Abstract

Using the Non-Abelian Batalin-Vilkovisky formalism introduced recently, we present a generalization of the Yang-Mills gauge transformations, to include antisymmetric tensor fields as gauge bosons. The Freedman-Townsend transformation for the two-form gauge field is automatically recovered. New characteristic classes involving this two-form field and the Yang-Mills one-form field are derived. We also show how to include, in an unified way, a gauge invariant coupling of the new gauge bosons to fermionic and bosonic matter.
1 Introduction

Yang-Mills gauge theories have provided the building principle for all the known interactions of nature. They enable the unified description of the weak and electromagnetic interactions as well as the strong interactions, guaranteeing the renormalizability and unitarity of the theory.

Gravity is a gauge theory of a different kind. This difference is precisely the main reason of why we do not have a theory of Quantum Gravity: In Yang-Mills gauge theories, the gauge symmetry at our disposal enable the formulation of unitary and renormalizable models. The gauge symmetry of General Relativity, on the other hand, is not enough to have renormalizability.

The most promising road to quantize gravity is provided by String Theory. This is so because in string theories the amount of gauge symmetries increases enormously. So much that the model is not only renormalizable, but finite.

It is hard to get physical information from string theories. Partially this is due to the way the theory has been studied up to now. The First Quantization do not exhibit the symmetries of the interacting theory explicitly and so part of the beauty is hidden. In fact the Second Quantized version of the model do exhibit an enormous symmetry. However the known formulations are still background dependent and explicitly refer to the first quantized version. Most of the powerful methods of physical analysis available in conventional Quantum Field Theory (instantons, background field method) require a background independent formulation of the theory which exhibit explicitly the whole symmetry involved.

For this reason it would be desirable to have a principle to built String Field Theories directly at the Second Quantize level. Such a principle will certainly be a much enlarged gauge symmetry principle encompassing Yang-Mills symmetries, invariance under general coordinate transformations and many other yet unknown symmetries.

One of the motivations of this work is to explore new kinds of gauge symmetries based in the algebraic structure built in the Batalin-Vilkovisky(BV) quantization method.

In a series of papers we have been uncovering a rich algebraic structure that generalizes the Batalin-Vilkovisky method of quantization. New nilpotent operators and generalized antibrackets emerge. This induces, in a standard manner, a master equation and a local symmetry transformation. The structure is reminiscent of the one discussed by Sen and Zwiebach in the context of String Field Theory.

As we mentioned above, it is interesting to consider extensions of the Yang-Mills gauge principle because these new local symmetries may be a part of the Gauge Principle of String Field Theory. From the physical point of view, the new gauge fields involved in these generalizations may provide corrections to the Standard Model predictions which reflect, at low energies, the existence of a string at high energies.

Presently, we want to exhibit an explicit realization of this algebraic scheme to provide unification of interacting higher forms gauge fields with values on a Lie algebra.

As a concrete example of the formalism, we construct Characteristic Classes involving a two form non-abelian gauge field. The transformation of this field automatically coincides with the Freedman-Townsend transformation.

Finally, we explain how to include matter coupled in a gauge invariant way to these higher form gauge fields.

2 A review of the Non-abelian BV structure

The conventional antibracket of the Batalin-Vilkovisky formalism can be viewed as being based on a 2nd-order odd differential operator $\Delta$ satisfying $\Delta^2 = 0$. In (super) Darboux coordinates it takes the
\[\Delta = (-1)^{\epsilon_A+1} \frac{\delta^r}{\delta \phi^A} \frac{\delta^r}{\delta \phi_A^*}; \tag{1}\]

where to each field \(\phi^A\) one has a matching “antifield” \(\phi_A^*\) of Grassmann parity \(\epsilon(\phi_A^*) = \epsilon(\phi^A) + 1\). The antifields are conventional antighosts of the Abelian shift symmetry that for flat functional measures leads to the most general Schwinger-Dyson equations \[\tag{3}\].

