Accurate calculation and measurement method for elongation of pre-stressed strand of Bridges

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Abstract: In the pre-stressed construction of bridge strands, tension stress is used as the control and strand elongation is used as the calibration and the elongation of strands directly affects the performance of the bridge structure. So, it must be very precise. In actual construction, due to inadequate grasp of basic theory by technicians, improper measurement methods or inaccurate selection of calculation formulas, neglecting the relationship between the actual elongation value and the tool/job clip withdrawal amount, resulting in the deviation between the actual measured value and the theoretical design value is beyond the standard, which is very unfavorable to the quality of the engineering structure. This article starts with the basic science theory, through numerical calculation and theoretical analysis, explains the work stress compensation retracting amount of the Work anchor and tool anchor, and the compensation stress calculation method due to the stress loss caused by the retraction. It shows the working principle of the pre-stressed equipment, and proposes and validates accurate calculation and measurement method of strand extension.

1. Introduction
The working principle of the pre-stressed concrete member is to rationally arrange the pre-stressed tendons and tension the concrete structure, and to apply the pre-stressing force to the concrete in the tension zone where the tensile strength is poor, so that the concrete in the tension zone is always under pressure or relatively affected by the load. Small tensile stress, give full play to the characteristics of high concrete compressive strength, so that the bearing capacity of concrete cross-section significantly improved. At present, pre-stressed concrete is widely used in high-rise buildings, underground buildings, Takamatsu structures, hydropower structures, offshore structures, airport runways, pressure vessels in nuclear power plants, and large-tonnage ship terminals [1-2].It is more than 80 years old since it was invented in the 1930s. The pre-stress can reduce or even counteract the concrete tensile stress caused by external load, so as to delay the occurrence and expansion of concrete cracks, so that the components can obtain higher crack resistance. Pre-stressed technology has the advantages of light weight, simple structure, easy installation, saving materials, reliable safety, reducing the main tensile stress and vertical shear force of concrete beams, and is widely used in the construction of highway bridges in China [3].For example, pre-stressed construction techniques have been used in the construction of reinforced concrete frame sections, tensioned steel bars, strands, and beam sections for prefabrication and reinforcement [4].

When the pre-stressed tension is applied, tension stress is used as the control, and the actual elongation of the pre-stressed strand is used as the check, which is called “double control”, and the
control of the elongation of the pre-stressed strand directly affects the application of the pre-stress and the mechanical properties of the structure [5]. If the value of tensile force is too low, the precompression stress on the concrete after various losses of the pre-stressed steel bar is too small, and the crack resistance and stiffness of the pre-stressed concrete component cannot be effectively improved. If the tensile force value is too high, the strands in the construction are overstretched. It is possible that the stress of the individual steel bars during the super-tensioning process exceeds its actual yield strength, resulting in greater plastic deformation or brittle fracture. At the same time, it also has a great influence on the components. During the use of the super-stretched components, the tensile force of the steel wire is greater than the design when the design load is reached, so the bearing capacity is reduced for the components [6-7]. Therefore, the elongation of the pre-stressed strand must be precise.

In this paper, the clip-type anchorage used in the construction of the elevated section of Hefei Metro Line 3 is taken as an example. Combining with the precautions in actual operation, numerical calculation and theoretical analysis of the theoretical elongation of the pre-stressed strand and the accurate measurement at the site are done.

2. Accurate calculation of theoretical elongation of strands

2.1. The calculation of elongation

When calculating the elongation of strands, the straight-line segment and the curved segment shall be dismantled, and the elongation of each segment shall be calculated in sections, and then cumulatively added. The calculation formula is as follows [8].

2.1.1. The theoretical elongation can be calculated according to eq. (1):

$$\Delta L = \frac{P_p L}{A_p E_p}$$

where:

- $P_p$ — average tension of pre-stressed tendons (N);
- $L$ — length of pre-stressed tendons (mm);
- $A_p$ — total cross-sectional area of pre-stressed tendons (mm²);
- $E_p$ — elastic modulus of pre-stressed tendons 1.95 × 10⁵ (N/mm²).

It can be known from eq. (1) that when the length, total sectional area and elastic modulus of pre-stressed tendons are determined, the theoretical elongation value directly determines the average tensile force.

2.1.2. Average tension $P_p$ (N) can be calculated according to eq. (2) [8]:

$$P_p = \frac{P(1-e^{-(kx+\mu\theta)})}{kx + \mu\theta}$$

where:

- $P$ — tensile force of tensioned end of pre-stressed tendons (N);
- $x$ — channel length (m);
- $\theta$ — curve channel tangent angle (rad); angle $\theta=\alpha \times \pi/180$;
- $k$ — hole friction coefficient $k$ (determination of the friction test);
- $\mu$ — coefficient of friction between the strand and the wall of the cell (determined by the friction test);
- $e$ — $e=2.718$.

