Merger-inspired rotation laws and the low-T/W instability in neutron stars

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ABSTRACT
Implementing a family of differential rotation laws inspired by binary neutron-star merger remnants, we consider the impact of the rotation profile on the low-T/W instability. We use time evolutions of the linearised dynamical equations, in Newtonian gravity, to study non-axisymmetric oscillations and identify the unstable modes. The presence and evolution of the low-T/W instability is monitored with the canonical energy and angular momentum, while the growth time is extracted from the evolved kinetic energy. The results for the new rotation laws highlight similarities with the commonly considered j-constant law. The instability sets in when an oscillation mode co-rotates with the star (i.e. whenever there is a point where the mode’s pattern speed matches the bulk angular velocity) and grows faster deep inside the co-rotation region. However, the new profiles add features, like an additional co-rotation point to the problem, which affect the onset of instability. The rotation laws influence more drastically the oscillation frequencies of the \( l = m = 2 \) f-mode in fast rotating models, but affect the instability growth time at any rotation rate. We also identify models where the low-T/W instability appears to be triggered by inertial modes. We discuss to what extent the inferred qualitative behaviour is likely to be of observational relevance.

Key words: methods: numerical – stars: neutron – stars: oscillation – star:rotation.

1 INTRODUCTION

With the "simultaneous" detection of gravitational and electromagnetic signals from binary neutron stars (Abbott et al. 2017) we truly entered the era multimessenger astronomy. The gravitational-wave signal from such mergers depends on a number of physical processes operating during the coalescence phase, the merger and the post-merger dynamics (Baiotti & Rezzolla 2017). The main parameters that determine the fate of the post-merger remnant are the total mass and the angular momentum. In essence, the remnant may continue to live as a stable compact star or undergo collapse, potentially delayed as the system loses angular momentum, leading to the distinction between hypermassive and supra-massive remnants (Baumgarte, Shapiro & Shibata 2000).

Hypermassive neutron stars may survive longer and avoid prompt core collapse because of differential rotation. Still, as their mass lies above the critical mass that can be supported by uniformly rotating stars, they become unstable as dissipative processes smooth out the differential rotation. Nonlinear simulations of hypermassive remnants from binary neutron star mergers have shown that the rotation profile may be non-trivial, especially during the early post-merger phase (Kastaun & Galeazzi 2015; Hanauske et al. 2017; Kastaun et al. 2017; De Pietri et al. 2019). The results suggest that the core of the remnant generally rotates slower than the outer layers, and these layers typically approach the Kepler velocity at larger distances, representing a disk of orbiting material (Kastaun et al. 2017). This profile is rather different from that commonly assumed in work on the dynamics of differentially rotating neutron stars, which tends to focus on the so-called j-constant law (the relativistic generalisation of a system with constant specific angular momentum, see for instance Hachisu (1986) and Komatsu et al. (1989a,b)). This then naturally leads to the question of how the differential rotation law impacts on the dynamics of the object.

A particularly interesting aspect of this question is associated with the fact that the gravitational-wave signal may be amplified by non-axisymmetric instabilities, developing on a dynamical timescale. It is well known that, in relativistic stars with realistic tabulated equation of state (EoS) the bar-mode instability only sets in at high rotation rates—when the star reaches \( \beta\nu \gtrsim 0.24 - 0.25 \) (Shibata
et al. 2000; Baiotti et al. 2007), where the rotation parameter is defined as \( \beta = T/[W] \), with \( T \) the kinetic and \( W \) the gravitational potential energy. The threshold for instability is only marginally lower than the Newtonian value, \( \beta_d \approx 0.27 \) (Chandrasekhar 1969), and it is not clear that remnants formed in a binary merger will get anywhere near this threshold.

However, differentially rotating stars may become dynamically unstable at a (perhaps significantly) lower rotation rate. This was first shown by Centrella et al. (2001) and soon after confirmed by Shibata et al. (2002, 2003) and Saijo et al. (2003). These numerical simulations demonstrated that an instability may set in already for \( \beta \approx 0.01 \). Given this low value, the mechanism is generally referred to as the low-T/W instability. Since the original work, the low-T/W instability has been found in many physical scenarios; in stellar core collapse (Ott et al. 2005, 2007; Scheidegger et al. 2008; Kuroda & Umeda 2010; Takiwaki et al. 2016; Takiwaki & Kotake 2018; Shibagaki et al. 2020), numerical evolutions of rapidly rotating cold neutron stars (Saijo & Yoshida 2006; Cerdá-Durán et al. 2007; Covino et al. 2010) and simulations of binary post-merger remnants (De Pietri et al. 2019). It has also been demonstrated that the interaction between magnetic field and differential rotation can decrease the amount of differential rotation and therefore suppress the low-T/W instability. This issue has been explored through both magnetohydrodynamical simulations (Camarda et al. 2009; Franci et al. 2013; Muhlberger et al. 2014) and equilibrium configurations of magnetised and differentially rotating stars (Fujisawa 2015).

The origin of the low-T/W instability is not yet well understood, although—as first proposed by Watts et al. (2005)—there is strong evidence that it sets in when an oscillation mode enters co-rotation with the bulk motion, i.e. when the pattern speed of a given oscillation mode matches the local angular velocity of the star. The low-T/W instability can then be viewed as related to local shear instabilities like the Papaloizou-Pringle instability in thick accretion discs (Papaloizou & Pringle 1984). In order to identify the instability, Saijo & Yoshida (2006) studied the behaviour of the canonical angular momentum (Friedman & Schutz 1975, 1978a,b) in the region of the co-rotation point, using both a linear method and hydrodynamical simulations. The relation between the co-rotation point and the low-T/W instability was also confirmed by Passamonti & Andersson (2015), using time evolutions of the linearised equations to establish the instability onset and estimate the growth time for sequences of differentially rotating polytropic stars (in Newtonian gravity). The results show that the \( l = m = 2 \) f-mode becomes unstable as soon as it co-rotates with the star and the growth time tends to increase gradually as the mode moves deeper into the co-rotation band. Moreover, the imaginary part of the f-mode can be described in terms of the stellar parameter and the mode pattern speed through an empirical formula. In contrast, the r-mode does not appear to suffer this instability, its pattern speed remains outside the co-rotation region. It only approaches the boundary of the region for highly differentially rotating models (Karino et al. 2001; Passamonti & Andersson 2015).

