Recent calculations of the two-loop electroweak effects enhanced by powers of the top mass have been implemented in the main electroweak libraries and have an important effect on the indirect determination of the Higgs mass $M_H$. I briefly review the main results of these calculations and discuss in detail the residual uncertainties and their impact on the global fit to $M_H$. The perspectives for the near future are also considered.

The overall agreement of the present precision data with the Standard Model (SM) is quite good\(^1\). The value of $M_t$ estimated by a global fit, for example, is $M_t = 161.1^{+8.2}_{-7.2}$ GeV, which compares well to the direct determination of $M_t$ at the Tevatron. One can try to obtain similar indirect constraints on the mass of the Higgs boson (see Fig. 1). However, the sensitivity of the various precision observables to $M_H$ is much milder than the one to the top mass. The extraction of the relevant information is therefore more difficult and delicate in this case, and requires a careful consideration of some two and three-loop effects and of the residual errors of the theoretical predictions.

Before going into details, it may be interesting to get a quick idea of the size and the importance of genuine two and three-loop corrections when we calculate the main precision observables. Table 1 shows the shifts induced by the known QCD and electroweak higher order effects on the one-loop prediction of $M_W$ and $\sin^2 \theta_{\text{eff}}^\text{lept}$ for a few values of $M_H$. The two-loop $O(\alpha_s)$ corrections have been calculated in\(^2\), the three-loop $O(\alpha_s^2)$ related to the leading top contribution in\(^3\), and the heavy top expansion of the complete $O(\alpha_s^2)$ effect in\(^4\). For what concerns the purely electroweak irreducible two-loop effects, only the first term $O(g^4 M_t^4/M_W^4)$, and second term $O(g^4 M_t^2/M_W^2)$, of an expansion in powers of the top mass are known. In particular, the second term is scheme dependent and the results in Table 1 refer to the $\overline{\text{MS}}$ scheme of Ref.\(^9\).

It is remarkable that all the effects listed in Table 1 have the same sign, corresponding to a screening of the leading one-loop contribution, $O(G_F M_t^2)$, which is due to the non-decoupling top quark effect in $\Delta \rho$. Without these

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higher order corrections the precision tests of the SM would point towards a much lighter top quark, about 20 GeV less, and would be in conflict with the Tevatron measurement of $M_t$. In fact, the bulk of these effects is directly connected to the top quark. Conversely, one may also note that their total is about 100 MeV and $5.5 \times 10^{-4}$ for $M_W$ and $\sin^2 \theta_{\text{eff}}$, respectively. This has to be compared with the present experimental accuracy of about 60 MeV and $2 \times 10^{-4}$. Notice that the enhancement of the screening due to the $O(g^4 M_t^2/M_W^4)$ correction is not only a feature of the MS scheme, since the trend is more or less common to all the popular schemes. This will be more clear in the following.

What is the effect of all this on the indirect determination of $M_H$? From Table 1 we see that among these higher order effects only the purely electroweak corrections slightly modify the one-loop $M_H$ slope of the predicted $M_W$ and effective sine. In other schemes their impact, because of a rearrangement of the reducible contributions, may be larger. Most of the influence of higher order effects on the Higgs mass determination is however indirect, through i) the screening of the Veltman correction which gives the bulk of the electroweak one-loop radiative corrections ($M_t$ and $M_H$ are indeed strongly correlated in the global fit, as we will see); ii) the reduction of the theoretical error that we expect when we include new higher order corrections. This last point can be visualized by the reduction in the size of the blue band in the $\chi^2$ vs $M_H$ plot of the global fit prepared by the LEP-SLD Electroweak Working Group (EWWG) (see Fig. 1), with respect to the same plot before the implementation of the latest electroweak higher order corrections.

