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An Experimental Study of Pressure Drop Characteristics and Flow Resistance Coefficient in a Fluidized Bed for Coal Particle Fluidization

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Abstract: Liquid–solid fluidized beds have a wide range of applications in metallurgical processing, mineral processing, extraction, and wastewater treatment. Great interest on their flow stability and heterogeneous fluidization behaviors has been aroused in research. In this study, various fluidization experiments were performed by adjusting the operating conditions of particle size, particle density, and liquid superficial velocity. For each case, the steady state of liquid–solid fluidization was obtained, and the bed expansion height and pressure drop characteristics were analyzed. The time evolution of pressure drop at different bed heights can truly reflect the liquid–solid heterogeneous fluidization behaviors that are determined by operating conditions. With the increase in superficial liquid velocity, three typical fluidization stages were observed. Accordingly, the flow resistance coefficient was obtained based on the experimental data of bed expansion height and pressure drop. The flow resistance coefficient experiences a decrease with the increase in the modified particle Reynolds number and densimetric Froude number.

Keywords: pressure drop; liquid–solid; fluidized bed; flow resistance coefficient; fluidization experiments

1. Introduction

Liquid–solid fluidized beds have been widely used in metallurgical processing, mineral processing, extraction, and wastewater treatment [1–9] for their excellent fluid–solid coupling as well as heat and mass transfer performance. Heterogeneous fluidization behaviors have recently aroused great interest for the purpose of operation stability and optimized operating conditions [10,11]. The transition from the homogeneous to heterogeneous regime occurs at some critical operating conditions. The intensity of turbulence is much higher in the heterogeneous regime, resulting in higher rates of heat, mass, and momentum transfer and better mixing. Such conditions are preferable in many applications. However, too much turbulence may be detrimental to the performance in operations such as flotation, causing higher rates of detachment of particles from the bubble surface. Therefore, an insight into the heterogeneous flow structures of liquid–solid fluidized systems is critical.

Generally, liquid–solid fluidization systems are homogeneous when compared with gas–solid systems. As a result, liquid–solid fluidization has received little attention despite its wide applications. Liquid–solid fluidized beds have unique characteristics, such as low solid–fluid density ratio, and thus, homogeneous fluidization phenomena have been frequently observed [12,13]. For small and light particles, particle entrainment occurs easily in the fluidized state. Severe heterogeneous liquid–solid fluidization phenomena (i.e.,
particle clustering, bubbling, and velocity fluctuations) occur only under some extreme conditions [14–22].

In the past few decades, various experiments and numerical simulations have been carried out to study liquid–solid flow characteristics. The earliest research on the liquid–solid heterogeneous fluidization phenomenon can be traced back to 1948 when Wilhelm and Kwauck performed the fluidization of lead particles in water [23]. Later, Di Felice [13] found that solid–fluid density ratio, particle morphology, particle size, and size distribution were responsible for heterogeneity. In recent years, experimental techniques have improved greatly for visualizing multiphase flow fields. Razzak et al. [24–26] measured the liquid phase holdup and velocity distribution using resistance tomography, whereas the solid phase holdup was measured using a pressure sensor. The solid phase holdup decreases with the increase in liquid superficial velocity. Similarly, Zbib et al. [27,28] measured the particle phase holdup by electrical resistance tomography in a liquid–solid fluidized bed. Reddy et al. [29] performed flow visualization experiments using particle image velocimetry. They found that a liquid–solid fluidized bed was a homogenous system for particle Reynolds number ($Re_p$) in the range of 51–759.

