Cosmological Acceleration from Virtual Gravitons

Leonid Marochnik
Physics Department, University of Maryland, College Park, MD 20742, USA

Daniel Usikov
36477 Buckeye St., Newark, CA 94560, USA

Grigory Vereshkov
Research Institute of Physics, Southern Federal University, 344090, Rostov–on–Don, Russia

Intrinsic properties of the space itself and quantum fluctuations of its geometry are sufficient to provide a mechanism for the acceleration of cosmological expansion (dark energy effect). Applying Bogoliubov–Born–Green–Kirkwood–Yvon hierarchy approach to self–consistent equations of one–loop quantum gravity, we found exact solutions that yield acceleration. The permanent creation and annihilation of virtual gravitons is not in exact balance because of the expansion of the Universe. The excess energy comes from the spontaneous process of graviton creation and is trapped by the background. It provides the macroscopic quantum effect of cosmic acceleration.

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I. INTRODUCTION

To explain the dark energy effect [1, 2] a cosmological constant, hypothetical fields or some modifications of physical laws have been proposed (see review [3]). We show that quantum fluctuations of the metric (gravitons) and their back reaction on the isotropic and homogeneous (on average) background provide the mechanism for cosmological acceleration. The dark energy effect is a consequence of the vacuum polarization and graviton creation by non–stationary gravitational field of the Universe.

The energy density of gravitons is a functional of the background geometry. In the non–empty Universe the background geometry is defined by all contributing cosmological subsystems — by gravitons, matter and radiation. The combination of conformal non–invariance with zero rest mass of gravitons (unique properties of the gravitational field) leads to a macroscopic quantum effect: condensation of gravitons in a quantum state with wavelength of the order of the distance to the horizon. In the process of the evolution of the Universe, the density of gravitons, as a result of their condensation, starts dominating over the sum total of the energy density of other subsystems of the cosmological media. The self–consistent state of background and gravitons, which evolves asymptotically, represents self–polarized vacuum in the de Sitter space. In the subsequent paper [4] we show that the regime of the de Sitter–like expansion is beginning to form in the current Universe which is consistent with dark energy observations [5].

We present three new exact solutions for the one–loop quantum gravity. Two of these provide cosmological expansion with acceleration. The de Sitter solution is one of these. All exact solutions can be found when the theory is presented as Bogoliubov–Born–Green–Kirkwood–Yvon (BBGY) hierarchy equations for moments of the graviton spectral function. The same results follow from direct calculations of quantum field operator functions, state vectors and the graviton spectral function.

We operate in the framework of one–loop quantum gravity because the theory cannot be renormalized in higher loops. Problems arising in two–loop theory are described, for example, in [6]. However, the effect of condensation of gravitons is created by general properties of gravitational field (conformal non–invariance and zero rest mass of gravitons). These properties will probably remain in the future comprehensive theory which includes quantum gravitation. The results of one–loop theory in which only the gravitational field is taken into account, are mathematically robust due to the finiteness of one–loop quantum gravitation [7]. In our case, the finiteness is provided by the compensation of diverged contributions of gravitons and ghosts to observable quantities.

*Electronic address: lmarochnik@gmail.com
†Electronic address: dusikov@electroglas.com
‡Electronic address: gveresh@gmail.com
The complete classic (non-quantum) theory of back reaction of scalar, vector and tensor fluctuations on the isotropic space–time background has been developed in [8]. In this paper, we consider the quantum theory of tensor fluctuations. Our model of the empty Universe consists of the background and gravitons only. In the self–consistent theory of gravitons, the macroscopic metric is described by regular Einstein equations

\[ R^k_i - \frac{1}{2} \delta^k_i R = \kappa \langle \psi | \hat{T}^k_{i(\text{grav})} + \hat{T}^k_{i(\text{ghost})} | \psi \rangle . \]  

(1)

The stress tensor of gravitons \( \hat{T}^k_{i(\text{grav})} \) and ghosts \( \hat{T}^k_{i(\text{ghost})} \) should be obtained by solving operator equations of motion and averaging over a quantum ensemble \( | \psi \rangle \). Note the average stress tensor of nontrivial ghost fields interacting with gravity must appear in the right hand side of (1) because there are no gauges that eliminate the diffeomorphism group degeneracy in the General Relativity. Our gauge selection was based on two principles. First, both background and gravitons should be considered in the same reference frame. Second, the gauge should provide automatically the group degeneracy in the General Relativity. Our gauge selection was based on two principles. First, both background and gravitons must appear in the right hand side of (1) because there are no gauges that eliminate the diffeomorphism effects of vacuum polarization and particle creation by background field are contained in equations (4) for gravitons and ghosts. These equations are linear in quantum fields but their coefficients depend on the non–stationary background.

