P-Wave Polarization of the $\rho$-Meson and the Dilepton Spectrum in Dense Matter

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Abstract

We study the $p$-wave polarization operator of the $\rho$-meson due to $\rho N$ interactions via the $N^*$ (1720) and $\Delta$(1905) resonances and compute the corresponding production rate for $e^+e^-$-pairs at finite temperature and baryon density. At high baryon density we find a significant shift of the spectrum to lower invariant masses.

1 Introduction

Heavy ion collisions at high energies have stimulated a general interest in hadron properties at high temperature and high baryon density. It is expected that at sufficiently high temperature a chiral symmetry restoring transition occurs where the constituent quark mass becomes very small [1]. The temperature scale for this transition is of the order of the Hagedorn temperature $T \simeq 160$ MeV. The equivalent density cannot be computed reliably at present since reliable lattice calculations of finite baryon density are not yet possible. However, one can perform simple estimates,

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e.g. by comparing the mean distance between nucleons with their geometrical size $R_N \simeq 0.8 \text{ fm}$. One finds that the transition should take place at a baryon density 5-10 times normal nuclear matter density.

The effects of chiral symmetry restoration have been widely explored and the spectrum of the pseudoscalar mesons has been computed within effective chiral models [2, 3, 4, 5]. Recent heavy-ion data from GSI are consistent, at least on a qualitative level, with the predicted strong reduction of the $K^-$ mass at finite baryon density [6]. For the $\pi$ meson the expected mass shift is small and does not lead to a clear experimental signature. On the other hand, for the properties of vector mesons in matter the situation is less clear. The theoretical foundation for the calculations is much less firm, and different models yield very different predictions. The additive constituent quark model implies that lower mass constituent quarks form lower mass bound states. A lowering of the $\rho$ mass in matter is also predicted by Brown and Rho using chiral symmetry and scale invariance [7, 8]. On the other hand, chiral symmetry associates the $\rho$ meson with its chiral partner the axial vector meson $a_1$, which is higher in mass. This may point to an increase of the $\rho$-mass when chiral symmetry is restored and chiral partners become degenerate [9].

Neutral vector mesons mix with photons and consequently decay (with a small branching ratio) into lepton pairs. It is plausible that short-lived vector mesons, like the $\rho$ meson, produced in heavy-ion collisions decay inside the hot and dense medium. Since the lepton pairs escape essentially without further interactions, they carry information on the conditions in the excited hadronic matter. Recent experiments indicate a qualitative change of the dilepton spectrum between $pA$- and $AA$-data. Both the CERES [10] and HELIOS-3 [11] data show a shift of strength in the dilepton invariant mass spectrum to lower masses, which can be interpreted in terms of a reduction of the in-medium $\rho$ meson mass [12, 13, 14].
In previous calculations one considered only the self energy of a $\rho$ meson at rest in nuclear matter. In the present paper we discuss the possibility that the $p$-wave polarization operator i.e. the momentum dependent self energy of the $\rho$-meson in matter, plays an important role. For $\pi N$ scattering, the (isospin symmetric) $s$-wave interaction is known to be much less important than that in $p$-wave states. This is partly due to chiral symmetry, which suppresses the $s$-wave interaction, but also due to the existence of the $\Delta$ resonance which is fairly close to the $\pi N$-threshold. The $\Delta$ resonance is responsible for the strong $p$-wave polarization of the pion in the nuclear medium. We explore the effect of the corresponding resonances in the $\rho N$ channel.

The data for $\pi N \to \rho N$ and $\gamma N \to \rho N$ interactions near threshold reveal two prominent resonances: the $N^*(J = 3/2^+, I = 1/2) 1720$ MeV and the $\Delta(J = 5/2^+, I = 3/2) 1905$ MeV baryons. In section 2 we discuss the decay characteristics and the resulting effective $p$-wave $\rho N$- and $\gamma N$-couplings of these resonances. In section 3 we employ this information to compute the properties of the $\rho$-meson in nuclear matter. Section 4 is devoted to a calculation of the resulting dilepton spectrum.

2 $N^*$- and $\Delta$-resonances near the $\rho N$- threshold

In the review of particle properties [15] two four-star baryon resonances with a large (> 50%) branching ratio into the $\rho N$ channel are listed (see Table 1). The relative angular momentum of the $\rho N$ final state is for the $N^*(1720)$ a pure $p$-wave, while for the $\Delta(1905)$ it is $p$- or $f$-wave. For simplicity we assume that the coupling of the $\Delta(1905)$ to the $\rho N$ channel is also pure $p$-wave.

