A New Method for Calculating Residual Astigmatism Produced by Toric Intraocular Lens Rotation

Yasuo Yae¹* and Toshiaki Kubota²

¹Yae Eye Clinic, 3-1 Kawasaki Hiji-machi Hayami-gun, Oita 879-1505, Japan.
²Department of Ophthalmology, Oita University, Faculty of Medicine, Iidaigaoa, Hasama-machi, Yufu-shi, Oita 879-5593, Japan.

Authors’ contributions

This work was carried out in collaboration between the two authors. Author YY designed the study, wrote the protocol, and wrote the first draft of the manuscript. Author TK managed the literature searches, analyzed the results, and supervised the study. Both authors read and approved the final manuscript.

ABSTRACT

Aims: To report a new method for calculating residual astigmatism when exact and inexact cylinder power toric intraocular lenses (IOLs) are rotated.
Methodology: Using Excel spreadsheets and \(\sin^2 \theta\) values, we expressed the cylinder powers of toric IOLs in percentage terms compared to corneal astigmatic powers and calculated values for various corneal astigmatisms and toric IOLs. From these values, we created charts of residual astigmatic powers and axes demonstrating the results of IOLs ranging from 50% to 150% and rotated 0º to 30º.
Results: Comparing residual astigmatic powers, the rank orders for each IOL were as follows: 100%<90%<80% at 5º of rotation, 90%<100%<80% at 10º of rotation, and 90%<80%<75%<100% at 15º of rotation. Thus, lower cylinder power IOLs perform better than higher power IOLs when rotated over 5º. Furthermore, the residual astigmatic powers of 125% IOLs were always higher than those of 100% IOLs; however, the residual astigmatic powers of 75% IOLs became lower than those of 100% IOLs when rotation exceeded 15º.
Conclusion: Our method shows that lower cylinder power IOLs are advantageous in environments where IOL rotation is likely and when inexact IOLs are utilized.

*Corresponding author: E-mail: opt-y@mbf.nifty.com;
Keywords: Astigmatism; toric; rotation; calculation.

1. INTRODUCTION

Toric intraocular lenses (IOLs) are relatively effective in correcting corneal astigmatisms [1, 2]. However, errors in measuring both the corneal astigmatic axis and postoperative IOL rotation may occur [3, 4]. Moreover, inexact IOLs are sometimes utilized because the cylinder powers of toric IOLs are limited, while corneal astigmatic powers are infinite. Therefore, we need precise calculation methods for residual astigmatism when higher, equal, or lower cylinder power IOLs are rotated compared to the corneal astigmatic power.

In general, the astigmatic distribution curve of a cylinder lens of f diopters (Cylinder (C)+f Diopter (D)×0°) is represented by f sin²θ [5]. In the present study, we used sin²θ and Excel spreadsheets to calculate the residual astigmatism produced by toric IOL rotation. Furthermore, we expressed the cylinder powers of toric IOLs in percentage terms compared to corneal astigmatic powers. Therefore, our method can be applied to any corneal astigmatism and to any cylinder power IOL.

Several calculation methods have been previously reported. Two main types of methods have been discussed, namely vector [6, 7] and matrix methods [8, 9]. While these methods are detailed and precise, the calculations must be performed one-by-one. In contrast, our method allows us to perform many precise calculations simultaneously in Excel spreadsheets using percentage terms and sin²θ. Furthermore, our method also allows for visually comprehensible charts to be obtained. Consequently, we are able to decide whether a higher, equal, or lower cylinder power IOL would be most appropriate.

We would typically use the website “AcryS of Toric Calculator” when preoperatively deciding which IOL would be most optimal [10]; however, this does not explain why a particular IOL is selected. In contrast, our method allows us to better understand the concept as it sheds light on the theoretical background behind the calculations through its tables and figures.

2. METHODOLOGY

Firstly, we defined the corneal astigmatism as C+2.0 D×0° (f=2.0), which is represented by 2 sin²θ (Fig. 1). Furthermore, we expressed the cylinder powers of toric IOLs in percentage terms compared to the corneal astigmatic powers. Namely, the cylinder power of a 100% toric IOL is 2.0 D, and that of a 75% toric IOL is 1.5 D if the corneal astigmatic power is 2.0 D (2.0 D×1.0=2.0 D, 2.0 D×0.75=1.5 D).

