On pricing kernels, information and risk

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Abstract

We discuss the finding that cross-sectional characteristic based models have yielded portfolios with higher excess monthly returns but lower risk than their arbitrage pricing theory counterparts in an analysis of equity returns of stocks listed on the JSE. Under the assumption of general no-arbitrage conditions, we argue that evidence in favour of characteristic based pricing implies that information is more likely assimilated by means of nonlinear pricing kernels for the markets considered.

Keywords: Arbitrage pricing theory, characteristic based models, size effect, value effect, linear pricing kernel, nonlinear pricing kernel

1 Introduction

In neo-classical finance, future securities prices are regarded as a function of risk and (realisable) present value. We compare arbitrage pricing theory (APT) risk-factor pricing models with characteristic based models (CBM), which offer alternative causal relationships for returns in a equity markets.

No-arbitrage (NA) refers to the reasonable idea that a (theoretic) riskless portfolio with possible positive return in the future, must cost something now. It can be shown to be equivalent to the existence of a positive linear pricing rule or pricing kernel [28, 42] and has been generalised to the concept of no-free-lunch-with-vanishing-risk [13], in an overarching valuation framework [1] that has facilitated the growth of a multi-trillion dollar derivatives market concerned with pricing future cashflows, risk-sharing and completing markets.

The general linear factor APT model formulation was introduced by Ross to explain returns in the simplest possible manner consistent with the assumption of NA [14, 43]. The Fama-and-French model followed thereafter, as special case, to incorporate specific risk premia which were not explained by the CAPM, namely value and size. In particular, Ball, Banz and Basu [2, 4, 5] were amongst the earliest authors to discuss empirical evidence that small-capitalised stocks
and stocks with low price-to-book values exhibit higher long-term performance. More than a decade of literature demonstrating the explanatory power of stock specific accounting variables followed before Haugen and Baker eventually crystallised the approach in the literature by describing a general characteristic based model for equity market returns [27]. This presented a different paradigm to the Fama-and-French time-series factors, which had been constructed to be consistent with a linear regression approach for explaining risk in both stocks and bonds and which could be combined with macro-economic variables in the APT framework.

In a comprehensive triple-sort testing framework to separate overlapping effects, Daniel and Titman compared characteristic based models and Fama-and-French APT models with attention to the same market factor, size and value variables [11]. They show that the return premia to size and value do not appear to originate from the covariance of stocks with intrinsic risk factors, but that cross-sectional price variation seems to be driven directly by stock characteristics. In particular, they demonstrate portfolios which have similar characteristics (size and book-to-market), but significantly different loadings to the Fama-and-French factors can have similar average returns.

One of the findings in [11] is that $\alpha$ is generally non-zero for both models, contradicting the FF model’s linear requirements. Similar results were obtained in [25]. A possible reconciliation of their findings is that $\alpha$ and the factor $\beta$’s are nonlinear functions of risk. Ferson and Harvey [22] used out-of-sample testing to illustrate the effectiveness of time-varying alpha and beta APT-type models.

We adopt the predictive form of the models used in [22] for a simplified comparison of conditional APT model\footnote{1} with corresponding CBM models\footnote{2} with respect to their abilities to forecast returns. This avoids the cumbersome triple-sorting as in [11]. In particular we compare the returns of portfolios whose weights (loadings) are updated monthly according to out-of-sample expected returns. This avoids the cumbersome triple-sorting as in [11]. In particular we compare the returns of portfolios whose weights (loadings) are updated monthly according to out-of-sample expected returns according to equations (2) and (4) below. The comparable APT and CBM implementations are constructed using stocks listed on the South African Johannesburg Stock Exchange (JSE) between 1994 and 2007. We demonstrate that characteristic-based models have provided higher returns at lower volatility than risk-based models over a relatively short time horizons.

Since we restrict our attention to specific sets of information in our comparisons, the out-performance suggests that CBM are more efficient in pricing in those types of information.

Thus, the characteristic approach not only provides a empirical tool for exposing the phenomenology and structure of a given market, is it is also useful

\footnotetext{1}{I.e. with time-varying alpha and beta’s} 
\footnotetext{2}{The authors of this report are not aware of other investigations with this particular innovation in the literature.}
for making ex-ante stock return predictions [27]. This makes the characteristic approach interesting both practically and theoretically.

