Simulated Predictions for HI at z = 3.35 with the Ooty Wide Field Array (OWFA) - II : Foreground Avoidance

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ABSTRACT

Considering the upcoming OWFA, we use simulations of the foregrounds and the z = 3.35 HI 21-cm intensity mapping signal to identify the (k⊥, k∥) modes where the expected 21-cm power spectrum P(k⊥, k∥) is substantially larger than the predicted foreground contribution. Only these uncontaminated k-modes are used for measuring P(k⊥, k∥) in the “Foreground Avoidance” technique. Though the foregrounds are largely localised within a wedge, we find that the small leakage beyond the wedge surpasses the 21-cm signal across a significant part of the (k⊥, k∥) plane. The extent of foreground leakage is extremely sensitive to the frequency window function used to estimate P(k⊥, k∥). It is possible to reduce the leakage by making the window function narrower, however this comes at the expense of losing a larger fraction of the 21-cm signal. It is necessary to balance these competing effects to identify an optimal window function. Considering a broad class of cosine window functions, we identify a six term window function as optimal for 21-cm power spectrum estimation with OWFA. Considering only the k-modes where the expected 21-cm power spectrum exceeds the predicted foregrounds by a factor of 100 or larger, a 5σ detection of the binned power spectrum is possible in the k ranges 0.18 ≤ k ≤ 0.3 Mpc−1 and 0.18 ≤ k ≤ 0.8 Mpc−1 with 1,000 – 2,000 hours and 104 hours of observation respectively.

Key words: Interferometric; cosmology: observations, diffuse radiation, large-scale structure of Universe

1 INTRODUCTION

Intensity mapping with the neutral hydrogen (H I) 21-cm radiation is a promising tool to study the large scale structures in the post-reionization Universe (Bharadwaj et al. 2001). It holds the potential of measuring the Baryon Acoustic Oscillation (BAO) that is imprinted in the H I 21-cm power spectrum, and the comoving scale of BAO can be used as a standard ruler to constrain cosmological parameters (Bharadwaj et al. 2009; Visbal et al. 2009). Higher order statistics such as the bispectrum holds the prospect of quantifying the non-Gaussianities in the H I 21-cm signal (Ali et al. 2005; Hazra & Sarkar 2012). Using the H I signal in cross-correlation with the WiggleZ galaxy survey data, the Green Bank Telescope (GBT) has made the first detection of the H I signal in emission at z ≈ 0.8 (Chang et al. 2010; Masui et al. 2013). Switzer et al. (2013) have constrained the auto-power spectrum of the redshifted H I 21-cm radiation from redshift z ∼ 0.8 with GBT.

The Giant Meterwave Radio Telescope (GMRT; Swarup et al. 1991) is sensitive to the cosmological H I signal from a range of redshifts in the post-reionization era (Bharadwaj & Pandey 2003; Bharadwaj & Ali 2005) and (Ghosh et al. 2011a,b) have carried out preliminary observations towards detecting this signal from z = 1.32. The Canadian Hydrogen Intensity Mapping Experiment (CHIME, Newburgh et al. 2014, Bandura et al. 2014) aims to measure the BAO in the redshift range 0.8 – 2.5. The future Tianlai (Chen 2012, 2015), SKA1-MID (Bull et al. 2015), HIRAX (Newburgh et al. 2016) and MeerKASS (Sanosts et al. 2017) also aim to measure the redshifted H I 21-cm signal from the post-reionization era. The Ooty Wide Field Array (OWFA) is an upgrade of the Ooty Radio telescope (ORT; Swarup et al. 1971) that aims to detect and measure H I from z = 3.35 (Subrahmanya et al. 2017a).

The ORT is a 530 m long (North-South) and 30 m wide (East-West) offset-parabolic cylinder, operating at a nominal frequency of νc = 326.5 MHz. The upgrade will re-
sult in two concurrent modes namely OWFA PI and PII. OWFA PI will be a linear array of $N_A = 40$ antennas each with a rectangular aperture $b \times d$, where $b = 30$ m and $d = 11.5$ m, arranged with a spacing $d$ along the North-South axis of the cylinder. In PII we have a larger number ($N_A = 264$) of antennas with smaller aperture and antenna spacing ($b = 30$ m and $d = 1.92$ m). The field-of-view (FoV) of OWFA PI and and PII are $1.8^\circ \times 4.8^\circ$ and $1.8^\circ \times 28.6^\circ$ respectively. The details of the antenna and hardware configuration can be found in Prasad & Subrahmanya (2011), Subrahmanya et al. (2017a) and Subrahmanya et al. (2017b). Theoretical estimates (Bharadwaj et al. 2015) predict that it should be possible to measure the amplitude of the 21-cm power spectrum with 150 hrs of observations using OWFA PII. A more recent study (Sarkar et al. 2017) indicates possible measurement of the 21-cm power spectrum in several different $k$ bins in the range $0.05 - 0.3$ Mpc$^{-1}$ with 1,000 hrs of observations. Sarkar et al. (2018b) have shown that the cross-correlation of the redshifted HI 21-cm signal with OWFA PII with the Lyman-$\alpha$ forest is detectable in a 200 hr-integration each in 25 independent fields-of-view.

The complex visibilities are the primary quantities measured by any radio-interferometric array like OWFA. It is possible to directly estimate the HI 21-cm power spectrum from the measured visibilities (Bharadwaj & Sethi 2001; Bharadwaj & Ali 2005). Sarkar et al. (2018a) have proposed and implemented a new technique to estimate the OWFA HI signal visibilities. Galactic and extragalactic foregrounds pose a severe challenge to the HI 21-cm signal detection (Ali et al. 2008; Ghosh et al. 2011b). The theoretical estimates (Ali & Bharadwaj 2014) predict that the visibilities measured at OWFA will be dominated by astrophysical foregrounds which are expected to be several orders of magnitude larger than the HI signal. The astrophysical foregrounds are all expected to have a smooth frequency dependence in contrast to the HI signal. With the increasing frequency separation ($\Delta v$), the HI signal is expected to decorrelate much faster ($\Delta v \lesssim 2$ MHz) than the foregrounds (Bharadwaj & Pandey 2003), on which most foreground removal techniques rely to distinguish between the foregrounds and the HI signal. Modelling foreground spectra is a challenging and is further complicated by the chromatic response of the telescope primary beam. Marthi et al. (2017) (from now Paper I) have introduced a Multi-frequency Angular Power Spectrum (MAPS) estimator and demonstrated its ability, using an emulator (PROWESS; Marthi 2017), to accurately characterize the foregrounds for OWFA PI.

Several studies have shown that the foreground contributions are expected to be locally confined within a wedge shaped region in the $(k_{\perp}, k_{\parallel})$ plane (Datta et al. 2010; Vedantham et al. 2012; Morales et al. 2012; Parsons et al. 2012; Trott et al. 2012). In this work we focus on a conservative strategy referred to as “foreground avoidance”. In this strategy only the $k$-modes where the predicted foreground contamination is substantially below the expected 21-cm signal are used for power spectrum estimation. Ideally, one hopes to use the entire set of $k$-modes outside the foreground wedge for estimating the 21-cm power spectrum. However, there are several factors which cause foreground leakage beyond the foreground wedge. The chromaticity of the various foreground components and also the individual antenna elements causes foreground leakage beyond the wedge. The exact extent of this wedge is still debatable (see Pober et al. 2014 for a detailed discussion). The large OWFA FoV makes it crucial to address the wide-field effects for the foreground predictions for OWFA. On a similar note, the Fourier transform along the frequency axis used to calculate the cylindrical power spectrum introduces artefacts due to the discontinuity in the measured visibilities at the edge of the band. It is possible to avoid this problem by introducing a frequency window function which smoothly falls to zero at the edges of the band. This issue has been studied by Vedantham et al. (2012) and Thyagarajan et al. (2013) who have proposed the Blackman-Nuttall (BN; Nuttall 1981) window function. While the additional frequency window does successfully mitigate the artefacts, it also introduces additional chromaticity which also contributes to foreground leakage beyond the wedge boundary.

In this paper we have used simulations of the foregrounds and the HI 21-cm signal expected for OWFA PII to quantify the extent of the foreground contamination outside the foreground wedge. The aim is to identify the $(k_{\perp}, k_{\parallel})$ modes which can be used for measuring the 21-cm power spectrum, and to assess the prospects of measuring the 21-cm power spectrum using the foreground avoidance technique. Our all sky foreground simulations (described in Section 2) incorporate the two most dominant components namely the diffuse Galactic synchrotron emission and the extragalactic point sources. This work improves upon the earlier work (Paper I) by introducing an all-sky foreground model. The simulated foreground visibilities (described in Section 3) incorporate the chromatic behaviour of both the sources and also the instrument. The actual OWFA primary beam pattern is unknown. We have carried out the entire study here using two different models for the primary beam pattern, we expect the actual OWFA beam pattern to be in between the two different scenarios considered here. We have used the “Simplified Analysis” of Sarkar et al. (2018a) to simulate the HI signal contribution to the visibilities (also described in Section 3). To estimate the 21-cm power spectrum from the the OWFA visibilities, in Section 4 we introduce and also validate a visibility based estimator which has been constructed so as to eliminate the noise bias and provide an unbiased estimate of the 3D power spectrum.

Our results (Section 5) show that the foreground leakage outside the wedge is extremely sensitive to the form of the frequency window function used for estimating the 21-cm power spectrum. While the leakage can be reduced by making the window function narrower, this is at the expense of increasing the loss in the 21-cm signal. It is necessary to balance these two competing effects in order to choose the optimal window function. In this paper we consider a broad class of cosine window functions each with a different number of terms. We introduce a figure of merit which allows us to quantitatively compare the performance of different window functions, and we use this to determine the optimal window function to estimate the 21-cm power spectrum using OWFA. Considering the optimal window function, we finally quantify the prospects of measuring the 21-cm power spectrum using OWFA. The results are discussed and summarized in Section 6.
2 SIMULATIONS

The radiation from different astrophysical sources other than the redshifted cosmological H\(_1\) 21-cm radiation are collectively referred to as foregrounds. The most dominant contributions to the foregrounds at 326.5 MHz, come from the diffuse synchrotron from our own galaxy (Diffuse Galactic Synchrotron Emission; DGSE) and the extragalactic radio sources (Extragalactic Point Sources; EPS). The free-free emission from our galaxy (Galactic Free-Free Emission; GFFE) and from external galaxies (Extragalactic Free-Free Emission; EGFF) are also larger than the H\(_1\) 21-cm signal. We exclude accounting the free-free emissions as a separate component in our analysis since they have power-law spectra similar to the other components (Kogut et al. 1996). They are easily subsumed by the uncertainty in the discrete continuum source contribution and they make relatively smaller contributions to the foregrounds.

