The case for non-gaussianity on cluster scales

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ABSTRACT
We address whether possible scale-dependent deviations from gaussianity in the primordial density field that are consistent with the cosmic microwave background observations could explain the apparent excess of early cluster formation at high redshift. Using two phenomenological non-gaussian models we find that at fixed normalisation to the observed local abundance of massive clusters, the protoclusters observed at \(z\sim 4\) are significantly more likely to develop in strongly non-gaussian models than in the gaussian paradigm. We compute the relative \(\frac{z}{\sim 1}\) evolution of X-ray cluster counts in the non-gaussian case with respect to the gaussian expectation, and the relative excess contribution to the CMB power spectrum due to the integrated thermal SZ effect. We find that both the observed hints of an unexpectedly slow evolution in the X-ray counts and the excess power at high \(\ell\) that may have been observed by CMB interferometers can also be reproduced in our non-gaussian simulations.

Key words: cosmology: theory – large-scale structure of the Universe – cosmic microwave background galaxies: clusters; general

1 INTRODUCTION
Non-gaussianities in the primordial perturbations of the cosmic density field are inherently difficult to generate in simple versions of inflation, and most inflationary models predict negligible amounts of relic non-gaussianity. Multiple field inflation or more speculative models such as those assuming non-homogeneous reheating processes or those seeded with topological defects can however readily rise to the challenge of spawning deviations from a gaussian initial density field. Of course, the amount of allowed primordial non-gaussianity on CMB scales has come under increasing pressure from the recent results of WMAP (Komatsu et al. 2003), and constraints are expected to tighten with upcoming data releases.

However current constraints from all-sky CMB maps only transfer to constraints at cluster or galaxy scales in the case of scale-invariant non-gaussianity. If non-gaussianity is strongly scale-dependent and arises only at cluster scales, it can simultaneously fit the WMAP bounds yet affect large-scale structure so as to induce predictable deviations from the gaussian paradigm. The caveat here is the “fine-tuning” of the physical scale at which non-gaussianity appears, which has to fall between \(\ell \sim 300\) and \(\ell \sim 2500\). (Models including defects such as linear cosmic strings as additional seeds for structure formation would naturally have such a scale, see Avelino et al. 1998). Our approach here is phenomenological, and we consider two hypothetical models which have such scale-dependent non-gaussianity, and, incidentally, are also to some extent motivated by theoretical considerations.

From the point of view of structure formation at low redshift and relatively small scales, issues for the gaussian \(\Lambda\)CDM paradigm include the early formation of protoclusters up to \(z \sim 4\) as traced by radio galaxies surrounded by Ly-\(\alpha\) emitters and Lyman-break galaxies, the observed apparently slow evolution of the comoving density of bright X-ray clusters, and the possible excess of angular power in CMB temperature fluctuations seen by CBI and ACBAR on cluster scales that most likely is due to the thermal SZ effect. It is well known that non-gaussianity may be able to naturally explain possible features associated with an excess (relative to Gaussian predictions) in the frequency at high redshift of galaxy protoclusters and clusters, by reasoning as follows. Rare peaks in the primordial density field define the locations where massive clusters will form, and the higher the peak the earlier the collapse to a virialized system. If the probability density function (PDF) of the density field shows an excess of high peaks compared to the gaussian case, (proto-)clusters of given mass form earlier, and systems above a given mass will be more abundant at fixed redshift. Note that more speculative topics such as the periodicity noted by Broadhurst et al. (1990), the presence of large regions underdense in galaxies Frith et al. (2003) or the high correlation length of very massive X-ray clusters could also hint at primordial non-gaussianity. Note also that scale-dependent non-gaussianity has been advocated by Avelino & Liddle (2004) and Chen et al. (2003) as a possible...
way to reconcile the high optical depth to reionization with realistic efficiencies for the formation of the first stars, and that its possible impact on the inner profiles of dark matter haloes has been addressed by Avila-Reese et al. (2003).

