The tightening of wide binaries in dSph galaxies through dynamical friction as a test of the Dark Matter hypothesis

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ABSTRACT
We estimate the timescales for orbital decay of wide binaries embedded within dark matter halos, due to dynamical friction against the dark matter particles. We derive analytical scalings for this decay and calibrate and test them through the extensive use of N-body simulations, which accurately confirm the predicted temporal evolution. For density and velocity dispersion parameters as inferred for the dark matter halos of local dSph galaxies, we show that the decay timescales become shorter than the ages of the dSph stellar populations for binary stars composed of 1 $M_\odot$ stars, for initial separations larger than 0.1 pc. Such wide binaries are conspicuous and have been well measured in the solar neighborhood. The prediction of the dark matter hypothesis is that they should now be absent from stellar populations embedded within low velocity dispersion, high density dark matter halos, as currently inferred for the local dSph galaxies, having since evolved into tighter binaries. Relevant empirical determinations of this will become technically feasible in the near future, and could provide evidence to discriminate between dark matter particle halos or modified gravitational theories, to account for the high dispersion velocities measured for stars in local dSph galaxies.

Key words: galaxies: dwarf — dark matter — stellar dynamics — gravitation — stars: binaries

1 INTRODUCTION
Over the past few years the dominant explanation for the large mass to light ratios inferred for galactic and meta galactic systems, that these are embedded within massive dark matter halos, has begun to be increasingly challenged. The lack of any direct detection of the elusive dark matter particles, in spite of extensive and dedicated searches, has led some to interpret the velocity dispersion measurements of stars in the local dSph galaxies, the extended and flat rotation curves of spiral galaxies (Milgrom & Sanders 2003, Sanders & Noordermeer 2007, Nipoti et al. 2007, Famaey et al. 2007, Gentile et al. 2007, Tiret et al. 2007, Sanchez-Salcedo et al. 2008), the large dispersion velocities of galaxies in clusters, the gravitational lensing due to massive clusters of galaxies, and even the cosmologically inferred matter content for the universe, not as indirect evidence for the existence of a dominant dark matter component, but as direct evidence for the failure of the current Newtonian and General Relativistic theories of gravity, in the large scale or low acceleration regimes relevant for the above. Numerous alternative theories of gravity have recently appeared, (TeVeS of Bekenstein 2004, and variations, Sanders 2005, Bruneton & Esposito-Farese 2007, Zhao 2007, F(R) theories e.g. Sobouti 2007) now mostly grounded on geometrical extensions of General Relativity, and leading in the Newtonian limit to laws of gravity which in the large scale or low acceleration regime, mimic the MOdified Newtonian Dynamics (MOND) fitting formulas.

It appears that no matter what dark matter potential is inferred in Newtonian dynamics, from observed rotation velocities or velocity dispersion measurements, it could in principle be accounted for by a suitably tuned alternative theory of gravity, resulting in an enhanced gravitational relevance for the normal baryonic matter content of a galactic system or collection of systems. However, even if the fit to the resulting orbits where equally good under both hypotheses, they reflect distinctly conflicting intrinsic physical realities, only one of which can be true; either the space in question is teeming with unseen exotic particles, or it is not.

Several recent studies have attempted to provide tests which might decide between the dark matter hypothesis and modified gravitational theories, focusing mostly on MOND vs. dark matter comparisons, not in terms of resulting orbits, but by considering other higher order gravitational effects. The derivative of the gravitational force leads to tidal forces, with their role in limiting the sizes of satellites and, in MOND, establishing escape velocities from satellites or galaxies subject to an external acceleration field. Sanchez-
Salcedo & Hernandez (2007) calculated tidal radii and escape velocities for local dSph galaxies under MOND and dark matter, and comparing to relevant observations, concluded a somewhat better fit under dark matter. Wu et al. (2008) calculated the escape velocity for the Milky Way under both MOND and dark matter, and concluded that under both hypotheses the LMC appears as a bound object, in spite of the recently high proper motion determined for this object, given current observational uncertainties. Sanchez-Salcedo et al. (2008) looked at the thickness of the extended HI disk of the Milky Way from both angles, and found a somewhat better fit to observations under MOND. Recently, Skordis et al. (2006) studied the cosmic microwave background, Halle et al. (2007) looked at the problem of the cosmic growth of structure, Zhao et al. (2006) studied gravitational lensing of galaxies and Angus et al. (2007) and Milgrom & Sanders (2007) studied dynamics of clusters of galaxies, all under MOND, all finding its description of the problem as a plausible option, within the observational errors of the relevant determinations. Most of the exploration in terms of alternatives to dark matter has traditionally focused on MOND, but recently it has been found at times to fail. For example, Zhao et al. (2006) find that when comparing across physical scales, it either requires the inclusion of dark matter (as also found in Sanchez-Salcedo & Hernandez 2007), or the structural parameters of the theory need to vary from one galaxy to another. The problem is more general than a MOND vs. dark matter comparison, but actually encompasses the need to decide between dark matter and any alternative theory of gravity which attempts to explain astrophysical observations in the absence of dark matter, e.g. MOND, or TeVeS or F(R) type theories.

