Virtual Photon Contribution
to Frictional Drag in double-layer Devices

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Abstract. The first order contribution to frictional drag in bi-layered fermion gas is examined. We discuss the relevance of single photon exchange in the evaluation of transresistance, which is usually explained by second order effects such as Coulomb and phonon drag. Since the effective e.m. interaction is unscreened, in the d.c. limit we obtain a finite (and large) contribution to transconductivity.

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1 Introduction

The double layer configuration of the fermion gas, especially under the influence of a strong magnetic field, allows one to study unusual transport phenomena which are interesting for the theory and are nowadays accessible to experiments. In 1976, Pogrebinskii and Price advanced the idea that a current driven in an electron gas should manifest as a potential difference in a separated electron system because of Coulomb scattering with a preferred direction of exchanged momentum. This Coulomb drag was described in terms of coupled transport equations. Later on the effect, which was originally analyzed for bulk electrons, was experimentally observed in double well heterostructures at low temperature, allowing for quantitative measurements.

The measured quantity is transresistance, the ratio of potential difference due to dragged charges and driving current. It was the main subject of several theoretical works based on transport equations or Kubo formula for conductivity. Transresistance resulting from Coulomb drag depends on temperature as $T^2$ and decreases as $d^{-4}$ in the distance between the wells. The theory was refined by including phonon exchange, which gave the main explanation of the observed deviations from the $T^2$ behaviour. A “current drag” effect was also proposed, and originates from the Van der Waals attraction between relative current flows.

With strong magnetic field, the bilayer geometry allows the observation of new FQHE phenomena and is an active research area.

In the Kubo formalism transconductivity is given by the retarded correlator of currents in different layers. As a consequence of charge conservation, it is a second order quantity in the electrostatic and phonon interlayer interaction. In this letter we discuss the effect of photon exchange, in the absence of external magnetic field, by considering the coupling of the photon field to currents, in the Coulomb gauge. The inclusion of photon exchange seems to us quite natural, given that photon drag arises as a first order effect. Though e.m. corrections are usually negligible, it was not obvious to us how they compared with second order effects.

The e.m. coupling in electron gas is the source of small though interesting effects. The effective e.m. interaction was evaluated in R.P.A. by Holstein et al. and displays the main feature of being unscreened at zero frequency. Reizer investigated its influence on the Fermi surface, the low temperature specific heat, and homogeneous transport. Gauge-invariant response
functions were studied by Kim et al. [10], and confirmed the Fermi liquid behaviour.
In this letter the photon polarization is evaluated in R.P.A. in the interlayer e.m. interaction, in the limit of thin layers. Single layer properties and disorder are accounted for in the diffusive regime. The results are discussed in the concluding section.

We model the system as two infinite parallel layers of electron gas, confined in narrow potential wells centered in \( z = 0 \) and \( z = d \), with negligible tunnelling because of low temperature and layer separation. The fermions in the two layers can be described by independent field operators. The Hamiltonian is

\[
H = H_1 + H_2 + H_{ph} + U_{Cou} + U_{em},
\]

where \( H_\ell \) is the kinetic energy of the electrons in layer \( \ell = 1, 2 \), \( H_{ph} \) is the energy of free e.m. field, \( U_{Cou} \) is the Coulomb interaction. The e.m. minimal coupling with the vector potential in the Coulomb gauge is

\[
H_{em} = \frac{1}{c} \int d^3x \bar{j}(x) \cdot A(x) - \frac{e}{2mc^2} \int d^3x \rho(x) A^2(x)
\]

where \( \rho \) is the electron charge density and \( \bar{j} \) is the derivative part of the charged current

\[
\bar{j} = j - \frac{e}{mc} \rho A
\]

that enters in the equation for charge conservation.

