Random Access in Distributed Source Coding

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Abstract

The lossless compression of a single source $X^n$ was recently shown to be achievable with a notion of strong locality; any $X_i$ can be decoded from a constant number of compressed bits, with a vanishing in $n$ probability of error. In contrast with the single source setup, we show that for two separately encoded sources $(X^n, Y^n)$, lossless compression and strong locality is generally not possible. More precisely, we show that for the class of “confusable” sources strong locality cannot be achieved whenever one of the sources is compressed below its entropy. In this case, irrespectively of $n$, the probability of error of decoding any $(X_i, Y_i)$ is lower bounded by $2^{-O(d_{loc})}$, where $d_{loc}$ denotes the number of compressed bits accessed by the local decoder. Conversely, if the source is not confusable, strong locality is possible even if one of the sources is compressed below its entropy. Results extend to any number of sources.

I. INTRODUCTION

The amount of data generated in many applications such as astronomy and genomics has highlighted the growing need for compression schemes that allows to interact and manipulate data directly in the compressed domain [1], [2], [3], [4], [5], [6], [7]. Indeed, traditional compression schemes such as Lempel-Ziv [8], [9] are suboptimal in this regard since the recovery of even a single message symbol necessitates to decompress the entire dataset. Accordingly, this paper focuses on providing random access in the compressed domain, where short fragments of data can be recovered without accessing the entire compressed sequence.

For the single source setup, [10], [11] showed that a strong notion of locality holds: for any rate above entropy there exists an encoder and a local decoder which probes a constant number $d_{loc}$ (independent of $n$) of compressed symbols, and yet achieves vanishing error probability as $n$ grows. Note that the naive concatenation scheme where the source is decomposed into $n/b$ consecutive blocks of some size $b$, each of which independently compressed at a desired rate $R > H(X)$, is not strongly local. Indeed, any $X_i$ is independent of all $n/b$ sub-block codewords, except one which reveals $b$ message symbols, and $X_i$ in particular. Hence, only “weak” locality holds in the sense that for the local decoder error probability to vanish, the number $d_{loc}$ of probed symbols—here equal to the sub-block codeword length $b \cdot R$—must grow with $n$.

In this paper we address the question whether strong locality extends to the Slepian-Wolf distributed compression of two sources $X^n$ and $Y^n$: given $(R_1, R_2)$ within the Slepian-Wolf rate region, is it possible to design a fix-length compressor and a local decompressor with $d_{loc} = O(1)$ and whose error probability is $o(1)$ as $n$ grows?

Obviously, if each source is compressed above its entropy then strong locality holds simply by duplicating the results of [10], [11] separately for each of the sources. Note also that the naive concatenation scheme—wherein $(X^n, Y^n)$ is decomposed into consecutive sub-blocks of size $b$ each of which encoded via Slepian-Wolf coding—achieves weak locality at any $(R_1, R_2)$ within the Slepian-Wolf rate region. So the interesting question is whether strong locality holds when at least one of the sources is compressed below its entropy.

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Our main result says that strong locality is generally impossible. More precisely, suppose $p_{XY}$ is "confusible," in the sense that, for every $x_1$ and $x_2$ in $\mathcal{X}$ there exists $y \in \mathcal{Y}$ such that $p_{XY}(x_1, y) > 0$ and $p_{XY}(x_2, y) > 0$. In this case, if $R_1 < H(X)$, the probability of wrongly decoding any particular $(X_i, Y_i)$ is lower bounded by $2^{-\Theta(d_{loc})}$. Moreover, this conclusion holds even if the decoder tries to decode only $X_i$ with the full cooperation of the $Y$-transmitter that provides $Y^n$ uncompressed. Conversely, if $p_{XY}$ is not confusible, then strong locality is possible for some $R_1 < H(X)$ and $R_2 = H(Y)$.

As a consequence, when the source is confusible, the naive scheme is order optimal in the tradeoff between local error probability and number of probes. However, observe that both the encoding and the decoding procedures of the naive scheme are tied to the sub-block length $b$ which governs the error probability of the local decoder. In particular, there is no point for the local decoder to probe more than $b$ bits; even if both codewords are entirely probed, that is $d_{loc} = n(R_1 + R_2)$, the error probability remains the same as if $d_{loc} = n(R_1 + R_2)$. Thus, to lower the error probability of the local decoder, the encoding procedure must be modified as well. We address this issue through a hierarchical compression scheme whose local decoder achieves an error probability that decreases as $d_{loc}$ increases, without modifying the encoding. Specifically, for any $(R_1, R_2)$ within the Slepian-Wolf rate region, and for every $1 > \eta > 2^{-2O(\log n)}$, the local decoder achieves $P_{e}^{(\text{loc})} \leq \eta$ with $d_{loc} = \text{poly}(\log(1/\eta))$.

A. Literature on locally decodable compression

Local decoding has been studied extensively in the context of compressed data structures by the computer science community; see, e.g., [12], [13], [14], [15], [16], [17] and the references therein. Most of these results hold under the word-RAM model which assumes that operations (memory access, arithmetic operations) on $w$-bit words take constant time. The word size $w$ is typically chosen to be $\Theta(\log n)$ bits, motivated in part by on-chip type of applications where data transfer happens through a common memory bus for both data and addressing (hence $w = \Theta(\log n)$ bits), and partly by the fact that certain proof techniques work only when $w = \Omega(\log n)$.

In the word-RAM model, it is possible to compress any sequence to its empirical entropy and still be able to locally decode any message symbol in constant time [12], [13]. Most approaches modify the Lempel-Ziv class of algorithms to provide efficient local decodability [18], [19], [20]. Similar results also hold for compression of correlated data et al. [21], and efficient recovery of short substrings of the message [22], [23], [24], [18]. However, all of these schemes require the local decoder to probe at least $O(\log n)$ compressed bits to recover any source symbol.

In this work, the decoding cost is measured by the number of compressed bits that need to be accessed in order to recover a single source symbol, sometimes referred to as the local decodability [25], or the bit-probe complexity in the literature [26].

