Estimation of the nozzle with the stepped divergent section in critical flow regime

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Abstract. A sonic nozzle with a toroidal inlet region, cylindrical throat and a stepped diffuser with cylindrical sections were considered. An analytical method was proposed for estimating the required pressure drop for the nozzle operating in a critical flow regime. The results of analytical estimation of the pressure buildup in the nozzle steps were compared with the ones of numerical simulation. It is shown that replacing the conical divergent section of ISO 9300 standard nozzle with easy-to-manufacture stepped divergent section leads to a twofold increase in the required pressure drop for the nozzle operating in the critical flow regime, which in many cases is acceptable.

1. Introduction

The independence of volumetric gas flow rate through the sonic nozzle from pressure at a pressure drop above critical is widely used in metrology. This fact allows reliable flow measurements over a wide range of pressure [1]. The divergent section of the sonic nozzle serves to increase the gas pressure when it moves from the throat of the nozzle to the outlet section. The ISO 9300 Standard [2] specifies rather rigid parameters for the divergent section of the nozzle: any steps, discontinuities, irregularities and lack of concentricity on the element of the cone surface should not exceed 1% of the local diameter, and the divergent section downstream of the point of tangency with the torus shall form a frustum of a cone with a half-angle between 2.5° and 6°. Obviously, these conditions cause significant technological difficulties in the industry of sonic nozzles, especially with a throat diameter of about 1 mm or less. Such requirements are justified by the fact that the sonic nozzles are inherent in a whole range of interesting from the scientific point of view but problematic phenomena despite the deceptively simplicity of the flow. These include the phenomenon of failure of the critical regime [3-5], the effect of thermal inertia on the discharge coefficient \( C_d \) [6-8] and the phenomenon of laminar-turbulent transition [9, 10].

This paper considers the sonic nozzle, which differs from ISO 9300 Standard [2] by the simplified form of the output divergent section (figure 1). The maximum flow rate provided by the sonic nozzles is determined by the parameters of stagnated flow before the nozzle inlet and the nozzle throat diameter. Therefore, replacing the conical part of the ISO 9300 nozzle with an easy-to-manufacture stepped divergent section should not affect the flow characteristics of the nozzle, but due to increase in the total pressure loss during the stagnation of flow in the stepped diffuser, the minimum pressure drop
corresponding to the critical flow regime will grow as compared to the conical one. However, if such a simplified sonic nozzle provides a constant flow rate, a deliberate increase in the required pressure drop in many cases can be quite acceptable due to a reduction in the cost of manufacture.

2. Analytical estimation of pressure buildup in divergent section
Prediction of the conditions for reaching the critical regime of nozzle operation with a stepped divergent section is complicated by the presence of separation regions after sudden expansion at the joint of the cylindrical sections of nozzle. The required static pressure drop for the nozzle operating in the critical regime was estimated by an analytical solution of the system of momentum, continuity, conservation of energy, and the state of ideal gas equations, taking into account the law of variation in cross-sectional areas. The gas parameters in the nozzle throat were determined by gas-dynamic functions.

Parameters of the stagnated gas flow at the nozzle inlet were set: $P^* = 100$ kPa, $T^* = 293$ K and $\rho^* = 1.188$ kg/m$^3$ as the boundary conditions.

The system of these equations was solved for each section joint of the cylindrical sections of nozzle. In the equation of motion, it was assumed that the pressure is constant over the cross section in the region of the stepwise change in area. This assumption usually holds well for subsonic flows with a small flow curvature.

3. Numerical simulation
In numerical simulation, the problem solved in a steady axisymmetric formulation under the assumption of no heat exchange with the external medium. The considered geometric model of the nozzle with a stepped divergent section and throat diameter of 10 mm is presented in figure 1. The model was supplemented by a small upstream straight-run pipe section.

Two-dimensional grid system with rectangular domains was generated. The computational mesh was adapted in the process of numerical solution by the Mach number gradient. The cell size adjacent to the wall was maintained at $y^+ < 1$ and was provided with a step-by-step adaptation in the solution process according to the chosen computation algorithm (EWT-$\varepsilon$) for the viscous section of the boundary layer. A structured grid system of about 80,000 grid points was employed in the computations after adaptation. The solution was tested for stability to mesh refinement.

