Feedback Control and Counting Statistics in Quantum Transport

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- Quantum Transport
  - Example: particle counting.
  - Moments, cumulants, generalized density operators.
  - Quantum dots.

- Feedback control
  - Introduction, various kinds of feedback control.
  - Wiseman-Milburn control.
  - Information and thermodynamics, Maxwell demon control.

- (Talk on Tuesday)
  - Time-versus-number feedback control.
  - A recent experiment with quantum dots.
Feedback control
Feedback control

Centrifugal governor, Boulton and Watt

Stochastic cooling of particle collider beam, S. van der Meer (1972), Nobel Prize (1984) - discovery of W and Z bosons.

Photodetector signal corrects laser diode pump current, S. Machida, Y. Yamamoto (1986).

Feedback control of a single-atom trajectory, A. Kubanek, M. Koch, C. Sames, A. Ourjoumtsev, P. W. H. Pinkse, K. Murr and G. Rempe (2009).
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Feedback control

Challenge for Quantum Systems

- Unitary dynamics versus measurement process.
- How to model the feedback loop?

H. M. Wiseman, G. J. Milburn, *Quantum measurement and Control* (2009)
Passive versus active feedback

- (Passive) Coherent feedback control of quantum transport

C. Emary, J. Gough, PRB 90, 205436 (2014).
Passive versus active feedback

- (Passive) Coherent feedback control of quantum transport

  ![Diagram](image)

  C. Emary, J. Gough, PRB 90, 205436 (2014).

- (Passive) Cavity in waveguide

  ![Diagram](image)

  J. Kabuss, D. O. Krimer, S. Rotter, K. Stannigel, A. Knorr, and A. Carmele, PRA 92, 052321 (2015).
Passive versus active feedback

- (Passive) Coherent feedback control of quantum transport
- (Active): feedback control of electron counting statistics.

C. Emary, J. Gough, PRB 90, 205436 (2014).

J. Kabuss, D. O. Krimer, S. Rotter, K. Stannigel, A. Knorr, and A. Carmele, PRA 92, 052321 (2015).

TB; Phys. Rev. Lett. 105, 060602 (2010); T. Wagner, P. Strasberg, J. C. Bayer, E. P. Rugeramigabo, TB, R. J. Haug; nature nanotechnology (2016).
Active feedback master equation

General setup

- Open system Hamiltonian. $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_{\text{res}} + \mathcal{H}_T$.
  - $\mathcal{H}_S$ system.
  - $\mathcal{H}_{\text{res}}$ reservoir.
  - $\mathcal{H}_T$ system-reservoir coupling.

G. Schaller, *Open Quantum Systems Far From Equilibrium* (2014)
Active feedback master equation

Quantum jumps

- $n$-resolved density operator $\rho(t) \equiv \sum_{n=0}^{\infty} \rho^{(n)}(t)$
- Stochastic trajectories $\{x_n\} = x_n, ..., x_1$: jumps of type $l_i$ at time $t_i$.
- ‘path integral’

$$\rho^{(n)}(t) = \sum_{l_1=1,\ldots,l_n=1}^{M} \int_0^t dt_n \ldots \int_0^{t_2} dt_1 \rho(t|\{x_n\})$$.
Active feedback master equation

Without feedback

- \( n \)-resolved density operator \( \rho(t) \equiv \sum_{n=0}^{\infty} \rho^{(n)}(t) \)
- Stochastic trajectories \( \{x_n\} = x_n, \ldots, x_1 \): jumps of type \( l_i \) at time \( t_i \).
- ‘path integral’

\[
\rho^{(n)}(t) = \sum_{l_1=1,\ldots, l_n=1}^{M} \int_{0}^{t_1} dt_n \cdots \int_{0}^{t_2} dt_1 \rho(t|\{x_n\}).
\]

Without feedback:

- \( \rho(t|\{x_n\}) \equiv S_{t-t_n} \mathcal{J}_{l_n} S_{t_n-t_{n-1}} \mathcal{J}_{l_{n-1}} \cdots \mathcal{J}_{l_1} S_{t_1} \rho_{in} \).
- Reduced density matrix \( \dot{\rho}(t) = \mathcal{L}\rho(t) \),
  \( \mathcal{L} \equiv \mathcal{L}_0 + \mathcal{J} \), \( S_t \equiv e^{\mathcal{L}_0 t} \).

