Optimization of finite-range effective interaction for in-medium cross sections

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Abstract

In order to incorporate the finite range effect into effective interactions, a modification of the Skyrme force by introducing a cut-off factor for high momentum transfers is proposed. The parameters of the cut-off factor are determined by fitting the microscopic in-medium cross-sections of Li and Machleidt over a wide range of energy and nuclear density. Results for the SkM* and SLy4 forces are presented.

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I. INTRODUCTION

Effective interactions, like Skyrme \cite{1} or Gogny \cite{3} forces, associated with the mean-field theory have made it possible to describe a large body of nuclear properties along the whole nuclear chart \cite{4}. These forces are constantly refined to reproduce more and more precisely experimental observations \cite{5}. However, these effective forces encounter difficulties for describing quantities that are sensitive to the short range correlations such as nucleon-nucleon collisions cross-sections \cite{6,7}. For example, the nucleon-nucleon cross-sections calcu-
olated from Skyrme forces do not interpolate correctly between free-space and the medium, and reaches large values at low densities due to the unrestricted high momentum transfers. These difficulties become more dramatic in the applications of the extend mean-field approach to nuclear dynamics [3]. Recently, it has been pointed out that serious difficulties arise when an effective Skyrme force is employed in the collision term of the extended Time-Dependent Hartree-Fock theory [3]. More generally, such shortcomings are expected in transport theories that go beyond the mean-field approximation by including two-body correlations through residual interactions. In particular, in the applications of Boltzmann-type semi-classical transport models [10–12] or molecular dynamics calculations [13], the effective interactions are rarely adjusted for in-medium cross-sections, which may strongly influence the collision dynamics. In this work, we discuss an approximate treatment of incorporating the effect of finite range into Skyrme parameterization of the effective interactions, and optimize the parameters by fitting the nucleon-nucleon cross-sections to the microscopic in-medium cross-sections of Li and Machleidt [14].

II. EFFECTIVE FORCES WITH FINITE RANGE.

Following ref. [8], we propose a modification of Skyrme interactions by introducing a cut-off factor $C (| < q^2 > |)$ in the two-body interaction matrix elements of the form,

$$< ij|v|kl >= < ij|v_S|kl > C (| < ij|q^2|kl > |)$$

where $v_S$ represents $t_0$ and $t_3$ terms of the Skyrme interaction,

$$v_S = t_0 (1 + x_0 P_σ) δ (r) + \frac{1}{6} t_3 (1 + x_3 P_σ) [ρ (R)]^α δ (r)$$

with $r = r_1 - r_2$ and $R = (r_1 + r_2)/2$. We assume that the cut-off factor $C$ is a function of a matrix $< ij|q^2|kl >$, which is determined by the expression,

$$< ij|δ(r)|kl >= < ij|q^2|kl > = < ij|q^2δ(r) + δ(r)q^2|kl >$$
where $q = (p_1 - p_2)/2$ represents the relative momentum operator. This quantity provides a measure for the relative momentum in finite systems, and for the special case of nuclear matter, the function $C$ represents a cut-off in momentum space. The expression (1) is very convenient for practical applications, not only for nuclear matter but also for finite systems, since it does not introduce additional numerical effort as compared to Skyrme forces.

We can calculate the nucleon-nucleon cross-sections associated with the modified effective force (1). In the local density approximation, the expression of the cross-section is given by,

$$
\left( \frac{d\sigma}{d\Omega} \right)_{xy} = \frac{1}{4\pi} R_{xy} (\rho) \left[ C \left( |\langle q^2 \rangle| \right) \right]^2
$$

where $R_{xy} (\rho)$ account for the $t_0$ and $t_3$ part and involves density dependence, while $C$ comes from the cut-off factor. In this expression, the labels $xy$ denote the cross-section in the proton-neutron channel $(np)$, the proton-proton channel $(pp)$ and the neutron-neutron channel $(nn)$, which are given by [16],

$$
R_{pn} (\rho) = \frac{\pi m^2}{4\hbar (2\pi\hbar)^3} \left[ t_0 (1 - x_0) + \frac{t_3}{6} (1 - x_3) \rho^a \right]^2 + 3 \left[ t_0 (1 + x_0) + \frac{t_3}{6} (1 + x_3) \rho^a \right]^2
$$

and

$$
R_{pp} (\rho) = \frac{\pi m^2}{4\hbar (2\pi\hbar)^3} \left[ t_0 (1 - x_0) + \frac{t_3}{6} (1 - x_3) \rho^a \right]^2
$$

with a similar expression for the neutron-neutron cross-section. The spin-isospin averaged total cross-sections is given by $\sigma_{tot} = (\sigma_{nn} + \sigma_{pp} + 2\sigma_{pn})/4$, where $\sigma_{xy} = 2\pi \int \sin \Theta d\Theta (d\sigma/d\Omega)_{xy}$ denotes the total cross-section in the corresponding channel. In the following, we refer to the density-dependent term associated with the total cross-section as $R_{tot}$.