Another topic I would like to briefly touch before expanding on the main
subject of this talk is related to something mentioned in the two nice review talks given at this conference by F. Teubert and W. Hollik. Unlike a few years ago, we have now a very strong experimental evidence for electroweak radiative corrections beyond the (trivial) running of the electromagnetic coupling contained in $\Delta \alpha$. If we argue that, after the discovery of the top quark, the fermionic sector of the SM is well-established and that its couplings with the vector bosons have been tested in many experiments, we can wonder what is the evidence for the purely bosonic contributions that contain virtual bosons and represent the core of the gauge structure and of the Higgs mechanism of the SM. They involve all the tri-linear couplings as well as the Higgs boson and represent a gauge-invariant subset of contributions which is unambiguous but numerically subleading. Already a few years ago, it was possible to show that the single measurement of the effective sine combined with the lower bound on the top mass $M_t > 131$ GeV provided evidence for the bosonic corrections of the theory at the level of 4$\sigma$ (see also). Today the same analysis, based on a conservative limit $M_t > 164$ GeV and on the present measurement of $\sin^2 \theta_{\text{eff}}^{\text{lep}}$ which is twice as accurate as it was at the time of the, establishes the necessity of purely bosonic contributions at the level of 8.9$\sigma$.

As my aim is to investigate the indirect Higgs boson mass determination, it will be sufficient to consider the three most precisely measured quantities: $\sin^2 \theta_{\text{eff}}^{\text{lep}}, M_W$, and $\Gamma_l$, the leptonic partial width of the $Z^0$ boson. All one-loop effects as well as some higher order effects have extensively been studied in (see for a comprehensive list of references). To go beyond that, the strat-
Table 2: Comparison of the predictions for $\sin^2 \theta_{\text{eff}}^{\text{lep}}$ and for $M_W$ (in GeV) in the three different schemes of $\text{SM}$ and by ZFitter before the inclusion of the $O(g^4 M_t^2/M_W^2)$ corrections. $M_t = 175$ GeV.

| $M_H$ | OSI | OSI | MS | ZFitter | OSI | OSI | MS | ZFitter |
|-------|-----|-----|----|---------|-----|-----|----|---------|
| 65    | .23131 | .23111 | .23122 | .23116 | 80.411 | 80.422 | 80.420 | 80.420 |
| 300   | .23212 | .23203 | .23203 | .23197 | 80.312 | 80.316 | 80.319 | 80.320 |
| 1000  | .23280 | .23282 | .23272 | .23264 | 80.215 | 80.213 | 80.221 | 80.224 |

Table 3: Same as in Table 2 but after the inclusion of the $O(g^4 M_t^2/M_W^2)$ corrections.

| $M_H$ | OSI | OSI | MS | OSI | OSI | MS |
|-------|-----|-----|----|-----|-----|----|
| 65    | .23132 | .23132 | .23130 | 80.404 | 80.404 | 80.406 |
| 300   | .23209 | .23212 | .23209 | 80.308 | 80.307 | 80.309 |
| 1000  | .23275 | .23277 | .23275 | 80.216 | 80.215 | 80.216 |

ey is obviously to look for possible large higher order effects, and corrections enhanced by powers of heavy masses are prime candidates. In particular, it is possible to organize the two-loop electroweak corrections to the various observables in asymptotic series of the heavy top mass and retain only the first two terms, $O(g^4 M_t^4/M_W^4)$ and $O(g^4 M_t^2/M_W^2)$. This is suggested by the dominance of the top quadratic contribution among electroweak effects at the one-loop level. In practice, one needs to calculate at this order the Thomson scattering, the $W$ and $Z$ propagators, the muon decay, and the leptonic $Z^0$ decay. Some details of these calculations are given in 6, 7, 14 and more will appear in 8. The calculation has been performed in three different electroweak schemes: the $\overline{\text{MS}}$ scheme of 9, and two very different implementation of the on-shell scheme of 15, which are defined in 7. The results of the precise determination of $\sin^2 \theta_{\text{eff}}^{\text{lep}}$ and $M_W$ obtained by incorporating these and all other known two and three-loop radiative corrections are shown in Tables 2 and 3. Table 2 reports the results before the implementation of the $O(g^4 M_t^2/M_W^2)$ effects, while Table 3 shows the results including them. We observe that i) the predictions of $\sin^2 \theta_{\text{eff}}^{\text{lep}}$ ($M_W$) in Table 3 are generally higher (lower) than in Table 2, which corresponds to an enhancement of the screening of the leading top contribution; ii) the scheme ambiguity of the predictions is much smaller in
Table 3 than in Table 2. This is consistent with the notion that the different schemes are equivalent up to $O(g^4)$ corrections not enhanced by powers of heavy masses or by large logarithms, and that the residual corrections should be correspondingly suppressed.

