Application of Multi-variable Grey Model Method in Power Load Forecasting

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Abstract. Electric power has become one of the most important energy sources in today's society. It is of great significance for power grid enterprises and people's livelihood to accurately grasp the historical data of power load and to mine the potential value of data. In this paper, by making use of the multi-variable grey model, the power load forecasting is studied. Firstly, this paper establishes the multi-variable grey model with grey system theory. Secondly, the correlation between four temperature factors and load is analyzed, which shows that four temperature factors should be considered as the influencing factors of load. Finally, multi-variable grey model is applied to forecast the summer load of some region in Ningbo. The results show that the prediction has better fitting and prediction accuracy.

1. Introduction
Load forecasting [1,2] refers to the process of determining the future load in a certain precision sense, taking into account some important relevant factors and social influences, using a systematic approach. Accurate load forecasting can economically and reasonably arrange the start and stop of the generators of the power grid, maintain the safety and stability of the grid operation, reduce the unnecessary rotating reserve capacity, reasonably arrange the equipment maintenance plan, and ensure the normal production and life of the society. The level of load forecasting has become one of the notable signs of measuring whether a power company's management is modernizing.

For a long time, domestic and foreign scholars have conducted in-depth research on power load forecasting problems, and proposed many effective forecasting methods, such as regression analysis method [3], expert system method, fuzzy prediction method [4], time series method [5], and so on. Based on the continuous development of artificial intelligence methods, in recent 20 years, a large number of artificial intelligence methods have emerged and applied in power load forecasting. Artificial intelligence methods have strong processing capacity for non-linear problems, which can overcome the shortcomings of traditional models. Modern prediction methods are mainly as follows: artificial neural network model [6], support vector machine model [7], wavelet analysis [8], group intelligence optimization method [9], etc.

Grey system theory has many advantages, such as fewer sample data, simple theory, convenient operation, high short-term prediction accuracy and verifiability, and has been widely used [10-13]. For the power load, it is affected by many factors, such as the development of the national economy, the temperature, wind speed, economic structure and production scale changes, population growth, geographical environment and so on. Because there are many factors of electric power load, many of them are difficult to quantify directly, and it is difficult to find a clear corresponding relationship between various factors and load. Therefore, the influence factors of power load can be described by a
grey space, which can be modeled and predicted by using grey system theory. In this paper, the multi-variable grey model will be used to forecast the power load in summer.

2. Multi-variable Grey Model

2.1. Grey system theory

Some information is known, some information is unknown, then this system is a gray system. For the electric load system, many factors are difficult to know exactly, so the electric load system is a gray system. The phenomena displayed in the grey process are random and chaotic, but after all, they are ordered and bounded. Therefore, this data set has potential laws. Grey prediction is a method of establishing grey model for grey system and making prediction by using this law.

The actual social and economic system is complex and often contains many variables. These variables interact with each other, interrelate with each other and develop together, forming a complex system. In this case, the development of each variable is not isolated, and the changes of each variable are affected by other variables as well as other variables. From this point of view, this paper adopts a multi-variable grey model, which is denoted by MGM(1,n), aiming at describing all variables from a systematic point of view, explaining the relationship between power load, climate factors and other variables, and revealing the changing law of power load.

2.2. Establishment of Multivariable Grey Model

As a generalization of the model GM(1,1) and the model GM(1,n), the multi-variable grey model MGM(1,n)[14] is a first-order linear differential equations consisting of n variables, which can better reflect the influence of variables on the overall model.

Let \( x_i^{(0)}(k)(i=1,2,\ldots n) \) be \( n \) time series, and let \( x_i^{(j)}(k)(i=1,2,\ldots n) \) be \( n \) new sequences which are generated by an accumulation as follows:

\[
x_i^{(j)}(k) = \sum_{j=1}^{k} x_i^{(j)} (j) \quad (k=1,2,\ldots m; \quad i=1,2,\ldots n)
\]

The first-order ordinary differential equations of MGM(1,n) can be expressed as:

\[
\begin{align*}
\frac{dx_1^{(1)}}{dt} &= a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \ldots + a_{1n}x_n^{(1)} + b_1 \\
\frac{dx_2^{(1)}}{dt} &= a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \ldots + a_{2n}x_n^{(1)} + b_2 \\
& \quad \vdots \\
\frac{dx_n^{(1)}}{dt} &= a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \ldots + a_{nn}x_n^{(1)} + b_n
\end{align*}
\]

(2.1)

