Intermittent dislocation flow in viscoplastic deformation

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The viscoplastic deformation (creep) of crystalline materials under constant stress involves the motion of a large number of interacting dislocations [1]. Analytical methods and sophisticated ‘dislocation-dynamics’ simulations have proved very effective in the study of dislocation patterning, and have led to macroscopic constitutive laws of plastic deformation [2–9]. Yet, a statistical analysis of the dynamics of an assembly of interacting dislocations has not hitherto been performed. Here we report acoustic emission measurements on stressed ice single crystals, the results of which indicate that dislocations move in a scale-free intermittent fashion. This result is confirmed by numerical simulations of a model of interacting dislocations that successfully reproduces the main features of the experiment. We find that dislocations generate a slowly evolving configuration landscape which coexists with rapid collective rearrangements. These rearrangements involve a comparatively small fraction of the dislocations and lead to an intermittent behavior of the net plastic response. This basic dynamical picture appears to be a generic feature in the deformation of many other materials [10–12]. Moreover, it should provide a framework for discussing fundamental aspects of plasticity, that goes beyond standard mean-field approaches that see plastic deformation as a smooth laminar flow.

Whenever dislocation glide is the dominant plastic deformation mechanism in a crystalline material, we observe a constant strain-rate regime usually described by Orowan’s relation $\dot{\gamma} \sim \rho_m b v$. Here, the plastic strain-rate of the material $\dot{\gamma}$ is simply related to average quantities such as $\rho_m$, the density of mobile dislocations, and $v$, their average velocity along the slip direction (parallel to the Burgers vector $b$) [1]. Transmission electron micrographs of plastically deformed materials display, on the other hand, complex features such as cellular structures and fractal patterns [2–3], which are the fingerprint of a complex multiscale dynamics not appropriately accounted for by the mean-field character of Orowan’s relation. In addition, rapid slip events [10] have been observed in the plastic deformation of various metals and alloys [11–12], and in the Portevin-LeChatelier effect [13]. We believe that formulating plastic deformation as a nonequilibrium statistical mechanics problem [14] requires a substantial understanding of basic collective dislocation dynamics.

Experimentally, the complex character of collective dislocation dynamics can be revealed by acoustic emission measurements. The acoustic waves recorded in a piezoelectric transducer disclose the pulse-like changes of the local displacements in the material during plastic deformation, whereas a smooth plastic flow would not be detected [15]. Thus, this method is particularly useful for inspecting possible fluctuations in the dislocation velocities and densities.

Ice single crystals can be used as a model material to study glide dislocation dynamics by acoustic emission [16–17] due to the following reasons: (i) Transparency allows direct verification that acoustic emission activity is not related to microcracking. (ii) Within the range of temperature and stress corresponding to our experimental conditions, fusional creep is not a significant mechanism of inelastic deformation [18] which, in hexagonal ice single crystals, occurs essentially by dislocation glide on the basal planes along a preferred slip direction. (iii) An excellent coupling between sample and transducer can be obtained by fusion/freezing.

Uniaxial compression creep experiments are performed on artificial ice single crystals, employing several steps of constant applied stress. We observe an intense acoustic activity, exhibiting a strong intermittent character (see the inset of Figure 1). We measure the energy associated to each acoustic burst and analyze its statistical properties. In Fig. 1 we show that the probability distribution of energy burst intensities exhibits a power law behavior spanning several decades. This fact is an indication that a large number of dislocations move cooperatively in an intermittent fashion. A similar behavior has been observed in the Portevin-
LeChatelier effect \[13\], a plastic instability where the intermittent flow is ruled by the interaction of dislocations and diffusing solute atoms. The intermittency observed in our case is different as we only have interacting dislocations subject to an external stress, without any other alien element interfering with their dynamics.

