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The influence of the bound vortex on the aerodynamics of curved wind turbine blades

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Abstract. Passive load alleviation on wind turbine blades can be achieved through geometric bend-twist coupling, for example by sweeping the blade backwards. In order to obtain the correct load distribution of a curved blade with in-plane sweep and/or out-of-plane dihedral, the influence of the blade shape on the aerodynamics must be modelled correctly. This includes the influence of the curved bound vortex, and it is especially important when designing a wind turbine blade with aeroelastic tailoring. In this paper, the background for modelling the curved bound vortex influence will be described in detail and a modified method is proposed. The proposed method of bound vorticity modelling is compared for curved and straight translating wings as well as wind turbine blades with results from a panel code and a Navier-Stokes solver. From this comparison, the advantages of the current modification with respect to the other lifting-line implementations are shown. The method proposed in the present work is general and applicable to any lifting-line like model.

1. Introduction
In-plane backward swept wind turbine blades can achieve passive load alleviation with geometric bend-twist coupling. In order to obtain the correct load distribution of such a curved blade, the influence of the blade shape on the aerodynamics needs to be modelled correctly. This is especially important when designing a blade with aeroelastic tailoring. The inaccurate modelling of the aeroelastic forces could overestimate the benefits of curved blades. This is especially hazardous when numerical optimization is involved in the design loop, since all the converged designs can be unrealistic (e.g. featuring a large backwards sweep). In a previous work [1], two parameters were identified as important for the aerodynamics of swept blades: the shifted position of the trailed vorticity and the curved bound vorticity. If only the trailed vorticity contribution is considered, while neglecting the curved bound vorticity, the load redistribution effect of the swept blade could not be correctly predicted. In that work, it was mentioned that the curved bound vortex could be modelled by adding the difference between the bound vorticity induction of the curved blade and an imaginary straight blade to the velocity triangle at each blade section. The method in that work is different from the method proposed here.

In this work, the background for modelling the curved bound vortex influence will be described in detail. A method based on the idea of splitting the problem into ‘inner’ and ‘outer’ parts, that originates from perturbation theory and utilizes 2-D unsteady airfoil theory is proposed. Results from the proposed method are compared with the results from a panel code and a Navier-Stokes
solver, for both curved and straight translating wings as well as wind turbine blades. To highlight the effect of different details in the implementation of the lifting-line method, results from other lifting-line implementations will be shown. The method proposed here is general and applicable to any lifting-line like model, with advantages over the previous methods.

2. Description of the proposed model

2.1. Common lifting-line implementation

A lifting-line like model is commonly implemented in the following form. The lifting-line, representing the concentrated bound vorticity, is located at the 1/4 chord line of the blade. This is where the trailed vortices emanate from, forming the vortex system illustrated in Figure 1. The induced velocities are evaluated at the lifting-line, and are due to the trailed vorticity with a possible contribution from the shed vorticity in the unsteady case. This model assumes a straight blade with a high aspect ratio, so there is no influence from the bound vorticity to itself.

![Figure 1. Illustration of the vortex system used for calculating the induced velocity for the highlighted section of a curved wing (left). Sketch of the rectangular wing and the curved wing geometries (right). The wings are both with span of 80 m and constant chord length of 6.5 m, consisting of NACA0012 airfoil sections.](image)

Building on the original work by Prandtl [2], there have been multiple works on the modern adaptation of the lifting-line method, such as Phillips and Snyder [3] for translating wings and van Garrel [4] for wind turbine blades. These adaptations are numerical methods where the bound vortex is discretized into multiple sections. Each section is assumed to be a line vortex segment with constant strength. The numerical solution is obtained from either a time-marching method where the wake geometry is updated and evolved, or a prescribed wake geometry known as the frozen wake method. Some adaptations [3] include the influence of curved bound vortex by calculating the contribution of induced velocity on the bound vortex (1/4 chord line) itself. For each section, there is contribution from all other bound vortex segments excluding the own segment because there is no induction from a straight vortex segment on itself.

There are two disadvantages of this method of including non-straight bound vortex influence. Firstly, there is a logarithmic singularity when evaluating on the curved bound vortex itself, shown analytically in [5]. If this numerical method is applied, the induction will then be dependent on the discretization due to the singular nature of the problem. As the curved vortex is discretized finer, more of the logarithmic singularity will show up in the solution. Essentially the problem arises due to the simplification made in the lifting-line framework: the bound vorticity is concentrated into a singular line/curve at the quarter chord. A common solution is to introduce a vortex core. Unfortunately, the result will then be dependent on the choice of the core size.

