On Symmetry and Quantification: 
A New Approach to Verify Distributed Protocols

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Abstract. Proving that an unbounded distributed protocol satisfies a given safety property amounts to finding a quantified inductive invariant that implies the property for all possible instance sizes of the protocol. Existing methods for solving this problem can be described as search procedures for an invariant whose quantification prefix fits a particular template. We propose an alternative constructive approach that does not prescribe, a priori, a specific quantifier prefix. Instead, the required prefix is automatically inferred without any search by carefully analyzing the structural symmetries of the protocol. The key insight underlying this approach is that symmetry and quantification are closely related concepts that express protocol invariance under different re-arrangements of its components. We propose symmetric incremental induction, an extension of the finite-domain IC3/PDR algorithm, that automatically derives the required quantified inductive invariant by exploiting the connection between symmetry and quantification. While various attempts have been made to exploit symmetry in verification applications, to our knowledge, this is the first demonstration of a direct link between symmetry and quantification in the context of clause learning during incremental induction. We also describe a procedure to automatically find a minimal finite size, the cutoff, that yields a quantified invariant proving safety for any size.

Our approach is implemented in IC3PO, a new verifier for distributed protocols that significantly outperforms the state-of-the-art, scales orders of magnitude faster, and robustly derives compact inductive invariants fully automatically.

1 Introduction

Our focus in this paper is on parameterized verification, specifically proving safety properties of distributed systems, such as protocols that are often modeled above the code level (e.g., [49, 63]), consisting of arbitrary numbers of identical components that are instances of a small set of different sorts. For example, a client server protocol [1] CS(i, j) is a two-sort parameterized system with parameters i ≥ 1 and j ≥ 1 denoting, respectively, the number of clients and servers. Protocol correctness proofs are critical for establishing the correctness of actual system implementations in established methodologies such as [42, 69]. Proving safety properties for such systems requires the derivation of inductive invariants
that are expressed as state predicates quantified over the system parameters. While, in general, this problem is undecidable [8], certain restricted forms have been shown to yield to algorithmic solutions [17]. Key to these solutions is appealing to the problem’s inherent symmetry. In this paper, we exclusively focus on protocols whose sorts represent sets of indistinguishable domain constants. The behavior of this restricted class of protocols remains invariant under all possible permutations of the domain constants. We leave the exploration of other features, such as totally-ordered sorts, integer arithmetic, etc., for future work.

Our proposed symmetry-based solution is best understood by briefly reviewing earlier efforts. Initially, the pressing issue was the inevitable state explosion when verifying a finite, but large, parameterized system [12, 29, 37, 60, 66, 68]. Thus, instead of verifying the “full” system, these approaches verified its symmetry-reduced quotient, mostly using BDD-based symbolic image computation [19, 20, 56]. The Murϕ verifier [60] was a notable exception in that it a) generated a C++ program that enumerated the system’s symmetry-reduced reachable states, and b) allowed for the verification of unbounded systems by taking advantage of data saturation which happens when the size of the symmetry-reduced reachable states become constant regardless of system size.

The idea that an unbounded symmetric system can, under certain data-independence assumptions, be verified by analyzing small finite instances evolved into the approach of verification by invisible invariants [9, 10, 25, 65, 70]. In this approach, assuming they exist, inductive invariants that are universally-quantified over the system parameters are automatically derived by analyzing instances of the system up to a cutoff size $N_0$ using a combination of symbolic reachability and symmetry-based abstraction. Noting that an invariant is an over-approximation of the reachable states, the restriction to universal quantification may fail in some cases, rendering the approach incomplete. The invisible invariant verifier IIV [10] employs some heuristics to derive invariants that use combinations of universal and existential quantifiers, but as pointed out in [58], it may still fail and is not guaranteed to be complete.

The development of SAT-based incremental induction algorithms [18, 27] for verifying the safety of finite transition systems was a major advance in the field of model checking and has, for the most part, replaced BDD-based approaches. These algorithms leverage the capacity and performance of modern CDCL SAT solvers [11, 28, 55, 57] to produce clausal strengthening assertions $A$ that, conjoined with a specified safety property $P$, form an automatically-generated inductive invariant $Inv = A \land P$ if the property holds. The AVR hardware verifier [38–40] was adapted in [53] to produce quantifier-free inductive invariants for small instances of unbounded protocols that are subsequently generalized with universal quantification, in analogy with the invisible invariants approach, to arbitrary sizes. The resulting assertions tended, in some cases, to be quite large, and the approach was also incomplete due to the restriction to universal quantification.

In this paper we introduce IC3PO, a novel symmetry-based verifier that builds on these previous efforts while removing most of their limitations. Rather than search for an invariant with a prescribed quantifier prefix, IC3PO con-
structively discovers the required quantified assertions by performing symmetric incremental induction and analyzing the symmetry patterns in learned clauses to infer the corresponding quantifier prefix. Our main contributions are:

- An extension to finite incremental induction algorithms that uses protocol symmetry to boost clause learning from a single clause $\varphi$ to a set of symmetrically-equivalent clauses, $\varphi$'s orbit.
- A quantifier inference procedure that expresses $\varphi$'s orbit by an automatically-derived compact quantified predicate $\Phi$. The inference procedure is based on a simple analysis of $\varphi$'s syntactic structure and yields a quantified form with both universal and existential quantifiers.
- A systematic finite convergence procedure for determining a minimal instance size sufficient for deriving a quantified inductive invariant that holds for all sizes.

We also demonstrate the effectiveness of IC3PO on a diverse set of benchmarks and show that it significantly advances the current state-of-the-art.

The paper is structured as follows: §2 presents preliminaries. §3 formalizes protocol symmetries. The next three sections detail our key contributions: symmetry boosting during incremental induction in §4, relating symmetry to quantification in §5, and checking for convergence in §6. §7 describes the IC3PO algorithm and implementation details. §8 presents our experimental evaluation. The paper concludes with a brief survey of related work in §9, and a discussion of future directions in §10.

2 Preliminaries

Figure 1 describes a toy consensus protocol from [6] in the TLA+ language [49]. The protocol has three named sorts $S = [\text{node}, \text{quorum}, \text{value}]$ introduced by the constants declaration, and two relations $R = \{\text{vote}, \text{decision}\}$, introduced by the variables declaration, that are defined on these sorts. Each of the sorts is understood to represent an unbounded domain of distinct elements with the relations serving as the protocol’s state variables. The global axiom (line 3) defines the elements of the quorum sort to be subsets of the node sort and restricts them further by requiring them to be pair-wise non-disjoint. We will refer to node (resp. quorum) as an independent (resp. dependent) sort. The protocol transitions are specified by the actions CastVote and Decide (lines 6-7) which are expressed using the current- and next-state variables as well as the definitions didNotVote and chosenAt (lines 4-5) which serve as auxiliary non-state variables. Lines 8-10 specify the protocol’s initial states, transition relation, and safety property.

Viewed as a parameterized system, the template of an arbitrary $n$-sort distributed protocol $\mathcal{P}$ will be expressed as $\mathcal{P}(s_1, \ldots, s_n)$ where $S = [s_1, \ldots, s_n]$ is an ordered list of its sorts, each of which is assumed to be an unbounded uninterpreted set of distinct constants. As a mathematical transition system, $\mathcal{P}$ is defined by a) its state variables which are expressed as $k$-ary relations on its

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1 The description in [6] is in the Ivy [63] language and encodes set operations in relational form with a member relation representing $\in$. 
The symmetry group of ˆ\(3\) Protocol Symmetries

We use primes (e.g., \(\phi\)) to denote the application of \(\phi\) to the state variables.

In the sequel, we will use \(\hat{\mathcal{P}}\) and \(\hat{T}\) as shorthand for \(\mathcal{P}(s_1, \ldots, s_n)\) and \(T(s_1, \ldots, s_n)\). Quantifier-free formulas will be denoted by lower-case Greek letters (e.g., \(\varphi\)) and quantified formulas by upper-case Greek letters (e.g., \(\Phi\)). We use primes (e.g., \(\varphi'\)) to represent a formula after a single transition step.

3 Protocol Symmetries

The symmetry group of \(\hat{\mathcal{P}}\) is \(G(\hat{\mathcal{P}}) = \times_{s \in \mathcal{S}} Sym(s)\), where \(Sym(s)\) is the symmetric group, i.e., the set of \(|s|!\) permutations of the constants of the set.
In what follows we will use $G$ instead of $G(\mathcal{P})$ to reduce clutter. Given a permutation $\gamma \in G$ and an arbitrary protocol relation $\rho$ instantiated with specific sort constants, the action of $\gamma$ on $\rho$, denoted $\rho^\gamma$, is the relation obtained from $\rho$ by permuting the sort constants in $\rho$ according to $\gamma$; it is referred to as the $\gamma$-image of $\rho$. Permutation $\gamma \in G$ can also act on any formula involving the protocol relations. In particular, the invariance of protocol behavior under permutation of sort constants implies that the action of $\gamma$ on the (finite) initial state, transition relation, and property formulas causes a syntactic re-arrangement of their subformulas while preserving their logical equivalence:

$$
\text{Init}^\gamma \equiv \text{Init} \quad \bar{T}^\gamma \equiv \bar{T} \quad \bar{P}^\gamma \equiv \bar{P}
$$

Consider next a clause $\varphi$ which is a disjunction of literals, namely, instantiated protocol relations or their negations. The orbit of $\varphi$ under $G$, denoted $\varphi^G$, is the set of its images $\varphi^\gamma$ for all permutations $\gamma \in G$, i.e., $\varphi^G = \{ \varphi^\gamma | \gamma \in G \}$. The $\gamma$-image of a clause can be viewed as a syntactic transformation that will either yield a new logically-distinct clause on different literals or simply re-arrange the literals in the clause without changing its logical behavior (by the commutativity and associativity of disjunction). We define the logical action of a permutation $\gamma$ on a clause $\varphi$, denoted $\varphi^{L(\gamma)}$, as:

$$
\varphi^{L(\gamma)} = \begin{cases} 
\varphi^\gamma & \text{if } \varphi^\gamma \not\equiv \varphi \\
\varphi & \text{if } \varphi^\gamma \equiv \varphi 
\end{cases}
$$

and the logical orbit of $\varphi$ as $\varphi^{L(G)} = \{ \varphi^{L(\gamma)} | \gamma \in G \}$. With a slight abuse of notation, logical orbit can also be viewed as the conjunction of the logical images:

$$
\varphi^{L(G)} = \bigwedge_{\gamma \in G} \varphi^{L(\gamma)}
$$

To illustrate these concepts, consider ToyConsensus(3, 3, 3) from (1). Its symmetries in cycle notation are as follows:

$$
\text{Sym(node)} = \{((), (n_1 n_2), (n_1 n_3), (n_2 n_3)), (n_1 n_2 n_3), (n_1 n_3 n_2))\}
$$

$$
\text{Sym(value)} = \{((), (v_1 v_2), (v_1 v_3), (v_2 v_3)), (v_1 v_2 v_3), (v_1 v_3 v_2))\}
$$

$$
G = \text{Sym(node)} \times \text{Sym(value)}
$$

The symmetry group (3) of ToyConsensus(3, 3, 3) has 36 symmetries corresponding to the $6 \text{node}_3 \times 6 \text{value}_3$ permutations. The permutations on quorum$_3$ are implicit and based on the permutations of node$_3$ since quorum$_3$ is a dependent sort. Now, consider the example clause:

$$
\varphi_1 = \text{vote}(n_1, v_1) \lor \text{vote}(n_1, v_2) \lor \text{vote}(n_1, v_3)
$$

