From Basu-Harvey to Nahm equation via 3-Lie bialgebra

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Abstract

Using the concept of 3-Lie bialgebra; we construct the Bagger-Lambert-Gustavson (BLG) model on the Manin triple $\mathcal{D}$ of the especial 3-Lie bialgebra $(\mathcal{D}, A_G, A^*_G)$ which is in correspondence with Manin triple of Lie bialgebra $(\mathcal{D}, G, G^*)$. We have shown that the Nahm equation (with Lie bialgebra $G$) can be obtained from the Basu-Harvey equation as a boundary condition of BLG model (with 3-Lie bialgebra $\mathcal{D}$) and vice versa.

1 Introduction

Bagger-Lambert-Gustavsson (BLG) theory is one of the exciting works of the three-dimensional superconformal field theories that is written as an action for multiple M2-branes \cite{1,2}. It has $N = 8$ supersymmetries and SO(8) global symmetry \cite{3}. The main difference of this model with the others is the utilization of 3-Lie algebra inspired by Basu-Harvey work \cite{4} (see also Ref. \cite{5}). BLG theory has two defects, i) it only describes two M2-branes \cite{6} and ii) there is only one example for it as $A_4$ \cite{3}, which have been removed by decreasing the number of supersymmetry to $N = 6$ \cite{7} and considering Lorentzian metric \cite{8}, respectively.

Study of BPS equations has a very important role in M-theory and string theory \cite{9-11}. One can consider these equations as generalizations of Basu-Harvey equation \cite{1} and determine the configuration of the M2-M5 bound states with half of the supersymmetries in BLG model \cite{12}. The BPS conditions for the D2-D4 configuration are described by Nahm equation \cite{13}. M2-M5 bound states in M-theory are completely related to the bound states in string theory for D2-D4 hence the BPS equations in the BLG theory must be closely related to the Nahm equation \cite{14} as the relation between D2-brane to M2-brane \cite{15}. It is possible to obtain Nahm equation from Basu-Harvey equation of the Lorentzian BLG theory \cite{16} but the inverse is impossible. However, it seems that for the special examples, the inverse is possible by employing the 3-Lie bialgebra, the Lie bialgebra and one to one correspondence between them.

Information about BLG theory can be increased by more studies in 3-Lie algebras. In this respect, identification of 3-Lie bialgebras would help us as follows \cite{17}. According to algebraic work in Ref. \cite{17} without any role for geometry, it is easy to obtain a reciprocating relation, that has an important and applicable role in M-theory.

An outline of the paper is as follows. In section 2, using the definition of 3-Lie bialgebra given in Ref. \cite{17}, we have considered an especial example and shown that there is a correspondence between this 3-Lie bialgebra and Lie bialgebra. In section 3, we have constructed a BLG model on the Manin triple of this especial 3-Lie bialgebra and obtained Basu-Harvey equation from it and shown that the Nahm equation can be obtained from it and vice versa.

2 3-Lie bialgebra

By remembering the definition of Lie bialgebra \cite{18} (see for a review Refs. \cite{19,22}) we give a short review of the definition of 3-Lie bialgebra \cite{17}.

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Definition: [17] 3-Lie algebra $A$ with the co commutator map $\delta: A \to \otimes^3 A \otimes A$ is a 3-Lie bialgebra if:

1) $\delta$ is a 1-cocycle of $A$ with value in $\otimes^3 A$, i.e:

$$
\delta([T_a, T_b, T_c]) = ad^{(3)}_{T_a \otimes T_c} \delta(T_a) - ad^{(3)}_{T_c \otimes T_a} \delta(T_c) + ad^{(3)}_{T_a \otimes T_a} \delta(T_c),
$$

(1)

where

$$
ad^{(3)}_{T_a \otimes T_c} = ad_{T_a \otimes T_c} \otimes 1 \otimes 1 + 1 \otimes ad_{T_a \otimes T_c} \otimes 1 + 1 \otimes 1 \otimes ad_{T_a \otimes T_c},
$$

(2)

where $\{T_a\}$s are bases of 3-Lie algebra $A$ and $ad_{T_a \otimes T_c} = [T_a, T_b, T_c]$, [21].