Here we explore a physical mechanism which might prove to be of help in the present dark matter vs. modified gravity debate. For massive bodies orbiting within dark matter halos, the individual two-body interactions between the massive body and each dark matter particle result in a net frictional drag which opposes the motion of the massive body. This is the well known dynamical friction effect. The resulting timescales for dynamical friction in galactic systems are typically in the Gyr range, and no direct observation of orbital decay exists. However, the problem has been studied extensively, and it is generally accepted that dynamical friction will result, for example, in the eventual in-spiraling of the Magellanic clouds onto the Milky Way. Dynamical friction is often considered as one of the main mechanisms responsible for the hierarchical merger of galaxies in many current standard cosmological structure formation models.

Studies of dynamical friction in dSph galaxies have been used to constrain the density profiles of dark matter halos, for example, when considering the decay timescales of orbits of globular clusters. Through the explicit dependence of the problem on the distribution function of dark matter particles, it has been shown that orbital decay timescales longer than the lifetimes of the system can be obtained for dark halo profiles characterized by constant density cores rather than divergent cusps, e.g. Hernandez & Gilmore (1998), Read et al. (2006). Indeed, even though the orbits resulting from a gaseous mass distribution under a modified gravity theory and those resulting from the dark matter hypothesis might be equally good fits to observations, the dynamical friction problem will be distinctly different under both scenarios. This has been realized, and recently several authors (Sanchez-Salcedo et al. 2006, Nipoti et al. 2008) have calculated dynamical friction decay timescales for globular clusters in dSph galaxies under the dark matter hypothesis, and under the MOND paradigm, where the stellar population itself acts as a dragging background on the globular clusters. The conclusion is that in the latter, dynamical friction timescales are shorter, perhaps uncomfortably shorter than the system lifetimes, something that is easily avoided in the dark matter scenario if the dark halos have central constant density cores.

In going to the stellar regime, the two components of a binary stellar system, if embedded within a dark matter halo, will experience dynamical friction. This will result in the progressive tightening of the orbit, leading to the eventual merger of the two stars in the absence of other effects. As is the case in the standard dynamical friction problem of the orbital decay of a massive body, it is the ratio of the orbital velocity to the velocity dispersion of the dark matter particles that largely determines the strength of the frictional drag and therefore the speed of the decay. One finds that as the velocity dispersion of the particles responsible for the dynamical friction drag becomes much larger than the velocity of the body undergoing this effect, the drag rapidly tends to zero. Also, this frictional drag is found to scale with the local density of dark matter particles. For the large velocity dispersion values estimated for the Milky Way halo, of order $220/3^{1/2}$ km/s, the effect on binary stars is negligible. However, it is plausible that in going to the highest density and lowest velocity dispersion dark matter halos inferred, those of the local dSph galaxies, dynamical friction on stellar scales might become relevant. Under a modified theory of gravity without dark matter, there would be no particular background dragging down the binary orbit, and hence no orbital tightening for binary stars.

The physical problem we will be treating is very similar to the study of the orbital evolution of a supermassive black hole binary due to interactions with a background stellar population. The above has been studied extensively, for example in the paper by Quinlan (1996), with more recent in the works of Merritt (2000), the comprehensive review of Merritt & Milosavljevic (2005) and references therein, and that of Sesana et al. (2007). In both problems we have a gravitationally bound binary with components of approximately equal mass, $M_1 \approx M_2$, embedded within a distribution of much lighter particles, $m \ll M_{1,2}$. The difference lies in the fact that the supermassive black hole binary is always modeled as residing at the center of a galaxy, while the binary stars studied here have a certain orbit within their host galaxy. The above results in the black hole binary eventually depleting somewhat the central region of stars, and further orbital evolution being determined (in the absence of gas, a key ingredient in the binary black hole case, which is absent from dSph local galaxies) by the rate at which external stars can replenish the central regions. For the solar mass binaries moving slowly through their hypothetical dark matter halos as considered in this paper, the underlying assumption of always finding them within a constant density background of dark matter particles is valid.

Here we explicitly calculate the dynamical friction problem for binaries within dark matter halos. We find that in-
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2 ANALYTIC CALCULATIONS

The standard approach to the study of dynamical friction problems usually starts from the Chandrasekhar formula. This considers the specific case of a massive body moving in a straight line through an infinite, constant density medium of small particles having a Maxwellian distribution function. Although numerical studies have generally validated its use in the case of circular orbits within finite systems, in our case we deviate significantly from the underlying assumptions, making it necessary to develop a detailed treatment from even more basic principles. The problem is that generally one considers a massive body orbiting on large circular orbits, of radius comparable to that of the halo of particles. The orbital period of the massive body is so long that by the time it completes a revolution, it returns to a region where the distribution function has long since returned to its undisturbed form, making the hypothesis leading to the Chandrasekhar formula of a fixed density and distribution function for the background medium valid.

