Anomalous Hall unidirectional magnetoresistance

M. Mehraeen and Steven S.-L. Zhang

Department of Physics, Case Western Reserve University, Cleveland, OH 44106, USA

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We predict an anomalous-Hall unidirectional magnetoresistance (AH-UMR) in conducting bilayers composed of a ferromagnetic layer and a nonmagnetic layer, which does not rely on the spin Hall effect in the normal metal layer—in stark contrast to the well-studied unidirectional spin-Hall magnetoresistance—but, instead, arises from the anomalous Hall effect in the ferromagnetic layer. Physically, it is the charge-spin conversion induced by the anomalous Hall effect that conspires with the structural inversion asymmetry to generate a net nonequilibrium spin density in the ferromagnetic layer, which, in turn, modulates the resistance of the bilayer when the direction of the applied current or the magnetization is reversed. The dependences of the AH-UMR effect on materials and geometric parameters are analyzed and compared with other nonlinear magnetoresistances. In particular, we show that, in magnetic bilayers where anomalous Hall and spin Hall effects are comparable, the overall UMR may undergo a sign change when the thickness of either layer is varied, suggesting a new scheme to quantify the spin Hall or anomalous Hall angle via a nonlinear transport measurement.

INTRODUCTION

Originating from the interplay between magnetism and relativistic spin-orbit interaction, the anomalous Hall effect in solid-state systems with broken time-reversal symmetry has been of enduring interest for more than a century [1, 2]. One class of materials that has received particular attention in studies of this effect are conducting ferromagnets [2], such as ferromagnetic metals.

The anomalous Hall effect in ferromagnetic metals has several salient properties. Due to the coupling of spin and orbital degrees of freedom, the effect not only generates a transverse charge current—which is perpendicular to both the magnetization and the applied electric field, but also gives rise to a transverse spin current. And, in ferromagnetic metals with strong exchange interaction, conduction-electron spins are well aligned with the local magnetization, making the coupled spin and charge currents controllable by varying the direction of the magnetization. Furthermore, the mobility of conduction electrons in a ferromagnetic metal is, in general, spin-dependent, enabling mutual conversion between spin and charge currents mediated by the anomalous Hall effect.

These properties have been shown to spawn unconventional magnetoresistances in the linear response regime. For instance, both a bulk anisotropic magnetoresistance and planar Hall resistance may result from two consecutive transverse scatterings of spin-polarized conduction electrons, due to the anomalous Hall effect [3]. In geometrically confined systems—such as ferromagnetic-metal thin films or layered structures, the anomalous Hall induced anisotropic magnetoresistances may acquire distinctive angular dependencies, owing to the modulation of the bulk spin and charge currents caused by interfacial spin accumulation and the resulting diffusive spin current [4–9].

In the nonlinear response regime—where the Onsager reciprocal relations no longer hold, the role of the anomalous Hall effect is yet to be explored. In this work, we unveil a unidirectional magnetoresistance (UMR) driven by the anomalous Hall effect in conducting ferromagnet/nonmagnet bilayer systems, whereby the resistance can be altered by reversing the direction of either the magnetization or the applied electric field. Hereafter, we shall refer to this nonlinear magnetoresistance as anomalous-Hall unidirectional magnetoresistance (AH-UMR).

The underlying physics of the AH-UMR can be understood as follows. In a single ferromagnetic-metal layer, the spin current induced by the anomalous Hall effect creates spin accumulations of opposite orientations at the top and bottom surfaces, but there is no net nonequilibrium spin density due to inversion symmetry, as shown schematically in Fig. 1a. This, however, is no longer the case when a nonmagnetic-metal layer is attached to the ferromagnetic-metal layer, as the spin accumulation at the interface would “leak” into the nonmagnetic layer, resulting in a net nonequilibrium spin density in the ferromagnetic layer, which conspires with the spin asymmetry in electron mobility to produce the AH-UMR effect, as depicted in Figs. 1b and 1c.