Here, we use simulations of large-scale structure to check the impact of strongly positively and negatively skewed non-gaussianity on the above observables. We evolve two primordial cosmic density fields with a scale-dependent non-gaussianity which is at least of the same order as the gaussian component at the scales probed by the simulation box. We employ the concordance cosmological parameters and normalise the models so that they give a similar abundance of massive clusters at $z = 0$. The two non-gaussian models differ by the sign of their primordial skewness: we choose a model with primordial voids (Griffiths et al. 2003) to realize negative skewness, and a model with a chi-square PDF of the density field to have positive skewness (this case has been worked out in the scale-independent case by Peebles 1999). We compare to the gaussian ΛCDM case they bracket from the point of view of cluster formation.

The paper is organised as follows. In Section 2 we review recent observational hints of non-gaussianity on cluster scales. Section 3 describes the setup of the simulations and compares to observations. We conclude in Section 4.

2 INDICATIVE OBSERVATIONS

We describe here the three observational hints which we can probe using simulations without recourse to any model for galaxy formation.

2.1 High redshift assembling structures

Willick (2000) was among the first to set constraints on possible primordial non-gaussianity using the presence of high-redshift massive systems. Even if some of the mass estimates he used for the $z \approx 0.8$ cluster MS 1054-03 were probably biased (Hoeft et al. 2000; Neumann & Arnaud 2000) because the system might not be dynamically relaxed, the precedent was set.

On the theoretical side, Robinson et al. (2000) have shown that given $\Omega_{\text{matter},0}\sim 1$ for instance, Sadat et al. 1999; see Borgani et al. 2001 for evidence for a low-density universe). However, at fixed cosmological parameters, fixed scaling relations and assuming normalisation to the local cluster abundance, increasing the amount of skewness in the primordial density field changes the $z \lesssim 1$ evolution in the abundance of massive clusters.

Deep ROSAT, Chandra and XMM X-ray observations have revealed a significant number of distant luminous clusters (Rosati et al. 2000). At redshifts $z \lesssim 0.8$, statistics of X-ray clusters have become sufficiently good to provide realistic estimates of their cosmological abundance. Vikhlinin et al. 2003; Voevodkin & Vikhlinin (2004) derive cosmological constraints from the evolution of the cluster baryon mass fraction which they relate to the total cluster mass, thereby skipping the usual uncertainties in the scaling relations. Using the 160 deg$^2$ ROSAT survey, they obtain results in line with the evolution of the mass function expected in a concordance gaussian CDM cosmology as they find moderately negative evolution of the cluster abundance to $z \sim 0.6$.

Recently, Vaucouleurs et al. 2003 prefer an $\Omega_{\text{matter},0} = 1$ universe from the reduction of the same data, but caution that a variation in the scaling of the $M - T$ relation with redshift might bias the analysis. In this work we will rederive the evolution of the density of luminous clusters from the ~200-member cluster catalogue of Vikhlinin et al. 1999 (updated using spectroscopic redshifts in Mullis et al. 2003; we will refer to this catalog as VM) and we will compare it with predictions from gaussian and non-gaussian models, at fixed scaling relations.

2.2 Evolution of the X-ray cluster luminosity function

The X-ray window is ideal for studying the evolution of massive clusters because the selection function effects can be much better understood than in the optical. No direct mass measurement is needed and one can use the observed relations between the X-ray luminosity and temperature ($L - T$) and between the temperature and mass ($M - T$) to select massive members from a catalogue of X-ray clusters. Inevitably however, results on the evolution of the abundance of X-ray clusters will depend on the adopted scaling relations.

Evolution of the abundance of X-ray luminous clusters has become one of the major tools for constraining the present-day cosmological matter density $\Omega_{\text{matter},0} = 1$ (for instance, Sadat et al. 1999; see Borgani et al. 2001 for evidence for a low-density universe). However, at fixed cosmological parameters, fixed scaling relations and assuming normalisation to the local cluster abundance, increasing the amount of skewness in the primordial density field changes the $z \lesssim 1$ evolution in the abundance of massive clusters.

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2.3 Small-scale CMB power spectrum

Majumdar & Subrahmanyan (2000) used the upper bounds on the rms CMB temperature fluctuations observed at arcmic scales by ATCA as constraints for structure formation models. They ruled out the possibility of a high $\Omega_{\text{matter},0}$ assuming COBE normalisation and gaussian fluctuations of the primordial density field. More recently, high resolution CMB experiments such as CBI (Mason et al. 2003) or ACBAR (Kuo et al. 2004) find an excess of small-scale ($\ell \approx 3000$ or $\theta \approx$ few arcmin) power with respect to the expectations from primary CMB fluctuations.

