Advertising and Entry Deterrence: 
An Exploratory Model

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In this model, the effects of advertising are infinitely durable, fixed (and sunk) costs give rise to economies of scale, post-entry behavior is noncooperative, and pre-entry expectations are rational. Despite the obvious resemblance to work on the use of investment in production capacity to deter entry, here the incumbent monopolist never finds it optimal to advertise more if entry is possible than if it is not. This result and other features of this model indicate the dangers of analyzing advertising with analogies to other sorts of investments. The results make clear the need for more theoretical work on advertising and entry deterrence.

A basic defect in the classic Bain/Sylos limit-pricing model of entry deterrence stems from its assumption that the potential entrant believes that the established firm would maintain its output constant if entry occurred. The problem is that if entry did occur it would not generally be rational for the established firm to carry out this threat; thus the threat is not credible (see, e.g., Scherer 1980, pp. 246–48). Recent work on entry deterrence has begun to correct this defect by exploring the consequences of ruling out such irrational post-entry behavior and of assuming that potential entrants have rational expectations.¹ A leading result in that literature is that in the presence of economies of scale an established monopoly facing the threat of entry

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¹ Dixit (1982) has recently provided a clear summary of this research. He also discusses complementary work on “reputation effects,” which are ignored here.
might find it possible and optimal to deter that entry credibly by investing more in specialized, durable production capacity than it would if no such threat were present. ² It appears that both the irreversibility of the investment and the presence of economies of scale are necessary for this result.

It is generally acknowledged that the effects of advertising persist over time, and it is often argued that there are important economies of scale in advertising. ³ Reasoning by analogy with investment in production capacity, as is commonly done in this context, one might conjecture (as I did initially) that in any plausible model that assumes durability and scale economies, it will sometimes be optimal for an established monopolist to overadvertise in order to deter entry credibly.

This essay presents an exploratory model, patterned on those focusing on investment in production capacity and constructed to study the role of advertising in entry deterrence. In this model, the effects of advertising are infinitely durable, fixed costs give rise to economies of scale, post-entry behavior is noncooperative, and pre-entry expectations are rational. Yet the incumbent monopoly never finds it optimal to advertise more if entry is possible than if it is not. Optimal entry deterrence always involves advertising less than if the threat of entry were absent. Certain disadvantages of size appear in this model’s post-entry equilibria, and it may be better to enter second than first. These results and others presented below make plain the dangers of analyzing investments in advertising largely by analogy to investments in production capacity. They also suggest that the strategic implications of investments in advertising are highly sensitive to the effects of advertising on buyer behavior and to the nature of post-entry equilibrium. ⁴

I. Assumptions

I deal throughout with only two firms, X and Y. Perhaps because of stochastic factors in the product development process, firm X appears on the market first. When firm Y subsequently appears, it may or may

² The seminal paper here is Spence (1977), which focuses on durable investment but does not deal with credibility. The result stated in the text seems due to Dixit (1980); see also Schmalensee (1981) and Dixit (1982). The importance of investment durability is made clear by Eaton and Lipsey (1981).

³ There is a good deal of controversy about both durability and scale economies (see Comanor and Wilson 1979).

⁴ Spence (1977, 1980) and Cubbin (1981) have dealt with advertising and entry deterrence, but their models suffer from the same basic problem as the Bain/Sylos analysis. Baldani and Masson (1981) construct a model with assumptions like mine in many respects but with very different predictions; see n. 23, below.
not elect to enter. Both firms produce the same product if they produce anything.

There is a continuum of potential buyers with linear individual demand curves. The total mass of these buyers is normalized to be unity for convenience, and the units in which prices and quantities are measured are chosen so that if a fraction $\theta$ of potential buyers are aware of the product their total demand is given by

$$q = \theta (1 - p) \text{ or } p = 1 - (q/\theta), \quad (1)$$

where $p$ is the market price and $q$ is the total quantity demanded. The unit cost of production, $\gamma$, is a constant less than one for both firms. None of what follows seems to depend critically on demand linearity or cost constancy.

In order to make any sales, firm X must invest a fixed amount, $f$, to design leaflets and prepare to print them. This cost is forever sunk.\(^5\) It then prints leaflets and sends them at random to potential buyers, exactly as in Butters (1977). There is a constant printing and distribution cost per leaflet sent. Buyers who receive one or more leaflets are aware forever of the location and telephone number of firm X and the attributes of its product. (No price information is sent; buyers can use telephones at low cost to learn prices.) Those who receive no leaflets never learn about this product. The effects of this introductory advertising are thus infinitely durable.

Butters (1977) notes that if $L$ leaflets are sent at random to $B$ buyers, and both $L$ and $B$ are large, then the fraction of buyers informed is approximately equal to $[1 - \exp(-L/B)]$. Treating this fraction as exact and letting $\alpha$ equal $B$ times the printing and distribution cost of a single leaflet, we obtain the cost of informing a fraction $x$ of potential buyers:

$$c(x) = f - \alpha \ln (1 - x). \quad (2)$$

We will use $x$ for the fraction informed by firm X and $y$ for the fraction informed by firm Y, which faces the same advertising cost function. Note that (2) implies a U-shaped average cost function for real advertising, $x$. Much of what follows holds for all cost functions of this sort.