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1 Actually D.M. Manley et al. [16] need another $3/2^+$-state at 1879 MeV to fit the $\pi N \to \pi \pi N$ data. This resonance has a large $\rho N$ branching ratio $\Gamma_{\rho N}/\Gamma_{\text{tot}} \simeq 40\%$ in the fit of Manley et al., but is not listed by the particle data group.
Table 1: Resonance properties according to the Review of Particle Properties.

In the calculation of the lepton-pair production rate we need the \( \gamma N \) decay channel of the resonances. This is much smaller than predicted by the Vector Meson Dominance (VMD) model of Sakurai [17]. We therefore adopt the modified VMD model of Kroll, Lee and Zumino [18]

\[
\mathcal{L}_{VMD} = \frac{e}{2g_{\rho\pi\pi}} F_{\mu\nu} \rho^3_{\mu\nu},
\]

which is explicitly gauge invariant and allows one to fix the coupling strengths in the \( \rho N \) and \( \gamma N \) channels independently. Here \( F_{\mu\nu} \) is the electro-magnetic field strength tensor, \( \rho^3_{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu \) the tensor associated with the neutral \((i = 3)\) \( \rho \) meson (we retain only the abelian terms), \( e \) is the electromagnetic and \( g_{\rho\pi\pi} \) the \( \rho\pi\pi \) coupling constant. For the \( \gamma N \) channel the relative angular momentum is not known. We assume that the lowest multipole dominates. The interaction Lagrangian for the \( N^\ast \) resonance is then of the form

\[
\mathcal{L}_{N^\ast} = \frac{f_{N^\ast N\rho}}{m_{\rho}} \left[ \Psi_{N^\ast}^\dagger (\vec{S} \times \vec{q}) \cdot \vec{\rho}^i \tau^i \Psi_N + h.c. \right] + \mu_{N^\ast} \left[ \Psi_{N^\ast}^\dagger (\vec{S} \times \vec{q}) \cdot \vec{A} \tau^3 \Psi_N + h.c. \right],
\]

where \( \vec{\rho}^i \) is the \( i \)th isospin component of the \( \rho \) meson field, \( \vec{A} \) the photon field, and \( \vec{S} \) the transition spin operator which connects \( J = 1/2 \) (\( N \)) and \( J = 3/2 \) (\( N^\ast \)) baryon states [19, 20]. We note that with this Lagrangian only transverse \( \rho \) mesons can excite the \( N^\ast(1720) \) resonance. Relativistic corrections may give rise to a non-vanishing coupling for longitudinal \( \rho \) mesons. In Fig. [4] the diagrams contributing to the \( N^\ast N\gamma \) form factor are shown. Note that for a real photon \((q^2 = 0)\), the \( \rho \) meson contribution vanishes, due to the structure of the \( \gamma \rho \) mixing term. The coupling...
constant $f_{N^*N\rho}$ and the transition magnetic moment $\mu_{N^*}$ are fixed by fitting the $\rho N$ and $\gamma N$ partial widths of the $N^*(1720)$.

\[ N^* \rightarrow N \gamma \]

Figure 1: The electromagnetic $N^*N$ transition form factor in VMD.

The corresponding interaction Lagrangian of the $\Delta(1905)$ is more complicated, due to the higher spin $J = 5/2$. We employ the following form

\[ \mathcal{L}_\Delta = \frac{f_{\Delta N\rho}}{m_\rho} \left[ \Psi_\Delta^\dagger R_{ij} \rho_i^k T^k \Psi_N + h.c. \right] \]

\[ + \mu_\Delta \left[ \Psi_\Delta^\dagger R_{ij} \rho_i A_j T^3 \Psi_N + h.c. \right], \]

where $R_{ij}$ is the transition spin operator which connects $J = 1/2$ and $J = 5/2$ baryon states and $T^k$ is the transition isospin operator which connects isospin 1/2 with isospin 3/2 states. The transition spin operator $R_{ij}$ is a tensor of rank 2, and a $6 \times 2$ matrix in spin space. One can construct a spin-5/2 object by coupling two spin-1 objects $\epsilon^s_i$ to the nucleon spinor $\chi_{1/2}^{m_s}$

\[ |X_{ij}^M\rangle = \sum_{r,s,t,m_s} \left( \frac{5}{2} M \right) \left( 2r \frac{1}{2} m_s \right) \left( 2s1t \right) \epsilon^s_i \epsilon^s_j |\chi_{1/2}^{m_s}\rangle. \]

