Efficiency and maximal CP-asymmetry of scalar triplet leptogenesis

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Abstract

We study thermal leptogenesis induced by decays of a scalar SU(2)\textsubscript{L} triplet. Despite the presence of gauge interactions, unexpected features of the Boltzmann equations make the efficiency close to maximal in most of the parameter space. We derive the maximal CP asymmetry in triplet decays, assuming that it is generated by heavier sources of neutrino masses: in this case successful leptogenesis needs a triplet heavier than \(2.8 \cdot 10^{10}\) GeV and does not further restrict its couplings, allowing detectable \(\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma\) rates in the context of supersymmetric models. Triplet masses down to the TeV scale are viable in presence of extra sources of CP-violation.

1 Introduction

Majorana neutrino masses can be mediated by tree-level exchange of three different kinds of new particles: I) fermion singlets \cite{1}; II) scalar SU(2)\textsubscript{L} triplets \cite{2}; III) fermion SU(2)\textsubscript{L} triplets \cite{3}. Indeed these particles can have renormalizable couplings to lepton doublets \(L\) and Higgs \(H\), generating the unique Majorana neutrino mass operator \((\overline{L}H)^2\). Low energy experiments can see neutrino masses and reconstruct the coefficients of the \((\overline{L}H)^2\) operator, but cannot tell their origin. More information about the unknown high-energy theory that generates neutrino masses can be obtained assuming that the observed baryon asymmetry of the Universe is produced via thermal leptogenesis \cite{4} in decays of the lightest of these three kinds of particles.

Leptogenesis has been extensively studied only in case I) \cite{1, 3}. The main concern with the other possibilities is that gauge scatterings can keep SU(2)\textsubscript{L} triplets close to thermal equilibrium, conflicting with the third Sakharov condition \cite{6}. Since gauge scatterings are slower than the expansion rate of the Universe only at temperatures \(T \gtrsim 10^{15}\) GeV \cite{7, 8}, it was generally expected that successful triplet leptogenesis is possible only around that energy scale. However, first estimates of the leptogenesis efficiency in scalar triplet decays \cite{9, 10, 11} as well as the full calculation for fermion triplet \cite{12} have shown that thermal leptogenesis is efficient enough even at lower temperatures.

In this paper we present the first full calculation of thermal leptogenesis in the decays of a SU(2)\textsubscript{L} triplet scalar \(T\). We start deriving the maximal CP asymmetry \(\varepsilon_L\) in triplet decays generated by
any other source of neutrino masses much heavier than $T$ (e.g. additional triplets $T'$, right-handed neutrinos, etc). We next derive and solve the full set of Boltzmann equations describing the thermal evolution of the relevant abundances. We find that thermal leptogenesis from scalar triplets proceeds in a qualitatively different way from the alternative scenarios studied so far. In particular, a quasi-maximal efficiency can be obtained even if gauge scatterings keep the triplet abundance very close to thermal equilibrium.

The CP asymmetry induced by the heavier sources of neutrino masses decreases with the triplet mass $M_T$, so that successful leptogenesis needs $M_T > 2.8 \times 10^{10}$ GeV. More general sources of CP violation allow thermal leptogenesis even down to $M_T \sim$ TeV: such light triplets can be tested at collider experiments [13].

Our paper is organized as follows. Section 2 contains the technical details necessary for this analysis. Results are discussed in section 3 and summarized in the conclusions.

2 Scalar triplet

We start by presenting the model under consideration. The relevant parts of the scalar triplet Lagrangian are

$$\mathcal{L} = \mathcal{L}_{SM} + |D_{\mu}T|^2 - M_T^2 |T^a|^2 + \frac{1}{16} \left( \lambda^{g'}_g L_i g' L_j T^a + M_T \lambda_H H^i \tau^a_H T^a + \text{h.c.} \right),$$

where $3 \times 3$ flavour matrices are denoted in bold-face, $g, g' = \{1, 2, 3\}$ are generation indices, $\epsilon$ is the permutation matrix and $\tau^a$ are the usual SU(2)$_L$ Pauli matrices. The hypercharges are $Y_L = -1/2$, $Y_H = 1/2$ and $Y_T = 1$. The triplet Lagrangian can be supersymmetrized introducing two chiral superfields $T$ and $\bar{T}$ with superpotential couplings:

$$W = W_{MSSM} + M_T T \bar{T} + \frac{1}{2} \left( \lambda^{g'}_L L_i g' T + \lambda_H H_d H_d T + \lambda_H H_u H_u \bar{T} \right).$$

(2)

Triplet exchange mediates the dimension-5 neutrino mass operator $(L_H u)^2$ such that the triplet contribution $m_T$ to the neutrino mass matrix $m_\nu$ is

$$m_T = \lambda_L \lambda_H v^2_u M_T$$

where $v = 174$ GeV and the subscript $u$ is present only in the SUSY version of the model.