We begin by analyzing firm X’s behavior if subsequent entry by Y is impossible. Section III describes post-advertising noncooperative duopoly equilibria, treating $x$ and $y$ as exogenous. Basic properties of the profit functions generated by these equilibria are shown in Section

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\(^{5}\) Baumol and Willig (1981) have stressed the importance of the distinction between fixed and sunk costs.
IV to produce nonstandard reaction functions in \((x, y)\) space and in Section V to drive the results on entry deterrence described above.

II. Monopoly Equilibrium

Suppose that X has just informed \(x\) potential buyers with its initial introductory advertising, and subsequent entry is impossible. Then X will optimally maintain its price and quantity constant forever. The present value of its net revenue is

\[
\pi(x) = x(1 - p)(p - \gamma)/r = q[1 - (q/x) - \gamma]/r, 
\]

where \(r\) is the relevant discount rate. If \(q\) is chosen optimally, price, quantity, and present value are given by

\[
p^* = (1 + \gamma)/2, \quad q^* = x(1 - p^*) = x(1 - \gamma)/2, \quad \pi^m(x) = x(1 - \gamma)^2/4r,
\]

respectively. It will simplify formulae in what follows to work with rescaled profits, \(\Pi^m(x) = r\pi^m(x)/(1 - \gamma)^2 = x/4\).

Taking into account the cost of introductory advertising, the present value of firm X is equal to \(\pi^m(x)\) minus the cost of advertising:

\[
w^m(x) = \pi^m(x) - c(x) = [x(1 - \gamma)^2/4r] + \alpha \ln (1 - x) - f. \tag{4}
\]

It will be convenient to define \(A = \alpha r/(1 - \gamma)^2, F = fr/(1 - \gamma)^2\), and \(C(x) = c(x)r/(1 - \gamma)^2\) and to work instead with

\[
W^m(x) = \Pi^m(x) - C(x) = (x/4) + A \ln (1 - x) - F. \tag{5}
\]

Conditional on having prepared to print leaflets, the optimal value of \(x\) can also be obtained by maximizing

\[
V^m(x) = W^m(x) + F = (x/4) + A \ln (1 - x). \tag{6}
\]

Differentiation of (5) or (6) establishes that if firm X elects to enter and subsequent entry is impossible, the optimal fraction informed is given by

\[
x^* = M = 1 - 4A. \tag{7}
\]

Thus if \(A\) exceeds \(1/4\), it is so expensive to inform potential buyers that firm X will not enter this market even if \(F = 0\). Accordingly I shall restrict \(A\) to the range \(0 < A < 1/4\). In what follows, I generally use \(M\) instead of \(A\) to characterize the variable costs of advertising.

Using the general notation developed above, next consider how the possibility of subsequent entry affects the optimal choice of \(x\) in this model. We proceed as above and consider first the equilibria that emerge after introductory advertising has been completed.
III. Post-Advertising Duopoly Equilibria

Suppose that both \( x \) and \( y \) have been fixed at some positive values. Because introductory advertising leaflets were sent at random, the total number of buyers informed of either product is given by

\[
\theta = 1 - (1 - x)(1 - y) = x + y - xy. \tag{8}
\]

Of the informed buyers, \( x(1 - y) \) know only of firm X, \( y(1 - x) \) know only of Y, and \( xy \) are aware of the products of both sellers. While actual advertising messages are of course not sent totally at random, a late entrant, like Y, will generally reach more buyers who already know of established sellers, like X, the more those sellers have spent to introduce their products. Our task in this section is to find the firms’ post-advertising profits as functions of \( x \) and \( y \).

It would perhaps be most natural to assume that post-advertising equilibria are noncooperative in prices. But, because the two firms produce identical products and some buyers always know this, no Nash-Bertrand pure-strategy equilibria exist in this model.\(^6\) We assume instead that post-entry equilibria are Nash-Cournot: each firm selects its output in order to maximize its own profit, taking as given the total output of the other firm. Both firms can accurately forecast these equilibria. Sellers are assumed unable to sort buyers by how much they know or by any other characteristic. The precise micro foundations of this equilibrium concept are not completely clear in this model or in general. (I have in mind perfectly informed buyers playing an arbitrage role by getting price quotations over the telephone from both sellers, but this does not seem to have been formalized.) It is also the case that our results seem to depend critically on the post-advertising equilibrium concept adopted.\(^7\) But the Nash-Cournot concept seems the most natural one that can be used in this model.

\(^6\) Consider the optimal choice of firm Y’s price, \( p_y \), as a function of X’s price, \( p_x \). If \( p_x \) exceeds the monopoly price, \( p^* = (1 + \gamma)/2 \), it is optimal for Y to set \( p_y = p^* \) and to sell to all \( y \) buyers who are aware of it. If \( p_x \) is slightly below the monopoly price, Y does better by undercutting X slightly and selling to the same set of buyers. Finally, if \( p_x \) is low enough, Y is better off selling to the \( y(1 - x) \) buyers who are aware of it but not of X at the monopoly price than it is undercutting X. The condition for this is \((1 - \gamma)^2 y(1 - x)^2/4r \geq y(1 - x)(p_x - \gamma)/r \) or \( p_x < [(1 + \gamma) - (1 - \gamma)x]/2 \). Symmetric reasoning yields the optimal \( p_x \) as a function of \( p_y \), and a sketch of these reaction functions makes clear the nonexistence of Bertrand equilibrium.

