OfedQIT: Communication-Efficient Online Federated Learning via Quantization and Intermittent Transmission

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Abstract—Online federated learning (OFL) is a promising framework to collaboratively learn a sequence of non-linear functions (or models) from distributed streaming data incoming to multiple clients while keeping the privacy of their local data. In this framework, we first construct a vanilla method (named OFedAvg) by incorporating online gradient descent (OGD) into the de facto aggregation method (named FedAvg). Despite its optimal asymptotic performance, OFedAvg suffers from heavy communication overhead and long learning delay. To tackle these shortcomings, we propose a communication-efficient OFL algorithm (named OFedQIT) by means of a stochastic quantization and an intermittent transmission. Our major contribution is to theoretically prove that OFedQIT over $T$ time slots can achieve an optimal sublinear regret bound $O(\sqrt{T})$ for any real data (including non-IID data) while significantly reducing the communication overhead. Furthermore, this optimality is still guaranteed even when a small fraction of clients (having faster processing time and high-quality communication channel) are active at each time. FedAvg has emerged as the de facto optimization method in this federated setting, where a global model is constructed via local stochastic gradient descent (SGD) at clients and model averaging at a central server. In [12], a communication-efficient algorithm (named FedPAQ) was proposed by integrating periodic averaging, partial client participation, and quantization. Also, both FedAvg and FedPAQ can yield an theoretical performance guarantee. However, it was shown in [13], [14] that FedAvg suffers from the so-called ‘client-drift’ when the degree of data heterogeneity is high (i.e., non-IID data), resulting in unstable and slower convergence. This problem becomes more serious as the fraction of participating clients becomes smaller. Many works have put much effort into addressing the above problem. SCAFFOLD [14] and Variance Reduced Local-SGD (VRL-SGD) [15] employed extra control variates to accelerate the convergence by reducing the variance of stochastic gradients. FedProx [18] added a proximal term to each local optimization in order to suppress the discrepancy among the local models. FedNova [16] normalized the magnitude of local updates across the clients in the network so that the model averaging less distracts the global loss.

In many real-world applications, machine learning tasks are expected to be conducted in an online fashion. For instance, online learning is necessary when data is generated as a function of time (e.g., time-series predictions) [19]–[21] and when large-scale data makes it hard to carry out data analytic in batch form [22]. Recently, this challenging problem has been actively investigated using random-feature based kernel learning for various network architectures such as centralized, fully decentralized, and federated learning [23]–[26]. In centralized network, online multiple kernel learning (OMKL) (a.k.a. Raker) was proposed in [23], [27] using online gradient descent (OGD) [28] and Hedge algorithm [29]. This algorithm seeks a sequence of functions (or models) from continuous streaming data and the resulting ones are non-sequential data. That is, all data are placed at the clients in the beginning of a learning process. In this framework, numerous algorithms (based on deep neural networks (DNNs)) have been proposed [9]–[17]. While there is a bunch of work on distributed optimization in the context of machine (or function) learning, federated learning algorithms have further considered two key challenges known as data and system heterogeneity [5], [17]. In attempt to handle system heterogeneity, low client participation is a popular approach where a small fraction of clients (having faster computation capability) are active at each time. FedAvg has emerged as the de facto optimization method in this federated setting, where a global model is constructed via local stochastic gradient descent (SGD) at clients and model averaging at a central server. In [12], a communication-efficient algorithm (named FedPAQ) was proposed by integrating periodic averaging, partial client participation, and quantization. Also, both FedAvg and FedPAQ can yield an theoretical performance guarantee. However, it was shown in [13], [14] that FedAvg suffers from the so-called ‘client-drift’ when the degree of data heterogeneity is high (i.e., non-IID data), resulting in unstable and slower convergence. This problem becomes more serious as the fraction of participating clients becomes smaller. Many works have put much effort into addressing the above problem. SCAFFOLD [14] and Variance Reduced Local-SGD (VRL-SGD) [15] employed extra control variates to accelerate the convergence by reducing the variance of stochastic gradients. FedProx [18] added a proximal term to each local optimization in order to suppress the discrepancy among the local models. FedNova [16] normalized the magnitude of local updates across the clients in the network so that the model averaging less distracts the global loss.

I. INTRODUCTION

Federated learning is an emerging distributed learning framework, in which a central server coordinates the machine learning training process with a massive number of clients without sharing the training data in the clients (i.e., local data) [1]–[5]. The key advantage of federated learning is to ensure data privacy, namely, clients do not require to share their own local data which may contain sensitive personal information. Instead, they separately train local models using the local data, and using the resulting ones, the central server optimizes a global model [5]. Specifically, federated learning learns a global model by conducting the two steps iteratively: i) local model optimizations at clients; ii) global model optimization (e.g., model averaging) at the central server. There are myriad of applications such as learning from wearable devices [6], location-based services [7], and human activity recognition [8].

In traditional federated learning, it is widely assumed that features do not posses an associated inherent ordering (i.e.,
used to predict the label of a newly incoming (previously unseen) data in real time. Also, it was proved that OMKL achieves an optimal sublinear regret bound compared with the best static function in hindsight. In [30], RFF-DOKL was proposed for fully decentralized network, which is constructed by combining OGD and a diffusion strategy for consensus constraint. This work was enhanced in [24], where distributed OMKL (DOMKL) was proposed by harnessing the principles of an online alternating direction method of multipliers and a distributed Hedge algorithm. Both methods can guarantee optimal asymptotic performances compared with the associated best static functions, respectively. Clearly, due to the advantage of using multiple kernels, DOMKL significantly outperforms RFF-DOKL. Especially in [26], online federated learning (OFL), which is closely related to the subject of this paper, was investigated, wherein a communication-efficient randomized algorithm (named PM-KOFL) was proposed. This algorithm achieves an optimal performance asymptotically while having much lower communication overhead than a vanilla method. We would like to remark that the aforementioned work has some limitations such as widely-adopted DNN learning, the impact of quantization, and the impact of data and system heterogeneity (i.e., the major challenges of federated learning) have not been carefully studied. Thus, it is still an open problem to construct a communication-efficient (DNN-based) OFL algorithm which can achieve an optimal sublinear regret bound when data and system heterogeneity are taken into account. This is the primary motivation of our work.

