance control solution \( u = u^* + \delta u \) can be obtained by combining the two parts. The introduction of control strategy module is mainly to coordinate the calculation sequence and reference model before trajectory optimization and tracking guidance, so that it can cope with the situation of non-convergence of trajectory optimization and large deviation of guidance and tracking error.

**Figure 2.** Unified architecture of optimal control problem for trajectory optimization and tracking guidance.

Taking the problem of trajectory optimization and guidance in the ascent stage of hypersonic vehicle as an example, the above unified framework can be used to give a performance index function \( \min J_1 = -m_f \), \( m_f \) is the residual mass of the vehicle, that is the trajectory energy optimization. Comprehensive index function of optimal energy and minimum tracking error for tracking guidance, that is \( \min J_2 = \frac{1}{2} \delta x^T \left( t_f \right) F \delta x \left( t_f \right) + \frac{1}{2} \int_{t_0}^{t_f} \left( \delta x^T Q \delta x + \delta u^T R \delta u \right) dt \). Similarly, the above unified architecture can also be adopted in other flight phases of the flight vehicle.

5. Conclusion

The characteristics of hypersonic flight vehicle determine that the application of off-line trajectory design is very limited. The real demand for on-line trajectory optimization is very urgent. With the improvement of computer data processing capability, the real demand for on-line trajectory optimization is increasing, and has become an important theoretical research frontier. The online trajectory optimization and application based on intelligent algorithm presented in this paper is a beneficial discussion for the intelligence of flight vehicle. The system framework of online high-efficiency intelligent algorithm application, the basic composition of system application and the core implementation process of intelligent algorithm are given. It can be predicted that with the improvement of computer level, the application of intelligent algorithms on flight vehicle will greatly promote the coordinated development of trajectory optimization and guidance.
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