Non-gaussian models deviate more from gaussian models at high redshift (at fixed present day normalisation). Because the SZ temperature decrement of the CMB is in-
dependent of redshift, the angular power spectrum of the thermal SZ effect is sensitive to the high redshift abundance of clusters. This turns the integrated SZ power spectrum into another efficient means of distinguishing gaussian from non-gaussian models, in addition to the evolution of the abundance of massive clusters.

Given the known local abundance of clusters, gaussian models require low values of the normalisation ($\sigma_8 \approx 0.8$ if $\Omega_{\text{matter},0} = 0.3$) when they are fitted to the data. It is not clear yet if the observed excess is due to systematic effects or if it is a real signature coming from compact sources (point sources and/or galaxy clusters). In case this excess in the CMB power spectrum at $\ell > 3000$ is confirmed by future experiments to be due to galaxy clusters, it would imply a high normalisation ($\sigma_8 > 1$ for $\Omega_{\text{matter},0} \approx 0.3$) in the gaussian case. Alternatively, it would point towards a combination of the $M - T$ and $r_{\text{core}} - M$ relations (detailed in paragraph 3.3.3 below) which would be rather extreme (see for example Majumdar 2001). A more natural explanation however is an over-abundant population of clusters at high redshift, in turn a possible consequence of non-gaussianity of the primordial density field. We will consider this possibility in line with the evolution of the scaling relations that we assume to compute the X-ray cluster evolution.

3 THE CASE FOR NON-GAUSSIANITY

We first describe the simulations and characterize the models with simple statistics. Then, we directly measure in the simulations the three observables of the previous Section.

3.1 Simulations setup

We consider two non-gaussian models: the first, called $V$ is a model with compensated primordial voids (Griffiths et al. 2003; Mathis et al. 2003), the second (hereafter model $C^2$) is a model where the PDF of the primordial field density is the square of a gaussian. In theory the models could be associated with, respectively, a possible first-order transition occurring during inflation, and to versions of multiple scalar field inflation with isocurvature-type initial perturbations in a massive scalar field playing the role of CDM (such an ICDM model was worked out in detail by Peebles 1999). While the first-order phase transition model yielding primordial voids would be truly gaussian on CMB scales, this is not the case for the ICDM model. Tuning the ICDM model initially suggested so that it has the correct scale-dependent non-gaussianity is beyond the scope of this paper. However in conformity with our phenomenological approach, we will refrain in the following from linking these $V$ and $C^2$ realisations of the initial density field to a specific inflationary scenario. Rather, we simply take them as possible representations of the primordial density field on cluster scales that could arise in generic inflationary models. For reference, we also consider a gaussian model $G$.

All three models share the same $\Lambda$CDM cosmological parameters $\Omega_{\Lambda,0} = 0.3$, $\Omega_{\text{baryon},0} = 0.7$ and $h = 0.7$. We impose the same initial matter power spectrum in the $V$ and $G$ models: $n_h = 1$ with an adiabatic CDM transfer function. In the $C^2$ model, the initial power spectrum is that used by Peebles 1999 with $n_s = -1.8$ with an isocurvature CDM transfer function. We ensure that the $z = 0$ cluster mass functions of the non-gaussian models match reasonably well those of the gaussian $\Lambda$CDM model with $\sigma_8 = 0.9$. This is shown in Fig. 4 where we compare the mass functions in our three simulated models to the local optical cluster mass function of Bahcall et al. 2003 and to the X-ray cluster mass function of Vikhlinin et al. 2003. Differences with respect to the gaussian case are limited to 25 percent up to $1.6 \times 10^{15} h^{-1} M_{\odot}$ cluster masses. At masses $\sim 10^{15} h^{-1} M_{\odot}$, differences reflect cosmic scatter. In practice, the match to the $\Lambda$CDM $G$ model was achieved in the $C^2$ model by normalising the initial power computed from the realized field so that linear extrapolation yields $\sigma_8 = 0.65$ today. In $V$, the underlying gaussian field before “addition” of the voids has a normalisation similar to that of $G$. Note that the parameters of the distribution of void sizes in the $V$ model was set so as to match at present day the large voids seen in local galaxy surveys (El-Ad & Piran 2006; Hove & Vogeley 2003). As such, $V$ would also be a natural solution to the presence of large regions in the nearby Universe which are completely empty of bright galaxies (Peebles 2001).
The simulations start at high redshift \( z_{\text{init}} = 300 \) and follow collisionless structure formation using \( 128^3 \) particles in a \( 100 \ h^{-1} \) Mpc box with periodic boundary conditions. We employ the publically available code N-body code GADGET without hydrodynamics \(^1\) and set the equivalent Plummer softening length to one tenth of the mean interparticle separation. The mass of a dark matter particle is \( 3.94 \times 10^{10} \ h^{-1} \) \( \text{M}_\odot \).

