Exploring the effects of pressure on the radial accretion of dark matter by a Schwarzschild supermassive black hole

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ABSTRACT

Based on the numerical solution of the time-dependent relativistic Euler equations on to a fixed Schwarzschild background space–time, we estimate the accretion rate of radial flow towards the horizon of a test perfect fluid obeying an ideal gas equation of state. We explore the accretion rate in terms of the initial density of the fluid for various values of the inflow velocity in order to investigate whether or not sufficiently arbitrary initial conditions allow a steady state accretion process depending on the values of the pressure. We extrapolate our results to the case where the fluid corresponds to dark matter and the black hole is a supermassive black hole seed. Then we estimate the equation of state parameters that provide a steady state accretion process. We found that when the pressure of the dark matter is zero, the black hole’s mass grows up to values that are orders of magnitude above $10^9 M_\odot$ during a lapse of 10 Gyr, whereas in the case of the accretion of the ideal gas dark matter with non-zero pressure the accreted mass can be of the order of $\sim 1 M_\odot/10$ Gyr for black holes of $10^6 M_\odot$. This would imply that if dark matter near a supermassive black hole acquires an equation of state with non-trivial pressure, the contribution of accreted dark matter to the supermassive black hole growth could be small, even though only radial accretion is considered.

Key words: accretion, accretion discs – black hole physics – dark matter.

1 INTRODUCTION

The problem of formation and evolution of supermassive black holes (SMBHs) in the centre of giant elliptic and disc galaxies remains unsolved. The black hole growth is usually related to its coexistence with the surrounding matter, both baryonic and dark matter. Some models consider these black holes are the result of the evolution of seed black holes of various initial masses that grow through accretion, for instance, if there are seed black holes as the result of primordial gas clouds collapse at the early evolution of galaxies, the seeds would have masses of the order of $10^3 – 10^5 M_\odot$ (e.g. Eisenstein & Loeb 1995; Koushiappas et al. 2004), whereas when the seeds are considered to be the result of the collapse of relativistic massive stars the seeds should have started with masses of the order of hundreds of solar masses (Heger et al. 2003); also combined models of the collapse of primordial gas and dark matter are assumed as black hole seeds of intermediate masses (Umeda et al. 2009). The study of bolometric quasar luminosity function at various distances indicates that the accreted mass of SMBHs should be baryonic (Small & Blandford 1992; Hopkins, Richards & Hernquist 2007), however no final answer about the amount of accreted dark matter by SMBHs is available, for instance in Peirani & Freitas (2008), it is shown that dark matter contributes with at most 10 per cent of the total accreted mass, which together with observations of the bolometric quasar luminosity confirms that baryons are the main matter component that feeds the black hole through accretion.

Standard analyses of SMBHs growth are based on the study of the evolution of phase space distributions, for instance, if SMBHs are primarily fed by collisionless dark matter, or stars, the key problem to overcome is how to refill the loss cone (e.g. Lightman & Shapiro 1977; Zhao, Haehnelt & Rees 2002). Previous work has shown that the time-scale for this is too long for collisionless matter to contribute significantly to black hole growth (Read & Gilmore 2003), though box orbits in a triaxial potential might provide an interesting caveat to this argument (Holley-Bockelman & Sigurdsson 2006). Another possibility is runway growth at the centre of dense star clusters. This could produce intermediate mass black holes, which would be seed black holes that grow through later gas accretion (Ebisuzaki et al. 2001; Portegies & McMillan 2002), which is expected to be fuelled by mergers (e.g. Volonteri & Rees 2005). Concerning the non-radial accretion process, particles have to overcome the angular momentum barrier in order to get the gas to the very centre of a galaxy (e.g. King & Pringle 2006).
Mechanisms proposed include the ‘bars within bars’ model (Shlosman, Frank & Begelman 1989; Begelman, Volonteri & Rees 2006) and direct collapse of cold clouds (Loeb & Rasio 1994).

