Production of doubly strange hypernuclei via \( \Sigma^- \) doorways in the \( ^{16}\text{O}(K^-, K^+) \) reaction at 1.8 GeV/c

Toru Harada\textsuperscript{a, *}, Yoshiharu Hirabayashi\textsuperscript{b}, Atsushi Umey\textsuperscript{a}

\textsuperscript{a} Research Center for Physics and Mathematics, Osaka Electro-Communication University, Neyagawa, Osaka 572-8530, Japan
\textsuperscript{b} Information Initiative Center, Hokkaido University, Sapporo 060-0811, Japan
\textsuperscript{c} Nishina Center for Accelerator-Based Science, RIKEN, Wako 351-0198, Japan

\textbf{A R T I C L E I N F O}

Article history:
Received 5 March 2010
Received in revised form 10 May 2010
Accepted 17 May 2010
Available online 25 May 2010

\textbf{A B S T R A C T}

We examine theoretically production of doubly strange hypernuclei, \( ^{16}\text{C} \) and \( ^{16}\Lambda\Lambda \), in double-charge exchange \( ^{16}\text{O}(K^-, K^+) \) reactions using a distorted-wave impulse approximation. The inclusive \( K^+ \) spectrum at the incident momentum \( p_K^- = 1.8 \text{ GeV/c} \) and scattering angle \( \theta_{lab} = 0^\circ \) is estimated in a one-step mechanism, \( K^- p \rightarrow K^+ \Sigma^- \) via \( \Sigma^- \) doorways caused by a \( \Sigma^- \)–\( \Lambda\Lambda \) coupling. The calculated spectrum in the \( \Sigma^- \) bound region indicates that the integrated cross sections are on the order of \( 7\text{–}12 \text{ nb/sr} \) for significant \( 1^- \) excited states with \( ^{14}\text{C}(0^+, 2^+) \otimes s_{\Sigma^-} \) configurations in \( ^{16}\Sigma^- \) via the doorway states of the spin-stretched \( ^{15}\text{N}(1/2^-, 3/2^-) \otimes s_{\Sigma^-} \) in \( ^{16}\text{C} \) due to a high momentum transfer \( q_{\Sigma^-} \approx 400 \text{ MeV/c} \). The \( \Sigma^- \) admixture probabilities of these states are on the order of \( 5\% \). However, populations of the \( 0^+ \) ground state with \( ^{14}\text{C}(0^+) \otimes s_{\Sigma^-}^2 \) and the \( 2^+ \) excited state with \( ^{14}\text{C}(2^+) \otimes s_{\Sigma^-}^2 \) are very small. The sensitivity of the spectrum on the \( \Sigma^- \)–\( \Lambda\Lambda \) coupling strength enables us to extract the nature of \( \Sigma^- \)–\( \Lambda\Lambda \) dynamics in nuclei, and the nuclear \( (K^-, K^+) \) reaction can extend our knowledge of the \( S = -2 \) world.

© 2010 Elsevier B.V. Open access under CC BY license.

\textbf{1. Introduction}

It is important to understand properties of \( \Sigma \) hypernuclei whose states are regarded as “doorways” to access multi-strangeness systems as well as a two-body \( \Sigma^- \)–\( \Lambda\Lambda \) system, and it is a significant step to extend study of strange nuclear matter in hadron physics and astrophysics [1]. Because the \( \Sigma \) hyperon in nuclei has to undergo a strong \( \Sigma^- N \rightarrow \Lambda\Lambda \) decay, widths of \( \Sigma \) hypernuclear states give us a clue to a mechanism of \( \Sigma \) absorption processes in nuclei. A pioneer study of \( \Sigma \) hypernuclei by Dover and Gal [2] has found that a \( \Sigma^- \)–nuclear potential has a well depth of \( 24 \pm 4 \text{ MeV} \) in the real part on the analysis of old emulsion data. However, our knowledge of these \( \Sigma^- \)–nucleus systems is very limited due to the lack of the experimental data [3]. Indeed, the missing-mass spectra of a double-charge exchange (DCX) reaction \( (K^-, K^+) \) on a \( ^{12}\text{C} \) target have suggested the \( \Sigma^- \) well depth of \( 14\text{–}16 \text{ MeV} \) [4,5]. Several authors [6] have used the unsettled \( \Sigma^- \)–nucleus (optical) potentials such as \( V_\Sigma = (-24\text{–}14) \text{ MeV} \) and \( W_\Sigma = (-6\text{–}3) \text{ MeV} \) in the Woods–Saxon potential to demonstrate the \( \Sigma^- \) production spectra in the nuclear \( (K^-, K^+) \) reactions. There remains a full uncertainty about the nature of doubly strange \( (S = -2) \) dynamics caused by the \( \Sigma^- \)–\( \Lambda\Lambda \) interaction in nuclei at the present stage. More experimental information is earnestly desired.

