Astigmatic transformation of the Bessel beam and the Gauss–Laguerre beam

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Abstract. In studies of the anisotropy may be useful the Bessel beams, which have a high sensitivity to the asymmetry of the wave front. Beams Gauss-Laguerre are capable of maintaining their structure in free space. Gauss-Laguerre modes have found their application in optical manipulation of micro-objects, quantum optics, optical communications. In this paper presents a numerical calculation and analysis of the passage of a Bessel beam of zero order and a Gauss-lager beam through polarization devices. Transverse intensity patterns are obtained for beam propagation after passing through wave plates with different types of polarization. This work provides detailed information on polarization transformations of light beams, which can be useful in acute focusing, with optical manipulation of micro-objects. The study of astigmatic transformations of Bessel beams can be useful in capturing and controlling microparticles.

1. Introduction

Anisotropic materials are widely used in optics [1, 2]. The propagation of laser modes high order in a medium with strong anisotropy leads to complex polarization-mode transformations [3-9].

The most noticeable anisotropic effects are manifested for Bessel beams [10-14] in connection with the special structure of their spatial spectrum. A similar transformation for the Gaussian mode is not so pronounced. For example, it was shown in [3, 15] that only half the energy of the Gaussian beam with circular polarization is transformed into a vortex beam. It was shown in Refs [16, 17] that for high-frequency Gauss-Laguerre modes in a non-paraxial regime we can see behavior close to Bessel modes, but only in the near zone of diffraction. At longer distances, the Gauss-Laguerre beam undergoes astigmatic distortions. A similar difference in the behavior of the Bessel and Gauss-Laguerre modes was noted earlier with a periodic self-reproduction of multimode beams in an isotropic medium [18, 19]. Due to special structure, laser Bessel beams are very sensitive to asymmetry of the wave front, which makes them useful for the study of optical anisotropy and astigmatism [20-27]. Changes in the distribution of intensity during propagation the Bessel beams are much more visible and they occur with smaller phase distortions than for beams with uniform intensity. In particular, a clearly visible visual distortion of the beam intensity structure was observed for Bessel beams propagating perpendicular to the axis of the anisotropic crystal [20-27], and also when passing through a cylindrical lens [25]. Similar, the transformation of the beam structure can be observed with an oblique incidence of a plane wave on axicon [28-31]. Gaussian modes are subjected to astigmatic transformations with the aim of formation of a vortex phase singularity, and also for visualization of the optical vortex order [32-38].

In this paper, we numerically calculate and analyze the propagation of a Bessel beam of zero order and Gauss-Laguerre beams of high radial order through an astigmatic lens. Transverse patterns of
intensity are obtained for propagation of the beams through the astigmatic lens at different distances. Also, the dependence of the polarization of the beam on the slopes of the optical axis is also considered.

2. The modelling for Bessel beam

We consider a paraxial model and use the Fresnel transformation as the propagation operator for the beams:

\[ G(u, v, z) = -\frac{ik}{2\pi z} \int_{-R}^{R} \int_{-R}^{R} g(x, y) \exp \left[ \frac{ik}{2z} \left( (x-u)^2 + (y-v)^2 \right) \right] \, dx \, dy, \] (1)

where \( k = \frac{2\pi}{\lambda} \) is wave number, \( \lambda \) is emission wavelength, \( z \) is the distance from the input plane, \( R \) is the input field size.

There is a zero-order Bessel beam in the input plane:

\[ B(x, y) = J_0(k\alpha_0 \sqrt{x^2 + y^2}), \] (2)

where \( \alpha_0 \) is a parameter appropriating to the beam scale.

2.1 The propagation of a Bessel beam in free space

It is known that the beam retains its properties up to \( z_{max} \):

\[ z_{max} = \frac{R}{\alpha_0} \] (3)

Calculation parameters are \( \lambda = 0.000633 \, \text{mm}, k = 9926.043 \, \text{mm}^{-1}, \alpha_0 = 0.003, R=1 \, \text{mm} \), \( z_{max} = 330.868 \, \text{mm} \).

