Emittance self-compensation in blow-out mode

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We report an unusual regime of emittance self-compensation in an electron bunch generated in blow-out mode by a radio-frequency photocathode gun. Simulations clearly show an initial growth and a subsequent self-compensation of projected emittance in a divergent electron bunch originating from the effects of: (i) strong space-charge forces of mirror charges on the cathode, (ii) an energy chirp in the bunch and (iii) substantial re-shaping of the electron bunch. Furthermore, we show analytically and numerically how a complex interplay between these effects leads to emittance self-compensation in free space – the effect that is normally observed only in a focusing magnetic field.

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The space-charge force in an ellipsoidal electron bunch with a uniform density distribution changes linearly in the bunch in any direction. Because of this nature of the space-charge force, there is no emittance growth associated with it. Such a bunch is an ideal object in accelerator physics.[1] J. Luiten and co-workers[2] proposed a simple method for the formation of ellipsoidal electron bunches with a uniform density distribution. In the method, demonstrated experimentally by P. Musumeci and co-workers[3] an initially short, pancake-like bunch expands to a fully-fledged uniformly charged ellipsoidal bunch thanks to its space-charge force, similarly to blowing out soap bubbles. Hence, the method is named as blow-out mode. The initial density distribution in the bunch must have a half-circular profile in the radial direction whereas in the longitudinal direction the distribution can be arbitrary as long as the bunch is short enough. Mathematically, the required initial bunch distribution has the form

\[ f(s)\sqrt{1 - r^2/R^2}, \]

where \( f(s) \) is the distribution along the longitudinal bunch coordinate \( s \), and \( \sqrt{1 - r^2/R^2} \) describes the transverse distribution with \( R \) being the centre-to-edge distance.

Furthermore, J. Luiten and co-workers found that the desired uniform ellipsoidal distribution is formed when the accelerating field \( E_{\text{acc}} \) is much larger than the space-charge field of the image charge induced on the cathode, i.e. \( |E_{\text{acc}}| \gg |\sigma|/\epsilon_0 \). Here, \( \sigma \) is the surface charge density of the bunch and \( \epsilon_0 \) is vacuum permittivity.

However, the goal of our study was to find the conditions for the maximum 4D brightness in blow-out mode, which implies the minimum normalised transverse emittance. Our extensive numerical simulations show that the lowest emittance for a given charge is achieved when \( E_{\text{acc}} \approx 0.35 \sigma/\epsilon_0 \). Furthermore, we found that in this regime of a strong space-charge force, natural self-compensation of bunch emittance occurs. The unusual feature of this emittance self-compensation is that it occurs in a divergent electron bunch, which is opposite to what is known about classical emittance compensation.[4] Note that the desired uniform ellipsoidal distribution can be simultaneously achieved.

Figure 1 reports the results of massive simulations for the emittance of an electron beam generated in blow-out mode in an APEX-like continuous wave radio-frequency gun operated at 352 MHz with a peak accelerating field of 35 MV/m. Note that the results were independently cross-checked using three different codes: ASTRA, GPT and RFtrack. The discrepancy between the results is less than 10% and below the results from ASTRA simulations are shown. In the simulations, starting from the Fermi-Dirac distribution of electrons in a copper cathode, electron bunches are ejected from the cathode via photo-emission using the mode.[5] The initial electron bunch distribution has the form required for the blow-out mode

\[ \exp(-t^2/2\sigma_t^2) \sqrt{1 - r^2/R^2}, \]

where \( \sigma_t \) is the initial rms duration of the electron bunch. From the Fermi-Dirac distribution, the calculated thermal emittance per unit length is 0.4 mm-mrad/mm – the value demonstrated experimentally.[6] After the emission, the electron bunches move solely in the RF field of the gun, no solenoid is present. The laser pulse energy allows for extraction of 16 pC charge if the space-charge field is neglected. The initial electron bunch radius and duration are scanned in a wide range of parameters, even for such small radii that the electron emission is suppressed by the space-charge force. The minimum critical radius allowing for full extraction of 16 pC corresponds to around 130 \( \mu \)m. In this critical regime, the accelerating field is fully screened by the space-charge field.

Figure 1 shows a clear minimum of the emittance for the bunch duration of 30 fs (shortest duration in the simulations) and the bunch radius of around 220 \( \mu \)m. The resulting lowest emittance \( \varepsilon_{\text{min}} \) for the 16 pC extracted bunch is 25% above the thermal level. In this mode.

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FIG. 1. Upper plot: colour map of emittance as a function of the initial rms bunch duration (changes from 30 fs to 1 ps) and bunch radius \( R \) (note that \( R \) is the centre-to-edge distance!). Bottom plot: emittance (left ordinate axis) and \( E_{\text{SC}}/E_{\text{acc}} \) (right ordinate axis) as a function of \( R \) for the initial rms duration of 30 fs. The space-charge field on the cathode is estimated as \( E_{\text{SC}} = \sigma/\epsilon_0 \). For \( R < 130 \mu \text{m} \), the shaded area on the left, less than 16 pC of charge is extracted because of the formation of the virtual cathode due to the strong longitudinal space-charge force \( eE_{\text{SC}} \).
FIG. 2. Upper left plot: transverse and longitudinal rms normalised emittance as a function of distance from the cathode. Upper right plot: bunch density distribution projected onto the $xz$-plane at $z = 0.2$ mm. The colour coding shows the relative longitudinal momentum. The three slices used in the analysis of the transverse phase space are highlighted by means of a more intense hue of the colour. The second and third rows show the $x-p_x$ phase space of the three slices. The linear $x-p_x$ correlation calculated for all electrons in the bunch was removed to reveal the fine structure in the phase space. The abscissa coordinate is normalised to the corresponding rms bunch size, which is calculated for each longitudinal distance from the cathode. Note that the phase space distribution at the position $E'$ is visually identical to that in the position E. Hence, it is not shown.

