Spontaneous Parity Violation in SUSY Strong Gauge Theory

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Abstract

We suggest simple models of spontaneous parity violation in supersymmetric strong gauge theory. We focus on left-right symmetric model and investigate vacuum with spontaneous parity violation. Non-perturbative effects are calculable in supersymmetric gauge theory, and we suggest two new models. The first model shows confinement, and the second model has a dual description of the theory. The left-right symmetry breaking and electroweak symmetry breaking are simultaneously occurred with the suitable energy scale hierarchy. The second model also induces spontaneous supersymmetry breaking.
1 Introduction

The standard model (SM) is regarded as an effective theory below a TeV-scale. There must be a fundamental theory beyond the SM, and a trial of searching it is a big challenge at today’s experiments. There are some mysteries in the SM, and one of them is a question, “why is the SM chiral gauge theory?”. There is no explanation why weak interaction is $SU(2)_L$, and left-right symmetry is broken. There have been a lot of trials of explaining its origin, and one of reliable candidates is a left-right symmetric model with a gauge symmetry, $SU(2)_L \times SU(2)_R \times U(1)$ [1–3]. Note that some grand unified theories (GUTs), such as $SO(10)$ GUT, contains this left-right symmetric gauge group. Anyhow, if $SU(2)_R \times U(1)$ is spontaneous broken to $U(1)_Y$, this is an origin of breaking of left-right parity symmetry in the SM. For this purpose, we must extend a Higgs sector which contains new Higgs fields with quantum charges of $SU(2)_R \times U(1)$.

On the other hand, supersymmetry (SUSY) is one of the most promising candidate beyond the SM, since the existence of dark matter candidate and a success of gauge coupling unification. In the minimal set up of the supersymmetric SM (MSSM), the left-right symmetry can play a important role to avoid R-parity violation, since $U(1)_{B-L}$ symmetry (a part of left-right symmetric models) is often related to the R-charge as $R = (-1)^{B-L}$ [4]. In this case, the $SU(2) \times U(1)$ gauge symmetry is broken down to $U(1)_Y$ by the triplet Higgs fields with $B - L = \pm 2$, and the double-charged Higgs fields are predicted. Furthermore, it also resolves the strong CP problem and the SUSY CP problem [5, 6].

The SUSY gauge theory has an important feature, that is, from the theoretical point of view, non-perturbative effects can be calculated in SUSY strong gauge theory. We know that the strong gauge dynamics plays important role in particle physics. The spontaneous breaking of the chiral symmetry in QCD is a typical example of the spontaneous symmetry breaking. This idea of the strong gauge dynamics applying to the electroweak symmetry breaking as a scale up of QCD is so-called technicolor model [7, 8]. The electroweak scale is given by the dynamical scale of the technicolor (around TeV) scale, so that the scenario is a natural solution of the hierarchy problem. In the case of non-SUSY theory, however, it is unclear for us to obtain non-perturbative vacuum structure in technicolor models, where we usually take non-trivial dynamical assumptions for flavor sector, e.g. walking or conformal technicolor models [9,10] or topcolor models [11]. In the framework of SUSY, non-perturbative effects can be calculable in a strong gauge theory. For example, we can calculate non-perturbative effects in a superpotential by using a dual description, which makes us know correct vacuum structure in a strong dynamics. The hierarchical flavor structures of this model may be suggested by the higher order coupling with non-trivial scale dimension $d$ of $\bar{\psi}\psi$ condensation which gives quark/lepton masses. This value of $d$ is often communicated with R-charge and the dimension of the chiral operator. For the recent work on the flavor structure of SUSY topcolor model,
see Refs. [12][13]. There are some related models that provide Yukawa hierarchy through the anomalous dimension in which the MSSM sector can couple to the superconformal sector [14].

If the SUSY is the underlying theory of our nature, it must be broken at intermediate scale between Planck scale and electroweak scale. An idea of a dynamical SUSY breaking [15] is one of the most attractive scenarios which can explain why the SUSY broken scale is much smaller than Planck scale. As for the dynamical SUSY broken models, they are based on a dynamics of $\mathcal{N} = 1$ SQCD with various number of color $N_C$ and flavor $N_F$. For example, modified moduli space model is represented in case of $N_F = N_C$ [16][17], and case of $N_F > N_C$ suggests model with meta-stable vacua [18].

In this paper, we suggest simple models of spontaneous parity violation in SUSY strong gauge theory. We focus on left-right symmetric model and investigate vacuum with spontaneous parity violation. Non-perturbative effects are calculable in SUSY gauge theory, and we suggest two new models. In both models, left-right symmetry breaking (and also additional $U(1)$) are triggered by vacuum expectation values (VEVs) of $SU(2)_R$ triplets or doublets Higgs fields.

# 2 A model of spontaneous parity violation

In this section we show a model of spontaneous parity violation in the SUSY theory. We first show a basic idea, and next try to modify the model by use of strong gauge dynamics. We will construct a model whose dimensional scales are all originated from the strong gauge dynamics.

