Toward $e^+e^- \rightarrow \pi^+\pi^-$ annihilation inspired by higher $\rho$ mesonic states around 2.2 GeV

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Very recently, the BABAR Collaboration indicated that there exist an explicit enhancement structure near 2.2 GeV when focusing on the $e^+e^- \rightarrow \pi^+\pi^-$ process again, which inspires our interest in studying the production of higher $\rho$ mesonic states. Since the branching ratio of $\pi^+\pi^-$ channel of $D$-wave $\rho$ states are much smaller than $S$-wave states, we choose $\rho(1900)$ and $\rho(2150)$ as the intermediate states in $e^+e^- \rightarrow \pi^+\pi^-$, where $\rho(1900)$ and $\rho(2150)$ are treated as $\rho(3S)$ and $\rho(4S)$ states, respectively. Our result indicates that the BABAR’s data of $e^+e^- \rightarrow \pi^+\pi^-$ around 2 GeV can be depicted well, which shows that this enhancement structure near 2.2 GeV existing in $e^+e^- \rightarrow \pi^+\pi^-$ can be due to the contribution from two $\rho$ mesons, $\rho(1900)$ and $\rho(2150)$. Additionally, this conclusion can be enforced by the consistence of the extracted values of $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)$ of $\rho(1900)$ and $\rho(2150)$ in the whole fitting processes and the corresponding theoretical calculations. The present study of $e^+e^- \rightarrow \pi^+\pi^-$ data may provide valuable information to establish the $\rho$ meson family.

I. INTRODUCTION

$e^+e^-$ annihilation process can be as an ideal platform to study vector particles. A typical example is the observation of $J/\psi$ charmonium [1]. By adopting initial state radiation method which plays crucial role to the observation of charmoniumlike state $Y(4260)$ from the $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ process [2], the BABAR Collaboration measured the $e^+e^- \rightarrow \pi^+\pi^-$ process [3] by the collected 232 fb$^{-1}$ experimental data, by which the cross section information of $e^+e^- \rightarrow \pi^+\pi^-$ from the $\pi^+\pi^-$ threshold to center-of-mass energy of 3 GeV was obtained. Very recently, the BABAR Collaboration focused on $e^+e^- \rightarrow \pi^+\pi^-$ again, and indicated that there exists an explicit enhancement structure near 2.2 GeV in the $\pi^+\pi^-$ invariant mass spectrum [4]. This phenomenon stimulates our interest in studying $e^+e^- \rightarrow \pi^+\pi^-$ since it has a close relation to establish light vector mesons around 2 GeV, which is one part of whole study of light hadron spectroscopy.

In fact, the $\pi^+\pi^-$ final state determines that $e^+e^- \rightarrow \pi^+\pi^-$ is a clean process to explore light vector $\rho$ mesons with positive $G$ parity. Generally, light vector mesons can be grouped into isovector $\rho$ meson family, isoscalar $\omega$ and $\phi$ meson families. If checking the mass spectrum of light vector meson [5–7], we may find that some higher $\rho$, $\omega$, and $\phi$ states accumulate around 2 GeV mass range, which may result in the difficulty of distinguishing them when analyzing some annihilation processes of $e^+e^-$ into light mesons. It is obvious that the pollution from $\omega$ and $\phi$ mesons can be avoided for $e^+e^- \rightarrow \pi^+\pi^-$, which is the reason why we are dedicated to the study of $e^+e^- \rightarrow \pi^+\pi^-$ by combing with higher $\rho$ mesons around 2 GeV.

In Ref. [6], Lanzhou group once performed the mass spectrum analysis and calculated these two-body Okuba-Zweig-Lizuka (OZI) allowed decays of $\rho$ meson family. By combining with these reported $\rho$-like states collected in Particle Data Group (PDG) [8], the possible assignment of these $\rho$-like states into the $\rho$ meson family was suggested [6], which is crucial step of constructing $\rho$ meson family. However, it is not the end of whole story.

After releasing the detailed data of cross section of $e^+e^- \rightarrow \pi^+\pi^-$ [3, 4] by BABAR, we may continue to carry out the study of the production of higher $\rho$ mesonic states around 2 GeV via $e^+e^- \rightarrow \pi^+\pi^-$. Based on the results given in Ref. [6], we may select suitable higher $\rho$ mesons around 2 GeV as the intermediate states in $e^+e^- \rightarrow \pi^+\pi^-$. And then, by fitting the cross section of $e^+e^- \rightarrow \pi^+\pi^-$ around 2.2 GeV under our theoretical approach, we may get the information of the contribution of these $\rho$ mesonic states to the cross section of $e^+e^- \rightarrow \pi^+\pi^-$, which is valuable to further test the suggestive assignment of the $\rho$ meson family [6]. We hope that our effort presented in this work can improve our understanding of constructing the $\rho$ meson family, especially for these higher $\rho$ mesonic states around 2 GeV.

