Planetary Period Oscillations in Saturn’s Magnetosphere: Cassini Magnetic Field Observations Over the Northern Summer Solstice Interval

G. Provan¹, S. W. H. Cowley¹, T. J. Bradley¹, E. J. Bunce¹, G. J. Hunt², and M. K. Dougherty²

¹Department of Physics and Astronomy, University of Leicester, Leicester, UK, ²Blackett Laboratory, Imperial College London, London, UK

Abstract

We determine properties of Saturn’s planetary period oscillations from Cassini magnetic measurements over the ~2-year interval from September 2015 to end of mission in September 2017, spanning Saturn northern summer solstice in May 2017. Phases of the northern system oscillations are derived over the whole interval, while those of the southern system are not discerned in initial equatorial data due to too low amplitude relative to the northern, but are determined once southern polar data become available from inclined orbits beginning May 2016. Planetary period oscillation periods are shown to be almost constant over these intervals at ~10.79 hr for the northern system and ~10.68 hr for the southern, essentially unchanged from values previously determined after the periods reversed in 2014. High cadence phase and amplitude data obtained from the short-period Cassini orbits during the mission’s last 10 months newly reveal the presence of dual modulated oscillations varying at the beat period of the two systems (~42 days) on nightside polar field lines in the vicinity (likely either side) of the open-closed field boundary. The modulations differ from those observed previously in the equatorial region, indicative of a reversal in sign of the radial component oscillations, but not of the colatitudinal component oscillations. Brief discussion is given of a possible theoretical scenario. While weak equatorial beat modulations indicate a north/south amplitude ratio >5 early in the study interval, polar and equatorial region modulations suggest a ratio ~1.4 during the later interval, indicating a significant recovery of the southern system.

1. Introduction

One of the principal features of Saturn’s magnetosphere is the ubiquitous presence of modulations near the ~10.5 hr planetary rotation period, which occur despite the close axisymmetry of the planetary magnetic field (Burton et al., 2010). The effects of these modulations, termed “planetary period oscillations” (PPOs), are observed in the magnetospheric magnetic field, plasma properties, boundaries, and dynamics, plasma waves, energetic particle, and related energetic neutral atom fluxes, together with auroral ultraviolet, infrared, and radio emissions (e.g., Andrews et al., 2011, 2010; Arridge et al., 2011; Badman et al., 2012; Carberry, 2017; Carberry et al., 2011; Clarke, Andrews, Arridge, et al., 2010; Clarke, Andrews, Coates, et al., 2010; Cowley & Provan, 2017; Jackman et al., 2016; Gunnert et al., 2011; Lamy, 2011; Lamy et al., 2013; Nichols, Cecconi, et al., 2010; Nichols, Cowley, et al., 2010; Provan et al., 2012; Ramer et al., 2017; Thomsen et al., 2017; Ye et al., 2016). Observations have shown that two rotating modulations are generally present having closely spaced but distinct periods, one associated with the northern polar hemisphere and the other with the southern (Andrews et al., 2010; Cowley et al., 2016; Gunnert et al., 2009; Kurth et al., 2008; Southwood, 2011), and that these periods vary slowly by up to ~±1% about ~10.7 hr over Saturn’s seasons (Andrews et al., 2012; Andrews, Coates, et al., 2010; Galopeau & Lecacheux, 2000; Gunnert et al., 2010, 2011; Lamy, 2011; Provan et al., 2013, 2016).

As revealed by previous studies, the PPO magnetic perturbations, the subject of the present paper, take the form sketched in Figure 1, where Figures 1a–1c show the northern PPO system and Figures 1d–1f the southern PPO system (Andrews, Cowley, et al., 2010; Bradley et al., 2018; Hunt et al., 2014, 2015, 2018; Southwood & Cowley, 2014; Southwood & Kivelson, 2007). Magnetic perturbation fields are shown by the blue lines and symbols, while associated electric currents are shown by the green lines and symbols, where the circled dots and crosses indicate vectors pointing out of and into the plane of the diagrams, respectively. Figures 1a and 1d show views of the two polar ionospheres looking down from the north (“through” the planet in the case of the southern ionosphere). The arrowed red lines in these diagrams show the sense of the driving flows in the...
polar atmosphere and ionosphere (Hunt et al., 2014, 2015; Jia et al., 2012; Jia & Kivelson, 2012). These diagrams also show the azimuthal phases employed to define position with respect to these field/current systems, $\Psi_N$ for the northern (N) system and $\Psi_S$ for the southern (S).

The other diagrams in Figure 1 show the currents and fields in two principal orthogonal meridian planes, the $\Psi_{N,S} = 90^\circ - 270^\circ$ meridian in Figures 1b and 1e, which best displays the currents, and the $\Psi_{N,S} = 0^\circ - 180^\circ$ meridian in Figures 1c and 1f, which best displays the perturbation fields. The black arrowed lines indicate the background magnetic field, to a first approximation axisymmetric about the planet’s spin axis (vertical arrowed line) with angular velocity $\Omega_s$. (Adapted from Hunt et al., 2018).

As depicted in Figure 1, the PPO phases $\Psi_{N,S}$ are defined such that the quasi-uniform perturbation fields of both systems point radially outward in the equatorial magnetosphere at $\Psi_{N,S} = 0^\circ$, the values then increasing monotonically with time (modulo 360$^\circ$) at a given position as the systems rotate with their particular periods,
anticlockwise with the planet in Figures 1a and 1d. These magnetic field phases and periods have been monitored near-continuously throughout the Cassini mission since 2004, with results to the end of 2015 being published in a sequence of papers by Andrews et al. (2008, 2012) and Provan et al. (2013, 2016). PPO periods have also been derived over this interval using Cassini radio data, specifically employing modulations of the powerful Saturn kilometric radiation (SKR) emissions (e.g., Gurnett et al., 2011; Kurth et al., 2007, 2008; Lamy, 2011; Ye et al., 2016). Comparison of the periods derived from the magnetic and radio data have shown some divergences on occasion but have generally been found to be in excellent agreement (Andrews et al., 2008; Andrews, Coates, et al., 2010; Cowley & Provan, 2015, 2016; Fischer et al., 2015; Provan et al., 2014, 2016). These analyses have shown that at the start of the Cassini mission, corresponding to late Saturn southern summer, the two periods were well separated at ~10.8 hr for the southern system and ~10.6 hr for the northern. The southern system was also dominant by a factor ~2.5 in amplitude over the northern system in the equatorial magnetic data. The two periods then slowly converged to a near-common value ~10.68 hr over a two-year interval centered near vernal equinox in mid-August 2009, with the two systems then having near-equal amplitudes. There then followed a ~2.5-year interval in which sharp changes in relative amplitude took place at intervals of ~100–200 days between southern-dominant, northern-dominant, and near-equal amplitude oscillations, accompanied by small changes in the periods, with the southern system period ~10.70 hr remaining longer than the northern ~10.64 hr. In mid-2013 the periods coalesced with the two systems in antiphase, moving upward from ~10.66 to 10.70 hr over the following year. The periods then separated again with the northern period becoming enduringly longer than the southern for the first time during the Cassini era, moving to ~10.78 hr by the end of 2015, while the southern system period remained near ~10.70 hr. The northern system was dominant over the southern system at this time, by a factor of more than 2 in equatorial amplitude.

In this paper we present an analysis of the magnetic data over an ~2-year interval from September 2015, thus providing modest overlap with the results of Provan et al. (2016), to the end of the Cassini mission in mid-September 2017. These data encompass high-cadence measurements of the PPO phases and amplitudes from the Cassini F ring orbits (and the approach thereto) from November 2016 to April 2017, together with the proximal orbits of the Cassini Grand Finale from May to September 2017, thus spanning Saturn northern summer solstice in mid-May 2017. Knowledge of the PPO phases and periods is of particular importance for the analysis and interpretation of the close-periapsis data acquired during the unique end-of-mission Cassini passes. We show that throughout this interval the PPO periods vary only modestly about those reported by Provan et al. (2016) to the end of 2015, that is, ~10.79 hr for the northern system and ~10.68 hr for southern system. However, we also report and analyze new PPO-related phenomena observed in F ring and proximal orbit data, namely, the dual modulation of fields on nightside polar field lines in the vicinity of the OCB, as well as the presence of differing north/south system amplitude ratios in differing magnetic field components in the southern equatorial region.

2. Data Set and Analysis Procedures

2.1. Magnetic Data Analysis Procedures

The procedures employed to derive PPO properties from the Cassini magnetic data have previously been detailed by Andrews et al. (2012) and Provan et al. (2013, 2014), such that only a brief outline is given here, focused on the specific results employed in this paper. It is first evident from the definition of the azimuthal phase functions $\Psi_{N, S}$ discussed in section 1 that the radial component of the PPO perturbation field as depicted in Figure 1 will vary to a first approximation as $\sim \cos \Psi_{N, S}$, such that it maximizes for both systems at $\Psi_{N, S} = 0^\circ$. Since the colatitudinal field components vary in antiphase with the radial for the northern system (Figure 1c) and in phase for the southern system (Figure 1f), these will vary as $\sim - \cos \Psi_N$ and $\sim \cos \Psi_S$, respectively. Similarly, the azimuthal field components for both systems will vary as $\sim - \sin \Psi_N, S$ in lagging quadrature with the radial component in the equatorial quasi-uniform field region, reversing sign on the other side of the principal PPO-related field-aligned currents to vary as $\sim - \sin \Psi_{N, S}$ in leading quadrature with the radial component in the two polar quasi-dipolar field regions (Provan et al., 2009). The orientations of the two systems within the magnetosphere at any time are defined by the azimuthal angles $\phi_{N, S}(t)$, which give the azimuth of the $\Psi_{N, S} = 0^\circ$ meridians with respect to the noon meridian at time $t$, increasing positive in the sense of planetary rotation. If an observer is located at azimuth $\phi$ with respect to the noon meridian,
similarly defined, then the value of the PPO phases $\Psi_{NS}$ at that location are given by $\Psi_{NS}(r, \varphi, t) = \Phi_{NS}(t) - \varphi$. This expression is taken to be valid for radial distances up to 12 $R_S$ from the planetary center. At larger radii we take account of radial phase propagation delay with gradient $G$ through the phase function

$$R(r) = \begin{cases} 0 & \text{for } r \leq 12 \ R_S \\ G(r - 12) & \text{for } r \geq 12 \ R_S, \end{cases}$$

(1)

where $r$ is radial distance in $R_S$ and $G$ is taken to be 3° per $R_S$ based on the results of Arridge et al. (2011) and Provan et al. (2012). The corresponding radial phase speed is ~200 km/s (see Provan et al., 2012, their equation (4)). In general, we therefore write

$$\Psi_{NS}(r, \varphi, t) = \Phi_{NS}(t)/C_0 - R(r)/C_0,$$

(2)

The PPO period of rotation about the planetary axis is then given by

$$\tau_{NS} = 360 \left( \frac{d\Phi_{NS}}{dt} \right).$$

(3)

where $\Phi_{NS}(t)$ is expressed in degrees. From the above discussion the field components of the two PPO systems may then be written as the real (or imaginary) parts of

$$B_{iNS} = B_{iON} \exp\left(\Phi_{NS}(t) - R(r) - \varphi_{iNS}\right).$$

(4)

where $i$ denotes one of the spherical polar field components $i = (r, \ \theta, \ \phi)$ as employed throughout this paper, $B_{iON}$ are the oscillation amplitudes, and the angles $\gamma_{iNS}$ denote the fixed relative phases between the three field components, chosen as relative to the $r$ component. From the above definition of the northern and southern quantities $\Phi_{NS}(t)$ in terms of the orientation of the meridian where the $r$ component maximizes, we then have

$$\gamma_{rNS} = 0^\circ,$$

(5)

while from the relative phases of the colatitudinal fields in Figure 1 as discussed above we have

$$\gamma_{\thetaNS} = 180^\circ,$$

(6a)

and

$$\gamma_{\phiNS} = 0^\circ.$$

(6b)

Similarly, for the azimuthal field in the equatorial quasi-uniform field region

$$\gamma_{\phiNS} = 90^\circ,$$

(7a)

while in the two polar quasi-dipolar field regions

$$\gamma_{\phiNS} = -90^\circ,$$

(7b)

(or equivalently 270°, modulo 360°).

To determine the PPO oscillation parameters (i.e., amplitude and phase), we employ 1-min averaged data from the Cassini magnetic field experiment (Dougherty et al., 2004), from which the Burton et al. (2010) internal planetary field is first subtracted to form the “residual field” data set. Suitable data segments from each spacecraft orbit are then selected for PPO analysis, which are band-pass filtered between 5 and 20 hr to extract the PPO-related signal. The latter band corresponds to a factor of ~2 in period either side of the expected ~10 hr signal, which is found in practice to well characterize the PPO-related field oscillations. We note that the detailed choice of internal field model employed in this process will have little effect on the resulting PPO parameters. Suitability of data segment principally involves selection of “quiet” intervals in which other physical effects, such as crossings through the center of the ring current, do not contribute significant power to the filter band unconnected with the PPO phenomenon. The filtered data for each component $i$ are then fitted to the rotating function

$$10.1029/2018JA025237

Journal of Geophysical Research: Space Physics

PROVAN ET AL. 3862
where \( \Phi_g(t) \) is a suitably-chosen “guide phase” corresponding to an oscillation, which is close in period to the observed oscillation; \( r(t) \) is the radial distance of the spacecraft; \( \varphi(t) \) is its azimuth as defined above (equivalent to local time [LT]); and the fit parameters are the amplitude \( B_0 \) and the relative phase \( \psi_r \). The guide phases employed in this study correspond to fixed periods \( \tau_g \) so that

\[
\Phi_g(t) = \frac{360r}{\tau_g},
\]

where for definiteness we take \( t = 0 \) at 00 UT on 1 January 2004 throughout. Comparing the arguments of the sinusoids in equations (8) and (4) we have, assuming initially the presence of either a pure northern or a pure southern oscillation

\[
\psi_{N,S} = \psi_i - \gamma_{N,S} = \Phi_g - \Phi_{N,S},
\]

where we term the quantities \((\psi_i - \gamma_{N,S})\) the “N-format” and “S-format” relative phases, respectively, in which the constant phase shifts between the components \( \gamma_{N,S} \) given by equations (5) to (7a) and (7b) are applied to the individual measured component phases \( \psi_i \) so as to bring all three components to a common phase value for a pure northern or pure southern system oscillation, equal to that of the \( r \) component (for which by definition \( \gamma_{N,S} = 0^\circ \)). If the \( \psi_{N,S} \) data are then fitted to continuous functions \( \psi_{N,S}(t) \), the northern and southern periods are given from equation (3) by

\[
\tau_{N,S} = \frac{\tau_g}{1 - \frac{d\psi_{N,S}}{dt}}.
\]

As in the equatorial region in Figure 1, however, the northern and southern oscillations may in general be superposed, such that dual modulation must be taken into account in the analysis. In this case the observed combined oscillation for component \( i \) can be written as the real (or imaginary) part of

\[
\bar{B}_i = B_{0iN} \exp(i\Phi_{N}(t) - r(r) - \varphi - \gamma_{N}) + B_{0iS} \exp(i\Phi_{S}(t) - r(r) - \varphi - \gamma_{S}).
\]

We now write the combined oscillation \( \bar{B}_i \) as

\[
\bar{B}_i = \bar{B}_i(t) \exp(i\Phi_i(t) - r(r) - \varphi),
\]

so that from equations (8) and (13) the N- and S-format relative phases become

\[
\psi_i - \gamma_{N,S} = \Phi_g - \Phi_i - \gamma_{N,S},
\]

where we wish to evaluate \( \Phi_i(t) \) and \( \bar{B}_i(t) \) using equation (12). Substituting equation (13) into equation (12), writing the ratio of the northern and southern system amplitudes for component \( i \) as

\[
k_i = \frac{B_{0iN}}{B_{0iS}},
\]

and canceling the common radial and azimuthal phase factors on both sides yields

\[
\bar{B}_i(t) \exp(i\Phi_i(t) = B_{0iS}(k_i \exp(i(\Phi_{N}(t) - r_{N})) + \exp(i(\Phi_{S}(t) - \gamma_{S}))).
\]

To determine the deviation of the N-format phases from the northern phase \( \Phi_{N} \) due to the presence of a southern oscillation (as most appropriate to a northern-dominated oscillation with \( k_i > 1 \)), or equivalently the deviation of the S-format phases from the southern phase \( \Phi_{S} \) due to the presence of a northern oscillation (as most appropriate to a southern-dominated oscillation with \( k_i < 1 \)), we take the complex conjugate of equation (16) and multiply throughout either by \( \exp(i(\Phi_{N}(t) - r_{N}) \) in the first case, or by \( \exp(i(\Phi_{S}(t) - \gamma_{S}) \) in the second. Substitution of the resulting equivalent expressions for \( \Phi_i(t) \) into equation (14) then yields the N-format phases as
\[
\psi_i - \gamma_N = (\Phi_i - \Phi_N) + \tan^{-1} \left[ \frac{\frac{1}{k_i} \sin(\Delta \Phi - \Delta \gamma_i)}{1 + \frac{1}{k_i} \cos(\Delta \Phi - \Delta \gamma_i)} \right],
\]

