Evaluation of input geological parameters and tunnel strain for strain-softening rock mass based on GSI

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The regression analysis method is being widely adopted to analyse the tunnel strain, most of which ignore the strain-softening effect of the rock mass and fail to consider the influence of support pressure, initial stress state, and rock mass strength classification in one fitting equation. This study aims to overcome these deficiencies with a regression model used to estimate the tunnel strain. A group of geological strength indexes (GSI) are configured to quantify the input strength parameters and deformation moduli for the rock mass with a quality ranging from poor to excellent. A specific semi-analytical procedure is developed to calculate the tunnel strain around a circular opening, which is validated by comparison with those using existing methods. A nonlinear regression model is then established to analyse the obtained tunnel strain, combining twelve fitting equations to relate the tunnel strain and the factors including the support pressure, GSI, initial stress state, and critical softening parameter. Particularly, three equations are for the estimation of the critical tunnel strain, critical support pressure, and tunnel strain under elastic behaviour, respectively; and the other nine equations are for the tunnel strain with different strain-softening behaviours. The relative significance between the GSI, the initial stress and the support pressure on the tunnel strain is assessed.

List of symbols

| Symbol | Description |
|--------|-------------|
| η      | Softening parameter |
| σᵣ, σ₀ | Radial and tangential stresses |
| εᵣᵢ(ᵣ₀), εθᵢ(ᵣ₀) | Radial and tangential strains at r = rᵢ(ᵣ₀) |
| εᵣᵢ₋₁, εθᵢ₋₁ | Radial and tangential strains at r = rᵢ₋₁ |
| ε₊plasᵢ, ε₋plasᵢ | Radial and tangential plastic strains |
| Δε₊plasᵢ, Δε₋plasᵢ | Radial and tangential plastic strain increments |
| Δε₊elasᵢ, Δε₋elasᵢ | Radial and tangential elastic strain increments |
| uᵢ(ᵣ₀) | Radial displacement at r = rᵢ(ᵣ₀) |
| p₀      | Initial ground stress |
| μ       | Poisson's ratio |
| σ₁, σ₃ | Major and minor principal stresses at failure, respectively |
| GSI     | Geological Strength Index |
| GSP, GSR | Peak and residual values of the Geological Strength Index |
| ψ       | Dilatancy angle |
| Kψ(η)   | Dilatancy coefficient |
| φ       | Friction angle |
| σci     | Uniaxial compression strength of intact rock |
| m₁b, s, α | Strength parameters of the Hoek–Brown rock mass |
| ω       | m₁b, s and α |

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assumptions. Some assumptions are considered prior to the analysis:

1. A circular opening, with a radius of $R_0$, is under a hydrostatic stress field of $\sigma_0$ asymmetrically distributed around it; the radial stress $\sigma_r$ and the tangential stress $\sigma_\theta$ correspond to the minor and major principal stresses $\sigma_3$ and $\sigma_1$, respectively;
2. Plane strain condition is considered as the deformation along the longitudinal direction of the tunnel is virtually uniform;
3. Material of the rock mass is isotropic, continuous, and initially elastic. Near underground excavations where confinement is reduced, most rock mass exhibits post-peak strength loss, which is called strain-softening property. The rock mass presents strain-softening (SS) behaviour; the elastic-perfectly-plastic (EPP) and elastic-brittle-plastic (EBP) behaviours are also considered, which are taken as special cases of the SS behaviour. The SS, EPP, and EBP rock masses are shown in Fig. 1. A support pressure $p_i$ is evenly imposed around the tunnel. $\sigma_{r_p}$ and $\sigma_{\theta_p}$ represent the radial and tangential stresses at the elasto-plastic boundary, respectively. Within a SS rock mass, $\sigma_{r_1}$ and $\sigma_{\theta_1}$ are the radial and tangential stresses at the plastic softening-residual boundary, respectively. The radii of the plastic softening and residual areas are symbolised as $R_{plas}$ and $R_r$, respectively. For the EPP and EBP rock masses, the radius of plastic area is represented as $R_p$.
4. The softening parameter $\eta$ characterises the softening quantity in the rock mass and is calculated as the gap between the tangential and radial plastic strains for the axisymmetric problem:

$$\eta = \varepsilon^{plas}_r - \varepsilon^{plas}_\theta$$

(1)

The critical value of $\eta$ is denoted as $\eta^*$, which occurs at the moment that the rock mass strength decays to its residual value. Specially, $\eta$ has values of $\infty$ and 0 for the EPP and EBP rock masses, respectively.
5. The Mohr–Coulomb failure criterion is considered for the plastic potential function\textsuperscript{22,23}

\[ g(\sigma_r, \sigma_\theta, \psi) = \sigma_0 - \frac{1 + \sin \psi}{1 - \sin \psi} \sigma_r \]  

where \( \psi \) is the dilatancy angle and herein is taken as nil.

6. The Hoek–Brown (H-B) failure criterion is satisfactory in the quick estimate of the rock mass strength\textsuperscript{24}:

\[ \sigma_1 = \sigma_3 + \sigma_\text{ci} \left( \frac{m_b \sigma_3}{\sigma_\text{ci}} + s \right)^a \]  

where \( \sigma_\text{ci} \) represents the uniaxial compression strength of the intact rock; \( m, s \) and \( a \) are strength parameters of the Hoek–Brown rock mass. Because of the axisymmetric condition, the radial stress \( \sigma_r \) and the tangential stress \( \sigma_\theta \) correspond to the minor and major principal stresses \( \sigma_3 \) and \( \sigma_1 \), respectively. Equation (3) can be transformed as:

\[ f(\sigma_r, \sigma_\theta, \eta) = \sigma_\theta - \sigma_r - \sigma_\text{ci} \left( \frac{m_b \sigma_r}{\sigma_\text{ci}} + s \right)^a \]  

According to the geological observations in the field, Reference\textsuperscript{10,24,25} constructed the relation between the strength parameters \( (m_b, s \text{ and } a) \) and GSI. The empirical equations are listed as follows:

\[ m_b = m_i \exp \left( \frac{\text{GSI} - 100}{28 - 14D} \right) \]  

\[ s = \exp \left( \frac{\text{GSI} - 100}{9 - 3D} \right) \]  

\[ a = \frac{1}{2} + \frac{1}{6} \left( e^{-\text{GSI}/15} - e^{-20/3} \right) \]  

where \( D \) is a coefficient influenced by the disturbance from blast impact and the stress relaxation. An optimised blasting operation with an accurate drilling control technique is assumed during the tunnel excavation, thereby, the damage to the tunnel wall is negligible and \( D \) is regarded as 0 by Hoek\textsuperscript{26}. \( m_i \) in Eq. (5) characterises the friction between the composition minerals.

**Strength classification of rock mass.** The strength classification systems, such as the RMR, Q, and GSI, were successfully applied to many tunnel excavations. Various empirical equations by the systems are feasible to characterise the strength and deformation behaviours of the rock mass. Herein, GSI is incorporated to quantify the rock mass properties. Advantages of the GSI are demonstrated in three aspects: GSI is directly correlated to the strength constants in the Hoek–Brown failure criterion\textsuperscript{24}; GSI can be estimated by RMR and Q systems, thus some strength parameters related to RMR can also be represented by GSI; and the residual strength of the strain-softening rock mass could be calculated from the peak value of GSI based on the equation proposed\textsuperscript{27}.

