Abstract

The BUU transport model is applied to study strangeness and pion production in nucleus-nucleus collisions. Starting from proton induced reactions, we further investigate Si+Au, Au+Au and Pb+Pb collisions in the energy range between 2 and 40 A·GeV and compare with data and with other transport calculations. The $q\bar{q}$ annihilation, or resonance, channel simulated by the string model in meson-nucleon collisions at $\sqrt{s} > 2$ GeV is introduced. The importance of this channel for a good description of the proton-nucleus data on $K^+$ production is demonstrated. We, furthermore, show that the meson-meson collisions contribute significantly to the $K\bar{K}$ production in heavy-ion collisions above 5 A GeV and improve an agreement with data on the $K^+ / \pi^+$ ratio. Finally, we study the influence of the in-medium modifications of the FRITIOF model on the pion and kaon production.
1 Introduction

High-energy heavy-ion collisions offer a unique possibility to study nuclear matter at high densities and temperatures under laboratory conditions. The maximum compression is expected at the beam energy around 30 A·GeV. The most intriguing phenomenon which can happen in highly compressed nuclear matter is the transition to the quark-gluon plasma. Pioneering work at large baryon densities was done at the AGS in Brookhaven where the energy range up to 15 A·GeV was explored (see [1] and refs. therein). The future facility at GSI will provide beams from 2 A·GeV up to 40 A·GeV. One believes that an indirect signal for the quark-gluon plasma is the strangeness enhancement, which was first suggested by Rafelski and Müller [2]. The enhancement should then be seen in the most abundant strange particles, the kaons. At the AGS and the SPS energies the \( K^+/\pi^+ \) ratio was studied and, indeed, a maximum in the ratio was found at about 30 A·GeV [3]. Theoretical calculations with different transport codes — RQMD [4, 5], HSD [6] and UrQMD [6] — have recently been performed. These calculations were not able to reproduce the peak in the ratio, which was either at a wrong energy (RQMD, UrQMD) or not present at all (HSD). In the case of HSD and UrQMD this discrepancy was due to overpredicted pion production, while the kaon yield was well described [6]. With an eye on the planned CBM experiment at GSI it is therefore important to check whether the mentioned difficulties are genuine difficulties in the transport approach or consequences of a particular numerical implementation.

In the present work we study pion and kaon production at the energies 2 - 40 A·GeV within the BUU model [7, 8]; we stress that this is a numerical implementation independent of those employed in [3, 4, 6]. First, we systematically increase the system size and show its effect on particle production. In particular, an analysis of the centrality dependence of the pion and kaon production from Au+Au collisions at 10.7 A·GeV is performed in comparison to the data [9]. Then we study the \( K^+/\pi^+ \) ratio in central Au+Au and Pb+Pb collisions as a function of the beam energy and compare our results to the experimental data and to other models. Special emphasis is put on the strangeness production and we will show in detail the most important production mechanisms at different energies.

The structure of the paper is as follows. In Section 2 we describe our BUU model. In Section 3 we study the influence of the system size on the pion and kaon production. In Section 4 we show the excitation functions of pions, kaons, Λ and Σ hyperons in central heavy-ion collisions. In Section 5 we discuss a medium modification of the FRITIOF string model and its influence on pion and kaon production. The summary and outlook are given in Section 6.
2 The BUU model

Our calculations are based on the BUU model described in Refs. [7, 8]. In the high-energy range ($\sqrt{s} > 2$ GeV) we adopt the treatment of Falter et al [10]. Thus we will drop details concentrating only on the main ingredients and modifications.

The model treats a nucleus-nucleus collision explicitly in time as a sequence of baryon-baryon, meson-baryon and meson-meson collisions. If not specified explicitly, the calculation is always done in the cascade mode, i.e. particles propagate freely between the two-body collisions. The baryon-baryon collisions at the invariant energy $\sqrt{s} < 2.6$ GeV are treated via a resonance scenario, whereas at $\sqrt{s} > 2.6$ GeV a FRITIOF string model [11] is applied. In the case of the meson-baryon collisions the resonance (FRITIOF) model is used at $\sqrt{s} < (>) 2$ GeV. In the most part of calculations we use an energy dependent strangeness suppression factor from Ref. [12]:

$$\gamma \equiv \frac{P(s)}{P(u)} = \begin{cases} 0.3 & \text{for } \sqrt{s} \geq 20\text{GeV} \\ 0.4 & \text{for } \sqrt{s} \leq 5\text{GeV} \\ 0.433 - \frac{1}{150}(\sqrt{s}[\text{GeV}])^{-1} & \text{for } 5\text{GeV} < \sqrt{s} < 20\text{GeV} \end{cases}$$

Sometimes we will also apply an energy independent strangeness suppression factor $\gamma = 0.3$ which will be mentioned explicitly in the text.

The most important modifications are the implementation of the strangeness production channels in meson-meson collisions and the possibility for a baryon and a meson to annihilate into a resonance with the invariant mass more than 2 GeV, whose decay is simulated by the string model. These two modifications will be explained in detail below. In Section 4 we show how the meson-meson collisions and the effective resonance channel influence the $K^+ / \pi^+$ ratio (see Fig. 19).

2.1 Meson-meson reactions

Our BUU model explicitly propagates $\pi, \eta, \rho, \sigma, \omega, \phi, K$ and the $K^*$-mesons. Charmed mesons are also included, but they are not relevant for the energies under consideration and thus we will not mention them further. At the beam energy up to 2 A GeV production of mesons heavier than pions is negligible and the only relevant meson-meson channel of strangeness production is $\pi\pi \leftrightarrow K\bar{K}$, which was included in the BUU model earlier [8]. At higher energies heavier mesons are produced more abundantly and, therefore, one also has to take into account the strangeness production in other meson-meson collisions, e.g. $\pi\rho \leftrightarrow K\bar{K}$. The problem is, however, that the cross sections of these processes are not measured experimentally. In Ref. [13] the cross sections of the processes $\pi\pi \rightarrow K\bar{K}$, $\pi\rho \rightarrow K\bar{K}$ and $\rho\rho \rightarrow K\bar{K}$ were calculated. We use a parametrisation of $\pi\pi \rightarrow K\bar{K}$
from Ref. [14], which is based on these calculations:

$$\sigma_{\pi\pi \rightarrow K\bar{K}} = C \cdot 6.075 \left(1 - \frac{(2m_K)^2}{s}\right)^{0.76} \text{ (mb)}, \quad (2)$$

where the factor $C$ is the combination of the Clebsch-Gordan coefficients for the respective isospin channels:

$$C = \sum_{I=0,1} |\langle i_1i_2m_1m_2 | i_1i_2IM \rangle|^2 |\langle i_3i_4m_3m_4 | i_3i_4IM \rangle|^2, \quad (3)$$

where $i_k$ and $m_k$ are the total isospin and the third isospin component of the particle $k$. Incoming and outgoing particles are enumerated by the pair of indices 1,2 and 3,4 respectively. For simplicity we take the same cross section for $\rho\rho \rightarrow K\bar{K}$ and $\pi\rho \rightarrow K\bar{K}$ since the isospins of the incoming particles are the same; this is not exactly the result from Ref. [13]. For all other reactions with two nonstrange mesons in the incoming channel we assume a constant value of 2 mb for the cross section. The back reactions are included and their cross sections are calculated according to detailed balance.

