SUPER-RESOLVED LOCALISATION WITHOUT IDENTIFYING LOS/NLOS PATHS

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ABSTRACT
This paper focuses on the problem of localising a transmitting mobile station (MS) using multiple cooperative base stations (BSs). There are two technical difficulties: one is the data association between intermediate parameters and scatters; the other is to identify Line-of-Sight (LoS) and Non-Line-of-Sight (NLoS) paths. Our main contribution is a unified approach bypassing both data association and LoS/NLoS path identification. This is achieved by introducing virtual scatters and then a direct localisation formulation. Modern super-resolution technique is then adapted to cast the localisation problem as convex programming and solve it. Our approach addresses long-standing issues not completely solved in the literature, guarantees a global convergence, achieves good localisation accuracy and demonstrates its robustness to noise in our numerical tests.

Index Terms—ADC, compressed sensing, mixed LoS/OLoS/NLoS conditions, super-resolution.

1. INTRODUCTION
This paper considers the problem of localising a transmitting mobile station (MS) based on measurements of several base stations (BSs). This problem has become vital in various applications such as the location based services and the Internet of things. In the literature, such a localisation is typically achieved by using the following two types of information. It has been assumed that BSs are equipped with multiple antennas and therefore BSs can estimate Direction of Arrival (DoA) of the received signals, and that localisation is performed in a cooperative setting and hence Time of Arrival (ToA)/Time Difference of Arrival (TDoA) information can be estimated and shared across BSs.

There are two technical difficulties in solving the localisation problem. The first one comes from data association between DoA/ToA information and locations of scatters. A typical approach is that BSs first estimate DoA/ToA and then infer MS location based on them. When there are multiple scatters in the scene, the task of associating DoA/ToA estimates to scatters is an NP-hard problem. In the literature, direct localisation [1–4] has been developed to address this issue by considering the map from locations directly to received signals and eliminating the intermediate variables DoA/ToA. Data association can be avoided.

The other difficulty is a consequence of different impacts of Line-of-Sight (LoS) and Non-Line-of-Sight (NLoS) paths. Lots of efforts have been made to distinguish these two types of paths and study the localisation problem under different conditions. The identification of LoS/NLoS paths is typically achieved by constructing probabilistic models [6,7], applying statistical estimation methods [8,10], and/or exploring the temporal correlations in the propagation environment [11]. However, mistaken identifications cannot be avoided completely. When they happen the localisation performance deteriorate.

Localisation mechanism varies based on the propagation environment. In the literature, typically three conditions are discussed: LoS condition where all paths are LoS; Obstructed-Line-of-Sight (OLoS) condition where all paths are NLoS; and NLoS condition where there exist both LoS and NLoS paths. Triangulation has been applied for localisation in all three conditions. Triangulation is straightforward in LoS condition. There are two approaches in NLoS condition. One is to identify LoS and NLoS paths and then only use LoS paths for triangulation [8,10]. The other does not require LoS/NLoS path identification [12–14]. Instead, geometric areas (typically circular areas) are obtained based on ToA/DoA estimation at BSs. The MS must lie in the intersection of these areas. This idea can be also applied to OLoS condition. However, in OLoS condition, the intersection is typically an area rather than a point. Extra assumptions are typically added to allow path Direction of Departure (DoD) information at BSs. Most of the above works rely on identification of LoS/NLoS paths and estimation of ToA/DoA information.

Our main contribution is a unified approach that bypasses both data association and the middle step of identifying LoS/NLoS paths. This is achieved by three key elements. Firstly, virtual scatters are added to LoS paths so that all LoS paths can be treated mathematically in the same way as for NLoS paths. There is no need for LoS/NLoS path identification. Secondly, a direct localisation approach is developed where the intermediate variables DoA/ToA get eliminated. Different from [1–4] where the idea of direct localisation has been applied, we formulate the localisation problem as a linear de-mixing problem so that super-resolution technique can be applied. Thirdly, modern super-resolution technique is adapted where a convex optimisation problem is formulated and solved. Our unified approach avoids the error propagation due to LoS/NLoS path identification, eliminates data association, guarantees a global convergence, achieves good localisation accuracy and demonstrates its robustness to noise in our numerical tests.

