Color dipole phenomenology of
diffractive electroproduction of light
vector mesons at HERA

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Abstract

We develop the color dipole phenomenology of diffractive photo- and electroproduction \( \gamma^* N \rightarrow V(V') N \) of light vector mesons \( V(1S) = \phi^0, \omega^0, \rho^0 \) and their radial excitations \( V'(2S) = \phi', \omega', \rho' \). The node of the radial wave function of the \( 2S \) states in conjunction with the energy dependence of the color dipole cross section is shown to lead to a strikingly different \( Q^2 \) and \( \nu \) dependence of diffractive production of the \( V(1S) \) and \( V'(2S) \) vector mesons. We discuss the restoration of flavor symmetry and universality properties of production of different vector mesons as a function of \( Q^2 + m_V^2 \). The color dipole model predictions for the \( \rho^0 \) and \( \phi^0 \) production are in good agreement with the experimental data from the EMC, NMC, ZEUS and H1 collaborations. We present the first direct evaluation of the dipole cross section from these data.

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1 Introduction

Diffractive electroproduction of vector mesons

\[ \gamma^* p \rightarrow V p, \quad V = \rho^0, \omega^0, \phi^0, J/\Psi, \Upsilon \]  

at high energy \( \nu \) offers a unique possibility of studying the pomeron exchange at high energies. Particularly important is the observation that the transverse size of the photon shrinks with the increase of its virtuality \( Q^2 \). This property can conveniently be quantified in the mixed \((r, z)\) lightcone technique, in which the high energy hadrons and photons are described as systems of color dipoles with the transverse size \( r \) frozen during the interaction process. Interaction of color dipoles with the target nucleon is quantified by the color dipole cross section \( \sigma(\nu, r) \) whose evolution with the energy \( \nu \) is described by the generalized BFKL equation (for a related approach see also \([12]\)). The shrinkage of the photon with \( Q^2 \) together with the small-size behavior of the dipole cross section \( \sim r^2 \) leads to what has come to be known as a scanning phenomenon: the \( V(1S) \) vector meson production amplitude is dominated by the contribution from the dipole cross section at the dipole size \( r \sim r_S \), where \( r_S \) is the scanning radius

\[ r_S \approx \frac{A}{\sqrt{m_V^2 + Q^2}}. \]  

This scanning property makes the vector meson production an ideal laboratory for testing the generalized BFKL dynamics \([\text{gBFKL hereafter}]\). At large \( Q^2 \) and/or for heavy vector mesons, the amplitude of reaction (1) becomes short-distance dominated and is perturbatively calculable in terms of the short-distance behavior of the vector mesons wave function. However, the asymptotic short distance formulas are not yet applicable at the moderate \( Q^2 \lesssim 20 \text{GeV}^2 \) of interest in the present fixed target and HERA experiments where the scanning radius \( r_S \) is still large due to a large scale parameter \( A \approx 6 \) in (2) as derived in \([6]\). For this reason, the onset of the short-distance dominance is very slow and there emerges a unique possibility of studying the transition between the soft and hard interaction regimes in a well controlled manner. Furthermore, the scanning phenomenon allows to directly test the steeper subasymptotic energy dependence of the dipole cross section at smaller dipole size \( r \), which is one of interesting consequences of the color dipole gBFKL dynamics \([11, 14]\).
The scanning phenomenon has particularly interesting implications for the diffractive production of the $2S$ radially excited vector mesons

$$\gamma^*p \rightarrow V'p, \quad V'(2S) = \rho', \omega', \phi', \Psi', \Upsilon'. \quad (3)$$

Here one encounters the node effect: a tricky and strong cancellation between the large and the small size contributions to the production amplitude i.e., those above and below the node position $r_n$ in the $2S$ radial wave function $[2, 13, 15]$ respectively. The node effect is the only dynamical mechanism that gives a strong natural suppression of the photoproduction of excited vector mesons $V'(2S)$ vs. $V(1S)$ mesons. For instance, it correctly predicted $[2, 13]$ the strong suppression of real photoproduction of the $\Psi'$ compared to the $J/\Psi$ observed in the NMC experiment $[14]$ and confirmed recently in the high statistics E687 experiment $[17]$. In anticipation of the new experimental data on real and virtual $V'$ photoproduction from HERA, it is important to further explore the salient features of the node effect in the framework of the color dipole gBFKL dynamics. At moderate $Q^2$, the scanning radius $r_S$ is comparable to $r_n$. First, for this reason even a slight variation of $r_S$ with $Q^2$ leads to a strong change of the cancellation pattern in the $V'(2S)$ production amplitude and to an anomalous $Q^2$ dependence for the electroproduction of the radially excited vector mesons $[2, 13, 15]$. Second, the cancellation pattern is sensitive also to the dipole-size dependence of the color dipole cross section $\sigma(\nu, r)$ which in the gBFKL dynamics changes which energy $\nu$ leading to an anomalous energy dependence for producing the $V'(2S)$ vector mesons as compared to a smooth energy dependence for the $V(1S)$ ground state vector mesons. This anomalous $Q^2$ and energy dependence of the $V'(2S)$ production offers a unique signature of the $2S$ radial excitation vs. the D-wave state. Third, at very small $Q^2$, the $V'(2S)$ production amplitude can be of opposite sign with respect to that of the $V(1S)$ production amplitude (the overcompensation scenario of ref. $[15]$) to then conform to the same sign at larger $Q^2$ (the undercompensation scenario of ref. $[15]$). Here we wish to emphasize that the relative sign of the $V'$ and $V$ production amplitudes is experimentally measurable using the so-called Söding-Pumplin effect ($[18, 19]$, see also [20]).

In this paper we develop the color dipole phenomenology of diffractive photo- and electroproduction of the $1S$ ground state and of the $2S$ radially excited vector mesons. As stated above, for a large scanning radius, the large distance contribution to the production ampli-
tude is not yet negligibly small in the so far experimentally studied region of $Q^2$, in particular in the $2S$ meson production. In this paper we show that the $Q^2$ and energy dependence of the diffractive production of vector mesons offers a unique possibility of studying how the color dipole cross section changes from the large nonperturbative to the small perturbative dipole size. The problem can be attacked both ways. First, we present detailed predictions using the color dipole cross section \[\sigma(\nu, r)\], which gives a very good quantitative description of the proton structure function from very small to large $Q^2$. Second, we can invert the problem and evaluate the color dipole cross section from the corresponding experimental data. Such an evaluation of the dipole cross section is presented here for the first time.

The paper is organized as follows. In section 2 we formulate the color dipole factorization for vector meson production amplitudes. In section 3 we present our numerical results. We find good agreement with the experimental data from the fixed target and HERA collider experiments. The subject of section 4 is the anomalous $Q^2$ and energy dependence of electroproduction of $2S$ radially excited vector mesons. In section 5 we discuss the scaling relations between production cross sections for different vector mesons and the restoration of flavor symmetry in the variable $Q^2 + m_V^2$. We comment on how the scanning phenomenon enables a direct comparison of the spatial wave functions of the $\rho^0$ and $\omega^0$ mesons. The first evaluation of the dipole cross section from the experimental data is presented in section 6. In section 7 we summarize our main results and conclusions. In the Appendix we describe the lightcone parameterization of wave functions of $V(1S)$ and $V(2S)$ vector mesons used in our analysis.

2 Color dipole factorization for vector meson production

The Fock state expansion for the relativistic meson starts with the quark-antiquark state which can be considered as a color dipole. The relevant variables are the dipole moment $r$ which is the transverse separation (with respect to the collision axis) of the quark and antiquark and $z$ - the fraction of the lightcone momentum of the meson carried by a quark. The interaction of the relativistic color dipole with the target nucleon is described by the energy dependent color dipole cross section $\sigma(\nu, r)$. The many gluon contributions of higher
Fock states $q\bar{q}g\ldots$ become very important at high energy $\nu$. The crucial point is that in the leading $\log \frac{1}{x}$ the effect of higher Fock states can be reabsorbed into the energy dependence of $\sigma(\nu, r)$, which satisfies the generalized BFKL equation \cite{10,11}. The flavor blind (one should really say flavor tasteless) dipole cross section unifies the description of various diffractive processes. To apply the color dipole formalism to deep inelastic and quarkonium scattering and diffractive production of vector mesons one needs the probability amplitudes $\Psi_{\gamma^*}(\vec{r}, z)$ and $\Psi_V(\vec{r}, z)$ to find the color dipole of size $r$ in the photon and quarkonium (vector meson), respectively. The color dipole distribution in (virtual) photons was derived in \cite{9,10}. In terms of these probability amplitudes, the imaginary part of the virtual photoproduction of vector mesons in the forward direction ($t = 0$) reads

$$\text{Im}M = \langle V|\sigma(\nu, r)|\gamma^*\rangle = \int_0^1 dz \int d^2r \sigma(\nu, r)\Psi^*_V(r, z)\Psi_{\gamma^*}(r, z)$$

whose normalization is $d\sigma/dt|_{t=0} = |M|^2/16\pi$. For small size heavy quarkonium the probability amplitude $\Psi_V(\vec{r}, z)$ can safely be identified with the constituent quark-antiquark quarkonium wave function. The color dipole factorization \cite{11} takes advantage of the diagonalization of the scattering matrix in the $(r, z)$ representation, which clearly holds even when the dipole size $r$ is large, i.e. beyond the perturbative region of short distances. Due to this property and to the fact that in leading $\log \frac{1}{x}$ the effect of higher Fock states is reabsorbed in the energy dependence of the dipole cross section $\sigma(\nu, r)$, as a starting approximation we can identify the probability amplitude $\Psi_V(\vec{r}, z)$ for large size dipoles in light vector mesons with the constituent quark wave function of the meson. This provides a viable phenomenology of diffractive scattering which is purely perturbative for small size mesons and/or large $Q^2$ and small scanning radius $r_S$ and allows a sensible interpolation between soft interactions for large dipoles and hard perturbative interactions of small dipoles. For light quarkonia and small $Q^2$, this implies the assumption that small-size constituent quarks are the relevant degrees of freedom and the spatial separation of constituent quarks is a major dynamical variable in the scattering process. \cite{11} The large-$r$ contribution to the production amplitude \cite{12} depends on the both dipole cross section for large-size dipoles and the amplitudes of distribution of large-size color dipoles and/or the nonperturbative wave functions of light\footnote{See also earlier works on the color dipole analysis of hadronic diffractive interactions which used constituent quark wave functions for the color dipole distribution amplitudes \cite{22}.}.
vector mesons at large $r$, both of which are poorly known at the moment. Still, testing the predictions from such a minimal model is interesting for its own sake and can shed a light on the transition between the soft and hard scattering regimes which is still far from understood. An analysis of sensitivity to models of the nonperturbative wave functions of vector mesons and of how one can disentangle the effects of large $r$ behavior of the wave function and of the dipole cross section, goes beyond the scope of the present exploratory study.

