Theoretical uncertainties for weak decays: higher dimension operators

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Abstract

A brief review of recent results on computing contributions of higher dimension operators to weak effective $\Delta S = 1, 2$ hamiltonians for light quarks is presented.

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1. Introduction

I describe some of our recent results on computing contributions of higher dimension operators to effective $\Delta S = 1, 2$ hamiltonians within the Standard Model. These corrections are due to higher order terms in heavy quark mass expansion ($m_c^{-1}$) and require thorough numerical study before being safely neglected.

2. Corrections to $K \to \pi\pi$ decays: $\Delta S = 1$ effective hamiltonian.

The short-distance analysis after removing the $W$-boson and the heavy quarks results in the effective $\Delta S = 1$ hamiltonian of the following form [1, 2]

\[
H_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{6} [z_i(\mu) + \tau y_i(\mu) Q_i] + h.c. \tag{1}
\]

where $G_F$ is the Fermi constant, $V$ stands for the Cabibbo-Kobayashi-Maskawa mixing matrix, $\tau = -V_{td} V_{ts}^*/V_{ud} V_{us}^*$, $z_i(\mu)$ and $y_i(\mu)$ are Wilson coefficients, and $\{Q_i| i = 1, ..., 6\}$ is the basis of local operators of dimension six in mass units (four-quark operators).

In Eq. (1) contributions of electroweak penguin operators [3, 4] are omitted. For more precise comparison of theoretical predictions with experiment the existing analysis cannot be considered complete and following points require further investigation:

- Perturbation theory for kaon nonleptonic decays can be improved by using more accurate effective hamiltonian. Within the standard approach only leading terms in the inverse masses of heavy particles are kept while a proper account of nonleading corrections in the inverse mass of charmed quark [5] can be important. Further corrections to the effective hamiltonian appear because the top quark is heavier than other quarks and $W$-boson. This results in an incomplete GIM cancellation and in appearance of a new operator in the effective hamiltonian [6].

- High precision theoretical estimates for kaon decays stumble at a necessity to calculate mesonic matrix elements of local four-quark operators entering the effective hamiltonian [7]. The only method of computation entirely based on first principles seems to be numerical simulations on the lattice (see, e.g. [7]) though so far
even this approach has not given unambiguous and stable results. Several semi-
phenomenological techniques have been developed and applied for computation of
matrix elements, \textit{e.g.} \[8\], though the precision still need to be essentially improved.
Recently a regular method to evaluate the mesonic matrix elements has been ex-
plotted \[8\] where the effective hamiltonian is represented in terms of the chiral theory
variables \[10\] and parameters of the chiral representation are determined via QCD
sum rules \[11, 12, 13\] for an appropriate three-point Green’s function.

- Perturbation theory does not take into account soft light quarks and gluons with
small virtual momenta. They are entirely hidden in matrix elements of local four-
quark operators. Well known factorization procedure for evaluation of these matrix
elements \[14\] accounts only for the ”factorizable” part of the interaction \[15, 16, 17\].
”Unfactorizable” contributions, for example, those corresponding to annihilation
of a quark pair from the four-quark operator into soft gluons \[18\] are omitted.
The calculation of these contributions and the generalization of the matrix element
estimates beyond the factorization framework can be systematically done within the
approach of ref. \[19\].

After decoupling heavy particles (\(W\)-boson, \(t\)-, \(b\)-, and \(c\)-quarks) from the light sector of
the theory, Eq. (1) corresponds to the leading order in inverse masses of these particles.
The removal of the \(c\)-quark however is not very reliable and, in general, requires a special
investigation. It is not heavy enough in comparison with a characteristic mass scale in the
sector of light \(u\)-, \(d\)-, and \(s\)-quarks, for example, with the \(\rho\)-meson mass. The nonleading
terms in the \(1/m_c\) expansion can, therefore, be important and require a quantitative
consideration. To compute corrections of order \(1/m_c\), the tree level hamiltonian before
decoupling of the \(c\)-quark is used. It reads

\[
H^\text{tr}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (Q^u_2 - (1 - \tau)Q^c_2) + \text{h.c.} \tag{2}
\]

where \(Q^b_2 = 4(\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma_\mu d_L)\), \(q_L(R)\) stands for left(right) handed quark.
OPE and restricting oneself to the first order terms in $\alpha_s$ and $m_c^{-2}$ one finds

$$H_{\Delta S = 1} = H^{(6)} + H^{(8)}. \quad (3)$$

The first addendum in Eq. (3) $H^{(6)}$ corresponds to leading contributions in $1/m_c$ and coincides with Eq. (1). Second addendum in Eq. (3) is the $1/m_c$ correction $\bar{H}^{(8)}$.