There is a key difference between the AH-UMR and other types of UMR effects previously studied in various magnetic systems [10–19], which stems from utterly different sources of nonequilibrium spin density in the presence of an external electric field. For the AH-UMR, it emanates from the anomalous Hall effect in the ferromagnetic layer itself, whereas for other UMRs—such as the unidirectional spin Hall magnetoresistance (USMR) [10] and the unidirectional Rashba magnetoresistance [18]—the nonequilibrium spin density is engendered by the spin Hall effect in the nonmagnetic layer [10–13, 15, 19] or by the Rashba-Edelstein effect due to the spin-momentum locked surface states [14, 16–18, 20].
FIG. 1. Schematics of the AH-UMR effect. (a) In a single ferromagnetic-metal (FM) layer, the spin accumulation $\mu_{s,F}$ (indicated by the small yellow arrows) has an antisymmetric distribution about the center line of the layer, with no net nonequilibrium spin density (spatially-averaged $\mu_{s,F}$) induced. So the overall AH-UMR is also zero. (b) The presence of a neighboring nonmagnetic-metal (NM) layer induces a net nonequilibrium spin density $\delta\mu_{s,F}$ (indicated by the large yellow arrow) in the FM layer, giving rise to a finite AH-UMR. (c) Reversing the electric field direction flips the direction of the anomalous Hall current $\mathbf{j}_{AH}$ and thus the sign of $\delta\mu_{s,F}$, thereby changing the sign of the AH-UMR.

In what follows, we will first examine the coupled spin and charge transport induced by the anomalous Hall effect in a ferromagnetic bilayer structure, based on a generalized drift-diffusion model of spin-polarized electrons. A UMR coefficient will be defined to properly characterize the nonlinear transport phenomenon. And an analytical expression of the AH-UMR coefficient will be derived, which reveals how the AH-UMR effect depends on specific materials and geometric parameters of the bilayer. We will then generalize our results to bilayer structures comprised of a ferromagnetic metal and a heavy-metal, wherein both the AH-UMR and USMR are present. And we predict, for particular choices of materials combinations, that the total UMR would exhibit a sign reversal when the thickness of either the ferromagnetic-metal or the heavy-metal layer is varied, which suggests a new scheme to quantify the spin Hall and anomalous Hall angles experimentally through a UMR measurement. We will conclude with some materials considerations on both direct and indirect detections of the AH-UMR as well as the enhancement of the effect.

RESULTS

Spin-dependent drift-diffusion model

Consider a thin-film ferromagnetic layer located at $0 < z < d_F$ placed on top of a nonmagnetic layer at $-d_N < z < 0$, as shown in Figs. 1b,c. Applying an in-plane electric field $\mathbf{E}$, the generalized spin-dependent Ohm’s law—taking into account the anomalous Hall effect in the ferromagnetic layer—can be written as [3, 21]

$$
\mathbf{j}^\alpha(z) = \sigma^\alpha(z)\mathbf{E}^\alpha(z) + \alpha\theta_{AH}\mathbf{j}_m^\alpha(z) \times \mathbf{m},
$$

where $\mathbf{j}^\alpha(z)$ is the local current density carried by spin-$\alpha$ electrons ($\alpha = \pm$), $\theta_{AH}$ is the anomalous Hall angle and $\mathbf{m} = \mathbf{M}/M$ is a unit vector denoting the direction of the magnetization. In Eq. (1), we have defined an effective local electric field felt by the spin-$\alpha$ electrons (in units with $e = 1$) as $\mathbf{E}^\alpha(z) \equiv \mathbf{E} + \nabla\mu^\alpha(z)$, where $\mu^\alpha(z)$ is the spin-dependent chemical potential, which is related to the nonequilibrium part of the electron density $n^\alpha(z)$ through

$$
\mu^\alpha(z) = [N^\alpha(\epsilon_F)]^{-1} n^\alpha(z) - \phi(z),
$$

with $N^\alpha(\epsilon_F)$ the density of states of spin-$\alpha$ electrons at the Fermi level and $\phi(z)$ the spin-independent part of the chemical potential.

The local conductivity varies linearly with the total local electron density—the sum of the densities of both equilibrium and nonequilibrium electrons, i.e.,

$$
\sigma^\alpha(z) = \nu^\alpha [n_0^\alpha + n^\alpha(z)],
$$

where $n_0^\alpha$ and $\nu^\alpha$ are the equilibrium-electron density and mobility of spin-$\alpha$ electrons, respectively. Local charge neutrality is assumed, i.e., $n^+(z) + n^-(z) = 0$, for the metallic system in question.