The components of the unit vectors $\epsilon^s_i$ are given by

\[ \epsilon^1 = -\frac{1}{2} (1, i, 0), \quad \epsilon^0 = (0, 0, 1), \quad \epsilon^{-1} = \frac{1}{2} (1, -i, 0). \]

The transition spin operator is defined by

\[ \langle X_{ij}|N\rangle = \langle \Delta|T_{ij}|N\rangle, \]
where $|\Delta\rangle$ is a six component vector corresponding to the possible spin projections of spin-5/2. As is well known for spin-3/2 resonances, one rarely needs to know the explicit form of the transition spin operator. The projection operator

$$Q_{ij,kl} = \sum_M |X^M_{ij}\rangle\langle X^M_{kl}|$$

(9)

turns out to be a more useful quantity. After some tedious algebra one finds

$$Q_{ij,kl} = \frac{1}{2} (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - \frac{1}{5}\delta_{ij}\delta_{kl}$$

$$- \frac{1}{10} (\delta_{ik}\sigma_j\sigma_l + \delta_{il}\sigma_j\sigma_k + \delta_{jk}\sigma_i\sigma_l + \delta_{jl}\sigma_i\sigma_k).$$

(10)

With the help of this operator processes involving spin-1/2 to spin-5/2 transitions are easily computed.

The partial width for the decay of the $N^*$ resonance into the $\rho N$ channel is given by

$$\Gamma_{N^*\to\rho N} = \frac{2}{3} \cdot 3 \frac{f^2_{N^*N\rho}}{m^2_\rho} \int \frac{d^4 q}{(2\pi)^4} \frac{m_\rho}{q^2 + m^2_N} \frac{4\pi m_\rho \Gamma_{\rho\to\pi\pi}}{\sqrt{q^2 + m^2_N} (q^2 - m^2_\rho)^2 + m^2_\rho \Gamma^2_{\rho\to\pi\pi}}$$

$$\delta(m_{N^*} - \sqrt{q^2 + m^2_N - q^0})$$

$$= \frac{4\pi f^2_{N^*N\rho}}{\sqrt{m^2_\rho} \pi} \int_{2m_\pi}^{m_{N^*} - m_N} dm \frac{m_\rho}{m_{N^*}} \cdot m |\vec{q}_1|^3 \frac{m_\rho \Gamma_{\rho\to\pi\pi}}{(m^2 - m^2_\rho)^2 + m^2_\rho \Gamma^2_{\rho\to\pi\pi}}$$

(11)

where $|\vec{q}_1| = \left[ \frac{(m^2_{N^*} - m_N^2 - m^2_\rho)^2 - 4m^2_\rho m^2_{N^*}}{4m^2_{N^*}} \right]^{1/2}$.

Here we include the proper phase space dependence for the $\rho$ meson decay into two pions,

$$\Gamma_{\rho\to\pi\pi}(m) = \Gamma^{(0)}_{\rho\to\pi\pi} \left( \frac{k}{k_\rho} \right)^3,$$

(12)

where $\Gamma^{(0)}_{\rho\to\pi\pi} = 150$ MeV, $k = \sqrt{m^2/4 - m^2_\pi}$ and $k_R = \sqrt{m^2_\rho/4 - m^2_\pi}$. By fitting the partial width $\Gamma_{N^*\to\rho N} \simeq 100$ MeV we find $f_{N^*N\rho} = 7.2$, which is close to the coupling constant of the magnetic $\rho NN$ interaction [21]

$$\mathcal{L} = iN g_{\rho\pi\pi} \frac{K_\rho}{2m_N} (\vec{\sigma} \times \vec{k}) \cdot \vec{\tau}^i \vec{\tau}^i N$$

(13)
with $K^V_\rho = 6.6 \pm 0.4$, which implies $f_{NN\rho} = (m_\rho/2m_N)(g_{\rho\pi\pi}/2)K^V_\rho = 8.1$.

