Emergent anyons are the key elements of the topological quantum computation and topological quantum memory. We study a two-component fermion model with conventional two-body interaction in an open boundary condition and show that several subsets in the low-lying excitations obey the same fusion rules as those of the toric code model. Those string-like non-local excitations in a given subset obey mutual semionic statistics. We show how to peel off one of such subset from other degenerate subsets and manipulate anyons in cold dipolar Fermi atoms or cold dipolar fermionic heteronuclear molecules in optical lattices by means of the established techniques.
Introductions

Anyons are long wistful objects in two-dimensional condensed matter systems [1, 2]. Interesting to explore anyons is currently renewed because of the potential application of anyons in topological quantum computation and topological quantum memory [3-4]. The researches mainly focus on two topics: non-abelian fractional quantum Hall states [5-7] and particular lattice spin models, e.g., Kitaev toric code model [9], Levin-Wen model [8] and Kitaev honeycomb-lattice spin model [9].

Experimentally exciting, manipulating and detecting abelian anyons have been suggested or tried for the toric code model [10, 11] and for the insulating phase of Kitaev honeycomb-lattice model [12]. Although these non-trivial tries are interesting, the reliable evidence for the existence of anyons still lacks since neither the toric code model nor honeycomb-lattice spin model is easy to be realized owing to those unconventional interactions between constitution particles, e.g., cold atoms [13].

A widely interesting question is: do we have a simple system with conventional two-body interactions whose non-local elementary excitations are anyons? We here study a two-component fermion system which is a set of decoupled Ising chains [14]. These chains form a two-dimensional square lattice with each chain along the horizontal diagonal direction (See Fig. 1). The fermions within a chain interact with on-site repulsive and nearest neighbor attractive potentials. Nussinov and Ortiz [15] have found the decoupled Ising chains are of the same spectrum as the toric code model and Liven-Wen model. However, the topological order is trivial when the plain lattice is curled up to a torus.

We here show that although the topological order of decoupled Ising chains with a periodic boundary condition is trivial, when the in-chain coupling constants are special chosen with respect to the chemical potential, there are low-lying non-local excitations in an open boundary condition which may obey anyonic statistics. We will see that the low-lying excitations of this model are classified by several kinds of closed subsets. One kind of them is local, including a single hole, a double occupant and a spin-flip. Other two kinds consist of a local fermion and two string-like non-local excitations. The fusion rules and exchange phase factor in later two are exactly the same as those of the excitations in the toric code model. The statistics of the non-local excitations themselves are bosonic while it is mutual semionic. We find that the ground state in this model is stable for a dipole-dipole long range interactions if inter-chain couplings are negligible. Thus, this system can be realized in cold dipolar Fermi atoms, e.g., rare-earth atoms of Ytterbium [16], or the cold fermionic het-
We restrict on the nearest neighbor attractive interaction while on-site is repulsive, i.e., squares. This is a set of decoupled Ising chains along the horizontal diagonals of squares. With an open boundary condition, the low-lying excitations above a given ground state are created by anyons by means of the established cold atom and molecular techniques.

**Two-component fermion model in square lattice** We consider a simple Hamiltonian for two-component fermion model in square lattice

\[
H = - \sum_{\langle ij \rangle_{hd}} J_s (2n_{s,i} - 1)(2n_{s,j} - 1) + U \sum_i (2n_{\uparrow,i} - 1)(2n_{\downarrow,i} - 1),
\]

where \(n_{s,i} = c_{s,i}^\dagger c_{s,i}\) with \(c_{s,i}\) being annihilation operators of (pseudo)spin-\(s\) fermions. The symbol \(\langle ij \rangle_{hd}\) means the sum is over nearest neighbors along the horizontal diagonals of squares. This is a set of decoupled Ising chains along the horizontal diagonals of squares. We restrict on the nearest neighbor attractive interaction while on-site is repulsive, i.e., \(J_s > 0\) and \(U > 0\). In the case, the ground states of this Hamiltonian are \(2^n\)-fold degenerate, i.e., every individual chain is ferromagnetic, i.e., for a set of spins \(\{s_1, \cdots, s_n, \cdots, s_n\}\),

\[
|G_{\{s\}}\rangle = \prod_{a=1}^n |G_{s_a}\rangle = \prod_{a,i} c_{s_a,i_a}^\dagger |0\rangle,
\]

where \(n\) is the number of chains, \(i_a\) is the site index in the \(a\)-th chain. Restricted to the open boundary condition, the low-lying excitations above a given ground state \(\ket{G_{\{s\}}}\) for a given site \(i_a\), reads (See Fig. 1 for a given ground state \(\ket{G_{\{1\}}}\) which will be defined later.)

