How many body QM and QFT in 4 dimensions emerge from single particle QM in 5 dimensions

Roman Sverdlov,
IV IISER Mohali, Knowledge city, Sector 81, SAS Nagar, Manauli

Abstract

The goal of this paper is to reduce many body quantum mechanics to single particle one in $\mathbb{R}^5$. This is done by treating $x^5$ coordinate as "space filling curve" that would "fill" configuration space up to some coarse graining. After working it out for the case of many body QM, this idea is then extended to QFT.

1. Introduction

It is often assumed that the key problem in interpretation of quantum mechanics is the collapse of wave function (see [10]). After all, in collapse free situation, we can say that $\psi$ is simply a "classical field" obeying "wave mechanics". Since we don’t have problem with interference of electromagentic field, we shouldn’t have a problem with interference of $\psi$ either. On the other hand, if we do have a collapse, we are tempted to "drop" the classical field interpretation since we "don’t understand" what might cause said collapse. Instead, we are tempted to say that, in some sense, we always had a point particle and the only issue is that the "probability amplitude" of finding said particle is complex valued. The conceptual likening of $\psi$ to "probability" rather than "field" is what ultimately leads to a paradox since classically the probabilities are always positive real, and obey simple addition rule.

The above line of argument, however, can be avoided. One can say that $\psi$ is, indeed, a "classical field" and it simply "happens" that the probability of collapse coincides with $|\psi|^2$. After all, the probability of coin falling on a certain side is the function of the weight attached to that particular side. This doesn’t imply that weight is a version of probability. Clearly, the weight is still a physical parameter; it simply "affects" probability. The same might be true for $\psi$. One difference is that we "know" the mechanism in which the weight of a coin affects the probability of its landing on a certain side, yet we "don’t know" the mechanism of $\psi$ affecting a collapse probability: after all, we don’t know why collapse happens to begin with! This situation, however, is not much different from Kepler who "didn’t know" why planets move according to certain orbits and yet could describe what these orbits were. Lack
of explanation didn’t force Kepler to conclude that different kind of logic was in order, and neither should we.

However, we are stopped from making the above argument by the fact that $\psi$ is a function over configuration space $\mathbb{R}^{3n+1}$ (where $n$ is the number of particles) rather than ordinary space. After all, all of the ”classical” fields are functions over ordinary $\mathbb{R}^4$. At the same time, classical probabilities are in fact functions over $\mathbb{R}^{3n+1}$, which brings us right back to the ”probability” interpretation of $\psi$ that we were trying to avoid. In other words, if $\psi$ is a function over $\mathbb{R}^4$, then it can be viewed as ”classical field”, the latter can be complex valued, so we have no problem. On the other hand, if $\psi$ is a function over $\mathbb{R}^{3n+1}$, then it can no longer be classical field, thus it is probability amplitude, thus our intuition demands it should be positive real, which creates a big problem. From this point of view, the difference between $\mathbb{R}^4$ and $\mathbb{R}^{3n+1}$ is the key aspect that would distinguish Keplerian mechanics that makes ontological sense, and quantum logic that does not.

Now, since $3 \times 1 + 1 = 4$, we claim that single particle quantum mechanics is the only one that makes sense. Indeed, the role of configuration space in creating a problem has also been acknowledged by others as well (see [1]). Thus, our goal is to model $n$-particle quantum mechanics as an ”exotic case” of single particle one. It turns out we can do that, if said ”single particle” QM is introduced in $\mathbb{R}^5$ rather than $\mathbb{R}^4$. We will use the extra coordinate $y = x^5$ as a ”space filling curve” that would ”fill” $\mathbb{R}^{3n}$. This will only be done up to some coarse graining. In other words, the ”space filling curve” does not hit every single point in $\mathbb{R}^5$, but it does hit a $\delta x$- neighborhood of every such point, where $\delta x$ is small, but still finite number. This means that we won’t be able to reproduce an exact $n$-particle quantum mechanics, but we would be able to reproduce an approximate version of it, up to very high degree of approximation.

Once we have completed this step, we will then extend this idea to QFT. In order to do it, we will follow the statement made by Hatfield, [5], that ”relativistic” QFT can be viewed as non-relativistic many body problem involving ”mattress” (or, in other words, a system of coupled harmonic oscillators). This logically implies that, if we convinced the reader that any ”non-relativistic” many-body system is reducible to single particle QM in $\mathbb{R}^5$, the same should be true for ”mattress” and, therefore, ”relativistic” QFT. The ”space filling curve” argument can work even if field is continuous as long as we have some alternative way of limiting its degrees of freedom. For example, we can impose the upper bound on its derivatives, which is the option we will take in this paper. This will result in upper bounds both in particle numbers as well as their momenta, leading to non-unitarity.

The non-unitarity is the price we have to pay for our use of ”space filling curve” that we need in order to ”solve” the conceptual paradox of ”probability amplitudes” discussed earlier. Such non-unitarities can’t be falsified as long as we claim that they are ”much smaller” than non-unitarities attached to quantum measurement (another logical option would be to claim that ”our” non-unitarities somehow ”create” quantum measurement, but at the moment we don’t have specific proposals on how this might take place).
Defining $n$-particle states in $\mathbb{R}^4$ from single particle states in $\mathbb{R}^5$

Let us now describe in more detail the model that would accomplish the above. First, we will introduce extra dimension, $y = x^5$. Thus, a point in space takes the form

$$(x^0, \ldots, x^5) = (t, \vec{x}, y), \quad \vec{x} = (x^1, x^2, x^3), \quad t = x^0, \quad y = x^5$$

(1)

