Search for two-scale localization in disordered wires in a magnetic field

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The supersymmetry technique [A. V. Kolesnikov and K. B. Efetov, Phys. Rev. Lett. 83, 3689 (1999)] predicts a two-scale behavior of wavefunction decay in disordered wires in the crossover regime from preserved to broken time-reversal symmetry. We have tested this prediction by a transmission approach, relying on the Borland conjecture that relates the decay length of the transmittance to the decay length of the wavefunctions. Our numerical simulations show no indication of two-scale behavior.

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In a remarkable paper [1], Kolesnikov and Efetov have predicted that the decay of wavefunctions in disordered wires is characterized by two localization lengths, if time-reversal symmetry is partially broken by a weak magnetic field. Using the supersymmetry technique [2] it was demonstrated that the far tail of the wavefunctions decays with the length $\xi_2$ characteristic for completely broken time-reversal symmetry—even if the flux through a localized area is much smaller than a flux quantum. At shorter distances the decay length is $\xi_1 = \frac{1}{2} \xi_2$. It was suspected that previous studies by Pichard et al. [3] found single-scale decay because of the misguiding theoretical expectation of such behavior. This expectation was also the basis for the interpretation of the experiments by Khavin, Gershenson, and Bogdanov [4] on submicron-wide wires.

The prediction of Kolesnikov and Efetov calls for a test by means of a dedicated experiment or computer simulation. It is the purpose of this work to provide the latter. We target the key feature of the two-scale localization phenomenon, which is the doubling of the asymptotic decay length at infinitesimally weak magnetic fields.

Our numerical simulations are based on a transmission approach. We rely on the Borland conjecture [5] (believed to be true generally [6]), that relates the asymptotic decay of the transmittance $T(L)$ to the asymptotic decay of the wavefunction $\psi(L)$. According to the Borland conjecture, the Lyapunov exponent $\alpha = -\lim_{L \to \infty} \frac{1}{L} \ln T(L)$ is identical to the inverse localization length $\xi^{-1} = -\lim_{L \to \infty} L^{-1} \ln |\psi(L)|$. Moreover, $\xi$ and $\alpha$ are self-averaging, meaning that the statistical fluctuations become smaller and smaller as $L \to \infty$. Our numerical simulations show that the crossover from $\xi = \xi_1$ to $\xi = \xi_2$ does not occur until the flux $\Phi_0$ through a wire segment of length $\xi_1$ is of the order of a flux quantum $\Phi_0 = h/e$. For our longest wires ($L \gtrsim 150 \xi_1$) the crossover according to Ref. [6] should have occurred at $\Phi_0 \approx \exp(-L/8 \xi_1) \approx 10^{-20}$. We consider various possible reasons for the disagreement with Ref. [6] (finite number of modes, anomalously localized states), but believe that none of these provides a satisfactory explanation.

Our first set of results is obtained from the numerical calculation (by the technique of recursive Green functions [7]) of the transmission matrix $T$ for a two-dimensional Anderson Hamiltonian with on-site disorder. In units of the lattice constant $a = 1$, the width of the wire is $W = 13$ and the wavelength of the electrons is $\lambda = 5.1$, resulting in $N = 5$ propagating modes through the wire. The localization lengths $\xi_1 = (N + 1) \lambda$ and $\xi_2 = 2N \lambda$ are determined by the scaling parameter $l$ of quasi-one dimensional localization theory, which differs from the

FIG. 1. Average logarithmic transmittance $\langle \ln T \rangle$ as a function of wire length $L$ for the Anderson model with $N = 5$ propagating modes. The two dashed lines have the slopes predicted for preserved ($\beta = 1$) and broken ($\beta = 2$) time-reversal symmetry. From bottom to top the data corresponds to fluxes $\Phi_0 = 0, 0.0005, 0.005, 0.05$ (four indistinguishable solid curves), 0.5, 1, 2.5, 5, 10, 15, 20, 25, 40, 50, 75, 125 (two indistinguishable solid curves). The inset shows $\ln T$ for an individual realization with $\Phi_0 = \frac{1}{3} \Phi_0$ (solid curve) and the slope of the ensemble-averaged result (dashed line).
transport mean-free path by a coefficient of order unity \[8\]. The average of the transmittance \(T = \text{tr} t t^\dagger\) in the metallic regime, fitted to \((T) = N(1 + L/l)^{-1}\), yields \(l = 65\). This gives a localization length \(\xi_1 = 390\) for preserved time-reversal symmetry (symmetry index \(\beta = 1\)) and a localization length \(\xi_2 = 650\) for broken time-reversal symmetry (\(\beta = 2\)).

Fig. 2 shows the ensemble-averaged logarithm of the transmittance \((\ln T)\) as a function of wire length \(L\) for various values of the magnetic field \(B\) (or flux \(\Phi_\xi = W\xi_1 B\)). We find a smooth transition between the theoretical expectations for preserved and broken time-reversal symmetry. Most importantly, we find an asymptotic slope \(s(B) = \lim_{L \to \infty} L^{-1} \langle \ln T \rangle\) that interpolates smoothly between the values \(s = -2/\xi_1\) for \(B = 0\) and \(s = -2/\xi_2\) for large \(B\). There is no indication of a crossover to the slope \(s = -2/\xi_2\) for smaller values of \(B\), even for very long wires \((L \gtrsim 150\xi_1)\). According to the theory of Ref. [1], the crossover should occur at a length \(L_{\text{cross}}\) given by

\[
L_{\text{cross}}/\xi_1 = 8 \ln(\sqrt{12\Phi_0}/4\pi\Phi_\xi) + O(1),
\]

which is well within the range of our simulations \((L_{\text{cross}} \approx 14\xi_1\) for \(\Phi_\xi \approx 0.05\Phi_0\)). The absence of two-scale behavior in the transmittance of an individual, arbitrarily chosen realization is demonstrated in the inset of Fig. 1 for \(\Phi_\xi = \frac{1}{2}\Phi_0\). The self-averaging property of the Lyapunov exponent is evident.

