A full-sky prediction of the Sunyaev–Zeldovich effect from diffuse hot gas in the local universe and the upper limit from the WMAP data

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ABSTRACT
We use the Point Source Catalogue Redshift Survey galaxy redshift catalogue combined with constrained simulations based on the IRAS 1.2-Jy galaxy density field to estimate the contribution of hot gas in the local universe to the Sunyaev–Zeldovich (SZ) effect on a large scale. We produce a full-sky HEALPIX map predicting the SZ effect from clusters as well as diffuse hot gas within 80 h⁻¹ Mpc. Performing cross-correlation tests between this map and the WMAP data in pixel, harmonic and wavelet space we can put an upper limit on the effect. We conclude that the SZ effect from diffuse gas in the local universe cannot be detected in current cosmic microwave background (CMB) data and is not a large-scale contaminating factor (ℓ < 60) in studies of CMB angular anisotropies. We derive an upper limit for the mean temperature decrement of ΔT < 0.33 μK at the 2σ confidence level for the 61-GHz frequency channel. However, for future high-sensitivity experiments observing at a wider range of frequencies, the predicted large-scale SZ effect could be of importance.

Key words: methods: data analysis – methods: statistical – techniques: image processing – cosmology: cosmic microwave background – cosmology: observations.

1 INTRODUCTION
The first high-resolution full-sky observation of the cosmic microwave background (CMB) provided by the WMAP satellite has significantly improved estimates of the angular power spectrum and cosmological parameters (see Bennett et al. 2003, and references therein). It has also put stringent limits on the level of primordial non-Gaussianity and point-source contribution in the CMB (Komatsu et al. 2003). Also the hot gas in some nearby clusters was seen through the Sunyaev–Zeldovich (SZ) effect.

In this paper we investigate whether CMB observations, at the frequencies observed by the WMAP satellite, could be used to study the large-scale properties of the local universe (<80 h⁻¹ Mpc) through the SZ effect of diffuse hot gas as well as nearby clusters and/or if this effect could contaminate, to some level, the CMB at the largest scales. This is done through cross-correlation tests between an all-sky estimate of the SZ thermal effect induced by the hot gas in the local universe and combinations of the WMAP observations at different frequencies. We also put limits on the contribution of the SZ effect in the local universe to the WMAP data. Uncertainties in the reconstructed positions of local cosmic structures in the currently available constrained simulations are significantly larger than 2°, which corresponds to the smallest angular scale that we consider in our cross-correlation analysis. To circumvent this problem we adopt an alternative reconstruction procedure that uses a further set of constraints provided by the observed positions of Point-Source Catalog Redshift Survey (PSCz) galaxies.

Recently there have been a number of papers focusing on cross-correlation tests to search for the SZ effect in the WMAP data; Myers et al. (2004) found a detection of the SZ effect out to a 1° angular distance of the cluster centres for clusters in the Automated Plate Measurement (APM), ACO (Abell, Corwin & Olowin 1989) and Two Micron All Sky Survey (2MASS) galaxy catalogues; Hernandez-Monteagudo, Genova-Santos & Atrio-Barandela (2004) performed a cross-correlation test between 2MASS (out to z ≈ 0.1) and WMAP for small patches on the sky and found a strong detection; Afshordi, Loh & Strauss (2004a) detected the thermal SZ effect at small angular scales by cross-correlating the WMAP data with the 2MASS galaxy catalogue; Afshordi, Lin & Sanderson (2004b) found an 8σ detection of the SZ effect for 116 X-ray clusters; Fosalba, Gaztanaga & Castander (2003) and Scranton et al. (2004)
reported a detection of the SZ effect of high-redshift clusters by cross-correlating the WMAP data with large-scale structures traced by the Sloan Digital Sky Survey (SDSS). All these detections of the SZ effect were found mainly on small angular scales and mostly due to high-redshift clusters. On large angular scales, cross-correlations between WMAP observations and large-scale structures have been found (Fosalba, Gaztanaga & Castander 2003; Fosalba & Gaztanaga 2004; Boughn & Crittenden 2004; Nolta et al. 2004; Scanton et al. 2004; Afshordi, Loh & Strauss 2004a; Vielva, Martínez-González & Tucci 2004b; Padmanabhan et al. 2004). These correlations have been interpreted as a detection of the Integrated Sachs–Wolfe effect (ISW) and are not compatible with the signal expected from the SZ effect. Some attempts to detect the SZ effect on large angular scales have also been performed. Scaramella (1994) used the distribution of Abell/ACO clusters to predict the SZ effect in the local universe and its detectability; Diego, Silk & Sliwa (2003) searched for cross-correlations between the WMAP data and the ROSAT diffuse X-ray background maps but found no correlations; Hernandez-Monteagudo & Rubino-Martin (2004) used a set of optical and X-ray based catalogues and detected a cross-correlation due to the SZ effect at small scales but failed to detect an SZ signal caused by superclusters at larger scales; Huffenberger, Seljak & Makaroff (2004) put limits on the power spectrum of the SZ effect by optimally combining cross power spectra between different WMAP channels; limits have also been put on gravitational lensing as well as the SZ effect by cross-correlating large-scale structure traced by SDSS and the WMAP data (Hirata et al. 2004).