Moelmer, Zoller, Hegerfeldt, Carmichael,... 1980s
Active feedback master equation

With feedback

- $n$-resolved density operator $\rho(t) \equiv \sum_{n=0}^{\infty} \rho^{(n)}(t)$
- Stochastic trajectories $\{x_n\} = x_n, \ldots, x_1$: jumps of type $l_i$ at time $t_i$.
- ‘path integral’

$$\rho^{(n)}(t) = \sum_{l_1=1,\ldots,l_n=1}^M \int_0^t dt_n \ldots \int_0^{t_2} dt_1 \rho(t|\{x_n\}).$$

With feedback:

$$\rho(t|\{x_n\}) = S(t|\{x_n\}) J_{l_n}(t_n|\{x_{n-1}\}) S(t_n|\{x_{n-1}\}) \ldots \times J_{l_2}(t_2|\{x_1\}) S(t_2|\{x_1\}) J_{l_1}(t_1) S(t_1) \rho_{in}.$$

- Future time evolution conditioned upon full trajectory.
Active feedback master equation

Feedback protocols

0. Open loop control,

$$J_I(t|\{x_n\}) = J_I(t).$$

1. Feedback conditioned on the previous jump,

$$J_I(t|\{x_n\}) = J_{I_{lln}}(t - t_n).$$

- Wiseman-Milburn quantum feedback control, $J_{I_{lln}}(t - t_n) = e^{K_I} J_I$.
  Wiseman, Milburn ('90s); Pöltl, Emary TB (2011); Schaller, Kiesslich, Emary, TB (2011); Kiesslich, Emary, Schaller, TB (2012); Daryanoosh, Wiseman, TB (2016).
- Waiting time feedback control TB, Emary (2016).
- Delayed quantum control, Emary (2013).

2. Time-versus-number feedback,

$$J_I(t|\{x_n\}) = J_I(t, n).$$
Wiseman-Milburn feedback
Wiseman-Milburn feedback

The idea

- Dissipation originates from quantum jumps.
- Fight dissipation by acting immediately after each jump: feedback control.
Wiseman-Milburn feedback
Counting fields and feedback

\[ \dot{\rho} = (\mathcal{L}_0 + e^{i\chi} \mathcal{J})\rho, \] counting field $\chi$. (R. J. Cook 1981).

FEEDBACK: complex number $e^{i\chi} \rightarrow$ superoperator $e^K$ (Wiseman,... 1990s).
Wiseman-Milburn feedback

Stabilization: the effective Hamiltonian (C. Emary, 2011)

- Quantum jump $\rho \to J\rho$.
- Non-jump $\rho \to e^{L_0\tau} \rho$. Mixes up everything.
Wiseman-Milburn feedback

Stabilization: the effective Hamiltonian

- Quantum jump $\rho \rightarrow \mathcal{J} \rho$.
- Non-jump $\rho \rightarrow e^{\mathcal{L}_0 \tau} \rho$. Mixes up everything.
- FEEDBACK IDEA: No mixing if $\mathcal{L}_0 \rho_k = \lambda_k \rho_k$.
- Compensate quantum jumps by rotation into $\rho_k$ via $e^{\mathcal{K}} \mathcal{J} \rho = \rho_k$.

$\sim$ eigenvalue problem to determine $\mathcal{K}$ in $\mathcal{L}_0 e^{\mathcal{K}} \mathcal{J} \rho_k = \lambda_k \rho_k$. 
Wiseman-Milburn feedback

Stabilization: the effective Hamiltonian

- Effective, non-Hermitian ‘Hamiltonian’ $\mathcal{H}_{\text{eff}}$ via

$$\mathcal{L}_0 \rho = -i(\mathcal{H}_{\text{eff}} \rho - \rho \mathcal{H}_{\text{eff}}^\dagger).$$

- Eigenstates

$$\rho_k = |\psi_k\rangle \langle \tilde{\psi}_k|$$

of $\mathcal{L}_0$ are pure density matrices.

- The $|\psi_k\rangle$ are (right) eigenstates of $\mathcal{H}_{\text{eff}}$.