On the other hand, non-standard models consider the SMBHs grow through the direct accretion of self-interacting or collisional dark matter (Ostriker 2000; Hu et al. 2006). In this case, the self-interaction cross-section is tuned to ensure that the loss cone is refilled by diffusion on an interestingly short time-scale. Collisional dark matter has also been studied at galactic scale as a possible solution to the galactic core problem (Spergel & Steinhardt 2000) and the corresponding gravitational collapse (e.g. Moore et al. 2000). The collisional dark matter also finds its candidates from extensions to the standard model of particles, for example singlets (e.g. Holz & Zee 2001).

In this paper we present a totally different approach that considers both the collisional and the collisionless dark matter. It consists in the solution of the fully relativistic time-dependent Euler equations for a fluid on a Schwarzschild black hole space–time, where the fluid plays the role of dark matter. We model the collisional and collisionless dark matter as a relativistic ideal gas with and without pressure, respectively, in order to study the possibility that the pressure itself suffices to stop the accretion and pull the system towards a stationary state that allows the SMBH to have the observed masses.

Figure 1. Accretion mass rate and the accreted mass $M_{\text{ac}}$ for a pressureless fluid for various values of the initial coordinate velocity $v' = 0.0, -0.1, -0.2, -0.3$. In all the cases, the initial density used is $\rho = 10^{-3}$. The different types of lines indicate that the accretion is being measured at various spherical surfaces, and the fact that they behave similarly shows that the trend of the accretion rate is consistent at the various detectors.

Table 1. Fit parameters describing the mass growth function for a wide set of initial density and inward velocity of the test fluid for the pressureless case. We also show the extrapolation value corresponding to the dark matter density $\rho = 10^{-26}$ and $\rho = 10^{-30}$ in geometric units, that corresponds to the two particular cases we analyse below.

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We measure the accretion rate of dark matter into the black hole parametrized by the equation of state, initial dark matter density and initial infall velocity.

Given that we only consider radial accretion, our results for both the collisional and collisionless cases are upper bounds to more general accretion situations where non-trivial impact parameters of the dark fluid particles are considered, for instance those where rotation of dark matter particles is considered.

The paper is organized as follows. In section 2 we define the relativistic Euler equations describing the fluid on a Schwarzschild black hole, whereas in section 3 we explain the numerical methods we use; in section 4 we show our results for the extrapolation to the SMBH plus dark matter system, and in Section 5 we summarize our conclusions.

2 RELATIVISTIC EULER EQUATIONS

2.1 In general

In order to track the evolution of a perfect fluid on a non-flat space–time it is necessary to write down the general relativistic Euler equations. For a generic fixed space–time, these can be derived from the local conservation of the stress-energy tensor \(T^{\alpha\beta}\) and the local conservation of the number of barions:

\[
\nabla_\alpha (T^{\alpha\beta}) = 0, \tag{1}
\]

\[
\nabla_\alpha (pu^\alpha) = 0, \tag{2}
\]

where \(p\) is the proper rest-mass density, \(u^\alpha\) is the four-velocity of the fluid and \(\nabla_\alpha\) is the covariant derivative consistent with the four-metric of the space–time (Misner et al. 1973).

The stress energy tensor representing a perfect fluid in general relativity is given by

\[
T^{\alpha\beta} = pu^\alpha u^\beta + pg^{\alpha\beta}, \tag{3}
\]

where \(p\), \(h\) and \(g^{\alpha\beta}\) are respectively the pressure, the specific enthalpy and the four-metric tensor of the space–time.

We choose to write down the space–time metric in the 3+1 decomposition fashion for a coordinate system \((t, x^i)\), that is (Misner et al. 1973)

\[
dx^2 = - (\alpha^2 - \beta^i \beta^i) dt^2 + 2 \beta^i dx^i dt + \gamma_{ij} dx^i dx^j, \tag{4}
\]

where the Latin indices \(i, j\) label the three spatial coordinates of the space–time, \(\alpha\) is called the lapse function and together with the shift vector \(\beta^i\) determine the foliation of the space–time in space–like hypersurfaces; \(\gamma_{ij}\) is the induced three-metric that relates proper distances on the spatial hypersurfaces (Misner et al. 1973).

