Effect of bath temperature on the quantum decoherence

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Abstract

The dynamics of a qubit under the decoherence of a two level fluctuator (TLF) in addition to its coupling to a bosonic bath is investigated theoretically based on a unitary transformation. With the merit of our unitary transformation, non-Markov effect can be taken into account. It shows that quantum decoherence of the qubit can either be reduced or be increased with increasing bath temperature $T$ or with increasing TLF-bath coupling.

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Quantum computation reveals advantages over classical one for its highly efficient parallel calculation and therefore attracts a wide interest among scientists\[1, 2\]. Today, the biggest obstacle is the quantum decoherence due to environmental noise, especially the intrinsic slow noise caused by the two level fluctuators (TLFs)\[3\]. To overcome this difficulty, great effort has been made to design a well behaved qubit, and many methods have been proposed such as optimum working point\[4\], dynamical decoupling\[5\], and etc.\[6, 7\]. In this paper, we focus on the temperature dependent properties of a qubit under the influence of such noise. It is commonly believed that environmental disturbance can be reduced by decreasing the bath temperature and the system-bath coupling. However, an interesting paper shown that the decoherence can be reduced by increasing the thermal bath temperature for quantum systems coupled directly only to a few modes\[8\]. In that paper, the authors introduced a full quantum model in which the concerned two-state system (TSS-A) is only coupled to another TSS-B, with the latter being dissipated by a multi-mode thermal bath (See Fig. 1). Therefore, TSS-B and the multi-mode bath together can be seen as a non-Markov environment of TSS-A. In this paper, we treat the model more rigorously without making rotating-wave approximation (RWA) and Markov approximation, and find that the decoherence of TSS-A can either be reduced or increased with increasing bath temperature \(T\) depending the couplings conditions. When A-B coupling is smaller than B-bath coupling (condition 1), decoherence reduction with temperature happens as shown in Ref. \[8\]. When A-B coupling is larger than B-bath coupling (condition 2), only decoherence enhancing with temperature is expected. Further more, we find that, the decoherence of TSS-A can also be reduced with increasing B-bath coupling when condition 1 is satisfied.

The model is given as \((\hbar = 1)\)\[8, 9, 10, 11, 12, 13\]:

\[
H = H_A + H_{AB} + H_B
\]

with

\[
H_A = \frac{\Delta_A}{2} \sigma_x^A, \quad H_{AB} = g_0 \sigma_x^A \sigma_x^B, \quad H_B = \frac{\Delta_B}{2} \sigma_x^B + \sum_k \omega_k b_k^\dagger b_k + \frac{\sigma_x^B}{2} \sum_k g_k (b_k^\dagger + b_k),
\]

where TSSs are characterized by pseudospin-1/2 operators \(\sigma_x^A\) and \(\sigma_x^B\) as usual, \(b_k\) and \(b_k^\dagger\) are the annihilation and creation operators of the bath mode, \(g_0\) and \(g_k\) are the coupling constants. Here we use the convention used in Ref. \[8\], where only transverse coupling are included. The bath is fully defined by the spectral density \(J(\omega) = \sum_k g_k^2 \delta(\omega - \omega_k)\).
Suppose the TSSs are the double quantum dots (DQDs) manufactured with GaAs, we have the following spectral density \([14, 15]\),

\[
J(\omega) = \alpha_{pz} \omega \left( 1 - \frac{\omega_d}{\omega} \sin \frac{\omega}{\omega_d} \right) e^{-\omega^2/2\omega_d^2}.
\] (3)