To derive the Kubo formula for conductivity [11], one perturbs the Hamiltonian with a term \( \delta H = \frac{1}{c} \int d^3x \delta A^\text{ext}(x,t) \cdot \bar{J}(x) \), which couples the total current to a weak external electric field. Linear response gives the conductivity tensor. Under the assumption that the Hamiltonian (1) is time-independent the response is function of the time difference only. Its Fourier transform is

\[
\sigma_{ij}(x,x',\omega) = \frac{i}{\hbar \omega} \pi_{ij}^\text{Ret}(x,x',\omega) - \frac{i e}{m \omega} \langle \rho(x) \rangle \delta_{ij} \delta_3(x-x')
\]

where \( i, j \) are space directions, \( \pi_{ij}^\text{Ret}(\omega) \) is the connected retarded current-current correlator. This is going to be the object of our discussion.
It is convenient to use imaginary time ($i\tau = \tau$) and to do perturbation theory in terms of time-ordered Green functions

$$
\pi_{\mu\nu}^C(x, x', \tau - \tau') = -\langle T J_\mu(x, \tau) J_\nu(x', \tau') \rangle
$$

(5)

where the four-dimensional notation means $J_\mu = (c_\rho, J_\rho)$, $\mu = 0, 1, 2, 3$ (since we use the Matsubara formalism, the metric is Euclidean). Brackets indicate equilibrium thermal average. The relation between the two kinds of correlators is a standard result [12] and it is briefly recalled here. The Fourier transform of $\pi_{\mu\nu}^C(x, x', \tau)$ is a sum over a discrete number of frequencies (Matsubara) due to the periodicity under the shift $\tau \rightarrow \tau + \frac{\bar{h}}{\beta}$

$$
\pi_{\mu\nu}^C(x, x', \tau) = \frac{1}{\bar{h} \beta} \sum_n e^{-i\omega_n \tau} \pi_{\mu\nu}^C(x, x', \omega_n).
$$

(6)

The correlator of interest $\pi^{\text{Ret}}(\omega)$ can be written in terms of $\pi^C(\omega_n)$ by analytic continuation. For real $\omega$:

$$
\pi_{\mu\nu}^{\text{Ret}}(\omega) = \int_{-\infty}^{\infty} d\omega' \frac{f_{\mu\nu}(\omega')}{\omega - \omega' + i\epsilon}
$$

$$
\pi_{\mu\nu}^C(\omega_n) = \int_{-\infty}^{\infty} d\omega' \frac{f_{\mu\nu}(\omega')}{i\omega_n - \omega'}
$$

(7)

A delicate issue is the presence of disorder. We can consider various approximations depending on the model describing the experimental device. One possibility is that the disorder is described by impurities and that the averaging is performed on the final result, thus allowing a correlation between the layers. This approach can be simplified by the independent averaging on the two layers. In the present approach we follow the last point of view in order to see the consequences of the direct electromagnetic interaction of the two layers. This simplifying dynamics has the consequence that translational invariance is valid in the plane $r = (x, y)$. Confinement is in the $z$ direction.

We then Fourier transform with a bidimensional wave-vector $\mathbf{q}$. The static and homogeneous limit of the transconductivity is:

$$
\sigma_{ij}(z, z') = \lim_{\omega \to 0} \lim_{q \to 0} \frac{i}{\bar{h} \omega} \pi_{ij}^{\text{Ret}}(\mathbf{q}, z, z', \omega)
$$

(8)

where $z$ and $z'$ belong to different layers. Moreover in the limit of thin layer we have to impose

$$
\sigma_{3j} = \sigma_{j3} = 0,
$$

(9)
because there is no flow of charge in the z-direction. Also the conservation of
the electromagnetic current requires the above condition in the limit of thin
layers.
In the next section, the current-current correlator, and therefore the conduc-
tivity, will be related to the polarization tensor of the e.m. field.

2 The polarization tensor

The thermal Green functions for the photon field are

\[ \mathcal{D}_{ij}(\mathbf{x},\tau,\mathbf{x}',\tau') = -\frac{1}{\hbar} \langle TA_i(\mathbf{x},\tau)A_j(\mathbf{x}',\tau') \rangle \] (10)

