Spacetime representation of electromagnetic radiation in a (2+1)-dimensional universe

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Abstract. Electromagnetic radiation’s nature embodying both classical and quantum phenomena suggests the fundamental interest of understanding its status within any view of spacetime. Maxwell’s equations have well proven their correctness as a formal representation of electromagnetic radiation. A speculative physical model of the light quantum developed within a (2+1)-dimensional view of spacetime is compared to the two mutual induction laws of Maxwell’s Equations.

1. Introduction
Questioning the essential nature of spacetime is not straightforward. Whatever technical theories anyone may follow, everyone nevertheless seems personally and socially to operate habitually within the long shared human presumption of events manifesting themselves within a Newtonian arena. Einstein and Minkowski revised the technical scientific understanding of how spacetime bookkeeping works, and quantum theory added an element of uncertainty to the bookkeeping, but all have mostly left spacetime at three spatial and one temporal dimensions. More recently, highly technical and speculative string theories have suggested as alternatives some or other increased dimensionality.

However, theories which consider a picture based on reduced spacetime dimensionality are much less common, and seem to be based on the theorists adopting a separable objective viewpoint from which they can see and describe spacetime.[1] Perhaps Archimedes could have moved the Earth if he had had a lever long enough and a place to stand (and a fulcrum). But the ability of the human mind to objectify the Universe as a whole may lead to error if there is not really a place to stand outside of the Universe to view it.

I have for some time been developing an idea not dependent on having a “place to stand”, and have found it useful heuristically if not yet a fully integrated theory. In its emphasis on physical motion and change it may be categorized as Machian,[2] and can be simplistically stated as being fundamental spacetime strictly founded on no more than three dimensions, two spatial plus an additional one spatial (or spatiotemporal). This view has been the subject of a few colloquia presentations and various informal discussions but has been so far unpublished. In private discussion the question has been raised, given the foundational importance and success of Maxwell’s equations, how to think of them in a way consistent with both their usual interpretation and this new view. The DICE2014 Spacetime - Matter - Quantum Mechanics conference presents an appropriate opportunity briefly to consider such an explication.
2. A thought experiment

Reading Maxwell’s original writings, one can find him puzzling through the data and abstracted relationships from which he synthesized the now well-established “Equations”.[3] Despite the widespread existence now of physical proofs—such as fantastically complex smartphones, for example—of the correctness of the ideas of Maxwell, of relativity, of quantum mechanics, and so on, it is still possible to puzzle about the physical world. One interesting puzzle, philosophical as well as physical, is the status of space and time. A phenomenon which Maxwell’s equations formalize, electromagnetic radiation of energy (“EMR”), seems a good choice for special attention in this puzzle. Particularly, the quanta of that fundamental phenomenon must have each in itself an intimate relationship with the essential nature of the spatial, temporal world, or it would not count as fundamental.

This is a more than philosophical question. For example the important cultural enterprise of teaching physics, especially as physics incorporates the attoworld, may be more or less efficient or successful depending on the conceptual understanding of EMR.[4][5] And the work of engineers, whose thinking may be more concrete than physicists’, may be similarly affected. Consider one very common representation used to express conceptual understanding, the simple transverse waveform shown in Figure 1. Some version of this probably is found in every introductory physics textbook.

![Figure 1. Electromagnetic radiation waveform](image)

This is a formal graphical representation of varying correlated values with no explicit reference to space or to time. It is not especially a representation of a physical entity in spacetime without bringing in additional, unseen concepts. One can use a thought experiment to examine exactly what ideas might be needed.

Einstein famously cited a youthful thought experiment involving chasing a light beam as the beginning of what became the special relativity world view.[6] As a variation on that thought experiment, rather than chase a light beam imaginatively looking at it, ride on a single light quantum in free space and see as it “sees”. In such case there is, for example, no waveform to be seen. If the light quantum is alone in free space, there is nothing to experience except the present moment’s electromagnetic field, ie the raw quantum itself, without even any way to distinguish any moment from any other. Lacking data storage, a history to which to refer, one cannot say more. (One might want to say that one would be experiencing also the present moment’s energy, but I omit direct consideration of energy in this paper.)

