We propose a method based on the redshift dependence of the Alcock–Paczynski (AP) test to measure the expansion history of the universe. It uses the isotropy of the galaxy density gradient field to constrain cosmological parameters. If the density parameter $\Omega_m$ or the dark energy equation of state $w$ are incorrectly chosen, the gradient field appears to be anisotropic with the degree of anisotropy varying with redshift. We use this effect to constrain the cosmological parameters governing the expansion history of the universe. Although redshift-space distortions (RSD) induced by galaxy peculiar velocities also produce anisotropies in the gradient field, these effects are close to uniform in magnitude over a large range of redshift. This makes the redshift variation of the gradient field anisotropy relatively insensitive to the RSD. By testing the method on mock surveys drawn from the Horizon Run 3 cosmological $N$-body simulations, we demonstrate that the cosmological parameters can be estimated without bias. Our method is complementary to the baryon acoustic oscillation or topology methods as it depends on $D_{AH}$, the product of the angular diameter distance and the Hubble parameter.

**Key words:** cosmological parameters – dark energy – large-scale structure of universe

**Online-only material:** color figures

1. INTRODUCTION

Since the discovery of cosmic acceleration (Riess et al. 1998; Perlmutter et al. 1999), the idea of the cosmological constant or dark energy has been used in cosmology as a theoretical explanation of the phenomenon. To constrain theories further, it is crucial to increase the amount of observational data and to improve the statistical methods for measuring the cosmological parameters governing the expansion of the universe.

The Alcock–Paczynski (AP) test (Alcock & Paczynski 1979) is a pure geometric probe of the cosmic expansion history based on a comparison of the observed tangential and radial directions of objects that are known to be isotropic. A few methods have been proposed to use the AP test for cosmological purposes. The most widely adopted method is the one using anisotropic clustering (Ballinger Peacock & Heavens 1996; Matsubara & Suto 1996), which has been used for the 2 degree Field Quasar Survey (Outram et al. 2004), the WiggleZ dark energy survey (Blake et al. 2011), the SDSS-II LRG survey (Chuang & Wang 2012), and the SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS) (Reid et al. 2012; Lopez-Corredoira 2013; Anderson et al. 2013; Beutler et al. 2013; Chuang et al. 2013; Sanchez et al. 2013; Linder et al. 2014; Samushia et al. 2014). The main caveat of this method is that, because the radial distances of galaxies are inferred from redshifts, AP tests are inevitably limited by redshift-space distortions (RSD; Ballinger Peacock & Heavens 1996), which lead to apparent anisotropy even if the adopted cosmology is correct. This effect must then be accurately modeled for the two-point statistics of galaxy clustering.

A second interesting approach measures the symmetry properties of galaxy pairs (Marinoni & Buzzi 2010). Unfortunately, this method is also seriously limited by peculiar velocities. The RSD effect in the apparent tilt angles of galaxy pairs is found to be dependent on both the redshift and underlying cosmology, making it difficult to model accurately (Jennings et al. 2011).

Ryden (1995) and Lavaux & Wandelt (2012) proposed another method using the apparent stretching of voids. The advantage of this method is that the void regions are easier to model compared with high-density regions. However, this method also has limitations in that it utilizes only low-density regions of the large-scale structure and requires much larger samples compared to other methods.

In this paper, we propose a new method that overcomes these limitations. It uses the distortion in the apparent density gradient field constructed from the galaxy distribution. The density gradient vectors are expected to be isotropic if the correct cosmology is adopted, while an anisotropic distribution implies a wrongly assumed cosmology. Similar to the two-point statistics, this method is also based on the distribution of galaxies. However, our method samples the density gradient vectors uniformly within the survey volume, and thus the high- and low-density regions are equally utilized. In contrast, methods of two-point statistics and galaxy pairs assign more weight to the high-density regions, while the void method only utilizes low-density regions.

The observed density gradient field is also affected by RSD, which perturbs galaxy positions along the line of sight (LOS) and produce spurious gradients. However, we find that these anisotropies can be distinguished from those induced by wrongly assumed cosmological parameters by looking at the redshift dependence of the anisotropies. We conduct a proof-of-concept test of our method on the Horizon Run 3 (HR3) mock surveys.