The residual uncertainty due to uncalculated higher order effects of electroweak origin can be estimated by the scheme dependence and by the scale dependence in the $\overline{\text{MS}}$ scheme. In the comparison of five different implementations of one-loop and leading higher order corrections led to predictions of $\sin^2 \theta_{\text{eff}}$ and $M_W$ which differed by at most $2.8 \times 10^{-4}$ and $32$ MeV, respectively. This is reflected in Table 2, where the maximum spreads are $2.0 \times 10^{-4}$ and $11$ MeV. Judging by the results in Table 3, these scheme ambiguities have provided a realistic estimate of the $O(g^4 M_t^2/M_W^2)$ effects. Indeed, if we assign to each value in Table 2 an error equal to the scheme ambiguity for that observable, all the results in Table 3 are within the range defined by this error, or close to it. This method can obviously provide only an order of magnitude estimate of the uncalculated effects, but it gives a clear indication that the residual $O(g^4)$ corrections should be quite smaller than the previous term of the heavy top expansion. Using this criterion in Table 3 (and in the more complete tables of), we obtain $\delta \sin^2 \theta_{\text{eff}} = \pm 4 \times 10^{-5}$ and $\delta M_W = \pm 2$ MeV, which can be considered as estimates of the uncalculated higher order electroweak effects. The scale dependence of the $\overline{\text{MS}}$ results leads to analogous estimates.

It remains to estimate the uncertainty due to higher order QCD corrections. For the three observables we are considering, only gluonic corrections to quark loops are involved. Fortunately, a lot of work has been done a few years ago, and analyses based on very different methods have led to consistent results. The largest part of the QCD corrections concerns the leading $O(G_{\mu} M_t^2)$ one-loop contribution to $\Delta \rho$. These corrections are quite large when $\Delta \rho$ is expressed in terms of the pole mass of the top. Because of its long-distance sensitivity, the latter is not a good expansion parameter and induces a QCD expansion plagued by large and rapidly growing coefficients. A high-scale mass definition, like the $\overline{\text{MS}}$ one, implies a QCD series with much smaller coefficients and presumably smaller higher corrections. If we adopt the estimate of the first reference of for the error on $\Delta \rho$, we obtain $\delta M_W \approx \pm 3$ MeV and $\delta \sin^2 \theta_{\text{eff}} \approx \pm 2 \times 10^{-5}$. As there are additional QCD contributions, we may enlarge the theoretical error to $\delta M_W = \pm 5$ MeV and $\delta \sin^2 \theta_{\text{eff}} = \pm 3 \times 10^{-5}$.

Table 4 summarizes the various parametric and intrinsic errors in the calculation of the three observables. It should be noted that in all three cases the uncertainty induced by the unknown Higgs mass is much larger than the
experimental error, despite the fact that their $M_H$ dependence is in all cases only logarithmic. Higher order electroweak uncertainties are at the level of the error induced by the experimental measurement of $M_Z$.

The results of the calculation of the $O(g^4 M_t^2/M_W^2)$ effects on $M_W$, $\sin^2 \theta_{\text{eff}}$, and $\Gamma_{\text{lept}}$, for $f \neq b$, have been now implemented, together with other new results less important for the $M_H$ determination, in the latest versions of TOPAZ0 and ZFitter, which are routinely used for the global fits by the EWWG. The numerical results of the two codes are in very good agreement with the ones of and among themselves. The MS scheme results have also been implemented in and again there is good agreement.

