Higher spin gauge theories and bulk locality: a no-go result

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Abstract: We present a no-go result on consistent interactions among higher-spin gauge fields in AdS_{d+1} space-times. We show that crossing symmetry with higher-spin gauge fields in the spectrum necessitates the presence of pathological 1/□ non-localities in the bulk. This suggests the breakdown of the field theory picture for interacting higher-spin gauge theories at the classical level.

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1 Introduction

In this work we study the problem of constructing consistent interactions for higher-spin gauge theories on anti-de Sitter space-times. Higher-spin gauge theories have gathered increased attention in the recent decades, owing in part to the hypothesis that a higher-spin symmetry may govern the high energy regime of a UV-complete theory of gravity [1]. This was motivated from a string theory perspective by D. Gross in the late 80’s [2].

By now the free propagation of higher-spin particles on maximally symmetric backgrounds is rather well understood [3–13],1 borne out of the early 1930’s works of Majorana, Dirac, Fierz, Pauli and Wigner [20–24]. But the question of whether mass-less higher-spin particles can interact in a consistent manner is a highly non-trivial one. In flat space, there are various no-go results [25–37] against the existence of consistent interactions.2 Constant curvature backgrounds have shown promise, with, for instance, the possibility of a minimal coupling of higher-spin gauge fields to gravity [35, 50, 51] and the existence of a well defined higher-spin algebra [1, 52, 53]. These positive results were further substantiated by the conjectured holographic dualities between higher-spin gauge theories on anti-de Sitter backgrounds and free conformal field theories [54–58], and the existence of Vasiliev’s system [53, 59] as a framework to describe consistent (semi-)classical interacting theories of higher-spin gauge fields.

An important issue in the quest for consistent non-trivial interacting theories of higher-spin gauge fields is that of locality, which on AdS backgrounds has yet to be fully addressed. It is well known that interacting theories of higher-spin gauge fields are non-local in the sense that a Lagrangian/equations of motion would involve an arbitrary number of derivatives. It is then crucial to understand whether the functional class of non-localities allows for a well defined notion of non-trivial interactions, a problem which we review below in §1.1.

We address this issue in the present work. In particular, we report a no-go result against the existence of consistent interactions among higher-spin gauge fields on AdS_{d+1}. Like analogous locality-based no-go results in flat space [33, 34, 36, 37], we find that interacting theories of higher-spin gauge fields on AdS necessarily contain non-localities of the pathological 1/□-type. Our findings suggest that a field theory description of interacting higher-spin gauge theories on AdS backgrounds at the classical level may not be well defined.

1.1 Review: The problem of functional class

We conclude the introduction by underlining the problem of functional class in classical field theory, and in particular for theories of higher-spin gauge fields. The key point is a result of

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1For other backgrounds see: [14–19].

2This problem has also recently been explored using non-covariant methods [38–45], inspired by the results of [46–49].

3We subscribe to the standard field theory doctrines which ask that “consistent” theories are free of: propagating negative-energy (ghost) excitations, algebraic inconsistencies of field equations, discontinuities in the number of degrees of freedom and so on.

4With the exception of purely topological higher-spin gauge theories, which do not give rise to propagating degrees of freedom. This includes: Three-dimensional purely Chern-Simons higher-spin gauge theories [60–63]. See also [64] in five-dimensions and [65] in generic odd dimensions.
G. Barnich and M. Henneaux [66] that, if there is no restriction placed on the functional class, there is no non-trivial interaction.

To illustrate the problem, without loss of generality, one can consider the simple example of a theory of a massive scalar field $\Phi$ in flat space (see [33, 67–69] in the context of higher-spin gauge theories, also on AdS space times):

$$S = \int \left[ \frac{1}{2} \partial_{\mu} \Phi \partial^{\mu} \Phi + m^2 \Phi^2 + \Phi J(\Phi, \Phi) + \mathcal{O}(\Phi^4) \right] + \int_{\partial} \mathcal{B},$$

with an arbitrary interaction term parametrised by the bi-linear functional $J(\Phi, \Phi)$ and possible higher-order interaction terms $\mathcal{O}(\Phi^4)$. $\mathcal{B}$ is an appropriately fixed boundary functional which, although usually discarded in flat space, we keep to illustrate features of the discussion for more general backgrounds.

With no restriction on the functional class of field-redefinitions (i.e. with no locality requirement or constraint on the degree of non-locality) one can, order by order, remove any interaction term. Starting at cubic order, the cubic interaction in (1.1) can be removed with the liberty to perform the following wildly non-local field re-definition of the $1/\Box$-type:

$$\Phi \to \Phi + \frac{1}{\Box - m^2} J(\Phi, \Phi),$$

which then replaces it with some higher-order interaction term that is at least quadrilinear in the scalar

$$\int \Phi J(\Phi, \Phi) \overset{(1.2)}{\to} \int \left( \frac{1}{\Box - m^2} J(\Phi, \Phi) \right) J(\Phi, \Phi) + \ldots .$$

With the cubic interactions re-defined away, with no functional class restriction one can proceed iteratively in the same manner, order-by-order, until no interaction term remains.

We note that, at each order, this procedure leaves at most a (non-local) boundary term. For the re-definition (1.2) we have:

$$S \overset{(1.2)}{\to} \int \frac{1}{2} \partial_{\mu} \Phi \partial^{\mu} \Phi + m^2 \Phi^2 + \mathcal{O}(\Phi^4)$$

$$+ \int_{\partial} n \cdot \partial \left[ \frac{1}{2} \left( \frac{1}{\Box - m^2} J(\Phi, \Phi) \right) \Phi \right] + \int_{\partial} \delta \mathcal{B},$$

with $n_\mu$ the normal vector to the boundary. The newly generated boundary term in the above may be compensated by the appropriate choice of the original boundary term $\mathcal{B}$. Ultimately, the latter should be fixed requiring a consistent boundary value problem in the bulk while it is usually discarded in flat space.

In classical field theories we thus see that it is important to identify, and restrict to, the functional class of field re-definitions which does not lead to the systematic removal of interactions illustrated above. Such re-definitions are referred to as admissible. This problem is particularly relevant for theories which contain an arbitrary number of derivatives and thus, assuming manifest covariance, an arbitrary number of time-derivatives: The pathologies associated to theories with Lagrangians depending non-trivially on more than first time-derivatives are well known, perhaps most notably the instability of the Hamiltonian [70, 71]. A crucial
consistency requirement is thus that the theory is first-order in time under (admissible) field re-
definitions. A necessary condition for which is the absence of couplings of the $1/\Box$-type in the
Lagrangian/equations of motion, as they would entail field re-definitions of the non-admissible
$1/\Box$-type (1.2) to lower the number of time-derivatives.

The above condition is often relaxed in theories where higher-order derivative terms can
be treated perturbatively, in regimes where their effects are (infinitesimally) small. This is
for instance the case for generic $\alpha'$ corrections to Einstein gravity. However, in the absence
of such a separation of scales, the ability to render a theory first order in time via admissible
field re-definitions is all the more important. An example of a class of theories in which
higher-derivative terms cannot be treated as sub-leading is given by theories of higher-spin
gauge fields, which are conjectured to describe a would-be tensionless ($\alpha' \rightarrow \infty$) limit of string
theory [2, 72, 73].

Higher-spin gauge theories
In the present work we consider the above problem of functional class for theories of higher-spin
gauge fields on anti-de Sitter space-times. The putative covariant Lagrangian of an interacting
higher-spin gauge theory is expected to be un-bounded in the number of derivatives:
In generic space-time dimensions, the minimal spectrum of consistent interacting theories of higher-spin
gauge fields consists of a tower of gauge fields $\varphi_s$ unbounded in spin $s$ and a scalar $\varphi_0$ [1, 92].
This is governed by an action of the schematic form (see §2.2 for precision)
\[
S_{\text{min HS}} = \int_{\text{AdS}} \sum_{s=0}^{\infty} \varphi_s (-m_s^2 + ...) \varphi_s + \sum_{s_3 \leq s_2 \leq s_1} g_{s_1,s_2,s_3} \nabla^{s_3} \varphi_{s_1} \nabla^{s_1} \varphi_{s_2} \nabla^{s_2} \varphi_{s_3}
+ \sum_{l=0}^{\infty} \sum_{m=0}^{l} \lambda_{l,m} \varphi_0 \nabla_{\mu_1} ... \nabla_{\mu_l} \varphi_0 \Box^m (\varphi_0 \nabla^{\mu_1} ... \nabla^{\mu_l} \varphi_0) + ..., \tag{1.5}
\]
where the ... in the kinetic terms denote the off-shell completion, given by the Fronsdal action
(2.26). On the second line, for concision we display only the schematic form of the quartic
scalar self-interaction, with the ... denoting quartic interactions among other quadruplets of
fields in the spectrum and also higher-order couplings.