Without loss of generality, let us fix the electric field $\mathbf{E}$ in the $x$ direction, and study the variation of the chemical potential in the $z$ direction, whereby the effective field becomes $\mathbf{E}^\alpha = (E_x, 0, d\mu^\alpha/dz)$. We also set the magnetization to point in the $y$ direction—as does the spin polarization, due to the large exchange coupling—in which case the longitudinal UMR would reach a maximum. In this setup, we may cast Eq. (1) in its component form as follows

$$
\begin{align*}
\begin{array}{c}
j_x^\alpha(z) = \sigma_0^\alpha(z)E_x - \alpha\theta_{AH}\mu_{x,F}^\alpha(z), \\
\tau_{sf}\frac{d}{dz}j_z^\alpha(z) = \frac{n^\alpha(z) - n^{-\alpha}(z)}{\tau_{sf}}.
\end{array}
\end{align*}
$$

where $\sigma_{0,F}^\alpha = \nu_0^\alpha n_0^\alpha$ is the bulk Drude conductivity of the spin-$\alpha$ channel. From Eqs. (3) and (4a), one can see that a correction to the longitudinal conductivity may arise if there is a net nonequilibrium spin density in the ferromagnetic layer.

The spin-up and spin-down conduction channels are mixed in the presence of spin-flip scattering, which leads to the generalized current-continuity equation [22]
where \( \tau_{sf} \) is the spin-flip relaxation time. Combining Eqs. (2), (4b) and (5), we obtain a set of differential equations for the charge and spin chemical potentials

\[
\frac{d^2 \mu_{c,F}(z)}{dz^2} + p_\sigma \frac{d^2 \mu_{s,F}(z)}{dz^2} = 0, \quad (6a)
\]

\[
\frac{d^2 \mu_{s,F}(z)}{dz^2} - \frac{\mu_{s,F}(z)}{\lambda_F^2} = 0, \quad (6b)
\]

where \( \mu_{c,F}(z) \equiv \left[ \mu_c^+(z) + \mu_c^-(z) \right]/2 \) and \( \mu_{s,F}(z) \equiv \left[ \mu_s^+(z) - \mu_s^-(z) \right]/2 \) are the charge and spin chemical potentials, \( p_\sigma \equiv (\sigma_{s,F}^0 - \sigma_{s,F}^-)/(\sigma_{s,F}^+ + \sigma_{s,F}^-) \) is the conductivity spin asymmetry, \( p_{\sigma} \equiv (\sigma_{0,F}^0 - \sigma_{0,F}^-)/(\sigma_{0,F}^+ + \sigma_{0,F}^-) \) is the spin asymmetry in the density of states at the Fermi level, and \( \lambda_F = \sqrt{\sigma_{0,F}(1 - p_F^2) \tau_{sf}}/2N_F(\epsilon_F)(1 - p_F^2) \) is the spin diffusion length, with \( N_F(\epsilon_F) = N_F^+(\epsilon_F) + N_F^-(\epsilon_F) \) is the density of states at the Fermi level.

In a nonmagnetic layer with negligible spin Hall effect, the regular spin-dependent Ohm’s law is retained, i.e., \( j^\alpha(z) = \sigma^\alpha_N(z)E^\alpha_N(z) \), with the effective field \( E^\alpha_N \equiv (E, 0, d\mu^\alpha_N/dz) \). And the set of equations satisfied by the charge and spin chemical potentials in the layer read

\[
\frac{d^2 \mu_{c,N}(z)}{dz^2} = 0, \quad (7a)
\]

\[
\frac{d^2 \mu_{s,N}(z)}{dz^2} - \frac{\mu_{s,N}(z)}{\lambda_N^2} = 0, \quad (7b)
\]

where \( \lambda_N = \sqrt{\tau_{sf}\sigma_{0,N}/2N_N(\epsilon_F)} \) is the spin diffusion length with \( N_N(\epsilon_F) = N_N^+(\epsilon_F) + N_N^-(\epsilon_F) = 2N_N^+(\epsilon_F) \) the density of states of electrons at the Fermi level. Note that the spin chemical potential still satisfies the diffusion equation, the chemical potential is not coupled to the spin counterpart due to the absence of conductivity spin-asymmetry (i.e., \( p_\sigma = 0 \)).