This paper is organized as follows. Section 2 briefly introduces the idea of the Alcock–Paczynski test. Section 3 introduces the simulation data used in our analysis. In Sections 4 and 5, the method is tested on the HR3 mock surveys. We then summarize and conclude in Section 6.
Let us consider an object in the universe for which the ratio between its size along and across the line of sight (LOS) is known. In a given cosmology, its observed redshift span Δz and angular size Δθ are related with the comoving sizes by

\[ \frac{\Delta r_{\parallel}}{\Delta r_{\perp}} = \frac{c}{H(z)} \Delta z, \quad \frac{\Delta r_{\perp}}{\Delta r_{\parallel}} = (1 + z)D_A(z)\Delta \theta, \] (1)

respectively, where \( H \) is the Hubble parameter and \( D_A \) is the angular diameter distance. For simplicity, let us consider a flat universe composed of a matter component with the present density parameter \( \Omega_m \) and a dark energy component with a constant equation of state (EoS) \( w \). The present value of the Hubble parameter and the intrinsic size of the object does not need to be known.

From Equations (1) and (3), we find that the AP test depends on the quantity

\[ F(z) \equiv \frac{1}{c} D_A(z)H(z). \] (4)

\( \Delta z \) and \( \Delta \theta \) are observed quantities, and do not change. If we adopt a wrong cosmology to convert galaxy redshift \( z \) to comoving distance \( r \), the ratio \( \Delta r_{\parallel}/\Delta r_{\perp} \) will change and the degree of distortion is given by

\[ \frac{[\Delta r_{\parallel}/\Delta r_{\perp}]_{\text{wrong}}}{[\Delta r_{\parallel}/\Delta r_{\perp}]_{\text{true}}} = \frac{[D_A(z)H(z)]_{\text{true}}}{[D_A(z)H(z)]_{\text{wrong}}}. \] (5)

For any object with known fixed ratio of the sizes along and across the LOS one can use this relation to constrain the cosmological parameters governing the expansion of the universe.

In Figure 1, we provide a demonstration of the deviations from isotropy for incorrectly chosen cosmological parameters. We assume that the true cosmology corresponds to a flat \( \Lambda \)CDM with \( \Omega_m = 0.26 \). Now we measure objects using the redshift–distance relations in four different cosmologies.

1. “SCDM”: Standard Cold Dark Matter cosmology with \( \Omega_m = 1.0 \).
2. “de Sitter”: de Sitter Universe with \( \Omega_m = 0, w = -1 \).
3. “Quintessence”: quintessence-like dark energy component, \( \Omega_m = 0.26 \) and \( w = -0.5 \).
4. “Phantom”: phantom-like dark energy component, \( \Omega_m = 0.26 \) and \( w = -3.0 \).

The left panel of Figure 1 shows \( F(z) \) in these cosmologies (normalized by its value in the correct cosmology \( F_{\text{fid}}(z) \)). Note that \( F_{\text{fid}}/F \) characterizes the magnitude of the distortion. For instance, in the case of de Sitter cosmology \( F_{\text{fid}}/F \approx 1.3 \) at \( z = 1.0 \), meaning that there is a 30% stretch of size in the radial direction relative to tangential direction.

In the right panel of Figure 1, we show the apparent shape of two squares in the four cosmologies as measured by an observer located at the origin. The distortions in different cosmologies are clearly shown. SCDM and Quintessence cosmologies result in \( F(z) < F_{\text{fid}}(z) \), i.e., apparent compression along the LOS. The opposite trend is observed in the de Sitter cosmology with a stretch along the LOS.

More importantly, Figure 1 highlights the redshift dependence of AP distortion. In the de Sitter cosmology, the radial stretch becomes stronger with increasing redshift, while in the SCDM cosmology the trend is opposite. As will be discussed later, this fact is of essential importance for our method, making anisotropies induced by AP distinguishable from those induced by RSD.

In this paper, the AP test is applied to the density gradient field, which should be statistically isotropic on all scales when the conversion from observed galaxy redshifts to comoving distances is correctly made. Any anisotropy in the density gradient field and the variation of the degree of anisotropy with redshift is evidence for incorrectly adopted cosmology. Sensitivity of the anisotropy to the cosmological parameters comes through the product \( D_A(z)H(z) \).
the BOSS LOWZ sample. The BOSS LOWZ sample is usually restricted to $z > 0.15$ where the galaxy number density is more or less uniform (Tojeiro & Percival 2011; Tojeiro et al. 2012; Parejko et al. 2013).

