MAGNETOHYDRODYNAMIC SIMULATION OF THE X2.2 SOLAR FLARE ON 2011 FEBRUARY 15. I. COMPARISON WITH THE OBSERVATIONS

S. Inoue1, K. Hayashi2, T. Magara3, G. S. Choe4, and Y. D. Park3

1 School of Space Research, Kyung Hee University, Yongin 446-701, Korea; inosato@khu.ac.kr
2 W. W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305, USA
3 Korea Astronomy and Space Science Institute, Daejeon 305-348, Korea

Received 2013 December 3; accepted 2014 April 10; published 2014 June 6

ABSTRACT

We performed a magnetohydrodynamic (MHD) simulation using a nonlinear force-free field (NLFFF) in solar active region 11158 to clarify the dynamics of an X2.2-class solar flare. We found that the NLFFF never shows the dramatic dynamics seen in observations, i.e., it is in a stable state against the perturbations. On the other hand, the MHD simulation shows that when the strongly twisted lines are formed at close to the neutral line, which are produced via tether-cutting reconnection in the twisted lines of the NLFFF, they consequently erupt away from the solar surface via the complicated reconnection. This result supports the argument that the strongly twisted lines formed in NLFFF via tether-cutting reconnection are responsible for breaking the force balance condition of the magnetic fields in the lower solar corona. In addition to this, the dynamical evolution of these field lines reveals that at the initial stage the spatial pattern of the footpoints caused by the reconnection of the twisted lines appropriately maps the distribution of the observed two-ribbon flares. Interestingly, after the flare, the reconnected field lines convert into a structure like the post-flare loops, which is analogous to the extreme ultraviolet image taken by the Solar Dynamics Observatory. Eventually, we found that the twisted lines exceed a critical height at which the flux tube becomes unstable to the torus instability. These results illustrate the reliability of our simulation and also provide an important relationship between flare and coronal mass ejection dynamics.

Key words: magnetohydrodynamics (MHD) – Sun: activity – Sun: coronal mass ejections (CMEs) – Sun: flares – Sun: magnetic fields

Online-only material: color figures

1. INTRODUCTION

Solar active region (AR) 11158 emerged in the eastern solar hemisphere on February 11 and consists of bipolar fields showing complicated behaviors, including strongly sheared and twisted motions (Jiang et al. 2012). Eventually, it produced one X-class flare and several M-class flares on 2011 February (Sun et al. 2012; Tziotziou et al. 2013; Nindos et al. 2012; Inoue et al. 2013; Aschwanden et al. 2014). Fortunately, solar physics satellites, such as the Solar Dynamics Observatory (SDO) and Hinode (Kosugi et al. 2007), provide rich data with unprecedented spatial and temporal resolutions obtained by multi-wavelength observations, enabling us to further understand the dynamics of these solar flares (e.g., Schrijver et al. 2011; Kosovichev 2011; Asai et al. 2012; Liu et al. 2012; Jing et al. 2012; Inoue et al. 2013; Toriumi et al. 2013).

In general, solar flares are widely considered as release phenomena of free magnetic energy accumulated in the solar corona (Shibata & Magara 2011); it is therefore important to understand the three-dimensional (3D) magnetic field to know where the free energy is accumulated in the AR. Unfortunately, we cannot directly measure the coronal magnetic field even by state-of-art ground- or space-based observations, which provide only a two-dimensional (2D) vector magnetic field on the photosphere. In this kind of situation, nonlinear force-free field (NLFFF) extrapolation from the vector field is a solid tool that allows us to show a 3D view of the magnetic field (see Wiegelmann & Sakurai 2012 for details). The Helioseismic and Magnetic Imager (HMI; Scherrer et al. 2012; Schou et al. 2012; Hoeksema et al. 2014) on board SDO and the Solar Optical Telescope (SOT; Tsuneta et al. 2008) on board Hinode provide the vector field data, which were already applied to extrapolate the 3D magnetic field under the NLFFF approximation in earlier studies (Wiegelmann et al. 2012; Jiang & Feng 2013; Inoue et al. 2013; Aschwanden et al. 2014). The NLFFF shows that the strongly twisted and sheared field lines are found to reside close to the polarity inversion line (PIL) before the flare and are confirmed to relax after the flare.

Some earlier studies analyzed the NLFFF and suggested the dynamics of M6.6- or X2.2-class flares by comparing the NLFFF with multi-wavelength observations or NLFFF configurations before and after the flares. Wang et al. (2012) suggested the possibility of tether-cutting reconnection (Moore et al. 2001) in X2.2-class solar flares through multi-wavelength analysis. Sun et al. (2012) analyzed the NLFFF, showing its energy distribution in space and temporal evolution, as well as suggesting evidence of the tether-cutting reconnection on the variation of the field line connectivity before and after the flare. Liu et al. (2012) analyzed the NLFFF just before and after the M6.6-class solar flare and found that the strong coronal current system underwent an apparent downward collapse and a small current-carrying loop was produced closer to the surface, suggesting that the tether-cutting reconnection is a suitable model. More recently, Liu et al. (2013) estimated the twist of a field line and investigated the connectivity of a field line; consequently, the obtained results were consistent with the tether-cutting magnetic reconnection model.

