Ecological Game of Animal Populations in Different Environments

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Abstract. Under different ecological conditions, the results of animal evolution are different. As global warming and the environment deteriorates, the rate of species extinction is accelerating. Therefore, in this paper, the game between animal populations and ecology is studied, which provides a basis for more scientific protection of animals and environment. First, the animal's environmental needs are studied: By establishing the relationship between the heat gained by the animal and the required land area, there comes a fact that the more energy the animal consumes, the more land it needs. The land area is related to environmental factors such as temperature and precipitation. Therefore, given the natural conditions of a certain place, the land area can be obtained through the regression equation. Secondly, considering the long-term development, based on the ecological game model, the differential dynamic system is established. In different food supply situations, three game results of population and ecology are obtained: continuous growth, continuous reduction, and cyclical fluctuations.

1. Introduction
It is known that the growth of animals is supplied by the energy in food, and different food conditions produce different growth trends of the animals. The energy needed for cattle was studied and determined in reference[1]. The growth reaction method studied the relationship between the weight change of cattle and energy intake to determine how much energy the cow needs. Animals consume energy from food, and food production is affected by different climatic conditions. The effect of temperature on corn yield is introduced in reference[2] The effects of accumulated temperature, precipitation and sunshine time on cotton yield are studied in reference[3], which explains that environmental factors affect the growth of animals by affecting their food.

Firstly, the function relationship between heat obtained by the animal and the required land area is gotten. Based on the ecological game of animals, the differential dynamic system model is established, and the three development laws are obtained.

2. The model

2.1. Heat-area curve
Assume the animal finally gets the heat $H$, the conversion rate is $r$, then the calorie intake is

$$P = H \times r$$

The heat of wheat is 3170Kcal per kilogram, so the required grain yield is

$$M = \frac{P}{3170}$$
According to the data, the yield per hectare of wheat is about 5250 kilogram. Then the area required to obtain H heat is

\[ D = \frac{M}{5250} \]

The relationship between H and D is finally established:

\[ H = 1.66 \times 10^7 \times r \times D \]

Different populations have different conversion rates, taking 20%, 30%, 40% as examples, the three straight lines are shown in figure 1. And each of the production substrates has different energy conversion factors[4].

2.2. The land area

The population needs the land area D, its exercise amount is x (Kcal), the body weight is y (Kg), the annual average temperature is T (°C), the average annual precipitation is W(mm/year), and there is the following relationship:

\[ D = a_0 + a_1 x + a_2 \frac{1}{T} + a_3 \frac{1}{W} + a_4 y \]

Where \( a_k \) are coefficients. Taking climate conditions in 40 regions including New York, London, Beijing, Wuhan, etc. (shown in appendix), then the regression equation is calculated as follows:

\[ D = 0.4392 + 0.1216 x + \frac{0.4703}{T} + \frac{2.104}{W} + 0.1032 y \]

The above formula shows that as the animal body weight increases, the required land area increases. In a livable environment, an increase in temperature and precipitation will increase the amount of food, so the area required for the population will decrease. When the temperature and precipitation tend to zero, the area is infinite. When the temperature is negative, the temperature contributes a negative value to the land and the population needs to migrate to sustain life. Make residual analysis on the model, the result is shown in figure 2, which means the data is highly representative and the model confidence is high, too.
2.3. Ecological game model

The good growth state of animals is a witness to the harmonious development of man and nature[5]. To explore the relationship between animal survival and the environment, based on the ecological game model, a differential dynamic model is established. In a certain area, assuming that \( x \) is the quantity of food and \( y \) is the number of animals, as can be seen from the previous description, food production depends on external growth conditions, so \( x \) is a reflection of environmental conditions. Then there are two-dimensional differential dynamic equations:

\[
\begin{align*}
x' &= ax + by \\
y' &= cx + gy
\end{align*}
\]

\[
X' = AX
\]

\[
X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad A = \begin{pmatrix} a & b \\ c & g \end{pmatrix}
\]

\( a, b, c, d \) are coefficients, which are closely related to the ecological environment, that is, temperature and rainfall.

\[
y = f(W, T)
\]

The calculation formulas of \( a, b, c, d \) are established as follows:

\( a \) indicates the effect of the amount of food on the growth rate of food. If the temperature is too low or too high, it is not suitable for grain growth. So with the increase of temperature, \( a \) shows a trend of rising first and then decreasing, which is a quadratic function, and the precipitation has the same effect. So the formulas can be written as:

\[
a = 0.0002(-0.005108W^2 + 10.75W + 721.4) + 0.0007(-7.501t^2 + 427.7t + 54.6)
\]

The 3D graphics is shown in Matlab as figure 3:

Figure 3. Function image of \( a \).