Additionally, computational fluid dynamics (CFD) simulations have great advantages in investigating the hydrodynamic characteristics of liquid–solid fluidized beds, particularly for the purpose of scaling up. The accuracy of a CFD model can be validated by experimental data. An entire flow field can be captured using a CFD simulation, including turbulence intensity, particle concentration, particle velocity, and particle clusters, which are beneficial to clarify the fluidization mechanism. Some studies consider the local heterogeneity of liquid–solid flow in CFB when calculating the drag force. Liu et al. [30] showed that compared with experimental data, the multi-scale resistance coefficient model predicted better particle concentration distribution by taking into account the effect of mesoscale cluster structure. In addition, Xie et al. proposed an effective resistance closure method for fluidization of coarse coal particles in transition liquid–solid two-phase flow by clarifying the relationship between resistance coefficient and Reynolds number, Froude number and Stokes number. The output prediction is in good agreement with the experimental data [11].

The effects of operating conditions, such as liquid superficial velocity, particle properties, and fluidized bed structures on fluidization behaviors have been widely investigated in recent years [31–35]. It is also generally accepted that liquid–particle and particle–particle interactions play an important role in determining liquid–solid fluidization characteristics [36–40]. However, there is still limited research on heterogeneous fluidization phenomena [41,42]. Moreover, the flow stability and flow resistance in a liquid–solid fluidized bed have received limited attention [28,43,44].

This study aims to investigate the heterogeneous fluidization and flow resistance in a square liquid–solid fluidized bed where coarse coal particles and water are the solid and liquid phases, respectively. Various fluidization experiments were carried out by adjusting the operating conditions such as particle size, particle density, and liquid superficial velocity. The bed-expansion height and pressure drop under different conditions were measured and used to analyze the flow stability and resistance. According to the measured experimental data, relationships between the flow resistance coefficient and the modified particle Reynolds number ($Re_p$) and densimetric Froude number ($Fr_p$) are obtained. To the best of our knowledge, the experimental data of the bed expansion height and pressure drop obtained can provide insight into the fluidization mechanism of a liquid–solid fluidized bed.

2. Experimental Design and Procedures

The lab-grade fluidized bed unit shown in Figure 1 is constructed of clear plexiglass of 100 mm in length, 20 mm in width, and 500 mm in height. A rectangular distributor is arranged at the bottom of the fluidized bed. The distributor has an opening ratio of 40% and a pore size of 1 mm. The coal particle samples for the solid phase were purchased from Pingdingshan Coal Industry Group Co. Ltd., located in Henan, China. Raw coal was crushed by a laboratory jaw crusher and then sieved to produce two large size fractions
(0.7 ± 0.1 and 1.25 ± 0.25 mm). Two methods were employed to determine the particle size of each sample: classical sieve analysis and microscopic static image analysis. These were further subdivided according to different densities using the ZnCl₂ heavy liquid method. Five density fractions of coal samples (1500 ± 50 and 1700 ± 50 kg/m³) were obtained. As a result, four different coal particle samples were used in this study. The physical properties of the coal particles are given in Table 1.

![Figure 1. Schematic of the liquid–solid fluidized bed experimental setup.](image)

| Scheme | Particle Density/kg/m³ | Particle Size/mm | Condition/State |
|--------|------------------------|------------------|-----------------|
| 1#     | 1500 ± 50              | 0.7 ± 0.1        | Fluidized bed   |
| 2#     | 1700 ± 50              | 0.7 ± 0.1        | Initial bed height: 100 mm |
| 3#     | 1500 ± 50              | 1.25 ± 0.25      | Superficial velocity: 0.008–0.04 m/s |
| 4#     | 1700 ± 50              | 1.25 ± 0.25      |                 |

The fluidization experiment was carried out as follows. The prepared coal sample is initially placed on the distribution plate with an initial filling height of 100 mm. Water is delivered to the bottom of the fluidized bed by a peristaltic pump (WT600-3J) manufactured by Longer Precision Pump Co., Ltd. (Baoding, China). The flow error of the peristaltic pump is less than ±0.5%. The inlet speed at the bottom of the fluidized bed is controlled by the speed of the peristaltic pump. When the superficial velocity of the water exceeds the minimum fluidization velocity, the coal particles leave the distribution plate and are suspended in the fluidized bed. Then, a ruler is used to record the expansion height of the bed. When the particle fluidization reaches a steady state, the bed expansion height is measured three times, and the average value is taken as the final experimental measurement value.