2.1 Decay rates and CP-asymmetry

From now on we work explicitly with the non-supersymmetric version of the model. The tree-level triplet decay rates are

$$\Gamma(T \rightarrow LL) = \frac{M_T}{16\pi} \text{Tr} \lambda_L \lambda_L^\dagger = B_L \Gamma_T, \quad \Gamma(T \rightarrow H \bar{H}) = \frac{M_T}{16\pi} \lambda_H \lambda_H^\dagger = B_H \Gamma_T,$$

where $B_L$ and $B_H$ are the tree-level branching ratios to leptons and Higgs doublets, respectively, and $\Gamma_T$ is the total triplet decay width. Assuming that these are the only decay modes, i.e. $B_L + B_H = 1$, and taking into account CPT-invariance, a single parameter $\varepsilon_L$ determines CP-violation in $T, \bar{T}$ decays to be:

$$\Gamma(T \rightarrow LL) = \Gamma_T (B_L + \varepsilon_L/2), \quad \Gamma(T \rightarrow H \bar{H}) = \Gamma_T (B_H - \varepsilon_L/2),$$

$$\Gamma(T \rightarrow \bar{L} \bar{L}) = \Gamma_T (B_L - \varepsilon_L/2), \quad \Gamma(T \rightarrow H \bar{H}) = \Gamma_T (B_H + \varepsilon_L/2),$$

(5)
Figure 1: One-loop diagrams contributing to the asymmetry in scalar triplet decays.

where $\varepsilon_L$ is the CP-asymmetry (i.e. the average lepton number produced per decay):

$$
\varepsilon_L \equiv \frac{2 \Gamma(T \to LL) - \Gamma(T \to \bar{LL})}{\Gamma_T + \Gamma_{\bar{T}}},
$$

(6)

The overall factor 2 arises because $\bar{T} \to LL$ generates 2 leptons. Defining, as usual, the efficiency factor to be unity in the limit where the triplets decay strongly out-of-equilibrium, the lepton to photon number density ratio produced is

$$
n_L/n_\gamma \equiv \varepsilon_L \eta \frac{n_T + n_{\bar{T}}}{n_\gamma} \bigg|_{T \gg M_T},
$$

(7)

where $n_T$ is the total triplet number density $n_T = n_{T^-} + n_{T^0} + n_{T^0}$. After partial conversion of the lepton asymmetry to a baryon asymmetry by sphalerons this leads to

$$
n_B/n_\gamma = -0.029\varepsilon_L \eta.
$$

(8)

One triplet $T$ alone can mediate the whole observed light neutrino mass matrix via eq. (3) (see e.g. [14] for models of this type). However, in this case the CP-asymmetry $\varepsilon_L$ is generated only at higher loops and is highly suppressed. To get a sizable CP-asymmetry extra couplings are needed. The minimal option is that the CP-asymmetry is generated by extra contributions to neutrino masses, mediated e.g. by heavier right-handed neutrinos (fig. 1a [15, 10]), or by additional heavier triplets (fig. 1b [16, 9]) or by any other heavier particle which induces the dimension-5 operator as shown in fig. 1c. In this case the neutrino mass $m_\nu$ is given by the sum of the triplet contribution $m_T$, plus an extra contribution $m_H$ mediated by these $H$eavier particles: $m_\nu = m_T + m_H$. Assuming that these particles are substantially heavier than the scalar triplet, the neutrino mass contribution to the CP-asymmetry is

$$
|\varepsilon| = \frac{1}{4\pi} \frac{M_T}{v^2} \sqrt{B_L B_H} \frac{|\text{Im Tr} m_T^\dagger m_H|}{\Tilde{m}_T^2},
$$

(9)

where $\Tilde{m}_T^2 \equiv \text{Tr} m_T^\dagger m_T$. We stress that, as for the decay of a right-handed neutrino [17], this result is independent of the nature of the heavier particle contributing to the neutrino masses (fig. 1a or fig. 1b or whatever) because the heavier particle effects on the asymmetries can be fully encoded in $m_H$, i.e. in term of the unique dimension-5 neutrino mass operator they induce (fig. 1c).