Let

\[
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\]

\[
B = (b_1, b_2, \ldots, b_n)^T
\]

and take
\[
X^{(0)}(k) = \left(x_1^{(0)}(k), x_2^{(0)}(k), \ldots, x_n^{(0)}(k) \right)^T
\]

\[
X^{(1)}(k) = \left(x_1^{(1)}(k), x_2^{(1)}(k), \ldots, x_n^{(1)}(k) \right)^T
\]

Then, (2.1) can be rewritten as:

\[
\frac{dX^{(i)}}{dt} = AX^{(i)} + B
\]

(2.2)

Here \( A \) and \( B \) are parameters to be identified. Thus, the continuous time response of equation (2.1) is

\[
X^{(i)}(t) = e^{at}X^{(i)}(0) + \int_0^t e^{a(t-\tau)}A^{-1}(t)B \, d\tau
\]

(2.3)

In order to identify the parameters \( A \) and \( B \), the differential quotient in the equation (2.1) is replaced by the difference quotient with equal time interval. Then, equation (2.1) can be discretized to get

\[
x_i^{(0)}(k) = \sum_{j=1}^{M} \frac{a_j}{2} \left(x_j^{(1)}(k) + x_j^{(0)}(k-1) \right) + b_i
\]

where we note that

\[x_i^{(0)}(k) - x_i^{(0)}(k-1) = x_i^d(k)\]

Let \( a_i = (a_{i1}, a_{i2}, \ldots, a_{in}, b_i) \), \( i = 1, 2, \ldots, n \). The estimated value \( \hat{a}_i \) of \( a_i \) can be obtained by the least square method:

\[
\hat{a}_i = \left[ \hat{a}_{i1} \hat{a}_{i2} \ldots \hat{a}_{in} \hat{b}_i \right]^T = \left( L^T L \right)^{-1} L^T Y_i, \quad i = 1, 2, \ldots, n
\]

Among them,

\[
L = \begin{bmatrix}
\frac{1}{2}(x_1^{(0)}(2) + x_1^{(0)}(1)) & \frac{1}{2}(x_1^{(0)}(2) + x_1^{(0)}(1)) & 1 \\
\frac{1}{2}(x_2^{(0)}(3) + x_2^{(0)}(2)) & \frac{1}{2}(x_2^{(0)}(3) + x_2^{(0)}(2)) & 1 \\
\vdots & \vdots & \vdots \\
\frac{1}{2}(x_n^{(0)}(m) + x_n^{(0)}(m)) & \frac{1}{2}(x_n^{(0)}(m) + x_n^{(0)}(m)) & 1
\end{bmatrix}
\]

\[
Y_i = \left(x_1^{(0)}(1), x_2^{(0)}(3), \ldots, x_n^{(0)}(m) \right)^T
\]

At the same time, the estimated values \( \hat{A} \) and \( \hat{B} \) of parameters \( A \) and \( B \) can be obtained:

\[
\hat{A} = \begin{bmatrix}
\hat{a}_{11} & \hat{a}_{12} & \hat{a}_{1n} \\
\hat{a}_{21} & \hat{a}_{22} & \hat{a}_{2n} \\
\hat{a}_{n1} & \hat{a}_{n2} & \hat{a}_{nn}
\end{bmatrix}
\]

\[
\hat{B} = \left( \hat{b}_1, \hat{b}_2, \hat{b}_n \right)^T
\]

Thus, by (2.3), the multi-variable grey model \( MGM(1,n) \) can be calculated as
\[
\begin{align*}
\hat{x}^{(0)}(k) &= e^{d(k-1)}X^{(0)}(1) + A^{d}(e^{d(k-1)} - I)B, \quad k = 2, 3, \ldots \\
\hat{x}^{(0)}(1) &= X^{(0)}(1) = \hat{x}^{(0)}(1) = X^{(0)}(1) \\
\hat{x}^{(0)}(k) &= \hat{x}^{(0)}(k) - \hat{x}^{(0)}(k-1), \quad k = 2, 3, \ldots 
\end{align*}
\] (2.4)

where \( \hat{x}^{(0)}(k) \) is the predicted value.

3. Correlation Analysis of Ningbo Power Data
The variation of power load is closely related to meteorological conditions. Among the meteorological factors, temperature has the greatest impact on load. How to inspect the impact of meteorological factors on power load and how to inspect the comprehensive impact of these factors on power load are the urgent problem to be solved in current load characteristic analysis and load forecasting. In this paper, the data of power load and temperature in summer in Ningbo are used to carry out correlation analysis, and the results are as follows.

| Maximum temperature | Minimum temperature | Average temperature | 8 am Temperature |
|---------------------|---------------------|---------------------|-----------------|
| 0.8615              | 0.8656              | 0.8550              | 0.8542          |

As can be seen from Table 1, the four indicators of maximum temperature, minimum temperature, average temperature, and 8 am temperature are highly correlated with the load of 11 am in summer. In the load forecasting of this paper, these four temperature factors should be considered as the influencing factors of load.