\[
\mathcal{H}_{i_a} |G_{\{s\}}\rangle = (c_{s_a,i_a}^\dagger + c_{s_a,i_a}) |G_{\{s\}}\rangle = c_{s_a,i_a} |G_{\{s\}}\rangle;
\]

\[
\mathcal{D}_{i_a} |G_{\{s\}}\rangle = (c_{s_a,i_a}^\dagger + c_{s_a,i_a}) |G_{\{s\}}\rangle = c_{s_a,i_a}^\dagger |G_{\{s\}}\rangle;
\]

\[
\mathcal{F}_{i_a} |G_{\{s\}}\rangle = i \mathcal{H}_{i_a} \mathcal{D}_{i_a} |G_{\{s\}}\rangle = i c_{s_a,i_a} c_{s_a,i_a}^\dagger |G_{\{s\}}\rangle;
\]

\[
\mathcal{W}_{P} |G_{\{s\}}\rangle = \prod_{i_s \leq i_a} \mathcal{F}_{i_s} |G_{\{s\}}\rangle = \prod_{i_s \leq i_a} i c_{s_{b_s},i_{b_s}} c_{s_{b_s},i_{b_s}}^\dagger |G_{\{s\}}\rangle;
\]

\[
\mathcal{W}_{P,P'} |G_{\{s\}}\rangle = \prod_{i_s \leq i_a} \mathcal{F}_{i_s} \mathcal{D}_{i_a} |G_{\{s\}}\rangle = \prod_{i_s \leq i_a} i c_{s_{b_s},i_{b_s}} c_{s_{b_s},i_{b_s}}^\dagger |G_{\{s\}}\rangle,
\]

where \(P\) and \(P'\) denote two plaquettes on the right and left of \(i_a\), respectively. \(\bar{s} = \uparrow (\downarrow)\). The order of sites is defined by \(i_{b} < i_{a}\) if \(i_{b}\) is on the left hand of \(i_{a}\) for \(b = a\) or \(i_{b}\) is in a chain lower than the chain with \(i_{a}\). \(\mathcal{H}, \mathcal{D}, \mathcal{F}\) create a hole, a double
occupant, and a spin-flip. \( W, W^d \) and \( W^h \) create a half-infinite string of spin-flips, a spin-flip string with a double occupant and a spin-flip string with a hole, respectively, since the spins of fermions at sites \( i_b < i_a \) are flipped from their ground state configuration while those at \( j_c > i_a \) keep in their ground state configuration. The excitation energies of these local and non-local excitations in turn are 

\[
4J_{s_a} + 2U, \quad 4J_{\bar{s}_a} + 2U, \quad 4J_{\Uparrow} + 4J_{\Downarrow}, \quad 2J_{\Uparrow} + 2J_{\Downarrow}, \quad 2J_{\Uparrow} + 2J_{\Downarrow} + 2U \text{ and } 2J_{\Uparrow} + 2J_{\Downarrow} + 2U
\]

respectively. The finite energies of the string-like excitations mean they are deconfined. Note that the ground state may be instable against an on-site energy \( V_B \sum_i (2n_{\uparrow}(i) - 1) \) if \( V_B > 0 \) and if \( V_B < 0 \), a pair of string-like excitations may be linearly confined and the statistics of the string-like excitations becomes ill-defined. These excitations are highly degenerate due to the degeneracy of the ground states. For example, if \( \{s_1, \ldots, s_a \ldots, s_n\} \rightarrow \{s_1, \ldots, \bar{s}_a \ldots, s_n\} \), \( H \leftrightarrow D \) and \( (\leq, <) \rightarrow (\geq, >) \) in (\ref{eq:fusion}).

One can also flip spin in other chains to get a new degenerate excitation.

**Fusion rules** Note that \( O^2 = I \) for \( O = H, D, F, W, W^d, W^h \). The fusion rules of these excitations are given in Table 1. The fusion rules of the closed subset \( \{I, D_{i_a}, W_P, W^h_{P,P'}\} \) (or \( \{I, H_{i_a}, W_P, W^d_{P,P'}\} \)) are exactly the same as the fusion rules in Kitaev toric code model if we identify \( D_{i_a}, W_P, \) and \( W^h_{P,P'} \) as \( \psi, e \) and \( m \) \cite{Kitaev2003, Bombin2008}. The subset \( \{I, F_{i_a}, W^h_{P,P'}, W^d_{P,P'}\} \) has similar fusion rules but \( F \) is bosonic. \( \{I, H_{i_a}, D_{i_a}, F_{i_a}\} \) is also a closed subset and has similar fusion rules but with two fermions (\( H, D \)) and one boson (\( F \)).

