Parametrically enhanced bandpass filters

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Funding information
Defense Advanced Research Projects Agency, Grant/Award Number: SPAR Program

Abstract
This paper proposes a technique to enhance the quality factor (Q) of a resonator, which can then be used to realise bandpass filters (BPFs) with reduced insertion loss (IL) and improved passband selectivity. The main idea behind this technique is to introduce a negative resistance to a resonator, through the parametric amplification effect, to compensate losses from non-ideal passive components (e.g. capacitors, inductors, and microstrips). Two bandpass filters enhanced by such resonators are designed and realised on printed-circuit boards (PCBs) to demonstrate the characteristics of low insertion loss and good passband selectivity. In addition, these prototype parametrically enhanced filters offer reasonably good linearity and similar noise figures (NFs) to their corresponding pure passive counterparts.

1 | INTRODUCTION

Various mobile communication systems have been rapidly adopted in recent years for diverse applications. Consequently, full and effective use of frequency resources is critical. One prospective approach to address this challenge is to employ narrowband and highly selective filters. However, these type of filters usually come with a significant drawback, namely, large insertion loss (IL) especially when implemented in miniaturised format.

Consider a typical fifth-order coupled-resonator bandpass filter (BPF) of 2.5% fractional bandwidth, with quality factors (Qs) of typical capacitors and microstrips being taken into account (Qc = 2900 @ 50 MHz and αm = 1.2 dB/m @ 1 GHz), the corresponding IL can be as much as 12 dB as shown in Figure 1. In order to reduce the IL, waveguide cavities or high-temperature superconductors (HTS) can be used to realise such type of filters. Another option is the use of piezoelectric materials, for example SAW/BAW filters. All of these approaches have their unique advantages and drawbacks.

A number of tunable filters [1–4] and active filtering techniques [5–11] have also been proposed where different combinations of both passive and active elements are employed in the filter design. The main objective is to keep the filtering characteristics controlled by the passive elements while using the active elements to tune the filter operation or neutralise all component losses. For active BPFs to be a useful alternative to their passive counterparts, their noise figures should be comparable to those of the competing passive filters. Moreover, they should also be “linear enough” for the applications of interest. Recently, a new loss compensation technique has been proposed [12] and successfully applied to realise a low-loss BPF with good noise figure and reasonable linearity. This particular technique utilises the negative resistance originated from the parametric amplification effect [13–19], which has been used in realizing non-magnetic non-reciprocal components [20,21], to compensate losses caused by passive components with finite quality factors, for example capacitors, inductors, and microstrips.

It is the purpose of this paper to provide a detailed study of, as well as to propose an improvement to, this technique. A quality factor enhanced (Q-enhanced) resonator operating at 2 GHz is first designed and implemented on a printed-circuit board (PCB). A fifth-order coupled-resonator BPF of 2.5% fractional bandwidth utilizing this resonator is then fabricated to demonstrate the reduced IL characteristic. Finally, a method to improve the passband selectivity (equivalently, the parametric enhancement bandwidth) is introduced and verified through a prototype third-order coupled-resonator BPF. Simulations and experimental results for both prototypes are reported to validate the proposed idea.

2 | THEORY

2.1 | Negative-resistance parametric amplification

Parametric amplification is a RF-to-RF conversion process, which operates by pumping a non-linear reactance with a large-signal RF source to either produce mixing products with gain
or to generate an equivalent negative resistance. For either case, Manley–Rowe relations [22] can be applied to predict the process behaviour. In this work, only the negative-resistance parametric amplification is of interest.

Assuming a three-wave mixing case where power flow in/out of a reactance only at three frequencies ($f_{\text{signal}}$, $f_{\text{idler}}$, and $f_{\text{pump}}$) is allowed (Figure 2), Manley–Rowe relations predict

\[
\frac{P_{\text{signal}}}{f_{\text{signal}}} + \frac{P_{\text{pump}}}{f_{\text{pump}}} = 0 \quad \text{(1a)}
\]

\[
\frac{P_{\text{idler}}}{f_{\text{idler}}} + \frac{P_{\text{pump}}}{f_{\text{pump}}} = 0 \quad \text{(1b)}
\]

where $f_{\text{idler}} = f_{\text{pump}} - f_{\text{signal}}$. These equations indicate that when power at the pump frequency ($f_{\text{pump}}$) is supplied ($P_{\text{pump}}$ is positive) to a non-linear reactance, $P_{\text{signal}}$ and $P_{\text{idler}}$ will be negative. That is, the reactance delivers power to the signal generator rather than absorbing it. Therefore, a power gain at the signal frequency ($f_{\text{signal}}$) is possible. Specifically, when large power at $f_{\text{pump}}$ causes a capacitance variation of

\[
C(t) = C_0 + 2\gamma C_0 \cos(\omega_{\text{pump}} t) \quad \text{(2)}
\]

