Premium calculation on health insurance implementing deductible

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Abstract. Implementation of deductibles on insurance contracts is one of many ways to solve the adverse selection and moral hazard problems that often arise in health insurance. Deductibles will make the amount of expected loss for insurance companies lower. However, the deductible will also affect the amount of net premium. The higher the deductible, the lower the net premium that insurance company will get. If the claims amount are very large, then the insurance company would not be able to pay the claims because the amount of net premium that have been earned is small. Thus, the net premium principle is less suitable to use when the insurance companies. Then, they must find more suitable premium calculation method and in this paper, Proportional Hazard (PH) transform principle proposed by Wang being analyzed as an alternative premium when the insurer applies deductibles. The amount of PH transform premium will be higher than net premium and it has more slightly decrease when deductibles are applied compared to the amount of net premium.

Keywords: Deductible, health insurance, premium, proportional hazard transform

1. Introduction

There are problems that often arise in health insurance, namely adverse selection and moral hazard [1]. Adverse selection is a situation where participants who follow health insurance are only individuals who are at high risk. An example condition of adverse selection is when individuals who have high risk disguise themselves as individuals who have low risk with the aim of getting lower premiums [2]. Then, moral hazard is a behavior of individual health insurance participants who are less likely to avoid the risks that occur because they feel they have been protected by health insurance owned.

These problems need to be solved in order to reduce the possibility of health insurance companies suffering huge losses. One way to solve the problem is to apply deductible [1]. Deductible is the minimum loss that the insured can claim to the insurance company. Implementation of the deductible will affect the amount of premiums obtained by health insurance companies and generally will cause the premiums that received by insurer will decrease. Thus, health insurance companies need to be careful in determining the premium principle that used when applies deductibles.

This paper will focus on discussing the suitable premium principle that can be used by health insurance companies when apply deductibles. The premium that defined in this paper is pure premium which has not considered expense, commission and inflation. There are two principles of premium calculation that will be discussed, namely net premium principle and PH transform premium principle. Then, it will be compared to determine which one is more suitable for use by health insurance companies.
that implement deductibles. Furthermore, there will be data simulations to see the effect of deductibles and the premium principle that used by health insurance companies against the premiums to be earned.

2. Theoretical analysis
Consider the loss variable $X$ used in health insurance. There are several characteristics that explain the distribution of the loss variable $X$ [3], that are:

a. The cumulative distribution function

The cumulative distribution function denoted by $F_X(x)$ is a probability that $X$ is less than or equal to a given value $x$ or defined by

$$F_X(x) = \Pr(X \leq x). \quad (1)$$

b. The survival function

The survival function denoted by $S_X(x)$ is a probability that $X$ greater than a given value $x$ or defined by

$$S_X(x) = \Pr(X > x) = 1 - \Pr(X \leq x) = 1 - F_X(x). \quad (2)$$

The probability density function

The probability density function denoted by $f_X(x)$ is the first derivative of the cumulative distribution function $X$ or the negative value of the first derivative of the survival function $X$.

The definition is

$$f_X(x) = \frac{d}{dx} F_X(x) = -\frac{d}{dx} S_X(x). \quad (3)$$

The density function is only defined at the points where the value of the first derivative exists.

c. The hazard rate

The hazard rate state the rate of an observation experiences an event immediately after $x$ or denoted by

$$h_X(x) = \frac{f_X(x)}{S_X(x)} = \frac{\frac{d}{dx} (1 - S_X(x))}{S_X(x)} = -\frac{S_X'(x)}{S_X(x)} = -\frac{d}{dx} [\ln S_X(x)]. \quad (4)$$

Based on equation 4, we obtained the other form of survival function $S_X(x)$, that is

$$S_X(x) = e^{-\int_0^x h(u)du}. \quad (5)$$

Then, the loss variable $X$ will have the expected value $E(X)$ or defined by

$$E(X) = \int_{-\infty}^{\infty} x f(x)dx \quad (6)$$

or we can have

$$E(X) = \int_0^{\infty} S(x)dx \quad (7)$$

using partial integral formula.

