Numerical simulations of flow past a circular cylinder

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Abstract. Flow past a circular cylinder is numerically investigated by solving the incompressible Navier-Stokes equations with a stabilized finite element method. The computations are carried out over the Reynolds number (Re) range $1 \times 10^4$ to $4 \times 10^5$. In this Re range, drag on cylinder drops rapidly with increase in Re. This phenomena, known as drag crisis, is associated with transition of boundary layer from laminar to turbulent state. The computations successfully capture the drag crisis phenomena and the accompanying flow field features like Laminar Separation Bubble (LSB). The computed flow field was used to study the statistical properties of the flow as well as its time-dependent features. It is found that appearance of LSB and the overall process of transition is intermittent. The frequency of appearance of LSB as well as the duration of its stay, in the critical Re regime, increases with increase in Re. The LSB is tracked and its intermittency is evaluated using fluctuations in surface pressure and Reynolds stress.

1. Introduction
Flow past a circular cylinder can be divided into four different regimes: subcritical, critical, supercritical and transcritical [1]. In the subcritical regime, the boundary layers separate in the laminar state at about 80° from the front stagnation point. The early separation of boundary layer leads to high a value of drag coefficient ($C_D$) of about 1.2. In the critical regime, the boundary layer is laminar when it first separates from the surface of cylinder. Due to the instabilities, the separated shear layer undergoes a transition to turbulent state and reattaches to the surface of cylinder [2, 3]. The boundary layer separates in turbulent state at a higher angle compared to laminar boundary layer. As a result, the wake is narrower and base pressure higher. This leads to a very rapid reduction in the drag coefficient ($C_D$) with increase Reynolds number (Re). This phenomena is popularly known as drag crisis. The turbulent reattachment of shear layer leads to the formation of LSB [4] and [5] experimentally established that formation of LSB is responsible for the drag crisis. In the supercritical regime, the boundary layer immediately transitions from laminar to turbulent state at a critical angle from front stagnation point. Also, the $C_D$ increases with increase in Re. In the transcritical regime, the formation of LSB ceases,
the boundary layer separates in turbulent state, and the $\overline{C_D}$ remains nearly constant with increase in $Re$.

In an experimental investigation [6] found that the variation of $\overline{C_D}$ with $Re$ during the drag crisis, in the critical regime, takes place in two stages. In the first stage, at the lower $Re$ of the critical regime, the LSB is present only one of the shoulders of the cylinder. At higher $Re$, the LSB appears on both sides of the cylinder. [3] carried out three-dimensional large eddy simulations (LES) for smooth cylinder and with a with trip located at $\theta = 55^\circ$ from the stagnation point on one of the sides. They observed that the smooth cylinder undergoes a single stage drag crisis while the one with a trip experiences a two-staged drag crisis similar to what is seen in experimental studies. This suggests that the asymmetry in the transition of the flow on the two sides of the cylinder is a consequence of a slight asymmetry in either the geometry or the flow conditions at the inlet. [7] reported two kinds of boundary layer reattachment during the critical regime: symmetric and asymmetric. Weakening of vortex shedding activity was also observed during drag crisis. The probability of appearance of one and two bubble states was presented for various scenarios of transition. It was observed that for asymmetric reattachment, one bubble state has a higher probability at the onset of the critical regime, while the two bubble state shows increased probability at $Re$ close to the end of critical regime.[8] performed three-dimensional large eddy simulations with high span wise grid resolution. They observed asymmetric reattachment in the critical regime.

Despite the various studies in the critical regime, the exact dynamics of the formation of the LSB on the circular cylinder is not well understood. Does the LSB appear suddenly at a certain critical $Re$? Or does it have an intermittent nature at the lower $Re$ end of the critical regime and later exists at all times? We explore this in the present work. For the same we use the concept of intermittency which is extensively used in the description of transition from laminar to turbulent states. We denote the intermittency factor by $I_f$. It is the fraction of time during which the flow is in a turbulent state [9]. $I_f$ lies between 0 and 1; $I_f = 0$ implies that the flow is laminar while $I_f = 1$ means that the flow is turbulent all the time. In the present work, we utilize $I_f$ to quantify the fraction of time during which LSB exists in the flow.