Here we focus on diffuse hot gas in the local low-redshift universe (ζ < 0.05) and perform cross-correlation tests of our predicted full-sky SZ map with the WMAP data on large angular scales (> 2'). A similar test was performed in Banday & Gorski (1996) for the COBE data using a model for a predicted X-ray-emitting halo surrounding the Local Group, but no detection was found. First we test cross-correlations in pixel space for the full sky as well as for limited parts of the sky where the effect is expected to be largest. Then we perform a similar test in harmonic space for different multipole ranges. Finally, we use tests built on wavelets which have proven to be a very powerful tool for detecting cross-correlations (Vielva et al. 2004b). Note that some of the clusters included in our analysis have been detected in some of the previously mentioned papers, but only at small angular scales. Here we are interested only in the large angular scale effect introduced by the local hot gas.

The outline of the paper is as follows. In Section 2 we describe the data sets used, the WMAP data and the PSCz and IRAS galaxy catalogues and in Section 3 we describe how we obtain a prediction of the SZ effect from these data sets. Then in Section 4 we outline the cross-correlation tests applied in pixel, harmonic and wavelet space and present the results. Finally, these results are discussed in Section 5.

2 DESCRIPTION OF THE DATA SETS

In this work, we have used the PSCz galaxy catalogue combined with constrained simulations based on the IRAS 1.2-Jy galaxy density field to infer the matter densities and hence the gas densities in the local universe. From this we have made a prediction of the SZ effect for which we have been looking using the latest CMB data from the WMAP experiment. In this section, we summarize the properties of these three data sets.

2.1 The WMAP data

The WMAP experiment (Bennett et al. 2003, and references therein) observed the sky at five frequencies, three of which are interesting to studies of the CMB, the Q band at 41 GHz, the V band at 61 GHz and the W band at 94 GHz. In this work, we have used the foreground cleaned V- and W-band maps (publicly available at the Lambda website) which have been co-added according to

\[ M^* = \frac{V + AW}{1 + A}, \]

where A is a constant. As the channel Q may be more contaminated by foreground residuals than the other two channels at the scales considered in this analysis, we have chosen to exclude it from the analysis (as was done also by the WMAP team e.g. Hinshaw et al. 2003, for the largest scales). We find that the value of A optimizing the ratio of the SZ effect to the variance of the CMB and noise is A = 0.59. We also use the difference map M^2 = W − V which does not have a CMB component. Note that we use the index s for the optimal map and d for the difference map. Furthermore, the galactic cut and point-source mask Kp2 (which we extend by 2' along the rim of the cut) used by the WMAP team (and available at the Lambda website) have been applied to avoid possible sources of contamination. Note that the unobserved region of the PSCz catalogue is already contained within this mask. The maps are all in healpix2 pixelization (Górski, Hivon & Wandelt 1998) with resolution parameter Nside = 512 corresponding to pixels with edges of about 7 arcmin.

2.2 The PSCz catalogue and the galaxy density field

The PSCz redshift catalogue lists the angular positions, redshifts and fluxes of ~15 500 IRAS PSC galaxies selected with a flux 60 μJy larger than 0.6 Jy. A detailed description of the selection criteria, star–galaxy separation algorithm and the procedures adopted to exclude galactic cirrus, are given in Saunders et al. (2000), to which we refer the interested reader. For our purposes the most interesting features of the IRAS PSCz catalogue is its very large sky coverage (~84 per cent of the sky, overlapping very well with the part of the sky outside the Kp2 sky cut used by WMAP) and depth (the median redshift is zc ~ 8500 km s⁻1). In this work we consider a subsample of galaxies within a radius of 80 h⁻¹ Mpc for which distances have been obtained from redshift using the iterative procedure of Branchini et al. (1999). The flux-limited nature of the catalogue causes the number of objects to decrease with distance. The average galaxy–galaxy separation of the sample has been computed by Branchini et al. (1999)

\[ l(r) = \begin{cases} \langle l_0 \rangle & \text{if } r \leq 6 h^{-1} \text{ Mpc} \\ A(l_0)(r)^{-0.36} \left(1 + \frac{r}{r_*} \right)^{0.61} & \text{if } r > 6 h^{-1} \text{ Mpc} \end{cases} \]

where r is the proper distance in h⁻¹ Mpc, r* = 87 h⁻¹ Mpc, \( \langle l_0 \rangle = 2.17 h^{-1} \text{ Mpc} \) and the parameter A guarantees continuity at \( r = 6 h^{-1} \text{ Mpc} \). To obtain a continuous density field from galaxy positions with constant sampling errors throughout the sample, we have divided our spherical volume into spherical shells 10 h⁻¹ Mpc thick and have smoothed the galaxy density field using a set of Gaussian filters of increasing radius. The latter have been fixed by imposing a constant angular smoothing of about 2', as required by