### 3.2 Non-gaussian signatures in the simulations

This specific choice of the strongly non-gaussian models \( V \) and \( C^2 \) is attractive in the sense that the skewness of the primordial density field approximately brackets that of a gaussian field. This is illustrated in Fig. 2 where we plot the PDF of the rescaled overdensity field \( \delta/\sigma \) directly measured it in the three simulations at \( z = 50 \) over \( 8 \ h^{-1} \) Mpc scales. Dotted, solid and dashed lines respectively correspond to the \( V \), \( G \) and \( C^2 \) cases. For comparison, a gaussian has been over-plotted in dash-dotted line. While the PDF of the density of the \( V \) simulation is strongly negatively skewed due to the presence of the voids, that of the \( C^2 \) simulation is positively skewed as expected. (Note that the \( C^2 \) and \( G \) simulations are still within the "linear regime" over \( 8 \ h^{-1} \) Mpc scales by \( z = 50 \), while the expanding primordial voids in the \( V \) simulation have already fully emptied regions as large as 8 \( h^{-1} \) Mpc.)

Such skewed primordial PDFs directly impact the \( z = 4 \) halo mass function as shown in Fig. 2. There, we find early structure formation to be a common feature of \( C^2 \) and \( V \) models. In the \( V \) model, this is the result of two effects: fragmentation of the shell surrounding the voids, and non-linear motions in the background which are induced by the void underdensities. In the \( C^2 \) model, it is the effect of the relative primordial overabundance of \( > 3\sigma \) peaks with respect to the gaussian case.

The effect of the three different initial conditions is also clearly mirrored, although differently, in the undensity probability function \( UPF_{\delta/\sigma} (r) \) \cite{Weinberg1994}. We define the UPF as the probability that the matter underdensity \( \delta \) in a randomly placed sphere of comoving radius \( r \) has a value less that -0.8, following \cite{Hoyle2003}. Fig. 4 shows the undensity probability function at \( z = 0 \) for the gaussian \( G \), the \( C^2 \) and the \( V \) models (solid, dashed and dotted lines respectively). Obviously, the void model has a larger probability of underdensities compared to the gaussian case, while the \( C^2 \) model has very few regions with a radius larger than \( 1 \ h^{-1} \) Mpc where \( \delta \leq -0.8 \), a consequence of the \( \chi^2 \) distribution of the PDF of the primordial field. A comparison to results from local galaxy surveys is nevertheless out of the present scope: this would require gathering all the spheres of adequate overdensities within the regions with respect to the centre of mass. We then construct the PDF of \( \sigma_v \) gathering all the spheres of adequate overdensity. (This procedure was suggested by \cite{RobinsonBaker2000} in the case of the sampling of a density field.) Note that this method does not require the overdensities within the spheres to have virialized.

### 3.3 Comparison to observations

We first compute statistics relevant to the protoclusters observed at \( z \approx 4 \). Then, we choose two models for the cluster scaling relations and employ them to compare the observed evolution of the X-ray luminosity function. Finally, we estimate the expected thermal SZ angular power on cluster scales.

#### 3.3.1 Finding high-redshift protoclusters

Comparing the "protocluster" observations of \cite{Miley2004} to simulations of structure formation requires suitable statistics. We proceed as follows. At \( z = 4 \), we randomly throw a large number (30000) of spheres of proper radius \( r_{\text{throw}} \) in the simulation and compute for each the overdensity with respect to the background and the one-dimensional velocity dispersion \( \sigma_v \) of all the dark matter particles with respect to the centre of mass. We then construct the PDF of \( \sigma_v \) gathering all the spheres of adequate overdensity. (This procedure was suggested by \cite{RobinsonBaker2000} in the case of the sampling of a density field.) Note that this method does not require the overdensities within the spheres to have virialized.

