Cosmological constraints from H II starburst galaxy apparent magnitude and other cosmological measurements

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ABSTRACT
We use H II starburst galaxy apparent magnitude measurements to constrain cosmological parameters in six cosmological models. A joint analysis of H II galaxy, quasar angular size, baryon acoustic oscillations peak length scale, and Hubble parameter measurements result in relatively model-independent and restrictive estimates of the current values of the non-relativistic matter density parameter Ω_m0 and the Hubble constant H_0. These estimates favour a 2.0–3.4σ (depending on cosmological model) lower H_0 than what is measured from the local expansion rate. The combined data are consistent with dark energy being a cosmological constant and with flat spatial hypersurfaces, but do not strongly rule out mild dark energy dynamics or slightly non-flat spatial geometries.

Key words: cosmological parameters – dark energy – cosmology: observations.

1 INTRODUCTION
The accelerated expansion of the current Universe is now well-established observationally and is usually credited to a dark energy whose origins remain murky (see e.g. Ratra & Vogeley 2008; Martin 2012; Coley & Ellis 2020). The standard ΛCDM model of cosmology (Peebles 1984) describes a universe with flat spatial hypersurfaces predominantly filled with dark energy in the form of a cosmological constant Λ and cold dark matter (CDM) together comprising ∼95 per cent of the total energy budget. While spatially flat ΛCDM is mostly consistent with cosmological observations (see e.g. Alam et al. 2017; Farooq et al. 2017; Planck Collaboration VI 2020; Sclicni et al. 2018), there are indications of some (mild) discrepancies between standard ΛCDM model predictions and cosmological measurements. In addition, the quality and quantity of cosmological data continue to grow, making it possible to consider and constrain additional cosmological parameters beyond those that characterize the standard ΛCDM model.

Given the uncertainty surrounding the origin of the cosmological constant, many workers have investigated the possibility that the cosmological ‘constant’ is not really constant, but rather evolves in time, either by positing an equation-of-state parameter w ≠ −1 (thereby introducing a redshift dependence into the dark energy density) or by replacing the constant Λ in the Einstein–Hilbert action with a dynamical scalar field φ (Peebles & Ratra 1988; Ratra & Peebles 1988). Non-flat spatial geometry also introduces a time-dependent source term in the Friedmann equations. In this paper, we study the standard spatially flat ΛCDM model as well as dynamical dark energy and spatially non-flat extensions of this model.

One major goal of this paper is to use measurements of the redshift, apparent luminosity, and gas velocity dispersion of H II starburst galaxies to constrain cosmological parameters.1 An H II starburst galaxy (hereinafter ‘HIIG’) is one that contains a large H II region, an emission nebula sourced by the UV radiation from an O- or B-type star. There is a correlation between the measured luminosity (L) and the inferred velocity dispersion (σ) of the ionized gases within these HIIG, referred to as the L–σ relation (see Section 2) which has been shown to be a useful cosmological tracer (see Melnick, Terlevich & Terlevich 2000; Siegel et al. 2005; Plionis et al. 2011; Chávez et al. 2012, 2014, 2016; Terlevich et al. 2015; González-Morán et al. 2019, and references therein). This relation has been used to constrain the Hubble constant H_0 (Chávez et al. 2012; Fernández Arenas et al. 2018), and it can also be used to put constraints on the dark energy equation-of-state parameter w (Terlevich et al. 2015; Chávez et al. 2016; González-Morán et al. 2019).

HIIG data reach to redshift z ~ 2.4, a little beyond that of the highest redshift baryon acoustic oscillation (BAO) data, which reach to z ~ 2.3. HIIG data are among a handful of cosmological observations that probe the largely unexplored part of redshift space from z ~ 2 to z ~ 1100. Other data that probe this region include quasar angular size measurements that reach to z ~ 2.7 (Gurvits, Kellermann & Frey 1999; Chen & Ratra 2003; Cao et al. 2017; Ryan, Chen & Ratra 2019, and references therein), quasar flux measurements that reach to z ~ 5 (Risaliti & Lusso 2015, 2019; Yang, Banerjee & Colgáin 2019; Khadka & Ratra 2020a,b; Zheng et al. 2020, and references therein), and gamma-ray burst data that reach to z ~ 8 (Lamb & Reichart 2000; Samushia & Ratra 2010; Demianski et al. 2019, and references therein). In this paper, we also use quasar angular size measurements (hereinafter ‘QSO’) to constrain cosmological model parameters.

1For early attempts, see Siegel et al. (2005), Plionis et al. (2009, 2010, 2011) and Mania & Ratra (2012). For more recent discussions, see Chávez et al. (2016), Wei, Wu & Melia (2016), Yennapureddy & Melia (2017), Zheng, Melia & Zhang (2019), Ruan et al. (2019), González-Morán et al. (2019), Wan et al. (2019), and Wu et al. (2020).

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While H IIG and QSO data probe the largely unexplored $z \sim 2.3$–2.7 part of the Universe, current H IIG and QSO measurements provide relatively weaker constraints on cosmological parameters than those provided by more widely used measurements, such as BAO peak length scale observations or Hubble parameter [hereinafter ‘$H(z)$’] observations (with these latter data being at lower redshift but of better quality than H IIG or QSO data). However, we find that the H IIG and QSO constraints are consistent with those that follow from BAO and $H(z)$ data, and so we use all four sets of data together to constrain cosmological parameters. We find that the H IIG and QSO data tighten cosmological parameter constraints relative to the $H(z) +$ BAO only case.

Using six different cosmological models to constrain cosmological parameters allows us to determine which of our results are less model dependent. In all models, the H IIG data favour those parts of cosmological parameter space for which the current cosmological expansion is accelerating.\(^2\) The joint analysis of the H IIG, QSO, BAO, and $H(z)$ data results in relatively model-independent and fairly tight determination of the Hubble constant $H_0$ and the current non-relativistic matter density parameter $\Omega_{m0}$\(^3\). Depending on the model, $\Omega_{m0}$ ranges from a low of 0.309\(^{+0.018}_{-0.004}\) to a high of 0.319 \pm 0.013, being consistent with most other estimates of this parameter (unless indicated otherwise, uncertainties given in this paper are \pm 1\sigma). The best-fitting values of $H_0$, ranging from 68.18\(^{+0.75}_{-0.27}\) km s\(^{-1}\) Mpc\(^{-1}\) to 69.90 \pm 1.48 km s\(^{-1}\) Mpc\(^{-1}\), are, from the quadrature sum of the error bars, 2.01–3.40\sigma lower than the local $H_0 = 74.03 \pm 1.42$ km s\(^{-1}\) Mpc\(^{-1}\)-measurement of Riess et al. (2019)\(^4\) and only 0.06–0.60\sigma higher than the median statistics $H_0 = 68 \pm 2.8$ km s\(^{-1}\) Mpc\(^{-1}\)-estimate of Chen & Ratra (2011a). These combined measurements are consistent with the spatially flat ΛCDM model, but also do not strongly disallow some mild dark energy dynamics, as well as a little non-zero spatial curvature energy density.