We then postulate $n$ string-like objects that ”stretch out” in $y$ direction while ”twisting around” in $(x^1, x^2, x^3)$ along the way. They change their shape in time. Thus, with flow in time, they trace out two dimensional surface in $\mathbb{R}^5$ given by

$$(\vec{x}_1, \ldots, \vec{x}_n) = (\vec{x}_1(t, y), \ldots, \vec{x}_n(t, y))$$

(2)

At any given time $t = t_0$, we can define

$$(\vec{x}_{1t_0}, \ldots, \vec{x}_{nt_0})(y) = (\vec{x}_1(t_0, y), \ldots, \vec{x}_n(t_0, y))$$

(3)

The left hand side is a ”curve” in $\mathbb{R}^n$ with ”parameter” $y$. After all, a curve is something that ”takes” an element of $\mathbb{R}$ and ”returns” an element of $\mathbb{R}^m$. Now, the $t = t_0$ ”snapshots” of above $n$ strings, if ”taken together” would ”take” a single value of $y = y_0$ and ”return” $(\vec{x}_1(y_0, t_0), \ldots, \vec{x}_n(y_0, t_0))$. Geometrically, if we state that $y = y_0$, this means that we will intersect the above strings with $y = y_0$ hyperplane. Such intersection would produce a configuration of $n$ points, which is what we are seeking. The ”curve” (or, equivalently, a function $\mathbb{R} \to \mathbb{R}^m$) ”takes” the value of ”height” at which hyperplane is positioned, and ”returns” the configuration of points produced by the intersection with said hyperplane with the strings.

We would like to assume that the above curve is ”space filling”. In order to do that, we have to assume that the space is compact,

$$x^1 + L_1 = x^1, \quad x^2 + L^2 = x^2, \quad x^3 + L_3 = x^3, \quad x^5 + L_5 = x^5$$

(4)

In this case, the total number of approximate configurations we would like to reproduce is

$$\#\{\text{Desired Configurations}\} = \left( \frac{L_1 L_2 L_3}{(\delta x)^3} \right)^n$$

(5)

where $\delta x$ is our desired precision of defining $\vec{x}$. If we, furthermore, assume that

$$\left| \frac{\partial \vec{x}_k}{\partial y} \right| \geq A$$

(6)

and the strings are sufficiently random, then the number of configurations we would, in fact, reproduce is

$$\#\{\text{Emergent Configurations}\} = A L_5$$

(7)

Now, in order for the curve to be ”space filling”, we would like any given desired configuration to match at least one emergent one. Besides, we would also like to make a statistical argument that the number of emergent configuration matching any given desired configuration is approximately the same. This can be done if

$$\#\{\text{Emergent Configurations}\} \gg \{\text{Desired Configurations}\}$$

(8)
or, in other words,

$$AL_5 \gg \left( \frac{L_1 L_2 L_3}{(\delta x)^3} \right)^n$$

(9)

The fact that we have multiple ways of reproducing a given "desired" configuration implies that we will identify it with a superposition of many different values of $y$,

$$|x_1, \ldots, x_n; t\rangle_{\delta x} = \int dy |t, y\rangle \exp \left( -\frac{1}{2(\delta x)^2} \sum_{k=1}^{n} |\bar{y}_k - \bar{x}_k(t, y)|^2 \right)$$

(10)

where exponential is a "finite" version of $\delta$-function. For example, suppose that

$$(\bar{x}_1(t_0, y_1), \ldots, \bar{x}_n(t_0, y_1)) = (\bar{x}_1(t_0, y_2), \ldots, \bar{x}_n(t_0, y_2)) = (\bar{v}_1, \ldots, \bar{v}_n)$$

(11)

In this case, it is easy to see that

$$\lim_{\delta x \to 0} |v_1, \ldots, v_n; t_0\rangle_{\delta x} = |y_1, t_0\rangle + |y_2, t_0\rangle$$

(12)

Now, the right hand side of Eq (10) amount to single particle QM in $\mathbb{R}^2$, while we would like to have single particle QM in $\mathbb{R}^5$. In order to obtain the latter, we define

$$|t, y\rangle = \int d^3x |t, x, y\rangle$$

(13)

which would lead to

$$|x_1, \ldots, x_n; t\rangle_{\delta x} = \int d^3x dy |t, x, y\rangle \exp \left( -\frac{1}{2(\delta x)^2} \sum_{k=1}^{n} |\bar{y}_k - \bar{x}_k(t, y)|^2 \right)$$

(14)

At this point the reader might ask a question. From the "formal" point of view, Eq (14) is well defined regardless of whether $(\bar{x}_1, \ldots, \bar{x}_n)$ is "space filling" or not. So why does it have to be space filling? In order to answer this question, lets pretend that it isn’t. In this case, we can select $(\bar{u}_1, \ldots, \bar{u}_n)$ that is simultaneously "far away" from every single $(\bar{x}_1(t_0, y), \ldots, \bar{x}_n(t_0, y))$ at a given $t = t_0$. At the same time, nothing stops us from selecting $(\bar{v}_1, \ldots, \bar{v}_n)$ that is close to said curve at some $y$: after all, a curve has to pass by something. In this case, it is easy to see from Eq (14) that

NOT Space Filling up to $\delta x \implies$

$$\implies \delta x \langle u_1, \ldots, u_n; t_0 | u_1, \ldots, u_n; t_0 \rangle_{\delta x} \ll \delta x \langle v_1, \ldots, v_n; t_0 | v_1, \ldots, v_n; t_0 \rangle_{\delta x}$$

(15)