\( b \) indicates the effect of the number of populations on food growth rate. When the temperature increases, the external conditions have a greater effect on the growth of food in the early stage, so the
influence of the population on it is relatively weak. Later, temperature is not suitable for grain to grow, so the influence of the population becomes larger. Therefore, $b$ shows a trend of decreasing first and then increasing, which is a quadratic function, and precipitation has the same effect. So the formulas can be written as:

$$b = 0.0002(0.005108W^2 - 10.75W - 721.4) + 0.0007(+7.501t^2 - 427.7t - 54.6)$$

The 3D graphics is shown in Matlab as figure 4:

![Figure 4. Function image of $b$.](image)

$c$ indicates the effect of the amount of food on the growth rate of food. When the food is sufficient, the growth rate of the population has nothing to do with the quantity of food. Therefore, when the temperature rises, the quantity of food increases first and then decreases, so the effect on the growth rate of the population first decreases and then rises, and it is a quadratic function as follows.

$$c = 0.00065(0.005108W^2 - 10.75W - 721.4) + 0.0003(+7.501t^2 - 427.7t - 54.6)$$

The 3D graphics is shown in Matlab as figure 5:

![Figure 5. Function image of $c$.](image)

g indicates the effect of the number of populations on the population growth rate. It is not directly related to temperature and precipitation, but is determined by the number of populations. If the food is sufficient, the population growth rate increases due to reproduction. When the food is scarce, the population growth rate decreases due to competition.

The graphics is shown in Matlab as figure 6:

![Figure 6. Function image of $g$.](image)
According to the quantity of food, the growth trend of the population is divided into three categories.

2.3.1. Adequate food
The growth rate of food increases in proportion to the amount of food, that is, \(a\) is always greater than 0. As food is always sufficient, the rate of food growth increases as the number of animals increases, that is \(b>0\), and the number of animals increases, too, that is, \(c>0\). The growth rate of animals is proportional to the number of animals because of reproduction, that is \(d>0\). Therefore, \(a, b, c, d\) are 2, 1, 1, 2, respectively, two-dimensional system of ordinary differential equations are obtained:

\[
\begin{align*}
  x' &= 5x - y \\
  y' &= x + 8y
\end{align*}
\]

Which can be expressed as a matrix:

\[
A = \begin{pmatrix} a & b \\ c & g \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 1 & 8 \end{pmatrix}
\]

Set \(|A-r^*E|=0\), the eigenvalues can be solved as \(r_1 = 7.62, r_2 = 5.38\). Let \(x_0\) denote the initial quantity of food, \(y_0\) denote the initial number of animals, \(D_{x_0}\) denotes the amount of food increase, and \(D_{y_0}\) denote the number of animals increased.Set \(x_0=1, y_0=3, D_{x_0}=14, D_{y_0}=60\), by solving the following equations:

\[
\begin{align*}
  x_1 + x_2 &= 1 \\
  7.62x_1 + 5.38x_2 &= 14 \\
  y_1 + y_2 &= 3 \\
  7.62y_1 + 5.38y_2 &= 60
\end{align*}
\]

The result is shown as: \(x_1=4, x_2=-3, y_1=20, y_2=-17\), so the equation solution can be expressed as:

\[
\begin{align*}
  x &= 4e^{7.62t} - 3e^{5.38t} \\
  y &= 20e^{7.62t} - 17e^{5.38t}
\end{align*}
\]

The function image shown in Figure 7 is drawn in MATLAB:

![Figure 7](image)

Figure 7. Quantity-time diagram when food is sufficient.

2.3.2. Lack of food
According to the previous analysis, \(a>0\); due to insufficient food, its quantity decreases with the increase of the number of animals, that is, \(b<0\); the number of animals increases with the increase of food, that is, \(c>0\); Due to food shortage, the internal competition of animals is intensified, so the growth rate of animals will decrease, that is, \(d<0\). Therefore, \(a, b, c, d\) are 1, -3, 3, -6, respectively, and two-dimensional ordinary differential equations are obtained.
\[ \begin{align*}
x' &= x - 3y \\
y' &= 3x - 6y
\end{align*} \]

The solution can be expressed as:
\[ \begin{align*}
x &= 62e^{-0.7t} + 63e^{-4.3t} \\
y &= 60e^{-0.7t} - 98e^{-98t}
\end{align*} \]

The function image is shown as figure 8:

![Figure 8](image)

Figure 8. Quantity-time diagram when food is insufficient.

2.3.3. The critical state
At the critical state, the number of animals and food is in a dynamic equilibrium. In order to support more animal growth, the growth rate of food increases with the number of animals increasing, that is \(b>0\). As the number of food decreases, the number of animals will decrease, that is, \(c < 0\). There is competition among the animals, so its number will decrease, that is, \(d < 0\). Therefore, \(a, b, c, d\) are 1, -5, 2, -1, respectively. So the differential equations are shown as follows:
\[ \begin{align*}
x' &= x - 5y \\
y' &= 2x - y
\end{align*} \]

Then the solution can be expressed as:
\[ \begin{align*}
x &= 2\sin t + 4\cos t \\
y &= 2\sin t + 2\cos t
\end{align*} \]

The function image is shown as follows:

![Figure 9](image)

Figure 9. Quantity-time diagram when in critical state.