Table 2. Mass accreted by a black hole since 10 Gyr ago for the present to two black seed masses. The initial density profile of the dark matter is \(\rho = 100 M_\odot pc^{-3}\), which in geometric units, for black hole seed masses \(M_{(i)} = 10^4 M_\odot\) and \(M_{(ii)} = 10^6 M_\odot\), correspond to \(\rho = 1.17 \times 10^{-30}\) and \(\rho = 1.17 \times 10^{-28}\) in geometric units, respectively. The values of the mass accreted for these densities are taken from the extrapolation in Table 1.

| Seed \(M_{(i)} = 10^4 M_\odot\) | Seed \(M_{(ii)} = 10^6 M_\odot\) |
|-----------------------------|-------------------------------|
| \(v' = 0.0\) | \(3.04 \times 10^7 M_{(i)}\) | \(3.82 \times 10^7 M_{(ii)}\) |
| \(v' = -0.1\) | \(2.49 \times 10^{17} M_{(i)}\) | \(2.07 \times 10^{16} M_{(ii)}\) |
| \(v' = -0.2\) | \(1.17 \times 10^{14} M_{(i)}\) | \(3.28 \times 10^{13} M_{(ii)}\) |
| \(v' = -0.3\) | \(1.96 \times 10^{13} M_{(i)}\) | \(7.82 \times 10^{12} M_{(ii)}\) |

With this information, the unknown quantities of Euler equations are the following primitive variables: the rest-mass density of the fluid \(\rho\), its pressure \(p\) and its velocity measured by an Eulerian observer \(v'\) defined in terms of the spatial part of the four-velocity of the fluid by \(v' = u^\alpha W + \beta^i x^i\), where \(W\) is the Lorentz factor\( W = 1/\sqrt{1 - \gamma_{ij} v'^i v'^j}\). Given that there are five unknowns and four equations, the system is closed with an equation of state \(p = (\rho / \epsilon)\), where \(\epsilon\) is the specific internal energy, defined by \(\epsilon = 1 + \epsilon / \rho\).

Even though one can write down relativistic Euler equations straightforwardly in terms of the primitive variables, we work with conservative variables that allow one to write down the relativistic Euler equations as a set of flux balance laws:

\[
\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x^i} = S, \tag{5}
\]

where \(q\) is the vector of conservative variables, \(F\) are the fluxes in the three spatial directions, \(S\) is a source vector. This approach is adequate to implement the finite volume methods described below.

For the perfect fluid above, a flux balance system of equations is (e.g. Martí et al. 1991; Font et al. 1994; Banyoul et al. 1997; Font et al. 2000)

\[
u = \begin{bmatrix} D \\ J_1 \\ \tau \\ \sqrt{\gamma} (\rho W^2 - p - \rho W) \end{bmatrix}, \tag{6}
\]

\[
F^\ell = \begin{bmatrix} \alpha (v^\ell - \frac{\epsilon}{\gamma}) D \\ \alpha (v^\ell - \frac{\epsilon}{\gamma}) J_1 + \alpha \sqrt{\gamma} \rho p J_\ell \\ \alpha (v^\ell - \frac{\epsilon}{\gamma}) \tau + \alpha \sqrt{\gamma} v^\ell p \\ 0 \end{bmatrix}, \tag{7}
\]

\[
S = \begin{bmatrix} \alpha \sqrt{\gamma} T^{\alpha\beta} g_{\alpha\beta} R^\mu_{\mu} \Gamma^\alpha_{\ell \mu} \\ \alpha \sqrt{\gamma} (\rho u^i p^\alpha \partial_{\alpha} \alpha - \alpha \rho u^i p^\alpha \Gamma^\alpha_{\ell \mu} ) \end{bmatrix}, \tag{8}
\]

where \(\gamma = det(\gamma_{ij})\) is the determinant of the three-metric and \(\Gamma^\alpha_{\ell \mu}\) are the Christoffel symbols.