The problem of locally decodable source coding of random sequences was first studied by [27], [25]. They showed that any compressor with $d_{loc} = 2$ cannot achieve a rate below the trivial rate $\log |\mathcal{X}|$, and any linear source code that achieves $d_{loc} = \Theta(1)$ necessarily operates at a trivial compression rate ($R = 1$ for binary sources). Later, [10] showed that rate $H(X) + \varepsilon$ and local decodability $d_{loc} = \Theta(\frac{1}{\varepsilon} \log \frac{1}{\varepsilon})$ is achievable. They also provided a converse result for non-dyadic sources: $d_{loc} = \Omega(\log(1/\varepsilon))$ for any compression scheme that achieves rate $H(X) + \varepsilon$. A similar scheme was given in [11] for Markov sources and they could compress the source to the entropy rate and achieve $d_{loc}(1) = \Theta(\frac{1}{\varepsilon} \log \frac{1}{\varepsilon})$. A common feature of the code construction in both papers is the use of the bitvector compressor of Buhrman et al. [23] which in turn is a nonexplicit construction based on expander graphs.

All the above papers on the bit-probe model consider fixed-length block coding. Variable-length source coding was investigated by Pananjady and Courtade [29] who gave upper and lower bounds on the achievable rate for the compression of sparse sequences under local decodability constraints. The works [30], [31] considered simultaneous local decodability and update efficiency. In particular, [30] designed a compressor whose average-case local decodability (defined as the expected number of bits that need to be probed to recover any $X_i$) and the average-case update efficiency (the expected number of bits
that need to be read and written in order to update a single $X_i$ both scale as $O\left(\frac{1}{\epsilon^2} \log \frac{1}{\epsilon}\right)$. In fact, our scheme for distributed compression with locality is inspired by the multilevel compression scheme in [30]. The paper [31] designed a compression scheme whose worst-case local decodability and update efficiency scales as $O(\log \log n)$. More recently, [32] implemented a version of the concatenation scheme and evaluated its performance on practical datasets.

B. Paper organization

In Section II we introduce notions of localities and formally define the problem. In Section III we present our results. In Sections IV and V we prove the results, and in Section VI we discuss the extension to more than two sources. In Section VII we draw concluding remarks.

II. Preliminaries and Statement of the Problem

A. Distributed compression without locality (Slepian-Wolf coding)

Let $(X^n, Y^n)$ be $n$ independent copies of a pair of random variables $(X, Y) \sim p_{XY}$ defined over some alphabet $\mathcal{X} \times \mathcal{Y}$. Without loss of generality, we assume that $\mathcal{X} = \{x : p_X(x) > 0\}$ and $\mathcal{Y} = \{y : p_Y(y) > 0\}$. Sequences $X^n$ and $Y^n$ represent two sources of information separately encoded into binary codewords $C^n R_1 (X^n)$ and $C^n R_2 (Y^n)$ at rate $R_1$ and $R_2$, respectively. Upon receiving these codewords, a receiver outputs sources estimates $(\hat{X}^n, \hat{Y}^n)$ and makes an error with probability

$$P_e \stackrel{\text{def}}{=} \Pr[(\hat{X}^n, \hat{Y}^n) \neq (X^n, Y^n)].$$

The rate region is the closure of the set of rate pairs $(R_1, R_2)$ for which $P_e \to 0$ as $n \to \infty$, and is given by:

**Theorem 1 ([33], [34]).** The rate region of a source $p_{XY}$ is the set of pairs $(R_1, R_2)$ that satisfy

$$R_1 \geq H(X|Y)$$

$$R_2 \geq H(Y|X)$$

$$R_1 + R_2 \geq H(X, Y).$$

Moreover, for any $(R_1, R_2)$ in the interior of the rate region, and any $\epsilon > 0$, there exist a sequence of coding schemes operating at rates at most $R_1 + \epsilon$ and $R_2 + \epsilon$ such that

$$P_e \stackrel{\text{def}}{=} \Pr[(\hat{X}^n, \hat{Y}^n) \neq (X^n, Y^n)] \leq 2^{-n(E-\epsilon)},$$

where $E$ is a constant that depends on $R_1, R_2$ and $p_{XY}$.

B. Distributed compression with locality

1) Local decoder: Given encodings $C^n R_1 (X^n)$ and $C^n R_2 (Y^n)$, a local decoder takes as input $i \in [n]^{\dagger}$ and probes/reads a fixed set $\mathcal{I}_i$ of components from $C^n R_1 (X^n)$ and $C^n R_2 (Y^n)$, which we simply denote as $C_{\mathcal{I}_i}$. The worst-case local decodability and error probability are defined as

$$d_{\text{loc}} \stackrel{\text{def}}{=} \max_{1 \leq i \leq n} d(i),$$

where $d(i) \stackrel{\text{def}}{=} |\mathcal{I}_i|$, and

$$P_e^{(\text{loc})} \stackrel{\text{def}}{=} \max_{1 \leq i \leq n} \Pr[(\hat{X}_i, \hat{Y}_i) \neq (X_i, Y_i)].$$

$^{\dagger}[n] \stackrel{\text{def}}{=} \{1, 2, \ldots, n\}$

$^{\ddagger}$In the sequel, we will use the same $C_{\mathcal{I}_i}$ notation for both the two-source setup and the single source setup. For the former, $C_{\mathcal{I}_i}$ refers to the set of codewords components from $C^n R_1 (X^n)$ and $C^n R_2 (Y^n)$ used to identify $(X_i, Y_i)$, and for the latter $C_{\mathcal{I}_i}$ refers to the set of codeword components from $C^n R_2 (Y^n)$ to identify $X_i$ only.
A few words about \( \mathcal{I} \). Note first that \( \mathcal{I} \) is allowed to contain different sets of components from \( C^{nR_1} \) and \( C^{nR_2} \). It should be non-adaptively chosen, that is, conditioned on the index \( i \), set \( \mathcal{I} \) should be independent of \( (X^n, Y^n) \). Note that to \( d_{\text{loc}} \) adaptive queries correspond (at most) \( 2^{d_{\text{loc}}} \) non-adaptive queries. Therefore, a lower bound on the probability of error for nonadaptive local decoders (the main contribution of this paper) translates into a corresponding lower bound for adaptive decoders. Finally, notice that even though \( \mathcal{I} \) is non-adaptively chosen, it could still be a random set, in which case \( d(i) \) is defined as the essential supremum of \( |\mathcal{I}_i| \).

