Effect of thermalized charm on heavy quark energy loss

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The recent experimental results on the flow of $J/\psi$ at LHC show that ample amount of charm quarks is present in the quark gluon plasma and probably they are thermalized. In the current study we investigate the effect of thermalized charm quarks on the heavy quark energy loss to leading order in the QCD coupling constant. It is seen that the energy loss of charm quark increases considerably due to the inclusion of thermal charm quarks. Running coupling has also been implemented to study heavy quark energy loss and we find substantial increase in the heavy quark energy loss due to heavy-heavy scattering at higher temperature to be realized at LHC energies.

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INTRODUCTION

The genesis of a novel state of matter, the quark-gluon plasma (QGP) presumably formed in the recent experiments like Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) provides us a scope for rich investigative physics about hot and/or dense Quantum Chromodynamics (QCD) matter [1]. One of the excellent probes of QGP, formed in such collider experiments, is the quenching of jet as first anticipated by Bjorken [2]. The jet quenching or in other words energy loss of the fragmenting partons at high transverse momentum \( p_T \) is the quenching of jet as first anticipated by Bjorken [2]. The jet quenching physics about hot and/or dense Quantum Chromodynamics (QCD) matter [1]. One of the excellent probes of QGP, like Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) provides us a scope for rich investigative physics about hot and/or dense Quantum Chromodynamics (QCD) matter [1].

The motion of a heavy quark in QCD medium looks similar to that of a test particle in the plasma. Hence, its motion can be described as a typical Brownian motion problem where quarks are executing random walk in the medium. Kinematics of the test particle in the plasma is described by Boltzmann equation. In the current scenario

I. COLLISIONAL ENERGY LOSS

The motion of a heavy quark in QCD medium looks similar to that of a test particle in the plasma. Hence, its motion can be described as a typical Brownian motion problem where quarks are executing random walk in the medium. Kinematics of the test particle in the plasma is described by Boltzmann equation. In the current scenario
since there are no external force and inhomogeneity present in the system, Boltzmann equation reduces to,

$$\frac{\partial f_p}{\partial t} = -C[f_p],$$

(1)

R.H.S of the above equation represents the collision integral. In the present paper we consider a high energy heavy (Q) quark of mass $m_1$, energy $E_p$ and momentum $p$ propagating through a plasma consisting light quarks ($q$), gluons ($g$) and charm quark in equilibrium at a temperature $T$. The injected heavy quark has small fluctuating part ($\delta f_p, f_p^0 \gg \delta f_p$) which vanishes suffering collisions with other particles of the medium. In the present work we are interested only in the 2 $→$ 2 ($P + K \rightarrow P' + K'$) scattering processes. The explicit form of the collision integral then becomes,

$$C[f_p] = \frac{1}{2E_p} \int \frac{d^3k}{(2\pi)^3 2E_k} \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{d^3k'}{(2\pi)^3 2E_{k'}} \delta f_p[PSF] \times (2\pi)^4 \delta^4(P + K - P' - K') \frac{1}{2} \sum_{\text{spin}} |\mathcal{M}|^2,$$

(2)

where, $[PSF]$ denotes the phase space factor. $|\mathcal{M}|^2$ in the above equation contains the information about the interaction concerned. In a $P+K \rightarrow P'+K'$ scattering process, the energy and momentum variables are denoted as $X = (E_X, \vec{X})$, (where $X = P, K, P', K'$). The thermal phase space here contains the information of elastic scattering processes (Fig.1). The form of the phase space changes with the nature of the scatterings. In case of heavy and light quark scatterings the possibility of back scatterings can be excluded whereas for the scatterings between two heavy quarks one has to consider back scatterings. Details of phase-space factor for different processes will be discussed later in the present paper.

