Multi-connected momentum distribution and fermion condensation

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Abstract. The structure of the ground state beyond the instability point of the quasiparticle system with Fermi-step momentum distribution is studied within the model of a Fermi liquid with a strong repulsive interaction. A ground state rearrangement occurs as the interaction strength is increased beyond a definite critical value. Numerical investigation of the initial stage of this structural transition shows that there are two temperature regions, corresponding to different scenarios of the rearrangement. While for temperature \( T > T_0 \) the behaviour of the system is the same as that in the case of the fermion condensation, for \( T < T_0 \) the intermediate structure with multiconnected quasiparticle momentum distribution arises. The transition of this structure to the fermion condensate at increasing interaction strength is discussed.

1. Introduction

The question of an applicability of the Landau Fermi liquid theory [1] for describing properties of strongly correlated Fermi systems has been discussed since a long time. It is known that this theory is not valid for one dimensional (1D) systems [2]. For such systems, the concept of Luttinger liquid [3] with a single-particle Green function containing no quasiparticle pole is usually introduced, instead of the quasiparticle picture. The frontiers of the non-Fermi liquid view cover also strongly correlated 2D liquids [4, 5], since HTC materials with quasi-2D structure possess properties in contradiction with the expected ones in the Landau theory. However the recently measured electronic spectra of such materials [6, 7, 8] show evidence for the presence of a quasiparticle pole in the single-electron propagator. At the same time, new possibilities were found within the quasiparticle approach in refs. [9–12], where quasiparticles with momentum distributions differing from the ones assumed in the Landau theory are introduced. This new class of systems with the presence of a fermion condensate, as predicted in refs. [11, 12], possesses a rich variety of properties [12–15], some of which are characteristics of a non-Fermi liquid behaviour. As discussed within different models [12–14, 16], a state with the fermion condensate arises as a result of the rearrangement of the ground state of the system. This rearrangement in a system of quasiparticles, whose momentum distribution has the shape of a Fermi sphere with an occupation number slightly smoothed at \( T > 0 \), takes place when some parameters are varied and the pertinent stability condition is violated. In the present work we consider the model of a homogeneous Fermi liquid, which also displays a rearrangement of the ground state of the quasiparticle system, and investigate the scenario of the initial stage of the rearrangement.
2. Theoretical and computational aspects

2.1. Two-connected Fermi sphere and the fermion condensate

Let us start by recalling the Landau relation between the quasiparticle momentum distribution \( n_p(T) \) and the quasiparticle spectrum \( \varepsilon_p(T) \),

\[
n_p(T) = \left\{ 1 + \exp \frac{\varepsilon_p(T) - \mu(T)}{T} \right\}^{-1},
\]

(\( \mu(T) \) is the chemical potential), which results from a variational equation \( \delta F/\delta n_p = 0 \) (\( F \) is the free energy of the system) with the usual expression for the entropy [17].

On one hand, eq. (1) is simply the Fermi-Dirac quasiparticle distribution over energies. On the other hand, this relationship is an equation for the quasiparticle distribution over the phase space. In fact, the quasiparticle energy is the variational derivative of the ground state energy functional \( E_0[n] \) with respect to the quasiparticle distribution, \( \varepsilon_p(T) = \delta E_0/\delta n_p(T) \), and, therefore, it is itself a functional of \( n_p(T) \).

It is postulated in the Landau theory that in a homogeneous isotropic Fermi liquid, like in a Fermi gas, the quasiparticle momentum distribution at \( T=0 \) has the shape of a fully occupied Fermi sphere \( n_F(p) = \theta(p_F - p) \) (the maximum momentum \( p_F \) is related to the density \( \rho \) of the system by the well known relation \( \rho = p_F^3/(3\pi^2) \)). The low-temperature behaviour of the quasiparticle spectrum corresponding to such a momentum distribution has the form [17]:

\[
\varepsilon_p(T) - \mu(T) = \xi(p) + O(T^2).
\]

The function \( \xi(p) \) increases monotonically in the vicinity of the Fermi momentum and changes its sign at \( p=p_F \). Its slope at this point, which is the group velocity of the quasiparticles on the Fermi surface \( v_F = d\xi(p)/dp|_{p=p_F} \), is determined by one of the phenomenological parameters of the Fermi liquid theory, the effective mass \( M^* = p_F/v_F \).

