Shimmy analysis of steering system with independent suspension based on coupled feedback model

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Abstract. Shimmy of steering system causes the snaking and deteriorates the handling stability and safety of vehicles, which has not been solved effectively for the ambiguity of shimmy characteristic. In order to uncover the mechanism of shimmy, a 5-DOF dynamic model of the steering system with independent suspension is established, and the coupled feedback model of the steering system is further obtained based on the dynamic model. Then the frequency characteristics of feedback are analysed, and the work and damping energy dissipation of each feedback link in a unit period are calculated. Based on the range of natural frequency and unit periodic energy of steering system, the range of wheel shimmy frequency and velocity interval is determined. The analysis results show that the shimmy frequency is the first natural frequency of steering system. The amplitude of wheel shimmy is calculated based on energy conservation, and the energy transfer of steering system is elaborated. The mechanism of shimmy was uncovered based on the energy transfer of coupled feedback system. The theoretical analysis was verified by numerical simulation. The results provide theoretical support for the mechanism research and active control of shimmy.

1. Introduction
Shimmy brings about the continuous vibration of wheels about the kingpin, deteriorates the handling stability, ride comfort, and safety [1, 2]. The uncertainty of the shimmy mechanism leads to these problems can not be thoroughly solved in engineering, which affects the safe driving of vehicles seriously, even nose landing gear of aircraft[3]. The shimmy can be avoided during design stage or usage stage by design and active control [4], while the clear mechanism of shimmy is the basis of these efforts. Different methods had been applied to research the mechanism of shimmy and the influence factor.

The shimmy is usually described as a bifurcation or instability of the steering dynamic system, and nonlinear dynamics and bifurcation theory are widely used in the shimmy research [5, 6, 7]. And the influence of sideslip force, clearance of pairs, and dry friction were analysed by numerical simulation during the analysis. The simulation can explicit the influence of different factors, while the mechanism of shimmy still was covered. Guo et al. [8] revealed the mechanism of shimmy by the effect of negative damping, and focused on the influence of sideslip characteristic. The results shown the negative damping of alignment torque and sideslip force in low frequency range might lead to shimmy. Besselink [9]...
applied the method of energy flow in the research of shimmy, and the dynamic characteristic of tire was mainly analysed. Then Ran [10, 11] treated the contact area as a 2 DOF motion based on Von Schlippe model, and analysed the effect of velocity on the input energy of steering system. Jiang et al. [12] treated the tires as delayed feedback blocks, and the simulation results shown that the phase difference between alignment torque and shimmy motion was the reason of shimmy. Similar thought emerged in the research of Mi [6] and Yang et al [13].

In this paper, the steering system with independent system is taken as the research object. Firstly, a 5 DOF dynamic model of steering system is established. And the coupling feedback model of steering system is further established based on the dynamic model. Secondly, the work and energy consumption of feedback block are calculated, and the frequency range of shimmy is obtained according to the characteristic of feedback block. Then the natural frequency of steering system is calculated to obtain the velocity range of shimmy. And the amplitude of shimmy is obtained based on the energy conservation, and the relationship of amplitude, frequency, and velocity range is revealed. The mechanism of shimmy is uncovered based on the energy transfer of coupled feedback system.

2. 5 DOF model of steering system

2.1. Structure of steering system

The structure of 5 DOF steering system with independent suspension was shown in Figure 1. The forward direction of vehicle was set as X-axis, the vertically up direction was set as Z-axis, and the Y-axis conform to right-hand coordinate system. The structures of steering system in Y-Z plane and X-Y plane were shown in (a) and (b) of Figure 1. The symbol $\phi_1$ and $\phi_2$ indicated the rotary motion of left and right tire around kingpin, respectively, $\theta_1$ and $\theta_2$ indicated the swing motion of left and right tires, respectively. And $\phi_3$ indicated the swing motion of middle rod. The parameters of steering system was shown in Table 1.