A crucial question is now whether the approximation based on the Heavy Top Expansion (HTE) described above and now used for the fits is reliable. The second term $O(g^4 M_t^2/M_W^2)$ of the two-loop HTE seems to be quite important wrt the first (see Fig. 2 and Table 1), so the convergence of the HTE may be legitimately questioned. An important point to take into account in this respect is that this is true mainly for a light Higgs, where the approximation of keeping $M_W = 0$ and $M_H \neq 0$ manifestly fails. The result of, which was based on such approximation, becomes therefore meaningless, and no hierarchy among the first and the second term of the asymptotic expansion should be expected. This is illustrated in Fig. 3. Moreover, in the way they are compared in Fig. 2 and in Table 1, no reducible contribution induced by resummation of
one-loop effects is included in the $O(g^4 M_t^4)$ term. Such separation between irreducible and reducible terms is not possible for the second term of the HTE, that, as we have seen, depends very strongly on the scheme. This tells us that reducible contributions (products of one-loop integrals) are very important there. But we know that the HTE works very well at one-loop level and that the leading quadratic term is dominant, so there is some indication (no proof) that the first two terms of the HTE should give a reasonable approximation. Concerning the irreducible contributions, the two-loop calculation of the HTE is based on two-point functions only. Unlike the case of three and four pointfunctions, the HTE seems to work quite well for self-energies, as has been demonstrated in the case of QCD corrections in up to three loops.

The preceding heuristic arguments are certainly not sufficient. A first direct test of the HTE for the two-loop electroweak corrections can be obtained by comparing the results of with the calculation of Bauberger and Weiglein (BW) calculated in the two-loop self-energies contributing to $\Delta r$ (i.e. to the prediction of $M_W$) which contain the Higgs boson together with fermions through a direct numerical evaluation of Feynman diagrams, which did not involve any heavy mass expansion. This subset of diagrams is gauge invariant but not ultraviolet finite. It cannot therefore be used to calculate observables at fixed $M_H$, but gives information on the $M_W$ slope. Even on the slope, however, the information from BW is not complete at $O(g^4)$, since purely bosonic contributions as well as boxes and vertices containing the Higgs

Figure 2: $M_H$ dependence of the two-loop electroweak corrections to $\Delta \rho$ in the MS scheme. The result of the heavy top expansion up to its first (second) term is shown in the upper (lower) curve.
Table 5: Comparison of the top-bottom contributions to the two-loop correction $\Delta r_{(2),\text{subtr}}$ from the calculations of (OSII scheme) and BW. QCD corrections are not included, and $M_W = 80.37$ GeV is employed in the evaluation of radiative corrections. The second and third columns give the two-loop result from Refs. 7, respectively. The last column gives the differences that are induced in $M_W$.

| $M_H$ (GeV) | $\Delta r_{(2),\text{OSII}}^{\text{subtr}}$ ($10^{-4}$) | $\Delta r_{(2),\text{BW}}^{\text{subtr}}$ ($10^{-4}$) | diff ($10^{-4}$) | $\delta M_W$ (MeV) |
|---|---|---|---|---|
| 100 | -0.73 | -1.01 | -0.28 | 0.4 |
| 300 | -3.10 | -3.97 | -0.87 | 1.4 |
| 600 | -5.69 | -6.63 | -0.94 | 1.5 |
| 1000 | -9.44 | -10.45 | -1.01 | 1.6 |

Despite these potential faults, the work of BW is an important step towards the goal of a complete two-loop calculation. From our point of view, moreover, it allows a nice partial check of the HTE for what concerns the $M_H$ slope because BW did not use any mass expansion. Indeed, one can isolate the top-bottom contributions from BW and compare them with the results of the OSII scheme of after expanding $\Delta r$ in the numerator in order to follow the procedure of BW. For simplicity, we computed radiative corrections at a fixed $M_W = 80.37$ GeV and removed all QCD corrections. The subtraction point can be chosen at $M_H = 65$ GeV, defining $\Delta r^{\text{subtr}}(M_H) = \Delta r(M_H) - \Delta r(65)$. The results are shown in Table 5. We observe that the projected discrepancies for $M_W$ are very small, well within the theoretical error reported in Table 4, even in the completely unrealistic case of $M_H = 1$ TeV, and that the maximum difference in the size of the two-loop correction is less than 10%. The HTE seems therefore to work quite well also at the two-loop level, at least for what concerns the diagrams containing top and Higgs.