Then, the bed expansion ratio can be calculated as follows:

\[ \chi = \frac{H - H_0}{H_0} \times 100\% \]
where \( \chi \) is the bed expansion ratio, \( H_0 \) is the initial bed height, and \( H \) is the bed height after fluidization.

Additionally, the pressure drop continues to fluctuate with flow time, even though the bed height is stable. In this study, four sections, namely, \( z = 50 \) mm, \( z = 100 \) mm, \( z = 150 \) mm, and \( z = 200 \) mm are selected, and the time evolutions of the pressure drop are recorded. The differential pressure gauge is composed of two pressure-measuring copper tubes, digital display pressure gauge (Xima AS510) manufactured by Dongguan Wanchuang Electronic Products Co., Ltd. (Dongguan, China), and rubber hose. The accuracy of the pressure gauge is \( \pm 3 \) Pa and the resolution is \( \pm 1 \) Pa. When measuring the pressure drop, the pressure drop at the end of two copper tubes is directly read by the digital differential pressure gauge. The total duration of pressure drop measurement is 30 s, and a data point is recorded every 0.5 s. Distilled water from the laboratory was employed as the fluidizing medium. The density and viscosity of water are 998.2 kg/m\(^3\) and 0.001 Pa, respectively. The experiments were performed at a temperature of 20 °C under atmospheric conditions.

In this study, 31 groups of fluidization experiments (see Figure 2) were performed based on the developed fluidized bed equipment. Figure 2 shows the bed expansion ratio at different superficial velocities for four different coal particles. Two different fluidization characteristics by particle size are observed. For small coal particles (0.7 ± 0.1 mm), the bed expansion ratio almost increases linearly with the superficial velocity. Generally, small particles achieve homogeneous fluidization easier, and the bed height increases uniformly and does not change abruptly. For large coal particles with sizes of 1.25 ± 0.25 mm, however, the linear relationship between bed expansion ratio and superficial velocity is only observed at low superficial velocity (<0.025 m/s). When the superficial velocity exceeds minimum fluidization velocity, the bed expansion ratio meets a significant increase, deviating from the linear growth trend. We guess that this trend may also be applicable to small particles if the superficial velocity is high enough. As shown in Figure 2, the growth trend of the bed expansion ratio is accelerated for small particles when the superficial velocity is higher than 0.02 m/s. Additionally, the bed expansion ratio decreases with the increase in particle density and particle size. When compared with particle density, the effect of particle size is more significant. When the superficial velocity is 0.0208 m/s, the bed expansion ratio decreases from 190% to 37.5% as the particle size increases from 0.7 ± 0.1 to 1.25 ± 0.25 mm. For both particle sizes, particle density has a limited effect on the bed expansion ratio at low superficial velocity. The particle density effect can only be highlighted at high superficial velocity. It should be noted that the local solid holdup is still fluctuating even though the whole bed height is stable. This fluctuation is reflected by the pressure drop of the cross section and will be analyzed in Section 3.1.
3. Results and Discussion

3.1. Pressure Drop Characteristics

In this section, the pressure drop was measured at different bed heights when the steady state of liquid–solid fluidization was obtained, for example, when the average bed expansion height remained constant. Generally, the pressure drop continues to fluctuate even though the bed height is stable. Therefore, the pressure drop can be written as

$$\Delta p = \Delta p + \Delta p' \quad (2)$$

$$\Delta p = \frac{1}{t_1} \int_{0}^{t_1} \Delta p dt \quad (3)$$

Then, the fluctuation intensity of the pressure drop can be defined as:

$$I = \sqrt{\frac{\sum_{i=1}^{N} \Delta p'^2}{N}} / \Delta p \quad (4)$$

where $\Delta p'$ is the fluctuating pressure drop and $\Delta p$ is the time-averaged pressure drop.

