Regularization Shortcomings for Continual Learning

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December 9, 2019

1 Abstract

In classical machine learning, the data streamed to the algorithms is assumed to be independent and identically distributed. Otherwise, if the data distribution changes through time, the algorithm risks to remember only the data from the current state of the distribution and forget everything else. Continual learning is a sub-field of machine learning that aims to find automatic learning processes to solve non-iid problems. The main challenges of continual learning are two-fold. Firstly, to detect concept-drift in the distribution and secondly to remember what happened before a concept-drift. In this article, we study a specific case of continual learning approaches: the regularization method. It consists of finding a smart regularization term that will protect important parameters from being modified to not forget. We show in this article, that in the context of multi-task learning for classification, this process does not learn to discriminate classes from different tasks. We propose theoretical reasoning to prove this shortcoming and illustrate it with examples and experiments with the "MNIST Fellowship" dataset.

2 Introduction

Continual Learning is a sub-field of machine learning which learns from a data distribution that is not static. Its goal is to learn the global optima to an optimization problem where the data distribution changes through time. In this paper, we study the case where the data distribution has a finite set of states and the change in the data distribution is signaled by a task label. In continual learning, this setting is called "multi-task continual learning" \cite{17}. Each state of the distribution is assimilated to a task. The task label is available at train time but not at test time. The continual algorithm should then learn on a sequence of tasks and find the optima corresponding to the whole sequence.

In this paper, we study a widely used approach for continual learning: regularization. We show that in the classical setting of disjoint classification continual learning, this approach has theoretical limitations and should not be used alone.

3 Related works

Continual learning approaches can be classified by the way they handle memories. We can distinguish four main types of approaches:

- **Dynamic architecture:** In order to maintain past knowledge, the neural networks can automatically be adapted to create new neurons that will learn new skills instead of modifying already trained ones \cite{23} \cite{5} \cite{16}. In this case, the memory is composed of the old neurons that are not modified anymore.

- **Rehearsal:** In order to maintain knowledge from past learning experiences, the algorithms save a subset of old data as memory \cite{21} \cite{19} \cite{1} \cite{2} \cite{8} \cite{28} \cite{7} \cite{3} \cite{26}.

- **Generative Replay:** Instead of saving samples, the method consists in learning generative models that will produce artificial samples as memory of past learning experiences \cite{25} \cite{13} \cite{27} \cite{14}.

- **Regularization:** Regularization defines a loss that will constrain weight updates to retain knowledge from previous tasks \cite{9} \cite{30} \cite{17}, or distill knowledge \cite{9} from old models to a new one to remember past learning experiences \cite{16} \cite{24}.
In this paper, we assume that the data stream is composed of disjoint tasks learnt sequentially one by one (with $N >= 2$). Tasks are noted $T_t$ for task $t$ and $\mathcal{T}_t$ is the associated dataset. The task label $t$ is a simple integer indicating the task index. We call the full sequence of tasks as the continuum noted $C_N$.

The whole dataset combining all the tasks is noted $\mathcal{T}_{data}$. While learning task $T_1$, the algorithm has access to data from $T_1$ only. We study a disjoint set of classification tasks where classes of each task only appear in this task and never again. We assume at least two classes by per task (in other cases a classifier can not learn).

Let $f$ be a function parametrized by $\theta$. We would like to learn such that $f(x; \theta)$ estimates class probability $p(y|x; \theta)$ for each $x$.

