Split Left-Right Symmetry and Scotogenic Quark and Lepton Masses

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Abstract

A model of split left-right symmetry is proposed, where the first family of quarks and leptons transforms under $SU(2)_R$ but the heavier two families do not. The Higgs scalar sector consists only of an $SU(2)_L$ doublet and an $SU(2)_R$ doublet. The $u, d$ quarks and the electron, as well as the neutrinos, are massless at tree level, but become massive radiatively through their interactions with a simple dark sector. Verifiable consequences include $Z'$ and Higgs properties, neutrino mixing and rare decays, etc.
Introduction: In the standard model (SM) of particle interactions, neutrinos are massless. It is thus reasonable to think that they may become massive through a simple mechanism beyond the SM. In 2006, this was accomplished [1] by postulating the interactions of neutrinos with dark matter, such that Majorana neutrino masses are generated in one loop, i.e. the scotogenic mechanism. Consider now the other fermion masses. They appear at tree level through their interactions with the one Higgs boson. Yet those of the first family, i.e. $m_u, m_d, m_e$, are much smaller than the rest. To understand this, a possible approach is the extension of the SM to the left-right model, where $(u,d)_R$ and $(\nu_e,e)_R$ are now doublets under $SU(2)_R$. To pair these up with the existing $(u,d)_L$ and $(\nu_e,e)_L$, a scalar bidoublet is required. Suppose this is withheld [2, 3, 4, 5, 6], then these fermions are massless at tree level. [The analog for neutrino mass is the withholding of a scalar triplet [7] in the SM.] They may then be generated in one loop, again using the scotogenic mechanism, as proposed in this paper. As for the heavier two families, they may still acquire masses as in the SM, in contrast to a previous proposal that all obtain scotogenic masses [8]. This notion that families are split in their left-right assignments leads to observable consequences involving $Z'$ and Higgs properties, neutrino mixing, rare decays, etc.

Model of split left-right symmetry: The particle content of this model is shown in Table 1. Note that the notion of an extra gauge $SU(2)$ under which the quarks and leptons do not transform is not new [9, 10]. Here the first family transforms but not the other two. The $Z^D_2$ symmetry is imposed to enable scotogenic masses for $u,d,e$ and 4 neutrinos. It is equivalent [11] to assigning $L = -1$ to $\eta_{L,R}$, $B = 1/3$ to $\zeta_{L,R}$, and $L = B = 0$ to $N$, then using $R$ parity, i.e. $(-1)^{3B+L+2j}$ for $Z^D_2$. The $SU(2)_L \times SU(2)_R \times U(1)_X$ symmetry is broken by $\langle \phi^0_L \rangle = v_L$ and $\langle \phi^0_R \rangle = v_R$. The mass-squared matrix spanning $(W^0_L, W^0_R, B)$ is given by

$$M^2 = \frac{1}{2} \begin{pmatrix}
g_L^2 v_L^2 & 0 & -g_L g_X v_L^2 \\
0 & g_R^2 v_R^2 & -g_R g_X v_R^2 \\
-g_L g_X v_L^2 & -g_R g_X v_R^2 & g_X^2 (v_R^2 + v_L^2)
\end{pmatrix}. \tag{1}$$
| fermion/scalar | $SU(3)_C$ | $SU(2)_L$ | $SU(2)_R$ | $U(1)_X$ | $Z^D$ |
|---------------|----------|----------|----------|--------|------|
| $(u, d)_L, (c, s)_L, (t, b)_L$ | 3 | 2 | 1 | 1/6 | + |
| $(u, d)_R$ | 3 | 1 | 2 | 1/6 | + |
| $c_R, t_R$ | 3 | 1 | 1 | 2/3 | + |
| $s_R, b_R$ | 3 | 1 | 1 | −1/3 | + |
| $(\nu_e, c)_L, (\nu_\mu, \mu)_L, (\nu_\tau, \tau)_L$ | 1 | 2 | 1 | −1/2 | + |
| $(\nu_e, c)_R$ | 1 | 1 | 2 | −1/2 | + |
| $\mu_R, \tau_R$ | 1 | 1 | 1 | −1 | + |
| $\Phi_L = (\phi_L^+, \phi_L^0)$ | 1 | 2 | 1 | 1/2 | + |
| $\Phi_R = (\phi_R^+, \phi_R^0)$ | 1 | 1 | 2 | 1/2 | + |
| $\eta_L = (\eta_L^+, \eta_L^0)$ | 1 | 2 | 1 | 1/2 | − |
| $\eta_R = (\eta_R^+, \eta_R^0)$ | 1 | 1 | 2 | 1/2 | − |
| $\zeta_L = (\zeta_L^{2/3}, \zeta_L^{-1/3})$ | 3 | 2 | 1 | 1/6 | − |
| $\zeta_R = (\zeta_R^{2/3}, \zeta_R^{-1/3})$ | 3 | 1 | 2 | 1/6 | − |
| $N_{1R}, N_{2R}, N_{3R}$ | 1 | 1 | 1 | 0 | − |

Table 1: Fermion and scalar content of split left-right model.

