Heavy-meson masses via Dick interquark potential

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Abstract

We study Dick quark-antiquark potential (up to a color factor) $V_D(r) = -\frac{\alpha_s}{7} + gf \sqrt{\frac{N}{2(3N-1)}} \ln[\exp(2mr) - 1]$ in the heavy meson sector. This potential emerges from an effective dilaton-gluon coupling inspired from string theory and proves to be linearly rising at large distances which ensures quark confinement. The semi-relativistic wave equation which appears in the theory of relativistic quark-antiquark bound states is used. This equation is cast into a constituent second order Schrödinger-like equation with the inclusion of relativistic corrections up to order $(v/c)^2$ in the quark speeds. The resulting equation is solved for Dick potential via the Shifted-$l$ expansion technique (SLET). The obtained results show that the spin-averaged energy levels of heavy-mesons are well explained and agree with other potential models or QCD sum rules predictions. Moreover, as a by-product, our analysis assign to the dilaton a mass around 56.9 MeV lying within the range of many theoretical scenario of dilatonic mass.

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1 Introduction.

The problem of calculating spin-averaged energy levels heavy-mesons is a very old subject, but still an important theme in the existing literature. At present much experimental material on the masses of ground and excited states of heavy quarkonia has been accumulated. By comparing theoretical predictions with experimental data, one can obtain information on the form of the potential of the quark-antiquark interaction. This information illuminates the most important features of quantum chromodynamics (QCD) dynamics: the asymptotic freedom and confinement. At present it is not possible to obtain the potential of the interquark interaction in the whole range of distances from the basic principles of QCD. Consequently, the corresponding interquark potential has to be determined phenomenologically. Therefore, to explain the meson masses within the experimental limits, serious attempts to build a quantitative potential model for quarks were made by many authors. The number of papers on quark potential model is so large that one cannot quote even a substantial fraction of them, but simply call attention to review [1], which contains many references on the subject.

To understand confinement in gauge theories, much attention has been focused on quark models based on QCD, because of their success in understanding the spectroscopy of mesons, especially the new ones. At short distances, QCD suggests a Coulomb-type potential \(-\frac{\alpha_s}{r}\), where \(\alpha_s\) is the quark-gluon fine-structure constant. At long distances one expects a confining potential. Lattice gauge theory [2], and string models [3] lead one to expect a linear confining potential. Now it is firmly established that the combination of linear confining potential, plus coulombic-type short distance potential, plus one-gluon-
exchange forces provides a good fit to meson mass spectra [4]. However, one would be hard pressed to say that they are well understood. Too many of the $Q\bar{Q}$ states predicted by the quark potential model have yet to be seen-only the ground state S, and P wave multiplets are filled. Therefore, until confinement is better understood, more states have been sited, and their properties measured, we cannot say that the subject is completed.

Recently, it has been pointed out by Dick [5,6,7] that an interquark potential is constructed through the inclusion of a scalar field (dilaton) in gauge theories. This potential proves to be rising at large distances which ensures quark confinement, and at the same time it incorporates the asymptotic freedom at small distances. Therefore, the aim of this paper is to dedicate more efforts to understand this new confinement generating mechanism through the investigation of the phenomenological application of Dick interquark potential $V_D(r)$ in the heavy mesons sector. This problem will be approached as in a previous work [8]. Therein, it has been demonstrated that the shifted-$l$ expansion (SLET), where $l$ is the angular momentum, provides a powerful, systematic and analytic technique for determining the bound states of the semi-relativistic wave equation consisting of two quarks of masses $m_1$, $m_2$, and total energy $M$ in any spherically symmetric potential, even one which has no small coupling constant parameter. It simply consists of using $1/\tilde{l}$ as a pseudo-perturbation parameter, where $\tilde{l} = l - \beta$, and $\beta$ is a suitable shift. This shift is vital for it removes the poles that would emerge, at lowest orbital states with $l = 0$, in our proposed expansion below. This method yields very accurate and rapidly converging energy eigenvalue series. It also handles highly excited states which pose problems for vari-
ational methods [9]. Moreover, relativistic corrections are included in a consistent way.

In this spirit, this paper is organized as follows: Section 2 recalls the conversion of the semi-relativistic equation into an equivalent Schrödinger-type equation, and there the Shifted-$l$ expansion technique (SLET) for this equation with any spherically symmetric potential is introduced. Section 3 is devoted to describe the phenomenological application of Dick quark-antiquark interaction potential. We present, in this section, our numerical results of the spin-averaged energy levels of charmonium, bottomonium and $b\bar{c}$ families, in connection to the dilaton mass, and then we draw our conclusion.