Inoue et al. (2013) also estimated the magnetic twist of the NLFFF before the X2.2-class flare in AR 11158, reconstructed using the magnetohydrodynamic (MHD) relaxation method (Inoue et al. 2014), and found that the strongly twisted lines having more than a half-turn twist but less than one turn are related to this flare because their footpoints correspond well to the location where the two-ribbon flares in the Ca II image...
Figure 1. (a) Photospheric vector field (left), and EUV image with 94 Å (middle), with 171 Å (right) observed at 00:00 UT on 2011 February 15, taken by HMI and AIA on board SDO, are shown, respectively. These sizes are in the range of 216 × 216 Mm², and observed tangential components reside in the black dotted square (79.2 ≦ x ≦ 136, 8, 86.4 ≦ x ≦ 144) (Mm), while other areas are fixed by the potential field. The value of $B_z$ is normalized by 2500 G; the non-dimensional value 0.25 corresponds to 625 G. (b) Field lines of the NLFFF are plotted over each image. (c) Field lines of the NLFFF that is reconstructed using the whole vector field, not partial reconstruction such as in panel (b), are plotted over each image.

(A color version of this figure is available in the online journal.)

2. OBSERVATIONS AND NUMERICAL METHOD

2.1. Observations

To extrapolate the NLFFF, we use the vector magnetic field shown in the left panel of Figure 1(a), observed at 00:00 UT.
on 2011 February 15, approximately 120 minutes before the X2.2-class flare detected by SDO/HMI. The vector magnetic field covers a 216 × 216 Mm² region, divided by a 600 × 600 grid that is provided by the description of the cylindrical equal area projected and remapped vector magnetic field. It was obtained using the very fast inversion of the Stokes vector algorithm (Borrero et al. 2011) based on the Milne–Eddington approximation. A minimum energy method (Metcalf 1994; Metcalf et al. 2006; Leka et al. 2009) was used to resolve the 180° ambiguity in the azimuth angle of the magnetic field. We also use extreme-ultraviolet (EUV) images observed with 94 Å on board SDO and Ca II H 3968 Å images during the flare taken by Hinode, in order to compare with the NLFFF and MHD simulation. Ca II images were obtained using the Broadband Filter Imager of the SOT on board Hinode. The field of view corresponds to 111.58 × 111.58 with a resolution of 1024 × 1024 at 01:50:18 UT on February 15.

2.2. Numerical Method of the Nonlinear Force-free Extrapolation

We carry out the NLFFF and MHD simulation according to the following equations:

\[ \rho = |B| \quad (1) \]

or

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) + \xi \nabla^2 (\rho - \rho_0), \quad (2) \]

\[ \frac{\partial \mathbf{v}}{\partial t} = - (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho} \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{v}, \quad (3) \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta_i \mathbf{J}) - \nabla \phi, \quad (4) \]

\[ \mathbf{J} = \nabla \times \mathbf{B}, \quad (5) \]

\[ \frac{\partial \phi}{\partial t} + c_h^2 \nabla \cdot \mathbf{B} = - \frac{1}{\mu_0} \mathbf{J} \cdot \nabla \phi, \quad (6) \]

where the NLFFF calculation employs Equation (1) as the density evolution in order to ease the relaxation by equalizing the Alfvén speed in space as shown in Inoue et al. (2013). \( \rho \) is the mass density, \( \rho_0 \) is an initial value of the density where the density evolution described by Equation (2) follows from Aulanier et al. (2005, 2010), \( \mathbf{B} \) is the magnetic flux density, \( \mathbf{v} \) is the velocity, \( \mathbf{J} \) is the electric current density, and \( \phi \) is a scalar potential. The last equation (Equation (6)) is used to reduce deviation from \( \nabla \cdot \mathbf{B} = 0 \) and was introduced by Dedner et al. (2002).

The length, magnetic field, density, velocity, time, and electric current density are normalized by \( L_* = 216 \text{ Mm}, B_* = 2500 \text{ G}, \rho_* = |B^*|, V_{A*}^2 = B^*/(\mu_0 \rho_*), J_*^2 = B^*/(\mu_0 L_*), \) respectively. The non-dimensional viscosity \( \nu \) is set as a constant \( (1.0 \times 10^{-3}) \), and the coefficients \( c_h^2, c_p^2 \) in Equation (6) also fix the constant values, 0.04 and 0.1, respectively. The subscript \( i \) of \( \eta \) corresponds to NLFFF or MHD. These equation sets, normalized values, and parameters are mostly identical to those in previous studies (Inoue et al. 2013), except for the density evolution described in Equation (2).