2. THE PROBLEM
We focus on the following localisation problem by adopting a commonly assumed setup in the literature [8,12,13]. We consider a wireless communication system where $J$ BSs cooperate to jointly estimate the location of one MS. The MS is assumed to transmit with an omnidirectional antenna and all BSs are equipped with a uniform linear antenna array (ULA) with $N_R$ antenna elements.
The synchronisation between MS and BSs can be avoided by using round-trip ToA information, or using TDoA information with synchronised BSs. It is typically assumed that via control signalling, the MS and the involved BSs are synchronised and the transmitted waveform from the MS is known to BSs. Thus BSs can estimate both DoA and ToA information. In our signal model, we only consider either LoS or single-bounced signals from the MS to the BSs. This is motivated by the fact that signals scattered twice or more times typically suffer from great propagation losses and are thus less perceptible. It is noteworthy that the above setup is a simplification of actual systems. For example, the assumption of synchronisation and the complete discard of multiple-bounced signals may be problematic in practice. Nevertheless, the above setup is widely adopted in the literature for the purpose of highlighting the technical approach/idea without being drowned into great technical details.

The received signal $y_j(t) \in C^{N_H}$ at the $j$-th BS is given by

$$y_j(t) = \sum_{k=1}^{K} \gamma_{j,k} x(t - \tau_{j,k}) a(\theta_{j,k}),$$

where the number $K$ denotes the number of received paths at the BS, $\gamma_{j,k} \in C$ is an unknown coefficient modelling the signal attenuation along the $k$-th path, $x(t) \in C$ is the transmitted signal, $\tau_{j,k} > 0$ is the delay experienced in the $k$-th path, $\theta_{j,k}$ is the DoA of the $k$-th path, $a(\theta) \in C^{N_H}$ reflects the phase differences in the received signals due to the BSs and takes the form of

$$a(\theta) = \left[ 1, e^{j2\pi L \sin(\theta)}, \ldots, e^{j2\pi L \sin(N_R-1)} \right]^T,$$

where $L$ is the wavelength of the carrier, and $L$ is the distance between the adjacent array elements at the BS.

For the purpose of localisation, it is very important to note the different impacts of the LoS path and NLoS paths. Denote the location of the MS by $l_t = [l_{t,x}, l_{t,y}]^T$ where $l_{t,x}$ and $l_{t,y}$ are the horizontal and vertical coordinates, respectively. Assume that there is a LoS path between the MS and one BS located at $l_{BS} = [l_{BS,x}, l_{BS,y}]^T$. Without loss of generality, assume that the position of the BS $l_{BS}$ is given and the directions of the uniform arrays are aligned with the horizontal axis. The ToA and DoA information is given by

$$\tau(l_t) = \|l_t - l_{BS}\|_2/c,$$

$$\theta(l_t) = \arctan(l_{t,y} - l_{BS,y})/l_{t,x},$$

respectively, where $c$ is the speed of light. There are two unknown variables in the MS location $l_{t,x}$ and $l_{t,y}$. They can be resolved from the ToA and DoA information defined in (3) and (4).

The information provided by an NLoS path is very different. Let $l_s = [l_{s,x}, l_{s,y}]^T$ be the location of a scatter, which is unknown to the BS. Then the ToA and DoA information is given by

$$\tau(l_s, l_t) = \|l_s - l_t\|_2/c + \|l_t - l_{BS}\|_2/c,$$

$$\theta(l_s, l_t) = \arctan(l_{s,y} - l_{BS,y})/l_{s,x} - \arctan(l_{t,y} - l_{BS,y})/l_{t,x},$$

respectively. Note that in this case ToA $\tau(l_s, l_t)$ depends on both $l_t$ and $l_s$, but DoA $\theta(l_s, l_t)$ only depends on $l_t$.

Due to their different impacts, LoS and NLoS paths are conventionally treated separately. The typical approach is to separate LoS path from NLoS paths and then use LoS path for localisation. In the case that there always exists a LoS path from MS to BS, the identification of LoS path can be achieved based on the fact that the LoS path experiences the minimum delay. However, in practice, there is no guarantee that a LoS path exists from the MS to a particular BS. The minimum delay path may correspond to an NLoS path. See [5–11] for other more sophisticated methods to identifying LoS and NLoS paths. Note that mistakes in identification cannot be avoided completely and they lead to localisation performance deterioration.