b) the dual map $\delta : \otimes^3 A^* \to A^*$ is a 3-Lie bracket on $A^*$ (dual space of $A$) with

$$
(\hat{T}^a \wedge \hat{T}^b \wedge \hat{T}^c, \delta(T_d)) = (\delta^a \delta^b \delta^c, T_d) = ([\hat{T}^a, \hat{T}^b, \hat{T}^c], T_d),
$$

(3)

such that it satisfies the fundamental identity. In the above relation $\{\hat{T}^a\}$ and $\{\ , \ , \ \}$ are the basis for the dual space $A^*$ and natural pairing between $A$ and $A^*$, respectively. In this way $A^*$ constructs a 3-Lie algebra. The 3-Lie bialgebra can be shown either by $(A, A^*)$ or $(A, \delta)$.

Definition: [17] The Manin triple $(D, A, A^*)$ is a triple of 3-Lie algebras $(D, A, A^*)$ so that there is a nondegenerate, symmetric and ad-invariant metric on $D$ (Drinfeld double algebra) with the properties:

a) $A$ and $A^*$ are 3-Lie subalgebras of $D$,
b) $D = A \oplus A^*$ as a vector space,
c) $A$ and $A^*$ are isotropic, i.e.

$$
(T_a, \hat{T}^b) = \delta^b_a, \quad (T_a, T_b) = (\hat{T}^a, \hat{T}^b) = 0.
$$

Using relations (1), as the fundamental identity, Eq. (3) and $\delta(T_a) = f^{bcd}_a T_b \otimes T_c \otimes T_d$, the fundamental and mix fundamental identities for the 3-Lie bialgebra $(A, A^*)$ can be obtained in terms of structure constant (of $A$ and $A^*$) $f_{abc\,d}$ and $\hat{f}^{abc\,d}$ as follows [17]:

$$
f_{abc\,d} f_{bcd\,g} - f_{bac\,d} f_{acd\,g} + f_{cbd\,a} f_{gab\,d} - f_{bda\,a} f_{gab\,c} = 0,
$$

(4)

$$
f_{abc\,d} f_{bcd\,g} - f_{bac\,d} f_{acd\,g} + f_{cbd\,a} f_{gab\,d} - f_{bda\,a} f_{gab\,c} = 0,
$$

(5)

$$
f_{abc\,d} \hat{f}^{def\,g} = f_{gbc\,d} \hat{f}^{def\,g} + f_{gbc\,d} \hat{f}^{deg\,a} - f_{gbc\,d} \hat{f}^{deg\,b} + f_{gac\,e} \hat{f}^{deg\,e} - f_{gac\,e} \hat{f}^{deg\,d} + f_{gac\,e} \hat{f}^{deg\,g} = 0.
$$

(6)

2.1 An example

Now, we will consider an especial example of 3-Lie bialgebra which is constructed of a 3-Lie algebra $A_G$ in relation to Lie algebra $G$. The 3-Lie algebra $A_G$ (mentioned in [8] for the first time) has commutation relations as follows:

$$
[T_-, T_a, T_b] = 0, \quad [T_+, T_i, T_j] = f_{ij\,k} T_k, \quad [T_i, T_j, T_k] = f_{ij\,k} T_-, \quad (7)
$$

where $\{T_i\}$s are the basis of the Lie algebra $G$ $(\{T_i, T_j\} = f_{ij\,k} T^k$ with $i, j, k = 1, 2, ..., \text{dim} G)$ and $f_{ij\,k}$ is its structure constant[2]. Furthermore, $T_-$ and $T_+$ are new generators and we have $a = +, -, i$. Now we propose that there exists a 3-Lie algebra structure on $A_G$ with similar commutation relations:

$$
[\hat{T}^-, \hat{T}^a, \hat{T}^b] = 0, \quad [\hat{T}^+, \hat{T}^i, \hat{T}^j] = \hat{f}_{ij\,k} T^k, \quad [\hat{T}^i, \hat{T}^j, \hat{T}^k] = \hat{f}_{ij\,k} T^-,
$$

(8)

such that $G^*([\hat{T}^+, \hat{T}^i], [\hat{T}^-, \hat{T}^a]) = \hat{f}_{ij\,k} T^k$ with $i, j, k = 1, 2, ..., \text{dim} G^*$ is a Lie algebra.

