Corrigendum: Experimental evidence of dissipative spatial solitons in an optical passive Kerr cavity

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Experimental evidence of dissipative spatial solitons in an optical passive Kerr cavity

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Abstract. We report on the first experimental evidence of localized structures (dissipative solitons) in a one-dimensional optical Fabry–Pérot passive Kerr cavity. The Kerr-like medium is a non-instantaneous, diffusive ultra-thin film of liquid crystal inserted in a low-finesse cavity. Solitons with oscillating tails are experimentally observed in this system that can lock together to form complexes of solitons. The numerical simulations carried out on an infinite-dimensional map describing the intra-cavity field dynamics fully agree with the experimental observations.

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1. Introduction

Localized structures (LSs) belong to the class of dissipative structures occurring in far from equilibrium spatially extended systems [1–10]. In practice, LSs appear as spikes, interfaces, spots, pulses, dissipative solitons, cavity solitons or autosolitons [11]. They have been predicted in such diverse domains in nonlinear science as biology [12, 13], chemistry [14, 15], fluids [16], mathematics [17] and optics [18–21]. Nowadays, optical solitons constitute an active research activity in optics not only because of the richness of their nonlinear dynamics but also because they are promising candidates for basic units of all-optical information storage [22, 23] and processing schemes [24]. Among spatially extended nonlinear optical devices, passive Kerr resonators were deeply studied theoretically and many transverse patterns have been predicted to occur in these systems, including rolls, hexagons and dissipative solitons. However, except for one experiment on pattern formation in a Fabry–Pérot cavity with a liquid crystal (LC) slice [25], no experimental evidence, especially of localized states, has been given until now for close agreement between theory and experiments.

The aim of this paper is to experimentally evidence the occurrence of optical solitons in a specific passive cavity filled with a nonlinear Kerr medium. The latter is an ultra-thin slice of LC whose nonlinear optical response time is many orders of magnitude lower than the intra-cavity field. For a detailed description of LC as nonlinear optical media (Kerr-like), see [26, 27]. The cavity configuration is of Fabry–Pérot type and the cavity length is very much larger than the LC thickness. Using a cavity linear phase shift stabilization, we show the formation of spatial solitons with oscillating tails in our setup conditions. Numerical simulations carried out for an infinite-dimensional map describing the intra-cavity field dynamics fully agree with the experimental observations.

2. Numerical simulation procedure

One of the main questions when looking for experimental evidence of passive cavity solitons is the question of the soliton existence in real experiments when taking into account specific features of the setup such as the non-instantaneous response time of the nonlinear medium, the finesse of the cavity and the non-locality of the medium. Indeed, theoretical studies that predict passive cavity solitons, in a Kerr medium, have mainly been carried out starting from the well-known Lugiato–Lefever (LL) model [28]. The latter was derived for an instantaneous
medium without diffusion and a ring cavity whose high finesse lies in the mean field approximation limit. Here, our nonlinear medium has a non-instantaneous response time of the order of seconds to be compared to the cavity field evolution, which is around nanoseconds. It is diffusive, non-local and noisy. The cavity has finesse around 12 that does not really allow for mean field approximation. So, the first step is to determine how the solitons are affected by all our experimental features, which are far from the assumptions of the LL model, and whether they still exist in real experimental conditions.

The experimental setup is a Fabry–Pérot resonator composed of a Kerr medium (an LC slice LC in figure 1) inserted between two plane mirrors (M₁ and M₂ in figure 1). The numerical simulations of the dynamics of this system are based on an extension of the model of the single Kerr slice with feedback first introduced by Akhmanov et al [29] and Firth and D’Alessandro [30] and were shown to be in very good agreement with our medium features [31]. The refractive index \( n \) of the LC is ruled by the following equation:

\[
\left( \tau \frac{\partial}{\partial t} + l_d^2 \frac{\partial^2}{\partial x^2} + 1 \right) n = |F|^2 + |B|^2 + \sqrt{\varepsilon} \xi,
\]