In other words, we “will” produce a "version of" $n$-particle QM, but said version will “not” be normalized properly. Thus, the purpose of "space filling" property is the appropriate normalization of resulting $n$-particle QM. At the same time, the "space filling" nature of said curve is necessary but not sufficient for the latter. After all, it is possible that the curve “visits” the $\delta x$-neighborhood of $(\bar{u}_1, \ldots, \bar{u}_n)$ less times than it “visits” the $\delta x$-neighborhood of $(\bar{v}_1, \ldots, \bar{v}_n)$ (here, when we are talking about "visit" we are picturing us traveling along $t = t_0$ snapshot of a curve, in $y$-direction). In this case, the inequality (15) will still occur
Despite the "space filling" nature of said curve. Nevertheless, if a curve "visits" any given point a "very large" number of times, as guaranteed by Eq 9, we would statistically expect that the number of times curves "visit" the neighborhoods of any given point are compatible. Thus,

\[(\text{Space Filling up to } \delta x) + (\text{Eq 9}) \implies \delta x \langle u_1, \ldots, u_n; t_0 | u_1 \ldots u_n; t_0 \rangle_{\delta x} \approx \delta x \langle v_1, \ldots v_n; t_0 | v_1, \ldots, v_n; t_0 \rangle_{\delta x} \]

which is what we want.

**Emergence of Schrodinger's equation in \( \mathbb{R}^{3n+1} \) from the one in \( \mathbb{R}^5 \)**

We will now rewrite Eq 14 in terms of wave functions \( \Psi: \mathbb{R}^5 \to \mathbb{C} \) and \( \psi_{\delta x}: \mathbb{R}^{3n+1} \to \mathbb{C}, \)

\[
\psi_{\delta x}(\vec{x}_1, \ldots, \vec{x}_n; t) = \int d^3xdy \, \Psi(t, \vec{x}, y) \exp\left(-\frac{1}{2(\delta x)^2} \sum_{k=1}^{n} |\vec{x}_k - \vec{x}_k(t, y)|^2\right) \tag{17}
\]

Suppose we would like \( \psi_{\delta x} \) to obey Schrodinger’s equation,

\[
i \frac{\partial \psi_{\delta x}}{\partial t} \approx -\frac{1}{2m} \sum_{k=1}^{n} \nabla^2_k \psi_{\delta x} + V \psi_{\delta x} \tag{18}
\]

for some potential \( V: \mathbb{R}^{3n+1} \to \mathbb{R} \). In order to reduce the above to "single particle" QM in \( \mathbb{R}^5 \), we need to find some other potential, \( U: \mathbb{R}^5 \to \mathbb{C} \) such that Schrodinger’s equation acting on \( \Psi: \mathbb{R}^5 \to \mathbb{C}, \)

\[
i \frac{\partial \Psi}{\partial t} = -\frac{1}{2\mu} \nabla^2 \Psi + U \Psi \tag{19}
\]

We purposely skip \( \partial_5^2 \) term when we define

\[
\nabla^2 = \partial_1^2 + \partial_2^2 + \partial_3^2 \tag{20}
\]

This will lead to different values of \( x_5 \) "not talking to each other" thus creating the effect of "parallel universes" which will ultimately produce configuration space we are seeking (we have different configurations on each of the "parallel universes"). But, since we want to claim that this is truly one single universe (after all, this is the agenda of our work), we can include \( \partial_5^2 \) with "very small" coefficient \( \epsilon^2 \):

\[
\nabla^2 = \partial_1^2 + \partial_2^2 + \partial_3^2 + \epsilon^2 \partial_5^2 \tag{21}
\]

We can then say that the measurements somehow "reverse" its effect and they occur often enough for its effects not to "add up" between the measurements. This part is up to speculation. In any event, we claim to "predict" Eq 18 as an emergent approximation of Eq 19. We claim that \( U: \mathbb{R}^5 \to \mathbb{C} \) that "fulfills" this purpose is