2 SLET for the semi-relativistic wave equation with any spherically symmetric potential

There are different methods to calculate the bound state energies of the semi-relativistic wave equation [10], and references therein. However, the method we present here is chosen for different reasons; ease of implementation, high accuracy, and substantial computation time reduction is achieved. An expansion in the powers of $(v/c)^2$ up to two terms in the semi-relativistic equation which is a combination of relativistic kinematics with some static interaction potential yields [8]:

\[
\left[- \frac{1}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + \gamma(r) + \frac{E_{n\ell}V(r)}{\eta}\right] R_{n\ell}(r) = \left(\frac{E_{n\ell}^2}{2\eta} + E_{n\ell}\right) R_{n\ell}(r),
\]

where $\gamma(r) = V(r) - V^2(r)/2\eta$, $E_{n\ell} = M - m_1 - m_2$, $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass, and $\eta = \nu/\mu^2$, where $\nu$ is a useful parameter defined as $\nu = m_1^3 m_2^3/(m_1^3 + m_2^3)$. In
this reduction formalism the spherically symmetric potential \( V(r) \) which represents the interaction between the two particles remains unspecified.

With the shifted angular momentum \( l = \bar{l} + \beta \), Eq.(1) becomes

\[
- \frac{1}{2\mu} \frac{d^2 R_{n\ell}(r)}{dr^2} + \bar{l}^2 \left[ 1 + \frac{(2\beta + 1)/\bar{l} + \beta + 1)/\bar{l}^2}{2\mu r^2} + \frac{\gamma(r)}{Q} + \frac{E_{n\ell}V(r)}{Q\eta} \right] R_{n\ell}(r) = \left( \frac{E_{n\ell}^2}{2\eta} + E_{n\ell}\right) R_{n\ell}(r),
\]

(2)

where \( n \) in this paper is the radial quantum number, and \( Q \) is a constant that scales the potential \( V(r) \) at large-\( l \) limit, and is set for any specific choice of \( l \) and \( n \), equal to \( \bar{l}^2 \) at the end of the calculations.

Following our earlier work [8], we present here the relevant formulas obtained in the SLET framework for semi-relativistic motion of a particle bound in radially symmetric potential. The calculations, and the results are very simple. The leading-order binding energy is given as:

\[
E_o = V(r_0) - \eta + \sqrt{\eta^2 + \frac{\eta Q}{\mu r_0^2}}.
\]

(3)

Expanding all the quantities in powers of \( 1/\bar{l} \) as described in Ref. [8], one finally gets the second-and third-order corrections \( E_2 \), and \( E_3 \) of the energy \( E_{n\ell} \) as:

\[
E_2 = \frac{Q\alpha(1)}{r_0^2 \left( 1 + \frac{E_0 - V(r_0)}{\eta} \right)},
\]

(4)

\[
E_3 = \frac{Q\alpha(2)}{r_0^2 \left( 1 + \frac{E_0 - V(r_0)}{\eta} \right)},
\]

(5)

where \( \alpha(1) \) and \( \alpha(2) \) appearing as a correction to the leading order of the energy expression are given in the appendix of Ref.[11].
Collecting these terms, and carrying out the mathematics immediately gives an expression for the energy eigenvalues, that is

$$E_{n\ell} = E_0 + \frac{\alpha_{(1)}}{r_0^2 \left( 1 + \frac{E_0 - V(r_0)}{\eta} \right)} + \frac{\alpha_{(2)}}{r_0^2 \left( 1 + \frac{E_0 - V(r_0)}{\eta} \right)} \bar{l} + O \left[ \frac{1}{r^2} \right],$$

(6)

where $\bar{l} = l - \beta$, $\beta$ is chosen so that the next contribution to the leading term in the energy eigenvalue series to vanish, i.e., $E_1 = 0$, which implies that (see Ref.8)

$$\beta = -1/2 - \mu(n + 1/2)\omega,$$

(7)

with

$$\omega = \frac{1}{\mu} \left[ 3 + r_0 V''(r_0)/V'(r_0) - \mu r_0^4 V'(r_0)^2/(Q\eta) \right]^{1/2},$$

