Dynamic Behavior Analysis of Compliant Micromechanisms

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Abstract. A dynamic behavior analysis of a compliant four-bar micromechanism is explored in this work. The Pseudo-Rigid-Body Model (PRBM) is used to model the compliant mechanism, and to analyze the large deflection of flexible segment. This model and Lagrange Equations are used to develop the dynamic equations. An example is given with all required parameters of a mechanism. The PRBM is also modelled in ADAMS, and the lagrangian model results are compared to the results obtained from ADAMS. Finally, a commercial finite element code capable of nonlinear analysis is used to model the mechanism with its flexible segment. By this way a comparison of PRB model and real model is proposed.

1. Introduction

The popularity of compliant mechanisms is expected to grow since the compliance grants the designer greater freedom in the number of possible solutions for a given problem. However, this freedom in design is often offset by difficulties encountered in the analysis of the compliant members. The use of compliance presents several advantages including part count reduction and more precise motion. Compliant mechanisms have numerous applications in MicroElectroMechanical Systems (MEMS). Many compliant mechanisms have been designed and analyzed [3], but less attention has been paid to dynamic analysis.

Compliant four-bar mechanisms are sometimes synthesized for motion generation tasks [5]. Mankame and Anathasuresh [6] have explored the use of contact interactions to enhance the functionality of compliant mechanisms. Also a compliant configuration has been selected for in depth study [7]. These mechanisms have applications in robotics and biologics [8]. There are some works which have determined the limit positions of Compliant Mechanisms [9].

Unlike rigid-body mechanisms, existence of compliant member causes nonlinearity of dynamic equations in these mechanisms. The Pseudo-Rigid-Body Model (PRBM) is a method of analysis that allows the large deformations to be modeled using rigid-body kinematics.

Having a good understanding of dynamic behavior of this kind of mechanisms will help us in the future works. A four-bar compliant micromechanism such as the one shown in Figure 1(a) is considered in this work. As the figure shows, one of the members is compliant and others are rigid. Two ways are chosen to make the mechanism move; first, by applying a torque on point D and second, by giving an initial angular velocity.

In Section 2 the pseudo-rigid-body model of the compliant micromechanism shown in Figure 1(a) is given. Also the dynamic model of this mechanism is proposed by expressing the lagrange’s equations. The parameters of an example are given in Section 3 and the results obtained from the lagrangian model are shown in some figures. Finally, Section 4 is the comparison section. The comparison of lagrangian results and the results obtained from ADAMS is presented. Also a
comparison of the results of pseudo-rigid-body model and the real one which is modeled in ANSYS is shown in this section.

![Figure 1](image)

**Figure 1.** (a) Four-bar Compliant micromechanism, (b) The pseudo-rigid-body model

## 2. Dynamic model

This section presents the derivation of a closed-form dynamic model for a mechanism shown in Figure 1(b). First, the compliant mechanism is modeled as a rigid-body mechanism using the PRBM. This configuration consists of three rigid links and one spring. Converting the mechanism to its rigid-body counterpart greatly simplifies kinematic and dynamic analysis by allowing the use of rigid-body modeling techniques. Lagrange’s Method is then used to obtain an equation of motion for the mechanism.

### 2.1. The pseudo-rigid-body model

The PRBM for the four-bar spring configuration is shown in Figure 1(b). The mechanism is converted to its rigid-body counterpart by using the PRBM for a cantilever beam, as described below.

The flexible segment of length $l$ is replaced by two rigid links with lengths $r_2$ and $r'_2$. The length of link 3 is determined by the relation for the pseudo-rigid-body link's characteristic radius,

$$r_2 = \gamma l$$

where $\gamma$ is the characteristic radius factor. The length of $r'_2$ is then

$$r'_2 = l - r_2$$

Since $r'_3$ is fixed to ground and has no motion, there is no need to enter it in analysis.