The prefactor \(1/\hbar\) ensures the same dimension as the Coulomb interaction \(\mathcal{D}_{00}\). The propagators for free photons and the bare Coulomb interaction are
best written in momentum space:

\[ \mathcal{D}_{ij}^{(0)}(k,\omega_n) = -\left( \delta_{ij} - \frac{k_ik_j}{k^2} \right) \frac{4\pi c^2}{\omega_n^2 + c^2k^2}, \quad \mathcal{D}_{00}^{(0)}(k,\omega_n) = \frac{4\pi}{k^2} \] (11)

and are the components of a tensor \(\mathcal{D}_{\mu\nu}^{(0)}\), with \(\mathcal{D}_{0i}^{(0)} = \mathcal{D}_{ij}^{(0)} = 0\). When the interaction with matter is included, the dressed photon propagator and the
effective Coulomb interaction are components of a tensor \(\mathcal{D}_{\mu\nu}\) which differs
from the bare one by polarization insertions:

\[ \mathcal{D}_{\mu\nu}(\mathbf{x},\mathbf{x}',\omega_n) = \mathcal{D}_{\mu\nu}^{(0)}(\mathbf{x},\mathbf{x}',\omega_n) - \mathcal{D}_{\mu\nu}^{(0)}(\mathbf{x}_1,\mathbf{x}_2,\omega_n) \mathcal{P}_{\rho\sigma}(\mathbf{x}_1,\mathbf{x}_2,\omega_n) \mathcal{D}_{\rho\sigma}^{(0)}(\mathbf{x}_3,\mathbf{x}',\omega_n) \] (12)

Summation and integration of repeated variables are understood hereafter.
The identification of the polarization insertion of the Coulomb interaction
with the connected density-density correlator is well known from textbooks:

\[ \mathcal{P}_{00}(\mathbf{x},\mathbf{x}',\omega_n) = \frac{1}{\hbar c^2} \pi_{00}^{c}(\mathbf{x},\mathbf{x}',\omega_n) \] (13)

A perturbative analysis shows the further exact relations among the polar-
ization insertion of the photon propagator and the current-current correlator:

\[ \mathcal{P}_{ij}(\mathbf{x},\mathbf{x}',\omega_n) = -\frac{e}{mc^2} \langle \rho(\mathbf{x}) \rangle \delta_{ij} \delta_3(\mathbf{x} - \mathbf{x}') + \frac{1}{\hbar c^2} \pi_{ij}^{c}(\mathbf{x},\mathbf{x}',\omega_n) \] (14)
\[ P_{0i} = \frac{1}{\hbar c^2} \pi_{0i}^C, \quad P_{i0} = \frac{1}{\hbar c^2} \pi_{i0}^C \] (15)

Therefore, the conductivity tensor is proportional to the retarded photon polarization:

\[ \sigma_{ij}(x, x', \omega) = i\frac{e^2}{\omega} P_{\text{Ret}}^{ij}(x, x', \omega) \] (16)

In our approximation, after the average on disorder, both \( \sigma \) and \( P \) are functions of the difference \( r - r' \), which is traded for the variable \( \mathbf{q} \). The dependence in \( z \) and \( z' \) will be simplified with the limit of thin layers.

### 3 The Dyson equation

To evaluate in some approximation the polarization, we start by writing an exact Dyson equation in coordinate space. Due to the geometry of the problem, we find it convenient to rearrange the graphs according to the two layer configuration:

\[ P_{\mu
u}(x, x', \omega_n) = P_{\mu\nu}^*(x, x', \omega_n) + P_{\mu\rho}^*(x, x'', \omega_n) D_{\rho\sigma}^{(0)}(x'' - x^n, \omega_n) P_{\sigma\nu}(x^n, x', \omega_n) \] (17)

where \( D_{\rho\sigma}^{(0)} \) is an interlayer bare propagator (points in different layers) and \( P^* \) is the irreducible polarization tensor, given by the sum of e.m. polarization insertions that cannot be disconnected by cutting a single interlayer photon or Coulomb line.