2.1 The Diffuse Galactic Synchrotron Emission

The diffuse galactic synchrotron emission (DGSE) arises from the energetic charged particles (produced mostly by supernova explosions) accelerating in the galactic magnetic field (Ginzburg & Syrovatskii 1969). Various observations at 150 MHz (Bernardi et al. 2009; Ghosh et al. 2012; Iacobelli et al. 2013; Choudhuri et al. 2017) show that the angular power spectrum of brightness temperature fluctuations of the DGSE is well described by a power law \( C_\ell = A\ell^{-\gamma} \), at the angular scale of our interest. The frequency spectrum of the DGSE has been measured to be a power law (Rogers & Bowman 2008) \( T_\nu \propto \nu^{-\alpha} \) with \( \alpha = 2.52 \) in the frequency range 150 to 408 MHz. Based on these observations, we model the multi-frequency angular power spectrum (MAPS; Datta et al. 2007) of the DGSE as

\[
C_\ell (\nu, \nu') = A \left( \frac{1000}{\ell} \right)^\gamma \left( \frac{\nu'}{\nu} \right)^\alpha \left( \frac{\nu'}{\nu'} \right)^\alpha, \tag{1}
\]

where, \( A \) is the amplitude at the reference frequency \( \nu' = 150 \text{ MHz} \). Here we use \( A = 513 \text{ mK}^2 \) and \( \gamma = 2.34 \) (adopted from Ghosh et al. 2012). The values of the three parameters \( A, \gamma \) and \( \alpha \) have been held constant in our simulations. In reality the spectral index \( \alpha \) can vary with the line of sight (De Oliveira-Costa et al. 2008). \( A \) and \( \gamma \) have been found to have different values in different patches of the sky (e.g. La Porta et al. 2008, Choudhuri et al. 2017). These variations will introduce additional angular and frequency structures in addition to the predictions of our simulations.

We simulate the DGSE using the package Hierarchical Equal Area isoLatitude Pixelization of a sphere (HEALPix; Górski et al. 2005), where we set \( N_{\text{side}} = 1024 \), equivalent to \( N_{\text{pix}} = 12582912 \) pixels of size 3.435°. We assume that the brightness temperature fluctuations of the DGSE are a Gaussian Random Field (GRF) and used the SYNFAST routine of HEALPix to generate different statistically independent realizations of the brightness temperature fluctuations at the nominal frequency \( \nu_c \). We scale the brightness temperature fluctuations generated at \( \nu_c \) to other frequencies to simulate the DGSE maps throughout the observing bandwidth of OWFA. The left panel of Figure 1 shows a particular realization of the simulated DGSE maps and the right panel shows a comparison of \( C_\ell (\nu_c, \nu_c) \) values estimated from the simulations (in points) and the input model (in solid line). We use 20 statistically independent realizations of the DGSE simulations to estimate the mean values and 1 – \( \sigma \) error bars shown here.

2.2 Extragalactic Point Sources

The extragalactic point sources (EPS) are expected to dominate the 326.5 MHz sky at most of the angular scales of our interest. These sources are a mix of normal galaxies, radio galaxies, quasars, star-forming galaxies, and other objects, which are unresolved by the OWFA. We model the differential source count \( dN/dS \) of the sources using the fitting formula given by Ali & Bharadwaj (2014).

\[
\frac{dN}{dS} = \begin{cases} 4000 \left( \frac{S}{3mJy} \right)^{-1.64} (\text{Jy} \cdot \text{Sr})^{-1} & 3 \text{ mJy} \leq S \leq 3 \text{ Jy} \\ 134 \left( \frac{S}{10 \mu \text{Jy}} \right)^{-2.24} (\text{Jy} \cdot \text{Sr})^{-1} & 10 \mu \text{Jy} \leq S \leq 3 \text{ mJy}, \end{cases}
\tag{2}
\]

where they fit the 325 MHz differential source counts measured by Sirothia et al. (2009). This is consistent with the WENSS 327 MHz differential source count, (Figure 9 of Rubart, M. & Schwarz, D. J. 2013). For the sources below 3 mJy, they fit the 1.4 GHz source counts from extremely deep VLA observations (Biggs & Ivison 2006) and extrapolate it to 326.5 MHz. Here we assume that the sources with flux \( S > S_{\text{min}} = 3 \text{ mJy} \) make the major contribution to foregrounds and only consider sources with \( S > S_{\text{min}} \). We assume that such sources can be spectrally modelled as a power law \( S_{\nu} \propto \nu^{\alpha} \), where for each source we randomly assign a value of \( \alpha \) drawn from a Gaussian distribution with mean of \( \alpha_0 = -2.7 \) and \( \sigma_{\alpha} = 0.2 \) (Oliveri et al. 2018). The angular clustering of radio sources at low flux densities is not well known. To make an estimate, we use the angular correlation function \( w(\theta) \) measured from NVSS, which can be approximated as \( w(\theta) \approx (1.0 \pm 0.2) \times 10^{-3} \theta^{-0.8} \) (Overzier et al. 2003), for which the angular power spectrum (APS) \( w_\ell \) has been calculated to be \( w_\ell \approx 1.8 \times 10^{-4} \ell^{-1.2} \) (Blake et al. 2004; Oliveri et al. 2018).

The EPS contribution to the brightness temperature fluctuations can be decomposed into two parts, namely (a) the Poisson fluctuations due to the discrete nature of the sources, and (b) a fluctuation due to the angular clustering of the sources. The simulations were carried out using HEALPix with the same specifications as mentioned in Section 2.1. We use the differential source counts (eq. 2) to estimate the mean number of sources \( \bar{N} = 0.25 \) expected at each pixel of the map. We expect a total of \( N_{\text{tot}} = \bar{N} \times N_{\text{pix}} = 3145728 \) sources in the sky map. To implement this in a simulation with discrete sources (also described in Paper I), we first consider a situation with \( 100 \times \bar{N} = 25 \) sources at each pixel. We construct a source table containing \( 100 \times N_{\text{tot}} \) whose flux values are drawn randomly from the differential source count distribution (eq. 2) and whose spectral index values \( \alpha \) are assigned.
randomly as discussed earlier. The first $100 \times \bar{N}$ sources in the table are associated with the first pixel, the next $100 \times \bar{N}$ sources are associated with the second pixel and so on. We then generate a realization of the source distribution by randomly selecting $N_{\text{tot}}$ sources from the $100 \times N_{\text{tot}}$ sources in the source table. Each source in the final source distribution has equal probability of occurring in any one of the pixels. The resulting brightness temperature distribution has only the Poisson component.

To introduce the angular clustering of the sources, we generate realizations of Gaussian random fluctuations $\delta \nu_p$ at pixel $p$, with the angular power spectrum $w_{\bar{v}}$. We now expect $\bar{N} \times (1 + \delta_p)$ number of sources at each pixel. In order to implement this in a simulation with discrete sources, we consider a situation with $100 \times \bar{N} \times (1 + \delta_p)$ sources at each pixel and rounded these values to the nearest integer. Considering the source table mentioned earlier, now the first $100 \times \bar{N} \times (1 + \delta_p)$ sources in the table are associated with the first pixel ($p = 1$), the next $100 \times \bar{N} \times (1 + \delta_p)$ sources are associated with the second pixel ($p = 2$) and so on. We then generate a realization of the source distribution by randomly selecting $N_{\text{tot}}$ sources from the $100 \times N_{\text{tot}}$ sources in the source table. Each source in the final distribution has a probability $(1 + \delta_p)$ of occurring in pixel $p$. The resulting brightness temperature distribution now has both the Poisson fluctuation and the angular clustering of the sources.

We now consider $C_\ell(\equiv C_\ell(\nu_c, \nu_c))$ which is the angular power spectrum of the brightness temperature fluctuations at frequency $\nu_c$. In Figure 2, $C_\ell$ estimated from the simulations are compared with the analytical predictions (Ali & Bharadwaj 2014) for the DGSE contribution, the Poisson component of the EPS contribution, and total EPS contribution (Poisson + angular clustering) and also the total predicted $C_\ell$ (DGSE + EPS). We generate 20 statistically independent realizations of the foreground to estimate the mean values and $1 - \sigma$ error bars. For all values of $S_c$, the DGSE contribution dominates at large angular scales i.e. small $\ell$, and the EPS dominates at small angular scales i.e. large $\ell$. We see that DGSE dominates at $\ell < 600$ for $S_c = 3$ Jy.

3 VISIBILITY SIMULATIONS

The ORT is an offset-parabolic cylinder of length 530 m (North-South) and breadth $b = 30$ m (East-West). OWFA will have $N_A = 264$ antennas each of length $d = 1.92$ m arranged end to end along the North-South axis of the cylinder (Figure 2 of Paper I). We may think of each OWFA antenna as a rectangular aperture of dimension $b \times d$ illuminated by four end to end linear dipoles (Figure 4 of paper I). The OWFA visibilities $\nu_i(\mathbf{U}_a, \nu_a)$ are measured at baselines $\mathbf{U}_a = (ad/\lambda) \times \hat{z}$ for $1 \leq a \leq N_A - 1$, where we use a Cartesian coordinate system which is tied to the telescope with the $z$ and $y$ axes respectively along the length and breadth of the telescope. We consider $N_\nu = 312$ frequency channels $\nu_a$ of channel width $\Delta \nu_a = 0.125$ MHz spanning a bandwidth of $B_{\nu_a} = 39$ MHz. The reader is referred to Table 1 of Paper I for further details of the OWFA specifications. Here the label $t = 1, 2, \ldots, N_t$ in $\nu_i(\mathbf{U}_a, \nu_a)$ denotes distinct measurements of the visibilities each corresponding to a different time stamp. Following Paper I, we express the measured visibilities as

$$\nu_i(\mathbf{U}_a, \nu_a) = g_{\alpha i}(\mathbf{U}_a, \nu_a) M(\mathbf{U}_a, \nu_a) + N_i(\mathbf{U}_a, \nu_a)$$

(3)

where $M(\mathbf{U}_a, \nu_a)$ refers to model visibilities originating from the sky signal, $g_{\alpha i}^{(\alpha)}$ and $g_{\beta i}^{(\beta)}$ are the complex gains for the antennas $\alpha$ and $\beta$ respectively, and $N_i(\mathbf{U}_a, \nu_a)$ is the additive system contribution noise. For the purpose of this work we have assumed that the measured visibilities are perfectly calibrated and we have set the gain values to unity ($g_{\alpha i}^{(\alpha)} = g_{\beta i}^{(\beta)} = 1$).

