Modulated superconductivity near Pomeranchuk instabilities in the spin channel

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We study the competition between a Pomeranchuk instability in the spin channel with angular momentum $\ell = 1$ and an attractive interaction, favoring Cooper pair formation. We find, at mean-field approximation, that superconductivity strongly suppress the Pomeranchuk instability. Moreover, we have found a metastable modulated superconducting phase with similar characteristics of the FFLO state.

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A Fermi liquid is, except in one dimension, a very stable state of matter. At least two types of instabilities, related with attractive interactions, are known: Pomeranchuk\textsuperscript{2} and superconducting instabilities. Pomeranchuk instabilities occur in the presence of two-body interactions containing a strong attractive component in the forward scattering channel with a definite angular momentum. In the context of Landau theory, the instability sets in when one or more dimensionless Landau parameters $F_\ell^{\sigma a}$, with angular momentum $\ell$ in the charge ($s$) or spin ($a$) channel, acquire large negative values.

Pomeranchuk instabilities in a charge sector spontaneously break rotational symmetry. In particular, an instability in the $F_2^1$ channel produces an ellipsoidal deformation of the Fermi surface.\textsuperscript{11} From a dynamical point of view, the resulting anisotropic ground state is a non-Fermi liquid, due to overdamped Goldstone modes that wipe out quasi-particle excitations.\textsuperscript{2}

In the spin channel, the ferromagnetic Stoner instability occurs when $F_0^0$ acquires large negative values, producing a divergence in the magnetic susceptibility. This phase transition preserves rotational symmetry, however it breaks time-reversal symmetry. Higher order angular momentum interactions produce anisotropic as well as isotropic phases. Several examples were studied in detail in refs. \textsuperscript{11} and \textsuperscript{12}. The $\ell = 1$ channel have special interest. When $F_1^a < 0$, an ordered isotropic and time-reversal invariant phase is possible. This phase, called $\beta$-phase in ref. \textsuperscript{11} dynamically generates a spin-orbit coupling. This is a very interesting possibility, since it allows the generation of spin-orbit couplings from many-body correlations, different from the usual one-particle relativistic effect.

On the other hand, superconductivity is developed in the presence of a small attractive interaction in the particle-particle (BCS) channel. The superconducting (SC) state is generally characterized by a complex order parameter which breaks gauge symmetry, $\Delta_{\sigma,\sigma'}(\vec{r},\vec{r}') = \langle \psi_{\sigma'}^\dagger(\vec{r}) \psi_{\sigma'}(\vec{r}') \rangle$, where the operator $\psi_{\sigma'}^\dagger(\vec{r})$ creates an electron with spin $\sigma$ at the position $\vec{r}$. The usual classification of $\Delta_{\sigma,\sigma'}(\vec{r},\vec{r}')$ as s-wave, d-wave, p-wave, etc, resides in the irreducible representations of the lattice point group. Also, the absence of spin-orbit interactions allows the additional differentiation between singlet and triplet order parameters. However, the SC state could break lattice translation and/or rotational symmetry. In that case, this classification is no more possible. One particular example is an oscillating order parameter like $\Delta_{\sigma,\sigma'}(\vec{r},0) = \Delta_{\sigma,\sigma} \cos(\vec{q} \cdot \vec{r})$, where $\vec{q}$ is an ordering wave-vector. In a modulated superconducting state, proposed by Fulde, Ferrell, Larkin and Ovchinnikov\textsuperscript{16-18} (FFLO state), the spatial modulation of the Cooper pairing is due to a mismatch of Fermi surfaces, produced by an external magnetic field (by Zeeman effect). FFLO states have also been proposed to occur in other scenarios like in imbalanced cold atoms with different species\textsuperscript{19} and in heavy fermions systems when different orbitals hybridize under external pressure.\textsuperscript{19} Also, for generally non-local BCS potentials, a modulated SC order parameter can coexist with charge density waves.\textsuperscript{20} Recently, a striped order parameter, called “Pair Density Wave, (PDW)”, was proposed\textsuperscript{21,22} to explain anomalous transport properties observed in cuprates superconductors.

Although Pomeranchuk and BCS instabilities are generated by attractive interactions, they are competing ones. A superconducting gap suppresses Fermi surface deformation. A detailed example of this effect was studied in refs. \textsuperscript{19} and \textsuperscript{20} where the competition between d-wave Pomeranchuk instabilities and d-wave superconducting order parameter was considered.

In the spin channel, a stronger competition is expected, since, in general, magnetic order is expelled from a superconductor state. However, it is possible to have instabilities with higher angular momentum that preserve time-reversal invariance. In this paper, we analyze an example of this class of systems. In particular, the $\beta$-phase introduced in ref. \textsuperscript{11} opens the possibility of the formation of Cooper pairs with zero helicity and finite momentum, possibly producing a modulated superconducting state.