As in Ref. \([16, 17]\), we apply a unitary transformation to the Hamiltonian, \(H' = \exp(S)H\exp(-S)\), with the generator \(S \equiv \sum_k \frac{g_k}{2\omega_k} \xi_k (b_k^\dagger - b_k) \sigma_z^B\). Note that this unitary transformation only affects \(H_B\) of the total Hamiltonian. After the transformation, \(H_B\) can be decomposed into three parts: \(H'_B = H'_0 + H'_1 + H'_2\), with

\[
H'_0 = \frac{\eta \Delta_B}{2} \sigma_x^B + \sum_k \omega_k b_k^\dagger b_k - \sum_k \frac{g_k^2}{4\omega_k} \xi_k (2 - \xi_k),
\] (4)

\[
H'_1 = \frac{\sigma_x^B}{2} \sum_k g_k (1 - \xi_k)(b_k^\dagger + b_k) + \frac{i\sigma_y^B}{2} \eta \Delta_B X,
\] (5)

\[
H'_2 = \frac{\sigma_x^B}{2} \Delta_B (\cosh X - \eta) + \frac{i\sigma_y^B}{2} \Delta_B (\sinh X - \eta X),
\] (6)

where \(X \equiv \sum_k \frac{g_k}{\omega_k} \xi_k (b_k^\dagger - b_k)\) and \(\eta\) is the thermodynamic average of \(\cosh X\),

\[
\eta = \exp \left[ -\sum_k \frac{g_k^2}{2\omega_k} \xi_k^2 \coth(\beta \omega_k/2) \right],
\] (7)

which insures \(H'_2\) contains only the terms of two-boson and multi-boson non-diagonal transitions and its contribution to physical quantities is \((g_k^2)^2\) and higher. Therefore, \(H'_2\) will be omitted in the following discussion.

The introduced parameter \(\xi_k\) is an important point here. If \(\xi_k = 0\) for all \(k\), that is, without the transformation, the perturbation expansion would be similar to the standard weak-coupling expansion (Bloch-Redfield theory). Besides, if \(\xi_k = 1\) for all \(k\), then our transformation is the usual polaronic transformation and the perturbation expansion is for the small parameter \(\Delta\) which is equivalent to the NIBA\([21]\). Here, we choose \(\xi_k\)’s as\([22]\):

\[
\xi_k = \frac{\omega_k}{\omega_k + \eta \Delta_B},
\] (8)

\(H'_1\) is of rotating-wave form,

\[
H'_1 = \sum_k V_k (b_k^\dagger \sigma_x^B + b_k \sigma_x^B),
\] (9)

with \(V_k = g_k \eta \Delta_B / (\omega_k + \eta \Delta_B)\) and \(\sigma_x^B \equiv (\sigma_x^B \mp i\sigma_y^B)/2\). It is easy to check that \(H'_1 |g_0\rangle = 0\), where \(|g_0\rangle\) is the ground state of \(H'_0\). Note that, the transformed Hamiltonian \(H'_B = H'_0 + H'_1\)
is of similar form of that of RWA, but $\Delta_B$ and $g_k/2$ are renormalized by $\eta \Delta_B$ and $V_k$ due to the contributions of anti-rotating terms. Here, $\eta \Delta_B$ gives a rough approximation of the renormalized qubit frequency and $(1 - \eta) \Delta_B$ is the corresponding Lamb shift of TSS-B in the sense of Quantum Optics $^{[18]}$.

At this point, we write out the master equation for this nonlinear system by treating $H_0 = H_A + H'_B$ as unperturbed part, and $H_{AB}$ as perturbation. Suppose $g_0 \ll \Delta_A, \Delta_B$, to the extent of second order of $g_0$, the reduced master equation of TSS-A is $^{[19, 20]}$:

\[
\frac{\partial \rho_A(t)}{\partial t} = -i [H_A, \rho_A(t)] - \int_0^t dt' X(t, t').
\]  