We discuss our findings in the context of pricing risk under assumptions of no-arbitrage and the weaker no-free-lunch-with-vanishing-risk. In Section 2 we review the two model types and evolution of ideas, as motivated by empirical evidence for equity returns. In Section 3 we review the market context for our investigation and in Section 4 we present the outcomes of our analysis of the risk-return profiles for portfolios constructed via the different asset price models. We conclude in Section 5.

2 Models for equity returns and risk premia

The idea that NA drives pricing is coupled to the perspective that risk factors exogenous to the market can move prices and that news arrives often and randomly and gets priced in instantaneously. In particular, this theory assumes that prices incorporate the impact of exogenous factors fully. If this is the case, then endogenous factors should be computable from the reflection of exogenous factors in price changes (see for example [34, 52]). Notably, APT does not specify its risk factors and macroeconomic, fundamental or statistical factors may be used, provided the dependence on these is linear.

Endogenous characteristics (attributes), such as book-to-price or earnings-per-share, may also be used to construct portfolios that represent the risk factors used in a linear pricing rule [42]. Fama and French [15, 19] developed a model to compensate for two such attributes which have been correlated with returns, namely value (measured via book-to-price) and size.

The initial idea of characteristic based models was that company specific characteristics could directly explain most of the expected return differentials. However, to facilitate consistent pricing of uncertain future cash-flows of different types of securities, factor mimicking portfolios that mirror the roles of characteristics, were devised. The use of stock characteristics to generate a covariance structure was a key innovation in asset pricing [16], since it allowed a parsimonious interpretation without the rejection of linear pricing models and the resulting 3-factor FF risk-based model became a standard against which others were compared.

2.1 APT risk factor vs. characteristic based models

We describe the two classes of models which are compared in order to identify how information and risk are reflected in equity prices.

For an APT-type risk factor model, we implement the following to explain and predict returns:

\[ R_{i,t} = \alpha_t^F + \sum_j \beta_{i,j,t} f_{j,t-1} + \epsilon_{i,t}. \] (1)
Here, the unexplained component $\epsilon_{i,t}$ relates to the factor loading (coefficient/weight) of the $i$-th stock with respect to the return on the $j$-th price factor $f_{j,t}$, at time $t$, and the risk factors are Fama-and-French model risk factors, derived from size and value information, which are described in the next section. The expectation of return at time $t - 1$ for time $t$ is:

$$E_{t-1}[R_{i,t}] = E_{t-1}[\alpha^f_i] + \sum_k E_{t-1}[\beta_{i,j,t}] f_{j,t-1}$$

(2)

Haugen and Baker discuss characteristic-based models (as class of models) for equity market returns in [27]. In our analysis we implement the form which they propose:

$$R_{i,t} = \alpha^C_i + \sum_k \delta_{k,t} \theta_{i,k,t-1} + \epsilon_{i,t},$$

(3)

where $\theta_{i,t-1}$ is the $i$-th observable stock characteristic at time $t - 1$, such as book-to-price, and the payoffs to the $k$-th characteristic at time $t$ is $\delta_{k,t}$. The expected return at time $t - 1$ for time $t$ is:

$$E_{t-1}[R_{i,t}] = E_{t-1}[\alpha^C_i] + \sum_k E_{t-1}[\delta_{k,t}] \theta_{i,k,t-1} + \epsilon_{i,t}.$$ 

(4)

For the comparison, the characteristic factors incorporated were size (market value) and value (book-to-price) attributes.

We note that the forms used in Eqn. (1) - Eqn. (4) should not be confused with general form considered in the Daniel and Titman [11, 12], where the risk model is purely explanatory and has no time-lags:

$$E_{t-1}[R_{i,t}] = \alpha_t + \sum_k \beta_{k,t} \theta_{i,k,t-1} + \sum_j \beta_{i,j,t-1} f_{j,t-1}.$$ 

(5)

Furthermore, the formulation of Eqn. (5) requires additional regressions to estimate expectations of both prior characteristic and risk factor loadings.