This paper is organized as follows. In Section 2, we introduce the data we use. Section 3 describes the models we analyse, with a description of our analysis method in Section 4. Our results are in Section 5, and we provide our conclusions in Section 6.

### 2 DATA

We use a combination of $H(z)$, BAO, QSO, and H IIG data to obtain constraints on our cosmological models. The $H(z)$ data, spanning the redshift range 0.070 \leq z \leq 1.965, are identical to the $H(z)$ data used in Ryan, Doshi & Ratra (2018), Ryan et al. (2019) and compiled in table 2 of Ryan et al. (2018); see that paper for description. The QSO data compiled by Cao et al. (2017; listed in table 1 of that paper) and spanning the redshift range 0.462 \leq z \leq 2.73, are identical to that used in Ryan et al. (2019); see these papers for descriptions.

\(^2\)This result could weaken, however, as the H IIG data constraint contours could broaden when H IIG data systematic uncertainties are taken into account. We do not incorporate any H IIG data systematic uncertainties into our analysis; see below.

\(^3\)The BAO and $H(z)$ data play a more significant role than do the H IIG and QSO data in setting these and other limits, but the H IIG and QSO data tighten the BAO + $H(z)$ constraints. We note, however, that the $H(z)$ and QSO data, by themselves, give lower central values of $H_0$ but with larger error bars. Also, because we calibrate the distance scale of the BAO measurements listed in Table 1 via the sound horizon scale at the drag epoch ($r_s$, about which see below), a quantity that depends on early-Universe physics, we would expect these measurements to push the best-fitting values $H_0$ lower when they are combined with late-Universe measurements like H IIG (whose distance scale is not set by the physics of the early Universe).

### Table 1. BAO data.

| $z$ | Measurement\(^a\) | Value | Ref. |
|-----|------------------|-------|-----|
| 0.38 | $D_M(r_s,\Omega_{b0},\Omega_{c0})$ | 1512.39 | Alam et al. (2017)\(^6\) |
| 0.38 | $H(z)(r_s,\Omega_{b0},\Omega_{c0})$ | 81.2087 | Alam et al. (2017)\(^6\) |
| 0.51 | $D_M(r_s,\Omega_{b0},\Omega_{c0})$ | 1975.22 | Alam et al. (2017)\(^6\) |
| 0.51 | $H(z)(r_s,\Omega_{b0},\Omega_{c0})$ | 90.9029 | Alam et al. (2017)\(^6\) |
| 0.61 | $D_M(r_s,\Omega_{b0},\Omega_{c0})$ | 2306.68 | Alam et al. (2017)\(^6\) |
| 0.61 | $H(z)(r_s,\Omega_{b0},\Omega_{c0})$ | 90.9029 | Alam et al. (2017)\(^6\) |
| 0.122 | $D_M(r_s,\Omega_{b0},\Omega_{c0})$ | 539 \pm 17 | Carter et al. (2018) |
| 0.81 | $D_M(r_s,\Omega_{b0},\Omega_{c0})$ | 10.75 \pm 0.43 | DES Collaboration (2019b) |
| 1.52 | $D_M(r_s,\Omega_{b0},\Omega_{c0})$ | 3843 \pm 147 | Ata et al. (2018) |
| 2.34 | $D_M(r_s,\Omega_{b0},\Omega_{c0})$ | 8.86 | de Sainte Agathe et al. (2019)\(^c\) |
| 2.34 | $D_M(r_s,\Omega_{b0},\Omega_{c0})$ | 37.41 | de Sainte Agathe et al. (2019)\(^c\) |

\(^a\) $D_M(r_s,\Omega_{b0},\Omega_{c0})$, $D_M(r_s,\Omega_{b0},\Omega_{c0})$, $r_s$, and $\Omega_{b0}$ have units of Mpc, while $H(z)(r_s,\Omega_{b0},\Omega_{c0})$ has units of km s\(^{-1}\) Mpc\(^{-1}\), and $D_M(r_s,\Omega_{b0},\Omega_{c0})$ is dimensionless.

\(^6\)The six measurements from Alam et al. (2017) are correlated; see equation (20) of Ryan et al. (2019) for their correlation matrix.

\(^c\)The two measurements from de Sainte Agathe et al. (2019) are correlated; see equation (27) below for their correlation matrix.

Our BAO data (see Table 1) have been updated relative to Ryan et al. (2019) and span the redshift range 0.38 \leq z \leq 2.34. Our H IIG data are new, comprising 107 low-redshift (0.0088 \leq z \leq 0.16417) H IIG measurements, used in Chávez et al. (2014), and 46 high-redshift (0.636427 \leq z \leq 2.42935) H IIG measurements, used in González-Morán et al. (2019).\(^4\) These extinction-corrected measurements (see below for a discussion of extinction correction) were very kindly provided to us by Ana Luisa González-Morán (private communications, 2019 and 2020).

To use BAO measurements to constrain cosmological model parameters, knowledge of the sound horizon scale at the drag epoch ($r_s$) is required. We compute this scale more accurately than in Ryan et al. (2019) using the approximate formula (Aubourg et al. 2015)

\[
    r_s = \frac{55.154 \exp \left[-72.3(\Omega_{m0} h^2 - 0.0006)^2\right]}{(\Omega_{m0} h^2)^{0.1207} (\Omega_{b0} h)^{0.2531}}. \tag{1}
\]

Here $\Omega_{m0} = \Omega_{m0} + \Omega_{b0} = \Omega_{m0} - \Omega_{i0}$ with $\Omega_{m0}$, $\Omega_{b0}$, $\Omega_{c0}$, and $\Omega_{i0} = 0.0014$ (following Carter et al. 2018) being the current values of the CDM + baryonic matter, CDM, baryonic matter, and neutrino energy density parameters, respectively, and the Hubble constant $H_0 = 100 h$ km s\(^{-1}\) Mpc\(^{-1}\)-Here, and in what follows, a subscript of ‘0’ on a given quantity denotes the current value of that quantity. Additionally, $\Omega_{m0} h^2$ is slightly model dependent; the values of this parameter that we use in this paper are the same as those originally computed in Park & Ratra (2018, 2019a,c) and listed in table 2 of Ryan et al. (2019).