\[
U(t, \vec{x}, y) = -\frac{1}{2m} \sum_{k=1}^{n} \left| \frac{\partial \vec{x}_k(t, y)}{\partial t} \right|^2 + V(\vec{x}_1(t, y), \ldots, \vec{x}_n(t, y)) \tag{22}
\]
One should note that $\vec{x}_k$ on the right hand side are distinct from $\vec{x}$ on the left hand side. Thus, $U$ is independant of $\vec{x}$,
\[
\frac{\partial U}{\partial x^1} = \frac{\partial U}{\partial x^2} = \frac{\partial U}{\partial x^3} = 0
\] (23)
which is a reflection of the fact that $y$ is the only thing that is of consequence while $\vec{x}$ is an "extra" variable we don’t need, whose sole purpose is to take us from single particle QM in $\mathbb{R}^2$ to single particle QM in $\mathbb{R}^5$, as was stated in the discussion around Eq 13. Now, the compactness condition in Eq 14 implies that
\[
\int d^3 x \nabla^2 \Psi = 0
\] (24)
Therefore,
\[
i \frac{\partial}{\partial t} \int d^3 x \Psi(t, \vec{x}, y) = \int d^3 x U(t, \vec{x}, y) \Psi(t, \vec{x}, y)
\] (25)
Now, due to Eq 23, we can say
\[
U(t, \vec{x}, y) = U(t, y)
\] (26)
and pull it out of integral. Thus, we obtain
\[
i \frac{\partial}{\partial t} \int d^3 x \Psi(t, \vec{x}, y) = U(t, y) \int d^3 x \Psi(t, \vec{x}, y)
\] (27)
where
\[
U(t, y) = -\frac{1}{2m} \sum_{k=1}^{n} \left| \frac{\partial \vec{x}_k(t, y)}{\partial t} \right|^2 + V(\vec{x}_1(t, y), \cdots, \vec{x}_n(t, y))
\] (28)
Therefore,
\[
\int d^3 x \Psi(t_2, \vec{x}, y) = \left[ \exp\left( -i \int_{t_1}^{t_2} dt U(t, y) \right) \right] \int d^3 x \psi(t_1, \vec{x}, y)
\] (29)
If we substitute this into Eq 17, we obtain
\[
\psi_{\delta x}(\vec{x}_1, \cdots, \vec{x}_n; t_2) = \int d^3 x dy \left\{ \left[ \exp\left( -\frac{1}{2(\delta x)^2} \sum_{k=1}^{n} |\vec{x}_k - \vec{x}_k(t_2, y)|^2 \right) \right] \right. \times
\left. \left[ \exp\left( -i \int_{t_1}^{t_2} dt U(t, y) \right) \right] \int d^3 x' \psi(t_1, \vec{x}', y) \right\}
\] (30)
For the same reasons as mentioned earlier (see, for example, Eq 23), the integrand is independant of $x$ (its only dependence is $x'$); thus, the integration over $x$ simply gives a constant factor of $L_1 L_2 L_3$:
\[
\psi_{\delta x}(\vec{x}_1, \cdots, \vec{x}_n; t_2) = L_1 L_2 L_3 \int d y \left\{ \left[ \exp\left( -\frac{1}{2(\delta x)^2} \sum_{k=1}^{n} |\vec{x}_k - \vec{x}_k(t_2, y)|^2 \right) \right] \right. \times
\left. \left[ \exp\left( -i \int_{t_1}^{t_2} dt U(t, y) \right) \right] \int d^3 x' \psi(t_1, \vec{x}', y) \right\}
\] (31)
As we have previously said, the space is compact; in particular, $0 \leq y \leq L_5$. Therefore, if we assume that $(\vec{x}_1, \ldots, \vec{x}_n)$ is well behaved it should be possible to write

$$\forall\{y_1, y_2\} \in S_k \left[ \forall t \in [t_1, t_2] (U(t, y_1) \approx U(t, y_2)) \wedge \right.$$

$$\left. \exp \left( -\frac{1}{2(\delta t)^2} \sum_{k=1}^{n} |\vec{x}_k - \vec{x}_k(t_2, y)|^2 \right) \approx 1 \right]$$

in such a way that

$$\exp \left( -\frac{1}{2(\delta t)^2} \sum_{k=1}^{n} |\vec{x}_k - \vec{x}_k(t_2, y)|^2 \right) \approx 1$$

The sets $S_k$ don’t have to be connected. For example, it is possible that $L_5/3$ and $2L_5/3$ are elements of $S_1$ while $L_5/4, L_5/2$ and $3L_5/4$ are not. The only clue regarding the structure of $S_l$-s is the fact that first and second derivatives of $\vec{x}_k$ are bounded above. We can now rewrite Eq 31 as

$$\psi_{\vec{x}}(\vec{x}_1, \ldots, \vec{x}_n; t_2) = L_1 L_2 L_3 \sum_{l=1}^{N} \int_{S_l} dy \left\{ \left[ \exp \left( -\frac{1}{2(\delta x)^2} \sum_{k=1}^{n} |\vec{x}_k - \vec{x}_k(t_2, y)|^2 \right) \right] \right.$$

$$\left. \times \left[ \exp \left( -i \int_{t_1}^{t_2} dt \ U(t, y) \right) \right] \int d^3 x' \psi(t_1, \vec{x}', y) \right\}$$

Now, lets select arbitrary $\{y_1, \ldots, y_N\}$ satisfying $y_i \in S_l$, and define

$$(\vec{\gamma}_1, \ldots, \vec{\gamma}_N)(t) = (\vec{x}_1, \ldots, \vec{x}_N)(y_i, t)$$

Now, Eq 33 tells us that certain things in Eq 34 can be thrown away as they approximate unity. Furthermore, Eq 35 allows us to substitute $\vec{\gamma}$-s into $\vec{x}$-s, which leads to

$$\psi(\vec{x}_1, \ldots, \vec{x}_n; t_2) \approx L_1 L_2 L_3 \sum_{l=1}^{N} \int_{S_l} dy \left( \exp(iS(\vec{\gamma}_1, \ldots, \vec{\gamma}_N)) \int d^3 x' \psi(t_1, \vec{x}', y) \right)$$

where

$$S(\vec{\gamma}_1, \ldots, \vec{\gamma}_N) = \frac{m}{2} \sum_{k=1}^{N} \left| \frac{d\vec{x}_k}{dt} \right|^2 - V(\vec{x}_1, \ldots, \vec{x}_N)$$

Now, within any given $S_l$, the value of $e^{iS}$ is independent of the choice of $y$ within $S_l$. Thus, we can pull it out of integral in Eq 36 to obtain

$$\psi(\vec{x}_1, \ldots, \vec{x}_n; t_2) \approx L_1 L_2 L_3 \sum_{l=1}^{N} \left( \exp(iS(\vec{\gamma}_1, \ldots, \vec{\gamma}_N)) \int_{S_l} dy \int d^3 x' \psi(t_1, \vec{x}', y) \right)$$

Now let us define

$$w_l = \int_{S_l} dy , \psi_l(t) = \frac{1}{w_l} \int_{S_l} dy \int d^3 x' \psi(t, \vec{x}', y)$$

$$7$$
Then we can rewrite Eq 38 becomes

\[
\psi(\vec{x}_1, \cdots, \vec{x}_n; t_2) \approx L_1L_2L_3 \sum_{l=1}^{N} w_l \psi_l(t_1) \exp(iS(\vec{\gamma}_1, \cdots, \vec{\gamma}_N))
\]  