\( \tau \) is the response time of the LC (\( \approx 2.23 \) s) and \( l_d \) is its diffusion length (\( \approx 10 \) \( \mu \)m) [32]. \( \varepsilon (= 0.01) \) scales the noise amplitude and \( \xi(x, t) \) are Gaussian stochastic processes of zero mean and delta correlation introduced to model thermal noise [32]. The temporal deterministic evolution of the refractive index \( n \) is computed by a variable step of Runge–Kutta order 8 solver (dop853\(^2\)) for the simulations without noise (\( \varepsilon = 0 \)). This code uses an explicit Runge–Kutta method of order 8(5,3) due to Dormand and Prince with step size control and dense output [33]. When equation (1) is stochastic (\( \varepsilon \neq 0 \)), we use a stochastic Runge–Kutta solver of the order of 2 with additive noise [34]. In this latter case, the temporal step (\( \Delta t \)) is equal to 0.005\( \tau \). We perform the spatial derivatives using the Fourier space and so the FFTW3 Library\(^3\). In all the simulations the spatial step (\( \Delta x \)) is 0.4 \( l_d \). The forward and backward intra-cavity fields \( F \)

\(^2\) See the Ernst Hairer homepage http://www.unige.ch/hairer/software.html
\(^3\) See the fftw3 homepage http://www.fftw.org/

Figure 1. The experimental setup. OI, optical isolator; BS, beam splitter; LC, liquid crystal slice; D₁ and D₂, photodetectors; L₁ and L₂, lenses of focal length \( f \); \( p \) and \( s \) are the polarized components of the pump (solid line) and probe (dashed line) beams, respectively; M₁ and M₂ are the real cavity mirrors but the optical Pérot–Fabry cavity is delimited by M₁ and M₂’ mirrors and its length is \( d \).
and $B$ (figure 1) have to be evaluated differently from the case of feedback configuration [30]. Since our experimental setup is a Fabry–Pérot resonator, no analytical expressions for the intra-cavity fields $F$ and $B$ can be obtained. So, they have to be numerically calculated using an iterative mapping over the cavity round trips [35]. Note that very recently Lugiato and Prati [36] have derived a simple and elegant model for Fabry–Pérot cavities valid when the length of the nonlinear medium inside the cavities is much smaller than the wavelength. Unfortunately, our experimental setup is not in this configuration. For the sake of simplicity, we consider that the nonlinear medium is close to the mirror $M_1$. The propagation of an incoming input field $F_0$ through the cavity satisfies the following two equations depending on whether the field passes through the LC slice

$$\frac{\partial F}{\partial z} = i \chi n F$$

or diffracts along the free space

$$\frac{\partial F}{\partial z} = -\frac{i}{2k_0} \frac{\partial^2 F}{\partial x^2}.$$  (3)

$k_0$ is the field wave number and $\chi$ accounts for the Kerr nonlinear effect of the LC. $\chi > 0$ for a focusing medium and $\chi < 0$ for a defocusing one. Our sample is of focusing type. Diffraction within the LC is neglected since its thickness $L$ (50 $\mu$m) is very small compared to the cavity length $d$ ($\simeq$ a few mm). The profile of the forward input field is assumed to be Gaussian so that $F_0(x) = F_0 \exp(-x^2/w^2)$, with $w$ being the beam radius at the sample. A last element to consider is the linear phase shift $\phi = 4\pi [d + (n_e - 1) L]$ accumulated over one round-trip for the intra-cavity fields, with $n_e$ being the extraordinary index of the LC.

