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ORIGINAL ARTICLE

Modeling and numerical analysis of a fractional order model for dual variants of SARS-CoV-2

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Abstract This paper considers the novel fractional-order operator developed by Atangana-Baleanu for transmission dynamics of the SARS-CoV-2 epidemic. Assuming the importance of the non-local Atangana-Baleanu fractional-order approach, the transmission mechanism of SARS-CoV-2 has been investigated while taking into account different phases of infection and various transmission routes of the disease. To conduct the proposed study, first of all, we shall formulate the model by using the classical operator of ordinary derivatives. We utilize the fractional order derivative and the model will be extended to a model containing fractional order derivatives. The operator being used is the fractional differential operator and has fractional order $\alpha$. The model is analyzed further and some basic aspects of the model are investigated besides calculating the basic reproduction number and the possible equilibria of the proposed model. The equilibria of the model are examined for stability purposes and necessary conditions for stability are obtained. Stability is also necessary in terms of numerical setup. The theory of non-linear functional analysis is employed and Ulam-Hyers’s stability of the model is presented. The approach of newton’s polynomial is considered and a new numerical scheme is developed which helped in presenting an iterative process for the proposed ABC system. Based on this scheme, sample curves are obtained for various values of $\alpha$ and a pattern is derived between the dynamics of the infection and the order of the derivative. Further simulations are presented which show the cruciality and importance of various parameters and the impact of such parameters on the dynamics and control of the disease is presented. The findings of this study will also provide strong conceptual insights into the mechanisms of contagious diseases, assisting global professionals in developing control policies.

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1. Introduction

The "severe acute respiratory syndrome coronavirus 2" (SARS-CoV-2) which causes the Coronavirus Disease 2019 (COVID-19) was initially discovered in Wuhan, China. SARS-CoV-2 is a mono-stranded RNA virus with rapidly mutating genetic material [1]. Only a small number of mutations can result in more severe disease with higher transmissibility and infectivity, according to the study of a huge number of patterns [2,3]. In the past two years, several SARS-CoV-2 variants have been identified, including the B.1.1.7 (alpha) variant discovered in the UK, the highly contagious and deadly B.1.617.2 (delta) variant discovered in India in December 2020 [1], and the more recent B.1.1.529 (Omicron) variant discovered for the first time in South Africa. The World Health Organization (WHO) has classified these variants as variants of concern (VOC), and they were found practically in every nation of the world [1]. The increase in COVID-19 infectivity where the recently reported variations such as Delta and Omicron are predominant and are linked to their circulation. Further, it is anticipated that the Omicron VOC will be transmitted to many other nations of the world, however, some medical professionals claim that the Omicron VOC caused sickness is less severe than other forms [2]. Despite this, many COVID-19 control strategies have shown to be quite successful against serious diseases caused by any of these variations. These control strategies comprise the vaccines but are not restricted to, the mRNA-1273 Moderna vaccine, the J & J vaccine, the BNT162b2 Pfizer-BioNTech vaccine, and several more [4,5]. All SARS-CoV-2 variants including the Delta and Omicron forms are reported to be significantly protected by these vaccines [6]. For epidemiologic research on the effectiveness of the COVID-19 vaccinations against VOCs that are now available, see [7,8] and references cited therein.

Dual infection occurs when a person got an infection from more than two strains of a virus at the same time. The virus can disturb the immune responses of the host and as a result, an increasing pattern in the viral population could be observed. The likelihood of simultaneous infections with two COVID-19 mutations is firmly supported by the reports [9–11]. Brazilian researchers discovered two cases of people whose infections are due to two distinct strains of COVID-19 at the same time [12,13]. In both situations, two teenage girls had the symptoms of like flue that typically appears during mild-to-moderate conditions. Both the patients are not severely ill and thus they were not hospitalized. In fact, these two identified variants of the virus circulating in Brazil since the start of this epidemic. Similarly, in Rio de Janeiro, a person was detected carrying both the older strain and the new P.2 strain. Additionally, it has been verified that an old woman in Belgium who was unvaccinated was discovered to have both the alpha and beta forms of COVID-19 infection [10]. On the same day that she tested positive for both variations, she also experienced rapidly deteriorating respiratory symptoms that ultimately resulted in her death. Additionally, patients who were both immuno-compromised and immuno-competent, and resided in various geographical locations developed dual infections with both omicron and delta [14–16]. A huge number of cases are reported in the last decades where people whose HIV illness is due to two or more strains of the disease [17–20].
models containing the arbitrary order derivatives, the readers are advised to see [38–42] and references cited therein. In the study [43], the author proposed a co-infected model for hepatitis and cancer using the core ideas of fractional calculus and investigated the key findings.

Atangana and Baleanu made a recent contribution to fractional calculus by presenting operators for solving problems based on fractional integrals and derivatives employing the Mittag-Leffler function [44]. It is investigated that the function of Mittag-Leffler is more suitable for describing nature-related problems compare with the power function. It should be remembered that the Mittag-Leffler function was introduced to provide an answer to a common question in complex analysis, specifically to portray the process of the analytic continuation of power law series outside the disc of their convergence. Since 2016, the Atangana-Baleanu operators have sparked a surge in fractional calculus research. In the fields of mathematics, engineering, and science, this work is expanding at an outstanding rate. The Atangana-Baleanu derivative is a type of non-local fractional derivative with a non-singular kernel and has many applications in various fields.

In order to better understand how SARS-CoV-2 and its various variations behave, a number of mathematical models have been formulated and investigated. Haq et al. [41] developed the Grünwald-Letnikov non-standard weighted average finite difference method and solved the proposed optimal control system for COVID-19. The work carried out in [25] studied the dynamics of SARS-CoV-2 using asymptotic and asymptomatic classes besides vaccination. The dynamics of COVID-19 was investigated in [26] by using the well-known Caputo operator. It is prominent to mention that these works assumes some control parameters and reflects the effect of these control variables on the infected compartments. A few studies indicate the dynamics of the vaccinated class while these control variables on the infected compartments. A few assumptions some control parameters and reflects the effect of the memory effect which is a crucial and more realistic component of the study’s main findings and provides additional guidance.