![Figure 1. Structure of steering system with independent suspension](image)

1-vehicle frame, 2-suspension, 3-upper arm, 4-lower arm, 5-wheel, 6-knuckle arm, 7-tie rod, 8-middle rod, 9-gearbox

2.2. Dynamic model of steering system

According to the structure in Figure 1, the dynamic model of steering system could be obtained based on Lagrangian equation:
Where the sideslip force $F_{yi}$ was nonlinear, and can be expressed as:

$$
F_{yi} = S_y + D \sin \left\{ \arctan\left[ B(\alpha_i - S_y) \right] \right\}
$$

(2)

Where where $\alpha_i$ is side slip angle of left and right tire, respectively. The parameters, $S_x$, $S_y$, $B$, $C$, $D$, and $E$ are constant and obtained from the experimental data. We take $S_x=0$, $S_y=0$ and other parameters are calculated by equations with vertical load [13].

And the non holonomic constrained equation between the shimmy angle $\phi_i$ and $\alpha_i$ can be given by[6]:

$$
\begin{align*}
\alpha_i' &+ \frac{V}{\sigma} \alpha_i + \frac{a - R \gamma}{\sigma} \alpha_i - \frac{R \gamma}{\sigma} \phi_i = 0 \\
\alpha_i' &+ \frac{V}{\sigma} \alpha_i + \frac{a - R \gamma}{\sigma} \alpha_i - \frac{R \gamma}{\sigma} \phi_i = 0
\end{align*}
$$

(3)

**Table 1.** Parameters of Steering System

| Symbol | Parameters | Value |
|--------|------------|-------|
| $J_1$, $J_2$ | Mass moment of inertia of left and right tire w.r.t kingpin | 7 kg m² |
| $J_3$ | Mass moment of inertia of middle rod w.r.t $Z$ | 5 kg m² |
| $J_4$, $J_5$ | Mass moment of inertia of left and right wheel w.r.t. $X$ | 6 kg m² |
| $J_0$ | Mass moment of inertia of wheel w.r.t. $Y$ | 4 kg m² |
| $c_1$, $c_2$ | Equivalent torsional stiffness of tie rod | 1.9 kN m rad⁻¹ |
| $k_3$ | Equivalent torsional stiffness of mid rod | 3.9 kN m rad⁻¹ |
| $c_3$ | Equivalent torsional damper of mid rod | 30 N m s rad⁻¹ |
| $k_4$, $k_5$ | Equivalent torsional stiffness of suspension | 15 kN m rad⁻¹ |
| $c_4$, $c_5$ | Equivalent torsional damper of suspension | 1130 N m s rad⁻¹ |
| $c_{1e}$, $c_{2e}$ | Torsional damper of left and right wheels rotation around kingpin | 40 N m s rad⁻¹ |
| $\gamma$ | Caster angle | 3° |
| $e$ | Pneumatic trail | 0.09 m |
| $l$ | Distance of lower arm hinge point and tire centre | 0.8 m |
| $h$ | Distance of kingpin axis and tire centre | 0.2 m |
| $R$ | Radius of tire | 0.4 m |
| $a$ | Half length of tire | 0.22 m |
| $\sigma$ | Relaxation length of tire | 0.4 m |
| $k_z$ | Vertical stiffness of tire | 2200 kN m⁻¹ |
| $k_y$ | Lateral stiffness of tire | 35 kN m⁻¹ |

### 2.3. Coupled feedback model of steering system

In order to establish the coupled feedback model of steering system, it is necessary to linearize the nonlinear block in Equation (1). And the previous research shown that the linearization at the equilibrium point will not bring significant deviation.

The nonlinear sideslip tire force in Equation (2) can be linearized as:

$$
F_{yi} = BCD \alpha_i
$$

(4)

Taking Equation (4) into (1), and making Laplace transform for (1) and (2):
The rotation motion of tires around kingpin can be treated as forward block, and the alignment torque and other torques can be treated as feedback block, then the coupled feedback model of steering system can be obtained from Equation (5), which is shown in Figure 2.

