Study on Reliability Modeling of NC Machine Tool Based on Self-expansion Method

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Abstract: With the improvement of the reliability of NC machine tools, it is becoming more and more difficult to obtain a large number of fault data. To deal with this problem, the improved Bootstrap sampling method is proposed. The failure data, which is gained by using seven a type of NC machine tools to monitor failure data for half year, is used respectively for the improved Bootstrap sampling method and the classic series research method. The results have been analyzed and calculated, and show that the Bootstrap sampling method’s relative error is obviously lower than the classic series research method’s. The improved Bootstrap sampling method is proved to be feasible in reliability modeling.

1. Introduction
The reliability modeling of CNC machine tools is the premise for the reliability evaluation of machine tools. Under the continuous improvement of the manufacturing industry, the reliability of machine tools has also been greatly improved. Most of the machine tool fault data is small sample data which is difficult to obtain. In the case of small sample data using classical statistical method, the results will show a large deviation. This paper uses the improved Bootstrap sampling method to make up for the lack of data, which improves the accuracy of modeling and has practical significance for reliability modeling.

Currently, there are many methods for data modeling. For the fault data of small sample types, Shen Guixiang first calculates the Weibull distribution model, and then uses the parameter deviation correction method to correct the Weibull distribution model. Although the modified Weibull model is closer to the empirical distribution function, the fault data that less than Three cannot be corrected[1].

Anderson-Cook proposed to combine the expert judgment of Bayes method with system data, component data and subsystem data and applied it to reliability modeling of strategic missile systems[2]. Ren Lina and others proposed a comprehensive evaluation method for Bayes reliability model of CNC machine tools based on model parameters, Monte Carlo simulation errors, BGR-diagnosis principles, DIC information standards and the interval length estimated by reliability indicators. It provides a reference for the model selection of Bayesian incomplete maintenance model[3]. Qian Hao proposed the Bootstrap combined with the Bayes calculation method and used the improved Bootstrap-Bayes calculation method. Through comparative analysis, the feasibility of the reliability modeling method based on Bootstrap-Bayes was verified, and the reliability of the CNC machine tool was assessed using Visual Basic 6.0 software, which makes the process more accurate and fast and more conducive to data comparison analysis[4]. Sun Huiling has improved the Bayes Bootstrap method. The principle is self-expansion the sample data without changing the original
sample data, and then use the Bayes Bootstrap method to estimate the extended data. However, this method only calculates the interval estimate of the parameter $\mu$ and does not consider the parameter $\sigma$. This paper uses the improved Bootstrap sampling method to increase the number of fault data of each group, and uses the classical statistical method to model, and finally analyzes and explains the specific sample data. This method has certain engineering application significance for the small sample reliability modeling of CNC machine tool, which is useful for engineering application of small sample reliability modeling of CNC machine tools.

2. Reliability Model of Classic Algorithm

(1) Collection of reliability data for CNC machine tools.

The collected fault data is first classified, counted in the qualified data, and then the data is arranged and grouped.

(2) Establishing Weibull distribution model.

Although the exponential distribution is simple and convenient, the failure efficiency during the early failure period is not constant, which affects the accuracy of the modeling. Normal distribution is usually suitable for fatigue analysis models of metal materials. The Weibull distribution has a wide applicability, not only for parameters of various shapes, but also for expressing multiple reliability indicators. Therefore, the Weibull distribution is used in the modeling of classical statistics in this paper.

Failure Distribution Density of Two Parameter Weibull Distribution:

$$f(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^\beta\right]$$  \hspace{1cm} (1)

Two parameter Weibull distribution failure distribution function:

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\alpha}\right)^\beta\right]$$  \hspace{1cm} (2)

The two-parameter Weibull distribution reliability function:

$$R(t) = \exp\left[-\left(\frac{t}{\alpha}\right)^\beta\right]$$  \hspace{1cm} (3)

Two parameter Weibull distribution loss efficiency function:

$$\lambda(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1}$$  \hspace{1cm} (4)

In the formula: $t$ is time; $\alpha$ is the scale parameter; $\beta$ is shape parameters.

(3) Using software to evaluate reliability indicators.

Calculate Mean Time Between Failures ($MTBF$), Scale Parameters ($\alpha$) and Shape parameter ($\beta$) using Matlab programming.