**Correlation between RMR and GSI.** In the latest version, the relationship between GSI and RMR is:

\[ \text{GSI} = \text{RMR} - 5, \text{RMR} > 23 \]  

It is noted that Eq. (8) is specialised for the dry condition of the rock mass and thus is not applicable to the weak rock mass with the RMR below 18.

**Residual value of GSI.** The guideline for the GSI was presented\textsuperscript{25}, which are to characterise the peak strength parameters of the EPP rock mass. Considering the strain-softening effect, Reference\textsuperscript{27} extended the GSI framework to consider the residual strength. In their study, through the in-situ block shear test at a number of real
To correlate the deformation modulus of rock mass, the peak and residual values (strength parameters) are denoted with \( E_p \) and \( E_r \) within the plastic area of the EBP rock mass. The deformation modulus \( E_r \) and strength parameters, such as \( \sigma_{ci} \) and \( m_i \), also need to be determined. A number of compression tests show that \( E_p \) deteriorates for the rock mass beyond the peak state\(^{28,29}\). It is proposed that \( \sigma_{ci} \) wanes from its peak value to the residual during the softening stage since the rock mass quality is weakened, and the variations of \( E_r \) and \( \sigma_{ci} \) also obey Eq. (10). Therefore, \( E_r \), \( \sigma_{ci} \), and \( m_i \) within the plastic softening area are all assumed to obey Eq. (10). As observed in Eq. (10), the prerequisite for obtaining \( E_r \), \( \sigma_{ci} \), \( m_i \), \( s \), and \( a \) in the softening area is to predict the peak and residual values (\( E_p^b \), \( E_t \), \( \sigma_{ci} \), \( m_i \), \( s_i \), \( a_i \)). Based on GSP and GSI, the derivation of \( E_p^b \), \( E_t \), \( \sigma_{ci} \), \( m_i \), \( s_i \), \( a_i \), \( s_i \), \( a_i \) is presented in the following.

**Within the plastic elastic and plastic residual areas.** Empirical equations to determine \( E_r \) were proposed with GSI and RMR.

**Reference**\(^3\)\(^1\):  
\[
E_r = 2RMR - 100
\]  
(11)

**Reference**\(^3\)^{30}:  
\[
E_r = 10^{[RMR-10]/40}
\]  
(12)

**Reference**\(^3\)^{31}:  
\[
E_r = 0.1 \left( \frac{RMR}{10} \right)^3
\]  
(13)

Simplified Hoek and Diederichs equation\(^3\)^{32}:  
\[
E_r = 100 \left( \frac{1 - D/2}{1 + e^{(75+25D - GSI)/11}} \right)
\]  
(14)

With GSP and GSI\(^p\) listed in Table 1, the calculated \( E_p^b \) and \( E_r^b \) from Eqs. (11)–(14) are shown in Table 2. In Table 3, \( E_p^b \) and \( E_r^b \) can be estimated as the average values from Eqs. (11)–(14).

**Strength constant \( m_i \).** In the previous works, such as Reference\(^{33-35}\), \( m_i \) was approximated by two methods. One is to determine the classification of \( m_i \) from the rock type, such as in Hoek and Brown\(^34\). The other method is to estimate \( m_i \) from the rock mass quality. Although the latter method tends to be subjective, it enables to establish a direct relationship between \( m_i \) and the rock mass strength classification\(^34\). Therefore, the latter method is utilised in this study to correlate \( m_i \) with GSI. The test data of \( m_i \) for different GSI by Hoek and Brown\(^33\) and Hoek and Marinors\(^36\) is listed in Table 4. The data for estimating \( m_i \) by GSI can be best-fitted by,

\[
m_i = 0.7375 GSI^{0.7586}
\]  
(15)
Table 2. Calculated values of $E_r$ and $E_r$ by Eqs. (11) to (14).

| GSI | Equation (11) | Equation (12) | Equation (13) | Equation (14) | GSI | Equation (11) | Equation (12) | Equation (13) | Equation (14) |
|-----|---------------|---------------|---------------|---------------|-----|---------------|---------------|---------------|---------------|
| 25  | 3.162         | 2.700         | 1.050         | 17.883        | 20.069 | 2.381         | 1.575         | 1.126         |
| 30  | 4.217         | 4.288         | 1.645         | 21.897        | 2.645 | 1.946         | 1.295         |
| 35  | 5.623         | 6.400         | 2.567         | 23.403        | 2.885 | 2.291         | 1.452         |
| 40  | 7.499         | 9.113         | 3.986         | 24.623        | 3.094 | 2.599         | 1.592         |
| 45  | 10.000        | 12.500        | 6.138         | 25.585        | 3.271 | 2.861         | 1.712         |
| 50  | 13.335        | 16.638        | 9.341         | 26.320        | 3.412 | 3.072         | 1.809         |
| 55  | 17.783        | 21.600        | 13.965        | 26.852        | 3.518 | 3.232         | 1.883         |
| 60  | 23.714        | 27.463        | 20.365        | 27.205        | 3.590 | 3.340         | 1.934         |
| 65  | 31.623        | 34.300        | 28.719        | 27.205        | 3.631 | 3.401         | 1.962         |
| 70  | 42.170        | 42.188        | 38.828        | 27.399        | 3.642 | 3.418         | 1.970         |
| 75  | 56.234        | 51.200        | 50.000        | 27.453        | 3.642 | 3.418         | 1.970         |

Table 3. Estimated values of $E_r$ and $E_r$.

| GSI | $E_r$ (MPa) | GSI | $E_r$ (MPa) |
|-----|-------------|-----|-------------|
| 75  | 54.359      | 27.453 | 3.010 |
| 70  | 43.296      | 27.399 | 2.998 |
| 65  | 33.660      | 27.205 | 2.955 |
| 60  | 25.385      | 26.852 | 2.878 |
| 55  | 18.337      | 26.320 | 2.764 |
| 50  | 12.328      | 25.585 | 2.615 |
| 45  | 7.160       | 24.623 | 2.429 |
| 40  | 5.149       | 23.403 | 2.209 |
| 35  | 3.648       | 21.897 | 1.962 |
| 30  | 2.537       | 20.069 | 1.694 |
| 25  | 1.728       | 17.883 | 1.417 |

Table 4. Values of $m_i$ with different GSI: (a) Hoek and Brown; (b) Hoek and Marino.

| (a) GSI | 75 | 50 | 30 | 75 | 75 | 65 | 20 | 24 |
|--------|----|----|----|----|----|----|----|----|
| $m_i$  | 25 | 12 | 8  | 16.3 | 17.7 | 15.6 | 9.6 | 10 |

| (b) GSI | 20 | 5  | 13 | 28 |
|--------|----|----|----|----|
| $m_i$  | 8.0 | 2.0 | 5.0 | 11.0 |

Figure 2. Fitting for $m_i$. 
The coefficient of determination \( R^2 \) reaches 81.38%, which indicates that the fitting line is in agreement with the test results. By Eq. (15) (see Fig. 2), the calculated \( m_i^p \) and \( m_i^r \) with different \( GSI_p \) and \( GSI_r \) are presented in Table 5.

