The Body Center Cubic Quark Lattice Model
(A Modification and Further Development of the Quark Model)

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Abstract

We assume that the u quarks and the d quarks constitute a body center cubic quark lattice in the vacuum. Using energy band theory, we deduce an excited quark spectrum (from the quark lattice). Using the accompanying excitation concept, we deduce a baryon spectrum (including S, C, b, I, Q, and mass) from the quark spectrum. With a phenomenological binding energy formula, we deduce a meson spectrum (including S, C, b, I, Q, and mass) from the quark spectrum. The baryon and meson spectra agree well with experimental results. The BCC Quark Model predicts many new quarks (u′(3), d′(6)), baryons (Λ^0(4280), Λ^+_b(6600), Λ_b^0(9960)), and mesons (K(3597), D(5996), B(9504), η(5926), Υ(17805), T(1603) with I=2). The quarks u′(3) and d′(6) and the meson T(1603) have already been discovered.

I Introduction

The Quark Model [1] has successfully deduced intrinsic quantum numbers (I, S, C, b, and Q) for all baryons and mesons. 1) It cannot, however, deduce the mass spectrum of baryons or the mass spectrum of mesons. 2) Some intrinsic quantum numbers and masses of the quarks are entered “by hand” [2]. 3) The Quark Model needs too many elementary quarks. 4) It needs too many parameters [3]. 5) Confinement is a very
plausible idea, but to date its rigorous proof remains outstanding [4]. 6) The large mass
differences of the quarks \(m_u = 1 - 5 \text{ Mev}, m_d = 3 - 9 \text{ Mev}, m_s = 75 - 170 \text{ Mev}, m_c =
1150 - 1350 \text{ Mev}, m_b = 4000 - 4400 \text{ Mev}, m_t = 174 \text{ Gev} \) [5] have already broken down
part of the mathematical foundation of the quark model (the flavored SU(6), SU(5),
might be SU(4) symmetries). 7) The quark masses of the Quark Model are not large
enough to build stable baryons and mesons. Thus, the confinement assumption of the
Quark Model cannot be maintained. Therefore, the Quark Model needs modification
and further development.

The BCC Quark Model (the Body Center Cubic Quark Lattice Model) [6] is a
good renovation of the Quark Model. It needs only 2 elementary quarks (u and d) and
deduces an excited (from the vacuum) quark spectrum (including S, C, b, I, Q, and M)
[7]. Using the quark spectrum, it can deduce the baryon spectrum [8] and the meson
spectrum [9], which both agree well with experimental results [10]. It also predicts some
new quarks, baryons, and mesons.

II  Fundamental Hypotheses

From the Dirac sea concept and the Quark Model, there are infinite u-quarks and d-
quarks in the vacuum. There are super strong attractive forces among the quarks. The
forces make and hold the densest structure – the body center cubic quark lattice [11].
Thus, we assume:

**Hypothesis I :** There is only one family of elementary quarks in the vacuum state.
They have the baryon number \(B = 1/3\), spin \(s = 1/2\), isospin \(I = 1/2\), \(S = C = b = 0\),
and one of the three color charges. The quarks with \(I_z = +1/2\) are called u quarks,
and the quarks with \(I_z = -1/2\) are called d quarks. There are super strong attractive
interactions among the quarks (colors). The forces make and hold an infinite body center
cubic (BCC) quark (u and d) lattice in the vacuum.
Hypothesis II: After a quark \( q \) is excited from the vacuum \( (q \rightarrow q^*) \), it can move in the vacuum space (the vacuum BCC quark lattice) freely; the nearest quarks are accompanying excited simultaneously by the excited quark \( q^* \) as well. Since the super forces are short range and saturable (a 3 different colored quark system is a colorless one), there are only 2 accompanying excited quarks, \( u' \) and \( d' \), for each excited quark \( (q^*) \). The 2 quarks, \( u' \) and \( d' \), are just a primitive cell of the BCC quark lattice. We call the excitation of the primitive cell the accompanying excitation.

The accompanying excited quark \( u' \) (or \( d' \)) does not leave its position in the quark lattice and cannot move freely in space since its excited energy is not large enough. Their color charges, electric charges, and baryon numbers, however, are temporarily excited from the vacuum state under the influence of the excited quark \( q^* \). Thus the excited energy of the accompanying excited quarks \( (u' \) and \( d') \) is much smaller than the excited energy of the quark \( q^* \). We have estimated that they are “very small and can be treated as a perturbation energy (0-order approximation, \( m_{u'} = m_{d'} = 0 \)” in our earlier paper \[12\]. For convenience, we assume furthermore that

\[
m_{u'} + m_{d'} = \alpha M_p = 938/137 = 7 \text{MeV},
\]

where \( M_p \) is the mass of proton, \( \alpha = e^2/\hbar c = 1/137 \) – the fine structure constant. Comparing them with theoretical and experimental results, we find that the masses of the quarks \( (u \) and \( d) \) of the Quark Model \( (m_u = 1 \text{ to } 5 \text{ MeV}, \ m_d = 3 \text{ to } 9 \text{ MeV}) \) \[5\] are just the quantities \( m_{u'} \) and \( m_{d'} \), and that the accompanying excited quarks \( (u' \) and \( d') \) have the same intrinsic quantum numbers \( (B, S, s, I, I_Z \) and \( Q) \) with the quarks \( (u \) and \( d) \). Thus, the accompanying excited quark \( u' \) is the quark \( u \) \[5\] with

\[
B = 1/3, \ S = 0, \ s = I = 1/2, \ I_z = 1/2, \ Q = 2/3, \ m_{u'} = 3 \text{ MeV};
\]

and the accompanying excited quark \( d' \) is the quark \( d \) \[5\] with

\[
B = 1/3, \ S = 0, \ s = I = 1/2, \ I_z = -1/2, \ Q = -1/3, \ m_{d'} = 6 \text{ MeV}.
\]
The accompanying excitation is temporary for a cell (u′ and d′). When the quark \( q^* \) is excited from the vacuum, the primitive cell (in which \( q^* \) is located) undergoes an accompanying excitation, which is due to \( q^* \); but when \( q^* \) leaves the cell, the excitation of the cell disappears. Although the truly excited cells are quickly changed— one following another—with the motion of \( q^* \), an accompanying excited primitive cell (u′ and d′) always appears to accompany \( q^* \), just as an electric field always accompanies the original electric charge.