## 2.1 Basic structure

We first explain the brief introduction to models of spontaneous parity violation. A model shown here is based on a gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We only focus on a Higgs sector which contains the following $(SU(2)_{L,R})$ doublets,

$$
\varphi_L = (2, 1, 1), \quad \tilde{\varphi}_L = (2, 1, -1), \quad \varphi_R = (1, 2, 1), \quad \tilde{\varphi}_R = (1, 2, -1),
$$

(2.1)

and the triplets

$$
\Phi_L = (3, 1, 0), \quad \Phi_R = (1, 3, 0).
$$

(2.2)

We consider a renormalizable superpotential $W_{LR}$ which has left-right parity symmetry, $L \leftrightarrow R$, as

$$
W_{LR} = m(\tilde{\varphi}_L \varphi_L + \tilde{\varphi}_R \varphi_R) + \frac{M}{2} \text{Tr}(\Phi_L^2 + \Phi_R^2) + h(\tilde{\varphi}_L \Phi_L \varphi_L + \tilde{\varphi}_R \Phi_R \varphi_R),
$$

(2.3)

where the coupling $h$ is $O(1)$ and $m$ and $M$ have mass dimension. The scale of $m$ and $M$ are related to the spontaneous parity breaking. The vacuum of this model should satisfy F-flatness
conditions, and for the moment we ignore the gauge indexes in the equations, for simplicity. The F-flatness conditions are given by

\[ F_\varphi I = \tilde{\varphi}_I (m + h \Phi_I), \quad F_\Phi I = M \Phi_I + h \tilde{\varphi}_I \varphi_I, \] (2.4)

where \( I = L, R \). The first equation induces two types of the solution; One is a vacuum of

\[ \langle \varphi_I \rangle = \langle \varphi_I \rangle = 0 \quad \text{and} \quad \langle \Phi_I \rangle = 0. \]

The other is a vacuum of

\[ \langle \varphi_I \varphi_I \rangle = \frac{M m}{h^2} \quad \text{and} \quad \langle \Phi_I \rangle = -\frac{m}{h}. \]

Remind that we can always take the above two solutions independently of the index \( I = L, R \), therefore, \( SU(2)_R \) broken vacuum can be easily obtained by choosing \( \langle \Phi_L \rangle = 0 \) and \( \langle \Phi_R \rangle = -\frac{m}{h} \). These two mass scales of \( m \) and \( M \) can be taken large enough to satisfy the current experimental bound.

This structure can be also reproduced in a model with charged triplets [20], in which the doublets \( \varphi_I, \tilde{\varphi}_I \) are replaced by triplet fields,

\[ \Omega_L = (3, 1, 2), \quad \tilde{\Omega}_L = (3, 1, -2), \]

\[ \Omega_R = (1, 3, 2), \quad \tilde{\Omega}_R = (1, 3, -2), \]

and then the superpotential in Eq.(2.4) becomes

\[ W_{LR} = m_\Omega \text{Tr}(\Omega_L \tilde{\Omega}_L + \Omega_R \tilde{\Omega}_R) + \frac{M_\Phi}{2} \text{Tr}(\Phi^2_L + \Phi^2_R) + h \text{Tr}(\Omega_L \Phi_L \tilde{\Omega}_L + \Omega_R \Phi_R \tilde{\Omega}_R). \] (2.7)

A similar analysis can show an existence of the left-right asymmetric vacua also in this setup. One can really find the suitable left-right breaking vacua from the analysis including the MSSM matter fields and D-flatness condition [21]. This triplet model has several advantages compared to the doublet model. For example, the triplet fields have parity-even charges and the R-parity does not break. It is also possible to have a tiny neutrino mass via see-saw mechanism through the triplet Higgs fields [22].

Above two models contain initial mass scales (parameters), which are necessary to give a parity breaking vacua and independent of weak scale (but just larger than the scale). These models are simple and complete in a sense, however, we would like to consider more attractive models which have no initial mass parameters. In the following subsections, we try to modify the above models and achieve the spontaneous parity breaking without mass parameters. Where all dimensional scales are originated from the strong gauge dynamics.

### 2.2 Model improvement

In order to avoid the ad-hoc mass scales, we introduce a new gauge symmetry which becomes strong at a large scale of \( \Lambda \), and the parity symmetry is expected to be broken by the strong

* In Ref. [19], they induce the left-right breaking vacua without the neutral triplet Higgs fields, \( \Phi, \tilde{\Phi} \), where the left-right breaking vacua can be found at 1-loop level potential.
SUSY dynamics. Also, we will consider a situation where left-right Higgs sectors are coupled with each other through Yukawa-type interactions.