It is admitted that \( m_i \) is the inherent characteristic of the intact rock. In this respect, \( m_i \) corresponds to \( GSI = 100 \). But from many references, it is found that generally a greater \( GSI \) gives rise to a larger value of \( m_i \). Hence, in the analysis, a rough and immature relation between \( GSI \) and \( m_i \) is proposed as shown in Eq. (15) is proposed. The aim of Eq. (15) is to solve the tunnel strain as one of the input parameter in the latter. And according to Eq. (15), the tunnel strain is greater in comparison to a constant \( m_i \) with no reduction. Then, the tunnel design will be conservative and safe. In this respect, Eq. (15) is reasonable. Furthermore, the sensitive analysis for the influence of multiple mechanical parameters on the tunnel strain has also been undertaken. It is found that in comparing with other input parameters such as the deformation modulus and the compressive rock strength, the effect of \( m_i \) on the rock deformation is trivial. In this aspect, although Eq. (15) is subjective, it seems to be not very important factor that affect the results in this analysis.

### Strength constants \( m_p \), \( s \), and \( a \)

According to Eqs. (5) to (7), when the disturbance factor \( D \) is 0, \( m_p^p \), and \( m_p^r \) can be obtained from \( GSI_p \), \( GSI^p \), \( m_i^p \), and \( m_i^r \); and \( s^p \), \( s^r \), \( a^p \), \( a^r \) can be calculated from \( GSI_p \) and \( GSI^r \). The estimated result is listed in Table 6.

### Compressive strength of intact rock \( \sigma_{ci} \)

Here, \( \sigma_{ci} \) by GSI is calculated in three steps.

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### Table 5. Estimated values of \( m_i^p \) and \( m_i^r \).

| GSIp | \( m_i^p \) | \( GSI^p \) | \( m_i^r \) |
|------|-------------|-------------|-------------|
| 75   | 19.507      | 27.453      | 9.101       |
| 70   | 18.512      | 27.399      | 9.087       |
| 65   | 17.500      | 27.205      | 9.038       |
| 60   | 16.469      | 26.852      | 8.949       |
| 55   | 15.417      | 26.320      | 8.814       |
| 50   | 14.342      | 25.585      | 8.627       |
| 45   | 13.240      | 24.623      | 8.380       |
| 40   | 12.108      | 23.403      | 8.063       |
| 35   | 10.942      | 21.897      | 7.666       |
| 30   | 9.734       | 20.069      | 7.176       |
| 25   | 8.477       | 17.883      | 6.575       |
| 20   | 7.157       | 15.298      | 5.840       |

### Table 6. Estimated values of \( m_p^p \), \( s^p \), \( a^p \) and \( m_p^r \), \( s^r \), \( a^r \).

| GSIp | \( m_p^p \) | \( s^p \) | \( a^p \) | \( GSI^p \) | \( m_p^r \) | \( s^r \) | \( a^r \) |
|------|-------------|--------|--------|-------------|-------------|--------|--------|
| 75   | 7.988       | 62.177 | 0.501  | 27.453      | 0.682       | 0.316  | 0.527  |
| 70   | 6.341       | 35.674 | 0.501  | 27.399      | 0.680       | 0.314  | 0.527  |
| 65   | 5.014       | 20.468 | 0.502  | 27.205      | 0.671       | 0.307  | 0.527  |
| 60   | 3.947       | 11.744 | 0.503  | 26.852      | 0.656       | 0.295  | 0.528  |
| 55   | 3.090       | 6.738  | 0.504  | 26.320      | 0.634       | 0.278  | 0.529  |
| 50   | 2.405       | 3.866  | 0.506  | 25.585      | 0.605       | 0.257  | 0.530  |
| 45   | 1.857       | 2.218  | 0.508  | 24.623      | 0.568       | 0.230  | 0.532  |
| 40   | 1.421       | 1.273  | 0.511  | 23.403      | 0.523       | 0.201  | 0.535  |
| 35   | 1.074       | 0.730  | 0.516  | 21.897      | 0.471       | 0.170  | 0.539  |
| 30   | 0.799       | 0.419  | 0.522  | 20.069      | 0.413       | 0.139  | 0.544  |
| 25   | 0.582       | 0.240  | 0.531  | 17.883      | 0.350       | 0.109  | 0.550  |
| 20   | 0.411       | 0.138  | 0.544  | 15.298      | 0.284       | 0.082  | 0.560  |

### Table 7. Estimated values of \( \sigma_{cm}/\sigma_{ci} \) proposed by Asef et al.14.

| RMR | \( \sigma_{cm}/\sigma_{ci} \) |
|-----|-----------------------------|
| 20  | 0.147                       |
| 30  | 0.142                       |
| 40  | 0.142                       |
| 50  | 0.166                       |
| 60  | 0.200                       |
| 70  | 0.250                       |
| 80  | 0.400                       |
| 90  | 0.666                       |
| 100 | 1.000                       |
1. Estimation of $\sigma_{cm}/\sigma_{ci}$
Considering different RMR, the reduction factor $\sigma_{cm}/\sigma_{ci}$ was proposed by Wilson\textsuperscript{37} to characterise the rock mass strength decreasing from its peak value to the residual. Assuming RMR-5 equals to GSI (see Eq. (8)), the estimated $\sigma_{cm}/\sigma_{ci}$ by Asef et al.\textsuperscript{14} are listed in Table 7. Other fitting equations for $\sigma_{cm}/\sigma_{ci}$ in the literature are presented in Eqs. (16) to (22):

Reference\textsuperscript{34}:

$$\frac{\sigma_{cm}}{\sigma_{ci}} = \sqrt{e^{\left(\frac{RMR - 100}{9}\right)}}$$

Reference\textsuperscript{38}:

$$\frac{\sigma_{cm}}{\sigma_{ci}} = e^{(0.0765RMR-7.65)}$$

Reference\textsuperscript{39}:

$$\frac{\sigma_{cm}}{\sigma_{ci}} = e^{\left(\frac{RMR-100}{24}\right)}$$

Reference\textsuperscript{40}:

$$\frac{\sigma_{cm}}{\sigma_{ci}} = e^{\left(\frac{RMR-100}{20}\right)}$$

Reference\textsuperscript{41}:

$$\frac{\sigma_{cm}}{\sigma_{ci}} = e^{\left(\frac{RMR-100}{18.75}\right)}$$

Reference\textsuperscript{42}:

$$\frac{\sigma_{cm}}{\sigma_{ci}} = \frac{RMR}{RMR + 6(100 - RMR)}$$

Reference\textsuperscript{26}:

$$\frac{\sigma_{cm}}{\sigma_{ci}} = 0.019e^{0.05GSI}, 5 \leq GSI \leq 35$$

$\sigma_{cm}$ was given values from 5 to 95 with 10 intervals, which is to compute $\sigma_{cm}/\sigma_{ci}$ through Eqs. (16) to (22). The obtained $\sigma_{cm}/\sigma_{ci}$ by Eqs. (16) to (22), by Asef et al.\textsuperscript{14}, and the field data retrieved from realistic construction sites\textsuperscript{42} are plotted in Fig. 3. With the estimated $\sigma_{cm}/\sigma_{ci}$, the best-fitting equation is expressed as:

$$\frac{\sigma_{cm}}{\sigma_{ci}} = 0.0103e^{0.0476GSI}$$

The coefficient of determination $R^2$ is 95.84%, which indicates the prediction by Eq. (23) is acceptable.