Given \(\Delta\) as above, one can define an odd (statistics-changing) antibracket \((F,G)\) from the failure of \(\Delta\) to act like a derivation:

\[\Delta(FG) = F(\Delta G) + (-1)^{\epsilon_G}(\Delta F)G + (-1)^{\epsilon_G}(F, G). \tag{2}\]

The antibracket so defined automatically satisfies the following relations. First, it has an exchange symmetry of the kind

\[(F,G) = (-1)^{\epsilon_F \epsilon_G + \epsilon_F + \epsilon_G}(G,F). \tag{3}\]

It also acts like a derivation in the sense of a generalized Leibniz rule:

\[\Delta(FG,H) = (F,G)H + (-1)^{\epsilon_G(\epsilon_F+1)}G(F,H)\]
\[\Delta(F,G,H) = F(G,H) + (-1)^{\epsilon_F(\epsilon_H+1)}(F,H)G, \tag{4}\]

and it satisfies a Jacobi identity,

\[\sum \text{cycl.} (-1)^{(\epsilon_F+1)(\epsilon_H+1)}(F, (G,H)) = 0. \tag{5}\]

In addition, there is a useful relation between the \(\Delta\) operator and its associated antibracket:

\[\Delta(F,G) = (F, \Delta G) - (-1)^{\epsilon_G}(\Delta F, G). \tag{6}\]

Recently \[\tag{8}\] it was shown that the antibracket formalism is open to a natural generalization. In a path-integral formulation, this generalization can be derived by considering general field transformations \(\phi^A \to g^A(\phi', a)\), where \(a\) represent certain collective fields \[\tag{8}\]. The idea is to impose on the Lagrangian path integral the condition that certain Ward identities are preserved throughout the quantization procedure. If one imposes the most general set of Ward identities possible – the Schwinger-Dyson equations – through an unbroken Schwinger-Dyson BRST symmetry \[\tag{8}\], one can recover the antibracket formalism of Batalin and Vilkovisky by integrating out certain ghosts \(c^A\) (the antifields \(\phi_A^*\) being simply the antighosts corresponding to \(c^A\)). For flat functional measures this corresponds to local shift transformations of the fields \(\phi^A\). If the measure is not flat, or if one wishes to impose a more restricted set of Ward identities through the BRST symmetry, the \(\Delta\) operator and the associated antibracket will differ from those of the conventional Batalin-Vilkovisky formalism. In ref. \[\tag{8}\] it was shown how the Batalin-Vilkovisky \(\Delta\)-operator (1) can be viewed as an Abelian operator corresponding to the Abelian shift transformation \(\phi^A \to \phi^A - a^A\). The analogous non-Abelian \(\Delta\)-operator for general transformations \(\phi^A \to g^A(\phi', a)\) was derived in ref. \[\tag{8}\]:

\[\Delta G \equiv (-1)^{\epsilon_i} \left[ \frac{\delta^r}{\delta \phi^A} \frac{\delta^r}{\delta \phi_i^*} G \right] u^i_A + \frac{1}{2} (-1)^{\epsilon_i+1} \left[ \frac{\delta^r}{\delta \phi_j^*} \frac{\delta^r}{\delta \phi_i^*} G \right] \phi_k^* U_{ji}^k, \tag{7}\]

where the \(U_{ji}^k\) are the structure coefficients for the supergroup of transformations. They are related to the field transformations \(g^A(\phi', a)\) by the relation

\[\frac{\delta^r u^A_i}{\delta \phi^B} u^B_j - (-1)^{\epsilon_i \epsilon_j} \frac{\delta^r u^A_i}{\delta \phi^B} u^B_j = -u^A_i U_{ij}^k, \tag{8}\]

\(^1\text{Taking for convenience that the supergroup is semi-simple, with } (-1)^{\epsilon_i} U_{ij}^k = 0.\)
where
\[ u^A_i(\phi) = \frac{\delta^r g^A(\phi, a)}{\delta a^i} \bigg|_{a=0}. \]

