Algebraic thinking obstacles of elementary school students: A Hermeneutics-phenomenology study

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Abstract. The main purpose of this research is to explore learning obstacles on algebraic thinking that occurs in elementary school students. It is important to understand learning obstacles to anticipate a didactic teacher during learning process and conduct an innovation in developing new learning designs. Therefore, this study is conducted by utilizing a qualitative research with hermeneutics and phenomenology paradigms. The findings suggest that there are three types of obstacles are ontogenic obstacle, epistemological obstacle and didactical obstacle. This information is expected to be a material for developing didactic design of algebraic thinking. Furthermore, Algebraic thinking habit in elementary school is expected to reduce the difficulty when studying algebra formally.

1. Introduction
The transition from arithmetic to algebra has a chance of creating cognitive gap in students. To overcome such gaps, algebraic thinking activities can be performed. Algebraic thinking can be interpreted as a process in which students generalize mathematical ideas from a set of particular instance, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways [1]. Students generalize the mathematical ideas they receive through examples, opinions, and suggest the ideas by using the right way.

Algebraic thinking needs to be done since elementary school. Some studies show that algebra is one of difficult subject for secondary school students, who students especially in studying algebraic counting operations [2-6]. Elementary and secondary school mathematics curriculums should facilitate students to learn algebra, so that it includes attention to other mathematical domains [7, 8]. Therefore, through algebraic thinking habituation since elementary school level, the difficulty at the time of the formal algebra can be reduced.

The scope of algebraic thinking in primary schools includes generalizations based on patterns, facts, phenomena or existing data, solving problems and communicating ideas through symbols, tables, diagrams, or other media to clarify circumstances or problems [9, 10]. On the other hand, before the students learn at the formal study of algebra, students should already have a lot of experience in making representation, abstraction and generalize the relationship between numbers and arithmetic operations [8]. Algebraic thinking is a thinking activity that is proven to be difficult for
students, and it is now widely recognized that students need an earlier opportunity to engage in algebraic reasoning in elementary school [11, 12].

Previous research shows that students are able to develop algebraic thinking in elementary school. The results of research on seven year old primary school students found that representational activity and problem solving in algebraic thinking activity [13]. Students develop algebraic thinking by making generalizations of the solutions obtained. Moreover, research results prove that grade 4 students have begun to develop algebraic thinking [14]. The development of algebraic thinking begins by revealing the generalization of numerical relationships in various representations. It is reinforced by the finding that non symbolic algebraic thinking Begins to occur when students are at the age of seven to eight years [15-17].

In grades 3-5, algebraic ideas should emerge and be investigated as students: (a) identify or build numerical and geometric patterns; (b) describe patterns verbally and represent them with tables or symbols; (c) look for and apply relationships between varying quantities to make predictions; (d) make and explain generalizations that seem to always work in particular situations; (e) use graphs to describe patterns and make predictions; (f) explore number properties; and (g) use invented notation, standard symbols, and variables to express a pattern, generalization, or situation [13].

Development of algebraic thinking of elementary school students in various countries is different. Studies conducted in US, Singapore, Korea, China and Australia indicates that the primary school curriculum about algebra is different [11, 13, 18-20]. In Indonesia, algebraic thinking learning in primary schools is not stated explicitly in the curriculum but it is implied in some basic mathematical competencies that use algebraic thinking processes. The ability of algebraic thinking is less good. The obstacles are caused by students' self and the learning that students experience [21].

Brousseau classifies didactical obstacle to three types, namely, ontogenic obstacle, didactic obstacle, and epistemological obstacle. Knowledge, people and their milieu being what they are, it is inevitable that these interactions lead to conceptions which are erroneous (or correct locally but not generally). However, these conceptions are controlled by conditions of the interaction that can be more or less modified [22]. Therefore, a thorough analysis of types and causes of learning obstacles is needed. The discovery of learning obstacles experienced by students is a new challenge for teachers to find solutions to overcome the problems.

2. Method
This research is preliminary qualitative study with design didactical research (DDR). This research in which hermeneutics and phenomenology paradigms were used to analyze learning obstacles. The subjects of this study were 40 students of fifth and sixth grade of elementary school in Garut, and whom 9 of them were interviewed after the test. Instruments are created based on 3-5 level algebraic thinking indicators consisting of learning obstacles, interviews, book analysis and learning. Test questions were used to uncover students' errors in algebra thinking. After that, the causes of error were investigated through interviews, analysis of instructional materials and learning analysis.

3. Results and discussion

3.1. Analysis of student answers
The students worked on algebraic thinking questions about pictogram, bar charts and line charts. After the students had done the test, they were interviewed to clarifying the answer. Based on this, researchers can find some learning obstacles

3.1.1. Students did not recognize the concept of the drawing diagram. The error occurred because students didn’t know the concept of image diagram, thus causing student made error in representing data in the form of picture diagram (pictogram). Image diagram is one of the ways to model the problem situation that students must master in algebraic thinking.
As for the questions on the test “Dimas is listing vehicles owned by classmates in their homes. There are 10 people owning cars, 17 people have motorcycles, and 9 people have bicycles. Help Dimas create a pictogram based on the data! Student’s response can be seen in picture 1 below:

Figure 1. Respons students who didn’t know the concept of pictogram.

Figure 1 shows that students find it difficult to find a meaning of learning about something because of the limitations of the way of thinking. These difficulties is categorized as an ontogenic obstacle type [23]. The response in Figure 1 is classified into the conceptual type. Students cannot make pictogram due to their limited understanding of the concept. On the other hand, students can already represent data in the form of different images. Furthermore, it can be said that the obstacle is a type of ontogenical conceptual.