The \( (K^-, K^+) \) reaction is one of the most promising ways of studying double strange systems such as \( \Sigma \) hypernuclei for the forthcoming J-PARC experiments [3]. One expects that these experiments will confirm the existence of \( \Sigma \) hypernuclei and establish properties of the \( \Sigma^- \)–nucleus potential, e.g., binding energies and widths. This reaction can also populate a \( \Lambda\Lambda \) hypernucleus through a conventional DCX two-step mechanism as \( K^- p \rightarrow \pi^0 \Lambda \) followed by \( \pi^0 p \rightarrow K^+ \Lambda \) [7–9], as shown in Fig. 1(a). Such an inclusive \( K^- \) spectrum in the \( \Lambda\Lambda \) bound region is rather clean with much less background experimentally. Early theoretical predictions for two-step \( ^{16}\text{O}(K^-, K^+) \) reactions at the incident momentum \( p_K^- = 1.1 \text{ GeV/c} \) and scattering angle \( \theta_{lab} = 0^\circ \) [7,8] have indicated small cross sections for the \( \Lambda\Lambda \) states, for example, \( \sim 0.1 \text{ nb/sr} \) for the \( 0^+ \) \( (s_{\Sigma^-}^2) \) ground state and \( \sim 2 \text{ nb/sr} \) for the \( 2^+ \) \( (s_{\Sigma^-}^2) \) excited state in \( ^{16}\Lambda\Lambda \) when we took 0.61 mb/sr and 0.32 mb/sr as the laboratory cross sections at \( 0^\circ \) for \( K^- p \rightarrow \pi^0 \Lambda \) and \( \pi^0 p \rightarrow K^+ \Lambda \), respectively.

It should be noticed that another exotic production of \( \Lambda\Lambda \) hypernuclei in the \( (K^-, K^+) \) reactions is a one-step mechanism, \( K^- p \rightarrow K^+ \Sigma^- \) via \( \Sigma^- \) doorways caused by a \( \Sigma^- \)–\( \Lambda\Lambda \) tran-
probability in the

2. Calculations

2.1 Theoretical Framework

Fig. 1. Diagrams for DCX nuclear \((K^-, K^+)\) reactions: (a) a two-step mechanism, \(K^- p \rightarrow \pi^0 A\) followed by \(\pi^0 p \rightarrow K^+ A\), and (b) a one-step mechanism, \(K^- p \rightarrow K^+ \Xi^-\) via \(\Xi^-\) doorways caused by the \(\Xi^- p - \Lambda A\) coupling.

sition, as shown in Fig. 1(b). The \(\Xi N - \Lambda A\) coupling induces the \(\Xi^-\) admixture and the \(\Lambda A\) energy shift \(\Delta E_{\Lambda A} = E_{\Lambda A} - 2B_{\Lambda A}\) in the \(\Lambda A\)-nuclear states [10–14], and its coupling strength is also related to widths of \(\Xi\)-hypernuclear states [15, 16]. For a viewpoint of \(S = -2\) studies, it is very important to extract quantitative information concerning the \(\Xi N - \Lambda A\) coupling from spectroscopy of the \(\Xi\) and \(\Lambda A\) hypernuclei [17,18].

In this Letter, we study theoretically production of a doubly strange hypernucleus in the DCX \((K^-, K^+)\) reaction on an \(^{16}\text{O}\) target at \(p_K = 1.8\) GeV/c and \(\theta_{\text{lab}} = 0^\circ\) within a distorted-wave impulse approximation (DWIA). Thus we focus on the \(\Lambda A - \Xi\) spectrum for \(^{16}\text{C}\) and \(^{15}\text{C}\) in the \(\Xi^-\) bound region considering the one-step mechanism, \(K^- p \rightarrow K^+ \Xi^-\) via \(\Xi^-\) doorways caused by the \(\Xi N - \Lambda A\) coupling in the nuclear \((K^-, K^+)\) reaction, rather than the two-step mechanism as \(K^- p \rightarrow \pi^0 A\) followed by \(\pi^0 p \rightarrow K^+ A\) [7,8]. These different mechanisms are well separated kinematically. The forward cross section for the \(K^- p \rightarrow K^+ \Xi^-\) elementary process is at its maximum at \(p_K = 1.8 - 1.9\) GeV/c, whereas the \(K^- p \rightarrow \pi^0 A\) reaction at \(p_K = 1.1\) GeV/c leads to the maximal cross section for the \(\pi^0 p \rightarrow K^+ A\) process. This present study is the first attempt to evaluate a production spectrum of the \(\Lambda A - \Xi\) hypernucleus via the \(\Xi N - \Lambda A\) coupling from the inclusive \((K^-, K^+)\) reaction, and to extract the \(\Xi^-\) admixture probability in the \(\Lambda A\) hypernucleus from the spectrum. We also discuss a combination of the two-step processes in the \((K^-, K^+)\) reactions within the eikonal approximation.

2.2 Calculations

Let us consider the DCX \((K^-, K^+)\) reaction on the \(^{16}\text{O}\) target at 1.8 GeV/c within a DWIA and examine the production cross sections and wave functions of the doubly strange hypernucleus. To fully describe the one-step process via \(\Xi^-\) doorways, as shown in Fig. 1(b), we perform nuclear \(\Lambda A - \Xi\) coupled-channel calculations [13,14], which are assumed to effectively represent the coupling nature in omitting other \(\Lambda \Sigma\) and \(\Sigma \Sigma\) channels for simplicity. Here we employ a multichannel coupled wave function of the \(\Lambda A - \Xi\) nuclear state for a total spin \(J_B\) within a weak coupling basis. It is written as

\[
|\psi_{J_B}(A_1 A_2 A_3)\rangle = \sum_{J_f J_1 J_2} \left[ (|\phi_J(1^{14}\text{C})\rangle, |\psi_{J_1}(r_{A_1})\rangle, |\psi_{J_2}(r_{A_2})\rangle) \right]_{J_f}.
\]