Equations (2, (3), (4) and quantization rules (5) have been obtained by the path integral [9, 10]. They have been approximated by solving operator equations of motion and averaging over a quantum ensemble \( | \psi \rangle \). Note the average stress tensor of nontrivial ghost fields interacting with gravity must appear in the right hand side of (1) because there are no gauges that eliminate the diffeomorphism

\[ 3H^2 = \kappa \varepsilon_g = \frac{1}{16} D + \frac{1}{4} W_1 , \quad -2\dot{H} - 3H^2 = \kappa p_g = \frac{1}{16} D + \frac{1}{12} W_1 , \]  

(2)

where \( H = \dot{a}/a \) is the Hubble function and \( a(t) \) is the scale factor. Here \( D \) and \( W_1 \) are moments of the spectral distribution function of gravitons that is renormalized by ghosts. The moments are:

\[ D = \dot{W}_0 + 3H\dot{W}_0 , \]

\[ W_m = \sum_k \frac{k^{2m}}{a^{2m}} \left( \sum_\sigma \langle g | \psi_{k\sigma}^+ \psi_{k\sigma} | g \rangle - \langle gh | \tilde{\theta}_k \theta_k | gh \rangle \right) , \quad m = 0, 1, 2, ..., \infty . \]  

(3)

Here and later the dots are time derivatives. Heisenberg’s equations for Fourier components of the transverse 3–tensor graviton field and Grassman ghost field are:

\[ \ddot{\psi}_{k\sigma} + 3H \dot{\psi}_{k\sigma} + \frac{k^2}{a^2} \psi_{k\sigma} = 0 , \]

\[ \ddot{\theta}_k + 3H \dot{\theta}_k + \frac{k^2}{a^2} \theta_k = 0 . \]  

(4)

Taking account of normalization of fields in accordance with (2) and (3), canonical commutation relations for gravitons and anticommutation relations for ghosts read

\[ \frac{a^3}{4\pi} \left[ \psi_{k\sigma}^+, \psi_{k'\sigma'} \right]_- = -i\hbar \delta_{kk'} \delta_{\sigma\sigma'} , \]

\[ \frac{a^3}{8\pi} \left[ \dot{\theta}_k, \dot{\theta}_{k'} \right]_+ = -\frac{a^3}{8\pi} \left[ \theta_k, \theta_{k'} \right]_+ = -i\hbar \delta_{kk'} . \]  

(5)

Equations (2, (3), (4) and quantization rules (5) have been obtained by the path integral [8, 9, 10]. They have been obtained from the class of synchronic gauges that automatically provide one–loop finiteness of observables. One–loop effects of vacuum polarization and particle creation by background field are contained in equations (4) for gravitons and ghosts. These equations are linear in quantum fields but their coefficients depend on the non–stationary background metric. Correspondingly, in the background equation (2) we keep the average values of bilinear forms of quantum fields only. In this model, quantum particles interact through a common self–consistent field only.

We would like to point out that our model contains the graviton energy stress tensor but the short wavelength approximation is not used. The separation of the total metric into background and gravitons is not done on the basis of space scales hierarchy but on the basis of symmetry criteria. The metric of the isotropic 3–space belongs to 3–scalar representation of the group \( O(3) \) while the gravitons are represented by transverse and traceless 3–tensor. The mathematically rigorous method of separating the background and the gravitons, which ensures the existence of the graviton energy stress tensor is based on averaging over graviton polarizations: \( \langle \psi_{\alpha}^2 \rangle = 0 \) if all polarizations are equivalent in the quantum ensemble. The properties of the theoretical model are quite specific, but it is due to this that we can study the effect of condensation of long wavelength gravitons in the isotropic universe.
III. BBGKY HIERARCHY AND EXACT SELF–CONSISTENT SOLUTIONS