(17)

or equivalently the S-format phases as

\[
\psi_i - \gamma_S = (\Phi_i - \Phi_S) + \tan^{-1} \left[ \frac{-k_i \sin(\Delta \Phi - \Delta \gamma_i)}{1 + k_i \cos(\Delta \Phi - \Delta \gamma_i)} \right],
\]

(18)

where \(\Delta \Phi(t)\) is the beat phase of the two oscillations defined (as in previous related papers) as

\[
\Delta \Phi(t) = \Phi_N(t) - \Phi_S(t),
\]

(19a)

with corresponding beat period \(\tau_B\) given by

\[
\frac{1}{\tau_B} = \frac{1}{360} \left| \frac{d \Delta \Phi}{dt} \right| = \left| \frac{1}{\tau_N} - \frac{1}{\tau_S} \right|,
\]

(19b)

and \(\Delta \gamma_i\) is the difference in the constant phase offsets for field component \(i\) in the two systems

\[
\Delta \gamma_i = \gamma_{N,i} - \gamma_{S,i}.
\]

(20)

The first terms on the RHSs of equations (17) and (18) are the same as equation (10) for pure northern or southern oscillations, respectively, while the second terms describe the beat-modulated phase deviations about those smoothly varying phases. In the latter expressions the numerator and denominator are evaluated separately in the arctangent to define the function over the full 360° range. Thus, from equations (17) and (18) the beat-related N-format phase modulation about the northern phase is given by

\[
\delta \psi_{N,i} = (\psi_i - \gamma_N) - (\Phi_i - \Phi_N) = \tan^{-1} \left[ \frac{\frac{1}{k_i} \sin(\Delta \Phi - \Delta \gamma_i)}{1 + \frac{1}{k_i} \cos(\Delta \Phi - \Delta \gamma_i)} \right],
\]

(21)

while equivalently the beat-related S-format phase modulation about the southern phase is given by

\[
\delta \psi_{S,i} = (\psi_i - \gamma_S) - (\Phi_i - \Phi_S) = \tan^{-1} \left[ \frac{-k_i \sin(\Delta \Phi - \Delta \gamma_i)}{1 + k_i \cos(\Delta \Phi - \Delta \gamma_i)} \right].
\]

(22)

Multiplying equation (16) by its complex conjugate and taking the square root also yields the amplitude function (defined positive definite) as

\[
\hat{B}_i(t) = B_{i0N} \left( 1 + \frac{1}{k_i^2} + \frac{2}{k_i} \cos(\Delta \Phi - \Delta \gamma_i) \right)^{1/2} = B_{i0S} \left( \frac{1}{k_i^2} + 2k_i \cos(\Delta \Phi - \Delta \gamma_i) \right)^{1/2}.
\]

(23)

We note that when the northern period exceeds the southern, as during the interval studied here, the beat phase \(\Delta \Phi(t)\) given by equation (19a) decreases continuously with time. We also note that in the dual-modulated equatorial region in Figure 1, for example, we have from equations (5) to (7a) and (7b) \(\Delta \gamma_N = \Delta \gamma_S = 0°\) so that these components are similarly beat modulated, while \(\Delta \gamma_N = 180°\) so that this component is beat modulated in antiphase with \(r\) and \(\varphi\). Of course, this is already qualitatively obvious from Figures 1c and 1f. These figures show directly that when the two systems are in phase, that is, \(\Phi_N(t) = \Phi_S(t)\) modulo 360° such that \(\Delta \Phi(t) = 0°\) modulo 360°, the \(r\) and \(\varphi\) components of the two systems are in phase with each other and add while the \(\theta\) components of the two systems are in antiphase and part cancel. Oppositely, when the two systems are in antiphase, that is, \(\Phi_N(t) = \Phi_S(t) \pm 180°\) modulo 360° such that \(\Delta \Phi(t) \pm 180°\) modulo 360°, the \(\theta\) components of the two systems are in phase and add while the \(r\) and \(\varphi\) components of the two systems are in antiphase and part cancel. We will recall these results in the discussion of the present data set in section 3.3.

We further note that given these phase modulations, the apparent beat-modulated oscillation periods that would be computed from the phases of field component \(i\) using modified equation (3) are then from equation (17)
or equivalently from equation (18)

\[
t_f' = \frac{\tau_N}{1 + \frac{1}{\tau_N} \left( \frac{1 + \frac{\tau}{\tau_S} \cos(\Delta\Phi - \Delta\gamma)}{1 + \frac{k_i^2 + k_s \cos(\Delta\Phi - \Delta\gamma)}{1 + k_i^2 + 2k_s \cos(\Delta\Phi - \Delta\gamma)}} \right)}.
\]

or equivalently from equation (18)

\[
t_f' = \frac{\tau_S}{1 - \frac{1}{\tau_N} \left( \frac{k_i^2 + k_s \cos(\Delta\Phi - \Delta\gamma)}{1 + k_i^2 + 2k_s \cos(\Delta\Phi - \Delta\gamma)}} \right)}
\]

**2.2. Cassini Orbit Over the Study Interval**

The nature of the magnetic data employed to determine the PPO phases, whether “equatorial” on one side of the center of the principal PPO-related field-aligned current sheets or “polar” on the other side (a terminology employed throughout this paper), across which the PPO-related azimuthal field component changes sign (Figure 1), evidently depends on the nature of the Cassini orbit. In Figure 2 we thus provide an overview of relevant orbit parameters over the study interval, together with the consequences for the nature of the PPO data available. This figure spans the ~2-year interval from 1 September 2015 to 30 September 2017 inclusive, or equivalently \( t = 4,261 \pm 5,022 \) days as shown at the bottom of the figure, where as in section 2.1 (and in previous related studies) \( t = 0 \) corresponds to 00 UT on 1 January 2004, essentially the start of the active Cassini science mission. Year boundaries are shown at the top of the figure, together with the time of northern summer solstice at \( t = 4,893 \) days (24 May 2017). The solar latitude increased from \( \sim 25.15^\circ \) at the start of the interval to peak at \( \sim 26.73^\circ \) at solstice, and then declined to \( \sim 26.67^\circ \) at the end of the interval, such that near-solstice solar conditions prevailed throughout. Cassini revolution (Rev) numbers defined from apoapsis to apoapsis are also shown at the top of the figure, plotted at the times of periastris and numbered every 10 Revs. The interval spans from Rev 221 to the end of mission at Rev 293. For ease of comparison this interval modestly overlaps that of the related study by Provan et al. (2016), which ended at \( t = 4,383 \) days (end of 2015), the last orbit providing data to that study being Rev 229. The black lines in Figure 2a show the latitude range encompassed on each Rev, plotted at the time of periastris, where the blue and red solid circles show the latitudes of periastris and apoapsis, respectively. Figure 2b similarly shows the radial range of each Rev on a log scale, Figure 2c the LT of periastris (blue) and apoapsis (red), and Figure 2d the period of each Rev plotted at the time of periastris. Figure 2e gives an overview of the nature of the PPO-related magnetic data provided by these Revs as described in section 2.3.

Figures 2a–2d show that over this study the Cassini orbit evolved in three main intervals. The first corresponds to Revs 221 to 231, from the start of the study interval to \( t \approx 4,400 \) days (January 2016), when the orbit was confined close to the equatorial plane, where it had been since Rev 214 at \( t \approx 4,100 \) days (March 2015). Periastris over most of the interval was at \( \sim 2.5 \) RS in the predusk sector, with apoapsis at \( \sim 35 \) RS predawn, and an orbit period of \( \sim 14 \) days. The second interval corresponds to Revs 232–239, \( t \approx 4,000 - 4,600 \) days (January–August 2016), when the inclination of the orbit was successively increased to \( \sim 50^\circ \), with apoapsis located near maximum northern latitudes at dawn and periastris located near maximum southern latitudes at dusk. Periastris increased from \( \sim 4.5 \) to \( \sim 10 \) RS over this interval, while apoapsis first increased from \( \sim 35 \) to \( \sim 55 \) RS for Revs 234–236 and then declined again to \( \sim 30 \) RS by Rev 239. The orbit period began and ended near \( \sim 14 \) days, but peaked at \( \sim 32 \) days for Rev 235. In the third interval, corresponding to Revs 240–293, \( t \approx 4,600 - 5,000 \) days (August 2016 to September 2017), the orbit changed in a sequence of five steps, across which the inclination was modestly increased to \( \sim 60^\circ \), but where periastris and apoapsis (i.e., the line of apsides) migrated successively closer to the equatorial plane. Apoapsis remained near \( \sim 20 \) RS (Titan’s orbit) throughout, with LTs declining modestly from post-midnight toward midnight, while periastris was successively reduced from \( \sim 8 \) toward \( \sim 1 \) RS at the end of mission, with LTs correspondingly declining from postnoon toward noon. The two final extended sequences of near-identical orbits are termed the “F ring” orbits, Revs 251–270 (December 2016 to April 2017), with periastris at \( \sim 2.5 \) RS, and the “proximal” orbits, Revs 271–293 (April–September 2017), with periastris a little above 1 RS. The PPO data from these (and directly adjacent) orbits for which the period was reduced to \( \sim 7 \) days for the F ring orbits, and to \( \sim 6.5 \) days for the proximal orbits, are subject to detailed analysis in section 3.
2.3. Orbit Implications for PPO-Related Magnetic Field Data

The consequences of this orbit evolution for the varying nature of the PPO-related magnetic data available for analysis are summarized in Figure 2e. During the first orbit interval (Revs 221–231), the data available are evidently closely equatorial, from which PPO amplitudes and phases for all three field components can...
generally be determined by fitting equation (8) to filtered residual data as described in section 2.1. This is indicated by the solid green horizontal bar in Figure 2e. As in previous related studies, the analysis interval is confined to near-periapsis segments in the quasi-dipolar magnetosphere within ~12 Rs, thus inside the region where the equatorial azimuthal component switches sign as mentioned in section 1 (Figures 1b and 1e). These data segments typically last for ~1.5 days, encompassing ~3 PPO oscillations. As indicated in section 1, such equatorial data are in general dual modulated by both northern and southern PPO systems. However, Provan et al. (2016) showed previously that during the present interval, starting at Rev 221, the equatorial oscillations were dominated by the northern system, with southern modulations being undetectable, implying a lesser amplitude by a factor of ~5 or more (Provan et al., 2013). As the plane of the orbit subsequently tilted as described above, similar amplitude and phase determinations could be obtained from filtered near-equatorial near-periapsis data segments of diminishing length up to Rev 241, thus encompassing the second and the beginning of the third orbit interval discussed above. However, the phases and amplitudes of the r component oscillations are evidently compromised for Rev 233 and later due to passage across the center of the ring current, whose magnetic signature dominates the power in the filter band for these near-equatorial data segments. The green bar in Figure 2e then changes to a green long-dashed bar indicating that the r component data, but not θ or φ, are excluded from further analysis. We note that additional individual phase determinations may also be excluded throughout the study interval if the fit to equation (8) is poor, that is, if the root-mean-square (RMS) deviation is comparable to or exceeds the fit amplitude, due either to low PPO amplitudes or to the simultaneous presence of other phenomena that contribute power to the filter band but are not well described by equation (8), resulting in poor fits. For example, such field variations could result from changes in solar wind dynamic pressure, or to storm-related reconnection events on the nightside. Beyond this, on Revs 242–248, equatorial PPO parameters can only be determined from short part-oscillation segments of unfiltered azimuthal component data as the spacecraft rapidly traverses the equatorial region (e.g., Andrews et al., 2012; Provan et al., 2014), indicated by the short-dashed green bar in Figure 2e. Both r and θ component data are dominated by ring current variations during such passes and are thus excluded. After Rev 248 the crossing interval becomes too short even for this analysis, such that equatorial data become unavailable for a number of subsequent Revs.

Tilting of the orbit plane during the second and third orbit intervals discussed above, however, also allows access to polar field regions beyond the center of the principal PPO-related field-aligned current sheets. Northern polar parameters are available from fits to filtered field data from Rev 232 in mid-February 2016 to Rev 293 at the end of mission, as indicated by the blue bar in Figure 2e. Initially, for Revs 232–237, data are obtained from two segments either side of apoapsis, inbound on the dayside and outbound in the predawn sector, separated by an interval in the northern dawn-sector magneto sheath, as indicated by the initial dash-dot segment of the blue bar. Azimuthal component phases determined from the inbound data, but not the outbound, have been excluded due to proximity to the PPO-related field-aligned current sheets, which typically gives rise to weak and irregular signatures in this component. As apoapsis moved from dawn toward midnight, however, extended ~2- to 4-day (~4–9 oscillation) northern polar intervals spanning apoapsis become available for analysis, later shifting wholly into the postapoapsis inbound segment in the postmidnight sector for the F ring and proximal orbits as the line of apsides rotated into the equatorial plane (Figure 2a). Phases from all three field components are generally obtained to Rev 253, shown by the solid blue bar in Figure 2e, while after this Rev, the phases of the azimuthal field component are again excluded due to proximity to the PPO-related field-aligned currents, as indicated by the blue long-dashed bar.

PPO parameters from filtered southern polar data also become available after Rev 235 (early May 2016) until the end of the F ring orbits at Rev 270 (mid-April 2017), as indicated by the red long-dashed bar in Figure 2e. Again, the azimuthal component phases have been excluded due to proximity to (but poleward of) the estimated center of the PPO-related field-aligned current layer. These data segments are initially located near periapsis in the dusk to midnight sector when the spacecraft line of apsides was highly inclined, but later moved into the postperiapsis outbound orbit segment in the midnight and postmidnight sector as the line of apsides rotated toward the equatorial plane. The data segments are relatively short throughout, typically ~1 day in length (~2 oscillations). During the proximal orbits, however, the equivalent southern segment of the outbound trajectory is estimated to have been located near the inner edge of the principal PPO-related field-aligned current layers (see section 2.4), such that the data are expected once more to be equatorial in nature (in the sense indicated above), though located south of the planet’s equatorial plane. PPO
amplitudes and phases have then been obtained from fits to filtered data for all three field components, as shown by the green bar in Figure 2e spanning proximal Revs 271–292.

Figure 2e thus provides an overview of the varying nature of the PPO data available over the interval, directly reflected in the phase and amplitude measurements presented in section 1. While only equatorial data are evidently available during the initial orbit interval, both equatorial and north and south polar data are available for much of the second interval and at the start of the third, up to the F ring orbit sequence. Only north and south polar data are available during the F ring orbits, reverting to north polar and (southern hemisphere) equatorial region data for the proximal orbits.

2.4. Data Examples

We now exemplify the data employed over the analysis interval, specifically for Revs 246 and 247 (pre-F ring) spanning the apoapsis of Rev 246 and the periapsis of Revs 247, 269 (F ring), and 283 (proximal). We note that an example of near-equatorial data, for Rev 221, was presented previously in Figure A3 of Provan et al. (2016) and is not repeated here.

2.4.1. Trajectories in the $\rho$-$Z$ Plane

In Figure 3 we first show the trajectories of these Revs mapped into a magnetic meridian in cylindrical ($\rho,Z$) coordinates, where $Z$ is distance along the spin/magnetic axis of the planet and $\rho$ is perpendicular distance from that axis. These trajectories are superposed on field lines of an axisymmetric model magnetic field (arrowed black lines), consisting of the three-term Burton et al. (2010) internal planetary field together with a Connerney et al. (1981) model ring current field. The parameters of the ring current model are similar to those determined from fits to Saturn data by Bunce et al. (2007), but tailored here to better represent the nightside magnetosphere in the outer region depicted in the figure, as appropriate to our data (Figure 2).

Specifically, the current disk parameters are inner radius $\rho_1 = 6.75$ $R_S$, outer radius $\rho_2 = 25$ $R_S$, half-width (Z direction) $Z = 2.5$ $R_S$, and current parameter $\mu_0 I_0 = 40$ nT. The total current flowing in the disk is $\sim 12.6$ MA. The specific field lines shown are those mapped from the southern ionosphere at $5^\circ$ intervals of southern colatitude between zero (the Z axis) and $30^\circ$ (mapping equatorially to $\sim 3.6$ $R_S$). The blue dashed field line represents the OCB mapping into the northern ionosphere at $\sim 13.0^\circ$ colatitude and into the southern at $\sim 14.3^\circ$ colatitude, in approximate agreement with the results of Jinks et al. (2014). The purple shaded region, also bounded by field lines, indicates the approximate location of the principal PPO-related field-aligned current layer for both northern and southern PPO systems, lying on closed field lines from just inside the OCB to $12$ $R_S$ in the equatorial plane, in line with the discussion in section 1. In the northern ionosphere these currents map between $\sim 14.3^\circ$ and $\sim 17.3^\circ$ colatitude, and in the southern ionosphere between $\sim 15.8^\circ$ and $\sim 19.1^\circ$, in approximate agreement with the results of Hunt et al. (2014, 2015) and Bradley et al. (2018). The central purple dashed line indicates the approximate position where the PPO-related azimuthal field switches sign, thus marking the boundary between the inner “equatorial region” and outer “polar region” field oscillations as indicated in section 2.2. This field line maps from $\sim 15.3^\circ$ colatitude in the northern ionosphere to $\sim 16.9^\circ$ colatitude in the southern, passing through the equator at $17$ $R_S$, in approximate agreement with the results of Andrews, Cowley, et al. (2010) in the nightside equatorial magnetosphere.