4. THE DENSITY GRADIENT FIELD DISTORTED BY AP AND RSD

For each mock survey, we embed the volume into a $250 \times 250 \times 500$ grid, and estimate the density gradient vectors at each cell from

$$\rho(r) = \sum_i m_i W(r - r_i, h),$$

(6)

and

$$\nabla \rho(r) = \sum_i m_i \nabla W(r - r_i, h),$$

(7)

where $\rho(r)$ is the halo mass density at position $r$, $m_i$ is the mass of the $i$th halos, and $W$ is the smoothing kernel, for which we choose the third order B-spline functions having nonzero values within a sphere of radius $2h (h^{-1} \text{Mpc})$; Gingold & Monaghan (1977; Lucy 1977). We adopt a variable radius of the smoothing kernel so that the kernel includes 20 of the nearest neighbor halos within $2h$. In our sample of halos for which the mean comoving number density of halos is $3 \times 10^{-4} (h^{-1} \text{Mpc})^{-3}$, the typical value of $h$ is 12.5. We find that the results of our method is rather insensitive to the choice of halo mass density or halo number density; therefore, in this paper, we only present results based on the halo mass density field.

To quantify the anisotropy, we use the angle between the density gradient vector and the LOS direction, $\theta$, where we define

$$\mu \equiv |\cos \theta| = \frac{|r \cdot \nabla \rho(r)|}{|r| \times |\nabla \rho(r)|},$$

(8)

For an isotropic field with gradient vectors uniformly sampled within the survey volume, $\mu$ follows a uniform distribution within $[0,1]$. To characterize the isotropy of the whole gradient field, we look at the mean value of gradient vectors

$$\bar{\mu} = \frac{1}{n_{\text{vector}}} \sum_{i=1}^{n_{\text{vector}}} \mu_i,$$

(9)

where $n_{\text{vector}}$ is the total number of gradient vectors. An isotropic field has $\bar{\mu} = 0.5$, while a compression or stretch along LOS results in $\bar{\mu} > 0.5$ or $\bar{\mu} < 0.5$, respectively.

In Figure 2, we present histograms of $\mu$ measured from the mock surveys. We adopt the correct cosmology and the four wrong cosmologies mentioned in Section 2, and compute values of $\mu$ in two different redshift ranges $z = 0.17–1.0$ and $z = 1.0–1.4$. Left/right panel shows the results without/with the effect of RSD, respectively.

4.1. AP Effect without RSD

In the left panel of Figure 2, we calculate values of $\mu$ without considering the RSD effect. Thus, if the obtained distribution is not uniform, then we are using the wrong cosmology to calculate galaxy distances.

As expected, we find that the correct choice of cosmology leads to uniformly distributed $\mu$ with $\bar{\mu} \approx 0.5$, while in wrong cosmologies this uniform distribution is not obtained. In SCDM and quintessence cosmologies, the apparent compression of structures along the LOS enhances the distribution at large $\mu$, resulting in $\bar{\mu} > 0.5$. Similarly, de Sitter and Phantom cosmologies give $\bar{\mu} < 0.5$.

Comparing the low-redshift and high-redshift histograms, we find redshift dependence of the anisotropy when incorrect cosmologies are adopted. For instance, a choice of the de Sitter cosmology results in $\bar{\mu} = 0.4874$ and 0.4801 for $0.17 < z < 1.0$ and $1.0 < z < 1.4$, respectively. Namely, the high-redshift
region shows a larger deviation from 0.5, the isotropic case, as expected from Figure 1. This results in the detection of redshift dependence at 48σ CL. Similarly, SCDM, phantom, and quintessence cosmologies each show redshift dependence at 8.1σ, 29σ, and 7.0σ CLs.

### 4.2. AP Effect with RSD

As the distances of galaxies are estimated from their redshifts in the actual situation, there exists a systematic bias in the distribution of galaxies. On small scales, high-density regions are stretched along the LOS due to the random motions of galaxies. On large scales, the large-scale peculiar velocity field produces LOS compression of filaments and walls and radial elongation of voids. On the smoothing scales we are interested in, the latter effect is more important. To incorporate the RSD effects in the distribution of galaxies in our mock survey samples, we change the radial distances of galaxies using the formula:

\[
r = \int_0^{z_{\text{cosmo}}+\Delta z} \frac{dz'}{H(z')} \text{,} \quad \Delta z = \frac{v_{\text{LOS}}}{c} (1 + z_{\text{cosmo}}),
\]

where \(z_{\text{cosmo}}\) is the cosmological redshift of the galaxy and \(v_{\text{LOS}}\) is the LOS component of the proper galaxy peculiar velocity. The distribution of \(\bar{\mu}\) after taking into account the RSD effects, is shown in the right panel of Figure 2.

