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General self-similarity properties for Markov processes and exponential functionals of Lévy processes. (English) J. Theor. Probab. 35, No. 4, 2083-2144 (2022)

Summary: Positive self-similar Markov processes are positive Markov processes that satisfy the scaling property and it is known that they can be represented as the exponential of a time-changed Lévy process via Lamperti representation. In this work, we are interested in what happens if we consider Markov processes in dimension 1 or 2 that satisfy self-similarity properties of a more general form than a scaling property. We characterize them by proving a generalized Lamperti representation. Our results show that, in dimension 1, the classical Lamperti representation only needs to be slightly generalized. However, in dimension 2, our generalized Lamperti representation is much more different and involves the exponential functional of a bivariate Lévy process. We briefly discuss the complications that occur in higher dimensions. We present examples in dimensions 1, 2 and 3 that are built from growth-fragmentation, self-similar fragmentation and Continuous-state Branching processes in Random Environment. Some of our arguments apply in the context of a general state space and show that we can exhibit a topological group structure on the state space of a Markov process that satisfies general self-similarity properties, which allows to write a Lamperti-type representation for this process in terms of a Lévy process on the group.

MSC:
60G18 Self-similar stochastic processes
60G51 Processes with independent increments; Lévy processes
60J25 Continuous-time Markov processes on general state spaces

Keywords: self-similar Markovian processes; Lévy processes; Lamperti representation; Lévy processes on Lie groups; exponential functionals of Lévy processes

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