Phase structure of $SU(2)$ Yang-Mills theory with global center symmetry

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Abstract

Retaining only the ‘timelike’ component $A_0$ of the vector potential a skelet model with explicit global center symmetry is constructed for $SU(2)$ Yang-Mills theory. It is shown that the $A_0$ gluon vacuum is equivalent with the 4-dimensional Coulomb-gas. In 1–loop approximation, the effective theory of the skelet model exhibits non-local self-interaction. A coupling constant $\lambda$ is found that governs the loop expansion. For $\lambda < 1$ the effective theory does not confine (the string tension vanishes), whereas for $\lambda = 1$ confinement (with non-vanishing string tension) takes place. In the deconfined phase the $A_0$ gluons exhibit non-zero rest mass and global center symmetry is broken by the vacuum state. Confinement sets on for $\lambda = 1$ when the rest mass of $A_0$ gluons vanishes and the global center symmetry of the vacuum is restored.

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I. INTRODUCTION

Lattice QCD results indicate that confinement and global center symmetry are strongly correlated\[1, 2, 3, 4\]. Simulating pure Yang-Mills systems on the lattice it was found: 
(i) there exists a critical temperature $T_c$ for quarkless QCD at which phase transition takes place; (ii) the free energy of a static $q\bar{q}$ pair rises linearly with increasing separation distance and the string tension obtained from the Wilson loop and that from the correlation of Polyakov lines agree well for $T < 0.6T_c$; (iii) the confining phase maintains global center symmetry, but it is broken in the deconfined phase. It is the most important message of the lattice results for us that the transition from the confined to the deconfined phase seems to be accompanied by breaking of the global center symmetry. In the present article we investigate the continuum $SU(2)$ gauge theory with global center symmetry. We show that the confined phase (with non-vanishing string tension) may exist with global center symmetry, whereas in the deconfined phase (with vanishing string tension) this symmetry is broken. In connection with the phase structure of the theory we also show that only a few of the infinitely many parameters are of significance.

Our treatment can be outlined as follows. We start with the generating functional of the Green’s functions of the lattice regularized $SU(2)$ Yang-Mills theory. In order to ensure global center symmetry the $SU(2)$ invariant Haar measure is used in the path integral\[5\], and the gauge fixing is performed without violating global center symmetry. This leads to a cut-off dependent periodic effective potential. For the sake of simplicity we neglect the spatial components of the vector potential remaining invariant under global center transformations. The ‘skelet’ $SU(2)$ Yang-Mills theory obtained in this way contains only the timelike $A_0$ components of the vector potential being sensitive to the global center transformations.

We show that the skelet model is equivalent with the macrocanonical ensemble for a $D = 4$ gas consisting of infinitely heavy point charges interacting via Coulomb interaction. This structure of the vacuum is completely induced by global center symmetry. In addition to the strong coupling constant the model contains infinitely many other parameters: the fugacities of the charges $\pm 1, \pm 2$, etc.

We expand the effective potential at its minima and derive the effective action
in 1-loop approximation. The 1-loop order effective action contains a non-local self-interaction. A parameter $\lambda$ is found governing the loop expansion.

In the framework of the effective theory we derive an expression for the string tension. The string tension is determined through the propagator of the gauge field in the effective theory. According to the value of the loop expansion parameter $\lambda$ the propagator of the effective theory has qualitatively different behaviour. For $\lambda < 1$ it exhibits simple poles whereas for $\lambda = 1$ a double pole occurs. In the first case the string tension vanishes, while it has a non-vanishing value in the second case.

II. SKELELT MODEL

The generating functional $Z[j]$ of the SU(2) Yang-Mills theory is by definition the vacuum to vacuum transition amplitude in the presence of the external source $j_{\mu}^a$:

$$Z[j] \sim \int \mathcal{D}A_i^a \int \mathcal{D}A_0^a \exp \left\{ -\frac{1}{4g^2} \int d^4xF_{\mu\nu}^aF_{\mu\nu}^a - S_{gf} - \int d^4xj_\mu^aj_\mu^a - S_{gh} \right\} \quad (2.1)$$

with the gauge fixing term $S_{gf}$ and the ghost term $S_{gh}$. We modify the generating functional of the usual perturbative treatment in order to ensure its invariance under global center transformations following [3, 4, 5], but we take global center transformations into account in a rather explicit way and use gauge conditions which do not violate global center symmetry.

1. Under an arbitrary finite SU(2) gauge transformation $h(x)$ the spatial and the timelike components of the vector potential transform as follows[4, 5]:

$$\left( A_i^a \frac{\tau^a}{2} \right)^h = h(x) \left( A_i^a \frac{\tau^a}{2} \right) h^{-1}(x) + h(x) \partial_i h^{-1}(x),$$

$$\left( \exp \left\{ iA_0^a \frac{\tau^a}{2} \right\} \right)^h = \left( \exp \left\{ iA_0^a \frac{\tau^a}{2} \right\} \right) h(x). \quad (2.2)$$

The center of a group consists of the elements commuting with any element of the group. The center of the group SU(2) consists of the elements $z_0 = 1$ and $z_1 = -1$. Global center transformation means the same center transformation at every point of 3-space at a given time slice. This transformation takes advantage
of the fact that we can perform finite gauge transformations at any time slice separately. In the continuum limit this leads to field configurations discontinuous in time. The global center transformation $z_1 = -1 = \exp \left\{ i2\pi \frac{x^3}{2} \right\}$ is not trivial. Parametrizing the general group element $h(\alpha \vec{n}) = \exp \left\{ i\alpha \vec{n} \cdot \vec{\tau}_2 \right\}$ by the colour vector $\vec{\alpha} = \alpha \vec{n}$ ($\alpha \in [0, 2\pi]$, and $\vec{n}$ arbitrary unit vector in the solid angle $4\pi$), we get from Eq. (2.2) for the effect of the center transformation:

$$h(\alpha \vec{n}) z_1 = h(-(2\pi - \alpha)\vec{n}).$$  

(2.3)

The colour vector $\alpha \vec{n}$ flips to $-(2\pi - \alpha)\vec{n}$ due to the non-trivial center transformation $z_1$. In order to treat the field configurations obtained by non-trivial global center transformation explicitly, we allow for global center transformations $z_1^{k_t}$ at each time slice $t$ ($k_t = 0$ for identity transformation and $k_t = 1$ for $z_1 = -1$). Taking the parameter of the local gauge transformation from the sphere $S_{2\pi}$ of radius $2\pi$, $\vec{\alpha}_{z_{k_t}} \in S_{2\pi}$, and taking the sum over $k_t = 0, 1$ we cover the $SU(2)$ group twice. The multiple counting of the gauge equivalent intermediate vacuum states shall be removed by dividing the path integral by the appropriate multiple of the volume of the gauge group.

The physical vacuum to vacuum transition amplitude $Z[0]$ is constructed in Hamiltonian formalism starting with the gauge $A_a^0 \equiv 0$ ($a = 1, 2, 3$) \cite{4, 5} (see Appendix A). The whole interval $T$ of time evolution is divided into small intervals of length $a$. At each time slice the projection to colour singlet vacuum states is performed by the operator:

$$P_\theta = \sum_{k_t=0,1} \int D_H (hz^{k_t}) (hz^{k_t}) = \sum_{k_t=0,1} \int D_H h(z^{k_t})$$

$$= \sum_{k_t=0,1} \prod_{\vec{x}} \int_{S_{2\pi}} d_H \vec{\alpha}_{zt} h \left( (\alpha_{zt} + 2\pi k_t(-1)^{k_t})\vec{n}_{zt} \right),$$  

(2.4)

with the gauge invariant Haar measure:

$$d_H \vec{\alpha} = d^3\alpha \frac{1}{4\pi^2} \frac{\sin^2 \left( \frac{1}{2} \alpha \right)}{\alpha^2}.$$  

(2.5)

($d^3\alpha$ the flat measure). Finally the path integral is divided by the multiple of the volume of the gauge group:

$$\sum_{\{k_t=0,1\}} \prod_{\vec{x}} \int_{S_{2\pi}} d_H \vec{\alpha}_{zt} \equiv \int G \mathcal{D}_H h.$$  

(2.6)
Later we make use of the integral
\[
\int_G \mathcal{D}H(aA^0a) \Phi[aA^0a] = \sum_{\{k_i=0,1\}} \prod_{\vec{x}_t} \int_{S_2} dH(aA^0a) \Phi[aA^0a + 2\pi k_i], \quad (2.7)
\]
with arbitrary functional \(\Phi[aA^0a]\). We can reintroduce the zeroth component of the vector potential via
\[
A^a_{0\vec{x}_t} = gA^a_{0\vec{x}_t}/T. \quad (2.8)
\]

2. We introduce the gauge fixing conditions \(F^a[A^h_{0}, A^j_{0}]\) in a gauge invariant way by inserting
\[
1 = \Delta[A] \int_G \mathcal{D}h \prod_{a=1}^3 \delta[F^a[A^h]] \quad (2.9)
\]
in the path integral. The functional \(\Delta[A]\) is invariant under gauge transformations including global center transformations as well. Making use of the gauge (and center) invariance of the integration measure, and the \(\Delta\) functional, we can factorize out the multiple of the group volume \(\int_G \mathcal{D}h\) and can divide with it. Thus we find for the physical vacuum to vacuum transition amplitude Eq. (A.7):
\[
\mathcal{Z}[0] \sim \int_G \mathcal{D}(aA^a_0) \int \mathcal{D}A^a_0 \Delta[A]_{F=0} \left( \prod_{a=1}^3 \delta[F^a[A^h]] \right) e^{iS[A^h_{(A_0)},A^j]}, \quad (2.10)
\]
with the action:
\[
S[A^h_{(A_0)}, A^j] = -\frac{1}{4g^2} \int_{-T/2}^{T/2} d^3x \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu}, \quad (2.11)
\]
and with the generalized field strength tensor:
\[
\tilde{F}^{0a}_{\vec{x}_t} = -\frac{1}{d} \left( A^h_{\vec{x}_t+a} - A^j_{\vec{x}_t} \right), \quad \tilde{F}^{ij} = F^{ij}. \quad (2.12)
\]
For infinitesimal gauge transformations the generalized field strength tensor takes the usual form.

3. We complete our definition of the vacuum to vacuum transition amplitude by a particular center invariant choice of the gauge conditions. According to (2.3) the colour vector \(A^a_0\) does not change its orientation in colour space due to center
transformation. Further on colour vectors \( aA_0 \vec{n} \) and \((aA_0 - 2\pi)\vec{n}\) are center equivalent. Therefore the gauge conditions

\[
F^1[A] = aA_0 \vec{x}_t \equiv 0, \quad F^2[A] = aA_0^2 \vec{x}_t \equiv 0,
\]
\[
F^3[A] = \mathcal{F}(aA_0^3) \vec{x}_{t+a} - \mathcal{F}(aA_0^3) \vec{x}_t \equiv 0 \tag{2.13}
\]
do not violate global center symmetry for \( \mathcal{F}(\alpha) \) periodic, \( \mathcal{F}(\alpha) = \mathcal{F}(\alpha + 2\pi) \). The gauge condition \( F^3 = 0 \) is the discretized version of the condition \( a\partial_0 \mathcal{F}(aA_0) = 0 \), and can be considered as the center invariant generalization of the temporal gauge \( \partial_0 A_0 = 0 \). This is a generalization allowing for finite jumps of the zeroth component of the vector potential at any of the time slices due to global center transformation. As for small fields we would like to recover the temporal gauge, we require \( \mathcal{F}(\alpha) \sim \alpha \) for \( \alpha \to 0 \). The gauge condition \( F^3 = 0 \) exhibits as a rule many solutions, many Gribov copies exist. To avoid this complication we require that the Gribov copies are those field configurations obtained by global center transformation. It is achieved if the equation \( \mathcal{F}(\beta) - \mathcal{F}(\alpha) = 0 \) has only the single solution \( \beta = \alpha \) for \( \alpha, \beta \in [0, 2\pi] \). For example the function \( \mathcal{F}(\alpha) = \tan \left( \frac{1}{2} \alpha \right) \) satisfies all these requirements. In the derivation of the vacuum to vacuum transition amplitude, however, we have to make use of the properties of the function \( \mathcal{F}(\alpha) \), but we do not have to specify it. The sums over the global center equivalent configurations (over \( k_i \)'s) are just those over the Gribov copies in the path integral.