To do so, we learn the first task $T_0$ by searching a set of parameters $\theta$ that minimizes an empirical risk $R(\theta)$.

$$R(\theta) = \mathbb{E}_{(x,y) \in \mathcal{T}_0} [\ell_{\mathcal{T}_0}(f(x; \theta), y)]$$  \hspace{1cm} (1)

where $\ell_{\mathcal{T}_0}(\cdot)$ is the loss function used to learn $T_0$, $x$ a data-point from $T_0$ and $y$ its class label. We use gradient descent based method to optimize $R(\theta)$, we compute the gradient such that for all parameters $\theta_j = \theta[j]$:

$$\nabla_{\theta_j} = \nabla_{\theta_j}(x, y) = \frac{\partial \ell_{\mathcal{T}_0}(f(x; \theta_j), y)}{\partial \theta_j}$$  \hspace{1cm} (2)

and apply a gradient-based update to $\theta$ similar to:

$$\theta'_j \leftarrow \theta_j - \eta \nabla_{\theta_j}$$  \hspace{1cm} (3)

This update is repeated until convergence to a local optima of $R(\theta)$ on $T_0$. $\theta_0^*$ denote the local optima reached by $R(\theta)$ on $T_0$. The learning procedure can be summarized as finding:

$$\theta_0^* = \arg\min_{\theta} \mathbb{E}_{(x,y) \in \mathcal{T}_0} [\ell_{\mathcal{T}_0}(f(x; \theta), y)]$$  \hspace{1cm} (4)

where $x$ is the data point and $y$ the class label.

Then, for another task $T_t$, the same process is applied to reach on optima $\theta_t^*$ but this time, since we are in a continual learning setting, $\theta_t^*$ should be an optima for all tasks $T_t$, $\forall t' \in [0, t - 1]$.
In this paper, we consider only the multi-task setting with no test label. It means that an optima $\theta^*$ for $T_0 \cup T_1$, is a set of parameters which at test time will, for any data point $x$ from $T_0 \cup T_1$, classify correctly without knowing if $x$ comes from $T_0$ or $T_1$.

Therefore, in our continual learning setting, the loss to optimize when learning a given task $t$ is not anymore $\ell(f(x; \theta), y)$ for $(x, y) \in T_t$ but an augmented loss:

$$\ell_{c_t}(f(x; \theta), y) = \ell_{t}(f(x; \theta), y) + \lambda \Omega(t \forall t' < t)$$  \hspace{1cm} (5)$$

where $\ell_{c_t}(.)$ is the continual loss at task $T_t$ and $\ell_{t}(.)$ the loss to optimize to learn only $T_t$. The $\ell_{c_t}(.)$ then optimizes the loss $\ell_{t}(.)$ of the actual task and a complementary loss from previous tasks, $\Omega(t \forall t' < t)$ designed to learn without forgetting. $\lambda$ characterizes the importance of remembering loss with respect to the learning loss.

### 4.2 Problem

In continual learning, a classical approach to learning without forgetting is to define $\Omega$ as a regularization term to maintain knowledge from $T_0$ \forall $t' < t$ in the parameters $\theta$ such as while learning a new task $T_t$:

$$\forall x \in T_0, f(x; \theta_0^*) \approx f(x; \theta)$$

In the regularization method, to keep $\ell_{T_0}(f(x; \theta), y)$ low $\forall x \in T_0$ while learning $T_1$, the regularization term $\Omega_0$ is defined as a memory of $f(x; \theta_0^*)$. This memory term depends on the learned parameters $\theta_0^*$, on $\ell_{T_0}$ the loss computed on $T_0$ and the actual parameters $\theta$. $\Omega_0$ memorizes the optimal state of the model at $T_0$ and generally the importance of each parameters with regard to the loss $\ell_{T_0}$.

When learning the second task $T_1$, the new loss to optimize is then:

$$\ell_{T_1}(f(x; \theta), y) = \ell_{T_1}(f(x; \theta), y) + \lambda_0 \Omega_0(\theta_0^*, \ell_{T_0}, \theta)$$  \hspace{1cm} (6)$$

Eq. (6) is composed of two components, the actual loss (blue square) which optimizes empirical risk on the task $T_1$ and a surrogate loss: the regularization term (red square) which regularizes the loss to maintain the optimum found at (4) without accessing $T_0$. $\lambda_0$ is the weight assigned to $T_0$ regularization (importance term). It is a classical hyper-parameter used to fine-tune the loss.

