Work extraction via quantum nondemolition measurements of qubits in cavities: Non-Markovian effects.

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We show that frequent nondemolition measurements of a quantum system immersed in a thermal bath allow the extraction of work in a closed cycle from the system-bath interaction (correlation) energy, a hitherto unexploited work resource. It allows for work even if no information is gathered or the bath is at zero temperature, provided the cycle is within the bath memory time. The predicted work resource may be the basis of quantum engines embedded in a bath with long memory time, such as the electromagnetic bath of a high-Q cavity coupled to two-level systems.

I. Introduction

Information acquired by an observer on the system can be commuted into work in a single-bath engine \cite{1, 2}. To this end, the observer (“Maxwell’s demon”) must manipulate the system according to the results of measurements performed on it. The measurements have therefore to be selective, i.e., their outcomes must be read and discriminated to determine the observer’s course of action \cite{3, 10}. The balance of work and information is embodied by the Szilard-Landauer (SL) principle \cite{1, 2} whereby work obtainable from a measurement must not exceed the energy cost of erasing its record from the observer’s memory. Notwithstanding ongoing efforts to expand information-based thermodynamics (IT) so as to include measurement-cost effects \cite{10, 12} beyond the original SL balance (see Appendix), an important aspect has been little addressed thus far \cite{13}. How essential is the thermodynamic paradigm of system-bath separability \cite{14, 15} to the analysis of work extraction by measurements?

This question is here investigated in the context of non-Markovian quantum thermodynamics “under observation” \cite{14, 21}. Namely, we show that frequent measurements can induce changes in system-bath correlations, unaccounted for by the separability paradigm, and thereby change the tradeoff of information and work. Consequently, selective (read) measurements can extract more work in a closed cycle than what the SL principle allows. This effect requires the cycle to be completed within a non-Markovian (bath-memory) time-scale.

We further show that non-selective quantum nondemolition (QND) measurements of the system energy (which we shall also denote as unread measurements, i.e., measurements whose outcomes do not matter) enable the system to do work in a cycle, although they provide no information on the system, nor do they change its state (by their definition as QND measurements) and thus do not act as a “demon.” Work is shown to be obtainable within the bath memory time even at zero temperature \(T = 0\), i.e., for the vacuum state of the bath. This finding is in stark contrast to the expectation that the extractable work at \(T = 0\) should vanish by the SL principle \cite{1–8} or its current generalizations \cite{10, 11} that are valid in the Markovian limit.

The predicted effects cannot be ascribed to quantum coherence in the system, which is the source of work in recently explored quantum heat engines (QHEs) \cite{22}, since coherence is absent from this scenario. Rather, these effects follow up on the anomalous temperature effects of frequent QND measurements on non-Markovian time scales \cite{16, 21}. Here, we show that these effects may well determine the performance of quantum engines that operate on such time scales. Clearly, one could instead consider energy pumping into the system by non-QND measurements, e.g., projections onto the \(x-y\) plane of the qubit Bloch sphere. Yet we are interested in minimal intrusion into the system that would have no effect on an isolated qubit (in the absence of a bath).

To this end, we analyze the following simple yet unconventional protocol: A brief unread QND measurement of the energy of a thermalized qubit neither changes its state nor yields information, yet inevitably decorrelates it from its bath and therefore requires energy investment that is absent in SL considerations (Sec. II). A post-measurement cycle produced by sinusoidal modulation of the qubit frequency is shown (both analytically and numerically) to yield work (despite the fact that no information is obtained) from the unread measurement, provided the cycle is completed within the bath memory time (Sec. III). Post-measurement system-bath correlation change is shown to be the work resource. The analysis culminates in a revised work-information relation (Sec. IV). A demonstration of consistency with the second law (Sec. V) is followed by a discussion of the cost of multiple cycles (Sec. VI) and an analysis of feasible cavity-based experimental scenarios (Sec. VII). The findings are summarized in Sec. VIII.
II. System-bath decorrelation via QND measurements

We consider a QND measurement of the energy of a thermalized qubit by a quantum probe (P) consisting of two degenerate yet distinguishable states ($H_P = 0$) (e.g., photon-polarization states). This situation is modeled by the Hamiltonian

$$H_{tot} = H_S + H_B + H_{SB} + H_{SP}(t)$$  \hspace{1cm} (1)

where $S$ labels a two-level system (TLS) with energy states $|e\rangle$ and $|g\rangle$, $B$ denotes a (bosonic or fermionic) bath, $H_{SB}$ is the coupling of $S$ to a bath operator $B$ and $H_{SP}(t)$ is the S-P interaction that effects the measurement. We choose the coupling Hamiltonians to have the form: $H_{SB} = \sigma_z B$, where $\sigma_z$ does not commute with $H_S$ and neither does $B$ with $H_B$, in order to describe S-B equilibration, i.e., transitions between the levels of $S$ and $B$.

Since the measurement is to have a QND effect on $S$ \cite{16}, in order for the density matrix of $S$, $\rho_S$, to retain the same $\sigma_z$-diagonal form it had in equilibrium, one should choose a projective measurement in the $\sigma_z$- (qubit-energy) basis, i.e., $H_{SP} \propto \sigma_z$. The impulsive QND measurement of the energy of $S$ by $P$ is well described by $\rho_S \longrightarrow Tr_P \left( U_C \rho_S \otimes \rho_P U^\dagger_C \right)$

$$= |e\rangle \langle e| \rho_S |e\rangle \langle e| + |g\rangle \langle g| \rho_S |g\rangle \langle g|$$  \hspace{1cm} (5)

e.i., the diagonal elements of $\rho_S$ are unchanged, and the off-diagonal elements are erased. Since the system is entangled with the bath, the effect of the measurement on $\rho_{SB}$ is:

$$\rho_{SB} \longrightarrow Tr_P \left( U_C \rho_{SB} \otimes \rho_P U^\dagger_C \right)$$

$$= |e\rangle \langle e| \rho_{SB} |e\rangle \langle e| + |g\rangle \langle g| \rho_{SB} |g\rangle \langle g| \equiv B_{ee} |e\rangle \langle e| + B_{gg} |g\rangle \langle g|$$  \hspace{1cm} (6)

where $B_{ee(gg)}$ are bath states correlated to $|e\rangle$ and $|g\rangle$ respectively. Thus, the post-measurement $\rho_{SB}$ is block-diagonal in the energy states of the system. It can be shown \cite{16,19} to be close to a product state of $\rho_S$ and $\rho_B$. We assume at this stage that the state of $P$ is unread (averaged out) after this measurement. The entire measuring process can be summarized as

$$\rho_{tot} = \rho_P \otimes \rho_{SB} \rightarrow \rho_{SP} \otimes \rho_B; \quad Tr_P \rho_{tot} \rightarrow \rho_S \otimes \rho_B$$  \hspace{1cm} (7)

If B effects were treated classically, or S-B correlations were ignored, this measurement would have no effect at all, since it does not change the energy-diagonal state of $S$. Yet, because of the non-commutativity of $H_{SP}$ and $H_{SB}$, an impulsive NSM does change the S+B "supersystem". Such change is absent from Markovian treatments wherein the measurement is assumed slow enough to warrant energy conservation of the supersystem \cite{23}, in contrast to the present fast one that breaks this conservation. This crucial point is beyond the system-bath separability paradigm \cite{15,24}, and stems from the fact that at equilibrium $S$ is correlated with $B$: they are in a Gibbs state $\rho_{eq} = e^{-\beta(H_{SB}+H_B)}$, $\beta$ being the inverse temperature, and their mean correlation energy is negative to ensure stable equilibrium: $\langle H_{SB} \rangle_{eq} \leq 0$ \cite{18,19}. The impulsive NSM changes the Gibbs state and its mean observables into their post-measured counterparts \cite{18}

$$\langle H_{SB} \rangle_{eq} \leq 0 \rightarrow \langle H_{SB} \rangle = \frac{1}{2} \langle H_{SB} \rangle_{eq} + \frac{1}{2} \text{Tr} [\hat{B} \sigma_z \sigma_x \sigma_z \rho_{eq}] = 0,$$  \hspace{1cm} (8)

where we have used the identity $\sigma_z \sigma_x \sigma_z = -\sigma_x$. Concurrently, $\langle H_{SP} \rangle_{eq} = 0 \rightarrow \langle H_{SP} \rangle \neq \langle H_{SB} \rangle_{eq}$, and $\langle H_P \rangle = 0$ both before and after the measurement. Hence, the decrease in the S+P correlation energy $\langle H_{SP} \rangle$ reflects an equal increase in the correlation energy $\langle H_{SB} \rangle$.

The foregoing expressions have the following meaning: Any NSM must increase the mixedness and thus the
entropy of $\rho_{S+B}$ [23]. Since prior to the measurement $S+B$ was in a maximal-entropy (Gibbs) state among all states with the same mean energy $\langle H_{SB} \rangle$, the entropy increase implies an increase of $\langle H_{SB} \rangle$. Since the QND NSM changes $\rho_{S+B}$ into $\rho_S \otimes \rho_B$, yet leaves $\rho_S$ and $\rho_B$ unchanged, it changes neither $\langle H_S \rangle$ nor $\langle H_B \rangle$. The work invested in performing the impulsive NSM is therefore

$$\Delta E_{meas} = -\Delta \langle H_{SP} \rangle = -\langle H_{SB} \rangle_{eq} > 0. \quad (9)$$

Equation (9) reveals the discrepancy between the energy cost (work investment) required for a brief QND measurement and that expected by the SL principle: the SL work investment in the measurement is $W_{SL}^{meas} = 0$, since it does not account for system-bath correlations. As shown in what follows, Eq. (9) can be a useful work resource only on non-Markovian time-scales at any temperature of the bath, including $T = 0$. By contrast, the measurement cost in current Markovian treatments [14, 15] vanishes at $T = 0$ (see Appendix).

### III Work extraction in a non-Markovian cycle

As shown below, it is crucial to ensure that the cycle duration $t_{cycle}$ satisfies $t_e \geq t_{cycle}$, i.e., work extraction must occur on the non-Markovian (memory) time-scale $t_e$ of $S+B$. Furthermore, it should be much longer than the duration $\tau_m$ of the measurement that triggers the cycle (Sec. II).

If these conditions hold, the maximal work extractable in a post-measurement optimal cycle is (see App.)

$$(W_{NSM}^{\text{ext}})_{\text{Max}} = \Delta E_{meas} - T \Delta s_{meas}, \quad (10)$$

where $\Delta E_{meas}$ is given by Eq. (9), and $\Delta s_{meas}$ is the NSM-induced entropy increase of the supersystem $S+B$. This entropy increase reflects the destruction of $S-B$ correlations (off-diagonal elements of $\rho_{SB}$) and the lack of information gain by the NSM.

The bound on the maximal extractable work discussed above does not suffice to demonstrate that work is indeed obtainable from $S$ in a post-measurement cycle: we must show that the extracted work $W_{cycle}^{\text{ext}} > 0$ in a feasible cycle. This work may be extracted by a classical, coherent (zero-entropy) off-resonant piston (harmonic oscillator) that is dispersively coupled to $S$ via $\sigma_Z$, and modulates its energy levels [24], allowing $S$ to undergo a closed cycle, after which $\rho_S(t_{cycle}) = \rho_S(0)$. The piston coherent excitation-change then expresses the extracted work (see Discussion).