3.1.2. Students’ mistake in interpreting questions. The question: Today Miss. Tiwi will make a bar chart of the results of the Fourth grade Mathematics Test. The total number of students in Fourth grade is 25 people. Based on the test results, 2 people got score of 5, 6 people got score of 6, 14 people got score of 8 and 3 people got score of 10. Let’s help Miss. Tiwi to make a bar chart! Based on the bar chart you have created!

Figure 2. Incorrect response of students in understanding the problem.
In Figure 2, students are less thorough or careless in reading the question. Students failed to understand the use of commas on the problem so mistaken in doing the modelling into the form of bar charts. In addition, students cannot make effective diagrams by determining the right scale on the vertical line. Based on this, student errors belong to ontogenical instrumental and epistemological conceptual. Ontogenical instrumental occurs due to limited understanding to the concept of bar charts, especially regarding technical reading the problems. Meanwhile, epistemological obstacle comes from a person's understanding that is limited to certain concepts and difficulties when faced with different conditions with the same content [23].

3.1.3. Students’ mistaken in determining the order of description of horizontal and upright lines. The given problem was the same as the problem in figure 2.

![Figure 3. Students’ mistaken in determining the sequence of description of horizontal and upright lines.](image)

In Figure 3, we can see two types of obstacles that is students who are less precise or careless in determining the numbers on the vertical lines as the values and horizontal lines which represent the number of students. In addition, the determination of the numbers on the horizontal line did not match the concept. Therefore, the error is categorized as conceptual ontogenical obstacle and didactical obstacle. Didactic obstacle is caused by incorrect of learning trajectory. In addition, the complexity of the material also determines the success of learning. The material is sorted from simple material to complex material [23].

In addition, didactical situation must pay attention of mental acts, ways of thinking and ways of understanding (Wot and WoT) [24]. In consist of learning objectives for students, learning activity
plans, and prediction of the learning process in the classroom. Moreover, didactical situation should contain for unity, flexibility and chorency that included in metapedadidactic activity. The activity during discussion of the material, definition prediction of response and anticipation that is flexible to the didactic and pedagogical situations that occur, as well as teachers’ ability to maintain relationships between students, didactical situation and lessons in a triangular didactic [23].

3.1.4. Student did not know the concept of line charts. Problems given to students: Is the table of sprouts height observation results:

| Day | Date       | Height (cm) |
|-----|------------|-------------|
| 1   | 1 March 2018 | 0           |
| 2   | 2 March 2018 | 1           |
| 3   | 3 March 2018 | 2           |
| 4   | 4 March 2018 | 3           |
| 5   | 5 March 2018 | ...         |

Based on the above table, the height of the sprouts on March 4 2018 is.... Then, how the observation results of sprout plants on the fifth day? What is the height of sprouts on the fifth day? What is the height of sprouts on the sixth day? What is the height of sprouts on the seventh day? How to find it? Let’s draw line chart of the observation until seventh day!

![Figure 4](image1.png)

Figure 4. Responses of students did not know the concept of line chart.

Figure 4 the students did not understand the concept of line chart. Thus, the students made mistake in representing data in the form of line chart. Such errors are categorized the conceptual ontogenical obstacle.

3.1.5. Students’ mistaken in determining growth patterns

![Figure 5](image2.png)

Figure 5. Student incorrect responses in determining growth patterns.

In figure 5 student did not learn to do problem solving about pattern. It is because the textbook that is used does not facilitate students to do so. Almost all students do the similar mistakes. The obstacles are categorized as didactical obstacles. Didactical obstacle was found by doing analysis of
instructional materials in which did not included validation activities and Institutional activities. It is part of didactic situation theory which gives the value of truth of knowledge learned in the classroom. Moreover, it is associated with concepts, symbols and knowledge that may be used at different situations and other purposes [22]. Teaching material includes the situation action and formulation

3.1.6. Students’ mistake in choosing the right modelling. Questions were given to students: Adi’s bottle leaked so the water dripped. In the first minute, the water dripped 60 times. The 4th minute, water dripped 45 times. The 8th minute, water dripped 30 times. In 12th minute, the drip was getting more rarely, that is only 15 times. Until in the 16th minute, Adi closed the leak hole and water stopped dripping. How does the diagram best describe the leak of Adi’s water bottle?

![Figure 6](image)

**Figure 6.** Incorrect response in choosing the appropriate modelling.

In Figure 6, it is known that the students is limited to a particular concept of a bar chart, so when it comes to draw a diagram, the student's mistake in representing the data in the form of an image diagram belongs to conceptual epistemological obstacle.

4. Conclusion
Lastly, we can draw the conclusion that there are three types of obstacles, namely, ontogenic obstacle, epistemological obstacle and didactical obstacle. The obstacles in elementary school students’ algebraic thinking occurs due to the limitations of understanding both technically and conceptually. In addition, the missed learning steps in teaching materials that were examined by using hermeneutics and phenomenology paradigm also cause students not to get a proper didactic situation. The didactic situation is given not only to the transfer of knowledge but also to the transposition of knowledge. In addition, learning obstacle encountered due to lack of experience in solving problems. This is evident from the lack of experience of institutionalization in the learning process. Based on this, it needs to conduct further research to create a didactic situation design that overcomes the learning obstacles of algebraic thinking in elementary school.

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