In a strongly correlated Fermi system, a quasiparticle momentum distribution minimizing the energy functional \( E_0[n(p)] \) at \( T=0 \) may be located out of the corner point \( n_F^{(0)}(p) \) of the functional space \( [n] \) and the low-temperature behaviour of the corresponding quasiparticle spectrum may differ from eq. (2). For instance, it was found in ref. [18] that quasiparticle energies which are equal to the chemical potential in a finite region of the momentum space can exist. In refs. [3, 10], some model functionals \( E_0[n(p)] \) were introduced which, for certain values of the parameters, reach their absolute minimum for a momentum distribution characterized by a two-connected Fermi sphere

\[
n_F^{(1)}(p) = \theta(p_1 - p) - \theta(p_2 - p) + \theta(p_3 - p).
\]

A quite different quasiparticle ground state corresponds to systems with fermion condensate [11–14]. Let us elucidate the main idea of the concept of a fermion condensate. A homogeneous and isotropic quasiparticle system with the fermion condensate is described by a singular solution of eq. (1) which corresponds to a quasiparticle spectrum linear in the temperature \( T \) within a finite region of momenta [11, 12]:

\[
\varepsilon_p(T) - \mu(T) = T\nu_0(p) + o(T), \quad p_i < p < p_F.
\]
Contrary to the Fermi liquid formula (2), there is no \( T \)-independent term in eq. (4). This means that at \( T=0 \) the quasiparticle spectrum has a plateau \( \varepsilon_p \equiv \mu \) in the region \( p_i<p<p_f \). At \( T>0 \) the slope of the plateau is linear in \( T \), and its position with respect to the chemical potential \( \mu(T) \) is determined by the function \( \nu_0(p) \) which is connected with the momentum distribution of quasiparticles in the condensate. Indeed, the singular solution of eq. (1), which can be easily obtained upon substitution the formula (4) into eq. (1), has the form \( n_p(T)=n_0(p)+O(T) \), where

\[
n_0(p) = \left\{ 1 + \exp(\nu_0(p)) \right\}^{-1}, \quad p_i<p<p_f
\]  

is the momentum distribution of the condensate quasiparticles at \( T=0 \). Outside the condensate region, \( n_0(p)=1 \) at \( p<p_i \), and \( n_0(p)=0 \) at \( p>p_f \). One can find the explicit form of \( n_p(T) \) and \( \varepsilon_p(T) \) provided one knows the functional dependence \( \varepsilon_p(T)[n_p(T)] \). A set of model functionals was suggested in refs. [12–14, 16], each of them possessing the minimum at the singular solution, within well defined regions of functional parameters. In the present work we indicate the possibility of a scenario, in which the transition to the fermion condensate occurs through an intermediate structure, corresponding to a multiconnected quasiparticle momentum distribution.

2.2. The effective functional

The initial stage of the rearrangement, which is studied in this paper, is characterized by variations of the quasiparticle momentum distribution within a relatively thin layer around \( p\sim p_F \) (see results below). Under these conditions one can use the concept of effective functional, which is widely used in many-body theory. We consider the simple effective quasiparticle functional for the ground state energy:

\[
E_0[n_p(T)] = \int \frac{p^2}{2M} n_p(T) d\tau + \frac{1}{2} \int V(p-p') n_p(T) n_p'(T) d\tau d\tau' 
\]  

with the effective repulsive interaction

\[
V(p-p') = \frac{V_0}{(p-p')^2 + \alpha^2}.
\]

