Bearing-Only Target Tracking Based on Piecewise Backward Smoothing Kalman Filter

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Abstract. In order to solve the problem that the Unscented Kalman Filter algorithm is sensitive to the initial value selection and the noise is easy to expand the error in the recursive process, a piecewise backward smoothing Kalman filter method is proposed in this paper. To reduce the influence of noise, the observation data is optimized in sections, and the forward and reverse use of historical data is increased. The simulation results under different initial distance error, different observation angle variance and different initial velocity error show that the proposed method is effective in underwater bearings only target tracking, and the effect of Unscented Kalman filter is obviously improved, the estimation error is greatly reduced, and the estimation accuracy and stability are improved.

Keywords. Unscented Kalman filter; bearing only; backward smoothing; target tracking.

1. Introduction
As a target tracking method, bearings only target motion analysis algorithm has a very wide range of applications in aviation, underwater and other environments. In the case of passive measurement, it estimates the target's motion characteristics based on the noise contaminated azimuth data measured by the sensor. Its advantage is that it can observe the target in the hidden state, estimating the motion parameters of the target, so that it can secretly plan and attack the target. Therefore, bearings only target motion analysis is the most basic target tracking method in underwater target attack, and it is also the most challenging recognized problem in the field of target tracking algorithm.

The research hotspot of bearings only target motion analysis methods is focused on two kinds of methods: batch processing and recursive estimation. Due to the simple model and small data storage, recursive estimation is especially suitable for computer applications, and has become the development direction and research hotspot of bearings only target motion algorithm [1-4]. Because the single station bearing only target tracking is a nonlinear model, the effect of linear Kalman filter is not good. The recursive estimation methods used in this area are: Extended Kalman filter (EKF) [5], unscented Kalman filter (UKF) [6], particle filter [7] and its improved algorithm [8, 9], etc.

However, the common difficulty of recursive estimation is that it is very dependent on the starting point of the estimation, and the accuracy of the starting point has a great influence on the estimation of the moving elements of the target; in addition, the noise is easy to gradually expand in the iterative process, resulting in the situation that the filtering process does not converge or the convergence is slow. To reduce the error of state prediction and improve the accuracy of this algorithm, Refs. [10-15] use various backward smoothing methods to reverse predict the state of $k$-1 from the state estimation of $k$, and then reverse modify the state estimation of $k$-1, so as to recurrence the state estimation of $k$-1. On the idea of this kind of algorithm, an improved method is proposed to segment the starting point and
filtering process of inverse smoothing. By optimizing the local key points in segments, the error of target motion trajectory estimation can be reduced.

2. Single Station Bearings Only Motion Mode

Only two-dimensional plane motion of the target is considered. And the target moves in a uniform straight line. The coordinate system is shown in figure 1. The initial position of the observation station is at the origin of the coordinate, and the target moves in the xOy plane.

As shown in figure 1, \(D_0\) is the initial distance of the target, \(K_m\) is the heading of the target, \(V_m\) is the speed of the target, \(\beta_0, \beta_1, \ldots, \beta_i\) is the target orientation corresponding to time \(t_0, t_1, \ldots, t_i\), \((x_{wi}, y_{wi})\) is the coordinate corresponding to position \(O_i\) of the observation platform at time \(t_i\), and \(D_i\) is the distance between the target corresponding to time \(t_i\) and the observation platform. Obviously, for a target moving in a uniform straight line, \((D_0, K_m, V_m, \beta_0)\) can uniquely determine the trajectory of the target.

Define \(V_{mx}\) as the component of target velocity \(V_m\) in x axis, \(V_{my}\) as the component of target velocity \(V_m\) in y axis, \(x_{m0}\) as x axis coordinate of target initial position, \(y_{m0}\) as y axis coordinate of target initial position. Then the following formula can be obtained:

\[
\begin{align*}
V_{mx} &= V_m \sin K_m \\
V_{my} &= V_m \cos K_m \\
x_{m0} &= D_0 \sin \beta_0 \\
y_{m0} &= D_0 \cos \beta_0
\end{align*}
\]  

(1)