2.2 Equations for the spherically symmetric case

We now specialize to the case of a Schwarzschild black hole, for which the line element in Eddington–Finkelstein coordinates \(x^i = [t, r, \theta, \phi]\) is written for the line element (4) with the following gauge and three-metric:

\[
\alpha = \frac{1}{\sqrt{1 + \frac{2M}{r}}},
\]

\[
\beta^i = \begin{bmatrix} 2M \left( \frac{1}{1 + \frac{2M}{r}} \right) \sin^2 \theta, 0, 0 \end{bmatrix},
\]

\[
\gamma_{ij} = \begin{bmatrix} 1 + \frac{2M}{r}, r^2, r^2 \sin^2 \theta \end{bmatrix}, \tag{9}
\]

where \(M\) is the mass of the black hole.
Note that the radial is the only non-trivial component of the shift vector \( \beta^r \) and also the only non-zero velocity is \( v^r \). This implies that there are only three non-trivial balance law equations. In terms of these geometric quantities, and provided the space–time is spherically symmetric, the system of flux balance equations reads

\[
\mathbf{u} = \begin{bmatrix}
D \\
J_r \\
\tau
\end{bmatrix},
\]

Figure 2. Accretion rate of an ideal gas, for two values of the initial coordinate velocity \( v^r \) and two values of the adiabatic constant \( \Gamma \). In all the cases, the initial density and the specific internal energy used are \( \rho = 10^{-4} \) and \( \epsilon = 0.5 \), respectively. The different types of lines indicate that the accretion is being measured at various spherical surfaces, and it is remarkable that the accretion mass rate approaches the same value, as happens in the \( p = 0 \) case. However, in the present case, due to the non-zero pressure the result is non-trivial because outward and inward flow are expected to happen.
\[
F' = \begin{bmatrix}
\alpha \left( v' - \frac{\rho' v}{\rho} \right) D \\
\alpha \left( v' - \frac{\rho' v}{\rho} \right) J_r + \alpha \sqrt{T} p \\
\alpha \left( v' - \frac{\rho' v}{\rho} \right) \tau + \sqrt{T} \alpha v' p
\end{bmatrix},
\]
\[
S = \begin{bmatrix}
0 \\
\alpha \sqrt{T} T^{\nu\rho} \beta_{\nu \rho} \Gamma_{\mu \nu} - \alpha \sqrt{T} T^{\nu \rho} \beta_{\nu \rho} \Gamma_{\mu \nu} \\
\alpha \sqrt{T} (T^{\nu \mu} \partial_r \alpha - \alpha T^{\nu \mu} \Gamma_{\mu \nu})
\end{bmatrix}
\]
which obey the balance law
\[
\frac{\partial u}{\partial t} + \frac{\partial F'(u)}{\partial r} = S.
\] (11)

Finally, we adopt an ideal gas equation of state, in which the pressure is given in terms of the rest-mass density and specific internal energy by
\[
p = (\Gamma - 1) \rho \epsilon,
\] (12)
where \(\Gamma\) is the adiabatic index or the ratio of specific heats.

### 3 Numerical Methods

**Solution of the equations.** In order to solve the system of equations (10) and (11), we provide initial data for the rest-mass density and velocity of the fluid, and use the equation of state (12) to calculate the initial pressure in case it is non-zero. In all our production runs, we use a constant density profile as initial data and various values of the constant density and velocity.