The following BBGKY hierarchy has been obtained from \( \mathfrak{B} \) by a standard procedure

\[
\dot{D} + 6HD + 4W_1 + 16HW_1 = 0 ,
\]

\[
\dot{W}_m + 3(2m + 3)H\dot{W}_m + 3 \left( 4m^2 + 12m + 6 \right) H^2 + (2m + 1)\dot{H} \dot{W}_m + 2 \left( 2m^2 + 9m + 9 \right) H^3 + 6(m + 2)H\dot{H} + \ddot{H} \right) W_m + 4\dot{W}_{m+1} + 8(m + 2)HW_{m+1} = 0 , \quad m = 1, ..., \infty .
\]

In the infinite BBGKY chain \( \mathfrak{B} \), each equation connects two neighboring moments \( \mathfrak{B} \) of spectral function. The system of equations \( \mathfrak{B} \), \( \mathfrak{C} \) has at least three exact self–consistent solutions. Two of them are the following

\[
D = -48 \left[ \pm \frac{K^2}{a^2} \left( \ln \frac{a}{a_0} + \frac{1}{2} \right) - C \right] , \quad W_1 = \pm 24 \frac{K^2}{a^2} \left( \ln \frac{a}{a_0} + \frac{1}{4} \right) ,
\]

\[
W_m = -24(\mp 1)^m \frac{K^{2m}}{a^{2m}} \ln \frac{a}{a_0} , \quad m \geq 2 ; \quad H^2 = \pm \frac{K^2}{a^2} \ln \frac{a}{a_0} + C \quad (7)
\]

where \( K, a_0, C \) are arbitrary constants. Evolution scenarios for \( \mathfrak{B} \) are simplest with \( C = 0 \). The first solution (upper signs in \( \mathfrak{B} \)) describes the Universe that was collapsing in the infinitely remote past to the state with the minimal scale factor \( a_{min} = a_0 \), and then began to expand with acceleration \( \ddot{a}/a = K^2/2a^2 \). Asymptotically, it becomes logarithmically slow and reads

\[
a(t) \to Kt \ln^{1/2}(Kt/a_0) , \quad t \to \infty .
\]

The second solution (lower signs in \( \mathfrak{B} \)) corresponds to the Universe creation from a singularity, expanding to the maximal scale factor \( a_{max} = a_0 \), then subsequently collapsing and ending in a final singularity.

The third solution describes the graviton vacuum in the de Sitter space. It reads

\[
H = \frac{1}{6} \sqrt{W_1} , \quad a = a_0 e^{Ht} , \quad D = -\frac{8}{3}W_1 , \quad W_m = -\frac{m(2m^2 + 9m + 9)}{2(m + 2)}H^2W_m , \quad m \geq 1 .
\]

One can show that this solution is stable against small perturbations.

From the recurrence relation for moments \( \mathfrak{B} \), we can evaluate graviton and ghost characteristic wave lengths as follows

\[
\lambda \sim \frac{a}{\bar{k}} \sim \sqrt{\frac{W_1}{|W_2|}} = \frac{1}{H} \sqrt{\frac{3}{10}} = const .
\]

Quantum fluctuations of these wave lengths dominate in the formation of observables. As it is seen from \( \mathfrak{B} \), in the process of the Universe exponential expanding characteristic values of \( \bar{k} \) rapidly shift to the region of exponentially large wave numbers, and all observables behave as constants with time. We will see below that this situation takes place if the graviton and ghost spectra are flat in \( \bar{k} \) — space of conformal wave numbers, and divergent contributions from graviton and ghost integrals compensate each other, so values of observables are generated by finite differences of these integrals.

IV. GRAVITON AND GHOST STATE VECTORS IN DE SITTER SPACE

Below we show that the de Sitter solution \( \mathfrak{B} \) follows from direct calculations of quantum field operator functions, state vectors and graviton spectral function, i.e. independently of BBGKY–approach. Due to the exact solution \( \mathfrak{B} \), there is a vacuum state vector such that exact solutions of operator equations \( \mathfrak{B} \) become self–consistent in the de Sitter space averaged over this vector. These exact solutions are

\[
\psi_{k\sigma} = \frac{1}{a} \sqrt{\frac{2\pi\hbar}{k}} \left[ c_{k\sigma} f(x) + c_{-k-\sigma}^+ f^*(x) \right] ,
\]

