A novel measurement of initial-state gluon radiation in hadron collisions using Drell-Yan events

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A study of initial-state gluon radiation (ISR) in hadron collisions is presented using Drell-Yan (DY) events produced in proton-antiproton collisions by the Tevatron collider at a center-of-mass energy of 1.96 TeV. This paper adopts a novel approach which uses the mean value of the $Z/\gamma^*$ transverse momentum ($p_T^{DY}$) in DY events as a powerful observable to characterize the effect of ISR. In a data sample corresponding to an integrated luminosity of 9.4 fb$^{-1}$ collected with the CDF II detector, $\langle p_T^{DY}\rangle$ is measured as a function of the $Z/\gamma^*$ invariant mass. It is found that these two observables have a dependence, $\langle p_T^{DY}\rangle = -8 + 2.2 \ln m_{DY}^2$ [GeV/c$^2$], where $m_{DY}$ is the value of the $Z/\gamma^*$ mass measured in units of GeV/c$^2$. This linear dependence is observed for the first time in this analysis. It may be exploited to model the effect of ISR and constrain its impact in other processes.

I. INTRODUCTION

Since the discovery of the Higgs boson [1, 2] in elementary particle physics, the search for physics beyond the Standard Model (SM) has become the main focus of attention. To identify small deviations from the SM expectations due to beyond the SM (BSM) physics, a precise understanding of the SM processes is required. At hadron colliders SM interactions are commonly accompanied by clusters of final-state hadrons (jets) generated from initial-state gluon radiation (ISR). A large fraction of these hadrons have low transverse momentum (pr) and are difficult to simulate and measure correctly. An accurate modeling of ISR is essential in BSM searches at the LHC, since many relevant models of BSM physics predict the production of undetectable particles (such as those expected from dark-matter candidates) whose presence can be inferred by triggering on single isolated jets produced by ISR. A detailed understanding of ISR effects would also benefit a precise measurement of the top-quark mass by enabling accurate modeling of the top-quark transverse momentum distribution.

In hadron collisions, quantum-chromodynamics (QCD) gluon emissions from the interacting partons are conventionally classified into two categories: hard and soft/collinear emissions. The hard QCD emissions are approximately described by perturbative QCD [3]. The soft/collinear QCD emissions are mostly non-perturbative in nature and they are absorbed into the parton distribution functions (PDFs). Parton-shower algorithms have been developed to approximate the physics of the non-perturbative emissions using the DGLAP equations [4,5]. The algorithms simulate the general features of non-perturbative QCD, but their accuracy may be insufficient for high-precision analyses [6] or some BSM particle searches. A systematic approach to study non-perturbative emissions is required.

The PDFs and QCD radiation are closely associated. The DGLAP equations describe the evolution of PDFs, which have been studied extensively in lepton-nucleon inelastic-scattering experiments [9]. The DGLAP equations describe the change in the quark density function.
of incoming quarks due to QCD processes:

\[
\frac{dq(x, Q^2)}{d \ln Q^2} = \int_x^1 \frac{dy}{y} \alpha_s(Q^2) P_{q\to qg} \left( \frac{x}{y}, Q^2 \right) q(y, Q^2). \tag{1}
\]

Here, \(\alpha_s\) is the strong coupling constant which is a function of the energy scale, \(Q^2\). The term \(q(y, Q^2)\) is the quark density function with momentum fraction \(y (> x)\) in the proton, and \(P_{q\to qg}\) is the QCD splitting function that gives the probability for the incoming quark to split into a quark and a gluon. Since all of these QCD processes are described by the DGLAP equations which have a logarithmic \(Q^2\) dependence, this study investigates the effects of QCD ISR as a function of \(Q^2\).

The Drell-Yan (DY) production of lepton pairs (\(p\bar{p} \rightarrow Z/\gamma^* \rightarrow ee, \mu\mu\)) is ideal to study QCD ISR. At the Born level, DY lepton pairs are produced with zero transverse momentum. The emission of one or more gluons from the initial-state quarks gives rise to a transverse momentum for the DY lepton pair. Thus, a transverse momentum of the DY lepton pair is a good observable to study the effect of QCD ISR. The final state of the DY lepton pair is free from final-state QCD radiation. This simplifies the interpretation of the measurements. In addition, the parton-parton energy scale \(Q^2\) is characterized by the squared dilepton mass \(m_{\ell\ell}^2\).