The standard expression for the extractable work (i.e., the negative of the invested work) over a cycle is [14, 24]:

$$W_{cycle}^{\text{ext}} = -\int_{0}^{t_{cycle}} \text{Tr} \{\rho_S H_S \} dt = -\int_{0}^{t_{cycle}} s(t) \dot{\omega}(t) dt. \quad (11)$$

Here $\omega(t)$ is the level-separation (frequency) of the piston-driven TLS, $s(t)$ is the polarization (population difference) of its energy-states $|e\rangle$, $|g\rangle$ and the cyclic integral is over a closed trajectory in the frequency-polarization plane.

According to the standard (Markovian) expression of the second law in open quantum systems [14, 15, 24], no work is extractable from the system: conversely, only the piston can do work on the system. This rule can be proved to be strictly obeyed if the bath-induced evolution is Markovian (App.). Yet, our analytical results and numerical simulations (Fig.1) show that while the system interacts with a bath on non-Markovian time scales, net work can be performed in a cycle by the system, i.e., the piston can be coherently amplified. Namely, under non-Markovian dynamics, we can ensure $W_{cycle}^{\text{ext}} > 0$ in Eq. (11) by choosing $\dot{\omega}(t)$ to oscillate out of phase with $s(t)$. For weak $S$-$B$ coupling and $T = 0$ we will explicitly show that $W_{cycle}^{\text{ext}} > 0$ is indeed possible only in the non-Markovian limit on the modulation rate, $\Omega \gg \frac{1}{\tau_m}$. More elaborate analysis, whereby the piston field “dresses” the qubit states with which the bath interacts [26], yields qualitatively similar results.

We wish to evaluate the work performed by a qubit in contact with a bath at temperature $T$ after a non-selective measurement of its energy, over a period (cycle) of its Stark-shift modulation of the form

$$\omega(t) = \omega_a + \delta \sin \Omega t. \quad (12)$$

To this end we shall use the results of the weak-coupling, non-Markovian master equation [13, 27, 28], whereby

$$s(t) = e^{-J(t)} \left( \int_0^{\pi} \Delta R(t') e^{J(t')} dt' + s(0) \right). \quad (13)$$

Here the relaxation integrals $J(t) = J_g(t) + J_e(t)$, $J_{g(e)}(t) = \int_0^{\pi} R_{g(e)}(t') dt'$ and $\Delta R(t) = \frac{1}{2} (R_g(t) - R_e(t))$, depend on the effect of the non-Markovian (B) bath: these are the bath-induced transition rates $R_e(t)$ ($|e\rangle \rightarrow |g\rangle$) and $R_g(t)$ ($|g\rangle \rightarrow |e\rangle$). Both $J(t)$ and $R(t)$ are partly oscillatory on non-Markovian time scales, reflecting the partial reversibility of S-B dynamics. They are proportional to the square of the system-bath coupling strength, $\eta$.

Since the coupling is assumed weak, $s(t)$ can be expanded as $s(t) \approx s(0)(1 - J(t)) + J(t) + O(\eta^3)$, where $\Delta J(t) = \int_0^t \Delta R(t') dt'$. The first term in the expansion, $s(0)$, does not contribute to the work (integration over $s(0) \cos(\Omega t) dt = 0$). The other terms are $O(\eta^3)$, so we set $s(0) \approx -1/2$, at $T \approx 0$. The universal formula [13, 27] yields the relaxation integrals $J(t)$ and $\Delta J(t)$ through $J_{g(e)}(t)$ as the spectral overlap of the bath response $G_T(\omega)$ and the modulation spectrum $F_\Omega(\omega)$.
\[
J_{e(g)}(t) = \int_{-\infty}^{\infty} d\omega G_T(\omega) F_i(\omega) + \omega = \\
\frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega G_T(\omega) \int_{0}^{t} dt' e^{i(\omega + \omega_a)t'} \epsilon(t')^2, \tag{14}
\]

where the modulated phase factor \(\epsilon(t') = e^{i \int_{t_0}^{t'} dt'' \omega(t'')}\). Using the Bessel expansion of \(\epsilon(t)\) for our \(\omega(t) = e^{-i\omega T} = \sum_{n=-\infty}^{\infty} i^n J_n(-\frac{\delta}{\Omega}) e^{i n t_0 T}\), and assuming a weak modulation, \(\frac{\delta}{\Omega} \ll 1\), the expression for work in a cycle reduces to

\[
W_{\text{ext}} \approx \frac{\delta}{2\pi} \int_{-\infty}^{\infty} G_0(\omega) \frac{2\pi}{(\omega^+)^2} \times \left(\text{sinc}\left(\frac{2\pi}{\Omega}(\omega^+ + \Omega)\right) + \text{sinc}\left(\frac{2\pi}{\Omega}(\omega^+ - \Omega)\right)\right) d\omega \tag{15}
\]

where \(G_0(\omega)\) is the zero-temperature bath response, and \(\omega^+ = \omega + \omega_a\). This expression may be approximated as

\[
W_{\text{ext cycle}}^{\text{cycle}} \approx -\delta \int_{0}^{2\pi} J_g(t) \Omega \cos \Omega dt. \tag{16}
\]

It shows that in the strongly non-Markovian limit \(\Omega \gg \delta \sim \omega_0\), the sign of \(W_{\text{ext cycle}}^{\text{cycle}}\) oscillates with \(\Omega\), for a fixed \(\omega_0\), and thus allows for either positive or negative work extraction, as opposed to the Markovian limit (App.). Hence the work invested by the NSM (Eq. (9)) can be partly extracted in a non-Markovian cycle.