The model visibility which originates from the sky signal is given by (Perley et al. 1989)

$$M(\mathbf{U}_a, \nu_a) = Q_{\nu_a} \int d\Omega_\theta T(\hat{n}, \nu_a) A(\Delta n, \nu_a) e^{-2\pi i \mathbf{U}_a \cdot \Delta n}$$

(4)

where, $Q_{\nu_a} = 2k_B/\lambda_a^2$ is the conversion factor from brightness temperature to specific intensity in the Raleigh - Jeans limit, $T(\hat{n}, \nu_a)$ is the brightness temperature distribution on the sky, $d\Omega_\theta$ is the elemental solid angle in the direction of the unit vector $\hat{n}$ which points to an arbitrary direction $(\alpha, \delta)$ on the sky with $\hat{n} = \sin(\delta) \hat{z} + \cos(\delta) \cos(\alpha) \hat{x} + \sin(\alpha) \hat{y}$ and $\Delta n = \hat{n} - \hat{m}$ where $\hat{m}$ is the unit vector in the direction...
Figure 2. The angular power spectrum $C_\ell$ of the brightness temperature fluctuations of the foreground components. The hollow and filled circles show the mean EPS and the total foreground (i.e. EPS+DGSE) contributions to $C_\ell$ with $\pm$ error bars estimated from 20 statistically independent realizations of the simulations, the analytical predictions are shown in different line-styles as indicated in the figure. The shaded region bounds the $\ell$ range probed by OWFA PII.

$(\alpha_0, \delta_0)$ of the phase center of the telescope. $A(\Delta \mathbf{n}, \nu_n)$ here denotes the primary beam pattern for the telescope, and throughout the present work we have assumed $\mathbf{m}$ to point along $(\alpha_0, \delta_0) = (0,0)$ which is perpendicular to the telescope’s aperture and along the $x$ axis. The model visibilities $M(U_a, \nu_n)$ can further be considered to be the sum of two parts

$$M(U_a, \nu_n) = F(U_a, \nu_n) + S(U_a, \nu_n).\quad (5)$$

the foreground and the H\textsc{i} signal respectively.

The foreground contribution $F(U_a, \nu_n)$ is highly sensitive to the telescope’s primary beam pattern (Berger et al. 2016). The actual OWFA primary beam pattern $A(\Delta \mathbf{n}, \nu_n)$ is currently unknown, and we have considered two different possibilities for the predictions presented here. The first model for $A(\Delta \mathbf{n}, \nu_n)$ (Table 1) is based on the simplest assumption that the OWFA antenna aperture is uniformly illuminated by the dipole feeds, which results in the “Uniform” sinc-squared primary beam pattern considered in several earlier works (Ali & Bharadwaj 2014, Paper I, Chatterjee & Bharadwaj 2018b). In reality, the actual illumination pattern is expected to fall away from the aperture centre resulting in a wider field of view as compared to the Uniform illumination. In order to assess how this affects the foreground predictions and foreground mitigation, we have considered a “Triangular” illumination pattern (Figure 3) for which we have a broader sinc-power-four primary beam pattern (Table 1).

Considering both the Uniform and the Triangular beam patterns, Figure 3 shows the variation of $A(\Delta \mathbf{n}, \nu)$ with $\delta$ (i.e. along the North-South direction) for fixed $\alpha = 0$ and $\nu = \nu_n$. Comparing the two beam patterns we find that the Uniform main lobe subtends $\sim \pm 28.6^\circ$ whereas this is approximately double $\sim \pm 57^\circ$ for Triangular. The number of side lobes is also found to decrease from Uniform to Triangular. The Uniform and Triangular beam patterns represent two extreme cases, and the actual OWFA beam pattern will possibly be somewhere in between these two extreme cases both in terms of the extent of the main lobe and the number of side lobes.

We use the simulated foreground maps (described in Section 2) to compute the foreground contribution $F(U_a, \nu_n)$ to the model visibilities,

$$F(U_a, \nu_n) = Q_{\nu_n} \Delta \Omega_{\text{pix}} \sum_{p=0}^{N_{\text{pix}}-1} T(\alpha_p, \delta_p, \nu_n) \times A(\alpha_p, \delta_p, \nu_n) e^{-2\pi i U_a (\sin \delta_p)},\quad (6)$$

where $\Delta \Omega_{\text{pix}}$ is the solid angle subtended by each simulation pixel, $(\alpha_p, \delta_p)$ the (RA, DEC) of the $p$-th pixel. The sum here runs over all the pixels ($N_{\text{pix}}$ in number) in the simulation.

Considering $S(U_a, \nu_n)$, the H\textsc{i} signal contribution to the model visibilities, we have simulated these using the flat-sky approximation (FSA). An earlier work (Chatterjee & Bharadwaj 2018b) has carried out a full spherical harmonic analysis for OWFA to find that the differences from the FSA are at most within 10% at the few smallest baselines and they are much smaller at the other larger baselines. Using $\Delta \mathbf{n} = \theta$ which is now a 2D vector on the plane of the sky, eq. (4) now reads

$$S(U_a, \nu_n) = Q_{\nu_n} \int d^2 \theta T(\theta, \nu_n) A(\theta, \nu_n) e^{-2\pi i \mathbf{U}_a \cdot \mathbf{\theta}}.\quad (7)$$

where $S(U_a, \nu_n)$ is the Fourier transform of $[Q_{\nu_n} T(\theta, \nu_n) A(\theta, \nu_n)]$. We can express this a convolution (Ali & Bharadwaj 2014) where,

$$S(U_a, \nu_n) = Q_{\nu_n} \int d^2 \mathbf{U} \tilde{a}(U_a - \mathbf{U}, \nu_n) \tilde{T}(\mathbf{U}, \nu_n),\quad (8)$$

where $T(U', \nu)$ is now the Fourier transform of $T(\theta, \nu)$, and the aperture power pattern $\tilde{a}(U, \nu) = \int d^2 \theta e^{-2\pi i \mathbf{U} \cdot \mathbf{\theta}} A(\theta, \nu)$ (Table 1).

In a recent work Sarkar et al. (2018a) have proposed an analytic technique to simulate $S(U_a, \nu_n)$ the H\textsc{i} signal contribution to the visibilities which is based on the FSA. Here we have used the “Simplified Analysis” presented in Section 2 of Sarkar et al. (2018a). This uses the eigenvalues and the eigenvectors of the predicted two-visibility correlation matrix $S_2(U_a, \nu_n, \nu_n') = \langle S(U_a, \nu_n) S^*(U_a, \nu_n') \rangle$ to simulate multiple statistically independent realizations of $S(U_a, \nu_n)$.

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The Simplified Analysis used here ignores the correlation between the H\textsc{i} signal at adjacent baselines and also the non-ergodic nature of the H\textsc{i} visibility signal along the frequency axis, both of these have however been included in the “Generalized Analysis” presented in Sarkar et al. (2018a). We note that it is necessary to diagonalize the entire covariance matrix between the visibilities at all the baselines and frequency channels in order to incorporate the correlations between the H\textsc{i} signal at the adjacent baselines. This is computationally intensive and we have avoided this by adopting the Simplified Analysis which considers each baseline separately significantly reducing the dimension of the covariance matrix.

The two-visibility correlation $S_2(U_\alpha, \nu_\alpha, \nu'_\alpha)$ is related to the 21-cm brightness temperature power spectrum $P_T(k)$ (Bharadwaj & Sethi 2001; Bharadwaj & Ali 2005). For OWFA we have (Ali & Bharadwaj 2014),

$$S_2(U_\alpha, \nu_\alpha, \nu'_\alpha) = \frac{Q^2}{4\pi} \int \frac{d^3k}{(2\pi)^3} |\tilde{a}(U_\alpha - \frac{k_\perp r}{2\pi}, \nu_\alpha)|^2 \times P_T(k_\perp, k_\parallel) e^{i\epsilon k_\perp (\nu'_\alpha - \nu_\alpha)}$$

where $k_\perp$ can be associated with the baselines $U$ available at OWFA as $k_\perp = 2\pi r/\lambda$, where $r = 6.84$ Gpc is the comoving distance to $z = 3.35$ and $r' = |dr/d\nu|_{\nu=\nu_\alpha} = 11.5$ Mpc MHz$^{-1}$ sets the conversion scale from the frequency separation to comoving distance in the radial direction.

The H\textsc{i} 21-cm brightness temperature power spectrum $P_T(k_\perp, k_\parallel)$ is modelled as (Ali & Bharadwaj 2014)

$$P_T(k_\perp, k_\parallel) = \bar{T}^2 b_{H_\perp}^2 \bar{\sigma}_{H_\perp}^2 [1 + \beta \mu^2] P(k),$$

where $\mu = k_\parallel/k$, $T = 4.05k(1 + z)^2 \left(\frac{0.27}{h(z)}\right) \left(\frac{h_{07}}{h_0}\right)$, $b_{H_\perp} = 2$ is the linear bias, $\bar{x}_{H_\perp} = 2.02 \times 10^{-2}$ is the mean neutral hydrogen fraction and $P(k)$ is the power spectrum of the underlying dark matter density distribution. The term $(1 + \beta \mu^2)$ arises due to the effect of HI peculiar velocities, and $\beta = f(\Omega)/b_{H_\perp}$, is the linear redshift distortion parameter, where $f(\Omega)$ is the dimensionless linear growth rate. We use $\beta = 0.493$ and $f(\Omega) = 0.986$ throughout this paper.

We consider the noise contribution $\sigma_N(U_\alpha)$ in each visibility is an independent complex Gaussian random variable with zero mean. The real part (or equivalently the imaginary part) of the noise contribution has a r.m.s. fluctuation, $\sigma_N(U_\alpha) = \sqrt{\frac{k_B T_{sys}}{\eta A}} \frac{\Delta T}{\sqrt{\Delta \nu}}$, where $T_{sys}$ is the total system temperature, $k_B$ is the Boltzmann constant, $A = b \times d$ is the physical collecting area of each antenna, $\eta$ is the aperture efficiency (Table 1) with $\lambda^2/\eta A = \int A(\theta, \nu) d^2\theta$ and $\Delta t = 16$ s is the correlator in-

\begin{table}
| Illumination | Uniform | Triangular |
|-------------|---------|------------|
| $A(\Delta n, \nu)$ | $\text{sinc}^2(\pi \tilde{\nu} \Delta n_u) \text{sinc}^2(\pi \tilde{\nu} \Delta n_z)$ | $\text{sinc}^4(\pi \tilde{\nu} \Delta n_u/2) \text{sinc}^4(\pi \tilde{\nu} \Delta n_z/2)$ |
| $\tilde{a}(U, \nu)$ | $(1/\tilde{b}) \Lambda(u/\tilde{b}) \Lambda(v/\tilde{b})$ | $(64/\tilde{b}) \tilde{G}(u/\tilde{b}) \tilde{G}(v/\tilde{b})$ |
| $\Lambda(x) = \begin{cases} 1 - |x| & \text{for } |x| < 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$ | $G(x) = \begin{cases} 1/6 - |x|^2 + |x|^3 & \text{for } |x| < 1/2 \\ 1/3 - |x| + |x|^2 - |x|^3/3 & \text{for } |x| \geq 1/2 \\ 0 & \text{for } |x| \geq 1 \end{cases}$ |
| FWHM, $\eta, \tilde{\eta}$ | $24^\circ \times 1.55^\circ$, 1, 32.49 | $35^\circ \times 2.25^\circ$, 9/16, 19.86 |
| $V_0, \epsilon$ | $(4/9) Q^2_{\nu_\alpha}/\tilde{b}^2$, 4 | $(302/315)^2 Q^2_{\nu_\alpha}/\tilde{b}^2$, 20 |

Table 1. Here $\Delta n_u$ and $\Delta n_z$ are respectively the y and z components of $\Delta n$. $\tilde{a}(U, \nu)$ is the aperture power pattern of the telescope (eq. 8) with $U = (u, v)$. FWHM is the full width at half-maximum of $A(\Delta n, \nu)$, $\eta$ (eq. 11) and $\tilde{\eta}$ (eq. A3) are the aperture efficiency and a dimensionless factor respectively. $V_0$ and $\epsilon$ are introduced in eq. (17) and eq. (19) respectively.