In this paper, we will discuss about the loss variable $X$ that followed exponential’s distribution with parameter $\lambda$. The density function can be defined by $f_X(x) = \lambda e^{-\lambda x}$, $x > 0, \lambda > 0$. Based on the density function, the following distribution function forms can be found.
Then, the survival function is denoted by

$$F_X(x) = \Pr(X \leq x) = \int_0^x f(w)dw = \int_0^x \lambda e^{-\lambda w}dw = 1 - e^{-\lambda x}, \quad 0 \leq x < \infty, \lambda > 0. \quad (8)$$

Then, the survival function is denoted by

$$S_X(x) = 1 - F_X(x) = 1 - \left(1 - e^{-\lambda x}\right) = e^{-\lambda x}, \quad 0 \leq x < \infty, \lambda > 0. \quad (9)$$

The hazard rate is denoted by

$$h_X(x) = \frac{f_X(x)}{S_X(x)} = \frac{\lambda e^{-\lambda x}}{e^{-\lambda x}} = \lambda, \quad \lambda > 0 \quad (10)$$

and the expected value is denoted by

$$E(X) = \int_0^\infty S(x)dx = \int_0^x \lambda e^{-\lambda w}dw = \frac{1}{\lambda}. \quad (11)$$

When health insurance companies apply deductibles of $d$, then loss variable $X$ will be modified to left censored and shifted variable denoted by $(X - d)_+$ [3], that is

$$(X - d)_+ = \begin{cases} \ 0, & X \leq d \\ \ X - d, & X > d \end{cases} \quad (12)$$

where expected value for left censored and shifted variable is

$$E[(X - d)_+] = \int_0^d 0 f(x) \, dx + \int_d^\infty (x - d)f(x) \, dx. \quad (13)$$

Then, the expected value of the left censored and shifted variable can be used to calculate the premiums obtained by health insurance companies. The pricing rule based on the expectation of loss is referred to net premium principle. Net premium principle is one of pricing rules that most commonly used by health insurance companies. The premium obtained by health insurance companies under net premium principle is defined by $\pi_N(X) = E(X)$, where $\pi_N(X)$ called net premium. When health insurance companies applies deductible, so the amount of premium changes to

$$\pi_N[(X - d)_+] = E[(X - d)_+] \quad (14)$$

where the right side is defined by equation 10. The other variable related to the implementation of deductible is right censored variable or limited loss variable and defined by

$$Y = X \wedge d = \begin{cases} \ X, & X < d \\ \ d, & X \geq d \end{cases} \quad (15)$$

and the expected for limited loss variable is

$$E[(X \wedge d)] = \int_0^d (x)f(x) \, dx + \int_d^\infty df(x) \, dx \quad (16)$$

or we can have

$$E(X) = \int_0^d S(x)dx \quad (17)$$

using partial integral formula.

There is a difference between amount of premium earned under full coverage to amount of premium earned under deductible coverage. Implementation of the deductible cause a decrease in the value of the expected loss. Therefore, the amount of net premium earned by health insurance companies will also
decrease. The decrease can be seen based on the value of Loss Elimination Ratio (LER). The Loss Elimination Ratio (LER) is defined as the ratio of the decrease in expected loss under a deductible \( d \) to the expected loss full coverage \([1]\) and is given by

\[
LER_d = \frac{E(X) - [E(X) - E(X \lor d)]}{E(X)} = \frac{E(X \lor d)}{E(X)} = \frac{\int_{0}^{d} S(x) \, dx}{\int_{0}^{\infty} S(x) \, dx}.
\] (18)

Based on equation 17, we can conclude that if the insurance company apply deductible of \( d \), then their expected loss will decrease in the amount of LER. It can be proved that the greater deductible, the lower expected loss will be. If the health insurance company apply deductible and still use net premium principle to calculate their premiums, it will causes the probability of loss occur will be higher. Therefore, health insurance companies must bear other factors in calculating the amount of premiums, beside the expected of loss.