FIG. 3. Upper plot: the ratio of the longitudinal to transverse bunch size and bunch emittance vs distance from the cathode. Inset demonstrates the linear $z^{-1}$ dependence of the inverse aspect ratio. Middle and bottom plots: the coefficients of the linear and cubic correlations between $x$ and $p_x$. Labels correspond to the phase space portraits in Fig. 2. Disappears after a few mm from the cathode thanks to the compensation effect. Appreciate almost linear phase space distributions in the plots A-I of Fig. 2.

Now, we turn our attention to the relative motion of electron slices due to the linear component of $E_r$. First, very near the cathode, $z_b \ll R$, all electrons experience the same linear space-charge field $E_{lin}^r = (\sigma/\epsilon_0)(z_b R^2)$ and all slices get the same transverse kick as it can also be seen from the plot of $p_1$ in Fig. 3. Note that the image charge gives a focusing effect and $E_{lin}^r = 0$ for the bunch being just on the cathode, $z_b = 0$.

As the bunch flies away and expands in the region OA, it takes an egg-like shape, see the top right plot in Fig. 2 primarily because of the pulling effect of the longitudinal space-charge force of the image charge on the cathode. There is also a weak focusing force in the radial direction.

FIG. 4. Linear component of the radial space-charge field for different uniform distributions of the electron bunch density. The true egg-like bunch is modelled by the sum of halves of two ellipsoidal bunches with different axial semi-axes.
due to the image charge. For the tail, this focusing is stronger than for the head simply because of the shorter distance to the cathode. In addition, a head-tail energy correlation – called energy chirp – develops due to the longitudinal space-charge force.

In the region AC, \( z > R/2 \), the transverse bunch dynamics is dominated by its own linear space-charge field and different slices acquire different amounts of \( x - p_x \) correlation, see plots B and C in Fig. 2 and the plot for \( p_1 \) in Fig. 3. The difference in the magnitude of the correlations for the tail and head slices (in other words, the tilt angles of the slices in the phase space) is due to the acquired momentum \( p_t \propto E_{in}^r/v_z \) that changes (linearly as we shall see) along the bunch because of both \( E_{in}^r \) and \( v_z \). The space-charge field of the egg-like bunch Fig. 2 can be calculated in the closed analytical form but unfortunately it is too cumbersome. Therefore, in Fig. 4 we show a graphical illustration of how \( E_{in}^r \) changes linearly in the bunch. As a result, the tail slice experiences stronger defocusing than the head one. In addition, there is almost a linear energy chirp in the bunch.

The difference in defocusing strength leads to a relative rotation between the slices in the \( x - p_x \) plane. Simultaneously, as the bunch dynamics evolves further, the egg-like shape transforms into a proper ellipsoidal shape (not shown in plots) because the tail expands faster than the head due to tail’s larger divergence. The field \( E_{in}^r \) becomes constant along the bunch. By the end of the region AC, the bunch density drops by 2 orders of magnitude because of bunch expansion.

Between the positions C and E, see Fig. 2, the bunch aspect ratio remains approximately constant whereas the bunch shape is nearly ellipsoidal. In this region, the projected emittance reduces almost to the thermal value as the tail slices rotate towards the head slices in the phase space due to the larger correlation in the tail. Specifically, consider two electrons having the same \( x \)-momentum \( p_x \) but located in the tail, superscript \( t \), and in the head, superscript \( h \). After propagating a distance \( l \), the separation between the electrons can be estimated in the ballistic approximation as \( (p_{x,t}^2/p_{x,h}^2 - p_{x,t}/p_{x,h})l \). The difference is negative because of the head-tail energy chirp \( (p_{x,h}^2 > p_{x,t}^2) \) and the tail electron catches up with head one. This energy chirp effect leads to the self-compensation of the projected emittance in the region CD.

The region EG corresponds to a transition through the gun exit aperture. A clear and simple treatment of the wakefield excitation-relaxation mechanism which occurs twice: in the gun (regions AC and CE) and in the exit aperture followed by drift space (regions E’G and GI).

To sum up, the discovered regime of emittance self-compensation is observed in a wide range of bunch charges from 160 fC to 16 pC. This regime is also observed for different lengths of the accelerating gun region deliberately designed to study possible limitations. The emittance self-compensation is, in general, robust except that the initial radial distribution of the bunch density must be close to a half-circular distribution to keep space-charge forces linear. The effect of self-compensation in blow-out mode with strong space-charge forces seems to be universal. The extent and position of self-compensation depend on specific settings of the gun. The discovered regime allows generating bunches with the lowest possible emittance for a given charge in blow-out mode. As an example, we tracked the generated bunch distribution through a 15 MeV linear accelerator and found the emittance to be preserved on a 60-nm scale for 16 pC bunches. A small solenoid at the position of the emittance minimum collimates the bunch that is then further accelerated by a 352 MHz booster.

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