Robust Task Clustering for Deep Many-Task Learning

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Abstract
We investigate task clustering for deep-learning based multi-task and few-shot learning in a many-task setting. We propose a new method to measure task similarities with cross-task transfer performance matrix for the deep learning scenario. Although this matrix provides us critical information regarding similarity between tasks, its asymmetric property and unreliable performance scores can affect conventional clustering methods adversely. Additionally, the uncertain task-pairs, i.e., the ones with extremely asymmetric transfer scores, may collectively mislead clustering algorithms to output an inaccurate task-partition. To overcome these limitations, we propose a novel task-clustering algorithm by using the matrix completion technique. The proposed algorithm constructs a partially-observed similarity matrix based on the certainty of cluster membership of the task-pairs. We then use a matrix completion algorithm to complete the similarity matrix. Our theoretical analysis shows that under mild constraints, the proposed algorithm will perfectly recover the underlying “true” similarity matrix with a high probability. Our results show that the new task clustering method can discover task clusters for training flexible and superior neural network models in a multi-task learning setup for sentiment classification and dialog intent classification tasks. Our task clustering approach also extends metric-based few-shot learning methods to adapt multiple metrics, which demonstrates empirical advantages when the tasks are diverse.

1 Introduction
This paper leverages knowledge distilled from a large number of learning tasks [1, 2], or \textit{M}any \textit{T}ask \textit{L}earning (\textsc{MatL}), to achieve the goals of (1) improving the overall performance of all tasks, as in \textit{m}ulti-\textit{t}ask \textit{l}earning (\textsc{Mtl}); and (2) rapid-adaptation to a new task by using previously learned knowledge, similar to \textit{f}ew-shot \textit{l}earning (\textsc{Fsl}) and transfer learning. Previous work on multi-task learning and transfer learning used small numbers of related tasks (usually ∼10) picked by human experts. By contrast, \textsc{MatL} tackles hundreds or thousands of tasks [1, 2], with unknown relatedness between pairs of tasks, introducing new challenges such as task diversity and model inefficiency. \textsc{MatL} scenarios are increasingly common in a wide range of machine learning applications with potentially huge impact.

Examples include reinforcement learning for game playing – where many numbers of sub-goals are treated as tasks by the agents for joint-learning, e.g. [2] achieved the state-of-the-art on the Ms. Pac-Man game by using a multi-task learning architecture to approximate rewards of over 1,000 sub-goals (reward functions). Another important example is corporate AI cloud services – where many clients submit various tasks/datasets to train machine learning models for business-specific purposes. The clients could be companies who want to know opinion from their customers on products and services, agencies that monitor public reactions to policy changes, and financial analysts who analyze news as it can potentially influence the stock-market. Such \textsc{MatL}-based services thus need to handle the diverse nature of clients’ tasks.

Task-clustering is a popular technique in \textsc{MatL} to handle the task diversity. Task-clustering seeks to share common knowledge across similar tasks within a cluster, while preserving task-specific knowledge among different clusters, given a carefully designed metric of task similarities. Standard task similarities include model parameter similarity (e.g., [3, 4, 5]), and co-training objectives (e.g., [6, 1]). However, these

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are mainly defined on convex models, leveraging the fact that the global optimal solution could serve as a unique model signature. Moreover, these methods usually cannot handle tasks with varying sets of class labels, which is common in real-world scenarios. We adopt a different measure of task similarity based on cross-task transfer performance. The cross-task transfer performance is a matrix whose \((i, j)\)-entry is the estimated performance \(S_{ij}\) of the \(i\)-th (source) task model in the \(j\)-th (target) task. Based on this measure, various numbers of labels among different tasks could be handled by transferring the common representation function, e.g., in text classification, only the representation function of the \(i\)-th model that encodes a sentence to a hidden representation could be transferred to the \(j\)-th task.

Although cross-task transfer performance can provide critical information of task similarities, we cannot directly use it as input to conventional clustering methods, because: (i) the matrix \(S\) is asymmetric due to the fact that the performance of transferring task \(i\) to task \(j\) is usually different from that of transferring task \(j\) to task \(i\); (ii) estimated cross-task performance is often unreliable due to small data size or inaccurate class labels, and (iii) more importantly, it is hard to decide whether we should put uncertain task-pairs in the same cluster, i.e., the pairs of tasks \((i, j)\) in which the transfer performance \(S_{ij} (S_{ji})\) is high but \(S_{ji} (S_{ij})\) is low. When the number of the uncertain task pairs is large, they can collectively mislead the clustering algorithm to output an incorrect task-partition.

To address the aforementioned challenges, we propose a novel task-clustering algorithm based on the theory of matrix completion \([7]\). Given a set of \(n\) tasks, the basic idea is to first construct a partially-observed \(n \times n\) similarity matrix, where the observed entries are generated based on reliable task pairs, i.e., the pairs of tasks \((i, j)\) with both \(S_{ij}\) and \(S_{ji}\) are either high enough or low enough. We then complete this generated partially-observed matrix using a robust matrix completion approach, and generate the final task partition by applying spectral clustering to the completed similarity matrix. In this way, the proposed approach has a 3-fold advantage. First, by filtering out uncertain task pairs, the proposed algorithm will be less sensitive to noise, leading to a more robust performance. Second, by converting an asymmetric transfer performance matrix to a symmetric similarity matrix, many widely-used similarity-based clustering algorithms (like spectral clustering), can be applied for task-partitioning. Third, our method carries a strong theoretical guarantee, showing that the full similarity matrix can be perfectly recovered if the number of observed correct entries in the partially observed similarity matrix is at least \(O(n \log^2 n)\).

Our results show that proposed method can discover task-clusters for flexible neural network models giving us significantly better multi-task learning algorithms on sentiment classification and intent classification tasks. Our task-clustering approach also extends metric-based few-shot learning methods to adapt multiple metrics, which demonstrates empirical advantage in a diverse task setting.

2 Related Work

**Task/Dataset Clustering on Model Parameters** This class of task clustering methods measure the task relationships in terms of model parameter similarities on individual tasks. Given the parameters of convex models, task clusters and cluster assignments could be derived via matrix decomposition \([3]\) or k-means based approach \([4]\). The parameter similarity based task clustering method for deep neural networks \([5]\) applied low-rank tensor decomposition of the model layers from multiple tasks. This method is infeasible for our MATL setting because of its high computation complexity with respect to the number of tasks and its inherent requirement on closely related tasks because of its parameter-similarity based approach.

**Task/Dataset Clustering on Training Objectives** Another class of task clustering methods joint assign task clusters and train model parameters for each cluster that minimize training loss within each cluster by K-means based approach \([6]\) or minimize overall training loss combined with sparse or low-ranker regularizers with convex optimization \([1][8]\). Deep neural networks have flexible representation power and they may overfit to arbitrary cluster assignment if we consider training loss alone. Also, these methods require identical class label sets across different tasks, which does not hold in most of the real-world MATL settings.