Using Pythia8 simulations, it is found that the truncated mean of \(p_T^2\) distributions \((p_T)\) for the DY process and other DY-type process (such as \(W\) boson or top-quark pair production) can be described with a function linear in the logarithm of the energy scale of the hard process with universal slope, as a consequence of the DGLAP evolution. This universality is illustrated in Fig. 1 which shows the prediction for \((p_T)\) of different processes, exhibiting a common dependence on the energy scale \(Q^2\). Since the recoil of the hard process due to ISR is the main factor of \((p_T)\), it can be used as a powerful observable to probe the effect of ISR.

This paper presents a novel approach to characterize the effect of ISR, which can be observed in the average \(p_T\) of DY lepton pairs as a function of the energy scale \(Q^2\). The measurements of the average \(p_T\) at various energy scales are performed using proton-antiproton (\(p\bar{p}\)) collision data at center-of-mass energy \(\sqrt{s} = 1.96\) TeV produced by the Tevatron collider and collected with the CDF II detector. The measured values are corrected for QED final-state radiation (FSR) effects and the exper-imental effects such as detector acceptance and resolution. Using such measurements a relationship between the \(p_T\) and mass-scale observables in the DY process can be established. This may improve modeling of ISR phenomenology as well as constrain its impact on the measurement of other processes in hadron colliders.

II. THE EXPERIMENTAL APPARATUS

The CDF II detector is a solenoidal magnetic spectrometer surrounded by calorimeters and muon detectors, which operated at the Tevatron proton-antiproton collider from 2001 until 2011. CDF II uses a cylindrical coordinate system with the positive \(z\)-axis along the proton beam direction. For particle trajectories, the polar angle \(\theta\) is relative to the proton direction and the azimuthal angle \(\phi\) is oriented about the beamline axis with \(\pi/2\) being vertically upwards. Detector coordinates are specified as \((\eta, \phi)\), where \(\eta\) is the pseudorapidity defined as \(-\ln \tan(\theta/2)\). The detector is described in detail in Ref. [11].

The beam pipe is surrounded by a 2 m long silicon vertex-tracker covering a pseudorapidity range of \(|\eta| < 2\) [12]. The central charged-particle tracking detector, which is a 3.1 m long open-cell drift chamber, extends radially from 0.4 to 1.4 m covering the range \(|\eta| < 1.0\) [13]. Both trackers are positioned in a 1.4 T axial magnetic field produced by a superconducting solenoid surrounding the outer radius of the drift chamber. Outside the solenoid is a central barrel calorimeter in the region \(|\eta| < 1.1\) [14, 15]. The forward endcap regions are covered by plug calorimeters in the regions 1.1 < \(|\eta| < 3.5\) [16]. Muon detectors are the outermost charged-particle trackers and cover the region \(|\eta| < 1.5\) [17].

The data were collected using a three-level electronics system (trigger). The first level, relying on special-

\[p_T^2 = \int_{x=1}^{\infty} \frac{1}{y} \frac{d\alpha_s}{d\ln Q^2} \left( \frac{x}{y}, Q^2 \right) q(y, Q^2).\]
purpose processors, and the second level, using a mixture of dedicated processors and fast software algorithms, reduce the event accept-rate to a level manageable by the data acquisition system. The accepted events are processed online at the third-level trigger [13] with fast reconstruction algorithms [19], and are recorded for offline analysis.

III. EVENT SELECTION

The DY candidate muon-pairs were accepted online by a single-muon trigger [11] with track $p_T$ threshold of 18 GeV/$c$. Electron candidate [11] pairs were accepted online by single- and double-electron triggers [11][20]. The single-electron trigger requires at least one electron candidate with transverse energy $E_T$ greater than 18 GeV and an associated track with $p_T$ larger than 9 GeV/$c$. For the double-electron trigger, events are accepted when containing at least two electron candidates without requiring an associated track.

Events collected are further required to pass the following offline-selection criteria. Muon candidates are required to have matching track elements in the muon chambers (except when the track extrapolates outward to an uninstrumented region (gap) of the muon detector), and an energy deposition in the calorimeters consistent with that for a minimum-ionizing particle [11]. Track-quality selections are also imposed on the candidates. The selection of muon pairs requires two oppositely-charged muon candidates with $|\eta| < 1.5$. The muon candidate with leading (sub-leading) $p_T$ is required to be larger than 20 (12) GeV/$c$.