---

**FIG. 1:** (Color on line) The ground state (taking \( |G_{\Uparrow}\rangle \) as an example), and the low-lying excitations in a set of decoupled Ising spin chains which form a square lattice. From left to right and up to down, they are \( |G_{\Uparrow}\rangle, |H_{i_a}\rangle, |D_{i_a}\rangle, |F_{i_a}\rangle, |W_P\rangle, |W^h_{P,P'}\rangle, |W^d_{P,P'}\rangle \) and \( |W_P\rangle \). Up- and down-arrows label the fermion with spin-up and spin down. Empty circle is unoccupied site and up-down arrow is double occupied. The white plaquette \( P^a \) has \( (G^a_{P^a}, G^h_{P^a}) = (1, 1) \), the yellow has \( (-1, 1) \), the red has \( (1, -1) \), and the grey has \( (-1, -1) \).
a subset. The local excitations
Statistics in a given subset
chains so that one can circle around another. Therefore, the ir (mutual) exchange statistics
Ground state is of all
Each chain. If we curl up the plain lattice to a torus, the topo logical order is completely
A string-like excitation may be thought as a spin flipping domain wall,
Boundary conditions
Discrete gauge symmetry
The conserved quantities are simply $Q_{s,i} = 2n_{s,i} - 1$, (in fact, they
are $n_{s,i}$), which have eigen values ±1 at each site. This generates $Z_2 \times Z_2$ gauge symmetry.
For a plaquette $P$, one can label the plaquette by $G^s \equiv (2n_{ia} , 2n_{ja} - 1)$ for a pair of
nearest neighbors $(i_a , j_a)$ in the $a$-th chain, which also have eigen values ±1. Obviously, the
ground state is of all $G^s = 1$. The excitations are also of all $G^s = 1$ except for the right
plaquette $P$ and left plaquette $P'$ of $i_a$ where $(G^s_i , G^s_{i_a} , G^s_{i_{a'}} , G^s_{i_a'}) = (-1, 1, -1, 1)$ for $H_{ia}$,
$(1, -1, 1, -1)$ for $D_{ia}$, $(-1, -1, -1, -1)$ for $F_{ia}$, $(1, 1, -1, -1)$ for $W_P$, $(\mp 1, \mp 1, \mp 1, \mp 1)$ for
$W^h_{P,P'}$ and $(\mp 1, \mp 1, \mp 1, \mp 1)$ for $W^d_{P,P'}$. To distinguish latter two, one uses $(Q_{s_{ia},i_a} , Q_{s_{ia},i_a}) =
(-1, -1)$ for $W^h_{P,P'}$ and $(1, 1)$ for $W^d_{P,P'}$. In this sense, these string-like excitations are also
called $Z_2 \times Z_2$ vortices.

Boundary conditions
A string-like excitation may be thought as a spin flipping domain wall,
a topological defect (See Fig. 1, late four). The open boundary condition allows odd number
domain walls while the periodic boundary condition restricts the wall number to even in
each chain. If we curl up the plain lattice to a torus, the topological order is completely
trivial and the system is identical to a one-dimensional one. In the open boundary condition,
two walls as ends of a string, e.g., $\cdots \uparrow \downarrow \circ \downarrow \cdots$ and $\cdots \downarrow \downarrow \uparrow \cdots$, may locate at different
chains so that one can circle around another. Therefore, their (mutual) exchange statistics
can be well defined.

Statistics in a given subset
We now study the statistics of the non-local excitations within
a subset. The local excitations $H, D$ are the fermionic while $F$ is bosonic. For a string-like
excitation, the site of its domain wall, $i_a$, may be used to label its 'position', e.g., $W_P = W_{ia}$,
etc. $W$ is bosonic because it is a string of $F$. $W^d,h$ themselves are bosonic, e.g.,

$$W^h_1 W^h_2 = (H_1)(D_1 H_2) = (H_1 D_1 H_2)(H_1) = W^h_2 W^h_1.$$ 

Since $W$ and $W^d,h$ are distinguishable, the exchange between them does not make sense.