From the above considerations it should be clear that discrepancies in the calculation of $\Delta r^{\text{subtr}}(1\text{TeV})$ are unlikely to provide a good estimate of the
overall theoretical error at more realistic values of \( M_\text{H} \), where the \( \chi^2 \) distribution of the fits is centered. However, a consistency check is possible. It consists in considering all the two-loop contributions calculated in [7] and [24], including also the light fermions-Higgs contributions of BW, without which their result would not be well-defined. In that case, we find [25] a maximum discrepancy in the calculation of \( \Delta r_{\text{subtr}}(M_\text{H}) \) at \( M_\text{H} = 1 \) TeV, corresponding to \( \delta M_\text{W} = 1.7 \) MeV. If one takes out from the calculation of [7] the square of the one-loop bosonic contribution (a term neglected by BW), one finds an additional \(-2.5\) MeV contribution, bringing the total difference to \( \delta M_\text{W} = 0.8 \) MeV. A more comprehensive comparison of the HTE with the work of BW (also \( \sin^2 \theta_{\text{lep}} \) and \( \Gamma_l \) have been calculated in the same way [26]) will be presented in [25]. Again, we conclude that deviations from [7] appear to be within the range of Table 4, and that the purely bosonic terms should not be neglected if one wants to go beyond the HTE: real improvement on the HTE awaits a complete two-loop calculation.

We move on to the fit of the Higgs boson mass. Understanding the main features of the global fit to \( M_\text{H} \) can be facilitated by the use of simple formulas [16] that summarize the precise calculation of [7]. In the \( \overline{\text{MS}} \) scheme with \( \alpha_s(M_\text{Z}) = 0.118 \) and expressing \( M_t, M_\text{W}, \) and \( M_\text{H} \) in GeV and \( \Gamma_l \) in MeV, we find

\[
\sin^2 \theta_{\text{lep}}^{\text{eff}} \frac{0.23151}{\text{0.23151}} = 1 + 0.00226 \ln \frac{M_\text{H}}{100} + 0.0426 \left( \frac{\Delta \alpha_h}{0.028} - 1 \right) - 0.012 \left( \frac{M_t^2}{175^2} - 1 \right) \tag{1}
\]

\[
\frac{M_\text{W}}{80.383} = 1 - 0.00072 \ln \frac{M_\text{H}}{100} - 1.0 \times 10^{-4} \ln^2 \frac{M_\text{H}}{100} -0.00643 \left( \frac{\Delta \alpha_h}{0.028} - 1 \right) + 0.00676 \left( \frac{M_t^2}{175^2} - 1 \right) \tag{2}
\]

\[
\frac{\Gamma_l}{84.013} = 1 - 0.00064 \ln \frac{M_\text{H}}{100} - 0.00026 \ln^2 \frac{M_\text{H}}{100} -0.00567 \left( \frac{\Delta \alpha_h}{0.028} - 1 \right) + 0.00954 \left( \frac{M_t^2}{175^2} - 1 \right) \tag{3}
\]

