Genetic algorithm approach for large scale quadratic programming of probabilistic supplier selection and inventory management problem

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Abstract. This paper is focused on the effectiveness of existing metaheuristic optimization method, genetic algorithm (GA), in providing solutions to a large-scale integer quadratic programming of a probabilistic supplier selection and inventory management problem. The term “probabilistic” in this case refers to any problem that involves some uncertain parameters, approached by random variables (probabilistic parameter). We used the existing mathematical model of probabilistic supplier selection problem and inventory management provided in our previous works. This was done for optimization problem in a small-scale, which could be solved efficiently either by analytical or numerical method. We resolved this model with an extensive number of decision variable, indicated by the number of supplier and time period sufficient to use GA, conducted to analyse if the solution of decision variable, is reliable or not for application to the larger problem. We generated some random data to simulate the problem and the results, running over hundreds of computational experiments. The results showed that the decision obtained by the genetic algorithm was significantly different from the global optimal solution generated by the generalized reduced gradient (GRG) performed in LINGO 18.0. In conclusion, GA is not preferred in solving large-scale problems as regards supplier selection and inventory management.

1. Introduction
A mathematical optimization approach is a powerful tool used to optimize industrial processes, such as logistics and supply chain. Presently, finding the best tool to determine the optimal decision in logistics and supply chain especially for manufacturing industries, is a future challenge due to the blooming of data which has led to the advancement of its collection and storage. Recently, cloud storage has created an avenue for massive data collection and storage, utilized by many global industries from production to selling data sets. Following the growth of available data, coupled with the existence of high-performance computer, decision-makers can now optimize their industries simultaneously on an extensive scale. Thus, implying that if a mathematical optimization is applied, then the decision maker faces a large-scale optimization problem to solve, which becomes a new challenge for the big current data era.

In manufacturing industries, there are many sub-processes which should be optimized from raw material procurement planning, production planning, inventory management and distribution process to the end user. For raw material supplier selection process, there are some classical models that have been
developed such as the linear programming model [1,2]. The more advanced models were developed to solve more complex problem such as a model of supplier selection problem as regards full truck loading was solved in probabilistic environment [3] and a model to solve dynamic supplier selection considering discounts [4]. Unravelling complex supplier selection problem, there are several published research articles that have been found useful. An example could be seen with the model developed under sustainability and risk criteria [5], while an advanced model was formulated under disruption risks [6], with another model refined with extended TOPSIS under Pythagorean fuzzy environment [7]. To illustrate how an optimization model approach of supplier selection problem is applied to real problems, several literatures covers applications in steel, manufacturing and welding industry [8,9].

Another subprocess in industrial manufacturing workflow is inventory management, which is related to a problem of managing raw material/product stock in the warehouse/inventory. There are also many approaches which can be applied to solve dynamical systems, an example is in a state space linear model formulated for inventory control, which was solved using linear quadratic Gaussian. Another model was developed for inventory controlling of imperfect delivery process [10], as advanced models were utilized for inventory management, which involved bounded batch size; solved using sliding mode method[11]. To integrate and solve inventory management and supplier selection problems in one model, some researches were conducted to develop existing approaches. This approach was applied in mathematical optimization of a quadratic programming model [12], while an extended model was formulated to solve a similar problems in an unknown environment [13].

In mathematical optimization theory, there are so many approaches to solving optimization problems from classical algorithms like interior point, steepest-descent, etc., to heuristic optimization approaches like genetic algorithm, ant-colony, particle swarm, etc. As regards genetic algorithm, dozens of research articles have shown its superiority in problem-solving. This genetic algorithm has been deployed in hemodialysis schedule optimization [14], fusion processing of plasma physics [15], cellulase production optimization [16] and design optimization of a high-voltage open-air substation lightning protection system [17]. It has also been deployed in the design optimization of hybrid power supply [18] and time-pickup optimization in radiation detectors [19]. Recently, some genetic algorithms were developed based on classical genetic algorithm and their applications [20], hence improving it by parallelizing and accelerating the execution of the cellular applications to optimize energy broadcast problem. Similarly, a genetic algorithm with dynamic mutation was developed to solve advanced travelling salesman problem [21], whereas an advanced model was developed with hybrid-fuzzy logic which was applied to optimize resource in 5G VANETs [22]. Furthermore, a conservative genetic algorithm was formulated to optimize E-commerce logistics distribution[23], and in other works, a self-tuning multi-objective genetic algorithm was developed for support vector machine classification [24]. Similarly also, an expedited genetic algorithm was formulated for medical image denoising [25], while another novel example was seen where it improved local search, applied to satellite range scheduling systems in monitoring environmental conditions [26]. These extensive lists of literatures, provide us with information that genetic algorithm is a perfect tool to solve many optimization problems.