(10)

with

\[
X(t, t') \equiv \text{Tr}_B \left[ H_{AB}, e^{-iH_0(t-t')} \left[ H_{AB}, \rho_A(t') \otimes \rho_B \right] e^{iH_0(t-t')} \right]
\]

\[
= g_0^2 G_1(t - t') \sigma_z^A e^{-iH_A(t-t')} \left[ \sigma_z^A, \rho_A(t') \right] e^{iH_A(t-t')}
\]

\[
+ g_0^2 G_2(t - t') e^{-iH_A(t-t')} \left[ \rho_A(t'), \sigma_z^A \right] e^{iH_A(t-t')} \sigma_z^A,
\]

where, we have assumed that the density matrix of A-B can be decoupled as $\rho_A \otimes \rho_B$ with $\rho_B = \exp(-\beta H'_B)/\text{Tr}_B \left[ \exp(-\beta H'_B) \right]$ being the thermal equilibrium density matrix of TSS-B. $G_1(t)$ and $G_2(t)$ are the correlation functions $\langle \sigma_z^B(t) \sigma_z^B \rangle_{\beta}$ and $\langle \sigma_z^B \sigma_z^B(t) \rangle_{\beta}$, in which $\langle \cdots \rangle_{\beta}$ represents the average with thermodynamic probability $\rho_B$. According to the fluctuation dissipation theory (FDT), $G_1(t)$ and $G_2(t)$ can be expressed by the Green’s function of frequency $\langle \langle \sigma_z^B; \sigma_z^B \rangle \rangle_{\omega}$. By solving the equation chain of motion, we can get the frequency domain of $G_1(t)$ and $G_2(t)$ as $G_1(\omega) = G(\omega)/(1 + e^{-\beta \omega})$ and $G_2(\omega) = G(\omega)/(1 + e^{\beta \omega})$ with (See Appendix)

\[
G(\omega) = \frac{\gamma(|\omega|)/\pi}{|\omega| - \eta \Delta_B - R(|\omega|)^2 + \gamma^2(|\omega|)}.
\]  

(11)

where, $|\cdots|$ mean absolute value, $R(\omega)$ and $\gamma(\omega)$ are the real and imaginary parts of $\sum_k V_k^2(2n_k + 1)/(\omega - \eta \omega^* - \omega_k)$,

\[
R(\omega) = \varphi \int_0^\infty d\omega' \frac{(\eta \Delta_B)^2 J(\omega') \coth(\beta \omega'/2)}{(\omega' + \eta \Delta_B)^2 (\omega - \omega')},
\]  

(12)

\[
\gamma(\omega) = \pi(\eta \Delta_B)^2 J(\omega) \coth(\beta \omega/2)/(\omega + \eta \Delta_B)^2,
\]  

(13)

where $\varphi$ means the Cauchy principal value.

The master equation Eq. (10) which is a $2 \times 2$ matrix equation can be solved exactly by the Laplace transform method since the convolution theorem can be applied to the equation
of each matrix element. Suppose the system is in the upper eigenstate of \( \sigma_z \) at the initial time \( t=0 \), \( \rho_A(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \). We can get the Laplace transform of population difference \( \langle \sigma_z^A(t) \rangle \equiv \text{Tr}_A(\sigma_z^A \rho_A(t)) \) as

\[
\langle \sigma_z^A(P) \rangle = \frac{P + 2F}{P^2 + 2PF + \Delta_A^2},
\]

where, \( F \equiv g_0^2 \int_{-\infty}^{\infty} d\omega G(\omega)/(P + i\omega) \). After the inverse Laplace transformation and a parameter change of \( P = 0^+ + i\omega \), we have the population difference \( P(t) \equiv \langle \sigma_z^A(t) \rangle \),

\[
P(t) = \text{Re} \left\{ \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{[\omega - 2\Sigma_F(\omega) - 2i\Gamma_F(\omega)]e^{-i\omega t}}{\omega^2 - 2\omega\Sigma_F(\omega) - \Delta_A^2 - 2i\omega\Gamma_F(\omega)} \right\},
\]

where, Re means the real part of a complex number, \( \Gamma_F(\omega) = \Gamma(\omega) + \Gamma(-\omega) \) and \( \Sigma_F(\omega) = \Sigma(\omega) - \Sigma(-\omega) \) are the imaginary and real parts of \( F(\omega) = g_0^2 \int_{-\infty}^{\infty} d\omega' \frac{\text{G}(\omega)}{\omega - \omega'} \),

\[
\Gamma(\omega) \equiv \pi J'(\omega), \Sigma(\omega) \equiv \varphi \int_0^{\infty} d\omega' \frac{J'(\omega')}{\omega - \omega'},
\]

where we have applied \( \frac{1}{\omega + i\delta} = \frac{1}{\omega} - i\pi\delta(\omega) \), \( \varphi \) means the Cauchy principal value, and \( J'(\omega) \equiv g_0^2 G(\omega)\theta(\omega) \).