Fig. 5 shows the resulting PDF at \( z = 4 \). The upper
left panel corresponds to throwing spheres with \( r_{\text{throw}} = 5 \) h\(^{-1}\) Mpc comoving and gathers the PDF of \( \sigma_\delta \) measured in the spheres irrespective of their environment (overdensity with respect to the background). The 3 to 5 Mpc proper extent reported by Miley et al. (2004) maps on to a 5 to 9 h\(^{-1}\) Mpc comoving sphere radius. The upper right panel of the same figure corresponds to throwing spheres with \( r_{\text{throw}} = 9 \) h\(^{-1}\) Mpc again selected in all environments.

The lower row of Fig. 5 is an addition to the upper row, for spheres whose dark matter overdensity \( \delta_{DM} \in [1, 3] \). Dotted, solid and dashed lines are for the \( V, G \) and \( C^2 \) simulations respectively. The Miley et al. (2004) 325 km s\(^{-1}\) velocity dispersion measure for the system of 21 Ly-\( \alpha \) emitters at \( z = 4.1 \) is shown as the vertical dash-dotted line. Fig. 5 repeats the same statistics at \( z \sim 2 \), where other such protoclusters have been observed. The 200 and 400 km s\(^{-1}\) velocity dispersion estimates of Kurk et al. (2003) are shown with vertical dash-dotted lines. (For the purpose of comparison we have kept the 5 and 9 h\(^{-1}\) Mpc comoving radii at \( z = 2 \).)

It is clear from these figures that at \( z = 4 \), there is no 5 h\(^{-1}\) Mpc comoving radius sphere in the gaussian simulation \( G \) whose inner velocity dispersion is within 100 km s\(^{-1}\) of what is observed, even regardless of environment. The situation is much more favourable for the other two non-gaussian models which we explore, where a non-zero PDF overlaps with the observations. The hypothesis here is clearly the absence of velocity bias between the simulated dark matter and the Ly-\( \alpha \) emitters. For \( r_{\text{throw}} = 9 \) h\(^{-1}\) Mpc, none of our simulations matches the observations.

### 3.3.2 Simulating the abundance of luminous X-ray clusters

We first select the most luminous clusters \( (L_X > L_* ) \) from the \( VM \) catalog and we build their density \( n(> L_*) \) as a function of redshift:

\[
n(> L_*) = \sum_i \frac{1}{V_i}
\]

where \( L_* \) is the threshold luminosity and the sum is over all the clusters above \( L_* \) in the redshift bin \( i \) (we take \( \Delta z = 0.1 \)). \( V_i \) is the search volume defined by the product of the volume in the shell of \( \Delta z = 0.1 \) at redshift \( z \) times the effective sky coverage \( f(L, z) \). For the 160 deg\(^2\) ROSAT survey used here, \( f(L, z) \) was computed in Vikhlinin et al. (1998). In figure 7, we show results for a threshold \( L_* = 2 \times 10^{43} h^{-2} \) erg s\(^{-1}\) cm\(^{-2}\). Note that because we employ \( H_0 = 100 h \) km s\(^{-1}\) Mpc\(^{-1}\) throughout, the original luminosities given for \( h = 0.5 \) in the \( VM \) catalog have to be rescaled. For example, \( 2 \times 10^{43} h^{-2} \) erg s\(^{-1}\) cm\(^{-2}\) correspond to an original \( 0.8 \times 10^{44} \) erg s\(^{-1}\) cm\(^{-2}\) for \( h = 0.5 \).

Fig. 9 compares the observed \( n(> L_*) \) with our non-gaussian simulations (dotted and dashed lines are for \( V \) and \( C^2 \) respectively) and the gaussian \( G \) simulation (solid lines).