We remind that when the lightest particle is a right-handed neutrino $N_1$ the CP-asymmetry in its decays is given by an analogous formula [18, 19]:

$$
|\varepsilon_1| = \frac{3}{16\pi} \frac{M_N}{v^2} \frac{|\text{Im Tr} m_1^\dagger m_{H_1}|}{\Tilde{m}_1} \leq \frac{3}{16\pi} \frac{M_1}{v^2} (m_{\nu_3} - m_\eta),
$$

(10)

\footnote{In the case of fig. 1a, our result in eq. (10) differs by a factor 1/2 from the first calculation performed in [10].}

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where, to make contact with the standard notation, the contribution to the neutrino mass matrix mediated by $N_1$ has been denoted as $\tilde{m}_1$. The bound of eq. (10) holds assuming that the heavier particles are two right-handed neutrinos [19]. In the case of generic heavy particles $m_{\nu_3} - m_{\nu_1}$ gets replaced by $m_{\nu_3}$.

In a similar way one obtains an upper bound on the neutrino mass contribution to the triplet CP-asymmetry of eq. (9):

$$|\varepsilon_L| \leq \frac{1}{4\pi} \frac{M_T}{v^2} \sqrt{B_L B_H} \Sigma_{m_{\nu_i}^2}.$$  \hspace{1cm} (11)

The result is different from the singlet case because $m_T$ is a generic matrix, while $\tilde{m}_1$ has rank 1. As a result $\varepsilon_L$ increases for larger $m_{\nu_i}$ [10], unlike $\varepsilon_1$ which decreases for larger quasi-degenerate neutrino masses. If extra information on how $m_p$ decomposes as the sum of $m_T$ and $m_H$ is available (as e.g. happens when considering particular neutrino mass models) the bound (11) can be strengthened, by replacing its last factor with $\min(\Sigma_{m_{\nu_i}^2}, \tilde{m}_H^2)$ where $\tilde{m}_H^2 \equiv \text{Tr} m_H^T m_H$.

When all terms in the effective Lagrangian are perturbative, this CP asymmetry is smaller (often much smaller) than the generic absolute maximal value allowed by unitarity (i.e. by demanding positivity of all decay widths in eq. (5)):

$$|\varepsilon_L| < 2 \min(B_L, B_H).$$  \hspace{1cm} (12)

Complex soft terms in supersymmetric triplet models are a concrete example of an extra source of CP-asymmetry that (unlike the neutrino mass contribution) is not suppressed for small $M_T$ [20, 21]. Alternatively one can add extra terms to the triplet Lagrangian, obtaining more complex phases.

2.2 Boltzmann equations

We denote with $n_p$ the number density of the type ‘p’ particles. Boltzmann equations describe the evolution as function of $z \equiv M_T/T$ of the total $T, \bar{T}$ density $\Sigma_T = (n_T + n_{\bar{T}})/s$ and of the asymmetries $\Delta_p = (n_p - n_{\bar{p}})/s$ stored in $p = \{T, L, H\}$. They are:

$$sHz \frac{d\Sigma_T}{dz} = -\left(\frac{\Sigma_T}{\Sigma_{eq}} - 1\right)\gamma_D - 2\left(\frac{\Sigma_T}{\Sigma_{eq}} - 1\right)\gamma_A,$$  \hspace{1cm} (13a)

$$sHz \frac{d\Delta_L}{dz} = X - 2\gamma_D B_L (\frac{\Delta_L}{\Sigma_{eq}} + \frac{\Delta_T}{\Sigma_{eq}}),$$  \hspace{1cm} (13b)

\hspace{1cm} (4)
\[
\begin{align*}
\text{s}H\frac{d\Delta H}{dz} & = X - 2\gamma_D B_H(\frac{\Delta H}{\gamma_{eq}^H} - \frac{\Delta T}{\gamma_{eq}^T}), \\
\text{s}H\frac{d\Delta T}{dz} & = -\gamma_D \left(\frac{\Delta T}{\gamma_{eq}^T} + B_L \frac{\Delta L}{\gamma_{eq}^L} - B_H \frac{\Delta H}{\gamma_{eq}^H}\right),
\end{align*}
\]

where \( H \) is the Hubble constant at temperature \( T \), \( s \) is the total entropy density, \( Y_X = n_X/s \), a suffix \( eq \) denotes equilibrium values, \( \gamma_P \) is the space-time density of type '\( P \)' processes computed in thermal equilibrium, and

\[
X = \gamma_D \varepsilon_L (\frac{\Sigma_T}{\Sigma_{eq}^T} - 1) - 2(\frac{\Delta L}{\gamma_{eq}^L} + \frac{\Delta H}{\gamma_{eq}^H})(\gamma_{T1} + \gamma_{T2}).
\]

Notice that because of hypercharge (or electric charge) conservation only three out of the four Boltzmann equations are independent. There exists a sum rule

\[
2\Delta_T + \Delta_H - \Delta_L = 0,
\]

satisfied by eqs (13). An important comment to be made here is that, since triplets are not self-conjugated (unlike right-handed neutrinos), there is a Boltzmann equation (13d) for \( \Delta_T \). As we discuss later, this structure of Boltzmann equations allows new effects which are absent in the heavy neutrino leptogenesis.