There are also efforts in the literature to perform localisation without distinguishing LoS/NLoS paths. The basic intuition is as follows. Based on the ToA/DoA estimates for each path, a geometric area of the possible locations of the MS can be inferred. If the intersection of all these geometric areas is a single point, then the MS must locate there. This approach suffers from the difficulty to infer the exact geometric areas for NLoS paths and rapid performance deterioration as noise increase.

### 3. OUR SOLUTION

#### 3.1. The Virtual Scatter

As detailed in Section 2, the impacts of LoS and NLoS paths are different. The localisation process may fail if an NLoS path is mistakenly identified as a LoS path or vice versa. In the literature, many efforts were put to distinguish them.

A key element of our approach is to introduce the concept of virtual scatter so that there is no need to distinguish LoS and NLoS paths. This is achieved by introducing virtual scatter in the LoS path between the MS and the BS. See Fig. 1(a) for an illustration. With the virtual scatter, the ToA and DoA information of both LoS and NLoS paths can be consistently presented by (3) and (4), respectively. The mathematical models for LoS and NLoS paths are therefore unified. This unified model eventually leads to a simplified and tractable formulation in (5) without handling LoS and NLoS paths separately.

It is worth to comment more on the virtual scatter. When there is only one LoS path existing in the localisation problem, the location of the virtual scatter can be arbitrary on the line segment of that particular LoS path. This arbitrariness will not affect the localisation performance as the ultimate goal is to locate the MS. When there are multiple LoS paths in the localisation problem, the most parsimonious solution (in number of scatters) is unique where a single virtual scatter is created at the intersection of all LoS paths.
paths, i.e., the location of the MS. This idea is illustrated in Fig. [12]. A similar idea was briefly mentioned in [4] but not carefully engineered into the localisation algorithm.

3.2. Direct localisation as linear de-mixing

There are different ways to perform localisation. Many works in the literature [12, 14] take the approach of first estimating ToA and DoA information and then using the estimated ToA and DoA for localisation. As the same target creates different paths and gives rise to different ToA and DoA information different BSs. Data association is needed. However, data association is known to be NP-hard and error propagation is inevitable.

Another way is to embed the geometry information into an optimisation formulation, also known as direct localisation [1, 4]. Rather than estimating ToA and DoA, the unknown variables are chosen as the location of MS \( l_t \) and that of scatter \( l_s \). In other words, the signal model is the ‘direct’ map from \((l_t, l_s)\) to \( y \). This strategy avoids data association.

It is challenging to solve the inverse problem based on the nonlinear map from \((l_t, l_s)\) to \( y \). There is a huge literature on estimating the intermediate variables ToA \( \tau \) and DoA \( \theta \), because of nice structures of the associated problem. In direct localisation, these structures cannot be directly explored. To avoid directly handling the nonlinear map from \((l_t, l_s)\) to \( y \), we formulate the forward model into a linear form. Consider the map from locations of the MS and scatter pairs: \( l_t, l_s \) to \( y \), we formulate the map from locations of the MS and scatter pairs: \( l_t, l_s \) to \( y \).

Localisation problem is then reduced to a problem of de-mixing a linear combination of atoms from the set of \( B_j(l_t, l_s) \).

3.3. Super-resolution formulation

There are two approaches to solve the demixing problem in [6]. One approach relies on discretising the space \((l_t, l_s)\). By such a discretisation, one obtains a final size dictionary of number of discrete grid points, the de-mixing problem (8) can be well approximated by signals from a small number of grid points. The de-mixing problem (8) can be well approximated by signals from a small number of grid points. By contrast, super-resolution approach works on the continuous parameter space, avoids the leakage effect, and its complexity mainly depends on the number of data samples \( N \), which can be many orders of magnitude less than the number of grid points \( N_d \).

In the super-resolution framework, the locations are encoded using atomic measures

\[
\mu_j = \sum_{k=1}^{K} \gamma_j \delta(l_t, l_s)_k, \quad j = 1, \ldots, J, \tag{9}
\]

where \( \delta(l_t, l_s) \) is Dirac function at \((l_t, l_s)\). The subscript \( j \) emphasises that the attenuation coefficients \( \gamma_j \) may be different for different BSs. The received signal at the \( j \)-th BS is then

\[
Y_j = \Psi_j \mu_j := \int_{(l_t, l_s) \in \mathbb{R}^2} B_j(l_t, l_s) d\mu_j(l_t, l_s), \tag{10}
\]

where \( \Psi_j \) is a linear operator mapping the atomic measure \( \mu_j \) to \( Y_j \in \mathbb{C}^{N_t \times N_s} \), and \( B_j(l_t, l_s) = \alpha(\theta_j) [x(0 - \tau_j), \ldots, x(N - 1 - \tau_j)] \).

