Wake flow stabilization with DBD plasma actuators for low Re numbers

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Abstract. We propose to study the stability properties of a wake air flow forced by electrohydrodynamic (EHD) actuators. EHD actuators can add momentum to the flow around an object in regions close to the wall. Hence, we adopt the hypothesis of the forcing actuation modeled as a slip-wall condition.

We study the flow around a cylinder modified by EHD actuators, dielectric barrier discharge (DBD) type, by means of particle image velocimetry (PIV) measures. For the sake of simplicity we observed flows at low Reynolds(Re) numbers (∼200) where the flow is mainly two-dimensional. As the forcing frequencies, characteristic of DBD, are much higher than the natural shedding frequency of the flow (kHz vs. Hz), we consider in this work only the forcing actuation as stationary. The actuators are disposed symmetrically near the boundary layer separation point. Thus, flow symmetry is conserved, as it remains weakly non parallel. This aspect allows us to perform stability analysis of the flow and to determine its characteristics which are governed by a strong mean flow correction. We also present results on the global mode evolution under forcing.

The main motivation of this work is to optimize the forcing on the flow, leading to a more effective energy consumption of EHD actuators.

1. Introduction
The simply geometry and the complex behaviour that exhibits the flow around a cylinder represents, for low Reynolds numbers (Re = U₀D/ν < 200), a prototypical 2D wake flow. The regime is identified with the Bénard von Kármán (BvK) vortex street, when the flow breaks its time continuous invariance at the bifurcation Re = 48.5. In this case, the velocity field in the whole flow domain oscillates with the same global frequency and its harmonics, related to the appearance of a sufficiently large region of absolute instability in the near wake. As the oscillation is spatially evolving, its envelope is called the global mode of the instability. Previous studies Zielinska et al. [1] and Wesfreid et al. (1996) [2], characterized the global mode behaviour determining scale laws for its nonlinear evolution as functions of the Reynolds number.

As a self-sustained hydrodynamical oscillator, the flow can be altered by forcing parameters that may be varied smoothly and accurately to give rise different regimes. In this sense, we study this benchmark in order to characterize flow control in wakes by means of electrohydrodynamical actuators (EHD). EHD actuators for flow control have been receiving special attention in the last years, as reviewed by Moreau (2007) [3]. Among all the types of low-energy plasma actuators, one can notice a group of actuators that produces surface discharges. With these devices the goal is...
usually to use an electric wind produced by the plasma in order to modify the properties of the boundary layer close to the wall. A particular type of discharge is the surface dielectric barrier discharge (DBD). This discharge has been perfected for the first time in air at atmospheric pressure by Masuda (1979) [4] for ionic charging of particles. Roth (1998) et al. used it for airflow applications at the end of the 1990s [5], characterizing the momentum injected for flat plate and airfoil flows. Later, Thomas et al. (2006) [6] applied the DBD for the bluff body flow control. In their results, the authors showed drastic reductions of the flow separation and the associated Karman vortex shedding. On the other hand, some hypothesis on the dynamics of the DBD forcing arose from a computational study validated with PIV experiments in Gonzalez et al. (2009) [7]. From a stability analysis for a bluff body flow, with no vortex shedding, the forcing is introduced numerically as a moving surface boundary condition. In this line, the work by Gronskis (2009) [8] analysed numerical models for DBD forced flows for a cylinder at low Reynolds numbers ($<200$).

We intend with this work to optimize the DBD control device by analysing the stability properties of the flow. For this reason, we simplify first by studying the problem at low Reynolds numbers. Global modes evolves as the mean flow is modified by external parameters. Barkley (2006) [9] introduced hypothesis to consider the mean flow as a base flow for 2D stability analysis. In this way, the mean flow represents a marginal stability state and defines the frequency and amplitude of the wake oscillations. The nonlinear saturation of the oscillatory instability is achieved by Reynolds stresses from the mean flow modification. Khor et al. (2008) [10] and Leontini et al. (2010) [11] support these hypothesis from experimental and numerical data for Re up to 600. These ideas have been applied to analyse forced wakes. Previous studies on the stability properties of forced wakes, Thiria and Wesfreid (2007) [12] [2009] [13], confirmed that a nonlinear critical behaviour takes place under forcing. Indeed, a bifurcation scenario reappears as the forcing action stabilizes the wake fluctuations, in their case, a rotary oscillation. Figure 1, adapted from [12], shows how the global modes develop under forcing. The scheme is represented by a vertical line, for Re $> Re_c$, that corresponds to forcing from the lower branch up to a critical value, when the wake reaches, or recovers, its basic state.

In this context, we pursue the characterization of the DBD control actuator to complete an understanding which will eventually lead to a more effective energy consumption. We organize the work as it follows: we describe the experimental setup to produce the DBD discharge as well as the method to measure the velocity fields of the flow, we present and discuss our results, concerning the global modes evolution, the stability properties of the wake under forcing; at last we resume some conclusions and formulate perspectives for future works.