Here we will think of a stellar binary in which the components revolve around each other in a circular orbit. This binary is embedded within an infinite medium of small dark matter particles (the large scale DM halo) which produce a constant background density, \( \rho_0 \), and obey a isotropic Maxwellian distribution function \( f_0(v) \propto \exp(-v^2/2\sigma^2) \) everywhere. The presence of the binary alters the distribution function and density distribution, leading to the build up of a perturbation, which will result in the frictional drag. We shall treat the problem through the first order perturbation of the distribution function.

To get a first idea of what the binary will do to the initially constant density dark matter halo, we shall begin by looking at the effect of a single star, at rest with respect to the background. This single star will contribute a perturbation to the potential felt by the dark matter given by:

\[
\Phi_1(r) = -\frac{GM}{r},
\]

where \( M \) is the mass of the star and \( r \) is a radial coordinate centered on the star in question. The problem evidently has spherical symmetry. We shall now approximate the distribution function as: \( f(r, v) = f_0(v) + \epsilon f_1(r, v) \), to which there will correspond a perturbed total density \( \rho(r) = \rho_0 + \epsilon \rho_1(r) \) due to an overall potential, \( \Phi(r) = \Phi_0(r) + \epsilon \Phi_1(r) \). The full distribution function will satisfy the Boltzmann equation:

\[
\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \nabla \Phi \cdot \nabla f = 0.
\]

We will now look for a stationary solution and keep only terms to first order in the perturbation. Taking \( \Phi_0 = 0 \), the familiar Jeans swindle, we obtain for the radial component:

\[
\frac{1}{\rho_0} \frac{\partial f_1}{\partial v} = \frac{GM}{\sigma^2} \frac{\partial f_0}{\partial v}.
\]

Using the explicit dependence of \( f_0(v) \), we can write the right hand side as of equation(3) as:

\[
-\frac{v}{\sigma^2} \frac{GM}{\sigma^2} f_0,
\]

after which we can integrate equation (3) over velocity space to yield

\[
\frac{d\rho}{dr} = -\frac{GM}{\sigma^2} \rho_0.
\]

The above equation can be readily solved to yield the density perturbation induced upon the dark matter distribution by the single star at rest as:

\[
\rho_1(r) = \frac{GM}{\sigma^2} \rho_0 = \left( \frac{v_e}{\sigma} \right)^2 \rho_0,
\]

where we have introduced an equivalent circular velocity due to the star, \( v_e = GM/r \). To first order, we can think of the response of the dark matter halo to the presence of a wide binary, where the orbital velocity is lower than \( \sigma \), as being composed of two density enhancements, centered upon each of the stars in the binary, each described by equation (5). Indeed, a close analogy can be found when calculating the dynamical friction due to a gaseous medium. Sanchez-Salcedo & Brandenburg (1999) find that in the sub-sonic case, when
the perturber moves at speeds lower than the sound speed of the medium, the response is in fact an approximately spherical perturbation centered on the perturber.

Hence, the resulting density enhancement will produce no gravitational force on the stars in the binary, being a spherical density enhancement centered upon each of the stars. However, it is a mistake to conclude that there will be no dynamical friction, because the identity of the dark matter particles making up each density enhancement is not fixed. Each dark matter particle only briefly forms part of the density response of the dark matter halo. In the absence of the binary, the angular momentum of the dark halo will be zero, whilst in its presence a steady state ensues, where two density enhancements, as given by equation (5), will follow it. As these two enhancements have to be constantly replenished, we can calculate the angular momentum loss for each star in the binary star as:

$$\dot{L} = \frac{M_0 V_0 R_o}{\tau},$$

(6)

In the above equation $M_e$ is the mass of each one of the density enhancements centered on the stars in the binary, $V_0$ and $R_o$ are the binary orbital velocity and orbital radius, and $\tau$ is a characteristic timescale over which the density enhancement is being replenished. To estimate $M_e$ we have to integrate equation (5), out to a certain maximum radius, $R_{max}$. This can not be larger that $R_o$, as then the two density enhancements would overlap, and the reaction of the dark halo to the presence of one of the stars in the binary would be erroneously calculated as being still determined by the distant component. $R_{max}$ could be smaller than $R_o$, in cases where inertial forces introduce a smaller truncation, this we shall consider in the appendix. As a first approximation we will take $R_{max} = R_o$, to integrate equation (5), to obtain $M_e$ as:

$$M_e = 2\pi \frac{GM}{\sigma^2} R_o^3 \rho_0 = \frac{3}{2} M (\frac{v_h}{\sigma})^2,$$