The choice of the cut-off $C$ factor is not unique. For simplicity, we may parameterize the cut-off in terms of Gaussian or exponential functions. These cut-off functions, respectively, lead to a Gaussian and Breit-Wigner shapes in the momentum space,

$$
C(q, q') = e^{-\beta^2 (q^2 + q'^2)/\hbar^2}
$$
\( \mathcal{C}(q, q') = \frac{1}{1 + \beta^2 (q^2 + q'^2)/\hbar^2} \)

where \( q = (p_1 - p_2)/2 \) and \( q' = (p_3 - p_4)/2 \) represent the relative momenta before and after collisions. For elastic collisions, we have \( q = q' \), thus the energy is given by \( E_{lab} = 2q^2/m \).
FIG. 1. Spin-isospin averaged cross-section as function of energy at the normal nuclear density. Left panel: the result obtained from the SkM$^*$ force (solid line) and Gogny force (dashed line) are compared with the microscopic cross-sections of Li and Machleidt (circles). Right panel: total cross-sections calculated for the SkM$^*$ force with bare nucleon mass, using a Gaussian cut off (thick line) and a Breit-Wigner cut-off (thick line). In both cases, the results with $\beta = 0.5$ fm (dashed line), $\beta = 0.7$ fm (solid line) and 1 fm (dot-dashed line) are presented.

Figure 1 shows a typical example of energy dependence of in-medium cross-sections at normal nuclear matter density $\rho_0$. The results on the right panel are obtained with the SkM$^*$ force \cite{15} and using a Gaussian (thin lines) or a Breit-Wigner (thick lines) cut-off for different values of $\beta$. These results are compared to the microscopic calculations of Li and Machleidt, which are shown by circles. Left panel of figure 1 presents the energy dependence of cross-sections that are obtained using the full Skyrme SkM$^*$ force and the Gogny force \cite{3}. From this figure, we first remark that the standard effective forces, like Skyrme or Gogny forces, provide a poor approximation for the energy dependence of the nucleon-nucleon cross-sections. We, also, see that calculations with the Gaussian or the Breit-Wigner cut-off are considerably improved and provide an approximate description for the cross-sections of Li and Machleidt \cite{14}. However, a cut-off factor with a single $\beta$ value is not able to reproduce the behavior of the cross section for a wide-range of energy. For example, with a Gaussian factor, $\beta = 1$ fm gives a good approximation for the low energy part, while $\beta = 0.7$ fm is better for the high energy part.
In order to improve the description of the cross-section in the whole energy range, we propose the following cut-off factor,

$$\mathcal{C}(q, q') = \frac{1 + \beta_1^2(q^2 + q'^2)/\hbar^2}{1 + \beta_2^2(q^2 + q'^2)/\hbar^2}$$  \hspace{1cm} (9)
| Density | $R_{tot}(\rho)$ | $\beta_1$ | $\beta_2$ |
|---------|----------------|-----------|-----------|
| $\rho_0$ | $pp$ | 243.2 | 1.15 | 5.15 |
|        | $np$ | 209.7 | 0.28 | 1.49 |
|        | $tot$ | 226.5 | 0.60 | 2.80 |
| $\rho_0/2$ | $pp$ | 212.6 | 0.52 | 2.38 |
|        | $np$ | 417.9 | 0.31 | 1.87 |
|        | $tot$ | 315.3 | 0.37 | 2.03 |
| $\rho_0/3$ | $pp$ | 197.2 | 0.36 | 1.64 |
|        | $np$ | 585.4 | 0.38 | 2.29 |
|        | $tot$ | 391.3 | 0.38 | 2.09 |

