BLACKBODY DISTRIBUTION FOR
WORMHOLES

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ABSTRACT

By assuming that only (i) bilocal vertex operators which are diagonal with respect to the basis for local field operators, and (ii) the convergent elements with nonzero positive energy of the density matrix representing the quantum state of multiply-connected wormholes, contribute the path integral that describes the effects of wormholes on ordinary matter fields at low energy, it is obtained that the probability measure for multiply-connected wormholes with nondegenerate energy spectrum is given in terms of a Planckian probability distribution for the momenta of a quantum field $\frac{1}{2}a_i^2$, where the $a_i$’s are the Coleman parameters, rather than a classical gaussian distribution law, and that an observable classical universe can exist if, and only if, such multiply-connected wormholes are allowed to occur.
1 INTRODUCTION

In this paper we shall re-explore some consequences arising from the spectrum dependence of the wormhole dynamics. In particular, we shall concentrate on the effects that the insertion of multiply-connected wormholes [1,2] may have on ordinary matter fields at low energies in the asymptotic regions from a statistical standpoint. It will be seen that the sums over the number of equal-state wormholes and over the different wormhole states, required to obtain the wormhole distribution function, are not independent of each other. Thus, the present study supersedes earlier work on nonsimply connected wormhole dynamics done [1] without imposing any dependence between the two sums, and without explicitly taking into account the relative probability for the states. This relative probability, which moreover appears as a prefactor in the bi-local interaction expression for nonsimply connected wormholes [1], should quite naturally enter the path integral expressing the effects of wormholes on ordinary matter, just at the same footing as the semiclassical probability for wormholes does.

2 PROBABILITY MEASURE FOR QUANTUM GRAVITY

In the dilute wormhole approximation [3], the effects of a single wormhole on ordinary matter fields in the asymptotic regions can be expressed in the path integral for the expectation value of a given observable $O$ by inserting a factor which, if the wormhole quantum state is given by a density matrix, is $-\frac{1}{2}C_{ij}\beta^2\epsilon_{mn}^{-1}$. Adapting the formalism given by Klebanov, Susskind and Banks [4] to this case,
we have then

\[ <O> \propto \int dg O e^{-I(g, \lambda)} \left( \frac{1}{2} \sum_{i,j} C_{ij} \beta_i(x) \beta_j(y) \epsilon^{-1}_{mn} \right), \tag{1} \]

where \( C_{ij} \propto e^{-S_w} \), in which \( S_w \) is the Euclidean action for the wormhole, \( \epsilon_{mn} = E_{m}^{(f)} - E_{n}^{(g)} \), with the \( E \)'s being the energy levels for the matter field \( (f) \) and gravitational field \( (g) \) harmonic oscillators, respectively, and \( \epsilon^{-1}_{mn} \) the relative probability for the state \( \Psi_{mn} \); \( \lambda \) collectively denotes parameters such as coupling constants, particles masses, the cosmological constant, etc.; \( \beta_i = \frac{1}{2 \pi} \int d^4x g(x) \frac{1}{2} K_i(x) \), where \( K_i \) denotes the vertex operator and the index \( i \) labels the elements of a basis for the local field operators at the point \( x \) on the large region. This interpretation ensures the grouping of the \( \alpha_i \) parameters with the coupling constants of a generic Lagrangian \([4]\). Indices \( i,j \) are independent of the quantum numbers \( n,m \) which label the off shell energy spectrum \([1]\). All dependence of the path integral on that spectrum is incorporated through the relative probability factor \( \epsilon_{mn}^{-1} \) because a unique invariant quantum theory of wormhole must satisfy the boundary requirement that the possible different kinds of wormholes and wormhole states ought all to have an equal asymptotic (classical) behaviour, thus rendering the insertion amplitude to join their ends onto the asymptotic regions independent of the wormhole spectrum.

It is worth noticing that, besides the usual insertion amplitude depending only on the vertex function at each insertion point on the low-energy large regions which corresponds to wormhole pure states, for mixed states the wormhole effective interaction factor also depends on the absolute value \([1,5]\) of the eigenenergies \( \epsilon_{mn} \). This additional dependence arises because the wormhole is now off shell and has, therefore, a true energy spectrum. Clearly, for Planck-size wormholes, which
is the case we shall consider throughout this paper, we have $\epsilon_{mn} = m - n$.