The orbit of $\varphi_1$ consists of 36 syntactically-permutted clauses. However, many of these images are logically equivalent yielding the following logical orbit of just 3 logically-distinct clauses:

$$
\varphi_1^{L(G)} = [\text{vote}(n_1, v_1) \lor \text{vote}(n_1, v_2) \lor \text{vote}(n_1, v_3)] \land \\
[\text{vote}(n_2, v_1) \lor \text{vote}(n_2, v_2) \lor \text{vote}(n_2, v_3)] \land \\
[\text{vote}(n_3, v_1) \lor \text{vote}(n_3, v_2) \lor \text{vote}(n_3, v_3)]
$$

\[\text{We assume familiarity with basic notions from group theory including permutation groups, cycle notation, group action on a set, orbits, etc., which can be readily found in standard textbooks on Abstract Algebra [33].} \]
4 SymIC3: Symmetric Incremental Induction

SymIC3 is an extension of the standard IC3 algorithm [18, 27] that takes advantage of the symmetries in a finite instance \(P\) of an unbounded protocol \(P\) to boost learning during backward reachability. Specifically, it refines the current frame, in a single step, with all clauses in the logical orbit \(\varphi^L(G)\) of a newly-learned quantifier-free clause \(\varphi\). In other words, having determined that the backward 1-step check \(F_{i-1} \land \overline{T} \land \overline{\varphi}'\) is unsatisfiable (i.e., that states in cube \(\overline{\varphi}\) in frame \(F_i\) are unreachable from the previous frame \(F_{i-1}\)), SymIC3 refines \(F_i\) with \(\varphi^L(G)\), i.e., \(F_i := F_i \land \varphi^L(G)\), rather than with just \(\varphi\). Thus, at each refinement step, SymIC3 not only blocks cube \(\overline{\varphi}\), but also all symmetrically-equivalent cubes \([\overline{\varphi}]^\gamma\) for all \(\gamma \in G\). This simple change to the standard incremental induction algorithm significantly improves performance since the extra clauses used to refine \(F_i\) a) are derived without making additional backward 1-step queries, and b) provide stronger refinement in each step of backward reachability leading to faster convergence with fewer counterexamples-to-induction (CTIs). The proof of correctness of symmetry boosting can be found in Appendix B.1.

5 Quantifier Inference

The key insight underlying our overall approach is that the explicit logical orbit, in a finite protocol instance, of a learned clause \(\varphi\) can be exactly, and systematically, captured by a corresponding quantified predicate \(\Phi\). In retrospect, this should not be surprising since symmetry and quantification can be seen as different ways of expressing invariance under permutation of the sort constants in the clause. To motivate the connection between symmetry and quantification, consider the following quantifier-free clause from our running example and a proposed quantified predicate that implicitly represents its logical orbit:

\[
\varphi_2 = \neg\text{decision}(v_1) \lor \text{decision}(v_2) \\
\Phi_2 = \forall X_1, X_2 \in \text{value}_3: (\text{distinct } X_1, X_2) \rightarrow [\neg\text{decision}(X_1) \lor \text{decision}(X_2)] 
\]  

(6)

As shown in Table 1, the logical orbit \(\varphi_2^L(G)\) consists of 6 logically-distinct clauses corresponding to the 6 permutations of the 3 constants of the \(\text{value}_3\) sort. Evaluating \(\Phi_2\) by substituting all \(3 \times 3 = 9\) assignments to the variable pair \((X_1, X_2) \in \text{value}_3 \times \text{value}_3\) yields 9 clauses, 3 of which (shown faded) are trivially true since their “distinct” antecedents are false, with the remaining 6 corresponding to each of the clauses obtained through permutations of the 3 \(\text{value}_3\) constants. Similarly, we can show that the 3-clause logical orbit \(\varphi_1^L(G)\) in (5) can be succinctly expressed by the quantified predicate:

\[
\Phi_1 = \forall Y \in \text{node}_3, \exists X \in \text{value}_3: \text{vote}(Y, X) 
\]  

(7)

which employs universal and existential quantification. And, finally, \(\varphi_3\) and \(\Phi_3\) below illustrate how a clause whose logical orbit is just itself can also be expressed as an existentially-quantified predicate.

\[
\varphi_3 = \text{decision}(v_1) \lor \text{decision}(v_2) \lor \text{decision}(v_3) \\
\Phi_3 = \exists X \in \text{value}_3: \text{decision}(X)
\]  

(8)
We will first describe basic quantifier inference for protocols with independent sorts. This is done by analyzing the syntactic structure of each quantifier-free clause learned during incremental induction to derive a quantified form that expresses the clause’s logical orbit. We later discuss extensions to this approach that consider protocols with dependent sorts, such as ToyConsensus, for which the basic single-clause quantifier inference may be insufficient.

5.1 Basic Quantifier Inference

Given a quantifier-free clause $\varphi$, quantifier inference seeks to derive a compact quantified predicate that implicitly represents, rather than explicitly enumerates, its logical orbit. The procedure must satisfy the following conditions:

Correctness – The inferred quantified predicate $\Phi$ should be logically-equivalent to the explicit logical orbit $\varphi^{L(G)}$.

Compactness – The number of quantified variables in $\Phi$ for each sort $s \in S$ should be independent of the sort size $|s|$. Intuitively, this condition ensures that the size of the quantified predicate, measured as the number of its quantifiers, remains bounded for any finite protocol instance, and more importantly, for the unbounded protocol.

SymIC3 constructs the orbit’s quantified representation by a) inferring the required quantifiers for each sort separately, and b) stitching together the inferred quantifiers for the different sorts to form the final result. The key to capturing the logical orbit and deriving its compact quantified representation is a simple analysis of the structural distribution of each sort’s constants in the target clause.

Let $\pi(\varphi, s)$ be a partition of the constants of sort $s$ in $\varphi$ based on whether or not they appear identically in the literals of $\varphi$. Two constants $c_i$ and $c_j$ are identically-present in $\varphi$ if they occur in $\varphi$ and swapping them results in a logically-equivalent clause, i.e., $\varphi^{(c_i, c_j)} \equiv \varphi$. Let $\#(\varphi, s)$ be the number of con-
constants of $s$ that appear in $\varphi$, and let $|\pi(\varphi, s)|$ be the number of classes/cells in $\pi(\varphi, s)$. Consider the following scenarios for quantifier inference on sort $s$:

A. $\#(\varphi, s) < |s|$ (infer $\forall$)

In this case, clause $\varphi$ contains a strict subset of constants from sort $s$, indicating that the number of literals in $\varphi$ parameterized by $s$ constants is independent of the sort size $|s|$. Increasing sort size simply makes the orbit longer by adding more symmetrically-equivalent but logically-distinct clauses. An example of this case is $\varphi_2$ and $\Phi_2$ in (6). The quantified predicate representing such an orbit requires $\#(\varphi, s)$ universally-quantified sort variables corresponding to the $\#(\varphi, s)$ sort constants in the clause, and expresses the orbit as an implication whose antecedent is a “distinct” constraint that ensures that the variables cannot be instantiated with identical constants.

B. $\#(\varphi, s) = |s|$

When all constants of a sort $s$ appear in a clause, the above universal quantification yields a predicate with $|s|$ quantified variables and fails the compactness requirement since the number of quantified variables becomes unbounded as the sort size increases. Correct quantification in this case must be inferred by examining the partition of the sort constants in the clause.

I. Single-cell Partition i.e., $|\pi(\varphi, s)| = 1$ (infer $\exists$)

When all sort constants appear identically in $\varphi$, $\pi(\varphi, s)$ is a unit partition. Applying any permutation $\gamma \in \text{Sym}(s)$ to $\varphi$ yields a logically-equivalent clause, i.e., the logical orbit in this case is just a single clause. Increasing the size of sort $s$ simply yields a wider clause and suggests that such an orbit can be encoded as a predicate with a single existentially-quantified variable that ranges over all the sort constants. For example, the partition of the $\text{value}_3$ sort constants in $\varphi_1$ from (4) is $\pi(\varphi_1, \text{value}_3) = \{\{v_1, v_2, v_3\}\}$ since all three constants appear identically in $\varphi_1$. The orbit of this clause is just itself and can be encoded as:

$$\Phi_1(\text{value}_3) = \exists X \in \text{value}_3 : \text{vote}(n_1, X)$$

Also, since $\#(\varphi_1, \text{node}_3) < |\text{node}_3|$, universal quantification (as in Section 5.1.A) correctly captures the dependence of the clause’s logical orbit on the $\text{node}_3$ sort to get the overall quantified predicate $\Phi_1$ in (7).

II. Multi-cell Partition i.e., $|\pi(\varphi, s)| > 1$ (infer $\forall \exists$)

In this case, a fixed number of the constants of sort $s$ appear differently in $\varphi$ with the remaining constants appearing identically, resulting in a multi-cell partition. Specifically, assume that a number $0 < k < |s|$ exists that is independent of $|s|$ such that $\pi(\varphi, s)$ has $k + 1$ cells in which one cell has $|s| - k$ identically-appearing constants and each of the remaining $k$ cells contains one of the differently-appearing constants. It can be shown that the logical orbit in this case can be expressed by a quantified predicate with $k$ universal quantifiers and
a single existential quantifier. For example, the partition of the \texttt{value}_3 constants in the clause:

$$\varphi_4 = \neg \text{decision}(v_1) \lor \text{decision}(v_2) \lor \text{decision}(v_3)$$

is \(\pi(\varphi_4, \text{value}_3) = \{\{v_1\}, \{v_2, v_3\}\}\) since \(v_1\) appears differently from \(v_2\) and \(v_3\).