Next we take the average over disorder, and make the simplification of retaining the same Dyson equation for averaged correlators. In such approximation, the equation is written in \( \mathbf{q} \)-space:

\[ P_{\mu
u}(q, z, z', \omega_n) = P_{\mu\nu}^*(q, z, z', \omega_n) + P_{\mu\rho}^*(q, z, z'', \omega_n) D_{\rho\sigma}^{(0)}(q, z'' - z^n, \omega_n) P_{\sigma\nu}(q, z^n, z', \omega_n) \] (18)

We put \( \mathbf{k} = (q, k_3) \) in the bare e.m. propagators and antitransform to \( z - z' \):

\[ D_{\mu\nu}^{(0)}(q, z - z', \omega_n) = \int_{-\infty}^{\infty} \frac{dk_3}{2\pi} D_{\mu\nu}^{(0)}(k, \omega_n) e^{ik_3(z - z')} \] (19)
With the definition of the auxiliary function

\[ \tilde{D}(q, z - z', \omega_n) = 2\pi \frac{e^{-\frac{1}{c}|z-z'|\sqrt{\omega_n^2 + q^2c^2}}}{\sqrt{\omega_n^2 + q^2c^2}} \]  \hspace{1cm} (20)

we evaluate

\[ D_{00}^{(0)} = 2\pi \frac{e^{-q|z-z'|}}{q} \]
\[ D_{ab}^{(0)} = -\delta_{ab}\tilde{D} - \frac{qabcdef^2}{\omega_n^2}(\tilde{D} - D_{00}^{(0)}) \]
\[ D_{33}^{(0)} = \frac{q^2 c^2}{\omega_n^2}(\tilde{D} - D_{00}^{(0)}) \]
\[ D_{3a}^{(0)} = D_{03}^{(0)} = 2\pi i \text{sign}(z - z') \frac{qabcdef^2}{\omega_n^2} \left( e^{-\frac{1}{c}|z-z'|\sqrt{\omega_n^2 + c^2q^2}} - e^{-q|z-z'|} \right) \]
\[ D_{0j}^{(0)} = 0, \]  \hspace{1cm} (21)

where the last equation is imposed by the gauge fixing condition \( \partial_j A_j = 0 \).

To proceed further, we consider the limit of thin layers. Since the confining well in the \( z \)-direction causes a strong splitting in the energy eigenvalues and the dynamics is translation invariant in the plane \( z = \text{const} \), we allow only single particle states of the form (for the unperturbed basis of the Hilbert space)

\[ u_\ell(r, z) = \phi_\ell(z)\psi_\ell(r) \]  \hspace{1cm} (22)

The motion in \( z \)-direction is thus frozen and all bilinear local operators contain a factor \( |\phi_\ell(z)|^2 \). The limit of thin layer is performed by

\[ |\phi_\ell(z)|^2 \rightarrow \delta(z - z_\ell). \]  \hspace{1cm} (23)

It is customary to introduce two canonical basis of fermion operators that create or remove a particle with position \( (x, y) = r \) and spin \( \sigma \) in layer \( \ell = 1, 2 \). With \( z_1 = 0 \) and \( z_2 = d \), we have the relevant limit operators:

\[ \rho(x) = \sum_\ell \rho_\ell(r)\delta(z - z_\ell), \quad \rho_\ell(r) = -e \sum_\sigma \psi^\dagger_\ell\sigma(r)\psi_\ell\sigma(r) \]  \hspace{1cm} (24)
\[ J_a(x) = \sum_\ell J_{a\ell}(r)\delta(z - z_\ell), \quad J_{a\ell}(r) = j_{a\ell}(r) - \frac{e}{mc}\rho_\ell(r)A_a(r, z_\ell) \]  \hspace{1cm} (25)
\[ j_{a\ell}(r) = \frac{\hbar e}{2m} \sum_{\sigma} \psi_{\ell\sigma}^\dagger(r) \partial_a \psi_{\ell\sigma}(r) - (\partial_a \psi_{\ell\sigma}^\dagger(r)) \psi_{\ell\sigma}(r) \]  

(26)

In the thin layer limit the component \( J_3 \) is assumed to vanish.

The index \( a \) will hereafter denote transverse space components \((x, y)\), while \( i \) is used for components \((x, y, z)\), and a greek index includes the imaginary time component.