The properties of modes suffering the low-T/W instability have been studied by Saijo & Yoshida (2016); Yoshida & Saijo (2017) as an eigenvalue problem. In these studies the perturbation approach was restricted to the equatorial plane and compared to three-dimensional Newtonian hydrodynamical simulations. The results confirmed the relation between the growth time and the location of the co-rotation radius Passamonti & Andersson (2015), not only for the fundamental mode but also for pressure modes. Moreover, the results indicated a mode amplification between the co-rotation radius and the star’s surface, suggesting that the unstable mode suffers an over-reflection at the co-rotation point. This analysis was later extended to stars described by equations of state with different stiffness (Saijo 2018).

In the literature, differential rotation has mainly been described in terms of the \( \eta \)-constant rotation law (see Paschalidis & Stergioulas 2017). However, more recent work (see Galeazzi et al. (2012) and Uryu et al. (2016, 2017)) introduces new classes of rotation laws inspired by the differential rotation profile in merger remnants. These multi-parameter rotation laws can be used to approximate the rotation properties of hypermassive neutron stars (Kastaun & Galeazzi 2015; Hanauke et al. 2017; Kastaun et al. 2017; De Pietri et al. 2019). The impact of these new rotation laws on the low-T/W instability and various oscillation modes has not been explored so far. Given this, we set out to consider these issues by adapting the approach of Passamonti & Andersson (2015). We study non-axisymmetric oscillations by evolving in time the linearised hydrodynamical equations in Newtonian gravity, for differentially rotating stellar models described by one of the new rotation laws proposed by Uryu et al. (2017). With this set-up we can describe the main features of the differential rotation observed in numerical simulations of binary neutron-star remnants. Due to the implicit mathematical form of the rotation law we have modified the original self-consistent method introduced by Hachisu (1986). We extract the mode frequencies and identify the unstable modes that drive the low-T/W instability by Fast Fourier Transformation (FFT) and monitor the canonical energy and angular momentum at the co-rotation point. The growth time of the unstable modes is determined by the kinetic energy time evolution.

In Section 2 we summarize the formalism we have used, provide the relevant perturbation equations and describe the stellar models. Section 3 provides the results and Section 4 concludes the paper with a brief discussion of the implications.

2 FORMALISM

2.1 The Newtonian equations

In Newtonian gravity the equations required to study differentially rotating stars are the Euler equation, the mass conservation equation and the Poisson equation for the gravitational potential:

\[
\frac{\partial}{\partial t} + \nabla \cdot \mathbf{v} = -\nabla(h + \Phi),
\]

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}),
\]

\[
\nabla^2 \Phi = 4\pi G \rho,
\]

where \( G \) is the gravitational constant. In these equations, the scalar fields \( \rho, h \) and \( \Phi \) represent, respectively, the mass
density, the specific enthalpy and the gravitational potential, while $v$ is the fluid velocity.

The system of equations is completed by an equation of state for the matter. In this work we are mainly interested in qualitative features, so it makes sense to consider a polytropic model;

$$P = k \rho^{\gamma},$$

where $k$ is a constant and the adiabatic index is given by

$$\gamma \equiv \frac{d \log P}{d \log \rho} = 1 + \frac{1}{n},$$

with $n$ is the polytropic index. All results in this paper are obtained for $n = 1$. We do not expect that changing the stiffness of the model will have significant qualitative impact on the results.

In a barotropic fluid, pressure and enthalpy are related by

$$h = \int \frac{dP}{\rho},$$

which for a polytropic model leads to:

$$h = \frac{\gamma P}{\gamma - 1 \rho}.$$

### 2.2 Equilibrium solutions

We consider sequences of axisymmetric and differentially rotating configurations determined from two different rotation laws. Each equilibrium model is a solution to (Hachisu 1986):

$$\nabla (h + \Phi) - \frac{\Omega^2}{2} \nabla \varpi^2 = 0,$$

which we can rewrite as

$$\nabla (h + \Phi) = \frac{1}{2} (\Omega \nabla j - j \nabla \Omega).$$

In order to obtain equation (9) we have used the definition of the specific angular momentum $j = \Omega \varpi^2$, where $\Omega$ is the angular velocity and $\varpi = r \sin \theta$ is the radial distance from the rotation axis.

The main new development in this paper relates to the implementation of the differential rotation law introduced by Uryu et al. (2017),

$$\Omega = \Omega_c \left[ 1 + \left( \frac{j}{B^2 \Omega_c} \right)^p \right] \left( 1 - \frac{j}{A^2 \Omega_c} \right).$$

Here, the quantity $\Omega_c$ denotes the angular velocity at the rotation axis and $A$ and $B$ are parameters which control the shape and the degree of differential rotation (see figure 1 for examples).

