Correlation effects on the weak response of nuclear matter

Omar Benhar\textsuperscript{a,b}, Nicola Farina\textsuperscript{b,a}

\textsuperscript{a}INFN, Sezione di Roma, I-00185 Roma, Italy
\textsuperscript{b}Dipartimento di Fisica, “Sapienza” Università di Roma, I-00185 Roma, Italy

Abstract

The consistent description of the nuclear response at low and high momentum transfer requires a unified dynamical model, suitable to account for both short- and long-range correlation effects. We report the results of a study of the charged current weak response of symmetric nuclear matter, carried out using an effective interaction obtained from a realistic model of the nucleon-nucleon force within the formalism of correlated basis functions. Our approach allows for a clear identification of the kinematical regions in which different interaction effects dominate.

Key words: nuclear matter, neutrino scattering, effective interaction
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1. Introduction

Short-range nucleon-nucleon correlations have long been recognized to play a prominent role in the electromagnetic response of nuclei. Comparison between the longitudinal responses at momentum transfer $|\mathbf{q}| \gtrsim 1.5 \text{ fm}^{-1}$, measured by inclusive electron scattering experiments, and the predictions of independent particle models shows that the data in the region of the quasi-free peak are sizably overestimated \cite{1, 2}. On the other hand, the results of approaches based on Nuclear Many-Body Theory (NMBT) and realistic models of the NN interaction \cite{3, 4}, in which the effects of short-range correlations are taken into account, provide a satisfactory description of a large body of data \cite{5}.

Long-range correlations, leading to the excitation of collective modes, are also known to be important. They become in fact dominant in the
region of low momentum transfer, in which the space resolution of the probe, \( \lambda \sim |q|^{-1} \), becomes much larger than the average NN separation. Most theoretical studies of this kinematical regime have been carried out within the Tamm Dancoff (TD) \( [6] \) and Random Phase Approximation (RPA) \( [7] \), using effective particle-hole interactions.

The development of a theoretical scheme allowing for a consistent treatment of short- and long-range correlations requires the construction of effective interactions based on realistic models of the NN force. This goal has been pursued by the authors of Ref.\( [8] \), who used the formalism of correlated basis functions (CBF) and the cluster expansion technique \( [9, 10] \) to obtain an effective interaction from a truncated version of the state-of-the-art Argonne \( v_{18} \) potential \( [11] \). The resulting effective interaction and the associated effective transition operators have been applied in a systematic study of the weak response of nuclear matter \( [8] \).

The development of a unified description of the nuclear response to weak interactions at low and high energy is important, its quantitative understanding being needed in a variety of different contexts. The region of neutrino energy \( E_\nu \sim 0.5 - 3 \text{ GeV} \) \( [12] \) is relevant to the analyses of long-baseline neutrino experiments, such as K2K and MiniBOONE (see, e.g., Ref.\( [13] \) and references therein), while nuclear interactions of low energy neutrinos, carrying energies of the order of tens of MeV, are believed to determine supernovae evolution \( [14] \) and neutron star cooling \( [15] \).

The effective interaction proposed in Ref.\( [8] \) has been recently improved with the inclusion of the effects of three- and many-nucleon forces \( [16] \). The resulting interaction has been used to calculate the nucleon-nucleon scattering cross section in nuclear matter, needed to obtain the transport coefficients from the Boltzmann-Landau equation \( [16, 17] \).

In this paper, we report the results of a study of the charged current weak response of symmetric nuclear matter, carried out using the effective interaction of Ref.\( [16] \). The main purpose of our work is identifying different interaction and correlation effects, all described within the same dynamical model, and determining the kinematical regions in which they play the dominant role.