As mentioned in Section 1, H IIG can be used as cosmological probes because they exhibit a tight correlation between the observed luminosity ($L$) of their Balmer emission lines and the velocity dispersion ($\sigma$) of their ionized gas (as measured from the widths of the emission lines). That correlation can be expressed in the form

\[
    \log L = \beta \log \sigma + \gamma, \tag{2}
\]

where $\gamma$ and $\beta$ are the intercept and slope, respectively, and $\log L$ = log$_{10}$ $L$ here and in what follows. To determine the values of $\beta$ and $\gamma$, it is necessary to establish the extent to which light from an H IIG is extinguished as it propagates through space. A correction

\(^4\)A total of 15 from González-Morán et al. (2019), 25 from Erb et al. (2006), Masters et al. (2014), and Maseda et al. (2014), and 6 from Terlevich et al. (2015).
must be made to the observed flux so as to account for the effect of this extinction. As mentioned above, the data we received from Ana Luisa González-Morán have been corrected for extinction. In González-Morán et al. (2019), the authors used the Gordon et al. (2003) extinction law, and in doing found
\[
\beta = 5.022 \pm 0.058,
\]
and
\[
\gamma = 33.268 \pm 0.083,
\]
respectively. These are the values of \(\beta\) and \(\gamma\) that we use in the \(L-\sigma\) relation, equation (2).

Once the luminosity of an H II G has been established through equation (2), this luminosity can be used, in conjunction with a measurement of the flux \(f\) emitted by the H II G, to determine the distance modulus of the H II G via
\[
\mu_{\text{obs}} = 2.5 \log L - 2.5 \log f - 100.2
\]
(see e.g. Terlevich et al. 2015, González-Morán et al. 2019, and references therein).5 This quantity can then be compared to the value of the distance modulus predicted within a given cosmological model
\[
\mu_{\text{th}}(p, z) = 5 \log D_L(p, z) + 25,
\]
where the luminosity distance \(D_L(p, z)\) is related to the transverse comoving distance \(D_A(p, z)\) and the angular size distance \(D_X(p, z)\) through \(D_L(p, z) = (1 + z)D_A(p, z) = (1 + z)^2D_X(p, z)\). These are functions of the redshift \(z\) and the parameters \(p\) of the model in question, and
\[
D_X(p, z) = \begin{cases} 
\frac{c}{H_0 \sqrt{\Omega_{k0}}} \sinh \left[ \sqrt{\Omega_{k0}} H_0 D_c(p, z)/c \right] & \text{if } \Omega_{k0} = 0, \\
\frac{c}{H_0 \sqrt{\Omega_{k0}}} \sin \left[ \sqrt{\Omega_{k0}} H_0 D_c(p, z)/c \right] & \text{if } \Omega_{k0} > 0, \\
\frac{c}{H_0 \sqrt{|\Omega_{k0}|}} & \text{if } \Omega_{k0} < 0. 
\end{cases}
\]
(7)

In the preceding equation,
\[
D_c(p, z) \equiv c \int_0^z \frac{dz'}{H(p, z')},
\]
\(\Omega_{k0}\) is the current value of the spatial curvature energy density parameter, and \(c\) is the speed of light (Hogg 1999).

As the precision of cosmological observations has grown over the last few years, a tension between measurements of the Hubble constant made with early-Universe probes and measurements made with late-Universe probes has revealed itself (for a review, see Riess 2019). Whether a given cosmological observation supports a lower value of \(H_0\) (i.e. one that is closer to the early-Universe Planck measurement) or a higher value of \(H_0\) (i.e. one that is closer to the late-Universe value measured by Riess et al. 2019) may depend on whether the distance scale associated with this observation has been set by early- or late-Universe physics. It is therefore important to know what distance scale cosmological observations have been calibrated to, so that the extent to which measurements of \(H_0\) are pushed higher or lower by these different distance calibrations can be clearly identified.

The \(H_0\) values we measure from the combined \(H(z)\), BAO, QSO, and H II G data are based on a combination of both early- and late-Universe distance calibrations. As mentioned above, the distance scale of our BAO measurements is set by the size of the sound horizon at the drag epoch \(r_d\). The sound horizon, in turn, depends on \(\Omega_{\text{b}} h^2\), which was computed by Park & Ratra (2018), Park & Ratra (2019a,c) using early-Universe data. Our H II G measurements, on the other hand, have been calibrated using cosmological model independent distance ladder measurements of the distances to nearby giant H II regions (see González-Morán et al. 2019 and references therein), so these data qualify as late-Universe probes. The distance scale of our QSO measurements is set by the intrinsic linear size \((l_0)\) of the QSOs themselves, which is a late-Universe measurement (see Cao et al. 2017). Finally, our \(H(z)\) data depend on late-Universe astrophysics through the modelling of the star formation histories of the galaxies whose ages are measured to obtain the Hubble parameter (although the differences between different models are not thought to have a significant effect on measurements of \(H(z)\) from these galaxies; see Moresco et al. 2018, 2020).

### 3 COSMOLOGICAL MODELS

The redshift \(z\) is related to the scale factor \(a\) as \(1 + z \equiv a_0/a\) and the Hubble parameter is \(H \equiv a'/a\), where the overdot denotes the time derivative. In this paper, we consider three pairs of flat and non-flat cosmological models.6 The data we use are at \(z < 2.73\) and in what follows we ignore the insignificant contribution that radiation makes to the late-time cosmological energy budget.

In the \(\Lambda\)CDM model, the Hubble parameter is
\[
H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{\Lambda0}(1 + z)^2 + \Omega_{\Lambda}},
\]
(9)
where \(\Omega_\Lambda\) is the (constant) dark energy density parameter. In the flat \(\Lambda\)CDM model, the parameters to be constrained are conventionally chosen to be \(H_0\) and \(\Omega_{m0}\). In this model \(\Omega_{k0} = 0\), which implies \(\Omega_\Lambda = 1 - \Omega_{m0}\). In the non-flat \(\Lambda\)CDM model the parameters to be constrained are \(H_0\), \(\Omega_{m0}\), and \(\Omega_\Lambda\), and the curvature energy density parameter is a derived quantity, being related to the non-relativistic matter and dark energy density parameters through \(\Omega_{k0} = 1 - \Omega_{m0} - \Omega_\Lambda\).