2. Preliminaries

In this part of the manuscript, we will look at some definitions and notions of integral and differential operators. In the context of fractional calculus, some key definitions include Riemann-Liouville, Erdelyi-Kober, Riesz, Jumarie, Hadamard, Weyl, Grunwald-Letnikov and Caputo, for detail explanation see [56,57] and references cited therein. The initial conditions produced by the Riemann-Liouville definition are physically undesirable [58]. The definition due to Caputo, which is given by

\[ \frac{d(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{1}{\Gamma(\Psi)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi), \]

it has physical significance since the initial constraints are stated in terms of integer-order derivatives [59]. The key drawback of this definition was the singularity in the kernels and thus to overcome this issue, Caputo-Fabrizio introduced the following definition

\[ \frac{d(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{M(\Psi)}{1-\Phi(t)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi) \exp \left[ -\frac{\Phi(t)}{1-\Phi(t)} (t-\Psi) \right] d\Psi. \]

Differential operators based on the generalized Mittag-Leffler function were proposed by Atangana and Baleanu in 2016. The intention was to present fractional differential operators with non-singular and nonlocal kernels. The Atangana-Baleanu fractional derivative is defined as

\[ \frac{ABC(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{AB(t)}{1-\Phi(t)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi) \exp \left[ -\frac{\Phi(t)}{1-\Phi(t)} (t-\Psi) \right] d\Psi. \]

The exponential decay, power-law and Mittag-Leffler kernels for the fractal-fractional derivative are provided by:

\[ \frac{FFE(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{M(\Psi)}{1-\Phi(t)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi) \exp \left[ -\frac{\Phi(t)}{1-\Phi(t)} (t-\Psi) \right] d\Psi, \]

\[ \frac{FFE(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{M(\Psi)}{1-\Phi(t)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi) \exp \left[ -\frac{\Phi(t)}{1-\Phi(t)} (t-\Psi) \right] d\Psi, \]

\[ \frac{FFE(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{M(\Psi)}{1-\Phi(t)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi) \exp \left[ -\frac{\Phi(t)}{1-\Phi(t)} (t-\Psi) \right] d\Psi, \]

where

\[ \frac{d(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \lim_{t \to 0} \frac{f(t)}{t^{1-\psi}} \left( \frac{t^{1-\psi}}{t^{1-\psi}} \right) \]

The fractal-fractional integral with power-law, exponential decay and Mittag-Leffler kernel are as below;

\[ \frac{FFE(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{M(\Psi)}{1-\Phi(t)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi) \exp \left[ -\frac{\Phi(t)}{1-\Phi(t)} (t-\Psi) \right] d\Psi, \]

\[ \frac{FFE(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{M(\Psi)}{1-\Phi(t)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi) \exp \left[ -\frac{\Phi(t)}{1-\Phi(t)} (t-\Psi) \right] d\Psi, \]

\[ \frac{FFE(t)}{dt} \int_0^t \frac{f(\Psi)}{\Gamma(\Psi)} \frac{d\Psi}{(t-\Psi)} = \frac{M(\Psi)}{1-\Phi(t)} \int_0^t \frac{d\Psi}{(t-\Psi)} f(\Psi) \exp \left[ -\frac{\Phi(t)}{1-\Phi(t)} (t-\Psi) \right] d\Psi, \]

3. Model formulation

We revisit the dynamic model studied in [45] and introduce a fractional-calculus. The whole population of the community
is divided into the following compartments: the vulnerable/susceptible compartment $S(t)$, strain 1 SARS-CoV-2 infected population $I_1(t)$, strain 2 SARS-CoV-2 infected population $I_2(t)$, population infected with both the strains $I_{12}(t)$, people who have survived from one or both SARS-CoV-2 strains: $R(t)$. From this point forward, strain 1 will refer to the original SARS-CoV-2 strain, while strain 2 will refer to other versions, such as omicron and delta variants, which are more contagious than the original SARS-CoV-2 strain and are hence the variants of concern (VoCs). All of the parameters appearing in the model are well-described in Table 1 while the dynamics of the disease is governed by the following system of equations:

$$
\frac{dS}{dt} = \Lambda - \frac{\lambda_1 S(t) I_1(t)}{N_0} - \frac{\lambda_2 S(t) I_2(t)}{N_0} - \frac{\lambda_{12} S(t) I_{12}(t)}{N_0} + \delta V(t) - (\eta + \gamma)S(t),
$$

$$
\frac{dI_1}{dt} = \left(\psi_1 + \psi_2 + \gamma\right)I_1(t) - \frac{\lambda_1 I_1(t)}{N_0} - \left(\psi_1 + \psi_2 + \gamma\right)I_1(t),
$$

$$
\frac{dI_2}{dt} = \left(\psi_1 + \psi_2 + \gamma\right)I_2(t) - \frac{\lambda_2 I_2(t)}{N_0} - \left(\psi_1 + \psi_2 + \gamma\right)I_2(t),
$$

$$
\frac{dI_{12}}{dt} = \left(\psi_1 + \psi_2 + \gamma\right)I_{12}(t) - \frac{\lambda_{12} I_{12}(t)}{N_0} - (\psi_1 + \psi_2 + \gamma)I_{12}(t),
$$

$$
\frac{dR}{dt} = \psi_1 I_1(t) + \psi_2 I_2(t) + \gamma I_{12}(t) - \gamma R(t),
$$

$$\frac{dV}{dt} = \eta S(t) - (\delta + \gamma) V(t).$$

(7)