| $H_{ia}$ | $D_{ia}$ | $F_{ia}$ | $W_P$ | $W^h_{P,P'}$ | $W^d_{P,P'}$ | $W_{P'}$ |
|----------|----------|----------|--------|--------------|--------------|----------|
| $H_{ia}$ | $I$      | $F_{ia}$ | $D_{ia}$ | $W^d_{P,P'}$ | $W_{P'}$     | $W^h_{P,P'}$ |
| $D_{ia}$ | $F_{ia}$ | $I$      | $H_{ia}$ | $W^h_{P,P'}$ | $W_P$        | $W^d_{P,P'}$ |
| $F_{ia}$ | $D_{ia}$ | $H_{ia}$ | $I$    | $W_{P'}$    | $W^d_{P,P'}$ | $W^h_{P,P'}$ |
| $W_P$    | $W^d_{P,P'}$ | $W^h_{P,P'}$ | $W_{P'}$ | $I$          | $D_{ia}$      | $H_{ia}$     |
| $W^h_{P,P'}$ | $W^d_{P,P'}$ | $W_P$    | $W_{P'}$ | $I$          | $D_{ia}$      | $H_{ia}$     |
| $W^d_{P,P'}$ | $W_{P'}$    | $W^h_{P,P'}$ | $H_{ia}$ | $F_{ia}$     | $I$          | $D_{ia}$     |

Table 1: Fusion rules of excitations.
However, because $\mathcal{W}_h^h, d$ fuses to a fermion while themselves are bosons, when $\mathcal{W}$ circles around $\mathcal{W}_h^h, d$ or vice versa, a minus sign is acquired. In general, this fact can be proved by applying the consistent conditions, i.e., the pentagon and hexagon equations \cite{[18]}. For the present case, one can take Kitaev’s graphical proof \cite{[9]}. For simplicity, denote \{ $I, \mathcal{H}, \mathcal{W}, \mathcal{W}^d$ \} (or \{ $I, \mathcal{D}, \mathcal{W}, \mathcal{W}^h$ \}) = \{ $I, \psi, e, m$ \}. Since $e^2 = m^2 = I$, we can create two pairs of $e$ and two pairs of $m$ from a given ground state at an initial time $T_0$ (See Fig. 2). These excitations move along the paths as shown in Fig. 2. At certain time $T_1$, they fuse to four fermions $\psi$. As time flies, while blue and black fermions stay alone, green and red fermions exchange their positions. Finally, at $T_2$, the fermions split to $m$ and $e$ pairs which, at the end ($T_f$), fuse back to the ground state. As two fermions exchange, this process contributes a minus sign $R_{\psi\psi} = -1$ to the ground state comparing to a process without fermion exchange. Now, examine this process in non-local excitation exchanges. As shown in Fig. 3, this fermion exchange is corresponding to four exchanges: $R_{em}, R_{ee}, R_{mm}$ and $R_{me}$. That is,

$$R_{me}R_{ee}R_{mm}R_{em} = R_{me}R_{em} = R_{\psi\psi} = -1,$$

(4)

since $R_{ee} = R_{mm} = 1$ as $e$ and $m$ themselves are bosonic. The minus sign when $e$ and $m$ doubly exchange, or equivalently, $e$ encircles $m$, $R_{me}R_{em} = -1$, proves the mutual statistics between $e$ and $m$ is semionic. Note that for the subset \{ $I, \mathcal{F}, \mathcal{W}^d, \mathcal{W}^h$ \}, since $\mathcal{F} = \mathcal{W}^d\mathcal{W}^h$ is bosonic, $R_{\mathcal{W}^d\mathcal{W}^h}R_{\mathcal{W}^h\mathcal{W}^d}(= R_{\mathcal{F}\mathcal{F}} = 1)$ is trivial.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{(Color on line) Two pairs of $m$ (dashed lines) and two pairs of $e$ (solid lines) created from the ground state at $T_0$; they fuse into four fermions $\psi$ (colored ellipses) at $T_1$; then they split back to four pairs at $T_2$ and annihilate to the ground state at $T_f$. The green and red fermions exchange, which differ a minus sign from none of exchange. The arrow indicates the time direction.}
\end{figure}

Peeling off a subset from degeneracy So far, we only say string-like excitations obey mutual semionic statistics but not call them anyons or semions. The reason for this is there are
FIG. 3: (Color on line) Divide the fermion exchange $R_{\psi\psi}$ into exchanges between the same non-local excitations, $R_{ee}$ and $R_{mm}$, and double exchange $e$ and $m$, $R_{me}R_{em}$, which is equal to moving $e$ (or $m$) around $m$ (or $e$).

many degenerate non-local states which are not in the same subset. For example, flipping any chain’s spin for $W_p$ results in a degenerate excitation with $W_p$ but already out of the subset of $W_p$. In this sense, these excitations can not be identified as individual quasiparticles.