Before doing any numerical calculation, we would like to summarize the approximations we have made. Three approximations are made: The first one is the omission of \( H_2' \), which is a 4th order approximation to the B-bath coupling. \( H_2' \) contains only multi-boson non-diagonal transition (like \( \delta kkb'k' \) and \( \delta^\dagger kkb'k' \)) whose contribution is also small in low temperature. The second one is to approximate \( (\delta kkb'k' + \delta^\dagger kkb'k') \approx (2n_k + 1)\delta_{kk'} \) when calculating the correlator in the Appendix. The third one is the Born approximation for deriving the master equation (10). Therefore, our treatment is applicable in low temperature for \( \alpha \ll 1 \) and \( g_0 \ll \Delta_A, \Delta_B \).

Now, we are in position of calculating \( P(t) \). We first use Eq. (3),(7) and (8) to get \( \eta \) self consistently, then calculate \( P(t) \) numerically by using Eq. (3),(11)-(13) and (14)-(15). We report \( P(t) \) as a function of time in Fig. 2 and Fig. 3 for \( \Delta_A = \Delta_B = 0.1\omega_l \), \( g_0 = 0.1\Delta_A \) and \( \omega_d = 0.05\omega_l \). In Fig. 2, \( \alpha \) is larger than \( g_0/\Delta_A \) which is set to be \( \alpha = 0.3 \), it shows that the decoherence is reduced by increasing the bath temperature \( T \) as predicted in Ref. [8]. However, in Fig. 3, where \( \alpha = 0.01 \), the coherence is not meliorated but rather damaged with increasing \( T \). In Fig. 4, we set a fixed detuning between A-B: \( \delta = \Delta_A - \Delta_B = 0.05\omega_l \), the coherence is damaged with increasing \( T \) only because of the finite detuning \( \delta \) (compare
with Fig. 2). In Fig. 5, B-bath coupling constant $\alpha$ is varied with other parameter fixed, we can see the decoherence can also be reduced by increasing B-bath coupling.

Now we are hoping for a bit more physical insight into the problem. For the sake of simplicity, we discuss it under the RWA to $H_{AB}$, which is believed to be valid for the near resonance case ($\Delta_A \approx \Delta_B$). If RWA is applied to the interacting Hamiltonian $H_{AB}$, note that no RWA to B-bath interaction, according to the same procedure, the population difference $P(t)$ would be,

$$P(t) = \text{Re} \left\{ \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega t}}{\omega - \Delta_A - \Sigma(\omega) - i\Gamma(\omega)} \right\}$$  \hspace{1cm} (16)

with $\Sigma(\omega)$ and $\Gamma(\omega)$ also defined by Eq. (15). One can find that, this expression has exact the same form of the population difference of SBM under RWA, only the spectral density $J(\omega)$ is replaced by $J'(\omega) = g_0^2 G(\omega) \theta(\omega)$\cite{16, 17, 23}. Therefore, from the point of view of the qubit, it feels like under the dissipation of a bosonic bath with an structured spectral density $J'(\omega)$. If we assume the poles of $G(\omega)$ to be $\omega_B \pm i\gamma(\omega_B)$, with the renormalized frequency $\omega_B$ being the solution of equation: $\omega - \eta\Delta_B - R(\omega) = 0$ (Because of the property of the piezoelectric bath $J(\omega)$, only one solution is expected and its value is always less than $\Delta_B$). We can simplify $J'(\omega)$ as,

$$J'(\omega) = \frac{g_0^2 \gamma(\omega_B)/\pi}{|\omega - \omega_B|^2 + \gamma^2(\omega_B)}.$$ \hspace{1cm} (17)

Note that this approximation would loss some non-Markov effects of the bath on the TLF. However, we will see that the main result of our numerical calculation can be explained, since the vital point is the non-Markov effect of the TLF+bath environment on the qubit.