ANSYS Fluent 18.2 software was used for numerical simulation. The mathematical formulation included the equations of conservation of mass, energy, and the Reynolds-averaged Navier-Stokes equations with the closure of k-$\varepsilon$ RNG turbulence model, corrected for the compressibility of the

Figure 1. Scheme of ISO 9300 nozzle with conical and stepped (dashed line) divergent section.
medium. The solution was a splitting algorithm (Density Based solver) with an implicit scheme. In the case of spatial discretization, a second order accuracy scheme was used to approximate the integrals of momentum, kinetic energy of turbulence and rate of its dissipation, the pressure integral. Calculation of the gradient vectors was carried out by the gradient least squares method (LSCBGE). As a criterion for convergence solution, residual errors of differential equations for the velocity and turbulent characteristics and normalized residual of the iterative process (normalized difference of the weighted mean values of the flow parameters between two consecutive iterations) were used.

The iterative process was considered complete when all relative residuals come up to $< 1 \cdot 10^{-5}$. An additional convergence condition was the balance of gas flow rate with accuracy up to 0.001%. To describe the characteristics of the model medium, the equation of density variation using ideal gas law and equation of variation of dynamic viscosity by Sutherland's law [11] were employed. The conditions repeated the boundary conditions of the analytical statement (total pressure $P^* = 100$ kPa, total temperature $T^* = 293$ K), as well as the turbulence intensity of 2% and the hydraulic diameter were set as boundary conditions on the inlet section. In the outlet section, a static pressure was 80.5 kPa obtained from the analytical estimation of the required pressure drop for the nozzle in the critical flow regime. In order to verify the independence of volumetric flow rate through the nozzle with stepped divergent section from a pressure drop higher than the required one, three more calculations were performed with a static pressure in the nozzle outlet section of 80, 79 and 75 kPa.

4. Results
The nozzle reaching the operating regime occurs at $\Delta P = P^* - P_4 = 19.492$ kPa according to the solution of equations of the analytical approach. Table 1 gives the values of gas characteristics in the nozzle throat and in the cylindrical sections of the stepped diffuser obtained from the results of analytical approach and numerical simulation (CFD results are presented in the outlet edge of the step where the profiles become maximally uniform, figure 2). It was obtained from numerical simulation that the lengths of the recirculation regions do not exceed half the length of the corresponding step (regions with a negative value of the streamwise component of the velocity vector are schematically rubricated in figure 2). Figure shows the flow joined the wall in each section. In addition, for sections with a high-speed flow at the outlet, the velocity profiles are relatively uniform in cross-section. Therefore, the assumptions about the uniformity of the flow velocity over the cross section adopted in the analytic approach are performed rather well.

The analytical estimation of the recovery of gas parameters in a stepped diffuser agree well with the numerical simulation results: the residuals between the values of static pressure, static temperature and density do not exceed 0.63%, 1.8% and 1.65%, respectively (table 1). Obviously, the analytical solution is invariant to the $Re$ number, therefore the obtained results will be valid for stepped nozzles with any diameter of the throat while maintaining the similarity of geometry.

Numerical simulation results showed that an increase in the pressure drop by 0.5, 1.5 and 5.5 kPa did not affect the volume flow rate through the nozzle.

**Table 1.** The gas characteristics in the nozzle throat and cylindrical sections of stepped diffuser.

|                | «*» | «1» | «2» | «3» | «4» |
|----------------|-----|-----|-----|-----|-----|
| **Analytical approach** |     |     |     |     |     |
| $P$, kPa        | 100 | 52.8| 71.0| 76.8| 79.4| 80.5|
| $T$, K          | 293.1| 244.1| 271.4| 279.0| 282.1| 283.5|
| $\rho$, kg/m$^3$ | 1.188| 0.754| 0.911| 0.959| 0.979| 0.988|
| **CFD (at $\Delta P = 19.5$ kPa)** |     |     |     |     |     |
| $P$, kPa        | 100 | 52.8| 70.6| 76.9| 79.1| 80.5|
| $T$, K          | 293.1| 244.0| 273.3| 282.1| 285.8| 288.7|
| $\rho$, kg/m$^3$ | 1.188| 0.753| 0.900| 0.950| 0.964| 0.972|
5. Conclusions
It was shown that replacing the conical divergent section of ISO 9300 nozzle with the toroidal inlet section by easy-to-manufacture stepped diffuser with cylindrical sections ensures a constant flow rate when the nozzle reaches the operation regime. The required pressure drop for starting the sonic nozzle with a stepped diffuser, as well as the static pressure buildup in the divergent section with a high degree of accuracy, can be estimated from solving the system of momentum, continuity, conservation of energy, the state of ideal viscous gas equations and the law of change in the cross-sectional areas.

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