$$H^{(8)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (1 - \tau) \frac{\alpha_s}{4 \pi m_c^2} \left( \sum_{i=1}^{7} C_i^{(8)} Q_i^{(8)} + \sum_{i=1}^{4} C_i^{(7)} m_s Q_i^{(7)} \right) + h.c. \quad (4)$$

where a basis $\{Q_i^{(8)}|i = 1, \ldots, 7\}$ ($\{Q_i^{(7)}|i = 1, \ldots, 4\}$) of the local operators with dimension eight (seven) in mass units is chosen in the form

$$Q_1^{(8)} = \bar{s}_L (\hat{D} G_{\mu \alpha} G_{\nu \mu} \sigma_{\alpha \nu} + G_{\nu \mu} \sigma_{\alpha \nu} \hat{D} G_{\mu \alpha}) d_L,$$
$$Q_2^{(8)} = ig_s \bar{s}_L (J_\mu \gamma_\alpha G_{\alpha \mu} - \gamma_\alpha G_{\alpha \mu} J_\mu) d_L,$$
$$Q_3^{(8)} = \bar{s}_L (P_\alpha G_{\mu \alpha} \gamma_\nu G_{\nu \mu} + \gamma_\nu G_{\nu \mu} G_{\mu \alpha} P_\alpha) d_L,$$
$$Q_4^{(8)} = g_s \bar{s}_L (G_{\mu \nu} \sigma_{\mu \nu} \hat{J} + \hat{J} G_{\mu \nu} \sigma_{\mu \nu}) d_L,$$
$$Q_5^{(8)} = i\bar{s}_L (G_{\mu \nu} \sigma_{\mu \nu} \gamma_\alpha G_{\alpha \beta} P_\beta - P_\beta \gamma_\alpha G_{\alpha \beta} G_{\mu \nu} \sigma_{\mu \nu}) d_L,$$
$$Q_6^{(8)} = \bar{s}_L (D^2 \hat{J}) d_L,$$
$$Q_7^{(8)} = i\bar{s}_L (\hat{D} G_{\nu \mu} G_{\nu \mu} - G_{\nu \mu} \hat{D} G_{\nu \mu}) d_L,$$
$$Q_1^{(7)} = \bar{s}_R (G_{\mu \nu} \sigma_{\alpha \beta} G_{\alpha \beta}) d_L,$$
$$Q_2^{(7)} = \bar{s}_R (G_{\mu \nu} G_{\nu \mu}) d_L,$$
$$Q_3^{(7)} = i\bar{s}_R (G_{\nu \mu} \alpha \beta \gamma_{\nu \mu}) d_L,$$
$$Q_4^{(7)} = \bar{s}_R (J_\mu P_\mu + P_\mu J_\mu) d_L. \quad (5)$$

Here $P_\mu = i\partial_\mu + g_s A_\mu$ is the momentum operator in the presence of external field $A_\mu \equiv A_\mu^a t^a$, $t^a$ are generators of the color group $SU(3)$, $G_{\mu \nu} \equiv G_{\mu \nu}^a t^a$ is the gluon field strength tensor, $J_\mu \equiv \sum_{q=u,d,s} (\bar{q} \gamma_\mu t^a q)$, and $\sigma_{\mu \nu} = i [\gamma_\mu, \gamma_\nu] / 2$. In derivation of Eq. (4) $u$ - and $d$-quark are considered massless, and the first order in strange quark mass was kept with the use of equations of motion $\bar{s} \hat{P} = m_s \bar{s}$, $\hat{P} d = 0$, $[P_\mu, G_{\mu \nu}] = i D_\mu G_{\mu \nu} = -ig_s J_\mu$. Straightforward calculation gives the following values for coefficients $C_i^{(j)}$ to the leading order in $\alpha_s$

$$C_2^{(8)} = -2C_1^{(8)} = 4C_3^{(8)} = -8C_4^{(8)} = 2C_6^{(8)} = 8C_7^{(8)} = -\frac{16}{15}, \quad C_5^{(8)} = 0,$$
\[ C_1^{(7)} = C_2^{(7)} = -\frac{2}{5}, \quad C_3^{(7)} = \frac{6}{5}, \quad C_4^{(7)} = 0. \]  