\(^7\) For instance, Jeffrey Baldani and Robert Masson have pointed out (in a personal communication) that if each firm takes as given the other’s sales to the perfectly informed buyers, rather than its total sales, the features of equilibrium that drive our main results vanish. (In particular, it may be optimal to overadvertise to deter entry.) In order for their equilibrium concept to make sense, however, sellers must be able to charge different prices to those with different knowledge about the market, and this does not seem especially plausible.
There are two types of Cournot equilibria in this model, depending on whether or not both firms make sales to the \( xy \) perfectly informed buyers. If both firms sell to these buyers, the situation is exactly equivalent on the margin to one in which both firms have informed all \( \theta \) knowledgeable buyers. The perfectly informed buyers ensure that all buyers pay the same price, and price and aggregate quantity must satisfy equation (1). Thus as long as both firms make positive sales to perfectly informed buyers, firm X’s profit function is \( \pi^X = q_x\{[(1 - [(q_x + q_y)/\theta)] - \gamma]/r \). Firm X’s Cournot reaction function is then just
\[
q_x = [\theta(1 - \gamma) - q_y]/2. \tag{9}
\]
Firm Y’s reaction function for this region simply interchanges the subscripts, and the unique solution to this pair of linear equations is
\[
q_x = q_y = \theta(1 - \gamma)/3. \tag{10}
\]
In order for this to describe an equilibrium, at least half the informed buyers must know of X and at least half must know of Y. That is, we must have \( x \geq \theta/2 \) and \( y \geq \theta/2 \), or, using equation (8),
\[
y/(1 + y) \leq x \leq y/(1 - y). \tag{11}
\]
Now suppose that only one firm makes sales to the perfectly informed buyers. (As long as at least one firm receives a price less than unity for its output, at least one firm must sell to these buyers.) This means that one of the inequalities in (11) must be violated, since (10) gives the unique equilibrium if both hold. Suppose that the right-most inequality is violated, so that \( y \) is smaller than \( x \). Equilibrium then involves firm Y selling to all \( y \) buyers who are aware of it, with firm X selling to the \( x(1 - y) \) who know only of it. Thus only firm Y sells to the perfectly informed buyers. At such a point, any increases in \( q_y \) would involve some sales to these individuals, so that (9) must hold at equilibrium. As it is easy to show that there can be only one price in a Cournot equilibrium,\(^8\) the other condition equates prices received by the two firms:
\[
p_x = 1 - [q_x/[x(1 - y)]] = p_y = 1 - (q_x/y). \tag{12}
\]
One can show directly that at the unique solution to (9) and (12), firm Y will wish neither to decrease its output (and allow firm X to make sales to perfectly informed buyers) nor to increase its output (and

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\(^8\) Suppose the contrary, and assume that equilibrium involves \( q \)’s such that \( p_x > p_y \). Then X must sell only to the \( x(1 - y) \) buyers who know only of it, and Y sells to all \( y \) who are aware of it. It is then clearly optimal for X to obtain the monopoly price, \( p^* \), from its captive audience by producing the corresponding monopoly quantity, \( x(1 - y)(1 - \gamma)/2 \). But Y can raise its price (by lowering its output) and increase its profits until it too obtains the monopoly price from its customers.
receive less per unit than X does). Firm Y’s demand is in fact kinked at
the equilibrium point; it is more elastic for quantity decreases than for
increases.

Solving for the profits earned at these equilibria and multiplying by
\( r/(1 - \gamma)^2 \) as above, we obtain the rescaled profit function for post-
advertising Cournot duopoly equilibrium:

\[
\Pi^x(x, y) = \begin{cases} 
\theta_y(1 - x)/(2\theta - x)^2, & 0 \leq x \leq y/(1 + y): \text{Region A} \\
\theta/y, & y/(1 + y) \leq x \leq y/(1 - y): \text{Region B} \\
\theta_x^2(1 - y)^2/(2\theta - y)^2, & y/(1 - y) \leq x \leq 1: \text{Region C}.
\end{cases}
\]

(13)

By symmetry, firm Y’s rescaled profit, \( \Pi^y \), is given by this same
function with arguments transposed. We define \( W^x, W^y, V^x \), and \( V^y \)
just as in (5) and (6) and use subscripts to denote partial derivatives. Note
that if \( x \) is in Region B, so is \( y \), while if \( x \) is in Region A, \( y \) is in Region C.
That is, \( x \leq y/(1 + y) \) implies \( y \geq x/(1 - x) \).