The symbol \( d\tau \) in eq. (6) means integration over \( d^3 p'/(2\pi)^3 \) and summation over spin indices. Calculating the variational derivative of \( E_0 \) with respect to quasiparticle distribution \( n_p(T) \) one obtains the quasiparticle spectrum

\[
\varepsilon_p(T) = \frac{p^2}{2M} + \int V(p-p') n_p'(T) d\tau.
\]

The functional dependence given by eqs.(7), (8) together with eq. (1) and the normalization condition

\[
\int n_p(T) d\tau = \rho
\]

are the set of equations to be solved for the quasiparticle distribution \( n_p(T) \) and the spectrum \( \varepsilon_p(T) \). The value \( \alpha=0.07p_F \) was used in the calculations of the present work and
the behaviour of the system as the parameter \( V_0 \) is varied was studied. The dimensionless parameter \( \gamma = MV_0/(4\pi^2p_F) \) will be used in the discussion below.

2.2. Numerical aspects

Eq. (8) together with the expression (1) represents the nonlinear integral equation for the function \( \varepsilon_p(T) \). This equation was solved numerically by an iterative procedure with weighting factors. The 5-point Newton-Cotes quadrature formula with 5-point filter on output was used for numerical folding the distribution \( n_p(T) \) with the effective interaction \( V(p, p') \). The momentum grid had a step \( h_p = 5 \cdot 10^{-5}p_F \). The accuracy of the numerical solution was determined upon substitution it into the initial equation. The permissible error, that is the maximum discrepancy between the left and the right hand sides of eq. (8), was fixed at \( 10^{-8} \varepsilon_F \). The number of iterations necessary for reaching this accuracy is about \( 3 \cdot 10^4 \) provided the iteration weight \( w = 0.001 \) is taken, which is optimal for a stability of the iterative procedure. It is worth to note that the results are independent of the point in the functional space taken as an initial one for the iterative procedure. For example, the same solution for \( \gamma = 0.50 \) at \( T = 10^{-7} \) was obtained by starting from i) the solution for \( \gamma = 0.50 \) at \( T = 10^{-5} \), ii) the solution for \( \gamma = 0.48 \) at \( T = 10^{-7} \) (here and below the temperature \( T \) is taken in units of \( \varepsilon_0F = p_F^2/2M \)).

3. Results

3.1. Instability of the Fermi-step momentum distribution

We begin from estimating the value \( \gamma_c^{(0)} \) of the interaction strength at which the necessary condition for stability of the system with the quasiparticle distribution \( n_F^{(0)}(p) \) at \( T = 0 \) is violated. This condition is fulfilled \([11, 12]\) provided the variation of the ground state energy \( E_0 \) under any admissible variations of the distribution \( n(p) \),

\[
\delta E_0 = \int \left[ \varepsilon(p)-\mu \right] \delta n(p) \frac{d^3p}{(2\pi)^3},
\]

is positive. Admissible variations of \( n_F^{(0)}(p) \) are of the same sign as the difference \( p-p_F \), as dictated by Pauli principle. Hence, upon substitution of the energy \( \varepsilon(p_F) \) for the chemical potential \( \mu \) in eq. (10) one can reformulate the necessary stability condition as a requirement for the value

\[
s(p) = 2M \frac{\varepsilon(p)-\varepsilon(p_F)}{p^2-p_F^2}
\]

to be positive for each value of the momentum \( p \) \([11, 12]\). Therefore, if the function \( s(p) \) has the first zero close to \( p_F \), the violation of the stability condition means that a bend appears in the curve \( \varepsilon(p) \) in the vicinity of the Fermi momentum. For the distribution \( n_F^{(0)}(p) \) the derivative \( d\varepsilon/dp \) can be easily calculated in the dimensionless form

\[
\zeta^{(0)}(p) = \frac{M}{p_F} \frac{d\varepsilon}{dp} = \frac{p}{p_F} + \frac{\gamma p_F}{p} - \frac{\gamma (p^2+p_F^2+\alpha^2)}{4p^2} \ln \left( \frac{(p+p_F)^2+\alpha^2}{(p-p_F)^2+\alpha^2} \right).
\]