Secondly, this purely intuitive method of including the bound vorticity is physically incorrect. The lifting-line method is formulated as a singular perturbation problem [6], which splits the aerodynamic problem into an ‘inner’ problem (the airfoil) and an ‘outer’ problem (the wake). The two domains are connected with matched asymptotic expansions, and the combined solution
is referred as the formal lifting-line theory [7]. The lifting-line methods that only use the information of the 1/4 chord points are categorized as 1st order methods with the assumption of straight blade and high aspect ratio. In order to relax these assumptions to model curved blades, the 2nd order lifting-line models are needed. According to Johnson [7], the most important characteristic for the 2nd order models is to use flow information at the 3/4 chord points.

2.2. Proposed modification

The method proposed in the present work is not derived analytically from the singular perturbation problem to the 2nd order. However, the idea of splitting the problem into ‘inner’ and ‘outer’ problem will be used. The method is similar to Weissinger’s L-method [8] but utilizes the airfoil look-up table approach. From the authors’ point of view the present model is also physically more intuitive.

The model relies on the building blocks provided by unsteady 2-D thin airfoil aerodynamics. Here, it is shown that the solution of the 2-D system is driven by the normal flow component through the airfoil which would have been there if the disturbance due to the airfoil itself was not present. For an airfoil performing heaving and pitching motion, this component is linearly varying along the chord [7]. From the solution of the thin-airfoil problem, it can be seen that the magnitude of the loading can be determined correctly by considering only the conditions (angle of attack) at one specific location along the chord, namely the 3/4 chord point [9]. In other words, this shows that as long as disturbances to the 2-D inflow can be considered linear along the chord, the correct solution is obtained by evaluating the disturbance (angle of attack) at this location. The other detail needed for a fully consistent implementation is to note the less known key result from 2-D thin airfoil theory [10]. The direction of the local lift and drag forces should be defined by the relative velocity at the 1/4 chord point. This is of key importance to get the induced drag component correct for non-rotating wings, and to get the tangential forces correct for rotors.

Considering this 2-D model the ‘inner’ part of the total 3-D model, then the key point is how to obtain this local 2-D disturbance velocity for each local spanwise section along the blade. For this, we recall that the 2-D problem is driven by the velocities that would have been there if the velocities induced by the 2-D airfoil itself were not present. Therefore, the induction taking into account the 3-D part of the problem is, for each spanwise location, equal to the induced velocities from the total 3-D vortex system minus the induced velocities from the 2-D section itself. And according to the results from thin airfoil theory, this velocity difference should be evaluated at the 3/4 chord point.

The 3-D velocity at the collocation points (3/4 chord points) is composed of bound, trailed and shed vorticity induction, blade motion and onset flow.

\[
    w_{3D} = w_{onset} + w_{motion} + w_{bound} + w_{trail} + w_{shed}
\]

The corresponding local 2-D velocities are

\[
    w_{2D} = w_{onset} + w_{motion} + w_{bound} + w_{shed}
\]

The induced velocity that takes the three-dimensionality into account in the core of each 2-D section, which is for the calculation of angle of attack, is therefore

\[
    \Delta w_{2D\rightarrow3D} = w_{3D} - w_{2D} = w_{3D} - w_{3D} + w_{shed} - w_{shed} + w_{bound} - w_{bound} - w_{shed}
\]

So, with the present model, the influence of the curved bound vortex is included in the modelling by adding the difference in induction between the 3-D and an imaginary 2-D bound vorticity (infinitely long vortex segment) to the sectional unsteady aerodynamics, see Figure 1.
Therefore, the result from the present model reassuringly tends to those of the underlying 2-D model when the wing aspect ratio tends to infinity. The induction at the 3/4 chord point is only used to calculate the effective angle of attack and therefore to obtain the circulation at the current section. The induced velocity vector of the 3-D curved bound vortex induction minus the own local 2-D induction in Equation 3 is assumed to be constant along the chord. It is also included in the total velocity at 1/4 chord point, which defines the local lift and drag direction. This is to avoid the logarithmic singularity problem of curved bound vortex induction evaluated on itself.

The modified lifting-line model described above in Equation 3 (noted as \( LL-mod \)) is different from the formalized 2\(^{nd} \) order lifting-line model derived by Johnson [7]. In the implementation of Johnson (noted as \( LL-Johnson \)), the trailed vorticity induction is calculated at the 3/4 chord point, but the bound vorticity induction is neglected. Thus it is expected that there will be an error for a non-straight blade. This will be tested numerically in Section 4.1 and Section 4.3.

