The Tool for the In-Depth Understanding of Electromechanical Wave’s Propagation and Inter-Generator Oscillations

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Abstract

When dealing with wave propagation in power systems, one usually thinks of electro-magnetic waves whose predictable characteristics are successfully implemented in several existing protection applications. However, when dealing with large interconnected power systems, different kind of events are considered under the term “disturbance,” such as trip of a large generating unit or a highly-loaded transmission line outage. After such disturbance, synchronous generators connected to the system react with a rotor angle deviation, which is not simultaneous all over the system. This invokes the so-called electromechanical wave whose propagation speed depends on transmission system impedances and the inertia of the involved generators. When the electromechanical wave reaches the boundaries of the system, it is reflected back towards the origin and the sum of inclined and reflected waves results in inter-generator oscillations. In this paper, a description of an educational tool that provides a dynamic simulation of the power-system model with arbitrary topology is given. The tool sets-up both steady-state conditions as well as second-order differential equations required to calculate rotor angle deviations.
of all included generators with respect to time after selected disturbance occurs. The three-dimensional visualization of the results enables one to analyse the situation in detail and especially to gain a physical understanding of the phenomenon.

Keywords

Power-system oscillations; dynamic simulations; large power systems; electromechanical wave

I. Introduction

One of the essential parts of undergraduate power engineering study is getting familiar with the whole assortment of the power-system dynamic phenomena, ranging from electromagnetic system responses and sub-synchronous resonance to different kinds of stability issues (first-swing stability, frequency stability, voltage stability, etc.). Most of these subjects are covered in many publications taking different approaches, which enables the soon-to-be electrical engineer to find a suitable representation of the effect under question which best suits his/her needs. However, according to the authors’ knowledge, not many graphically-inspired explanations of power-system small-signal stability problems can be found in the literature. In this way, only those students with highly developed mathematical skills may gain a satisfactory level on knowledge and physical understanding in this field. Namely, authors usually provide merely mathematically correct derivations of eigenvalues/eigenvectors together with derived indices ([1]–[3]), which are rather difficult to follow and draw a connection between power-system related physics and mathematical derivations. Some attempts were made in the past to graphically represent electromechanical wave propagation ([4]–[6]), that inspired the authors of this work.

On the one hand, the authors are aware that solid mathematical background is an essential part of understanding complicated effects that constantly take place in the power system. On the other hand, the engineer’s internalised physical perception appears even more valuable, especially as nowadays engineers often rely too much on results provided by several simulation tools available on the market. With this in mind, a tool for in-depth understanding of the electromechanical wave’s propagation and inter-generator oscillations was developed. Currently, the tool is in the initial stage of development, but it already shows several positive effects on understanding the physics of electromechanically related phenomena in the power system.

The paper is organized as follows. First, some facts about investigated electromechanical phenomena are provided, together with corresponding equations used within the tool. Next, early-phase graphical user interface is presented and an example of handling the tool. As several improvements are to
be made in the future, those are listed in discussed as well. Finally, the conclusions are drawn.

II. Theoretical background – short overview

Small-signal stability of a power-system is a very important issue that has to be fully understood by the electrical engineer. Its analysis is based on linearization of non-linear system of differential equations, providing mathematical relationship between several system variables. Several terms arise from such analysis [1]:

- vector of state variables \( \mathbf{x} \)
- vector of control variables \( \mathbf{u} \)
- vector of output variables \( \mathbf{y} \)
- state matrix \( \mathbf{A} \)
- control matrix \( \mathbf{B} \)
- output matrix \( \mathbf{C} \)
- coefficient matrix \( \mathbf{D} \)

All listed terms can be found in the following two matrix expressions:

\[
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\
\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}
\] (1)

From the two basic formulas in (1), expressions for eigenvalues and eigenvectors can be derived which enables one to investigate the oscillatory nature of the system. In general, complex conjugate pair of eigenvalues are referred to as “oscillatory modes,” whereas real eigenvalues determine “monotonous modes.” From eigenvalues, only the frequency and damping of each mode can be seen. On the other hand, for each mode two eigenvectors can be obtained as well: right eigenvector (mode shape) describes the mode observability and left eigenvector provides the information about the mode controllability.

