Large extra dimensional theories attempt to solve the hierarchy problem by assuming that the fundamental scale of the theory is at the electroweak scale. This requires the size of the extra dimensions to be stabilized at a scale which is determined by the effective four dimensional Planck mass and the number of extra dimensions. In this paper we concentrate upon the dynamical reasons to stabilize them by providing a running mass to the radion field. We show that it is possible to maintain the size of the extra dimensions once it is stabilized throughout the dynamics of inflation.

I. INTRODUCTION

Longstanding hierarchy problem, meaning why the Higgs field has a mass $m_H \sim 1$ TeV, and not the Planck mass $M_P \sim 10^{18}$ GeV as suggested by renormalization corrections, finds a natural solution in supersymmetry where the stability of the Higgs mass is provided by the symmetry between bosons and fermions. There, the loop contributions from both the sectors cancel the quadratic divergences to all orders keeping the Higgs mass at the electroweak scale. So far this has been the preferred solution. Recently, however, this problem has been addressed without invoking this supersymmetry, but, by recognizing the fundamental scale to be the higher dimensional Planck mass, $M_*$, In this case $M_*$ takes a value close to the electroweak scale $M_W \sim 1$ TeV, and, the four dimensional Planck mass is then obtained via dimensional reduction by assuming the extra dimensions are compact. Thus, the volume of the extra dimensions $V_d$, the effective four dimensional Planck mass, and, the fundamental scale are all related to each other by a simple relationship

$$M_P^2 = M_*^{2+d} V_d,$$

(1)

where $d$ is the number of extra dimensions. This automatically determines the present common size of all the extra dimensions at $b_0$. For two extra dimensions, and, $M_*=1$ TeV, the required size is of the order of $0.2 \text{ mm}$ $^3$. This suggests that the extra dimensions are indeed quite large and thus require a trapping potential which can stabilize them at their present size. Usually this problem is known as a radion stabilization problem, and in fact its stabilization is reminiscent to the problem of moduli and dilaton stabilization in low energy effective string and M-theories $^3$. Notice, that in the models with large extra dimensions the Standard Model particles are trapped in a four dimensional hypersurface (a3-brane), thus, they are not allowed to propagate in the bulk. However, it is generically assumed that besides gravity, the Standard Model singlets may propagate in the bulk. Among them the inflaton can be a candidate, which is less favored to be a brane field $^3$.

It is also quite unnatural to think that the size of the extra dimensions can be fixed during the evolution of the Universe right at the electroweak scale. In fact, just at the electroweak scale the size of the Universe is given by $\sim 10^{-16}$ cm. Thus, in order to solve the hierarchy we need to inflate the size of the extra dimensions $^2$. This also tells us that the effective four dimensional theory which can be obtained by dimensional reduction must have a new scalar entity, which is known as a radion. The radion also corresponds to the size of the extra dimensions as mentioned earlier. For simplicity, we just take this radion to be homogeneous and treat it only as a zeroth mode. It has been noticed that the radion has a very light mass $m_r \sim 10^{-12}$ eV for two large extra dimensions $^2$.

Now the prime question that appears is how do we stabilize the large extra dimensions while keeping all the virtues and predictions of the big bang and the inflationary cosmology. This is a non-trivial issue, however, we can get some insight from the fact that the expansion in the extra dimensions inevitably introduces expansion in the four dimensions $^4$. This still does not guarantee that the extra dimensions can stabilize themselves after expanding their size to the millimeter scale. The other important factor which one has to keep in mind is that this expansion must also provide adequate scale invariant density perturbation to form the structure formation of the Universe. In fact the theory of large extra dimensions naturally sets the cut-off for any mass scale, hence, at most the inflating fields in four dimensions can acquire a mass $\sim$ TeV. So keeping all these constraints in mind, it is not so easy task to construct a simple model for the early Universe. There have been many attempts in this direction, in Ref. $^5$, it has been proposed that inflation occurs before the stabilization of the extra dimensions with the radion field playing the role of inflaton. In Ref. $^6$, it has been suggested that the initial displacement of the brane from the stable point can lead to inflation in 3 spatial dimensions.

In this paper we raise the issue of stabilizing the extra dimensions at a millimeter scale with extremely light radion mass. We argue that this is possible provided there exists a radion potential which has a minimum corresponding to the size of the large extra dimensions. We then show that it is possible to maintain their large size throughout the dynamics of inflation. This is the main goal of the present work. In the next section we set-up our framework and discuss the initial phase of radion
dominated inflation and the stabilization mechanism. In the subsequent sections we describe the radion dynamics, density perturbations, and, reheating of the Universe. In the last section we briefly summarize our scenario.