The intensity of the backward output optical beam ($B_{\text{out}}$ in figure 1) is the quantity that is experimentally recorded. It is our dynamical variable in this study and especially its related probability density functions. Indeed, using equations (2) and (3), it reads

$$|B_{\text{out}}(x, t)|^2 = T_2 |e^{i\chi n(x, t)} e^{i\frac{\chi}{2} \nabla^2} (e^{i\chi n(x, t)} F(x))|^2$$  (4)

with $T_2$ being the mirror 2 intensity transmission. We have set $\sigma = d/k_0 l_D^2$, where $d$ is the slice–mirror distance (figure 1).

Since our nonlinear medium is non-instantaneous, the system presents two different time scales. Indeed, the dynamical time $\tau$ of the refractive index $n$ is many orders of magnitude lower than that of the intra-cavity fields $F$ and $B$. This leads to the following procedure for integration. Cavity fields are first calculated, assuming $n$ constant, until they reach their stationary values. Then, $n$ is evaluated using equation (1) with the stationary values of $F$ and $B$. In figure 2(a), we depict typical intra-cavity field $F$ evolution versus cavity round-trip numbers. As can be seen from figure 2(b), $F$ converges to its asymptotic value with a relative accuracy of $10^{-8}$.

3. Numerical evidence of localized structure with our experimental features

A Kerr Pérot–Fabry cavity is known to display two different regimes corresponding to stationary homogeneous solutions of the set of equations (1)–(3) [28, 37]. The first one is a monostable evolution of the intra-cavity field $F$ versus the input field $F_0$, while the second depicts a hysteresis cycle. In the first case, MI (also called Turing instability) destabilizes the stationary solution above a primary threshold depending on the linear phase shift $\phi$ (figures 3(a) and (c)). On the other hand, in the second regime, localized patterns connecting a Turing
Figure 2. (a) Temporal evolution of the intra-cavity field $F$ versus cavity round trip number $n$. (b) Convergence of $F$ to its stationary value. $F_0 = 0.125$, $\varphi = 0$ rad.

Figure 3. Left column: MI; right column: soliton. (a, b) Transverse profiles with (a) stationary periodic pattern and (b) soliton with oscillating wings. (c, d) Transverse profiles of the far fields. (a) $\varphi = 0.3$ rad, $F_0 = 0.26$ and (b) $\varphi = -1.18$ rad, $F_0 = 0.3$, $d = 5$ mm, $R_1 = 81.4\%$ and $R_2 = 81.8\%$, and plane wave profile for the input field $F_0$. The continuous part of the signal is removed in the spectra for a better reading.

Subcritical bifurcation branch of solutions and a homogeneous solution arise [18, 37]. Numerical simulations carried out with our experimental features, assuming a plane wave input profile and neglecting the noise source, show the existence of stable LSs. The latter can be viewed as homoclinic orbits in an appropriate phase space. They are found for negative linear phase shifts $\varphi$, as can be seen from figures 3(b) and (d). A local perturbation on the $F_0$ plane wave field has been taken as the initial condition to connect locally the lower branch of the bistable cycle to the upper branch in order to seed the solitons. The latter are found numerically within a wide domain of $\varphi \simeq [-2.7; -0.25]$ rad. As can be seen from figure 4, they always possess decreasing oscillating wings whose oscillation amplitude depends on $\varphi$ and $F_0$. The threshold values for transverse instabilities strongly depend on the cavity detuning $\varphi$, but typical values are, e.g.,

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Figure 4. Soliton oscillating wings for two pairs of parameters: (a) $\varphi = -1.75$, $F_0 = 0.75$; (b) $\varphi = -0.5$, $F_0 = 0.12$. Other parameter values are the same as in figure 3.

Figure 5. Soliton profile for Gaussian wave pumping, $w = 1400 \mu m$ (a) without noise and (b) with noise. $\varphi = -1.18$ and, $F_0 = 0.45$. Both simulations were carried out without initial local perturbation.

$F_0 = 0.114$ ($\varphi = 0$) for MI and $F_0 = 0.111$ ($\varphi = -0.5$) for the soliton assuming a plane wave input pumping field and no noise.