Now, Eq.(1) in the relaxation time approximation can be expressed as,

$$\frac{\partial \delta f_p}{\partial t} = -C[f_p] = -\delta f_p \Gamma(p).$$

(3)

where $\Gamma(p)$ can be recognized as the particle interaction rate, given by

$$\Gamma(p) = \frac{1}{2E_p} \int \frac{d^3k}{(2\pi)^3 2k} \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{d^3k'}{(2\pi)^3 2E_{k'}} [PSF](2\pi)^4 \delta^4(P + K - P' - K') \frac{1}{2} \sum_{\text{spin}} |\mathcal{M}|^2.$$

(4)

One obtains heavy quark energy loss ($-dE/dx$) by averaging over the interaction rate times the energy exchange per scattering and dividing by the velocity of the injected heavy quark,

$$\frac{dE}{dx} = \frac{1}{\bar{v}_p} \int d\Gamma \omega.$$

(5)

The factor $\omega$ in Eq.(5) is essential in making $dE/dx$ finite within resummation perturbation theory. The calculation of interaction rate mentioned in Eq.(4), using tree level diagram suffers quadratic infrared divergence. Whereas, in case of energy loss the extra $\omega$ factor softens the infrared divergence to only logarithmic. However, the resummed effective boson propagator makes $\Gamma$ to be only logarithmically divergent which in turn makes $-dE/dx$ finite $[12, 13]$. The energy loss of a heavy-quark propagating through a hot quark-gluon plasma can be calculated either from the field theory approach or using effective kinetic theory. In the present paper we follow the latter one.

In order to have detailed discussion of charm quark energy loss it is necessary to have an idea of number of scattering centers present in the bath. From the last equation it is evident that to estimate energy loss one has to have the information about the interaction rate. Now, it is well known that $\Gamma = n \sigma v$, where $n$ is the density of the plasma particles, $\sigma$ is the collision cross section and $v$ is the velocity of the particle, which is equal to the velocity of light in
Next, we introduce an energy variable in Eq. (7), as |
\begin{align*}
\text{gluon has momentum of the order of } gT
\end{align*}
where, $\bar{\omega}$ possibility of back scatterings. The factor is then given by [10], quark and anti-quark scatterings. In the phase space factor (PSF) for the current process one has to consider the hard thermal loop (HTL) resummation method to incorporate the in-medium effects.

The expression has been obtained by inserting the factor $\bar{\omega}$ in Eq.(4) and multiplying by a factor of 2 to consider both quark and anti-quark scatterings. In the phase space factor (PSF) for the current process one has to consider the possibility of back scatterings. The factor is then given by [10],

$$PSF = f_{E_k}(1 - f_{E_{k'}})(1 - f_{E_{k'}}) + (1 - f_{E_k})f_{E_{k'}}f_{E_{k'}}$$

$$\approx \frac{df_{E_k}}{dE_k} q_0 T \left( \frac{T}{q_0} + 1 \right),$$

where, $f_{q_0}$ is the boson distribution function and is given by $f_{q_0} = (\exp(q_0/T) - 1)^{-1}$ and $q_0 = \omega$.

In the present section first we present the derivation of heavy quark energy loss in the soft regime defined earlier as $|q| \ll |q^*|$. The soft contribution to the energy loss is evaluated in the region of phase space where the exchanged gluon has momentum of the order of $gT$. In this kinematical region, the gluon propagator has to be modified using the hard thermal loop (HTL) resummation method to incorporate the in-medium effects.

We start our derivation from Eq. (7). Using the 3-momentum delta function we eliminate the $p'$ integral appearing in Eq. (7),

$$\int d^3p' \delta^{(3)}(p + k - p' - k') = 1.$$

Next, we introduce an energy variable $\omega$ in the integrand of Eq. (7) in the following way,

$$\delta(E_p + E_k - E_{k'} - E_{k'}) = \int_{-\infty}^{\infty} \delta(E_p - E_{k'} - \omega) \delta(\omega - E_{k'} + E_k) d\omega.$$
Two delta functions of the energy variable can then be written as,

\[
\delta(E_p - E_{p'} - \omega) = \frac{E_{p'}}{E_p} \delta \left( \omega - \bar{v}_p q - \frac{t}{2E_p} \right),
\]