The absence of tunnelling ensures the conservation of charge in each layer, which reads:

\[ \frac{1}{\hbar} [H, \rho_{\ell}(r)] = \text{div}_{xy} J_{\ell}(r). \]  

(27)

In the thin layer limit the Dyson equations (11) become algebraic, with all delta functions factoring out or allowing to make the double integrations in the \( z \) direction:

\[ P_{\mu\nu}(\ell\ell') - P_{\mu\nu}(\ell\ell') = P_{\mu\nu}(\ell_1\ell_2) D^{(0)}(\ell_1\ell_2) P_{\sigma\nu}(\ell_2\ell') \]  

(28)

The variables \( q \) and \( \omega_n \) are omitted for brevity. Recall that the Dyson equation was constructed with the requirement that \( D^{(0)} \) connects different layers, then \( \ell_1 \neq \ell_2 \). In the formula we put

\[ D^{(0)}_{\mu\nu}(\ell\ell') = D^{(0)}_{\mu\nu}(q, z_\ell - z_{\ell'}, \omega_n), \quad P_{\mu\nu}(\ell, \ell') = P_{\mu\nu}(q, \omega_n, z_\ell, z_{\ell'}) \]  

(29)

They are respectively the entries of two \( 4 \times 4 \) matrices \( D^{(0)}(\ell\ell') \) and \( P(\ell\ell') \). Therefore, the Dyson equations correspond to 4 matrix equations \((\ell, \ell' = 1, 2)\):

\[ P(\ell\ell') = P^*(\ell\ell') + P^*(\ell_1) D^{(0)}(\ell_1\ell_2) P(2\ell') + P^*(\ell_2) D^{(0)}(\ell_1\ell_2) P(1\ell') \]  

(30)

The structure of the polarization tensor is greatly limited by symmetry and charge conservation. The latter implies the following exact relations:

\[ i\omega_n c P_{\ell\nu}(q, \omega_n, \ell, \ell') = q_\alpha P_{\alpha\nu}(q, \omega_n, \ell, \ell'), \quad \nu = 0, 1, 2, 3 \]  

(31)

The same holds when the indices are exchanged. These relations correspond to the Ward identity relating the vertex functions for Coulomb and e.m. coupling to the electron field. In absence of external magnetic field rotational symmetry requires the tensor structure

\[ P_{ab}(q, \omega_n, \ell, \ell') = \delta_{ab} A(q, \omega_n, \ell, \ell') + \frac{q_a q_b}{q^2} B(q, \omega_n, \ell, \ell') \]  

(32)
From charge conservation we find:

\[ \mathcal{P}_{0a} = \mathcal{P}_{a0} = i \frac{\omega_n}{cq^2} q_a \mathcal{P}_{00}, \quad A + B = - \frac{\omega_n^2}{c^2 q^2} \mathcal{P}_{00}. \] (33)

4 The Interlayer Polarization in RPA

We shall solve Dyson’s equation in RPA and for identical layers. In this approximation \( \mathbf{P}^\star \) is deprived of all interlayer interaction lines and therefore

\[ \mathbf{P}^\star(\ell \ell') = \delta_{\ell \ell'} \mathbf{P}^{(0)} \] (34)

where \( \mathbf{P}^{(0)}_\ell \) is the exact polarization matrix of the single isolated layer with its internal dynamics. The index \( \ell \) keeps track of the charge of the carriers. However in the present approximation scheme (no charged impurities and no interlayer interaction) the charge appears always with even powers. Thus the index \( \ell \) can be neglected.

With this simplification, Dyson’s equations eqs (30) are:

\[ \mathbf{P}(12) = \mathbf{P}^{(0)} \mathbf{D}^{(0)}(12) \mathbf{P}(22) \]
\[ \mathbf{P}(22) = \mathbf{P}^{(0)} + \mathbf{P}^{(0)} \mathbf{D}^{(0)}(21) \mathbf{P}(12). \] (35)

After the analytic continuation, the sub-matrix \( P(22)_{ab} \) is proportional to the conductivity tensor of layer 2, while \( P(12)_{ab} \) is proportional to transconductivity among layers 1 and 2. Eqs. (35) states that these two quantities are related by the electromagnetic interaction.