Using $e^2 = g_L^{-2} + g_R^{-2} + g_X^{-2}$, and assuming $g_L = g_R$ with $x = \sin^2\theta_W$, the physical gauge bosons are

$$
\begin{pmatrix}
A \\
Z \\
Z'
\end{pmatrix} =
\begin{pmatrix}
\sqrt{x} & \sqrt{x} & \sqrt{1 - 2x} \\
\sqrt{1 - x} & -x/\sqrt{1 - x} & -\sqrt{x(1 - 2x)/(1 - x)} \\
0 & \sqrt{(1 - 2x)/(1 - x)} & -\sqrt{x/(1 - x)}
\end{pmatrix}
\begin{pmatrix}
W^0_L \\
W^0_R \\
B
\end{pmatrix}.
$$

(2)

The photon $A$ is massless, whereas the $2 \times 2$ mass-squared matrix spanning $(Z, Z')$ is given by

$$
\mathcal{M}^2_{Z, Z'} = \frac{e^2}{2}
\begin{pmatrix}
v_L^2/(1 - x) \\
v_L^2/(1 - x)\sqrt{1 - 2x} \\
v_R^2/(1 - x)\sqrt{1 - 2x} \\
(1 - x)v_L^2/(1 - 2x) + xv_R^2/(1 - x)(1 - 2x)
\end{pmatrix}.
$$

(3)

The $Z - Z'$ mixing is then about $x\sqrt{1 - 2x}v_L^2/(1 - x)^2v_R^2$ which is constrained experimentally to be less than $10^{-4}$, implying thus $v_R > 9.3$ TeV. Whereas $Z$ couples to the current

$$
J_Z = J_{3L} - xJ_{em}
$$

(4)

with strength $e/\sqrt{x(1 - x)}$ as in the SM, $Z'$ couples to the current

$$
J_{Z'} = xJ_{3L} + (1 - x)J_{3R} - xJ_{em}
$$

(5)
with strength $e/\sqrt{x(1-x)(1-2x)}$. Thus neutral flavor changing $Z'$ couplings occur through $J_{3R}$.

The charged $W^\pm_L$ and $W^\pm_R$ bosons have masses given by

$$M^2_{W_L} = \frac{e^2 v_L^2}{2x}, \quad M^2_{W_R} = \frac{e^2 v_R^2}{2x}. \tag{6}$$

Whereas $W^\pm_L$ couples the $(u,c,t)$ quarks to the $(d,s,b)$ quarks through the $3 \times 3$ unitary CKM matrix, $W^\pm_R$ couples only one linear combination of up quarks to one linear combination of down quarks.

**Scotogenic masses for u, d, e and neutrinos**: The gauge structure of the heavier two families of quarks and leptons is identical to that of the SM. Hence they acquire tree-level masses from $\langle \phi^0_L \rangle = v_L = 174$ GeV as usual. However, $(u,d)_L$ and $(\nu_e,e)_L$ cannot pair up with $(u,d)_R$ and $(\nu_e,e)_R$ at tree level because the usual left-right scalar bidoublet is absent. Nevertheless, with the help of the postulated dark sector, they acquire one-loop radiative masses as shown in Figures 1 to 4.

![Figure 1: Scotogenic u quark mass.](image)

The most general $3 \times 3$ mass matrix linking $(u,c,t)_L$ to $(u,c,t)_R$ may be chosen as

$$M_u = \begin{pmatrix} m_{uu} & 0 & 0 \\ m_{cu} & m_{cc} & 0 \\ m_{tu} & 0 & m_{tt} \end{pmatrix}, \tag{7}$$

where $m_{uu}, m_{cu}, m_{tu}$ are radiative contributions from Fig. 1. The corresponding $3 \times 3$ mass
matrix linking \((d, s, b)_L\) to \((d, s, b)_R\) is then of the form

\[
\mathcal{M}_d = \begin{pmatrix}
  m_{dd} & m_{ds} & 0 \\
  m_{sd} & m_{ss} & m_{sb} \\
  m_{bd} & m_{bs} & m_{bb}
\end{pmatrix},
\]  

(8)

where \(m_{dd}, m_{sd}, m_{bd}\) are radiative contributions from Fig. 2. After diagonalization by unitary transformations \(U_L^\dagger\) on the left and \(U_R\) on the right, the mismatch \(U_L^\dagger(u)U_R(d)\) becomes the well-known observed charged-current mixing matrix \(U_{CKM}\) as usual.