**Few Shot Learning** FSL \([9][10]\) aims to learn classifiers for new classes with only a few training examples per class. Bayesian Program Induction \([11]\) represents concepts as simple programs that best explain observed examples under a Bayesian criterion. Siamese neural networks rank similarity between inputs \([12]\). Matching Networks \([13]\) maps a small labeled support set and an unlabeled example to its label, obviating the need for fine-tuning to adapt to new class types. These approaches essentially learn one metric for
Figure 1: The convolutional neural networks used in this work: (a) a single-task CNN; (b) a multi-task CNN (MTL-CNN). The encoder component takes the sentence as input and outputs a fixed-length sentence embedding vector, and the classifier component predicts class labels with the sentence embedding.

3 Challenges on Learning with Heterogeneous Tasks

Previous multi-task learning and few-shot learning research usually work on homogeneous tasks, e.g. all tasks are binary classification problems, or tasks are close to each other (picked by human experts) so the positive transfer between tasks is guaranteed. With a large number of tasks in a many-task setting, the above assumption may not hold. Therefore it is necessary for the MATL algorithms to adapt to heterogeneous tasks. Specifically, in this paper we hope to handle the following challenges:

Tasks with Varying Numbers of Labels When tasks are heterogeneous, different tasks could have different numbers of labels; and the labels might be defined in different label spaces without relatedness. Most of the existing multi-task and few-shot learning methods will fail in this setting.

Tasks with Positive and Negative Transfers Since tasks are not guaranteed to be similar to each other in the many-task setting, they are not always able to help each other when trained together. This problem is called negative transfer between tasks. From our observation, one important reason for negative transfer is the Differences between Conditional Distributions, which means that the heterogeneous tasks could vary in terms of the conditional distribution $P(y|x)$, instead of the input distribution $P(x)$, where $x$ is the input and $y$ is the label. For example, in dialog services, the sentences “What fast food do you have nearby” and “Could I find any Indian food” may belong to two different classes “fast_food” and “indian_food” for a restaurant recommendation service in a city; while for a travel-guide service for a park, those two sentences could belong to the same class “food_options”, as illustrated in Figure 1(b). In this case the two tasks could hurt each other when trained jointly with a single representation function, since the first task turns to give similar representations to both sentences while the second one turns to distinguish them in the representation space.

Our solution to the above problem is shown in Section 4, where we deal with the negative transfer problem with a novel task clustering algorithm; and we handle the problem of varying numbers of labels with (1) evaluation of task similarity based on the transferability of learned representations during task clustering; and (2) metric-learning and nearest neighbor based classifier when we need to handle new tasks in the few-shot learning setting.
4 Methodology

We have a set of $n$ tasks $\mathcal{T} = \{T_1, T_2, \cdots, T_n\}$, each task $T_i$ consists of a train/validation/test data split $\{D_i^{\text{train}}, D_i^{\text{valid}}, D_i^{\text{test}}\}$. We consider text classification tasks, comprising labeled examples $\{(x, y)\}$, where the input $x$ is a sentence or document (a sequence of words) and $y$ is the label. We first train each classification model $M_i$ on its training set $D_i^{\text{train}}$, which yields a set of models $\mathcal{M} = \{M_1, M_2, \cdots, M_n\}$. We use convolutional neural network (CNN), which has reported results near state-of-the-art on text classification [15] [16]. CNNs also train faster than recurrent neural networks [17], making large-$n$ MTL scenarios more feasible. Figure 1 shows the CNN architecture. Following [18, 15], the model consists of a convolution layer and a max-pooling operation over the entire sentence. The model has two parts: an encoder part and a classifier part. Hence each model $M_i = \{M_i^{\text{enc}}, M_i^{\text{cls}}\}$. The above broad definitions encompass other classification tasks (e.g. image classification) and other classification models (e.g. LSTMs [17]).

We propose a task-clustering framework for both multi-task learning (MTL) and few-shot learning (FSL) settings. In this framework, we have the MTL and FSL algorithms summarized in Algorithm 1 & 2, where our task-clustering framework serves as the initial step in both algorithm.

4.1 Cross-Task Transfer-Performance Matrix Estimation

Using single-task models, we can compute performance scores $s_{ij}$ by adapting each $M_i$ to each task $T_j (j \neq i)$. This forms an $n \times n$ pair-wise classification performance matrix $S$, called the transfer-performance matrix. Note that $S$ is asymmetric since usually $s_{ij} \neq s_{ji}$.

When all tasks have identical label sets, we can directly evaluate the model $M_i$ on the training set of task $j$, $D_j^{\text{train}}$, and use the accuracy as the cross-task transfer score $s_{ij}$.

When tasks have different label sets, we freeze the encoder $M_i^{\text{enc}}$ from $M_i$, on top of which we use $D_j^{\text{train}}$ to train a classifier layer. This gives us a new task $j$ model, and we test this model on $D_j^{\text{valid}}$ to get the accuracy as the transfer-performance $s_{ij}$. The score shows how the representations learned on task $i$ can be adapted to task $j$, thus indicating the similarity between tasks.

Algorithm 1: ROBUST-TC-MLT: Multi-Task Learning based on Task Clustering

**Input**: A set of $n$ tasks $\mathcal{T} = \{T_1, T_2, \cdots, T_n\}$; number of clusters $K$

**Output**: $K$ task clusters $C_{1,K}$ and cluster-models $\Lambda = \{\Lambda_1, \Lambda_2, \cdots, \Lambda_K\}$

1. **Robust Task Clustering**: $C_{1,K} = \text{ROBUSTTC}(T, K)$ (Algorithm 3)

2. **Cluster-Model Training**: Train one multi-task model $\Lambda_i$ on each task cluster $C_i$ (Section 4.3)

Algorithm 2: ROBUST-TC-FSL: Task Clustering for Few-Shot Learning

**Input**: $N$ training tasks $\mathcal{T} = \{T_1, T_2, \cdots, T_n\}$; number of clusters $K$; target few-shot learning task $T_{trg}$

**Output**: A classification model for the target task $M_{trg}$, $K$ task clusters $C_{1,K}$ and cluster-models $\Lambda = \{\Lambda_1, \Lambda_2, \cdots, \Lambda_K\}$

1. **Learning Cluster-Models on Training Tasks**: $C_{1,K}, \Lambda = \text{ROBUSTTC-MLT}(T, K)$ (Algorithm 1)

2. **Few-Shot Learning on Cluster-models**: Train a model $M_{trg}$ on task $T_{trg}$ with the method in Section 4.3

Algorithm 3: ROBUSTTC: Robust Task Clustering based on Matrix Completion

**Input**: A set of $n$ tasks $\mathcal{T} = \{T_1, T_2, \cdots, T_n\}$, number of task clusters $K$

**Output**: $K$ task clusters $C_{1,K}$

1. **Learning of Single-Task Models**: Train single-task models $M_i$ for each task $T_i$