The logical orbit of this clause is:

$$\varphi_4^{L(G)} = [\neg \text{decision}(v_1) \lor \text{decision}(v_2) \lor \text{decision}(v_3)] \land$$

$$[\neg \text{decision}(v_2) \lor \text{decision}(v_1) \lor \text{decision}(v_3)] \land$$

$$[\neg \text{decision}(v_3) \lor \text{decision}(v_2) \lor \text{decision}(v_1)]$$ (9)

and can be compactly encoded with an outer universally-quantified variable corresponding to the sort constant in the singleton cell, and an inner existentially-quantified variable corresponding to the other \(|\text{value}_3| - 1\) identically-present sort constants. A “distinct” constraint must also be conjoined with the literals involving the existentially-quantified variable to exclude the constant corresponding to the universally-quantified variable from the inner quantification. \(\varphi_4^{L(G)}\) can thus be shown to be logically-equivalent to:

$$\Phi_4 = \forall Y \in \text{value}_3, \exists X \in \text{value}_3 : \neg \text{decision}(Y) \lor [\text{distinct } Y X \land \text{decision}(X)]$$ (10)

### Combining Quantifier Inference for Different Sorts

The complete quantified predicate \(\Phi\) representing the logical orbit of clause \(\varphi\) can be obtained by applying the above inference procedure to each sort in \(\varphi\) separately and in any order. This is possible since the sorts are assumed to be independent: the constants of one sort do not permute with the constants of a different sort. This will yield a predicate \(\Phi\) that has the quantified prenex form \(\forall^* \exists^* < \text{CNF expression } >\), where all universals for each sort are collected together and precede all the existential quantifiers.

It is interesting to note that this connection between symmetry and quantification suggests that an orbit can be visualized as a two-dimensional object whose height and width correspond, respectively, to the number of universally- and existentially-quantified variables. A proof of the correctness of this quantifier inference procedure can be found in Appendix B.2.

### 5.2 Quantifier Inference Beyond \(\forall^* \exists^*\)

We observed that for some protocols, particularly those that have dependent sorts such as \texttt{ToyConsensus}, the above inference procedure violates the compactness requirement. In other words, restricting inference to a \(\forall^* \exists^*\) quantifier prefix causes the number of quantifiers to become unbounded as sort sizes increase. Recalling that the \(\forall^* \exists^*\) pattern is inferred from the symmetries of a single clause, whose literals are the protocol’s state variables, suggests that inference of more complex quantification patterns may necessitate that we examine the structural distribution of sort constants across sets of clauses. While this is an interesting possible direction for further exploration of the connection between symmetry and quantification, an alternative approach is to take advantage of the \texttt{formula structure} of the protocol’s transition relation. For example,
the transition relation of ToyConsensus is specified in terms of two quantified
sub-formulas, didNotVote and chosenAt, that can be viewed, in analogy with
a sequential hardware circuit, as internal auxiliary non-state variables that act
as “combinational” functions of the state variables. By allowing such auxiliary
variables to appear explicitly in clauses learned during incremental induction,
the quantified predicates representing the logical orbits of these clauses (according
to the basic inference procedure in Section 5.1) will implicitly incorporate
the quantifiers used in the auxiliary variable definitions and automatically have
a quantifier prefix that generalizes the basic $\forall^*\exists^*$ template.

Revisiting ToyConsensus— When SymIC3 is run on the finite instance Toy-
Consensus(3,3,3), it terminates with the following two strengthening assertions:

$$\begin{align*}
A_1 &= \forall N \in \text{node}_3, V_1, V_2 \in \text{value}_3 : \text{distinct}(V_1, V_2) \rightarrow \neg \text{vote}(N, V_1) \lor \neg \text{vote}(N, V_2) \quad (11) \\
A_2 &= \forall V \in \text{value}_3, \exists Q \in \text{quorum}_3 : \neg \text{decision}(V) \lor \text{chosenAt}(Q, V) \quad (12)
\end{align*}$$

which, together with $\hat{P}$, serve as an inductive invariant proving that $\hat{P}$ holds
for this instance. Both assertions are obtained using the basic quantifier inference
procedure in Section 5.1 that produces a $\forall^*\exists^*$ quantifier prefix in terms of
the clause variables. Note, however, that $A_2$ is expressed in terms of the auxiliary
variable chosenAt. Substituting the definition of chosenAt yields an assertion
with a $\forall\exists\forall$ quantifier prefix exclusively in terms of the protocol’s state variables.

6 Finite Convergence Checks

Given a safe finite instance $\hat{P} \triangleq \mathcal{P}(|S_1|, \ldots, |S_n|)$, let $\text{Inv}_{|S_1|, \ldots, |S_n|}$ denote the
inductive invariant derived by SymIC3 to prove that $\hat{P}$ holds in $\hat{P}$. What remains
is to determine the instance size $|S_1|, \ldots, |S_n|$ needed so that $\text{Inv}_{|S_1|, \ldots, |S_n|}$ is also
an inductive invariant for all sizes. If the instance size is too small, $\hat{P}$ may not
include all protocol behaviors and $\text{Inv}_{|S_1|, \ldots, |S_n|}$ will not be inductive at larger
sizes. As shown in the invisible invariant approach [9, 10, 58, 65, 70], increasing
the instance size becomes necessary to include new protocol behaviors missing
in $\hat{P}$, until protocol behaviors saturate. We propose an automatic way to update
the instance size and reach saturation by starting with an initial base size and
iteratively increasing the size until finite convergence is achieved.

The initial base size can be chosen to be any non-trivial instance size and
can be easily determined by a simple analysis of the protocol description. For
example, any non-trivial instance of the ToyConsensus protocol should have
$|\text{node}| \geq 3$, $|\text{quorum}| \geq 3$, and $|\text{value}| \geq 2$.

Our finite convergence procedure can be seen as an integration of symmetry
saturation and a stripped-down form of multi-dimensional mathematical induc-
tion, and has similarities with previous works on structural induction [35, 47]
and proof convergence [25]. To determine if $\text{Inv}_{|S_1|, \ldots, |S_n|}$ is inductive for any size,
the procedure performs the following checks for $1 \leq i \leq n$:

a) $\text{Init}([s_1],..,[s_n] + 1) \rightarrow \text{Inv}_0([s_1],..,[s_n] + 1)$ (13)

b) $\text{Inv}_0([s_1],..,[s_n] + 1) \wedge T([s_1],..,[s_n] + 1) \rightarrow \text{Inv'}_0([s_1],..,[s_n] + 1)$ (14)

where $\text{Inv}_0([s_1],..,[s_n] + 1)$ denotes the application of $\text{Inv}_0([s_1],..,[s_n])$ to an instance in which the size of sort $s_1$ is increased by 1 while the sizes of the other sorts are unchanged.$^3$

If all of these checks pass, we can conclude that $\text{Inv}_0([s_1],..,[s_n])$ is not specific to the instance size used to derive it and that we have reached cutoff, i.e., that $\text{Inv}_0([s_1],..,[s_n])$ is an inductive invariant for any size. Intuitively, this suggests that adding a new protocol component (e.g., client, server, node, proposer, acceptor) does not add any unseen unique behavior, and hence proving safety till the cutoff is sufficient to prove safety for any instance size. While we believe these checks are sufficient, we still do not have a formal convergence proof. In our implementation, we confirm convergence by performing the unbounded induction checks a) $\text{Init} \rightarrow \text{Inv}_0([s_1],..,[s_n])$, and b) $\text{Inv}_0([s_1],..,[s_n]) \wedge T \rightarrow \text{Inv'}_0([s_1],..,[s_n])$ noting that they may lie outside the decidable fragment of first-order logic.

On the other hand, failure of these checks, say for sort $s_1$, implies that $\text{Inv}_0([s_1],..,[s_n])$ will fail for larger sizes and cannot be inductive in the unbounded case, and we need to repeat $\text{SymIC3}$ on a finite instance with an increased size for sort $s_1$, i.e., $P_{\text{new}} \equiv P([s_1],..,[s_i] + 1,...,[s_n])$, to include new protocol behaviors that are missing in $P$.

Recall from (11) and (12), running $\text{SymIC3}$ on $\text{ToyConsensus}(3,3,3)$ produces $\text{Inv}_{3,3,3} = A_1 \land A_2 \land \overset{\prime}{P}$. $\text{Inv}_{3,3,3}$ passes checks (13) and (14) for instances $\text{ToyConsensus}(4,4,3)$ and $\text{ToyConsensus}(3,3,4)$, indicating finite convergence.$^4$

$\text{Inv}_{3,3,3}$ passes standard induction checks in the unbounded domain as well, establishing it as a proof certificate that proves the property as safe in $\text{ToyConsensus}$.

7 IC3PO: IC3 for Proving Protocol Properties

Given a protocol specification $P$, IC3PO iteratively invokes $\text{SymIC3}$ on finite instances of increasing size, starting with a given initial base size. Upon termination, IC3PO either a) reaches convergence on an inductive invariant $\text{Inv}_0([s_1],..,[s_n])$ that proves $P$ for the unbounded protocol $\overset{\prime}{P}$, or b) produces a counterexample trace $Cex([s_1],..,[s_n])$ that serves as a finite witness to its violation in both the finite instance and the unbounded protocol. The detailed pseudo code of IC3PO is available in Appendix A.

We also explored a number of simple enhancements to IC3PO, such as strengthening the inferred quantified predicates whenever safely possible to do during incremental induction by a) dropping the “distinct” antecedent, and b) rearranging the quantifiers if the strengthened predicate is still unreachable from the previous frame. We describe these enhancements in Appendix C. The results presented in this paper were obtained without these enhancements.

$^3$ Sort dependencies, if any, should be considered when increasing a sort size.

$^4$ Since $\text{quorum}$ is a dependent sort on $\text{node}$, it is increased together with the $\text{node}$ sort.
Implementation — Our implementation of IC3PO is publicly available at https://github.com/aman-goel/ic3po. The implementation accepts protocol descriptions in the Ivy language [63] and uses the Ivy compiler to extract a quantified, logical formulation \( P \) in a customized VMT [22] format. We use a modified version [5] of the pySMT [34] library to implement our prototype, and use the Z3 [24] solver for all SMT queries. We use the SMT-LIB [14] theory of free sorts and function symbols with datatypes and quantifiers (UFDT), which allows formulating SMT queries for both, the finite and the unbounded domains. For a safe protocol, the inductive proof is printed in the Ivy format as an independently check-able proof certificate, which can be further validated with the Ivy verifier.

8 Evaluation

We evaluated IC3PO on a total of 29 distributed protocols including 4 problems from [53], 13 from [46], and 12 from [2]. This evaluation set includes fairly complex models of consensus algorithms as well as protocols such as two-phase commit, chord ring, hybrid reliable broadcast, etc. Several studies [16,32,42,46,53,63] have indicated the challenges involved in verifying these protocols.

All 29 protocols are safe based on manual verification. Even though finding counterexample traces is equally important, we limit our evaluation to safe protocols where the property holds, since inferring inductive invariants is the main bottleneck of existing techniques for verifying distributed protocols [30,31,63].

We compared IC3PO against the following 3 verifiers that implement state-of-the-art IC3-style techniques for automatic verification of distributed protocols:

– I4 [53] performs finite-domain IC3 (without accounting for symmetry) using the AVR model checker [39], followed by iteratively generalizing and checking the inductive invariant produced by AVR using Ivy.

