Positive Correlations in Tunneling through coupled Quantum Dots

G. Kießlich, H. Sprekeler, A. Wacker, and E. Schöll
Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany
E-mail: kieslich@physik.tu-berlin.de

Abstract.
Due to the Fermi-Dirac statistics of electrons the temporal correlations of tunneling events in a double barrier setup are typically negative. Here, we investigate the shot noise behavior of a system of two capacitively coupled quantum dot states by means of a Master equation model. In an asymmetric setup positive correlations in the tunneling current can arise due to the bunching of tunneling events. The underlying mechanism will be discussed in detail in terms of the current-current correlation function and the frequency-dependent Fano factor.

PACS numbers: 72.70+m,73.23.Hk,73.40.Gk,73.63.Kv,74.40.+k

1. Introduction
Resonant tunneling of electrons through bound states in mesoscopic double barrier devices still attracts wide research activities. Not only the current flow is of interest. Due to the granularity of the electron charge the tunneling current is a discrete stochastic process which leads to shot noise in the current signal [1]. The spectral power density gives information about temporal correlations which are not available by conductance measurements alone. In the pioneering theoretical work of L. Y. Chen et al. [2] a suppression of the zero-frequency spectral power density $S_0$ with respect to the uncorrelated value $S_P = 2eI$ (Poissonian noise, $e > 0$ is the elementary charge, $I$ is the average current) in tunneling through double barriers was obtained. This refers to negative correlations in the tunneling current due to Pauli’s exclusion principle: if the bound state becomes occupied by tunneling of an electron, then a subsequent electron “has to wait” until the state becomes empty. Hence, negative correlations are a fingerprint of the fermionic statistics of electrons. Contrarily, boson statistics lead to positive correlations, e.g. in the intensity-intensity correlation function of emission of photons from a black radiation source due to temporal bunching of emission events (3 and references therein).

For tunneling through quantum dots (QDs) Coulomb interaction generates an additional source of correlations. It was shown in Ref. [4, 5] that for an asymmetric setup of the tunneling rates in a system of two capacitively coupled QD states the current-voltage characteristic exhibits negative differential conductance. In this regime positive correlations in the tunneling current are present (super-Poissonian shot noise). Here, we give a detailed examination of this effect and show that its origin is related
to bunching of tunneling events due to Coulomb blocking. Positive correlations in tunneling current were also investigated in a double barrier resonant tunneling diode [3], in coupled metallic QDs [4], and in a single-electron transistor setup [5].

2. Model

We consider two QD single-particle states with energies $E_{1/2}$ which are coupled to emitter/collector contacts by tunneling rates $\Gamma_{E/C}^i$ ($i = 1, 2$), respectively. The emitter/collector contact is assumed to be in local equilibrium and the occupation is given by the Fermi functions $f_{E/C}(E)$ and chemical potentials $\mu_{E/C}$, respectively. An applied bias voltage $eV = \mu_E - \mu_C$ drives the QD system out of equilibrium. The tunneling energy $E$ is given by the difference of energies before and after a tunneling process considering the charging energy $U$ which is present if both QD states are occupied. The time evolution of the occupation probabilities $P_i(0)$ is given by the difference of energies before and after a tunneling process considering the charging energy $U$ which is present if both QD states are occupied. The time evolution of the occupation probabilities $P_i$ for the Fock states $\nu = (n_1, n_2)$ ($n_i \in \{0, 1\}$, $i = 1, 2$) is described by the Master equation $\dot{P}_i = M_i^P$. The $4 \times 4$ matrix $M$ consists of the transition rates for tunneling into or out of the QD system which contain the occupation of the contacts and the tunneling rates (for details see [5]).

The stationary current is given by $I = \sum_\nu \left( \frac{1}{\Delta \nu} \rho(\nu) \right)_\nu$ with the stationary occupation probability given by $M^P \rho = 0$ and the operator for the current at the emitter barrier $\delta \nu$, respectively. The current-current correlation function at the emitter barrier for $t \geq 0$ is $C_{EE}(t) \equiv \langle I_E(t) I_E(0) \rangle - I^2 = \sum_\nu \left( \frac{1}{\Delta \nu} T(t) \frac{1}{\Delta \nu} \rho(\nu) \right)_\nu + \delta(t)eI - I^2$ with the time translation operator $T(t) \equiv \exp M t$ so that $P(t) = T(t)P(0)$ holds [3]. Then, the spectral power density reads (Wiener-Khintchine theorem) $S_{EE}(\omega) = 2 \int_0^\infty dt C_{EE}(t) \cos \omega t$. As a measure for correlations at the emitter barrier the Fano factor is defined as $\alpha(\omega) \equiv \frac{S_{EE}(\omega)}{\langle I_E^2 \rangle}$ which is unity for uncorrelated tunneling, $< 1$ for negative correlations, and $> 1$ for positive correlations in the tunneling current (super-Poissonian shot noise). Note that $S_{EE}(0) = S_{CC}(0) = S_{EC}(0) = S_{CE}(0)$, but $S(\omega) \neq S_{EE}(\omega)$ for $\omega \neq 0$.