Since the reorientation of the LC director for achieving the refractive index change is purely optical, the intensity of the pumping beam to induce transverse nonlinear effects has to be powerful enough [38], e.g. 170 W cm$^{-2}$ in our experiments [39]. The intensity threshold values for nonlinear effects (MI or soliton instabilities) are more than one order of magnitude higher than the intensity for optical reorientation of the LC director [40]. Thus, the Gaussian beam cannot be sufficiently transversely expanded as an input plane wave. In fact, in our experiment, the input beam profile is Gaussian $F_0 \exp(-x^2/w^2)$ where the beam diameter $2w$ is on the LC cell. So, we investigate the influence of the Gaussian input pump on soliton formation. In this case, solitons are still evidenced (figure 5(a)). However, no initial localized perturbation is necessary to initiate the soliton except when a specific location of the incoming localized structure is desired. Even when no initial perturbation is seeded, a soliton is created at the center of the Gaussian profile or in its very close vicinity provided that the input amplitude $F_0$ is larger than the upper value of the bistable cycle. The soliton amplitude and its transverse extension do not depend on the Gaussian waist $w$. Only the number of solitons and their location are $w$ dependent. Modulations in the wings of the soliton are still present besides the soliton with an exponential decrease of modulation amplitude. Finally, to achieve numerical simulations in the exact conditions of our experiments, we also take into account the noise source and show that localization of light survives (figure 5(b)). Indeed, solitons are obtained but not necessarily close to the center of the Gaussian beam. Noise acts as initial perturbations that randomly seed solitons in the transverse space. Depending on $\varphi$ and $F_0$ values, modulation in the wings of solitons can
be visible or not depending on their amplitude with respect to the noise level. It turns out, from
the above investigations, that localized patterns can be observed even in the realistic conditions
of our experimental setup. We now proceed, in the following, to experimentally evidence the
generation of localized patterns in this device.

4. Experimental setup

The experiments have been carried out in a Kerr slice medium inserted in an optical Fabry–Pérot
resonator. Two plane mirrors $M_1$ and $M_2$ define the physical cavity but the optical one is
delimited by $M_1$ and $M'_2$ (which is the image of $M_2$ through the 4f lens arrangement) and
its optical length is $d$ (figure 1). The intensity reflection coefficients of mirrors $M_1$ and $M_2$
are $R_1 = 81.4\%$ and $R_2 = 81.8\%$, respectively, so that the cavity finesse is estimated to be 15.
The experimental recording of the Airy function gives a finesse of 11.6 (figure 6), indicating
the presence of supplementary losses due to a nonlinear medium and the lenses’ transmission
coefficients. The nonlinear Kerr medium is a 50 $\mu$m thick layer of E7 LC homeotropically
anchored. Due to thermal fluctuations that induce random motion of the molecular axis
around the mean azimuth director, the local variations of the birefringence in the LC induce
an additive noise contribution. The cavity is pumped with a beam delivered by a single-
mode frequency doubled Nd$^{3+}$:YVO$_4$ laser ($\lambda_0 = 532$ nm), which is shaped by means of two
cylindrical telescopes. The resulting diameters ($\sim 400 \mu$m $\times$ 2800 $\mu$m) of the 'cigar' transverse
laser beam differ by a factor of 14 in the vertical ($y$) and horizontal ($x$) directions, yielding
an aspect ratio of about 14. The smallest waist (e.g. vertical) is chosen such that only one roll
or soliton can develop in its direction and the system may be considered as monodimensional.
An intra-cavity 4\textit{f} lens arrangement (L\textsubscript{1} and L\textsubscript{2} in figure 1) allows for short optical cavity length \(d = 3\) mm giving the fictitious mirror M\textsubscript{2}' that is the image of the real one M\textsubscript{2}. A 4\textit{f} lens arrangement is introduced to generate very short equivalent optical cavity lengths (some millimeters) or else negative ones. This length is not physically achievable due to the stand of the tilted LC cell that exceeds 25 mm. The fictitious mirror delimits the equivalent optical cavity length (between mirrors M\textsubscript{1} and M\textsubscript{2}'). The beam is monitored at the output of the M\textsubscript{2} mirror in the forward propagation direction (\(B_{\text{out}}\) in figure 1). Near and far fields are simultaneously recorded. Two control parameters are accessible in the experiments: the maximum intensity \(I_0 = |F_0|^2\) of the incident laser beam and the cavity detuning via the linear phase shift \(\phi\).