We find that the degree of the anisotropies produced by RSD is very large. In all cosmologies, we find \(\bar{\mu} > 0.5\) with CL > 40, which means that RSD overwhelms AP. This makes it impossible for us to correctly determine the right cosmology by simply requiring that \(\bar{\mu} = 0.5\) of the distribution.

On the other hand, we note that the redshift dependence of \(\bar{\mu}\) is not significantly affected by RSD. In the correct cosmology, the difference between the \(\bar{\mu}\)'s of nearby and farther volumes is as small as 0.0001, on the same level of statistical fluctuation. On the other hand, de Sitter, SCDM, Phantom, and Quintessence cosmologies all show an evident redshift dependence of \(\bar{\mu}\) at 50σ, 10.4σ, 33σ, and 5.5σ CLs, respectively, which are close to the CLs of the corresponding results with no RSD effect. The fact that the effect of RSD is large but its redshift dependence is small makes our method still applicable for the data with RSD. Even with RSD, we can still correctly determine the true cosmology by using the relative change of the gradient field anisotropy with redshift.

### 5. LIKELIHOOD OF THE GALAXY DENSITY GRADIENT FIELD

In the last section, we showed that incorrect cosmologies result in redshift dependent \(\bar{\mu}\), a phenomenon less affected by RSD. Inspired by this fact, we construct the following likelihood...
function to discriminate between different cosmologies

\[
\chi^2 \equiv \sum_{i=1}^{n_{\text{bin}}} \left( \frac{\bar{\mu}_i - \bar{\mu}_{\text{whole}}}{\sigma_{\bar{\mu}_i}} \right)^2.
\]  

(11)

We split the sample into \(n_{\text{bin}}\) redshift bins having an equal comoving volume, compute the values of \(\bar{\mu}\) in each redshift bin, and quantify to what extent they deviate from the \(\bar{\mu}\) averaged over all redshift bins. The value of \(n_{\text{bin}}\) shall be chosen according to the redshift range of the sample.

Figure 3 shows the \(\chi^2\)'s calculated based on 15 redshift bins measured in one of our mock surveys with a redshift range from 0.17 to 1.4. Three different cosmologies, the correct, de Sitter, and Phantom cosmologies, are adopted.

The results without and with RSD effects are shown in left and right panels, respectively. In both cases, we find that the correct estimation of \(\bar{\mu}\) can be found regardless of RSD. The constrained regions of the two cases are close to each other, meaning that RSD effects can be effectively removed since we are concerned with only the redshift dependence of \(\bar{\mu}\).

To remove the remaining weak redshift dependence of RSD effects completely, we modify the \(\chi^2\) function as follows:

\[
\chi^2 \equiv \sum_{i=1}^{n_{\text{bin}}} \left( \frac{\bar{\mu}_i - \bar{\mu}_{\text{whole}} - (\Delta\bar{\mu}_i - \Delta\bar{\mu}_{\text{whole}})}{\sigma_{\bar{\mu}_i}} \right)^2.
\]  

(13)

As an approximation, we use \(\Delta\bar{\mu}\) computed in the correct cosmology in Equation (12). Figure 4 demonstrates that the redshift dependence of \(\Delta\bar{\mu}\) is insensitive to the change in cosmological parameters. In principle, \(\Delta\bar{\mu}\) can be numerically estimated accurately for any trial cosmology.

Figure 5 shows the likelihood of cosmological models in the \(\Omega_m\) and \(w\) space obtained by computing Equation (13) in the four individual mock surveys (panels on the left) and their average (right panel). We find that the correct estimation of \(\Omega_m\) and \(w\) is achieved in both cases with and without RSD. The constrained regions of the two cases are close to each other, meaning that RSD contamination is completely removed.

We find that \(\Omega_m\) and \(w\) are positively degenerated with each other. This is expected. For instance, reducing \(\Omega_m\) and having a more phantom-like dark energy produce similar influences on the expansion history of the universe. Roughly, we find that \(\Omega_m\) and \(w\) are constrained to 0.25 ± 0.05 and −1.0 ± 0.1 (68.3% CL), respectively, using a sample like ours and our AP test method alone.