with \(\phi_J(1^{14}\text{C}) = A_1|\phi_J(1^{14}\text{C}), \psi(p)\rangle_J\), where \(r_{A_1}\) (\(r_{A_2}\)) denotes the relative coordinate between the \(14\text{C}\) core-nucleus and the \(\Lambda\) (proton), and \(r_{A_2}\) (\(r_{\Xi}\)) denotes the relative coordinate between the center of mass of the \(14\text{C} - \Lambda\) \((15\text{N})\) subsystem and the \(\Lambda\) \((\Xi^-)\). Thus \(\psi_{J_1}, \psi_{J_2}\) and \(\psi(p)\) describe the relative wave functions of shell model states (that occupy \(20\text{C}\) core-nucleus states) and \(\Lambda\) (proton) orbitals. \(\phi_J(1^{14}\text{C})\) is a wave function of the \(14\text{C}\) core-nucleus state, and \(A\) is the antisymmetrized operator for nucleons. The energy difference between \(15\text{N} - \Xi^-\) and \(14\text{C} + \Lambda + \Lambda\) channels is \(\Delta M = M(1^{15}\text{N}) + m_{\Xi^-} - M(1^{14}\text{C}) - 2m_{\Lambda} = 18.4\) MeV, whereas the \(15\text{N}, 14\text{C}\), \(m_{\Lambda}\) and \(m_{\Xi^-}\) are masses of the \(15\text{N}\) nucleus, the \(14\text{C}\) nucleus, the \(\Xi^-\) and \(\Lambda\) hyperons, respectively. We take the \(15\text{N}\) core-nucleus states with \(J = 0 - 1\) states \((\pi, \rho)\) pions with \(\Xi^-\) cross-sections that are selectively formed by non-spin-flip processes in the forward \(K^- p \rightarrow K^+ \Xi^-\) reaction, we consider a spin \(S = 0, \Lambda A\) pair in the hypernucleus. If the \(\Lambda A\) component is dominant in a bound state, we can identify it as a state of the \(\Lambda A\) hypernucleus \(^{15}\text{C}\) in which the \(\Xi^-\) admixture probability can be estimated by

\[
P_{\Xi^-} = \sum_{J_f J_1 J_2} \langle \psi_{J_1 J_2}(\Xi^-) | \psi_{J_f J_2}(\Xi^-) \rangle.
\]

under the normalization of

\[
\sum_{J_f J_1 J_2} \langle \psi_{J_1 J_2}(\Xi^-) | \psi_{J_f J_2}(\Xi^-) \rangle + \sum_{J_f J_1 J_2} \langle \psi_{J_1 J_2}(\Xi^-) | \psi_{J_f J_2}(\Xi^-) \rangle = 1.
\]

After we set up the \(^{15}\text{C}\) and \(^{15}\text{N}\) configurations in our model space with Eq. (1), we calculate the wave functions of \(\psi_{J_f J_2}(\Xi^-)\) and \(\psi_{J_f J_2}(\Xi^-)\) taking into account their channel coupling. Thus, the complete Green's function \(G(\omega)\) [19] describes all information concerning \((^{15}\text{C} \otimes \Lambda) + (^{15}\text{N} \otimes \Xi^-)\) coupled-channel dynamics, as a function of the energy transfer \(\omega\). It is numerically obtained as a solution of the \(N\)-channels radial coupled equations with a hyperon–nucleus potential \(U\) [20,21], which is written in an abbreviated notation as

\[
G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega) U G(\omega)
\]

where \(G^{(0)}(\omega)\) is a free Green's function. In our calculations, for example, we deal with \(N = 28\) for the \(J_B = 1\) state. The nuclear optical potentials \(U_y (Y = \Xi\) or \(A\) can be written as

\[
U_y(r) = V_y f(r, R, a) + \imath W_y f(r, R', a') + \imath W_y^0 g(r, R', a'),
\]

where \(f\) is the Woods–Saxon (WS) form, \(f(r, R, a) = \left(1 + \exp\left(-r - R/a\right)\right)^{-1}\), and \(g\) is the derivative of the WS form, \(g(r, R', a') = -4a'/d(r)\). The spin–orbit potentials are neglected. In \(^{15}\text{N} - \Xi^-\) channels, we assume the strength parameter of \(V_{\Xi^-} = \)
be obtained by a two-body method [19]. In the one-step mechanism, the Racah algebra [29]:

$$W_{\Lambda \Lambda}$$

have no criterion for a choice of doorway, it is given [21] as

where parameters of $$\Lambda$$-$$\Xi$$ in the WS-type to give widths of $$\Xi^-$$-quasibound states in recent calculations [5,6,23]. In $$^{14}\text{C}-\Lambda\Lambda$$ channels, we should use a $$^{12}\text{C}-\Lambda$$ potential, which can be constructed in folded potential models [24]:

$$U_A(r) = \int \rho_A^r(\mathbf{r}_A) [U_{\Lambda A}(|\mathbf{r} - \mathbf{r}_A|)] d\mathbf{r}_A.$$  \hspace{1cm} (6)

where $$\rho_A^r(\mathbf{r}_A) = \sum_i \rho_i^r(n_i M_{\text{h}]} \pi_j^r(|\mathbf{m}'\mathbf{n}'|)^2 \rho_A^r(\mathbf{r}_A)^2$$ and $$\lambda_A = 1 - \frac{V_A}{(M_{14C} + m_A)}.$$ $$U_{\Lambda A}$$ and $$V_{\Lambda A}$$ denote an optical potential for $$^{14}\text{C}$$-$$\Lambda$$ as given in Eq. (5) and a $$\Lambda\Lambda$$ residual interaction, respectively. Here we neglected $$V_{\Lambda\Lambda}$$ for simplicity. The real part of $$U_{\Lambda\Lambda}$$ leads to $$B_A = 12.2 \text{ MeV}$$ for the $$(0s_A)$$ state and $$B_A = 1.6 \text{ MeV}$$ for the $$(0p_A)$$ state in $$^{12}\text{C}$$ [25], and its imaginary part exhibits a flux loss of the wave functions through the core excitations of $$^{14}\text{C}$$-$$\Lambda$$. We assume $$W_{\Lambda} \approx \frac{1}{2} W_N$$ and $$W_{\Lambda A} \approx \frac{1}{2} W_{\Lambda A}$$ where parameters of $$W_N$$ and $$W_{\Lambda A}$$ for nucleon were obtained in Ref. [26] because the well depth of the imaginary potential for $$\Lambda$$ is by a factor of 4 weaker than that for nucleon in $$\text{g-matrix}$$ calculations [27].