**Hypothesis III**: Due to the effect of the vacuum quark lattice, fluctuations of energy \( \varepsilon \) and of intrinsic quantum numbers (such as the strange number \( S \)) of an excited quark may exist. The fluctuation of the Strange number, if it exists, is always \( \Delta S = \pm 1 \)\(^{[13]} \). From the fluctuation of the Strange number, we will be able to deduce new quantum numbers, such as Charmed number \( C \) and Bottom number \( b \).

The excited quark \( q^* \) moving in space, in fact, is moving in a periodic field of the body center cubic quark lattice. According to the energy band theory \([14]\), it will be in one of the energy bands [with a degeneracy (deg) on a rotary fold (R) symmetry axis] of the periodic field. Thus, it will have different quantum numbers and energy (mass) when it is in the different energy band.

**Hypothesis IV**: The quantum numbers and masses of the excited quarks are determined as follows (except for the 6 energy bands of the first Brillouin zone):

1. Baryon number \( B \): When a quark is in a vacuum state, \( B = 0 \). If, however, it is excited from the vacuum state, it has

\[
B = 1/3. \tag{4}
\]

2. Isospin number \( I \): \( I \) is determined by the energy band degeneracy deg \([14]\), where

\[
\text{deg} = 2I + 1. \tag{5}
\]
3. Strange number $S$: $S$ is determined by the rotary fold $R$ of the symmetry axis with

$$S = R - 4,$$  

(6)

where the number 4 is the highest possible rotary fold number of the BCC lattice.

4. Electric charge $Q$: After obtaining $B$, $S$ and $I$, we can find $Q$ from the Gell-Mann–Nishijimam relationship:

$$Q = I_z + \frac{1}{2}(S + B).$$  

(7)

5. Charmed number $C$ and Bottom number $b$: If a degeneracy (deg) of an energy band is smaller than the rotary fold $R$

$$\text{deg} < R \text{ and } R - \text{deg} \neq 2,$$  

(8)

then formula (R) will instead be

$$\bar{S} = R - 4.$$  

(9)

From Hypothesis III ($\Delta S = \pm 1$), the real value of $S$ is

$$S = \bar{S} + \Delta S = S_{\text{Axis}} \pm 1, \quad \text{if } \text{deg} < R \text{ and } R - d \neq 2.$$  

(10)

The “Strange number”, $S$, in (10) is not completely the same as the Strange number in (R). In order to compare it with the experimental results, we would like to give it a new name under certain circumstances. Based on Hypothesis III, the new names will be the Charmed number and the Bottom number:

if $S = +1$, which originates from the fluctuation $\Delta S = +1$, then we call it the Charmed number $C$ ($C = +1$);
if $S = -1$, which originates from the fluctuation $\Delta S = +1$,
and if there is an energy fluctuation,
then we call it the Bottom number $b$ ($b = -1$). \hfill (12)

Similarly, we can obtain charmed strange quarks $q^*_c$ and $q^*_\Xi_c$.[7]

6. We assume that the excited quark’s static mass is the minimum energy of the
energy band that represents the excited quark:

$$m_{q^*} = \text{Minimum (energy) of the energy band} \hfill (13)$$

7. The 6 energy bands of the first Brillouin zone represent the quark $q^*_N(931)$ [7].

Hypothesis V A meson is made of an excited quark $q^*_i$ and an excited antiquark $\bar{q}^*_j$.
The quantum numbers of the baryons and the mesons are determined by sum laws from
the constituent quarks:

$$B = \sum B_q, \quad S = \sum S_q, \quad C = \sum C_q, \quad b = \sum b_q, \quad Q = \sum Q_q, \quad \vec{I} = \sum \vec{I}_q. \hfill (14)$$

III The Spectrum of the Quarks

When a excited quark is moving in the vacuum, it is moving, in fact, in the periodic
field ($V(\vec{r})$) of the vacuum quark lattice. Since the quark is a Fermion, its motion
equation should be the Dirac equation. According to the renormalization theory [16],
the bare mass ($m_q$) of the quark is much larger than the empirical values of the excited
energies of the quark in the energy bands. Thus, we use the Schrödinger equation instead
of the Dirac equation (our results will show that this is a very good approximation)

$$\frac{\hbar^2}{2m_q} \nabla^2 \Psi + (\varepsilon - V(\vec{r})) \Psi = 0, \hfill (15)$$
where $V(\vec{r})$ denotes the strong interaction periodic field of the vacuum quark lattice with body center cubic symmetries, and $m_q$ is the bare mass of the elementary quark.

Using the energy band theory \[14\] and the free particle approximation (taking $V(\vec{r}) = V_0$ constant and making the wave functions satisfy the body center cubic periodic symmetries), we have

$$\frac{\hbar^2}{2m_q}\nabla^2 \Psi + (\varepsilon - V_0) \Psi = 0,$$

(16)

where $V_0$ is a constant potential. The solution of Eq. (16) is a plane wave

$$\Psi_\varepsilon(\vec{r}) = \exp\{-i(2\pi/a_x)[(n_1 - \xi)x + (n_2 - \eta)y + (n_3 - \zeta)z]\},$$

(17)

where the wave vector $\vec{k} = (2\pi/a_x)(\xi, \eta, \zeta)$, $a_x$ is the periodic constant of the quark lattice, and $n_1$, $n_2$, and $n_3$ are integers satisfying the condition $n_1 + n_2 + n_3 = \pm$ even number or 0. The wave functions must satisfy the symmetries of BCC periodic field.

The 0-order approximation of the energy (in \[12\], we assume $m_u = m_d = 0 \to V_0 = 940$ Mev; now we find $m_u' = 3$ Mev, $m_d' = 6$ Mev $\to V_0 = 931$ Mev) is

$$\varepsilon^{(0)}(\vec{k}, \vec{n}) = 931 + 360E(\vec{k}, \vec{n})\text{ Mev, } E(\vec{k}, \vec{n}) = (n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \zeta)^2.$$  

(18)