The relevant processes contributing to triplet leptogenesis are:

- Decays and inverse decays. \( \gamma_D \) is the total decay space-time density of \( T \) plus \( \bar{T} \) decays, given by the usual expression:

\[
\gamma_D = s\Gamma_T \Sigma_{eq}^T K_1(z)/K_2(z),
\]

where \( K_{1,2} \) are Bessel functions.

- \( \Delta T = 2 \) scatterings. \( \gamma_A \) is the space-time density of the SU(2) \( \otimes U(1)_Y \) gauge scatterings \( \bar{T}T \rightarrow \text{SM particles} \) shown in fig. 2. This is a new effect not present with right-handed neutrinos. The contributions of the various final states to the reduced cross section\(^2\) \( \tilde{\sigma}_A \) are

\[
\begin{align*}
\tilde{\sigma}_A(T\bar{T} \rightarrow F\bar{F}) & = \frac{6g_2^4 + 5g_Y^4}{2\pi} r^3, \\
\tilde{\sigma}_A(T\bar{T} \rightarrow H\bar{H}) & = \frac{g_2^4 + g_Y^4/2}{8\pi} r^3, \\
\tilde{\sigma}_A(T\bar{T} \rightarrow W^aW^b) & = \frac{g_2^4}{\pi} \left[ r(5 + 34/x) - \frac{24}{x^2}(x - 1) \ln \frac{1 + r}{1 - r} \right], \\
\tilde{\sigma}_A(T\bar{T} \rightarrow YY,WW) & = \frac{3g_2^4(g_2^2 + 4g_Y^2)}{2\pi} \left[ r(1 + 4/x) - \frac{4}{x}(x - 2) \ln \frac{1 + r}{1 - r} \right],
\end{align*}
\]

where \( F \) denotes SM fermions, \( r = \sqrt{1 - 4/x} \) and \( x = s/M_T^2 \). The low-energy behavior \( (\tilde{\sigma}_A \propto r^3 \) in the first two cases and \( \tilde{\sigma}_A \propto r \) in the last two cases) is dictated by conservation of angular momentum. Notice that by summing over SU(2) \( L \) indices we include all ‘coannihilation’ processes among different \( X \) components.

\(^2\)We remind that reduced cross sections for \( \bar{T} \rightarrow 2 \) are defined as \( \tilde{\sigma} = \sum\int dt |A|^2/8\pi s \) where here \( s,t \) are the usual Mandelstam variables and the sum runs over initial and final spins and gauge indices. The reaction densities are obtained as

\[
\gamma = \frac{T}{64\pi^2} \int_{s_{min}}^{\infty} ds s^{1/2} K_1\left(\frac{s}{T}\right) \tilde{\sigma}(s).
\]
• $\Delta L = 2$ scatterings. Unlike in the singlet fermion case, the scalar triplet generates the $LL \leftrightarrow \bar{H}H$ density rate $\gamma_{T_s}$ only by $s$-channel exchange; and generates the $LH \leftrightarrow \bar{L}H$ density rate $\gamma_{T_l}$ only by $t$-channel exchange. The reduced cross sections are

$$
\hat{\sigma}_{T_s} = \frac{3xM_T^2}{4\pi v^4} \left[ \frac{M_T}{1-x} + m_H \right]^2, \quad (17a)
$$

$$
\hat{\sigma}_{T_l} = \frac{3M_T^2}{4\pi v^4} \left[ m_H^2 x + 4m_H M_T \left( 1 - \frac{\ln(1+x)}{x} \right) + 2M_T^2 \left( -\frac{1}{1+x} + \frac{\ln(1+x)}{x} \right) \right]. \quad (17b)
$$

Note that in $\hat{\sigma}_{T_s}$ we included an extra factor 2 by hand which we took out in the coefficient of $\hat{\sigma}_{T_s}$ in eq. (14), so that $\hat{\sigma}_{T_s}$ becomes equal to $\hat{\sigma}_{T_l}$ in the low energy limit, where neutrino masses encode all $L$-violating effects:

$$
\hat{\sigma}_{T_s,T_l} \simeq \frac{m_\nu^2 3s}{v^2 4\pi}. \quad (18)
$$

In Boltzmann equations one must subtract from $\gamma_{T_s}$ the contribution due to on-shell $T$-exchange, already taken into account by successive decays and inverse-decays. The subtracted reaction density is $\gamma_{T_s}^{\text{sub}} \equiv \gamma_{T_s} - B_L B_H \gamma_D$ and can be more conveniently computed by replacing the $T$ propagator with its off-shell part, as described in [5]. Once this is done, $\gamma_{T_s}^{\text{sub}}, \gamma_{T_l} \ll \gamma_D$ at $T \sim M_T$, unless the couplings $\lambda_{L,H}$ are big enough that $\gamma_{T_s,T_l}$ are sizable, producing a strong wash-out of the baryon asymmetry.