3.1. Model in Atangana-Baleanu-Caputo sense

It has recently been discovered that the theory of fractional-calculus is rich in applications, and researchers acquired more precise findings using fractional systems compare to modeling the same problems with the ordinary derivatives. Thus, we will shift model (7) into the new framework that use fractional derivative with a generalized Mittag-Leffler kernel as:

$$
\frac{d^\alpha S}{dt^\alpha} = \Lambda - \frac{\lambda_1 S(t) I_1(t)}{N_0} - \frac{\lambda_2 S(t) I_2(t)}{N_0} - \frac{\lambda_{12} S(t) I_{12}(t)}{N_0} + \delta V(t) - (\eta + \gamma)S(t),
$$

$$
\frac{d^\alpha I_1}{dt^\alpha} = \left(\psi_1 + \psi_2 + \gamma\right)I_1(t) - \frac{\lambda_1 I_1(t)}{N_0} - \left(\psi_1 + \psi_2 + \gamma\right)I_1(t),
$$

$$
\frac{d^\alpha I_2}{dt^\alpha} = \left(\psi_1 + \psi_2 + \gamma\right)I_2(t) - \frac{\lambda_2 I_2(t)}{N_0} - \left(\psi_1 + \psi_2 + \gamma\right)I_2(t),
$$

$$
\frac{d^\alpha I_{12}}{dt^\alpha} = \left(\psi_1 + \psi_2 + \gamma\right)I_{12}(t) - \frac{\lambda_{12} I_{12}(t)}{N_0} - (\psi_1 + \psi_2 + \gamma)I_{12}(t),
$$

$$\frac{d^\alpha R}{dt^\alpha} = \psi_1 I_1(t) + \psi_2 I_2(t) + \gamma I_{12}(t) - \gamma R(t),
$$

$$\frac{d^\alpha V}{dt^\alpha} = \eta S(t) - (\delta + \gamma) V(t).$$

(8)

Under the starting approximation

$$S(0) = S_0^0, I_1(0) = I_{10}^0, I_2(0) = I_{20}^0, I_{12}(0) = I_{120}^0, R(0) = R_0^0, V(0) = V_0^0 \geq 0.$$

4. Deterministic analysis

In the subsequent part of the manuscript, we intend to present detail qualitative analysis of the deterministic model (7).

4.1. Deterministic Basic reproduction number

The infection-free fixed point for the deterministic system is of the form

$$Q_0 = \left(S^0, I_1^0, I_2^0, I_{12}^0, R_0^0, V_0^0\right) = \left(\frac{\Lambda}{(\eta + \delta + \gamma)}, 0, 0, 0, 0, \frac{\Lambda}{(\eta + \delta + \gamma)}\right)$$

The well-known threshold parameter $R_0$ of system (7) is calculated by using the next generation matrix techniques [51]. The transfer matrices are given by:

$$F = \begin{pmatrix}
\frac{\psi_1}{N_0} & 0 & 0 \\
0 & \frac{\psi_1}{N_0} & 0 \\
0 & 0 & \frac{\psi_1}{N_0} + \frac{\psi_2}{N_0} + \frac{\gamma}{N_0} \\
\frac{\psi_2}{N_0} & 0 & 0 \\
0 & \frac{\psi_2}{N_0} & 0 \\
0 & 0 & \frac{\psi_2}{N_0} + \frac{\gamma}{N_0} \\
\end{pmatrix}; \\
V = \begin{pmatrix}
\psi_1 + \psi_2 + \gamma & 0 & 0 \\
0 & \psi_1 + \psi_2 + \gamma & 0 \\
0 & 0 & \psi_1 + \psi_2 + \gamma \\
\end{pmatrix}.$$

(9)

The basic reproduction number of the model (7) is given by

$$R_0 = \rho(FV^{-1}) = \max\{R_{01}, R_{02}, R_{012}\},$$

where $R_{01}, R_{02}$ and $R_{012}$ are the corresponding reproduction numbers for SARS-CoV-2 strains 1 and 2, as well as for co-infection with both strains, and are provided by:

$$R_{01} = \frac{\zeta_1 (\gamma + \delta)}{(\eta + \gamma + \delta) (\psi_1 + \psi_2 + \gamma)},$$

$$R_{02} = \frac{\zeta_2 (\delta + \gamma)}{(\eta + \gamma + \delta) (\psi_1 + \psi_2 + \gamma)},$$

$$R_{012} = \frac{\zeta_{12} (\gamma + \delta)}{(\eta + \gamma + \delta) (x_1 + x_2 + \gamma)}.$$
where, \( |J - \lambda I| = 0 \) gives the following characteristic polynomial
\[
(\gamma + \lambda)(\gamma + \eta) + \gamma(\gamma + \delta)(\gamma + \phi_1 + \phi_2 + \gamma)(1 - \frac{\gamma(\phi_1 + \phi_2 + \gamma)}{\eta(\eta + \phi_1 + \phi_2 + \gamma)}) \ldots
\]
\[
\times \left( \frac{\gamma(\phi_1 + \phi_2 + \gamma)}{\eta(\eta + \phi_1 + \phi_2 + \gamma)} \right)(\gamma + \phi_1 + \phi_2 + \gamma)(1 - \frac{\gamma(\phi_1 + \phi_2 + \gamma)}{\eta(\eta + \phi_1 + \phi_2 + \gamma)}) = 0.
\]

The eigenvalue are \( \lambda_1 = -\gamma, \lambda_2 = -\gamma + \ delta, \lambda_3 = -\delta + \gamma \), and the solutions of the equations:
\[
(\lambda + (\phi_1 + \phi_2 + \gamma)(1 - \beta_01))
\]
\[
= 0, \quad (\lambda + (\phi_1 + \phi_2 + \gamma)(1 - \beta_02))
\]
\[
= 0, \quad (\lambda + (\phi_1 + \phi_2 + \gamma)(1 - \beta_012)) = 0.
\]

Referring to the Routh-Hurwitz condition, Eq. (11) will have roots with negative real portions iff
\[
\beta_0 = \max\{\beta_{01}, \beta_{02}, \beta_{012}\} < 1.
\]

4.3. The endemic equilibria and its analysis

4.3.1. The boundary equilibrium point

When \( \beta_0 = \max\{\beta_{01}, \beta_{02}, \beta_{012}\} > 1 \), then model (7) has three endemic equilibria (or boundary fixed points) \( E_1, E_2 \) and \( E_3 \) as follows:

1. Strain 1 only: \( E_1 = (S_1^*, I_2^*, R_3^*, V_1^*) \)
2. Strain 2 only: \( E_2 = (S_2^*, I_2^*, R_3^*, V_1^*) \)
3. Co-infection only: \( E_3 = (S_1^*, I_2^*, R_3^*, V_1^*) \)

where:
\[
\begin{align*}
S_1^* &= \frac{\Lambda(\phi_1 + \phi_2 + \gamma)}{\gamma(\phi_1 + \phi_2 + \gamma)(1 - \beta_01)}, \\
R_3^* &= \frac{\frac{\gamma(\phi_1 + \phi_2 + \gamma)}{\eta(\eta + \phi_1 + \phi_2 + \gamma)}}{\gamma(\phi_1 + \phi_2 + \gamma)(1 - \beta_01)} - 1, \\
V_1^* &= \frac{1}{\gamma(\phi_1 + \phi_2 + \gamma)(1 - \beta_01)}, \\
I_2^* &= \frac{\gamma(\phi_1 + \phi_2 + \gamma)}{\eta(\eta + \phi_1 + \phi_2 + \gamma)} - 1.
\end{align*}
\]

4.4. Numerical solution by RK4 method

The method of RK4 is a numerical method for the solution of differential equations and can be easily obtained from Taylor series after truncating the series up to fourth order derivatives. As the name suggest, if \( h \) denotes the step-size, this method has
Applying the definition of 3, we can write
\[
S(t) - S(0) = \frac{1}{\rho_2} N_1(\Phi_1, t, s) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_1(\Phi_1, \xi, S_0(\xi)})d\xi,
\]
\[
I_1(t) - I_0(0) = \frac{1}{\rho_2} N_2(\Phi_1, t, I_1) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_2(\Phi_1, \xi, I_0(\xi)})d\xi,
\]
\[
I_2(t) - I_0(0) = \frac{1}{\rho_2} N_3(\Phi_1, t, I_2) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_3(\Phi_1, \xi, I_2(\xi)})d\xi,
\]
\[
I_2(t) - I_0(t) = \frac{1}{\rho_2} N_4(\Phi_1, t, I_2) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_4(\Phi_1, \xi, I_2(\xi)})d\xi,
\]
\[
R(t) - R(0) = \frac{1}{\rho_2} N_5(\Phi_1, t, R) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_5(\Phi_1, \xi, R(\xi)})d\xi,
\]
\[
V(t) - V(0) = \frac{1}{\rho_2} N_6(\Phi_1, t, V) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_6(\Phi_1, \xi, V(\xi)})d\xi,
\]
\[
(16)
\]
where
\[
M_1(\Phi_1, t, S(t)) = A - \frac{1}{\rho_2} N_1(\Phi_1, t, I_0(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_1(\Phi_1, \xi, S_0(\xi)})d\xi,
\]
\[
M_2(\Phi_1, t, I_1(t)) = \frac{1}{\rho_2} N_2(\Phi_1, t, I_1(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_2(\Phi_1, \xi, I_0(\xi)})d\xi,
\]
\[
M_3(\Phi_1, t, I_2(t)) = \frac{1}{\rho_2} N_3(\Phi_1, t, I_2(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_3(\Phi_1, \xi, I_2(\xi)})d\xi,
\]
\[
M_4(\Phi_1, t, V(t)) = \frac{1}{\rho_2} N_4(\Phi_1, t, V(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_4(\Phi_1, \xi, V(\xi)})d\xi,
\]
\[
(17)
\]
If \( S, I, I_2, R \) and \( V \) contains their upper greatest value or bound, then \( M_1, M_2, M_3, M_4, M_5 \) and \( M_6 \) must fulfill the Lipschitz condition. Assuming that \( S \) and \( S' \) are two different functions, we obtain
\[
\|M_1(\Phi_1, t, S) - M_1(\Phi_1, t, S')\| = \|\frac{1}{\rho_2} N_1(\Phi_1, t, S) - \frac{1}{\rho_2} N_1(\Phi_1, t, S')\| = \|\rho_1\|\|S - S'\|
\]
\[
(18)
\]
Taking into account
\[
\eta_1 := \left( \frac{1}{\rho_2} N_1(\Phi_1, t, S) - \frac{1}{\rho_2} N_1(\Phi_1, t, S') \right),
\]
\[
M_1 = \max_{\eta_1} \|I_1(t)\|, M_2 = \max_{\eta_2} \|I_2(t)\|,
\]
\[
M_3 = \max_{\eta_3} \|I_3(t)\|, M_4 = \max_{\eta_4} \|I_4(t)\|
\]
one reaches
\[
\|M_1(\Phi_1, t, S) - M_1(\Phi_1, t, S')\| \leq \eta_1 \|S - S'\|
\]
(19)

In a similar way, we can get the following
\[
\|M_2(\Phi_1, t, I_1) - M_2(\Phi_1, t, I_1')\| \leq \eta_2 \|I_1 - I_1'\|
\]
\[
\|M_3(\Phi_1, t, I_2) - M_3(\Phi_1, t, I_2')\| \leq \eta_3 \|I_2 - I_2'\|
\]
\[
\|M_4(\Phi_1, t, I_2) - M_4(\Phi_1, t, I_2')\| \leq \eta_4 \|I_2 - I_2'\|
\]
\[
\|M_5(\Phi_1, t, R) - M_5(\Phi_1, t, R')\| \leq \eta_5 \|R - R'\|
\]
\[
\|M_6(\Phi_1, t, V) - M_6(\Phi_1, t, V')\| \leq \eta_6 \|V - V'\|
\]
(20)