In this paper we investigate a communication-efficient OFL algorithm which are robust to data and system heterogeneity. We first construct a vanilla method (named OFedAvg) for OFL framework, in which decentralized clients separately optimize their local models via OGD and harnessing the similarity of SGD and OGD, a central server updates a global model via averaging) into the vanilla method (OFedAvg). We highlight that all clients are participated at every time (i.e., long learning delay [12], [14]. To overcome these shortcomings, we propose OFedQIT as a communication-efficient OFL algorithm. The proposed method is constructed by incorporating a stochastic quantization and an intermittent transmission (or intermittent averaging) into the vanilla method (OFedAvg). We highlight the major contributions of this paper below:

- Theoretically we prove that OFedQIT over $T$ time slots can achieve an optimal sublinear regret bound $O(\sqrt{T})$ for any real-world data (including non-IID data or heterogeneous data). When $T$ is sufficiently large (i.e., online learning proceeds in a sufficient time), our algorithm yields the almost same performance as OFedAvg while significantly reducing the communication overhead (about 99% reduction). Thus, OFedQIT can realize the centralized counterpart (i.e., all data are gathered at the central server) without sharing local data.
- Furthermore, we prove that the asymptotic optimality of OFedQIT is still guaranteed even when a small fraction of clients, for example having faster local-processing time or high-quality communication channel, are active at once. This implies that our algorithm can attain an optimal performance asymptotically while providing the robustness to data and system heterogeneity. Namely, OFedQIT completely addresses the shortcomings of OFedAvg.
- Beyond the asymptotic analysis, we demonstrate the effectiveness of our algorithm on various online learning tasks with real-world datasets. Notably, OFedQIT can almost achieve the performance of OFedAvg while reducing the communication overhead by 90%. Due to the attractive performance, privacy-preserving, and robustness to data and system heterogeneity, the proposed OFedQIT would be very useful in practice.

The remaining part of this paper is organized as follows. In Section [II] we formally define OFL framework and introduce a vanilla method (named OFedAvg). The proposed OFedQIT is described in Section [III]. In Section [IV] we prove the asymptotic optimality of our algorithm for OFL framework under data- and system-heterogeneity. Beyond the asymptotic analysis, in Section [V] we demonstrate the superiority of OFedQIT via experiments with real datasets. Some concluding remarks are provided in Section [VI].

**Notations:** Bold lowercase letters denote the column vectors. For any vector $w$, $w^T$ is the transpose of $w$ and $\|w\|$ is the $\ell_2$-norm of $w$. Also, $\langle \cdot, \cdot \rangle$ and $\text{E}[\cdot]$ represent the inner product in Euclidean space and the expectation over an associated probability distribution, respectively. To simplify the notations, for any positive integers $T$, $L$, we let $[T] \triangleq \{1, \ldots, T\}$ and $[T]_L \triangleq \{t \in [T] : t \mod L = 0\} \subseteq [T].$ We define a projection $\psi_L : [T] \to [T]_L + 1$ such that

$$\psi_L(t) \triangleq \lfloor (t-1)/L \rfloor \times L + 1 \text{ for some } t \in [T]. \quad (1)$$

Also, $k$ and $t$ are used to indicate the indices of clients and time, respectively.

**II. Preliminaries**

In this section, we formally define an online federated learning (OFL) framework and then construct a vanilla method (named OFedAvg) by properly integrating some existing techniques.

**A. Online Federated Learning (OFL)**

We consider a distributed learning framework consisting of $K$ clients indexed as $k \in [K]$ and a central server. The objective of OFL is to learn a sequence of global functions (or models) from distributed sequential data incoming to clients under the coordination of the central server. Specifically, at every time $t - 1$, the central server broadcasts the latest global model (denoted by $w_t$) to the centralized $K$ clients. Accordingly, our learned function at time $t - 1$ is defined as $f(x; w_{t-1})$, from which each client $k \in [K]$ can estimate the label of a newly incoming (and previously unseen) data $x_{k,t+1}$.
at time $t$ in real time. Focusing on the local operations, at time $t$, each client $k$ has the current global model $w_t$ and receives an incoming data $(x_{k,t}, y_{k,t})$, where $x_{k,t} \in \mathcal{X} \subseteq \mathbb{R}^N$ and $y_{k,t} \in \mathcal{Y} \subseteq \mathbb{R}$ denote the feature and the label, respectively. Using them, the client $k$ optimizes the local information (denoted as $m_{k,t} \in \mathbb{R}^D$ for $k \in [K]$) and sends it back to the central server. From $\{m_{k,t} : k \in [K]\}$, the central server constructs an updated global model such as
\[
w_{t+1} = \Psi(m_{1,t}, \ldots, m_{K,t}),
\]
and then broadcasts it to the $K$ clients. The mapping $\Psi(\cdot)$ should be carefully designed by taking the structure of a learned function into account. As aforementioned, one representative mapping is to average the aggregated local models, called FedAvg [5].

**Stochastic modeling of system heterogeneity:** As pointed out in the existing works on federated learning [5], [12], [14], the following key challenges should be also taken into account in our OFL framework: i) data heterogeneity; ii) system heterogeneity (time-varying partial client participation). Clients frequently generate and collect data in a non-identically distributed way across the network, thereby resulting in heterogeneous data (or non-IID data) [18]. For example, mobile phone users have varied use of language in the context of a next word-prediction task. In addition, the computation capabilities and network connectivity (e.g., 4G, 5G, and WiFi) of clients in the network may differ due to the variability in hardware, (wireless) communication channel conditions, and power [18]. There is a significant variability in terms of the system characteristics on each client in the network. In particular, the clients with limited computation capabilities or poor channel conditions, known as stragglers, can result in long delays. This is referred to as system heterogeneity. In an attempt to handle such heterogeneity, partial client participation is a popular approach for federated learning [5], [18]. Specifically, the clients (or stragglers) that fail to perform local update within a specified time window or to send local information via a communication channel are simply dropped, which results in that a small fraction of clients are participated in federated learning at once. Throughout the paper, the impact of system heterogeneity is probabilistically modeled as follows. Each client $k \in [K]$ in the network can be activated with probability $0 < p_k < 1$, independently at every time and from other clients, where $p_k$ indicates the activation (or participation) probability. The clients with higher computation capabilities tend to have higher participation probabilities. The hyperparameters $\{p_k : k \in [K]\}$ are fixed a priori, based on the processing and communication capabilities of clients. For practicality, it is assumed that $p_k$ is known only at the associated client $k$, and unknown at the other clients and the central server. Throughout the paper, $S_t \subseteq [K]$ represents the index subset of active clients at time $t$. Also, in convention, full client participation implies that $p_k = 1$ for $\forall k \in [K]$ (i.e., $S_t = [K]$ for $\forall t$). Clearly, when $p_k < 1$ for some $k$, $S_t$ is a random variable due to our stochastic modeling.

**Performance metric:** Algorithms (or methods) for OFL framework are evaluated in terms of learning accuracy and communication overhead:

- **Learning accuracy:** Following online learning frameworks [26]–[29], the learning accuracy of an algorithm with the outputs $\{w_t\}$ is measured by the cumulative regret over $T$ time slots:
\[
\text{regret}_T = \sum_{t=1}^{T} \sum_{k=1}^{K} \mathcal{L}(f(x_{k,t}; w_t), y_{k,t}) - \min_{w \in \mathbb{R}^D} \sum_{t=1}^{T} \sum_{k=1}^{K} \mathcal{L}(f(x_{k,t}; w), y_{k,t}),
\]
where $\mathcal{L}(\cdot, \cdot)$ is a loss (or cost) function that measures the error between true labels and those predicted by $f(\cdot; w_t)$’s. The upper bound on regret is referred to as regret bound. This metric compares the cumulative loss of the algorithm with that of the static optimal function in hindsight.