To conclude, we comment on various small additional effects. We neglect all processes that give corrections of relative order $\alpha \lesssim$ few %. We included RGE corrections to gauge couplings and neutrino masses: the result roughly is $m_\nu(\text{High scale}) \sim (1.2 \div 1.3)m_\nu$ [22, 5]. We assumed that the lepton asymmetry is concentrated in a single flavour (this e.g. typically happens if $m_T \approx m_\nu$ and neutrinos are hierarchical): to fully include flavor one needs to evolve a $3 \times 3$ density matrix of lepton asymmetries as described in [22]. At $T \lesssim 10^{11}$ GeV sphalerons and SM Yukawa couplings redistribute the asymmetries to left-handed quarks and right-handed fermions respectively. These effects can be taken into account inserting appropriate $O(1)$ redistribution factors (see e.g. [22]); apart from generating the baryon asymmetry of eq. [5], they have small impact on the dynamics of leptogenesis.

## 3 Results for triplet scalar leptogenesis

The efficiency $\eta$ depends on 3 parameters which can be chosen to be $M_T$, $\lambda_L^2 \equiv \text{Tr}(\lambda_L \lambda_L^\dagger)$ and $\tilde{m}_T$, the contribution to neutrino masses mediated by triplet exchange. We choose $\lambda_L$ because in supersymmetric models it controls the renormalization induced Lepton Flavour Violating (LFV) signals due to triplet interactions. If $m_T$ dominates neutrino masses, these signals are of Minimal Flavour Violation [23] type; if furthermore neutrinos are hierarchical then $\tilde{m}_T = (\Delta m^2_{\text{atm}})^{1/2} \equiv m_{\text{atm}} \approx 0.05$ eV.

Boltzmann equations for leptogenesis in scalar triplet decays show two main qualitative differences with respect to the well known case of right-handed neutrinos. Firstly, gauge scatterings keep the triplet abundance $\Sigma_T$ close to thermal equilibrium such that the final lepton asymmetry does not depend on the initial conditions. This effect is present also in the case of leptogenesis from Majorana fermion triplets [12], and tends to reduce the efficiency. It affects the efficiency in a negligible way only if the decay rate is much faster than the expansion rate because in this case the triplets decay before annihilating. Secondly, unlike for Majorana triplets, due to the fact that the scalar triplets have two independent types of decay, and due to the related fact that there is one more Boltzmann equation, the wash-out from decay can be avoided even if the decay is much faster than the expansion rate. Lepton
number is violated by the contemporaneous presence of $\lambda_L$ and $\lambda_H$, so that the lepton asymmetry is washed-out only when both partial decay rates to leptons and Higgses are faster than the expansion rate. Otherwise, even for a fast total decay rate, a quasi maximal efficiency can be obtained in large portions of the parameter space.

To demonstrate how a large efficiency $\eta \sim 1$ arises, we plot in fig. 3 the interaction rates as function of temperature (left panel) as well as the evolution of the abundances of $\Sigma_T$, $\Sigma_T - \Sigma_T^{eq}$, $\Delta_L$, $\Delta_H$, $\Delta_T$. Asymmetries are plotted in units of $\varepsilon_L$. We take $M_T = 10^{10}$ GeV, $\tilde{m}_T = (\Delta m^2_{atm})^{1/2} \approx 0.05$ eV, $\lambda_L = 0.1$ giving $\lambda_H \approx 2 \cdot 10^{-4}$ and efficiency $\eta \approx 0.38$.

