A Large Electron EDM and Minimal Flavor Violation

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Abstract

The latest data from the ACME Collaboration have put a stringent constraint on the electric dipole moment (EDM) $d_e$ of the electron. Nevertheless, the standard model (SM) prediction for $d_e$ is many orders of magnitude below the new result, making it a powerful probe for physics beyond the SM. We carry out a model-independent study of $d_e$ in the SM with right handed neutrinos and its extension with neutrino seesaw mechanism under the framework of minimal flavor violation. We find that $d_e$ crucially depends on whether neutrinos are Dirac or Majorana particles. In the Majorana case, $d_e$ can reach its experimental bound and constrains the scale of minimal flavor violation to be above a few hundred GeV or more. We also explore the effects on $d_e$ of extra $CP$-violating sources in the Yukawa couplings of the right-handed neutrinos. Such new sources can have important effects on $d_e$. 
To understand the origin of CP violation is one of the outstanding challenges of modern particle physics. It may hold the key to solve the problem of why our Universe is dominated by matter over anti-matter. Although CP violation has been observed in flavor violating processes such as the mixing of neutral K- and B-mesons with their respective antiparticles and the decays of these particles \[1\], no laboratory experiments have yet revealed evidence of CP violation in flavor conserving transitions \[2, 3\]. The best constraints for the latter class of processes are available from measurements on the electric dipole moments (EDMs) of fermions, such as the electron and neutron. A nonzero EDM would break the \(T\) and \(CP\) symmetries. Since the standard model (SM) predictions for the electron and neutron EDMs are way below their current experimental bounds, EDM searches are powerful probes for new sources of \(T\) and \(CP\) violation beyond the SM.

Recently the ACME Collaboration \[4\] looking for the electron EDM, \(d_e\), using the polar molecule thorium monoxide has reported a new result of \(d_e = (-2.1 \pm 3.7_{\text{stat}} \pm 2.5_{\text{syst}}) \times 10^{-29}\,e\,cm\), corresponding to an upper bound of \(|d_e| < 8.7 \times 10^{-29}\,e\,cm\) at 90% confidence level. This is an improvement over the previous strictest bound by an order of magnitude, but still much higher than the SM expectation for \(d_e\), of order \(10^{-44}\,e\,cm\) \[3\]. Thus there is ample room between the present limit and SM value of \(d_e\) where new physics may manifest itself.

Beyond the SM, \(d_e\) can greatly exceed its SM prediction and even reach its measured bound \[3\]. Such enhancement of \(d_e\) may hail from various origins depending on the specifics of the new physics models. Therefore it is desirable to do an analysis of \(d_e\) beyond the SM which treats some general features of the physics without going into detailed model structures. In the literature, there is indeed a framework, that of minimal flavor violation (MFV) \[6, 7\], in which such an analysis can be carried out. Under the assumption of MFV, the sources of all flavor-changing neutral currents (FCNC) and CP violation are contained in renormalizable Yukawa couplings defined at the tree level. This offers a systematic method to organize and study possible SM-related new flavor- and \(CP\)-violating interactions.

In this letter, adopting the MFV hypothesis, we examine \(d_e\) in the SM with three right-handed neutrinos and its extension with neutrino seesaw mechanism. We look at contributions to tree-level dipole interactions, and only a finite number of terms pertain to \(d_e\). We find that its size depends considerably on whether neutrinos are Dirac or Majorana fermions. In the Majorana case, \(d_e\) can reach its experimental bound, which therefore limits the MFV scale to be above a few hundred GeV or more.

In the SM slightly expanded with the addition of right-handed neutrinos, the renormalizable Lagrangian relevant for lepton masses is given by

\[
\mathcal{L}_m = -\bar{L}_{i,L}(Y_\nu)_{ij} \nu_{j,R} \overline{H} - \bar{L}_{i,L}(Y_e)_{ij} E_{j,R} H - \frac{1}{2} \nu_{i,R}^T (M_\nu)_{ij} \nu_{j,R} + \text{H.c.} ,
\]

where summation over \(i, j = 1, 2, 3\) is implicit, \(L_{i,L}\) denote left-handed lepton doublets, \(\nu_{i,R}\) and \(E_{i,R}\) represent right-handed neutrinos and charged leptons, respectively, \(Y_\nu, Y_e\) are matrices for the Yukawa couplings, \(M_\nu\) is the right-handed neutrinos’ Majorana mass matrix, \(H\) is the Higgs doublet, and \(\overline{H} = i \tau_2 H^*\). The Higgs’ vacuum expectation value \(v \approx 246\,\text{GeV}\) as usual breaks the electroweak symmetry and makes SM particles massive. The \(M_\nu\) term in \(\mathcal{L}_m\) facilitates the type-I seesaw mechanism for producing small neutrino masses \[8\].