To make proper comparison between the models, the non-gaussian abundances have been slightly rescaled to have exactly the same \( z = 0 \) abundance of \( L_* \) clusters as in the \( G \) simulation. To take into account the uncertainties in the cluster scaling relations we have compared two different assumptions on the evolution of the scaling relations. Lower and upper lines are respectively for a no evolution model \( L \propto M^2 \) and for a model where \( L \propto M^2(1 + z)^3 \). The gaussian model can be compatible with the observed density of luminous clusters which is flat at least up to \( z = 0.6 \) only if there is a strong evolution in the \( L - M \) relation up to this redshift. It is interesting to compare our Fig. 9 with figure 9 in Mullis et al. (2004). Mullis et al. (2004) present a curve similar to ours also using the 160 deg\(^2\) sample, but they choose a higher \( L_* = 10^{43} h^{-2} \) erg s\(^{-1}\). They similarly find a decrease in the density of luminous clusters at \( z > 0.6 \). When they fit the observed density with an evolving Schechter luminosity function normalised to match the \( z = 0.2 \) observed luminosity function of X-ray clusters, they recover this trend at \( z > 0.6 \). Their result is independent of any assumption (such as gaussianity) on the evolution of the mass function. In the non-gaussian case the \( L - M \) would need not to evolve or even to evolve negatively to explain the \( z = 0.6 \) drop. Whether this high-redshift drop is real or is due to the lack of statistics is an issue which has to be addressed with deeper and wider surveys.

In fact, the \( L - M \) relation has been poorly determined so far, but we can use the recent attempts made to constrain the evolution of the \( L - T \) and \( M - T \) relation to constrain the evolution of the \( L - M \) relation. Vikhlinin et al. (2002) constrain the evolution of the \( L - T \) relation as \( L \propto T^{\alpha}(1 + z)^{6} \) with \( \alpha = 2.64 \) and \( \gamma = 1.5 \pm 0.3 \). Holden et al. (2002) find a softer value of \( \gamma = 0.3 \pm 0.2 \). The BMW cluster survey (Moretti et al. 2001) found evidence for a non-evolving \( L - T \) relation (or even a negative evolution).
For the $M-T$ relation, Vikhlinin et al. (2003) find $T \propto M^\psi(1+z)^\gamma$ with $\psi = 0.5 \pm 0.4$. The spherical collapse model predicts $\psi = 1$ for $\Omega_{\text{matter},0} = 1$. Combining the previous constraints one can derive that $L \propto M^{\alpha \chi}(1+z)^{2-3}$.

Rosati et al. (2002) gave a comprehensive review of the situation and suggest that a weak evolution in the $L-T$ relation ($\gamma < 1$) is a reasonable assumption. If both the $L-T$ and the $M-T$ relation are non-evolving then the $L-M$ relation must also be a non-evolving relation. With the current uncertainties in the observed cluster scaling relations we stress that it is unclear if the observed flatness in $n(> L_c)$ is due to an evolving $L-M$ or if it is due to an overdensity of high redshift clusters (when compared with gaussian models). Again, our purpose here is mainly to bring out the differences between gaussian and non-gaussian models at fixed scaling relations.

We can use the following simple arguments to infer the redshift dependence of the $L-M$ relation, if we assume the validity of a theoretical mass function model and place ourselves in the gaussian case. We use the fact that the luminosity function is compatible with no evolution up to redshift $z \approx 1$ (for instance figure 9 in Rosati et al. 2002).

We simulated the luminosity function at different redshifts for a flat cosmological model ($\sigma_8 \approx 0.9$, $\Omega_{\text{matter},0} = 0.3$). The luminosity function can be derived from the mass function through the relation:

$$\frac{dn}{dL} = \frac{dn}{dM} \frac{dM}{dL}.$$  \hspace{1cm} (2)

Given the mass function (for example from the Press-Schechter formalism or its extensions to the non-gaussian cases), the luminosity function only depends in the assumed $L-M$ relation. Requiring that the luminosity function must be similar at low, intermediate and high redshifts we can constrain the redshift dependence of the $L-M$ relation. In a simple test we have assumed the form:

$$L = L_0 M^\beta(1+z)^\zeta,$$  \hspace{1cm} (3)