Also in the input plane is a cylindrical lens with a transmission function:

\[ L(x) = \exp \left\{ -ik \frac{x^2}{2f} \right\} \] (4)

where \( f \) is the focal length of lens.

Thus, the input function in (1) has the following form:

\[ g(x, y) = B(x, y) \cdot L(x) \] (5)

The zero-order Bessel function was considered \( J_0(\alpha r) \), where \( r = \sqrt{x^2 + y^2} \), \( \alpha = k\alpha_0 = 29.7 \, \text{mm}^{-1} \), \( x \) and \( y \) change from -1 to 1 in increments, grid spacing is \( \frac{1}{32} \) mm.

The simulation took place in the Matlab R 2014a.

The following distribution is obtained by modeling the zero-order Bessel function \( J_0(\alpha r) \) (figure 1).

Then the Fresnel transformation (1) was performed from the Bessel function (2) in the absence of a cylindrical lens (4).

The output parameters were set: \( u \) and \( v \) also change from -1 to 1 in \( \frac{1}{32} \) increments, \( k = \frac{2\pi}{0.000633} \).

Distance \( z = 200 \, \text{mm} \). In these conditions, the distribution shown in figure 2 is obtained.

A cylindrical lens (4) was used to study the astigmatism of the beam. The focal length \( f = 150 \, \text{mm} \) was chosen for the calculations.

The results of beam propagation with astigmatism are shown in figures 3-5.

The aim of the experiment was to analyze the beam propagation at different distances. At a distance of 150 mm, the picture is shown in figure 3, at 250 mm - in figure 5.

As can be seen from the simulated patterns of intensity distribution of Bessel beams transformed by cylindrical lens, the most severe astigmatic distortions are exposed to the beams formed by the lens, standing at a larger distance. Large distances will guarantee a strong aberration. This indicates a high sensitivity of the beam to increase the distance to the lens.
3. Research of astigmatic transformations of light beams in an anisotropic medium

An experiment was conducted with Gauss-Laguerre modes to study the behavior of the Gauss-Laguerre beam as it passed through various anisotropic materials: a linear polarizer, a quarter-wave plate, and a half-wave plate. For the clarity of the experiment, a Gauss-Laguerre beam 4 of radial order and 2 angular order was chosen. The polarization of the beam depends on the angles $\theta$ (the angle at which the fast axis of the plate or the polarizer is oriented) and $\alpha$ (the angle of inclination to the axis $O_x$). In the experiment, these angles changed in search of a dependence of the type of polarization on these angles.

The input is fed by a beam described by the Gauss-Laguerre function:

$$S_{mn}(r, \theta) = \frac{2\sqrt{\pi(n-m)!}}{\alpha(n!)^3}\left(\frac{r}{a}\right)^m\times\exp\left[-\left(\frac{r}{a}\right)^2/2\right]L_n^m\left(\frac{r}{a}\right)^2\exp[\pm im\theta],$$

where $L_n^m(x) = (-1)^m \frac{d^m}{dx^m}[L_{n+m}(x)]$ - generalized Lagrange polynomial.

The scheme of the experiment is shown in Figure 6:
Figure 6. The scheme of experiment.