2. Estimation of $\sigma_{cm}$ and $\sigma_{ci}$. 
Reference\(^4\) claimed that \(\sigma_{cm}\) can be described as a function of RMR:

\[
\sigma_{cm} = 0.5e^{0.06RMR}
\]

(24)

Combing Eqs. (23) and (24), the solution for \(\sigma_{ci}\) is derived as:

\[
\sigma_{ci} = \frac{0.5e^{0.06RMR}}{0.0387 + 0.00474e^{18.9086}}
\]

(25)

\(\sigma_{ci}^p\) and \(\sigma_{ci}^r\) with different values of GSI\(^p\) and GSI\(^r\) are calculated by Eq. (25), and the result is presented in Table 8.

### Semi-analytical procedure

#### Governing equation.

For the case of plane strain, the equilibrium equation is:

\[
\frac{\partial\sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0
\]

(26)

In terms of small strain case, the displacement compatibility is:

\[
e = \frac{du}{dr}, \quad \epsilon_\theta = \frac{u}{r}
\]

(27)

#### Stresses and strains in the plastic softening zone.

The generalised H-B failure criterion\(^3\) is:

\[
\sigma_1 = \sigma_3 + \sigma_{ci}(m_b\sigma_3/\sigma_{ci} + s)^a
\]

(28)

where \(\sigma_1\) and \(\sigma_3\) are the major and minor principal stresses, \(\sigma_{ci}\) is the uniaxial compression strength of intact rock, \(m_b\) and \(s\) are the strength constants, respectively. According to Eq. (28), the yielding function of the rock mass surrounding a circular opening is:

\[
f(\sigma_0, \sigma_1, \sigma_\theta, \eta) = \sigma_0 - \sigma_1 - \sigma_{ci}(\eta)(m_b(\eta)\sigma_1/\sigma_{ci} + s(\eta))^a
\]

(29)

First, \(\sigma_1, \sigma_2\), the radial stress at the elastic–plastic boundary is solved by combing Eq. (26) with Eq. (29) through Runge–Kutta method.

A constant radial stress increment is assumed for each annulus, i.e.:

\[
\Delta\sigma_r = \sigma_{r(i)} - \sigma_{r(i-1)}
\]

(30)

where \(\sigma_{r(i)}\) and \(\sigma_{r(i-1)}\) are the radial stresses at the inner and outer boundaries of each annulus (i.e. \(r = r_{i}\)) and \(r_{i+1}\).

The plastic strain is expressed as:

\[
\left\{ \begin{array}{l}
\epsilon_{r(i)} \\
\epsilon_{\theta(i)}
\end{array} \right\} = \left\{ \begin{array}{l}
\epsilon_{r(i-1)} \\
\epsilon_{\theta(i-1)}
\end{array} \right\} + \left\{ \begin{array}{l}
\Delta \epsilon_{r(i)}^{plas} \\
\Delta \epsilon_{\theta(i)}^{plas}
\end{array} \right\} + \left\{ \begin{array}{l}
\Delta \epsilon_{r(i)}^{plas} \\
\Delta \epsilon_{\theta(i)}^{plas}
\end{array} \right\}
\]

(31)

where \(\epsilon_{r(i)}\) and \(\epsilon_{\theta(i)}\) are the radial and tangential strains at \(r = r_{i}\); \(\Delta \epsilon_{r(i)}^{plas}\) and \(\Delta \epsilon_{\theta(i)}^{plas}\) are the radial and tangential plastic strain increments; \(\Delta \epsilon_{r(i)}^{elas}\) and \(\Delta \epsilon_{\theta(i)}^{elas}\) are the radial and tangential elastic strain increments.

According to Hooke’s law, the elastic strain increments can be correlated to the stress increments, i.e.:
The relation between $\epsilon_{\eta}^{\text{plas}}$ and $\epsilon_{r}^{\text{plas}}$ can be given as:

$$\epsilon_{\eta}^{\text{plas}} = -K_\psi(\eta)\epsilon_{\eta}^{\text{plas}}$$  \hspace{1cm} (33)

where $K_\psi(\eta)$ is the dilatancy coefficient and can be written as:

$$K_\psi(\eta) = \frac{1 + \sin \psi(\eta)}{1 - \sin \psi(\eta)}$$  \hspace{1cm} (34)

where $\psi$ is the dilatancy angle, it should be noted that $\psi$ is not equal to the friction angle $\varphi$ when the non-associated flow rule is employed.

In order to solve the strain components, Eq. (27) can be rewritten as:

$$\begin{align*}
\epsilon_{r(i)} &= \frac{\Delta u_{r(i)}}{\Delta r_{(i)}} \quad \epsilon_{\theta(i)} = \frac{u_{\theta(i)}}{r_{(i)}}
\end{align*}$$  \hspace{1cm} (35)

where $u_{(i)}$ is the radial displacement at $r = r_{(i)}$ substituting Eqs. (32) and (33) into Eqs. (31) and (36), one gains:

$$\begin{align*}
\epsilon_{\theta(i)} &= \frac{\Delta u_{\theta(i)}}{\Delta r_{(i)}} = -K_\psi(\eta)\epsilon_{\theta(i)} + A_{(i-1)} \cdot \frac{r_{(i)}}{r_{(i-1)}} - 1 \\
\epsilon_{r(i)} &= \frac{\Delta u_{r(i)}}{\Delta r_{(i)}} = -K_\psi(\eta)\epsilon_{\theta(i)} + \frac{r_{(i)}}{r_{(i-1)}} - 1 \\
A_{(i-1)} &= \frac{(1 + v)}{E} \{ \Delta \sigma_{r(i)}(1 - v - K_\psi v) + [\sigma_{\theta(i-1)} + \sigma_{r(i)} + H(\sigma_{\theta(i)}, \eta_{(i-1)})] (K_\psi - K_\psi v - v) + \epsilon_{r(i)} + K_\psi \epsilon_{\theta(i-1)}
\}
\end{align*}$$

In accordance with Reference4, the relation between $r_{(i)}$ and $r_{(i-1)}$ can be derived as:

$$\frac{r_{(i)}}{r_{(i-1)}} = \frac{2H(\sigma_{\theta(i)} + \sigma_{r(i-1)})/2, \eta_{(i-1)}}{2H(\sigma_{\theta(i)} + \sigma_{r(i-1)})/2, \eta_{(i-1)}} - \Delta \sigma_{r}$$  \hspace{1cm} (39)

As illustrated in Eqs. (36), (37) and (39), $\epsilon_{\theta(i)}$, $\epsilon_{r(i)}$ and $r_{(i)}/r_{(i-1)}$ are independent of the radius $R_p$, or $R_c$. This means that with no need to obtain the value of $R_c$, stress and strain components in the plastic softening zone can be solved first.