For the $\gamma N$ partial width we find

$$\Gamma_{N^*\rightarrow N\gamma} = \frac{1}{3\pi}(\mu_{N^*})^2 \frac{m_N}{m_{N^*}} |\vec{q}_2|^3$$

(14)

where $|\vec{q}_2| = \left(\frac{m_{N^*}^2 - m_N^2}{2m_{N^*}}\right)$. The corresponding partial widths of the $\Delta(1905)$ are given by

$$\Gamma_{\Delta\rightarrow N\rho} = \frac{1}{3\pi^2} \frac{f_{\Delta N\rho}^2}{m_\rho^2} \frac{m_N}{m_\Delta} m |\vec{q}_1|^3 \frac{m_\rho \Gamma_{\rho\pi\pi}}{(m^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_{\rho\pi\pi}^2}$$

(15)

and

$$\Gamma_{\Delta\rightarrow N\gamma} = \frac{1}{30\pi}(\mu_\Delta)^2 \frac{m_N}{m_\Delta} |\vec{q}_2|^3,$$

(16)

where $m_{N^*}$ should be replaced by $m_\Delta$ in $q_1$ and $q_2$. Using $\Gamma_{\Delta\rightarrow N\rho} = 210$ MeV we find $f_{\Delta N\rho} = 9.0$.

The values of $\mu_{N^*}$ and $\mu_\Delta$ are determined by fitting the PDG estimates [13] for $\Gamma_{\gamma N}/\Gamma_{\text{tot}}$ obtained by analyzing the $\gamma N \rightarrow \pi N$ reaction. It is convenient to introduce the ratio of the transition magnetic moment to its value in VMD $r_{N^*} = \mu_{N^*}(g_{\rho\pi\pi}/e)(m_\rho/f_{N^*N\rho})$ and $r_\Delta = \mu_\Delta(g_{\rho\pi\pi}/e)(m_\rho/f_{\Delta N\rho})$. Using the maximal $\gamma N$ branching ratios we find $r_{N^*} \simeq 1/\sqrt{30}$ and $r_\Delta \simeq 1/\sqrt{5}$, respectively. Thus the $\gamma N$ branching ratios of the $N^*(1720)$ and $\Delta(1905)$ resonances are suppressed by factors of approximately 1/30 and 1/5 compared to the VMD results.

It is instructive to estimate the cross section for photoproduction of $\rho$ mesons off protons

$$\sigma_{\gamma N \rightarrow \rho N} \approx \frac{\pi}{k_{\text{cm}}^2} (\Gamma_{N^*\rightarrow \gamma N} \Gamma_{N^*\rightarrow \rho N}) \frac{m_{N^*}^2}{(s - m_{N^*}^2)^2 + \Gamma_{N^*}^2 m_{N^*}^2},$$

$$+ \frac{\pi}{k_{\text{cm}}^2} \frac{3}{2} \frac{\Gamma_{\Delta\rightarrow \gamma N} \Gamma_{\Delta\rightarrow \rho N}}{(s - m_\Delta^2)^2 + \Gamma_\Delta^2 m_\Delta^2}.$$

(17)

Here $\Gamma_{N^*}$ and $\Gamma_\Delta$ denote the full widths of the baryon resonances. The resulting photoproduction cross section at low energies is smaller than the measured one [22].
The difference is probably due to other partial waves, which may be described in terms of other baryon resonances or the exchange of mesons in the t-channel \cite{23}. Thus, the p-wave self energy of a $\rho$ meson in nuclear matter cannot be obtained by a naive extrapolation of the photoproduction data. Forthcoming experiments on the production of $\rho$ and $\omega$ mesons e.g. at CEBAF will shed new light on the mechanism for vector meson production near threshold.

3 $\rho N$ interactions at finite $\vec{q}$

In this section we estimate the strength of the $\rho$ meson p-wave optical potential and explore its consequences for the production of lepton pairs in relativistic nucleus-nucleus collisions.