(Non-hermitian part of $\mathcal{H}_{\text{eff}}$ contains the back-action of the measurement process.)
Wiseman-Milburn feedback: charge qubit

C. Pöltl, C. Emary, TB; Phys. Rev. B 84, 085302 (2011).

\[ \Gamma_L \quad T_C \quad \Gamma_R \]

- Effective Hamiltonian

\[ \mathcal{H}_{\text{eff}} \equiv \mathcal{H}_S - \frac{i \Gamma_L}{2} |0\rangle\langle 0| - \frac{i \Gamma_R}{2} |R\rangle\langle R| \]

- Pure eigenstates \(|0\rangle\langle \tilde{0}|, |\psi_{\pm}\rangle\langle \tilde{\psi}_{\pm}|\)
Wiseman-Milburn feedback: charge qubit

Control Stabilization

Electron tunnels into $|L\rangle\rangle$ which is instantaneously rotated into $|\psi_{\pm}\rangle\rangle$ (alternatively, $|\psi_{-}\rangle\rangle$).

$t\rightarrow$ effective *single* level system, decay rate $\gamma_R^{(\pm)} \equiv -2\Im\epsilon_{\pm}$
Wiseman-Milburn feedback: charge qubit

Controlled master equation

\[ \dot{\rho} = (\mathcal{L}_0 + \mathcal{C}_L \mathcal{J}_L + \mathcal{J}_R) \]

\[ \mathcal{C}_L \equiv e^{-2i\theta_C \mathbf{n}_0 \cdot \Sigma}, \quad \Sigma \rho \equiv [\sigma, \rho] \]

- Rotation in Liouville-space about angle $\theta_C$ around direction $\mathbf{n}_0 = (\sin \theta, 0, \cos \theta)$, Pauli matrix vector $\sigma$.
- Every qubit rotation as possible control operation, J. Wang and H. M. Wiseman, Phys. Rev. A 64, 063810 (2001).
Wiseman-Milburn feedback: charge qubit

Controlled master equation

\[ \dot{\rho} = (\mathcal{L}_0 + C_L \mathcal{J}_L + \mathcal{J}_R) \]

\[ C_L \equiv e^{-2i\theta_C n_0 \cdot \Sigma}, \quad \Sigma \rho \equiv [\sigma, \rho] \]

- Rotation in Liouville-space about angle \( \theta_C \) around direction \( n_0 = (\sin \theta, 0, \cos \theta) \), Pauli matrix vector \( \sigma \).
- Every qubit rotation as possible control operation, J. Wang and H. M. Wiseman, Phys. Rev. A 64, 063810 (2001).
- Project out empty state \((\Gamma_L \to \infty)\).
- Bloch representation of stationary state \( \rho_{\text{stat}} = (\langle \sigma_x \rangle, \langle \sigma_y \rangle, \langle \sigma_z \rangle) \).
Wiseman-Milburn feedback: charge qubit

Right eigenvalues of $H_{\text{eff}}$:

\[
\varepsilon_0 = -i\frac{\Gamma_L}{2}, \quad \varepsilon_\mp = \frac{1}{4} \left( -i\Gamma_R \mp \sqrt{16T_C^2 - \Gamma_R^2} \right)
\]

Eigenstates of the free Liouvillian

For $\Gamma_R > 4T_C$:

\[
\langle \sigma_x \rangle = 0, \quad \langle \sigma_y \rangle = \frac{4T_C}{\Gamma_R}, \quad \langle \sigma_z \rangle = \mp \frac{\sqrt{\Gamma_R^2 - 16T_C^2}}{\Gamma_R}
\]

For $\Gamma_R < 4T_C$:

\[
\langle \sigma_x \rangle = \mp \frac{\sqrt{16T_C^2 - \Gamma_R^2}}{4T_C}, \quad \langle \sigma_y \rangle = \frac{\Gamma_R}{4T_C}, \quad \langle \sigma_z \rangle = 0
\]
Maxwell’s thought experiment
- Two volumes of gas in equilibrium separated by sliding door.
- Demon opens/closes door, separates fast from slow gas molecules.
- Decrease of entropy. "... to show that the second law of thermodynamics has only a statistical certainty".

Concept of a transport device that acts like Maxwell’s demon.

*Colloquium*: The physics of Maxwell’s demon and information; K. Maruyama, F. Nori, and V. Vedral, Rev. Mod. Phys. 81, 1 (2009).
Maxwell demon type feedback
Single electron transistor

- Modify tunnel rates, e.g. $\Gamma_R$, depending on dot occupation.

