The entropy puzzle and the quark combination model

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We use two available methods, the Duhem-Gibbs relation and the entropy formula in terms of particle phase space distributions, to calculate the entropy in a quark combination model. The entropy of the system extracted from the Duhem-Gibbs relation is found to increase in hadronization if the average temperature of the hadronic phase is lower than that of the quark phase. The increase of the entropy can also be confirmed from the entropy formula if the volume of the hadronic phase is larger than 2.5-3.0 times that of the quark phase. So whether the entropy increases or decreases during combination depends on the temperature before and after combination and on how much expansion the system undergoes during combination. The current study provides an example to shed light on the entropy issue in the quark combination model.

The quark combination model [1, 2] (QCM) or its variants such as the quark recombination or coalescence model are used to describe multi-hadron processes in high energy electron-positron, hadron-hadron and nucleus-nucleus collisions. Recently they are successful in explaining the data at Relativistic Heavy Ion Collider (RHIC), which showed that the combination is more preferable than the fragmentation for the hadronization mechanism in intermediate transverse momentum region in heavy ion collisions at high energies [3, 4]. The basic idea of the QCM is that hadrons are combined from quarks and anti-quarks, i.e. two quarks merge into a meson (a 2-to-1 process) and three quarks into a baryon (a 3-to-1 process). One of the most frequently asked questions about the validity of the QCM is: does the entropy decrease in such 2-to-1 and 3-to-1 processes? The question arises naturally from the common sense that the entropy is normally reduced in a particle number decreasing process. The entropy problem is essential to nucleus-nucleus collisions since the issue of the entropy production is closely related to thermalization and has been the concern of the researchers for many years. There have been previous attempts or arguments about the entropy in the QCM-like models in literature [4, 5], but no systematic investigation has yet been made. In this paper we will address the entropy problem in the course of the quark combination in heavy ion collisions. As an example we will use a specific version of the QCM which is developed by the Shandong group [10, 13]. The advantage of the Shandong QCM is its exclusive nature and simplicity, i.e. it can give all hadrons in an event. Considering that the entropy can be regarded as a measure of degrees of freedom, the exclusive nature of Shandong QCM is suitable and applicable to the entropy problem.

The building blocks for the QCM are constituent quarks and anti-quarks of the light flavors u,d and s with the momentum distributions \( f_i(p) = f_i(y, p_T) \) [12, 10]. All hadrons come out with fixed momenta from combination of their constituent quarks. Note that our QCM (or any of QCM-like models) does not deal with how the constituent quarks acquire masses which is closely related to the physics of chiral phase transition, although the QCM-like models can be regarded as a phenomenological and microscopic description of the phase transition. There are no gluons in the model because the role of a gluon can be replaced by a pair of quarks. Actually hadronization is a dynamic process which takes place locally at a freeze-out temperature, while our QCM (or any of QCM-like models) lacks the real time feature but treats hadronization in a global or an effective way. In this sense the QCM is similar to the statistical models.

Suppose we have \( N_i \) quarks or anti-quarks with the flavor \( i = u, d, s, \bar{u}, \bar{d}, \bar{s} \), the total number of quarks and anti-quarks are then \( N_q = N_{\bar{q}} = N_q + N_{\bar{q}} \) with \( N_q = N_u + N_d + N_s \) and \( N_{\bar{q}} = N_{\bar{u}} + N_{\bar{d}} + N_{\bar{s}} \). In mid-rapidity and high energies almost all quarks and anti-quarks are excited from vacuum in pairs, we have \( N_q \approx N_{\bar{q}} \). In lower energies there are remanents of nucleons participating in the collisions which are left in the mid-rapidity region and leads to \( N_q > N_{\bar{q}} \). Since the strange quarks are heavier they are less produced and we can use a factor \( \lambda_s = N_s/N_u \approx N_s/N_d < 1 \) to characterize the suppression of strange quarks relative to up and down quarks. The procedure to randomly combine all quarks and anti-quarks into hadrons is quite straightforward: we line up all these quarks and anti-quarks randomly into a 1-dimensional queue and let them combine into the ground state hadrons following the combination rule [10–13]. The momentum distribution of a specific hadron is the convolution of those of its constituent quarks. Here the ground state hadrons include \((3/2)^+\) decuplet and \((1/2)^+\) octet baryons and \(0^-\) pseudoscalar and \(1^-\) vector mesons. The short-life resonances are allowed to decay into the long-life or final state hadrons which are recorded in the detectors. The experimental data are used to fix the inputs of the model. The charged multiplicity can fix \( N_q \) and \( N_{\bar{q}} \). The ratio of multiplicity of kaons to pions can fix \( \lambda_s \). Note that we focus on the unit rapidity region at the central rapidity \(|y| < 0.5\). One of the most important element of the model is the transverse momen-
tum spectra of thermal quarks which are parameterized by the blast wave model [17].