\[
\theta_k = \frac{1}{a} \sqrt{\frac{4\pi\hbar}{k}} \left[ \alpha_k f(x) + \beta_k^* f^*(x) \right] ,
\]

\[
(10)
\]
where \( f(x) = (1 - i/x)e^{-ix} \), \( x = k\eta \), \( \eta = \int dt/a \). Substituting (10) into (5), we get commutation/anticommutation relations for operator constants:

\[
[c_{k\sigma}, c_{k'\sigma'}^+] = \delta_{kk'}\delta_{\sigma\sigma'},
\]

\[\alpha_k, \alpha_k' = \delta_{kk'}, \quad [\beta_k, \beta_k'] = -\delta_{kk'} .\]

In accordance with (11), the space of graviton states is constructed over the standard Fock basis \( |n_{k\sigma}\rangle \). Note that quantum occupation numbers \( n_{k\sigma} = 0, 1, 2, \ldots \), cannot be interpreted as real graviton numbers. In non–stationary space quanta with arbitrary momentum are not in the mass shell. So, formally, these are virtual particles, and they behave as real particles asymptotically only (when \( k\eta \gg 1 \)). Quanta with the characteristic wave length \( \eta \) are essentially virtual particles. Thus, occupation numbers \( n_{k\sigma} \) are parameters of polarized vacuum. The vacuum state vector is a product of superposition of states with different occupation numbers:

\[
|g\rangle = \prod_{k\sigma} C_{n_{k\sigma}} |n_{k\sigma}\rangle, \quad \sum_{n_{k\sigma}} |C_{n_{k\sigma}}|^2 = 1 .
\]

where \( C_{n_{k\sigma}} \) is the amplitude of the state with the occupation number \( n_{k\sigma} \).

According to (2), (3), (11), the ghosts that correspond to the set of synchronic gauges redefine the moments of spectral distribution function additively. In the one–loop quantum gravity they should provide finiteness values of the observables. From the physical point of view, ghost fields act as compensators. We will show later that ghosts provide exact zero energy of quantum (and quasi–classical) gravitational waves, which cannot exist in de Sitter space.

Grassman units are extracted from ghost operator constants multiplicatively. They read

\[
\alpha_k = u a_k, \quad \bar{\alpha}_k = u a_k^+, \quad \beta_k = \bar{u} b_k, \quad \bar{\beta}_k = u b_k^+ .
\]

By definition, \( \bar{u}u = -u\bar{u} = 1 \), so operators \( a_k, a_k^+, b_k, b_k^+ \), which were introduced in (11), satisfy standard Bose commutation relations. Then, the procedure to construct the ghost space of states is obvious. The ghost state vector is

\[
|gh\rangle = \prod_k \sum_{n_k} A_{n_k} |n_k\rangle \prod_k \sum_{n_k} B_{n_k} |\bar{n}_k\rangle, \quad \sum_{n_k} |A_{n_k}|^2 = \sum_{n_k} |B_{n_k}|^2 = 1 .
\]

V. CALCULATION OF OBSERVABLES

Further, we make use of the de Sitter solution \( a = -(H\eta)^{-1} \) and variable of integration \( x = k\eta \). Substituting (10) into (3) and averaging over state vectors (13), (15) one gets

\[
W_m = \frac{2\pi H^2}{a^2} \int_0^\infty dx x^{2m+1} \left\{ U_{k(wave)}|f(x)|^2 + U_{k(cr)}|f^*(x)|^2 + U_{k(ann)}|f(x)|^2 \right\} ,
\]

where

\[
U_{k(wave)} = \sum_{\sigma} \langle g| c_{k\sigma}^+ c_{k\sigma}|g\rangle - \langle gh| a_k^+ a_k |gh\rangle - \langle gh| b_k^+ b_k |gh\rangle ;
\]

\[
U_{k(cr)} = \frac{1}{2} \sum_{\sigma} \langle g| c_{k\sigma}^+ c_{-k-\sigma}|g\rangle - \langle gh| a_k^+ b_k^+ |gh\rangle ;
\]

\[
U_{k(ann)} = \frac{1}{2} \sum_{\sigma} \langle g| c_{-k-\sigma} c_{k\sigma}|g\rangle - \langle gh| b_{-k} a_k |gh\rangle = U_{k(cr)}^* .
\]

Here \( U_{k(wave)} \) is the spectral parameter of quantum waves, which are real gravitons if \( k\eta \gg 1 \), and \( U_{k(cr)} \), \( U_{k(ann)} \) are spectral parameters of quantum fluctuations that emerge in processes of graviton creation from the vacuum and graviton annihilation to the vacuum.