Obviously, \((V_{mx}, V_{my}, x_{m0}, y_{m0})\) can also uniquely determine the target’s trajectory. Target motion analysis is the process of analyzing and solving parameters \((D_0, K_m, V_m, \beta_0)\) or \((V_{mx}, V_{my}, x_{m0}, y_{m0})\) or their subsets. When the target is moving in a straight line at a constant speed, the trajectory can be uniquely determined according to \((D_0, K_m, V_m)\) when \(\beta_0\) is known. Bearings only target motion analysis is the process of estimating the target motion trajectory by using the known quantities such as the azimuth observation sequence \((\beta_0, \beta_1, \ldots, \beta_i)\) and the position coordinates \((x_{wi}, y_{wi})\) of the observation platform at the time of \(t_i\) to solve \((D_0, K_m, V_m)\) or \((V_{mx}, V_{my}, x_{m0}, y_{m0})\).

The target state is expressed as: \(X(k) = [x_m \quad v_x \quad y_m \quad v_y]^T\). For bearings only target motion analysis, the target view measurement is the azimuth of the target, which is expressed as: \(Z(k) = \beta_k\).

The state space model of the system is:

\[
X(k+1) = \Phi X(k) + \Gamma w(k) \tag{2}
\]

\[
Z(k) = HX(k) + v(k) \tag{3}
\]
Among them, \( \Phi = \begin{bmatrix} 1 & T_0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) is the state transition matrix; \( \Gamma = \begin{bmatrix} \frac{T_0^2}{2} & 0 \\ T_0 & 0 \\ 0 & \frac{T_0^2}{2} \\ 0 & T_0 \end{bmatrix} \) is the noise transition matrix. \( H \) is the observation matrix, which is related to the selection of observation measurement. \( w(k) \) is the system noise with mean value of 0 and variance of \( Q \). And \( v(k) \) is the observation noise with mean value of 0 and variance of \( R \). Equation (2) is the state equation of the system, and (3) is the observation equation.

When the observation azimuth is observed, the observation equation can be written as follows:

\[
Z(k) = \arctan \frac{y_m(k) - y_w(k)}{x_m(k) - x_w(k)} + v(k)
\] (4)

3. Unscented Kalman Filter

Obviously, the observation equation applied to bearings only target motion analysis is nonlinear. There are two ways to solve the problem of nonlinear filtering with Kalman filtering: one is to linearize the nonlinear part of the system equation and ignore or approach the higher-order term; the other is to linearize the nonlinear distribution and nonlinear function by sampling.

The UKF adopts the second kind of approach mentioned above. The sampling form is deterministic sampling, not random sampling of particle filter. The method of unscented transformation (UT) is used to realize the nonlinear transfer of mean and covariance.

3.1. Unscented Transformation

The idea of UT is: in the original state distribution, deterministic sampling is carried out in a certain way to ensure that the selected sampling points have the same mean and variance as the original state. These selected sampling points are brought into the nonlinear function, and the corresponding point set is obtained. Then use these point sets to find the mean and covariance after transformation.

The algorithm framework of UT is as follows (take the unscented transformation of symmetric distribution sampling as an example). Let \( y = f(x) \) be a nonlinear transformation, and the state vector \( x \) be a random variable of \( n \) dimension with mean \( \bar{x} \) variance \( P \).

3.1.1. \( 2n + 1 \) Sampling Points are Obtained According to a Sampling Strategy. That is, the set of sigma points \( \{X^{(i)}\}, i = 0, \cdots, 2n \). In symmetric sampling, the sampling strategy is:

\[
\begin{cases}
X^{(0)} = \bar{x}, i = 0 \\
X^{(i)} = \bar{x} + \sqrt{(n + \lambda)P}, i = 1, \cdots, n \\
X^{(i)} = \bar{x} - \sqrt{(n + \lambda)P}, i = n + 1, \cdots, 2n
\end{cases}
\] (5)

Among them, \( \sqrt{(n + \lambda)P} \) is the \( i \)th column of \( (n + \lambda)P \). \( \lambda = \alpha^2 (n + \kappa) - n \) is used to adjust the distance from each point to the mean value and reduce the total prediction error. The \( \alpha \) determines the distribution of sigma points. \( \kappa \) is a parameter to be selected with no limit value. Generally, the \((n + \lambda) P\) should be semi positive matrix.