– UPDR is the implementation of the PDR'/UPDR algorithm [44] for verifying distributed protocols, from the mypyvy [4] framework.

– fol-ic3 [46] is a recent technique implemented in mypyvy that extends IC3 with the ability to infer inductive invariants with quantifier alternations.

All experiments were performed on an Intel (R) Xeon CPU (X5670). For each run, we used a timeout of 1 hour and a memory limit of 32 GB. All tools were executed in their respective default configurations. We used Z3 [24] version 4.8.9, Yices 2 [26] version 2.6.2, and CVC4 [13] version 1.7.

8.1 Results

Table 2 summarizes the experimental results. Apart from the number of problems solved, we compared the tools on 3 metrics: run time in seconds, proof size measured by the number of assertions in the inductive invariant for the unbounded protocol, and the total number of SMT queries made. Each tool uses SMT queries differently (e.g., I4 uses QF_UF for finite, UF for unbounded). Comparing the number of SMT queries still helps in understanding the run time behavior.
| Protocol (#29)          | Human | IC3PO | I4 | UPDR | fold-ic3 |
|------------------------|-------|-------|----|------|---------|
| tla-consensus          | 1     | 0    | 1 | 1    | 1       |
| tla-commit             | 3     | 1    | 2 | 31   | unknown |
| i4-lock-server         | 2     | 1    | 2 | 37   | 2       |
| ex-quorum-leader-elec  | 3     | 3    | 5 | 129  | 32      |
| pty-toy-consensus-foral| 4     | 3    | 4 | 105  | unknown |
| tla-simple             | 8     | 6    | 3 | 285  | 4       |
| ex-lockserve-automaton | 2     | 7    | 12| 504  | 3       |
| tla-simplesregular     | 9     | 8    | 4 | 346  | unknown |
| pty-sharded-kv         | 5     | 10   | 8 | 590  | 4       |
| pty-lockserv           | 9     | 11   | 12| 702  | 3       |
| tla-twophase           | 12    | 14   | 10| 984  | unknown |
| i4-learning-switch     | 8     | 14   | 9 | 589  | 22      |
| ex-simple-decentralized-lock | 5 | 19 | 15 | 2219 | 14  |
| i4-two-phase-commit    | 11    | 27   | 11| 2541 | 4       |
| pty-consensus-woo-decide| 5  | 50   | 9 | 1886 | 1144   |
| pty-consensus-forall   | 7     | 90   | 10| 3445 | 1006   |
| pty-learning-switch    | 8     | 127  | 13| 3388 | 387    |
| i4-chord-ring-maintenance | 18 | 229 | 12| 6418 | timeout |
| pty-sharded-kv-no-lost-keys | 2 | E   | 3 | 2  | 57  |
| ex-naive-consensus     | 4     | E    | 6 | 4  | 239 |
| ex-simple-election     | 3     | E    | 7 | 4  | 268 |
| pty-toy-consensus-epr  | 4     | E    | 9 | 4  | 370 |
| ex-toy-consensus       | 3     | E    | 10| 3  | 209 |
| pty-client-server-aec  | 5     | E    | 17| 6  | 868 |
| pty-hybrid-reliable-broadcast | 8 | E    | 157| 4 | 1747|
| pty-firewall           | 2     | E    | 2 | 3  | 131 |
| ex-majorityset-leader-election | 5 | E  | 72 | 7 | 1552 |
| pty-consensus-epr      | 7     | E    | 1300| 9 | 29601|

| No. of problems solved (out of 29) | 29 | 13 | 14 | 23 |
|------------------------------------|----|----|----|----|
| Uniquely solved                    | 3  | 0  | 0  | 0  |
| For 10 cases solved by all         | 232| 2221|667 |2711 |
| Time                               | 85 | 186| 52 | 114 |
| SMT                                | 12160| 22890|45911|27168|

Table 2: Comparison of IC3PO against other state-of-the-art verifiers

IC3PO solved all 29 problems, while 10 protocols were solved by all the tools. The 5 rows at the bottom of Table 2 provide a summary of the comparison. Overall, compared to the other tools IC3PO is faster, requires fewer SMT queries, and produces shorter inductive proofs even for problems requiring inductive invariants with quantifier alternations (marked with $\exists E$ in Table 2).

We did a more extensive comparison between the two finite-domain incremental induction verifiers IC3PO and I4 (Appendix D), performed a statistical analysis using multiple runs with different solver seeds to account for the effect of randomness in SMT solving (Appendix E), compared the inductive proofs produced by IC3PO against human-written invariants (Appendix F), and performed a preliminary exploration of distributed protocols with totally-ordered domains and ring topologies (Appendix G).
8.2 Discussion

Comparing IC3PO and I4 clearly reveals the benefits of symmetric incremental induction. For example, I4 requires 7814 SMT queries to eliminate 443 CTIs when solving ToyConsensus(3,3,3), compared to 192 SMT calls and 13 CTIs for IC3PO. Even though both techniques perform finite incremental induction, symmetry-aware clause boosting in IC3PO leads to a factorial reduction in the number of SMT queries and yields compact inductive proofs.

Comparing IC3PO and UPDR reveals the benefits of finite-domain reasoning methods compared to direct unbounded verification. Even in cases where existential quantifier inference isn’t necessary, symmetry-aware finite-domain reasoning gives IC3PO an edge both in terms of run time and the number of SMT queries.

Comparing IC3PO and fol-ic3, the only two verifiers that can infer invariants with a combination of universal and existential quantifiers, highlights the advantage of IC3PO’s approach over the separators-based technique [46] used in fol-ic3. The significant performance edge that IC3PO has over fol-ic3 is due to the fact that a) reasoning in IC3PO is primarily in a (small) finite domain compared to fol-ic3’s unbounded reasoning, and b) unlike fol-ic3 which enumeratively searches for specific quantifier patterns, IC3PO finds the required invariants without search by automatically inferring their patterns from the symmetry of the protocol.

Overall, the evaluation confirms the main hypothesis of this paper, that it is possible to use the relationship between symmetry and quantification to scale the verification of distributed protocols beyond the current state-of-the-art.

9 Related Work

Introduced by Lamport, TLA+ is a widely-used language for the specification and verification of distributed protocols [15, 59]. The accompanying TLC model checker can perform automatic verification on a finite instance of a TLA+ specification, and can also be configured to employ symmetry to improve scalability. However, TLC is primarily intended as a debugging tool for small finite instances and not as a tool for inferring inductive invariants.

Several manual or semi-automatic verification techniques (e.g., using interactive theorem proving or compositional verification) have been proposed for deriving system-level proofs [21, 36, 42, 43, 62, 69]. These techniques generally require a deep understanding of the protocol being verified and significant manual effort to guide proof development. The Ivy [63] system improves on these techniques by graphically displaying CTIs and interactively asking the user to provide strengthening assertions that can eliminate them.

Verification of parameterized systems using SMT solvers is further explored in MCMT [67], Cubicle [23], and paraVerifier [52]. Abdulla et al. [7] proposed \textit{view abstraction} to compute the reachable set for finite instances using forward reachability until cutoff is reached. Our technique builds on these works with the capability to automatically infer the required quantified inductive invariant using the latest advancements in model checking, by combining symmetry-aware clause...
learning and quantifier inference in finite-domain incremental induction. The use of derived/ghost variables has been recognized as important in [48, 58, 61]. IC3PO utilizes protocol structure, namely auxiliary definitions in the protocol specification, to automatically infer inductive invariants with complex quantifier alternations.

Several recent approaches (e.g., UPDR [45], QUIC3 [41], Phase-UPDR [32], fol-ic3 [46]) extend IC3/PDR to automatically infer quantified inductive invariants.

Unlike IC3PO, these techniques rely heavily on unbounded SMT solving. Our work is closest in spirit to FORHULL-N [25] and I4 [53, 54]. Similar to IC3PO, these techniques perform incremental induction over small finite instances of a parameterized system and employ a generalization procedure that transforms finite-domain proofs to quantified inductive invariants that hold for all parameter values. Dooley and Somenzi proposed FORHULL-N to verify parameterized reactive systems by running bit-level IC3 and generalizing the learnt clauses into candidate universally-quantified proofs through a process of proof saturation and convex hull computation. These candidate proofs involve modular linear arithmetic constraints as antecedents in a way such that they approximate the protocol behavior beyond the current finite instance, and their correctness is validated by checking them until the cutoff is reached. I4 uses an ad hoc generalization procedure to obtain universally-quantified proofs from the finite-domain inductive invariants generated by the AVR model checker [39].

10 Conclusions and Future Work

IC3PO is, to our knowledge, the first verification system that uses the synergistic relationship between symmetry and quantification to automatically infer the quantified inductive invariants required to prove the safety of symmetric protocols. Recognizing that symmetry and quantification are alternative ways of capturing invariance, IC3PO extends the incremental induction algorithm to learn clause orbits, and encodes these orbits with corresponding logically-equivalent and compact quantified predicates. IC3PO employs a systematic procedure to check for finite convergence, and outputs quantified inductive invariants, with both universal and existential quantifiers, that hold for all protocol parameters. Our evaluation demonstrates that IC3PO significantly is a significant improvement over the current state-of-the-art.

Future work includes exploring methods to utilize the regularity in totally-ordered domains during reachability analysis, investigating techniques to counter undecidability in practical distributed systems verification, and exploring enhancements to further improve the scalability to complex distributed protocols and their implementations. As a long-term goal, we aim towards automatically inferring inductive invariants for complicated distributed protocols, such as Paxos [50,51], by building further on this initial work.
Data Availability Statement and Acknowledgments

The software and data sets generated and analyzed during the current study, including all experimental data, evaluation scripts, and IC3PO source code are available at https://github.com/aman-goel/nfm2021exp. We thank the developers of pySMT [34], Z3 [24], and Ivy [63] for making their tools openly available. We thank the authors of the I4 project [53] for their help in shaping some of the ideas presented in this paper.

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Appendices

We include additional/supplementary material in the appendices, as follows:

Appendix A: IC3PO Pseudo Code (detailed)
   − Presents the detailed pseudo code of IC3PO and SymIC3

Appendix B: Proof of Correctness
   − Provides a correctness proof for symmetry-aware clause boosting during incremental induction (Section 4), and a correctness proof for quantifier inference (Section 5)

Appendix C: Simple Enhancements to the SymIC3 Algorithm
   − Describes simple enhancements to SymIC3 learning as briefly mentioned in Section 7

Appendix D: Effect of Symmetry Learning in Incremental Induction
   − Evaluates the effect of symmetry-aware learning in finite-domain incremental induction with a detailed comparison between IC3PO and I4

Appendix E: Statistical Analysis with Multiple SMT Solver Seeds
   − Provides a statistical analysis of the experiments from Section 8 through multiple runs for each tool with different solver seeds

Appendix F: Comparison against Human-Written Invariants
   − Compares IC3PO’s automatically-generated quantified inductive invariants against human-written invariant proofs on several metrics

Appendix G: Ordered Domains, Ring Topology, and Special Variables
   − Describes an extension to IC3PO that allows handling totally-ordered domains, as well as further details relating to ring topology and special variables, along with a preliminary evaluation

Appendix H: Finite Instance Sizes used in the Experiments
   − Lists down the instance sizes for IC3PO and I4 for each protocol in the evaluation (Section 8)
Appendix A  IC3PO Pseudo Code (detailed)

This section presents the detailed pseudo code of IC3PO and SymIC3.