Let us introduce a new gauge dynamics $SU(2)_H$, and fours on the gauge symmetry $SU(2)_H \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. A SUSY $SU(2)$ gauge theory with certain number of flavors becomes strong at low energy, and has a VEV of composite scalar fields through a non-perturbative deformed moduli space \cite{23}. We introduce fields $L^\alpha_a$ and $R^\alpha_a$, which are charged under both $SU(2)_H$ and $SU(2)_L,R$, where $a, b = 1, 2$ ($\alpha = 1, 2$) denotes $SU(2)_L,R$ ($SU(2)_C$) indexes. Under $(SU(2)_H, SU(2)_L, SU(2)_R, U(1)_{B-L})$, they are given by

$$L = (2, 2, 1, 0), \quad R = (2, 1, 2, 0).$$

(2.8)

This field content is also used in a SUSY version of the minimal technicolor model \cite{24}. In order to realize spontaneous parity violation (and also for a cancellation of $B - L$ gauge anomaly), we introduce triplet fields of left-right Higgs sector as,

$$\Omega_L = (1, 3, 1, 2), \quad \bar{\Omega}_L = (1, 3, 1, -2),$$

$$\Omega_R = (1, 1, 3, 2), \quad \bar{\Omega}_R = (1, 1, 3, -2),$$

$$\Phi_L = (1, 3, 1, 0), \quad \Phi_R = (1, 1, 3, 0).$$

(2.9), (2.10), (2.11)

We also introduce additional gauge singlet fields $S_L$ and $S_R$ which connect between strong gauge sector and left-right Higgs sector. Then, a tree level superpotential is given by

$$W_{tree} = \lambda (S_L LL + S_R RR) + a \left\{ S_L \text{Tr}[\Omega_L \bar{\Omega}_L] + S_R \text{Tr}[\Omega_R \bar{\Omega}_R] \right\} + b \left\{ S_L \text{Tr}[\Phi_L^2] + S_R \text{Tr}[\Phi_R^2] \right\} + y \left\{ \text{Tr}[\Omega_L \Phi_L \bar{\Omega}_L] + \text{Tr}[\Omega_R \Phi_R \bar{\Omega}_R] \right\} + W(S_L, S_R),$$

(2.12)

where the coupling constants $\lambda$, $a$, $b$ and $y$ are $O(1)$ coefficients, and $W(S_L, S_R)$ is the superpotential which contains only $S_L$ and $S_R$. Here we impose $Z_3$ discrete symmetry, whose charged are given by

$$\omega : L, S_L, \Omega_L, \bar{\Omega}_L, \Phi_L,$$

$$\omega^2 : R, S_R, \Omega_R, \bar{\Omega}_R, \Phi_R,$$

(2.13), (2.14)

which restrict couplings such as $S_L RR$. We must take a setup that the superpotential has no global $U(1)$ symmetry which is important not to have the massless goldstone boson nor axions after spontaneous symmetry breaking. It is assumed that the global $U(1)_R$ symmetry will be broken explicitly by the SUSY breaking terms. For the $SU(2)_H$ gauge theory, it has $N_F = 2$ fundamental (vector-like) matter fields, and becomes strong at low-energy scale $\Lambda$. Then, below the scale of $\Lambda$, all $SU(2)_H$-charged matter fields are confined, and light degrees of freedom are represented by composite fields as

$$B_L \sim \frac{L^0_1 L^2_3 e^{\alpha_3}}{\Lambda}, \quad B_R \sim \frac{R^0_2 R^2_3 e^{\alpha_3}}{\Lambda}, \quad \Pi = \begin{pmatrix} \Pi^0_u & \Pi^{-}_d \\ \Pi^+_u & \Pi^0_d \end{pmatrix} \sim \frac{L^0_3 R^3 e^{\alpha_3}}{\Lambda},$$

4
where \( \Pi \) field is a bi-doublet under the \( SU(2)_L \) and \( SU(2)_R \). Due to the strong dynamics of \( SU(2)_H \), a quantum constraint is given by

\[
\text{det} \Pi - B_L B_R = \Pi_u \Pi_d - \Pi^+ \Pi^- - B_L B_R = f^2,
\]

where \( f = \Lambda/4\pi \). If we set this scale \( \Lambda \sim 1 \text{ TeV} \), it is shown that \( f \sim 100 \text{ GeV} \). Thus, the scale of electroweak symmetry breaking can be related to the strong \( SU(2)_H \) dynamics, which will be discussed later after including the electroweak Higgs fields. Anyhow, a low energy effective theory below a TeV scale can be described by these composite fields. We apply a naive dimensional analysis \([25, 26]\) for these fields and canonical normalization for all the composite fields, we can obtain the following effective superpotential,

\[
W_{\text{eff}} = \lambda f (S_L B_L + S_R B_R) + a \left\{ S_L \text{Tr}[\Omega_L \tilde{\Omega}_L] + S_R \text{Tr}[\Omega_R \tilde{\Omega}_R] \right\} + b \left\{ S_L \text{Tr}\Phi_L^2 + S_R \text{Tr}\Phi_R^2 \right\}
\]

\[
+ y \left\{ \text{Tr}\Omega_L \Phi_L \tilde{\Omega}_L + \text{Tr}\Omega_R \Phi_R \tilde{\Omega}_R \right\} + W(S_L, S_R)
\]

\[
+ X(\Pi_u^0 \Pi_d^0 - \Pi^+ \Pi^- - B_L B_R - f^2), \tag{2.15}
\]

where \( X \) denotes the Lagrange multiplier to constrain the quantum deformed moduli spaces.