In the XCDM parametrization, dark energy is modeled as an ideal, spatially homogeneous X-fluid with equation of state \(w_X = p_X/\rho_X\), where \(p_X\) and \(\rho_X\) are the X-fluid’s pressure and energy density, respectively.7 In the XCDM parametrization, the Hubble parameter is
\[
H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{k0}(1 + z)^2 + \Omega_{X0}(1 + z)^{3(1+w_X)}},
\]
(10)
where \(\Omega_{X0}\) is the present value of the X-fluid energy density parameter. From this equation, it can be seen that when \(w_X = -1\) XCDM

5For each H II G in our sample, we have the measured values and uncertainties of \(\log f\), \(\log \sigma\) and \(z\).

6Observational constraints on non-flat models are discussed in Farooq, Mania & Ratra (2015), Chen et al. (2016), Yu & Wang (2016), Rana et al. (2017), Ooba, Ratra & Sugiyama (2018a,b,c), Yu, Ratra & Wang (2018), Park & Ratra (2018), Park & Ratra (2019a,b,c), Park & Ratra (2020), Wei (2018), DES Collaboration (2019a), Ruan et al. (2019), Coley (2019), Jesus et al. (2019), Handley (2019), Wang et al. (2020), Zhai et al. (2020), Li, Du & Xu (2020), Geng et al. (2020), Kumar et al. (2020), Elstathiou & Gratton (2020), Di Valentino, Melchiorri & Silk (2020), Gao, Chen & Zheng (2020), and references therein.

7It should be noted, however, that the XCDM parametrization cannot sensibly describe the evolution of spatial inhomogeneities, and therefore is, unlike the \(\Lambda\)CDM and \(\phi\)CDM models, physically incomplete. It is possible to extend this parametrization by allowing for an additional free parameter \(c_X^2 = dp_X/d\rho_X\) and requiring that \(c_X^2 > 0\).
reduces to $\Lambda$CDM. In the non-flat case, the model parameters to be constrained are $H_0$, $\Omega_{m0}$, $\Omega_{k0}$, and $w_X$, with $\Omega_{X0} = 1 - \Omega_{m0} - \Omega_{k0}$ as a derived parameter (we do not report constraints on its value in this paper). In the spatially flat case, the parameters to be constrained are $H_0$, $\Omega_{m0}$, and $w_X$, with $\Omega_{X0} = 1 - \Omega_{m0}$.

In the flat and non-flat $\phi$CDM models, dark energy is modeled as a dynamical scalar field $\phi$, with a potential energy density given by

$$V(\phi) = \frac{1}{2} \kappa m_p^2 \phi^2 - \alpha, \quad (11)$$

where $m_p$ is the Planck mass, $\alpha$ is a non-negative scalar, and

$$\kappa = \frac{8}{3m_p^2} \left( \frac{\alpha + 4}{\alpha + 2} \right)^{\frac{2}{3}} \left( \frac{\alpha}{\alpha + 2} \right)^{\frac{1}{2}}. \quad (12)$$

(Peebles & Ratra 1988; Ratra & Peebles 1988; Pavlov et al. 2013).8 Note that when $\alpha = 0$ the $\phi$CDM model reduces to the $\Lambda$CDM model. In the spatially homogeneous approximation, valid for the cosmological tests we consider in this paper, the dynamics of the scalar field is governed by two coupled non-linear ordinary differential equations, the first being the scalar field equation of motion

$$\dot{\phi} + 3 \left( \frac{\dot{a}}{a} \right) \phi - \frac{1}{2} \alpha \kappa m_p^2 \phi^{-\alpha - 1} = 0, \quad (13)$$

and the second being the Friedman equation

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3m_p^2} (\rho_m + \rho_\phi) - \frac{k}{a^2}. \quad (14)$$

In equation (14), $-k/a^2$ is the spatial curvature term (with $k = 0, -1, +1$ corresponding to $\Omega_{k0} = 0, >0, <0$, respectively), and $\rho_m$ and $\rho_\phi$ are the non-relativistic matter and the scalar field energy densities, respectively, where

$$\rho_\phi = \frac{m_p^2}{32\pi} \left( \phi^2 + \kappa m_p^2 \phi^{-\alpha} \right). \quad (15)$$

It follows that the Hubble parameter in $\phi$CDM is

$$H(z) = H_0 \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_\phi(z, \alpha)}, \quad (16)$$

where the scalar field energy density parameter

$$\Omega_\phi(z, \alpha) = \frac{1}{2H_0^2} \left( \phi^2 + \kappa m_p^2 \phi^{-\alpha} \right), \quad (17)$$

as can be determined from equations (13) and (14). For non-flat $\phi$CDM the parameters to be constrained are $\alpha$, $H_0$, $\Omega_{m0}$, and $\Omega_{k0}$, and for flat $\phi$CDM the parameters to be constrained are $\alpha$, $H_0$, and $\Omega_{m0}$.

4 DATA ANALYSIS METHODOLOGY

We perform a Markov chain Monte Carlo (MCMC) analysis with the PYTHON module EMCEE (Foreman-Mackey et al. 2013) and maximize the likelihood function, $L$, to determine the best-fitting values of the parameters $p$ of the models. We use flat priors for all parameters $p$. For all models, the priors on $\Omega_{m0}$ and $h$ are non-zero over the ranges 0.1 $\leq \Omega_{m0} \leq 0.7$ and 0.50 $\leq h \leq 0.85$. In the non-flat $\Lambda$CDM model, the $\Omega_X$ prior is non-zero over 0.2 $\leq \Omega_X \leq 1$. In the flat and non-flat $\phi$CDM parametrizations the prior range on $w_X$ is $-2 \leq w_X \leq 0$, and the prior range on $\Omega_{k0}$ in the non-flat $\phi$CDM parametrization is $-0.7 \leq \Omega_{k0} \leq 0.7$. In the flat and non-flat $\phi$CDM models the prior range on $\alpha$ is 0.01 $\leq \alpha \leq 3$ and 0.01 $\leq \alpha \leq 5$, respectively, and the prior range on $\Omega_{k0}$ is also $-0.7 \leq \Omega_{k0} \leq 0.7$.