To lowest order in the density, the $\rho$ meson self energy in nuclear matter is given by

$$\Sigma_\rho = 4\pi f_{\rho N} \rho_B,$$

(18)

where $f_{\rho N}$ is the $\rho$-nucleon forward scattering amplitude and $\rho_B$ is the baryon density. We approximate the scattering amplitude with two resonance contributions, the $N^*(1720)$ and $\Delta(1905)$. In reference \cite{24} a complementary approach was chosen, where the interactions of the pions, present in a physical $\rho$ meson, with the nuclear medium were taken into account. It was shown that such interactions for a $\rho$ meson at rest in nuclear matter leads to a strong increase of the width but only a small mass shift. Similar results were obtained by Asakawa et al. \cite{25} and Chanfray and Schuck \cite{26} and recently, using a different scheme, by Klingl and Weise \cite{27}. Here we are interested in the momentum dependence of the self energy. A mass shift, due to other processes like the exchange of a scalar object in the t-channel, is possible \cite{23}.

The $N^*(1720)$ has a total width of 150 MeV and decays into a $\rho$ meson and
a nucleon in a relative p-wave, while the $\Delta(1905)$ has a total width of 350 MeV and decays into the $\rho N$ channel in a p- or f-wave state \[15\]. Since the p- and f-wave fractions of the $\Delta(1905) \rightarrow \rho N$ branching ratio are not known, we assume, as mentioned in the section 2, that the p-wave channel dominates and neglect the f-wave contribution. This approximation does not significantly affect the results.

Figure 2: The $\rho$ meson self energy diagrams in matter.

The $\rho$ meson self energy in nuclear matter is now, to lowest order in density, given by the diagrams shown in Fig. 2, which are easily evaluated in the static approximation \[24\]. The in-medium self energy of transverse $\rho$ mesons due to resonance excitation is then given by\[19\]

$$\Sigma^R_{\rho}(\omega, \vec{q}) = \frac{4}{3} \frac{f_{N*}^{2} N\rho}{m_{\rho}^{2}} F(q^{2})q^{2} \rho_{B} \frac{(\varepsilon_{q}^{N*} - m_{N})}{\omega^{2} - (\varepsilon_{q}^{N*} - m_{N})^{2}} + \frac{2}{5} \frac{f_{\Delta N\rho}^{2} N\rho}{m_{\rho}^{2}} F(q^{2})q^{2} \rho_{B} \frac{(\varepsilon_{q}^{\Delta} - m_{N})}{\omega^{2} - (\varepsilon_{q}^{\Delta} - m_{N})^{2}},$$

(19)

where

$$\varepsilon_{q}^{N*} = \sqrt{q^{2} + m_{N*}^{2}} - \frac{i}{2} \Gamma_{N*}, \quad \varepsilon_{q}^{\Delta} = \sqrt{q^{2} + m_{\Delta}^{2}} - \frac{i}{2} \Gamma_{\Delta},$$

(20)

and $F(q^{2}) = \Lambda^{2}/(\Lambda^{2} + q^{2})$ is a form factor with $\Lambda = 1.5$ GeV. In eq. (20) $\Gamma_{N*}$ and $\Gamma_{\Delta}$ are the full widths, modified by the phase space appropriate for resonances

\[2\]We consider only transverse $\rho$ mesons, because the longitudinal ones are not strongly affected by the p-wave self energy, since they do not couple to the $N^*(1720)$ in leading order.
Figure 3: The $\rho$ meson spectral function at $q = 750$ MeV in vacuum (full line) and in nuclear matter at $\rho_B = 2\rho_0$ (dashed line). The arrows show the position of the unperturbed levels in the zero width limit.

embedded in the $\rho$ meson self energy diagrams. This point will be discussed in more detail below. In order to be consistent with the low-density expansion, we employ the vacuum values for the $\rho$ meson and baryon resonance widths. The vector-meson propagator $D_\rho(\omega, \vec{q})$ can be represented with the help of the spectral function $A(\omega, \vec{q})$

$$D_\rho(\omega, \vec{q}) = \int d\omega' \frac{A(\omega', \vec{q})}{\omega - \omega' + i\varepsilon}$$  \hspace{1cm} (21)

$$A(\omega, \vec{q}) = -\frac{1}{\pi} \frac{\text{Im} \Sigma(\omega, \vec{q})}{[\omega^2 - (m_\rho)^2 - \text{Re} \Sigma(\omega, \vec{q})]^2 + [\text{Im} \Sigma(\omega, \vec{q})]^2},$$  \hspace{1cm} (22)