\[
Y_{\text{var}} = j\omega_{\text{signal}} C_0 - \omega_{\text{signal}} \omega_{\text{idler}}^2 C_0^2 \left( \frac{1}{Y_{\text{idler}}} - \frac{1}{\omega_{\text{idler}} C_0} \right) \quad \text{(3)}
\]

where $Y_{\text{idler}}$ is the admittance seen by the non-linear capacitor at the idler frequency ($f_{\text{idler}}$) (see Appendix for more details).
The second term in Equation (3) is equivalent to a negative conductance \( G \) when \( Y_{\text{idler}} \) is made resonant with \( C_0 \) at \( f_{\text{idler}} \), or equivalently, \( Y_{\text{idler}} = \frac{\omega_{\text{idler}} C_0}{G_i} = G_i \) where \( G_i \) accounts for the resonator loss at \( f_{\text{idler}} \). In other words, the pumped non-linear capacitor acts like a normal capacitor in parallel with a negative resistor at \( f_{\text{signal}} \). Power at \( f_{\text{idler}} \) is not utilised in this work, however it must exist in order to satisfy the Manley–Rowe relations given in Equation (1).

2.2 Q-enhanced resonators

As discussed in the previous section, a pumped non-linear capacitor has its admittance equivalent to a capacitor in parallel to a negative resistor at the signal frequency. Therefore, it can be used to construct “lossless” resonators (e.g. LC tanks) even though they are composed of non-ideal lossy components. However, it is necessary to mention that these reduced-loss resonators (or Q-enhanced resonators) should resonate at both signal and idler frequencies. Before any discussion on their implementations, the power gain, bandwidth, and noise properties of Q-enhanced resonators should first be investigated.

Figure 3(a) depicts a Q-enhanced parallel LC resonator at the signal frequency. Since there is now a negative resistor in parallel with the LC tank, it is expected that the return loss can be reduced to zero (complete reflection) even though the inductor and capacitor are lossy. Specifically, the magnitude of \( S_{11} \) in this case can be derived from Equation (3) as

\[
[S_{11}]_{\text{db}} = 20 \log_{10} \left| 1 + \gamma^2 Q_i Q_s - \frac{2G_s}{(G_s + Y_0)} \right| \quad (4a)
\]

\[
Q_s = \frac{\omega_{\text{signal}} C_0}{Y_0 + G_s} \quad (4b)
\]

\[
Q_i = \frac{\omega_{\text{idler}} C_0}{G_i} \quad (4c)
\]

where \( Q_s \) and \( Q_i \) are the loaded and unloaded resonator quality factors at signal and idler frequencies, respectively. The term \( G_s \) accounts for the resonator losses at signal frequency. It can be seen from Equation (4) that the magnitude is always less than 0 dB when there is no pumping (\( \gamma^2 Q_i Q_s = 0 \)) since \( 0 < G_s/(G_s + Y_0) < 1 \). However, it becomes 0 dB by properly choosing the value of \( \gamma^2 Q_i Q_s \) through pumping (e.g. \( \sim 0.09 \) if \( Y_0 = 10G_s \)). In other words, the resonator loss is completely mitigated. It is important to realise that a Q-enhanced resonator is not equivalent to a true lossless resonator. Firstly, the lossless characteristic of a Q-enhanced resonator only occurs at the signal frequency. Secondly, its noise figure at signal frequency is always greater than 0 dB even without loss.

For the two-port configuration of the proposed resonator as shown in Figure 3(b), its available power gain and noise figure can be expressed as

\[
g_a = \frac{Y_0}{Y_0 + G_s} \left( 1 - \frac{1}{1 - \gamma^2 Q_i Q_s} \right) \quad (5a)
\]

\[
F = \left( 1 + \frac{G_s}{Y_0} \right) \left( 1 + \frac{\omega_{\text{signal}}}{\omega_{\text{idler}}} \gamma^2 Q_i Q_s \right) \quad (5b)
\]

where \( Q_s \) and \( Q_i \) are defined in Equations (4b) and (4c). Derivations for these expressions are given in the Appendix.

Notice that a few insights can be obtained from Equation (5). Firstly, the gain is always less than unity if there is no pumping power as \( G_s \) is always greater than zero in practice. However, it can be made unity by choosing a suitable value of \( \gamma^2 Q_i Q_s \). In other words, the signal power is amplified by a factor of \( g = 1/(1 - \gamma^2 Q_i Q_s)^2 \) as compared with the no pumping case. Secondly, the noise figure not only depends on the resonator loss at the signal frequency but also on the gain, or equivalently, pumping power. The larger the pumping power, the greater the noise figure. An approximate optimal solution for a reasonable gain and a not too large noise figure is given by
\[ r^2 Q_i Q_s \approx 1 - \frac{\omega_{\text{signal}}}{\omega_{\text{idler}}} \]  

(6)

As an example, if \( f_{\text{signal}}/f_{\text{idler}} \) is 0.57, a maximum of approximately 5 dB resonator loss is possible to be recovered without introducing too much noise.

