We report on magnetoconductance measurements of metallic networks of various sizes ranging from 10 to $10^6$ plaquettes, with anisotropic aspect ratio. Both Altshuler-Aronov-Spivak (AAS) $h/2e$ periodic oscillations and Aharonov-Bohm (AB) $h/e$ periodic oscillations are observed for all networks. For large samples, the amplitude of both oscillations results from the incoherent superposition of contributions of phase coherent regions. When the transverse size becomes smaller than the phase coherent length $L_\varphi$, one enters a new regime which is phase coherent (mesoscopic) along one direction and macroscopic along the other, leading to a new size dependence of the quantum oscillations.

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tte is $B = 100\, G$. All networks with number of plaquettes $N$ varying from 10 to $10^6$ have the same aspect ratio $L_x/L_y = 10$ (see figure 1). As a consequence, their resistances are similar and of the order of 100 $\Omega$. Measurements have been performed at 400 $mK$; this allows to stay in the linear regime with a relatively high current ($\approx 4\, nA$) and optimizes the signal to noise ratio without heating the electrons. At this temperature, the phase coherence length, determined from standard weak localization measurements on a 120 nm wide wire fabricated on the same wafer, is about $L_\phi \approx 6\, \mu m$, the diffusion constant $D \approx 105\, cm^2 s^{-1}$ and the thermal length $L_T = \sqrt{\hbar D/k_B T} \approx 0.45\, \mu m$ \textsuperscript{10}. 

In figure 2 we show typical data for the magnetoresistance of a square network with 3000 plaquettes. At low field (figure 2a), oscillations with a period $B = 50\, G$, corresponding to $\phi_0/2$ per plaquette are identified as the AAS oscillations. At fields typically higher than the field which suppresses weak localization, we observe a different type of oscillations. These oscillations have a periodicity of $B = 100\, G$, corresponding to $\phi_0$; these are AB oscillations. In order to emphasize the different periodicity of these magnetoconductance oscillations, we display their Fourier spectra in figures 2c (low field) and 2d (high field): in the high field regime, the main peak clearly appears at $0.01\, G^{-1}$, whereas in the low field regime, it appears at $0.02\, G^{-1}$. To our knowledge, this is the first time that both AAS and AB oscillations are observed on such large samples.

We now concentrate on the variation of the amplitude of the AB as well as AAS oscillations versus the number of plaquettes $N$. To measure the AB oscillations we sweep the magnetic field from 7000 $G$ to 13000 $G$, whereas for the AAS oscillations we cover a field range of $\pm 1200\, G$. To extract precisely the amplitude of the AB oscillations, we take the Fourier transform over 20 periods after subtraction of a smooth background to remove low frequency fluctuations. We also measure the background noise by repeating the measurement exactly in the same conditions but at fixed magnetic field, and taking again the Fourier transform. The amplitude of the AB signal is then obtained from the Fourier spectrum after subtraction of the background spectrum integrated over the same frequency range, in a similar way used for persistent current measurements \textsuperscript{11}. This procedure is only necessary for very large networks (typically larger than $10^5$ plaquettes) since for smaller networks the noise is negligible. For the determination of the AAS amplitude such a procedure is not necessary, as the background noise is always negligible. However, the second harmonic ($\phi_0/2$) of the AB oscillations has the same frequency as the first harmonic ($\phi_0/2$) of the AAS oscillations. For small networks (typically $N \leq 100$) this contribution cannot be neglected. In order to extract the AAS signal, we therefore determine first the amplitude of the second harmonic of the AB oscillations at high field and then subtract this amplitude from the first harmonic of the oscillations measured at low field \textsuperscript{12}.

In figure 3 we display the amplitude of magnetoconductance oscillations (AAS and AB) extracted from the Fourier spectra as a function of the number $N$ of plaquettes. For large networks ($N \gtrsim 300$), the amplitude of the AB oscillations clearly decreases as $1/\sqrt{N}$, whereas the amplitude of the AAS oscillations are independent on the number of plaquettes as naively expected. More surprising is the behavior observed for small networks: when they contain typically less than $N \approx 300$ plaquettes, the amplitude of the AB oscillations varies faster than $1/\sqrt{N}$. At the same time the AAS amplitude now decreases as $N^{-1/2}$. All networks with number of plaquettes $N$ varying from 10 to $10^6$ have the same aspect ratio $L_x/L_y = 10$ (see figure 1). As a consequence, their resistances are similar and of the order of 100 $\Omega$. Measurements have been performed at 400 $mK$; this allows to stay in the linear regime with a relatively high current ($\approx 4\, nA$) and optimizes the signal to noise ratio without heating the electrons. At this temperature, the phase coherence length, determined from standard weak localization measurements on a 120 nm wide wire fabricated on the same wafer, is about $L_\phi \approx 6\, \mu m$, the diffusion constant $D \approx 105\, cm^2 s^{-1}$ and the thermal length $L_T = \sqrt{\hbar D/k_B T} \approx 0.45\, \mu m$ \textsuperscript{10}. 