The last equation is the Lipschitzian condition that has held for all the mappings. Going in a repetition mode, the equation in (16) becomes
\[
S_n(t) - S(0) = - \Phi_1 B(\Phi_1) M_1(\Phi_1, 1, S_{n-1}(1)) + \frac{\Phi_1}{B(\Phi_1)} \Gamma(\Phi_1)
\]
\[
\times \int_0^{\text{t}} (1 - (\xi)^{2, N_1(\Phi_1, \xi, S_{n-1}(\xi)})d\xi, I_{n-1}(t)
\]
\[
- I_1(0)
\]
\[
= \frac{1}{\rho_2} N_2(\Phi_1, t, I_{n-1}(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_2(\Phi_1, \xi, I_{n-1}(\xi)})d\xi, I_{2n}(t)
\]
\[
- I_2(0)
\]
\[
= \frac{1}{\rho_2} N_3(\Phi_1, t, I_{2n-1}(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_3(\Phi_1, \xi, I_{2n-1}(\xi)})d\xi, I_{3n}(t)
\]
\[
- I_3(0)
\]
\[
= \frac{1}{\rho_2} N_4(\Phi_1, t, I_{3n-1}(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_4(\Phi_1, \xi, I_{3n-1}(\xi)})d\xi, I_{4n}(t)
\]
\[
- I_4(0)
\]
\[
= \frac{1}{\rho_2} N_5(\Phi_1, t, I_{4n-1}(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_5(\Phi_1, \xi, I_{4n-1}(\xi)})d\xi, I_{5n}(t)
\]
\[
- I_5(0)
\]
\[
= \frac{1}{\rho_2} N_6(\Phi_1, t, I_{5n-1}(t)) + \frac{\rho_1}{\rho_2} \times \int_0^{\text{t}} (1 - (\xi)^{2, N_6(\Phi_1, \xi, I_{5n-1}(\xi)})d\xi,
\]
\[
(21)
\]

\[\text{together with} \quad S(0) = S^0, I_1(0) = I_0^1, I_2(0) = I_0^2, R(0) = R^0 \quad \text{and} \quad V(0) = V^0. \quad \text{Whenever the repeating terms divination is considered, we get}
\]
\[
\Pi_{n+1} = S_n - S_{n-1} - \frac{\Phi_1}{B(\Phi_1)} M_1(\Phi_1, t, S_{n-1}(\xi)) - \frac{\Phi_1}{B(\Phi_1)} \Gamma(\Phi_1)
\]
\[
\times \int_0^{\text{t}} (1 - (\xi)^{2, N_1(\Phi_1, \xi, S_{n-1}(\xi)})d\xi,
\]
\[
(22)
\]
It is important to see that
\[
S_n = \sum_{i=0}^{n} \Omega_{k,i}, \quad I_n = \sum_{i=0}^{n} \Omega_{k,i}, \quad I_{2n} = \sum_{i=0}^{n} \Omega_{k,i}, \quad R_n = \sum_{i=0}^{n} \Omega_{k,i}, \quad V_n = \sum_{i=0}^{n} \Omega_{k,i}.
\]

Furthermore, on implication of (19)–(20) and choosing that
\[
\Omega_{k,i} = S_{n-1} - S_{n-2}, \quad \Omega_{k,i+1} = I_{n-1} - I_{n-2}, \quad \Omega_{k,i+2} = I_{2n-1} - I_{2n-2}, \quad \Omega_{k,i+3} = R_{n-1} - R_{n-2}, \quad \Omega_{k,i+4} = V_{n-1} - V_{n-2},
\]
we reach
\[
\begin{align*}
\sqrt[1]{\Omega_{k,i}} & \leq \frac{1}{\sqrt{1 - \eta}} \|\Omega_{k,i-1}\| \leq \frac{4}{\sqrt{1 - \eta}} \|\Omega_{k,i-1}\|, \\
\sqrt[1]{\Omega_{k,i}} & \leq \frac{1}{\sqrt{1 - \eta}} \|\Omega_{k,i-1}\| \leq \frac{4}{\sqrt{1 - \eta}} \|\Omega_{k,i-1}\|, \\
\sqrt[1]{\Omega_{k,i}} & \leq \frac{1}{\sqrt{1 - \eta}} \|\Omega_{k,i-1}\| \leq \frac{4}{\sqrt{1 - \eta}} \|\Omega_{k,i-1}\|, \\
\sqrt[1]{\Omega_{k,i+1}} & \leq \frac{1}{\sqrt{1 - \eta}} \|\Omega_{k,i+1}\| \leq \frac{4}{\sqrt{1 - \eta}} \|\Omega_{k,i+1}\|, \\
\sqrt[1]{\Omega_{k,i+2}} & \leq \frac{1}{\sqrt{1 - \eta}} \|\Omega_{k,i+2}\| \leq \frac{4}{\sqrt{1 - \eta}} \|\Omega_{k,i+2}\|, \\
\sqrt[1]{\Omega_{k,i+3}} & \leq \frac{1}{\sqrt{1 - \eta}} \|\Omega_{k,i+3}\| \leq \frac{4}{\sqrt{1 - \eta}} \|\Omega_{k,i+3}\|, \\
\sqrt[1]{\Omega_{k,i+4}} & \leq \frac{1}{\sqrt{1 - \eta}} \|\Omega_{k,i+4}\| \leq \frac{4}{\sqrt{1 - \eta}} \|\Omega_{k,i+4}\|.
\end{align*}
\]
Such that
\[
|S - S| \leq \zeta_1 \gamma_1, \quad |I_1 - I_1| \leq \zeta_2 \gamma_2, \quad |I_2 - I_2| \leq \zeta_3 \gamma_3, \quad |I_{12} - I_{12}| \leq \zeta_4 \gamma_4,
\]
\[
|R - R| \leq \zeta_5 \gamma_5, \quad |V - V| \leq \zeta_6 \gamma_6.
\]