One of the factors that can be considered in calculating the amount of premiums earned by health insurance companies is risk loading \([4]\). Risk loading is used in the calculation of premiums as a consideration for health insurance companies to face the risks they had. One of the premium calculation principle that considering risk loading is Proportional Hazard (PH) transform.

PH transform premium principle transformed the original loss distribution to a loss distribution that contains risk loading. The transformation obtained by transformation of the hazard rate proportional to \( \rho \), that is

\[
h_X(x) = \frac{1}{\rho} h_X(x), \rho > 1.
\] (19)

Based on equation 5, we obtained another expression for equation 19

\[
S_\tilde{X}(x) = [S_X(x)]^{1/\rho}, \rho > 1.
\] (20)

The amount of premium under PH transform premium principle introduced by Wang \([4]\) calculated by

\[
\pi_{PH}(X) = E(\tilde{X}) = \int_{0}^{\infty} S_\tilde{X}(x) \, dx = \int_{0}^{\infty} [S_X(x)]^{1/\rho} \, dx, \rho \geq 1
\] (21)

under full coverage. Note that \( \rho = 1 \) is a special case of PH transform premium which will produce the same value with net premium. Then, the amount of PH transform premium under deductible coverage can be obtained by

\[
\pi_{PH}(X - d, \rho) = \int_{d}^{\infty} [S_X(x)]^{1/\rho} \, dx, \rho \geq 1
\] (22)

and LER under PH transform premium principle is defined by

\[
LER_\tilde{X} = \frac{E(\tilde{X}) - [E(\tilde{X}) - E(\tilde{X} \lor d)]}{E(\tilde{X})} = \frac{E(\tilde{X} \lor d)}{E(\tilde{X})} = \frac{\int_{0}^{d} S_\tilde{X}(x) \, dx}{\int_{0}^{\infty} S_\tilde{X}(x) \, dx} = \frac{\int_{0}^{d} S_X(x) \, dx}{\int_{0}^{\infty} S_X(x) \, dx} \frac{1}{\rho} \frac{1}{\rho} \int_{0}^{\infty} S_\tilde{X}(x) \, dx.
\] (23)

3. Results and discussion

We will compare amount of premium earned under net premium principle and PH transform premium principle. The bigger premium, the better principle will be. Data on 400 insureds having inpatient and outpatients benefits was analyzed. The claim amount, that is loss variable \( X \) were fitted to 4 classes of insureds depending on the sex and the type of benefit. The first step is determined the best models using
Kolmogorov-Smirnov. The result is data fitted to exponential distribution with parameter $\lambda$. The values of estimated parameters for exponential distribution or $\frac{1}{\lambda}$ can be seen in table 1.

Then, we compute the amount of premium obtained by the health insurance companies under deductible coverage that written in Indonesian Rupiah based on equation 14 and equation 22 and the value of LER based on equation 18 and equation 23. The premium and LER calculated using our loss variable X that fitted to exponential distribution with parameter $\lambda$ as seen in table 1 above. Then, we will have the premium and LER for each gender (men and women) and benefit (inpatient and outpatient). We use different deductibles for inpatient and outpatient benefit, because their amount of claim is different. The amount of claim for inpatient benefit is higher than outpatient benefit so we apply higher deductibles on inpatient benefit.

The amount of net premium, PH transform premium and the values of LER for each benefit and gender for different amount of deductibles can be seen on table 2 and table 3.

Based on table 2 and table 3, we can make the graphs that can illustrated comparison between amount of premium under net premium principle and PH transform premium principle.