(8)

and $Q$ satisfies

$$Q = \frac{\mu}{2\eta} \left[ r_0^2 V'(r_0) \right]^2 (1 + \xi),$$

(9)

with

$$\xi = \sqrt{1 + [2\eta/r_0 V'(r_0)]^2}.$$

(10)

On the other hand, $r_0$ is chosen to minimize $E_0$, such that,

$$\frac{dE_0}{dr_0} = 0 \quad \text{and} \quad \frac{d^2E_0}{dr_0^2} > 0,$$

(11)

and then one can get

$$1 + 2\ell + \mu(2n + 1)\omega = r_0^2 V'(r_0) \left( \frac{2\mu}{\eta} + \frac{2\xi\mu}{\eta} \right)^{1/2},$$

(12)
which is an explicit equation in $r_0$. It is convenient to summarize the above procedure in the following steps: (a) Calculate $r_0$ from Eq.(12) and substitute it in Eq.(9) to find $Q$ (b) Substitute $Q$ in Eq.(3) to obtain $E_0$. (c) Finally, one can obtain $E_0$ and then calculate $E_{nl}$ from Eq.(6). However, one is not always able to calculate $r_0$ in terms of the potential coupling constants since the analytical expressions become algebraically complicated, although straightforward. Therefore, one has to appeal to numerical computations to find $r_0$, and hence $E_0$.

3 Application, Results and Discussion

The dilaton $\phi$ is a scalar field predicted by superstring theory [12]. The mechanism and the form of the dilaton potential are unknown, although it is believed that they could be related to nonperturbative sector of the theory. Recently it was observed in [5] (see also [6,7]) that a string inspired coupling of a dilaton $\phi$ to 4d $SU(N_c)$ gauge fields $A_\mu = T^a A^a_\mu$, with $T^a$ the $(N_c^2 - 1)$ $SU(N_c)$ generators, yields a phenomenologically interesting potential $V(r)$ for the quark-antiquark interactions. Dick interquark potential was obtained as follow: First start from the following effective field theory with the dilaton-gluon coupling $G(\phi)$ and the dilaton potential $W(\phi)$:

$$L(\phi, A) = -\frac{1}{4G(\phi)} F^a_{\mu\nu}F_{a\mu\nu} - \frac{1}{2}(\partial_\mu \phi)^2 + W(\phi) + J^a_\mu A^\mu_a$$

(13)
then construct $G(\phi)$ under the requirement that the Coulomb problem still possesses analytical solutions. The coupling $G(\phi)$ and the potential $W(\phi)$ that emerged are:

$$G(\phi) = \text{const.} + \frac{f^2}{\phi^2}, \quad W(\phi) = \frac{1}{2}m^2\phi^2$$  \hspace{1cm} (14)

where $f$ is a scale parameter characterizing the strength of the scalar-gluon coupling and $m$ represents the dilaton mass.

Next, consider the equations of motion of the fields $A_\mu$ and $\phi$ and solve them for static point like color source described by the current density $J_\mu^a = \rho_\alpha \eta^{\mu\alpha}$. After some straightforward algebra, Dick shows that the interquark potential $V_D(r)$ is given by (up to a color factor),

$$V_D(r) = -\frac{\alpha_s}{r} + g f \sqrt{\frac{N_c}{2(N_c - 1)}} \ln[\exp(2mr) - 1]$$  \hspace{1cm} (15)

Eq.(15) is remarkable since at large values of $r$ it leads to a linear confining potential $V_D(r) \sim 2gf m \sqrt{\frac{N_c}{2(N_c - 1)}} r$. This derivation provides a challenge to monopole condensations as a new quark confinement scenario. Therefore, it is well justified to dedicate more efforts to the investigation of this effective coupling function $G(\phi)$ and to the phenomenological application of Dick potential $V_D(r)$. The method of obtaining results from the theory requires us to choose several numerical inputs, examination of the previous section shows that there are five parameters to be calculated, namely, $m_c$, $m_b$, $m$, $f$ and $\alpha_s$. A few comments about the choice of the parameters are in order. The numerical values of the charmed-quark mass $m_c = 1.89$ GeV and the bottom-quark mass $m_b = 5.19$ GeV are chosen so that, the QCD coupling constant $\alpha_s(\mu)$ at any renormalization scale can be
calculated from the world average experimental value $\alpha_s(m_z) = 0.117$ via [13]

$$\alpha_s(\mu) = \frac{\alpha_s(m_z)}{1 - (11 - \frac{2}{3} n_f)\alpha_s(m_z)/2\pi\ln(m_z/\mu)},$$  \hspace{1cm} (16)

then one has,

$$\alpha_s(m_c) = 0.31, \quad \alpha_s(m_b) = 0.2.$$  \hspace{1cm} (17)