The forms used in Eqn. (1) - Eqn. (4) are consistent with our investigation to test the ability of stock characteristics to act as predictors in comparison with analogous risk factors. In particular, we consider the ability of the factor mimicking portfolios to predict future returns (via dynamically changing loadings) rather than the ability of the loadings themselves to act as predictors (independent of the factor-mimicking portfolios). This latter formulation has been explored elsewhere [25].

These equations are well suited for multi-factor regression analysis with out-of-sample testing using small frequently reformed data sets for rolling time-frames: In order to compare the risk-return profiles of CBM models described in Eqn. (3) with corresponding risk factor models described in Eqn. (1), ex-ante predictions from Eqn. (2) and Eqn. (4) were used to evaluate the risk-return profiles of the two models out-of-sample. The expectations: $E[\alpha^f_i]$, $E[\alpha^C_i]$, $E[\beta_{i,j}]$ and $E[\delta_{k}]$ were assumed to be slowly varying functions of time. This allowed us to estimate $E[\alpha^f_i]$ and $E[\beta_{i,j}]$ using time-series analysis and $E[\alpha^C_i]$ and $E[\delta_{k}]$ using cross-sectional analysis.
2.1.1 The Fama and French APT model

We review the construction of the three Fama-and-French risk factors (FF) which are proxied by factor mimicking portfolios based on book-to-price and size information. First, six portfolios are constructed via the intersection of three book-to-price categories ($H \equiv$ High, $M \equiv$ Medium and $L \equiv$ Low) with two size categories ($B \equiv$ Big and $S \equiv$ Small). These portfolios are designated $HS$, $MS$, $LS$, $HB$, $MB$, and $LB$ [15, 17, 19, 11]. These portfolios are then used to capture the disentangled size and value effects as factor mimicking portfolios:

\[ R_{SMB} = \frac{1}{3} ((R_{HS} + R_{MS} + R_{LS}) - (R_{HB} + R_{MB} + R_{LB})), \]  
\[ R_{HML} = \frac{1}{2} ((R_{HB} + R_{HS}) - (R_{LB} + R_{LS})). \]

Here the risk factor variation $SMB$ is the excess return for small relative to large capitalized stocks (corrected for value), while the variation of $HML$ is the excess return of high value stocks relative to low value stocks (adjusted for size-effect contributions).

The resulting FF risk factor model for the $i$-th stock takes on the usual explanatory form

\[ R_{i,t} - R_{rfr,t} = \alpha_i + \beta_{i,Mkt}(R_{Mkt,t} - R_{rfr,t}) + \beta_{i,HML}R_{HML,t} + \beta_{i,SMB}R_{SMB,t} + \epsilon_{i,t}, \]

where the risk-free rate of return is denoted by $R_{rfr,t}$ at time $t$, the bias is $\alpha$ and the factor loadings are given by the respective $\beta$’s. This is not the same form as the return-prediction mode for APT risk factors used in Eqn. 2. It is the predictive formulation that is investigated here. We note further that it is the predictive power of the characteristics relative to the factor loadings that is explicitly tested for in Daniel and Titman [11, 12], rather than the predictive power of the factor mimicking portfolio’s as loaded by the $\beta$’s.

2.2 Model implications: Arbitrage and linear vs nonlinear pricing kernels

From a pragmatic point of view it may be that characteristic based models make better predictions, given the vagaries and noise of financial markets and financial data, simply because they are better representations of available measurements.

Calibration to APT risk-factors is rewarded with linearity of the pricing kernel. A linear pricing kernel yields the price of an asset as a scalar product of a representation of future payoffs (including contingent claims) with numbers which calibrate how the payoffs impact current asset prices. It’s equivalence to the reasonable assumptions of NA pricing follows by translating the NA into concise mathematical assumptions.

In particular, by combining NA with some simplifying assumptions on the nature of investment returns in a finite (and complete) state-space, an extremely
elegant pricing framework can be derived from a foundational theorem of early 20th century Hilbert space theory \[41\]. The Riesz Representation theorem provides the mathematical foundation for the following key equivalences in the Fundamental Theorem of Asset Pricing and the Pricing Representation theorems \[10, 14\]:

1. the market does not admit arbitrage opportunities
2. there exists a positive linear pricing rule for the relationship between asset prices and future state-dependent payoffs,
3. there exists a pricing measure under which risk-neutral discounted securities prices are martingales
4. there exists of a positive pricing kernel which provides the connection between positive probabilities of events occurring and the payoffs of those events.

