Probabilistic multi-objective optimization approach to solve production planning and raw material supplier selection problem under probabilistic demand value

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Abstract. This article is addressed to study the development of a probabilistic multi-objective optimization model that can be used to optimize the production planning and raw material procurement in a manufacturing industry where the demand value is unknown. First, the unknown demand value is assumed to be a random variable with some known probability distribution. Then, we formulate the multi-objective optimization model with two objective functions which are the total procurement cost that is minimized and the total production number that is maximized. Some related constraints that should be satisfied are also be formulated. We solve this multi-objective optimization problem by finding the Pareto solution. The calculation is performed in LINGO 18.0. To simulate and observe how the optimal decision is made, a computational simulation using generated data was performed. From the results, the optimal decision is obtained (the number of the raw material that should be purchased from each supplier and the number of the product that should be produced).

1. Introduction

Raw material procurement in manufacturing industries plays as an important component in determining of the company’s performance. Another component which is also important is production planning. There are many approaches in order to maintain these two components. The most used approaches are mathematical optimization since it can produce a solution which optimizes some objective functions [1], [2]. Then, many research articles have been published about problem solving in raw material procurement and production planning problem by using optimization approach as in [3–7]. For more detail example, an integer linear optimization approach was used in [6] whereas a throughput accounting approach was used in [7]. Besides the development of the mathematical model, some related researches were conducted by case study such as product mixing problem on paper mill [8].

One of the powerful mathematical tools to solve an optimization cases in many fields is mathematical programming. There are several classes on mathematical programming from the simplest linear programming to the most complex multi-objective non-linear programming. The term “multi-objective” means the objective function that will be optimized contains two or more functions. Multi-objective programming approaches have been used in many problems e.g. energy optimization [9]–[14], mechanical vibration optimization [15], optimization on water distribution [16], biopharmaceutical
industry [17], MA CFB reactor optimization [18], high-speed train cabin ventilation designing [19], robust scheduling of electric vehicles aggregator [20], etc. A multi-objective optimization problem can be solved by calculating the Pareto solution. To calculate the Pareto solution, the weighting method can be used as a simplest way [21]. Many research articles were published to show how the Pareto solution can be used as a decision. For example, Pareto optimal solution was used in battery cell designing [22], re-insurance optimization [23]–[25] and radar designing [26].

Table 1. Notations used in the model

| Category       | Notation | Interpretation                                                                 |
|----------------|----------|--------------------------------------------------------------------------------|
| Index          | \( r \)  | index of raw material type: 1, 2, ..., \( R \)                                 |
|                | \( s \)  | index of supplier name: 1, 2,..., \( S \)                                    |
|                | \( p \)  | index of product type: 1, 2, ..., \( P \)                                    |
| Decision variable | \( Y_p \) | (initial decision variable which decided before the demand value is revealed) |
|                | \( YR_p \) | the number of the product \( p \) decided to be produced                     |
|                | \( X_{sr} \) | (recourse decision variable which decided after the demand value is revealed) |
|                | \( Z_s \) | the number of the product \( p \) decided to be procured                     |
|                | \( S_s \) | Amount of raw material \( r \) purchased from supplier \( s \)               |
|                | \( Z_s \) | A binary variable for supplier \( s \) (0 if there is no purchased raw material or 1 if there is some purchased raw material) |
| Parameter      | \( UP_{sr} \) | Price for one unit of raw material \( r \) that purchased from supplier \( s \) |
|                | \( YRP_{p} \) | Unit price for product \( p \) that should be procured after the demand value is revealed to meet the demand volume |
|                | \( O_s \) | Cost for ordering some raw material from supplier \( s \)                     |
|                | \( TC_s \) | Transport cost for one truck from supplier \( s \) to the manufacturer         |
|                | \( p^d_{sr} \) | penalty cost for one unit defected raw material \( r \) from supplier \( s \) |
|                | \( d_{sr} \) | Percentage of defected product purchased from supplier \( s \)               |
|                | \( p^l_{sr} \) | Penalty cost for one unit of late delivered raw material \( r \) from supplier \( s \) |
|                | \( l_{sr} \) | Percentage of the late delivered (shortage) raw material \( r \) from supplier \( s \) |
|                | \( r_{p} \) | The amount of raw material \( r \) which required to be used to produce one unit of product \( p \) |
|                | \( \hat{D}_p \) | Demand value of product \( p \) which is uncertain parameter approached as a random variable |
|                | \( MH_{pm} \) | Duration or hour of machine \( m \) which is required to be used to produce one unit product \( p \) |
|                | \( MC_m \) | Maximum duration capacity of machine \( m \) that can be operated for production activities |
|                | \( C \) | Truck’s capacity to transport raw material from supplier to the manufacturer |
|                | \( SC_{sr} \) | Maximum amount of raw material \( r \) that can be purchased from supplier \( s \) |