Proposition: $(A_G, A_G^*)$ is a 3-Lie bialgebra and the structure constants $f_{abc\,d}$ and $\hat{f}^{abc\,d}$ (details of which are given in the following) satisfy the relation (6) if and only if $(G, G^*)$ is Lie bialgebra, i.e. $G^*$ is a dual Lie

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1 Note that in general, the vector space $D$ is not a 3-Lie algebra [17].

2 Note that the indices of $f_{ij\,k}$ are lowered and raised by the ad-invariant metric $g_{ij}$ of the Lie algebra $G$. 
algebra of the Lie algebra $\mathcal{G}$, i.e., their structure constants $f_{ij}^{k}$ and $\tilde{f}_{ij}^{k}$ satisfy the following Jacobi and mixed Jacobi identities [18, 20]

\begin{align}
&f_{ij}^{k} f_{kl}^{m} - f_{ik}^{m} f_{jl}^{k} + f_{jk}^{m} f_{il}^{k} = 0, \\
&\tilde{f}_{ij}^{k} \tilde{f}_{kl}^{m} - \tilde{f}_{ik}^{m} \tilde{f}_{jl}^{k} + \tilde{f}_{jk}^{m} \tilde{f}_{il}^{k} = 0, \\
&-f_{ij}^{k} \tilde{f}_{lm}^{k} + f_{ik}^{m} \tilde{f}_{jm}^{l} - f_{jk}^{m} \tilde{f}_{im}^{l} + f_{lm}^{k} \tilde{f}_{ik}^{j} = 0.
\end{align}

**Proof:** By expanding the indices $g = i, +, -$ in two sides of the relation (1) and using the following equalities:

\begin{align}
&f_{+ij}^{k} = f_{ij}^{k}, \\
&\tilde{f}_{+ij}^{k} = \tilde{f}_{ij}^{k}, \\
&f_{-ab}^{c} = 0, \\
&f_{abc}^{+} = 0,
\end{align}

and by taking $a \equiv i, b \equiv j, d \equiv l, e \equiv m, c = f \equiv k$, we arrive at (11), in this way we see that $(\mathcal{A}_{0}, \mathcal{A}_{0}^{*})$ is a 3-Lie bialgebra if and only if $(\mathcal{G}, \mathcal{G}^{*})$ is a Lie bialgebra, i.e., $\mathcal{A}^{*} = \mathcal{A}_{0}^{*}$. One can consider the Manin triple $(\mathcal{D}, \mathcal{A}_{0}, \mathcal{A}^{*}_{0})$ for this example, i.e. for this example $\mathcal{D}$ is a 3-Lie algebra.

### 3 From Basu-Harvey to Nahm equation

In the previous section, we constructed and showed the existence of a one to one correspondence between the special 3-Lie bialgebra and Lie bialgebra. In other words, a special example of 3-Lie bialgebra can be constructed by Lie bialgebra. As is well known, Bagger, Lambert and Gustavsson [1–3] have used 3-Lie algebra instead of Lie algebra for describing multiple M2-branes. In our method, we should construct the BLG theory constructed by Lie bialgebra. As is well known, Bagger, Lambert and Gustavsson [1–3] have used 3-Lie algebra instead of Lie algebra for describing multiple M2-branes. In our method, we should construct the BLG theory constructed by the 3-Lie bialgebra valued on Manin triple are the ob-

\begin{align}
&T^{−}, T^{+}, T^{i} = f_{ij}^{k} T^{k}, \\
&T^{−}, T^{+}, T^{j} = \tilde{f}_{ij}^{k} T^{k}, \\
&T^{−}, T^{+}, T^{k} = \tilde{f}_{ij}^{k} T^{j}, \\
&T^{−}, T^{+}, T^{l} = \tilde{f}_{ij}^{k} T^{l}.
\end{align}

