New Technique for Finding the Maximization to Transportation Problems

Haleemah Jawad Kadhim\textsuperscript{1}, Mushtak A. K. Shiker\textsuperscript{2}, Hussein A H Al-Dallal\textsuperscript{3}
\textsuperscript{1,2}Department of mathematics, collage of education for pure sciences, university of Babylon, Babil- Iraq.
\textsuperscript{3}General Directorate for Education in Najaf, The Iraqi Ministry of Education, Najaf-Iraq
\textsuperscript{1}halima.kazem@student.uobabylon.edu.iq , \textsuperscript{2}mmttmhh@yahoo.com , \textsuperscript{3}pure.mushtaq@uobabylon.edu.iq

Abstract. Transportation problems (TP) are one of the important problems in linear programming problems (LPP) that generally address the problems of transporting and distributing goods with the aim of achieving the largest profit or the lowest cost depending on the type of problem. In this research study, a new technique was proposed to solve transportation problems with an objective function of the type of maximization that is used to achieve the highest possible profit. This technique was obtained by relying on a published research paper that deals with the same problem but with an objective function of the miniaturization type. The efficiency of this new technique was tested in terms of the type of results obtained when it was used to solve many transportation problems in life, and some of them were mentioned in this paper. After that, the solution results were compared using the proposed technique with the use of the three well-known classical methods which are NWCM, LCM, and VAM. Whereas, the results using the new technique were the required results that represent the optimal solution or close to the optimal solution.

Keywords. Operations Research, Optimization Problems, Transportation Model, Maximization of Transportation Problems, IBFS, VAM.

1. Introduction

In this fast-moving world the need for commodities increases day by day. Accordingly, the importance of transportation plays a big role in society. The profits and fortunes of firms that move goods from one place to another are determined by transportation. The transportation problem model is one of the well-known models in operations research that is concerned with finding the number of products transferred from a group of distributors to a group of warehouses through the road network so that the demand in the warehouses is met, but with the largest possible profit or the lowest possible cost depending on the type of problem. TP was first proposed by Frank L. Hitchcock (1875-1957) in 1941 in his paper “The distribution of a product from several sources to numerous localities”. In 1947, Tjalling C. Koopmans presented his paper entitled "Optimum utilization of the transportation system". The two aforementioned studies are the main achievements in developing various approaches to solving transportation problems [1- 2]. Transportation models focus mainly on the optimal method that achieves the maximum profit or the lowest cost according to the type of problem with which the homogeneous goods are to be transported from many factories (supply centers) to many warehouses (destinations). In this problem, the main goal is to find the optimal schedule for shipping the commodity while meeting...
the constraints of both supply and demand. In this study, transportation problems with an objective function of the type of maximization were addressed, and based on a detailed study of what researchers have done to develop solutions to this problem, a new technique has been proposed to solve these balanced and unbalanced problems [3-4]. The proposed new technique through which the initial solution (and sometimes even gives the optimal solution) can be found, through which the optimal solution is easily reached. In recent years several methods have been proposed to find IBFS for a transportation model [5-9]. After testing the new technique proposed in this paper it may be used to solve many transportation problems in public life, its efficiency had been proven by giving the required results that are better or equal to the results obtained by using the three well-known classical methods: NWCM, LCM and VAM. What distinguishes the new technique is that it includes clear and easy solution steps that can save a lot of time and effort when using it to solve various transportation problems. The authors introduced many articles to find the solutions in variant fields such as transportation problems, reliability, optimization, and so on. For examples, to find the optimal solution of nonlinear systems and optimization problems we used the trust region techniques [10-13], conjugate gradient techniques [14-18], line search techniques [19-22], and projection technique [23-25], and some article in reliability [26-31], but in this work we introduce a new technique to finding the maximization to transportation problems.

2. The Steps of the New Technique

The following solution steps are specific to the technique suggested in this paper:

Step 1) Building TP schedule.

Step 2) Transform the problem from max into min by subtracting every value from the largest value in TP schedule.

Step 3) Convert the unbalanced TP to balanced TP.

Step 4) At every row define the two minimum values and output the difference between these two values (called the penalty). And at every column define the two maximum values and output the difference between these two values (called the penalty).

Step 5) Determine which row or column corresponds to the largest difference obtained in Step 4.