In this article, we develop a probabilistic multi-objective optimization model which we used to solve an optimization on the production planning and raw material procurement problem that can be applied in many manufacturing industries. The discussed problem is containing an unknown parameter which is demand value for a simple problem, commonly the demand value is known, but for the problem discussed in this paper, the demand value is unknown. But, we assumed that the decision maker has data of this demand value, so the probability distribution can be made. Then, we the solution of the developed model can applied by the decision maker. The solution is the number of the raw material the number of the product that should be produced.
2. Mathematical model
Consider a product mixing and raw material procuring optimization problem with \( P \) product types which planned to be produced using \( R \) raw material types procured from \( S \) supplier alternatives.

2.1. Mathematical notation
Suppose that the decision maker will maximize the amount of the product and minimize the total occurred cost. Let the used notations in the formulated model are shown in table 1.

2.2. Mathematical model
The objectives are maximizing the amount of the planned production volume for each product type and minimizing the total occurred cost from raw material procurement to production cost. Then, we have the following multi-objective optimization problem:

\[
\begin{align*}
\text{max } Z_1 &= \sum_{p=1}^{P} Y_p \\
\text{min } Z_2 &= \sum_{s=1}^{S} \sum_{r=1}^{R} \left[ X_{sr} \cdot UP_{sr} \right] + \sum_{s=1}^{S} \left[ O_s \cdot Z_s \right] + \sum_{s=1}^{S} \left[ TC_s \cdot S_s \right] + \sum_{s=1}^{S} \sum_{r=1}^{R} \left[ p_{sr} \cdot d_{sr} \cdot X_{sr} \right] \\
&\quad + \sum_{s=1}^{S} \sum_{r=1}^{R} \left[ P_{sr} \cdot l_{sr} \cdot X_{sr} \right] + \sum_{p=1}^{P} \left[ YRP_p \cdot YR_p \right]
\end{align*}
\]

The first objective function, (1) describes the amount of all products which will be planned to be produced. The second objective function, (2) describes the total occurred cost where the first term is the total buying cost for all raw materials, the second term is the ordering cost for all suppliers, the third term is the raw material’s transport cost from all suppliers, the fourth term is the total penalty cost for defected products, the fifth term is the total penalty cost for late delivered products, and the last term is the occurred cost for procuring the shortage product that will be used to meet the demand (if there is some demanded product which is not met by the product produced by the manufacturer). The constraints that should be satisfied are designed as follows:

- The received raw material volume (purchased volume minus defect volume and late delivered volume) has to satisfy (greater or equal to) the raw material volume which is needed by the manufacturer for the production process i.e. to be used to produce the planned product:
  \[
  \sum_{s=1}^{S} X_{sr} - \sum_{s=1}^{S} \left[ (d_{sr} + l_{sr})X_{sr} \right] \geq \sum_{p=1}^{P} \left[ r_{yp}Y_p \right], \forall r;
  \]

- The product volume which will be manufactured plus the product volume procured after the demand is revealed has to satisfy the (random) demand value:
  \[
  Y_p + YR_p \geq D_p, \forall p, \forall p;
  \]

- The machine’s working duration has to satisfy it’s maximum capacity:
  \[
  \sum_{p=1}^{P} MH_{pm}Y_p \leq MC_m, \forall m;
  \]
• The delivery load has to satisfy the truck’s capacity:

\[
\frac{\sum_{r=1}^{R} X_{sr}}{C} \leq S_s, \forall s;
\]

• The amount of the raw material purchased from each supplier has to satisfy the supplier’s maximum capacity:

\[X_{sr} \leq SC_{sr}, \forall s, \forall r;\]

• Auxiliary binary variable indicated a supplier is selected or not is formulated as follows:

\[\sum_{r=1}^{R} X_{sr} \leq M \cdot Z_s, \forall s;\]