A formulation of \( \eta_{\text{NLFFF}} \) is given by

\[ \eta_{\text{NLFFF}} = 5\eta_0 + \eta_1 \frac{|\mathbf{J} \times \mathbf{B}| |\mathbf{v}|^2}{|\mathbf{B}|^2}, \quad (7) \]

which is slightly different from Inoue et al. (2013), where \( \eta_0 = 1.0 \times 10^{-2} \) and \( \eta_1 = 1.0 \times 10^{-3} \). The vector field shown in Figure 1(a) is preprocessed in accordance with Wiegelmann et al. (2006), and a potential field applied to an initial condition of the NLFFF calculation is extrapolated using the Green function method (Sakurai 1982). The observed tangential components of the magnetic field reside in the region enclosed by a dashed square, as shown in the left panel of Figure 1(a); the outside region is fixed by the potential field because we focused on the dynamics of the magnetic field in an early phase near the central area and constructed a field close to an equilibrium state numerically. Note that, according to the mathematical formula, this state mixing of the force-free field and the potential field cannot keep an equilibrium. Nevertheless, we employed this method in this study in order to avoid an effect from inconsistent force-free \( \alpha \) residing outside areas where the weak magnetic fields are dominant.

2.3. Numerical Method of the MHD Simulation

Next, we perform MHD simulation under the zero \( \beta \) approximation (Mikic et al. 1989 or Amari et al. 1996) due to the low-\( \beta \) (0.01–0.1) coronal plasma where the time evolution of density is according to Equation (1), which is similar to Amari et al. (1996), i.e., the equations employed by the MHD simulation are identical to that of the NLFFF extrapolation method in Inoue et al. (2014) or Inoue et al. (2013), except the velocity limit is not imposed and the resistivity formulation is different.

The initial condition on the density is given by \( \rho_0 = |\mathbf{B}|_{t=0} \) in all MHD (also NLFFF) calculations. In this case, the density is modeled by Equation (1); therefore, we cannot compare it with observational data exactly or discuss the timescale of the dynamics of the magnetic field, as the Alfvén timescale depends on this model. On the other hand, this density profile might be more reasonable than the constant density profile, e.g., \( \rho = 1.0 \), because the real coronal density is stratified due to gravity, and the Alfvén wave propagates effectively in the upper corona where the magnetic field is driven. In addition to these, Inoue & Kusano (2006) showed the dynamics of a flux tube using two types of density. One is modeled by \( \rho = 1.0 \) at all times, i.e., the density variation is not considered. Another is obtained from a continuous Equation (2). Consequently, both dynamics of the magnetic field are nearly identical at the early eruptive phase. We will discuss this again later in this paper. A resistivity formula is given as an anomalous resistivity as follows:

\[ \eta_{\text{MHD}} = \begin{cases} \eta_0, & J < j_e, \\ \eta_0 + \eta_2 \left( \frac{J - j_e}{j_e} \right)^2, & J > j_e. \end{cases} \quad (8) \]

where \( \eta_2 = 5.0 \times 10^{-4} \) and \( j_e \) is the threshold current, set to 30 in this study.

In this simulation, a numerical box with dimensions of \( 216 \times 216 \times 216 \text{ Mm}^3 \) is given by \( 1 \times 1 \times 1 \) as a non-dimensional value. The grid number is assigned as \( 300 \times 300 \times 300 \), which is \( 2 \times 2 \) binning from the original data. Regarding the boundary conditions of NLFFF (Run A) and MHD relaxation processes (Run C), the physical values of all boundaries are fixed (fixed
boundary condition), except for the Neumann-type boundary condition, on which the convenient potential $\phi$ is imposed. On the other hand, in MHD simulations (Run B, and Runs D–F), tangential components of the magnetic field are released on all of the boundaries (released boundary conditions). The numerical scheme is quite identical to that in Inoue et al. (2014) or Inoue et al. (2013). The kind of simulation (NLFFF or MHD), equation of the density evolution, initial conditions, and boundary conditions used in each run are summarized in Table 1.

3. RESULTS

3.1. Results of the Nonlinear Force-free Extrapolation

3.1.1. 3D Magnetic Structure of the NLFFF

We first show the 3D magnetic structure of the NLFFF and compare it with EUV images taken by SDO/AIA. We call this NLFFF calculation Run A, and all of the runs in this study are summarized in Table 1. The upper panels in Figure 1(a) show the vector field map obtained from HMI and EUV images with 94 Å and 171 Å, respectively. The 3D field lines are plotted over each image shown in the middle panels in Figure 1(b). In the lower corona, we found that the strongly sheared field lines are formed above the PIL located between the sunspots in the central area. On the other hand, although the large loops extending to the upper corona can roughly capture the corona loops in AIA images, these reconstructed field lines seem to be deviating from them as the altitude increases.

We perform further NLFFF reconstruction on the basis of the whole vector field, not partial reconstruction such as in Figure 1(b), in order to further explore the reason for this problem. Figure 1(c) shows the NLFFF reconstructed from the whole vector field, which does not have any noticeable difference from that in Figure 1(b). Consequently, the potential field and NLFFF do not work effectively well on the reconstruction of the magnetic field in the upper corona. It is already suggested by Inoue et al. (2014) that it might be difficult to reconstruct these underlying field lines exactly because they cannot keep a steady state since they are further expanding to the upper area pointed out by Magara (2011).

