Abstract

We present a study of the electrical and thermal conductivities of the \textit{d}-density wave (DDW) state in an external magnetic field \( B \) in the low temperature regime and in the presence of impurities. We show that in the zero temperature limit, \( T \to 0 \), the Wiedemann-Franz (WF) law remains intact. For finite \( T \) the WF law violation is possible and it is enhanced by the external field.

Key words: \textit{d}-density wave, magnetic field, Wiedemann-Franz law

PACS: 71.10.-w, 74.25.Fy, 11.10.Wx, 74.72.-h

A recent experiment of Hill et al. [1] that measured electrical and thermal conductivities of the optimally electron doped copper-oxide superconductor \( \text{Pr}_{2-x}\text{Ce}_x\text{CuO}_4 \) (PCCO) in its normal state found striking deviations from the Wiedemann-Franz (WF) law. The hole overdoped system \( \text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta} \) (Tl-2201) was also studied recently by Proust et al. [2].

They verified that in the overdoped Tl-2201 WF law holds perfectly. The WF law is one of the basic properties of a Fermi liquid, reflecting the fact that the ability of a quasiparticle to transport heat is the same as its ability to transport charge, provided it cannot lose energy through collisions. The WF law states that the ratio of the heat conductivity \( \kappa \) to the electrical conductivity \( \sigma \) of a metal is a universal constant:

\[
L_0 \equiv \frac{\kappa}{\sigma T} = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2,
\]

where \( k_B \) is the Boltzmann's constant, \( e \) is electron's charge and \( L_0 = 2.45 \times 10^{-8} \text{W}\Omega\text{K}^{-2} \) is Sommerfeld’s value for the Lorenz ratio \( L \equiv \kappa/(\sigma T) \). To be more precise, one should also specify the temperature range where the WF law holds. Strictly speaking this law is proven only in the limit \( T \to 0 \) and for a small concentration of impurities [3]. One of the possible theoretical interpretations of the WF law breakdown is that the quasiparticle fractionalizes into separate spin and charge. This separation can be investigated using various models and approaches.

An examination of the WF law was done by Yang and Nayak (YN) [4] and also by Kim and Carbotte (KC) [5] within the phenomenological \textit{d}-density wave (DDW) picture. The DDW scenario proposed in Ref. [6] is based on the assumption that the pseudogap phenomenon [7] in high-\( T_c \) cuprates is the result of the development of another order parameter called DDW order that has \textit{d}-wave symmetry and can be described by the mean-field Hamiltonian

\[
H_{\text{DDW}} = \int_{\text{RBZ}} \frac{d^2 k}{(2\pi)^2} \chi_\uparrow^+(t, \mathbf{k}) [H_0(\mathbf{k}) - \mu] \chi_\downarrow(t, \mathbf{k}),
\]

where

\[
H_0(\mathbf{k}) = \varepsilon(\mathbf{k})\sigma_3 - D(\mathbf{k})\sigma_2,
\]

the spinors \( \chi_\uparrow \) and \( \chi_\downarrow \) are

\[
\chi_\uparrow(t, \mathbf{k}) = \left( c_\uparrow(t, \mathbf{k}) \quad c_\downarrow(t, \mathbf{k} + Q) \right),
\]

the single particle energy is \( \varepsilon(\mathbf{k}) = -2t(\cos k_x a + \cos k_y a) \) with \( t \) being the hopping parameter, \( \mu \) is the chemical potential, \( D(\mathbf{k}) = \frac{D_0}{2}(\cos k_x a - \cos k_y a) \) is...
the $d$-density wave gap and $Q = (\pi/a, \pi/a)$ is the wave vector at which the density-wave ordering takes place and $\sigma_i$ are Pauli matrices. The integral is over the reduced Brillouin zone. The units $\hbar = k_B = c = 1$ are chosen.

One of the unusual features of the DDW state is that for a half-filled band the chemical potential of the nodal quasiparticles participating in the electrical and thermal transport can be small or even zero, i.e. $|\mu| < k_BT$, that violates the usual conditions of the WF law validity. Indeed, exactly in the limit $\mu = 0$ the WF law is strongly violated in the extremely clean limit [4]. There is no WF law violation in the $T = 0$ limit for finite $\mu$ and/or $\Gamma$ [4,5]. For finite temperatures the WF violation depends on the impurity scattering [5]: in the Born limit (for a constant impurity scattering rate $\Gamma$) there is no change in the WF law, but in the unitary limit for the frequency dependent scattering rate the WF law is violated, but only for $|\mu|$ smaller that the DDW gap. When $\mu$ is increased sufficiently, the Lorenz number becomes approximately equal to its conventional value and its temperature dependence is small.

While in general the validity of the DDW pseudogap scenario is still questionable, it is important to scrutinize all its theoretical consequences. One of the opportunities is to study possible WF law violations using the DDW model, so that these results can be compared with the experimental results of Refs. [1,2]. The presence of the external magnetic field is an essential ingredient of the experiments [1,2], so that if the DDW state exists, it would show up in the magnetic field in the underdoped regime at low $T$ when the superconductivity is destroyed. In this paper we present the study [8] of the WF law for the DDW model including the external magnetic field for a constant impurity scattering rate paying special attention to the regime $|\mu| \lesssim k_BT$ where the violation of the WF law is expected.

We show that the strongest violation of the WF law is possible for $\mu = 0$ and only at finite temperatures. This is the case shown in Fig. 1. We observe that while the results of [1] suggest that the WF law is violated at $T \to 0$, there is no violation of the WF law in this limit in the DDW scenario of the pseudogap. Since in Fig. 3 of Ref. [1] the electrical conductivity is a constant, the line $\kappa_e(T)/T$ directly represents the normalized Lorenz number $L(T)/L_0$. For finite temperatures there is then some similarity between Fig. 1 (see also other figures in Ref. [8]) and Fig. 3 of [1] where as $T$ increases the thermal conductivity crosses from the region with $\kappa_e(T)/L_0 < \rho_0$ to the region with $\kappa_e(T)/T > L_0/\rho_0$ resembling the character of the WF law violation seen in the experiment. It is obvious from Fig. 1 that such a behavior of $L(T)$ is due to the presence of the magnetic field. This confirms our claim that to interpret theoretically the experiment [1] one should take into account the influence of the external field.

Our main conclusions concerning the WF law can be summarized as follows.

1) We have shown that in the DDW state in the presence of impurities the WF law holds in $T \to 0$ limit for an arbitrary field $B$ and chemical potential $\mu$. This is checked within the bubble approximation, i.e. not including the impurity vertex.

2) For finite temperatures $T \lesssim |\mu|$, the WF law violation is possible and in zero field the thermal conductivity dominates over the electrical conductivity, i.e. $L(T)/L_0 > 1$.

3) For $T \lesssim |\mu|$ in the nonzero field the WF law violation becomes even stronger than in zero field and depending on the temperature both regimes $L(T)/L_0 \ll 1$ and $L(T)/L_0 > 1$ are possible.

4) For $T \ll |\mu|$ there is no WF violation even in the presence of magnetic field.

This work was supported by the research project 20-65045.01 of the Swiss NSF. The work of V.P.G. was supported by the SCOPES-projects 7UKP062150.00/1 and 7 IP 062607 of the Swiss NSF and by the Grant No. PHY-0070986 of NSF (USA).

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