3. Results and Discussion

We consider the case where both energies of the QD single-particle states are equal: $E_1 = E_2$, the tunneling rates to the emitter are equal: $\Gamma_{E}^1 = \Gamma_{E}^2 \equiv \Gamma_E$, and a bias voltage $V$ is applied such that both single-particle states $(1, 0)$ and $(0, 1)$ can be occupied but the occupancy of the doubly-occupied state $(1, 1)$ is energetically forbidden. Then, the zero-frequency Fano factor is derived analytically to be

$$\alpha(0) = 1 + \frac{2 \left[ \gamma_1^2 + \gamma_2^2 - \gamma_1 \gamma_2 (2 + \gamma_1 + \gamma_2) \right]}{\gamma_1 + \gamma_2 + \gamma_1 \gamma_2}$$

with the ratios of collector and emitter tunneling rates $\gamma_i \equiv \Gamma_i^C/\Gamma_E$ ($i = 1, 2$). The dependence of the Fano factor on $\gamma_1$ and $\gamma_2$ is depicted in Fig.1a. The Fano factor is symmetric with respect to an interchange of $\gamma_1$ and $\gamma_2$. Two different regions corresponding to $\alpha \leq 1$ and $\alpha > 1$ are labeled by $N$ and $P$, respectively. The region $N$ indicates negative correlations in the tunneling current for $\gamma_i \geq 1$ ($i = 1, 2$). In this regime, mutual Coulomb blocking of the single-particle states $(1, 0)$ and $(0, 1)$
Positive Correlations in Tunneling through coupled Quantum Dots

is negligible since the levels are mostly empty in time average due to the stronger collector coupling. If either one ratio or both ratios become smaller than unity the tunneling is positively correlated (region $P$ in Fig. 1a) except for $\gamma_1 \approx \gamma_2$ where $\alpha$ approaches unity. Hence, the conditions for positive correlations are: $\gamma_1 < 1$ or $\gamma_2 < 1$, and $\gamma_1 \neq \gamma_2$. To illustrate the effect of positive correlations we consider a realisation of time-dependent currents $I_1(t)$ and $I_2(t)$ through the levels 1 and 2, respectively, in Fig. 1b. There, it is assumed that $\gamma_1 \ll \gamma_2 = 1$. If an electron jumps from the emitter into level 1 (black peak) this level becomes occupied. Until the level becomes empty by tunneling of the electron out in the collector (grey peak) no electrons can jump in level 2 (Coulomb blocking). Only if level 1 is empty, a current $I_2$ is flowing. Hence, a bunching of tunneling events in current $I_2(t)$ occurs.

In Fig. 2 the frequency-dependent Fano factor $\alpha(\omega)$ and in the insets of Fig. 2 the current-current correlation function $C_{EE}(t)$ at the emitter barrier are shown. The dashed curve $\alpha(\omega)$ represents the case $\gamma_1 = \gamma_2 = 1$. It exhibits a minimum at $\omega = 0$ and approaches unity for $\omega \to \pm \infty$. The corresponding correlation function in the left inset is negative for all times, i.e. only negative correlations are present (region $N$ in Fig.1a). By lowering $\gamma_1$ below one a maximum at $\omega = 0$ and two minima symmetric to $\omega = 0$ arise (dotted curve: $\gamma_1 = 0.4$, $\gamma_2 = 1$; full curve: $\gamma_1 = 0.2$, $\gamma_2 = 1$). The correlation function (full curve in the right inset of Fig. 2) belonging to the full curve $\alpha(\omega)$ ($\alpha > 1$, $P$-region in Fig. 1a) has now two contributions with different signs: positive (dotted curve) and negative (dashed curve). The positive part of the correlation function is due to the bunching of tunneling events as discussed with respect to Fig. 1b and the negative part is due to anti-bunching caused by Pauli’s exclusion principle which is still present in the bunches of tunneling through the current carrying level. Therefore, the frequency-dependent Fano factor in Fig. 2 consists of a sum of two Lorentzians with either positive or negative sign and different FWHM corresponding to the two time scales related to the respective collector tunneling rates.
Positive Correlations in Tunneling through coupled Quantum Dots

4. Conclusions

We discussed the effect of positive correlations in the tunneling current through two single-particle states in a regime where mutual Coulomb blocking leads to bunching of tunneling events. We showed that the current-current correlation function at one barrier consists of the sum of a negative and a positive term for an asymmetric choice of the tunneling rates of both QD states. Then, the corresponding spectral power density is a sum of two Lorentzians with different FWHM and different signs which gives rise to a maximum in the zero-frequency Fano factor and which can become larger than one (super-Poissonian shot noise).

Acknowledgments

This work was supported by Deutsche Forschungsgemeinschaft in the framework of SFB 296.

[1] Y. M. Blanter and M. Büttiker, Phys. Rep. 336, 1 (2000).
[2] L. Y. Chen and C. S. Ting, Phys. Rev. B 43, 4534 (1991).
[3] M. Büttiker, in Quantum Noise in Mesoscopic Physics, edited by Y. Nazarov (Kluwer Academic Publishers, Dordrecht, Boston, London, 2003).
[4] G. Kießlich, A. Wacker, E. Schöll, A. Nauen, F. Hohls, and R. J. Haug, phys. status solidi (c) 0, 1293 (2003).
[5] G. Kießlich, A. Wacker, and E. Schöll, cond-mat/0303025 (2003).
[6] G. Iannaccone, G. Lombardi, M. Macucci, and B. Pellegrini, Phys. Rev. Lett. 80, 1054 (1998).
[7] M. Gattobigio, G. Iannaccone, and M. Macucci, Phys. Rev. B 65, 115337 (2002).
[8] S. S. Safonov, A. K. Savekenko, D. A. Bagrets, O. N. Jouravlev, Y. V. Nazarov, E. H. Linfield, and D. A. Ritchie, cond-mat/0303027 (2003).
[9] S. Hershfield, J. H. Davies, P. Hyldgaard, C. J. Stanton, and J. W. Wilkins, Phys. Rev. B 47, 1967 (1993).