Figure 3. The left panel shows the Uniform (red solid line) and Triangular (blue dashed line) illumination patterns considered here. The right panel shows the corresponding primary beam patterns (Table 1) along the North-South direction.
tegration time. The OWFA baselines are highly redundant (Ali & Bharadwaj 2014; Subrahmanya et al. 2017b) and the factor \(1/\sqrt{N_t - a}\) in \(\sigma(U_s)\) accounts for the redundancy in the baseline distribution. We expect \(T_{\text{sys}}\) to have a value around 150 K, and we use this value for the estimates presented here.

## 4 3D POWER SPECTRUM ESTIMATION

We now discuss how the measured visibilities \(V^{(i)}(U_a, \nu_a)\) are used to estimate the 3D power spectrum \(P(k_{\perp}, \psi_{\parallel})\) (3DPS). Considering a particular baseline \(U_a\) and frequency \(\nu_a\), the different time-stamps \(V^{(i)}(U_a, \nu_a)\) contain the same sky signal, only the system noise is different. We first average over the different time-stamps to reduce the data volume

\[
\tilde{V}(U_a, \nu_a) = \frac{1}{N_t} \sum_{i=1}^{N_t} V^{(i)}(U_a, \nu_a). \tag{12}
\]

The visibilities \(\tilde{V}(U_a, \nu_a)\) are then Fourier transformed along the frequency axis to obtain the visibilities \(v^f(U_a, \tau_m)\) in delay space (Morales & Hewitt 2004)

\[
v^f(U_a, \tau_m) = (\Delta \nu_c) \sum_n e^{2\pi i m \nu_c f} F(\nu_a) \tilde{V}(U_a, \nu_a). \tag{13}
\]

where the delay variable \(\tau_m\) takes values \(\tau_m = m/B_{\text{bw}}\) with \(-N_t/2 < m \leq N_t/2\). The Fourier transform here assumes that the frequency signal is periodic across the frequency bandwidth \(B_{\text{bw}}\). The visibilities \(\tilde{V}(U_a, \nu_a)\), however, do not satisfy this requirement. This introduces a discontinuity at the edge of the frequency band, resulting in foreground leakage outside the foreground wedge. This can be avoided (Vedantham et al. 2012) by introducing \(F(\nu)\) (eq. 13) which is a frequency window function that smoothly falls to zero at the edges of the band making the product \([F(\nu_a) \tilde{V}(U_a, \nu_a)]\) effectively periodic over the bandwidth. Earlier works (Vedantham et al. 2012; Thyagarajan et al. 2013) show that the Blackman-Nuttall (BN) (Nuttall 1981) window function is promising candidate for power spectrum estimation, and this is expected to reduce the foreground leakage by 7–8 orders of magnitude. However, as we shall see later, the BN window function fails to reduce the foreground leakage to a level below the H1 signal expected at OWFA. In order to investigate if this problem can be overcome by considering other filters, we have considered a broader set of cosine window functions

\[
F(\nu_a) = \sum_{p=0}^{N_p-1} (-1)^p A_p \cos \left(\frac{2\pi p \nu}{N_c} \right), \quad \tag{14}
\]

each having different coefficients \(A_p\) and number of terms \(N_p\). Here we have considered the Blackman-Harris 4-term window function (BH4), and a family of Minimum Sidelobe (MS) window functions (MS5, MS6 and MS7). Of these, the BN (Paul et al. 2016) and BH4 (Eastwood et al. 2019) have been used extensively in recent observational studies. Table 2 shows the coefficients (Albrecht 2001) of these window functions considered here.

The left panel of Figure 4 shows the different window functions \(F(\nu)\) considered here. As discussed earlier, we see that the window function smoothly goes down to zero towards the edge of the band. An immediate consequence of introducing the window function \(F(\nu)\) is the loss of signal, primarily towards the edge of the frequency band. Considering the window functions \(F(\nu)\) in the order shown in Table 2, we see that the \(F(\nu)\) gets successively narrower as we move from BN to MS7. We expect the suppression at the edge of the band to be more effective as the window function gets narrower, however this comes at an expense of increasing signal loss.

Considering the delay space visibilities \(v(U_a, \tau_m)\) without the window function (i.e. \(F(\nu) = 1\) in eq. 13), we have (Choudhuri et al. 2016)

\[
v^f(U_a, \tau_m) = \frac{1}{B_{\text{bw}}} \sum_{m'} \hat{f}(\tau_m - \tau_{m'}) v(U_a, \tau_{m'}). \tag{15}
\]

We see that \(v^f(U_a, \tau_m)\) is related to \(v(U_a, \tau_m)\) through a convolution with \(\hat{f}(\tau_m)\) which is the Fourier transform of the frequency window \(F(\nu)\). This convolution smooths out the signal over the width of \(\hat{f}(\tau_m)\). Considering the H1 signal, the delay space visibilities \(v(U_a, \tau_m)\) and \(v(U_a, \tau_{m'})\) at two different delay channels \(\tau_m\) and \(\tau_{m'}\) are predicted to be uncorrelated (e.g. Choudhuri et al. 2016). The convolution in eq. (15) however introduces correlations in \(v^f(U_a, \tau_m)\) at two different values of the delay channel, the extent of this correlation is restricted within the width of \(\hat{f}(\tau_m)\). The right-hand panel of Figure 4 shows the amplitude of \(\hat{f}(\tau_m)\) for the different window functions considered here. We see that \(\hat{f}(\tau_m)\) peaks at \(m = 0\), and the values of \(\hat{f}(\tau_m)\) are very small beyond the primary lobe which is typically a few delay channels wide. This primary lobe of \(\hat{f}(\tau_m)\) gets successively wider as we move from BN to MS7 i.e. the window function \(F(\nu)\) gets successively narrower. The BN window function has the narrowest \(\hat{f}(\tau_m)\) and \(v^f(U_a, \tau_m)\) will be correlated upto \(m \approx 4\), whereas this extends to \(m \approx 7\) for MS7 which is the widest in delay space. The finite width of \(\hat{f}(\tau_m)\) also leads to a loss of H1 signal at the smallest \(\tau_m\) values which correspond to the largest frequency separations. Figure 4 illustrates the fact that \(\hat{f}(\tau_m)\) widens and this loss in H1 signal increases as we move from the BN to the MS7 window function.

The delay space visibility \(v^f(U_a, \tau_m)\) is related to the H1 21-cm brightness temperature fluctuation \(\Delta T_b(k_{\perp}, \psi_{\parallel})\) where \(k_{\perp} = 2\pi U_a/r\) and \(\psi_{\parallel} = 2\pi \tau_m r'/r\) (Morales & Hewitt 2004), and we can use this to estimate \(P(k_{\perp}, \psi_{\parallel})\) the 3D power spectrum of the sky signal. Considering the auto-correlation of \(v^f(U_a, \tau_m)\) we have

\[
\langle |v^f(U_a, \tau_m)|^2 \rangle = C_F^{-1} \left[ P(k_{\perp}, \psi_{\parallel}) + P_N(k_{\perp}, \psi_{\parallel}) \right],
\]

with

\[
C_F^{-1} = \frac{V_0 B_{\text{bw}} A_F}{r^2 r'}, \tag{17}
\]

where \(A_F = \sum_{\nu_c} |F(\nu_c)|^2\), \(V_0 = Q_{\nu_c} \int d^2 U |U| (U)\) (Table 1) and the noise power spectrum

\[
P_N(k_{\perp}, \psi_{\parallel}) = C_F \left( \frac{\Delta \nu_c}{N_c} \right)^2 \sum_{m=0}^{N_t-1} \sum_{i=0}^{N_t-1} |\tilde{V}^{(i)}(U_a, \nu_c)|^2 |\hat{f}(\nu_c)|^2. \tag{18}
\]

The angular brackets \(\langle \cdot \rangle\) here denote an ensemble average over different random realizations of the H1 21-cm signal. We can use \(|v^f(U_a, \tau_m)|^2\) to estimate the
Table 2. The coefficients of the different window functions used in this work.

![Graph showing window functions and frequency space](image)

Figure 4. The left panel shows the window functions $F(\nu_n)$ (mentioned in the legend) as a function of channel number $n$ and the right panel shows the $f(\tau)$ for a small number of delay channels. $f(\tau)$ is normalized to unity at the central delay channel.

H: 21-cm power spectrum $P(k_{\perp a}, k_{|m|})$ except for the term $P_N(k_{\perp a}, k_{|m|})$ which arises due to the system noise (eq. 3) in the measured visibilities. This introduces a positive noise bias which needs to be accounted for before we can use eq. (16) to estimate $P(k_{\perp a}, k_{|m|}).$

In addition to the auto-correlation considered in eq. (16), for OWFA the signal at the adjacent baselines ($U_a$ and $U_{a \pm 1}$) are also correlated (Ali & Bharadwaj 2014). We note that this correlation is restricted to the adjacent baselines and the baselines at larger separations are uncorrelated. Considering the correlation between adjacent baselines, we have

$$
\langle v_i(U_a, \tau_m) v_i^*(U_{a \pm 1}, \tau_m) \rangle = (\epsilon C_P)^{-1} P(k_{\perp a}, k_{|m|}),
$$

where $k_{\perp a} = \hat{z}(U_a + U_{a \pm 1})$ and $\epsilon^{-1} V_0 = Q^2 \epsilon \int d^2U |\hat{a}(U) \hat{a}^*(d/\lambda_\nu) \hat{z}(U)|^2$ (Table 1). The system noise in two adjacent baselines is uncorrelated and there is no noise bias in this case.

We use eqs. (16) and (19) to define the 3D power spectrum estimator $\tilde{P}_i(U_a, \tau_m)$ where $a$ can take both integer and half integer values. Integer values of $a$ refers to the auto-correlations and we have

$$
\tilde{P}_i(U_a, \tau_m) = C_P \left[ |v_i(U_a, \tau_m)|^2 - \left( \frac{\Delta \nu}{N_v} \right)^2 \sum_{n=0}^{N_v} \sum_{t=0}^{T_v} |v(t) U_{a,v_n}|^2 |F(v_n)|^2 \right],
$$

(20)

The second term in the right-hand side of eq. (20) is introduced to exactly subtract out the noise bias in eq. (16). In addition to the noise bias, this term also subtract out a part of the signal, however the fraction of the total visibility correlation signal that is lost is of the order of $1/N_v$ which is extremely small for a long observation. For example we have $N_v \sim 10^7$ for $t_{obs} = 1,000$ hrs of observation with an integration time of $\Delta t = 16 \text{ s}$.