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1. Obtainable from the LAMBDA website: http://lambda.gsfc.nasa.gov/
2. healpix homepage: http://www.eso.org/science/healpix/
our cross-correlation analysis. As a consequence the galaxy distribution within the spherical shells with external radii of 20, 30, 40, 50, 60, 70 and 80 \( h^{-1} \) Mpc have been interpolated on a cubic grid and smoothed with Gaussian filters of radii (FWHM) 0.7, 1.05, 1.4, 1.75 2.1, 2.45 and 2.8 \( h^{-1} \) Mpc. Equation (2) shows that this filtering procedure guarantees at least one object per resolution element, and thus keeps the shot noise errors at an acceptable level.

### 2.3 Constrained simulation

In this work we also use a third data set generated by the cosmological hydrodynamic simulation of our local universe performed by Dolag et al. (2005). The simulation is based on a previous numerical experiment performed by Mathis et al. (2002) aimed at reproducing the mass distribution within 12,000 km s\(^{-1}\) traced by the IRAS galaxies in the 1.2-Jy redshift survey (Fisher et al. 1995). Similar constrained simulations of the local universe have also been studied by Bistolas & Hoffman (1998), Kravtsov, Klypin & Hoffman (2002) and Klypin et al. (2003). The original galaxy distribution was smoothed with a Gaussian filter of radius 5 \( h^{-1} \) Mpc and traced back in time to \( z = 50 \) using the method of Kolatt et al. (1996). This initial field was used as Gaussian constraint for an otherwise random realization of a flat \( \Lambda \)CDM cosmology with \( \Omega_m = 0.3 \), Hubble constant of \( H_0 = 70 \) km s\(^{-1}\) Mpc\(^{-1}\) and a rms density fluctuation \( \sigma_8 = 0.9 \). The initial density field, specified in a box of 240 \( h^{-1} \) Mpc with a high-resolution region covering a sphere of 80 \( h^{-1} \) Mpc, has been evolved forward in time using the latest version of the GADGET code (Springel, Yoshida & White 2001) including gas physics with smoothed particle hydrodynamics in the high-resolution region. As shown by Mathis et al. (2002), the simulated mass distribution at \( z = 0 \) reproduces well the main characteristics of the most prominent nearby cosmic structures, including clusters like Virgo and Coma, the Perseus-Pisces and the Great Attractor complexes. The hydrodynamic re-simulation of these initial conditions has been used to study the propagation of cosmic rays through the local universe (Dolag et al. 2005). In this work, we have considered the simulated mass and gas distribution at \( z = 0 \) within the high-resolution spherical region of radius 80 \( h^{-1} \) Mpc and applied the same procedure adopted for the PSC\(\text{z} \) data sets, with the purpose of smoothing the mass density and gas density and temperature fields on an angular smoothing scale of 2\(^{\circ}\). To minimize any boundary effects in the smoothing procedure we have assigned gas density and temperature to dark matter density also in each resolution element beyond 80 \( h^{-1} \) Mpc according to the gas versus mass relations measured in the high-resolution region containing the simulated gas particles. The end product consists of values of the dark matter density, gas density and temperature specified at the same locations as the PSC\(\text{z} \) galaxy density field, within the same set of spherical shells and smoothed with the same Gaussian filters specified in Section 2.2.

The uncertainties connected with the electron densities and temperatures as obtained from simulations are mostly connected to the precision with which we manage to reconstruct the underlying dark matter structure. The reason for this is that the parameter of interest here is the Compton \( y \) parameter (described in detail later) which is basically the pressure of the gas which mostly depends on the dark matter potential. It was demonstrated in Mathis et al. (2002) that the mass of the most prominent clusters in the simulation agrees well with the observed mass, indicating that the well-known problem of IRAS galaxies undersampling high-density regions (Strauss et al. 1992) has been properly accounted for when setting the initial condition of the constrained simulation. In Dolag et al. (2005) it was further shown that the temperature of these clusters also agrees well (within 30 per cent) with the observations. Finally, White, Hernquist & Springel (2002) demonstrated that the integrated Compton \( y \) parameter from simulations only changes by up to 20 per cent even if various additional physical processes (cooling, star formation and feedback) are included into the simulations.