The black dotted lines in Figure 3 show the spacecraft trajectories comprising one orbit of the planet, with arrowed colored portions showing the segments selected for PPO analysis as described in section 2.3. Overplotted open black circles show start of day markers, with the first on each orbit shown being numbered with the time in days (see also the data plots in Figures 4–6 introduced below). Corresponding dates are given in the figure caption. The color scheme of the colored portions follows Figure 2e, green for equatorial region data interior to the purple dashed line and blue and red for northern and southern polar data, respectively, exterior to the purple dashed line. For Revs 246 and 247 in Figure 3a all three regions are represented, with the northern polar inbound segment (blue) being well displaced from the OCB on open field lines, the southern outbound segment (red) straddling the OCB nearer the outer edge of the current layer, and a rapid near-periapsis traversal of the equatorial region (green) at $\sim 6$ $R_S$ between. For F ring Rev 269 in Figure 3b the northern inbound segment lies closer to but still outside of the OCB, while the southern outbound segment lies on outer closed field lines just inside of the outer part of the current layer. The near-periapsis equatorial traversal at $\sim 2.5$ $R_S$ is now so fast that PPO data cannot be retrieved as indicated in Figure 2e. For proximal Rev 283 in Figure 3c the northern inbound segment has moved inside the OCB near the outer edge of the current layer, while the southern outbound segment lies just inside the center of the current layer indicated by the
purple dashed line, such that the data are expected to be equatorial in nature (green) as discussed in section 2.3.

2.4.2. Revs 246 and 247

Figure 4 shows data from Revs 246 and 247 spanning one whole orbit from and to a radial distance of 20 $R_S$ over the interval $t = 4,681.84 - 4,691.43$ days, where apoapsis marking the boundary between Revs 246 and 247 is indicated at the top of the figure. Figures 4a–4f show the residual and band-pass filtered (5–20 h) data for each of the spherical polar field components, where the residual data panels all span 30 nT (though with varying ranges), and the filtered data panels all span ±4 nT, such that they are simply comparable. Figure 4g shows the radial distance of the spacecraft, while Figures 4h and 4i show the mapping of the spacecraft along model field lines into the northern and southern ionospheres, respectively, using the same model as employed in Figure 3. Each panel shows the range $0^\circ$–$25^\circ$ colatitude from the corresponding pole, and the mapped colatitude is shown solid when the spacecraft is in the corresponding hemisphere (north of the equator in Figure 4h and south of the equator in Figure 4i), and dotted when in the opposite hemisphere. The purple band in these figures shows the colatitude range of the principal PPO-related field-aligned current as in Figure 3, while the purple dotted line shows the latitude where the PPO azimuthal field switches sign from equatorial to polar directions. The blue dotted line indicates the approximate position of the OCB, also as in Figure 3. Figures 4j–4l show the spacecraft trajectory mapped into the $X$-$Y$, $X$-$Z$, and $Y$-$Z$ planes in KGS coordinates, where $Z$ is again the planet’s spin/magnetic axis, the $X$-$Z$ plane contains the Sun (positive $X$), and $Y$ completes the right-handed triad pointing to dusk. The black circles indicate start of day markers, and the solid and dashed lines show the Kanani et al. (2010) magnetopause model and the Masters et al. (2008) bow shock model (just visible in the corners of Figures 4j and 4k), respectively, for a typical solar wind dynamic pressure of 0.03 nPa.

The colored vertical dotted lines in Figure 4 show the intervals selected for PPO analysis as outlined in section 2.3 (Figure 2e). The pair of blue dotted lines spanning apoapsis show the selected north polar interval,
Figure 4. Plot of magnetic field and Cassini position data spanning \( t = 4,681.84 \) to \( 4,691.43 \) days from prior to apoapsis on Rev 246 to prior to apoapsis on Rev 247, constituting a full orbit around the planet from and to a radial distance of 20 \( R_S \), where the time of apoapsis is indicated at the top of the plot. Figures 4a–4f show the residual field (Burton et al., 2010 internal planetary field removed) and 5–20 hr band-pass filtered field for each of the three spherical polar field components as indicated. Figure 4g shows the radial distance of the spacecraft \( (R_S) \). Figures 4h and 4i show the model field-aligned mapping of the spacecraft into the northern and southern ionospheres, respectively, over the colatitude ranges 0°–25° from each pole, where the solid and dotted lines indicate that the spacecraft was located in the corresponding or the opposite hemisphere, respectively. Following Figure 3, the blue dotted lines indicate the OCB, while the purple bands and dotted lines indicate the principal PPO-related field-aligned current and its center where the PPO azimuthal component switches sign. The colored vertical dotted lines show the intervals selected for PPO analysis, blue for the north polar region spanning apoapsis, red for the postperiapsis south polar region, and green for the near-periapsis north-south crossing of the equatorial region. Least squares best fits of equation (8) (guide period 10.68 hr) to the selected data segments are shown by the correspondingly colored superposed lines, fitted to the filtered data for each component in the two polar regions (blue and red), and to unfiltered azimuthal field data only in the equatorial region (green). The fit parameters (phase and amplitude) are given in Table 1. Figures 4j–4l show the spacecraft trajectory mapped into the \( X-Y, X-Z, \) and \( Y-Z \) planes in KGS coordinates (see text), the black circles indicate start of day markers, and the solid and dashed lines show the Kanani et al. (2010) magnetopause model and the Masters et al. (2008) bow shock model (just visible in the corners of Figures 4j and 4k), respectively, for a typical solar wind dynamic pressure of 0.03 nPa. The blue, red, and green trajectory segments correspond to the selected data intervals in Figures 4a–4i, and to the blue, red, and green trajectory segments in Figure 3a.
corresponding to the blue trajectory segment in Figure 3a and in Figures 4j–4l. The oscillatory blue lines superposed on the filtered data in this interval show the best least squares fit of equation (8) to these data, using a guide phase given by equation (9) with a guide period of 10.68 hr. The fit parameters for each field component are given in Table 1, together with the RMS deviation between the data and the best fit.

**Figure 5.** Plot of magnetic field and trajectory data spanning \( t = 4,847.00 \rightarrow 4,854.17 \) days from apoapsis to apoapsis of F ring Rev 269. The format is the same as Figure 4, except that the equatorial crossing is now sufficiently brief that no PPO analysis is undertaken of these data.
model. The fits for all three field components are seen to be good, with oscillation amplitudes significantly larger than the RMS deviations in each case. The N-format phases ($\psi_i / \gamma_i N$) for each field component relative to the guide phase are also similar to each other as expected, lying in the band $-260^\circ \pm 25^\circ$. (The N- and S-format phases quoted here and below correspond to the $\psi_i$ values shown in Table 1 from which...
Fit Analysis Results for the Selected Intervals of Revs 246/7, 269, and 283 Shown in Figures 4–6

| Rev and region | 246/247 N polar | 246/247 equatorial | 246/247 S polar | 269 N polar | 269 S polar | 283 N polar | 283 equatorial |
|----------------|-----------------|-------------------|----------------|------------|------------|------------|--------------|
| Start time/days | 4,683.2         | 4,688.3           | 4,689.2        | 4,848.5    | 4,851.25   | 4,939.0    | 4,942.4      |
| End time/days   | 4,686.7         | 4,688.8           | 4,690.5        | 4,849.5    | 4,852.2    | 4,940.5    | 4,943.0      |
| $B_0$/nT        | 0.06            | -                 | 0.32           | 0.15       | 0.09       | 0.10       | 0.17         |
| $\psi$/deg     | 238             | -                 | 43             | 180        | 5          | 218        | 358          |
| $B_0$/nT        | 0.26            | -                 | 0.47           | 0.41       | 0.90       | 0.48       | 1.54         |
| $\psi$/deg     | 105             | -                 | 58             | 64         | 53         | 46         | 81           |
| $B_0$/nT        | 0.06            | 0.51              | 0.46           | 0.08       | 0.41       | 0.20       | 0.28         |
| $\psi$/deg     | 170             | 63                | 199            | 125        | 24         | 273        | 25           |

*a* Relative to a guide phase given by equation (9) with a guide period of 10.68 hr.

...
northern polar segment between the blue dotted lines is now located inside the OCB near the outer boundary of the PPO-related current layer, shown by the blue trajectory segment in Figure 3c and in Figures 4j–4l. The amplitudes of the \( r \) and \( \theta \) components are again much larger than the RMS deviations with closely similar N-format phases \( \approx 220^\circ \pm 5^\circ \) correspondingly being employed in further analysis, while the azimuthal component amplitude is again weaker and now smaller than the RMS deviation with a very different N-format phase \( \approx 0^\circ \) (see Table 1), and has been excluded from further analysis as for all the proximal Revs (Figure 2e). Due to the orbit evolution, however, the southern trajectory segment now lies within the inner edge of the principal PPO-related current layer (Figure 3e), such that the data correspond to the equatorial region with the vertical dotted lines and model fits hence colored green. In this case the enhanced amplitudes of all three field components well exceed the RMS deviations, and thus are considered as valid determinations. With regard to the phases, however, we note that the relationship between the \( \theta \) and \( \varphi \) component phases is consistent with northern-dominant equatorial oscillations with an N-format phase \( \approx 280^\circ \pm 15^\circ \), while the phase of the \( r \) component differs by \( \approx 80^\circ \). Understanding of these data requires analysis of the overall data set, presented in the next section.

3. Analysis Results

3.1. Magnetic Field PPO Phases

The N- and S-format phases \( (\theta, \varphi) \) obtained from the magnetic data as discussed in section 2 are shown versus time in Figures 7a and 7c, respectively, over the same interval as in Figure 2. Year markers and Rev numbers are again shown at the top of the plot. Two cycles of phase are shown on the vertical axes to aid visualization of data continuity, with each phase value being plotted twice. Phases determined from \( r, \theta \), and \( \varphi \) component data are shown by red, green, and blue symbols, respectively, while polar and equatorial region data are shown by open and solid circles, respectively. Specifically, northern polar data are shown in Figure 7a and southern polar data in Figure 7c, while equatorial data, in general modulated by both systems, are shown in both figures, in N and S formats, respectively. We recall from section 2.1 that the \( r \) component phases, which define the orientation of the two PPO systems (Figure 1), are plotted as measured throughout, while those of the \( \theta \) and \( \varphi \) components are shifted according to equations (5) to (7a) and (7b) such that for a pure northern oscillation, they would coincide with those of the \( r \) component in N format, while for a pure southern oscillation, they would coincide with those of the \( r \) component in S format. Andrews et al. (2012) and Provan et al. (2013) demonstrated previously that dual-modulated data are clustered about the northern phase in N format and about the southern phase in S format, so that both can be discerned from such data provided that one system is not too weak relative to the other. Otherwise only the phase of the dominant system can be determined. Given typical measured phase uncertainties of \( \approx 10^\circ \), a typical limit is that the amplitudes should not differ by more than a factor of \( \approx 5 \) (Provan et al., 2013). The N-format phases in Figure 7a are shown relative to a guide phase with a guide period of 10.8 hr (equations (9) and (10)), while the southern values in Figure 7c are similarly shown relative to a guide phase with a guide period of 10.7 hr. The implied period of the oscillations then relates to the temporal gradient of the N- and S-format phases (equation (11)), with values that fall (or rise) with time indicating PPO periods that are smaller (or larger) than the guide periods.

The nature of the phase data in Figures 7a and 7c follows expectations based on the discussions in sections 2.2 and 2.3 summarized in Figure 2e, as outlined by the vertical dashed lines shown in the figure. In the initial interval of near-equatorial orbits to \( t \approx 4,425 \) days (Revs 221–231) the in general three-component phases form a tightly banded set of slowly falling values in N format in Figure 7a, thus implying a northern-dominant equatorial oscillation with a period slightly smaller than the 10.8 hr guide period (\( \approx 10.78 \) hr as shown below). In S format in Figure 7c these same data form rapidly rising lines of \( \theta \) and \( (r, \varphi) \) data separated by \( \approx 180^\circ \), this being the signature of northern-dominant oscillations plotted in S format relative to a guide phase with a significantly smaller 10.7 hr guide period. As indicated previously by Provan et al. (2016), analysis (in that case to the end of 2015) fails to reveal any southern system modulation in these S-format data, thus indicating that the southern system amplitude is less than the northern by a factor of at least \( \approx 5 \).

In the second data interval \( t \approx 4,425–4,700 \) days (Revs 232–248) marked by the vertical dashed lines in Figure 7, both equatorial and polar data are available, though the polar data are confined to the northern hemisphere prior to \( t \approx 4,500 \) days (Rev 235). These data are also relatively sparse before \( t \approx 4,600 \) days.
Figure 7. (a–d) Plots showing measured and modeled magnetic PPO phases and periods over the same interval as in Figure 2, with year markers and Rev numbers again shown at the top of the plot. Vertical dashed lines delineate the main intervals of differing phase data types as indicated at the top of the plot. Figure 7a shows N-format phase data (deg) relative to a guide phase of 10.80 hr period, where two cycles of phase are shown on the vertical axis with each data point being plotted twice. The red, green, and blue symbols show the phases of the $r$, $\theta$, and $\phi$ spherical polar field components, respectively, with open circles showing northern polar data and solid circles equatorial data. The sequence of blue straight-line segments shows the northern PPO phase model relative to the guide phase, joining the center-points of running linear fits to 125-day data segments taken at 12.5-day intervals. All of the phase data shown in the figure are used in the fits, except for the $r$ component data from the proximal orbits (solid red circles for Revs 271–293). Figure 7b shows the best fit variances $V$ to the 125-day data segments for linear fit slopes corresponding to modulation periods between 10.5 and 10.9 hr obtained at intervals of 0.001 hr, gray scaled as shown at the top of the figure. The blue solid line traces a variance minimum taken to be the northern PPO period. The blue squares show the periods determined from the straight line segments joining the model phase values in Figure 7a, not necessarily precisely the same values as the periods determined from the best linear fits themselves. Figure 7c, similar to Figure 7a, shows S-format phase data relative to a guide phase corresponding to a 10.70-hr period, where the open circles correspond to southern polar data and the solid circles to equatorial data. The southern PPO phase model relative to the guide phase similarly derived from running 125-day linear fits evaluated every 12.5 days is shown by the sequence of red straight-line segments. Fits are indeterminate in the initial interval to $\sim$4,500 days, while in the following interval the northern-dominated equatorial data (solid circles) for Revs 235–248 are excluded from the fit, as well as the northern-dominated $\theta$ and $\phi$ component equatorial data from the proximal orbits (solid green and blue circles for Revs 271–292). Figure 7d, similar to Figure 7b, shows the best fit variances for linear slopes corresponding modulation periods between 10.5 and 10.9 hr, also gray scaled according to the scale at the top of the plot. The red solid line marking a continuous variance minimum after $t = 4,500$ days is taken to be the southern period, while the red squares show the periods obtained from the straight-line segments joining the 12.5-day phase values in Figure 7c.
(before Rev 240) due to the lengthy orbital period (Figure 2d). Figure 7a shows that in this interval the equatorial data are in reasonable agreement with the N-format northern polar data, both continuing to fall slowly with time relative to the 10.8 hr guide phase. The gradient of the fall decreases after \( t = 4,550 \) days, however, indicative of a small shift in northern period to slightly closer to the guide period after this time \((-10.79 \) h). In Figure 7c the equatorial data continue to form rapidly rising lines of \( \theta \) and \( \phi \) component data separated by \(-180^\circ\), again indicative of northern-dominant oscillations similar to the initial interval, though the data are too sparse and variable to provide a clear determination or limit on the north/south amplitude ratio. However, the southern polar data available after \( t \approx 4,500 \) days form a tightly banded set of slowly falling values indicative of a southern system period slightly smaller than the 10.7 hr guide period \((-10.68 \) hr), which remains nearly constant to the end of the interval.