Step 6) Assign the largest possible amount of demand and supply units to the cell with the lowest value in the specific row or column chosen in Step 5. If the highest differences are repeated, then the cell with the lowest value in the row or column corresponding to the largest difference is chosen. When there is a redundancy in the minimum cells, the cell that takes the maximum possible amount of allocation is chosen. If a repeat occurs in the maximum allotted quantity in the minimum cells, the cell corresponding to the largest is chosen from among the demand or supply.

Step 7) After allocating the maximum possible quantity for the selected cell, the row (or column) in which the supply (or demand) was consumed is removed.

Step 8) Repeat the solution steps from Step 4 until all supply and demand have been exhausted.

Step 9) Put the values of the decision variables $x_{ij}$ into the TP schedule in Step 1.

Step 10) Apply the objective function to find the solution to the problem.

3. Numerical Examples

Example 1. There are three factories in different locations A, B and C. These factories supply shipments to three different warehouses I, II and III. Table 1 below shows the net profits for each product unit, along with the availability of companies and warehouse requirements. Find the appropriate allocation to maximize the overall return.

| TABLE 1 | The TP matrix of example 1 |

\[ \begin{array}{ccc|c|c|c|c|c|c} \hline |   &   &   &   &   &   &   \\ \hline |   &   &   &   &   &   &   \\ \hline |   &   &   &   &   &   &   \\ \hline \end{array} \]
Solution: The mathematical model for Example 1 is:

Objective Function: \[ Z = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} \]

Restrictions:

\[
\begin{align*}
\text{Supply Restrictions} & \quad \text{Demand Restrictions} \\
x_{11} + x_{12} + x_{13} &= 65 & x_{11} + x_{21} + x_{31} &= 100 \\
x_{21} + x_{22} + x_{23} &= 110 & x_{12} + x_{22} + x_{32} &= 70 \\
x_{31} + x_{32} + x_{33} &= 75 & x_{13} + x_{23} + x_{33} &= 80
\end{align*}
\]

The above TP table is balanced because total supply = total demand = 250, and it must be converted into a miniaturization problem table as in the following Table 2:

**TABLE 2** The TP matrix of example 1 converted into a miniaturization problem

| Origin | Destination | Supply |
|--------|-------------|--------|
| A      | I 24        | II 28   | III 21 | 65  |
| B      | I 27        | II 25   | III 26 | 110 |
| C      | I 23        | II 22   | III 29 | 75  |

Demand 100 70 80 250

According to the algorithm of the new technique, the following Table 3 is obtained:

**TABLE 3** The TP matrix of example 1 according to the new technique

| Origin | Destination | Supply | Penalty |
|--------|-------------|--------|---------|
| A      | I 0         | II 5   | III 65  | 65 4 4 4 |
| B      | I 100       | II 2   | III 5   | 110 1 1 2 |
| C      | I 0         | II 6   | III 75  | 75  6 - - |

Demand 100 70 80 250
\[ Z = (65 \times 28) + (100 \times 27) + (5 \times 25) + (5 \times 26) + (75 \times 29) = 6950 \]

**Example 2.** What is the appropriate allocation to maximize TP profit in following Table 4?

| Origin | Destination | Supply |
|--------|-------------|--------|
| A      | 4           | 10     | 7      | 40    |
| B      | 11          | 12     | 16     | 35    |
| C      | 18          | 3      | 14     | 51    |
| D      | 11          | 15     | 6      | 37    |
|        | Demand      |        |        | 54    | 61    | 48    | 163   |

**Solution:** The mathematical model for Example 2 is:

Objective Function: \[ Z = \sum_{i=1}^{4} \sum_{j=1}^{3} c_{ij} x_{ij} \]

Restrictions:

| Supply Restrictions | Demand Restrictions |
|---------------------|---------------------|
| \( x_{11} + x_{12} + x_{13} + x_{14} = 40 \) | \( x_{11} + x_{21} + x_{31} + x_{41} = 54 \) |
| \( x_{21} + x_{22} + x_{23} + x_{24} = 35 \) | \( x_{12} + x_{22} + x_{32} + x_{42} = 61 \) |
| \( x_{31} + x_{32} + x_{33} + x_{34} = 51 \) | \( x_{13} + x_{23} + x_{33} + x_{43} = 48 \) |
| \( x_{41} + x_{42} + x_{43} + x_{44} = 37 \) | |

The above TP table is balanced because total supply = total demand = 91, and it must be converted into a miniaturization problem table as in the following Table 5:
According to the algorithm of the new technique, the following Table 6 is obtained:

**TABLE 6** The TP matrix of example 2 according to the new technique

| Origin | Destination | Supply | Penalty |
|--------|-------------|--------|---------|
|        | I           | II     | III     |
| A      | 0           | 14     | 27      | 8       | 13      | 11     | 40     | 3       | 3       | 3       | 3       |
| B      | 0           | 7      | 0       | 6       | 35      | 2      | 35     | 4       | 4       | 4       | 4       |
| C      | 51          | 0      | 0       | 15      | 0       | 4      | 51     | 4       | -       | -       | -       |
| D      | 3           | 7      | 34      | 3       | 0       | 12     | 37     | 4       | 4       | 9       | -       |

| Demand | 54 | 61 | 48 | 163 |
|--------|----|----|----|-----|
| Penalty| 7  | 7  | 1  |     |
|        | 7  | 2  | 1  |     |
|        | -  | 2  | 1  |     |
|        | -  | 2  | 9  |     |

\[ Z = (27 \times 10) + (13 \times 7) + (35 \times 16) + (51 \times 18) + (3 \times 11) + (34 \times 15) = 2382 \]

4. **Comparison the Results**

The methods that used to find the optimal solution of TP differ mainly in terms of preference the solutions at the beginning. A good solution that has a beginning will produce the value of the objective function greater because the type of objective function in this problem covered in this study is of the type of maximization. The results of solving the examples presented in this paper have been compared to demonstrate the efficiency of the new technique. The results obtained were compared with the results of the three classic solution methods, namely NWCM, LCM and VAM. We can see from table 7 that the optimal solution obtained by the new technique is equal to other three methods regarding to example 1, but it better than NWCM and LCM but it also equal to VAM regarding to example 2. It should be noted that VAM is the best method of the classic methods for finding a solution, but it is characterized by the difficulty and complexity of its steps to find the solution. What distinguishes the new method is the simplicity and ease of its steps compared to VAM. In other words, we reached the same solution that obtained by VAM, but in fewer and easier steps, which leads to less effort and time to get the solution, and that is indicated the efficiency and goodness of the new technique.

**TABLE 7** Comparison of the new technique with NWCM, LCM, and VAM

| Name | NWCM | LCM | VAM | The New Technique |
|------|------|-----|-----|-------------------|
| Ex 1 | 6560 | 6950| 6950| 6950              |
| Ex 2 | 1062 | 2376| 2382| 2382              |

4. **Conclusion**
In this work, a new technique is proposed to find a solution to the TP with an objective function of maximization. By comparing the results of the new technique with the results of the three classic methods (NWCM, LCM, VAM), Table (7) shows that the new technique gives better results or equal to the results of the other three methods. Note that we employed the new proposed technique to solve many balanced and unbalanced TP examples in the case of maximization and in most cases the desired results were obtained compared with the classical methods. It can be concluded that the new technique gives favorable and appropriate results and has easy solution steps in terms of understanding and application and thus a lot of time and effort is saved to obtain the optimal solution or near of it.

5. References

[1] Schrijver, A. 2005 On the history of combinatorial optimization (Till 1960). *Handbooks in operations research and management science*, 12, pp 1–68.

[2] Hussein H A and Shiker M A K 2020 A modification to Vogel’s approximation method to Solve transportation problems, *J. Phys.: Conf. Ser.* 1591 012029.

[3] Mahdi M M et al. 2021 Solving systems of nonlinear monotone equations by using a new projection approach, *J. Phys.: Conf. Ser.* 1804 012107.

[4] Hussein H A, Shiker M A K and Zabiba M S M 2020 A new revised efficient of VAM to find the initial solution for the transportation problem, *J. Phys.: Conf. Ser.* 1591 012032.

[5] Zabiba M S M, Al-Dallal H A H, Hashim, Hashim K H, Mahdi M M and Shiker M A K 2021 A new technique to solve the maximization of the transportation problems. “in press”, *ICCEPS - April*.

[6] Hussein H A and Shiker M A K 2020 Two New Effective Methods to Find the Optimal Solution for the Assignment Problems, *Journal of Advanced Research in Dynamical and Control Systems*, 12: 7, p 49- 54.

[7] Mahdi M M and Shiker M A K 2020 A new projection technique for developing a Liu-Storey method to solve nonlinear systems of monotone equations, *J. Phys.: Conf. Ser.* 1591 012030.

[8] Dwail H H et al. 2021 A new modified TR algorithm with adaptive radius to solve a nonlinear systems of equations, *J. Phys.: Conf. Ser.* 1804 012108.