The basics of the MFV framework for quarks and leptons are given in Refs. \[6, 7\]. For the lepton sector it presupposes that \(\mathcal{L}_m\) in Eq. \(1\) is formally invariant under a global group \(U(3)_L \times U(3)_\nu \times U(3)_E = G_L \times U(1)_L \times U(1)_\nu \times U(1)_E\) with \(G_L = SU(3)_L \times SU(3)_\nu \times SU(3)_E\). The three generations of \(L_{i,L}, \nu_{i,R}\), and \(E_{i,R}\) transform as fundamental representations of \(SU(3)_{L,\nu, E}\) in \(G_L\), respectively. Since it is still unknown whether light neutrinos are Dirac or Majorana particles, we will deal with the two possibilities separately.
For Dirac neutrinos, the $M_\nu$ part in Eq. (1) is absent. We will work in the basis where $Y_e$ is already diagonal,

$$Y_e = \frac{\sqrt{2}}{v} \operatorname{diag}(m_e, m_\mu, m_\tau),$$

(2)

and the fields $\nu_{i,L}, \nu_{i,R}, E_{i,L}$, and $E_{i,R}$ refer to the mass eigenstates. We can express $L_{i,L}$ and $Y_\nu$ in terms of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix $U_{\text{PMNS}}$ as

$$L_{i,L} = \left( U_{\text{PMNS}} \right)_{ij} \nu_{j,L}, \quad Y_\nu = \frac{\sqrt{2}}{v} U_{\text{PMNS}} \hat{m}_\nu, \quad \hat{m}_\nu = \operatorname{diag}(m_1, m_2, m_3),$$

(3)

where $m_{1,2,3}$ are the light neutrino eigenmasses.

If neutrinos are Majorana fermions, some modifications to $Y_\nu$ are needed. The presence of the $M_\nu$ term in Eq. (1) with $M_\nu \gg M_D = vY_\nu/\sqrt{2}$ activates the seesaw mechanism involving the $6 \times 6$ neutrino mass matrix $M$ given by, in the $(U^\dagger_{\text{PMNS}} v_L^c, v_R^c)^T$ basis,

$$M = \begin{pmatrix} 0 & \tilde{M}_D \\ \tilde{M}_D & M_\nu \end{pmatrix},$$

(4)

and leading to the light neutrinos’ mass matrix

$$m_\nu = -\frac{v^2}{2} Y_\nu M_\nu^{-1} Y_\nu^\dagger U_{\text{PMNS}} \hat{m}_\nu U_{\text{PMNS}}^T.$$  

(5)

This allows one to choose $Y_\nu$ to be

$$Y_\nu = \frac{i\sqrt{2}}{v} U_{\text{PMNS}} \hat{m}_\nu^{1/2} O M_\nu^{1/2},$$

(6)

where $O$ is a matrix satisfying $OO^T = I$, which is a $3 \times 3$ unit matrix, and $M_\nu = \operatorname{diag}(M_1, M_2, M_3)$. As will be seen later, $O$ offers a potentially significant new source of CP violation beyond that in $U_{\text{PMNS}}$.

To derive nontrivial FCNC and CP-violating interactions, one assembles an arbitrary number of the Yukawa matrices to devise the representations $\Delta_\ell \sim (8, 1, 1), \Delta_\phi \sim (1, 8, 1), \Delta_\phi \sim (1, 1, 8), \Delta_\phi \sim (3, 3, 1)$, and $\Delta_\ell \sim (3, 1, 3)$ under $G_\ell$, combines them with two lepton fields to arrive at the $G_\ell$-invariant objects $\tilde{L}_L \Delta_\ell \tilde{L}_L$, $\bar{\nu}_R \Delta_\phi \nu_R$, $\tilde{E}_R \Delta_\phi \tilde{E}_R$, $\bar{\nu}_R \Delta_\ell \bar{\nu}_R$, and $\tilde{E}_R \Delta_\ell \tilde{L}_L$, attaches appropriate numbers of the Higgs field $H$ and SM gauge fields to form singlets under the SM gauge group, and also contracts all Lorentz indices. Since fermion EDMs flip chirality, only the last combination, $\tilde{E}_R \Delta_\ell \tilde{L}_L$, is of interest to our examination of $d_e$. For $\Delta_\ell$, we can construct it to be $\Delta_\ell = Y_\ell^\dagger \Delta_\ell$, where $\Delta_\ell$ is discussed next.