In fig. 1, where the curves \( \zeta^{(0)}(p) \) are shown for different values \( \gamma \), one can see that the contact with zero takes place at the point \( p = p_c \simeq 0.97p_F \) for \( \gamma = \gamma_c^{(0)} \simeq 0.415 \). Note that
the proximity of $p_\gamma$ to $p_F$ justifies the substitution of the function $s(p)$ with the function $\zeta^{(0)}(p)$. Therefore, the ground state with the quasiparticle momentum distribution $n_F^{(0)}(p)$ becomes unstable at $\gamma > \gamma_c^{(0)}$ and its rearrangement takes place.

3.2. Multiconnected momentum distribution

To see how the ground state is arranged right beyond the transition point, let us look at the fig. 2, where the results of the calculations of $n_p(T)$ for $\gamma = 0.45$ at different $T$ are shown. At $T = 2 \cdot 10^{-3}$, the quasiparticle momentum distribution $n_p(T)$ has the shape characteristic of a system with fermion condensate (we shall discuss that in detail below). At $T < 2 \cdot 10^{-3}$ a downfall appears in the distribution, which deepens as temperature decreases, and finally, at $T = 10^{-7}$, one can hardly distinguish $n_p(T)$ from the two-connected Fermi sphere defined in eq. (3). The quasiparticle spectrum $\varepsilon_p(T)$ corresponding to such distribution calculated at $T = 10^{-7}$ is shown in fig. 3. The spectrum of a fermion condensate has a shape with a plateau at a value exactly equal to the chemical potential $\mu$ at $T = 0$ and getting little sloping at $T > 0$ [11, 12]. Here we have a quite different behaviour, and the quasiparticle spectrum of the two-connected Fermi-like distribution equals $\mu$ at three points, which are the border $p_1$ of the inner sphere and the borders $p_2$ and $p_3$ of the spherical layer. The deviation of $\varepsilon_p(T)$ from $\mu$ reaches the value $\sim 2 \cdot 10^{-4} \varepsilon_F$ at the point of the minimum of the spectrum and the value $\sim 2 \cdot 10^{-6} \varepsilon_F$ at the point of its maximum. Despite the latter value is small, it is still higher than the estimated numerical error by two orders of magnitude.

The behaviour of the two-connected Fermi sphere with increasing interaction strength $\gamma$ is displayed in fig. 4. The spherical layer appearing beyond the transition point has, from the start, a finite thickness, while the gap between the layer and the inner sphere develops starting from a vanishing small width. The outer layer gets thicker and moves away from the inner sphere as the parameter $\gamma$ increases. What then happens with such a distribution? To understand that, let us investigate the stability of such Fermi sphere divided into two layers. We calculate the function $\zeta^{(1)}(p)$ for the momentum distribution in the form of eq. (3) and see when and where the critical change of the sign of that function occurs. An elementary calculation gives

$$\zeta^{(1)}(p) = \frac{M}{p_F} \frac{d\varepsilon}{dp} = \frac{p}{p_F} + \sum_{i=1}^{3} (-1)^{i-1} \left\{ \frac{\gamma p_i}{p} - \frac{\gamma (p^2 + p_i^2 + \alpha^2)}{4p^2} \ln \left( \frac{(p+p_i)^2 + \alpha^2}{(p-p_i)^2 + \alpha^2} \right) \right\}. \quad (13)$$

The function $\zeta^{(1)}(p)$ calculated for different values of $\gamma$ is displayed in fig. 4. The points of the maximum absolute values of the derivative $dn/dp$ at $T = 10^{-7}$ were taken as the boundary momenta $p_1, p_2, p_3$ for that calculation. One can easily realize that inside the region $\gamma_c^{(0)} < \gamma < \gamma_c^{(1)} \simeq 0.452$, the two points where $\zeta^{(1)}(p)$ changes sign are located in such a way that the corresponding local minimum and maximum of $\varepsilon(p)$ lie in the domains where $n(p)$ equals 0 and 1 respectively. This means that the sign of the difference $\varepsilon(p) - \mu$ coincides with that of the possible variations $\delta n(p)$ allowed by Pauli principle. Hence the considered distribution satisfies the stability condition.