### Table 1. Estimated parameters

| Claims                  | Estimated Parameter $\frac{1}{\lambda}$ |
|-------------------------|----------------------------------------|
| Outpatient Men          | 232,628                                |
| Outpatient Women        | 203,715                                |
| Inpatient Men           | 3,410,175                              |
| Inpatient Women         | 3,274,930                              |

### Table 2. Net premiums $\pi_N [(X-d)_+]$ and the values of LER.

| Deductible $(d)$ | Outpatient Men | Outpatient Women | Inpatient Men | Inpatient Women |
|-----------------|----------------|------------------|---------------|---------------|
| $\pi_N$ ($\pi_N$) | $L_{ER, X}$ | $L_{ER, X}$ | $L_{ER, X}$ | $L_{ER, X}$ |
| 0               | 232,628       | 0                | 203,715       | 0             |
| 20,000          | 213,463       | 0.082            | 184,665       | 0.093         |
| 40,000          | 195,878       | 0.157            | 167,397       | 0.178         |
| 60,000          | 179,741       | 0.227            | 151,743       | 0.255         |
| 80,000          | 164,933       | 0.291            | 137,553       | 0.324         |
| 200,000         | 348,943       | 0                | 305,574       | 0             |
| 400,000         | 339,085       | 0.055            | 286,214       | 0.093         |
| 600,000         | 320,196       | 0.108            | 268,081       | 0.178         |
| 800,000         | 302,360       | 0.157            | 251,097       | 0.255         |

### Table 3. PH transform premiums $\pi_{PH} [(X - d)_+]$ $(\rho = 1.5)$ and the values of LER.

| Deductible $(d)$ | Outpatient Men | Outpatient Women | Inpatient Men | Inpatient Women |
|-----------------|----------------|------------------|---------------|---------------|
| $\pi_{PH}$ ($\pi_{PH}$) | $L_{ER, X}$ | $L_{ER, X}$ | $L_{ER, X}$ | $L_{ER, X}$ |
| 0               | 348,943       | 0                | 305,574       | 0             |
| 20,000          | 339,085       | 0.055            | 286,214       | 0.093         |
| 40,000          | 320,196       | 0.108            | 268,081       | 0.178         |
| 60,000          | 302,360       | 0.157            | 251,097       | 0.255         |
| 80,000          | 277,451       | 0.204            | 235,189       | 0.324         |
As an example, for outpatient men when $d = 0$, we will have

$$\pi_N = \frac{1}{\lambda} = 232,628$$

and

$$LER_x = \frac{\int_0^\infty s(x) \, dx}{\int_0^\infty s(x) \, dx} = 0.$$ 

Then,

$$\pi_{PH} = \int_0^\infty \left[ e^{-\frac{x}{232,628}} \right]^{\frac{1}{1.5}} \, dx = 232,628$$

and

$$LER_x = \frac{\int_0^\infty \frac{1}{s(x)} \, dx}{\int_0^\infty \frac{1}{s(x)} \, dx} = 0.$$ 

We can conclude from figure 1 and figure 2 that deductible will make the expected loss is decreased because the health insurance company will not pay the amount of losses below the deductible. The losses under deductible that not paid by the insurance company will make the people who are at high risk acknowledge the actuarial risk that they have to insurance companies because they will demur if their amount of coverage is reduced. On the other hand, the insured will avoid the risk that will occur because health insurance companies will not fully cover their claim. Therefore, adverse selection and moral hazard problems will be more solved.

**Figure 1.** Comparison between amount of net premium and PH transform premium for outpatient benefit.
4. Conclusion
In this paper, we looked the impact of deductibles to amount of premium earned by health insurance companies. The implementation of deductible will reduce the amount of premium that received by insurer. The ratio of decrease in premium has been calculated with LER and yields results indicating that there has significant reduction premium under deductible coverage. Health insurance companies need to use the alternative pricing rule, that is PH transform premium principle.

PH transform premium principle had a simple way to calculate the amount of premium so can understood easily. Then, we compared two pricing rules under deductible insurance, that are net premium principle and PH transform premium principle. PH transform premium principle produces a higher premium and a smaller LER than net premium. The decrease of LER and amount of premium is smoother under PH transform premium principle. So that, PH transform is profitable for health insurance companies when applies deductible. Then, the suggestion that can be given through this study is considering age factor in calculate the amount of premium earned by health insurance companies.

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