The supersymmetric vacua is given by the following F-flatness conditions

\[
\frac{\partial W}{\partial S_L} = \lambda f B_L + a \text{Tr}[\Omega_L \tilde{\Omega}_L] + b \text{Tr}[\Phi_L^2] + \frac{\partial W(S_L, S_R)}{\partial S_L} = 0,
\]

\[
\frac{\partial W}{\partial S_R} = \lambda f B_R + a \text{Tr}[\Omega_R \tilde{\Omega}_R] + b \text{Tr}[\Phi_R^2] + \frac{\partial W(S_L, S_R)}{\partial S_R} = 0,
\]

\[
\frac{\partial W}{\partial \Pi_{u,d}^0} = X \Pi_{d,u}^0 = 0, \quad \frac{\partial W}{\partial \Pi^\pm} = -X \Pi^\mp = 0,
\]

\[
\frac{\partial W}{\partial \omega_L^0} = \omega_L^0(aS_L + \frac{y\delta_L}{\sqrt{2}}) = 0, \quad \frac{\partial W}{\partial \omega_R^0} = \omega_R^0(aS_R + \frac{y\delta_R}{\sqrt{2}}) = 0, \tag{2.16}
\]

\[
\frac{\partial W}{\partial \omega_L^0} = \omega_L^0(aS_L + \frac{y\delta_L}{\sqrt{2}}) = 0, \quad \frac{\partial W}{\partial \omega_R^0} = \omega_R^0(aS_R + \frac{y\delta_R}{\sqrt{2}}) = 0,
\]

\[
\frac{\partial W}{\partial \delta_L} = 2bS_L \delta_L + \frac{y\omega_L^0 \omega_L^0}{\sqrt{2}} = 0, \quad \frac{\partial W}{\partial \delta_R} = 2bS_R \delta_R + \frac{y\omega_R^0 \omega_R^0}{\sqrt{2}} = 0,
\]

\[
\frac{\partial W}{\partial B_L} = \lambda f S_L - B_R X = 0, \quad \frac{\partial W}{\partial B_R} = \lambda f S_R - B_L X = 0,
\]

with quantum constraint, \( \Pi_u^0 \Pi_d^0 - \Pi^+ \Pi^- - B_L B_R = f^2 \). Here, we have used the following notation for the triplet fields as

\[
\Omega_L = \begin{pmatrix} \omega_L^+ \omega_L^- \omega_L^0 \omega_L^0 \omega_L^- \omega_L^+ \end{pmatrix}, \quad \Omega_R = \begin{pmatrix} \omega_R^+ \omega_R^- \omega_R^0 \omega_R^0 \omega_R^- \omega_R^+ \end{pmatrix}, \quad \Phi_L = \begin{pmatrix} \delta_L^0 \delta_L^- \delta_L^+ \delta_L^+ \delta_L^- \delta_L^0 \end{pmatrix}, \tag{2.17}
\]
in which superscripts show the electro-magnetic charge, \( Q = T_L + T_R + (B - L)/2 \) and the field contents for \( \tilde{\Omega}_L, \tilde{\Omega}_R \) and \( \Phi_R \) are understood as well.

We have assumed that charged fields do not take non-zero magnitudes of VEVs, and thus have dropped the charged fields, for simplicity. We are interested in the left-right asymmetric vacua which are given by a solution of \( \langle \omega^0_L \rangle, \langle \tilde{\omega}^0_L \rangle = 0 \) and \( \langle \omega^0_R \rangle, \langle \tilde{\omega}^0_R \rangle \neq 0 \). Through the equation \( F_{\Pi} = 0 \), we can show that the electroweak symmetry is not broken (\( \langle \Pi \rangle = 0 \)), if \( \langle X \rangle \neq 0 \). Thus, in order to avoid the unwanted large scale electroweak symmetry breaking, we take \( \langle X \rangle \neq 0 \) here, and we will consider the electroweak symmetry breaking later.

As shown in Eq.\((2.16)\), one can obtain the similar structure given in Eq.\((2.4)\). In this case, from the equation of \( F_{\omega} = 0 \) or \( F_{\omega^c} = 0 \), the solutions for left-right Higgs sectors are classified into two types of solutions, that is, \( \langle \omega^0_L,R \rangle = 0 \) or \( \neq 0 \). For example, if we have a solution \( \langle \omega^0_R \rangle = 0 \), we obtain \( \langle \delta^0_R \rangle = 0 \), where the \( SU(2)_R \) symmetry is not broken. On the other hand, if we have another solution of \( \langle \delta^0_R \rangle = -\sqrt{2}a\langle S_R \rangle \), we obtain the VEVs as \( \langle \omega^0_R \tilde{\omega}^0_R \rangle = \frac{2ab\langle S_R \rangle^2}{y^2} \).

