NON-LINEAR EVOLUTION AND PARTON DISTRIBUTIONS AT LOW X.

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We suggest a new procedure for extrapolating the parton distributions from HERA to much higher energies. The procedure suggested consists of two steps. First, we solve the non-linear evolution equation. Second, we introduce a correcting function for which we write a DGLAP-type linear evolution equation. The nonlinear equation is solved numerically and estimates for the saturation scale, as well as for the gluon density at HERA and LHC energies are made.

1 Introduction

In the present talk we present a new approach for extrapolation of the parton densities to low \( x \). By low \( x \) we mean values up to \( 10^{-7} \), which correspond to the energy range potentially covered by the HERA and LHC.

A conventional approach to parton density evolution is based on the linear DGLAP equation which describes the gluon radiation leading to an increase in the number of partons. However, at low \( x \) the parton cascade become dense and the recombination processes start to be important. We are convinced that such system cannot be described by a linear evolution any more.

Our motivation is based on two main problems of the DGLAP evolution. First, it predicts a very steep growth of parton distributions at low \( x \) violating the unitarity constraints. The second problem is common for all perturbative series which are asymptotic. In application to DIS processes it leads to twist OPE break down at low \( x \), when the high twists become of the same order as the leading one. We believe that a non-linear evolution is a solution to both problems! The non-linear evolution accounts for the saturation effects due to high parton density and it sums high twist contributions.

2 New approach for extrapolating the parton distributions

In the colour dipole picture the evolution is applied to the imaginary part of the dipole elastic scattering amplitude \( N(r_\perp, x; b) \) for the dipole of the size \( r_\perp \) elastically scattered at the impact parameter \( b \). The amplitude \( N \) is the major unknown to be determined. Our approach to the problem is based on two steps in which we obtain \( N \) as a sum of two terms: \( N = \tilde{N} + \Delta N \). As the first step,
the function $\tilde{N}$ is found as a solution of the non-linear evolution equation (1). This equation is valid in the leading $\ln(x)$ approximation of QCD. Moreover, it does not describe correctly the evolution at very short distances. In order to improve we perform the second step of our program: the correcting function $\Delta N$ is introduced to incorporate the correct DGLAP kernel at short distances. For $\Delta N$ we write down a DGLAP-type linear evolution equation.

The non-linear evolution equation for the function $\tilde{N}$ has the form:

$$\frac{d\tilde{N}(r_{01}, x)}{d \ln x} = \frac{C_F \alpha_s}{\pi^2} \int d^2 r_2 \frac{r_0^2}{r_{02}^2 r_{12}^2} \left( 2 \tilde{N}(r_{02}, x) - \tilde{N}(r_{02}, x) \tilde{N}(r_{12}, x) \right)$$  \hfill (1)

The above equation was derived in quite different approaches to high density QCD and hence we believe it to be a very reliable tool. The equation (1) is derived for the large number of colors $N_c$ and for a constant $\alpha_s$. We have dropped the $b$-dependence implying the large $b$ limit. Having assumed $b$ to be large we will allow to ourselves to extrapolate back to $b = 0$. The non-linear equation (1) is a subject to certain initial conditions, which we set at $x_0 = 10^{-2}$. The initial conditions are taken in the Glauber-Mueller form.

3 Numerical solution

The equation (1) is solved numerically by the method of iterations for $b = 0$. The solutions are obtained for the fixed $\alpha_s = 0.25$ and for the one loop running $\alpha_s$. The figure presents examples of the solutions as a function of distance. The solutions display a step like behavior consistent with all theoretical predictions. At short distances the function $\tilde{N}$ tends to zero, while at large distances it approaches unity, which is the unitarity bound.
4 Results

Basing on the above solutions several physical quantities can be determined.

- The saturation scale $Q_s(x)$ is estimated as a typical scale where the function $\tilde{N}$ undergoes the step like transition shown above. Though no exact mathematical definition of the saturation scale is known we proposed several reasonable definitions which in average produce our predictions depicted in the figure 2.

- The solutions found display the scaling phenomena. Namely, the function $\tilde{N}$ is not a function of two independent variables $x$ and $r_\perp$, but a function of a single variable $\tau = r_\perp Q_s(x)$: $\tilde{N}(x, r_\perp) \simeq \tilde{N}(r_\perp Q_s(x))$. The scaling holds with a few percent accuracy in a broad kinematic domain below $x = 10^{-2}$.

- The gluon density was defined according to the Mueller formula 3, which relates the gluon density to the dipole elastic scattering amplitude. Examples of our predictions are shown in the Fig. 3. At small $x$ the gluon density is damped by a factor 2-3 comparing to the DGLAP predictions 4.

- The structure function $F_2$ is computed. The experimental data below $x = 10^{-2}$ is reproduced with less than 20% error for $Q^2 \leq 50$ GeV$^2$. It is important that the data is reproduced without use of the full DGLAP kernel.

- The LO BFKL equation, which is the linear part of the equation (1), is solved numerically by the same method of iterations. The obtained results are compared in the Fig. 4 with the corresponding solutions of the non-linear
The solution of the BFKL equation rapidly diverges from the solution of the non-linear equation. We conclude that the shadowing effects become important before the BFKL dynamics actually takes place.

5 Summary and plans for the future

A new method for extrapolating the parton densities to high energies was proposed. As the first step in our program we solved (numerically) the non-linear evolution equation (1). From the solutions obtained we estimated the saturation scale as a function of $x$. We found an approximate scaling behavior of the solutions. Both $xG$ and $F_2$ are found to be significantly damped at high energies. We predicted the LO BFKL dynamics unlikely to be ever seen. We work currently on single diffractive dissociation from the non-linear evolution.

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