\[
    f_i(y, p_T) \propto \int_0^1 d\xi \sqrt{\frac{\rho^2}{T_f^2} + m_i^2} I_0 \left( \frac{p_T \sinh \rho}{T_f} \right) \\
    \times K_1 \left( \frac{p_T \cosh \rho}{T_f} \right),
\]

where \( \xi = r/R_{\text{max}} \) with \( R_{\text{max}} \) the maximum radius of the thermal source, \( T_f = 165 \text{ MeV} \) is the local temperature, \( \rho = \tanh^{-1}(\beta_i \xi^n) \) with \( \beta_i \), the radial velocity on the surface of the fireball and \( n = 0.3 \) a parameter, \( I_0 \) and \( K_1 \) are the modified Bessel functions. The average radial velocity is given by \( \langle \beta_i \rangle = 2\beta_i/(n + 2) \). By fitting the data for transverse momentum spectra of hadrons we can fix the parameters \( \beta_i \) or equivalently \( \langle \beta_i \rangle \), see Tab. II. With these inputs the available data for the transverse momentum spectra and rapidity density of hadrons have been well described [14–16]. The reason we only consider thermal quarks is that the entropy is a bulk property which is dominated by the small \( p_T \) region.

Equipped by the above settings, we are able to calculate the entropy of the system before and after hadronization. Given all particle momentum distributions, one way to obtain the entropy of a system is through the Duhem-Gibbs relation [18].

\[
    S = \frac{1}{T} (E + PV - \sum_i \mu_i N_i),
\]

where \( S, E, P, V \) and \( T \) are the entropy, energy, pressure, volume, and temperature of the system respectively, and \( \mu_i \) and \( N_i \) are the chemical potential and number for the quark/hadron species \( i \) respectively. Note that the system will undergo expansion during hadronization which is indeed a dynamic and real time process and takes place in local volume. The global volume of an expanding system is not well defined in a Lorentz invariant way. Here the volume \( V \) is only an effective parameter to characterize the average particle occupancy in spatial dimension. The chemical potential \( \mu_i \) can be determined from the multiplicity ratio of the particle \( i \) and to its anti-particle \( \bar{i} \) via \( N_i/N_{\bar{i}} = e^{\mu_i/T} \) where \( T \) is set to 170 MeV for all particles. The quantities on the right hand side of Eq. 2 are all known with:

\[
    E = \sum_i \int d^3p \epsilon_i f_i(p), \\
    PV = \sum_i \int d^3p p^2 \frac{3\epsilon_i}{3\epsilon_i} f_i(p), \\
    N_i = \int d^3p f_i(p),
\]

where \( p \equiv |\mathbf{p}| \) and \( \epsilon_i = \sqrt{p^2 + m_i^2} \) denotes the scalar momentum and energy respectively. For constituent quarks we choose \( m_u = m_d = 330 \text{ MeV} \) and \( m_s = 550 \text{ MeV} \).

In Tab. I we list the total \( E, PV, \mu N \equiv \sum_i \mu_i N_i \) and \( TS \) per unit rapidity in the central rapidity region \(|y| < 1 \) for the quark and hadronic phases. We can see that \( (PV)_h \) (for hadrons) is slightly smaller than \( (PV)_q \) (for quarks) because

\[
    \frac{p_1^2}{3\epsilon_1} + \frac{p_2^2}{3\epsilon_2} \approx \frac{(p_1 + p_2)^2}{3(\epsilon_1 + \epsilon_2)} \\
    = \frac{[p_1\epsilon_2 - p_2\epsilon_1]^2 + 2p_1p_2\epsilon_1\epsilon_2}{3\epsilon_1\epsilon_2(\epsilon_1 + \epsilon_2)} \left[ 1 - \frac{\mathbf{P}_1 \cdot \mathbf{P}_2}{p_1p_2} \right] \\
    \approx \frac{(p_1\epsilon_2 - p_2\epsilon_1)^2}{3\epsilon_1\epsilon_2(\epsilon_1 + \epsilon_2)} \gtrsim 0,
\]