Radii of plastic softening and residual zones.  \hspace{0.5cm} IN the plastic residual zone, by incorporating Eq. (29) into Eq. (26), one obtains:

$$\frac{\partial \sigma_{r}}{\partial r} = \frac{\sigma_{\text{ci}}(m_{b}^{\prime} \sigma_{r}^{\prime} + s')^{a'} - (m_{b}^{\prime} \sigma_{r}^{\prime} + s')^{a'}}{m_{b}^{\prime}(1 - a')}$$  \hspace{1cm} (40)

where $m_{b}^{\prime}$ and $s'$ are the strength parameters in the residual zone. The boundary conditions for Eq. (41) are: (1) $r = R_p$, $\sigma_{r} = \sigma_{\text{ci}}$ and (2) $r = R_c$, $\sigma_{r} = \sigma_{\text{ci}}$. Hence, the following equation can be derived from Eq. (40):

$$R_c = R_0 \exp \left[ \frac{(m_{b}^{\prime} \sigma_{r}^{\prime} + s')^{a'} - (m_{b}^{\prime} \sigma_{r}^{\prime} + s')^{a'}}{m_{b}^{\prime}(1 - a')} \right]$$  \hspace{1cm} (41)

Equation (41) illustrates that $R_c$ can be obtained by use of $m_{b}^{\prime}$, $s'$ and $\sigma_{\text{ci}}$. In fact, from Eq. (40), the relation between $R_c$ and $R_p$ can be derived as follows:

$$\frac{R_c}{R_p} = \prod_{i=1}^{j} \frac{2H(\sigma_{\theta(i)}^{\prime}, \eta_{(i-1)}) + \Delta \sigma_{r}}{2H(\sigma_{\theta(i)}^{\prime}, \eta_{(i-1)}) - \Delta \sigma_{r}}$$  \hspace{1cm} (42)

where $j$ is the number of the annulus immediately outside the plastic softening-residual boundary. Equation (42) shows that $R_p$ can be solved by $R_c$. In addition, when only the plastic softening zone is formed, Eq. (42) can be rewritten into:
Table 9. Parameters of the tunnel cases for verification.

|     | C1  | C2  | C3  | C4  | C5  | C6  |
|-----|-----|-----|-----|-----|-----|-----|
| ψ* | φ'/2| φ'/4| φ'/8| 0.25| 0.25| 0.25|
| ψpp | 30 | 30 | 30 | 30 | 30 | 24.81 |
| ψdr | 22 | 22 | 22 | 22 | 22 | 15.69 |
| η* | 0.008 | 0.008 | 0.008 | 0.008 | 0.008 | 0.017 |
| E/GPa | 10 | 10 | 10 | 10 | 3.837 | 3.837 |
| μ | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| R/ln | 3 | 3 | 3 | 3 | 3 | 7 |
| σ/MPa | 1 | 1 | 1 | 1 | 1 | 0.744 |
| σ/MPa | 20 | 20 | 20 | 20 | 20 | 12 |

\[
\frac{R_p}{R_0} = \prod_{i=1}^{j} \frac{2H(\sigma'_{\tau(i)} \eta_{(i-1)}) + \Delta \tau}{2H(\sigma'_{\tau(i)} \eta_{(i-1)}) - \Delta \sigma} \tag{43}
\]

Radial displacement of plastic softening and residual zones. Essentially, after obtaining \( R_p, u_{(i)} \) in the plastic softening zone can be solved by Eq. (38). As for \( u \) in the plastic residual zone, it can be obtained in a closed-form as shown in Eq. (44)\(^4\). Since the plastic softening zone is considered herein, \( R_0 \) and \( \sigma_0 \) of Eq. (44) are substituted for \( R_p \) and \( \sigma_0 \) of Eq. (38) presented in the elastic-brittle-plastic solution\(^4\).

\[
\frac{u}{r} = \frac{1}{2G} \frac{1}{R_0} \left[ D_1 f_1 (r) + D_2 f_2 (r) + D_3 f_3 (r) + 2 R_0 \mu u \right] \tag{44}
\]

where \( G \) is the shear modulus, \( G = E/2(1 + \mu) \).

\[
A^{H-B} = (m_b \sigma_{\tau}^{\alpha_1} p_1 + s \sigma_{\tau}^{\alpha_2})/\sigma_{\tau} = m_b \sigma_{\tau}^{\alpha_1}/4
\]

\[
D_1 = (K_\psi - \mu K_\psi - \mu A^{H-B} + (K_\psi + 1)(1 - 2\mu)(p_1 - \sigma_1),
\]

\[
D_2 = (K_\psi + 1)(1 - 2\mu)A^{H-B} + 2(K_\psi - \mu K_\psi - \mu)B^{H-B},
\]

\[
D_3 = (K_\psi + 1)(1 - 2\mu)B^{H-B},
\]

\[
f_1 (r) = r_{K_\psi + 1} / (K_\psi + 1),
\]

\[
f_2 (r) = \frac{r_{K_\psi + 1} / (K_\psi + 1)}{\ln \left( \frac{r}{R_0} \right) - \frac{1}{K_\psi + 1}}
\]

\[
f_3 (r) = \frac{r_{K_\psi + 1} / (K_\psi + 1)}{\ln \left( \frac{r}{R_0} \right) - \frac{2}{K_\psi + 1} \ln \left( \frac{r}{R_0} \right) + \frac{2}{(K_\psi + 1)^2}}
\]

Verification

The strength parameters for a group of tunnel excavation cases are used to verify the proposed semi-analytical procedure (Table 9). The cases are from the References\(^2,3,6\). Figure 4 demonstrates the distribution of the normalised radial displacement predicted by the semi-analytical procedure and a self-similar method\(^2\) for the SS rock masses. In this aspect, the above can also serve as a verification of the EPP and EPB rock masses.

In some existing studies, efforts were given to calculate the tunnel strain \( \varepsilon_t \) for the EPP rock mass with a wide range of qualities\(^13-16\). Particularly, Reference\(^16\) established a regression model with 20 < RMR < 90:
Figure 4. Variations of dimensionless support pressure $p_i/\sigma_0$ versus dimensionless radial displacement $u_0E_r/2R_0(1 + \mu)(\sigma_0 - \sigma_{r2})$ for case C1, C2, C3 and C4.

Figure 5. Variations of dimensionless support pressure $p_i/\sigma_0$ versus plastic radius $R_r/R_0$, or $R_p/R_0$ for case C5.

Figure 6. Ground reaction curve for case C6.
Based on Eqs. (8), (45) can be transferred to the following equation,

\[ u_0 = \begin{cases} 
24711 \times 0.43p_i \times RMR^{-2.42}, & \sigma_0 = 2.7 \text{ MPa} \\
157513 \times 0.59p_i \times RMR^{-2.71}, & \sigma_0 = 5.4 \text{ MPa} \\
696395 \times 0.65p_i \times RMR^{-2.99}, & \sigma_0 = 8.1 \text{ MPa} \\
3973329 \times 0.66p_i \times RMR^{-3.37}, & \sigma_0 = 10.8 \text{ MPa} \\
18531047 \times 0.67p_i \times RMR^{-3.72}, & \sigma_0 = 13.5 \text{ MPa} 
\end{cases} \]  

Then \( GSI_p \) was varied between 40 and 65 with 5 intervals to compare the proposed method with that by Reference 16. For each \( GSI_p \) value, \( \sigma_0 \) ranges from 2.7 to 13.5 MPa, \( p_i = 0 \) and \( R_0 = 5 \text{ m} \). \( \varepsilon_{\theta_Basarir} \) is obtained by dividing \( u_0 \) by \( R_0 \). The comparison of \( \varepsilon_{\theta} \) obtained from the semi-analytical procedure and \( \varepsilon_{\theta_Basarir} \) by the scheme in Reference 16 shows good agreement with the coefficient of determination \( R^2 \) up to 90.8% (see Fig. 7). Then the rationality of the input geological parameters (\( E_r, E_r', \sigma_c, \sigma_c', m_i, m_i', m_b, m_b', s', s_i, a', a_i' \)) in this study can be validated to some extent.

