Population effects on the Red Giant Clump absolute magnitude, and distance determinations to nearby galaxies

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ABSTRACT

The red giant clump has been recently argued to be a reliable distance indicator for the galaxies in the Local Group. The accuracy of distance determinations based on this method, however, depends on the possible presence of systematic magnitude differences ($\Delta M_{RC}$) between the local clump revealed by the Hipparcos colour-magnitude diagram, and the clump stars observed in distant galaxies.

In this paper, we re-address the problem of these systematic ‘population’ effects. First, we present tables with the theoretically-predicted $I$-band clump magnitude as a function of age and metallicity. Simple equations, taken from basic population synthesis theory, are provided for the easy computation of the mean clump magnitude for any given galaxy model. We use our models to explain in some detail what determines the distribution of masses, ages, and metallicities of clump stars in a galaxy. Such an approach has so far been neglected in the analysis of clump data related with distance determinations. We point out that, in galaxies with recent/ongoing star formation (e.g. the disks of spirals), the age distribution of clump stars is strongly biased toward younger ($\sim 1 \text{--} 3 \text{ Gyr}$) ages, and hence toward higher metallicities. Obviously, this does not happen in galaxies with predominantly old stellar populations (e.g. ellipticals and bulges).

We construct detailed models for the clump population in the local (Hipparcos) sample, the Bulge, Magellanic Clouds, and Carina dSph galaxy. In all cases, star formation rates and chemical enrichment histories are taken from the literature. The Hipparcos model is shown to produce distributions of metallicities, colours, and magnitudes, that are similar to those derived from spectroscopic and Hipparcos data. The Bulge, Magellanic Clouds, and Carina dSph models are used to analyse the values of $\Delta M_{RC}$ for these different stellar systems. We show how the clump–RR Lyrae data from Udalski (1998a) is well reproduced by the models. However, despite the similarity between the models and data, the models indicate that the linear $\Delta M_{RC}$ versus [Fe/H] relations that have been derived from the same data (Udalski 1998a, 2000; Popowski 2000) are not general. In fact, the distribution of clump stars has several factors hidden in it – e.g. the age-metallicity relation, the rate of past star formation – that cannot be described by such relations.

The model behaviour is also supported by empirical data for open clusters by Sarajedini (1999) and Twarog et al. (1999). We argue that Udalski’s (1998b) data for LMC and SMC star clusters do not allow a good assessment of the age dependence of the clump magnitude. Moreover, we remark that similar analyses of cluster data should better include clump stars with ages $1 \text{--} 2 \text{ Gyr}$, which turn out to be very important in determining the mean clump in galaxies with recent star formation.

Finally, we provide revised clump distances to the Bulge, Magellanic Clouds, and Carina dSph, and further comment on their reliability. The largest $\Delta M_{RC}$ values are found for the Magellanic Clouds and Carina dSph, which turn out to be located at distance moduli $\sim 0.2 \text{--} 0.3$ mag longer than indicated by works which ignore population effects. The Galactic Bulge, instead, may be slightly closer (up to 0.1 mag in distance modulus) than indicated by previous works based on the red clump, the exact result depending on the use of either scaled-solar or $\alpha$-enhanced stellar models.

Key words: Hertzsprung-Russell (HR) diagram – stars: horizontal branch – stars: luminosity function, mass function – solar neighbourhood – Magellanic Clouds – galaxies: stellar content
1 INTRODUCTION

The red giant clump has been recently claimed to provide a very accurate standard candle. Once the mean I-band magnitude of the red clump, \( I^{\text{RC}} \), is measured in a nearby galaxy, its absolute distance modulus \( \mu_0 = (m - M)_0 \) is easily derived by means of

\[
\mu_0 = I^{\text{RC}} - I^{\text{RC}}_1 - A_I + \Delta M^{\text{RC}}_1. \tag{1}
\]

In this equation, \( I^{\text{RC}}_1 \) is the reference value (zero point) provided by nearby clump stars whose distances are known from \textit{Hipparcos} trigonometric parallaxes, \( A_I \) is the interstellar extinction to the red clump population of an external galaxy, and \( \Delta M^{\text{RC}}_1 = M^{\text{RC}}_1(\text{Hipp}) - M^{\text{RC}}_1(\text{galaxy}) \) is the population effect, i.e. the difference of the mean red clump absolute magnitude between the local and external samples of stars.

Determining \( M^{\text{RC}}_1 \) and \( I^{\text{RC}} \) is, usually, less of a problem. Both in the \textit{Hipparcos} database of nearby stars, and in CMDs covering even a small fraction of a nearby galaxy, one typically finds several hundreds of clump stars, clearly identifiable by their position in the CMD. Then, by performing a non-linear least-square fit of the function

\[
N(I) = a + bI + cI^2 + d \exp\left[\frac{(I^{\text{RC}} - I)^2}{2\sigma_I^2}\right], \tag{2}
\]

to the histogram of stars in the clump region per magnitude bin [i.e. the luminosity function \( N(I) \), the value of \( I^{\text{RC}} \) and its associated standard error are easily determined (Stanek & Garnavich 1998)]. The parameter \( \sigma_I \) gives a good indication of how sharp the luminosity distribution of clump stars is, whereas \( a, b, c \) (also derived from the fitting procedure) are constants of less interest.

By applying this procedure to the \textit{Hipparcos} database of nearby stars (closer than, say, 70 pc), \( M^{\text{RC}}_1 \) has been determined with accuracy of hundredths of magnitude (Paczyński & Stanek 1998; Stanek, Zaritsky & Harris 1998). Similar accuracies are obtained for \( I^{\text{RC}} \) in nearby systems like the Galactic Bulge (Paczyński & Stanek 1998), the Magellanic Clouds (Udalski et al. 1998), and M 31 (Stanek et al. 1998). In all cases, the accuracy is limited mainly by the calibration of the photometry, rather than by number statistics of clump stars.

Therefore, the main concerns in the use of clump stars as standard candles are in the determination of the extinction \( A_I \), and the population effects \( \Delta M^{\text{RC}}_1 \). The Magellanic Clouds provide emblematic examples of the problems associated with both determinations. In fact, the first applications of the red clump method to the LMC have provided extremely short distances (e.g. \( \mu_0^{\text{LMC}} = 18.1 - 18.2 \) mag; see Udalski et al. 1998 and Stanek et al. 1998) if compared with the values commonly accepted a few years ago (\( \mu_0^{\text{LMC}} \approx 18.5 \pm 0.1 \) mag; see Westerlund 1997), and in strong disagreement with the latest values derived from classical methods as the Cepheids P-L relation (e.g. \( \mu_0^{\text{LMC}} = 18.7 \pm 0.1 \) mag cf. Feast & Catchpole 1997). These ‘short’ LMC distances have been suspected to arise from errors in estimating either \( A_I \), or \( \Delta M^{\text{RC}}_1 \), or both.

With respect to \( A_I \), Romaniello et al. (1999) and Zaritsky (1999) recently claimed that the \( A_I \) values used for LMC clump stars in Stanek et al. (1998) are overestimated by about 0.2 mag, thus resulting in an underestimate of \( \mu_0^{\text{LMC}} \) by the same amount. Both argue to obtain more reliable estimates of \( A_I \), either using high-quality multi-band HST photometry as Romaniello et al. (1999), or determining the reddening directly from red stars (from 5500 to 6500 K) in the observed fields, instead of using mean reddening values (Zaritsky 1999).

In this paper, we address the subtle problem of determining the population effect \( \Delta M^{\text{RC}}_1 \). This factor has been initially neglected (Paczyński & Stanek 1998; Udalski et al. 1998; Stanek et al. 1998), after noticing that \( I^{\text{RC}} \) is remarkably constant, as a function of the \( V-I \) colour, in several stellar systems like the Bulge (Paczyński & Stanek 1998), M 31 (Stanek et al. 1998), LMC and SMC (Udalski et al. 1998; Stanek et al. 1998), especially in the colour range 0.8 < \( V-I \) < 1.5 which defines the local clump from \textit{Hipparcos} (Paczyński & Stanek 1998). However, Cole (1998) and Girardi et al. (1998, hereafter GGWS98) called attention to the non-negligible \( \Delta M^{\text{RC}}_1 \) values (up to 0.6 mag according to Cole 1998) expected from theoretical models of clump stars, and claimed a red clump distance to the LMC larger by about \( \Delta M^{\text{RC}}_1(\text{LMC}) = 0.2 - 0.3 \) mag. GGWS98, and later Girardi (1999), provided simulations of the red clump in model galaxies which naturally showed the effect of an almost constant \( M^{\text{RC}}_1 \) with \( V-I \), and presented significant \( \Delta M^{\text{RC}}_1 \) values at the same time.

In two successive papers, Udalski (1998ab) aimed at empirically determining the dependence of \( M^{\text{RC}}_1 \) on stellar parameters. The result was a very modest dependence on both metallicity and age. Namely, \( M^{\text{RC}}_1 = (0.09 \pm 0.03) \left[ \text{[Fe/H]} + \text{const.} \right] \), was obtained by comparing red clump and RR Lyrae stars in the Bulge, LMC, SMC, and Carina dSph, and assuming \( M^{\text{RRLy}}_V = (0.18 \pm 0.03) \left[ \text{[Fe/H]} + \text{const.} \right] \) (Udalski 1998a). Moreover, by comparing the clump in several LMC and SMC clusters, Udalski (1998b) claimed negligible changes in \( M^{\text{RC}}_1 \) for cluster ages ranging from 2 to 10 Gyr. More recently Udalski (2000) fits a \( M^{\text{RC}}_1 = (0.13 \pm 0.07) \left[ \text{[Fe/H]} + \text{const.} \right] \) relationship to the nearby clump stars with spectroscopic \([\text{Fe/H}]\) determinations. All together, these results would imply quite modest \( \Delta M^{\text{RC}}_1 \) values (less than 0.1 mag for the LMC), thus still supporting a ‘short distance scale’, as opposed to the ‘long distance scale’ provided by, e.g. Cepheids and RR Lyrae calibrated by subdwarf fitting to globular clusters (see Feast 1999 and Carretta et al. 2000 for recent reviews).

Udalski’s (1998ab, 2000) results have been extensively used in subsequent works regarding the clump method for distance determinations. Zaritsky (1999) and Popowski (2000), for instance, attempted to reduce the errors in the red clump method associated with reddening estimates, but assumed that Udalski’s (1998ab, 2000) conclusions were valid. However, there are some potential drawbacks in Udalski’s analyses that we think should be considered in more detail. These drawbacks, in fact, are among the main points we wish to address in this paper.