![Graph](image)

**FIG. 1.** Statistical properties of the acoustic energy bursts recorded in ice single crystals under constant stress. The main figure shows the distribution of energy bursts for the different loading steps. The cutoffs depend slightly on the applied stress, but the power law exponent remains the same. The fit yields an exponent $\tau_E = 1.60 \pm 0.05$. Inset, a typical recorded acoustic signal. The creep experiment is performed at $T = -10^\circ C$ under different constant compression stresses ranging from $\sigma = 0.58\text{MPa}$ to $\sigma = 1.64\text{MPa}$. The stress is applied at an angle with respect to the $c$ axis, giving rise to a small resolved shear stress acting on the glide plane ($\sigma = 0.030\text{MPa} - 0.086\text{MPa}$). The frequency bandwidth of the transducer was 10 to 100kHz. The amplitude range between the minimum and the maximum recordable thresholds is 70dB, that is, a range of seven orders of magnitude in energy.

Multiscale properties and pattern formation are ubiquitous in plastic materials and we expect that the large dynamical fluctuations observed in ice compression experiments are also a prevalent feature of plastic deformation micromechanics. This general picture implies that the experimental phenomenology should be reproducible in simple numerical simulations of discrete dislocation dynamics that preserve the relevant characteristics of the system under study. As in previous dislocation dynamics models \[14\], \[15\], we consider a two-dimensional cross-section of the crystal (that is, the $xy$ plane), and randomly place $N$ straight-edge dislocations gliding along a single slip direction parallel to their respective Burgers vectors $b$ (that is, the $x$ direction). This implies that we have point-like dislocations moving along fixed lines parallel to the $x$ axis. This simplification effectively describes materials like ice crystals that, owing to their strong plastic anisotropy, deform by glide on a single slip system. An edge dislocation with Burgers vector $b$ located at the origin gives rise to a shear stress $\sigma_s$ at a point $r = (x, y)$ of the form

$$\sigma_s(r) = \frac{b\mu}{2\pi(1-\nu)}\frac{x(x^2-y^2)}{(x^2+y^2)^2}, \quad (1)$$

where $\mu$ is the shear modulus and $\nu$ is the Poisson ratio \[3\]. This long-range stress is responsible for mutual interactions among dislocations which are important in all dislocation dynamics models \[14\], \[15\]. Under a constant external stress $\sigma_e$, dislocation $i$ performs an overdamped motion along the $x$ direction described by the following equation

$$\eta \frac{dx_i}{dt} = b_i(\sum_{m \neq i} \sigma_s(r_m - r_i) - \sigma_e), \quad (2)$$

where $\eta$ is the effective friction and $b_i$ is the Burgers vector. Throughout the simulations, we will be dealing with dimensionless quantities after setting the distance scale $b = 1$, and the time scale $t_o \equiv \eta b / (\mu/2\pi(1-\nu)) = 1$.

Other essential ingredients commonly introduced in most dislocation dynamics models \[14\], \[15\] are the mechanisms for dislocation (i) annihilation and (ii) multiplication. (i) When the distance between two dislocations is of the order of a few Burgers vectors, the high stress and strain conditions close to the dislocation core invalidate the results obtained from a linear elasticity theory (equation (1)). In these instances, phenomenological nonlinear reactions describe more accurately the real behavior of dislocations in a crystal. In our model, we annihilate two dislocations with opposite Burgers vectors when the distance between them is shorter than $2b$.

(ii) The activation of Frank-Read sources \[16\] is accepted as the most relevant dislocation multiplication mechanism under creep deformation. The activation of Frank-Read sources have been observed in ice crystals, along with more specific multiplication processes \[16\]. Because an accurate multiplication mechanism cannot be simulated in a two-dimensional point-dislocation model, we have implemented various phenomenological procedures. A simple way of taking into account the new dislocation loops generated at different sources within the crystal is the random introduction of opposite sign dislocation pairs in the cross-section under consideration. The rate and frequency of this creation process depend solely on the external stress $\sigma_e$. We have checked that other multiplication rules in which the creation rate depends on the local stress yield similar results.
FIG. 2. *Snapshot of the total stress field and dislocations arrangement in a numerical simulation with* $v_\sigma = 0.025$.