changing $\beta$ and $\zeta$ and looking for the combination which yields a similar luminosity function at different redshifts ($\zeta$ does not show a particular dependence with $L_0$). We have found that only models close to the relation $\zeta = 1.5\beta - 1.1$ produce a non-evolving luminosity function. If $\beta$ is derived from the observed $T \propto M^{0.65-0.66}$ and $L \propto T^{2.5-3}$ then $\beta \approx 1.4 - 2$. The corresponding $\zeta$ for a non-evolving luminosity function reads $\zeta \approx 1 - 2$. This suggests that models with strong evolution ($\zeta > 2$) are not favoured by the observed non-evolving luminosity function. On the other hand, there must be some mild evolution in the $L-M$ relations ($\zeta \geq 1$) in order to reproduce the high redshift luminosity function. This reasoning holds for a gaussian primordial density field. In any case, if one finds using direct measurements of the mass and luminosity that $\zeta < 1$, it would then be difficult to explain the non-evolving luminosity function within the gaussian paradigm on cluster scales.
### 3.3.3 Estimating the unresolved thermal SZ contribution

Figure 8 illustrates the sensitivity of the SZ power spectrum at small angular scales to the abundance of the high redshift population of clusters. In practice, the computation of the cluster SZ thermal power spectrum involves an integral of the mass function where each mass must be assigned a temperature. To compute the components of the mass function where each mass must be assigned a temperature, we follow the formalism of Diego et al. (2002). We write the \( M - T \) scaling relation as:

\[
T \propto M^\beta (1 + z)\psi.
\]

In this paragraph, the gas distribution inside the cluster needs modelling as well. For simplicity, we assume a \( \beta \)-model with \( \beta = 2/3 \) and core radius \( r_{\text{core}} \). We normalise the total emission to the total mass and the temperature. The angular power spectrum scales quadratically with the constant in front of equation 4 and we set it so that \( T = 8.5 \) keV for a \( M = 10^{15} h^{-1} M_\odot \) cluster at \( z = 0 \). The results depend only mildly on \( \psi \) and we fix it to \( \psi = 0.6 \). Our free parameters are therefore the redshift dependence of \( \psi \) and the mass and redshift-dependence of the core radius \( r_{\text{core}} \) which we parametrise as:

\[
r_{\text{core}} = r_0 M^{1/3} (1 + z)
\]

Fig. 8 shows the cluster thermal SZ power spectrum for the gaussian (dotted line) and non-gaussian models (solid lines), taking \( \psi = 1 \) and \( r_{\text{core}} = 0.08 h^{-1} \) Mpc (for \( M = 10^{15} h^{-1} M_\odot \) at \( z = 0 \)). The diamonds and vertical lines report the ACBAR and CBI observations, the star is the extrapolation of the primary CMB power spectrum, and the upper left curve is a fit to the WMAP measurements of the temperature power spectrum. In this plot, contrary to the previous paragraph, we did not require non-gaussian and gaussian models to have exactly the same abundance of massive clusters at \( z = 0 \), but respect the normalisation given in Fig. 4. The non-gaussian predictions clearly exceed the observations at \( \ell \sim 2500 \), while the gaussian case underpredicts the power. To see how this depends on the free parameters of the modelling, Fig. 8 repeats the exercise for \( \psi = 0 \) and \( r_{\text{core}} = 0.15 h^{-1} \) Mpc. The effect of changing these parameters is significant (mainly the increase in \( r_{\text{core}} \)): predictions for the non-gaussian models agree with the observed power while the gaussian case has too little power with such a set of parameters. At \( \ell = 2000 \), we note that the thermal SZ power spectrum shown in Fig. 4 for the gaussian case agrees within a factor 1.5 with that obtained by Zhang et al. (2002) using hydrodynamical simulations.

In fact, rather than the absolute level, it is the relative excess of the thermal SZ power spectrum in the non-gaussian models compared to the gaussian case which is interesting. Typically, at scales \( \ell \gtrsim 2000 \) non-gaussian models have more than an order of magnitude more power than the gaussian case. If \( \sigma_T = 0.9 \) and if the excess power seen by CBI and ACBAR is mostly due to the cluster thermal SZ effect, this makes a strong case for non-gaussianity. Even if the excess factor we find for the power in the non-gaussian case were too high to fit observations given the local abundance of clusters, much room would be left for mildly non-gaussian models with only reduced departure from gaussianity (the \( V \) and \( C^2 \) simulations used here have significant non-gaussianity).
and a $\beta$-model with $r_{\text{core}}=0.08\ h^{-1}\ Mpc$. The diamonds and vertical lines report the observations and the star is the extrapolation of the primary CMB power spectrum. Positive evolution in $M-T$ and compact clusters increase the power at small scales.