where in the last line is due to that quarks with the same momentum direction combine. We also see that \( \mu N \) is very small compared to \( PV \) and \( E \) since in high energy collisions almost all quarks in central rapidity are excited in vacuum. Note that the energy per unit rapidity are not exactly equal for the two phases and the difference is within a few percent. The effect of the energy non-conservation is the result of the momentum conservation imposed during the course of a specific combination, because energy and momentum conservation cannot be simultaneously fulfilled with fixed quark masses. For example two light constituent quarks with the total mass 660 MeV combine into a pion of mass 140 MeV, the energy of pion is less than that of two quarks for small momenta. But the non-conservation effect is small on average. With \( (PV)_q \approx (PV)_h \) and \( V_q < V_h \) we obtain \( P_q > P_h \) and then \( P_q - B = P_h \) (\( B \) is the bag constant), which is consistent to the condition for phase transition. Finally we see that \( TS \) per unit rapidity is almost constant for two phases at each collisional energy up to a few percent. If we assume a sudden hadronization the temperature should be the same for the two phases, then we have the approximate conservation of the total entropy in hadronization. Actually the hadronization time is finite during which the system expands and cools down. Therefore the total entropy \( S \) of the hadronic phase (with lower temperature) is larger than that of the quark phase (with higher temperature). All above observations are about the directly produced hadrons right after the combination. For the final state hadrons after resonance decays we also see that \( E, PV, \mu N \) and \( TS \) do not change much as compared to the directly produced hadrons. This observation that the resonance decays cannot compensate much to the total entropy is different from previous arguments that resonant decays might be important.

Another way of obtaining the entropy is directly through the formula,

\[
    S = \sum_i d_i \int \frac{d^3rd^3p}{(2\pi)^3} \left\{ -g_i(\mathbf{r}, \mathbf{p}) \ln g_i(\mathbf{r}, \mathbf{p}) \\
    \pm [1 \pm g_i(\mathbf{r}, \mathbf{p})] \ln [1 \pm g_i(\mathbf{r}, \mathbf{p})] \right\},
\]

where \( d_i \) are degeneracy factors for quark or hadron species \( i \), the signs +/- are for bosons/fermions, and \( g_i(\mathbf{r}, \mathbf{p}) \) are phase-space distributions satisfying
Table I: Total \( E, PV, \mu N \equiv \sum_i \mu_i N_i \) and \( TS \) per unit rapidity in the central rapidity region \( |y| < 1 \) for the quark and hadronic phases. The unit is GeV. The direct hadrons are those directly from combination without any resonance decays. The final hadrons are long-life hadrons including the contributions from resonance decays, namely \( \pi^\pm, K^\pm, K^0, \bar{K}^0, p, \bar{p}, n \) and \( \pi \).

| \( \sqrt{s_{NN}} \) | \( E \) | \( PV \) | \( N_\mu \) | \( TS \) | \( E \) | \( PV \) | \( N_\mu \) | \( TS \) |
|----------------|-----|-----|------|-----|-----|-----|------|-----|
| 17.3           | 459.4 | 112.9 | 10.8 | 561.5 | 456.5 | 91.5 | 15.4 | 532.6 |
| 62.4           | 609.6 | 155.1 | 1.48 | 763.7 | 615.0 | 132.4 | 2.0 | 745.5 |
| 130            | 873.6 | 226.1 | 0.34 | 1099.4 | 861.0 | 191.5 | 0.46 | 1052.0 |
| 200            | 939.7 | 248.1 | 0.21 | 1187.6 | 951.0 | 216.5 | 0.29 | 1167.2 |

Table II: Radial flow parameters in the transverse momentum spectra for quarks at four collisional energies. The unit for energies is GeV.

| \( \sqrt{s_{NN}} \) | \( \langle \beta_r \rangle \) for \( u,d \) | \( \langle \beta_r \rangle \) for \( s \) |
|----------------|----------|----------|
| 17.3           | 0.38     | 0.416    |
| 62.4           | 0.483    | 0.5      |
| 130            | 0.537    | 0.55     |
| 200            | 0.562    | 0.587    |

\[
\int \frac{d^3p}{(2\pi)^3} d_i g_i(\mathbf{r}, \mathbf{p}) = f_i(\mathbf{p}).
\]

In the classical limit where \( g_i(\mathbf{r}, \mathbf{p}) \ll 1 \), one reaches,

\[
S \approx \sum_i d_i \int \frac{d^3r d^3p}{(2\pi)^3} \left[ -g_i(\mathbf{r}, \mathbf{p}) \ln g_i(\mathbf{r}, \mathbf{p}) + g_i(\mathbf{r}, \mathbf{p}) \right].
\]

(6)

We can take pions which are most populated particles as an example to estimate the order of magnitude of the phase space distribution. The multiplicity rapidity density of \( \pi^+ \) in most central collisions (centrality 0-5\%) at 200 GeV \( \Lambda \) is about 320. Suppose most pions are limited inside a momentum volume of the size \((\Delta p)^3 \sim 2n p_T^2 \cosh(y) (\Delta p_T \Delta y) \sim 10 \text{ GeV}^3\) and inside the spatial volume of about \( V_h \approx 2200 \text{ fm}^3 \), then we get \( g_{\pi}(\mathbf{r}, \mathbf{p}) \sim 0.027 \). This provides an upper limit, for other particles \( g_i(\mathbf{r}, \mathbf{p}) \ll 1 \), therefore Eq. (6) is a quite good approximation of Eq. (5).