By setting the cross sections constant we did not take into account the p-wave suppression of e.g. $\pi\rho \rightarrow K\bar{K}$ near threshold. Due to the spin of the $\rho$-meson the strangeness production in that reaction is suppressed up to the threshold of $\pi\rho \rightarrow K\bar{K}^*$ and $\pi\rho \rightarrow \bar{K}\bar{K}^*$. Including this suppression, however, has only a small effect on our results (see discussions in section 4 and Fig.20).

Since the elementary reactions are not accessible experimentally, we test our choice of the cross sections by looking on the yields of kaons in heavy-ion collisions. Although this will give us only a rough estimate of our cross sections due to presence of the baryon-baryon and the meson-baryon channels of the kaon production, there is no other way to get more reliable cross sections for meson-meson reactions.

### 2.2 Annihilation processes

Fig. 1 (see dashed lines) shows that we underestimate the strangeness production in the region just above the FRITIOF threshold ($\sqrt{s} > 2$ GeV). We see that directly above the threshold the cross section without the annihilation descends to almost zero.

This is due to the fact that the FRITIOF model is only capable to produce two excited hadrons, which fragment separately. Thus it is not possible to describe

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3In Figs. 1,3,5,7 the statistical errorbars are shown for the theoretical curves. They are calculated assuming a Poisson distribution, i.e. by dividing the plotted physical value by $\sqrt{N}$, where $N$ is the total accumulated number of events which is used to construct the value. In all other figures the statistical errors of theoretical results are either negligibly small or visible from the histogram representation.
Drell-Yan like processes in which, e.g., a quark from an incoming baryon and an antiquark from an incoming meson annihilate (c.f. Fig. 2). An example for such a process is $\pi N \rightarrow Y K$, where $Y$ stands for a hyperon.

For that reason we have included the annihilation channel phenomenologically. In the case of a reaction of a baryon with a meson we check if an annihilation between a quark and an antiquark is possible; we split each of the interacting hadrons into their constituents and check whether a quark and antiquark with the same flavour exist. If they exist we annihilate the quark and antiquark with probability (4) and (5), neglecting any particles that might be created in this process. In order to make up for this neglect we put all the energy and momentum of both incoming hadrons into the remaining quark content of the baryon and the meson. The fragmentation of this hadron is then done according to the Lund Model. UrQMD [15] interprets meson-baryon reactions in a similar way. RQMD [16] also includes baryon resonances with mass $> 2$ GeV whose decay is described by the Lund model.

The probability for the annihilation is chosen such that we agree with experiment for the strangeness production in $\pi p \rightarrow$ strange particles (see solid lines in Fig. 1):

$$\text{Prob(annihilation)} = \max(0.85 - 0.17 \cdot \frac{\sqrt{s}}{\text{GeV}}, 0).$$

(4)

For the constant strangeness suppression factor $\gamma = 0.3$ the probability for the annihilation processes is readjusted:

$$\text{Prob(annihilation)}_{\gamma=0.3} = \max(1.2 - 0.2 \cdot \frac{\sqrt{s}}{\text{GeV}}, 0).$$

(5)

There are two main reasons for the increase of the strange particle production by including the annihilation. First, we include new channels, as discussed above. Another point is that the invariant energy per string decay is higher. If we have two strings instead of one, the two strings decay separately and therefore it will often occur that every string alone is below the threshold for strangeness production. By putting all the energy into one string, the invariant energy becomes higher and the production of strangeness more probable.

### 3 System-size dependence

In order to clarify the reaction mechanisms in heavy-ion collisions, it is instructive, first, to understand the proton and light ion induced reactions. For larger mass numbers of colliding nuclei, the effect of secondary hadron-hadron collisions becomes more and more important, which drives the system towards thermal equilibrium and enhances the maximum baryon density reached in the collision process. Thus by increasing the system size we can also see how particle spectra evolve with increasing density and if our model within the standard parameters is
able to reproduce the experimental measurements. In the discussion of numerical results it is assumed, if the opposite is not stated explicitly, that the meson-meson cross sections and the $q\bar{q}$ annihilation are included as described in the previous section.

First, we study the proton-induced reactions $p+Be$ and $p+Au$ at the beam momentum of 14.6 GeV/c measured at BNL-AGS \cite{17}. Fig. 3 shows rapidity distributions of produced $\pi^\pm$ and $K^\pm$ for the $p+Be$ system. In this case of light target, the pions and kaons are produced mostly in the first-chance nucleon-nucleon (NN) collisions and have only a small probability to rescatter afterwards. Therefore, their rapidity distributions are centered near the NN center-of-mass (c.m.) rapidity $y_{NN} = y_{beam}/2 = 1.72$. The pion yield is underestimated by $\sim 20\%$, whereas the kaon and antikaon yields are well described by BUU for the $p+Be$ system.

According to Ref. \cite{17}, we have fitted the calculated transverse mass spectra with an exponential function

$$\frac{d^2\sigma}{2\pi m_\perp dm_\perp dy} = a \exp\{-m_\perp/T\} \quad (6)$$

at various rapidities $y$. Fig. 4 presents the inverse slope parameter $T$ as a function of rapidity for $\pi^+$ and $K^+$ in the case of $p+Be$ collisions. The pion inverse slope parameter is well reproduced by BUU except for the very forward and the very backward rapidities in the NN c.m. system. The calculated kaon inverse slope parameter overestimates the data by $\sim 20\%$ at midrapidity.

Fig. 5 shows the rapidity distributions of $\pi^\pm$ and $K^\pm$ for the $p+Au$ collisions. These distributions are shifted to smaller rapidities $y < y_{NN}$ with respect to the case of $p+Be$ reaction (Fig. 3) due to contribution of the secondary NN and $\pi N$ collisions to the meson production and rescattering of the produced mesons on the target nucleons. The $K^-$ rapidity distribution is narrower and is shifted somewhat less than the $K^+$ distribution, since an antikaon is always produced together with a kaon, while a kaon can be also produced in association with a hyperon which requires less c.m. energy \cite{17}. Thus, the secondary NN and $\pi N$ collisions contribute more to the $K^+$ than to the $K^-$ production. BUU describes the experimental pion and kaon rapidity distributions within $\sim 20\%$. In Fig. 6 we present the rapidity dependence of the inverse slope parameter $T$ of the $K^+$ and $\pi^+$ transverse mass spectra for the $p+Au$ reaction. There is a good agreement between BUU and the data except for the very forward rapidity in the $\pi^+$ case where we overpredict the experiment by $\sim 25\%$.

In agreement with the data, we observe little change in the value ($\sim 150$ MeV) of the inverse slope parameter for $\pi^+$ and $K^+$ with increasing target mass (c.f. Figs. 4 and 5). The $K^+$ yield at $y = y_{NN}$ shows a factor of two enhancement, while the $K^-$ yield at $y = y_{NN}$ stays practically unchanged both in BUU and in the data (c.f. Figs. 3 and 5). We attribute this behaviour to the stronger absorption of $K^-$ in the heavier target. The experimental pion yield at $y = y_{NN}$

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is the same for both systems, whereas in BUU we observe a slight enhancement of the pion yield at \( y = y_{NN} \) with increasing target mass.