2. **Evaluation of Transfer-Performance Matrix**: get performance matrix $S$ (Section 4.1)

3. **Score Filtering**: Filter the uncertain scores in $S$ and construct symmetric matrix $Y$ with Eq. (4)

4. **Matrix Completion**: Complete the similar matrix $X$ from $Y$ with Eq. (2)

5. **Task Clustering**: $C_{1,K} = \text{SpectralClustering}(X, K)$
4.2 Robust Task Clustering by Matrix Completion

To address the uncertainty of task pairs, we propose a novel task clustering approach based on matrix completion [7]. Given a set of $n$ tasks, the key idea of the proposed algorithm is to first construct a partially observed $n \times n$ similarity matrix, where only the entries associated with the reliable task relationships are marked as observed. We then apply a matrix completion approach to fill up the unobserved entries. Finally, spectral clustering [19] is applied to the completed similarity matrix to generate the final clusters. Below, we describe our algorithm (summarized in Algorithm 3) in detail.

First, we use only reliable task pairs to generate a partially observed similarity matrix. Specifically, if $S_{ij}$ and $S_{ji}$ are high enough, then it is likely that tasks $\{i, j\}$ belong to a same cluster and share significant information. Conversely, if $S_{ij}$ and $S_{ji}$ are low enough, then they tend to belong to different clusters. To this end, we need to design a mechanism to determine if a performance is high or low enough. Since different tasks may vary in difficulty, a fixed threshold is not suitable. Hence, we define a dynamic threshold using the mean and standard deviation of the target task performance, i.e., $\mu_j = \text{mean}(S_{ij})$ and $\sigma_j = \text{std}(S_{ij})$, where $S_j$ is the $j$-th column of $S$. We then introduce two positive parameters $p_1$ and $p_2$, and define high and low performance as $S_{ij}$ greater than $\mu_j + p_1 \sigma_j$ or lower than $\mu_j - p_2 \sigma_j$, respectively. When both $S_{ij}$ and $S_{ji}$ are high enough, we set their pairwise similarity to 1, and when both $S_{ij}$ and $S_{ji}$ are low enough, we set their pairwise similarity to 0. Other task pairs are treated as uncertain task pairs and are marked as unobserved, with no influence on our clustering method, leading to a more robust clustering performance. The resulting partially observed similarity matrix $Y$ is given by:

$$Y_{ij} = \begin{cases} 
1 & \text{if } S_{ij} > \mu_j + p_1 \sigma_j \text{ and } S_{ji} > \mu_i + p_1 \sigma_i \\
0 & \text{if } S_{ij} < \mu_j - p_2 \sigma_j \text{ and } S_{ji} < \mu_i - p_2 \sigma_i \\
\text{unobserved} & \text{otherwise}
\end{cases}$$

(1)

Note that the generated similarity matrix $Y$ is now symmetric, unlike the asymmetric matrix $S$.

Given the partially observed matrix $Y$, we can reconstruct the full similarity matrix $X \in \mathbb{R}^{n \times n}$. We first note that the similarity matrix $X$ should have a low-rank (proof deferred to appendix). Additionally, since the observed entries of $Y$ are generated based on high and low enough performance, it is safe to assume that most observed entries are correct and only a few would be incorrect. So, we introduce a sparse matrix $E$ to capture the observed incorrect entries in $Y$. Combining the two observations, $Y$ can be decomposed into the sum of two matrices $X$ and $E$, where $X$ is a low rank matrix storing similarities between task pairs, and $E$ is a sparse matrix that captures the errors in $Y$. The matrix reconstruction problem can be cast as a convex optimization problem

$$\min_{X, E} \|X\|_* + \lambda \|E\|_1 \quad \text{s.t.} \quad \mathbf{P}_\Omega(X + E) = \mathbf{P}_\Omega(Y),$$

(2)

where $\|\cdot\|_*$ denotes nuclear norm, the convex surrogate of rank function. $\Omega$ is the set of observed entries in $Y$, and $\mathbf{P}_\Omega: \mathbb{R}^{n \times n} \mapsto \mathbb{R}^{n \times n}$ is a matrix projection operator defined as

$$[\mathbf{P}_\Omega(A)]_{ij} = \begin{cases} 
A_{ij} & \text{if } (i,j) \in \Omega \\
0 & \text{otherwise}
\end{cases}$$

The following theorem shows the perfect recovery guarantee for the problem $[2]$. \footnote{Proof deferred to supplementary document.}

**Theorem 4.1.** Let $X^* \in \mathbb{R}^{n \times n}$ be a rank $k$ matrix with a singular value decomposition $X^* = U \Sigma V^T$, where $U = (u_1, \ldots, u_k) \in \mathbb{R}^{n \times k}$ and $V = (v_1, \ldots, v_k) \in \mathbb{R}^{n \times k}$ are the left and right singular vectors of $X^*$, respectively. Similar to many related works of matrix completion, we assume that the following two assumptions are satisfied:

1. The row and column spaces of $X$ have coherence bounded above by a positive number $\mu_0$.
2. Max absolute value in matrix $UV^T$ is bounded above by $\mu_1 \sqrt{r}/n$ for a positive number $\mu_1$.

Suppose that $m_1$ entries of $X^*$ are observed with their locations sampled uniformly at random, and among the $m_1$ observed entries, $m_2$ randomly sampled entries are corrupted. Using the resulting partially observed matrix as the input to the problem $[3]$, then with a probability at least $1 - n^{-3}$, the underlying matrix $X^*$ can be perfectly recovered.
1. \( \mu(E) \xi(X) \leq \frac{1}{4k+5} \),

2. \( \frac{\xi(X) - (2k-1)\mu(E) \xi(X)}{1-2(k+1)\mu(E) \xi(X)} < \lambda < \frac{1-(4k+5)\mu(E) \xi(X)}{(k+2)\mu(E)} \),

3. \( m_1 - m_2 \geq C[\max(\mu_0, \mu_1)]^4 n \log^2 n \),

where \( C \) is a positive constant; \( \xi(\cdot) \) and \( \mu(\cdot) \) denotes the low-rank and sparsity incoherence \[20\].

Theorem 4.1 implies that even if some of the observed entries computed by (1) are incorrect, problem (2) can still perfectly recover the underlying similarity matrix \( X^* \) if the number of observed correct entries is at least \( O(n \log^2 n) \). For MATL with large \( n \), this implies that only a tiny fraction of all task pairs is needed to reliably infer similarities over all task pairs. Moreover, the completed similarity matrix \( X \) is symmetric, due to symmetry of the input matrix \( Y \). This enables analysis by similarity-based clustering algorithms, such as spectral clustering.