However one can see in fig. 4 that at $\gamma > \gamma_c^{(1)} \simeq 0.452$, the situation changes. With the displayed behaviour of the function $\zeta^{(1)}(p)$, there are regions where $\varepsilon(p) - \mu > 0$, but with $n(p) = 1$. This means that the necessary stability condition is violated, since allowed
variations $\delta n(p)$ exist which lower the ground state energy. This results in the second rearrangement of the ground state of the system emerging at $\gamma=\gamma_c^{(1)}$. It is shown in fig. 5 how the quasiparticle distribution is arranged beyond the second transition point $\gamma_c^{(1)}$. For definitness, the results of the calculations for $\gamma=0.46$ at different $T$ are displayed. The calculations show that the new layer appears for the above mentioned value of the coupling constant and the quasiparticle distribution $n_p(T)$ at $T=10^{-7}$ is very close to the three-connected Fermi sphere

$$n_F^{(2)}(p) = \theta(p_1 - p) - \theta(p_2 - p) + \theta(p_3 - p) - \theta(p_4 - p) + \theta(p_5 - p).$$

(14)

The calculation shows that the appearance of new layers at increasing values of the parameter $\gamma$ does not stop at the level of three-connected Fermi sphere. Figs. 6 and 7, where the quasiparticle distributions $n_p(T)$ are calculated for $\gamma = 0.48$ and 0.50, show further divisions into layers of the momentum space. In particular, the distribution $n_p(T)$ for $\gamma=0.50$ approaches a multi-connected Fermi sphere with lowering $T$, which is arranged as follows: the inner small occupied sphere with the radius $\sim 0.85p_F$ is surrounded by four spherical occupied layers with thickness $\sim 0.3 - 0.4p_F$ divided by spherical empty layers with thickness $\sim 0.1 - 0.2p_F$. The quasiparticle spectra $\varepsilon_p(T)$ for $\gamma=0.50$ are shown in fig. 8. At $T=10^{-7}$, the spectrum crosses the line $\varepsilon=\mu$ nine times, at the boundary of the inner sphere and at the boundaries of the spherical layers.

4. Discussion

4.1. Two scenarios of the rearrangement

Thus beyond the first transition point $\gamma_c^{(0)}$, the scenario of the rearrangement of the quasiparticle ground state at low $T<10^{-3}$ is the succession of transitions with increasing $\gamma$. Each one of them results in the appearance of a new spherical layer in momentum space. Let us compare this scenario of the rearrangement and some distinctive features of the ground state under consideration with those of the fermion condensation.