1 procedure IC3PO(\(P, \sigma_0\))  
   \(-\) \(P \triangleq [S, R, Init, T, P]\), and \(\sigma_0\) is the initial base size
   2 reuse \(\triangleq \{\}\)  
   3 \(\sigma \triangleq \sigma_0\)
   4 \(Inv, Cex \leftarrow \text{SymIC3}(\hat{P}, \text{reuse})\)  
   5 if \(Cex\) is not empty then  
      6 return Violated, \(Cex\)  
      7 else  
         8 for each \(s_i \in S\) do  
             9 if not \(\text{IsInductiveInvariantFinite}(Inv, P(\sigma + \lfloor s_i \rfloor))\) then  
                10 reuse \(\triangleq \{\Phi \mid \Phi \in Inv \text{ and } \text{Init } \rightarrow \Phi \text{ and } \text{Init } \wedge T \rightarrow \Phi' \text{ in } P(\sigma + \lfloor s_i \rfloor)\}\)  
                11 \(\sigma \triangleq \sigma + \lfloor s_i \rfloor\)  
                12 go to Line 4  
         13 if not \(\text{IsInductiveInvariantUnbounded}(Inv, P)\) then  
              14 < never occurred >  
              15 return Error, Increase \(\sigma_0\)  
         16 return Safe, \(Inv\)  

Algorithm 1 presents the detailed pseudo code of IC3PO. Let \(\sigma : S \rightarrow \mathbb{N}\) be a function that maps each sort \(s_i \in S\) to a sort size \(|s_i|\). Given a protocol specification \(P\) and an initial base size \(\sigma_0\), IC3PO invokes SymIC3 on the finite protocol instance \(\hat{P} \triangleq P(\sigma)\), where \(\sigma\) is initialized to \(\sigma_0\) (lines 2-4). Upon termination, SymIC3 either a) produces a quantified inductive invariant \(Inv\) that proves the property for \(\hat{P}\), or b) a counterexample trace \(Cex\) that serves as a finite witness to its violation in both \(\hat{P}\) and the unbounded protocol \(P\) (lines 4-6). If the property holds for \(\hat{P}\), IC3PO performs finite convergence checks (Section 6) to check whether or not the invariant extends beyond \(\hat{P}\) and the unbounded protocol \(P\) (lines 8-12), by checking whether or not \(Inv\) is an inductive invariant for the larger finite instance \(\hat{P}^i \triangleq P(\sigma^i + [s_i])\) for each \(s_i \in S\), where \(\sigma^i + [s_i] \triangleq [s_i] \text{ except } ![s_i] = \sigma([s_i]) + 1\). If all finite checks pass, \(Inv\) is checked whether an inductive invariant in the unbounded domain (lines 13-15) using the standard induction checks a) \(\text{Init } \rightarrow Inv\), and b) \(Inv \wedge T \rightarrow Inv'\) in the unbounded domain. If all these checks pass, IC3PO emits the unbounded invariant \(Inv\), that holds for the unbounded \(P\) and is a proof certificate for the safety property (line 16). Otherwise, it re-starts SymIC3 on a finite instance with an increased size \(\sigma^i + [s_i]\) (lines 11-12), while seeding in all the strengthening assertions in \(Inv\) that are safe to learn in the first frame for the new SymIC3 iteration (line 10).

Algorithm 2 describes the symmetric incremental induction algorithm. The procedure first checks whether the property can be trivially violated (lines 19-22), and if not, starts recursively deriving and blocking counterexamples-to-induction (CTI) from the topmost frame (lines 24-35). Given a solver model \(m\),
procedure SymIC3(\(\hat{P}\), reuse) 
- \(\hat{P}\) \(\triangleq\) \([S, R, \hat{I}_{init}, \hat{T}, \hat{P}]\)
- reuse is a set of seed assertions that are safe to learn in the frame \(F_1\)

\[
F \leftarrow \emptyset, Cex \leftarrow \emptyset 
\]
- \(\hat{P}, F, Cex\) are global data structures

if SAT \(\vdash [\hat{I}_{init} \wedge \neg \hat{P}]\) : model \(m\) then
- - initial states check

\[
\text{state} \leftarrow \text{StateAsCube}(m) 
\]
- - get a single state from model \(m\), in cube form

Cex.extend(state)
- - property is trivially violated

return \(\emptyset, Cex\)
- - return the counterexample

\[
F.\text{extend}(\hat{I}_{init}) 
\]
- - setup the initial frame

while \(\top\) do

\[
N \leftarrow F.\text{size}() - 1 
\]

if SAT \(\vdash [F_N \wedge \hat{T} \wedge \neg \hat{P}']\) : model \(m\) then
- - check the topmost frame for counterexample-to-induction (CTI)

\[
\text{state} \leftarrow \text{StateAsCube}(m) 
\]
- - found a CTI

if SymRecBlockCube(state, \(N\)) then
- - try recursively blocking the CTI

\[
\text{return } \emptyset, Cex 
\]
- - failed to block CTI, return the counterexample

else
- - no CTI in the topmost frame

\[
F.\text{extend}(\hat{P}) 
\]
- - add a new frame

if \(N = 0\) then
- - add reusable seed assertions to the frame \(F_1\)

\[
F[1].\text{add}(\text{reuse}) 
\]

if ForwardPropagate() then
- - propagate inductive assertions forward

\[
\text{return } F_{\text{converged}}, \emptyset 
\]
- - frames converged, return \(F_{\text{converged}}\) as the inductive invariant

procedure SymRecBlockCube(\(cti, i\))
- - \(cti\) can reach \(\neg \hat{P}\) in \(F.\text{size}() - i\) steps

\[
\text{Cex.extend}(cti) 
\]
- - add the CTI to the counterexample

if \(i = 0\) then
- - check if reached the initial states

\[
\text{return } \top 
\]
- - reached initial states, property is violated

else
- - \(cti\) is unreachable from the previous frame

\[
\text{uc}' \leftarrow \text{MinimalUnsatCore}(F_{i-1} \wedge \hat{T}, cti') 
\]
- - get MUS from UNSAT query

\[
\varphi \leftarrow \neg \text{uc} 
\]
- - negate uc to get the quantifier-free clause

\[
\Phi \leftarrow \text{SymBoost}\{\varphi\} 
\]
- - symmetry-aware clause boosting with quantifier inference

\[
\Phi \leftarrow \text{AntecedentReduction}(\Phi, i) 
\]
- - antecedent reduction (optional), Appendix C.1

\[
\Phi \leftarrow \text{EprReduction}(\Phi, i) 
\]
- - EPR reduction (optional), Appendix C.2

\[
\text{Learn}(\Phi, F_i) 
\]
- - learn \(\Phi\) in frame \(i\)

\[
\text{return } \bot 
\]

Algorithm 2: Symmetric Incremental Induction

a state cube is derived as a single state represented as a cube, i.e., a conjunction of literals assigning each state variable with a value based on its assignment in \(m\) (lines 20, 27, 41). Lines 32-33 add the seed assertions in the given \(reuse\) set to the first frame \(F_1\). SymIC3 differs from the standard IC3 algorithm majorly
procedure SymBoost∀∃(ϕ) - - ϕ is the quantifier-free clause
52 Vγ ← {} , V3 ← {} - - a set of universally/existential quantified variables
53 body ← ϕ - - starting with ϕ, body is recursively generated
54 - - Vγ, V3 and body are global data structures
55 for each sort s that appears in clause ϕ do
56 π(ϕ, s) ← PartitionDistribution(ϕ, s)
57 if #(ϕ, s) < |s| then
58 (Vγ, V3, body) ← Infer∀(ϕ, π(ϕ, s)) - - infer ∀ for sort s, refer §5.1.A
59 else if |π(ϕ, s)| = 1 then
60 (Vγ, V3, body) ← Infer∃(ϕ, π(ϕ, s)) - - infer ∃ for sort s, refer §5.1.B.I
61 else if all but a few scenario then
62 (Vγ, V3, body) ← Infer∀∃(ϕ, π(ϕ, s)) - - infer ∀∃ for sort s, refer §5.1.B.II
63 else
64 < never occurred >
65 (Vγ, V3, body) ← Infer∀∃(ϕ, π(ϕ, s)) - - infer ∀∃ by default (may not be compact, though correct for the current instance)
66 Φ ← ∀Vγ. ∃V3. body - - stitch quantifiers for different sorts as ∀... ∃... < body >
67 return Φ - - Φ is the quantified predicate to learn in a SymIC3 frame

Algorithm 3: Symmetry-aware Clause Boosting with Quantifier Inference

in symmetry-aware quantified learning (line 46) and simple enhancements (lines 47-48).

The core of the SymIC3 algorithm is the SymBoost∀∃ algorithm, presented in Algorithm 3. SymBoost∀∃ is a simple and extendable procedure to perform symmetry-aware clause boosting and quantifier inference, as explained in detail in Sections 4 and 5. Starting from a given quantifier-free clause ϕ, the algorithm constructs a symmetrically-boosted quantified predicate Φ (line 67) by iteratively inferring quantifiers for each sort s (lines 55-65), and stitching them together (line 66). The algorithm maintains a set of universal and existential variables (line 53) and a body (line 54), that are iteratively modified based on the quantifier inference for each sort. For each sort s, the algorithm first generates π(ϕ, s) (line 56) based on how constants in sort s appear in the literals of ϕ (whether identically or not). The next step is to infer quantifiers using #(ϕ, s) and π(ϕ, s) (lines 57-65): a) infer universal quantifiers when #(ϕ, s) < |s|, b) otherwise if all constants of s appear in ϕ identically, infer existential quantifier, c) otherwise if all but a few scenario, infer ∀∃ based on the partitioning of constants in π(ϕ, s), and d) otherwise, infer ∀ by default (this case has not occurred). Changing the iteration order in line 55 doesn’t result in any difference, and is ensured during the recursive building of the body. At the end, a single quantified predicate Φ is derived by stitching together the quantified variables in Vγ and V3 with the body as ∀... ∃... < body > (line 66).
Appendix B  Proof of Correctness

Appendix B.1  Correctness Proof for Symmetric Incremental Induction

This section provides a correctness proof for symmetry-aware clause boosting during incremental induction (Section 4).

