Single-photon router: enhancing the transfer rate by the boundary

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We study the transport property of single photon scattered by a two-level system (TLS) in a T-shaped waveguide, which are made of an infinite coupled-resonator waveguide (CRW) and a semi-infinite CRW. The spontaneous emission of the TLS directs single photons form one CRW to the other. Although the transfer rate is different for the wave incident from different CRW, the transfer rate could be unit for the wave incident from the semi-infinite CRW due to the break of the translational symmetry by the presence of boundary.

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I. INTRODUCTION

Quantum channel and node are the building blocks of quantum network. Photons are natural carriers in quantum channels due to their robustness in preserving quantum information during propagation. Quantum channels are made of structures that guides waves (called waveguides), which means that controlling photon transport coherently in confined geometry is of both fundamental and practical importance to build quantum networks. Due to the negligible interaction between individual photons, quantum devices at single-photon level has been proposed based on the interaction of confined propagating fields with a single atom [2,10]. Considering that a quantum network has more than one quantum channel, a multichannel quantum router [11,12] for single photons has been explored to transfer single photons from one quantum channel to the other.

It is well-known that there are two ways to change nontrivially the transport properties of particles in quantum channels: the incorporation of a single impurity or slight structural variations. The single-photon routing proposed in Ref. [11,12] has been make use of the arrangement of the elastic scatterer — the configuration of the single atom. In this paper, we propose a single-photon routing scheme using a two-level system (TLS) embedded in two quantum channels. Different from previous studies in Refs. [11,12] where two infinite one-dimensional (1D) coupled-resonator waveguides (CRWs) are used as two quantum channels, we consider a slight structural variation of the two 1D CRWs, i.e., one 1D CRW is infinite and the other is semi-infinite, which form a T-shaped waveguide [13,10]. We have study the single-photon scattering process with waves incident from either CRW. The spontaneous emission of the TLS routes single photon from one CRW to the other. The probability for finding single photon in each waveguide is different for waves incident from different CRW due to the break of the translational symmetry by the presence of boundary.

It is found that the probability for finding single photon in the infinite CRW could reach one for waves incident from the semi-infinite CRW.

This paper is organized as follows: In Sec. II the model of a TLS inside a T-shaped waveguide are introduced. In Sec. III the single-photon scattering process is studied for waves incident from either infinite CRW or semi-infinite CRW. Finally, we conclude with a brief summary of the results.

II. MODEL SETUP

A 1D CRW is made of single-mode cavities that are coupled to each other through the evanescent tails of adjacent fields, which results in a coherent hopping of photons between neighbouring cavities. Here, we consider two noninteracting 1D CRWs which forms a T-shaped waveguide. As sketched in Fig. 1, coupled resonators on the red (green) line construct the infinite (semi-infinite) CRW, which is called CRW-a (CRW-b) hereafter. The cavity modes of the two 1D CRWs are described by the annihilation operators $a_j^\dagger$ and $b_j^\dagger$, respectively, and subscripts $j_a = -\infty, \cdots, +\infty$ and $j_b = 1, \cdots, +\infty$. We model each CRW as a series of quantum harmonic oscillators with a nearest-neighbor interaction, then the free propagation of the photons in the T-shaped waveguide is described by the Hamiltonian

$$H_C = \sum_{d=a,b} \sum_{j_a} [\omega_d d_j^d d_{j_d}^d - \xi_d (d_{j_d}^d d_{j_d+1}^d + h.c.)], \quad (1)$$

In the above equation, we have assumed that all cavities in the CRW $a$ ($b$) have the same frequency $\omega_a$ ($\omega_b$) and the hopping energies $\xi_a$ ($\xi_b$) between any two nearest-neighbour cavities in the CRW $a$ ($b$) are the same. We note that there is no interaction among the $j_a = 0$ and $j_b = 1$ cavities.