More generally at task $t$, the regularization function can be written $\Omega_t(\theta^*, \ell_{T_t}, \theta)$ with $\theta^*$ a set of optimal weights for previous tasks (or at a previous point in time), $\ell_{T_t}$ is a function dependant on the loss computed on previous tasks, it could be for example a matrix pondering weights importance in previous tasks [9, 22, 50]. $\theta$ is the vector of actual model parameters.

The general augmented loss is then at task $t$:

$$\ell_{c_{t+1}}(f(x; \theta), y) = \ell_{T_{t+1}}(f(x; \theta), y) + \lambda \Omega_t(\theta^*, \ell_{c_t}, \theta)$$  \hspace{1cm} (7)$$

Where $\lambda$ is the importance attributed to past tasks toward the new task.

### 4.3 Regularization methods

To illustrate the regularization method described in previous section, we link several famous regularization approaches to our formalism:

- **Elastic Weight Consolidation (EWC)** [9]

EWC is one of the most famous regularization approaches for continual learning. It proposes a method to learn continually by adding a regularization surrogate term to the loss. The loss augmented with a regularization term is at task $t$:

$$\ell_{c_t}(\theta) = \ell_{T_t}(f(x; \theta), y) + \frac{\lambda}{2} * F_{t-1}(\theta_{t-1}^* - \theta)^2$$  \hspace{1cm} (8)$$

We can then by identification, extract our function $\Omega_t(\theta^*, \ell_D, \theta)$

$$\Omega_t(\theta^*, \ell_{c_{t-1}}, \theta) = \frac{1}{2} * F_{t-1}(\theta_{t-1}^* - \theta)^2$$  \hspace{1cm} (9)$$

$F_t$ is a tensor of size $card(\theta)$, specific to task $t$, characterizing the importance of each parameter $\theta_k$. This is an example of what we denote as $\ell_{c_{t-1}}$, a set of values computed on past data to optimize the regularization loss.
effectiveness on past tasks. In EWC, the $F_t$ tensor is implemented as a diagonal approximation of the Fisher Information Matrix:

$$ F_t = \mathbb{E}_{(x,y) \in T_t} \left[ \left( \frac{\partial \log p(y)}{\partial \theta} \right)^2 \right] $$

(10)

where $\hat{y} \sim P(f(x; \theta))$

- **Kronecker factored online Laplace approximation for overcoming catastrophic forgetting** [22]

This approach is very similar to EWC but approximates the Fisher matrices with a Kronecker factorization [18] to improve the expressiveness of the posterior over the diagonal approximation. However, the Kronecker factorization saves more values than the diagonal approximation.

- **Incremental Moment Matching (IMM)** [11]

This paper proposes two regularization approaches for continual learning which differ into the computation of the mean $\theta_{0:t}$ and the variance $\sigma_{0:t}$ of the parameters on all tasks.

- **Mean based Incremental Moment Matching (mean-IMM)**

$$ \theta_{0:t} = \sum_{i=0}^{t} \alpha_i \theta_i^* $$

(11)

$$ \sigma_{0:t} = \sum_{i=0}^{t} \alpha_i (\sigma_i + (\theta_i^* - \theta_{0:t})^2) $$

(12)

$\alpha_i$ are importance hyper-parameters to balance past task weight into the loss function. They sum up to one.

- **Mode based Incremental Moment Matching (mode-IMM)**

$$ \theta_{0:t} = \sigma_{0:t} \sum_{i=0}^{t} (\alpha_i \sigma_i^{-1} \theta_i^*) $$

(13)

$$ \sigma_{0:t} = (\sum_{i=0}^{t} \alpha_i \sigma_i^{-1})^{-1} $$

(14)

$\sigma_i$ is computed as the Fisher matrix (eq. 10) at task $i$.