Figure 3 shows the time evolution of the pressure drop at different bed heights and superficial velocities when the particle size is 0.7 ± 0.1 mm and particle density is 1500 kg/m$^3$. The fluctuation of the pressure drop is very small at low flow rates (see Figure 3A). The calculated fluctuation intensities of the pressure drops are 0.097, 0.049, 0.029, and 0.031, respectively, indicating that the particle fluidization is homogeneous. With the increase in superficial velocity, however, the heterogeneous fluidization phenomenon was first observed at the top of the bed, accompanied by significant fluctuations in the pressure drop (see Figure 3B,C). The possible reason is liquid void motion. Nijssen and Kramer et al. also observed the formation of voids through experiments and numerical simulation in the study of liquid–solid heterogeneous fluidization behavior [10]. Small liquid voids are generated at the bottom of the fluidized bed for low superficial liquid velocity. These liquid voids will grow in the process of rising up, and finally break up at the top of the fluidized bed. The liquid void motion and breakage lead to changes in the local solid phase holdup, which is reflected by fluctuations in the pressure drop. When the
superficial liquid velocity further increases to 0.0188 m/s (see Figure 3D), the fluctuation in the pressure drop mainly appears at the bottom of the fluidized bed. Of course, this is exactly the opposite phenomenon when compared with that at low superficial liquid velocity. High superficial liquid velocity is often accompanied by high bed expansion height. However, high superficial liquid velocity may only influence particle fluidization at the bottom of the fluidized bed, and homogeneous fluidization is observed at the top of the fluidized bed.

![Figure 3](image)

**Figure 3.** The time evolution of the pressure drop at different bed heights and superficial velocities when the particle size is $0.7 \pm 0.1$ mm and particle density is 1500 kg/m$^3$: (A) $v_s = 0.0125$ m/s; (B) $v_s = 0.0146$ m/s; (C) $v_s = 0.0167$ m/s; (D) $v_s = 0.0188$ m/s.

When the particle density increases to 1700 kg/m$^3$, the fluidization becomes more stable at the same superficial liquid velocity (see Figure 4). The fluctuation intensity of the pressure drop is less than 0.065 when the superficial liquid velocity increases from 0.0125 m/s to 0.0167 m/s. That is to say, heavy particle fluidization is insensitive to the superficial liquid velocity. In this study, we observed a large fluctuation in the pressure drop only when the superficial liquid velocity increases to 0.0208 m/s. The pressure drop shows a periodic trend throughout the fluidized bed, indicating that liquid–solid fluidization reaches a steady state.
Figure 4. The time evolution of the pressure drop at different bed heights and superficial velocities when the particle size is 0.7 ± 0.1 mm and particle density is 1700 kg/m$^3$: (A) $v_g = 0.0125$ m/s; (B) $v_g = 0.0167$ m/s; (C) $v_g = 0.0208$ m/s.

Figure 5 shows the time evolution of the pressure drop at different bed heights and superficial velocities when the particle size further increases to 1.25 ± 0.25 mm. In determining fluidization behaviors, gravity dominates for large and heavy particles, while drag force dominates for small and light particles. With the increase in particle size, the fluidized bed also becomes more stable at the same superficial liquid velocity (less than 0.025 m/s), although a significant decrease in bed expansion height is observed. Similarly, when the superficial liquid velocity increases to 0.0292 m/s, a very chaotic pressure drop trend is monitored, indicating that the local particle concentration varies greatly.

