Supplementary Materials

Synthesis of ZSM-5 zeolites from biomass power plant ash for removal of ionic dyes from aqueous solution: Equilibrium isotherm, kinetic and thermodynamic analysis

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1. Figures

Fig. S1 Characterization of the A-2: XRD (a), FTIR (b), $^{29}$Si MAS NMR (c), and SEM-EDX (d) analysis.
Fig. S2 XRD analysis of synthesized zeolites under different conditions: 120 °C/24-72 h (a), 150 °C/12-60 h (b), 180 °C/6-48 h (c), and 210 °C/3-36 h (d).
Fig. S3 Images of zeolite growth process at 120 °C for diverse times: 24 h (a), 48 h (b), and 72 h (c).
Fig. S4 Particle size distribution of the ZSM-5 zeolites: Z-1 (a), Z-2 (b), Z-3 (c), and Z-4 (d).
**Fig. S5** Digital photographs (a) and SEM images (b) coupled with EDX elemental mapping images of Si, Al, O and C after zeolite adsorbing MB (1) and CR (2) dyes.
The corresponding relation of temperature is as below:

25 °C — 298 K, 45 °C — 318 K and 65 °C — 338 K

Fig. S6 The variation of MB adsorption quantity with ZSM-5 zeolite adsorbent under diverse conditions: 25 mg/L (a), 50 mg/L (b), 75 mg/L (c), 100 mg/L (d), 200 mg/L (e), 300 mg/L (f), 400 mg/L (g) and 500 mg/L (h).
Fig. S7 The variation of CR adsorption quantity with ZSM-5 zeolite adsorbent under diverse conditions: 25 mg/L (a), 50 mg/L (b), 75 mg/L (c), 100 mg/L (d), 200 mg/L (e), 300 mg/L (f), 400 mg/L (g) and 500 mg/L (h).
Fig. S8 The fitting curves of the Van’t Hoff (a) and the Arrhenius (b) plots at 25 mg/L initial dye concentration.
2. Tables

Table S1 Chemical composition of A-1 and A-2 (Weight %).

| Sample | SiO₂  | CaO  | Al₂O₃ | Fe₂O₃ | K₂O  | MgO  | Na₂O  | P₂O₅ | TiO₂* |
|--------|-------|------|-------|-------|------|------|-------|------|-------|
| A-1    | 44.41 | 23.84| 10.80 | 3.63  | 3.99 | 3.76 | 1.27  | 2.02 | 1.05  |
| A-2    | 93.00 | 1.04 | 0.28  | 0.15  | 2.70 | 0.61 | 0.07  | 1.46 | -     |

* Not detected in A-2.
**Table S2** The growth states of ZSM-5 zeolite under different hydrothermal conditions.

| T/°C | 3   | 6   | 12  | 24  | 36  | 48  | 60  | 72  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|
| 120  | ×   | ×   | ×   | +   | +   | ○   | -   | -   |
| 150  | ×   | ×   | +   | +   | ○   | -   | -   | ×   |
| 180  | ×   | +   | +   | ○   | -   | -   | ×   | ×   |
| 210  | +   | ○   | -   | -   | -   | ×   | ×   | ×   |

**NOTE:** × — Unnecessary to consider,  
+ — Crystal growing,  
○ — Crystal mature,  
- — Crystal degradation or transition
Table S3  Labeling of synthesized ZSM-5 zeolites.

| Sample labeling | Temperature, °C | Time, h |
|-----------------|----------------|--------|
| Z-1             | 120            | 48     |
| Z-2             | 150            | 36     |
| Z-3             | 180            | 24     |
| Z-4             | 210            | 6      |
### Table S4 Textural properties of ZSM-5 zeolite samples.

| Sample | $S_{\text{BET}}$ (m$^2$/g) | $S_{\text{micro}}$ (m$^2$/g) | $S_{\text{ext}}^a$ (m$^2$/g) | $V_{\text{micro}}$ (cm$^3$/g) | $V_{\text{meso}}$ (cm$^3$/g) | $D_{\text{micro}}$ (nm) | $D_{\text{meso}}$ (nm) | $d_{50}^b$ (µm) |
|--------|-----------------------------|-------------------------------|-----------------------------|-------------------------------|-----------------------------|------------------------|------------------------|---------------|
| Z-1    | 317.10                      | 166.03                        | 151.07                      | 0.085                         | 0.088                       | 0.373                  | 3.41                   | 2.98           |
| Z-2    | 302.27                      | 131.93                        | 170.34                      | 0.070                         | 0.077                       | 0.374                  | 2.88                   | 1.70           |
| Z-3    | 254.98                      | 194.45                        | 60.53                       | 0.099                         | 0.080                       | 0.368                  | 4.11                   | 2.24           |
| Z-4    | 312.42                      | 161.45                        | 150.97                      | 0.083                         | 0.079                       | 0.371                  | 3.06                   | 3.91           |

$^a$ Calculated from the subtraction between $S_{\text{BET}}$ and $S_{\text{micro}}$.