The asymptotic decay length \(\xi(B) = -2/s(B)\) is plotted versus magnetic field in Fig. 2, together with the weak-localization correction \(\delta T = T(B = \infty) - T(B)\) at \(L = \xi_1\). For both quantities, breaking of time-reversal symmetry sets in when \(\Phi_\xi\) is comparable to \(\Phi_0\). The transition from \(\beta = 1\) to \(\beta = 2\) is completed for \(\Phi_\xi \approx 100\Phi_0\).

Our second set of results is obtained from a computationally more efficient model of a disordered wire, consisting of a chain of chaotic cavities (or quantum dots) with two leads attached on each side. This so-called ‘domino’

model [9] is similar to Efetov’s model of a granulated metal [10] and to the Iida-Weidenmüller-Zuk model of connected slices [11]. The length \(L\) is now measured in units of cavities, and the mean free path \(l = 1\). The scattering matrices of each cavity are randomly drawn from an ensemble (proposed by Życzkowski and Kuś [11]) that interpolates (by means of a parameter \(\delta\)) between the circular orthogonal (\(\beta = 1\), \(\delta = 0\)) and unitary (\(\beta = 2\), \(\delta = 1\)) ensembles of random-matrix theory. The relationship between \(\delta\) and \(\Phi_\xi/\Phi_0\) is linear for \(\delta \ll 1\).

We increased the number of propagating modes to \(N = 50\), because it is conceivable that the two-scale localization becomes manifest only in the large \(N\)-limit, or that only in this limit the critical flux \(\Phi_\xi\) for the transition from \(\xi_1\) to \(\xi_2\) becomes \(\ll \Phi_0\). (In the experiments of Ref. [1] \(N \approx 10\), so our simulations are in the experimentally relevant range of \(N\).) Because of the much larger value of \(N\), we restricted ourselves for larger values of the magnetic flux to \(L \gtrsim 25\xi_1\), which should be sufficient to observe the localization length \(\xi_2\) for \(\Phi_\xi/\Phi_0 \gtrsim 10^{-2}\). For
smaller values of the flux, we increased the wire length to \( L \simeq 100\xi_1 \). The data is presented in Fig. 2. It is qualitatively similar to the results for the \( N = 5 \) Anderson model. Instead of two-scale behavior, we only see a single decay length which crosses over smoothly from \( \xi_1 \) to \( \xi_2 \) with increasing \( \delta \). Again, the crossover of \( \xi \) coincides with the crossover of the weak-localization correction, so there is no anomalously small crossover flux for the localization length.

The logarithmic average \( \langle \ln T \rangle \) is the experimentally relevant quantity since it is representative for a single realization (see Fig. 1, inset). The average transmittance \( \langle T \rangle \) itself is not representative, because it is dominated by rare occurrences of anomalously localized states \([12]\). Since Kolesnikov and Efetov \([13]\) studied the average of wavefunctions themselves, rather than the average of logarithms of wavefunctions, it is conceivable that their findings are the result of such rare occurrences. For completely broken or fully preserved time-reversal symmetry the average transmittance is given by

\[
\ln\langle T \rangle = -L/2\xi_\beta - \frac{\beta}{2}\ln L/\xi_\beta + O(1). \tag{2}
\]

The order 1 terms are also known \([13,14]\) and contribute significantly for \( L \lesssim 30\xi_1 \). (This is the numerically accessible range, because anomalously localized states become exponentially rare with increasing wire length.) We have plotted the full expressions in Fig. 3 (dashed curves), together with the numerical data for the \( N = 5 \) Anderson model. Again we find a smooth crossover between preserved and broken time-reversal symmetry. There is no transition with increasing wire length to a behavior indicative of completely broken time-reversal symmetry, even though the flux \( \Phi_\xi \) is much larger than required (according to Eq. \([1]\)) to observe this crossover for the wavefunctions.

In conclusion, we have presented a numerical search for the two-scale localization phenomenon predicted by Kolesnikov and Efetov \([1]\), with negative result: The asymptotic decay length of the transmittance is found to be given by \( \xi_1 \) and not by \( \xi_2 \), as long as the flux through a localization area is small compared to the flux quantum. How can one reconcile this numerical finding with the result of the supersymmetry theory? We give three possibilities. (1) One might abandon the Borland conjecture and permit the asymptotic decay length of the transmittance (Lyapunov exponent) to differ from the asymptotic decay length of the wavefunction (localization length). Since the Borland conjecture has been the cornerstone of localization theory for more than three decades, this seems a too drastic solution. (2) One could argue that the wires in the simulation are too narrow or too short — although they are in the experimentally relevant range of \( N \) and \( L \). (3) One could attribute the two-scale localization phenomenon to anomalously localized states, that are irrelevant for a typical wire. We can not fully exclude these two remaining possibilities, but they would severely diminish the experimental relevance of the phenomenon.

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