A total variation (TV) norm and a group total variation (GTV) norm are defined for atomic measures to promote certain sparse structures of locations. In practice, there are typically a small number of MS and scatter pairs compared with the signal dimension \( N_t \times N_s \). The total variation norm is defined to promote the sparsity of the MS and scatter pairs:

\[
||\mu_j||_{TV} := \sup_{||\alpha(l_t, l_s)||_2 \leq 1} \text{Re} \int \alpha(l_t, l_s) d\mu_j(l_t, l_s) = \sum_k ||\gamma_j||_2 \tag{11}
\]

It is an analogy to \( \ell_1 \)-norm for finite dimensional vector space [13]. Note that for given MS and scatter pair located at \((l_t, l_s)\), it likely generates signals received at multiple BSs. Motivated by the group sparsity concept commonly used in the literature of compressed sensing, the GTV norm is defined as

\[
||\mu||_{GTV} := \sup_{||\alpha(l_t, l_s)||_2 \leq 1} \sum_j \text{Re} \int \alpha(l_t, l_s) d\mu_j(l_t, l_s) = \sum_k ||\gamma_k||_2 \tag{12}
\]

where \( \mu = [\mu_1, \ldots, \mu_J]^T \), \( \alpha(l_t, l_s) = [\cdots, \alpha_j(l_t, l_s), \cdots]^T \), and \( \gamma_k = [\gamma_{k,1}, \ldots, \gamma_{k,J}]^T \). Clearly it is an analogy to \( \ell_2 \)-norm [19].

With above definitions, the localisation problem in the noise free case can be then formulated as

\[
\min_{\mu} \sum_{j=1}^{J} ||\mu_j||_{TV} + \lambda ||\mu||_{GTV} \quad \text{s.t.} \quad Y_j = \Psi_j \mu_j, \quad \forall j \in [J], \tag{13}
\]

where \( \lambda > 0 \) is a regularisation constant, and \( [J] := \{1, \ldots, J\} \). For the noisy case, either replace the equality \( Y_j = \Psi_j \mu_j \) with \( ||Y_j - \Psi_j \mu_j||_F \leq \epsilon_j \) where the noise power is known at most \( \epsilon_j > 0 \), or turn the constrained optimisation (13) into a Lasso type unconstrained optimisation. Note that in order to ensure the convexity of (13), we do not restrict the number of possible \( l_t \) to one. Nevertheless, all trials in our simulations return a single MS location.

3.4. Solving the super-resolution problem

It is highly non-trivial to solve the convex optimisation problem (13). The biggest hurdle is that (13) is a sparse encoding using a dictionary with infinite many atoms, hence an optimisation problem
Algorithm 1 ADCG in multiple cooperative BSs system

**Input**: Candidate set $\mathcal{S}_0 = \emptyset$, threshold, $Y_{j,l} = 1, \ldots, J$.

**Output**: Locations of MS and scatters : $\mathcal{S}_K$.

**Iteration $k$**
- Compute gradient of loss : $g_{j,k} = \nabla \| r_{j,k} \|_2$, where $r_{j,k} = \mathcal{P}_{j,k} \mu_{k-1} - Y_j$
- Compute next source : $(l_1, l_2) = \arg \min \sum_{j=1}^J (B_j(l_1, l_2) g_{j,k})$
- Update the candidate set : $\mathcal{S}_k = \mathcal{S}_{k-1} \cup \{l_1, l_2\}$
- Coordinate descent
  - Compute weights : $\gamma = \arg \min \sum_{j=1}^J \| Y_j - B_j (S_k) \gamma_j \|_2^2 + \lambda_1 \sum_{j=1}^J \| \gamma_j \|_1 + \lambda_2 \| \gamma \|_2$
  - Prune support : $S_k = \{l_1, l_2\} \in \mathcal{S}_k : \| \gamma_{1,1} \|_2 > \text{threshold}\$
  - Locally improve support : $S_k = \text{gradient descent} (\{l_1, l_2\}, \gamma)$

with infinite many unknown variables. To overcome the technical difficulty, the method alternating descent conditional gradient (ADCG) [20] is adopted and slightly modified here. See Algorithm 1 for a highly level description. It is also noteworthy that the gradient computations in Algorithm 1 are related to the specific waveform $x(t)$ used in [1] and non-trivial. Details are omitted due to space constraint and will be presented in the journal paper.