**Theorem 6.1.** Under the condition $J$, the considered model of arbitrary order (8) is H-U stable.

**Proof.** By Theorem (5.1), the proposed AB fractional problem (8) has unique root $(S, I_1, I_2, I_{12}, R, V)$ fulfilling equations of model (10). Then as follows

\[
\begin{align*}
\|S - S\| &= \frac{1 - \phi}{\mu_{k+1}} \left[ \|M_1(\Phi_1, t, S) - M_1(\Phi_1, t, \tilde{S})\| + \|M_2(\Phi_1, t, I_1) - M_2(\Phi_1, t, \tilde{I}_1)\| d\xi \right] \\
&= \left[ \frac{1 - \phi}{\mu_{k+1}} + \frac{\phi}{\mu_{k+1}} \right] \Phi_1 \|S - \tilde{S}\| \\
&= (29)
\end{align*}
\]

\[
\begin{align*}
\|I_1 - I_1\| &= \frac{1 - \phi}{\mu_{k+1}} \left[ \|M_3(\Phi_1, t, I_1) - M_3(\Phi_1, t, \tilde{I}_1)\| d\xi \right] \\
&= \left[ \frac{1 - \phi}{\mu_{k+1}} + \frac{\phi}{\mu_{k+1}} \right] \Phi_3 \|I_1 - \tilde{I}_1\| \\
&= (30)
\end{align*}
\]

\[
\begin{align*}
\|I_2 - I_2\| &= \frac{1 - \phi}{\mu_{k+1}} \left[ \|M_4(\Phi_1, t, I_2) - M_4(\Phi_1, t, \tilde{I}_2)\| d\xi \right] \\
&= \left[ \frac{1 - \phi}{\mu_{k+1}} + \frac{\phi}{\mu_{k+1}} \right] \Phi_4 \|I_2 - \tilde{I}_2\| \\
&= (31)
\end{align*}
\]

\[
\begin{align*}
\|S - \tilde{S}\| &\leq \gamma_1 \Delta_1 \\
\|I_1 - \tilde{I}_1\| &\leq \gamma_2 \Delta_2 \\
\|I_2 - \tilde{I}_2\| &\leq \gamma_3 \Delta_3 \\
\|I_{12} - \tilde{I}_{12}\| &\leq \gamma_4 \Delta_4 \\
\|R - \tilde{R}\| &\leq \gamma_5 \Delta_5 \\
\|V - \tilde{V}\| &\leq \gamma_6 \Delta_6.
\end{align*}
\]

Similarly, we have the followings

\[
\begin{align*}
\|I_1 - I_1\| &\leq \gamma_1 \Delta_1 \\
\|I_2 - I_2\| &\leq \gamma_2 \Delta_2 \\
\|I_{12} - I_{12}\| &\leq \gamma_3 \Delta_3 \\
\|R - R\| &\leq \gamma_4 \Delta_4 \\
\|V - V\| &\leq \gamma_5 \Delta_5.
\end{align*}
\]

So the derivation is achieved.

7. **Iterative Solution by Newton Polynomial**

In this section, we presented numerical schemes based on the Newton polynomial [53] for our model. In [54,55] Atangana and Seda proposed new COVID-19 models and solved by Newton polynomial. In both numerical analysis and image processing, Newton’s interpolation, a traditional polynomial interpolation method, is crucial. In majority of the traditional methods, the interpolation functions being used are specific to the given data. The primary focus of this research is on Newton’s polynomial interpolation as this has many advantages over other polynomial interpolation techniques. Further, this type of interpolation has a fast convergence rate, simply to implement, mathematically safe and easy in many aspects like in integration, differentiation. Beside all of these, one can compute arbitrary order derivatives of such polynomial very easily. By selecting the appropriate parameter values, the value of the Newton-type polynomial interpolant function in the interpolant region can be changed. The shape of the interpolation curves or surfaces can also be changed depending on the actual geometric design requirements.

We start with Mittag-Leffler kernel

\[
\begin{align*}
\beta_S(\phi, t; s, I_1, I_2, I_{12}, R, V) &= \lambda - \frac{\lambda}{\mu_{k+1}} \left( \frac{\lambda}{\mu_{k+1}} \right)^{\phi - 1} + \lambda \psi_1 (t - \phi) S(t) - \phi \psi_3 (t - \phi) S(t) \\
\beta_{I_1}(\phi, t; s, I_1, I_2, I_{12}, R, V) &= \frac{\lambda}{\mu_{k+1}} \left( \frac{\lambda}{\mu_{k+1}} \right)^{\phi - 1} + \frac{\lambda}{\mu_{k+1}} \psi_2 (t - \phi) I_1(t) - \psi_3 (t - \phi) I_1(t) \\
\beta_{I_2}(\phi, t; s, I_1, I_2, I_{12}, R, V) &= \frac{\lambda}{\mu_{k+1}} \psi_2 (t - \phi) I_2(t) - \psi_3 (t - \phi) I_2(t) \\
\beta_{I_{12}}(\phi, t; s, I_1, I_2, I_{12}, R, V) &= \frac{\lambda}{\mu_{k+1}} \psi_2 (t - \phi) I_{12}(t) - \psi_3 (t - \phi) I_{12}(t) \\
\beta_R(\phi, t; s, I_1, I_2, I_{12}, R, V) &= \frac{\lambda}{\mu_{k+1}} \psi_2 (t - \phi) R(t) - \psi_3 (t - \phi) R(t) \\
\beta_V(\phi, t; s, I_1, I_2, I_{12}, R, V) &= \frac{\lambda}{\mu_{k+1}} \psi_2 (t - \phi) V(t) - \psi_3 (t - \phi) V(t)
\end{align*}
\]

More simply, we can write as follows;

\[
\begin{align*}
\beta_S(\phi, t; s, I_1, I_2, I_{12}, R, V) &= S^* (t, S, I_1, I_2, I_{12}, R, V) \\
\beta_{I_1}(\phi, t; s, I_1, I_2, I_{12}, R, V) &= I_1^* (t, S, I_1, I_2, I_{12}, R, V) \\
\beta_{I_2}(\phi, t; s, I_1, I_2, I_{12}, R, V) &= I_2^* (t, S, I_1, I_2, I_{12}, R, V) \\
\beta_{I_{12}}(\phi, t; s, I_1, I_2, I_{12}, R, V) &= I_{12}^* (t, S, I_1, I_2, I_{12}, R, V) \\
\beta_R(\phi, t; s, I_1, I_2, I_{12}, R, V) &= R^* (t, S, I_1, I_2, I_{12}, R, V) \\
\beta_V(\phi, t; s, I_1, I_2, I_{12}, R, V) &= V^* (t, S, I_1, I_2, I_{12}, R, V)
\end{align*}
\]