However, small-signal stability analysis is a sort of steady-state calculation. One obtains a systematic overview of all possible system modes, where at all times one should be aware that different events in the system might invoke different combination of modes, each of them in a different extent. Several conditions can bring to power-system oscillations:

- **Sudden events** - in this category, events such as tripping of highly-loaded transmission line, generator tripping, etc. can be found. As a result, several modes are invoked, depending on the type, location and specifics of the event.
- **Continuous excitation of certain modes** - within this category, a low-energy repetitive stimulation is meant. The stimulation is said to have the frequency, close to at least one of resonant frequencies. An example of such excitation is a slow transition of hydro-generating units with a
Francis turbine through a rough zone, which causes oscillations of a turbine shaft due to vortex effect [7].

At this stage, the presented tool enables the simulation of sudden event only, which reflects as a sudden change of voltage phase angle on a so-called disturbance bus. The important phenomenon to understand is the transition from no-fault condition towards oscillating condition. After the sudden event, all system generators do not react simultaneously to this change. The so-called electromechanical wave propagates through the system, originating in the disturbed bus. The specifics of such electromechanical wave are determined by system impedance conditions and generator’s inertia as well. The following equation for the electromechanical wave propagation speed can be found in [5], derived from theoretical model of a power system with spatially distributed parameters (transmission line impedance as well as generator inertia):

\[ v = \sqrt{\frac{\omega V^2 \sin \Theta}{2h \cdot |z|}} \]  

(2)

where \( \omega \) is the nominal system frequency, \( \Theta \) is the line impedance angle in radians, \( V \) is the voltage amplitude in p.u., \( h \) is the inertia constant in seconds and \( |z| \) is the line impedance in p.u. per line length.

After the electromechanical wave reflects back from the system outer borders, the incident and reflected waves sum-up to oscillations, which can be analysed via eigen-properties of the system.

A. Power-system model

For the purpose of the tool, the power-system is modelled as a rectangular \( n \) by \( m \) grid of interconnected busses (see Fig. 1). The connection between two busses represents a transmission line, modelled with a series reactance \( X_{\text{line}} \). Each bus has a parallel branch as well, representing a generation unit (synchronous generator modelled with a constant voltage source \( E' \) behind a transient reactance \( X_d' \)). The phase angle of the internal voltage source is considered as a rotor angle and follows the swing equation, described in the following subsection.

Depending on the desired fault location, the so-called disturbance bus does not have any parallel branches connected, as is clearly evident from Fig. 1.
B. System of equations

When dealing with the power-system oscillations, the time-dependency of rotor angles $\delta_{r,i}(t)$ and voltage phase angles $\delta_i(t)$ of the corresponding $i$-th machine (or bus) are of main concern. The initial values prior to any disturbance of all angles ($\delta_{r,i}^0$ for rotor angle and $\delta_i^0$ for voltage phase angle) are determined via load-flow calculation. For clearer visualization, only time-dependent deviations from initial angle values are depicted, where:

$$\Delta \delta_{r,i}(t) = \delta_{r,i}(t) - \delta_{r,i}^0$$
$$\Delta \delta_i(t) = \delta_i(t) - \delta_i^0$$

The rotor angle of each synchronous machine with respect to time is determined by a second-order differential equation, commonly known as the swing equation:

$$2H_i \cdot \dot{\omega}_{r,i,pu}(t) \approx \Delta P_{i,pu}$$

(4)

where $H_i$ stands for the inertia constant of the $i$-th generating unit in seconds, based on the generator rated apparent power $S_r$, $\omega_{r,i,pu}$ is the angular mechanical speed of the $i$-th machine in p.u. based on the machine rated frequency $\omega_r$ and $\Delta P_{i,pu}$ the power imbalance in p.u. on the $S_r$ base as seen by the generator. The angular rotor speed can be written in terms of a rotor angle:

$$\omega_{r,i,pu} = \dot{\delta}_{r,i}$$

(5)
The active power flow $P_{\text{line},ij}(t)$ across the transmission line between busses $i$ and $j$ with a reactance $X_{\text{line},ij}$ follows the expression:

$$P_{\text{line},ij}(t) = \frac{1}{X_{\text{line},ij}} \cdot \sin\left(\delta_i(t) - \delta_j(t)\right)$$

(6)

whereas the active power of the generator equals:

$$P_{\text{gen},i}(t) = \frac{1}{X_{d,i}} \cdot \sin\left(\delta_{r,i}(t) - \delta_{r,i}(t)\right)$$

(7)

For each bus the rule of active-power preservation is applied.