This generalizes the effective hamiltonian for \( \Delta S = 1 \) decays beyond the leading order in \( 1/m_c \) expansion.

To complete our treatment of the local effective hamiltonian we consider the case of a heavy top quark. GIM cancellation is not perfect and the quark-gluon operator \( m_s Q^{(5)} \) appears in the effective hamiltonian already in the first order in \( \alpha_s \). This additional contribution \([6]\) reads

\[ \Delta H^{(6)} = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{uq} C^{(5)}(\mu) m_s Q^{(5)}(\mu), \]

\[ C^{(5)}(\mu) = \frac{1}{16\pi^2} (F(x_c) - F(x_t)) \eta(\mu), \quad x_q = \frac{m_q^2}{M_W^2}, \]

\[ F(x_q) = \frac{1}{3} \frac{1}{(x_q - 1)^4} \left( \frac{5}{2} x_q^4 - 7 x_q^3 + \frac{39}{2} x_q^2 - 19 x_q + 4 - 9 x_q^2 \ln x_q \right), \]

\[ F(x_c) \sim F(0) = \frac{4}{3}. \]  

The renormalization group factor has the form

\[ \eta(\mu) = \left( \frac{\bar{\alpha}_s(m_b)}{\bar{\alpha}_s(M_W)} \right)^{\gamma^{(5)}/2\beta_5} \left( \frac{\bar{\alpha}_s(m_c)}{\bar{\alpha}_s(m_b)} \right)^{\gamma^{(5)}/2\beta_4} \left( \frac{\bar{\alpha}_s(\mu)}{\bar{\alpha}_s(m_c)} \right)^{\gamma^{(5)}/2\beta_3} \]

where \( \gamma^{(5)} = -28/3 \) is the anomalous dimension of the operator \( m_s Q^{(5)} \) \([20]\), \( \beta_{nf} = 11 - \frac{2}{3} n_f, n_f \) is the number of active quarks flavors.

Contributions of \( u \)- and \( c \)-quarks to the real part of the effective hamiltonian cancel each other via GIM mechanism and the operator \( m_s Q^{(5)} \) contributes to imaginary parts of amplitudes and, therefore, can be important in the analysis of direct CP violation. However, its contribution is suppressed numerically because \( \eta(\mu) < 1 \) and the function \( F(x) \) changes slowly. Indeed, at the point \( \Lambda_{QCD} = 0.3 \) GeV, \( \mu = 1 \) GeV, \( m_t = 130 \) GeV, one has \( C^{(5)} = 0.001 \), while the numerical value of the Wilson coefficient of the dominant penguin operator \( Q_6 = -8 \sum_{q=u,d,s} (\bar{s}_L q_R)(\bar{q}_R d_L) \) is \( y_6 = 0.1 \).

Thus, the complete form of the effective hamiltonian up to the first order in \( m_s, 1/m_c \) and \( \alpha_s \) with \( m_t \sim M_W \) is now available.
As an example of using the above Hamiltonian we consider $K \rightarrow \pi\pi$ decays. For this end we have to extract an information about the matrix elements of the local operators $Q^{(j)}$ between mesonic states. We start with operators $m_s Q_i^{(7)}$ and $m_s Q_i^{(5)}$. These operators contain explicitly the strange quark mass and, therefore, in the leading order of the chiral expansion they correspond to tadpole terms in the chiral weak Lagrangian [21] that do not generate any observable effect and can be neglected in the leading order of chiral symmetry breaking [22, 23].