2) **Strong vs. weak locality:** A rate pair \((R_1, R_2)\) is said to be achievable with **strong locality** if

\[
P_e^{(\text{loc})} = o(1) \quad \text{and} \quad d_{\text{loc}} = \Theta(1) \quad \text{as} \quad n \to \infty.
\]

That is, by probing only a constant number (independent of \( n \)) of symbols, the error probability of the local decoder goes to zero as the blocklength increases. By contrast, \((R_1, R_2)\) is said to be achievable with **weak locality** if

\[
P_e^{(\text{loc})} = o(1) \quad \text{and} \quad d_{\text{loc}} = \omega(1) \quad \text{as} \quad n \to \infty.
\]

Weak locality is always achievable through the naive concatenation scheme that decomposes source sequences \( X^n \) and \( Y^n \) into consecutive blocks of size \( b \)

\[
X^b(j) \overset{\text{def}}{=} X_{(j-1)b+1}^{jb} = (X_{(j-1)b+1}, X_{(j-1)b+2}, \ldots, X_{jb})
\]

\[
Y^b(j) \overset{\text{def}}{=} Y_{(j-1)b+1}^{jb} = (Y_{(j-1)b+1}, Y_{(j-1)b+2}, \ldots, Y_{jb})
\]

for \( j = 1, 2, \ldots \) and compresses each block \((X^b(j), Y^b(j))\) independently using a Slepian-Wolf code operating at the desired \((R_1, R_2)\). Given \( i \in [n] \), the local decoder accesses block \( j = \lfloor i/b \rfloor \) of each of the sources (thereby reads \( b(R_1 + R_2) \) compressed bits), and outputs the estimates of the \( i \)-th bit of \( X^n \) and \( Y^n \). By letting \( d_{\text{loc}} = b(R_1 + R_2) \) in Theorem 1 we get:

**Corollary 1** (Naive concatenation). For any source \( p_{XY} \) and any \((R_1, R_2)\) in the interior of the rate region, the concatenation scheme achieves weak locality:

\[
P_e^{(\text{loc})} \leq 2^{-\Theta(d_{\text{loc}})}.
\]

C. **Statement of the problem**

By contrast with weak locality, whether strong locality is generally achievable is much less clear. In fact, it is only recently that strong locality was shown to be achievable for the single source setup at any lossless compression rate \( R > H(X) \) \cite{35, 36}. For the Slepian-Wolf setup at hand, this result implies that strong locality holds for any \((R_1, R_2)\) such that \( R_1 > H(X) \) and \( R_2 > H(Y) \). In this regime, sources can be encoded using the single source strongly local codes of \cite{35, 36}, separately for source \( X^n \) and source \( Y^n \)—and ignore dependency between \( X^n \) and \( Y^n \). Does this conclusion extend to the entire lossless rate region, when at least one of the sources is encoded at a rate below its entropy?

III. **Main results**

Our main result stated next answers the above question in the negative: if the source is “confusable”, strong locality is impossible whenever one of the sources is compressed below its entropy.

**Definition 1** (Source confusability). Source \( p_{XY} \) is said to be \( \mathcal{X} \)-confusable if for every \( x_1, x_2 \in \mathcal{X} \), there exists \( y \in \mathcal{Y} \) such that \( p_{XY}(x_1, y) > 0 \) and \( p_{XY}(x_2, y) > 0 \).

Hence, any source with full support, that is \( p_{XY}(x, y) > 0 \) for all \((x, y) \in \mathcal{X} \times \mathcal{Y} \), is both \( \mathcal{X} \)- and \( \mathcal{Y} \)-confusable. An example of an \( \mathcal{X} \)-confusable source which does not have full support is \( p_{XY} \) where \( p_X = \text{Bernoulli}(p) \), \( 0 < p < 1 \), and where \( p_{Y|X} \) is a \( Z \) channel with crossover parameter \( 0 < \varepsilon < 1 \). If
$p_{Y|X}$ is the perfect binary channel, then $p_{XY}$ is not confusable, and this is the only possibility if $\mathcal{X}$ and $\mathcal{Y}$ are binary.

The set of all codewords components probed by a local decoder to estimate $(X_i, Y_i)$, or only $X_i$ for the single source setup, is simply denoted as $C_{X_i}$.

**Theorem 2.** Suppose source $p_{XY}$ is $\mathcal{X}$-confusable. Suppose $X^n$ is encoded into codeword $C^nR_1$ with $R_1 < H(X)$. Then

$$\max_{1 \leq i \leq n} \Pr[\hat{X}_i(C_{X_i}, Y^n) \neq X_i] \geq 2^{-\Theta(d_{loc})},$$

where $\hat{X}_i(C_{X_i}, Y^n)$ denotes any estimator of source symbol $X_i$ given observations $C_{X_i}$ and $Y^n$, where $C_{X_i}$ denotes any subset of the components $C^nR_1$ with cardinality at most $d_{loc}$.

This result says that if the source is $\mathcal{X}$-confusable, then strong locality (even when restricted to the $X$-source only) is impossible whenever $R_1 < H(X)$; not even the full cooperation of the $Y$-transmitter through the transmission of the uncompressed source $Y^n$ allows to achieve strong locality. A particular version of this theorem for doubly symmetric sources, where $p_X$ is the Bernoulli($1/2$) distribution and where $p_{Y|X}$ corresponds to a BSC$(\rho)$ for some crossover parameter $0 < \rho < 1/2$, was proved in [37].

The $\mathcal{X}$-confusability property turns out to be necessary for Theorem 2 to hold:

**Theorem 3.** Suppose source $p_{XY}$ is not $\mathcal{X}$-confusable. Then, it is possible to achieve strong locality at some $R_1 < H(X)$ and $R_2 = H(Y)$.

**Proof:** If $p_{XY}$ is not $\mathcal{X}$-confusable, then there exists $x_1, x_2 \in \mathcal{X}$ such that, for any $y \in \mathcal{Y}$, either $p_{XY}(x_1, y) > 0$ or $p_{XY}(x_2, y) > 0$ (recall that without loss of generality $p_Y(y) > 0$ for any $y \in \mathcal{Y}$). Therefore, conditioned on $X \in \{x_1, x_2\}$, the knowledge of $Y$ reveals whether $X = x_1$ or $X = x_2$.