### 4.3 Multi-Task Learning Based on Tasks Clusters

For each cluster \( C_k \), we train a model \( \Lambda_k \) with all tasks in that cluster to encourage parameter sharing. We call \( \Lambda_k \) the cluster-model. When evaluated on the MTL setting, with sufficient data to train a task-specific classifier, we only share the encoder part and have distinct task-specific classifiers (Figure 1(b)). These task-specific classifiers provide flexibility to handle varying number of labels.

### 4.4 Few-Shot Learning Based on Tasks Clusters

We only have access to a limited number of training samples in few-shot learning setting, so it is impractical to train well-performing task-specific classifiers as in the multi-task learning setting. Instead, we make the prediction of a new task by linearly combining prediction from learned clusters.

\[
p(y|x) = \sum_k \alpha_k P(y|x; \Lambda_k).
\]

where \( \Lambda_k \) is the learned (and frozen) model of the \( k \)-th cluster, \( \{\alpha_k\}_{k=1}^K \) are adaptable parameters.

We use some alternatives to train cluster-models \( \Lambda_k \), which could better suit (and is more consistent to) the above FSL method\[2\]. When all tasks have identical label sets, we train a single classification model on all the tasks like in previous work \[1\], the predictor \( P(y|x; \Lambda_k) \) is directly derived from this cluster-model. When tasks have different label sets, we train a metric-learning model like \[13\] among all the tasks in \( C_k \), which consist a shared encoding function \( \Lambda_k^{enc} \) aiming to make each example closer to examples with the same label compared to the ones with different labels.

Then we use the encoding function to derive the predictor by

\[
P(y = y_l|x; \Lambda_k) = \frac{\exp \{ \Lambda_k^{enc}(x_l) ^\top \Lambda_k^{enc}(x) \}}{\sum_{l'} \exp \{ \Lambda_k^{enc}(x_{l'}) ^\top \Lambda_k^{enc}(x) \}}
\]

where \( x_l \) is the corresponding training sample for label \( y_l \).

### 5 Experiments

#### 5.1 Experiment Setup

**Data** We test our methods by conducting experiments on three text classification datasets. In the data-preprocessing step we used NLTK toolkit\[4\] for tokenization. For MTL setting, all tasks are used for clustering and model training. For FSL setting, the task are divided into training tasks and testing tasks (target tasks), where the training tasks are used for clustering and model training, the testing tasks are few-shot learning ones used to for evaluating the method in Eq. 3.

\[2\] We tried these alternatives under the MTL settings, which perform worse than MTL-CNN.

\[3\] http://www.nltk.org/
1. **Amazon Review Sentiment Classification** First, following [1], we construct a multi-task learning setting with the multi-domain sentiment classification [21] data set. The dataset consists of Amazon product reviews for 23 types of products (see Appendix 3 for the details). For each domain, we construct three binary classification tasks with different thresholds on the ratings: the tasks consider a review as positive if it belongs to one of the following buckets: 5 stars, >=4 stars or >=2 stars. These review-buckets then form the basis of the task-setup for MATL, giving us $23 \times 3 = 69$ tasks in total. For each domain we distribute the reviews uniformly to the three tasks. For evaluation, we select tasks from 4 domains (Books, DVD, Electronics, Kitchen) as the target tasks (12 tasks) out of all 23 domains. For FSL evaluation, we create five-shot learning tasks on the selected target tasks. The cluster-models for this evaluation are standard CNNs Figure 1(a), and we share the same output layer to evaluate the probability in Eq. (3) as all tasks have the same number of labels.

2. **Diverse Real-World Tasks: User Intent Classification for Dialog System** The second dataset is from an on-line service which trains and serves intent classification models to various clients. The dataset comprises recorded conversations between human users and dialog systems in various domains, ranging from personal assistant to complex service-ordering or a customer-service request scenarios. During classification, intent-labels are assigned to user utterances (usually sentences). We use a total of 175 tasks from different clients, and randomly sample 10 tasks from them as our target tasks. For each task, we randomly sample 64% data into a training set, 16% into a validation set, and use the rest as the test set (see Appendix 3 for details). The number of labels for these tasks vary from 2 to 100. Hence, to adapt this to a FSL scenario, we keep one example for each label (one-shot), plus 20 randomly picked labeled examples to create our training data. We believe this is a fairly realistic estimate of labeled examples one client could provide easily. Since we deal with various number of labels in the FSL setting, we chose matching networks [13] as the cluster-models.

3. **Extra-Large Number of Real-World Tasks** Similar to the second dataset, we further collect 1,491 intent classification tasks from the on-line service. This setting is mainly used to verify the robustness of our task clustering method, since it is difficult to estimate the full transfer-performance matrix $S$ in this setting (1,491 $^2$–2.2M entries). Therefore, in order to extract task clusters, we randomly sample task pairs from the data set to obtain 100,000 entries in $S$, which means that only about $100K/2.2M \approx 4.5\%$ of the entries in $S$ are observed. The number of 100,000 is chosen to be close to $n \log^2 n$ in our theoretical bound in Theorem 4.1 so that we could also verify the tightness of the bound empirically. To make the best use of the sampled pairs, in this setting we modified the Eq. 1 so that each entry $Y_{ij} = 1$ if $S_{ij} \geq \mu_j$ and $Y_{ij} = 0$ otherwise. In this way we could have determined number of entries in $Y$ as well, since all the sampled pairs will correspond to observed (but noisy) entries in $Y$. We only run MTL setting on this dataset.

**Baselines** For MTL setting, we compare our method to the following baselines: (1) **single-task CNN**: training a CNN model for each task individually; (2) **holistic MTL-CNN**: training one MTL-CNN model (Figure 1(b)) on all tasks; (3) **holistic MTL-CNN (target only)**: training one MTL-CNN model on all the target tasks. For FSL setting, the baselines consist of: (1) **single-task CNN**: training a CNN model for each task individually; (2) **single-task FastText**: training one FastText model [22] with fixed embeddings for each individual task; (3) **Fine-tuned the holistic MTL-CNN**: fine-tuning the classifier layer on each target task after training initial MTL-CNN model on all training tasks; (4) **Matching Network**: a metric-learning based few-shot learning model trained on all training tasks. We initialize all models with pre-trained 100-dim Glove embeddings (trained on 6B corpus) [23].

As the intent classification tasks usually have various numbers of labels, to our best knowledge the proposed method is the only one supporting task clustering in this setting; hence we only compare with the above baselines. Since sentiment classification involves binary labels, we compare our method with state-of-the-art logistic regression based task clustering method (ASAP-MT-LR) [1]. We also

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4Data downloaded from [http://www.cs.jhu.edu/~mdredze/datasets/sentiment/](http://www.cs.jhu.edu/~mdredze/datasets/sentiment/) unavailable due to their ambiguous nature [21].