The compliance of the flexible segment is represented by a torsional spring at the new pin (called the characteristic pivot) joining links 3 and ground. The torsional spring constant $K$ for a cantilever beam with a force at the free end is given by

$$K = \gamma K_{\theta} \left( \frac{EI}{l} \right)$$

where $E$ is the Young’s Modulus of the material, $I$ is the area moment of inertia, $l$ is the beam length and $K_{\theta}$ is the pseudo-rigid-body model stiffness coefficient, which has the value of 2.65 [3] and $\gamma = 0.85$ [4].

### 2.2. Formulating the Lagrangian

Angle $\theta_2$ is selected as the generalized coordinate, corresponding to the generalized force $Q_{\theta_2}(t)$ to simplify the derivation. Note that because $\theta_2$ is an angle, its corresponding generalized force, $Q$, has the dimension of moment.

The Lagrangian $\ell$ is formed by taking the difference of the scalar quantities of kinematic energy $T$ and potential energy $V$ of the system.
\[ \ell = T - V \] (4)

One way to formulate T is to separate the motion of the mechanism inertias in to both translation and rotation, as illustrated in Figure 2(b). The center of mass of each link translates along a predefined path as the mechanism moves, and each link rotates about its center of mass.

Figure 2. (a) The pseudo-rigid-body model with parameters, (b) Translational and rotational motions of the mechanism links.

The first three terms of the kinetic energy expression represent the translational energy of the system, and the last three represent the rotational energy:

\[ T = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} m_4 v_4^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{2} J_3 \dot{\theta}_3^2 + \frac{1}{2} J_4 \dot{\theta}_4^2 \] (5)

where \( m_i \) is the mass, \( v_i \) is the velocity of the center of the mass, \( J_i \) is the mass moment of inertia and \( \dot{\theta}_i \) is the angular velocity of links.

the mass moments inertia of links are

\[ J_i = \frac{1}{12} m_i r_i^2 \] (6)

and

\[ v_2 = \frac{1}{2} r_2 \dot{\theta}_2, \quad v_4 = \frac{1}{2} r_4 \dot{\theta}_4 \] (7)

\[ v_3^2 = r_2^2 \dot{\theta}_2^2 + \frac{1}{4} r_3^2 \dot{\theta}_3^2 + r_2 r_3 \dot{\theta}_2 \dot{\theta}_3 \cos(\theta_2 - \theta_3) \] (8)

Assuming that the mechanism has a planar motion its potential energy is

\[ V = \frac{1}{2} K(\theta_2 - \theta_{20}) \] (9)

2.3. Lagrange’s Equations

Using lagrange’s formulation, the equation of motion for the system is expressed as

\[ \frac{d}{dt} \left( \frac{\partial \ell}{\partial \dot{\theta}_2} \right) - \frac{\partial \ell}{\partial \theta_2} = Q_{\theta_2} \] (10)

Expanding out the above equation, motion equation for the system becomes
\[
\frac{1}{3} m_1 r_1^2 + \frac{1}{3} m_1 r_1^2 h_{32}^2 + \frac{1}{3} m_4 r_4^2 h_{42}^2 + m_3 r_2 r_3 h_{32} \sin(\theta_2 - \theta_3) \ddot{\dot{\theta}_2} + \frac{1}{3} m_3 r_3^2 h_{32}^2 + \frac{1}{3} m_4 r_4^2 h_{42}^2 - \frac{1}{2} m_3 r_2 r_3 h_{32} \sin(\theta_2 - \theta_3) + \frac{1}{2} m_3 r_3 r_3 h_{32} (1 - h_{32}) \cos(\theta_2 - \theta_3) \dot{\theta}_2^2 + K(\theta_2 - \theta_2) = Q_{\theta_2}
\]

(11)

where

\[
Q_{\theta_2} = M(t) h_{42}
\]

(12)

\[
h_{42} = \frac{r_2 \cos(\theta_3 - \theta_2)}{r_4 \cos(\theta_3 - \theta_4)} \quad , \quad h_{32} = \frac{r_2 \cos(\theta_3 - \theta_2)}{r_3 \cos(\theta_3 - \theta_4)}
\]