2. Experimental Setup

The experimental set-up consists of a flow around a circular cylinder at low Reynolds number, $Re \sim 200$.

The velocity field measurements were undertaken with the cylinder placed in a closed loop wind tunnel with a test section of 50 $\times$ 50cm$^2$. The EHD actuator was mounted on the surface of a polymethyl methacrylate cylinder tube with external diameter $D = 20$ mm. The tube wall was 2 mm thick.

The electric circuit is composed of a signal generator, an audio amplifier and an ignition coil to give a discharge at $V_{EHD} = 10$ kV and a frequency of $f_{EHD} = 5.6$ kHz. The signal amplitude is then modulated by bursting the signal as showed by Figure 2. A second frequency arises $f_{Burst} = 1/T_{Burst}$ and the electrical energy delivered by the device is characterized by a duty cycle ($DC = T_1/T_{Burst}$). Special care was taken to assure a stationary input as $f_{Burst} \gg f_{flow}$, being $f_{flow}$ the vortex shedding frequency ($\sim Hz$, as the Strouhal number $St = fD/U_0 \simeq 0.2$). In short, the flow control parameter is the Duty Cycle (DC) input that modulates the "ionic
Figure 1. The reverse flow region behind the cylinder evolves nonlinear with $Re$, as it is modified by the mean flow. A new bifurcation scenario appears under forcing conditions.

Figure 2. Schematic of the EHD actuator: Electrodes disposed flush-mounted. Electric circuit and input signal detail.
wind” momentum.
Quantitative measurements were performed using 2D Particle Image Velocimetry (PIV) on a
vertical plane placed at mid-span of the cylinder. Image acquisition and PIV calculation were
done using a LaVision system, composed of an ImagerPro 1600 × 1200 CCD camera with a
12-bit dynamic range capable of recording double-frame pairs of images at ∼ 8Hz and a two
rod Nd:YAG (15mJ) pulsed laser synchronized by a customized PC using LaVision DaVis 7.1
software. Laser sheet width was about 1mm in the test section. The whole 300mm × 200mm
imaging region (about 12 × 8D) gives a spatial resolution of 0.06D as showed in 3. All image
acquisitions where done using the double-frame mode with a time lapse between the two frames
(dt) set to 7ms.
Given the flow natural frequency, its dynamic behaviour is well resolved by the PIV measures
for the selected acquisition frequency.

3. Results
3.1. On global modes modification
The modifications introduced by the plasma actuator can be described firstly by the mean flow
field, obtained from 400 snapshots which contains al least 30 shedding periods. We observe,
in Figure 4 that the near wake region behind the cylinder enlarges under increasing voltage.
The streamlines get close further downstream when it is applied the higher EHD actuation,
resulting that the mean flow resembles more to the stationary solution, characterized by a long
recirculation zone as it in Figure 1 scheme.

Wake flows are characterized by oscillations of a propagating wave which amplitude grows
from the origin, the cylinder in our case, reaches a maximum and decays afterwards. The spatial
envelope of this coherent oscillation gives the amplitude of the global mode and, for practical
purposes, it is reasonable to consider that the flow’s first harmonic does, by far, the dominant
contribution to the global mode.

Previous works by [1] and [2] studied scaling laws for the global mode in wake flows near
the threshold Re. A typical contour is presented on Figure 5(a) where its maximum amplitude,
A_{max} at (X_{max}, Y_{max}) coordinates, is highlighted as it represents an important parameter for
scaling. The authors proposed that both A_{max} and X_{max} follow scaling laws of the following
form:

\[ A_{max} \simeq (Re - Re)^{1/2}, X_{max} \simeq (Re - Re)^{-1/2} \]  \hspace{1cm} (1)
Figure 4. Mean flow velocity vector field and velocity modulus in contours.

Figure 5. Scaling laws for Global modes near the threshold, from [2]
Hence, the curves for different Reynolds numbers collapse into an universal curve. On the other hand, a scaling law, Figure 5(b), that resumes the behaviour as the parameter $Re$ approaches the threshold ($Rc \approx 46$ in cylinders).

In analogy, we determine the global mode evolution for DBD stationary forcing as we increase the duty cycle (DC) parameter. We observe in Figure 6 that increments on DC leads to decrements in the mode amplitude and accordingly its position displaces downstream, proving the flow stabilization. From a certain threshold, the behaviour is quite different as shows the curve corresponding to $DC=28$.

3.2. Scaling law
From the global mode evolution, we can determine a critical value for the forcing parameter DC. From Figure 7(a), we confirm the hypothesis for the von Karman modes evolution when they are forced toward stabilization. The ratio $A_{max}/X_{max}$ decreases almost linearly from $DC=10$ up to $DC=22$ and we can estimate a critical value from the linear approximation ($DC_C \approx 27.5$).

With this value we construct the scaling law for $(DC_C - DC)$ in Figure 7(b).