The $$\Lambda\Lambda$$-$$\Xi$$ coupling potential $$U_X$$ in off-diagonal parts of $$U$$ is the most interesting object in this calculation [10-16]. It can be obtained by a two-body $$\Xi N$$-$$\Lambda\Lambda$$ potential $$v_{\Xi N,\Lambda\Lambda}^X(r, r')$$ with the $$^{1}\Sigma_0$$, isospin $$T = 0$$ state. Here we use a zero-range interaction $$v_{\Xi N,\Lambda\Lambda}^X(r, r') = \frac{1}{2} v_{\Xi N,\Lambda\Lambda}^X \delta_\Sigma_0 (r - r')$$ in a real potential for simplicity, where $$v_{\Xi N,\Lambda\Lambda}^X$$ is the strength parameter that should be connected with volume integral $$\int v_{\Xi N,\Lambda\Lambda}^X(r') d^3r' = \frac{1}{2} v_{\Xi N,\Lambda\Lambda}^X$$ [13,14,16].

Thus the matrix elements can be easily estimated by use of Racah algebra [29]:

$$U_X(r) = \langle [\Phi_j(15N) \otimes \gamma_j^X(\mathbf{r}_j)]_J \bigg| \sum_i v_{\Xi N,\Lambda\Lambda}(v_i \mathbf{r}_i, r) \bigg| [\Phi_j^f(14C) \otimes \gamma_j^{f*}(\mathbf{r}_j)]_J \rangle$$

$$\times \sqrt{\frac{1}{2}} v_{\Xi N,\Lambda\Lambda}^X \delta_{\Sigma_0} c_{\Xi \Lambda} \langle J'f' | Jf \rangle.$$  \hspace{1cm} (7)

where $$\gamma_{j\ell} = \{Y_{\ell} \otimes X_{j\ell}\}_{j\ell}$$ is a spin-orbit potential and the $$c_{\Xi \Lambda}$$ is a purely geometrical factor [29]; $$\delta_{\Sigma_0}$$ is the $$\Sigma_0$$-$$\Lambda$$ separation energy. The factor $$\sqrt{1/2}$$ comes from the procedure handling a transition between $$\pi^+$$ and $$\Lambda\Lambda$$ states in the nucleus.

The inclusive $$K^-$$ double-differential laboratory cross section of the $$\Lambda\Lambda$$-$$\Xi$$ production in the nuclear ($$K^-$$ $$\to$$ $$K^+$$ $$\Xi^-$$) reaction can be written within the DWIA [32,33] using the Green's function method [19]. In the one-step mechanism, $$K^-$$ $$\to$$ $$K^+$$ $$\Xi^-$$ via $$\Xi^-$$-doorways, it is given [21] as

$$\left( \frac{d^2\sigma}{d\Omega dE_K} \right)_{\text{lab}} = \frac{1}{|J_A|} \sum_{\alpha'\alpha} \left( - \frac{1}{\pi} \right) \times \text{Im} \left[ \int d\mathbf{r}' d\mathbf{r} \bar{\gamma}_\alpha' \gamma_{\alpha}^{*}(\mathbf{r}) \left( C_{\alpha'}^{*}(\mathbf{r}, \mathbf{r}') \right)^2 \right]$$

for the target with a spin $$J_A$$ and its $$z$$-component $$M_z$$, where $$|J_A| = 2J_A + 1$$, and a kinematical factor $$\beta$$ [34] that expresses the transition from the two-body $$K^-$$ $$\to$$ proton laboratory system to the $$K^-$$ $$\to$$ $$\Xi^-$$ laboratory system [2]. The production amplitude $$F_{\text{2D}}$$ is

$$F_{\text{2D}}(r) = \int f_{K^-} p \to K^+ \Xi^- \gamma_{\nu}^{+}(\frac{M_C}{M_B}) \gamma_{\nu}^{+}(\frac{M_C}{M_A}) \times \langle \bar{\psi}_K(r) | \psi_{J A M_z} \rangle,$$  \hspace{1cm} (8)

where $$f_{K^-} p \to K^+ \Xi^-$$ is a Fermi-averaged amplitude for the $$K^-$$ $$\to$$ $$K^+$$ $$\Xi^-$$ reaction in nuclear medium [2], and $$\gamma_{\nu}^{+}$$ and $$\gamma_{\nu}^{+}$$ are the distorted waves for outgoing $$K^+$$ and incoming $$K^-$$ mesons, respectively; the factors of $$M_C/M_B$$ and $$M_C/M_A$$ take into account the recoil effects, where $$M_A$$ and $$M_B$$ and $$M_C$$ are masses of the target, the final state and the core-nucleus, respectively. $$(\alpha | \bar{\psi}_K(r) | \psi_{J A M_z})$$ is a hole-state wave function for a struck proton in the target, where $$\alpha$$ denotes the complete set of eigenstates for the system. It should be recognized that the $$\Lambda\Lambda$$-$$\Xi$$ coupled-channel Green's function with the spreading potential provides an advantage of estimating contributions from sources both as $$\Lambda\Lambda$$ components in $$\Xi^-$$-nucleus eigenstates [16] and as $$\Xi^-$$ $$\to$$ $$\Lambda\Lambda$$ quasi-scattering processes in the nucleus [15].