These formulas are very accurate within 1\( \sigma \) from the central values of their inputs: \( 170 \lesssim M_t \lesssim 181 \) GeV, 0.0273 \( \lesssim \Delta \alpha_h \lesssim 0.0287 \), and for \( 75 \lesssim M_\text{H} \lesssim 350 \) GeV. In this range they reproduce the exact results of the calculation with maximal errors of \( \delta \sin^2 \theta_{\text{lep}} \approx 1 \times 10^{-5} \), \( \delta M_\text{W} \approx 1 \text{MeV} \) and \( \delta \Gamma_l \approx 3 \text{KeV} \), which are all very much below the experimental accuracy. More complete expressions for \( \sin^2 \theta_{\text{lep}}^{\text{eff}} \) and \( M_\text{W} \), including also the \( \alpha_s \) dependence, can be found in [7].
By comparing the coefficients of $\ln M_H$ in Eqs. (1) and (2), we see that $\sin^2 \theta_{\text{lept}\text{eff}}$ is 3 times more sensitive to $\ln M_H$ than $M_W$, 6.6 times more sensitive to $\Delta \alpha_h$, almost 2 times more sensitive to $M_t$ and $\alpha_s$. Despite the recent progresses in the measurement of $M_W$, it is therefore clear that most of the present sensitivity to $M_H$ still comes from the effective sine. Indeed, the world average $\sin^2 \theta_{\text{lept}\text{eff}} = 0.23157 \pm 0.00018$, can be used alone to obtain an upper bound on $M_H$ roughly comparable to the one of the global fit. Using $M_t = (173.8 \pm 5)$ GeV and combining in quadrature the errors on $M_t$, $\sin^2 \theta_{\text{lept}\text{eff}}$, $\Delta \alpha_h$ (the conservative value of $\delta \alpha$), one finds from Eq. (1) $\ln M_H / 100 = 0.042 \pm 0.638$, which corresponds to $M_H = 104^{+39}_{-25}$ GeV or $M_H < 297$ GeV at 95% C.L. The theoretical error can be included in this estimate as a systematic error. In fact, this simple exercise can be repeated in three different schemes as done in [2], the respective central values can be averaged, and the error expanded to cover the range of the three calculations. This gives $M_H < 300$ GeV. The QCD uncertainty can then be taken into account using the estimate discussed above, $\delta \sin^2 \theta_{\text{lept}\text{eff}} \approx \pm 3 \times 10^{-5}$. This shifts the upper bound by about 6%, leading to $M_H < 318$ GeV at 95% C.L., which can be compared to the global fit result of $M_H < 262$ GeV. Like the EWWG, I am not taking into account the existence of a direct lower bound on $M_H$ from LEP, $M_H > 89.8$ GeV, which can be thought to play a role in deriving the $M_H$ fit.

The very high sensitivity of $\sin^2 \theta_{\text{lept}\text{eff}}$ to the inputs has its disadvantages, however. In particular, this observable depends very strongly on the precise value of the electromagnetic coupling at the $Z^0$ scale. We have seen in Table 2 that indeed the present error on $\Delta \alpha_h$ may shift $\sin^2 \theta_{\text{lept}\text{eff}}$ by $2.3 \times 10^{-4}$, more than the error associated to the world average. Even taking into account the new estimates of $\Delta \alpha_h$ or future low-energy measurements of $R_h$, this factor constitutes a major limitation of the resolving power on $M_H$. In addition, the experimental situation for the $\sin^2 \theta_{\text{lept}\text{eff}}$ measurement, although better than a year ago, is far from satisfactory, given the unresolved discrepancy between LEP and SLD asymmetries.

The measurement of $M_W$, on the other hand, can be considered complementary to the one of the effective sine. At present, it easy to see from Eq. (2) that, if we try to determine $M_H$ from $M_W$ alone, $\delta x = \delta \ln M_H / 100 \approx 1.05$. The $W$ mass has started to play a role in the global fit to $M_H$, but is still far from competing with the effective sine, for which $\delta x = 0.638$ (notice that $e^{1.05} \sim 2.9$ and that $e^{0.64} \sim 1.9$ are the relevant quantities). However, assuming an error of 35 MeV on $M_W$ and of 2.5 GeV on $M_t$, and that the measurement of $\sin^2 \theta_{\text{lept}\text{eff}}$ will not improve significantly in the next few years, one finds that $\delta x \approx 0.55$ for both the effective sine and for $M_W$ ($e^{0.55} \sim 1.7$). This seems to be a quite reasonable scenario for the near future, as the LEP200 experiments
are still going to improve the $M_W$ measurement and the Run II at Tevatron, expected to start next year, should decrease significantly the error on the top mass. One can therefore conclude (see also) that in a few years time the measurement of $M_W$ will provide the same sensitivity to the Higgs mass of the effective sine, allowing an important check.