### 3 METHOD FOR OBTAINING THE SUNYAEV–ZELDOVICH EFFECT FROM THE GALAXY CATALOGUE

In this section, we describe how we use the galaxy catalogue to derive an estimate of the gas density and temperature in the local universe. From this estimate, we use a spherical projection to make a prediction for the SZ effect. The projection is made on to a HEALPIX pixelized sphere with resolution parameter \( N_{\text{side}} = 64 \) corresponding to pixel edges of roughly 0.9.

#### 3.1 Mapping the local hot gas

The goal of the smoothing procedure specified in the previous sections is to obtain a (Gaussian FWHM) 2\(^{\circ}\) resolution map of the predicted, cumulative SZ effect generated by all cosmic structures within 80 \( h^{-1} \) Mpc. As the SZ distortions are fully specified by the density and temperature of the gas along the line of sight, one may think that reliable SZ maps can be obtained directly from the constrained hydrodynamical simulations. This is certainly the case for most statistical analyses but not for cross-correlation studies like the one we wish to perform here. Indeed, the constraints in the simulations are only effective above a (Gaussian) resolution scale of 5 \( h^{-1} \) Mpc, corresponding to an angular scale \( > 3.5 \)\(^{\circ}\) within our spherical volume. As the actual positions of cosmic structures within each resolution element are essentially random, we cannot directly cross-correlate the SZ maps obtained from the hydrodynamical simulation with the temperature fluctuations of the CMB with the desired angular resolution of 2\(^{\circ}\). The PSC\(\text{z} \) galaxy density maps described in Section 2.2 can be used to circumvent this problem. The mass overdensity field, \( \delta_m \) in the constrained simulation has been obtained from the IRAS 1.2-Jy galaxy density field assuming that IRAS galaxies trace the underlying mass distribution. On the other hand Teodoro, Branchini & Frenk (2003) have shown that the IRAS 1.2-Jy and PSC\(\text{z} \) galaxy density fields are in very good agreement (apart from a monopole mismatch that does not affect our analysis) and thus that a tight relation exists between \( \delta_{\text{PSCz}} \) and \( \delta_m \). The existing monotonic and almost deterministic relation between these two fields means that we can assign a unique value of \( \delta_{1:2Jy} \) to a spatial location characterized by a value of \( \delta_{\text{PSCz}} \) through simple rank ordering techniques. Using the fact that, at each point of the hydrodynamical simulation, the mass density at \( \delta_{1:2Jy} \) is associated to the electron density \( n_e \) and the temperature \( T \) of the gas, we can map the gas physical properties at the correct spatial location. In other words, we are able to assign the simulated gas properties to the cosmic structures traced by PSC\(\text{z} \) galaxies whose angular positions are known with a precision far better than the 2\(^{\circ}\) required to perform our correlation analysis.

#### 3.2 The SZ effect from line of sight integrals

The SZ effect (Sunyaev & Zeldovich 1972) arises due to the inverse Compton scatter of CMB photons against the hot and diffuse electron gas trapped in the potential well of the cosmic web and of each cluster of galaxies. In the latter case these electrons are also
Our prediction of the Compton $y$ parameter from the local SZ effect (the colour scales indicate $\log[y]$). The plot is in galactic coordinates with galactic longitude $l = 0$ in the middle and increasing leftwards.

In Fig. 1 we show the resulting map of the Compton $y$ parameter. We can see that the most dominant structure is the Virgo cluster on the upper right side. We have also plotted the power spectrum (corrected for the 2' beam) of this map at two different frequencies in Fig. 2. The dashed line corresponds to the Q (41 GHz) frequency and the dotted line to the W (94 GHz) frequency. Because of the very limited redshift range being considered, the power spectrum starts to fall off at the smallest scales. Clearly, the closer the clusters are the larger is the scale to which they add power and we confirmed this by calculating the power spectrum for the SZ template when even lower redshift ranges were considered. As expected, in this case the spectrum falls off at even smaller multipoles. We see that the predicted SZ power spectrum is considerably lower than the CMB power spectrum (solid line) and could not be distinguished from the cosmic variance at these scales.

4 UPPER LIMITS ON THE SZ EFFECT FROM THE WMAP DATA: CROSS-CORRELATIONS IN PIXEL, HARMONIC AND WAVELET SPACE

Having obtained a map of the predicted SZ effect from gas in the local universe, the next step is to make a cross-correlation test between CMB data and the SZ map. We perform this cross-correlation in pixel, harmonic and wavelet space, using the two WMAP channels, V and W that are less foreground contaminated. We apply the test both on the optimally combined map which is dominated by the CMB and on the difference map in which the CMB disappears and only components which depend on the frequency remain.