In this paper, we employed genetic algorithm to solve a supplier selection problem and inventory management, using random parameters with some known probability distribution function. Due to the presence of these random parameters, the mathematical model of this problem produced a probabilistic optimization problem, where the number of variables in the equivalent deterministic model grew exponentially. Further on we analyzed the performance of the genetic algorithm compared with analytical method (gradient based), in small-scale problems with adequate size groups. Based on the results, we predicted the relevancy of genetic algorithm in large-scale problem solutions.

2. Mathematical model

2.1. Problem definition and methodology
A simulated scenario of a manufacturing/retail industry was set up, faced with a supplier selection problem and inventory management for one of its products. The products were assumed as non-perishable and will not expire during the review/optimization time periods either. Furthermore, there are several supplier alternatives to supply the product, where the unit price and supplier’s capacity may differ between these suppliers. The unit price was assumed to be including transport cost from supplier to the inventory system (warehouse), and that the ordered product at any time period will be received at the corresponding time period (as long as there are no delays in delivery). The decision maker has to find the optimal supplier by calculating the optimal product amount from each supplier, along with the optimal amount of the product stored in the warehouse, so that the total cost incurred from the problem is minimal.

For such problems with a small number of supplier alternatives and only few time periods, the corresponding optimization problem will not pose as a herculean task. Followed up with the use of a computer and classical optimization methods like gradient based, the problem can be solved easily. For complex multi-level problems which could consist of supplier alternatives or hundreds of review time periods, the corresponding optimization problem involves a very large-scale decision variable of numbers. For example, if the problem contains only two supplier alternatives and three review time periods, the optimization problem will have only $(2 \times 3 + 3) = 9$ variables, that is 6 variables for each product decision from two suppliers, for three review time periods and 3 variables for decision of product amount stored in the inventory for each of the three periods (a relative small problem). Let the number of supplier alternatives be $S$, number of the review time period as $T$, making the number of variables as $(S \times T + T)$. Therefore, if a problem contains 20 supplier alternatives and 100 review time periods (which could be hourly/daily/weekly/etc.), then the number of decision variable will be 2100, which is sufficiently a large problem.

To solve a large-scale optimization problem within a short time, a high-performance computer and a suitable method are needed. In optimization theory, there are many methods which one can use, and notably is the genetic algorithm which is a heuristic method. Observing the genetic algorithm suitability for supplier selection and inventory management problem, we analyzed it by conducting research using
a methodology illustrated in figure 1. The Generalized Reduced Gradient (GRG) method was performed in LINGO 18.0, which was used to calculate the global optimal solution of the problem. Furthermore, the solution generated by GA will be compared to this global solution, to observe its suitability in solving large problems or not.

### 2.2. Mathematical model

The mathematical notations used in the model are listed as follows:

- $S$: number of the supplier alternatives,
- $T$: number of the review time periods,
- $X_{ts}$: product amount (unit) ordered from supplier $s$ at period $t$,
- $I_t$: product amount (unit) stored in the warehouse at period $t$,
- $U_{ts}$: unit price of product,
- $H_t$: holding cost of product,
- $r_t$: set point for the inventory level,
- $\hat{D}_t$: random variable representing the demand value which is uncertain,
- $C_s$: supplier $s$ maximum capacity to supply the product,
- $M$: warehouse’s maximum capacity to store the product,
- $E[\cdot]$: expectation value notation.

To simplify the research, we resolved the mathematical optimization model of supplier selection and inventory management, using random parameters formulated in our previous work [27], i.e.

$$
\min Z = E[Z] = E\left[\sum_{s=1}^{S} \sum_{t=1}^{T} U_{ts} X_{ts} + \sum_{t=1}^{T} H_t I_t + \sum_{t=1}^{T} (I_t - r_t)^2\right]
$$

subject to:

1. $I_{t-1} + \sum_{s=1}^{S} X_{ts} - I_t \geq \hat{D}_t, \forall t$
2. $X_{ts} \leq C_s, \forall t \in T, \forall s$
3. $I_t \leq M, \forall t$
4. $X_{ts}, I_t$ integer and nonnegative, $\forall t, \forall s$

where the objective function and the constraints appeared in the model are explained as follows. The objectives that will be minimized, contains the expectation value of the total cost accrued to the problem, which covers purchasing and holding cost. The last term in the objectives represents a terminal constraint to bring the inventory to a reference point $r_t$, which is given by the decision maker. Constraint in inequality (2) ensures that for any $t$, the available product amount satisfies the demand amount, where $\hat{D}_t$ denotes a random variable representing the demand value which is uncertain. Constraint in equality (3) ensures that the product amount ordered from $s$ will not exceed the maximum capacity, while constraint in equality (4) ensures that the stored product amount will not exceed the maximum warehouse capacity. Lastly, constraint in (5) is applied to assign integers and non-negative values for all decision variables.