A subtle one is that \( M^{\text{RC}}_1 \) is assumed either not to depend on ages (as in Udalski 1998a, 2000), or not to depend on metallicities (as in Udalski 1998b). Thus, the presence of an age-metallicity relation (AMR) in the observational data could be masking a possible dependence of \( M^{\text{RC}}_1 \) on both parameters, provided that this dependence were in the sense

* Throughout this paper we will refer to the Cousins \( I \)-band
of producing brighter clumps at both lower metallicities and younger ages. And these are exactly the trends indicated by theoretical models.

We should also mention the particular selection of Magellanic Cloud clusters in Udalski (1998b). They are limited to the 2–10 Gyr age interval, that is not the complete range of possible ages of clump stars. Moreover, quite large ‘distance corrections’ were adopted for some SMC clusters in Udalski (1998b); see comments in Girardi (1999), which could also be the source of systematic errors in his analysis of the \( M_1^{RC} \) dependence on age. It is worth noticing that, although Udalski (1998b) finds evidence that the clump fades by 0.3–0.4 mag at later ages, this particular result has been ignored in later works, where the clump is simply assumed not to depend on age. At the same time Tworog et al. (1998) and Sarajedini (1999), using data from open clusters with main-sequence fitting distances, reached conclusions apparently in contradiction with Udalski’s ones, i.e. favouring larger dependences of \( M_1^{RC} \) on either age or metallicity.

Another point of perplexity to us, is that Udalski (1998,ab, 2000) and later Popowski (2000) decide to express the population dependence of the clump magnitude by means of simple linear relationships between \( M_1^{RC} \) and [Fe/H], and between \( M_1^{RC} \) and age. In our opinion, these choices are not justified. To this respect, we notice that similar relations are largely used in stellar astrophysics; but they are, in most cases, well supported by theory. A classical example of this kind comes from RR Lyrae stars, for which a linear \( M_V \) versus [Fe/H] relation is well documented and widely accepted. However, we should keep in mind that RR Lyrae are very particular objects, which occupy a quite narrow range in both physical (\( T_{eff} \)) and population (age, metallicity) properties. Clump stars, on the contrary, do not have such tight constraints: they are almost ubiquitous, with a very large range of ages, metallicities, and \( T_{eff} \). Clearly, nature has much more freedom to play with the luminosity of clump stars, than it has with RR Lyrae. Why should we expect both types of stars to behave in a similar way? Then, why should we expect linear relations to hold for the properties of clump stars?

To clarify these points, in the following we explore the several systematic effects on \( M_1^{RC} \) indicated by stellar evolutionary models. In Sect. 2, the metallicity and age effects are described with some detail. We present simple formulas for computing the mean clump magnitude in a galaxy, then allowing the readers to determine the appropriate value for the population correction \( \Delta M_1^{RC} \) for any given history of star formation and chemical enrichment. We then present a careful comparison between models and the local clump from Hipparcos (Sect. 3), which is particularly helpful to illustrate the different effects involved in the problem, and to understand Udalski’s (2000) results. The clump behaviour in Galactic, LMC and SMC clusters is briefly commented on in Sect. 4. We proceed by modelling the clump in several nearby galaxies, and comparing the results with the data discussed by Udalski (1998a) and Popowski (2000) (Sect. 5). Overall, we find that systematic effects (at the level of \( \lesssim 0.3 \) mag) are not only present in any determination of red clump distances to nearby galaxies, but are also hidden in many of the previous analyses of clump data discussed in this regard. Our final conclusions regarding the red clump distance scale to nearby galaxies are presented in Sect. 6.

2 THE CLUMP AS A FUNCTION OF AGE AND METALlicity

As in previous works, we will base the discussion of model behaviour mostly on the Girardi et al. (2000) set of evolutionary tracks and isochrones. These models have been extensively discussed in GGWS98 and Girardi (1999). It is worth remarking that different models in the literature – although presenting systematic luminosity differences for the core helium burning (CHeB) stars that have passed through the helium flash – present similar behaviours for this luminosity as a function of either age or metallicity (Castellani et al. 2000). The model behaviour is also supported by empirical data for clusters (see Sect. 3). Moreover, the formulas provided in this section allow the reader to check our results using any alternative set of stellar models.

2.1 Simple models for the mean clump magnitude

In our previous works (GGWS98 and Girardi 1999) we have discussed the behaviour of the clump as a function of mass, age and metallicity, considering mainly the sequences of zero-age horizontal branch models (ZAHB), i.e. of the stellar configurations at the beginning of quiescent CHeB. This time, however, we prefer to discuss the behaviour of the mean clump as a function of age and metallicity.

The properties of the mean clump are defined as follows. For a given isochrone of age and metallicity (\( t, Z \)), we perform the following integrals over the isochrone section corresponding only to CHeB stars:

\[
\langle M_\lambda(t,Z) \rangle = -2.5 \log \left[ \frac{1}{N_{cl}(t,Z)} \int_{M_{\text{CHeB}}} \phi(m_\lambda) 10^{-0.4 M_\lambda} \, dm_\lambda \right] \tag{3}
\]

where \( M_\lambda \) is the absolute magnitude in the pass-band \( \lambda \), \( m_\lambda \) is the initial mass of the star at each isochrone point, and \( \phi(m_\lambda) \) is the Salpeter IMF (number of stars by initial mass interval \([m_{\text{CHeB}}, m_\lambda + dm_\lambda] \)). \( N_{cl} \) is the number of clump stars (at age \( t \)) per unit mass of stars initially born. It is simply given by the integral of the IMF by number, along the CHeB isochrone section, i.e.

\[
N_{cl}(t, Z) = \int_{M_{\text{CHeB}}} \phi(m_\lambda) \, dm_\lambda . \tag{4}
\]

In our case, the IMF is normalised such as to produce a single-burst stellar population of total initial mass of 1 \( M_\odot \) (i.e. \( \int \phi(m_\lambda) \, dm_\lambda = 1 \, M_\odot \)), and a mass-to-light ratio of \( M/L_V = 0.2 \) at an age of \( 10^8 \) yr. The details of this normalisation can be found in Girardi & Bica (1993) and Salasnich et al. (2000). It is worth remarking that none of the results presented in this paper depends on the particular choice of IMF normalisation. However, having an IMF normalised to unit mass turns out to be a convenient choice.

From eq. (3), accurate values for the mean clump colours and magnitudes can be obtained. Table 1 presents the values of \( \langle M_\lambda \rangle \), \( \langle V-I \rangle = \langle M_V \rangle - \langle M_\lambda \rangle \), and \( N_{cl} \) obtained for ages ranging from 0.5 to 12 Gyr, and metallicities from \( Z = 0.0004 \) to 0.03. More extensive tables, suitable to perform accurate interpolation in the quantities \( (t, Z) \), are available in computer-readable form in http://pleiadi.astro.it. Part of this information is also illustrated in Fig. 1.

Another useful quantity included in Table 1 is the mean initial mass of clump stars,
Mean clump properties, as a function of age and metallicity, from Girardi et al. (2000) isochrones.

\[
\langle m_i(t,Z) \rangle = \frac{1}{N_{\text{cl}}(t,Z)} \int \chi^\text{Clump} m_i \phi(m_i) \, \text{d}m_i. \tag{5}
\]

Figure 3 presents the I-band luminosity function (LF) for several single-burst stellar populations, compared to the mean magnitudes calculated according to eq. (5). It is worth noticing that:

(i) Since the range of clump magnitudes at a given \((t, Z)\) is small, it does not really matter whether the \(\langle M_\lambda \rangle\) integral is performed over luminosities (as in eq. (5)) or magnitudes. The resulting \(\langle M_\lambda \rangle\) values are always accurate to within \(\sim 0.01\) mag. In fact, what limits the accuracy of eq. (5) is, mainly, the coarseness of the library of stellar evolutionary tracks used to construct the isochrones.

(ii) The intrinsic dispersion of clump magnitudes is slightly larger for younger ages and lower metallicities. For high metallicities and old ages, the LF of clump stars is characterized by a sharp spike that coincides with the ZAHB bin, and a decreasing tail for higher luminosities. Thus, the mean values \(\langle M_\lambda \rangle\) are always brighter than the maximum of the LF. This offset is of order 0.1 mag, for the several ages and metallicities considered in the figure.

Therefore, reducing the clump of a single generation of stars to a single point of magnitude \(\langle M_\lambda(t,Z) \rangle\), should be seen as a useful approximation, rather than the detailed behaviour indicated by models.

So far, the equations and data refer to the clump as expected in single-burst stellar populations, i.e. in star clusters of given age and metallicity \((t, Z)\). In order to compute the mean clump magnitude for a given galaxy model, of total age \(T\), we need to perform the following integral:
Table 1. (continued)

| $t$ (Gyr) | $N_{cl}(10^{-4})$ | $(M_\odot)$ | $(M_\odot)$ | $(M_\odot)$ |
|----------|------------------|------------|------------|------------|
| 0.5      | 34.30            | 2.670      | -0.113     | -1.001     |
| 0.6      | 40.10            | 2.492      | 0.174      | -0.718     |
| 0.7      | 44.90            | 2.352      | 0.586      | -0.538     |
| 0.8      | 48.50            | 2.236      | 0.508      | -0.392     |
| 0.9      | 51.80            | 2.139      | 0.627      | -0.276     |
| 1.0      | 54.80            | 2.056      | 0.719      | -0.188     |
| 1.1      | 56.70            | 1.985      | 0.791      | -0.120     |
| 1.2      | 54.30            | 1.930      | 0.770      | -0.148     |
| 1.3      | 53.50            | 1.874      | 0.674      | -0.251     |
| 1.4      | 27.80            | 1.788      | 0.575      | -0.371     |
| 1.5      | 23.20            | 1.739      | 0.552      | -0.401     |
| 1.6      | 20.60            | 1.698      | 0.548      | -0.410     |
| 1.7      | 18.80            | 1.662      | 0.542      | -0.421     |
| 1.8      | 17.10            | 1.629      | 0.542      | -0.424     |
| 1.9      | 16.30            | 1.600      | 0.541      | -0.429     |
| 2.0      | 15.30            | 1.574      | 0.540      | -0.433     |
| 2.1      | 14.10            | 1.526      | 0.556      | -0.422     |
| 2.2      | 13.30            | 1.484      | 0.573      | -0.409     |
| 2.3      | 12.80            | 1.447      | 0.591      | -0.393     |
| 2.4      | 12.30            | 1.413      | 0.610      | -0.375     |
| 2.5      | 11.00            | 1.383      | 0.619      | -0.369     |
| 2.6      | 10.70            | 1.357      | 0.628      | -0.363     |
| 2.7      | 10.50            | 1.332      | 0.635      | -0.357     |
| 2.8      | 10.30            | 1.310      | 0.643      | -0.352     |
| 2.9      | 10.00            | 1.289      | 0.653      | -0.343     |
| 3.0      | 9.67             | 1.269      | 0.665      | -0.332     |
| 3.1      | 9.14             | 1.241      | 0.681      | -0.316     |
| 3.2      | 8.29             | 1.216      | 0.696      | -0.303     |
| 3.3      | 7.93             | 1.194      | 0.705      | -0.295     |
| 3.4      | 7.68             | 1.174      | 0.714      | -0.286     |
| 3.5      | 7.43             | 1.155      | 0.724      | -0.277     |
| 3.6      | 6.47             | 1.127      | 0.738      | -0.264     |
| 3.7      | 6.16             | 1.101      | 0.756      | -0.245     |
| 3.8      | 5.87             | 1.080      | 0.771      | -0.230     |
| 3.9      | 5.59             | 1.061      | 0.784      | -0.216     |
| 4.0      | 5.32             | 1.044      | 0.795      | -0.205     |
| 4.1      | 5.02             | 1.013      | 0.819      | -0.179     |
| 4.2      | 4.71             | 0.985      | 0.840      | -0.155     |
| 4.3      | 4.65             | 0.961      | 0.859      | -0.131     |
| 4.4      | 4.42             | 0.939      | 0.880      | -0.102     |