The $\Delta$-operator of eq. (7) can be shown to be nilpotent \[7\], and it gives rise to a new non-Abelian antibracket by use of the relation (2). Explicitly, this antibracket takes the form \[7\]
\[
(F, G) \equiv (-1)^{\epsilon_1(\epsilon_A+1)} \frac{\delta F}{\delta \phi_i^*} u^A_i \frac{\delta G}{\delta \phi^A} - \frac{\delta F}{\delta \phi^A} u^A_i \frac{\delta G}{\delta \phi_i^*} + \frac{\delta F}{\delta \phi_i^*} \phi^k U^k_j \frac{\delta G}{\delta \phi^*_j}. \tag{10}
\]

In ref. \[7\] this non-Abelian antibracket was derived directly in the path integral (by integrating out the ghosts $c^A$), but it can readily be checked that it is related to the associated $\Delta$-operator (7) in the manner expected from (2). Because this particular non-Abelian $\Delta$-operator is of 2nd order, the corresponding antibracket automatically satisfies all the properties (3-6).

From the $\Delta$ operator we can derive the following master equation:
\[
\Delta e^S = 0 = \Delta S + \frac{1}{2}(S, S), \epsilon(S) = 0 \tag{11}
\]
which is invariant under the following transformation:
\[
\delta S = \Delta \epsilon + (\epsilon, S) \tag{12}
\]
where $\epsilon$ has ghost number equal to $-1$.

In the next section we will show a connection of this formalism to the usual representation of gauge theories in term of forms.

### 3 Connection to Gauge Theories

It has already been noticed by Witten\[14\] that the transformation that leaves the master equation invariant is similar to the Yang-Mills transformations written in term of forms. In this context the master equation itself can be thought as a sort of "field strength". In this section we will show that this analogy can be very concrete indeed, providing us with an obvious way to generalize Yang-Mills theories.

From now on what we called $S$ in the previous sections will be denoted by $A$. This is the "BV gauge field". It has $\epsilon(A) = 0$ and will be, in general, a function of the space coordinates $x^\mu$ and certain "internal variables" which can be Grassmann even ($y^a$) or odd variables($y^*_a$). Its exact nature will be described below. The "BV gauge field" and some other fields that will be introduced soon have to be thought as "superfields" and so can be expanded in a Taylor series in the internal variables.

Let us introduce the "field strength" $F$;
\[
F = \Delta A + \frac{1}{2}(A, A), \tag{13}
\]
Consider the transformation that leaves $F = 0$ invariant:
\[
\delta A = \Delta \epsilon + (\epsilon, A) \tag{14}
\]
\[
\epsilon(\epsilon) = -1 \tag{15}
\]
These transformations form a closed algebra:
\[
[\delta_\alpha, \delta_\beta] = \delta_{(\beta, \alpha)} \tag{16}
\]
It is easy to show that:
\[ \delta F = (\epsilon, F), \]  
(17)

Now, notice the crucial point that all these nice relations, equations (13-17), are based in only three properties:

1) \( \Delta^2 = 0 \),

2) \( \Delta \) is a second order differential operator, both in the space time variables and in the internal variables and

3) \( \epsilon(A) = 0 \) mod 2.

1) and 2) implies that the antibracket derived from \( \Delta \) using equation (2) will satisfy the Jacobi identity equation (5).

To make contact with the Yang-Mills field we choose the following form of \( \Delta \), which will acts on functions of \( x^\mu \) (space-time coordinates), \( \theta^\mu \) (an anticommuting variable), \( y^A \) and \( y^*_a \) (internal coordinates):
\[
\Delta_{NAG} \equiv (-1)^i \left[ \frac{\delta^r}{\delta y^A} \frac{\delta^r}{\delta y^*_i} G \right] u^A_i + \frac{1}{2} (-1)^{i+1} \left[ \frac{\delta^r}{\delta y^*_j} \frac{\delta^r}{\delta y^*_i} G \right] y^*_j f^k_{ij}, \]
(18)

\[
\epsilon(y^A) = 0 \quad \epsilon(y^*_a) = -1
\]
(20)

\( \partial_\mu \) means derivation with respect to \( x^\mu \).