The essential features of the global fit to $M_H$ can be easily reproduced using the three most precise measurements ($\sin^2 \theta_{\text{lept}}^{\text{eff}}, M_W$, and $\Gamma_l$) and Eqs. (1-3). Because of the strong correlation between $M_t$ and $M_H$, apparent in Eqs. (1-3), also observables insensitive to the Higgs boson have an indirect effect on the $M_H$ fit. $R_b$, in particular, still points to a much lighter top ($M_t = 151 \pm 25$ GeV) than most other data. We can take this effect into account by using a lower $M_t = 171.3$ instead of 173.8 GeV as input. The $M_H$ fit obtained is very close to the global one and is shown in the first plot of Fig. 3. Without including the theoretical errors, the 95% C.L. upper bound on $M_H$ is about 235 GeV and its central value 84 GeV. The exclusion of the SLD result for $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ from the world average gives $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23189 \pm 0.00024$, which leads to $M_H \lesssim 385$ GeV. I am not arguing here in favor of this exclusion. This last result simply shows that even with a value of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ 1.5σ higher there would be strong indication for a light Higgs boson.

The effect of various theoretical errors on the $M_H$ fit is shown in the second plot of Fig. 3 and in Fig. 4. We see that the electroweak scheme dependence has a very small effect on the fit. More important is the effect of the QCD uncertainty. Considering that most of it is linked to the leading $O(G_\mu M_t^2)$
contribution to $\Delta \rho$, and in particular to the top quark mass definition, we can implement it as a simple shift of $M_t$. The conservative values in Table 4 correspond to a systematic shift $\delta M_t = \pm 0.9$ GeV, displayed in the first plot of Fig. 4. The effect on the present fit is then an increase of about +15 GeV of the upper $M_H$ bound. The QCD and electroweak uncertainties are combined in the second plot of Fig. 4, forming the analogue of the blue band in Fig. 1, with which there is very good agreement. The upper bound on $M_H$ is now 260 GeV. The result of the same analysis carried out without implementing the $O(g^4 M_t^4/M_W^2)$ corrections is also considered. It is clear that in that case the central value and upper bound of $M_H$ are significantly larger, about 30 and 90 GeV, respectively.

Finally, the future scenario considered above is analyzed in Fig. 5 where the central value and 95% C.L. upper bounds on $M_H$ are reported as a function of the central value of the measurement of $M_W$ for different values of $M_t$, under the assumptions that i) the measurements of $\sin^2 \theta_{eff}$ and $\Gamma_l$ will not change significantly; ii) the errors on $M_W$ and $M_t$ will decrease to 30 MeV and 2.5 GeV, respectively. The value $\Delta \alpha_h = 0.0278 \pm 0.0003$, corresponding to the conservative scenario of the second of Ref. 28, has been used and the intrinsic theoretical errors neglected. The conclusion is that under these two assumptions a determination of $M_H$ within about 80% with a confidence level of 95% will be possible. The results if the central values of $M_W$ and $M_t$ should not change are also marked, for the two cases $M_t = 173.8$ and 171.3 GeV.

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Figure 5: Future scenarios for the indirect determination of $M_{H}$, assuming no change in the measurements of $\sin^2 \theta^{\text{eff}}$ and in $\Gamma_{t}$, and $4M_{W} = 30 \text{ MeV}$, $4M_{t} = 2.5 \text{ GeV}$.

The horizontal line marks the exclusion potential of LEP200, the dashed lines the 95% C.L. upper bounds on $M_{H}$ and the solid lines the $M_{H}$ central values for a given $(M_{W}, M_{t})$ central value.

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