We evolve the initial data using a finite volume approximation of equations (10), together with high resolution shock capturing methods involving approximate Riemann solvers that are able to track the potential shocks formed during the evolution of relativistic fluid equations (e.g. LeVeque 1992). We specially use the HLL Riemann solver and a constant piecewise reconstruction of variables at the inter cell boundaries (for a detailed description, see Toro 1997).

We also implement an atmosphere that prevents the rest-mass density to be small enough as to provoke that relativistic Euler equations diverge or develop unphysical results, because when the density is extremely small the enthalpy diverges which provokes the numerical implementation to fail, which combines with round-off errors through arithmetic operations, cumulate during the evolution and finally produce approximations that are not accurate. The atmosphere we use is such that \(\rho = \max(\rho, 10^{-10})\), which is a value that allows the convergence of our numerical methods. The need of the atmosphere is in fact one of the reasons why it is not – at the moment – trivial to simulate evolutions of fluids with ultra low densities and the reason why our results on dark matter are actually extrapolations of numerical experiments with rather higher densities of matter.

**Boundaries and boundary conditions.** We carry out our simulations on a finite domain \(0 < r \leq r_{\max}\). On the one hand, given that we use Eddington–Finkelstein coordinates, which penetrate the horizon, we apply the excision technique (Seidel & Suen 1992), that is a chunk of the domain well inside the black hole’s horizon is removed from the numerical domain from \(0 < r \leq r_{\text{exc}}\) in order to avoid the black hole’s singularity and the steep gradients of metric functions near there; since the light cones inside the horizon point towards the singularity there is no need to impose boundary conditions at \(r = r_{\text{exc}}\) instead, the fluid simply gets out the domain towards the singularity through such boundary. We choose the excision radius to be at \(r_{\text{exc}} = 2\)M, that is half the Schwarzschild radius, which is both far enough from the singularity at \(r = 0\) and provides a buffer zone \(M < r < 2M\) for the fluid inside the horizon to flow smoothly from the horizon towards the excised chunk of the domain.

On the other hand, we implement an artificial outer boundary at a finite distance from the black hole at \(r = r_{\max}\), and apply simple boundary conditions through the extrapolation of the conservative variables. We have found that such boundary condition makes the artificial boundary to behave appropriately, that is we have verified that our calculations do not depend on the location of the external boundary, which is the requirement that must be fulfilled by a boundary–boundary condition.

**Diagnostics.** In order to diagnose our simulations on the fly we implement detectors located at various radii in the numerical domain, that is we define spheres where we calculate scalar quantities. In particular, we track the accretion mass rate as a function of time. In order to do so we calculate the accretion mass rate specialized for spherical flow on a spherically symmetric space–time by
\[
\dot{M}_{\text{acc}} = -4\pi r^2 \rho W \left( v' - \frac{\beta}{\alpha} \right),
\] (13)
at various spherical surfaces, including the black hole’s event horizon.

Since we are very interested in checking the validity of our numerical results, we also monitor both the conservative and the primitive variables of our system of equations in the whole numerical domain. Then we apply a self-convergence test of our variables in order to verify that our calculations carried out under a discretized domain converge to a correct solution in the continuum limit. That is, we want to make sure that our implementation is consistent, which in practice means that when increasing the numerical resolution, our numerical solution converges fast enough towards a continuous solution of the continuous equations, which is a highly non-trivial test (LeVeque 1992).

**Units.** We use geometrical units \(G = c = 1\), and time and spatial units of \(M\); a typical dark matter density \(\rho_{DM} \sim 100 \text{M}_\odot \text{pc}^{-3}\) corresponds to \(\rho_{DM} \sim 10^{-20} M^{-2} \sim 10^{-30} M^{-2}\) for black hole seed masses of \(10^6\) and \(10^8 \text{M}_\odot\), respectively. However, due to numerical restrictions in the use of ultra low fluid densities – as corresponds to dark matter – in geometrical units when solving the relativistic.
Table 3. Parameters fitting the accreted mass for the initial internal energy $\epsilon = 0.5$, 1.0 and the adiabatic constant $\Gamma = 1.1, 1.2$. We also show the extrapolation value corresponding to the dark matter density $\rho = 10^{-26}$ and $\rho = 10^{-30}$ in geometric units, that correspond to two particular cases we analyse below. The anzatz we use to fit the mass accreted is $M_{\text{ac}} = A \rho^B + C$.