Let $\mathcal{X} = \{1, 2, \ldots, |\mathcal{X}|\}$, and suppose without loss of generality that $(x_1, x_2) = (1, 2)$. Define the new source $U^n$ over the reduced alphabet $\{2, 3, \ldots, |\mathcal{X}|\}$ as

$$U_i = \begin{cases} 2 & \text{if } X_i \in \{1, 2\} \\ X_i & \text{if } X_i = \{1, 2\}. \end{cases}$$

Clearly, $H(U) < H(X)$ and $(U, Y)$ determine $(X, Y)$. We can therefore compress $U^n$ and $Y^n$ independently at rates $R_1 = H(U) < H(X)$ and $R_2 = H(Y)$ using the compressors of [11], [10] to achieve strong locality. \hfill \square

From Corollary [1] the naive concatenation scheme achieves a local error probability that decays as $2^{-\Theta(d_{loc})}$, and this is order optimal by Theorem 2 for non-confusable sources. However, a weakness of the concatenation scheme is that its local decoding procedure is tied to a specific value of $d_{loc}$ which is equal to the sub-block length $b$. In particular, if $b = \Theta(1)$, then, because the concatenation scheme encodes each sub-block independently, it is impossible to recover $(X^n, Y^n)$ with vanishing probability of error as $n$ grows, even after reading the entire compressed sequences(!) Our second contribution is a compression scheme whose local decoder has an error probability that decreases with the number of probed symbols. The performance of this scheme is given in the following theorem:

**Theorem 4.** For any $(R_1, R_2)$ in the interior of the rate region, there exists a rate $(R_1, R_2)$ distributed compression scheme such that for every $1 > \eta > 2^{-2\Theta(\log n)}$ specified at the decoder, the local decoder probes $d_{loc} = \poly(\log(1/\eta))$ bits and achieves $P_{e}^{(loc)} \leq \eta$.

The above results easily generalize to more than two sources, see Section VI.

Note: The present paper differs from the paper [37] submitted to ISIT 2022 mainly in that it establishes the impossibility of strong locality (Theorem 2) for the most general class of sources (confusable sources), and not only for the specific class of doubly symmetric binary sources. In fact, the arguments used in [37] do not extend beyond sources with full-support. The arguments used here are not only more general, but also more direct than those in [37]. Theorem 3 is new and [37] contains mostly a sketch of the proof of
Theorem 4 Theorems 5, 6 and 7 that extend the above results to more than two sources (see Section VI) did not formally appear in [37].

IV. LOCAL DECODING ERROR PROBABILITY LOWER BOUND

In this section we prove Theorem 2.

A. Preliminaries

Lemma 1 (Reverse hypercontractivity [38]). Let $(V, W)$ be two random variables with full support distribution $p_{V, W}$ over some finite alphabet $\mathcal{V} \times \mathcal{W} - p_{V, W}(v, w) > 0$ for all $(v, w) \in \mathcal{V} \times \mathcal{W}$. Then, for every $A \subset \mathcal{V}^n$ and $B \subset \mathcal{W}^n$, we have

$$\Pr[V^n \in A, W^n \in B] \geq (\Pr[V^n \in A])^\alpha (\Pr[W^n \in B])^\beta$$

for some finite constants $\alpha, \beta$.\(^3\)

One key element in proving Theorem 2 is to consider the following coupling. Given $p_{X, Y}$, define random variable $\tilde{X}$ so that

$$X - Y - \tilde{X}$$

forms a Markov chain and so that\(^4\)

$$p_{X|Y} = p_{\tilde{X}|Y}.$$ Observe that if $p_{XY}$ is $\mathcal{X}$-confusable, then for any given $(x, \tilde{x}) \in \mathcal{X} \times \mathcal{X}$ there exists $y$, with $p_Y(y) > 0$ (recall that without loss of generality $p_Y(y) > 0$ for any $y \in \mathcal{Y}$), for which

$$p_{X,\tilde{X}}(x, \tilde{x}|y) = p_{X|Y}(x|y)p_{\tilde{X}|Y}(\tilde{x}|y) > 0.$$ Hence, we have:

Lemma 2. If $p_{XY}$ is $\mathcal{X}$-confusable, then $p_{X,\tilde{X}}$ has full support.

Lemma 3 (Rate-distortion). Let $X^n$ be an i.i.d. sequence encoded into binary codeword $C^n_R$ at rate $R < H(X)$. Suppose the code has a local decoder with worst-case local decodability $d_{loc} = \max_{1 \leq i \leq n} d(i) \in [n]$. Then, there exists a constant $\delta > 0$ that depends only on $R$ such that for any estimator $\hat{X}_i(C_{T_i})$ of $X_i$ we have

$$\delta \leq \max_{1 \leq i \leq n} \Pr[\hat{X}_i(C_{T_i}) \neq X_i],$$

and for any maximizer index $i$ there exists $c \in \{0, 1\}^{d(i)}$ such that

$$\Pr[\hat{X}_i(C_{T_i}) \neq X_i, C_{T_i} = c] \geq \delta 2^{-d_{loc}}.$$

Proof of Lemma 3. The converse to Shannon’s lossy source coding theorem implies that if $R < H(X)$, there exists a $\delta = \delta(R) > 0$ such that

$$\mathbb{E} d_H(X^n, \hat{X}^n) = \sum_{i=1}^{n} \Pr[\hat{X}_i(C_{T_i}) \neq X_i] \geq n\delta$$

where $d_H(X^n, \hat{X}^n)$ denotes the Hamming distance between $X^n$ and $\hat{X}^n$. Hence, there is at least one index $1 \leq i \leq n$ such that

$$\delta \leq \Pr[\hat{X}_i(C_{T_i}) \neq X_i].$$

\(^3\)More precisely, $\alpha$ and $\beta$ are in $(1, \infty)$, but for the purpose of this paper the exact values of $\alpha$ and $\beta$ (as functions of $p_{V, W}$) are irrelevant.