Two conditions for \( W_m \), the absence of divergences and absence of \( t \) dependence, should be considered jointly. Because of \( k = x/\eta \) dependence, \( U_{k(wave)} \) term in (10) is time independent only if \( U_{k(wave)} \) does not depend on
If, however, \( U_{k(\text{wave})} = \text{const}\, \langle k \rangle \neq 0 \) then this integral does not exist because of the \(|f(x)|^2 \to 1\) condition at \( x \to \infty \). This is the reason why an exact compensation of graviton and ghost contributions in \( U_{k(\text{wave})} \) is a mandatory requirement. The compensation condition leading to \( U_{k(\text{wave})} = 0 \) is

\[
|C_{n+1}| = |A_n| = |B_{n-1}| .
\]  

(17)

This result has a simple physical interpretation. Quantum gravitational waves, whose equation of state is not \( p = -\varepsilon \), cannot exist in de Sitter space with the self–consistent geometry.

Analogously, spectral parameters \( U_{k(\text{cr})} = U \), \( U_{k(\text{ann})} = U^* \) cannot depend on \( k \) also. But corresponding integrals in (16) do not lead to obligatory divergences because oscillating functions \( \sim e^{\pm 2i\xi} \) are integrated at \( x \to \infty \). Such integrals may be redefined to have certain finite values. Thus, in (16) we have flat graviton and ghost spectra, \( U_{k(\text{wave})} \equiv 0 \), \( U_{k(\text{cr})} = U_{k(\text{ann})}^* = U \) and

\[
U = \left( \sum_n C_{n+1}C_n \sqrt{n+1} \right)^2 - \left( \sum_n A^*_{n+1}A_n \sqrt{n+1} \right) \left( \sum_n B^*_{n+1}B_n \sqrt{n+1} \right) ,
\]

where \( P_n \) is any universal distribution and \( \tilde{n} = \sum_{n=0}^{\infty} nP(n) \) is the average number of gravitons with wave lengths that are near the characteristic value \( \tilde{n} \).

From (17) and (18) it follows that \( U = 0 \) if all amplitudes are real. We have \( U \neq 0 \) if some amplitude are complex and phase dependencies of occupation numbers are different in graviton and ghost sectors. Observable quantities are proportional to \( U + U^* = 2N_g \), where \( N_g \) is a generalized parameter of graviton vacuum state. To use some model distributions (Poisson distribution, for example) one can see that \( N_g \sim \tilde{n} \).

Integrals in (16) should be redefined. They can be calculated in the following way

\[
\lim_{\zeta \to 0} \int_0^\infty dx x^{2m+1} e^{-(\zeta-2i)x} = \mp (-1)^m \frac{(2m+1)!}{2^{2m+1} \pi^2} ,
\]

\[
2i \lim_{\zeta \to 0} \int_0^\infty dx x^{2m} e^{-(\zeta-2i)x} = (-1)^{m+1} \frac{(2m)!}{2^{2m}} .
\]

(19)

The physical interpretation of (19) is as follows. In any instant of time, the procedure of redefining the integrals in (19) selects the contribution of virtual gravitons with wave lengths that are near characteristic value \( \tilde{n} \) only, and eliminates contributions of all other gravitons. This redefining procedure provides the existence of the exact solution of BBGKY hierarchy.

Zero moment \( W_0 \) is not included in the equations of the theory. Therefore, to avoid a logarithmic infrared singularity, the function under the integral sign in (19) is differentiated initially, then derivatives are combined as \( D = W_0 + 3HW_0 \), and redefined integrals are calculated. The results are

\[
D = -\frac{12\pi \hbar N_g}{\pi^2} H^4 , \quad W_m = \left( -1 \right)^{m+1} \frac{(2m+1)!}{2^{2m}} (2m-1)! (2m+1) \left( m+2 \right) \times \frac{2\pi \hbar N_g}{\pi^2} H^{2m+2} , \quad m \geq 1.
\]

(20)

It is easy to verify that expressions (20) satisfy to recurrence relations (8), following from the BBGKY hierarchy immediately.