II. EXTRA DIMENSIONS FROM THE POINT OF VIEW OF FOUR DIMENSIONS

Let us begin by considering that there exists a scalar field in the bulk. This scalar field essentially plays a double role: providing effective mass to the radion during inflation, and, also acting as an inflaton which essentially reheats the Universe before nucleosynthesis. Since, the actual mass of the radion is very small, for instance, for two large extra dimensions it is $\sim O(10^{-2})$ eV, we can not expect the radion field to reheat the Universe to the extent that we obtain from a bulk scalar field/fields. On the other hand, we require the bulk scalar fields in order to produce density perturbations during inflation. Following this wisdom, we invoke scalar fields $\chi_i$, living in the bulk along with gravity. From the point of view of four dimensions the extra dimensions are assumed to be compactified on a $d$ dimensional Ricci flat manifold with a radii $b(t)$, which have a minimum at $b_0$. The higher dimensional metric then reads

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)dy^2,$$

where $x$ denotes the three spatial dimensions, and $y$ collectively denote the extra dimensions. The scale factor of the four dimensional space-time is denoted by $a(t)$. After dimensional reduction the effective four dimensional action reads

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_p^2}{16\pi} R + \frac{1}{2} \partial_{\mu}\sigma \partial^{\mu}\sigma - U(\sigma) 
+ \frac{1}{2} g^{\mu\nu} \partial_\mu \chi_i \partial^{\mu} \chi_i - \exp(-d\sigma/\sigma_0)V(\chi_i) \right].$$

The four dimensional metric is then determined by $g_{\mu\nu}$, where $\mu, \nu = 0, 1, 2, 3$. At this moment we are not so specific about any matter potential $V(\chi_i)$, we shall keep the discussion general until the next section, where we specify a hybrid inflationary model in order to produce the required density perturbations. The radion field is expressed as $\sigma(t)$, which can be written in terms of the radii of the extra dimensions

$$\sigma(t) = \sigma_0 \ln \left( \frac{b(t)}{b_0} \right), \quad \sigma_0 = \left[ \frac{d(d+2)M_p^2}{16\pi} \right]^{1/2}. $$

From the above equation, it is evident that $\sigma_0$ is proportional to the four dimensional Planck mass. For $b(t) \sim (\text{TeV})^{-1}$, and, $b_0 \sim 1\text{mm}$, the modulus of the radion field takes a very large initial value. The radion field has a potential which is given by $U(\sigma)$ in Eq. (3).

In this paper we shall not shed any light upon its origin, rather we shall assume that such a potential with a minimum is essential to stabilize the large extra dimensions. It has been well known that there can be at least three sources which might contribute to $U(\sigma)$ such as a cosmological constant in $4 + d$ dimensional theory. Second source might be the curvature of the extra dimensions. However, such a contribution can be made zero if the extra dimensions are compactified on a torus, and, the third possibility could be due to the Casimir force. All these known possibilities can only help to stabilize the extra dimensions close to the electroweak scale and thus they do not serve our purpose in solving the hierarchy problem. New avenues could be opened if the extra dimensions were not commutative and a Casimir repulsive force could be generated due to a non-trivial topology of the extra dimensions. Even then we require to fine tune the non-commutative parameter and thus again it is far from convincing. At large there is no proper understanding of how such a non-trivial potential for the radion field can be generated, and here we do not make any suggestion in this direction. Inspite of having a minimum in the radion potential, it is still a model dependent issue to stabilize the radion around its minimum. However, as we feel that the issue of radion stabilization is interwined with that of inflation, and, generation of density perturbation in four dimensions, it is still required to have a simple model where one can address all these issues together in a single framework.

In this paper we provide a dynamical argument which suggests that the radion field can have a running mass during the inflationary epoch. When the radion field approaches a critical value, it decouples from the rest of the inflationary dynamics and rolls down the potential to reach the minimum. When inflation finally comes to an end, the running mass of the radion field settles down to its bare mass which can be as small as eVs.