The cubic coupling constants $g_{s_1,s_2,s_3}$ were determined in [41, 93, 94] up to field re-
definitions, while the quartic and higher-order couplings are currently unknown in an orthogonal basis (see [95–97]). An important open question is whether or not the asymptotic behaviour of higher-order coupling constants (like the $\lambda_{l,m}$ in (1.5)) subverts the triviality argument of [66]
described in the previous section. A necessary condition for which is the absence of $1/\Box$-type
non-localities in the re-summation of derivative expansions, such as the one displayed in (1.5)
for the quartic scalar self-interaction. For previous studies in this direction, see: [69, 95–99].

\[\text{5} \text{For work in this direction, see: [54, 55, 74–89].}\]
\[\text{6} \text{This problem has also been addressed in open string field theory by T. Erler and D. Gross [90], whose action}\]
\[\text{[91] also contains an infinite number of time and space derivatives.}\]
\[\text{7} \text{Exceptions include: Three-dimensional purely Chern-Simons higher-spin gauge theories [60–63]. See also}\]
\[\text{[64] in five-dimensions and [65] in generic odd dimensions. More generally, any purely topological higher-spin}\]
\[\text{gauge theories, which do not give rise to propagating degrees of freedom.}\]
We provide an answer to this question in the present work. In particular, we report a no-go result against consistent non-trivial interactions among higher-spin gauge fields that are described, in the free limit, by the Fronsdal action (2.26) on an AdS background (for exceptions see footnote 7). We show that crossing symmetry with higher-spin gauge fields in the spectrum necessitates the presence of $1/\Box$ non-localities in an interacting Lagrangian (1.5) at the quartic order, and such theories thus fall under the triviality argument of [66] described in the previous section.

Let us also briefly mention parallel studies of the functional class problem in higher-spin gauge theories within the framework of Vasiliev’s equations. In contrast to the metric-like Lagrangian description (1.5) employed in the present work, Vasiliev’s equations [53, 59] provide a fully non-linear framework to describe higher-spin gauge theories at the level of the equations of motion, using the so-called unfolded, first-order, formalism [100–102] (for reviews see [103, 104]). Recently, in three and four space-time dimensions, the $g_{s_1,s_2,s_3}$ couplings were extracted from the unfolded framework in [98],\(^8\) and were shown to be divergent. This indicates that $1/\Box$ non-localities also arise within the unfolded description of higher-spin gauge theories [69, 99].\(^9\) In contrast, the results of the present work indicate that $1/\Box$-type non-localities are unavoidable in any interacting field theory description of higher-spin gauge theories on AdS spaces at the classical level, in general dimensions, as a consequence of basic consistency requirements. They also suggest that the $1/\Box$-type non-localities of the unfolded description uncovered in [69, 98, 99, 110] are not solely an artifact of the unfolded formalism itself.

1.2 Outline and summary of results

This work consists of two main sections: §2 Noether procedure and §3 Locality.

In §2 we fix the complete non-linear metric-like action of the minimal higher-spin gauge theory on $\text{AdS}_{d+1}$ up to field re-definitions, using the Noether procedure and global higher-spin symmetries. We first review the Noether procedure in a generic setting. We highlight that particular solutions for interaction terms at a given order in fields are given by exchange amplitudes generated by lower order couplings, and that homogeneous solutions can be identified with scattering observables.

In §2.1 we derive constraints placed by global symmetries on homogeneous solutions to the Noether consistency conditions. For theories on $\text{AdS}_{d+1}$, we show that these are equivalent to Ward identities for conformal correlation functions on the AdS boundary.

In §2.2 we turn to higher-spin gauge theories on $\text{AdS}_{d+1}$. Assuming that the higher-spin gauge symmetry arises from the gauging of a rigid higher-spin symmetry, we show that symmetry fixes all couplings in a putative higher-spin gauge theory action uniquely up to field re-definitions. We present the result for the minimal higher-spin theory on $\text{AdS}_{d+1}$.

In §3 we study the problem of functional class (that was reviewed in §1.1) for higher-spin gauge theories on AdS backgrounds. In §3.1 we first review the problem in flat space, and how it leads to obstructions to the existence of interacting higher-spin gauge theories as field theories on flat backgrounds.

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\(^8\)See [68, 105] for the details on the extraction of the back-reaction in a redundant basis of interactions.

\(^9\)For further works on the problem of functional class within this framework, see [106, 107]. For early discussions see: [51, 108] which are based on the results presented in [109].
In §3.2, using the results of §2, we show that obstructions to non-trivial interactions among higher-spin gauge fields are also present on AdS backgrounds. In §3.3 we show that the obstructions in AdS can be attributed to the properties of solutions to crossing in the presence of higher-spin symmetry. In §3.4 we discuss in further detail the assumptions of our analysis.

In the concluding section 4 we summarise and discuss how the results of the present work fit into the existing literature on higher-spin gauge theories and holography.

2 Noether procedure

The Noether procedure is a systematic scheme to solve for interacting field theories as deformations of free actions governed by gauge and global symmetries (see e.g.: [66, 111]): Given the latter, one postulates the existence of a fully non-linear action and gauge symmetries, which are expanded on a given background in weak fields as

$$ S = S^{(2)} + \sum_{n > 2} S^{(n)}, \quad \delta \xi = \delta^{(0)} \xi + \sum_{n > 0} \delta^{(n)} \xi. $$

The notation \((n)\) signifies that the corresponding term is power \(n\) in the weak fields.

The requirement of gauge invariance translates into an infinite set of coupled equations:

$$ \delta^{(0)} \xi S^{(2)} = 0, $$
$$ \delta^{(1)} \xi S^{(2)} + \delta^{(0)} \xi S^{(3)} = 0, $$
$$ \delta^{(2)} S^{(2)} + \delta^{(1)} \xi S^{(3)} + \delta^{(0)} \xi S^{(4)} = 0, $$
$$ \vdots $$
$$ \delta^{(n-2)} \xi S^{(2)} + \left( \sum_{k=3}^{n-1} \delta^{(n-k)} \xi S^{(k)} \right) + \delta^{(0)} \xi S^{(n)} = 0, $$
$$ \vdots, $$

whose most general solution at a given order can be written as

$$ S^{(n)} = S^{(n)}_h + S^{(n)}_p, $$

where \(S^{(n)}_h\) solves the homogeneous equation

$$ \delta^{(0)} \xi S^{(n)}_h \approx 0, $$

and \(S^{(n)}_p\) is the particular solution to the original non-homogeneous equation (2.2e), which contains the information about the lower order solutions.

A particular solution that has a neat physical interpretation is given by \textit{minus} the exchange amplitudes generated by the lower order couplings. For instance, at quartic order

$$ S^{(4)}_p = -A^{(4)} \equiv - \left( A^s + A^t + A^u \right), $$

where

\(A^s\), \(A^t\), and \(A^u\) are the exchange amplitudes.
where the sanserif font denotes the s-, t- and u-channels. That this solves the inhomogeneous quartic consistency condition (2.2c) can be seen from [33, 49, 85]

$$\delta^{(0)}_\xi \left(-A^{(4)}\right) \approx \delta^{(1)}_\xi S^{(3)}.$$  (2.6)

A further attractive feature of the above choice of particular solution is that the corresponding homogeneous solution

$$S_h^{(n)} = A^{(n)} + S^{(n)},$$  (2.7)

is then directly related to “scattering-like” observables of the theory. This link is especially significant for theories on AdS\(_{d+1}\), where one can draw upon the dual interpretation of such observables as correlation functions of single-trace operators on the \(d\)-dimensional conformal boundary. We make this relationship more concrete in the following, and in particular show that conformal correlation functions in \(d\)-dimensions provide homogeneous solutions to the Noether procedure in AdS\(_{d+1}\).

2.1 The homogeneous solution, global symmetries and AdS/CFT

Beyond quadratic order \((n > 2)\), the homogeneous equation

$$\delta^{(0)}_\xi S_h^{(n)} \approx 0,$$  (2.8)

does not have a unique solution. Further constraints are placed on the homogeneous solution \(S_h^{(n)}\) if the gauge symmetry (2.1) arises from the gauging of a global symmetry. In particular, this requires the Noether consistency conditions (2.2) to hold also for parameters \(\xi\) associated to global symmetries, which are obtained by imposing the Killing equations

$$0 = [\delta_\xi \phi],_{\phi=0} = \delta^{(0)}_\xi \phi,$$  (2.9)

with solutions given by Killing tensors \(\xi = \bar{\xi}\), and \(\phi\) collectively denotes the field content. In particular, it implies that the homogeneous solution (2.8) is also invariant under global transformations,\(^\text{11}\)

$$\delta^{(1)}_\xi S_h^{(n)} \approx 0.$$  (2.12)

This constraint originates from the restriction of the \((n+1)\)-th order consistency condition (2.2) to global gauge parameters \(\hat{\xi}\):

$$0 = \delta^{(1)}_{\hat{\xi}} \left(S_h^{(n)} + S_p^{(n)}\right) + \sum_{k=2}^{n-1} \delta^{(n+1-k)}_{\hat{\xi}} S^{(k)} \approx \delta^{(1)}_{\hat{\xi}} S_h^{(n)}.$$  (2.13)

\(^{10}\)The intuition being that it is of the form “exchange”+“contact”.