In accordance with (2 and 20), the energy density and the pressure of virtual gravitons (that generate the back reaction of quantum fluctuations on background geometry) are

\[
\varepsilon_g = \frac{1}{12\pi} W_1 = \frac{3\hbar N}{8\pi^2} H^4 , \quad p_g = -\varepsilon_g .
\]

(21)

One can also obtain the equation (21) by other methods. The vacuum amplitude of graviton field in the one–loop approximation can be presented via a path integral, which, as it is known [14], can be calculated exactly in the de Sitter space. One can calculate this path integral under boundary conditions that correspond to the state vector [13], [14], under the assumption that the de Sitter geometry is self–consistent. A variation of result of this calculation over the scale factors leads to the convolution of Einstein equations (1) and thus to the expression \( (\varepsilon_g - 3p_g)/4 \equiv \varepsilon_g \) that coincides with (21). The same result can be obtained by Schwinger–DeWitt method using the effective action formalism.
VI. ENERGY BALANCE

Equation (21) is a consequence of the balance equation for the energy that is emerging to the space due to graviton creation and vanishing due to graviton annihilation. The characteristic energy of gravitons in these processes is \( \hbar \omega \sim \hbar H \). Total probabilities of graviton creation and annihilation processes (normalized to the unit volume) \( w_{cr} \) and \( w_{ann} \) are proportional to the phase volume of one graviton \( \omega^3/3\pi^2 \sim H^3/3\pi^2 \). The exponent of the background–graviton coupling constant is unity if \( \omega \sim H \). Thus, we obtain for \( w_{cr} \) and \( w_{ann} \) the following estimations

\[
 w_{cr} = \frac{\alpha}{3\pi^2} H^3 (\overrightarrow{N}_k + 1) (\overrightarrow{N}_{-k} + 1), \quad w_{ann} = \frac{\alpha}{3\pi^2} H^3 \overrightarrow{N}_k \overrightarrow{N}_{-k}.
\]  

(22)

where \( \alpha = O(1) \), \( \overrightarrow{N}_{\pm k} \sim N_g/2 \) is the average number of gravitons with wavelengths that are near characteristic value \( \omega \). Finally, we get the balance equation in the form

\[
 \varepsilon_g = \hbar \omega (w_{cr} - w_{ann}) = \frac{\alpha}{3\pi^2} \hbar H^4 (\overrightarrow{N}_k + \overrightarrow{N}_{-k} + 1) \simeq \frac{\alpha}{3\pi^2} \hbar N_g H^4.
\]  

(23)

This estimate with accuracy of numerical factor of the order of unity coincides with (21). From (22) and (23) one can see that the non-zero effect occurs because of quantum spontaneous process of the particle creation.

If non-gravitational contributions to the vacuum energy are compensated exactly (due to, e.g., a consequence of Supersymmetry), it follows from (2) and (21)

\[
 H^2 = \frac{8\pi^2}{3\hbar N_g}.
\]  

(24)

According (24), \( H^2 \sim \hbar^{-1} \), and it clearly demonstrates that the acceleration of the Universe expansion is a macroscopic quantum effect in the graviton vacuum at the scale of the Universe (analogous to such known macroscopic quantum effects as super-fluidity and super-conductivity). Substituting the observable value of Hubble constant into (24), we get an estimate of the total number of virtual gravitons under the de Sitter horizon \( N_g \sim 10^{123} \). This could be a possible interpretation of the \( \varepsilon_{DE}/\varepsilon_{Planck} \sim 10^{-123} \) ratio.

VII. CONCLUSION

Virtual gravitons with wavelength of the order of the horizon must appear and disappear in the graviton vacuum because of massless and conformal non–invariance of the graviton field. A non-zero balance of energy is due to the pure quantum process of spontaneous graviton creation, in other words, due to the uncertainty relation. Numerical integration of the system of equations (2), (6) shows several types of solutions with acceleration. In this paper, we considered the physics of this effect by the example of the exact de Sitter solution, which is the only one that corresponds to the state of equilibrium of graviton vacuum, and this is the reason why this exact solution exists.

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