Algorithm for constructing trajectories of maneuvering object based on bearing-only information using the Basis Pursuit method

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Abstract. The possibility of using the Basis Pursuit signal recovery method in the object tracking algorithm as a method of filtering and extrapolating the trajectory of the observed object in a one-dimensional measurement space is considered. The simulation results are presented for the trajectory processing algorithm using the proposed filtering method for the data obtained during the object direction-finding in comparison with the block algorithm based on the maximum likelihood method and the Kalman filter.

Introduction

The problem of determining the angular position of an object relative to the observer using bearing-only information is widely presented in the literature [1 ... 4]. The urgency of this task is due to the need to use bearing-only observation systems and the lack of location information in the environment surveillance systems. A similar situation arises due to various reasons: the need for secrecy of the observer, requirements for radio and sound pollution of the environment, etc. The main problem arising in the synthesis of tracking algorithms (as well as in other similar problems) is inaccurate knowledge of the motion model of the object of direction-finding. Lack of information about an object leads to the need to impose strict restrictions on the mathematical models describing its movement. This leads to a loss of stability and additional errors in the estimation of the angular coordinates of the observed object, especially in the case of algorithms based on Kalman filtering. In this regard, it becomes necessary to design robust algorithms that ensure stable tracking of objects when they are maneuvering. To solve this problem, in most cases, the authors of publications suggest using the method of multiple model filtering [5-7]. However, due to the finiteness of the set of models of the observed object motion, there will be no significant gain in robustness. One of the most productive approaches to achieving robustness is the use of nonparametric estimation methods. These methods include a wide class of spectral methods. The article is devoted to consideration of one of such approaches - the Basis Pursuit (BP) algorithm used in the problem of signal recovery [8]. The BP algorithm, within the framework of this article, is used as a filter in the object tracking algorithm based on the method of joint probabilistic data association (JPDA) [9,10].
The paper considers the construction of the observed object trajectory based on the bearing-only measurements in the horizontal plane, according to the results of which it is necessary to estimate the angular coordinate of the object: relative bearing (RB) to it from the carrier. Linear phased antenna arrays in hydro- and radar systems can act as direction-finding. The aim of the work was to analyze the possibility of using the BP algorithm to solve the filtering and extrapolation problem in object tracking in the presence of bearing-only information.

The article is structured as follows. The first section contains the problem formulation, and describes the input and output data for the object tracking algorithm. In the second section, the mathematical description of the object tracking algorithm is briefly presented, where the main tracking stages and the algorithms for each stage are presented. The third section describes realization of a trajectory parameter filter based on the BP method. In the fourth section, mathematical modeling of the algorithm is presented using the example of processing data from a hydroacoustic station with a linear antenna array. The final section presents the main conclusions from the results obtained.

1. Statement of the problem of object tracking in the direction-finding mode in the horizontal plane

The solution to the problem of tracking the observed objects is usually divided into two stages [10]:

- tentative track formation and detection of the trajectory;
- tracking (included tracking cancelled).

At the stage of the trajectory detection, an assumption is made about the possible presence of a new object of observation and a hypothesis of the law describing its motion is formed. At the stage of detection, on the basis of a preselected criterion, a decision is taken on the detection of the observed object and the hypothesis (model) of its motion is refined. The tentative track formation and detection algorithms are described in details in various literature sources [9-12] and will not be considered in the present article.

As a rule, it takes 5-7 cycles of the tracking algorithm to implement the trajectory detection. In case of a positive decision about the presence of the object, the stage of its tracking begins.

Let us consider the formulation of the problem of tracking $J$ trajectories, in the presence of $M$ objects of observation in the line of sight of a linear antenna. The number of trajectories $J$ can be either less than the number of objects $M$, if no signals from them are detected, or more in the case of tracking false objects (Fig. 1).

The input data are received cyclically, while the data processing cycle is understood as the time period of information accumulation in the problem of calculating the spatial-frequency spectrum of the signal. In this case, the tracking task can cover several cycles of input data $N$ (batch processing) with an overlap value of $N$-1, or use data only from the current cycle (for example, approaches based on Kalman filtering). The task of tracking is to estimate the trajectory parameters of objects observed at the current moment in time to predict the values of these parameters at the next moment in time (data reception cycle).