We next clarify how \(S\) can regain the energy deposited by the measurement, using our analytical results and simulations (Fig. 1–main panel): The source of work is seen to be only the change of \(\langle H_{SB}\rangle\), the system-bath correlation energy. The rapid variation of the extractable work with the cycle duration \(t_{\text{cycle}}\) (Fig. 1a) proves that work retrieval from \(\langle H_{SB}\rangle\) is limited to non-Markovian time scales: \(t_{\text{cycle}} < t_c\) should be shorter than \(t_c\), the bath memory time, to ensure work performance by the system enabled by an unread QND measurement. The reason for this anomalous effect is the (partly reversible) S+B dynamics expressed by the oscillatory relaxation integrals \(J_g(t)\) and \(J_e(t)\) on non-Markovian time scales triggered by the measurement.

**IV The revised work-information relation**

We are now in a position to address the fundamental questions that motivate this paper: What is the difference between the maximal work extracted in a cycle via a selective (read) measurement, \((W_{\text{ext cycle}}^{\text{cycle}})_{\text{max}}\), and its non-selective (unread) counterpart, \((W_{\text{ext cycle}}^{\text{cycle}})_{\text{max}}\)? How do they differ from their SL counterparts?

Let us define the measurement basis as \(|j\rangle\), \(j = e, g\). Then \(p_j\) is the probability of finding the state \(\rho_{S+B}\) in the state \(j\), and \(\rho_{S+B}^j\) is the state of the supersystem after the measurement. The maximum extractable work by a nonsel ective measurement is given by Eq. (10). Its counterpart for selective measurement is \((W_{\text{ext cycle}}^{\text{cycle}})_{\text{max}} = \sum p_j \Delta E_j - T \sum p_j \Delta J_j\), where \(\Delta E_j\) and \(\Delta J_j\) are the respective changes when the state \(j\) is measured, and \(\Delta E_{\text{meas}} = \sum p_j \Delta E_j\). The difference between work extraction based on selective and nonselective measurement is

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**FIG. 1. Work extraction by measurements.** Main panel: Simulations of the first-cycle evolution of the energy of the system (solid thick blue line), the bath (solid thin purple line), the classical piston (dashed thin green line), and the system-bath correlations (dashed thick brown line). The piston periodically modulates the TLS frequency, \(\omega(t) = \omega_0 + \delta \sin \Omega t\). The period of the modulation starts with an unread measurement of the TLS energy. The parameters for the curves are \(\omega_0 = 1\), \(\delta = 1/4\), \(\Omega = 5/2\). The S-B coupling spectrum is a Lorentzian centered at \(\omega_0\) of width \(t_c^{-1}\) (inverse correlation or memory time of the bath), with \(\omega_0 - \omega_a = 3/7\), \(t_c = 10\) and the inverse bath temperature is \(\beta = 3.74\). The cycle starts with a non-selective measurement at time \(\omega_0 t_m = 1\) and ends at \(\omega_0 t_{\text{end}} = 3.51\). The measurement invests \(\Delta E_{\text{meas}} = -\langle H_{SB}(t = t_m)\rangle\) in the system. Here the piston \(\langle H_{\text{piston}}\rangle\) is seen to have gained energy during the cycle, but not at the expense of \(\langle H_S\rangle\) which returns to its initial value. Since \(\langle H_S\rangle\) has also gained energy, the source of work is the change in \(\langle H_{SB}\rangle\), the system-bath correlation. The simulations imply that \(W_{\text{ext cycle}} = W_{\text{ext cycle}}^{\text{cycle}} - \Delta E_{\text{meas}} < 0\), although \(W_{\text{ext cycle}}^{\text{cycle}} = \langle H_{\text{piston}}(t_{\text{end}})\rangle - \langle H_{\text{piston}}(t_m)\rangle > 0\). Inset: extractable work in a cycle \(W_{\text{ext cycle}}^{\text{cycle}}\) normalized to the maximum \((W_{\text{ext cycle}}^{\text{cycle}})_{\text{max}}\) as a function of the cycle duration \(t_{\text{cycle}}\). It is seen that \(W_{\text{ext cycle}}^{\text{cycle}} > 0\) (work done by the system) requires \(t_{\text{cycle}} \geq t_c\), \(t_c\) being the bath memory time. Same parameters as in main panel 1.
The last step follows from the equality of the Shannon entropy $\mathcal{H}$ of the system and its von Neumann entropy $\mathcal{J}(\rho_S)$ when $\rho_S$ is diagonal, as in our case, if we set $k_B = 1$ and $\mathcal{H} = \ln 2^{H_{\text{Shannon}}}$. $H_{\text{Shannon}}$ being the standard definition of the Shannon entropy. Explicitly, in our case $\mathcal{H} = -p \ln p - (1 - p) \ln (1 - p)$. The foregoing expression may be recast in the form

$$\left( W_{\text{ext}}^{\text{sel}} \right)_{\text{Max}} = \left( W_{\text{ext}}^{\text{NSM}} \right)_{\text{Max}} + W_{\text{SL}}.$$  \hspace{1cm} (18)

Here the first term on the r.h.s. is Eq. \[10\] and $W_{\text{SL}} = T\mathcal{H}(p)$ denotes the standard SL work extraction: the energy required to reset the probe to its initial state.

Equation (18) is our main result: it implies that the maximal work extractable via a selective measurement is higher than what is expected from the SL relation between work and information: the S-B correlations increase this extractable work by $(W_{\text{ext}}^{\text{NSM}})_{\text{Max}}$. In the standard (Markovian) case, where the system-bath interaction energy is assumed negligible, $(W_{\text{ext}}^{\text{NSM}})_{\text{Max}} = 0$ and $(W_{\text{ext}}^{\text{sel}})_{\text{Max}} = T\mathcal{H}(p)$. Hence, in the Markovian case, the selective measurement (in this scenario) does not yield any work beyond the Landauer cost $W_{\text{SL}} = T\mathcal{H}(p)$ of resetting (“cleaning”) the memory of the probe after each measurement. In the non-Markovian case, the measurement process requires higher investment of work, since it changes the system-bath correlation energy, but also allows more work to be extracted than in the Markovian case.

