OBDD-based reliability evaluation for power communication networks

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OBDD-based reliability evaluation for power communication networks

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Abstract. The current stability assessments for communication network mostly use the static model without effectively taking common cause failure (CCF) into consideration. This paper has proposed an assessment method based on CCF. This method has taken the advantage of the high computational efficiency of ordered binary decision diagram (OBDD). The method determines the order of variables with the help of breadth-first search (BFS)), and then it uses the factorization to construct the OBDD structure of the network. The reliability expression of the fault independent case is obtained through the OBDD structure. Finally, the network reliability expression can be obtained via implicit substitution. This method is easy to use and experiment shows that this method evaluates the reliability of network more objectively.

1. Introduction
With the development of smart grid, power system is becoming more and more dependent on power communication network. Electric power communication network carries more and more business of electric power dispatching of power grid companies, business management and enterprise information construction, etc., it can be said that the normal operation of power communication network is directly related to the stable operation of the power grid [1]. Therefore, the research on reliability evaluation of power communication network has great significance.

As a dedicated communication network of power communication system, the power communication network itself has the common nature of ordinary communication networks, but also has its own characteristics. Therefore, reliability evaluation of power communication network can use the reliability evaluation method of general network as a reference [2]. Traditional network reliability assessment methods, such as the state enumeration method [3], the disjoint product method [4], may result in a "combinatorial explosion" in the case of large network sizes. In reference [6], an algorithm of ordered binary decision tree is proposed based on the binary decision tree. The algorithm can effectively solve the "combined explosion" problem in the traditional method and quickly calculate the reliability of the network.

However, the OBDD method assumes that the network component failure statistics are independent and does not take into adequate account of 0the characteristics of the power communication network. The power communication network usually covers a wide range, and in many cases the working environment is worse. Therefore, it is necessary to consider the correlation of network component failures, which is a typical case of the CCF [7]. Considering the reality of power communication network, this paper proposes a calculation method of network reliability. First, regardless of the
common cause failure of the network, the Boolean function of the network reliability is obtained according to the OBDD method, and then the implicit replacement [8] method is used to calculate the network reliability considering the common cause failure.

2. OBDD

2.1. Concept of OBDD
OBDD is a development of BDD by adding variable ordering and simplified rules. In the OBDD structure, there are two kinds of nodes, non-terminal nodes and terminal nodes, and a terminal node have only two components, called 0 nodes and 1 node respectively, which represent the two results of Boolean function subtypes. The non-terminal node \( v \) is identified by the variable name \( \text{var}(v) \) and has two output edges. The output edge corresponding to the value of 0 is called 0-edge, and which is represented by the dotted line, when \( v \) is 1, the corresponding output side is 1-side, which is represented by the solid line. In the OBDD structure, the variables on the path from the root node to the end node are arranged in order of variable order, and each variable appears only once.

2.2. Variable Order of OBDD
The time of OBDD’s operation depends on the size of the OBDD, and the size of OBDD is heavily dependent on the variable order [9]. And determining the optimal order is a NP-complete problem [10]. The study of reference [6] and [9] show that breadth first search is a more efficient algorithm among many OBDD sorting algorithms. This method is employed in this paper.

2.3. OBDD and Power Communication Network
The "function" and "failure" states in the network are equivalent to "0" and "1" states in the Boolean function. OBDD said the minimal path sets based on Boolean functions, and according to the OBDD decomposition of disjoint characteristics, the calculation of reliability of network by each of the variables in the Shannon decomposition the way to solve the recursive:

\[
f = x \cdot f_{x=1} + \overline{x} \cdot f_{x=0}
\]  

(1)

2.4. Edge Expansion Diagrams
Edge expansion diagrams method is based on the network edge’s unreliability and the node’s reliability. Then for the edges associated to source points, expand each edge \( e_i \) on each side to get a new subnet \( G \times e_i \). Repeat the above steps for each subnet \( G \times e_i \), until reaching the terminal node \( t \). Among them, in case edge expansion of \( e_i \) refers to combining source point in the network \( G \) s and other terminal node of \( e_i \) form the new source point in the subnet \( G \times e_i \). Delete all the edges associated to the source point, and use the merge of isomorphism child nodes and the delete of redundant nodes in the extension process and to avoid redundant computation.
Figure 1 illustrates the working process of the edge expansion diagrams method, which use the method of combining isomorphic subgraphs and removing redundant nodes to reduce the size of the OBDD. The specific construction process of OBDD can be referred to in the reference [11].

As we can see from Figure 1, subnets $G \times e_0$ and $G \times e_1$ can be obtained when we along the edge $e_0$ and $e_1$ expansion in the network $G$, and then respectively expansion operation on the subnet, and so on, until all reach the terminal node. In the above decomposition process, $G \times e_1 \times e_3 \times e_4 \times e_5$, $G \times e_1 \times e_4 \times e_6$ and $G \times e_1 \times e_2 \times e_5 \times e_6$ are isomorphic subgraphs. In the process of constructing OBDD, only one of the same isomorphic subgraph is needed, which can greatly reduce the redundancy calculation. In addition, by deleting the redundant nodes and their connected edges, you can further avoid unnecessary expansion, thereby reducing the amount of calculation. As can be seen from the working process in the graph, the OBDD structure of the network with the EED construction can effectively reduce the redundant computation.

3. Network Reliability
The reliability of a communication network refers to the ability of a network to accomplish specified tasks at a given time. The connected reliability of network can be divided into two - terminal reliability, K - terminal reliability and full - end reliability. The difference between them is the number of nodes with ensured connection. This paper considers the reliability of the two - terminal networks. Power communication networks usually cover a wide range of area, and some areas may be hostile (high temperature, humidity, earthquakes, etc.), which may result in failure correlation of local network units. Therefore, it is necessary to consider the effect of the total failure on the communication network.