| $\rho = 10^{-4}$ | $\rho = 10^{-6}$ | $\rho = 10^{-8}$ | $\rho = 10^{-10}$ (Floor) | $\rho = 10^{-30}$ | $\rho = 10^{-30}$ |
|------------------|------------------|------------------|-------------------|------------------|------------------|
| $C$ = 0.080384 ± 0.0001781 | $C$ = 0.080384 ± 0.0001781 | $C$ = 0.080384 ± 0.0001781 | $C$ = 0.080384 ± 0.0001781 | $C$ = 0.080384 ± 0.0001781 | $C$ = 0.080384 ± 0.0001781 |
| $B$ = 0.96003 ± 0.0002526 | $B$ = 0.96003 ± 0.0002526 | $B$ = 0.96003 ± 0.0002526 | $B$ = 0.96003 ± 0.0002526 | $B$ = 0.96003 ± 0.0002526 | $B$ = 0.96003 ± 0.0002526 |
| $A$ = -1.45148 ± 0.0002033 | $A$ = -1.45148 ± 0.0002033 | $A$ = -1.45148 ± 0.0002033 | $A$ = -1.45148 ± 0.0002033 | $A$ = -1.45148 ± 0.0002033 | $A$ = -1.45148 ± 0.0002033 |
| $C$ = -1.22404 ± 0.0006492 | $C$ = -1.22404 ± 0.0006492 | $C$ = -1.22404 ± 0.0006492 | $C$ = -1.22404 ± 0.0006492 | $C$ = -1.22404 ± 0.0006492 | $C$ = -1.22404 ± 0.0006492 |
| $A$ = -3.14018 ± 0.0000141 | $A$ = -3.14018 ± 0.0000141 | $A$ = -3.14018 ± 0.0000141 | $A$ = -3.14018 ± 0.0000141 | $A$ = -3.14018 ± 0.0000141 | $A$ = -3.14018 ± 0.0000141 |
Euler equations, and we study the accretion of fluid densities numerically tractable and then extrapolate our results to the density corresponding to dark matter and extract our conclusions.

4 RESULTS

4.1 Preparing the initial conditions

In order to solve Euler equations (10) and (11), we have to provide initial data at \( t = 0 \), for \( p \) and \( \nu' \) that for each value of \( \Gamma \) would imply different values for \( \rho \). In principle, there is no prescription for \( \rho \) and \( \nu' \) which would be arbitrary. Since we are interested in searching for steady state scenarios, we are inclined to set rather unbiased out of equilibrium initial conditions in order to see whether or not the system relaxes and approaches a sort of late-time attractor solution.

We have found that an initial constant density profile corresponds to a non-steady state condition, and instead produces an immediate dynamical behaviour. We also choose different values of \( \nu' \), because we want to explore the space parameter in this direction too. We have found that the initial radial velocity is only restricted in the sense that for big initial inward velocities the fluid may reach ultra relativistic speeds when approaching the event horizon. Finally, the pressure is obtained from the equation of state.

An initial parameter for which there is no prescription is that of the internal energy of the gas \( \epsilon \) in the equation of state. Its initial value determines the pressure at the initial time with the property that when \( \epsilon \) approaches zero the initial conditions corresponds to the \( p = 0 \) case.

4.2 Case \( p = 0 \)

In Fig. 1, we show the numerical results of the mass accretion rate and the accreted mass for different values of the initial inward velocity with an initial constant density profile \( \rho = 10^{-4} \); the accretion rate is measured at three different surfaces located at \( r = 2M, 14M, 29M \) in order to make sure the trend of the accretion rate is not an artefact due to backscattering near the horizon.