Because the momentum transfer is very high in the nuclear ($$K^-$$ $$\to$$ $$K^+$$) reaction at 1.8 GeV/c, i.e., $$q_{\Xi^-} \simeq 360-430 \text{ MeV/c}$$, the distorted waves for outgoing $$K^+$$ and incoming $$K^-$$ [Eq. (9)] are calculated with the help of the eikonal approximation [32,35]. As the distortion parameters, we use total cross sections of $$\sigma_{K^- N} = 28.9 \text{ mb}$$ for $$K^-$$ $$\Xi^-$$ scattering and $$\sigma_{K^+ N} = 19.4 \text{ mb}$$ for $$K^+$$ $$\Xi^-$$ scattering [6], and $$\alpha_{K^- N} = \alpha_{K^+ N} = 0.$$ We take 35 $$\mu$$b/sr as the laboratory cross section of $$\sigma d\sigma/dQ \approx \hat{A} f_{K^-} p - K^+ \Xi^-$$ including the kinematical factor $$\hat{A}$$ [9,5]. For the target nucleus $$^{16}\text{O}$$ with $$J_A = 0^+$$, we assume the wave functions for the proton hole-states in the relative coordinate, which are calculated with central (WS-type) and spin-orbit potentials [22], by fitting to the charge rms radius of 2.72 fm [36]. For the energies (widths) for proton-hole states, we input 12.1(0.0), 18.4(2.5) and 36(10) MeV for the 0p$$_{1/2}^+_1$$, 0p$$_{3/2}^+$$ and 0s$$_{1/2}^+$$ states, respectively.

Three parameters, $$V_S$$, $$W_Z$$ and $$v_{\Xi N,\Lambda\Lambda}^X$$, are very important for calculating the inclusive spectra with the one-step mechanism. These parameters are strongly connected each other for the shape of the spectrum and its magnitude, as well as for the $$\Xi^-$$-$$\Lambda\Lambda$$ conversion and widths of the $$\Xi^-$$ states. Several authors [10,16,14] investigated the effects of the $$\Xi^-$$-$$\Lambda\Lambda$$ coupling in light nuclei evaluating the volume integrals for $$K^-$$-dependent $$\Xi^-$$-$$\Lambda\Lambda$$ effective interactions based on Nijmegen potentials [28], in which these values are strongly model dependent; for example, 250,9, 370.2, 501.5, 582.1 and 873.9 MeVfm$$^3$$ for NHC-D, NSC97e, NSC44a, NSC-F and NSCD4d potentials ($k_t = 1.0 \text{ fm}^{-1}$), respectively [14,28]. The $$\Xi^-$$ $$\to$$ $$\Lambda\Lambda$$ conversion cross section of (v0) $$\Xi^-$$ $$\to$$ $$\Lambda\Lambda$$ $$\simeq 7.9 \text{ mb}$$ also yields to be about 544 MeVfm$$^3$$ [16]. To see the dependence of the production on the $$\Xi^-$$-$$\Lambda\Lambda$$ coupling strength, here, we choose typical values of $$v_{\Xi N,\Lambda\Lambda}^X = 250$$ and 500 MeV, which approximate the volume integrals of NHC-D and NSC44a, respectively. We take the spreading potential of Im$$U_Z$$ to be $$U_Z \simeq -3 \text{ MeV}$$ at the $$^{15}\text{N} + \Xi^-$$ threshold [5,6,14,18]. It should be noticed that this spreading potential expresses nuclear core breakup processes.
caused by the $\Xi^-$ p → ΛΛ conversion in the $^{15}$N nucleus, and its effect cannot be involved in $U_X$.

3. Results and discussion

Now let us discuss the inclusive spectrum in the $^{16}$O(K$^-$, K$^+$) reaction at 1.8 GeV/c (0°) in order to examine the dependence of the spectrum on the parameters of $V_S$ and $v_{SN,AA}$. We consider contributions of the ΛΛ→Ξ nuclear bound and resonance states to the $\Xi^-$ p → ΛΛ conversion processes in the $\Xi^-$ bound region.