This is the suitable symmetry breaking of \( SU(2)_R \times U(1)_{B\!-\!L} \rightarrow U(1)_Y \). Thus, as long as \( \langle S_R \rangle \neq 0 \) these two types of the solutions have different structure. In this case, we can take a different vacuum structure for \( L \) and \( R \) sector as, \( \langle \delta^0_L \rangle = 0 \) and \( \langle \delta^0_R \rangle = b\langle S_R \rangle/y \). Where, VEVs of other fields are given by

\[
|\langle B_L \rangle| = \left( \frac{hy^2}{hy^2 + 2a^2b} \right)^{1/6} f,
|\langle B_R \rangle| = \left( \frac{hy^2 + 2a^2b}{hy^2} \right)^{1/6} f,
|\langle X \rangle| = \left( \frac{\lambda^{3/2}}{(3hc)^{1/4}} \right) f.
\]
Notice that we have only one energy scale $f$, which can appear as SUSY dynamics and its value can be taken arbitrary. Thus, taking larger energy scale of $f$ (than the electroweak symmetry breaking), it is possible to have a large scale spontaneous left-right parity violation. The order estimation gives the scale of VEVs as $\langle S_L \rangle \sim \langle S_R \rangle \sim \langle \delta_R \rangle \sim f$, $\langle \omega_R \bar{\omega}_R \rangle \sim f^2$, and $\langle B_L \rangle \sim \langle B_R \rangle \sim f$. The $SU(2)_R$ symmetry is broken down to $U(1)_R$ by $\langle \delta^0_R \rangle$, and further breakdown of the symmetry $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ by the VEVs of $\omega_R \bar{\omega}_R$. Therefore, these vacua can derive left-right asymmetric solution as in Ref. [21]. It should be noted that, in this model, left and right Higgs sectors are not independent of each other due to the quantum constraint, so the asymmetric solution (vacua) is non-trivial.

Now let us investigate mass spectra of the Higgs supermultiplet. In the model, two symmetry breaking scales of $SU(2)_R$ and $U(1)_{B-L}$ are the same with the VEVs of $S_L$ and $S_R$. So all the triplet mass scale may be given by $f$. Similarly the masses of the singlets $B_{L,R}$ and $S_{L,R}$ are same scale $f$. As for the composite Higgs fields $\Pi$, they can have effective mass of order $f$ with Higgs bi-doublet after including the electroweak Higgs field (see Eq.(2.23)).

Here we comment on some topics. The first is about field content. While we construct the model using the triplet fields, we can also apply these mechanism in the doublet fields as shown in the previous section. The next is about the electroweak symmetry breaking. At the present stage, we do not add an elementary Higgs fields which should have VEVs for the electroweak symmetry breaking and inducing quark/lepton masses. As shown in Ref. [27], the electroweak Higgs field can couple to the $\Pi$ which gives rise to a SUSY mass term for Higgs fields. This may be a alternative solution to so-called $\mu$-problem as shown next subsection (see, Eq.(2.23)). For the realistic model, soft SUSY breaking terms should be included in the model and can lead to the electroweak symmetry breaking vacua as the usual MSSM. The soft masses $m_{\text{soft}}$ of order a few hundred GeV are small enough comparing to the dynamical scale $f$, so that SUSY breaking effects can be treated as a perturbation based on the naive dimensional analysis. Thus, it is possible to analyse the spectra including the soft SUSY breaking terms, and this situation leads entirely different structure from Ref. [27].

It is also interesting to consider the dynamical electroweak symmetry breaking due to this SUSY strong gauge dynamics. In this setup, there are the composite electroweak Higgs fields $\Pi$, and its VEVs itself can break the electroweak symmetry breaking. It is the same as the scenario of the technicolor models with SUSY extension [24]. Naively, we can consider a setup that these fields can directly couple to the elementary Higgs fields, and the quark/lepton mass matrices and mixing can be obtain through the ordinary Yukawa couplings without dangerous FCNC processes. These mechanisms can be achieved by taking the different vacuum. In the next section, we discuss the possible electroweak symmetry breaking in above models.
2.3 Electroweak symmetry breaking

We include the Higgs bi-doublets field $H$ and an additional singlet $S$. The superpotential is given by

$$W_H = \tilde{h}_y L R H + \alpha S H^2 + \kappa S^3. \quad (2.22)$$

Then, below the dynamical scale $\Lambda$, the effective superpotential becomes

$$W_{eff} = h_y f \Pi H + \alpha S H^2 + \kappa S^3. \quad (2.23)$$

The F-flatness conditions for $H$, $\Pi$, and $S$ are given by

$$\frac{\partial W}{\partial H} = f h_y \Pi + \alpha S H, \quad \frac{\partial W}{\partial \Pi} = X \Pi + f h_y H, \quad (2.24)$$

$$\frac{\partial W}{\partial S} = \alpha H^2 + 3 \kappa S^2.$$

Assuming $\Pi^\pm = 0$, the VEVs of the fields $S$ and $\Pi$ are given by the following relations,

$$\langle \Pi^0_u \Pi^0_d \rangle = -\frac{3\kappa (fh_y)^6}{\alpha^3 \langle X \rangle^4}, \quad \langle S \rangle = \frac{(fh_y)^2}{\alpha \langle X \rangle}. \quad (2.25)$$