The half-integer values of $a$ refers to the correlations between the adjacent baselines and we have

$$
\tilde{P}_i(U_a, \tau_m) = \epsilon C_P Re \left[ v_i(U_{a+1/2}, \tau_m) v_i^*(U_{a-1/2}, \tau_m) \right].
$$

(21)

where $Re[\cdots]$ refers to the real part of $[\cdots]$. For both the integer and half-integer values of $a$ we have

$$
P(k_{\perp a}, k_{|m|}) = \langle \tilde{P}_i(U_a, \tau_m) \rangle
$$

(22)

where $k_{\perp a} = 2\pi U_a/r$ and $k_{|m|} = 2\pi \tau_m/\epsilon$. The variance of

\[ \text{Table 2. The coefficients of the different window functions used in this work.} \]

| Coeff. | BN | BH4 | MS5 | MS6 | MS7 |
|--------|----|-----|-----|-----|-----|
| $A_0$  | $3.6538 \times 10^{-1}$ | $3.5875 \times 10^{-1}$ | $3.2321 \times 10^{-1}$ | $2.9355 \times 10^{-1}$ | $2.7122 \times 10^{-1}$ |
| $A_1$  | $4.891775 \times 10^{-1}$ | $4.8829 \times 10^{-1}$ | $4.7149 \times 10^{-1}$ | $4.5193 \times 10^{-1}$ | $4.3344 \times 10^{-1}$ |
| $A_2$  | $1.3659 \times 10^{-1}$ | $1.4128 \times 10^{-1}$ | $1.7553 \times 10^{-1}$ | $2.0414 \times 10^{-1}$ | $2.1800 \times 10^{-1}$ |
| $A_3$  | $1.06411 \times 10^{-2}$ | $1.1680 \times 10^{-2}$ | $2.8496 \times 10^{-2}$ | $4.7926 \times 10^{-2}$ | $6.5785 \times 10^{-2}$ |
| $A_4$  | $1.2613 \times 10^{-3}$ | $5.0261 \times 10^{-3}$ | $5.0768 \times 10^{-3}$ | $1.07618 \times 10^{-2}$ |
| $A_5$  | $1.3755 \times 10^{-4}$ | $7.7001 \times 10^{-4}$ |
| $A_6$  | $1.3680 \times 10^{-5}$ |
the power spectrum estimator $P^U(U_m, \tau_m)$ defined here is calculated in Appendix A, these variance calculations can be used to theoretically predict the errors in the estimated power spectrum.

4.1 Validating the estimator

To validate the HI signal simulations and the 3D power spectrum estimator we have carried out simulations of the HI signal visibilities using the prescription described in Section 3 considering both the Uniform and the Triangular illuminations. For both cases we have simulated $N_s = 1,000$ statistically independent realizations of the HI signal visibilities including the system noise component. To reduce the data volume and the computation, we have considered a total observation time of $t_{\text{obs}} = 1,000$ hours with an integration time of $\Delta t = 1$ hour for the Uniform illumination, and $t_{\text{obs}} = 10,000$ hours with $\Delta t = 10$ hours for the triangular illumination respectively. In both cases we have $N_s = 1,000$ which implies that we have $1/N_s = 0.1\%$ loss in the visibility correlation due to the term which cancels out the noise bias. As mentioned earlier, we expect this loss to be even smaller in actual observations where $N_s$ will be much larger. The upper panels of Figure 5 show the spherical averaged input model 21-cm brightness temperature power spectrum $P_T(k)$ as a function of $k$. The figure also shows the binned input model power spectrum where we have considered $P_T(k)$ at the $(k_\perp, k_\parallel)$ modes corresponding to the OWFA baselines and delay channels, and binned these into 20 equally spaced logarithmic bins.

The simulated HI 21-cm signal visibilities $S(U_m, \nu_\alpha)$ considered here only contain the auto-correlation signal, as mentioned earlier the correlations between the adjacent baselines have not been incorporated here. We have used the simulated visibilities in eq. (20) to estimate the power spectrum. The upper panels of Figure 5 show the binned power spectrum $P(k)$ estimated from the simulations, the left and the right panels show the results for Uniform and Triangular illuminations respectively. The $N_s$ realizations of the simulations were used to estimate the mean and the $1 - \sigma$ error bars shown in the figure. In both the cases we find that the estimated power spectra are in good agreement with the input power spectrum. The error-bars at the smallest $k$ bins are somewhat large due to the cosmic variance, though we see that a detection is possible here. At large $k$ the errors exceed the expected power spectrum, and a detection is not possible within the $t_{\text{obs}}$ considered here. In both the illuminations we see that the errors are relatively small in the $k$ range $0.05 - 0.3 \text{Mpc}^{-1}$ which is most favourable for measuring the power spectrum with OWFA (Sarkar et al. 2017). The lower panels of Figure 5 show the dimensionless ratio

\[ \Delta = \frac{\delta P(k) \sqrt{N_s}}{\sigma} \]

Here $\delta P(k)$ is the difference between the estimated and the input model power spectrum. Ideally we expect this to have a spread of the order of $\sigma/\sqrt{N_s}$ around zero arising from statistical fluctuations. The normalized dimensionless ratio $\Delta$ is thus expected to have a variation of order unity provided the estimator provides an unbiased estimate of the power spectrum. We find that the values of $\Delta$ in the lower panels are distributed within $\pm 5$ at all the bins except for that at the smallest $k$ value. The power spectrum is possibly underestimated at the lowest few baselines because the estimator ignores the convolution with the aperture power pattern which is included in the visibility signal (see eq. 9 and also Choudhuri et al. 2014). This deviation is however seen to be well within the $1 - \sigma$ error-bars for $t_{\text{obs}} = 1,000$ hours of observation (upper left panel of 5). Overall we conclude that our simulations validate the power spectrum estimator presented here.

5 RESULT

We first focus on the HI signal and foreground predictions, and we have not included the noise contribution here. Considering the Uniform illumination and the BN window function, the left panel of Figure 6 shows the predicted cylindrical power spectrum $P(k_\perp, k_\parallel)$ averaged over 20 statistically independent realizations of the simulations for the HI signal, the individual DGSE and EPS foreground components and the total sky signal. We see that the foregrounds are largely confined within the “Foreground Wedge” (Datta et al. 2010). The foreground contamination would be restricted to $k_\parallel = 0$ if the foregrounds were spectrally flat i.e. the visibilities $V(U, \nu)$ were independent of frequency. However, the fact that the baselines $U = d\nu/c$ change with frequency introduces a frequency dependence in $V(U, \nu)$ even if the sky signal is frequency dependent. The foreground simulations here include both the $\nu$ scaling of $U$ as well as the intrinsic $\nu$ dependence of the sky signal, and as a consequence the foreground contribution to $P(k_\perp, k_\parallel)$ extends out along $k_\parallel$ onto a wedge which is expected to be bounded by 

\[ k_\parallel = \left[ \frac{r \sin(\theta_l)}{\nu/\nu_c} \right] k_\perp \]

in the $(k_\parallel, k_\perp)$ plane (Datta et al. 2010; Vedantham et al. 2012; Morales et al. 2012; Parsons et al. 2012; Trott et al. 2012) where $\theta_l$ is the largest angle (relative to the telescope’s pointing direction) from which we have a significant foreground contamination. Here we consider $\theta_l = 90^\circ$ as the horizon limit. We see (Figure 6) that there is a very large foreground contribution at $k_\parallel = 0$, and the foregrounds beyond this are largely contained within a wedge. The dotted line in the figure shows the wedge boundary predicted by eq. (24); we see that the boundary of the simulated foreground wedge is located beyond the dotted line. The primary beam pattern $A(\Delta \mathbf{n}, \nu)$, the intrinsic frequency dependence of the sources introduced through the spectral index $\alpha$ and $F(\nu)$ all introduce additional frequency dependence (or chromaticity) in $V(U, \nu)$ which enhance the extent of the foreground wedge beyond that predicted by eq. (24). We also notice that there are several structures visible inside the foreground wedge.

The right panels show vertical sections through the left panels i.e. they show $P(k_\perp, k_\parallel)$ as a function of $k_\parallel$ for fixed $k_\perp$ values. We have chosen $k_\perp = 0.095$ and $0.34 \text{Mpc}^{-1}$ (dashed and solid lines respectively) for which the horizontal lines show the corresponding wedge boundaries predicted by eq. (24). Considering the foregrounds, the $k_\parallel$ dependence of $P(k_\perp, k_\parallel)$ shows two peaks, the first at $k_\parallel = 0$ and the second at the wedge boundary. The second peak corresponds to
Figure 5. Considering the power spectrum, the upper panels show a comparison of the spherically averaged input model, the binned input model and that estimated from the simulations. \( N_r = 1,000 \) statistically independent realizations of the simulation were used to estimate the mean and \( 1 - \sigma \) error bars shown here. The points in the bottom panels show \( \Delta \) (eq. 23), which quantifies the deviation between the binned input model and the simulated power spectrum.

**Figure 6.** This shows the predictions of the component-wise contributions to the 3DPS \( P(k_\perp, k_\parallel) \) from the \( \text{H}_i \) signal, the DGSE, the EPS and the total 3DPS. Panels in left show the cylindrical power spectrum \( P(k_\perp, k_\parallel) \). The dotted lines mark the approximate wedge boundaries (eq. 24). Panels in right show vertical sections through the left panels for fixed \( k_\perp = 0.095 \) Mpc\(^{-1}\) (in dashed lines), 0.34 Mpc\(^{-1}\) (in solid lines). The horizontal solid lines and dotted lines in the right panels indicate the approximate wedge boundaries (eq. 24) for the above mentioned \( k_\perp \) modes.

what is known as the “pitch fork” effect (Thyagarajan et al. 2015; Thyagarajan et al. 2015), which is seen to be more prominent at the larger baseline. The foreground wedge is found to extend by \( \Delta k_\parallel \approx 0.1 \) Mpc\(^{-1}\) beyond the horizontal lines. In addition to this, we find oscillatory structures within the wedge where the \( k_\parallel \) values of the dips correspond to the nulls in the primary beam pattern \( \text{i.e. replace } \theta_i \) in eq. 24 with \( \theta_1, \theta_2 \ldots \) the angular positions of the various nulls of the primary beam pattern). Considering large \( k_\parallel \) beyond the wedge boundary, in all cases we find that \( P(k_\perp, k_\parallel) \) drops to a small value which does not change very much with \( k_\parallel \). This small value of \( P(k_\perp, k_\parallel) \) arises due to the foreground
leakage beyond the wedge. For DGSE the value of \( P(k_{\perp}, k_{||}) \) decreases with increasing \( k_{\perp} \). This reflects the fact that the DGSE contribution decreases with increasing \( \ell \) (\( C_\ell \propto \ell^{-7} \)). In contrast, the EPS contribution, which is Poisson dominated, does not change much with \( k_{\perp} \).