To peel off a designed subset, we need to set barriers between individual degenerate ground states without changing their energies. To control the electron spin of each individual chain is not easy. However, it becomes possible in a cold atom (molecule) system because the 'spin' we are studying actually labels the different hyperfine states of atoms in the cold atom context. Once an atom is in a given hyperfine state, local fluctuations from the environment can not switch it to others. Therefore, we can peel off a given string-like excitation from others by preparing the ground state. For example, we can apply a magnetic field so that only the atoms with a given hyperfine state are loaded into the lattice and then turn off the magnetic field after the system is stable at the ground state. A global ferromagnetic ground state $|G_\uparrow\rangle = |G_{\{\uparrow, \uparrow, \cdots, \uparrow\}}\rangle$ is prepared. All excitations in (3) then can be prepared by creating, annihilating the fermions or changing fermions from $\uparrow$ to $\downarrow$ hyperfine states by means of recently developed stimulated Raman spectroscopy or photoemission spectroscopy technique [19]. Here annihilating and creating a fermion do not mean removing fermions from or reloading them into lattice sites. It can be turned into other hyperfine states which are almost not coupled to the 'spin' $\uparrow$ and $\downarrow$ hyperfine states or reverse. The non-local excitations may be controllably prepared. We may merely prepare excitations in a given subset and they are barricaded from their degenerate states. The non-local excitations obeying mutual simonic statistics in this subset are now identified as mutual semions.

Recently, a realization of this model in superconducting circuits has been proposed [20]. It is also a possible way to peel off a semionic subset.
Cold fermions with dipole-dipole interaction For cold fermions in optical lattice, off-site interaction between the cold atoms can be induced by their dipole-dipole interaction. Recently, the degenerate Fermi gases of rare-earth atoms of Ytterbium (Yb) have been obtained [16]. They are possible candidate to be a practical system of our model because the fermionic isotopes $^{171}$Yb and $^{173}$Yb are stable in nature and their metastable state $^3P_2$ has a large magnetic dipole moment $3\mu_B$. Deeply bound cold fermionic heteronuclear molecules have much larger dipole moment, e.g., the electric dipole moment of $^{40}{^8}$K$^{87}$Rb in its absolute bound ground state is $0.3\,ea_B$ [17]. Load the fermions in an optical lattice and polarize all dipoles along the horizontal diagonal of squares by using an external field. The interaction potential is $V_d(\mathbf{r}, \mathbf{r'}) = d^2 \frac{1-\cos^2 \Theta}{R^3}$ where $\Theta$ is the angle between $\mathbf{R} = \mathbf{r} - \mathbf{r'}$ and $d$ (the dipole moment of an atom). The interactions along the diagonal become attractive. The repulsive interaction is restricted in the region with $\Theta > \Theta_c$ for $\cos^2 \Theta_c = 1/3$. The interacting Hamiltonian can be written as

$$V = -\sum_{\langle ij \rangle_{hd,s}} |V_{ij,s}| n_{s,i} n_{s,j} - \sum_{\langle \langle ij \rangle \rangle_{hd,s}} |V_{ij}| n_{s,i} n_{s,j} - \sum_{ij,0<\Theta<\Theta_c,s} |V_{ij}| n_{s,i} n_{s,j} + \sum_{ij,\Theta>\Theta_c,s} V_{ij} n_{s,i} n_{s,j}$$

where $\langle \langle ij \rangle \rangle_{hd}$ denotes the sum along the horizontal diagonal other than the nearest neighbors. It is very easy to stabilize the ground state because one may increase the distance between the horizontal chains or adjust the optical lattice potentials so that the inter-chain couplings become weak.

Strictly speaking, the anyons emerging from these dipolar particle systems are logarithmic confinement in thermodynamic limit, i.e., $E_{\text{pair}} - E_g \propto \ln L$ as $L \to \infty$ for $L$ the distance between a pair of the non-local excitations. This weak divergence in a practical optical lattice may be abided, e.g., if the short range model we proposed has $E_{\text{pair}} - E_g \sim 1$, this logarithmic excitation energy is 4.5 for $L = 50$. Even $L = 1000$, this energy only increases about 15 times.

Conclusions In conclusions, we have proved that it is possible to find a non-trivial mutual anyonic statistics in a fermionic system with conventional two-body interaction in open boundary condition. How to peel up a semionic quasiparticle from many degenerate states and manipulate them were discussed.

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