\( u^A_i(y) \) defines the infinitesimal transformation of the internal bosonic variables \( y^A \):
\[
\delta y^A = u^A_i(y) \lambda^i
\]
(21)

\( \lambda \) is an infinitesimal parameter. This transformation can be non-linear.

Here \( \theta(x^\mu) \) anticommutate (commute) with the Grassman odd (even) variables in the internal sector. This guarantees that \( \Delta^2 = 0 \). It can be readily seen that \( \Delta \) is a second order differential operator. Notice that \( \theta^\mu \partial_\mu \) must acts as a right derivative.

From this nilpotent operator, we derive the following antibracket:
\[
(F, G) \equiv (-1)^{i_1} \frac{\delta^r}{\delta y^A} u^A_i \frac{\delta^r}{\delta y^*_i} G - \frac{\delta^r}{\delta y^*_j} u^A_i \frac{\delta^r}{\delta y^*_i} G + \frac{\delta^r}{\delta y^*_k} y^*_j f^k_{ij} \delta^r G
\]
(22)

Now, let us make explicit the connection with the Yang-Mills field. Choose:
\[
A = A^{a}_\mu(x)y^a_\theta^\mu
\]
(23)

Since we must have \( \epsilon(A) = 0 \), we assign \( \epsilon(\theta) = 1 \) and \( \epsilon(y^*_a) = -1 \). Then we get, from (13-17):
\[
\delta A^{a}_\mu(x) = \partial_\mu \epsilon^a + f^{ab}_c \epsilon^c A^{d}_\mu
\]
(24)

\[
F = (\partial_\mu A^{a}_\nu - \partial_\nu A^{a}_\mu - f^{ab}_c A^{c}_\mu A^{d}_\nu) y^*_a \theta^\nu \theta^\mu
\]
(25)

\[
= F^{a}_{\mu\nu} y^*_a \theta^\nu \theta^\mu
\]
(26)

which are the transformation and the field strength of the Yang-Mills field.

We see that for a very particular \( A \) satisfying the condition \( \epsilon(A) = 0 \), we already get the structure of the Yang-Mills field. But this condition can be met by many more monomials in the \( \theta, y^A, y^*_a \) variables, providing us with generalizations of the Yang-Mills symmetry principle, through equations (13-17).

Before presenting some of these generalizations, let us prove a very useful identity, corresponding to the Bianchi identity in the Yang-Mills case.
4 Bianchi Identity

Let us write $F$ in the following form:

$$F = e^{-A} \Delta e^A$$  \hspace{1cm} (27)

We get the following identity, using the nilpotency of $\Delta$:

$$\Delta (Fe^A) = 0$$  \hspace{1cm} (28)

That is:

$$\Delta F + (F, A) = 0$$  \hspace{1cm} (29)

This is the Bianchi identity.

5 Enhanced Gauge Symmetry

Since the gauge transformations defined above form a closed algebra, they open the road to build gauge invariant Lagrangians involving interacting antisymmetric tensors of arbitrary order.

In this letter, we will consider in detail just the simplest generalization of Yang-Mills provided by the Non-Abelian BV formalism. It will be clear how to proceed in more complex situations.