3.1.2. Physical Properties of the NLFFF

Next, we show the physical properties of the NLFFF, focusing in particular on how much the reconstructed field is in a force-free state, its stability, and how much magnetic twist is accumulated in it. Figure 2(a) shows a distribution of the force-free $\alpha$ where the vertical and horizontal axes represent the values of the force-free $\alpha$ estimated on opposite footpoints of each field line that are calculated within the dashed square in Figure 1(a). Like Inoue et al. (2013), these force-free $\alpha$ values are estimated at 720 km approximately, i.e., 1 grid size in this case above the photosphere, and the field lines are traced from the region where $B_z < -250$ G. This result seems to show slightly larger scatter than that in our previous result (Inoue et al. 2013; see Figure 2(a)) even though many points are aligned close to the $y = x$ line plotted in green. This might be due to the fine force-free $\alpha$ compared to the previous one residing in the vector field reduced by an $8 \times 8$ binning process.

We perform MHD simulations using a reconstructed NLFFF as an initial condition to find a state in the details of the NLFFF where the tangential components of the magnetic field are released on the bottom boundary, employing constant resistivity. These calculations are designated as Run B. Figure 2(b) shows the temporal evolution of the kinetic energy $\int (\rho |v|^2/2) dv$, where $dv$ is a volume element) for Run B. We first observe an increase of the kinetic energy because the NLFFF cannot meet an equilibrium state exactly in addition to the configuration further deviating from the NLFFF during the simulation due to the release boundary condition at the bottom surface. On the other hand, after that, this profile decreases as time goes on, which suggests that the reconstructed field moves back to the potential field even on the disturbances derived from the velocity. Two insets in Figure 2(b) represent the 3D view of the field lines at $t = 0$ and $t = 15$, marked by the circles. We clearly see that the twisted field lines at $t = 15$ become looser than the initial state $t = 0$, i.e., the NLFFF is in a stable state and never shows the drastic dynamics as seen in the observations.

We also investigate the magnetic twist that is defined here by

$$T_n = \frac{1}{4\pi} \int \alpha dl,$$

where $\alpha$ and $dl$ are the force-free $\alpha = J \cdot B / |B|^2$ and a line element of a field line, respectively. We trace the field lines starting from the photospheric region with $|B_z| > 0.2$ (500 G) within the dashed square in the left panel of Figure 1(a), in order to focus on the twisted lines in the strong magnetic field. Figure 2(c) shows the 3D field lines over the $B_z$ distribution. Orange lines represent the strongly twisted lines over the half-turn twist that are traced inside of the red contours ($T_n = 0.5$) shown in Figure 2(d). Figure 2(e) represents the twist distribution on the positive polarity. This result indicates that most of the twists have a value of less than one turn, suggesting that they are stable against the kink mode instability. These results are quite similar to our previous result in Inoue et al. (2013) even though some parts of the NLFFF model are different.

3.2. Formation of the Strongly Twisted Lines in the NLFFF

From the above section showing the physical properties of the NLFFF, we found that the NLFFF never shows dramatic dynamics or eruption away from the solar surface, which is contrary to the observation (Schrijver et al. 2011). Therefore,

| Run   | Type                  | Density             | Resistivity       | Initial Condition | Boundary Condition |
|-------|-----------------------|---------------------|-------------------|-------------------|-------------------|
| Run A | NLFFF                 | Equation (1)        | Equation (7)      | Potential field   | Fix               |
| Run B | MHD simulation        | Equation (1)        | Constant          | NLFFF             | Release           |
| Run C | MHD relaxation        | Equation (1)        | Anomalous         | NLFFF             | Fix               |
| Run D | MHD simulation        | Equation (1)        | Anomalous         | $t = 1$ in Run C  | Release           |
| Run E | MHD simulation        | Equation (2)        | Anomalous         | $t = 1$ in Run C  | Release           |
| Run F | MHD simulation        | Equation (1)        | Constant          | $t = 1$ in Run C  | Release           |
there might be a mechanism that excites the stable NLFFF into a dynamically unstable phase, causing the observed eruptive phenomena. Thus, we perform the MHD relaxation process using the NLFFF as an initial condition, which is similar to the NLFFF calculation in Run A, except a velocity limit is released and an anomalous resistivity described in Equation (8) is set in this calculation, in order to form further strongly twisted lines with more than a one-turn twist. Because the anomalous resistivity plays a role in enhancing the reconnection in the strong current region, we can expect it to generate through reconnection more strongly twisted lines than that formed in the NLFFF. Because the magnetic field is fixed at the bottom boundary during this calculation, more strongly twisted lines are formed without changing the original horizontal components on the vector field, i.e., the map of the observed force-free $\alpha$ on the vector field is not changed. We call this calculation Run C.

Figure 3 shows the temporal evolution of the kinetic energy for Run C. We clearly see that the kinetic energy starts to accelerate after $t = 0.5$ and then increases as time goes on. The result in Run C indicates that the solution is deviating from the NLFFF, suggesting that it converts into the dynamic phase through the reconnection due to the anomalous resistivity.

Figure 3 shows the temporal evolution of the kinetic energy for Run C. The vertical dashed line indicates $t = 1$, at which the magnetic field is set up as an initial condition in Run D.