In Fig. 2(a), we show the calculated spectra in the $\Xi^-$ bound region without the ΛΛ→Ξ coupling potential when we use $V_S = -24$ MeV or $-14$ MeV with the Coulomb potential. The calculated spectra are in agreement with the spectra obtained by previous works [6]. In the case of $V_S = -24$ MeV, we find a broad peak of the $[^{15}\text{N}(1/2^-) \otimes s_{S^-}]_1^-$ quasibound state in $^{16}$C is located at $B_{S^-} = 13.4$ MeV with a sizable width of $\Gamma = 3.5$ MeV, and a clear peak of the $[^{15}\text{N}(1/2^-) \otimes p_{S^-}]_2^-$ quasibound state at $B_{S^-} = 3.7$ MeV with $\Gamma = 3.1$ MeV. Integrated cross sections indicate $\sigma(0^\circ) / d\Omega \simeq 28$ nb/sr for the 1$^-$ state and 77 nb/sr for the 2$^+$ state in $^{16}$C. In the case of $V_S = -14$ MeV, which is favored in recent calculations [6,13,14,18], we have the $[^{15}\text{N}(1/2^-) \otimes s_{S^-}]_1^-$-state at $B_{S^-} = 6.8$ MeV with $\Gamma = 3.8$ MeV and the $[^{15}\text{N}(1/2^-) \otimes p_{S^-}]_2^-$-state at $B_{S^-} = 0.5$ MeV with $\Gamma = 1.1$ MeV. The integrated cross sections indicate $\sigma(0^\circ) / d\Omega \simeq 6$ nb/sr for the 1$^-$ state and 9 nb/sr for the 2$^+$ state. Note that the $\Xi^-$ p → ΛΛ conversion processes that can be described by the absorption potential I$\mu U_{S^-}$ must appear above the $^{15}\text{C} + n$ decay threshold at $\omega = 360.4$ MeV (which corresponds to $B_{AA} = 16.7$ MeV). We confirm that no clear signal of the $\Xi^-$ bound state is measured if $V_S$ is sallow such as $V_S \leq 14$ MeV and/or $W_S$ is sizably absorptive ($-W_S \geq 3$ MeV at the $^{15}$N + $\Xi^-$ threshold) in $U_{S^-}$. Nevertheless, the production of these $\Xi^-$ states as well as $\Xi^-$ states coupled to a $^{15}$N(3/2$^-$) nucleus is essential in this model because these states act as doorways when we consider the ΛΛ states formed in the one-step mechanism. We also expect to extract properties of the Ξ–nucleus potential such as $V_S$ and $W_S$ from the $\Xi^-$ continuum spectra in the $(K^-, K^+)$ reactions on nuclear targets, as already discussed for studies of the $\Xi^-$–nucleus potential in nuclear $(\pi^-, K^+)$ reactions [37,38].

On the other hand, the ΛΛ→Ξ coupling plays an important role in making a production of the ΛΛ states via $\Xi^-$ doorways below the $^{15}$N + $\Xi^-$ threshold. The positions of their peaks must be slightly shifted downward by the energy shifts $\Delta B_{AA}$ due to the coupling potential in Eq. (7). When $v_{SN,AA} = 500$ MeV (250 MeV), we obtain $\Delta B_{AA} = 1.17$ MeV (0.15 MeV) and the $\Xi^-$ admixture probability $P_{S^-} = 5.24\%$ (0.87%) in the $[^{14}\text{C}(0^+) \otimes s_{S^-}]_1^-$ excited state and $\Delta B_{AA} = 0.38$ MeV (0.09 MeV) and $P_{S^-} = 0.58\%$ (0.14%) in the $[^{14}\text{C}(0^+) \otimes s_{S^-}]_1^+$ ground state. The value of $P_{S^-}$ in the 1$^-$ state is by a factor of 6–9 as large as that in the 0$^+$ state. These values are strongly connected with the magnitude of the peak for the ΛΛ states in the spectrum.

In Fig. 2(b), we show the calculated spectra with the ΛΛ→Ξ coupling potential when $V_S = -14$ MeV. We recognize that the shape of these spectra is quite sensitive to the value of $v_{SN,AA}$. It is obvious that no $\Xi^-$→ΛΛ coupling cannot describe the spectrum of the ΛΛ states below the $^{14}$C + Λ + Λ threshold. The calculated spectrum for $v_{SN,AA} = 500$ MeV has a fine structure of the ΛΛ excited states in $^{16}$C. We find that significant peaks of the 1$^-$ excited states with $^{14}$C + Λ + Λ threshold. The calculated spectrum for $v_{SN,AA} = 500$ MeV leads to a preferential population of the spin-stretched $\Xi^-$–doorways states followed by the $[^{15}\text{N}(1/2^-, 3/2^-) \otimes s_{S^-}]_1^-$ transitions, to which a sum of their continuum states may contribute predominately in the $(K^-, K^+)$ reactions. Fig. 3 also displays partial-wave decomposition of the calculated inclusive spectrum for $^{16}$C in the ΛΛ bound region when $V_S = -14$ MeV and $v_{SN,AA} = 500$ MeV. The integrated cross sections at $\theta_{AA} = 0^\circ$ for the 1$^-$ excited states with $^{14}$C$(0^+)$ and $^{14}$C$(2^+)$ are respectively

$$\frac{d\sigma}{d\Omega}(\Xi^{16}_S(1^-)) \simeq 7$ nb/sr and 12 nb/sr,

where the $\Xi^-$ admixture probabilities of these states amount to $P_{S^-} = 5.2\%$ and 8.8\%, respectively. It should be noticed that the cross sections are on the same order of magnitude as those for the 1$^-$ and 2$^+$ quasibound states that are located at $B_{S^-} = 6.8$ MeV and 0.5 MeV, respectively, in the $^{16}$C hypernucleus. Therefore,
such $\Lambda\Lambda$ excited states below the $^{14}\text{C} + \Lambda + \Lambda$ threshold will be measured experimentally at the J-PARC facilities [3].