The charged-lepton mass matrix may be organized as

\[
\mathcal{M}_e = \begin{pmatrix}
  m_{ee} & 0 & 0 \\
  m_{\mu e} & m_{\mu\mu} & 0 \\
  m_{\tau e} & 0 & m_{\tau\tau}
\end{pmatrix},
\]

(9)

where \(m_{ee}, m_{\mu e}, m_{\tau e}\) are radiative contributions from Fig. 3. As for neutrinos, there are three \(\nu_L\)s which belong to \(SU(2)_L\) doublets and one \(\nu_R\) which belongs to an \(SU(2)_R\) doublet. They
acquire scotogenic masses in a $4 \times 4$ matrix. In Fig. 4, one linear combination of the three $\nu_L$'s pairs up with the one $\nu_R$ to form a Dirac mass.

![Diagram](image1)

Figure 4: Scotogenic Dirac neutrino mass.

In Fig. 5, the three $\nu_L$'s obtain radiative Majorana masses as in the original scotogenic model [1]. In Fig. 6, the analog mechanism is used for the one $SU(2)_R$ neutrino $\nu_R$. This

![Diagram](image2)

Figure 5: Scotogenic Majorana $\nu_L$ mass.

mechanism is akin to that [13] for a singlet neutrino in the SM with an extra gauge $U(1)$. 

![Diagram](image3)

Figure 6: Scotogenic Majorana $\nu_R$ mass.
New gauge bosons: All quarks and leptons transform under $SU(2)_L$ as in the SM. This means that the known gauge bosons, $A, W^\pm_L, Z$, have interactions as in the SM. As for the new gauge bosons, $W^\pm_R, Z'$, their interactions are different from all previously proposed left-right models, because only one copy each of $(u, d)_R$ and $(\nu_e, e)_R$ transforms under $SU(2)_R$. Note that these fermions are not mass eigenstates themselves, as shown in Eqs. (7,8,9). They are linear combinations of mass eigenstates, i.e.

\[
\begin{pmatrix}
u_e \\ d_L \\ u_L \\ e_R \\ \nu_e_R \end{pmatrix}_R \rightarrow \begin{pmatrix}
u_e' \\ d'_L \\ u'_L \\ e'_R \\ \nu_e_R \end{pmatrix}_R = \begin{pmatrix} U^\nu_{eL}(u) & U^{uL}_{eR} & U^d_{eR} & U^e_{eR} & 0 \\ U^u_{uR} & U^{uL}(u) & U^d_{uR} & U^e_{uR} & 0 \\ U^d_{dR} & U^{dL}(d) & U^{uR} & U^e_{dR} & 0 \\ U^e_{eR} & U^{eL}(e) & U^d_{eR} & U^{uR}(e) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},
\]

(10)

and similarly for $(\nu_e, e)_R$, except that $(\nu_e)_R$ now has 4 components. With the natural expectation that radiative quark and lepton masses are much smaller than their tree-level counterparts and using the observed hierarchy of masses among families, it is reasonable to find $|U^u_{eR}(u)| \ll |U^{uL}(u)| \ll |U^{uL}_{eR}(u)| \sim 1$ and $|U^{dL}(d)| \ll |U^{dL}_{eR}(d)| \ll |U^{dL}_{dR}(d)| \sim 1$, etc.

The new charged $W^\pm_R$ gauge boson couples $u'_R$ to $d'_R$ and $\nu'_R$ to $e'_R$, whereas $Z'$ couples to $\bar{u}_R^c \gamma u'_R - \bar{d}_R^c \gamma d'_R$ and $\bar{\nu}_R^c \gamma \nu'_R - \bar{e}_R^c \gamma e'_R$ through $J_{3R}$ in Eq. (5). This means that flavor-changing neutral currents, absent at tree level in the SM, are now present through $Z'$ exchange. Their effects are however suppressed because of the expected small deviations of $u'_R$ from $u_R$, $d'_R$ from $d_R$, $e'_R$ from $e_R$, and $\nu'_R$ from $(\nu_e)_R$.