• The non-negativity and integer constraint are formulated as follows:

\[X_{spr}, Y_p \geq 0 \text{ and integer, } W_s \in \{0,1\}.\]

Consider a two-objectives optimization problem (1)-(2) subject to constraints no. (1)-(7). Since the feasible solution set, if non empty, is closed and bounded then (1) can be replaced by \( \min Z_1 \leftarrow (-Z_1) = -\sum_{p=1}^{P} Y_p. \) A solution vector \( x^o \) is called as Pareto solution if there does not exist another point \( x \) so that \( Z_j(x) \leq Z_j(x^o) \), \( \forall i = 1,2 \) and \( Z_j(x) < Z_j(x^o) \) for \( i = 1 \) or \( 2 \) and the set of all Pareto solution is called Pareto set which can be generated by using weighting method [21]. The Pareto set of this bi-objective optimization problem can be calculated by

\[
\min Z = w_1(-Z_1) + w_2 Z_2, \quad (3)
\]

subject to: \( w_1 + w_2 = 1, 0 \leq w_1, w_2 \leq 1. \)

3. Numerical experiment

Consider the raw material procurement and production planning problem with three raw materials R1, R2 and R3, four suppliers S1, S2, S3 and S4, and three products P1, P2 and P3 which modelled as (1)-(2). The value of parameters are shown in the appendix. Let the probability distribution function of \( \hat{D}_p \) is

\[
\begin{align*}
0.05, & \quad D_1 = 100; & 0.05, & \quad D_2 = 80; & 0.05, & \quad D_3 = 120 \\
0.06, & \quad D_1 = 120; & 0.06, & \quad D_2 = 90; & 0.06, & \quad D_3 = 130 \\
0.10, & \quad D_1 = 140; & 0.10, & \quad D_2 = 100; & 0.10, & \quad D_3 = 140 \\
0.12, & \quad D_1 = 160; & 0.12, & \quad D_2 = 110; & 0.12, & \quad D_3 = 150 \\
0.15, & \quad D_1 = 180; & 0.15, & \quad D_2 = 120; & 0.15, & \quad D_3 = 160 \\
0.20, & \quad D_1 = 200; & 0.20, & \quad D_2 = 130; & 0.20, & \quad D_3 = 170 \\
0.15, & \quad D_1 = 220; & 0.15, & \quad D_2 = 140; & 0.15, & \quad D_3 = 180 \\
0.10, & \quad D_1 = 240; & 0.10, & \quad D_2 = 150; & 0.10, & \quad D_3 = 190 \\
0.05, & \quad D_1 = 260; & 0.05, & \quad D_2 = 160; & 0.05, & \quad D_3 = 200 \\
0.05, & \quad D_1 = 280; & 0.05, & \quad D_2 = 170; & 0.05, & \quad D_3 = 210 \\
0.00, & \text{ others.} & 0.00, & \text{ others.} & 0.00, & \text{ others.}
\end{align*}
\]
Figure 1. (a) The optimal decision for raw material procurement for each R1, R2, and R3 (b) The optimal decision for the amount of product P1, P2, and P3 planned to be produced

Figure 2. Demand value and product shortage volume for scenarios 1, 2, 3, 9, and 10

We have solved (3) using $w_1 = w_2 = 0.5$ which means that a fifty-fifty impact is applied to the two objective functions (1)-(2). The optimization process was calculated in LINGO 18.0 software interfaced with Ms Excel 2016 as the input and output form. We have solved the problem in a common PC with 2.3 GHz of processor and 6 GB of memory. The computation time is limited to 10 minutes. The value
of E[Z] is 164768.5 and the other results are shown in figure 1 and figure 2. It can be seen from figure 1.a that the manufacturer should not purchasing raw material from supplier S3 and S4. The other optimal decision is illustrated in figure 1.b which shows how many units that should be produced for product type P1 and P2 where no unit of product P3 should be produced.

After the demand’s value is revealed, the number of the demanded product that is not satisfied by the produced product is revealed and it can be satisfied by purchasing from other manufacturer or it can be released which means that the decision maker is decided to be not satisfying the demanded product. For example, let us see the scenario 1 (SC1) in figure 2. After the demand value is revealed, let the demand value for product P1 is 100 units then it is satisfied by the produced product which is 120 units. The demand value for product P2 is 80 units and it is satisfied by the produced product P2 which is 90 units. The demand value for product P3 is 120 units and it is not satisfied by the produced P2 since there is no P2 which is produced. Then, the options for the decision maker is not satisfying this demand value or satisfying it by purchasing 120 units P2 with unit price 600. The total number of scenarios is ten scenarios where five of them are shown in figure 2.