Considering the H\(_i\) signal (Figure 6) we find that, the foreground contribution is \( \sim 10^{10} \) times larger at \( k_{||} = 0 \) and other points within the wedge boundary. We also find that the foreground leakage remains \( \sim 10^{10} \) times larger than the H\(_i\) signal beyond the wedge boundary. This implies that the BN window is not a suitable choice for H\(_i\) power spectrum detection with OWFA. This leads us to investigate the possibility of using higher term window functions for H\(_i\) power spectrum detection with OWFA. To this end we make a comparative study of the expected foreground leakage for the set of window functions discussed earlier (eq. 14 and Table 2).

To identify the \((k_{\perp}, k_{||})\) modes which can be used for the H\(_i\) power spectrum detection we introduce the ratio

\[
R(k_{\perp}, k_{||}) = \frac{P_T(k_{\perp}, k_{||})}{P_T(k_{\perp}, k_{||})},
\]

where \( P_T(k_{\perp}, k_{||}) \) is the theoretically expected H\(_i\) 21-cm signal power spectrum (eq. 10) and \( P_T(k_{\perp}, k_{||}) \) is the foreground leakage contribution. Figure 7 shows \( R(k_{\perp}, k_{||}) \) for the different higher term window functions. The left and right panels show the results for the Uniform and the Triangular illumination respectively. We have only shown the points where \( R(k_{\perp}, k_{||}) > 1 \) i.e. the H\(_i\) signal exceeds the foreground leakage. For both the illumination patterns we find that the largest values of \( R(k_{\perp}, k_{||}) \), which are in the range 50–500, are located at the lowest \((k_{\perp}, k_{||})\) modes just beyond the wedge boundary. The values of \( R(k_{\perp}, k_{||}) \) and the region where \( R(k_{\perp}, k_{||}) > 1 \) both increase as we increase the number of terms in the window function. In all cases we have \( R(k_{\perp}, k_{||}) < 1 \) at large \((k_{\perp}, k_{||})\) where the H\(_i\) signal is small. In comparison to the Uniform illumination, the region where \( R(k_{\perp}, k_{||}) > 1 \) is found to be somewhat smaller for the Triangular illumination because of the larger FoV.

We have assumed that the \((k_{\perp}, k_{||})\) region where \( R(k_{\perp}, k_{||}) \geq R_t \) can be used to detect the H\(_i\) 21-cm signal power spectrum. \( R_t \) here is a threshold value which has to be set sufficiently high so as to minimize the possibility of residual foreground contamination. We discuss the criteria for deciding the value of \( R_t \) later in this section. We see that the \((k_{\perp}, k_{||})\) region corresponding to different values of \( R_t \) are somewhat smaller for the Triangular illumination as compared to the Uniform illumination. The OWFA illumination pattern is unknown, but we expect the actual OWFA predictions to be somewhere between the Uniform and the Triangular predictions. The \((k_{\perp}, k_{||})\) range which simultaneously satisfies \( R(k_{\perp}, k_{||}) \geq R_t \) for both the Uniform and the Triangular illuminations can safely be used to detect the H\(_i\) 21-cm signal power spectrum. The \( R(k_{\perp}, k_{||}) \geq R_t \) regions for \( R_t = 10, 50 \) and 100 for the Triangular illumination are shown by the solid, dashed and fine-dotted contours respectively in both the left and right panels. The Triangular illumination considered here represents the worst possible scenario for the illumination pattern of the OWFA antennas. We do not expect the allowed \((k_{\perp}, k_{||})\) range for the actual OWFA beam pattern to be smaller than that predicted for the Triangular illumination. Thus for any value of \( R_t \), throughout we have used the Triangular illumination to determine the allowed \((k_{\perp}, k_{||})\) range.

From Figure 7 we see that the allowed \((k_{\perp}, k_{||})\) region and the peak \( R(k_{\perp}, k_{||}) \) values increase as we increase the number of terms in the window function. It thus appears to be advantageous for H\(_i\) 21-cm signal detection to increase the number of terms in the window function. This would indeed be true if the power spectrum estimated at the different \((k_{\perp}, k_{||})\) modes were uncorrelated. However, the convolution in eq. (15) causes the H\(_i\) signal at different \( k_{||} \) modes to be correlated. We see that \( f(\tau_{m}) \) gets wider (right panel of Figure 4) causing the \( k_{||} \) extent of the correlations to increase as we increase the number of terms in the window function. The system noise contribution at the different \( k_{||} \) modes are also expected to be correlated because of the convolution. Further, the window function \( F(u) \) gets narrower (left panel of Figure 4) and the loss in the H\(_i\) signal at the edge of the frequency band also increases as we increase the number of terms. It is therefore not obvious whether it is advantageous for H\(_i\) 21-cm signal detection to increase the number of terms in the window function. Rather, it would be more appropriate to ask as to which of the different window functions considered here is best suited for H\(_i\) signal detection. In order to quantitatively address this issue we consider a figure of merit namely the Signal to Noise Ratio (SNR) for measuring \( A_{H_i} = b_{H_i}^2 \tau^2_i \) which is the amplitude of the H\(_i\) 21-cm signal power spectrum (eq. 10). We have used the Fisher-matrix formalism where the SNR for the measurement of \( A_{H_i} \) is given by

\[
\text{SNR}^2 = \sum_{a,m,m'} \frac{\partial P(k_{\perp}, k_{||})}{\partial \ln A_{H_i}} C^{-1}_{a,m,m'} \frac{\partial P(k_{\perp}, k_{||})}{\partial \ln A_{H_i}}.
\]

Here we have assumed that the entire allowed \((k_{\perp}, k_{||})\) range where \( R(k_{\perp}, k_{||}) \geq R_t \) is combined to estimate \( A_{H_i} \). Considering \( \Delta A_i^I (U_a, \tau_m) = A_i^I (U_a, \tau_m) - \langle A_i^I (U_a, \tau_m) \rangle \) the error in the 21-cm power spectrum, the correlation between different \( k_{||} \) mode arising from the convolution in eq. (15) can be quantified through the covariance matrix

\[
C_a(m, m') = \langle |\Delta A_i^I (U_a, \tau_m)| |\Delta A_i^I (U_{a'}, \tau_{m'})| \rangle.
\]

Here we have used simulations to estimate \( C_a(m, m') \) for the different window functions. Considering a range of different \( t_{\text{obs}} \) and the two different illuminations, for each combination we have generated \( N_t = 1,000 \) statistically independent realizations of the OWFA visibilities incorporating the H\(_i\) signal and system noise. To reduce the data volume and computation, we have considered an integration time of \( \Delta t = 10 \) hours. We have used the 1,000 statistically independent estimates of the 21-cm power spectrum to estimate \( C_a(m, m') \) for different window functions.

Figure 8 shows the predicted SNR values as a function of the observing time \( t_{\text{obs}} \) and threshold \( R_t \). The four columns respectively correspond to the four higher term window functions, whereas the two rows respectively correspond to the Uniform and Triangular illuminations. Our aim here is to identify the optimal window function. Considering BH4, we find that the SNR values are considerably lower compared to the three other window functions and BH4 is not a good choice. We find that for the entire \( R_t \) range considered here (1 \( \leq R_t \leq 500 \)) the SNR values do not differ much between the MS6 and MS7 window functions. The SNR values
for the MS5 window function also are comparable to those for MS6 and MS7 for \( R_t \lesssim 30 \), however the SNR values for MS5 drop rapidly for larger \( R_t(>30) \). Figure 8 therefore indicates that BH4 can definitely be excluded, however all three MS5, MS6 and MS7 exhibit comparable performance if one wishes to use a threshold \( R_t < 30 \). For a higher threshold \( R_t > 30 \) MS5 also is excluded, however both MS6 and MS7 exhibit comparable performance.

In order to quantify the small differences in the SNR predictions of the window functions, we consider the ratio of the SNRs for the different window functions with respect to that for MS6 which we take as reference

\[
\mathcal{R}(R_t, t_{\text{obs}}) = \frac{\text{SNR}(R_t, t_{\text{obs}})}{\text{SNR}(R_t, t_{\text{obs}})_{\text{MS6}}}. \tag{28}
\]

A value \( \mathcal{R}(R_t, t_{\text{obs}}) > 1 \) tells us that the corresponding window function performs better than MS6 whereas the converse is true if \( \mathcal{R}(R_t, t_{\text{obs}}) < 1 \). The left, middle and right panels of Figure 9 show \( \mathcal{R}(R_t, t_{\text{obs}}) \) as a function of \( t_{\text{obs}} \) for \( R_t = 10, 50 \) and 100 respectively. As expected, the \( \mathcal{R}(R_t, t_{\text{obs}}) \) values always remain significantly below 1.0

**Figure 7.** This shows a the ratio \( R(k_\perp, k_\parallel) \) (eq. 25) for different window functions (mentioned in figure) considered here. The left and right panels show the results for the Uniform and Triangular illumination respectively. \( R(k_\perp, k_\parallel) \geq 10, 50 \) and 100 regions for the Triangular illumination are shown by the solid, dashed and fine-dotted contours respectively in both the left and right panels.

**Figure 8.** This shows a comparison of predicted SNRs for different higher term window functions considered here (mentioned in the figure legend). The upper and lower panels show the predictions for the Uniform and Triangular illuminations respectively. The SNR values 5, 10 and 20 are shown by the solid, dashed and dotted contours respectively.

**Figure 9.** This shows a comparison of predicted SNRs for different window functions with respect to that for MS6 which we take as reference
for BH4 and this is excluded. Note that $\mathcal{R}(R_t, t_{obs})$ values for BH4 is not visible at the middle and right panels due to the very small allowed $(k_\perp, k_\parallel)$ region at these $R_t$ values. Considering MS5 next, for $R_t = 10$ we find that $\mathcal{R}(R_t, t_{obs}) > 1.0$ provided $t_{obs} \leq 1,000$ hours, however $\mathcal{R}(R_t, t_{obs}) < 1$ if $t_{obs} > 1,000$ hours and it declines steadily with increasing $t_{obs}$. For $R_t = 50$ and 100, we have $\mathcal{R}(R_t, t_{obs}) < 1$ irrespective of $t_{obs}$. Considering MS7, we find that $0.9 < \mathcal{R}(R_t, t_{obs}) < 1.0$ for all the three $R_t$ values shown here. This is a direct consequence of the fact that the extent of the correlation between the $k_j$ modes increases (Figure 4) with an increase in the number of terms in the window function. Although the allowed $(k_\perp, k_\parallel)$ increases if we increase the number of terms, the enhanced correlation causes the SNR to degrade beyond MS6. The Uniform and Triangular illuminations both show very similar results. Our analysis suggests that the MS5 window is optimal at small $R_t$ (e.g. $R_t < 50$) and small $t_{obs}$ (e.g. $t_{obs} < 1,000$ hours), barring this situation the MS6 window function is optimal for H1 power spectrum estimation with OFWA.