For H$tG$, the likelihood function is

$$L_{H \& G} = e^{-\chi^2_{H \& G}/2}, \quad (18)$$

where

$$\chi^2_{H \& G}(p) = \sum_{i=1}^{153} \frac{[\mu_{\theta}\phi(p, z_i) - \mu_{\theta}\phi(\mu_{\theta}(z_i))]^2}{\epsilon_i^2}. \quad (19)$$

and $\epsilon_i$ is the uncertainty of the $i$th measurement. Following González-Morán et al. (2019), $\epsilon$ has the form

$$\epsilon = \sqrt{\epsilon_{stat}^2 + \epsilon_{sys}^2}, \quad (20)$$

where the statistical uncertainties are

$$\epsilon_{stat}^2 = 6.25 \left( \frac{\epsilon_{\log f}}{2} + \frac{\beta^2 \epsilon_{\log \sigma}^2 + \epsilon_{\mu}(\log \sigma)^2 + \epsilon_{\mu}^2}{\epsilon_{\mu}} \right) \epsilon_i^2. \quad (21)$$

Following González-Morán et al. (2019), we do not account for systematic uncertainties in our analysis, so the uncertainty on the H$tG$ measurements consists entirely of the statistical uncertainty (so that $\epsilon = \epsilon_{stat}$).9 The reader should also note here that although the theoretical statistical uncertainty depends on our cosmological model parameters [through the theoretical distance modulus $\mu_{\theta} = \mu_{\theta}(p, z)$], the effect of this model dependence on the parameter constraints is negligible for the current data.10

For $H(z)$, the likelihood function is

$$L_H = e^{-\chi^2_H/2}, \quad (22)$$

where

$$\chi^2_H(p) = \sum_{i=1}^{31} \frac{[H_{\theta}^\phi(p, z_i) - H_{\theta}(z_i)]^2}{\epsilon_i^2}, \quad (23)$$

and $\epsilon_i$ is the uncertainty of $H_{\theta}(z_i)$.

For the BAO data, the likelihood function is

$$L_{BAO} = e^{-\chi^2_{BAO}/2}, \quad (24)$$

and for the uncorrelated BAO data (lines 7-9 in Table 1) the $\chi^2$ function takes the form

$$\chi^2_{BAO}(p) = \sum_{i=1}^{7} \frac{[A_{\phi}(p, z_i) - A_{\phi}(z_i)]^2}{\epsilon_i^2}. \quad (25)$$

8Observational constraints on the $\phi$CDM model are discussed in Chen & Ratra (2004), Samushia, Chen & Ratra (2007), Yashar et al. (2009), Samushia et al. (2010), Chen & Ratra (2011b), Campanelli et al. (2012), Farooq & Ratra (2013), Farooq, Crandall & Ratra (2013), Avsajanishvili et al. (2015), Soli, Gómez-Valent & de Cruz Pérez (2017), Zhai et al. (2017), Sangwan, Tripathi & Jassal (2018), Solia Peracaula, de Cruz Pérez & Gómez-Valent (2018), Solia Peracaula, Gómez-Valent & de Cruz Pérez (2019), Ooba, Ratra & Sugiyama (2019), Singh, Sangwan & Jassal (2019), and references therein.

9A systematic error budget for H$tG$ data is available in the literature; however, see Chávez et al. (2016).

10In contrast to our definition of $\chi^2$ in equation (19), González-Morán et al. (2019) defined an $H_0$-independent $\chi^2$ function in their equation (27) and weighted this $\chi^2$ function by $1/\epsilon_{stat}^2$ (where $\epsilon_{stat}$ is given by their equation 15), which we do not do. This procedure is discussed in the literature (Melnick et al. 2017; Fernández Arenas et al. 2018), and when we use it we find that it leads to a reduced $\chi^2$ identical to that given in González-Morán et al. (2019) (being less than 2 but greater than 1) without having a noticeable effect on the shapes or peak locations of our posterior likelihoods (hence providing very similar best-fitting values and error bars of the cosmological model parameters). As discussed below, with our $\chi^2$ definition we find reduced $\chi^2$ values $\sim 2.75$. González-Morán et al. (2019) note that an accounting of systematic uncertainties could decrease the reduced $\chi^2$ values towards unity.
where $A_{\text{th}}$ and $A_{\text{obs}}$ are, respectively, the theoretical and observational quantities as listed in Table 1, and $\epsilon_i$ corresponds to the uncertainty of $A_{\text{obs}}(z_i)$. For the correlated BAO data, the $\chi^2$ function takes the form

$$
\chi^2_{\text{BAO}}(p) = |A_{\text{th}}(p) - A_{\text{obs}}(z_i)|^2 C^{-1}[A_{\text{th}}(p) - A_{\text{obs}}(z_i)],
$$

where superscripts $T$ and $-1$ denote the transpose and inverse of the matrices, respectively. The covariance matrix $C$ for the BAO data, taken from Alam et al. (2017), is given in equation (20) of Ryan et al. (2019), while for the BAO data from de Sainte Agathe et al. (2019),

$$
C = \begin{bmatrix}
0.0841 & -0.183396 \\
-0.183396 & 3.4596
\end{bmatrix}.
$$

For QSO, the likelihood function is

$$
\mathcal{L}_{\text{QSO}} = e^{-\chi^2_{\text{QSO}}/2},
$$

and the $\chi^2$ function takes the form

$$
\chi^2_{\text{QSO}}(p) = \sum_{i=1}^{120} \left( \frac{\theta_{\text{th}}(p, z_i) - \theta_{\text{obs}}(z_i)}{\epsilon_i + 0.1 \theta_{\text{obs}}(z_i)} \right)^2,
$$

where $\theta_{\text{th}}(p, z_i)$ and $\theta_{\text{obs}}(z_i)$ are the theoretical and observed values of the angular size at redshift $z_i$, respectively, and $\epsilon_i$ is the uncertainty of $\theta_{\text{obs}}(z_i)$ (see Ryan et al. 2019 for more details).

For the joint analysis of these data, the total likelihood function is obtained by multiplying the individual likelihood functions (that is, equations 18, 22, 24, and 28) together in various combinations. For example, for $H(z)$, BAO, and QSO data, we have

$$
\mathcal{L} = \mathcal{L}_H \mathcal{L}_\text{BAO} \mathcal{L}_\text{QSO}.
$$

We also use the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) to compare the goodness of fit of models with different numbers of parameters, where

$$
AIC = -2 \ln \mathcal{L}_{\text{max}} + 2n \equiv \chi^2_{\text{min}} + 2n,
$$

and

$$
BIC = -2 \ln \mathcal{L}_{\text{max}} + n \ln N \equiv \chi^2_{\text{min}} + n \ln N.
$$

In these two equations, $\mathcal{L}_{\text{max}}$ refers to the maximum value of the given likelihood function, $\chi^2_{\text{min}}$ refers to the corresponding minimum $\chi^2$ value, $n$ is the number of parameters of the given model, and $N$ is the number of data points (for example, for H IIG we have $N = 153$, etc.).