- **Communication overhead (per time):** The communication cost of an algorithm is measured by the number of bits of local information conveyed from $K$ clients to the central server (in short, uplink communication overhead). For example, if every client sends a $D$-dimensional real-valued vector to the central server, the corresponding communication overhead is equal to $32KD$ bits. Here, 32-bit quantization is typically assumed to represent a real-value precisely. The communication overhead from the central server to the $K$ clients (i.e., downlink communication overhead) is not of concern as it is relatively very small.

**B. Vanilla Method**

We construct a vanilla method for OFL framework by incorporating online gradient descent (OGD) [28] into FedAvg (the de facto algorithm in federated learning) [4]. This method is referred to as OFedAvg. To learn a sequence of global models $w_1, w_2, \ldots$ in an online fashion, at every time $t$, OFedAvg performs with the following two steps (see Algorithm 1):

i) **Local update:** At time $t$, each client $k$ is aware of a current global model $w_t$ (which is conveyed from the central server at time $t - 1$) and receives an incoming data $(x_{k,t}, y_{k,t})$. Using them, it optimizes the local model via OGD [28]:
\[
w_{k,t+1} = w_t - \eta \nabla \mathcal{L}(f(x_{k,t}; w_t), y_{k,t}),
\]
with a step size (or learning rate) $\eta > 0$ and the initial value $w_1 = 0$, where $\nabla \mathcal{L}(f(x_{k,t}; w_t), y_{k,t})$ is the gradient at the point $w_t$. Then, every client $k$ sends the updated local model $m_{k,t} = w_{k,t+1} \in \mathbb{R}^D$ back to the central server.

ii) **Global update:** From the aggregated local models $\{w_{k,t+1} : k \in [K]\}$, the central server constructs a global model via FedAvg [5] as follows:
\[
w_{t+1} = \frac{1}{K} \sum_{k=1}^{K} m_{k,t+1}
= w_t - \eta \frac{1}{K} \sum_{k=1}^{K} \nabla \mathcal{L}(f(x_{k,t}; w_t), y_{k,t}).
\]

Then, it distributes the updated global model $w_{t+1} \in \mathbb{R}^D$ to the $K$ clients. In OFedAvg, the communication overhead is equal to $32DK$ bits.

**Algorithm 1 Vanilla Method (OFedAvg)**

1: **Input:** $K$ clients and learning rate $\eta$.
2: **Output:** A sequence of function (or global) parameters \{ $w_{t+1} : t \in [T]$ \}.
3: **Initialization:** $w_1 \leftarrow 0$ and $w_{k,1} \leftarrow 0$ for $\forall k \in [K]$.
4: **Iteration:** $t = 1, 2, \ldots, T$.
   - At the client $k \in [K]$:
     - Receive streaming data $(x_{k,t}, y_{k,t})$.
     - Update local parameter $w_{k,t+1}$ via $\Delta$.
     - Transmit $m_{k,t} = w_{k,t+1}$ to the central server.
   - At the central server:
     - Receive $m_{k,t}$ from the $K$ clients $k \in [K]$.
     - Update the global parameter $w_{t+1}$ via $\mathcal{A}$.
     - Broadcast $w_{t+1}$ to the $K$ clients.

**III. THE PROPOSED OFEDQIT**

We present a communication-efficient OFL algorithm by means of a stochastic quantization and an intermittent transmission. The proposed method is referred to as OFEDQIT. We obtain some intuitions to devise OFEDQIT from our theoretical analysis in Section IV. The superiority of our algorithm will be verified analytically and experimentally in Sections IV and V, respectively. In particular, OFEDQIT almost achieves the learning accuracy of the vanilla method in Section II-B, while reducing the communication overhead significantly and providing the robustness to system heterogeneity. Our algorithm consists of the following two steps (see Algorithm 2):

**i) Local update:** At time $t \in [T]$, each client $k \in [K]$ optimizes the local model using the incoming data $(x_{k,t}, y_{k,t})$ and the up-to-date global model. First, the parameter of a local function is updated via either globally or locally:

$$g_{k,t} = \begin{cases} w_{t}, & t - 1 \in [T] \setminus \emptyset \\ \theta_{k,t}, & \text{else}, \end{cases}$$  \hspace{1cm} (5)

Using the incoming data and via OGD, the local model is optimized as

$$\theta_{k,t+1} = g_{k,t} - \eta \nabla k_{t},$$  \hspace{1cm} (6)

where $\eta$ is a learning rate and

$$\nabla k_{t} = \Delta \nabla L(f(x_{k,t}; g_{k,t}), y_{k,t}).$$  \hspace{1cm} (7)

To reduce the communication overhead, our algorithm takes a stochastic quantization and intermittent transmission. The intermittent transmission is also illustrated in Fig. 1. Specifically, at every $L$ time slots (i.e., when $t \in [T] \setminus \emptyset$), each client $k$ transmits the quantized and weighted local information $m_{k,t}$ to the central server, where

$$m_{k,t} = \Delta \sum_{t=1}^{L} \frac{\nabla k_{t-L+1}}{p_k} = Q_{s,b}(\bar{m}_{k,t}),$$  \hspace{1cm} (8)

where $\bar{m}_{k,t} \Delta \sum_{t=1}^{L} \frac{\nabla k_{t-L+1}}{p_k}$ denotes the unquantized version of $m_{k,t}$ and a $(s,b)$-stochastic quantizer $Q_{s,b}()$ is described in Definition 1. We remark that although the above quantizer in this paper is assumed for the tractability of our theoretical analysis, the other types of quantizers (i.e., $1$-bit quantizer) can be naturally incorporated. As computed in Definition 1, in order to send the local update $m_{k,t}$ to the central server, $(32b + D + 1 + \log(s + 1))$ bits are required.

**ii) Intermittent global update:** This step is conducted periodically only when $t \in [T] \setminus \emptyset$ (see Fig. 1). Using the received information \{ $m_{k,t} : k \in S_{t}$ \}, at each time $t \in [T] \setminus \emptyset$, the central server constructs the global parameter $w_{t}$ by simply averaging them:

$$w_{t+1} = w_{t-L+1} - \frac{\eta}{K} \sum_{k \in S_{t}} m_{k,t} \in [\bar{m}_{k,t}],$$  \hspace{1cm} (9)

where $\mathbb{I}_{\{k \in S_{t}\}}$ stands for an indicator function. Then, the central server broadcasts the updated global model $w_{t+1}$ to all the clients.