This paper is organized as follows. After Introduction, we will give a concise review of $\rho$ mesons with mass around 2 GeV, and introduce the possible assignment to them (see Sec. II). In Sec. III, we will present our theoretical framework of calculating $e^+e^- \rightarrow \pi^+\pi^-$ with these discussed $\rho$ mesons as intermediate states. When fitting the BABAR data, we finally extract the magnitude of different $\rho$ meson contributions to the $e^+e^- \rightarrow \pi^+\pi^-$ cross section around 2.2 GeV. And then, we give the numerical result in Sec. IV. This paper ends with the summary.

II. THE SITUATION OF $\rho$ MESONS AROUND 2 GEV

There are many $\rho$-like states reported by experiments. Among these states, $\rho(770)$ is a well established ground state with very broad width. As shown in PDG [8], $\rho(1450)$ can be assigned as the first radial excited state of $\rho(770)$. By the analysis of the mass spectrum [9] and the study of total decay width [6] and the branching ratio of the $\rho(1700) \rightarrow 2\pi, 4\pi$
and $e^+e^- \to \omega\eta^0$ process [11], we may find that $\rho(1700)$ as a candidate of the $\rho(1^{D_1})$ meson state is suitable. It is the research status of some low-lying $\rho$-like states.

$\rho(1900)$ was firstly observed by the DM2 Collaboration, which corresponds to a dip around 1.9 GeV with analyzing the process $e^+e^- \to 6\pi$ [12]. After that, there were many experiments relevant to $\rho(1900)$ which include the FENICE Collaboration [13], the E687 Collaboration [14, 15], the BABAR Collaboration [16, 17], and the CMD3 Collaboration [18]. It is worth nothing that $\rho(1900)$ was identified from 6\pi peak exactly at the $\rho\rho$ threshold [18]. Thus, Bugg suggested that this state is likely to be a $\rho(3S_1)$ captured by the very strong $\rho\rho$ $S$-wave but could be a nonresonant cusp effect [19].

In 2013, Lanzhou group [6] indicated that $\rho(1900)$ can be regarded as $3S_1$ state since the obtained total width overlaps with the BABAR’s data [20]. Here, the main decay channels of $\rho(1900)$ are $\pi\pi, \pi\omega(1260), \pi\eta(1170), \pi\sigma(1300)$, and $\pi\eta(1420)$ [6]. Therefore, $\rho(1900)$ with a large branching ratio of 4\% can be understood.

Clegg and Donnachi jointly analyzed the data on 6\pi states produced in the $e^+e^-$ annihilation and diffractive photoproduction, and indicated that there exists a resonance with peak near 2.1 GeV [21–23], which corresponds to $\rho(2150)$. Later, $\rho(2150)$ was assigned as the third radial excitation of $\rho(770)$ by fitting the pion form factor [24]. In addition, other experiments like GAMS [25, 26], Crystal Barrel [27–30] and BABAR [31] confirmed the observation of $\rho(2150)$ in different processes.

According to the analysis of mass spectrum [19, 32–34], $\rho(2150)$ can be a good candidate of $\rho(4S_1)$ meson state. The study of OZI-allowed two-body strong decay behaviors of $\rho(2150)$ supports this assignment [6], since the SPEC’s data [30] can be reproduced [6]. Here, the dominant channels of $\rho(2150)$ are $\pi\pi, \pi\sigma(1260), \pi\omega$ and $\pi\eta(1170)$ [6], which can explain why $\rho(2150)$ was observed in $\pi^+\pi^-, \omega\eta^0, \eta\pi\pi, f_1(1285)\pi\pi$, and $\omega\rho\pi$ experimentally.