In this paper, the assumption of network reliability is based on the following conditions: (1) the lifetime of the links in the same common cause group has the same probability distribution; (2) the nodes in the network are reliable. If the node's unreliability is considered, the edge replacement policy [6] can be used to further process it.
3.1. Common Cause Failure Calculation
Assuming that the failure process is independent of each other and obeys the Poisson distribution within the same common group, the time interval of any two failure events in the same failure process follows the exponential distribution. The probability of failure of any failure process is:

\[ P_j = \exp(-\lambda_j t) \]  

Where \( P_j \) represents the probability that \( j \) links do not fail at the same time in the common cause group, the \( \lambda_j \) is a parameter related to the failure process, indicating the failure rate of \( j \) links within the common cause group.

Assuming that a system is composed of \( n \) components, there may be many combinations in the failure process of the communication network. Therefore, the probability of the proper functioning of a link at time \( t \) is the probability that the failure of the link will not occur

\[ R_i(t) = \prod_{j=1}^{n} P_j^{C_j} \]

Since the link lifetime of the same group has the same probability density function, the probability that \( m \) links work normally at random moment can be expressed as:

\[ R_n^{(m)}(t) = P\{S_1 \cap S_2 \cap ... \cap S_m \cap t\} = P\{S_1; t\} ... P\{S_m; t | S_1 \cap ... \cap S_{m-1} \cap t\} = R_n^{(1)}(t) R_n^{(1)}(t) ... R_n^{(1)}(t) \]

So:

\[ R_n^{(m)}(t) = \prod_{j=n-m+1}^{n} R_j^{(1)}(t) \]  

3.2. Reliability Calculation Algorithm
Considering the common cause failure, the network reliability calculation method mainly includes the following steps.

1) First, regardless of the case of common cause failure, the order of variables is determined according to breadth first search method; then, according to the variable order, the OBDD of the network is constructed according to the CAE method what was mentioned above; finally, network reliability expression is generated by traversing the OBDD structure in a recursive way. Figure 2 shows the pseudo-code for constructing the OBDD.

```plaintext
OBDDconstruct(network G)
{
    if (source is merged into target) {
        return bdd_none;
    }
    remove redundant nodes from network G;
    if G in network_hash[
        return OBDD(G) from network_hash;
    ]
    for (each adjacent edge of source) {
        execute edge expansion to decompose G into subgraphs;
        call OBDDConstruct recursively for each subgraphs;
        use BDD Operation to get OBDD(G) from subgraphs;
    }
    insert the OBDD into network_hash;
    return OBDD(G);
}
```

Figure 2 Pseudo-code for Constructing the OBDD

2) There are \( n \) links in the same common cause group of the communication network, and each link has the same probability distribution in the case of independent failure, that is \( p_1 = p_2 = ... = p_n = P(t) \), which can simplify the reliability expression.

3) Then, the implicit substitution method is used to replace the \( P^{(k)}(t) \) in step (3) with \( R_n^{(k)} \)
failure probability corresponding to each process is $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and link reliability of $P(t)$. The OBDD generated by fig.1 shows that the network reliability is as follows:

$$R_s = 3 P^3(t) + 6 P^4(t) - 9 P^5(t) - 17 P^6(t) + 34 P^7 - 20 P^8 + 4 P^9$$

Consider the common cause failure of the network reliability expression $R_c$:

$$R_c = 3 R_0^3(t) + 6 R_0^4(t) - 9 R_0^5(t) - 17 R_0^6(t) + 34 R_0^7(t) - 20 R_0^8(t) + 4 R_0^9(t)$$

The above formula $R_0^3(t) - R_0^4(t) - R_0^5(t) - R_0^6(t)$ can be calculated from the formula (3), (4).

4. Experimental Results

Figure 3 is a topological model of a communication network, where the gray nodes represent sources and terminals respectively. Suppose that the network contains three common groups, where $e_1, e_2, e_5, e_6$ belong to the common group 1, the failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$; $e_3, e_4, e_7, e_8$ belong to the common group 2, the failure rates $\mu_1, \mu_2, \mu_3, \mu_4$; $e_9, e_{10}, e_{11}$ belong to the common group 3, with the failure rates $\omega_1, \omega_2, \omega_3$.

![Figure 3 Network Topology Model]

Table 1 Failure Parameters

| Invalid Parameter | probability | Invalid Parameter | probability |
|-------------------|-------------|-------------------|-------------|
|                   | 0.004/h     |                   | 0.001/h     |
|                   | 0.0022/h    |                   | 0.0005/h    |
|                   | 0.0014/h    |                   | 0.0034/h    |
|                   | 0.0009/h    |                   | 0.0024/h    |
|                   | 0.003/h     |                   | 0.0012/h    |
|                   | 0.002/h     |                   | 0.0007/h    |

The experimental results are shown in Fig. 4, where the dotted line $R_c(t)$ represents the network reliability considering the common cause failure, and the solid line $R_s(t)$ represents the network reliability without considering the common cause failure.
Figure 4 Network Reliability

From the curve of reliability of the communication network in Fig. 4, it can be seen that the reliability considering CCF is lower than that without considering CCF, therefore the former case is closer to the reality.

5. Conclusion

In order to evaluate the reliability of power communication network more objectively, this paper proposes a calculation method of network reliability considering common cause failure, and uses OBDD to simplify the function of Boolean function expression and improve the calculation efficiency. The experimental results show that this method can evaluate the reliability of the network more objectively, and provide a more effective method to consider the common cause failure. However, this paper does not give a clear estimate of the failure rate parameters, the next step will be its in-depth study.

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