Using the formulae (5), (6), (7), (11), and (12) for quantum numbers and energy (13) of the quarks, we can find the quark spectrum \[7\]:

| q | B | S | C | b | I | Q   | q | B | S | C | b | I | Q   |
|---|---|---|---|---|---|-----|---|---|---|---|---|---|-----|
| u | 0 | 0 | 0 | 0 | 0 | 0   | d | 0 | 0 | 0 | 0 | 0 | 0   |
| u' | 1/3 | 0 | 0 | 0 | 1/2 | 2/3 | d' | 1/3 | 0 | 0 | 0 | 1/2 | -1/3 |
| q^+_N | 1/3 | 0 | 0 | 0 | 1/2 | 2/3,-1/3 | q^+_S | 1/3 | -1 | 0 | 0 | 0 | -1/3 |
| q^+_\Delta | 1/3 | 0 | 0 | 0 | 3/2 | 5/3, 2/3, -1/3 | q^*_6 | 1/3 | 0 | 0 | -1 | 0 | -1/3 |
| q^+_\Sigma | 1/3 | -1 | 0 | 0 | 1 | 2/3,-1/3,-4/3 | q^+_C | 1/3 | 0 | 1 | 0 | 0 | 2/3 |
| q^+_\Xi | 1/3 | -2 | 0 | 0 | 1/2 | -1/3,-4/3 | q^*_9 | 1/3 | -3 | 0 | 0 | 0 | -4/3 |
| q^+_\Xi_C | 1/3 | 0 | 1 | 0 | 1 | 5/3,2/3,-1/3 | q^*_9_C | 1/3 | -2 | 1 | 0 | 0 | -1/3 |
| q^+_{\Xi C} | 1/3 | -1 | 0 | 0 | 1/2 | 2/3,-1/3 | --- | --- | --- | --- | --- | --- | --- |
The elementary quarks (u and d) in the BCC vacuum quark lattice, \( m_u = m_d = 0 \).

The accompanying excited quarks \( u' \) and \( d' \), \( m_{u'} = 3 \text{ Mev} \), and \( m_{d'} = 6 \text{ Mev} \).

\( S = C = b = 0; I = \frac{1}{2}; I_{x, u'} = \frac{1}{2}; I_{x, d'} = -\frac{1}{2} \); \( Q_{u'} = \frac{2}{3} \), \( Q_{d'} = -\frac{1}{3} \), spin \( s = \frac{1}{2} \).

The energy band excited states \( q^* \) (from the vacuum) of the quarks (u and d):

| \( q_N^* (931) \# \) | \( q^*_u (931) \# \) | \( q^*_d (931) \# \) | \( q^*_S (1111) \# \) | \( q^*_C (2271) \# \) | \( q^*_b (5531) \# \) |
|------------------|------------------|------------------|------------------|------------------|------------------|
| \( q_N^* (1201), \ q^*_\Delta (1291) \) | \( q^*_S (1391), \ 2q^*_\Xi (1291) \) | \( q^*_C (2441), \ q^*_b (9951) \) |
| \( q_N^* (1471), \ 2q^*_\Delta (1651) \) | \( q^*_S (2011), \ 3q^*_\Xi (1471) \) | \( q^*_C (2531), \ q^*_b (15811) \) |
| \( 2q_N^* (1831), \ q^*_\Delta (2011) \) | \( q^*_S (2451), \ 2q^*_\Xi (1651) \) | \( q^*_C (2961) \) |
| \( 4q_N^* (1921), \ q^*_\Delta (2371) \) | \( q^*_S (2551), \ 2q^*_\Xi (1831) \) | \( q^*_C (6591), \ q^*_\Xi (2541) \) |
| \( 2q_N^* (2191), \ 4q^*_\Delta (2731) \) | \( 3q^*_S (2641), \ q^*_\Xi (1921) \) | \( q^*_C (13791), \ q^*_\Xi (2631) \) |
| \( 2q_N^* (2551), \ 3q^*_\Delta (3091) \) | \( q^*_S (2731), \ 2q^*_\Xi (2011) \) | \( q^*_\Xi (3161) \) |
| \( 3q_N^* (2641), \ 2q^*_\Xi (2191) \) | \( q^*_\Xi (2441) \) |
| \( 2q_N^* (2731) \) | \( q^*_S (1201), \ 2q^*_\Xi (2371) \) | \( q^*_\Sigma_c (2531), \ q^*_\Omega_c (2651) \) |
| \( 3q^*_S (1651), \ 3q^*_\Xi (2551) \) | \( q^*_\Sigma_c (2961), \ q^*_\Omega_c (3471) \) |
| \( 5q^*_S (1921), \ 8q^*_\Xi (2731) \) |
| \( 2q^*_S (2011) \) |
| \( q^*_S (2371), \ q^*_\Omega (1651) \) |
| \( 3q^*_S (2551), \ q^*_\Omega (2451) \) |
| \( 2q^*_S (2641), \ q^*_\Xi (3071) \) |
| \( 3q^*_S (2731), \ q^*_\Xi (3711) \) |

**IV The Baryon Spectrum**

Since the baryon number of the system \((q^* u' d')\) is

\[
B_{(q^* u' d')} = B_{q^*} + B_{u'} + B_{d'} = 1
\]  

(21)

from \((2)\), \((3)\), \((4)\), and \((14)\), the 3 quark system \((q^* u' d')\) is a baryon. The mass of the baryon \((q^* u' d')\) equals

\[
M_{(q^* u' d')} = m_{q^*} + m_{u'} + m_{d'}.
\]  

(22)
Using the sum laws (14) and (22), from (19) and (20), we can find the baryon spectrum (including S, C, b, I, Q, and mass). We compare the theoretic results with the experimental results [17] in Table 1 – 4. In the comparison, we do not take into account the angular momenta of the baryons. We assume that the small differences of the masses in the same group of baryons (the same kind of baryons with roughly the same masses but different angular momenta) originate from their angular momenta. If we ignore this effect, their masses should be the same. We use the average of the masses to represent the mass of the group. We show quantum numbers of a baryon by its name.

Table 1. The Ground States of the Baryons

| Theory   | Experiment | $\Delta M / M$ % | Theory   | Experiment | $\Delta M / M$ % |
|----------|------------|-----------------|----------|------------|-----------------|
| N(940)   | N(939)     | 0.1 %           | $\Omega^-$ (1660) | $\Omega^-$ (1672) | 0.7 %           |
| $\Lambda$(1120) | $\Lambda$(1116) | 0.4 %         | $\Lambda^+_c$ (2280) | $\Lambda^+_c$ (2285) | 0.2 %           |
| $\Sigma$(1210) | $\Sigma$(1193) | 1.4 %         | $\Lambda^0_b$ (5540) | $\Lambda^0_b$ (5624) | 1.5 %           |
| $\Xi$(1300) | $\Xi$(1318) | 1.4 %         |          |            |                 |