It is an important fact that an EMR packet carries no past with it. But maybe oneself is carrying data storage of some kind. If so, one can have a list of field strength values in sequence as one has “seen” them. One can relate the entries to each other, and note that they consist of a variety of values which repeat periodically. But one is not required to treat the data sequence as temporal, as a history. It can alternatively, and sufficiently, be treated solely as spatial, as values “here” and “behind” rather than “now” and “past”. Do the data values go with clicks of
a surveyor’s wheel, or with ticks of a clock? It is merely an issue of adding place stamps rather than time stamps to the raw data items. There is no need to use both, and given that the idea of physical existence presumes spatiality that may be chosen as more basic.

In any case, riding on the light quantum, since there is a behind one might allow oneself to think of there being a “forward”, at least in the sense of continuing the periodic sequence. However, there is no “ahead” nor “future”. In this thought experiment one is not allowed to bring in the whole complicated extra apparatus of an usual space and time arena. So nothing exists until the quantum gets there, which consists merely of field strength values being added in sequence to the list. Whether space or time, the light quantum seems to be creating its own.

Before leaving the thought experiment, consider the light quantum itself, taken as an object separate from an observer. What might be said about what it is like? In the attempted view of an observer in a rest frame, a photon moving by at c is like a passing instantaneous presence, it cannot be “seen” coming or going since it is itself the criterion of visibility. In one instant, it need not have any more than a two-dimensional form. The person riding on the photon has no reason to suggest it is any other form, and any alternative would not be meaningful without an external reference. The two-dimensionality is like a physical realization of length contraction at the extreme of relative velocities.[7]

Can this understanding be justified? Is there a simple physical model which reasonably well coheres both with the understanding produced by the thought experiment and with all the usual scientific ideas about EMR?

3. A visualizable simple model
An heuristic image, a visualized interpretation of equation or graph, can be useful. After all, even scientists spend most of their time processing information using their non-technical mental habits. A good model works because it offers imagery involving recognizable physical things in manifest spacetime which preserve important technical scientific relationships. The Bohr model of the atom, for example, was quite productive despite its flaws. And its derivative vocabulary, of “orbits”, energy “jumps”, and “spin”, is still, for better or worse, widely used in teaching.

What picture to aid thought might suit the EMR quantum discussed above, knowing only what has been seen so far? What are the formal features of the Maxwell equations (below) with which the model should correlate? First of all, how to understand the “∂t” has already been indicated. It is no more than the infinitesimal of the third dimensional forward spatial (or spatiotemporal) movement, correlating with the instantaneity of the light quantum.

Part of the beauty of Maxwell’s equations is their simplicity, and their remaining features are references to electric and magnetic fields. What model, what shape, might realize the characteristic phenomena of electromagnetism to match with those remaining mathematical features? Following the principle that simplicity is a virtue, and trying not to think anything extra, allow the electromagnetic fields to have spatial existence, which they certainly do as ordinarily understood. This suggests a picture of the instantaneous EMR event as embodied by a two dimensional form of some kind, spatially oscillating radially as successive field strength states periodically occur. Consistent with a principle of spatial isotropy, and with orthogonality of \( \vec{E} \) and \( \vec{B} \), this would be two rather than one-dimensional, and discoid as its default shape. The sequence of periodic variations corresponds to the accumulating of successively \( \partial t \)-forward positions. Note that, at least in the case of a single light quantum, this need not be thought of as occupying successive positions in some spatial arena, which would assume an unnecessary separate concept. Rather, the discoid is constructing the third dimension of its own existence.

Merely by reducing the instantaneous state to a two dimensional form, and allowing oscillation of that form to proceed as (not in) a third dimension, one finds one has already a realization of a particle-wave hybrid: orderly variation of a two dimensional particle constituting three (but not four) dimensional wavelike behavior. Figure 1 represents just such a phenomenon.
The issue of course is how well a minimal two-plus-one dimensional understanding of fundamental spacetime could cohere with the great corpus of physics, both theory and experiment. Simplicity may be elegant without being correct. That Maxwell’s equations legitimately include the case of a single quantum is ok, see for example [8] for one analysis. And so consideration continues with a speculative rough sketch.

4. Trying the model

Only two of Maxwell’s equations will be considered here, the Faraday and Ampere-Maxwell laws.[9]

\[
\nabla \times \vec{E} = -\partial \vec{B} / \partial t \quad \text{Ampere-Maxwell law (simplified):} \quad \nabla \times \vec{B} = \partial \vec{E} / \partial t
\]

Conceptually, Maxwell’s equations for EMR, shown in derivative form here, include only electric and magnetic fields and characteristics of their activity and interrelationship. Considering the basic understanding of the fields as developed from the 19th Century to now, how might one optimize correlation of that knowledge with the so-far uncharacterized 2-D discoidal entity?