The main point of this paper is to report on the com-
two-body interactions in this channel

\[ H = \int d^2 x \psi^\dagger (x) \left( \epsilon_0 - \mu \right) \psi (x) + \]
\[ + \frac{1}{2} \int d^2 x d^2 x' f_n^\dagger (x - x') V^{a \mu} (x) V^{a \mu} (x') \, . \]  

The Fourier transform of \( f_n^\dagger (x) \) is given by \( f_n^\dagger (k) = f_n^\dagger (|k| r) \), defining, in this way, an effective interaction range \( r = \sqrt{\kappa |f_n^\dagger|} \). For a particle-hole symmetrical system we consider the following expansion of the dispersion relation around a circular Fermi surface,

\[ \epsilon (k) - \mu = \bar{\nu}_F |\vec{k} - \vec{k}_F| + \frac{b}{(v_F k_F)^2} \left( \bar{\nu}_F \cdot [\vec{k} - \vec{k}_F] \right)^3 \, , \]  

where the dimensionless parameter \( b \) measures the effective curvature of the band near the Fermi surface and we have ignored terms proportional to \((k - k_F)^3\).

The \( \beta \)-phase is defined by the mean-field Hamiltonian

\[ H_{MF} = \int \frac{d^2 k}{(2 \pi)^2} \psi^\dagger (k) \left( \epsilon (k) - \mu - \bar{n} \vec{\sigma} \cdot \hat{k} \right) \psi (k) \, , \]  

where \( \bar{n} \) is determined self-consistently by

\[ \bar{n} = - \frac{1}{2} f_n^\dagger (0) \int \frac{d^2 k}{(2 \pi)^2} \psi^\dagger (k) (\vec{\sigma} \cdot \hat{k}) \psi (k) \, . \]  

This mean-field theory is valid when \( k_F \sqrt{\kappa |f_n^\dagger|} \gg 1 \), i.e., when the range of the interaction is much larger than the interparticle distance.

In references 9 and 10, eq. (5) was solved by considering a Fermi liquid ground state. When \( \bar{n} \neq 0 \) the spectrum of \( H_{MF} \) splits into two opposite chiralities with dispersions \( c^\dagger = \epsilon (k) - (\mu + \bar{n}) \) and \( c^\dagger = \epsilon (k) - (\mu - \bar{n}) \), as shown in figure (2). The Fermi momentum of each branch is given by \( k_F^\pm = k_F \pm q / 2 \) and the relation between \( q \) and \( \bar{n} \) is computed using eq. (6),

\[ \frac{v_F q}{2} = \bar{n} - \frac{b}{(v_F k_F)^2} \bar{n}^3 + O(\bar{n}^5) \, , \]  

(we have renormalized the chemical potential to keep the density constant).

The ordered phase is characterized by a spontaneously generated spin-orbit coupling with the global rotation invariance unbroken. The spin and orbital angular momentum are not conserved independently, however the total angular momentum \( \vec{J} = \vec{L} + \vec{S} \) is conserved. In the helicity basis \( \zeta \equiv (\zeta_1, \zeta_2) \), where the operator \( \vec{k} \cdot \vec{\sigma} \) is diagonalized with eigenvalues \pm 1, the mean-field Hamiltonian takes the simpler form

\[ H_{MF} = \int \frac{d^2 k}{(2 \pi)^2} \zeta^\dagger (k) \left( \epsilon (k) - \mu - \bar{n} \sigma^z \right) \zeta (k) \, , \]  

with the self-consistent equation

\[ \bar{n} = - \frac{1}{2} f_n^\dagger (0) \int \frac{d^2 k}{(2 \pi)^2} \zeta^\dagger (\sigma^z \zeta) \, . \]
To find a self-consistent solution, it is necessary to solve the single-particle excitations read

\[ \omega \pm = \left( \frac{\epsilon_{k+q/2}^0 - \epsilon_{k-q/2}^0}{2} \right) \pm \xi, \]  

where \( \Theta \) is the usual Heaviside function and single-particle excitations read

\[ \xi = \sqrt{\left( \frac{\epsilon_{k+q/2}^0 + \epsilon_{k-q/2}^0}{2} \right)^2 + |\Delta_q|^2}. \]  

It is simple to realize that the s-wave BCS order parameter \( n = 0, \Delta_q = \Delta_0 \) is always a solution of eqs. (11) and (12). On the other hand, in the absence of superconductivity, \( \Delta_0 = 0 \), a solution with finite \( n \sim q \) has lower energy for \( F_1^0 < -2 \), leading to the \( \beta \)-phase. In this regime and for small values of the BCS coupling constant, the \( \beta \)-phase has always smaller energy than the uniform superconducting one. For stronger superconducting coupling, the \( \beta \)-phase is suppressed and the mean-field ground state is a uniform superconducting phase, as shown in figure (1). The first order line that separates these two phases was evaluated by equating the mean-field Hamiltonians, \( \langle H \rangle_{\Delta_0} = \langle H \rangle_{\beta} \), computed in the two possible solutions: uniform superconductor \( \{ n = 0, \Delta_q = \Delta_0 \} \) and the \( \beta \)-phase \( \{ n \sim (v_F k_F/\hbar)^{1/2} \sqrt{1 - 2|F_1^0|}, \Delta_q = 0 \} \), respectively. Note that the band curvature at the Fermi surface \( (b) \) is essential to stabilize the \( \beta \)-phase.