Electron candidates are either reconstructed in the central electromagnetic calorimeter or in the forward region covered by the plug electromagnetic calorimeter [11]. Two levels of identification criteria are used for central-electron candidates. High-quality criteria select ‘tight’ central electron (TCE) candidates by requiring a matching good-quality track in the tracking chamber and a shower profile consistent with that of an electron in the electromagnetic calorimeter. Less stringent criteria select ‘loose’ central electron (LCE) candidates by relaxing some track- and shower-quality requirements. Plug-electron identification uses plug electromagnetic calorimeter information and requires an associated track reconstructed in the silicon tracker.

Electron pairs are classified into three topologies depending on where each electron candidate is reconstructed, central-central (CC, 40% of selected candidates), central-plug (CP, 46%), and plug-plug (PP, 14%). For CC-topology pairs, at least one electron candidate should pass TCE identification. The two electron candidates are required to be oppositely charged, and the candidate with leading (sub-leading) $E_T$ is required to have $E_T > 25$ (15) GeV. In the CP-topology, the central-electron candidate is required to pass TCE criteria. Both electron candidates are required to have $E_T > 20$ GeV.

The plug-electron pair candidates (PP-topology) are required to pass the plug-electron identification criteria and to have $E_T > 25$ GeV. In addition, PP-topology pairs are requested to be contained in the same side plug detector to reduce the QCD background, because the QCD multijet events tend to have larger $\eta$ difference between the two leptons than DY events. The plug-electron candidates have a poor charge identification because electron tracks in the forward region are constructed only using silicon detector information due to the small coverage of the drift chamber. Thus, the opposite-charge requirement is not applied to CP- and PP-topology pairs. In order to reduce misreconstructed events and contributions from background processes, the transverse energy imbalance ($E_T$) [21] associated to the selected dielectron events is required to be lower than 40 GeV.

Dielectron events produced at the Z-mass peak may be reconstructed at much lower masses if the electron loses a significant portion of its energy by radiating photons. It is difficult to simulate photons from the QED FSR process at small dielectron masses ($m_{ee}$), where the effect is comparatively more important due to this migration. To suppress the migration effects, the following additional selection criteria are applied to the events with dielectron invariant mass $m_{ee} < 80$ GeV/$c^2$ based on a study using a DY simulated sample:

Reject if $|\Delta\phi(e_1,e_2) - \pi| < 0.25$ and

\[
\begin{align*}
\Delta p_T(e_1,e_2) &> 15 \text{ GeV}/c, \text{ when } E_T < 15 \text{ GeV} \\
\Delta p_T(e_1,e_2) &> 10 \text{ GeV}/c, \text{ when } E_T > 15 \text{ GeV}.
\end{align*}
\]

Here, $e_1$ and $e_2$ are the leading and the sub-leading electrons in $p_T$, $\Delta\phi(e_1,e_2)$ is the azimuthal opening angle of electron pairs, and $\Delta p_T(e_1,e_2)$ is the scalar $p_T$ difference of the two electrons.

Differences in acceptance and efficiency between simulation and data are corrected by applying scale factors to the event yields reconstructed in simulation. The trigger and identification scale factors for electrons are obtained using unbiased electron samples in data as functions of pseudorapidity, $E_T$, number of extra vertices in the event, and the time when the events were collected. The scale factors for the muons depend on the detector topology of the muon pair and data taking time. For the selected electron- and muon-pair events of the simulation, the number of primary vertices and the distribution of the vertices along the beamline are weighted to match their corresponding distributions in data.

As this analysis focuses on QCD ISR mostly from non-perturbative QCD emissions, all candidates are required to have the dilepton system transverse momentum $p_T^{\ell\ell} < 100$ GeV/$c$. This selection also has the benefit of removing events with poorly reconstructed muons, which tends to have large measured $p_T^{\mu\mu}$ values.
The simulated events are dominated by the $Z/\gamma^* \to \mu\mu$ signal process, and there are also contributions from $Z/\gamma^* \to \tau\tau$, diboson ($WW$, $WZ$, $ZZ$), $t\bar{t}$, $W+jets$, and QCD multijet processes, which are considered as backgrounds. They are all estimated using a fit to the experimental dielectron mass distribution following Ref. [20]. The sum of the expected SM contributions agrees with the data as shown in Fig. 3 and Table II. The effect of the QCD ISR is probed by measuring the truncated mean of $Z/\gamma^*$ $p_T$ distribution in bins of the lepton-pair mass. Several corrections relevant for this measurement are described in this section.