2. Numerical method and computational details

2.1. The finite element formulation

The flow is modeled by the incompressible Navier-Stokes equations in the primitive variables. A stabilized finite element formulation [10] is utilized to discretize the equations. The second-order-accurate-in-time, Crank-Nicholson scheme is employed for time integration. In this work the streamline-upwind/Petrov-Galerkin (SUPG) and pressure-stabilizing/Petrov-Galerkin (PSPG) method is utilized [10] to stabilize the computations. The linear algebraic equations resulting from the finite element discretization are solved using a matrix-free implementation of the Generalized Minimal RESidual (GMRES) technique [11] in conjunction with diagonal preconditioners. To handle large scale computations, the solution method is implemented on a distributed memory parallel computing machine. For more details on the finite element formulation and its parallel implementation the reader is referred to the article by [12].

Computations are carried out in both two- and three-dimensions. Detailed results for the 2D computations were presented in our earlier study [2]. In this work, we focus on 3D computations; the results for 2D computations are presented mostly for comparison, and to bring out the three-dimensional effects. At the $Re$ considered in this study, the flow involves small scale flow structures. It is not possible to resolve the flow at all the scales via a Direct Numerical Simulation (DNS), with the present computational resources, as the number of grid points required is extremely large. We present two sets of computations. In the first set, the smaller unresolved flow structures are modeled using a sub-grid scale model. Large Eddy Simulation (LES) is employed; the effect of small scales is represented by a constant coefficient ($C_s = 0.1$)
2.2. Problem set-up and the finite element mesh

The two-dimensional mesh employed for computations is shown in figure 1. The mesh consists of a structured part close to the cylinder. The remaining domain consists of an unstructured mesh, which is created using Delaunay triangulation. The structured part of the mesh is adequately fine to resolve the boundary layer, its separation and the subsequent rolling up of the separated shear layer into vortices, in the range of $Re$ being considered. The height of the first element from the surface of the cylinder is $5 \times 10^{-6} D$ in the radial direction, where $D$ is the diameter of the cylinder. The 2D mesh consists of 115,847 nodes and 230,840 triangular elements.

The mesh for the three-dimensional computations is generated by stacking sections of the two-dimensional mesh along the span. The spanwise extent of the domain is $L_z = 1 D$. Fifty one uniformly spaced sections of the two-dimensional mesh, described above, are utilized to construct the three-dimensional mesh. The resulting 3D mesh consists of 5,908,197 nodes and 11,542,000 6-noded wedge elements. A time step of $\Delta t = 5 \times 10^{-4}$ is utilized for all the computations. All the results presented in the paper are in terms of the non-dimensional time.

3. Results

Results are presented for the flow past a cylinder for $1 \times 10^4 \leq Re \leq 4 \times 10^5$. In all the cases the unsteady computations are carried out for sufficiently long time such that the flow achieves a fully developed unsteady state. For estimating the statistical quantities, the data for the transient during the unsteady development of the flow, following the initiation of the computation, is discarded.

Figure 2 shows the variation of $\overline{CD}$ with $Re$ from the present computations along with their comparison from earlier studies. We observe that both two- and three-dimensional simulations predict drag crisis. The range of $Re$ for which the flow lies in the critical regime is very similar from the two sets of computations. This suggests that the phenomenon itself is largely two-dimensional. Of course, the three-dimensional effects are quite significant as indicated by the difference in $\overline{CD}$ in the sub- and super-critical regime from the 2D and 3D computations. However, the mechanism of transition in a span-averaged sense appears to be two-dimensional. Compared to the measurements from experiments, the present results predict the drag-crisis at a slightly lower $Re$. Results are also shown in Figure 2 for the Large Large Eddy Simulation.
Figure 2. Flow past a cylinder: variation of the mean drag coefficient with $Re$. Also shown are results from earlier studies. The data from experiments by [14] has been taken from [15].