**Remark 1:** The proposed OFEDQIT in Algorithm 2 generates a sequence of global model intermittently, i.e., $w_{L+1} \rightarrow w_{2L+1} \rightarrow w_{3L+1} \rightarrow \cdots$. Specifically, during the $L$ time slots $2L + 1 \leq t \leq 3L$, the identical global model $w_{2L+1}$ (which is updated at time $2L$) is used. In general, for any $t \in [T]$, the global model corresponds to $w_{\psi_{L}(t)}$.

**Definition 1:** $(s, b)$-stochastic quantizer For any positive integer $b$, let $I_{1}, I_{2}, \ldots, I_{b}$ be the partition of the indices of the vector $u \in \mathbb{R}^{D}$ (i.e., the partition of $|D|$) such that

$$I_{s} = \left\{ \sum_{t=1}^{\ell-1} |I_{t}| + 1, \ldots, \sum_{t=1}^{\ell} |I_{t}| \right\}.$$  \hspace{1cm} (10)

Note that in our experiments, the uniform-size partition is considered, i.e., $|I_{t}| \approx \frac{|D|}{b}$ for all $t \in [b]$. Given a vector $u \in \mathbb{R}^{D}$, let $u_{I_{s}}$ be the subvector of $u$, which is formed by taking the entries of $u$ whose indices belong to $I_{s}$. For any $u \in \mathbb{R}^{D}$, a stochastic quantizer $Q_{s,b} : \mathbb{R}^{D} \rightarrow \mathbb{R}^{D}$ is defined as

$$Q_{s,b}(u)_{i} = \|u_{I_{s}}\| \cdot \text{sign}(u_{i}) \cdot \xi_{i}(u),$$  \hspace{1cm} (11)

for $i \in I_{s}$, where $u_{i}$ is the $i$-th component of $u$ and $\xi_{i}(u)$ is a random variable with

$$\xi_{i}(u) = \begin{cases} q/s, & \text{with prob. } 1 - \left( \frac{|u_{i}|}{\|u_{I_{s}}\|} - q \right) \\ (q + 1)/s, & \text{otherwise}, \end{cases}$$

for $q \in [0, s)$ is an integer such that $|u_{i}|/\|u_{I_{s}}\| \in [q/s, (q+1)/s)$ for all $i \in I_{s}$. The parameters $s$ and $b$ denote the quantization-level for each entry of $u$ and the number of partitions to indicate magnitudes, respectively. Then, the quantized information $Q_{s,b}(u)$ is represented by $32b + D + 1 + \log(s + 1)$ bits. Particularly when $b = 1$, this quantizer is reduced to the $s$-level stochastic quantizer in [12]. Closely following the
Central server

![Diagram showing the process of communication between the central server and client k.](Image)

**Algorithm 2** The Proposed OFedQIT

1. **Input:** $K$ clients, quantization parameter $(s, b)$, transmission period $L$, activation ratio $p_k$ and learning rate $\eta$.
2. **Output:** A sequence of function (or global) parameters \( \{w_{t+1} : t \in [T_L]\} \) (equivalently, \( \{w_{\theta_{k,t}} : t \in [T]\}\)).
3. **Initialization:** \( w_1 \leftarrow 0 \) and \( \theta_{k,1} \leftarrow 0 \) for \( \forall k \in [K] \).
4. **Iteration:** \( t = 1, 2, \ldots, T \).
   - At the client $k \in [K]$
     - Receive streaming data \((x_{k,t}, y_{k,t})\).
     - Update local parameter \( \theta_{k,t+1} \) via (6).
   - At the central server only when \( t \in [T_L] \):
     - Transmit \( m_{k,t} \) from some part of clients \( k \in S_t \).
     - Update the global parameter \( w_{t+1} \) via (9).
     - Broadcast \( w_{t+1} \) to the $K$ clients.

For our analysis, we make the following assumptions, which are usually assumed for the analysis of online convex optimization and online learning [27]–[30]:

**Assumption 1.** For any fixed \((x_{k,t}, y_{k,t})\), the loss function \( \mathcal{L}(f(x_{k,t}; w), y_{k,t}) \) is convex with respect to $w$ and differentiable. The loss function is $G$-Lipschitz continuous, i.e., \( \|\nabla \mathcal{L}(f(x_{k,t}; w), y_{k,t})\| \leq G \).

**Assumption 2.** The optimal parameter $w_*$ is assumed to be bounded, i.e., \( \|w_*\| \leq W \).

**Assumption 3.** For any fixed data $x_{k,t}$ and $y_{k,t}$, the resulting loss function is $\beta$-smooth, i.e., \( \|\nabla \mathcal{L}(f(x_{k,t}; w_1), y_{k,t}) - \nabla \mathcal{L}(f(x_{k,t}; w_2), y_{k,t})\| \leq \beta \|w_1 - w_2\| \).

Assumptions 1-2 are generally required for the analysis of online learning and online convex optimization [27]–[30]. Also, Assumption 3 is used for the analysis of intermittent transmission. Recall that \( \{w_{t+1} : t \in [T_L]\} \) is the sequence of outputs of our Algorithm 1 and accordingly, from the notation...
in [1], \( w_{\psi_L(t)} \) denotes the common global model for any \( t \in [T] \). For ease of exposition, let \( \mathbb{E}[\|w_{\psi_{[(t-1)/L]-1},L+1}\|^{2}] \) denote the conditional expectation at time \( t \) given the latest global parameter. Also, we let

\[
P \triangleq \max\{1/p^2_1, \ldots, 1/p^2_K\},
\]

which can determine the impact of the slowest client (e.g., straggler).

**Theorem 1:** Under Assumption 1 - Assumption 2, when full client participation is considered (i.e., \( p_k = 1, \forall k \in [K] \)), the vanilla method (called OFedAvg) in Algorithm 1 with \( \eta = \sqrt{W/GT} \) achieves the following regret bound:

\[
\text{regret}_T = \sum_{t=1}^{T} \sum_{k=1}^{K} \mathcal{L}(f(x_{k,t}; w_t), y_{k,t})
- \sum_{t=1}^{T} \sum_{k=1}^{K} \mathcal{L}(f(x_{k,t}; w), y_{k,t})
\leq K \sqrt{W GT}.
\]

Namely, a sublinear regret bound \( O(\sqrt{T}) \) is achieved, where all the constants are removed by \( O \).

**Proof:** The proof is completed exactly following the proof of [20] Theorem 1.

We would like to emphasize that the above analysis is only valid when full client participation is considered. For more practical case of partial client participation, a more rigorous analysis should be conducted as in our main result below.

