Pair distribution function of strongly coupled quark gluon plasma in a molecule-like aggregation model

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Pair distribution function for delocalized quarks in the strongly coupled quark gluon plasma (sQGP) as well as in the states at intermediate stages of crossover from hadronic matter to sQGP are calculated using a molecule-like aggregation model. The shapes of the obtained pair distribution functions exhibit the character of liquid. The increasing correlation length in the process of crossover indicates a diminishing viscosity of the fluid system.

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I. INTRODUCTION

Relativistic heavy-ion collision experiments at RHIC have found evidences for a new state of matter, the quark-gluon plasma (QGP). The collective flow phenomena, the transport properties and other theoretical developments indicate that the new state of matter is not a weakly coupled gas as expected, but a strongly coupled ideal fluid, referred to as sQGP, probably in a wide temperature region $T_c < T < 2T_c$. What is the microscopic dynamics responsible for the small viscosity of the strongly coupled quark gluon plasma and how is the evolution of the property of this fluid in the process of crossover from hadronic gas to sQGP are still open questions.

Many attempts have been done to explain the small viscosity. Theoretically, people obtain transport coefficient perturbatively by using Kubo formula in the scope of thermal field theory or solve the transport equation from kinetic theory. They found that it is not enough to account for the small viscosity if only considering the perturbative interaction. While ref. claimed that they can reproduce the small viscosity for a gluon gas if including $gg \rightarrow ggg$ bremsstrahlung in the pQCD calculation within relativistic kinetic theory. Another way to reach small viscosity is to solve the strongly coupled super-symmetric Yang-Mills gauge theory by AdS/CFT duality. In phenomenology, Ref. developed a new picture of QGP with multiple colored bound states and the increased re-scattering among bound states were expected to reduce the viscosity of QGP.

Quantitative investigation on the liquid property of plasma is possible by considering two particle correlation function in coordinate space or its Fourier transform — the structure function in momentum space. The characteristic behavior of pair correlation function reveals the intrinsic feature of plasma in different phase. For example, in the case of liquid state the pair correlation function exhibits a pronounced peak and one or two small and broad additional peaks. The first peak corresponds in coordinate space to the near-neighbor “shell” of atoms because of short-range order. The next near-neighbor “shell” in the liquid is much less prominent and the next outer shell may hardly visible due to the lack of long-range order. In the case of a solid crystalline phase, where a long-range order exists, a number of sharp peaks with comb-like shape are observed. In the ideal gas phase, where no order is present, the pair correlation function shows no clear structure. A sketch of the pair correlation function for liquid and gas is shown in Fig. 1.

Though it is not easy to obtain the viscosity of a liquid quantitatively from its pair correlation function, the tendency of the viscosity could be inferred via the shape evolution of the latter. As stated above the pair correlation function of liquid exhibits a number of peaks with diminishing heights. The location of the last visible peak can be taken as the range of effective interaction in the liquid. The increasing of the latter results in a diminishing mean free path, which in turn causing the reduction of viscosity. Therefore, the larger distance the peaks of pair distribution function with reduced amplitudes shows, the less viscous the liquid is. Ref. calculated the correlation function and structure function for QGP perturbatively by using hard thermal loop approximation and it failed to represent the typical character of liquid state since only weakly coupled QGP is considered.

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FIG. 1: Sketch of the pair correlation function vs. distance between two atoms in the gas and liquid phase.
Recently, a dynamical percolation model based on the molecule-like aggregation is proposed to describe the crossover transition between hadronic matter and QGP [20]. In this model as the increase of temperature, partons inside hadrons are delocalized to a certain extent, tunneling between neighboring hadrons through bonds, to form a sort of grape-shape QGP (gQGP), which is a special form of sQGP. Inside the gQGP, partons move around with large cross section and short mean free path, resulting in a quark-gluon matter possessing the property of near-perfect fluid. In this article we investigate the pair correlation function of the delocalized quarks from this molecule-like aggregation model and discuss the information about the structure of the grape shape QGP during the process of crossover.