(LES) with a static Smagorinsky model to account for the subgrid scales. Compared to results from model-free computations, these results are in closer agreement with the experimental data. Since the model-free computations also capture the essential trend, therefore, all the analysis in this work is carried out for data from model-free computations. Both, the computational and experimental data in Figure 2 shows that the variation of $C_D$ with $Re$ during the transition is smooth, and not abrupt. We explore the changes in the flow during transition.

The instability of the separated shear layer and its subsequent rolling up into vortices plays a major role in the reattachment of the boundary layer and formation of LSB. To study this we need to filter out the activity due to the Karman shedding that has a much lower frequency compared to the activity of the shear layer. For the same, consider the time variation of the span-averaged surface coefficient of pressure $C_P(\theta, t)$ at different $Re$. We compute a moving average of $C_P(\theta, t)$: $\bar{C}_P(\theta, t)$ over a time window of size $T_k/10$, where $T_k$ is the time period of the Karman shedding. This averaging filters out the shear layer activity while retaining the vortex shedding effects. By subtracting $\bar{C}_P(\theta, t)$ from the original signal, we compute the rms of the fluctuations that correspond to shear layer activity: $C'_PC'_P(\theta, t)$. Figure 3 shows a space-time diagram of the variation of $C'_PC'_P(\theta, t)$ for $Re = 1.5 \times 10^5$. The flow is in the transitional regime at this $Re$ and undergoing drag-crisis. Figure 3 shows the streamlines for the moving time-averaged flow that are also span averaged at two time instants that are marked in figure 3. These figures reveal the co-relation between the presence/absence of LSB with the level of fluctuations due to shear layer activity as observed in $C'_PC'_P(\theta, t)$. At $t = t_1$ the fluctuations in the figure 3 are high at $\theta \approx 105^\circ$ but relatively low at $\theta \approx 250^\circ$. Correspondingly a LSB can be seen in the time-averaged streamlines shown in figure 1(d) on the upper shoulder of the cylinder. At $t = t_2$, an LSB appears on both the upper and lower shoulder of the cylinder. Correspondingly, high fluctuations in $C'_PC'_P(\theta, t)$ can be seen at $\theta \approx 105^\circ$ and $\theta \approx 250^\circ$. This demonstrates that high fluctuations in $C'_PC'_P(\theta, t)$ correspond to increased shear layer activity and formation of LSB. As seen from figure 1(b), the high fluctuations in $C'_PC'_P(\theta, t)$ appear in bursts. The space-time
Figure 3. $Re = 1.5 \times 10^5$ flow past a cylinder: space-time diagram of the $rms$ of the fluctuations subjected to high pass filter, (a) $C_P' C_P' (\theta, t)$. Shown in (b) are the close-up views of the streamlines for the span- and moving time-averaged flow ($\bar{\psi}$) at the time instants $t_1 = 2.0$ and $t_2 = 5.6$ marked in (a).

diagram is utilized to compute the Intermittency factor, $I_f$. An LSB exists on the shoulder if $C_P' C_P' (\theta = 105^\circ$ or $250^\circ, t)$ exceeds a certain threshold value. Figure 4 shows the variation of $1 - I_f$ and $C_D$ with $Re$. This figure clearly shows the correlation between $1 - I_f$ and $C_D$. In the subcritical regime $I_f \approx 0$, indicating LSB does not appear in the flow. In the early stages of critical regime, $I_f$ starts increasing from a zero value and reaches unity towards the end of the critical regime. This shows that LSB appears intermittently during the regime of drag crisis and its frequency of appearance increases towards the end of drag crisis. Thus it can be said that in the transitional regime during the drag crisis, the flow fluctuates between laminar and turbulent states. The laminar state is associated with larger drag and is devoid of LSB. The turbulent state has a lower drag and is associated with an LSB. The average drag at any $Re$ depends on the relative time spent in the two states.
4. Conclusions
The numerical simulations capture the drag-crisis. The Laminar separation bubble (LSB) is found to exhibit an intermittent behaviour. This nature of LSB is utilized to explain the gradual change in $\overline{C_D}$ in the transition regime during drag crisis as opposed to a sudden decrease in drag at a certain Re.

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