The third and fourth data intervals marked by the vertical dashed lines in Figure 7 correspond to those of the F ring (Revs 249–270) and proximal (Revs 271–293) orbits, respectively. We note that, strictly, the F ring orbits correspond to Revs 251–270 only, but since the orbits and data of the two preceding Revs 249 and 250 are so similar and contribute equally to analyses of the end of mission interval, we incorporate them here and later into the F ring category for ease of further discussion. As indicated in Figure 3b, the F ring orbit phase data consist of northern and southern polar data shown in Figures 7a and 7c, respectively, which overall continue to show slowly falling values. However, the increased cadence of these data due to the decreased orbit period after \( t \approx 4,700 \) days also reveals the presence of \(-40-day\) modulations in these phases in both hemispheres which we show below is at the beat period of the northern and southern PPO oscillations. These modulations thus unexpectedly show the dual presence of both northern and southern period oscillations in these polar data, contrary to the previous results of Andrews et al. (2012) and Hunt et al. (2015), who found no such dual modulation in the polar data they studied (high-latitude data from 2006 to 2009) to within a 10% limit by amplitude. The nature of these modulations is also at variance with those observed in near-equatorial data in earlier studies (Andrews et al., 2012; Cowley et al., 2015; Provan et al., 2011, 2013), as described in section 2.1. In the equatorial modulations the phase deviations have opposite senses for the \((r, \varphi)\) and \(\theta\) components due to the opposite polarization of the \(\theta\) component relative to \((r, \varphi)\) in the two PPO systems (Figure 1). In terms of the analysis in section 2.2, for equatorial data \(\gamma_\theta = 0^\circ\) while \(\gamma_\varphi = 180^\circ\) (equations (5) to (7a) and (7b) and (20)), thus switching the signs of the trigonometric functions in the expressions describing the beat modulations on the right side of equations (17) and (18). In the polar data in Figures 7a and 7c, however, the \(r\) (red) and \(\theta\) (green) component phases are seen to be modulated approximately in phase with each other, implying similar \(\Delta \gamma_\theta\) and \(\Delta \gamma_\varphi\) values, to be determined from the analysis in section 3.4.

Across the transition between F ring and proximal orbits at \( t \approx 4,860 \) days, the character of the northern polar data in Figure 7a remains unchanged, while the phase data obtained from the proximal southern orbit segments shown in S format in Figure 7c (Revs 271–293) change significantly from the previous southern polar data. While the \(r\) (red) component phases in Figure 7c continue generally to fall along the same line as the previous S-format southern polar data, indicative of a southern-dominant oscillation, the \(\theta\) (green) and \(\varphi\) (blue) component data form rapidly rising lines separated by \(-180^\circ\) similar to those observed earlier in the interval mentioned above, again indicative of a dominant northern oscillation. These data are shown on expanded scales for greater visibility in Figure S1b of the supporting information, where further discussion is provided. Unique among the PPO data studied to date, therefore, the proximal orbit phase data indicate the presence of an \(r\) component oscillation, which is dominated by the southern PPO system, but \(\theta\) and \(\varphi\) components, which are dominated by the northern system, thus elucidating the discussion of the phase data for proximal Rev 283 in section 2.4. In terms of section 2.1, these data thus provide unequivocal evidence of oscillations for which the north/south amplitude ratios \(k_i\) are different among the three field components. Close inspection of the proximal orbit \(\theta\) and \(\varphi\) component data in Figure 7c shows, however, that the rising phase values deflect toward the line of the \(r\) component values when each crosses that line, this being the signature of dual modulation by a weaker southern PPO oscillation (Andrews et al., 2012; Provan et al., 2013).

These inferences are confirmed in Figure 7a, where these same southern hemisphere data, taken to be equatorial in nature line with Figure 3c and hence shown as solid circles, are now plotted in N format. While the \(r\) component data (red) now form steeply falling lines indicative of an oscillation with a period significant less
than the 10.8 hr guide period (i.e., the southern system period as shown in Figure S1a of the supporting information), the N-format values of the $\theta$ and $\varphi$ component phases (green and blue) agree closely with the northern polar values (open circles) thus confirming the presence of dominant northern PPO system oscillations in these southern hemisphere equatorial region data. However, these $\theta$ and $\varphi$ component phases also show that ~40-day modulations about the overall slowly falling trend which, as expected for equatorial data as indicated above, are now oppositely modulated for the $\theta$ and $\varphi$ components (these being the counterparts in N format of the “deflections” in the S-format data noted in Figure 7c). Inspection shows that the equatorial $\theta$ component is deflected in the same sense as the polar $\theta$ component, while the equatorial $\varphi$ component is deflected opposite to the polar $\varphi$ component. These phase relationships will be further analyzed in section 3.4. We finally remark that while the nature of the equatorial region data obtained on the proximal orbits is certainly unique, the location from which it was obtained, shown in Figure 3c, is also unique. With the exception of the small amount of phase data obtained from unfiltered $\varphi$ component data on rapid near-periapsis crossings on highly inclined orbits, by far the majority of analyzed equatorial data have been obtained on orbits located close to the equatorial plane in the quasi-dipolar magnetosphere within a radial distance of ~12 Rs, well inside the principal region of PPO-related field-aligned current flow (Figures 1 and 3). The southern hemisphere proximal data, however, were obtained within the inner part of the current layer well south of the equator at radial distances ~9–15 Rs (Figure 3c). Further discussion is given in section 4.

3.2. PPO Phase Models and Period Determinations

We now employ the data in Figures 7a and 7c to obtain continuous models of the northern and southern PPO phases and periods, that is, the functions $\psi_{N}(t)$ and $\tau_{N}(t)$ in equations (10) and (11) in section 2.1. As in previous related studies these are determined from piecewise running linear fits to the N- and S-format phase data (Andrews et al., 2012; Provan et al., 2013, 2016), the methodology of which will only be briefly outlined here. Specifically, we determine the values of $\alpha$ and $\beta$ in the linear function

$$\psi'(t) = \alpha t + \beta,$$

which gives the minimum value of the variance $V$ between the function and a selected segment of N- or S-format phase values containing $L$ elements, written for convenience here simply as $\psi_l$ at center time $t_l$. Following the directional statistics methodology of Mardia and Jupp (2000), we define the variance $V$ as

$$V = \frac{1}{L} \sum_{l=1}^{L} \left(1 - \cos(\psi_l - \psi'(t_l))\right) = \frac{1}{L} \sum_{l=1}^{L} \left(1 - \cos(\psi_l - (\alpha t_l + \beta))\right),$$

such that the value is zero for a perfect fit, but approaches unity when for every value of the phase difference $\psi_l - \psi'(t_l)$ there exists another in the data set that differs by 180°. For a given value of slope $\alpha$, directly related to the oscillation period $\tau$ associated with the fit through equation (11), the value of the constant $\beta$, which gives the lowest value of $V$, is found by determining its value with suitable resolution within a suitable range of slopes $\alpha$ or equivalently period $\tau$ values. Here we have employed values of $\alpha$ corresponding to periods between 10.5 and 10.9 hr with a resolution of 0.001 hr (i.e., 3.6 s).

It is evident from the discussion in section 3.1 that the behavior of the fits will depend on the length of the data segments employed and the nature of the data included. In particular, running fits to short data segments may respond to beat-related modulations, especially for the high-cadence northern and southern polar data where the $\tau$ and $\theta$ component phases are modulated approximately in phase with each other, while running fits to sufficiently long data segments will tend to average over such modulations, as well as scatter in the data, then giving an unmodulated but smoothed estimate of the northern and southern PPO periods. In previous studies Andrews et al. (2012) employed running fits to phase data from 2005 to 2011 taken 25 points at a time corresponding to intervals of ~200 days (similar to related SKR studies, e.g., Lamy, 2011), while Provan et al. (2016) employed running fits to data from 2012 to 2015 taken 5 Revs (~10–15 phase points) at a time corresponding to intervals of ~100–150 days. Such intervals are generally much longer than the PPO beat periods, for example, ~25 days under late southern summer conditions 2005–2008, though becoming comparable with the beat period under postequinox conditions 2010–2012 (see, e.g., Cowley & Provan, 2015, and Cowley et al., 2015). In our equivalent analyses here we
have employed running 125-day intervals evaluated every 12.5 days, thus similar to the intervals employed in previous related studies. Such intervals, corresponding to ~3 beat periods, were found to be the shortest, hence having the highest temporal resolution, for which the fit results did not respond significantly to the beat period. Results are shown in Figure 7b for fits to the northern phase data and in Figure 7d for fits to the southern phase data. Specifically, these plots show values of the fit variance \( V \), gray scaled as shown at the top of the figure, for fit slopes corresponding to periods between 10.5 and 10.9 hr as indicated above.

For the northern system fits we have included all of the N-format phase data shown Figure 7a with the exception of the proximal orbit equatorial region \( r \) component data (solid red circles for Revs 271–292), since analysis in section 3.4 shows that these data contain no measureable northern system contribution and serve only to increase the variance of the fit. We note, however, that apart from the fit variance, the northern system fit results are hardly changed whether these data are included or not, or indeed whether any of the proximal orbit equatorial phases are included or not, in the latter case leaving only the northern polar phases in the proximal orbit interval fit. The results are thus robust to detailed choices, within uncertainty values discussed below. For the southern phase data in Figure 7c we do not include the equatorial data between Revs 235 and 248 where southern polar data are also available, since these data are dominated by the northern oscillation as described in section 3.1 above, and again serve only to increase the variance of the fits. During the proximal orbit interval we also use only the \( r \) component phase data, and not the northern-dominated \( \theta \) and \( \phi \) component data (solid green and blue circles), though investigation again shows that the fitted phases and periods do not change much if all the proximal orbit phase data are included, at the expense of significantly increased minimum variance values.

The corresponding PPO periods are defined by the sequence of adjacent minima in \( V \) marked by the blue line for the northern period in Figure 7b and the red line for the southern period beyond \( t \approx 4,500 \) days in Figure 7d, there being no clear minimum in southern fit \( V \) using the available equatorial data prior to this time as indicated above, consistent with the results of Provan et al. (2016). Short-lived sequences of subsidiary minima in \( V \) also occur within the period band, in particular due to aliasing arising from the inescapable modulo 360° ambiguity in the phase difference between one Rev and the next (see Appendix B of Provan et al., 2016). However, such effects depend on the cadence of the data, that is, mainly on the orbit period, such that the aliased periods lie within the band shown when the orbit period is longer than \( \sim 15 \) days, for example, during \( t \approx 4,400 – 4,600 \) days (Figure 2d), but outside the band for shorter orbit periods, for example, after \( t \approx 4,600 \) days, where these effects are seen largely to disappear.

Overall, however, it is evident from these results that the northern PPO period varied only modestly over the study interval and of the beat period are shown in Figures S2a and S2c in the supporting information. Consistent with these results, we note that Carbary et al. (2017) have recently reported observations of modulations in energetic electrons during the F ring and proximal orbits (\( t \approx 4,720 – 5,100 \) days) with a near-constant period of \( 10.79 \pm 0.01 \) hr, most clearly observed in northern hemisphere data. However, they did not detect the southern system period in southern hemisphere (or northern) energetic electron data.

Comparing the periods in Figures 7a and 7c with those determined from earlier data by Provan et al. (2016), we note that these values had hardly changed following the reversal in northern and southern periods, which took place after mid-2014. During the first half of 2015, for example, the northern period determined from both magnetic field and SKR data was found to be \( \sim 10.76 – 10.77 \) hr, while the southern period was found to be \( \sim 10.69 – 10.70 \) hr. Although southern phases and periods could not be discerned in the equatorial phase data after \( t \approx 4,250 \) days (late August 2015) near the start of the interval investigated here, southern periods determined from SKR data after this time varied between \( \sim 10.69 \) and \( \sim 10.70 \) hr to \( t \approx 4,400 \) days (end of 2015; Provan et al., 2016).
The blue line in Figure 7a and the red line in Figure 7c are formed from the sequence of straight-line segments joining the center-points of each of the 125-day best fit lines at 12.5-day cadence whose period (slope) is shown by the corresponding blue and red lines in Figures 7b and 7d, thus defining our models of the northern and southern phases (equation (10)). The slopes of these linear segments define periods through equation (11), which are not in general precisely the same as those of the individual linear fits, but are shown by the overplotted blue and red squares in Figures 7b and 7d. These differ from the fit values (solid blue and red lines) typically by $\pm 0.001$ hr ($\pm 18$ s) for the northern data prior to $\pm 4,600$ days, decreasing to $\pm 0.005$ hr ($\pm 9$ s) for the higher cadence northern and southern data after this time. A time series plot of these difference values is provided in Figure S2b in the supporting information.

Following the analysis of Cowley and Provan (2016), the uncertainties in the individual phase and period values may be estimated, for minimum variance $V$ small compared with unity, as

$$\delta \psi \approx \sqrt{\frac{2V}{T}}, \tag{28a}$$

which gives the RMS deviation (in radians) of the phase data from the fitted line, and

$$\delta \tau \approx \frac{T^2}{2\pi} \sqrt{\frac{6V}{L}}, \tag{28b}$$

where $\tau$ is the period, $T$ is the length of the data interval employed in the fit (here 125 days), and $L$ is the number of phase data points in the fit as in equation (27). Time series plots of the number of data points $L$ in each fit, the minimum variance $V$, the related RMS phase deviation $\delta \psi$ (equation (28a)), and the period uncertainty $\delta \tau$ (equation (28b)) are given for both the N- and S-format data fits in Figure S3 in the supporting information. The minimum variance values are typically $\pm 0.05$, with means of $\pm 0.19$ for the northern data and $\pm 0.14$ for the southern, such that the phase uncertainties given by equation (28a) are typically $\pm 0.005$ to $0.007$, with means of $\pm 0.005$ for the northern data and $\pm 0.007$ for the southern. We note that much of this phase variability is related to the beat modulation effects to be discussed in sections 3.3 and 3.4, rather than simply being due to random measurement uncertainties. With $\pm 30$ over much of the interval investigated, with means of $\pm 36$ for the northern data and $\pm 21$ for the southern, we find from equation (28b) typical uncertainties in the individual period values of $\delta \tau \approx 0.002$ to $0.003$ hr, with means of $\pm 0.0025$ hr (i.e., $\pm 9$ s) for both northern and southern data, comparable with the model deviations noted above.

It can be seen from Figure 7 that the two PPO periods are closely constant after $t \approx 4,700$ days, an interval spanning the high-cadence data from the F ring and proximal orbits. It may then be useful to determine best fit constants for single linear fits to these near-linearly varying phase data, specifically those from Revs 249–293 (as discussed in detail in sections 3.3 and 3.4 below), using the same exclusions for the proximal orbit data as indicated above. The best fit slopes over this $\pm 310$-day interval correspond to periods of 10.792 hr for the northern system and 10.679 hr for the southern, with uncertainties in these values given by equation (28b) of order $\pm 0.0005$ hr ($\pm 2$ s). The minimum variances $V$ are 0.167 over 134 phase values for the northern system and 0.192 over 61 phase values for the southern system, associated with RMS phase deviations of $\pm 35^\circ$ from equation (28a). An uncertainty in the period of the above order corresponds to an uncertainty in the phase of an $\pm 10.75$ hr modulation of $\pm 1^\circ$ over $\pm 310$ days. The corresponding beat period is $42.5 \pm 0.3$ days. The modulo $360^\circ$ linear phase model for the northern system relative to a guide phase given by equation (9) with a guide period of 10.80 hr (as in Figure 7a) is given by

$$\psi_N(t) = (287.4 - 0.593(t (\text{days}) - 4700)) \ \text{deg}, \tag{29a}$$

while the phase model for the southern system relative to a guide phase with a guide period of 10.70 hr (as in Figure 7c) is given by

$$\psi_S(t) = (131.3 - 1.588(t (\text{days}) - 4700)) \ \text{deg}, \tag{29b}$$

valid for $t \geq 4,700$ days (from mid-November 2016) to the end of mission (mid-September 2017). The fits themselves are shown in Figure S4 of the supporting information, together with the data from which they were derived, in the same formats as Figures 7a and 7c. The differences between the linear models in equations (29a) and (29b) and the full models in Figures 7a and 7c are generally very small over this interval.
3.3. Beat Modulations of Phase and Amplitude

We now show that the phase modulations evident in the high-cadence data from the F ring and proximal orbits in Figures 7a and 7c are related to the beat phase of the northern and southern PPO oscillations, as implied in the above discussion, and that related modulations are also present in the PPO amplitude data. An initial overview is provided in Figure 8, where we show data on an expanded scale covering \( t = 4, 700 - 5, 010 \) days (13 November 2016 to 18 September 2017 inclusive), spanning the F ring (plus Revs 249 and 250 as indicated in section 3.1) and proximal Revs 249–293. Figure 8a shows the modulo 360° PPO beat phase \( \Delta \Phi \) derived from equation (19a) using the 125-day fit northern and southern PPO phase models shown in Figures 7a and 7c, respectively. Because the northern period is longer than the southern throughout the interval of this study, the beat phase decreases monotonically with time. Times when the two systems are in phase, \( \Delta \Phi = 0°/360° \) modulo 360°, are indicated by the vertical dashed lines, while times when the two systems are in antiphase, \( \Delta \Phi = 180°/360° \) modulo 360°, are shown by the vertical dotted lines.