Under these conditions, we created (Table 1) to explain our calculations. In the table, sin²θ is calculated using the actual formula (Microsoft Excel, Redmond, WA, USA) as follows: $\sin(\theta \times \pi/180) \times \sin(\theta \times \pi/180)$

Through this formula, we can calculate values using degrees rather than radians; moreover, any errors in the calculation results were less than $1 \times 10^{-10}$ D.
The astigmatic power curve of a cylinder (C) +2.0 diopter (D)×0° corneal astigmatism

Table 1. Residual astigmatism when a corneal astigmatism of C+2.0 D×0° is corrected by a toric intraocular lens of C-2.5 D rotated 15°

| θ (Angle from the horizontal axis) (degree) | CA (Corneal astigmatism) | TI (125% (C-2.5D) IOL rotated 15°) | CA+TI (Residual astigmatism) |
|--------------------------------------------|--------------------------|------------------------------------|-----------------------------|
|                                            | C+2.0 D×0°=2 sin²θ (diopter) | C-2.5D×15°=2.5sin²(θ-15) (diopter) | C+2.0×0° + C-2.5D×15°=2 sin²θ-2.5sin²(θ-15) (diopter) |
| 0                                          | 0.000                    | -0.167                             | -0.167                      |
| 15                                         | 0.134                    | -0.000                             | 0.134                       |
| 30                                         | 0.500                    | -0.167                             | 0.333                       |
| 45                                         | 1.000                    | -0.625                             | 0.375                       |
| 60                                         | 1.500                    | -1.250                             | 0.250                       |
| 75                                         | 1.866                    | -1.875                             | -0.009                      |
| 90                                         | 2.000                    | -2.333                             | -0.333                      |
| 105                                        | 1.866                    | -2.500                             | -0.634                      |
| 120                                        | 1.500                    | -2.333                             | -0.833                      |
| 135                                        | 1.000                    | -1.875                             | -0.875                      |
| 150                                        | 0.500                    | -1.250                             | -0.750                      |
| 165                                        | 0.134                    | -0.625                             | -0.491                      |
| 180                                        | 0.000                    | -0.167                             | -0.167                      |

C=cylinder. D=diopter. IOL=intracocular lens. θ=angle from the horizontal axis. CA=C+2.0D×0° corneal astigmatism. TI=125% (C-2.5D) toric IOL rotated 15°

The residual astigmatic power is represented by the amplitude of the sine wave, the axes of which are the peak and trough of the sine wave, respectively (cf. Fig. 2).

The approximate residual astigmatic axes at 45° and 135° are shown in square.

The approximate residual astigmatic power=0.375D(-0.875D)=1.250D. ("0.375") and ("-0.875") are shown in square.
In (Table 1), column 2 indicates the values for a C+2.0D×0° corneal astigmatism in 15° increments, while column 3 indicates the values for a 125% (C-2.5D) toric IOL in the same increments when rotated 15°. In column 4, the residual astigmatism is calculated for a C+2.0D×0° corneal astigmatism (column 2)+125% (C-2.5D) toric IOL when rotated 15° (column 3). We then created (Fig. 2) from (Table 1).

**Fig. 2. The residual astigmatism observed when a cylinder (C)+2.0 diopter (D)×0° corneal astigmatism is corrected by a C-2.5D toric intraocular lens (IOL) rotated 15° (cf. Table 1)**

CA represents a cylinder (C)+2.0 diopter (D)×0° corneal astigmatism, TI represents a C-2.5D×15° toric IOL, and CA+TI represents the residual astigmatism. The approximate residual astigmatic power is 1.250D=0.375D-(-0.875D), which is derived from (Table 1). Area A is magnified in (Fig. 3).

Next, we demonstrate how to calculate the precise residual astigmatic power and axes. The residual astigmatic power is represented by the amplitude of the sine wave, while the residual astigmatic axes are the peak and the trough of the sine wave, respectively. Therefore, in (Table 1 and Fig. 2), the approximate residual astigmatic power is 1.250 (0.375-(-0.875)) D, and the approximate residual astigmatic axes are 45° and 135°. However, more precise calculations can be performed by calculating in 1° increments. Therefore, we created (Table 2 and Fig. 3) from area a in (Fig. 2). (Table 2 and Fig. 3) reveal that a more precise value for the residual astigmatic power is 1.2608 (=0.3804-(-0.8804)) D, and more precise residual astigmatic axes are 41° and 131°.