Approximately, the integral in Eq. (16) can be evaluated as $P(t) = \sum_p a_p e^{-a_p \gamma_p t} \cos \omega_p t$, where, $\gamma_p = \Gamma(\omega_p)$, $a_p$ is the weight of each pole and $\omega_p$ is the solution of equation

$$\omega - \Delta_A - \Sigma(\omega) = 0.$$ \hspace{1cm} (18)

The above equation usually has two solutions $\omega_{\pm}$, and correspondingly, $a_p$ can be evaluated as $a_+ = \frac{\Delta_A - \omega_+}{\omega_+ - \omega_-}$ and $a_- = \frac{\omega_+ - \Delta_A}{\omega_+ - \omega_-}$\cite{24}. In the limit of $\alpha_{pz} \to 0$, that is $\gamma(\omega_B) \to 0$, $J'(\omega) \to g_0^2 / \pi \delta(\omega - \omega_B)$ \ , then $\omega_p$ can be solved as, $\omega_p = \frac{1}{2} \left[ (\Delta_A + \omega_B) \pm \sqrt{(\Delta_A - \omega_B)^2 + 4g_0^2} \right]$. Suppose the resonance condition $\Delta_A = \omega_B \lesssim \Delta_B$ is satisfied, then $\omega_p = \Delta_A \pm g_0$ (also see the inset of Fig. 3), which fully agree with the result of the simple Jaynes-Cummings model\cite{25}. In this case, the decoherent rate is $\gamma_A = \gamma_p = \pi J'(\omega_p)$. From Eq. (11) and Eq. (13),

$$\gamma_A \approx \frac{g_0^2 \gamma(\Delta_A)}{g_0^2 + \gamma^2(\Delta_A)},$$ \hspace{1cm} (19)
where, $\gamma(\Delta_A) = \gamma_0 \coth(\beta \Delta_A/2)$ with $\gamma_0 \equiv \pi (\eta \Delta_B)^2 J(\omega_B)/(\omega_B + \eta \Delta_B)^2 \approx \pi J(\Delta_A)/4$. When $T$ is large, we have $\gamma(\Delta_A) \gg g_0$, therefore, monotonic increasing function of the coherence with temperature is expected as in Ref. [8]. Nevertheless, when $T$ is large, other decoherence mechanism and multi-phonon process may take place, and our treatment is no longer suitable. Therefore, we are interested in whether temperature can indeed reduce decoherence at low temperature. When $T$ is small, $g_0$ and $\gamma(\Delta_A)$ would compete with each other in the denominator of Eq. [19]. Which one would dominate depends on the coupling condition, e.g., $\gamma(\Delta_A)$ would dominate when $g_0 \ll \alpha_{pz} \Delta_A$ and $g_0$ dominate when $\alpha_{pz} \Delta_A \ll g_0$. Therefore, for small $T$, $\gamma_A$ can be expressed as

$$\gamma_A = \begin{cases} 
\frac{g_0^2}{\gamma_0 \coth(\beta \Delta_A/2)}, & g_0 \ll \alpha_{pz} \Delta_A \ll \Delta_A \approx \Delta_B \\
\gamma_0 \coth(\beta \Delta_A/2), & \alpha_{pz} \Delta_A \ll g_0 \ll \Delta_A \approx \Delta_B
\end{cases} \quad (20)$$

That is, decoherence reduction with increasing $T$ and $\alpha_{pz}$ happens only when $g_0 \ll \alpha_{pz} \Delta_A \ll \Delta_A \approx \Delta_B$, which is the region Markov approximation applies [26]. When $\alpha_{pz} \Delta_A \ll g_0$ non-Markov effect becomes important, and different temperature dependent behavior comes out.