In order to calculate the entropy via Eq. (5) from the known momentum distributions \( f_i(\mathbf{p}) \) in the QCM, we make an approximation or give an estimate for \( g_i(\mathbf{r}, \mathbf{p}) \) by \( g_i(\mathbf{r}, \mathbf{p}) \approx [(2\pi)^3/(d_i V)] f_i(\mathbf{p}) \), where \( V = V_q, V_h \) are the effective volumes of quark and hadronic phases respectively. We can then obtain the entropy ratio \( R = S_h/S_q \) as functions of volume ratio \( x = V_h/V_q \). The volumes \( V_h \) of the hadronic phase is taken to be the chemical freezout values of the fireball which are extracted from the thermal model \([19,20]\) at four collisional energies, \( V_h \text{(fm}^3\text{)} = 1125 (17.3 \text{ GeV}), 1620 (62.4 \text{ GeV}), 1900 (130 \text{ GeV}), 2200 (200 \text{ GeV}) \). The results are shown in Fig. 1.

For the above set of values of \( V_h \), only if \( x > 2.5 - 3.0 \), which corresponds to about 1.35-1.44 for the ratio of the fireball radii in the hadronic to quark phase, should the entropy of the hadronic phase be larger than that of the quark one.

As a contrast we also consider an ideal case for constituent quarks whose phase space distribution follows Fermi-Dirac distribution, \( g_i(\mathbf{r}, \mathbf{p}) = 1/\exp(\varepsilon_i/T) + 1 \). The hadron spectra can be determined by combination of quarks. Then we use the Duhem-Gibbs relation (2) and entropy formula (5) to obtain the entropies for quark and hadronic phases. The result from the Duhem-Gibbs relation is \( \langle E, PV, TS \rangle = (0.61,0.145,0.755) \) (quark), \( (0.59,0.136,0.726) \) (hadron) for unit volume of quark phase. We can also see that \( TS \) is almost a constant, same as in the real case. The result from the entropy formula shows the same behavior as in Fig. 1 except that the entropy of the hadronic phase is larger than that of the quark one happens when \( x \gtrsim 16 \).

A few comments about our approach and results are needed. Our results show that the increase of the entropy requires an effective and adequate expansion of the fireball during hadronization, which also implies that hadronization takes finite time to complete. Actually in hydrodynamic simulation of the evolution of the fireball, hadronization is indeed a dynamic process with non-trivial space-time profile, i.e. it does not take place uniformly in the same space-time but on the freezeout hypersurface, so different part of the fireball hadronizes in
different time. Our current approach does not take such a space-time picture but just provide an averaged effect which is more simple and transparent than a real hydrodynamic simulation. If combination is treated locally one can still compute the entropies before and after hadronization for each space-time cell using Eq. (2) or Eqs. (5-6). The total entropy is a sum over all cells and the result is similar to the ideal case in our paper. This implies that the entropy can be described in a global and effective way as in our current model. In other words the entropy is a global quantity which should be insensitive to the local fine structure. As we have emphasized in the beginning that we do not address in our QCM the entropy issue in the context of phase transition. We just made a few comments about it. If local equilibrium is reached for a closed system the entropy would not change during the transition from the quark to hadronic phase, it is the entropy density that changes (decreases) during the transition accompanied by the volume expansion. If the system is not in local equilibrium the phase transition is not well defined (it is indeed a crossover) but still one can obtain the total entropy which should increase. Such a study in the QCM is independent of whether the entropy increases or decreases beyond the combination process.

In summary, we have investigated the issue of the entropy in the framework of the quark combination-like model. As an example for such types of models we used the one developed by Shandong group whose exclusive nature makes a transparent calculation feasible. We used two available methods to calculate the entropies for the quark and hadronic phases, one from the Duhem-Gibbs relation, another from the entropy formula in terms of particle phase space distributions. We found that the total entropy from the Duhem-Gibbs relation always increases in hadronization if the average temperature of the hadronic phase is lower than that of the quark phase. The increase of the entropy during hadronization can also be confirmed from the entropy formula if the volume of the hadronic phase is larger than $2.5-3.0$ times that of the quark phase. This implies that the expansion of the fireball takes place during hadronization and it takes finite time for the quark phase to hadronize. So whether the entropy increases or decreases during combination depends on the temperature before and after combination and on how much expansion the system undergoes during combination. The current study provides an example to shed light on the entropy issue in the quark combination model.

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