3.1.2. Calculate the Corresponding Weights of Sampling Points. According to the Sampling Strategy, the weights of corresponding step-by-step sampling are
\begin{equation}
\begin{cases}
\omega_m^{(0)} = \frac{\lambda}{\lambda+n+\lambda} \\
\omega_c^{(0)} = \frac{\lambda}{\lambda+n+\lambda} + (1 - \alpha^2 + \beta) \\
\omega_c^{(i)} = \omega_c^{(i)} = \frac{\lambda}{2(\lambda+n+\lambda)}, i = 1, \ldots, 2n
\end{cases}
\end{equation}

Among them, the value of subscript \( m \) is the weight used for mean weighting, the value of subscript \( c \) is the weight used for covariance weighting, and \( i \) means the \( i \)th sampling point. \( \beta \) is a non-negative parameter to be selected. It is used to introduce the dynamic difference of the higher-order term in \( f(\cdot) \). When the information of the higher-order term is not used, its value is 2.

3.1.3. The Nonlinear Transformation of \( f(\cdot) \) Is Applied to Every Point in the Sampled Sigma Point Set \( \{X^{(i)}\} \). And the transformed point set \( \{Y^{(i)}\} \) is obtained.

\begin{equation}
Y^{(i)} = f(X^{(i)}), i = 0, \ldots, 2n
\end{equation}

3.1.4. The Point Set \( \{Y^{(i)}\} \) is Weighted after the Transformation. The weights are the corresponding \( \omega_m^{(i)} \) and \( \omega_c^{(i)} \), as follows:

\begin{align}
\bar{Y} &= \sum_{i=0}^{2n} \omega_m^{(i)} Y^{(i)} \\
\bar{P}_{YY} &= \sum_{i=0}^{2n} \omega_c^{(i)} (Y^{(i)} - \bar{Y})(Y^{(i)} - \bar{Y})^T
\end{align}

In this way, the sigma points obtained by traceless transformation have the following properties: the sample mean value and the sample variance of the sigma point set are the same as those of the state vector.

3.2. Steps of Unscented Kalman Filter
UKF is mainly divided into two parts. One is to update the state, and the other is to update the observation. The UKF can be constructed according to the nonlinear system described in (2) and (3). The basic processes of UKF filtering at time \( k \) are as follows:

According to (5) and (6), the sampling points at \( k \) time, and their corresponding weights are obtained.

\begin{equation}
X^{(i)}(k|k) = \left[ \begin{array}{c} \hat{X}(k|k) + \sqrt{(n+\lambda)}P(k|k) \\ \hat{X}(k|k) - \sqrt{(n+\lambda)}P(k|k) \end{array} \right]^{T}
\end{equation}

The equation of state was used for prediction:

\begin{equation}
X^{(i)}(k+1|k) = \Phi X^{(i)}(k|k)
\end{equation}

where \( i = 0, \ldots, 2n \).

Calculate the prediction mean value and covariance matrix according to equation (8).

\begin{align}
\hat{X}(k+1|k) &= \sum_{i=0}^{2n} \omega_m^{(i)} X^{(i)}(k+1|k) \\
\bar{P}(k+1|k) &= \sum_{i=0}^{2n} \omega_c^{(i)} \left( X^{(i)}(k+1|k) - \hat{X}(k+1|k) \right)^2 + Q^T
\end{align}

According to the predicted value, the new sigma point set is obtained by using traceless transformation again.
\[ X^{(i)}(k+1|k) = \begin{bmatrix} \hat{X}(k+1|k) \\ \hat{X}(k+1|k) + \sqrt{(n+\lambda)}P(k+1|k) \\ \hat{X}(k+1|k) - \sqrt{(n+\lambda)}P(k+1|k) \end{bmatrix}^T \]  

According to the new sigma point set obtained in (13), the observed measurement is predicted by substituting equation (3):

\[ Z^{(i)}(k+1|k) = H X^{(i)}(k+1|k) \]  

The mean value and covariance of the prediction can be calculated according to the observed measurement prediction value obtained in (14).

\[ \hat{Z}(k+1|k) = \sum_{i=0}^{2n} \omega_i^{(i)} Z^{(i)}(k+1|k) \]  

\[ P_{ZZ}(k+1|k) = \sum_{i=0}^{2n} \omega_i^{(i)} \left( Z^{(i)}(k+1|k) - \hat{Z}(k+1|k) \right)^T \]  

\[ P_{XZ}(k+1|k) = \sum_{i=0}^{2n} \omega_i^{(i)} \left( X^{(i)}(k+1|k) - \hat{X}(k+1|k) \right)^T \]  