As a first step, consider the following supersymmetric transformation [1, 3]:

\begin{align}
&\delta X^{\mu}_{A} = i \bar{\psi}^{A} \Gamma^{\mu} \epsilon, \\
&\delta \psi_{A} = D_{\mu} X^{\mu}_{A} \Gamma^{\mu} \gamma_{5} - \frac{1}{2} X^{\mu}_{B} X^{\nu}_{C} X^{\pi}_{D} F^{BCD}_{\mu} \Gamma_{IJK} \delta_{IJK}, \\
&\delta (\bar{\psi}_{\mu})^{A}_{B} = i \bar{\psi}^{A} \Gamma^{\mu} \psi_{B},
\end{align}

where $(\bar{\psi}_{\mu})^{A}_{B} = F^{CD}_{\mu} A_{CD}^{\mu}$. $I, J, \ldots = 1, 2, \ldots, 8$ and $\mu, \nu, \ldots = 0, 1, 2$ are the world volume coordinate for M2-brane. $\Gamma_{I}$ s, $X^{I}$ and $\Psi$ are Dirac matrices, 3-Lie algebra valued transverse coordinates of M2-brane and Majoiaiara spinor, respectively with $\Gamma^{IJK} = -\Psi$, and correspondingly $\Gamma^{IJK} = \epsilon$, as condition for spinor and supersymmetric parameter. $D_{\mu}$ is covariant derivative which is identified as $D_{\mu} X^{I}_{A} = \partial_{\mu} X^{I}_{A} + F^{IJK}_{\mu} A_{JKCD}^{\mu} X^{CD}_{B}$. The equations of motion of BLG model constructed by the 3-Lie bialgebra valued on Manin triple are the obtained conditions from a closed form of the property of the algebra of supersymmetric transformation with the following Lagrangian [2, 3]:

\begin{align}
L = & -\frac{1}{2} \bar{\psi}_{\mu} D_{\mu} X^{I}_{A} \psi^{A} + \frac{i}{2} \bar{\psi}_{\mu} \Gamma^{\mu} \psi^{A} + \frac{i}{4} F^{IJKLM} X^{I} X^{J} X^{K} X^{L} \psi^{M} \\
& - \frac{1}{12} F^{IJKLMN} D_{\mu} X^{I} X^{J} X^{K} X^{L} \psi^{M} + \frac{1}{2} \omega^{\alpha} \bar{\psi}_{\mu} F^{ABCD}_{\mu} A_{\alpha}^{AB} A_{\alpha}^{CD} X^{\alpha}.
\end{align}
Evaluation of the supersymmetric boundary condition results in the supercurrent which has been obtained previously as follows [23]:

\[ J^\mu = -\varepsilon D_\nu X^I_A \Gamma^\nu \Gamma^\mu \Psi^A - \frac{1}{6} \varepsilon X^I_A X^K_B X^L_C F^{A B C D} \Gamma^{I J K} \Gamma^\mu \Psi_D. \]  

(16)

Setting to zero normal component of the boundary of the supercurrent preserves maximum supersymmetric boundary condition [12]. Assuming the \( x^2 \) direction as the boundary, leads to vanishing \( J^2 \) for remaining maximal unbroken supersymmetry and we will have the following relation:

\[ 0 = \left( -\varepsilon D_\nu X^I_A \Gamma^\nu \Gamma^2 \Psi^A - \frac{1}{6} \varepsilon X^I_A X^K_B X^L_C F^{A B C D} \Gamma^{I J K} \Gamma^2 \Psi_D \right) \big|_{\partial M}. \]

(17)

In order to solve this equation, one should note the presence of M2-brane which breaks the Lorentz invariance of M-theory from \( SO(1, 10) \) to \( SO(1, 2) \times SO(8) \) and half of supersymmetry [12]. In this way, \( SO(1, 10) \) breaks to \( SO(1, 2) \times SO(8) \rightarrow SO(1, 1) \times SO(4) \times SO(4) \) where \( SO(1, 1) \) is world sheet of string, one of \( SO(4) \) as transverse space for M5-brane and the other as transverse space for both of M2-brane and M5-brane. The scalar fields \( X^I \) are decomposed as \( X^V = \{ X^3, X^4, X^5, X^6 \} \) corresponding to \( SO(4) \) symmetry and \( Y^P = \{ X^7, X^8, X^9, X^{10} \} \) corresponding to the other \( SO(4) \) symmetry, subsequently the equation (17) turns into the following form [12]:

\[ 0 = -\varepsilon D_\nu X^I_A \Gamma^\nu \Gamma^2 \hat{\Psi}^A \\
-\varepsilon \left( D^2 X^I_A \Gamma^2 \delta^D \right) \hat{\Psi}^A \\
-\varepsilon \left( D^2 X^I_A \Gamma^2 \hat{\Psi}^A \right) \\
-\varepsilon \left( \frac{1}{2} X^V A \Gamma^2 \hat{\Psi}^A \right) \\
-\varepsilon \left( \frac{1}{2} X^V A \Gamma^2 \hat{\Psi}^A \right)
\]