(7)

where we have introduced an equivalent circular velocity due to the dark halo at the binary scale, $v_h^2 = GM_h/R_o$, with $M_h = (4\pi/3)\rho_0 R_o^3$, the dark halo mass within a sphere of radius $R_0$ under unperturbed conditions. To estimate $\tau$, we can think of the dissipation of the density enhancement to be of order $R_0/\sigma$. However, in the absence of the binary, the density enhancement will first stop turning, and only later dissipate. In order to estimate the rate at which the binary loses angular momentum, it is more exact to think of $\tau$ as given in terms of the binary orbital period, $\tau = \alpha/\Omega$, with $\Omega$ the orbital frequency of the binary and $\alpha$ a dimensionless scaling factor. With these hypothesis, we can obtain the rate of loss of angular momentum for each component of the binary from equation (6) as:

$$\dot{L} = \frac{4\pi}{\alpha} \left( \frac{GM}{\sigma} \right)^2 \rho_0 R_0,$$

(8)

which we can write in terms of the angular momentum of each star in the binary $L_b = (2GM^3 R_0)^{1/2}$ and the particle crossing time for the dark matter particles over the radius of the binary, $\tau_c = R_0/\sigma$, as:

$$\dot{L} = \frac{3}{2^{5/2}\alpha} \left( \frac{L_b}{\tau_c} \right) \left( \frac{v_h^2 v_c}{\sigma^3} \right).$$

If we now make the assumption that the orbital decay of the binary proceeds slowly in terms of the orbital period, describing a very closed spiral, we can think of the orbit as circular throughout the evolution. This allows to obtain the temporal evolution of the binary radius by deriving the expression for $L_b$, with respect to time, and equating it to equation (8),

$$\left( \frac{GM^3}{2R_0} \right)^{1/2} = \frac{4\pi}{\alpha} \left( \frac{GM}{\sigma} \right)^2 \rho_0 R_0,$$

(9)

giving:

$$\frac{R_0}{\tau} = \frac{\left( \frac{2^{5/2}\pi}{\alpha} \right) \left( \frac{GM^3 M^{1/2} R_0^3}{\sigma^2} \right) R_0^{3/2}}{\frac{3}{2} V_0 \left( \frac{\sigma}{\tau} \right)},$$

or

$$\dot{R}_0 = \frac{3}{\alpha} V_0 \left( \frac{v_h}{\sigma} \right)^2 = \frac{4\alpha}{\alpha} V_0 \left( \frac{\tau_c}{\tau} \right).$$

The last expression for $\dot{R}_0$ is given in terms of the gravitational free fall time of the unperturbed background. Since $\tau_c$ scales with the radius, and for the whole halo $\tau_{ff} \sim \tau_c$, given that the radius of the binary is much smaller than the size of the dark matter halo, we see that the rate of radial decay will be much slower than the orbital velocity of the binary. In the following section we shall see that $\alpha$ is of order $10^{-3}$, and thus the in-spiraling will therefore precede along tight spirals, as assumed previously. To order of magnitude, the parenthesis in the last expression can also be replaced by the ratio of the binary radius to the dark halo size. The solution to equation (9) is trivial, and gives the temporal evolution of the binary radius as:

$$t = \left( \frac{\alpha}{2^{5/2}\pi} \right) \frac{\sigma^2}{\rho_0 M^{1/2} G^{3/2}} \left( \frac{1}{R_0(t)} \frac{1}{R_0(t = 0)} \right).$$

What remains now is to evaluate $\alpha$. This will be done in the following section, where high resolution numerical simulations are used to derive $R_0$ for a few specific cases, from which we calibrate $\alpha$. A more detailed treatment of the dark halo response to the presence of the rotating binary, with full account of the inertial forces present, is included in the appendix. The calculations there shown, together with the numerical results of the following section, will be seen to validate the assumptions going into the derivation of equation (9).

3 NUMERICAL CALCULATIONS

We have carried out a series of numerical experiments to directly compute the rate at which the binary separation decays under the influence of dynamical friction.

The standard setup consists of a binary with identical components, with mass $M_1 = M_2 = 1 M_{\odot}$, placed in a circular orbit with semi-major axis $a = 2 R_0$. The three-dimensional volume in the vicinity of the binary, out to a radius $R > a$, is filled with dark matter particles of mass $m \ll M_{1,2}$ which, in the absence of the binary, produce a constant mass density $\rho_{DM} = 1 M_{\odot} pc^{-3}$ and have an equilibrium Maxwellian distribution function with isotropic
velocity dispersion $\sigma=5\,\text{km/s}$, in accordance with the assumptions in our analytic derivation in §2. These parameters in core radius and density are in the typical ranges estimated for the central regions of the dark matter halos of local dSph galaxies, where the bulk of the stellar populations reside, as inferred through modeling of the observed kinematics of their stars e.g. Koch et al. (2007). The corresponding typical values for the velocity dispersion in the better studied local dwarf spheroidals with the earliest discovery dates, are of around $\sigma=10\,\text{km/s}$. However, we will calculate the case for $\sigma=5\,\text{km/s}$, rather corresponding to the more recently discovered of such systems, e.g. Gilmore et al. (2007) and references therein. This is done to set ourselves in the astrophysical case where equation (9) predicts the highest decay rates.