Next, we present results for Si+Au collisions at the beam momentum of 14.6 A GeV/c, which were studied experimentally in Ref. [1]. This reaction has earlier been studied theoretically in Refs. [16, 18]. In Ref. [16] the RQMD model has been employed, which includes known nonstrange baryon resonances with \( m < 2 \) GeV. RQMD also describes the production of baryon resonances with \( m > 2 \) GeV in high-energy collisions; their decay is simulated by a string model. Shapes of the \( \pi^\pm \) and \( K^\pm \) transverse momentum spectra are quite well described by RQMD. No conclusions on the agreement of the absolute yields of the produced particles with data have been drawn in [16] due to the absolute normalization on the experimental \( \pi^+ \) spectra. However, ratios \( K^+/\pi^+ \) and \( K^-/\pi^- \) computed within RQMD agree well with data. In Ref. [18] a relativistic hadronic cascade (ARC) model has been used; a pure hadronic scenario without string excitation was assumed. A good agreement between ARC and E-802 data on proton \( m_\perp \) spectra, proton, \( \pi^+ \) and \( K^+ \) rapidity distributions was reached within the resonance model, i.e. when, e.g. a three-pion production channel in a NN collision is simulated as \( NN \rightarrow \Delta\Delta\pi \) rather than directly as \( NN \rightarrow NN\pi\pi\pi \). Inverse slope parameters of the \( m_\perp \) spectra for protons and pions are well described by ARC, but underestimated by \( \sim 20\% \) for kaons.

We have considered only central collisions Si+Au corresponding to 7% of the inelastic cross section selected on multiplicity of charged particles [1]. In the theoretical calculations we selected the central collisions in the same way. Fig. 7 shows the calculated \( \pi^+ \) and \( K^+ \) rapidity distributions which were divided by the projectile mass (28) in order to be able to directly compare them with the rapidity distributions from the proton induced reactions.

By comparing the data points in Figs. 7 and 5 we see that the pion yields at \( y = y_{NN} \) are, practically, the same in p+Au and Si+Au systems. This feature is not reproduced by BUU: there is an enhancement of the pion yield per projectile nucleon in the system Si+Au with respect to the p+Au system in our calculations. This may indicate a problem with pion production (or reabsorption) in the heavy system. The experimental \( K^\pm \) yields per nucleon are higher in the Si+Au case than in the p+Au case, which is well reproduced by BUU.

In Fig. 8 we present the inverse slope parameters of the \( K^+ \) and \( \pi^+ \) transverse mass spectra. Despite of the big errorbars plus systematic errors of \( \pm 10\% \) which are not included into the errorbars of the experimental data [1], we see that BUU underpredicts the inverse slope parameter for \( K^+ \)'s by about 25 % and for \( \pi^+ \)'s by 15 %. The calculated inverse slope parameter stays, practically, constant \( T \simeq 150 - 160 \) MeV for all three systems p+Be, p+Au and Si+Au both for \( \pi^+ \)'s and \( K^+ \)'s, whereas the experimental data show higher \( T \simeq 200 \) MeV for \( K^+ \)'s in the Si+Au system.

Studying strangeness production in more detail, we have also performed the BUU calculations using the constant energy-independent strangeness suppression
factor \( \gamma = 0.3 \) (dashed lines in Figs. 3, 5, 7). The \( K^+ \) rapidity distributions favour the energy-dependent strangeness suppression factor, while the \( K^- \) spectra are better described with \( \gamma = 0.3 \).

In order to demonstrate an effect of the \( q\bar{q} \) annihilation on the \( K^+ \) production (see discussion in the previous section) we also show in Fig. 5 the results without the annihilation. In the p+Au system the secondary \( \pi N \) and \( \rho N \) collisions play already an important role. Thus, including the annihilation improves an agreement with the data (c.f. solid and dotted lines in the lower left panel of Fig. 5).

The heaviest colliding system measured at AGS is Au+Au at the beam energies of 2-10.7 A GeV [9, 19, 20]. Before discussing the beam energy dependence (see next section), we will consider the centrality dependence of the pion and kaon production for Au+Au collisions at the top AGS energy of 10.7 A GeV [9].

In Ref. [9] the collision centrality was determined by using two criteria: (i) The energy deposited in the zero-degree calorimeter \( E_{ZCAL} \), which gives an estimate of the projectile participant number \( N_{pp} \) as

\[
N_{pp} = 197 \times \left( 1 - \frac{E_{ZCAL}}{E_{kin, beam}} \right),
\]

where \( E_{kin, beam} = 2123 \) GeV is the kinetic energy of the beam. The smaller \( E_{ZCAL} \) is, the larger is the size of the participant zone, which selects geometrically more central events. (ii) The multiplicity of particles with velocity \( \beta > 0.8 \) in the New Multiplicity Array (NMA) \( mult_{NMA} \). The velocity cut filters out the slow protons, whereas the produced mesons (mostly pions) are accepted. Thus, the larger \( mult_{NMA} \) corresponds to the larger energy transfer from the longitudinal motion of colliding nuclei to the meson production. In average, events with smaller impact parameter \( b \) have larger \( mult_{NMA} \). However, at fixed \( b \) the multiplicity \( mult_{NMA} \) fluctuates stochastically event-by-event depending on the amount of stopping of the counterstreaming nuclear matter in the interaction zone.

According to Ref. [9] we, first, divided BUU events using \( E_{ZCAL} \) into eight classes with increasing \( E_{ZCAL} \) (decreasing centrality) from the first to the eighth class (see Table II in [9]). Second, by modelling the NMA acceptance, we subdivided each of the first three \( E_{ZCAL} \) event classes to the three \( mult_{NMA} \) classes with decreasing \( mult_{NMA} \) (decreasing centrality) from the first to the third multiplicity class (see Table III in [9]).

Fig. 9 shows the proton rapidity distributions for various combinations of \( E_{ZCAL} \)- and \( mult_{NMA} \)-event classes. We see that in each case BUU overestimates stopping. Agreement with experiment can be improved by taking into account the in-medium corrections to the FRITIOF model (see Sect V).

Figs. 10 and 11 show the rapidity spectra of pions and kaons for different event classes selected by applying the \( E_{ZCAL} \) cut only. The \( K^+ \) rapidity spectra are very well described for all \( E_{ZCAL} \) classes. For the \( \pi^+ \) rapidity spectra we see deviations
from the data: In the most central collisions there are excessive pions in BUU produced mainly at midrapidity. With decreasing centrality the deviation from the data disappears gradually, and in the most peripheral collisions there is even an underprediction of the pion multiplicity by BUU. These results are consistent with Figs. 3, 5, 7 where one can also observe a tendency to overpredict pion production with increasing size of the participant zone.