Like the invariance of \( \hat{\text{Init}} \), \( \hat{T} \), and \( \hat{P} \) under any permutation \( \gamma \in G \) (refer (2)), the logical orbit of a clause \( \varphi \) is also invariant under such permutations, i.e.,

\[
\left[ \varphi^{L(G)} \right]^{\gamma} \leftrightarrow \varphi^{L(G)}
\]

**Lemma 1.** For any SymIC3 frame \( F_i \), \( F_i^{\gamma} \equiv F_i \) for any \( \gamma \in G \).

**Proof.** Recall that \( \hat{\text{Init}}^{\gamma} \equiv \hat{\text{Init}} \) and \( \hat{P}^{\gamma} \equiv \hat{P} \). The condition \( F_i^{\gamma} \equiv F_i \) is trivially true for \( i = 0 \) since \( F_0 = \hat{\text{Init}} \). When \( i > 0 \), the condition is true during frame initialization since each frame is initialized to \( \hat{P} \). When blocking a cube \( \neg \varphi \) in \( F_i \), incremental induction with symmetry boosting refines \( F_i \) with the complete logical orbit \( \varphi^{L(G)} \) of \( \varphi \). Since \( \left[ \varphi^{L(G)} \right]^{\gamma} \equiv \varphi^{L(G)} \), the logical invariance of \( F_i \) under \( \gamma \), continues to be preserved in all backward reachability updates. \( \Box \)

The following theorem establishes the correctness of symmetry-aware clause boosting in incremental induction.

**Theorem 1.** If a quantifier-free cube \( \neg \varphi \) is unreachable from frame \( F_{i-1} \), i.e., \( F_{i-1} \wedge \hat{T} \wedge \neg[\varphi]^{'} \) is unsatisfiable, then \( F_{i-1} \wedge \hat{T} \wedge \neg[\varphi^{L(G)}]^{'} \) is also unsatisfiable.

**Proof.** Let \( Q \triangleq F_{i-1} \wedge \hat{T} \wedge \neg[\varphi]^{'} \) and assume that \( Q \) is unsatisfiable. Consider any permutation \( \gamma \in G \) and the corresponding permuted formula \( Q^{\gamma} \triangleq F_{i-1}^{\gamma} \wedge \hat{T}^{\gamma} \wedge \neg[\varphi^{L(G)}]^{'} \). Since permuting the sort constants simply re-arranges the protocol’s state variables in a formula without affecting its satisfiability, \( Q \) and \( Q^{\gamma} \) must be equisatisfiable, and hence \( Q^{\gamma} \) is unsatisfiable.

Noting that \( \hat{T} \) and \( F_{i-1} \) are invariant under \( \gamma \in G \) (from (2) and Lemma 1), we obtain \( Q^{\gamma} = F_{i-1}^{\gamma} \wedge \hat{T}^{\gamma} \wedge \neg[\varphi^{L(G)}]^{'} \) proving that if cube \( \neg \varphi \) is unreachable from frame \( F_{i-1} \), then its image under any \( \gamma \in G \) is also unreachable. Therefore, \( F_{i-1} \wedge \hat{T} \wedge \neg[\varphi^{L(G)}]^{'} \) is unsatisfiable. \( \Box \)

Appendix B.2  Correctness Proof for Quantifier Inference

This section provides a correctness proof sketch for quantifier inference (Section 5).

**Theorem 2.** Given a finite instance \( \hat{P} \), let \( \varphi \) be such that \( 0 < \#(\varphi, s) < |s| \) for some sort \( s \in S \). Let \( \Phi(s) \) be the quantified predicate obtained by applying SymIC3’s quantifier inference for \( s \). \( \Phi(s) \) is logically equivalent to \( \varphi^{L(Sym(s))} \).
Proof. Let \( \gamma \) be any permutation in \( \text{Sym}(s) \), and let \( n \triangleq \#(\varphi, s) \). Let \( \hat{\varphi} \) be the clause obtained by replacing in \( \varphi \) each constant \( c_I \in s \) by a corresponding variable \( V_i \) of sort \( s \).

Let \( A \triangleq [ (V_1 = c_1) \land \cdots \land (V_n = c_n) ] \rightarrow \hat{\varphi} \). By the transitivity of equality, \( A \equiv \varphi \). Let \( B \triangleq \bigwedge_{\gamma \in \text{Sym}(s)} A^\gamma \). Since \( A \equiv \varphi \), therefore, \( B \equiv \varphi^L(\text{Sym}(s)) \), and can be re-written as:

\[
B = \bigwedge_{\gamma \in \text{Sym}(s)} [ (V_1 = c_1) \land \cdots \land (V_n = c_n) ] \rightarrow \hat{\varphi}^\gamma \quad (15)
\]

\[
= \bigwedge_{\gamma \in \text{Sym}(s)} ( (V_1 = c_1) \land \cdots \land (V_n = c_n) ) \rightarrow \hat{\varphi} \quad (16)
\]

\[
= \forall V_1 \ldots V_n. (\text{distinct } V_1 \ldots V_n) \rightarrow \hat{\varphi} \quad (17)
\]

\[
= \Phi(s) \quad (18)
\]

(15) \& (16) are equal since \( \hat{\varphi} \) does not contain any constant of sort \( s \), and hence \( [\hat{\varphi}]^\gamma \equiv \hat{\varphi} \). (16) \& (17) are equal since the antecedents in (16) cover all possible assignments of variables \( (V_1, \ldots, V_n) \) to \( n \) distinct constants of sort \( s \). There are total \( \binom{n}{2} \times n! \) possible assignments of the variables in (17) to \( n \) distinct constants of sort \( s \), one each corresponding to the \( \binom{n}{2} \times n! \) permutations in \( \text{Sym}(s) \) that yield a logically-distinct antecedent in (16). (17) \& (18) are equal since given \( \#(\varphi, s) < |s| \). Since \( B \equiv \varphi^L(\text{Sym}(s)) \), therefore \( \Phi(s) \equiv \varphi^L(\text{Sym}(s)) \). \( \square \)

Theorem 3. Given a finite instance \( \hat{\mathcal{P}} \), let \( \varphi \) be such that all constants of a sort \( s \in \mathcal{S} \) appear identically in the literals of \( \varphi \). Let \( \Phi(s) \) be the quantified predicate obtained by applying \( \text{SymIC}3 \)'s quantifier inference for \( s \). \( \Phi(s) \) is logically equivalent to \( \varphi^L(\text{Sym}(s)) \).

Proof. Let \( \gamma \) be any permutation in \( \text{Sym}(s) \). Since given all constants in sort \( s \) appear identically in the literals of \( \varphi \), therefore \( \pi(\varphi, s) \) consists of a single cell, and any permutation \( \gamma \in \text{Sym}(s) \) does not result in a new logically-distinct clause, i.e., \( \varphi^\gamma \equiv \varphi \). As a result, \( \varphi^L(\text{Sym}(s)) \equiv \varphi \).

Without loss of generality, \( \varphi \) can be written as:

\[
\varphi = \varphi_{\text{others}} \lor \bigvee_{c_1 \in s} \varphi_s(c_1) \quad (19)
\]

where \( \varphi_{\text{others}} \) is the disjunction of literals in \( \varphi \) that do not contain any constant of sort \( s \), and \( \varphi_s(c_1) \) is the disjunction of literals in \( \varphi \) that contain a constant \( c_1 \in s \). Note that \( \varphi_{\text{others}} \) can be \( \bot \).

Let \( \hat{\varphi}_s \) be the clause obtained by replacing in \( \varphi_s(c_1) \) each constant \( c_1 \in s \) by a variable \( V \) of sort \( s \). Note that since all constants of sort \( s \) appear identically in the literals of \( \varphi \), therefore \( \hat{\varphi}_s \) is the same for each \( c_1 \in s \). The clause \( \varphi \) can
therefore be re-written as:

$$\varphi = \varphi_{\text{others}} \lor \bigvee_{c_i \in \mathcal{s}} (V = c_i) \rightarrow \widehat{\varphi_s}$$  \hspace{1cm} (20)

$$= \varphi_{\text{others}} \lor \exists V. \widehat{\varphi_s}$$  \hspace{1cm} (21)

$$= \Phi(s)$$  \hspace{1cm} (22)

(19) & (20) are equal due to the transitivity of equality. (20) & (21) are equal since expanding the existential quantifier as a disjunction over all possible assignments of the variable $V$ gives the expression in (20). (21) & (22) are equal since $\#(\varphi, \mathcal{s}) = |\mathcal{s}|$ and $|\pi(\varphi, \mathcal{s})| = 1$, and hence $\text{SymIC3}$ infers $\Phi(s)$ as (21).

Since $\varphi \equiv \varphi_{L(\text{Sym}(s))}$, therefore $\Phi(s) \equiv \varphi_{L(\text{Sym}(s))}$. \hfill \Box
Appendix C  Simple Enhancements to the IC3PO Algorithm

This section describes simple enhancements to SymIC3 learning as mentioned in Section 7.

Appendix C.1  Antecedent Reduction

Antecedent reduction strengthens a quantified predicate $\Phi$ by dropping the antecedent (distinct $\ldots$) and checking the unsatisfiability of the query $[F_{i-1} \land \hat{T} \land \neg \Phi^\prime]$. For example, $\Phi_2$ from (6) can possibly be strengthened by dropping (distinct $X_1, X_2$) from the antecedent to get $\Phi_{new}$, if the query $[F_{i-1} \land \hat{T} \land \neg \Phi_{new}^\prime]$ is unsatisfiable, where

$$\Phi_{new} = \forall X_1, X_2 \in \text{value.} \neg \text{decision}(X_1) \lor \text{decision}(X_2)$$

If instead, the query is satisfiable, the original predicate $\Phi_2$ should be learnt.

Appendix C.2  EPR Reduction

With the quantifier inference employed by SymBoost$\exists$ (Algorithm 3), SymIC3 can produce predicates with alternating quantifiers, which can result in quantifier-alternation cycles. For example, our running example already includes a quantifier alternation from $\text{quorum} \rightarrow \text{node}$ (Figure 1, line 3). Consider an example predicate:

$$\Phi = \forall Y \in \text{node}, \exists Z \in \text{quorum. member}(Y, Z)$$

The quantified predicate $\Phi$ adds the arc $\text{node} \rightarrow \text{quorum}$, generating a quantifier-alternation cycle:

$$\text{quorum} \rightarrow \text{node} \rightarrow \text{quorum}$$

Even though there are no undecidability concerns while reasoning over the finite instance $\hat{P}$ (since the sort domains are finite), it is desirable to avoid quantifier-alternation cycles and derive the invariant in the EPR fragment [64] of FOL. Restricting to the EPR fragment allows robustly checking the inductive invariant over the unbounded protocol $P$. Note that IC3PO performs invariant construction as well as finite convergence checks both in a finite domain (as detailed in Section 7).