\(^4\)Hence, $(X, Y)$ and $(\tilde{X}, Y)$ form a coupling.
Expanding the right-hand side and assuming a worst-case local decodability of $d_{\text{loc}} \in [n]$ we have
\[
\delta \leq \Pr[\hat{X}_i(C_{I_i}) \neq X_i] = \sum_{c \in \{0,1\}^{d(i)}} \Pr[\hat{X}_i(C_{I_i}) \neq X_i, C_{I_i} = c] \\
\leq 2^{d_{\text{loc}}} \max_{c \in \{0,1\}^{d(i)}} \Pr[\hat{X}_i(C_{I_i}) \neq X_i, C_{I_i} = c].
\]

This concludes the proof. \( \square \)

**B. Proof of Theorem 2**

Suppose $p_{XY}$ is confusable. Suppose $X^n$ is compressed at rate $R_1 < H(X)$, and that the local decoder has locality $d_{\text{loc}}$. From Lemma 3 there exists an index $i \in [n]$ and a local codeword $c \in \{0,1\}^{d(i)}$, with $d(i) \leq d_{\text{loc}}$, such that
\[
\Pr[X_i \neq \bar{x}, C_{I_i} = c] \geq \delta 2^{-d_{\text{loc}}} \geq \delta |\mathcal{X}|^{-d_{\text{loc}}},
\]
where
\[
\bar{x} \overset{\text{def}}{=} \arg \max_x \Pr[X_i = x|C_{I_i}(X^n) = c]
\]
is the MAP estimate given $C_{I_i} = c$. Moreover, it must be the case that
\[
\Pr[X_i = \bar{x}|C_{I_i} = c] \geq \frac{1}{|\mathcal{X}|},
\]
for otherwise the probabilities would not sum to one. From (3) and (4) it then follows that
\[
\Pr[X_i = \bar{x}, C_{I_i} = c] = \Pr[X_i = \bar{x}|C_{I_i} = c] \Pr[C_{I_i} = c] \geq \delta \frac{2^{-d_{\text{loc}}}}{|\mathcal{X}|} \geq \delta |\mathcal{X}|^{-d_{\text{loc}}-1}.
\]

The key to proving Theorem 2 is the following lemma:

**Lemma 4.** Fix source $p_{XY}$. Suppose $X^n$ is encoded into codeword $C^n R(X^n)$ at some rate $R \geq 0$. Fix $i \in [n]$ and let $\hat{X}_i(C_{I_i}, Y^n)$ be an estimator of $X_i$ with the knowledge of both $C_{I_i}(X^n)$ and $Y^n$. Then, for any realization $c$ of $C_{I_i}(X^n)$, we have:
\[
\Pr[\hat{X}_i(C_{I_i}, Y^n) \neq X_i, C_{I_i}(X^n) = c] \geq \Pr[X_i = \bar{x}, \tilde{X}_i \neq \bar{x}, C_{I_i}(X^n) = C_{I_i}(\tilde{X}^n) = c]
\]
where
\[
\bar{x} \overset{\text{def}}{=} \arg \max_x \Pr[X_i = x|C_{I_i}(X^n) = c].
\]

**Proof of Theorem 2.** Suppose the source is confusable and suppose $X^n$ is compressed at rate $R_1 < H(X)$. From Lemma 4 we have
\[
\Pr[\hat{X}_i(C_{I_i}, Y^n) \neq X_i, C_{I_i}(X^n) = c] \geq \Pr[X_i = \bar{x}, \tilde{X}_i \neq \bar{x}, C_{I_i}(X^n) = C_{I_i}(\tilde{X}^n) = c].
\]
Now, since $p_{XY}$ is confusable, we have that $p_{X\tilde{X}}$ has full support by Lemma 2 and therefore from Lemma 1 there exist finite constants $\alpha, \beta$ such that
\[
\Pr[X_i = \bar{x}, \tilde{X}_i \neq \bar{x}, C_{I_i}(X^n) = C_{I_i}(\tilde{X}^n) = c] \geq \left(\Pr[X_i = \bar{x}, C_{I_i}(X^n) = c]\right)^{\alpha} \left(\Pr[\tilde{X}_i \neq \bar{x}, C_{I_i}(\tilde{X}^n) = c]\right)^{\beta} = 2^{-\Theta(d_{\text{loc}})},
\]
where the equality follows from (5). We therefore conclude that
\[
\Pr[\hat{X}_i(C_{I_i}, Y^n) \neq X_i] \geq 2^{-\Theta(d_{\text{loc}})},
\]
which completes the proof. \( \square \)
Proof of Lemma 4. For any estimator \( \hat{X}_i(C_{i}, Y^n) \) of \( X_i \), we have

\[
\Pr(\hat{X}_i(C_{i}, Y^n) \neq X_i, C_{i}(X^n) = c) \geq \Pr(\hat{E}(C_{i}, Y^n) \neq E(X_i), C_{i}(X^n) = c) \\
= \sum_{y^n} \Pr(\hat{E}(C_{i}, Y^n) \neq E(X_i), C_{i}(X^n) = c | Y^n = y^n) \Pr(Y^n = y^n) \\
\geq \sum_{y^n} \Pr(Y^n = y^n) \min\{\Pr[X_i = \bar{x}, C_{i}(X^n) = c | Y^n = y^n], \Pr[X_i \neq \bar{x}, C_{i}(X^n) = c | Y^n = y^n]\} \tag{6}
\]

where \( \hat{E}(C_{i}, Y^n) \) is an estimator of the binary random variable \( E(X_i) \), defined to be equal to zero if \( X_i = \bar{x} \) and one if \( X_i \neq \bar{x} \); and where the right-hand side of the second inequality is the error probability of the optimal (MAP) estimator \( \hat{X} \) with the foreknowledge of \( Y^n \).

By multiplying the minimum on the right-hand side by

\[
\max\{\Pr[X_i = \bar{x}, C_{i}(X^n) = c | Y^n = y^n], \Pr[X_i \neq \bar{x}, C_{i}(X^n) = c | Y^n = y^n]\},
\]

which is at most one, from (6) we get

\[
\Pr(\hat{X}_i(C_{i}, Y^n) \neq X_i, C_{i}(X^n) = c) \geq \sum_{y^n} \Pr(Y^n = y^n) \left( \min\{\cdot\} \times \max\{\cdot\} \right) \\
= \sum_{y^n} \Pr(Y^n = y^n) \Pr[X_i = \bar{x}, C_{i}(X^n) = c | Y^n = y^n] \times \Pr[X_i \neq \bar{x}, C_{i}(X^n) = c | Y^n = y^n] \\
= \sum_{y^n} \Pr(Y^n = y^n) \Pr[X_i = \bar{x}, C_{i}(X^n) = c | Y^n = y^n] \times \Pr[\hat{X}_i = \bar{x}, C_{i}(\hat{X}^n) = c | Y^n = y^n] \\
= \Pr[X_i = \bar{x}, \hat{X}_i \neq \bar{x}, C_{i}(X^n) = C_{i}(\hat{X}^n) = c]
\]

where the second equality follows from the Markov chain \( X - Y - \hat{X} \). This yields the desired result. \( \square \)

V. AN EXPLICIT LOCAL DECODER

We now describe a locally decodable compression scheme that has the properties guaranteed by Theorem 4.