Figure 8b shows the deviation of the northern polar phase data from the 125-day fit northern phase model in Figure 7a, where to guide the eye, the solid blue line shows the results of a running 3-Rev linear fit to these data, the shortest practicable data segment for such fits, also relative to the 125-day fit model. The corresponding 3-Rev fit modulated period (fit slope) is shown by the solid blue line in Figure 8d, where the blue dashed line shows the 125-day fit values from Figure 7b. The 3-Rev fit phases and periods derived from the northern polar data are shown directly in Figures S5a and S5b in the supporting information, in the same format as Figures 7a and 7b. Figure 8c shows the corresponding northern polar oscillation amplitudes. Examination of the relative phases in Figure 8b and the amplitudes in Figure 8c clearly shows that the modulations are related to the beat phase of the two PPO oscillations, thus showing the presence of a southern PPO oscillation in these northern polar data, on both sides of the OCB (Figures 3b and 3c). Specifically, the phase values of both the \( r \) (red) and \( \theta \) (green) components rise with time when the two systems are in phase, and fall with time when they are in antiphase. Bearing in mind that the beat phase is a monotonically decreasing function of time, this means that to a lowest approximation the phase modulations vary as \(~ + \sin \Delta \Phi\), as will be confirmed using quantitative analysis in section 3.4. Correspondingly, the effective modulated northern period in Figure 8d exhibits maxima when the PPO systems are in phase and minima when in antiphase, thus varying to a first approximation as \(~ + \cos \Delta \Phi\).

With \( \Delta \Phi \) varying as \(~ + \cos \Delta \Phi\), from equation (21) we note that for a northern-dominant oscillation (large \( k_j \)) the N-format phase modulation about the northern phase is given by

\[
\delta \phi_{N} = \frac{1}{k_i} \sin(\Delta \Phi - \Delta \gamma_j), \tag{30}
\]

with the implication that the phasing of the superposed southern-period oscillations is such that \( \Delta \gamma_r - \Delta \gamma_{r,s} \approx 180° \). Similar to equation (24) for large \( k_j \) (or equivalently from differentiation of equation (30)), the apparent period modulations about the northern PPO period are given by

\[
\delta \tau_{N} = \tau_{r} - \tau_{N} = \frac{t_{N} - t_{S}}{t_{S}} \left( \tau_{N} - \tau_{S} \right) \frac{1}{k_i} \cos(\Delta \Phi - \Delta \gamma_j), \tag{31}
\]

which corresponds to maxima when the two systems are in phase and minima when in antiphase, as in Figure 8d, provided \( \Delta \gamma_r - \Delta \gamma_{r,s} \approx 180° \). Examination of the amplitudes in Figure 8c also shows that to the extent modulations can be discerned despite the scatter in the data, smaller values occur when the two systems are in phase, and larger when in antiphase, again consistent with \( \Delta \gamma_r - \Delta \gamma_{r,s} \approx 180° \) as seen from equation (23). From equation (20) \( \Delta \gamma_r \) is given by

\[
\Delta \gamma_r = \gamma_{N} - \gamma_{S}, \tag{32}
\]

and since by definition \( \gamma_{N} = 0° \) (equation (5)), the implication is that the relative phase of the radial component of the southern oscillation in the northern polar region is \( \gamma_{S} \approx 180° \), that is, oppositely directed to the radial component of the southern oscillations in the equatorial and southern polar regions as depicted in Figure 1f, for which by definition \( \gamma_{S} = 0° \) also (equation (5)). Similarly \( \Delta \gamma_{\theta} \) is given by

\[
\Delta \gamma_{\theta} = \gamma_{N} - \gamma_{S}, \tag{33}
\]

and since \( \gamma_{N} \approx 180° \) (equation (6a)), the implication is that the relative phase of the colatitudinal component
Figure 8. (a–h) Overview of beat-modulated PPO phases and amplitudes for the interval $t = 4,700 - 5,010$ days (13 November 2016 to 18 September 2017 inclusive), spanning F ring to proximal Revs 249–293. Rev markers and year boundaries are shown at the top of the plot in a similar format to Figures 2 and 7. Figure 8a shows the modulo 360° value of the PPO beat phase $\Delta \Phi$ (deg) derived from the 125-day fit phase models for the northern and southern PPO systems shown in Figures 7a and 7c, respectively. Times when the systems are in phase, $\Delta \Phi = 0°/360°$ modulo 360°, are shown by the vertical dashed lines, while times when they are in antiphase, $\Delta \Phi = 180°$ modulo 360°, are shown by the vertical dotted lines. Figure 8b shows the modulations of the northern polar phases (deg) in Figure 7a relative to the 125-day fit northern system phase model, red open circles for the $r$ component and green open circles for the $\theta$ component. The blue solid line shows the result of a running 3-Rev linear fit to these data also relative to the 125-day fit model (Figure S5a in the supporting information). Figure 8c shows the corresponding northern polar oscillation amplitudes (nT). Figure 8d shows the modulated periods (h) corresponding to the 3-Rev fits to the northern polar data in Figure 8b (blue line) and the southern system data in Figure 8e (red line). The dashed lines show the corresponding 125-day fit values from Figures 7b and 7d. Figure 8e shows the southern polar phase data (deg) from the pre-F ring and F ring orbits relative to the 125-day southern system phase fit from Figure 7c, red open circles for the $r$ component and green open circles for the $\theta$ component, as well as the $r$ component phases of the southern equatorial region data from the proximal orbits shown by the red solid circles. The red line shows a 3-Rev linear fit model relative to the 125-day southern phase model (figure S5c). Figure 8f shows the corresponding amplitudes (nT). Figure 8g shows the southern equatorial region phase data (deg) from the proximal orbits for the $\theta$ and $\phi$ component oscillations relative to the 125-day northern phase model from Figure 7a (green and blue solid circles, respectively), together with individual running 3-Rev fits (Figures S6a and S6c) also relative to the 125-day northern phase model (corresponding green and blue lines). Figure 8h shows the corresponding amplitudes (nT).
of the southern oscillation in the northern polar region is $\gamma_{0S}=0^\circ$; that is, it has the same sense as the colatitudinal component in the equatorial and southern polar regions as depicted in Figure 1f (equation (6b)). The implication of these results for the overall geometry of the PPO-related perturbation fields in these regions will be discussed in section 4.

Figure 8e similarly shows the southern polar phase data from the F ring orbits relative to the 125-day southern system phase from Figure 7c (red and green open circles), as well as the southern-dominated $r$ component data (only) of the southern hemisphere equatorial region data from the proximal orbits (red solid circles). (The northern system-dominated $\theta$ and $\varphi$ component data from the southern passes of the proximal orbits are treated separately below.) We also show the results of a running 3-Rev linear fit to these data (red line) relative to the 125-day southern phase model, the corresponding period (fit slope) of which is shown by the red solid line in Figure 8d together with the 125-day fit period from Figure 7d shown by the red dashed line. The 3-Rev fit phases and periods derived from these data are shown directly in Figures S3c and S3d in the supporting information, in the same format as Figures 7c and 7d. Figure 8f shows the associated amplitudes. Concentrating first on the southern polar data (open circles), it can be seen that although these data are rather scattered, the modulations in both phase and amplitude again clearly relate to the beat phase. The phase values for the $r$ and $\theta$ components now both fall with time when the two systems are in phase, and rise with time when they are in antiphase, opposite to the northern polar data. Again, bearing in mind that the beat phase is a monotonically decreasing function of time, this means that to a first approximation, the phase modulations vary as $- + \sin \Delta \Phi$. Correspondingly, the effective modulated southern period in Figure 8d exhibits minima when the PPO systems are in phase and maxima when in antiphase, thus varying to a first approximation as $- - \cos \Delta \Phi$. From equation (22) we note that for a southern-dominant oscillation (small $k_i$) the $S$-format phase modulation about the southern phase is given by

$$\delta \psi_S \approx -k_i \sin(\Delta \Phi - \Delta \gamma_i), \quad (34)$$

with the implication that the phase of the superposed northern period oscillations is again such that $\Delta \gamma_r = -\Delta \gamma_i \approx 180^\circ$. Similarly from equation (25) for small $k_i$ (or equivalently from differentiation of equation (34)) the apparent period modulations about the southern PPO period are given by

$$\delta \tau_S = \tau_i - \tau_S \approx \frac{\tau_S}{\tau_N} (\tau_N - \tau_S) k_i \cos(\Delta \Phi - \Delta \gamma_i), \quad (35)$$

which corresponds to minima when the two systems are in phase and maxima when in antiphase, as in Figure 8d, provided $\Delta \gamma_r = -\Delta \gamma_i \approx 180^\circ$. Examination of the F ring southern polar amplitudes in Figure 8f (open circles) again shows that the values are small when the two systems are in phase, and larger when in antiphase, consistent with $\Delta \gamma_r = -\Delta \gamma_i \approx 180^\circ$ as seen from equation (23). Parameter $\Delta \gamma_i$ is again given by equation (32), and since by definition $\gamma_{IS} = 0^\circ$ for the dominant southern oscillation in the southern polar region (equation (5)), the implication is that the relative phase of the radial component of the northern oscillation in the southern polar region is $\gamma_{IS} \approx 180^\circ$, that is, again directed opposite to the radial component of the northern oscillations in the equatorial and northern polar regions as depicted in Figure 1f, for which by definition $\gamma_{NS} = 0^\circ$ (equation (5)). Similarly, with $\gamma_{0S} = 0^\circ$ for the dominant southern $\theta$ component oscillation in equation (33) (equation (6b)), the implication is that the relative phase of the colatitudinal component of the northern oscillation in the southern polar region is $\gamma_{IS} \approx 180^\circ$; that is, it has the same phase as the colatitudinal component in the equatorial and northern polar regions as depicted in Figure 1f (equation (6a)). Thus, the phase relationships of the secondary oscillations in the two polar regions are the same in both hemispheres relative to their primary oscillations in the equatorial region and opposite hemispheres; that is, the colatitudinal components have the same phase as the primary oscillations while the radial components are reversed in sense.

With regard to the relative amplitudes of the northern and southern oscillations in the two polar regions, we note that rough values of $(1/k)$ in the northern polar region and $k$ in the southern polar region may be estimated from the modulations in the northern and southern periods in Figure 8d using equations (31) and (35). Since the amplitudes of the period modulations in both cases correspond to about one third of the total difference between the northern and southern periods, as can be seen directly in Figure 8d, this means that $(1/k)$ in the north and $k$ in the south both have values $\approx 0.3$. However, since, for example, it can be seen in Figures 8b and 8e that the amplitudes of the $\theta$ component phase oscillations are generally somewhat
larger than those of the \( r \) component, these should be taken only as rough overall values, which imply that in both polar regions the modulations from the “opposite” hemisphere make a significant but not a dominant contribution. More precise values will be derived for individual field components in section 3.4.

With regard to the proximal orbit \( r \) component data shown by the red solid circles in Figures 8e and 8f, no beat phase effects are apparent in either the phase or amplitude (see also Figures 5Sc and 5Sd in the supporting information). Correspondingly, the 3-Rev fit periods in Figure 8d show only irregular variations about the 125-day fit values during this interval. The implication is that if a northern oscillation is present in these data, its amplitude relative to the southern system must be small, likely less than a factor \( k \approx 0.1 \) (see Figure S8 in the supporting information).

In Figure 8g we show equatorial region phase data from the southern hemisphere passes of the proximal orbits for the northern system-dominated \( \theta \) and \( \varphi \) oscillations relative to the 125-day northern phase model from Figure 7a (solid green and blue circles, respectively), while in Figure 8h we show the corresponding amplitudes. Unlike the equivalent \( r \) component data in Figures 8e and 8f (solid red circles), clear beat-related modulations are again present in both the phases and amplitudes showing the presence of significant southern system contributions. However, unlike both the northern and southern polar data in Figures 8b–8f, the modulations now have opposite senses for the two field components, such that in Figure 8g we show individual running 3-Rev fits to the phase data from the two field components relative to the 125-day northern phase model (corresponding green and blue solid lines). The 3-Rev fit profiles derived from these data are shown directly in Figures S6a–S6d in the supporting information. For the \( \theta \) component the phases rise when the two systems are in phase and fall when in antiphase, thus varying approximately as \( \sim + \sin \Delta \Theta \), while the amplitudes are smaller when the systems are in phase and larger when in antiphase. These behaviors are the same as for the northern polar \( \theta \) component in Figures 8b and 8c, and again, both imply \( \Delta \gamma_{\varphi} = 180^\circ \). Since \( \gamma_{0N} \approx 180^\circ \) for the dominant northern system oscillation (equation (6a)), the implication from equation (33) is that the relative phase of the secondary southern \( \theta \) component oscillation is given by \( \gamma_{0S} = 0^\circ \), consistent with the expected value for the southern system in the equatorial region (equation (6b)). For the \( \varphi \) component, however, the phases fall when the two systems are in phase and rise when in antiphase, thus varying approximately as \( \sim + \sin \Delta \Phi \), while the amplitudes are larger when the systems are in phase and smaller when in antiphase. These behaviors are the opposite of those of the northern polar \( r \) component in Figures 8b and 8c, and now from equations (30) and (23), both imply \( \Delta \gamma_{r} = 0^\circ \). Since from equation (20)

\[
\Delta \gamma_{r} = \gamma_{0N} - \gamma_{0S},
\]

and since \( \gamma_{0N} \approx 90^\circ \) for the dominant northern system oscillation in the equatorial region (equation (7a)), the implication is that the relative phase of the secondary southern \( \varphi \) component oscillation is given by \( \gamma_{0S} = 90^\circ \), again consistent with the expected value for the southern system in the equatorial region (equation (7b)). Thus, the modulations observed in these southern equatorial region data are of the same form as those observed previously in the equatorial region as discussed in section 2.1 (e.g., Andrews et al., 2012; Cowley et al., 2015; Provan et al., 2011, 2013), except for the reversal in temporal gradient of the beat phase given by equation (19a) resulting from the reversal in northern and southern periods after 2013/2014.

### 3.4. Beat Phase Modulation Analysis

We now follow the qualitative discussion of the beat phase modulations of phase and amplitude in section 3.3 with a quantitative analysis, which determines best fit values of the phase difference constants \( \Delta \gamma_i \) which relate to the phasing of the secondary modulations; the north/south amplitude ratios \( k_i \) which relate to their amplitude; and from these the individual amplitudes of the contributing oscillations. The \( \Delta \gamma_i \) and \( k_i \) parameters are determined from fitting either equation (21) to the N-format phase modulations relative to the 125-day northern phase model (Figures 8b and 8g), or, completely equivalently, equation (22) to the S-format phase modulations about the 125-day southern phase model (Figure 8e). We then use these values with equation (23) to determine the northern and southern oscillation amplitudes \( B_{0N} \) and \( B_{0S} \) from fits to the amplitude data. The best fit to the phase modulations is determined from the minimum value of the variance \( V \) between the phase data and the phase model equations (21) or (22), defined as in equation (27), while the best fit to the amplitude data is determined from the least squares difference between the data and equation (23).
3.4.1. Northern Polar Data

We begin with the northern polar data, which since it is northern system dominated, the phase data are most conveniently treated by fitting equation (21) to the N-format phases relative to the 125-day fit northern phase. Since the physical regions from which these data were obtained are somewhat different for the F ring and proximal data, lying outside the expected location of the OCB for the F ring orbits (Figure 3b) and inside the OCB but poleward of the principal PPO-related field-aligned current layer for the proximal orbits (Figure 3c), here we analyze these data separately to examine similarities and differences. Results for the F ring orbits (Revs 249–270) are shown in Figure 9, while corresponding results for the proximal orbits (Revs 271–293) are shown in Figure 10, together spanning the same interval as in Figure 8. Two-parameter fits to the r and θ phase data (Figure 8b) for the F ring orbits are shown in Figures 9a and 9f, respectively, where color-coded variance V between the data and the model is plotted versus phase difference constant Δγr,θ on the vertical axis and amplitude ratio kρ,φ on the horizontal axis. The variance color code is shown between these figures. On the horizontal axes we plot kρ,φ between zero and unity in the left half of the plot, and k′ρ,φ = 1/kρ,φ between unity and zero on the right, thus appropriately covering the full range of kρ,φ between zero and infinity. The white crosses show the position of the minimum V in each plot, at Δγr and kρ,φ values given above each figure. To gain a view of how well defined the minima in V are, in Figures 9b and 9g, we plot V versus Δγr,θ at the value of kρ,φ at the minimum, while in Figures 9c and 9h, we show V versus kρ,φ at the value of Δγr,θ at the minimum. The position of the minimum is marked in each plot by the red vertical dotted line. Using the fit parameters so determined in equation (23), the least squares fits to the amplitude data (Figure 8c) are shown for the r and θ components in Figures 9d and 9i, respectively, where we plot the amplitude versus beat phase ΔΦ together with the best fit model. Figures 9e and 9j then show the r and θ component amplitudes plotted versus the model values, together with the cross-correlation coefficient (XCC). The dotted line in these figures is simply a line of unit slope representing perfect correlation. Figures 10a–10j show corresponding results for the proximal orbit northern polar data in the same format.