Generally, under the high-gain condition, the bandwidth of negative-resistance parametric amplification depends mainly on the transformed unloaded quality factor at idler frequency with a transformation ratio of \( f_{\text{signal}}/f_{\text{idler}} \). Mathematically, it is given by (see Appendix)

\[ (g - 2)b^2 \approx \frac{1}{\left(\frac{\omega_{\text{signal}}}{\omega_{\text{idler}}}\right)^2 Q_i^2} \]  

(7)

where \( b \) is the fractional bandwidth and \( g \) is the normalised transducer gain. Notice that, for the high-gain condition to be valid, \( g \) should be at least greater than 2. Hence, it is expected that the amplification bandwidth will be wider for resonators with lower quality factor at the idler frequency. Moreover, the bandwidth decreases when the normalised gain increases.

After the study of some basic characteristics of Q-enhanced resonators, it is time to discuss about their implementations. The highlighted portion of Figure 3(c) depicted one possible configuration of such Q-enhanced resonators. It consists of a transmission line of impedance \( Z_2 \) and a varactor-loaded transmission line of impedance \( Z_1 \). The key principles behind this resonator are as follows: (1) \( Z_2 \)-line acts as a quarter-wave resonator at \( f_{\text{idler}} \); (2) \( Z_1 \)-line acts as a half-wave resonator at \( f_{\text{idler}} \); and (3) \( Z_2 \)-line (capacitive) and loaded \( Z_1 \)-line (inductive) together form a parallel tank that resonates at \( f_{\text{signal}} \). With these three conditions and the input admittances given in Figure 3(d), the corresponding design equations are given as follows

\[ \theta_2(\omega_{\text{idler}}) = \frac{\pi}{2} \]  

(8a)

\[ \theta_1(\omega_{\text{idler}}) = \tan^{-1}\left(\frac{2}{\omega_{\text{idler}} C_0 Z_1}\right) \]  

(8b)

and the line impedance \( Z_2 \) can be obtained by

\[ \frac{Z_2}{Z_1} = \frac{2 \tan \theta_1(\omega_{\text{signal}}) - \omega_{\text{signal}} C_0 Z_1 \tan^2 \theta_1(\omega_{\text{signal}})}{1 - \tan^2 \theta_1(\omega_{\text{signal}}) - \omega_{\text{signal}} C_0 Z_1 \tan \theta_1(\omega_{\text{signal}})} \]  

(8c)

When a single port of reference impedance \( Z_0 \) is connected to the tapping point, this resonator acts like an open circuit at the signal frequency and a short circuit at the idler frequency. The corresponding simulated (using Keysight ADS) scattering parameter \( S_{\text{11}} \) is shown in Figure 4(a). In reality, both transmission lines and the varactor have losses,
and the resonator will therefore have return loss greater than 0 dB as depicted by the dotted lines in Figure 4(b).

When pumping the varactor with a suitable power at pump frequency (~17.4 dBm), the return loss can be reduced back to 0 dB at the signal frequency (2 GHz in this case). This loss compensated property has been predicted by Equation (4) where \(|S_{11}|_{dB}\) can be made equal to zero by choosing a proper value of \(r^2Q_1Q_2\).

Also mentioned previously is that the amplification bandwidth will be wider if the quality factor \(Q_1\) is made smaller. This relation can be roughly demonstrated by reducing the quality factor of the varactor to half of its original value (change to \(Q_c = 1450 @ 50\) MHz), the gain-bandwidth product is larger but greater pumping power (~21 dBm) is required to completely mitigate the IL. Figure 4(b) shows the corresponding amplification of \(|S_{11}|\) around the signal frequency, which has a slightly wider bandwidth than the original one \((Q_c = 2900 @ 50\) MHz) under the same zero return loss condition.

3 | PARAMETRICALLY ENHANCED BPFS

3.1 | Uniform enhancement case

With the discussion of Q-enhanced resonator properties in the previous section, BPFS with reduced IL, or equivalently, parametrically enhanced BPFS can now be realised. Specifically, a fifth-order coupled-resonator BPFS shown in Figure 3 (c) is one major focus of this paper. It consists of five Q-enhanced resonators that are coupled through six capacitors and can be designed by following the standard filter design procedure [23]. Circuit parameters obtained for a filter of 2.5% fractional bandwidth and operating at 2 GHz are \(Z_1 = 100\) \(\Omega\), \(\theta_1 = 8.2^\circ\), \(\theta_2 = 51.4^\circ\), \(C_0 = 1.8\) pF, and \(Z_2\)'s of the five resonators are, respectively, 66.6, 55.6, 55.1, 55.6, and 66.6 \(\Omega\). In addition, the values of the six coupling capacitors are 0.3731, 0.0759, 0.0579, 0.0579, 0.0759, and 0.3731 pF, respectively. Notice that the capacitance value \(C_0\)
is chosen based on the commercially available varactor SMV1409 from Skyworks. It has a capacitance of around 1.8 pF when in reverse biased at 4.5 Vdc.