III. RADION DYNAMICS AND INFLATION

As we have discussed very briefly in our last section, the radion field plays a very significant role in the early Universe, in this paper we devise a simple scheme which provides a running mass to the radion field. This comes quite naturally if there exists a scalar field in the bulk, which upon dimensional reduction couples to the radion field as shown in the effective four dimensional action Eq. (3). If we assume that the scalar field $\sigma$, and, at least one of the bulk fields; say $\chi$ is rolling down the potential, then the slow-roll equations yield

$$3H\dot{\sigma} \approx \frac{d}{\sigma_0} e^{-d\sigma/\sigma_0} V(\chi),$$

$$3H\dot{\chi} \approx -e^{-d\sigma/\sigma_0} V'(\chi),$$

$$H^2 \approx \frac{8\pi}{3M_p^2} \left[ e^{-d\sigma/\sigma_0} V(\chi) + U(\sigma) \right].$$
where the dot denotes time derivative and prime denotes the derivative with respect to $\chi$ field. Notice, here we have assumed that among many bulk fields only a single field which we call inflaton is taking part in a slow-roll inflation while the rest of them are trapped in some false vacuum. Thus, the potential $V(\chi)$, evaluated at those points, has become an effective function of $\chi$ alone, however, the bulk of the potential is coming from the false vacuum. On the other hand, the form of $U(\sigma)$ must be such that there exists a global minimum at $\sigma = 0$. For simplicity we can model the potential to be $V(\chi) \approx M^2 d_M^2$, hence, the total energy density is still bounded by the upper limit $\approx M^4$ in our world. From Eq. (6), it is quite evident that in order to obtain $b_0$ as large as 1 mm, the extra dimensions need to grow at least 35 e-foldings in size. As we have discussed, if we assume that inflation proceeds in $\chi$ direction, then, from Eqs. (6) and (7), we obtain a simple relation between the expansion rates of the observable and the extra dimensions $\chi[7]$. Which yields

$$\frac{d+2}{2} \dot{b} - \frac{\ddot{a}}{a}.$$  

(8)

The set of slow-roll equations satisfy power law inflation in the observable world for arbitrary number of extra dimensions. The scale factor is given by

$$a(t) \sim t^{(d+2)/d},$$  

(9)

however, during this period the extra dimensions grow much slower as

$$b(t) \sim t^{2/d}.$$  

(10)

It is noticeable that the extra dimensions do not inflate for $d \geq 2$ extra dimensions. From the point of view of four dimensions the radion field inflates our Universe and during this process it rolls down the exponential potential to reach the critical value $|\sigma_0|$. About this point the exponential term in front of $V(\chi)$ in Eq. (6), becomes weaker, and this helps the effective mass of the radion field to catch up with the Hubble expansion

$$m_{r,\text{eff}}^2 \approx m_r^2 + \frac{V(\chi)}{\sigma_0^2} \sim O(1) H^2.$$  

(11)

Here again, we have neglected the contribution from $U(\sigma)$ compared to $V(\chi)$, because the bare mass of the radion field which is of the order of electroweak scale is much smaller than the Hubble parameter. This is also the reason why we have approximated the effective mass to that of the Hubble parameter in the above equation Eq. (11). At this point the radion field can not anymore support inflation. However, inflation is still continuing, but, now due to the vacuum expectation value of the matter fields. This is not an assumption, but, as we shall see later, it is very important to maintain the mass of the radion of the order of Hubble expansion while not disturbing the effective mass parameters of $\chi$ field. The simplest way to realize inflation is from the vacuum dominated phase rather than the slow-roll motion of all $\chi_i$ fields. This can be satisfied if we assume hybrid potential for $\chi_i$ fields. In fact, in the next section we shall notice that the necessity to assume hybrid model has a completely different motivation. The large effective mass for the radion field helps to decouple its dynamics from the rest of the fields. If $H$ changes slowly, then the field $\sigma$, for $m_{\text{eff}} \sim H$, approaches the minimum configuration, $\sigma = 0$, exponentially fast. We remind the readers that the minimum value of $\sigma$ field is provided by the radion potential $U(\sigma)$. The radion field evolves like

$$\sigma(t) \approx \sigma_0 e^{(-m_{r,\text{eff}}(Ht)/3H)} \sim \sigma_0 e^{(-Ht/3)}.$$  

(12)