6. DISCUSSION AND CONCLUSIONS

In this paper, we propose a novel method to use the Alcock–Paczynski test applied to the galaxy density gradient field. If an incorrect cosmology is chosen to compute the distances of galaxies from redshifts, the gradient field appears to be anisotropic with the degree of anisotropy varying with redshift. RSD effects also produce large anisotropy in the gradient field, but maintain a roughly uniform magnitude at all redshifts. By focusing on the redshift dependence on the degree of anisotropy, we are able to derive correct estimations of cosmological parameters in spite of contamination induced by RSD.
Figure 4. Redshift dependence of the RSD effect, characterized by the quantity $\Delta\bar{\mu} = \bar{\mu}_{\text{RSD}} - \bar{\mu}_{\text{No RSD}}$ and measured in the correct (blue dots) and two incorrect cosmologies with parameters moderately deviated from the correct values. The lines are linear fits to the points. We find that the slopes of these lines are rather small; thus, RSD does not introduce significant redshift dependence into $\bar{\mu}$. Fitted lines are roughly parallel to each other, suggesting a small cosmological dependence of the redshift dependence of the RSD effects.

(A color version of this figure is available in the online journal.)

Figure 5. Likelihood contours (68.3%, 95.4%, and 99.7%) in the $\Omega_m$-$\omega$ plane, obtained from the one-fourth sky HR3 mock surveys with a redshift range from 0.17 to 1.4. Left panel shows the results of four individual mock surveys. Right panel shows the average. No RSD and with RSD contours are plotted as gray filled regions and blue lines, respectively. The green cross marks the true cosmology. We achieve unbiased estimations of $\Omega_m$ and $\omega$ regardless of RSD.

(A color version of this figure is available in the online journal.)

Our method is a new attempt to apply the AP test to the large-scale structure of the universe. It is complementary to the existing AP tests using the two-point correlation function, galaxy pairs, and large-scale voids, or to the methods for measuring the cosmic expansion history, e.g., Type Ia supernovae, baryon acoustic oscillations, and topology (Park & Kim 2010) in three ways. First, it uses the galaxy density gradient field, a new approach to apply the AP test. Second, since our method is using the redshift dependence of $\bar{\mu}$, and not its absolute magnitude, we are measuring the first derivative of $DAH$, while other methods mainly focus on $DAH$. Our method, which utilizes the redshift dependence of the AP effect, can be combined with other methods to take full advantage of the cosmological information encoded in the large-scale structure data. It should also be pointed out that our method allows us to use given observational data down to scales of about $20\ h^{-1}\ Mpc$, much smaller than that of the currently popular BAO method (about $100\ h^{-1}\ Mpc$). Third, Figures 3 and 4 show that the redshift dependence of $\bar{\mu}$ has only a very weak dependence on RSD, which can be effectively removed by using mock data. The change of $\bar{\mu}$ with redshift is dominated by the systematic effects of the assumed cosmology, and the cosmological parameter estimation does not suffer from the bias due to the RSD.

One might worry about the systematic effects due to the galaxy sample’s variation with redshift. For a sample of galaxies whose bias changes with redshift, the LOS density gradient may be somewhat affected by the sampling variation. However, the selection effects of the density tracers will not affect the results of our method much because of the following reasons. (1) Our method uses only the “local isotropy” of the density field of the tracer. Unless the target selection varies significantly over the scale of smoothing ($<50\ h^{-1}\ Mpc$), our method will not be affected by the variation of bias in the galaxy distribution. (2) As can be seen in Figure 3, using our mocks, the systematic change in the selection of the tracer galaxies does not affect the shape of $\bar{\mu}(z)$ much when a volume-limited (constant comoving number density) sample with the minimum mass cut varying with redshift is used. The shape of the blue line on the right panel (with RSD) is almost the same as that in the left panel (no RSD) even though the mock galaxies at $z \sim 0.17$ have much larger masses ($\sim 1.4 \times 10^{14}\ M_\odot$) than those at $z \sim 1.4\ (\sim 6 \times 10^{13}\ M_\odot)$.

When dealing with real observational data, the sampling bias can vary more widely than in our mock sample and it will be important to accurately model the observed galaxies to remove
the small residual RSD effects on the isotropy of smoothed density field. It is also needed to handle various observation-related effects such as survey geometry, selection bias, fiber collisions, etc. We will report results of such investigations in forthcoming studies.

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