On the other hand, it is extremely difficult to populate the $0^+$ ground state with $^{14}\text{C}(0^+) \otimes s_\Lambda^2$ at $\omega \simeq 352.3$ MeV ($B_{\Lambda\Lambda} \simeq 24.9$ MeV) and also the $2^+$ excited state with $^{14}\text{C}(2^+) \otimes s_\Lambda^2$ at $\omega \simeq 359.6$ MeV ($B_{\Lambda\Lambda} \simeq 17.5$ MeV) in the one-step mechanism via $\pi^-$ doorways in the $(K^-, K^+)$ reactions. The high momentum transfer of $q_\pi \simeq 400$ MeV/c necessarily leads to the non-observability with $\Delta L = 0$. Thus the integrated cross section of the $0^+$ state is found to be about 0.02 nb/sr, of which the $g$ dependence is approximately governed by a factor of $\exp(-\frac{1}{2}(bq_\pi)^2)$ where a size parameter $b = 1.84$ fm. There is no production in the $2^+$ state with $^{14}\text{C}(2^+) \otimes s_\Lambda^2$ under the angular-momentum conservation in the $^{16}\text{O}(K^-, K^+)$ reactions by the one-step mechanism. The contribution of these states to the $\Lambda\Lambda$ spectrum in the one-step mechanism is completely different from that in the two-step mechanism as obtained in Refs. [7,8].

In the $(K^-, K^+)$ reaction, $\Lambda\Lambda$ hypernuclear states can be also populated by the two-step mechanism, $K^- p \to \pi^0 A$, followed by $\pi^0 p \to K^+ A$ [7–9], as shown in Fig. 1(a). Following the procedure by Dover [7,9], a crude estimate can be obtained for the contribution of this two-step processes in the eikonal approximation using a harmonic oscillator model. The cross section at $0^0$ for quasielastic $\Lambda\Lambda$ production at $p_K^- = 1.8$ GeV/c in the two-step mechanism, which is summed over all final state, is given by [9] as

$$\sum_f \left( \frac{da_f^{(2)}}{d\Omega_f^2} \right)_{0^0} \approx 2\pi \frac{\xi}{p_f^3} \left( \frac{1}{r_f^2} \right) \left( \frac{\alpha}{d\Omega_f^2} \right)_{0^0} \times \left( \frac{\alpha}{d\Omega_f^2} \right)_{0^0} \pi^0 p \to K^+ A \cdot N_{\text{eff}}^P. \quad (11)$$

where $\xi = 0.022–0.019$ mb$^{-1}$ is a constant nature of the angular distributions of the two elementary processes, $p_T \approx 1.68$ GeV/c is the intermediate pion momentum, and $\left( 1/r_f^2 \right) \approx 0.028$ mb$^{-1}$ is the mean inverse-square radial separation of the proton pair. $N_{\text{eff}}^P \approx 1$ is the effective number of proton pairs including the nuclear distortion effects [7]. The elementary laboratory cross section $(\alpha d\sigma/d\Omega_f)_{0^0}$ is estimated to be $1.57–1.26$ mb/sr for $K^- p \to \pi^0 A$ and $0.070–0.067$ mb/sr for $\pi^0 p \to K^+ A$ depending on the nuclear medium corrections. This yields

$$\sum_f \left( \frac{da_f^{(2)}}{d\Omega_f^2} \right)_{0^0} \simeq 0.06–0.04 \mu b/sr, \quad (12)$$

which is half smaller than $\sim 0.14$ mb/sr at 1.1 GeV/c. Considering a high momentum transfer $q \simeq 400$ MeV/c in the $(K^-, K^+)$ reactions by comparison with the $\pi^0$ production reaction [39], we expect that the production probability for the $\Lambda\Lambda$ bound states does not exceed 1% in the quasielastic $\Lambda\Lambda$ production, so that an estimate of the $\Lambda\Lambda$ hypernucleus in the two-step mechanism may be on the order of 0.1–1 nb/sr. This cross section is smaller than the cross section for the $\Lambda\Lambda$ $1^-$ states we mentioned above in the one-step mechanism. Consequently, we believe that the one-step mechanism acts in a dominant process in the $(K^-, K^+)$ reaction at 1.8 GeV/c ($0^0$) when $v_{2n,\Lambda\Lambda} = 400–600$ MeV. This implies that the $(K^-, K^+)$ spectrum provides valuable information concerning $\Xi N - \Lambda\Lambda$ dynamics in the $S = -2$ systems such as $\Lambda\Lambda$ and $\Xi$ hypernuclei, which are often discussed in a full coupling scheme [40].

4. Summary and conclusion

We have examined theoretically production of doubly strange hypernuclei in the DCX $^{16}\text{O}(K^-, K^+)$ reaction at 1.8 GeV/c within DWIA calculations using coupled-channel Green’s functions. We have shown that the $S = -2$ admixture in the $\Lambda\Lambda$ hypernuclei plays an essential role in producing the $\Lambda\Lambda$ states in the $(K^-, K^+)$ reaction.

In conclusion, the calculated spectrum for the $^{16}\text{C}$ and $^{16}\text{C}/\Lambda\Lambda$ hypernuclei in the one-step mechanism $K^- p \to K^+ \Xi^-$ via $\Xi^-$ doorways predicts promising peaks of the $\Lambda\Lambda$ bound and excited states in the $^{16}\text{O}(K^-, K^+)$ reactions at 1.8 GeV/c ($0^0$). It has been shown that the integrated cross sections for the significant $1^-$ excited states in $^{16}\text{C}$ are on the order of 7–12 nb/sr depending on the $\Xi N - \Lambda\Lambda$ coupling strength and also the attraction in the $\Xi$–nucleus potential. The $S = -2$ admixture probabilities are on the order of 5–9%. The sensitivity to the potential parameters indicates that the nuclear $(K^-, K^+)$ reactions have a high ability for the theoretical analysis of precise wave functions in the $\Lambda\Lambda$ and $\Xi$ hypernuclei. New information on $\Lambda\Lambda$–$\Xi$ dynamics in nuclei from the $(K^-, K^+)$ data at J-PARC facilities [3] will bring the $S = -2$ world development in nuclear physics.