A TLS (an atom, a quantum dot, or a superconducting qubit) described by the free Hamiltonian

$$H_A = \omega_A |\epsilon\rangle \langle \epsilon| \quad (2)$$

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is placed at the node of the T junction, where \( \omega_A \) is its energy splitting. The TLS characterized by a ground state \(| g \rangle\) and an excited state \(| e \rangle\) is coupled at a rate \( g_a \) and \( g_b \) to the \( j_a = 0 \) and \( j_b = 1 \) cavities. To describe the interaction between a TLS and quantized electromagnetic modes, we employ the standard model — the Jaynes-Cummings Hamiltonian

\[
H_{AC} = g_a \sigma_+ a_0 + g_b \sigma_+ b_1 + h.c. \tag{3}
\]

where the operators \( \sigma_+ \) and \( \sigma_- \) are the ladder operators of the TLS. The dynamics of the total system is governed by Hamiltonian \( H = H_C + H_A + H_{AC} \), which is the sum of three above terms in Eqs. (1), (2) and (3).

The Hamiltonian in Eq. (3) can be exactly diagonalized as \( H_C = \sum_{d=a,b} E_{kd} d_{kd}^{\dagger} d_{kd} \) by introducing the Fourier transformations \( a_{kd} = \frac{1}{\sqrt{\epsilon}} \int d j_a a_{j_a} e^{i k_d j_a} \) and \( b_{kd} = \frac{1}{\sqrt{\epsilon}} \int d j_b b_{j_b} \sin (k_d j_b) \) for the waveguides a and b respectively. The dispersion relation of both CRWs

\[
E_{kd} = \omega_d - 2 \xi_d \cos k_d \tag{4}
\]

is a cosine function of the wavenumber \( k_d \) (\( d = a, b \)), which indicates that each CRW possesses an energy band with bandwidth \( 4 \xi_d \). Consequently, two quantum channels (i.e., two broad continua of propagating modes) are formed by the confined propagating fields of these two CRWs.

### III. QUANTUM ROUTING SINGLE PHOTONS

Since the operator \( N = \sum_{d=a,b} \sum_{j_d} d_{j_d}^{\dagger} d_{j_d} + \sigma_+ \sigma_- \) commutes with Hamiltonian \( \tilde{H} \), the total number of excitations is a conserved quantity. To find the scattering equation in the single-excitation subspace, we assume the eigenstate of the full Hamiltonian

\[
|E\rangle = \sum_{d=a,b} \sum_{j_d = -\infty}^{+\infty} U_{j_d}^{[d]} d_{j_d}^{\dagger} |g0\rangle + U_e |e0\rangle \tag{5}
\]

where \(|0\rangle\) is the vacuum state of the T-shaped waveguide, \( U_{j_d}^{[a]} \) (\( U_{j_d}^{[b]} \)) is the probability amplitudes of single-photon states in the \( j_a \)th (\( j_b \)th) cavity of the CRW-a (CRW-b), and \( U_e \) is the atomic excitation amplitude. The eigenvalue gives rise to a series of coupled stationary equations for all amplitudes

\[
\begin{align}
EU_{j_d}^{[a]} &= \omega_a U_{j_d}^{[a]} - \xi_a (U_{j_d}^{[a]} + U_{j_d + 1}^{[a]}) + g_a U_e \delta_{j_d 0} \tag{6a} \\
EU_{j_d}^{[b]} &= \omega_b U_{j_d}^{[b]} - \xi_b (U_{j_d}^{[b]} + U_{j_d + 1}^{[b]}) + \delta_{j_d 0} g_b U_e \tag{6b} \\
EU_e &= \omega_A U_e + g_a U_{j_d 0}^{[a]} + g_b U_{j_d 1}^{[b]} \tag{6c}
\end{align}
\]

where \( \delta_{mn} = 1 \) \((0)\) for \( m = n \) \((m \neq n)\). Removing the atomic amplitudes in the above equation leads to a system of equations for describing the wave propagation of single photons in the T-shaped waveguide

\[
\begin{align}
(E - \omega_a) U_{j_d}^{[a]} &= -\xi_a (U_{j_d - 1}^{[a]} + U_{j_d + 1}^{[a]}) \\
&\quad + \delta_{j_d 0} \left[ V_a (E) U_{j_d 0}^{[a]} + G (E) U_{j_d 1}^{[a]} \right] \tag{7a} \\
(E - \omega_b) U_{j_d}^{[b]} &= -\xi_b (U_{j_d - 1}^{[b]} + U_{j_d + 1}^{[b]}) \quad (j \geq 2) \tag{7b} \\
(E - \omega_b) U_0^{[b]} &= -\xi_b (U_{-1}^{[b]} + U_{1}^{[b]}) \tag{7c}
\end{align}
\]