During the simulation, each star feels the gravitational effect of the companion and of all the dark matter particles directly (no approximations, other than a gravitational softening length to avoid excessive accelerations in the case of close encounters, are used). The dark matter particles feel only the presence of the two stars (i.e., they are not self-gravitating), allowing for a large number of particles to be used, since the computation time scales with the number of dark matter particles, $N_{\text{DM}} = 4\pi R^3 \rho_{\text{DM}} / 3 m$. This is substantially different in terms of computational requirements than, for example, the interaction of galaxies (Hernandez & Lee 2004) or individual stars (Lee & Ramirez-Ruiz 2007), in which the self gravity of the various components and gas dynamics are important ingredients one needs to consider when doing detailed modeling.

The numerical integration is performed with a Runge-Kutta algorithm accurate to fourth order, and we have checked that in the limit of vanishing dark matter mass density, the binary remains stable, with the separation remaining constant.

The distribution of orbital separations in local binaries has been well studied, e.g. Griffin (2006), with detailed modeling yielding orbital separation distributions well fitted by a uniform distribution in log(a) for a in the interval $10\,\text{Au}$ to 0.5 pc e.g. Eggleton et al. (1989). We run four cases, uniformly spaced in log ($R_0$), at initial values of $a=(10^{-4}, 10^{-3}, 10^{-2}, 10^{-1})$ pc. The upper value was chosen to be consistent with high values found for local samples, in fact, we remain a factor of 5 below the upper range of the distribution of Eggleton et al. (1989). It is not known what the intrinsic binary separations in the dSphs might be, but we take it as plausible that the local sample might serve as an initial zero order approximation.

In the absence of the binary, the dark matter placed inside the sphere of radius $R$ will leak out of it isotropically (since the initial spatial distribution is homogeneous, and the velocities are isotropic) at a rate given by

$$M_{\text{leak}} = (8\pi)^{1/2} \rho_{\text{DM}} \sigma R^3,$$

so that their number would drop to zero if nothing were done. In fact we remove particles when they cross the outer boundary at $R$, and inject new particles over the surface of the sphere at twice the rate given in equation (10). Half of them are removed essentially immediately because their velocity vector points outward, while the remaining move in. We have verified that the expression in equation (10) is correct numerically by computing a test case in which the stars are absent. The number (and mass) density of the dark matter particles remains constant with the algorithm described above to within sampling errors given by discretization effects.

Ideally one would hope to have a sphere around the binary that is much larger than the separation between the stars so as to allow for the correct trajectories of the particles that are injected. The computational cost at a fixed particle number density scales as $R^2$ and so we have chosen a trade-off, with $R = 5a$. This choice sensitive to the velocity dispersion, $\sigma$. At the start of the calculation there will be an initial transient because the binary suddenly feels the effect of a homogeneous distribution of matter around it. After a crossing time, $\tau_c$, the majority of the initial particles will have left the sphere, and it will be filled with ones that have been injected at its boundary. For the trajectory of an infalling particle to be computed realistically, the escape velocity at radius $R$ must be small compared with the typical velocity (the dispersion), i.e., we must have:

$$v_{\text{esc}}(R) = \left(\frac{4GM}{R}\right)^{1/2} \leq \sigma, \quad (11)$$

where $2M$ is the total mass in the stellar binary. For our parameters, with $\sigma = 5\,\text{km/s}$, and $R = 5a$ we find that equation (11) is satisfied for $a \geq 10^{-2}\,\text{pc}$. At smaller radii not taking the infall trajectories properly could modify the interaction with the binary. However, as will be seen below, this falls in the range of parameter space where the decay rate is already very small, and thus does not alter our conclusions significantly. We did nevertheless, for small binary separations ($a = 10^{-4}, 10^{-3}\,\text{pc}$) perform simulations where $R$ was greater (up to $R = 15a$) in order to carefully check for numerical convergence of the actual decay rate.

A large number of dark matter particles is required in order to successfully model the effect of dynamical friction on the binary and reduce the level of noise due to discretization. We have computed the evolution of the binary for fixed values of the initial separation, $a$, at increasing levels of resolution, with $N_{\text{DM}}$ typically ranging from $2 \times 10^3$ to up to $10^6$. The number of particles required for numerical convergence of the solution correlates with the choice of $a$, and is $N \approx 10^5$ for the largest binary separations (0.1 pc) we considered.

