Dark Energy: Is It of Torsion Origin?
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Abstract

"Dark Energy" is a term recently used to interpret supernovae type Ia observation. In the present work we give two arguments on a possible relation between dark energy and torsion of space-time.

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1 Introduction

Recently, the exotic term "Dark Energy" is frequently used to interpret supernovae type Ia observations [1], [2]. These observations indicate very clearly that the Universe is in an accelerating expansion phase. This implies the existence of a large scale "repulsive force", causing the observed accelerating expansion phenomena.

It is well known that the interaction responsible of the behavior of the large scale structure and evolution of the Universe is mainly gravitational. Unfortunately, gravity as we understand it in the solar, and comparable, systems cannot account for supernovae type Ia observations. The reason is that gravity theories, including general relativity (GR), deals with gravity as an "attractive force".

In the literature, it is widely accepted that inserting a cosmological term in Einstein’s field equations can solve the problem of supernovae type Ia observation. But the cosmological constant itself has many problems, concerning its value which is still controversial [3]. So, it is necessary to seek other interpretations, to solve this problem.

Einstein, in constructing his theory of GR, has used a geometric property, "the curvature", to interpret gravity. Several applications and predictions of this theory show its success over about eight decades. I consider this success as a success of using geometry in solving physical problems, rather than a success of GR itself. So, when new problems concerning interpretation of large scale phenomena appear, I prefer to revisit geometry, seeking a solution. Any geometric structure, characterized by a linear connection, has two important geometric entities: "Curvature" and "Torsion". Einstein has used the curvature to interpret, successfully, attractive gravity. What is the role of torsion?

The aim of the present work is to review briefly some of the properties of the torsion, and to show that it gives rise to a repulsive force, in contrast to the attractive force implied by the curvature. For this reason in section 2, I briefly review a geometric structure with simultaneously non-vanishing curvature and
torsion. In section 3, I give some properties of the torsion giving rise to a repulsive force. The paper is discussed and concluded in section 4.

2 Geometries with Curvature and Torsion: The PAP-Geometry

Torsion tensor is the antisymmetric part of any non-symmetric linear connection. Geometries with curvature and torsion are frequently used (see references [4], [5]) and are classified in the literature as "Riemann-Cartan" geometry. In what follows, we are going to review very briefly the main features of a version of Absolute Parallelism (AP)-geometry, "The Parameterize Absolute Parallelism (PAP)-Geometry", in which both torsion and curvature are simultaneously non-vanishing. We have chosen this type of geometry since calculations in its context are very easy (for details, see reference [6]).

The structure of a 4-dimensional PAP-space is defined completely by a set of 4-contravariant linearly independent vector fields $\lambda^\mu$. This geometry is characterized by the linear non-symmetric connection,

$$\nabla^\alpha_{\mu\nu} = \left\{ \alpha_{\mu\nu} \right\} + b \gamma^\alpha_{\mu\nu},$$

(3)

where $\left\{ \alpha_{\mu\nu} \right\}$ is Christoffel symbol of the second kind, $\gamma^\alpha_{\mu\nu}$ is the contorsion and $b$ is a dimensionless parameter. The connection (3) is metric and has a non-vanishing torsion ($\hat{\Lambda}^\alpha_{\mu\nu} = b\Lambda^\alpha_{\mu\nu} \overset{\text{def}}{=} \nabla^\alpha_{\mu\nu} - \nabla^\alpha_{\nu\mu}$). The curvature tensor corresponding to (3) is, in general, non-vanishing and defined by

$$B^\alpha_{\mu\nu\sigma} = R^\alpha_{\mu\nu\sigma} + b \hat{Q}^\alpha_{\mu\nu\sigma},$$

(4)

where,

$$R^\alpha_{\mu\nu\sigma} \overset{\text{def}}{=} \left\{ \alpha_{\mu\nu} \right\}_{\sigma} - \left\{ \alpha_{\mu\sigma} \right\}_{\nu} + \left\{ \epsilon_{\mu\sigma} \right\}_{\nu} - \left\{ \epsilon_{\mu\nu} \right\}_{\sigma}$$

(5)

is the Riemann-Christoffel curvature tensor and the tensor $\hat{Q}^\alpha_{\mu\nu\sigma}$ is defined by

$$\hat{Q}^\alpha_{\mu\nu\sigma} \overset{\text{def}}{=} \gamma^\alpha_{\mu\nu,\sigma} - \gamma^\alpha_{\mu\sigma,\nu} - b(\gamma^\beta_{\mu\sigma} \gamma^\alpha_{\beta\nu} - \gamma^\beta_{\mu\nu} \gamma^\alpha_{\beta\sigma}).$$

(6)

The stroke and the (+)-sign are used to characterize covariant derivatives using the linear connection (3), while the stroke and the (-)-sign are used to characterize covariant derivatives using the dual connection $\tilde{\nabla}^\alpha_{\mu\nu}(= \nabla^\alpha_{\nu\mu})$.

The path equations corresponding to (3) can be written as,

$$\frac{d^2 x^\mu}{d\tau^2} + \left\{ \frac{\mu}{\alpha,\beta} \right\} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = -b \Lambda^\alpha_{\alpha,\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau},$$

(7)

where $\tau$ is a scalar parameter.