The coupling between the TLS and CRWs gives rise to the energy-dependent deltalike potentials with strength \( V_d (E) = \frac{g_d^2}{\epsilon} \) \((E - \omega_A)\) and the effective dispersive coupling strength \( G (E) = \frac{g_a g_b}{\epsilon} \) \((E - \omega_A)\) between two CRWs, which are highly localized.

In this paper, we are interested in routing single photons from one CRW to the other, i.e., the TLS acts as a multichannel quantum router, which means we aim to find the propagating Bloch waves reflected, transmitted, and transferred from the TLS. According to the results in Refs. [11, 12], we set \( \omega_a = \omega_b = \omega \) and \( \xi_a = \xi_b = \xi \) in the following discussion. Since the boundary in the CRW-b breaks the translational symmetry, we solve Eq. (7) for waves incoming from both CRWs separately.

### A. Single photons incident from the infinite CRW-a

The overlap of the spatial profile of the cavity modes allows photons to hop from one cavity to another. For a photon of wave vector \( k \) incident in the CRW-a along the \( j_a \) axis onto the T-shaped waveguide, it is first absorbed by the TLS, which transmits from its ground state to its excited state. Since the excited state is coupled to the continua of states, the excited TLS will emit a photon spontaneously into the propagating state of either CRW-a or CRW-b. Then a scattering process of single photons is completed, i.e., waves encountering the TLS result in reflected, transmitted, and transferred waves with the same energy \( E = \omega - 2 \xi \cos k \). However, the boundary of the CRW-b forces the photon within a certain region of
where at the boundary cavity ω is a function of the atomic energy splitting coupling strength g. Eξ g parameters are set as follow: gα = gb = 0.3. (b) The coefficient Tα(E) as a function of the atomic energy splitting ωA, where the parameters are set as follow: gα = gb = 0.15, E = 10 − √2 for solid line, gα = gb = 0.3, E = 10 for dotted line, gα = gb = 0.25, E = 10 + √2 for dashed line. All the parameters are in units of ξ, and we always set ω = 10.

space. We search for a solution of Eqs. (4) in the form

\[ U_{jα} = \begin{cases} e^{ikjα} + re^{-ikjα}, & j < 0 \\ te^{ikjα}, & j > 0 \end{cases} \]  

(8a)

\[ U_{jβ} = \begin{cases} tβe^{ikjβ}, & jβ > 1 \\ \frac{-2gβgβk}{vβ(E - ωA) + gβ2 sin(2k) + i(gα2 + 2gβ2 sin2k)} \end{cases} \]  

(8b)

where r, t, and tβ are the reflection, transmission, and transfer amplitudes respectively, and A is the amplitude at the boundary cavity jβ = 1. Substitution of Eqs. (4) into Eqs. (7), after simple algebra, we obtain the relation t = r + t between the transmission amplitude and the transfer amplitude with the following expressions

\[ t = \frac{vβ(E - ωA) + sin(2k)gα2 + i2gβ2 sin2k}{vβ(E - ωA) + gβ2 sin(2k) + i(gα2 + 2gβ2 sin2k)}, \]  

(9a)

\[ tβ = \frac{-2gβgβk}{vβ(E - ωA) + gβ2 sin(2k) + i(gα2 + 2gβ2 sin2k)}, \]  

(9b)

where the group velocity vβ = 2ε sin k. It can be verified that the scattering amplitudes satisfy |t|² + |r|² + |tβ|² = 1, which indicates probability conservation for the photon. It can be found in Eq. (9) that when gβ = 0, tβ = 0, and the transmission amplitude t is the same as the one obtained in Ref. [3], where the system show the resonant scattering at energy E = ωA and Γα(E) = gα2/vβ is regarded as the width of the resonance. However, there is no frequency shift of the atom. The total reflection in Ref. [3] is the interference between the spontaneous emission from two-level systems and the propagating modes in the 1D continuum. It can be found in Eq. (9b) that the coupling between the waveguide and the TLS that plays the important role on transferring single photon from one quantum channel to the other.