![Figure 2. Coupled feedback model of steering system](image)

As shown in Figure 2, \( M_{z1}' \) and \( M_{z2}' \) were the interference torque around Z axis, and the \( G_1 \) and \( G_2 \) indicated the forward block of rotation motion of two tires around kingpin, \( G_3 \) and \( G_4 \) indicated the forward block of two tires motion in Y axis. Similarly, \( G_5 \) and \( G_6 \) indicated the forward block of two tie rods. \( H_1 \) and \( H_6 \) represented the feedback block of middle rod, \( H_2 \) and \( H_5 \) indicated the feedback block of gyroscopic torque, \( H_3 \) and \( H_4 \) indicated the feedback block of sideslip force and alignment torque. And the details of different block in Figure 2 can be given by:

\[
\begin{align*}
G_1 &= \frac{1}{J_{o2} + (c_{o3} + k_3 + c_{o4})} \frac{V}{R} \left[ (c_{o3} + k_3 + c_{o4}) \right] \\
G_2 &= \frac{1}{J_{o2} + (c_{o3} + k_3 + c_{o4})} \frac{V}{R} \left[ (c_{o3} + k_3 + c_{o4}) \right] \\
G_3 &= \frac{J_{o2} V}{R} \left[ (c_{o3} + k_3 + c_{o4}) \right] \\
G_4 &= \frac{J_{o2} V}{R} \left[ (c_{o3} + k_3 + c_{o4}) \right] \\
G_5 &= \frac{1}{J_{o2} + (c_{o3} + k_3 + c_{o4} + k_3 + k_4) + (c_{o3} + k_3 + c_{o4} + k_3 + k_4)} \\
G_6 &= \frac{1}{J_{o2} + (c_{o3} + k_3 + c_{o4} + k_3 + k_4)} \\
H_1 &= \frac{V}{R} \left[ (c_{o3} + k_3 + k_4) \right] \\
H_2 &= \frac{V}{R} \left[ (c_{o3} + k_3 + k_4) \right] \\
H_3 &= \frac{V}{R} \left[ (c_{o3} + k_3 + k_4) \right] \\
H_4 &= \frac{V}{R} \left[ (c_{o3} + k_3 + k_4) \right] \\
H_5 &= \frac{V}{R} \left[ (c_{o3} + k_3 + k_4) \right] \\
H_6 &= \frac{V}{R} \left[ (c_{o3} + k_3 + k_4) \right]
\end{align*}
\]

3. Energy analysis of steering system

3.1. Frequency character of feedback block

Due to the symmetry of steering structure, the frequency character of the alignment torque feedback block \( H_3 \) and \( H_4 \) can be given by
Similarly, the frequency character of the gyroscopic torque and middle rod torque feedback block $G_3H_2$ and $G_5H_1$ can be given by

\[
A_i(\omega) = \sqrt{(1 + J_i^2 \omega^2)^2 + (\frac{J_i^2}{R} + BCDRa)^2} \\
\phi_i(\omega) = -\arctan\left(\frac{J_i^2}{R} + BCDRa\right) - \arctan\left(\frac{J_i^2}{R} + BCDRa\right)
\]

(9)

3.2. Work of feedback block per unit period

According to literature [11], the rotation motion of left and right tire around kingpin can be assumed as:

\[
\begin{align*}
\varphi_1(t) &= A_1 \sin \omega t \\
\varphi_2(t) &= A_1 \sin (\omega t + \phi)
\end{align*}
\]

(11)

Where $A_1$ is the amplitude of motion.

Then the motion around Y and Z axis can be given by:

\[
\begin{align*}
\theta_1(t) &= G_1 A_1 \sin (\omega t + \phi_1) \\
\theta_2(t) &= G_1 A_1 \sin (\omega t + \phi_2 + \phi_3) \\
\phi_2(t) &= G_1 \Gamma \sin (\omega t + \xi)
\end{align*}
\]

(12)

Where $T = \sqrt{(A_1 + B \cos \phi)^2 + \sin^2 \phi}$, $\xi = \arctan\left(\frac{A_1 + A \cos \phi}{\sin \phi}\right)$.