A surprising and important implication of (13) is that \( \Pi^y \) is positive
for \( x \) in Region A or Region B. Unless \( x \) is large relative to \( y \), firm X
benefits from increases in firm Y’s advertising. One can think of two
effects being at work here. First, for fixed \( x \), increases in \( y \) decrease the
fraction of X’s customers who know only of it. One would expect this
to reduce \( \Pi^x \) by reducing the set of customers for which X is the only
possible supplier. Second, increases in \( y \) reduce the importance of X
to firm Y. This tends to make it optimal for Y to react more mildly to
X’s presence. (Thus, if \( x \) is in Region A, one can show that market
price is a decreasing function of \( x/\theta \).) This second effect is intimately
related to the “judo economics” of Gelman and Salop (1982). They
find that a potential entrant that can credibly limit its output may be
able to enter profitably against an established monopoly despite demand
and cost disadvantages. In this model, the second of these
effects dominates the first, so that there are net disadvantages of large
(relative) size. (If \( x > y \), one can show that \( \Pi^x/\Pi^y < x/y \).) This is cer-
tainly not true for all models of this general sort.9

Figure 1 shows the average return on informed buyers, \( \Pi^x(x, y)/x \),
for several values of \( y \). By symmetry, this graph also shows \( \Pi^y(x, y)/y \)
for the same values of \( x \). The kink in the curves for \( y = 0.1 \) and 0.3
occurs as X passes from Region B to Region C. The locus of those
kinks, which forms a lower envelope for average returns, is given by
\( \Pi^x(x, y)/x = 2/[9(1 + y)] \). If the rescaled average cost function for

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9 I am indebted to Jean Tirole for clarifying my thinking on this point and for presenting (in a personal communication) a rather different model in which the first effect dominates. See also n. 7, above, and n. 23, below.
informed buyers, \( C(x)/x \), is anywhere below this envelope, entry deterrence is clearly impossible.

As intuition suggests, firm X is best off when \( y = 0 \). But, as figure 1 shows, if \( y \) is positive, increases in \( y \) serve to raise X’s average return in Regions A and B. Such increases enhance the advantages of small scale (relative to Y) that X can enjoy in those regions. Figure 1 suggests that for at least some average cost functions, \( C(x)/x \), entry deterrence might involve advertising less than would otherwise be optimal in order to keep average returns below average costs in those regions. To see if this suggestion is correct, we must employ (13) in an explicit analysis of duopoly choices of \( x \) and \( y \).

**IV. Advertising Reaction Functions**

Suppose that both firm X and firm Y have designed leaflets and prepared to print them, so that we can ignore their fixed and sunk costs, \( F \). In this section we also ignore the actual sequence in which \( x \) and \( y \) are chosen. Sunk costs and timing considerations are brought back into the analysis in Section V. Under these assumptions, I shall first investigate firm X’s optimal response, \( x^*(y) \), to exogenously fixed values of \( y \). By symmetry, Y’s optimal response to X, \( y^*(x) \), is given by the identical function. We then look for Nash equilibria in real advertising levels and describe the behavior of profits along reaction functions.

Figure 2 depicts the geometry of the situation. One can show that \( \Pi^x_{xy} \) is positive in Region A and negative in Regions B and C, so that the shifts in \( \Pi^x_{xy} \) shown in figure 2 are qualitatively valid everywhere.
The limiting values of $\Pi_x$ at regional boundaries are given in figure 2 and used in what follows. This derivative equals $(1 - y)/9$ throughout Region B.

If marginal costs of $x$ are given by $C_x^a(x)$, figure 2 shows that $x^*$ is in Region C if $y = 0.25$ and $x^*$ is in Region B if $y = 0.35$. It should be clear from the geometry of the figure that the transition between these two regions involves a discontinuous drop in $x^*$. For this cost function, there will exist some value of $y$, call it $D$, such that the first-order condition has two solutions, one in Region B and one in Region C, that yield equal values of $V$. As $y$ is increased beyond $D$, $x^*$ drops discontinuously from Region C to Region B. If the $C_x$ schedule were flatter than $C_x^a$, firm X’s transition from Region C might involve a discontinuous drop to Region A or to the boundary between A and B. In contrast, as long as $C_x$ slopes upward, the discontinuity in $\Pi_x$ at the A/B boundary does not induce a discontinuity in $x^*(y)$. With marginal cost schedule $C_x^b$ in figure 2, for instance, $x^*$ is clearly equal to $y/(1 + y)$ for both values of $y$ shown. Since $\Pi^v(x, y)$ is continuous, one can use the Region B portion of (13) to compute firm X’s present value.

Except when $x^*$ is on the A/B boundary, it is determined as a solution to the first-order condition $C_x = \Pi_x^1$. By the second-order condition, the slope of X’s reaction function has the sign of $\Pi_x^{xy}$. Given the sign pattern of this derivative (as described above), it follows that $dx^*/dy$ is positive in Region A and negative in Regions B and C. If $x^*$ is on the A/B boundary, as in figure 2 with marginal cost schedule $C_x^b$, it is clear that $dx^*/dy$ is positive.