5In conversational dialog systems, intent-labels are used to guide the dialog-flow.
try another approach where we run our MTL/FSL methods on top of the (ASAP-Clus-MTL/FSL) clusters (as their entire formulation is only applicable to convex models).

**Hyper-Parameter Tuning** In all experiments, we set both $p_1$ and $p_2$ parameters in (1) to 0.5. This strikes a balance between obtaining enough observed entries in $Y$, and ensuring that most of the retained similarities are consistent with the cluster membership. For MTL settings, we tune parameters like the window size and hidden layer size of CNN, learning rate and the initialization of embeddings (random or pre-trained) based on average accuracy on the union of all tasks’ dev sets, in order to find the best identical setting for all tasks. Finally we have the CNN with window size of 5 and 200 hidden units. The learning rate is selected as 0.001; and all MTL models use random initialized word embeddings on sentiment classification and use Glove embeddings as initialization on intent classification, which is likely because the training sets of the intent tasks are usually small. We also used the early stopping criterion based on the previous condition.

For the FSL setting, hyper-parameter selection is difficult since there is no validation data (which is a necessary condition to qualify as a $k$-shot learning). So, in this case we preselect a subset of training tasks as validation tasks and tune the learning rate and training epochs (for the rest we follow the best setting from the MTL experiments) on the validation tasks. During the testing phase (i.e. model training on the target FSL tasks), we fix the selected hyper-parameter values for all the algorithms.

**Out-of-Vocabulary in Transfer-Performance Evaluation** In text classification tasks, transferring an encoder with fine-tuned word embeddings from one task to another may not work as there can be a significant difference between the vocabularies. Hence, while learning the single-task models (line 1 of Algorithm 3) we always use the CNNs with fixed set of pre-trained embeddings.

### 5.2 Sentiment Classification on Amazon Product Reviews

#### Improving Observed Tasks (MTL Setting)

Table 1 shows the results of the 12 target tasks when all 69 tasks are used for training. Since most of the tasks have a significant amount of training data, the single-task baselines achieve good results. Because the conflicts among some tasks (e.g. the 2-star bucket tasks and 5-star bucket tasks require opposite labels on 4-star examples), the holistic MTL-CNN does not show accuracy improvements compared to the single-task methods. It also lags behind the holistic MTL-CNN model trained only on 12 target domains, which indicates that the holistic MTL-CNN cannot leverage large number of background tasks. Our ROBUSTTC-MTL method based on task clustering achieves a significant improvement over all the baselines.

The ASAP-MTLMR (best score achieved with five clusters) could improve single-task linear models with similar merit of our method. However, it is restricted by the representative strength of linear models so the overall result is lower than the deep learning baselines.

#### Adaptation to New Tasks (FSL Setting)

Table 1(b) shows the results on the 12 five-shot tasks by leveraging the learned knowledge from the 57 previously observed tasks. Due to the limited training resources, all the baselines perform poorly.
Table 2: Accuracy on the 10 dialog intent classification tasks.

(a) MTL setting (i.e. training on all 175 tasks).

| Model                                | Avg Acc |
|--------------------------------------|---------|
| Single-task CNN                     | 58.47   |
| Holistic MTL-CNN                    | 62.42   |
| Holistic MTL-CNN (target only)      | 62.45   |
| **ROBUSTTC-MTL**                     |         |
| clus=10                             | 64.41   |
| clus=20                             | **68.11**|
| clus=30                             | 66.74   |

(b) FSL setting (one-shot + 20 examples).

| Model                                | Avg Acc |
|--------------------------------------|---------|
| Single-task CNN w/pre-trained emb    | 34.46   |
| Single-task FastText w/pre-trained emb| 23.87   |
| Fine-tuned holistic MTL-CNN          | 30.36   |
| Matching Network [13]                | 25.14   |
| **ROBUSTTC-FSL**                     |         |
| clus=10                             | 34.64   |
| clus=20                             | **37.59**|
| clus=30                             | 36.82   |
| clus=num_of_tasks (no clustering)    | 34.43   |
| Adaptive ROBUSTTC-FSL (clus=20)      | **42.97**|

Our ROBUSTTC-FSL gives far better results compared to all baselines (>6%). It is also significantly better than applying Eq. (3) without clustering (78.85%), i.e. using single-task model from each task instead of cluster-models for \( P(y|x; \cdot) \).

Comparison to the ASAP Clusters  Our clustering-based MTL and FSL approaches also work for the ASAP clusters, in which we replace our task clusters with the task clusters generated by ASAP-MTLR. In this setting we get a slightly lower performance compared to the ROBUSTTC-based ones on both MTL and FSL settings, but overall it performs better than the baseline models. This show that, apart from the ability to handle varying number of class labels, our ROBUSTTC model can also generate better clusters for MTL/FSL of deep networks, even under the setting where all tasks have same number of labels.

It is worth to note that from Table 1(a), training CNNs on the ASAP clusters gives better results compared to training logistic regression models on the same 5 clusters (86.07 vs. 85.17), despite that the clusters are not optimized for CNNs. Such result further emphasizes the importance of task clustering for deep models, when better performance could be achieved with such models.

5.3 User Intent Classification from Diverse Real-World Online Services

Table 2(a) & (b) show the MTL & FSL results on dialog intent classification, which demonstrates trends similar to the sentiment classification tasks. Note that the holistic MTL methods achieve much better results compared to single-task CNNs. This is because the tasks usually have smaller training and development sets, and both the model parameters learned on training set and the hyper-parameters selected on development set can easily lead to over-fitting. ROBUSTTC-MTL achieves large improvement (5.5%) over the best MTL baseline, because the tasks here are more diverse than the sentiment classification tasks and task-clustering greatly reduces conflicts from irrelevant tasks.

Although our ROBUSTTC-FSL improves over baselines under the FSL setting, the margin is smaller. This is because of the huge diversity among tasks – by looking at the training accuracy, we found several tasks failed because none of the clusters could provide a metric that suits the training examples. To deal with this problem, we hope that the algorithm can automatically decide whether the new task belongs to any of the task-clusters. If the task doesn’t belong to any of the clusters, it would not benefit from any previous knowledge, so it should fall back to single-task CNN. The new task is treated as “out-of-cluster” when none of the clusters could achieve higher than 20% accuracy (selected on dev tasks) on its training data. We call this method Adaptive ROBUSTTC-FSL, and it gives more than 5% performance boost over the best ROBUSTTC-FSL result.