(40)

The above expression would reproduce path integral (where by “path” we mean the one in configuration space) if we make the following assumptions:

**Assumption 1:** \( \psi_l(t_1) = \psi(\vec{x}_{l1}, \cdots, \vec{x}_{ln}; t_1) \)

**Assumption 2:** \( w_i = w_j \) for all \( i \) and \( j \)

Assumption 1 is less trivial than it seems. After all, the \( y \)-s contributting to \( \psi_l(t) \) are the ones for which \((\vec{x}_{l1}(y, t), \cdots, \vec{x}_{ln}(y, t)) \approx (\vec{\gamma}_1, \cdots, \vec{\gamma}_N) \) for all \( t \in [t_1, t_2] \), while the \( y \)-s contributting to \( \psi(\vec{x}_{l1}, \cdots, \vec{x}_{ln}; t_1) \) are the ones for which approximation holds only at \( t = t_1 \). In order for the two things to be equal up to somel constant, we have to make sure that it “doesn’t matter” which \( y \) we take ”as long as” the above holds at \( t = t_1 \); thus, an ”additional restriction” for the ”remainder” of the interval \([t_1, t_2]\) doesn’t change much. From the point of view of our ”agenda” this makes sense: after all, as far as \( t \) is concerned, these different choices of \( y \) would ”represent” almost the same configuration. Thus, if \( \psi \) corresponds to the function over configuration space it would make sense why one would expect \( \psi \) to be almost the same configuration. However, this has become a lot more problematic once we realize that we don’t view configuration space as fundamental, but rather as emergent. In fact, the value of \( \psi \) over a configuration involves averaging of its values of \( y_1 \) and \( y_2 \). Therefore, we would get an extra parameter, \( \psi(y_2, t_1) - \psi(y_1, t_1) \) (where \( \vec{x}(y_2, t_1) \approx \vec{x}(y_1, t_1) \)) that is ”orthogonal” to what we are trying to depict in terms of wave function in configuration space. In other words, the latter becomes incomplete. Yet, our experiments show that the latter is complete. If we view configuration space as something emergent, its our task to provide a ”mechanism” as to how ”orthogonal” quantities, such as \( \psi(y_2, t_1) - \psi(y_1, t_1) \) become irrelevant.

The mechanism we would like to propose involves random ”swapping” of the values of \( \psi \). For the aesthetic purposes, we will try to reduce information any point has to process by introducing sixth coordinate, \( x_0 = z \). Since this coordinate is being introduced only for convenience purpose, the strings \( \vec{x}(y, t) \) will be translated into membranes \( \vec{x}(y, z, t) \) satisfying

\[
\frac{\partial \vec{x}}{\partial z} = 0
\]  

(41)

Also, in order for the algorithm to have finitely many steps, we will imagine discretized space (it should be understood that discrete picture we are about to describe has continuum limit).

And finally we will propose the following algorithm of swappng:

1. A point \((\vec{x}, y, z = 0)\) picks a random number, \( r \)
2. If the point \((\vec{x}, y, z = r)\) is ”blocked”, then nothing happens
3. If the point \((\vec{x}_0, y_0, z = r)\) is not blocked, then the following two steps take place:
   a) For all \( z \), \( \psi(t_0^+, \vec{x}_0, y_0, z) = \psi(t_0^-, \vec{x}_0, y_0 - r, z) \)
   b) For all \( z \), \( \psi(t_0^+, \vec{x}_0, y_0 - r, z) = \psi(t_0^-, \vec{x}_0, y_0, z) \)
4) Whether or not \((\vec{x}_0, y_0, z = r)\) is “blocked” or not is determined by the following criteria:

a) \((\vec{x}_0, y_0, z = r)\) is “blocked” at \(t^+_0\) if and only if there is at least one membrane that blocked it

b) The probability that membrane number \(k\) has blocked the point \((\vec{x}_0, y_0, z = r)\) is

\[
1 - e^{-\frac{1}{2(\delta x)^2}|\vec{x}_k(\vec{y}_0 - r, z, t_0) - \vec{x}_k(\vec{y}_0, z, t_0)|^2}
\]

It is easy to see that the probability of a point “not” being blocked by a given point becomes an exponential, and the probability of \(not\) being blocked by any of the points becomes a product of exponentials,

\[
\prod_{k=1}^{n} e^{-\frac{1}{2(\delta x)^2}|\vec{x}_k(\vec{y}_0 - r, z, t_0) - \vec{x}_k(\vec{y}_0, z, t_0)|^2} = \exp \left(-\frac{1}{2(\delta x)^2} \sum_{k=1}^{n} |\vec{x}_k(\vec{y}_0 - r, z, t_0) - \vec{x}_k(\vec{y}_0, z, t_0)|^2 \right)
\]

(42)

The above coincides with part of the definition of \(S_k\) given in Eq 33. Therefore, we have just shown that \(S_k\), as defined in Eq 33, consists of \(y\)-s that “swap around” the values of \(\psi\) with each other. This is the ultimate answer to the question as to why the averaged value of \(\psi\) is the only degree of freedom we can detect. Once we have established this, the truthfulness of Assumption 1 is a mere statistician consequence.

As far as Assumption 2 is concerned, it is actually \(not\) true on a level of \(few\) \(S\)-s, but it \(becomes\) true as a statistical outcome of a very large number of \(S\)-s. Since the criteria for \(S_k\) is very restrictive, we would, indeed, expect the number of \(S\)-s to be very large, thus justifying Assumption 2.