**Regression model for tunnel strain**

The strain \( \varepsilon_{\theta} \) can be fitted as a function of \( GSI_p, \sigma_0 \) and \( p_i/\sigma_0 \) by a nonlinear regression method. The equations for \( \varepsilon_{\theta} \) in the plastic and elastic areas differ from each other:

\[ \varepsilon_{\theta} = f_1 (GSI_p, \sigma_0, p_i/\sigma_0), \varepsilon_{\theta} > \varepsilon_{\theta_2}, p_i < \sigma_{r2}, \text{ plastic area} \]  

\[ \varepsilon_{\theta} = f_2 (GSI_p, \sigma_0, p_i/\sigma_0), \varepsilon_{\theta} \leq \varepsilon_{\theta_2}, p_i \geq \sigma_{r2}, \text{ elastic area} \]  

In Eq. (38), the critical strain \( \varepsilon_{\theta_2} \) and the critical support pressure \( \sigma_{r2} \) need to be determined prior to solving \( \varepsilon_{\theta} \). Combining Eqs. (26) and (27), fitting equations for \( \sigma_{r2} \) and \( \varepsilon_{\theta_2} \) can be written as:

\[ \sigma_{r2} = f_3 (GSI_p, \sigma_0) \]  

\[ \varepsilon_{\theta_2} = f_4 (GSI_p, \sigma_0) \]  

The Taylor series polynomial regression (PR) can be adopted to solve \( f_1, f_3 \) and \( f_4 \). Particularly for \( f_1 \), a nonlinear function can be constructed as:

**Figure 7.** Comparison between \( \varepsilon_{\theta} \) and \( \varepsilon_{\theta_Basarir} \).
For $f_2$ and $f_3$, the variable $y (e_0$ or $\sigma_p$) depends on $x_1$ (GSP) and $x_2 (\sigma_0)$, as:

$$y = a_1 + b_1 x_1 + b_2 x_2 + c_1 x_1 + c_2 x_2 + c_3 x_1 x_2 + c_4 x_1 + c_5 x_2 + c_6 x_1 x_2 + c_7 x_1 + c_8 x_2 + c_9 x_1 x_2$$

For $f_4$ and $f_5$, the variable $y (e_0$ or $\sigma_p$) depends on $x_1$ (GSP) and $x_2 (\sigma_0)$, as:

$$y = a_1 + b_1 x_1 + b_2 x_2 + c_1 x_1 + c_2 x_2 + c_3 x_1 x_2 + d_1 x_1^3 + d_2 x_2^3 + d_3 x_1 x_2^2 + d_4 x_1^2 x_2$$

To obtain the coefficients in Eqs. (40) to (42), $e_0$ for a large number of tunnelling cases are calculated by the proposed iterative procedure. The input geological parameters (GSP, $E_p$, $E_r$, $\sigma_p$, $\sigma_r$, $m_f$, $m_s$, $\phi$, $\delta$, and $\theta$) for the calculation are given in Tables 1, 3, 6, and 8. Nine values for $\eta$ within cases A1 to A9 are listed in Table 10. $e_0$ varies from 5 to 50 MPa with intervals of 5 MPa. $p_i/\sigma_0$ ranges from 0 to 1 MPa and 10 to 20 values are selected for different combination of $p_i$ and $\sigma_0$. $f_2$, $f_3$, and $f_4$ are correlated to the peak geological parameters in the elastic zone. The regression model is composed of twelve equations: three equations are for $f_1$, $f_2$, and $f_3$, and nine equations are for $f_4$. Then the coefficients can be determined with the Levenberg Marquardt iteration algorithm (see Tables 11 and 12), which is validated through the analysis of variance ANOVA. The predictions with the twelve equations match well with those by the semi-analytical procedure.

### Parametric study

#### Variation of tunnel strain with different critical softening parameters.

Values of $e_0$ are calculated by the proposed regression model, which are plotted for Cases A1 to A9 versus GSP, $\sigma_0$, and $p_i/\sigma_0$, respectively, as in Figs. 8 and 9. In Fig. 8, GSP is variable, $\sigma_0$ is 30 MPa and $p_i/\sigma_0$ is 0.1, and in Fig. 9, $p_i/\sigma_0$ is variable, GSP is 30 and $\sigma_0$ is 5 MPa. When GSP is 70 or 75, and $p_i/\sigma_0$ is 0.3, $e_0$ maintains constant. The reason is that GSP and $p_i/\sigma_0$ are relatively large, so that the rock mass takes elastic deformations and is independent of $\eta$. With plastic deformations in the rock mass, $e_0$ decreases to a substantial constant with the increase in $\eta$. The decrease of $e_0$ is induced by the shrinkage of the plastic residual area. If $\eta$ is nil, all rock mass within the plastic area is characterised with the residual strength; and the maximum $e_0$ is therefore reached; as $\eta$ increases, $e_0$ falls rapidly since the softening area expands; and $e_0$ becomes stable when the softening zone dominates in the plastic area. The expansion of the plastic residual area is the critical factor enhancing the deformation within the rock mass. In the practical engineering, the measures to decrease the plastic residual zone can substantially improve the tunnel stability. Furthermore, $e_0$ falls quickly and becomes constant within a small $\eta$ for excellent quality rock mass, whereas $e_0$ for the weak rock mass decreases slowly and the decline process is prolonged until a plateau is reached (see Fig. 9). Hence, the rock mass deformation decreases more suddenly with a better quality rock while $\eta$ increases.

### Table 10. Critical plastic softening parameter $\eta$.

| Case | $\eta$ | $f_1$ | $f_2$ |
|------|--------|-------|-------|
| A1   | 0      | 0.005 | 0.01  |
| A2   | 0.025  | 0.05  | 0.1   |
| A3   | 0.5    | 1     | $\approx$ |

### Table 11. Coefficients in $f_1, f_2$, and $f_3$.

| $a_1 (10^{-5})$ | $a_2$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ | $c_8$ | $c_9$ | $f_1$ | $f_2$ |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 42.5845         | -0.28589 | 0.01052 | 0.00273 |       |       |       |       |       |       |       |       |       |
| 2.88088         |       | 0.0338 | -0.01925 |       |       |       |       |       |       |       |       |       |
| 1.1807          |       | 0.91007 | 0.47982 |       |       |       |       |       |       |       |       |       |
| 0.075412        | -0.00195 | -3.57994 |       |       |       |       |       |       |       |       |       |       |
| 0.043128        |       | 0.00938 |       |       |       |       |       |       |       |       |       |       |
| 0.035988        |       | -0.01975 |       |       |       |       |       |       |       |       |       |       |
| 0.000485        |       |       |       |       |       |       |       |       |       |       |       |       |
| 0.000479        |       |       |       |       |       |       |       |       |       |       |       |       |
| 0.000468        |       |       |       |       |       |       |       |       |       |       |       |       |
| 0.000488        |       |       |       |       |       |       |       |       |       |       |       | 5.4923 |
Difference of tunnel strain between the EPP and EBP rock masses. $\varepsilon_0$ for the EPP rock mass is symbolised by $\varepsilon_{0\text{-EPP}}$. The increase ratio of $\varepsilon_0$ for the EBP rock mass in comparison to the EPP counterpart is denoted by $\Delta\varepsilon/\varepsilon_{0\text{-EPP}}$. $\Delta\varepsilon/\varepsilon_{0\text{-EPP}}$ versus GSIp for variations in $\sigma_0$ and $p_i/\sigma_0$ is plotted in Fig. 10.