We can now describe the qualitative properties of $x^*(y)$ for any upward-sloping $C_x$ schedule. If $y = 0$, $x^* = M$ by definition. As $y
increases from zero, \( x^* \) initially declines smoothly in Region C. As \( y \) increases beyond a critical value, \( D \), \( x^* \) drops discontinuously to a value below \( y/(1 - y) \) and thus out of Region C. If this drop yields \( x^* \) above \( y/(1 + y) \), and thus in Region B, further increases in \( y \) lower \( x^* \) continuously until \( y = 1 \) or the A/B boundary is encountered at \( x^* = y/(1 + y) \). If the A/B boundary is encountered for \( y < 1 \), \( x^* \) increases in \( y \) thereafter. If the discontinuous transition from Region C drops \( x^* \) to the A/B boundary or into Region A, all further increases in \( y \) cause \( x^* \) to increase.

Figure 3 shows the pair of reaction functions, \( x^*(y) \) and \( y^*(x) \), for the particular cost function assumed here with \( M = 0.5 \). The discontinuity at \( D \) involves a drop to the A/B boundary in this case; it is never optimal to operate in Region B. For large \( y \), \( x^*(y) \) moves from the A/B boundary into the interior of Region A.

Let us next consider Nash equilibria in this model. No such equilib-

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10 For the \( c(x) \) function assumed here, \( x^*(1) \) is always in Region A or on the A/B boundary. Use of the limiting values of \( \Pi \) shown in Fig. 2 establishes that sufficient conditions for \( x^* \) not to lie in B or C are \( A/(1 - y(1 + y)) > (1 - y)/9 \) and either \( y \geq \frac{1}{2} \) or \( A/(1 - y(1 + y)) \leq 7(1 - y)/27 \). The first of these is satisfied for \( y > (1 - 9A)/(1 + 9A) \), and the second is satisfied for \( y > (7 - 27A)/14 \). Both of these critical values are always less than one. Hence, if \( y \) is close enough to unity, \( x^*(y) \) must be either in Region A or on the A/B boundary.

11 The profit function in (13) is such that points on the reaction function cannot generally be obtained analytically. Proceeding as in n. 10, the program employed first ascertain the regions in which solutions to the first-order condition exist, and it checks to see if \( x^* \) may be on the A/B boundary. It then computes all solutions to \( C_x = \Pi \). In Regions A and C, this requires evaluation of the relevant roots of fourth-degree polynomials. A direct comparison of profits is used to determine \( x^* \) if multiple candidate values are thus identified.
rium in $x$ and $y$ exists in figure 3. In general, one can show that Nash equilibria must involve both firms operating in Region B.\textsuperscript{12} A necessary condition for $(x, y)$ to be a Nash equilibrium is thus that both firms' first-order conditions for that region be satisfied:

$$(1 - y)/9 = A/(1 - x) \text{ and } (1 - x)/9 = A/(1 - y).$$

These conditions are identical, however, so that in general if there is one Nash equilibrium there will be a continuum of such equilibria.\textsuperscript{13} But (14) is only necessary; the necessary and sufficient condition requires both firms to be in Region B, with $x = x^*(y)$ and $y = y^*(x)$. By the symmetry of the situation, there exist equilibria if and only if the symmetric point $x = y = 1 - \sqrt{9A}$ is an equilibrium. Numerical evaluation of reaction functions corresponding to different values of $M$ indicates that this condition is satisfied for $M \geq E$, where $E$ is between 0.82 and 0.83. Figure 4 shows the reaction functions for $M = 0.86$. All points $[x^*(y), y]$ for $D < y < T_1$ are Nash equilibria, as indicated. For this value of $M$, it is never optimal to operate in Region A, and the discontinuity at $D$ leads to operation in Region B.

Finally, let us indicate how $V$ varies along the reaction function. We noted in Section III that $\Pi_y$ is negative in Region C and positive in Regions A and B. Thus by the envelope theorem, for any upward-sloping $C_x$ schedule, $V^x$ is reduced by increases in $y$ for $y < D$, minimized at $y = D$, and increased by increases in $y$ for $y > D$. It is easy to show that $D$ is always less than $M$, as in figures 3 and 4.\textsuperscript{14}

\textsuperscript{12} There can be no equilibria with either firm on the A/B boundary, since this would imply that the other was on the B/C boundary, and it is never optimal to operate there. Suppose an equilibrium exists with $x$ in Region A and $y$ in Region C. Then if $z = x/(1 - x)$, equilibrium involves $z < y$. Writing down the corresponding first-order conditions and solving both for $A/(1 + z)(2y + z)^2/y$, one finds that an equilibrium of this sort, $T(z; y) = z^3 + z^2[y(1 - y)] + z[3y^2(1 - y) + 3y] + 2y^2[y(1 - y) - 1] = 0$. To show that no equilibria outside Region B exist, it suffices to show that this equation, treated as a cubic in $z$ for given $y$, has no real roots between zero and $y$. Since the maximum value of $T(y, y)$ is negative, $T$ is always negative when $z = y$. Since $T$ must be positive for large $z$, $T(z; y) = 0$ thus has one real root larger than $y$. At $z = 0$, $T$ is negative and decreasing in $z$, it attains a local maximum at some negative $z$, and it is clearly negative for large negative values of $z$. It thus follows that $T(x; y) = 0$ has either a pair of negative roots or a pair of complex roots. Since it has no real roots between zero and $y$, the proof is complete.

