Integrated Model for Design of SWS and Beam-Wave Interaction Analysis of a Planar THz Sheet-beam TWT

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Abstract—A computationally efficient, integrated, and dynamic model has been developed for the design of a planar Slow Wave Structure (SWS) and beam-wave interaction analysis of a planar THz Traveling Wave Tube (TWT) with sheet beam. A Staggered Double Vane-Slow Wave Structure (SDV-SWS) is used for its numerous advantages over other types of SWSs. The integrated model determines RF performance of a planar TWT directly from the given beam voltage and center frequency by performing three different tasks, (i) determining geometrical parameters of an SDV-SWS of maximum possible bandwidth and high interaction impedance, (ii) determining RF circuit parameters of an SDV-SWS, and (iii) performing beam-wave interaction analysis of a planar TWT. The model was developed by adopting a numerically computing environment, MATLAB. Also, highly accurate numerical techniques with double precision were used, e.g., Sixth Order Runge Kutta Method was used for electron beam dynamic. The model was used to design and simulate a 0.22 THz Sheet Beam TWT of 100 W output power. The energy balance factor was achieved within ±0.001% over a very wide dynamic range from even 100 dB below saturation to more than 10 dB above saturation. The power growth of the forward wave achieves exactly 1 dB/dB. The program is fast enough for interactive use on a standard computer with a basic configuration. The model has been compared with the published works using a 3D electromagnetic field simulator for demonstrating its accuracy.

1. INTRODUCTION

With the changing time, there is a need for higher bandwidth, speed, and signal to noise ratio in communication and sensing applications. The terahertz frequency band of 0.1 THz to 10 THz in electromagnetic spectrum is being explored for various applications in the field of science and technology [1–3], and hence a major focus is towards the improvisation and fabrication of vacuum electronic devices (VED). Among many VEDs, Traveling Wave Tube (TWT) Amplifiers are widely used for high power applications because of their wide bandwidth and high linearity. For the past few decades, multiple modeling techniques for TWT with different Slow-Wave Structures (SWS) have been proposed [4, 5]. An integrated-dynamic approach for design and beam-wave interaction analysis of terahertz TWT with staggered double-vane slow-wave structure (SDV-SWS) with sheet beam has been presented in the paper. SDV-SWS is preferred over other structures because of its numerous advantages related to wide bandwidth, high impedance, low loss, easy fabrication, etc. Figure 1 shows a schematic diagram of a part of SDV-SWS for a planar THz TWT. In the next section, different modules of the proposed integrated model are presented. In the third section, the simulated results are presented to demonstrate the self-consistency and accuracy of the proposed integrated model.
2. INTEGRATED-DYNAMIC MODEL

The proposed integrated model, as shown in Figure 2, is a combination of three models determining, respectively, the geometrical parameters of an SDV-SWS of large bandwidth and high interaction impedance (by Model A), RF circuit parameters like dispersion and impedance characteristics of the structure (by Model B), and the RF performance of a TWT by executing beam-wave interaction analysis (by Model C).

![Flow Chart representing integrated model.](image)

2.1. Model A — Geometrical Analysis

Based on the analytical approach discussed in [6], the model determines the geometrical parameters of an SDV-SWS of maximum possible bandwidth and high interaction impedance, for a given beam voltage and center frequency. The beam is selected in synchronization with the first space harmonic of the forward wave of phase shift per pitch equal to \((2.5\pi)\) at the central frequency. The geometrical parameters of the SDV-SWS for a 0.22 THz TWT operating at 20 kV Beam Voltage are shown in Table 1. A rectangular sheet beam of 100 µm × 600 µm is used on the basis of the geometrical beam
Table 1. Geometrical parameters of slow wave structure.

| Symbol | Quantity            | Value  |
|--------|---------------------|--------|
| \( p \) | Pitch               | 463 µm |
| \( w \) | Waveguide Width     | 770 µm |
| \( h \) | Waveguide Height    | 824 µm |
| \( 2a \) | Tunnel Height       | 118 µm |
| \( l \)  | Vane Height         | 353 µm |
| \( g \)  | Gap between two vanes | 139 µm |

Tableau 1. Paramètres géométriques de la structure à ondes lentes.