A key benefit of this approach is that it facilitates a relatively simplified methodology to pricing derivative claims which have nonlinear payoffs.

No-arbitrage has been generalised to more realistic continuous (infinite-dimensional) and incomplete market settings \[28, 29\] to the assumption of no-free-lunch-with-vanishing-risk (NFLVR) \[13\]. The latter can be interpreted as the condition whereby it is impossible to devise a potentially profitable trading strategy which never loses money.

While the failure of NA or NFLVR negates the existence of any reliable market pricing kernel, the validity of a CBM does not necessarily imply the existence of any arbitrage opportunities. In fact it follows from arguments similar to those used in \[14\], that it is also possible to construct a linear pricing kernel in the case of a finite dimensional market which admits a zero-alpha CBM.

In the literature, stock specific characteristics have been interpreted to be non-risk determinants of asset prices \[8\]. Our view is that the effectiveness of these variables points to a reality of non-linear pricing kernel \[3\]. For example, Daniel and Titman find \(\alpha \neq 0\) for both FF and CBM type models investigated \[11\]. Such results imply that \(\alpha\) must be a nonlinear function of risk factors for a risk pricing perspective to be preserved.

Departures from linearity may also be interpreted as manifesting themselves in the \(\beta\) of a factor-pricing model, where for example, the \(\beta\)’s may be functions of the characteristics that underpin the factors. In this case, the model can remain linear in the factors, but becomes non-linear in the characteristics in an auto-regressive manner via factor coefficients \[22\].

The notion of a nonlinear pricing kernel is consistent with the more general reflection of information in prices. Motivated by the same considerations

\[3\] Giving up linearity of the pricing kernel poses a further challenges for finance theory. In particular, if it is possible to construct a NA characteristic based model which offers more comprehensive and consistent market modelling, then the result that firms are independent of their financial structure \[36, 37\] may require reinterpretation.
as Ross, Fama and French in their development of the linear APT risk factor approach, Bansal and Viswanathan (1993) and Harvey and Kirby (1995) investigated the theory of nonlinear pricing kernels in [3, 30]. Ferson and Harvey showed how to calibrate the nonlinear (conditional) APT approach in [22], while Chernov developed a more general nonparametric calibration of nonlinear kernel models in [9].

From an investors perspective, decisions are rewarded partly as compensation for taking on the risk, which can be proxied by appropriately specified factors, as well as for removing possible mispricings (arbitrage opportunities) identified by their investment model.

We note that behavioural economists have not been convinced by the risk-based explanation of the FF model [32] and have suggested, for example, that behavioural biases in the forward estimation of earnings could explain the value premium found by Fama and French [15]. Similar criticism could be directed to characteristic based models.

Thus, as asset managers chase alpha by following almost identical strategies to identify possible mispricings, the collective impact of their investment strategies in the same assets can impact price formation. This would be consistent with the extensive literature emanating from theorists on the SA market, who have promoted characteristic approaches ([47, 48, 49] and subsequent investigations in that journal).

We also note that the size effect can explained as a portfolio level phenomena by following arguments given in the stochastic portfolio theory (SPT) of Fernholz [20, 21]. While the SPT approach does not offer insight into the existence or nature of pricing kernels, it does expose how capital entering and leaving the market impacts price as a function of cross-border cash-flow risk [31].

In the next sections we present a period in the SA market for which characteristic models have offered higher returns for lower volatility, with the possible interpretation that APT models were not able to price in as much information available to investors as CBM.

3 Data, market context and related investigations

Our investigation is based on monthly data for stocks listed on the Johannesburg Stock Exchange (JSE) between 1994 and 2007.

3.1 Data and market context

Our data set runs from 31 Jan 1994 to 30 April 2007, complementing previous studies and including the bull-market post 2003, a period in which the market factor made a considerable contribution, as well as the peak in a value cycle in mid-2005.

We consider three distinct universes of stocks by using market value to select the largest 50, 100 and 250 stocks respectively, each month. The Top 250
stock universe was the largest universe which could be kept sufficiently constant in stock number for the study. We regarded this as the simplest approach to creating a representative universe of JSE as a whole for the out-of-sample comparison of the APT and CBM model. In this paper we report primarily on the Top 250 stock universe since the results are similar across all universes including, yet surprisingly, the relatively small Top 50 universe.