Now, we her the interesting possibility of a modulated solution \( n \sim q \neq 0, \Delta_q \neq 0 \). The \( k \)-integrals in equations (11) and (12) are strongly constrained by the Heaviside functions. The main contribution to eq. (11) comes from the region \( \omega_q < 0 \). Written in polar coordinates \( (k, \theta) \), with \( \cos \theta = (\vec{k} \cdot \vec{q})/kq \), and first integrating over \( k \), we get

\[ n = \frac{|F_1^0|}{2} \int_{\theta_-}^{\theta_+} \frac{d\theta}{2\pi} \left[ C_0(x) - \frac{b}{(v_F k_F)^2} n^2 C_1(x) \right] + O((\frac{n}{v_F k_F})^5), \]  

with

\[ C_0(x) \sim 1 - (x/2)^{3/2}, C_1(x) = 1 + x \ln(1+x/5) \] and \( F_1^0 = f_1^0 N(0) \), where \( N(0) \) is the density of states at the Fermi surface.

On the other hand, the integral in eq. (12) has the usual ultraviolet divergence of the BCS gap equation. A convenient way to deal with this integral is to sum and subtract \( \Theta(\omega_-) \) in the second term and then to subtract the identity

\[ -1 + g \int \frac{d^2k}{(2\pi)^2} \frac{1}{2\beta} = 0 \]  

from the first term of eq. (12). \( \xi = \sqrt{\epsilon_{k+q/2}^0 + |\Delta_0|^2} \) and \( \Delta_0 \) is the uniform superconducting gap in the absence of...
we note that for the need to solve eqs. (16) and (19) self-consistently. Firstly, This imposes a lower limit for modulated superconductivity appears as a metastable phase for finite values of ∆0 and |Fg| > 2. The precise location of the onset of metastability depends on the relation between the band-width, in which interactions are relevant (the energy cut-off), and the band-curvature at the Fermi surface.

In this way, the above integral is controlled. The coupling constant g and the ultraviolet cut-off are both contained in the definition of ∆0 (eq. 17). As before, near the Fermi surface, where interactions are important, Θ(ω−) = 0. Performing the remaining integral over k, the gap equation is written as

\[ \ln \left| \frac{\Delta_0}{\Delta_q} \right| = \Gamma(x), \]  

(19)

where \( x = \Delta_q/\bar{n} \), and we have defined the function

\[ \Gamma(x) = 2 \int_{-\theta(x)}^{\theta(x)} \frac{d\theta}{2\pi} \sinh^{-1}\left( \frac{\sqrt{(1 + \cos \theta)^2 - x^2}}{x^2} \right), \]  

(20)

with \( \theta(x) = \cos^{-1}(x - 1) \) for \( x \leq 2 \) and \( \Gamma(x > 2) = 0 \). Therefore, to look for modulated superconductivity we need to solve eqs. (16) and (19) self-consistently. Firstly, we note that for \( x > 2 \) the only solution, \( \{\bar{n} = 0, \Delta_q = \Delta_0\} \), corresponds to the uniform superconducting phase. This imposes a lower limit for modulated superconductivity since, for obtaining a non-trivial solution, we must have \( \Delta_q < 2\bar{n} < \Delta_0 \). However, for any value of the parameter \( \Delta_0 > 2\bar{n} \), the uniform superconducting mean-field energy is always lower than the modulated one, \( \langle H \rangle_{\Delta_0} < \langle H \rangle_{\Delta_q} \). On the other hand, there is a region of the phase diagram, shown over the dashed line in figure (1), in which \( \langle H \rangle_{\Delta_q} < \langle H \rangle_{\beta} \). Therefore, modulated superconductivity appears as a metastable phase for finite values of ∆0 and |Fg| > 2. The precise location of the onset of metastability depends on the relation between

\[ \frac{N(0)}{2} \ln \left| \frac{\Delta_0}{\Delta_q} \right| = \int \frac{d^2k}{(2\pi)^2} \frac{1}{2\xi} \left\{ \Theta(-\omega_+) + \Theta(\omega_-) \right\}. \]  

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