What we have shown is that Schrodinger’s equation in \(\mathbb{R}^5\) (or \(\mathbb{R}^6\) if we include the extra coordinate for ”swapping”) as described in Eq 19 and 22 would produce ”path sum” in \(\mathbb{R}^{3n+1}\), as given in Eq 40 (the strict match with ”path sum” is based on Assumptions 1 and 2 just discussed) which, through conventional calculations, leads to Schrodinger’s equation in \(\mathbb{R}^{3n+1}\) given in Eq 18. The ultimate link between postulated Schrodinger equation in \(\mathbb{R}^5\) (or \(\mathbb{R}^4\)) and the emergent one in \(\mathbb{R}^{3n+1}\) is what we were trying to show. Our claim that the former has realistic meaning while the latter does not. Thus, producing the link between these two things, ultimately ”provides” realistic meaning for \(n\)-particle quantum mechanics.

**Emergence of second quantization from single particle QM in \(\mathbb{R}^5\)**

So far we have shown that \(n\)-particle quantum mechanics can be represented as single particle one in \(\mathbb{R}^5\). Now, according to Hatfield, one can think of QFT as a special case of many body non-relativistic QM; namely, a system of coupled oscillators that Hatfield referred to as ”mattress”. Thus, we can ”apply” what we done for many body QM in the previous section ”to” QFT which would produce realistic interpretation.

One problem one can notice, however, is that a lot of the things we have done in the previous section are non-relativistic and non-local. For example, a ”potential” given in Eq 22 assumes \(t = const\) hyperplane, over which potential is being computed. Besides, the
very fact that Schrödinger’s equation is being used (instead of Klein Gordon equation) is also non-relativistic in itself. And finally, the "swapping" procedure described at the end of previous section is superluminal and, therefore, non-relativistic. The way we explain relativity is that Lagrangian that is being substituted into non-relativistic framework (per QFT generalization of Eq[22] which continues to be non-relativistic even in QFT case) happens to have contracted indexes (even though it didn’t have to!) As a result we are guaranteed to produce predictions consistent with QFT where indexes contract, and we know that such predictions "happen" to be Lorentz covariant. In other words, we don’t believe that Lorentz covariance is geometric property of spacetime; rather we believe it is physical property that is a consequence of specific Lagrangian.

The ultimate definition of quasi-relativity is that "large" enough ant (it has to be "large" in order to perceive mattress as continuum) does not have a way of finding out the reference frame of the mattress. Such ant, however, would know its own frame, \((t', x', y', z')\). Thus, it will have uncountably many "theories" regarding all possible speeds of the frame of the mattress, \((t, x, y, z)\) relative to its own frame. Each such "theory" will amount to the interpretation of what constitutes \(\partial'_{0}\) as well as \(\nabla'\). For example, according to "theory" that the ant is stationary relative to the mattress, \(\partial'_{0}\) is exclusively due to kinetic energy, while \(\nabla'\) is exclusively due to Hook’s law. On the other hand, other "theories" would say that \(\partial'_{0}\) as well as \(\nabla'\) are both due to some kind of combination of kinetic energy and Hook’s potential; the coefficients in said linear combinations are functions of the "velocity of the mattress" postulated in these theories. Thus, the problem is not that any of these theories can be falsified; rather, the problem is that none of them can be proven.

It turns out that we don’t have to postulate a discrete mattress: the construction that we use to "allow" \(n\)-particle case can be generalized to allow any non-trivial shape (such as continuous field trajectory as opposed to simply \(n\)-point set). At the same time, the coarse graining spoken of earlier implies that we can’t "capture" the prediction of ultraviolet behavior of QFT. This is ok: after all, even in conventional QFT, one has to introduce ultraviolet cutoff in order to perform calculations. We will claim that said cutoff has a literal meaning, attached to the coarse graining we are using. Incidentally, Hatfield also claimed cutoff has literal meaning: the inverse of the spacing between lattice points (****). In our case, we would no longer be able to exactly repeat Hatfield’s claim since we view field as continuous, but we will still attach cutoff to our version of coarse graining. Another difference between us and Hatfield is that, in case of mattress, as long as a given harmonic exists, it can have arbitrarily high excitations. That is, while some momenta are "cut off", we can still have arbitrarily many particles of "allowed" momenta. In our case, however, the coarse graining will limit the number of particles of any given momentum we can create, which would compromise unitarity as well. At the same time, as long as non-unitarity is small, it can’t be falsified: after all, we have other non-unitary processes, such as quantum measurement.

Now that we have established that we can violate relativity, let us proceed. We will first start by describing Fock states as wave functions in \(\mathbb{R}^\infty\) and, after that, we will convert it into \(\mathbb{R}^5\). From conventional QFT, we know that Fock space correspond to the excitation states on infinite dimensional harmonic oscillator: each momentum corresponds to each of the
dimensions in said harmonic oscillator. Now, in case of QM, Harmonic oscillator corresponds to \( \psi(x, t) \); this means that in QFT it should correspond to \( \psi(f, t) \), where \( f: \mathbb{R}^3 \to \mathbb{C} \) and \( \psi(f, t) \) is a probability amplitude that \( \phi(x^0 = t, \vec{x}) = f(\vec{x}) \). Now, by analogy with Eq 17 (where \( \vec{x}(t, y) \) is being replaced by \( \phi(t, \vec{x}', y) \)), we define the relation between \( \psi(f, t) \) and \( \psi(t, \vec{x}, y) \) in the following way:

\[
\psi(f, t) = \int d^3 x dy \left[ \psi(t, \vec{x}, y) \exp \left(-\frac{1}{2\epsilon} \int d^3 x' (f(\vec{x}') - \chi(t, \vec{x}', y))^2 \right) \right]
\]