According the frequency character in 3.1, the work per unit period of sideslip feedback block can be given by:

\[
W_{o1} = \int_0^{2\pi} M_z \dot{\phi}_1(t) \, dt = \int_0^{2\pi} A_1^2 A_2(\omega) \sin(\omega t + \phi_1) \cos(\omega t) \, dt = \pi A_1^2 A_2(\omega) \sin(\phi_1(\omega))
\]

(13)

Considering the phase difference between the rotation motion of left and right tire, the work per unit period of gyroscopic and middle rod can be given by:

\[
W_{o2} = \int_0^{2\pi} M_z \dot{\phi}_2(t) \, dt = \pi A_1^2 A_2(\omega) A_3(\omega) \sin(\phi_2(\omega) + \phi_3(\omega))
\]

(14)

\[
W_{o3} = \int_0^{2\pi} M_z \dot{\phi}_3(t) \, dt = \pi A_1^2 T A_3(\omega) \sin(\xi + \phi_3(\omega))
\]

(15)
6

\[ W_{\text{in}} = \int_{0}^{2\pi} M_{c} \phi_{1}(t) dt = \pi A_{1}^{2} T A_{1}(\omega) \sin(\xi + \phi_{1}(\omega) - \phi) \]  

(16)

3.3. Energy consumption of damping per unit period

As shown in Figure 2, the damping of the steering system included the damp in kingpin, suspension, and the middle rod. According to the assumption (11), the energy consumption of kingpin damping can be given by:

\[ W_{\text{out1}} = \int_{0}^{2\pi} c_{s} \phi_{1}(t) \phi_{1}(t) dt = A_{1}^{2} c_{s} \pi \omega \]  

(17)

The energy consumption of suspension damping can be given by:

\[ W_{\text{out2}} = \int_{0}^{2\pi} c_{s} \phi_{2}(t) \phi_{2}(t) dt = c_{s} \pi \omega \left( A_{1} A_{2}(\omega) \right)^{2} \]  

(18)

And the energy consumption of middle rod and tie rod can be given by:

\[ W_{\text{out3}} = \int_{0}^{2\pi} c_{s} \phi_{3}(t) \phi_{3}(t) dt = c_{s} \pi \omega \left( A_{1} A_{2} T \right)^{2} \]  

(19)

\[ W_{\text{out4}} = \int_{0}^{2\pi} c_{s} \left( \phi_{3}(t) - \phi_{2}(t) \right)^{2} dt = \pi c_{s} \omega \left[ A_{1}^{2} + A_{2}^{2} T^{2} - 2 A_{1} A_{2} T \cos(\xi) \right] \]  

(20)

Similarly, the amplitude of the energy consumption was directly related to the frequency characteristic of shimmy vibration. And the amount of energy in system determined the state of steering system.

3.4. Energy analysis of steering system

According to the results of section 3.2 and 3.3, the energy of steering system can be given:

\[ W_{\text{total}} = W_{\text{in}} - W_{\text{out}} \]  

(21)

Where

\[ \begin{cases} W_{\text{in}} = 2W_{\text{in1}} + 2W_{\text{in2}} + W_{\text{in3,1}} + W_{\text{in3,2}} \\ W_{\text{out}} = 2W_{\text{out1}} + 2W_{\text{out2}} + W_{\text{out3}} + 2W_{\text{out4}} \end{cases} \]  

(22)

We take the velocity \( V=60 \) km/h, \( A_{1}=0.1 \) rad, and \( \phi=2\pi \) considering the snaking of vehicle when the shimmy occurred. Then we obtained the work of different feedback block per unit period, as shown in Figure 3 (a). Similarly, the energy consumption can also be obtained, as shown in Figure 3(b). And the total energy of system was shown in Figure 3(c), where the \( \omega_{c} \) was the corresponding frequency when \( W_{\text{total}}=0 \).

(a)                             (b)                           (c)

Figure 3. Energy of unit period

As shown in Figure 3(a), the main part of the feedback work is the alignment torque block, and the work increased firstly and then decreased with the frequency. While the energy consumption of damping
increased with the frequency increased, and the damping in kingpin and middle rod accounted for a large proportion according to the details of Figure 3(b). For the energy of steering system, when the $\omega<\omega_c$, the total energy of system was positive, and when the $\omega>\omega_c$, the total energy of system was negative.

4. Mechanism analysis of shimmy

4.1. Frequency and velocity interval of shimmy

According to the results of section 3.4, the total energy of system was related to the frequency of steering system, and the frequency was always same as the natural frequency of steering system.