### Table I: The suggested assignment to these observed $\rho$-like states and the information of their main decay channels from Ref. [6].

| State  | Assignment | Main decay channels       |
|--------|------------|--------------------------|
| $\rho(770)$ | $\rho(1S)$ | $\pi\pi$               |
| $\rho(1450)$ | $\rho(2S)$ | $\pi\pi, \pi\sigma(1260), \pi\omega, \pi\eta(1170)$ |
| $\rho(1900)$ | $\rho(3S)$ | $\pi\pi, \pi\sigma(1260), \pi\eta(1170), \pi\eta(1300), \pi\omega(1420)$ |
| $\rho(2150)$ | $\rho(4S)$ | $\pi\pi, \pi\sigma(1260), \pi\omega, \pi\eta(1170)$ |
| $\rho(1700)$ | $\rho(1D)$ | $\pi\sigma(1260), \pi\eta(1170)$ |
| $\rho(2000)$ | $\rho(2D)$ | $\pi\rho(1300), \rho\rho, \pi\sigma(1670), \pi\sigma(1260)$ |
| $\rho(2270)$ | $\rho(3D)$ | $\pi\rho(1300), \pi\rho(1800)$ |

In PDG [8], there are two $\rho$-like states are listed as further state, which are $\rho(2000)$ and $\rho(2270)$. $\rho(2000)$ was observed in the $p\bar{p} \to \pi\pi$ reaction with the mass around 1988 MeV [35]. Later, a combined fit was presented to the data of $p\bar{p} \to \omega\eta^0$ and $\omega\pi$, by which the existence of $\rho(2000)$ was confirmed [27]. The analysis of the Regge trajectory shows $\rho(2000)$ as the first radial excitation of $\rho(1700)$ [6, 30]. The dominant channels of $\rho(2000)$ include $\pi\pi(1300), \rho\rho, \pi\sigma(1670)$ and $\pi\sigma(1260)$ indicated in Ref. [6].

In the reaction $\gamma p \to \omega\pi\pi\pi^0$, a spin-parity analysis shows the existence of a resonance with $J^P = 1^-$ in the $\omega\pi\pi$ final state, which has mass around 2.28±0.05 GeV [22]. And then, the Crystal Barrel experiment fitted the $\omega\rho\pi$ data from the $p\bar{p}$ annihilation, where $\rho(2270)$ was confirmed [27]. The Regge trajectory analysis gives that $\rho(2270)$ is a good candidate of the second radial excitation of $\rho(1700)$ [6, 30]. The decay information of $\rho(2270)$ was also provided in Ref. [6].

In Table I, we collect the information of these reported $\rho$-like states, and their assignments and dominant decay channels.

### III. DEPICTING THE CROSS SECTION OF $e^+e^- \to \pi^+\pi^-$ AROUND 2 GEV

In this section, we focus on the $e^+e^- \to \pi^+\pi^-$ process at center-of-mass energy around 2 GeV. Due to the constraint of conservation of $G$ parity, $e^+e^- \to \pi^+\pi^-$ can be applied to study $\rho$-like states. As shown in Fig. 1, there exist two mechanisms working together for $e^+e^- \to \pi^+\pi^-$. The first one is $e^+e^-$ direct annihilation into $\pi^+\pi^-$, where the virtual photon couples with the final state $\pi^+\pi^-$, which provides the background contribution. The second one is that $e^+e^- \to \pi^+\pi^-$ occurs via intermediate $\rho$ states.

![FIG. 1: (Color online.) The diagrams for depicting the $e^+e^- \to \pi^+\pi^-$ process. Here, (a) is direct annihilation process while (b) corresponds to the intermediate $\rho$ state contribution.](image.png)

When trying to reproduce the line shape of the cross section of $e^+e^- \to \pi^+\pi^-$ process around 2 GeV reported by the BABAR Collaboration recently [4], we need to choose suitable intermediate $\rho$ meson states. The collected information in Table I shows that $\rho(1900), \rho(2000), \rho(2150)$ and $\rho(2270)$ should be considered in our calculation.

In this work, we adopt effective Lagrangian approach to calculate these discussed processes shown in Fig. 1. The effec-
tive Lagrangian involved in the concrete work include \[36\]
\[
\mathcal{L}_{\pi\pi} = i e A^\mu (\bar{\pi}_\mu \pi - \bar{\pi}_\mu \pi),
\]
\[
\mathcal{L}_{\rho\rho} = i g_{\rho\pi\pi} f_\rho^2 (\bar{\rho}_\mu \rho - \bar{\rho}_\mu \pi\pi),
\]
\[
\mathcal{L}_{\eta\eta} = \frac{-m^2_\rho p^\mu A_\mu}{f_\rho^2},
\] (1)