3. Conclusions
- The more energy the animal population needs, the larger land area is required.
- The game between animals and ecology has three outcomes: continuous increase, continuous decrease, and cyclical fluctuations.
4. Appendices

Table 1. Climate conditions in 40 regions

| Number | City            | Temperature (°C) | Precipitation (mm/year) | Wheat production (Kg/hm²) |
|--------|-----------------|------------------|-------------------------|--------------------------|
| 1      | Guilin          | 19.000           | 1221.3                  | 10370.0                  |
| 2      | Wenzhou         | 17.900           | 1742.4                  | 10788.0                  |
| 3      | Guangzhou       | 22.000           | 1736.1                  | 10011.0                  |
| 4      | Nanchang        | 17.500           | 1624.4                  | 9872.0                   |
| 5      | Hangzhou        | 16.200           | 1454.6                  | 9678.0                   |
| 6      | Nanning         | 21.800           | 1309.8                  | 9680.0                   |
| 7      | Wuhan           | 16.400           | 1269.0                  | 9578.0                   |
| 8      | Shanghai        | 15.600           | 1164.5                  | 9520.0                   |
| 9      | Guiyang         | 15.500           | 1117.7                  | 9498.0                   |
| 10     | Chengdu         | 16.700           | 927.5                   | 8987.0                   |
| 11     | Xuzhou          | 13.900           | 831.6                   | 8540.0                   |
| 12     | Qingdao         | 12.100           | 720.0                   | 8422.0                   |
| 13     | Shenyang        | 7.600            | 690.3                   | 7867.0                   |
| 14     | Luoyang         | 14.200           | 602.3                   | 8267.0                   |
| 15     | Weifang         | 11.200           | 588.3                   | 7951.0                   |
| 16     | Beijing         | 11.600           | 571.8                   | 7889.0                   |
| 17     | Changchun       | 4.500            | 570.4                   | 7019.0                   |
| 18     | Xi'an           | 13.200           | 553.3                   | 7753.0                   |
| 19     | Tianjin         | 11.900           | 544.3                   | 7720.0                   |
| 20     | Harbin          | 3.700            | 524.4                   | 6789.0                   |
| 21     | Shuizhuang      | 8.800            | 517.1                   | 7246.0                   |
| 22     | Yan'an          | 6.500            | 510.7                   | 6992.0                   |
| 23     | Taiyuan         | 9.100            | 431.1                   | 6757.0                   |
| 24     | Hohhot          | 3.400            | 397.9                   | 6578.0                   |
| 25     | Sining          | 5.400            | 373.8                   | 6632.0                   |
| 26     | Lanzhou         | 9.400            | 311.7                   | 6676.0                   |
| 27     | Philippine river| 6.483            | 50.8                    | 5326.0                   |
| 28     | Libreville      | 25.975           | 2841.7                  | 18870.0                  |
| 29     | Washington      | 12.855           | 105.5                   | 6535.0                   |
| 30     | New York        | 12.725           | 1262.1                  | 9729.0                   |
| 31     | London          | 11.567           | 592.0                   | 7289.0                   |
| 32     | Sydney          | 18.508           | 1126.9                  | 11150.0                  |
| 33     | Singapore       | 27.390           | 2150.0                  | 16457.0                  |
| 34     | Moscow          | 5.079            | 691.0                   | 7457.0                   |
| 35     | Nuuk            | -1.375           | 754.0                   | 7221.0                   |
| 36     | Vancouver       | 5.995            | 934.4                   | 7821.0                   |
| 37     | Paris           | 10.000           | 649.8                   | 7611.0                   |
| 38     | Tokyo           | 15.300           | 1466.7                  | 11176.0                  |
| 39     | Berlin          | 9.400            | 583.8                   | 7354.0                   |
| 40     | Roman           | 15.500           | 733.1                   | 8176.0                   |

References
[1] Li, K, Sun, G.Q, Song, E.I, Liu, G.F, Tan, X.W, Wan, F.C. (2011) Advances in research on
determination methods of cattle energy requirements. Journal of Animal Ecology, 32(01): 5-8.

[2] Wang, Q. (2009) Effect of temperature on corn growth and yield. Journal of ecology, 28(2): 255-260.

[3] Wang, T.X, Dong, S.T. (2018) The impact of waiting change on china's cotton yield-- An empirical analysis based on C-D-C model. Jiangsu Agricultural Science, 46(24): 383-386.

[4] Han, J.H, Guo, J. (2005) Food energy and energy conversion factor. New observation on nutrition health, (02): 68-75.

[5] Tang, Z.P, Du, X, Sun J.P. (2013) Bayesian Game Analysis of Wildlife Ecological Protection in China. Ecological Economy, (12): 171-174.