We observe metastable structure formation (dipoles and walls) and the associated stress field which goes from light blue (low values) to dark blue (high values). The complex low-stress pathways joining dislocations are the result of the anisotropic elastic interactions. Dislocations moving at low velocities (lower than $v_\sigma$) are depicted in magenta, while those moving at higher velocities are depicted in black. Most dislocations in walls are moving slowly, whereas isolated ones tend to move at higher speed. In the simulations, an initial number of $N_0 = 1500$ dislocations are randomly distributed on a square cell of size $L = 300$. Their Burgers’ vectors are randomly chosen to be $+b$ or $-b$ with equal probability. In the absence of external stress, we first let the system relax until it reaches a metastable arrangement. The number of remaining dislocations is then $N \sim 700$. At this point, we apply a constant shear stress and study their evolution. To avoid the discontinuities arising from truncating long-range elastic interactions in Eq. (1), we resort to the Ewald summation method. We have thus exactly taken into account the interaction of a dislocation with all the infinite periodic images of another dislocation in the same cell.
by each moving dislocation \cite{15}. In particular, it is due to the superposition of the waves emitted the acoustic signal that is detected experimentally. The high amplitude pulses detected in the experiment cannot be ascribed to uncorrelated emissions from each individual dislocation, but rather to the cooperative motion of several dislocations occurring, for example, after the activation of a Frank-Read source, or if a dislocation cluster breaks apart. To gain further insight into this behavior, in Figures 3(b) and 3(c), we show the evolution of the total number of dislocations as well as that of the fast moving dislocations, that is, dislocations moving faster than if they were moving under the only action of the external stress, \( v_i > v_\sigma \) (other threshold values yield equivalent results). After the injection of a few new dislocations or the annihilation of a dislocation pair, several other dislocations start to move and rearrange, not necessarily in the close vicinity of the triggering event. To quantify this effect, we measure the collective velocity \( V = \sum_{i} v_i \) of the fast moving dislocations and define the acoustic energy as \( E = V^2 \) \cite{17}. In the inset of Figure 4, we see that the signal \( E(t) \) consists of a succession of intermittent and pronounced bursts, each one signalling the onset of collective dislocation rearrangements. The slow and continuum motion of dislocation structures \( (v_i < v_\sigma) \) is not considered as it only sets up a background noise signal which cannot be detected experimentally. In Fig. 4 we show that the distribution of energy bursts, obtained by sampling the signal over different times and realizations, decays very slowly, spanning various decades. For intermediate values, the distribution shows an algebraic decay with an exponent \( \tau_E = 1.8 \pm 0.2 \), in reasonable agreement with experiments (see Fig. 3). The maximum number of dislocations we can handle in our simulations severely restricts the maximum value of the signal in the system. Consequently, the extension of the power law regime grows with the number of dislocations, but is eventually bounded by the different nature of the extremes statistics.

The broadly distributed intermittency resulting from the statistical analysis can be interpreted as a nonequilibrium transport phenomenon with scaling properties. Scaling behavior is usually associated with the proximity of a critical point and is characterized by a high degree of universality. Critical exponents only depend on a few fundamental parameters such as the effective dimensionality, and the basic symmetries of the system. Specifically, the reasonable quantitative agreement between our model and the experimental data is due to the strong plastic anisotropy present in ice single crystals, that can thus be well described by a two dimensional model. Although the quantitative results we obtain should be restricted to the case of single slip systems, the general features of the observed inter-

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FIG. 3. Statistical properties of dislocation velocities and density obtained in numerical simulations. a) Probability density of the individual dislocation velocities \( |v_i| \) in the stationary state. The black line is a least-squares fit of the intermediate data points. We obtain an exponent \( \nu = 2.5 \) for the scaling of the intermediate velocities. b) Time evolution of the total number of dislocations in the cell. c) Fraction of fast moving dislocations for a given run of the simulations.

In the initial stage of the simulations, the system relaxes slowly and the average strain-rate decreases until it reaches a constant value which depends on the applied stress, that is the stationary creep regime. By monitoring the activity in a single run, we observe that most dislocations are arranged into metastable structures (such as walls and cells) moving at a very slow rate (see Figure 2 and the Supplementary Information). A smaller fraction of dislocations, however, move intermittently at much higher velocities, giving rise to sudden increases of the plastic strain. In Figure 3(a), we plot the probability density of finding an individual dislocation moving with a velocity \( |v_i| \). Already at the level of individual dislocations, the velocity distribution \( P(|v_i|) \) is quite broad and exhibits power law behavior for velocity values larger than the external stress induced velocity \( v_\sigma = b\sigma_e / \eta \) (we have considered two values of \( v_\sigma \) : 0.0125 and 0.025).