Thus the problem is reduced to the estimation of the matrix elements of the operators $Q_i^{(8)}$. At present there is no regular method to calculate them within QCD except direct simulations on the lattice. To estimate at least the scale of nonleading $1/m_c$ corrections we work with a simplified model that uses factorization. One selects operators containing scalar quark currents which can be written as $(\bar{s}_L G_{\mu\nu} \sigma_{\mu\nu} q_R)(\bar{q}_R q_L)$ and $(\bar{s}_L q_R)(\bar{q}_R G_{\mu\nu} \sigma_{\mu\nu} d_L)$. This step seems to be justified because in the case of dimension six operators the similar "penguin-like" structures are strongly enhanced and dominate the others. The last simplification consists in the substitution $\bar{q} g_s G_{\mu\nu} \sigma_{\mu\nu} q \rightarrow m_0^2 \bar{q} q$ where $m_0^2$ determines the scale of nonlocality of the quark condensate and is defined by the equation $\langle \bar{q} g_s G_{\mu\nu} \sigma_{\mu\nu} q \rangle = m_0^2 \langle \bar{q} q \rangle$, $m_0^2 (1 \text{ GeV}) = 0.8 \pm 0.2 \text{ GeV}^2$ [24, 25]. This substitution is valid in the chiral limit for the operator $\bar{q} g_s G_{\mu\nu} \sigma_{\mu\nu} q$. We suppose that it is justified also in our case at least for estimates of the order of magnitude. All above assumptions about the factorization procedure in the case of dimension six operators become exact within the many color limit of QCD, $N_c \rightarrow \infty$, to the leading order in $N_c$ [26].

Thus, only operator $Q_i^{(8)}$ has a nonvanishing matrix element that reads

$$\langle \pi\pi | Q_i^{(8)} | K \rangle = \frac{m_0^2}{4} \langle \pi\pi | Q_6 | K \rangle.$$  

We should note that the coefficients $C_i^{(8)}$ are finite to the leading order in $\alpha_s$ and independent of renormalization scheme. We can therefore use the leading order values of mesonic matrix elements that is consistent up to the considered level of accuracy. The next-to-leading $\alpha_s$ corrections to Wilson coefficients depend on the renormalization scheme and
matching between them and mesonic matrix elements is necessary to make physical am-
plitudes independent of the renormalization scheme. Thus, account for the first order
$1/m_c$ corrections results in the shift of coefficients in front of the penguin operator $Q_6$
\begin{equation}
z_6 \to \left( z_6 + \frac{\alpha_s m_0^2}{4\pi 4m_c^2} C_4^{(8)} \right), \quad y_6 \to \left( y_6 - \frac{\alpha_s m_0^2}{4\pi 4m_c^2} C_4^{(8)} \right).
\end{equation}
Using the numerical values $z_6 = -0.015$, $y_6 = -0.102$ at the point $\Lambda_{\text{QCD}} = 0.3$ GeV,
$\mu = 1$ GeV, $m_t = 130$ GeV one finds numerically relative corrections to the Wilson
coefficients to be
\begin{align*}
z_6 &\to z_6(1 - 0.1), \\
y_6 &\to y_6(1 + 0.01).
\end{align*}
The main correction appears in the real part of the Wilson coefficient of the penguin
operator $Q_6$. Parametrically, the contribution of dimension eight operators can be as large
as 50\% ($m_0^2/m_c^2 \sim 0.5$) of the leading term. However much smaller value has been found
within the simplest factorization framework for meson matrix elements of nonleading
$1/m_c$ contributions to the kaon decay amplitudes. Large violation of the factorization for
matrix elements of dimension eight operators seems to be likely and the numerical value
of nonleading $1/m_c$ correction can be estimated only when a self-consistent method to
calculate these matrix elements within QCD will be available.