G. Schaller, C. Emary, G. Kießlich, TB; Phys. Rev. B \textbf{84}, 085418 (2011).
Maxwell demon type feedback

Single junction

- Integrate out the dot for $\Gamma_L \gg \Gamma_R \equiv \Gamma$.
- Rates $\gamma = \Gamma f_L (1 - f_R)$, $\tilde{\gamma} = \bar{\Gamma} f_R (1 - f_L)$.
- $p_n(t)$ probability for $n$ charges transfered to right reservoir.

$$\dot{p}_n = \gamma p_{n-1} + \tilde{\gamma} p_{n+1} - (\gamma + \tilde{\gamma}) p_n$$
Maxwell demon type feedback

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- Rates $\gamma = \Gamma f_L (1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R (1 - f_L)$.
- $p_n(t)$ probability for $n$ charges transferred to right reservoir.

$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

Without feedback:

- Identical microscopic forward and backward rates $\Gamma = \bar{\Gamma}$.
- Local **detailed balance** condition with affinity $A$,

$$\frac{\gamma}{\bar{\gamma}} = e^A, \quad A \equiv \frac{\mu_L - \mu_R}{k_B T}$$
Maxwell demon type feedback

Single junction

- Integrate out the dot for $\Gamma_L \gg \Gamma_R \equiv \Gamma$.
- Rates $\gamma = \Gamma f_L (1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R (1 - f_L)$.
- $p_n(t)$ probability for $n$ charges transferred to right reservoir.

$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

With feedback:

- Different forward and backward rates $\Gamma \neq \bar{\Gamma}$ (‘by hand’)
- Violates local detailed balance condition,

$$\frac{\gamma}{\bar{\gamma}} = e^{A + \ln \frac{\Gamma}{\bar{\Gamma}}}, \quad A \equiv \frac{\mu_L - \mu_R}{k_B T}$$
Maxwell demon type feedback

Single junction

- Integrate out the dot for $\Gamma_L \gg \Gamma_R \equiv \Gamma$.
- Rates $\gamma = \Gamma f_L (1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R (1 - f_L)$.
- $p_n(t)$ probability for $n$ charges transferred to right reservoir.
- $\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$
- Elevate (violated) local detailed balance to modified *exchange fluctuation theorem*

\[
\frac{p_n(t)}{p_{-n}(t)} = e^{(A + \ln \frac{\Gamma}{\bar{\Gamma}})n}, \quad A \equiv \frac{\mu_L - \mu_R}{k_B T}.
\]
Maxwell demon type feedback

Single junction: Interpretation

- Rates $\gamma = \Gamma f_L (1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R (1 - f_L)$.

$$\frac{p_n(t)}{p_{-n}(t)} = e(\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}) n, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

- Stationary charge current $\mathcal{J} = \gamma - \bar{\gamma}$ (set $-e = 1$).

- With feedback $\Gamma \neq \bar{\Gamma} \Rightarrow$ finite $\mathcal{J}$ even for zero voltage drop.
Maxwell demon type feedback

Single junction: Interpretation

Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.

$$\frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}\right)n}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

- Only $n$ (number of transferred charges) thermodynamically relevant.
- Shannon entropy $S \equiv -\sum_n p_n \ln p_n$.
- Decompose $\dot{S} = \dot{S}_e + \dot{S}_i$ with $\dot{S}_i \geq 0$, J. Schnakenberg, Rev. Mod. Phys. 48, 571 (1976); M. Esposito, C. Van den Broek, Phys. Rev. E 82, 011143 (2010).

$$\lim_{t \to \infty} \frac{p_n(t)}{p_{-n}(t)} = e^{\dot{S}_it}, \quad \dot{S}_i = \mathcal{A} \mathcal{J} + \ln \frac{\Gamma}{\bar{\Gamma}} \mathcal{J}, \quad \mathcal{J} \equiv \gamma - \bar{\gamma}$$

$\dot{S}_i =$ dissipated electric power per $k_B T$ plus information current.
Maxwell demon type feedback

Single junction: Interpretation

- Rates \( \gamma = \Gamma f_L(1 - f_R), \bar{\gamma} = \bar{\Gamma} f_R(1 - f_L) \).

\[
\frac{p_n(t)}{p_{-n}(t)} = e^{(A + \ln \frac{\Gamma}{\bar{\Gamma}})n}, \quad A \equiv \frac{\mu_L - \mu_R}{k_B T}.
\]

- Information gain via feedback modifies exchange fluctuation relation.