At the interface of the bilayer \( (z = 0) \), we assume both the current density and chemical potential for each conduction channel are continuous (see Supplementary Information for a more detailed discussion on the boundary conditions), i.e.,

\[
\mu_{N}^0(0^-) = \mu_{N}^0(0^+), \quad (8a)
\]

\[
\mu_{N}^0(0^-) = \mu_{N}^0(0^+), \quad (8b)
\]

And open boundary conditions are imposed at the two outer surfaces, i.e., \( j_z^0(-d_N) = j_z^0(d_F) = 0 \).

### Spin accumulation and nonlinear transport

The total charge current density, given by \( j^+(z) + j^-(z) \), can be divided into two parts: \( j = j^{(1)} + j^{(2)} \), with a linear component \( j^{(1)} \) that is proportional to \( E_x \) and a nonlinear one that is quadratic in \( E_x \). The latter can be expressed as

\[
j_x^{(2)}(z) = (\nu^+ - \nu^-)\mu_s(z)\tilde{H}(N^+, N^-)E_x \quad (9)
\]

where \( \tilde{H}(N^+, N^-) \) is the harmonic mean of the density of states. And the spin accumulation \( \mu_s \) is linear in \( \theta_{AH}E_x \), resulting in \( j_x^{(2)} \propto E_x^2 \). Note that \( j_x^{(2)} \) only emerges in the ferromagnetic layer wherein \( \nu^+ \neq \nu^- \).

In order to properly quantify the nonlinear charge current, we introduce a UMR coefficient \( \zeta(z) \) as

\[
\zeta(z) \equiv \frac{\sigma_{xx}(z, E_x) - \sigma_{xx}(z, -E_x)}{\sigma_{0}E_x}, \quad (10)
\]

where \( \sigma_{ij} = j_i/E_j \) is the conductivity tensor, and \( \sigma_0 \) is the bulk Drude conductivity. The UMR coefficient \( \zeta \) is so defined that its magnitude is independent of the external electric field. The dimension of \( \zeta \) is length per Volt, the inverse of which sets the scale of the electric field for which the nonlinear longitudinal conductivity—given by \( j_x^{(2)}/E_x \)—becomes comparable to its linear counterpart.

The spatial distribution of the AH-UMR coefficient \( \zeta_{AH} \) is displayed in Fig. 2a, along with that of the spin chemical potential \( \mu_s \) (or the spin accumulation) in the bilayer structure. There is a clear correlation between the two quantities: \( \zeta_{AH} \) goes to zero wherever \( \mu_s \) vanishes. Furthermore, the AH-UMR completely comes from the ferromagnetic layer, in which the electron mobility is spin-dependent. Within the nonmagnetic layer, for

![FIG. 2. Spatial dependences of the spin accumulation and AH-UMR coefficient in the bilayer system. (a) z-dependences of the spin accumulation \( \mu_s \) and AH-UMR coefficient \( \zeta_{AH} \) in the FM and NM layers. Note the absence of the AH-UMR in the NM. (b) Plots of the spin accumulation and AH-UMR coefficient in the FM for different thicknesses of the NM layer. Parameters used: \( \lambda_F = 10 \) nm, \( \lambda_N = 20 \) nm, \( \theta_{AH} = 0.05 \), \( \epsilon_F = 5 \) eV, \( \sigma_{0,F} = \sigma_{0,N} = 0.033 \) (\( \mu \Omega \) cm\(^{-1} \)), \( p_\sigma = 0.7 \), and \( p_{\sigma} = 0.2 \).](image-url)
which $\nu^+ = \nu^-$, $\zeta_{\text{AH}}$ vanishes everywhere despite the remnant $\mu_s$ near the interface. These observations are in full agreement with Eq. (9).

Although the nonmagnetic layer in question neither plays an active role as a spin polarizer nor accommodates any nonlinear charge transport, it is still indispensable to the generation of a net AH-UMR in its neighboring ferromagnetic layer. In the absence of the nonmagnetic layer, the spin accumulation $\mu_s$ and thus the local AH-UMR coefficient $\zeta_{\text{AH}}$ have antisymmetric distributions about the center line of the ferromagnetic layer, as shown by the dashed lines in Fig. 2b. In this case, the total (spatially-averaged) AH-UMR is zero, as a result of the lack of a net nonequilibrium spin density.