The energy dependence of the dipole cross section is quantified in terms of the dimensionless rapidity $\xi$, which in deep inelastic scattering equals $\xi = \log \frac{1}{x}$. Considerations of intermediate masses in diagrams for exclusive production of vector mesons show that to the considered leading log $1/x$ approximation one must take $\xi = \log \frac{1}{x_{\text{eff}}}$, where

$$x_{\text{eff}} = \frac{Q^2 + m_V^2}{2m_{p\nu}} ,$$

and $m_V$ is a mass of the vector meson. The pomeron exchange dominance holds when the Regge parameter is large,

$$\omega = \frac{1}{x_{\text{eff}}} = \frac{2m_{p\nu}}{(Q^2 + m_V^2)} \gg 1 .$$

Hereafter we write the amplitudes in terms of $\sigma(x_{\text{eff}}, r)$. The spin independence of the dipole cross section $\sigma(x_{\text{eff}}, r)$ in (4) leads to the $s$-channel helicity conservation: the transversely polarized photons produce transversely polarized vector mesons and the longitudinally polarized vector mesons are produced by longitudinal (to be more precise, the scalar one) photons. More explicitly, the form of the forward production amplitudes for the transversely (T) and the longitudinally (L) polarized vector mesons in terms of the lightcone radial wave function $\phi(r, z)$ of the $q\bar{q}$ Fock state of the vector meson reads [6]

$$\text{Im} \mathcal{M}_T(x_{\text{eff}}, Q^2) = \frac{N_c C_V \sqrt{4\pi\alpha_{\text{em}}}}{(2\pi)^2} \cdot \int d^2 r \sigma(x_{\text{eff}}, r) \int_0^1 \frac{dz}{z(1-z)} \left\{ m_V^2 K_0(\varepsilon r) \phi(r, z) - [z^2 + (1-z)^2] \varepsilon K_1(\varepsilon r) \partial_r \phi(r, z) \right\}$$

$$= \frac{1}{(m_V^2 + Q^2)^2} \int d^2 r \frac{\sigma(x_{\text{eff}}, r)}{r^2} W_T(Q^2, r^2)$$

$$\text{Im} \mathcal{M}_L(x_{\text{eff}}, Q^2) = \frac{N_c C_V \sqrt{4\pi\alpha_{\text{em}}}}{(2\pi)^2} \frac{2\sqrt{Q^2}}{m_V} .$$
\[ \cdot \int d^2 r \sigma(x_{\text{eff}}, r) \int_0^1 dz \left\{ [m_q^2 + z(1-z)m_V^2] K_0(\varepsilon r) \phi(r, z) - \varepsilon K_1(\varepsilon r) \partial_r \phi(r, z) \right\} \]

\[ = \frac{1}{(m_V^2 + Q^2)^2} \frac{2\sqrt{Q^2}}{m_V} \int \frac{d^2 r}{r^2} \frac{\sigma(x_{\text{eff}}, r)}{r^2} W_L(Q^2, r^2) \]  

(8)

where

\[ \varepsilon^2 = m_q^2 + z(1-z)Q^2, \]  

(9)

\[ \alpha_{\text{em}} \] is the fine structure constant, \( N_c = 3 \) is the number of colors, \( C_V = \frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{1}{3}, \frac{2}{3} \) for \( \rho^0, \omega^0, \phi^0, J/\Psi \) production, respectively and \( K_{0,1}(x) \) are the modified Bessel functions. The detailed discussion and parameterization of \( \phi(r, z) \) is given in the Appendix, here we only mention that the form of \( \phi(r, z) \) we use has the hard-QCD driven short distance behavior and gives the electromagnetic form factor of mesons which has the correct QCD asymptotic behavior. At large \( r \) we follow the conventional spectroscopic models [23] and constrain the parameters of the wave functions by the widths of the leptonic decays \( V, V' \rightarrow e^+e^- \), the radii of the vector mesons and the 2S-1S mass splitting. The terms \( \propto \phi(r, z) K_0(\varepsilon r) \) and \( \propto \partial_r \phi(r, z) \varepsilon K_1(\varepsilon r) \), i.e., \( \partial_r \phi(r, z) \partial_r K_0(\varepsilon r) \), in the integrands of (7) and (8) derive from the helicity conserving and helicity nonconserving transitions \( \gamma^* \rightarrow q\bar{q} \) and \( V \rightarrow q\bar{q} \) in the \( A_{\mu} \Psi\gamma_{\mu} \Psi \) and \( V_{\mu} \Psi\gamma_{\mu} \Psi \) vertices (see Bjorken et al. [24] and [4], the technique of calculation of traces in the spinorial representation of the relevant Feynman amplitudes is given in [4] and need not be repeated here; for the related Melosh transformation analysis see the recent Ref. [25]). The latter are the relativistic corrections, for the heavy quarkonium the nonrelativistic approximation [2] has a rather high accuracy, the relativistic corrections become important only at large \( Q^2 \) and for the production of light vector mesons. Eqs. (7), (8) give the imaginary part of the production amplitudes; one can easily include small corrections for the real part by the substitution [27],

\[ \sigma(x_{\text{eff}}, r) \Rightarrow \left(1 - i \frac{\pi}{2} \frac{\partial}{\partial \log x_{\text{eff}}} \right) \sigma(x_{\text{eff}}, r) = \left[ 1 - i \cdot \alpha_V(x_{\text{eff}}, r) \right] \sigma(x_{\text{eff}}, r) \]  

(10)

For brevity, in the subsequent discussion we suppress the real part of the production amplitude; it is consistently included in all the numerical calculations.

The color dipole cross section is flavor blind. The only kinematical sensitivity to the vector meson produced comes via the rapidity variable, see Eq. (5). For small \( r \), in the leading \( \log \frac{1}{x} \) and leading \( \log \frac{1}{r^2} \), i.e., leading \( \log Q^2 \), the dipole cross section can be related
to the gluon structure function $G(x, \bar{Q}^2)$ of the target nucleon through

$$\sigma(x, r) = \frac{\pi^2}{3} r^2 \alpha_s(r) G(x, \bar{Q}^2), \quad (11)$$

where the gluon structure function enters at the factorization scale $\bar{Q}^2 \sim \frac{B}{r^2}$ (for the origin of the large scale factor $B \sim 10$, see [28]). The integrands of (7), (8) are smooth at small $r$ and decrease exponentially at $r > 1/\epsilon$ due to the exponential decrease of the modified Bessel functions. Together with the $\propto r^2$ behavior of the color dipole cross section (11), this implies that the amplitudes (7), (8) receive their dominant contribution from $r \approx r_S$. (Eq. (2) assumes that the scanning radius $r_S$ is substantially smaller than the radius $R_V$ of the vector meson.) Then, a simple evaluation gives

$$\text{Im} \mathcal{M}_T \propto r_S^2 \sigma(x_{\text{eff}}, r_S) \propto \frac{1}{Q^2 + m_V^2} \sigma(x_{\text{eff}}, r_S) \propto \frac{1}{(Q^2 + m_V^2)^2} \quad (12)$$

and

$$\text{Im} \mathcal{M}_L \approx \sqrt{Q^2} m_V, \mathcal{M}_T \propto \sqrt{Q^2} m_V r_S^2 \sigma(x_{\text{eff}}, r_S) \propto \sqrt{Q^2} m_V \frac{1}{(Q^2 + m_V^2)^2} \quad (13)$$

respectively. † The prediction of the dominance of the longitudinal cross section at large $Q^2$ is shared by all the models of diffractive leptonproduction, starting with the vector dominance model ([3, 4, 7], for the excellent review of early works on photo- and electroproduction of vector mesons and on vector dominance model see Bauer et al. [29]) and is confirmed by all the experiments on leptonproduction of the $\rho^0$ at large $Q^2$ [30, 31, 32]. The first factor $\propto r_S^2 \propto 1/(Q^2 + m_V^2)$ in (12) comes from the overlap of wave function of the shrinking photon and that of the vector meson. The familiar vector dominance model (VDM) prediction is $M_T \propto \frac{1}{(m_V^2 + Q^2)} \sigma_{\text{tot}}(\rho N)$, whereas in our QCD approach a small $\sigma(x_{\text{eff}}, r_S) \propto r_S^2 \propto 1/(Q^2 + m_V^2)$ enters instead of $\sigma_{\text{tot}}(V N)$. In (12), (13) we show only the leading $Q^2$ dependence, suppressing the phenomenologically important departure form the law $\sigma(x, r) \propto r^2$, whose large $Q^2$ dependence can be related to scaling violations in the gluon density (see (11) and the discussion below). We recall that the shrinkage of the virtual photons and/or the decrease of the scanning radius $r_S$ with $Q^2$ is the origin of color transparency effects in diffractive leptonproduction of vector mesons off nuclei [2, 4, 5, 33]. The important confirmation of the

† Unless otherwise specified, for each flavor, $m_V$ will always be the mass of the ground state 1S vector meson.
quantitative predictions [4, 5] of color transparency effects based on the same technique as used here came from the E665 experiment [34].

More accurate analysis of the scanning phenomenon can be performed in terms of the weight functions $W_{T,L}(Q^2, r^2)$ which are sharply peaked at $r \approx A_{T,L}/\sqrt{Q^2 + m_V^2}$, in the relevant variable $\log r$ the width of the peak in $W_L(Q^2, r^2)$ at half maximum equals $\Delta \log r \approx 1.2$ for the $J/\Psi$ production and $\Delta \log r \approx 1.3$ for the $\rho^0$ production and varies little with $Q^2$ [6].

The values of the scale parameter $A_{T,L}$ turn out to be close to $A \sim 6$, which follows from $r_S = 3/\varepsilon$ with the nonrelativistic choice $z = 0.5$; in general $A_{T,L} \geq 6$ and increases slowly with $Q^2$. This $Q^2$ dependence of $A_{T,L}$ comes from the large-size asymmetric $q\bar{q}$ configurations when, for instance, the antiquark and the quark in the photon and in the vector mesons carry a very large and a very small fraction of the meson momentum respectively (or the other way around). A comparison of the integrands in eqs. (7) and (8) shows that the latter contains an extra factor $z(1 - z)$ which makes considerably smaller the contribution from asymmetric configurations to the longitudinal meson production. For completeness, we quote the results of [3]: $A_{T,L}(J/\Psi; Q^2 = 0) \approx 6$, $A_{T,L}(J/\Psi; Q^2 = 100 \text{GeV}^2) \approx 7$, $A_L(\rho^0; Q^2 = 0) \approx 6.5$, $A_L(\rho^0; Q^2 = 100 \text{GeV}^2) \approx 10$, $A_T(\rho^0; Q^2 = 0) \approx 7$, $A_T(\rho^0; Q^2 = 100 \text{GeV}^2) \approx 12$.

An alternative formulation of the slow onset of the purely perturbative regime can be seen as follows: at very large $Q^2$ when the scanning radius is very small, the dipole cross section $\sigma(x_{eff}, r)$ and the vector meson production amplitudes are proportional to the gluon density $G(x_{eff}, Q^2)$ at the factorization scale $Q^2 = \tau(Q^2 + m_V^2)$ (see also Refs. [3, 7] which use a different technique of the momentum-space wave functions, related to the color dipole factorization by the Fourier-Bessel transform; the detailed comparison with the work of Brodsky et al. [7] will be presented below in Section 6). The large values of $A_{T,L}$ previously quoted, reflect into very small values of $\tau$ [3]: in the interesting region of $Q^2 \gtrsim 10 \text{GeV}^2$ one finds $\tau_{T,L}(J/\Psi) \approx 0.2$, $\tau_L(\rho^0) \approx 0.15$ and $\tau_T(\rho^0) \approx 0.07-0.1$, which is different and substantially smaller than the values $\tau = 0.25$ suggested in [3] and $\tau = 1$ suggested in [7]. Very large $Q^2$ values are needed for reaching the perturbatively large $\bar{Q}^2$ and for the applicability of the pQCD relationship (11).