Later on after application of fractional integration with Mittag-Leffler kernel law and plugging Newton polynomial in type of equations, we evaluate our model as follows;

\[
S^{\phi + 1} = e^{\phi \psi_1} S(t) + \psi_2 \int_s^t \frac{S(t)}{\mu_{k+1}} S^{\phi + 1}(t, S^{\phi + 1}, E^{\phi + 1}, E^{\phi + 1}, R^{\phi + 1}, V^{\phi + 1}) \mu_{k+1} \\
+ \mu_{k+1} \sum_{j=1}^N \left( \frac{\lambda}{\mu_{k+1}} \psi_2 (t - \phi) S(t) - \psi_3 (t - \phi) S(t) \right) S(t)
\]
Simulations of $S(t)$, $I_1(t)$, $I_2(t)$, $I_{12}(t)$, $R(t)$, $V(t)$ for the deterministic model by RK4 method. The parameter values used are: 

\begin{align*}
K &= 20; \\
f_1 &= 0.5; \\
f_2 &= 0.45; \\
f_{12} &= 0.42; \\
\theta_1 &= 0.05; \\
\theta_2 &= 0.05; \\
\psi_1 &= 0.05; \\
\psi_2 &= 0.05; \\
\vartheta_1 &= 0.05; \\
\vartheta_2 &= 0.05; \\
\gamma_1 &= 0.05; \\
\gamma_2 &= 0.05; \\
\delta &= 0.01; \\
\eta &= 0.01,
\end{align*}

so that $R_S^0 = \max\{R_S^0, R_S^0, R_S^0\} = \max\{2.4851, 2.2366, 2.0875\} = 2.4851 > 1.$

Fig. 1 Simulations of $(S(t), I_1(t), I_2(t), I_{12}(t), R(t), V(t))$ for the deterministic model by RK4 method. The parameter values used are: $\Lambda = 20; \xi_1 = 0.5; \xi_2 = 0.45; \xi_{12} = 0.42; \phi_1 = 0.051; \phi_2 = 0.051; \psi_1 = 0.05; \psi_2 = 0.05; \vartheta_1 = 0.05; \vartheta_2 = 0.05; \gamma_1 = 0.05; \gamma_2 = 0.05; \delta = 0.01; \eta = 0.01$, so that $R_S^0 = \max\{R_S^0, R_S^0, R_S^0\} = \max\{2.4851, 2.2366, 2.0875\} = 2.4851 > 1.$
The values of parameter being used are; \( \Lambda = 20; \xi_1 = 0.5, \xi_2 = 0.45; \xi_{12} = 0.42; \phi_1 = 0.051; \phi_2 = 0.05; \psi_1 = 0.05; \psi_2 = 0.05; \vartheta_1 = 0.05; \vartheta_2 = 0.05; \gamma_1 = 0.05; \gamma_2 = 0.05; \eta = 0.01; \delta = 0.01, \) so that \( \mathbb{R}_0^* = \max \{ \mathbb{R}_0^s, \mathbb{R}_0^c, \mathbb{R}_0^{12} \} = \max \{ 2.4851, 2.2366, 2.0875 \} = 2.4851 > 1. \)

\[
\begin{align*}
\Gamma_1^{1+1} & = \frac{1 - \phi_1}{\mathbb{R}_0^i} \left( t_0, S_0, I_0, E_0, R_0, V_0 \right) \\
& + \frac{\phi_1}{\mathbb{R}_0^i} \left( t_0, S_0, I_0, E_0, R_0, V_0 \right) \int \left\{ \Gamma_1(t_{i-1}, S_{i-1}, I_{i-1}, E_{i-1}, R_{i-1}, V_{i-1}) - \Gamma_1(t_{i-2}, S_{i-2}, I_{i-2}, E_{i-2}, R_{i-2}, V_{i-2}) \right\} \Delta \\
& + \frac{\phi_1}{\mathbb{R}_0^i} \left( t_0, S_0, I_0, E_0, R_0, V_0 \right) \sum_{i=1}^{\text{inf}} \left\{ -2 \Gamma_1(t_{i-1}, S_{i-1}, I_{i-1}, E_{i-1}, R_{i-1}, V_{i-1}) \right\} \Delta, \\
\end{align*}
\]

\[
\begin{align*}
\Gamma_2^{1+1} & = \frac{1 - \phi_2}{\mathbb{R}_0^i} \left( t_0, S_0, I_0, E_0, R_0, V_0 \right) \\
& + \frac{\phi_2}{\mathbb{R}_0^i} \left( t_0, S_0, I_0, E_0, R_0, V_0 \right) \int \left\{ \Gamma_2(t_{i-1}, S_{i-1}, I_{i-1}, E_{i-1}, R_{i-1}, V_{i-1}) - \Gamma_2(t_{i-2}, S_{i-2}, I_{i-2}, E_{i-2}, R_{i-2}, V_{i-2}) \right\} \Delta \\
& + \frac{\phi_2}{\mathbb{R}_0^i} \left( t_0, S_0, I_0, E_0, R_0, V_0 \right) \sum_{i=1}^{\text{inf}} \left\{ -2 \Gamma_2(t_{i-1}, S_{i-1}, I_{i-1}, E_{i-1}, R_{i-1}, V_{i-1}) \right\} \Delta, \\
\end{align*}
\]
Fig. 3 Numerical simulation for COVID-19 epidemic model via Mittag-Leffler Generalized Function for t = 600 days at $\Phi_t = 0.60, 0.70, 0.80, 0.90, 0.1$.