$^b$ Particle size accounted for 50% of total volume, illustration in Fig. S4.
| Dye | MB | CR |
|-----|----|----|
| 25  | 45 | 65 |
| 25  | 45 | 65 |
| 25  | 45 | 65 |
| 25  | 45 | 65 |
| 25  | 45 | 65 |

| Parameter | MB | CR |
|-----------|----|----|
| Temperature, °C | 25 | 45 | 65 | 25 | 45 | 65 |
| Removal efficiency, R% | 89.84 | 91.32 | 95.04 | 78.76 | 72.24 | 65.76 |

### a. Isothermal model parameters

| Model | Parameter | MB | CR |
|-------|-----------|----|----|
| Langmuir | $q_{\text{max}}, \text{mg/g}$ | 120.00 | 130.46 | 160.24 | 87.49 | 70.31 | 35.88 |
| | $K_L, \text{L/mg}$ | 0.0852 | 0.0921 | 0.1324 | 0.0972 | 0.0843 | 0.1124 |
| | $R^2$ | 0.9990 | 0.9961 | 0.9922 | 0.9589 | 0.9588 | 0.9959 |
| | $R_L$ | 0.3195 | 0.3028 | 0.2320 | 0.2915 | 0.3218 | 0.2625 |
| Freundlich | $K_F, (\text{mg/g})/(\text{L/mg})^{1/n}$ | 32.5171 | 34.7753 | 44.5263 | 30.3936 | 25.2063 | 16.9815 |
| | $1/n$ | 0.2295 | 0.2349 | 0.2341 | 0.1811 | 0.1733 | 0.1271 |
| | $R^2$ | 0.9370 | 0.9439 | 0.9393 | 0.8416 | 0.8413 | 0.9325 |
| Temkin | $K_T, \text{L/g}$ | 1.7238 | 1.8124 | 2.6397 | 3.1361 | 3.1768 | 21.5983 |
| | $R^2$ | 0.9800 | 0.9853 | 0.9826 | 0.8833 | 0.8798 | 0.9486 |
| Sips | $q_{\text{max}}, \text{mg/g}$ | 120.74 | 134.54 | 164.55 | 82.68 | 66.16 | 34.96 |
| | $K_S, \text{L/mg}$ | 0.0838 | 0.0835 | 0.1193 | 0.0982 | 0.0820 | 0.1072 |
| | $\gamma$ | 0.9701 | 0.8753 | 0.8801 | 1.9433 | 1.9445 | 1.2658 |
| | $R^2$ | 0.9989 | 0.9970 | 0.9919 | 0.9966 | 0.9965 | 0.9988 |
| R–P | $K_{\text{RP}}, \text{L/g}$ | 10.7033 | 13.9605 | 24.6669 | 5.9554 | 3.9353 | 3.4024 |
| | $\alpha, (\text{L/mg})^{1/\beta}$ | 0.0962 | 0.1353 | 0.1956 | 0.0312 | 0.0233 | 0.0784 |
| | $\beta$ | 0.9864 | 0.9576 | 0.9552 | 1.1396 | 1.1552 | 1.0336 |
| | $R^2$ | 0.9990 | 0.9974 | 0.9932 | 0.9755 | 0.9804 | 0.9977 |

### b. Kinetic model parameters

| Model | Parameter | MB | CR |
|-------|-----------|----|----|
| PFO | $q_{\text{e,cal}}, \text{mg/g}$ | 29.76 | 43.84 | 18.80 | 30.47 | 25.03 | 49.91 |
| | $q_{\text{e,exp}}, \text{mg/g}$ | 22.46 | 22.83 | 23.76 | 19.69 | 18.06 | 16.44 |
| | $k_1, \text{min}^{-1}$ | 0.1171 | 0.1736 | 0.1158 | 0.1339 | 0.1249 | 0.1988 |
| | $R^2$ | 0.9693 | 0.9607 | 0.9289 | 0.9878 | 0.9348 | 0.9155 |
### PSO

|                  | q<sub>e,cal</sub>, mg/g | q<sub>e,exp</sub>, mg/g | k<sub>2</sub>, g/(g·min) | R<sup>2</sup> |
|------------------|-------------------------|-------------------------|--------------------------|--------------|
|                  | 25.69                   | 24.88                   | 25.65                    | 21.78        |
|                  | 24.08                   | 21.78                   | 20.11                    |              |
|                  | 22.46                   | 22.83                   | 23.76                    | 19.69        |
|                  | 18.06                   | 0.0056                  | 0.0098                   | 0.0105       |
|                  | 18.06                   | 0.0051                  | 0.0059                   | 0.0069       |
|                  | 0.9920                  | 0.9928                  | 0.9966                   | 0.9934       |
|                  |                         |                         |                          | 0.9941       |
|                  |                         |                         |                          | 0.9899       |