More precise values for residual astigmatic axes at 41° and 131° are shown in square. More precise values for residual astigmatic power=0.3804-(-0.8804)=1.2608D. ("0.3804") and ("-0.8804") are shown in square.
Table 2. More precise values for residual astigmatic axes and powers (cf. Fig. 3)

| θ (Angle from the horizontal axis) (degree) | CA (Corneal astigmatism) C+2.0D×0° =2sin²θ (diopter) | TI (125% (C-2.5D) IOL rotated 15°) C-2.5D×15°=-2.5sin²(θ-15)(diopter) | CA+TI (Residual astigmatism) C+2.0D×0°+C-2.5D×15° = 2 sin²θ-2.5sin²(θ-15) (diopter) |
|-------------------------------------------|-------------------------------------------------|---------------------------------|----------------------------------|
| 39                                       | 0.7921                                           | -0.4136                         | 0.3785                           |
| 40                                       | 0.8264                                           | -0.4465                         | 0.3799                           |
| 41                                       | 0.8608                                           | -0.4804                         | 0.3804                           |
| 42                                       | 0.8955                                           | -0.5153                         | 0.3802                           |
| 43                                       | 0.9302                                           | -0.5510                         | 0.3792                           |
| 44                                       | 0.9651                                           | -0.5876                         | 0.3775                           |
| 45                                       | 1.0000                                           | -0.6250                         | 0.3750                           |
| 46                                       | 1.0392                                           | -0.6696                         | 0.3728                           |

C=cylinder. D=diopter. IOL=intracocular lens. θ=angle from the horizontal axis. CA=C+2.0 D×0° corneal astigmatism. TI=125% (C-2.5D) toric IOL rotated 15°. CA+TI=residual astigmatism.

Fig. 3. Magnified area a (cf. Fig. 2)

CA represents a cylinder (C)+2.0 diopter (D)×0° corneal astigmatism, TI represents a C-2.5D×15° toric IOL, and CA+TI represents the residual astigmatism. The peak value of the residual astigmatism is 0.3804D at 41°.
Furthermore, we are able to calculate almost any corneal astigmatism and any toric IOL through percentage expressions, because the residual astigmatic power is proportional to the corneal astigmatic power. Similar to the aforementioned calculations, we calculated the residual astigmatism of each angle (θ) from 0° to 180° for each IOL (75%, 80%, 90%, 100%, and 125%) and each IOL rotation (0°, 5°, 10°, 15°, 20°, 25°, and 30°). From these calculations, we created (Table 3 and Fig. 4). In addition, we calculated the residual astigmatism of each angle (θ) from 0° to 180° for each IOL (75%, 80%, 90%, 100%, and 125%) and each IOL rotation (0°, 5°, 10°, 15°, 20°, 25°, and 30°), and obtained (Fig. 5). We are unaware of previous reports that contain graphs similar to (Fig. 4 and Fig. 5). (Fig. 5) is useful when considering oblique (OB) astigmatisms.

Table 3. Percentages of the residual astigmatic powers for each intraocular lens rotation

| Angle of IOL rotation (degree) | 75% IOL (percentages) | 80% IOL (percentages) | 90% IOL (percentages) | 100% IOL (percentages) | 125% IOL (percentages) |
|-------------------------------|-----------------------|-----------------------|-----------------------|------------------------|------------------------|
| 0                             | 25.00                 | 20.00                 | 10.00                 | 0.00                   | 25.00                  |
| 5                             | 29.20                 | 25.36                 | 19.32                 | 17.44                  | 31.70                  |
| 10                            | 39.10                 | 36.94                 | 34.42                 | 34.72                  | 46.18                  |
| 15                            | 51.32                 | 50.44                 | 50.12                 | 51.76                  | 63.04                  |
| 20                            | 64.30                 | 64.36                 | 65.66                 | 68.40                  | 80.44                  |
| 25                            | 77.36                 | 78.20                 | 80.80                 | 84.52                  | 97.74                  |
| 30                            | 90.14                 | 91.64                 | 95.38                 | 100.00                 | 114.56                 |

IOL = intraocular lens. Cylinder power of n% IOL = n/100 × corneal astigmatic power (n = 75, 80, 90, 100, and 125).

Each value (from row 1 to row 7) is represented in percentage terms compared to the corneal astigmatic power. Minimum values for each angle are shown in square.