Consequently, for the domain presently under experimental study, $Q^2 + m_V^2 \lesssim 10-20 \text{GeV}^2$, the production amplitudes receive substantial contribution from semiperturbative and non-
perturbative $r$. In [21, 6] this contribution was modeled by the energy independent soft cross section $\sigma^{(npt)}(r)$. The particular form of this cross section successfully predicted [21] the proton structure function at very small $Q^2$ recently measured by the E665 collaboration [35] and also gave a good description of real photoabsorption [6]. As an example, in Fig. 1 we present an evaluation of the vector meson-nucleon total cross section

$$\sigma_{tot}(VN) = \frac{N_c}{2\pi} \int_0^1 \frac{dz}{z^2(1-z)^2} \int d^2r \left\{ m_q^2 \phi(r,z)^2 + [z^2 + (1-z)^2][\partial_r \phi(r,z)]^2 \right\} \sigma(x_{eff}, r). \quad (14)$$

The total cross section $\sigma_{tot}(\rho^0 N)$ so found, is close to $\sigma_{tot}(\pi N)$, and the rise of $\sigma_{tot}(VN)$ with the c.m.s energy $W$ is consistent with the observed trend of the hadronic total cross sections [36]. In the color dipole picture the smaller values of $\sigma_{tot}(\phi N)$ and $\sigma_{tot}(\phi' N)$ derive from the smaller radius of the $s\bar{s}$ quarkonium. In the simple model [21, 6] the rise of $\sigma_{tot}(VN)$ is entirely due to the gBFKL rise of the perturbative component $\sigma^{(pt)}(x_{eff}, r)$ of the dipole cross section. The rate of rise is small for two reasons: i) at moderate energy, $\sigma^{(pt)}(x_{eff}, r)$ at large $r$ is much smaller than the soft cross section $\sigma^{(npt)}(r)$, ii) at large $r$ the subasymptotic effective intercept of the gBFKL pomeron is small [11, 14]. The detailed description of the dipole cross section used in the present analysis is given in [21, 6] and will not be repeated here. It is partly shown below in Fig. 16. The reason why we focus here on this particular model is that its success in phenomenological applications makes it a realistic tool for the interpolation between soft and hard scattering regions. Once the vector mesons wave functions are fixed from their spectroscopic and decay properties, all the results for diffractive real and virtual photoproduction of vector mesons to be reported here do not contain any adjustable parameters.

### 3 Diffractive $\rho^0$ and $\phi^0$ production: predictions and comparison with experiment

The most interesting prediction from the color dipole dynamics is a rapid decrease of production amplitudes (12), (13) at large $Q^2$. The broadest region of $Q^2$ was covered in the recent NMC experiment [31] where special care was taken to minimize the inelastic production background which plagued earlier data on $\rho^0$ and $J/\Psi$ production. In Fig. 2 we compare our predictions for $\rho^0$ and $\phi^0$ production with the NMC data and the data from the HERA
shown here is the observed polarization-unseparated cross section 
\[ \sigma(\gamma^* \rightarrow V) = \sigma_T(\gamma^* \rightarrow V) + \epsilon \sigma_L(\gamma^* \rightarrow V) \]
for the value of the longitudinal polarization \( \epsilon \) of the virtual photon taken from the corresponding experimental paper (typically, \( \epsilon \sim 1 \)). The quantity which is best predicted theoretically is 
\[ \frac{d\sigma(\gamma^* \rightarrow V)}{dt}|_{t=0} \]
for the value of the longitudinal polarization \( \epsilon \) of the virtual photon taken from the corresponding experimental paper (typically, \( \epsilon \sim 1 \)).

Eqs. (7),(8) describe the pure pomeron exchange contribution to the production amplitude. While at HERA energies secondary Reggeon exchanges can be neglected since the Regge parameter \( \omega \) is a very large, at the lower energy of the NMC experiment, \( \langle \nu \rangle = (90-140) \text{GeV} \), the Regge parameter \( \omega \) is small and non-vacuum Reggeon exchange cannot be neglected. The fit to \( \sigma_{\text{tot}}(\gamma p) \) can, for instance, be cast in the form
\[ \sigma_{\text{tot}}(\gamma p) = \sigma_{\text{IP}}(\gamma p) \cdot \left(1 + \frac{A}{\omega^\Delta}\right) \]
where the term \( A/\omega^\Delta \) in the factor \( f = 1 + A/\omega^\Delta \) represents the non-vacuum Reggeon exchange contribution. The Donnachie-Landshoff fit gives \( A = 2.332 \) and \( \Delta = 0.533 \) [34].

We do not know how large this non-vacuum contribution to \( \rho^0 \) production is at large \( Q^2 \); for a crude estimation we assume the Reggeon/pomeron ratio to scale with \( \omega \), which is not inconsistent with the known decomposition of the proton structure function into the valence (non-vacuum Reggeon) and sea (pomeron) contributions. Then, for the NMC kinematics we find \( f = 1.25 \) at \( \omega \approx 70, Q^2 = 3 \text{GeV}^2 \) and \( f = 1.8 \) at \( \omega \approx 9, Q^2 = 20 \text{GeV}^2 \). This departure of \( f \) from unity provides a conservative scale for the theoretical uncertainties at moderate values of \( \omega \). Anyway, the \( Q^2 \) dependence of the Reggeon correction factor \( f \) is weak compared with the very rapid variations of \( M_T \) and \( M_L \) with \( Q^2 \). The correction for the secondary exchanges, 
\[ \sigma(\gamma^* \rightarrow \rho^0) = f^2 \sigma_{\text{IP}}(\gamma^* \rightarrow \rho^0), \]
brings the theory to a better agreement with the NMC data. The dipole cross section of [21, 3] correctly describes the variation of the \( \rho^0 \) production cross section by 3 orders in magnitude from \( Q^2 = 0 \) to \( Q^2 = 16.5 \text{GeV}^2 \). For \( \phi^0 \) production, \( f \equiv 1 \) due to the Zweig rule and the pure pomeron contribution correctly reproduces the magnitude of \( \sigma(\gamma^* \rightarrow \phi^0) \) and its variation by nearly three orders in the magnitude from \( Q^2 = 0 \) to \( Q^2 = 11.3 \text{GeV}^2 \).

The specific prediction from the gBFKL dynamics is a steeper subasymptotic growth with energy of the dipole cross section \( \sigma(\nu, r) \) at smaller dipole size \( r \), which by virtue of
the scanning phenomenon translates into a steeper rise of $\sigma(\gamma^* \rightarrow V)$ at higher $Q^2$ and/or for heavy quarkonia. This consequence of the color dipole dynamics was first explored in [6]; the $\rho^0$ wave function parameters used in [6] are slightly different from those used here but the difference in $\sigma(\gamma^* \rightarrow \rho^0)$ is marginal. The agreement of our high-energy results with the HERA data is good for both $Q^2 = 0$ (Fig. 3) and large $Q^2$ (Fig. 2) and confirms the growth of the dipole cross section with energy expected from the gBFKL dynamics.

The above high-$Q^2$ data are dominated by the longitudinal cross section; real photoproduction ($Q^2 = 0$) measures the purely transverse cross section. In Fig. 3 we present our results with and without secondary Reggeon corrections ($d\sigma_{\text{IP}}(\gamma \rightarrow \rho^0)/dt|_{t=0}$ and $d\sigma(\gamma \rightarrow \rho^0)/dt|_{t=0} = f^2 d\sigma_{\text{IP}}(\gamma \rightarrow \rho^0)/dt|_{t=0}$ respectively) as a function of energy. The Reggeon correction factor $f^2$ brings the theory to a better agreement with the low energy $\rho^0$ production data [38]. Real photoproduction of $\rho^0$ is dominated by the soft contribution, the growth of the production cross section is driven by the rising gBFKL component of the dipole cross section. Our predictions for high energy agree well with the recent ZEUS data [39, 40].

The $\phi^0$ production is pomeron dominated which implies $f \equiv 1$. We find good agreement with the fixed target [41] and ZEUS [42] data on real photoproduction of the $\phi^0$, although the error bars are large (Fig. 4). Because in $\phi^0$ photoproduction the relevant dipole sizes are smaller than in the $\rho^0$ case, (see the radii of $\rho^0$ and $\phi^0$ in Table 1), we predict a steep energy dependence of the $\phi^0$ production forward cross section: $d\sigma(\gamma \rightarrow \phi^0)/dt|_{t=0}$ is predicted to grow by a factor $\approx 2.5$ from $3.75 \mu b/GeV^2$ at $\nu = 175 GeV$, i.e., $W = 18 GeV$, up to $\sim 8.84 \mu b/GeV^2$ at $W = 170 GeV$ at HERA. At $W = 70 GeV$ we have $\sigma(\gamma \rightarrow \phi^0) = 0.87 \mu b$ which agrees with the first ZEUS measurement $\sigma(\gamma \rightarrow \phi^0) = 0.95 \pm 0.33 \mu b$ [42]. More detailed predictions for the energy and $Q^2$ dependence of $d\sigma(\gamma^* \rightarrow V)/dt|_{t=0}$ are presented in Fig. 5 and clearly show a steeper rise with energy at larger $Q^2$ (see also [6]).

In Fig. 6 we show our predictions for

$$R_{LT} = \frac{m_V^2 d\sigma_L(\gamma^* \rightarrow V)}{Q^2 d\sigma_T(\gamma^* \rightarrow V)}.$$  \hspace{1cm} (16)

The steady decrease of $R_{LT}$ with $Q^2$ which implies a diminution of the dominance of the longitudinal cross section is a very specific prediction of the color dipole approach. It follows from a larger contribution from large size dipoles to the production amplitude for the transversely polarized vector mesons and larger value of the average scanning radius, i.e.,
$A_T \gtrsim A_L$ \[3\]. This prediction can be checked with the higher precision data from HERA; the available experimental data \[30, 31, 32\] agree with $R_{LT} < 1$ but have still large error bars.

The $Q^2$ dependence of the observed polarization-unseparated cross section depends on the longitudinal polarization $\epsilon$ of the virtual photon. To a crude approximation the color dipole dynamics predicts

$$\sigma(\gamma^* \rightarrow V) = \sigma_T(\gamma^* \rightarrow V) + \epsilon\sigma_L(\gamma^* \rightarrow V) \propto \frac{1}{(Q^2 + m_V^2)^4} \left(1 + \epsilon R_{LT} \frac{Q^2}{m_V^2}\right)$$ \(17\)

If one approximates \(17\) by the $(Q^2 + m_V^2)^{-n}$ behavior, one finds $n \sim 3$ vs. $n \sim 1$ in the naive VDM. In \(17\) we suppressed the extra $Q^2$ dependence which at large $Q^2$ comes from the scaling violations in the gluon density factor $\propto G^2(x, \tau(Q^2 + m_V^2))$, see \(11\). For these scaling violations, at fixed $x_{eff}$ and asymptotically large $Q^2$ we expect $n \lesssim 3$. In Fig. 7a we present our predictions for $\rho^0$ and $\phi^0$ production at $W = 100$ GeV as a function of $Q^2 + m_V^2$, assuming for the longitudinal polarization $\epsilon = 1$ as in the ZEUS kinematics \[32\]. These cross sections can be roughly approximated by the $\propto (Q^2 + m_V^2)^{-n}$ law with the exponent $n \approx 2.4$ for the semiperturbative $r_S$ region $1 \lesssim Q^2 \lesssim 10$ GeV$^2$. At fixed $W$, $x_{eff}$ varies with $Q^2$ and for the $x_{eff}$ dependence of $\sigma(x_{eff}, Q^2)$ we predict $n \approx 3.2$ for the perturbative $15 \lesssim Q^2 \lesssim 100$ GeV$^2$ where $r_S$ is small. We strongly urge a careful analysis of the $Q^2$ dependence in terms of the natural variable $Q^2 + m_V^2$ (for more discussion see section 5 below). For the sake of completeness, in Fig.5 we present also our predictions for the energy dependence of the polarization-unseparated production cross section $\sigma = \sigma_T + \epsilon\sigma_L$ for the typical $\epsilon = 1$.