From a symmetry perspective, a net current-induced spin density is allowed, when and only when a system lacks inversion symmetry. For the present case, the nonmagnetic layer introduces structural inversion asymmetry, and makes a net nonequilibrium spin density achievable in the ferromagnetic layer next to it. Physically, it "absorbs" spin accumulation at the interface from the ferromagnetic layer, leaving a net nonequilibrium spin density in the latter, as illustrated by the dash-dotted lines in Fig. 2b.

**Spatially-averaged AH-UMR**

By taking the spatial average of the overall UMR coefficient over the thickness of the bilayer, $ar{\zeta} \equiv \int_{-d_N}^{d_F} dz \, \zeta(z)/(d_N + d_F)$, we find that, up to $\mathcal{O}(\theta_{\text{AH}})$, the spatially-averaged AH-UMR coefficient reads

$$
\bar{\zeta}_{\text{AH}} = p_F \theta_{\text{AH}} \frac{\lambda_F}{\epsilon_F} \mathcal{G} \left( \frac{d_F}{\lambda_F}, \frac{d_N}{\lambda_N}, \frac{\sigma_{0,F}}{\sigma_{0,N}}, \frac{\lambda_F}{\lambda_N} \right),
$$

where $p_F = (p_a - p_N)$ characterizes the overall spin asymmetry of electron mobility for the ferromagnetic layer, and the thickness dependence of the AH-UMR is encapsulated in the dimensionless $\mathcal{G}$ function as

$$
\mathcal{G}(s, t; u, v) = 3 \frac{\left( \frac{u}{(1-\frac{u}{2})^2} \right) \tanh(s) \tanh\left( \frac{s}{2} \right)}{1 + (1 - \frac{u}{2}) \left( \frac{u}{v} \right) \tanh(s) \coth(t)}. \quad (12)
$$

For simplicity, we have adopted the free-electron model whereby $N_F^2 = 3n_{0,F}^2/2\epsilon_F$ with $\epsilon_F$ the Fermi energy of conduction electrons in the ferromagnet. Equations (11) and (12) are the main results of the paper.

Several remarks regarding the AH-UMR are in order.

1) The AH-UMR coefficient, to leading order, is proportional to the anomalous Hall angle $\theta_{\text{AH}}$, in contrast to the USMR effect, which is proportional to the spin Hall angle of the heavy metal layer [12].

2) The AH-UMR coefficient is also linear in $p_F$, as is the USMR [12]. This is not surprising, as the conversion of a net nonequilibrium spin density to a (nonlinear) charge current relies entirely on the spin asymmetry in electron scatterings.

3) The ratio $\lambda_F/\epsilon_F$ has the same dimension as the AH-UMR coefficient (with $e = 1$). In fact, the prefactor of the averaged UMR coefficient—the thickness independent part in Eq. (11)—sets the scale of the maximum AH-UMR that one can obtain for a given ferromagnet. For a typical transition metal with $p_F = 0.2$, $\theta_{\text{AH}} = 0.02$, $\lambda_F = 100$ nm, and $\epsilon_F = 5$ eV, the upper bound of the AH-UMR coefficient, $\bar{\zeta}_{\text{AH}}$, is of the order of $1 \text{ A/V}$.

4) Information about how other geometric and material parameters of a magnetic bilayer would shape the AH-UMR is all encoded in the dimensionless $\mathcal{G}$ function given by Eq. (12). The first two variables, $d_F/\lambda_F$ and $d_N/\lambda_N$, indicate that the dependences of the AH-UMR on the thicknesses of the ferromagnetic and nonmagnetic layers must scale with their respective spin diffusion lengths, as plotted in Fig. 3.

5) Another remarkable property of the AH-UMR—revealed by the $\mathcal{G}$ function—is that it increases monotonically with the ratio $\lambda_F/\epsilon_F$, which would be useful for guiding the search for magnetic bilayers with a sizable AH-UMR effect.