For the $b\bar{c}$ quarkonia, we obtain $\alpha_s(4\mu_{bc}) = 0.22$, where $\mu_{bc}$ is the reduced mass [14]:

$$\mu_{bc} = \frac{m_b m_c}{m_b + m_c},$$  \hspace{1cm} (18)

On the other hand, the potential parameters $m$ and $f$ are considered free in our analysis and are obtained by fitting the spin-averaged $c\bar{c}$, and $b\bar{b}$ mesons. An excellent fit with the available experimental data can be seen to emerge when the following values are assigned

$$m = 56.9 \text{ MeV}, \quad gf \sqrt{\frac{N_c}{2(N_c - 1)}} = 430 \text{ MeV}.$$  \hspace{1cm} (19)

The results of our calculation for the spin-averaged energy levels of interest are given in Tables (1,2). In all cases, where comparison with experiment is possible, agreement is very good. We also present, in Table 3, the results for the $b\bar{c}$ quarkonia. Our estimate for the $B_c$ mass, the lowest pseudoscalar S-state of the spectra, is compatible with the experimental value reported in [15]. As to the higher states masses, they compare favorably with other predictions based on QCD sum-rules [16] or other potential models [17]. In conclusion, to the best of our knowledge, this is the first time that Dick interquark potential of Eq.(15), is tested and used successfully to fit the spin-averaged $c\bar{c}$, $b\bar{b}$, and $b\bar{c}$ systems. This potential will certainly open a new window in the potential model applications,
essentially those with a Coulomb plus a confining potential. On the other hand, since a unique theory/scenario for the dilaton mass is still lacking [18], our estimate for the mass of the dilaton $m = 56.9 \, MeV$, resulting from a fit to existing experimental data of heavy quarkonium, lies in the range given in [19]. As suggested by the authors of Ref.[20], the possibility to identify the dilaton particle to a fundamental scalar, invisible to present day experiment, should not be ruled out.
Table 1: Calculated spin-averaged data (in units of GeV) $M_{n\ell}$ of charmonium $c\bar{c}$ energy levels in the semi-relativistic wave equation using Dick potential

| State, $n\ell$ | $M_{n\ell}$, SLET | $M_{n\ell}$, Exp. | State, $n\ell$ | $M_{n\ell}$, SLET | $M_{n\ell}$, Exp. |
|----------------|-------------------|------------------|----------------|-------------------|------------------|
| 1S             | 3.073             | 3.068            | 1P             | 3.548             | 3.525            |
| 2S             | 3.662             | 3.663            | 2P             | 3.871             | -                |
| 3S             | 4.027             | 4.028            | 1D             | 3.787             | 3.788            |

Table 2: Calculated spin-averaged data (in units of GeV) $M_{n\ell}$ of bottomonium $b\bar{b}$ energy levels in the semi-relativistic wave equation using Dick potential

| State, $n\ell$ | $M_{n\ell}$, SLET | $M_{n\ell}$, Exp. | State, $n\ell$ | $M_{n\ell}$, SLET | $M_{n\ell}$, Exp. |
|----------------|-------------------|------------------|----------------|-------------------|------------------|
| 1S             | 9.450             | 9.446            | 1P             | 9.903             | 9.900            |
| 2S             | 10.014            | 10.013           | 2P             | 10.206            | 10.260           |
| 3S             | 10.292            | 10.348           | 1D             | 10.129            | -                |

Table 3: Calculated spin-averaged data (in units of GeV) $M_{n\ell}$ of $b\bar{c}$ energy levels in the semi-relativistic wave equation using Dick potential

| State, $n\ell$ | $M_{n\ell}$, SLET | $M_{n\ell}$, Exp. | State, $n\ell$ | $M_{n\ell}$, SLET | $M_{n\ell}$, Exp. |
|----------------|-------------------|------------------|----------------|-------------------|------------------|
| 1S             | 6.322             | 6.40 ±0.39±0.19  | 1P             | 6.767             | -                |
| 2S             | 6.876             | -                | 2P             | 7.072             | -                |
| 3S             | 7.161             | -                | 1D             | 6.994             | -                |
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