The various components of \( \mathbf{P}^{(0)} \) fulfill the relations of rotational symmetry and charge conservation, such as (index \( \ell \) is here forgotten)

\[ P^{(0)}_{ab} = A^{(0)} \delta_{ab} + B^{(0)} \frac{q_a q_b}{q^2}, \quad \mathcal{P}_{0a} = \mathcal{P}_{a0} = i q_a \frac{\omega_n}{q c^2} \mathcal{P}^{(0)}_{00}, \]
\[ A^{(0)} + B^{(0)} = \frac{q_a q_b}{q^2} P^{(0)}_{ab} = - \frac{\omega_n^2}{c^2 q^2} \mathcal{P}^{(0)}_{00}. \] (36)

\( P^{(0)}_{ab} \) is directly linked to the conductivity tensor \( \sigma^{(0)}_{ab} \) of the isolated layer.

The coupled Dyson equations (25) provide a matrix equation for the interlayer polarization:

\[ \mathbf{P}(12) = \mathbf{Q} + \mathbf{Q} \mathbf{D}^{(0)}(21) \mathbf{P}(12) \] (37)
where \( Q = P^{(0)}D^{(0)}(12)P^{(0)} \).

Before solving the equation, let us evaluate the components \( Q_{ab} \), which give the lowest order approximation in the interlayer interaction to transconductivity. From eq. (21) we get

\[
Q_{ab} = P^{(0)}_{a0}D^{(0)}_{00}(12)P^{(0)}_{0b} + P^{(0)}_{ac}D^{(0)}_{cd}(12)P^{(0)}_{db}.
\]  

(38)

By using charge conservation for the polarization we obtain after some algebra

\[
Q_{ab} = -\tilde{D}
\left[
A^{(0)^2}(\delta_{ab} - \frac{q_a q_b}{q^2}) + \frac{q_a q_b}{q^2} (P^{(0)}_{00})^2 \frac{\omega_n^2}{c^2 q^4} (\omega_n^2 + c^2 q^2)
\right]
\]  

(39)

where \( \tilde{D} \) is the function in eq. (20) with \(|z - z'| = d\). From the isotropic part of \( Q_{ab} \) we obtain transresistance to first order in interlayer interaction: \( \sigma(12) = (2\pi/c)\sigma^{(0)^2} \). The experiments are carried under the condition that no current flows in the driven layer. The measured quantities are the driving current \( J^{(1)} \) and the electric field \( E^{(2)} \) that builds up in layer two to balance the drag field. Transresistance is the ratio

\[
\rho(12) = \frac{E^{(2)}}{J^{(1)}} = -\frac{\sigma(12)}{\sigma(11)\sigma(22) - \sigma(12)\sigma(21)}
\]  

(40)

Under the condition \( \sigma(12) << \sigma^{(0)} \), we approximate the layer conductance \( \sigma(\ell\ell) \) in presence of the other by the value \( \sigma^{(0)} \) of the isolated layer.

We then find \( \rho(12) \approx \sigma(12)/\sigma^{(0)^2} \) and obtain a universal result for transresistivity:

\[
\rho(12) = \frac{2\pi}{c} = \alpha R_H
\]  

(41)

where \( R_H = \hbar/e^2 \) is the Hall resistance and \( \alpha \) is the fine structure constant corrected for the interlayer medium.

While \( Q \) represents the inter-layer polarization with a single one-photon exchange, the solution of \( 37 \) is the polarization with an effective interlayer interaction, in RPA. \( P^{(0)} \) corresponds to disorder averaged single layer polarization where the second layer is absent. If we consider also the second layer, then our approximation neglects those contributions to the disorder average that correlate various \( P^{(0)} \) insertions. This last approximation was used also
in the evaluation of Coulomb and phonon drag. One computes:

\[
P^{(12)00}(\mathbf{q}, \omega_n) = \frac{P^{(0)}_{00}(\mathbf{q}, \omega_n)\sqrt{\omega_n^2 + c^2 q^2} e^{-\frac{\omega_n^2}{c} \sqrt{\omega_n^2 + c^2 q^2}}}{1 - P^{(0)}_{00} \frac{4\pi^2}{c^2 q^2} (\omega_n^2 + c^2 q^2) e^{-2\frac{\omega_n^2}{c} \sqrt{\omega_n^2 + c^2 q^2}}}
\]  

(42)

In the diffusive regime, we use

\[
P^{(0)}_{00}(\mathbf{q}, \omega) = \Gamma e^2 \frac{Dq^2}{Dq^2 - i\omega}
\]  