Simulated scattering parameters are plotted in Figure 5(a). As seen from this figure, the filter has an IL of about 12.3 dB when assuming the varactors have a quality factor of 2900 at 50 MHz and the transmission lines have an attenuation constant of 1.2 dB/m at 1 GHz. On the other hand, when pumping all five resonators with 17.4 dBm power at 5.5 GHz, almost zero IL at the passband is resulted. Moreover, the five sharp poles of return loss roughly reappear again (similar to the lossless case in Figure 1(a)). The out-of-band rejection at 40 MHz away from the centre frequency of 2 GHz has improved from 25 dB to almost 40 dB. Since the idler frequency is 5.5–2 GHz = 3.5 GHz, a significant degradation of rejection at around this frequency can be observed (Figure 5(b)). However, it is still 80 dB below and should not be a concern. Notice that 5.5 GHz is chosen as the pump frequency because of the limitation on achievable value of $Z_2/Z_1$.

The analysis in the previous section is only valid when the signal power is much smaller than the pump power. This means that the signal does not cause any capacitance variation. Large signal power will affect the pumped filter responses and result in gain compression. In this particular design, the input P1dB is around 5.5 dBm (Figure 5(c)).

Besides having reduced IL, the pumped filter may have a smaller noise figure as compared with the same filter without pumping. This is because each resonator has a significant gain and the resulted total noise figure follows the well-known expression of

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots$$

This effect is obvious in the simulated results shown in Figure 5(d). Without pumping, the filter has a noise figure of around 12.3 dB. However, when it is pumped, the noise figure reduced to 7.6 dB, a total of 4.7 dB improvement.
3.2 | Selectivity enhancement case

Notice that the passband response shown in Figure 5(a) is relatively round in shape. In other words, near both edges of the passband, the resonator quality factor is not enhanced enough to completely recover the IL. This is because the Q-enhanced resonator used in this example is single-tuned at $f_{\text{filter}}$ and its resonant bandwidth is relatively narrow depending on the quality factor at this frequency.

In order to improve the bandwidth of the enhancement region, an obvious choice is to reduce the resonator quality factor at $f_{\text{filter}}$. However, this is undesired since extra noise will be introduced to the filter. A better alternative is to have the Q-enhanced resonator multiple-tuned at $f_{\text{filter}}$. One way to do this is by introducing additional coupling at $f_{\text{filter}}$ between adjacent resonators (see Figure 6(b)). The basic principle is to split the idler resonance frequency through this additional coupling. By properly choosing the coupling strength, the compensation bandwidth (equivalently, the passband selectivity) can be improved as shown in Figure 7. Notice that this approach requires a way to provide two distinct coupling values simultaneously, one at $f_{\text{signal}}$ and the other at $f_{\text{filter}}$.

4 | EXPERIMENTS

An individual Q-enhanced resonator operating at 2 GHz is first designed and fabricated on a two-layer PCB to verify the loss reduction property. Figures 8 and 9 depict its detailed schematic and layout. Here, the two transmission lines are implemented in microstrip format. In addition, the variable capacitor shown in Figure 3(c) is replaced by a diode ring so that the resonator can be pumped differentially to enhance isolation of the pumping source from the filter. The differential feed is a bandpass balun-filter to reject all frequencies except the pump frequency, which is 5.5 GHz in this case. Details on designing this balun-filter are well documented in existing literatures [24,25], and therefore, will not be discussed here. Since this is not a true double-balanced structure, the balun-filter does exhibit some loading effects to the resonator, and should be taken into consideration when designing the resonator.

Loading effects from the balun-filter and excess inductances introduced by the grounding vias are compensated by tuning the transmission-line impedances. All varactors are reversed biased with 4.5 Vdc so that their capacitance matches the desired capacitance of 1.8 pF. Measured results (see Figure 10) indicate that there is 0.25 dB IL at 2 GHz. On the contrary, it can be reduced to almost 0 dB by pumping 18 dBm power at 5.3 GHz to the resonator. The downshift of $f_{\text{pump}}$ is due to the downshift of $f_{\text{filter}}$, which is roughly 3.3 GHz for this prototype design.