Eqn.(1) supersedes Eqn.(4.11) of Ref. 1 which was prepared for performing the sum over the states first.

The multiply-connected wormhole dynamics derived from (1) will be re-considered in what follows. According to Coleman [6], we first sum over any number of wormholes, all in the state with energy $\epsilon_{mn}$. Then (1) exponentiates to give

$$\int dg O e^{-I(g,\lambda)} e^{\frac{1}{2} \sum_{ij} C_{ij} \beta_i \beta_j (m-n)^{-1}}.$$  \hspace{1cm} (2)

The exponent in (2) is still bi-local. However, it can be made local by using the transformation employed by Coleman [6]. Eqn. (2) can then be re-written

$$\int dg O e^{-I(g,\lambda)} \prod_p d\alpha_p e^{-\frac{1}{2} (m-n) D_{ij} \alpha_i \alpha_j} e^{-\beta_l \alpha_l},$$  \hspace{1cm} (3)

with

$$D_{ij} = C_{ij}^{-1} = e^{S_w},$$  \hspace{1cm} (4)

through which the position-independent parameter $\alpha$ enters the formalism.

Now, unlike the case considered by Coleman in which there was no true wormhole spectrum, one should next sum over the quantum numbers $m$ and $n$; i.e. over all possible wormhole states. However, since multiply-connected wormhole physics at the inner (Planck) region contributes only through the quantum-number combination $m - n$ appearing in the second exponent of (3), in summing over $m$ and $n$ independently, there will be an overcounting (in the sum over the number of wormholes) associated with all those values of $m$ and $n$ that result in the same value of their difference $m - n$. This overcounting, which would actually lead to summing again over any number of wormholes, can be however gauged off by simply replacing the independent sums over $m$ and $n$ by a single sum over the
discrete index defined by $k = m - n$. Negative values of $k$ would be associated with divergent path integrals [1]. However, such negative values of $k$ are here ruled out because, once the path integral has been made local in (3), there could no longer be gravitational excitation energy equal to or greater than the matter excitation energy, and therefore $k > 0$, since, otherwise, relative to an observer in one asymptotic region, the resulting negative energy density could convert any decreasing cross-sectional area into an increasing cross-sectional area near the wormhole throat, and this would necessarily represent a bi-local insertion, in contradiction with the local character of the path integral after applying the Coleman’s transformation leading to (3). This assumption may also be connected with the feature [7] that Euclidean nonsimply-connected wormholes describable as a nonfactorizable density matrix are squeezed states which require violation of the weak energy condition, but simply connected wormholes do not. Making the path integral local somehow renders the wormhole manifold divided into two disconnected parts, each now describable as an unsqueezed pure state which should not violate weak energy condition [8]. On the other hand, the lower limit in this summation should be taken to be 1 rather than 0 because there always are wormhole instantons (of the Tolman-Hawking type [3]) which are off shell even in the pure gravity limit [9]. We have then

$$
\sum_{k=1}^{\infty} \int dg O e^{-I(g,\lambda)} \int \prod_p d\alpha_d e^{-\frac{1}{2}k D_{ij} \alpha_i \alpha_j} e^{-\alpha_l \beta_l}
$$

$$
= \int \prod_p d\alpha_d \int dg O e^{-I(g,\lambda + \alpha)} \left( e^{\frac{1}{4}D_{ij} \alpha_i \alpha_j} - 1 \right)^{-1},
$$

where the third exponent $-\alpha_l \beta_l$ in the l.h.s. of (5) has been inserted in the path integral of the large universe in the r.h.s.. Hence, the path integral can be written
in the form
\[
< O > \propto \int d\alpha P(\alpha)Z(\alpha) < O >_{\lambda+\alpha},
\] (6)
where
\[
P(\alpha) = \frac{1}{e^{\frac{1}{2}D_{ij}\alpha_i\alpha_j} - 1},
\] (7)
\[
Z(\alpha) = \int d\gamma e^{-I(g,\lambda+\alpha)}.
\] (8)

The probability measure will be then
\[
\mu(\alpha) = P(\alpha)Z(\alpha).
\] (9)

