PROPERTIES OF SHORT-WAVELENGTH OBLIQUE ALFVÉN AND SLOW WAVES

J. S. Zhao1,2,3, Y. Voitenko3, M. Y. Yu5,6, J. Y. Lu7, and D. J. Wu1

1 Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China; jszhao@pmo.ac.cn
2 Key Laboratory of Solar activity, National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China
3 Key Laboratory of Modern Astronomy and Astrophysics, Nanjing University, Nanjing 210093, China
4 Solar-Terrestrial Centre of Excellence, Space Physics Division, Belgian Institute for Space Aeronomy, Avenue Circulaire 3, B-1180 Brussels, Belgium
5 Institute for Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou 310027, China
6 Institute for Theoretical Physics I, Ruhr University, D-44780 Bochum, Germany
7 College of Math and Statistics, Nanjing University of Information Science and Technology, Nanjing 210044, China

Received 2014 May 2; accepted 2014 August 5; published 2014 September 12

ABSTRACT

Linear properties of kinetic Alfvén waves (KAWs) and kinetic slow waves (KSWs) are studied in the framework of two-fluid magnetohydrodynamics. We obtain the wave dispersion relations that are valid in a wide range of the wave frequency \( \omega \) and plasma-to-magnetic pressure ratio \( \beta \). The KAW frequency can reach and exceed the ion-cyclotron frequency at ion kinetic scales, whereas the KSW frequency remains sub-cyclotron. At \( \beta \sim 1 \), the plasma and magnetic pressure perturbations of both modes are in anti-phase, so that there is nearly no total pressure perturbations. However, these modes also exhibit several opposite properties. At high \( \beta \), the electric polarization ratios of KAWs and KSWs are opposite at the ion gyroradius scale, where KAWs are polarized in the sense of electron gyration (right-hand polarized) and KSWs are left-hand polarized. The magnetic helicity \( \sigma \sim 1 \) for KAWs and \( \sigma \sim -1 \) for KSWs, and the ion Alfvén ratio \( R_{Ai} \ll 1 \) for KAWs and \( R_{Ai} \gg 1 \) for KSWs. We also found transition wavenumbers where KAWs change their polarization from left-handed to right-handed. These new properties can be used to discriminate KAWs and KSWs when interpreting kinetic-scale electromagnetic fluctuations observed in various solar-terrestrial plasmas. This concerns, in particular, identification of modes responsible for kinetic-scale pressure-balanced fluctuations and turbulence in the solar wind.

Key words: magnetohydrodynamics (MHD) – plasmas – solar wind – turbulence – waves

Online-only material: color figure

1. INTRODUCTION

Kinetic Alfvén waves (KAWs) have recently been receiving much attention in connection to understanding turbulence at kinetic scales in the solar wind and the near-Earth space environment (Chaston et al. 2008; Podesta 2013). KAWs can be generated by the MHD Alfvénic turbulence through an anisotropic cascade (Howes et al. 2008; Bian et al. 2010; Zhao et al. 2013), or through non-local coupling of the MHD Alfvén waves (Zhao et al. 2011, 2014). These processes provide a pathway for the turbulence to dissipate via KAWs’ damping (Schekochihin et al. 2009). KAWs have been found in many in situ spacecraft measurements (Chaston et al. 2008, 2009; Huang et al. 2012; Podesta 2013). Identification of KAWs is usually accomplished by analyzing characteristic wave parameters, such as the ratio of electric to magnetic perturbations (Chaston et al. 2008, 2009), magnetic compressibility (Podesta & TenBarge 2012), magnetic helicity (Howes et al. 2010; Podesta et al. 2011; He et al. 2012), or the profile of wave dispersion (Sahraoui et al. 2009, 2010; Roberts et al. 2013).

On the other hand, kinetic slow waves (KSWs) have been found indirectly by analyzing the compressible turbulent fluctuations in the solar wind turbulence (Howes et al. 2012; Klein et al. 2012). KSWs have been used in the interpretation of recent observations of small-scale pressure balanced structures (PBSs) in the solar wind (Yao et al. 2011, 2013), which exhibit an anti-correlation between the plasma and the magnetic pressure fluctuations. Since such small-scale PBSs can also be associated with KAWs, it is of great interest to investigate in more detail properties of the KAW and KSW modes at parallel and perpendicular kinetic scales, especially their differences.

Hollweg (1999) derived a two-fluid dispersion equation for KAWs and KSWs, and investigated properties of low-frequency KAWs, \( \omega \ll \omega_{ci} \), where \( \omega_{ci} \) is the ion-cyclotron frequency. Another often made restricting assumption (see, e.g., Shukla & Stenflo 2000, and many others) was that the plasma beta \( \beta \ll 1 \) (\( \beta \) is the plasma/magnetic pressure ratio). These restrictions make the applicability of the obtained result to the solar wind, where \( \beta \) is often \( \sim 1 \) and the wave frequency \( \omega \) that can approach and exceed \( \omega_{ci} \) (Huang et al. 2012; Sahraoui et al. 2012), problematic. For the quasi-perpendicular Alfvén waves with frequencies reaching and extending above \( \omega/\omega_{ci} = 1 \), we will still use the same term KAW. The reason is that the wave dispersion and wave properties do not change much when the wave frequency crosses \( \omega_{ci} \).

In the present study we relax the two above restrictions and study KAWs and KSWs in a wide range of wave and plasma parameters. A two-fluid plasma model is used to simplify derivations of the wave dispersion and wave properties. The two-fluid plasma model has been proven to provide a good description for non-dissipative KAWs (Hollweg 1999; Bellan 2013). We suggest that the kinetic-scale PBSs observed in the solar wind can be interpreted not only in terms of KSWs, but also in terms of KAWs, and hence both these modes can contribute to PBSs. A more detailed analysis using the new mode properties we obtained in this paper is needed to reveal the dominant mode in every particular event.

In Section 2, we derive the dispersion relation of the waves for the two-fluid model. Sections 3 and 4 discuss the properties of KAWs and KSWs, respectively. A discussion and conclusion is given in Section 5. The detailed derivation of the general dispersion equation is presented in Appendix A, the wave polarization and correlation properties are given in Appendix B,
and Appendix C gives the analytic expressions of the linear wave dispersion relations and the linear responses in the low-β plasmas.

2. DISPERSION RELATION AND LINEAR RESPONSE

We shall start with the linear two-fluid equations

\[ m_α n_0 δ_v_α = n_0 q_0 (δ_E + δ_v_α × B_0) − ν P_α, \]

\[ δ_t δ_n_α = −∇ · (n_0 δ_v_α), \]

\[ \nabla × δ_B = μ_0 δ_J + \frac{1}{c^2} δ_t δ_E, \]

\[ \nabla × δ_E = −δ_t δ_B, \]

where the subscript \( α = i, e \) represents ions and electrons, respectively, \( m_α \) is the mass, \( q_0 \) is the charge, \( P_α = T_α δ_v_α \) is the thermal plasma pressure, \( T_α \) is the temperature, \( δ_v_α \) is the perturbed velocity, \( B_0 = B_0 \hat{e}_z \) is the uniform external magnetic field, \( δ_J \) is the perturbed current density, \( δ_E \) and \( δ_B \) are the electric and magnetic field perturbations, respectively, \( n_α = n_0 + δ_n_α \), and \( n_0 \) and \( δ_n_α \) are the background and perturbed number densities, respectively. We also assume \( ω_{pi}/ω_α \gg 1 \), so that the displacement current in Ampere’s law (3) can be neglected.