Table 1. Polarization of the Gauss-Laguerre beam at an angle $\theta$ with $\alpha = 0^\circ$.

| $\theta$ | Linear polarizer | Quarter-wave plate | Half-wave plate |
|---------|------------------|---------------------|-----------------|
| $-30^\circ$ | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) |
| $30^\circ$ | ![Image](image4.png) | ![Image](image5.png) | ![Image](image6.png) |
| $45^\circ$ | ![Image](image7.png) | ![Image](image8.png) | ![Image](image9.png) |
| $60^\circ$ | ![Image](image10.png) | ![Image](image11.png) | ![Image](image12.png) |
| $90^\circ$ | ![Image](image13.png) | ![Image](image14.png) | ![Image](image15.png) |

Table 2. Polarization of the Gauss-Laguerre beam at an angle $\theta$ with $\alpha = 30^\circ$.

| $\theta$ | Linear polarizer | Quarter-wave plate | Half-wave plate |
|---------|------------------|---------------------|-----------------|
| $-30^\circ$ | ![Image](image16.png) | ![Image](image17.png) | ![Image](image18.png) |
| $30^\circ$ | ![Image](image19.png) | ![Image](image20.png) | ![Image](image21.png) |
| $45^\circ$ | ![Image](image22.png) | ![Image](image23.png) | ![Image](image24.png) |
| $60^\circ$ | ![Image](image25.png) | ![Image](image26.png) | ![Image](image27.png) |
| $90^\circ$ | ![Image](image28.png) | ![Image](image29.png) | ![Image](image30.png) |

At zero angle $\alpha$, the intensity of the output beam from the linear polarizer decreases with increasing angle $\theta$. As expected, the linear polarizer and the half-wave plate produced linear polarization at the output, the quarter-wave plate gave a circular and elliptical polarization. However, with an increase in the angle $\alpha$, the quarter-wave plate increasingly gave the beam with linear polarization.

As a result of the experiment, the dependence the direction and type of polarization of the output beam on the angles $\theta$ and $\alpha$ was obtained.

In the work, the Gauss-Laguerre beam of the 4 order was modeled with a wavelength of 532 nm, a radius (Waist radius) of 100 $\mu$m, passing through a cylindrical lens. For the experiment we use a plane-concave lens measuring 30 mm $\times$ 32 mm, a focus $f = -70$ mm, a radius of curvature of 20 mm, a thickness of 2 mm.
Table 3. Polarization of the Gauss-Laguerre beam at an angle $\theta$ with $\alpha = 60^\circ$.

| Linear polarizer | $\theta = -30^\circ$ | $\theta = 30^\circ$ | $\theta = 45^\circ$ | $\theta = 60^\circ$ | $\theta = 90^\circ$ |
| Linear polarizer | ![Image] | ![Image] | ![Image] | ![Image] | ![Image] |
| Quarter-wave plate | ![Image] | ![Image] | ![Image] | ![Image] | ![Image] |
| Half-wave plate | ![Image] | ![Image] | ![Image] | ![Image] | ![Image] |

Table 4. Distortions of the Gauss-Laguerre beam as a function of the distance to the detector.

| $z$ (mm) | 50 | 100 | 200 |
|---------|----|-----|-----|
| ![Image] | ![Image] | ![Image] | ![Image] |
| ![Image] | ![Image] | ![Image] | ![Image] |
| ![Image] | ![Image] | ![Image] | ![Image] |

Table 4 shows the distortion of a beam passed through a cylindrical plane-convex lens at various distances $z$. As you can see, the beam is quite sensitive to increasing the distance to the detector. With increasing distance, the beam sharply changes its scalability. The degree of astigmatism increases in proportion to distance.

4. Conclusion
A simulation of the propagation of a zero-order Bessel beam through a cylindrical lens was carried out. Dependences of the degree of astigmatism of the beam on the distance to the lens are found.

The Gauss-Laguerre modes are modeled through anisotropic media in order to determine the effect of polarization on the degree of anisotropy. The simulation showed that there is a dependence of the degree of astigmatism on the angle $\theta$ and $\alpha$. The effect of anisotropy is also subject to dependence on the magnitudes of these angles.
In this paper, we analyzed the dependence of the propagation of a fourth-order Gauss-Laguerre beam from the distance to the detector. Based on these results, further studies of astigmatism, the behavior of light beams when passing through astigmatic lenses, and also to determine the anisotropy characteristics of crystals can be carried out.

5. References
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