The first, obvious, correlation is of field strengths with spatial extent. Then the field partial differentials in the wave equations may be taken as describing characteristics of changing spatial features. From the result of the thought experiment and this one simple conceptual correlation, everything else including the cross products follows. That \( \vec{E} \) by itself is more associated with localizable linear phenomena than with areal, and \( \vec{B} \) vice versa, and that a 2-D discoid’s essential physical features are just circumference and area (radius being a mathematical summary), together suggest correlating \( \vec{E} \) with circumference and \( \vec{B} \) with area. (I will use “\( \Theta \)” for circumference and “\( A_m \)” for area.)

As always seeking a minimum interpretation, find \( \vec{B} \) to be the usual supposed magnetic field lines perpendicular to and (by simplest hypothesis) filling the discoid surface’s area “\( A_m \)”.

Suitable to the two-dimensionality, the \( \vec{B} \) field lines, though having direction, are constituted actually as instantaneous 2-D transverse sections of field lines, which is not the same as points. An interesting question which will not be considered in this short paper is whether the \( \vec{B} \) field lines might be found actually curving back outside of \( \Theta \) and closing themselves, perhaps giving some evanescent or virtual \( \vec{B} \) presence outside of \( \Theta \) (in contextual circumstances when “outside” might make a difference). But here it is sufficient to leave them as merely 2-D slices with a direction + or – sense associated with them. The obvious choice for relevant spatial change of \( \vec{B} \), distinct from sense direction changes, is greater or lesser area of \( A_m \) as field strength varies periodically. But keep in mind that for induction effects, the change that counts is relative motion of the field lines of \( \vec{B} \) versus an electric charge or analogue to such.

Looking at \( \Theta \), its relevant spatial change can be as periodic increase and decrease of circumferential length, its own expansion and contraction occurring with \( A_m \)’s. This seems an appropriate phenomenon to count as \( \partial \vec{E} \), the displacement current which according to the Ampere-Maxwell law varies directly with \( A_m \) and has the same direction as its curl. Similar to the \( \vec{B} \) slices, one must allow \( \Theta \) to have no transverse thickness, leaving radial “size” to be as it may be. Thereby they may be said to have physical characteristics matching the formal representations “\( \vec{B} \)” and “\( \vec{E} \)”.

Considering further, one can continue to see this configuration agreeing with Maxwell’s equations. The displacement current has just been identified. Using the right hand rule and calling the thumb direction along \( \Theta \) the + direction, curling one’s fingers transversely around the circumference \( \Theta \) matches the \( \vec{B} \) lines slices within \( A_m \) and gives their + direction. The outmost \( \vec{B} \) slices are effectively co-extensive with the circumferential displacement current and thereby provide loci of non-zero circulation of \( \vec{B} \) to justify the curl \( \nabla \times \vec{B} \) in the Ampere-Maxwell law left side which, by the same right hand rule, has as it should the same direction as \( \partial \vec{E} \).
As for the Faraday law, there is not much to choose from to be circulation of $\vec{E}$ except the displacement current, the curvature of which varies from tighter at $\Theta_{\text{min}}$ to slighter at $\Theta_{\text{max}}$. Curving one’s right hand fingers along $\Theta$ finds one’s thumb, the + direction of the $\vec{E}$ curl, pointing in the + direction of the $\vec{B}$ which is of course the direction of the $\vec{B}$ lines whose $\partial \vec{B}$ would be lines being effectively cut as $\Theta$ expands and contracts. Since the Faraday law includes the “−” changing the sign of the right side, one uses the left hand rule to find the direction of the current induced by the cut lines, and one finds one’s thumb pointing in the − direction of the displacement current, opposing it consistent with the usual understanding. So the model’s basic geometry seems to be acceptable.