We consider all perturbed variables \( δf \) in the form of plane waves, \( δf ∝ δf_0 e^{−i(ωt + k · r)} \), where \( ω \) is the wave frequency and \( k \) is the wave vector. The waves are further assumed to propagate in the \((x, z)\) plane, that is, \( k = k_x \hat{e}_x + k_z \hat{e}_z \).

The derivation of the general dispersion equation (A17) is given in Appendix A. Using corresponding approximations, the dispersion equation (A17) can be reduced to previously derived equations (Shukla & Stenflo 2000; Zhao et al. 2010; Chen & Wu 2011). Note that the two-fluid MHD plasma theory has several limitations as compared to the kinetic plasma theory.

Most importantly, the two-fluid MHD cannot describe kinetic wave–particle interactions, like Landau damping, transit-time damping, and cyclotron damping. Also, some wave modes, like the ion Bernstein mode, can only be found in the kinetic theory. As a result, the highly oblique fast wave transforms into the ion Bernstein mode at \( ω \gg ω_{ci} \) in the kinetic theory, whereas in the fluid theory it continuously extends from \( ω < ω_{ci} \) to the electron cyclotron frequency \( ω \rightarrow ω_{ce} \) (Sahraoui et al. 2012). As far as the highly oblique KAWs are concerned, the wave properties given by the two-fluid MHD are consistent nearly with those given by the kinetic theory (Sahraoui et al. 2012; Hunana et al. 2013). In particular, the KAW dispersion relation is nearly the same in the two theories in the low-beta plasmas (Hunana et al. 2013). The frequency of the quasi-perpendicular slow mode also rises slowly as compared to the fast mode. Therefore, we focus on the quasi-perpendicular Alfvén and slow modes, but not the fast mode.

For quasi-perpendicular propagation, \( k_z \gg k_x \), the cubic dispersion equation in \( ω^2 \) (Equation (A17)) can be reduced to the quadratic equation for \( ω^2 \ll k^2 (V_f^2 + V_i^2) \):

\[ \omega^2 [1 + \lambda_α^2 k_x^2 + \lambda_e^2 k_z^2 + (1 + \lambda_α^2 k_z^2)^2 \beta] − ω^2 (1 + 2β + ρ^2 k_x^2) V_f^2 k_x^2 + β V_f^4 k_z^2 = 0, \]

which describes the dispersion relation of the Alfvén and slow waves, \( ω^2 ∼ V_f^2 k_x^2 \) and \( ω^2 ∼ V_i^2 k_z^2 \). Here \( β = V_f^2/V_i^2 \). The terms of the order \( Q = m_i/m_e \) or smaller are neglected in Equation (5). The straightforward solutions to this equation are

\[ ω^2 = \frac{V_f^2 k_z^2 (1 + 2β + ρ^2 k_x^2)}{2(1 + \lambda_α^2 k_x^2 + \lambda_e^2 k_z^2 + (1 + \lambda_α^2 k_z^2)^2 \beta)} \]

\[ \times \left[ 1 ± \sqrt{1 − 4β (1 + \lambda_α^2 k_x^2 + \lambda_e^2 k_z^2 + (1 + \lambda_α^2 k_z^2)^2 \beta) \left(1 + 2β + ρ^2 k_x^2\right)^2} \right] \]

where \( +^{+} \) stands for KAWs and \( −^{−} \) for KSWs. The dispersion relation derived by Hollweg (1999) is recovered from (6) in the low-frequency limit \( \lambda_α^2 k_z^2 \ll 1 \) (hence \( ω^2 \ll ω_{ci}^2 \)) and \( (1 + \lambda_α^2 k_z^2)(1 + β) \simeq 1 + β + \lambda_α^2 k_z^2 \).

Figure 1 compares numerical solutions of the general dispersion relation (A17) and the dispersion relations (6) obtained for the quasi-perpendicular wave propagation. At such oblique propagation, the fast mode has significantly higher frequencies than the other two modes. The remaining KAW and KSW modes behave differently. Similarly to the fast mode, the KAW mode frequency increases monotonously with increasing wavenumber and exceeds \( ω_{ci} \) at some wavelength close to the ion gyroradius. On the contrary, the KSW frequency slows down its increase above the ion gyroradius scale and never crosses \( ω = ω_{ci} \). From this figure one can see that the quasi-perpendicular dispersion relations (6) are also valid down to \( θ \sim 60^° \).

The physical quantities associated with the KAW and KSW dispersion relations can be easily obtained from Equations (A1)–(A6) and (A11)–(A15):

\[ \delta B = −i \frac{ω_{ci}}{ω} Y_1 δ B_\alpha \hat{e}_α + δ B_\alpha \hat{e}_α + \frac{k_y}{k_z} \frac{ω_{ci}}{ω} Y_1 δ B_\alpha \hat{e}_\alpha, \]

\[ δ E = \frac{ω}{k_z} \left(1 + \lambda_α^2 k_x^2 + Y_2 \frac{T_e}{T_i} \right) \bar{T}_i δ B_\alpha \hat{e}_α + i \frac{ω_{ci}}{k_x} Y_1 δ B_\alpha \hat{e}_\alpha \]

\[ + \frac{ω}{k_z} \left(1 + \lambda_α^2 k_x^2 + Y_2 \frac{T_e}{T_i} - \frac{1}{\bar{T}_i} \right) \bar{T}_e δ B_\alpha \hat{e}_\alpha, \]

Figure 1. Comparison of the dispersion relations (6) (thin dashed lines are for Alfvén waves; thick dot-dashed lines are for slow waves) with the wave modes obtained from the general dispersion equation (A17) (thick solid lines are for fast waves; thick dashed lines are for Alfvén waves; thick dot-dashed lines are for slow waves). The two propagation angles are \( θ = 60^° \) and \( 89^° \), and \( β = 0.1 \). (A color version of this figure is available in the online journal.)
The number density and the parallel magnetic field are related and solid lines represent the propagating angle temperatures, $T_e = T_i$, are used here and following figures.

These relations (7)–(11) can be used in the diagnostics of experimentally observed wave phenomena.

$\omega / \omega_{ci} = \gamma + 1$, where $\gamma = 2$ for extremely oblique propagation, $\theta = 89.9^\circ$. This is consistent with the result in Sahraoui et al. (2012) that $\omega / \omega_{ci} < 1$ at all scales as the propagating angle $\theta > \theta_{crit} = \cos^{-1}(Q) \approx 89.9^\circ$.