where $\Sigma = -im_\rho \Gamma_\rho + \Sigma_\rho^R$. We retain only the imaginary part of the $\rho$ meson self energy in vacuum. The resulting spectral function is shown in Fig. 3 at $q = 750$ MeV and $\rho_B = 2\rho_0$ together with the spectral function of a $\rho$ meson in vacuum, which has an energy $\omega_q = 1.1$ GeV. Note that at finite momentum the strength in
medium is moved down to energies far below the unperturbed $N^*N^{-1}$ and $\rho$ meson peaks. At finite momenta one expects three states to contribute to the spectral function, corresponding to the $\rho$ meson, and the two resonance-hole states $N^*N^{-1}$ and $\Delta N^{-1}$. The arrows in Fig. 3 indicate their unperturbed energies in the zero-width limit. Due to the interactions, the levels mix, so that e.g. the lowest level is not a pure $N^*N^{-1}$ state, but it acquires some $\rho$ meson strength. Furthermore, the levels are pushed apart (level-level repulsion), and the strength is accumulated in the upper and lower levels, leaving very little in the middle one. This effect, combined with the strong smearing of the spectral strength due to the large widths of the $\rho$ meson and the baryon resonances are responsible for the fact that there is no structure in the spectral function at finite density corresponding to the original $\rho$ meson peak.

We stress that it is important to take the energy dependence of the widths properly into account. The free width of the $\rho$ meson is completely dominated by decay into two pions. We account for the energy dependence of this decay process by employing the standard p-wave form (eq. (12)). The energy dependence of the widths of the baryon resonances, is more complicated because there are several important decay channels. We assume that the phase space is given by the decay of the resonance into the $\pi N$ channel. The approximate treatment of the phase space is reasonable close to threshold and is not expected to affect the results at large energies where all widths are large anyway. Furthermore, we must express the phase space available for the decay of a resonance embedded in a self energy diagram in terms of the $\rho$ meson energy and momentum. This is done in an approximate way using

$$\Gamma_{N^*}(\omega, \vec{q}) = \Gamma_{N^*}^{(0)} \left( \frac{p}{p_{N^*}} \right)^3,$$

(23)
where
\[
p = \sqrt{\left( (s - m_N^2 - m_{\pi}^2)^2 - 4m_N^2m_{\pi}^2 \right) / 4s},
\]
\[
p_{N^*} = \sqrt{\left( (m_{N^*}^2 - m_N^2 - m_{\pi}^2)^2 - 4m_N^2m_{\pi}^2 \right) / 4m_{N^*}^2}.
\]
and \(\Gamma_{N^*}^{(0)} = 150\) MeV. An analogous expression is obtained for the \(\Delta(1905)\). The invariant mass \(s\) is computed in the static approximation for the nucleon, \(s = (\omega + m_N)^2 - q^2\).

![Diagram of lepton pair production](image)

Figure 4: The processes contributing to lepton pair production in the model. The bottom diagram illustrates the scattering processes implied by the resonance-hole contributions.

The effect of the energy dependence is clearly seen in the in-medium spectral function. The low lying peak is relatively narrow because here the phase space for the resonance decays is relatively small. On the other hand, the structures at higher
energy are smeared out by the large widths. The low-energy peak carries about 20% of the strength at $q = 750$ MeV. As we show below, this gives rise to a qualitative change of the lepton-pair spectrum.

The processes contributing to the $e^+e^-$ production are shown in Fig. 4a-d. The corresponding rate for the production of $e^+e^-$ pairs per unit volume, unit time and a given invariant mass is given by

$$\frac{dN_{e^+e^-}}{d^3xdtdm} = \frac{8\alpha^2}{3\pi^2g_{\rho\pi\pi}^2m}\int_0^\infty dq \frac{q^2}{\sqrt{q^2 + m^2}}F(\sqrt{q^2 + m^2}, q) \frac{1}{\exp \beta \sqrt{q^2 + m^2} - 1}, \quad (24)$$

where $\alpha = e^2/4\pi$ and $g_{\rho\pi\pi} = 6.0$ is the $\rho\pi\pi$ coupling constant. We include only the contribution of the transverse $\rho$ mesons, which are affected by the p-wave self energy and treat the electrons and positrons as ultra relativistic, i.e., we neglect the electron mass. The function $F(\omega, q)$ is given by