Figure 8. WMAP temperature angular power spectrum (upper left curve) compared with the cluster thermal SZ power spectrum computed in non-gaussian simulations (solid lines for $V$ and $C^2$) and gaussian (dotted line) models for clusters. The $M-T$ relation assumes here $\psi=1$ and the $\beta$-model $r_{\text{core}}=0.08\ h^{-1}\ Mpc$. The diamonds and vertical lines report the observations and the star is the extrapolation of the primary CMB power spectrum. Positive evolution in $M-T$ and compact clusters increase the power at small scales.

Figure 9. Same as Fig. 8 but for a $M-T$ relation with $\psi=0$ and a $\beta$-model with $r_{\text{core}}=0.15\ h^{-1}\ Mpc$.

4 CONCLUSIONS

While CMB-scale observations have started to constrain the possible amounts of non-gaussianity in the primordial cosmic density field, models with scale-dependent non-gaussianity could remain attractive for several aspects of structure formation if they predict significant non-gaussianity on cluster scales. Here, we have addressed whether such non-gaussianity with reasonable scaling relations can indeed explain the observables at high and low redshifts.

We have confronted three recent observational hints of early cluster formation and reduced late-time X-ray cluster abundance evolution to the predictions of two phenomenological models with strong non-gaussianity on galaxy to supercluster scales and opposite amounts of primordial skewness. Our approach using collisionless simulations was mainly illustrative; for instance, we did not fit the parameters of the models we employed so that they precisely match the present-day galaxy power spectrum, but we have simply normalised the models so that they approximately reproduce the abundance of local clusters predicted in the concordance $\Lambda$CDM cosmology.

Rather, we probed whether strongly non-gaussian models such as a model with primordial voids with negatively skewed overdensity distribution, and a $\chi$-square model with one degree of freedom with positively skewed overdensity distribution would help resolve three possible difficulties faced by the gaussian $\Lambda$CDM paradigm.

Firstly, we have found the protocluster observed by Milev et al. (2003) to be significantly easier to form in the two non-gaussian models we have simulated than in the $\Lambda$CDM case, where its estimated velocity dispersion seems difficult to reach.

Secondly, we have compared the slow bright X-ray cluster abundance evolution observed in the 160 deg$^2$ ROSAT survey at $z<0.6$ to the predictions of the non-gaussian models using two models for the redshift evolution of the $L-M$ relation. To reproduce the data, the simulated gaussian model required significant $L-M$ evolution, which is observationally disfavoured. In the case of the two non-gaussian models, both evolving and non-evolving $L-M$ relations provided predictions bracketing the data.

Thirdly, we tackled the excess of power in CMB temperature maps seen by the interferometers at $\ell \sim 2500$. Using a simple $\beta$-model for the intracluster gas distribution, we found that the thermal SZ contribution to the CMB temperature power spectrum is typically an order of magnitude higher in the non-gaussian models than in the gaussian case, irrespective of the two normalisations we have tried for the core radius of the intracluster gas and for the evolution of the $T-M$ relation. If the observed excess of power is due to the thermal SZ effect, non-gaussianity may be required for $\Lambda$CDM to also match the local abundance of clusters (an alternative being an adjustment of the scaling relations). In fact, more “moderate” amounts of non-gaussianity than the ones considered here might suffice, and the room between the two strongly non-gaussian models we have simulated and a gaussian primordial density field should be explored in future work.

To summarise, we have found that cluster-scale non-gaussianity can simultaneously explain an important constraint at high redshift, and two observables at lower redshift when we use proper scaling relations.

New constraints on possible non-gaussianity of the primordial density field on relatively small scales will come from large weak lensing surveys. They are expected by Amara & Refregier (2003) to yield constraints at a level similar to those set by current all-sky CMB maps on very large scales. In addition, both the inner density profiles of large DM haloes and the abundance of substructure are sensitive to the sign of the skewness of the primordial gravitational potential field (Avila-Reese et al. 2003). Finally, new measurements of the 2 and n-point correlation function of local galaxy clusters of different masses will also set tight constraints. If they yield negligible cluster scale non-gaussianity...
then cluster formation and evolution might need substantial revisions. If not, simulations such as those presented here will need to be generalised.

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