It is simple to see that $A = Y_\nu^\dagger Y_\nu$ and $B = Y_\nu^\dagger Y_\nu$ transform as $(8, 1, 1)$. Then formally $\Delta_\ell$ consists of an infinite number of terms, $\Delta_\ell = \sum \xi_{ijk} \cdots A^iB^jA^k \cdots$. Using the Cayley-Hamilton identity, one can resum the infinite series into a finite number of terms

$$\Delta_\ell = \xi_1 I + \xi_2 A + \xi_3 B + \xi_4 A^2 + \xi_5 B^2 + \xi_6 AB + \xi_7 BA + \xi_8 AB + \xi_9 BA^2 + \xi_{10} BAB + \xi_{11} AB^2 + \xi_{12} ABA^2 + \xi_{13} A^2B^2 + \xi_{14} B^2A^2 + \xi_{15} B^2AB + \xi_{16} AB^2A^2 + \xi_{17} B^2A^2B.$$  

(7)

We assume that the coefficients $\xi_{ijk\cdots}$ in the infinite series have negligible imaginary parts, so that the coefficients $\xi_e$ in Eq. (7) are complex mainly due to imaginary parts among the traces of the
matrix products $A^iB^jA^k \cdots$ from the application of the Cayley-Hamilton identity. Such imaginary contributions are reducible to $\text{Im} \text{Tr}(A^iB^jB^k) = (i/2) \text{Det}[A, B]$ which is a Jarlskog invariant and very small compared to unity $[9]$. This implies that, with all $\xi_{ijk\ldots}$ being of $\mathcal{O}(1)$, the contributions of $\xi_r$ to $d_e$ are suppressed by a factor of $m^2_\mu m^2_\tau/v^4$ compared to the contribution from $\mathcal{A}^2B^2$ which has the least number of suppressive factor $Y_e$ among the products in Eq. (7) that can potentially contribute to $d_e$. Hereafter we neglect the impact of the imaginary part of $\xi_r$.

The Lagrangian for the EDM $d_l$ of a lepton $l$ is $\mathcal{L}_d = -(i\xi_l/2)\bar{l}\sigma^{\mu\nu}\gamma_5F_{\mu\nu}$, where $F_{\mu\nu}$ is the photon field strength tensor. In the MFV framework, this arises from operators constructed using $\Delta_e$ plus the lepton, Higgs, and gauge fields. At lowest order, the operators are $[7]$

$$O^{(el)}_{RL} = g E_R^\dagger Y_e^\dagger L_1 \sigma_{\mu\nu} H^\dagger L_2 B^\mu_\nu, \quad O^{(e2)}_{RL} = g E_R^\dagger Y_e^\dagger L_2 \sigma_{\mu\nu} H^\dagger \tau_a L_2 W^\mu_\nu a,$$

where $W$ and $B$ denote the usual SU(2)$_L \times U(1)_Y$ gauge fields with coupling constants $g$ and $g'$, respectively, $\tau_a$ are Pauli matrices, and $\Delta_{ei}$ have the same form as in Eq. (7), but generally unequal $\xi_r$ which are real as well. One can express the effective Lagrangian containing these operators as

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \left( O^{(el)}_{RL} + O^{(e2)}_{RL} \right) + \text{H.c.},$$

where $\Lambda$ is the MFV scale. In general the operators in $\mathcal{L}_{\text{eff}}$ have different coefficients which have been absorbed by $\xi_r$ in their respective $\Delta$'s.