5 RESULTS

5.1 H IIG constraints

We present the posterior 1D probability distributions and 2D confidence regions of the cosmological parameters for the six flat and non-flat models in Figs 1–6, in grey. The unmarginalized best-fitting parameter values are listed in Table 2, along with the corresponding $1\sigma$, $2\sigma$, and other cosmological data.

From the fit to the H IIG data, we see that most of the probability lies in the part of the parameter space corresponding to currently accelerating cosmological expansion (see the grey contours in Figs 1–6). This means that the H IIG data favour currently accelerating cosmological expansion, in agreement with supernova Type Ia, BAO, $H(z)$, and other cosmological data.

From the H IIG data, we find that the constraints on the non-relativistic matter density parameter $\Omega_m$ are consistent with other

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11We plot these figures using the PYTHON package GETDIST (Lewis 2019), which we also use to compute the central values (posterior means) and uncertainties of the cosmological parameters listed in Table 3.

12Although a full accounting of the systematic uncertainties in the H IIG data could weaken this conclusion.
estimates, ranging between a high of 0.300^{+0.106}_{-0.083} (flat XCDM) and a low of \(\Omega_m = 0.210^{+0.043}_{-0.036}\) (flat \(\phi\)CDM).

The HiTIG data constraints on \(H_0\) in Table 3 are consistent with the estimate of \(H_0 = 71.0 \pm 2.8\)\,(stat.) \pm 2.1\,(sys.) km s^{-1} Mpc^{-1}\) made by Fernández Arenas et al. (2018) based on a compilation of HiTIG measurements that differs from what we have used here. The HiTIG \(H_0\) constraints listed in Table 3 are also consistent with other recent measurements of \(H_0\), being between 0.90\(\sigma\) (flat XCDM) and 1.56\(\sigma\) (Non-flat \(\phi\)CDM) lower than the recent local expansion rate measurement of \(H_0 = 74.03 \pm 1.42\) km s^{-1} Mpc^{-1}\) (Riess et al. 2019), and between 0.78\(\sigma\) (Non-flat \(\phi\)CDM) and 1.13\(\sigma\) (flat XCDM) higher than the median statistics estimate of \(H_0 = 68 \pm 2.8\) km s^{-1} Mpc^{-1}\) (Chen & Ratra 2011a), with our measurements ranging from a low of \(H_0 = 70.60^{+1.68}_{-1.84}\) km s^{-1} Mpc^{-1} (non-flat \(\phi\)CDM) to a high of \(H_0 = 71.85 \pm 1.96\) km s^{-1} Mpc^{-1} (flat XCDM).

As for spatial curvature, from the marginalized 1D likelihoods in Table 3, for non-flat XCDM, non-flat XCDM, and non-flat \(\phi\)CDM, we measure \(\Omega_k = 0.094^{+0.237}_{-0.360}\), \(\Omega_k = 0.011^{+0.457}_{-0.460}\), and \(\Omega_k = 0.291^{+0.348}_{-0.113}\), respectively. From the marginalized likelihoods, we see that non-flat \(\Lambda\)CDM and XCDM models are consistent with all three spatial geometries, while non-flat \(\phi\)CDM favours the open case at 2.58\(\sigma\). However, this seems to be a little odd, especially for non-flat \(\phi\)CDM, considering their un marginalized best-fitting \(\Omega_m\)'s are all negative (see Table 2).

The fits to the HiTIG data are consistent with dark energy being a cosmological constant but do not rule out dark energy dynamics. For flat (non-flat) XCDM, \(w_X = -1.180^{+0.560}_{-0.330}\) \((w_X = -1.125^{+0.337}_{-0.324})\), which are both within 1\(\sigma\) of \(w_X = -1\). For flat (non-flat) \(\phi\)CDM, 2\(\sigma\) upper limits of \(\alpha < 2.784 (\alpha < 4.590)\), with the 1D likelihood functions, in both cases, peaking at \(\alpha = 0\).

Current HiTIG data do not provide very restrictive constraints on cosmological model parameters, but when used in conjunction with other cosmological data they can help tighten the constraints.

### 5.2 \(H(z)\), BAO, and HiTIG (HzBH) constraints

The HiTIG constraints discussed in the previous subsection are consistent with constraints from most other cosmological data, so it is appropriate to use the HiTIG data in conjunction with other data to constrain parameters. In this subsection, we perform a full analysis of \(H(z)\), BAO, and HiTIG (HzBH) data and derive tighter constraints on cosmological parameters.

The 1D probability distributions and 2D confidence regions of the cosmological parameters for the six flat and non-flat models are shown in Figs 1–6, in red. The best-fitting results and uncertainties are listed in Tables 2 and 3.

When we fit our cosmological models to the HzBH data, we find that the measured values of the matter density parameter \(\Omega_m\) fall within a narrower range in comparison to the HiTIG-only case, uncertainties. A similar procedure, based on \(\Omega_m = 1 - \Omega_k\), is used to measure \(\Omega_m\) in the flat \(\Lambda\)CDM model.
Cosmological constraints from HIIG data

Figure 3. 1σ, 2σ, and 3σ confidence contours for flat XCDM. The black-dotted line is the zero-acceleration line, which divides the parameter space into regions associated with currently accelerated (below left) and currently decelerated (above right) cosmological expansion. The magenta lines denote $w_X = -1$, i.e. the flat ΛCDM model.

Figure 4. Same as Fig. 3 but for non-flat XCDM, where the zero acceleration lines in each of the three subpanels are computed for the third cosmological parameter set to the HIIG data only best-fitting values listed in Table 2. Currently accelerated cosmological expansion occurs below these lines. The cyan dash–dotted lines represent the flat case, with closed spatial hypersurfaces either below or to the left. The magenta lines indicate $w_X = -1$, i.e. the non-flat ΛCDM model.

being between $0.314 \pm 0.015$ (non-flat ΛCDM) and $0.323^{+0.014}_{-0.016}$ (flat φCDM).

Similarly, the measured values of $H_0$ also fall within a narrower range when our models are fit to the HzBH data combination [and are in better agreement with the median statistics estimate of $H_0$ from Chen & Ratra 2011a than with the local measurement carried out by Riess et al. 2019; this is because the $H(z)$ and BAO data favour a lower $H_0$ value] being between $H_0 = 68.36^{+1.05}_{-0.86}$ km s$^{-1}$ Mpc$^{-1}$ (flat φCDM) and $70.21 \pm 1.33$ km s$^{-1}$ Mpc$^{-1}$ (non-flat ΛCDM). We assume that the tension between early- and late-
Figure 5. $1\sigma$, $2\sigma$, and $3\sigma$ confidence contours for flat $\phi$CDM. The black dotted zero-acceleration line splits the parameter space into regions of currently accelerated (below left) and currently decelerated (above right) cosmological expansion. The $\alpha = 0$ axis is the flat $\Lambda$CDM model.