3. Corrections to $K^0 - \bar{K}^0$ mixing: $\Delta S = 2$ effective lagrangian.

The effective local $\Delta S = 2$ lagrangian for the $K^0 - \bar{K}^0$ mixing in the leading approx-
imation in the heavy charmed quark mass $m_c$ is well known \[27, 28\]. The corresponding
hadronic matrix element of the effective local $L_{\Delta S=2}$ lagrangian between $K^0$ and $\bar{K}^0$
states $\langle \bar{K}^0(k')|L_{\Delta S=2}|K^0(k) \rangle$ has been intensively studied during several last years with different
techniques (e.g. \[17, 19, 23, 31, 31, 32\]). However it has been pointed out in \[33\] that
the local effective hamiltonian does not exhaust the physics of $\Delta S = 2$ transitions. It
cannot account for the long distance contribution which is present in the initial Green’s
function for the matrix element of the $K^0 - \bar{K}^0$ mixing and is connected with the propa-
gation of the light $u$-quark round the loop of the box diagram. This contribution is purely
nonperturbative and ultimately depends on infrared properties of QCD.
Nevertheless there is one point which can be essentially improved just within perturbation theory for the standard model. It consists in the calculation of corrections in the inverse mass of charmed quark to the local part of the effective lagrangian [34]. These corrections are represented by local operators with dimension eight in mass units.

Because the $\Delta S = 1$ lagrangian has the form

$$L_{\Delta S=1} = \frac{G_F}{\sqrt{2}} J_{\mu} J_{\mu}^+,$$

(10)

where $J_{\mu} = \bar{Q}_L \gamma_{\mu} V q_L$ is the weak charged hadronic current, $Q = (u, c, t)^T$, $q = (d, s, b)^T$, the matrix element $M$ of the transition is represented by

$$\langle \bar{K}_0(k')|K_0(k)\rangle_{out} = i(2\pi)^4 \delta(k-k')M,$$

$$M = \frac{i}{2} \int dx \langle \bar{K}_0(k')|T L_{\Delta S=1}(x)L_{\Delta S=1}(0)|K_0(k)\rangle.$$

(11)

Eqs. (10-11) are valid for the $t$-quark much lighter than the $W$-boson. This is not the case anymore but for our purpose it is inessential and in the following we neglect the $t$-quark admixture and restrict ourselves to the simplified model with two generations.

The effective $\Delta S = 2$ lagrangian can be written in the form

$$L_{\Delta S=2} = \left(4G_F \sin \theta_c \cos \theta_c \right)^2 (L_H + L_L)$$

(12)

where $\theta_c$ is the Cabibbo angle. Here

$$L_H = i \int T_H(x)dx, \quad T_H = T_{cc} - T_{cu} - T_{uc}$$

is the heavy part of the whole effective $\Delta S = 2$ lagrangian containing loops with virtual heavy $c$-quark in the intermediate state, while

$$L_L = i \int T_L(x)dx, \quad T_L = T_{uu}$$

describes the light part of the transition. We introduced useful notations

$$T_{cu}(x) = T \bar{s}_L \gamma_{\alpha} u_L \bar{c}_L \gamma_{\alpha} d_L(x) \bar{s}_L \gamma_{\beta} c_L \bar{u}_L \gamma_{\beta} d_L(0)$$
and so on. Both $L_H$ and $L_L$ separately require some regularization because they are ultraviolet divergent. The dimensional regularization is not convenient in this case due to the presence of the $\gamma_5$-matrix. The Pauli-Willars regularization introduces a regulator mass that makes difficult to perform explicit calculations. We will use the regularization which is free of these shortcomings in our particular case. Namely, let me define the regularized quantities $L_{H,L}^R$ by the equation

$$L_{H,L}^R = i \int T_{H,L}(x)(-\mu^2 x^2)^\epsilon dx$$

where $\epsilon$ is a regularization parameter and $\mu$ represents the mass scale analogous to one of dimensional regularization.

Now for the heavy part of the effective $\Delta S = 2$ lagrangian $L_H$ we develop a regular expansion in the inverse charmed quark mass in the following form (from now on we omit the index " $R$" )

$$16\pi^2 L_H = C_0(\mu, m_c)O_0(\mu) + \sum_j C_j(\mu, m_c)O_j(\mu)$$

where $C_j$ are coefficient functions depending on the heavy quark mass $m_c$ and $O_j$ are the local operators built from the light ($u, d, s$) quark fields only. If we split the whole lagrangian into the sum

$$L_H = L_H^{(0)} + L_H^{(1)}$$

then $16\pi^2 L_H^{(0)} = -m_c^2(\bar{s}_L\gamma_\alpha d_L)^2$ is the well known result of Gaillard and Lee [14]. The rest part of Eq. (13) contains local operators which have dimension eight in mass units. A convenient form of the operator basis is