Let us introduce the gauge field:

$$A = \phi + A^a_\mu y^*_a + \frac{1}{2} A^{ab}_\mu \theta_\mu y^*_a y^*_b$$  \hspace{1cm} (30)

We can compute the field strength $F$:

$$F = F^\mu_{\nu} + \bar{F}^a_{\mu \nu} y^*_a \theta_\nu + \frac{1}{2} F^{ab}_{\mu \nu \lambda} y^*_a y^*_b \theta_\nu \theta_\lambda$$  \hspace{1cm} (31)

$$F^\mu_{\nu} = \phi^\mu_{\nu}$$  \hspace{1cm} (32)

$$\bar{F}^a_{\mu \nu} = F^a_{\mu \nu} + f^a_{ji} A^{ij}_{\mu \nu}$$  \hspace{1cm} (33)

$$F^{ab}_{\mu \nu \lambda} = (A^{ij}_{\mu \nu \lambda} + f^b_{ji} A^{ij}_{\mu \nu \lambda})_{ab, \mu \nu \lambda}$$  \hspace{1cm} (34)

()$_{xy}$ means antisymmetrization with respect to the indices $xy$.

Here $F^a_{\mu \nu}$ is the usual Yang-Mills field strength.

The gauge parameter is:

$$\epsilon = \epsilon^a(x) y^*_a + \frac{1}{2} \epsilon^{ab}_{\mu \nu} y^*_a y^*_b \theta^\mu$$  \hspace{1cm} (35)

We get the following gauge transformations of the component fields:

$$\delta \phi = 0$$  \hspace{1cm} (36)

$$\delta A^a_{\mu}(x) = \partial_{\mu} \epsilon^a(x) + \epsilon^i(x) f^{a}_{ij} A^j_{\mu}(x) + \frac{1}{2} \epsilon_{\mu \nu}^{ij}(x) f^{a}_{ji}$$  \hspace{1cm} (37)

$$\delta A^{ab}_{\mu \nu} = \frac{1}{2} (\epsilon^{ab}_{\mu \nu} - \epsilon^{ab}_{\nu \mu}) - (f^a_{ji} \epsilon^i A^{jb}_{\mu \nu})_{ab} + (f^a_{ji} \epsilon^i A^{jb}_{\mu \nu})_{ab, \mu \nu}$$  \hspace{1cm} (38)

Also, we obtain:

$$\delta F^a_{\mu} = 0$$  \hspace{1cm} (39)

$$\delta \bar{F}^a_{\mu \nu} = -f^a_{ji} \epsilon^j \bar{F}^{ij}_{\mu \nu}$$  \hspace{1cm} (40)

$$\delta F^{ab}_{\mu \nu \lambda} = (f^a_{kim} \epsilon^i \bar{F}^{ab}_{\mu \nu \lambda})_{k \nu} + f^b_{ji} \epsilon^j \bar{F}^{ab}_{\mu \nu \lambda}$$  \hspace{1cm} (41)

\footnote{The condition $\epsilon^a(A) = 0$ permits also to incorporate additional terms, some of them containing odd monomials in $y^*_a$ and $\theta$, whose coefficients will be fermionic fields. We will not discuss such fields here.}
It is easy to check that the following action is gauge invariant in four dimensions:

$$S = \int d^4x \{ a_1 \tilde{F}_\mu^i \tilde{F}_{\mu i} + a_2 \tilde{F}_\mu^i \tilde{F}_{\mu i} + a_3 A_{\mu
u}^{ab} \epsilon^{\mu\nu\rho\lambda} \tilde{F}_i^{\rho i} f_{iab} \}$$  \hspace{1cm} (42)

\(\tilde{F}_\mu^i = \epsilon_{\mu\nu\lambda\rho} \tilde{F}_\nu^i\) is the dual of \(\tilde{F}_\mu^i\).

To prove the invariance of the last term, we have to use the Bianchi identity.

\(B_{\mu\nu}^i = A_{\mu
u}^{ab} f_{iab}\) transforms exactly as the Freedman-Townsend two-form does \([11]\) and when \(\epsilon^a(x) = 0\), the last term of our Lagrangian is similar but not equal to the Freedman-Townsend action for the antisymmetric tensor field. However, this is not in contradiction with reference \([12]\) because even when \(\epsilon^a(x) = 0\), the Yang-Mills field transforms, according to equation (37).