**Theorem 2:** Under Assumption 1 - Assumption 3, OFedQIT in Algorithm 2 achieves the following regret bound:

\[
\text{regret}_T = \sum_{t=1}^{T} \sum_{k=1}^{K} \mathbb{E}_t \left[ \mathcal{L}(f(x_{k,t}; w_{\psi_{L}(t)}), y_{k,t}) \right]
- \sum_{t=1}^{T} \sum_{k=1}^{K} \mathcal{L}(f(x_{k,t}; w), y_{k,t})
\leq W K \sqrt{\frac{2}{2\eta} + \frac{\eta GT ((M + K)P + 3\eta K L \beta)}{2}},
\]

where \( M \) is the upper bound on the variance of \((s,b)\)-stochastic quantizer, given as

\[
M \triangleq \min \left( \frac{|D/b|}{s^2}, \sqrt{\frac{|D/b|}{s}} \right).
\]

Setting \( \eta = O(1/\sqrt{T}) \), an optimal sublinear regret bound \( O(\sqrt{T}) \) is achieved.

**Proof:** The proof is provided in Section IV-A.

More specifically, with \( \eta = \sqrt{W K/GT(M + K)P} \) (i.e., \( O(1/\sqrt{T}) \)), the upper bound in Theorem 1 can be represented as

\[
\text{regret}_T \leq \sqrt{GLWK (M + K) PT}.
\]

Also, since \( D \gg bs^2 \) in many real-world applications, \( M \) is usually equal to \( \sqrt{|D/b|/s} \). Accordingly, the upper bound in (13) is given as

\[
\text{regret}_T \leq \sqrt{GLWK (\sqrt{D/bs^2} + K) PT}.
\]

Based on this, we can obtain some interesting results on the regret bounds of various algorithms, which are stated in the following corollaries.

**Corollary 1:** Under Assumption 1 - Assumption 3, OFedQIT with \( s = \infty \) (i.e., no quantization) and \( \eta = \sqrt{W/GT P} \), which is also called **FedQGD**, achieves the following regret bound:

\[
\text{regret}_T \leq K \sqrt{GLWT}.
\]

**Corollary 2:** Under Assumption 1 - Assumption 3, OFedQIT with \( L = 1 \) (i.e., contiguous transmission) and \( \eta = \sqrt{W K/GT(M + K)P} \), which is also called **FedQGD**, achieves the following regret bound:

\[
\text{regret}_T \leq \sqrt{GWK (\sqrt{D/bs^2} + K) PT}.
\]

The above regret bounds are also summarized in Table I.

**Remark 3:** We provide some discussions on our theoretical analysis with respect to data heterogeneity, especially when a very small fraction of clients in a network are active at each time (in short, lower client-participation regime). We emphasize that in this challenging regime, the proposed OFedQIT can achieve an optimal asymptotic performance for any dataset (i.e., irrespective of the degree of heterogeneity of incoming data to the decentralized clients). As discussed in conventional federated learning [13], [14], in non-asymptotic cases (i.e., \( T \) is finite) and lower client-participation regime, the performances of OFedQIT as well as OFedAvg might be worse as the degree of data heterogeneity becomes higher. One might expect that in this regime, our algorithm would be enhanced by incorporating the existing techniques [14]–[16], [18] to correct for data heterogeneity in conventional federated learning. Yet, to validate this argument, the regret analysis of the resulting algorithms in OFL should be conducted, which is not straightforward and should be further investigated. Thus, it is left for an interesting future work.
A. Proof of Theorem 2

We provide the proof of our main theorem. To concise the expressions and explain the key ideas of our proof clearly, it is assumed that $T$ is a multiple of $L$. In a general case, we need to take some exceptions for the few time indices $[T/L] \cdot L + 1, [T/L] \cdot L + 2, \ldots, T$. Clearly, the number of these extra-terms is less than $L$. Thus, when $T$ is sufficiently large, the impact of exceptional terms can be negligible. Supporting lemmas (Lemmas 1-3) for this proof are provided in the last part of this section.

We first focus on time $t \in [T]_L$ where recall that $[T]_L \overset{\triangle}{=} \{ t \in [T] : t \mod L = 0 \}$. At this time, every client is aware of the up-to-date global parameter $w_{t-L+1}$ and aims at learning an updated global parameter $w_{t+1}$. Leveraging the global update in (9) and taking the conditional expectations in both sides, we obtain the following key equality:

$$
\mathbb{E}_{t+1} \left[ \| w_{t+1} - w_* \|^2 \right]
= \| w_{t-L+1} - w_* \|^2 + \mathbb{E}_{t+1} \left[ \left\| \frac{\eta}{K} \sum_{k \in S_t} \nabla_{k,t}^L \right\|^2 \right]
- \frac{2\eta}{K} \mathbb{E}_{t+1} \left[ \sum_{k \in S_t} \left( \nabla_{k,t}^L \right)^T (w_{t-L+1} - w_*) \right] .
$$

(18)

where for ease of exposition, we let

$$
\nabla_{k,t}^L \overset{\triangle}{=} \sum_{\ell=1}^L \frac{\nabla_{k,t-L+\ell}}{p_k} \text{ and } \hat{\nabla}_{k,t}^L \overset{\triangle}{=} Q_{s,b} \left( \nabla_{k,t}^L \right) .
$$

(19)

Note that the randomness in the above is caused by a stochastic quantization and the random selection of active clients. From Lemma 1, we obtain the following inequality:

$$
\langle \nabla_{k,t-L+\ell}, w_{t-L+1} - w_* \rangle
\geq \mathcal{L} \left( f(x_{k,t-L+\ell}; w_{t-L+1}), y_{k,t-L+\ell} \right)
- \mathcal{L} \left( f(x_{k,t-L+\ell}; w_*), y_{k,t-L+\ell} \right)
- \frac{\beta}{2} \| w_{t-L+1} - g_{k,t-L+\ell} \|^2 .
$$

(20)

Harnesing the probabilistic modeling of our system heterogeneity, we can get

$$
\mathbb{E}_{t+1} \left[ \sum_{k \in S_t} \left( \nabla_{k,t}^L \right)^T (w_{t-L+1} - w_*) \right]
= \sum_{k=1}^K \sum_{\ell=1}^L \left( \nabla_{k,t-L+\ell} \right)^T (w_{t-L+1} - w_*) .
$$

due to the following equality:

$$
\mathbb{E} \left[ \sum_{k \in S_t} \nabla_{k,t}^L \right] = \mathbb{E} \left[ \sum_{k \in S_t} \nabla_{k,t}^L | S_t \right]
= \sum_{k=1}^K \sum_{\ell=1}^L \mathbb{P}(k \in S_t) \nabla_{k,t-L+\ell} \frac{1}{p_k}
= \sum_{k=1}^K \sum_{\ell=1}^L \nabla_{k,t-L+\ell} .
$$