Continuous-time model returns (log-returns) were used. The proxy for the cash asset was the current 3 month NCD (negotiable cash deposit). This is the closest SA equivalent to US 3-month treasury bills. South African NCD’s are quoted at discount yield (NACQ). This required that we construct a monthly price index using the publicly quoted yield time-series for the instruments by computing the face value and re-balance monthly.

Zero-valued prices and returns were treated as missing data in all the sorts and regressions so that the lack of meaningful numbers would not bias the analysis for other stocks and variables where data existed. In particular, we did not use interpolation methods, nor did we systematically exclude data for some stocks due to missing data for others. Infinite characteristic values were also treated as missing data in all the sorts and regressions.

The data preprocessing follows Haugen and Baker [27] by first winsorising the data at 3 standard deviations and then z-scoring the resulting data. Missing data is excluded at the level of the algorithms employed, and as such we have avoided excluding entire data rows if missing data is encountered because of data sparsity (rather than the occurrence of non-trading days).

A synthetic market weighted index was computed from market values. The CAPM $\beta$’s were computed relative the synthetic market index. Such a market index was computed for each of the three universes and re-balanced monthly. This was useful in that it ensured that the market index was defined in terms of the same universe of stocks as the study itself as well as ensuring meaningful comparison within the 3 stock universes.

We restricted ourselves to a small set of input variables for the characteristic models: a total return index, historic dividend yield, price-to-book, volume traded, market value and earnings yield. The data was sourced from Thomson-DataStream. Although a more extensive list of characteristics was available and commercial applications of such CBM models include forward broker information the intention here is to focus on value and size in the context of $HML$ and $SMB$ constructions for the Johannesburg Stock Exchange.

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4 Our data treatment avoids the use of liquidity screens, used in preceding analysis of the same [23, 40, 52].

5 A typical stable commercial characteristic based model for the South African market could include most of the following factors (perhaps less if volatility becomes a concern): total return, log-size, price, book-to-price, cash-to-price, dividend yield, earnings yield, 1-year forward earnings yield, 2-year forward earnings yield, 1-year forward earnings growth, earnings torpedo (change from latest earnings to next consensus earnings), neglect (negative log of number of analysts covering a stock), earnings revision, earnings downgrade, earnings upgrade, broker recommendation (buy, hold, sell), low price, payout ratio, 3-month momentum, 6-month momentum, 9-month momentum, 1-month momentum (smoothed), currency plays (dummy for USD/ZAR exposure).
Two momentum characteristics were also computed using the total return index. Specifically, a long-term 12-month momentum, lagged 1 month prior to formation date, and a short-term 3-month momentum signal, also lagged 1 month prior to formation date, were computed. The limited length of the data sets and the relative sparsity of the sort data made momentum signals longer than 2 years impractical.

A unique problem with the South African market is the dominant effect that dual listed stocks have. Dual listed stocks whose primary listings were not on the Johannesburg Stock Exchange were included and the book-to-price and earnings yield of the primary listed entities where used in the analysis.

3.2 Related investigations

There have been several studies relating to characteristic based models on JSE listed stocks since [24, 47, 49]. More recently, we considered the impact of foreign portfolio investment, as a risk factor for emerging markets [51], via the stochastic portfolio theory factorisation of Fernholz [20].

In [24, 47, 49], the authors argued that the cross-section of returns on the JSE is explained by characteristics (attributes) rather than risk factors for correlated price-to-earnings and size risk factors. Our approach is different however, since [49] constructed the following: a portfolio SLL or small-less-large, driven by the difference between small and large cap returns, as a risk factor capturing the size effect, and a portfolio LLH or low-less-high, driven by the difference between low and high value stock returns. Their factors are uncorrected for possible correlation, which is a key ingredient in the FF risk factor constructions [19, 11].