With a successful implementation of the Q-enhanced resonator, the proposed parametrically enhanced fifth-order BPF shown in Figure 3(c) can now be realised. Here, five resonators are put in parallel to each other and coupled through T-type transmission-line inverters. Using this particular inverter type can facilitate the filter implementation on a single PCB layer. The final layout and a picture of the fabricated prototype are depicted in Figure 11.

Same as the single resonator case, all varactors are reversed bias with 4.5 Vdc. As seen from the measured results shown in Figure 12, the filter has 12.79 dB IL at 2 GHz. After pumping
each resonator with 18 dBm power at 5.3 GHz (a total pumping power of 26 dBm), the IL reduces to only 0.23 dB. Notice that there is a rejection degradation at around the idler frequency of 3.3 GHz. The measured P1dB is about 13 dBm, which is larger than the simulated value. Finally, the measured noise figure changed from 12.6 to 9.07 dB after pumping, a total of 3.5 dB improvement (Figure 13). This prototype is an example of the uniform enhancement case of which the passband is relatively round in shape.

Next, a third-order filter of 160-MHz bandwidth and operating at 2 GHz is fabricated to demonstrate the selectivity enhancement case. A transmission-line loaded with short-circuit stub is used as a J-inverter (see Figure 14(a)). The tapping position of the two resonator-to-resonator J-inverters have been properly tuned to provide the correct coupling at both $f_{\text{signal}}$ as well as $f_{\text{signal}}$. Experimental results depicted in Figure 14(b) indicate that there is a slight shift of 30 MHz to 2.03 GHz for the passband. The discrepancy between the actual diode and its circuit model is the main cause of this shift. Notice that all varactors are reverse biased with four Vdc. The IL is 3.3 dB at 2.03 GHz without pumping and is 0.22 dB when pumping 24 dBm power at 5.35 GHz to each resonator. This indicates that $f_{\text{signal}}$ of the three resonators is actually about 3.3 GHz (Figure 15).

The output third-order intermodulation product (OIP3) is measured to be 26.5 dBm, which is obtained from the calculation given in Figure 5(a). Furthermore, the measured output 1-dB power compress point (OP1dB) is 17 dBm. Finally, the noise figure is 3.7 dB in average over the passband (see Figure 5(b)). Hence, the parametric amplification process does not introduce too much noise in this case given that the original filter (no pump power) has an IL of 3.3 dB.

5 | CONCLUSION

A method to implement BPFs with reduced IL and improved passband selectivity has been proposed. It is based on the parametric amplification effect to create an equivalent negative resistance in a resonator to compensate its loss due to non-ideal passive components, or equivalently, to enhance its quality factor. A prototype consisting of a single resonator has been realised on a two-layer PCB to validate the proposed idea.
Experimental results show that the loss of this resonator can be reduced by pumping a suitable amount of power to its non-linear components. Two parametrically enhanced BPFs, one fifth-order with 2.5% fractional bandwidth and one third-order with 8% fractional bandwidth, utilizing this Q-enhanced resonator have also been designed and fabricated. Measured results indicate that both filters are lossless when pumped with a certain amount of power.

It is worth to make a comparison with other transistor-based active filters at this point. Table 1 below summarises the reported data of several active filters from other researchers, as well as the data obtained in this work, in terms of the operating frequency, fractional bandwidth, noise figure, power consumed, and the OIP3. It appears that parametrically enhanced filters can achieve a low noise figure comparable to the best transistor approaches. They offer a much higher OIP3 in general while consume slightly more power than the transistor approaches. As one of the most important roles of filters is to reject strong interferences, a high OIP3 can be crucial for some applications. In view of this, future research will focus on how to optimise the trade-off between power consumption, efficiency, and linearity. Generally, the proposed method is best

![Figure 15](https://via.placeholder.com/150)

**Figure 15** (a) Measured third-order IMD and (b) measured noise figure

| Ref.  | $f_c$ (GHz) | BW (%) | Order | NF (dB) | $P_{in}$ (dBm) | OIP3 (dBm) |
|-------|-------------|--------|-------|---------|----------------|------------|
| [6]   | 9.07        | 7.5    | 2     | –       | –              | 10         |
| [7]   | 1.9         | 2      | 3     | 6.2     | 3.7            | –          |
| [8]   | 1.8         | 2      | 2     | 6.8     | –              | 12         |
| [9]   | 1.88        | 5      | 2     | 2.4     | 14.8           | 8          |
| [10]  | 5.52        | 3.77   | 2     | 1.9     | 16.2           | –          |
| [11]  | 2.42        | 0.5    | 4     | 9.0     | 5.9            | –4         |
| This work | 2.0   | 2.5    | 5     | 9.1     | 18             | 22         |
| This work | 2.03  | 8      | 3     | 3.7     | 24             | 26.5       |

*Per resonator basis.
suit to enhanced filters with less than 10% fractional BW and more than 3 dB IL.

ACKNOWLEDGMENTS
This work is supported by DARPA SPAR program.