This graph was created from (Table 3). The cylinder power of an n% IOL is n/100 × the corneal astigmatic power (n = 75, 100, and 125). Point B is the intersection of a 100% IOL and a 75% IOL (between 14° and 15°). The corneal astigmatic power is equal to 100% residual astigmatic power.

3. RESULTS

In (Table 3), the residual astigmatic powers produced by each toric IOL rotation can be compared. Namely, the rank orders of each IOL are 100% < 90% < 80% < 75% < 125% at 5° of rotation, 90% < 100% < 80% < 75% < 125% at 10° of rotation, and 90% < 80% < 75% < 100% < 125% at 15° of rotation. Therefore, 100% IOLs are the best selection for rotations within 5°, while 90% to 100% IOLs are best for rotations from 5° to 10°, and 80% to 90% IOLs are best for rotations over 10°. Moreover, (Fig. 4) reveals that the residual astigmatic powers of 125% IOLs are always higher than those of 100% IOLs. In contrast, those of 75% IOLs become lower than those of 100% IOLs when the IOLs rotation exceeds the point of intersection (point B: between 14° and 15°). Accordingly, the curves of the residual astigmatic powers of lower cylinder power toric IOLs have gentler slopes than those of higher cylinder power toric IOLs. This means that low cylinder power IOLs are a better choice than high cylinder power IOLs when rotated.
Fig. 4. The residual astigmatic powers of 75%, 100%, and 125% intraocular lens (IOL) rotations

In (Fig. 5), corrections with lower cylinder power IOLs (50%) result in the residual astigmatic axis being close to the corneal astigmatic axis, while corrections with higher cylinder power IOLs (150%) result in it being close to the axis of the IOLs. In addition, it is demonstrated that IOLs closer to 100% produce more oblique (OB) astigmatism. In general, OB astigmatism is not very significant if its power is small; however, the power of OB astigmatism should be minimized [11]. Therefore, low cylinder power toric IOLs are also better from this viewpoint, when exact IOLs are not available for the corneal astigmatism.

The cylinder power of an n% IOL is n/100 x the corneal astigmatic power (n=50, 75, 100, 125, and 150). In a 100% IOL (a), there is no axis when there is no rotation; however, the axis is approximately 45° when rotated by even 0.1°.
Fig. 5. The residual astigmatic axes of each intraocular lens (IOL) rotation

4. DISCUSSION

The advantage of our method is that the table (Table 3) and chart (Fig. 4) prepared beforehand are useful references that allow us to select the most appropriate toric IOLs. This method was facilitated by expressing the cylinder power of toric IOLs in percentage terms compared to the corneal astigmatic power and by using Excel tables; therefore, the calculations are simple. Consequently, (Table 3 and Fig. 4) indicate that a low cylinder power IOL would be a better choice than a high cylinder power IOL when rotated. This opinion is also shared by Felipe et al. [9]. On the other hand, some methods have recommended high cylinder power IOLs, contrary to our opinion [12, 13]. However, we must remember that slightly higher cylinder power IOLs will cause OB astigmatisms instead of becoming against-the-rule (ATR) astigmatisms when correcting with-the-rule (WTR) astigmatism (Fig. 5). Furthermore, the residual astigmatic powers corrected by high cylinder power IOLs are larger than those corrected by low cylinder power IOLs (Fig. 4). Kobayashi et al. reported that eyes with OB astigmatism had significantly lower visual performance than eyes with WTR or ATR astigmatism [11]. Moreover, the influence of IOL rotation becomes more severe with greater corneal astigmatism and when higher cylinder power IOLs are selected. In fact, one report suggested that the error was particularly relevant in patients in whom higher cylinder power IOLs were implanted [2]. Consequently, we recommend low cylinder power IOLs when correcting corneal astigmatisms.

Clinically, ATR astigmatic change in corneal astigmatisms with advancing age is a documented fact [14, 15]. Therefore, ATR astigmatism calculations are important. However, we have only calculated WTR astigmatisms in order to make the charts easy to understand.
This is because the differences in calculation results between the two astigmatisms are only the residual astigmatic axes. Thus, the residual astigmatic powers are the same for the two types of astigmatisms. For this reason, (Fig. 4) can also be applied to ATR astigmatisms.