Figure 3 presents the electric polarization of KAWs. The waves are polarized elliptically (almost linearly in low-$\beta$ plasmas), $P_{SE, B_0} = \delta E_z / (i \delta E_x) < 1$. At relatively small $\rho k_\perp$, the polarization parameter is positive, $P_{SE, B_0} > 0$, in the kinetic and high-$\beta$ regimes, which corresponds to the right-hand polarization, as was first shown by Gary (1986) and Hollweg (1999). However, at larger $\rho k_\perp > 1$ there are several transition points where $\delta E_z$ passes through zero and $\delta E_z / \delta E_x$ and $\delta E_z / \delta E_x$ change their signs. These polarization reversals are discussed in more detail below.

Figures 4 and 5 show the magnetic polarization and the magnetic helicity $\sigma$ of KAWs. At the ion scale $\rho k_\perp \sim 1$ we observe quite small $i \delta B_z / \delta B_x \sim 0.01$, but the values of $i \delta B_z / \delta B_x$ are larger. For $\rho k_\perp \lesssim 1$ the KAW helicity is right-handed ($\sigma < 0$) in the inertial regime but becomes left-handed at larger $\beta$ (it also becomes left-handed in the inertial range at larger $\rho k_\perp$).

3. KAW PROPERTIES

KAWs behave differently in different $\beta$ regimes, namely, the inertial regime $\beta < m_e / m_i$, the kinetic regime $m_e / m_i < \beta < 1$, and the high-$\beta$ regime $\beta \gg 1$. Thus, we will investigate the KAW properties for the representative values $\beta = 10^{-4}$, typical in the Earth’s ionosphere and the solar flare loops, $\beta = 10^{-2}$, typical in the Earth’s magnetosphere and the solar corona, and $\beta = 1$, typical in the solar wind at $\sim 1$ AU.

From Figure 2 one can see that the KAW frequency is larger than the ion-cyclotron frequency, $\omega > \omega_{ci}$, when $k_\perp \rho = 1$ and $\theta = 87^\circ$ or $89^\circ$, but $\omega / \omega_{ci} < 1$ for extremely oblique propagation, $\theta = 89.9^\circ$. These relations (7)–(11) can be used in the diagnostics of experimentally observed wave phenomena.
Figure 3. Electric polarization ratios $\delta E_y/(i\delta E_x)$ and $\delta E_z/\delta E_x$ for KAWs. The dotted, dashed, and solid lines represent the propagating angles $\theta = 87^\circ$, $89^\circ$, and $89.99^\circ$, respectively.

Figure 4. Magnetic polarization ratios $i\delta B_x/\delta B_y$ and $i\delta B_z/\delta B_y$ for KAWs. The dotted, dashed, and solid lines represent the wave propagation angles $\theta = 87^\circ$, $89^\circ$, and $89.99^\circ$, respectively.
...ion cross-helicity becomes nearly zero, \( \delta v_i \) velocity mismatch appears, Figure 6. The Astrophysical Journal

...observe the pressure balance \( \theta \) oblique KAWs (\( \lambda \)ek) = \( \beta \) low changes its sign, which occur when \( \rho k_\perp \) = \( 10^{−4} \) at arbitrary \( \beta \). \( C_{PBn} \approx −1 \) also holds for the arbitrary propagation angle in the high-\( \beta \) plasmas. Note several transition points in the inertial regime where \( C_{PBn} \) changes its sign, which occur when \( k_\perp^2/k_i^2 = (1 + \lambda^2 k_i^2) \beta = 0 \). \( C_{PBn} \) nearly follows the approximate expression \( C_{PBn} \approx \rho^2 k_\perp^2 (1 + \lambda^2 k_i^2 + \lambda^4 k_i^4) / \beta (1 + \rho^2 k_\perp^2) \), which is valid for the low \( \beta \).

Figure 7 presents the ion Alfvén ratio \( R_{Ai} \) and the ion cross helicity \( \sigma_{Ci} \) of KAWs. \( R_{Ai} \) and \( \sigma_{Ci} \) can be rewritten as \( R_{Ai} = \delta v_\perp / \delta v_\parallel \) and \( \sigma_{Ci} = 2(\delta v_\perp \cdot \delta v_\parallel) / (\delta v_\parallel^2 + \delta v_\perp^2) \), where \( \delta v_\parallel \) is the magnetic perturbation in units of Alfvén speed. The ion cross-helicity becomes nearly zero, \( \sigma_{Ci} \approx 0 \), as the strong velocity mismatch appears, \( \delta v_i \gg \delta v_B \) (for the kinetic scale \( \lambda \)ek, \( \rho k_\perp \gg 1 \) in the inertial regime) or \( \delta v_B \gg \delta v_i \) (for KAWs with \( \rho k_\perp > 1 \) in the high-\( \beta \) plasmas).

In the following sub-sections we consider the electric field polarization and its reversal in more detail.

3.1. Electric Polarization and Its Reversal

In the limit \( k_\perp \gg k_z \), from expression (8) we get the polarization ratio

\[
\frac{\delta E_\parallel}{\delta E_\perp} = \frac{1}{\omega_{ci} \lambda^2 k_\perp^2} \frac{(1 + \lambda^2 k_i^2) \omega^2}{\nu^2 k_i^2} - 1,
\]

(12)

The sense of the wave polarization can change when the numerator or denominator of this expression passes through zero, which corresponds to \( \delta E_\parallel = 0 \) or \( \delta E_\perp = 0 \), respectively.
The $\delta E_x = 0$ transition occurs at

$$\left( \frac{\omega^2}{\omega_c^2} - 1 \right) = \frac{T_i}{T_e} \left( 1 + \lambda_z^2 k_z^2 \right) \frac{\omega^2}{V_A^2 k_z^2} > 0,$$

which implies high-frequency $\omega > \omega_c$ waves at the transition point. For the low-frequency waves ($\omega \ll \omega_c$), there are no such transition points. In the low-$\beta$ limit this transition can occur as

$$\lambda_z^2 k_z^2 = \frac{\left( 1 + \lambda_z^2 k_z^2 \right) \left( 1 + \rho^2 k_z^2 \right) T_i}{\rho^2 k_z^2}.$$  \hspace{1cm} (14)

The $\delta E_y = 0$ transition occurs at

$$\frac{\omega^2}{V_A^2 k_z^2} = \frac{1}{1 + \lambda_z^2 k_z^2} < 1.$$  \hspace{1cm} (15)

This transition implies sub-Alfvénic phase velocities of KAWs and is possible only due to finite $\lambda_z^2 k_z^2$. In the low-$\beta$ limit (15) gives the transition wavenumber

$$\rho^2 k_z^2 = \frac{m_i}{m_e} \left( \frac{1}{\tan^2 \theta} - \beta \right).$$  \hspace{1cm} (16)

With growing $\rho^2 k_z^2$, transition from left- to right-hand polarization occurs. For this transition to occur, the wave propagation angle should be less than a certain value,

$$\theta < \theta_{ct} = \arctan \frac{1}{\sqrt{\beta}}.$$  \hspace{1cm} (17)

Otherwise, the wave is always right-hand polarized. Only in this last case does the conclusion by Gary (1986) and Hollweg (1999) hold that KAWs are right-hand polarized.