(18)

where \( \Gamma_2 \Psi_A = \bar{\Psi}_A \) and \( \hat{\mu} = 0, 1 \), also the terms ordered in each line in terms of their Lorentzian structures and preserving \( SO(1, 1) \times SO(4) \times SO(4) \) symmetry vanishes separately. In order to solve the equations, the type of boundary condition (the Dirichlet or Neumann condition) must be specified. Assuming half of the scalars to obey Dirichlet conditions [3] \( D_\hat{\mu} Y^P = 0, Y^P = 0 \) is obtained as the simplest solution for these conditions and after substituting this solution in (18) we have the following relations:

\[ 0 = \varepsilon D_\nu Y^P \Gamma^2 \hat{\Psi}, \]  

(19)

\[ 0 = \varepsilon D_\nu X^V \Gamma^2 \hat{\Psi}, \]  

(20)

\[ 0 = \varepsilon \left( D^2 X^I_A \delta^D \Gamma^2 \hat{\Psi} + \frac{1}{6} F^{A B C D} X^V A X^W B X^T C \Gamma^V U W \right) \hat{\Psi}_D. \]  

(21)

For solving these equations suppose \( \frac{1}{6} (1 - \Gamma^{013456}) \hat{\Psi} = Q_\epsilon \hat{\Psi} = 0 \) or \( (1 - \Gamma^{013456}) \epsilon = Q_\epsilon \epsilon = 0 \) on the supersymmetric parameter, then (20) holds automatically [12]. Using \( \Gamma^V = \frac{1}{6} \epsilon^{VUWZ} \Gamma^{UWZ} \Gamma^{3456} \) the equation (21) takes the following relation:

\[ 0 = \left( D^2 X^I_A \epsilon^{ZV U W} \delta^D A + X^V A X^U B X^W C F^{A B C D} \right) \epsilon \Gamma^{V U W} \hat{\Psi}_D, \]

(22)

which for the scalar fields \( X^V \) is the Basu-Harvey type equations:

\[ 0 = D^2 X^V + \frac{1}{6} \epsilon^{VUWZ} X^U B X^W C X^Z D F^{B C D A}. \]  

(23)

3 In Ref. [12] there is another way of breaking Lorentzian symmetry \( SO(1, 2) \times SO(8) \rightarrow SO(1, 1) \times SO(8) \) where we don’t describe it here.
4 We will not describe here the Neumann boundary condition, for a review see [12].
we expand it as follows:

\[ 0 = D_2X^Y_A + \frac{1}{6}\epsilon^{UVW}X^U_B X^W_D F_{BCD}, \]  
\[ 0 = D_2X^Y_\pm + \frac{1}{6}\epsilon^{UVW}X^U_B X^W_D F_{BCD}, \]  
\[ 0 = D_2X^Y_i + \frac{1}{2}g_{YM}\epsilon^{UVW}X^U_j X^W_k f_{ijk} + \frac{1}{2}g_{YM}\epsilon^{AUW}X^U_j X^W_k f_{ijk}, \]  
\[ 0 = D_2X^Y_i + \frac{1}{2}g_{YM}\epsilon^{UVW}X^U_j X^W_k f_{ijk} + \frac{1}{2}g_{YM}\epsilon^{AUW}X^U_j X^W_k f_{ijk}. \]

where we have used \( i, j, k \) for representation of bases of Lie bialgebra \( D_G \). Using relation (13), one can add up relations (24)-(27) to obtain the following relation:

\[ 0 = \partial_\sigma X^I_A + \frac{1}{2}\epsilon^{JKL}X^J_B X^K_C F^{BC}_A, \]

\[ \partial_\sigma X^I_i = \frac{1}{2}\epsilon^{JKL}X^J_K X^K_i f_{ijk} + \frac{1}{2}\epsilon^{JKL}X^J_K X^K_i f_{ijk}, \]

where \( F^{ABC}_C \) is the structure constant of \( D_G \) (the Drinfeld double of Lie algebra \( G \)) (20) and \( A, B, C = i, \bar{i} \). As it has been done by the algebraic structure in this work, it can be done inversely, i.e., by this method, one can obtain the Nahm equation from the Basu-Harvey equation and vice versa.