[9] Dreeb N K, et al. 2021. Using a new projection approach to find the optimal solution for nonlinear systems of monotone equation, *J. Phys.: Conf. Ser.* 1818 012101.

[10] Shiker M A K and Sahib Z 2018 A modified trust-region method for solving unconstrained optimization. *Journal of Engineering and Applied Sciences*, 13: 22, p 9667–9671.

[11] Dwail H H and Shiker M A K 2020 Reducing the time that TRM requires to solve systems of nonlinear equations, *IOP Conf. Ser.: Mater. Sci. Eng.* 928 042043.

[12] Dwail H H and Shiker M A K 2020 Using a trust region method with nonmonotone technique to solve unrestricted optimization problem, *J. Phys.: Conf. Ser.* 1664 012128.

[13] Dwail H H and Shiker M A K 2021 Using trust region method with BFGS technique for solving nonlinear systems of equations, *J. Phys.: Conf. Ser.* 1818 012022.

[14] Mahdi M M and Shiker M A K 2020 Three-term of new conjugate gradient projection approach under Wolfe condition to solve unconstrained optimization problems, *Journal of Advanced Research in Dynamical and Control Systems*, 12: 7, p 788–795.

[15] Wasi H A and Shiker M A K 2021 Proposed CG method to solve unconstrained optimization problems, *J. Phys.: Conf. Ser.* 1804 012024.

[16] Wasi H A and Shiker M A K 2020 A new hybrid CGM for unconstrained optimization problems, *J. Phys.: Conf. Ser.* 1664 012077.

[17] Wasi H A and Shiker M A K 2021 Nonlinear conjugate gradient method with modified Armijo condition to solve unconstrained optimization, *J. Phys.: Conf. Ser.* 1818 012021.

[18] Wasi H A and Shiker M A K 2021 A modified of FR method to solve unconstrained optimization, *J. Phys.: Conf. Ser.* 1804 012023.

[19] Hashim K H and Shiker M A K 2021 Using a new line search method with gradient direction to solve nonlinear systems of equations, *J. Phys.: Conf. Ser.* 1804 012106.
[20] Hashim K H, et al. 2021 Solving the Nonlinear Monotone Equations by Using a New Line Search Technique, J. Phys.: Conf. Ser. 1818 012099.

[21] Hashim L H, et al. 2021 An application comparison of two negative binomial models on rainfall count data, J. Phys.: Conf. Ser. 1818 012100.

[22] Hashim L H, et al. 2021 An application comparison of two Poisson models on zero count data, J. Phys.: Conf. Ser. 1818 012165.

[23] Shiker M A K and Amini K 2018 A new projection-based algorithm for solving a large scale nonlinear system of monotone equations, Croatian operational research review, 9: 1, p 63-73.

[24] Mahdi M M and Shiker M A K 2020 Three terms of derivative free projection technique for solving nonlinear monotone equations, J. Phys.: Conf. Ser. 1591 012031.

[25] Mahdi M M and Shiker M A K 2020 A New Class of Three-Term Double Projection Approach for Solving Nonlinear Monotone Equations, J. Phys.: Conf. Ser. 1664 012147.

[26] Hassan Z A H and Mutar E K 2017 Geometry of reliability models of electrical system used inside spacecraft, Second Al-Sadiq International Conference on Multidisciplinary in IT and Communication Science and Applications (AIC-MITCSA), pp. 301-306.

[27] Hassan Z A H and Balan V 2017 Fuzzy T-map estimates of complex circuit reliability, International Conference on Current Research in Computer Science and Information Technology (ICCIT-2017), IEEE, Special issue, p. 136-139.

[28] Hassan Z A H and Balan V 2015 Reliability extrema of a complex circuit on bi-variate slice classes, Karbala International Journal of Modern Science, 1: 1, p. 1-8.

[29] Hassan Z A H and Shiker M A K 2018 Using of generalized baye’s theorem to evaluate the reliability of aircraft systems. Journal of Engineering and Applied Sciences, (Special Issue13), 10797–10801.

[30] Abdullah G and Hassan Z A H 2020 Using of particle swarm optimization (PSO) to addressed reliability allocation of complex network, J. Phys.: Conf. Ser. 1664: 012125.

[31] Abdullah G and Hassan Z A H 2020 Using of Genetic Algorithm to Evaluate Reliability Allocation and Optimization of Complex Network, IOP Conf. Ser.: Mater. Sci. Eng. 928 042033.