2. Formalism

In the kinematical regime in which the non relativistic approximation is applicable, the nuclear response to a weak probe delivering momentum \( \mathbf{q} \) and
energy \( \omega \) can be written in the form

\[
S(q, \omega) = \frac{1}{N} \sum_n \langle 0 | O^\dagger(q) | n \rangle \langle n | O(q) | 0 \rangle \delta(\omega + E_0 - E_n) .
\]  

(1)

where \( N \) is the particle number and \(|0\rangle\) and \(|n\rangle\) denote the initial and final nuclear states, respectively. The transition operators are obtained expanding the weak nuclear current in powers of \(|q|/m\), \(m\) being the nucleon mass.

In the case of charged current interactions, at leading order one finds the Fermi and Gamow-Teller operators, whose expressions in coordinate space are

\[
\langle R' | O^F(q) | R \rangle = \delta(R - R') g_V \sum_{i=1}^N e^{iqr_i} \tau_i^+ = \delta(R - R') \sum_{i=1}^N O^F_i(q) ,
\]

(2)

\[
\langle R' | O^GT(q) | R \rangle = \delta(R - R') g_A \sum_{i=1}^N e^{iqr_i} \sigma_i \tau_i^+ = \delta(R - R') \sum_{i=1}^N O^GT_i(q) .
\]

(3)

In the above equations, \( r_i \) specifies the position of the \( i \)-th particle, \( \sigma_i \) describes its spin and \( \tau_i^+ \) is the isospin rising operator.

In our formalism nuclear matter is described using correlated states, obtained from the Fermi (FG) gas states through the transformation \[9, 10\]

\[
|n\rangle = \frac{F|n\rangle}{(n|F|F|n\rangle)^{1/2}} ,
\]

(4)

where \(|n\rangle\) is a determinant of single particle states representing \( N \) noninteracting nucleons at density \( \rho = 2k_F^3/3\pi^2 \), \( k_F \) being the Fermi momentum. The operator \( F \), embodying the correlation structure induced by the nonperturbative components of the NN interaction, is written in the form

\[
F(1, \ldots, N) = S \prod_{j>i=1}^N f_{ij} ,
\]

(5)

where \( S \) is the symmetrization operator, accounting for the fact that, in general, \([f_{ij}, f_{ik}] \neq 0\).

The two-body correlation function \( f_{ij} \) features an operatorial structure reflecting the complexity of the NN potential:

\[
f_{ij} = \sum_{TS} [f_{TS}(r_{ij}) + \delta_{S1}t_{TS}(r_{ij})S_{ij}] \ P_{TS} .
\]

(6)
In the above equation \( r_{ij} = |\mathbf{r}_i - \mathbf{r}_j| \), \( P_{TS} \) is the operator projecting onto two-nucleon states of total spin and isospin \( S \) and \( T \), respectively, and 
\[
S_{ij} = 3(\mathbf{\sigma}_i \cdot \mathbf{r}_{ij})(\mathbf{\sigma}_j \cdot \mathbf{r}_{ij})/r_{ij}^2 - (\mathbf{\sigma}_i \cdot \mathbf{\sigma}_j).
\]
The shapes of the radial functions \( f_{TS}(r_{ij}) \) and \( f_{tT}(r_{ij}) \), are determined through functional minimization of the expectation value of the nuclear hamiltonian in the correlated ground state, carried out at the two-body level of the cluster expansion \[18\].

The CBF formalism naturally leads to the appearance of an effective interaction, acting on the FG basis states. At lowest order of CBF perturbation theory \( V_{\text{eff}} \) is defined by
\[
\langle H \rangle = \frac{(0|F^\dagger HF|0)}{(0|F^\dagger F|0)} = \frac{(0|F^\dagger (T + V) F|0)}{(0|F^\dagger F|0)} = (0|T + V_{\text{eff}}|0),
\]
where \( H = T + V \) is the nuclear hamiltonian, \( T \) is the kinetic energy operator and \( V = \sum_{j>i} v_{ij} \), \( v_{ij} \) being the NN potential.

As the above equation suggests, in principle the approach based on the effective interaction allows one to obtain any nuclear matter observables using perturbation theory in the FG basis. However, the calculation of the expectation value of the hamiltonian in the correlated ground state, needed to extract \( V_{\text{eff}} \) from Eq.(7), involves severe difficulties.