Discussion on Clustering-Based FSL  The single metric based FSL method (Matching Network) achieved success on homogeneous few-shot tasks like Omniglot and miniImageNet [13] but performs poorly in both of our experiments. This indicates that it is important to maintain multiple metrics for few-shot learning problems with more diverse tasks, similar to the few-shot NLP problems investigated in this paper. Our clustering-based FSL approach maintains diverse metrics while keeping the model simple with a small number of \( K \) parameters. It is worthwhile to study how and why the NLP problems make few-shot learning more difficult/heterogeneous; and how well our method can generalize to non-NLP problems like miniImageNet. We will leave these topics for future work.
5.4 Large-Scale User Intent Classification with Task-Pair Sampling

Table 3 shows the MTL results on the extra-large dialog intent classification dataset. Compared to the results on the 175 tasks, the holistic MTL-CNN achieves larger improvement (6%) over the single-task CNNs, which is a stronger baseline. Similar as the observation on the 175 tasks, here the main reason for its improvement is the consistent development and test performance due to holistic multi-task training approach: both the single-task and holistic multi-task model achieve around 66% average accuracy on development sets. Unlike the experiments in Section 5.3, we did not evaluate the full transfer-performance matrix $S$ (which is time consuming), but only a subset corresponding to $\sim 4.5\%$ of all task-pairs. However, significant improvement is achieved by applying the RobustTC-MTL algorithm. Note that this result is achieved by sampling about $n \log^2 n$ task pairs, so it not only confirms the empirical advantage of our multi-task learning algorithm, but also verifies the correctness of our theoretical bound in Theorem 4.1.

Table 3: Accuracy of Multi-Task Learning on the 1,491 dialog intent classification tasks.

| Model               | Avg Acc |
|---------------------|---------|
| Single-task CNN     | 60.49   |
| Holistic MTL-CNN    | 66.42   |
| **ROBUSTTC-MTL**    |         |
| clus=30             | 69.62   |
| clus=40             | **70.50** |
| clus=50             | 69.87   |

6 Conclusion

In this paper, we propose a robust task-clustering method that not only has strong theoretical guarantees but also demonstrates significantly empirical improvements when equipped by our MTL and FSL algorithms. Our empirical studies verify that (i) the proposed task clustering approach is very effective in the many-task learning setting especially when tasks are diverse, and (ii) cross-task transfer performance can serve as a powerful task similarity measure. Our work opens up many future research directions like supporting on-line many-task learning with computation on similarities between small proportion of task pairs (based on our theoretical results). It is also worth investigating the combination of our clustering approach with the recent learning-to-learn methods (e.g. [14]). This will help us enhance our MTL and FSL methods by learning meta-learner for each task cluster.

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Appendix A: Proof of Low-rankness of Matrix X

We first prove that the full similarity matrix $X \in \mathbb{R}^{n \times n}$ is of low-rank. To see this, let $A = (a_1, \ldots, a_k)$ be the underlying perfect clustering result, where $k$ is the number of clusters and $a_i \in \{0,1\}^n$ is the membership vector for the $i$-th cluster. Given $A$, the similarity matrix $X$ is computed as

$$X = \sum_{i=1}^k a_i a_i^\top = \sum_{i=1}^k B_i$$

where $B_i = a_i a_i^\top$ is a rank one matrix. Using the fact that $\text{rank}(X) \leq \sum_{i=1}^k \text{rank}(B_i)$ and $\text{rank}(B_i) = 1$, we have $\text{rank}(X) \leq k$, i.e., the rank of the similarity matrix $X$ is upper bounded by the number of clusters. Since the number of clusters is usually small, the similarity matrix $X$ should be of low rank.

Appendix B: Proof of Theorem 4.1

We then prove our main theorem. First, we define several notations that are used throughout the proof. Let $X = UV^\top$ be the singular value decomposition of matrix $X$, where $U = (u_1, \ldots, u_k) \in \mathbb{R}^{n \times k}$ and $V = (v_1, \ldots, v_k) \in \mathbb{R}^{n \times k}$ are the left and right singular vectors of matrix $X$, respectively. Similar to many related works of matrix completion, we assume that the following two assumptions are satisfied:

1. **A1**: the row and column spaces of $X$ have coherence bounded above by a positive number $\mu_0$, i.e.,

$$\sqrt{n/r} \max_i \|P_U(e_i)\| \leq \mu_0 \text{ and } \sqrt{n/r} \max_i \|P_V(e_i)\| \leq \mu_0,$$

where $P_U = UU^\top$, $P_V = VV^\top$, and $e_i$ is the standard basis vector, and

2. **A2**: the matrix $UV^\top$ has a maximum entry bounded by $\mu_1 \sqrt{n/r}$ in absolute value for a positive number $\mu_1$.

Let $T$ be the space spanned by the elements of the form $u_i y^\top$ and $x v_i^\top$, for $1 \leq i \leq k$, where $x$ and $y$ are arbitrary $n$-dimensional vectors. Let $T^\perp$ be the orthogonal complement to the space $T$, and let $P_T$ be the orthogonal projection onto the subspace $T$ given by

$$P_T(Z) = P_U Z + ZP_V - P_U ZP_V.$$

The following proposition shows that for any matrix $Z \in T$, it is a zero matrix if enough amount of its entries are zero.

**Proposition 1.** Let $\Omega$ be a set of $m_0$ entries sampled uniformly at random from $[1, \ldots, n] \times [1, \ldots, n]$, and $P_\Omega(Z)$ projects matrix $Z$ onto the subset $\Omega$. If $m > m_0$, where $m_0 = C_\Omega^2 \mu_0 r \beta \log n$ with $\beta > 1$ and $C_\Omega$ being a positive constant, then for any $Z \in T$ with $P_\Omega(Z) = 0$, we have $Z = 0$ with probability $1 - 3n^{-\beta}$.

**Proof.** According to the Theorem 3.2 in [7], for any $Z \in T$, with a probability at least $1 - 2n^{2-2\beta}$, we have

$$\|P_T(Z)\|_F - \delta \|Z\|_F \leq \frac{n^2}{m} \|P_T P_\Omega(Z)\|_F^2 = 0 \quad (5)$$

where $\delta = m_0/m < 1$. Since $Z \in T$, we have $P_T(Z) = Z$. Then from (5), we have $\|Z\|_F \leq 0$ and thus $Z = 0$. \qed

In the following, we will develop a theorem for the dual certificate that guarantees the unique optimal solution to the following optimization problem

$$\min_{X, E} \quad \|X\|_* + \lambda \|E\|_1$$

subject to $P_\Omega(X + E) = P_\Omega(Y)$.