$$
I^{(3)}_1(t) = \frac{1-\Phi}{\Delta} + \sum_{j=1}^{10} \frac{t^j}{\Gamma(j+1)} \left( J^j(\mu, S^0, I^0, R^0, V^0) + \frac{\Phi(t^j)}{\Delta} \sum_{i=1}^{10} \sum_{k=1}^{10} \left( A_{ij}(t, j-1), S^{j-1}, I^{j-1}, R^{j-1}, V^{j-1} \right) \sum_{k=1}^{10} \left( B_{ij}(t, j-1), S^{j-1}, I^{j-1}, R^{j-1}, V^{j-1} \right) \Delta. \right)
$$

$$
R^{(3)} = \frac{1-\Phi}{\Delta} + \sum_{j=1}^{10} \frac{t^j}{\Gamma(j+1)} \left( J^j(\mu, S^0, I^0, R^0, V^0) + \frac{\Phi(t^j)}{\Delta} \sum_{i=1}^{10} \sum_{k=1}^{10} \left( A_{ij}(t, j-1), S^{j-1}, I^{j-1}, R^{j-1}, V^{j-1} \right) \sum_{k=1}^{10} \left( B_{ij}(t, j-1), S^{j-1}, I^{j-1}, R^{j-1}, V^{j-1} \right) \Delta. \right)
$$
Fig. 4 Numerical simulation for COVID-19 epidemic model \( \Phi \) via Mittag-Leffler Generalized Function for \( t = 800 \) days at \( \Phi_1 = 0.60, 0.70, 0.80, 0.90, 0.91 \).
\[ V^{n+1} = \frac{d}{dt}V^n + \sum_{i=1}^{3} \left[ V^n(t_i; S_i, I_i, R_i, V^n) + \phi_1(t_i) \right] + \sum_{i=1}^{3} \left[ V^n(t_i; S_i, I_i, R_i, V^n) + \phi_1(t_i) \right] \]

\[ \Lambda = \left[ (a - \mu + 1)^\alpha \left[ 2(a - \mu)^2 + (3\Phi_1 + 10)(a - \mu) \right] + 2a^2 + 9\Phi_1 + 12 \right] \]

\[ \Sigma = \left[ (a - \mu + 1)^\alpha (a - \mu + 3 + 2\Phi_1) \right] \]

\[ \Pi = \left[ (a - \mu + 1)^\alpha - (a - \mu)^\alpha \right] \]

Where

Fig. 5 Numerical simulation for COVID-19 epidemic model via Mittag-Leffler Generalized Function for \( t = 600 \) days at \( \Phi_1 = 0.55, 0.65, 0.75, 0.85, 0.95. \)
7.1. Graphical results

This section deals with explaining the disease behavior predicted by the system (8) via graphical illustrations. The primary goal is to examine how memory index and other important parameters affect the dynamics and, ultimately, the possibility of controlling the ongoing SARS-CoV-2 epidemic. The proposed SARS-CoV-2 model (8) is numerically solved with the help of an iterative scheme based on Newton’s Polynomial. The simulation results are based on the parameter values listed in Table 1. The effect of fractional order derivatives is graphically shown in Figs. 3–5 by varying the value of $\Phi_1$ (i.e., memory index). As shown in Fig. 3a, the susceptible population increases for a while before decreasing for all values of $\Phi_1$ to a particular positive density. During the simulations, we found a similar behavior irrespective of the values of $\Phi_1$. The behavior of the individuals infected with strain 1 is shown in Fig. 3b by changing the values of $\Phi_1$ within the range. It was found that for smaller values of $\Phi_1$, the peaks of the strain 1 infected curves occurred more slowly and slightly less frequently. We followed the same procedure and Fig. 3c is obtained showing the dynamics of the infected population due to strain 2. Similarly, Fig. 3d depicts the dynamics of Co-infected individuals of strain 1 and strain 2 for varying values of $\Phi_1$. The Co-infected individuals show decline irrespective of $\Phi_1$ to a specific value as shown in Fig. 3d. Fig. 3e analyses the dynamics of the recovered or removed population for various values of $\Phi_1$. The curve for the recovered individuals initially increases and then shows stability. Fig. 3f shows the cases of those who received vaccinations; they increased gradually for the first few days before bending to reach stability. The compartment as mentioned above also reaches its equilibrium points as time evolves.

8. Conclusion

In this work, we have presented a new mathematical model for two variants of SARS-CoV-2 with vaccine effects in the framework of the ABC fractional derivative. The literature has suggested that arbitrary operator analysis is the best method for examining the infection’s dynamic spread. The fractionalization order is $\Phi_1$ and assumptions were given to the dimension consistency between the remaining parameters. It should be noted that the infection problem under investigation, which is of the non-integer order type, more accurately depicts the dynamics of the infection than does the variance of the natural order. The system’s equilibrium, time dependence, and fundamental results are computed. The fixed-point theory is used to demonstrate that the solution concept exists. To verify theoretical findings on the extinction and persistence of SARS-CoV-2 variants in the population, numerical simulations were run. We use an approximation to evaluate the ABC operator-based, considered globalized system. Results of approximate solutions are obtained for various arbitrary parameter counts. Specific vaccine procedures and treatments for infectious diseases are very important as the process of receiving vaccinations are seen as an effective tool for eradicating SARS-CoV-2 in the human community. By this, the general situation and public health sectors must tribute close interest to this problem for the prevention of SARS-CoV-2 infection in the human population. Additionally, government agencies should educate citizens in every region and offer vaccinations, proper care in hospitals, schools, and colleges, as well as additional privatized health care facilities. This discussion will be more useful and important than the pre-related investigations because it will make use of the asymptomatic carriers and immunize with ABC generalized operators. According to our analysis, the findings and results of the model using the generalized derivatives are much more precise and more accurately describe real-world phenomena.

In this research, we have explored the dynamics of two variants of SARS-CoV-2 along with vaccination. The cross-immunity between the variations was not represented by the model. This might be an expansion of the model that makes more sensible assumptions about the worrying new versions. Further research on the co-infections of SARS-CoV-2 variations with other diseases, such as malaria, tuberculosis, Hepatitis B virus, and influenza to name a few, is warranted by the appearance of different SARS-CoV-2 variants. Therefore, we may take into account a reliable stochastic model for SARS-CoV-2 variations and co-infection with some other illnesses. There is still room to study this line of inquiry. Additionally, we were unable to fit our model to actual SARS-CoV-2 data since there was a lack of accurate information and data about developing variants of concern. With more accurate and trustworthy knowledge of the dynamics of various kinds of concern, we expect to accomplish this in the near future.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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