## 4 Anomalies in electroproduction of $2S$ radially excited vector mesons

Here the keyword is the node effect - the $Q^2$ and energy dependent cancellations from the soft (large size) and hard (small size) contributions to the production amplitude of the $V'(2S)$ radially excited vector mesons. When the value of the scanning radius $r_S$ is close to the node $r_n \sim R_V$, these cancellations must exhibit a strong dependence on both $Q^2$ and energy due to the different energy dependence of the dipole cross section at small
(r < R_V) and large (r > R_V) dipole sizes. It must be made clear from the very beginning that when strong cancellations of the large and small region contributions are involved, the predictive power becomes very weak and the results strongly model dependent. Our predictions for the production of the V'(2S) radial excitations which we report here serve mostly as an illustration of the unusual Q^2 and energy dependence possible in these reactions. (Manifestations of the node effect in electroproduction on nuclei were discussed earlier, see [15] and [43].)

In the nonrelativistic limit of heavy quarkonia, the node effect will not depend on the polarization of the virtual photon and of the produced vector meson. Not so for light vector mesons. The wave functions of the transversely and longitudinally polarized photons are different, the regions of z which contribute to the M_T and M_L are different, and the Q^2 and energy dependence of the node effect in production of the transverse and longitudinally polarized V'(2S) vector mesons will be different.

Let us start with the transverse amplitude. Two cases can occur [15], the undercompensation and the overcompensation scenario. In the undercompensation case, the production amplitude \langle 2S|\sigma(x_{eff}, r)\gamma^* |\gamma \rangle is dominated by the positive contribution coming from r \lesssim r_n and the V(1S) and V'(2S) photoproduction amplitudes have the same sign. With our model wave functions this scenario is realized for transversely polarized \rho' and \phi' (we can not, however, exclude the overcompensation scenario). As discussed in [15], in the undercompensation scenario a decrease of of the scanning radius with Q^2 leads to a rapid decrease of the negative contribution coming from large r \gtrsim r_n and to a rapid rise of the V'(2S)/V(1S) production ratio with Q^2. The stronger the suppression of the real photoproduction of the V'(2S) state, the steeper the Q^2 dependence of the V'(2S)/V(1S) production ratio expected at small Q^2. With our model wave functions, the \rho'(2S)/\rho^0 and \phi'(2S)/\phi^0 production ratios for the transverse polarization are predicted to rise by more than one order of magnitude in the range Q^2 \lesssim 0.5\, GeV^2, see Fig. 8; the V(2S) and V(1S) production cross sections become comparable at Q^2 \gtrsim 1\, GeV^2, when the production amplitudes are dominated by dipole size r \ll r_n [15].

For the longitudinally polarized \rho'(2S) and \phi'(2S) mesons, our model wave functions predict overcompensation; at Q^2 = 0 GeV^2 the amplitude is dominated by the negative
contribution from $r \gtrsim r_n$. Consequently, with the increase of $Q^2$, i.e. with the decrease of the scanning radius $r_S$, one encounters the exact cancellation of the large and small distance contributions. Our model wave functions lead to this exact node effect in the dominant imaginary part of the production amplitude at some value $Q^2_n \sim 0.5\text{GeV}^2$ for both the $\rho'_L(2S)$ and $\phi'_L(2S)$ production (see Fig. 6). The value of $Q^2_n$ is slightly different for the imaginary and the real part of the production amplitude but the real part is typically very small and this difference will be hard to observe experimentally. Here we can not insist on the precise value of $Q^2_n$ which is subject to the soft-hard cancellations, our emphasis is on the likely scenario with the exact node effect at a finite $Q^2_n$.

We wish to emphasize that only the experiment will be able to decide between the overcompensation and undercompensation scenarios. For instance, let the $\rho^0$ and $\rho'(2S)$ be observed in the $\pi\pi$ photoproduction channel. The Söding-Pumplin effect of interference between the direct, non-resonant $\gamma p \to \pi\pi p$ production and the resonant $\gamma p \to \rho^0(\rho') p \to \pi\pi p$ production amplitudes leads to the skewed $\rho^0$ and $\rho'$ mass spectrum. The asymmetry of the $\rho^0(\rho')$ mass spectrum depends on the sign of the $\rho^0(\rho')$ production amplitudes ([13], the detailed theory has been worked out in [14]). The Söding-Pumplin technique has already been applied to the $\rho'(1600)$ mass region in $\gamma p \to \pi^+\pi^-$ at 20 GeV studied in the SLAC experiment [20]. Their fit to the $\rho'(1600)$ mass spectrum requires that the sign of the $\rho'$ production amplitude be negative relative to that of the $\rho$. Although the interpretation of this result is not clear at the moment, because there are two $\rho'(1450)$ and $\rho'(1700)$ states which were not resolved in this experiment, the Söding-Pumplin technique seems promising.

With the further increase of $Q^2$ and decrease of the scanning radius one enters the above described undercompensation scenario. Although the radii of the $s\bar{s}$ and $u\bar{u}, d\bar{d}$ vector mesons are different, the $Q^2$ dependence of $\rho'(2S)/\rho^0$ and $\phi'(2S)/\phi^0$ production cross section ratios will exhibit a similar pattern. For both the transverse and longitudinally polarized photons, these ratios rise steeply with $Q^2$ on the scale $Q^2 \sim 0.5\text{GeV}^2$. At large $Q^2$ where the production of longitudinally polarized mesons dominates, the $\rho'(2S)/\rho^0$ and $\phi'(2S)/\phi^0$ cross section ratios level off at $\sim 0.3$ (see Fig. 8). This large-$Q^2$ limiting value of the $\rho'(2S)/\rho^0$ and $\phi'(2S)/\phi^0$ cross section ratios depend on the ratio of $V'(2S)$ and $V(1S)$ wave functions at the origin, which in potential models is subject to the detailed form of the confining potential.
It is interesting that due to the different node effect for the $T$ and $L$ polarizations, we find $R_{LT}(2S) \ll R_{LT}(1S)$, see Fig. 6.

In Fig. 7b we present our predictions for the $Q^2$ dependence of the polarization-unseparated cross section $\sigma(\gamma^* \rightarrow V'(2S)) = \sigma_T(\gamma^* \rightarrow V'(2S)) + \epsilon \sigma_L(\gamma^* \rightarrow V'(2S))$ at the HERA energy $W = 100\text{GeV}$ assuming $\epsilon = 1$. In Fig. 9 we show the $Q^2$ dependence of the polarization-unseparated forward cross section ratios $d\sigma(\gamma^* \rightarrow \rho'(2S))/d\sigma(\gamma^* \rightarrow \rho^0)$ and $d\sigma(\gamma^* \rightarrow \phi'(2S))/d\sigma(\gamma^* \rightarrow \phi^0)$ at $W = 100\text{GeV}$. Due to its smallness, the anomalous properties of $\sigma_L(2S)$ at small $Q^2$ are essentially invisible in the polarization-unseparated $V'(2S)$ production cross section shown in Figs. 7b, 9 and 10. In contrast to $\sigma(\gamma^* \rightarrow V(1S))$, which falls monotonically and steeply from $Q^2 = 0\text{GeV}^2$ on, the $\sigma(\gamma^* \rightarrow V'(2S))$ shown in Fig. 7b exhibits a weak rise at small $Q^2$. At $Q^2$ large enough that the scanning radius $r_s < R_V$ and the node effect becomes negligible, we predict very similar dependence on $Q^2 + m_V^2$ of the $V'(2S)$ and $V(1S)$ production.

Color dipole dynamics uniquely is the source of such a tricky $Q^2$ dependence of the $V'(2S)/V(1S)$ production ratio. We already mentioned about the experimental confirmation of the node effect predicted in $\Psi'$ production. Further experimental confirmations of the node effect, in particular of the unique overcompensation scenario which is possible for light vector mesons, would be extremely interesting. The available experimental data on real photoproduction of radially excited light $V'(2S)$ mesons confirm $\sigma(\gamma \rightarrow V'(2S))/\sigma(\gamma \rightarrow V(1S)) \ll 1$, but are still of a poor quality and the branching ratios of the $V'$ decays are not yet established (for the review see and the Review of Particle Properties). For instance, the FNAL E401 experiment at $\nu \approx 100\text{GeV}$ found $\sigma(\gamma \rightarrow \phi'(1700, K^+K^-)) = 8.0 \pm 2.7(\text{stat}) \pm 1.4(\text{syst})\text{ nb}$ to be compared with $\sigma(\gamma \rightarrow \phi) \approx 0.55\mu\text{b}$ (see Fig.4). In the $\rho$ family, the very spectroscopy of the $\rho'$ mesons is not yet conclusive. There are two $\rho'$ states, $\rho'(1450)$ and $\rho'(1600)$, the $2S$ and $D$-wave assignment for these states is not yet clear. The first high energy data on the $\rho'(1450)$ and $\rho'(1700)$ leptoproduction were reported by the E665 collaboration. These E665 data refer to the coherent production on Ca target. For the $\rho'(1700)$, they exhibit a strong rise of $R_{21} = \sigma(\rho' \rightarrow 4\pi)/\sigma(\rho \rightarrow 2\pi)$ with $Q^2$ by more than one order in magnitude from $(0.004 \pm 0.004)$ at $Q^2 = 0.15\text{GeV}^2$ to $(0.15 \pm 0.07)$ at $Q^2 = 4.5\text{GeV}^2$. Such a steep $Q^2$ dependence is perfectly consistent
with our expectations for the production of radially excited 2S light vector mesons. For the \( \rho'(1450) \) there is a weak evidence of a nonmonotonic \( Q^2 \) dependence: \( R_{21} = (0.035 \pm 0.011) \) at \( Q^2 = 0.15 \text{ GeV}^2 \) followed by decrease down to \( R_{21} = (0.012 \pm 0.004) \) at \( Q^2 = 0.3 \text{ GeV}^2 \) and then to an increase and leveling off to \( R_{21} = (0.08 \pm 0.04) \) at larger \( Q^2 \geq 2 \text{ GeV}^2 \). Such a \( Q^2 \) dependence of \( R_{21} \) would be natural for a D-wave state which has a nodeless radial wave function. If these E665 observations will be confirmed in higher statistics experiments, then the color dipole interpretation of the \( Q^2 \) dependence would strongly suggest the 2S and D-wave state assignments for the \( \rho'(1700) \) and \( \rho'(1450) \), respectively. We remind the reader that, for a quantitative comparison with the predictions of our model (shown in Fig. 10), the E665 results for \( R_{21} \) must be corrected for the branching ratio \( B(\rho' \to 4\pi) \), which is still experimentally unknown.