### 2.3. Issues of evaluating on the 3/4 chord line

For the contribution of induced velocity due to trailed vorticity, the method of placing the collocation point (\( CP \)) on the 3/4 chord line in the proposed modification has more numerical instabilities compared to having \( CP \) on the 1/4 chord line. For every trailed vorticity, there will be more induction at the \( CP \) because the collocation points are shifted a bit ‘into’ the trailed vortex system. Especially when the chordwise length of a section is much larger than its spanwise length, the induction due to a trailed vortex filament of this section at the 3/4 chord point is almost doubled comparing to that at the 1/4 chord point. In this case the trailed vorticity almost seems to be an infinite vortex line seen from the 3/4 chord point, instead of the semi-infinite line seen from the 1/4 chord point.

This issue was one of the main motivations for other researchers to place the \( CP \) on the 1/4 chord line. Since the focus of this work is on the modelling of curved bound vortex influence instead of explaining the whole modified lifting-line model, details of this issue will not be discussed. In this paper, the focus is only on the steady-state results. Thus, the approach of decreasing the relaxation factor and number of iterations per time step (even with no iteration) is applied. After marching for many time steps, the solution of the described method reaches a steady-state, which can be monitored through the convergence of the load distribution.

### Table 1. The comparison of different lifting-line variants used in this paper. The differences are the methods of modelling the contribution of the bound vorticity and the trailed vorticity on the induced velocities. All models place the bound vorticity at the 1/4 chord line.

| Bound vorticity induction evaluated at: | Trailed vorticity induction evaluated at: |
|----------------------------------------|------------------------------------------|
| \( LL-origin \)                        | 1/4 c                                    |
| \( LL-Johnson \)                       | 3/4 c                                    |
| \( LL-P&S \)                           | 1/4 c                                    |
| \( LL-mod \)                           | 3/4 c (3D - 2D)                          |
| \( LL-test \)                          | 3/4 c (3D - 2D)                          |

### 2.4. Details of the implementation

The in-house aerodynamic solver MIRAS [11], developed at the Technical University of Denmark (DTU), was selected for the integration of the different lifting-line variants. In the standard lifting-line model of MIRAS, the induction is evaluated at the 1/4 chord points and the curved...
bound vortex influence is not considered. This baseline method is noted in this document as \textit{LL-origin}. The method of including the contribution from the bound vortex induction calculated on itself by Phillips and Snyder \cite{3} (noted as \textit{LL-P\&S}), as well as the method proposed by Johnson (\textit{LL-Johnson}) are implemented. Finally, the enhanced method described in the preceding sections is implemented in MIRAS and labelled as \textit{LL-mod}. All the lifting-line variants in this work are implemented with a time-marching free wake method as well as the airfoil look-up table approach. The differences between these lifting-line variants are summarized in Table 1.

3. Description of the models used for comparison

In order to assess the performance of the proposed method, two additional numerical approaches were considered. In particular, both the built-in panel method of MIRAS and the Navier-Stokes solver EllipSys3D \cite{12, 13, 14} were used. In the panel method, the solid bodies are simulated with a distribution of quadrilateral surface dipoles and sources. A Neumann condition of non-penetration is used by forcing the normal component of the velocity to be zero at the walls. EllipSys3D is a finite volume code that solves the incompressible Navier-Stokes equation on a structured grid. In the present study, the \textit{k-\omega SST} turbulence model was used \cite{15}. The results of this higher fidelity numerical method are labelled in this document as \textit{CFD}.

4. Results

4.1. Curved and straight translating wings

For a clean comparison to highlight the curved bound vortex effects, a straight and a curved translating wing both with constant chord are used for comparison. The planforms and dimensions of the wings are shown in Figure 1. The results of the different lifting-line implementations considered in this study are depicted in Figure 2, and are compared with the prediction from the panel method.

The inviscid flow is assumed and the corresponding lift coefficient of the NACA0012 airfoil for the lifting-line models is $C_l = 2\pi \times 1.078\sin \alpha$. The simulations are with the condition of uniform wind speed of 10 m/s and geometric angle of attack of 5\degree. The vortex core size is set to be $1 \times 10^{-4}$ m. The wing span is uniformly discretized into 50 sections. For the panel method, the number of sections around the airfoil is 150.