Expanding the Lagrangian in Eq. (3), one can identify the contributions to the charged lepton EDMs, $d_{E_l}$. They are proportional to $\text{Im}(Y_e^\dagger\Delta_{ei})_{kk}$. Since $Y_e$ are diagonal, $Y_e$ has the form in Eq. (3) or (4), and A and B are Hermitian, not all of the terms in Eq. (7) will yield nonzero contributions to $d_{E_i}$. For instance, $(Y_e^\dagger A)_{kk} = \sqrt{2} m_{E_k} A_{kk}/v$ is a real number and thus does not contribute to $d_{E_k}$. One needs to have terms in $\Delta_{ei}$ which are not Hermitian in order to have imaginary parts in $(Y_e^\dagger\Delta_{ei})_{kk}$. We find that only the terms proportional to $\mathcal{A}^2B^2$ and $\mathcal{A}^2B^2$ are pertinent to $d_{E_k}$.

Thus for the electron we have

$$d_e = \frac{\sqrt{2} ev}{\Lambda^2} \left[ \xi^\ell_{12} \text{Im}(Y_e^\dagger\mathcal{A}^2B^2)_{11} + \xi^\ell_{16} \text{Im}(Y_e^\dagger\mathcal{A}^2B^2)_{11} \right],$$

where $\xi^\ell_r = \xi^\ell_{12} - \xi^\ell_{16}$. If neutrinos are Dirac particles, with $Y_\nu$ from Eq. (3) we obtain

$$d_e^D = \frac{32 e m_e}{\Lambda^2} \left[ \xi^\ell_{12} + \frac{2(m^2_\mu + m^2_\tau)}{v^2} \xi^\ell_{16} \right] \left( \frac{m^2_\mu - m^2_\tau}{\sqrt{2}} \right) \frac{\left( m^2_\mu - m^2_\tau \right)\left( m^2_\mu - m^2_\tau \right)\left( m^2_\mu - m^2_\tau \right)}{v^8} J_{\ell},$$

where $J_{\ell} = \text{Im}(U_{e\ell}U^*_{\ell\mu}U^*_{\mu\nu}U_{\nu\rho})$ is the Jarlskog invariant for $U_{\text{PMNS}}$. For Majorana neutrinos, if $\nu_{i,R}$ are degenerate, $\tilde{M}_\nu = \tilde{M}_\nu \mathbb{1}$, and $O$ is a real orthogonal matrix, from Eq. (6) we have $A = (2/v^2)M U_{\text{PMNS}} \tilde{m}_\nu U^\dagger_{\text{PMNS}}$ and consequently

$$d_e^M = \frac{32 e m_e \tilde{M}^3}{\Lambda^2 v^8} \left( \frac{m^2_\mu - m^2_\tau}{v^2} \right) \left( m^2_\mu - m^2_\tau \right) \left( m^2_\mu - m^2_\tau \right) J_{\ell},$$

after neglecting the $\xi^\ell_{16}$ term. Clearly $d_e^M$ is substantially enhanced relative to $d_e^D$ due to $\tilde{M} \tilde{m}_\nu \gg m^2_\mu$.

In the preceding cases, $d_e$ arises from the $CP$-violating Dirac phase $\delta$ in $U_{\text{PMNS}}$, and the two Majorana phases $\alpha_{1,2}$ therein do not enter. However, if $\nu_{i,R}$ are nondegenerate, nonzero $\alpha_{1,2}$ can
lead to an extra effect on \( d_e \) even with a real \( O \neq \mathbf{1} \). With a complex \( O \), the phase in it may induce an additional contribution to \( d_e \), whether or not \( \nu_{i,R} \) are degenerate. The formulas for \( d_e \) in these scenarios are more complicated and are not shown here, but we will explore some of them numerically later.

The contributions to the electron EDM treated above have high powers in Yukawa couplings. Since in the MFV framework all \( CP \)-violation effects originate from the Yukawa couplings, the high orders in them actually reflect the fact that nonvanishing EDMs in the SM start to appear in multi-loops.

To evaluate \( d_e \), we need to know the \( U_{PMNS} \) elements. In the standard parametrization of Ref. [1], it depends on three mixing angles \( \theta_{12,13,23} \) and the phase \( \delta \) for Dirac neutrinos. We adopt their values from the latest fit to the global data on neutrinos with a normal hierarchy (NH) or inverted hierarchy (IH) of masses in Ref. [11], which also provides results on squared-mass differences of the neutrinos. For Majorana neutrinos, \( U_{PMNS} \) includes a phase matrix \( P = \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) \) multiplied from the right [1]. Since \( \alpha_{1,2} \) are still unknown, we will select specific values for them in our illustrations.