The energy dependence of the \( \rho'(2S), \phi'(2S) \) real photoproduction is shown in Fig. 10 and has its own peculiarities. In the color dipole gBFKL dynamics, the negative contribution to the 2S production amplitude coming from large size dipoles, \( r \gtrsim r_n \), has a slower growth with energy than the positive contribution coming from the small size dipoles, \( r \gtrsim r_n \). For this reason, in the undercompensation regime the destructive interference of the two contributions becomes weaker at higher energy and we predict a growth of the \( V'(2S)/V(1S) \) cross section ratios with energy. Taking only the pure pomeron contributions into account, we find for the forward cross section ratio \( d\sigma(\gamma \to \rho'(2S))/d\sigma(\gamma \to \rho^0) = 0.041 \) at \( W = 15 \text{ GeV} \), which at HERA energies increases to 0.063 and 0.071 at \( W = 100 \text{ GeV} \) and \( W = 150 \text{ GeV} \), respectively. Whereas in \( \rho \) and \( \rho' \) production one must be aware of the non-vacuum Reggeon exchange contributions at lower energy, in the pomeron dominated \( \phi', \phi \) real photoproduction we find a somewhat faster rise of \( d\sigma(\gamma \to \phi'(2S))/d\sigma(\gamma \to \phi^0) \) with energy from 0.054 at \( W = 15 \text{ GeV} \) to 0.089 and 0.099 at \( W = 100 \text{ GeV} \) and \( W = 150 \text{ GeV} \), respectively.

If the leptoproduction of the longitudinally polarized \( V'_{L}(2S) \) will be separated experimentally, we will have a chance of studying the \( Q^2 \) and energy dependence in the overcompensation scenario. Start with the moderate energy and consider \( Q^2 \) very close to \( Q^2_n \) but still \( \lesssim Q^2_n \). In this case the negative contribution from \( r \gtrsim r_n \) takes over in the \( V'_{L}(2S) \) production amplitude. With increasing energy, the positive contribution to the produc-
tion amplitude rises faster and ultimately takes over. At some intermediate energy, there will be an exact cancellation of the two contributions to the production amplitude and the longitudinal $V'_L(2S)$ production cross section shall exhibit a minimum at this energy (the minimum will partly be filled because cancellations in the real and imaginary part of the production amplitude are not simultaneous). With our model wave functions, we find such a nonmonotonic energy dependence of the $\rho'_L(2S)$ and $\phi'_L(2S)$ production at $Q^2 \approx 0.5\text{ GeV}^2$, which is shown in Figs. 8 and 10. At higher $Q^2$ and smaller scanning radii $r_S$ the energy dependence of $V'_L(2S)/V_L(1S)$ production ratio becomes very weak.

Finally, a brief comment on the $t$-dependence of the differential cross sections is in order. For the $1S$ vector mesons we expect the conventional diffractive peak with smooth and gentle energy dependence. For the radially excited vector mesons the $t$-dependence can be anomalous. The point is that the large size contribution to the $V'(2S)$ meson production amplitude has steeper $t$-dependence that the small size contribution. The destructive interference of these two amplitudes can lead to two effects: i) the diffraction slope in the $V'(2S)$ meson production will be smaller than in the $V(1S)$ meson production, ii) the effective diffraction slope for the $V'(2S)$ meson production decreases towards small $t$ contrary to the familiar increase for the $V(1S)$ meson production. High statistics data on the $\rho', \phi'$ production at HERA are needed to test these predictions. More detailed discussion of the diffraction slope will be presented elsewhere.

5 Scaling relations between production of different vector mesons

The color dipole cross section is flavor blind and only depends on the dipole size. The results (12),(13) for the production amplitudes strongly suggest the restoration of flavor symmetry, i.e., a similarity between the production of different vector mesons when compared at the same value of the scanning radius $r_S$ and/or the same value of $Q^2 + m_V^2$. Such a comparison must be performed at the same energy, which also provides the equality of $x_{eff}$ at equal $Q^2 + m_V^2$. Evidently, the value of $Q^2$ must be large enough so that the scanning radius $r_S$
is smaller than the radii of vector mesons compared.

In order to illustrate the above point we present in Figs. 11 and 12 the ratio of forward production cross sections $R((J/\Psi)/\rho^0; Q^2) = d\sigma(\gamma^* \to J/\Psi)/d\sigma(\gamma^* \to \rho^0)$ and $R(\phi^0/\rho^0; Q^2) = d\sigma(\gamma^* \to \phi^0)/d\sigma(\gamma^* \to \rho^0)$ as a function of the c.m.s energy $W$ at different $Q^2$ (here we use for the $J/\Psi$ production cross section the values obtained from a recent calculation [48], which practically coincide with those of ref. [6], the slight difference being due to a somewhat different $J/\Psi$ wave function). Here we compare the polarization-unseparated cross sections $\sigma = \sigma_T + \epsilon \sigma_L$, taking for the definiteness $\epsilon = 1$ which is typical of the HERA kinematics. These ratios exhibit quite a strong $Q^2$ dependence, which predominantly comes from the $Q^2$ dependence of the factor $(Q^2 + m_{V_1}^2)/(Q^2 + m_{V_2}^2)^n$, which changes rapidly when the two vector mesons have different masses. The energy dependence of the cross section ratios taken at the same $Q^2$ derives from the different energy dependence of the dipole cross section which enters at different radii $r_{Si} \approx 6 \sqrt{Q^2 + m_{Vi}^2}$ in the numerator and denominator of the $V_1/V_2$ cross section ratio,

$$R(V_1/V_2; Q^2) = \frac{\sigma(\gamma^* \to V_1)}{\sigma(\gamma^* \to V_2)} \propto \frac{\sigma^2(\nu, r_{S1})}{\sigma^2(\nu, r_{S2})}.$$ 

In the HERA energy range we predict $R((J/\Psi)/\rho^0; Q^2 = 0) = \sigma(\gamma \to J/\Psi)/\sigma(\gamma \to \rho^0) = 0.022$ at $W = 70$ GeV and 0.028 at $W = 150$ GeV, which agrees with the experimentally observed ratio $0.0034 \pm 0.0014$ of H1 ($W \sim 70$ GeV) [43, 37] and $0.0045 \pm 0.0023$ of ZEUS ($W = 150$ GeV) [39, 40, 50]. Notice the rise of $R((J/\Psi)/\rho^0; Q^2)$ by more than 3 orders in the magnitude from $Q^2 = 0$ to $Q^2 = 100$ GeV$^2$. Our result for the ratio $R(\phi^0/\rho^0; Q^2 = 0) = d\sigma(\gamma \to \phi^0)/d\sigma_{IP}(\gamma \to \rho^0)$ shown in Fig. 11 is substantially smaller than the factor $2/9$ expected from the naive VDM, in a very good agreement with the experiment ([41] and references therein). This suppression is a natural consequence of the color dipole approach and derives from the smaller radius of the $s\bar{s}$ quarkonium and smaller transverse size of the $s\bar{s}$ Fock state of the photon as compared to the radius of the $\rho^0$ and size of the $u\bar{u}, d\bar{d}$ Fock states of the photon, respectively, cf. Table 1. For increasing $Q^2$s, the ratio $R(\phi^0/\rho^0; Q^2)$ overshoots the VDM ratio $2/9$ and rises by one order of magnitude from $Q^2 = 0$ to $Q^2 = 100$ GeV$^2$. In Figs. 11 and 12 we compare only pure pomeron contributions to the production cross
section; at smaller values of the energy and of the Regge parameter \( \omega \), the \( \phi^0/\rho^0 \) and \((J/\Psi)/\rho^0\) production ratios will be further suppressed by the factor \( f^2 \).

The remarkable restoration of flavor symmetry in the natural scaling variable \( Q^2 + m_V^2 \) is demonstrated in Figs. 13 and 14, where we present a ratio \( R(i/k; Q^2 + m_V^2) \) of the same cross sections taken at equal \( Q^2 + m_V^2 \) rather than equal \( Q^2 \). A marginal variation of the \( R((J/\Psi)/\rho^0; Q^2 + m_V^2) \) and \( R(\phi^0/\rho^0; Q^2 + m_V^2) \) in this scaling variable must be contrasted with the variation of the \( R((J/\Psi)/\rho^0; Q^2 = 0) \) and \( R(\phi^0/\rho^0; Q^2 = 0) \) by the three and one orders in the magnitude previously mentioned, respectively, over the same span of \( Q^2 \) values \( 0 < Q^2 < 100 \text{ GeV}^2 \). The origin of the slight departures from exact scaling in the variable \( Q^2 + m_V^2 \) comes from a well understood difference between the scales \( A_{T,L} \) and \( \tau_{T,L} \) for production of different vector mesons. The same difference of \( A_{T,L} \) and \( \tau_{T,L} \) brings in the energy dependence of \( R(i/k; Q^2 + m_V^2) \). This is a specific prediction from the preasymptotic gBFKL dynamics. The radii of the \( \phi^0 \) and \( \rho^0 \) mesons do not differ much and for this reason we find a precocious scaling in \( Q^2 + m_V^2 \). The energy dependence of the \( \phi^0/\rho^0 \) ratio also turns out very weak. The radii of the \( \rho^0 \) and \( J/\Psi \) differ much more strongly and the ratio \( R((J/\Psi)/\rho^0; Q^2 + m_V^2) \) exhibits a somewhat stronger dependence on energy and \( Q^2 + m_V^2 \).

For the same reason, we predict a substantial departure of \( R(i/k; Q^2 + m_V^2) \) from the short-distance formula

\[
R(i/k; Q^2 + m_V^2) = \frac{m_i \Gamma_i(e^+e^-)}{m_k \Gamma_k(e^+e^-)}, \tag{18}
\]

which is shown in Fig. 13 by horizontal lines. The formula (18) can readily be derived generalizing the asymptotic-\( Q^2 \) considerations [3], for the further discussion of the crucial rôle of the scaling variable \( Q^2 + m_V^2 \) in this comparison see below Section 6.

The case of the \( \omega^0, \omega' \) virtual photoproduction is very interesting. Is the \( \rho^0 - \omega^0 \) mass degeneracy accidental? Does it imply also similar spatial wave functions in the \( \rho \) and \( \omega \) families? The scanning property of diffractive production allows a direct comparison of spatial wave functions of the \( \rho^0 \) and \( \omega^0 \). If the \( \omega^0 - \rho^0 \) degeneracy extends also to the spatial wave functions, then we predict

\[
\frac{\sigma(\gamma^* \rightarrow \omega^0)}{\sigma(\gamma^* \rightarrow \rho^0)} = \frac{1}{9} \tag{19}
\]

independent of energy and \( Q^2 \). On the other hand, if the radii of the \( \rho^0 \) and \( \omega^0 \) are different, for instance \( R_\omega < R_\rho \), then the \( \omega^0/\rho^0 \) production ratio must exhibit the \( Q^2 \) dependence
reminiscent of the $\phi^0/\rho^0$ ratio. Similarly, a comparison of the $\omega'$ and $\rho'$ production can shed light on the isospin dependence of interquark forces in vector mesons.

6 Determination of color dipole cross section from vector meson production data

Inverting Eqs. (12), (13) one can evaluate $\sigma(x_{\text{eff}}, r)$ from the vector meson production data. It is convenient to cast Eqs. (12), (13) in the form

$$\text{Im} \mathcal{M}_T = g_T \sqrt{4\pi \alpha_{\text{em}}} C_V \sigma(x_{\text{eff}}, r_S) \frac{m_V^2}{m_V^2 + Q^2}$$

$$\text{Im} \mathcal{M}_L = g_L \sqrt{4\pi \alpha_{\text{em}}} C_V \sigma(x_{\text{eff}}, r_S) \frac{\sqrt{Q^2}}{m_V} \cdot \frac{m_V^2}{m_V^2 + Q^2}$$

In (20), (21) the coefficient functions $g_{T,L}$ are defined so as to relate the amplitude to $\sigma(r_S)$ at the well defined scanning radius (2) with $A \equiv 6$. The major point of this decomposition is that at large $Q^2$ and/or small $r_S \ll R_V$, the coefficients $g_{T,L}$ will be very smooth functions of $Q^2$ and energy. The smooth $Q^2$ and energy dependence of $g_{T,L}$ mostly reflects the smooth and well understood $Q_2$ dependence of scale factors $A_{T,L}$. Such a procedure is somewhat crude and the $\text{Im} \mathcal{M}_{T,L} - \sigma(x_{\text{eff}}, r_S)$ relationship is sensitive to the assumed $r$ dependence of the dipole cross section $\sigma(x_{\text{eff}}, r)$. Using the dipole cross section [6] the shape of which changes significantly from $\omega = \frac{1}{x_{\text{eff}}} = 30$ up to $\omega = 3 \cdot 10^6$, we have checked that this sensitivity is weak. In Fig. 15 we present the $Q^2$ dependence of the $g_{T,L}$ for different production processes at $W = 15$ GeV and $W = 150$ GeV. The variation of the resulting coefficient functions $g_{T,L}$ from small to large $W$ does not exceed 15%, which is a conservative estimate of the theoretical uncertainty of the above procedure.