In addition, (Fig. 4) can be used for both clockwise and counterclockwise IOL rotations. While we calculated only counterclockwise rotations, we could similarly calculate the clockwise rotation by changing the two formulas C-2.5D×15° and -2.5sin²(θ-15) in (Table 1) to C-2.5D ×(-15°) and -2.5 sin²(θ+15), respectively.

In our report, we did not mention surgically induced astigmatism (SIA) [16,17]. However, SIA does not contradict our method, because our method can also be used to calculate total astigmatisms. From the viewpoint of SIA, there is an opinion that recommends higher cylinder power IOLs [13]. In contrast, we recommend lower cylinder power IOLs, even if we consider including SIA, because slightly higher cylinder power IOLs cause OB astigmatism (Fig. 5).

To the best of our knowledge, there have been no methods for calculating residual astigmatisms using Excel spreadsheets, sin²θ, and percentage expressions of cylinder power IOLs compared to the corneal astigmatic power.

5. CONCLUSION

In conclusion, we were able to obtain precise and visually comprehensible charts that are useful for selecting toric IOLs, particularly when exact IOLs are not available.

CONSENT

Not applicable.

ETHICAL APPROVAL

Not applicable.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Sun XY, Vicary D, Montgomery P and Griffiths M. Toric intraocular lenses for correcting astigmatism in 130 eyes. Ophthalmology. 2000;107:1776-81.
2. Visser N, Berendschot TT, Bauer NJ, Jurich J, Kersting O, Nuijts RM. Accuracy of toric intraocular lens implantation in cataract and refractive surgery. J Cataract Refract Surg. 2011;37:1394-1402.
3. Koshy JJ, Nishi Y, Hirnschall N, Crnej A, Gangwani V, Maurino V, et al. Rotational stability of a single-piece toric acrylic intraocular lens. J Cataract Refract Surg. 2010;36:1665-70.
4. Jampaulo M, Olson MD, Miller KM. Long-term Staartoric lens rotation stability. Am J Ophthalmol. 2008;146:550-3.
5. Schwartz SH. Cylindrical lenses and the correction of astigmatism. In: Schwartz SH, editor. Geometrical and visual optics. A clinical introduction. New York: McGraw-Hill Education. 2013;127–148.
6. Alpins N. Astigmatism analysis by the Alpins method. J Cataract Refract Surg. 2001;27:31-49.
7. Tseng SS and Ma JJ. Calculating the optimal rotation of a misaligned toric intraocular lens. J Cataract Refract Surg. 2008;34:1767-72.
8. Harris WF. Astigmatism. Ophthalmic Physiol Opt. 2000;20:11-30.
9. Felipe A, Artigas JM, Diez-Ajenjo A, Garcia-Domene C and Alcocer P. Residual astigmatism produced by toric intraocular lens rotation. J Cataract Refract Surg. 2011;37:1895-1901.
10. AcrySof® Toric IOL Web Based Calculators. Available: http://www.acrysoftoriccalculator.com. Accessed 2 Nov 2012.
11. Kobashi H, Kamiya K, Shimizu K, Kawamorita T, Uozato H. Effect of axis orientation on visual performance in astigmatic eyes. J Cataract Refract Surg. 2012;38:1352-9.
12. Hill W, Potvin R. Monte Carlo simulation of expected outcomes with the AcrySof® toric intraocular lens. BMC Ophthalmol. 2008;8:22.
13. Ernest P and Potvin R. Effects of preoperative corneal astigmatism orientation on results with a low-cylinder-power toric intraocular lens. J Cataract Refract Surg. 2011;37:727-32.
14. Hayashi K, Hirata A, Manabe S, Hayashi H. Long-term change in corneal astigmatism after sutureless cataract surgery. Am J Ophthalmol. 2011;151:858-65.
15. Asano K, Nomura H, Iwano M, Ando F, Niino N, Shimokata H, et al. Relationship between astigmatism and aging in middle-aged and elderly Japanese. Jpn J Ophthalmol. 2005;49:127-33.
16. Hill W. Expected effects of surgically induced astigmatism on AcrySoft Toric intraocular lens results. J Cataract Refract Surg. 2008;34:64-7.
17. Borsaio E, Mehta JS, Maurino V. Surgically induced astigmatism after phacoemulsification in eyes with mild to moderate corneal astigmatism. J Cataract Refract Surg. 2006;32:565-72.

© 2014 Yae and Kubota; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sciencedomain.org/review-history.php?iid=523&id=23&aid=5333

377