(43)

where \( D \) is the diffusion constant and \( \Gamma \) is related to the single layer conductivity: \( \sigma^{(0)} = \Gamma e^2 D \). In this approximation, the limits \( q \to 0, \omega \to 0 \) of trans-conductivity yield a small correction to (29) for transresistance:

\[
\rho(12) = \frac{2\pi}{c} \frac{1}{1 - \sigma^{(0)2} (2\pi/c)^2}.
\]  

(44)

5 Discussion

In the present work we have considered the virtual photon contribution to the frictional drag in two parallel layers. In contrast to the Coulomb drag the contribution of the single virtual photon survives in the limit of \( q \to 0 \) and \( \omega \to 0 \). The RPA approximation allows to get the result in terms of the full single layer polarization even in presence of disorder. The result is valid also at finite temperature, which enters in the single layer conductivity. Eq. (39) gives the contribution of one single-photon-exchange and eq. (42) yields the sum over all possible single-photon-exchanges. The result is large, and seems not to compare with the experimental values. This is peculiar to the e.m. contribution: the idealization of thin and infinite layers is the same used in previous works devoted to Coulomb or phonon drag. We also made the same approximation of neglecting correlations due to disorder among polarization insertions in the evaluation of the effective interaction.

This peculiarity goes along with that of being unscreened, and the possibility of significative changes in more realistic models, due to finite size effects. We leave this analysis to a future investigation.

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Figure 1: The first order contribution to the current-current correlator: coupling mediated by (a) screened Coulomb interaction, (b) four-dimensional electromagnetic propagator. \( \rho \) is the electron density, \( j \) the paramagnetic current density and \( J = j - \left( \frac{-e}{m_c} \right) \rho A \). The number inside the bubbles indicates the subsystem. The intralayer interaction is included within RPA in each bubble. (a) vanishes in the DC limit and corresponds to the \( \mu = \nu = 0 \) component of (b). The space-like components of (b) are nonvanishing in this limit.

References

[1] T.J.Gramila, J.P.Eisenstein, A.H.MacDonald, L.N.Pfeiffer, K.W.West, Mutual friction between parallel two-dimensional electron systems, Phys. Rev. Lett. 66, 1216 (1991).

[2] A.Kamenev, Y.Oreg, Coulomb drag in normal metals and superconductors: Diagrammatic approach, Phys. Rev. B 52, 7516 (1995).

[3] K. Flensberg, B. Y.-K. Hu, A.-P. Jauho, and J. M. Kinaret, Linear-response theory of Coulomb drag in coupled electron systems, Phys. Rev. B 52, 14761 (1995).

[4] H.C.Tso, P.V.Vasilopoulos, F.M.Peeters, Direct Coulomb and Phonon mediated coupling between spatially separated electron gases, Phys. Rev. Lett. 68, 2516 (1992);
[5] M.Bønsager, K.Flensberg, B.Y.K.Hu, A.H.MacDonald, *Frictional drag between quantum wells mediated by phonon exchange*, Phys. Rev. B 57 (1998) 7085.

[6] A.G.Rojo, G.Mahan, *Current drag from the Van der Waals interaction*, Phys. Rev. Lett. 68, 2075 (1992).

[7] A.G.Rojo, *Electron-drag effects in coupled electron systems*, J. Phys.: Condens. Matter 11 (1999) R31-52.

[8] T.Holstein, R.E.Norton and P.Pincus, *de Haas-van Alphen Effect and the Specific Heat of an Electron Gas*, Phys. Rev. B 8, 2649 (1973).

[9] M.Yu.Reizer, *Gauge Invariance and the electron-electron interaction in a normal metal*, Phys. Rev. B 44 (1991) 5476.

[10] Y.B.Kim, A.Furusaki, X.G. Wen, P.Lee, *Gauge-invariant response functions of fermions coupled to a gauge field*, Phys. Rev. B 50 (1994) 17915.

[11] G. D. Mahan, *Many-Particle Physics*, Kluwer Academic/Plenum Publishers, New York 2000.

[12] G. Rickayzen, *Green’s Functions and Condensed Matter*, Academic Press, London, 1984.