| (a) $\eta^*$ | $\infty$ | 1 | 0.5 | 0.1 | 0.05 |
|-------------|---------|---|-----|-----|-----|
| $a_1$ | 0.37576 | −0.74117 | −0.61639 | 1.08615 | 3.27504 |
| $b_1$ | −0.3432 | −0.24502 | −0.2742 | −0.43415 | −0.58165 |
| $b_2$ | −16.0768 | −16.14255 | −16.5117 | −16.45784 | −19.47774 |
| $b_3$ | 0.32939 | 0.41235 | 0.46672 | 0.53739 | 0.53157 |
| $c_1$ | 0.00788 | 0.00246 | 0.00225 | 0.00656 | 0.00967 |
| $c_2$ | 19.49741 | 21.77123 | 22.57404 | 21.46441 | 27.19204 |
| $c_3$ | −0.00289 | −0.00176 | −0.00156 | −0.00562 | −0.00485 |
| $c_4$ | 0.33925 | 0.34786 | 0.36874 | 0.38357 | 0.45668 |
| $c_5$ | −0.07987 | −0.22695 | −0.26771 | −0.70855 | −0.82604 |
| $c_6$ | −0.00862 | −0.02094 | −0.02172 | −0.01372 | −0.00686 |
| $d_1$ | −0.00059963 | −0.01182 | −0.00343 | 0.00441 | 0.00765 |
| $d_2$ | 2.24755 | 0.707777 | −0.862592 | 4.88805 | 10.3286 |
| $d_3$ | 4.96784 | 15.5397 | 17.8538 | 8.07328 | −8.19222 |
| $d_4$ | −0.01828 | 0.29111 | 0.37644 | 0.83708 | 0.87071 |
| $d_5$ | −0.00565 | −0.02268 | −0.0153 | 0.00261 | 0.00517 |
| $d_6$ | 15.49841 | −18.33127 | −18.88734 | −19.65351 | −25.08435 |
| $d_7$ | 9.11563 | 8.73084 | 10.4683 | 1.56558 | −2.26792 |

| (b) $\eta^*$ | 0.025 | 0.01 | 0.005 | 0 |
|-------------|------|------|-------|----|
| $a_1$ | 3.37806 | −1.23767 | −0.45896 | 3.37629 |
| $b_1$ | −0.56164 | −0.16611 | −0.15858 | −0.47128 |
| $b_2$ | −21.86517 | −21.45617 | −19.50907 | −16.89839 |
| $b_3$ | 0.48318 | 0.52196 | 0.38419 | 0.25426 |
| $c_1$ | 0.0088 | −0.00075226 | −0.00132 | 0.00759 |
| $c_2$ | 33.78561 | 37.16971 | 26.33782 | 28.57907 |
| $c_3$ | −0.00656 | −0.00627 | −0.00162 | −0.00173 |
| $c_4$ | 0.51334 | 0.43875 | 0.29709 | 0.15241 |
| $c_5$ | −0.96887 | −0.66871 | −0.00379 | 0.38674 |
| $c_6$ | −0.00277 | −0.00527 | −0.0063 | 0.00259 |
| $d_1$ | 0.01421 | 0.00686 | −0.0018 | −0.00575 |
| $d_2$ | 14.2824 | 10.6465 | 4.79792 | 2.97141 |
| $d_3$ | −19.9824 | 2.54789 | 19.5277 | 4.91271 |
| $d_4$ | 0.85951 | 0.57685 | −0.19027 | 0.13331 |
| $d_5$ | 0.0037 | 0.00075557 | −0.01386 | 0.02578 |
| $d_6$ | −30.75648 | −32.48237 | −20.64928 | −36.72157 |
| $d_7$ | −4.78882 | 2.51696 | 2.35951 | −5.6103 |
| $e_1$ | 68.6 | 81.9 | 25.7 | 109.2 |
| $e_2$ | 97.7 | 19.3 | 94.2 | 39.7 |
| $e_3$ | −7.28793 | −3.79109 | 12.6335 | 32.7694 |
| $e_4$ | 19.6994 | 4.32378 | −20.7783 | 4.66801 |
| $e_5$ | −0.18899 | −0.13045 | 0.17124 | 0.25735 |
| $e_6$ | −10.6367 | −2.62886 | 13.3079 | 2.50107 |
| $e_7$ | 0.00376 | −0.00163 | −0.00931 | 0.00429 |

Table 12. Coefficients in $f_1$: (a) when $\eta^* = \infty$, 1, 0.5, 0.1, 0.05; (b) when $\eta^* = 0.025$, 0.01, 0.005, 0.
When $p_i/\sigma_0$ is 0.1, 0.2 and 0.3, $\Delta\varepsilon/\varepsilon_{\text{EPP}}$ decreases as $GSI_p$ increases (see Fig. 10b–d). Hence, while $p_i$ exceeds 0.1, the effect of $\eta^*$ on $\varepsilon$ for the weakest rock mass ($GSI_p = 25$) is the greatest, which should be highlighted. While $p_i$ is 0, and $\sigma_0$ ranges from 10 to 20 MPa, $\Delta\varepsilon/\varepsilon_{\text{EPP}}$ rises but then decreases with the increase in $GSI_p$ (Fig. 10a). The maximum $\Delta\varepsilon/\varepsilon_{\text{EPP}}$ appears while $GSI_p$ is around 45 or 50. In this case, the influence of $\eta^*$ on $\varepsilon$ for the moderate rock mass ($GSI_p = 45, 50$) is the largest. For $GSI_p$ is 50 and $\sigma_0$ is 20 MPa, $\Delta\varepsilon/\varepsilon_{\text{EPP}}$ reaches almost 10.64 for $p_i/\sigma_0$ is 0 but drops to 1.77 for $p_i/\sigma_0$ is 0.1 (see Fig. 10a,b). This means that the growth of $p_i$ effectively weakens the softening effect on the deformation for moderate quality rock mass with high initial stress. Furthermore, when $GSI_p$ is greater than 55 and $p_i/\sigma_0$ exceeds 0.1, $\Delta\varepsilon/\varepsilon_{\text{EPP}}$ for most cases is 0, which means $\varepsilon$ by EPP and EBP rock masses are equivalent (see Fig. 10b–d). This is because that the rock mass undergoes an elastic deformation. Therefore, if $p_i/\sigma_0$ reaches 0.1, the rock mass deformation is inconsiderable and irrespective of $\eta^*$ for the excellent rock mass quality ($GSI_p \geq 55$).