\(^{11}\)Recall that the Lie algebra bracket \([,]\) is inherited from that of the gauge algebra via

$$\delta^{(0)}_{[\xi_1, \xi_2]} = \delta^{(0)}_{\xi_1} \delta^{(1)}_{\xi_2} - \delta^{(0)}_{\xi_2} \delta^{(1)}_{\xi_1},$$  (2.10)

where \(\delta^{(0)}_\xi\) and \(\delta^{(1)}_\xi\) are the first two terms in the expansion (2.2) of the non-linear gauge transformation. The rigid symmetries also close at the level of \(\delta^{(1)}_{[\xi_1, \xi_2]}\):

$$\delta^{(1)}_{[\xi_1, \xi_2]} = \delta^{(1)}_{\xi_1} \delta^{(1)}_{\xi_2} - \delta^{(1)}_{\xi_2} \delta^{(1)}_{\xi_1} + \text{(trivial)},$$  (2.11)

meaning that \(\delta^{(1)}_\xi\) provides a representation of the rigid symmetries carried by the field content.
In the following we first discuss the condition (2.12) for the quadratic and cubic action \((n = 2\) and \(n = 3\)), before moving on to general \(n\) and the relation to scattering observables.

At order \(n = 2\), (2.12) is a condition on the full quadratic action
\[
\delta_{\xi}^{(1)} S^{(2)} = 0,
\]
endowing it with an invariant scalar product. This constrains the field content \(\{\varphi\}\) of the theory, with the kinetic terms encoded in \(S^{(2)}\)
\[
S^{(2)} = \sum_{\{\varphi\}} S^{(2)} [\varphi],
\]
and the action of \(\delta^{(1)}_{\xi}\) as in (2.14) rotates them into each other.

The global symmetries are closely related to the cubic couplings of the theory, as can be seen from the full cubic order consistency requirement (2.2b)
\[
\delta_{\xi}^{(1)} \xi S^{(2)} = -\delta_{\xi}^{(0)} \xi S^{(3)},
\]
However, this relationship does not constrain couplings that are exactly invariant under \(\delta^{(0)}\),
\[
\delta_{\xi}^{(0)} S^{(3)} = 0,
\]
which includes those expressed purely in terms of curvatures.\(^{12}\) As discussed above, further constraints come from invariance (2.12) under global symmetries
\[
\delta_{\xi}^{(1)} S^{(3)} \approx 0,
\]
whose efficacy naturally depends on the size of the symmetry algebra. As we shall see in the following section, for higher-spin gauge theories governed by an infinite-dimensional higher-spin algebra, this is sufficient to fix \(S^{(3)}\) and all higher-order couplings (2.12) uniquely up to field re-definitions.

At general order \(n\) recall that, in choosing the particular solution to be the exchange amplitude generated by all lower-order couplings (2.5), the \(n\)-th order homogeneous solution, on-shell, acquires the interpretation as a scattering observable:
\[
S^{(n)}_{\text{h}} = S^{(n)} + A^{(n)}.
\]
For theories on \(\text{AdS}_{d+1}\) backgrounds, with the appropriate boundary conditions on the bulk fields, the form (2.19) of the homogeneous solution is the generating function of \(n\)-point correlation functions of single-trace operators in the dual CFT\(_d\) \([112, 113]\).\(^{13}\) In particular, the boundary value \(\bar{\varphi}\) of the bulk field \(\varphi\) sources its dual single-trace operator \(\mathcal{O}\), such that
\[
\langle \mathcal{O}_{1} \ldots \mathcal{O}_{n} \rangle_{\text{connected}} \left( \text{N.D.O.F.} \gg 1 \right) \left( -1 \right)^{n} \delta_{\bar{\varphi}_n} \ldots \delta_{\bar{\varphi}_1} S^{(n)}_{\text{h}} [\varphi_i, \varphi_i]_{\partial\text{AdS}} = \bar{\varphi}_i,
\]
\(^{12}\)In pure gravity for example, these come from (Riemann)\(^2\) and (Riemann)\(^3\) terms.
\(^{13}\)For totally symmetric bulk fields of spin-\(s\) and mass \(m^{2}R^{2} = \Delta (\Delta - d) - s\), the conformal-symmetry-preserving boundary condition
\[
\varphi^{(1\ldots i)} (y, w) \sim \bar{\varphi} (y)^{(1\ldots i)} w^{d+s-\Delta}, \quad \text{as} \quad w \to 0,
\]
sources a single-trace operator on the boundary of scaling dimension \(\Delta\) and spin-\(s\). (2.20) is given in Poincaré co-ordinates \(x^\mu = (y^\prime, w), \ i = 1, \ldots, d\) with the AdS boundary located at \(w = 0\) and the \(y^\prime\) parametrise the boundary directions.
where the number of boundary degrees of freedom, \( N_{\text{d.o.f.}} \), is taken to be large in accordance with weak fields in the bulk.\(^{14}\)

We see that, in AdS, the construction of consistent bulk interactions can be mapped to the classification of consistent conformal correlators, which serve as the scattering observables of the bulk theory. In particular, in the dual CFT picture the global symmetry constraints (2.12) are equivalent to Ward identities

\[
\delta_{\xi_s}^{(1)} S_h^{(n)} \approx 0 \iff 0 = \sum_i \langle \mathcal{O}_1 ... [Q_{\xi_s}, \mathcal{O}_i] ... \mathcal{O}_n \rangle, \tag{2.23}
\]

where \( Q_{\xi_s} \) is the charge associated to the Killing tensor \( \xi_s \).

### 2.2 Higher-spin gauge theories

Having discussed the generalities behind the construction of interacting theories using the Noether procedure, we now turn to the application of this approach to the case where higher-spin gauge fields are present in the spectrum. In particular, under the assumption that the corresponding higher-spin gauge symmetry arises from the gauging of a higher-spin symmetry algebra, we show that interactions in higher-spin gauge theories on \( \text{AdS}_{d+1} \) are fixed uniquely at all orders up to field re-definitions.

The two-derivative kinetic term for a totally symmetric spin-\( s \) gauge field \( \varphi_s \) on an \( \text{AdS}_{d+1} \) background was determined by Fronsdal in 1978 [3]: The \( n = 2 \) homogeneous consistency condition (2.2) for invariance under linearised spin-\( s \) gauge transformations:

\[
\delta_{\xi_s}^{(0)} S^{(2)} [\varphi_s] = 0, \quad \delta_{\xi_s}^{(0)} \varphi_s (x, u) = (u \cdot \nabla) \xi_s (x, u), \tag{2.25}
\]

admits the unique two-derivative solution

\[
S^{(2)} [\varphi_s] = \frac{s!}{2} \int_{\text{AdS}_{d+1}} \varphi_s (x, \partial_u) \mathcal{G}_s (x, u), \tag{2.26}
\]

where \( \mathcal{G}_s \) is the generalisation of the linearised Einstein tensor to spin-\( s \) gauge fields,

\[
\mathcal{G}_s (x; u) = \left( 1 - \frac{1}{4} u^2 (\partial_u \cdot \partial_u) \right) \mathcal{F}_s (x; u, \nabla, \partial_u) \varphi_s (x, u), \tag{2.27}
\]

and \( \mathcal{F}_s \) is the so-called Fronsdal operator

\[
\mathcal{F}_s (x; u, \nabla, \partial_u) = \Box - m^2 - u^2 (\partial_u \cdot \partial_u) - (u \cdot \nabla) \left( \frac{1}{2} (u \cdot \nabla) (\partial_u \cdot \partial_u) \right), \tag{2.28a}
\]

\[
m^2 R^2 = (s + d - 2) (s - 2) - s. \tag{2.28b}
\]

\(^{14}\)Recall that

\[
\frac{R_{d-1}}{G} \sim N_{\text{d.o.f.}}, \tag{2.22}
\]

where \( R \) is the AdS radius and \( G \) the gravitational constant.

\(^{15}\)Here, and throughout, we employ generating function notation to package the space-time indices

\[
\varphi_s (x, u) = \frac{1}{s!} \varphi_{\mu_1 ... \mu_s} u^{\mu_1} ... u^{\mu_s}, \quad \xi_{s-1} (x, u) = \frac{1}{(s - 1)!} \xi_{\mu_1 ... \mu_{s-1}} u^{\mu_1} ... u^{\mu_{s-1}}. \tag{2.24}
\]
This also requires to impose the following algebraic trace constraints on the field $\varphi_s$ and the gauge parameter $\xi_{s-1}$:

$$
(\partial_u \cdot \partial_u)^2 \varphi_s (x, u) = 0, \quad (\partial_u \cdot \partial_u) \xi_{s-1} (x, u) = 0.
$$

(2.29)

The free Fronsdal formulation (2.26) serves as a starting point to construct theories of interacting higher-spin gauge fields using the Noether consistency conditions (2.2). The free theory alone encodes the global symmetries that govern the spectrum, which can be extracted from the free theory Noether currents [35]. The higher-spin symmetry algebra closing on totally symmetric gauge fields is unique in generic dimensions [92], and is given by the Eastwood-Vasiliev algebra [52, 53]. This constrains the spectrum via the global symmetry requirement (2.8)

$$
\delta^{(1)} \bar{S}^{(2)} = 0.
$$

(2.30)

The minimal higher-spin-symmetric spectrum consists of a tower of even spin totally symmetric gauge fields $\varphi_s$, $s = 2, 4, 6, \ldots$ and a parity even scalar $\varphi_0$. This is the so-called type A higher-spin theory [53, 59], and is the unique higher-spin theory in generic dimensions consisting of only totally symmetric fields. Its quadratic action reads

$$
S^{(2)} = \sum_{s \in 2\mathbb{N}} S^{(2)} [\varphi_s],
$$

(2.31)

which is a sum of free Fronsdal actions (2.26). The spectrum (2.31) forms a closed sub-sector of any theory of totally symmetric higher-spin gauge fields on $\text{AdS}_{d+1}$. For simplicity we thus henceforth restrict to the theory of this minimal spectrum i.e. the type A higher-spin theory.