### 6 Characteristic Classes

It is useful to introduce the following notation:

$$F_2 = \tilde{F}_\mu^a t^a \theta^\mu \theta^\nu$$  \hspace{1cm} (43)

$$F_3 = F_{\mu
u\lambda}^{ab} [t_a, t_b] \theta^\mu \theta^\nu \theta^\lambda$$  \hspace{1cm} (44)

\(t_a\) are the generators of the Lie algebra of the gauge group \(G\), satisfying:

$$[t_a, t_b] = if_{ab}^c t_c$$  \hspace{1cm} (45)

In terms of these variables the transformation of the field strengths are the following:

$$\delta F_2 = [\alpha, F_2]$$  \hspace{1cm} (46)

$$\delta F_3 = [\alpha, F_3] + [\alpha_1, F_2]$$  \hspace{1cm} (47)

$$\alpha = \alpha^a t_a$$

$$\alpha_1 = \alpha_{a\mu}^b [t_a, t_b] \theta^\mu$$  \hspace{1cm} (49)

We readily check that the following object is gauge invariant:

$$C_k = \int d^{2k+3}x d^{2k+3} \theta \mathrm{tr} F_2^k F_3$$  \hspace{1cm} (50)

Moreover, using the Bianchi identity:

$$dF_2 - \frac{1}{4} F_3 + [F_2, A_1] = 0$$  \hspace{1cm} (51)

$$A_1 = A_{\mu a}^a t_a \theta^\mu$$

$$d = \theta^\mu \partial_\mu$$  \hspace{1cm} (53)

we get:

$$d \mathrm{tr} F_2^k = \frac{k}{4} \mathrm{tr} F_2^{k-1} F_3$$  \hspace{1cm} (54)

This implies that \(C_k\) is a closed form. This is the analog of the Chern Class in \(d=5,7,9,..\) dimensions. On the other hand:

$$CS_k = \int d^{2k}x d^{2k} \theta \mathrm{tr} F_2^k$$  \hspace{1cm} (55)

is also gauge invariant, giving the analog of the Chern-Simons class.
7 Matter Fields

To introduce matter fields in the formalism we borrow the transformation rule for the field strength $F$:

$$\delta F = (\epsilon, F)$$  \hspace{1cm} (56)

It also forms a representation of the same closed algebra:

$$[\delta_\alpha, \delta_\beta] = \delta_{(\beta,\alpha)}$$  \hspace{1cm} (57)

This can be proven using the Jacobi identity.

From now on we will call "BV matter field" a function(superfield) $\phi$ of the space-time and internal coordinates whose transformation rule is:

$$\delta \phi = (\epsilon, \phi), \ \epsilon(\epsilon) = -1$$  \hspace{1cm} (58)

$\epsilon(\phi)$ is arbitrary.

To built gauge invariant Lagrangians, involving BV matter fields, we need the concept of covariant derivative.

In the present formalism this is done using what is called in the BV formalism the "quantum BRST transformation"

$$DG = \Delta G + (G, A)$$  \hspace{1cm} (59)

Indeed, under a gauge transformation $D\phi$ transforms as $\phi$ does:

$$\delta A = \Delta \epsilon + (\epsilon, A)$$

$$\delta \phi = (\epsilon, \phi)$$

implies

$$\delta(D\phi) = (\epsilon, D\phi)$$  \hspace{1cm} (62)

which is the fundamental property of the covariant derivative.

We also get:

$$D^2\phi = (\phi, F)$$  \hspace{1cm} (63)

So if $F$ vanishes $D$ is nilpotent.

We will say that $\Psi(x, y, \theta, y^*)$ is fermionic (bosonic) if $\epsilon(\Psi(x, y, \theta, y^*))$ is 1(0), mod 2.