**Theorem 1.** Suppose we observe $m_1$ entries of $X$ with locations sampled uniformly at random, denoted by $\Omega$. We further assume that $m_2$ entries randomly sampled from $m_1$ observed entries are corrupted, denoted by $\Delta$. Suppose that $P_\Omega(Y) = P_\Omega(X + E)$ and the number of observed correct entries $m_1 - m_2 > m_0 =
Theorem 1. First, according to Theorem 1, when

\[ (a) \quad Q = P_\Omega(Q), \quad (b) \quad P_T(Q) = UV^T, \quad (c) \quad \|P_{T^\perp}(Q)\| < 1, \quad (d) \quad P_\Delta(Q) = \lambda \text{ sgn}(E), \quad \text{and (e) } \|P_{\Delta^\perp}(Q)\| < \lambda. \]

Proof. First, the existence of Q satisfying the conditions (a) to (e) ensures that \((X, E)\) is an optimal solution. We only need to show its uniqueness and we prove it by contradiction. Assume there exists another optimal solution \((X + N_X, E + N_E)\), where \(P_\Omega(N_X + N_E) = 0\). Then we have

\[
\|X + N_X\|_* + \lambda\|E + N_E\|_1 \geq \|X\|_* + \lambda\|E\|_1 + \langle Q_E, N_E \rangle + \langle Q_X, N_X \rangle
\]

where \(Q_E\) and \(Q_X\) satisfying \(P_\Delta(Q_E) = \lambda\text{ sgn}(E), \|P_{\Delta^\perp}(Q_E)\| \leq \lambda, P_T(Q_X) = UV^T\) and \(\|P_{T^\perp}(Q_X)\| \leq 1\). As a result, we have

\[
\lambda\|E + N_E\|_1 + \|X + N_X\|_* \\
\geq \lambda\|E\|_1 + \|X\|_* + \langle Q + P_{\Delta^\perp}(Q_E) - P_{\Delta^\perp}(Q), N_E \rangle + \langle Q + P_{T^\perp}(Q_X) - P_{T^\perp}(Q), N_X \rangle \\
= \lambda\|E\|_1 + \|X\|_* + \langle Q, N_E + N_X \rangle + \langle P_{\Delta^\perp}(Q_E) - P_{\Delta^\perp}(Q), N_E \rangle + \langle P_{T^\perp}(Q_X) - P_{T^\perp}(Q), N_X \rangle \\
= \lambda\|E\|_1 + \|X\|_* + (P_{\Delta^\perp}(Q_E) - P_{\Delta^\perp}(Q), P_{\Delta^\perp}(N_E)) + (P_{T^\perp}(Q_X) - P_{T^\perp}(Q), P_{T^\perp}(N_X))
\]

We then choose \(P_{\Delta^\perp}(Q_E)\) and \(P_{T^\perp}(Q_X)\) to be such that \(\langle P_{\Delta^\perp}(Q_E), P_{\Delta^\perp}(N_E) \rangle = \|P_{\Delta^\perp}(N_E)\|_1\) and \(\langle P_{T^\perp}(Q_X), P_{T^\perp}(N_X) \rangle = \|P_{T^\perp}(N_X)\|_*\). We thus have

\[
\lambda\|E + N_E\|_1 + \|X + N_X\|_* \\
\geq \lambda\|E\|_1 + \|X\|_* + (\lambda - \|P_{\Delta^\perp}(Q)\|_1)\|P_{\Delta^\perp}(N_E)\|_1 + (1 - \|P_{T^\perp}(Q)\|_1)\|P_{T^\perp}(N_X)\|_*
\]

Since \((X + N_X, E + N_E)\) is also an optimal solution, we have \(\|P_{\Omega^\perp}(N_E)\|_1 = \|P_{T^\perp}(N_X)\|_1\), leading to \(P_{\Omega^\perp}(N_E) = P_{T^\perp}(N_X) = 0\), or \(N_X \in T\). Since \(P_{\Omega}(N_X + N_E) = 0\), we have \(N_X = N_E + Z\), where \(P_{\Omega}(Z) = 0\) and \(P_{\Omega^\perp}(N_E) = 0\). Hence, \(P_{\Omega^\perp T\Omega}(N_X) = 0\), where \(|\Omega^\perp \cap \Omega| = m_1 - m_2\). Since \(m_1 - m_2 > m_0\), according to Proposition 1, we have, with a probability \(1 - 3n^{-3}\), \(N_X = 0\). Besides, since \(P_{\Omega}(N_X + N_E) = P_{\Omega}(N_E) = 0\) and \(\Delta \subset \Omega\), we have \(P_{\Delta}(N_E) = 0\). Since \(N_E = P_{\Delta}(N_E) + P_{\Delta^\perp}(N_E)\), we have \(N_E = 0\), which leads to the contradiction. 

Given Theorem 1 we are now ready to prove Theorem 3.1.

Proof. The key to the proof is to construct the matrix \(Q\) that satisfies the conditions (a)-(e) specified in Theorem 1. First, according to Theorem 1 when \(m_1 - m_2 > m_0 = C_2^\theta m_0 \log n\), with a probability at least \(1 - 3n^{-3}\), mapping \(P_T P_\Omega P_T(Z) : T \mapsto T\) is an one to one mapping and therefore its inverse mapping, denoted by \((P_T P_\Omega P_T)^{-1}\) is well defined. Similar to the proof of Theorem 2 in [20], we construct the dual certificate \(Q\) as follows

\[ Q = \lambda \text{ sgn}(E) + \epsilon_\Delta + P_\Delta P_T P_\Omega P_T^{-1}(UV^T + \epsilon_T) \]

where \(\epsilon_T \in T\) and \(\epsilon_\Delta = P_\Delta(\epsilon_\Delta)\). We further define

\[ H = P_\Omega P_T P_\Omega P_T^{-1}(UV^T) \]

\[ F = P_\Omega P_T P_\Omega P_T^{-1}(\epsilon_T) \]

Evidently, we have \(P_\Omega(Q) = Q\) since \(\Delta \subset \Omega\), and therefore the condition (a) is satisfied. To satisfy the conditions (b)-(e), we need

\[ P_T(Q) = UV^T \Rightarrow \epsilon_T = -P_T(\lambda \text{ sgn}(E) + \epsilon_\Delta) \]

\[ \|P_{T^\perp}(Q)\| < 1 \Rightarrow \mu(E)(\lambda + \|\Delta\|_\infty) + \|P_{T^\perp}(H)\| + \|P_{T^\perp}(F)\| < 1 \]

\[ P_\Delta(Q) = \lambda \text{ sgn}(E) \Rightarrow \epsilon_\Delta = -P_\Delta(H + F) \]

\[ \|P_{\Delta^\perp}(Q)\|_\infty < \lambda \Rightarrow \xi(X)(1 + \|\epsilon_T\|) < \lambda \]