$\langle M_\lambda(gal) \rangle = \frac{1}{N_{cl}(gal)} \int_{t=0}^{T} \psi(t) \langle M_\lambda(t, Z) \rangle \, dt,$

where

$N_{cl}(gal) = \int_{t=0}^{T} N_{cl}(t, Z) \psi(t) \, dt.$

The function $\psi(t)$ is the star formation rate (SFR, in $M_\odot$ by unit time), at a moment $t$ in the past, for the galaxy model considered. Last but not least, also the age-metallicity relation (AMR) $Z(t)$ should be specified. Notice that we average the magnitudes in eq. (6) instead of luminosities as in the previous eq. (4). The reason for this is that, in this way, we get a quantity similar to the $I^{RC}$ derived in empirical works by means of eq. (3).

2.2 Comparison between different methods

From eq. (3) and the numbers tabulated in Table 1, the reader can easily derive the theoretical mean clump magnitudes for any galaxy model, once the SFR $\psi(t)$ and AMR $Z(t)$ are provided. This is a simple and almost direct way of deriving $M^{RC}_{I}$ from theoretical models. We will refer to it, hereafter, as method 1. There are some additional details about this method, which are worth mentioning:

(i) Some interpolation among the age and metallicity values presented in the tables (e.g. Tab. 1) may be required. In this case, our experience is that the most accurate interpolations in metallicity are obtained using $\log Z$ or $[\text{Fe}/\text{H}] \approx \log(Z/0.019)$ as the independent variable, instead of $Z$.

(ii) For galaxy models with ongoing star formation, the
integral of eq. (6) should be given a lower-age cut-off of at least 0.5 Gyr. This because CHeB stars of younger ages, although few, have much higher luminosities, so that they cannot be considered clump stars (see Fig. 1). The final results for $M_{RC}^I$ depend slightly on the choice of this cut-off. In the models discussed in this section, we will assume it equal to 0.5 Gyr.

A second, more refined approach, comes from a complete population synthesis algorithm: synthetic CMDs (or simply the stellar LF) are simulated for a given galaxy model, and then $M_{RC}^I$ is derived by fitting eq. (2) to the synthetic data. This is the approach followed by GGWS98, and will hereafter be referred as method 2. Its advantage is that the synthetic CMDs contain all the information about the distribution of stellar luminosities and colours, which can then be easily compared to actual observations of clump stars. (Additional effects such as sample size, photometric errors, or differential reddening, could also be simulated if required.) However, method 2 is much more demanding, because it requires tools – a population synthesis code, and a non-linear least squares fitting routine – which are not needed in method 1.

For our purposes, the interesting point is to test whether the complexity of method 2 pays for the additional effort with a higher accuracy than method 1. Thus, we verify whether both methods lead to similar results for $M_{RC}^I$. Using our population synthesis code (Girardi, unpublished), we generate two different galaxy models. The first, model A, assumes a constant SFR from $T = 10$ Gyr ago until now, and a metallicity only slightly increasing with the galaxy age $T - t$, i.e. $Z(t) = 0.008 + 0.011 \cdot (1 - 0.1 t(Gyr))$. This represents a relatively young and metal-rich galaxy population, as found in the discs of spirals. The second, model B, assumes a predominantly old population, i.e. with constant SFR from 8 to 12 Gyr ago, and the complete range of metallicities given by $Z(t) = 0.0004 + 0.0296 \cdot (1 - 0.25 \cdot (t(Gyr) - 8))$.

For both models A and B, the mean clump magnitude is computed either with method 1 (table 1 plus eq. 6) or with method 2 (synthetic CMD plus eq. 2). The results are presented in Table 2, and in Fig. 3.

As seen, method 1 provides $M_{RC}^I$ values about 0.05 mag brighter than method 2. This offset results, mainly, from the
luminosity of the clump LF maximum always being slightly fainter than the mean clump brightness (see Fig. 2): whereas the maximum is more easily accessed by method 2, method 1 always accesses the mean.

Anyway, the important result is that both methods provide almost the same magnitude difference between the two galaxy models, i.e. \( \Delta M_{\text{RC}}^{\text{B}}(A - B) \) is equal to \(-0.153 \) cf. Method 1, and \(-0.137 \) cf. Method 2 (already on the base of of this simple exercise, we should reasonably expect that the red clump is about \(0.14 - 0.15\) mag brighter in a late spiral’s disk than in an old elliptical galaxy). Thus, the results from the two methods turn out to be very similar. In the following we will always apply method 2, based on synthetic CMDs, since it permits to compare in more detail the LF of the synthetic red clump for a given stellar system with the corresponding observational data. Moreover, any value of \( \Delta M_{\text{RC}}^{\text{B}} \) will refer to the difference between the ‘local’ clump simulation and a synthetic model for the stellar system under scrutiny. Our results can be checked by any reader, by using the more simple method 1.

### 3 AN ANALYSIS OF THE HIPPARCOS CLUMP

The clump stars in the Solar Neighbourhood (or ‘the Hipparcos clump’ as hereafter referred) are fundamental for distance determinations based on clump stars, because they provide the only empirical zero point for eq. 1 that can be measured with good accuracy. The ESA (1997) catalog contains \(\sim 1500\) clump stars with parallax error lower than 10 per cent, and hence standard errors in absolute magnitude lower than \(0.21\) mag. The sample defined by this accuracy limit is complete up to a distance of about \(125\) pc. Accurate \(BV\) photometry is available for these stars (and also \(I\) for \(\sim 1/3\) of them), and the interstellar absorption is small enough to be neglected (Paczyński & Stanek 1998). The Lutz-Kelker bias should not affect their mean absolute magnitude determination by more than \(0.03\) mag (GGWS98).

Therefore, the intrinsic photometric properties of the Hipparcos clump can be said to be known with high accuracy, when aspects such as data quality and number statistics are considered.

However, the same can not be said about the population parameters of these stars, i.e. their distributions of masses, ages, and metallicities. Since it was only after Hipparcos that a significant number of clump stars was identified in the Solar Neighbourhood (Perryman et al. 1997), few works attempted to describe these stars in terms of their parent populations. This aspect, of course, is of fundamental importance in the present work.

A comprehensive but short discussion of the nearby clump stars’ masses, ages, and metallicities, is presented by Girardi (2000). From this latter work, we select and develop the following points which are more relevant to the present study. More specifically, we discuss general aspects in the mass, age, and CMD distribution of clump stars that may be applied to any galaxy, then focusing on the specific case of the Hipparcos clump.

#### 3.1 The mass, age and metallicity distributions: general aspects

The mass distribution of core-helium burning (CHeB) stars in a galaxy of total age \(T\), is roughly proportional to the IMF \(\phi(m_i)\), to the core-helium burning lifetime \(t_{\text{He}}(m_i)\) of each star, and to the star formation rate (SFR) at the epoch of its birth, \(\psi(t(m_i))\) (where \(t\) is the stellar age, and not the galaxy age). There is also a low-mass cut-off, given by the lowest mass to leave the main sequence at ages lower than \(T\). Since \(t_{\text{He}}(m_i)\) presents a peak at about \(2\) \(M_\odot\) (the transition from low to intermediate masses), and the IMF shows a peak at the lowest masses, a double-peaked mass distribution turns out for CHeB stars (Girardi 1999). This is shown in the left panel of Fig. 1 for the case of a constant SFR from 0.1 to 10 Gyr ago, and a Salpeter IMF. This distribution contrasts with the more natural idea of a clump mass distribution roughly following the IMF – which would present only a single peak at the lowest possible masses, i.e. at \(0.8-1.2\) \(M_\odot\).

Moreover, from these simple considerations, it turns out that the intermediate-mass stars from say 2 to 2.5 \(M_\odot\), are not severely under-represented in the mass distribution, with respect to the low-mass ones. Stars with 2 to 2.5 \(M_\odot\) are close enough to the clump region in the CMD, to be considered as genuine clump stars (GGWS98). For the case of a constant SFR from now up to ages of 10 Gyr, they would make about 20 per cent of the clump. Again, this contrasts with the common idea that the clump is formed only by low-mass stars.

Let us now consider the age distribution expected for clump stars. The number of evolved stars of a certain type and age is proportional to: (1) their birth rate at the present time, and (2) their lifetime at the evolved stage under consideration (see e.g. Tinsley 1980). The birth rate of clump stars, \(dN_{\text{cl}}/d\tau_1\), can be estimated as

![Figure 3. I-band luminosity functions for galaxy Models A and B (see text). The continuous histogram represents the synthetic LF as obtained from a population synthesis code, whereas the dashed line is the result of a least-squares fit of eq. 1. For both models vertical lines indicate the mean \(M_{\text{RC}}^{\text{B}}\) as obtained either with method 1 (continuous) or with method 2 (dashed).](image-url)
\[ \frac{dN_{\text{cl}}}{dt} = \psi(t = \tau_H) \phi(m_{\text{TO}}) \left| \frac{dm_{\text{TO}}}{d\tau_H} \right| \]  

(8)

where \( m_{\text{TO}} \) is the turn-off mass corresponding to a given main sequence lifetime \( \tau_H \), and \( \psi(t = \tau_H) \) is the star formation rate at the epoch of stellar birth.

For clump stars, the age distribution is obtained by multiplying this birth rate by the CHeB lifetime \( \tau_H \). In the right panel of Fig. 4, we show the result for the case of a galaxy with a constant SFR over all its lifetime. This age distribution turns out to be far from constant: it peaks at an age of 1 Gyr, and decreases monotonically afterwards. In the particular case here illustrated, half of the clump stars have ages lower than 2 Gyr. This result is in sharp contrast with the common idea that clump stars trace equally well the intermediate-age and old components of a galaxy.