The results in Figures 9a–9c and 10a–10c show that the northern polar r component is relatively weakly modulated in both intervals, with kρ = 0.26 (i.e., kr = 3.8) for the F ring orbits and kρ = 0.25 (i.e., kr = 4.0) for the proximal orbits, with the minima in V being quite weakly defined in both cases. Phase parameter Δγθ = γN − γS has a best fit value of 272° for the F ring data, indicating in this case that the southern r contribution is nearly in quadrature with the corresponding southern equatorial and polar oscillations (since γN = 0° for the dominant northern oscillations), and a value of 208° for the proximal data, with the southern r contribution nearer to antiphasic with the southern equatorial and polar oscillations as indicated overall in section 3.3. The results in Figures 9d and 9e show, however, that there is no matching amplitude modulation in the somewhat scattered F ring data (XCC between data and best fit model of ~0.13), while in Figures 10d and 10e we see a clear matching amplitude modulation for the proximal data with minima near ΔΦ ≈ 0°/360° and maxima near ΔΦ ≈ 180° (XCC value +0.36), thus supporting the phase measurement in this case. By comparison, the θ component phase results in Figures 9f–9h for the F ring data and Figures 10f–10h for the proximal data are more clear-cut, with well-defined minima in V indicating larger similar kθ values of 0.53 and 0.58 for the F ring and proximal data respectively, that is, kθ values of 1.9 and 1.7, respectively, and similar Δγθ = γN − γS values of 180° and 198°, respectively, indicating in this case that the southern θ contributions are in phase with the corresponding southern equatorial and polar oscillations (since γN = 180° for the dominant northern oscillations), as also indicated in section 3.3. Clear corresponding amplitude modulations with minima again near ΔΦ = 0°/360° and maxima near ΔΦ ≈ 180° are also seen in Figures 9i and 9j for the F ring data and Figures 10i and 10j for the proximal data, with XCC values between data and best fit models of +0.59 and +0.52, respectively. We note at this point that the values of the main fit parameters for all the data sets analyzed in this section are collected together for easy reference in Table 2. We also give the fit parameters for the northern polar data taken as a whole, which are generally intermediate between the values for the F ring and proximal orbit data shown here in Figures 9 and 10. Corresponding data plots in the same format are shown in Figure S7 in the supporting information.

Overall, the results for the northern polar data first show that the north/south amplitude ratios are clearly different for the two poloidal field components, with the r component data being more northern-dominated (factor of ~4) than the θ component data (factor of ~2). With regard to the amplitudes themselves, for the dominant northern system, the best fit amplitudes for the r and θ components are $B_{θ,N} ≈ 0.38$ nT and...
Figure 9. (a–o) Results of beat-modulation model fitting to the northern system dominated phase and amplitude data from the northern polar region, over the interval spanning F ring Revs 249–270 (see Figure 2). Figures 9a and 9f show two-parameter fits to the $r$ and $\theta$ component N-format phase data relative to the 125-day fit northern phase (as in Figure 8b), respectively, where color-coded variance $V$ between the data and the model (equation (21)) is plotted versus phase difference constant $\Delta \gamma_r, \theta$ on the vertical axis and amplitude ratio $k_r, \theta$ on the horizontal axis. The variance color code is shown between these figures. The horizontal axis employs $k_r, \theta$ directly between zero and unity on the left half of the plot and $k_0 r, \theta = 1/k_r, \theta$ between unity and zero on the right, thus covering the whole range $0 \leq k_r, \theta \leq \infty$. The white crosses show the position of the minimum $V$ in each plot, with the values of $\Delta \gamma_r, \theta$ and $k_r, \theta$ (or $k_0 r, \theta$) being shown above each plot. Figures 9b and 9g show $V$ versus $\Delta \gamma_r, \theta$ at the value of $k_r, \theta$ at the minimum, while Figures 9c and 9h show $V$ versus $k_r, \theta$ at the value of $\Delta \gamma_r, \theta$ at the minimum, where the position of the minimum is marked by the vertical red dotted line. Figures 9d and 9i show least squares fits to the amplitude data for the $r$ and $\theta$ components, respectively, using the best fit parameters $\Delta \gamma_r, \theta$ and $k_r, \theta$ in equation (23), where the amplitude data are plotted versus beat phase $\Delta \Phi$ together with the best fit model. Figures 9e and 9j show corresponding plots of the measured versus the model amplitude values together with the cross-correlation coefficients, where the dotted lines are simply lines of unit slope. Figures 9k to 9o show overall fits of the models to the data versus time over the same interval and in a similar format to Figure 8. Figure 9k shows the modulo 360° beat phase, while the vertical dashed and dotted lines show the times when the two systems are in phase and in antiphase, respectively. Figure 9l shows the $r$ component phase data (red solid circles) relative to the 125-day fit northern phase (as in Figure 8b) together with the best fit model shown by the black line, while Figure 9m shows the corresponding amplitude data and best fit model. Figure 9n shows the $\theta$ component phase data (green solid circles) relative to the 125-day fit northern phase model (as in Figure 8b) together with the best fit model shown by the black line, while Figure 9o shows the corresponding amplitude data and best fit model.
Figure 10. (a–o) Results of beat-modulation model fitting to the northern polar phase and amplitude data from proximal Revs 271–293. The format is the same as Figure 9.
Table 2
Fit Results for Beat Modulation of Phase and Amplitude Data for Revs 249–293

| Data interval/parameter | N polar data<sup>a</sup> | N polar data<sup>a</sup> | N polar data<sup>b</sup> | S polar data<sup>b</sup> | S equatorial data<sup>b</sup> |
|-------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|                         | F ring Revs<sup>f</sup>  | Prox Revs<sup>f</sup>    | F ring and Prox Revs<sup>f</sup> | F ring Revs<sup>b</sup> | Prox Revs<sup>b</sup> |
| \( \Delta_{\gamma}/\text{deg} \) | 272                      | 208                      | 235                      | 139                      | 270                      |
| \( k_r \)                | (1/0.26) = 3.8           | (1/0.25) = 4.0           | (1/0.21) = 4.8           | 0.23                     | 0.12                     |
| \( B_{\text{ION}}/\text{nT} \) | 0.32                     | 0.43                     | 0.38                     | 0.15                     | 0.12                     |
| \( B_{\text{IOS}}/\text{nT} \) | 0.08                     | 0.11                     | 0.08                     | 0.65                     | 1.01                     |
| \( k_{\theta} \)         | 180                      | 198                      | 189                      | 205                      | 185                      |
| \( \Delta_{\theta}/\text{deg} \) | (1/0.53) = 1.9          | (1/0.58) = 1.7           | (1/0.53) = 1.9           | 0.98                     | (1/0.69) = 1.4           |
| \( B_{\text{ION}}/\text{nT} \) | 0.30                     | 0.28                     | 0.30                     | 0.51                     | 0.78                     |
| \( B_{\text{IOS}}/\text{nT} \) | 0.16                     | 0.16                     | 0.16                     | 0.52                     | 0.54                     |
| \( k_{\theta} \)         | -                        | -                        | -                        | -                        | -                        |
| \( k_{p} \)              | -                        | -                        | -                        | -                        | -                        |
| \( B_{\text{ION}}/\text{nT} \) | -                        | -                        | -                        | -                        | -                        |
| \( B_{\text{IOS}}/\text{nT} \) | -                        | -                        | -                        | 1.48                     | -                        |

<sup>a</sup>See Figure 9.  <sup>b</sup>Revs 249–270 (Revs 249 and 250 plus F ring Revs).  <sup>c</sup>See Figure 10.  <sup>d</sup>Revs 271–293.  <sup>e</sup>See Figure 57 (supporting information).  <sup>f</sup>Revs 249–293.  <sup>g</sup>See Figure 11.  <sup>h</sup>See Figures S8 and 12.  <sup>i</sup>Revs 271–292.

\( B_{\text{ION}} \approx 0.30 \text{ nT} \) (for simplicity here we quote the overall F ring plus proximal fit values in Table 2), while for the secondary southern system the amplitudes are \( B_{\text{IOS}} \approx 0.08 \text{ nT} \) and \( B_{\text{IOS}} \approx 0.16 \text{ nT} \). Thus, the two poloidal component amplitudes are of comparable magnitude for the dominant northern system, while for the secondary southern system the \( \theta \) component has half that value (such that \( k_{\theta} \approx 0.5 \)) and the \( r \) component half of the southern \( \theta \) component value (such that \( k_r \approx 0.25 \)). With the exception of the rather scattered F ring \( r \) component data, the results also second show that the modulation phasing in both phase and amplitude are such that \( \Delta_{\gamma} \approx \Delta_{\theta} \approx 180^\circ \) to within a few tens of degrees (as opposed, e.g., to being within a few tens of degrees of 0°/360°). As discussed in section 3.3, this means that for the \( r \) component the superposed southern system oscillations in the northern polar region are in antiphase with the corresponding oscillations in the equatorial and southern polar regions, while for the \( \theta \) component they are in phase.

The overall model fits to the data are shown in time series format in Figures 9k–9o for the F ring data and Figures 10k–10o for the proximal data, in a similar format and over the same combined interval as in Figure 8. Figures 9k and 10k show the modulo 360° beat phase in the two intervals considered, where the vertical dashed and dotted lines show the times when the two systems are in phase and antiphase, respectively. Figures 9l and 10l show the \( r \) component phase data relative to the 125-day northern phase model (Figure 7a) together with the best fit models (black solid lines), while Figures 9m and 10m similarly show the \( r \) component amplitude data and best fit models (black solid lines). Figures 9n and 9o and 10n and 10o show corresponding results for the \( \theta \) component phase and amplitude data, respectively. Overall, the model fits to the observed modulations are seen to be reasonably good, particularly for the \( \theta \) component due to the larger modulations resulting from amplitude ratio \( k_{\theta} \) being closer to unity than for \( k_r \).

### 3.4.2. Southern Polar Data

In Figure 11 we show corresponding results for model fits to the phase and amplitude data from the southern polar region obtained on F ring Revs 249–293, in a similar format to Figure 9. Since these oscillations are southern system dominated, it is now most convenient to fit equation (22) to the S-format phase data relative to the 125-day southern phase fit, as shown in Figure 8e. Phase model fit results are shown in Figures 11a and 11f for the \( r \) and \( \theta \) component phases, respectively, where the minimum variances shown by the white crosses occur at \( \Delta_{\gamma} \approx 139^\circ \) and \( k_r \approx 0.23 \) for the \( r \) component, and \( \Delta_{\theta} \approx 205^\circ \) and \( k_{\theta} \approx 0.98 \) for the \( \theta \) component (Table 2). Both \( \Delta_{\gamma}, \theta \) values are again near 180° within a few tens of degrees as discussed in section 3.3 (as opposed, e.g., to 0°/360°), again showing that the \( \theta \) component oscillation of the northern system has the same phase as that in the equatorial and northern polar region, while the \( r \) component of the northern system is reversed in sense. Amplitude ratio \( k_{\theta} \) is near to unity (near-equal southern and northern system amplitudes), while \( k_r \) remains small (southern dominated), associated with much larger beat phase modulations in the \( \theta \) component than in \( r \). Better-defined variance minima then occur for the \( r \) component in Figures 11g and 11h than for the \( r \) component in Figures 11b and 11c. The best fit models to the...
Figure 11. (a–o) Results of beat-modulation model fitting to the southern polar phase and amplitude data from F ring Revs 249–270. The format is the same as Figure 9 except that for these southern-dominated data the phases in Figures 11l and 11n are shown relative to the 125-day fit southern phase model (as in Figure 8e) and are fit using equation (22).
amplitude data (Figure 8f) shown in Figures 11d–11j correspondingly show minima when the systems are in phase and maxima when in antiphase, with XCC values between data and model of +0.75 for the \( r \) component and +0.62 for \( \theta \). The best fit amplitudes are \( B_{r0S} \approx 0.65 \) nT and \( B_{r0N} \approx 0.52 \) nT for the southern system, and \( B_{\theta0N} \approx 0.15 \) nT and \( B_{\theta0S} \approx 0.51 \) nT for the northern system (Table 2). Similar to the northern polar region, therefore, in the southern polar region, the amplitudes of the southern system \( r \) and \( \theta \) component oscillations are again near equal, while the amplitude of the northern system \( \theta \) component is significantly larger than that of the \( r \) component. In this case, however, the amplitude of the northern system \( \theta \) component is comparable with that of the southern system components, while the amplitude of the northern system \( r \) component is again about one quarter of those values. The best fit models of the phase and amplitude shown in time series format in Figures 11I–11o again show reasonable fits to the data, particularly for the \( \theta \) component due to the larger modulations rastering between \( \pm 90^\circ \) associated with an amplitude ratio near to unity.

### 3.4.3. Southern Equatorial Region Data

Turning to the phase and amplitude data obtained in the southern equatorial region on the proximal orbits, we begin by noting that the southern dominated \( r \) component phase data plotted in Figure 8e are found to show no significant organization with respect to the beat phase, with the analysis implying \( k_r \approx 0.1 \), at the limit of resolution. Related results are thus not shown here, though the fit parameters are included in Table 2, and a corresponding plot in the same format as Figures 9–11 is shown in Figure S8 in the supporting information. We only note that the well-determined amplitude of the southern system \( r \) component oscillations shown in Figure 8e is \( B_{r0S} \approx 1.01 \) nT, with a northern system contribution of \( B_{r0N} \approx 0.12 \) nT having the nature of an upper limit.

In Figure 12, however, we show results for the \( \theta \) and \( \phi \) component southern equatorial region proximal orbit data in the same format as Figure 9. Since these oscillations are dominated by the northern system (Figures 7 and 8), despite their southern location, the phases are most conveniently analyzed by fitting equation (21) to the N-format phases relative to the 125-day fit northern phase (Figure 8g). The best fit to the \( \theta \) component phase data shown by the white cross in Figure 12f is given by \( \Delta \gamma_\theta = 185^\circ \) again close to \( -180^\circ \) as indicated in section 3.3, the oscillations again being strongly beat-modulated by a southern system oscillation with a modestly smaller amplitude such that \( k_\phi \approx 0.69 \) (i.e., \( k_r \approx 1.4 \)). The best fit to the \( \phi \) component phase data shown by the white cross in Figure 12a, however, is given by \( \Delta \gamma_\phi \approx 310^\circ \), now closer to \( -0^\circ \)/360° as indicated in section 3.3, and is near-equally modulated by a southern system oscillation such that \( k_\phi \approx 0.72 \) (i.e., again \( k_r \approx 1.4 \)), thus much more strongly so than for the \( r \) component data in the two polar regions. The minima in the variance values are well defined for both components, as can be seen in Figures 12b, 12c, 12g, and 12h. As indicated in section 3.3, these phase relationships and near-equal \( k \) values are consistent with previous observations in the equatorial region, as depicted in Figure 1. The best fits to the amplitude data (Figure 8h) shown in Figures 12g, 12e, 12j, and 12j give values for the larger northern system oscillations of \( B_{r0N} \approx 0.78 \) nT and \( B_{r0S} \approx 1.48 \) nT, while for the southern system oscillations \( B_{r0S} \approx 0.54 \) nT and \( B_{r0N} \approx 0.65 \) nT (Table 2). Thus, the \( \phi \) component amplitude is close to twice the \( \theta \) component amplitude for both systems. The relative phasing is such that for the \( \theta \) component, amplitude minima occur when the two systems are in phase and maxima when in antiphase (with data and model XCC value +0.63), as in the other cases shown here, while for the \( \phi \) component maxima occur when the two systems are approximately in phase and minima when in antiphase (with XCC value +0.49), in line with expectation for equatorial data on the basis of Figure 1. The overall fits between the modulated phase and amplitude data and the model values in the time series plots in Figures 12I–12o are again seen to be quite good.