The \( \Delta[A] \) functional defined above takes the following explicit form (see Appendix B):

\[
\Delta[A]|_{F\equiv 0} = \exp \left\{ TV \ln(4\pi^2) - \sum_{x_t} \frac{1}{2} \ln \sin^2 \left( \frac{1}{2} aA_0 \vec{x}_t \right) + \sum_{x_t} \ln \left| \mathcal{F}'(aA_0^3) \vec{x}_t \right| \right\},
\]
\[
\mathcal{F}'(\alpha) = \frac{d}{d\alpha} \mathcal{F}(\alpha). \tag{2.14}
\]

Inserting this into the path integral \( \mathcal{Z}[0] \), we carry out the integration over \( A_0^1 \) and \( A_0^3 \) and also the sums over \( k_i \)'s (see Appendix C). Making use of the global center invariance of the function \( \mathcal{F}(\alpha) \) and that of the action, we can replace \( \delta(F^3) \) by (see Eq. (2.5)):

\[
\frac{2 \sin^2 \left( \frac{1}{2} aA_0^3 \vec{x}_t \right)}{|\mathcal{F}'(aA_0^3)\vec{x}_t|} \delta (G_{\vec{x}_{t+a}} - G_{\vec{x}_t}) \tag{2.15}
\]
with \( \mathcal{G}(\alpha) = \frac{1}{2}(\alpha - \sin \alpha) \). This allows us to recover the Haar measure and eliminate the dependence on the function \( \mathcal{F}(\alpha) \). Shifting the argument of the Dirac delta by a field independent value \( c_{xt} \) and integrating over it by Gaussian weights, we obtain (Appendix C):

\[
\mathcal{Z}[0] = \int \mathcal{D}u \int \mathcal{D}A^a_i \exp \left\{ iS[A^h(u)^j, A^j] + \sum_{xt} \ln 2 + \sum_{xt} \ln \sin^2 \left( \frac{1}{2} au \right) \right. \\
- \frac{1}{2g^2\xi} \sum_{xt} \left( \frac{1}{2} (au - \sin(au))_{xt+a} - \frac{1}{2} (au - \sin(au))_{xt} \right)^2 \right\}, \\
(2.16)
\]

where \( au \equiv aA_0^3 \in [-2\pi, 2\pi] \).

We see that retaining global center symmetry has lead to the periodic tree level effective potential due to the Haar measure, as expected\(2, 3, 4\).

We see from Eq. (2.2) that the spatial components of the vector potential remain unaltered under global center transformations. For our goal is to investigate the consequences of the global center symmetry we neglect the spatial components of the vector potential by setting \( A^a_i = 0 \) in the present work.

In this way we obtained the ‘skelet’ model which only contains those degrees of freedom being sensitive to the global center transformations. In Euclidean space the generating functional of the Green’s functions takes the form:

\[
\mathcal{Z}[q] = \mathcal{Z}[q]/\mathcal{Z}[0], \\
(2.17)
\]

with

\[
\mathcal{Z}[q] = \int \mathcal{D}ue^{-S_s-S_{gf}-\int d^4xqu}, \\
(2.18)
\]

and \( a^3j_0^3 \equiv q \) the external source coupled to \( u \), where

\[
S_s = \frac{1}{2g^2} \int d^4x(\partial^i u)^2 + \mathcal{V}[u], \\
(2.19)
\]

\[
S_{gf} = \frac{1}{2g^2\xi} \int d^4x \frac{1}{a^2} (\partial_0 \mathcal{G}(au))^2, \\
(2.20)
\]

and the tree level effective potential is given by:

\[
\mathcal{V} = \int d^4x \mathcal{V}(u(x)), \\
\mathcal{V}(u) = -\frac{1}{a^4} \ln \sin^2 \left( \frac{1}{2} au \right) - \ln 2. \\
(2.21)
\]
As the action $S_s + S_{gf}$ exhibits $u \to -u$ symmetry, we can rewrite the integrals:

$$\prod_x \int_{-2\pi/a}^{2\pi/a} du_x \ldots \to \prod_x 2 \int_0^{2\pi/a} du_x \ldots = e^{TV a^{-4} \ln 2} \prod_x \int_0^{2\pi/a} du_x \ldots$$

(2.22)

The additional exponential factor just cancels the zero frequency term of the Fourier expansion of the tree level effective potential.

As the tree level effective potential has minima at $au = \pm \pi$, the field configurations close to these contribute most significantly to the path integral. Therefore we use the expansion

$$G(\alpha) = G(\pm \pi) + \frac{dG}{d\alpha}{\bigg|}_{\alpha = \pm \pi} (\alpha \mp \pi) + \mathcal{O}((\alpha \mp \pi)^3)$$

$$= \alpha \mp \frac{1}{2} \pi + \mathcal{O}((\alpha \mp \pi)^3),$$

(2.23)

which choosing $\xi = 1$ leads to:

$$S_{gf} = \frac{1}{2g^2} \int d^4 x (\partial_0 u)^2.$$  

(2.24)

Then we obtain for the generating functional:

$$Z[q] = \int D u e^{-S[u] - \int d^4 x q u},$$

(2.25)

with the action

$$S[u] = \frac{1}{2g^2} \int d^4 x \left[ (\partial^i u)^2 + (\partial_0 u)^2 \right] + V_0[u]$$

(2.26)

and with the tree level effective potential

$$V_0 = V - TV a^{-4} \ln 2 = \int d^4 x V_0(u).$$

(2.27)

The Fourier expansion of the periodic effective potential is given by:

$$V_0(u) = \frac{1}{a^4} \sum_{\sigma \neq 0} v_\sigma \cos(\sigma au),$$

(2.28)

with $v_\sigma = v_{-\sigma} = |\sigma|^{-1}$. The tree level effective potential is periodic in the interval $-2\pi \leq au \leq 2\pi$ with minima at $au = \pm \pi$, and it tends to $+\infty$ for $au = 0, \pm 2\pi$. 

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Having performed the Fourier expansion of the effective potential and the Taylor expansion of \(\exp\{-V_0\}\), we carry out the Gaussian path integral over the field \(u\) and obtain[5]:

\[
Z_C[q] = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \prod_{j=1}^{n} \sum_{x_j, \sigma(j) \neq 0} v_{\sigma(j)} \right) z_C[q],
\]

with

\[
z_C[q] = (\text{Det}D)^{-1/2} \exp \left\{ \frac{1}{2} \sum_{xy} \left( \sum_{j=1}^{n} i\sigma(j)\delta_{xxj} - q_x \right) a^2 D_{xy}, \right. \]
\[
\left. \cdot \left( \sum_{k=1}^{n} i\sigma(k)\delta_{yxk} - q_y \right) \right\}. \tag{3.2}
\]

For vanishing external source

\[
Z_C[0] = (\text{Det})^{-1/2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \prod_{j=1}^{n} \sum_{x_j, \sigma(j) \neq 0} v_{\sigma(j)} \right) \exp \left\{ -\frac{1}{2} \sum_{x_j, x_k} \sigma(j)D_{x_j, x_k}\sigma(k) \right\} \tag{3.3}
\]

is identical with the macrocanonical partition sum of the \(D=4\) dimensional Coulomb gas with infinitely heavy (static) point charges \(\sigma = \pm 1, \pm 2, \ldots\). The Fourier coefficients \(-v_\sigma\) of the periodic tree level effective potential play the role of the fugacities of the charges \(\sigma\). The free \(u\)-propagator,

\[
a^2D(x_j, x_k) \rightarrow \frac{a^2g^2}{\pi^2(x_j - x_k)^2} \quad \text{for} \quad a \to 0 \tag{3.4}
\]

represents the Coulomb interaction between the static charges. In addition to the strong coupling constant \(g\) the model contains infinitely many other parameters, the fugacities \(v_\sigma\).

We conclude that the requirement of global center symmetry leads to the equivalence of the \(A_0\) gluon vacuum with a \(D = 4\) dimensional Coulomb gas of static charges.
IV. EFFECTIVE THEORY

For now on we use lattice regularization which is rather natural from the point of view of the definition of the path integral and of the dependence of the effective potential on the lattice size \( a \). Although the calculations have been done in lattice regularized form, for the sake of simplicity we write the expressions in their continuous form if it is not misleading. The main steps of obtaining the effective theory are as follows:

1. The effective potential at the tree level has minima at \( au = \pm \pi \). Therefore the vacuum expectation value \( \langle u(x) \rangle \equiv \bar{u} \) does not vanish as a rule, but corresponds to one of the minima of the effective potential. Let us shift the field variable

\[
u(x) = \bar{u} + \eta(x), \tag{4.1}\]

and write the generating functional of the Green’s functions as:

\[
Z[q] = e^{a\bar{u} - \frac{4}{a^4} \int d^4x \eta \eta} \int D\eta \exp \left\{ \frac{1}{2g^2} \int d^4x \eta \eta \right\} - \frac{a}{4} \int d^4x \eta \eta - \frac{1}{a^4} \int d^4x V_0(\bar{u} + \eta). \tag{4.2}\]

2. Now we can linearize the exponential of the path integral in terms of \( \eta(x) \) by introducing the auxiliary field \( h(x) \) by the definition:

\[
\exp \left\{ \frac{1}{2g^2} \int d^4x \eta \eta \right\} = (\text{Det} D)^{1/2}.
\]

\[
\cdot \int Dh \exp \left\{ \frac{1}{2a^2} \int d^4x y h(x)a^2 D(x, y)h(y) + \int \frac{d^4x}{a^2} h(x)a\eta(x) \right\}. \tag{4.3}\]

The generating functional has now the form:

\[
Z[q] = (\text{Det} D)^{1/2} e^{a\bar{u} - \frac{4}{a^4} \int d^4x q} \int Dh e^{\frac{1}{2a^2} \int d^4x y h(x)a^2 D(x, y)h(y)}.
\]

\[
\cdot \int D\eta e^{a\bar{u} \int d^4x (\eta - \eta)} \int d^4x V(\bar{u} + \eta). \tag{4.4}\]

In order to clarify the meaning of the auxiliary field let us go back to Minkowski space, which roughly means the replacements \( d^4x \to -id^4x, \eta \to i\eta, q \to iq \).
\[ h \to ih, \text{ and forget about lattice regularization. Then the naive integration over } D\eta \text{ leads to the Dirac-delta functional:} \]

\[
\delta \left[ h(x) - q(x) + \sum_j \sigma(j) a \delta(x - x_j) \right].
\] (4.5)

According to that the auxiliary field \( h(x) \) can be interpreted as the effective charge density being the sum of the external charge density and the polarisation charge density. The latter arises due to the Coulomb charges in the vacuum.