Higher-spin symmetry is an incredibly powerful constraint at the interacting level. Alone, it fixes all interactions and their couplings completely up to field re-redefinitions. This can be seen by analysing the global symmetry constraint

$$
\delta^{(1)} \bar{S}^{(n)} \approx 0,
$$

(2.32)

for higher-spin Killing tensors $\bar{\xi}$. As explained at the beginning of §2, with the particular solution given by exchange amplitudes generated by lower order couplings, the homogeneous solution $\bar{S}^{(n)}_h$ takes the role of a scattering observable when evaluated on-shell. The constraint (2.32) forces such observables to take the unique form of correlators of a free scalar conformal theory on the $d$-dimensional boundary, as first shown in $d = 3$ by Maldacena and Zhiboedov [115] and later extended to general $d$ in [92, 116]. In particular, for fixed external legs of spins $s_1 - s_2 - \ldots - s_n$, evaluated on-shell with boundary condition (2.20), we have

$$
\bar{S}^{(n)}_h = \langle J_{s_1} \cdots J_{s_n} \rangle_{\text{conn}},
$$

(2.33)

---

16 Its uniqueness can be seen by requiring closure of the algebra of charges constructed from the free theory Noether currents.

17 See also the earlier work [1] on the simpler $\text{AdS}_4$ case.

18 This can also be understood from the CFT perspective where, in the free $\lambda \to 0$ limit of the boundary theory, the sector of single-trace bi-linears that are dual to a tower of higher-spin gauge fields in the bulk is closed under the OPE [114].
where \( J_{s_i} \) is the single-trace operator of twist \( \tau = \Delta = d - 2 \) on the \( d \)-dimensional boundary that is sourced by the bulk field \( \varphi_n \). For \( s_i > 0 \) these are conserved: \( \partial \cdot J_{s_i} \approx 0 \).

The explicit form of (2.33) can be expressed in terms of Bessel functions [93] \(^{19}\)

\[
\langle J_{s_1}(y_1;z_1)J_{s_2}(y_2;z_2) \ldots J_{s_n}(y_n;z_n) \rangle_{\text{conn.}} = \frac{N_{\text{d.o.f.}}}{(y_{12})^{\Delta/2}(y_{23})^{\Delta/2} \ldots (y_{n1})^{\Delta/2}} \times \left( \prod_{i=1}^{n} c_{s_i} q_i^{1-\frac{\Delta}{2}} \Gamma \left( \frac{\Delta}{2} \right) \frac{J_{s_i-2}}{y_i^{s_i}} \right) Y_1^{s_1} Y_2^{s_2} \ldots Y_n^{s_n} + \text{perm.},
\]

where \( y_{ij}^2 = (y_i - y_j)^2 \), \( q_i = 2H_i \partial Y_{i+1} \cdot \partial Y_{i+2} \) and the six three-point conformally covariant building blocks read explicitly \( i \triangleq i + 3 \)

\[
Y_i = \frac{z_i \cdot y_i(i+1)}{y_i^{s_{i+1}}}, \quad Y_i = \frac{z_i \cdot y_i(i+2)}{y_i^{s_{i+2}}},
\]

\[
H_i = \frac{1}{2y_i^{s_{i+1}}(i+2)} \left( z_{i+1} \cdot z_{i+2} + \frac{2z_{i+1} \cdot y_{i+1}(i+2)}{y_i^{s_{i+2}}}, \frac{z_{i+2} \cdot y_{i+2}(i+1)}{y_i^{s_{i+2}}}, \ldots \right).
\]

The coefficients \( c_{s_i} \) are given explicitly, for canonically normalised two-point functions, by [93]

\[
(c_{s_i})^2 = \sqrt{\pi} \frac{2^{-d-s_i}}{N_{\text{d.o.f.}} s_i!} \frac{\Gamma(s_i + d - 3)}{\Gamma(s_i + d - 2)\Gamma(d-2)^2},
\]

where \( N_{\text{d.o.f.}} \) labels the number of degrees of freedom on the boundary, which for weak fields in the bulk is taken to be large.

Combined with the particular solution (2.5), the complete solution to the Noether procedure is thus dictated uniquely by higher-spin symmetry to be

\[
S^{(3)}_{s_1,s_2,s_3} = \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\text{connected}}
\]

\[
S^{(n)}_{s_1,s_2, \ldots, s_n} = \langle J_{s_1} \ldots J_{s_n} \rangle_{\text{connected}} - A^{(n)}_{s_1,s_2, \ldots, s_n} \text{ for } n > 3
\]

where \( A^{(n)}_{s_1,s_2, \ldots, s_n} \) is the tree-level exchange amplitude with external legs of spins \( s_1 \sim s_2 \sim \ldots \sim s_n \), generated by lower-order couplings. Note that, in this higher-spin symmetric case, the particular solution is unique as a consequence of the uniqueness of the homogeneous solution.

We thus see that the action of the minimal higher-spin gauge theory is fixed uniquely by (2.39) with quadratic action (2.31). At cubic order, the couplings for any triplet of fixed spins take, up to field re-defnitions, the local form

\[
S^{(3)}_{s_1,s_2,s_3} = \int_{\text{AdS}} V_{s_1,s_2,s_3},
\]

\(^{19}\)As usual we employ the generating function notation

\[
J_s(y;z) = \frac{1}{n!} J_{s_1 \ldots s_n} (y) z^{s_1} \ldots z^{s_n},
\]

where \( z^i \) is a null auxiliary vector \( z^2 = 0 \) that packages the boundary indices \( i = 1, \ldots, d \) and enforces tracelessness.
with \[41, 93\]

\[
V_{s_1,s_2,s_3} = g_{s_1,s_2,s_3} \left[ \nabla^{\rho_1} \cdots \nabla^{\rho_{s_1}} \varphi_{\mu_1} \cdots \mu_{s_1} \cdots \nabla^{\mu_1} \cdots \nabla^{\mu_{s_2}} \varphi_{\rho_1} \cdots \rho_{s_2} \cdots \nabla^{\nu_1} \cdots \nabla^{\nu_{s_3}} \varphi_{\nu_1} \cdots \nu_{s_3} \cdots \right] \tag{2.41a}
\]

\[
g_{s_1,s_2,s_3} = \frac{1}{\sqrt{N_{d.o.f.}}} \frac{\pi^{d-3}}{2} \frac{3d-1+s_1+s_2+s_3}{2} \prod_{i=1}^{3} \frac{\Gamma(s_i + \frac{d-1}{2})}{\Gamma(s_i + 1)} , \tag{2.41b}
\]

and where the ... denote lower-derivative terms of order \(\mathcal{O}(\Lambda)\).

At cubic order there is thus no apparent issue of locality, with the couplings (2.40) involving a finite number of derivatives (2.41a) with finite coupling constants (2.41b). However, the couplings at quartic and higher-orders are expected involve an arbitrary number of derivatives, even for fixed external spins.\(^{20}\) For example, the quartic self-interaction of the scalar \(\varphi_0\) is expected to have a derivative expansion of the form \(95\)

\[
V_{0,0,0} = \sum_{l=0}^{\infty} \sum_{m=0}^{l} \lambda_{l,m} \varphi_0 \nabla_{\mu_1} \cdots \nabla_{\mu_{l+1}} \varphi_0 \nabla^{\mu_1} \cdots \nabla^{\mu_{2l+1}} \varphi_0 . \tag{2.42}
\]

The status of locality on AdS backgrounds at these higher orders has yet to be clarified in the literature. In particular, an important open issue is whether or not the asymptotic behaviour of coefficients in the derivative expansions of higher-order couplings (such as the \(\lambda_{l,m}\) in (2.42)) avoids the triviality problem outlined in §1.1.\(^{21}\) A necessary condition for which is that the re-summation of the derivative expansion does not yield \(1/\Box\)-type non-localities. With knowledge of the full solution (2.39) to the Noether procedure, we investigate this issue at quartic order \((n = 4)\) in the following section.

### 3 Locality

In the previous section we fixed the complete action for the minimal higher-spin gauge theory expanded about AdS\(_{d+1}\), using the Noether procedure and the assumption of global higher-spin symmetries. In the present section we investigate its locality properties, focusing on the quartic couplings. For earlier works directed at the study of locality in higher-spin gauge theories on AdS backgrounds at the quartic order, see: \([69, 95–98]\).