REFERENCES
1. Shikokawa, N., et al.: Ultra-narrowband HTS filter with 2.5-wavelength hairpin resonators in 7 GHz band. In: 2006 Asia-Pacific Microwave Conference Proceedings, pp. 789-792 (2006)
2. Rauscher, C.: Varactor-tuned active notch filter with low passband noise and signal distortion. IEEE Trans. Microwave Theory Tech. 49(8) 1431–1437 (2001)
3. Trabhsi, H., Cruehon, C.: A varactor-tuned active microwave bandpass filter. IEEE Microwave Guided Wave Lett. 2(6), 231–232 (1992)
4. Lockerbie, I., Kumar, S.: A broadband tunable composite filter using active devices. In: Proceedings on IEEE WESCANEX Conference, pp. 196–200 (1993)
5. Snyder, R.V., Jr., Bozarth, D.L.: Analysis and design of a microwave transistor active filter. IEEE Trans. Microwave Theory Tech. 18(1), 2–9 (1970)
6. Chang, C.Y., Itoh, T.: Microwave active filters based on coupled negative resistance method. IEEE Trans. Microwave Theory Tech. 38(9), 1879–1884 (1990)
7. Sabouret, S.: A GaAs MMIC active filter with low noise and high gain. In: IEEE MTT-S International Microwave Symposium Digest, Vol. 3, pp. 1177–1180 (1998)
8. Chang, C.Y., Itoh, T.: Microwave active filters based on coupled negative resistance method. IEEE Trans. Microwave Theory Tech. 38(9), 1879–1884 (1990)
9. Sabouret, S.: A GaAs MMIC active filter with low noise and high gain. In: IEEE MTT-S International Microwave Symposium Digest, Vol. 3, pp. 1177–1180 (1998)
10. Snyder, R.V., Jr., Bozarth, D.L.: Analysis and design of a microwave transistor active filter. IEEE Trans. Microwave Theory Tech. 18(1), 2–9 (1970)
11. Chang, C.Y., Itoh, T.: Microwave active filters based on coupled negative resistance method. IEEE Trans. Microwave Theory Tech. 38(9), 1879–1884 (1990)
12. Sabouret, S.: A GaAs MMIC active filter with low noise and high gain. In: IEEE MTT-S International Microwave Symposium Digest, Vol. 3, pp. 1177–1180 (1998)
13. Sabouret, S.: A GaAs MMIC active filter with low noise and high gain. In: IEEE MTT-S International Microwave Symposium Digest, Vol. 3, pp. 1177–1180 (1998)
14. Johannessens, P., Ku, W., Andersen, J.: Theory of nonlinear reactance amplifiers. IEEE Trans. Magn. 3(3), 376–380 (1967)
15. Hines, M.: The virtues of Nonlinearity–Detection, frequency conversion, parametric amplification and harmonic generation. IEEE Trans. Microwave Theory Tech. 32(9), 1097–1104 (1984)
16. Engelbrecht, R.: Parametric energy conversion by nonlinear admittances. Proc. IRE. 50(3), 312–321 (1962)
17. Smith, V., et al.: Low noise microwave parametric amplifier. IEEE Trans. Magn. 21(2), 1022–1028 (1985)
18. Smith, V., et al.: Low noise microwave parametric amplifier. IEEE Trans. Magn. 21(2), 1022–1028 (1985)
19. Heffner, H., Wade, G.: Gain, band width, and noise characteristics of the variable-parameter amplifier. J. Appl. Phys. 29(9), 1321–1331 (1958)
20. Estep, N.A., et al.: Magnetic-free non-reciprocity and isolation based on parametrically modulated coupled-resonator loops. Nat. Phys. 10(12), 923–927 (2014)
21. Qin, S., Xu, Q., Wang, Y.E.: Nonreciprocal components with distributely modulated capacitors. IEEE Trans. Microwave Theory Tech. 62(10), 2260–2272 (2014)
22. Blackwell, I., Kotzebue, K.: Semiconductor-dioide parametric amplifiers. Prentice-Hall Englewood (1961)
23. Mathaei, G.L., Young, L., Jones, E.M.T.: Microwave filters impedance matching networks and coupling structures. McGraw-Hill, New York (1980)
24. Yeung, L.K., Wu, K.L.: Dual band balanced-to-unbalanced bandpass filter. US Patent 7,541,888, 2 June 2009
25. Yeung, L.K., Wu, K.L.: A dual-band coupled-line balun filter. IEEE Trans. Microwave Theory Tech. 55(11), 2406-2411 (2007)

How to cite this article: Yeung LK, Zou X, Wang Y. Parametrically enhanced bandpass filters. IET Microw. Antennas Propag. 2021;15:229–240. https://doi.org/10.1049/mia2.12039