In order to confirm this suggestion that we note in the previous section, we observe the 3D magnetic structure and estimate the twist of the magnetic field lines at $t = 1$, marked by the vertical dashed line in Figure 3, which is just beginning to convert into the dynamic phase. Figures 4(a) and (b) show the 3D view of the magnetic field at $t = 0$, i.e., NLFFF and $t = 1$. The twisted lines in orange at $t = 1$ look slightly different from the initial NLFFF state, even though the overlying field lines surrounding these twisted lines almost keep their original configurations during this period in Run C. We further see the distribution of the
magnetic twist and 3D magnetic structure in Figures 4(c) and (d) to understand the more detailed magnetic structure close to the PIL in the central area. Figure 4(c) shows that most of the twist values in NLFFF are less than one turn as discussed in the above section; on the other hand, the strongly twisted lines with more than one turn are generated at \( t = 1 \) as shown in Figure 4(d), indicating the locations of their footpoints marked by dashed circles in the upper panel, and showing the field lines traced from there in the lower panel. Interestingly, the locations of these footpoints are close to those of the sunquake detected by helioseismology (Zharkov et al. 2011), which was triggered at the footpoint of the erupting flux rope at an early flare phase. In addition, the change of the field line topology from \( t = 0 \) to \( t = 1 \) is reminiscent of the tether-cutting reconnection (Liu et al. 2012; Wang et al. 2012; Liu et al. 2013).

Therefore, this result might suggest that the tether-cutting reconnection plays an important role in breaking the equilibrium state before the flare.

3.3. MHD Simulation of Eruptive Magnetic Fields

3.3.1. 3D Dynamics of Eruptive Magnetic Fields

We suggested in the above section that the magnetic field with a strongly twisted line over one turn can break the equilibrium condition and convert the NLFFF into the dynamic phase. However, its time evolution is not consistent in that simulation

because the three components of the magnetic field are fixed on the boundaries, that is clearly inconsistent with the boundary condition to Equation (4), meaning that the dynamics is not shown properly. In this section, we therefore perform the MHD simulation using the magnetic field shown in Figure 4(b), which is breaking the equilibrium state, as an initial condition where the tangential components are released on all the boundaries to discuss its time evolution. This calculation is called Run D.

We first show an overview of the 3D dynamics of the magnetic field lines for Run D with vertical velocity \( (v_z) \) in Figure 5. This result shows that the twisted field lines with more than a half-turn twist, being formed at \( t = 1 \) in Figure 4(b) (in orange), erupt away from the solar surface even though the tangential components of the magnetic field are released on all the boundaries. Because of the release boundary condition, the tangential components are deviating from that of the original vector field over time, which is relaxing toward them close to the potential field. The NLFFF obtained from Run A never shows these dynamics; this result thus indicates that the strongly twisted lines over the one-turn twist formed in the NLFFF are required in this AR to break the quasi-equilibrium state.

3.3.2. Comparison with Observational Data

We compare these simulation results with observational data to confirm the reliability of our simulation. Figure 6(a) shows the EUV images in 171 Å taken by SDO/AIA. Figure 6(b) plots the contour of \(|B_z| = 0.25\) over the AIA images. EUV is strongly enhanced at the PIL between the sunspots located in the central area; on the other hand, we can also observe another enhanced location, marked by a red circle, away from the central area. This location corresponds to the edge of the negative polarity on the east side in Figure 6(b). From Figure 5, we can see that
one footpoint of the twisted field lines also jumps from the central area to the edge of the negative polarity after \(t = 2.5\), as suggested from Figure 6. Therefore, these observational results might support our simulation.

Next we compare our simulation results with the Ca \(\text{ii}\) image taken by \textit{Hinode}. We focus on investigating the reconnection of only the twisted lines with more than one-third turn \((T_n = 0.3)\) because Inoue et al. (2011, 2012, 2013) suggested that these twisted lines close to the PIL are related to the two-ribbon flares where the Ca \(\text{ii}\) image is strongly enhanced. Furthermore, in this study we focus on the early eruptive phase as shown in Figure 7(a) because in the late phase the tangential components of the magnetic field on the bottom boundary are deviating gradually from the observational one due to the released boundary condition.

Following Toriumi et al. (2013), the reconnected field lines are estimated using the spatial variance of the field line connectivity, which is allowed only by the magnetic reconnection. The formulation is defined by the following equation:

\[
\delta(x_0, t_n) = |x_1(x_0, t_{n+1}) - x_1(x_0, t_n)| \quad (|T_n| \geq 0.3),
\]

where \(x_1(x_0, t_n)\) is a location of one footpoint of each field line at time \(t_n\), which is traced from another footpoint at \(x_0\). Eventually, we calculate

\[
\Delta(x_0, t) = \int_0^t \delta(x_0, t_n)dt_n,
\]

where the enhanced region in \(\Delta(x_0, t)\) indicates memories in which the reconnection took place dramatically in the twisted lines. The left panel in Figure 7(b) exhibits the Ca \(\text{ii}\) image taken by \textit{Hinode}/SOT at the early flare phase, and the middle and right panels show the distribution of \(\Delta(x_0)\) at \(t = 2.0\) and \(t = 4.0\), respectively. Interestingly, the distribution of \(\Delta(x_0)\) is similar to that of the observed two-ribbon flares seen in the left panel of Figure 7(b). This result clearly shows that the two-ribbon flare is enhanced due to the reconnection of twisted lines, which is consistent with classical flare models (see Shibata & Magara 2011).