It is to be noted that the PAP-geometry covers both Riemannian geometry ($b = 0$) and conventional AP-geometry ($b = 1$), as special cases.
3 Dark Energy-Torsion Relation

In this section we give two arguments on a probable relation between torsion and dark energy. The first is a kinematical argument and the second is a dynamical one.

The Kinematical Argument: In the case \( b = 1 \) (the AP-case [6]), the case of vanishing curvature, equation (4) gives,

\[
R^\alpha_{\mu\nu\sigma} \equiv -Q^\alpha_{\mu\nu\sigma}.
\]  

(8)

Although the tensors \( R^\alpha_{\mu\nu\sigma} \) and \( Q^\alpha_{\mu\nu\sigma} \) appear to be mathematically equivalent, they have the following differences:

1- The Riemann-Christoffel curvature tensor is made purely from Christoffel symbols (see (5)), while the tensor \( Q^\alpha_{\mu\nu\sigma} \) (given by (6) with \( b = 1 \)) is made purely from the contortion or from the torsion via the relation

\[
\gamma^\alpha_{\mu\nu} = \frac{1}{2}(\Lambda^\alpha_{\mu\nu} - \Lambda^\alpha_{\mu\nu} - \Lambda^\alpha_{\nu\mu}).
\]

The first tensor is non-vanishing in Riemannian geometry, while the second vanishes in the same geometry.

2- The non-vanishing of \( R^\alpha_{\mu\nu\sigma} \) is the measure of the curvature of the space, while the addition of \( Q^\alpha_{\mu\nu\sigma} \) to it, causes the space to be flat. So, one is causing an inverse effect, on the properties of space-time, compared to the other. For this reason we call \( Q^\alpha_{\mu\nu\sigma} \) (or \( \hat{Q}^\alpha_{\mu\nu\sigma} \)) "The Curvature Inverse of Riemann-Christoffel Tensor" [6], or in other words "The Additive inverse of the Curvature Tensor". Note that both tensors are considered as curvature tensor, but one of them cancels the effect of the other, if both existed in the same geometric structure.

Let us now approach, geometrically, the problem of dark energy. It is well known that Riemann-Christofel curvature tensor (5) satisfies Bianchi second identity, which can be written in the contracted form

\[
(R^\mu_{\nu} - \frac{1}{2}\delta^\mu_{\nu}R):\mu = 0.
\]

(9)

This identity is interpreted, physically as a generalization of a law of conservation. The quantity between brackets in (9) represents the conserved quantity.
We can attribute to this quantity the property of energy associated with curvature, the "Curvature Energy". Similarly, using (8) & (9), we can write,

\[(Q^\mu_\nu - \frac{1}{2}\delta^\mu_\nu Q); \mu = 0.\]  

(10)

where \(Q^\mu_\nu\) is the only non-vanishing contracted form of (6) (with \(b = 1\)) and \(Q\) is its scaler. Now, using the same argument, (10) can be considered as conservation of another type of energy represented by the quantity between brackets, and since \(Q^\mu_\nu\) and its contraction \(Q\) are made purely from the contortion (or the torsion), as clear from (6), we call this type of energy the "Torsion Energy".

If we assume that the effects of gravity and anti-gravity are not exactly equal in the same system, then space-time curvature can be represented by the tensor (4). The existence of anti-gravity gives rise to a repulsive force, which can be used to interpret SN type Ia observation. This can be achieved by adjusting the parameter \(b\).

It is clear that, "Torsion Energy" follows a conservation law, similar to that of the curvature energy (for details see reference [7]).

The Dynamical Argument for the existence of a repulsive force, corresponding to the torsion of space-time, consider the linearized form of (7) which can be written as [8],

\[\Phi_T = \Phi_N(1 - b) = \Phi_N + \Phi_\Sigma,\]

(11)

where,

\[\Phi_\Sigma \overset{\text{def}}{=} -b\Phi_N,\]

(12)

\(\Phi_N\) is the Newtonian gravitational potential and \(\Phi_T\) is the total gravitational potential due the presence of gravity and anti-gravity. It is clear from (11) that the Newtonian potential is reduced by a factor \(b\) due to the existence of the torsion energy. It is obvious from (12) that \(\Phi_\Sigma\) and the Newtonian potential have opposite signs. Then one can deduce that \(\Phi_\Sigma\) is a repulsive potential, in contrast to the attractive potential \(\Phi_N\).

4 Discussion and Concluding Remarks

In the present work we have chosen a version of the 4-dimensional AP-geometry to represent the physical world including space and time. On the present work, we can draw the following remarks:

1- It is clear that torsion energy, defined in the previous section, can solve the problem of SN type Ia observations, since it gives rise to a repulsive force. This can be achieved by adjusting the parameter \(b\). One can now replace the exotic term "dark energy" by the term "torsion energy". The later has a pure geometric origin.

2- Curvature and torsion corresponds to two different types of energy. The energy corresponding to the first gives rise to an attractive force, while the energy corresponding to the second is repulsive. We believe that torsion energy
is what has been discovered recently by the SN type Ia observation [1]. Both
ergies obey the same conservation.
3- The results obtained in the present work can be obtained, with some efforts,
using other geometries with curvature and torsion, since such geometries possess
similar features [9].
4- From the geometerization point of view, one can conclude that "Dark Energy"
is nothing but "Torsion Energy", responsible for repulsion in the Universe. For
more details and discussions cf. [10], [11], [12]& [13].

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