One of the scenarios of transition to the turbulent mode of the flow of liquid crystals

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Abstract. The article presents the results of an experimental study of the transition to a turbulent flow regime of thin layers of nematic liquid crystals (4-octyl-4'-cyanobiphenyl) with a thickness of 20–125 μm at temperature of 39° C. The cell we used was a “sandwich”, which was assembled from three translucent glass plates separated by narrow strips of gaskets. Shear effects on the cell were carried out by the pendulum method with a frequency of 190 Hz, the phoner method was used. The analysis of the amplitude dependence of the optical signal, its spectrum, as well as the amplitude dependence of the width of the spectrum showed that low-frequency modes appear in the fluctuation spectrum, which lead to the appearance of periodic convection in the nematic liquid crystal layer.

1. Introduction
The question of what state the medium is in after reaching the equilibrium stability threshold is interesting for various fields of physics. And this confirms a number of scientific studies conducted recently [1-13]. Not so long ago, it was believed that for most isotropic systems the establishment of either turbulent or fully ordered field states is most typical. In the first case, any combination of elementary excitations is unstable, since a spatio-temporal disorder is established in the volume — a turbulent state. In the second case, in the process of nonlinear interaction, various elementary excitations self-consistent with each other, as a result of which a regular (in time and space) formation is formed — a structure. Although the problem of the relation between chaos and order has always attracted the attention of physicists, the theory of nonlinear structures (self-organization) and the theory of turbulence existed independently of each other. Recent years have made it possible to demonstrate the achievements of nonlinear dynamics and essentially new approaches [1–13] to the experimental study of the turbulent state, as well as to approach understanding of the problem of the connection between turbulence and structures. Consider several scenarios of stochastization (turbulization).

The Landau — Hopf theory considers a sequence of normal bifurcations that generates the limiting $u[x, \varphi(t),...\varphi(t)]$ quasiperiodic flow with a period of $2\pi$ in all arguments $\varphi(t)=\omega t+\alpha c$ with certain frequencies $\omega_1,...,\omega_n$, which occupies in the phase volume a region that corresponds to the initial phases $\alpha_1,...,\alpha_n$. This space is ergodic, because the trajectory that “wraps” on it with time passes in the vicinity of any point of the designated volume. In this case, the correlation functions of the velocities in this case do not tend to zero at infinity, at first they quickly decrease $N^{-1/2}$, and the time $T$ to the next maximum is very large: $T \sim c^{\alpha}$ ($\alpha \approx 1$).
The Ruelle — Takens theorem suggests, after three normal bifurcations, the appearance of a strange attractor. This scenario assumes structural stability: in every sufficiently small neighborhood of a phase flow with an $n$-dimensional invariant torus at $n \geq 3$ there is an open set of phase flows with a strange attractor. The proof of the theorem of Ruelle and Takens is based on the possibility of approximating the torus $T^3$ by a flow with a closed path trailing around the torus, into which the Cantor attractor can be embedded. Thus, in this scenario, the destruction of three frequency motions is realized by nonlinear synchronization (occurrence of resonances) of its high harmonics. The development of such a scenario is too soft for the transition to turbulence. In the experiments performed, stochasticization is more similar to the destruction of the two-frequency movement of $T^2$ beats, when during their synchronization, and then bifurcation of the doubling period, a cycle is formed, or the stable and saddle cycles merge and disappear. For example, in the case of the Feigenbaum scenario, a strange attractor is observed due to the appearance of an infinite sequence of bifurcations with a double period.

The Pomeau — Manneville theory assumes the emergence with time of a periodic and turbulent (stochastic) flow following the inverse tangential bifurcation of the merger and disappearance of the stable and unstable fixed points of the Poincare map, i.e. stable and unstable periodic phase trajectories. The implementation of the above scenarios became possible when conducting experiments under the influence of the Couette circular flow obtained in the gap between two coaxial rotating cylinders. Thermal convection was obtained by heating the liquid layer from below. It was previously believed that with increasing Reynolds number, axisymmetric perturbations should lose stability. It was found that the first change in stability (bifurcation) is the transition to Taylor roller toroidal vortices. Experiments specially carried out to verify the scenarios of transition to Taylor vortices showed that at a certain Reynolds number, Taylor vortices lose stability, since azimuthal bending waves arise on them. With an increase in the Reynolds number during subsequent bifurcations, independent frequencies still arise, then stochastic motion with a continuous spectrum is observed.