The linearization of these equations (not provided due to the fact that this is a quite well known approach, provided in many books and other publications) results in the following matrix equation, solved by MATLAB’s internal function “ode45”:

$$
\begin{bmatrix}
\Delta \delta_{r,1}(t) \\
\Delta \delta_{r,2}(t) \\
\vdots \\
\Delta \delta_{r,x}(t)
\end{bmatrix}
= K_1 \times H \times K_2 \cdot
\begin{bmatrix}
\Delta \delta_{r,1}(t) \\
\Delta \delta_{r,2}(t) \\
\vdots \\
\Delta \delta_{r,x}(t)
\end{bmatrix}
$$

(8)

where matrixes $K_1$ and $K_2$ contain elements with model reactances and initial phase-angle differences, whereas matrix $H$ contains inertia constants of all generation units. The sign “×” represents the scalar product of two matrixes. Solving (8) provides rotor-angle deviations with respect to time for periods after the disturbance.

C. Disturbance model

As already mentioned in Section 2, two different kinds of events can trigger power-system oscillations: sudden events and continuous excitation of certain modes. The latter can be modelled either in the form of forced oscillation (where e.g. additional load can be modelled with an active power following a sinus wave with respect to time with the appropriate frequency corresponding to desired mode frequency [8]) or special set-up of rotor-angles initial conditions (initial misplacements of all generator rotor angles according to mode shape corresponding to eigenvalue with the desired mode frequency [9]).

In the future plans, the generalization of disturbance modelling is expected. However, at current stage it is possible to model the disturbance merely via a sudden voltage phase-angle change. Consequently, less idealized electromechanical wave such as in [4] can be observed.
D. Graphical interface

As already mentioned, the tool is intended as a supplement during the educational process on undergraduate level of power engineering study. In order to increase the level of understanding among students, it is of great importance to present a high-quality visualization of the electromechanical oscillations that are a subject of investigation. This task needs further work and will be a priority in the following period. However, at this moment in time the graphical user interface (GUI) as was initially set up in Matlab is described.

The largest portion of the main window (see Fig. 2) is reserved for symmetrical $n$ by $m$ grid of squares, connected with red/grey links. This is a representation of a simplified symmetrical electricity grid with $n$ by $m$ busses, where each square represents the high voltage (HV) generator bus and each link between squares represents the transmission line. Red-coloured transmission line stands for disconnected status, whereas grey-coloured symbolizes connected status. Every generator bus has an ID number, consisting of combined $x$ and $y$ coordinate of its position within the grid (this is something similar than identification of elements within the two-dimensional matrix).

To each of the generator busses a single synchronous generator is connected, modelled with a constant voltage source behind a transient reactance $x_d'$, where the phase angle of the voltage source follows a swing equation and can therefore be considered as a rotor angle of that particular machine with the inertia constant $H$. Transmission lines are modelled as a series impedance $Z = R + jX$, therefore ignoring capacitances. $R$ stands for ohmic resistance and $X$ stands for reactance.

By default, all generators have the same values of $x_d'$ and $H$ (in the case from Fig. 2 these values are $x_d' = 20\%$ or 0.2 p.u. based on generator rated apparent power and its rated voltage and $H = 5$ seconds), shown in the section “Gen parameters”. In a similar manner, initially all transmission lines have the same default value of ohmic resistance $R$ and reactance $X$ (in the case from Fig. 2 these values are $R = 0 \Omega$ and $X = 20 \Omega$, so no ohmic losses are considered), shown in the section “Line parameters.”

With respect to the model of the power system itself, the user has the following possibilities:

- one may simply consider default values for power-system elements and model the n by m square system (dimensions can be set by entering appropriate values in the section “Dimensions,” 9 by 4 in the case from Fig. 2) or
- one may click on individual line and in this way switch it on or off. In addition, after individual line is activated by a click, its parameters can be changed arbitrary in the “Line parameters” section. Similarly, every click on the generator bus enables the individualization of its parameters.