Figure 6. Same as Fig. 5 but for non-flat $\phi$CDM, where the zero-acceleration lines in each of the subpanels are computed for the third cosmological parameter set to the HIIG data only best-fitting values listed in Table 2. Currently accelerating cosmological expansion occurs below these lines. The cyan dash–dotted lines represent the flat case, with closed spatial geometry either below or to the left. The $\alpha = 0$ axis is the non-flat $\Lambda$CDM model.

Universe measurements of $H_0$ is not a major issue here, because the 2D and 1D contours in Fig. 1 overlap, and so we compute a combined $H_0$ value (but if one is concerned about the early versus late-Universe $H_0$ tension then one should not compare our combined-data $H_0$’s here, and in Sections 5.3 and 5.4, directly to the measurements of Riess et al. 2019 or of Planck Collaboration VI 2020).

In contrast to the HIIG-only cases, when fit to the HzBH data combination the non-flat models mildly favour closed spatial hypersurfaces. This is because the $H(z)$ and BAO data mildly favour closed spatial hypersurfaces; see e.g. Park & Ratra (2019b) and Ryan et al. (2019). For non-flat $\Lambda$CDM, non-flat XCDM, and non-flat $\phi$CDM, we find $\Omega_{k0} = -0.029^{+0.049}_{-0.048}$, $\Omega_{k0} = -0.082^{+0.135}_{-0.119}$, and...
cases, and are in better agreement with the median statistics (Chen & Ratra 2011a) estimate of $H_0$ than with what is measured from the local expansion rate (Riess et al. 2019). Compared with Ryan et al. (2019), the central values are lower except for the non-flat XCDM model.

For non-flat XCDM, non-flat XCDM, and non-flat $\phi$CDM, we measure $\Omega_m = 0.029^{+0.056}_{-0.063}$, $\Omega_k = -0.078^{+0.124}_{-0.091}$, and $\Omega_w = -0.103^{+0.111}_{-0.091}$, respectively. These results are consistent with their un marginalized best fits (see Table 2), where the best fit to the non-flat XCDM model favours open spatial hypersurfaces, and the best fits to the non-flat XCDM parameterization and the non-flat $\phi$CDM model both favour closed spatial hypersurfaces. Note that the central values are larger than those of Ryan et al. (2019), especially for non-flat XCDM (positive instead of negative). In all three models, the constraints are consistent with flat spatial hypersurfaces.

The fit to the H fbq data combination provides slightly stronger evidence for dark energy dynamics than does the fit to the H fbq data combination. For flat (non-flat) XCDM, $w_X = -0.911^{+0.122}_{-0.098}$ ($w_X = -0.826^{+0.085}_{-0.088}$), with the former barely within 1σ of $w_X = -1$ and the latter almost 2σ away from $w_X = -1$. For flat (non-flat) $\phi$CDM, $\alpha = 0.460^{+0.116}_{-0.094}$ ($\alpha = 0.854^{+0.539}_{-0.094}$), with the former 1.0σ and the latter 1.4σ away from the $\alpha = 0$ cosmological constant. In comparison with Ryan et al. (2019), central values of $w_X$ are larger and smaller for flat and non-flat XCDM models, respectively, and that of $\alpha$ is larger for both flat and non-flat $\phi$CDM models.

### 5.4 $H(z)$, BAO, QSO, and H fbq (H fbq) constraints

Comparing the results of the previous two subsections, we see that when used in conjunction with $H(z)$ and BAO data, the QSO data result in tighter constraints on $\Omega_m$, $\Omega_k$ (in non-flat XCDM), $w_X$ (in non-flat XCDM), and $H_0$ (in flat XCDM), while the H fbq data result in...
Table 3. 1D marginalized best-fitting parameter values and uncertainties (±1σ error bars or 2σ limits) for all models from various combinations of data.

| Model                  | Data set             | $\Omega_{\text{m}}$ | $\Omega_{\Lambda}$ | $\Omega_{k0}$ | $w_X$ | $\alpha$ | $H_0$ |
|------------------------|----------------------|----------------------|---------------------|---------------|-------|----------|--------|
| Flat ΛCDM              | HiG                  | 0.289$^{+0.053}_{-0.071}$ | $-$ | $-$ | $-$ | $-$ | 71.70 ± 1.83 |
|                        | (z) + BAO + HiG      | 0.319$^{+0.014}_{-0.015}$ | $-$ | $-$ | $-$ | $-$ | 69.23 ± 0.74 |
|                        | (z) + BAO + QSO      | 0.316$^{+0.014}_{-0.015}$ | $-$ | $-$ | $-$ | $-$ | 68.60 ± 0.68 |
|                        | (z) + BAO + QSO + HiG| 0.315$^{+0.013}_{-0.012}$ | $-$ | $-$ | $-$ | $-$ | 69.06$^{+0.63}_{-0.62}$ |
| Non-flat ΛCDM          | HiG                  | 0.275$^{+0.081}_{-0.078}$ | $>$ 0.501$^b$ | 0.094$^{+0.237}_{-0.363}$ | $-$ | $-$ | 71.59$^{+1.80}_{-1.81}$ |
|                        | (z) + BAO + HiG      | 0.314 ± 0.015 | 0.714$^{+0.054}_{-0.049}$ | $-$ | $-$ | 70.21 ± 1.33 |
|                        | (z) + BAO + QSO      | 0.313$^{+0.013}_{-0.015}$ | 0.658$^{+0.069}_{-0.060}$ | $-$ | $-$ | 68.29 ± 1.47 |
|                        | (z) + BAO + QSO + HiG| 0.310 ± 0.013 | 0.711$^{+0.055}_{-0.048}$ | $-$ | $-$ | 69.70$^{+1.12}_{-1.11}$ |
| Flat XCDM              | HiG                  | 0.300$^{+0.106}_{-0.083}$ | $-$ | $-$ | $-$ | $-$ | 71.85 ± 1.96 |
|                        | (z) + BAO + HiG      | 0.315$^{+0.016}_{-0.017}$ | $-$ | $-$ | $-$ | $-$ | 70.05 ± 1.54 |
|                        | (z) + BAO + QSO      | 0.322$^{+0.015}_{-0.016}$ | $-$ | $-$ | $-$ | $-$ | 66.98$^{+2.30}_{-2.30}$ |
|                        | (z) + BAO + QSO + HiG| 0.312 ± 0.014 | $-$ | $-$ | $-$ | $-$ | 69.90 ± 1.48 |
| Non-flat XCDM          | HiG                  | 0.275$^{+0.084}_{-0.125}$ | $-$ | 0.011$^{+0.405}_{-0.460}$ | $-$ | $-$ | 71.71$^{+1.07}_{-2.08}$ |
|                        | (z) + BAO + HiG      | 0.318 ± 0.019 | $-$ | $-$ | $-$ | $-$ | 69.83$^{+1.50}_{-1.62}$ |
|                        | (z) + BAO + QSO      | 0.320 ± 0.015 | $-$ | $-$ | $-$ | $-$ | 66.29$^{+1.90}_{-2.35}$ |
|                        | (z) + BAO + QSO + HiG| 0.309$^{+0.015}_{-0.014}$ | $-$ | $-$ | $-$ | $-$ | 69.68$^{+1.49}_{-1.64}$ |
| Flat φCDM              | HiG                  | 0.210$^{+0.043}_{-0.002}$ | $-$ | $-$ | $-$ | $-$ | <2.784 |
|                        | (z) + BAO + HiG      | 0.323$^{+0.014}_{-0.016}$ | $-$ | $-$ | $-$ | $-$ | <0.411 |
|                        | (z) + BAO + QSO      | 0.324$^{+0.014}_{-0.015}$ | $-$ | $-$ | $-$ | $-$ | <0.460 |
|                        | (z) + BAO + QSO + HiG| 0.319 ± 0.013 | $-$ | $-$ | $-$ | $-$ | <0.411 |
| Non-flat φCDM          | HiG                  | <0.321 | $-$ | 0.291$^{+0.348}_{-0.113}$ | $-$ | $-$ | <4.590 |
|                        | (z) + BAO + HiG      | 0.322$^{+0.015}_{-0.016}$ | $-$ | $-$ | $-$ | $-$ | <0.538 |
|                        | (z) + BAO + QSO      | 0.319$^{+0.013}_{-0.015}$ | $-$ | $-$ | $-$ | $-$ | <0.854 |
|                        | (z) + BAO + QSO + HiG| 0.313$^{+0.012}_{-0.014}$ | $-$ | $-$ | $-$ | $-$ | <0.926 |