These aspects of the age distribution are so important that it is worth commenting on them in more detail:

(i) The continuous decline in the clump age distribution at ages larger than about 2 Gyr, comes, essentially, from the continuous decrease, with the stellar age, of the birth rate of post-main sequence stars. It can be easily understood as follows: between 0.8 and 2 \( M_\odot \), the main sequence lifetime scales approximately as \( \tau_H \propto m_{\text{TO}}^{-3.5} \) (cf. Girardi et al. 2000 tables), whereas the lifetime of CHeB stars \( \tau_{\text{He}} \) is roughly constant at about 10^6 yr. Together with a Salpeter IMF (\( \propto m_{\text{TO}}^{-2.35} \)), this implies that \( dN_{\text{cl}}/d\tau_H \) (and also the age distribution function) scales as \( \propto t^{-0.6} \) for ages \( t \gtrsim 2 \) Gyr.

(ii) At ages of about 1 Gyr, \( \tau_{\text{He}} \) has a local maximum (about 2.5 \( \times 10^6 \) yr, corresponding to the star with \( m_{\text{TO}} = 2 \ M_\odot \)), which sensibly increases the amplitude of the age distribution at that age.

(iii) At younger and decreasing ages, \( \tau_{\text{He}} \) decreases slightly faster than the birth rate increase of CHeB stars, then causing a decrease in the age distribution function. Even if the number distribution of CHeB stars does not become negligible as we go to younger ages, the ones with ages lower than 0.5 Gyr can hardly be classified as clump stars, since they become much brighter than the clump. This justifies defining a cut-off in the age distribution function for ages \( \tau_H < 0.5 \) Gyr.

Finally, it should be noticed that, for the case of a constant SFR, the number of clump stars per unit mass of born stars \( N_{\text{cl}} \), as obtained from complete population synthesis models (eq. 6), is simply proportional to the age distribution function of clump stars we have here obtained from simple considerations about stellar lifetimes and birth rates:

\[ N_{\text{cl}} \propto \phi(m) \tau_{\text{He}} N_{\text{cl}}/dt, \]

This result is just expected, and can be well appreciated by comparing the right panel of Fig. 4 with the upper panel of Fig. 1. The similarity between these two figures, gives us even more confidence in the numbers derived from eq. 4.

The hypothesis of constant SFR, above illustrated, is just a particular case of a most common one, namely that of continued SFR for most of a galaxy’s history which in general applies to the disc of spiral and irregular galaxies. When the SFR has not been constant, the age distribution function of clump stars can be simply evaluated by multiplying the SFR, at any given age, by the curves in Fig. 4.

### 3.2 Simulating the Hipparcos clump

After these introductory considerations of general validity, let us consider the specific case of the local Hipparcos clump. The local SFR is almost certainly not constant, but there are good indications that it has been continuous from today back to at least 9 Gyr ago (e.g. Carraro 2000). This continuity renders the age and mass distribution of clump stars qualitatively similar to the cases illustrated in Fig. 4.

Rocha-Pinto et al. (2000b), from a sample of nearby dwarfs with ages determined from their chromospheric activity, find a SFR history marked by several (and statistically significant) ‘bursts’ up to the oldest ages. This work also indicates a volume-corrected age-distribution function which is higher for the youngest stars, as can be seen in Fig. 4. These properties are also supported by other recent works, based on the analysis of Hipparcos CMD. Namely, Bertelli & Nasi (2000) favour increasing rates of star formation in the last few Gyr, whereas Hernandez, Valls-Gabaud & Gilmore (2000b) also find evidences for different bursts in a sample of Hipparcos stars limited to ages lower than 3 Gyr.

In the following, we adopt the results from Rocha-Pinto et al.’s (2000ab) data in order to simulate the local CMD. This work covers the complete range of ages we are interested in. Before proceeding, we should remind that that stars born at older ages in the local disc, are now distributed over higher scale heights in relation to the Galactic plane. Therefore, in our simulations of the Hipparcos sample, we should use the volume-limited age distribution of unevolved dwarfs.
not corrected by any scale-height factor, rather than the cumulative SFR in the so-called ‘solar cylinder’. Rocha-Pinto et al. (2000b) kindly provided us with the uncorrected age distribution we need. Moreover, we have also the possibility of using the local AMR, derived by Rocha-Pinto et al. (2000a) from the same data. Both functions are illustrated in Fig. 5.

Figure 6 shows the final CMD simulated from these data. For the sake of simplicity, we have not simulated – the same applies also to the simulations presented in Sect. 3.1 – observational errors (as discussed in GGWS98 the inclusion of observational errors increases the width $\sigma_M$ of the clump stars’ LF, leaving almost unchanged the value of $M_{RC}^I$). Fig. 6 shows the resulting LF of clump stars from this simulation, together with the result of fitting eq. 1.

Even if in this paper we are discussing population effects (i.e., the value of $\Delta M_{RC}$ for different stellar systems) on the red clump brightness, and we will use theoretical models only in a differential way, it is nevertheless interesting to note that the absolute value of $M_{RC}^I = -0.171$ for the local clump as derived from our simulations is satisfactorily close to the observed one, $M_{RC}^I = -0.23 \pm 0.03$ (Stanek et al. 1998).

### 3.3 The observed metallicity distribution

Let us now compare the results from our Hipparcos simulation, with additional data for nearby clump stars. Their metallicity distribution has been the subject of some unexpected results in the recent past.

There have been few attempts to derive typical metallicities for Hipparcos clump stars. The first one has been a quite indirect method by Jimenez et al. (1998). They applied the concept that red giants become redder at higher metallicities to derive, solely from the colour range of Hipparcos clump stars, an estimate of their metallicity range, obtaining $-0.7 < [\text{Fe/H}] < 0.0$. This approach was based on the fact that in Jimenez et al. (1998) models the clump at a given metallicity is very concentrated on the CMD, and has a mean colour very well correlated with metallicity. In this regard, however, we should consider that the galaxy models considered by Jimenez et al. were characterized by a SFR strongly decreasing with the galaxy age (i.e. increasing with the stellar age). This implies that they were considering, essentially, the behaviour of the old clump stars, with masses of about $0.8 - 1.4 M_\odot$. Intermediate-mass clump stars with mass $\gtrsim 1.7 M_\odot$ were even absent in their simulations. As discussed in the previous Sect. 3.1, this turns out to be an incomplete description of the clump stars, at least in galaxy systems with recent star formation as the Solar Neighbourhood.

In contrast, GGWS98 considered all the interesting mass range of clump stars, and models with constant SFR up to 10 Gyr ago, obtaining clumps with a somewhat more extended distribution in colour than Jimenez et al. (1998). They demonstrate that a galaxy model with mean solar metallicity and a very small metallicity dispersion ($\sigma_{[\text{Fe/H}]} = 0.1$ dex), shows a clump as wide in colour as the observed...
we understand such a small metallicity dispersion, coming out for clump stars in a so complex stellar environment as the Solar Neighbourhood? The key to the answer is in our discussion of the age distribution of clump stars in Sect. 3.3. Actually, nearby clump stars are (in the mean) relatively young objects, reflecting mainly the near-solar metallicities developed in the local disc during the last few Gyr of its history, rather than its complete chemical evolution history.

It is also interesting to notice that the age distribution of nearby clump stars (mostly K giants), turns out to be very different from that of low-main sequence stars (e.g. the G dwarfs). This because the long-lived G dwarfs have an age distribution simply proportional to the SFR, whereas K giants have it ‘biased’ towards intermediate-ages (1–3 Gyr). This difference reflects into their metallicities: since younger stars are normally more metal rich, giants should necessarily be, in the mean, more metal-rich than G dwarfs. G dwarfs in the Solar Neighbourhood are already known to present a relatively narrow distribution of [Fe/H] – the relative lack of low-metallicity stars, with respect to the predictions from simple closed-box models of chemical evolution, being known as the G-dwarf problem. Then, even more narrow should be the distribution of [Fe/H] among K giants. And this is, in fact, exactly what is suggested by the observations of Fig. 8.

3.4 The magnitude and colour distribution

In the above subsection, we presented arguments and data which go against the interpretation of clump $V-I$ colours as being mainly due to metallicity differences. This point is also crucial for distance determinations. Were the clump colour determined by metallicity only, the observed constancy of the $I$-band magnitude with colour inside the clump, in different galaxies, could be indicating that $M_I$ is virtually independent of metallicity, and hence an excellent standard candle (cf. Paczynski & Stanek 1998; Stanek et al. 1998; Udalski 1998a). This latter conclusion comes from a simplified analysis, where the possibility that the clump magnitude and colour depend significantly also on the age, has been neglected.

Let us briefly discuss the specific results we get from our models. In them, $M_I^{RC}$ presents a non negligible dependence on both age and metallicity. Nonetheless, our simulations of the Hipparcos clump turn out to present an almost-horizontal clump feature, without any detectable systematic effect on $M_I$ as a function of colour or [Fe/H]. In fact, dividing our simulation of Fig. 6 into ‘blue’ ($V-I < 1.1$) and ‘red’ ($V-I > 1.1$) samples, we obtain values of $M_I^{RC} = -0.159$ and $M_I^{RC} = -0.170$, respectively.

How can the galaxy models present almost-horizontal clumps, whereas $M_I^{RC}$ changes systematically as a function of both age and metallicity ? The answer is not straightforward, since a number of effects enter into the game. First, $M_I$ does not change monotonically with age: it is fainter than the mean (by up to 0.4 mag) at both the $\sim 1$ Gyr and $\sim 10$ Gyr age intervals, which represent the bluest and reddest clump stars for a given metallicity (provided that $Z$ and $t$ are not, respectively, too low and too old, to cause the appearance of a blue HB). Due to this effect alone, the clump for a given metallicity would describe an arc in the CMD, without a detectable mean slope in the $M_I$ versus $V-I$ CMD (see GGWS08). Second, an age-metallicity re-
Distances from the Red Giant Clump

Figure 8. Metallicities of clump stars in the Hipparcos catalogue (see text). For the stars shown in the CMD of the upper left panel, the [Fe/H] distribution (upper right panel) turns out to be well described by a Gaussian of mean \( \langle [\text{Fe/H}] \rangle = -0.12 \) dex and dispersion 0.18 dex. The two lower panels show the same stars in the [Fe/H] versus \( V-I \), and \( M_I \) versus [Fe/H] diagrams.

lution tends to flatten the \( M_I \) vs. age relation in the age interval \( t \gtrsim 2 \) Gyr, since older clump stars tend to become dimmer due to their larger age, but brighter due to the lower metallicity. Third, the presence of a \( \sim 0.2 \) dex metallicity dispersion among stars of the same age, may cause substantial blurring in the diagrams of Fig. 8. Which effect prevails depends on the exact shape of the SFR and AMRs. The important point is that, due to these effects, the possible correlations between colour, magnitude, and metallicity of clump stars, may become small enough to escape detection.