We can additionally strengthen the learning to be within the EPR fragment, by pushing out existential quantifiers and avoid generation of quantifier-alternation cycle. For example, the EPR-reduced version $\Phi_{epr}$ of $\Phi$ is

$$\Phi_{epr} = \exists Z \in \text{quorum, } \forall Y \in \text{node. member}(Y, Z)$$

If we consider both $\Phi$ and its negation $\neg \Phi$ (as needed during induction checks), EPR-reduction basically flips the quantifier-alternation arcs. For example, the quantifier-alternation graph with the EPR-reduced predicate $\Phi_{epr}$ (instead of $\Phi$) is:

$$\text{quorum} \rightarrow \text{node} \leftarrow \text{quorum}$$
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$\neg \Phi_{epr}$ adds the arc $\text{node} \leftarrow \text{quorum}$.

Logically, pushing out the existential quantifier results in a reduced/stricter formula, with $\Phi_{epr} \rightarrow \Phi$, but $\Phi \not\rightarrow \Phi_{epr}$ (hence we call it EPR “reduction”). Intuitively, this difference is analogous to the difference in the statements:

$\text{Likes}_{\forall \exists} := \text{Everyone likes someone}$ $\quad \text{Likes}_{\exists \forall} := \text{Someone is liked by everyone}$

where $\text{Likes}_{\exists \forall} \rightarrow \text{Likes}_{\forall \exists}$, but $\text{Likes}_{\forall \exists} \not\rightarrow \text{Likes}_{\exists \forall}$.

We can add EPR reduction in the incremental induction procedure with SymIC3, that enables learning the EPR-reduced form $\Phi_{epr}$ instead of $\Phi$ only when it is safe, i.e., only when $\neg \Phi_{epr}$ is still unreachable from the previous incremental induction frame $F_{i-1}$. We do so by checking the unsatisfiability of the finite domain (and hence decidable) query $[ F_{i-1} \land \top \land \neg \Phi_{epr}']$. If the query is unsatisfiable, we learn the strengthened EPR-reduced predicate $\Phi_{epr}$. Else, the original form, i.e., $\Phi$, is learnt.

Note- Both simple enhancements presented in this section were left disabled in IC3PO for all experiments in this paper to focus the evaluation on the main paper contents. Initial investigation with these enhancements shows significant benefits in performance and robustness, with hardly any overhead.
Appendix D Effect of Symmetry Learning in Incremental Induction

This section evaluates the effect of symmetry-aware clause boosting in finite-domain incremental induction with a detailed comparison between IC3PO and I4.

Table 3 compares the effect of symmetry-aware learning in incremental induction for the problems solved by both IC3PO and I4. The table compares the number of SMT solver calls made and counterexamples-to-induction (CTI) encountered during the incremental induction procedure, as well as the number of assertions in the final (quantified) inductive invariant. SymIC3’s symmetry boosting helps IC3PO to make orders of magnitude fewer SMT solver calls compared to I4 and solve the problem after discovering many fewer CTIs.

Overall, Table 3 justifies the runtime speedups observed in Table 2, and confirms the benefits of symmetry-aware learning.

| Protocol                   | IC3PO        | I4          |
|----------------------------|--------------|-------------|
|                            | #SMT | #CTI | #Inv | #SMT | #CTI | #Inv |
| tla-consensus              | 13   | 0    | 1    | 7    | 0    | 1    |
| i4-lock-server             | 31   | 1    | 2    | 35   | 2    | 2    |
| ex-quorum-leader-election  | 117  | 7    | 5    | 15429| 847  | 14   |
| tla-simple                 | 273  | 23   | 3    | 1319 | 41   | 3    |
| ex-lockserv-automaton      | 568  | 51   | 12   | 1731 | 156  | 15   |
| pyv-sharded-kv             | 572  | 25   | 8    | 2101 | 170  | 15   |
| pyv-lockserv               | 676  | 58   | 12   | 1606 | 142  | 15   |
| i4-learning-switch         | 567  | 32   | 9    | 26345| 1310 | 11   |
| ex-simple-decentralized-lock| 2155 | 87   | 15   | 5561 | 490  | 22   |
| i4-two-phase-commit        | 2131 | 68   | 11   | 4045 | 288  | 16   |
| pyv-consensus-wo-decide    | 1866 | 141  | 9    | 41137| 2451 | 42   |
| pyv-consensus-foral        | 3423 | 247  | 10   | 156838| 10316| 44   |
| pyv-learning-switch        | 3352 | 112  | 13   | 51021| 3639 | 49   |

\[
\sum #SMT &= 15744 \quad (19.5x\ better) \\
\sum #CTI &= 852 \quad (23.3x\ better) \\
\sum #Inv &= 110 \quad (2.3x\ better)
\]

Table 3: Comparison of different incremental induction metrics between IC3PO and I4 for the problems solved by both

#SMT: number of solver queries, #CTI: number of counterexamples-to-induction
#Inv: number of assertions in the final (quantified) inductive invariant
Appendix E  Statistical Analysis with Multiple SMT Solver Seeds

This section provides a statistical analysis of the experiments from Section 8 through multiple runs for each tool with different solver seeds.

Different tools perform best with different SMT solvers (e.g., I4 uses a combination of Yices 2 [26] and Z3 [24], fol-ic3 uses Z3 and CVC4 [13], while UPDR and IC3PO use Z3). For the results presented in Table 2, a fixed SMT solver seed (i.e., seed = 1) was used for all tools. To get an idea of the effect of randomness in SMT solving, we performed 10 runs with different solver seeds for each tool on all protocols, and compared the runtime mean and standard deviation.

| Protocol (#29)                  | IC3PO | I4   | UPDR | fol-ic3 |
|--------------------------------|-------|------|------|---------|
|                               | #     | Time | σ    | #     | Time | σ    | #     | Time | σ    | #     | Time | σ    | #     | Time | σ    |
| tla-consensus                  | ✓ 0   | 0    | ✓ 5  | ✓ 0   | 0    | ✓ 1  | ✓ 2   | ✓ 1   | ✓ 10 | ✓ 21  | ✓ 11  |
| tla-tcommit                    | ✓ 1   | 0    | ✓ 2  | ✓ 1   | ✓ 0  | ✓ 1  | ✓ 2   | ✓ 1   | ✓ 10 | ✓ 21  | ✓ 11  |
| i4-lock-server                 | ✓ 1   | 0    | ✓ 2  | ✓ 1   | ✓ 0  | ✓ 1  | ✓ 2   | ✓ 1   | ✓ 10 | ✓ 21  | ✓ 11  |
| ex-quorum-leader-election      | ✓ 3   | 0    | ✓ 32 | ✓ 10  | 1    | ✓ 21 | ✓ 11  |
| pyv-toy-consensus-forall       | ✓ 3   | 1    | ✓ 6  | ✓ 1   | ✓ 11 |
| tla-simple                     | ✓ 34  | 93   | ✓ 5  | ✓ 2   | ✓ 3  |
| ex-lockserv-automaton          | ✓ 9   | 3    | ✓ 32 | ✓ 21  | ✓ 11 |
| tla-simpleregular              | ✓ 8   | 4    | ✓ 79 | 22    |
| pyv-sharded-kv                 | ✓ 8   | 1    | ✓ 4  | ✓ 6   | ✓ 22 |
| pyv-locksev                    | ✓ 11  | 4    | ✓ 3  | ✓ 15  | ✓ 8  |
| tla-two-phase                  | ✓ 15  | 3    | ✓ 99 | 16    |
| i4-learning-switch             | ✓ 20  | 8    | ✓ 22 | ✓ 6   | ✓ 22 |
| pyv-toy-consensus-epr          | ✓ 14  | 8    | ✓ 4  | ✓ 47  |
| pyv-consensus-wo-decide        | ✓ 79  | 167  | ✓ 19 | ✓ 9   |
| pyv-consensus-fee              | ✓ 40  | 9    | ✓ 1226 | 398 |
| pyv-consensus-fee              | ✓ 135 | 72   | ✓ 1042 | 398 |
| pyv-learning-switch            | ✓ 161 | 66   | ✓ 387 | 17   |
| i4-chord-ring-maintenance      | ✓ 8 1289 | 1191 | ✓ 387 | 17   |
| pyv-sharded-kv-no-lost-keys    | ✓ 2   | 0    | ✓ 5  |
| ex-naive-consensus             | ✓ 5   | 1    | ✓ 80 | 17    |
| pyv-client-server-ae           | ✓ 1   | 0    | ✓ 630 | 130 |
| ex-simple-election             | ✓ 172 | 522  | ✓ 38 | 8     |
| pyv-toy-consensus-epr          | ✓ 14  | 8    | ✓ 47 | 12    |
| ex-toy-consensus               | ✓ 11  | 5    | ✓ 22 | 4     |
| pyv-client-server-db-ae        | ✓ 32  | 30   | ✓ 6 | 1     |
| pyv-hybrid-reliable-broadcast  | ✓ 6 157 | 211  | ✓ 6 2264 | 740 |
| pyv-firewall                   | ✓ 2   | 0    | ✓ 6  |
| ex-majorityset-leader-election | ✓ 63  | 47   | ✓ 5  |
| pyv-consensus-epr              | 2 1968 | 943  | 5 768 | 404  |

No. of problems solved (out of 29) | 29   | 13   | 14  | 25    |
Uniquely solved                    | 3    | 0    | 0   | 0     |
For 11 cases solved by all: ∑ Time | 470  | 2727 | 795 | 2752  

Table 4: Statistical comparison of IC3PO against other state-of-the-art verifiers

# number of runs where successfully solved (out of 10) (√ means 10, ✗ means 0),

Time: runtime mean (in seconds), σ: runtime standard deviation (in seconds)

5 We used Yices 2 version 2.6.2, Z3 version 4.8.9 and CVC4 version 1.7.
Appendix F  Comparison against Human-Written Invariants

Figure 2 compares IC3PO’s automatically-generated inductive invariants against the human-written proofs on several metrics. Our evaluation shows IC3PO produces compact proofs of sizes comparable to the manually-written inductive invariants, even shorter than the human proofs on several occasions. As a side benefit, IC3PO’s inductive invariants are pretty-printed in the Ivy format [3], and thus, can also be independently checked/validated through Ivy.