Let us recall the main features of the fermion condensation phenomenon. One of them is the plateau in the quasiparticle spectrum $\varepsilon_p(T)$, with a value exactly equal to the chemical potential $\mu$ at $T=0$, in accordance with eq. (4), and which gets a finite slope at increasing temperature. Unlike systems with the fermion condensate, the spectrum $\varepsilon_p(T=0)$ for the multi-connected Fermi-like distribution equals the chemical potential in a finite number of points, which are the boundaries of the spherical layers. The low-temperature expansion of such a spectrum has the form of eq. (2), typical of the Landau theory, with the non-monotone function $\xi(p)$ changing its sign several times, unlike the monotone one for the usual Fermi liquid. At $T=0$, the ground state with the multi-connected Fermi-like quasiparticle distribution is not macroscopically degenerated, unlike the ground state of a fermion condensate. At the same time, there exist singularities of the density of states connected with the maxima and the minima of the function $\varepsilon_p(T=0)$. These singularities gradually disappears with increasing $T$ up to $T_0 \sim 2 \cdot 10^{-3}$, where the last twist of the spectrum corresponding to the outer layer is smoothed. At $T>T_0$ the difference $\varepsilon_p(T)-\mu(T)$ becomes linear in temperature, like that for systems with a fermion condensate.
The other feature of systems with fermion condensate is the shape of the distribution \( n(p) \) given by eq. (5). Within the region occupied by the fermion condensate \( 0 < n(p) < 1 \), this corresponds to non-zero entropy of the fermion condensate at \( T=0 \). The contradiction with the Nernst theorem disappears providing correlations (e.g. superfluid) are taken into account, which immediately rearrange the ground state due to its degeneracy and re-establishes a zero entropy at \( T=0 \). The entropy of the state with the multi-connected Fermi-like quasiparticle momentum distribution equals zero at zero temperature because \( n(p) \) takes only the values 0 and 1. The multi-layered distribution changes quickly with increasing \( T \): the sharp boundaries of the layers are smoothed and the layers combine all together at \( T=T_0 \) into a monotonically decreasing curve which is very similar to the momentum distribution of a system with the fermion condensate at that temperature. Being equal zero at \( T=0 \), the entropy of the system with the multi-connected Fermi-like distribution increases sharply with temperature due to quick smoothing of the function \( n_p(T) \). The calculation show that at \( T=T_0 \) the entropy reaches the value \( S_0 \sim \Omega_0/\Omega \), the ratio of the phase volume of multi-layered region \( \Omega_0 \) to that of the whole system \( \Omega \). Just this value of the entropy would be characteristic at \( T \sim 0 \) for a system with the fermion condensate occupying the phase volume \( \Omega_0 \). At \( T>T_0 \) the entropy becomes linear in temperature, like that of a system with a fermion condensate [12, 16].

All these features of the momentum distribution, entropy, quasiparticle spectrum and density of states for a system with a multi-connected Fermi-like quasiparticle distribution, as well as the problem of validity of such a quasiparticle pattern, will be studied in detail in a separate paper.

The scenario of the fermion condensation is characterized by the single critical value \( \gamma_c \) of the coupling constant, at which the fermion condensate arises in the system. The phase volume of the fermion condensate increases as \( \gamma \) further increases, however the shape of the momentum distribution does not modify and no further qualitative changes occur [11, 12]. The scenario of the rearrangement found for the model under consideration is different for different temperatures. At \( T=0 \) it is characterized by a sequence of critical values \( \gamma_c(i) \), each one corresponding to the appearance of a new ground state with a larger number of connectivity. The number of critical constants decreases as \( T \) increases, so that only one of them, \( \gamma_c(0) \), survives at \( T>T_0 \). This means that for the considered model, the scenario of formation of the multi-connected Fermi sphere at \( T=0 \) gradually transforms into that of the fermion condensation with increasing temperature.

### 4.2. Interplay between the multiconnected distribution and the fermion condensation

Unfortunately, the computation time sharply increases with increasing the phase volume \( \Omega_0 \) of the layered distribution. This is the reason why the calculations of the present work are carried out up to the value of the interaction strength \( \gamma \leq 0.5 \), corresponding to the initial stage of the rearrangement. What happens in the system at larger values of \( \gamma \)? To have a feeling of that, let us try to use the mechanical analogy, treating the momentum \( p \) as a spatial coordinate [11, 12]. Then the problem of minimizing the ground state energy functional (6) at \( T=0 \) can be interpreted in the language of mechanics as a search for the statical equilibrium of the spatial distribution \( \nu(r) \) of particles, which move in the external harmonic field \( U(r) = kr^2/2 \) with the stiffness \( k=1/M \) and interact among each others by the repulsive force (7), the particle number being fixed by the
normalization condition (9). Unless the solution $\nu(r)$ of the mechanical problem exceeds $2/(2\pi)^3$, at least in one point, it can not be accepted as a solution $n(p)$ because the latter should satisfy the additional restriction due to the Pauli principle. With strengthening interparticle repulsion, the considered mechanical system, obviously, expands and rarefies. As soon as the distribution $\nu(r)$ is everywhere less than $2/(2\pi)^3$, one can conclude that it corresponds to the solution $n(p)$ of the initial quantum problem. Since one can expect that the distribution $\nu(r)$ should be smoothed and monotone, the fermion condensate seems to appear in the system at $T=0$ for large values of the interaction strength $\gamma$. This transition will be investigated in further publications.