Table 2. The Unflavored Baryons $N$ and $\Delta$ ($S=C=b=0$)

| Theory   | Experiment | $\Delta M / M$ % | Theory   | Experiment | $\Delta M / M$ % |
|----------|------------|-----------------|----------|------------|-----------------|
| $\bar{N}$(1255) | $\bar{\Delta}$ (1255) | 1.9 |          |            |                 |
| $\bar{N}$(1480) | $\bar{N}$(1498) | 1.2 |          |            |                 |
| $\bar{N}$(1660) | $\bar{N}$(1689) | 1.7 | $\bar{\Delta}$(1660) | $\bar{\Delta}$(1640) | 1.2 |
| $\bar{N}$(1915) | $\bar{N}$(1923)* | 3.9 | $\bar{\Delta}$(1953) | $\bar{\Delta}$(1923) | 1.6 |
| $\bar{N}$(2200) | $\bar{N}$(2220) | 0.9 |          |            |                 |
| $\bar{N}$(2380) | ? | $\bar{\Delta}$(2380) | $\bar{\Delta}$(2420) | 1.7 |
| $\bar{N}$(2628) | $\bar{N}$(2600) | 1.1 | 3$\bar{\Delta}$(2650) | ? |
| $6\bar{N}$(2740) | $\bar{N}$(2700)* | 1.5 | 4$\bar{\Delta}$(2750) | $\bar{\Delta}$(2740)* |
Table 3. The Strange Baryons Λ and Σ ($S = -1$)

| Theory   | Experiment | $\frac{\Delta M}{M} \%$ | Theory   | Experiment | $\frac{\Delta M}{M} \%$ |
|----------|------------|--------------------------|----------|------------|--------------------------|
| $\Lambda(1120)$ | $\Lambda(1116)$ | 0.36%                    | $\Sigma(1210)$ | $\Sigma(1193)$ | 1.4%                     |
| $\Lambda(1400)$ | $\Lambda(1463)$ | 4.3%                     | $\Sigma(1350)$ | $\Sigma(1385)$ | 2.5%                     |
| $\Lambda(1660)$ | $\Lambda(1653)$ | 0.4%                     | $\Sigma(1660)$ | $\Sigma(1714)$ | 3.2%                     |
| $\Lambda(1930)$ | $\Lambda(1830)$ | 5.5%                     | $\Sigma(1930)$ | $\Sigma(1928)$ | 10%                      |
| $\Lambda(2020)$ | $\Lambda(2105)$ | 4.0%                     | $\Sigma(2020)$ | $\Sigma(2030)$ | 50%                      |
| $\Lambda(2420)$ | $\Lambda(2350)$ | 3.0%                     | $\Sigma(2380)$ | $\Sigma(2353)$# | 1.2%                     |
| $\Lambda(2560)$ | $\Lambda(2585)^*$ | 1.0%                     | $\Sigma(2650)$ | $\Sigma(2620)$ | 1.1%                     |
| $\Lambda(2650)$ | ?                      |                         | $\Sigma(2740)$ | ?                      |                          |

# $\Sigma(2353)^# = 1/2(\Sigma(2250)+\Sigma(2455))$

Table 4. The Ξ, Ω, $\Lambda_c^+$, $\Sigma_c$, and $\Omega_C$ Baryons

| Theory   | Experim. | $\frac{\Delta M}{M} \%$ | Theory   | Experim. | $\frac{\Delta M}{M} \%$ |
|----------|----------|--------------------------|----------|----------|--------------------------|
| $\Xi(1300)$ | $\Xi(1318)$ | 1.4 %                    | $\Omega(1660)$ | $\Omega(1672)$ | 0.7 %                    |
| $\Xi(1480)$ | $\Xi(1530)$ | 3.3 %                    | $\Omega(2460)$ | $\Omega(2367)$ | 3.9 %                    |
| $\Xi(1660)$ | $\Xi(1690)$ | 1.8 %                    | $\Omega(3080)$ | ?                      |                          |
| $\Xi(1840)$ | $\Xi(1820)$ | 1.1 %                    | $\Xi(1930)$ | $\Xi(1950)$ | 1.1 %                    |
| $\Xi(2020)$ | $\Xi(2030)$ | 1.0 %                    | $\Lambda_c^+(2280)$ | $\Lambda_c^+(2285)$ | 0.22 %                   |
| $\Xi(2200)$ | $\Xi(2250)$* | 0.5 %                    | $\Lambda_c^+(2540)$ | $\Lambda_c^+(2609)$ | 2.3 %                    |
| $\Xi(2380)$ | $\Xi(2370)^*$ | 0.4.2 %                  | $\Lambda_c^+(2970)$ | ?                      |                          |
| $\Xi(2560)$ | ?                      |                         | $\Lambda_c^0(5540)$ | $\Lambda_c^0(5624)$ | 1.5%                     |
| $\Xi(2740)$ | ?                      |                         | $\Lambda_c^0(9960)$ | ?                      |                          |
| $\Xi_C(2550)$ | $\Xi_C(2523)$ | 1.1 %                    | $\Sigma_c(2495)$ | $\Sigma_c(2488)$ | .28 %                    |
| $\Xi_C(2640)$ | $\Xi_C(2730)$ | 3.2 %                    | $\Sigma_c(2970)$ | ?                      |                          |
| $\Xi_C(3170)$ | ?                      |                         | $\Omega_C(2660)$ | $\Omega_C(2704)$ | 1.6 %                    |
| $\Omega_C(3480)$ | ?                      |                         | $\Omega_C(3480)$ | ?                      |                          |

In Table 1 – Table 4, we see that the theoretical baryon spectrum agrees well with the experimental results [17].
V  The Meson Spectrum

According to the Quark Model, a meson is composed of a quark and an antiquark. Using the quark quantum numbers (19) and the sum law (14), we can find the quantum numbers (S, C, b, I, Q) of the mesons. Using a phenomenological formula of the meson binding energy (23) and the mass of the quarks (20), we can find the masses [M(q_iq_j)=m_{q_i}+m_{q_j}+E_{bind}] of the mesons. The binding energy $E_{B(i,j)}$ of a quark ($q_i^*$) and an antiquark ($\bar{q}_j^*$) in the meson $M(q_i^*\bar{q}_j^*)$ is

$$E_{B(i,j)} = -1723 + 100\{N + [2-|S| + \delta_{ng}f(I,S,C)](1-\delta_{ij}) + 2\delta_{ng} + (1-\tilde{m}) - \delta_{SP}N\} + 25A. \quad (23)$$