![Figure 2](image-url)  
**Figure 2.** Comparison of the lift distribution of the straight wing (left), the \textit{LL-origin} and \textit{LL-P\&S} method give identical results. The offset of the lift between the curved wing and a straight wing (right).

The lift distribution of the rectangular wing is plotted in Figure 2 on the left, and the offset of the lift between the curved and straight wings is plotted on the right. For the straight wing,
the result from LL-origin and LL-P&S are identical, as it was expected. For the offset in the lift between curved and straight wing, the modification method introduced here (LL-mod) is predicting a very similar trend as the panel method. For the curved wing, the lift is lower near the center of the wing and higher near the tip, compared to the straight wing. This is clearly visualized by the crossing of the horizontal zero-value line in Figure 2. The original lifting-line method and the method formalized by Johnson (LL-Johnson) did not reproduce this trend. Instead, they predicted the lift of the curved wing to be higher than the straight wing over the entire span. The method of including bound vortex influence at itself (LL-P&S) with 1/4 chord collocation point can predict the ‘redistribution’ of lift. However, it overestimates the magnitude of offset at the mid-span as well as near the wingtip. The good agreement between the proposed model and panel method justifies the derivation of the current modified lifting-line model.

4.2. Singularity of bound vortex self induction

As described in Section 2.2, the method of calculating the induction at the bound vortex (1/4 chord line) itself will have singular behaviour which shows itself by the inability of the solution to converge toward one solution with increasing resolution of discretization. This will be shown numerically for the curved and straight translating wings by varying the number of sections from 30 to 200, with uniform spacing.

For all the methods, the contribution from bound vortex and trailed vortex are combined linearly to get the total induction. For the method that includes the curved bound vortex influence, there are three different approaches. The first one is simply neglecting this influence, as done in LL-origin and LL-Johnson. The second method is to calculate the influence of the curved bound vortex at the 1/4 chord line on itself, as in LL-P&S. Finally, the third option is to calculate the induction by curved bound vortex at 3/4 chord points and subtract the 2-D influence of the current section, as in LL-mod.

Since the focus of this parametric study is on the singular behaviour introduced by the methods of calculating curved bound vortex induction, the method of calculating the trailed vortex induction should be kept the same. For this purpose, the collocation point is set to 1/4 chord points. Then, the different lifting-line methods to be tested are LL-origin, LL-P&S. And an additional method, labelled as LL-test, was also considered. For LL-test, the curved bound induction is calculated at 3/4 chord point, as in LL-mod, but the trailed vortex induction is computed on the lifting-line. This method is not physical and is only to investigate the singular behaviour introduced by curved bound vortex modelling. For each method, similar to Figure 2, the offset of the lift of the curved wing compared to the straight wing with different resolution of discretization is plotted in Figure 3 and Figure 4.

For the method of LL-origin, there is no bound vortex influence included, therefore there will be no singular behaviour due to it. This is then the baseline for this investigation. For the method of LL-test in Figure 4, the behaviour is similar to that of LL-origin. However, for the method of LL-P&S, it could be clearly visualized that the offset of lift at the mid-span is changing with the number of sections. This behaviour is due to the characteristic of this method of calculating bound vortex self induction having singularities.

For the three different methods, the change of the offset at mid-span with respect to that with 30 sections, which has the least singular effect, is compared in Figure 4. For the method LL-P&S, the offset of the lift force is strongly dependent on the discretization. For the other two methods, the variation of the offset is much smaller and only due to numerical convergence instead of the inherent mathematical singularity. Even though the singular behavior is partially mitigated due to the numerical implementation of the lifting-line methods, the remaining strength is still strong enough to influence the final results.

In summary, this numerical investigation confirms the disadvantage of the LL-P&S method compared to the proposed method. The result is dependent on the discretization due to its
4.3. Curved and straight wind turbine blades
The different lifting-line implementations considered in the present study are also assessed for a straight and a curved wind turbine blade. In this case, the results are compared with the results of a higher fidelity Navier-Stokes solver, labelled as CFD.

