Oliveira de Silva, Diogo; Quilodrán, René
Global maximizers for adjoint Fourier restriction inequalities on low dimensional spheres.
(English) Zbl 1458.35011
J. Funct. Anal. 280, No. 7, Article ID 108825, 74 p. (2021).

Summary: We prove that constant functions are the unique real-valued maximizers for all \(L^2 - L^{2n}\) adjoint Fourier restriction inequalities on the unit sphere \(S^{d-1} \subset \mathbb{R}^d, d \in \{3, 4, 5, 6, 7\}\), where \(n \geq 3\) is an integer. The proof uses tools from probability theory, Lie theory, functional analysis, and the theory of special functions. It also relies on general solutions of the underlying Euler-Lagrange equation being smooth, a fact of independent interest which we establish in the companion paper [the authors, Forum Math. Sigma 9, Paper No. e12, 40 p. (2021; Zbl 1467.45006)]. We further show that complex-valued maximizers coincide with nonnegative maximizers multiplied by the character \(e^{i \xi \cdot \omega}\), for some \(\xi\), thereby extending previous work of M. Christ and S. Shao [Adv. Math. 230, No. 3, 957–977 (2012; Zbl 1258.35007)] to arbitrary dimensions \(d \geq 2\) and general even exponents.

MSC:
35A23 Inequalities applied to PDEs involving derivatives, differential and integral operators, or integrals
35B38 Critical points of functionals in context of PDEs (e.g., energy functionals)
33C10 Bessel and Airy functions, cylinder functions, \(\text{\_}F\_1\)
42B10 Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type
42B37 Harmonic analysis and PDEs
45C05 Eigenvalue problems for integral equations
51M16 Inequalities and extremum problems in real or complex geometry

Keywords:
sharp Fourier restriction theory; Tomas-Stein inequality; convolution of singular measures; Bessel integrals

Software:
QUADPACK; Matlab; DLMF; Algorithm 858; BESSELINT

Full Text: DOI arXiv

References:
[1] Abi-Khuzam, F., Inequalities and asymptotics for some moment integrals, J. Inequal. Appl., Article 257 pp. (2017), 8 pp. - Zbl 1375.26035
[2] Antoneli, F.; Forger, M.; Gaviria, P., Maximal subgroups of compact Lie groups, J. Lie Theory, 22, 4, 949-1024 (2012) - Zbl 1261.22007
[3] Ball, K., Cube slicing in \(\setminus (\text{\_}n!\text{\_}n^n\setminus)\), Proc. Am. Math. Soc., 97, 3, 465-473 (1986) - Zbl 0601.52005
[4] Borwein, J.; Chan, O-Y., Uniform bounds for the complementary incomplete gamma function, Math. Inequal. Appl., 12, 1, 115-121 (2009) - Zbl 1177.33009
[5] Borwein, J.; Simmanson, C., A closed form for the density functions of random walks in odd dimensions, Bull. Aust. Math. Soc., 93, 2, 330-339 (2016) - Zbl 1347.60049
[6] Borwein, J.; Straub, A.; Vignat, C., Densities of short uniform random walks in higher dimensions, J. Math. Anal. Appl., 437, 1, 668-707 (2016) - Zbl 1333.60086
[7] Borwein, J.; Straub, A.; Wan, J.; Zudilin, W., Densities of short uniform random walks, Can. J. Math., 64, 5, 961-990 (2012), With an appendix by Don Zagier - Zbl 1296.33011
[8] Brislawn, C., Kernels of trace class operators, Proc. Am. Math. Soc., 104, 4, 1181-1190 (1988) - Zbl 0695.47017
[9] Brislawn, C., Traceable integral kernels on countably generated measure spaces, Pac. J. Math., 150, 2, 229-240 (1991) - Zbl 0724.47014
[67] Landau, L. J., Bessel functions: monotonicity and bounds, J. Lond. Math. Soc. (2), 61, 1, 197-215 (2000) · Zbl 0948.33001

[68] Mann, L. N., Gaps in the dimensions of transformation groups, Ill. J. Math., 10, 532-546 (1966) · Zbl 0142.21701

[69] Mordhorst, O., The optimal constants in Khintchine’s inequality for the case \(2 < p < 3\), Colloq. Math., 147, 2, 203-216 (2017) · Zbl 1370.26047

[70] NIST Digital Library of Mathematical Functions. http://dlmf.nist.gov/, Release 1.0.23 of 2019-06-15. F.W.J. Olver, A.B. Olde Daalhuis, D.W. Lozier, B.I. Schneider, R.F. Boisvert, C.W. Clark, B.R. Miller, B.V. Saunders (Eds.).