**UMR sign reversal and spin/anomalous Hall angle quantification**

In a magnetic bilayer consisting of a ferromagnetic metal and a heavy metal, both anomalous Hall and spin Hall effects, in principle, may contribute to the total UMR measured in the bilayer. And their contributions turn out to be additive, i.e., $\bar{\zeta}_{\text{tot}} = \bar{\zeta}_{\text{AH}} + \bar{\zeta}_{\text{SH}}$ (see Supplementary Information for the full expression of $\bar{\zeta}_{\text{tot}}$). The ratio of the two UMR contributions due to spin-dependent scattering—provided electron-magnon...
scattering is suppressed by applying a magnetic field or lowering the temperature [15] — takes a rather neat form

\[
\frac{\tilde{\zeta}_{\text{AH}}}{\tilde{\zeta}_{\text{SH}}} = \frac{\theta_{\text{AH}} \lambda_F \tanh \left( \frac{d_F}{2\lambda_F} \right)}{\theta_{\text{SH}} \lambda_N \tanh \left( \frac{d_N}{2\lambda_N} \right)}. \tag{13}
\]

It is worthy to note that this ratio depends on only a few parameters, namely the spin/anomalous Hall angle, the spin diffusion length, and the thickness of each layer.

The simple relation between \( \tilde{\zeta}_{\text{AH}} \) and \( \tilde{\zeta}_{\text{SH}} \) nonetheless has a remarkable physical consequence: the total UMR coefficient of such a magnetic bilayer may exhibit qualitatively different thickness dependences, depending on the relative signs of the anomalous Hall and spin Hall angles. When \( \theta_{\text{AH}} \) and \( \theta_{\text{SH}} \) have the same sign, the associated contributions simply add up (see the blue curves in Fig. 4). It becomes more intriguing when \( \theta_{\text{AH}} \) and \( \theta_{\text{SH}} \) are of opposite signs. In this case, the total UMR inevitably undergoes a sign reversal as the thickness of either layer is varied (see the red curves in Fig. 4).

At the sign reversal point, the ratio of the Hall angles fulfills the following condition:

\[
\frac{\theta_{\text{SH}}}{\theta_{\text{AH}}} = - \frac{\lambda_F \tanh \left( \frac{d_F}{2\lambda_F} \right)}{\lambda_N \tanh \left( \frac{d_N}{2\lambda_N} \right)}, \tag{14}
\]

which can be used to experimentally quantify the spin (anomalous) Hall angle of the heavy-metal (ferromagnetic-metal) layer, provided the Hall angle and spin diffusion length of the other layer—which serves as a reference layer—are known.

Proposal for detecting AH-UMR

As compared to the USMR, there are more options of materials systems for probing the AH-UMR effect. For the USMR, the nonmagnetic heavy-metal layer plays a central role in creating nonequilibrium spin density — via the spin Hall effect — in the adjacent ferromagnetic layer. But this is not the case for the AH-UMR. For the AH-UMR, it is the ferromagnetic layer that serves as the spin polarizer and, hence, the choice of its neighboring layer is not necessarily limited to nonmagnetic materials with a strong spin Hall effect.

For instance, the AH-UMR effect, in principle, can also be hosted in bilayers comprised of a ferromagnetic metal and a normal metal, such as Cu, Al, or Ag. Given the weak spin Hall effect in the normal-metal layer, the AH-UMR is expected to dominate over the USMR in these systems, making the detection of the former more straightforward. There is, perhaps, also a downside to these metallic bilayers — the AH-UMR therein is likely to be much smaller than that in magnetic bilayers with heavy-metals, as normal metals with a weak spin Hall effect are oftentimes also poor “spin sinks” with long spin diffusion lengths [23], which would diminish the AH-UMR effect (especially when the ratio \( \frac{\lambda_F}{\lambda_N} \) is small [24], as we discussed in a previous section).

The shortcoming of normal metals with long spin diffusion lengths may be compensated for by choosing a ferromagnetic layer with low carrier density. To see this, let us insert Eq. (9) together with \( \sigma_0 = e \sum_s n_0 \nu_s \) into Eq. (10), which yields

\[
\tilde{\zeta}_{\text{AH}} \propto \frac{(\nu^+ - \nu^-)}{\nu^+ + \nu^-} \theta_{\text{AH}} n_0. \tag{15}
\]

The above relation conveys a valuable piece of information: the lower the equilibrium carrier density of the ferromagnet, the larger the UMR coefficient. Thus, a more sizable AH-UMR is expected to arise in bilayers consisting of a normal-metal and a ferromagnetic semiconductor [e.g., (Ga,Mn)As] [11] whose carrier density is usually two to three orders of magnitude smaller than that of a ferromagnetic metal.