Where $N = |S_{i or j}| + 1.5|C_{i or j}| + 3|b_{i or j}|$, S is the strange number, C is the charmed number, and b is the bottom number. If $q_i^*$ and $\bar{q}_j^*$ have the same S, C, or b, $N_{i or j} = N_i$. $\delta_{ng} = 1$ if $q_i^*$ or $\bar{q}_j^*$ is not ground state (or quark that is born from the single band of the $\Delta$-axis and the $\Lambda$-axis). $f(I,S,C) = -1.5\text{SI}_q\text{SI}_{\bar{q}} + \Delta I - 2\frac{S}{\Delta S}$, $\text{SI}_q$ is strange number $S$ times $I$ of $q_i^*$; $\text{SI}_{\bar{q}}$ is strange number $S$ times $I$ of $\bar{q}_j^*$. $\Delta I = |I_i - I_j|$, $\Delta S = |S_i - S_j|$, $\Delta C = |C_i - C_j|$, $\tilde{m} = m_qm_{\bar{q}}/m_qm_{\bar{q}}$. $\delta_{SP} = 1$ if $i = j$ and both quarks ($q_i^*$ and $\bar{q}_j^*$) are born from the single binds; $\delta_{SP} = 0$ in all other cases. $A = G_q - G_{q\bar{q}} - SI_q + SI_{\bar{q}}$, $G = S + 1.5C + 3b$. Although the above formula looks very complex, we can simplify it into seven cases [1]; thus, it becomes very simple and easy to use. For example, for ground quark pairs, from (23) we have

$$E_B(i, i) = -1723 + 100N_i + 50(G_q - SI_q). \quad (24)$$

Using (24), we find the mesons that are composed of the ground quark pairs:

$$q_N^*(931)q_N^*(931) = \pi(139)[\pi(139)] \quad q_C^*(2271)q_C^*(2271) = J/\Psi(3044)[J/\Psi(3097)]$$
$$q_S^*(1111)q_S^*(1111) = \eta(549)[\eta(547)] \quad q_b^*(5531)q_b^*(5531) = \Upsilon(9489) [\Upsilon(9460)]. \quad (25)$$

In the fashion similarly to (24) and (25), we can find the meson spectrum [4].
We compare the theoretical results with the experimental results using Table 5 - 7. In the comparison, as is the case with the baryon spectrum, we do not take into account the angular momenta but show the quantum numbers of a meson with its name.

Table 5  Light Unflavored Mesons (S=C=b=0) and Bottom Mesons (b=±1)

| Light Unflavored Mesons (I = 0) | Light Unflavored Mesons (I = 1) |
|---------------------------------|---------------------------------|
| The BCC Quark Model             | Exper.                          | The BCC Quark Model | Exper. |
| $q_i(m_k)q_j(m_l) = \eta(m)$   | M(m)                            | $q_i(m_k)q_j(m_l) = \pi(m)$ | M(m) |
| $q_5(1111)q_S^*(1111) = \eta(549)$ | $\eta(547)$                   | $q_N^()(931)q_N^*(931) = \pi(139)$ | $\pi(139)$ |
| $q_N^*(931)q_N^*(1201) = \eta(780)$ | $\omega(782)$                  | $q_N^*(931)q_N^*(1201) = \pi(780)$ | $\rho(770)$ |
| $q_N^*(1201)q_N^*(1201) = \eta(813)$ | $f_0(800)$                    | $q_N^*(931)q_N^*(1291) = \pi(960)$ | $a_0(980)$ |
| $q_S^*(1391)q_S^*(1391) = \eta(952)$ | $\eta'(958)$                 | $q_N^*(931)q_N^*(1651) = \pi(1282)$ | $\pi(1248)$ |
| $q_A^*(1291)q_A^*(1291) = \eta(967)$ | $f_0(800)$                    | $q_N^*(931)q_N^*(1831) = \pi(1342)$ | $\pi(1340)$ |
| $q_N^*(931)q_N^*(1471) = \eta(1021)$ | $\phi(1020)$                  | $q_N^*(931)q_N^*(1921) = \pi(1423)$ | $\pi(1450)$ |
| $q_S^*(1111)q_S^*(1391) = \eta(1204)$ | $h_1(1170)$                  | $q_N^*(931)q_N^*(2011) = \pi(1603)$ | $\pi(1600)$ |
| $q_N^*(1471)q_N^*(1471) = \eta(1269)$ | $\eta(1283)$                  | $q_N^*(931)q_N^*(2191) = \pi(1664)$ | $\pi(1687)$ |
| $q_N^*(931)q_N^*(1831) = \eta(1342)$ | $\eta(1375)$                 | $q_N^*(1111)q_S^*(1921) = \pi(1861)$ | $\pi(1800)$ |
| $q_N^*(931)q_N^*(1921) = \eta(1422)$ | $\eta(1428)$                  | $q_N^*(931)q_N^*(2371) = \pi(1924)$ | $\chi(2000)$ |
| $q_A^*(1651)q_A^*(1651) = \eta(1565)$ | $\eta(1525)$                  | $q_N^*(931)q_N^*(2641) = \pi(2065)$ | $\pi(2040)$ |
| $q_N^*(931)q_N^*(2191) = \eta(1664)$ | $\eta(1656)$                  | $q_N^*(931)q_N^*(2731) = \pi(2146)$ | $\pi(2125)$ |
| $q_N^*(1111)q_S^*(2011) = \eta(1768)$ | $\eta(1735)$                  | $q_N^*(931)q_N^*(2731) = \pi(2244)$ | $\pi(2250)$ |
| $q_N^*(1831)q_N^*(1831) = \eta(1852)$ | $\eta(1850)$                  | $q_N^*(1111)q_S^*(2551) = \pi(2434)$ | $\pi(2400)$ |
| $q_N^*(931)q_N^*(2551) = \eta(1985)$ | $\eta(1930)$                  | $q_N^*(931)q_N^*(2641) = \eta(2065)$ | $\eta(2030)$ Bottom (b=±1) Meson Exper. |
| $q_N^*(931)q_N^*(2641) = \eta(2065)$ | $\eta(2030)$                  | $q_N^*(931)q_N^*(2641) = \eta(2146)$ | $\eta(2199)$ |
| $q_N^*(931)q_N^*(2731) = \eta(2146)$ | $\eta(2199)$                  | $q_N^*(931)q_N^*(2731) = \eta(2423)$ | $\eta(2510)$ |
| $q_S^*(1111)q_S^*(2641) = \eta(2341)$ | $\eta(2313)$                  | $q_N^*(5531)q_N^*(931) = B(5164)$ | B(5279) |
| $q_S^*(1111)q_S^*(2731) = \eta(2423)$ | $\eta(2510)$                  | $q_N^*(5531)q_N^*(2641) = B(5655)$ | B(5732) |
| $q_S^*(2731)q_S^*(2731) = \eta(3178)$ | $\eta(3178)$                  | $q_N^*(5531)q_N^*(1471) = B(5896)$ | ? |
| $q_S^*(2641)q_S^*(2641) = \eta(3479)$ | $\eta(3479)$                  | $q_N^*(5531)q_N^*(1111) = B_S(5319)$ | B_S(5370) |
| $q_S^*(4271)q_S^*(4271) = \eta(5926)$ | $\eta(5926)$                  | $q_N^*(5531)q_N^*(1391) = B_S(5674)$ | B_S(5850) |
| $q_S^*(5531)q_S^*(2271) = B_C(6691)$ | $B_C(6400)$                   | $q_N^*(5531)q_N^*(2271) = B_C(6691)$ | $B_C(6400)$ |