\textsuperscript{13} This is a property of the particular $C(x)$ function assumed here. If $C(x) = F + Ax/(1 - x)$, which has roughly the same shape, there is at most one Nash equilibrium for any value of $A$. For this cost function, numerical evaluation of the reaction function indicates that such equilibria exist for all $A \leq 0.035$.

\textsuperscript{14} Since reaction functions are negatively sloped in Region C, $J < M$, where $J$ is the limiting value of $x^*(y)$ as $y$ approaches $D$ from below. (See figs. 3 and 4.) Since the point $x = J, y = D$ is in $X$'s Region C, it must be that $J > D/(1 - D) > D$, so that $M$ also exceeds $D$ as asserted.
V. Advertising by the First Entrant

We can now analyze the optimal choice of $x$ by firm X, the first entrant, on the assumption that it knows that Y will appear after $x$ has been chosen (and advertising leaflets have been sent to potential buyers) and contemplate entry. As in the production capacity literature, firm X can credibly threaten not to decrease its investment, $x$, when firm Y appears, since such decreases are technically impossible. ( Buyers never forget in this model.) In the production capacity literature, investment by an entrant reduces the attractiveness of investment by the incumbent, so the latter never wishes to expand capacity in response to entry. Here things are more complicated because X and Y may benefit from each other’s advertising, and reaction functions are not monotonic in $x$ and $y$. For some initial values of $x$, Y’s entry may make it optimal for X to inform more buyers, and this may induce more advertising by Y. If X cannot commit in advance not to increase $x$, and it initially selects an $x$ such that further increases may be optimal, the advertising game becomes quite complex. Both X and Y must in general analyze possible sequences of $x$ and $y$ over time.

In order to avoid solving such dynamic games, we consider two polar special cases. In the first case, firm X can credibly commit not to increase $x$ after firm Y’s entry, so that $x$ will in fact never change. (Under some conditions, destruction of the materials necessary to print more leaflets may serve to accomplish this.) This case should overstate X’s ability to deter entry. In the second case, firm X cannot make such commitments and is restricted to credible $x$’s, defined as
those values of $x$ such that $X$ will not wish to increase advertising if $Y$ enters on its reaction function. Formally, $x_0$ is credible if and only if

$$x^*[y^*(x_0)] \leq x_0.$$  

(15)

This constraint generally rules out small values of $x_0$. (In fig. 3, $x$ is credible unless it is less than $T$. In fig. 4, there are two ranges of credible $x$’s: $D \leq x \leq T_1$ and $x \geq T_2$.) If firm $X$ selects a credible $x$, firm $Y$’s best response if it enters is $y^*(x)$, and the game is over. Limiting $X$ to credible advertising levels seems likely to overstate the effects of $X$’s inability to commit not to increase advertising after $Y$’s entry. The second case should thus understate $X$’s ability to deter entry.

If $F$ is large enough, entry will be blockaded in the sense of Bain (1956) in both cases. That is, if firm $X$ sets $x = M$, as if entry were impossible, and $V^y[M, y^*(M)]$ is less than $F$, firm $Y$’s entry will be deterred.\footnote{The point $x = M$ is always credible. By the shapes of reaction functions in this model, if $x^*(1) \leq M$, it follows that $x = M$ is credible, since firm $X$ will never want to raise $x$ no matter what $Y$ does. Note 10 establishes that $x^*(1)$ is either in Region A, with $x^* < y(1 + y) = \frac{1}{2}$, or on the A/B boundary with $x^* = \frac{1}{2}$. Referring to fig. 2, $x^*$ is on the boundary if and only if $(5 - y)27 \geq A[y(1 - y)],$ or $A \leq 2/27.$ But $A \leq 2/27$ implies $M = 1 - 4A \geq 19/27 > \frac{1}{2}.$ Thus $x^*(1) < M$ for $A \leq 2/27.$ For all larger $A,$ $X$’s Region A first-order condition when $y = 1$ is $(2 + 3z - z^3)/(2 + z)^2 = A(1 + z),,$ where $0 \leq z = x/(1 - x) \leq 1.$ At $z = 0,$ the left-hand side of this equation equals $\frac{1}{4},$ while the right-hand side equals $A,$ which is less than $\frac{1}{4}.$ The left-hand side is decreasing in $z,$ so that $A(1 + z) < \frac{1}{4}$ at a solution. Substituting for $z,$ this last inequality establishes that $x^*(1) < M$ for $A > 2.27$ as well, and the proof is complete. (In fig. 3, $A > 2/27,$ while $A < 2/27$ in fig. 4.)} The upper boundary in figure 5 is derived by setting $F$
equal to this critical value and scaling by X’s present value net of leaflet preparation costs. If $F \geq V^m(M)$, the market is not profitable for firm X even if it is free from the threat of entry.