- General scenario T. Sagawa and M. Ueda ('08), J. M. P. Parrondo ('11); D. Abreu, U. Seifert ('12); T. Munakata, M. L. Rosinberg ('12); H. Tasaki ('13); J. M. Horowitz, M. Esposito ('14); D. Hartich, A. C. Barato, U. Seifert ('14); J. M. Horowitz ('15); J. M. Horowitz, K. Jacobs ('15)

- Example \( \langle e^{-\beta(W - \Delta F) - I} \rangle = 1 \) modified Jarzynski relation.
Feedback controlled tunnel barrier
Single electron transistor

- Rate equation \( \dot{\rho} = \mathcal{L}\rho, \rho = (\rho_0, \rho_1)^T. \)
- Explicitely break local detailed balance:

\[
\mathcal{L} = \sum_{\alpha=L,R} \begin{pmatrix}
-\Gamma_\alpha f_\alpha & \bar{\Gamma}_\alpha (1 - f_\alpha) e^{i\chi} \\
\Gamma_\alpha f_\alpha & -\bar{\Gamma}_\alpha (1 - f_\alpha)
\end{pmatrix}
\]

- Fluctuation relation (affinity \( A \equiv V/k_B T, \) voltage \( V \equiv \mu_L - \mu_R \))

\[
\lim_{t \to \infty} \frac{p_n(t)}{p_{-n}(t)} = e^{(A + \ln \frac{\Gamma_R}{\Gamma_L} + \ln \frac{\bar{\Gamma}_R}{\bar{\Gamma}_L}) n}.
\]

M. Esposito, G. Schaller; EPL 99, 30003 (2012).
Feedback controlled tunnel barrier

Single electron transistor

- Fluctuation relation (affinity $\mathcal{A} \equiv V/k_B T$, voltage $V \equiv \mu_L - \mu_R$)

$$\lim_{t \to \infty} \frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A}+\ln\frac{\Gamma_R}{\Gamma_R}+\ln\frac{\bar{\Gamma}_L}{\Gamma_L}\right)n}.$$ 

M. Esposito, G. Schaller; EPL 99, 30003 (2012).

- Term $\ln\frac{\Gamma_R}{\Gamma_R} + \ln\frac{\bar{\Gamma}_L}{\Gamma_L} = -V^*/k_B T$
as offset-voltage

G. Schaller, C. Emary, G. Kießlich, TB; Phys. Rev. B 84, 085418 (2011).
Hardwiring the demon

Key idea

- Microscopic model for larger system: SET + detector.
- Reduced SET dynamics described by effective model as above.
Hardwiring the demon
Thermoelectric device

- Energy to current converter.
- Different temperatures in different parts of the system.
- Energy-dependent tunnel rates.

R. Sánchez, M. Büttiker, Phys. Rev. B 83, 085428 (2011); Europhys. Lett. 100, 47008 (2012).
Hardwiring the demon

- Single level SET (bottom, two reservoirs L/R) and detector (top, one reservoir).
- States $|0E\rangle$, $|0F\rangle$, $|1E\rangle$, $|1F\rangle$.
- Energies $0$, $\epsilon_s$, $\epsilon_d$, $\epsilon_s + \epsilon_d + U$.
- Energy dependent rates.

P. Strasberg, G. Schaller, TB, M. Esposito, Phys. Rev. Lett. 110, 040601 (2013).
Hardwiring the demon

P. Strasberg, G. Schaller, TB, M. Esposito, Phys. Rev. Lett. 110, 040601 (2013).

Detector requirements:
- Fast $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
- Precise $U \gg k_B T_D$.

SET requirements:
- Spatial asymmetry $\Gamma_R^U \gg \Gamma_L^U, \Gamma_L \gg \Gamma_R$. 

Detector requirements:
- Fast $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
- Precise $U \gg k_B T_D$.

SET requirements:
- Spatial asymmetry $\Gamma_R^U \gg \Gamma_L^U, \Gamma_L \gg \Gamma_R$. 

Maxwell demon limit

Reduced SET dynamics

- Full rate equation \( \dot{\rho}_{ij} = \sum_{i'j'} W_{ij,i'j'} \rho_{i'j'} \)

- Separation of time scales for fast detector \( \Gamma_D, \Gamma^U_D \gg \Gamma_\alpha, \Gamma^U_\alpha \).
  - Technical trick: use conditional stationary probabilities M. Esposito, Phys. Rev. E 85, 041125 (2012).