3. Self-expansion Method

The central idea of the self-expansion method is to expand the sample data without changing the original sample data. The self-expansion method comes from the improved Bayes Bootstrap method. Improved Bayes Bootstrap methods are usually divided into two types. The first is to improve the empirical function and reconstruct the more reasonable empirical distribution function[6]. The second is to improve the Bootstrap sampling method for small samples. The purpose is to adjust the sampling method and increase the sample capacity[7]. This paper proposes an improvement to the Bootstrap sampling method. The new method is to increase the number of samples based on the Bootstrap sampling method.
3.1. The basic idea of the bootstrap method

(1) Sample data $X = (x_1, ..., x_n)$, $x_i \sim F(x)$, $i = 1, 2, ..., n$.

Empirical distribution function of sample data composition:

$$F_n(x) = \begin{cases} 
0 & x < x_{(1)} \\
\frac{k}{n} & x_{(k)} \leq x \leq x_{(k+1)} \\
1 & x \geq x_{(n)} 
\end{cases} \quad (5)$$

(2) Take $N$ samples from $F_n$:

- The computer generates random numbers $\eta$ in interval $0 \sim M$;
- Command $i = \eta \% n$;
- Sample $x_i$ with a subscript of $i$ was found in the observations as a regenerative sample $x^*$. Then $x^*$ is the required random sample. Self-expansion sample is $X^*(j) = (x^*_1, ..., x^*_n)$ in the formula $j = 1, ..., n$.

(3) Calculate:

$$R^*(X^*, F_n) = \hat{\theta} (F_n^*) - \hat{\theta} (F_n) \equiv R_n$$ \quad (6)

In the formula: $F_n^*$ is an empirical distribution function for a self-expansion sample. Replace $\theta (F_n)$ with $\hat{\theta} (F_n)$ approximation.

(4) Under given conditions of $F_n$. Using the distribution of $R_n$ to approximate the distribution of $T_n$. Then the distribution and characteristic values of unknown parameter $\theta$ can be obtained by corresponding statistical methods[5].

3.2. The self-expansion method

Assume that $x_1, x_2, ..., x_n$ is a simple random fault sample data for a machine tool. Divide $n$ fault sample data into $K$ groups. Group distance $m = 1 + 3.322 \log (n)$. The grouping of fault sample data for CNC machine tools is $\theta_1 = (x_1, ..., x_n)$, $\theta_2 = (x_{m+1}, ..., x_{2m})$, ..., $\theta_m = (x_{n-m+1}, ..., x_n)$.

(1) The data in $\theta_i = (x_1, ..., x_n)$ is arranged from small to large to get $\theta_i = (x_{(1)}, ..., x_{(n)})$.

The observation value $x_{(i)}$ is expanded to give the following neighborhood:

$$\Pi_i = \left[ x_{(i)} - \left( x_{(2)} - x_{(1)} \right) / q, x_{(i)} + \left( x_{(2)} - x_{(1)} \right) / q \right]$$

$$\Pi_j = \left[ x_{(j)} - \left( x_{(j)} - x_{(j-1)} \right) / q, x_{(j)} + \left( x_{(j)} - x_{(j-1)} \right) / q \right] \quad (7)$$

$$\Pi_m = \left[ x_{(m)} - \left( x_{(m)} - x_{(m-1)} \right) / q, x_{(m)} + \left( x_{(m)} - x_{(m-1)} \right) / q \right]$$

In the formula: $i = 2, ..., n-1$, $q \geq 2$.

(2) An extension data can also be obtained in other neighborhoods, so each set of data can expand $m$-1 data.

(3) Repeat the above two steps to calculate each grouping.

(4) The extended data of group $K$ were combined with the original sample data as regenerative samples, and the parameters were solved by classical statistical method.
4. Example validation

A total of 61 fault data were obtained from the observation of seven certain types of CNC machine tools for a period of six months. The numbers of seven CNC machine tools were $Y_1$, $Y_2$, ..., $Y_7$, as shown in table 1. Mix the seven sets of data and calculate $MTBF$, $\alpha$ and $\beta$ using classical statistical methods and use this as a benchmark. Then use the self-expansion method and directly use the classical statistical method to calculate the seven sets of data and compare them with the benchmark.