According to the covariance prediction, the Kalman gain matrix is calculated.

\[ K(k+1) = P_{XZ}(k+1|k)P_{ZZ}^{-1}(k+1|k) \]  

System status update:

\[ \hat{X}(k+1|k+1) = \hat{X}(k+1|k) + K(k+1)[Z(k+1) - \hat{Z}(k+1|k)] \]  

System covariance update:

\[ P(k+1|k+1) = P(k+1|k) - K(k+1)P_{ZZ}(k+1|k)K^T(k+1) \]  

According to the above steps, the unscented Kalman filter can be estimated. It can be seen that unscented Kalman filter performs ut transformation near the estimated points, so that the mean value and variance of the obtained sigma point set are the same as the original statistical characteristic strip. The state probability density function is approximated by nonlinear mapping of sigma point set. This is a statistical approximation, not a solution of the equation.

4. Improved Method

Due to the recursive nature of Unscented Kalman filter, it is very sensitive to the selection of initial value, and the latter filter depends on the result of the former filter. Therefore, a segmented inverse smoothing method is proposed, which can replace the initial points in the original data by segmented inverse filtering, so as to modify some points in the filtering and reduce the error of filtering results.

4.1. Principle of Inverse UKF Filtering

Expand equation (2):
For a uniform linear moving target, it can be considered that the target speed is constant, that is:

\[
\begin{align*}
    v_x(k + 1) &= v_x(k) \\
    v_y(k + 1) &= v_y(k)
\end{align*}
\]  

(22)

(23)

Move the left and right of equation (21) and substitute equations (22) and (23) into the collation to get:

\[
\begin{align*}
    x_m(k + 1) - T_0 v_x(k + 1) - \frac{T_0^2}{2} w_x &= x_m(k) \\
    v_x(k + 1) - T_0 w_x &= v_x(k) \\
    y_m(k + 1) - T_0 v_y(k + 1) - \frac{T_0^2}{2} w_y &= y_m(k) \\
    v_y(k + 1) - T_0 w_y &= v_y(k)
\end{align*}
\]  

(24)

Equation (24) can be expressed as a matrix:

\[
\Psi X(k + 1) - \Gamma w(k) = X(k)
\]  

(25)

Among them:

\[
\Psi = \begin{bmatrix}
1 & -T_0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -T_0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(26)

\[
\Gamma = \begin{bmatrix}
\frac{T_0^2}{2} & 0 \\
0 & T_0 \\
0 & \frac{T_0^2}{2} \\
0 & T_0
\end{bmatrix}
\]

(27)

A new set of state space models can be obtained by combining equation (25) with equation (3). This state space model satisfies the nonlinear system model required by unscented Kalman filter. This spatial model is used in the recursive process of UKF, which has theoretical basis. According to this new state space model, the target state is recursive from \(K\) time to \(k\)-1 time. The process is as follows:

According to equation (9), obtain the sigma point set sampling at time \(k\), and obtain the corresponding weight according to equation (6).

Carry out one-step inverse prediction of sigma point set, and calculate the prediction mean and covariance:

\[
X^{(i)}(k - 1|k) = \Psi X^{(i)}(k|k) , i = 0, \cdots , 2
\]  

(28)

\[
\bar{X}(k - 1|k) = \sum_{i=0}^{2n} \omega_m^{(i)} X^{(i)}(k - 1|k)
\]

(29)

\[
P(k - 1|k) = \sum_{i=0}^{2n} \omega_c^{(i)} \left( X^{(i)}(k - 1|k) - \bar{X}(k - 1|k) \right)^T + Q
\]

(30)
After UT transformation, a new sigma point set is obtained

\[
X^{(i)}(k-1|k) = \begin{bmatrix} \tilde{X}(k-1|k) \\ \tilde{X}(k-1|k) + \sqrt{(\alpha + \lambda)P(k-1|k)} \\ \tilde{X}(k-1|k) - \sqrt{(\alpha + \lambda)P(k-1|k)} \end{bmatrix}^T \tag{31}
\]