- \( \sim \) SET rate equation \( \dot{\rho}_i = \sum_{i'} V_{ii'} \rho_{i'} \).
Maxwell demon limit

Reduced SET dynamics

- Full rate equation \( \dot{\rho}_{ij} = \sum_{i'j'} W_{ij,i'j'} \rho_{i'j'} \)
- Separation of time scales for fast detector \( \Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U \).
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- \( \rightsquigarrow \) SET rate equation \( \dot{\rho}_i = \sum_{i'} V_{ii'} \rho_{i'} \).

Reduced fluctuation theorem for SET with information current \( I \)

\[
\lim_{t \to \infty} \frac{p_n(t)}{p_{-n}(t)} = \exp [\mathcal{A} n + I \times t], \quad I \times t \equiv \ln \frac{f_L^U f_R^U \Gamma_L^U \Gamma_R}{f_R^U f_L^U \Gamma_R^U \Gamma_L} n, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.
\]
Maxwell demon limit

Reduced SET dynamics

- Full rate equation \( \dot{\rho}_{ij} = \sum_{i'j'} W_{ij,i'j'} \rho_{i'j'} \)

- Separation of time scales for fast detector \( \Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U \).
  
  ▶ Technical trick: use conditional stationary probabilities M. Esposito, Phys. Rev. E 85, 041125 (2012).

- \( \rightsquigarrow \) SET rate equation \( \dot{\rho}_i = \sum_{i'} V_{ii'} \rho_{i'} \).

Reduced fluctuation theorem for SET with information current \( I \)

\[
\lim_{t \to \infty} \frac{p_n(t)}{p_{-n}(t)} = \exp \left[ A n + I \times t \right], \quad I \times t \equiv \ln \frac{f_U^L f_R^U \Gamma_U^L \Gamma_R^L}{f_R^U f_L^U \Gamma_R^U \Gamma_L^R} n, \quad A \equiv \frac{\mu_L - \mu_R}{k_B T}.
\]

- Reduction to previous model (‘feedback by hand’):

\[
f_\alpha^U / f_\alpha = 1 \rightsquigarrow I \times t = (\ln \Gamma_L^U \Gamma_R^U / \Gamma_R^U \Gamma_L^R) n
\]
Maxwell demon limit

Energetics: first law

- Cycle between system energies:
  \[ \epsilon_d \rightarrow \epsilon_s + \epsilon_d + U \rightarrow \epsilon_s \rightarrow 0 \rightarrow \epsilon_d. \]
- Net energy \( U \) transferred from SET to detector.
Maxwell demon limit

Energetics: first law

For $\epsilon_S \gg U$, modification of first law $I_L^E + I_R^E = -I_D^E \approx 0$ negligible.
Maxwell demon limit

Summary of demon conditions

- Separation of time scales:
  - Fast demon $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.

- Separation of energy scales:
  - No back-action, precision:
    
    $k_B T \gg U \gg k_B T_D$.
  - Almost no work done:
    
    $\epsilon_S \gg U$.

- Spatial/energy lever condition
  - $\Gamma_\alpha \neq \Gamma_\alpha^U, \Gamma_L \neq \Gamma_R^U, \alpha = L,R$.

\[
\lim_{t \to \infty} \frac{p_n(t)}{p_{-n}(t)} \approx \exp \left[ \left( A + \ln \frac{\Gamma_L^U \Gamma_R^U}{\Gamma_R^U \Gamma_L} \right) n \right], \quad A \equiv \frac{\mu_L - \mu_R}{k_B T}.
\]
Maxwell demon limit

Where is the demon?
Maxwell demon limit

Where is the demon?

Maxwell’s demon

A feedback mechanism that shuffles particles against a chemical or thermal gradient by adjusting barriers without requiring work.

‘Hardwiring’ of the feedback mechanism.

‘Information’ is really physical: rates in term $\ln \left( \frac{\Gamma_L^U \Gamma_R}{\Gamma_R^U \Gamma_L} \right)$. 
Summary

- Transport master equations
  - Example: particle counting.
  - Moments, cumulants, generalized density operators.
  - Quantum dots.

- Feedback control
  - Introduction, various kinds of feedback control.
  - Wiseman-Milburn control.
  - Information and thermodynamics, Maxwell demon control.

Co-workers: G. Schaller (Berlin); C. Emary (Newcastle); C. Pöltl (Strasbourg); M. Esposito, P. Strasberg (Luxembourg)