Finally, we show the field line structure after the flare. Figure 8(a) shows the field lines over the flare ribbons obtained from the MHD simulation from which all of the field lines are traced. Our simulation shows the post-flare loops on the flare ribbons as often seen in observed EUV or Ca \(\text{ii}\) images. We compare the simulation results with the EUV image with 94 Å taken by \textit{SDO}/AIA shown in Figure 8(b). Before showing the
4. DISCUSSION

4.1. Formation of the Coronal Mass Ejection

In the previous section, the MHD simulation showed that the twisted lines initially embedded in the lower corona successfully erupt from the solar surface. However, it is still unclear whether these twisted lines grow to become coronal mass ejection (CME) or not. In order to clarify this, we discuss the decay index profile related to the torus instability (Kliem & Török 2006), which is an important factor in determining whether the flux tube can escape from the lower corona or not. The decay index refers to an estimation of a criterion for torus instability and is given by the following equation:

\[ n(z) = -\frac{z}{B} \frac{\partial |B|}{\partial z}, \tag{11} \]

where the threshold is at approximately \( n_c = 1.5 \) (Török & Kliem 2007). Because it is difficult to separate an external field from the NLFFF, following Fan & Gibson (2007) or Aulanier et al. (2010), the decay index on the potential field is plotted in Figure 9(a), marked by a cross in the inset, where the strong twisted lines of the NLFFF are embedded. From this result, the threshold is located at approximately \( h = 0.2 \), corresponding to 43.2 Mm. Figure 9(b) shows the time evolution of selected twisted field lines with more than a half turn where at time \( t = 0.0 \) the twisted line exists in the lower area corresponding to less than \( h = 0.2 \) at the top of the box. This indicates that the twisted line is stable against torus instability. On the other hand, as time goes on, these cross over the critical height as a result of the initial eruption. This result might give an important scenario related to the dynamics of the flare CME.

We present a more detailed temporal evolution of the same selected twisted lines in Figure 9(c), projecting them at each time on a 2D plane with a red line tracing the temporal evolution of the loop top. The dashed line indicates the critical height of the torus instability. Figure 9(d) shows the time variation of the loop top, i.e., its ascending velocity upon appearance, with the vertical dashed line where the twisted line is passing marking the critical height. This result does not exhibit the sudden acceleration of the twisted line in this study, as shown in Aulanier et al. (2010) and Fan (2010), even though the top of the twisted line exceeds the critical height and its velocity increases slowly after time \( t = 5.0 \).

We point out two possible causes of this problem. One is that the whole box is not large enough to trace the twisted lines for a long time. In addition, we set the rigid wall, which is not able to expel the overlying field lines to the outside of the numerical domain, at all boundaries; consequently, the magnetic pressure accumulates close to the boundary because the twisted lines keep pushing the overlying field lines. Another is due to the bottom boundary condition, i.e., due to the release of the tangential components. Torus instability is driven by the poloidal field of the flux tube generated by the toroidal current in it. However, the release condition for the tangential components relaxes the twisted lines, which means that the hoop force driving the twisted lines upward is loosening gradually. These two factors preventing the twisted lines from accelerating might work effectively on their dynamics. In other words, these twisted lines might have the potential for a solar eruption if
settling these problems on the size of the numerical box and the boundary condition. Therefore, for future works, it is important to understand how to bring the three observed components of the magnetic field as a boundary condition into an induction equation, i.e., we would require a data-driven simulation as presented in Cheung & DeRosa (2012).

4.2. Depending on the Density Profile

We check the dynamics of the density formula dependence by replacing Equation (1) by Equation (2) where \( \xi \) is given by \( 1.0 \times 10^{-4} \). This calculation is called Run E. Figure 10(a) shows the temporal evolution of the kinetic energy for Run D and Run E, respectively. Both of them have a lot of similar profiles, except we can see a little bit of deviation at the late phase. On the other hand, Figure 10(b) shows the temporal evolution of the iteration number, which shows that Run E requires much more calculation time than Run D. The 3D field line structures marked by each circle in Figure 10(a) are shown in Figures 10(c) and (d). As a consequence, we conclude that the dynamics is almost the same between them, but the cost performance of Run D is much better than that of Run E, as shown in Inoue & Kusano (2006). This conclusion is also in agreement with the previous work by Amari et al. (1996) and their series of papers.

4.3. Depending on the Resistivity Formula

We further perform an MHD calculation (Run F) replacing the anomalous resistivity with a constant one to also show the dynamics of the dependence on the resistivity formula. Figure 11(a) shows the temporal evolution of the maximum value of the current density \( |J|_{\text{max}} \) measured above 3600 km, which corresponds to five grids above the photosphere, and (b) kinetic energy for Run D and Run F in red and blue, respectively. The 3D field line structures with 2D \( |J| \) map at \( t = 5.0 \) are plotted in (c) Run D and (d) Run F. Field line format is the same as in Figure 5. (A color version of this figure is available in the online journal.)