If entry is not blockaded, it is necessary to make some assumption about X’s post-entry options. Let us consider the first case, in which the $x$ it selects can never be changed. Given the threat of firm Y’s entry, firm X can either select the Stackelberg value of $x$, which maximizes $V^x[x, y^*(x)]$ in anticipation of Y’s arrival, or it can attempt to deter Y’s entry. Since $V^y[x, y^*(x)]$ is minimized at $x = D$ and increases in $x$ for $x > D$, and since $D < M$, it is clear that X will never set $x > M$ in an attempt to deter entry. Such overinvestment in introductory advertising would make entry more attractive to Y, not less. Entry deterrence thus must involve setting $x$ between $D$ and $M$. (See figs. 3 and 4.) If $F < V^y[D, y^*(D)]$, deterrence is impossible, because the minimum value of $W^x$ that X can impose is nonnegative. The lowest curve in figure 5 presents a numerical evaluation of this critical level of $F$, scaled as before.\(^\text{16}\)

If $F$ is between the upper and lower critical levels, so that deterrence is neither unnecessary nor impossible, X’s optimal strategy is found by comparing the profits yielded by optimal deterrence with those obtained at Stackelberg equilibrium (with X as leader). One can show that $V^m(x)$ is concave, so that the most profitable deterrence strategy involves setting $x$ as close to $M$ as possible. This is accomplished by choosing the unique $x$ between $D$ and $M$ such that $F = V^y[x, y^*(x)]$. At this point, (6) gives X’s present value. The Stackelberg return, to which this must be compared, is found by maximizing $V^y[x, y^*(x)]$. In fairly extensive computations, deterrence was always more profitable than Stackelberg equilibrium when $F$ was between the upper and lower critical levels.\(^\text{17}\) It is thus apparently always optimal in this model to deter entry when deterrence is both possible and necessary and $x$ can commit never to increase its advertising after Y’s entry. But, in sharp contrast to models involving investment in production capacity, deterrence is here accomplished by underinvestment relative to the monopoly level, not overinvestment.

\(^{16}\) For $M = 0.02, 0.04, \ldots, 0.98$, we searched over $x = 0.002, 0.004, \ldots, 0.998$ to find the minimum value of $V^y[x, y^*(x)]$. Since the true minimum was thus never found exactly, the lowest curve in fig. 5 is too high by an unknown but probably small amount. For small $M$ the search over $x$ involves a coarse grid in the relevant region, and the results are correspondingly less reliable. (Indeed, some model properties known to hold globally were violated in the output for $M = 0.02$ and $M = 0.04$.) While some of the general results suggested by these computations and described in what follows may be susceptible to analytical proof, this model is both so special and so algebraically complex that the slight strength this would add to our conclusions hardly seems worth the substantial effort that would be required.

\(^{17}\) See n. 16. The Stackelberg profit was taken to be the largest value of $V^y[x, y^*(x)]$ encountered, and this was compared to $V^m(x)$ for all $x$’s examined between $D$ and $M$. 
If deterrence is impossible, and apparently only then, firm X’s best policy is to act as a Stackelberg leader in anticipation of firm Y’s entry. For all values of $M \geq 0.06$ for which computations were performed, this policy called for X to select the largest $x$ less than $D$ at which the relevant functions were evaluated. \(^{18}\) Thus Y’s post-entry profits are apparently generally minimized at X’s Stackelberg point. While Y is indifferent between $x$’s just above and just below $D$, X prefers the latter because they induce sharply larger values of $y$ from which it benefits. At least for $M \geq 0.06$, and possibly more generally, $y$ exceeds $x$ at X’s Stackelberg point, as figures 3 and 4 suggest. In this case, X’s position as first mover enables it to capture advantages associated with being small relative to its rival; $V^x$ exceeds $V^y$ at all Stackelberg points computed.

The results differ somewhat in the second case in which X is restricted to credible $x$’s. In order for deterrence to be possible in this case, $F$ must exceed the minimum value of $V^y[x, y^*(x)]$ over all credible $x$’s. In figure 3, $x$’s below $T$ are not credible, so that if $F$ is less than $V^y[T, y^*(T)]$, it is not possible to deter entry credibly. In figure 4, however, points arbitrarily close to $D$ are credible, and the credibility constraint does not reduce the set of $F$’s for which deterrence is possible. The middle curve in figure 5 is a numerical evaluation of the upper bound of the region in which credible deterrence is impossible. \(^{19}\) Note that at $M = E$, where Nash equilibria in $x$ and $y$ come into existence and the geometry changes from that of figure 3 to that of figure 4, this curve drops discontinuously to the lower curve, with which it coincides for all $M > E$.

If deterrence is neither impossible nor unnecessary, we must again compare deterrence profits with those obtained by Stackelberg leadership. The Stackelberg return is found by maximizing $V^X[x, y^*(x)]$ over credible values of $x$. In all cases for which computations were performed, $V^m(x)$ exceeded this return for all credible $x$’s between $D$ and $M$ (see n. 16). Thus, even if firm X is limited to credible values of $x$, it is apparently optimal to deter entry whenever possible. As before, deterrence must always involve advertising less than a protected monopolist would; advertising more would make entry more attractive, not less.

Finally, if deterrence is impossible, and, again, apparently only then, X’s optimal policy is Stackelberg leadership. The credibility constraint (15) is apparently always binding: none of the unconstrained

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\(^{18}\) See n. 16. Results for $M = 0.02$ and 0.04 are suspect for the reasons noted there. The finding reported in the text suggests that X’s Stackelberg point is strictly undefined, as there is no “largest $x$ less than $D$.”