The remaining equations are given by

$$\frac{\partial W}{\partial S^L} = \lambda f B_L + \frac{\partial W(S^L, S^R)}{\partial S^L} = 0, \quad (2.26)$$

$$\frac{\partial W}{\partial S^R} = \lambda f B_R + a \omega^0_{R^*} \omega^0_R + b (\delta^0_{R^*})^2 + \frac{\partial W(S^L, S^R)}{\partial S^R} = 0, \quad (2.27)$$

with quantum constraint

$$\Pi^0_u \Pi^0_d - B_L B_R = -f^2. \quad (2.28)$$

Now let us consider the superpotential $W(S^L, S^R)$ as

$$W(S^L, S^R) = MS^L S^R + h(S^2_3 + S^3_3), \quad (2.29)$$

where we introduce the vector-like mass term $MS^L S^R$ for the singlet fields. We here introduce additional mass scale of $M$ as the singlet mass, since it is very difficult to induce two different scales, left-right symmetry breaking and electroweak scales, only from $f(\Lambda)$. Although it is contradict our motivation, “all scale must be introduced dynamically”, it might be a minimum setup of reproducing suitable scales of left-right symmetry breaking and electroweak scales. We will soon know they are suitably induced from $M$ and $f(\Lambda)$. 


Actually this new parameter (scale) \( M \) can be regarded as the SUSY breaking effects. Since, without the term \( MS_L S_R \), the superpotential in Eq. (2.12) has a global \( U(1)_R \) symmetry where both the \( L \)- and \( R \)-subscripted fields have their charge 2/3. Thus it could be natural to assume the \( U(1)_R \) symmetry is broken by the soft SUSY breaking terms. For example, an introduction of a spurion field \( \xi = F_\xi \theta^2 \) with \( U(1)_R \)-charge of 2 can induce the term \( MS_L S_R \) through \( D \)-term interaction \( [\xi^\dagger S_L S_R]_D \) as well as SUSY breaking terms.

Assuming the hierarchy between \( f \) and \( M \), which is reasonable assumption and makes us easily see the vacuum structure, we find three different types of vacuum solutions; (i): \( \langle B_L \rangle \sim \langle B_R \rangle \sim M^2/f \) and \( \langle X \rangle \sim f^2/M \), (ii): \( \langle B_L \rangle = \langle B_R \rangle = 0 \) and \( \langle X \rangle \sim f \), (iii): \( \langle B_L \rangle \sim \langle B_R \rangle = f \) and \( \langle X \rangle \sim M \). The cases of (i) and (ii) are out of our interest. It is because, in case of (i), the triplet Higgs fields get the mass from the VEVs of \( \langle S_{L,R} \rangle \sim \langle B_{R,L} X \rangle/f \sim M \), so that the VEVs of \( \Pi \) are estimated as \( \langle \Pi \rangle \sim M^2/f \) from Eq. (2.25), which means the VEV of \( \Pi \) is too large since we assume \( M \gg f \). As for the case of (ii), obviously the vacua do not suggest the left-right symmetry breaking. Thus, let us take the case of (iii).

From Eqs. (2.18), the left-right breaking scale is given the scale of \( M \), while the VEVs of \( S \) and \( H \) are of order \( f^2/M \). Thus we can easily obtain the hierarchy between the left-right breaking scale and electroweak scale. Furthermore, the composite Higgs VEVs is of order \( \langle \Pi \rangle \sim f^3/M^2 \). Taking a numerical example as \( a = 1, b = -1/3, \lambda = y = h = h_y = 1, \kappa = -1/3, \quad f = \frac{\Lambda}{4\pi} = 1.5 \text{ TeV}, \text{ and } M = 15 \text{ TeV} \), the VEVs for the case (iii) are given by

\[
|\langle B_L \rangle| \sim |\langle B_R \rangle| \sim 1.5 \text{[TeV]}, \quad |\langle X \rangle| \sim 15 \text{[TeV]},
\]

This vacuum has correct electroweak symmetry breaking vacua with large scale left-right symmetry breaking. It is because the electroweak symmetry breaking vacua is dominated by \( \langle H \rangle \sim f^2/M \), and the left-right symmetry scale is given by the mass scale of \( M \).