3. We expand the tree level effective potential in the Fourier series (2.28) and the exponential \( e^{-\mathcal{V}} \) in Taylor series, and perform the path integral over \( \eta \). In order to integrate over all group elements in terms of the new variable \( \eta_x \), we choose the interval of integration as \( a\eta_x \in [-\pi, \pi] \). Then we obtain:

\[
\mathcal{Z}[q] = e^{\alpha a^{-4} \int d^4 x q} \int D\eta e^{\frac{1}{2a^4} \int d^4 x d^4 y h(x) a D(x,y) h(y) + \ln z[h - q]},
\] (4.6)

where

\[
e^{\ln z[h - q]} = (\text{Det} D)^{1/2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \prod_{j=1}^{n} \int \frac{d^4 x_j}{a^4} \sum_{\sigma(j) \neq 0} v_{\sigma(j)} \right) I_{\sigma}^{(n)} [h - q],
\] (4.7)

with

\[
I_{\sigma}^{(n)}[\xi] = e^{i \sum_{j=1}^{n} \sigma(j) a \bar{u} \int_{-\pi}^{\pi} d(a\eta_x) e^{(\xi_x + i \sum_{j=1}^{n} \sigma(j) \delta_{xx_j}) a \eta_x}}
\]

\[
= \prod_{x} 2 \sinh (\pi \xi_x) \cdot \prod_{j=1}^{n} e^{i \sigma(j) (a \bar{u} + \pi) \frac{\xi_{x_j}}{\xi_{x_j} + i \sigma(j)}},
\] (4.8)

and \( \xi_x = h_x - q_x \). Here we used the correct discretized expressions of the lattice regularized theory. We can write:

\[
\ln z[\xi] = \ln I_0[\xi] - \ln z_0[\xi],
\] (4.9)

with

\[
\ln I_0[\xi] = \sum_x \ln \text{Det}(a^2 D) + \sum_x [\ln \sinh (\pi \xi_x) - \ln \xi_x].
\] (4.10)
The functional \( z_0[\xi] \) has the form of the macrocanonical partition sum of an ideal gas:

\[
- \ln z_0 = \ln \left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \sum_x \sum_{\sigma \neq 0} v_\sigma \frac{\xi_x}{\xi_x + i\sigma} e^{i\sigma(a\bar{u} + \pi)} \right)^n \right\} \\
= - \sum_x \sum_{\sigma \neq 0} v_\sigma \frac{\xi_x}{\xi_x + i\sigma} e^{i\sigma(a\bar{u} + \pi)}. \tag{4.11}
\]

According to that we can interpret the Coulomb gas of charges \( \sigma \) as an ideal gas of quasiparticles in the external field \( h_x \).

4. To the next we perform the path integral over \( h(x) \) by the saddle point method taking into account the tree level and the 1–loop contributions. The solution of the saddle point equation

\[
\frac{\delta}{\delta h_x} \left\{ \frac{1}{2} \sum_{xy} h_x a^2 D_{xy} h_y + \ln z[h - q] \right\} = 0 \tag{4.12}
\]

is a local functional \( h_{0x}[q] \). Performing the saddle point integration over the auxiliary field \( h_x \), the generating functional can be written as:

\[
Z[q] = e^{-a\bar{u} \sum_x q_x Z_0[q] Z_1[q]}, \tag{4.13}
\]

where \( Z_0[q] \) and \( Z_1[q] \) are the tree level and 1-loop contributions, respectively:

\[
Z_0[q] = \exp \left\{ \frac{1}{2} \sum_{xy} h_{0x}[q] a^2 D_{xy} h_{0y}[q] + \ln z[h_0[q] - q] \right\}, \tag{4.14}
\]

\[
Z_1[q] = \left[ \text{Det}(a^2 Q) \right]^{-1/2}, \tag{4.15}
\]

with

\[
a^2 Q_{xy} = a^2 D_{xy} + \frac{\delta^2 \ln z[h_0[q] - q]}{\delta h_{0x}[q] \delta h_{0y}[q]}. \tag{4.16}
\]

5. The generating functional \( W[q] = \ln Z[q] \) of the connected Green’s functions takes the following form:

\[
W[q] = \text{const.} + W_0[q] + W_1[q], \tag{4.17}
\]
where $W_0[q]$, and $W_1[q]$ are the tree level and 1-loop contributions, respectively, for the Coulomb gas,

$$W_0[q] = -a\bar{u} \sum_x q_x + \frac{1}{2} \sum_{xy} h_{0x}[q] a^2 D_{xy} h_{0y}[q] + \ln z[h_0[q] - q], \quad (4.18)$$

$$W_1[q] = -\frac{1}{2} \text{Tr} \ln(a^2 Q). \quad (4.19)$$

6. Let us derive the effective action in 1-loop order defined as the Legendre transform of the generating functional $W[q]$:

$$\Gamma[\chi] = -\sum_x a(\chi_x + \bar{u}) q_x - W[q], \quad (4.20)$$

where the classical field $\chi_x$ is given by

$$a\chi_x + a\bar{u} = -\frac{\delta W[q]}{\delta q_x} \quad (4.21)$$

as the functional of the external source $q_x$.

7. In order to make our treatment self-consistent we require

$$\langle au_x \rangle = a\bar{u}, \quad \text{i.e.} \quad a\chi_x |_{q=0} = 0. \quad (4.22)$$

Furthermore we require that the effective potential has minimum for the constant field configuration $u_x = \bar{u}$. In this way we ensure that the action is expanded at the classical solutions.

The solution of the saddle point equation (4.12) and the $\bar{u}$ values satisfying the self-consistency equation (4.22) were found (Appendix D). It was established in 1-loop order that the effective potential has extrema at $au = 0, \pm \pi, \pm 2\pi$. $a\bar{u} = \pm \pi$ correspond to the minima of the tree level effective potential. Therefore we perform the expansion of the theory around these configurations and later check that the 1-loop effective potential has minima for them. It is a consequence of global center symmetry and that of the charge conjugation symmetry $u \rightarrow -u$, that the minima of the effective potential remain at the same place if quantum fluctuations are included.

The solution of the saddle point equation (4.12) is given by

$$h_{0x} - q_x = -(Q_0^{-1}Dq)_x + \frac{1}{6} f_4 \left( Q_0^{-1}(Q_0^{-1}Dq)^3 \right)_x + O(q^5), \quad (4.23)$$

with

$$(a^2 Q_0)_{xy} = a^2 D_{xy} + f_2 \delta_{xy}. \quad (4.24)$$
8. We construct the effective action $\Gamma[\chi]$ by inverting the functional dependence given by Eq. (4.21) and inserting it in the r.h.s. of Eq. (4.20) retaining terms up to the fourth order in the classical field.

Although the skelet model contains infinitely many parameters, the fugacities $v_\sigma$ and the strong coupling constant $g$, the effective action in 1-loop order can be expressed in terms of the parameters

$$f_2 = 2 \cdot 2! \sum_{\sigma > 0} v_\sigma(\sigma)^{-2} > 0, \quad f_4 = -2 \cdot 4! \sum_{\sigma > 0} v_\sigma(\sigma)^{-4} < 0,$$

and the new coupling constant

$$\lambda = -\frac{f_4}{f_2 g^2} > 0.$$

This is a significant reduction of the number of relevant parameters of the theory.

Calculating the 1-loop contribution $W_1[q]$ to the generating functional of the connected Green’s functions in Appendix E, we show that it is of order $\lambda$. Therefore we investigate the skelet model for $0 < \lambda \leq 1$, where the loop expansion seems to be justified. The 1–loop contribution turned out to be convergent only if $s \equiv 2f_2/g^2 < 1/8$, so $s$ is a small parameter in the model. The expressions listed below are obtained by simple algebraic manipulations from the tree and 1–loop contributions to the generating functional, $W_0[q]$ and $W_1[q]$, resp., given in Appendix E.

The classical field Eq. (4.21) as the functional of the external source is given by:

$$\chi_x = -f_2 \left( (1 + \lambda - f_2 D^{-1} + O(\lambda^2, s^2))q \right)_x$$

$$-\frac{1}{6} f_4 q_x^3 + \frac{1}{6} f_4 f_2(D^{-1}q^3)_x + \frac{3}{4} f_4 f_2 q_x^2(D^{-1}q)_x + O(\lambda^2, q^5).$$

(4.27)

Here $f_2 D^{-1}$ is of the order $s$. For we did not go beyond the 1–loop approximation, the operator in the linear term on the r.h.s. is known only up to $O(\lambda)$ terms. Therefore we must take its inverse with the same accuracy. Then we get for the inverse relation

$$q_x = -\left( \frac{1 - \lambda}{f_2} + D^{-1} \right) \chi_x$$

$$+ \frac{f_4}{6f_2^3}(1 - 4\lambda)\chi_x^3 - \frac{f_4}{4f_2^3} \chi_x(D^{-1}\chi)_x + O(\lambda^2, \chi^5),$$

(4.28)
which was found by iteration first solving the equation without the self-interaction terms. The effective action takes now the form:

$$\Gamma[\chi] = \frac{1}{2} \sum_{x,y} \chi_x \left( \frac{1 - \lambda}{f_2} \delta_{xy} + D^{-1}_{xy} \right) \chi_y - \frac{f_4(1 - \lambda)^4}{24f_2^4} \sum_x \chi_x^4. \tag{4.29}$$

Here we included only the leading order local terms of the self-interaction, taking into account that $f_2 D^{-1} \sim O(s)$.

We see that $f_2 > 0$, and $f_4 < 0$ are the correct signs for the effective potential having minimum at $\chi_x \equiv 0$. That justifies the expansion made.

The effective action describes a non-local theory with quartic self-interaction. We read off the inverse propagator of the $A_0^3$ gluons from the quadratic term of the effective action:

$$\tilde{D}_0^{-1}(p) = \frac{1 - \lambda}{f_2} + \tilde{D}^{-1}(p) + O(\lambda^2, s^2), \tag{4.30}$$

with the lattice regularized inverse propagator of the bare theory\[8\]:

$$f_2 \tilde{D}^{-1}(p) = \frac{s}{a^2} \sum_{\mu=0}^3 (1 - \cos(ap^\mu)). \tag{4.31}$$

A rest mass is generated due to the periodic effective potential, i.e. the global center symmetry of the effective action. This rest mass can be read off by comparing the $p_\mu \to 0$ limits of the inverse propagator with the general form $g^{-2}(p^2 + m^2)$:

$$m^2 = a^{-2} \frac{2}{s} (1 - \lambda) = a^{-2} \frac{f_4}{f_2^2} \frac{1 - \lambda}{\lambda} \geq 0. \tag{4.32}$$

In the neighbourhood each of its minima the effective potential at 1–loop order has the following form (neglecting the off-diagonal non-local terms):

$$\mathcal{V}_1[\chi] = \frac{1}{2g^2} m^2 a^2 \sum_x (a\chi_x)^2 - \frac{f_4(1 - \lambda)^4}{4!f_2^4} \sum_x (a\chi_x)^4. \tag{4.33}$$

It is interesting to note that the rest mass as well as the quartic self-interaction vanish for $\lambda = 1$ in 1-loop approximation. We show in the next chapter that the propagator exhibits rather different analytic properties for $\lambda \neq 1$ and $\lambda = 1$, which lead to a non-vanishing string tension in the latter case.
V. STRING TENSION

Let us determine the free energy of a static quark-antiquark pair positioned on the \( z \) axis (the quark at \( z = 0 \), the antiquark at \( z = L \)). In the \( SU(2) \) Yang-Mills theory the Wilson operator is given by:

\[
    w(L; [A]) = \text{Tr} \left( e^{-i \int_0^T dx \frac{1}{2} g A_0^a \tau^a} \bigg| \bigg|_{z=0} e^{-i \int_0^L dz \frac{1}{2} g A_0^a \tau^a} \bigg| \bigg|_{x^0=0} \right) 
\]

which leads in the skellet model to the expression:

\[
    w(L; [u]) = \text{Tr} \exp \left\{ i \int_0^T dx \frac{1}{2} \left( u(x^0, \vec{0}, 0) - u(x^0, \vec{0}, L) \right) \tau^a \right\},
\]

i.e.

\[
    w(L; [\chi]) = 2 \cos \left( \frac{a}{2} \sum_x Q_x (\bar{u} + \chi_x) \right) = 2 \cos \left( \frac{a}{2} \sum_x Q_x \chi_x \right),
\]

with the source

\[
    Q_x = \delta_{x0} \delta_{y0} (\delta_{zL} - \delta_{z0}).
\]

The free energy of the static pair is defined by the vacuum expectation value of the Wilson operator:

\[
    e^{-F(L)} = \frac{\int \mathcal{D}[\chi] w(L; [\chi]) e^{-\Gamma[\chi]} \mathcal{D}[\chi] e^{-\Gamma[\chi]}}{\int \mathcal{D}[\chi] e^{-\Gamma[\chi]}}
\]

with the approximate form of the effective action:

\[
    \Gamma[\chi] = \frac{1}{2} \sum_{x,y} \chi_x (\mathcal{D}^{-1})_{xy} \chi_y.
\]

The Gaussian integrals can be performed easily:

\[
    e^{-F(L)} = 2 e^{-\frac{1}{8} \sum_{x,y} Q_x \mathcal{D}_{xy} Q_y}.
\]

Then we express the propagator in coordinate space through its Fourier-transform and take the sums over \( x \) and \( y \). This results in the following expression for the free energy:

\[
    F(L) = \frac{1}{2} \sum_{x,y} \frac{d^3p}{(2\pi)^3} \frac{\sin^2 \left( \frac{1}{2} p_x L \right)}{D^{-1}(\vec{p} \cdot \vec{p}_z; p^0 = 0)}
\]
where the 3-momentum integral is taken over the Brillouin zone.