Then at task $t$, with $\theta_{0:t-1}$ and $\sigma_{0:t-1}$ we can compute:

$$ \Omega_t(\theta^*, \ell_{t-1}, \theta) = \frac{1}{2} \sigma_{0:t-1}(\theta_{0:t-1} - \theta)^2 $$

(15)

- **Synaptic Intelligence (SI)** [30]

In this method the regularization protocol is similar but the importance is computed differently:

$$ \Omega_t = M_t(\theta_i^{t-1} - \theta)^2 $$

(16)

$M_t$ is a tensor of size $\text{card}(\theta)$ specific to task $t$ characterizing the importance of each parameter $\theta_k$ over the all past tasks such as:

$$ M_t = \sum_{0 < i < t} \frac{m_i}{\Delta_i^2 + \xi} $$

(17)

$M_t$ sum over $m_i$ characterizing the importance of each parameter on task $i$ only with $\Delta_i = \theta_i^* - \theta_{i-1}^*$.

$$ m_i = \int_{T_{i-1}}^{T_i} \nabla_{\theta} \delta \theta(t) dt $$

(18)

With $\delta \theta(t)$ the parameter update at time step $t$. 

4.4 Preliminary Definition / Axiom

Definition 1. Linear separability
Let $S$ and $S'$ be two sets of points in an $n$-dimensional Euclidean space. $S$ and $S'$ are linearly separable if there exists $n+1$ real numbers $\omega_1, \omega_2, \ldots, \omega_n, k$ such that $\forall x \in S, \sum_{i=1}^{n} \omega_i x_i > k$ and $\forall x \in S', \sum_{i=1}^{n} \omega_i x_i < k$

where $x_i$ the $i$-th component of $x$. This means that two classes are linearly separable in an embedded space if there exists a hyperplane separating both classes of data points.

Definition 2. Interferences
In machine learning, interferences are unsolved conflicts between two (or more) objective functions leading to prediction errors. Optimizing one of the objective function increases the error on the other one. In continual learning, interferences happen often after a concept drift in the data distribution. The loss on previous data is increased with the optimization of the loss for the new data leading to interferences and catastrophic forgetting.

Axiom 4.1. $\forall(S, S')$ set of discrete points in $R^n$ linearly separable by a hyperplane $q$, $q$ cannot be learned without access to $S'$.

This can be summarized as: “The decision boundary between two classes can not be found if there is no access to data from both”.

Axiom 4.2. $\forall(S, S')$ set of discrete points in $R^n$ linearly separable by a hyperplane $q$. If $S$ and $S'$ are accessible then $q$ can be found.

Axiom 4.3. $\forall(S, S')$ two datasets not linearly separable, it is not possible to learn a function $g$ that is a projection of $S$ and $S'$ into a space were they are linearly separable without access to one of them.

This can be summarized as: "Two classes representations cannot be disentangled if there is no access to data from both.”

In those axiom, the concept of "not having access" to a certain dataset can both be applicable to not being able to sample data point from the distributions and to not have a model of the dataset. It can be generalized to not having access to the data distribution.

4.5 Proposition
We would like to prove that in continual learning with disjoint classification tasks, it is not possible to learn correctly only with a regularization based memory. The main point is that to correctly learn to discriminate classes over different tasks the model needs access to both data distributions simultaneously. In regularization methods, the memory only characterizes the model and the important parameters. This memorization might give insight on some characteristics but it is not a model of the data distributions of past tasks. If we take again the cat/dog examples, a model that needs to discriminate white cats from black cats will learn to discriminate black features from white features and this can be saved in $\Omega$ but $\Omega$ will not save the full characteristics of a cat because the model will never learn it.

Proposition 4.4. While learning a sequence of disjoint classification tasks, if the memory $\Omega$ of the past tasks is only dependant on the learning criterion and does not model the past distribution, it is not possible to learn new tasks without interference.

