THE STRONG COUPLING CONSTANT AT LOW $Q^2$

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We extract an effective strong coupling constant using low-$Q^2$ data and sum rules. Its behavior is established over the full $Q^2$-range and is compared to calculations based on lattice QCD, Schwinger-Dyson equations and a quark model. Although the connection between all these quantities is not known yet, the results are surprisingly alike. Such a similitude may be related to quark-hadron duality.

1. The strong coupling constant

A peculiar feature of strong interaction is asymptotic freedom: quark-quark interactions grow weaker with decreasing distances. Asymptotic freedom is expressed in the vanishing of the QCD coupling constant, $\alpha_s(Q^2)$, at large $Q^2$. Conversely, the fact that $\alpha_s(Q^2)$, as calculated in pQCD, becomes large when $Q^2 \to \Lambda_{QCD}^2$ is often linked to quark confinement. Since it is not expected that pQCD holds at the confinement scale and since the condition $\alpha_s(Q^2) \to \infty$ when $Q \to \Lambda_{QCD}$ is far from necessary to assure confinement, it is interesting to study $\alpha_s(Q^2)$ in the large distance domain.

Experimentally, moments of structure functions are convenient objects to extract $\alpha_s$. Among them, $\Gamma_{p-n}^{p-n}$ is the simplest to use. In pQCD, it is linked to the axial charge of the nucleon, $g_A$, by the Bjorken sum rule:

$$\Gamma_{1}^{p-n} \equiv \int_{0}^{1} dx (g_{1}^{p}(x) - g_{1}^{n}(x)) = \frac{1}{6} g_{A} [1 - \frac{\alpha_{s}}{\pi} - 3.58 \left(\frac{\alpha_{s}}{\pi}\right)^{2} - 20.21 \left(\frac{\alpha_{s}}{\pi}\right)^{3} - 130.0 \left(\frac{\alpha_{s}}{\pi}\right)^{4} - 893.38 \left(\frac{\alpha_{s}}{\pi}\right)^{5} + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}}{Q^{2i-2}}] \, \, \, (1)$$

where $g_1^p$ ($g_1^n$) is the first spin structure function for the proton (neutron). The $\mu_i(Q^2)/Q^{2i-2}$ are higher twist corrections and become important at lower $Q^2$. This series, usually truncated to leading twist and to 3rd order, can be used to fit experimental data and to extract $\alpha_s$. The higher twists can be computed with non-perturbative models or can be extracted from data, although with limited precision at the moment. This imprecise knowledge and the breakdown of pQCD at low $Q^2$ prevent a priori the extraction of $\alpha_s$ at low $Q^2$. However, an effective strong coupling constants
was defined by Grunberg\textsuperscript{4} in which higher twists and higher order QCD radiative corrections are incorporated. Eq. 1 becomes by definition:

\[ \Gamma_{p-n}^{p-n} = \frac{1}{6g_A}[1 - \frac{\alpha_{s,g_1}}{\pi}]. \]  

(2)

This definition yields many advantages: the coupling constant is extractable at any \( Q^2 \), is well-behaved when \( Q^2 \to \Lambda_{QCD} \), is not renormalization scheme (RS) dependent and is analytic when crossing quark thresholds. The price to pay for such benefits is that it becomes process-dependent (hence the subscript \( g_1 \) in Eq. 2). However, as pointed out by Brodsky \textit{et al.}\textsuperscript{5}, effective couplings can be related to each other, at least in the pQCD domain, by “commensurate scale equations”. These relate, using different \( Q^2 \) scales, observables without RS or scale ambiguity. Thus, one effective coupling constant is enough to characterize the strong interaction.

Among the possible observables available to define an effective coupling constant, \( \Gamma_{p-n}^{p-n} \) has unique advantages. The generalized Gerasimov-Drell-Hearn (GDH)\textsuperscript{6,7} and Bjorken sum rules predict \( \Gamma_{p-n}^{p-n} \) at low and large \( Q^2 \), and \( \Gamma_{p-n}^{p-n} \) is experimentally known between these two domains. Hence, \( \alpha_{s,g_1} \) can be extracted at any \( Q^2 \). In particular, it has a well defined value at \( Q^2 = 0 \). Furthermore, we will see that \( \alpha_{s,g_1} \) might best be suited to be compared to the predictions of theories and models.

2. Experimental determination of \( \alpha_{s,g_1} \)

A measurement of \( \Gamma_{p-n}^{p-n} \) at intermediate \( Q^2 \) was reported recently\textsuperscript{8} and was used to extract \( \alpha_{s,g_1} \).\textsuperscript{9} The results are shown by the triangles in Fig. 1, together with \( \alpha_{s,g_1} \) extracted from SLAC data\textsuperscript{10} at \( Q^2 = 5 \text{ GeV}^2 \) (open square). Note that the elastic contribution is not included in \( \Gamma_{p-n}^{p-n} \).