In conclusion, without making RWA and Markov approximation, the dynamics of a qubit coupled with nonlinear bath was investigated through a perturbation method based on a unitary transformation. We find that: 1. The decoherence of TSS-A can either be reduced or increased with increasing bath temperature $T$. The couplings condition controls the temperature dependent behavior. Decoherence reduction with $T$ happens only when the A-B coupling is smaller than B-bath coupling. 2. Finite detuning between TSS-A and TSS-B would destroy this decoherence reduction effect. It would benefit by setting $\Delta_A$ a little smaller than $\Delta_B$, since the effective level spacing of TSS-B $\omega_B$ is always smaller than original $\Delta_B$ because of the dressing of phonons. 3. Keeping $g_0 \ll \alpha_{pz} \Delta_A \ll \Delta_A \approx \Delta_B$ but increasing $\alpha_{pz}$ also reduces the decoherence of TSS-A.

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**APPENDIX A: THE CALCULATION OF CORRELATION FUNCTIONS**

For the sake of simplicity, we can apply a unitary transformation to \( H_B' \) with the generator \( \tilde{S} = i\pi \sigma_y^B / 4 + i\pi \sum_k \omega_k b_k^\dagger b_k \), the Hamiltonian \( H_B'' \) becomes,

\[
H_B'' = \frac{\eta \Delta_B}{2} \sigma_z^B + \sum_k \omega_k b_k^\dagger b_k + \sum_k V_k (b_k^\dagger \sigma_-^B + b_k \sigma_+^B),
\]

where the definition of \( \sigma_\pm^B \) becomes \( \sigma_\pm^B \equiv (\sigma_x^B \pm i\sigma_y^B) / 2 \). Correspondingly, \( G_1(t) \) and \( G_2(t) \) become \( \langle \sigma_x^B(t) \sigma_x^B \rangle_\beta' \) and \( \langle \sigma_x^B(t) \sigma_x^B \rangle_\beta'' \).

The equation of motion is given by

\[
\omega \langle\langle A|B\rangle\rangle_\omega = \langle\langle A, B, t\rangle\rangle_n'' + \langle\langle [A, H_B'']|B\rangle\rangle_\omega'',
\]

where, \( \langle\langle A|B\rangle\rangle_\omega \) represents the Fourier transformation of the Green’s function \(-i\theta(t)\langle\langle A, B, t\rangle\rangle_\beta'\). Then, we can get the equation chain of \( \langle\langle \sigma_x^B|\sigma_x^B \rangle\rangle_\omega'', \langle\langle i\sigma_y^B|\sigma_x^B \rangle\rangle_\omega'' \), \( \langle\sigma_x(b_k^\dagger + b_k)|\sigma_x^B \rangle \) and \( \langle\sigma_x(b_k^\dagger - b_k)|\sigma_x^B \rangle \). Since, \([\sigma_x^B b_k^\dagger, b_k \sigma_+^B] = 2b_k^\dagger b_k + 1 \sigma_+^B \) \( [\sigma_x^B b_k, b_k^\dagger \sigma_-^B] = -2(b_k^\dagger b_k + 1) \sigma_-^B \), we have,

\[
[\sigma_x^B (b_k^\dagger + b_k), (b_k^\dagger \sigma_-^B + b_k \sigma_+^B)] = (2n_k + 1) i\sigma_y^B,
\]

\[
[\sigma_x^B (b_k^\dagger - b_k), (b_k^\dagger \sigma_-^B + b_k \sigma_+^B)] = (2n_k + 1) \sigma_x^B,
\]

where, we have substituted \( b_k^\dagger b_k \) by its thermodynamic average value \( n_k \) and omitted all the \( b_k^\dagger b_k \) and \( b_k b_k \) terms. This approximation makes the equation chain self-closed. By solving these equations, we get

\[
\langle\langle \sigma_x^B|\sigma_x^B \rangle\rangle_\omega'' = \frac{1}{\omega - \eta \Delta - \sum_k V_k^2 (2n_k + 1) / \omega - \omega_k} + \frac{1}{\omega + \eta \Delta - \sum_k V_k^2 (2n_k + 1) / \omega + \omega_k}
\]