Here we briefly show the mass spectra in the supersymmetric model. As already explained, the charged triplet Higgs masses \( \omega_{L,+}^+ \omega_{R,+}^+ \) are given by the VEVs of \( S_{L,R} \sim M \). On the other hand, the neutral components \( \delta \) are mixed with \( S_{L,R} \) or composite fields \( B_{L,R} \), where their VEVs are \( M, M \) and \( f \), respectively. As we expected, they have a TeV scale mass. For the neutral components of doublet Higgs \( H_u^0 \) and \( H_d^0 \), they can mix with \( \Pi_u^0, \Pi_d^0, \text{ and } S \). And, the mass matrix of these fields \( (S, H, \Pi) \) are given

\[
\begin{pmatrix}
  \langle S \rangle & \langle H \rangle & 0 \\
  \langle H \rangle & \langle S \rangle & f \\
  0 & f & 0
\end{pmatrix}
\]

The mass eigenvalues of this mass matrix are roughly estimated by \( (f, f, f/M^2) \). Thus, there exists possibly one light Higgs component in the model.

For the realistic electroweak symmetry breaking, the following Fermi relation should be satisfied,

\[
(176)^2 \text{GeV}^2 = \langle \Pi_u^0 \rangle^2 + \langle \Pi_d^0 \rangle^2 + \langle H_u^0 \rangle^2 + \langle H_d^0 \rangle^2.
\]
Notice that these two symmetry breakings occur by the strong gauge dynamics in both cases with and without SUSY breaking effect. We can expect the realistic electroweak symmetry breaking vacua is obtained even after additional soft SUSY breaking terms are induced, and its reason is shown just below. As for oblique electroweak contributions [28], the dynamical breaking models can give a certain value of the electroweak $S$-parameter in general, which is strongly constrained in conventional technicolor models. In our model, SUSY strong dynamics also give a contribution to it. However the leading order contribution is small comparing to previous technicolor models. It is because the electroweak symmetry breaking scale is given by $f^2/M$, and which means there is further suppression in the case of $f \sim 1[\text{TeV}]$. This naive dimensional analysis can also apply to the soft SUSY breaking terms. This is the reason why desirable vacuum is expected to be obtained after including SUSY breaking effects.

3 Model II (IR free model)

In the previous section, we considered the strongly coupled gauge dynamics and light degrees are confinement. Next, we consider another example for spontaneous parity violation based on the doublet model. We use the fact that strong SUSY dynamics which has various vacuum structures can exhibit meta-stable SUSY breaking [18]. The simplest way to construct a model with spontaneous parity violation is to consider both the left and right gauge sectors individually as the model explained in Eq. (2.1), and combine these two sectors with different vacuum alignment. Let us consider the gauge group $SU(3)_{H_I} \times SU(2)_I \times U(1)_{B-L}$, where the gauge $SU(3)_{H_I}$ is additional strong interaction and $I$ means L(eft) or R(ight). The chiral multiplets $Q^a$ and $\tilde{Q}^a$, $(a=1,2)$ have following quantum numbers

$$Q^a \sim (3,2,1), \quad \tilde{Q}^a \sim (\bar{3},2,-1),$$

and a common vector-like mass $m$. This theory is $N_C = 3$ and $N_F = 4$ in SQCD, and has a magnetic dual description valid at the low energy scale [23]. Below the scale of $m \ll \Lambda$, the magnetic dual theory has dual matter fields $\varphi^a$, $\tilde{\varphi}^a$ and meson fields $\Phi^{ab}$ with the quantum number

$$\varphi^a \sim (2,-1), \quad \tilde{\varphi}^a \sim (2,-1), \quad \Phi^{ab} \sim 1 + \text{Adj}.$$  

In the case the magnetic dual fields are equivalent to the s-confinement fields given by the mesons and baryons.

The general dual superpotential is given by

$$W = h \text{Tr}(\tilde{\varphi}\varphi) + \alpha \Lambda m \text{tr}(\Phi) + \frac{c}{\Lambda} \text{det}\Phi,$$  

where the couplings, $h$, $\alpha$, and $c$ are $\mathcal{O}(1)$ parameters, and tr means the trace over gauge and flavor indexes. If we neglect the third term in Eq. (3.35), where the scale $\Lambda$ is much larger
than the VEVs of $\Phi$, SUSY is broken by the rank conditions [18]. This SUSY breaking vacua is regarded as meta-stable vacua, since there is a SUSY vacuum in Eq. (3.35).

We see more details of this model. At first, let us consider a effective superpotential

$$W = h \text{Tr}(\tilde{\varphi}\Phi\varphi) + \alpha \Lambda m \text{tr}(\Phi).$$  \hspace{1cm} (3.36)