Daniel and Titman recommend the use multi-factor regressions on triple sorted portfolios as a step for dealing with the errors-in-variables problem which portfolio sort based testing procedures have [12]. Understanding of the method is important because it provides a key motivation for our review of the FF model for South African data. We note that [49] used single factor regressions in the context of double sorted portfolios. However, the theoretical context of their contribution was not that of discriminating between risk and characteristic-based models but rather that of identifying a more appropriate form of asset pricing model specification based on characteristics (attributes).

The sample studied by Fraser and Page [24] was from 1973 through 1997 and examined financial and industrial stocks with a focus on price-to-earnings, dividend yield and market value. They argued that price-to-earnings was the key determinant of the cross-sectional price differences at that time. The sample

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Footnotes:

6This included dual-listed counters such as Anglo American, BHP-Billiton, Richmont, SAB-Miller, Old Mutual, Liberty Life and Investec. The list was constructed using the JSE listed ISIN codes to find the primary listing exchange and tickers. With this information the correct primary listing characteristics could be sourced. By using price ratios in the primary listing's currency one avoids additional uncertainties arising when converting the characteristics, such as earnings and book-value, to local currencies.

7The use of multi-factor regressions in the context of triple sorted portfolio intercept tests was also presented in [25].
studied by [47] ran 1990 through 2000 and also focused on price-to-earnings and market value. They considered a more extensive set of characteristics with commercial applications in fund management in view. This provided one of the first screens of characteristics and aligned with their stated objective of finding a more appropriate asset pricing model for the JSE.

In our investigation, we recover an apparent value effect in three different stock universes, namely the Top 50, Top 100 and Top 250 stocks, as reformed monthly. The size effect was more prominent in the Top 50 and Top 250 stock universes. Prior work on the size effect on US common stocks [4] was extended to the JSE by Page and Palmer [40]. Similarly, the effect of price-to-earnings [5] and price-to-book [46] has been extended to the JSE [24, 47]. Various ad hoc investigations on price premia in the same market are also documented in the literature.

We add to this discussion in the context that the South African market has hierarchical aspects: ALSI 40 (largest 40 stocks) attracts international investors while simultaneously being important to domestic investors, who also focus on the ALSI (largest 165 stocks). We promote a nonlinear pricing kernel perspective to reconcile the findings in favour of direct characteristic based pricing with no-arbitrage pricing paradigms.

4 Results and discussion

4.1 Risk and return comparisons between model types

We compare a 3-factor APT model, using our FF factors, and a CAPM model with two characteristic based models (CBM), one without momentum (CBM #2 in Table 1) and one with momentum (CBM #6 in Table 1).

For Figure 3, returns for each model were sorted into five quintiles, with highest expected returns binned into the 1\textsuperscript{st} quintile. Best fit lines for corresponding realised returns and realised volatilities were then plotted.

The CAPM and FF risk models yielded ex-post returns which were less than those from two CBM, but at higher volatilities. This is evident in Figure 3 for the Top 250 universe, but holds for all three stock universes studied and demonstrates that one could generate higher returns with lower risk by using CBM and selecting stocks whose returns fall into the 1\textsuperscript{st} quintile. A more rigorous statistical treatment using the Daniel and Titman [11, 12] approach was given in [25] and is consistent with the visual results shown here.

Risks, measured by volatility, vary little across the quintiles for CAPM and FF models, but are significantly higher in the 4\textsuperscript{th} and 5\textsuperscript{th} quintiles for their characteristic based counterparts. This result is robust across the three different

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...The sorts were carried out using a symmetric quantiling algorithm that kept the sort symmetric around the middle quantile for odd numbers of quantiles, and symmetric around the two middle quantiles for even numbers of quantiles. The algorithm was also tailored to ignore missing data and cope with listings, delistings and illiquid instruments that may have had no trade or fundamental data available on a given month. This quantiling algorithm was then used to sort stocks into quintiles.
universes of stocks. We note further that loadings to the market factors in the APT and CAPM models were higher than the corresponding market factor loadings in the CBM.

Hence, we claim that CBM have been effective even in fairly small concentrated data sets, such as the ALSI 40 universe, as well as being effective in the larger universes. This demonstrates that investors have not necessarily rewarded for risk in the sense of APT or CAPM on the JSE. These results holds across the Top 250, Top 100 and Top 50 stock universes, providing a key finding for JSE for the pre-global crisis period considered and corroborating the findings of [26].