APPENDIX
The aim of this appendix is to provide some basic mathematical details to understand the negative-resistance parametric amplification effect. In-depth details can be found in Ref. [22]. The equivalent admittance given in Equation (3) can be obtained by considering the circuit shown in Figure A1(a). Here, a non-linear capacitor is connecting to other components through two hypothetical filters—one allows only the signal frequency \( f_{\text{signal}} \) passing whereas the other allows only the idler frequency \( f_{\text{idler}} \). In addition, its capacitance is made varying according to Equation (2) by a separate pumping source of frequency \( f_{\text{pump}} = f_{\text{signal}} + f_{\text{idler}} \). Notice that the term \( \frac{Y_{\text{vav}}}{\gamma} \) is not shown in the figure. With the assumption of pump power much greater than signal power, the pumped non-linear capacitor can be described by a small-signal admittance matrix such that

\[
\begin{pmatrix}
I_{\text{signal}} \\
I_{\text{idler}}
\end{pmatrix} =
\begin{pmatrix}
\omega_{\text{signal}} C_0 & \omega_{\text{signal}} C_0 \\
-\omega_{\text{idler}} C_0 & -\omega_{\text{idler}} C_0
\end{pmatrix}
\begin{pmatrix}
V_{\text{signal}} \\
V_{\text{idler}}
\end{pmatrix}
\]

(A1)

By eliminating both \( I_{\text{idler}} \) and \( V_{\text{idler}} \), the equivalent admittance of the non-linear capacitor at \( f_{\text{signal}} \) as given in Equation (3) is resulted. For convenience, it is repeated here

\[
Y_{\text{vav}} = \omega_{\text{signal}} C_0 - \omega_{\text{idler}} C_0 - \frac{\omega_{\text{signal}} \omega_{\text{idler}} C_0^2}{\gamma}
\]

(A2)

Notice that the term \( \frac{Y_{\text{vav}}}{\gamma} - \omega_{\text{idler}} C_0 \) is equal to a real-valued conductance \( G_0 \) which account for the loss of idler tank due to finite quality factors of \( L_i \) and \( C_0 \) at resonance.

Now, in view of the signal frequency, the equivalent circuit can be redrawn as the one shown in Figure A1(b) and the transducer power gain can be derived according to this circuit. Firstly, the available power from the signal current source is given by

\[
P_{\text{source}} = \frac{|I_s|^2}{4Y_0}
\]

(A3)
At resonance, the signal tank \((L_s \text{ and } C_o)\) becomes open circuit and the power dissipated on the load is

\[
P_{\text{load}} = \frac{|I_g|^2 G_t}{(Y_0 + G_t + G_s + G_i)^2} \tag{A4}
\]

where \(G = -\omega_{\text{signal}} \alpha_{\text{dcl}} \gamma^2 C_0^2 G_i\) is the negative conductance introduced by the pumped non-linear capacitor, and \(G_s\) accounts for the loss of signal tank due to finite quality factors of \(L_s\) and \(C_o\). Therefore, the transducer gain is given by the ratio of these two expressions as

\[
g_t = \frac{4 Y_0 G_t G_i^2}{(1 + G/G_i)^4} = \frac{4 Y_0 G_t G_i^2}{(1 - \gamma^2 Q_0 Q_i)^4} \tag{A5}
\]

where \(G_t = Y_0 + G_t + G_s\) is the total conductance of the circuit at \(f_{\text{signal}}\). In addition, \(Q_d = \alpha_{\text{dcl}} C_o / G_i\) and \(Q_i = \omega_{\text{dcl}} C_o / G_i\) are, respectively, the loaded and unloaded quality factors of signal and idler resonant tanks. For conjugate matching, the load conductance \(G_t\) should be equal to \(Y_0 + G_s + G_i\). In this case, the (available) gain is derived to be.

\[
g_a = \frac{Y_0}{Y_0 + G_s} \left( \frac{1}{1 - \gamma^2 Q_o Q_i} \right) \tag{A6}
\]

where \(Q_o\) and \(Q_i\) are loaded quality factor respectively for the signal tank with the load impedance excluded and the unloaded quality factor for the idler tank, as defined in Equations (4b) and (4c). This gives the expression shown in Equation (5a).