In the presence of a large number of dislocations, the acoustic signal that is detected experimentally is due to the superposition of the waves emitted by each moving dislocation \cite{15}. In particular, it has been shown that a single dislocation performing a sudden movement with velocity \( v_0 \) for a short time \( \tau \) gives rise to an acoustic wave whose amplitude is proportional to \( v_0 \) \cite{15}. The high amplitude pulses detected in the experiment cannot be ascribed to uncorrelated emissions from each individual dislocation, but rather to the cooperative motion of several dislocations occurring, for example, after the activation of a Frank-Read source, or if a dislocation cluster breaks apart. To gain further insight into this behavior, in Figures 3(b) and 3(c), we show the evolution of the total number of dislocations as well as that of the fast moving dislocations, that is, dislocations moving faster than if they were moving under the only action of the external stress, \( v_i > v_\sigma \) (other threshold values yield equivalent results). After the injection of a few new dislocations or the annihilation of a dislocation pair, several other dislocations start to move and rearrange, not necessarily in the close vicinity of the triggering event. To quantify this effect, we measure the collective velocity \( V = \sum_{i} v_i \) of the fast moving dislocations and define the acoustic energy as \( E = V^2 \). In the inset of Figure 4, we see that the signal \( E(t) \) consists of a succession of intermittent and pronounced bursts, each one signalling the onset of collective dislocation rearrangements. The slow and continuum motion of dislocation structures \( (v_i < v_\sigma) \) is not considered as it only sets up a background noise signal which cannot be detected experimentally. In Fig. 4 we show that the distribution of energy bursts, obtained by sampling the signal over different times and realizations, decays very slowly, spanning various decades. For intermediate values, the distribution shows an algebraic decay with an exponent \( \tau_E = 1.8 \pm 0.2 \), in reasonable agreement with experiments (see Fig. 3). The maximum number of dislocations we can handle in our simulations severely restricts the maximum value of the signal in the system. Consequently, the extension of the power law regime grows with the number of dislocations, but is eventually bounded by the different nature of the extremes statistics.

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intermittent flow regime appear to be generic in plastic deformation \[1,2\]. The particular universality class will depend on the effective dimensionality of each specific material as well as the particular deformation mechanism, and one can still expect to obtain the right critical exponents from simplified models if the relevant symmetries are correctly taken into account.

The close-to-criticality nonequilibrium framework motivates possible analogies with elastic manifolds driven in random media such as fluid flow in porous media, vortices in superconductors, and charge density waves \[20\]. In all of these systems, a critical value of the driving force separates a static regime from a moving one, and scaling is observed only close to this point. The intermittent behavior close to criticality is in these cases associated to static random heterogeneities which exert space dependent pinning forces on the moving objects. On the contrary, the present dislocation dynamics model, as well as the ice experiment, does not contain any quenched source of pinning forces. Dislocation themselves, through the various structures such as dipoles and walls, are self-generating a pinning force landscape in which the dynamics is virtually frozen; i.e. a slow dynamics state. Creation and annihilation mechanisms, often triggered by the presence of unsettled dislocations, allow the system to jump between slow dynamics states through bursts of activity. This behavior is reminiscent of driven-dissipative self-organizing systems that achieve criticality in the limit of a very slow driving \[21\].

The emerging scenario poses many basic theoretical questions. Is there a critical stress value below which the system decays into a slow dynamics state? Can the large dynamical fluctuations be associated with a diverging response function and thus, be related to a critical point? Can we derive scaling laws relating the various observed exponents as suggested by the theory of critical phenomena? The answers to all these questions pave the way to the nonequilibrium statistical theory of dislocation motion that is needed for a deeper understanding of the micromechanics of plastic deformation.

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