In all cases the mass accretion rate increases rapidly and never during our evolutions enter in a steady state. In order to measure how fast the accreted mass grows, we fit the accreted mass with an initial constant density profile corresponds to a non-steady state condition, and instead produces an immediate dynamical behaviour. We also choose different values of \( \nu' \), because we want to explore the space parameter in this direction too. We have found that the initial radial velocity is only restricted in the sense that for big initial inward velocities the fluid may reach ultra relativistic speeds when approaching the event horizon. Finally, the pressure is obtained from the equation of state.

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In all the combinations of these parameters, we found that the accretion process reaches a stationary accretion mass rate. We show in Fig. 2 a representative sample of our results indicating a linear accreted mass in time after a transient lapse, where the initial relax towards a sort of stationary late-time attractor solution. For completeness in Fig. 3, we show as an example the mass accretion rate with two different initial specific internal energies. This shows the role played by the initial internal energy, that is, in the limit of \( \epsilon \)

### Table 4

| Seed \( M_{(i)} = 10^7 M_\odot \) | Seed \( M_{(i)} = 10^9 M_\odot \) |
|--------------------------------|--------------------------------|
| \( \nu' = -0.1 \) | \( 9.21 \times 10^{-9} M_{(i)} \) | \( 1.11 \times 10^{-6} M_{(i)} \) |
| \( \nu' = -0.2 \) | \( 1.72 \times 10^{-8} M_{(i)} \) | \( 2.16 \times 10^{-6} M_{(i)} \) |
| \( \nu' = -0.1 \) | \( 6.31 \times 10^{-9} M_{(i)} \) | \( 6.59 \times 10^{-7} M_{(i)} \) |
| \( \nu' = -0.2 \) | \( 1.1 \times 10^{-8} M_{(i)} \) | \( 1.14 \times 10^{-6} M_{(i)} \) |
| \( \nu' = -0.1 \) | \( 5.2 \times 10^{-9} M_{(i)} \) | \( 6.11 \times 10^{-7} M_{(i)} \) |
| \( \nu' = -0.2 \) | \( 1.1 \times 10^{-8} M_{(i)} \) | \( 1.29 \times 10^{-6} M_{(i)} \) |
| \( \nu' = -0.1 \) | \( 3.28 \times 10^{-9} M_{(i)} \) | \( 3.48 \times 10^{-7} M_{(i)} \) |
| \( \nu' = -0.2 \) | \( 7.80 \times 10^{-9} M_{(i)} \) | \( 7.90 \times 10^{-7} M_{(i)} \) |
Figure 4. Accretion mass rate for various values of $\Gamma$ for the configurations corresponding to the initial parameters $\rho = 10^{-4}$, $v' = -0.1$, $\epsilon = 0.5$, 1. In the insets we show a zoom-in of the accretion mass rate that illustrates the various behaviours at the initial transient stage that depends on $\Gamma$.

approaching zero as can be seen in (12), the pressure tends to zero too and one recovers the unstable case.

Our results on the accretion mass rate and accreted mass are summarized in Table 3, where, as in the pressureless case, we also extrapolate our fitting results to the ultra low density corresponding to dark matter. The domain we use in these numerical experiments is $M < r < 1001M$ and, we make sure the running time $t = 3000M$ is adequate for this grid size and our code converges. We then use these extrapolated parameters to estimate the accreted mass during 10 Gyr, which can be found in Table 4.

From our results it calls the attention that approaching a stationary accretion process is very sensitive to the value of $\Gamma$, as can be seen in the Tables 2 and 4, where the resulting estimates in the accreted mass can change by about 15 orders of magnitude when changing $\Gamma$ from 1 to 1.1, which corresponds to the system of dust and our ideal gas dark matter model with pressure, respectively. In order to show that this is not an artefact of our numerical approach, we show in Fig. 4 that the accretion mass rate changes monotonically with $\Gamma$ for two configurations chosen from the collection of cases we experienced with.