We want a scheme that achieves the following: For any fixed \( \delta > 0 \) and \( (R_1, R_2) \) within the Slepian-Wolf rate region,

- The sequences \( (X^n, Y^n) \) are independently compressed to rates \( (R_1 + \delta, R_2 + \delta) \) respectively.
- For any \( i \in [n] \) and \( 1 > \eta > 2^{-2^O(\log n)} \) specified at the receiver, the local decoder probes \( \text{poly}(\log(1/\eta)) \) compressed bits, and outputs \( (\hat{X}_i, \hat{Y}_i) \) which satisfies

\[
P_e^{(\text{loc})} = \Pr[(\hat{X}_i, \hat{Y}_i) \neq (X_i, Y_i)] \leq \eta.
\]

Our coding scheme is inspired by that in [30], and is a hierarchical compression scheme. The compressed bits consist of various blocks and are spread across multiple “levels” \( 1 \leq \ell \leq \ell_{\text{max}} \). At level \( \ell = 0 \), each compressed block is obtained by encoding constant-sized substrings of the source using a standard Slepian-Wolf code. The compressed bits at level 0 is obtained by applying the naive concatenation scheme defined in Section II-B2 with \( b = O(1) \). This guarantees that any pair of source symbols can be recovered with \( 2^{-\Theta(b)} = O(1) \) probability of error. The compressed bits at the higher level can be viewed as additional refinement bits that are probed only when we desire a lower probability of error. By probing blocks corresponding to higher levels, we obtain a more reliable estimate of \( (X_i, Y_i) \). The compressed blocks at level \( \ell \) are obtained by using a random binning scheme applied to blocks of size \( b_{\ell} \), where \( b_{\ell} \) is growing with \( \ell \). However, the rates for higher levels is chosen to decay with \( \ell \). The key challenge is to choose the parameters carefully so that the additional bits corresponding to higher levels essentially lead to no loss in terms of the overall compression rates.
1) \textbf{Parameters:} We choose\footnote{Since we only aim to get order-optimal results, we have not attempted to optimize over the various parameters.} a sufficiently small \( \epsilon_0 > 0 \), positive integers
\[
b_0 = n_0
\] which are constants independent of \( n \), and
\[
k_0^{(1)} = [(R_1 + \epsilon_0)b_0] \quad \text{and} \quad k_0^{(2)} = [(R_2 + \epsilon_1)b_1]
\] such that the probability of error of a Slepian-Wolf code for sequences of length \( b_0 \) satisfies
\[
\Pr[(\hat{X}^{b_0}, \hat{Y}^{b_0}) \neq (X^{b_0}, Y^{b_0})] \leq 2^{-\beta \epsilon_0 b_0} \leq \delta'
\] where \( \beta > 0 \) depends on \( p_{XY}, R_1, R_2 \) only and \( 0 < \delta' < 1 \) is a parameter that determines an upper bound on the probability of local decoding error that can be achieved.

For each \( \ell = 1, 2, \ldots, \ell_{\text{max}} \), define
\[
\epsilon_{\ell} = \epsilon_{\ell-1}/2 = \epsilon_0/2^\ell \\
b_{\ell} = 16b_{\ell-1} = 16^\ell b_0 \\
n_{\ell} = b_{\ell}n_{\ell-1} = 4^{\ell+1}b_0^{\ell+1} \\
k_{\ell}^{(1)} = \epsilon_{\ell} n_{\ell} \left( \beta + |X| + \frac{b_{\ell}}{n_{\ell}} \log \frac{e^{2\ell}}{\epsilon_0} \right) \\
k_{\ell}^{(2)} = \epsilon_{\ell} n_{\ell} \left( \beta + |Y| + \frac{b_{\ell}}{n_{\ell}} \log \frac{e^{2\ell}}{\epsilon_0} \right)
\] (9)

2) \textbf{Encoder:} The encoder works as follows:

- **Level 0:** At this level, each encoder partitions the \( n \) symbols it has (i.e., \( X^n \) or \( Y^n \)) into blocks of \( b_0 \) consecutive symbols each, and compresses this using a standard Slepian-Wolf code where the compressed lengths are given by (8). To set up some notation, let us define for \( i = 1, 2, \ldots, n/b_0 \), the \( i \)th (source) block at level 0 to be \( X^{b_0}(0, i) = X_{(i-1)b_0+1}^{ib_0} \), and \( Y^{b_0}(0, i) = Y_{(i-1)b_0+1}^{ib_0} \). Likewise, let \( U_{b_0}^{(1)}(0, i) \) and \( V_{b_0}^{(2)}(0, i) \) denote the respective compressed blocks at level 0, i.e., these are obtained by applying the Slepian-Wolf random binning encoder [39, Chapter 10] on \( X^{b_0}(0, i) \) and \( Y^{b_0}(0, i) \) respectively. For every sequence in \( X^{b_0} \) (resp. \( Y^{b_0} \)), we assign a codeword of length \( k_0^{(1)} \) (resp. \( k_0^{(2)} \)) independently and uniformly at random and this is used to encode each \( X^{b_0}(0, i) \) (resp. \( Y^{b_0}(0, i) \)). The compressed blocks are therefore random hashes of the respective source blocks.

- **Level \( \ell \), for \( \ell > 0 \):** At higher levels, the compression works similarly. The respective encoders obtain \( U_{b_0}^{(1)}(\ell, i) \) and \( V_{b_0}^{(2)}(\ell, i) \) by taking random hashes of \( X^{n_{\ell}}(\ell, i) = X_{(i-1)n_{\ell}+1}^{in_{\ell}} \) and \( Y^{n_{\ell}}(\ell, i) = Y_{(i-1)n_{\ell}+1}^{in_{\ell}} \) respectively (for \( \ell = 1, 2, \ldots, \ell_{\text{max}} \)). In other words, for every sequence of length \( n_{\ell} \), we assign a codeword of length \( k_{\ell}^{(1)} \) (to encode \( X^n \)) or \( k_{\ell}^{(2)} \) (to encode \( Y^n \)) independently and uniformly at random.