The \(s_1-s_2-s_3-s_4\) quartic coupling, evaluated on-shell with boundary condition (2.20), takes the form

\[
S_{s_1,s_2,s_3,s_4}^{(4)} = \langle \mathcal{J}_{s_1} \mathcal{J}_{s_2} \mathcal{J}_{s_3} \mathcal{J}_{s_4} \rangle_{\text{connected}} - A_{s_1,s_2,s_3,s_4}^{(4)} , \tag{3.1}
\]

which we depict in figure 1. Explicitly, (for notation see (2.35))

\[
\langle \mathcal{J}_{s_1}(y_1;z_1) \mathcal{J}_{s_2}(y_2;z_2) \mathcal{J}_{s_3}(y_3;z_3) \mathcal{J}_{s_4}(y_4;z_4) \rangle_{\text{conn.}} = N_{d.o.f.} \left( \prod_{i=1}^{4} c_i q_i^{-1} \Gamma \left( \frac{d}{2} - 1 \right) J_{\frac{d}{2} - 2} \left( \sqrt{-1} \right) \right) \frac{J_{\frac{d}{2} - 2} \left( \sqrt{-1} \right)}{Y_1^{s_1} Y_2^{s_2} Y_3^{s_3} Y_4^{s_4} + \text{perm.}} , \tag{3.2}
\]

\(^{20}\)This expectation is for instance motivated by the spin-dependent lower-bounds \([47, 117, 118]\) on the number of derivatives in on-shell cubic-vertices.

\(^{21}\)Proposals for the required behaviour of the \(\lambda_{l,m}\) couplings in (2.42) were put forward in \([69, 95, 98]\).
and the tree-level exchange amplitude is given by

\[
A^{(4)}_{s_1,s_2,s_3,s_4} = -(A^s_{s_1,s_2,s_3,s_4} + A^t_{s_1,s_2,s_3,s_4} + A^u_{s_1,s_2,s_3,s_4}),
\]

(3.3)

where, for instance, \(A^s_{s_1,s_2,s_3,s_4}\) is the sum of the exchanges in the s-channel

\[
A^s_{s_1,s_2,s_3,s_4} = \sum_{s \in 2N} A^s_{s_1,s_2}|s|s_3,s_4.,
\]

(3.4)

The exchange \(A^s_{s_1,s_2}|s|s_3,s_4\) of a spin-s gauge field generated by the cubic couplings (2.40) was computed for any \(s\) in [94] (see the earlier [95, 119] for the 0-0-0-0 case).

Figure 1.

\[\langle J_{s_1}(y_1) J_{s_2}(y_2) J_{s_3}(y_3) J_{s_4}(y_4)\rangle_{\text{conn.}} = \sum_{s=0}^{\infty} \sum_{s_1,s_2,s_3,s_4} \sum_{s_1,s_2|s|s_3,s_4} + \text{crossed}\]

In the present section we study the locality properties of the quartic couplings (3.1), focusing on the possibility of 1/□-type non-localities. We shall often refer to the latter as a non-local obstruction: As explained in §1.1, their absence is necessary to have a well-defined notion of a non-trivial interacting classical field theory.

Before we proceed, let us state our assumptions:

A1. Uniform convergence of infinite summations over derivatives.

A2. The summation over spin (3.4) in the exchange diagrams does not generate additional single-particle exchanges, in any channel.\(^{22}\)

While these assumptions may not be needed from an S-matrix perspective, for a field theory they provide necessary conditions for 1/□-type non-localities to arise only from cubic graphs. In other words, in forgoing these assumptions one would anyway inevitably encounter non-local obstructions to interactions among higher-spin gauge fields, and they can thus be applied without any loss of generality. For this reason, these assumptions are often implicit in the field theory literature. For our purposes, these assumptions are useful on a practical level:

\(^{22}\)Let us emphasise that we are not ruling out that, considering for ease of illustration flat space amplitudes, a sum over poles in one channel can give a sum over poles in another i.e. crossing symmetry. Rather, this assumption constrains the contact contribution in each individual spin-s exchange amplitude, such that they cannot generate poles in any channel upon summing over spin s.
As will become clear, they simplify the analysis by narrowing the search for $1/\Box$-type non-localities to specific components of the homogeneous and particular solutions. We discuss these assumptions in more detail at the end of §3.2.

It is instructive to first review the problem in flat space [33, 36, 37], where non-local obstructions to interactions among higher-spin gauge fields arise.

### 3.1 Review: Locality in flat space

In flat space, the defining feature of tree-level exchange amplitudes is meromorphicity and simple poles in the Mandelstam variables

$$ s = - (p_1 + p_2)^2, \quad t = - (p_1 + p_3)^2, \quad u = - (p_1 + p_4)^2. \tag{3.5} $$

In particular, for the exchange amplitude $A_{s,m}^s$ of a spin-$s$ field of mass $m^2$ in the $s$-channel we have

$$ A_{s,m}^s = g_s \tilde{g}_s \frac{(t-u)^s}{m^2 - s} + \text{Poly}_{s-1} (t-u,s), \tag{3.6} $$

where $g_s$ and $\tilde{g}_s$ are cubic couplings and $\text{Poly}_{s-1} (x,y)$ is a polynomial of degree $s-1$ in $x$ and $y$. The pole in the Mandelstam invariant $s$ is the non-local part of the exchange, which, signifying the exchange of the single-particle state, is of the $1/\Box$-type. The polynomial in $t-u$ and $s$ originates from contact terms.

In flat space, the necessary condition for the absence of non-local obstructions is then the existence of a homogeneous solution (2.7) which cancels the poles in the particular solution (2.5), leaving the full solution for the quartic coupling (2.3) (being the sum of the homogeneous and particular solutions) an entire function in the Mandelstam variables.

In investigating whether this condition is upheld by flat space higher-spin gauge theories, the field theory assumptions $A_1$ and $A_2$ allow us, without loss of generality, to sharply distinguish pole and contact parts of the homogeneous and particular solutions. In particular, they ensure that a contact term in one channel remains a contact term when expanded in other channels. This allows one to initiate the investigation of locality by focusing on the pole structure of the homogeneous and particular solutions, to see if the necessary requirement that the full solution for the coupling (2.3) is an entire function of the Mandelstam variables is fulfilled.

For simplicity let us consider the case of quartic solutions in the minimal higher-spin gauge theory with external fields of spins $s_1$-0-0-0, where $s_1$ is an even integer. The particular solution reads

$$ S_p^{(4)} = -A_{s_1,0,0,0} = - \left( A_{s_1,0,0,0} + A_{s_1,0,0,0}^t + A_{s_1,0,0,0}^u \right), \tag{3.7} $$

where in, say, the $s$-channel

$$ A_{s_1,0,0,0}^s = \sum_{s \in \mathbb{Z}^2} A_{s_1,0,0,0}^s, \tag{3.8} $$

and, dropping contact terms (as per the assumptions), the gauge field exchanges are given explicitly by

$$ A_{s_1,0,0,s}^s = g_{s_1,0,s} g_{s,0,0} \frac{(\partial_{u_1} \cdot p_2)^{s_1}}{s} \left( \frac{t-u}{4} \right)^s \varphi_{s_1} (p_1, u_1) \varphi_0 (p_2) \varphi_0 (p_3) \varphi_0 (p_4). \tag{3.9} $$
The most general solution to the homogeneous equation\textsuperscript{23}
\[ \delta^{(0)}_{\xi} S^{(4)}_h \approx 0, \quad (3.10) \]
up to contact terms that may be dropped under the assumption A1, has the form \[33\]
\[ S^{(4)}_h = \left( \partial_{u_1} \cdot p_2 \right)^{s_1} \sum_{n \geq 0} g_{s_1,0,0,0}^{(n)} \left( -1 \right)^{\frac{s_1+n}{2}-1} \left( \frac{t-u}{4} \right)^{n+s_1-2} \varphi_{s_1} (p_1, u_1) \varphi_0 (p_2) \varphi_0 (p_3) \varphi_0 (p_4) \]
+ t- and u-channels.

The coefficients \( g_{s_1,0,0,0}^{(n)} \) are unfixed by the quartic order Noether consistency condition (3.10), however at this point we may ask if there exist coefficients that do not give rise to \( \frac{1}{\Box} \)-type non-localities in the full solution for the coupling (2.3): Combining the particular and homogeneous solutions together, the complete solution for the \( s_1-0-0-0 \) quartic coupling is given by (up to an entire function in the Mandelstam variables)

\[ S^{(4)}_{s_1,0,0,0} = \left( \partial_{u_1} \cdot p_2 \right)^{s_1} \sum_{n \geq 0} g_{s_1,0,0,0}^{(n)} \left( -1 \right)^{\frac{s_1+n}{2}-1} \left( \frac{t-u}{4} \right)^{n+s_1-2} \varphi_{s_1} (p_1, u_1) \varphi_0 (p_2) \varphi_0 (p_3) \varphi_0 (p_4) \]
+ t- and u-channels \( (3.11) \)

To avoid the appearance of \( \frac{1}{\Box} \) non-localities in (3.11), we require the cancellation of poles in each of the \( s \), \( t \) and \( u \) channels. For \( s_1 \leq 4 \), this yields the following condition relating cubic and quartic coupling constants \[36\]:

\[ g_{s_1,0,0,0}^{(n)} = \left( -1 \right)^{\frac{s_1+n}{2}} g_{s_1,0,n+s_1-2} g_{n+s_1-2,0,0} \cdot \quad (3.12) \]

However, we see that for \( s_1 > 4 \) there is no solution for \( g_{s_1,0,0,0}^{(n)} \) that would give the cancellation of all poles in (3.11). Since the spectrum of an interacting higher-spin gauge theory necessarily contains gauge fields of spins \( s_1 > 4 \), one then concludes that such theories in flat space suffer from non-local obstructions to non-trivial interactions.