The experimentally measured forward cross production section equals

$$\frac{d\sigma(\gamma^* \rightarrow V)}{dt} |_{t=0} = \frac{f^2}{16\pi} \cdot \left[ (1 + \alpha_{V,T}^2) \mathcal{M}_T^2 + \epsilon(1 + \alpha_{V,L}^2) \mathcal{M}_L^2 \right]$$

The difference between $\alpha_{V,L}$ and $\alpha_{V,T}$ for the longitudinal and transverse cross sections and the overall effect of the real part is marginal and can safely be neglected compared to other uncertainties. Then, making use of the above determined $g_{T,L}$ and combining Eqs. (20), (21) and (22), we obtain

$$\sigma(x_{\text{eff}}, r_S) = \frac{1}{f} \cdot \frac{1}{C_V} \cdot \frac{Q^2 + m_V^2}{m_V^2} \cdot \frac{2}{\sqrt{\alpha_{\text{em}}}} \cdot \left( g_T^2 + \epsilon g_L^2 \cdot \frac{Q^2}{M_V^2} \right)^{-1/2}.$$
Here $\epsilon$ is the longitudinal polarization of the photon the values of which are taken from the corresponding experimental publications. In (22),(23) $f$ is the above discussed factor which accounts for the non-vacuum Reggeon contribution to the $\rho^0$ production, for $\phi^0$ and $J/\Psi$ production, $f \equiv 1$. In the case the experimental data are presented in the form of the $t$-integrated cross section, we evaluate $\frac{d\sigma(\gamma^* \rightarrow V)}{dt} \bigg|_{t=0} = B\sigma_{tot}(\gamma^* \rightarrow V)$ using the diffraction slope $B$ as cited in the same experimental publication.

In Fig. 16 we show the results of such an analysis on the low energy [41] and ZEUS [42] $\phi^0$ real photoproduction data, on the $\rho^0$ and $\phi^0$ NMC electroproduction data [51], on the $\rho^0$ HERA real and virtual photoproduction (H1 [37], ZEUS [39, 40, 32]), on the fixed target data on real photoproduction (EMC [51], E687 [52]), on the EMC $J/\Psi$ electroproduction data ([53]) and on the HERA real photoproduction $J/\Psi$ data (H1 [49], ZEUS [50]). The error bars are the error bars in the measured cross sections as cited in the experimental publications.

The experimental data on the vector meson production give a solid evidence for a decrease of $\sigma(x_{eff}, r_S)$ by one order of magnitude from $r_S \approx 1.2$ fm in $\phi^0$ real photoproduction down to $r_S \approx 0.24$ fm in the electroproduction of $\rho^0$ at $Q^2 = 23$ GeV$^2$ and of $J/\Psi$ at $Q^2 = 13$ GeV$^2$.

In the region of overlapping values of $r_S$ there is a remarkable consistency between the dipole size dependence and the absolute values of the dipole cross section determined from the data on the $\rho^0$, $\phi^0$ and $J/\Psi$ production, in agreement with the flavor independence of the dipole cross section. A comparison of determinations of $\sigma(x_{eff}, r)$ at fixed-target and HERA energy confirms the prediction [11, 14, 6] of faster growth of the dipole cross section at smaller dipole size, although the error bars are still large.

The above determination of $\sigma(x_{eff}, r_S)$ is rather crude for the several reasons.

i) First, a comparison of the NMC [31] and early EMC data [54] on the $\rho^0$ production suggests that the admixture of inelastic process $\gamma^* p \rightarrow V X$ could have enhanced the EMC cross section by as large a factor as $\sim 3$ at $Q^2 = 17$ GeV$^2$. The value of $\sigma(\gamma^* \rightarrow V)$ thus overestimated, leads to $\sigma(x_{eff}, r_S)$ overestimated by the factor $\sim \sqrt{3}$, which may be a reason why the EMC $J/\Psi$ electroproduction data [53] lead consistently to somewhat larger values of $\sigma(x_{eff}, r_S)$. Still, even this factor of $\sim \sqrt{3}$ uncertainty is much smaller than the more
than the one order of magnitude by which $\sigma(x_{\text{eff}}, r_S)$ varies over the considered span of $r_S$. In the recent NMC data [31] a special care has been taken to eliminate an inelastic background and the values of $\sigma(x_{\text{eff}}, r_S)$ from the $\rho^0$ and $\phi^0$ production data are consistent within the experimental error bars.

ii) There are further uncertainties with the value of the diffraction slope $B(\gamma^* \to V)$ and the curvature of the diffraction cone which affect the extrapolation down to $t = 0$. The experimental situation with the diffraction slopes is quite unsatisfactory; in the case of the $J/\Psi$ and of the light vector mesons at large $Q^2$, one can not exclude even a $\sim 50\%$ uncertainty in the value of $B(\gamma^* \to V)$. However, this uncertainty in $B(\gamma^* \to V)$ corresponds to $\lesssim 25\%$ uncertainty in our evaluation of $\sigma(x_{\text{eff}}, r_S)$, which is sufficient for the purposes of the present exploratory study.

iii) In addition, there is also the above evaluated conservative $\lesssim 15\%$ theoretical inaccuracy of our procedure.

iv) Finally, there is a residual uncertainty concerning the wave function of light vector mesons. As a matter of fact, if the dipole cross section were known, the diffractive production $\gamma^* p \to V p$ would be a unique local probe of the vector meson wave function at $r \approx r_S$ [4]; this may well become one of the major applications of vector meson production. To this aim, the consistency of $\sigma(x_{\text{eff}}, r)$ determined from different reactions indicates that wave functions of vector mesons are reasonably constrained by modern spectroscopic models and by the leptonic width.

This is the first direct determination of the dipole cross section from the experimental data and our main conclusions on the properties of the dipole cross section are not affected by the above cited uncertainties. In Fig. 16 we show also the dipole cross section from the gBFKL analysis [21, 6], which gives a good quantitative description of structure function of the photon at small $x$. We conclude that the color dipole gBFKL dynamics provides a unified description of diffractive production of vector mesons and of the proton structure function.

Finally, a comparison of the color dipole analysis of diffractive electroproduction [1, 5, 6] with the related momentum space analysis of Refs. [3, 7] is in order. At a very large $Q^2$ and/or very short scanning radius, $r_S \ll R_V$, the electroproduction probes the wave
function of vector mesons and/or the z-distribution amplitude at a vanishing transverse size, integrated over the z with the certain z-dependent factor which emerges in Eqs. (7),(8). The wave function at the vanishing 3-dimensional separation of the quark and antiquark can be related to the width of the leptonic decay, $V \rightarrow e^+e^-$. The form of the z-dependent factor is mostly dictated by $r_S^2 \sim 1/(4z(1-z)Q^2 + m_V^2)$ which emerges in the integrands of (7),(8) after the r integration, and for the asymptotical $Q^2$ when $4z(1-z)Q^2 \gg m_V^2$, Brodsky et al. [7] introduced the moment of the longitudinal distribution amplitude

$$\eta_V = \frac{\int_0^1 dz \frac{1}{2z(1-z)} \Psi_V(r = 0, z)}{\int_0^1 dz \Psi_V(r = 0, z)}. \quad (24)$$

One must be careful with the interpretation of $\eta_V$, though, because for the very asymmetric $q\bar{q}$ configurations, $z(1-z) \lesssim M_V^2/Q^2$, the scanning radius stays large even for $Q^2 \rightarrow \infty$; for instance, precisely these asymmetric configurations dominate the cross section of the diffraction dissociation of photons, $\gamma^*p \rightarrow Xp$, into the continuum states $X$ [9,10]. With these reservations, we can combine the representations (20),(21), the pQCD relationship (11) and the formula (2) for the scanning radius, and cast the production amplitude in the form (here we focus on the dominant longitudinal amplitude)

$$M_L = \frac{8\pi^2}{3} f_V \sqrt{4\pi \alpha_{em}} \eta_V m_V \sqrt{Q^2/m_V} \frac{1}{(Q^2 + m_V^2)^2} \alpha_s(Q^2)G(x,\bar{Q}^2) \frac{2f_V\eta_V m_V \sqrt{Q^2}}{9/m_V} \left(\frac{6}{A}\right)^2 r_S^2 \sigma(r_S), \quad (25)$$

where

$$f_V^2 = \frac{3}{8\pi \alpha_{em}^2} \Gamma(V \rightarrow e^+e^-) m_V. \quad (26)$$

Then, we can present our results for $M_L$ in terms of this parameter $\eta_V$. The first line of Eq. (25) gives the asymptotic-$Q^2$ form of $M_L$ in terms of the gluon structure function of the proton, the second line is equivalent to it at large pQCD factorization scale $\bar{Q}^2$ and serves as a working definition of $\eta_V$ at moderately large $Q^2$ and/or moderately small scanning radius $r_S$. The above finding that $g_{T,L}$ only weakly depend on $Q^2$ and energy, already suggests the $\eta_V$ defined by the second line of Eq. (25) will be approximately constant, and now we show this is indeed the case.