If we introduce equation (10) in the equilibrium equation (9) and integrate, we find the following expression:

$$h + \Phi = C - \frac{\Omega^2}{2} \left[ 1 + \frac{1}{1 + p} \left( \frac{j}{B^2 \Omega_c} \right)^p + \frac{p}{2 + p} \frac{1}{A^2 B^2} \left( \frac{j}{\Omega_c} \right)^{p+1} \right],$$

where the integration constant $C$ is determined by imposing the required boundary condition at the pole. At this location, the enthalpy and the specific angular momentum both vanish, leading to $C = \Phi_p$. Therefore, equation (11) becomes

$$h = \Phi_p - \Phi - \frac{\Omega^2}{2} j \left[ 1 + \frac{1 - p}{1 + p} \left( \frac{j}{B^2 \Omega_c} \right)^p + \frac{p}{2 + p} \frac{1}{A^2 B^2} \left( \frac{j}{\Omega_c} \right)^{p+1} \right].$$

It is worth noting that, for $B \to \infty$ equation (10) reduces to the well known $j$-constant law

$$j = A^2 (\Omega_c - \Omega),$$

which has an explicit expression in term of the star’s angular velocity:

$$\Omega = \frac{\Omega_c A^2}{A^2 + \varpi^2}$$

a result that follows after using the definition of the specific angular momentum.

The main practical difference from implementations based on the $j$-constant law, relates to the fact that equation (10) is an implicit expression for $\Omega$. This means that we have to modify the usual iterative method, described in Hachisu (1986). We do this by the following steps:

1. We choose the polytropic index, $n = 1$, the rotation parameters ($\rho, A, B$) and the desired axis ratio.
2. From an initial guess density we solve the Poisson equation.
3. We find the value of $\hat{\Omega}$ at the equator, where $\hat{\Omega} = \Omega / \Omega_c$, by solving the following equation with a root finder routine,

$$\hat{\Omega} = \left( 1 - \frac{\varpi^2}{A^2} \hat{\Omega}^p \right) \left( 1 + \frac{\varpi^2}{B^2} \hat{\Omega} \right).$$

This relation is easily obtained from equation (10).

4. We determine $\Omega_c$ by solving equation (12) at the equator, where (for a barotropic model) the enthalpy vanishes.
5. We solve equation (12) with a root finder routine to determine the enthalpy at each point.
6. We find the density profile, $\rho$, from the equation of state and repeat the iteration until the solution converges to the specified accuracy.

### 2.3 Stellar models

In order to determine the equilibrium configuration using the rotation law (10) we need to fix three parameters: the index $p$ and the two constants $A$ and $B$ which control the degree of differential rotation. The rotation rate at the center, $\Omega_c$, is determined indirectly via the iterative method we outlined above. The parameter $A$, which is also present in the $j$-constant rotation law, controls the ratio between the equatorial and axial rotation rate. The parameter $B$ and the index $p$ in equation (10) mainly affect the shape of the rotation profile and the position of a maximum away from the rotation axis (see figure 1).

We consider two sets of solutions with rotation properties similar to the remnants of binary neutron-star mergers from nonlinear numerical evolutions (Kastaun & Galeazzi 2015; Hanauke et al. 2017; Kastaun et al. 2017; De Pietri et al. 2019). For the first set of models we choose $p = 1$, $A = 0.316$ and three different values of $B$, namely $B = 0.2,$
Table 1. This table lists key quantities for two sequences of differentially rotating equilibrium configurations. The stellar models are described by an $n = 1$ polytropic equation of state and the $j$-constant rotation law. The first column gives the parameter $A$ that controls the degree of differential rotation. In the second and third columns, we provide, respectively, the ratio of polar to equatorial axes and the star’s mass. The fourth column displays the central angular velocity $\Omega_c$, while in the last column we provide the rotation parameter $\beta = T/|W|$, the ratio between the rotational kinetic energy and gravitational potential energy. The first row refers to the non-rotating model which is common to all the rotating sequences presented in this work. All quantities are expressed in dimensionless units, where $G$ is the gravitational constant, $\rho_m$ represents the maximum mass density and $R_{eq}$ is the equatorial radius.

| $A/R_{eq}$ | $R_p/R_{eq}$ | $M/(\rho_m R_{eq}^3)$ | $\Omega_c/(G\rho_m)^{1/2}$ | $\beta \times 10^2$ |
|------------|-------------|----------------------|--------------------------|-----------------|
| 1.0        | 1.273       | 1.432                | 0.000                    | 0.00            |
| 0.32       | 0.9         | 1.277                | 0.794                    | 2.41            |
| 0.32       | 0.7         | 1.295                | 0.859                    | 13.59           |
| 0.32       | 0.5         | 1.241                | 4.347                    | 11.31           |
| 0.32       | 0.3         | 1.143                | 3.909                    | 15.54           |
| 0.32       | 0.1         | 1.066                | 4.046                    | 17.04           |
| 0.60       | 0.9         | 1.211                | 0.794                    | 2.41            |
| 0.60       | 0.7         | 1.097                | 1.396                    | 8.05            |
| 0.60       | 0.5         | 1.034                | 1.844                    | 14.91           |
| 0.60       | 0.3         | 0.930                | 2.049                    | 21.75           |
| 0.60       | 0.1         | 0.823                | 1.954                    | 24.13           |

Table 2. Same physical quantities as in Table 1, but for models with $p = 1$ and $A = 0.32$. The first column now reports the parameter $B$. | $B/R_{eq}$ | $R_p/R_{eq}$ | $M/(\rho_m R_{eq}^3)$ | $\Omega_c/(G\rho_m)^{1/2}$ | $\beta \times 10^2$ |
|------------|-------------|----------------------|--------------------------|-----------------|
| 0.20       | 0.9         | 1.296                | 1.046                    | 1.89            |
| 0.20       | 0.7         | 1.346                | 1.939                    | 6.01            |
| 0.20       | 0.5         | 1.285                | 2.524                    | 10.23           |
| 0.20       | 0.3         | 1.198                | 2.803                    | 13.59           |
| 0.20       | 0.1         | 1.161                | 2.962                    | 14.64           |
| 0.15       | 0.9         | 1.311                | 0.909                    | 1.74            |
| 0.15       | 0.7         | 1.355                | 1.680                    | 5.48            |
| 0.15       | 0.5         | 1.302                | 2.192                    | 9.22            |
| 0.15       | 0.3         | 1.232                | 2.516                    | 12.11           |
| 0.15       | 0.1         | 1.184                | 2.662                    | 13.53           |
| 0.13       | 0.9         | 1.319                | 0.838                    | 1.64            |
| 0.13       | 0.7         | 1.358                | 1.545                    | 5.14            |
| 0.13       | 0.5         | 1.312                | 2.022                    | 8.58            |
| 0.13       | 0.3         | 1.251                | 2.332                    | 11.22           |
| 0.13       | 0.1         | 1.269                | 2.480                    | 12.52           |