We find that after a relaxation time of order $\tau_c$, the simulation settles to a steady state solution in which the decay rate is constant. When seen in a reference frame rotating with the binary, this steady state is characterized by two static density enhancements, very closely centered on each of the stars. Figure 1 then shows the density enhancement as a function of distance to a single star, where we have averaged over spheres centered on the position of each star, in the rotating frame of the binary. The straight line gives the prediction of equation (5), strictly, derived for a static star. We see that out to the radius of the binary, $R_0 = 0.0005\,\text{pc}$ in this case, the actual response of the dark halo does not deviate from the analytic prediction beyond sampling errors, which it matches in both the amplitude and the radial scaling. We have also checked that the form and amplitude of these density enhancements are in fact constant with time.

When calculating the decay rate over long periods of time, at very low resolutions (i.e., number density of particles) the result is not converged, showing fluctuations in...
the decay and sometimes even reversals. As the number of particles in the computational volume increases the decay is smoother and becomes monotonic. The correct solution when converged is in fact attained from below in the decay rate, making our estimates conservative absolute lower bounds.

We have hence found through direct N-body simulation that the expectations of section (2) are verified, in the regime where $V_0 \leq \sigma$. The response of the dark halo is in accordance with the assumptions leading to the derivation of equation (9). It would therefore appear to follow that the mechanism of angular momentum loss for the binary has been correctly identified, and that the numerically calculated decay rates should also follow the scalings of equation (9). A careful computation of a few particular cases was then used to calibrate the $\alpha$ parameter appearing in equation (9).

The results of section (2), and in particular equation (9), indicate that the binary decay rate should scale linearly with the background matter density and with the square root of the total binary mass. We have tested both of these predictions by varying $\rho_{DM}$ by a factor of 3 to both larger and smaller values, at a fixed particle number density, and the total mass $2M$ by a factor of 4 (also to higher and lower values) and find that this is indeed the case, with $\dot{a} \propto \rho_{DM} M^{1/2}$ (in all cases we maintained a binary mass ratio of unity).

In figure 2 we show with filled circles the best estimate of the decay rate for four values of the initial separation of the binary. We have spanned four orders of magnitude in initial separation, and run each simulation long enough to clearly establish the decay rate, although the actual value of $R_0$ decreased only by a small factor. The solid line gives the prediction of equation (9), a decay rate which scales with $R_0^{3/2}$, where we have calibrated $\alpha = 1.07 \times 10^{-3}$ to obtain a best fit to the numerically estimated decay rates. The excellent agreement of the simplified analytical calculation, over four orders of magnitude against the simulations is encouraging of the physical scenario presented in section (2) as an explanation to the dynamical friction of binaries, in the range of parameters relevant to dSph galaxies. This highlights the inaccuracies incurred by assuming that only the direct gravitational force of the induced wake determines the dynamical friction problem; the sustaining of the wake itself, even if spherically symmetrical about the perturber, also results in a drag of linear or angular momentum. This becomes particularly obvious in the sub-sonic regime ($V < \sigma$), where the wake becomes spherical and the direct gravitational drag vanishes.

It is interesting to compare equation (9) and the findings of this section with results for the orbital tightening of binary black holes. Recently, such studies include the effects of gas, and of the depletion of stars from the central regions, where the black hole binary resides e.g. Merrit & Milosavljevic (2005). However in the first such studies, e.g. the thorough numerical work of Quinlan (1996), the tightening of a black hole binary was calculated including only the presence of a constant background of stars, essentially the situation we model here. It is reassuring that these could be highly accurately fitted by a function which satisfies equation (9), plus the inclusion of a further factor of $(V_0/\sigma) ln(\sigma^2/V_0^2)$. It is possible that we have missed this factor, because over the range of orbital separations we modeled numerically it varies only by a factor of a few. The analytical developments of section (2) provide a physical understanding to the dominant scalings of the functional fits to the numeri-

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**Figure 1.** Dark matter density profile around a single star as a function of distance from it, after a steady state has been achieved, with error bars showing sampling errors. The straight line is the analytical prediction of equation (5).

**Figure 2.** The dots show four numerically calculated decay rates, for a halo density of $1M_{\odot}pc^{-3}$ and a dark matter velocity dispersion of $5km/s$. The solid line gives the analytical prediction of equation (9), for $\alpha = 1.07 \times 10^{-3}$. The vertical dotted curve gives the binary radius beyond which $R_0/(2R_0)$ becomes shorter than 10 Gyr.
cally calculated black hole binary decay rates of e.g. Quinlan (1996), who considered a wider range of parameters. The validity of equation (9) is restricted to a certain range in $V_0/\sigma$ (see the appendix), over which there is excellent agreement with the numerical simulation performed here, and to those of black hole binary numerical simulations over the range of parameters where they can be compared. The missing factor noted above becomes relevant only in the range where $V_0/\sigma$ becomes larger. It is in this ‘supersonic’ regime where the simplifying assumptions behind equation (9) break down, most importantly ignoring the truncation of the density enhancements due to inertial forces, and the imposing of a stationary solution. These effects are hard to consider analytically, but from comparison to the black hole experiments, should be credited as the origin of the additional factor $(V_0/\sigma)\ln(\sigma^2/V_0^2)$ in Quinlan (1996).