Proof. We are here in the context of learning with a deep neural network. We can decompose the model into a non-linear feature extractor $g$ and a linear output layer to predict a class $y$:

$$ y = \arg\max_r (\sigma(A \cdot g(x) + b)) $$ (19)

The output layers allows for each class $i$ to learn a hyperplane $A[i, \cdot]$ with bias $b[i]$ that separate all classes from the class $i$, such as:

$$ \forall i \in [1, N]$$

$$ \forall (x, y) \in T_i, \arg\max_r (A[i, \cdot] x + b[i]) = y $$ (20)

However if we look at the classes independently the model should only require that:

$$ \forall i \in [1, N], $$

$$ \forall (x, y) \in T_i, \begin{cases} if \ y = i, A[i, \cdot] x + b[i] > 0 \\ if \ y \neq i, A[i, \cdot] x + b[i] < 0 \end{cases} $$ (21)

The projection $g(.)$ ensure the linear separability of classes and the output layer learns the separation.
We now study how to learn new tasks \( T_t \) for \( 0 < t < N \). There are two different cases, first if \( g \) is already a good projection for \( T_t \), i.e. classes are already linearly separable in the embedded space and secondly if \( g \) needs to be adapted, i.e. classes are not yet linearly separable in the embedded space and new features need to be learned by \( g \) to fix it. We refer as features, intrinsic characteristics of data that a model needs to detect to distinguish a class from another.

- **First case**: Classes are linearly separable
  
  For all task \( \forall(t) \in [[1, N - 1]], T_{t-1} \) happens before \( T_t \) in the continuum.

  At task \( T_t \) (for \( t \geq 1 \)), since we are in a regularization setting, we have access to \( \Omega_{t-1} \) which contains classification information from previous tasks including \( T_{t-1} \). However, by hypothesis, \( \Omega_{t-1} \) does not model the data distribution from \( T_{t-1} \) and therefore it does not model data distribution from \( T_{t-1} \) classes.

  Consequently following axiom [4.3], it is not possible to find a linear boundary between classes from \( T_t \) and classes \( T_{t-1} \) even if by hypothesis this boundary exists.

- **Second case**: \( g \) needs to be updated.

  Let \( \delta_{t-1} \) be the set features already learned by \( g_{t-1} \) the feature extractor from previous task. \( \Omega_{t-1} \) should keep \( \delta_{t-1} \) unchanged while learning \( T_t \). Either, \( \delta_{t-1} \) allows to solve \( T_t \) and we are in case 1, either a new set of features \( \delta_t \) needs to be learned while learning \( T_t \). In the second case, the set \( \delta_t \) contains features to solve \( T_t \), but features \( \delta_{t-1:t} \) that distinguish classes from \( T_{t-1} \) to classes from \( T_t \) should also be learned.

  Then two cases raise, \( \delta_{t-1:t} \not\subset \delta_t \) or \( \delta_{t-1:t} \subset \delta_t \).

  - \( \delta_{t-1:t} \subset \delta_t \), is impossible by hypothesis, as the task are disjoint. If, \( \delta_{t-1:t} \subset \delta_t \) it would means that data from \( T_{t-1} \) are in \( T_t \).

  - if \( \delta_{t-1:t} \not\subset \delta_t \), then supplementary features need to be learned to distinguish \( T_{t-1} \) and \( T_t \) but since \( \Omega \) does not give access to \( T_{t-1} \) data distribution, from axiom [4.3] we can not learn the set of features \( \delta_{t-1:t} \) to discriminate \( T_{t-1} \) from \( T_t \) and solve the continual problem.

Therefore, it is not possible to learn proper boundaries whether the feature space is already adapted to it or not. There will have in any way conflict between loss optimization leading to interferences in the decision boundaries either because classes are not linearly separable or because a separation hyperplane can’t be found.

**Corollary 4.1.** Proposition [4.3] still holds if task are only partially disjoint, i.e. only some classes appears only once in the continual curriculum.