The set of equations (6)-(9) is the main result of this work. It differs from the similar result obtained by Coleman in that the probability distribution for the effective coupling constants \(\alpha\) is given here by Eqn. (7) which reflects an essentially quantum uncertainty about the values of \(\alpha\): instead of the classical measure obtained for wormholes in a pure quantum state, one obtains a probability distribution which closely resembles the Planck formula for black-body radiation for the most general mixed wormhole states. In what follows, we shall use the shorthand notation \(D\alpha^2\) for \(D_{ij}\alpha_i\alpha_j\). Actually, we will see in the next section that among all possible combinations of indices \(i,j\) in (7), only the diagonal combinations \(i = j = l\) are allowed by quantum requirements, so that the Coleman probability becomes gaussian and (7) reduces to
\[
P(\alpha) \equiv P(\alpha_l) = \frac{1}{e^{\frac{1}{2}D\alpha_l^2} - 1}.
\] (10)
3 DOES QUANTUM GRAVITY REQUIRE AN EXTRA QUANTIZATION?

The classical gaussian probability law, $P_c(\alpha)$, discovered by Coleman [4,6]

$$P_c(\alpha) = e^{-\frac{1}{2}D\alpha^2}. \quad (11)$$

does not represent a true probability distribution, but just an unrenormalized probability. However, one may still introduce a normalized probability distribution for Coleman statistics however by defining

$$p_c(\alpha) = \frac{e^{-\frac{1}{2}D\alpha^2}}{\sum_\alpha e^{-\frac{1}{2}D\alpha^2}}. \quad (12)$$

A true probability measure can now be derived from (12) by performing the statistical average for the path integral $Z(\alpha)$, i.e.

$$\bar{Z}(\alpha) = \sum_\alpha Z(\alpha)p_c(\alpha). \quad (13)$$

Clearly, if we want to obtain $\mu(\alpha)$ as given by (9) and (7) from (13), one must necessarily require that neither $\alpha$ nor $Z(\alpha)$ can vary continuously, but are integral multiples of some minimum values $\alpha_0$ and $Z_0 \equiv Z(\alpha_0)$, such that $\alpha^2 \equiv \alpha_n^2 = n\alpha_0^2$, and $Z(\alpha) \equiv Z_n = nZ_0$. In this case,

$$\mu \equiv \bar{Z}(\alpha) = \frac{Z(\alpha)\sum_{n=0}^{\infty} ne^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = \frac{Z(\alpha)}{e^{\frac{1}{2}D\alpha^2} - 1} = Z(\alpha)P(\alpha), \quad (14)$$

where for the sake of generality, we have dropped off the subscript 0 for $\alpha$, $x = \frac{1}{2}D\alpha^2$, and $Z(\alpha) = \frac{1}{2}\alpha^2$. We shall show in what follows that the last identification is actually a necessary condition for a consistent derivation of $Z(\alpha)P(\alpha)$ from (13).

Thus, starting with the Coleman probability for the $\alpha$ parameters, we have recovered the main result of this work. It follows that the statistical character
of the most general quantum state for wormholes appears to lead to an essential discontinuity which is over and above that is already contained in Coleman theory.

For large \( D\alpha^2 \), we recover then the Coleman probability measure \( P_c(\alpha)Z(\alpha) \); in turn, for small \( D\alpha^2 \), one obtains the semiclassical probability law, \( \mu(\alpha) = D^{-1} \). Thus, if we interpret \( D^{-1} \) as an equilibrium temperature, and the \( \frac{1}{2}\alpha^2 \) as the momenta of a quantum field, Coleman distribution and the probability for Euclidean gravity (here, \( e^{-S_w} \)) appear to play the role of, respectively, the Wien law and the Rayleigh-Jeans law for average energy. This interpretation is made more precise if we calculate the quantity

\[
\omega = \int Dd\mu = \ln \frac{\left(\frac{2(\mu+Z)}{\alpha^2}\right)^{\frac{2(\mu+Z)}{\alpha^2}}}{\left(\frac{2\mu}{\alpha^2}\right)^{\frac{2\mu}{\alpha^2}}} + \text{Const.},
\]