The amplitudes corresponding to the diagrams in Fig. 1 can be written as
\[
\mathcal{M}_{\text{Dir}} = \left[ \bar{v}(p_2, m_\rho)(ie\gamma^\mu)u(p_2, m_\rho) \right] \frac{-i g_{\rho\pi\pi}}{q^2} \left[ i e(p_4^\mu - p_3^\mu) F_\rho(q^2) \right],
\]
\[
\mathcal{M}_\rho = \left[ \bar{v}(p_2, m_\rho)(ie\gamma^\mu)u(p_2, m_\rho) \right] \frac{-i g_{\rho\pi\pi}}{q^2} \left( -e \frac{m^2_\rho}{f_\rho^2} \right)
\times \frac{-i g_{\rho\pi\pi}}{q^2} \left[ i g_{\rho\pi\pi}(p_4^\mu - p_3^\mu) \right].
\] (2)

Here, \( F_\rho \) is the time-like form factor of charged pion and \( q = p_1 + p_2, \rho \) denote intermediate \( \rho \) meson states. \( m_\rho \) and \( \Gamma_\rho \) are resonance parameters, which can be fixed by the corresponding experimental data \[8\]. The total amplitude of \( e^+ e^- \rightarrow \pi^+ \pi^- \) is superposition of different contributions
\[
\mathcal{M}_{\text{Total}} = \mathcal{M}_{\text{Dir}} + \sum_i e^{i\theta_i} \mathcal{M}_\rho_i,
\] (3)

where \( \theta_i \) denotes the phase angle between the amplitudes from direct annihilation and the intermediate \( \rho \) state contribution. \( g_{\rho\pi\pi} = g_{\rho\pi\pi} m^2_\rho / f_\rho \) and \( f_\rho \) represents decay constant of some \( \rho \) meson. With the above amplitude, the differential cross section of \( e^+ e^- \rightarrow \pi^+ \pi^- \) can be calculated directly, i.e.,
\[
\frac{d\sigma}{dE} = \frac{1}{\pi s} \left| \mathcal{M}_{\text{Total}} \right|^2.
\] (4)

When fitting the cross section for \( e^+ e^- \rightarrow \pi^+ \pi^- \), we can treat \( \theta_i \) and \( g_{\rho\pi\pi} \) as free parameters. In addition, the form factor of charged pion is not determined since the form factor in the time-like range is a complex function. In general, the form factor changes slowly when \( q^2 \) is far away from threshold. For simplicity, we assume the form factor is a constant in the center-of-mass energy considered here. Thus, we argue that the form factor of charged pion can be absorbed into phase angle. In the present work, the form factor is taken as \( F_\rho(s) = \frac{e^{-bs}}{\sqrt{s}} \), where \( a \) and \( b \) are free parameters.

By the effective Lagrangian listed in Eq. (1), the dilepton and \( \pi^+ \pi^- \) decay widths of these intermediate \( \rho \) meson state can be expressed as
\[
\Gamma_{e^+ e^-} = \frac{e^4 m^4_\rho}{12 \pi^2 f^2_\rho},
\]
\[
\Gamma_{\pi^+ \pi^-} = \frac{g^2_{\rho\pi\pi}(m^2_\pi - 4 m^2_\rho \rho)^{3/2}}{48 \pi^2 m^2_\rho},
\] (5)

respectively. Thus, we may further define the production of dilepton decay width and the branching ratio of \( \pi^+ \pi^- \) mode of the discussed \( \rho \) meson
\[
\Gamma_{e^+ e^-} \mathcal{B}(\pi^+ \pi^-) = \frac{e^4 g^2_{\rho\pi\pi}(m^2_\pi - 4 m^2_\rho \rho)^{3/2}}{576 \pi^2 m^2_\rho \Gamma_\rho},
\] (6)

where \( m_\rho \) and \( m_\pi \) are the masses of intermediate \( \rho \) states and final state pion, respectively.