Let us present a more technical explanation of the behavior discussed above. In general, Boltzmann equations reduce to thermal equilibrium (giving a vanishingly small efficiency $\eta$) when the source terms in their right-handed sides have coefficients much larger than the expansion rate. From their explicit form one can see that when $B_L \rightarrow 0$ or when $B_H \rightarrow 0$ the combinations of $\Delta_L$, $\Delta_H$, $\Delta_T$ that are forced to vanish become linearly dependent, so that one combination gets not washed out and can store a large lepton asymmetry. This behavior is possible because $B_L$ and $B_H$ can be different from each other, which results in the additional Boltzmann equation, eq. (13d). We can analytically explain the

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An aside comment: Boltzmann equations for decays of right-handed sneutrinos have a similar form as leptogenesis from decays of scalar triplets. In the sneutrino case the new effect discussed here is typically negligible, because unbroken
Figure 4: Iso-curves of the efficiency $\eta$ in the $(\lambda_L, M_T)$ plane at fixed $\tilde{m}_T = 0.05\text{ eV}$ (left panel) and $\tilde{m}_T = 10^{-3}\text{ eV}$ (right panel). The diagonal line corresponds to $\text{BR}(T \to LL) = \text{BR}(T \to HH) = 1/2$ and shading covers regions with $\lambda_H > 1$.

numerical results in fig. 3. In this example $\Gamma(\tilde{T} \to LL) \gg H$ and $\Gamma(T \to HH) \lesssim H$. Then:

a) Eq. 13a is dominated by the $\gamma_D$ term which puts $\Sigma_T$ close to $\Sigma^e_T$; gauge scatterings have a negligible effect.

b) The washout terms in eq. 13c can be neglected thanks to $B_H \ll 1$, and a large $\Delta_H$ asymmetry develops: it does not depend on $\gamma_D$, because it is proportional to $\gamma_D$ times $\Sigma_T/\Sigma^e_T - 1$ (which is inversely proportional to $\gamma_D$). Indeed the approximate analytical solution is $\Delta_H(T) \approx \varepsilon_L[\Sigma^e_T(T \gg M_T) - \Sigma^e_T(T)]$ where $\Sigma^e_T(T \gg M_T)$ is a constant and $\Sigma^e_T(T \ll M_T) \approx 0$.

c) Due to the $2\Delta_T + \Delta_H - \Delta_L = 0$ sum rule, an equally large $2\Delta_T - \Delta_L$ asymmetry is also produced.

d) Subsequently, as all triplets decay, $\Delta_T$ goes to zero so that the large $2\Delta_T - \Delta_L$ becomes a large $\Delta_L$ asymmetry, with efficiency of order unity.

To show the dependence of the efficiency in the 3 parameters $M_T$, $\lambda_L$ and $\tilde{m}_T$, various isocurve plots can be considered. Fig. 4 shows $\eta$ in the $(\lambda_L, M_T)$ plane for $\tilde{m}_T = m_{\text{atm}} = 0.05\text{ eV}$ (left panel) and for $\tilde{m}_T = 10^{-3}\text{ eV}$ (right panel). Fig. 5 shows $\eta$ in the $(\tilde{m}_T, M_T)$ plane for $\lambda_L = 0.1$ (left panel) and $\lambda_L = 0.001$ (right panel). For $B_L = B_H = 1/2$ (represented in the plot by the diagonal line) the efficiency is dominantly determined by $\gamma_D$ and $\gamma_A$: $\Delta L = 2$ scatterings are relevant only above $\sim 10^{14}\text{ GeV})$. By comparing their rates with the Hubble rate

$$\frac{\gamma_D}{H n_\gamma} \bigg|_{T \approx M_T} \approx \frac{\Gamma_T}{H} \bigg|_{T \approx M_T} \approx \frac{\tilde{m}_T}{10^{-3}\text{ eV}},$$

$$\frac{\gamma_A}{H n_\gamma} \bigg|_{T \approx M_T} \approx \frac{10^{14}\text{ GeV}}{M_T},$$

supersymmetry forces equal branching ratios for the 2 different sneutrino decay modes.
one can understand our numerical results: for $\tilde{m}_T = 0.05$ eV decays are more important than gauge scatterings and $\eta \approx 10^{-3}$ almost independently on $M_T$; the reverse happens for $\tilde{m}_T = 10^{-3}$ eV and $\eta$ decreases with $M_T$. For $B_L \gg B_H$ ($B_H \gg B_L$), the efficiency is larger thanks to $\gamma_{DL} < H n_\gamma$ ($\gamma_{DH} < H n_\gamma$). In conclusion, numerical results shows the features anticipated above: i.) The efficiency is maximal, $\eta \sim 1$, when either $B_L \gg B_H$ or $B_H \gg B_L$ and minimal when $B_L = B_H = 1/2$. ii.) Even when $B_L = B_H = 1/2$ the efficiency does not depend much on $M_T$, especially for larger $\tilde{m}_T \gtrsim 10^{-3}$ eV, remaining relatively large even for $M_T \sim$ TeV.