and interpret it as the entropy of the system when it contains just one large universe. For \( N \) large universes, we had then \( N\mu = P\nu \) (with \( P \) an integer), and it is finally obtained

\[
\Omega = N\omega = \ln \frac{(P+N)^{P+N}}{N^NP^P} + \text{Const.},
\]

where we have used \( Z = \nu = \frac{1}{2}\alpha^2 \). Now, if we interpret (16) as an entropy, the probability measure (14) is immediately recovered from just the condition [10] that \( \Omega \) is a maximum, which corresponds to a system in thermal equilibrium at temperature \( D^{-1} \). We can see now why \( Z = \frac{1}{2}\alpha^2 \) is a necessary condition to achieve full consistency in this interpretation. Furthermore, if \( Z(\alpha) = \frac{1}{2}\alpha^2 \), it also follows that only diagonal elements \( i = j = l \) may enter the quantity \( \alpha_i\alpha_j \), since according to (8) the path integral \( Z(\alpha) \) must only depend on just one index labelling the basis for the local field operators. Hence, \( Z(\alpha) \equiv Z(\alpha_l) \) and the probability distribution reduces to (10).

In what follows we shall explore some possible implications that the existence
of multiply-connected wormholes may have in the convergence problem of Euclidean quantum gravity. It has been recently emphasized by Hawking [11] that the divergence problem of Euclidean path integral becomes even more severe in the presence of wormholes. In this case, besides the known divergences arising from the unboundedness of the Euclidean action, there appears a very serious divergence in the probability measure on the space of effective coupling constants which is connected to the Coleman mechanism for the vanishing of the cosmological constant. None of these divergences should be expected to have any relevance if the wormholes are off shell. The path integral \( Z(\alpha) \) becomes then discontinuous as well, and the relevant quantum state for the large universe with an arbitrary number of multiply-connected wormholes is given by the probability measure itself, that is, by the statistical average of the path integral \( Z(\alpha) \). The point now is to check that the so-defined quantum state is finite under all circumstances. That this is actually the case can be readily seen by performing the integral

\[
\Theta = \int_0^{\infty} dZ(\alpha) \mu(\alpha) = \frac{\pi^2}{6} D^{-2},
\]

which is in fact finite.

Convergence will be preserved even if we just integrate over \( \alpha \) as the Riemann zeta function \( \zeta(z) \) is regular for all values of \( z \) other than unity [12]. Moreover, introducing the concept of probability measure density,

\[
\rho_\mu(\alpha) = \mu(\alpha) Z(\alpha)^2,
\]

and integrating over \( Z(\alpha) \), one obtains

\[
U_\mu = \int_0^{\infty} dZ(\alpha) \rho_\mu(\alpha) = \frac{\pi^4}{15} D^{-4}.
\]

Clearly, Eqn. (19) plays the role of a Stefan-Boltzmann law for quantum
gravity, and states that the total probability measure density is given by the fourth power of the nucleation rate of baby universes per Planck volume per Planck time. Actually, Eqns. (17) and (19) express the very remarkable result that an observable universe can exist if, and only if, it is connected to a nonzero number of multiply-connected wormholes. This result can quite naturally be connected with the recent proposal [2,7] of a universal decoherence in the matter field sector by which all quantum correlations contained in the pure state-vector evolving according to the Schroedinger equation, are traced off by the action of multiply connected wormholes.

In summary, this paper shows that, if we assume bilocal vertex operators diagonal with respect to the indices that label the elements of the basis for local field operators on the large regions, and consider that only convergent density matrix elements contribute the path integral for the quantum state of multiply-connected wormholes, the effects of such multiply-connected wormholes with nondegenerate energy spectrum on ordinary matter at low energies are given in terms of a Planckian probability distribution law for the momenta of a quantum field $\frac{1}{2}\alpha_i^2$. Under these assumptions, it has been also seen that for such a distribution law to admit a consistent interpretation, it is required that both, the $\alpha$ parameters themselves and the path integral for the large universes should vary discontinuously, and that this path integral be also given by $\frac{1}{2}\alpha^2$. It appears, moreover, that all divergences of quantum gravity could be remedied according to this interpretation which parallels closely Planck theory of black-body radiation.
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