The input data for the tracking algorithm are:

- set of signal marks $\{I\}_k$, where $k \geq 1$ – is the index of the data reception cycle. The mark refers to the measured RB to the intended object and its standard deviation [10]. In this case, the mark can be either false - not corresponding to any object, or true, otherwise.
- set of trajectory forms $\{J\}_k$. This set contains the forms of the trajectories detected at the previous stages of the trajectory analysis, as well as the forms of the trajectories tracked at the previous cycles of the algorithm. The form is a set of trajectory parameters estimated at the tracking stage. If, at the stage of trajectory detection, new trajectories were initiated, then the number of forms increases, and if at the previous $k$-1 stage of tracking a decision is made to cancel the trajectories from tracking, the number of forms decreases (Fig. 1). For the case of a direction-finding operating in the horizontal plane, the form contains an
estimate of the RB for the proposed object and the magnitude of its change, as well as the corresponding root mean square errors.

Fig. 1. Traced trajectories and values of RB of the marks «+» – RB value for all marks in the corresponding data reception cycles, «» – RB value for marks corresponding to tracked trajectories.

The output of the algorithm is a set of trajectory forms \( \{ \mathbf{J}_k \} \). The forms contain the coordinate information obtained in the current machining cycle. In addition, the trajectory forms are deleted from the set, according to which the decision to deactivate the tracking was made.

For trajectory processing, a state equation is traditionally used, where the trajectory of the observation object is described by a polynomial of no higher than the first degree with respect to the motion coordinate with a disturbing effect in the form of random acceleration, represented by white Gaussian noise with zero mean and some variance. This model corresponds to the general motion of an object along a straight line with a noise-induced unintentional maneuver. However, even with more complex target behavior, this model is used quite often, and the resulting inconsistency with real motion is compensated by increased noise variance [10].

In the case of arbitrary motion of the observation object, the trajectory equation can be written in the form [13, 14]:

\[
x_k^m = f_m \left( x_{k-1}^m \right) + w_k^m, \tag{1}
\]

where: \( x = \{ \alpha \quad \mathbf{\Pi}_d \quad \mathbf{\Pi}_o \}^T \) – state vector of the \( m \)-th object (\( m=1...M \)) at \( k \)-th time instant, describing its model params, \( \alpha \) – RB to the object, \( \mathbf{\Pi}_d \) – params of the dynamic models of the object (moments of inertia, mass, coordinates of the mass center, etc.), \( \mathbf{\Pi}_o \) – other parameters of motion and location relative to the observer (for example, the rate of relative bearing change), \( w_k^m \) – centered, generating white noise, describing random disturbances during the object motion, with the correlation function \( M \{ w_i^m w_n^m \} = \delta_{k,n} Q_i^m \), where: \( Q_i^m \) – the covariance matrix of the generating noise of the \( j \)-th trajectory; \( \delta_{k,n} \) – Kronecker symbol; \( M \{ \} \) – expectation sign; \( f_m (\bullet) \) – function that defines the model of the object motion.

As described above, in each cycle, a set of measurements is received in the form of marks, which are RB values in this case. Here the observation equation can be written in the form of measurements \( \alpha_k^i \) of every \( i \)-th mark at time instant \( k \):
\[ \tilde{\alpha}_k^i = Hx_k^m + v_k^i, \quad i = 1 \ldots I_k, \quad j = 1 \ldots J_k \]  

where \( H = \{1 \ 0 \ldots 0\}^T \) – measurement matrix being the same for \( \forall \{i, m, k\} \), connecting the \( i \)-th measurement with the state vector corresponding to the \( m \)-th tracking object; \( v_k^i \) – white noise of observation with specified root mean square (RMS) values of RB measurement – \( \tilde{\sigma}_\alpha \).

All of the above is also true for false marks, and the difference will only be in the form of a function \( f_m(\bullet) \). Therefore, formally, we include all false marks in the set of indices \( i \). Absence of index \( m \) in \( \alpha_k^i \) is due to complicated dependence \( \{m\} \leftrightarrow \{i\} \), which is ambiguous and, therefore, the index \( m \) is present in \( \tilde{\alpha}_k^i \) as the relation between \( i \) and \( m \). Here one \( m \) can correspond to several \( i \).

The converse is also true. In what follows, we will consider only the case when there is a one-to-one correspondence for \( \{m\} \rightarrow \{i\} \).

Thus, based on \( \{\tilde{\alpha}_k^i\} \) you need to generate filtered \( \{\hat{\alpha}_k^i\} \) and extrapolated \( \{\hat{\alpha}_{k+1}^i\} \) estimates of RB, as well as the corresponding root mean square errors.

2. Mathematical description of the object tracking algorithm

Many methods have been developed to solve the problem of object tracking affected by noise [1-7, 9-13]. These methods are usually divided into five stages:

1. extrapolation of the \( j \)-th trajectory to the current (\( k \)-th) moment of time and determination of the position of the identification strobe center;
2. determination of the size of the identification strobe for the \( j \)-th trajectory;
3. identification of marks with trajectories (selection of one mark from all received marks \( \{1\}_k \), which corresponds to the \( j \)-th trajectory);
4. estimation of the parameters of the \( j \)-th trajectory (RB and its standard deviation), taking into account the parameters of the mark identified with it;
5. making a decision on the status of the trajectory (tracked or no longer tracked).