Long time attractor behaviour of the solutions. As an example of how the state variables approach a stationary regime in the case of non-zero pressure, we show in Fig. 5 snapshots of $\rho$, $p$ and $v'$. It can be noted that the initial data corresponding to constant density, pressure and velocity, after a transient lapse approach a time-independent regime. It can also be observed that the pressure is bigger near the horizon – because it is linear with the rest-mass density – which prevents the fluid from falling quickly into the hole, an effect that does not happen in the $p = 0$ case and which we consider to be responsible for slowing down the accretion rates.

4.4 Test of the code

In order to validate our numerical results we present a very stringent test which is related to the convergence of our numerical solution.
Figure 6. The order of self-convergence of $\rho$, for two different physical cases. (1) left-hand panel: $p = 0$, $\rho = 10^{-4}$ and $v^r = -0.1$ and (2) right-hand panel: $\Gamma = 1.2$, $\rho = 10^{-4}$, $v^r = -0.1$, $\epsilon = 0.5$. We calculate the convergence using the $L_1$ norm of the differences between the value of the density for three different resolutions, $\Delta x_1 = 0.1$, $\Delta x_2 = \Delta x_1/2$ and $\Delta x_3 = \Delta x_2/2$. Then, the self-convergence factor, $Q$, for these three resolutions is calculated from the following expression as $20\epsilon = \Delta E_1/\Delta E_2$, where $\Delta E_1 = L_1(\rho_1 - \rho_2)$ and $\Delta E_2 = L_1(\rho_2 - \rho_3)$. The dominant approximation is that of the reconstruction of variables at the interface cells, which is a piecewise constant and therefore first order accurate. This is consistent with the self-convergence of this plot.

Since we do not have an exact solution of the equations, we perform a Cauchy type of self-convergence test on the state variables. As a sample of such test we show in Fig. 6 the self-convergence of the rest-mass density for two of our runs. We find that the order of self-convergence is 1, which is consistent with the order of approximation (first order) of our variable reconstruction at the cell interfaces of the Riemann solver.

5 CONCLUSIONS

We have numerically studied the spherical accretion of a perfect fluid with an ideal gas equation of state on a fixed Schwarzschild background space–time in penetrating coordinates and supported our results with self-convergence tests. Our procedure consists in the numerical solution of the time-dependent relativistic Euler equations on the black hole space–time. As a first approximation, due to well known numerical difficulties, we have extrapolated our results to the physical values corresponding to the accretion of dark matter on to an SMBH. We have explored a wide range of parameters related to density, pressure, internal energy, adiabatic constant and inward initial velocity of the fluid.

For the parameter space explored, we have found that when the pressure of the dark matter is zero, within the window of our simulations there is no steady state accretion process, and also that the current masses of SMBHs should be orders of magnitude bigger than those observed if they are assumed to be fed by spherical accretion of dark matter. However, for the case of non-zero pressure, we have found in all the experiments we have analysed that a steady state accretion is reached and behaves like a late-time attractor solution which is reached sooner for bigger values of the adiabatic index.

We found that for initial black hole seed of already supermassive size ($10^9 M_\odot$), the accreted mass during 10 Gyr is of the order of solar masses. Either this implies that dark matter may obey an equation of state with non-zero pressure or, in a conservative case, the accretion of dark matter on to SMBHs contributes with a small fraction compared to possible accretion of baryonic matter as suggested by the bolometric luminosity of quasars (Peirani & Freitas 2008), where a bound of 10 per cent of the accretion is allowed to be dark matter.

This spherically symmetric accretion model is a first step towards a full study of the SMBH plus dark matter system, which in a complete version should involve the introduction of tidal forces due to angular components of the fluid that will break the spherical symmetry, back reaction effects or full non-linear evolution of the black hole, transfer of angular momentum to the black hole, etc.

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