Also, we can derive the following inequality:

$$
\mathbb{E}_{t+1} \left[ \left\| \frac{\eta}{K} \sum_{k \in S_t} \nabla_{k,t}^L \right\|^2 \right]
= \frac{\eta^2}{K^2} \mathbb{E} \left[ \left\| \sum_{k \in S_t} \nabla_{k,t}^L - \nabla_{k,t}^L + \nabla_{k,t}^L \right\|^2 \right]
= \frac{\eta^2}{K^2} \mathbb{E} \left[ \left\| \sum_{k \in S_t} \left( \nabla_{k,t}^L - \nabla_{k,t}^L \right) \right\|^2 \right]
+ \frac{\eta^2}{K^2} \mathbb{E} \left[ \left\| \sum_{k \in S_t} \nabla_{k,t}^L \right\|^2 \right]
\leq \frac{\eta^2}{K^2} \mathbb{E} \left[ \left\| \sum_{k=1}^K \left( \nabla_{k,t-L+\ell} \right) \right\|^2 \right]
+ \frac{K}{p_k} \sum_{k=1}^L \nabla_{k,t-L+\ell} \right\|^2 .
$$

(21)

where $(v)_+$ denotes the vector whose $i$-th component is equal to $|v_i|$ (i.e., the positive value of the $i$-th component of $v$), (a) is satisfied as $Q_{s,b}$ is an unbiased quantizer (i.e., $\mathbb{E} [Q_{s,b}(u) - u] = 0$), (b) is because by construction of a stochastic quantization, $(\nabla_{k,t} - \nabla_{k,t})$ and $(\hat{\nabla}_{k,t} - \nabla_{k,t})$ for $k \neq l$ are independent random variables, i.e., $\mathbb{E}[(\nabla_{k,t} - \nabla_{k,t})(\hat{\nabla}_{l,t} - \nabla_{l,t})] = 0$ for $k \neq l$, (c) is from (19) and due to the bounded variance of the stochastic quantizer in Definition 1, (d) is from the Cauchy-Schwarz inequality and Assumption 1 and (e) is from the definition of $P$ in (12). Combining (18),
we used the fact that
\[
\mathbb{E} \left[ \|w_{t+1} - w_*\|^2 | w_{t-2L+1}, \ldots, w_{t-L+1} \right] = \mathbb{E} \left[ \|w_{t+1} - w_*\|^2 | w_{t-L+1} \right] = \mathbb{E}_{t+1} \left[ \|w_{t+1} - w_*\|^2 \right],
\]
namely, \(w_{t+1}\) is conditionally independent from \(w_{t-2L+1}\) given \(w_{t-L+1}\). Note that (24) implies the upper bound on the cumulative loss of the \(L\) consecutive time slots (i.e., time slots \(t - L + 1, t - L + 2, \ldots, t\) for some \(t \in [T]_L\)). Summing (24) over all \(t \in [T]_L\) and by telescoping sum, we have:

\[
\sum_{t=1}^{T} \sum_{k=1}^{K} \mathbb{E}_t \left[ \mathcal{L} \left( f(x_{k,t}; w_{\psi_t(t)}), y_{k,t} \right) \right] - \sum_{t=1}^{T} \sum_{k=1}^{K} \mathcal{L} \left( f(x_{k,t}; w_*), y_{k,t} \right) \\
\leq \frac{K}{2\eta} \left( \|w_{t-1} - w_*\|^2 - \mathbb{E}_{t+1} \left[ \|w_{t+1} - w_*\|^2 \right] \right) + \frac{\eta GL^2 \left( (M + K)P + 3\eta KL \beta \right)}{2}.
\]

(25)

where (a) is from Assumption 3 and \(\|w_{T+1} - w_*\|^2 \geq 0\). This completes the proof of Theorem 1.

**Lemma 1:** For any \(\beta\)-smooth convex function \(h\) and any \(x, y, z\) in the domain of \(h\), the following inequality holds:

\[
\langle \nabla h(x), z - y \rangle \geq h(z) - h(y) - \beta \|z - x\|^2.
\]

**Proof:** From the smoothness and convexity of \(h\), we obtain the following two inequalities:

\[
\langle \nabla h(x), z - x \rangle \geq h(z) - h(x) - \frac{\beta}{2} \|z - x\|^2
\]

\[
\langle \nabla h(x), x - y \rangle \geq h(x) - h(y).
\]

Combining them, we can complete the proof. ■

**Lemma 2:** Letting \(v_i, v_j \in \mathbb{R}^d\), the following inequality holds: For any \(\alpha > 0\),

\[
\|v_i + v_j\|^2 \leq (1 + \alpha)\|v_i\|^2 + \left(1 + \frac{1}{\alpha}\right)\|v_j\|^2.
\]

**Proof:** The proof directly follows the equality:

\[
\|v_i + v_j\|^2 = (1 + \alpha)\|v_i\|^2 + \left(1 + \frac{1}{\alpha}\right)\|v_j\|^2
\]

\[
- \left(\sqrt{\alpha}v_i - \frac{1}{\sqrt{\alpha}}v_j\right)^2.
\]

■

**Lemma 3:** For \(\ell \geq 2\), the following inequality holds:

\[
\mathbb{A}(\mathbb{F}) - \mathbb{G}(\mathbb{F})^2 + B \|\nabla k, t - \ell - 2\|^2
\]

\[
\leq \sum_{\tau=0}^{\ell-2} B\|\nabla k, t - \ell - 2 - \tau\|^2 A^\tau,
\]

(26)

where \(A = (1 + 1/(L - 1))\) and \(B = L\eta^2\).

**Proof:** The proof will be done by induction. When \(\ell = 2\), the inequality in (26) is trivial because \(w_t = \mathbb{G}_t\) and accordingly we have:

\[
\mathbb{A}(\mathbb{F}) - \mathbb{G}(\mathbb{F})^2 + B \|\nabla k, t\|^2 = B \|\nabla k, t\|^2.
\]

(27)

Suppose that the inequality in (26) holds for \(2 \leq m < \ell\), i.e.,

\[
\mathbb{A}(\mathbb{F}) - \mathbb{G}(\mathbb{F})^2 + B \|\nabla k, t + m - 2\|^2
\]

\[
\leq \sum_{\tau=0}^{m-2} B\|\nabla k, t + m - 2 - \tau\|^2 A^\tau.
\]

(28)
Focusing on $m + 1$, then, we have:

$$
A\|\mathbf{w}_t - \mathbf{g}_k,t+m-1\|^2 + B\|\nabla k,t+m-1\|^2
\leq A\left(A\|\mathbf{w}_t - \mathbf{g}_k,t+m-2\|^2 + \|\nabla k,t+m-2\|^2\right)
+ B\|\nabla k,t+m-1\|^2

(b)

$$
= \sum_{\tau=0}^{m-2} B\|\nabla k,t+m-\tau\|^2 A^\tau
+ B\|\nabla k,t+m-1\|^2

(\tau)

$$
= \sum_{\tau=0}^{(m+1)-2} B\|\nabla k,t+(\tau+1)\|^2 A^\tau

$$

where (a) is from Lemma 2 with $a = \frac{1}{\tau+1}$ and (b) is due to the hypothesis assumption in 28. This completes the proof.