The above analysis indicates that KAWs can be both left- and right-hand polarized.

### 3.2. The Low-$\beta$ Low-frequency Limit

For KAWs in low-$\beta$ plasmas, $\beta \ll 1$, we get

$$\frac{\delta E_y}{\delta E_x} = i \beta \frac{\omega}{\omega_c} \frac{1}{1 + \rho^2 k_z^2} \left( 1 + \lambda_z^2 k_z^2 \right) \frac{1}{\left( \beta + \frac{m_i}{m_e} \rho^2 k_z^2 \tan^2 \theta \right)},$$

\hspace{1cm} (18)

At small $\lambda_z^2 k_z^2 \ll 1$ (i.e., in the low-frequency range) the denominator of the above expression is dominated by the term $(1 + \lambda_z^2 k_z^2)$, and we arrive at

$$\frac{\delta E_y}{\delta E_x} = i \beta \frac{\omega}{\omega_c} \frac{1}{1 + \rho^2 k_z^2} \frac{1}{\left( \beta + \frac{m_i}{m_e} \rho^2 k_z^2 \tan^2 \theta \right)}.$$  \hspace{1cm} (19)

which indicates that the electric polarization depends on the magnitude of $k_z$ compared to the ion kinetic scales. In the limit $\beta \tan^2 \theta \gg 1$, our expression (19) simplifies to

$$\frac{\delta E_y}{\delta E_x} = i \beta \frac{\omega}{\omega_c} \frac{1}{1 + \rho^2 k_z^2}.$$  \hspace{1cm} (20)

In this limit the waves are always right-hand polarized. In principle, this conclusion agrees with previous results (Gary 1986; Hollweg 1999). In the long-wavelength limit $\rho^2 k_z^2 \ll 1$, Equation (19) reduces to the approximate analytical formula Equation (46) by Hollweg (1999).

$$\frac{\delta E_x}{\delta E_y} = i \beta \frac{\omega}{\omega_c}.$$  \hspace{1cm} (21)

### 4. KSW Properties

Before discussing properties of the oblique slow waves in the two-fluid MHD, it is instructive to mention some of their known properties in the kinetic theory. For general oblique
propagation, the slow/sound wave extends to the frequency larger than the ion-cyclotron frequency as shown in Figure 1 of Krauss-Varban et al. (1994) for the propagation angle $\theta = 30^\circ$. At larger propagation angles, large wavenumbers are required for the waves to reach the ion-cyclotron frequency, and for quasi-perpendicular propagation slow waves remain sub-cyclotron in the wide range of perpendicular wavenumbers, up to large values of $\rho^{-1}k_{\perp}^2$. In the two-fluid MHD, the frequency of quasi-perpendicular KSWs always remains sub-cyclotron:

$$\frac{\omega}{\omega_{ci}} = \frac{k_{\perp}}{\rho} \sqrt{\frac{\rho^2k_{\perp}^2}{1 + \rho^2k_{\perp}^2}} < 1 \quad \text{for} \quad k_{\perp}/\rho \leq 1.$$

The KSW dispersion is shown in Figure 8 in terms of the phase velocity $\omega/(V_T k_{\perp})$, which also exhibits a depression at high $\beta$ in the long-wavelength limit.

Figure 9 presents the electric polarization of KSWs. The polarization parameter $P_{E_B} = E_y/(i\delta E_x)$ < 0 means the left-hand KSW polarization. $\delta E_x$ becomes the dominant component for the waves at the ion gyroradius scale ($\rho k_{\perp} \sim 1$). The KSW electric field polarization ratios can be approximated as $i\delta E_y/\delta E_x \simeq \beta k_{\perp}/(\rho k_{\perp}^2 \sqrt{1 + \rho^2k_{\perp}^2})$, and $\delta E_z/\delta E_x = k_{\perp}/k_{\perp}$.

Figure 10 presents the magnetic polarization of KSWs. For the long-wavelength waves ($\rho k_{\perp} \ll 1$), the compressible magnetic field perturbation $\delta B_z$ is the dominant component, $|\delta B_x/\delta B_z| \gg 1$. However, $\delta B_z$ becomes as important as $\delta B_x$ when the wavelength approaches the ion gyroradius scale, where $\delta B_x \simeq \delta B_z \gg \delta B_y$. The magnetic field polarization ratios nearly follow the approximate relations $i\delta B_x/\delta B_z = -(k_{\perp}/k_{\perp})\sqrt{1 + 1/\rho^2k_{\perp}^2}$ and $i\delta B_z/\delta B_y = \sqrt{1 + 1/\rho^2k_{\perp}^2}$.

The helicity $\sigma$ in the low-$\beta$ plasmas can be written as $\sigma = -2\rho k_{\perp}(1 + \rho^2k_{\perp}^2)^{1/2}/(1 + 2\rho^2k_{\perp}^2)$, which indicates that $\sigma$ decreases from 0 to $-$1 as $\rho k_{\perp}$ increases from $10^{-3}$ to 10. This behavior is seen from Figure 11.

Figure 12 presents the plasma/magnetic pressure correlation $C_{PBn}$ and the compressibility $C_{Bn}$ of KSWs. The behavior of these functions is in accordance with the theoretical predictions that $C_{PBn} \simeq -1$ and $C_{Bn} \simeq (1 + \rho^2k_{\perp}^2)/(1 + 2\rho^2k_{\perp}^2)/\beta^2$ in the low-$\beta$ plasma. Note that $C_{PBn}$ and $C_{Bn}$ in the high-$\beta$ plasmas can be approximated by the same expressions.

Figure 13 presents the ion Alfvén ratio $R_{bi}$ and the cross-helicity $\sigma_{ci}$ of KSWs. $\delta v_{ij}$ is nearly equal $\delta v_B$ in the long-wavelength waves $(\rho k_{\perp} \ll 1)$ in high-$\beta$ plasmas. In other cases $\delta v_i$ dominates over $\delta v_B$, $\delta v_i > \delta v_B$. The corresponding
expressions in the low-\(\beta\) plasmas are \(R_{\text{Ai}} \simeq (1 + 2\beta^2k_\perp^2) / \beta\) and \(\sigma_{\text{ci}} \simeq -2\sqrt{\beta / (1 + \beta^2k_\perp^2)}\).