\[
\delta(\omega - E_{k'} + E_k) = \frac{E_{k'}}{E_k} \delta \left( \omega - \bar{v}_k q' + \frac{t}{2E_k} \right),
\]

where, \( \bar{q} = \vec{p} - \vec{p}' = \vec{k} - \vec{k}' \) and \( \bar{v}_k = \vec{k}/E_k \). From now onwards it is convenient to change the integration variables from \( k, k' \) to \( k \) and \( q \) respectively. Using the expressions of the \( \delta \) functions given above we write the energy loss of the heavy quark given in Eq. (7) as,

\[
\left( -\frac{dE}{dx} \right)_{\text{soft}}^{QQ \rightarrow QQ} = \frac{(2\pi)^4}{(2\pi)^3 E_p 2v_p} \int d^3k \int d^3q \int_{-\infty}^{\infty} d\omega \left[ \frac{E_{k'} E_k}{E_p E_k E_k} \right] [\text{PSF}] \delta \left( \omega - \bar{v}_p q - \frac{t}{2E_p} \right) \delta \left( \omega - \bar{v}_k q' + \frac{t}{2E_k} \right) \omega |\mathcal{M}|^2.
\]

In order to proceed further, it is necessary to introduce the form of the interaction. In the Coulomb gauge the matrix element \( \mathcal{M} \) for the \( QQ \rightarrow QQ \) process can be expressed as follows [7],

\[
\mathcal{M} = g^2 D_{\mu
u}(q) \bar{u}(p', s') \gamma^\mu u(p, s) \bar{u}(k', \lambda') \gamma^\nu u(k, \lambda),
\]

where, \( g^2 = 4\pi\alpha_s \) is the color charge and \( \alpha_s \) is the strong coupling constant. In the above mentioned gauge, only non-vanishing components of the bosonic propagator are,

\[
\Delta^{00}(Q) = \Delta_L(q_0, q),
\]

\[
\Delta^{ij}(Q) = \Delta_T(\delta^{ij} - \bar{q}^i q^j).
\]

\( \Delta_L \) and \( \Delta_T \) are the longitudinal and transverse components of the boson propagator and are given by [32],

\[
\Delta_L(Q) = \frac{-1}{q^2 + m_D^2 \left( 1 - \frac{t}{2} \log \left( \frac{x_1 + \frac{1}{2}}{x_2} \right) \right)},
\]

\[
\Delta_T(Q) = \frac{-1}{q_0^2 - q^2 - \frac{m_D^2}{2} \left( x_2 + \frac{x_1 - x_2}{x_2} \log \left( \frac{x_1 + \frac{1}{2}}{x_2} \right) \right)},
\]

where, \( m_D \) is the Debye mass and \( x = \omega/q \).

Following Eq. (17) the non-zero components of the matrix element are [1],

\[
\mathcal{M} = g^2 \Delta_L(q) \bar{u}(p', s') \gamma^0 u(p, s) \bar{u}(k', \lambda') \gamma^0 u(k, \lambda)
\]

\[
+ g^2 \Delta_T(q) (\delta^{ij} - \bar{q}^i q^j) \bar{u}(p', s') \gamma^i u(p, s) \bar{u}(k', \lambda') \gamma^j u(k, \lambda).
\]

The matrix element given above has to be squared, averaged over initial spin \( s \) of the jet and summed over final spins. After evaluating the Dirac traces, we obtain,

\[
\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 = 32 g^4 E_p^2 \left[ |\Delta_L(Q)|^2 E_k^i + 2E_k \left| \Delta_L(Q) E_k^i \right| \right]
\]

\[
+ \left| \Delta_T(Q) \right|^2 \bar{k}^i \left[ \bar{v}_p \bar{k} - (\bar{q} \bar{k}) (\bar{q} \bar{v}_p) \right] \Re \left( \Delta_L(Q) \Delta_T(Q)^* \right).
\]