In figure 2 we show typical data for the magnetoresistance of a square network containing 3000 plaquettes: a) low field data; b) high field data; c), d) Fourier amplitudes of a), b) respectively.
The dimensionless conductance $g = G/(2e^2/h)$ of the network is then related to the conductivity by Ohm’s law $g \propto \sigma L_y/L_x$. Combined with eq. (1) and given that $\text{Vol} \propto L_x L_y$, this yields for the amplitudes of the conductance oscillations $\Delta g_{AAS}$ and $\delta g_{AB}$:

$$\delta g_{AB}^2 = \frac{2\pi L_T^2}{3 L_x^2} \Delta g_{AAS}$$

which is the key relation from which we now discuss our results, bearing in mind that temperature, and thus $L_T$ and $L_\phi$ are fixed parameters.

Let us first consider large networks with both dimensions larger than the phase coherence length: $L_x, L_y \gg L_\phi$. Since interfering time reversed trajectories extend over a typical size $L_\phi$, they do not feel the boundaries of the system and therefore $\Delta \sigma$ is size independent. Therefore, the AAS amplitude varies as $\Delta g_{AAS} \propto L_y/L_x$ and since this ratio is constant, this amplitude is independent on $N$:

$$\Delta g_{AAS} \propto N^0$$

In this regime we also see from eq. (2) that $\delta g_{AB}^2 \propto L_y/L_x^3$, which leads to:

$$\delta g_{AB} \propto N^{-1/2}$$

This is exactly what is observed for large networks: when the number of plaquettes is larger than $\approx 300$, electrons diffuse on what they feel as a two-dimensional network.

For smaller networks, the transverse dimension $L_y$ eventually becomes smaller than the phase coherence length: we enter a regime where the network becomes transversally coherent whereas it remains longitudinally incoherent: $L_y \ll L_\phi \ll L_x$. In this case, we have the usual quasi-1D scaling $\Delta \sigma_{AAS} \propto L_\phi/L_y$. Therefore we find $\Delta g_{AAS} \propto 1/L_x$ and $\delta g_{AB}^2 \propto 1/L_x$, which leads to:

$$\Delta g_{AAS} \propto N^{-1/2}$$
$$\delta g_{AB} \propto N^{-3/4}$$

This is precisely what is observed for small networks on figure 3.

It remains now to check whether the position of the crossover observed on figure 3 agrees with our estimate of the phase coherence length. The crossover occurs for a size $N \approx 300$ corresponding to $L_y \approx 3.8 \mu m$. This length has to be compared with the coherence length $L_\phi \approx 6 \mu m$ measured at $T = 400 mK$. This comparison, which cannot be more than qualitative, supports our analysis.

To summarize, the dimensional crossover observed for the scaling of the AB oscillations corresponds to the different scaling $L_y/L_x^3 \rightarrow L_\phi/L_x^3$ of the variance of the conductance fluctuations, with $N \propto L_x L_y$. At this point, it is useful to compare these dependences with the case of a 1D chain where the number $N$ of rings...
FIG. 4: (color online). $\Delta g_{AAS}/(\delta g_{AB})^2$ as a function of the number of plaquettes $N$ for square networks (squares) and diamonds networks (diamonds).

scales linearly with the length $L_x$ of the chain, so that $\Delta g_{AB}^2 \propto 1/L_x^3 \propto 1/N^3$ and $\Delta g_{AAS} \propto 1/N$. Since the conductance $g$ scales as $1/N$, this yields for the relative fluctuations $\Delta g_{AAS}/g \propto N^0$ and $\Delta g_{AB}/g \propto 1/\sqrt{N}$ as was observed experimentally.

An interesting way of checking our analysis comes from equation (2): we can see that the ratio $\Delta g_{AB}^2/\Delta g_{AAS} \propto L_T^2/L_x^2$ is proportional to $1/N$ and more importantly is independent on $L_\phi$. This fundamental relation between $\Delta g_{AB}$ and $\Delta g_{AAS}$ is clearly shown on figure 4 where we have plotted the ratio $\Delta g_{AB}^2/\Delta g_{AAS}$ as a function of the number of plaquettes $N$: one sees that it follows perfectly the predicted $1/N$ behavior, with no dimensional crossover. This is a definitive check of our interpretation of the experimental data in terms of dimensional crossover.

In conclusion, we have measured both Aharonov-Bohm $\phi_0$ periodic oscillations and Altshuler-Aronov-Spivak $\phi_0/2$ periodic oscillations in metallic networks containing 10 to 10$^6$ plaquettes. Ensemble averaging can lead to different size dependences for small and large networks. The crossover takes place when the width of the network is of the order of the phase coherence length; this behavior does correspond to a dimensional crossover between effectively one- and two-dimensional networks. In this new one-dimensional regime we observed, we have shown that the amplitude of the AB oscillations varies as $N^{-3/4}$ and the AAS oscillations as $N^{-1/2}$, a behavior which has never been observed up to now. Moreover, we have been able to probe experimentally the fundamental relation between AB and AAS magnetooconductance oscillations due to their common physical origin.

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