In magnetic bilayers comprised of nonmagnetic materials with strong spin-orbit coupling, the coexistence of the AH-UMR and USMR poses a challenge to differentiate the two. But, interestingly, they may also conspire to bring about a sign reversal of the overall UMR when the thickness of either layer is varied, a distinct transport signature that would not appear when either effect stands alone. This can be experimentally verified by contrasting the thickness dependences of the total UMR in two ferromagnetic-metal|heavy-metal bilayers, either with different heavy-metal layers whose spin Hall angles are of opposite signs (such Pt and \( \beta\)-Ta [25]) or with different ferromagnetic-metal layers whose anomalous Hall angles are of opposite signs (such as Fe and Ni [26], or Fe and Gd [27]). We anticipate that the results of such
comparative measurements will resemble what are shown in Fig. 4, with one bearing a sign change and the other not. On a related note, a sign change of the UMR was recently observed in single-crystalline Fe/Pt bilayers as the thickness of the Fe layer was increased [28], implying possible competition between the AH-UMR and the USMR.

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Supplementary Information

Anomalous Hall unidirectional magnetoresistance

A. CONTRIBUTION OF BOUNDARY RESISTANCE

The presence of a boundary resistance at the bilayer interface modifies the boundary conditions on the charge and spin chemical potentials as \[22\]

\[
\begin{align*}
\mu_{c,F}(0^+) - \mu_{c,N}(0^-) &= -\gamma r_b J^y_z(0), \\
\mu_{s,F}(0^+) - \mu_{s,N}(0^-) &= r_b J^y_z(0),
\end{align*}
\]

where \(J^y_i(z) \equiv j^+_i(z) - j^-_i(z)\) is the spin current density, in which the spin quantization axis is taken to be along the magnetization direction \(m = y\) and \(i = x, y, z\) indicates the direction of electron momentum flow. Furthermore, \(\gamma\) is the interfacial spin asymmetry coefficient and \(r_b\) is the boundary resistance for a unit surface of the interface.

Resolving the continuity equations presented in the main text with the modified boundary conditions, the total UMR coefficient \(\zeta\) of the system may be calculated. This is comprised of the spin-Hall and anomalous-Hall UMR coefficients, \(\zeta = \zeta_{AH} + \zeta_{SH}\). Taking the spatial average defined as \(\bar{\zeta} \equiv \int_{d_N} dz \frac{\zeta}{d_N + d_F}\), to first order in the spin and anomalous Hall angles, we obtain

\[
\bar{\zeta} = 3 \frac{p_\sigma - p_N}{\epsilon_F} \left( \frac{\sigma_{0,F} \lambda_F}{\sigma_{0,F} d_F + \sigma_{0,N} d_N} \right) \frac{\tanh \left( \frac{d_F}{\lambda_F} \right) \left[ \theta_{AH} \lambda_F \tanh \left( \frac{d_F}{2\lambda_F} \right) + \theta_{SH} \lambda_N \tanh \left( \frac{d_N}{2\lambda_N} \right) \right]}{1 + (1 - p_\sigma^2) \tanh \left( \frac{d_F}{\lambda_F} \right) \left[ \frac{\sigma_{0,F} \lambda_N}{\sigma_{0,N} \lambda_F} \right] \coth \left( \frac{d_N}{\lambda_N} \right) + r_b \frac{\sigma_{0,F}}{\lambda_F} },
\]

where, as in the main text, we note the relation between the anomalous-Hall and spin-Hall UMR coefficients

\[
\frac{\bar{\zeta}_{AH}}{\bar{\zeta}_{SH}} = \frac{\theta_{AH} \lambda_F \tanh \left( \frac{d_F}{2\lambda_F} \right)}{\theta_{SH} \lambda_N \tanh \left( \frac{d_F}{2\lambda_N} \right)}.
\]

Taking into account the interfacial resistance, we have verified that, for a typical boundary resistance of \(r_b \sim 1 \Omega\) m \[22, 23\], the effect on the transport coefficients is negligible. Thus, we may safely neglect the interfacial resistance in the present study.