In this work we follow the procedure developed in Ref. \[8\], whose authors derived the expectation value of \( V_{\text{eff}} \) carrying out a cluster expansion of \( \langle H \rangle \) of Eq.(7), and keeping only the two-body cluster contribution. The resulting expression is
\[
(0|V_{\text{eff}}|0) = \sum_{j<i} \langle ij|f_{12} \left[-\frac{1}{m}(\nabla^2 f_{12}) + v_{12} f_{12}\right]|ij\rangle_a,
\]
where the laplacian operates on the relative coordinate and the subscript \( a \) indicates that the two-nucleon state \( |ij\rangle_a \) is antisimmetryzed.

We have computed the effective potential defined by the above equation using the truncated form of the Argonne \( v_{18} \) potential referred to as \( v'_8 \) \[19\]. Three-nucleon forces, whose inclusion is needed to reproduce the saturation properties of nuclear matter, have been described following the approach of Ref. \[20\], in which the main effect of three- and many-body interactions is taken into account through a density dependent modification of \( v_{ij} \) at intermediate range. The Euler Lagrange equations for the correlation functions \( f_{TS} \) and \( f_{tT} \) have been solved using correlation ranges taken from Ref. \[21\].
The calculation of transition matrix elements between correlated states is also non-trivial. While in the FG model the Fermi and Gamow-Teller operators can only induce transitions to one particle-one hole (1p1h) states, in the presence of correlations more complex final states yield nonvanishing contributions to the response. In addition, the numerical determination of the matrix elements requires the use of the cluster expansion technique.

In this work we only include transitions to correlated 1p1h states and approximate the transition matrix element according to

\[
M_{ph} = \frac{(ph|F^\dagger OF|0)}{(ph|F^\dagger F|ph)^{1/2}(0|F^\dagger F|0)^{1/2}} \approx (ph|(1+\sum_{j>i} g_{ij})O(1+\sum_{j>i} g_{ij})|0),
\]

where \( g_{ij} = f_{ij} - 1 \) and \( O = \sum_i O_i \), \( O_i \) being the Fermi or Gamow-Teller operator (see Eqs.(2) and (3)).

The authors of Ref.[8] used a different truncation scheme to obtain \( M_{ph} \) from the cluster expansion formalism. Our choice is motivated by the requirement of consistency with the definition of the effective interaction, Eq.(8). However, we have verified that the difference between the numerical results obtained from the two different prescriptions never exceeds few percent.

**3. Effects of short-range correlations**

The calculation of \( M_{ph} \) has been performed on a cubic lattice, with a discrete set of \( N_h \) states specified by the hole momentum \( h_i \), satisfying the conditions \( h_i < k_F \) and \( h_i + q > k_F \). The size of the basis has been determined requiring that the response of a system of noninteracting nucleon computed on the lattice agreed with the analytical result of the FG model. Obviously, the response obtained from the discrete set of final states consists of a collection of delta function peaks. A smooth function of \( \omega \) has been obtained using a gaussian representation of the energy conserving \( \delta \)-function of finite width \( \sigma \). For sufficiently small values of \( \sigma \), the results become independent of \( \sigma \).

Figure[1] shows a comparison between correlated and FG matrix elements for the case of Fermi transitions. Note that the effect of the sizable quenching produced by short-range correlations is enhanced in the calculation of the response, whose definition involves the correlated matrix element squared (see Eq.(11)). Similar results are obtained for the Gamow-Teller transitions.