Therefore, clump models can assume a quite large variety of shapes, depending on the SFR and AMR. The position of an individual star in the CMD, in general cannot be unequivocally interpreted as the result of a given age or metallicity. There are, however, important exceptions to this rule: A quite striking feature of the models is the presence of a ‘secondary clump’ feature located about 0.4 mag below the blue extremity of the clump, accompanied by a bright plume of clump stars directly above it. These features are described in detail in GGWS98 and Girardi (1999). They are caused by the intermediate-mass clump stars, i.e. those just massive enough for starting to burn helium in non-degenerate conditions, and are the signature of \( \lesssim 1 \)-Gyr old populations with metallicities \( Z \gtrsim 0.004 \) (Girardi 1999). The important point here is that these structures are present in the Hipparcos CMD, and about as populated as they are in the models presented in Fig. 8. It evidences that some key aspects of the formalism and stellar models we use are correct, and that the adopted SFR and AMR constitute a reasonably good approximation.

3.5 The magnitude as a function of [Fe/H]

From our simulation, we can also derive the metallicity and age distribution of local clump stars. They are presented in Fig. 9.

Although the total range of metallicities allowed by the model is very large \( (-0.7 \lesssim [\text{Fe/H}] \lesssim 0.3, \text{ see Fig. 5}) \), the distribution for clump stars turns out to be very narrow: a Gaussian fit to the [Fe/H] distribution produces a mean \( \langle [\text{Fe/H}] \rangle \) = +0.03 dex and dispersion \( \sigma_{[\text{Fe/H}]} = 0.17 \) dex. (Actually, the [Fe/H] distribution presents an asymmetric tail at lower metallicities, which causes the straight mean of [Fe/H] to be \( \sim 0.04 \) dex, i.e. slightly lower than the center of the Gaussian.) This distribution is almost identical to the observed one (Fig. 8), except for an offset of +0.15 dex.

The small [Fe/H] dispersion displayed in the lower panel of Fig. 8 is easily understood when we look at the clump age distribution in the upper panel. Not surprisingly (cf. the discussion in Sect. 3.3), the mean age of nearby clump stars turns out to be \( \langle t \rangle \) = 2.5 Gyr, whereas the peak of the distribution is at just 1 Gyr. At these ages, the local disc metallicity had already grown to \( [\text{Fe/H}] \sim 0.0 \) dex (cf. Fig. 5), and the bulk of clump stars is expected to have similar metallicities. Therefore, this main aspect of the metallicity distribution – the small [Fe/H] dispersion – is very well accounted for by the models.
from the offsets of +0.15 dex in [Fe/H], and +0.1 mag in $V-I$. Also the models do not show any significant correlation of [Fe/H] with either $V-I$ or $M_I$. The only particularity is the presence of two main sequences of clump stars in the [Fe/H] versus $V-I$ plane, which are due, essentially, to two main groups of clump stars: the ‘old’ ones which follow the age-metallicity relation and span the $V-I$ interval from 1.0 to 1.2, and [Fe/H] from 0.0 to −0.6; and the youngest ones which have [Fe/H] $\gtrsim$ 0.0 and concentrate at $V-I \sim 0.9 - 1.2$. With a reasonable distribution of observational errors (i.e., typical errors on the [Fe/H] values are of about 0.15 dex), these two sequences can give origin to a distribution in which no general [Fe/H] versus $V-I$ relation is apparent.

Udalski (2000) used the data for local clump stars to fit $M^{RC}_I$ to data in different [Fe/H] bins. The results were then used to derive the ‘metallicity dependence’ of $M^{RC}_I$. We can now check whether our models produce a similar relation from synthetic data. There is however, an important difference in our interpretation of this relation: in the models, it does not assume the character of a general relation, as was intended to be measured in Udalski (2000). Instead, the $M^{RC}_I$ versus [Fe/H] relation derived in this way cannot be considered universal, because it represents the result of a very particular distribution of clump ages and metallicities (displayed in Fig. 8).

Table 3 presents the $M^{RC}_I$ values we derive for different [Fe/H] bins in the models. They are divided in groups composed by ‘metal-poor’, ‘intermediate’, and ‘metal-rich’ bins. For each group, the final slope of the $M^{RC}_I$ versus [Fe/H] relation is also presented, both for the cases in which all bins have been included, and ignoring the metal-rich bin as in Udalski (2000). The first group represents the same bin limits as in Udalski (2000), and results in a very flat $M^{RC}_I$ versus [Fe/H] relation. The second group has bins shifted by +0.15 dex, in order to account for the offset in our models’ metallicities; in this case, a marginal slope of $0.14 \pm 0.06$ mag/dex is detected, which increases to $0.24$ mag/dex if we consider only the metal-poor and intermediate bins.

These two former groups present too few stars in the metal-poor bin. We tried to improve upon this point, selecting bin limits such as to separate the peak and wings of the metallicity distribution shown in Fig. 1. The result is a mean slope of $0.13 \pm 0.05$ mag/dex.

Udalski (2000) gets a slope for the $M^{RC}_I$ versus [Fe/H] relation of about 0.2 mag/dex using all three bins in their fit, and of $0.13 \pm 0.07$ mag/dex considering only the metal-poor and intermediate bins. Similar results (Table 3) are obtained with our models which include an offset in the metallicity scale. However, we do not give any strong weight to the final slope resulting from the models. In fact, the present exercises convinced us that the obtained slope may depend somewhat on the way the bins are defined. Moreover, the simulation of observational errors in the models, could probably lead to slightly different results. What we have done in this section should be considered, rather than a model calibration, just a check of whether Udalski’s results can be understood with present theoretical models of clump stars.
Figure 10. The distribution of stars from our \textit{Hipparcos} model (see Fig. 6) in the [Fe/H] versus $V - I$ (left panel) and $M_I$ versus [Fe/H] (right panel) planes. Compare with the observational distribution of Fig. 8.

Table 3. $M_I^{RC}$ for \textit{Hipparcos} model, separated in [Fe/H] bins, and the derived slope of $M_I^{RC}$ versus [Fe/H].

| Group | [Fe/H] bin limits | $M_I^{RC}$ | $\sigma$ | clump fraction | $\langle$[Fe/H]\rangle | slope |
|-------|-------------------|------------|----------|---------------|-----------------|-------|
| 1     | −0.60, −0.25      | −0.135     | 0.039    | 0.12          | −0.38           | −0.07 ± 0.06 (a) |
|       | −0.25, −0.05      | −0.176     | 0.082    | 0.26          | −0.14           | 0.00 (b)         |
|       | −0.05, +0.20      | −0.167     | 0.049    | 0.58          | 0.07            |                   |
| 2     | −0.45, −0.10      | −0.221     | 0.081    | 0.14          | −0.22           | 0.14 ± 0.06 (a)  |
|       | −0.10, +0.10      | −0.170     | 0.059    | 0.46          | 0.01            | 0.24 (b)         |
|       | +0.10, +0.35      | −0.164     | 0.039    | 0.30          | 0.18            |                   |
| 3     | −0.55, −0.05      | −0.218     | 0.072    | 0.18          | −0.21           | 0.13 ± 0.05 (a)  |
|       | −0.05, +0.15      | −0.167     | 0.052    | 0.48          | 0.05            | 0.32 (b)         |
|       | +0.15, +0.45      | −0.164     | 0.038    | 0.21          | 0.23            |                   |

Note: (a) Fit using all 3 bins; and (b) mean slope using only the metal-poor and intermediate bin.

4 THE CLUMP IN STAR CLUSTERS

We have seen in the previous section how theoretical models are able to reproduce the main observational features of the \textit{Hipparcos} red clump. Another crucial test for the reliability of theoretical models involves the use of star clusters. Since star clusters are made of stars all with the same age and initial chemical composition, they constitute template single-burst stellar populations with which it is possible to compare the $\Delta M_I^{RC}$ values derived from theory. With a large sample of clusters of different ages and metallicities, we can directly test the predicted metallicity and age dependence of this key quantity. Galactic open clusters are particularly useful in this respect, since they are well studied objects with a large age range, and reasonably accurate distance and age estimates.

To this aim, we have adopted the data (ages, $M_I^{RC}$ and [Fe/H]) by Sarajedini (1999) for a sample of 8 galactic open clusters, plus the data from Twarog et al. (1999) for NGC 2506. The clusters span the age range between 1.9 and 9.5 Gyr, while the metallicities range between [Fe/H] = −0.39 and [Fe/H] = 0.15. Since we want to test the theoretical $\Delta M_I^{RC}$ values, for each cluster we have computed the observational $\Delta M_I^{RC}$ value using $M_I^{RC}(\text{Hipp.}) = −0.23 ± 0.03$ (Stanek & Garnavich 1998) for the local red clump. The same quantity has been derived from the theoretical models, using for the local clump the $M_I^{RC}$ value given in Table 4. There is a further detail we considered; since Sarajedini (1999) measured the luminosity of the peak of the red clump stellar distribution, and not its mean value, we have derived from our simulations (see discussion in Sect. 2) the $M_I$ values corresponding to the peak of the red clump LF, for the relevant age and metallicity range, rather than the mean values defined in Sect. 2. We remark that the peak
values are systematically lower than mean values by about 0.06 mag.

In Fig. 12 we show the comparison between observed and theoretical \( \Delta M_\text{RC} \) values. It is immediately evident how the observations show a marked dependence of \( \Delta M_\text{RC} \) on the clusters' age and [Fe/H] values. Moreover, it is clear that observations are well reproduced by theoretical models (with the sole exception of NGC6791, the oldest cluster in the plot). This lends further support to the use of theoretical models for computing \( \Delta M_\text{RC} \).

Before concluding this section we just mention that the data by Sarajedini (1999) and Twarog et al. (1999) are inconsistent with the ones by Udalski (1998b), who has shown how the \( M_\text{RC} \) value for a sample of 15 star clusters in the LMC and SMC is largely independent of age and metallicity, in the age interval from 2 to 10 Gyr. However, it is clear that the determination of metallicity and age for LMC and SMC clusters is subject to larger uncertainties than in the case of their Galactic counterparts. Moreover, additional uncertainties due to depth effects and geometric corrections (values as high as ±0.2 mag being applied for some SMC clusters) can also spuriously modify the observed relationship between red clump brightness, metallicity and age.

