SUSY CP phases and asymmetries at colliders

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Abstract. In the Minimal Supersymmetric Standard Model, physical phases of complex parameters lead to CP violation. We show how triple products of particle momenta or spins can be used to construct asymmetries, that allow us to probe these CP phases. To give specific examples, we discuss the production of neutralinos at the International Linear Collider (ILC). For the Large Hadron Collider (LHC), we discuss CP asymmetries in squark decays, and in the tri-lepton signal. We find that the CP asymmetries can be as large as 60%.

1. Introduction
Supersymmetry (SUSY) [1] is a very well motivated theory to extend the Standard Model (SM) of particle physics [2]. SUSY models are not only favored by gauge coupling unification and naturalness considerations, but are also attractive from the cosmological point of view. For example, the lightest SUSY particle (LSP) is a good dark matter candidate, if it is stable, massive and weakly interacting [3, 4]. Most interestingly, SUSY models provide a number of new parameters, among some having physical phases which cause manifest CP violation [5]. Remarkably in the SM, the single CP phase in the quark mixing matrix, which is currently confirmed in \( B \) meson experiments [6,7], cannot explain the observed baryon asymmetry of the universe [8]. Additional sources of CP violation in models beyond the SM are required [9].

In the MSSM, a set of remaining complex parameters is obtained after absorbing unphysical phases of parameters by redefining particle fields. In the literature, the complex parameters are usually chosen to be the higgsino mass parameter \( \mu \), the U(1) and SU(3) gaugino mass parameters \( M_1 \) and \( M_3 \), respectively, and the trilinear scalar coupling parameters \( A_f \)

\[
\mu = |\mu|e^{i\varphi_\mu}, \quad M_1 = |M_1|e^{i\varphi_{M_1}}, \quad M_3 = |M_3|e^{i\varphi_{M_3}}, \quad A_f = |A_f|e^{i\varphi_{A_f}}.
\]  

(1)

The SUSY CP phases of these parameters lead to theoretical predictions for the electric dipole moments (EDMs) of electron, neutron and that of the atoms \(^{199}\text{Hg}\) and \(^{205}\text{Tl}\), which can be (sometimes orders of magnitude) beyond the current experimental upper bounds [6,10,11]. These strong bounds suggest that either the SUSY CP phases are severely suppressed (in particular \( \varphi_\mu < 0.1\pi \)), the SUSY particles are very heavy (e.g. the first two generations of sfermions with \( m_{\tilde{f}} > 10 \text{ TeV} \)), or different loop-contributions to the EDMs cancel accidentally. However, these solutions require a fine-tuning of the SUSY parameters, and would be unnatural. The need of tuning the SUSY parameters and phases to fulfill the EDM constraints is referred to as the \textit{SUSY CP problem} in the literature [12,13]. In order to analyze the problem, it is necessary...
to independently measure the SUSY phases at colliders. In particular CP-odd observables are needed to find or exclude direct evidence of CP violation.

In this talk we will concentrate on a particular class of CP-odd\(^1\) (T-odd) observables, which can be defined with the help of triple products [16]. Triple products can lead to large CP asymmetries, since they already appear at tree level due to spin correlations [17]. We will only give a few selected examples to motivate the use of triple products, and cannot give a thorough review of the vast amount of literature in the field. Therefore we will discuss the production of neutralinos [18–20] at the International Linear Collider (ILC) [21]. For the Large Hadron Collider (LHC) [22], CP asymmetries in top squark decays [23–25] and in the tri-lepton signal [26, 27] are discussed.