Calculating the string tension we replaced the propagator $D$ by $D_0$, i.e. neglected the quartic self-interaction vanishing for $\lambda = 1$.

The momentum integral can be performed noticing that the integrand exhibits poles on the complex $p_z$ plane for vanishing transverse momentum $\vec{p}_\perp = 0$. Therefore it is a good approximation to set $\vec{p}_\perp = 0$:

$$F(L) = \frac{T}{4\pi a} \mathcal{T},$$

where the remaining one-dimensional integral,

$$\mathcal{T} = f_2 a \int_{-\pi/a}^{\pi/a} dp_z \frac{\sin^2 \left( \frac{1}{2} p_z L \right)}{1 - \lambda + s - s \cos(ap_z)} = \frac{f_2}{s} \int_{-\pi}^{\pi} dv \frac{\sin^2 \left( \frac{1}{2} L v \right)}{\zeta - \cos v}$$

with $v = ap_z$, $\zeta = 1 + (1 - \lambda)/s = 1 + m^2 a^2/2 \geq 1$, has significantly different properties for $\lambda < 1$ and $\lambda = 1$.

Expressing the denominator in terms of exponentials, the momentum integral takes the form $\mathcal{I} = \mathcal{I}_0 + \mathcal{I}_+ + \mathcal{I}_-$, where

$$\mathcal{I}_0 = \frac{f_2}{2s} \int_{-\pi}^{\pi} dv \frac{\cos v}{\zeta - \cos v}, \quad \mathcal{I}_\pm = -\frac{f_2}{4s} \int_{-\pi}^{\pi} dv \frac{e^{\pm i \alpha}}{\zeta - \cos v}.$$ \hspace{1cm} (5.11)

The integrand has simple poles on the complex $v$ plane at $v = iv_\pm = i \ln(\zeta \pm \sqrt{\zeta^2 - 1})$ ($v_+ > 0, v_- < 0$) corresponding to the zeros of the equation $\zeta = \cos iv_\pm = \cosh v_\pm$. We calculate these integrals by closing the integration path through the lines $C_1$: $(\pi + i0) \rightarrow (\pi + iR)$, $C_2$: $(\pi + iR) \rightarrow (-\pi + iR)$, $C_3$: $(-\pi + iR) \rightarrow (-\pi + i0)$ for $\mathcal{I}_0$ and $\mathcal{I}_+$, and through $C'_1$: $(\pi - i0) \rightarrow (\pi - iR)$, $C'_2$: $(\pi - iR) \rightarrow (-\pi - iR)$, $C'_3$: $(-\pi - iR) \rightarrow (-\pi - i0)$ for $\mathcal{I}_-$, where $R \rightarrow \infty$. We show that the contributions of these lines to the integral vanish for $L/a = \text{integer}$, i.e. in the lattice regularized theory:

$$\frac{f_2}{2s} \left( \int_{C_1} + \int_{C_3} \right) \frac{dv}{\zeta - \cos v} = 0,$$

and using $\cos(\mp \pi \pm i\alpha) = -\cosh \alpha$,

$$-\frac{f_2}{4s} \left( \int_{C_1} + \int_{C_2} \right) \frac{dv}{\zeta - \cos v} e^{i\alpha} = -\frac{f_2}{4s} \int_{0}^{\infty} \frac{id\alpha}{\zeta + \cosh \alpha} e^{-\frac{4}{\alpha} i \pi} = 0.$$ \hspace{1cm} (5.13)
for \( L/a = \text{integer} \). Similarly the contributions from \( C'_{1} \) and \( C'_{3} \) cancel for \( \mathcal{I}_- \), and

\[
\frac{f_2}{2s} \int_{C_2} \frac{dv}{\zeta - \cos v} \sim e^{-R} \to 0,
\]

\[-\frac{f_2}{4s} \int_{C_2} \frac{dv}{\zeta - \cos v} e^{i\frac{L}{a}v} \sim e^{-\frac{L}{a}R-R} \to 0,
\]

\[-\frac{f_2}{4s} \int_{C'_{2}} \frac{dv}{\zeta - \cos v} e^{-i\frac{L}{a}v} \sim e^{-\frac{L}{a}R-R} \to 0 \] (5.14)

for \( R \to \infty \). Then we can deform the integration path into the circles \( C_+ \), and \( C_- \) of radius \( \rho \to 0 \) centered at the pole \( iv_+ \) for \( \mathcal{I}_0 \) and \( \mathcal{I}_+ \), and at \( iv_- \) for \( \mathcal{I}_- \), resp.:

\[
\mathcal{I}_0 = \frac{f_2}{2s} \int_{C_+} \frac{dv}{\zeta - \cos v},
\]

\[
\mathcal{I}_+ = -\frac{f_2}{4s} \int_{C_+} \frac{dv}{\zeta - \cos v} e^{i\frac{L}{a}v},
\]

\[
\mathcal{I}_- = -\frac{f_2}{4s} \int_{C_-} \frac{dv}{\zeta - \cos v} e^{-i\frac{L}{a}v}. \] (5.15)

We write \( v = iv_+ + \rho e^{i\varphi} \), and \( \zeta - \cos v = i \sinh v_+ \rho e^{i\varphi} + \frac{1}{2} \zeta \rho^2 e^{2i\varphi} + \mathcal{O}(\rho^3) \).

Now we have to consider the cases \( \lambda < 1 \) and \( \lambda = 1 \) independently.

- For \( \lambda < 1 \) we have \( \zeta > 1 \) and \( v_\pm \neq 0 \), and the nominator is of the order \( \rho \). Retaining in the denominator the terms of the order \( \rho \), i.e. writing \( \exp \left\{ \pm i\frac{L}{a} \rho e^{i\varphi} \right\} \approx 1 \), we get:

\[
\mathcal{I}_0 = \frac{f_2}{2s} \int_{0}^{2\pi} \frac{\rho \hat{d} \varphi e^{i\varphi}}{\sinh v_+ \rho e^{i\varphi}} = \frac{f_2}{2s} \frac{\pi}{\sinh v_+},
\]

\[
\mathcal{I}_+ = -\frac{f_2}{4s} \int_{0}^{2\pi} \frac{\rho \hat{d} \varphi e^{i\varphi}}{\sinh v_+ \rho e^{i\varphi}} e^{-\frac{L}{a}v_+} = -\frac{f_2}{2s} \frac{\pi}{\sinh v_+} e^{-\frac{L}{a}v_+},
\]

\[
\mathcal{I}_- = \frac{f_2}{4s} \int_{0}^{2\pi} \frac{\rho \hat{d} \varphi e^{i\varphi}}{\sinh v_- \rho e^{i\varphi}} e^{\frac{L}{a}v_-} = \frac{f_2}{2s} \frac{\pi}{\sinh v_-} e^{\frac{L}{a}v_-}. \] (5.16)

The free energy of the static quark-antiquark pair is then given by:

\[
\mathcal{F}(L) = \frac{T}{4\pi a} \frac{\pi f_2}{s} \left( \frac{1}{\sinh v_+} - \frac{1}{2 \sinh v_+} e^{-\frac{L}{a}v_+} + \frac{1}{2 \sinh v_-} e^{\frac{L}{a}v_-} \right). \] (5.17)

There is an attraction between the quark and the antiquark, which tends exponentially to zero for increasing separation distance \( L \). A kind of Debye screening
takes place governed by the rest mass \( m \) of the \( A_0 \) gluons through \( v_\pm \) and \( \zeta \). Consequently the string tension vanishes:

\[
\kappa = \lim_{L \to \infty} \frac{\mathcal{F}(L)}{LT} = 0. \tag{5.18}
\]

- For \( \lambda = 1 \) we have \( \zeta = 1 \) and \( v_\pm = 0 \), and the nominator of the integrand on the r.h.s. of the integrals Eq. (5.13) becomes of the order \( \rho^2 \). The simple poles at \( v = iv_\pm \) merged into a single double pole at \( v = 0 \) for \( \lambda \to 1 \). Then we have to retain the terms up to the same order \( \rho^2 \) in the denominator, writing \( \exp \left\{ \pm i_L \rho e^{i \varphi} \right\} \approx 1 \pm \frac{i_L \rho e^{i \varphi}}{\rho} \). The terms of the order \( 1/\rho \) vanish, because they contain the integral of the function \( e^{-i \varphi} \) for a period. Therefore \( \mathcal{I}_0 = 0 \) and we get for the free energy:

\[
\mathcal{F}(L) = \frac{T}{4\pi a} (\mathcal{I}_+ + \mathcal{I}_-) = \frac{TL f_2^2}{4a^2 s}. \tag{5.19}
\]

We see that the free energy exhibits the ‘area law’. Now we obtain a non-vanishing string tension:

\[
\kappa = a^{-2} \frac{f_2^2}{4s} = a^{-2} \frac{g^2}{8} = a^{-2} \frac{f_4}{8f_2}, \tag{5.20}
\]

where we used \( \lambda = 1 \), i.e. \( -f_4 = f_2 g^2 \) in the last equation. The simple proportionality of the string tension \( \kappa \) to the strong coupling \( g^2 \) will be modified by taking the quartic self-interaction as perturbation into account.

It is interesting to note that the 1–loop effective potential (4.33) becomes constant (the coefficient of the quartic self-interaction term vanishes) for the limit \( m \to 0 \) which means that the vacuum state does not break the global center symmetry. Oppositely, in the case \( \lambda < 1 \) the effective potential has two minima and the vacuum expectation value \( \langle u \rangle \) must be one of them, so that the vacuum state violates global center symmetry. The result obtained is an indication rather than a proof for the existence of the confining phase in the model, because the 1–loop approximation can be problematic for \( \lambda \approx 1 \). Anyhow, the ‘area law’ of the Wilson loop is a consequence of the treatment which does not violate center symmetry. Therefore we see that global center symmetry is of extraordinary importance for the existence of a phase with non-vanishing string tension even in the continuum theory.
VI. CONCLUSIONS

Summarizing we constructed the skelet model for $SU(2)$ Yang-Mills theory with global center symmetry retaining only the $A_0^3$ component of the vector potential. Due to the treatment not violating global center symmetry a cut-off dependent periodic effective potential occurs in the skelet model. The minima of this potential are fixed by charge conjugation and global center symmetry. It has been shown that the $A_0$ gluon vacuum is equivalent with the 4–dimensional Coulomb gas. This structure of the vacuum is entirely induced by global center symmetry.

The effective action for the skelet model has been derived in 1–loop approximation. In this approximation the effective potential contains only a finite number of parameters which are particular combinations of the strong coupling constant and the fugacities of the static charges. The parameter $\lambda$ is found which governs the loop expansion. Depending on its value, the vacuum can be in either deconfined or confined phase.

For $\lambda < 1$ the string tension vanishes, and the renormalized rest mass of the $A_0$ gluons given by Eq. (4.32) is positive. Consequently a test $SU(2)$ charge is screened at a distance $1/m$ due to Debye-screening. The effective potential is periodic and its curvature at the minima is defined by the renormalized mass $m$. The ground state is characterized by the non-vanishing expectation value of the gluon field (either $a\bar{u} = \pi$ or $-\pi$) and the global center symmetry is broken. Thus the theory describes the deconfined phase for $\lambda < 1$.