After the model is set, the user can save it for future needs (in the section “Save configuration” in Fig. 2) so the numerous clicking does not have to be repeated every time the tool is restarted. Loading of previously saved network
configurations can also be performed at any time (in the section “Load configuration” in Fig. 2).

![Fig. 2. Main window of the graphical user interface (GUI)](image)

In order to initiate the disturbance, one ought to first set the disturbance location by entering its appropriate coordinates (section “Disturbance” in Fig. 2). Also, the time of disturbance start and ending in seconds can be set in the same section. By clicking the button in section “Simulate,” one initiates the calculation, which will be described in the continuation.

### III. Example of use

Even though in reality the interconnected transmission systems are highly meshed, a representation of an electromechanical travelling wave is most appropriate in the radial system. An example of a radial 34-generator ring system is shown in Fig. 3, where the connection between the first and the 35th bus is disconnected (opened ring). The bus No. 1 is considered as the disturbance bus. Parameters of all transmission lines are equal and the same goes for synchronous generators.
The selection of the network topology, consistent with the one from Fig. 3, should be done according to Fig. 4. It is obvious that most of the lines are disconnected, except for the border lines around the symmetrical electricity grid (so lines from elements (1,1) up to (1,10), then continuing to (10,10), down to (10,1) and finally to (3,1)).

By leaving all settings on default and running the simulation, one is presented with the animation of voltage phase angles deviations (from their steady-state values) through time. Six snapshots taken in different moments are provided in Fig. 5. In the moment marked with the letter “a”, the initial conditions are shown together with the disturbance which is to be followed by the system response. A sudden phase-angle jump with an amplitude of 15° is modelled on the disturbance bus. The 34-generator ring system busses are laid on the dashed thick solid line in the coloured plane.
Fig. 4. Selection of topology for 34-generator ring system, opened at both ends

In the moment marked with the letter “b”, the phase angle on the disturbance bus already bounces towards the negative values, whereas phase angles of busses (1,2) to (1,7) already form a wave, moving in the direction away from the disturbance. When the wave is close enough to bus (1,10), it appears as if the wave changes its direction (moment marked with the letter “c”). However, this is merely an appearance due to square representation of the grid (electrical conditions along the ring system are identical). The same situation appears at moments marked with the letters “d” and “e.” Once the wave reaches the last bus in series, it is reflected back towards the origin and as a result the summation of inclined and reflected wave pose as a power system oscillation (moment marked with the letter “f”). One must note that the there are several oscillations present with different frequencies.
IV. Further work

The presented tool is in the initial stage of development. There are several issues that have to be solved and improved in the future. Also, quite a few additional features are expected to be added in the following stages of development. So the plans for the future can be summarized in the following bullets:

- Improve the graphics - the visualization of the system oscillations is the key factor for setting-up the tool in the first place. This is why several different possibilities will be offered to the user.

- Adding reverse-time simulation - one of very interesting and promising ideas for the localization of disturbance which results in
electromechanical oscillations is setting up a simplified model and run the dynamic simulation in the reverse time, being driven by captured PMU measurements.

- **Adding module for testing different algorithms** - several research activities can gain a lot from such a tool, especially in the algorithm testing phase. This is why a testing module is foreseen that would enable one to implement different algorithms in a simple manner.

- **Inclusion of the model into an extensive WAMPAC testing platform** - referring to previous bullet, such a testing environment could be included into larger Wide Area Monitoring, Protection and Control (WAMPAC) testing platform.

- **Adding different kinds of a disturbance** - as was already written in Section 2.3, the generalization of disturbance modelling approach is expected by adding additional possibilities to invoke electromechanical phenomena (for example possibility of performing an outage of an arbitrary element, forced oscillations, etc.).

V. Conclusion

In this paper the tool for in-depth understanding of electromechanical wave’s propagation and inter-generator oscillations in its initial phase of development is presented. The intention of the authors is to enhance the student’s physical perception of electromechanical phenomena taking place in a power system. Quite large amount of work is foreseen for the future, especially building a special graphical representation in a form of animation. Several other generalizations are planned as well and the inclusion of the tool within a larger Wide Area Monitoring, Protection and Control (WAMPAC) testing platform.

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