$^a$km s$^{-1}$ Mpc$^{-1}$.

$^b$This is the 1σ lower limit. The 2σ lower limit is set by the prior, and is not shown here.

There is not much evidence in support of dark energy dynamics in the HzbQH case, with Λ consistent with this data combination.

5.5 Model comparison

From Table 4, we see that the reduced χ$^2$ for all models is relatively large (being between 2.25 and 2.75). This could probably be attributed to underestimated systematic uncertainties in the HiG data. This is suggested by González-Morán et al. (2019), who also found relatively large values of χ$^2$/ν in their cosmological model fits to the HiG data (though not as large as ours, because they compute a different χ$^2$, as explained in footnote 10 in Section 4). They note that an additional systematic uncertainty of ∼0.22 could bring their χ$^2$/ν down to ∼1. As mentioned previously, we do not account for HiG systematic uncertainties in our analysis.

One thing that is clear, regardless of the absolute size of HiG or QSO systematics (and ignoring the large values of χ$^2$/ν), is that the flat ΛCDM model remains the most favoured model among the six

16Underestimated systematic uncertainties might also explain the large reduced χ$^2$ of QSO data (Ryan et al. 2019).
In this paper, we have constrained cosmological parameters in six flat and non-flat cosmological models by analysing a total of 315 observations, comprising 31 $H(z)$, 11 BAO, 120 QSO, and 153 H IIG measurements. The QSO angular size and H IIG apparent magnitude measurements are particularly noteworthy, as they reach to $z \sim 2.7$ and $z \sim 2.4$, respectively (somewhat beyond the highest $z \sim 2.3$ reached by BAO data) and into a much less studied area of redshift space. While the current H IIG and QSO data do not provide very restrictive constraints, they do tighten the limits when they are used in conjunction with BAO + $H(z)$ data.

By measuring cosmological parameters in a variety of cosmological models, we are able to draw some relatively model-independent conclusions (i.e. conclusions that do not differ significantly between the different models). Specifically, for the full data set (i.e. the HzBQH data), we find quite restrictive constraints on $\Omega_m$, a reasonable summary perhaps being $\Omega_m = 0.310 \pm 0.013$, in good agreement with many other recent estimates. $H_0$ is also fairly tightly constrained, with a reasonable summary perhaps being $H_0 = 69.5 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is in better agreement with the results of Chen & Ratra (2011a) and Planck Collaboration VI (2020) than that of Riess et al. (2019). The HzBQH measurements are consistent with the standard spatially flat $\Lambda$CDM model, but do not strongly rule out mild dark energy dynamics or a little spatial curvature energy density. More and better-quality H IIG, QSO, and other data at $z \sim 2–4$ will significantly help to test these extensions.

6 CONCLUSIONS

In this paper, we have constrained cosmological parameters in six flat and non-flat cosmological models by analysing a total of 315 observations, comprising 31 $H(z)$, 11 BAO, 120 QSO, and 153 H IIG measurements. The QSO angular size and H IIG apparent magnitude measurements are particularly noteworthy, as they reach to $z \sim 2.7$ and $z \sim 2.4$, respectively (somewhat beyond the highest $z \sim 2.3$ reached by BAO data) and into a much less studied area of redshift space. While the current H IIG and QSO data do not provide very restrictive constraints, they do tighten the limits when they are used in conjunction with BAO + $H(z)$ data.

By measuring cosmological parameters in a variety of cosmological models, we are able to draw some relatively model-independent conclusions (i.e. conclusions that do not differ significantly between the different models). Specifically, for the full data set (i.e. the HzBQH data), we find quite restrictive constraints on $\Omega_m$, a reasonable summary perhaps being $\Omega_m = 0.310 \pm 0.013$, in good agreement with many other recent estimates. $H_0$ is also fairly tightly constrained, with a reasonable summary perhaps being $H_0 = 69.5 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is in better agreement with the results of Chen & Ratra (2011a) and Planck Collaboration VI (2020) than that of Riess et al. (2019). The HzBQH measurements are consistent with the standard spatially flat $\Lambda$CDM model, but do not strongly rule out mild dark energy dynamics or a little spatial curvature energy density. More and better-quality H IIG, QSO, and other data at $z \sim 2–4$ will significantly help to test these extensions.

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DATA AVAILABILITY

The H IIG data underlying this article were provided to us by the authors of González-Morán et al. (2019). These data will be shared...
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