### 4. NUMERICAL SIMULATIONS

#### 4.1. Simulation Setup

The transmitted signal can be chosen for different purpose. In this paper, we take one block of orthogonal frequency division multiplexing (OFDM) as an example, since it is used to combat multi-path fading and achieve high spectral efficiency. Suppose the transmitted signal from MS is

$$x(t) = \sum_{n=0}^{N-1} s(n) e^{i2\pi n \Delta f t},$$

where $s(n)$ is the data symbol, $\Delta f = 1/T$ is the sub-carrier frequency, and $T$ is the duration of each block.

By taking the Fourier transform on the received signal at the BS $j$, the observation at $n$-th sub-carrier can be written as

$$Y_j(n) = \int_{t=0}^{T} e^{-i2\pi n \Delta f t} y_j(t) \, dt.$$  \hfill (15)

In practice, the available data are uniform time-samples of $y_j(t)$. In this case, the integrals can be replaced by summation of time instances. The matrix form of measurement at receiver $j$ is

$$Y_j = \sum_{k=1}^{K} \gamma_{j,k} B_j(\theta_{j,k}, \tau_k) = \sum_{k=1}^{K} \gamma_{j,k} \mathbf{a}(\theta_{j,k}) b^T(\tau_{j,k}),$$

where the vector $b(\tau_{j,k}) \in \mathbb{C}^N$ associated with time delay for $k$-th propagation path is expressed as

$$b(\tau_{j,k}) = \left[ s(0), \ldots, s(N-1) e^{-i2\pi \Delta f \tau_{j,k}(t_{l,k}-t_i)}/(K+1) \right]^T.$$  \hfill (15)

In this section, we apply the super-resolution based approach to estimate the localisation of MS under the 4 cooperative base stations which are horizontally located at

$$(0,0) \text{ km}, (0,1) \text{ km}, (1,0) \text{ km}, (1,1) \text{ km}.$$  

We consider the scenario that the MS, scatters and BSs are located in a $1\text{km} \times 1\text{km}$ area. In simulation, the transmitted OFDM signal contains $N = 32$ sub-carrier frequencies, the sub-carrier frequency spacing is $\Delta f = 10kHz$, the speed of light is $c = 3 \times 10^8 \text{ m/s}$, carrier frequency $f_c = 2GHz$.

#### 4.2. Results and Analysis

Fig. 2(a) depicts the root mean square error (RMSE) of proposed super-resolved localisation scheme under unknown propagation conditions. The RMSE illustrates the estimated spatial resolution of both MS and scatters, and it is defined as RMSE $= \left( \sum_{k=1}^{K} \| \tilde{l}_{s,k} - l_{s,k} \|_2 + \| l_i - \tilde{l}_i \|_2/(K+1) \right)$, where $l_{s,k}, \tilde{l}_{s,k}$ are estimated locations of scatters $l_{s,k}$ and MS $l_i$ respectively. The output of the proposed approach contains one MS and $K$ scatters, the correspondence of estimated MS and ground truth is easily determined. The correspondence of scatter can be determined by finding ground truth which is nearest to estimated scatter. For each RMSE, we run a total of 300 Monte Carlo trials, and randomly generate locations of MS and scatters. In the simulation, the OLoS condition refers to all the BSs have the OLoS condition, and the same concept can be applied to NLoS condition. For each path, we add extra Additive White Gaussian Noise (AWGN) to noise environment. The simulation shows that proposed approach is robust to noise. It is observed that the localisation of MS and scatters is of high precision. Even in the worst case (SNR = -10dB), the spatial resolution in NLoS, mixed and OLoS condition are approximately $1.17m, 1.61m$ and $5.14m$. The proposed algorithm has better performance in NLoS condition. This is due to the NLoS condition containing both LoS paths and scattered paths, which has more measurements than mixed and OLoS condition.

Fig. 2(b) illustrates one trial of mixed condition with the SNR $= 0$ dB. In this case, one BS faces to NLoS condition, and the rest of BSs are OLoS condition. Without any prior knowledge of the propagation conditions, the proposed unified localisation approach can be applied in mixed propagation environments. The approach can not only estimate the localisation of the MS to a high precision, but also determine locations of scatters.
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