Sensitive analysis. Figure 11 illustrates the sensitivity analysis concerning the tunnel strain $\varepsilon_0$, showing the relative significance of the most significant input data (i.e. $GSI_p$, $\sigma_0$ and $p_i/\sigma_0$) on this final output (i.e. $\varepsilon_0$). Three base cases with different rock mass qualities are given in Table 13. In the sensitive analysis, $\sigma_0$ varies between 5 and 30 MPa with even intervals of 5 MPa. $p_i/\sigma_0$ ranges from 0 to 0.225 with 0.025 intervals. $GSI_p$ ranges from 25 to 75 with 5 intervals. $GSI_p$, $\sigma_0$ or $p_i/\sigma_0$ is represented by the variable $m$. $GSI_p$, $\sigma_0$ or $p_i/\sigma_0$ in cases B1 to B3 is represented by $m_{\text{base}}$. $\varepsilon_0$ calculated by cases B1 to B3 is represented by $\varepsilon_0_{\text{base}}$.

In comparison with the EBP rock mass, $\varepsilon_0/\varepsilon_{0,\text{base}}$ of the EPP rock mass with the moderate and weak rock qualities tends to be closer to the line for $\varepsilon_0/\varepsilon_{0,\text{base}}$ is 1 (see Fig. 11b,c). In this respect, $\varepsilon_0$ for the EBP rock mass is more sensitive to the change in $GSI_p$, $p_i/\sigma_0$ and $\sigma_0$. However, for the excellent quality rock mass, $\varepsilon_0/\varepsilon_{0,\text{base}}$ of EBP rock mass coincides with that of EPP rock mass (Fig. 11a). This is attributed to that the rock mass exhibits the elastic behaviour, and thus $\varepsilon_0$ is independent of the plastic parameters. In this respect, the influence of $GSI_p$, $p_i/\sigma_0$ or $\sigma_0$ on $\varepsilon_0$ by EPP and EBP rock masses are equivalent.

Among the input parameters $GSI_p$, $\sigma_0$ and $p_i/\sigma_0$, the change in $GSI_p$ gives rise to the greatest change in $\varepsilon_0$. Especially for the excellent rock mass, $\varepsilon_0/\varepsilon_{0,\text{base}}$ by $GSI_p$ is considerably higher than $\sigma_0$ and $p_i/\sigma_0$ (Fig. 11a). Therefore, $GSI_p$ is of vital importance in controlling $\varepsilon_0$. The relative significance of $p_i/\sigma_0$ and $\sigma_0$ varies with different conditions. For the EBP rock mass, when $p_i/\sigma_0$ decreases and $\sigma_0$ increases with an equivalent variation, $\varepsilon_0/\varepsilon_{0,\text{base}}$ affected
by $p_i/\sigma_0$ is always higher than that by $\sigma_0$; and it becomes remarkably higher while $p_i/\sigma_0$ decreases to a small value. Hence, for the EBP rock mass, when $p_i/\sigma_0$ decreases and $\sigma_0$ increases, the influence of $p_i/\sigma_0$ on $\varepsilon_\varnothing$ is larger than that of $\sigma_0$. For all the other conditions, the influence of $\sigma_0$ on $\varepsilon_\varnothing$ is greater than that of $p_i/\sigma_0$. For instance, for the EPP rock mass, the change in $\sigma_0$ causes a larger variation in $\varepsilon_\varnothing$ for the EBP rock mass, when $p_i/\sigma_0$ increases and $\sigma_0$ decreases with the equivalent variation, a decrease of $\sigma_0$ yields a higher reduction of $\varepsilon_\varnothing$. As the weak rock mass shows the EPP behaviour\textsuperscript{33}, the reduction of $\sigma_0$ exerts greater influence than the increase in $p_i/\sigma_0$ in controlling the rock deformation for the weak rock mass. In the tunnelling engineering, the reduction of $\sigma_0$ and the increase of $p_i/\sigma_0$ can be obtained by relieving the stress and installing the rigid support, respectively.

Conclusions

Various GSI were considered to quantify the input geological parameters for the strain-softening rock masses with various qualities. A specialised numerical scheme was presented to calculate the tunnel strain around a circular opening within the rock mass. The proposed semi-analytical procedure and the input geological parameters were validated through comparison of the tunnel strain obtained by the semi-analytical procedure with that predicted by the previous studies. With the obtained input geological parameters, more accurate quantification of the tunnel strain was obtained by a semi-analytical procedure. A regression model, composed of 12 fitting equations, was further proposed: 3 equations were to calculate the critical tunnel strain, the critical support pressure and the tunnel strain with elastic behaviour, and 9 equations were for the tunnel strain with different strain-softening behaviours. The model provides practical guidelines to assess the deformations of the rock mass prior to the tunnel construction. Following conclusions can then be drawn:

The tunnel strain wanes to a constant value with the critical softening parameter keeps increasing, which is mainly ascribed to the shrinkage of the plastic residual area. Reversely, the rock deformation is mainly raised due to the expansion of the plastic residual area. In the practical engineering, the measures to decrease the plastic residual area can substantially improve the tunnel stability.

While the support pressure exceeds a certain value ($p_i/\sigma_0 \geq 0.1$), the critical softening parameter makes the most significant influence on the tunnel strain for the weakest rock mass (GSP = 25). In comparison, with no support pressure ($p_i/\sigma_0 \geq 0$) and relatively high initial stress ($\sigma_0 \geq 10$ MPa), the influence of the critical softening parameter for the moderate rock mass (GSP is around 45 or 50) is the most significant. While the support pressure that acted on the good rock mass quality (GSP $\geq 55$) exceeds a certain value, the rock mass deformation becomes inconsiderable.

While the rock mass exhibits a strain-softening behaviour, the tunnel strain for the EBP rock mass can be affected by the change in the rock mass quality, the support pressure and the initial stress state. Among the three

Figure 10. Variation of $\Delta\varepsilon_{\varnothing}/\varepsilon_{\varnothing-\text{EPP}}$ versus GSI $p$ : (a) $p_i/\sigma_0 = 0$; (b) $p_i/\sigma_0 = 0.1$; (c) $p_i/\sigma_0 = 0.2$; (d) $p_i/\sigma_0 = 0.3$. 
input geological parameters (i.e. $GSI_p$, the support pressure, and the initial stress), $GSI_p$ is of vital importance in controlling the tunnel strain. The relative significance of the support pressure and initial stress varies with different conditions. For the EBP rock mass, with the support pressure decreases and the initial stress increases, the tunnel strain is mostly influenced by the variation in the support pressure. For all other conditions, the initial stress state becomes the critical factor.

Data availability
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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**Figure 11.** Sensitive analysis of $GSI_p$, $\sigma_0$ and $p/\sigma_0$ on $\varepsilon_\theta$: (a) cases B1; (b) case B2; (c) case B3.

| Case B1 | Case B2 | Case B3 |
|---------|---------|---------|
| $GSI_p$ | 70      | 50      | 30      |
| $\sigma_0$ (MPa) | 20      | 20      | 20      |
| $p/\sigma_0$ | 0.15    | 0.15    | 0.15    |

**Table 13.** $GSI$, $\sigma_0$ and $p/\sigma_0$ for cases B1 to B3.
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Author contributions
Contributor roles taxonomy: Y.D. and L.C.: conceptualization, methodology, validation, investigation and writing-original draft; Q.S.: data curation, formal analysis; Y.D.: visualization, project administration; J.Z., Z.G. and L.C.: writing—review and editing.
Competing interests
The authors declare no competing interests.

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