The NSM effect discussed above allows work extraction at $T = 0$ and without any information gain: both features take us beyond the SL \[1, 11\] information-work balance or its IT extensions\[10, 11\]. Namely, even at $T = 0$, $(H_{SB})_{\text{Eq}} < 0$ and its change by $\Delta E_{\text{meas}}$ powers the cycle, yielding $(W_{\text{ext}}^{\text{sel}})_{\text{Max}} = (W_{\text{ext}}^{\text{NSM}})_{\text{Max}} > 0$ (Fig. 2). The entire work then originates from the non-Markovian change of system-bath correlations.

Whereas $W_{\text{ext}}^{\text{NSM}}$ has been shown above to exceed the SL bound, it can be argued that $W_{\text{ext}}^{\text{sel}}$ can be even higher. To this end, consider that in the case of selective measurements, the work extraction will be the weighted sum of that obtained by each measurement result

$$W_{\text{ext}}^{\text{sel}} = \rho_{ee}(0)W_e + \rho_{gg}(0)W_g = \int \left[ J_g(t)\rho_{gg}(0)\dot{\omega}_g(t) - J_e(t)\rho_{ee}(0)\dot{\omega}_e(t) \right] dt.$$  \hspace{1cm} (19)

where the two modulations $\dot{\omega}_e(t)$ may be different. Suppose we choose $\dot{\omega}_e(t)$ so that they totalize the maximum work $W_{\text{ext}}^{\text{sel}} = (W_{\text{ext}}^{\text{sel}})_{\text{Max}}$. If the corresponding modulations happen to coincide, $\dot{\omega}_e(t)_{\text{Max}} = \dot{\omega}_g(t)_{\text{Max}}$, then the resulting expression is the same as $(W_{\text{ext}}^{\text{NSM}})_{\text{Max}}$ (Eq. \[11\]). If, on the contrary, $\dot{\omega}_e(t)_{\text{Max}} \neq \dot{\omega}_g(t)_{\text{Max}}$, then the two bounds differ. Clearly, we may then have

$$(W_{\text{ext}}^{\text{sel}})_{\text{Max}} > (W_{\text{ext}}^{\text{NSM}})_{\text{Max}},$$  \hspace{1cm} (20)

since by choosing $\dot{\omega}_e(t)$ and $\dot{\omega}_g(t)$ to be out of phase throughout the cycle, the two terms in Eq. (19) acquire the same sign, i.e., add up. By contrast, in

$$W_{\text{ext}}^{\text{NSM}} = \int \left[ J_g(t)\rho_{gg}(0) - J_e(t)\rho_{ee}(0) \right] \dot{\omega}(t) dt,$$  \hspace{1cm} (21)

the two terms have opposite signs, so that $(W_{\text{ext}}^{\text{sel}})_{\text{Max}}$ can exceed $(W_{\text{ext}}^{\text{NSM}})_{\text{Max}}$, q.e.d.

V Consistency with the second law

The second law is upheld (in the sense that perpetual motion becomes forbidden) only when we account for the energy and entropy cost of changing the “supersystem” state $\rho_{S+B}$ from its correlated Gibbs form at equilibrium $\rho_{\text{Eq}}$ to its post-measurement product-state form $\rho_S \otimes \rho_B$. This cost becomes evident only in a description of the evolution in terms of the total Hamiltonian and the corresponding state $\rho_{\text{rot}}$ that encompass the degrees of freedom of the probe and the supersystem, P+B.
Explicitly,
\[ H_{\text{tot}} = H_0 + H_{BP} + H_{SP}, \quad (22) \]

where \( H_{SP} \) is the system-probe coupling term, \( H_{BP} \) allows a fast decorrelation between the system and probe, and \( H_0 \) describes the “supersystem”, system+bath. Using the fact that the total Hamiltonian is cyclic, \( H_{\text{tot}}(\tau) = H_{\text{tot}}(0) \), the total extractable work in a cycle (by \( P+S+B \)) can be written as

\[
W_{\text{tot}}^{\text{ext}}(\tau) = -\int_0^\tau \text{Tr} \left[ U(t) \rho_{\text{tot}}(0) U^\dagger(t) \right] \dot{H}_{\text{tot}}(t) dt = -\text{Tr} \left[ \rho_{\text{tot}}(\tau) H_{\text{tot}}(\tau) - \rho_{\text{tot}}(0) H_{\text{tot}}(0) \right] + \int_0^\tau \text{Tr} \left[ \rho_{\text{tot}}(t) H_{\text{tot}}(t) \right] dt. \quad (23)
\]

Upon inserting the expression for \( \dot{\rho}_{\text{tot}}(t) \) and calculating the trace we find that the second term on the RHS is zero. The first term thus represents the energy change of the supersystem. Since initially the supersystem and the probe were at thermal equilibrium, any energy change should be positive. We then find:

\[
W_{\text{tot}}^{\text{ext}}(\tau) = -\text{Tr} \left[ U(\tau) \rho_{\text{tot}}(0) U^\dagger(\tau) H_{\text{tot}}(0) - \rho_{\text{tot}}(0) H_{\text{tot}}(0) \right]. \quad (24)
\]

Here the first term is the final mean energy and the second is the initial one.