Figure 1. Rotation profiles obtained from equation (10) for different stellar parameters. In the upper panel, we show three models having $p = 1$, $A = 0.317$ and $B$, respectively given by $B = 0.20$, 0.15 and 0.13. In the lower panel, the star has $p = 1.8$, $A = 0.6$ and the following three values of $B$: 0.20, 0.15 and 0.13. For comparison we also show the solution corresponding to the $j$-constant rotation law.

0.15 and 0.13 (note that the rotation parameters $A$ and $B$ are given in dimensionless units, $A = A/R_{eq}$ and $B = B/R_{eq}$, where $R_{eq}$ is the equatorial radius, but for clarity, we will continue to use $A$ and $B$ in the text). For these values we construct three sequences of differentially rotating stars, from the non-rotating models up to the extreme case of an axis ratio $R_p/R_{eq} = 0.05$, where $R_p$ and $R_{eq}$ are, respectively, the stellar radius at the pole and at the equator. The second family of models has $p = 1.8$, $A = 0.6$ and the three values $B = 0.4$, 0.316, 0.28. The rotation profiles for these models are shown in figure 1 together with the $j$-constant law for $A = 0.316$ and 0.6. By varying the rotational parameters we have constructed models with different maximum angular velocity, $\Omega_{\text{max}}$, and degree of differential rotation. For instance, figure 1 shows that the quantity $\Omega_{\text{max}}/\Omega_c$ varies between 1 and 2.8, while $\Omega_{eq}/\Omega_c$ is practically equal for all models with $p = 1$. Meanwhile, for the case with $p = 1.8$ the $j$-constant law leads to a slightly smaller value of $\Omega_{eq}/\Omega_c$ compared to the other three models. The main quantities of the models we use are reported in Tables 1, 2 and 3.
Table 3. Same physical quantities as in Table 2, but for models with $p = 1.8$ and $A = 0.6$.

| $\omega$ | $\bar{R}$ | $\bar{R}_{\nu}$ | $\bar{M}/(\bar{R}_{\nu}^3)$ | $\Omega_c/(G\bar{M})^{1/2}$ | $\beta \times 10^{2}$ |
|----------|-----------|-----------------|-------------------------|-------------------------|-------------------------|
| 0.40     | 0.9       | 1.201           | 0.546                   | 2.64                    |                         |
| 0.40     | 0.7       | 1.068           | 0.949                   | 8.82                    |                         |
| 0.40     | 0.5       | 1.008           | 1.240                   | 16.27                   |                         |
| 0.40     | 0.3       | 0.954           | 1.366                   | 22.82                   |                         |
| 0.40     | 0.1       | 0.819           | 1.253                   | 24.02                   |                         |
| 0.316    | 0.9       | 1.227           | 0.418                   | 2.59                    |                         |
| 0.316    | 0.7       | 1.165           | 0.743                   | 8.51                    |                         |
| 0.316    | 0.5       | 1.205           | 1.003                   | 15.01                   |                         |
| 0.316    | 0.3       | 1.019           | 1.046                   | 19.80                   |                         |
| 0.316    | 0.1       | 0.908           | 1.013                   | 20.92                   |                         |
| 0.28     | 0.9       | 1.248           | 0.359                   | 2.46                    |                         |
| 0.28     | 0.7       | 1.241           | 0.650                   | 7.94                    |                         |
| 0.28     | 0.5       | 1.228           | 0.856                   | 13.66                   |                         |
| 0.28     | 0.3       | 1.073           | 0.918                   | 17.71                   |                         |
| 0.28     | 0.1       | 0.974           | 0.917                   | 18.81                   |                         |

2.4 Perturbation equations

Turning to the dynamical aspects of the problem, we now briefly review our approach to the oscillation modes and the properties of the low-T/W instability (more details can be found in Passamonti & Andersson (2015)).

We study non-axisymmetric oscillations of differentially rotating stars by using as dynamical variables the enthalpy $\delta h$ and the velocity perturbation $\delta v$, which, for an inertial observer and in spherical coordinates $(r, \theta, \phi)$, obey the linearised equations

\[
\left( \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi} \right) \delta v = -\nabla \delta h - 2\Omega \times \delta v - (\delta v \cdot \nabla) r \sin \theta \hat{e}_\phi, \tag{16} \\
\left( \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi} \right) \delta h = -\frac{\partial}{\partial h} \hat{\nabla} \cdot (\rho \delta v), \tag{17} 
\]

where $\hat{e}_\phi$ is the $\phi$-component of the orthonormal basis. We use the Cowling approximation, i.e. neglect the gravitational potential perturbation $\delta \Phi$ during the numerical evolution. The benefit of this is that we do not have to solve the linearised Poisson equation (an elliptic equation), speeding up the evolution. The drawback is that we lose accuracy in the calculation of the $f$-mode frequencies (Karino 2003), but this is not a major issue in this exploratory study. Our current results would anyway change when we consider the problem in General Relativity (Xie et al. 2020, in preparation).

The accuracy of the Cowling approximation for this kind of problem has been already discussed in Passamonti & Andersson (2015), in the context of the full set of Newtonian perturbation equations.