5. Conclusion

In summary, the structure of the ground state of the homogeneous Fermi liquid with the strong interparticle repulsion is studied within the framework of the effective functional approach. The numerical investigation of the simple functional with strong repulsive effective interaction showed that at fixed value of the interaction radius there exist the critical value of the interaction strength $\gamma_c^{(0)}$, beyond which the ground state with the Fermi-step quasiparticle momentum distribution becomes unstable and the rearrangement of the ground state takes place. The scenario of the initial stage of that rearrangement with increasing $\gamma$ was found to be different for different regions of temperature. At $T=0$, there exist the set of critical constants $\gamma_c^{(i)}$ corresponding to the succession of transitions, each resulting in emerging the new spherical layer of the quasiparticle momentum distribution $n(p))$. The quasiparticle spectrum $\varepsilon(p)$ corresponding to such the layered distribution does not have the plateau, unlike the fermion condensate, and equals the chemical potential in the finite number of points, which are the boundaries of the layers. The ground state with the multi-connected Fermi-like distribution possesses no macroscopic degeneracy and the entropy of this state is zero at zero temperature. With increasing temperature the layers are quickly smoothed, so that at $T\sim T_0\sim 2\cdot10^{-3}$, there is no more reminiscence of the critical constants $\gamma_c^{(i)}$ with the exception of $\gamma_c^{(0)}$. At $T>T_0$ the scenario of the rearrangement is that of the fermion condensation. The qualitative analysis showed that the found structure with the multi-connected quasiparticle momentum distribution is the intermediate one, yielding the place for the fermion condensate with increasing the interaction strength.

Acknowledgments

We are indebted to V A Khodel for the unceasing interest for the present research and countless fruitful discussions as well as S A Artamonov, A E Bulatov, E E Saperstein, V R Shaginyan and S V Tolokonnikov for valuable discussions. This research was partially supported by Grant No. 96-02-19293 from the Russian Foundation for Basic Research. M V Z would like to thank INFN (Catania, Italy), where the main part of the work was done, for the kind hospitality.
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Figure captions

1. The function $\zeta(0)(p)$ calculated for $\gamma = 0.410, 0.415, 0.420$.

2. The quasiparticle momentum distributions $n_p(T)$ calculated for $\gamma = 0.45$ at different temperatures.

3. The quasiparticle spectrum $(\varepsilon_p(T) - \mu)/\varepsilon_p^0$ calculated for $\gamma = 0.45$ at $T = 10^{-7}$.

4. The quasiparticle momentum distributions $n(p)$ and the function $\zeta^{(1)}(p)$ calculated for different values of the parameter $\gamma$.

5. The same as for Fig. 2, for $\gamma = 0.46$.

6. The same as for Fig. 2, for $\gamma = 0.48$.

7. The same as for Fig. 2, for $\gamma = 0.50$.

8. The quasiparticle spectra $(\varepsilon_p(T) - \mu)/\varepsilon_p^0$ calculated for $\gamma = 0.50$ at $T = 10^{-4}$ (solid line), $T = 10^{-5}$ (long dashes), $T = 10^{-6}$ (short dashes), and $T = 10^{-7}$ (dots).
Fig. 1

\[ \zeta^{(0)}(p) \]

\[ 0.415 \]

\[ 0.420 \]

\[ p/p_F \]
Fig. 3

\[ \frac{(\varepsilon(p) - \mu)}{\varepsilon_F^0} \times 10^4 \]
Fig. 4
Fig. 5
Fig. 6
Fig. 7
\[ \frac{(\varepsilon(p) - \mu)}{\varepsilon_F^0} \times 10^4 \]