\( \Gamma_{p-n}^{p-n} \) is related to the generalized GDH sums:

\[ \Gamma_{1}^{p-n} = \frac{Q^2}{16\pi^2\alpha}(GDH^p - GDH^n) \]  

(3)

where \( \alpha \) is the QED coupling constant. Hence, at \( Q^2 = 0 \), \( \Gamma_{1}^{p-n} = 0 \) and

\[ \alpha_{s,g_1} = \pi. \]  

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At \( Q^2 = 0 \), the GDH sum rule implies:

\[ \Gamma_{1}^{p-n} = \frac{Q^2}{16\pi^2\alpha}(GDH^p - GDH^n) = -\frac{Q^2}{8}\left(\frac{\kappa_p^2}{M_p^2} - \frac{\kappa_n^2}{M_n^2}\right) \]  

(5)

where \( \kappa_p \) (\( \kappa_n \)) is the proton (neutron) anomalous magnetic moment. Combining Eq. 2 and 5, we get the derivative of \( \alpha_{s,g_1} \) at \( Q^2 = 0 \):

\[ \frac{d\alpha_{s,g_1}}{dQ^2} = \frac{3\pi}{4g_A} \times \left(\frac{\kappa_n^2}{M_n^2} - \frac{\kappa_p^2}{M_p^2}\right). \]  

(6)
Relations 4 and 6 constrain $\alpha_{s,g_1}$ at low $Q^2$ (dashed line in Fig. 1). At large $Q^2$, $\Gamma_1^{p-n}$ can be estimated using Eq. 1 at leading twist and $\alpha_s$ calculated with pQCD. $\alpha_{s,g_1}$ can be subsequently extracted (gray band).

These data and sum rules give $\alpha_{s,g_1}(Q^2)$ at any $Q^2$. A similar result is obtained using a model of $\Gamma_1^{p-n}$ and Eq. 2 (dotted line). The Burkert-Ioffe model is used because of its good match with data.

One can compare our result to effective coupling constants extracted using different processes. $\alpha_s,\tau$ was extracted from $\tau$-decay data from the OPAL experiment (inverted triangle). It is compatible with $\alpha_{s,g_1}$. The Gross-Llewellyn Smith sum rule (GLS) can be used to form $\alpha_{s,F_3}$. The sum rule relates the number of valence quarks in the hadron, $n_v$, to the structure function $F_3(Q^2,x)$. At leading twist, it reads:

$$\int_0^1 F_3(Q^2,x)dx = n_v \left[ 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.21 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 \right] (7)$$

We expect $\alpha_{s,F_3} = \alpha_{s,g_1}$ at high $Q^2$, since the $Q^2$-dependence of Eq. 1 and 7 at leading twist are identical. The GLS sum was measured by the CCFR collaboration and the resulting $\alpha_{s,F_3}$ is shown by the star symbols.

Figure 1. Extracted $\alpha_{s,g_1}(Q)/\pi$ using JLab data (up triangles), the GLS sum rule (stars), the world $\Gamma_1^{p-n}$ data (open square), the Bjorken sum rule (gray band) and the Burkert-Ioffe Model. $\alpha_s,\tau(Q)/\pi$ from OPAL is given by the reversed triangle. The dashed line is the GDH constrain on the derivative of $\alpha_{s,g_1}/\pi$ at $Q^2=0$.\n\n
3. Comparison with theory

Just like effective coupling constants extracted experimentally, there are also many possible theory definitions for the coupling constant and, contrariwise to the experimental quantities, the relations between the various definitions are not well known. Furthermore, the connection between the experimental and the theoretical quantities is not clear. Hence, the remainder of this paper is to be understood as a candid comparison of quantities \textit{a priori} defined differently, in order to see if they share common features.

Calculations of $\alpha_s$ using Schwinger-Dyson equations (SDE), lattice QCD or quark models are available. Different SDE results are shown in Fig. 2. The pioneering result of Cornwall\cite{cornwall} is shown by the blue band in the top left panel. The more recent SDE results from Fisher \textit{et al}., Bloch \textit{et al}., Maris and Tandy, and Bhagwat \textit{et al}.

are shown in top left, top right, bottom left and bottom left panels respectively. There is a good match between the data and the result from Fisher \textit{et al}.

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and Cornwall do not match the data. The Godfrey and Isgur curve in the top right panel of Fig. 2 is the coupling constant used in the framework of hadron spectroscopy\cite{godfrey}. $Q^2$-behavior of coupling constants can also be compared regardless of their absolute magnitudes by normalizing them to $\pi$ at $Q^2 = 0$ (These curves are not shown here). The Godfrey-Isgur, Cornwall and Fisher \textit{et al}. $Q^2$-behavior match well the data. The normalized curves from Maris-Tandy, Bloch \textit{et al}.

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are slightly below the data (by typically one sigma) for $Q > 0.6$ GeV.