Because the pre-stressed tendons are mostly arranged in a curved line, the calculation of $P_p$ is usually performed in sections. For $\theta$ in equation (2), it can be seen from its symbolic interpretation that the sum of the included angles of the tangents of the curve hole section should be equal to the corresponding central angle of the curve. According to the geometric relationship: $\theta=L/R$(rad), The
middle L is the length (m) from the tensioned end to the calculated cross-sectional curve, and R is the radius of curvature (m) of the curve in the segment.

2.1.3. **Tension of tension can be calculated according to eq. (3):**

\[ P_i = P_{i-1} \times e^{-(kx + \mu \theta)} \]  

where:

- \( P_i \) —— Tension at the end of paragraph i (N);
- \( P_{i-1} \) —— Tension at the end of paragraph i-1 (N).

In the calculation, it is gradually calculated from the outside to the inside from the tensioning end under the anchor. If the two ends are symmetrically stretched, then from the one end to the midpoint of the strand, multiply the length of the segment and multiply it by 2; if it is unsymmetrical, pull the end from the tension end to the end of the anchor. Segment calculations are then accumulated [9].

2.2.** Calculation Example**

The longitudinal pre-stressed B1 cable of the 30m simple beam of the station between the Dazhong road and Xiangcheng Road on Line 3 of Hefei Subway uses the 1860-9Φs15.2 strand and the tensile stress is 1302Mpa. As shown in Figure 1, the AB segment, CD segment, and EF segment are straight segments, while the BC segment and DE segment are curved segments. Because it is the asymmetrical structure of the left side anchoring the right side tension, it can only be calculated from the A side to the F side segment by segment.

![Fig 1. Schematic diagram of the 30m support beam at the intersection of the Dazhong Road Station and Xiangcheng Road Station on Line 3 of Hefei Subway](image)

2.2.1. **According to the equation (1-3), the calculation of elongation at AB section of simply supported beam is as follows:**

\[ P_{AB} = 1302\text{MPa} \times 9\text{mm}^2 \times 140 = 164020\text{N} \]

\[ P_p = P_{AB} \times \left[ 1 - e^{-(kx + \mu \theta)} \right] / (kx + \mu \theta) \]
\[ = 1640520 \times \left[ 1 - e^{-0.0025 \times 0.7042 + 0.25 \times 0} \right] / (0.0025 \times 0.7042 + 0.25 \times 0) \]
\[ = 163939076.7793 \text{N} \]
\[ \Delta L_{AB} = \frac{P_p L}{A_p E_p} = 163939076.7793 \text{N} \times 0.7042 \text{mm} / 140 \text{mm}^2 \times 9 \times 195 \text{ N/mm}^2 = 4.6978 \text{ mm} \]

According to equation (1) (2), when the tensile stress of the simple section AB is 1302 MPa, the tensile force of the tensioned end of the pre-stressed tendon is 164020N, and the average tensile force of the pre-stressed tendon is 163939076.7793N and the elongation of strands is 4.968 mm.

2.2.2. **The elongation of BC segment:**

\[ P_{BC} = 164020 \times e^{-(kx + \mu \theta)} = 1637634.4053 \text{N} \]

\[ P_p = P_{BC} \times \left[ 1 - e^{-(kx + \mu \theta)} \right] / (kx + \mu \theta) \]
\[ = 1637634.4053 \times \left[ 1 - e^{-0.0025 \times 2.126 + 0.25 \times 0.2094} \right] / (0.0025 \times 2.126 + 0.25 \times 0.2094) \]
\[ = 1591311.96 \text{ 80 N} \]
\[ \Delta L_{BC} = \frac{P_p L}{A_p E_p} = 1591311.96 \text{ 80 N} \times 2.126 \text{mm} / 14 \text{ 0mm}^2 \times 9 \times 195 \text{ N/mm}^2 = 13.7693 \text{ mm} \]
According to equation (1) (2) (3), it can be known that the tensile force of the tensioned end of the pre-stressed tendon of the simply supported beam BC is 1637634.4053 N, the average tensile force of the pre-stressed tendon is 1591311.9680 N, and the elongation is 13.7693 mm.