**IV. NUMERICAL RESULTS**

In the following, we will fit the cross section for \( e^+ e^- \rightarrow \pi^+ \pi^- \) measured by the BABAR Collaboration \[4\], where an event accumulation near 2.2 GeV exists in the \( \pi^+ \pi^- \) invariant mass spectrum. In our realistic analysis, we choose \( \rho(1900) \) and \( \rho(2150) \) as intermediate resonances, which are assigned as \( \rho(3S) \) and \( \rho(4S) \) states, respectively. As seen in Table II, \( \pi^+ \pi^- \) is the most dominant decay channel for the \( S \)-wave \( \rho \) mesons but not important to the \( D \)-wave \( \rho \) mesons. In order to reduce the number of fitting parameters, \( \rho(2000) \) and \( \rho(2270) \) that are treated as \( D \)-wave meson states \[6\] will not be considered in the following study. When fitting the experimental data under our theoretical framework, there are six free parameters, \( a, b, \theta_1, \theta_2, \Gamma_{e^+ e^-} \mathcal{B}(\pi^+ \pi^-) \rho(1900) \) and \( \Gamma_{e^+ e^-} \mathcal{B}(\pi^+ \pi^-) \rho(2150) \). Here, the subscripts in \( \Gamma_{e^+ e^-} \mathcal{B}(\pi^+ \pi^-) \rho(1900) \) and \( \Gamma_{e^+ e^-} \mathcal{B}(\pi^+ \pi^-) \rho(2150) \) is applied to distinguish the \( \rho(1900) \) and \( \rho(2150) \) contributions. Additionally, \( [m_{3/2}(1900), \Gamma_{3/2}(1900)] \) and \( [m_{3/2}(2150), \Gamma_{3/2}(2150)] \) as the resonance parameters of \( \rho(1900) \) and \( \rho(2150) \), respectively, are input parameters which are taken from PDG (see Table II).

In the fitting process, we found that \( \rho(2150) \) play dominant role to reproduce the line shape of \( e^+ e^- \rightarrow \pi^+ \pi^- \) around 2.2 GeV \[4\]. We can find two solutions (solution A and solution B), both of which can reproduce the BABAR’s data well. In Table III, we list these obtained fitting parameters. And then, in Fig. 2, we further present the fitted results and the comparison with the experimental data. It is worth mentioning that the intermediate resonance \( \rho(1900) \) has obvious contribution if describing the line shape corresponding to the center-of-mass energy \( \sqrt{s} < 2.2 \) GeV.

![FIG. 2: (Color online.) The fitted result of the cross section of \( e^+ e^- \rightarrow \pi^+ \pi^- \). Here, the black dots with error bar is BABAR result \[4\]. We present two solutions (Solution A (left) and Solution B (right)), which can depict the data well.](image-url)
TABLE II: The information of four intermediate ρ state involved in the $e^+e^- \rightarrow \pi^+\pi^-$ process around 2.2 GeV. The second and the third columns are resonance parameter. $R$ is the parameter in SHO wave function (see Eq. (9)) which was given in Ref. [6]. $\mathcal{B}(\pi^+\pi^-)$ is branching ratio of $\pi^+\pi^-$ mode calculated via the QPC model [6]. $\Gamma_{e^e}$ is dilepton decay width calculated in this work by Eq. (7). By the numerical results listed in the fifth and the sixth columns, the results of $\Gamma_{e^e}\mathcal{B}(\pi^+\pi^-)$ can be obtained.

| state   | $M_{\exp}$ (MeV) | $\Gamma_{\exp}$ (MeV) | $R$ (GeV$^{-1}$) [6] | $\mathcal{B}(\pi^+\pi^-)$ [6] | $\Gamma_{e^e}$ (keV) | $\Gamma_{e^e}\mathcal{B}(\pi^+\pi^-)$ (keV) |
|---------|------------------|------------------------|-----------------------|-------------------------------|----------------------|----------------------------------------|
| $\rho(1900)$ | 1909 ± 17 ± 25 [16] | 160 ± 20 [16] | 3.85 ± 4.28          | 0.1450 ± 0.3509               | 0.1958 ± 0.1578     | 0.0284 ± 0.0554                        |
| $\rho(2150)$ | 2150 ± 17 [6]       | 230 ± 50 [30]         | 4.74 ± 4.98          | 0.3889 ± 0.3936               | 0.8888 ± 0.806     | 0.0345 ± 0.0274                        |
| $\rho(2000)$ | 2000 ± 30 [37]      | 260 ± 45 [37]         | 4.34 ± 4.80          | 0.0740 ± 0.0573               | 0.0204 ± 0.0160    | 0.0015 ± 0.0009                        |
| $\rho(2270)$ | 2265 ± 40 [30]      | 325 ± 80 [30]         | 4.40 ± 4.80          | 0.0510 ± 0.0315               | 0.0163 ± 0.0129    | 0.0008 ± 0.0004                        |

the widths of these discussed ρ mesons decaying into the $\pi^+\pi^-$ channel by the quark pair creation (QPC) model. Under the framework of the potential model, the general expression of dilepton decay width of the discussed ρ state [9] is

$$\Gamma_{e^e} = \frac{4\pi}{3}a^2m_p\lambda^2_{p},$$

(7)

where $M_p$ denotes decay amplitude, which is defined as $M_p = \sqrt{2}\nu_p$, and $\lambda_{p} = (4/3)^{1/2}\nu_p$ for $S$-wave and $D$-wave ρ mesons, respectively. Here, factor $V_{\rho}$ and $V'_{\rho}$ read as

$$V_{\rho} = \frac{m_1m_2}{m_1} \left(2\pi e^{-i\phi_{\rho}}(p) \left(\frac{m_1m_2}{E_1E_2}\right)^{1/2} \right),$$

$$V'_{\rho} = \frac{m_1m_2}{m_1} \left(2\pi e^{-i\phi_{\rho}}(p) \left(\frac{m_1m_2}{E_1E_2}\right)^{1/2} \right),$$