**Proof.** The main difference with the previous task is \( T_0 \cap T_1 \neq \emptyset \) However, since we are in a partially disjoint setting, we can define \( T_t \) a subset of \( T_t \) \( \left( T_t \cap T_1 = T_t \right) \) such as classes from \( T_0 \) and classes from \( T_1 \) are disjoint.

Then the previous demonstration can be re-applied on \( T_0 \) and \( T_1 \). Then \( T_t \) and \( T_{t-1} \) suffer from interference showing a shortcoming of regularization methods for this case.

### 4.6 Practical examples

To illustrate the proposition from previous section, we present two insightful examples of regularization limitations.

**The task separability problem:**

In the first case of proposition [4.3] proof, we have already a perfect feature extractor that allows us to learn only the output layer in a continual fashion. Then, the goal is to learn the hyperplane for each task that disentangle the different classes. Figure [4.1] shows an example with two classes per tasks. Figure [4.2] shows the first task. Since we have a perfect feature extractor, there is only the hyperplane to learn (the dashed line). Then Figure [4.3], we see we discarded data from \( T_0 \) but kept the model (our \( \Omega_0 \)) and learn the new hyperplane separating new data. However, we see that the two hyperplane are not enough to discriminate all classes since the two models can’t separate red dots from yellow squares and blue dots from green squares. This illustrates the problem of learning a solution able to separate classes only by remembering the model optimizing the past tasks, even if the feature space is potentially able to discriminate tasks and classes linearly.

**The latent features problem:**

In the second case of proposition [4.3] proof, the feature extractor needs to be adapted to learn a projection into a latent-space where classes are linearly separable.

Then, we propose a simulation of what could happen while learning two sequential tasks with a regularization method. Figure [2] we see the evaluation of latent space while learning \( T_0 \), the latent space is at first entangled, and then while learning \( g_0 \) learn a projection of data into a latent-space where data are linearly separable. \( T_0 \) is then over, we compute a regularization term dependant on the learned solution, here we suppose we save the
learned model. Then we learn $T_1$ (Figure 3), the risk involved in only remembering the model but not the data is that the projection $g_1$ will only learn to disentangle the new data and not the old one from the new one. This could lead to interference as seen in the right part of Figure 3 where at the end of the task, data from $T_0$ and $T_1$ are entangled into the latent space. At test time, it will not be possible for the model to discriminate between red dots and yellow squares.

In some approaches, the solution found to discriminate $T_0$ from $T_1$ with a regularization method is to use a task label at test time. The task label at test time allows solving the global problem with only partial solutions. However, assuming the task label available at test time is a strong assumption in continual learning. This paper does not support in any way the use of test label for continual learning, we consider as important to continue research in approaches that do not need labels to alert on concept drift and find more autonomous learning processes.

In the next section, we illustrate the regularization shortcomings with experiments with the MNIST Fellowship dataset [12].

Figure 3: At task $T_1$, feature space of $T_1$ before learning $T_1$ (Left), Feature space of $T_1$ after learning $T_1$ with a possible decision boundary (Right). New data are plot by yellow and green squares and hold data that are not available anymore to learn a t shown with pale red and blue dots.
5 Experiences

To illustrate the limitations presented earlier in this paper, we experiment with the "MNIST-Fellowship" dataset proposed in [12]. This dataset is composed of three datasets (Fig. 4): MNIST [10], Fashion-MNIST [29] and KMNIST [4], each composed of 10 classes, those datasets should be learned sequentially one by one. We choose this dataset because it gathered three easy datasets for prototyping machine learning algorithms but solving those three quite different datasets is still harder than solving only one.

Our goal is to illustrate the limitation of regularization based methods, in disjoint settings. So, we propose three different experiments with the MNIST-Fellowship dataset.

• 1. Disjoint setting: all tasks have different classes (i.e. from 0 to 29).
• 2. Joint setting: all tasks have the same classes (i.e. from 0 to 9) but different data.
• 3. Disjoint setting with test label: All tasks have different classes but at inference, we know from which task a data-point is coming from.