$$F(\omega, q) = m_\rho \Gamma_\rho |d_\rho(\omega, q) - 1|^2 - \text{Im}\Sigma^{N^*}_\rho(\omega, q)|d_\rho(\omega, q) - r_{N^*}|^2 - \text{Im}\Sigma^\Delta_\rho(\omega, q)|d_\rho(\omega, q) - r_\Delta|^2, \quad (25)$$

where

$$d_\rho(\omega, q) = \frac{m^2 - r_{N^*}\Sigma^{N^*}_\rho - r_\Delta\Sigma^\Delta_\rho + im_\rho \Gamma_\rho}{m^2 - m_\rho^2 - \Sigma^{N^*}_\rho - \Sigma^\Delta_\rho + im_\rho \Gamma_\rho}. \quad (26)$$

In the limit $r_{N^*} = r_\Delta = 1$ eq. (25) reduces to the standard VMD result $F(\omega, q) = m_\rho^4 \text{Im}D_\rho$, where the rate is proportional to the imaginary part of the $\rho$ meson propagator. The realistic values of $r_{N^*}$ and $r_\Delta$ are much smaller than unity (see above).

Because the resonances are unstable, the resonance-hole annihilation diagrams de facto correspond to scattering processes, like the one shown in Fig. 4e. Consequently, these processes produce $e^+e^-$ pairs at all invariant masses $m > 2m_e$. The $\rho$ mesons in these diagrams correspond to the full propagator, including the self energy contributions in Fig. 2. For consistency the $\gamma\rho$ transition vertex is also dressed.
by the same diagrams (see Fig. 5). This gives rise to the terms proportional to \( r_{N^*} \) and \( r_\Delta \) in the numerator of \( d_\rho \). The term proportional to \( \Gamma_\rho \) is due to the two-pion loop diagram in Fig. 5. We neglect the temperature dependence of the self energy and the vertex renormalization. The main effect of the finite temperature is that a fraction of the baryons are in the form of resonances, mainly the \( \Delta(1232) \). It is difficult to correct for this, since the couplings of these resonances to the \( \rho \) meson and higher mass baryon resonances are not known. A crude estimate of the effect can be obtained by identifying the density in eq. 19 with the nucleon density. This is tantamount to setting the unknown coupling constants to zero.

In Fig. 6 the resulting production rate for \( e^+e^- \) pairs is shown for various densities, at a temperature of \( T = 140 \) MeV. For comparison the rate due to \( \pi^+\pi^- \) annihilation computed with the \( \rho \) meson propagator at \( \rho_B = 0 \) is also shown. The rates include an integral over all pair momenta, but no corrections for experimental acceptance. Clearly there is a strong enhancement of the population of lepton pairs with invariant masses around 300-500 MeV. At finite baryon density and finite temperature the dilepton spectrum in the mass range above the \( \rho \) resonance is modified equally due to \( \pi\pi \) (figs.4a,b), \( N^*(1720) \)-hole and \( \Delta \)-hole (figs.4c,d) annihilation. In the mass region of interest, below the \( \rho \) meson peak, only the \( \pi\pi \) and \( N^*(1720) \)-hole self energies of the \( \rho \) meson remain important. For invariant masses below \( m = 400 \) MeV the \( N^* \)-hole contribution dominates the lepton pair spectrum. This justifies
the crude treatment of the f-wave channel in the width of the $\Delta(1905)$.

In Fig. 7 we show the dilepton production rate at twice nuclear matter density and temperature $T = 140$ MeV with cuts in the transverse momentum of the lepton pair. The $\rho N$ interaction in the p-wave channel has a large effect on large transverse momenta ($q > 500$ MeV), whereas the shape of the spectrum at small transverse momenta ($q < 500$ MeV) is fairly close to that at vanishing density. The strong momentum dependence of the spectrum implies that a binning of the data in both momentum and invariant mass of the lepton pairs would be a crucial test of the p-wave $\rho$-nucleon interactions in nuclear matter.

We have also computed the corresponding photon production rate, which is due only to the resonance-hole annihilation processes. The so obtained photon
Figure 7: The rate for lepton pair production at $T = 140$ MeV and $\rho_B = 2$ for momenta $q < 500$ MeV (dashed line) and $q > 500$ MeV (long-dashed line). For comparison the rate for $\rho_B = 0$ is also shown (full line).

production rate is comparable to the rates of the dominating processes ($\pi\pi \rightarrow \rho\gamma$ and $\pi\rho \rightarrow \pi\gamma$) in the calculation of Kapusta et al. at photon energies between 0.5 and 1 GeV and far below those rates at higher energies. Consequently, we do not expect the p-wave mechanism to be in conflict with the photon spectra in heavy-ion collisions.