Udalski (1998b) has also noticed that in the oldest clusters of his sample the clump is 0.3 – 0.4 mag fainter than the mean in the 2 – 10 Gyr interval. This observation could be simply reflecting the gradual fading of the clump, that occurs in the models for ages larger than 3 Gyr (see Fig. 12).

We conclude that the empirical evidence for a negligible dependence of the clump magnitude on age (Udalski 1998b), is weak compared to the evidence that it depends on age (Fig. 12). A larger sample of Magellanic Clouds cluster data may certainly improve upon the present-day results.

We also remark that in galaxies with recent star formation, a large fraction of the clump stars should have ages in the 1 – 2 Gyr interval (as discussed in Sect. 3.1). Clusters younger than 2 Gyr, however, are not present in Udalski’s (1998b) and Sarajedini’s (1999) samples. It would be extremely interesting, in future empirical works, to test the age dependence of \( M_\text{RC} \) in samples containing also younger clusters.

5 THE CLUMP IN OTHER GALAXY SYSTEMS

In Sects. 2 and 3, the main factors determining the mean clump magnitude in a galaxy model have been extensively reviewed. In the present section we proceed computing \( M_\text{RC} \) for a series of models representing nearby galaxy systems, whose mean clump magnitudes have been used in the past for distance determinations (by galaxy systems we mean composite stellar populations, whose stars cover relatively large intervals of age and metallicity). All results from this section are summarized in Table 4.

There are just two further technical details we need to mention before proceeding: (i) In all models, we assume the relation \([\text{Fe/H}]=\log(Z/0.019)\), which is appropriate for populations with scaled-solar distributions of metals. The case of stars with enhancement of \(\alpha\)-elements will be discussed in Sect. 5.5. (ii) For the few cases in which tracks with metallicities lower than \(Z=0.0004\), or higher than \(Z=0.03\) are required, we use these two limiting values. This is done to avoid risky extrapolations of the model behaviours. Anyway, apart from the case of the Carina dSph galaxy, stars with such extreme metallicities represent just a tiny fraction of the clump stars in our models.
Table 4. $M_I^{RC}$ and mean [Fe/H] values for the clump in nearby galaxy systems.

| System                  | SFR(t)               | AMR                        | $M_I^{RC}$ | $\Delta M_I^{RC}$ | [Fe/H]^{RC} | comm. |
|-------------------------|----------------------|----------------------------|------------|-------------------|-------------|-------|
| Solar Neighbourhood     | Rocha-Pinto et al. (2000b) | Rocha-Pinto et al. (2000a) | −0.171     | 0.000             | −0.04       | (•)   |
| (Hipparcos)             |                      |                            |            |                   |             |       |
| Baade’s Window          | 8 – 12 Gyr old       | McWilliam & Rich (1994)    | −0.087     | −0.084            | −0.22       | (•)   |
| (solar-scaled)          | Mollá et al. (2000)  | Mollá et al. (2000)        | −0.063     | −0.108            | −0.36       | (1)   |
| Baade’s Window          | 8 – 12 Gyr old       | McWilliam & Rich (1994)    | −0.161     | −0.010            | −0.22       | (•)   |
| (α-enhanced)            | Mollá et al. (2000)  | Mollá et al. (2000)        | −0.148     | −0.023            | −0.36       | (1)   |
| Carina dSph             | Hernández et al. (2000a) | Pagel & Tautvaisiene (1998) | [Fe/H] = −1.7 | +0.287            | −1.7        | (•)   |
|                        | Hurley-Keller et al. (1998) best model | Pagel & Tautvaisiene (1998) | [Fe/H] = −1.7 | −0.271            | +0.100      | −1.7   |
| SMC                     | Pagel & Tautvaisiene (1998) | Pagel & Tautvaisiene (1998) | −0.457     | +0.286            | −0.77       | (3, •) |
| LMC bar                 | Holtzman et al. (1999; their fig. 2) | Pagel & Tautvaisiene (1998) | −0.371     | +0.200            | −0.39       | (•)   |
| LMC outer fields        | Holtzman et al. (1999; their fig. 4) | Pagel & Tautvaisiene (1998) | −0.373     | +0.202            | −0.39       | (3)   |
|                         | Holtzman et al. (1999; their fig. 11) | Pagel & Tautvaisiene (1998) | −0.386     | +0.215            | −0.40       | (3)   |
|                         | Holtzman et al. (1999; their fig. 3) | Pagel & Tautvaisiene (1998) | −0.360     | +0.189            | −0.37       | (3)   |
|                         | Holtzman et al. (1999; their fig. 5) | Pagel & Tautvaisiene (1998) | −0.360     | +0.189            | −0.37       | (3)   |
|                         | Holtzman et al. (1999; their fig. 12) | Pagel & Tautvaisiene (1998) | −0.396     | +0.225            | −0.38       | (3)   |
| LMC northern fields     | Dolphin (2000)       | Dolphin (2000)             | −0.281     | +0.110            | −0.88       | (4)   |

(*•) The most representative or ‘preferred’ value of $\Delta M_I^{RC}$ for distance determinations.

(1) Theoretical model for the bulge.

(2) The clump LF is double, resulting in a bad fit of eq. [1]. Their second and third best models give identical results.

(3) SFR comes from a theoretical model.

(4) This model produces a CMD with far too many old metal-poor stars.

5.1 The Baade’s Window clump

Determining the ages of Bulge stars is difficult, and so its age distribution has been somewhat uncertain. Whereas there are some indications for the presence of intermediate-mass stars (see Rich 1999 for a review), studies of Baade’s Window population (e.g. Frogel 1988; Holtzman et al. 1993; Ortolani et al. 1995; Ng et al. 1996) generally indicate that the bulk of star formation occurred at old ages. Only the very central regions of the Bulge show unequivocal evidences of recent star formation (e.g. Frogel, Tiede & Kuchinsky 1999; Figer et al. 1999).

Let us then assume that the bulk of stellar populations in the Bulge are older than 5 Gyr. After this age, the clump magnitude fades slowly (only ~ 0.025 mag/Gyr), and $N_c(t)$ becomes almost flat (i.e. the age distribution of clump giants become simply proportional to the SFR, as seen in Fig. [1]). Therefore, the assumptions about the actual SFR history affect much less $M_I^{RC}$ for such a population, than in the case of galaxies with ongoing star formation.

We compute two models for the Bulge (or Baade’s Window), illustrated in Fig. [3]. The first assumes an ‘old’ Bulge, with constant SFR between 8 and 12 Gyr ago, and the McWilliam & Rich (1994) distribution of [Fe/H] values at any age. The latter has been obtained from spectroscopic analysis of Bulge K giants. Our second model comes from the recent ‘bulge model’ by Mollá, Ferrini & Gozzi (2000): it represents still a predominantly old Bulge, but with an ever-decreasing SFR going up to the present days. The [Fe/H](t) relation is derived from their chemical evolution models, and is shown (Mollá et al. 2000) to produce a distribution of [Fe/H] values very similar to the observational one by McWilliam & Rich (1994).

In both cases, Bulge models show an almost-horizontal clump – in very good agreement with the observational CMD presented by Paczyński & Stanek (1998) – very extended in colour and almost evenly populated along the colour sequence (Fig. [3]). Also in both cases, Bulge $M_I^{RC}$ values turn out to be very similar, and about 0.1 mag higher (i.e. fainter) than the local Hipparcos ones (Table [1]).

Notice that the mean metallicity of clump stars is rather similar in the local and Bulge samples (Table [1]). Therefore, the fainter clump we find in Bulge models is, essentially, the result of their higher mean ages, compared to local clump stars.

5.2 The clump in the Carina dSph

The Carina dSph galaxy represents an interesting case, because of its very low metallicity, of mean [Fe/H] = −1.9 and 1σ dispersion of 0.2 dex (Mighell 1997). Moreover, its CMD clearly indicates episodic star formation (Smecker-Hane et al. 1996), with main bursts occurring ~ 15, 7, and 3 Gyr ago (Hurley-Keller, Mateo & Nemec 1998).

Weighting these 3 different episodes of star formation in different proportions, and with different durations, Hurley-Keller et al. (1998) find several solutions for the SFR history of Carina. We have tested their best model (first entry in their table 7), assuming metallicity values equal to
Figure 13. The same as Fig. 6, but for the Bulge, as derived from two different models: (upper panel) an ‘old’ model with constant SFR between 8 and 12 Gyr and the McWilliam & Rich (1994) [Fe/H] distribution at any age, and (lower panel) a model following the SFR and AMR from the bulge theoretical model from Mollá et al. (2000).

Figure 14. The same as Fig. 7, but for the Bulge models presented in Fig. 13.

Z = 0.0004 ([Fe/H] = −1.7) at any age. Actually, we have tested Hurley-Keller et al.’s three best models, obtaining always identical results for $M_{RC}^I$.

In addition, we have tested the SFR history derived by Hernandez, Gilmore and Valls-Gabaud (2000a). They apply an objective numerical algorithm to find the SFR which best fits the observed CMD, without imposing artificial or subjective constraints on it. Their solution is characterized by periods of marked star formation separated by lower (but not null) activity. Similar results have been obtained by Mighell (1997), who also uses a non-parametric approach.

We present, in Figs. 15 and 16, the synthetic CMDs and LFs, respectively, obtained from Hernandez et al.’s (2000a) solution, and from Hurley-Keller et al.’s (1998) best model. It can be noticed (Table 4) that the two models provide $M_{RC}^I$ values differing by 0.18 mag. Actually, there is an obvious problem in the LF fit obtained from Hurley-Keller et al.’s model: since it presents a kind of dual clump – resulting from their assumption of discrete bursts of star formation separated by periods of quiescence – the LF is badly suited for a fit with a Gaussian function. Such a fit turns out to favour the fainter clump, and clearly does not represent in a satisfactory way the magnitude distribution of clump stars. Moreover, there is no evidence of a dual clump in the observational data (e.g. figure 6 in Udalski 1998a). The clump obtained from Hernandez et al.’s (2000a) SFR, on the contrary, turns out to be very compact in the CMD, in better agreement with the data. For these reasons, this latter model is to be preferred in the present work.

5.3 The clump in the SMC

In the literature for the SMC, we did not find quantitative assessments of the SFR, derived directly from SMC data. Qualitative descriptions can be found in e.g. Westerlund (1997) and Hatzidimitriou (1999).

Pagel & Tautvaisiene (1998) describe both the SFR and AMR of the SMC population by means of a chemical evolution model. Their AMR is shown to describe quite well the data for SMC star clusters. Their SFR, however, is not derived directly from stellar data (as in the cases previously discussed), and hence should be looked upon with some caution. It is characterised by strong star formation in the last 4 Gyr of the SMC history, an almost negligible SFR between 4 and 10 Gyr ago, and more pronounced SFR at 10–12 Gyr.