The vertical dotted line in this last figure gives the initial value of $R_0$ beyond which the typical timescale over which the orbit is substantially modified, $\tau_{dmc} = R_0/(2\dot{R}_0)$, becomes larger than 10 Gyr, $R_0 = 0.067$ pc. The value of 10 Gyr was chosen to reflect a typical value for the ages of the old component (in some cases the only component) of the stellar populations in local dSph galaxies, as inferred through the direct statistical modeling of their resolved HR diagrams, e.g. Hernandez et al. (2000), Lanfranchi et al. (2006). We therefore see that for wide binaries, the decay timescales in dSph galaxies can become shorter than the lifetimes of these systems. Although in the low density and high dispersion velocity conditions inferred for the dark matter halo in the solar neighbourhood the effects of dynamical friction on any stellar binary are absolutely negligible, we have shown that the effect should become relevant in local dSph.

4 DISCUSSION AND CONCLUSIONS

The direct study of the distribution of binary separations in even the closest local dSph currently remains beyond the scope of technical feasibility, even with the HST, but only very slightly so (G. Gilmore, private communication). It is certain that with the next generation of space telescopes, the undertaking of detailed determinations of binary separation distributions in local dSph galaxies will become possible.

The interpretation of whatever such studies reveal might be ambiguous. If no binaries with separations larger than the limits we derived in this paper are found, from the point of view of the dark matter hypothesis, the results will be seen as a new and independent validation of the physical reality of dark matter. From the point of view of modified theories of gravity, the absence of wide binaries will be interpreted as a reflection of initial conditions and a different intrinsic mechanism for binary star formation, to what gives rise to the solar neighborhood or local Galactic halo population, perhaps unlikely given the universality of a kindred distribution, the stellar IMF. If on the other hand plentiful wide binaries were to be found in local dSph galaxies, the dark matter scenario would be very seriously challenged. Also, given the linear dependence of the decay rates on the local dark matter densities of equation (9), a strong galactic radial gradient in the maximum binary separations would be expected in any hypothetical strongly cusped dark matter halo, of the type suggested by recent cosmological computer simulations, e.g. Navarro et al. (1997).

The possibility of appreciably altering an initial distribution of binary separations, of forming new wide binaries through stellar capture, or of continuous replenishing through the partial disruption of tighter binaries, as it happens in globular clusters, is in this case not available. Although the velocity dispersions of stars are comparable to what one finds in globular clusters, in dSph galaxies these do not reflect the potential produced by the stars themselves, but the overwhelming dynamical dominance of the dark matter (under such a hypothesis). The stars therefore move as fast as they do in globular clusters, but the volume densities are very substantially lower. The number of stars in a typical dSph is of order $10^{7-8}$, about 10 times larger than in globular clusters, the volume occupied by the stellar population in a dSph however, is of order $(2kpc)^3$, some million times greater than the case in globular clusters, of order $(20pc)^3$. This makes the usual mechanisms operating in globular clusters completely inefficient. Also, for the test to be meaningful, one requires the use of main sequence stars, as strong mass loss over the red giant phase and beyond would result in the orbital widening of the binary, e.g. Valls-Gabaud (1988).

We have also assumed the binary is at rest. In reality, it will find itself on an orbit within the dSph galaxy, and will therefore see the dark matter particles as approaching with a certain velocity, which will change in direction and magnitude over the course of the orbit. Observational determinations of the orbits of stars in dSph galaxies generally concur in assigning relatively elliptical orbits to these stars. The typical ratios for the maximum to minimum radii for the orbits being 0.3. Over such elliptical orbits, the binary will spend the greater fraction of its orbital period moving very slowly near its apocenter. For a small fraction of the time, it will be dashing past its pericenter. This means that the calculations we have performed will be valid over most of the galactic orbital period of the binary. During the short lived, fast moving phase, the orbital decay due to dynamical friction could be reduced, as the outer regions of the density enhancements will be ‘blown away’ to some extent. In any case, even during this fast moving phase, the galactic orbital speeds will still be close to $\sigma$, so it is safe to assume the effects of considering the galactic orbit for the binary will be minor on the final dynamical friction timescales we have estimated here. More important than the above effect, and making our estimates of decay rates fall somewhat below the true values, is the lack of self gravity for the dark matter particles in the simulations from which we calibrated $\alpha$. If this effect were included, the density enhancements about each star would become more massive and coherent, resulting in increased dynamical friction effects.