In the following section we extend the above discussion to AdS backgrounds. As exemplified by the flat space analysis above, a key point to clarify is whether the \( \frac{1}{\Box} \) non-localities present in the exchange amplitudes (3.3) that constitute the particular solution can be compensated by the homogeneous solution (3.2).

### 3.2 Locality in AdS

As explained in the previous section, the \( \frac{1}{\Box} \) non-locality of a single exchange diagram generated by local cubic vertices is universal, and is associated to the exchange of the single-particle

\textsuperscript{23}In flat space we cannot at present draw upon a global symmetry constraint of the type (2.14), owing to the current absence of a flat higher-spin algebra. See however \[41\] for a candidate and also \[120\] for previous work on the issue of flat space higher-spin algebras.
state. This is supplemented by local contact terms, arising from the integration over space-time. In flat space, this non-locality generates simple poles in the Mandelstam variables, while the contact terms lead to polynomials (3.6).

In AdS, on the other hand, the $1/\Box$ non-locality is represented by a contribution to its CPWE from the single-trace operator dual to the exchanged single-particle state. In particular, as verified explicitly in [94],

$$A_{s_1,s_2|s_3,s_4}^{s} = c_{J_{s_1} J_{s_2} J_{s_3} J_{s_4}} W_{(s_1,s_2|s_3,s_4)} + \text{contact},$$

where the OPE coefficients $c_{J_{s_1} J_{s_2} J_{s_3} J_{s_4}}$ coincide with those in the free scalar theory (2.35),

$$c_{J_{s_1} J_{s_2} J_{s_3}} = N_{\text{d.o.f.}} c_{s_1} c_{s_2} c_{s_3},$$

(3.14)

with the $c_{s_i}$ given explicitly in (2.38). The conformal partial wave $W_{(s_1,s_2|s_3,s_4)}$ re-sums the contributions induced by the spin-$s$ conserved current $J_s$ in the $s$-channel with external conserved operators $J_i$ in a free scalar theory. The local contact terms in (3.13) are given by conformal partial waves associated only to the exchange of double-trace operators, in any channel.

To summarise: A necessary condition to avoid non-local obstructions to interacting fields in AdS is that the CPWE of the full quartic solution (3.1) is free from single-trace contributions, in all channels.

Before we proceed, let us reiterate the assumptions. Like in the flat space case, the field theory assumptions $A1$ and $A2$ create, without loss of generality, a sharp distinction between single-trace and double-trace contributions. In particular, this means that the sum of all exchange diagrams in the $s$-, $t$- or $u$-channels (3.4) introduces no new single-trace contributions. I.e. That the contact terms from each individual exchange diagram (3.13) do not generate single-trace contributions in the sum over spin (3.4) when expanded in any channel. This allows us to neglect the contact contributions in the exchange diagrams (3.13) in the search for $1/\Box$-type non-local obstructions in the full quartic solution (3.1). This is discussed in more detail at the end of this section.

We first study the example of quartic solutions with external scalars (0-0-0-0) in the minimal higher-spin theory on AdS$_{d+1}$, which turns out to already be enough to illustrate a problem of locality for interacting higher-spin gauge fields on anti-de Sitter backgrounds. Afterwards for completeness we also extend the study to the $s_1$-0-0-0 case for arbitrary integer $s_1$.

**Four scalars**

As explained in the previous section, the issue of bulk locality can be investigated by focusing on the single-trace contributions in the CPWEs of the homogeneous (3.2) and particular (3.3) solutions. Since the spectrum of single-trace operators under consideration belong to that of a free CFT, there is a large twist degeneracy. For example, each single-trace operator $J_s$ ($s = 0, 2, 4, ...$) in the higher-spin multiplet has the same twist $\tau = d - 2$. In such a scenario it is useful to introduce twist conformal blocks [121], which encode the contribution to a four-point
correlator from operators of a given twist. More explicitly, in the case of the four-point function (3.2) in the free scalar theory, the twist conformal block for \( \tau = d-2 \) is the re-summation

\[
\mathcal{H}_{(s_1,s_2)|s_3,s_4)} = \sum_{s=0}^{\infty} c_{\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, \mathcal{J}_4} c^{\mathcal{J}_4} \mathcal{W}_{(s_1,s_2)|s_3,s_4)},
\]

(3.15)
of contributions from each operator \( \mathcal{J}_s \). For the 0-0-0-0 four-point function, the re-summation (3.15) was carried out directly in [122] (see also [123]), and for contributions in the \( s \)-channel

\[
\mathcal{H}_{(0,0)(d-2)(0,0)} (y_1, y_2; y_3, y_4) = \frac{1}{c} \frac{1}{(y_{12}^2 y_{34}^2)^{d/2}} \left[ u^{d-1} + \left( \frac{u}{v} \right)^{d-1} \right],
\]

(3.16)
in terms of the cross-ratios

\[
u = \frac{y_{12}^2 y_{34}^2}{y_{13}^2 y_{24}^2}, \quad v = \frac{y_{11}^2 y_{23}^2}{y_{13}^2 y_{24}^2},
\]

(3.17)
and where \( c \) is proportional to the central charge of the theory.

Accordingly, the homogeneous (3.2) and particular (3.3) solutions with identical external scalars receive contributions from the twist block (3.16). The homogeneous solution in this case is the four-point conformal correlation function of the scalar single-trace operator \( \mathcal{O} := \mathcal{J}_0 \),

\[
\mathcal{S}^{\text{h}}_{0,0,0,0} = \langle \mathcal{O} \mathcal{O} \mathcal{O} \mathcal{O} \rangle_{\text{conn.}},
\]

(3.18)
where [124]

\[
\langle \mathcal{O} (y_1) \mathcal{O} (y_2) \mathcal{O} (y_3) \mathcal{O} (y_4) \rangle_{\text{conn.}} = \frac{1}{c} \frac{1}{(y_{12}^2 y_{34}^2)^{d/2}} \left[ u^{d-1} + \left( \frac{u}{v} \right)^{d-1} \right] + \frac{1}{c} \frac{1}{(y_{12}^2 y_{34}^2)^{d/2}} u^{d-1} \left( \frac{u}{v} \right)^{d-1}.
\]

(3.19)
In the second equality we simply identified the \( s \)-channel twist block (3.16). The right-most contribution denotes contributions from (higher-twist) double-trace operators in the \( s \)-channel.

The particular solution is given by the exchange amplitude (3.3) with identical external scalars,

\[
\mathcal{S}^{\text{p}}_{0,0,0,0} = -\mathcal{A}_{0,0,0,0} = -(\mathcal{A}^{\text{h}}_{0,0,0,0} + \mathcal{A}^{\text{t}}_{0,0,0,0} + \mathcal{A}^{\text{u}}_{0,0,0,0}).
\]

(3.20)
Since the single-trace contribution from an individual exchange diagram coincides with that in the dual CFT correlator (c.f (3.13)), the twist block (3.16) also re-sums the single-trace contributions in the particular solution (3.20). For each of the \( s \)-, \( t \)- and \( u \)-channel exchanges we have:

\[
\mathcal{A}^{\text{h}}_{0,0,0,0} = \mathcal{H}_{(0,0)(d-2)(0,0)} (y_1, y_2; y_3, y_4) + \ldots
\]

(3.21a)
\[
\mathcal{A}^{\text{t}}_{0,0,0,0} = \mathcal{H}_{(0,0)(d-2)(0,0)} (y_1, y_3; y_2, y_4) + \ldots
\]

(3.21b)
\[
\mathcal{A}^{\text{u}}_{0,0,0,0} = \mathcal{H}_{(0,0)(d-2)(0,0)} (y_1, y_4; y_3, y_2) + \ldots,
\]

(3.21c)
where the ... denote terms which, under the the field theory assumptions $A1$ and $A2$, encode only double-trace (contact) contributions in any channel, which may be neglected in this analysis (see discussion at the beginning of §3.2).

The re-summation of the single-trace contributions by the twist block (3.16) allows to effortlessly combine corresponding parts of exchanges (3.21). In particular, it leads to the result\(^{24}\)

$$S_{0,0,0,0}^p = -2\langle OOOO\rangle_{\text{conn.}} + \ldots$$

(3.22)

In combination with the homogeneous solution (3.18), it is then straightforward to see that in the full solution for the quartic coupling (3.23) the single-trace contributions in the exchange amplitude are not completely cancelled by those in the homogeneous solution:

$$S_{0,0,0,0}^{(4)} = \langle OOOO\rangle_{\text{conn.}} + S_{0,0,0,0}^p = -\langle OOOO\rangle_{\text{conn.}} + \ldots,$$

(3.23)

as indicated by the presence of the $\tau = d - 2$ twist block (3.16) in the connected CFT correlator (3.19).

The result (3.23) gives evidence to conclude that, like in the flat space case, the quartic self-interaction of the scalar in the minimal higher-spin gauge theory contains $1/\Box$-type non-localities. In particular, as explained in §1.1, it indicates that there may not exist a proper functional class to define an interacting classical field theory in AdS.

**One spinning leg**

The non-local obstruction is not restricted to the quartic self interaction of the scalar, and can be observed from the same analysis of any of the quartic solutions (3.1). In this section we give the extension to one external leg of spin $s_1$.