For Dirac neutrinos, we perform a scan of the empirical ranges of the parameters from Ref. [11] in order to maximize \( d_e^D \) in Eq. (11). The result in the NH or IH case is

\[
\frac{d_e^D}{e \text{ cm}} = 1.3 \times 10^{-99} (\xi_{12} + 1.0 \times 10^{-4} \xi_{16}) (\text{GeV}^2/\Lambda^2) e \text{ cm.}
\]

This is negligible compared to the most recent experimental upper bound [3].

If neutrinos are Majorana fermions, we now demonstrate that, in contrast, \( d_e \) can be sizable. We start by considering the simplest possibility that \( \nu_{i,R} \) are degenerate, \( M_L = M \mathbf{1} \), and the \( O \) matrix in Eq. (6) is real. In this scenario, \( d_e \) is already given in Eq. (12), which depends on the choice for one of \( m_{1,2,3} \) after the mass data are included. Scanning again the empirical parameter ranges in Ref. [11] to maximize \( d_e^M \), we obtain for \( m_1 = 0 \) \( (m_3 = 0) \) in the NH (IH) case

\[
\frac{d_e^M}{e \text{ cm}} = 4.7 (0.52) \times 10^{-23} \left( \frac{\mathcal{M}}{10^{15} \text{ GeV}} \right)^3 \left( \frac{\text{GeV}}{\Lambda} \right)^2, \tag{13}
\]

where \( \Lambda = \Lambda / |\xi_{12}|^{1/2} \) and the value of \( \mathcal{M} \) is specified below. Then \( |d_e^{\text{exp}}| < 8.7 \times 10^{-29} \text{ e cm} \) [4] implies

\[
\Lambda > 0.74 (0.24) \text{ TeV} \left( \frac{\mathcal{M}}{10^{15} \text{ GeV}} \right)^{3/2}. \tag{14}
\]

Since \( d_e \) in Eq. (13) is proportional to \( \mathcal{M}^3 \), one might naively think that \( d_e \) can easily reach its measured bound, which would therefore constrain \( \Lambda \) to a very high level with a very large \( \mathcal{M} \). However, there are restrictions on \( \mathcal{M} \). One is from consideration on the convergence of the series in Eq. (7) which supposedly includes arbitrarily high powers of \( A \) and \( B \). If the biggest eigenvalue of \( A \) exceeds 1, the coefficients \( \xi_{i} \) might not converge to finite numbers. However, the expansion quantities may not necessarily be \( A \) and \( B \), depending on the origin of MFV. It may be that it emerges from calculations of SM loops, in which case the expansion quantities may be more naturally be \( A/(16\pi^2) \) and \( B/(16\pi^2) \), restricting the eigenvalues of \( A \) to be below \( 16\pi^2 \). One may also consider instead the perturbativity condition for the Yukawa couplings, \( Y \mu_{ij} < \sqrt{4\pi} \) [12], implying a cap on the eigenvalues of \( A \) at \( 4\pi \). If we take the possible requirement that the eigenvalues of \( A \) not exceed 1 or \( 4\pi \), then \( \mathcal{M} = 6.2 \times 10^{14} \text{ GeV} \) or \( 7.7 \times 10^{15} \text{ GeV} \), respectively, in Eq. (13), which are roughly the expected seesaw scales in some grand unified theories. Hence \( \Lambda > 360 \text{ (120) GeV} \) or \( 16 \text{ (5) TeV} \), respectively, from Eq. (14). These \( \mathcal{M} \) and \( \Lambda \) numbers would decrease if \( m_{1(3)} > 0 \).
For \( M_{\nu} = \mathcal{M} \mathbb{I} \) and \( O \) being complex, \( A = (2/v^2)\mathcal{M}U_{\text{PMNS}}\tilde{m}_{\nu}^{1/2}OO^\dagger\tilde{m}_{\nu}^{1/2}U_{\text{PMNS}}^\dagger \) and \( d_e^M \) is given in Eq. (10), with the \( \xi_{16}^\ell \) term again being neglected. In general we can write \( OO^\dagger = e^{2i\mathcal{R}} \) with a real antisymmetric matrix

\[
R = \begin{pmatrix}
0 & r_1 & r_2 \\
-r_1 & 0 & r_3 \\
-r_2 & -r_3 & 0
\end{pmatrix}.
\]