We now come to the calculation of the produced baryon asymmetry. The values of $\varepsilon_L$ necessary for obtaining the observed baryon asymmetry, $n_B/n_\gamma \approx 6.2 \cdot 10^{-10}$, can be obtained straightforwardly from fig.s 4, 5 using eq. (8). Notice that, while the efficiency is maximal when $B_L \rightarrow 0$ or $B_H \rightarrow 0$, in these limits the CP-asymmetry gets suppressed, in a way which depends on its origin. In the minimal model $\varepsilon_L \propto \sqrt{B_L B_H}$, see eq. (9). To illustrate this fact, fig. 6 shows the values of $|\varepsilon_L|/\sqrt{4B_L B_H}$ needed for successful leptogenesis for $\tilde{m}_T = m_{atm}$ (left panel) and $\tilde{m}_T = 10^{-3}$ eV (right panel). Using the upper bound on $|\varepsilon_L|$ of eq. (10) (valid assuming the minimal model where $\varepsilon_L$ is generated only by other sources of neutrino masses much heavier than the scalar triplet; we evaluate it assuming hierarchical neutrino masses) restricts the ranges of $M_T$ and $\lambda_L$ which can lead to successful leptogenesis to the region shaded in dark green. This allowed region covers a wide range of $\lambda_L$ values: the decrease of the maximal $|\varepsilon_L|$ when $B_L \rightarrow 0$ or $B_H \rightarrow 0$ is roughly compensated by the increase in the efficiency, such that successful leptogenesis does not need $B_L \approx B_H$. In this minimal model $\varepsilon_L \propto M_T$, so that successful leptogenesis needs a heavy enough triplet. Assuming a hierarchical spectrum of light neutrinos (i.e. setting $\sum_i m_{\nu_i}^2 = \Delta m_{atm}^2$ in eq. (11)) we find the model-independent bounds:

$$M_T > 2.8 \cdot 10^{10} \text{ GeV} \quad (\tilde{m}_T = 0.001 \text{ eV});$$

$$M_T > 1.3 \cdot 10^{11} \text{ GeV} \quad (\tilde{m}_T = 0.05 \text{ eV}).$$

(20a)  

(20b)
A stronger bound on the CP asymmetry holds in models where heavier sources of neutrino masses are predicted to give a small contribution $m_H$. A too small $m_H$ prevents successful leptogenesis, and the bounds of eqs. (20b) become stronger, by a factor well approximated by $(\sum_i m_{\nu_i}^2)^{1/2}/\tilde{m}_H$ where $\tilde{m}_H^2 \equiv \text{Tr} m_H^\dagger m_H$. For example, for $\tilde{m}_H = (\Delta m_{\text{sun}}^2)^{1/2} = 0.007$ eV this gives: $M_T > 8 \cdot 10^{11}$ GeV (with $\tilde{m}_T = 0.05$ eV). These precise constraints can be compared to the estimated constraint: $M_T > 10^{11-12}$ GeV [9, 10, 11].

For larger values of $\sum_i m_{\nu_i}^2$ the constraints of eqs. (20b) get relaxed by a $(\Delta m_{\text{atm}}^2/\sum_i m_{\nu_i}^2)^{1/2}$ factor: i.e. by one order of magnitude for quasi-degenerate neutrinos with $m_{\nu} \approx 0.5$ eV $\approx 10 m_{\text{atm}}$. In fact, by increasing the neutrino mass scale keeping $m_T$ fixed, the asymmetry increases but the efficiency remains unchanged [10].

We expect that adding supersymmetry does not significantly affect our results. More precisely, the interaction rates and the CP-asymmetry generated by neutrino masses become $O(2)$ times bigger, and the numerical coefficient in eq. (8) remains almost unchanged. This means that the constraint on $M_T$ in eq. (20a) conflicts with the gravitino constraint on the maximal big-bang temperature [24]. From this point of view, triplet leptogenesis is not better than the other two mechanisms that can mediate tree-level masses. In all 3 cases this incompatibly can be circumvented in many ways; in particular extra sources of CP-violation unrelated to neutrino masses (and related e.g. to soft supersymmetry-breaking terms) allow much larger asymmetries and the lower bound on $M_T$ can be considerably relaxed. In general the CP-asymmetry is bounded only by eq. (12), that does not depend on $M_T$: a light triplet can produce successful leptogenesis because its efficiency remains large enough. Fig. 6 shows that even $M_T \sim \text{TeV}$ is allowed: the region allowed by the unitarity bound is shaded in light green.