In order to stabilize the radion field we have to ensure that the radion field settles down to the bottom of the potential much faster before the end of vacuum dominated inflation. In the standard hybrid model this phase ends with a phase transition when the effective mass of the subsidiary field which triggers the phase transition becomes imaginary. This takes place when $\tau \approx 3H(\delta \ln \phi)/m^2 \approx 3 \cdot 10^{-22}$ sec, where $m$ is the mass term of the inflaton which we take around 10 GeV. Thus, by the time this phase transition takes place, the factor $H \tau \sim 10^8$ becomes quite large. This leads to an extremely large suppression to the exponential factor in Eq. (12). In fact the radion field rolls down towards the minimum of $U(\sigma)$ much faster compared to the inflationary time scale, the time taken is give by $t \sim 1/H \sim 1/M_* = 10^{-30}$ sec. This is perhaps the easiest way to freeze the dynamics of the large extra dimensions. It is noticeable that the effective mass for the radion remains of the order of Hubble expansion until the end of vacuum dominated phase. Once the matter fields begin oscillations and eventually settle down at their respective minima via reheating, the effective mass term for the radion approaches its bare mass $\sim m_r$. However, one might suspect that quantum corrections to the radion mass due to its coupling to the matter field, such as Higgs, could be large. Usually these corrections must be of the order of the bare mass of the radion field due to the Planck mass suppression. There could also be a possible contribution that might come from the Higgs vacuum, $v$, which has the form $V(v)/M_p^2 \leq (eV)^2$. We mention that this contribution is directly associated with the cosmological constant problem in the sense that it is expected that $V(v) < (eV)^2$, in order to be consistent with current limits. [17]
IV. DENSITY PERTURBATIONS

Any successful inflationary model has to pass through the test of density perturbation. As we have noted earlier, the last stage of inflation has to be supported by the vacuum energy density of \( V(\chi_1) \), whose upper bound is \( \sim M_p^2 M^2 \). Since, for our purpose, the masses of the fields have a natural cut-off scale \( \sim M_* \), it is extremely difficult to produce adequate density perturbation for a simple chaotic inflationary scenario \[3\]. For this reason hybrid model has been chosen because it may overcome a low level of density perturbation \[8\]. In this section we briefly review the model. Let us consider the 4 + \( d \) dimensional potential

\[
V(\chi_1, \chi_2) = \frac{M_p^d}{4\lambda} \left( M_*^2 - \frac{\lambda}{M_*^d} \chi_1^2 \right)^2 + \frac{m^2}{2} \chi_2^2 + \frac{g^2}{M_*^2} \chi_2^2 \chi_1^2, \tag{13}
\]

where \( \chi_2 \) is the inflaton field, and \( \chi_1 \) is the subsidiary field which is responsible for the phase transition. Notice, that the higher dimensional field has a mass dimension 1 + \( d/2 \), which leads to non renormalisable interaction terms. However, the suppression is given by the fundamental scale, instead of the four dimensional Planck mass. Upon dimensional reduction the effective four dimensional fields, \( \chi_i \), are related to their higher dimensional relatives by a simple scaling

\[
\chi_i = \sqrt{d_1} \chi_i. \tag{14}
\]

Therefore, eventhough the natural initial value for \( \chi_i \sim M_*^{1+d/2} \), in the effective four dimensional theory we may have \( \chi_i \sim M_p \). It is easy to check that the slow roll conditions for the inflation breakdown in hybrid model when \( \chi_2^2 \gtrsim M_*^{2+d}/2g^2 \). This provides a water fall solution to the fields when \( \chi_2 \sim M_p^d/\sqrt{2g} \).

Now, in order to perform the density perturbation calculation we need to know the follow on history of the Universe. Especially, we must notice when the observable world should be within the horizon at the beginning of inflation. This depends on the total number of e-foldings we have in general. In order to estimate how many e-foldings do we get in this model, we make a naive estimation while supposing that there are only two extra dimensions. It is clear that the extra dimensions ought to expand 35 e-foldings in order to reach the millimeter size, this tells us that the three spatial dimensions must grow up to 70 e-foldings during the first phase of inflation. The second phase of vacuum dominated inflation proceeds soon after this initial phase and lasts for at least another 40 – 60 e-foldings. So, total number of e-foldings can be as large as 130. We notice that there are two important differences from the standard hybrid inflationary cosmology. First, the inflationary scale is not given as usual by the grand unification scale; \( \sim 10^{16} \) GeV, but in our case the scale is much lower, and it is given by a geometric mean; \( \sqrt{M_* M^2} \sim 10^{12} \) GeV. Second, the reheat temperature of the Universe in our case is extremely low. Indeed, as we shall see below reheat temperature is certainly more than MeV, but can be as low as \( T_r \sim 100 \) MeV, which for the time being we take as granted while estimating when the interesting modes are crossing the present horizon. In fact, one can easily estimate that what matters is the density perturbations produced during the last 43 e-foldings of inflation \[15\].