II. A BRIEF REVIEW ON MOLECULE-LIKE AGGREGATION MODEL

In order to answer the question how to crossover from the hadronic to partonic phase in QCD without contradiction with color confinement, a basic assumption is proposed [20], which states that quarks in neighboring hadrons can be delocalized, i.e. tunnel between the hadrons to form clusters. After delocalization the clusters, which are molecule-like aggregations, are color singlets and the original hadrons turned to colored objects, called cells.

To exhibit the physical consequence of the assumption on molecule-like aggregation a toy model basing on percolation procedure is constructed [20]. In the simplified 2-D version of the model the initial system is set to be a nucleon gas consisting of $2 \times 197$ cells, i.e. hadrons, which are small circles of hard-core radius $r_c = 0.1$ fm distributed randomly in a big circle of radius $R = 7$ fm.

In calculating the probability for bond-formation between neighboring cells, or delocalization of quarks, a non-relativistic model used in nuclear force theory [21] is utilized. The model Hamiltonian for the 6 constituent quarks in the c.m. system of two nearby cells is assumed to be

$$\mathcal{H} = \sum_{i=1}^{6} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{cm} + \sum_{i<j} V_{ij}^C, \quad (1)$$

where $T_{cm}$ is the center-of-mass kinetic energy. When the quarks $i, j$ belong to a same cell, a square-confinement potential $V_{ij}^C = -a_c \vec{r}_{ij} \cdot \vec{r}_{ij}$ is assumed. When they belong to two nearby cells, the infinite potential between them will drop down, forming a potential barrier, and a parametrization $V_{ij}^C = -a_c \vec{r}_{ij} \cdot \vec{r}_{ij} \frac{1-e^{-\alpha r_{ij}}}{r_{ij}}$ is used, where $\alpha$ is a model parameter, proportional to temperature square from dimensional consideration.

In doing variational calculation for the ground state energy the trial wave function of the two-cell system in adiabatic approximation is chosen to be an antisymmet-

\[
|\Psi_0(S)\rangle = \mathcal{A} \prod_{i=1}^{3} \psi_L(r_i) \prod_{i=4}^{6} \psi_I(r_i) \prod_{i<j}^{B_1B_2A_1A_2} \chi_{ij}, \quad (2)
\]

where $\mathcal{A}$ is the anti-symmetrization operator, which permutes quarks between the two cells; $[\cdots]_{00}$ means that the spin, isospin and color of the two cells are coupled to a particular color singlet state with total spin and isospin equal zero. For the orbital motion, we have the left (right) single-quark orbital wave function $\phi_L(r_i) = (\frac{1}{2} - \frac{r_i}{2R}) e^{-\frac{r_i^2}{2R^2}}$, $\phi_R(r_i) = (\frac{1}{2} + \frac{r_i}{2R}) e^{-\frac{r_i^2}{2R^2}}$, where $\pm \frac{R}{2}$ are the cell-centers, $S$ is the distance between the two cells. $b$ is a baryon-size parameter. Delocalized orbit is defined as $\psi_L(r) = \frac{1}{\sqrt{N}}(\phi_L + \epsilon\phi_R)$, $\psi_R(r) = \frac{1}{\sqrt{N}}(\epsilon\phi_L + \phi_R)$, where $\epsilon$ is a variational parameter characterizing the degree of delocalization and $N$ a normalization factor. At each separation $S$, $\epsilon$ is determined by minimizing the energy $E(S) = \frac{\langle \Psi_0(S) | H | \Psi_0(S) \rangle}{\langle \Psi_0(S) | \Psi_0(S) \rangle}$. The model parameters are: $m_d = 603$ fm, $a_c = 101.14$ MeV/fm², $\mu = 0.063$ fm, with $\mu$ is left as a free parameter.