Figs. 12 and 13 show transverse mass spectra of $\pi^+$'s and $K^+$'s for the central collisions of Au+Au at 10.7 A GeV. The spectra are shown for various rapidities starting from the backward rapidity in the c.m. frame (upper line) to the midrapidity (lower line). The shapes of the $\pi^+$ spectra are well described by standard BUU, however, the $\pi^+$ yields are slightly overpredicted at small $m_\perp$ (see also upper left panel in Fig. 10). The agreement of BUU with the $K^+$ spectra is much poorer. The low $m_\perp$ part of the $K^+$ spectra is, typically, overestimated by BUU, whereas the high $m_\perp$ part of the spectra is underestimated by our calculations. Thus, BUU underestimates the inverse slope parameter of the $K^+$ transverse mass spectra, while the $K^+$ yield is well described (c.f. upper left panel in Fig. 11). This problem has been pointed out earlier in Ref. [21].

Fig. 14 shows a fiducial yield of $K^+$ and $\pi^+$ divided by the projectile participant number $N_{pp}$ as a function of $N_{pp}$. The fiducial yield is defined as follows [9]:

$$\text{fiducial yield} = \sum_{0.6<y<1.3} \frac{dN}{dy} dy,$$

(8)

where the $dN/dy$ are the rapidity distributions selected by the zero degree energy. The $K^+$ fiducial yield, as expected, agrees quite well with the data except for a slight underprediction at peripheral collisions. The $\pi^+$ fiducial yield increases with $N_{pp}$ faster than the data do. In the absence of secondary NN collisions the fiducial yields divided by $N_{pp}$ would be constant.

4 Excitation functions

In this section we show the excitation functions of pions, kaons, $\Lambda$- and $\Sigma$-hyperons from central Au+Au and Pb+Pb collisions in comparison to data, two other transport models HSD and UrQMD [6] and the statistical model [22]. The calculations performed with the transport models have all been done in the cascade mode, which makes the comparison easier. We selected the data sets for the Au+Au system at 1.96, 4.00, 5.93, 7.94 and 10.7 A GeV [19, 20] with 5 % of the most central events and for the Pb+Pb system at 30 and 40 A GeV [3, 23] with 7 % of the most central events. In the theoretical calculations we used a sharp impact parameter cut off at 3.5 fm for AGS energies and 4 fm for SPS energies. The influence of the centrality selection was tested at 10.7 A GeV by comparing calculations with a sharp cutoff to calculations which were done by
employing the centrality criteria described in section 3. No deviations were seen in the observables which will be presented in the following.

Fig. 15 shows the midrapidity yield of positive pions as a function of the beam energy. We see that all three models overpredict the pion yield in the considered beam energy range (2-40 A GeV). Our model (solid line) overpredicts the $\pi^+$ midrapidity yield by $\sim 10\%$ at 40 A GeV to $\sim 50\%$ at 2 A GeV. However, the shape of the experimental excitation function $dN/dy$ vs $E_{\text{Lab}}$ for $\pi^+$ is remarkably well described by BUU. The HSD model (dot-dashed line) produces the $\pi^+$ yields close to the BUU results excepting the points at 6, 8 and 10.7 A GeV, where HSD has $\sim 10\%$ more pions than BUU. The UrQMD model (dotted line) agrees well with the pion data at the smallest energy of 2 A GeV, but the pion yield grows too fast with energy within UrQMD producing the discrepancy $\sim 30\%$ with data at the highest considered energy of 40 A GeV.

Figs. 16, 17 and 18 show the midrapidity yields of $K^+$, $K^-$ and $(\Lambda + \Sigma^0)$, respectively, as functions of the beam energy. BUU quite well describes the $K^+$ midrapidity yield excepting the points at 4 and 6 A GeV, where BUU overestimates the data by 30-50%.

The $K^-$ midrapidity yield and $(\Lambda + \Sigma^0)$ midrapidity yield at $E_{\text{Lab}} < 40$ A GeV are overestimated by BUU. Using the constant strangeness suppression factor $\gamma = 0.3$ (dashed lines) reduces the yields of $K^+, K^-$ and hyperons. This leads to a better description of the $K^-$ yields, while in the cases of $K^+$ and $(\Lambda + \Sigma^0)$ it is hard to judge which strangeness suppression factor works better. In Fig 16 and Fig 18 we see that up to about 10 A GeV the $K^+$ and hyperon yields are better described with $\gamma = 0.3$. Above that energy, however, the energy dependent suppression factor works better. For the antikaons the strangeness exchange processes $KN \leftrightarrow \pi Y$ are important due to the strong in-medium modifications [24], which are not taken into account in our study.

The HSD results on the $K^+$ production are close to our calculation with $\gamma = 0.3$. The UrQMD model gives the $K^+$ yield at lower energies similar to our standard calculation, whereas at higher energies UrQMD produces substantially less kaons. The $K^-$ yield is rather well described by both models, HSD and UrQMD, except for the point at 30 A GeV. At lower energies the $(\Lambda + \Sigma^0)$ yields calculated within HSD and UrQMD are somewhat closer to the data than our standard calculation. At higher energies the HSD, UrQMD and our standard calculation give very close results for the $(\Lambda + \Sigma^0)$ yield.

Fig. 19 shows the ratio of midrapidity yields of $K^+$ and $\pi^+$ as a function of the beam energy. In the upper left panel of Fig. 19 we see that neither BUU nor HSD and UrQMD describe the ratio $K^+/\pi^+$ in the whole beam energy region. At the lowest beam energy of 2 A GeV BUU and UrQMD overpredict the ratio by a factor of two, whereas HSD agrees with data. Between 4 and 8 A GeV BUU is quite close to the data. However, the $K^+/\pi^+$ ratio excitation function levels off too early in BUU and, as a consequence, we underestimate the ratio by $\sim 25\%$ between 10 and 30 A GeV. The HSD results on the ratio $K^+/\pi^+$ have a similar
beam energy dependence, but the value of the ratio is smaller by $\sim 20\%$, which is close to our calculation with $\gamma = 0.3$. UrQMD produces a larger slope at lower energies overestimating the ratio at $E_{\text{Lab}} < 8\ \text{A GeV}$, but at higher energies the slope gets negative which causes a strong discrepancy with data in SPS energy regime. Overall, we observe that BUU has the best agreement with data on the $K^+/\pi^+$ ratio in the considered energy regime. At the beam energies of 4-6 A GeV this, however, comes about due to cancellation of the overestimation of the pion and kaon yields.

Fig. 20 shows the ratio of the midrapidity yields $K^-/K^+$ vs the beam energy. The BUU calculations with the energy dependent strangeness suppression factor and our "standard" meson-meson cross sections is shown as the upper boundary of the errorband. In order to estimate an effect of the p-wave suppression on the $\pi\rho \to KK$ and $\pi\omega \to K\bar{K}$ cross section (see section 2.1), we also performed a calculation by putting the cross section equal to zero below the $K^+\bar{K}$ production threshold, which is shown by the lower boundary of the errorband in Fig. 20. Thus the p-wave suppression could reduce the $K^-$ multiplicity by about 10%. The $K^+$ multiplicity is reduced less than 5% by this effect, since the fraction of $K^+$ coming from meson-meson collisions is less than the fraction of $K^-$ coming from meson-meson reactions. (For this reason our results on the $K^+/\pi^+$ ratio Fig. 19 are, practically, untouched by the p-wave suppression effect.)