Simulations of the SMC clump, using Pagel & Tautvaisiene (1998) results, are shown in Figs. 17 and 18. This model produces a compact clump in the CMD, but with some substructures which are due the discontinuous SFR history.

5.4 The clump in the LMC

Quantitative determinations of the SFR in the LMC abound in the literature (Bertelli et al. 1992; Vallenari et al. 1996; Holtzman et al. 1997; Stappers et al. 1997; Elson, Gilmore 1997).
& Santiago 1997; Geha et al. 1998). The most recent determinations are generally based on deep photometry of some few selected fields, and on fairly objective algorithms for reconstructing the SFR history (see e.g. Holtzman et al. 1999; Olsen 1999; Dolphin 2000 and references therein). A general result is that the SFR has increased in the last few Gyr (starting 2.5 − 4 Gyr ago). This increase roughly coincides with the start of a major period of formation of star clusters, and with a major increase in the stellar mean metallicities (see Olszewski, Suntzeff & Mateo 1996; Dopita et al. 1997).

In the present work, we use the SFR results from Holtzman et al. (1999). Their results correspond to two LMC regions (bar, and ‘outer’) and three different assumptions in the analysis. Namely, the following three cases have been tested by Holtzman et al.: (i) at any age, the metallicity follows a known AMR; (ii) at any age, the metallicity is not constrained; (iii) the metallicity follows a known AMR, and there has been no star formation between 4 and 10 Gyr ago. For any of the 6 different SFRs from Holtzman et al. (1999), we have to assume some AMR; we take the Pagel & Tautvaisiene (1998) one, which is known to reproduce reasonably well the AMR derived from LMC star clusters.

We show in Figs. 4 and 5 the simulations of the bar and the outer field, with the SFR derived according to item (i) above. For the same fields, (ii) and (iii) produce similar results (see Table 3).

Remarkable in the CMD of Fig. 5 is the complex structure of the predicted LMC clump. It presents a marked vertical structure on the blue side, starting about 0.4 − 0.5 mag below the mean clump, and extending to higher luminosities. This feature of the models (the ‘secondary clump’; or ‘vertical structure’) has been extensively discussed by Girardi (1999), and has been clearly observed in some outer LMC fields by Bica et al. (1998) and Piatti et al. (1999). Moreover, the simulated LMC clumps present a horizontal tail of stars departing to the blue, which is simply the beginning of the old and metal-poor horizontal branch. It is interesting to notice the extreme similarity between these simulated clumps (Fig. 4), and the detailed CMD of a large area in

![Figure 15](image-url)

**Figure 15.** The same as Fig. 6 but for the Carina dSph galaxy, as derived from the SFRs and AMRs from (upper panel) Hernandez et al. (2000a), and Hurley-Keller et al. (1998) best model (lower panel).

![Figure 16](image-url)

**Figure 16.** The same as Fig. 7 but for the Carina dSph galaxy models presented in Fig. 15.

![Figure 17](image-url)

**Figure 17.** The same as Fig. 8 but for the SMC galaxy, as derived from the SFRs and AMRs from Pagel & Tautvaisiene (1998).
Finally, we have also tested the SFR and AMR derived by Dolphin (2000) from a field in the northern LMC. Surprisingly, this simulation produces a double clump (Fig. 19), which is evidently the result of a large population of old metal-poor stars in Dolphin’s (2000) solution. The synthetic CMDs turn out not to reproduce the characteristics of the LMC clump, as noticed by Dolphin himself. Moreover, the mean $[\text{Fe/H}]$ of clump stars is $-0.88$ dex for this model, which is far too low if compared with typical values found for LMC field giants. For these reasons, we prefer not to use the results from this latter model in our analysis.

5.5 Considering the enhancement of $\alpha$-elements

All the models discussed above assume a scaled-solar distribution of metals. However, it is well established that in some stellar populations (e.g. the Galactic Halo) the group of $\alpha$-elements (mainly O, Ne, Mg, Si, Ca, Ti) is overall enhanced with respect to Fe in comparison with solar ratios (i.e. $[\alpha/\text{Fe}] > 0$). This is probably the case for Bulge giants, where measurements of Mg and Ti abundances provide an enhancement by about $+0.4$ dex (see McWilliam & Rich 1994; Barbuy 1999). This is usually considered to be evidence for the chemical enrichment in the Bulge having occurred in a relatively short time scale.

A ratio $[\alpha/\text{Fe}] = 0.4$ means that, at a given $[\text{Fe}/\text{H}]$ value, the metal content $Z$ is a factor of about 2.5 larger than given by the scaled-solar relation $[\text{Fe}/\text{H}] = \log(Z/0.019)$. Moreover, for nearly-solar metallicities, $\alpha$-enhanced models cannot be reproduced by scaled-solar ones by simply modifying the relationship between $Z$ and $[\text{Fe}/\text{H}]$ (Salaris & Weiss 1998; Salasnich et al. 2000). Therefore, it is worth exploring how the Bulge $M_{\text{RC}}$ would change if appropriate $\alpha$-enhanced models were adopted. To this aim, we repeated our Bulge simulations using the isochrones from Salasnich et al. (2000), for both scaled-solar and $\alpha$-enhanced ($[\alpha/\text{Fe}] \simeq 0.35$ dex) cases. The difference between both $M_{\text{RC}}$ values was then added to the value obtained from Girardi et al.’s (2000) scaled-solar models (see Table 4).

It turns out that $\alpha$-enhanced models produce a Baade’s Window clump about 0.08 mag brighter than the scaled-solar ones. This occurs because of two competing effects. For the same $[\text{Fe}/\text{H}]$ distributions centered at almost-solar values ($[\text{Fe}/\text{H}] \sim -0.2$ dex; see Table 4), $\alpha$-enhanced models have a much higher $Z$, and higher $Z$ causes lower clump brightness at a given age. However, since our models assume a constant helium-to-metals enrichment ratio of 2.25, much higher values of the helium content $Y$ are reached by the $\alpha$-enhanced models; due to the fact that an increase of $Y$ (at fixed age and $Z$) causes an increase of the clump brightness, the net effect of using $\alpha$-enhanced models is an increase of the clump luminosities with respect to the scaled-solar case. The final result is that Bulge models computed considering the enhancement of $\alpha$ elements, may have $M_{\text{RC}}$ values very similar to those of the Hipparcos sample.
The same as Fig. 7, but for the LMC models presented in Fig. 19.

Figure 20. The same as Fig. 9 but for the LMC models presented in Fig. 13.

In the other galaxies we are considering, α-enhancement should not be as important as in the Bulge. In the cases of the Solar Neighbourhood and the Magellanic Clouds, the bulk of clump giants are relatively young (<3 Gyr), and such stellar populations are expected to have a scaled-solar metal distribution. This is confirmed by spectroscopic observations of: (i) thin disk stars, that indicate almost scaled-solar ratios ([Mg/Fe] between 0.0 and 0.1 dex) for stars with [Fe/H] > −0.5 (Fuhrmann 1998); and (ii) giants in LMC clusters younger than 3 Gyr and with [Fe/H] ≈ −0.5, which have [O/Fe] ≈ +0.1 dex (Hill et al. 2000). These low levels of α-enhancement can, at least as a first approximation, be ignored.

The situation for the Carina dSph is not clear. The clump giants in this galaxy are neither too young, nor have been formed in a short time interval as the Bulge ones. Thus, it is not clear whether some degree of α-enhancement should be expected, and present observations do not give information on this. Thus, we prefer not to consider the possibility of α-enhancement for this galaxy.

5.6 The clump – RR Lyrae difference

Udalski (1998a) measured the mean clump apparent magnitude $I_0^{\text{RC}}$ in Baade’s Window, LMC, SMC, and Carina dSph galaxy. For the same fields, RR Lyrae data has provided a reference magnitude to compare the clump with. Assuming that RR Lyrae stars follow a $M_I^{\text{RR}} = \{0.18 \pm 0.03\}[\text{Fe/H}] + \text{const}$ relation and adopting empirical mean values for the metallicity of RR Lyrae in the different environments, Udalski (1998a) constructed the $I_0^{\text{RC}} - V_0^{\text{RRatGB}}$ parameter, where $V_0^{\text{RRatGB}}$ means the magnitude that RR Lyrae in each galaxy would have if they had the same [Fe/H] as the Bulge ones. Udalski’s (1998a) data for Baade’s Window has been later revised by Popowski (2000). Following the results by Paczyński et al. (1999) about a systematic difference between OGLE-I and OGLE-II photometry, he considers a clump dimmer by 0.035 mag with respect to Udalski (1998a) data and RR Lyrae stars brighter by 0.021 mag; this produces a change of $I_0^{\text{RC}} - V_0^{\text{RRatGB}}$ by +0.06 mag. Moreover, in order to solve the so-called ‘$V-I$ colour problem’ of Baade’s Window clump giants (Paczyński 1998), Popowski (2000) modifies the original extinction values and reddening curves for Baade’s Window by an amount which implies a final global revision by +0.17 mag for the $I_0^{\text{RC}} - V_0^{\text{RRatGB}}$ value.

By construction, $I_0^{\text{RC}} - V_0^{\text{RRatGB}}$ provides the differential behaviour of $M_I^{\text{RC}}$ in these four stellar systems, and it is therefore interesting to compare it with the $\Delta M_I^{\text{RC}} = M_I^{\text{RC}}(\text{Hipp}) - M_I^{\text{RC}}(\text{galaxy})$ values derived from our simulations. Before proceeding, we just mention that our ZAHB values for the metallicity of RR Lyrae in the different environments, Udalski (1998a) constructed the $\Delta M_I^{\text{RC}} = M_I^{\text{RC}}(\text{Hipp}) - M_I^{\text{RC}}(\text{galaxy})$ values derived from our simulations. Before proceeding, we just mention that our ZAHB values for the metallicity of RR Lyrae in the different environments, Udalski (1998a) constructed the $\Delta M_I^{\text{RC}} = M_I^{\text{RC}}(\text{Hipp}) - M_I^{\text{RC}}(\text{galaxy})$ values derived from our simulations. Before proceeding, we just mention that our ZAHB values for the metallicity of RR Lyrae in the different environments, Udalski (1998a) constructed the $\Delta M_I^{\text{RC}} = M_I^{\text{RC}}(\text{Hipp}) - M_I^{\text{RC}}(\text{galaxy})$ values derived from our simulations.
In Fig. 22 we show the run of \( \Delta M_{RC} \) as a function of the mean red clump [Fe/H] for the stellar systems in Tab. 4, the local \textit{Hipparcos} red clump and the open clusters of Fig. 12. If we consider, for example, metallicities around [Fe/H] = −0.1 one can easily notice the large dispersion of the \( \Delta M_{RC} \) values. On the basis of all the evidences discussed in this paper it is easy to understand that this dispersion is due to the different SFR and AMR of the stellar populations we are considering. This should also warn against the use of empirical linear relationships for deriving \( \Delta M_{RC} \) as a function of [Fe/H], \( \Delta M_{RC} \) depends in a complicated way on the properties of the stellar populations under scrutiny and any empirical calibration of this quantity is not universal, but reflects the particular SFR and AMR of the calibrating sample. Moreover, there is no physical reason at all for linear relationships to hold for \( \Delta M_{RC} \).