It is found from variational calculation that a maximum distance $S_0$ for delocalization exists for a fixed temperature, i.e. fixed $\mu$. Using $S_0$ as input, a bond percolation model is constructed. After the percolation procedure all the cells in the system are grouped into clusters. In each cluster, any two cells are connected by bonds with zigzag path, mimic the atoms in molecule, cf. Fig. 2(a).

When temperature increases, the potential barrier between two cells decreases, and the maximum distance $S_0$ for bond formation increases. The crossover from hadronic phase to partonic phase starts when an infinite cluster i.e. a cluster extending from one end of the big circle to the other end appears, cf. Fig. 2(b). The corresponding temperature is $T_c$. The colored partons are confined in the group of cells connected by bonds, being able to move around inside the big cluster via quantum tunneling through the potential barriers, resulting in a quark-gluon matter with the property of fluid inside the cluster. The crossover process is completed when all the cells belong to one infinite cluster, cf. Fig. 2(c), and

\[
\text{FIG. 2: (Color online) Continuously distributed cells connected by bonds form clusters. In (b) the big cluster extending from the left- to the right-boundary is an infinite cluster. In (c) all the cells are connected to an infinite cluster.}
\]
the corresponding temperature is denoted by \( T'_c \). In the following we will investigate the structure of sQGP and of the intermediate states during the crossover from hadronic phase to sQGP through the pair correlation function.

### III. Definition of Pair Correlation Function

In the liquid state theory, the pair correlation function \( g(r) \) is defined as the probability of finding two atoms in the liquid at a distance \( r \) from each other \([22][23]\). In two dimensional space the quantity \( 2\pi \rho g(r) dr \) is the mean number of atoms inside a ring of radius \( r \) with thickness \( dr \), centered on an “average” atom. In this expression, \( \rho \) is the number density of the bulk homogeneous liquid. From this definition we can calculate \( g(r) \) in two dimensional coordinate space in Monte Carlo simulation by the formula:

\[
g(r) = \frac{dN(r)}{2\pi \rho r dr}, \tag{3}
\]

where \( dN(r) \) is the number of atoms inside a ring with radius \( (r, r + dr) \) apart from the selected center atom. \( dN(r) \) is normalized by the uniform distribution, so that \( g(r) = 1 \) when there is no correlation. It needs to be noted that for a finite system, the boundary effect has to be taken into account since it will affect the normalization factor. Assume the selected central atom is near the boundary, then part of the ring with radius from \( r \) to \( r + dr \) may lie out of the finite system. Thus the normalization factor should be \( \theta dr \) instead of \( 2\pi dr \), where \( \theta \) is the angle of the arc inside the system. To eliminate the boundary effect, we add a weight \( w \) to the above formula, and define

\[
g(r) = \frac{dN(r) \cdot w}{2\pi \rho r dr}, \quad w = \frac{2\pi}{\theta}. \tag{4}
\]

Fig. 3 is the pair correlation function \( g(r) \) for a randomly distributed hadron gas. Fig. 3(a) shows the result from Eq. (3) and Fig. 3(b) is that after the boundary effect correction by Eq. (4). Evidently the correction is effective up to the radius 7 fm of the big circle. All the following calculations are restricted in this \( r \) range.

To calculate the pair correlation function for quarks in the above-mentioned molecule-like aggregation model, the usual definition about \( g(r) \) has to be modified since, due to color confinement the correlation between quarks does not happen along straight lines in geometrical space but are along the zigzag path of the connecting bonds. In order to reveal the structure information of the sQGP inside clusters, the correlation function must be defined as a function of the zigzag distance \( D \) along bonds between quarks, i.e. \( g(D) \). The zigzag distance \( D \) is referred to as “chemical distance” in the language of percolation theory \([24]\). The formula to calculate \( g(D) \) in 2 dimensional space is

\[
g(D) = \sum_R \frac{dN(D, r) \cdot w}{2\pi \rho r dr}. \tag{5}
\]

In the calculation, a quark is randomly selected as center and the distance \( D \) and \( r \) between this center and any other quarks in the same cluster are evaluated. \( dN(D, r) \) is the number of quarks in the small region \( (D, D + DR; r, r + DR) \). The normalization \( 2\pi \rho r dr \) for normal liquid is still used. The color confinement in the present case, which causes quarks to be able to communicate only through tunneling bonds, makes the resulting \( g(D) \) tends to a value smaller than unity.