La taille du tunnel est de 118 µm x 770 µm. Le courant de faisceau de 100 mA est utilisé pour obtenir une densité de courant de faisceau beaucoup moins de 200 Amp/Cm².

2.2. Model B — Electrical Parameter Analysis

En utilisant les paramètres géométriques donnés dans le Tableau 1, le modèle calcule les caractéristiques de dispersion et d’impédance de l’SDV-SWS sur toute la bande pour le premier harmonique de l’onde directe. L’équation matricielle déterminante (équation (29) de [7]) est utilisée pour la courbe de dispersion. La valeur moyenne de l’impédance d’interaction à travers le faisceau est calculée en utilisant (équation (30) de [7]). La Figure 3 montre la courbe de dispersion (vitesse de phase normalisée) et la courbe de l’impédance d’interaction (moyenne sur la section de faisceau) de l’onde de harmonique +1 de l’onde directe pour la TWT à 0.22 THz, qui sont trouvées presque similaires à celles de [8].

![Normalized phase velocity and interaction impedance (Ω) curves over the frequency band.](image)

Figure 3. Normalized phase velocity and interaction impedance (Ω) curves over the frequency band.

2.3. Model C — Beam-Wave Interaction Analysis

Pour l’analyse d’interaction faisceau-onde d’une TWT à THz avec un faisceau plat, un modèle non linéaire unidimensionnel est développé de la même façon que pour le modèle SUNRAY-1D de grande signaux [9]. Un certain nombre de modifications dans le modèle SUNRAY-1D ont été incorporées pour obtenir une meilleure précision numérique sans augmenter beaucoup le temps de simulation. Le modèle a incorporé la possibilité de sélectionner un nombre flexible de feuilles d’électrons rectangles (ND) dans un faisceau d’onde (Figure 4). De la même manière, le modèle a incorporé la possibilité de sélectionner un nombre flexible de pas d’intégration par onde.
Figure 4. (a) Electron beam representation in 32 rectangular sheets. (b) A single rectangular sheet.

(NZ) along a tube. The values of ND and NZ can be increased along the tube length for better convergence of RF performance over a section. This provides optimum speed and accuracy in simulation.

The space charge force between 2 rectangular sheets in a rectangular tunnel is calculated by solving Green’s Function for an infinite rectangular tunnel of height \((g + h/3)\). The space charge field due to each rectangular sheet is calculated using Equation (1) \[10\].

\[
E_{sc} = \frac{2\omega^2_p}{|\eta|ab} \int_{-\infty}^{+\infty} \text{sgn}(dz) \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \frac{2a}{k\pi} \frac{2b}{m\pi} \sin \frac{m\pi w}{2b} \sin^2 \frac{m\pi}{2} \\
\exp \left[-\frac{m\pi dz}{b}\right] \sin \frac{k\pi H}{2a} \sin^2 \frac{k\pi}{2} \exp \left[-\frac{k\pi dz}{a}\right] dz
\]

(1)

Here, \(a\) is the height, and \(b\) is the width of the rectangular tunnel. \(H\) and \(W\) are the height and width of the sheet electron beam, respectively, and \(dz\) is the distance between the two particles under analysis.

Figure 5 shows the variation in space charge force (SCF) between 2 rectangular sheets in a rectangular tunnel over one beam wavelength. It can be observed that the SCF tends to zero after half-wavelength distance. It signifies that the force of repulsion between two particles is negligible after half-wavelength distance \((\lambda_e/2 = 1.85 \times 10^{-4} \text{ m})\). Figure 5 also shows SCF for equivalent cylindrical discs. For a cylindrical beam, the technique proposed in [9] is used. The dimensions of a cylindrical beam and cylindrical tunnel are selected for the same cross-section areas as that of the sheet beam and rectangular tunnel, respectively.