Then we linearized the Equation(1)~(3), and obtained:

$$\dot{X} = AX + g(X)$$  \hspace{1cm} (23)

where $X \in \mathbb{R}^{12}$, and $A$ is the Jacobian matrix at equilibrium point, and the $g(X)$ is the high order term. And the eigenvalues of the Jacobian matrix can be calculated with:

$$\det(\lambda I - A) = 0$$  \hspace{1cm} (24)

Combining the parameters in Table 1, we can obtain the eigenvalues of $A$, $\omega_1=35.2$ rad/s, $\omega_2=51.1$ rad/s, $\omega_3=129.8$ rad/s, $\omega_4=599.7$ rad/s, $\omega_5=599.7$ rad/s, and they were also the natural frequency of steering system. While it was necessary to explicit the relationship of different natural frequencies and velocity range of shimmy. The total energy of steering system at different velocity and frequency can be obtained with the values and Equation (13)~(20). And the result was shown in Figure 4(a), where the red region was the negative region of steering system, and the green region was the positive region, and the corresponding range of frequency and velocity were marked as vermilion. As we can see, only the first natural frequency $\omega_1$ had intersection with the positive energy region. Therefore, we can believe that the frequency of shimmy was the first natural frequency $\omega_1$.

And we taken the value of $\omega_1$ as the frequency in Equation (11), the total energy of steering system was obtained and shown in Figure 4(b). The energy consumption was not affected by velocity according to the results in Figure 4(b), therefore the total energy of steering system increased firstly and then decreased with the increase of velocity. And the total energy was positive during 34 km/h~72 km/h, which can be considered as the velocity interval of shimmy. According to the result in Figure 5, when the steering system occurred shimmy, the amplitude of vibration would increase continuously, while it was not the case in practice. It can be explained by the nonlinear characteristic of sideslip force of tire. The error between the linear model and nonlinear model increased with the increasing of sideslip angle, which also implied that the amplitude of shimmy increased. And the input energy originated from sideslip force was less than the calculated values. Therefore the amplitude of shimmy can be stable in practice. This also shown that the calculation of amplitude should use nonlinear model of tire.

(a)                           (b)

Figure 4. Total energy of steering system at different velocity and frequency

4.2. Calculation of shimmy amplitude

When the shimmy occurred, the state of steering system satisfied
Then we calculated the shimmy amplitude with the Equation (1)–(3), (21), and (25) based on the conservation of energy. The velocity interval of shimmy was 34 km/h–72 km/h, the maximum amplitude of shimmy was 5.2°.

When the steering system was disturbed in the static equilibrium position and the velocity was within the range where the shimmy might occur, where $W_{\text{in}} > W_{\text{out}}$, the energy of the steering system will increase gradually with time, showing that the amplitude of the shimmy will increase, including the tire sideslip angle. And there was a non-linear relationship between the sideslip angle and the sideslip force, which resulted in the reducing of feedback block $W_{\text{in}}$ with the increase of the amplitude until the condition $W_{\text{in}}=W_{\text{out}}$ was met. Then the work brought by the feedback block was equal to the energy consumption by damping in steering system. The continuous reciprocating vibration occurred, i.e., shimmy.

4.3. Numerical simulation verification

Numerical simulation of the equation (1)–(3) was carried out with Runge-Kutta method combining with the parameters in Table 1, assuming that the left wheel encountered perturbation. The comparing results of theoretical and numerical simulation were shown in Figure 5. The velocity interval and the amplitude of shimmy were all consistent according to the Figure 5 (a). The amplitude of shimmy was 4.4° and the frequency was 5.6 Hz according to the results in Figure 5 (a) and (b), which means that the frequency of shimmy was consistent with theoretical results. Therefore the results of theoretical analysis based on the coupled feedback model were verified by the numerical simulation.

5. Conclusion

(1) A 5 DOF dynamical model of steering system with independent suspension was established firstly, and then the coupled feedback model of system was obtained based on the dynamical model.

(2) On this basis, the frequency characteristic of feedback block was analyzed. The instability velocity interval, frequency, and amplitude of shimmy were obtained based on conservation of energy. The relationship of them and mechanism of shimmy were uncovered further according to the results.

(3) The results obtained by theoretical analysis were verified by numerical simulation, which can provide theoretical support for the suppression and active control of shimmy in steering system.

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