Optimization of objective function (1) contains some random parameters, which its solution requires the application of wait and see approach. Generating a deterministic problem, the problem is converted into a deterministic equivalent model, for each scenario of the random variable’s realization. After which, it is solved by the application of genetic algorithm to generate the optimal decision.

### 2.3. Genetic algorithm (revisiting)

Genetic Algorithm (GA) works by introducing a chromosome vector containing all decision variables, further generating a population decision variable vector, before its implementation. At a GA’s iteration, a new population is generated using selection, mutation and crossover. Thus, the objective function of
the optimization problem is assigned to the GA’s fitness function. The algorithmic procedures in the GA are:

- Initialization of the population randomly.
- Fitness function evaluation and scaling.
- Selection process: ‘Parents’ are selected from population that will be used to produce a successive generation which is called ‘reproduction’. Furthermore, roulette wheel, tournament, stochastic uniform, remainder selection are some selection methods we can also use.
- Crossover: It is a genetic operator to carry out a reproduction. Arithmetic, intermediate, scattered, are several crossover alternatives that can be used.
- Mutation: Several methods like adaptive feasible and Gaussian are commonly used as mutation operator.
- Termination: There are many conditions which can be applied as termination criteria which includes; number of generations, allocated budget, highest ranking solution, manual inspection.

3. Numerical experiment
Assume the optimization problem (1) where the number of suppliers is 3 and the number of the time period is 3. In a bid to observe the result of the optimal objective function value and optimal decision generated by genetic algorithm, we ran hundreds of numerical experiments where the parameter value for optimization problem (1) refers to our previous published article [27]. However, the demand is random where $\hat{D}_t, \forall t = 1, 2, 3$ are identical and independent to the probability density function:

$$f_{\hat{D}_t}(d) = \begin{cases} 0.3, & \text{if } d = 100 \\ 0.3, & \text{if } d = 150 \\ 0.4, & \text{if } d = 200 \end{cases}$$

for all $t$. The remaining parameter values (unit price, holding cost, etc.) are available in our previous published article [28]. The parameter values for simulation used in the experiment are also shown in table 1. We conducted hundreds of numerical experiments, where the results were similar for all experiments. Hence to describe the optimization results, i.e. the optimal decision value generated by GA and the optimal decision generated by LINGO 18.0, which employs the GRG method are displayed in figure 2 and figure 3.

**Table 1. GA’s parameter values.**

| Parameter               | Value   | Parameter               | Value   |
|-------------------------|---------|-------------------------|---------|
| Population size         | 1000    | Migration Interval      | 1000    |
| Elite Count             | 100     | Migration fraction      | 0.2     |
| Crossover fraction      | 0.8     | Generation limit        | 10000   |
| Crossover function      | Arithmetic | Stall generation limit  | 1000    |
| Mutation function       | adaptive | Maximum computation time | 20 minutes |
| Function’s tolerance    | 1e-10000 | Selection function      | Roulette |
| Constraint’s tolerance  | 1e-10000 |                         |         |

Figure 2 shows the amount of the product which could be ordered from each supplier for each time period generated by GA and GRG method; (a) scenario-1 (b) scenario-2. The term ‘scenario-1’ refers to a condition where the realization of the demand is 100 units for each time period 1, 2, and 3. However, in contrast, scenario-2 refers to a condition where the demand value is 100 units for time period 1, 100 units for 2, and 150 units for 3. The decision generated by GRG is a global optimal solution, whereas
the decision generated by GA is a sub-optimal solution. Therefore, it can be seen that the amount of the ordered product generated by GA, largely differs from GRG which is the global optimal solution. Furthermore, examining the total number of products, GA generated around two times the solution generated by GRG.

In this experiment, the reference point for inventory management was 50 units for each time period. As regards scenario-1’s global solution generated by GRG, decision for the inventory was 48 units stored at time period 1 and 2, while 24 units was stored in time period 3 (figure 3). The decision for the inventory generated by GA was approximately 50% under or above the decision generated by GRG. For other scenarios, similar results were reached, implying that the decision generated by GA was significantly different from the global solution. Furthermore, the GRG method achieved the total expected cost of 25613, whereas GA achieved the total expected cost of 52185, which is about two times the total expected cost achieved by GRG. From these results, it means that the solution generated by GA is around 100% above the global optimal solution, implying that GA is not suitable for this problem.

4. Concluding remarks
In this paper, genetic algorithm was employed to solve probabilistic supplier selection problems including inventory management problem, using probabilistic demand modeled as quadratic programming. The corresponding probabilistic optimization problem was solved using deterministic equivalent optimization, which produced a wait-and-see solution. Furthermore, there was reported significant difference for the solution to a specified small problem, when genetic algorithm was compared to the global solution. Hence, it implies that genetic algorithm is not adequate in solving large Scale problems. Lastly, our subsequent research aims at discovering other metaheuristic methods,
capable of solving large-scale probabilistic supplier selection problems, which includes inventory management.

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