The calculation of the response requires the knowledge of the energies of the initial and final states. In the case of 1p1h final states, the argument of
the energy conserving $\delta$-function appearing in Eq.(1), reduces to $E_n - E_0 = e_{p} - e_{h}$, $e_{k}$ being the energy of a nucleon of momentum $k$ in nuclear matter. Within our approach, $e_{k}$ can be calculated using the effective interaction and the Hartree-Fock (HF) approximation:

$$e_{k} = \frac{k^2}{2m} + \sum_{|h| < k_F} \langle{hk|V_{\text{eff}}|hk}\rangle a = \frac{k^2}{2m} + U_{HF}(k).$$

The numerical results turn out to be in close agreement with those of Ref. [22], obtained within the CBF approach using the Fermi-Hyper-Netted-Chain (FHNC) summation technique and a realistic nuclear hamiltonian.

Figure 2 shows the nuclear matter response for the case of Fermi transitions. Panels (A), (B) and (C) correspond to the kinematical regions $|q| < k_F$, $k_F < |q| < 2k_F$ and $|q| > 2k_F$, respectively. The solid lines represent the results of the FG model, while the squares show the response obtained within the correlated HF approximation, i.e. using the correlated matrix elements of Eq.(9) and the energy spectrum of Eq.(10). In order to clearly identify the $\sim 30 \%$ quenching due to short-range correlations, whose size is consistent with the results of Fig. 1, we also show by diamonds the response obtained replacing the HF spectrum of Eq.(10) with the kinetic energy spectrum, i.e. neglecting $U_{HF}(k)$ in Eq.(10). It appears that interac-

Figure 1: Fermi transition matrix element at $|q| = 0.3$ fm$^{-1}$ (Eq.(9)) as a function of the magnitude of hole momentum $|h|$. The calculation has been carried out using a basis of 3040 ph states. The dashed horizontal line corresponds to the result of the FG model.
tion effects leading to the appearance of the mean field also play a significant role, pushing strength to energies well beyond the kinematical limit of the FG model. Panels (A), (B) and (C) correspond to momentum transfer $|q|=0.3, 1.8$ and $3.0 \text{ fm}^{-1}$ and basis size $N_h=3040, 2637$ and $2777$, respectively. The same qualitative pattern is obtained in the case of Gamow-Teller transitions.

4. Effects of long-range correlations

Equation (9) shows that in the correlated HF approximation the Fermi and Gamow-Teller transition operators are replaced by effective operators, acting on FG states, defined through

$$
(ph| (1 + \sum_{j>l} g_{ij}) O (1 + \sum_{j>l} g_{ij}) |0) = (ph| O_{\text{eff}} |0).
$$

Note that the above effective operators are different from those of Ref. [8], as we have chosen a different truncation scheme of the cluster expansions.

The FG ph states, while being eigenstates of the HF hamiltonian

$$
H_{HF} = \sum_k e_k,
$$

with $e_k$ given by Eq. (10), are not eigenstates of the full nuclear hamiltonian. As a consequence, there is a residual interaction $V_{\text{res}}$ that can induce transitions between different ph states, as long as their total momentum, $q$, spin and isospin are conserved.

We have included the effects of these transitions, using the Tamm Dancoff (TD) approximation, which amounts to expanding the final state in the basis of one $1p1h$ states according to [23]

$$
|f\rangle = |q, TSM\rangle = \sum_i c_i^{TSM} |p_i h_i, TSM\rangle,
$$

where $p_i = h_i + q$. $S$ and $T$ denote the total spin and isospin of the particle-hole pair and $M$ is the spin projection.

At fixed $q$, the excitation energy of the state $|f\rangle$, $\omega_f$, as well as the coefficients $c_i^{TSM}$, are determined solving the eigenvalue equation

$$
H |f\rangle = (H_{HF} + V_{\text{res}})|f\rangle = (E_0 + \omega_f)|f\rangle,
$$

where

$$
H_{HF} = \sum_k e_k,
$$

and

$$
V_{\text{res}} = \sum_{i>l} g_{ij} |p_i h_i, TSM\rangle \langle p_l h_l, TSM|
$$

with $g_{ij}$ given by Eq. (10).
where $E_0$ is the ground state energy. Within our approach this amounts to diagonalizing a $N_h \times N_h$ matrix whose elements are
\[
H_{ij}^{TSM} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i, TSM| V_{\text{eff}} | h_j p_j, TSM) ,
\]
(15)