As mentioned above, the dynamical regimes are completely defined by the value of \(\phi\). Thus, the observation of the soliton requires us to fix the value of \(\phi\) within the range \([-2.7; -0.5]\) rad. One experimental central issue here is that care must be taken to ensure that this value does not change at the time scale of the dynamics (\(\sim \tau\)). The difficulty of achieving this experimental requirement is that the cavity length (the physical one, between M\textsubscript{1} and M\textsubscript{2}, is about 450 mm) should remain constant to some tens of nanometers length variations during seconds, which is never the case due to mechanical vibrations, air fluctuations, etc. To overcome this experimental problem we implemented a stabilization of the optical cavity length. We use the same scheme as the one developed by Coen [41, 42] using a second beam as the probe and its Airy function variation for the stabilization of a fiber ring cavity. The probe beam is very weak (a few mW) and is s-polarized with respect to the p-polarization of the pump beam in order to avoid the nonlinear phase change induced by the pump on the probe beam, thus allowing for only linear cavity phase shift reading. Moreover, the probe beam is transversely shifted with respect to the pump beam (dashed line in figure 1) in a way not to produce any nonlinear effects even with p-to-s polarization conversion [43].

To fix the value of \(\phi\), we first simultaneously plot the two Airy functions of the pump and probe beams versus cavity detuning (figure 6). Then, we stabilize, using a proportional integrator electronic device (PI control on figure 1) connected to the PZT actuator mounted on mirror M2, the optical cavity length. The reference voltage level selected (13 V in the case of figure 6) is chosen such that the Airy curve of the probe varies rapidly. This sets the crossing points between the probe Airy function and the reference voltage level as phase references for the measurement of \(\phi\). Reporting one of these values on the Airy function of the pump then gives \(\phi\), as indicated in figure 6. Finally, by translating the probe detector D2, the fringes of the Airy function are shifted and then also the phase reference. Thus, the value of \(\phi\) can be continuously adjusted from positive to negative (figure 6) values to achieve the monostable regime or the bistable one, respectively. This procedure allows one to measure the linear phase shift with an accuracy of \(\pi/20\).

5. Turing instability and soliton experimental evidence

The two main dynamical regimes, (i) pattern formation via MI and (ii) the generation of LSs, have been experimentally investigated. Firstly, we fixed the value of the phase shift \(\phi\) in the range where the system exhibits MI. By increasing the pump intensity we have systematically observed the destabilization of the stationary solution (Gaussian background profile) above the threshold of Turing instabilities, in excellent agreement with analytical predictions. A typical experimental pattern arising from MI is shown in figures 7(a), (c), (e) and (g) (left panels). In these figures, transverse periodic oscillations are clearly experienced by the near-field profile.
Figure 7. (a) MI pattern ($\varphi \approx 0.3$ rad). (b) Experimental passive Kerr cavity soliton ($\varphi \approx -1.18$ rad measured in figure 6). (b') Intensity saturation of (b) to highlight rings in the soliton wings. (c, d) Dark lines: horizontal cut in the profiles of (a, b); gray line: the profile of (b'). (e, f) Far fields of (a, b). (g, h) Horizontal cut in the profiles of (e, f). $d = 5$ mm, $|F_0|^2 = 164$ W cm$^{-2}$.