Overall, we see that BUU overestimates the $K^-/K^+$ ratio independent on the strangeness suppression factor. This result is expected from the previous Figs. 16 and 17, where we see that the $K^+$ yield is rather well described by BUU, while the $K^-$ yield is overestimated substantially. Since the choice $\gamma = 0.3$ reduces both $K^+$ and $K^-$ yields, the ratio $K^-/K^+$ is practically independent on the strangeness suppression factor. The HSD and UrQMD models describe the experimental $K^-/K^+$ ratio quite well. This can be also traced back to Figs. 16 and 17.

Fig. 21 shows the ratio of the midrapidity yields $(\Lambda + \Sigma^0)/\pi$ vs the beam energy. This ratio has a peak near $E_{\text{Lab}} = 8\ \text{A GeV}$ which is reproduced by BUU and HSD models. As far as the absolute values of this ratio are concerned at small energies $E_{\text{Lab}} < 10\ \text{A GeV}$, we would like to remind that both pion and hyperon yields are overestimated by standard BUU (see Figs. 15 and 18). Thus, the agreement of standard BUU with data on $(\Lambda + \Sigma^0)/\pi$ at 8 and 10.7 A GeV is again a result of a mutual cancellation of the $(\Lambda + \Sigma^0)$ and $\pi$ excesses. The choice of $\gamma = 0.3$ which describes the $(\Lambda + \Sigma^0)$ midrapidity yield at small energies better (c.f. Fig. 18) leads to the underestimation of the $(\Lambda + \Sigma^0)/\pi$ ratio at the peak due to the overestimation of the pion yield.

All calculations discussed above were performed in the cascade mode. There is an option in our BUU model, which switches on a nuclear mean field potential. The nuclear mean field potential is necessary, in particular, for a description of the experimental data on collective in-plane and out-of-plane proton and neutron flows at 0.15-2 A GeV. At higher energies, however, the parametrisation of
the momentum-dependent interaction used in [25] leads to too strong repulsive in-plane flow (see also Ref. [26]). Nevertheless, in order to estimate the mean field effect on pion and kaon production, we have also done the calculation with the mean field potential (incompressibility \( K = 215 \) MeV, soft momentum-dependent mean field SM). The lower left panel of Fig. 19 shows the results of this calculation (dashed line) in comparison with our standard calculation in the cascade mode (solid line) and with experimental data. We see that at \( E_{\text{Lab}} < 40 \) A GeV the ratio \( K^+/\pi^+ \) is reduced due to the mean field potential, since the pion yield is relatively insensitive to the nuclear mean field, whereas the kaon yield is reduced. Indeed, a part of the kinetic energy of the counterstreaming nucleon flows transforms now to the potential energy. This reduces, generally, particle production. However, the kaon production is closer to its threshold than the pion production. Therefore, kaons are more strongly influenced by the mean field potential than pions.

As we described in Section 2, the meson-meson collisions and the \( q\bar{q} \) annihilation channel for the meson-baryon collisions are implemented in our BUU model. The lower right panel of Fig. 19 shows an effect of these implementations on the \( K^+/\pi^+ \) midrapidity ratio. The result of our standard calculation including both the meson-meson collisions and the \( q\bar{q} \) annihilation is shown by the solid line in Fig. 19. The dotted and dashed lines represent the calculations without the meson-meson collisions but with annihilation and without the annihilation but with the meson-meson collisions, respectively. The meson-meson collisions strongly enhance the \( K^+/\pi^+ \) ratio above 6 A GeV due to the increased \( KK \) production. An effect of the \( q\bar{q} \) annihilation channel is less pronounced: only a slight enhancement of the \( K^+/\pi^+ \) ratio is visible at 5-10 A GeV.

The upper right panel of Fig. 19 compares the BUU and the statistical model calculations for the \( K^+/\pi^+ \) ratio at midrapidity. Since we use a string model, which produces a multiparticle final state for the two colliding particles, the thermal equilibrium would be only reached if we also included the corresponding back reactions (c.f. Ref. [27]). However, this was out of scope of the present work. Nevertheless, there is a surprisingly good agreement between BUU and the statistical model. The statistical model is closer to the data, but it is also unable to describe the data points at 10 and 30 A GeV.

Finally, in Fig. 22 we show the inverse slope parameter \( T \) of the \( K^+ \) transverse mass spectra vs the laboratory energy. At the energies 2, 4, 6 and 8 A GeV the inverse slope parameter was obtained by fitting the exponential function \( \Theta \) to the \( m_\perp \)-spectrum of kaons in the rapidity range \( |(y - y_{NN})/y_{NN}| < 0.25 \) [20]. At 10.7 A GeV the rapidity range was \( |(y - y_{NN})/y_{NN}| < 0.125 \) [20]. At the SPS energies of 30 and 40 A GeV the rapidity range was taken as \( |y - y_{NN}| < 0.1 \) [3, 23]. Our model underestimates the inverse slope parameter by 30-40 % (see also Fig. 13). A similar problem was reported before in Ref. [21, 28] and ascribed to the lack of pressure due to missed nonhadronic degrees of freedom in the transport models. We speculate here that also the inclusion of multi-baryon collisions would tend to make the spectrum harder. The probability for such processes naturally increases
with a high power of baryon density.

In order to see an origin of the produced kaons we performed a channel decomposition of the $\bar{s}$-quark production for the central Au+Au collisions at 4, 10.7 and 20 A GeV. Fig. 23 shows the number of the produced $\bar{s}$-quarks vs time for four different channels: (i) baryon-baryon channel at high energies ($\sqrt{s} > 2.6$ GeV) or, in other words, the baryon-baryon reactions simulated by the FRITIOF string model (solid line), (ii) baryon-meson collisions at $\sqrt{s} > 2$ GeV simulated by the string model (dashed line), (iii) baryon-meson collisions at $\sqrt{s} < 2$ GeV, i.e. below the string model threshold (dash-dotted line), (iv) meson-meson collisions (dotted line). The low energy ($\sqrt{s} < 2.6$ GeV) baryon-baryon collisions do not contribute to the $\bar{s}$-production significantly at the considered beam energies. Thus, this channel is not shown in Fig. 23. We counted only the creation of the $\bar{s}$-quark and we did not consider reactions or decays, as e.g. $K^* \rightarrow K \pi$ where the $\bar{s}$-quark is only shifted from a $K^*$ to a $K$.

The baryon-baryon-string channel plays the dominant role in the whole beam energy region. This channel includes mainly the first-chance NN collisions between the projectile and the target nucleons. The meson-meson channel is not important at 4 A GeV, but its role grows quickly with energy and at 20 A GeV it includes already 25 % of the produced $\bar{s}$-quarks. A relative contribution of the baryon-meson-string channel also increases with energy, while the low energy baryon-meson collision relative contribution stays always very small and decreases with energy.

The time evolution of the $\bar{s}$-quark production can be better understood if one looks also on the central density time evolution shown in Fig. 24 for the central Au+Au collision at 10.7 A GeV. The central density reaches its maximum value $\sim 4.5\rho_0$ at $t \simeq 10.5$ fm/c and stays above $3\rho_0$ in the time interval $t = 7.5 - 14$ fm/c, where the $\bar{s}$-quark production just takes place. Here $\rho_0 = 0.16$ fm$^{-3}$ is the nuclear saturation density. Thus, strangeness is produced during the high-density stage of a heavy-ion collision. It is evident also from Fig. 23 that the $\bar{s}$-quark production from the meson-meson and the baryon-meson channels, which contain the secondary collisions, starts later than from the baryon-baryon channel.