A prime example is the possible rare decay $\mu \rightarrow eee$. Experimentally, its branching fraction is less than $1.0 \times 10^{-12}$ [14]. Here it is mediated by $Z'$ exchange:

\[
A(\mu \rightarrow eee) = \frac{e^2 U^e\mu_{eR}(e)}{x(1-x)(1-2x)M_Z^2} \bar{e} \gamma^\alpha \left[ - \frac{1}{2} (1-x) \right] \left( \frac{1 + \gamma_5}{2} \right) \mu \\
\times \bar{e} \gamma^\alpha \left[ \frac{x}{2} \left( 1 - \gamma_5 \right) + \left( \frac{1}{2} + \frac{3x}{2} \right) \left( \frac{1 + \gamma_5}{2} \right) \right] e.
\]

The resulting constraint [15] is

\[
[U^e\mu_{eR}(e)]^2 \left( \frac{M_Z^2}{M_{Z'}^2} \right)^2 \left( \frac{(1-x)^2}{4(1-2x)^2} \right)^2 [2 - 12x + 19x^2] < 1.0 \times 10^{-12}.
\]

(11)
For $x = \sin^2 \theta_W = 0.23$, 
\[
U_{R\mu}^e(e) \left( \frac{M_Z^2}{M_{Z'}^2} \right) < 2.83 \times 10^{-6} \tag{13}
\]
is obtained. Using $\mu-e$ conversion in nuclei, this bound may be improved \[16\] by about a factor of two. Other rare decays sensitive to $Z'$ are $s \rightarrow dl^+l^-, b \rightarrow dl^+l^-$, etc. The present collider bound on $M_{Z'}$ is a few TeV \[14\].

**Exotic Higgs decays**: The Higgs sector of this model is very simple. The part containing $\Phi_{L,R}$ only is given by
\[
V_{\Phi} = \mu_L^2 \Phi^\dagger_L \Phi_L + \mu_R^2 \Phi^\dagger_R \Phi_R + \frac{1}{2} \lambda_L (\Phi^\dagger_L \Phi_L)^2 + \frac{1}{2} \lambda_R (\Phi^\dagger_R \Phi_R)^2 + \lambda_{LR} (\Phi^\dagger_L \Phi_L)(\Phi^\dagger_R \Phi_R). \tag{14}
\]
After spontaneous symmetry breaking with $\langle \phi^0_L \rangle = v_L$ and $\langle \phi^0_R \rangle = v_R$, the only physical scalar bosons are $H_L = \sqrt{2} Re(\phi^0_L)$ and $H_R = \sqrt{2} Re(\phi^0_R)$, with $2 \times 2$ mass-squared matrix given by
\[
M_H^2 = \begin{pmatrix} 2\lambda_L v^2_L & 2\lambda_{LR} v_L v_R \\ 2\lambda_{LR} v_L v_R & 2\lambda_R v^2_R \end{pmatrix}. \tag{15}
\]
In the limit $\lambda_{LR} \rightarrow 0$, $H_L$ decouples from $H_R$ and appears to be the known SM Higgs boson. However, because some fermion masses are radiative in origin, Higgs decays will not be exactly those of the SM, as pointed out in Refs.\[17\] \[18\].

Consider for simplicity the $2 \times 2$ mass matrix linking $(\bar{e}_L, \bar{\mu}_L)$ to $(e_R, \mu_R)$:
\[
M_{e\mu} = \begin{pmatrix} m_{ee} & 0 \\ m_{\mu e} & m_{\mu \mu} \end{pmatrix}, \tag{16}
\]
where $m_{ee}, m_{\mu e}$ come from Fig. 3. It is diagonalized on the right and left by
\[
\begin{pmatrix} \cos \theta_R & \sin \theta_R \\ -\sin \theta_R & \cos \theta_R \end{pmatrix}, \quad \begin{pmatrix} \cos \theta_L & -\sin \theta_L \\ \sin \theta_L & \cos \theta_L \end{pmatrix}, \tag{17}
\]
where $\tan \theta_R = m_{\mu e}/m_{\mu \mu}$ and $\tan \theta_L = (m_e/m_{\mu}) \tan \theta_R$ with the mass eigenvalues $m_e = \cos \theta_R m_{ee}$ and $m_{\mu} = m_{\mu \mu}/\cos \theta_R$. 

