Supersymmetric Strings and Fermionic Zero-Modes

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Abstract

I review recent work concerning the microphysics of cosmic string solutions of $N = 1$ supersymmetric gauge field theories.

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I. INTRODUCTION

Topological soliton solutions of gauge field theories may have played an important role in the evolution of the universe. In addition to the well-known gravitational effects of such objects, the microphysics of topological defects can also have cosmological consequences.

In light of continued theoretical and experimental interest in supersymmetry (SUSY), it seems natural to explore the properties of topological solitons in SUSY field theories. In this short review I focus on such an analysis in a simple abelian model with one SUSY generator ($N = 1$).

II. A SUSY ABELIAN HIGGS MODEL AND COSMIC STRINGS

This theory consists of a vector superfield $V(A_{\mu}, \lambda, D)$ and three chiral superfields $\Phi_i(\phi_i, \psi_i, F_i)$, ($i = \pm, 0$), with $U(1)$ charges $q_i$. Here, $\phi_i$ are complex scalar fields and $A_{\mu}$ is a vector field. These correspond to the familiar bosonic fields of the abelian Higgs model. The fermions $\psi_{i\alpha}, \bar{\lambda}_{\alpha}$ and $\lambda_{\alpha}$ are Weyl spinors. The gauge symmetry is broken through an $F-$term generated by the holomorphic superpotential $W(\Phi_i) = \mu \Phi_0 (\Phi_+ \Phi_- - \eta^2)$, with $\eta$ and $\mu$ real. In this model the potential is minimized for $F_i = 0$ and $D = 0$.

Setting all the fermion fields to zero, there exists a Nielsen-Olesen cosmic string solution with ansatz

$$\phi_+ = \phi_- = \eta e^{i \varphi} f(r),$$
$$A_{\mu} = -\frac{2}{g} \frac{a(r)}{r} \delta_{\mu}^\varphi,$$
$$F_0 = \mu \eta^2 (1 - f(r)^2).$$

All other fields are zero and the profile functions obey the usual differential equations of the abelian-Higgs model with boundary conditions $f(0) = a(0) = 0$ and $\lim_{r \to \infty} f(r) = \lim_{r \to \infty} a(r) = 1.$
III. FERMION ZERO MODES AND SUSY TRANSFORMATIONS

The fermionic sector has Yukawa couplings

$$\mathcal{L}_Y = i g \sqrt{2} \left( \bar{\phi} \psi^+ - \bar{\phi} \psi^- \right) \lambda - \mu \left( \phi_0 \psi^+ + \phi_0 \psi^- + \phi \psi_0 \psi^- + \phi \psi_0 \psi^+ \right) + (c.c.) \quad (3.1)$$

and, as with a non-SUSY theory, non-trivial zero energy fermion solutions can exist around the string. Consider the fermionic ansatz

$$\psi_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_i(r, \varphi), \quad \lambda = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \lambda(r, \varphi).$$

If we can find solutions for the $\psi_i(r, \varphi)$ and $\lambda(r, \varphi)$ then, following Witten, we know that solutions of the form

$$\Psi_i = \psi_i(r, \varphi) e^{i \chi(z+t)}, \quad \Lambda = \lambda(r, \varphi) e^{i \chi(z+t)}, \quad (3.2)$$

with $\chi$ some function, represent left moving superconducting currents flowing along the string at the speed of light. Thus we must solve for $\psi_i(r, \varphi)$ and $\lambda(r, \varphi)$.

Consider performing an infinitesimal SUSY transformation with infinitesimal Grassmann parameters $\xi, \bar{\xi}$. This induces a gauge transformation and we undo this using a SUSY gauge parameter $\Lambda = ig \bar{\xi} \bar{\sigma}^\mu \theta A_\mu(y)$. Writing everything in terms of the background string fields and keeping only terms up to first order, the string fields are unchanged and the fermion fields become

$$\lambda_\alpha \rightarrow \frac{2na'}{gr} i(\sigma^z)^{\beta}_{\alpha \beta} \xi_\beta, \quad (3.3)$$

$$\left(\psi^\pm\right)_\alpha \rightarrow \sqrt{2} \left( if'\sigma^r \mp \frac{n}{r}(1 - a)f\sigma^e \right)_{\alpha \beta} \xi_\beta \eta e^{\pm in\varphi}, \quad (3.4)$$

$$\left(\psi^0\right)_\alpha \rightarrow \sqrt{2} \mu \eta^2 (1 - f^2) \xi_\alpha, \quad (3.5)$$

where $\sigma^r$ and $\sigma^e$ are the cylindrical Pauli matrices.

If we choose $\xi_2 = 0$ and $\xi_1 = -i \delta/(\sqrt{2}\eta)$, where $\delta$ is a complex constant, we obtain

$$\lambda_1 = \delta \frac{n \sqrt{2} a'}{\eta r}, \quad (3.6)$$

$$\left(\psi^\pm\right)_1 = \delta^* \left[ f' \mp \frac{n}{r}(1 - a)f \right] e^{i(\pm n - 1)\varphi}, \quad (3.7)$$

$$\left(\psi^0\right)_1 = -i \delta \mu \eta (1 - f^2). \quad (3.8)$$

It is these fermion solutions which are responsible for the string superconductivity. Similar expressions can be found when $\xi_1 = 0$. Note that these solutions can be checked in the special case of the Bogomolnyi limit and they agree with those found by previous authors.
IV. CONCLUDING REMARKS

Supersymmetric Abelian-Higgs models naturally possess cosmic string solutions which carry fermionic supercurrents. This result also holds in nonabelian models where the structure is considerably richer. \[5\]

In a cosmological context such strings may settle into vorton states and thus may be cosmologically catastrophic. A potential solution is that it appears that the zero modes responsible for superconductivity vanish when supersymmetry is broken. \[6\]

Finally, simple potentials of the type described here can also give rise to a separate cosmological regime of hybrid inflation. When such inflation ends the strings form. In such a scenario density fluctuations from both the inflationary epoch and the defect dynamics should be taken into account.

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REFERENCES

[1] S.C. Davis, A.-C. Davis and M. Trodden, Phys. Lett. B 405, 257 (1997).
[2] H. Nielsen and P. Olesen, Nucl. Phys. B 61, 45 (1973).
[3] E. Witten, Nucl. Phys. B 249, 557 (1985).
[4] J. Garriga and T. Vachaspati, Nucl. Phys. B 438, 161 (1995).
[5] S.C. Davis, A.-C. Davis and M. Trodden, “Cosmic Strings in $N = 1$ Supersymmetric Yang-Mills-Higgs Theories”, in preparation (1997).