### Intraparticle diffusion

|                  | k<sub>p1</sub>, mg/(g·min<sup>1/2</sup>) | |C<sub>1</sub> |, mg/g | 7.9498 | 8.7800 | 7.3589 | 3.2226 | 2.7777 | 3.3980 |
|------------------|------------------------------------------|-------------------------|--------------------------|--------------|
|                  | |C<sub>2</sub> |, mg/g | 12.47 | 15.91 | 15.91 | 2.50 | 3.05 | 0.11 |
|                  | k<sub>p2</sub>, mg/(g·min<sup>1/2</sup>) | |C<sub>2</sub> |, mg/g | 1.7147 | 1.2024 | 1.1623 | 0.6152 | 0.3428 | 0.5572 |
|                  | |C<sub>3</sub> |, mg/g | 11.85 | 15.91 | 16.84 | 16.32 | 16.19 | 13.66 |
|                  | k<sub>p3</sub>, mg/(g·min<sup>1/2</sup>) | |C<sub>3</sub> |, mg/g | 0.0454 | 0.0377 | 0.0294 | 0.2606 | 0.1074 | 0.2288 |

### c. Thermodynamic model parameters

|                  | ΔG<sup>0</sup>, kJ/mol | ΔH<sup>0</sup>, kJ/mol | ΔS<sup>0</sup>, J/mol·K | E<sub>a</sub>, kJ/mol |
|------------------|-------------------------|-------------------------|--------------------------|----------------------|
|                  | -5.40                   | 15.98                   | 71.14                    | 13.36                |
|                  | -6.22                   | -13.78                  | -35.35                   | 6.32                 |
|                  | -8.30                   | -3.25                   |                          |                      |
|                  | -3.25                   | -2.53                   |                          |                      |
|                  | -5.40                   | -1.83                   |                          |                      |
3. Equations

Adsorption quantity:

\[ q_t = \frac{V(C_0 - C_t)}{m} \]  

(1)

Removal efficiency:

\[ \%R = \frac{C_0 - C_t}{C_0} \times 100 \]  

(2)

Where \( q_t \) is the dye adsorption quantity of adsorbent at \( t \) time, mg/g; \( C_0 \) and \( C_t \) are the initial and \( t \) time concentrations of dye effluents, respectively, mg/L; \( V \) is the solution volume of dye effluents, L; and \( m \) is the mass of zeolite adsorbent, g.

Adsorption efficiency:

\[ \%AE = \frac{q_{e,exp}}{q_{e,exp}} \times 100 \]  

(3)

Where \( q_{e,exp} \) is the experimental equilibrium adsorption quantity, mg/g; \( x \) is the cycle number, \( x=0, 1, 2, 3, 4 \) and 5. When \( x=0 \), \( q_{e,exp} \) is equal to \( q_{e,exp} \).

Adsorption isothermal models:

Langmuir isotherm model is used to describe the maximum quantities of monolayer adsorption onto a homogeneous surface where the adsorption sites have equal affinity and energy without adsorbate-adsorbent interaction in the assumed conditions \(^1\). \( R_L \) is a significant constant (dimensionless) related to adsorption behaviors of Langmuir isotherm.

\[ q_e = q_{\text{max}} \frac{K_L C_e}{1 + K_L C_e} \]  

(4)

\[ R_L = \frac{1}{1 + K_L C_0} \]  

(5)

Where \( q_{\text{max}} \) is the maximum adsorption quantities, mg/g; \( K_L \) is the Langmuir equilibrium
constant, L/mg. If $R_L > 1$, the adsorption is unfavorable, and if $0 < R_L < 1$, the adsorption is favorable, while $R_L = 0$ or 1, the adsorption is irreversible and linear, respectively.