4.1 Estimating the SZ contribution in the WMAP data: Method

Before cross-correlating the WMAP data with the SZ prediction obtained, we must smooth both maps to a common resolution. As described already, the map of the SZ effect has been smoothed with a Gaussian beam of 2' FWHM. Due to the limited resolution of the PSCz map, this is also the highest resolution which can be used in the cross-correlation procedure. For the CMB data, we have deconvolved the maps for each frequency channel with the corresponding beams (being of the order 14 arcmin FWHM) and convolved it with a 2' FWHM Gaussian beam. Before performing the

$$\frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} = y_s \xi(x),$$

where $x = h\nu/kT_{\text{CMB}}$. The Comptonization parameter is

$$y_s = \int \frac{kT}{mc^2} n_e \sigma_T \, dl,$$

where $n_e$ and $T$ are the electron density and temperature, respectively, $\sigma_T$ is the Thomson cross section, and the integral is over a line of sight. The spectral form of this ‘thermal effect’ is described by the function $s(x) = [x \coth(x/2) - 4]$, which is negative (positive) at values of $x$ smaller (larger) than $x_0 = 3.83$, corresponding to a critical frequency $\nu_0 = 217 \text{ GHz}$. At the frequencies considered here, the temperature variation is always negative with values $s(x) = -1.55$ in the W channel, $s(x) = -1.80$ in the V channel and $s(x) = -1.91$ in the Q channel. Here we neglect the relativistic corrections to the SZ effect. These corrections, in fact, start to affect the spectral shape only for gas temperatures higher than the local universe ($T > 12 \text{ keV}$) and in any case at frequencies much higher than those of the WMAP channels.

To obtain a map of the SZ effect, we need to integrate equation (1) along the line of site in all the shells described in Section 2. Each of the shells are pixelized in a three-dimensional (3D) grid of $128 \times 128 \times 128$ pixels. Before making the line of sight integral, we made a 3D linear interpolation to obtain the shells on a $1280 \times 1280 \times 1280$ cubic grid. From this, we performed the following integration:

$$y_i = \frac{1}{\Omega} \frac{k}{mc^2} \sum_{\mu l} T^i/n^l \delta\Omega_i \delta l_i,$$

where $i$ is the pixel number on the HEALPIX grid, $j$ is the grid element in the interpolated 3D grid and the sum is performed over all elements $j$ contained in the solid angle spanned by the HEALPIX pixel $i$. The temperature and electron density at gridpoint $j$ are given by $T^j$ and $n^j$ (obtained from the procedure described in Section 3.1), $\delta\Omega_i$ and $\delta l_i$ are the mean solid angles and distances for each gridpoint $j$ and finally $\Delta\Omega = 4\pi/(12N^2_{\text{side}})$ is the solid angle of each HEALPIX pixel. As all shells were smoothed with a 3D filter corresponding to a Gaussian angular beam of FWHM 2', our final map has this resolution.

Figure 1. Our prediction of the Compton $y$ parameter from the local SZ effect (the colour scales indicate log $[y]$). The plot is in galactic coordinates with galactic longitude $l = 0$ in the middle and increasing leftwards.
deconvolution/convolution operation, the map was multiplied with the Kp2 galaxy and point-source mask, and a refilling procedure, described in Hansen et al. (2004b) and Cabella et al. (2004), was applied. In the refilling procedure, the parts of the map just outside the point-source holes where mirrored into the holes. The galactic cut was filled mirroring the data north and south of the cut. After the point-source holes where mirrored into the holes. The galactic

described in Hansen et al. (2004b) and Cabella et al. (2004), was

tended 2

testing for a possible detection whereas the harmonic space
approach has the advantage that different regions of the sky

can be tested for the cross-correlation test to the HEALPIX resolution $N_{\text{side}} = 64$ (that correspond to a pixel size of 1.8'). We neglected the contribution from the noise correlation matrix to the total correlation matrix $C$ for the optimal combination map as the noise contribution is negligible compared to the CMB (which was tested in simulations). The correlation matrix was calculated using the theoretical formula for the CMB two-point correlation function. On comparison with the total correlation matrix including noise obtained from 1000 Monte Carlo simulations, we found that the error was at most a few per cent. For the difference map, we found from Monte Carlo simulations that the pixel–pixel correlations in the noise introduced by the smoothing was limited to the two to three neighbouring pixels which justifies using the diagonal approximation.

In order to test the variance of $\hat{C}$ on maps with no SZ effect, we produced 1000 simulated maps. We estimated $\hat{C}$ on each of these maps by applying the following procedure.

(i) We generated 1000 realizations of the CMB from the best-fitting WMAP power spectrum, convolving with a beam and adding noise according to the specifications of the WMAP experiment.

(ii) We multiplied the map with the Kp2 mask and performed the refilling procedure described above for the data.