5. DISCUSSION AND CONCLUSION

Our study shows that the Alfvén wave frequency \(\omega\) at the ion gyroscale \(\rho k_\perp \sim 1\) is smaller than the ion-cyclotron frequency \(\omega_{\text{ci}}\) for extremely oblique propagation, say for \(\theta = 89.99\). In agreement with Sahraoui et al. (2012), the KAW frequency can reach \(\omega_{\text{ci}}\) for propagation angle \(\theta \leq 89.97\). At \(\rho k_\perp \sim 1\), \(\omega\) can reach and exceed \(\omega_{\text{ci}}\) for the propagation angles \(\sim 87^\circ\) or less. At smaller propagating angles, the frequency \(\omega \sim \omega_{\text{ci}}\) occurs at smaller wavenumbers. In the solar-terrestrial plasmas, the high-frequency KAWs may be generated through the Alfvénic turbulent cascade (Huang et al. 2012), excited kinetically by the field-aligned currents and ion beams (Voitenko & Goossens 2002, 2003), or by phase mixing combined with cyclotron sweep of Alfvén waves (Voitenko & Goossens 2006).

Our study investigated KAWs and KSWs in a wide range of \(\beta\), covering a wide range of conditions in most solar-terrestrial plasmas. Several mode properties of KAWs are totally different in the limits \(\beta < m_e/m_i\) (so-called inertial range) and \(\beta > m_e/m_i\) (kinetic range). So, we confirmed that the KAW phase velocity \(\omega/k_\perp\) in the inertial range decreases with increasing \(k_\perp\), but increases in the kinetic and high-\(\beta\) ranges (Goertz & Boswell 1979; Lysak & Lotko 1996; Stasiwicz et al. 2000). The electric polarization ratios of KAWs are also obviously different in these two distinct \(\beta\) ranges. Namely, except for the case of extremely oblique waves, at smaller \(k_\perp\) KAWs are left-hand polarized in the inertial range and right-hand polarized in the kinetic/high-\(\beta\) ranges. The magnetic helicity at these wavenumbers is right-handed for \(\beta < m_e/m_i\) and left-handed for \(\beta > m_e/m_i\). At higher \(k_\perp\) they undergo two polarization reversals defined by (13) and (15). These new polarization properties of KAWs, especially polarization reversals, are quite specific and can be used as a critical test for mode identification in the solar wind and terrestrial magnetosphere. In addition to the Landau dissipation caused by the parallel electric field fluctuations of KAWs, the perpendicular electric field can induce the occurrence of the cyclotron resonant damping as the frequency of KAWs reaches or exceeds the ion-cyclotron frequency (Voitenko & Goossens 2002, 2003). The cyclotron-resonant damping (Kennel & Wong 1967; Marsch 2006) does not require purely left-hand or purely right-hand polarizations of Alfvén waves. When the resonant-cyclotron condition \(\omega(k_\perp^2 - k_x^2)v_z = n\omega_{\text{ci}}\) is satisfied for the oblique waves, there are two resonance cases depending on the interaction with \(E_x\) or \(E_y\) (Hollweg & Markovskii 2002), where \(v_z\) is the parallel velocity of particles. \(E_x\) can cause cyclotron resonance if the particles stay in the phase with the waves, whereas the resonance relating to \(E_y\) strongly depends on the \(x\)-position of the particles (Hollweg & Markovskii 2002). Both Landau and cyclotron dampings are crucial when investigating the turbulence dissipation channels at kinetic scales. On the contrary, the KSW properties are nearly the same in all \(\beta\) ranges and can be approximately described by the approximate expressions for the low-\(\beta\) plasmas given in Section 4.

Our study provided a clear evidence of anti-correlation between the plasma and magnetic pressures for both KAWs and KSWs in high-\(\beta\) plasmas. This makes the total pressure fluctuations almost zero, \(\delta P_{\text{tot}} \simeq 0\), which resembles the main common property of the observed PBSs (Yao et al. 2011). Kellogg & Horbury (2005) and Yao et al. (2011) used the Cluster data, which have a high time resolution of 0.2 s for
the plasma number density and magnetic field, and found that the scales of PBSs extend down to the ion scale. Kellogg & Horbury (2005) interpreted these kinetic-scale PBSs in terms of KSWs. However, KAWs may be an alternative explanation since they also drive anti-correlated number density and magnetic field fluctuations. Again, one needs more mode properties to discriminate which mode dominates in PBSs, KSW or KAW.

Our results reveal three different properties that distinguish these modes at the ion gyroradius scale: (1) right-hand electric polarization for KAWs and left-hand for KSWs; (2) magnetic helicity $\sigma \sim 1$ for KAWs and $\sigma \sim -1$ for KSWs; (3) Alfvén ratio $R_{AI} \ll 1$ for KAWs and $R_{AI} \gg 1$ for KSWs. Besides, the large-scale Alfvén waves, permeating the solar wind, can nonlinearly excite simultaneous KSWs and KAWs (Zhao et al. 2014), which both can contribute to the observed PBSs. This implies that PBSs may be KSWs, or KAWs, or a mixture of KSWs and KAWs. A recent work (Hollweg et al. 2014) showed that the highly oblique slow mode has a small variation of $P_{tot}$ in the three-fluid plasmas consisting of fully ionized hydrogen and a heavy ion drifting along the background magnetic field. Therefore, one needs additional tests for a more careful identification of the wave modes producing the observed kinetic-scale PBSs.

In summary, we revealed several new mode properties of KAWs and KSWs accounting for the kinetic effects of the ion and electron thermal pressure and inertia. For KAWs, their frequency can reach and exceed the ion-cyclotron frequency at the ion kinetic scales, where both the thermal and the inertial ion effects are important. The polarization properties of KAWs are different in different $\beta$ ranges and depend on both the propagation angle $\theta$ and the normalized perpendicular wavenumber $\rho k_\perp$. It appeared that KAWs undergo two reversals of electric polarization defined by the zeros of the denominator (13) and the numerator (15) of (12). In particular, in low-$\beta$ plasmas less oblique KAWs (17) are left-hand polarized at longer wavelengths and right-hand polarized at shorter wavelengths. These properties are important for the turbulence cascade transition across the ion-cyclotron frequency, where it can be partially dissipated by the ion-cyclotron resonance, in such a way that the left-hand KAWs possess stronger dissipation as compared to the right-hand ones.

For KSWs, its frequency is always smaller than ion-cyclotron frequency and the mode is left-hand polarized. At the ion kinetic scales, $\rho k_\perp \sim 1$, the electric and magnetic KSW components obey $|\delta E_y| \ll |\delta E_x|$, $|\delta B_x| \ll |\delta B_y|$, and $|\delta B_z| \sim |\delta B_y|$. All these properties of KSWs can be described approximately by the reduced expression obtained in the low-$\beta$ limit.

These new properties are important for understanding short-wavelength Alfvén and slow modes and can be used in interpreting waves and turbulence at kinetic scales.
This research was supported by the Belgian Federal Science Policy Office via IAP Programme (project P7/08 CHARMM), by the European Commission FP7 Program (project 313038 STORM), by NSFC under grant nos. 11303099, 11373070, and 41074107, by MSTC under grant no. 2011CB811402, by NSF of Jiangsu Province under grant no. BK2012495, and by Key Laboratory of Solar Activity at NAO, CAS, under grant No. KLSA201304.