Predicted observations and their covariance:

\[
Z^{(i)}(k-1|k) = H X^{(i)}(k-1|k) \tag{32}
\]

\[
\tilde{Z}(k-1|k) = \sum_{i=0}^{2n} \omega_c^{(i)} (Z^{(i)}(k-1|k) - \tilde{Z}(k-1|k)) \tag{33}
\]

\[
P_{ZZ}(k-1|k) = \sum_{i=0}^{2n} \omega_c^{(i)} (Z^{(i)}(k-1|k) - \tilde{Z}(k-1|k))^T + R \tag{34}
\]

\[
P_{XZ}(k-1|k) = \sum_{i=0}^{2n} \omega_c^{(i)} (X^{(i)}(k-1|k) - \tilde{X}(k-1|k))^T \tag{35}
\]

System status update and covariance update:

\[
K(k-1) = P_{XZ}(k-1|k) P_{ZZ}^{-1}(k-1|k) \tag{36}
\]

\[
\tilde{X}(k-1|k-1) = \tilde{X}(k-1|k) + K(k-1)[Z(k-1) - \tilde{Z}(k-1|k)] \tag{37}
\]

\[
P(k-1|k-1) = P(k-1|k) - K(k-1)P_{ZZ}(k-1|k)K^T(k-1) \tag{38}
\]

Through the above steps, the reverse UKF recursion of the target state from K time to k-l time can be completed. Repeat this process for m times to realize the reverse estimation of the target state from k-time to K-M time. Since equation (25) for the inverse UKF is derived from the system state equation, the process is of theoretical significance. Through the inverse UKF filtering, the positive and negative bidirectional application of data is increased, which has a positive effect on reducing system noise.

4.2. Piecewise Inverse Smoothing Filter

Based on UKF filter, the segmented inverse smoothing filter proposed in this paper divides the target state \(\tilde{X}(0), \tilde{X}(1), \ldots, \tilde{X}(n)\) and observation value \(Z(0), Z(1), \ldots, Z(n)\) into segments every l points. The inverse UKF filtering is carried out in sections, and the predicted value of the target state at the starting point of the section is replaced by the original target state. Taking the target state of the new starting point in the segment as the initial value, UKF filtering in the segment is carried out again to get a new target state prediction. The specific steps are as follows:

According to the initial state \(\tilde{X}(0)\) and the observed measurement \(Z(0), \ldots, Z(l)\) of the target, 1 iterations of UKF filtering are carried out to obtain the target state estimation \(\tilde{X}(1), \ldots, \tilde{X}(l)\);

Take \(\tilde{X}(l)\) and \(Z(l)\) as initial values, substitute them into (26) to (38), perform inverse UKF filtering, repeat iterations l times, and get new target state estimates \(\tilde{X}'(0), \ldots, \tilde{X}'(l-1)\);

Replace the original target state \(\tilde{X}(0)\) with the new target state \(\tilde{X}'(0)\) as the new initial value, perform UKF filtering again, and get \(\tilde{X}''(1), \ldots, \tilde{X}''(l)\);
Take $\hat{X}''(l)$ as the initial state, and according to the $Z(l), \cdots, Z(2l)$, repeat the above three steps to get $\hat{X}''(l+1), \cdots, \hat{X}''(2l)$;

Repeat segmentation and reverse smoothing, and the final result of target state estimation is $(\hat{X}''(0), \hat{X}''(1), \cdots, \hat{X}''(n))$.

This kind of segmented inverse smoothing can reduce the influence of some noises by inverse UKF filtering. Some key nodes in the data are replaced by segmented reverse smoothing, which increases the forward and reverse application of observation data. The real-time performance of trajectory optimization can be adjusted by adjusting the segment length $L$.

5. Simulation Test

To verify the performance of the proposed method, the following simulations are carried out in different simulation environment conditions.

Condition 1: the target moves in a uniform straight line with the initial position coordinate of (100 m, -400 m); the target moves at a speed of 50 knots and a heading of -41 degrees. The initial position of the observer (0 m, 0 m) moves in a uniform straight line at a speed of 45 knots in the direction of -36 degrees for 50 s, and then in a uniform straight line at a speed of 45 knots in the direction of -63 degrees for 100 s. The sampling frequency of observer azimuth estimation is 10 Hz, and the standard deviation of azimuth estimation error is 2. The initial distance estimation error is 10%, and the initial velocity vector estimation error is 10%. The UKF and the method of piecewise reverse smoothing Kalman filter are respectively used to carry out 100 Monte Carlo simulation tests and get the average value of 100 test errors.