To implement stages 2, 3, 5, the authors used the methods widely known in the literature:

For stage 2 – the gating technique proposed by Bar-Shalom, presented in [9, 10].

For stage 3 – the JPDA method described in the literature [9, 10].

For stage 5 – the method of ratio between the calculated value of the strobe and the track initiation strobe presented in [15, 16].

Since the well-known methods are used in the algorithm for 2, 3 and 5 stages, we will go into detail on stages 1 and 4, namely on extrapolation and filtering. Since the extrapolation and filtering algorithms using one method are very close, hereinafter we will consider the trajectory filtering algorithms assuming that they will solve the extrapolation problem as well.

In the literature, filters that provide the process of tracking an observation object are usually divided into batch and recurrent filters.

The solution of the problem of ensuring the required tracking accuracy and resistance to disruption of highly maneuverable object tracking with the help of recurrent filtering algorithms does not bring the desired result due to the practical independence of the filter gains of the state vector elements. Batch filters are preferred over recurrent ones when tracking a maneuvering target using nonlinear filters, since they provide greater accuracy [17]. However, batch filtering algorithms have very significant drawbacks as compared to recurrent filters, namely:

- large number of calculations;
- the need to recalculate all filter weights when obtaining a new measurement;
- impossibility to use information about previous (relative to the considered data set) process values;
- delay in generating parameter estimates at the initial stage of trajectory tracking [13].
Despite all the shortcomings, to improve the filtering accuracy of a maneuvering object under conditions of the maneuver nature uncertainty, when recurrent filters can generate an estimate with a large error, it is proposed to apply the batch BP method, which is used in various tasks to restore signals from partially known data, as well as to isolate a signal from the noise. BP method is used to restore signals whose shape is undefined, irregular and non-periodic, accompanied by a sharp change in the intensity and periodic loss of the received signal, which means that, unlike filters traditionally used in TA, this method can track the trajectory variability if the object position changes arbitrarily.

The BP algorithm allows filtering and extrapolation of trajectory data for maneuvering purposes, while the application of widely used recurrent algorithms and a number of batch algorithms requires the use of a multi-model filtering method. Such methods use not one, but several motion models at a time. These methods at each data acquisition cycle consist in selection of such models from the bank, which comply with the nature of the observed object's movement, and the estimation of the state vector based on the use of all selected models. This approach leads to increase in the cost of computing resources, because they grow in direct proportion to the number of models in the bank. Based on this, the main advantages of BP can be distinguished:

- it does not require setting a motion model;
- it does not require a large number of computing resources, because it does not use a bank of filters and analysis of the probabilities of their correspondence to the nature of the observed objects motion;
- it allows filtering when data are omitted.

Since the BP method is a batch method, its correct application requires preliminary data accumulation. Therefore, in this work, we consider the process of tracking an object on the assumption that the decision about its initiation and detection has already been made based on the analysis of accumulated data (usually 5-7 marks identified with the trajectory) and we know the coordinates of its position at previous times.

By the time the filtering stage begins, as a result of identifying marks \( \{I\}_k \) and trajectories \( \{J\}_k \) for each trajectory \( j \), one of the marks \( \tilde{a}_k^j \) is selected if it hits the auto-capture strobe. At this stage, the estimates of the object positions are calculated, provided that the mark \( \tilde{a}_k^j \) is assigned to the trajectory \( j \). For this, the value \( \tilde{a}_k^j \) is combined with those obtained earlier \( \{\tilde{a}_1^i, \tilde{a}_2^i, ..., \tilde{a}_{t-1}^i\} \). Such a resulting sequence can be considered a periodic process even with a linear time dependence of the RB to the observed object due to the presence of measurement errors, which means that it can be analyzed using the BP method.

3. Application of the Basis Pursuit method in the trajectory tracking problem

Basis Pursuit belongs to the class of spectral methods [18, 19] and is one of the sparse approximation methods, when a sequence of measurements is represented as a finite linear combination of elementary functions selected from a large, in general case, linearly dependent set of functions. The difference from simple approximation is that not all functions from the set are involved in the expansion, but only some of them. The sparse approximation results in representation of the process as a superposition of the components of the "optimal" expansion in the basis [20]. The criterion for optimality of a basis is \( l_1 \)-minimization of the norm of its expansion coefficients. The Fourier transform is considered as the basis of the expansion in this problem. In our case, the process under consideration is the time dependence of the course angle of the observed object \( \tilde{\alpha}^j \), which can be expanded into a Fourier series:

\[
\tilde{\alpha}^j = \Phi a + n = \sum_{l=1}^{L} a_l \phi_l + n
\]  (1)
where: $\Phi$ – a vector containing an orthogonal set of harmonic functions; $\phi_l$ – a set of harmonic functions; $a_i$ – expansion coefficients; $n$ – some noise generated as a result of using only a part of the set of functions of the basis – $\phi_l$.