First setting (disjoint with no test label), is the hardest because all classes need to be discriminate from all the others, second setting (joint) is a bit easier because we don’t need to discriminate task from each other but the model needs to use the same output layer for all task which can produce interferences, the last setting (disjoint with test label) is the easiest, classes from different task don’t need to be compared and the output layer is different for each task.

We present results with EWC with diagonal Fisher Matrix [9] approaches and with Kronecker Factorization of the Fisher matrix [22]. We add an expert model which learned on the full dataset at once and a baseline model who learned continually without any memorization process.

In Experiment 1 (Fig. 5), illustrates that regularization methods are not very effective when there is no test label in the disjoint settings. Meanwhile experiment 3 (also on Fig. 5), shows that the test labels improve regularization methods performance. The low performance of regularization methods without test labels in disjoint settings illustrates the shortcomings in the continual learning of the output layer (task separability problem, Section 4.6).

In Experiment 2 (Fig. 6), only the feature extractor needs to be learned continually, as the classes are the same in all classes. The low performance of the models proposed, illustrates the shortcomings in the continual learning of the feature extractor (the latent features problem, Section 4.6).

All models have been trained with stochastic gradient descent with a learning rate of 0.01 and a momentum of 0.9. Even if continual learning does not support a-posteriori hyper-parameter selection, for fairness in comparison, the parameter lambda has been tuned. The best lambda upon [0.1; 1; 2; 5; 10; 20; 100; 1000] have been selected for each model.

Figures 5 and 6 show that learning continually is only efficient in the setting with task label and maintains performance on task 0. The two other settings seem to either have interference in the output layer or the feature extractor.

6 Discussion

The goal of this article is to propose a theoretical approach to the shortcomings of regularization methods. Regularization methods might have great characteristics for continual learning under certain conditions, but it is important to know their limitations to use the best of their capabilities. However, in the case of disjoint tasks, a great idea is to mix regularization with a replay method such as rehearsal or generative replay. In any way,
Figure 5: Experiment on disjoint classes without test label vs test label. Left, the mean accuracy of all 3 tasks, vertical dashed line are task transitions. Right, accuracy on the first task. Legends with ‘Lab_’ indicate experiments with task labels for test, the number at the end of the legend indicate the value of the importance \( \lambda \). ‘Lab_Ewc_Kfac_5.0’ is the experiment with test label, using Ewc with Kronecker factorization of the fisher matrix with a \( \lambda \) of 5.0. The expert model is trained with i.i.d. data from all task and the baseline model is finetuned on each new task without any continual process.

Figure 6: Experiment with joint classes. Left, the mean accuracy of all 3 tasks, vertical dashed line are task transitions. Right, accuracy on the first task.

to maximize discrimination between tasks with no test label it is mandatory to have good memorization of past tasks. Either by modeling their data distribution with generative models or samples or by adding surrogate losses that help the model to learn more variate representation of past tasks. Memorization is intrinsically linked to representation. Hence, adding surrogate loss to improve the learnt representation would then necessarily improve memorization and consequently continual learning.

In this paper, we supposes available a “task label” indicating each time a concept drift happens in the data distribution while learning. These settings make learning easier than when the concept drifts are not signaled in any way. In case of no task label available, it is even more important to have a robust memorization process to detect concept drift and face it without catastrophic forgetting.

7 Conclusion

In this paper, we show that regularization methods are dependant on the task label availability at inference. This dependence is reductive in the potential application of regularization methods in a real setting. Indeed, even if an algorithm has access to a learning task label, it is unlikely to still have the same supervision while in deployment. Nevertheless, we think that a strong baseline can be designed by associating regularization with other families of continual learning approaches.
Acknowledgments

Thanks to Clément Pinard, Thomas George, Hugo Caselles-Dupré and Alexandre Chapoutot for interesting discussions for the realization of this paper. Thanks also to Vyshakh Palli Thazha for proofreading this article.

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