Figure 6: Iso-curves of value of $\varepsilon_L/\sqrt{4 B_L B_H}$ needed to have successful leptogenesis in the $(\lambda_L, M_T)$ plane for two values of $\tilde{m}_T$ as indicated in the figure. The dark-green (grey) region is the allowed region assuming that the CP-asymmetry arises from heavier sources of neutrino masses, i.e. fulfilling eq. (17). The light green (grey) region is obtained assuming that the CP-asymmetry is bounded only by unitarity, eq. (12).
Notice that the fact that relatively large values of $\lambda_L$ are compatible with thermal triplet leptogenesis, see fig. 4, has an interesting consequence in SUSY models: RGE corrections imprint $\lambda_L^2$ in slepton masses. If $\lambda_L$ violates lepton flavour this effect induces LFV charged-lepton processes with possibly detectable rates. As usual, the predicted LFV rates depend also on sparticle masses which can be measured at colliders. Taking into account naturalness considerations and experimental bounds and hints, we give our numerical examples in the leading log approximation for $m_0 = M_{1/2} = 200 \text{GeV}$, $A_0 = 0$, $\tan \beta = 5$. The most relevant effects are then approximately given by (see also [25])

$$\text{BR}(\mu \rightarrow e\gamma) \approx 1.2 \cdot 10^{-4} r |\lambda_L^\dagger \cdot \lambda_L|^2 \ln^2 \frac{M_{\text{GUT}}}{M_T},$$

$$\text{BR}(\tau \rightarrow \mu\gamma) \approx 2 \cdot 10^{-5} r |\lambda_L^\dagger \cdot \lambda_L|^2 \ln^2 \frac{M_{\text{GUT}}}{M_T}$$

where $r \approx (\tan \beta/5)^2(200 \text{GeV}/M_{\text{SUSY}})^4$ equals 1 at our reference point. $\lambda_L^\dagger > 10^{-1}$ gives rise to detectable $\tau \rightarrow \mu\gamma$ rates, $\lambda_L^\dagger > 10^{-2}$ gives rise to detectable $\mu \rightarrow e\gamma$ rates.

4 Conclusions

We computed the efficiency factor $\eta$ that summarizes the dynamics of scalar triplet thermal leptogenesis. Despite the presence of gauge interactions, that tend to maintain the triplet abundancy very close to thermal equilibrium, one can have even maximal efficiency, $\eta \sim 1$, for any triplet mass $M_T$, even $M_T \sim \text{TeV}$. This happens when i) one of the decay rates ($T \rightarrow L\overline{L}$ or $T \rightarrow H H$) is faster than the annihilation rate; ii) the other one is slower than the expansion rate. Thanks to i) gauge scatterings are ineffective: triplets decay before annihilating. Thanks to ii) fast decays do not produce a strong washout of the lepton asymmetry (and consequently a small efficiency $\eta$), because lepton number is violated only by the contemporaneous presence of the two $T \rightarrow L\overline{L}$ and $T \rightarrow H H$ processes. Our numerical results are obtained by writing and solving the full set of Boltzmann equations, eq.s (11).

We obtained in eq. (9) an expression for the CP asymmetry in triplet decays, assuming that it is related to neutrino masses. Neutrino masses $m_{\nu} = m_T + m_H$ can be written as the sum of the triplet contribution $m_T$, plus the contribution $m_H$ from any other sources. The suffix $H$ indicates that we assume that other sources are much heavier than $M_T$, such that at energies $E \lesssim M_T$ all their effects are encoded in $m_H$. This assumption allows us to derive an upper bound on the triplet CP-asymmetry, eq. (13), analogous (but not equal) to the bound that holds in the right-handed neutrino case.

Combining $\eta$ with the maximal CP-asymmetry generated by neutrino masses allows us to derive a lower bound on the triplet mass $M_T$ which varies between $10^9 \text{ GeV}$ and $10^{12} \text{ GeV}$, depending on the neutrino mass contribution of both triplet and heavier source of neutrino mass, see eq.s (21a). This also leads on lower and upper bounds on the triplet Yukawa coupling to leptons, that in supersymmetric models induces LFV processes such as $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. Neutrino masses depend on the product $\lambda_L \lambda_H$ of triplet couplings to leptons and Higgs. Leptogenesis separately depends on $\lambda_L$ and $\lambda_H$, but adds no more information: the region $\lambda_L \sim \lambda_H$ is not singlet out, due to the unexpected behavior of the efficiency. As a result large values of $\lambda_L$, and therefore large rates of LFV processes, are allowed.

By relaxing the assumption on the origin of the CP-asymmetry, it can reach larger values (bounded only by unitarity) that can be realized e.g. in supersymmetric models with complex soft terms (‘soft leptogenesis’), especially if the triplet mass is not much heavier than the scale of SUSY breaking. In this context the competing effect of the CP-asymmetry and of the efficiency favors $\lambda_L \sim \lambda_H$, allowing successful triplet thermal leptogenesis even at $M_T \sim \text{TeV}$ provided that $\lambda_L/\lambda_H \lesssim (0.1 \div 10)$. 11
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