In this case the $\tau = d - 2$ twist block is given, say in the $s$-channel, by

$$\mathcal{H}(s_1,0,d-2|0,0) \left( y_1, y_2; y_3, y_4 \right)$$

$$= \frac{1}{c} \frac{1}{(y_{12}y_{34})^7} \left[ u^{\tau/2} \left( \frac{z_1 \cdot y_{13}}{y_{13}^7} - \frac{z_1 \cdot y_{12}}{y_{12}^7} \right)^{s_1} + \left( \frac{u}{v} \right)^{\tau/2} \left( \frac{z_1 \cdot y_{12}}{y_{12}^7} - \frac{z_1 \cdot y_{14}}{y_{14}^7} \right)^{s_1} \right]$$

(3.24)

$$= \sum_{s=0}^{\infty} c_{J_1} c_{J_2} c_{J_3} \mathcal{E} \mathcal{W}_{(s_1,0)(0,0)}(y_1, y_2; y_3, y_4).$$

The homogeneous solution (3.2) in this case is

$$S_{s_1,0,0,0}^{h} = \langle J_{s_1} OOO \rangle_{\text{conn.}},$$

(3.25)

---

\(^{24}\)To reach this result it is useful to note that $y_2 \leftrightarrow y_3$ and $y_2 \leftrightarrow y_4$ send $u \rightarrow 1/u$, $v \rightarrow v/u$ and $u \leftrightarrow v$ respectively.
with (obtained by expanding the Bessel functions in (3.2))

\[
\langle J_{s_1}(y_1; z_1) \mathcal{O}(y_2) \mathcal{O}(y_3) \mathcal{O}(y_4) \rangle_{\text{conn.}} = \frac{1}{c} \frac{1}{(y_{12}^2 y_{34}^2)^{\tau}} \left[ u^{\tau/2} \left( \frac{z_1 \cdot y_{13}}{y_{13}^2} - \frac{z_1 \cdot y_{12}}{y_{12}^2} \right)^{s_1} \right. + \left. \left( \frac{u}{v} \right)^{\tau/2} \left( \frac{z_1 \cdot y_{14}}{y_{14}^2} - \frac{z_1 \cdot y_{13}}{y_{13}^2} \right)^{s_1} \right]
\]

\[
= \mathcal{H}_{(s_1,0|d-2|0,0)} (y_1, y_2; y_3, y_4) + \frac{1}{c} \frac{1}{(y_{12}^2 y_{34}^2)^{\tau}} u^{\tau/2} \left( \frac{u}{v} \right)^{\tau/2} \left( \frac{z_1 \cdot y_{14}}{y_{14}^2} - \frac{z_1 \cdot y_{13}}{y_{13}^2} \right)^{s_1}
\]

where, as for the 0-0-0-0 case (3.19), the rightmost term encodes contributions from double-trace operators in the s-channel.

The particular solution reads

\[
S_{s_1,0,0,0}^p = -A_{s_1,0,0,0} = -\left(A_{s_1,0,0,0}^s + A_{s_1,0,0,0}^t + A_{s_1,0,0,0}^u \right),
\]

where, employing (3.13) and (3.24), one recovers again that the single-trace contributions are twice those in the homogeneous solution:

\[
A_{s_1,0,0,0}^s + A_{s_1,0,0,0}^t + A_{s_1,0,0,0}^u = \mathcal{H}_{(s_1,0|d-2|0,0)} (y_1, y_2; y_3, y_4) + \mathcal{H}_{(s_1,0|d-2|0,0)} (y_1, y_3; y_2, y_4) + \mathcal{H}_{(s_1,0|d-2|0,0)} (y_1, y_4; y_3, y_2) + \ldots
\]

\[
= 2 \langle J_{s_1} \mathcal{O} \mathcal{O} \mathcal{O} \mathcal{O} \rangle_{\text{conn.}} + \ldots
\]

As in the 0-0-0-0 case, the ... denote terms which give double-trace (contact) contributions in all channels, which may be neglected.

In the same way, one uncovers a non-local obstruction in the s_1-0-0-0 quartic coupling by combining (3.28) with the homogeneous solution (3.25)

\[
S_{s_1,0,0,0}^{(4)} = -\langle J_{s_1} \mathcal{O} \mathcal{O} \mathcal{O} \mathcal{O} \rangle_{\text{conn.}} + \ldots,
\]

which, like in the 0-0-0-0 case (3.23), contains single-trace contributions that harbingers the presence of 1/□ non-localities in the full s_1-0-0-0 quartic coupling.

### 3.3 The role of crossing symmetry

The appearance the of 1/□-type non-local obstructions to non-trivially interacting higher-spin gauge fields in AdS can be traced back to the behaviour of the tower of single-trace operators \( \mathcal{J}_s \) under crossing. To see this, it is instructive to consider terms in the correlator (3.19) that are independent solutions to the crossing equation, and their microscopic interpretation:

For a four-point function of scalar operators \( \mathcal{O} \) of dimension \( \Delta \),

\[
\langle \mathcal{O} \mathcal{O} \mathcal{O} \mathcal{O} \rangle = \frac{G(u,v)}{(y_{12}^2 y_{34}^2)^{\Delta}}.
\]

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crossing symmetry is the requirement
\[ f(u, v) = v^\Delta G(u, v) = u^\Delta G(v, u). \] 

(3.31)
The crossing equation (3.31) is straightforward to solve in free theories owing to the simplicity of the double light-cone limit \( u, v \to 0 \) in this context,\(^{25}\) which interpolates smoothly between the \( s \)- and \( u \)-channel OPE \cite{128}.\(^{26}\) This allows to expand the correlation function simultaneously in the \( s \)- and \( u \)-channel, with the generic free theory solution to (3.31) taking the form \cite{128}
\[ f(u, v) = \sum_{i,j} c_{ij} u^{\tau_i} v^{\tau_j}, \quad c_{ij} = c_{ji}, \] 

(3.32)
which sums over the twists \( \tau_i \) of the operators in the theory. The condition \( c_{ij} = c_{ji} \) is imposed by crossing symmetry. The general solution (3.32) exhibits that, in free theories, pairs of twist-trajectories are mapped into each other under crossing,
\[ \text{twist } \tau_1 \overset{\text{crossing}}{\leftrightarrow} \text{twist } \tau_2. \] 

(3.33)
This analysis for the correlation function (3.19) was carried out in \cite{128}. The following two functions were identified as independent solutions to crossing (3.31)
\[ f_{d-2}(u, v) = \frac{1}{c} \frac{d-2}{u^{d-1} v^{d-1}} \] 

(3.34a)
\[ f_{2(d-2)}(u, v) = \frac{1}{c} \left( \frac{d-2}{u^{d-1} v^{d-2}} + \frac{d-2}{u^{d-2} v^{d-1}} \right). \] 

(3.34b)
To discern the microscopic origin of the above, one recalls that in the light-cone limit \( u, v \to 0 \) with, say, \( u \ll v \ll 1 \), operators of twist \( \tau \) enter in the OPE as \( u^\tau \).\(^ {27}\) The solution (3.34b) therefore originates from the exchange of operators of twist \( \tau = d-2 \), which are the single-trace operators \( J_s \) of the higher-spin multiplet, together with the exchange of double-trace operators of twist \( \tau = 2 (d-2) \). These two twist-trajectories thus map into each other under crossing
\[ \text{Regge single-trace } \overset{\text{crossing}}{\leftrightarrow} \text{Regge double-trace}. \] 

(3.35)
The mapping of single-trace contributions to double-trace contributions under crossing is quite generic of CFTs in \( d > 2 \) \cite{128–130} (see also \cite{131–133}), and a characteristic of CFTs with a local bulk dual.

On the other hand, the solution (3.34a) is self-dual under crossing, with
\[ \text{Regge single-trace } \overset{\text{crossing}}{\leftrightarrow} \text{Regge single-trace}, \] 

(3.36)
which is typical of CFTs at/around a point of large twist degeneracy \cite{128, 134, 135}, such as the present case of theories with higher-spin symmetry. It is this property that appears to be responsible for the non-local obstruction to interactions among higher-spin gauge fields on an

\(^{25}\)See e.g. \cite{125–127}.

\(^{26}\)Often referred to as \( u \)- and \( v \)-channels, respectively, in the CFT literature.

\(^{27}\)Likewise, for \( v \ll u \ll 1 \) operators of twist \( \tau \) enter in the OPE as \( v^\tau \).
AdS background. Indeed, it is straightforward to see that the absence of such a contribution would mean that one instead has (and likewise for the $s_1$-0-0-0 case)

$$S_{p,0,0,0}^0 = -\langle OO\rangle_{\text{conn.}} + \ldots,$$

(3.37)

precisely cancelling the single-trace contributions in the homogeneous solution (3.18), and would thus avert the appearance of the non-local obstruction (3.23). This is, for instance, exemplified by the locality of bulk duals to large $N$ CFTs in which all single-trace operators of spin greater than two have parametrically large dimensions [136] – such as CFT duals to AdS supergravity – which do not have a large twist degeneracy, and thus do not exhibit the behaviour (3.36) under crossing.