Once we have identified the optimal window function, we next aim to fix a suitable $R_t$ for H1 21-cm power spectrum estimation. We have earlier discussed that the value of $R_t$ must be set sufficiently high to minimize the possibility of residual foreground contamination. Shorter observations (e.g. $t_{obs} < 1000$ hours) are expected to have a relatively large noise contribution, and it is possibly adequate to consider a less conservative threshold $R_t \approx 10$ along with the MS5 window function for H1 power spectrum estimation. For $t_{obs} \geq 1000$ hours where we target a more precise measurement of the H1 power spectrum, it is worth considering a more conservative threshold $R_t \geq 50$ and use the MS6 window function. The question is whether the SNR would fall significantly if we increase the value of the threshold $R_t$ in the range 50 to 100. The thin dashed line in the right panels of Figure 9 show the ratio $\mathcal{R} = \left[\frac{\text{SNR}(R_t = 100, t_{obs})_{\text{MS6}}}{\text{SNR}(R_t = 50, t_{obs})_{\text{MS6}}}\right]_{\text{SNR}}$. We find that the SNR values degrade at most by $\sim 8\%$ if we increase $R_t$ from 50 to 100. This indicates that one can set the value of the threshold $R_t$ as high as 100 without a significant loss of SNR. For $R_t = 100$, the residual foreground contamination is expected to be $\leq 1\%$ for every $(k_\perp, k_\parallel)$ modes that is used for H1 power spectrum estimation.

An earlier study (Sarkar et al. 2017) has predicted that a 5 $\sigma$ detection of the binned power spectrum is possible in the $k = \sqrt{k_\perp^2 + k_\parallel^2}$ range $0.05 \leq k \leq 0.3 \text{Mpc}^{-1}$ with 1,000 hours of observation, this however uses the entire available $(k_\perp, k_\parallel)$ region and does not take the foreground contamination into account. The fact is that a significant $(k_\perp, k_\parallel)$ range has to be excluded due to the foreground wedge and the residual foreground leakage. We next consider the revised SNR predictions for the binned H1 power spectrum taking into account the $(k_\perp, k_\parallel)$ modes which have to be excluded to avoid the foreground contamination. For these prediction we have used the MS6 window function and set a high threshold of $R_t = 100$, the results do not change very much if $R_t$ is varied in the range $R_t = 10$ and 100 (Figure 8). The range $k \leq 0.1 \text{Mpc}^{-1}$ is completely within the foreground wedge, and this is excluded from H1 power spectrum estimation. We have binned the allowed $k$ range ($0.1 < k < 2.0 \text{Mpc}^{-1}$) into 10 logarithmic bins and estimated the SNR prediction for different $t_{obs}$. The upper row of Figure 10 show the SNR predictions as a function of $k$ and $t_{obs}$, with the left and right panels corresponding to the Uniform and Triangular illuminations respectively. The middle row shows horizontal sections through the upper panels i.e. they show the SNR as a function of $k$ for fixed values of $t_{obs}$ (mentioned in the figure legend), and the lower row shows the percentage loss of SNR ($\Delta$SNR) due to the excluded $(k_\perp, k_\parallel)$ region. To calculate $\Delta$SNR we have used the SNR predictions considering the entire available $(k_\perp, k_\parallel)$ region (similar to Sarkar et al. 2017) as reference.

Considering the upper row of Figure 10, we see that the SNR predictions are similar for both the illuminations but the SNR values are $\sim 1.5$ times lower for the Triangular illumination in comparison to the Uniform illumination. Our results are also similar to those in Figure 3 of Sarkar et al. (2017) except that our prediction for the Uniform illumination are $\sim 1.5$ times lower due to the foreground contamination. We find that at low $t_{obs}$ the SNR peaks in the smallest $k$ bin ($\sim 0.18\text{Mpc}^{-1}$) and a 5 $\sigma$ measurement is possible at this $k$ bin with $t_{obs} \approx 600$ hours and 1,000 hours for the Uniform and Triangular illuminations respectively. A 5 $\sigma$ detection of the binned power spectrum is possible in the range $0.18 \leq k \leq 0.3 \text{Mpc}^{-1}$ with $t_{obs} \sim 1,000$ hours for the Uniform illumination, whereas this will require $t_{obs} \sim 2,000$ hours for the Triangular illumination. The peak SNR shifts towards larger $k$ bins for larger $t_{obs}$, and the peak is at $\sim 0.3 \text{Mpc}^{-1}$ for $t_{obs} = 10^4$ hours where a 15 $\sigma$ detection is possible. The shift in the peak SNR is clearly visible in the middle row of the figure. A 10 $\sigma$ detection is possible in the range $k \sim 0.2 – 0.4 \text{Mpc}^{-1}$ with $t_{obs} \sim 3,000$ hours and 4,000 hours for the Uniform and Triangular illuminations respectively. The SNR falls drastically at large $k$ ($> 0.8 \text{Mpc}^{-1}$), this is also noticeable in Figure 3 of Sarkar et al. (2017) and this is due to the fact that the H1 power spectrum fall at large $k$ (Figure 5) whereby these bins are dominated by the system noise contribution. The situation is further aggravated here because a considerable fraction of the available $(k_\perp, k_\parallel)$ region has to be excluded to avoid the foregrounds. Considering the lower row of the figure, we see that the fractional loss in the SNR ($\Delta$SNR) is $\sim 60\%$ at $k \geq 0.8 \text{Mpc}^{-1}$, and it increases rapidly to $\sim 80\%$ at the larger $k$ bins. The fractional loss in the SNR is in the range $40 – 60\%$ for $k$ in the range $0.18 \leq k \leq 0.8 \text{Mpc}^{-1}$ where there are prospects of a detection. We also note that ($\Delta$SNR) is minimum at $\leq 40\%$ at $k \sim 0.3 \text{Mpc}^{-1}$ where the SNR peaks for $t_{obs} \leq 10^4$ hours.

6 SUMMARY AND CONCLUSION

The ORT (Swarup et al. 1971) is currently being upgraded to operate as a radio interferometer, the Ooty Wide Field Array (OWFA; Subrahmanya et al. 2017b) and this work focuses on PII of OWFA. The array operates with a single linear polarization. The ORT (and also OWFA) feed system consists of linear dipoles arranged end to end along the long axis of the cylindrical parabolic reflector. Considering any particular dipole, its radiation pattern is minimum along the direction of the adjacent dipoles and we thus expect minimal coupling between the adjacent dipoles. The actual primary beam pattern $A(\Delta n, \nu)$ for OWFA is unknown. For
this study we use two extreme models for $A(\Delta n, \nu)$, the first one is based on the simplest assumption that the OWFA antenna aperture is uniformly illuminated by the dipole feeds (Uniform illumination) whereas the second one assumes a Triangular illumination pattern (Figure 3). We expect the actual OWFA illumination to be somewhat in between these two scenarios.

OWFA is sensitive to the HI 21-cm signal from $z = 3.35$, and measuring the cosmological 21-cm power spectrum is one of the main goals of this upcoming instrument. The cosmological HI 21-cm signal is faint and is buried in foregrounds which are several orders of magnitude brighter. The foregrounds processed through the chromatic response of the instrument produce spectral features which contaminate the HI signal, and this poses a severe challenge for detecting the 21-cm power spectrum. In this paper we have simulated the HI 21-cm signal and foregrounds expected for OWFA PII. Our aim here is to use these simulations to quantify the extent of the expected foreground contamination and assess the prospects of detecting the 21-cm power spectrum.

We have used all sky foreground simulations (described in Section 2) which incorporate the contributions from the two most dominant components namely the diffuse Galactic synchrotron emission and the extragalactic point sources. These were used to calculate the foreground contribution $F(U_n, \nu_n)$ to the model visibilities (eq. 5) expected at OWFA. These simulations incorporate the chromatic behaviour of both the sources and also the instrument. To simulate the HI signal contribution to the model visibilities $S(U_n, \nu_n)$ (eq. 8), we use the “Simplified Analysis” presented in Sarkar et al. (2018a). This is based on the flat-sky approximation, and also ignores the correlation between the HI signal at adjacent baselines and the non-ergodic nature of the HI visibility signal along the frequency axis. To estimate the 21-cm power spectrum from the measured visibilities, we introduce an estimator (eq. 20 and eq. 21) which has been constructed so as to eliminate the noise bias and provide an unbiased estimate of the 3D power spectrum $P(k_\perp, k_\parallel)$. We have validated this for both the Uniform and the Triangular illuminations using a large number of statistically independent realizations of HI simulations. These particular simulations also include the system noise, the foregrounds however are ignored. We find (Figure 5) that in the absence of foregrounds, for both the illuminations, the $k$ range $0.05−0.3 \text{ Mpc}^{-1}$ is most favourable for measuring the power spectrum with OWFA. This is consistent with the results of earlier work (Sarkar et al. 2017).

Considering the foregrounds, the contamination is primarily localized within a wedge shaped region of the $(k_\perp, k_\parallel)$ plane (Figure 6). The $k$-modes outside this “foreground wedge” are believed to be largely uncontaminated by the foregrounds. However, there is a relatively small fraction of the foreground which leaks out beyond the wedge. Though small, this foreground leakage may still exceed the expected HI signal in many of the $k$ modes outside the foreground wedge. For signal detection we focus on a strategy referred to as “foreground avoidance” where only the $k$-modes which are expected to be uncontaminated are used for measuring the 21-cm power spectrum. In this work we use simulations to identify the region of the $(k_\perp, k_\parallel)$ plane which is expected to be uncontaminated, and we use this to quantify the prospects of measuring the 21-cm power spectrum using OWFA.

Our simulations show that foreground leakage outside the wedge, though small, can still exceed the 21-cm power spectrum expected at OWFA. We find that the extent of foreground leakage is extremely sensitive to the frequency window function $F(\nu)$ (eq. 13) which is introduced...
PSfrag replacements

4.

We therefore need to choose the optimal window function by balancing between these two competing effects. We have used the Fisher matrix formalism to define the SNR (eq. 26) for measuring the amplitude of the 21-cm power spectrum, and we use this as a figure of merit to identify the optimal window function.

Our analysis (Figure 8) shows that the optimal choice of window function depends on the observing time $t_{\text{obs}}$ and the threshold value $R_t$. A threshold value $R_t$ implies that we only use the modes where $R(k_{\perp}, k_{\parallel}) \geq R_t$ for measuring the 21-cm power spectrum. We note that the value of $R_t$ must be set sufficiently high to minimize the possibility of residual foreground contamination. We find that the five term MS5 window function is optimal at small $t_{\text{obs}}$ ($\lesssim 1,000$ hours) and small $R_t$ ($\lesssim 30$), whereas the six term MS6 window function is optimal for larger values of $t_{\text{obs}}$ and $R_t$. Relative to MS6, the SNR is found to degrade slightly if we consider the seven term MS7 window function. The Uniform and Triangular illuminations both show very similar results.