Because the total dynamics is unitary, the entropy of \( \rho_{\text{tot}} \) is fixed. This implies that the final mean energy (first term) must be greater or equal to the initial one (second term), as the thermal-equilibrium initial state minimizes the mean energy at fixed entropy.

\[
W_{\text{tot}}^{\text{ext}} = -\int_0^\tau \text{Tr} \{ \rho_{\text{tot}} \dot{H}_{\text{tot}} \} dt = -\Delta E_{\text{meas}} + W_{\text{tot}}^{\text{ext}} \text{cycle} < 0. \quad (25)
\]

The negativity of Eq. 25 under a cyclic unitary evolution of the total Hamiltonian, starting from equilibrium of the supersystem and the probe, can be proved completely generally. It shows that the second law that forbids drawing work from a single bath only applies to the entangled evolution of S+B+P and that their standard separability assumption fails for sufficiently fast cycles.

VI Multiple cycles: Resetting cost for non-selective measurement?

It might be suspected that resetting (purifying) the probe is necessary if we wish to reuse it in successive cycles and that would add to the thermodynamic cost as per the SL principle. Clearly, the probe cannot circumvent the SL resetting cost \( W_{\text{SL}} \) when selective measurements are required (Eq. 18). Yet, this is not the case for a NSM, which requires no resetting because the bath can rapidly decorrelate \( P \) and the supersystem following the measurement (Eq. 7), but prior to the next cycle:

\[
\rho_{\text{tot}} \mapsto \rho_{S+P \otimes B} \mapsto \rho'_{P} \otimes \rho'_{S+B}. \quad (26)
\]

The absence of resetting cost after a NSM follows from a remarkable observation: a single probe qubit has the same NSM effect on any number of system cycles. This holds since it does not matter how each cycle changes the state of the probe, \( \rho_P \), because the resulting state commutes with the probe’s \( \sigma_x^P \) (Eq. 4). In particular, work is extractable even if \( P \) is in the fully mixed (infinite-temperature) state: for \( \rho_P = \frac{1}{2} I^P \), the CNOT leaves the probe qubit unchanged, i.e., it cannot be read out and yet the same probe qubit can still perform the required NSM on the system qubit as often as we like, i.e., our probe is never used up. This is because Eq. 26 still holds in this case: S+P become correlated by the measurement (after the previously described fast modulation period), but then the correlations between the system and the probe decay via thermal relaxation and revert to a product state (Fig. 3). After this relaxation the probe can be reused in the next cycle and have the same effect as in the first cycle. Hence, there is no need of resetting the probe for further use in consecutive cycles, provided it performs repeated NSM.

![System-Probe correlations](image)

FIG. 3. The role of probe-system correlations and their destruction by the bath. Decay of the off-diagonal system-probe (S-P) elements (correlations) \( \langle i|\rho_{\text{tot}}|j \rangle \) under \( H_{\text{tot}} \) (Eq. 1) where \( |i \rangle = |e, 1, n \rangle \) and \( |j \rangle = |g, 0, n \pm 1 \rangle \), the entries denoting the system, probe and bath quantum numbers, respectively. The parameters are the same as in Fig. 1. The probe frequency is \( 10/7 \omega_s \). The decay time of the correlations is that of the oscillations envelope, here \( \sim 4 \) modulation periods (4\( \tau_{\text{mod}} \)). After this time the probe can be reused for the next cycle.

VII Experimental Scenario

A feasible experimental test of these predictions may
The distinctive signature of this amplification is that it is restricted to $t_{\text{cycle}} = \frac{\pi}{\Omega} \leq t_c$: as $t_{\text{cycle}}$ starts exceeding $t_c$, amplification will revert to loss. The described process is akin to intracavity parametric conversion of external driving (probe pulses), resulting in signal ("piston" mode) amplification [22, 32], but it is unique in its reliance on system-bath correlations, and in its insensitivity to the probe noise.

VII Discussion

We have shown the possibility of extracting useful work from an open quantum system following either a non-selective (unread) QND measurement (NSM) (Eq. (16)) or a selective (read) measurement (SM) (Eq. (19)). In both cases, a modulator (piston) can take work and gain energy from the system (be coherently-amplified) in a closed cycle. This work originates neither from the probe free-energy [10, 11] nor from the heat energy of the bath (as in Szilard’s engine [1, 8]) but from a hitherto unexploited (and little-discussed) source: the inevitable change of the system-bath (S-B) correlation (interaction) energy (see [29]) by a brief QND measurement [16, 21]. Only non-Markovian supersystem (S+B) dynamics can yield extractable work following such a measurement, as opposed to its Markovian limit that ignores system-bath correlations (Fig. 1a).

When discussing these effects, certain misunderstandings must be dispelled: (i) The proposed work resource cannot be explained by viewing either the unread probe or the piston as a fictitious additional “bath”. Neither constitutes a proper heat bath: the piston is a zero-temperature and zero-entropy classical drive that only gains work and energy from the system, while the probe must act impulsively, unlike usual sources of noise of heat. (ii) Nor can one deny the cycle is triggered by a measurement: even if the measurement is unread, it is still a measurement, as evidenced by the S-P correlations (see Eq. (20)). (iii) Recently considered measurement-cost (Markovian) effects [10, 11] are beyond the scope of our scenario (see SI2).

The colloquial maxim “there are no free lunches” applies to the predicted effect, i.e., the surplus work is allowed only by extra investment of energy consistently with the first law (otherwise it would enable a “perpetuum mobile” machine [29, 33] and the second law is also upheld (Eq. (25)). Yet this effect may allow us to study the possibilities of transforming energy input (e.g., electromagnetic probe pulses which may be very noisy as argued in Sec. VI, similarly to [22]) into useful work [coherent signal (piston) amplification] via rapid modulations of thermalized quantum systems

The present engine model, in which the system is always coupled to a single bath and yet may perform useful work, is potentially important for systems totally embedded in a single bath, such as a cavity, so that conventional heat-engine (two-bath) thermodynamic cycles may be impossible to implement. Further investigation may include brief disturbances other than measurements,
e.g., phase flips of a TLS in a bath [34].