$$O_F = \bar{s}_L\gamma_\alpha d_L \bar{s}_L\gamma_\mu \bar{F}_{\mu \alpha} d_L, \quad O_A = \bar{s}_L\gamma(\mu D_\nu)d_L\bar{s}_L\gamma(\mu D_\nu)d_L,$$

$$O_B = \bar{s}_L\gamma_\mu D_\mu d_L\bar{s}_L\gamma_\nu D_\nu d_L,$$

$$O_C = \bar{s}_L\gamma_\alpha d_L\bar{s}_L(\gamma_\mu D_\mu D_\alpha + D_\alpha \gamma_\mu D_\mu)d_L - \frac{(m_s^2 + m_d^2)}{2}(\bar{s}_L\gamma_\alpha d_L)^2$$

where $\gamma(\mu D_\nu) = (\gamma_\mu D_\nu + \gamma_\nu D_\mu)/2$. The direct calculation gives

$$16\pi^2 L_H^{(1)} = -\frac{4}{3}(O_F + O_A) \left( \frac{1}{\epsilon} + \ln \left( \frac{4\mu^2 e^{-2\epsilon}}{m_c^2} \right) + \frac{4}{3} \right) - \frac{2}{3} O_A$$
where $C = 0.577\ldots$ is the Euler constant. After performing a renormalization (say, a minimal subtraction of the pole term) we will have the finite quantity and the parameter $\mu$ recalls the necessity to have the proper short distance contribution of $u$-quark. In this order of the expansion in $m_c^{-1}$ the dependence on this parameter is explicit contrary to the leading order that is finite and does not depend on $\mu$. The heavy and light parts of the whole lagrangian must be defined simultaneously in the coordinated way. The pole part of Eq. (14) is cancelled by the corresponding divergences of the light part due to GIM mechanism. Consider now the light part. The operator product expansion for the amplitude $T_L(x)$ in $x^2$ at $x^2 \to 0$ has the form

$$T_L(x) = \frac{1}{4\pi^4 x^6}(\bar{s}_L\gamma_\alpha d_L)^2 + \frac{1}{24\pi^4 x^4}(2\bar{O_F} + 2O_A + O_B + O_C).$$

(15)

The short distance contribution of the light part does cancel divergences of the heavy part. More technically we extract the short distance contribution of the light part by splitting the entire light part into a sum

$$L_L = i \int T_L(x)(-\mu^2 x^2)^\epsilon dx = i \int T_L(x)(f(x, \bar{x}) + \bar{f}(x, \bar{x}))-\mu^2 x^2)^\epsilon dx = L^{SH}_L + L^{LG}_L$$

(16)

where

$$L^{SH}_L = i \int T_L(x)f(x, \bar{x})(-\mu^2 x^2)^\epsilon dx, \quad L^{LG}_L = i \int T_L(x)\bar{f}(x, \bar{x})(-\mu^2 x^2)^\epsilon dx,$$

and $f(x, \bar{x}) + \bar{f}(x, \bar{x}) = 1$. The smooth generalization of the step $\theta$-function is chosen for functions $f(x, \bar{x})$ and $\bar{f}(x, \bar{x})$

$$f(x, \bar{x}) = \frac{\bar{x}^{2n}}{\bar{x}^{2n} + (-\bar{x}^2)^n}, \quad \bar{f}(x, \bar{x}) = \frac{(-x^2)^n}{\bar{x}^{2n} + (-\bar{x}^2)^n}$$

in such a way that the function $f(x, \bar{x})$ cuts out the short distances only (up to $\bar{x}$) and the function $\bar{f}(x, \bar{x})$ does the long distances. The short distance contribution of the light part is

$$16\pi^2 L^{SH}_L = (\bar{s}_L\gamma_\alpha d_L)^2 \frac{\pi/n}{\sin(\pi/n)} \frac{1}{\bar{x}^2} + \frac{2}{3} (2\bar{O_F} + 2O_A + O_B + O_C) \left(\frac{1}{\epsilon} + \ln \mu^2 \bar{x}^2\right).$$
Now the whole answer is

\[ 16\pi^2(L_H + L_L) = 16\pi^2 L_L^{\text{LG}} + (\bar{s}_L \gamma_\alpha d_L)^2 \left( -m_c^2 + \frac{\pi/n}{\sin(\pi/n)} \frac{4}{x^2} \right) \]

\[ - \frac{2}{3} (2O_\bar{F} + 2O_A + O_B + O_C) \left( \ln \left( 4 \frac{e^{-2C}}{m_c^2 x^2} \right) + \frac{4}{3} \right) - \frac{2}{3} O_A. \]