While, writing the above equation it has been assumed that leading order contributions come from the region where the exchanged energy \( \omega \) is small. Hence, \( E_{p'} \simeq E_p \) and \( E_k \simeq E_k \). The soft contribution to the heavy quark energy loss then reduces to,

\[
\left( -\frac{dE}{dx} \right)_{\text{soft}}^{QQ \rightarrow QQ} = \frac{g^4 C_F}{4\pi^3} \int dq \int dk \int_{-\omega}^{\omega} \frac{1}{E_k} \int_{v_q q}^{v_k q} \omega^2 (-) n_F(E_k) d\omega
\]

\[
\times \left\{ |\Delta_L(Q)|^2 \frac{E_k^2}{v_k} + |\Delta_T(Q)|^2 \bar{k}^i \left[ \bar{v}_p \bar{k} - (\bar{q} \bar{k})(\bar{q} \bar{v}_p) \right] \right\}.
\]
The energy loss $dE/dx$ of a charm quark as a function of its momentum for $T = 400$ MeV (left panel) and $T = 600$ MeV (right panel).

The above equation has been derived by using the following integrals over the angles of $k$,

$$
\int \frac{d\Omega_k}{4\pi} \delta(\omega - \hat{v}_k \cdot \hat{q}) = \frac{1}{2v_k q};
$$

$$
\int \frac{d\Omega_k}{4\pi} \delta(\omega - \hat{v}_k \cdot \hat{q}) \left[ \hat{v}_p \cdot \hat{k} - \frac{\omega}{v_k q} (v_k q).\hat{k} \right] = 0;
$$

$$
\int \frac{d\Omega_k}{4\pi} \delta(\omega - \hat{v}_k \cdot \hat{q}) \left[ \hat{v}_p \cdot \hat{k} - \frac{\omega}{v_k q} (v_k q).\hat{k} \right]^2 = \frac{1}{4v_k q} \left[ 1 - \frac{\omega^2}{(v_k q)^2} \right] \left[ v_p^2 - \frac{\omega^2}{(v_k q)^2} \right].
$$

The $\omega, q$ and $k$ integration cannot be performed analytically and have to be solved numerically.

It is to be noted that contribution from the integration domain where screening effect is absent can easily be extracted from Eq.(18) by putting screening mass zero in the denominator of the propagator [36]. Final result of heavy quark energy loss scattering off thermalized charms in the medium is obtained by adding both the soft and hard contributions,

$$
\left. \frac{-dE}{dx} \right|_{QQ\rightarrow QQ} = \frac{-dE}{dx} \bigg|_{QQ\rightarrow QQ}^{soft} + \frac{-dE}{dx} \bigg|_{QQ\rightarrow QQ}^{hard}
$$

In this regard it would be worthwhile to mention the contribution of scatterings of heavy quark with other particles of the medium. The results for scatterings with light quarks can be obtained from Eq.(20) in the limit $v_k \rightarrow 1$ and the result matches with the findings of Ref.[13]. The contribution of $Qg \rightarrow Qg$ scatterings can be read from Eqs.(1), (6) and (7) of Ref.[13]. Hence, the complete expression for heavy quark energy loss can be obtained by adding all possible scatterings mentioned in Eq.(20) and in Ref.[13].

II. RESULTS

In the present paper we calculate the energy loss of a heavy quark in a medium where, in addition to the light quarks and gluons, thermalized heavy quarks are also present. The heavy quark loses energy in the hot medium via all $t, s$ and $u$ channel processes. In Fig.(2) the total heavy quark energy loss due to scatterings with medium particles has been compared with the known result of light quarks and gluons scatterings [13]. We also plot the contribution of heavy quark scatterings off thermalized heavy quarks in the medium. In this calculation, the momentum has been scaled to an upper limit of $q_{max} = \sqrt{4E_p T}$ in compliance with the results in [12]. With our present calculation, we present plots of $-dE/dx$ with the momentum, considering the temperatures of 400 MeV and 600 MeV respectively relevant to the plasma temperature produced at LHC. For the plots the heavy quark mass has taken to be 1.25 GeV and strong coupling constant $\alpha_s = 0.3$.