First, the lepton energy and momentum are calibrated using muon pairs assumed to have the same kinematics with opposite-sign events. The procedure described in Ref. [24], the multijet background for dimuon events is determined from the data using events with same-sign muon pairs. The selected dimuon events are dominated by the $Z/\gamma^* \to \mu\mu$ signal process, and there are also contributions from $Z/\gamma^* \to \tau\tau$, diboson ($WW$, $WZ$, $ZZ$), $t\bar{t}$, $W+jets$, and QCD multijet processes, which are considered as backgrounds. The total cross sections corresponding to the size of simulated samples are normalized using NLO QCD calculations respectively [20 25 26]. Following the procedure described in Ref. [22], the multijet background for dimuon events is estimated from data using events with same-sign muon pairs having the same kinematics with opposite-sign events. The selected dimuon events are dominated by the $Z/\gamma^* \to \mu\mu$ signal process, and there are also contributions from $Z/\gamma^* \to \tau\tau$, diboson ($WW$, $WZ$, $ZZ$), $t\bar{t}$, $W+jets$, and QCD multijet processes, which are considered as backgrounds. As in the case of the dimuon final state, all processes are modeled using simulation except for QCD multijet production, which is estimated from data. The dielectron mass shape of QCD multijet events is obtained from data selected through inverse electron isolation requirements and normalized using a fit to the experimental dielectron mass distribution following Ref. [20].

The selected dimuon events are dominated by the $Z/\gamma^* \to \mu\mu$ signal process, and there are also contributions from $Z/\gamma^* \to \tau\tau$, diboson ($WW$, $WZ$, $ZZ$), $t\bar{t}$, $W+jets$, and QCD multijet processes, which are considered as backgrounds. They are all estimated using a fit to the experimental dielectron mass distribution following Ref. [20]. The sum of the expected SM contributions agrees with the data as shown in Fig. 3 and Table II.
TABLE I. The number of observed dimuon events and the number of estimated events with statistical uncertainties in the full CDF II sample, corresponding to 9.4 fb\(^{-1}\) of integrated luminosity, in five dimuon-mass ranges. The fractions for the individual SM processes are listed below the dashed line.

| \(m_{\mu\mu} (\text{GeV}/c^2)\) | [40, 60) | [60, 80) | [80, 100) | [100, 200) | [200, 350) |
|-----------------|--------|--------|--------|--------|--------|
| Data yield      | 16754  | 18471  | 24472  | 13089  | 562    |
| Estimated yield | 15975 ± 36 | 17786 ± 30 | 24424 ± 80 | 13184 ± 24 | 580 ± 5 |
| \(Z/\gamma^* \rightarrow \mu\mu\) | 91.45% | 97.42% | 99.86% | 98.99% | 97.78% |
| QCD multijet     | 2.82%  | 0.54%  | 0.01%  | 0.09%  | 0.04%  |
| \(Z/\gamma^* \rightarrow \tau\tau\) | 4.85% | 1.40% | 0.01% | 0.06% | 0.02% |
| Diboson          | 0.42%  | 0.40%  | 0.12%  | 0.52%  | 1.32%  |
| \(W+\text{jets}\) | 0.29%  | 0.07%  | 0.00%  | 0.04%  | 0.00%  |
| \(t\bar{t}\)      | 0.17%  | 0.16%  | 0.01%  | 0.31%  | 0.84%  |

TABLE II. The number of observed dielectron events and the number of estimated events with statistical uncertainties in the full CDF II sample, corresponding to 9.4 fb\(^{-1}\) of integrated luminosity, in five dielectron-mass ranges. The fractions for the individual SM processes are listed below the dashed line.

| \(m_{ee} (\text{GeV}/c^2)\) | [40, 60) | [60, 80) | [80, 100) | [100, 200) | [200, 350) |
|-----------------|--------|--------|--------|--------|--------|
| Data yield      | 11590  | 28856  | 49021  | 27956  | 1357   |
| Estimated yield | 11416 ± 53 | 28374 ± 54 | 49339 ± 158 | 27481 ± 51 | 1307 ± 15 |
| \(Z/\gamma^* \rightarrow ee\) | 85.88% | 93.78% | 99.70% | 94.67% | 85.12% |
| QCD multijet     | 7.58%  | 3.45%  | 0.18%  | 3.96%  | 10.98% |
| \(Z/\gamma^* \rightarrow \tau\tau\) | 4.70% | 1.95% | 0.01% | 0.06% | 0.12% |
| Diboson          | 0.33%  | 0.22%  | 0.08%  | 0.35%  | 1.15%  |
| \(W+\text{jets}\) | 1.03%  | 0.43%  | 0.02%  | 0.62%  | 1.48%  |
| \(W\gamma\)      | 0.43%  | 0.13%  | 0.01%  | 0.25%  | 0.87%  |
| \(t\bar{t}\)      | 0.07%  | 0.03%  | 0.00%  | 0.09%  | 0.28%  |