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1 Measurement cost in our scenario compared to Maxwell's demon models

A) Measurement cost for asymmetric detector

The SL principle has been extended to a detector/memory modeled by a quantum particle in an asymmetric double-well potential (the Sagawa-Ueda (SU) model[10]). Such asymmetry may reduce the expense of resetting the detector to its initial state at the cost of increasing the work required to perform the measurement, but the sum of the two costs remains the same as the cost set by the SL principle. In that case, the mean energy of the device (D) may be altered by the measurement, and in particular the device may exchange energy with the measured system. For such a device the resetting cost may differ from Landauer’s. SU assume a total Hamiltonian of the form:

\[ H(t) = H_D(t) + H_B + H_{DB}(t); \]
\[ H_D(0) = H_D(\tau) = H_D; \quad H_{DB}(0) = H_{DB}(\tau) = 0 \quad (A1) \]

The SU Hamiltonian does not describe the measurement itself, only the detector-bath interaction \( H_{DB}(t) \). By contrast, in our scenario the \( H_{SD}(t) \) term in Eq. (1) causes a change of \( \langle H_{SB} \rangle \) a S+B correlation term, which is missing from the SU analysis: they adopt the S+B correlation change.

Similarly, the work invested in the measurement in our scenario is performed not on D, as in the SU model, but on S+B, and is equal to the change in \( \langle H_{SB} \rangle \) in Eq. (S).

In the SU model, the measurement cost is given by Eq. (2) and is nonzero due to S+B correlation change. Similarly, the work invested in the measurement in our scenario performed on S+B, is equal to the change in \( \langle H_{SB} \rangle \) in Eq. (S).
change in the free energy of D due to the measurement process

$$\Delta F_D \equiv \Sigma_k p_k F_{k,D} - F_{0,D}$$ (A2)

where $p_k$ is the post-measurement probability for D to be found in subspace $H_k^D$. By contrast, in our case the final state of the D has the same energy as its initial state and $p_0 = 1$, giving: $\Delta F_D = 0$. Yet our measurement cost is given by Eq. (9) and is nonzero due to S+B correlation change.

Similarly, the work performed on D by the measurement is assumed by SU to be the negative of the change in energy of D, which in our scenario is given by Eq. (9) and is nonzero due to S+B correlation change.

$$W_{D,\text{meas}} = Tr \{(H_D + H_B) \rho_{DB} | \tau \} = 0$$ (A3)

Namely, in our scenario this change in the mean energy of D vanishes. By contrast, we have post-measured work extraction due to the change in S+B correlations by a measurement after time $\tau$:

$$W_{\text{ext}} = \langle H_{S+B}(\tau) \rangle_D - \langle H_{S+B}(0) \rangle_D \neq W_{D,\text{meas}} = 0.$$ (A4)

b) Measurement cost for degenerate detector

In Ref. [11] a detector D consisting of degenerate states, initially in thermal equilibrium, performs a selective measurement (SM) on S in order to extract work in a cycle as Maxwell’s demon. Since a SM reduces the entropy of S, the entropy of D must correspondingly rise. Hence, D must not be in a maximal entropy state prior to the measurement. Neither can its temperature be the same as that of S, i.e., $T$. The required lowering of the temperature and entropy of D are achieved by isothermally (quasistatically) lifting the degeneracy of the levels of D at a cost $\Delta E_D$. The maximal work extraction by D is the free energy lost by D

$$W_D = \Delta E_D + T\Delta S$$ (A5)

Although superficially looks similar to our Eq. (9), it is essentially different in that $\Delta E_D$ is determined by the detector temperature, which is irrelevant for the impulsive NSM used in our scenario to change $\langle H_{SB} \rangle$. In particular, $\Delta E_D = W_D = 0$ at $T = 0$ in Ref. [11], as opposed to our Eqs. (3) and (9), where the measurement cost does not change $\langle H_D \rangle = 0$, but $\langle H_{SB} \rangle$ changes even at $T = 0$. The temperature and entropy restrictions on D in Ref. [11] do not exist in our model (see Sec. VIII). The reason for these differences is that by venturing beyond the S-B separability paradigm we enable the entropy of S to be reduced at the expense of $\Delta S_{S+B}$ reflecting S-B correlation change by the measurement, whereas in Ref. [11] the bath B does not affect the entropy balance during the measurement under the S-B separability paradigm.

2 Maximal work in a post-measurement cycle

The post-measured S+B supersystem is thus in a nonequilibrium state that can be harnessed to perform work on its way back to equilibrium. The maximal work possible is extractable in a cycle that is thermodynamically reversible apart from the measurement “stroke”[34]. Were $\rho_{S+B}$ a thermal (Gibbs) state (for some temperature), we could use standard processes[24] to “close the cycle” by a reversible process, and the maximal extractable work would then be given by the difference in the Helmholtz free energy between $\rho_{S+B}$ and the equilibrium state[23]. However, since $\rho_{S+B}$ is not a Gibbs state, it is not clear that this upper bound on work is appropriate.

To find a thermodynamically reversible process that would bring the post-measured state back to equilibrium, we resort to a nonstandard procedure that allows maximal work extraction. Namely, we envision that the supersystem $S + B$ is embedded in a Markovian bath $B_M$, at the same temperature as $B$, $T = \frac{1}{2}$. The supersystem $S + B$ equilibrates with $B_M$ at time $t_{ EQ}$ say via coupling between $B$ and $B_M$. Since $B_M$ is Markovian we can neglect its correlation with $S + B$. Yet the correlations between S and B persist much longer, because B is non-Markovian, with correlation (memory) time $t_c \gg t_{ EQ}$.