Acknowledgements

The authors are obliged to T. Fukuda, Y. Akaishi, D.E. Lansky, T. Motoba and T. Nagae for many discussions. This work was supported by Grants-in-Aid for Scientific Research on Priority Areas (Nos. 17070002 and 20028010) and for Scientific Research (C) (No. 22540294).

References

[1] For example in: A. Gal, R.S. Hayano (Eds.), Special Issue on Recent Advances in Strangeness Nuclear Physics, Nucl. Phys. A 804 (2008) 1.
[2] C.B. Dover, A. Gal, Ann. Phys. 146 (1983) 309.
[3] T. Nagae, et al., J-PARC proposal E05, http://j-parc.jp/NuclPart/Proposal_e.html.
[4] T. Fukuda, et al., Phys. Rev. 58 (1998) 1306.
[5] P. Khaustov, et al., Phys. Rev. C 61 (2000) 054603.
[6] S. Takahara, H. Kobayashi, Y. Akaishi, Phys. Rev. C 51 (1995) 2656; H. Maekawa, K. Tsuchibahara, A. Omishii, Eur. Phys. J. A 33 (2007) 269; S. Hashimoto, M. Kohno, K. Ogata, M. Kawai, Prog. Theor. Phys. 119 (2008) 1005.
[7] C.B. Dover, Nucl. Phys. A 25 (1980) 521.
[8] A.J. Baltz, C.B. Dover, D.J. Millener, Phys. Lett. B 123 (1983) 9.
[9] T. Iijima, et al., Nucl. Phys. A 546 (1992) 588.
[10] Khin Swe Myint, Y. Akaishi, Prog. Theor. Phys. Suppl. 117 (1994) 251.
[11] S.B. Carr, I.R. Afnan, B.F. Gibson, Nucl. Phys. A 625 (1997) 143.
[12] T. Yamada, C. Nakamoto, Phys. Rev. C 62 (2000) 034319.
[13] Khin Swe Myint, S. Shinnmura, Y. Akaishi, Eur. Phys. J. A 16 (2003) 21.
[14] D.E. Lanskoy, Y. Yamamoto, Phys. Rev. C 69 (2004) 014303;
    D.E. Lanskoy, in: Proceedings of the XVIII International Workshop on Quantum
    Field Theory and High Energy Physics (QFTHEP-04), Saint-Petersburg, Russia,
    June 2004, p. 419.
[15] K. Ikeda, et al., Prog. Theor. Phys. 91 (1994) 747.
[16] C.B. Dover, A. Gal, D.J. Millener, Nucl. Phys. A 572 (1994) 85;
    D.J. Millener, C.B. Dover, A. Gal, Prog. Theor. Phys. Suppl. 117 (1994) 307.
[17] H. Takahashi, et al., Phys. Rev. Lett. 87 (2001) 212502.
[18] E. Hiyama, et al., Phys. Rev. C 66 (2002) 024007.
[19] O. Morimatsu, K. Yazaki, Prog. Part. Nucl. Phys. 33 (1994) 679, and references
    therein.
[20] T. Harada, Phys. Rev. Lett. 81 (1998) 5287;
    T. Harada, Nucl. Phys. A 672 (2000) 181.
[21] T. Harada, A. Umeya, Y. Hirabayashi, Phys. Rev. C 79 (2009) 014603.
[22] A. Bohr, M. Mottelson, Nuclear Structure, vol. 1, Benjamin, New York, 1969,
    p. 238.
[23] E. Hiyama, et al., Phys. Rev. C 78 (2008) 054316.
[24] G.R. Satchler, Direct Nuclear Reactions, Oxford University Press, New York,
    1983, p. 464.
[25] D.J. Millener, C.B. Dover, A. Gal, Phys. Rev. C 38 (1988) 2700.
[26] B.A. Watson, P.P. Singh, R.E. Segel, Phys. Rev. 182 (1969) 977.
[27] Y. Yamamoto, H. Bandô, Phys. Lett. B 214 (1988) 174.
[28] Y. Yamamoto, Th.A. Rijken, Nucl. Phys. A 804 (2008) 139, and references
    therein.
[29] N.K. Glendenning, Direct Nuclear Reactions, Academic Press, New York, 1983,
    p. 144.
[30] S. Cohen, D. Kurath, Nucl. Phys. A 101 (1967) 1.
[31] A.E.L. Dieperink, T. de Forest, Phys. Rev. C 10 (1974) 543.
[32] J. Hüfner, S.Y. Lee, H.A. Weidenmüller, Nucl. Phys. A 234 (1974) 429.
[33] E.H. Auerbach, et al., Ann. Phys. (N.Y.) 148 (1983) 381.
[34] T. Koike, T. Harada, Nucl. Phys. A 804 (2008) 231.
[35] T. Harada, Y. Hirabayashi, Nucl. Phys. A 744 (2004) 323.
[36] H. de Vries, C.W. de Jager, C. de Vries, At. Data Nucl. Data Tables 36 (1987) 459.
[37] E. Friedman, A. Gal, Phys. Rep. 452 (2009) 89.
[38] T. Harada, Y. Hirabayashi, Nucl. Phys. A 759 (2005) 143;
    T. Harada, Y. Hirabayashi, Nucl. Phys. A 767 (2006) 206.
[39] H. Hotch, et al., Phys. Rev. C 64 (2001) 044302.
[40] H. Nemura, S. Shinnmura, Y. Akaishi, Khin Swe Myint, Phys. Rev. Lett. 94 (2005)
    202502.