2. Triple products and their asymmetries

Triple products are built up from particle spin or momenta three-vectors,
\[
\mathcal{T} = (\vec{p}_a \times \vec{p}_b) \cdot \vec{p}_c,
\]
see a schematic picture in Fig. 1. Since each of the momentum (spin) vector changes its sign under a naive time transformation, \(t \rightarrow -t\), the triple product is T-odd. Thus T-odd asymmetries of the cross section \(\sigma\) can be defined
\[
A_T = \frac{\sigma(T > 0) - \sigma(T < 0)}{\sigma(T > 0) + \sigma(T < 0)} = \frac{\int \text{Sign}[\mathcal{T}]|\mathcal{T}|^2 d\text{Lips}}{\int |\mathcal{T}|^2 d\text{Lips}},
\]
with the amplitude squared \(|\mathcal{T}|^2\), and the Lorentz invariant phase-space element \(d\text{Lips}\), such that \(\int |\mathcal{T}|^2 d\text{Lips} = \sigma\). The triple product asymmetry is thus an angular distribution
\[
A_T = \frac{N_+ - N_-}{N_+ + N_-},
\]
with the number of events \(N_+ (N_-)\) of particle \(\vec{p}_c\) above (below) the plane spanned by \(\vec{p}_a \times \vec{p}_b\). The T-odd asymmetry \(A_T\) would also be CP-odd, if absorptive phases (e.g. from higher order final-state interactions or finite-widths effects) can be neglected. Since the absorptive phases do not change sign under charge conjugation, they can be eliminated in some cases by defining a genuine CP asymmetry
\[
A_{CP} = \frac{1}{2}(A_T - \bar{A}_T),
\]
of the corresponding asymmetry \(\bar{A}_T\) for the charge conjugated process.

\(^1\) Other classes of CP-odd observables would be rate asymmetries of cross sections, branching ratios and distributions [14]. In addition it should be noted that SUSY phases have large impact on the neutral MSSM Higgs sector. For recent reviews and references, see for example Ref. [15]
To start with an intuitive example of the use of triple products, we consider the production of the lightest neutralino pair at the ILC.

Indeed, the polarization of each neutralino normal to the production plane is CP-sensitive, and since, due to momentum conservation, the production takes place in a plane, an additional vector-degree of freedom perpendicular to this plane is needed to build up a triple product.

Now a triple product \( T = (\vec{p}_{\ell^+} \times \vec{p}_{\ell^-}) \cdot \vec{p}_{e^\pm} \), see Eq. (2), can be formed from the beam momentum \( \vec{p}_{e^\pm} \) and the two outgoing lepton momenta \( \vec{p}_{\ell^\pm} \).

In Fig. 2, we show the phase dependence of the cross section and the corresponding asymmetry \( A_T \) (3). It is remarkable that the maximal values \( A_T \approx \pm 9\% \) are not necessarily obtained for maximal CP phases. The reason is that the asymmetry \( A_T \) is proportional to the neutralino spin correlations, which are a product of a CP-odd term from the production, and a CP-even term from the decay. Since the CP-odd (CP-even) factor has as sine-like (cosine-like) dependence on the phases, the maxima of \( A_T \) are shifted towards \( \varphi_{M_1} = 0 \) in Fig. 2(b). It is interesting to note that the asymmetry can be sizable for small values of the phases, which is suggested by the EDM constraints. Note also that the variation of the cross section, Fig. 2(a), is more than 100%. In addition to the CP-sensitive asymmetry, the cross section may serve to constrain the phases. Note that the choice of longitudinal beam polarization \( (P_{e^-}, P_{e^+}) = (0.8, -0.6) \) almost doubles the size of the asymmetry and the cross section.

As a next example, we show that final state particle polarizations can give large asymmetries. The transverse polarization \( s_\tau \) of the tau in the neutralino decay

\[
\chi^0_2 \to \tilde{\tau}^\pm_1 + \tau^\mp
\]
for neutralinos, to analyze their CP properties through their subsequent decays [25, 35, 36].

We discuss the top squark decay at the LHC [23–25] and also CP-odd observables with transversely polarized beams at the ILC [32]. For a general discussion of the neutralino system with CP phases, see Ref. [33].

4. Numerical examples for the LHC

We discuss the top squark decay at the LHC [23–25]

$$\tilde{t}_m \rightarrow t + \tilde{\chi}^0_i; \quad m = 1, 2; \quad i = 2, 3, 4;$$

(9)

with the subsequent two-body decay chains of the neutralino and the top as shown in Fig. 4. Only the spin-spin correlations of the neutralino and the top are sensitive to the imaginary part of the product of the left and right $\tilde{t}_m - \tilde{\chi}^0_i$ couplings, $\text{Im}\{a_{mi}^t (b_{mi})^*\}$, which depend on the phases $\varphi_{\tilde{A}_t}$, $\varphi_\mu$ and $\varphi_{\tilde{M}_i}$ [23]. The final state particle momenta $\vec{p}_t, \vec{p}_\mu, \vec{p}_{\tilde{\chi}_i}, \vec{p}_{\tilde{\tau}_2}, \vec{p}_{\tilde{\tau}_3}$ can be used to define various triple products and their corresponding asymmetries. In the following, we concentrate on the asymmetries in the two-body decays of the neutralino [23, 24]. Triple product asymmetries in the three-body stop decay have been studied in Ref. [34].