In Fig. 2 we plot the probabilities for finding the photon in CRW-a and CRW-b, which are denoted by coefficients Tα = |t|² + |r|² (blue line), and Tβ = |tβ|² (red line), respectively. It can be observed that: 1) the maximum transfer rate is 50%; 2) Although the magnitudes of the probabilities are dependent on the atomic energy splitting, the energy splitting of the TLS is no longer the position of the peak; 3) The product of the coupling strengths determines the width of the lineshape. Comparing to the observations in Ref. [11, 12], the only difference is the resonant-scattering energy, i.e., the second observation here.

B. Single photons incident from the semi-infinite CRW-b

Now, we consider a plane Bloch wave of single photon is launched from the upper to the bottom along the semi-infinite CRW-b with the dispersion relation E = ω − 2ε cos k. When the traveling photon arrives at the node of the T junction, it is either absorbed by the TLS or reflected by the boundary. The fraction reflected by the boundary propagates along the positive jβ semi-axis. The portion absorbed by the TLS is reemitted into the waveguide. Since the CRW-a is an infinitely long 1D waveguide, the emitted radiation propagates into two directions, namely the forward and backward directions along the CRW-a. In the CRW-b, the TLS radiates the photon to the upper and down, the photon originally radiated to the down is retroreflected to the TLS, since the termination of the CRW-b imposes a hard-wall boundary condition on the field which behaves as a perfect mirror. The probability amplitudes in the asymptotic regions are given by

\[ U_{jα} = \begin{cases} tα^μe^{-ikjα}, & jα < 0 \\ tα^μe^{ikjα}, & jα > 0 \end{cases} \]  

(10a)

\[ U_{jβ} = A sin k, \; jβ = 1 \]  

(10b)

\[ U_{jβ} = e^{-ikjβ} + rβe^{ikjβ}, \; jβ > 1, \]  

(10c)

where tα (tβ) and rβ have the meaning of the forward (backward) transfer and reflected amplitudes in the TLS-free region. The continuity of the wavefunction implies that tα = tβ = tα. With the stationary-wave scattering equation (7), the transfer and reflected amplitudes is related to the wavenumber k by the expression

\[ tα = \frac{-2gαgβk}{vβ(E - ωA) + gβ2 sin(2k) + i(gα2 + 2gβ2 sin2k)}, \]  

(11a)

\[ rβ = \frac{2iξ sin k(E - ωA + gα2 e^{−ik}) - gα2}{2iξ sin k(E - ωA + gα2 e^{ik}) - gα2}. \]  

(11b)