It is a hint on the possibility of the existence of a phase with unbroken global center symmetry that the renormalized rest mass vanishes for $\lambda = 1$ in 1-loop approximation. Then the simple poles of the effective propagator of $A_0$ gluons on the complex momentum plane merge into a single double pole and this leads to a non-vanishing string tension. The test charges are not screened now and the effective potential becomes a constant. Therefore the global center symmetry broken for $\lambda < 1$ is restored for $\lambda = 1$ in the ground state. These are just the properties of the confined phase.

Finally some remarks on our procedure and the results obtained above:

- Our procedure of deriving the effective theory contains a non-perturbative resum-
formation of infinitely many vertices generated by the periodic effective potential, i.e. by global center symmetry. This results in a vacuum structure being equivalent with a 4–dimensional Coulomb gas. Due to reserving global center symmetry during the whole treatment we expect our approach to reveal the IR (long distance) features of $SU(2)$ Yang-Mills theory correctly. On the other hand, it is necessary to incorporate the spatial components of the gauge field for the extension of the model to the UV regime.

- Our gauge fixing does not violate global center symmetry and is done rather consistently than in our earlier treatment\cite{7}. We have shown that the periodic effective potential is completely determined by the Haar measure. It is independent of the explicit form of the periodic gauge condition $F^3$, as one expects\cite{2,3}.

- Part of the self-interaction (higher order in the field) and the terms higher order in the loop expansion were neglected. The qualitative features of the effective potential determined by global center symmetry are not sensitive to this approximation. The critical value of the parameter $\lambda$ for which confinement sets on can be modified by the self-interaction and higher order terms in the loop expansion.

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A Vacuum to vacuum transition amplitude

Let us denote by $\hat{H}$ the Hamiltonian of $SU(2)$ Yang-Mills theory in the gauge $A_0^a \equiv 0$ ($a = 1, 2, 3$). Dividing the interval $T$ of the whole time evolution into $N$ intervals of length $a$ and inserting the projector operator $P_0$ of the colour neutral physical states at each time slice, we write the physical vacuum to vacuum transition amplitude as
the product of infinitesimal physical transition amplitudes:

$$\langle 0 | e^{-iHT} | 0 \rangle_{\text{phys}} = \left( \prod_{i=1}^{N-1} \sum_{k_i=0,1} \prod_{x} dA_{\bar{x}t_i} \int_{S_{2x}} dH \left( \frac{g a a^q_{\bar{x}t_i}}{T} \right) \right) \prod_{k=0}^{N-1} \langle A_{\bar{x}t_{k+1}} | a \hat{H} | A_{\bar{x}t_{k}} \rangle, \quad (A.1)$$

where $h(\alpha_{\bar{x}t_i}) = h_0(\alpha_{\bar{x}t_i}) z_1^{k_i}$, $h_0 \in SU(2)$. Using the explicit form of the infinitesimal transition amplitude[4],

$$\left. \langle A_{\bar{x}t_{k+1}} | e^{-i a \hat{H}} | A_{\bar{x}t_{k}} \rangle \right|_{t=t_k} = \exp \left\{ -ia \frac{1}{4g^2} \int d^3y \bar{F}_{\mu
u} F^{\mu\nu a} \right\}, \quad (A.2)$$

and reintroducing the zeroth component of the vector potential by Eq. (2.8), we obtain:

$$\langle 0 | e^{-iHT} | 0 \rangle_{\text{phys}} = \int_{\mathcal{G}} \mathcal{D}_H (a A_{0a}) \int \mathcal{D}A^i a e^{iS[\bar{A}^{(A_0)}_j, A^j]}.$$  \hspace{1cm} (A.3)

Now we introduce the gauge conditions $F^a$ by inserting 1 in the form of (2.9). Dividing by the multiple of the group volume, we get:

$$\langle 0 | e^{-iHT} | 0 \rangle_{\text{phys}} = \frac{\int_{\mathcal{G}} \mathcal{D}_H (a A_{0a}) \int \mathcal{D}A^i a \delta \left[ F^a [A^{(A_0)}] \right] e^{iS[\bar{A}^{(A_0)}_j, A^j]}}{\int_{\mathcal{G}} \mathcal{D}h'}. \quad (A.4)$$

Making use of the fact that the integration measure, the functional $\Delta [A]$, and the action are invariant under any gauge transformation $h'$, we can write the denominator in the following form:

$$\int_{\mathcal{G}} \mathcal{D}h' \int_{\mathcal{G}} \mathcal{D}_H \left( a A^{(A_0)} \right) \int \mathcal{D}A^{(A_0)} \Delta [A^{(A_0)}] \left( \prod_{a=1}^{3} \delta [F^a [A^{(A_0)}]] \right) e^{iS[\bar{A}^{(A_0)}_j, A^{(A_0)}]}.$$  \hspace{1cm} (A.5)

Further on we make use of the transformation law of the parameters of the gauge transformations (see Eq. (2.2)):

$$h(A_0) h' = h(A^{(A_0)}').$$  \hspace{1cm} (A.6)

Then we can remove $h'$ from the integration variables $A^{(A_0)}_0$, and $A^{(A_0)}_j$:

$$\langle 0 | e^{-iHT} | 0 \rangle_{\text{phys}} = \int_{\mathcal{G}} \mathcal{D}_H (a A_{0a}) \int \mathcal{D}A^i a \Delta [A] \left( \prod_{a=1}^{3} \delta [F^a [A]] \right) e^{iS[\bar{A}^{(A_0)}_j, A^{(A_0)}]}.$$  \hspace{1cm} (A.7)
B $\Delta[A]$ functional

Let us write the general gauge transformation $h_{xt} \in SU(2)$ in the form:

$$h_{xt} = h_0(\beta \alpha_{xt}) z_1^{n_t};$$

(B.1)

where $a \beta_{xt} \in [0, 2\pi]$, and $n_t = 0, 1$ integer at the time slice $t$. (For the sake of simplicity we omit the lower indeces $(\vec{x}, t)$ of the variables if it is not confusing.)

According to Eq. (2.2), the following general expressions are obtained:

$$(A^h_0)^1 \frac{\sin \left( \frac{1}{2} aA_0^{a_0} \right)}{A_0^{a_0}} = \left( \beta^1 \cos \frac{aA_0}{2} + \beta^2 \sin \frac{aA_0}{2} \right) \frac{\sin \left( \frac{1}{2} a\beta \right)}{\beta},$$

(B.2)

$$(A^h_0)^2 \frac{\sin \left( \frac{1}{2} aA_0^{a_0} \right)}{A_0^{a_0}} = \left( \beta^2 \cos \frac{aA_0}{2} - \beta^1 \sin \frac{aA_0}{2} \right) \frac{\sin \left( \frac{1}{2} a\beta \right)}{\beta},$$

$$(A^h_0)^3 \frac{\sin \left( \frac{1}{2} aA_0^{a_0} \right)}{A_0^{a_0}} = \frac{\beta^3}{\beta} \cos \frac{aA_0}{2} \sin \frac{a\beta}{2} + \frac{A_0^3}{A_0} \sin \frac{aA_0}{2} \cos \frac{a\beta}{2},$$

and

$$(A^h_0)^1 = \frac{A_0^{a_0}(-1)^{n_t} + 2\pi n_t/a}{\sin \left( \frac{1}{2} \alpha_{nt} \right)} \left[ \beta^1 \cos \left( \frac{1}{2} \alpha_{nt} \right) + \beta^2 \sin \left( \frac{1}{2} \alpha_{nt} \right) \right] \frac{\sin \left( \frac{1}{2} a\beta \right)}{\beta},$$

(B.3)

$$(A^h_0)^2 = \frac{A_0^{a_0}(-1)^{n_t} + 2\pi n_t/a}{\sin \left( \frac{1}{2} \alpha_{nt} \right)} \left[ \beta^2 \cos \left( \frac{1}{2} \alpha_{nt} \right) - \beta^1 \sin \left( \frac{1}{2} \alpha_{nt} \right) \right] \frac{\sin \left( \frac{1}{2} a\beta \right)}{\beta},$$

$$(A^h_0)^3 = \frac{A_0^{a_0}(-1)^{n_t} + 2\pi n_t/a}{\sin \left( \frac{1}{2} \alpha_{nt} \right)} \left[ \frac{\beta^3}{\beta} \cos \left( \frac{1}{2} \alpha_{nt} \right) \sin \frac{a\beta}{2} \right.

+ \left. (-1)^{n_t} A_0^3 \sin \left( \frac{1}{2} \alpha_{nt} \right) \cos \frac{a\beta}{2} \right],$$

with $\alpha_{nt} = aA_0(-1)^{n_t}$, where we made use of the equations:

$$(aA_0^a)^{z_1} = aA_0^a + 2\pi n_t(-1)^{n_t} aA_0^a \frac{A_0^a}{aA_0},$$

$$\sin \left( \frac{1}{2} aA_0^{a_0} \right) = (-1)^{n_t} \sin \left( \frac{1}{2} \alpha_{nt} \right),$$

$$\cos \left( \frac{1}{2} aA_0^{a_0} \right) = (-1)^{n_t} \cos \left( \frac{1}{2} \alpha_{nt} \right).$$

(B.4)
The expressions for \( \sin \left( \frac{aA_0}{2} \right) \), i.e. for \( A_0 \) in terms of \( \beta \)'s we find by taking the sum of the squares of the expressions given by (B.2):

\[
\sin^2 \left( \frac{aA_0}{2} \right) = \left[ 1 - (1 + \cos^2 \theta \sin^2 \frac{aA_0}{2}) \right] \sin^2 \left( \frac{1}{2} a\beta \right) + \sin^2 \left( \frac{aA_0}{2} \right) + \frac{1}{2} \cos \theta \frac{A_0^3}{A_0} \sin(a\beta) \sin(aA_0)
\]

\[
\equiv B(a\beta, \theta), \quad (B.5)
\]

where the spherical coordinates

\[
\beta^1 = \beta \sin \theta \cos \varphi, \quad \beta^2 = \beta \sin \theta \sin \varphi, \quad \beta^3 = \beta \cos \theta \quad (B.6)
\]

have been introduced. Then we get:

\[
aA_0 = \arccos[1 - 2B(a\beta, \theta)]. \quad (B.7)
\]

The inverse of the functional \( \Delta[A] \) is given by:

\[
\Delta^{-1}[A] = \sum_{\{n_t\}} \int D_H(a \beta^a) \prod_{a=1}^3 \delta[F^a[A^{\beta z^{n_t}}]]
\]

\[
= \sum_{\{n_t\}} \left( \prod_{\tilde{x}_t} \int_0^{2\pi} d(a \beta_{\tilde{x}_t}) \int_0^\pi \sin \theta_{\tilde{x}_t} d\theta_{\tilde{x}_t} \int_0^{2\pi} d\varphi_{\tilde{x}_t} \frac{1}{4\pi^2} \frac{\sin^2 \left( \frac{1}{2} a\beta_{\tilde{x}_t} \right)}{(a \beta_{\tilde{x}_t})^2} \right) 
\]

\[
\cdot \prod_{a=1}^3 \delta[F^a[A^{\beta z^{n_t}}]] . \quad (B.8)
\]