The wind turbine blades that are used for the comparison are variants of the IEA-10.0-198 10 MW reference wind turbine (RWT) [16], which has a rotor diameter of 198 m. The hub radius is 2.8 m and the blade length is 96.2 m. The baseline straight blade is modified by aligning the half-chord line to a straight main axis. For the backwards swept blade, the planform is obtained from a Bézier curve which is parameterized with: sweep ratio $\bar{r}_s$, sweep magnitude $\Delta d$ and tip sweep angle $\Lambda_{tip}$. For a clean comparison, the pre-bend as well as cone angle of both blades are removed. A sketch is shown in Figure 5 to illustrate the parameterization. The swept blade used in this study is having the same parameters as Blade 1 in a previous work [1]. The sweep ratio is inherent singular behaviour in the method of calculating bound vortex self induction. This will limit the validity of the results obtained from this method.
50% with sweep magnitude of 10% and tip sweep angle of 20°. The cross-sections are orthogonal to the curved main axis of the half-chord line. The chord and twist distribution of the swept blade remains the same as the baseline blade, for the sections with the same y-coordinate. For the swept blade, the radial length in the y-coordinate remains the same, see Figure 5. Thus, the total effective radius is increased compared to the baseline.

The operational condition is the same as the previous study [1], with 8 m/s uniform inflow wind speed and zero yaw angle, operating at the optimum tip speed ratio of 10.58 with zero pitch angle. Thus, the curved and straight blades will both stay in the rotor plane.

![Figure 5. The parameterization of the swept blade [1] (left) and the top view of the swept blade as well as the baseline straight blade (right).](image)

Rotor-resolved meshes were used for the different CFD simulations. They were generated in two consecutive steps that were fully scripted in order to ensure a similar resulting grid quality. First, a structured mesh of the blade surface was generated with the openly available Parametric Geometry Library (PGL) tool [17]. A total of 128 cells were used in the spanwise direction, and the chordwise direction was discretized with 256 cells (with 8 of them lying on the trailing edge). Secondly, the surface mesh was radially extruded with the hyperbolic mesh generator Hypgrid [18] to create a volume grid. A total of 256 cells were used in this process, and the resulting outer domain was located at approximately 11 rotor diameters. A boundary layer clustering was taken into account, with an imposed first cell height of $1 \times 10^{-6}$ m. The resulting volume mesh accounted for a total of 14.2 million cells. An inlet/outlet strategy was followed for the boundary conditions of the outer limit of the CFD domain, and the flow was assumed to be fully turbulent.

For the lifting-line methods, each time step corresponds to 1.5° of azimuthal angle. Each simulation is calculated for 15 thousand time steps which correspond to 62.5 revolutions. The vortex core size is 0.1% of the local chord length. Each blade is discretized into 50 sections with cosine spacing. The airfoil data are from 2-D fully turbulent CFD results [16].

Firstly, the steady-state results of the baseline straight blade calculated from the four different lifting-line methods and CFD are compared in Figure 6. To be noted, the load distributions from both lifting-line and CFD are corresponding to aerodynamic force per unit length of y-coordinate. The results from all the lifting-line modifications are almost identical to each other because the blade is straight with a high aspect ratio.

Then, the steady-state results of the swept blade is also calculated from different lifting-line models and CFD. In order to clearly show the influence of the backward sweep on load distribution, the offset of the load distributions of the curved blade with respect to the baseline straight blade are shown in Figure 7. The results from the four different lifting-line methods form two groups. The method LL-origin and LL-Johnson are having similar results, and the results from LL-mod and LL-P&S almost coincide with each other except near the blade tip. From the CFD results of the loading offset between curved and straight blade, the pattern of a load redistribution could be observed. For the swept blade, near the spanwise location where the blade starts to sweep backwards, the in-plane and out-of-plane loads are decreased compared to the baseline until halfway to the tip. When moving further towards the tip, both in-plane and out-of-plane loads are then increased compared to the baseline until the blade tip. This pattern was also observed in the previous work [1].
It can be observed that for the method \textit{LL-origin} and \textit{LL-Johnson}, where the curved bound vortex influence is not modelled, the redistribution of the loading could not be predicted. Instead, these two models predict the loading on the curved blade being higher than the baseline straight blade throughout the whole span. For the two methods, the behaviour on modelling the curved wind turbine blades is similar to that of modelling curved translating wings, as expected.

For the methods of \textit{LL-mod} and \textit{LL-P&S}, the redistribution of the loading was captured. The two methods have similar results up to 80% of the span. However, when moving further towards the tip, the offset of the loading predicted by the \textit{LL-mod} method is higher than the prediction by \textit{LL-P&S} method, especially for the out-of-plane loading. The performance of the \textit{LL-P&S} method is better than expected, maybe because of the high aspect ratio of the blade. However, this may not justify using this method because the result is highly dependent on the resolution of the discretization, which will be shown in Section 4.4.