which is fast and efficient but is restricted in precision to
the leading logarithmic order in the approximation used.
This approach sums all emissions by evolving from large
to small \(Q^2\) by reformulating the DGLAP equations. Un-
certainties in this evolution and missing higher-order ef-
fects may introduce a mismodeling of the \(p_{\ell\ell}T\) distribution.
Mistuning of the intrinsic transverse momentum of the
incoming partons may also contribute to an inaccuracy.
Consequently, the simulated \(p_{\ell\ell}T\) distribution is adjusted
to agree with the data by reweighting the \(Z/\gamma^*\) boson \(p_T\)
at the generator level, where the correction is assumed
to be independent of the lepton flavor. This correction
is iteratively extracted from the reconstructed \(p_{\ell\ell}T\) distributions of both dielectron and dimuon events within the
\(Z\)-peak mass region of 66–116 GeV/c\(^2\). The correction
is a function of the \(p_T\) and the rapidity of the boson.
The \(p_T\) dependence of the weight is parametrized using a
continuous piecewise polynomial function of \(\ln(p_T)\) and
the rapidity dependence is parametrized using a linear
function.

The mean values of the \(Z/\gamma^*\) transverse momentum \(\langle p_T^{DY} \rangle\) in DY events are obtained from the average val-
ues of the reconstructed \(p_{\ell\ell}T\) distribution \(\langle p_{\ell\ell}T \rangle\) by mul-
tiplying corrections for acceptance, detector and QED
FSR effects. The mean values of the \(Z/\gamma^*\) mass \(\langle m_{DY} \rangle\) are also obtained from the average values of the recon-
structed lepton-pair mass \(\langle m_{\ell\ell} \rangle\) by the same procedure.
Combined corrections for the average \(p_T\) and mass of the
lepton pairs within a mass bin, denoted by \(R_{p_T}\) and \(R_m\) respectively, are derived from the simulated DY sample
and applied as shown below.

\[
\langle p_T^{DY} \rangle_{\text{data}} = R_{p_T} \times \langle p_{\ell\ell}T \rangle_{\text{MC}}
\]

where \(R_{p_T} = \frac{\langle p_T^{DY, \text{gen}} \rangle_{\text{MC}}}{\langle p_{\ell\ell}T \rangle_{\text{MC}}}\) and \(R_m = \frac{\langle m_{DY} \rangle_{\text{MC}}}{\langle m_{\ell\ell} \rangle_{\text{MC}}}\).
FIG. 4. Distributions of the reconstructed dimuon \( p_T \) in data (black circle) compared with distributions from the DY simulation after applying the \( Z/\gamma^* \) boson \( p_T \) corrections in each dimuon mass bin: (a) \([40 \text{ GeV/c}^2, 60 \text{ GeV/c}^2]\), (b) \([60 \text{ GeV/c}^2, 80 \text{ GeV/c}^2]\), (c) \([80 \text{ GeV/c}^2, 100 \text{ GeV/c}^2]\), (d) \([100 \text{ GeV/c}^2, 200 \text{ GeV/c}^2]\), (e) \([200 \text{ GeV/c}^2, 350 \text{ GeV/c}^2]\). Other backgrounds are subtracted from the data and the normalized residuals of the data and the DY simulation are shown in the lower panels. Statistical uncertainties (black bar) and systematic uncertainties (red shaded area) are shown.
FIG. 5. Distributions of the reconstructed dielectron $p_T$ in data (black circle) compared with distributions from the DY simulation after applying the $Z/\gamma^* \rightarrow e\gamma$ corrections in each dielectron mass bin: (a) $[40 \text{ GeV}/c^2, 60 \text{ GeV}/c^2]$, (b) $[60 \text{ GeV}/c^2, 80 \text{ GeV}/c^2]$, (c) $[80 \text{ GeV}/c^2, 100 \text{ GeV}/c^2]$, (d) $[100 \text{ GeV}/c^2, 200 \text{ GeV}/c^2]$, (e) $[200 \text{ GeV}/c^2, 350 \text{ GeV}/c^2]$. Other backgrounds are subtracted from the data and the normalized residuals of the data and the DY simulation are shown in the lower panels. Statistical uncertainties (black bar) and systematic uncertainties (red shaded area) are shown.
\begin{align}
\langle m_{\text{DY}} \rangle_{\text{data}} &= \mathcal{R}_m \times \langle m_{\ell\ell} \rangle_{\text{data}} \\
\text{where } \mathcal{R}_m &= \left( \frac{\langle m_{\text{DY}} \rangle_{\text{MC}}^{\text{gen}}} {\langle m_{\ell\ell} \rangle_{\text{MC}}^{\text{det}}} \right)^2.
\end{align}