Here we can write:

\[
\delta[F^1[A^{\beta z^{n_t}}]] = \prod_{\tilde{x}_t} \frac{\delta(\varphi - \varphi_{\nu})}{\frac{A^{\beta_{\tilde{x}_t}}(a \beta_{\tilde{x}_t})^{1/a}}{aA_0(1)^{n_t} + 2\pi n_t/a} \sin \left( \frac{1}{2} a\beta \right) \sin \theta \sin \left( \frac{1}{2} aA_0(1)^{n_t} - \varphi_{\nu} \right)}
\]

\[
= \prod_{\tilde{x}_t} \frac{\delta(\varphi - \varphi_{\nu})}{\frac{A^{\beta_{\tilde{x}_t}}(a \beta_{\tilde{x}_t})^{1/a}}{aA_0(1)^{n_t} + 2\pi n_t/a} \sin \left( \frac{1}{2} a\beta \right) \sin \theta} . \quad (B.9)
\]

where the equation

\[
\cos \left( \frac{1}{2} aA_0(1)^{n_t} - \varphi_{\nu} \right) = 0 \quad (B.10)
\]
has 1 root for \( \varphi_\nu \in [0, 2\pi] \) and \( n_t = 0 \) of the form \( \varphi_\nu = [aA_0(-1)^{n_t} - (2\nu + 1)\pi]/2 \) with \( \nu = 0 \) for \( \pi/2 \leq aA_0/2 \leq \pi \), and \( \nu = -1 \) for \( 0 \leq aA_0/2 \leq \pi/2 \). For \( n_t = 1 \) we can change the integration variable from \( \varphi \) to \( -\varphi \) and then we get the same root as for \( n_t = 0 \) (notice that \( |\sin(aA_0/2 - \varphi_\nu)| = 1 \));

\[
\delta[F^2[A^{hoz_1^{nt}}]] = \prod_{\nu = 0, 1} \sum_{\mu = 1, 2} \frac{\delta(\theta - \theta_\mu)}{\frac{\partial}{\partial \theta} \frac{1}{2\, aA_0} \sin \left( \frac{1}{2} aA_0 \right) \sin \left( \frac{1}{2} aA_0 \mu - \varphi_\nu \right)}
\]

\[
= \prod_{\nu = 0, 1} \sum_{\mu = 1, 2} \frac{\delta(\theta - \theta_\mu)}{\frac{\partial}{\partial \theta} \frac{1}{2\, aA_0} \sin \left( \frac{1}{2} aA_0 \right) \sin \left( \frac{1}{2} aA_0 \mu \right)}
\]

\[
= \prod_{\nu = 0, 1} \sum_{\mu = 1, 2} \frac{\delta(\theta - \theta_\mu)}{\frac{\partial}{\partial \theta} \frac{1}{2\, aA_0} \sin \left( \frac{1}{2} aA_0 \right) \sin \left( \frac{1}{2} aA_0 \mu \right)}
\]

where \( \sin \theta_\mu = 0 \) has 2 roots, \( \theta_\mu = 0, \pi \). Now we perform the integrals over the angles:

\[
\Delta^{-1}[A] = \sum_{\{n_t\}} \left( \prod_{\nu = 0, 1} \int_0^{2\pi} \frac{d(\alpha \beta)}{4\pi^2} \frac{1}{2} \sum_{\mu = 1, 2} \sin^2 \left( \frac{1}{2} aA_0 \right) \left( \frac{1}{2\, aA_0} \right) \frac{\delta(\theta - \theta_\mu)}{\frac{\partial}{\partial \theta} \frac{1}{2\, aA_0} \sin \left( \frac{1}{2} aA_0 \right) \sin \left( \frac{1}{2} aA_0 \mu \right)} \right) \delta[F^3[(A^{hoz_1^{nt}})^3]]_{\theta_\mu \varphi_\nu}.
\]

(B.12)

For \( \theta_\mu = 0, \pi \) and \( A_0^1 = A_0^2 = 0 \):

\[
B(\alpha \beta, \theta_\mu = 0, \pi) = \frac{1}{2} (1 - \cos \alpha_\mu) = \sin^2 \left( \frac{\alpha_\mu}{2} \right)
\]

(B.13)

with \( \alpha_\mu = a(A_0^3 + \beta \cos \theta_\mu) \), and then we obtain:

\[
aA_0^{ho} = \begin{cases} |\alpha_\mu| & \text{for } |\alpha_\mu| \in [0, 2\pi] \\ 2\pi - |\alpha_\mu| & \text{for } |\alpha_\mu| \in [2\pi, 4\pi] \end{cases}
\]

(B.14)

and

\[
aA_0^{ho^3} = A_0^{ho} \frac{\sin \left( \frac{\alpha_\mu}{2} \right)}{\sin \left( \frac{\alpha_\mu}{2\, A_0^0} \right)} = \begin{cases} \alpha_\mu & \text{for } |\alpha_\mu| \in [0, 2\pi] \\ \alpha_\mu - 2\pi \text{sgn} \alpha_\mu & \text{for } |\alpha_\mu| \in [2\pi, 4\pi] \end{cases}
\]

(B.15)
As the function $\mathcal{F}(\alpha)$ is invariant under global center transformations, the sums over $n_t$'s can be carried out, and the inverse of the $\Delta$ functional takes the following form:

$$
\Delta^{-1}[A] \bigg|_{A_0^1 = A_0^2 = 0} = \sum_{\{n_t\}} \left( \prod_{x,t} \int_0^{2\pi} d(a\beta_{xt}) \frac{1}{4\pi^2} \frac{1}{2} \sum_{\mu} \sin^2 \left( \frac{a}{2} \left( \frac{A_0^3}{a} + \beta \cos \theta_{\mu} \right) \right) \right) 

\cdot \delta \left( \mathcal{F}(aA_0^3 t + a) - \mathcal{F}(aA_0^3 t) \right) \bigg|_{\theta_{\mu}, A_0^1 = A_0^2 = 0} 

= \prod_{x,t} \int_0^{2\pi} d(a\beta_{xt}) \frac{1}{4\pi^2} \frac{1}{2} \sum_{\mu} \sin^2 \left( \frac{a}{2} \left( \frac{A_0^3}{a} + \beta \cos \theta_{\mu} \right) \right) 

\cdot \frac{1}{(aA_0^3 t |_{\theta_{\mu}})^2} \left[ \frac{1}{(2\pi - aA_0^3 t |_{\theta_{\mu}})^2} \right] 

\cdot \delta \left( \mathcal{F}(aA_0^3 t + a) - \mathcal{F}(aA_0^3 t) \right) \bigg|_{\theta_{\mu}, A_0^1 = A_0^2 = 0}. \tag{B.16}
$$

Inserting the expression $\mathcal{F}(\alpha)$ we find:

$$
\Delta^{-1}[A] \bigg|_{A_0^1 = A_0^2 = 0} = \prod_{x,t} \int_0^{2\pi} d(a\beta_{xt}) \frac{1}{4\pi^2} \frac{1}{2} \sum_{\mu} \sin^2 \left( \frac{a}{2} \left( \frac{A_0^3}{a} + \beta \cos \theta_{\mu} \right) \right) 

\cdot \frac{1}{(aA_0^3 t |_{\theta_{\mu}})^2} \left[ \frac{1}{(2\pi - aA_0^3 t |_{\theta_{\mu}})^2} \right] 

\cdot \delta \left( \mathcal{F}(aA_0^3 t + a) - \mathcal{F}(aA_0^3 t) \right) \bigg|_{\theta_{\mu}, A_0^1 = A_0^2 = 0}. \tag{B.17}
$$

We introduce the new integration variable: $\xi = a\beta \cos \theta_{\mu}$. Then the integrand becomes independent of $\mu$ and the sum over $\mu$ can be carried out:

$$
\sum_{\mu} \frac{1}{\cos \theta_{\mu}} \int_0^{2\pi \cos \theta_{\mu}} d\xi \ldots = \int_{-2\pi}^{2\pi} d\xi \ldots. \tag{B.18}
$$

Furthermore we have to use the field configurations satisfying $\mathcal{F}(aA_0^3 t + a) - \mathcal{F}(aA_0^3 t) = 0$, i.e. $aA_0^3 t |_{t+a} = aA_0^3 t+ |_{t+a}$, and $aA_0^3 t_{t+a} - 2\pi \text{sgn} A_0^3 |_{t+a}$. As $\mathcal{F}$ is periodic, the Dirac delta can be rewritten in both cases in the same way:

$$
\delta \left( \mathcal{F}(aA_0^3 t + \xi) - \mathcal{F}(aA_0^3 t) \right) = \sum_k \frac{\delta(\xi - \xi_k)}{|\mathcal{F}'(aA_0^3 t + \xi_k)|}, \tag{B.19}
$$

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According to the requirements satisfied by $F(\alpha)$, the argument of the Dirac delta has 2 zeros in the interval $\xi_{kt} \in [-2\pi, 2\pi]$: $\xi_{kt} = \xi_{t+a}, \xi_{t+a} - 2\pi \text{sgn}\xi_{t+a}$. For both zeros hold the equations

$$\sin^2 \left( \frac{1}{2}(aA^3_{0\ t} + \xi_{kt}) \right) = \sin^2 \left( \frac{1}{2}(aA^3_{0\ t+a} + \xi_{t+a}) \right),$$

$$|F'(aA^3_{0\ t} + \xi_{kt})| = |F'(aA^3_{0\ t+a} + \xi_{t+a})|. \quad (B.20)$$

In the case of the second zero we can make the integral transformation from $\xi_{t+a}$ to $\xi_{t+a} - 2\pi \text{sgn}\xi_{t+a}$, under which the rest of the integrand remains unaltered. Then we can take for both zeros:

$$\frac{1}{|aA^3_{0\ t+a} + \xi_{kt}|^2 + (2\pi - |aA^3_{0\ t+a} + \xi_{kt}|)^2} \rightarrow \frac{1}{|aA^3_{0\ t+a} + \xi_{t+a}|^2 + (2\pi - |aA^3_{0\ t+a} + \xi_{t+a}|)^2}. \quad (B.21)$$

Finally both zeros give the same contribution to the integral cancelling the factor $\frac{1}{2}$ before the sum over $\mu$. We integrate over the $\beta_{\vec{x}t}$’s successively according to their time argument $t$ simply writing $\xi_{kt} = \xi_{t+a}$:

$$\Delta^{-1}[A]|_{F=0} = \prod_{\vec{x}t} \left[ \frac{1}{4\pi^2} \frac{\sin^2 \left( \frac{1}{2}(aA^3_{0\ \vec{x}t} + \xi_{\vec{x}t/2}) \right)}{|F'(aA^3_{0\ \vec{x}t} + \xi_{\vec{x}t/2})|} \cdot \left( \frac{1}{|aA^3_{0\ \vec{x}t} + \xi_{\vec{x}t/2}|^2 + (2\pi - |aA^3_{0\ \vec{x}t} + \xi_{\vec{x}t/2}|)^2} \right) \right]. \quad (B.22)$$

Choosing $\xi_{\vec{x}t/2} \rightarrow 0$ for the pure gauge final state, we obtain:

$$\Delta^{-1}[A]|_{F=0} = \prod_{\vec{x}t} \left[ \frac{1}{4\pi^2} \frac{\sin^2 \left( \frac{1}{2}aA^3_{0\ \vec{x}t} \right)}{|F'(aA^3_{0\ \vec{x}t})|} \left( \frac{1}{(aA^3_{0\ \vec{x}t})^2 + (2\pi - |aA^3_{0\ \vec{x}t}|)^2} \right) \right]. \quad (B.23)$$

### C Explicit form of the vacuum to vacuum transition amplitude

We have made the jumps of the vector potential $A_0$ explicit in the path integral. This resulted in the sum over center equivalent copies of any field configuration. As the Haar
measure, the $\Delta$ functional, the action, and the gauge condition $F^3$ are invariant under global center transformations, only the arguments of the Dirac deltas with the gauge conditions $F^1$ and $F^2$ transform in a non-trivial way under global center transformation. Taking this into account, we obtain:

$$Z[0] =$$

$$= \sum_{\{n_t\}} \int_{S^2 n} D^3 H(aA_0^a) \int D A a \Delta[A]_{F=0} \left( \prod_{a=1}^3 \delta \left[ F^a \left[ A_0^a \left( 1 + \frac{2\pi n_t (-1)^{n_t}}{aA_0} \right) , A_t^a \right] \right) \right).$$

$$= \sum_{\{n_t\}} \int D_{flat}(aA_0^a) \int D A^a_t \exp \left\{ \sum_x \ln \sin^2 \left( \frac{1}{2} aA_0 \right) - \sum_x \ln (4\pi^2) \right\} \prod x (aA_0 + 2\pi n_t (-1)^{n_t})^2 \Delta[A]_{F=0}. \cdot \delta \left( aA_0^a \left( 1 + \frac{2\pi n_t (-1)^{n_t}}{aA_0} \right) \right) \cdot \delta \left( aA_0^a \left( 1 + \frac{2\pi n_t (-1)^{n_t}}{aA_0} \right) \right) \cdot \delta \left( F(aA_0^a t) - F(aA_0^a t) \right) e^{i S[A^0(A_0)]} \bigg|_{F=0} \cdot \exp \left\{ \sum_x \ln (4\pi^2) - \sum_x \ln \sin^2 \left( \frac{1}{2} aA_0 \right) + \sum_x \ln |F'(aA_0^a)| \right\}. \cdot \prod x \frac{1}{(aA_0)^2 + \frac{1}{(2\pi - aA_0)^2}}^{-1} \cdot \prod x \delta \left( aA_0^a \right) \delta \left( aA_0^a \right) \delta \left( F(aA_0^a t) - F(aA_0^a t) \right) e^{i S[A^0(A_0)]} \bigg|_{F=0}. \cdot$$