APPENDIX A

THE GENERAL DISPERSION EQUATION

From the momentum Equation (1), the ion and electron velocities are found as

\[ \Lambda_i \delta v_{ix} = -i \frac{e}{m_{i,0} \omega_{ci}} \delta E_x + \frac{1}{B_0} \delta E_y - \frac{\gamma T_i \omega k_{i,E}}{m_{i,0} \omega_{ci}} \delta \eta_i, \]  
\[ \Lambda_e \delta v_{ey} = - \frac{1}{B_0} \delta E_x - i \frac{e}{m_{e,0} \omega_{ce}} \delta E_y + \frac{\gamma T_e \omega k_{e,E}}{m_{e,0} \omega_{ce}} \delta \eta_e, \]
\[ \delta v_{dz} = i \frac{e}{m_{e,0} \omega_{ce}} \delta E_z + \frac{\gamma T_e k_{e,E}}{m_{e,0} \omega_{ce}} \delta \eta_e, \]  
\[ \Lambda_i \delta v_{ex} = i \frac{Q_i \omega}{B_0 \omega_{ci}} \delta E_x + \frac{1}{B_0} \delta E_y - \frac{\gamma T_i \omega k_{i,E}}{m_{i,0} \omega_{ci}} \delta \eta_i, \]
\[ \Lambda_e \delta v_{ey} = - \frac{1}{B_0} \delta E_x + i \frac{Q_e \omega}{B_0 \omega_{ce}} \delta E_y - \frac{\gamma T_e \omega k_{e,E}}{m_{e,0} \omega_{ce}} \delta \eta_e, \]
\[ \delta v_{ez} = -i \frac{e}{m_{e,0} \omega_{ce}} \delta E_z + \frac{\gamma T_e k_{e,E}}{m_{e,0} \omega_{ce}} \delta \eta_e, \]  
where \( Q \equiv m_e/m_i, \Lambda_i \equiv 1 - \omega^2/\omega_{ci}^2, \) and \( \Lambda_e \equiv 1 - Q^2 \omega^2/\omega_{ce}^2. \) Note that the quasi-neutrality condition, \( \delta n_i = \delta n_e \equiv \delta n, \) has been used in the last derivation. Using expressions (A1)–(A6), the current density \( \mathbf{j} = n_0 e (\mathbf{v}_i - \mathbf{v}_e) \) can be presented in the following form:

\[ \Lambda_i \Lambda_e, \delta J_z = -i \frac{n_0 e \omega}{B_0 \omega_{ci}} (\Lambda_i - Q \Lambda_e) \delta E_z + \frac{n_0 e}{B_0} (\Lambda_e - \Lambda_i) \delta E_y - \frac{\gamma T_i \omega k_{i,E}}{B_0} (\Lambda_i \delta \eta_i - Q \Lambda_e \delta \eta_e), \]
\[ \Lambda_e \Lambda_i, \delta J_y = - \frac{n_0 e}{B_0} (\Lambda_e - \Lambda_i) \delta E_y - i \frac{n_0 e \omega}{B_0 \omega_{ce}} (\Lambda_e + Q \Lambda_i) \delta E_y + i \frac{\gamma T_e \omega k_{e,E}}{B_0} (\Lambda_e \delta \eta_i + Q \Lambda_i \delta \eta_e), \]
\[ \delta J_z = i (1 + Q) \frac{n_0 e^2}{m_{e,0} \omega_{ce}} \delta E_z - \frac{e \gamma T_e k_{e,E}}{m_{e,0} \omega_{ce}} (\delta \eta_e - Q \delta \eta_i), \]
\[ \delta J_y = - \frac{i e}{m_{e,0} \omega_{ce}} \delta E_z + \frac{\gamma T_e k_{e,E}}{m_{e,0} \omega_{ce}} \delta \eta_e, \]
\[ \delta J_z = -i \frac{k_{z,E}}{\mu_0 \omega} \delta E_z - i \frac{k_{z,E}}{\mu_0 \omega} \delta E_z, \]  
\[ \delta J_y = - \frac{i k_{z,E}}{\mu_0 \omega} \delta E_z, \]  
where \( T = T_i + T_e \) and \( \tilde{T}_i, \tilde{T}_e \equiv T_i / T. \)

On the other hand, the current density can be expressed in terms of the perturbed electric field only,

\[ \delta J_z = i \frac{k_{z,E}}{\mu_0 \omega} \delta E_z - i \frac{k_{z,E}}{\mu_0 \omega} \delta E_z, \]
\[ \delta J_y = - \frac{i k_{z,E}}{\mu_0 \omega} \delta E_z, \]
\[ \delta J_x = i \frac{k_{x,E}}{\mu_0 \omega} \delta E_x - i \frac{k_{x,E}}{\mu_0 \omega} \delta E_x. \]  

From Equations (A1)–(A12), the electric field components can be expressed in terms of \( \delta n: \)

\[ \Pi \delta E_x = i k_{x,E} \frac{\gamma T}{e} \Pi \frac{\delta n}{n_0}, \]  
\[ \Pi \delta E_y = k_{y,E} \frac{\gamma T}{e} \Pi \frac{\delta n}{n_0}, \]  
\[ \Pi \delta E_z = i k_{z,E} \frac{\gamma T}{e} \Pi \frac{\delta n}{n_0}, \]  
where

\[ \Pi = (1 + Q)^2 (1 + Q + \lambda^2 k_z^2) \rho^4 - (1 + Q) \left[ 1 + Q + \lambda^2 k_z^2 \delta n \right] A_n V_n^2 k^2 \]  
\[ + (1 + Q) \lambda T_i / V_n, \]
\[ \Pi_x = (1 + Q) (1 + Q + \lambda^2 k_z^2 \delta n) - \left[ (1 + Q + \lambda^2 k_z^2 \delta n) \Lambda_i \delta \eta_i - Q \Lambda_e \delta \eta_e \right] \]  
\[ + (1 + Q) \lambda T_i (\delta \eta_i - Q \delta \eta_e) - (1 + Q) \lambda T_i (\delta \eta_i - Q \delta \eta_e), \]
\[ \Pi_y = (1 + Q) \left[ 1 + Q + \lambda^2 k_z^2 \delta n \right] \omega_{ci} \]  
\[ + (1 + Q) \lambda T_i (\delta \eta_i - Q \delta \eta_e), \]
\[ \Pi_z = (1 + Q) (1 + Q + \lambda^2 k_z^2 \delta n) - \lambda \Lambda_i V_n^2 k^2 \omega_{ci} \]  
\[ + (1 + Q)^2 V_n^2 k^2 \omega_{ci}, \]
\[ \Lambda_i \equiv 1 - Q^2 \omega^2 / \omega_{ci}^2. \]