The simulation results are shown in figures 2-5. After 100 experiments, it can be seen from the mean value of the error results that, under this simulation condition, the piecewise inverse smoothing Kalman filter method has obvious optimization effect on UKF. The proposed method reduces the estimation error in the estimation of target position coordinates, target velocity, target heading and target distance. Especially for the peak value of each index error, the effect of reducing is very obvious, which has an obvious effect on reducing the error range.

Figure 2. Position error comparison under condition 1.
To verify the effectiveness of the proposed method in various other conditions, especially in harsh conditions, the simulation conditions were added and 100 Monte Carlo simulation tests were carried out. To verify the influence of single index on the optimization results, the initial distance error, standard
deviation of observation angle and initial velocity (vector) error are added on the basis of condition 1. The new simulation conditions are as follows:

Condition 2: increase the initial distance error of observation to 20% on the basis of condition 1, and other conditions remain unchanged.
Condition 3: Based on condition 1, the standard deviation of observation angle error is increased to 5, other conditions remain unchanged.
Condition 4: on the basis of condition 1, the error of initial velocity vector is increased to 20%, the heading error will increase with the increase of velocity vector error, and other conditions will not change.

Average the errors of the two methods under various conditions within 150 s, count the average errors of the two methods respectively, and calculate the reduction rate of UKF error of the proposed method, and make the following table for comparison.

Comparing the average errors of the two methods in table 1, the proposed method has better optimization effect on UKF algorithm when the observation angle, initial distance error and initial speed error increase. In terms of the reduction rate of each index, the optimization effect of the proposed method on the target course estimation is more obvious, the maximum error can be reduced by 41.9%, and the judgment of the course is more accurate.

| Simulation condition | Condition 1 | Condition 2 | Condition 3 | Condition 4 |
|----------------------|-------------|-------------|-------------|-------------|
| Standard deviation of observation angle (°) | 2           | 2           | 5           | 2           |
| Initial distance error (%) | 10          | 20          | 10          | 10          |
| Initial speed error (%) | 10          | 10          | 10          | 20          |
| Average position error (m) | UKF 68.68   | 83.40       | 108.26      | 84.09       |
| After optimization | 56.54       | 69.85       | 98.26       | 73.88       |
| Reduction rate | 17.7%       | 16.2%       | 9.2%        | 12.1%       |
| Average speed error (m/s) | UKF 2.21    | 2.42        | 3.06        | 2.82        |
| After optimization | 1.78        | 1.94        | 2.74        | 2.43        |
| Reduction rate | 19.2%       | 19.9%       | 10.6%       | 13.9%       |
| Average heading error (°) | UKF 1.02    | 1.34        | 1.83        | 1.17        |
| After optimization | 0.59        | 0.88        | 1.33        | 0.76        |
| Reduction rate | 41.9%       | 34.7%       | 27.8%       | 35.2%       |
| Average distance error (m) | UKF 62.53   | 77.03       | 93.25       | 77.07       |
| After optimization | 51.00       | 64.45       | 83.63       | 67.29       |
| Reduction rate | 18.4%       | 16.3%       | 10.3%       | 12.7%       |

From different simulation conditions, under condition 1, the reduction rate of each error is lower, and the optimization effect is more obvious. The proposed method is not sensitive to the increase of the initial distance error, and can reduce the average error by more than 16%. The standard deviation of observation angle has a great influence on the results. When the standard deviation of observation angle increases, the errors of both methods increase, and the reduction rate after optimization is low, but it also remains above 9.2%.

6. Conclusion

Based on the average data of simulation tests under different conditions, the proposed piecewise inverse smoothing Kalman filter can reduce the position error, velocity error, heading error and distance error compared with the unscented Kalman filter. Moreover, when the standard deviation of observation
angle increases, the error of initial distance increases, and the error of initial velocity increases, the methods proposed can still be effective, reduce the errors of UKF, and improve the estimation accuracy. The complexity of this method is not large, and the real-time performance can be adjusted by adjusting the segment length according to the actual situation.

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