(15)

Since \( OO^\dagger \) is not diagonal, the Majorana phases in \( U_{\text{PMNS}} \) will generally also enter \( A \) if \( \alpha_{1,2} \neq 0 \). We focus first on the \( CP \)-violating effect of \( O \) by setting \( \alpha_{1,2} = 0 \). For illustration, we pick \( r_{1,2,3} = \rho \), employ the experimental central values \( m_2^2 - m_1^2 = 7.54 \times 10^{-5} \text{ eV}^2 \), \( m_3^2 - \frac{1}{2}(m_1^2 + m_2^2) = 2.44(2.40) \times 10^{-3} \text{ eV}^2 \), \( \sin^2 \theta_{12} = 0.308 \), \( \sin^2 \theta_{23} = 0.425(0.437) \), \( \sin^2 \theta_{13} = 0.0234(0.0239) \), and \( \delta = 1.39(1.35)\pi \) in the NH (IH) case from Ref. [11], demand the eigenvalues of \( A \) be below 1, and present in Fig. 1(a) the resulting \( d_e^M \Lambda^2 \) as a function of \( \rho \) for \( m_{1(3)} = 0 \). We also display the curves with \( \delta = 0 \) for comparison. We have checked that other \( |r_{1,2,3}| = \rho \) cases with different relative signs of \( r_{1,2,3} \) produce roughly similar figures and that for \( m_{1(3)} > 0 \) the results tend to diminish in magnitude. If the eigenvalues of \( A \) were required to be below \( 4\pi \) instead, \( d_e^M \Lambda^2 \) would be multiplied by \((4\pi)^3\). Hereafter we require the eigenvalues of \( A \) to be less than 1.

With \( \alpha_{1,2} = 0 \), the \( CP \)-violating effect of \( O \) can still occur even if it is real provided that \( \nu_{i,R} \) are nondegenerate, in which case \( A = (2/v^2)U_{\text{PMNS}}\tilde{m}_{\nu}^{1/2}OM_{\nu}O^\dagger\tilde{m}_{\nu}^{1/2}U_{\text{PMNS}}^\dagger \) according to Eq. (6). For instance, assuming \( O \) to be real, \( O = e^{i\mathcal{R}} \) with \( r_{1,2,3} = \rho \), and \( M_{\nu} = \mathcal{M} \text{diag}(1, 0.8, 1.2) \), we show the resulting \( d_e^M \Lambda^2 \) versus \( \rho \) in Fig. 1(b), where only the \( \delta \neq 0 \) curves are nonvanishing and the sinusoidal nature of \( d_e \) is visible. It is evident from these examples that \( O \) supplies a potentially significant new source of \( CP \) violation which can have a bigger impact than \( \delta \).

To see the effect of the Majorana phases, we entertain a couple of possibilities: (a) \( M_{\nu} = \mathcal{M} \mathbb{I} \) and \( O = e^{i\mathcal{R}} \) and (b) \( M_{\nu} = \mathcal{M} \text{diag}(1, 0.8, 1.2) \) and \( O = e^{i\mathcal{R}} \), both with \( r_{1,2,3} = \rho \). Fixing \( \alpha_1 = 0 \), we depict the resulting dependence on \( \alpha_2 \) in Fig. 2 for \( \rho = 0.5 \) and nonzero or zero \( \delta \). Clearly the Majorana phases yield an additional important \( CP \)-violating contribution to \( d_e \) beyond \( \delta \).

![FIG. 1: Dependence of \( d_e^M \times \Lambda^2 = \Lambda^2/\xi_{12}^\ell \) on \( \rho \) in the absence of Majorana phases, \( \alpha_{1,2} = 0 \), for (a) degenerate \( \nu_{i,R} \) and complex \( O \) and (b) nondegenerate \( \nu_{i,R} \) and real \( O \), as explained in the text. In all figures, the label N (I) refers to the NH (IH) case with \( m_{1(3)} = 0 \).](image-url)
analysis. The treatment of the quark contributions is similar to that of leptons in the Dirac neutrino from the lepton sector as well as the quark sector, which needs to be included for a consistent MFV.