(iii) We deconvolved with the beam and convolved with the 2° FWHM Gaussian beam, degrading the map to resolution parameter $N_{\text{side}} = 32$ and multiplying it with an extended galactic cut, following the same procedure as for the data.

(iv) We used this map to estimate $\hat{C}$ according to equation (9).

4.1.1 Cross-correlation in pixel space

We model the CMB map obtained by combining optimally the the V and W channels (see Section 2) as the sum of three components, CMB, noise and SZ, i.e.

$$ M_i^c = m_i^{\text{CMB}} + n_i + C_i^{\text{SZ}}, $$

where $i$ is pixel number on the HEALPix grid, $m_i^{\text{CMB}}$ is the pure CMB contribution, $n_i$ is the noise and $C_i^{\text{SZ}}$ is the predicted SZ effect normalized in such a way that $C = 1$ if the prediction is correct. Our aim is to estimate $C$ from the WMAP data. Consequently the difference map is modelled as

$$ M_i^d = n_i + C_i^{\text{SZ}}, $$

where again $m_i^{\text{SZ}}$ is normalized so that $C = 1$ is the expected value of $C$. We use a $\chi^2$-approach to estimate $C$

$$ \chi^2 = (m - Cm^{\text{SZ}})^T C^{-1}(m - Cm^{\text{SZ}}) $$

which is minimized for the best estimate $\hat{C}$

$$ \hat{C} = \frac{mC^{-1}m^{\text{SZ}}}{m^{\text{SZ}}C^{-1}m^{\text{SZ}}}. $$

The correlation matrix $C$ is given by $C_i^{ij} = \langle (m_i^{\text{CMB}} + n_i)(m_j^{\text{CMB}} + n_j) \rangle$ for the optimal combination map and $C_i^{ij} = \langle n_in_j \rangle$ for the difference map. Different sets of pixels will be included in the vector $m$ and the correlation matrix $C$ in order to test both the full sky as well as only the areas where the SZ effect is expected to be largest. It is important to say that the map resolution $N_{\text{side}} = 64$, corresponding to a pixel size of 0.9°, is much smaller that the smoothing length of 2° used to generate the SZ maps (see Section 3.1). This means that there is hardly any information at the pixel scale. This creates singularities in the pixel–pixel correlation matrix, as it corresponds to producing an overdetermined set of equations by adding equations with no extra information to a linear system. Thus for this case only, we degrade the maps used for the cross-correlation test to the HEALPix resolution $N_{\text{side}} = 32$ (that correspond to a pixel size of 1.8°).

4.1.2 Cross-correlation in harmonic space

In this second analysis we apply a spherical harmonic transform to the data and SZ map, obtaining a set of coefficients $a_{lm}$

$$ a_{lm} = \sum_i M_i Y_{lm}^i, $$

where $M_i$ is the map, $Y_{lm}^i$ are the spherical harmonic functions, all taken at pixel $i$. The sum is performed over all pixels $i$ outside the mask given by the extended Kp2 cut at resolution $N_{\text{side}} = 64$. We apply the procedure described in the previous section with the $a_{lm}$ as the elements of the vectors $m$ and the correlation matrix now being given by $C_{lm,l'm'} = \langle a_{lm} \ a_{l'm'} \rangle$. Here we use the diagonal approximation of the correlation matrix. To test the correlations at different scales, we performed the analysis in a set of different multipole ranges.
cross-correlation in wavelet space

The wavelet coefficients of a spherical function $T(\theta, \phi)$ can be defined as

$$w(R, \theta, \phi) = \int d\Omega \psi(\Delta \theta', R)T(\theta', \phi'),$$ (11)

where the integration is performed over the whole sphere in $(\theta', \phi')$, $\Delta \theta'$ is the angular distance between the points $(\theta, \phi)$ and $(\theta', \phi')$, $R$ is the wavelet scale and $\psi$ is the Spherical Mexican Hat Wavelet given by Martínez-González et al. (2002)

$$\psi(\Delta \theta, R) = \frac{1}{2\pi N(R)} \left[ 1 + \left( \frac{\gamma}{2} \right)^2 \left( 2 - \left( \frac{\gamma}{R} \right)^2 \right) e^{-\gamma^2/2R^2} \right],$$ (12)

where $\gamma = 2 \tan(\Delta \theta / 2)$ and $N(R) = R \sqrt{1 + R^2/2 + R^4/4}$. The starting point for the wavelet analysis is as earlier, the convolved and degraded CMB map. We construct the cross-correlation coefficients defined as

$$c_R = \sum_i w_i(R)w_i^{SZ}(R)$$ (13)

where $w_i$ and $w_i^{SZ}$ are the wavelet coefficient in pixel $i$ and scale $R$ for the observed (simulated) map and the SZ map, respectively. The sum is performed over all pixels outside the wavelet mask at scale $R$ defined as the Kp2 mask (without point sources) extended by 2$'$ as well as the angle 2.5$R$ (to account for inaccuracies introduced by the convolution inside the galactic cut, see Vielva et al. 2004a) around all edges.