2.2.3. The elongation of CD segment:
\[
P_{CD} = 1637634.4053 N \times e^{-(kx+\mu \theta)} = 1545871.4007 N
\]
\[
P_P = P_{CD} \times \left[ 1 - e^{-(kx+\mu \theta)} \right]/(kx + \mu \theta)
\]
\[
= 1545871.4007 N \times \left[ 1 - e^{-0.0025 \times 17.382 + 0.25 \times 0}/(0.0025 \times 17.382 + 0.25 \times 0) \right]
\]
\[
= 1512764.761 N
\]
\[
\Delta L_{CD} = \frac{P_P L}{A_p E_p} = 1512764.761 N \times 17.382 \text{mm} / 40 \text{mm}^2 \times 9 \times 195 N/mm^2 = 107.02 \text{mm}
\]

According to equation (1) (2) (3), it can be known that the tensile force of the tensioned end of the pre-stressed tendon of the simply supported beam CD is 1545871.4007 N, the average tensile force of the pre-stressed tendon is 1512764.761 N, and the elongation is 107.02 mm.

2.2.4. The same calculation method as above, the elongation of the other two sections can be calculated.
\[
\Delta L_{DE} = 5.2143 \text{mm}, \Delta L_{EF} = 9.9 \text{mm}
\]

The total theoretical elongation of the longitudinal pre-stressed B1 cable of the 30m simple beam of the intersection between the DAzhong station and Xiangcheng station on Line 3 of Hefei Subway can be calculated by summing up the elongation of each segment.

3. Accurate Measurement Method of Strand elongation

3.1. The concept and principle of several elongations and retractions

Before explaining the measurement of elongation, it is necessary to explain the concept and principle of several elongations and retractions, and to express the characteristics of the internal structure of pre-stresses for illustration.

3.1.1. Work length elongation

In the construction of strand tensioning, the post-clip type piercing type jack is often used. The working length of the strand in the jack is generally about 70cm. The working length refers to the distance from the center of the tool anchor to the center of the working anchor plate, the length of the work is the sum of the length of the jack and the thickness of the anchor.

3.1.2. Working anchorage strand retraction

The construction of strand tensioning is mainly using YCW hydraulic jacks. The contact point between
the jack and the working anchor is provided with a limiting plate that restricts the displacement of the working anchor clip during the tensioning process. When the strand is pulled, the working anchor clips move backwards following the strand tension and reach the bottom of the groove of the limiting plate, unrestraining the strand. When the strand is stretched to the design tensile force, the strand elastically shrinks when the jack is returned to the oil, and the working anchor clips follow the strand to displace in the anchor ring hole, which will cause the strand tension to be reduced. Reduction and compensation methods will be described later. After the tension is removed, remove the jack and measure the exposed length $C_2$ of the work anchor clip at the working anchor. Normally, at least three points are taken to obtain the average value. The depth of the groove of the jack limit plate is known as $C_1$, then the work anchor retraction can be calculated according to $C = C_1 - C_2$ [10-11].

Work anchor retraction not only affects the amount of elongation, but also affects the tensile stress. Generally, this value is selected by an empirical value after the process test and involved in the calculation of the actual measured elongation.

3.1.3. Tool anchor strand retraction

Tool anchor clips will be hammered by the operator before tension, when the tension is stretched to 10% (tension stress under the anchor), clips will slide inwards due to strand stress. When it is stretched to 20%, the clip will continue to slide inward, so that the 10%-20% elongation obtained by measuring the extension of the jack cylinder is 1-2mm longer than the actual elongation of the strand. If the elongation of 10%-20% $\sigma_k$ is taken as the elongation of 0%-10% $\sigma_k$, if the elongation of 10% to 20% is taken as the elongation of 0% to 10%, the elongation of the strand will have an error of 2 to 3 mm in the tension control section of 0%-20% $\sigma_k$. From 20% $\sigma_k$ to 100% $\sigma_k$, the clip of the strand has a slip of 2-3mm, and the elongation of the single-ended strand has an error of 3 to 4mm according to the minimum sliding amount. There is a total of about 6 to 8 mm of error when both sides are simultaneously pulled (the magnitude of the error depends on the degree of tightening of the tool anchor clip).

Measurement of the anchor retraction of tool anchor: When the tensile force of the jack reaches the initial tensile force of the strand (10% to 15% of the control stress), the loose pre-stressed strand has been pulled tight. At this time, the jack should be fully fixed, the length $B_1$ is accurately measured from the exposed end of the jack tool anchor plate to the exposed end of the strand. When the jack tensile force reaches the design tension of the strand pretensioning tension, measure the length $B_2$ from the exposed end face of the anchor anchor cup of the jack tool to the exposed strand end of the strand, and retract the tool anchor $B = B_1 - B_2$ [10]. Usually at least three measurement values are averaged.