We perform now the sums over $n_t$'s:

$$Z[0] = \int D_{flat}(aA_0) \int D A^a_t \left( \prod x \delta \left( F(aA_0^a t) - F(aA_0^a t) \right) \right). \cdot e^{i S[A^0(A_0)]} \bigg|_{F=0}. \cdot$$

In order to recover the effective potential induced by the Haar measure, we rewrite the Dirac delta on the r.h.s. and show that the physical vacuum to vacuum amplitude is independent of the choice of the gauge function $F(\alpha)$. The argument of the Dirac delta exhibits 2 zeros: $aA_0^a t = aA_0^a t+a$, and $aA_0^a t+a - 2\pi \text{sgn} A_0^a t$. Consequently, we can write:

$$\delta \left( F(aA_0^a t) - F(aA_0^a t) \right) = \cdots$$
induced tree level effective potential: 

\[ \frac{1}{|F'(aA^3_0 t)|} \left[ \delta \left( aA^3_0 t - aA^3_0 t + a \right) + \delta \left( aA^3_0 t - aA^3_0 t + a + 2\pi \text{sgn} A^3_0 t + a \right) \right]. \] (C.3)

We make the integral transformation \( A^3_0 t + a \rightarrow A^3_0 t + a - 2\pi \text{sgn} A^3_0 t + a \) in the second term. Under this transformation the rest of the integrand remains unaltered as \( F, F' \), and the action \( S \) remain invariant. Then both terms give the same contribution to the path integral and we can replace the Dirac delta as follows:

\[ \delta \left( F(aA^3_0 t + a) - F(aA^3_0 t) \right) \rightarrow \frac{2}{|F'(aA^3_0 t)|} \delta \left( aA^3_0 t - aA^3_0 t + a \right). \] (C.4)

Let us take now the function \( G(\alpha) = \frac{1}{2}(\alpha - \sin \alpha) \), being monotonically increasing as its first derivative \( G'(\alpha) = \sin^2 \frac{\alpha}{2} > 0 \). Then the equation \( G(\beta) = G(\alpha) \) has only a single solution \( \beta = \alpha \). Making use of this property, we can write:

\[ \delta \left( F(aA^3_0 t + a) - F(aA^3_0 t) \right) \rightarrow \frac{2\sin^2 \left( \frac{1}{2}aA_0 t \right)}{|F'(aA^3_0 t)|} \delta \left( G(aA^3_0 t) - G(aA^3_0 t + a) \right). \] (C.5)

Inserting this into the physical vacuum to vacuum transition amplitude, the terms \( \sum_x \ln |F'| \) cancel in the exponent of the integrand, and we recover the Haar measure induced tree level effective potential:

\[ Z[0] = \int D_{\text{flat}} A^3_0 \int DA^a_0 \left( \prod_x \delta \left( G(aA^3_0 x + a) - G(aA^3_0 x) \right) \right) \cdot e^{iS[A^0(\bar{A}_0), A^a] + \sum_x \ln 2 + \sum_x \ln \sin^2 \left( \frac{1}{2}aA^a_0 \right)}. \] (C.6)

Shifting the argument of the Dirac delta by a (vector potential independent) constant \( c_x \) and integrate over \( c_x \) by Gaussian weights, we obtain:

\[ \int Dc \exp \left\{ -\frac{a^2}{2g^2 \xi} \sum_x c^2_x \right\} \ldots \left( \prod_x \delta \left( G(aA^3_0 x + a) - G(aA^3_0 x) - c_x \right) \right) = \exp \left\{ -\frac{a^2}{2g^2 \xi} \sum_x \frac{1}{a^2} \left( G(aA^3_0 x + a) - G(aA^3_0 x) \right)^2 \right\} \ldots \] (C.7)

Then the physical vacuum to vacuum transition amplitude takes the form:

\[ Z[0] = \int D_{\text{flat}} A^3_0 \int DA^a_0 \cdot e^{iS[A^0(\bar{A}_0), A^a] + \sum_x \ln 2 + \sum_x \ln \sin^2 \left( \frac{1}{2}aA^a_0 \right)} \cdot \exp \left\{ -\frac{a^2}{2g^2 \xi} \sum_x \frac{1}{a^2} \left( G(aA^3_0 x + a) - G(aA^3_0 x) \right)^2 \right\}. \] (C.8)

The additional interaction term expressed through the function \( G(\alpha) \) is also defined completely by the Haar measure.

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D Solution of the self-consistency and the saddle point equations

We find the solution iteratively in two steps. At first we solve equations (4.12) and (4.22) in the tree approximation. Then we show that the functional $h_{0x}$ and the values $\bar{u}$ are not modified by the 1-loop corrections.

1. In tree approximation the self-consistency equation (4.22)

$$\frac{\delta \ln z[\xi]}{\delta \xi} \bigg|_{\xi_x = h_{0x}[q] = 0} = 0$$  \hspace{1cm} (D.1)

is consistent with the saddle point equation (4.12),

$$\sum_y a^2 D_{xy} h_{0y} + \frac{\delta \ln z[\xi]}{\delta \xi} \bigg|_{\xi_x = h_{0x}[q] - q_x} = 0, \hspace{1cm} (D.2)$$

only if the functional $h_{0x}[q]$ vanishes for vanishing external source, i.e. $h_{0x}[q = 0] = 0$. In light of that we expand $\ln z[\xi]$ in Taylor series at $\xi_x \equiv 0$:

$$\ln z[\xi] = \ln z[0] + \sum_x \sum_{k=1}^{\infty} \frac{1}{k!} f_k \xi^k, \hspace{1cm} (D.3)$$

where $f_k$ is the $k$-th functional derivative of $\ln z$ with respect to $\xi_x$ at $\xi_x \equiv 0$. We take into account the terms up to the order $O(\xi^5)$. It is easy to show that the odd derivatives of $\ln I_0$ vanish at $\xi_x \equiv 0$, $\delta^{2k+1}(\ln I_0)/\delta \xi^{2k+1}|_{\xi=0} = 0$. From Eq. (D.1) we get $f_1 = 0$, which then means

$$\frac{\delta \ln z_0[\xi]}{\delta \xi} \bigg|_{\xi=0} = \sum_{\sigma \neq 0} v_\sigma \frac{i\sigma}{(\xi_x + i\sigma)^2} e^{i\sigma(\bar{u}\sigma + \pi)} \bigg|_{\xi=0} = \frac{1}{i} \sum_{\sigma \neq 0} v_\sigma \frac{\cos(a\bar{u}\sigma) + \sin(a\bar{u}\sigma)}{\sigma} e^{i\sigma \pi} = 0. \hspace{1cm} (D.4)$$

Since $v_\sigma = v_{-\sigma}$, the real part of the sum is automatically zero. The imaginary part,

$$\sum_{\sigma \neq 0} v_\sigma \frac{(-1)^\sigma}{\sigma} \sin(a\bar{u}\sigma) = 0 \hspace{1cm} (D.5)$$
vanishes for \( \bar{u} \) satisfying \( \sin(a\bar{u}\sigma) = 0 \) for any integer \( \sigma \), i.e. for \( a\bar{u} = \nu\pi \), \( (\nu = \text{integer}) \). Then we get \( e^{ia\bar{u}\sigma} = (-1)^{\nu\sigma} \), and

\[
f_k = (-1)^{k+1} \sum_{\sigma \neq 0} \nu_\sigma (-1)^{\sigma(\nu+1)} \frac{k!}{(\sigma)^k}, \tag{D.6}
\]
i.e. all the odd derivatives of \( \ln z \) vanish at \( \xi = 0 \), \( f_{2k+1} = 0 \), and the even derivatives are given by:

\[
f_{2k} = (-1)^{k+1} 2 \sum_{\sigma > 0} \nu_\sigma (-1)^{\sigma(\nu+1)} \frac{(2k)!}{\sigma^{2k}}, \tag{D.7}
\]

Note that \( \nu = \pm 1 \) correspond to the minima of the effective potential in the tree approximation. For \( \nu = \pm 1 \) holds:

\[
f_{2k} = (-1)^{k+1} \cdot 2 \sum_{\sigma > 0} \nu_\sigma \frac{(2k)!}{\sigma^{2k}}, \tag{D.8}
\]
i.e. \( f_2 > 0 \), \( f_4 < 0 \), etc. We shall see that these are the correct signs ensuring that the effective potential at 1–loop order has minima at \( a\bar{u} = \pm \pi \).

We rewrite the saddle point equation (4.12):

\[
\sum_y [(a^2 Q_0)^{-1}]_{xy} a^2 D_{xy} \xi_y = -\sum_y a^2 D_{xy} q_y - \frac{1}{6} f_4 \xi_x^3 \tag{D.9}
\]
including terms up to the order \( \mathcal{O}(\xi^4) \). Introducing the matrix

\[
(a^2 Q_0)_{xy} = a^2 D_{xy} + f_2 \delta_{xy}, \tag{D.10}
\]
we can write:

\[
\xi_x = -\sum_{yz} [(a^2 Q_0)^{-1}]_{xy} a^2 D_{yz} q_z - \frac{1}{6} f_4 \sum_z [(a^2 Q_0)^{-1}]_{xz} \xi_z^3. \tag{D.11}
\]

We solve this equation by iteration:

\[
\xi_x = -\sum_{yz} [(a^2 Q_0)^{-1}]_{xy} a^2 D_{yz} q_z + \frac{1}{6} f_4 \sum_y [(a^2 Q_0)^{-1}]_{xy} \left( \sum_{zu} [(a^2 Q_0)^{-1}]_{yz} a^2 D_{zu} q_u \right)^3 + \mathcal{O}(q^5). \tag{D.12}
\]
(Note that the matrices $D_{xy}$ and $(Q_0)_{xy}$ are symmetric.) For the sake of simplicity we introduce the notations $(Aq)_x = \sum_y A_{xy}q_y$, $(qAq)_x = \sum_{xy} q_x A_{xy}q_y$ and do not write out the powers of $a$ (they can be introduced at any step to obtain dimensionless expressions), then:

$$\xi_x = -(Q_0^{-1}Dq)_x + \frac{1}{6} f_4(Q_0^{-1}(Q_0^{-1}Dq)^3)_x. \quad (D.13)$$

Including terms up to the order $O(q^4)$, we obtain:

$$\xi^2_x = (Q_0^{-1}Dq)_x^2 - \frac{1}{3} f_4(Q_0^{-1}Dq)_x(Q_0^{-1}(Q_0^{-1}Dq)^3)_x,$$
$$\xi^3_x = -(Q_0^{-1}Dq)_x^3,$$
$$\xi^4_x = (Q_0^{-1}Dq)_x^4 \quad (D.14)$$

for later use.