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### Table 6  K Mesons, D Mesons, and D_S Mesons

| The BCC Quark Model | Exper. | The BCC Quark Model | Exper. |
|---------------------|--------|---------------------|--------|
| $q_i(m_k)q_j(m_l)$ = K(m) | K(494) | $q_i(m_k)q_j(m_l)$ = D(m) | D(1866) | M(m) |
| $q_N^*(3931)q_S^*(1111)$ = K(494) | K(494) | $q_N^*(2271)q_N^*(3931)$ = D(1866) | D(1869) |
| $q_N^*(3931)q_S^*(1391)$ = K(899) | K(892) | $q_N^*(2441)q_N^*(931)$ = D(2029) | D(2010) |
| $q_N^*(3931)q_S^*(1651)$ = K(1310) | K(1270) | $q_N^*(2531)q_N^*(931)$ = D(2115) | ? |
| $q_N^*(3931)q_S^*(2011)$ = K(1463) | K(1423) | $q_N^*(2271)q_N^*(1471)$ = D(2349) | D(1(2420) |
| $q_N^*(3931)q_S^*(1921)$ = K(1556) | K(1580) | $q_N^*(2961)q_N^*(931)$ = D(2526) | D(2507) |
| $q_N^*(3931)q_S^*(2011)$ = K(1638) | K(1680) | $q_N^*(6591)q_N^*(931)$ = D(5996) | ? |
| $q_N^*(2191)q_S^*(1111)$ = K(1769) | K(1775) | $q_i(m_k)q_j(m_l)$ = D_S(m) |
| $q_N^*(3931)q_S^*(2551)$ = K(1804) | K(1820) | $q_i(m_k)q_j(m_l)$ = D_S(m) |
| $q_N^*(3931)q_S^*(2371)$ = K(1966) | K(1965) | $q_N^*(2271)q_S^*(1111)$ = D_S(2021) | D_S(1969) |
| $q_N^*(3931)q_S^*(2641)$ = K(2036) | K(2045) | $q_N^*(2271)q_S^*(1391)$ = D_S(2126) | D_S(2112) |
| $q_N^*(3931)q_S^*(2641)$ = K(2211) | K(2250) | $q_N^*(2961)q_S^*(1111)$ = D_S(2531) | D_S(2555) |
| $q_N^*(3931)q_S^*(2731)$ = K(2293) | K(2350) | $q_N^*(2271)q_S^*(2551)$ = D_S(3333) | ? |
| $q_N^*(3931)q_S^*(3091)$ = K(2621) | K(2500) | $q_N^*(6591)q_S^*(1111)$ = D_S(6151) | ? |
| $q_N^*(3931)q_S^*(4271)$ = K(3597) | ? |

### Table 7  Charmed Pair Mesons and Bottomed Pair Mesons

| The BCC Quark Model | Exper. | The BCC Quark Model | Exper. |
|---------------------|--------|---------------------|--------|
| $q_i(m_k)q_j(m_l) = M(m)$ | M(m) | $q_i(m_k)q_j(m_l) = M(m)$ | M(m) |
| $q_S^*(2551)q_S^*(2551)$ = η(2902) | η_c(2902) | η(2902) | Υ(9489) |
| $q_S^*(2271)q_S^*(2271)$ = J/Ψ(3044) | J/Ψ(3097) | $q_S^*(1111)q_S^*(10031)$ = η(9734) | χ(9888) |
| $q_S^*(2641)q_S^*(2641)$ = η(3394) | η(3394) | $q_S^*(1391)q_S^*(10031)$ = η(9955) | Υ(10023) |
| $q_S^*(2731)q_S^*(2731)$ = η(3535) | η(3510) | $q_S^*(2011)q_S^*(10031)$ = η(10443) | χ(10252) |
| $q_S^*(2441)q_S^*(2441)$ = η(3568) | η(3569) | $q_S^*(6591)q_S^*(6591)$ = η(10792) | Υ(10355) |
| $q_S^*(2271)q_S^*(2531)$ = ψ(3693) | ψ(3686) | $q_S^*(2451)q_S^*(10031)$ = η(10791) | Υ(10580) |
| $q_S^*(2531)q_S^*(2531)$ = η(3740) | ψ(3770) | $q_S^*(2551)q_S^*(10031)$ = η(10870) | Υ(10860) |
| $q_S^*(1111)q_S^*(4271)$ = η(3952) | ψ(4040) | $q_S^*(2641)q_S^*(10031)$ = η(10941) | ? |
| $q_S^*(2271)q_S^*(2561)$ = η(4014) | ψ(4160) | $q_S^*(4273)q_S^*(10031)$ = η(10612) | Υ(11020) |
| $q_S^*(2961)q_S^*(2961)$ = ψ(4554) | ψ(4415) | $q_S^*(5531)q_S^*(9951)$ = Υ(14329) | ? |
| $q_S^*(6591)q_S^*(2271)$ = η(7374) | ? | $q_S^*(9951)q_S^*(9951)$ = Υ(17805) | ? |
In Table 5 – Table 7, we see that the theoretical meson spectrum agrees well with the experiment results \[10\].