Note that if the decay (or the transverse polarization) of the top is not taken into account, the spin-spin correlations are lost. Then only CP asymmetries can be obtained, which are sensitive to the CP phases $\varphi_\mu$ and $\varphi_{\tilde{M}_i}$ alone, which enter solely from the neutralino decay. Still in that case, a three-body neutralino decay is required [25, 35], or a two body-decay chain with an intermediate Z-boson [36]. However, the sfermion then merely serves as a production channel for neutralinos, to analyze their CP properties through their subsequent decays [25, 35, 36].
Figure 4. Schematic picture of the two-body stop quark decay-chain.

Figure 5. (a) Contour lines in the $m_0-m_{1/2}$ plane of the asymmetry $A_T$, Eq. (3), of the triple product $T = \langle \vec{p}_t \times \vec{p}_{\ell_1} \rangle \cdot \vec{p}_{\ell_3}$ for the $\tilde{t}_1 \rightarrow t \tilde{\chi}_2^0$ decay chain as shown in Fig. 4, in the stop rest-frame, i.e., for boost $\beta_{\tilde{t}_1} = |\vec{p}_{\tilde{t}_1}|/E_{\tilde{t}_1} = 0$. The MSUGRA parameters are $\tan \beta = 10$, $|A_0| = 100$ GeV, with added phases $\varphi_{A_0} = 0.55\pi$, $\varphi_\mu = \varphi_{M_1} = 0$ at the weak scale. In the gray shaded areas the considered $\tilde{t}_1$ decay chain is kinematically forbidden, the upper left corner is excluded by $m_{\tilde{\chi}_2^0} > m_{\tilde{\ell}_R}$. (b) The boost dependence of the various asymmetries for different combinations of the triple product momenta, with MSUGRA parameters as in (a), and $m_0 = 100$ GeV, $m_{1/2} = 300$ GeV [24].
In Fig. 5(a), we now show contourlines of the asymmetry $A_T$ from the triple product $T = (\vec{p}_t \times \vec{p}_{\ell_1}) \cdot \vec{p}_{\ell_3}$ in the $m_0-m_{1/2}$ plane. We choose an MSUGRA-inspired scenario, with the input parameters $m_0$, $m_{1/2}$, $\tan \beta$, $A_0$, at the GUT scale, to obtain the low energy parameters. We then add the CP-violating phases $\varphi_{A_0}$, $\varphi_{\mu}$, $\varphi_{M_1}$. We see that the asymmetry can reach up to 30%, values which have also been found in Ref. [23]. However, the asymmetry has been evaluated in the rest-frame of the stops. At the LHC the stops will be boosted, and the corresponding asymmetries will be reduced [25]. In Fig. 5(b) we show their stop boost dependence. We also show other asymmetries, which are obtained using different combinations of momenta for the triple products.

Finally we want to shortly comment on the tri-lepton signal at the LHC. If a produced pair of a chargino and a neutralino both decay leptonically, three leptons and missing energy will form a distinctive signal with low QCD background [37]. A triple product of the three leptons has been analyzed at the Tevatron, however only small asymmetries have been obtained [26]. For the LHC, a systematic analysis of CP observables in the tri-lepton signal is planned [27].

5. Summary and conclusions
We have shown how triple products of particle momenta or spins can be used to define asymmetries, which are sensitive to the CP phases of the MSSM. Such asymmetries can be large, since they are already present at the tree level. For example, for neutralino production and decay at the ILC, and squark decays at the LHC, we have shown that the asymmetries reach up to 60% and 30%, respectively. As the asymmetries can be sensitive to small phases of order $0.1\pi$, they will be an ideal tool to measure or constrain SUSY CP phases at colliders, independently from low energy measurements of electric dipole moments. We hope that our theoretical studies motivate detailed experimental studies, taking into account backgrounds and event reconstruction efficiencies, to resolve the question whether SUSY CP phases can indeed be measured at the ILC and LHC.

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