Evidently, the resulting values of $\eta_V$ will depend on the pQCD factorization scale $\bar{Q}^2$. The scale parameter $\tau$ in the pQCD factorization scale $\bar{Q}^2 = \tau(Q^2 + m_V^2)$ was evaluated in
It is related to the scale parameter $B \approx 10$ in the pQCD formula (11) and the scale parameter $A$ in the scanning radius (2) as $\tau \sim B/A^2$, for production of the longitudinal vector mesons in DIS Ref. [6] finds $\tau_L(J/\Psi) \approx 0.2$ and $\tau_L(\rho^0) \approx 0.15$. Ref. [4] cites the asymptotic leading twist form of (24), with $m_V^2$ neglected in the denominator compared to $Q^2$ and with the pQCD factorization scale $\bar{Q}^2 = Q^2$, besides the more accurate definition of the pQCD scale $\bar{Q}^2$ we differ from Ref. [7] also by the factor 2 in Eq. (11). The scale $A$ in the scanning radius is given by the position of the peak in $W_L(Q^2, r^2)$, it varies with $Q^2$ slightly bringing the slight variation of the scale factors $A_{T,L}$ in, at large $Q^2$ it is reasonable to take $A_L(J/\Psi) = 6$ and $A_L(\rho^0, \phi^0) = 8$ [4]. With this choice of $A_L$, our results for the production amplitude $M_L$ correspond to the values of $\eta_V$ shown in Figs. 17 and 18. For the nonrelativistic quarkonium, in which $z \approx 1/2$, Eq. (24) gives $\eta_V \approx 2$. The $\Upsilon$ is a good approximation to the nonrelativistic quarkonium and we indeed find $\eta_V \approx 2$. Taking a fixed scale $A$, we neglected the slight variation of $\tau$ with $Q^2$, which propagates into the slight variation of $\eta_V$ with $Q^2$. Because the shape of the color dipole cross section varies with $x_{eff}$, the scale parameter $A$ varies also with $x$ slightly. Taking the $x$-independent $A$, we cause the slight mismatch of the $x$-dependence of the r.h.s. and l.h.s. of Eq. (24), which propagates into the weak $x_{eff}$ dependence of $\eta_V$. In Fig. 17 we show $\eta_V$ for the fixed energy $W = 150$ GeV relevant to the HERA experiments, here the combined $Q^2$ and $x_{eff}$ dependence of the scale parameter $A$ contribute to the variations of $\eta_V$. The issue of the $Q^2$ and $x$ dependence of the pQCD factorization scale $\bar{Q}^2$ in (24) deserves a dedicated analysis, here we only wish to focus on the fact that the so determined $\eta_V$ exhibits a remarkably weak variation with $Q^2$ and $x_{eff}$. Furthermore, Fig. 18 shows that the $x_{eff}$-dependence of $\eta_V$ becomes substantially weaker at larger $Q^2$. This testifies to an importance of the $Q^2 + m_V^2$ as a relevant scaling variable, which absorbs major mass corrections to the $Q^2$ dependence of the production amplitude (see also the discussion of the flavor symmetry restoration in Section 5). To this end we wish to notice that the expansion

$$\frac{1}{(Q^2 + m_V^2)^2} = \frac{1}{Q^4} \left(1 + \frac{2m_V^2}{Q^2} - \cdots\right)$$  

(27)

corresponds to the abnormally large scale $2m_V^2$ for the higher twist correction to the production amplitude of leading twist. For the light vector mesons, Brodsky et al. cite estimates $\eta_V = 3-5$, our results in Figs. 17 and 18 are very close to these estimates, as it must be
expected because the momentum-space technique of Brodsky et al. and our color dipole factorization technique are related by the Fourier-Bessel transform. With the present poor knowledge of the large dipole distributions in vector mesons and/or the wave functions of vector mesons, the variations of $\eta_V$ in Figs. 17 and 18 and the range of estimates for $\eta_V$ in [7] indicate the range of uncertainty in predictions leptoproduction amplitudes.

7 Conclusions

The purpose of this paper has been the phenomenology of diffractive photoproduction and electroproduction of ground state ($1S$) and radially excited ($2S$) light vector mesons in the framework of the color dipole picture of the QCD pomeron. In this picture, the $Q^2$ dependence of production of the $1S$ vector mesons is controlled by the shrinkage of the transverse size of the virtual photon and the small dipole size dependence of the color dipole cross section. Taking the same color dipole cross section as used in the previous successful prediction of the low $x$ structure function of the proton, we have obtained a good quantitative description of the experimental data on diffractive photoproduction and electroproduction of $1S$ vector mesons $\rho^0, \phi^0$ and $J/\Psi$. We have presented the first determination of the dipole cross section from these data and found a remarkable consistency between the absolute value and the dipole size and energy dependence of the dipole cross section extracted from the data on different vector mesons. This represents an important cross-check of the color dipole picture. The pattern we found for the energy dependence of the dipole cross section is consistent with flavor independence and with expectations from the gBFKL dynamics. The color dipole picture leads to the restoration of the flavor symmetry and to novel scaling relations between the production of different vector mesons when compared at the same $Q^2 + m_V^2$. Such relations are borne out by the available data and will be further tested when the higher precision data from HERA will become available. Regarding this $(Q^2 + m_V^2)$-scaling, perhaps still more interesting are the deviations from scaling, which originate from a substantial contribution of the large size dipoles even at very large $Q^2$.

The second class of predictions concerns the rich pattern of an anomalous $Q^2$ and energy dependence of the production of the $V'(2S)$ radially excited vector mesons, which depends entirely on the quantum mechanical fact that the $2S$ wave function has a node
which makes these anomalies an unavoidable effect. We find a very strong suppression of the \( V'(2S)/V(1S) \) production ratio in the real photoproduction limit of very small \( Q^2 \). For the longitudinally polarized \( 2S \) mesons we find a plausible overcompensation scenario leading to a sharp dip of the longitudinal cross section \( \sigma_L(2S) \) at some finite \( Q^2 = Q^2_n \sim 0.5 \text{GeV}^2 \). The position \( Q^2_n \) of this dip depends on the energy and leads to a nonmonotonic energy dependence of \( \sigma_L(2S) \) at fixed \( Q^2 \). Regarding the experimental choice between the overcompensation and undercompensation scenarios in the HERA experiments, the situation looks quite favorable because the sign of the \( \rho' \) production amplitude relative to that of the \( \rho^0 \) can be measured directly by the Söding-Pumplin method. At larger \( Q^2 \), the scanning radius becomes shorter, and we predict a steep rise of the \( 2S/1S \) cross section ratio, typically by one order of magnitude on the very short scale \( Q^2 \lesssim 0.5 \text{GeV}^2 \) in agreement with the present indications from the E665 data. The flattening of this \( 2S/1S \) ratio at large \( Q^2 \) is a non-negotiable prediction from the color dipole dynamics. Remarkably, the \( Q^2 \) dependence of the \( V' \) production offers a unique possibility of distinguishing between \( 2S \) radially excited and \( D \)-wave vector mesons.

Finally, in the color dipole framework, a comparison of the \( Q^2 \) dependence of the diffractive production of the \( \rho^0 \) and \( \omega^0 \) constitutes a direct comparison of the spatial wave functions of the two mesons. A comparison of the \( Q^2 \) dependence of the \( \omega' \) and \( \rho' \) production can shed light on the isospin dependence of the interquark forces in vector mesons.

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Appendix.

Here we present the parameterization of the wave functions of vector mesons in the lightcone mixed \((r, z)\) representation. Due to the fact that small size \(q\bar{q}\) configurations become important at large \(Q^2\), one needs to include the short distance hard QCD gluon exchange effects so as to make the electromagnetic form factors consistent with the QCD predictions. Here we follow a simple procedure suggested in \[6\], which uses the relativization technique of Refs. \[55, 56\]. We are perfectly aware of the fact the wave functions of light vector mesons are still unknown; in the present exploratory study our major concern is to have a parameterization which is consistent with the size of vector mesons as suggested by the conventional spectroscopic models and has the short distance behavior driven by the hard QCD gluon exchange \[56\].

Let the \(Vq\bar{q}\) vertex be \(\Gamma \bar{q}V_{\mu}\gamma_{\mu}q\) where the vertex function \(\Gamma\) is a function of the lightcone invariant variable \[55\]

\[
p^2 = \frac{1}{4}(M^2 - 4m_q^2) \tag{28}
\]

where \(M\) is the invariant mass of the \(q\bar{q}\) system

\[
M^2 = \frac{m_q^2 + k^2}{z(1 - z)}, \tag{29}
\]

\(k\) and \(m_q\) are the transverse momentum and quark mass, and \(z\) is a fraction of lightcone momentum of the meson carried by the quark \((0 < z < 1)\). In the nonrelativistic limit \(p\) is the 3-momentum of the quark and we have the familiar relationship between the vertex function and the momentum space wave function

\[
\Psi(p^2) \propto \frac{\Gamma(p^2)}{4p^2 + 4m_q^2 - m_V^2}. \tag{30}
\]

The hard gluon exchange Coulomb interaction \(\frac{4}{3}\frac{\alpha_s(d)}{d}\), where \(d\) is the 3-dimensional quark-antiquark separation and \(\alpha_s(d)\) is the running QCD coupling in the coordinate representation, is singular at the origin, \(d \to 0\), but becomes important only at short distances \(d\), much smaller than the radius \(R_V\) of the vector meson. For this reason, the hard gluon Coulomb interaction can be treated perturbatively. Namely, let \(\Psi_{soft}(d)\) be the wave function of the vector meson in the soft, non-singular potential. Solving the Schrödinger equation
at small $d$ to the first order in Coulomb interaction, one readily finds the Coulomb-corrected wave function of the form

$$\Psi(d) = \Psi_{\text{soft}}(d) + \Psi_{\text{soft}}(0) C \exp\left(-\frac{d}{2C a(d)}\right).$$

(31)

Here $a(d)$ is the "running Bohr radius" equal to

$$a(d) = \frac{3}{8m\alpha_s(d)}$$

(32)

where $m = m_q/2$ is the reduced quark mass. The parameter $C$ is controlled by the transition between the hard Coulomb and the soft confining interaction; we treat it as a variational parameter. (Similar analysis of the correction to the momentum space wave function for the short distance Coulomb interaction is reviewed in [56]). The 3-dimensional Fourier transform of the Coulomb-corrected wave function (31) reads

$$\Psi(p) = N_0 \left\{ (2\pi R^2)^{3/2} \exp\left[-\frac{1}{2}p^2 R^2\right] + C^4 \frac{64a^3(p^2)\pi}{(1 + 4C^2a^2(p^2)p^2)^2} \right\},$$

(33)

where $a(p^2)$ is still given by (32) with the running $\alpha_s(p^2)$ evaluated in the momentum representation.

The relativistic lightcone wave function $\Psi(z, k)$ is obtained from $\Psi(p)$ by the standard substitution of the light cone expression (28,29) for the nonrelativistic $p^2$ in (33) [55, 56]. The relativistic wave function thus obtained gives the correct QCD asymptotics $\propto \alpha_s(Q^2)/Q^2$ of the vector meson form factor, in perfect correspondence to the familiar hard QCD mechanism (for the review see [56]; the more detailed analysis of form factors will be presented elsewhere). Then, the lightcone radial wave function is the Fourier transform

$$\phi(r, z) = \int \frac{d^2k}{(2\pi)^2} \Psi(z, k) \exp(ikr).$$

(34)

With the conventional harmonic oscillator form of $\Psi_{\text{soft}}(d)$ we obtain the simple analytical formula

$$\phi_{1S}(r, z) = \Psi_0(1S) \left\{ 4z(1-z)\sqrt{2\pi R_{1S}^2} \exp\left[-\frac{m_q^2R_{1S}^2}{8z(1-z)}\right] \exp\left[-\frac{2z(1-z)r^2}{R_{1S}^2}\right] \exp\left[\frac{m_q^2R_{1S}^2}{2}\right] + C^4 \frac{16a^3(r)}{AB^3} r K_1(\beta r) \right\},$$

(35)

where $a(r)$ is given by Eq. (32), $\beta = A/B$, and

$$A^2 = 1 + \frac{C^2a^2(r)m_q^2}{z(1-z)} - 4C^2a^2(r)m_q^2$$

(36)
\[
B^2 = \frac{C^2 a^2(r)}{z(1-z)}
\] (37)

For the 1S ground state vector mesons we determine the parameters \(R_{1S}^2\) and \(C\) by the standard variational procedure using the conventional linear+Coulomb potential models [23]. We check that the resulting wave function are consistent with the experimentally measured width of the \(V \rightarrow e^+e^-\) decay (see Tab. 1). This is one of the major constraints because at very large \(Q^2\) and/or \(r_S \ll R_V\), the electroproduction amplitude is controlled by the wave function at the vanishing transverse size. For the heavy quarkonia, we check that the radii of the 1S states are close to the results of more sophisticated solution of the Schrödinger equation [23]. The radius of the \(\rho^0\) meson given by our wave function is consistent with the charge radius of the pion. Still another cross check is provided by \(\sigma_{\text{tot}}(\rho^0N)\) discussed in Section 3, which comes out very close to the pion-nucleon total cross section.