From Figures 3–5, it can be concluded that liquid–solid fluidization behavior is highly dependent on the superficial liquid velocity, particle size, and particle density. In most cases, the liquid–solid fluidized bed is in a homogeneous fluidization state. With the increase in particle size and particle density, the fluidized bed will become more stable. As the superficial liquid velocity increases, however, the homogeneous fluidization will change into heterogeneous fluidization. Particle fluidization behavior is controlled by the drag force, gravity, and the heterogeneous flow structure. For small and light particles, the drag force is much larger than gravity. As superficial liquid velocity continues to increase, heterogeneous flow structures, such as liquid voids, may evolve and determine the particle fluidization behaviors. For the large and heavy particles, gravity is greater than the drag force at low superficial liquid velocity. As a result, the particles are inclined to stay at the bottom of the fluidized bed. When the drag force increases to the same level as gravity, the liquid–solid flow is unstable and will produce oscillations. In the following, we will provide explanations by analyzing the relative magnitudes of the drag force, inertia force, and gravity.
Figure 5. The time evolution of the pressure drop at different bed heights and superficial velocities when the particle size is 1.25 ± 0.25 mm and particle density is 1500 kg/m$^3$: (A) $\nu_s = 0.0167$ m/s; (B) $\nu_s = 0.0208$ m/s; (C) $\nu_s = 0.025$ m/s; (D) $\nu_s = 0.0292$ m/s.

3.2. Drag Force Coefficient

In fluidized beds, the fluid will be accompanied by a pressure drop when passing through a granular bed due to frictional loss and inertia. Even though the fluidization reaches steady state, for example, when the bed height is nearly constant, the pressure drop may still fluctuate because of local changes in the bed voidage. As recommended by Ergun [45], the pressure drop in a fluidized bed is defined as:

$$\Delta p = f \frac{H v_0^2 \rho_l (1 - \epsilon)}{d_s \epsilon^3} \quad (5)$$

where $\Delta p$, $f$, $v_0$, $\rho_l$, $H$, $d_s$, and $\epsilon$ are the pressure drop, drag coefficient, flow velocity, fluid density, bed height, particle size, and bed voidage, respectively. For a fluidized bed, the pressure drop can be calculated as follows [46–48]:

$$\Delta p = (\rho_s - \rho_l) g (1 - \epsilon) H \quad (6)$$

As shown in Equation (6), the pressure drop is determined by the bed voidage and bed expansion height. Generally, the bed voidage is also one of the most important evaluation indexes for the fluidization performance. The Richardson–Zaki relation has been widely used to calculate the bed voidage [48]:

$$\epsilon^n = \frac{v_s}{v_l} \quad (7)$$
where \( v_s \) is the superficial velocity, which can be determined by the flow rate and pipe diameter; \( v_t \) is the terminal velocity of an isolated particle in an unbounded fluid. Stokes has derived the famous expression for the settling velocity of a sphere, now known as Stokes’ settling velocity \( v_{ts} \).

\[
v_{ts} = \frac{g}{18} (\rho_s - \rho_l) d_s^2 \tag{8}
\]

Phillip P. Brown and Desmond F. Lawler [49] obtained a new settlement velocity expression based on 480 data points, which can predict settlement velocity very accurately when the terminal Reynolds number is less than 4000.

\[
v_t = \frac{v_{ts}}{3.13 + Re_t^{0.682} + 0.409 Re_t^{1/3} + 0.00985 Re_t^{2/3}} \tag{9}
\]

In Equation (7), \( n \) is the Richardson–Zaki coefficient, which is highly dependent on the Reynolds number. A number of researchers have modified this correlation. In the literature, a collection of equations is given to estimate the Richardson–Zaki index \( n \) of which the most popular are presented in Table 2.

**Table 2. Richardson–Zaki index equations from literature.**

| References                  | Equation                              |
|-----------------------------|---------------------------------------|
| Classical Richardson–Zaki   | \( n = \begin{cases} 
        4.65 & \text{Re}_t < 0.2 \\
        4.4Re_t^{-0.03} & 0.2 \leq \text{Re}_t < 1 \\
        4.4Re_t^{-0.1} & 1 \leq \text{Re}_t < 500 \\
        2.4 & \text{Re}_t \geq 500 
\end{cases} \) | (10) |
| General expression          | \( n = c_1 Re_t^2 \)                  | (11) |
| Garside and Al-Dibouni      | \( \frac{m-n}{n-n} = \alpha Re_t^\beta \) | (12) |
| Khan and Richardson         | \( \frac{m-n}{n-n} = \alpha Ar^\beta \) | (13) |