[71] Natalini, P.; Palumbo, B., Inequalities for the incomplete gamma function, Math. Inequal. Appl., 3, 1, 69-77 (2000) · Zbl 0979.33001

[72] Nazarov, F. L.; Podkorytov, A. N., Ball, Haagerup, and distribution functions, (Complex Analysis, Operators, and Related Topics. Complex Analysis, Operators, and Related Topics. Oper. Theory Adv. Appl., vol. 113 (2000), Birkhäuser: Birkhäuser Basel), 247-267 · Zbl 0969.46014

[73] Oleszkiewicz, K.; Pełczyński, A., Polydisc slicing in \((\mathbb{C}^n)^m\), Stud. Math., 142, 3, 281-294 (2000) · Zbl 0971.32018

[74] Oliveira e. Silva, D.; Quilodrán, R., Smoothness of solutions of a restricted convolution equation on spheres (2019), Preprint at

[75] Oliveira e. Silva, D.; Thiele, C., Estimates for certain integrals of products of six Bessel functions, Rev. Mat. Iberoam., 33, 4, 1423-1462 (2017) · Zbl 1384.33014

[76] Oliveira e. Silva, D.; Thiele, C.; Zorin-Kranich, P., Band-limited maximizers for a Fourier extension inequality on the circle, Exp. Math. (2019)

[77] Pearson, K., Mathematical contributions to the theory of evolution. IX. A mathematical theory of random migration, (Draper’s Company Research Memoirs. Draper’s Company Research Memoirs, Biometric Series, vol. 3 (1906))

[78] Piessens, R.; de Doncker-Kapenga, E.; Überhuber, C. W.; Kahaner, D. K., QUADPACK: A Subroutine Package for Automatic Integration (1983), Springer-Verlag: Springer-Verlag Berlin · Zbl 0508.65005

[79] Sawa, J., The best constant in the Khintchine inequality for complex Steinhaus variables, the case \((p = 1)\), Stud. Math., 81, 1, 107-126 (1985) · Zbl 0612.60017

[80] Shao, S., On existence of extremizers for the Tomas-Stein inequality for \((\mathbb{S}^{n-1})\), J. Funct. Anal., 270, 3996-4038 (2016) · Zbl 1339.42011

[81] Steen, E. M., Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals (1993), Princeton University Press: Princeton University Press Princeton, NJ · Zbl 0821.42001

[82] Steinhaus, H., Sur les distances des points dans les ensembles de mesure positive, Fundam. Math., 1, 93-104 (1920) · Zbl 47.0179.02

[83] Stromberg, K., An elementary proof of Steinhaus's theorem, Proc. Am. Math. Soc., 36, 308 (1972) · Zbl 0278.28013

[84] Tomasz, P., A restriction theorem for the Fourier transform, Bull. Am. Math. Soc., 81, 2, 477-478 (1975) · Zbl 0298.42001

[85] Van Deun, J.; Cools, R., Algorithm S58: computing infinite range integrals of an arbitrary product of Bessel functions, ACM Trans. Math. Softw., 32, 4, 580-596 (2006), Associated computer program available online · Zbl 1230.65027

[86] Van Deun, J.; Cools, R., A Matlab implementation of an algorithm for computing integrals of products of Bessel functions, Mathematical Software. Mathematical Software, ICMS 2006. Mathematical Software, ICMS 2006, Lecture Notes in Comput. Sci., vol. 4151 (2006), Springer: Springer Berlin), 289-295 · Zbl 1370.65015

[87] Van Deun, J.; Cools, R., Integrating products of Bessel functions with an additional exponential or rational factor, Comput. Phys. Commun., 178, 8, 578-590 (2008), Associated computer program available online · Zbl 1196.65059

[88] Watson, G. N., A Treatise on the Theory of Bessel Functions (1944), Cambridge University Press/The Macmillan Company: Cambridge University Press/The Macmillan Company Cambridge, England/New York · Zbl 0094.04301

[89] Werner, D., Funktionalanalysis (2018), Springer-Verlag: Springer-Verlag Berlin · Zbl 1395.46001

[90] Zhou, Y., Wick rotations, Eichler integrals, and multi-loop Feynman Diagrams, Commun. Number Theory Phys., 12, 1, 127-192 (2018) · Zbl 1393.81029

[91] Zhou, Y., On Borwein’s conjectures for planar uniform random walks, J. Aust. Math. Soc., 107, 3, 392-411 (2019) · Zbl 1472.60081

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.