V. EXPERIMENTS

In this section, we demonstrate the effectiveness of the proposed OFedQIT in Algorithm 2 via experiments with real datasets on online classification and regression tasks. To simplify the notation, OFedQIT with $(s, b)$-stochastic quantization and $L$ intermittent transmission is denoted by OFedQIT$(s, b, L)$. As the baseline method, we adopt the vanilla algorithm (named OFedAvg) in Algorithm 1. We believe that it is reasonable benchmark method as OFedAvg is constructed by integrating the best-known online optimization method (dubbed OGD) 28 and commonly used aggregation method (dubbed FedAvg) 3. In addition, to the best of our knowledge, no other algorithm for OFL framework can be found. In our experiments, we consider a communication network consisting of $K = 100$ decentralized clients, where a system heterogeneity (or partial client participation) is captured by the activation probability $\{p_k : k \in [K]\}$ introduced in Section II. For simplicity, it is assumed in our experiments that $p_k = p$ for all $k \in [K]$. Thus, at every time, $pK$ clients are participated in average and the set of participated clients are changed independently at every time. A single parameter $0 < p < 1$ is specifically defined in Fig. 2 and Fig. 3, where $p = 1$ (i.e., full client participation) and $p = 0.1$ (i.e., 90% dropout on average) are respectively used to reflect the two extreme cases. Obviously, when other parameters belonging to $[0, 1]$ are considered, the performances lie between the corresponding solid and dashed lines. Regarding a hyperparameter $\eta$, it is proved in Section IV that $\eta = O(1/\sqrt{T})$ is asymptotically optimal for both OFedAvg and OFedQIT.

As underlying models, we consider the following parameterized functions:

- **Model I (CNN):** This convolutional neural network (CNN) model consists of two convolutional layers (each with 32 nodes and 64 nodes), one non-linear softmax layer with 10 nodes, and 3 bias nodes. Each convolutional layer is composed of multiple sequential operations including convolution, ReLU activation, and max-pooling. This model contains the overall 109 nodes and accordingly the number of parameters are computed as $D = 10 \cdot 32 + 289 \cdot 64 + 1601 \cdot 10 = 34,826$.

A cross-entropy loss function is considered.

- **Model II (DNN):** This deep neural network (DNN) model consists of two non-linear ReLU layer with 32
nodes, 3 bias nodes, and one non-linear softmax layer with \( c \) nodes, where \( c \) is the number of output labels and is determined according to the number of classes of the associated dataset. This model contains the overall 71 nodes and accordingly the number of parameters are computed as

\[
D = 2 \cdot 32 + 33 \cdot 32 + 33 \cdot c = 1, 120 + 33c.
\]

In our experiments, \( c = 3 \) and \( c = 4 \) are chosen for Room Occupancy Estimation and Cardiotocography datasets, respectively. A cross-entropy loss function is considered.

- **Model III (DNN)**: This DNN model consists of two non-linear ReLU layer with 32 nodes, one non-linear softmax layer with 1 node, and 3 bias nodes. There are overall 68 nodes and accordingly the number of the parameters are computed as

\[
D = 2 \cdot 32 + 32 \cdot 32 + 32 \cdot 1 = 1, 153.
\]

A least-square loss function is considered.

Among the above three models, a proper underlying model is adopted according to online learning tasks and datasets, which is clearly specified in Table III and Table IV. Finally, the real-world datasets for our experiments are described in Section V.A and Section V.B on online classification and regression tasks, respectively. They are also summarized in Table II. Due to the limited number of data in each dataset, the entire data samples can be reused. However, to avoid the identical data assignments to the \( K \) nodes, the reused data are randomly permuted. Given a dataset (possibly having the reused ones), in order build a decentralized learning framework, data samples are distributed to the \( K \) nodes in the following way. The whole data samples in each dataset are partitioned into the \( K \) subsets of the equal size \( T = \lceil \text{the total number of data} / K \rceil \), in which some remaining data samples can be excluded. The resulting partition is denoted as \( \{D_1, ..., D_K\} \), where \( |D_k| = T, \forall k \in [K] \) and \( D_k \cap D_{k'} = \emptyset \) for any \( k, k' \in [K] \) with \( k \neq k' \). Then, each client \( k \in [K] \) employs \( D_k = \{(x_{k,t}, y_{k,t}) : t = 1, ..., T\} \) as its local dataset. At time \( t \), each client \( k \) receives the data \( (x_{k,t}, y_{k,t}) \) from \( D_k \) in a sequential fashion. Via experiments, we verified that although there are numerous possible partitions of the equal size \( T \), they have little impact on the performance gaps between OFedAvg and OFedQIT. For the accuracy and MSE performances respectively in Fig.2 and Fig. 3, the partition \( \{D_1, ..., D_K\} \) is constructed as follows:

Letting \( \{(x_t, y_t) : t \in [KT]\} \) be the time-series data in a dataset, it is partitioned as

\[
D_k = \{(x_{K(t-1)+k}, y_{K(t-1)+k}) : t \in [T]\},
\]

for \( k = 1, 2, ..., K \). This partition can maintain the characteristic of time-series data.

**Performance evaluation**: We evaluate the performances of the proposed OFedQIT and the vanilla method (OFedAvg) by conducting the experiments on various online classification and regression tasks with real-world datasets in Section V.A and Section V.B. In the proposed OFedQIT(\( s, b, L \)), the so-called hyperparameters \( (s, b) \) for a stochastic quantization and \( L \) for an intermittent transmission should be optimized by taking into account the tradeoff between the accuracy (or MSE) performance and communication overhead. Also, the optimal parameters can be changed according to datasets (e.g., the degrees of data heterogeneity). Unfortunately, as similarly in conventional federated learning [11], [12], there is no systematic way to optimize such hyperparameters. Instead we resort to finding proper hyperparameters manually throughout experiments. We first determine the \( s = 1 \) (i.e., 1-level quantization per dimension) as this extreme quantization is sufficient to yield attractive performances in all our datasets while minimizing the communication overhead. Next for each dataset, the hyperparameters \( b \) and \( L \) are respectively optimized from the following predetermined candidates such as

\[
b \in \{1, 10, 10^2, 10^3\} \quad \text{and} \quad L \in \{1, 2, \cdots, 10\}.
\]