An iterative algorithm for filtering and extrapolating the trajectory of an object using the Basis Pursuit method can be represented as a sequence of the following steps:

1. Setting the initial conditions for finding the smoothing function:
   - the maximum number of time cycles for receiving the input data of the problem $K \geq 5$, which will be used to find the smoothing function (we say smoothing because determination of the trajectory coordinates will be made for all $K$ instants at a time);
   - the number of discarded upper coefficients of the Fourier series $n_f$, which determine the magnitude of the noise $n$ in expression (1), and, as a consequence, the degree of trajectory smoothing;
   - the number of iterations $n_i$ in the calculation cycle according to the Basis Pursuit method;
   - the maximum possible rate of RB change - $V_{\text{max}}$ - for an observation object during the maneuver.

2. Using the Basis Pursuit method to determine the Fourier transform coefficients for the trajectory, which provide filtering of the trajectory or extrapolation by the expressions:
   - values are initialized, $\mu > 0$, $d$;
   - the following calculations are performed cyclically:
     
     $$
     v \leftarrow \text{soft}(a + d, \lambda / \mu) - d
     $$

     $$
     d \leftarrow \frac{1}{\mu + p} \Phi^H (\hat{\alpha} - \Phi v),
     $$

     (2)

     $$
     a \leftarrow d + v
     $$

     - where $\text{soft}(x, T) = \max(1 - T/|x|, 0) \cdot x$; $p$ – multiplier in the expression $\Phi \Phi^H = pI$;
     - computations stop when the number of iterations $n_i$ reaches the specified value.

3. The obtained value of the observation object coordinates is checked for physical feasibility:
   - the rate of relative bearing change should not exceed the maximum value $V_{\text{max}}$.

If the condition in item 3 is not met, the number of discarded upper coefficients of the Fourier series $n_f$ decreases and items 2-3 are repeated.

4. Simulation results

Simulations were carried out for a linear antenna located on a mobile carrier (observer). Two objects moving in a straight line at a constant speed were under observation. During the episode, the carrier performed a maneuver.

Figures 2 (a and b) show the results of calculating the RB to observation objects depending on time for trajectory tracking algorithms using the Basis Pursuit filters and filtering based on the maximum likelihood function method. Figures 3 (a and b) show the results of calculating the same parameters for trajectory tracking algorithms using Kalman and BP filters.

The results of mathematical modeling showed stable tracking of objects when using all the considered algorithms for straight sections. During the carrier maneuvering the trajectory bends significantly due to tracking of the observation object only by the RB and to the impossibility of taking into account the influence of the observer's movement on the measurements obtained. The trajectories of maneuvering objects of observation have the same character when observed from a non-maneuvering observer. In this case, both the Kalman filter and the algorithm based on the use of a
non-recurrent filtering option based on the maximum likelihood function method did not allow steady tracking of the objects, and the trajectories were interrupted. The algorithm using the BP filter in this situation showed the best results and allowed objects to be tracked without breaking contact. No false trajectories were observed. This is due to the fact that the model of rectilinear motion, incorporated in the Kalman filter and in the algorithm based on the use of a non-recurrent filtering option, does not correspond to the model of true displacement. For such maneuvering objects, it is necessary to use multi-model filtering methods that require more cumbersome calculations and computational costs. In addition, in the Kalman filter, the prediction of the current value is greatly influenced by the weight of the sum of the previous values. The presence of an object maneuver leads to dynamic filtering errors, which significantly affect the trajectory forecast.

![Graph a)](image)

![Graph b)](image)

**Fig. 2.** Results of filtering RB trajectories. Red - non-recurrent filtering option based on the maximum likelihood function method, Blue - BP
5. Conclusion
The BP method proposed in this work allows us to move from the established practice of direct determination of motion parameters to finding a trajectory function (the ratio between the relative bearing and time) of an arbitrary form and approximating the results of discrete observations. The results obtained show a greater stability of such filtering in comparison with the considered methods of Kalman filtering and a batch filter based on the method of maximum likelihood, which makes it possible to increase the time of continuous trajectory tracking. This is due to the use of a nonparametric model with weak constraints on the noise model.
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