Now we can use expressions (A13)–(A15) to eliminate the electric field from the number density equation

\[ \left[ \Delta \lambda_i + \rho_T^2 k_z^2 \right] \rho^2 - \Delta_i V_n^2 k_z^2 \delta n = -i \frac{e \omega k_z}{B_0 \omega_{ci}} \delta E_x + \frac{e \omega k_z}{B_0} \delta E_y \]  
\[ + i \frac{e}{m_i} \lambda T_i \delta E_z, \]
\[ (A16) \]

which results in the general dispersion equation:

\[ \omega^6 (1 + Q) (1 + Q + \lambda^2 k_z^2)^2 \]  
\[ - \omega^4 \left[ (1 + Q) (1 + Q + \lambda^2 k_z^2) + (1 + Q + \lambda^2 k_z^2)^2 V_n^2 / V_A^2 \right] \]  
\[ + (1 + Q^3) \lambda^2 k_z^2 + (1 + Q^2) k_z^2 \]  
\[ + \omega^2 \left[ (1 + Q) (1 + Q^2 V_n^2 / V_A^2) + (1 + Q^2) \rho^2 k_z^2 \right] \]  
\[ - \beta V_n^2 k_z^2 k^4 = 0, \]  
\[ (A17) \]

where \( V_T = \sqrt{T/m_i}, \rho^2 = \rho_i^2 + \rho_e^2, \) \( \rho_i \) is the ion gyroradius, \( \rho_e \) is the electron gyroradius, and \( \lambda_i \) is the ion inertial length.

In deriving Equation (A17), we neglected the displacement current in Ampère’s law, but kept all other terms. Also, we treated the electrons and ions separately. This makes our derivation and results different from the derivations by Stringer (1963) and Bellan (2012). The above two authors used equations of the mass motion and the generalized Ohm’s law (Equations (A1) and (A2) in Stringer (1963)) with one-fluid variables \( \rho = \sum_{a=i,e} m_a n_a \) and \( \mathbf{v} = \sum_{a=i,e} m_a n_a \mathbf{v}_a / \sum_{a=i,e} m_a n_a \)
where some terms of order $Q$ were discarded. The resulting general dispersion equation is (Stringer 1963)

$$
\omega^6 \left( 1 + \lambda^2 k^2 \right)^2 - \omega^4 \left[ (1 + \lambda^2 k^2) + (1 + \lambda^2 k^2)^2 \right] V_A^2 / V_A^4 \\
+ (1 + 2Q) \lambda^2 k^2 + k^2 / k^2 V_A^2 k^2 \\
+ \omega^2 \left[ (1 + 2V_A^2 / V_A^2) + (1 + 2Q) \rho^2 k^2 \right] V_A^4 k^2 k^2 \\
- \beta V_A^2 k^2 k^2 = 0.
$$

(A18)

Some terms in our expression (A17) and in Stringer’s expression (A18) are different. The differences come from the different treatment of some minor terms $\sim Q$: all small terms are kept in our derivation but an incomplete set of terms was used by Stringer (1963). Since the major terms in Equations (A17) and (A18) are the same, the resulting dispersion relations for the fast, Alfvén, and slow modes are also nearly the same. However, behavior of some polarization ratios differ significantly.

**APPENDIX B**

**POLARIZATION AND CORRELATION**

The wave properties involving wave polarization and correlation are summarized in Krauss-Varban et al. (1994). Here we repeat these definitions for convenient discussion. The electric field polarization with respect to the ambient magnetic field is defined as

$$
P_{E,B_a} = \frac{\delta E_y}{i \delta E_x}.
$$

(B1)

The right-hand polarized mode corresponds to $\text{Re}(P_{E,B_a}) > 0$, and the left-hand polarized mode corresponds to $\text{Re}(P_{E,B_a}) < 0$. $\text{Im}(P_{E,B_a}) = \pm 1$ correspond to the right- or left-hand circularly polarized mode. Note that the definition $P_{E,B_a} = i \delta E_x / \delta E_y$ is used in Krauss-Varban et al. (1994).

The magnetic field polarization with respect to the ambient magnetic field is defined as

$$
P_{B,B_a} = \frac{\delta B_y}{i \delta B_x}.
$$

(B2)

and the magnetic field polarization with respect to the wave vector is

$$
P_{B,k} = \frac{i \delta B \cdot (\hat{z} \times \hat{e}_z)}{\delta B_y} = \frac{1}{P_{B,B_a} \cos \theta}.
$$

(B3)

The magnetic helicity is expressed as

$$
\sigma = \frac{k (A \cdot \delta B^2)}{\delta B^2} = \frac{2 \text{Re}(P_{B,k})}{1 + |P_{B,k}|^2},
$$

(B4)

where positive or negative helicity corresponds to a left- or right-hand sense of rotation with respect to $k$ (Gary 1986), respectively.

The magnetic field–density correlation corresponds to

$$
C_1 = \frac{\delta n / n_0}{\delta B / B_0}.
$$

(B5)

Correspondingly, the thermal-pressure–magnetic-pressure correlation is defined as

$$
C_{P_Bn} = \frac{\gamma (T_i + T_e) \delta n}{\delta B / B_0 / \mu_0} = \beta C_1.
$$

(B6)

hence, the total pressure perturbation is written as $\delta P_{tot} = (1 + \beta C_1) \delta B / B_0 / \mu_0$.

Compressibility,

$$
C_{Bn} = \frac{\delta n^2 / n_0^2}{\delta B^2 / B_0^2} = \sin^2 \theta |C_1|^2 \frac{|P_{BB,k}|^2}{1 + |P_{BB,k}|^2},
$$

(B7)

describes the relation between the total magnetic field perturbation and the number density.

The Alfvén ratio and cross helicity for $j$ species are defined as

$$
R_A^j = \frac{\mu_0 n_0 m_i |\delta v_j|^2}{|B|^2},
$$

(B8)

$$
\sigma_C^j = 2 \frac{\mu_0 n_0 m_i |\delta v_j \cdot \delta B^*|}{\mu_0 n_0 m_i |\delta v_j|^2 + |\delta B|^2},
$$

(B9)

which gives the correlation between the perturbed velocity and magnetic field.

**APPENDIX C**

**LINEAR DISPERSION AND WAVE PARAMETERS IN THE LOW-$\beta$ PLASMA**

In the low-beta plasma, $\beta \ll 1$, the linear dispersion and relations among field and plasma quantities are simpler than (6)–(7). For KAWs, the dispersion relation

$$
\omega^2 = V_A^2 k^2 \mathcal{R} / \mathcal{L}',
$$

(C1)

and

$$
\delta \mathcal{E} / \mathcal{V}_A = \frac{R_i \mathcal{L} - \rho^2 \lambda^2 \mathcal{L}^2}{\mathcal{R}^{1/2} \mathcal{L}'^{1/2}} \delta B \mathcal{e}_x + i \frac{k_z}{k_\perp} \left( \mathcal{L} \delta \mathcal{R}^2 - \mathcal{L} \delta \mathcal{L}^2 \right) / \mathcal{R}^{1/2} \mathcal{L}'^{1/2} \delta B \mathcal{e}_y
$$

$$
\delta B = -i \frac{k_z}{k_\perp} \left( \lambda^2 k_\perp^2 - \lambda^2 k_\parallel^2 \right) \delta B \mathcal{e}_z + i \frac{\mathcal{L} \delta \mathcal{R}^2 - \mathcal{L} \delta \mathcal{L}^2}{\mathcal{R}^{1/2} \mathcal{L}'^{1/2}} \delta B \mathcal{e}_x
$$