In [25], which implemented the triple-sort approach of Daniel and Titman [11] to compare the models, it was found that generally $\alpha \neq 0$ for the FF models. Assuming that the FF factors provided a robust model, this points to possibility that the stock $\alpha$’s were nonlinear functions of risk [22].

4.2 JSE results for the Fama and French risk factor model

The cumulative factor returns for the value ($HML$), size ($SMB$) and market ($Mkt$) factors are given in Figure 1 for the Top 250 universe of stocks. The market factor return was constructed using stock market capitalisation values and the factor was re-balanced monthly. The fundamental factor (market cap data) dates were shifted 1 quarter backward in time relative to the date the information is documented in the raw data, since the latter is done with a 1 quarter time lag. The $\beta$’s were estimated in 6 year rolling windows (72 months).

The study of the Top 250 stock universe uncovered a delayed size effect. Assessment of the $SMB$ factor returns [25] showed a peak 12 months after portfolio formation. By inspection, there was limited return advantage apparent in the time-series behaviour of the $SMB$ factor shown in Figure 1. Nevertheless, there was still a discernible size-effect. The value effect is clearly evident from 1998 through to mid 2005. The importance of the market factor after 2003 is visually apparent from the nominal performance of the factor mimicking portfolio in Figure 1.

4.3 Observations for characteristic based models

Results are summarised for 14 characteristic based models in Table 1 for the 250 stock universe studied. Table entries are the median characteristic payoffs.

As expected, we find a broad positive loading to book-to-price and a negative loading to size. This is in agreement with the analysis using the $HML$ and $SMB$ factor loading portfolios. The value effect increases with increase in universe size, while the size effect diminishes. This is surprising, since one would expect the size effect to be more meaningful in a larger universe of stocks. When controlling for dividend yield the size effect is diminished in the Top 50 universe but not significantly diminished within the Top 100 and Top 250 universes of stocks. The size effect is also diminished in the presence of momentum.
Figure 1: The Top 250 HML (High-Minus-Low) and SMB (Small-Minus-Big) factor mimicking portfolios formed from the 6 intersection portfolios as in Eqn’s. 5 and Eqn. 6 and reformed monthly using only the largest 250 stock each month out of 472 stocks listed on the JSE between 31 January 1994 and 30 April 2007 are shown. The HML (value) factor mimicking portfolio is given by the solid line, the SMB (size) factor mimicking portfolio is given by the dashed line and the market portfolio Mkt is given by the dotted line.
Table 1: Top 250 stock JSE characteristics based models. The characteristic factors are, from left to right, the model bias, book-to-price, market value, market factor, long term momentum, short term momentum, earnings yield, dividend yield and volume traded. Model #2 and #6 are compared with CAPM and the 3-factor Fama and French APT model in Figures 2 and 3. The dynamics of the payoffs for model #14 (the medians are presented here) is shown in Figure 2.

The book-to-price effect is correlated with the earnings yield (inverse of price-to-earnings), and it is found that the book-to-price effect can be reduced when controlling for earnings yield in the presence of size. This apparent multi-collinearity between book-to-price and earnings yield becomes less effective in the presence of momentum. This broadly corroborates the identified ability to substitute book-to-price with price-to-earnings and size on the JSE [47].

It is also noted the earnings yield and change in volumes traded provide little additional explanatory power when in the presence of price-to-book, market value, momentum and a market factor. Hence, there appears to have been little additional advantage in using volumes traded, given its high multi-collinearity with other factors. The market factor can be substituted with volume traded.

We do find subtle dependencies on the dividend yield: when controlling for the loading on the market portfolio one finds that the effect of the dividend yield changes sign in the Top 50 universe. Long-term momentum is generally positive except when controlling for the market portfolio. Short term momentum is negative except when controlling for dividends, earnings and volumes.

The payoffs to the various characteristics have time dependent oscillatory dynamics. This is graphically demonstrated in Figure 2 for CBM model #14 (whose median payoffs values are in Figure 1 for the full sample). For example, prior to December 2001 there was a positive payoff to market value.