(figures 7(a) and (c)) at the output of the passive cavity. We have compared the wavelength of the experimental structure (figures 7(a) and (c)) with the numerical solution obtained from equations (1)–(3) for the same parameters and with a noisy Gaussian profile. The measured experimental wavelength in the far field profile of figures 7(e) and (g), $\lambda_{\text{exp}} = 145$ $\mu$m, and the predicted one, $\lambda_{\text{num}} = 147$ $\mu$m, are in very good agreement (accuracy is less than 2%). This result confirms the presence of MI for positive detunings. We note that our experimental structure includes the harmonics of $\lambda_{\text{exp}}$ that explains the peak form of the modulation in figure 7(c).

To explore soliton occurrence in our system, we proceed in a similar way as above by fixing the value of $\varphi = -1.18$ rad giving rise to the bistability regime, and using the stabilizing procedure in our passive Kerr cavity (figure 6). Note that this bistable regime does not correspond to a subcritical Fréedericksz transition as reported in [44]. Indeed, in our experimental configuration we have a thresholdless Fréedericksz effect as described in [40, 45] since our LC director is oriented at 45$^\circ$ with respect to the optical field. We typically observe
either a dissipative localized spot alone (figure 7(b)) or a state of localized spots (figure 8). The typical profile of a single localized state (dark line in figure 7(d)) is in excellent agreement with that of figure 5(b) obtained numerically. The fit of the experimental profile of such a localized spot by a hyperbolic secant function gives a correspondence at 98% ($R^2$ value). The result of this regression is a good argument for evidence of solitons in our experimental setup. Moreover, the location of the localized spots depends on initial conditions, in contrast to MI that always appears in the central part of the Gaussian pumping profile. Thus, we can conclude that the experimental localized spots correspond to solitons. If we saturate the CCD camera and focus on the lower intensity part of the localized state, decreasing intensity rings (figure 7(b')) are highlighted in the wings. The corresponding profile then evidences decreasing oscillating wings (gray line in figure 7(d)). We have measured the spatial period by Fourier transform of a near field (figure 7(b')) of these damped oscillations and the agreement with the ones obtained numerically in figure 5(b) is rather satisfactory taking into account experimental measurement errors ($\lambda_{\text{exp}} = 48 \pm 10 \mu m$ and $\lambda_{\text{num}} = 57 \mu m$). Figures 7(f) and (h) show the one-dimensional (1D) nature of the field obtained experimentally by means of a spherical lens in our setup. Finally, we have performed several experiments to investigate the formation of multiple solitons that appear in the strongly nonlinear regime (far from threshold).

Two typical multiple soliton patterns have been observed in this regime: independent solitons (figure 8(a)) and a complex of solitons composed of solitons locked (figure 8(b)) via their oscillating decaying wings similar to the ones observed in [46, 47]. The latter are bound states of solitons where their separation distances are fixed by a multiple of the wavelength of the oscillations on the soliton tail. This locking phenomenon was reported in e.g. [48]. These multiple solitons are displayed in figure 8. Indeed, by increasing the incident pump intensity value above threshold a stable state of three solitons is formed, as can be seen from figure 8(a). A slight change in the linear phase shift $\varphi$ yields a new nonlinear state of a complex soliton composed of two solitons locked with oscillating wings (see figure 8(b)).

6. Conclusion

We have reported the first experimental evidence of spatial solitons in a passive Fabry–Pérot cavity filled with a nonlinear Kerr medium. Two operating regimes have been investigated, both numerically and experimentally: the Turing instability regime where transverse periodic states have been observed and a bistability regime where we have observed transverse soliton with oscillating tails. Our experimental findings are in very good agreement with numerical solutions obtained by integrating the governing equation of our device. Multiple solitons in the form of
either three independent solitons or a complex soliton composed of two locked solitons have been experimentally observed. Our experiments can easily be extended to 2D configuration and experimental and numerical investigations on the occurrence and stability of 2D solitons are in progress.

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