Fig. 2: Comparison of IC3PO’s inductive invariant against human-written proof. IC3PO is on x-axis, human-written on y-axis.
Appendix G  Ordered Domains, Ring Topology and Special Variables

This section describes an extension to IC3PO that allows handling totally-ordered domains, as well as further details relating to ring topology and special variables (along with a preliminary evaluation).

| Protocol (#13)                  | Human IC3PO | IC4 | UPDR | fol-ic3 |
|---------------------------------|-------------|-----|------|---------|
| ex-distributed-lock-abstract    | < 12        | 15  | 25   | 11      |
| ex-decentralized-lock           | < 4         | 25  | 5    | 65      |
| ex-distributed-lock-maxheld     | < 6         | 58  | 10   | 1866    |
| ppy-ticket                      | < 14        | 65  | 8    | 1896    |
| i4-database-chain-replication   | < 9         | 98  | 6    | 1382    |
| ex-decentralized-lock-abstract  | < 6         | 126 | 18   | 5069    |
| i4-distributed-lock             | < 7         | 155 | 10   | 3472    |
| ex-ring-not-dead                | < 2         | 10  | 2    | 161     |
| ex-ring                         | < 3         | 11  | 3    | 269     |
| ex-ring-id-not-dead-limited     | < 2         | 24  | 2    | 250     |
| ppy-ring-id-not-dead            | < 2         | 37  | 2    | 275     |
| ppy-ring-id                     | < 4         | 73  | 4    | 809     |
| i4-leader-election-in-ring      | < 6         | 323 | 5    | 2907    |

Table 5: Comparison of IC3PO against other state-of-the-art verifiers

Time: runtime in seconds, Inv: # assertions in the inductive invariant, SMT: # SMT solver queries made, <$> indicates protocol has a ring topology, < indicates protocol has a totally-ordered domain.

Ordered domains like *epoch*, *time*, etc. are not symmetric, which makes such domains unsuitable to directly apply a symmetry argument. Specifically, restricting an unbounded ordered domain to a finite size results in introducing boundary cases with a “max” element, complicating finite-domain behavior.

Even in the presence of ordered domains, symmetry-aware learning can still be applied to all the un-ordered domains while leaving the ordered domains as unbounded. As an initial exploration, we devised a hybrid procedure in IC3PO where ordered domains are handled in an unbounded fashion, in the same manner as in UPDR, while all other domains are handled in the SymIC3-style symmetry-aware and finite manner. We use UPDR’s *diagram-based abstraction* to infer quantifiers for the ordered domain, while using *SymBoost* (Algorithm 3) for the un-ordered domains.\(^6\)

For the protocols that involve a ring topology, a ring domain, generally composed of identical components arranged in a ring topology, retains domain symmetry since the position of each individual component in the ring is left uninitialized and can be arbitrarily permuted. Hence, SymIC3 can be directly

\(^6\) We refer the reader to [44] for a complete description of incremental induction with diagram-based abstraction.
applied. The same is true for protocols that have special components, like a special `start_node` that initially holds the lock in a distributed lock. Non-Boolean functions and variables are modeled in relational form with equality predicates. For example, permuting the predicate `(start_node = n_1)` with the permutation `(n_1, n_2)` gives the permuted predicate `(start_node = n_2)`. IC3PO exploits the symmetry in the sort domains, not symmetries over the protocol symbols (i.e., relations, functions and variables), and hence is unaffected by the presence of special protocol symbols.

Table 5 summarizes the experimental results for 13 protocols with totally-ordered domains, collected again from [2, 46, 53]. IC3PO solves all 13 problems and shows the advantages of symmetry-aware learning even when applied only to a subset of protocol’s domains. We believe additional exploration is needed for these cases, where the non-symmetric regularity in totally-ordered domains can be further utilized to improve learning during incremental induction.
## Appendix H  Finite Instance Sizes used in Experiments

Table 6 lists down the initial base instance sizes used for IC3PO runs in the evaluation (Section 8) for each protocol. The table also includes the final cutoff instance sizes reached, where the corresponding Inv generalizes/saturates to be an inductive proof for any size. Note again that IC3PO updates the instance sizes automatically, as described in Section 6.

| Protocol                                      | Finite instance sizes used for IC3PO |
|-----------------------------------------------|--------------------------------------|
| tla-consensus                                 | value = 2                            |
| tla-commit                                    | resource-manager = 2                 |
| i4-lock-server                                | client = 2, server = 1               |
| ex-quorum-leader-election                    | node = 2 → 3, nset = 2               |
| pyv-toy-consensus-forsall                     | node = 2 → 3, quorum = 1 → 3, value = 2 |
| tla-simple                                    | ∃ E node = 2, pcstate = 3, value = 2 → 3 |
| ex-locker-automaton                           | node = 2                            |
| tla-simple-regular                            | node = 2, pcstate = 4, value = 2 → 3 |
| pyv-sharded-kv                                | key = 2, node = 2, value = 2         |
| pyv-lockserv                                  | node = 2                            |
| i4-two-phase                                  | resource-manager = 2                 |
| i4-learning-switch                            | node = 2 → 3, packet = 1             |
| ex-simple-decentralized-lock                  | node = 2 → 4                         |
| i4-two-phase-commit                           | node = 4                            |
| pyv-consensus-wo-decide                       | node = 2 → 3, quorum = 1 → 3         |
| pyv-consensus-forsall                         | node = 2 → 3, quorum = 1 → 3, value = 2 |
| pyv-learning-switch                           | node = 2 → 4                         |
| i4-chord-ring-maintenance                     | node = 3 → 5                         |
| pyv-sharded-kv-no-lose-keys                   | key = 2, node = 2, value = 2         |
| ex-naive-consensus                            | node = 3, quorum = 3, value = 3      |
| pyv-client-server-ae                          | node = 2, request = 2 → 3, response = 2 |
| ex-simple-election                            | accep = 2 → 3, propose = 2, quorum = 1 → 3 |
| pyv-toy-consensus-ep                           | node = 2 → 3, quorum = 1 → 3, value = 2 |
| ex-toy-consensus                              | node = 2 → 3, quorum = 1 → 3, value = 2 |
| pyv-client-server-db-ae                       | db-request-id = 2 → 3, node = 2, request = 2 → 3, response = 2 |
| pyv-hybrid-reliable-broadcast                 | node = 2 → 3, quorum-a = 2 → 3, quorum-b = 2 |
| pyv-firewall                                  | node = 2 → 3                         |
| ex-majorityset-leader-election                | node = 2 → 3, nodeset = 2 → 3        |
| pyv-consensus-ep                              | node = 2 → 3, quorum = 1 → 3, value = 2 |

| Protocol                                      | Finite instance sizes used for IC3PO |
|-----------------------------------------------|--------------------------------------|
| ex-distributed-lock-abstract                  | ⊥ epoch = ∞, node = 2                |
| ex-decentralized-lock                         | ⊥ node = 2, time = ∞                 |
| ex-distributed-lock-maxheld                   | ⊥ epoch = ∞, node = 2                |
| pyv-ticket                                    | ⊥ thread = 2 → 3, ticket = ∞        |
| i4-database-chain-rollback                    | ⊥ key = 1, node = 2, operation = 2 → 3, transaction = ∞ |
| ex-decentralized-lock-abstract                | ⊥ node = 2 → 4, time = ∞             |
| i4-distributed-lock                           | ⊥ epoch = ∞, node = 2                |
| ex-ring-not-dead                              | ⊥ node = 3                            |
| ex-ring                                       | ⊥ node = 3                            |
| ex-ring-id-not-dead-limited                   | ⊥ id = 3, node = 3                   |
| pyv-ring-id-not-dead                          | ⊥ id = ∞, node = 3                   |
| pyv-ring-id-not-dead                          | ⊥ id = ∞, node = 3                   |
| i4-leader-election-in-ring                    | ⊥ id = ∞, node = 3                   |

Table 6: Finite instance sizes used for IC3PO

- \( a = x \) denotes sort \( a \) has both initial base size and final cutoff size \( x \)
- \( a = x \rightarrow y \) denotes sort \( a \) has initial size \( x \) and final cutoff size \( y \) (incrementally increased by IC3PO automatically)
- \( a = ∞ \) denote the totally-ordered sort \( a \) is left unbounded
- ⊥ indicates protocol has a ring topology, < indicates protocol has an ordered domain
- \( E \) indicates the protocol description has \( E \)

\[ a = x \rightarrow y \]
Table 7 lists down the instance sizes used for I4 runs in the evaluation (Section 8) for each protocol.

| Protocol                                      | Finite instance sizes used for I4 |
|-----------------------------------------------|-----------------------------------|
| tlac-consensus                                | value = 2                         |
| tlac-ccommit                                  | resource-manager = 2              |
| i4-lock-server                                | client = 2, server = 1            |
| ex-quorum-leader-election                    | node = 3, nset = 3                |
| pvv-toy-consensus-forall                      | node = 3, quorum = 3, value = 2   |
| tlac-simple                                   | node = 3, pctype = 3, value = 3   |
| ex-lockserv-automaton                         | node = 2                          |
| tlac-simpleregular                            | node = 3, pctype = 4, value = 3   |
| pvv-sharded-kv                                | key = 2, node = 2, value = 2      |
| pvv-lockserv                                  | node = 2                          |
| i4-learning-switch                            | resource-manager = 3              |
| ex-simple-decentralized-lock                  | node = 3, packet = 2              |
| i4-two-phase-commit                           | node = 4                          |
| pvv-consensus-wodecide                        | node = 3, quorum = 3              |
| pvv-consensus-forall                          | node = 3, quorum = 3, value = 2   |
| pvv-learning-switch                           | node = 4                          |
| i4-chord-ring-maintenance                     | node = 4                          |
| pvv-sharded-kv-no-lost-keys                   | key = 3, node = 3, value = 3      |
| ex-naive-consensus                            | node = 3, quorum = 3, value = 3   |
| pvv-client-server-ae                          | node = 3, request = 3, response = 3 |
| ex-simple-election                            | acceptor = 3, proposer = 2, quorum = 3 |
| pvv-toy-consensus-epr                         | node = 3, quorum = 3, value = 2   |
| ex-toy-consensus                              | node = 3, quorum = 3, value = 2   |
| pvv-client-server-db-ae                       | node = 3, quorum-a = 3, quorum-b = 3 |
| pvv-firewall                                  | node = 3                          |
| ex-majorityset-leader-election                | node = 3, nodeset = 3             |
| pvv-consensus-epr                             | node = 3, quorum = 3, value = 2   |
| ex-distributed-lock-abstract                  | epoch = 4, node = 2               |
| ex-decentralized-lock                         | node = 2, time = 4                |
| ex-distributed-lock-maxheld                   | epoch = 4, node = 2               |
| pvv-ticket                                    | thread = 3, ticket = 5            |
| i4-database-chain-replication                 | key = 1, node = 2, operation = 3, transaction = 3 |
| ex-decentralized-lock-abstract                | node = 4, time = 4                |
| i4-distributed-lock                           | epoch = 4, node = 2               |
| ex-ring-not-dead                              | node = 3                          |
| ex-ring                                       | node = 3                          |
| ex-ring-id-not-dead-limited                   | id = 3, node = 3                  |
| pvv-ring-id-not-dead                          | id = 4, node = 3                  |
| pvv-ring-id                                   | id = 4, node = 3                  |
| i4-leader-election-in-ring                    | id = 4, node = 3                  |

Table 7: Finite instance sizes used for I4

○ indicates protocol has a ring topology, < indicates protocol has an ordered domain

E indicates the protocol description has ∃