\(^{19}\) The procedure described in n. 16 was used, except that the search was restricted to values of $x$ passing the credibility test, given by inequality (15) in the text.
Stackelberg points computed satisfied it. In this case, \( V^x[x, y^*(x)] \) was always maximized at the smallest credible \( x \) examined (see n. 18). For \( M < E \), when no Nash equilibria exist, \( X \) thus apparently optimally chooses a point like \( T \) in figure 3 if \( Y \)'s entry is inevitable. As that figure suggests, \( y^*(x) \) was less than \( x \) at all such points examined, and \( V^3 \) exceeded \( V^x \). Since \( x \) cannot be credibly set below \( y^*(x) \) for \( M < E \), the credibility constraint apparently makes it better to enter second than first! For \( M > E \), values of \( x \) arbitrarily close to \( E \) become credible, as in figure 4, and computations indicate that \( X \) optimally selects the Nash equilibrium with the smallest possible value of \( x \).\(^{20}\) At such points \( V^x \) exceeds \( V^3 \), though \( y^*(x) \) exceeds \( x \). When Nash equilibria exist, there is thus apparently still an advantage in entering first even if later entry cannot be deterred. As before, that advantage turns on the ability to be small relative to the later entrant.

VI. Conclusions

The assumptions stated in Section I were chosen in an attempt to produce a simple model resembling as closely as possible models in which overinvestment in production capacity may credibly deter entry, while allowing advertising to have plausible effects on buyers' behavior. The resulting model turned out to be of surprising (to me, at least) complexity and to differ in basic ways from the models after which it was patterned. Here, as in those models, durability and scale economies can interact to produce blockaded entry. But if entry is not blockaded, optimal deterrence here involves underinvestment, whereas deterrence strategies involving production capacity employ overinvestment. Moreover, this model exhibits pervasive disadvantages of relative size that do not have analogues in most models not involving advertising.

The analysis presented here thus suggests that the strategic implications of investments in advertising may, under some conditions, contrast sharply with those of investments in productive capacity. At the very micro level, one can point to a number of differences between these types of investments. In this model, the most important difference seems to be the following.\(^{21}\) Investment in productive capacity by an incumbent generally discourages entry by making high

\(^{20}\) Again, this means that \( X \)'s Stackelberg point may be undefined, as there is in general no "Nash equilibrium with the smallest \( x \)."

\(^{21}\) Another difference worth mentioning is the following. Investment in productive capacity alters the relation between total cost and the total volume of output, while investment in advertising fundamentally affects the demand curves of individual buyers. When those buyers can vary the number of units they purchase, as in the model presented here, this formal difference may have basic strategic implications.
levels of output, and thus low market prices, relatively more attractive as a response to entry. But an incumbent’s investment in advertising in this model increases the set of customers who would not be tempted by an entrant’s product (because they are unaware of the entrant) at every level of entrant advertising. This makes the incumbent less eager to expand output or lower price in response to entry, since such reactions involve giving up secure profits that could be earned on sales to those loyal customers. With price discrimination ruled out, an incumbent’s investment in advertising tends to make entry more attractive by guaranteeing the entrant a friendlier welcome. Incomplete introductory advertising seems to function here much as output limits do in the analysis of Gelman and Salop (1982) to enhance the profitability of entry at (relatively) small scale.

I view this model as establishing a theoretical possibility, not as necessarily providing a useful description of most markets or indeed of any actual market. While I think its assumptions are natural and plausible, its predictions run counter to the intuitions of many economists and to some recent research in related areas. In particular, these predictions seem at odds with recent theoretical and empirical research that points to general advantages enjoyed by pioneering brands; see Schmalensee (1982) and the references there cited.22 One might expect that under some conditions advertising by an incumbent would hinder entry by making it harder for an entrant to attract buyers’ attention or, in the context of this model, by lowering the fraction of the entrant’s customers for whom it is the only known source of supply.23 The fact that the second of these effects in particular never operates here to discourage entry seems due both to the focus on introductory advertising and the attendant assumption of no product differentiation and to the particular post-advertising equilibrium concept employed.

More work is clearly required to see exactly what features of the model presented here drive its results and, more importantly, to construct more general models that can delineate the conditions under

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22 This comparison suggests the importance of the assumption here that advertising removes all uncertainty about product quality. It is worth noting that some recent empirical results of Leffler (1981, pp. 69–71) seem consistent with the present model’s predictions, though they are also consistent with other models.

23 In the model of Baldani and Masson (1981), pre-entry advertising by the incumbent increases its “goodwill,” and the higher the incumbent’s goodwill, the lower the return to subsequent advertising investments by the entrant. While Baldani and Masson present no model of individual buyer behavior, it seems sensible to interpret entry deterrence in their model as stemming from the entrant’s problem of attracting buyers’ attention. In the model of Jean Tirole mentioned in n. 7, above, products are differentiated, and the second of these effects makes possible entry deterrence: the more the incumbent advertises, the more elastic is the entrant’s demand curve, in effect.
which overadvertising can optimally and credibly deter entry. This paper should serve to indicate that those conditions are more restrictive than many economists might have thought, but it should not be interpreted as arguing that there do not exist plausible conditions under which incumbent sellers can use advertising to deter entry.

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