### 4. Theoretical Discussion

#### 4.1. Empirical Perturbation Field and Current Picture

The results obtained from the F ring and proximal orbit data in section 3 reveal the presence of a new PPO-related phenomenon, in which oscillations of a given PPO system, northern or southern, are detected in the polar region in the opposite hemisphere, poleward of the principal PPO field-aligned current layer but in the vicinity of the OCB, likely on either side as seen in Figures 3b and 3c. In retrospect it is perhaps not surprising that such dual modulation is found in the region just outside of the principal PPO current layer given that these currents lie on outer closed field lines where perturbations may be transmitted directly from one hemisphere to the other. The dual oscillations in these regions, observed in the midnight sector at down-tail
Figure 12. (a–o) Results of beat-modulation model fitting to the $\theta$ and $\phi$ component phase and amplitude data from the southern hemisphere equatorial region spanning proximal Revs 271–292. The format is the same as Figure 9, noting in particular that since these data are northern-dominated, the phases in Figures 12l and 12n are shown relative to the 125-day fit northern phase model (as in Figure 8g) and are fit using equation (21).
distances of ∼10–20 R₉, have been shown to produce clear beat modulations in the polar oscillation phases and amplitudes not observed in previous analyses of polar data from the 2008 sequence of highly inclined orbits (Andrews et al., 2012; Hunt et al., 2015), that are different in form from those observed in the equatorial region (e.g., Andrews et al., 2012; Cowley et al., 2015; Provan et al., 2011, 2012). The polarization of the oscillations from the opposite hemisphere in both polar regions is such that the \( \theta \) component oscillates approximately in phase with the \( \psi \) oscillations of that system in the equatorial and generating polar region, while the \( r \) component oscillates approximately in antiphase (with the possible quadrature exception of the northern polar F ring data). The beat modulations of the \( \theta \) component are thus the same as those observed in the dual-modulated equatorial region, with amplitude minima occurring when the two PPO systems are in phase (in the sense of the phase systems in Figure 1), and maxima when they are in antiphase, resulting from the opposite senses of the \( \theta \) component relative to \( r \) in the two systems. Due to the reversal in sign of the polar \( r \) component oscillations relative to those in the generating polar region and equatorial region, however, the polar beat modulations of the \( r \) component are reversed in sense compared with the equatorial region, now also showing amplitude minima when the two systems are in phase and maxima when in antiphase as for the \( \theta \) component, as seen in Figure 8. Corresponding phase modulations are also observed.

A second novel result has been the finding of clearly different north/south amplitude ratios in the oscillations of different field components (Table 2). In both polar regions the \( \theta \) component is much more strongly modulated than the \( r \) component, indicating a north/south ratio nearer to unity in \( \theta \) than in \( r \). In addition, analysis of the southern equatorial region proximal orbit data observed ∼10–15 R₉ down-tail (Figure 3c) demonstrates the presence of oscillations in the \( r \) component that are dominated by the southern PPO system (no detectable northern component within an ∼10% limit by amplitude), while the oscillations in the \( \theta \) and \( \phi \) components have larger contributions from the northern system than from the southern, with approximately the same north/south ratio \( k \approx 1.4 \) for each component.

Previous analyses of near-equatorial data (e.g., Provan et al., 2013, 2016) have reasonably assumed that the north/south amplitude ratio is approximately the same for all three field components following the initial results of Provan et al. (2011), with consistent results being obtained on that basis.

This follows from the expectation that the geometries of the two perturbation field systems should be similar relative to their generating hemispheres, with approximate mirror symmetry about the equatorial plane. Clearly, however, the north/south amplitude ratios differ significantly between the three field components for the data obtained from the regions newly explored on the end of mission orbits.

In Figures 13a and 13b we provide interpretive sketches of the overall field perturbations associated with the northern and southern PPO systems based on these results, which augment the related introductory sketches in Figures 1c and 1f. These again show the perturbation fields in a planetary meridian plane, with PPO phase \( \Psi_{N,S} = 0° \) on the right and \( \Psi_{N,S} = 180° \) on the left. To a first approximation, the fields away from this meridian may be visualized simply by displacing the perturbation pattern directly into and out of the plane of the diagram shown, thereby also introducing azimuthal field components into the picture. The principal feature of the northern system in Figure 13a is again the anticlockwise field circulation in the equatorial and north polar regions, associated by Ampère’s law with a current directed out of the plane of the diagram (northern green circled dot), while similarly, the principal feature of the southern system in Figure 13b is the clockwise field circulation in the equatorial and south polar regions, associated with a current directed into the plane of the diagram (southern green circled cross). Across these currents, the oscillations in the \( \phi \)
component reverse sign from quasi-uniform equatorial to quasi-dipolar polar as marked in the diagrams, as discussed in relation to Figure 1 in section 1. Our results then show that similar field circulations occur in the opposite polar hemispheres for each system associated with the same senses of current flow, each acting to weaken the quasi-uniform perturbation fields near the equator, and leading to a reversal in the sign of the \( r \) component south of the equator for the northern system and north of the equator for the southern system as also marked, but not for the \( \theta \) component. It is evident from comparison of these diagrams that when the two systems are in phase, a situation corresponding to the orientations of the two sketches in Figure 13, the fields of the two systems oppose each other in both polar regions, such that amplitude minima occur in both \( r \) and \( \theta \) field components under this condition as seen in Figures 8–11. When the two systems are in antiphase, however, with one pattern reversed relative to the other compared with those shown, the fields of the two systems augment each other in both polar regions, such that amplitude maxima then occur. We thus note from this geometry that there will consequently exist points where the \( r \) components of these systems are near zero, south of the equator for the northern system and north of the equator for the southern system as indicated, but where the \( \theta \) (and \( \varphi \)) perturbations remain similar in magnitude to those in the equatorial region. This situation corresponds to observations on the proximal orbit southern passes as mentioned above, where the \( r \) component oscillations are strongly dominated by the southern system with no detectable northern contribution, while the \( \theta \) and \( \varphi \) component oscillations have stronger northern contributions than southern.

We note that in Figure 13 we have also included a small inner region where the quasi-uniform equatorial perturbation field closes directly over the “opposite” polar region, that is, over the southern pole for the northern system and over the northern pole for the southern system. Such a perturbation field is then associated with a reversed current flow, into the diagram for the northern system in Figure 13a and out of the diagram for the southern system in Figure 13b. While we have no direct evidence for the existence of such fields in the Cassini data examined here, these are included in response to some theoretical considerations that follow.

### 4.2. Possible Theoretical Scenario

We now briefly discuss a theoretical scenario that may account for these observations, related to the transmission of stress along field lines from a PPO-generating hemisphere to the opposite responding hemisphere. For definiteness we consider the perturbations generated in the northern hemisphere and their interaction with the southern, but corresponding considerations apply in the reverse direction. In Figure 14a we show the flows (red arrowed lines), currents (green arrowed lines and symbols), and perturbation fields (blue arrowed lines) associated with the northern system in the northern ionosphere in a view looking down from the north with the \( \Psi_N = 90° \) meridian at the bottom of the diagram, this in essence reproducing Figure 1a. The outer black dashed circle passes through the center of the driving flow vortices where the main PPO-related field-aligned currents flow. The inner black dashed circle indicates the OCB. Figure 14b similarly shows the flows, perturbation fields, and currents driven in the southern polar region in a view looking down from the north “through” the planet. The format follows Figure 14a except that the arrowed red dashed lines indicate the “return” flow of the neutral atmosphere across the high-latitude polar region. Note that while the circled dots and crosses again represent current flow out of and into the plane of the diagram, respectively, these directions now correspond to current flow into and out of the southern ionosphere, respectively, due to the view “through” the planet from the north. Figure 14c shows the corresponding current system in and near the \( \Psi_N = 90° - 270° \) meridian, where the arrowed solid green lines indicate the principal field-aligned and ionospheric currents, while the dashed lines indicate the magnetospheric cross-field currents that transmit the stress from the northern polar driving neutral atmospheric flow to the magnetospheric plasma.

![Figure 14](https://example.com/figure14.png)
colatitude poleward of the center of these currents (e.g., Jinks et al., 2014). In line with the model calculations of Jia et al. (2012) and Jia and Kivelson (2012) the system is taken to be driven by a rotating twin-vortex flow in the upper neutral atmosphere, which drives a similar but reduced flow in the ionospheric plasma due to ion-neutral collisions. Consideration of the consequent ionospheric electric fields and currents shows that the field perturbations are directed opposite to the flow, towards $\Psi_N = 0^\circ$ equatorward of the black dashed line corresponding to the quasi-uniform equatorial perturbation field, reversing to point towards $\Psi_N = 180^\circ$ poleward of the black dashed line corresponding to the quasi-dipolar polar perturbation field. The principal field-aligned currents then flow out of the ionosphere near $\Psi_N = 90^\circ$ and into the ionosphere near $\Psi_N = 270^\circ$, these currents corresponding to the schematic current pointing out of the plane of the diagram in the northern region in Figure 13a (and in Figure 1c). In addition, currents of reversed sense flow at lower latitudes, mapping to the inner part of the magnetosphere.

The field-aligned currents shown in Figure 14a serve to transmit the rotating pattern of twin-vortex flows along field lines into the magnetosphere and opposite ionosphere, via the $\mathbf{j} \times \mathbf{B}$ force of the related cross-field closure currents (see Figure 1b). The effect in the southern hemisphere is sketched in Figure 14b in a similar format to Figure 14a, where we view the southern polar region through the planet from the north (as in Figure 1d). The red solid lines show the principal southern hemisphere ionospheric flows, the blue lines the perturbation fields, and the green lines and symbols the associated currents. Note that in this case the circled dots represent downward field-aligned currents with respect to the ionosphere, while the circled crosses represent upward field-aligned currents (directed into the plane of the diagram). In the first instance the effect is to transmit the same flow pattern, in attenuated form, into the southern ionosphere, though perhaps with some modest phase delay due to the finite travel time of Alfvénic disturbances along the planetary field lines (not represented in Figure 14b). The direct resultant effects on the flow and perturbation fields in the southern hemisphere are shown by the outer red and blue crescent-shaped lines on closed field lines on either side of the pole, which have the same circulation senses as those in the northern hemisphere. However, the related ionospheric currents are oppositely directed, associated with the transfer of momentum from the neutral atmosphere/ionosphere into the magnetosphere in the northern hemisphere, but from the magnetosphere into the ionosphere/neutral atmosphere in the southern hemisphere. The perturbation fields in the polar parts of these crescent-shaped systems correspond to closure of the quasi-uniform lower-latitude field over the southern polar hemisphere, in the manner shown in the inner southern hemisphere for the northern system in Figure 13a. The associated field-aligned current is directed from the $\Psi_N = 90^\circ$ meridian towards the $\Psi_N = 270^\circ$ meridian, hence into the plane of the diagram in the southern polar region in Figure 13a (inner southern circled green cross).

The rationale for showing the outer flow and field vortices as crescent-shaped, compared, for example, with the vortices in Figure 14a, is that the drag force transmitted along field lines to the opposite ionosphere will be communicated directly only on closed field lines, rather than on open or highly distended closed field lines stretching down the magnetospheric tail, though some related flow may also be transmitted to the open field region by related compressive forces. The southern ionospheric plasma flow will also excite a corresponding circulating flow in the southern neutral atmosphere due to ion-neutral collisions, whose initial effect is to reduce the ionospheric drag experienced by the magnetosphere, hence also the ionospheric current. However, while the induced ionospheric flow may thus be expected to be confined mainly to closed (or nondistended) field lines, the neutral atmospheric flow need not generally be so confined, with the poleward “return” flow spreading poleward onto open field lines, as indicated by the red dashed lines in Figure 14b. There the neutral atmosphere will drive a weaker ionospheric flow in a similar manner to that driven in the northern hemisphere in Figure 14a. Thus, on moving poleward in the southern hemisphere, for example, on the $\Psi_N = 90^\circ$ – $270^\circ$ meridian, there will be a transition from the region where the neutral atmospheric flow is driven by the plasma on outer closed field lines (red solid line) to a region where the neutral atmospheric flow drives a polar ionospheric flow (red dashed line), associated with a reversal in the sense of the perturbation field and current as shown. The sense of this polar perturbation field is the same as that in the outer polar southern hemisphere in Figure 13a and as observed in section 3, with an associated current flow now directed from the $\Psi_N = 270^\circ$ meridian to the $\Psi_N = 90^\circ$ meridian in Figure 14b, into the ionosphere at $\Psi_N = 270^\circ$ and out at $\Psi_N = 90^\circ$. The corresponding current flows out of the plane of the diagram in the southern hemisphere in Figure 13a, thus having the same sense as the principal PPO-related current in the generating northern hemisphere, but now flowing at higher latitudes, in the vicinity of the OCB. We note
that the polar neutral atmospheric flows that drive this system may not encroach all the way across the southern polar region to the pole itself, but may only form a more limited layer on outer open field lines. In this case the polar current system in Figure 14b would be truncated within the central polar region where dual PPO field modulations would be absent, while the currents in the outer part of the open field region, directed as shown, would be fed by an additional pair of field-aligned currents on either side of the pole, directed downward at Ψ_N = 90° and upward at Ψ_N = 270° (not shown in Figure 14b).

Figure 14c provides a summary of the overall currents envisaged associated with the northern PPO system in the Ψ_N = 90° − 270° meridian plane corresponding to the simpler polar case shown in Figure 14b, thus augmenting the initial diagram in Figure 1b. Here the dashed lines schematically indicate the magnetospheric cross-field closure currents whose \( j \times B \) force acts to transmit the twin-vortex flow from the northern atmosphere/ionosphere to both the magnetosphere and southern polar ionosphere/atmosphere. The blue symbols show the corresponding azimuthal perturbation fields in this meridian, consistent with Figure 14b.

5. Summary and Discussion

We have analyzed the PPO-related oscillations in Saturn’s magnetospheric magnetic field observed over the ~2-year interval from September 2015 to the end of the Cassini mission in September 2017, thus spanning northern summer solstice in May 2017. A principal purpose of the study is to determine the behavior of the phase and period of the northern and southern PPO systems over this interval to further understanding of the seasonal evolution of the PPO phenomenon over this trans-solstice interval, as well as to provide necessary input to studies of other phenomena observed on the unique F ring and proximal orbits during 2016–2017 that are influenced by the PPOs. This paper thus completes a set of studies that have reported related results covering the whole of the Cassini orbital mission, beginning under postsolstice southern summer conditions in 2004, extending through vernal equinox, and now to northern summer solstice (Andrews et al., 2008, 2012; Provan et al., 2013, 2016). In addition, the F ring and proximal orbit data have revealed some unique properties of the PPO oscillations newly analyzed here, and briefly discussed theoretically.

The nature of the PPO-related data obtained from magnetospheric field measurements depends on the region sampled and hence the spacecraft orbit, varying in past studies between the dual modulated oscillations observed on near-equatorial orbits within the quasi-dipolar magnetosphere where the properties of the two PPO systems can be determined from the resulting beat modulations, and the oscillations observed in the polar magnetosphere on highly inclined orbits which are dominated by the oscillations of the corresponding hemispheric system. During the interval studied here, spanning spacecraft Revs 221–293, the orbits were initially near-equatorial to the beginning of 2016, then became increasingly inclined with apoapses fixed near ~20 Rs after mid-2016 and periapses decreasing in a number of steps to the end of the Cassini mission in Saturn’s upper atmosphere on 15 September 2017. Careful selection of quiet data intervals with field variations dominated by PPO oscillations has then provided equatorial region phase and amplitude data on a Rev-by-Rev basis from the start of the study interval in September 2015 to November 2016 (Revs 221–248) and again on the southern passes of the proximal orbits from April to September 2017 (Revs 271–292). The inclined orbits also provide northern polar data from the region poleward of the principal PPO-related field-aligned currents from February 2016 to the end of mission in September 2017 (Revs 232–293), as well as southern polar data from May 2016 to April 2017 (Revs 235–270). In principle, these data should allow determination of the northern and southern system phases and hence periods over the whole study interval. However, it was found that the equatorial data over the interval to at least March 2016 were strongly dominated by northern system oscillations with no southern-related beat modulations being discernable, indicating that the northern amplitude in the equatorial region was then larger than the southern by a factor of at least ~5 (Provan et al., 2013). While northern PPO system phase and amplitude data were thus obtained over the whole study interval, southern PPO system data were not determined until southern polar data became available in May 2016, continuing to the end of F ring orbit data in April 2017. Southern system data then continue to be available to the end of mission from analysis of the southern equatorial region proximal orbit data (mapping interior to the center of the principal PPO-related field-aligned currents), which were found to contain significant southern oscillations in varying proportions relative to the northern in the three field components. As discussed briefly below, these data suggest a revival in the southern system amplitude relative to the northern during the later interval, from at least November 2016.
Models of the oscillation phase and period of the two PPO systems have been determined from the phase measurement data, as available, using running linear fits to 125-day data segments determined every 12.5 days. Phase and period uncertainties in individual fit determinations are ~30° and ~0.0025 hr (~9 s), respectively. The results show that the northern and southern system PPO periods were approximately unvarying over the study interval apart from small variations no more than ±0.01 hr (~30 s) occurring over intervals of ~100 days, with the northern system period increasing slightly from ~10.78 to ~10.79 hr in mid-2016, and the southern system period remaining near-constant at ~10.68 hr after that time. The northern value is consistent with the near-constant period of 10.79 ± 0.01 hr determined by Carbary et al. (2017) from energetic electron modulations during the F ring and proximal orbit intervals, particularly in northern hemisphere data, though these authors did not detect the southern system period determined here. Single linear fits to the magnetic data over essentially the same ~310-day interval give periods of 10.792 hr for the northern system and 10.679 hr for the southern system, with uncertainties of ~0.0005 hr (~2 s). These periods had thus remained almost unchanged from those determined previously in magnetic field and SKR data by Provan et al. (2016) following the reversal in periods that took place after mid-2014. We further note that such northern summer solstice values compare with ~10.59 hr for the northern PPO system and ~10.78 hr for the southern system in the earliest postsolestic southern summer Cassini data from late 2004 and early 2005 (see, e.g., Figure 11 of Provan et al., 2016, and related references). Thus, while the northern system period in our northern summer data had increased to values similar to the southern system period in the earlier southern summer data, the southern system period remained significantly elevated relative to the earlier northern system period. The beat period associated with our northern summer solstice interval is ~42.5 days, compared with ~24 days for the postsolestic southern summer interval.