4 Discussion and conclusions

We have computed the low mass enhancement of the lepton-pair production rate due to $\rho N$ interactions, where the nucleon is excited into the $N^*(1720)$ and $\Delta(1905)$ resonances. The $N^*(1720)$ resonance is very clearly seen in $\pi N \rightarrow \pi\pi N$ reactions with a resonant $\rho(\pi\pi)$ final states. Furthermore, the mass of the $N^*(1720)$ is close
to the sum of the $\rho$ meson and nucleon masses. Therefore, it seems natural that an important contribution to the $\rho$ meson self energy in baryonic matter is due to this resonance.

We note that the calculation presented here does not address the problem of low mass dileptons at finite temperature and vanishing baryon density. In this case the interactions with the surrounding mesons \[29\] are important. The time scale set by the dilepton invariant mass ($\tau \sim m^{-1}$) allows the quark antiquark pair to probe long distance interactions. Thus, low-mass lepton pairs explore non-perturbative gluon configurations at high temperatures. Because of the high density of hadrons in heavy-ion collisions at ultra-relativistic energies the studies within hadronic models should definitely be supplemented by further analysis in terms of quark and gluon degrees of freedom. Dynamical properties of constituent quarks at finite temperature may be reflected in the lepton pair spectrum \[30\].

The properties of $\rho$ mesons in matter can also be studied with QCD sum rules \[31\]. Recently, this approach has been extended to $\rho$ mesons at finite momentum \[32\]. Preliminary results indicate that the p-wave transverse $\rho$-polarization operator is attractive as in our calculation, and furthermore that the longitudinal self energy is weakly repulsive.

The experimentally observed low mass enhancement cannot be reproduced by transport models without modifications of the $\rho$ meson properties at high temperatures and densities. In this paper we consider the production rate for lepton pairs in a high temperature high density system in thermodynamic equilibrium. For a direct comparison with experiment a realistic description of the collision dynamics, as given e.g. by a transport simulation of nucleus-nucleus collisions, is needed. In recent work, Rapp, Chanfray and Wambach \[33\] have included the p-wave polarization of the $\rho$ meson discussed here in their dynamical model. They find quantitative
agreement with the CERES data. A substantial part of the low mass enhancement is in their model due to the p-wave mechanism.

The CERES and HELIOS-3 data can also be interpreted in terms of a model with reduced vector meson masses in matter \cite{12, 13, 14, 34}. There is an important difference between such a model with an s-wave self energy and our model with a p-wave $\rho$ meson self energy. The differential cross section for lepton pairs at large transverse momenta is strongly enhanced by the p-wave self energy, while the cross section at small transverse momenta is almost unchanged. This is a distinctive feature which allows one to experimentally discriminate between the two models. In reality probably both medium effects will be present, but the transverse momentum dependence can be used to pin down the relative importance of the s- and p-wave contributions to the $\rho$ meson self energy in nuclear matter.

![Figure 8: Same as Fig. 6 but for $T = 80$ MeV.](image-url)
We note that the relative enhancement of the low mass lepton pair spectrum due to the p-wave mechanism is stronger at lower temperatures. In Fig. 8 we show the production rate at $T = 80$ MeV, which together with a density in the range 2-3 $\rho_0$ represents the conditions probed in heavy-ion collisions at GSI energies (1-2 GeV/A). Because the temperatures are moderate, these experiments explore the properties of matter where the baryon density plays a key role. For such a system, the approximation where we neglect the temperature dependence of the polarization operator is a reasonable first approximation. Thus, the future dilepton experiment at GSI, HADES, will be sensitive not only to shifts of the $\rho$ meson mass, but also to the $\rho$ meson p-wave self energy in dense matter. Forthcoming experimental efforts both at GSI and CERN will provide more accurate data, which may pin down the mechanism for medium modifications of vector mesons in matter. Complementary information on the properties of vector mesons in cold matter can be obtained by studying $\rho$ meson production off nuclei with photon and hadron beams.

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