4. Concluding remarks
In this article, a bi-objective mathematical optimization model was developed that can be used to find the optimal decision for raw material procurement and product mixing planning problem. The optimal decision was calculated by using the Pareto solution scheme based on the weighting method. A numerical problem example was solved and it was resulted the optimal decision for the given problem. For further research, a model for multi-period case will be developed so that it will be a dynamic problem and an option of storing the raw material and product in the warehouse will be discussed. Furthermore, besides the demand value, the other parameters like unit price of raw material or transport cost can also be unknown. It will be a great challenge to solve with many parameters which their values are unknown.

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### Appendix

#### Table A. Unit price

| Suppliers | Raw Material | R1 | R2 | R3 |
|-----------|--------------|----|----|----|
| S1        |              | 10 | 20 | 40 |
| S2        |              | 12 | 22 | 40 |
| S3        |              | 11 | 22 | 41 |
| S4        |              | 12 | 20 | 45 |

#### Table B. Order cost and transport cost

| Supplier | Order Cost | Transport Cost |
|----------|------------|----------------|
| S1       | 120        | 100            |
| S2       | 100        | 100            |
| S3       | 100        | 110            |
| S4       | 120        | 110            |

#### Table C. Defect product penalty cost

| Supplier | Raw Material | R1 | R2 | R3 |
|----------|--------------|----|----|----|
| S1       |              | 5  | 5  | 4  |
| S2       |              | 10 | 5  | 8  |
| S3       |              | 8  | 10 | 12 |
| S4       |              | 12 | 10 | 12 |

#### Table D. Defect rate

| Supplier | Raw Material | R1 | R2 | R3 |
|----------|--------------|----|----|----|
| S1       |              | 0.02| 0.01| 0.09|
| S2       |              | 0.03| 0.01| 0.02|
| S3       |              | 0   | 0.02| 0.03|
| S4       |              | 0.06| 0.04| 0  |

#### Table E. Late delivery rate

| Supplier | Raw Material | R1 | R2 | R3 |
|----------|--------------|----|----|----|
| S1       |              | 0.02| 0.02| 0.05|
| S2       |              | 0.01| 0.02| 0  |
| S3       |              | 0.00| 0.05| 0.05|
| S4       |              | 0.02| 0.05| 0.03|

#### Table E. Late delivery penalty cost

| Supplier | Raw Material | R1 | R2 | R3 |
|----------|--------------|----|----|----|
| S1       |              | 5  | 4  | 2  |
| S2       |              | 5  | 4  | 3  |
| S3       |              | 4  | 2  | 4  |
| S4       |              | 3  | 2  | 2  |

#### Table F. Raw material required to produce the product

| Raw Material | Product | P1 | P2 | P3 |
|--------------|---------|----|----|----|
| R1           |         | 10 | 12 | 10 |
| R2           |         | 20 | 22 | 20 |
| R3           |         | 4  | 2  | 2  |

#### Table F. Shortage cost

| Product | Shortage cost | Raw Material | Shortage cost |
|---------|---------------|--------------|---------------|
| P1      | 4             | R1           | 2             |
| P2      | 3             | R2           | 3             |
| P3      | 4             | R3           | 2             |

#### Table G. Required machine working hour to produce product unit

| Product | Machine | M1 | M2 | M3 |
|---------|---------|----|----|----|
| P1      |         | 4  | 2  | 2  |
| P2      |         | 5  | 1  | 2  |
| P3      |         | 4  | 1  | 2  |

#### Table H. Machine working hour max. capacity

| M1 | M2 | M3 |
|----|----|----|
| 8500 | 7500 | 7000 |

#### Table I. Supplier maximum capacity to supply raw material

| Supplier | Raw Material | R1 | R2 | R3 |
|----------|--------------|----|----|----|
| S1       |              | 5000| 4500| 6000|
| S2       |              | 4000| 4500| 5000|
| S3       |              | 8000| 10000| 5000|
| S4       |              | 9500| 8000| 7500|