The stages of this nonstandard, optimal cycle are as follows (Fig. 5): (1) The initial equilibrium state $\rho_{B_M} \otimes \rho_{SB}$, where $\rho_{SB} = \rho_{EQ} = e^{-\beta H_{SB}}$, undergoes at time $t = 0$ a measurement of S (Eqs. (8) and (9)) that leaves $S + B + B_M$ in (approximately) the product state $\rho_{B_M} \otimes \rho_S \otimes \rho_B$. (2) We next stabilize $\rho_S \otimes \rho_B$ by making a sudden change of the S+B Hamiltonian: $H_{S+B} \rightarrow H_{S+B}'$, so that the overall state becomes $\rho_{B_M} \otimes \rho_S \otimes \rho_{B'}$. The change of work is $W_{\text{stab}} = \langle H_{S+B}' \rangle - \langle H_{S+B} \rangle$. We are guaranteed that such stabilization is possible[35, 36], but it may not be feasible if we only act on S (by modulating the qubit level-distance). (3) Subsequently, we change $H_{S+B}' \rightarrow H_{S+B}$ by modulation over time $\tau_{S+B} \gg t_{ EQ}$, i.e. quasistatically and isothermally as concerns $B_M$, until we attain the original equilibrium state $\rho_{B_M} \otimes e^{-\beta H_{S+B}}$ and thereby close the cycle. The work change during the isothermal stage is $W_{\text{isot}} = \Delta E_{\text{isot}} - T \Delta S_{\text{isot}}$.

The overall optimal cycle is described as follows: (i) In the first stroke, the energy cost of the measurement is (see Eq 9) $\Delta E_{\text{meas}} = \langle H \rangle_{\rho'} - \langle H \rangle_{\rho}$. The NSM increases the VN entropy: $\Delta S_{\text{meas}} = \langle H_{\rho'} - \rho \rangle - \langle H_{\rho} \rangle$. (ii)In the next (return) stroke, the stabilization (sudden) Hamiltonian change implies that work is performed by the system: $W_{\text{stab}} = \langle H \rangle_{\rho'} - \langle H \rangle_{\rho}$ and the entropy is unchanged. (iii) In the last stroke, the energy change of the supersystem is $\Delta E_{\text{isot}} = \langle H \rangle_{\rho} - \langle H \rangle_{\rho'}$, $\Delta S_{\text{isot}} = -\Delta S_{\text{meas}}$ and the extracted work during this stroke is $W_{\text{isot}} = -\Delta E_{\text{isot}} + T \Delta S_{\text{isot}}$. (iv) Fi-
nally, combining these results for all strokes one gets the expression in Eq. (10) for
\[ W_{\text{extracted}} = W_{\text{sudden}} + W_{\text{isotherm}} \]

3 No work can be extracted from a single Markovian-bath engine in a closed cycle

We consider the evolution of the TLS state, \( \rho_S(t) \), that is diagonal in the energy basis, with parametrically time-dependent energy levels \( E_c(t) - E_g(t) = \omega(t) \):

\[
\dot{\rho}_{ee}(t) = R_g(t)\rho_{gg} - R_c(t)\rho_{ee} \\
\dot{\rho}_{ee}(t) = -\dot{\rho}_{gg}(t)
\] (A6)

Let us now assume Markovian properties: A) \( R_{g(e)}(t) \geq 0 \); B) Gibbs probability distribution in a stationary state (detailed thermal balance) at temperature \( k_B T = \frac{1}{\beta} \)

\[ R_c(t)\rho_{ee}^g(t) = R_g(t)\rho_{gg}^g(t) , \]
\[ \rho_{jj}^g(t) = Z^{-1}(t) \exp\{-\beta E_j(t)\} \quad j \in (g,e) \] (A7)

\( Z(t) \) being the normalization constant.

To prove this result (which is consistent with known results) consider the following auxiliary expression

\[
\sum_j \dot{\rho}_{jj}(\ln \rho_{jj} - \ln \rho_{jj}^g) =
\]
\[ (R_g\rho_{gg} - R_e\rho_{ee}) \ln \frac{\rho_{ee}}{\rho_{gg}^g} - (R_g\rho_{gg} - R_e\rho_{ee}) \ln \frac{\rho_{gg}}{\rho_{gg}^g} =
\]
\[ R_g\rho_{gg}^g(x \ln y - x \ln x + x - y) +
R_e\rho_{ee}^g(y \ln y - y \ln y + y - x) \leq 0 \] (A8)

where \( x = \frac{\rho_{gg}}{\rho_{gg}^g} \) and \( y = \frac{\rho_{ee}}{\rho_{ee}^g} \). Notice, that \( R_g\rho_{gg}^g(x - y) + R_e\rho_{ee}^g(y - x) = 0 \) due to assumption B). The inequality in (A8) is obtained from the relation \( a \ln a - a \ln b + b - a \geq 0 \) (for \( a, b \geq 0 \) and assumption A). It implies the following inequality for the entropy \( S(t) = -k_B \sum_j \rho_{jj}(t) \ln \rho_{jj}(t) \):

\[
\dot{S} = -k_B \sum_j \dot{\rho}_{jj} \ln \rho_{jj} \leq -k_B \sum_j \dot{\rho}_{jj} \ln \rho_{jj}^g = \frac{1}{T} \dot{Q}
\]
(A9)

where we used the fact that \( \sum_j \dot{\rho}_{jj} \ln Z(t) = \ln Z(t) \sum_j \rho_{jj} = 0 \).

Since for a closed cycle the entropies and internal energies in the initial and final states of the system are equal, \( W = Q \leq 0 \) (which is the second law of thermodynamics). This means that we cannot extract work from a single Markovian bath engine.

FIG. 5. Work extraction by measurements from S-B correlations: Optimal cycle that consists of 3 stages (see text): 1-measurement, 2-stabilization, 3-modulation.