**TABLE I.** Parameters $\beta_1$ and $\beta_2$ associated with the SLy4 force, which are determined by fitting the microscopic cross-sections of Li and Machleidt at different densities. We also present the $R_{tot}(\rho)$ values for the same force at different densities.
\begin{table}
\centering
\begin{tabular}{lccc}
\hline
Density & $R_{tot}(\rho)$ & $\beta_1$ & $\beta_2$ \\
\hline
$\rho_0$ & $pp$ & 47.8 & 0.05 & 0.28 \\
 & $np$ & 324.9 & 0.61 & 3.18 \\
 & $tot$ & 186.4 & 0.44 & 2.03 \\
$\rho_0/2$ & $pp$ & 103.2 & 0.12 & 0.63 \\
 & $np$ & 506.3 & 0.45 & 2.60 \\
 & $tot$ & 304.8 & 0.35 & 1.92 \\
$\rho_0/3$ & $pp$ & 141.4 & 0.18 & 0.88 \\
 & $np$ & 620.4 & 0.42 & 2.52 \\
 & $tot$ & 380.9 & 0.35 & 1.91 \\
\hline
\end{tabular}
\caption{Same as table I for the SkM* force.}
\end{table}
By comparing the calculations with this cut-off factor with the microscopic \((pp)\) and \((np)\) cross-sections, we deduce the best corresponding \(\beta_1\) and \(\beta_2\) coefficients for each channel and for different nuclear densities. The extracted values of \(\beta_1\) and \(\beta_2\), that are associated with SLy4, and SkM* forces, are reported in table I and table II, respectively. We observe from these tables that for diluted systems the coefficients are less sensitive to nuclear density. Here, we consider the spin-isospin averaged cross-sections. For this purpose, by averaging over spin-isospin channel and also over different densities, we deduce average values of \(\beta_1 = 0.48\) fm and \(\beta_2 = 2.41\) fm for the SLy4 force, and \(\beta_1 = 0.29\) fm and \(\beta_2 = 1.66\) fm for the SkM* force. In table III, we present different parameters for modified Skyrme forces, which are indicated by \((\text{SLy4})^{\text{cut}}\) and \((\text{SkM*})^{\text{cut}}\).

![Figure 2](image)

**FIG. 2.** Spin-isospin averaged cross-section as a function of energy at different nuclear densities. Left panel: thick lines show the result from the modified force \((\text{SLy4})^{\text{cut}}\) with the new cut-off factor with \(\beta_1 = 0.39\) fm and \(\beta_2 = 2.05\) fm and a comparison with the cross-sections of Li and Machleidt indicated by circles. Right panel: same as left panel for the modified force SkM* force with \(\beta_1 = 0.29\) fm and \(\beta_2 = 1.66\) fm. In figure, thin and dashed lines show the results of the full SLy4 Skyrme force and the Gogny force, respectively.
Figure 2 shows the total nucleon-nucleon cross-sections as a function of bombarding energy at nuclear densities $\rho = \rho_0$, $\rho = \rho_0/2$ and $\rho = \rho_0/3$. The thick lines show the result of our calculations, left panel for (SLy4)$^{\text{cut}}$ and right panel for (SkM$^{*}$)$^{\text{cut}}$, with the parameters reported in table III. As seen from the figure, our calculations perfectly match the in-medium cross-sections of Li and Machleidt, which are indicated by circles. Figure 3 presents the total cross-section as a function of nuclear density at energies $E_{\text{lab}} = 50, 100, 250\, \text{MeV}$. Again, a perfect agreement with the microscopic cross-sections is found for a wide range of nuclear density. In figures 2 and 3, dashed lines and thin lines show the results of the Gogny force and the full SLy4 force, respectively.

![FIG. 3. Spin-isospin averaged cross-sections as a function of density at different energies. The results for the (SkM$^{*}$)$^{\text{cut}}$ force, the (SLy4)$^{\text{cut}}$ force and cross-sections of Li and Machleidt are shown by crosses, circles and solid lines, respectively. Dashed lines and thin lines lines represents the results obtained with the full Skyrme forces, Sly4 and SkM*, respectively.](image-url)
TABLE III. Summary of parameters for the modified Skyrme forces ($\text{SLy}4^\text{cut}$ and ($\text{SkM}^*\text{cut}$).

|       | $t_0$  | $t_3$   | $x_0$ | $x_3$ | $\alpha$ | $\beta_1$ | $\beta_2$ |
|-------|--------|---------|-------|-------|----------|-----------|----------|
| ($\text{SLy}4^\text{cut}$) | -2488.91 | 13777   | 0.834 | 1.354 | $\frac{1}{6}$ | 0.39 | 2.05 |
| ($\text{SkM}^*\text{cut}$)  | -2645   | 15595   | 0.09  | 0.0   | $\frac{1}{6}$ | 0.29 | 1.66 |

III. CONCLUSION

In this paper, in order to provide an approximate description of the in-medium nucleon-nucleon cross-sections, we discuss a modification of the effective Skyrme interactions by introducing a cut-off factor for large momentum transfer. We show that neither Gaussian nor Breit-Wigner cut-off in momentum space are appropriate for this purpose. We propose a new parameterization of the cut-off function and determine its parameters by fitting the microscopic in-medium cross-sections of Li and Machleidt, which interpolate correctly between the free-space and the medium and provide the best available in-medium cross-sections. The proposed modification approximately takes account for the finite range effect of the residual interactions, and provides a practical but realistic tool for application of transport models, in which the short range correlations are incorporated in the form of a binary collision term.

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