Here we denoted these doublets $\varphi$ and meson $\Phi$ as

$$(\varphi^T)^a = (\varphi^0, \varphi^-)^a, \quad \Phi^{ab} = \left( \frac{S + \delta^0}{\sqrt{2}}, \frac{\delta^+}{\sqrt{2}}, \frac{s - \delta^0}{\sqrt{2}} \right)^{ab},$$  \hspace{1cm} (3.37)

where superscript shows the electromagnetic charge of $SU(2)$ doublet and triplet, and the field $S$ is gauge singlet field. It is always possible to take the VEVs of $\varphi$ as $\langle \varphi^T \rangle^a = (\langle \varphi^0 \rangle, 0)^a$ by using the degrees of freedom of the gauge and flavor transformations. Then, the second term in the superpotential $\alpha \Lambda m \text{tr}(\Phi)$ is expressed as

$$\alpha \Lambda m \text{tr}(\Phi) = \sqrt{2} \alpha \Lambda m (S^{11} + S^{22}),$$  \hspace{1cm} (3.38)

and this mass parameter preserves the $SU(2)$ gauge symmetry but breaks the flavor $SU(2)$ symmetries to its diagonal form. The F-flatness conditions for $S$ and $\delta^0$ are given by

$$F_{S^{11,22}} = \sqrt{2} \alpha \Lambda m + \frac{(\tilde{\varphi}^0 \varphi^0)^{11,22}}{\sqrt{2}} = 0,$$

$$F_{(\delta^0)^{ab}} = \frac{(\tilde{\varphi}^0 \varphi^0)^{ab}}{\sqrt{2}} = 0.$$

From these equations, the minimum is given by

$$\langle \tilde{\varphi}^0 \varphi^0 \rangle^{ab} = -\frac{\alpha \Lambda m}{2} \neq 0.$$  \hspace{1cm} (3.39)

The above solutions mean that gauge group $SU(2) \times U(1)$ reduces to $U(1)$ as well as SUSY is broken in the vacuum.

Next, we find the true vacuum with the third term of Eq. (3.35). The true vacua can be obtained by solving the full superpotential in Eq. (3.35). We can find the supersymmetric vacuum at

$$\langle S^{11} \rangle \sim \langle S^{22} \rangle \sim (\Lambda^2 m)^{1/3},$$  \hspace{1cm} (3.40)

and doublet fields can not have their VEVs. This singlet VEV gives masses of the vector-like Higgs doublets. This leads to the SUSY solution, and the gauge group $SU(2) \times U(1)$ never breaks into its diagonal form.
Finally, let us consider the larger gauge group as $SU(3)_H \times SU(2)_L \times SU(3)_R \times U(1)_{B-L}$ with left-right symmetry, and combine above two results. We take a vacuum where the meta-stable vacua for right-sector, and the SUSY true vacua for left-sector. Then, we obtain parity breaking vacuum. Please note that since the full gauge theory of this model is chiral gauge model so our results do not conflict the Vafa-Witten theorem [29].

The flavor structure of these models are expected to be obtained by introducing elementary Higgs fields $H$ through the Yukawa couplings. Anyhow, notice again, the electroweak symmetry breaking and SUSY breaking are triggered simultaneously in this model. We can expect that the suitable soft SUSY breaking terms are induced from the right-sector.

4 Summary and discussions

We have suggested simple models of spontaneous parity violation in SUSY strong gauge theory. We have focused on left-right symmetric model and investigate vacuum with spontaneous parity violation. Non-perturbative effects are calculable in SUSY gauge theory, and we have suggested two new models. The first model showed confinement, and the second model had a dual description of the theory. The left-right symmetry breaking and electroweak symmetry breaking are simultaneously occurred with the suitable energy scale hierarchy. The second model had also induced spontaneous SUSY breaking.

In detail, the first model suggested that left-right sectors couple to new strong gauge fields which are confined at low energy. Scale of left-right symmetry breaking is determined by the dynamical scale of new strong gauge theory, where we consider a $\mathcal{N} = 1$ SQCD with $N_C = 2$ and $N_F = 4$. Our setup is related to the SUSY technicolor model, so it is easy to make the model couple to the electroweak Higgs. We can realize the left-right symmetry breaking and the electroweak symmetry breaking simultaneously with the suitable energy scale hierarchy. This structure has several advantages compared to the MSSM. The scale of the Higgs mass scale (left-right breaking scale) and that of VEVs are different, so the SUSY little hierarchy problems are absent. At same time, the SUSY breaking terms, which are expected to be smaller than dynamical scale, can be treated perturbatively by assuming the canonical Kahler potential for all the composite fields.

The second model also has the structure, that is, left-right Higgs fields couple to new strong gauge fields. The dual description of the model is possible, and one can find the $SU(2)_R$ broken vacua of meta-stable, which means this model also induces spontaneous SUSY breaking. The UV completion of this setup only includes vector-like left-right two Higgs fields, therefore this setup is a minimal for left-right symmetric scenario. The low energy effective theory of this model should be the MSSM with induced soft SUSY breaking terms, and all the spectra are calculable.

Finally, we comment on a recent Tevatron experiment [30] which reported the anomalous
excess in $W_{jj}$ dijet events and implied the existence of the resonance state with mass $M \sim 150[GeV]$. Their simple explanation can be given by new particle originated from new strong interaction like a technicolor model [31]. It is naively expected that there exists a possible particle candidate in our model. For example, in our first model, a new particle from the mixing of $\Pi$ and $H$ in Eq.(2.31) could have a mass around 150[GeV].

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