For axisymmetric stars, we can simplify the problem by using a Fourier expansion in the $\phi$ coordinate of equations (16)-(17) (Papaloizou & Pringle 1980). This way we end up with a two-dimensional problem, because the perturbation variables depend only on the spatial coordinates $(r, \theta)$ and the azimuthal index $m$. More specifically, any perturbation variable can be written in a similar way to the enthalpy:

\[
\delta h (t, r, \theta, \phi) = \sum_{m=0}^{\infty} [\delta h^+_m (t, r, \theta) \cos m\phi + \delta h^-_m (t, r, \theta) \sin m\phi]. \tag{18} 
\]

With this approach we numerically evolve, for any chosen $m$, a system of eight partial differential equations for $(\delta v^\pm, \delta h^\pm)$.

In Section 2.5 we will introduce the canonical energy and canonical angular momentum, the main diagnostics we use to identify the position where the instability develops. In order to determine these two quantities we need the Lagrangian displacement vector $\xi$ (Friedman & Schutz 1978a).

Therefore, we have to solve (at each time step)

\[
\delta v = \frac{\partial \xi}{\partial t} + v \cdot \nabla \xi - \xi \cdot \nabla v. \tag{19} 
\]

Finally, we need to impose the relevant boundary conditions. At the origin, $r = 0$, and on the rotational axis, $\theta = 0$, the solutions to equations (16)-(17) must remain regular. For non-axisymmetric oscillations with $m \geq 2$, this condition is realised by imposing

\[
\delta h = \delta \rho = 0, \quad \text{and} \quad \delta v = 0. \tag{20} 
\]

At the stellar surface we require that the Lagrangian perturbation of the enthalpy vanishes, i.e.,

\[
\Delta h = \delta h + \xi \cdot \nabla h = 0. \tag{21} 
\]

For a barotropic model, this is equivalent to the vanishing of the Lagrangian variation in pressure. We satisfy this boundary condition by imposing

\[
\delta h = -\xi \cdot \nabla h, \tag{22} 
\]

at the surface. All other variables are extrapolated at the surface grid point at each time step.

Finally, at the equator, $\theta = \pi/2$, the perturbation variables divide into two classes with opposite reflection symmetry. In the first class, the variables $\delta h^+, \delta v^+, \delta v^+_{\phi}$ are all even under reflection with respect to the equatorial plane, while $\delta v^-_{\phi}$ is odd. In contrast, for the second class $\delta h^-, \delta v^-, \delta v^-_{\phi}$ are odd and $\delta v^-_{\phi}$ is even.

2.5 Canonical energy and angular momentum

At the linear perturbation level, the development of non-axisymmetric instabilities in rotating bodies can be monitored in terms of the canonical energy and angular momentum (Friedman & Schutz 1975, 1978a,b). In an inviscid star, an unstable mode does not violate the energy and angular momentum conservation laws. This means that such a mode may only grow if both the canonical energy and angular momentum vanish. As explained in Passamonti & Andersson (2015), our numerical approach (based on time evolving the perturbation equations) is not accurate enough to directly monitor these two conditions, mainly because of the presence of other oscillation modes in the evolved data, the general issue of keeling track of a growing unstable mode and the numerical viscosity which contaminates the eigenfunction extraction. However, as shown by Saio & Yoshida (2006) and confirmed by (Passamonti & Andersson 2015) the canonical energy and angular momentum still...
Figure 2. Variation of the $2f$ mode frequency with $\beta = T/W$ for the models shown in figure 1. The mode frequencies of the three models with $p = 1$ and $A = 0.32$ are shown in the left-hand panel (see legend), while those for the three models with $p = 1.8$ and $A = 0.6$ are shown in the right-hand panel. The horizontal dotted line denotes the neutral point of the CFS instability (see Section 3.1).

This showed that the numerical error in the key quantities was significantly less than 1%, which means that the conclusions we draw from the results should be reliable.

3 RESULTS

In our previous work we studied the relation between co-rotating modes and the low-T/W instability for differentially rotating stars (Passamonti & Andersson 2015). The rotation profile was modelled by the $j$-constant rotation law. As suggested by Watts et al. (2005) we found—within the accuracy of the numerical framework—that the instability sets in when the f-mode enters the co-rotation region, i.e. whenever there is a point where the pattern speed of the mode matches the local rotation velocity of the star. This means that

$$\sigma = \Omega(\varpi_c),$$

where $\sigma = \omega/m$ is the pattern speed of the mode and $\varpi_c$ is the co-rotation point.

In this work, we consider the impact of the rotation law from (10) on the mode frequencies and the instability growth time. This question is interesting because the new rotation laws relates more directly to the dynamics of neutron star merger remnants and, as is evident from figure 1, a given mode may have two co-rotation points inside the star. If the mode entering co-rotation is a requirement for the instability to be triggered, then does the presence of additional co-rotation points impact on this?

First of all, we construct for each stellar model a sequence of differentially rotating stars—from the non rotating case up to very rapidly rotating configurations. For each member of the rotating sequence we evolve in time the linearised equations and extract the mode frequencies via a FFT on the evolved quantities. For unstable models, we identify the mode frequency—presumably related to the instability being triggered—and determine the presence of a

helps identify the location inside the star where the low-T/W instability develops.

The canonical energy is given by (Friedman & Schutz 1978a):

$$E_c = \frac{1}{2} \int dr \left[ \rho |\partial_r \xi_i|^2 - \rho |v^j \nabla_j \xi_i|^2 + \rho \xi^i \xi^j \nabla_i \nabla_j (h + \Phi) \right] + \frac{\partial}{\partial \rho} \left[ \rho |^2 - \frac{1}{4\pi G} |\nabla \delta \Phi|^2 \right],$$

while the canonical angular momentum follows from:

$$J_c = -\text{Re} \int d r \rho \partial_r \xi^i \left( \partial_i \xi_i + v^j \nabla_j \xi_i \right).$$

The integrals are calculated over the star’s volume, and Re denotes the real part. Note that these expressions are given in a coordinate basis, not the orthonormal basis used elsewhere in the paper.