The transition to the turbulent state of nematic liquid crystals (NLC) subjected to various external influences was also discussed. A comparative analysis of the mechanisms and conditions for the occurrence of instabilities in isotropic liquids and in LCs showed that in an isotropic liquid the transition to a turbulent state, for example, with thermoconvective instability, mainly depends on the geometric parameters of the system under consideration. In nematics, electrohydrodynamic instability is most studied, which is associated with a distortion of the director field, and the nature of the motion of liquid crystal molecules is nonlinear. One instability in the LC is observed when the Leslie viscosity coefficient is $\alpha > 0$, and the director lies in the plane defined by the velocity and velocity gradient vectors. At the same time, upon reaching the threshold value of the velocity gradient, the director of the LC overturns due to the appearance of a moment of friction forces, which is proportional to the coefficient of viscosity of Leslie $\alpha$, then the system proceeds to a turbulent state. Another instability arises in a homeotropically oriented NLC layer subjected to an elliptical shift. In this case, the generation of various defects is observed, their dynamics and role in determining the parameters of the corresponding convective structure were studied.

Manneville previously studied convective structures that arose in a nematic liquid crystal under the influence of a temperature gradient or elliptical shear strain. In the case of thermoconvective instability, the main attention was paid to the study of dynamic structures. It was shown that unsteady convective flow arises due to the isotropic interaction of fluctuations in the flow velocity and the orientation of the director field of a liquid crystal. To stabilize the flows, a magnetic field was used. The authors noted that stationary convective instability corresponds to direct bifurcation of the system of equations of hydrodynamics and thermal conductivity of liquid crystals, and on the other hand, unsteady convection is observed during reverse bifurcation. In the first case considered, bifurcation leads to the establishment of a stationary flow, i.e., to a stable state, in the second case leads to the occurrence of a limiting case — to an oscillation state. The possibility of observing regular convective hydrodynamic flows in an NLC under the influence of radiation pressure of acoustic waves with a transverse intensity distribution was demonstrated. Two sound waves that interfered partially absorbed the LC. An acoustic wave with a spatially periodic intensity distribution has an effect on NLC particles. These waves cause a stationary
convective flow of the LC, which in turn leads to a rotation of the NLC molecules. When there is no influence of acoustic waves on the NLC, the LC placed between the crossed polarizers does not let light normally incident on the LC layer. As a result of external influence, the NLC director is reoriented and the layer is clarified. The conditions for the occurrence of thermoconvective instability can be more accurately described if we take into account the interaction of director fluctuations and the order parameter modulus.

Thus, at present, there is no reliable unified theory describing the transition to a turbulent state of both isotropic media and anisotropic systems, such as liquid crystals. Therefore, this issue is important and relevant for study. Only the successes of nonlinear dynamics obtained in recent years, as well as fundamentally new approaches to the study of the turbulent state, allowed us to come closer to understanding the problem of the relationship between turbulence and structures. In this regard, liquid crystals, like anisotropic liquids, exhibit a wide range of structural transformations and instabilities due to an additional degree of freedom - the presence of a crystalline order. In this regard, it seems important to study the transition to a turbulent state and dynamic chaos, in particular with periodic low-frequency shear ($\omega<500$ Hz) in uniformly oriented thin layers of nematic LCs.

2. Materials and Methods

Thin layers (thickness $h=20–125 \mu m$) of a nematic liquid crystal of 4-octyl-$4'$-cyanobiphenyl (OCB) with a homeotropic orientation of molecules at a temperature of 39° C were experimentally investigated. The amplitude range of the periodic shift is $a<900 \mu m$, the exposure frequency is 190 Hz (the frequency at which the interference effect is minimal) was selected [14, 15].

The liquid crystal was placed in a cell, which was a design of two slides, separated by gaskets. The thickness of the liquid crystal was determined by the thickness of the gaskets. Between the slides was placed the third thin plate of glass cover, which floated in a liquid crystal. Using a glass waveguide, she connected to the membrane of a mechanism that performs periodic oscillations. The experimental cell was mounted on a microscope stage. Polarized light passed through the LC cell, was recorded and converted into an electrical signal by a microscope photoelectronic multiplier. Further, the received signal was sent to the device for studying the probabilistic characteristics of random processes, where the signal was directly recorded. Then, using the Fourier transform, the spectrum of the optical signal was calculated. The effective width of the optical spectrum was determined by the formula:

$$
\Delta \omega = \frac{1}{I_{max}} \int_0^{\infty} I(\omega) d\omega,
$$

where $I_{max}$ is the maximum value of the intensity of the optical signal in the spectrum.