2. Now we show that the functional $h_{0x}[q]$ is not modified by 1-loop corrections. Including the 1-loop term the self-consistency equation (4.22) is given by:

$$f_2\xi_x[0] + \frac{1}{6} f_4\xi^3_x[0] + \frac{1}{2} \frac{\delta}{\delta q_x} \text{Tr} \ln(a^2Q) \bigg|_{q=0} = 0. \quad (D.15)$$

Inserting the explicit form of the 1-loop contribution of the generating functional (Appendix E),

$$W_1[q] = -\frac{1}{2} \text{Tr} \ln(a^2Q) = \frac{f_4}{2f_2} \mu_2 \sum_y (Q_0^{-1}Dq)_y^2, \quad (D.16)$$

and writing $\xi_x[q] = \xi^{(0)}[q] + h\xi^{(1)}[q]$, we obtain for the term of order $h$:

$$\xi^{(1)}_x[q] \bigg|_{q=0} = -\frac{1}{2f_2} \frac{\delta}{\delta q_x} \text{Tr} \ln(a^2Q) \bigg|_{q=0} = \frac{1}{f_2} \frac{\delta}{\delta q_x} W_1[q] \bigg|_{q=0}$$

$$= \frac{f_4}{f_2^2} \mu_2 \left( (Q_0^{-1}Dq)_x^2 \right) \bigg|_{q=0} = 0. \quad (D.17)$$

This means that the initial value $h_{0x}[q = 0] = 0$ is not modified due to 1-loop corrections. Then the solution of the saddle point equation remains unaltered and also the value of $a ù$ is not modified.
E 1–loop contribution

Now we calculate the 1–loop contribution $W_1[q]$ to the generating functional of the skeleton model.

The 1–loop part of the generating functional of the connected Green’s functions can be rewritten:

$$W_1[q] = \text{const.} - \frac{1}{2} \text{Tr} \ln (a^2 Q) = \text{const.} - \frac{1}{2} \text{Tr} \ln \left[ (a^2 D) \left( 1 + f_2(\xi) D^{-1} \right) \right]$$

$$= \text{const.} - \frac{1}{2} \text{Tr} \ln \left( 1 + f_2(\xi) D^{-1} \right)$$

$$= \text{const.} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{Tr} \left[ f_2(\xi) D^{-1} \right]^n,$$  \hspace{1cm} (E.1)

where $f_2(\xi)$ is defined by $\frac{\delta^2}{\delta \xi \delta \xi} \ln z[\xi] = \delta_{xy} f_2(\xi_x)$. Making use of the Taylor expansion (D.3) with vanishing odd derivatives, we get for the trace:

$$\text{Tr}[f_2(\xi) D^{-1}]^n = \text{Tr} \left\{ f_2^n(D^{-1})^n + n f_2^{n-1}(D^{-1})^{n-1} \left[ \frac{1}{2} f_4(\xi^2 D^{-1}) + \frac{1}{24} f_6(\xi^4 D^{-1}) \right] \right. $$

$$+ \left. \frac{1}{2} n(n-1) f_2^{n-2}(D^{-1})^{n-2} \frac{1}{4} f_4^2(\xi^2 D^{-1})^2 + O(\xi^6) \right\}, \hspace{1cm} (E.2)$$

and for $W_1[q]$:

$$W_1[q] = \text{const.} + \frac{1}{4} \frac{f_4}{f_2} \sum_{n=1}^{\infty} (-1)^n f_2^n \sum_x \xi_x^2 \left[ (D^{-1})^n \right]_{xx}$$

$$+ \frac{1}{48} \frac{f_6}{f_2} \sum_{n=1}^{\infty} (-1)^n f_2^n \sum_x \xi_x^4 \left[ (D^{-1})^n \right]_{xx}$$

$$+ \frac{1}{10} \frac{f_4^2}{f_2} \sum_{n=1}^{\infty} (-1)^n (n-1) f_2^{n-2} \sum_{xy} \xi_x^2 (D^{-1})_{xy} \xi_y^2 (D^{-1})_{zy} \left[ (D^{-1})^{n-2} \right]_{yy}. \hspace{1cm} (E.3)$$

Let us make use of the fact that the diagonal matrix elements of the powers of the inverse propagator do not depend on the Euclidean coordinates due to translation invariance (in the infinite volume limit) and reexpress the sums over $n$ in terms of $\mu_2$:

$$\mu_2 = \frac{1}{2} \sum_{n=1}^{\infty} (-1)^n f_2^n \left[ (D^{-1})^n \right]_{xx}. \hspace{1cm} (E.4)$$

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The last term on the r.h.s. of $W_1$ we rewrite as follows:

\[
\frac{1}{16} f_4^2 \left( \xi^2 (D^{-1})^2 \xi^2 \right) \sum_{n=1}^{\infty} (-1)^n (n-1) f_2^{n-2} [(D^{-1})^n]_{uu}
\]

\[
= \frac{1}{16} f_4^2 \left( \xi^2 (D^{-1})^2 \xi^2 \right) (-1) \sum_{m=0}^{\infty} (-1)^m m f_2^{m-1} [(D^{-1})^m]_{uu}
\]

\[
= \frac{1}{16} f_4^2 \left( \xi^2 (D^{-1})^2 \xi^2 \right) (-1) \sum_{m=1}^{\infty} (-1)^m m f_2^{m-1} [(D^{-1})^m]_{uu}
\]

\[
= \frac{1}{16} f_4^2 \left( \xi^2 (D^{-1})^2 \xi^2 \right) (-1) \frac{\partial}{\partial f_2} \left[ -f_2 \sum_{n=1}^{\infty} (-1)^{n-1} f_2^{n-1} [(D^{-1})^n]_{uu} \right]
\]

\[
= \frac{1}{16} f_4^2 \left( \xi^2 (D^{-1})^2 \xi^2 \right) \frac{\partial}{\partial f_2} \left[ f_2 \sum_{n=1}^{\infty} (-1)^n f_2^n [(D^{-1})^n]_{uu} + f_2 \right]
\]

\[
= \frac{1}{8} f_4^2 \left( \frac{1}{2} + \frac{\partial}{\partial f_2} (f_2 \mu_2) \right) (\xi^2 (D^{-1})^2 \xi^2).
\] (E.5)

Here we can write:

\[
\frac{\partial}{\partial f_2} (f_2 \mu_2) = -\frac{\partial}{\partial f_2} \left( \frac{f_2^2}{g^2} \right) = -\frac{2 f_2}{g^2} = 2 \mu_2.
\] (E.6)

Finally the 1–loop contribution to the generating functional of the connected Green’s functions takes the form:

\[
W_1[\xi] = \text{const.} + \frac{1}{2} f_4 \mu_2 \sum_x \xi_x^2 + \frac{1}{24} f_2 \mu_2 \sum_x \xi_x^4
\]

\[
+ \frac{1}{8} f_4 \left( \frac{1}{2} + 2 \mu_2 \right) \sum_{xy} \xi_x^2 [(D^{-1})^2]_{xy} \xi_y^2.
\] (E.7)

Let us express this functional through the external source $q_x$:

\[
W_1[q] = \frac{1}{2} f_4 \mu_2 \sum_x \left[ \left( Q_0^{-1} D q \right)_x^2 - \frac{1}{3} f_4 (Q_0^{-1} D q)_x \left( Q_0^{-1} (Q_0^{-1} D q)^3 \right)_x \right]
\]

\[
+ \frac{1}{24} f_2 \mu_2 \sum_x (Q_0^{-1} D q)_x^4
\]

\[
+ \frac{1}{8} g_4^2 \sum_{xy} (Q_0^{-1} D q)^2 (D^{-1})^2_{xy} (Q_0^{-1} D q)^2,
\] (E.8)

where $g_4^2 = f_4^2 \left( \frac{1}{2} + 2 \mu_2 \right)$. 33
Now we see that the 1–loop contribution is determined by \( \mu_2 \):

\[
\mu_2 = \frac{1}{2} \sum_{n=1}^{\infty} (-1)^n f_2^n a^4 \int_B \frac{d^4 p}{(2\pi)^4} \left[ a^2 \tilde{D}^{-1}(p) \right]^n \\
= \frac{1}{2} \left\{ \sum_{n=0}^{\infty} (-1)^n \left( \frac{2f_2}{q^2} \right)^n a^4 \int_B \frac{d^4 p}{(2\pi)^4} \left[ \sum_{\mu=0}^{3} (1 - \cos(ap^\mu)) \right]^n - 1 \right\} \\
= \frac{1}{2} \left\{ a^4 \int_B \frac{d^4 p}{(2\pi)^4} \left[ 1 + 2f_2 g^{-2} \sum_{\mu=0}^{3} (1 - \cos(ap^\mu)) \right] - 1 \right\}. \tag{E.9}
\]

The geometric series converges for \( 8s \equiv 8 \cdot 2f_2 g^{-2} \leq 1 \), i.e. \( s \leq \frac{1}{8} \). We write the nominator of the integrand as: \( 4s \left( 1 - \tilde{\beta} \sum_{\mu=0}^{3} \cos(ap^\mu) \right) \) with \( \tilde{\beta} = s(1 + s)^{-1} \). It is well known that the momentum integral converges for \( 0 < \tilde{\beta} < \frac{1}{4} \) which is satisfied for \( s \leq \frac{1}{8} \).

Let us perform the integral over \( p^0 \) at first. The integrand develops simple poles on the complex \( p^0 \) plane if \( \vec{p} \to 0 \). We estimate the momentum integral by the pole contribution, and set \( \vec{p} = 0 \). Then the 3-momentum integration can be carried out trivially, and the 1–dimensional integral

\[
\mu_2 = \frac{1}{2} \left( -\frac{1}{(1 + 4s)\tilde{\beta} 2\pi} a \int_{-\pi/a}^{\pi/a} dp^0 \frac{1}{\zeta - \cos(ap^0)} - 1 \right) \tag{E.10}
\]

is left over, \( \zeta = (1 - 3\tilde{\beta})\tilde{\beta}^{-1} = s^{-1} + 1 \). It has simple poles at \( ap^0 = iv_\pm = i\ln \left( \zeta \pm \sqrt{\zeta^2 - 1} \right) \). We can take \( v_\pm \approx \pm \ln(2\zeta) \) considering \( \zeta \gg 1 \) for \( s \leq \frac{1}{8} \). The 1-dimensional integral is of the same type as the integral \( I_0 \) in Part V. With a similar treatment, we obtain:

\[
\mu_2 = \frac{1}{2} \left( \frac{1}{s \sinh v_+} - 1 \right) = \frac{1}{2} \left( \frac{1}{s\zeta} - 1 \right) \approx -\frac{1}{2}s. \tag{E.11}
\]

The parameter \( s \ll 1 \) is a small one in the theory.

The generating functional \( W[q] \) of the connected Green’s functions is the sum of the tree contribution \( W_0[q] = \ln Z_0 + \text{const.} \) and that of the 1-loop contribution \( W_1[q] \):

\[
W[q] = -a\bar{u} \sum_x q_x + \frac{1}{2} f_2 (qQ_0^{-1}Dq) + \frac{1}{24} f_4 \left( (Q_0^{-1}Dq)^4 \right) + \frac{1}{12} f_4 \left( q(1 - f_2 Q_0^{-1}Q_2)(Q_0^{-1}Dq)^3 \right) + \frac{1}{16} f_4 \sum_{xy} (Q_0^{-1}Dq)^2_x (D^{-1})^2_{xy}(Q_0^{-1}Dq)_{y}, \tag{E.12}
\]

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with

\[ Q_0 = D + f_2 = D(1 + f_2 D^{-1}), \]
\[ Q_1 = 1 + \lambda Q_0^{-1} D, \quad Q_2 = 1 + 2\lambda Q_0^{-1} D, \]  \tag{E.13}

and the parameter

\[ \lambda = \frac{f_4}{f_2^2} \mu_2 = -\frac{f_4}{2f_2^2} s = -\frac{f_4}{f_2 g^2}. \]  \tag{E.14}

The parameter \( \lambda \) governs the loop expansion of the generating functional \( W[q] \). The terms proportional to \( \lambda \) in \( Q_1 \) and \( Q_2 \) represent the 1-loop contributions. The last term on the r.h.s. of Eq. (E.12) is of the order \( f_4^2 f_2^{-2} s^2 \sim f_2^3 \lambda \) and was neglected. (We neglected also the term proportional to \( f_6 \).)

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