Figure 11. Temporal evolution of the (a) maximum current density (\( |J|_{\text{max}} \)) measured above 3600 km, which corresponds to five grids above the photosphere, and (b) kinetic energy for Run D and Run F in red and blue, respectively. The 3D field line structures with 2D \( |J| \) map at \( t = 5.0 \) are plotted in (c) Run D and (d) Run F. Field line format is the same as in Figure 5. (A color version of this figure is available in the online journal.)

We performed an MHD simulation combined with the NLFFF and showed an overview of the dynamics of the magnetic field at the early eruptive phase of the X2.2-class flare, and we compared with observational results. The NLFFF is in a stable state against the kink mode instability (also confirmed by Inoue et al. 2013), and even any other perturbations as long as reconnection does not occur in the NLFFF. On the other hand, when the tether-cutting reconnection in the NLFFF could form the strongly twisted lines with more than one-turn twist, then we found that it plays an important role in breaking the quasi-steady state in the NLFFF. Consequently, the NLFFF is converted into the dynamic stage and the twisted lines can erupt away from the solar surface. This scenario is already suggested by previous studies analyzing of the NLFFF in AR 11158 by Wang et al. (2012), Liu et al. (2012, 2013), and Inoue et al. (2013). As a result of comparing with the observations, the distribution of the observed value of 30, it works by \( t = 3 \) approximately. Nevertheless, temporal evolutions of the kinetic energy for Run D and Run F shown in Figure 10(b) have quite similar profiles, although we can see a little bit of deviation between them. Furthermore, 3D field line structures and color maps of \( |J| \) for Run D and Run E shown in Figures 10(c) and (d) are almost the same, too, even at \( t = 5.0 \). From these results, in this case the anomalous resistivity does not work effectively on the dynamics while the twisted lines are ascending. Note that the reconnect is very important in the early phase to form the strongly twisted lines in the lower corona that launch from the solar surface. However, after that launching, the reconnection would take place only accompanied by the ascension of the twisted lines but not play an important role in the acceleration of the twisted line, as pointed out by Inoue & Kusano (2006).

5. SUMMARY

We performed an MHD simulation combined with the NLFFF and showed an overview of the dynamics of the magnetic field at the early eruptive phase of the X2.2-class flare, and we compared with observational results. The NLFFF is in a stable state against the kink mode instability (also confirmed by Inoue et al. 2013), and even any other perturbations as long as reconnection does not occur in the NLFFF. On the other hand, when the tether-cutting reconnection in the NLFFF could form the strongly twisted lines with more than one-turn twist, then we found that it plays an important role in breaking the quasi-steady state in the NLFFF. Consequently, the NLFFF is converted into the dynamic stage and the twisted lines can erupt away from the solar surface. This scenario is already suggested by previous studies analyzing of the NLFFF in AR 11158 by Wang et al. (2012), Liu et al. (2012, 2013), and Inoue et al. (2013). As a result of comparing with the observations, the distribution of the observed...
two-ribbon flare that formed in the early phase is explained well by the map on the spatial variance of the footpoints due to the reconnection of the twisted lines. In addition to this, the post-flare loops seen in the AIA image are captured by the field lines after reconnection in the MHD simulation. Thus, these results could support the reliability of our simulation results.

However, some detailed dynamics of the twisted lines have not yet been revealed. One is how much the flux tube needs the magnetic twist just before launching from the solar surface in order to reach the threshold height of the torus instability. Although only one case (Run D) was performed and presented in this study, we have to investigate the dynamics of other twisted lines having initially different twist values, which is an extension of this study. Then we can expect to find the critical twist leading to the torus instability. Another is to reveal the complicated magnetic reconnection while the twisted line is ascending. A detailed analysis extending from this paper will definitely answer this question.

On the other hand, our simulation left some problems. Although the twisted lines cross over the threshold height of the torus instability, we could not see the sudden acceleration shown in previous theoretical studies done by, e.g., Török & Kliem (2007), Aulanier et al. (2010), and Fan (2010). In order to reproduce the CME, we should extend the size of the numerical domain, modify or develop an advanced boundary condition such as an induction equation that could accept the three observed components of the magnetic field as the boundary condition such as would be required of a data-driven simulation (Cheung & DeRosa 2012).

In addition, in this study the triggering reconnection to produce the twisted flux tube is induced by the anomalous resistivity. However, in general, photospheric motion or a new emerging flux drives preexisting strongly twisted lines into the dynamic phase (e.g., van Ballegooijen & Martens 1989; Feynman & Martin 1995). Therefore, in the future, we must develop a simulation considering this observational information. Doing so will allow further physical insight into the onset and dynamics of flare CME.

We are grateful to anonymous referees for helping us improve and polish this paper. S.I. was supported by the International Scholarship of Kyung Hee University. This work was supported by the Korea Astronomy and Space Science Institute under the R&D program (project No. 2013-1-600-01) supervised by the Ministry of Science, ICT, and Future Planning of the Republic of Korea. G.S.C. was supported by the National Research Foundation grant NRF-2010-0025403. The computational work was carried out within the computational joint research program at the Solar-Terrestrial Environment Laboratory, Nagoya University. Computer simulation was performed on the Fujitsu PRIMERGY CX250 system of the Information Technology Center, Nagoya University. Data analysis and visualization are performed using resources of the OneSpaceNet in the NICT Science Cloud. We are sincerely grateful to NASA/SDO and the HMI and AIA science team. Hinode is a Japanese mission developed and launched by ISAS/JAXA, with NAOJ as domestic partner and NASA and STFC (UK) as international partners. It is operated by these agencies in cooperation with ESA an NSC (Norway).