This shifted to a negative payoff to market value a year later, which can be attributed to the December 2001 crash in the USD/ZAR exchange rate. Similar pathologies can be seen for almost all the characteristic payoffs, demonstrating that price anomalies associated with payoffs to characteristics were reasonably short-term.
Figure 2: The Top 250 dynamics of the smoothed payoffs to the various characteristics used in CBM #14 from Table II against the change in Tsallis entropy (LOC), which is a measure of market wide localisation (the more localised the market the fewer opportunities there are because portfolio market values are more concentrated on fewer stocks). The dynamic behaviour of the characteristic payoffs is apparent.
On a practical note, using an extensive list of characteristics can lead to data-mining effects. This is particularly important in a concentrated market such as the JSE. This may arise when multi-collinearity swamps the model estimation process with noise effects. It is our view that a model with more than about 10 factors in the SA market will be prone to this flaw. It is also easy to select a particular window of data for which the model happens to work well. This type of error is difficult avoid in a bull market as almost all the characteristics become coupled to momentum.

We recommend that practitioners opting to use CBM carry-out tests for randomness in the model outputs in addition to the usual out-of-sample back-testing. We also recommend scenario analysis with characteristic shocks to further understand dynamics under noisy conditions due to the presence of oscillatory cycles (see Figure 2).

A related concern is that associated with the liquidity premium. Characteristic based models constructed with many too factors can result in the identification of a liquidity premium which cannot be traded. Finally, we note that equity premia have been shown to include exogenous factors such as currency flows during regional and global crises [51].

5 Conclusion

We have demonstrated that cross-sectional characteristic models have yielded portfolios with higher excess monthly returns but lower risk than their risk based factor model counterparts on the JSE between 1994 and 2007. The outcome is consistent with the Daniel-and-Titman triple sort comparison carried out in [25]. Thus, investors appear to have been rewarded for the self-consistent use of information as proxied by stock characteristics.

This may simply have been a reflection of the way information and risk were priced. In particular, CBM may have priced information more efficiently if stock specific accounting variables were used by many investors in that period. With such models popularised in academic literature, the price impact of investors using similar strategies may have influenced price fluctuations in traded assets.

We have focussed our attention on detecting how size and value information were reflected in stock prices and of the two models investigated, the cross-sectional characteristic model was the better predictor of returns. Extending the pricing kernel further to incorporate more prevailing market information, including other stock characteristics, exchange-traded derivative prices and credit, liquidity and other risks in an incomplete market, would more likely confirm a nonlinear price transition kernel.

Characteristic based models are typically nonlinear, but can still be consistent with assumptions of NFLVR for a market with non-unique risk-neutral pricing solutions [9][30]. At the same time, evidence of CBM information pricing does not imply that a market is free of arbitrage.

In the context of an aggregate influx of capital into the JSE [51] for the period investigated, this paper highlights that it was possible to select outperforming
Figure 3: Stocks were quintiled on expected return, with the highest in the first quintile. The upper graph has the average annualised returns in the top 250 stock universe for each of the 5 quintiled portfolio’s of two characteristic based models and two risk based models is given. The lower graph depicts the average volatilities. The first characteristic based model (CBM) uses market capitalisation, book-to-price and a market factor and corresponds to CBM #2 in the Table 1 and is denoted by solid squares with a thin dashed best fit line. The second CBM is model #6 from Table 1, it includes momentum signals and is denoted by dark circles with a solid line running from the upper left to lower right. The risk based models (RBM) are the 3-Factor APT model using the HML, SMB and Mkt factors and CAPM, denoted by right triangles and a solid dash best fit line and left triangles and the solid horizontal best fit line respectively. The CBM models provide realised returns commensurate with the quintile sorts from highest expected return quintile 1 through to the lowest expected return quintile 5. As such the CBM model are good predictors of portfolio wide return commonalities, the RBM models are not. The lower graph has the volatilities of two risk based asset pricing models and two characteristic based pricing models in the top 250 stock universe. The CBM models provide realised volatilities inversely related to the quintile sorts from highest expected return quintile 1 through to the lowest expected return quintile 5. As such the CBM models provide lower volatilities in quintiles with higher expected returns this is contrary to the RBM models which expect higher returns with relatively higher volatilities.
portfolios by incorporating information based on value and size criteria directly, as compared to pricing risk information via Fama-and-French time-series factors. This contributes to the understanding of how information and risk are priced within a more general Markovian perspective of markets.

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