Application of Inertia Ellipse in Code Marker Matching

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Abstract In close-range photogrammetry, 3D information acquisition is based on image matching. The application of code marker helps to improve the level of automatic matching and the matching accuracy. This paper investigates the application of inertia ellipse algorithm to code marker matching. We can calculate the inertia ellipse of a target with a certain boundary. First, the method is applied to a single code marker; the angle and scaling are valid. Then, the paper introduces the multi code markers matching method by the inertia ellipse. Rotation and scaling changes of homonymy images can be calculated by inertia ellipse algorithm. These parameters can be used for code marker matching in arbitrary attitude close-range photogrammetry.

Keywords close-range photogrammetry; inertia ellipse; code marker matching

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Introduction

In the absence of texture of the detected objects, the code markers help to improve the level of automatic matching and the matching accuracy. Simultaneously, convergent photography is often applied to improve mapping accuracy. The code marker to be measured is photographed from different angles and directions. Rotation, geometric distortion and gradation distortion of code marker are caused by the change of angle, orientation and distance of photography. These factors make code marker matching more difficult. For the digital close-range photogrammetry, the theory and realization of code marker matching is the key of automatic 3D visual measurement. Least squares image matching is a method of high precise image correlation, which utilizes the gray distribution of the image. If the object-gray is different, the area based image matching result is difficult. Feature based matching (primitive-based matching, in computer vision) utilizes the feature and their relation to matching the homologous points, e.g., bridging mode. It has become a hotspot to utilize geometry characteristics of homologous points for matching, but the elements of relative orientation or absolute orientation must be known in advance. Without any known parameters, the rotation angle and scaling of code marker is a favorable guarantee for reliable matching. Ellipse matching method, which calculates the moment of inertia to build the equivalent ellipse, rotation and scaling parameters of the object, are obtained by

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the two principal axes. Ellipse matching is especially useful for face detection, moving vehicle, video, etc. [4-8]. This paper researches the method of inertia ellipse algorithm for code-marker matching, and the experiments prove that the method is feasible.

1  Inertia ellipse principle [6,9]

Ellipse matching refers to the concept of rigid body inertia moment. It can express and describe targets on an image. The lengths and orientations of the two principal axes can provide information about the scaling and rotation of the object, respectively. The parameters of inertia moment can meet the geometric transform.

1.1  Calculating the inertia ellipse of matching targets

In geostatics, the inertia of a rigid body in rotation is ensured by its moment of inertia. If a rigid body is composed of \( N \) particles, whose qualities are \( m_1, m_2, \ldots, m_n \) and coordinates are \((x_1, y_1, z_1), (x_2, y_2, z_2), \ldots (x_n, y_n, z_n)\), its moment of inertia \( I \) around the axis \( L \) is defined as:

\[
I = \sum_{i=1}^{N} m_i d_i^2
\]  

(1)

Where \( d_i \) is the vertical distance from particle \( m_i \) to the axis \( L \). Suppose that \( L \) passes the coordinate origin, and \( \alpha, \beta, \gamma \) are defined as its three direct cosines, then Eq. (1) can be rewritten as:

\[
I = A\alpha^2 + B\beta^2 + C\gamma^2 - 2F\beta\gamma - 2G\alpha\gamma - 2Ha\beta
\]

(2)

Where \( A, B \) and \( C \) are the moment of inertia of the rigid body in the \( X, Y \) and \( Z \) directions. They are defined as:

\[
A = \sum m_i y_i^2 + z_i^2, \quad B = \sum m_i z_i^2 + x_i^2, \quad C = \sum m_i x_i^2 + y_i^2
\]

\( F, G \) and \( H \) are the inertial integral of the three coordinate axes. They can be defined as:

\[
F = \sum m_i y_i z_i, \quad G = \sum m_i z_i x_i, \quad H = \sum m_i x_i y_i
\]

Eq. (2) can be explained in geometry:

\[
Ax^2 + By^2 + Cz^2 + 2Fyz - 2Gzx - 2Hxy = 1
\]

(3)

This is a cone, in which one of its centers is the origin of coordinate. Supposed that \( r \) is the vector from origin to cone, and \( \alpha, \beta, \gamma \) show their direct cosine of the three coordinate axes. According to Eq. (2) and (3), the Eq. (4) can be obtained.

\[
r^2(A\alpha^2 + B\beta^2 + C\gamma^2 - 2F\beta\gamma - 2G\alpha\gamma - 2Ha\beta) = r^2 I = 1
\]

(4)

Where \( I \) always has a positive value, so \( r \) is limited. That is to say, the surface is closed. Therefore, this conical surface must be an ellipsoid called inertia ellipse. It has three mutually perpendicular principal axes of inertia. For an inertia ellipse with uniform weight, two principal axes with coplanar profile is an ellipse called inertia ellipse. The 2D image can be regarded as a plane rigid body, the inertial ellipse of all the parts can be obtained, which reflects the spatial distribution of the target on the image.