5 In-medium modification of the FRITIOF model

In the course of a heavy-ion collision the elementary hadron-hadron collisions happen at a finite baryon density. Therefore, the wave functions of incoming and outgoing particles are the in-medium plane waves rather than the vacuum ones.\(^4\)

\(^4\)Exchange particles expressed by the propagators get also in-medium modified. However, this last effect would be strongly dependent on the model used for the description of an elementary collision. Thus we will, for simplicity, neglect it by using the vacuum matrix elements.
In order to take into account the in-medium modifications of the incoming and outgoing particles in the FRITIOF events, we follow here the approach of [29] generalized to the processes with many-meson final states. Only the events with two colliding nonstrange baryons will be modified. The in-medium modifications of the meson-baryon and meson-meson collisions are neglected, since as we expect, they are small with respect to the baryon-baryon case (see below).

Let us consider the process

\[ B_1 B_2 \rightarrow B_3 B_4 M_5 M_6 \ldots M_N , \]

(9)

where \( B_1, B_2 \) and \( B_3, B_4 \) are the incoming and the outgoing baryons, respectively; \( M_5, M_6, \ldots, M_N \) are the produced mesons. The in-medium differential cross section of this process is given by the following expression

\[
d\sigma^{\text{med}} = \frac{(2\pi)^4 (2m_1^*) (2m_2^*) (2m_3^*) (2m_4^*)}{4I^* |T|^2} d\Phi_{N-2}(p_1^* + p_2^*; p_3^*, p_4^*, k_5^*, k_6^*; \ldots, k_N^*) ,
\]

(10)

where \(|T|^2\) is the matrix element squared in the normalization of Ref. [30] averaged over spins of initial particles and summed over spins of final particles,

\[
d\Phi_{N-2}(p_1^* + p_2^*; p_3^*, p_4^*, k_5^*, k_6^*; \ldots, k_N^*) = \delta^{(4)}(p_1^* + p_2^* - p_3^* - p_4^* - k_5^* - k_6^* - \ldots - k_N^*) \times \frac{d^3 p_3^*}{(2\pi)^3 2(p_3^*)^0} \frac{d^3 p_4^*}{(2\pi)^3 2(p_4^*)^0} \frac{d^3 k_5^*}{(2\pi)^3 2(k_5^*)^0} \ldots \frac{d^3 k_N^*}{(2\pi)^3 2(k_N^*)^0}
\]

(11)

is the \((N-2)\)-body phase space [31] with \((p_i^*)^0 = ((p_i^2 + m_i^*)^2)^{1/2} \), \( i = 1, 2, 3, 4 \) and \((k_i^*)^0 = ((k_i^2 + m_i^*)^2)^{1/2} \), \( i = 5, 6, \ldots, N \) being the zeroth components of the kinetic four-momenta, and the \( m_i^* \) being the effective masses of the particles involved. In Eq. (11) the flux factor is

\[
I^* = q(\sqrt{s^*}, m_1^*, m_2^*)\sqrt{s^*} ,
\]

(12)

where \( s^* \equiv (p_1^* + p_2^*)^2 \) and

\[
q(\sqrt{s^*}, m_1^*, m_2^*) = [(s^* + (m_1^*)^2 - (m_2^*)^2) / (4s^*) - (m_1^*)^2]^{1/2}
\]

(13)

is the c.m. momentum of incoming baryons.

The matrix element \(|T|^2\) entering into Eq. (10) can be extracted from the vacuum cross section by dividing out the vacuum phase space and multiplying by the vacuum flux factor. Thus, our final result for the in-medium total cross section of the process (9) is

\[
\sigma^{\text{med}}(\sqrt{s^*}) = F \sigma^{\text{vac}}(\sqrt{s}) .
\]

(14)

on the place of the in-medium ones.
The modification factor $F$ is

$$F \equiv \frac{m_1^* m_2^* m_3^* m_4^*}{m_1 m_2 m_3 m_4} \frac{I}{I^*} \Phi_{N-2}(\sqrt{s}; m_3^*, m_4^*, \ldots, m_N^*),$$

(15)

where $I = q(\sqrt{s}, m_1, m_2)\sqrt{s}$. In Eq. (14) $\sqrt{s}$ is the c.m. energy of the colliding baryons in vacuum, which is directly provided by the BUU calculations in the cascade modus performed in the present work. The in-medium c.m. energy $\sqrt{s^*}$ is then determined from the condition that the energy excess above threshold is the same as in vacuum, i.e.

$$\sqrt{s^*} = \sqrt{s} - m_3 - m_4 - \cdots - m_N + m_3^* + m_4^* + \cdots + m_N^* .$$

(16)

Since the modification factor $F$ is proportional to the product of the ratios of the Dirac mass to the bare mass for incoming and outgoing fermions, we expect that the meson-baryon and meson-meson channels will be modified relatively weaker.

In Eq. (10) we replaced the canonical four-momenta by the kinetic ones in the $\delta$-function entering the phase-space volume element (11). This is possible only if the vector fields cancel each other, which is valid in the case of the Walecka model (c.f. ref. [32]), but would be violated in a more sophisticated relativistic mean field model with momentum-dependent scalar and vector fields [33]. Taking into account the momentum dependence, in particular, for the vector field, which drops with momentum, is important for the description of the baryon flow in heavy ion collisions above 1 A GeV [34]. However, in the present exploratory work we will neglect the momentum dependence of the $\sigma$ and $\omega$ fields, which would strongly complicate the calculation of the phase space volume.

We evaluate the in-medium masses using a nonlinear version NL2 [35] of the relativistic mean field model and assuming that the nucleons and all nonstrange baryonic resonances are coupled to the scalar mean field $\sigma$ and to the vector mean field $\omega$ by the same universal coupling constants $g_\sigma$ and $g_\omega$ [36]. This gives the Dirac effective masses

$$m_B^* = m_B + g_\sigma \sigma$$

and the kinetic four-momenta

$$p_B^* = p_B - g_\omega \omega$$

(17)

(18)

of the nonstrange baryons. The hyperon coupling constants are (c.f. Ref. [32]):

$$g_\sigma^Y = \frac{2}{3} g_\sigma , \quad g_\omega^Y = \frac{2}{3} g_\omega ,$$

(19)

where $Y = \Lambda$ or $\Sigma$. The baryon single-particle energy is

$$\varepsilon(p_B) = g_\omega \omega^0 + \sqrt{(p_B^*)^2 + (m_B^*)^2} .$$

(20)
For the mesons $\pi$, $\rho$ and $\omega$ we neglect any in-medium modifications, while for the $K$ and $\bar{K}$ single-particle energies we use the model of Ref. [37] with parameters of Ref. [38]:

$$\omega(k, \rho) = \sqrt{(k^*)^2 + (m_K^*)^2} \pm V^0,$$  \hspace{1cm} (21)

where the upper (lower) sign corresponds to the $K$ ($\bar{K}$) case,

$$k^* = k \mp V$$ \hspace{1cm} (22)

is the kaon kinetic momentum,

$$m_K^* = \sqrt{m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s + V^2}$$ \hspace{1cm} (23)

is the kaon effective (Dirac) mass, and

$$V^\mu = \frac{3}{8(f_\pi^*)^2} j^\mu$$ \hspace{1cm} (24)

is the kaon vector field. $\rho_s$ and $j^\mu$ are the scalar density and the baryonic four-current, respectively. The parameters which appear in Eqs. (23), (24) are $\Sigma_{KN} = 450$ MeV, $f_\pi = 93$ MeV and $(f_\pi^*)^2 = 0.6(f_\pi^2)$ [38]. Within these parameters the following relation is expected to hold [38]

$$V^\mu \simeq \frac{1}{3} g_\omega \omega^\mu.$$ \hspace{1cm} (25)