The Basu-Harvey and Nahm equations can be considered as BPS bounds for M2-brane and D1-string, respectively. It would be possible to state a reciprocal relation between these equations as BPS bounds identified in 3-Lie bialgebra, i.e., we want to reach from BPS bound for M2-brane to BPS bound for D1-string and vice versa. BPS bound is a result of vanishing supersymmetric transformations of gauge and fermion fields that we obtained it for M2-branes ending M5-brane by using BLG Lagrangian and showed that the result was similar bound for D1-string ending D3-brane. The BLG Lagrangian (13) regardless of the fermion piece can be rewritten as (24):

\[ L = \frac{1}{2}D_\mu X^{A(I)}D^\mu X^{A(I)} - \frac{1}{12}F_{ABCD}F_{EFGD}X^{A(I)}X^{B(I)}X^{C(K)}X^{E(I)}X^{F(J)}X^{G(K)}, \]

and the equivalent indication for the energy can be obtained as follows:

\[ E = \frac{1}{2}Tr(\partial_\sigma X^{A(I)}\partial^\sigma X^{A(I)}) + \frac{1}{12}Tr([[X^{I(J)}, X^{I(K)}], [X^{I(I)}, X^{I(J)}, X^{I(K)}]]) + \frac{1}{2}Tr(\partial_\sigma X^{A(I)}) - \frac{1}{12}Tr([X^{I(I)}, X^{I(J)}, X^{I(K)}])^2 + \frac{1}{6}\epsilon^{JKL}Tr(\partial_\sigma X^{I(I)}, [X^{I(J)}, X^{I(K)}, X^{I(L)}]) \geq \frac{1}{6}\epsilon^{JKL}Tr(\partial_\sigma X^{I(I)}, [X^{I(J)}, X^{I(K)}, X^{I(L)}]) \]

i.e., in this way the Basu-Harvey equation is obtained as a BPS bound. But the above equation can be written as

\[ E \geq \frac{1}{2}\epsilon^{JKL}Tr(\partial_\sigma X^{K(I)}T^+ + \partial_\sigma X^{K(I)}T^- + \partial_\sigma X^{I(K)}T^+ + \partial_\sigma X^{I(K)}T^- + \partial_\sigma X^{I(K)}T^+ + \partial_\sigma X^{I(K)}T^- + \partial_\sigma X^{I(K)}T^+ \]

\[ + \partial_\sigma X^{I(K)}T^+ + \partial_\sigma X^{I(K)}T^- + \partial_\sigma X^{I(K)}T^+ + \partial_\sigma X^{I(K)}T^- + \partial_\sigma X^{I(K)}T^+ \]

\[ + X^{I(K)}X^{I(K)}X^{I(L)}[T^+, T^+, T^+] + X^{I(K)}X^{I(K)}X^{I(L)}[T^+, T^+, T^+] + X^{I(K)}X^{I(K)}X^{I(L)}[T^+, T^+, T^+] \]

\[ \geq \frac{1}{2g_{YM}^2} \int d\sigma \epsilon^{1JK}\partial_\sigma X^{I(I)}[X^{J}, X^{K}]. \]

Therefore, we have shown that there is a relation between Basu-Harvey equation as BPS bound obtained from BLG model for multiple membranes and the Nahm equation as BPS bound obtained from Yang-Mills action for multiple Dp-branes.
Conclusions

Using the concept of 3-Lie bialgebra, studied in arXiv:1604.04475, we have obtained Nahm equation from Basu-Harvey equation by studying the boundary condition of BLG model, the Manin triple $D$ of 3-Lie bialgebra $(D, A_G, A_{G^*})$. In this manner, it seems that the concept of 3-Lie bialgebra is a good idea. One can consider the BLG model on $D$ 3-Lie algebra and can obtain the $N = (4, 4)$ WZW like a model on Lie bialgebra $(G, G^*)$ using the correspondence of 3-Lie bialgebra $(A_G, A_{G^*})$ with Lie bialgebra $v$ and vice versa, i.e. $(D2 \leftrightarrow M2)$. Another problem is the investigation of the $N = 6$ BL model where is the superconformal model \cite{7} using 3-Leibniz bialgebra \cite{17} and studying the boundary condition.

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