which tell us that \( \psi(f, t) \) is a superposition over different \( \psi(x^0, x^5 = \alpha) \) for which \( \chi_{\alpha}(t, \vec{x}) \approx f(\vec{x}) \). Therefore, we can start by finding out infinite dimensional \( \psi(f, t) \) we are looking for, and after that we can "convert it" into a five dimensional statement about \( \psi(t, \vec{x}, y) \). Now, for the purposes of avoiding various mathematical problems, we will pretend that the number of dimensions of the space of functions is very large but finite. This can be done by postulating a large, but finite, set of "allowed" harmonics. At the same time, we also notice that, in order to have a perfect match with "quantum mechanical" harmonic oscillator, we postulating a large, but finite, set of "allowed" harmonics. At the same time, we also notice that, in order to have a perfect match with "quantum mechanical" harmonic oscillator, we would like Fourier components to be real, which is possible to do since we are assuming \( \phi \) is real. Thus, instead of using \( e^{i k x} \) and \( e^{-i k x} \), we use \( \cos(k x) \) and \( \sin(k x) \). This would give us two dimensional harmonic oscillator, while \( e^{i k x} \) and \( e^{-i k x} \) would correspond to the two states "rotating" in two opposite directions. Therefore, if we are to discretize the set of "allowed" values of \( \vec{k} \), we have to make sure to include \(-\vec{k}\) whenever we include \(+\vec{k}\). This means that the general quantum state is

\[
|\psi\rangle = \sum_{\{c\}, \{d\}} K_{c_1 \cdots c_n d_1 \cdots d_n} (a^\dagger p_{x_1})^{c_1} (a^\dagger p_{x_2})^{d_1} \cdots (a^\dagger p_{x_n})^{c_n} (a^\dagger p_{x_n})^{d_n} |0\rangle.
\]

Now, the wave function for the above two-dimensional harmonic oscillator is given by

\[
\lambda_{c d}(\omega; x, y) = \left[ \prod_{j=1}^n \left( \sqrt{\frac{\omega}{2}} (x + iy) + \frac{1}{\sqrt{2\omega}} \left( \frac{d}{dx} + i \frac{d}{dy} \right) \right)^{c_j} \right] \times \left[ \prod_{j=1}^n \left( \sqrt{\frac{\omega}{2}} (x - iy) + \frac{1}{\sqrt{2\omega}} \left( \frac{d}{dx} - i \frac{d}{dy} \right) \right)^{d_j} \right] e^{-\frac{\omega}{2}(x^2 + y^2)}.
\]

Therefore, the state given in 44 corresponds to

\[
\psi(f) = \sum_{\{c\}, \{d\}} \left[ K_{c_1 \cdots c_n d_1 \cdots d_n} \prod_a \lambda_{c_a d_a} \left( \sqrt{|\vec{p}|^2 + m^2}; \int d^3 x f(x) \cos(\vec{p} \cdot \vec{x}), \int d^3 x f(x) \sin(\vec{p} \cdot \vec{x}) \right) \right],
\]

By substituting Eq 43, we obtain

\[
\sum_{\{c\}, \{d\}} \left[ K_{c_1 \cdots c_n d_1 \cdots d_n} \prod_a \lambda_{c_a d_a} \left( \sqrt{|\vec{p}|^2 + m^2}; \int d^3 x f(x) \cos(\vec{p} \cdot \vec{x}), \int d^3 x f(x) \sin(\vec{p} \cdot \vec{x}) \right) \right] = \int d^3 x dy \left[ \psi(t, \vec{x}, y) \exp \left(-\frac{1}{2\epsilon} \int d^3 x' (f(\vec{x}') - \chi(t, \vec{x}', y))^2 \right) \right]
\]

\[
= \int d^3 x dy \left[ \psi(t, \vec{x}, y) \exp \left(-\frac{1}{2\epsilon} \int d^3 x' (f(\vec{x}') - \chi(t, \vec{x}', y))^2 \right) \right]
\]

11
By inspecting Eq 47 one can see that the only wave function present is the one living in \( \mathbb{R}^5 \) (namely, \( \psi(t, \vec{x}, y) \); yet, it provides a description of general state on Fock space. Now the reason it is "possible" is that states are reproduced only up to coarse graining, which is defined by the scale \( \epsilon \).

Now that we have convinced ourselves that \( \psi(t, \vec{x}, y) \), indeed, describes Fock space states, we can use analogies from Eq 19 and Eq 22 for the field case to describe their dynamics. As in Eq 19 \( \Psi \) obeys Schrödinger’s equation,

\[
i \frac{\partial \Psi}{\partial t} = -\frac{1}{2\mu} \nabla^2 \Psi + U\Psi
\]  

(48)

but this time potential \( U \) is different. While in Eq 22 the "potential" \( U \) was matching an action for point particles, this time it would match the action for fields. In both cases, the action is an integral taken over \( t = \text{const} \) hypersurface. In case of \( \phi^4 \) theory we would alve

\[
U(t, \vec{x}, y) = \int d^3 x' \left( \frac{1}{2} \left( \frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} |\vec{\nabla} \phi|^2 - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \right)
\]  

(49)

where \( \vec{\nabla} = (\partial_1, \partial_2, \partial_3) \). Also, in the same way as in many body QM case we had to supplemet Schrödinger’s equation by additional "swapping" (see algorithm on p.8), in the same way we will now describe a "swapping" algorithm for fields. The only difference is that "membranes" (see part 4) we will replace by specific values of \( \vec{x} \). After all, according to Hatfield, QFT is simply a QM problem with a "mattress"; thus, a membrane has to be replaced by lattice point, and, in continuum limit, lattice point corresponds to \( \vec{x} \). However, it should be understood that the algorithm relies on the discrete space, since in case of continuum a point would have infinitesimal effect; although it is possible to replace point with continuum region. With this being said, here is the algorithm:

1. A point \((\vec{x}, y, z = 0)\) picks a random number, \( r \)
2. If the point \((\vec{x}, y, z = r)\) is "blocked", then nothing happens
3. If the point \((\vec{x}_0, y_0, z = r)\) is not blocked, then the following two steps take place:
   a) For all \( z \), \( \psi(t_0^+, \vec{x}_0, y_0, z) = \psi(t_0^-, \vec{x}_0, y_0 - r, z) \)
   b) For all \( z \), \( \psi(t_0^+, \vec{x}_0, y_0 - r, z) = \psi(t_0^-, \vec{x}_0, y_0, z) \)
4. Whether or not \((\vec{x}_0, y_0, z = r)\) is "blocked" or not is determined by the following criteria:
   a) \((\vec{x}_0, y_0, z = r)\) is "blocked" at \( t_0^+ \) if and only if at least one other point \((\vec{x}, y_0, z = r)\) has blocked it
   b) The probability that a point \((\vec{x}, y_0, z = r)\) has blocked the point \((\vec{x}_0, y_0, z = r)\) is
\[
1 - e^{-\frac{1}{\pi^2} \int_0^L \phi(\vec{x}_0, y_0 - r, z, t_0) - \phi(\vec{x}, y_0, z, t_0) \, dz}
\]
Conclusion

In this work we have shown that both multiparticle QM (sections 2 and 3) as well as QFT (section 4) can be emergent from single particle QM in $\mathbb{R}^5$. This is demonstrated by the fact that all three chapters use Schrodinger’s equation (Eq 19 and Eq 48) with appropriately chosen potentials (Eq 22 and Eq 49) to produce $n$-particle first quantization and second quantization, respectively. This process had to be supplemented by ”swapping” as described in algorithms on p.8 and p.12. The ”swapping” can be seen as ”additional” collapse-like processes, since, just like wave function collapse, it ”interrupts” dynamical evolution. However, what distinguishes it from collapse is that ”swapping” is needed in order to produce ”collapse-free” unitary process. In other words, what ”looks like” collapse-free process in $n$-particle QM or Fock space QFT is actually an outcome of very frequent collapses occuring in single particle QM in $\mathbb{R}^5$. In either case, the collapse that accomplishes measurement is separate from the above, and is not included in this paper.

One thing that might strike the reader is that it seems like many body QM is ”richer” than single particle one, so ”how is it possible” to produce richer informational content by means of the poor one? The answer to this question is that the ”rich” informational content is being coarse grained. In case of many body QM, we are unable to describe the exact locations of particles, we can only describe their location up to some approximation, although that afore-given approximation can be made arbitrarily high. In case of QFT we end up having upper bounds on the particle numbers which lead to some non-unitary effects. These non-unitarities can be ”hidden” by the fact that the measurement process is ”even more” non-linear. Another logical way to argue is that the former is the cause of the latter, but at the present time we are unaware of any model that would accomplish that.

At the same time, our intuition about informational content is based on differentiable functions. On the other hand, if we allow non-differentiability, it has been shown that a curve can, in fact, perfectly fill the space, without resorting to approximations (see [2], [3] and [4]). Thus, it should be possible to rewrite what we have written in this paper in terms of such space filling curve. The additional bonus of doing so would be a possibility of using single variable integral along such curve as a way of coming up with rigorous definition of path integrals, which so far has not been solved. On the other hand, an argument in favor of sticking with well behaved functions is that we don’t have any experimental evidence that QFT is being reproduced exactly. On the other hand, the well behaved curves seem more physical. As far as this paper is concerned we were sticking with well behaved curves, but for future both avenues of research are worth exploring.

One ”selling point” of this work is that if we stick with an idea that Kaluza Klein model is the source of extra dimension, then we can argue that we haven’t introduced anything new. In fact, the ”modification” we are proposing amounts to relaxing a condition $\partial_5 g_{\mu\nu} = 0$ used in Kaluza Klein model. Thus, for example, $\phi$, which is $g_{55}$, would be different at each of the $x^5 = \text{const}$ hyperplane, which is effectively what we are suggesting. At the same time, however, we still need weaker version of said conditions. Possibly, we might have to say that even though $\partial_5 g_{\mu\nu} \neq 0$ we have to look at the ”special points” where $\partial_5 g_{\mu\nu} = \partial_5 \partial_\rho g_{\mu\nu} = 0$, since this has been used in computing curvature tensor. In any case, the point I am trying
to make is that many body QM as well as QFT can both be thought as first quantization in Kaluza Klein background, with "less restrictive" constraint than usually imposed. However, in light of the fact that Eq. 21 puts small coefficient of $\epsilon^2$ next to fifth derivative in Laplassian, the fifth coordinate is in some ways "longest" rather than shortest! Thus, we would have to introduce bimetric theory in order to make sense of this apparent contradiction.

As was explained in Introduction, reduction of multiparticle QM and QFT to single particle case is important in order for wave function to gain ontological meaning. While it is true that collapse is a problem even in single particle case, we already have some proposals on how to treat collapse (8 and 9), on the other hand, no proposals have been made regarding the very definition of $\psi$ in collapse free scenario. By reducing things to single particle case we fill this gap: we claim $\psi$ is a "classical field" and the collapse it is subjected to is "classical" phenomenon we simply "don’t understand" (yet have some really good clues such as 8 and 9). This fills the gap that has otherwise never been worked on, although occasionally acknowledged (see 11).

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