Taking into account relations (19) and (25) one can see that the vector field is, indeed, completely excluded from the energy-momentum conservation conditions for the strange particle production processes like $B_1B_2 \rightarrow B_3Y_4K$ or $B_1B_2 \rightarrow B_3B_4K\bar{K}$, which gives a possibility to simplify the in-medium calculations by just replacing the bare masses of particles by the Dirac masses and canonical four-momenta by the kinetic four-momenta.

We have calculated the modification factor $F(\sqrt{s}, \rho)$ as a function of the c.m. energy $\sqrt{s}$ and the baryon density $\rho$ for various outgoing channels with no more than four mesons in the final state. We assume that the incoming baryons are nucleons, but an outgoing baryon can be either nucleon or $\Lambda$-hyperon. For outgoing mesons we have considered all possible combinations of pions, $\rho$-mesons, kaons and antikaons with no more than one kaon and one antikaon in the final state. The upper panel of Fig. 25 shows the medium modification factor $F(\sqrt{s}, \rho_0)$ for some selected processes: $NN \rightarrow NN\pi$, $NN\pi\pi$, $NN\rho$, $N\Lambda K$, $NNK\bar{K}$. We see that the modification factor depends on the outgoing channel rather weakly. In particular, the addition of a pion does not change the factor. On the lower panel of Fig. 25 we demonstrate the density dependence of the
modification factor for the one-pion production channel. One can observe a strong decrease of the factor with the baryon density.

For the application to the FRITIOF model built in the BUU code, we have stored the factors $F$ on a two-dimensional grid $(\sqrt{s}, \rho)$. Once some final state is generated by FRITIOF, it is accepted with the probability $F$. In the case where at least one of incoming baryons is a resonance, we use the modification factor for incoming nucleons at the same $\sqrt{s}$. If an outgoing baryon is the $\Delta$-resonance, the factor for an outgoing nucleon is applied, shifted by the production threshold, i.e. $F(\sqrt{s} - m_\Delta + m_N)$. An analogous threshold correction is performed if $\Sigma$-hyperon and/or $K^*$ are produced.

In order to see an effect of the in-medium string model modifications in heavy-ion collisions, we have performed the calculation for the Au+Au system at 10.7 A GeV. The results of this calculation are shown by the dashed lines in Figs.

The proton rapidity spectra (Fig. 9) get now wider, in agreement with the experiment. This is expected since the inelastic NN cross section is reduced by the in-medium effects. The results with in-medium modifications for the most central events (upper left panel in Fig. 9) have a big statistical error due to a small number of events in this centrality class.

The $\pi^+$ rapidity spectra (Fig. 10) are reduced in a closer agreement with the data, except for the very peripheral collisions. The $K^+$ rapidity spectra (Fig. 11) are also reduced. This, however, makes the agreement with the $K^+$ data worse.

The transverse mass spectra of $\pi^+$'s (Fig. 12) and $K^+$'s (Fig. 13) are reduced more at high transverse masses, since the high-$m_\perp$ particles are emitted from hard collisions which happen at the high baryon density where the in-medium modifications are stronger. Thus, the $m_\perp$-spectra get steeper, which, again, leads to a worse description of the $K^+$ data. The $\pi^{++}$ transverse mass spectra at small $m_\perp$ are now better described, whereas at high $m_\perp$ the in-medium modifications result in a slight underestimation of the experiment.

Fig. 14 summarizes our findings on the in-medium modifications. The $\pi^+$ fiducial yield is described better with the in-medium modifications, except for the very peripheral collisions. The $K^+$ fiducial yield is underestimated at all collision centralities with the in-medium modifications.

6 Summary and outlook

In the present work, the BUU model developed earlier in Refs. [7, 8] is further improved by including the heavy ($m > 2$ GeV) resonance or $q\bar{q}$-annihilation channel in the meson-baryon collisions and by the new meson-meson channels ($\pi\rho \rightarrow K\bar{K}, \rho\rho \rightarrow K\bar{K}$) for the strangeness production. Moreover, an in-medium modification of the FRITIOF model by taking into account the effective (Dirac) masses of the incoming and outgoing particles is formulated and implemented in
BUU. The BUU model is applied to the nucleus-nucleus collisions at 2-40 A GeV.

By performing the systematic study of the pion and kaon production for various systems and collision energies we came to the following conclusions:

1. The $\pi^\pm$ and $K^\pm$ rapidity spectra and the inverse slope parameters of the transverse mass spectra from the proton-nucleus reactions are well described by BUU. The $q\bar{q}$-annihilation channel improves the agreement with the data on the $K^+$ production in proton-nucleus collisions (Fig. 5).

2. In the peripheral Au+Au collisions at 10.7 A GeV the $\pi^+$ rapidity spectra are slightly underestimated, whereas the $K^+$ rapidity spectra are well described. This result is consistent with a good agreement between BUU and the data on the proton-nucleus reactions. The proton rapidity spectra are too narrow, i.e. the stopping power is overestimated by BUU, for all collision centralities.

3. In the central nucleus-nucleus collisions (Si+Au at 14.6 A GeV/c, Au+Au at 2-10.7 A GeV, Pb+Pb at 30 and 40 A GeV) the $K^+$ yields are, overall, well described, the $K^-$ and hyperon yields are somewhat overestimated, the $\pi^+$ yields are overestimated in all central heavy-ion collisions under study. The inverse slope parameters of the $K^+$ transverse mass spectra are strongly underestimated (Figs. 13, 22). The pion slopes are well described (Fig. 12). Our BUU results on $\pi^+$ and $K^+$ agree, generally, with results of the HSD and UrQMD calculations from Refs. [6, 21]. The excitation function of the $K^+/\pi^+$ ratio is described, however, better and closer to the thermal model results due to the introduction of the new meson-meson channels of the strangeness production which increase the $K^+$ yield at the beam energy above 6 A GeV (Fig. 19).

4. The in-medium modification of the FRITIOF model, which reduces the particle production cross sections, was tested for the system Au+Au at 10.7 A GeV. This leads to the better description of the pion production, whereas the $K^+$ production is overdamped by the in-medium effects. The stopping power of the nuclear matter is reduced, which results in the better description of the proton rapidity spectra.