VI Predictions and Discussion

A Some New Particles

1. New Quarks

| \(u'(3)\) | \(q^*_N(3091)\) | \(q^*_S(2551)\) | \(q^*_S(4271)\) | \(q^*_b(10031)\) | \(q^*_b(9951)\) | \(q^*_b(15811)\) |
| \(d'(6)\) | \(q^*_\Delta(3091)\) | \(q^*_C(2961)\) | \(q^*_C(6591)\) | \(q^*_b(13791)\) | \(q^*_b(3071)\) | \(q^*_b(3711)\) |

2. New Baryons

| \(N(3100)\) | \(\Lambda^0(2560)\) | \(\Lambda^0(4280)\) | \(\Lambda^0_b(10040)\) | \(\Lambda^0_b(9960)\) | \(\Lambda^0_b(15820)\) |
| \(\Delta(3100)\) | \(\Lambda^-_C(2970)\) | \(\Lambda^-_C(6600)\) | \(\Lambda^-_C(13800)\) | \(\Omega^- (3080)\) | \(\Omega^- (3720)\) |

3. New Mesons

| \(q^*_N(931)q^*_S(4280)\) \(=K(3597)\) | \(q^*_N(931)q^*_b(9951)\) \(=B(9504)\) |
| \(\bar{q}^*_N(931)q^*_C(6591)\) \(=D(5996)\) | \(\bar{q}^*_S(1111)q^*_b(9951)\) \(=B_S(9659)\) |
| \(q^*_C(6591)q^*_S(1111)\) \(=D_S(6151)\) | \(q^*_S(4271)q^*_b(3271)\) \(=\eta(5926)\) |
| \(q^*_S(10031)q^*_b(10031)\) \(=\eta(17837)\) | \(q^*_N(931)q^*_\Delta(1291)\) \(=T(960)\) |
| \(q^*_C(13791)q^*_C(13791)\) \(=\psi(25596)\) | \(q^*_N(931)q^*_\Delta(1651)\) \(=T(1282)\) |
| \(q^*_b(9951)q^*_b(9951)\) \(=\Upsilon(17805)\) | \(q^*_N(931)q^*_\Delta(2011)\) \(=T(1603)\) |

The discovery of any one of the above mesons will provide strong support for the BCC Quark Model. The meson \(T(1603)\) with \(I = 2\) has been discovered \[18\] \((\chi(1600))\), although this still needs confirmation. The quarks \(u'(3)\) and \(d'(6)\) have already been discovered \([u(1–5) \text{ and } d(3–9)]\) \[5\].
B Discussion

1. Based on Dirac’s sea concept of the vacuum, the BCC Quark Model assumes that an infinite body center cubic quark (u and d) lattice is in the vacuum. The quark lattice is the foundation of the quark spectrum, the baryon spectrum, and the meson spectrum. Thus, it is the physical foundation of the BCC Quark Model. It should be the physical foundation of the Quark Model also since the Quark Model is an approximation of the BCC Quark Model. That the theoretical baryon spectrum and the theoretical meson spectrum agree well with the experimental results shows that the BCC quark lattice does exist in the vacuum.

2. After the discovery of super conductors, we could understand the vacuum material. In a sense, the vacuum material (skeleton – the BCC quark lattice) works like a superconductor. Since the energy gaps are so large (the energy gap of an electron is about 0.5 Mev and the energy gap of a proton is 939 Mev), under the transition temperature (much higher than the temperature at the center of the sun), there are no electric or mechanical resistances to any particle or to any physical body moving inside the vacuum material. The vacuum material is a super superconductor.

3. The Quark Model assumes 6 flavored elementary quarks (u, d, s, c, b, and t); there are the 6 flavored quark in the vacuum state; there are only the 6 flavored quarks in the excited states (from the vacuum) also. They have fixed intrinsic quantum numbers (I, S, C, b, and Q) and static masses that cannot be changed. Using 5 of the 6 flavored quarks (the t-quark with m=175 Gev is not needed), the Quark Model explains the baryon spectrum and the meson spectrum. It cannot deduce the mass spectrums of the baryons and the mesons and cannot simply explain the decays of the baryons and the mesons (it needs to give up the “elementary” concept of the 5 flavored quarks and the assumption that the quarks d, s, c, and b can decay into other quarks). The BCC quark model needs only 2 flavored quarks (u and d); there are only the 2 flavored quarks in
the vacuum state (the BCC quark lattice); all other quarks of the BCC Quark Model are excited quarks. There are many different energy-band excited states (including the quarks s, c, and b) with different intrinsic quantum numbers (I, S, C, b, and Q) and static masses (the quark spectrum). Using the excited quark spectrum, the BCC Quark Model can deduce the mass spectra of the baryons and the mesons, and it can explain easily the decays of the baryons and the mesons since the higher mass quarks are the higher energy excited states of the same quark. (The higher energy excited states decay into lower energy excited states and finally into the ground state is a physical law.)

4. A very important distinction between the BCC Quark Model and the Quark Model is the model of the baryons. A baryon is composed of 3 quarks in the Quark Model (proton=udd); but, in the BCC Quark Model, a baryon is made of an excited quark (q*) and 2 accompanying excited quarks (u' and d'). The two models give the same intrinsic quantum numbers (S,C,b, I, and Q) but a different mass for each baryon. The baryon model of the Quark Model is like a molecule (baryon) made by 3 atoms (quarks); the baryon model of the BCC Quark Model is like 1 earth (q*) with 2 moons (u' and d'). Which is correct?

5. The small masses (m_u = 1 to 5 Mev, m_d = 3 to 9 Mev) of the quarks are supported by a vast amount of experimental and theoretic results. We believe that there are actually the quarks u(3) and d(6) inside baryons. Although they are not mistakes, they are really too small to build a stable baryon. Using the sum laws (14) and the quark masses of the Quark Model, we find the theoretical masses of the most important baryons of the Quark Models. Similarly, using the sum laws and the quark spectrum (20), we find the masses of the same baryons in the BCC Quark Model. We list the theoretical results of the two models and the experimental results of the most important baryons as follows:
Comparing the theoretical results of the Quark Model with the experimental results \((26)\), we find that the theoretical baryon masses of the Quark Model are too small to match the experimental masses. Thus, there will be another quark with a large mass inside the baryon. According to the BCC Quark Model, this quark with a large mass really exists in a baryon – it is excited quark \(q^*\) \((26)\); the quarks with small masses are the accompanying excited quarks \((u'\) and \(d')\). The theoretical masses of the baryons of the BCC Quark Model agree well with the experimental results. Thus, the experimental results support the baryon model \((q^*u'd')\) of the BCC Quark Model. Since the mass of \(q^*_N\) (931 Mev) is closest to the mass of a proton or a neutron, the mass of the quark \(q^*_N\) (931 Mev) \([u^*(931), d^*(931)]\) was misidentified as the mass of a proton (938 Mev) and a neutron (940 Mev) in the experiments. Therefore, we only find the small masses of the quarks in the Quark Model. In fact, the small masses of \(u\) and \(d\) in the Quark Model are very important results – they are very strong evidence of the accompanying excited quarks \((u'\) and \(d')\).