In this case, the probabilities for finding the photon in CRW-a and CRW-b now are changed as transfer rate
Where $g_a = 0$, the transfer amplitude vanishes. According to the probability conservation, all the incident waves get perfectly reflected. Only the phase of the reflected amplitude varies with the incident plane wave. As $g_a = 0$, the system becomes a semi-infinite CRW with a TLS inside. The hard-wall boundary due to the CRW termination reflects all the incident waves. The absorption and emission of single photons by the TLS introduce the phase different from $\pi$ to the reflected amplitude, which is originated from the radiative properties of the TLS. It is well-known that spontaneous emission of a TLS depends on the electromagnetic vacuum environment that the atom is subjected to. Here, the boundary modifies the radiation field and acts back onto the TLS. According to the method of images [17], the radiated photon is reflected by the virtual $j_b = 0$ cavity. Consequently, the excited state of the TLS is dressed by its own radiation field with the Lamb shift $\Delta(E) = g_b^2 \cos k/\xi$ and the decay rate $\Gamma_b(E) = 2g_b^2 \sin^2 k/v_g$, where the group velocity $v_g = 2\xi \sin k$. In the weak-coupling limit $g_b \to 0^+$, the transition frequency of the TLS is renormalized as $\omega_A + \Delta(\omega_A)$. Actually, the changes in the radiative rates of the TLS can be qualitatively understood by the following consideration. First, let us explain how the factor two appear in $\Gamma_b$. In an infinite CRW, the TLS radiates amplitude $\alpha$ to the upper and downward. Therefore the total rate of energy loss is proportional to $2|\alpha|^2$. In the presence of a perfect mirror, light originally radiated to the downward is reflected back toward the TLS and interferes with the light originally radiated to the upper. For constructive interference, the total amplitude is $2\alpha$. And the total rate of energy loss is proportional to $4|\alpha|^2$, which leads to twice the infinite CRW. Now, come to the factor $\sin^2 k$. For a TLS inside an infinite CRW, $g_b$ is the coupling strength between the continuum and the TLS. Consequently, $2|\alpha|^2 = \frac{g_b^2}{v_g}$. However, for the TLS inside a semi-infinite CRW, the coupling strength between the continuum and the TLS is modified as $g_b \sin k$ by the Fourier transformation. Consequently, $4|\alpha|^2 = \Gamma_b$

With $g_a \neq 0$, the CRW-a provide an extra channel for the radiated photon. Transferring becomes possible after the incident photon is absorbed by the TLS. Therefore, the transfer rate should be related to both the coupling strength $g_a$ and the modified coupling strength $g_b \sin k$ by the boundary, which is why $T_b^a$ in Eq. (11b) has the product of $g_a$ and $g_b \sin k$ in its numerator. The coupling of the TLS to the extra channel introduce additional energy loss, which is characterized by the decay rate $\Gamma_a$. Hence, the decay rate of the TLS is the sum of $\Gamma_a$ and $\Gamma_b$, which construct the imaginary part of the denominator in Eqs. (9) and (11). Since the energy-level shift caused by the CRW-a is zero, the real part of the denominator in Eqs. (9) and (11) only contains the atomic transition energy $\omega_A$ and the Lamb shift $\Delta(E)$ introduced by the semi-infinite CRW. One can also observed that the transfer amplitudes in Eqs. (9b) and (11b) have the same expression. However, the transfer rate $T_b^a$ is twice larger than $T_a^b$. According to studies in the previous section, the maximum $T_b^a$ could be one. By comparison, we plot the probabilities for finding the photon in each CRW in Fig. 3; with the parameters same to Fig. 2. Obviously, the reflectance $R_b^a$ could be lower than 50%, even down to zero. Actually, it is not difficult to find that a peak of the transfer rate occurs when the incident energy satisfies the resonant condition $E = \omega_A + \Delta(E) = 0$ for the given coupling strengths. The results in Refs. [11, 12] told us that the decay rates should be equal to further improve the transfer rate, i.e., the incident energy should further satisfies $2g_b^2 \sin^2 k = g_a^2$ (called decay-match condition), which requires that $g_a \leq \sqrt{2}g_b$. The above discussion told us that when the coupling and hopping strengths are fixed, one can adjust $\omega_A$ to achieve the maximal value one of the transfer rate for the photon with a given incident energy.

IV. DISCUSSION AND CONCLUSION

In this work we have analytically studied the scattering process of single photons in a T-shaped waveguide with a TLS embedded in the node of the T junction, where the T-shaped waveguide is made of an infinite CRW (CRW-a) and a semi-infinite CRW (CRW-b). Unlike the X-shaped waveguide, boundaries introduce new physical features. First, the probability conservation has different expression, for example, $T_b^a + T_a^b + T_b^a = 1$ for waves incident from the infinite CRW, and $R_b^a + T_b^a = 1$ for waves incident from the semi-infinite CRW. Second, the probability for successfully transferring the photon is different: the maximum magnitude of the transfer rate is 50% for waves incident from the CRW-a, however, one for waves incident from the CRW-b.

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