The $K^+/\pi^+$ ratio is quite sensitive to the meson-meson cross sections and, within the reasonable choice of these cross sections, we have decreased the discrepancy between BUU and the data by about a factor of 2. The resulting disagreement $\sim 20\%$ can, thus, hardly to be considered as a signal of the “new physics”. The inverse slope parameter of the $K^+$ transverse mass spectra is a more serious problem for BUU. It would be worthwhile to study this topic in more detail by introducing, e.g. string-string and many-body collisions, which could both enhance the hard part of the kaon spectra.
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Figure 1: The cross section of the strangeness production in $\pi^+p$ collisions (upper panel) and $\pi^-p$ collisions (lower panel) as a function of beam energy in comparison to data from [39]. The vertical line corresponds to the threshold for the string model ($\sqrt{s} = 2$ GeV). Solid and dashed lines show results without and with $q\bar{q}$ annihilation channels (see subsection 2.2). Errorbars on calculations are statistical.
Figure 2: Quark Diagram for the process $\pi^+ p \rightarrow \Sigma^+ K^+$. 
Figure 3: Rapidity distributions of $\pi^\pm$ and $K^\pm$ for $p+$Be at 14.6 GeV/c in comparison to data from [17]. Solid and dashed lines show results with energy dependent (see Eq.(1)) and constant ($\gamma = 0.3$) strangeness suppression factor, respectively. Errorbars on calculations are statistical.
Figure 4: Rapidity dependence of the inverse slope parameter $T$ of the transverse mass spectra of $\pi^+$ and $K^+$ for $p+$Be at 14.6 GeV/c in comparison to data from [17].
Figure 5: Rapidity distributions of $\pi^\pm$ and $K^\pm$ for $p+Au$ at 14.6 GeV/c in comparison to data from [17]. Solid and dashed lines show results with energy dependent (see Eq.(1)) and constant ($\gamma = 0.3$) strangeness suppression factor, respectively. The dotted line in the lower left panel shows a calculation without the $q\bar{q}$ annihilation channel in meson-baryon collisions. Errorbars on calculations are statistical.
Figure 6: Rapidity dependence of the inverse slope parameter $T$ of the transverse mass spectra of $\pi^+$ and $K^+$ for $p+$Au at 14.6 GeV/c in comparison to data from [17].
Figure 7: Rapidity distributions of $\pi^{\pm}$ and $K^{\pm}$ for central collisions Si+Au at 14.6 A GeV/c in comparison to data from [1]. Solid and dashed lines show results with energy dependent (see Eq.(1)) and constant ($\gamma = 0.3$) strangeness suppression factor, respectively. The spectra are divided by 28 in order to be able to compare to proton induced reactions. Errorbars on calculations are statistical.
Figure 8: Rapidity dependence of the inverse slope parameter $T$ of the transverse mass spectra of $\pi^+$ and $K^+$ for Si+Au at 14.6 A GeV/c in comparison to data from [1].
Figure 9: Proton rapidity spectra from BUU events Au+Au at 10.7 A GeV double selected by the total multiplicity and the zero degree energy in comparison to data from [9]. The centrality decreases from the upper left corner to the lower right. The solid line shows the standard BUU calculation, whereas the dashed line is calculated with the medium modification described in section 5.
Figure 10: Rapidity spectra of $\pi^+$ from BUU events Au+Au at 10.7 A GeV selected by the zero degree energy in comparison to data from [9]. The centrality decreases from the upper left corner to the lower right. The solid line shows the standard BUU calculation, whereas the dashed line is calculated with the medium modification described in section 5.
Figure 11: Rapidity spectra of $K^+$ from BUU events Au+Au at 10.7 A GeV selected by the zero degree energy in comparison to data from [9]. The centrality decreases from the upper left corner to the lower right. The solid line shows the standard BUU calculation, whereas the dashed line is calculated with the medium modification described in section 5.
Figure 12: Transverse mass spectra for $\pi^+$ from Au+Au collisions at 10.7 A GeV for different slices of rapidity from the most central events selected by $E_{ZCAL}$. The rapidity slices range from 0.6-0.7 for the uppermost line to 1.3-1.4 for the lowermost line with a step 0.1. The spectra are multiplied by powers of 10: $10^0, 10^{-1}, \ldots, 10^{-7}$ from the uppermost to the lowermost line. Data are from [9].
Figure 13: Transverse mass spectra for $K^+$ from Au+Au collisions at 10.7 A GeV for different slices of rapidity from the most central events selected by $E_{ZCAL}$. The rapidity slices range from 0.5-0.6 for the uppermost line to 1.2-1.3 for the lowermost line. The spectra are multiplied by powers of 10: $10^0, 10^{-1}, ..., 10^{-7}$ from the uppermost to the lowermost line. Data are from [9].
Figure 14: Fiducial yields of $\pi^+$ and $K^+$ divided by the number of projectile participants $N_{pp}$ as functions of $N_{pp}$ from Au+Au collisions at 10.7 A GeV in comparison to data from [9].
Figure 15: Midrapidity yields for $\pi^+$ as a function of energy in comparison to results of HSD, results of UrQMD and data from [19, 3, 23].

Figure 16: Midrapidity yields for $K^+$ as a function of energy in comparison to results of HSD, results of UrQMD and data from [19, 3, 23].
Figure 17: Midrapidity yields for $K^-$ as a function of energy in comparison to results of HSD, results of UrQMD and data from [20, 3, 23].

Figure 18: Midrapidity yields for $\Lambda + \Sigma^0$ as a function of energy in comparison to results of HSD, results of UrQMD and data from [40, 41, 42, 43, 44].
Figure 19: $K^+ / \pi^+$ ratio at midrapidity as a function of the beam energy. Upper left panel - comparison of the BUU results with the results of the UrQMD and HSD model [6]. Upper right panel - comparison between BUU and statistical model [22]. Lower left panel - comparison of standard BUU (cascade mode) and BUU with nuclear mean field potential. Lower right panel - comparison of the standard BUU calculation with the calculations without $q\overline{q}$ annihilation and without meson-meson collisions. The data are from [3, 19, 23].
Figure 20: The $K^-/K^+$ ratio at midrapidity as a function of energy in comparison to results of HSD, results of UrQMD and data from [20, 3, 23]. The errorband on the BUU results indicate the uncertainty in the meson-meson channels (see discussions in sections 2.1 and 4).

Figure 21: The $(\Lambda + \Sigma_0)/\pi$ ratio at midrapidity, where $\pi = 1.5(\pi^+ + \pi^-)$, as a function of energy in comparison to data from [40, 41, 42, 43, 44].
Figure 22: Inverse slope parameter $T$ for $K^+$ as a function of energy in comparison to data from [20, 3, 23].
Figure 23: Contribution of different channels to strangeness production in Au+Au collisions with $b = 1$ fm at 4 A GeV (upper panel), 10.7 A GeV (middle panel) and 20 A GeV (lower panel).
Figure 24: Time evolution of the central baryon density in Au+Au collisions at 10.7 A GeV, b=0 fm.
Figure 25: In-medium modification factor $F$ of the $NN$ cross section vs c.m. energy. Upper panel shows $F$ at $\rho = \rho_0$ for various outgoing channels: $NN\pi$ – solid line, $NN\pi\pi$ – long-dashed line, $NN\rho$ – short-dashed line, $N\Lambda K$ – dotted line and $NNK\bar{K}$ – dash-dotted line. On the lower panel the factor $F$ is presented for the $NN\pi$ channel at $\rho = \rho_0$, $2\rho_0$ and $3\rho_0$ in the order from the upper to the lower line.