As for the previous tests, we aim at estimating the constant $C$ by minimizing a similar $\chi^2$ function to obtain the following best estimate $\hat{C}$

$$\hat{C} = \frac{c^{C} - 1 c^{SZ}}{c^{C} - 1 c^{C^{SZ}}},$$ (14)

where the elements of the vector $c$ are the coefficients $c_R$ of equation (13), the elements of the vector $c^{SZ}$ are given by $c^{SZ}_R = \sum_i (w^{SZ}_i(R))^2$ and the scale–scale correlation matrix $C_{RR} = \langle c^T c \rangle$ is obtained from 1000 Monte Carlo simulations. We also estimate $\hat{C}$ independently at each scale $R$ to detect possible correlations limited to certain scales.

4.2.2 Cross-correlation in harmonic space

In the previous section we showed that if we take the whole map into account, we do not find any significant cross-correlation between the CMB and the predicted SZ effect. If some multipole ranges are contaminated by the Galaxy or if for some reason the SZ prediction is too inaccurate at certain scales, the pixel space test could give a negative result even if cross-correlation can be seen at some multipoles. In this section, we check the possibility of detecting the SZ effect in certain limited multipole ranges. In Fig. 4 (left) we show the distribution of the estimates $\hat{C}$ for 1000 simulations of CMB in the 12 different multipole ranges indicated in Table 3, the grey bands correspond to the 1 and 2$\sigma$ levels. The crosses show the data values. Note that for some of the multipole ranges, we have excluded the first five multipoles which could have galactic residuals present (see i.e. Schwarz et al. 2004; Hansen, Banday & Gorski 2004a). The results of a similar test on the difference map are shown in the same figure (right plot).

As the CMB is dominating the variance on the largest scales whereas the noise dominates the variance on smaller scales, the map ‘contaminated’ with CMB (left plot) has larger variance than the difference map (right plot) for the large scales. For the smaller scales, we see that the optimal map containing CMB does better. As in the previous section, we do not find any significant detection in any multipole range, but note further that the error bars on some ranges are too large to draw any conclusions about the cross-correlation. Looking at the right plot, all the data points seem to be above zero, and there is also a 2$\sigma$ outlier for multipole range 12. This could at first sight appear to be a detection, but comparing this to simulations using a $\chi^2$ procedure it turns out that that 50 per cent of the simulations have a higher $\chi^2$ value.
Figure 3. The masks used in the pixel space analysis. The mask (a) shown in the upper figure excludes only the pixels inside the extended $K_p2$ cut. The mask (b) shown in the middle excludes in addition all pixels where the expected Compton parameter $y < 10^{-7}$ and finally the mask (c) shown in the lower figure excludes all pixels with $y < 10^{-6}$.

Table 1. Estimates of $\hat{C}$ in pixel space with 1σ errors.

| Mask              | a       | b       | c       |
|-------------------|---------|---------|---------|
| Combined map      | $0.40 \pm 1.03$ | $0.36 \pm 1.12$ | $-0.45 \pm 1.60$ |
| Difference map    | $-1.00 \pm 1.44$ | $0.00 \pm 1.44$ | $-1.69 \pm 1.69$ |
| Combining the two estimates | $-0.07 \pm 0.85$ | $0.23 \pm 0.93$ | $-0.46 \pm 1.66$ |

Table 2. The upper limit on $\Delta T$ in $\mu$K at 61 GHz at the 2σ confidence level.

| Mask              | a   | b   | c   |
|-------------------|-----|-----|-----|
| Combined map      | 0.49 | 4.71 | 25.7 |
| Difference map    | 0.38 | 5.21 | 15.8 |
| Combining the two estimates | 0.33 | 3.78 | 26.7 |
Figure 4. The estimate of $C$ in harmonic space using the co-added V+W map (left plot) and the difference map (right plot). The grey bands show the 1 and 2σ level set from 1000 Monte Carlo simulations of pure CMB maps, the dotted line shows the average of 1000 simulated CMB maps with the expected level of SZ effect added. The crosses show the result from cross-correlation with the WMAP data. The multipole ranges corresponding to the abscissa values are given in Table 3.