To make contact with the standard formulation of matter fields we consider the BV matter field,

$$\Psi(x, y, \theta, y^*) = \psi_A(x)y^A$$  \hspace{1cm} (64)

as an example.

Then we get using the equation (65),

$$\delta \psi_A = \epsilon^i \frac{\partial u_i^B}{\partial y^A} |_{y^A=0} \psi_B(x)$$

which coincides with the gauge transformation of the matter field $\psi$ which belongs to the representation of the gauge group defined by $u$. For linear representations of the gauge group $u_i^A$ is given by:

$$\delta y^A = \lambda^i y^B T^i_{BA}$$

$$u_i^A = y^B T^i_{BA}$$

$\lambda^i$ is an infinitesimal parameter and $T^i_{AB}$ are the Lie algebra generators.
Notice that the representation expanded by $u_i^A$ can be reducible, so this permits to incorporate various fields transforming differently under the gauge group.

The Higgs field can be incorporated as in the last equation (starting with a scalar BV matter field and taking $u_i^A$ in the appropriate representation of $G$) or if it belongs into the adjoint representation of the group, we can "unify" a fermion (in the $u_i^A$ representation of $G$) and a scalar expanding the adjoint representation of $G$, choosing a fermion BV matter field:

$$\Psi = \psi_A(x)y^A + \phi^a(x)y^*_a$$

$$\delta \psi_A = \frac{\partial u_B^a}{\partial y^A}|_{y^A=0} \epsilon^B$$

$$\delta \phi^a(x) = \epsilon^i f^a_{ij} \phi^j$$

The covariant derivative is, in terms of the components of the BV matter field:

$$D_\mu \psi_A(x) = \partial_\mu \psi_A(x) + A_i^\mu T_i^A \psi_B(x)$$

$$D_\mu \phi^k(x) = \partial_\mu \phi^k(x) - f^k_{ij} \phi^i(x) A_j^\mu$$

They coincide with the standard answer.

The simplest generalization of a matter field we can consider is the following:

$$\Phi = \phi_A(x)y^A + \phi^a_{\mu A}(x)y^A y^*_a \theta^\mu$$

$$\delta \phi_A(x) = \epsilon^i \frac{\partial u_B^a}{\partial y^A}|_{y^A=0} \phi_B$$

$$\delta \phi^a_{\mu A} = \epsilon^i \frac{\partial u_B^a}{\partial y^A}|_{y^A=0} \phi_B + \epsilon^i f^a_{ij} \phi^j_{\mu A}$$

The covariant derivative is, in this case:

$$D_\mu \phi = \partial_\mu \phi + \phi_{\mu A} u_i^A - \phi_{A \mu} u_i^i$$

$$\phi = \phi_{AB}^A$$

$$\phi^i = \phi_{\mu A} y^A$$

From the transformation law of the matter field, we get that

$$D_\mu \phi A D_\mu \phi B g^{AB}$$

is gauge invariant. $g^{AB}$ is an invariant tensor under transformations of $G$.

8 Conclusions

In this letter, we have presented a particular realization of the Non-Abelian BV formalism, which incorporates in a unified way all the ingredients of the standard model: Yang-Mills field, covariant derivatives, Higgs fields and fermions.

The whole structure offer ample scope for generalizations: By considering superfields in the internal ($y^A$, $y^*_a$ and external variables $\theta^\mu$ with suitable Grassman signatures, we can incorporate, gauge bosons of higher spin interacting with fermions and bosons expanding arbitrary representations of the Lorentz group. This is done in a systematic form, by enlarging the symmetry of the models.

We have offered some simple examples of these generalizations. In particular we have shown how to built Characteristic Classes involving a non-abelian two-form. We also constructed a gauge invariant Lagrangian coupling the two-form gauge field to the Yang-Mills field in four space-time dimensions. This differs from the Freedman-Townsend coupling, although the transformation law for the two-form gauge field is the same. The reason being that the one-form gauge field transforms differently.
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