There may be problems related to how well numerical calculations, presumably referencing two rather than three dimensions, might match data. Given that present three-dimensional calculations do work, how the (2+1)-D model may produce results properly agreeing with empirical observations will need to be considered. Of course the rough model reported here does not suggest that the usual three-dimensional world does not exist or that science is not done in it. The project is merely to try to figure out how it may be possible to systematize seeing the world without reifying the time construct. This has long been suggested as appropriate, but as far as I know only in general without many specifics. Presumably a great number of adjustments may be needed to coordinate the differences between a (2+1)-D world view and the usual manifest one. In the “Comments” section I mention a couple of the many intriguing ideas about how usual three-dimensional materials may arise from 2-D energy packets. But this is indeed not a project for one single person to contemplate.

A different issue here is that so far + and − have been arbitrary as I limited my discussion to the general interrelationships found in Maxwell’s equations. Explaining the decision among options how the + and − senses might correlate specifically with discoidal features or activities would require another discussion as long and detailed as I have just given. So I will merely mention that I use the obvious simple idea that field magnitudes correspond directly to area of $A_m$ and length of $\Theta$, and the less simple idea that there are two discoid oscillation periods per one light quantum wavelength period. The rest may be seen sketchily in the following brief discussion of light quantum activity using Figure 2. Note that given continuously mutual effects it is necessary to pick some one point and begin to describe the process from there.

![Figure 2](image-url)

**Figure 2.** Simplified and annotated form of Figure 1. Solid red line is sine; dotted green line is cosine, the first derivative of sine. In arbitrary units.

At E in Figure 2, the fields have magnitude nominally 0, at which polarity flips within a continuous process the rate of change of which is here at maximum. This could correspond to the induced counter-field change in the positive direction, which began at D and is now at E,
(i) having nullified the negative fields the absolute value of which had maximized at D, now
(ii) beginning to increase the discoid field again but of positive polarity. As the positive fields
increase, the induced counter field begins to oppose them and will bring the positive-direction
increase to a halt at F, where the lack of change leaves the fields to collapse and to begin to
decrease toward G. This is the beginning of a change in the negative direction the rate of which
will maximize at G and then taper off at H, at which point the process described just above
from D repeats.

5. Comments
In this discussion I used only the concept “circumference” not eg “boundary”. Electromagnetic
energy in the real, well-filled Universe as we ordinarily understand it certainly is affected
by surrounding objects which themselves, ex hypothesis, are ultimately constituted of
electromagnetic quanta in various configurations. But it should be an empirical question whether
discooidal form such as this paper postulates would be best understood as an oscillating self-
delimited object in a void or conversely as a pulsating void delimited by something like an
aetherial substance. An aether theorist—which it is heuristically very tempting, but I think not
necessary, to become with this view—might think of this raw electromagnetic energy as moving
through a universal, uniform, immobile, and isotropic elastic medium.

Of course the exact form might not be discoid, or not always, as the phenomenon of
polarization brings up. The representation of quanta as line-like and rotating in helical
orientations seems inconsistent with spatial isotropy, nor is it easy to think of paired $\vec{E}$ and
$\vec{B}$ as forming no more than a two dimensional stick. Spin is another puzzle which I have not
covered in this paper, nor the reduction to this simple geometry of the complex imagery of string
theory.

In any case, combination of individual light quanta with other entities would be likely to
require some spatial distortion of the quanta. Whether completely losing their identity in
merging with something, retaining an individuation while combining in a complex dance of
trajectories, or splitting into parts, the transitions between isolated two dimensionality to
functional three (but not four) dimensionality are an interesting spatetime puzzle. Heuristic
possibilities are not hard to imagine, such as something like “feelers” of $\vec{B}$ lines extending from
contiguous quanta as their geometries intertwined, then breaking under specific stresses. In
all of these cases, though, the “arrow of time”, of essential third spatiotemporal dimension
progression, would remain as part of the fundamental nature of spacetime.

6. Conclusion
Since the physical models of light developed during the heyday of the wave–particle debate
withered in the face of quantum theory, models of light quanta have been mostly nonphysical.
However, as part of activity related to understanding the place of time in scientific thinking, a
question was raised whether it is possible to model Maxwell’s equations as describing a physical,
electromagnetic entity in a spacetime of no more than three dimensions, two spatial plus an
additional one spatial (or spatiotemporal). The belief is that given the increasing geometrization
of all of the sciences, contributions should be made toward the development of a shared scientific
culture in which thinking spatially has at least as much status as thinking temporally A model
has been found which is at least heuristically successful and worth refining in future work.

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measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by
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