REFERENCES

Aschwanden, M., Sun, X., & Liu, Y. 2014, ApJ, 785, 34
Amari, T., Luciani, J. F., Aly, J. J., & Tagger, M. 1996, ApJL, 466, L39
Asai, A., Ishii, T. T., Isobe, H., et al. 2012, ApJL, 745, L18
Aulanier, G., Démoulin, P., & Grappin, R. 2005, A&A, 430, 1067
Aulanier, G., Török, T., Démoulin, P., & DeLuca, E. E. 2010, ApJ, 708, 314
Borrero, J. M., Tomczyk, S., Kubo, M., et al. 2011, SoPh, 273, 267
Cheung, M. C. M., & DeRosa, M. L. 2012, ApJ, 757, 147
Dedner, A., Kemm, F., Kröner, D., et al. 2002, JCoPh, 175, 645
Fan, Y. 2010, ApJ, 719, 728
Fan, Y., & Gibson, S. E. 2007, ApJ, 668, 1232
Feynman, J., & Martin, S. F. 1995, JGR, 100, 3355
Hoeksema, J. T., Liu, Y., Hayashi, K., et al. 2014, SoPh, 57
Inoue, S., Hayashi, K., Shiota, D., Magara, T., & Choe, G. S. 2013, ApJ, 770, 79
Inoue, S., & Kusano, K. 2006, ApJ, 645, 742
Inoue, S., Kusano, K., Magara, T., Shiota, D., & Yamamoto, T. T. 2011, ApJ, 738, 161
Inoue, S., Magara, T., Pandey, V. S., et al. 2014, ApJ, 780, 101
Inoue, S., Shiota, D., Yamamoto, T. T., et al. 2012, ApJ, 760, 17
Jiang, C., & Feng, X. 2013, ApJ, 769, 144
Jiang, Y., Zheng, R., Yang, J., et al. 2012, ApJ, 744, 50
Jing, J., Park, S.-H., Liu, C., et al. 2012, ApJL, 752, L9
Kliem, B., & Török, T. 2006, PhRvL, 96, 255002
Kosovichev, A. G. 2011, ApJL, 734, L15
Kosugi, T., Matsuzaki, K., Sakao, T., et al. 2007, SoPh, 243, 3
Leka, K. D., Barnes, G., Crouch, A. D., et al. 2009, SoPh, 260, 83
Lemen, J. R., Title, A. M., Akin, D. J., et al. 2012, SoPh, 275, 17
Liu, C., Deng, N., Lee, J., et al. 2013, ApJL, 778, L36
Liu, C., Deng, N., Liu, R., et al. 2012, ApJL, 745, L4
Magara, T. 2011, ApJ, 731, 122
Metcalf, T. R. 1994, SoPh, 155, 235
Metcalf, T. R., Leka, K. D., Barnes, G., et al. 2006, SoPh, 237, 267
Mikic, Z., Schnack, D. D., & van Hoven, G. 1989, ApJ, 338, 1148
Moore, R. L., Sterling, A. C., Hudson, H. S., & Lemen, J. R. 2001, ApJ, 552, 833
Nindos, A., Petsouros, S., & Wiegelmann, T. 2012, ApJL, 748, L6
Sakurai, T. 1982, SoPh, 76, 301
Scherrer, P. H., Schou, J., Bush, R. I., et al. 2012, SoPh, 275, 207
Schou, J., Scherrer, P. H., Bush, R. I., et al. 2012, SoPh, 275, 229
Schrijver, C. J., Aulanier, G., Title, A. M., Pariat, E., & Delannée, C. 2011, ApJ, 738, 167
Shibata, K., & Magara, T. 2011, LRSP, 8, 6
Shibata, K., & Magara, T. 2011, LRSP, 8, 6
Toriumi, S., Iida, Y., Bamba, Y., et al. 2013, ApJL, 773, 128
Török, T., & Kliem, B. 2007, AN, 328, 743
Tsuneta, S., Ichimoto, K., Matsuzaki, K., et al. 2008, SoPh, 249, 167
Tziotziou, K., Georgoulis, M. K., & Liu, Y. 2013, ApJ, 772, 115
van Ballegooijen, A. A., & Martens, P. C. H. 1989, ApJ, 343, 971
Wang, S., Liu, C., Liu, R., et al. 2012, ApJL, 745, L17
Wiegelmann, T., Inhester, B., & Sakurai, T. 2006, SoPh, 233, 215
Wiegelmann, T., & Sakurai, T. 2012, LRSP, 9, 5
Wiegelmann, T., Thalmann, J. K., Inhester, B., et al. 2012, SoPh, 281, 37
Zharkova, V. V., Green, L. M., Matthews, S. A., & Zharkova, V. V. 2011, ApJL, 741, L35