In Table 2, the particle Reynolds number \( \text{Re}_t \) under terminal settling conditions and the Archimedes number \( Ar \) can be determined as follows:

\[
\text{Re}_t = \frac{d_s v_t \rho_l}{\mu_l} \tag{14}
\]

\[
Ar = \frac{gd_s^3 \rho_l (\rho_s - \rho_l)}{\mu_l^2} \tag{15}
\]

Kramer et al. [54] obtained a new expression for \( n \) based on more ideal fluidization experiments:

\[
\frac{4.8 - n}{n - 2.4} = 0.043 Re_t^{0.75} \tag{16}
\]

\[
\frac{4.8 - n}{n - 2.4} = 0.015 Ar^{0.5} \tag{17}
\]

which improves the accuracy deviation of the Richardson–Zakie equation from 15% to 3%.

Figure 6 shows the relationship between bed voidage and superficial velocity for the four different coal particle samples. With the increase in superficial velocity, the bed voidage significantly increases, especially for the small and light particles. The maximum bed voidage is 0.74 when the particle size is 0.7 ± 0.1 mm, the particle density is 1500 kg/m\(^3\), and the superficial velocity is 0.0208 m/s. For the large and heavy particles, the bed voidage is 0.65 even though the superficial velocity increases to 0.0375 m/s. As discussed above, the linear relationship between bed expansion ratio and superficial velocity was only observed at low superficial velocities. It can be seen in Figure 6 that there is a power function relationship between bed voidage and superficial velocity throughout the entire range of velocities. The goodness of fit \((R^2)\) is higher than 0.988. The reliable and straightforward
voidage prediction model obtained by Kramer et al. [55] in a fluidization system is also a power function relationship.

The dimensionless drag force coefficient can be determined as follows:

\[ f = \left( \frac{\rho_s - \rho_l}{\rho_l} \right) \frac{gd_s}{v_0^2} \epsilon^3 \]  

(18)

In addition to the modified particle Reynolds number (Re_s), the densimetric Froude number (Fr_p) is also commonly used in liquid–solid fluidization. These values are defined as follows:

\[ Re_s = \frac{\rho_l d_s v_0}{\mu_l} \frac{1}{1 - \epsilon} \]  

(19)

\[ Fr_p = \frac{v_s}{\sqrt{(\rho_s/\rho_l - 1)gd_s}} \]  

(20)

By adjusting particle size, particle density, and apparent velocity, 31 fluidization experimental datasets (see Figures 2 and 6) were obtained. Figure 7 shows the relationship between the drag force coefficient and the modified particle Reynolds number. The drag force coefficient decreases as the particle Reynolds number increases. In this study, the least squares method is used to fit the experimental data. As reported in the literature, the relationship between drag coefficient and Reynolds number satisfies a power function [45,56–59]. Therefore, a power function is considered as the target fitting function in this study. The following drag coefficient functional relationship is obtained:

\[ f = 44.56Re_s^{-0.674} \quad R^2 = 0.9909 \]  

(21)
As shown, the goodness of fit ($R^2$) is higher than 0.99. Additionally, our experimental data are compared with those reported by Xie et al., who investigated the drag coefficient correlation for coarse coal particle fluidization in a transitional flow regime [11]. Although the experimental conditions are different, the calculated dimensionless drag coefficients agree well with the reported data, thus validating the rationality of our experimental data. In addition, the regularity of our experimental data is better. Therefore, the proposed drag force coefficient correlation may have wider application prospects.