We remark that further increases of DC parameter produces the flow to behave qualitatively different as we expected from our hypothesis. Figure 8 shows how Von Karman mode has been suppressed and fluctuations remain in the cylinder wall neighbourhood. Indeed, the momentum added from the cylinder wall induces higher velocities and the velocity profiles downstream prevent self-sustained oscillations. The instability is confined to a region very close to the wall, Figure 8(a) shows an instantaneous velocity field for $DC=28$ and Figure 8(b) the mean flow for the corresponding forcing.

3.3. Stability analysis
As the studied flow is weakly non parallel, we analyse the change on stability properties produced by the forcing on the wake. The analysis of the wake, therefore, is decomposed into several equivalent parallel-flow problems, studied through the inviscid Orr-Sommerfeld, or Rayleigh, equation:

$$ (kU(y) - \omega)(\phi'' - k^2 \phi) - kU''(y)\phi = 0 $$

(2)
where $U = U(y)$ is the local (for $x = x_1$) velocity profile depending on the transverse coordinate $y$, $k$ and $\omega$ are respectively the complex wavenumber and the complex frequency of the perturbation and $\phi$ is the associated mode.

An eigenvalue problem arises relating $\omega$ and $k$. The most unstable mode appears for $k = k_0$ where $\frac{\partial \omega}{\partial k} = 0$ and it manifests following the cusp map procedure described i.e. Triantafyllou (1987) [14]. The map in the $\omega$ complex plane generated from $k$ complex plane presents a pinch point as showed by Figure 9(c). This determines the mode growth rate $\omega_i$ and the real frequency $\omega_r$ for the most unstable mode at the $x_1$ station. The result allows to determine regions whether the flow is locally absolutely unstable (for $\omega_i > 0$) or convectively unstable ($\omega_i < 0$).

The regions are plotted on Figure 10 and we confirm that the global mode evolution is in agreement to a progressive vanish of the absolute instability region.

On the other hand, having one real frequency for each position $x$ do not allow us to determine directly the global frequency of the flow. From Hammond and Redekopp (1997) [15], the frequency selected is equal to the real part of the absolute frequency $\omega_0$ evaluated at the

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**Figure 7.** Scaling law for DC parameter.

**Figure 8.** Beyond the bifurcation a qualitatively different dynamics takes place in the flow.
(a) Mean Flow field

(b) Velocity profiles

(c) Results of the stability calculation for a local profile, a pinch point in the $\omega$-plane determines the mode complex frequency.

Figure 9. Linear stability analysis from the mean flow.

streamwise position $x_s$ defined as:

$$\frac{\partial \omega}{\partial x} \bigg|_{x_s} = 0$$

The saddle point $x_s$ does not lie necessarily on the real axis. Since derivatives of $\omega_0(x)$ are only known along the real $x$-axis, the location of the saddle point $x_s$ of $\omega_0(x)$ is found through use of the Cauchy–Riemann equations and analytic continuation to complex values of $x = x_r + ix_i$. The expansion is:

$$\omega_0(x_s) = \omega_0(x_r, x_i = 0) - \frac{\partial \omega_0}{\partial x_r} \bigg|_{x_i} x_i + O(x_i)^2$$

$$\omega_i(x_s) = \omega_i(x_r, x_i = 0) - \frac{\partial \omega_i}{\partial x_r} \bigg|_{x_i} x_i + O(x_i)^2$$

Examining the $\omega$ plane for every position in the $x$-axis, Figure 11(a) shows the evolution of $\omega_i$.
versus $\omega_r$. Expanding (4) produces a curves family (Figure 11(b)). One of these curves presents a pinch point, where the condition (3) is verified.

Performing the method for the forced cases, we determine how the global frequency selection is modified. As in other forced wakes [13], the global frequency diminishes as expected for a
stabilization. Experimental and numerical errors did not allow us to extract a scaling law for Strouhal, however these results confirm the physical assumptions on stabilization.

![Figure 12. Global frequency selection for forced cases.](image)

4. Conclusions and perspectives
We achieved the analysis of a plasma DBD forced wake flow for low Reynolds numbers. From PIV data of a cylinder flow at $Re \sim 200$, we made use of stability properties of wake flows in order to characterize the forcing. The mean flow modification yields a corresponding evolution of the global modes. In this way, we determine a scaling law in order to optimize the actuator forcing parameter, DC, its duty cycle. On the other hand, the region of absolute instability changes dramatically under forcing as we confirmed by solving the Rayleigh equation. In the same sense, the global frequency (St number) decreases when the forcing approaches the threshold determine by the vanish of the absolute instability region.

Further studies may complete the analysis by increasing Reynolds numbers, and by considering a non-stationary actuation. Indeed, as suggested by [12], the forcing will be more effective energetically when its frequency is around the natural shedding frequency. On the other hand, some recent methods as dynamical mode decomposition (see i.e. Schmid (2010) [16]) presents as a very promising tool for experimental data analysis sharing some properties of linear stability methods.

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