3. Results and Discussion

As a result of the experiments, the dependence of the optical signal on the amplitude of the effect was obtained (figure 1). As the amplitude of shear effects increases, the LC layer becomes more transparent (nicoles are crossed), the intensity of transmitted light reaches a maximum at an amplitude value of 40 $\mu m$, and then gradually decreases. The latter is associated on the one hand with the formation of an inhomogeneous distribution of the director over the thickness of the sample — the appearance of convective low-frequency flows of the “rolls” type, and on the other hand, as the amplitude increases, the continuity of the director field orientation breaks, and domain walls are generated that strongly scatter and reduce the transparency of the layer. So, for example, with an amplitude $a=95 \mu m$, transverse domains - “rolls” are formed, with a shear amplitude $a=10 \mu m$, longitudinal domains appear, which are destroyed at $a=140 \mu m$.

The study and interpretation of turbulence - stochastization of the system in this case presents certain difficulties that are caused by the object itself, because The LC is an anisotropic system and the relationship between the recorded optical signal and the parameters of the system itself, in particular, the behavior of the director’s orientation angles, is nonlinear. In addition, unlike an ordinary liquid in
which scattering can be caused by density fluctuations, for example, during critical opalescence [16], scattering in an LC is anisotropic and, as indicated above, can be associated simultaneously with various mechanisms. Therefore, to identify certain regular patterns of system behavior in this case is a considerable difficulty. Nevertheless, by studying the frequency spectra of the transmitted optical signal, some conclusions can be drawn.

\[ S(\omega) \]

**Figure 1.** The dependence of the optical signal intensity on the shear amplitude (4-octyl-4’-cyanobiphenyl (OCB), \( h=35 \mu m \), \( T=39^\circ C \), \( \omega=190 \) Hz).

Figure 2 shows a typical frequency dependence of the optical signal \( S(\omega) \) for a shear amplitude \( a=125 \mu m \).

\[ S, Hz^2 \]

**Figure 2.** Spectrum of the optical signal (4-octyl-4’-cyanobiphenyl (OCB), \( h=35 \mu m \), \( T=39^\circ C \)).

It follows that low-frequency fluctuation modes are excited in the LC system in the frequency range \( \omega<25 \) Hz, which are associated both with director deviations and the nature of the flows of the liquid crystal itself. The latter actually manifests itself in the optical spectrum through fluctuations of the optical anisotropy \( \delta(\Delta n) \). Given the relative smallness of the spectral representation of the variable part of the optical signal (figure 2) with respect to the constant (figure 1), in a first approximation, the nonlinearity of the LC system can be neglected. This becomes possible due to the smallness of the observed spectral densities, where the maximum relative spectral density \( S=10^{-4} \). Accordingly, it is possible to obtain estimates of the phase fluctuation, which is of the order of \( 10^{-2} \) rad, and the maximum
characteristic director fluctuations are of the order of $10^{-3}$ rad, which allows us to consider the relationship between the optical signal and director fluctuations linear. It follows from this that under the influence of an oscillating shift of the relative amplitude $a/h = 3.5$ ($a=125 \, \mu m$ and $\omega > 100 \, Hz$) in the NLC with the initial homeotropic orientation of the molecules, the low-frequency fluctuation spectrum is excited, which reflects the nature of the orientational motion of the director and convective flow NLC. In particular, the fact that two local maximums take place in the optical spectrum indicates the presence of periodic motions associated, for example, with periodic convection in the LC layer. The characteristic frequencies of this process are $\omega_1 = 2 \, Hz$ and $\omega_2 = 4 \, Hz$. Although the presence of the second frequency is most likely due to the nonlinearity of the type $I \sim \Delta \sin^2 \theta$, if $\theta \sim \theta_0 \exp(j \omega t)$. The latter reflects the frequency in the director’s behavior in time during convection. A further increase in the amplitude of the shift leads to the destruction of the uniform orientation of the director field, the formation of gaps in the orientation. The system becomes finely dispersed, i.e. breaks up into rapidly moving and oscillating clusters of size $d<<h$.

![Figure 3](image.png)

**Figure 3.** The dependence of the width of the spectrum on the amplitude of the shift (4-octyl-4'-cyanobiphenyl (OCB), $h=35 \, \mu m$, $T=39^\circ C$, $\omega=190 \, Hz$).

We discussed above the existing mechanisms of stochasticization of molecular systems, the formation of stationary and non-stationary periodic structures and their transition to a turbulent state. Based on the already known scenarios of the development of turbulence, it should be noted that, in contrast to the listed scenarios, in this case the transition process is accompanied by a continuous broadening of the spectrum (figure 3).

4. Conclusions
The transition to a turbulent state in thin layers of homeotropically oriented nematics under the influence of an oscillating shift is associated with the excitation of orientational and hydrodynamic fluctuations in the NLC system. As the amplitude of the periodic shift increases, they are accompanied by continuous broadening of the spectrum. In addition, low-frequency modes appear in the fluctuation spectrum, which lead to the formation of periodic convection flows in the NLC layer, and then the formation of various periodic structures, for example, such as “rolls”, is observed.

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