To derive the noise figure expression, current sources at both signal and idler frequencies will first be used to represent thermal noise as depicted in Figure A1(c). The output noise power at these two frequencies can be derived based on the \(Y\) parameters of the network, which are, respectively,

\[
N_i = \frac{i_t^2 G_t |y_{22} + y_{23}|^2}{(Y_0 + y_{11} + y_{12}) (y_{22} + y_{23}) - y_{12} y_{21}} \tag{A7a}
\]

\[
N_i = \frac{i_t^2 G_t |y_{21}|^2}{(Y_0 + y_{11} + y_{12}) (y_{22} + y_{23}) - y_{12} y_{21}} = \frac{i_t^2 G_t}{(Y_0 + G_t + G_i + G_s)^2} \tag{A7b}
\]

where \(y_{ij}\)’s are admittance matrix elements given in Equation (A1), \(y_{11} = Y_0 + G_t + G_s + j \omega L_s\), \(y_{12} = G_t + j \omega L_i\), and

\[
F = \frac{N_i + N_t}{g \kappa T_0 B} = \frac{1}{4kT_0BY_0} \left( \frac{i_t^2}{4} + \frac{\omega_{\text{signal}}^2 \gamma^2 C_0^2}{G_i} \right) \tag{A9}
\]

This expression can be further simplified to

\[
F = 1 + \frac{G_s}{Y_0} + \left( \frac{Y_0 + G_t + G_i}{Y_0} \right) \frac{\omega_{\text{signal}}^2 \gamma^2 C_0^2}{G_i} \tag{A10}
\]

by using Equation (A8). To isolate the contribution of the load, it is obvious that Equation (A10) can be written as

\[
F = \left( 1 + \frac{G_s}{Y_0} \right) \left( 1 + \frac{\omega_{\text{signal}}^2 \gamma^2 C_0^2}{G_i} \right) \tag{A11}
\]

where \(Q_o\) and \(Q_i\) are defined in Equations (4b) and (4c). This gives Equation (5b).

The normalised gain-bandwidth product relation given in Equation (7) can be obtained by first expressing \(G_s\) and \(G_i\) around the signal and idler frequencies, respectively, as

\[
Y_t \rightarrow G_t (1 + j 2 \delta Q_d) \tag{A12a}
\]
\[ Y_i^* \rightarrow G_i(1 - j2\delta_i Q_i) \quad (A12b) \]

with \( \delta_i = \Delta\omega_{\text{signal}}/\omega_{\text{signal}} \), \( \delta_i = \Delta\omega_{\text{idler}}/\omega_{\text{idler}} \) and the ratio of \( \delta_i/\delta_i \) is equal to \( \omega_{\text{idler}}/\omega_{\text{signal}} \). The transducer gain becomes

\[
g_t = \frac{4Y_0 G_l}{Y_i^*} \left( 1 - \frac{\omega_{\text{signal}}\omega_{\text{idler}} r^2 C_0}{Y_i Y_i^*} \right)^2
\]

\[
= \frac{4Y_0 G_l}{G_t^2} \left( 1 + j2\delta_i Q_{d_l} + \frac{G_t}{G_t - j2\delta_i Q_{d_l}} \right)^2 \quad (A13)
\]

by substituting Equations (A12a) and (A12b) into Equation (A5) and the definition of \( G \). Now Equation (A13) is rewritten in terms of normalised form

\[
g_t = \frac{1}{4Y_0 G_l/ G_t^2} \left( 1 + j2\delta_i Q_{d_l} \right)^2
\]

\[
= \frac{1}{2(1 + G/G_t)^2} \left\{ \frac{1}{(1 + j2\delta_i Q_{d_l})(1 - j2\delta_i Q_{d_l}) + G_t/G_t} \right\}^2 \quad (A15)
\]

which can be further written as

\[
g_t = \frac{1}{2 \left( 1 + \frac{G}{G_t} \right)^2} \left( 1 - j2\delta_i Q_{d_l} \right)^2
\]

\[
= \frac{1 + 4\delta_i^2 Q_{d_l}^2}{\left( 1 + j2\delta_i Q_{d_l} \delta_i Q_{d_l} - j2\delta_i Q_{d_l} \left( 1 - \frac{\delta_i Q_{d_l}}{\pi Q_{d_l}} \right) \right)^2} \quad (A16)
\]

where \( g = 1/(1 + G/G_t)^2 = 1/(1 - \gamma^2 Q_{d_l} Q_{d_l})^2 \) is normalised transducer gain \( g_t \). By using assumptions of \( \delta_i Q_{d_l} << \delta_i Q_{d} \) and \( \delta_i Q_{d} << \text{half} \), the above equation can be simplified to

\[
g_t \approx 1 - \frac{\omega_{\text{signal}}}{\omega_{\text{idler}}} Q_{d_l}^2 \quad (A17)
\]

\[
\frac{b}{2} \approx \frac{1}{\left( \frac{\omega_{\text{signal}}}{\omega_{\text{idler}}} Q_{d_l}^2 \right)}
\]

where \( b = 2\delta_i \) is the fractional bandwidth. Note that the above conditions are often met because the signal resonator is loaded with the source and the load impedance while the idler resonator is loaded only with the diode's own resistance. It is thus obvious that the bandwidth of the parametric amplification is limited by the idler resonator bandwidth under the high-gain (\( g > 2 \)) assumption.