3.2. Precise measurement of elongation

3.2.1. Direct method

The method of directly measuring the elongation of strands is referred to as "direct method" (Figure 3), that is, marks are made on the strand on the outside of the tool anchor and used as a benchmark for measurement. One advantage of using this method is that the length of the graded strands is measured using several strokes. The cumulative result is the measured elongation between the initial and final stresses values. The disadvantage is that it is inconvenient to measure, and when the strand is subjected to force, the tip has a divergence phenomenon, which affects the elongation of the strand.
Fig 3. Direct measurement of strand elongation

Using "direct method" to measure the elongation of strands:

\[ L_R = \left[ (L_{100\%} - L_{10\%}) + (L_{20\%} - L_{10\%}) \right] - \Delta L_D - \Delta L_A \] (4)

where:
- \( L \) —— the actual elongation of the strand;
- \( L_{20\%} \) —— the sum of the strokes of the jack pistons at both ends of the beam when the tensile stress is 20% \( \sigma_0 \);
- \( L_{100\%} \) —— the sum of the strokes of the jack pistons at both ends of the beam when the tensile stress is 100% \( \sigma_0 \) (the initial tensile stress).

3.2.2. Travel method

The measuring method for the elongation of the jack piston is abbreviated as “travel method”, as shown in Fig 4. When the strand begins to stretch to the initial stress (usually 10% of the tensile control stress), the elongation \( \Delta L_1 \) of the jack piston is measured, and when the strand stretched to the controlled tensile stress, the elongation \( \Delta L_2 \) is measured again. The elongation is \( \Delta L_2 - \Delta L_1 \), when control stress is 10% to 100%.

The elongation of the piston is not the elongation of the strand. Even if the compression of the anchors is ignored, the clip on the tool anchor still moves inward with respect to the anchor plate along with the strand when the tension is increased from 10% to 100%.

Assume that there is no relative slippage between the strand and the clip. When the strand exerts initial stress, the anchor plate has a positive pressure \( N \) and friction resistance \( F \) with the clip, where \( F = \mu N \) (\( \mu \) is the friction coefficient), this is an equilibrium condition. Then the calculation formula of tension \( T \) is \( T = N \sin \alpha + F \cos \alpha = N (\sin \alpha + \mu \cos \alpha) \). When both the angle \( \alpha \) and the friction coefficient \( \mu \) are constant, increasing the pulling force \( T \), \( N \) must increase. Therefore, the strand must move with the clip along the \( T \) direction to achieve a new balance. So, measuring the piston stroke should be corrected.

When measuring the elongation of strands using the "travel method":

\[ L_R = \left[ (L_{100\%} - L) + (L_{20\%} - L_{10\%}) \right] - \Delta L_D - \Delta L_A \] (5)

where:
- \( \Delta L_R \) —— the actual elongation of strands;
1234567890——the sum of the piston strokes at both ends of the beam when the tensile stress is 20%;

$L_{100\%}$ —— the sum of the piston strokes at both ends of the beam when the tensile stress is 100%;

$L_{10\%}$ —— the sum of the jack piston stroke at both ends of the beam when tensile stress is 10% (that is, the initial tensile stress, the recommended specification is 10% -25%);

$\Delta L_P$ —— unimpeded elongation of strands in the jacks at both ends of the beam (take theoretical calculation value);

$\Delta L_A$ —— the amount of anchorage compression and strand retraction at both ends of the beam (take the actual value of the process test).

3.2.3. Deviation rate of elongation of strand

When the pre-stress is pulled by the stress control method, it should be checked with the elongation value. The difference between the actual elongation and the theoretical elongation should be controlled within ±6%, that is 

$\frac{\Delta L_{LL} - \Delta L_T}{\Delta L_T} \leq 6\%$  [12].

3.3. Calculation of elongation when tensions are divided

In actual operation, it often happens that the maximum stroke of the jack cylinder does not reach the elongation of the strand, so that the maximum tension is not completed. In this case, the distance of each jack piston stroke should be recorded. The elongation of each tension is first calculated, and then which is added. However, when calculating the tensile elongation of each time, it includes an elongation of the working length of the strand in the tensioning jack corresponding to the tensile force of the tensioning stage. Therefore, the elongation of the working length in the tension jack at all levels should be subtracted from the measured elongation obtained by adding the tension elongation for each time.