(8)

Here, $m_1 = m_2 = 0.22$ GeV [9] is quark mass inside the ρ meson, and $E_1$ and $E_2$ are the energy of the corresponding quarks. And then, $\lambda_{\rho} = 2\int d^3p E |\phi_{\rho}(p)|^2$. The radial part of the spatial wave functions of these involved ρ meson states can be depicted by the radial part of simple harmonic oscillator (SHO) wave function $\phi_{\rho}(p)$, i.e.,

$$\phi_{\rho}(p) = (-1)^L e^{\frac{p^2}{2\nu}} \frac{2n!}{\sqrt{\Gamma(n + L + 3/2)}} (pR)^L \times L^{L+1/2}_{n} (\frac{p^2 R^2}{2}).$$

(9)

Here, $R$ that refers to the size of meson state is the parameter of SHO wave function and its possible value has been suggested by Ref. [6], which is summarized in Table II. Combined with the $\Gamma_{\pi^\pm}$ listed in Table II, $\Gamma_{e^e}\mathcal{B}(\pi^+\pi^-)$ can be directly calculated, which is also dependent on $R$ value (see the seventh column in Table II). Since the fitted results of $\Gamma_{e^e}\mathcal{B}(\pi^+\pi^-)_{\rho(1900)}$ and $\Gamma_{e^e}\mathcal{B}(\pi^+\pi^-)_{\rho(2150)}$ are comparable with the calculated results from the potential model, the fitted result corresponding to Solution A in Fig. 2 and Table III is more favorable.

V. SUMMARY

The $e^+e^- \rightarrow \pi^+\pi^-$ process is a good platform to study ρ-like states due to $G$-parity conservation. Inspired by the BABAR measurement of the cross section of the $e^+e^- \rightarrow \pi^+\pi^-$ process around 2 GeV, we study the contribution of higher radial excitations in the ρ meson family to $e^+e^- \rightarrow \pi^+\pi^-$. When re-producing the experimental data of $e^+e^- \rightarrow \pi^+\pi^-$ around 2.2 GeV, $\rho(2150)$ and $\rho(1900)$ as $\rho(4S)$ and $\rho(3S)$ play important role. Combining with former study of mass spectrum and decay behavior of ρ meson family [6], the present work enforces the assignment of $\rho(2150)$ and $\rho(1900)$ as $\rho(4S)$ and $\rho(3S)$, respectively, which is a crucial step in constructing ρ meson family.

In recent years, BESIII measured some processes of $e^+e^-$ annihilation into light mesons [38–42]. In the following, we may focus on other typical processes of $e^+e^-$ annihilation into light mesons, which have close relation to light vector mesonic states. Since there exist abundant ρ, ω and φ higher
excitations around 2 GeV, studying these processes of $e^+e^-$ annihilation into light mesons is helpful to better understand the contribution of these light vector meson to these processes. Obviously, the present work is a beneficial attempt on this issue.

We also feel that promoting experimental precision can inspire theoretical progress. The present work is a good example for this point. In the near future, BESIII and Belle II will be main force of the study of light hadron spectroscopy [43, 44]. We also expect more precise experimental data of processes of $e^+e^-$ annihilation into light mesons, which is valuable to construct light vector meson family.