Gluon bremsstrahlung and vertex corrections contribute to the running of $\alpha_s$. Modern SDE calculations include those\cite{modern} but it is \textit{a priori} not the case for the $\alpha_s$ used in the one gluon exchange term of the Godfrey and Isgur quark model, or for older SDE works. If so, pQCD corrections should be added to these calculations. The effect of those corrections (on $\alpha_{s,g_1}$) is given by the ratio of $\alpha_{s,g_1}$, extracted using Eq. 2 to $\alpha_{s,g_1}$, extracted using Eq. 1 at leading twist. For both Eq. 1 and 2, $\Gamma_{1}^{p-n}$ is given by a model\cite{model}. Since model and data agree well, no strong model dependence is introduced. The difference between results using Eq. 1 up to 4th and 5th order is taken as the uncertainty due to the truncation of the pQCD series. The resulting $\alpha_s$ are shown in the bottom right panel of Fig. 2.

Finally, we can compare lattice QCD data to our results. Many lattice results are available and are in general consistent. We chose to compare with the results of Furui and Nakajima\cite{furui}, see bottom left panel in Fig. 2.
They match well the data. The lowest $Q^2$ point is afflicted by finite size effect and should be ignored.

The match between our data and the various calculations might be surprising since these quantities are defined differently. We can try to understand this fact. Choosing $\Gamma_{p-n}^-$ minimizes the role of resonances, in particular it fully cancels the $\Delta_{1232}$ contribution which usually dominates the moments at low $Q^2$. By furthermore excluding the elastic contribution, we obtain a quantity for which coherent reactions (elastic and resonances) are suppressed and we are back to a DIS-like case in which the interpretation is straightforward. One can also possibly invoke the phenomenon of quark-hadron duality to explain why the extraction of $\alpha_s, g_1$, using a formalism developed for DIS$^{12}$, seems to also work at lower $Q^2$.

4. Conclusion

We have extracted, using JLab data at low $Q^2$ together with sum rules, an effective strong coupling constant at any $Q^2$. A striking feature is its loss of $Q^2$-dependence at low $Q^2$. We compared our result to SDE and lattice QCD calculations and to a coupling constant used in a quark model. Despite the unclear relation between these various coupling constants, data and calculations match in most cases, especially for relative $Q^2$-dependences. This could be linked to quark-hadron duality.

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Experimentally, moments of structure functions are convenient objects to extract $\alpha_s$. Among them, $\Gamma_1^{p-n}$ is the simplest to use. In pQCD, it is linked to the axial charge of the nucleon, $g_A$, by the Bjorken sum rule:

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2. Experimental determination of $\alpha_{s, g_1}$

A measurement of $\Gamma_1^{p-n}$ at intermediate $Q^2$ was reported recently and was used to extract $\alpha_{s, g_1}$. The results are shown by the triangles in Fig. 1, together with $\alpha_{s, g_1}$ extracted from SLAC data at $Q^2=5 \text{ GeV}^2$ (open square). Note that the elastic contribution is not included in $\Gamma_1^{p-n}$.

$\Gamma_1^{p-n}$ is related to the generalized GDH sums:

$$\Gamma_1^{p-n} = \frac{Q^2}{16\pi^2\alpha} (GDH^p - GDH^n)$$  

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where $\alpha$ is the QED coupling constant. Hence, at $Q^2=0$, $\Gamma_1^{p-n} = 0$ and

$$\alpha_{s, g_1} = \pi.$$  

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At $Q^2 = 0$, the GDH sum rule implies:

$$\Gamma_1^{p-n} = \frac{Q^2}{16\pi^2\alpha} (GDH^p - GDH^n) = -\frac{Q^2}{8} \left( \frac{\kappa_p^2}{M_p^2} - \frac{\kappa_n^2}{M_n^2} \right)$$  

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where $\kappa_p$ ($\kappa_n$) is the proton (neutron) anomalous magnetic moment. Combining Eq. 2 and 5, we get the derivative of $\alpha_{s, g_1}$ at $Q^2=0$:

$$\frac{da_{s, g_1}}{dQ^2} = \frac{3\pi}{4g_A} \times \left( \frac{\kappa_n^2}{M_n^2} - \frac{\kappa_p^2}{M_p^2} \right).$$  

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Relations 4 and 6 constrain $\alpha_{s, g_1}$ at low $Q^2$ (dashed line in Fig. 1). At large $Q^2$, $\Gamma_{\pi^-}^{\pi^-}$ can be estimated using Eq. 1 at leading twist and $\alpha_s$ calculated with pQCD. $\alpha_{s, g_1}$ can be subsequently extracted (gray band).

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We expect $\alpha_{s, F_3} = \alpha_{s, g_1}$ at high $Q^2$, since the $Q^2$-dependence of Eq. 1 and 7 at leading twist are identical. The GLS sum was measured by the CCFR collaboration14 and the resulting $\alpha_{s, F_3}$ is shown by the star symbols.

![Figure 1. Extracted $\alpha_{s, g_1}(Q)/\pi$ using JLab data (up triangles), the GLS sum rule (star), the world $\Gamma_{\pi^-}^{\pi^-}$ data (open square), the Bjorken sum rule (gray band) and the Burkert-Ioffe Model. $\alpha_{s, \tau}(Q)/\pi$ from OPAL is given by the reversed triangle. The dashed line is the GDH constrain on the derivative of $\alpha_{s, g_1}/\pi$ at $Q^2=0$.](image-url)
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