Propagation of a Gaussian-top-hat function

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ABSTRACT

We study the propagation of a particular field that we call Gaussian-top-hat that presents self-focusing and maintains its shape for some propagation distances.

Introduction

During the last two decades, research in optical fields that maintain their shape [1, 2] during propagation has generated a myriad of applications with a great impact in fields such as particle manipulation, medicine, imaging and materials processing, to name just a few. Just as important are fields that present self-focusing effects during propagation, i.e., the field energy concentrates in a small region for some propagation distances [3, 4].

In this contribution, we present a novel field, which we call Gaussian-top-hat, that presents interesting properties during propagation, such as self-focusing and propagation invariance. The field was conceived from the derivation of the Fresnel diffraction integral through the use of the squeeze operators in quantum mechanics and the fractional Fourier transform.

Propagation in terms of the Fractional Fourier transform and the squeeze operators

The propagation of light in free space is described by the paraxial wave equation

\begin{equation}
\frac{i k}{2} \frac{\partial^2 E(x, y, z)}{\partial z^2} = \frac{\partial^2 E(x, y, z)}{\partial x^2} + \frac{\partial^2 E(x, y, z)}{\partial y^2}, \quad (1)
\end{equation}

where \( k \) is the wavevector. We may define the operators \( \hat{a}_x = -i \partial / \partial a \), with \( a = x, y \) such that we rewrite the above equation as

\begin{equation}
\frac{i}{2} \frac{\partial^2 E(x, y, z)}{\partial z^2} = \hat{a}_x^2 + \hat{a}_y^2 E(x, y, z), \quad (2)
\end{equation}

with the simple formal solution

\begin{equation}
E(z) = \exp \left[ -i \frac{z}{2k} (\hat{a}_x^2 + \hat{a}_y^2) \right] E(0). \quad (3)
\end{equation}

We define annihilation and creation operators for the harmonic oscillator,

\begin{equation}
\hat{a}_x = (\hat{a} + i \hat{\theta} ) / \sqrt{2}, \quad \hat{a}_y = (\hat{a} - i \hat{\theta} ) / \sqrt{2}, \quad \alpha = x, y \quad (4)
\end{equation}

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where we have used the fact that the exponential is an eigenfunction of the derivative. By writing explicitly the Fourier transform inside the integral (10) we obtain

\[ E(x, y, z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{i}{2\pi} \int_0^z} e^{i(x-kz)} e^{-i(x-kz)^2} \, dx \, dy \, dz. \]

that for \( x = 0 \) propagates along the \( z \)-axis as

\[ E(0, z) = -2 e^{-i\pi/4} \sqrt{\frac{1}{2\pi}} \int_0^{\infty} \left( e^{-\frac{k}{\pi}} \right) e^{-\frac{k}{\pi} z} \, du. \]

or, equivalently [7],

\[ E(0, z) = \frac{1}{N} \left( 1 - \exp \left( -2 \sqrt{\frac{k\alpha}{i2z}} \right) \right) \]  

We plot the propagated field in figures 1 and 2 for the values \( a = 10 \) and \( a = 20 \), respectively. It may be observed that there is a self-focusing effect as well as the fact that the field is bounded for some propagation distances.

**Conclusion**

We have shown that the Gaussian-top-hat function presents interesting characteristics while it propagates, namely, it is bounded for some propagation distances and shows self-focusing effects.

**References**

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