2.6 Code description and tests

We use the code developed by Passamonti & Andersson (2015) to study the time evolution of the linearised equations. The numerical grid is two-dimensional in the coordinates $(r, \theta)$ which lie in the range: $0 \leq r \leq R(\theta)$ and $0 \leq \theta \leq \pi/2$. With a new definition of the radial coordinates $x = r/R(\theta)$ we adapt the grid to the star even when the fluid is highly deformed by rotation. The perturbation equations are discretized on the grid and updated in time with a Mac-Cormack algorithm. Finally, the numerical simulations are stabilised from high frequency noise with the implementation of a second order Kreiss-Oliger numerical dissipation $\varepsilon D_D \xi$, with $\varepsilon D \approx 0.01$. More technical details on the numerical implementation can be found in Passamonti & Andersson (2015) and Passamonti et al. (2009a,b).

Most of the results discussed in this paper were obtained using a $48 \times 90$ grid to cover the $\theta$ and $r$ coordinates, respectively. In order to test the accuracy of our instability growth time extraction we evolved some models on a $96 \times 180$ grid.

$\frac{\rho}{m}$
associated co-rotation radius. Finally, we monitor both the canonical energy density and the angular momentum density to check that these quantities have the expected growth on both sides of the co-rotation radius (Saijo & Yoshida 2006; Passamonti & Andersson 2015).

To establish the growth time $\tau$ of the unstable mode, we focus on the kinetic energy,

$$E_k = \frac{1}{2} \int dr \rho \delta \omega^2,$$

and assume that

$$E_k \sim e^{2\omega t}, \quad \text{where} \quad \omega t = \frac{2\pi}{\tau}.$$  \hspace{1cm} (26)

When the instability sets in the kinetic energy starts to grow exponentially and we determine $\omega t$ from a linear fit of the time evolved energy (see Passamonti & Andersson 2015, for more details).

3.1 The f-mode

Before we consider the low-T/W instability we explore the effects of the new differential laws on the f-mode frequencies. We focus on the quadrupole mode ($l = m = 2$), which is the most important for gravitational wave emission. Figure 2 shows the $^4f$ f-mode frequencies, measured in the inertial reference frame, for the eight sequences of models illustrated in figure 1. In the left-hand panel, all stars have $p = 1$ and $A = 0.317$ but different values of $B$ (see legend of figure 2). The sequence associated with the j-constant law has the lowest level of differential rotation and is simply obtained by setting $B = 100$. Similarly, we show in the right-hand panel of figure 2 the f-mode frequencies for models with $p = 1.8$ and $A = 0.6$ but different $B$. The fastest spinning stellar models of these sequences assume a “toroidal-like” configuration with a small axis ratio and a mass density whose maximum is shifted away from the rotation axis.

The $^2f$ mode is split by rotation, as expected, into two branches, which are prograde and retrograde with respect to the star rotation. In both panels of figure 2, we notice a similar behaviour of the mode frequencies with regard to $\beta = T/W$, the ratio between the rotational kinetic energy and gravitational potential energy. For lower rotation rates the splitting of the $^2f$ mode seems largely independent of the parameter $B$. In fact, figure 2 suggests that models with $p = 1$ and $A = 0.317$ have very similar f-mode frequencies when $\beta \leq 0.4$. Models with $p = 1.8$ and $A = 0.6$ shows the same behaviour for $\beta \leq 0.10 – 0.11$. The main lesson may be that, up to these rotation rates the f-mode frequency is adequately described by the usual j-constant prescription. For faster rotation rates, the parameter $B$ strongly affects the mode frequencies but apparently not in a unique way. For models with $p = 1$ and $A = 0.317$ we see that the mode frequencies decrease comparing the j-constant sequence to models with lower $B$. In particular, the prograde f-mode branch shows a gradual variation with $B$. Models with $p = 1.8$ and $A = 0.6$ have the same overall trend with a varying $B$, although for the $B = 0.4$ case the frequencies are slightly higher than the j-constant models.

The oscillation frequency of the retrograde f-mode generally decreases with rotation and may become negative for rapidly rotating models. The neutral point, where the inertial frame f-mode frequency passes through zero, marks the point at which an f-mode is first driven unstable by gravitational radiation via the well-known Chandrasekhar-Friedman-Schutz (CFS) mechanism (Chandrasekhar 1970; Friedman & Schutz 1975, 1978a). This secular instability occurs when a locally retrograde mode is dragged forward by the star’s rotation to the point where it is seen to be prograde by an inertial observer. The results in figure 2 demonstrate how the onset of the CFS instability changes with the rotational parameters, $B$ in particular. However, there does
not appear to be an obvious link between (say) a decrease in $B$ and an earlier onset of the instability.

### 3.2 Instability growth time

The current understanding is that the low-T/W instability sets in when the f-mode enters the co-rotation region. As this region is larger for models with a higher degree of differential rotation, we expect that the instability may set in even for very slowly rotating stars. A key feature of the new differential rotation law (10) is that the rotation rate may have a maximum displaced from the rotation axis. This characteristic is interesting because the star can then have two co-rotation radii for a given mode frequency (see for example figure 8) and it is worth exploring at which of the two positions the instability develops.

Watts et al. (2005) suggested that the growth rate of the instability should depend on the position of the mode relative to the boundary of the co-rotation region, being more rapid deep inside the co-rotation region and essentially vanishing at the co-rotation boundary (as the mode stabilizes). As in Passamonti & Andersson (2015), we find that the instability growth rate is, indeed, faster (larger $\omega_I$) when the f-mode is well inside the co-rotation region while it gradually increases (smaller $\omega_I$) towards the boundary of the co-rotation region. This accords, at least qualitatively, with expectations.