Freundlich isotherm model is an empirical equation used to describe non-ideal multilayer adsorption in a heterogeneous system\(^2\).

$$q_e = K_F c^{1/n}$$ \hspace{1cm} (6)

Where $K_F$ is the Freundlich equilibrium constant, (mg/g)/(L/mg)\(^{1/n}\); 1/n is the adsorption intensity. If 1/n>1, the adsorption is unfavorable. On the contrary, If 0<1/n<1, the adsorption is favorable, and it is irreversible when 1/n is equal to 0.

According to the trait of linear reduction of adsorption heat on the adsorbate-adsorbent molecule surfaces, Temkin isotherm model reflects that the bond energies between adsorbate and adsorbent, which is assumed to distribute uniformly\(^3\).

$$q_e = \frac{RT}{b_T} \ln K_T + \frac{RT}{b_T} \ln C_e$$ \hspace{1cm} (7)

Where $K_T$ is the Temkin equilibrium constant related to the bond energies between adsorbate and adsorbent, L/g; $b_T$ is the adsorption heat associated with adsorption rate in the initial stage, kJ/mol; $R$ is the gas constant, 8.314 J/(mol·K).

Combined with Langmuir and Freundlich models, Sips isotherm model which is a three-parameter equation can be improved to express the heterogeneity of adsorption interaction\(^4\). As revealed, when the amount of adsorbent is large with low concentration of adsorbate, Sips model will trend to be the Langmuir model, otherwise the Freundlich model.

$$q_e = q_{\text{max}} \frac{(K_S c_e)^\gamma}{1 + (K_S c_e)^\gamma}$$ \hspace{1cm} (8)

Where $K_S$ is the Sips equilibrium constant, L/mg; $\gamma$ is the heterogeneity parameter. Herein,
the closer the γ value is to 1, the more uniform the status is on the adsorbent surface.

Concerning on the absence of homogeneous adsorption in the Sips model, a three-parameter empirical equation of Redlich-Peterson isotherm model is put forward, which is suitable for either homogeneous or heterogeneous systems.\(^5\)

\[
q_e = \frac{K_{RP} C_e}{1 + \alpha C_e^\beta}
\]

Where \(K_{RP}\) is the Redlich-Peterson equilibrium constant, L/mg; \(\alpha\) \([(L/mg)^{1/\beta}]\) and \(\beta\) \((0<\beta<1)\) are the Redlich-Peterson isotherm parameters.

Adsorption kinetic models:

Generally speaking, PFO is assumed that one dye molecule would adhere onto one adsorption active site at the initial stage of adsorption process. Contrarily, PSO is assumed that one dye molecule would adhere onto two adsorption active sites on the whole adsorption process.\(^6\) In particular, the diffusion rates are primarily determined by intraparticle diffusion and liquid film diffusion steps.\(^7\) Thus, intraparticle diffusion model will make clear of the limiting step in adsorption stages. All the kinetic models were shown in a linear form as following:

\[
\log(q_e - q_t) = \log q_e - \frac{k_1 t}{2.303}
\]

\[
\frac{t}{q_t} = \frac{t}{q_e} + \frac{1}{k_2 q_e}
\]

\[
q_t = k_p t^{1/2} + C_i
\]

Where \(k_1\) is the adsorption rate constant of PFO model, min\(^{-1}\); \(k_2\) is the adsorption rate constant of PSO model, g/(mg·min); \(k_p\) is the intraparticle diffusion constant, mg/(g·min\(^{1/2}\)); \(C\) is the correlation coefficient related to boundary layer thickness, mg/g; \(i\) is the order of
diffusion step (i=1, 2 and 3); t is the adsorption time, min.

Adsorption thermodynamic models:

To evaluate the thermodynamic properties during the zeolite adsorption process of MB and CR dyes, the parameters of $\Delta G^0$, $\Delta H^0$, $\Delta S^0$ and $E_a$ were put forward from the Van’t Hoff and Arrhenius equations, which were given below:

\[
K_d = \frac{q_e}{C_e}
\]  
(13)

\[
\Delta G^0 = \Delta H^0 - T \Delta S^0
\]  
(14)

\[
\Delta G^0 = -RT \ln K_d
\]  
(15)

\[
\ln K_d = -\frac{\Delta H^0}{RT} + \frac{\Delta S^0}{R}
\]  
(16)

\[
\ln k_2 = \ln A - \frac{E_a}{RT}
\]  
(17)

Where $K_d$ is the adsorbate distribution coefficient, L/g; $\Delta H^0$ is the standard enthalpy, kJ/mol; $\Delta S^0$ is the standard entropy, J/(mol·K); $\Delta G^0$ is the standard Gibbs free energy, kJ/mol; $E_a$ is the activation energy, kJ/mol; $A$ is the Arrhenius factor; $T$ is the adsorption temperature, °C.

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