Given these results, the running 125-day linear fits to the phase data used to construct the phase and period models correspond to intervals of ~3 beat periods, such that the fits average across any beat modulations present. As indicated above, beat modulations are expected in the equatorial region data interior to the principal PPO-related field-aligned current layer where the oscillations of both northern and southern systems are present (Figure 1), but previous detailed analyses of oscillations observed in the poleward region in either hemisphere, specifically in data obtained from the late southern summer seasons of highly inclined orbits in 2006/2007 and 2008/2009, did not find evidence of dual modulation effects above a detection threshold of an ~10% secondary oscillation by amplitude, even though the oscillations were of near-equal equatorial amplitude during the second of these seasons (Andrews et al., 2012; Hunt et al., 2015). An important new result in the present work is then the finding of clearly observed beat modulations in the polar data in both the northern and southern hemispheres on the F ring orbits, and in the northern hemisphere on the proximal orbits, from November 2016 to September 2017 (Revs 249–293). These Revs had a sufficiently short orbit period of ~7 days that the beat effects were clearly delineated in the Rev-by-Rev phase and amplitude data, with ~6 Revs per beat. We note that unlike, for example, the 2008/2009 polar data, which were obtained more centrally on open field lines in the region of the dawn-dusk meridian and on the dayside, the F ring and proximal orbit data were obtained close to (likely on either side of) the OCB in both hemispheres, near the midnight meridian at downtail distances between ~10 and ~20 Rs.

These polar beats were found to be of different form from those observed previously in the equatorial region, the latter exhibiting maxima in the r and φ components when the two PPO systems are in phase (in the sense of the phase systems defined in Figure 1) and minima when in antiphase, while the θ component oppositely exhibits minima when the two systems are in phase and maxima when in antiphase due to its opposite sense relative to r in the two systems (e.g., Andrews et al., 2012; Cowley et al., 2015; Provan et al., 2011, 2012). For the polar beats newly observed here the θ component in both hemispheres behaves in the same way as in the equatorial region, with minima when the two systems are in phase and maxima when in antiphase, showing that the polar oscillations in this component from the “opposite” hemisphere have the same phase as those in the equatorial and generating polar regions. However, unlike the equatorial beats, the r component beats in both polar hemispheres occur in concert with the θ component beats, with minima when the two systems are in phase and maxima when in antiphase, showing that the polar oscillations in this component from the “opposite” hemisphere have the opposite sense to the r component oscillations in the equatorial and generating polar regions. Corresponding beat modulations of the oscillation phases are also observed. This shows that the oscillations of the r component, but not of the θ component, must change sign at some point south of the equator for the northern system, and north of the equator for the southern system. A theoretical
scenario has been outlined that may account for such oscillations, involving a differential response of the ionized and neutral components of the upper atmosphere in the “responding” hemisphere to driving principally on closed field lines from the “generating” hemisphere, in which the responding vortical flows of the ionized component are mainly confined to closed field lines, while the vortical flows of the neutral component extend further poleward to drive a flow extending to polar open field lines.

Analysis of the $\theta$ component modulations shows that in the southern polar region, the northern system amplitude was near-equal to that of the southern system (north/south amplitude ratio $k_{\theta} \approx 0.98$), while in the northern polar region, the southern system amplitude was $\approx 50\%$ of that of the northern system ($k_{\theta} \approx (1/0.53)$), overall suggesting that the northern system was somewhat stronger than the southern during this interval, as briefly addressed further below. The $r$ components in both polar hemispheres were more weakly modulated; however, such that the amplitude of the oscillations from the “opposite” hemisphere was $\approx 25\%$ of that of the given hemisphere in both cases ($k_r \approx (1/0.25)$ for the proximal orbits in the northern hemisphere and $k_r \approx 0.23$ for the F ring orbits in the southern). Unlike the near-equatorial region, therefore, where the north/south amplitude ratios appear similar in each field component (Provan et al., 2011), indicative of similar field geometries for the two systems, this is clearly not the case for the polar oscillations, where the oscillations from the “opposite” hemisphere have a weaker $r$ component relative to $\theta$ than the oscillations from the given hemisphere. Specifically, the data in Table 2 show that the northern system polar oscillations in the north and the southern system polar oscillations in the south have near-equal $r$ and $\theta$ component amplitudes, while for the oscillations from the “opposite” hemisphere, the $r$ component amplitude is less than that of the $\theta$ component by factors of $\approx 2–3$, showing that the perturbation field lines are more rounded in shape in that case (Figure 13).

Although as indicated above, no beats were observed in the equatorial data obtained from the beginning of our study interval in September 2015 to at least March 2016, indicative of northern system dominance over southern by a factor of at least $\approx 5$ in amplitude; clear beats were again present in the $\theta$ and $\varphi$ component data when equatorial region data once more became available on the southern hemisphere passes of the proximal orbits from April 2017 to the end of mission in September 2017. Although these data were obtained well south of the planetary equator near the midnight meridian at radial distances $\approx 10–15$ $R_p$, they are equatorial in nature by virtue of being observed in the region (just) interior to the center of the principal PPO-related field-aligned currents. Accordingly, the beats were found to be of the same form as those observed previously in equatorial data, with maxima in the $\varphi$ component when the two systems are in phase and minima when in antiphase, and vice versa for the $\theta$ component. The northern system oscillations were the strongest in both cases, with southern system contributions of $\approx 70\%$ of those of the northern in both cases (i.e., $k_\varphi \approx k_\theta \approx (1/0.7) \approx 1.4$). However, the $r$ component oscillations were dominated by the southern PPO system, with no northern contribution being discernable above a level of $\approx 10\%$ by amplitude. We suggest that this finding relates to the reversal in sign of the $r$ component oscillations between the two polar regions discussed above, in which the $r$ component of the northern system in particular reverses sign at some point south of the equator, thus passing through a region of small amplitudes across zero, while the $\theta$ and $\varphi$ components remain relatively less affected. These data thus again newly demonstrate unequal north/south amplitude ratios in the three field components, albeit in data from a region not explored in detail by previous data sets. Since it seems likely that the $\theta$ components of the two systems will remain at comparable values to those observed closer to the equator, being components essentially normal to the equatorial plane, these observations suggest that the equatorial north/south amplitude ratio during the interval of the proximal orbits (April to September 2017) was $k \approx 1.4$, thus being considerably closer to unity than earlier in our study interval between September 2015 and March 2016.

We finally point out that the polar region north/south amplitude ratios for the relatively unvarying $\theta$ component, mentioned above as indicative of a stronger northern system, are also consistent with such an overall north/south amplitude ratio. Suppose that the overall north/south amplitude ratio is $k$, as reflected in the relative amplitudes of the field components close to the equatorial plane. Suppose also that in either polar region, the amplitude of the field generated in that hemisphere is a factor $f_1$ times the equatorial field, while the amplitude of the field generated in the opposite hemisphere is $f_2$ (which we may suppose generally to be smaller than $f_1$). Then the north/south amplitude ratio in the northern polar region will be $k_{N} \approx (f_1/f_2)k$, while the north/south amplitude ratio in the southern polar region will be $k_{S} \approx (f_2/f_1)k$. Eliminating $k$ first gives
(f_1/f_2) = \sqrt{k_{R1}/k_{S1}} while eliminating the f factors similarly gives k = \sqrt{k_{R1}k_{S1}} the geometric mean of the northern and southern k factors. From Table 2 we have k_{R1} \approx (1/0.53), while k_{S1} \approx 0.98. This then gives the interhemispheric amplitude ratio (f_1/f_2) \approx 1.4, larger than unity as anticipated, such that in the near-OCB regions explored here the polar field in the "opposite" hemisphere is \sim 70\% of the polar field in the generating hemisphere. It also gives the near-equatorial amplitude ratio as k \approx 1.4, in good agreement with the equatorial k_\theta and k_\phi values derived from the southern hemisphere equatorial data obtained on the proximal orbits as indicated above. Thus, these data provide evidence of a revival in the southern system amplitude relative to the northern from being less than \sim 20\% of the northern system amplitude (i.e., k > 5) early in the study interval, September 2015–March 2016, as found previously by Provan et al. (2016), to becoming \sim 70\% of the northern system amplitude during the interval of the F ring and proximal orbits from at least November 2016 to September 2017. Since northern system equatorial amplitudes in the earlier interval are comparable with those determined from the southern equatorial region proximal orbit \theta and \phi component data here (see Table 2 and, e.g., Figure 6 of Provan et al., 2016), the indication is of an increase in southern system amplitude by a factor of at least \sim 3 between these intervals.

Acknowledgments
Work at the University of Leicester was supported by STFC Consolidated Grant ST/N000749/1, while work at Imperial College London was supported by STFC Consolidated Grant Consolidated Grant ST/N000692/1. M. K. D. was supported by a Royal Society ResearchProfessorship. T. J. B. was supported by STFC Quota Studentship Quota Studentship ST/N000692/1. We thank S. Kellock and the Cassini magnetometer team at Imperial College for access to processed magnetic field data. Calibrated magnetic field data from the Cassini mission are available from the NASA Planetary Data System at the Jet Propulsion Laboratory (https://pds.jpl.nasa.gov/).

References
Andrews, D. J., Bunce, E. J., Cowley, S. W. H., Dougherty, M. K., Provan, G., & Southwood, D. J. (2008). Planetary period oscillations in Saturn’s magnetosphere: Phase relation of equatorial magnetic field oscillations and SKR modulation. Journal of Geophysical Research, 113, A09205. https://doi.org/10.1029/2007JA012275
Andrews, D. J., Cecconi, B., Cowley, S. W. H., Dougherty, M. K., Lamy, L., Provan, G., & Zarka, P. (2011). Planetary period oscillations in Saturn’s magnetosphere: Evidence in magnetic field phase data for rotational modulation of Saturn kilometric radiation emissions. Journal of Geophysical Research, 116, A09206. https://doi.org/10.1029/2011JA016636
Andrews, D. J., Coates, A. J., Cowley, S. W. H., Dougherty, M. K., Lamy, L., Provan, G., & Zarka, P. (2010). Magnetospheric period oscillations at Saturn: Comparison of equatorial and high-latitude magnetic field periods with north and south SKR periods. Journal of Geophysical Research, 115, A12252. https://doi.org/10.1029/2010JA015666
Andrews, D. J., Cowley, S. W. H., Dougherty, M. K., Lamy, L., Provan, G., & Southwood, D. J. (2012). Planetary period oscillations in Saturn’s magnetosphere: Evolution of magnetic oscillation properties from southern summer to post-equinox. Journal of Geophysical Research, 117, A04224. https://doi.org/10.1029/2011JA017444
Andrews, D. J., Cowley, S. W. H., Dougherty, M. K., & Provan, G. (2010). Magnetic field oscillations near the planetary period in Saturn’s equatorial magnetosphere: Variation of amplitude and phase with radial distance and local time. Journal of Geophysical Research, 115, A04212. https://doi.org/10.1029/2010JA014729
Arridge, C. S., André, N., Khurana, K. K., Russell, C. T., Cowley, S. W. H., Provan, G., et al. (2011). Periodic motion of Saturn’s nightside plasma sheet. Journal of Geophysical Research, 116, A11205. https://doi.org/10.1029/2011JA016827
Badman, S. V., Andrews, D. J., Cowley, S. W. H., Lamy, L., Provan, G., Tao, C., et al. (2012). Rotational modulation and local time dependence of Saturn’s infrared H_3^+ auroral intensity. Journal of Geophysical Research, 117, A09228. https://doi.org/10.1029/2011JA017990
Bradley, T. J., Cowley, S. W. H., Provan, G., Hunt, G. J., Bunce, E. J., Wharton, S. J., et al. (2018). Field-aligned currents in Saturn’s nightside magnetosphere: Subcorotation and planetary period oscillation components during northern spring. Journal of Geophysical Research: Space Physics, 123. https://doi.org/10.1029/2017JA024885
Carbary, J. F., Cowley, S. W. H., Alexeev, I. I., Arridge, C. S., Dougherty, M. K., Nichols, J. D., & Russell, C. T. (2007). Cassini observations of the variation of Saturn’s ring current parameters with system size. Journal of Geophysical Research, 112, A10202. https://doi.org/10.1029/2007JA012275
Burton, M. E., Dougherty, M. K., & Russell, C. T. (2010). Saturn’s internal planetary magnetic field. Geophysical Research Letters, 37, L24105. https://doi.org/10.1029/2010GL045148
Carbary, J. F. (2017). Update on Saturn’s energetic electron periodicities. Journal of Geophysical Research: Space Physics, 122(1), 156–165. https://doi.org/10.1002/2016JA023405
Carbary, J. F., Mitchell, D. G., Kollman, P., Krupp, N., & Roussos, E. (2017). Energetic electron periodicities during the Cassini grand finale. Journal of Geophysical Research: Space Physics, 122, 12,229–12,235. https://doi.org/10.1002/2017JA024836
Clarke, K. E., Andrews, D. J., Arridge, C. S., Coates, A. J., & Cowley, S. W. H. (2010). Magnetopause oscillations near the planetary period at Saturn: Occurrence, phase, and amplitude. Journal of Geophysical Research, 115, A08209. https://doi.org/10.1029/2009JA014745
Clarke, K. E., Andrews, D. J., Coates, A. J., Cowley, S. W. H., & Masters, A. (2010). Magnetospheric period oscillations of Saturn’s bow shock. Journal of Geophysical Research, 115, A05202. https://doi.org/10.1029/2009JA015164
Connerney, J. E. P., Acauha, M. H., & Ness, N. F. (1981). Modeling the jovian current sheet and inner magnetosphere. Journal of Geophysical Research, 86, 8370–8384. https://doi.org/10.1029/JA086iA10p08370
Cowley, S. W. H., & Provan, G. (2015). Planetary period oscillations in Saturn’s magnetosphere: Comments on the relation between post-equinox periods determined from magnetic field and SKR emission data. Annales de Geophysique, 33(7), 901–912. https://doi.org/10.5194/angeo-33-901-2015
Cowley, S. W. H., & Provan, G. (2016). Planetary period oscillations in Saturn’s magnetosphere: Further comments on the relationship between post-equinox properties deduced from magnetic field and Saturn kilometric radiation measurements. Icarus, 272, 258–276. https://doi.org/10.1016/j.icarus.2016.02.051
Cowley, S. W. H., & Provan, G. (2017). Planetary period modulations of Saturn’s magnetotail current sheet during northern spring: Observations and modelling. Journal of Geophysical Research: Space Physics, 122, 6049–6077. https://doi.org/10.1002/2017JA023993
Cowley, S. W. H., Provan, G., & Andrews, D. J. (2015). Comments on “Magnetic phase structure of Saturn’s 10.7 hour oscillations” by Yates et al. Journal of Geophysical Research: Space Physics, 120(7), 5686–5690. https://doi.org/10.1002/2015JA021351
Southwood, D. J. (2011). Direct evidence of differences in magnetic rotation rate between Saturn’s northern and southern polar regions. *Journal of Geophysical Research*, 116, A01201. https://doi.org/10.1029/2010JA016070

Southwood, D. J., & Cowley, S. W. H. (2014). The origin of Saturn magnetic periodicities: Northern and southern current systems. *Journal of Geophysical Research: Space Physics*, 119, 1563–1571. https://doi.org/10.1029/2013JA019632

Southwood, D. J., & Kivelson, M. G. (2007). Saturn magnetospheric dynamics: Elucidation of a camshaft model. *Journal of Geophysical Research*, 112, A12222. https://doi.org/10.1029/2007JA012254

Thomsen, M. F., Jackman, C. M., Cowley, S. W. H., Jia, X., Kivelson, M. G., & Provan, G. (2017). Evidence for periodic variations in the thickness of Saturn’s nightside plasma sheet. *Journal of Geophysical Research: Space Physics*, 122, 280–292. https://doi.org/10.1002/2016JA023368

Ye, S.-Y., Fischer, G., Kurth, W. S., Menietti, J. D., & Gurnett, D. A. (2016). Rotational modulation of Saturn’s radio emissions after equinox. *Journal of Geophysical Research: Space Physics*, 121, 11, 714–11, 728. https://doi.org/10.1002/2016JA023281