In figure 3, we show the results for differentially rotating models with $p = 1$, $A = 0.317$ and different values for $B$ (see legend). As expected, the model with the highest degree of differential rotation ($B = 0.13$) is unstable for a lower rotation rate ($\beta \gtrsim 0.02$). We note that the instability rate decreases for increasing $B$ and that the fastest growth time is reached for a model with $\beta \simeq 0.125$ and described by the j-constant law. As observed by Saijo & Yoshida (2006) and confirmed by Passamonti & Andersson (2015), the canonical energy and angular momentum integrands generally grow during a low-T/W instability while they pass through zero at (or, at least, close to) the co-rotation point. Therefore, in order to confirm that the instability is driven by the $l = m = 2$ f-mode we monitor the canonical energy and angular momentum integrands, see equations (23)-(24). Typical results are provided in figure 4, which shows the canonical energy density for a selection of rotating models with $p = 1$, $A = 0.317$ and $B = 0.15$. It is clear that $E_c$ grows in the region close to the co-rotation point while it vanishes at $\varpi_{cor}$. Exactly as expected.

The instability growth rate for stellar models with $p = 1.8$ and $A = 0.6$ is shown in figure 5. Models with a higher degree of differential rotation develop the instability at lower values $\beta$, again as expected. However, we now find a different behaviour compared to the previous models ($p = 1$ and $A = 0.317$). In this case, the fastest unstable mode is not associated with the j-constant law but a model with $B = 0.316$ which reaches the fastest growth time for $\beta \simeq 0.18$. The position of the maximum of $\omega_I$ with respect to the rotation parameter $\beta$ no longer changes monotonically with $B$. It moves towards higher rotation rates up to the model with $B = 0.45$. As we further increase $B$, the peak of $\omega_I$ slowly moves back towards lower $\beta$, gradually approach the j-constant result. Moreover, the maximum growth rate $\omega_I$ now progressively decreases with increasing $B$, at least for models with $B > 0.316$ (see figure 5).

### 3.3 An unstable inertial mode?

The results we have presented so far are not too surprising given the expected connection between the modes and the co-rotation region. The dependence on the detailed rotation law, and the parameter $B$ in particular, has not been studied before, but the results are (perhaps unfortunately) not easily summarized in terms of a general trend.

If the current understanding is correct, the low-T/W instability can be triggered by any oscillation mode which has (or develops) a co-rotation point within the star. In this respect, the sequence of rotating models with $p = 1.8$ and $A = 0.6$ demonstrates an interesting feature, present for the models with $B = 0.28$ and 0.316. In the oscillation spectrum (as obtained from the FFT) we notice an different unstable mode. The frequency of this new mode appears to be proportional to the star’s rotation rate and drives the instability in the slowest rotating models. Because of the general features, we interpret the behaviour as associated with an inertial mode (Lockitch & Friedman 1999).

More specifically, for rotating models with $B = 0.28$ and $\beta \lesssim 0.08$, the results in figure 5 exhibit an irregular bump before the main maximum of $\omega_I$. A similar feature is present in models with $B = 0.316$ for $\beta \lesssim 0.12$. For these rotation rates, the analysis of the FFT shows that the instability is not associated with by the f-mode. In the left-hand panel of figure 6 we show the FFT for a selection of rotating models with $p = 1.8$, $A = 0.6$ and $B = 0.316$. For the slower rotating models, the main peak in the spectrum is not the f-mode. As the rotation increases, say for $\beta \gtrsim 0.11$, there is a tran-
Figure 6. This figure shows the dominant unstable oscillation modes and their relation to the co-rotating region for models with $p = 1.8$, $A = 0.6$ and $B = 0.316$. In the left-hand panel, we show the power spectrum density of the enthalpy perturbation $δh$ for a selection of models with different rotation rates expressed in terms of $β$ (see legend). When $β < 0.12$ the inertial mode appears to drive the instability, while for $β ≥ 0.12$ the $2f$ mode is dominant. In the right-hand panel we show the pattern speed of the inertial and $2f$ modes and their variation with the rotation rate $β$. The grey zone represents the region where a mode can be in co-rotation with the star. The inertial mode is always inside this region, while the $f$-mode is in co-rotation when $β ≥ 0.047$.

Figure 7. Canonical energy density for a sequence of rotating models with $p = 1.8$, $A = 0.6$, $B = 0.316$. From the upper-left panel in the clockwise direction, the results refer to models with, respectively, $β = 0.013, 0.039, 0.101, 0.150, 0.179$ and $0.198$. The white vertical line indicates the co-rotation radius for a specific oscillation mode. In the three upper panels the instability is driven by an inertial mode, while it is driven by the $2f$ mode in the three (faster rotating) models shown in the lower panels.
sition at which the \( \beta \) mode becomes the dominant unstable mode. Tracking the oscillation modes along the rotating sequence we obtain the results for the pattern speed shown in the right-hand panel of figure 6. The inertial mode always lies inside the co-rotation region while the \( \beta \) mode is not co-rotating in slowly rotating models. It enters the co-rotation region when \( \beta \geq 0.05 \) but does not become the dominant unstable mode until it is well inside the co-rotation region, i.e. when \( \beta \geq 0.11 \). This behaviour is new, and possibly unexpected given previous analyses suggesting that the inertial r-mode does not exhibit the low-T/W instability in models determined from the j-constant rotation law (Karino et al. 2001; Passamonti & Andersson 2015). The result certainly raises the question whether the r mode is unstable with the new rotation laws, an issue that we will address in future work.

As before, the canonical energy density confirms the association of the unstable mode frequencies and the co-rotation radius \( \varpi_{cor} \). For slower rotating models, the results in figure 7 show that \( E_c \) grows around the co-rotation radius \( \varpi_{cor} \) which in this case corresponds to the inertial mode. After the transition (with increasing \( \beta \)), \( E_c \) blows up at the co-rotation radius of the \( \beta \) oscillation mode.