According to FDT, \( G_1(t) \) and \( G_2(t) \) can be expressed by the corresponding retarded Green’s function \( \langle\langle \sigma_x^B|\sigma_x^B \rangle\rangle_\omega + i\theta^+ \), so we get

\[
G_1(\omega) = G(\omega) / (1 + e^{-\beta \omega}),
\]

\[
G_2(\omega) = G(\omega) / (1 + e^{\beta \omega}),
\]

with \( G(\omega) \equiv -1/\pi \text{Im} \langle\langle \sigma_x^B|\sigma_x^B \rangle\rangle_\omega + i\theta^+ \) which leads to Eq. (11).

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proportional to $1/\gamma(\omega_B)$. When B-bath coupling $\alpha_{pz}$ is large, $\gamma(\omega_B)$ is large. Therefore, the shape of $J'(\omega)$ is low and flat. Correspondingly, the correlation time is long and the Markov approximation is feasible.

**FIGURES CAPTIONS**

Fig. 1: diagrammatic sketch of a qubit coupled with a two level fluctuator

Fig. 2: $P(t)$ as a function of time in the case of $g_0 \ll \alpha_{pz}\Delta_A$, where the decoherence is reduced with $T$. Inset is the Fourier analysis of the main plot, where the half width of each plot characterizes the rate of decoherence. Since the integration of each plot $P(\omega)$ must be one as a result of the initial condition, the higher the peak is, the narrower it should be, therefore, the half width which indicate decoherence is also smaller. We can see the inset is consist with the main plot.

Fig. 3: $P(t)$ as a function of time in the case of $g_0 \gg \alpha_{pz}\Delta_A$, where the decoherence is enhanced with $T$. Inset: From the Fourier analysis of $P(t)$ one sees that two frequencies are dominating the dynamics and the peaks locate at $\Delta_A \pm g_0$ which agree with the result of Jaynes-Cummings model.

Fig. 4: Fourier analysis of $P(t)$ in the case of $g_0 \ll \alpha_{pz}\Delta_A$ with a finite detuning. The time domain of $P(t)$ is similar to the plots in Fig. 2, we only give the frequency domain here for clear and simplicity. The decoherence is enhanced with $T$ just because of the finite detuning compared to Fig.2.

Fig. 5: Fourier analysis of $P(t)$ with different B-bath coupling $\alpha_{pz}$ in the case of $g_0 \ll \alpha_{pz}\Delta_A$, the decoherence is reduced with increasing $\alpha_{pz}$.
$P(t)$

$T = 1 \Delta_A$, ($\eta = 0.77$)
$T = 5 \Delta_A$, ($\eta = 0.48$)
$T = 10 \Delta_A$, ($\eta = 0.24$)

$\Delta_A = 0.1 \omega_l$, $g_0 = 0.1 \Delta_A$, $\alpha = 0.3$, $\omega_d = 0.05 \omega_l$
\[ \Delta_A = 0.1\omega_p, \Delta_B = 0.05\omega_p, g_0 = 0.1\Delta_A, \alpha = 0.3, \omega_d = 0.05\omega_p \]

- \( T = 0.5\Delta_A, \) \( (\eta = 0.75) \)
- \( T = 1\Delta_A, \) \( (\eta = 0.71) \)
- \( T = 2\Delta_A, \) \( (\eta = 0.61) \)
\[ \Delta_A = \Delta_B = T = 0.1 \omega_1, g_0 = 0.1 \Delta_A, \omega_d = 0.05 \omega_1 \]

- \( \alpha = 0.3, \quad (\eta = 0.77) \)
- \( \alpha = 0.4, \quad (\eta = 0.71) \)
- \( \alpha = 0.5, \quad (\eta = 0.65) \)