4. Power load forecasting

4.1. Establishment of MGM(1,5)
In the following, this paper will use multi-variable grey model to forecast the summer load of some region in Ningbo at 11 a.m. Through the above correlation analysis, maximum temperature, minimum temperature, average temperature, and 8 am temperature should be considered as four variables in the prediction model. In addition, with load as a variable, the forecasting model in this paper should have 5 variables. Thus, \( n = 5 \). Let \( x_1, x_2, x_3, x_4, x_5 \) denote load, maximum temperature, minimum temperature, average temperature, and 8 am temperature. The multi-variable grey model MGM(1,5) to forecast the load at 11 am can be established as follows

\[
\begin{align*}
\frac{dx_1(t)}{dt} &= a_{11}x_1(t) + a_{12}x_2(t) + a_{13}x_3(t) + a_{14}x_4(t) + a_{15}x_5(t) + b_1 \\
\frac{dx_2(t)}{dt} &= a_{21}x_1(t) + a_{22}x_2(t) + a_{23}x_3(t) + a_{24}x_4(t) + a_{25}x_5(t) + b_2 \\
\frac{dx_3(t)}{dt} &= a_{31}x_1(t) + a_{32}x_2(t) + a_{33}x_3(t) + a_{34}x_4(t) + a_{35}x_5(t) + b_3 \\
\frac{dx_4(t)}{dt} &= a_{41}x_1(t) + a_{42}x_2(t) + a_{43}x_3(t) + a_{44}x_4(t) + a_{45}x_5(t) + b_4 \\
\frac{dx_5(t)}{dt} &= a_{51}x_1(t) + a_{52}x_2(t) + a_{53}x_3(t) + a_{54}x_4(t) + a_{55}x_5(t) + b_5
\end{align*}
\]

By making using of the matrix operation method in Section II, the following estimations \( \hat{A} \) and \( \hat{B} \) of \( A \) and \( B \) can be obtained
Thus, the $MGM(1,5)$ model that was finally established in this application is as follows:

\[
\begin{align*}
\frac{dx_1^{(i)}}{dt} &= -0.0983 x_1^{(i)} - 0.0182 x_2^{(i)} - 0.0407 x_3^{(i)} - 0.0023 x_4^{(i)} - 0.0063 x_5^{(i)} + 111.7176 \\
\frac{dx_2^{(i)}}{dt} &= 15.0939 x_1^{(i)} + 0.4892 x_2^{(i)} + 0.6413 x_3^{(i)} + 0.5940 x_4^{(i)} + 0.5010 x_5^{(i)} + 24.0888 \\
\frac{dx_3^{(i)}}{dt} &= -0.8054 x_1^{(i)} + 0.2136 x_2^{(i)} + 0.1705 x_3^{(i)} + 0.1037 x_4^{(i)} + 0.1872 x_5^{(i)} + 25.4516 \\
\frac{dx_4^{(i)}}{dt} &= -1.1687 x_1^{(i)} + 0.1260 x_2^{(i)} + 0.0486 x_3^{(i)} + 0.1280 x_4^{(i)} + 0.0366 x_5^{(i)} + 22.7369 \\
\frac{dx_5^{(i)}}{dt} &= -12.9034 x_1^{(i)} - 0.7629 x_2^{(i)} - 0.6641 x_3^{(i)} - 0.8363 x_4^{(i)} - 0.7320 x_5^{(i)} + 24.4804
\end{align*}
\]

According to formula (2.4), we can forecast the load of some region in Ningbo at 11 am in summer. The predicted and measured values are listed in Table 2.

| Date | Measured value(MW) | Fitted value (MW) | Date | Measured value(MW) | Fitted value (MW) |
|------|-------------------|------------------|------|-------------------|------------------|
| 1    | 114.78            | 122.01           | 13   | 183.41            | 191.57           |
| 2    | 116.13            | 118.24           | 14   | 186.14            | 188.30           |
| 3    | 116.12            | 118.29           | 15   | 181.44            | 182.01           |
| 4    | 124.65            | 122.01           | 16   | 174.95            | 173.32           |
| 5    | 133.68            | 129.01           | 17   | 173.86            | 163.09           |
| 6    | 154.66            | 138.53           | 18   | 192.53            | 152.27           |
| 7    | 168.05            | 149.62           | 19   | 180.44            | 141.85           |
| 8    | 160.01            | 161.16           | 20   | 152.77            | 132.73           |
| 9    | 165.05            | 172.05           | 21   | 150.31            | 125.67           |
| 10   | 169.73            | 181.25           | 22   | 138.94            | 121.19           |
| 11   | 172.96            | 187.92           | 23   | 122.84            | 119.52           |
| 12   | 171.29            | 191.45           | 24   | 136.99            | 120.57           |