8
In the SM, the Higgs coupling matrix is simply \((\sqrt{2}v_L)^{-1}\) times \(\mathcal{M}_{\mu\mu}\). After diagonalization, it is of the well-known form \((\sqrt{2}v_L)^{-1}[m_e\bar{e}e + m_\mu\bar{\mu}\mu]\). Here, because of the radiative mass, the Higgs coupling matrix is modified \([17]\), and becomes

\[
\begin{pmatrix}
  rm_{ee} & 0 \\
  rm_{\mu e} & m_{\mu\mu}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
  rm_e & (r-1)\tan\theta_R m_e \\
  (r-1)\sin\theta_R \cos\theta_R m_\mu & \left[1 + (r-1)\sin^2\theta_R\right]m_\mu
\end{pmatrix}.
\]

(18)

It is clear that for \(r = 1\), it reduces to the SM result. Here \(r \neq 1\) in general and the Higgs decays to \(\bar{e}e\) and \(\bar{\mu}\mu\) are changed. Furthermore, exotic decays to \(\bar{\mu}e\) and \(\bar{e}\mu\) are possible, contrary to the predictions of the SM.

Recently, ATLAS reports \([19]\) an observation of the \(\mu^+\mu^-\) mode at the level \(1.2 \pm 0.6\) relative to the SM prediction. Also, CMS has the result \([20]\) \(1.2 \pm 0.4 \pm 0.2\). This leaves much room for \(r \neq 1\), but more important would be the observation of \(\mu^\pm e^\mp\). Oviously, \(\tau\) may replace \(\mu\) in the above analysis, and modifies the Higgs decay to \(\tau^+\tau^-\) and allows the \(\tau^\pm e^\mp\) mode.

**Neutrino sector**: There are four neutrinos. They acquire scotogenic masses as shown in Figs. 4 to 6. The \(4 \times 4\) neutrino mass matrix spanning \((\bar{\nu}_R, \nu_e, \nu_\mu, \nu_\tau)\) is of the form

\[
\mathcal{M}_\nu =
\begin{pmatrix}
  m_R & m_D & 0 & 0 \\
  m_D & m_{ee} & m_{e\mu} & m_{e\tau} \\
  0 & m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\
  0 & m_{e\tau} & m_{\mu\tau} & m_{\tau\tau}
\end{pmatrix}.
\]

(19)

If \(m_D\) is very small, then this reduces to the usual case of three active Majorana neutrinos, with the possible exception of mixing with a mostly sterile singlet neutrino \(\nu_S = \bar{\nu}_R\). This may have relevance in models where a fourth neutrino is added to explain recent oscillation data \([21]\). On the other hand, if both \(m_{ee}\) and \(m_R\) are very small, then \(\nu_e\) pairs up with \(\nu_R\) to form a mostly Dirac fermion, in which case neutrinoless double beta decay \([22]\) would be suppressed, but the kinematic mass measurement \([23]\) of \(\nu_e\) would succeed. In either case, the lightest \(N\) is a possible dark-matter candidate \([24]\). However, since \(N\) has the additional interactions of Figs. 1 and 2, the constraint from dark-matter relic abundance is relaxed,
which in turn makes lepton flavor-changing radiative decays \cite{25} less restrictive. Hence $m_N$ may be much less than in the original scotogenic model \cite{1} of neutrino masses. This means that the color triplet scalars $\zeta_{L,R}$ may be light enough to be produced at the Large Hadron Collider and decay to quarks and $N$, in analogy to that of scalar quarks in supersymmetry.

**Concluding remarks**: In an extended gauge model with $SU(2)_R$, the particle content is chosen (Table 1) so that all neutrinos as well as the $u, d$ quarks and the electron are massless at tree level, whereas the second and third families of quarks and charged leptons acquire mass as in the SM. With the implementation of a dark sector, $u, d, e, \nu_e$ obtain one-loop radiative Dirac masses through their mutual interactions (Figs. 1 to 4) in analogy to the original scotogenic mechanism for the Majorana masses of the three left-handed neutrinos (Fig. 5) and similarly for the one right-handed neutrino (Fig. 6). This model of radiative masses for the lightest known particles has verifiable consequences in the appearance of $W_R^\pm$ and $Z'$ gauge bosons with flavor nondiagonal interactions. Rare decays such as $\mu \rightarrow eee$ through $Z'$ exchange are predicted, as well as exotic Higgs decays such as $\tau^\pm e^\mp$ and deviations from SM predictions in the $\tau^+\tau^-, \mu^+\mu^-$ and $e^+e^-$ modes.

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