The node of the radial wave function of the \(V'(2S)\) is expected at \(r_n \sim R_V\) far beyond the Coulomb region. For this reason, we only modify the soft component of the wave function and take the same functional form of the Coulomb correction as for the 1S state:

\[
\phi_{2S}(r, z) = \Psi_0(2S) \left\{ 4z(1-z)\sqrt{2\pi R_{2S}^2} \exp \left[ - \frac{m_q^2 R_{2S}^2}{8z(1-z)} \right] \exp \left[ - \frac{2z(1-z)r^2}{R_{2S}^2} \right] \exp \left[ \frac{m_q^2 R_{2S}^2}{2} \right] \right\}
\]

\[
\quad \left\{ 1 - \alpha \left[ 1 + \frac{m_q^2 R_{2S}^2}{4z(1-z)} + \frac{4z(1-z)}{R_{2S}^2} r^2 \right] \right\}
\]

\[
+ C \frac{16a^3(r)}{AB^3} \beta K_1 \{ \{ \right. 
\]

The new parameter \(\alpha\) controls the position \(r_n\) of the node. The two parameters \(\alpha\) and \(R_{2S}\) are determined from the orthogonality condition

\[
\frac{N_c}{2\pi} \int_0^1 \frac{dz}{z^2(1-z)^2} \int d^2r \cdot \left\{ m_q^2 \phi_i(r, z) \phi_k(r, z) + [z^2 + (1-z)^2][\partial_r \phi_i(r, z)][\partial_r \phi_k(r, z)] \right\} = \delta_{ik}
\] (39)

and from the \(2S-1S\) mass splitting evaluated with the same linear+Coulomb potential. For the heavy quarkonia, we can check the resulting \(V'(2S)\) wave function against the accurate data on the width of the \(V'(2S) \rightarrow e^+e^-\) decay, the agreement in all the cases is good. The so determined parameters, the quark masses used and some comparisons with the experiment are summarized in Table 1. It is \(Ca(r)\) which defines at which radii the interaction is
important. A posteriori, for light vector mesons $C$ is found small, the radius $C a(r)$ is indeed small and the resulting parameters are consistent with the assumption that the Coulomb interaction is a short-distance perturbation. Furthermore, for the light vector mesons we find $R_{1S} \approx R_{2S}$. For heavier mesons $C$ is larger and Coulomb effects are becoming more important and $R_{2S} > R_{1S}$ in the ratio closer to the one for the Coulomb system (see Table 1). Because our Ansatz for the relativistic wave function has the correct short-distance QCD behavior and gives a reasonable description of soft cross sections, we believe it provides a reasonable interpolation between the soft and hard regimes in the electroproduction of vector mesons.
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Figure captions:

Fig. 1 - The color dipole model predictions for the total cross section $\sigma_{\text{tot}}(VN)$ for the interaction of the light vector mesons $\rho, \rho', \phi$ and $\phi'$ with the nucleon target as a function of c.m.s. energy $W$.

Fig. 2 - The color dipole model predictions for the $Q^2$ dependence of the observed cross section $\sigma(\gamma^* \rightarrow V) = \sigma_T(\gamma^* \rightarrow V) + \epsilon \sigma_L(\gamma^* \rightarrow V)$ of exclusive $\rho^0$ and $\phi^0$ production vs. the low-energy NMC \cite{31} and high-energy ZEUS \cite{32} and H1 \cite{37} data. The top curve is a prediction for the $\rho^0$ production at $W = 70 \text{ GeV}$, the lower curves are for the $\rho^0, \phi^0$ production at $W = 15 \text{ GeV}$. The dashed curve for the $\rho^0$ shows the pure pomeron contribution $\sigma_P(\gamma^* \rightarrow \rho^0)$, the solid curve for the $\rho^0$ shows the effect of correcting for the non-vacuum Reggeon exchange as described in the text.

Fig. 3 - The color dipole model energy dependence predictions for forward real photoproduction of $\rho^0$ mesons compared with fixed target data \cite{38} and high energy datum from the ZEUS experiment at HERA collider \cite{39, 40}. The dashed curve is the pure pomeron exchange contribution, the solid curve shows the correction for the the non-vacuum Reggeon exchange as described in the text.

Fig. 4 - The color dipole model predictions for the energy dependence of real photoproduction of the $\phi^0$ mesons compared with fixed target \cite{41} and high energy ZEUS data (open square for the $\phi^0$ \cite{42}, solid circle for the $\rho^0$ \cite{39, 40}).

Fig. 5 - The color dipole model predictions of the forward differential cross sections $d\sigma_{L,T}(\gamma^* \rightarrow V)/dt|_{t=0}$ for transversely (T) (top boxes) and longitudinally (L) (middle boxes) polarized $\rho^0$ and $\phi^0$ and for the polarization-unseparated $d\sigma(\gamma^* \rightarrow V)/dt|_{t=0} = d\sigma_T(\gamma^* \rightarrow V)/dt|_{t=0} + \epsilon d\sigma_L(\gamma^* \rightarrow V)/dt|_{t=0}$ (bottom boxes) for $\epsilon = 1$ as a function of the c.m.s. energy $W$ at different values of $Q^2$.

Fig. 6 - The color dipole model predictions for the $Q^2$ and $W$ dependence of the ratio of the longitudinal and transverse differential cross sections in the form of the quantity $R_{LT} = \frac{m^2_V}{Q^2} \frac{d\sigma_L(\gamma^* \rightarrow V)}{d\sigma_T(\gamma^* \rightarrow V)}$, where $m_V$ is the mass of the vector meson. The solid and dashed curves are for $W = 15 \text{ GeV}$ and $W = 150 \text{ GeV}$. 

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Fig. 7 - The color dipole model predictions for the dependence on the scaling variable $Q^2 + m_V^2$ of the polarization-unseparated $d\sigma(\gamma^* \rightarrow V)/dt|_{t=0} = d\sigma_T(\gamma^* \rightarrow V)/dt|_{t=0} + \epsilon d\sigma_L(\gamma^* \rightarrow V)/dt|_{t=0}$ for $\epsilon = 1$ at the HERA energy $W = 100 GeV$.

Fig. 8 - The color dipole model predictions for the $Q^2$ and $W$ dependence of the ratios $\sigma(\gamma^* \rightarrow \rho'(2S))/\sigma(\gamma^* \rightarrow \rho^0)$ and $\sigma(\gamma^* \rightarrow \phi'(2S))/\sigma(\gamma^* \rightarrow \phi^0)$ for the (T) and (L) polarization of the vector mesons.

Fig. 9 - The color dipole model predictions for the $Q^2$ dependence of the ratio of the polarization-unseparated forward production cross sections $d\sigma(\gamma^* \rightarrow \rho'(2S))/d\sigma(\gamma^* \rightarrow \rho^0)$ and $d\sigma(\gamma^* \rightarrow \phi'(2S))/d\sigma(\gamma^* \rightarrow \phi^0)$ for the polarization of the virtual photon $\epsilon = 1$ at the HERA energy $W = 100 GeV$.

Fig. 10 - The color dipole model predictions of the forward differential cross sections $d\sigma_{L,T}(\gamma^* \rightarrow V')/dt|_{t=0}$ for transversely (T) (top boxes) and longitudinally (L) (middle boxes) polarized radially excited vector mesons $\rho'(2S)$ and $\phi'(2S)$ and for the polarization-unseparated $d\sigma(\gamma^* \rightarrow V')/dt|_{t=0} = d\sigma_T(\gamma^* \rightarrow V')/dt|_{t=0} + \epsilon d\sigma_L(\gamma^* \rightarrow V')/dt|_{t=0}$ for $\epsilon = 1$ (bottom boxes) as a function of the c.m.s. energy $W$ at different values of $Q^2$.

Fig. 11 - The color dipole model predictions for the energy dependence of the ratio of the polarization-unseparated forward production cross sections $d\sigma(\gamma^* \rightarrow \rho^0)/d\sigma(\gamma^* \rightarrow \rho^0)$ for the polarization of the virtual photon $\epsilon = 1$ at different values of $Q^2$.

Fig. 12 - The color dipole model predictions for the energy dependence of the ratio of the polarization-unseparated forward production cross sections $d\sigma(\gamma^* \rightarrow J/\Psi)/d\sigma(\gamma^* \rightarrow \rho^0)$ for the polarization of the virtual photon $\epsilon = 1$ at different values of $Q^2$.

Fig. 13 - Approximate scaling in the variable $Q^2 + m_V^2$, for the ratio of the polarization-unseparated forward production cross sections $d\sigma(\gamma^* \rightarrow \phi^0)/d\sigma(\gamma^* \rightarrow \rho^0)$ and $d\sigma(\gamma^* \rightarrow J/\Psi)/d\sigma(\gamma^* \rightarrow \rho^0)$ for the polarization of the virtual photon $\epsilon = 1$. The horizontal dotted straight lines show the ratio corresponding to Eq. (18).

Fig. 14 - Approximate scaling in the variable $Q^2 + m_V^2$, for the ratio of the polarization-unseparated forward production cross sections $d\sigma(\gamma^* \rightarrow \phi^0)/d\sigma(\gamma^* \rightarrow \rho^0)$ and $d\sigma(\gamma^* \rightarrow
\( J/\Psi/d\sigma(\gamma^* \rightarrow \rho^0) \). at c.m.s. energy \( W = 150 \text{GeV} \) (the polarization of the virtual photon \( \epsilon = 1 \)).

Fig. 15 - The \( Q^2 \) dependence of the coefficient functions \( g_{T,L} \) at \( W = 15 \text{ GeV} \) (dashed curve) and \( W = 150 \text{ GeV} \) (solid curve).

Fig. 16 - The dipole size dependence of the dipole cross section extracted from the experimental data on photoproduction and electroproduction of vector mesons: the NMC data on \( \phi^0 \) and \( \rho^0 \) production \[91\], the EMC data on \( J/\Psi \) production \[21, 53\], the E687 data on \( J/\Psi \) production \[52\], the FNAL data on \( \rho^0 \) production \[41\], the ZEUS data on \( \phi^0 \) production \[12\], the ZEUS data on \( \rho^0 \) production \[39, 40, 32\], the H1 data on \( \rho^0 \) production \[37\] and the average of the H1 and ZEUS data on \( J/\Psi \) production \[19, 51\]. The dashed and solid curve show the dipole cross section of the model \[21, 6\] evaluated for the c.m.s. energy \( W = 15 \) and \( W = 70 \text{ GeV} \) respectively. The data points at HERA energies and the corresponding solid curve are multiplied by the factor 1.5.

Fig. 17 - The \( Q^2 \) dependence of the parameter \( \eta_V \) in the representation (25) for the amplitude of leptoproduction of different vector mesons at fixed energy \( W = 150 \text{ GeV} \).

Fig. 18 - The \( x_{\text{eff}} \) dependence of the parameter \( \eta_V \) in the representation (25) for the amplitude of leptoproduction of different vector mesons at several values of \( Q^2 \).
Table 1: The parameters $R^2$, $C$, $m_q$ and $\alpha$ of the vector mesons wave function and some of the observables evaluated with these wave functions: the r.m.s. $R_V$, the leptonic width $\Gamma(e^+e^-)$ and the $V'(2S) - V(1S)$ mass splitting. The values of $\Gamma(e^+e^-)$ from the Particle Data Tables [45] is shown for the comparison.