Here, \( \langle p_T^{\text{DY}} \rangle_{\text{MC}}^{\text{gen}} \) and \( \langle m_{\text{DY}} \rangle_{\text{MC}}^{\text{gen}} \) are generator-level averages over the full phase space within a mass bin, and \( \langle p_T^{\text{DY}} \rangle_{\text{MC}}^{\text{det}} \) and \( \langle m_{\ell\ell} \rangle_{\text{MC}}^{\text{det}} \) are detector-level averages corresponding to the data. The corrections for \( p_T \) (\( \mathcal{P}_p \)) range from 0.75 to 1.00 and the corrections for \( m \) (\( \mathcal{R}_m \)) range from 0.82 to 1.01.

VI. SYSTEMATIC UNCERTAINTIES

The QCD ISR model, QED FSR model, energy and momentum corrections on leptons, and background normalizations are sources of systematic uncertainty of the measurements of \( \langle p_T^{\text{DY}} \rangle \) and \( \langle m_{\text{DY}} \rangle \). The uncertainties of \( \langle p_T^{\text{DY}} \rangle \) for the dimuon and dielectron final states are listed in Tables III and IV respectively. The QCD ISR model and QED FSR model uncertainties dominate.

The QCD ISR uncertainty is defined to be the uncertainty of the underlying \( Z/\gamma^* \)-boson \( p_T \) distribution used in the DY simulation. The simulated \( Z/\gamma^* \) boson \( p_T \) distribution is improved by reweighting the \( p_T \) distribution so as to agree with the data. Two sources of uncertainty of this correction are considered: the correction derived from events in the \( Z \)-mass region to mass values away from the \( Z \) peak, and the statistical uncertainty to the correction. The QCD ISR uncertainty is estimated using a set of simulated experiments in which the input values are varied within uncertainties from both sources.

The reconstructed \( Z/\gamma^* \) mass and \( p_T \) can be affected by the presence of QED FSR which is not fully contained in the reconstructed event products. The simulation accounts for this in the correction factors \( \mathcal{R}_p \) and \( \mathcal{R}_m \). The default model of QED FSR in PYTHIA6 is augmented with a QED shower algorithm which could be inaccurate for hard QED emissions. The PHOTOS QED generator [29] calculates single-photon radiation in the leading order of the electromagnetic interaction and simulates multiphoton radiation by iterating the single-photon calculation. To estimate the QED FSR uncertainty to the correction factors, the generator-level results for \( \langle p_T^{\text{DY}} \rangle \) and \( \langle m_{\text{DY}} \rangle \) before and after QED FSR are evaluated for PYTHIA6 and PHOTOS. The difference is taken as the QED FSR uncertainty.

The systematic uncertainty due to the electron energy and muon momentum scales is estimated using simulated samples by varying the scales within their uncertainties. The systematic uncertainty due to simulation inaccuracies in the energy and momentum resolution is evaluated by varying the simulation parameters that control the resolution smearing.

The uncertainties from the simulation-based description of the background processes are estimated by changing each cross section by its uncertainty, where the factorization, renormalization, and PDF uncertainties are considered. The 6% uncertainty on the determination of the integrated luminosity [20] is included. The background normalizations are treated as 100% correlated.

Using averages of \( m_{\text{DY}} \) over large bins may lead to biased results when representing \( \langle p_T^{\text{DY}} \rangle \) as a function of \( m_{\text{DY}} \) [30]. The additional systematic uncertainties accounting for this binning effect are considered for \( \langle m_{\text{DY}} \rangle \) measurement by taking the difference between \( \ln \langle m_{\text{DY}} \rangle \) and \( \langle \ln m_{\text{DY}}^2 \rangle \). This is the dominant systematic uncertainty for the \( \langle m_{\text{DY}} \rangle \) measurement, and ranges from 0.04% to 1.13%.

VII. RESULTS

The measurements of \( \langle p_T^{\text{DY}} \rangle \) and \( \langle m_{\text{DY}} \rangle \) in DY events are obtained for dimuon and dielectron final states. The dimuon and dielectron results are in good agreement as shown in Table V and Table VI. Results are combined using the best linear unbiased estimation [31] and are given in Table VII. Systematic uncertainties are combined accounting for all the correlations between final states and are listed in Table VII. Systematic uncertainties are correlated by approximately 12%.