The fitting diagram is shown in the following figure.
4.2. Model prediction
By making use of the multi-variable grey model $MGM(1,5)$, the power load as 11 am in the next 5 days in summer can be predicted. The results are compared with the actual measured values, as shown in Table 3 and Figure 2, respectively.

**Table 3.** Forecast of power load at 11 am in the next 5 days.

| Date | Measured value | Predictive value | Relative error |
|------|----------------|------------------|----------------|
| 25   | 128.16         | 130.99           | 2.21%          |
| 26   | 143.64         | 139.13           | 3.14%          |
| 27   | 146.14         | 143.15           | 2.05%          |
| 28   | 150.77         | 146.06           | 3.12%          |
| 29   | 150.40         | 148.85           | 1.03%          |

**Figure 1.** Comparison of measured values and fitted values at 11 am.

**Figure 2.** Comparison of measured values and predictive values at 11 am.

The above results show that the prediction has high accuracy. In the same way, we can predict the load at other time through the multi-variable grey model $MGM(1,5)$. Due to space limitations, this article will not list them here.

5. Summary and outlook
In this paper, the multi-variable grey model is used to forecast the power load in summer. The model takes into account the effects of the highest temperature, the minimum temperature, the average temperature and the 8 am temperature on the load, and better reflects the interaction and correlation between the load and the temperature. The results show that the method has good accuracy. This method can be used to predict power load, and provide reliable basis for the scientific and rational implementation of DSM in power sector, so as to improve the efficiency of terminal power consumption and optimize the allocation of resources.
References

[1] Kang C, Xia Q. Review of power system load forecasting and its development[J]. Automation of Electric Power Systems, 2004, 28(17):1-11.

[2] Hippert H S, Pedreira C E, Souza R C. Neural networks for short-term load forecasting: a review and evaluation[J]. IEEE Transactions on Power Systems, 2001, 16(1):0-55.

[3] Song K B, Back Y S, Hong D H, et al. Short-term load forecasting for the holidays using fuzzy linear regression method[J]. IEEE Transactions on Power Systems, 2005, 20(1):96-101.

[4] Papadakis S E, Theocharis J B, Bakirtzis A G. A load curve based fuzzy modeling technique for short-term load forecasting[J]. Fuzzy Sets & Systems, 2003, 135(2):279-303.

[5] Parrmann L D, Mohamed D N. Adaptive online load forecasting via time series modeling[J]. Electric Power Systems Research, 1995, 32(32):219–225.

[6] Li L, Huang W. A Short-Term Power Load Forecasting Method Based on BP Neural Network[J]. Applied Mechanics & Materials, 2014, 494-495:1647-1650.

[7] Fan Z, Zhong Q, Hong T, et al. The studying of combined power-load forecasting by error evaluation standard based on RBF network and SVM method[C]. International Conference on Chinese Control & Decision Conference. 2009.

[8] Zhai M Y. A new method for short-term load forecasting based on fractal interpretation and wavelet analysis[J]. Power System Technology, 2015, 69(1):241-245.

[9] Wei S, Ying Z. Short Term Load Forecasting Based on BP Neural Network Trained by PSO[C]. International Conference on Machine Learning & Cybernetics. 2007.

[10] Hsu C C, Chen C Y. Applications of improved grey prediction model for power demand forecasting[J]. Energy Conversion & Management, 2003, 44(14):2241-2249.

[11] Yin W, Meng P, Li Y. Application of improved grey prediction model in Jilin Province of the software Industry[J]. Advanced Materials Research, 2014, 998-999:1079-1082.

[12] Li M, Sha X. Application of improved GM(1,1) grey prediction model[J]. Computer Engineering & Applications, 2016.

[13] Lei M, Feng Z. A proposed grey model for short-term electricity price forecasting in competitive power markets[J]. International Journal of Electrical Power & Energy Systems, 2012, 43(1):531-538.

[14] Zhai J, Sheng J, Feng Y. The grey model MGM(1,n) and its application[J]. Systems Engineering Theory & Practice, 1997, 17(5):109-113.

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