Now, the $\xi_{12}$ term contributes not only to $d_e$, but also to the muon anomalous magnetic moment ($g_\mu - 2$), the radiative decay $\mu \to e\gamma$, and the $\mu \to e$ conversion. We have checked that the constraint on $\hat{\Lambda}$ from the $g_\mu - 2$ data is weaker than that from $|d_e^{\text{exp}}|$. On the other hand, the constraint from the present empirical limits on $\mu \to e\gamma$ and $\mu \to e$ conversion are stronger by up to several times if the $\xi_{12}$ term dominates. However, these latter transitions, which are CP conserving, in general also receive contributions from some of the other $\xi_e$ terms among the operators in Eq. (9) which can, in principle, reduce the impact of the $\xi_{12}$ term, thereby weakening the restriction on $\hat{\Lambda}$ from the data on these processes. It follows that $d_e$ provides the best CP-violating probe for $\hat{\Lambda}$.

Lastly, the ACME experiment can also probe the CP-violating electron-nucleon interaction described by $\mathcal{L}_{eN} = -i(C_S G_F/\sqrt{2})\bar{e}\gamma_5 e\bar{N}N$ if $d_e$ is negligible. The source of $C_S$ may come from the lepton sector as well as the quark sector, which needs to be included for a consistent MFV analysis. The treatment of the quark contributions is similar to that of leptons in the Dirac neutrino case, with the roles of $Y_\nu$ and $Y_e$ interchanged with those of $Y_u$ and $Y_d$ for up- and down-type quarks. Thus the lowest-order operators contributing to $C_S$ are given by

$$\mathcal{L}_{\ell q} = \frac{1}{\Lambda^2} \bar{E}_R Y_\ell^\dagger W_{\ell\ell} L L \bar{U}_R Y_u L_{qu} \xi_{12} \tau_2 Q_L + \frac{1}{\Lambda^2} \bar{E}_R Y_e^\dagger W_{e\ell} L L \bar{Q}_L \Delta_{qd} D_R + \text{H.c.} \quad (16)$$

where $\Delta_{qu,qd}$ are the quark counterparts of $\Delta_{qi}$. The contributions from $\Delta_{qi}'$ turn out to be dominant. To determine $C_S$, we need the matrix elements $\langle N | m_q \bar{q} q | N \rangle = g_q^N \bar{u}_N u_N v$. Employing the chiral Lagrangian estimate in Ref. [13], we have for $M_\nu = \mathcal{M} \mathbb{I}$ and $O$ being real

$$C_S = \frac{16\sqrt{2} J_f m_e \mathcal{M}^3}{\Lambda^2 G_F v^9} \left( m_\mu^2 - m_\tau^2 \right) \left( m_1 - m_2 \right) \left( m_2 - m_3 \right) \left( m_3 - m_1 \right) \times \left[ \left( g_d^N + g_s^N + g_t^N \right) \xi_{12}^{\ell_2} - \left( g_u^N + 2g_t^N \right) \xi_{12}^{\ell_1} \right]. \quad (17)$$

The experimental bound $|C_S| < 5.9 \times 10^{-9}$ reported by ACME [4] then implies, if $\mathcal{M} \approx 6 \times 10^{14}$ GeV and the maximal values of $g_q^N$ from Ref. [13] are used,

$$\hat{\Lambda} > 0.24 \ (0.077) \ \text{GeV} \quad (18)$$
in the NH (IH) case with $m_{1(3)} = 0$. This is far less stringent than the restriction from $d_e$ directly.

In conclusion, motivated by the newest measured bound on the electron EDM, $d_e$, we have explored its prediction under the hypothesis of minimal flavor violation. In this framework, $d_e$ can be close to its experimental limit if neutrinos are Majorana particles acquiring mass via the seesaw mechanism. Accordingly $d_e$ is sensitive to the MFV scale of order hundreds of GeV or more. We have demonstrated that $d_e$ has the potential to probe not only the Dirac phase in the lepton mixing matrix, but also the Majorana phases therein and extra $CP$-violation sources in the Yukawa couplings of the right-handed neutrinos. Finally, we would like to remark that a similar analysis on the neutron EDM yields a much weaker constraint on the MFV scale [14].

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