VIII. CONCLUSIONS

This analysis presents a novel approach to characterize the effect of QCD ISR and a measurement of the ISR activity in DY events produced in proton-antiproton collisions at \( \sqrt{s} = 1.96 \) TeV, corresponding to an integrated luminosity of 9.4 fb\(^{-1}\). The measurement of the average lepton-pair \( p_T \) distribution is performed as a function of the dilepton invariant mass to quantify the effect of ISR activity. The average value of the lepton-pair \( p_T \) is found to have a linear dependence on \( \ln m_{\text{DY}}^2 \). The dependence is described by the function, \( \langle p_T^{\text{DY}} \rangle = A + B \ln m_{\text{DY}}^2 \), where \( m_{\text{DY}} \) is the value of the \( Z/\gamma^* \)-mass measured in units of GeV/c\(^2\). The average symbol for \( m_{\text{DY}} \) is omitted as the additional systematic uncertainty for the binning effect is included. The two fitted parameters are found to be strongly anti-correlated (99.97% [32]).

\begin{footnote}
One can reduce the correlation between the two parameters for easier interpretation by using a modified fitting function \( \langle p_T^{\text{DY}} \rangle = (11.82 \pm 0.02) + (2.15 \pm 0.09) \ln \frac{m_{\text{DY}}^2}{m_{Z,\text{pole}}} \text{[GeV}/c], where \( m_{Z,\text{pole}} \) is 91.19 GeV/c\(^2\). In this case, the two parameters are positively correlated by approximately 12%.
\end{footnote}
TABLE III. Fractional statistical and systematic uncertainties of \( \langle p_T^{DY} \rangle \) (in %) for each dimuon mass bin. Individual sources of systematic uncertainties are listed below the dashed line.

| Mass bin (GeV/c²) | [40, 60] | [60, 80] | [80, 100] | [100, 200] | [200, 350] |
|------------------|----------|----------|-----------|------------|------------|
| Statistical uncertainty (%) | 0.96 | 0.73 | 0.22 | 0.95 | 3.79 |
| Systematic uncertainty (%) | 1.29 | 1.33 | 0.26 | 0.90 | 2.76 |
| ISR model (%) | 0.90 | 0.93 | 0.24 | 0.50 | 2.28 |
| QED FSR model (%) | 0.87 | 0.93 | 0.03 | 0.18 | 0.41 |
| Momentum scale (%) | 0.12 | 0.13 | 0.04 | 0.20 | 0.86 |
| Momentum resolution (%) | 0.07 | 0.06 | 0.08 | 0.68 | 1.20 |
| Background normalization (%) | 0.28 | 0.10 | 0.03 | 0.16 | 0.25 |

TABLE IV. Fractional statistical and systematic uncertainties of \( \langle p_T^{DY} \rangle \) (in %) for each dielectron mass bin. Individual sources of systematic uncertainties are listed below the dashed line.

| Mass bin (GeV/c²) | [40, 60] | [60, 80] | [80, 100] | [100, 200] | [200, 350] |
|------------------|----------|----------|-----------|------------|------------|
| Statistical uncertainty (%) | 1.38 | 0.70 | 0.16 | 0.72 | 3.44 |
| Systematic uncertainty (%) | 2.03 | 0.89 | 0.12 | 0.69 | 2.11 |
| ISR model (%) | 1.37 | 0.47 | 0.06 | 0.33 | 1.68 |
| QED FSR model (%) | 1.39 | 0.72 | 0.05 | 0.22 | 0.67 |
| Energy scale (%) | 0.13 | 0.08 | 0.02 | 0.07 | 0.19 |
| Energy resolution (%) | 0.02 | 0.02 | 0.08 | 0.10 | 0.15 |
| Background normalization (%) | 0.57 | 0.21 | 0.03 | 0.56 | 1.06 |

TABLE V. Average values and their uncertainties of the dimuon mass and \( p_T \) for \( \mu \mu \) events.

| Mass bin (GeV/c²) | \( \langle m_{DY} \rangle \pm \text{stat} \pm \text{syst} \) (GeV/c²) | \( \langle p_T^{DY} \rangle \pm \text{stat} \pm \text{syst} \) (GeV/c²) |
|------------------|---------------------------------|---------------------------------|
| [40, 60]         | 47.72 ± 0.05 ± 0.31             | 9.12 ± 0.09 ± 0.12             |
| [60, 80]         | 70.66 ± 0.04 ± 0.27             | 10.81 ± 0.08 ± 0.14            |
| [80, 100]        | 90.99 ± 0.01 ± 0.08             | 11.84 ± 0.03 ± 0.03            |
| [100, 200]       | 115.29 ± 0.18 ± 1.30            | 13.17 ± 0.12 ± 0.12            |
| [200, 350]       | 243.33 ± 1.63 ± 2.52            | 16.18 ± 0.61 ± 0.45            |