In order to identify the co-rotation radius of an unstable oscillation mode, we need to compare the mode pattern speed with the star’s rotation profile. For the new sets of rotating models, an oscillation mode may (at least in principle) have two co-rotation radii. In figure 8, we present the relation between the mode pattern speed and the rotation profile for a selection of rotating models with \( p = 1.8 \), \( A = 0.6 \) and \( B = 0.316 \). These are the models which were studied in figure 6. When \( \beta = 0.026 \), the instability is driven by the inertial mode which potentially has two co-rotation radii, in figure 8 indicated by two black circles. However, from the evolution of the canonical energy density the instability seems to develop mainly at one of these two points, i.e. \( \varpi_{cor} = 0.296 \). In figure 8 this point is denoted by the filled black circle, while the empty black circle corresponds to the co-rotation radius where we do not see any significant canonical energy density growth (\( \varpi_{cor} = 0.369 \)). Of course, with our numerical simulation approach we are not able to establish whether the instability at \( \varpi_{cor} = 0.369 \) does not develop at all or simply has a slower growth rate compared to the position \( \varpi_{cor} = 0.296 \). For models with \( \beta = 0.04 \) and 0.10 we find a similar behaviour (red and green circles in figure 8). When the rotation reaches \( \beta = 0.118 \) both the inertial and the \( \beta \) modes are unstable, but the \( \beta \) mode begins to dominate (blue circles in figure 8). Finally, when \( \beta = 0.15 \), the \( \beta \) mode is the fastest growing unstable mode and its co-rotation radii are shown as magenta circles in figure 8. For this rotating sequence, the results suggest that the inertial mode always develops the instability at the smaller co-rotation radius, while the \( \beta \) mode mainly grows at the larger \( \varpi_{cor} \). We do not yet have a clear explanation for this behaviour.

## 4 CONCLUDING REMARKS

We have explored the impact of the differential rotation law on the low-T/W instability (and the \( l = m = 2 \) \( \beta \) mode), using the numerical framework developed by Passamonti & Andersson (2015). We numerically evolved in time linear perturbations of differentially rotating stars in Newtonian gravity and extracted the information required to study mode frequencies and the properties of the low-T/W instability. We focussed on rotating configurations which are more complex than the standard j-constant law, implementing the three-parameter rotation law from Uryū et al. (2017). We focussed on models that capture the main rotational features observed in nonlinear simulations of hypermassive neutron-star merger remnants (Kastaun & Galeazzi 2015; Hanauske et al. 2017; Kastaun et al. 2017; De Pietri et al. 2019).

Our results confirm the relation between the instability and oscillation modes in co-rotation. When an \( \beta \) mode enters the co-rotation region the instability sets in and its growth time depends on the location of the co-rotation point. The instability grows more rapidly when the mode is well inside the co-rotation region, while it slows down near the boundaries. This behaviour was suggested by Watts et al. (2005) and confirmed by Passamonti & Andersson (2015). Our results generally support the previous evidence.

We have demonstrated that the \( \beta \) mode frequencies are influenced by the new rotation laws only beyond a specific rotation rate, expressed in terms of \( \beta = T/|W| \). This rotation threshold depends on the specific rotation law and its parameters. For example, for rotating models with \( p = 1 \),
\( A = 0.317 \) the variation of \( B \) has a clear effect on the \( f \) mode frequencies only when \( \beta \gtrsim 0.4 \). In contrast, the case with \( p = 1.8, A = 0.6 \) requires \( \beta \gtrsim 0.10 \) − 0.11. At this moment, we cannot identify a general trend from these two sets of rotating sequences. An analysis based on a larger parameter space would be required.

We also find that the rotation law influences the instability growth time at any rotation rate. Apparently, any parameter of the rotation law may affect the instability growth. For instance, all the rotating models with \( p = 1, A = 0.317 \) and different \( B \) have a larger growth time compared to the \( j \)-constant case with \( A = 0.317 \). The opposite result is obtained for rotating models with \( p = 1.8 \) and \( A = 0.6 \). In this case, the instability develops faster for a model with \( B = 0.316 \), while the \( j \)-constant law with \( A = 0.6 \) has the largest growth time.

For some rotating models, we also identified an unstable inertial mode. This mode triggers the instability when the star is slowly rotating and the \( f \)-mode is either not co-rotating or is still close to the boundary of the co-rotation region. For more rapidly rotating models the \( f \)-mode lies well inside the co-rotation band, and tends to dominate the instability. This interplay between \( f \) and inertial modes is present in stellar models where the maximum rotation rate is strong and clearly offset from the rotation axis. In order to determine a relation between the instability growth time and the stellar parameters we would need to carry out a more extensive analysis of the various models.

The Newtonian framework is not accurate enough to provide results and templates for gravitational-wave astronomy. We can only provide qualitative evidence. In this respect, we have shown that the instability growth time and the mode frequencies strongly depend on the rotation law. It is perhaps particularly interesting that inertial modes can trigger, for some rotating models, the low-T/W instability and hence amplify the gravitational-wave signal. Inertial modes (as well the gravity g-modes) can be excited in differentially rotating merger remnants through convective instabilities and therefore potentially drive the instability (De Pietri et al. 2018, 2019). However, it would be interesting to understand how the Cowling approximation affects this result. The \( 2f \) mode frequencies decrease by roughly 20–30% when the gravitational potential perturbation and the linearised Poisson equation are added to the hydrodynamical problem (Karino 2003; Passamonti & Andersson 2015). In this situation, the \( 2f \) mode may enter the co-rotation band at lower rotation rates and therefore become unstable, most likely being the dominant mode. The consequence could be that the inertial mode have less opportunity to power the instability.

Further work is needed to improve and confirm the conclusions for other rotation laws and more realistic equation of state. Further Newtonian work should provide a better insight into the nature of the low-T/W instability, but quantitative studies for realistic neutron star matter will require a fully relativistic analysis.

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