In our case, for two large extra dimensions the last 43 e-foldings are supported by adiabatic perturbations due to the slow roll of \( \chi_2 \) field \[8\].

\[
\frac{\delta \rho}{\rho} \sim \frac{g}{2\lambda^{3/2}} \frac{M_*^4}{m_{\chi_2}^2 M_p}, \tag{15}
\]

If we set in the last equation the values; \( M_* \sim 10^2 \) \( TeV \), and, \( m_{\chi_2} \sim m \sim 10 \) \( GeV \), we obtain the correct COBE normalization \( \delta \rho/\rho \sim (g/2\lambda^{3/2}) \times 10^{-5} \), up to the uncertainties in the coupling constants. So far, we have only discussed the adiabatic fluctuations of \( \chi_2 \) field, in fact there is another source of density perturbation in our case, which is due to the isocurvature fluctuations of the radion field. Since, we have noticed that the radion field settles down at the bottom of the potential during inflation with an effective mass \( \sim \mathcal{O}(H) \). So, the energy density of the radion field is effectively given by its quantum fluctuations, which is quadratic in nature. The perturbations generated in the density of the radion field will have a non-gaussian feature \[16\]. So, in principle we can expect a mixture of adiabatic and isocurvature fluctuations. In this paper we do not delve into the details of the mixed perturbation scenario.

V. REHEATING

After hybrid inflation ends the field \( \chi_1 \) goes to one of its minima: \( \chi_1 \pm = \pm M_*/\sqrt{2\lambda} \). The inflaton field gets a mass contribution: \( \sim g^2 M_*^2/\lambda \), which dominates over \( m_{\chi_2} \). Therefore, by using the same set of parameters we have introduced to explain density perturbations, we can estimate that the physical mass of the oscillating inflaton can be about an order of magnitude below \( M_* \). Thus, the decay of \( \chi_1 \) into Higgs fields; \( \phi \), is allowed for typical Higgs field masses in the range 100 – 200 GeV. Let us stress that this process will take place only within the brane, which marks the departure from the former Kaluza Klein (KK) theories, where the production of matter via inflaton decays occurs everywhere.

The reheating temperature has been estimated in Ref. \[3\]. Here we only recollect the main ideas. First, we calculate the decay rate of the inflaton field into two Higgses. That gives

\[
\Gamma_{\chi_1 \rightarrow \phi \phi} \sim \frac{M_*^4}{32\pi M_p^2 m_{\chi_1}}; \tag{16}
\]
With an inflaton mass $m_{\chi_1} \sim 0.1 \, M_4$, we obtain the reheating temperature to be: $T_r \geq 100 \, \text{MeV}$. As it is expected, this temperature is above the required temperature for which the big bang nucleosynthesis can successfully take place. It is worth mentioning that bulk reheating is much less efficient than the brane reheating, because the production of KK gravitons; $g_{KK}$, is very inefficient due to the Planck mass suppressed couplings, and, KK number conservation which for bids kinematically the decay of zero mode inflaton into graviton, or, radion, since, the process has to have an inflaton mode as a final product. One may also wonder upon the radion decay. In fact radion can also decay into two light fermions, through the coupling $(\sigma/\sigma_0) m_\psi \bar{\psi} \psi$, however, its decay rate is again Planck mass suppressed. Nevertheless, we mention that the last e-foldings of inflation, and entropy production during reheating is sufficient enough to substantially dilute their number density.

VI. CONCLUSION

We conclude our paper by summarizing our main results. We have noticed that non-trivial radion dynamics can help to stabilize the size of the large extra dimensions. In fact the radion mass is running throughout the inflationary evolution. The initial phase of inflation is governed by the exponential radion potential. The radion field rolls down the exponential potential and its effective mass becomes of the order of the Hubble expansion. When this happens, the dynamics of the radion field decouples from the rest of the matter fields. However, inflation continues with the help of vacuum dominated phase of the hybrid inflationary model. During inflation, the radion field rolls down within one Hubble time to its minimum, which helps to stabilize the size of the extra dimensions. Once, inflation comes to an end via the phase transition in the hybrid model, the effective radion mass drops down to its bare value which is $\sim 10^{-2}$ eV for the two large extra dimensions. In this model we require last 43 e-foldings of inflation in order to match the density perturbations from the COBE data. We have shown that the adiabatic fluctuations of $\chi_1$ field can produce the adequate density perturbations. We have also noticed that there exists another source of density perturbations. This is due to the quantum fluctuations of the radion field around the bottom of the potential. This kind of density perturbations leads to nongaussian spectrum. In fact we would expect to have mixture of both adiabatic and isocurvature fluctuations. It will be interesting to study the details of the density fluctuations, which might lead to a definitive prediction of our model.

ACKNOWLEDGMENTS

A. M. acknowledges the support of The Early Universe network HPRN-CT-2000-00152.