It is worth pointing out that most of the entropy of the compressed sequence lies in the level-0 codewords. For \( \ell > 0 \), the rates \( k_{\ell}^{(1)}/n_{\ell} \) and \( k_{\ell}^{(2)}/n_{\ell} \) are exponentially decaying functions of \( \ell \).

3) \textbf{Local decoder:} The local decoder takes two parameters as input: a location \( i \in [n] \), and \( \ell_d \in \{0, 1, 2, \ldots, \ell_{\text{max}}\} \). The first parameter specifies which \( (X_i, Y_i) \) the decoder wishes to recover. The second parameter specifies the number of bits to probe (which decides the probability of error). For a specified \( \ell_d \), the local decoder probes \( 2^{O(\ell_d^2)} \) compressed bits, and the probability of error is \( 2^{-2^{O(\ell_d^2)}} \). This statement will be made more precise shortly.

The decoder works by probing compressed bits up to level \( \ell_d \) as follows:
Lemma 6. For any given parameters \( (i, \ell_d) \), the number of bits probed by the local decoder is

\[
d_{\text{loc}}(\ell_d) \leq b_{\ell_d}^{\ell_d+1} 4 \ell_d(\ell_d+1)(R_1 + R_2 + \gamma_1 \varepsilon_0) \leq 2^{\gamma_2 \ell_d^2}
\]

where \( \gamma_1 \) is a constant that only depends on \( p_{XY}, R_1, R_2, \) while \( \gamma_2 \) may depend on \( p_{XY}, R_1, R_2, \varepsilon_0, b_0 \).

Proof. The total number of compressed bits probed is equal to

\[
d_{\text{loc}}(\ell_d) = \sum_{\ell=0}^{\ell_d} \frac{n_{\ell}}{n_\ell} (k_1^{(\ell)} + k_2^{(\ell)})
\]

\[
\leq n_{\ell_d}(R_1 + R_2 + \gamma_2 \varepsilon_0)
\]

\[
= b_{\ell_d}^{\ell_d+1} 4 \ell_d(\ell_d+1)(R_1 + R_2 + \gamma_1 \varepsilon_0)
\]

where \( \gamma_1 \) is a constant that only depends on \( p_{XY}, R_1, R_2 \).

Lemma 5. For any given parameters \( (i, \ell_d) \), the number of bits probed by the local decoder is

\[
d_{\text{loc}}(\ell_d) \leq b_{\ell_d}^{\ell_d+1} 4 \ell_d(\ell_d+1)(R_1 + R_2 + \gamma_1 \varepsilon_0) \leq 2^{\gamma_2 \ell_d^2}
\]

where \( \gamma_1 \) is a constant that only depends on \( p_{XY}, R_1, R_2, \varepsilon_0, b_0 \).

Proof. The total number of compressed bits probed is equal to

\[
d_{\text{loc}}(\ell_d) = \sum_{\ell=0}^{\ell_d} \frac{n_{\ell}}{n_\ell} (k_1^{(\ell)} + k_2^{(\ell)})
\]

\[
\leq n_{\ell_d}(R_1 + R_2 + \gamma_2 \varepsilon_0)
\]

\[
= b_{\ell_d}^{\ell_d+1} 4 \ell_d(\ell_d+1)(R_1 + R_2 + \gamma_1 \varepsilon_0)
\]

where \( \gamma_1 \) is a constant that only depends on \( p_{XY}, R_1, R_2 \).

Lemma 6. For any given parameters \( (i, \ell_d) \), the probability of error of decoding \( X^{n\ell_d}, Y^{n\ell_d} \) after decoding up to level \( \ell_d \) is upper bounded as follows

\[
P_e^{(\ell_d)} \leq 2^{-\beta \varepsilon_0 b_0}(\ell_d+1)^2 = 2^{-2\log d_{\text{loc}}(\ell_d)}
\]

Proof. We will derive the bound by obtaining an upper bound on \( P_e^{(\ell)} \) in terms of \( P_e^{(\ell-1)} \). For \( \ell = 0 \), we know that

\[
P_e^{(0)} \leq 2^{-\beta \varepsilon_0 b_0}
\]

for a suitable constant \( \beta > 0 \).

For decoding at level \( \ell > 1 \), there are two possible error events:

1) Event \( \mathcal{E}_1 \): More than \( \varepsilon b_\ell \) blocks were decoded incorrectly at level \( \ell - 1 \)

2) Event \( \mathcal{E}_2 \): There is an incorrect pair of sequences \( (\hat{x}^{n\ell}, \hat{y}^{n\ell}) \) that has the same level-\( \ell \) hash/codeword as the true sequence and matches the \( (\ell - 1) \)-level decoded sequence on at least \( (1 - \varepsilon) b_\ell \) blocks.

The overall probability of error is then

\[
P_e^{(\ell)} \leq P_{\mathcal{E}_1} + P_{\mathcal{E}_2 | \mathcal{E}_1^c}.
\]

We will bound the two terms separately. For the first term, observe that

\[
\Pr[\mathcal{E}_1] \leq \left( \frac{b_\ell}{\varepsilon b_\ell} \right) (P_e^{(\ell-1)}) \varepsilon b_\ell
\]

\[
\leq \left( \frac{e P_e^{(\ell-1)}}{\varepsilon} \right) \varepsilon b_\ell
\]
where in the last step, we have assumed that \( P_{e}^{(\ell-1)} \leq 2^{\frac{1}{2}\beta(\varepsilon_0b_0)^{2(\ell-1)^2}} \). Rewriting the right-hand side, we get

\[
\Pr[\mathcal{E}_1] \leq \exp_2 \left( -\beta(\varepsilon_0b_0)^{\ell+2} + \ell + 1 + \log \left( \frac{2^\ell}{\varepsilon_0^2} \right) \right)
\]