From the two plots in Fig.(2) it is evident that the contribution to the energy loss due to $QQ \rightarrow QQ$ scatterings increases with temperature like other two contributions. The observation is consistent with the earlier findings of
number densities in the current paper. Increase in temperature increases number densities which in turn increases interaction rate as well as particle energy loss. It is observed that by including the process $QQ \rightarrow QQ$ the total heavy quark energy loss increases by 5% and 8% for temperatures 400 MeV and 600 MeV respectively at momentum 25 GeV under inclusion of the heavy quark scattering.

In the above mentioned treatment we assume that $\alpha_s$ is fixed. However, it has been shown in many calculations \cite{18, 30, 31} that the energy loss of light as well as heavy quarks increases noticeably when $\alpha_s$ is running. In one of the parametrization of $\alpha_s$ \cite{30, 31}, the renormalization group approach at finite $T$ has been invoked and it has been assumed that $\alpha_s$ depends on both the momentum scale and $T$. Here, we use a standard parametrization of running coupling at finite temperature which obeys the perturbative ultraviolet nature and the 3D infrared fixed point. The form of running coupling is given by \cite{31},

$$
\alpha_s(k, T) = \frac{u_1 k}{1 + \exp \left( u_2 \frac{k}{T} - u_3 \right)} + \frac{v_1}{(1 + \exp \left( v_2 \frac{T}{k} - v_3 \right)) \ln \left( e + \left( \frac{k}{\Lambda_s} \right)^a + \left( \frac{k}{\Lambda_s} \right)^b \right)},
$$

(21)

where the values of the parameters have been taken from Refs.\cite{30, 31}. Running coupling constant has been implemented to calculate quark energy loss in \cite{18, 30} in presence of light quarks and gluons as bath particles. Using the expression of $\alpha_s$, we here evaluate the energy loss with charm quarks as particles of the medium.

In the left panel of Fig. 3, a comparative study has been presented of the charm quark energy loss due to scattering with another heavy quark with constant and running coupling constant for a temperature of 0.4 GeV. In the right panel of Fig. (3), similar comparisons have been performed for a fixed temperature of 0.6 GeV. In both of these panels, we find that the energy loss of the heavy quark under the inclusion of the t-channel $QQ \rightarrow QQ$ scattering increases with temperatures as already obtained in the constant $\alpha_s$ case. We also find that the energy loss is significantly higher when the coupling constant is taken to be running (Eq. (21)). These findings are consistent with the results obtained in Refs.\cite{18, 30}.

III. SUMMARY AND CONCLUSIONS

In this study we have illustrated the theory of heavy quark energy loss in a hot QCD plasma. More specifically our main concern in the current paper has been to find the impact of thermalized charms on the heavy flavor energy loss. In all the previous calculations of charm quark energy loss it has been assumed that the thermal bath is devoid of heavy quarks and consists of only light quarks and gluons. But recent ALICE data has revealed the fact that formed $J/\psi$ exhibits a large elliptic flow, which is possible only if charm quarks gets thermalized in the medium. We, in this work have revealed the fact that the number of heavy quarks in the bath is not negligible at temperatures relevant to the ongoing experiment at LHC. Hence, we have elucidated a consistent formalism of heavy quark energy loss, where all possible scatterings with the particles of the medium (heavy quarks, light quarks and gluons) have been taken into consideration. From the numerical study presented in the paper it is observed that the formalism has significant
effect on the charm quark energy loss. It has also been observed that with the increase in temperature heavy quark energy loss by scattering with heavy quarks in the bath increases thereby enhancing the total energy loss. In addition, the heavy quark energy loss increases significantly under inclusion of running coupling. The observation is consistent with the nature of number densities of plasma particles with temperature. Increase in temperature increases number densities of the bath particles which in turn increases interaction rate and charm quark energy loss as well.

In fact, this mechanism proves to be quite an efficient one for the energy transfer into the plasma which might have possible implications in the explanation of the regeneration of $J/\psi$ at LHC. Effects of current findings on different physical observables at LHC will be reported elsewhere.

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