TABLE VI. Average values and their uncertainties of the dielectron mass and \( p_T \) for \( ee \) events.

| Mass bin (GeV/c²) | \( \langle m_{DY} \rangle \pm \text{stat} \pm \text{syst} \) (GeV/c²) | \( \langle p_T^{DY} \rangle \pm \text{stat} \pm \text{syst} \) (GeV/c²) |
|------------------|---------------------------------|---------------------------------|
| [40, 60]         | 47.83 ± 0.05 ± 0.31             | 9.10 ± 0.13 ± 0.18             |
| [60, 80]         | 70.76 ± 0.04 ± 0.26             | 10.84 ± 0.08 ± 0.10            |
| [80, 100]        | 90.98 ± 0.01 ± 0.08             | 11.79 ± 0.02 ± 0.01            |
| [100, 200]       | 115.11 ± 0.13 ± 1.31            | 12.93 ± 0.09 ± 0.09            |
| [200, 350]       | 245.46 ± 1.29 ± 2.61            | 16.41 ± 0.56 ± 0.35            |
where the parameters A and B are obtained by a linear fit to be $-7.56 \pm 0.83$ GeV/$c$ and $2.15 \pm 0.09$ GeV/$c$, respectively. The linear dependence is an original finding of this work and may be exploited to improve the modeling of ISR. Since there is no theoretical prediction of this measurement, theorists may usefully make calculations of this measurement under various models of ISR. In addition, by means of a linear extrapolation this finding can be used to estimate the expected ISR activity at higher masses and is relevant to searches for BSM particles.

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TABLE VII. Average values and their uncertainties of the lepton-pair mass and $p_T$, combining ee and $\mu\mu$ events.

| Mass bin (GeV/$c^2$) | $\langle m_{DY}\rangle\pm\text{stat}\pm\text{syst}$ (GeV/$c^2$) | $\langle p_{T}^{DY}\rangle\pm\text{stat}\pm\text{syst}$ (GeV/$c^2$) |
|----------------------|-------------------------------------------------|-------------------------------------------------|
| [40, 60]             | 47.77 ± 0.04 ± 0.31                             | 9.12 ± 0.09 ± 0.12                              |
| [60, 80]             | 70.75 ± 0.03 ± 0.26                             | 10.84 ± 0.07 ± 0.10                             |
| [80, 100]            | 90.98 ± 0.00 ± 0.08                             | 11.80 ± 0.02 ± 0.02                             |
| [100, 200]           | 115.19 ± 0.11 ± 1.30                            | 13.01 ± 0.07 ± 0.09                             |
| [200, 350]           | 244.49 ± 1.02 ± 2.56                            | 16.32 ± 0.42 ± 0.36                             |

TABLE VIII. Statistical and systematic uncertainties of $\langle p_{T}^{DY}\rangle$ in the combined dielectron and dimuon final states. Fractional uncertainties are shown for the various dilepton mass bins and each systematic source.

| Mass bin (GeV/$c^2$) | [40,60] | [60,80] | [80,100] | [100,200] | [200,350] |
|----------------------|---------|---------|----------|-----------|-----------|
| Statistical uncertainty (%) | 0.96    | 0.62    | 0.14     | 0.57      | 2.56      |
| Systematic uncertainty (%) | 1.29    | 0.93    | 0.13     | 0.66      | 2.23      |
| ISR model (%) | 0.90    | 0.52    | 0.09     | 0.39      | 1.93      |
| QED FSR model (%) | 0.87    | 0.74    | 0.05     | 0.20      | 0.56      |
| Muon momentum scale (%) | 0.12    | 0.01    | 0.01     | 0.07      | 0.35      |
| Electron energy scale (%) | 0.00    | 0.07    | 0.01     | 0.05      | 0.11      |
| Muon momentum resolution (%) | 0.07    | 0.01    | 0.02     | 0.22      | 0.49      |
| Electron energy resolution (%) | 0.00    | 0.02    | 0.07     | 0.07      | 0.09      |
| Background normalization (%) | 0.28    | 0.20    | 0.03     | 0.43      | 0.73      |