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[1] J. Augustin et al. [SLAC-SP-017], Discovery of a Narrow Resonance in $e^+e^-$ Annihilation, Phys. Rev. Lett. 33, 1406-1408 (1974).
[2] B. Aubert et al. [BaBar], Observation of a broad structure in the $\pi^+\pi^-J/\psi$ mass spectrum around 4.26-GeV/$c^2$, Phys. Rev. Lett. 95, 142001 (2005).
[3] J. Lees et al. [BaBar], Precise Measurement of the $e^+e^-\rightarrow \pi^+\pi^-\gamma$ Cross Section with the Initial-State Radiation Method at BABAR, Phys. Rev. D 86, 032013 (2012).
[4] J. Lees et al. [BaBar], Resonances in $e^+e^-$ annihilation near 2.2 GeV, Phys. Rev. D 101, no.1, 012011 (2020).
[5] X. Wang, Z. Sun, D. Chen, X. Liu and T. Matsuki, Non-strange partner of strangeonium-like state Y(2175), Phys. Rev. D 85, 074024 (2012).
[6] L. He, X. Wang and X. Liu, Towards two-body strong decay behavior of higher $\rho$ and $\rho_3$ mesons, Phys. Rev. D 88, no.3, 034008 (2013).
[7] C. Pang, Y. Wang, J. Hu, T. Zhang and X. Liu, Study of the $\omega$ meson family and newly observed $\omega$-like state X(2240), Phys. Rev. D 101, 074022 (2020).
[8] M. Tanabashi et al. [Particle Data Group], Review of Particle Physics, Phys. Rev. D 98, no. 3, 030001 (2018).
[9] S. Godfrey and N. Isgur, Mesons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D 32, 189 (1985).
[10] A. Abele et al. [CRYSTAL BARREL Collaboration], 4$\pi$ decays of scalar and vector mesons, Eur. Phys. J. C 21, 261 (2001).
[11] K. Kittimanapun, Y. Yan, K. Khoonthongkée, C. Kobdaj and P. Suebka, $e^+e^\rightarrow \omega\phi$ reaction and rho(1450) and rho(1700) mesons in a quark model, Phys. Rev. C 79, 025201 (2009).
[12] A. Castro et al. [DM2 Collaboration], The $\pi, K$ Proton Electromagnetic Form-factors And New Related Dm2 Results, LAL-88-58.
[13] A. Antonelli et al. [FENICE Collaboration], Measurement of the total $e^+e^-$ hadrons cross-section near the $e^+e^-$ to $N\bar{N}$ threshold, Phys. Lett. B 365, 427 (1996).
[14] P. L. Frabetti et al. [E687 Collaboration], Evidence for a Narrow Dip Structure at 1.9-GeV/$c^2$ in $3\pi^+3\pi^-$ Diffractive photoproduction, Phys. Lett. B 514, 240 (2001).
[15] P. L. Frabetti et al., On the narrow dip structure at 1.9-GeV/$c^2$ in diffractive photoproduction, Phys. Lett. B 578, 290 (2004).
[16] B. Aubert et al. [BaBar Collaboration], The $e^+e^-\rightarrow 3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0)$ and $K^+K^-2(\pi^+\pi^-)$ cross-sections at center-of-mass energies from production threshold to 4.5GeV measured with initial-state radiation, Phys. Rev. D 73, 052003 (2006).
[17] B. Aubert et al. [BaBar Collaboration], Measurements of $e^+e^-\rightarrow K^+K^-\eta, K^+K^-\eta$ and $K^+K^-\pi^0$ cross-sections using initial state radiation events, Phys. Rev. D 77, 092002 (2008).
[18] E. P. Solodov [CMD-3 Collaboration], First results from the CMD3 Detector at the VEPP2000 Collider, arXiv:1108.6174 [hep-ex].
[19] D. V. Bugg, Comment on “Systematics of radial and angular-momentum Regge trajectories of light nonstrange $q\bar{q}$-states”, Phys. Rev. D 87, no. 11, 118501 (2013).
[20] T. Barnes, F. E. Close, P. R. Page and E. S. Swanson, Higher quarkonia, Phys. Rev. D 55, 4157 (1997).
[21] D. Bisello, J. C. Bizot, J. Buon, A. Cordier, B. Delcourt and F. Mane, Study of the Reaction $e^+e^-\rightarrow 3\pi^+3\pi^-$ in the Total Energy Range 1400-MeV to 2180-MeV, Phys. Lett. 107B, 145 (1981).
[22] M. Atkinson et al. [Omega Photon Collaboration], Evidence for a $\omega\pi^0\pi^0$ State in Diffractive Photoproduction, Z. Phys. C 29, 333 (1985).
[23] A. B. Clegg and A. Donnachie, $\rho'$s in 6 $\pi$ States From Materialization of Photons, Z. Phys. C 45, 677 (1990).
[24] M. E. Biagini, S. Dubnicka, E. Etim and P. Kolar, Phenomenological evidence for a third radial excitation of $\rho$(770), Nuovo Cim. A 104, 363 (1991).
[25] A. Alde et al. [IHEP-HIIN-LANL-LAPP-KEK Collaboration], Study of the $\omega_{p_0}$ system, Z. Phys. C 54, 553 (1992).
[26] D. Alde et al. [GAMS Collaboration], Partial wave analysis of the $\omega_{p_0}$ system at high masses, Nuovo Cim. A 107, 1867 (1994) [Z. Phys. C 66, 379 (1995)].
[27] A. V. Anisovich et al., $I = 0$ $C = +1$ mesons from 1920 to 2410 MeV, Phys. Lett. B 491, 47 (2000).
[28] A. V. Anisovich, C. A. Baker, C. J. Batty, D. V. Bugg, V. A. Nikonov, A. V. Sarantsev, V. V. Sarantsev and B. S. Zou, Resonances in $p\bar{p} \rightarrow \omega\rho$ in the mass range 1600-MeV to 2410-MeV, Phys. Lett. B 513, 281 (2001).
[29] A. V. Anisovich et al., Analysis of $p\bar{p} \rightarrow \pi^+\pi^-\pi^0$, $\eta\eta'$ and $\eta\eta'$ from threshold to 2.5 GeV, Phys. Lett. B 471, 271 (1999).
[30] A. V. Anisovich et al., Combined analysis of meson channels with $I = 1$, $C = -1$ from 1940 to 2410 MeV, Phys. Lett. B 542, 8 (2002).
[31] B. Aubert et al. [BaBar Collaboration], The $e^+e^-\rightarrow 2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-\pi^0)$, $K^+K^-\pi^0\pi^0$ and $K^+K^-\pi^0\pi^-\pi^0$ Cross Sections Measured with Initial-State Radiation, Phys. Rev. D 76, 092005 (2007) Erratum: [Phys. Rev. D 77, 119902 (2008)].
[32] A. V. Anisovich, V. V. Anisovich and A. V. Sarantsev, Systematics of $q\bar{q}$ states in the $(n,M^2)$ and $(J, M^2)$ planes, Phys. Rev. D 62, 051502 (2000).
[33] P. Masjuan, E. Ruiz Arriola and W. Broniowski, Systematics of radial and angular-momentum Regge trajectories of light nonstrange $q\bar{q}$-states, Phys. Rev. D 85, 094006 (2012).
[34] P. Masjuan, E. Ruiz Arriola and W. Broniowski, Reply to Comment on Systematics of radial and angular-momentum Regge...
trajectories of light nonstrange $q\bar{q}$-states, Phys. Rev. D 87, no. 11, 118502 (2013).