(13)

\[
h_{42}' = \frac{dh_{42}}{d\theta_2} \quad , \quad h_{32}' = \frac{dh_{32}}{d\theta_2}
\]

(14)

2.4. ADAMS Model

The basis of modelling in ADAMS is simulating the pseudo-rigid-body model which included rigid links related by revolute joints to each other. Length of the flexible segment is modified as it is shown in Figure 1(b). Considering the flexibility of the compliant segment a torsional spring is set on pivot (point E in Figure 1(b)). The stiffness of this spring is obtained from equation (3). According to the geometry and density of each link, masses automatically calculate by this software.

2.5. Finite Element Model

In this work, ANSYS is used as finite element software capable of nonlinear analysis. It should be noted that the mechanism is modeled like a real one. So, this model has one flexible segment and other two are rigid (Figure 1(a)). Beam element is used to model these links. The flexibility of the flexible segment is considered by its small cross section while the rigid links have bigger one. The revolute joints are modeled by coupling the degrees of freedom of nodes (B and C in Figure 1(a)).

3. Results

All necessary parameters of a mechanism are given in Table 1. Using these parameters we can yield to the results illustrated in following figures.

In Figure 3 two plots are shown which related to the four-bar mechanism illustrated in Figure 2(a) using the parameters given in Table 1, in addition to initial angular velocity of 100 rad/s in link 2 (\( \theta_{20} = 100 \) rad/s).

In another experience a torque is applied on link 4. This torque linearly changes with time by the following equation:

\[
M = 100 \times \text{time}
\]

(15)

The results are shown in Figure 4.

| Parameter | \( r_1 \) (\( \mu m \)) | \( r_2 \) (\( \mu m \)) | \( r_3 \) (\( \mu m \)) | \( r_4 \) (\( \mu m \)) | \( \theta_{20} \) (deg) | \( E_2 \) (\( N/mm^2 \)) | \( I_2 \) (\( \mu m^4 \)) | \( m_2 \) (\( \mu g \)) | \( m_3 \) (\( \mu g \)) | \( m_4 \) (\( \mu g \)) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Value     | 500             | 500             | 500             | 500             | 0               | 150*10^3        | 208.33          | 0.0986          | 1.2388          | 1.2388          |
4. Comparison
The base of this work is the lagrangian formulation of PRBM which is illustrated in Section 2 and the results are given in Section 3. For comparison, the pseudo-rigid-body model which is shown in Figure 1(b) is modeled in ADAMS. Figure 5(a) shows two graphs. The first one is obtained from lagrangian model and the second one from ADAMS.

It should be noted here that the displacement of joint B in Figure 1(b) is used for comparison.

On the other hand, a commercial finite element code capable of nonlinear analysis (ANSYS) is used to model the mechanism which is shown in Figure 1(a). Figure 5(b) shows the differences of the results of lagrangian model obtained using the pseudo-rigid-body model and the results obtained using finite element analysis.

In finite element model the mechanism is modeled with rigid and flexible segments and a non-linear solution is obtained while in lagrangian model the pseudo-rigid-body model is used to develop the lagrange's equations. PRB model has three rigid links and one spring, without a compliant link. Therefore, a linear solution is possible.

The correlation between these three modeling results proves that the pseudo-rigid-body model can predict the dynamic behavior of compliant micromechanisms.
5. Conclusion

In this work, a four-bar compliant micromechanism with flexible link has been presented. Dynamic analysis has been done and the results are depicted. First the pseudo-rigid-body model of compliant mechanism presented and then lagrange's equations developed. The PRB model has been also modeled in ADAMS. Finally, ANSYS as a commercial finite element code capable of non-linear analysis has been used to model the mechanism. The comparison between three models of the mechanisms is presented. It should be noted that the models are developed assuming no friction in joints.

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