In TD approximation, the response can be written as
\[
S(q, \omega) = \sum_{TSM} \sum_{n=1}^{N_h} \left| \sum_{i=1}^{N_h} (c_n^{TSM})_i (h_i p_i, TSM| O_{\text{eff}}(q)|0) \right|^2 \delta(\omega - \omega_n^{TSM}) ,
\]
(16)

where $(c_n^{TSM})_i$ denotes the $i$-th component of the eigenvector belonging to the eigenvalue $\omega_n^{TSM}$.

The diagonalization has been performed using a basis of $N_h \sim 3000$ ph states for each spin-isospin channel. The appearance of an eigenvalue lying outside the particle hole continuum, corresponding to a collective excitation reminiscent to the plasmon mode of the electron gas, is clearly visible in panel (A) of Fig. 3, showing the TD response at $|q| = 0.3$ fm$^{-1}$ for the case Fermi transitions. For comparison, the result of the correlated HF approximation is also displayed. Note that the sharp peak arises from the contributions of particle-hole pairs with $S = 1 T = 0$.

In order to identify the kinematical regime in which long range correlations are important, we have studied the TD response in the region $0.3 \leq |q| \leq 3.0$ fm$^{-1}$. The results show that at $|q| \lesssim 1.2$ fm$^{-1}$ the peak corresponding to the collective mode in the $S = 1 T = 0$ channel is still visible, although less prominent. However, it disappears if the exchange contribution to the matrix element of the effective interaction appearing in the rhs of Eq.(15) is neglected.

The transition to the regime in which short-range correlations dominate is illustrated in panels (B) and (C) of Fig. 3 showing the comparison between TD and HF responses at $|q| = 1.5$ and 2.4 fm$^{-1}$, respectively.

At $|q| = 1.5$ fm$^{-1}$ the peak no longer sticks out, but the effect of the mixing of ph states with $S = 1$ and $T = 0$ is still detectable, resulting in a significant enhancement of the strength at large $\omega$. At $|q| = 2.4$ fm$^{-1}$ the role of long range correlations turns out to be negligible, and the TD and correlated HF responses come very close to one another. The calculation of the response associated with Gamow-Teller transitions shows a similar pattern.
5. Conclusions

The CBF formalism employed in our work is ideally suited to construct an effective interaction starting from a realistic NN potential. The resulting effective interaction, which has been shown to provide a quite reasonable account of the equation of state of cold nuclear matter [16], allows for a consistent description of the weak response in the regions of both low and high momentum transfer, where different interaction effects are important.

The results of our calculations suggest that in addition to the HF mean field, which moves the kinematical limit of the transitions to 1p1h states well beyond the FG value, correlation effects play a major role, and must be taken into account. While at $|q| \lesssim 0.5$ fm$^{-1}$ long-range correlations, leading to the appearance of a collective mode outside the particle-hole continuum, dominate, at $|q| \gtrsim 2.0$ fm$^{-1}$ the most prominent effect is the quenching due to short-range correlations.

As a final remark, we emphasize that the results discussed in this paper, while being certainly interesting in their own right, should be seen as a step towards the development of a unified description of a variety of nuclear matter properties, relevant to the understanding of neutron-star structure and dynamics [16, 17].

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Figure 2: Nuclear matter response for the case of Fermi transitions. Solid lines: FG model. Squares: correlated HF approximation. Diamonds: same as the squares, but with the HF spectrum of Eq. (10) replaced by the kinetic energy spectrum. Panels (A), (B) and (C) correspond to $|q| = 0.3, 1.8$ and $3.0$ fm$^{-1}$, respectively.
Figure 3: Nuclear matter response calculated within the TD (squares) and correlated HF (diamonds) approximations, for the case of Fermi transitions. Panels (A), (B) and (C) correspond to $|q| = 0.3$, 1.5 and 2.4 fm$^{-1}$, respectively.