We propose a possible observational strategy based on the finding summarized above. Shorter observations (e.g. $t_{\text{obs}} < 1,000$ hours) are expected to have a relatively large noise contribution, and it is possibly adequate to consider a relatively low threshold $R_t \approx 10$ along with the MS5 window function. For longer observations $t_{\text{obs}} > 1,000$ hours where we target a more precise measurement of the 21-cm power spectrum, it is worth considering a more conservative threshold $R_t \gg 50$ and use the MS6 window function. Our investigations also show that the SNR does not fall much if $R_t$ is increased from 50 to 100, and we could equally well consider using a very conservative threshold of $R_t = 100$ where the contribution from foreground leakage is expected to be less than 1% of the 21-cm power spectrum.

The SNR values for measuring the amplitude of the 21-cm power spectrum (Figure 8) are approximately 1.5 times lower for the Triangular illumination in comparison to the Uniform Illumination. Using MS5 with $R_t \approx 30$, a $5-\sigma$ detection will take $\sim 180$ hours and $\sim 300$ hours with the Uniform and Triangular illuminations respectively. The same is increased to $\sim 200$ hours and $\sim 300$ hours if we use MS6 or MS7 with $R_t \approx 100$.

We have also considered the prospects of measuring the binned 21-cm power spectrum. The discussion here is restricted to MS6 with $R_t = 100$. We find that the range $k \lesssim 0.1\ Mpc^{-1}$ is completely within the foreground wedge (Figure 10) and has to be excluded. For low $t_{\text{obs}}$ the SNR peaks at the smallest $k \approx 0.18\ Mpc^{-1}$ bin and a $5-\sigma$ measurement is possible at this $k$ bin with $t_{\text{obs}} \approx 600$ hours and 1000 hours for the Uniform and Triangular illuminations respectively. A $5-\sigma$ detection of the binned power spectrum is possible in the $k$ range $0.18 < k < 0.3\ Mpc^{-1}$ with $t_{\text{obs}} \sim 1,000$ hours for the Uniform illumination, whereas this will require $t_{\text{obs}} \sim 2,000$ hours for the Triangular illumination. Considering $t_{\text{obs}} = 10^4$ hours, for both the illuminations the peak SNR shifts to larger $k$ values $0.3-0.4\ Mpc^{-1}$ and a $5-\sigma$ detection is possible in the range $0.18 < k < 0.8\ Mpc^{-1}$. We have used $\Delta \text{SNR}$ to quantify the fractional loss in SNR due to the foreground contamination, the comparison here is with respect to the situation where there are no foregrounds. We find that $\Delta \text{SNR}$ has values in the range $40-60\%$ for $k$ in the

![Figure 10](https://example.com/figure10.png)

**Figure 10.** This shows the SNR predictions for the binned power spectrum estimation using MS6 window function for the Triangular and Uniform illuminations. Here we have set $R_t = 100$. The upper panels show the predicted SNR as a function of $k$ and $t_{\text{obs}}$. The contours mark the SNR values 5, 10 and 15 (mentioned in the figure). The middle panels show horizontal sections through the upper panels for $t_{\text{obs}} = 1000$, 4000 and $10^4$ hours (mentioned in the figure legend). The horizontal dot-dashed line marks the SNR value 5. The lower panels show the percentage loss of SNR ($\Delta \text{SNR}$) due to the presence of the foregrounds for $t_{\text{obs}} = 10^4$ hours.

(Vedantham et al. 2012) to suppress the measured visibilities near the boundaries of the frequency band. Considering the extensively used (e.g. Paul et al. 2016) Blackman-Nuttall filter which has four terms, we find that the foreground leakage exceeds the expected 21-cm power spectrum at all the available $k$ modes, and it will not be possible to measure the 21-cm power spectrum using OWFA. In order to overcome this problem, we consider a set of cosine window functions with progressively increasing number of terms (Table 2 and Figure 4). The window function gets narrower resulting in better suppression at the edges of the band as we increase the number of terms. Using $R(k_{\perp}, k_{\parallel})$ which is the ratio of the expected 21-cm power spectrum to the foreground leakage contribution, we find that the $(k_{\perp}, k_{\parallel})$ region where $R(k_{\perp}, k_{\parallel}) > 1$ (i.e. the region where HI signal exceeds the foreground leakage) increases if we increase the number of terms in the window function (Figure 7). Taken at face value, this indicates that it is advantageous to increase the number of terms in the window function. It is however also necessary to take into consideration the fact that the HI signal in adjacent $k_{\parallel}$ modes get correlated due to $F(\nu)$ and the extent of this correlation increases as we increase the number of terms. The number of independent estimates of the 21-cm power spectrum thus gets reduced if we increase the number of terms. We therefore need to choose the optimal window function by balancing between these two competing effects.
range $0.18 \leq k \leq 0.8 \text{Mpc}^{-1}$ where there are good prospects of measuring the 21-cm power spectrum.

The exact beam pattern of OWFA is not known, but we expect this to be somewhere between the Uniform and Triangular illuminations considered here. We therefore expect the actual situation for measuring the 21-cm power spectrum to lie somewhere between the two different sets of predictions presented here. The present study indicates that “Foreground Avoidance” provides an effective technique for measuring the 21-cm power spectrum with OWFA. It is also predicted that a $5\sigma$ measurement of the 21-cm power spectrum should be possible within approximately a few hundred hours of observations despite the $k$ modes which have to be excluded due to foreground contamination. It is however necessary to note that the entire analysis presented here is based on 20 statistically independent realizations of our specific foreground model. While this model attempts to incorporate the salient features of the two dominant foreground components, it still remains to establish how robust the results are with respect to variations in the foreground model. Although the exact quantum of foreground leakage may vary depending on the foreground model, we do not expect this to be a very severe effect as we have adopted a pretty conservative threshold $R_1 = 100$ for a considerable part of our analysis. Calibration (Marthi & Chengalur 2014) is another issue which could affect the results presented here. In future work we plan to study the effect of calibration errors and also the effect of varying the foreground model.

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we have used several statistically independent
realizations of the signal to determine the variance ($\sigma^2$) of
the estimated power spectrum. Such a procedure is, by and
large, only possible with simulated data. We usually have
accessed to only one statistically independent realizations
of the sky signal, and the aim is to use this to not only estimate
the power spectrum but also predict the uncertainty in
the estimated power spectrum. Considering the power spectrum estima-
tor ($P^0(U_{\alpha},\tau_m)$), we theoretically calculate the variance
\[
\sigma^2_P(U_{\alpha},\tau_m) = \langle P^0(U_{\alpha},\tau_m) \rangle - \langle P^0(U_{\alpha},\tau_m) \rangle^2.
\] (A1)
which is used to predict the uncertainty in the estimated
power spectrum $P(k L, k_1)$. The entire analysis here is based
on the assumption that the H\textsc{i} signal is a Gaussian random
field.

Considering the power spectrum estimator for integer $a$
which corresponds to the auto-correlation we have
\[
\sigma^2_P(U_{\alpha},\tau_m) = [P(k_{L,\perp},k_{m}) + P_N(k_{L,\parallel},k_{m})]^2.
\] (A2)
We use $\langle |N^{(f)}(U_{\alpha},\nu)|^2 \rangle = \sigma^2_P(U_{\alpha})$ (eq. (11), eq. (17) and
(18) to simplify the noise power spectrum,
\[
P_N(k_{L,\perp},k_{m}) = \frac{\epsilon^2}{\sigma^2_P(U_{\alpha})} \left(\frac{2}{N} \langle \theta^2 \rangle \prod_{m=1}^{N} \langle A_m^2 \rangle \right),
\] (A3)
where $\epsilon = [\int A^2(\theta) d\theta^2]/[\int A(\theta)d\theta]^2$ is a dimensionless factor
(Chatterjee & Bharadwaj 2018a). It is worth noting that the second term in the righthand side of eq. (20) which has been introduced to subtract out the noise bias in eq. (16) is
ignored for calculating the variance. The signal contribution
from this term to the estimator is of the order of $\sim 1/N_s$
which is extremely small for a long observation.

Considering the correlations between the adjacent base-
lines, the variance of the power spectrum estimator (eq. (21))
can be calculated using,
\[
|\sigma_P| = \frac{1}{2} [P(k_{L,\perp},k_{m})]^2
+ \epsilon^2 \langle |\sigma_P(U_{\alpha+1/2},k_{m})| \rangle \langle |\sigma_P(U_{\alpha-1/2},k_{m})| \rangle
\] (A4)
where a take only half-integer values, $k_{L,\perp} = 2\pi U_{\alpha}/r$ and
$k_{m} = 2\pi \tau_m/r$.

Figure A1 shows the analytic prediction for the variance calculated using eq. (A2) (solid line) for a total ob-
servation time of $t_{\text{obs}} = 1,000$ hours with an integration
time $\Delta t = 1$ hour for the Uniform illumination. For com-
parison we also show (points) the variance estimated from
$N_s = 1,000$ statistically independent realizations of the simulated signal.
Here we have binned the variance $\sigma_P(U_{\alpha},\tau_m)$ at
the (k$_{L,\perp},k_{1}$) modes corresponding to the OWFA baseline
and delay channels into 20 equally spaced logarithmic bins
to compute $\sigma_P(k)$. The shaded region in the figure shows the
theoretically estimated error $\Delta \sigma_P = \sigma_P(k)/\sqrt{N_s}$ in $\sigma_P(k)$
for $N_s = 1000$ statistically independent realizations of the
H\textsc{i} signal. We see that the analytic predictions are in reason-
able good agreement with the values obtained from the simulations
over the entire k-range that we have considered
here, except the two smallest k bins. This discrepancy pos-
sibly arises because the estimator ignores the convolution
with the aperture power pattern which is included in the
simulated visibility signal (eq. (9)). From Figure A1 we also notice
that $\sigma_P(k)$ remains relatively small in the k-range

APPENDIX A: VARIANCE OF THE ESTIMATOR

In Section 4 we have used several statistically independent
realizations of the signal to determine the variance ($\sigma^2$) of

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APPENDIX A: VARIANCE OF THE ESTIMATOR

In Section 4 we have used several statistically independent
realizations of the signal to determine the variance ($\sigma^2$) of

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Figure A1. This shows the analytic prediction for the variance (eq. A2) for auto-correlation is compared with variance estimated from \( N_r = 1,000 \) realizations of the simulated signal visibilities.

0.05 – 0.3 Mpc\(^{-1}\), which is consistent with the findings of Sarkar et al. (2017). The \( k \)-modes larger than 0.7 Mpc\(^{-1}\) remains noise dominated and larger hours of observation is required to extract signal from these modes.