(17)

The long distance part of the whole light lagrangian \( L_L^{\text{LG}} \) cannot be calculated due to strong infrared problems and requires some low-energy model, lattice or chiral effective theory [35]. Numerical estimates for corrections depend on kaon-antikaon matrix elements of the local operators \( O_\bar{F} - O_C \) for which factorization was used

\[ \langle \bar{K}_0(k) | (\bar{s}_L \gamma_\alpha d_L)^2 | K_0^0(k) \rangle^{\text{fact}} = (1 + \frac{1}{N_c}) \left( \frac{f^2 K^2}{2} \right) \equiv Z, \]

\[ \langle \bar{K}_0(k) | O_F | K_0^0(k) \rangle^{\text{fact}} = -\delta^2 \left( \frac{f^2 K^2}{2} \right), \quad \langle \bar{K}_0(k') | O_A | K_0^0(k) \rangle^{\text{fact}} = -\delta^2 \frac{1}{N_c} \left( \frac{f^2 K^2}{2} \right) \]

where the parameter \( \delta^2 \) is defined by the relation [36]

\[ \langle 0 | \bar{s}_L \gamma_\mu \bar{F}_\mu \alpha | K_0^0(k) \rangle = -ik_\mu f_K \delta^2 \]

and all other matrix elements vanish. From Eq. (17) one finds

\[ 16\pi^2(L_H + L_L^{\text{SH}}) = Z (-m_c^2 + \frac{\pi/n}{\sin(\pi/n)} \frac{4}{3} \delta^2 (\ln \left( \frac{4 e^{-2C}}{m_c^2 x^2} \right) + \frac{4}{3} + \frac{1}{6} \delta^2). \]

Let me estimate \( \bar{x}^2 \). Eq. (15) gives

\[ T_L(x) = \frac{1}{4\pi^4 x^6} \left( 1 - \frac{\delta^2 x^2}{3} + o(x^2) \right). \]

(18)

Then at \( \delta^2 \bar{x}^2 = 3 \) where the expansion (18) blows up and at \( n = \infty \) we obtain the following representation for nonleading corrections

\[ -m_c^2 Z \left( 1 - \frac{4}{3} \frac{\delta^2}{m_c^2} - \frac{4}{3} \frac{\delta^2}{m_c^2} (\ln \left( \frac{4 \delta^2}{3 m_c^2} \right) - 2C + \frac{35}{24} \right). \]

Numerically, at \( \delta^2 = m_0^2/4 \) [28] we get

\[ -m_c^2 Z (1 - 0.2 + 0.3) = -m_c^2 Z (1 + 0.1). \]

Defining the value of \( \bar{x}^2 \) by the relation \( \delta^2 \bar{x}^2 = 1 \) we get

\[ -m_c^2 Z (1 - 0.5 + 0.1) = -m_c^2 Z (1 - 0.4). \]
4. Conclusions.

The corrections in the inverse mass of the charmed quark to the effective $\Delta S = 1, 2$ lagrangians can be important numerically for precise comparison theory with experiment. The $\Delta S = 2$ lagrangian reveals at this level an explicit dependence on the boarder between short and long distances. The OPE at $x^2 \to 0$ for the light part of the lagrangian is analyzed for determining the convergence scale. Within vacuum dominance approximation this scale is large enough and is given by the parameter $\delta$. The corrections can change the transition matrix element considerably depending on the concrete choice of the scale $\bar{x}$. This means that the long distance hadronic contribution is important to stabilize the result in the region where the expansion (15) fails, in other words the integral $L_{L}^{LG}$ changes quickly for $\bar{x}^2$ in the region $1/\delta^2 < \bar{x}^2 < 3/\delta^2$ in order to compensate corresponding changes of the short distance part (15).

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