Let us note that the mapping (3.36) of single-trace twist trajectories into each other under crossing – and thus the corresponding non-local obstruction to bulk interactions – is also present in theories with slightly broken higher-spin symmetry [128, 135] (such as weakly coupled CFTs), where one has an approximate twist degeneracy. The property (3.36) also holds in a generic free/weakly coupled CFT [128], and we thus expect our analysis to hold for any theory of higher-spin gauge fields on AdS.

To conclude this subsection we briefly compare our results to those in an earlier work [95], which carried out a similar investigation on the locality of the quartic scalar self-interaction in the minimal higher-spin theory. There, the absence of single-trace contributions in the CPWE of the complete quartic amplitude as a necessary condition for the existence of well-defined interactions in a higher-spin gauge theory was proposed. However, the non-local obstruction (3.23) that we observe was not uncovered in [95]. The arguments presented in this section clarify that in [95] the non-local obstruction was disguised as an expansion in double-trace conformal partial waves. In particular, our analysis shows that expressing the latter expansion of double-trace contributions in a different channel would reveal single-trace contributions, recovering the obstruction (3.23) that we see in this work.$^{28}$

### 3.4 Re-visiting the assumptions

The analysis of the preceding sections made use of the standard field theory assumption that contact terms in each exchange amplitude should not generate single-particle exchanges, in any channel, upon summing over spin (3.4). Although from a field theory perspective this assumption is not so out of the ordinary, since the problem of (non-)locality in higher-spin gauge theories is an intricate issue which has been long debated, we dedicate this subsection to giving it further justification.

$^{28}$The essence of this point is captured by the fact that it is possible to express the CFT correlator (3.19) as an expansion purely in double-trace conformal partial waves in the $s$, $t$- and $u$-channels,

$$\langle OO\rangle_{\text{conn.}} = \sum_{s,n=0}^\infty c_{s,n} W_{\text{conn}}^{s,n}(y_1, y_2; y_3, y_4) + y_2 \leftrightarrow y_1 + y_2 \leftrightarrow y_4,$$

(3.38)

which masks the presence of the single-trace contributions.

How to obtain the representation (3.38) for the correlator is shown in reference [95], section 5.3. One chooses $a = 0$ and $c = 1$ in the function defined in [95] which is labelled by the symbol $\mathcal{F}$. 

...
In the language of conformal partial waves, where single-particle bulk exchanges correspond to single-trace contributions in the CPWE, considering for simplicity the case of identical external scalars, for a given spin-$s$ exchange in the $s$-channel we have

$$A_{0,0|s,0,0} = c_{O_J} c_{J^s_O} W_{(0,0|s,0,0)} (y_1, y_2; y_3, y_4) + \sum_{n=0}^{s} \sum_{l=0}^{s} d_{l,n} W_{[O_O]_{l,n}} (y_1, y_2; y_3, y_4), \quad (3.39)$$

where the contributions from the double-trace operators $[O_O]_{l,n}$ arise from contact terms in the spin-$s$ exchange, with coefficients $d_{l,n}$. The above assumption in this language is then that the double-trace contributions do not generate single-trace contributions in the sum over exchanged spin $s$. In other words

$$A_{0,0,0,0} = \sum_{s=0}^{\infty} A_{0,0|s,0,0} = \sum_{s=0}^{\infty} c_{O_J} c_{J^s_O} W_{(0,0|s,0,0)} (y_1, y_2; y_3, y_4) + \ldots, \quad (3.40)$$

where the $\ldots$ do not contain single-trace contributions in any of the $s$, $t$, or $u$-channels.

In employing this assumption, we encountered non-local obstructions in the complete solution for the quartic scalar self-interaction, by evaluating it on-shell with boundary condition (2.20):

$$S^{(4)}_{0,0,0,0} = \langle O_O O_O \rangle_{\text{conn.}} - (A^s_{0,0,0,0} + A^t_{0,0,0,0} + A^u_{0,0,0,0})$$

$$= -\sum_{s=0}^{\infty} c_{O_J} c_{J^s_O} W_{(0,0|s,0,0)} (y_1, y_2; y_3, y_4) + \ldots. \quad (3.41)$$

The presence of the non-local obstruction is signalled by the single-trace contributions in the CPWE, which correspond to $1/\Box$-type non-localities. One may then ask if it is possible to relax the assumption on the sum over spins stated above, and contemplate the possibility that the double-trace terms in each exchange diagram (3.39) could generate precisely the single-trace contributions necessary to cancel the non-local obstruction in the quartic coupling (3.41) upon summation over the exchanged spin (3.40). However, in dropping this condition one would lose the ability to perform any (admissible) field re-definition:

Contact terms in exchange diagrams are sensitive to the off-shell completion of the cubic vertices mediating the exchange (i.e. the part of the vertices which is on-shell trivial), and are thus sensitive to field re-definitions. Consider, then, adding an arbitrary improvement to the cubic vertices,

$$\mathcal{V}_{0,0,s} \rightarrow \mathcal{V}_{0,0,s} + \delta \mathcal{V}_{0,0,s}, \quad \delta \mathcal{V}_{0,0,s} \approx 0. \quad (3.42)$$

At the level of each individual exchange (3.39), this generates double-trace contributions of the type on the RHS of (3.39). If one drops the above assumption, this improvement would generate single-trace contributions in the sum over spin (3.40)

$$A^s_{0,0,0,0} \rightarrow A^s_{0,0,0,0} + \delta A^s_{0,0,0,0}, \quad (3.43)$$

where the CPWE of $\delta A^s_{0,0,0,0}$ contains single-trace contributions in some channel [69]. While this could lead to the possibility of cancelling the non-local obstruction (3.41), since the CFT correlator $\langle O_O O_O \rangle_{\text{conn.}}$ should (as an observable) be invariant under field re-definitions, this
corresponds to allowing field redefinitions that generate single-trace contributions in the quartic contact amplitude

\[ S^{(4)}_{0,0,0,0} \rightarrow S^{(4)}_{0,0,0,0} - \left( \delta A^{s}_{0,0,0,0} + \delta A^{l}_{0,0,0,0} + \delta A^{u}_{0,0,0,0} \right). \]  

(3.44)

I.e. precisely those of the non-admissible 1/□-type.

We conclude that, should there be a field frame where the (off shell) cubic vertices precisely cancel the non-local obstruction (3.41), it would not be possible to perform any field re-definition. The latter scenario rather adopts a more S-matrix perspective. Moreover, one would have to verify that such cubic vertices do not contain 1/□ non-localities themselves. From a field theory point of view we consider it reasonable to assume that the contact terms in the exchange diagrams do not generate single-particle contributions. Other wise, as shown in (3.44) above, any field re-definition would be a non-admissible one.

4 Discussion

In the present work, we have argued that there are non-local obstructions to the existence of non-trivial interacting theories of higher-spin gauge fields on anti-de Sitter spaces at the classical level. The obstructions arise as a consequence of the basic consistency requirements of invariance under higher-spin symmetry and crossing. Our findings suggest that a field theory description of interacting higher-spin gauge theories may not be well defined.

In the following we discuss how our conclusion fits into the existing results on higher-spin gauge theories on anti-de Sitter space-times.

**Witten diagram tests of higher-spin holography**

One of the most significant checks of the holographic duality between higher-spin gauge theories on AdS and free CFTs [54–58] was given by demonstrating that the cubic couplings among higher-spin gauge fields on AdS recover the three-point correlation functions of single-trace operators in the dual free CFT [41, 142–145].

On the other hand, our results suggest no consistent non-trivial interactions among higher-spin gauge fields on AdS backgrounds. Naïvely, this appears to be in contradiction with the three-point function tests of higher-spin holography, which seem to require the existence of non-trivial cubic vertices in the bulk. The key subtlety here is that the non-local class of field redefinitions in the theory would shift the bulk interactions into (non-local) boundary terms, which can still generate the non-trivial dual CFT correlators.30

We emphasise that the encoding of dual CFT correlators in boundary terms of the bulk theory is by no means a peculiarity of holography with higher-spin gauge theories in AdS. For instance, the same phenomenon arises in the context of the holographic duality involving \( \mathcal{N} = 8 \) gauged supergravity [147] in the bulk: The latter has no \( A^3 \) couplings in four-dimensions but the dual CFT correlators are non-vanishing. In the recent paper [148] it was shown that the latter were generated by boundary terms implied by requiring preservation of supersymmetry on the boundary.

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29 For reviews see [137–141].

30 See [146] for some discussions on boundary terms in the context of higher-spin gauge theories in the unfolded formalism.
The flat space limit

The above observation may also shed some light on the flat limit of higher-spin gauge theories on AdS.\textsuperscript{31}

If, as our analysis suggests, higher-spin gauge theories on AdS are a sum of free Fronsdal actions plus boundary terms encoding the dual CFT correlators, in the flat space limit the boundary terms vanish and one is left with a sum of free Fronsdal actions in flat space – consistent with no-go results that indicate the absence of non-trivial interactions of higher-spin gauge fields in flat space.

Other tests of Higher Spin Holography

Our analysis also does not appear to be in contradiction with other tests of higher-spin holography (as far as we are aware), which so far have relied only on the spectrum. This includes the Flato-Fronsdal theorem [151, 152] and computations of one-loop vacuum energies [152–161].

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