Below, we will first show that there exist solutions \(\epsilon_T \in T\) and \(\epsilon_\Delta\) that satisfy conditions (7) and (9). We will then bound \(\|\epsilon_\Omega\|_\infty, \|\epsilon_T\|, \|P_{T^\perp}(H)\|, \text{ and } \|P_{T^\perp}(F)\|\) to show that with sufficiently small \(\mu(E)\) and \(\xi(X)\), and appropriately chosen \(\lambda\), conditions (8) and (10) can be satisfied as well.
First, we show the existence of $\epsilon_\Delta$ and $\epsilon_T$ that obey the relationships in \((\ref{2})\) and \((\ref{3})\). It is equivalent to show that there exists $\epsilon_T$ that satisfies the following relation

$$
\epsilon_T = -P_T(\lambda \text{sgn}(E)) + P_TP_\Delta(H) + P_TP_\Delta P_T(P_TP_\Omega P_T)^{-1}(\epsilon_T)
$$

or

$$
P_TP_\Omega P_T(P_TP_\Omega P_T)^{-1}(\epsilon_T) = -P_T(\lambda \text{sgn}(E)) + P_TP_\Delta(H),
$$

where $\Omega \setminus \Delta$ indicates the complement set of set $\Delta$ in $\Omega$ and $|\Omega \setminus \Delta|$ denotes its cardinality. Similar to the previous argument, when $|\Omega \setminus \Delta| = m_1 - m_2 > m_0$, with a probability $1 - 3n^{-\beta}$, $P_TP_\Omega P_T(Z) : T \mapsto T$ is an one to one mapping, and therefore $(P_TP_\Omega P_T(Z))^{-1}$ is well defined. Using this result, we have the following solution to the above equation

$$
\epsilon_T = P_TP_\Omega P_T(P_TP_\Omega P_T)^{-1}(-P_T(\lambda \text{sgn}(E)) + P_TP_\Delta(H))
$$

We now bound $\|\epsilon_T\|$ and $\|\epsilon_\Delta\|_\infty$. Since $\|\epsilon_T\| \leq \|\epsilon_T\|_F$, we bound $\|\epsilon_T\|_F$ instead. First, according to Corollary 3.5 in \((\ref{1})\), when $\beta = 4$, with a probability $1 - n^{-3}$, for any $Z \in T$, we have

$$
\|P_T\Delta P_T(P_T\Omega P_T)^{-1}(Z)\|_F \leq \|Z\|_F.
$$

Using this result, we have

$$
\|\epsilon_\Delta\|_\infty \leq \xi(X)(\|H\| + \|F\|) \leq \xi(X)(1 + \|P_T\Delta(H)\|_F + \|\epsilon_T\|_F + \|P_T\Delta(F)\|_F) \leq \xi(X)(2 + \|\epsilon_T\|_F) \leq \xi(X)(2 + (2k + 1)\|\epsilon_T\|_F)
$$

In the last step, we use the fact that $\text{rank}(\epsilon_T) \leq 2k$ if $\epsilon_T \in T$. We then proceed to bound $\|\epsilon_T\|$ as follows

$$
\|\epsilon_T\| \leq \mu(E)(\lambda + \|\epsilon_\Delta\|_\infty)
$$

Combining the above two inequalities together, we have

$$
\|\epsilon_T\| \leq \xi(X)\mu(E)(2k + 1)\|\epsilon_T\|_F + 2\xi(X)\mu(E) + \lambda\mu(E) \quad \|\epsilon_\Delta\|_\infty \leq \xi(X)(2 + (2k + 1)\mu(E)(\lambda + \|\epsilon_\Delta\|_\infty),
$$

which lead to

$$
\|\epsilon_T\| \leq \frac{\lambda\mu(E) + 2\xi(X)\mu(E)}{1 - (2k + 1)\xi(X)\mu(E)} \quad \|\epsilon_\Delta\|_\infty \leq \frac{2\xi(X)(2k + 1)\lambda\xi(X)\mu(E)}{1 - (2k + 1)\xi(X)\mu(E)}
$$

Using the bound for $\|\epsilon_\Delta\|_\infty$ and $\|\epsilon_T\|$, we now check the condition \((\ref{4})\)

$$
1 > \mu(E)(\lambda + \|\epsilon_\Delta\|_\infty) + \frac{1}{2} + \frac{k}{2}\|\epsilon_T\|
$$

or

$$
\lambda < \frac{1 - \xi(X)\mu(E)(4k + 5)}{\mu(E)(k + 2)}
$$

For the condition \((\ref{5})\), we have

$$
\lambda > \xi(X) + \xi(X)\|\epsilon_T\|
$$

or

$$
\lambda > \frac{\xi(X) - (2k - 1)\lambda^2\xi(X)\mu(E)}{1 - 2(k + 1)\xi(X)\mu(E)}
$$

To ensure that there exists $\lambda \geq 0$ satisfies the above two conditions, we have

$$
1 - 5(k + 1)\xi(X)\mu(E) + (10k^2 + 24k + 8)[(\xi(X)\mu(E))]^2 > 0
$$

14
and

\[ 1 - \xi(X)\mu(E)(4k + 5) \geq 0 \]

Since the first condition is guaranteed to be satisfied for \( k \geq 1 \), we have

\[ \xi(X)\mu(E) \leq \frac{1}{4k + 5}. \]

Thus we finish the proof. \( \square \)

Appendix C: Data Statistics

We listed the detailed domains of the sentiment analysis tasks in Table 4. We removed the musical_instruments and tools_hardware domains from the original data because they have too few labeled examples. The statistics for the 10 target tasks of intent classification in Table 5.

Table 4: Statistics of the Multi-Domain Sentiment Classification Data.

| Domains              | #train | #validation | #test |
|----------------------|--------|-------------|-------|
| apparel              | 7398   | 926         | 928   |
| automotive           | 601    | 69          | 66    |
| baby                 | 3405   | 437         | 414   |
| beauty               | 2305   | 280         | 299   |
| books                | 19913  | 2436        | 2489  |
| camera_photo         | 5915   | 744         | 749   |
| cell_phones_service  | 816    | 109         | 108   |
| computer_video_games | 2201   | 274         | 296   |
| dvd                  | 19961  | 2624        | 2412  |
| electronics          | 18431  | 2304        | 2274  |
| gourmet_food         | 1227   | 182         | 166   |
| grocery              | 2101   | 268         | 263   |
| health_personal_care | 5826   | 687         | 712   |
| jewelry_watches      | 1597   | 188         | 196   |
| kitchen_housewares   | 15888  | 1978        | 1990  |
| magazines            | 3341   | 427         | 421   |
| music                | 20103  | 2463        | 2510  |
| office_products      | 337    | 54          | 40    |
| outdoor_living       | 1321   | 143         | 135   |
| software             | 1934   | 254         | 202   |
| sports_outdoors      | 4582   | 566         | 580   |
| toys_games           | 10634  | 1267        | 1246  |
| video                | 19941  | 2519        | 2539  |

Table 5: Statistics of the User Intent Classification Data.

| Dataset ID | #labeled instances | #labels |
|------------|-------------------|---------|
| 1          | 497               | 11      |
| 2          | 3071              | 14      |
| 3          | 305               | 21      |
| 4          | 122               | 7       |
| 5          | 110               | 11      |
| 6          | 126               | 12      |
| 7          | 218               | 45      |
| 8          | 297               | 10      |
| 9          | 424               | 4       |
| 10         | 110               | 17      |