Table 3. Multipole ranges.

| bin  | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|------|----|----|----|----|----|----|----|----|----|----|----|----|----|
| $\ell_{\text{min}}$ | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 |
| $\ell_{\text{max}}$ | 10 | 20 | 30 | 40 | 63 | 10 | 20 | 30 | 40 | 63 | 20 | 30 | 40 |

4.2.3 Cross-correlation in wavelet space

Fig. 5 is analogous to Fig. 4 for different wavelet scales. We see that wavelets give much stronger limits on the constant $C$ at a single scale than does the harmonic space test, but still we do not find any detection at any scale. In addition, for the difference map, we can only put upper limits. Combining all the scales as described in Section 4.1.3 gives $C = 0.87 \pm 1.3$ for the combined map and $C = 0.17 \pm 1.69$ for the difference map, all at 1σ, in agreement with the pixel space analysis. This was expected as the results from cross-correlation tests in different spaces are highly correlated when all the information is included. Note again that the for the larger scales, the map containing CMB has the larger variance whereas for the smaller scales the difference map containing only noise has the largest variance.

5 CONCLUSION

We have used the galaxy density field obtained from the PSCz galaxy survey and hydrodynamical simulations based on the observed distribution of IRAS 1.2-Jy galaxies to make a full-sky prediction of the SZ effect from diffuse hot gas in the local universe ($<80\,h^{-1}\,\text{Mpc}$). We have studied whether this effect could be observed and studied by the CMB observations performed by the WMAP satellite and if
it could be a contaminating factor in the study of the CMB at large angular scales. We have cross-correlated our map of the predicted local SZ effect with the WMAP data at different frequencies taking into account the frequency dependence of the SZ effect. We have performed the cross-correlation test in three spaces, enabling us to test certain parts of the sky (pixel space test), certain multipole ranges (harmonic space test) and certain wavelet scales (wavelet test). Applying the pixel-based cross-correlation test, we checked the full sky including all pixels outside of an extended galactic cut as well as smaller areas of the sky where the SZ effect was expected at a higher level. None of these tests gave a significant detection. The harmonic space test has the advantage that cross-correlation occurring only at certain scales can be detected, but no detection was found for any multipole range. Finally, the wavelet transform is very efficient at amplifying structures on a specific scale and the cross-correlation test in wavelet space could reveal a possible correlation between the CMB and the SZ effect from local hot gas, but no correlation was found at any scale. We found that the different tests have a very similar capability of detecting possible cross-correlation when all the information is added together and the three tests are strongly correlated as the same information is used in three different spaces. But as the map of the SZ effect could be inaccurate at certain scales or certain parts of the sky due to limited knowledge of temperature and gas densities, a test of cross-correlation at different scales or at different spatial positions is necessary.

In general, we find the predicted effect too small to give a clear detection using the 1-yr WMAP data and only an upper limit can be set: at the 2σ confidence level, the SZ effect from diffuse hot gas in the local universe cannot exceed our predictions by more than 63 per cent, i.e. the mean temperature decrement at 61 GHz is ΔT < 0.33 μK. We find that the power spectrum of our SZ prediction in the WMAP channels is 3–4 orders of magnitude lower than the power spectrum of the CMB. The local SZ effect is hence far too small to influence the estimate of the cosmological parameters from the WMAP data. Our findings are in agreement with Hernandez-Monteagudo & Rubino-Martin (2004), concluding that only an upper limit can be set on the SZ effect on supercluster scales in the 1-yr WMAP data.

In this paper we have combined constrained simulations with data from the PSCz galaxy catalogue to obtain the temperature and gas densities. One could also obtain maps of gas in the local universe from constrained simulations directly. The problem with this approach is that the structures are slightly shifted with respect to their real positions. For this reason, such maps are not very useful for cross-correlation studies, but they can be utilized for studying general statistical properties of the SZ effect from the local universe (Dolag et al., 2005). This was also studied using random Hubble volume simulations by Schäfer et al (2004), where they were able to go deeper and with smaller scales than in the work presented here.

We also note that the WMAP frequency range is very limited. The Planck satellite experiment, scheduled for launch in 2007, will provide high-resolution maps of the CMB in the frequency range from 30 to 857 GHz. The SZ effect is zero at 217 GHz, negative below and positive above. By taking advantage of a set of frequencies below and above the zero point, we expect to greatly enhance the detection power. Furthermore, some of the Planck channels are expected to have a considerably lower noise level than the WMAP data which will improve the possibilities for seeing small signals such as the SZ effect from gas in the local universe. Finally, the huge range of frequency channels planned for the Planck experiment will allow the subtraction of galactic foregrounds with a much higher precision. In this way, the risks for false detections due to foreground residuals is greatly reduced. In addition, the SZ map prediction can be improved by merging all-sky galaxy redshift catalogues like PCSz used here with all-sky cluster catalogues like the Abell/ACO one. Not only CMB data will be improved in the future, but the galaxy surveys will also go deeper with a high resolution. By including all these effects, we expect that the SZ map data could be even be used for studying the properties of gas in the local universe. This will be investigated in a future paper.

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