4. Calculation of compensation stress of anchor compression and strand retraction

4.1. Introduction to working principles of clip-on anchors

Anchorage has self-anchorage performance. First of all, the part system of the jack is equipped with a limiting plate, there is a gap between the limiting plate and the anchoring ring, which is generally about 6mm [13]. Therefore, during the tensioning process, the clip can only recede 6 mm, it will be blocked by the limit plate. When the tension is pulled to the specified stress, the electric oil pump is returned to zero after the load is held for 2 minutes. When the anchoring operation is performed, due to the loss of tensile force, the originally stretched strand immediately retracts and drives the clip and drags the clip into the anchor hole. Because of the conical fit of the clip and the anchor ring, the strand is retracted and tightened by the clip pulled into the anchor until it can no longer be retracted. At this time, it has been anchored.

4.2. Calculation of controlling tension of retraction of compensation work anchor

According to the analysis of the retracting process of the anchor clips, the working anchors have an influence on tensile elongation and tensile force.

4.2.1. The influence value for the tensile force can be calculated according to eq. (6):

$$\Delta P = P \times \Delta L_A / \Delta L_T$$  \(6\)

where:

$\Delta L_T$ —— theoretical elongation;

$P$ —— control tension for design;

$\Delta L_A$ —— retraction of anchor strand at both ends of beam.
4.2.2. The value of work anchor retraction

The value of $\Delta L_f$ should be considered in two stages. In the pre-stressed construction of strand, tension testing must be carried out to ensure quality. Tensile process tests usually take one component or 2 to 3 tensions [10]. Large-area construction is carried out after completion of tensile test. In the tensile process test stage, due to the performance of the batch anchorage do not understand, the retraction of anchor strand at both ends of beam can only be estimated according to eq. (7):

$$\Delta L_d = C_1 \times 0.65 \times a \times 2$$  (7)

where:

- $C_1$——limit plate groove depth, generally take 6mm;
- $a$—— coefficient related to strand diameter, $\phi_j = 15.24 mm$ taking 1.0; $\phi_j = 12.7 mm$ taking 1.2.

After the tension test is completed, the tension compensation is calculated according to eq. (8)

$$\sigma_c = \sigma_{con} + \Delta L_d \times E_p / L$$  (8)

where:

- $L$——the theoretical length of strand.

4.2.3. Compensation for super tension control procedures

When the two ends are pulled, one end should be anchored first, and then the other end should be anchored after compensating the tensile force. Because when the A-end is anchored, due to the retraction of the work anchor clips, the hydraulic pressure at the B-end is reduced and the tensile force is reduced, which does not meet the requirements of design. Therefore, it should be anchored after the B-side added tension [10]. In the same way, when the B-side compensatory tensile anchorage is applied, the tensile force loss will still occur due to the retraction of the working anchor clip. To ensure that the design control tensile force meets the requirements, the modified design control tensile force (that is the design control tensile force plus the supplemental tensile force) PK should be calculated according to the equation (8), which can not be confused with the super tension, this value belongs to the compensation for the loss of pre-stress generated by the work anchor when the pre-stressed is pulled.

4.2.4. Precautions for on-site operation

In the actual operation of the project, the stroke volume of the jack is generally recorded at 10% $\sigma_{con}$ (initial tensile stress), 20% $\sigma_{con}$, 100% $\sigma_{con}$, 103 $\sigma_{con}$% (compensate for the super-tension). At the same time, the retract stroke volume of the jack is recorded.

Specific description: ① The retract stroke volume of the jack is the sum of $\Delta L_d$ and $\Delta L_f$; ② When using a compensatory over-expansion, the work anchor retraction is offset. When using the 100% $\sigma_{con}$ stroke amount calculation, the work length elongation should be subtracted. When using the 103% stroke amount calculation, the work length elongation and work anchorage retraction should be subtracted. Therefore, it is usually calculated with a stroke amount of 103% $\sigma_{con}$ in actual project.

5. Conclusion

Pre-stressed tension is the key process to ensure the structural quality of bridge strand pre-stressed construction engineering. This paper takes the clip-type anchorage used in the elevated section bridge construction on Line 3 of HeFei Subway as an example, and combines with the practical precautions, the theoretical elongation of the pre-stressed strand and accurate measurement of the scene were done a numerical calculation and theoretical analysis from the three aspects of the theoretical elongation of the strand, the elongation of the strand, and the calculation of the compensation stress of the anchor compression and strand retraction.
Combining with the actual engineering construction conditions, the theoretically calculated numerical value of this paper is highly compatible with the actual construction measurement. Therefore, the theoretical calculation method proposed in this paper is feasible, and it can be used for the preliminary theoretical analysis and simulation of actual construction.

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