6. Another important distinction between the BCC Quark Model and the Quark Model is the model of the unflavored mesons with \(I = 0\) (\(\eta, \omega, \phi, \ h,\) and \(f\)). The meson model of the Quark Model is the mixture of a 3 quark-antiquark pairs \((u\bar{u}, d\bar{d},\) and \(s\bar{s})\). For example,

\[
\eta(547) = \eta_8 \cos \theta_p - \eta_1 \sin \theta_p, \quad \eta'(958) = \eta_8 \sin \theta_p + \eta_1 \cos \theta_p, \\
\text{where } \eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}, \quad \text{and } \eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}.
\]

(27)

The Quark Model needs a parameter \(\theta_p\). For different mesons, \(\theta_p\) has the following
different values: for $\eta$ and $\eta', \theta_p = -10^0$; for $\phi$ and $\omega$, $\theta_p = 39^0$; for $f_2(1525)$ and $f_2(1270)$, $\theta_p = 28^0$; for $\phi_3(1850)$ and $\omega_3(1670)$, $\theta_p = 29^0$. The meson model of the BCC Quark Model is one quark-antiquark pair. For the same mesons, according to the BCC Quark Model, we have

$$
\begin{align*}
q^*_S(1111)q^*_S(1111) &= \eta(549) [\bullet \eta(547)] , \\
q^*_S(1391)q^*_S(1391) &= \eta(952) [\bullet \eta'(958)].
\end{align*}
$$

Comparing the two models, the meson model of the BCC Quark Model is better. It does not need the the mixture of the 3 quark-antiquark pairs or the parameter ($\theta_p$).

7. Since the quarks ($q^*_u$, $q^*_d$, $q^*_s$, $q^*_c$, $q^*_b$...) are the excited states of the elementary quarks, the SU(3), SU(4), SU(5),..., SU(N) symmetry groups (flavor) based on the quarks ($q^*_u$, $q^*_d$, $q^*_s$, $q^*_c$, $q^*_b$...) are a natural extension of SU(2) based on the elementary quarks (u and d). They do not need any assumptions. Although there are large mass differences among these quarks, the SU(N) symmetries of the Quark Model are still correct.

8. In the high-energy scattering cases, because the strong interactions (color) of the quarks are short range and saturable (a 3 different color system is a colorless system), we can only consider the 3 quark system ($q^*u'd'$) in a baryon case and the 2 quark system ($q^*_f$ and $q^*_f'$) in a meson case. We do not need to consider the whole BCC quark lattice. Thus, in high-energy scattering cases, the Quark Model is an excellent approximation of the BCC Quark Model.

VII Conclusions

1. There is a body center cubic quark (u, d) lattice in the vacuum. The vacuum material (skeleton – the BCC quark lattice) is a super superconductor with super-high energy gaps and a super-high transition temperature. It has the body center cubic periodic symmetries. We think that the body center cubic periodic symmetries might
be some of the “have not yet been identified” symmetries [19].

2. There are only 2 (not 6) flavored elementary quarks (u and d) in the vacuum state; other quarks (s, c, b...) can be deduced (energy band excited states). There are always 2 accompanying excited quarks, $u'(3)$ and $d'(6)$, accompanying each excited quark $q^*$. There is the excited quark spectrum [20] (many more than the 6 flavored quarks) in the excited states. These excited quarks are the energy band excited states of the elementary quarks.

3. The small masses of the quark u(1 to 5 Mev) and the quark d(3 to 9 Mev) are very strong evidence of the 2 accompanying excited quarks [$u'(3)$ and $d'(6)$]. They show that the accompanying excitation concept of the BCC Quark Model is correct.

4. Using the quark spectrum, we deduce the baryon spectrum and the meson spectrum. They agree well with experimental results. We can also explain the decay of the (excited) quarks, the baryons, and the mesons since the higher-mass quarks and the lower-mass quarks are all the excited states of the same elementary quarks.

5. According to the intrinsic quantum numbers (S, C, b, I, and Q), the 5 elementary quarks (u, d, s, c, and b) of the Quark Model are just the 5 ground states of the quark spectrum of the BCC Quark Model [20]. The Quark Model only uses these 5 quarks to explain the baryon spectrum and the meson spectrum. Therefore, the Quark Model is the 5-ground-state approximation of the BCC Quark Model. We will discuss the sixth quark (t) later since the baryon spectrum and the meson spectrum do not need it now.

6. The number of the flavored elementary quarks will change from 6 (u, d, s, c, b, and t) to 2 (u and d). The small quark masses of the Quark Model should be changed as follows: u(1 to 5) → u(931), d(3 to 9) → d(931), s(75 to 170) → s(1111), c(1150 to 1350) → c(2271), and b(4000 to 4400) → b(5531). The baryon model will change from 3 excited quarks to 1 excited quark $q^*$ with 2 accompanying excited quarks [$u'(3)$ and $d'(6)$]. The meson model of the unflavored mesons ($\eta, \omega, \phi, h, \text{and} f$) with isospin $I = 0$ will change from the mixture of the 3 quark pairs ($u\overline{u}, d\overline{d}, \text{and} s\overline{s}$) to 1 pair of
a quark and an antiquark. The confinement concept of the Quark Model should be replaced by the accompanying excitation concept of the BCC Quark Model. According to the accompanying excitation concept, any excited (from the vacuum) quark $q^*$ is always accompanied by the 2 accompanying excited quarks $u'$ and $d'$. They cannot be separated. Therefore, individually free quarks can never be seen.

7. Due to the existence of the vacuum material, all observable particles are constantly affected by the vacuum material (vacuum state quark lattice). Thus, some laws of statistics (such as fluctuation) cannot be ignored.

8. Although the Quark Model needs modification, its principles are still correct. The relevant principles are as follows: a baryon is composed of 3 quarks; a meson is made by a quark and an antiquark; the sum laws (the quantum numbers of a baryon (a meson) are the sum of the same quantum numbers of the quarks that compose the baryon (meson)); the confinement idea (individual free quarks can never been seen); and the decay assumption (the higher mass quarks can decay into lower mass quarks). After the modification, the Quark Model has a solid physical foundation. It is more reasonable, more easy to understand, more useful (it can now deduce the mass spectrum of the baryons and the mesons), and without a need for the parameters $m_s$, $m_c$, $m_b$, and $\theta_p$.

9. Quantum Chromodynamics, of course, does not require a change. In the high energy scattering cases, the Quark Model will work well and does not need a change now.

10. The BCC Quark Model is really a good modification of the Quark Model, but it is a phenomenological model. Furthermore, we need to consider the symmetry wave functions to find the angular momenta and parities of the quarks, the mesons, and the baryons (see the next paper by Xin Yu and Jiao-Lin Xu, “The symmetry wave functions of the Quarks in the BCC Quark Model”).

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