4.4. \textit{Discretization dependency for turbine simulation}

It has been shown for the curved translating wing that the offset of the loading was strongly dependent on the number of sections when using \textit{LL-P&S}. This strong dependency is related
to the singular behaviour of the curved bound vortex induction when calculated on itself. For the curved wind turbine blades, the extent of this dependency was further investigated. Similar to the investigation with the translating wings in Section 4.2, the load distributions of the swept and baseline blade are calculated with \textit{LL-P&S} and \textit{LL-test} using different resolutions of discretization. The only difference between the two lifting-line methods is the curved bound vortex modelling, as shown in Table 1. As has been explained in Section 4.2, the method of \textit{LL-test} is not physical and is only to investigate the singular behaviour introduced by curved bound vortex modelling. For the baseline blade, the load distributions from the two methods are almost identical. For better visualization, the offset of the out-of-plane and in-plane loading for the swept blade with respect to the baseline blade are compared.

![Offset of in-plane loading](image)

**Figure 8.** The offset of the in-plane loading for the swept blade with respect to the baseline blade, with different number of sections calculated from \textit{LL-test} (left) and \textit{LL-P&S} (right).

The behaviour of the offset of the in-plane and the out-of-plane loading are very similar, and for brevity only the in-plane loading offset is shown in Figure 8. For the results calculated with \textit{LL-test}, it could be inferred that with increasing number of sections, the result is approaching the converged value. For the results of the \textit{LL-P&S} method, the strong dependency on the number of sections can be seen. There will be a relatively large change in the solution with the increase of the number of sections. Especially for the offset of the loading near the spanwise location of 62 m. The decrease of the in-plane loading calculated with 150 sections is about 1.4 times of that calculated with 50 sections.

In summary, the \textit{LL-P&S} method might still give satisfactory results with 50 sections. However, when the number of sections is increased, the result does not converge due to the inherent problem with the singularity in the determination of the effect of the bound vorticity. This effect can dominate the results, and the method is therefore not generally trustworthy.

5. Conclusion

The proposed method for including curved bound vortex influence is derived and described. The different previous lifting-line implementations, including the method that considers the curved bound vortex self induction, are also described. A pure comparison of the different lifting-line methods against a panel method is conducted for straight and curved translating wings for inviscid flows. The use of the proposed method is justified based on the good agreements with the panel method. The singular behaviour due to the previous method of calculating the curved bound vortex induction at itself is confirmed numerically for the translating wings. The result from the previous method shows a strong dependency on the number of sections. In contrast, the newly proposed method does not have this singular behaviour.
For the application to straight and curved wind turbine blades, the different lifting-line methods are compared against a set of results obtained with a higher fidelity Computational Fluid Dynamics (CFD) solver. The results indicate the influence of curved bound vortex should be included, in order to correctly calculate the loading of curved wind turbine blades. For both the proposed method and the previous method of including the curved bound vortex influence, the results are in much better agreement with CFD compared to those that do not include the induced velocity from the bound vorticity. The difference between the performance of the proposed method and the previous method for wind turbine blades is smaller than for translating wings. Furthermore, similar to the case of translating wings, the singular behaviour of the previous method, which calculates the curved bound vortex induction on itself, is also confirmed numerically. The predicted result from the previous method is highly dependent on the number of spanwise sections in the discretization. Thus, despite the influence is less obvious when the number of spanwise sections is 50, use of the previous method is not recommended. In contrast, the newly proposed method does not have this issue and will not introduce any additional computational efforts or difficulties compared to the previous method. Thus, the authors recommend using the newly proposed method for calculation of curved blades in lifting-line methods.

Starting from this work, there are multiple future works. Firstly, the main issue of the proposed method is the increased numerical instabilities due to the calculation of the trailed vortex induction at the 3/4 chord points. For the unsteady simulations, this issue is unfavourable. One possible solution could be estimating the necessary relaxation factor based on the method by Pirrung et al. [19]. Secondly, only in-plane blade shape (sweep) is investigated in this study. The investigations on wind turbine blades with out-of-plane shapes (dihedral) as well as combine 3-D shapes are also of great interest. The results and conclusions will be shown in future publications. Thirdly, it can be shown that in analogy to the method for the curved bound vortex induction, the correct shed vortex induction should also use locally the 3-D shed effect minus the 2-D shed effect to be able to refer the results in a consistent manner to a 2D ‘inner’ model. This will be of key importance to get the correct unsteady aerodynamic behaviour for the lifting-line method.

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