If we consider only the stellar systems of Tab. 4 and try to fit a linear relationship to the points displayed in Fig. 22 (open circles) we would derive a slope of about 0.18, in agreement with the slope of the empirical corrections (0.19±0.05) derived by Popowski (2000) using the \( v_0^{\text{RRatGB}} \) values as a function of [Fe/H] for the same sample of objects; but we want to stress the point that this slope has no meaning whatsoever. It is just an accident that for this particular sample of galaxies there is a relationship close to a linear one between the clump brightness and the metallicity. In distance determinations, the individual values of \( \Delta M_{RC} \) derived from population synthesis simulations must be used, and not an average relation obtained from a linear fit to the real corrections.

6 CONCLUSIONS ABOUT THE CLUMP DISTANCE SCALE

In the previous two sections we have shown how theoretical models of stellar populations are able to reproduce most of the relevant observational features regarding red clump stars in different environments. In the following we will redetermine the red clump distances to the galactic and extragalactic systems previously discussed using \( v_0^{\text{RC}} \) values taken from the literature. It is not our intention to exhaustively discuss the uncertainties and conflicting results about the distances to these stellar systems; we only want to show the changes of the distance estimates using red clump stars when one is using the \( \Delta M_{RC} \) corrections predicted by stellar models.

6.1 The Bulge – Magellanic Clouds – Carina dSph distance scale

By applying the red clump method (see Eq. 1) and the population corrections \( \Delta M_{RC} \) displayed in Table 4, we derive here the distances to the Galactic Bulge, LMC, SMC and Carina dSph. The values for \( v_0^{\text{RC}} \) and \( A_I \) come from the literature, and \( M_I^{\text{RC}} = -0.23 \pm 0.03 \) (Stanek & Garnavich 1998) has been empirically obtained from local \textit{Hipparcos} red clump stars.

By using a dereddened \( v_0^{\text{RC}} = 14.32 \pm 0.04 \) for the Galactic Bulge (Udalski 1998a) and \( M_I^{\text{RC}} = -0.087 \) (from solar-scaled models; Table 4), using the McWilliam & Rich (1994) AMR, we get \( \mu^H_{\text{GB}} = 14.47 \pm 0.05 \). Analogous value is obtained using the SFR and AMR by Mollá et al. (2000).

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corresponds to a linear distance of 7.8 ± 0.2 Kpc, in good agreement with the generally accepted value of the distance to the Galactic center 8.0 ± 0.5 Kpc (Reid 1993). Our estimate, however, is slightly lower than the values obtained by Paczynski & Stanek (1998) and Stanek & Garnavich (1998), simply because we include a population effect which was then assumed to be negligible. If instead we correct \( I_{\text{RC}} \) by +0.035 mag as suggested by Paczynski et al. (1999) and Popowski (2000), and modify \( A_I \) by −0.11 mag as suggested by Popowski (2000), we get \( \mu_0^{\text{GB}} = 14.62 ± 0.05 \) (8.4 ± 0.2 Kpc). These results slightly change if we consider the α-enhanced model for the Bulge. In this case the population effect is negligible and we obtain, respectively, \( \mu_0^{\text{GB}} = 14.55 ± 0.05 \) (8.1 ± 0.2 Kpc) and \( \mu_0^{\text{GB}} = 14.70 ± 0.05 \) (8.7 ± 0.2 Kpc).

In the case of the LMC there is still much debate about the observational value of the red clump \( I_{\text{RC}} \) (Zaritsky 1999; Romaniello et al. 2000; Udalski 2000), the main reason being the extinction correction (see also Sec. 3.1.1). Romaniello et al. (2000) obtained \( I_{\text{RC}} = 18.12 ± 0.02 \), a value in agreement also with the results by Zaritsky (1999), while Udalski (2000) obtained \( I_{\text{RC}} = 17.94 ± 0.05 \). When adopting Romaniello et al.’s (2000) result, together with \( \Delta M_{\text{RC}} = 0.200 \) (considering the SFR from figure 2 of Holtzman et al. 1999; see Table 1), we get a LMC distance modulus \( \mu_0 = 18.55 ± 0.05 \) (51.3 ± 1.1 Kpc). The other SFR prescriptions by Holtzman et al. (1999) displayed in the Table do not significantly modify \( \Delta M_{\text{RC}} \). Our \( \mu_0 \) LMC value is in good agreement with the so-called ‘long’ distance scale. In case of assuming the Udalski (2000) dereddened red clump brightness one obtains \( \mu_0 = 18.37 ± 0.07 \) (47.2 ± 1.5 Kpc). Notice that this value is ‘longer’ by 0.13 mag with respect to the result obtained by Udalski (2000) using the same red clump brightness but his empirical correction for metallicity effects.

As for the SMC Udalski (1998a) gives \( I_{\text{RC}} = 18.33 ± 0.05 \) which, together with \( \Delta M_{\text{RC}} = 0.286 \) obtained from Table 1 provides \( \mu_0^{\text{SMC}} = 18.85 ± 0.06 \) (58.9 ± 1.6 Kpc).

The distance to Carina turns out to be \( \mu_0^{\text{Car}} = 19.96 ± 0.06 \) (98.2 ± 2.7 Kpc) when using \( I_{\text{RC}} = 19.44 ± 0.04 \) from Udalski (1998a) and \( \Delta M_{\text{RC}} = 0.287 \) from Table 1 (using the Hernandez et al. 2000a SFR).

6.2 Final comments

In this paper, we use an extended set of stellar models, standard population synthesis algorithms, and independent data about the distributions of stellar ages and metallicities, to derive the behaviour of the clump magnitude in different stellar systems. We also provide the basic equations and tables for a straightforward computation of the red clump mean brightness for any stellar system.

We are able to reproduce quite well a number of observational features of the clump in nearby galaxy systems. The most striking are:

(i) For the Hipparcos clump: a) the distribution in the \( M_I \) versus \( V-I \) diagram (a colour shift of 0.1 mag being probably due to a mismatch between two different empirical metallicity/age scales); b) the narrow and Gaussian-like [Fe/H] distribution; c) the absence of a correlation between \( V-I \) colour and [Fe/H]; d) the approximate slope of the empirical \( M_I^{\text{RC}} \) versus [Fe/H] relation.

(ii) For the Baade’s Window clump, the wide and nearly horizontal clump in the \( M_I \) versus \( V-I \) diagram.

(iii) For the LMC, the striking vertical structure (fainter secondary clump plus bright tail) at the blue side of the clump, and a blue plume of horizontal branch stars.

(iv) For the SMC and Carina dSph, the compact clump structure.

(v) For galactic open clusters older than 2 Gyr, the rate of change of the clump brightness with both age and metallicity.

(vi) For the Bulge, Magellanic Clouds, Carina dSph, the approximate slope of the empirical \( M_I^{\text{RC}} \) versus [Fe/H] relation.

We have shown that the models predict a complex dependence of the red clump magnitude on age, metallicity, and star formation history, which cannot be expressed by relations such as: (i) a linear \( M_I^{\text{RC}} \) versus [Fe/H] relation, or (ii) a linear (or constant) \( M_I^{\text{RC}} \) versus age relation. Present empirical linear \( M_I^{\text{RC}} \) versus [Fe/H] relations, used to describe the dependence of the red clump \( M_I^{\text{RC}} \) on the metallicity, are misleading, since they are originated by the particular age and metallicity distributions of the objects included in the calibrating sample, and do not have a general validity. Using such linear relations may produce spurious results, even when statistically good fits to the calibrating data are obtained.

To summarize, there are four main features indicated by the models, that cannot be expressed by present empirical relations:

(i) \( M_I^{\text{RC}} \) depends on both metallicity and age, and then on the underlying age-metallicity relation;

(ii) for a given metallicity, the \( M_I^{\text{RC}} \) versus age relation is complex and not monotonic;

(iii) at a given age, \( M_I^{\text{RC}} \) generally increases with [Fe/H], but not necessarily in a linear way;

(iv) stars of different ages have very different weights in determining \( M_I^{\text{RC}} \) in a galaxy, younger stars (if present) being dominant.

These features are, nowadays, better predicted by models, than expressed by empirical calibrations.

The results summarized above have been obtained using, essentially, models with scaled-solar metal abundances. However, the entire problem of describing the clump behaviour with respect to age and metallicity gets even more complicated if we take into account that some stellar populations may be characterized by different initial metal distributions. We have demonstrated the sensitivity of the clump brightness to the metals relative abundances with our simulations of the Baade’s Window clump using α-enhanced evolutionary tracks. This adds a further variable – degree of α-enhancement – to the problem. Moreover, if the metal content \( Z \) is above solar – a situation that can be met even for [Fe/H] = 0 if α-enhancement is present – also the helium content \( Y \) becomes important in determining the absolute clump magnitude (see GGWS98). Therefore, in these cases a fourth variable is to be considered: the helium-to-metal enrichment ratio.

The correct approach to using the red clump as a distance indicator is therefore to evaluate the population corrections \( \Delta M_{\text{RC}} \) – using population synthesis models – for
each individual object, provided that evaluations of the SFR and AMR do exist. In addition, informations on \([\alpha/Fe]\) and reasonable assumptions about the helium-to-metal enrichment ratio are needed. These occurrence, however, raises a fundamental question about the accuracy of the red clump as distance indicator. Udalski (1998b) emphasizes the advantages of the red clump with respect to other widely used standard candles such as Cepheid and RR Lyrae stars, these being mainly: (1) the existence of larger samples of red clump stars in galaxies with respect to Cepheids and RR Lyrae; (2) the very precise calibration of the absolute red clump brightness for the Solar Neighbourhood; (3) the existence of only a weak and empirically calibrated dependence of the red clump brightness on the metallicity. However, now that one has demonstrated that these empirical relations have no general validity, the red clump does not seem anymore to be a very reliable standard candle. At least, any determination of red clump distances requires the critical evaluation of the population effects, then implying that the red clump method cannot be meaningfully applied to objects for which there are no determinations of the SFR and AMR, unless errors up to \(\approx 0.3\) mag are to be accepted.

On the other hand, stellar evolution and population synthesis theory provide potentially important tools for the interpretation of clump data in nearby galaxies. We hope this paper has provided convincing examples of this.

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