### IV. Pair Correlation Function from Molecule-Like Aggregation Model

In our molecule-like aggregation model, the formation of infinite clusters is taken as the appearance of a new constituent i.e. sQGP in the system. The temperature where infinite clusters start to form is referred to as \( T_c \) and all the temperatures are scaled by it while investigating the quark pair correlation function inside clusters.

At each temperature, one quark is randomly selected from a randomly chosen cluster in the system as the center particle. The geometrical distance \( r \) and chemical distance \( D \) between this center quark and any other quarks inside this cluster are calculated. Pair correlation function \( g(D) \) is then evaluated by Eq. (5). Fig. 4 shows the
pair correlation function for $T = 0.475T_c$ and $T = T_c$. At $T = 0.475T_c$, the maximum bond length $S_0 = 0.19$ fm is less than the smallest distance between any two cells i.e. $S_0 < 2r_c$ and there is no bond formed between different cells. Thus the peak at $D < 0.2$ fm in Fig. 4 shows the intra-cell structure, which is analog to the unpenetrable distance in the usual liquid state theory. When $T = T_c$, infinite clusters appear with small probability, and we see from the right panel of Fig. 4 that small bumps are shown at larger $D$ beside the peak at $D < 0.2$ fm. To study the quark correlation induced by delocalization and the formation of bonds, we will not look at the trivial intra-cell correlation and only care the region where the new correlations appear. In the following figures, only $g(D)$ in the range $D > 0.2$ fm is drawn to focus on the structure of the pair correlation function induced by quark delocalization.

Fig.'s 5 show the $g(D)$ evolution at $T < T_c$. When $T$ is much less than $T_c$, i.e. long before crossover started, there are no clear correlation peaks, while when going near to crossover temperature, shoulder appears and quickly grows into the first pronounced peak at $T = 0.804T_c$. As $T$ increases further new shoulder emerges following this peak and the second peak forms when $T = T_c$, cf. the full line in Fig. 6.

From the evolution of the pair correlation function with temperature, we can get the following information about the structure of gQGP formed in the molecule-like aggregation model:

1. The first highest peak of the pair correlation function represents the correlation from the nearest neighboring quarks between different hadrons. The other small peaks are diminishing as $D$ increases because of the long range disorder. The shape of $g(D)$ around $T_c$ shows the typical short-range-order behavior of liquid state, which indicates that the quark matter in the molecule-like aggregation model possesses liquid structure.

2. During the process of crossover, i.e. $T_c < T < 1.39T_c$, the position of peaks shift to larger $D$ as the temperature increases, cf. Fig. 5 showing that the correlation length increases with temperature. This indicates that the viscosity is getting smaller and smaller when temperature increases.

V. CONCLUSION

The molecule-like aggregation model provides a clear picture for the structure of the matter formed in crossover, i.e. gQGP, which is a special form of sQGP, and the evolution of the structure in the process of crossover. In this picture, quarks are moving from one cell, which is initially hadron, to the other through bonds to form grape shape quark matter. Basing on this model the pair distribution function related to chemical distance $D$ for gQGP and its evolution in the whole process of crossover are investigated. The typical behavior of pair distribution function demonstrates that liquid structure exists for the gQGP. The temperature dependence of the correlation range qualitatively shows that the viscosity is getting lower as the temperature increases during the crossover process.

It is worth while noticing that the above conclusion on the behavior of pair correlation function in the process of crossover is mainly based on the molecule-like aggregation assumption itself, and does not rely much on the toy model used in the calculation. It could be expected that the qualitative features shown in Fig’s 2 and 6 will persist in a more realistic case.
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