The inertia ellipse is determined by the direction and the length of the two principal axes. The direction of the mainshaft can be calculated by the computational method of linear algebraic eigenvalue. The slopes of two principal axes (\( k \) and \( l \)) are:

\[
k = \frac{1}{2H}[(A - B) - \sqrt{(A - B)^2 + 4H^2}]
\]

(5)

\[
l = \frac{1}{2H}[(A - B) + \sqrt{(A - B)^2 + 4H^2}]
\]

(6)

The lengths of semi-long-principal axes (\( p \) and \( q \)) can be defined by:

\[
p = \sqrt{2/[(A + B) + \sqrt{(A - B)^2 + 4H^2}]}\]

(7)

\[
q = \sqrt{2/[(A + B) - \sqrt{(A - B)^2 + 4H^2}]}\]

(8)

1.2  Equivalent ellipse of matching targets

On the basis of the target’s inertia ellipse, we can calculate an equivalent ellipse of it for matching. All the translating, rotating and scaling parameters should be considered in matching. To acquire these parameters, it is necessary to define the center coordinate, the orientation angle (the angle between the long-mainshaft of the ellipse and the forward direction of the axis-\( X \)) and the long-mainshaft length of the equivalent ellipse.

First, to make the equivalent ellipse represent the spatial location of the target, define the barycentre of the target as the center coordinate (\( x_c, y_c \)) of the equivalent ellipse:

\[
x_c = \frac{1}{N} \sum_{i=1}^{N} x_i; \quad y_c = \frac{1}{N} \sum_{i=1}^{N} y_i
\]

(9)
Then, the orientation angle $\Phi$ of the equivalent ellipse is:

$$\Phi = \begin{cases} 
\tan^{-1}(k), & \text{if } A < B \\
\tan^{-1}(l), & \text{if } A > B 
\end{cases}$$

Lastly, to make the equivalent ellipse’s shape parameters the same as the inertia ellipse’s, the lengths of two semi-long-principal axes ($a$ and $b$) of the equivalent ellipse are proportional to those ($p$ and $q$) of the inertia ellipse. To make the equivalent ellipse approximate to the original target, the length of the mainshaft should be normalized using the target area $M$. Therefore, when $A < B$, the length of semi-long-mainshaft $a$ of the equivalent ellipse is:

$$a = \sqrt{2[(A + B) - \sqrt{(A - B)^2 + 4H^2}]} / M$$

## 2 Inertia ellipse matching approach

We use a common digital camera to take pictures in any photographic gesture in the experiment, and we did not calibrate the camera and the camera parameters we know nothing of.

### 2.1 Single code marker

We calculate inertia ellipse parameters of the single code marker. Figs.1(a) and 1(b) are the same code marker, (c) and (d) are the same code marker. The inertia ellipse parameters can be seen in Table 1. We rotate Fig.1(a) 31° and Fig.1(c) 22° (clockwise), and then overlap Fig.1(b) and Fig.1(d). The proportion of semi-long-principal axes $a$ of the equivalent ellipse between Fig.1(a) and Fig.1(b) is 1.03, Fig.1(c) and Fig.1(d) is 1.02.

### Table 1 Parameters of inertia ellipse

| Parameter | Fig.1(a) | Fig.1(b) | Fig.1(c) | Fig.1(d) |
|-----------|----------|----------|----------|----------|
| $A$       | 873929   | 537058   | 276239   | 2428983  |
| $B$       | 1511399  | 2291469  | 2790881  | 3012360  |
| $H$       | -502387  | 64992    | -325616  | -276515  |
| $k$       | 1.8187   | -27.0313 | 1.0443   | 2.5084   |
| $l$       | -0.5498  | 0.0370   | -0.9576  | -0.3987  |
| $p$       | 7.4793e-004 | 6.6026e-004 | 5.6772e-004 | 5.6590e-004 |
| $q$       | 0.0013   | 0.0014   | 6.3877e-004 | 6.5671e-004 |
| $\phi$    | -28.8036 | 2.1186   | -43.5299 | -21.7352 |
| $a$       | 30.5300  | 29.6907  | 51.2733  | 50.4024  |

### 2.2 Multi code markers

(1) Hough transform circle detection is carried out on two images. We get the coordinates and radii of circles of code markers. See Fig.2.

(2) According to the center point and radius of the code marker, we intercept the square window of three times radius length for each one code marker for every image.

(3) We calculate inertia ellipse and equivalent ellipse parameters of each code marker of the two images. Code markers are sorted by the long axle length of equivalent ellipse on Image1. The largest axle of code marker of sorting is taken. Similarly, the largest axle of code markers is obtained on Image 2. We rotate the code marker window of Image 2 toward the orientation of Image 1, then we judge whether they are the same code marker by correlative coefficient. If they are not the same, we select the next larger axle of code marker on Image 2 to match. See Fig.3.

(4) According to the position (152,184) (357,378) and (235,99) (275,373) of the two code markers on Image 1 and Image 2, we can rotate Image 1 38° (clockwise) and reduce 98% to match the Image 2. Matching results of code marker are shown in Fig.4.

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![Fig.1 Code marker](image1.png)

![Fig.2 Code marker circle coordinates](image2.png)
3 Discussion and conclusion

In close-range photogrammetry, rotation, scaling and distortion in images caused by the change of shooting picture angle, orientation and distance make image matching more difficult. In this paper, inertia ellipse is used for matching of code marker. The code-markers are matched correctly and the rotating and scaling parameters between two images are calculated. These parameters are valuable to automatic close-range photogrammetric code marker matching.

Although the application of inertia ellipse in code marker matching has achieved encouraging results, several priorities for future research remain.

● The first priority relates to matching according to the long axle length of equivalent ellipse order. The long axle length of different circular markers is nearly equal. We can rotate the code marker window of Image 2 toward the orientation of Image 1 repeatedly.

● The second research priority relates to code markers of larger deformation. There are significant differences in their long axle length of equivalent ellipse. We can find a better feature to replace the long axle length of equivalent ellipse.

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