[35] A. Hasan and D. V. Bugg, Amplitudes for $p\bar{p} \rightarrow \pi\pi$ from 0.36-GeV/c to 2.5-GeV/c, Phys. Lett. B 334, 215 (1994).

[36] D. Y. Chen, J. Liu and J. He, Reconciling the $Y(2240)$ with the $Y(2175)$, Phys. Rev. D 101, no. 7, 074045 (2020).

[37] D. V. Bugg, Comments on the $\sigma$ and $\kappa$, Phys. Lett. B 572 (2003) 1 Erratum: [Phys. Lett. B 595 (2004) 556].

[38] M. Ablikim et al. [BESIII Collaboration], Measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section between 600 and 900 MeV using initial state radiation, Phys. Lett. B 753, 629 (2016).

[39] M. Ablikim et al. [BESIII Collaboration], Measurement of $e^+e^- \rightarrow K^+K^-$ cross section at $\sqrt{s} = 2.00 - 3.08$ GeV, Phys. Rev. D 99, no. 3, 032001 (2019).

[40] M. Ablikim et al. [BESIII Collaboration], Cross section measurements of $e^+e^- \rightarrow K^+K^-K^+K^-$ and $\phi K^+K^-$ at center-of-mass energies from 2.10 to 3.08 GeV, Phys. Rev. D 100, no. 3, 032009 (2019).

[41] M. Ablikim et al. [BESIII Collaboration], Observation of a structure in $e^+e^- \rightarrow \phi\eta$ at $\sqrt{s}$ from 2.05 to 3.08 GeV, arXiv:2003.13064 [hep-ex].

[42] M. Ablikim et al. [BESIII], Observation of a Resonant Structure in $e^+e^- \rightarrow K^+K^\pi^0\pi^0$, Phys. Rev. Lett. 124, no. 11, 112001 (2020).

[43] M. Ablikim et al., Future Physics Programme of BESIII, Chin. Phys. C 44, no. 4, 040001 (2020).

[44] E. Kou et al. [Belle-II Collaboration], The Belle II Physics Book, PTEP 2019, no. 12, 123C01 (2019) Erratum: [PTEP 2020, no. 2, 029201 (2020)].