$$
\delta \mathcal{V}_j / \mathcal{V}_A = -i \lambda \frac{k_z}{k_\perp} \delta B \mathcal{e}_x - \frac{\mathcal{R}^{1/2} \delta B \mathcal{e}_y}{\mathcal{R}^{1/2} \mathcal{L}'^{1/2}}
$$

$$
- \frac{i}{\lambda} \frac{k_z}{k_\perp} \mathcal{R} \lambda \frac{k_\perp}{k_\parallel} \delta B \mathcal{e}_z
$$

$$
\delta \mathcal{V}_e / \mathcal{V}_A = \frac{k_z}{k_\perp} \left( \delta \mathcal{R}^2 - \delta \mathcal{L}^2 \right) \delta B \mathcal{e}_x - \mathcal{R} \lambda \frac{k_\perp}{k_\parallel} \delta B \mathcal{e}_y
$$

$$
- i \lambda \frac{k_z}{k_\perp} \delta B \mathcal{e}_z,
$$

(C2)

where

$$
\mathcal{R} = 1 + \rho^2 k_\perp^2, \quad \mathcal{R}_i = 1 + \rho_i^2 k_\perp^2, \quad \mathcal{L} = 1 + \lambda^2 k_\perp^2,
$$

$$
\mathcal{L}' = 1 + \lambda_i^2 k_\perp^2 + \lambda_i^2 k_\parallel^2.
$$

For KSWs, the dispersion relation

$$
\omega^2 = V_A^2 k^2 / \mathcal{R},
$$

(C3)
and

\[
\frac{\delta E}{V_T} = \frac{L \tilde{T}_i + R \tilde{T}_e/\beta}{R^{1/2}} \delta B_y \hat{e}_x - i \frac{k_z}{\rho k^2} \delta B_y \hat{e}_y + \frac{k_z}{k_\perp} \frac{L \tilde{T}_i + R \tilde{T}_e/\beta}{R^{1/2}} \delta B_y \hat{e}_z,
\]

\[
\frac{\delta B}{\rho k^2} = i \frac{k_z}{\rho k^2} \frac{R^{1/2}}{\beta} \delta B_y \hat{e}_x + \delta B_y \hat{e}_y - \frac{R^{1/2}}{\beta} \delta B_y \hat{e}_z,
\]

\[
\frac{\delta v_i}{V_T} = -i \frac{k_z}{\rho k^2} \left(1 + \frac{k^2}{k_\perp} \right) \delta B_y \hat{e}_x - \frac{R^{1/2}}{\beta} \delta B_y \hat{e}_y + i \frac{R}{\beta \rho k^2} \delta B_y \hat{e}_z,
\]

\[
\frac{\delta v_e}{V_T} = -i \frac{k_z}{\rho k^2} \delta B_y \hat{e}_x - \frac{L}{R^{1/2}} \delta B_y \hat{e}_y + i \frac{1}{\beta \rho k^2} \delta B_y \hat{e}_z. \quad (C4)
\]

REFERENCES

Bellan, P. M. 2012, JGR, 117, A12219
Bellan, P. M. 2013, JGR, 118, 4435
Bian, N. H., Kontar, E. P., & Brown, J. C. 2010, A&A, 519, 114
Chaston, C. C., Johnson, J. R., Wilber, M., et al. 2009, PhRvL, 102, 015001
Chaston, C. C., Salem, C., Bonnell, J. W., et al. 2008, PhRvL, 100, 175003
Chen, L., & Wu, D. J. 2011, PhPl, 18, 072110
Gary, S. P. 1986, JPlPh, 35, 431
Goertz, C. K., & Boswell, R. W. 1979, JGR, 84, 7239
He, J., Tu, C., Marsch, E., & Yao, S. 2012, ApJ, 749, 86
Hollweg, J. V. 1999, JGR, 104, 14811
Hollweg, J. V., & Markovskii, S. A. 2002, JGR, 107, 1080
Hollweg, J. V., Verscharen, D., & Chandran, D. G. 2014, ApJ, 788, 35

Howes, G. G., Bale, S. D., Klein, K. G., et al. 2012, ApJL, 753, L19
Howes, G. G., Cowley, S. C., Dorland, W., et al. 2008, JGR, 113, A05103
Howes, G. G., & Quataert, E. 2010, ApJ, 709, L49
Huang, S. Y., Zhou, M., Sahraoui, F., et al. 2012, GeoRL, 39, L11104
Hunana, P., Goldstein, M. I., Passot, T., et al. 2013, ApJ, 766, 93
Kennel, C. F., & Wong, H. V. 1967, JPlPh, 1, 75
Kellogg, P. J., & Horbury, T. S. 2005, AnGeo, 23, 3765
Klein, K. G., Howes, G. G., TenBarge, J. M., et al. 2012, ApJ, 755, 159
Krauss-Varban, D., Omidi, N., & Quest, K. B. 1994, JGR, 99, 5987
Lysak, R. L., & Lotko, W. 1996, JGR, 101, 5058
Marsch, E. 2006, LRSP, 3, 1
Podesta, J. J. 2013, SoPh, 286, 529
Podesta, J. J., & Gary, S. P. 2011, ApJ, 734, 15
Podesta, J. J., & TenBarge, J. M. 2012, JGR, 117, A10106
Roberts, O. W., Li, X., & Li, B. 2013, ApJ, 769, 58
Sahraoui, F., Belmont, G., & Goldstein, M. I. 2012, ApJ, 748, 100
Sahraoui, F., Goldstein, M. L., Belmont, G., Canu, P., & Rezeau, L. 2010, PhRL, 105, 131101
Sahraoui, F., Goldstein, M. L., Robert, P., & Khoyaintsev, Yu. V. 2009, PhRL, 102, 231102
Schekochihin, A. A., Cowley, S. C., Dorland, W., et al. 2009, ApJS, 182, 310
Shukla, P. K., & Stenflo, J. 2000, JPlPh, 64, 125
Stasiewicz, K., Bellan, P., Chaston, C., et al. 2000, SSRv, 92, 423
Stringer, T. E. 1963, JNuE, 5, 89
Voitenko, Y., & Goossens, M. 2002, SoPh, 206, 285
Voitenko, Y., & Goossens, M. 2003, SSRv, 107, 387
Voitenko, Y., & Goossens, M. 2006, SSRv, 122, 255
Yao, S., He, J.-S., Marsch, E., et al. 2011, ApJ, 728, 146
Zhao, J. S., Voitenko, Y., Wu, D. J., & De Keyser, J. 2014, ApJ, 785, 139
Zhao, J. S., Wu, D. J., & Lu, J. Y. 2010, JGR, 115, 12227
Zhao, J. S., Wu, D. J., & Lu, J. Y. 2011, ApJ, 735, 114
Zhao, J. S., Wu, D. J., & Lu, J. Y. 2013, ApJ, 767, 109