We have presented a new and independent test of the dark matter hypothesis. The empirical application is currently unavailable, but will become so in the near future. This gives even grater relevance to the continual development of increasingly advanced studies of the stellar populations of the local dSph galaxies, they might hold the clue to the dark matter mystery in more senses than formerly appreciated.
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APPENDIX

In this appendix we approximate for the density enhancement around each of the two rotating stars in a binary at rest within a dark matter halo, strengthening the physical understanding of the excellent agreement between the analytic expectations of § 2 and the numerical experiments of § 3. The notation is as in § 2, we shall proceed by writing the third Jeans equation in a reference frame which rotates with the binary with angular frequency Ω, linearizing with respect to the density enhancement, and looking for a stationary solution (in the rotating frame). The third Jeans equation in the rotating frame, under cylindrical coordinates, will read:

\[
\frac{\partial \rho}{\partial t} + \rho \frac{\partial}{\partial x_i} \left( v_i - \Omega \times r \right) = -\frac{\partial \Phi}{\partial x_i} - 2\rho \left( \frac{\partial}{\partial t} + \Omega \times \frac{\partial}{\partial x} \right) v_i - \frac{\partial \left( \rho^2 \right)}{\partial x_i} \tag{A1}\]

In the above equation \( \bar{X} \) refers to the mean (or expected) value of \( X \), the second term in the RHS gives the Coriolis force, and the centrifugal force is included in \( \Phi_{ij} = \epsilon \Phi_1 - \| \Omega \times \bar{r} \|^2 / 2 \), where we have again taken the undisturbed potential to be zero, and \( \Phi_1 \) is the potential due to each star.

We now take \( \rho = \rho_0 + \epsilon \rho_1, \bar{v}_i = \bar{v}_0 + v_i \) for all \( i \). Imposing a stationary solution with no flows beyond \( \bar{v}_0 = (0, \Omega R, 0) \), we get for \( j = \theta \):

\[
0 = -\frac{\rho_0 \partial \Phi_1}{R \partial \theta} - \sigma^2 \left( \frac{\partial \rho_1}{R \partial \theta} + \frac{\partial \rho_1}{R \partial \phi} + \frac{\partial \rho_1}{\partial z} \right) \tag{A2}\]

Limiting the analysis to any plane \( z = \text{cst} \), along any line \( R = \text{cst} \), we get for an isotropic velocity for the dark matter particles,

\[
\frac{\partial \rho_1}{\partial \theta} = -\frac{\rho_0 \partial \Phi_1}{\sigma^2 R \partial \theta} \tag{A3}\]

Equation (A3) is analogous to equation (4), and shows that the density enhancement on each star, even under the full rotation description, will follow the angular variations in \( \Phi_1 \), i.e. it will remain centered on the stars. This is a result of having assumed a static solution with no flows, valid for orbital velocities not substantially exceeding \( \sigma \), and seen to hold through the numerical simulations of § 3.

Under the same assumptions, for \( j = R \) equation (A1) evaluated at constant height and angular direction yields:

\[
0 = -\rho_0 \frac{GM}{(R - R_0)^2} + \frac{\rho_1 \Omega^2 R}{2} - 2\rho_1 \Omega^2 R - \sigma^2 \frac{\partial \rho_1}{\partial R} \tag{A4}\]

Writing \( R = R - R_0 \) we can write the previous equation as:

\[
\frac{\partial \rho_1}{\partial R} = -\rho_0 \frac{GM}{\sigma^2 R^2} - \rho_1 \frac{3}{2} \frac{\Omega}{\sigma} (R + R_0) \tag{A5}\]

We see that we have again obtained equation (4) the density enhancement centered on each star of § 2, with the addition of an extra term which scales with the distance to the center of the coordinate system, the center of the binary, and which is proportional to \( \Omega / \sigma^2 \). This last term effectively contributes with a radial exponential cut off to the
density enhancement calculated in § 2, due to the inertial forces of the spinning binary.

For small values of $R$ it is the first term on the RHS of equation (A5) which dominates, and we find the density enhancement of § 2. For large values of $R$ the second term dominates, imposing an effective cut off radius at a distance $R_{\text{cut}} \simeq \sigma/\Omega$. For the approach of § 2 to be valid, we need approximately $R_{\text{cut}} \geq R_0$. With $\Omega = V_0/R_0$, this condition reduces to:

$$\frac{\sigma}{V_0} \geq 1,$$

for the regime over which inertial forces do not dominate to the point of truncating the density enhancements within a typical distance $R_0$. Also, we need the dispersion timescales of the density enhancements due to particle streaming not to dominate over the turning timescale of binary orbit, i.e., approximately

$$\frac{\alpha R_0}{V_0} \lesssim \frac{R_0}{2\sigma}.$$

Combining these last two conditions we obtain the range of validity for the analytic decay rates of § 2 as:

$$\frac{1}{2\alpha} \geq \frac{\sigma}{V_0} \geq 1 \quad (A6).$$

For the values of $M$, $R_0$ and $\sigma$ used in § 3, we see that equation (A6) is satisfied (recall the fit to $\alpha = 1.07 \times 10^{-3}$) for all the numerical experiments, which hence, yielded results in good agreement with the expectations of § 2.