We provide the optimized parameters to each dataset in Table III and Table IV on online classification and regression tasks, respectively. In order to reflect the impact of system heterogeneity, we consider the two extreme environments such as \( p = 1 \) (i.e., full client participation) and \( p = 0.1 \) (i.e., 90% dropout on average). As expected, for the other parameters between 0.1 and 1, the accuracy and MSE performances of OFedQIT (or OFedAvg) lie between the corresponding solid and dashed lines in Fig. 2 and Fig. 3. We observe that as time index \( t \) grows, OFedQIT can approach the performances of OFedAvg in all our datasets, regardless of the degrees of system heterogeneity. These results are well-matched with our theoretical analysis in Section IV. Specifically in MNIST dataset, where Model I is adopted as an underlying learning model, the communication overheads of OFedAvg and OFedQIT are computed as \( 1.11 \times 10^8 \) bits and \( 5.08 \times 10^9 \) bits, respectively. Also, from [31], the communication overhead

| Datasets/ (Learning model) | Algorithms | Accuracy(T) | COR |
|---------------------------|------------|-------------|-----|
| Room Occupancy (Model II) | OFedAvg    | 0.962       | 92.8% |
| MNIST (Model I)           | OFedAvg    | 0.962       | 95.4% |
| Cardiotocography (Model II)| OFedAvg    | 0.927       | 97.6% |
|                          | OFedQIT(1, 10, 3) | 0.920 |

| Datasets/ (Learning model) | Algorithms | MSE(T) | COR |
|---------------------------|------------|--------|-----|
| Conductivity (Model III)  | OFedAvg    | 0.057  | 96.4% |
| Air quality (Model III)   | OFedAvg    | 0.0374 | 92.9% |
| Twitter (Model III)       | OFedAvg    | 0.02402| 99.3% |

**TABLE III**

**TABLE IV**

Comparisons of Accuracy and Communication Overhead on Online Classification Tasks When \( p = 0.1 \) (90% dropout)

Comparisons of MSE and Communication Overhead on Online Regression Tasks When \( p = 0.1 \) (90% dropout)
Fig. 2. Comparisons of accuracy performances of online classification tasks, where $K = 100$ and the degree of system heterogeneity ($p$) is set by either 1.0 (i.e., full client participation) or 0.1 (90% dropout).

Fig. 3. Comparisons of MSE performances of online regression tasks, where $K = 100$ and the degree of system heterogeneity ($p$) is set by either 1.0 (i.e., full client participation) or 0.1 (90% dropout).
reduction (COR) is computed as $\text{COR} = 95.4\%$. We can see that the proposed algorithm considerably reduces the communication overhead of OFedAvg while the accuracy gap between them is negligible as 0.008. The similar trends can be observed in the other datasets, which are respectively provided in Table III and Table IV on online classification and regression tasks. Our experiments demonstrate that OFedQIT with proper parameters $(s, b, L)$ yields the almost same performances as the vanilla method (OFedAvg) even with not-so-large number of incoming data (i.e., $T$ is finite) while reducing the communication overhead tremendously (e.g., higher than 90% reduction in all the datasets). We finally provide some discussions on the robustness of our algorithm to data heterogeneity. Since the real-world datasets are used in our experiments, we are not able to precisely measure the degrees of data heterogeneity to each dataset. Nevertheless, we can implicitly estimate them based on the optimized $L$ from the given candidates $\{1, 2, \ldots, 10\}$. Intuitively, as $L$ is larger, a larger number of local updates (without global update), may lead each client towards the optimal model of its local incoming data, as can be opposed to the global one. This effect gets worse as the degree of data heterogeneity becomes larger. As a consequence, choosing a smaller $L$ implies that the heterogeneity of the associated dataset is quite large. Among our datasets, it seems that Twitter data has the smallest heterogeneity, whereas Air quality has the largest one. From Fig. 2 and Fig. 3, we verify that the proposed OFedQIT with well-optimized parameters can provide the robustness to data heterogeneity. Therefore, our experiments demonstrate the effectiveness of OFedQIT in terms of attractive performances, lower communication overhead, and the robustness to data and system heterogeneity, which suggests the practicability of our algorithm in real-world OFL tasks.

A. Online Classification Tasks

For the experiments of online classifications, the following real datasets from UCI Machine Learning Repository are considered:

- **Room Occupancy Estimation** [33]: This dataset contains 10130 time series samples, in which features are extracted from non-intrusive environmental sensors such as temperature, light, sound, CO2 and PIR. The task is to estimate the precise number of occupants in a room, where labels have the four classes as $\{0, 1, 2, 3\}$.
- **MNIST** [34]: This dataset contains 60,000 samples with 10 handwritten single digits between 0 and 9. Each sample has 28x28 pixel grayscale input. The goal of this task is to classify a given image of a handwritten digit into one of 10 classes representing integer values from 0 to 9, inclusively.
- **Cardiotocography** [35]: This dataset contains 2126 time series fetal carditocograms (CTGs) samples. They are automatically processed and the respective diagnostic features are measured. The CTGs are also classified by three expert obstetricians and a consensus classification label is assigned to each of them. The objective is to classify a fetal state into one of 3 classes $(N, S, P)$.

B. Online Regression Tasks

For the experiments of online regressions, the following real datasets from UCI Machine Learning Repository are considered:

- **Conductivity** [36]: This dataset contains 11000 time-series samples of extracted from superconductors, of which each feature represents critical information to build superconductor such as density and mass of atoms. The goal is to estimate the critical temperature to create superconductor.
- **Air quality** [37]: This dataset contains 38563 time-series samples, of which features include hourly response from an array of 5 metal oxide chemical sensors embedded in a city of Italy. The goal is to estimate the concentration of polluting chemicals in the air.
- **Twitter** [38]: This dataset contains 98704 time-series buzz events from Twitter, each of which attribute is used to estimate the popularity of an issue. Higher value indicates more popularity.

VI. CONCLUSION

In this paper we proposed OFedQIT as a communication efficient online federated learning (OFL) algorithm. This method was devised by incorporating a stochastic quantization and an intermittent transmission into online gradient descent-based federated averaging. Our key contribution is to theoretically prove that OFedQIT achieves an optimal sublinear regret bound for any real-world data (including heterogeneous data) while having lower communication overhead. Furthermore, this asymptotic optimality was still guaranteed even when a small fraction of clients are active at once (e.g., 90% dropout), thereby manifesting the robustness to system heterogeneity. Via experiments, beyond the analytical contributions, we demonstrated the effectiveness of our algorithm on various online learning tasks with real-world datasets. The advantages of privacy-preserving, communication efficiency, robustness to data and system heterogeneity, and attractive performances suggest the practicability of our algorithm in practical OFL tasks. One interesting future work to extend our work into wireless OFL frameworks, by constructing wireless transmission/reception techniques suitable to the proposed OFedQIT. Another interesting future work is to enhance the regret bound of our algorithm (i.e., to elaborate $\alpha$ in Table I) via a sharper analysis.

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