| number | MTBF (h)          |
|--------|-------------------|
| Y1     | 945.5 1264.25 2332.5 302 63.5 2591.5 2894 215.5 639.5 |
| Y2     | 178 2154 645.5 374.5 2449.25 1246.58 318.08 1240 1419.5 1337 |
| Y3     | 2842.33 215.33 230.17 2491.17 1486 1017.27 838.67 837.33 |
| Y4     | 537.25 1274 1584 3062.08 862.38 1045.5 953.67 1027.67 |
| Y5     | 1040 3602.5 194 271.5 913 399 2304.5 |
| Y6     | 397.67 2591.83 141.5 2312 1382.5 239.5 2072.5 454.5 241.83 |
| Y7     | 153.5 1873.5 409 186 184 639 1037 655.5 1375 686 |

Taking 61 data as large sample data, using the classical statistical method to establish the Weibull distribution model, the distribution model parameters are as follows: $\alpha = 1204.5$, $\beta = 1.2458$, $MTBF^{*} = 1122.7h$. Using the self-expansion method to establish $Y_1$, $Y_2$, ..., $Y_7$ respectively Weibull distribution model of seven sets of data. Taking $Y_1$ as an example, using the self-expansion method to calculate the values of each parameter. First, extend the sample data of $Y_1$ machine tool by self-expansion method to obtain the newly generated fault data $Z_1 = (12.83, 63.5, 186.67, 189.5, 215.5, 302, 537.5, 639.5, 839.25, 908.167, 945.5, 1264.25, 2332.5, 2490.67, 2591.5, 2894)$. Then the fault data $Z_1$ is calculated by using the classical statistical method. $MTBF = 1323.9h$, $\alpha = 1135.7$, $\beta = 0.7701$. Therefore, the reliability function of machine tool $Y_1$ can be obtained:

$$R(t) = \exp\left[\frac{-t}{\alpha}\right] = \exp\left[\frac{-t}{1135.7}\right]^{0.77} \quad (8)$$

The lost efficiency function is:

$$\lambda(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} = \frac{0.77}{1135.7} \left(\frac{t}{1135.7}\right)^{-0.23} \quad (9)$$

Draws the reliability function and the lost efficiency function fitting curve according to formulae(8) and(9), as shown in Figures 1 and 2.
Tak ing the method of calculating the failure data of machine tool Y1 as an example. The self-expansion method and the classical statistical method were used to calculate the failure data of Y2~Y7 successively. The estimated values of the parameters are shown in table 2.

| number | classical statistical method | The self-expansion method |
|--------|-----------------------------|---------------------------|
|        | α   | β  | α   | β  |
| Y1     | 1342.00 | 0.84 | 1135.70 | 0.77 |
| Y2     | 1302.90 | 1.25 | 1246.80 | 1.40 |
| Y3     | 1417.60 | 1.09 | 1328.40 | 1.27 |
| Y4     | 1479.80 | 1.99 | 1384.30 | 2.49 |
| Y5     | 1316.30 | 0.96 | 1207.70 | 1.10 |
| Y6     | 1164.00 | 0.93 | 1059.90 | 1.00 |
| Y7     | 808.62  | 1.22 | 950.32  | 1.49 |

Put the parameters in table 2 into formula (10) to calculate the corresponding $MTBF$, The results of calculation $\Delta MTBF$ are shown in table 3.

$$MTBF = aT \left(1 + \frac{1}{\beta}\right)$$  \hspace{1cm} (10)

$$\Delta MTBF = \frac{MTBF - MTBF^*}{MTBF^*}$$  \hspace{1cm} (11)

As can be seen from table 3, the relative error of the results calculated using the self-expansion method was significantly smaller than that of the direct classical statistical method and the maximum relative error was reduced by $15.78\%$. It is proved that the reliability model established by the self-expansion method has less error and can obtain a more accurate reliability model. Moreover, the feasibility of the self-expansion method is verified, which has application significance in engineering.

5. conclusion

(1) Now most of NC machine tool fault data belong to the small sample data. The self-expansion method proposed in this paper is to expand the fault data into large sample data without changing the original small sample data, and then uses the classical statistical method for calculation, which not only simplifies the complexity of Bayes method for small sample modeling, but also improves the accuracy of calculation results.

(2) Based on the failure data of a machine monitoring as the research object, the self-expansion method was compared with the classical statistical method. The results showed that the relative error of the results calculated by the self-expansion method was obviously less than that calculated by the direct classical statistical method, and the maximum relative error was reduced by $15.78\%$. The
self-expansion method provides some guidance for small sample fault data in reliability engineering modeling.

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