Figure 5. Space charge force vs distance for the rectangular and equivalent circular beam.
Effective space charge force on a sheet is calculated by summing up the SCF on the sheet due to all other surrounding sheets up to half a beam wavelength. An efficient approach is developed for determining relative sheet positions. Also, an efficient approach is developed to use a look-up table for determining the exact value of SCF between 2 sheets.

Sixth order Runge-Kutta Method [11, 12] is used for tracking beam particles accurately in the presence of space charge field and the RF circuit field along tube length in small integration steps. The complete program is developed with double precision for high numerical accuracy over a wide dynamic range. It is ensured that the efficient value of ac beam current, as well as the effective value of space charge force on a sheet, remains zero for the dc beam propagating in the structure without RF circuit field. Figure 6 shows a fundamental component of the ac beam current and effective value of SCF on a sheet throughout the length of the tube, for a dc beam propagating through the structure without the RF circuit field.

![Figure 6](image)

**Figure 6.** Normalized AC current and effective SCF along with the structure for no drive power.

3. SELF-CONSISTENCY AND ACCURACY OF THE MODEL

The model described above is programmed in MATLAB, a numerical simulation tool, and runs on a computer system with at least 1 GB RAM and 500 MB storage space. The program is fast for interactive

![Figure 7](image)

**Figure 7.** Output power and phase transfer characteristic curves representing consistency over a wide range of input drive power.
use, as the time taken for a TWT with 24 rectangular sheets and 16 integration steps per wavelength takes approximately 100 seconds to simulate on an Intel Core i3 processor. The model takes operating frequency and operating voltage-current for electron beam as input, and generates the rest of the data.

For verifying the consistency of the code, the transfer characteristic curve (output power and phase versus drive power) over a wide dynamic range is simulated, as shown in Figure 7. It is found that the power transfer curve is consistent from $-100$ dB below saturation to beyond 10 dB above saturation. Also, there is no or very minute variation in the phase angle of the output voltage for a linear condition. The energy balance factor is found 100% for all drive powers, and the power growth rate is exactly 1 dB/dB over the linear region.

The accuracy and self-consistency of the program are also observed by varying the number of rectangular sheets (ND) as well as the integration steps (NZ) per wavelength. As shown in Figure 8 (with expanded $y$-scale), the output power gets converged with ND equal to 72. Also, the output power
tends to be the same for different values of NZ when ND is more than or equal to 72. It is significant to note that the energy balance factor remains consistent at 100% with a minute error of ±0.001%, for different values of ND and NZ.

Figure 9 shows variation in program run-time on an Intel Core i3 processor for varying ND and NZ. The program run time increases almost linearly with an increase in NZ but increases exponentially with an increase in ND.

Figure 10 shows profiles of the forward growing RF power and RF beam current along tube length for center frequency 0.22 THz, with 24 rectangular sheets and 16 integration steps per wavelength. The simulated power was compared against the published results using the 3D PIC simulator [13]. The energy balance factor profile is also shown, and it can be observed that it remains at 100% throughout the length of the tube.

Figure 10. Output power, energy balance and current profile of RF beam throughout the structure.

Figure 11. Saturated RF output power over full frequency band calculated using the code and compared with the published result [13].
Figure 11 shows a variation of RF output power at saturation over the frequency band of 190 GHz to 260 GHz. The figure also shows a comparison of simulated power with published simulated power using the 3D PIC Simulator [13]. The energy balance factor although not shown over the band achieves 100% at saturation over the full frequency band.

4. CONCLUSION

The integrated model for a planar THz TWT was developed to determine RF performance of a tube over the operating band for given beam parameters like beam voltage and beam current. The model first calculates the geometrical parameters of an SDV-SWS and then RF circuit parameters like dispersion and impedance curves, and then performs beam-wave interaction analysis. The self-consistency of the model was demonstrated over a very wide dynamic range with high numerical accuracy. The presented model is simple, interactive, accurate, easy to use, and reliable for the simulation of planar TWT using SDV-SWS with sheet beam. The model can also be used for the analysis of Backward Wave Amplifiers and Backward Wave Oscillators. The code is also extended for 3D analysis following approach as given in [14].

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