Since \( \varepsilon_0b_0 \) is large enough, the second term is at least twice that of the first. Therefore,

\[
\Pr[\mathcal{E}_1] \leq \exp_2 \left( -\beta(\varepsilon_0b_0)^{\ell+2} + \ell + 1 + \log \left( \frac{2^\ell}{\varepsilon_0^2} \right) \right) \leq \frac{2^{2\beta(\varepsilon_0b_0)^{\ell+2}}}{2}
\]

(10)

To compute the probability of the second error event, let us define \( \mathcal{E}_2^A \) (resp. \( \mathcal{E}_2^B \)) to be the event that there is an incorrect sequences \( \tilde{x}_n \) (resp. \( \tilde{y}_n \)) that has the same hash as the true sequence and matches the \( (\ell - 1) \)-level decoded sequence on at least \( (1 - \varepsilon_0)\beta \) blocks. We have,

\[
\Pr[\mathcal{E}_2^A | \mathcal{E}_1^c] \leq \left( \frac{b_0}{\varepsilon_0^{\ell} b_0^2} \right) |X|^{|\varepsilon_0n\ell - k^{(1)}_\ell|} \leq \left( \frac{e^{b_0}}{\varepsilon_0^{\ell}} \right) |X|^{|\varepsilon_0n\ell - k^{(1)}_\ell|}
\]

Substituting for \( k^{(1)}_\ell \) in the above and simplifying, we get

\[
\Pr[\mathcal{E}_2^A | \mathcal{E}_1^c] \leq 2^{-\beta_\epsilon n\ell} \leq \frac{2^{-\beta(\varepsilon_0b_0)^{\ell+2}}}{4}
\]

(11)

Similarly,

\[
\Pr[\mathcal{E}_2^B | \mathcal{E}_1^c] \leq 2^{-\beta_\epsilon n\ell} \leq \frac{2^{-\beta(\varepsilon_0b_0)^{\ell+2}}}{4}
\]

(12)

Combining (10), (11) and (12), we get

\[
P_{e}^{(\ell)} \leq \Pr[\mathcal{E}_1] + \Pr[\mathcal{E}_2^A | \mathcal{E}_1^c] + \Pr[\mathcal{E}_2^B | \mathcal{E}_1^c] \leq 2^{-\beta(\varepsilon_0b_0)^{\ell+2}}
\]

which completes the proof.

\[\square\]

VI. EXTENSION TO \( k > 2 \) SOURCES

We first extend Theorem [2] to a \( k \)-source distribution \( p_{X_1, \ldots, X_k} \) defined over alphabet \( \mathcal{X}_1 \times \cdots \times \mathcal{X}_k \). Let the source be \( \mathcal{X}_1 \)-confusable if for every \( x_1, x'_1 \in \mathcal{X}_1 \), there exist \( (x_2, x_3, \ldots, x_k) \) for which \( p_{X_1, \ldots, X_k}(x_1, \ldots, x_k) > 0 \) and \( p_{X_1, \ldots, X_k}(x_1', \ldots, x_k) > 0 \). Observe that this condition holds if and only if for every \( x_1, x'_1 \in \mathcal{X}_1 \), there exist an index \( i \geq 2 \) and \( x_i \in \mathcal{X}_i \) for which \( p_{X_1, \ldots, X_k}(x_1, x_i) > 0 \) and \( p_{X_1, \ldots, X_k}(x_1', x_i) > 0 \). Now if we repeat the same line of arguments as for the proof of Theorem [2] but with the side information \( Y^n \) replaced by all sources except \( X_1^n \), that is \( X_2^n, \ldots, X_k^n \), we get:

**Theorem 5** (Confusable, \( k \geq 2 \) sources). Suppose source \( p_{X_1, \ldots, X_k} \) is \( \mathcal{X}_1 \)-confusable. If \( X_1^n \) is compressed at rate \( R_1 < H(X_1) \), then

\[
\max_{1 \leq i < n} \Pr[\hat{X}_{1i}(C_{1i}(X_1^n), X_2^n, \ldots, X_k^n) \neq X_1^n] \geq 2^{-\Theta(d_{\text{loc}})}
\]

where \( \hat{X}_{1i}(C_{1i}(X_1^n), X_2^n, \ldots, X_k^n) \) is any estimator of the \( i \)-th symbol of source \( X_1^n \) given at most \( d_{\text{loc}} \) components \( C_{1i}(X_1^n) \) of \( C_{1i}(X_1^n) \) and \( (X_2^n, \ldots, X_k^n) \).
Similarly, Theorem 3 immediately generalizes to

**Theorem 6** (Non-confusable, \(k \geq 2\) sources). Suppose source \(p_{X_1, \ldots, X_k}\) is not \(X_1\)-confusable. Then, it is possible to achieve strong locality at some \(R_i < H(X_i)\) and \(R_i = H(X_i), i \in \{2, \ldots, k\} \).

The coding scheme of Section V easily extends to more than two sources, with the same encoding scheme for each source, and an identical local decoder:

**Theorem 7** (Hierarchical coding scheme, \(k \geq 2\) sources). For any \((R_1, R_2, \ldots, R_k)\) in the interior of the Slepian-Wolf rate region, there exists a rate \((R_1, R_2, \ldots, R_k)\) distributed compression scheme such that for every \(1 > \eta > 2^{-\Omega(\log n)}\), the local decoder achieves \(d_{\text{loc}} = \text{poly}(\log(1/\eta))\) and \(P_e(\text{loc}) \leq \eta\).

**VII. Concluding Remarks**

In contrast with the single source set up, we showed that for multiple sources lossless compression and strong locality can generally not be accommodated. For the broad class of confusable sources, for strong locality to hold all sources must be compressed at rates above their respective entropies. On the other hand, if the distribution is not confusable, an arguably peculiar situation, strong locality may hold even if compression rates are below individual entropies. For this case, the characterization of all rate pairs for which strong locality can be achieved remains an open problem.

Our compression scheme is able to achieve \(d_{\text{loc}} = \text{poly}(\log(1/\eta))\) for any target probability of local decoding error \(\eta\) specified at the decoder. Note that from our lower bound, \(d_{\text{loc}} = \Omega(\log(1/\eta))\) and our scheme is suboptimal by a polynomial factor. Designing an improved scheme that achieves this lower bound is left as future work.

In this paper, we only considered the problem of local decodability in the context of distributed compression. One may also require provisioning of local substitutions/insertions/deletions of source symbols in the compressed domain. This is an interesting problem that warrants more attention.

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