The densimetric Froude number is widely used to evaluate the fluidization characteristics of a liquid–solid fluidization system. As early as 1948, Wilhelm and Kwauk proposed that the two fluidization states, namely particulate fluidization and aggregative fluidization, can be distinguished according to the Froude number [17]. The Froude number expresses the influence of gravity on flow; its physical meaning is the ratio of inertial force to gravity. A plot of the drag force coefficient versus Froude number is shown in Figure 8. In this study, the Froude number ranges from 0.12 to 0.43. This means that the fluidization of coal particles does not belong to particulate fluidization or aggregative fluidization but is in the transition stage between the two. Generally, the drag force coefficient decreases with the increase in Froude number. The relationship between drag coefficient and Froude number also satisfies a power function. However, these functional relationships are related to particle density and particle size.

As can be seen from Equations (18) and (20), both particle size and density have an impact on drag force coefficient and Froude number. In the case of a constant apparent flow rate, the drag force coefficient increases with the increase in particle size and density, while Froude number decreases with the increase of particle size and density.

In Figure 8, the experimental comparison of 1# and 2# shows that the particle sizes are both 0.7 mm, and the particle density increases from 1500 to 1700 m$^3$/kg. Compared with curve 1#, curve 2# deviates to the lower left, indicating that the influence of particle density on Froude number is greater than the drag force coefficient. The comparison of test 1# and 3# shows that the particle density is the same, and the particle size increases from 0.7 mm to 1.25 mm. Compared with curve 1#, curve 3# moves to the lower left with a larger amplitude, indicating that the influence of particle size on Froude number is greater than the drag force coefficient.
Figure 8. The relationship between drag force coefficient and modified particle Froude number.

4. Conclusions

In this study, the heterogeneous fluidization and flow resistance in a square liquid–
solid fluidized bed were investigated. Various fluidization experiments were performed by
adjusting the operating conditions of particle size, particle density, and liquid superficial
velocity. For each case, the bed expansion height and the pressure drop were measured
and used to analyze the flow stability and flow resistance.

In most cases, the liquid–solid fluidized bed is in a homogeneous fluidization state. With
the increase in particle size and particle density, the fluidized bed will become more stable. As
the superficial liquid velocity increases, however, the homogeneous fluidization will change
to heterogeneous fluidization, which is reflected by fluctuations in the pressure drop.

The relationships between the flow resistance coefficient and the Reynolds number
and Froude number both satisfy a power function. The relationship between the drag
coefficient and Froude number is dependent on particle density and particle size, while
the relationship between the drag coefficient and Reynolds number is independent of
particle density and particle size. The obtained experimental data provide insight into the
fluidization characteristics of liquid–solid fluidized beds. In future work, attention will
focus on the fluidization mechanism of liquid–solid systems.

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Nomenclature

- **Ar**: Archimedes number, [-]
- **d**<sub>s</sub>: Diameter of particles, [mm]
- **f**: Drag force coefficient, [-]
- **Fr**: Froude number, [-]
- **Fr<sub>p</sub>**: Densimetric or particle Froude number, [-]
- **g**: Gravitational acceleration, [m s<sup>−2</sup>]
- **H**: Bed expansion height, [mm]
- **H<sub>0</sub>**: Initial bed expansion height, [mm]
- **I**: Fluctuation intensity of the pressure drop, [-]
- **n**: Richardson–Zaki coefficient, [-]
- **p**: Pressure, [Pa]
- **Re<sub>t</sub>**: Particle Reynolds number, [-]
- **Re<sub>s</sub>**: Modified particle Reynolds number, [-]
- **t**: Time, [s]
- **ν<sub>s</sub>**: Superficial velocity, [m s<sup>−1</sup>]
- **ν<sub>t</sub>**: Terminal velocity, [m s<sup>−1</sup>]
- **ν<sub>ts</sub>**: Stokes’ settling velocity, [m s<sup>−1</sup>]
- **ρ<sub>s</sub>**: Solid density, [kg m<sup>−3</sup>]
- **ρ<sub>l</sub>**: Liquid density, [kg m<sup>−3</sup>]
- **µ**: Viscosity, [Pa s]
- **ε**: Voidage, [-]
- **χ**: Bed expansion ratio, [-]

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