Effect of Internal Heat Generation on Benard-Marangoni Convection in Micropolar Fluid with Feedback Control

N F M Mokhtar, I K Khalid, N M Arifin

1,3Centre of Foundation Studies, Universiti Putra Malaysia, 43400 UPM, Serdang, Malaysia
2,3Department of mathematics, Universiti Putra Malaysia, 43400 UPM, Serdang, Malaysia

norafadzillah.mokhtar@gmail.com

Abstract. The effect of uniform distribution of internal heat generation on the linear stability analysis of the Benard-Marangoni convection in an Eringen’s micropolar fluids with feedback control is investigated theoretically. The upper free surface is assumed to be non-deformable and the lower boundary is taken to be rigid and isothermal with fixed temperature and span-vanishing boundaries. The eigenvalue is solved numerically using the Galerkin method. The influence of the internal heat generation; $Q$ and feedback control; $K$ in micropolar fluids with various parameters on the onset of stationary convection has been analysed.

1. Introduction

The theory of micropolar fluid was developed by Eringen [1]-[3], in which the fluid elements possess rotator motion, in addition to unusual translator motion. The translator motion is characterized by the velocity vector, while the rotator motion is characterized by the gravitation vector. Physically speaking, micropolar fluids may be thought of the fluids containing dumb-bell shaped molecule, e.g., polymeric fluids suspension, blood, muddy fluids like crude oils etc. These fluids differ from the classical fluids in the sense that they support couple stresses due to rotation motion, in addition to force stress due to translator motion and they are asymmetric in nature. The former is absent in the classical fluids. Rayleigh-Benard situation in Eringen’s [4]-[8] micropolar fluids has been investigated by many authors. The main result from all of these studies is that for heating from below stationary convection is the preferred mode. The instability of convection driven by buoyancy is referred to as the Rayleigh-Benard convection. The classical Rayleigh problem on the onset of convective instabilities in a horizontal layer of fluid heated from below has its origin in the experimental observations of Benard [9] and [10]. Rayleigh’s paper is the pioneering work for almost all modern theories of convection. Rayleigh [11] showed that Benard convection, which is caused by buoyancy effects, will occur when the Rayleigh number exceeds a critical value.

The problem of convective instability of a micropolar fluids layer confined between two rigid boundaries and pointed out that the analysis of the instability finds applications in the area of geophysics was studied by Walzer [12]. The onset of convection for a heat conducting micropolar fluids layer between two rigid boundaries is investigated by Rama Rao [13]. The instability of a rotating micropolar fluids also has been investigated by Sastry & Rao [14] and Qin & Kaloni [15]. The universal stability of magneto-micropolar fluids was studied by Ahmad & Shahinpoor [16]. The effect...
of throughflow on Marangoni convection in micropolar fluids is investigated by Murty & Rao [17]. The magnetoconvection in micropolar fluids is studied Siddheshwar and Pranesh [18]. Tang and Bau [19] and Howle [20] have shown that the critical Rayleigh number for the onset of Rayleigh-Benard convection can be delayed.

The nonlinear temperature distribution in a horizontal fluid layer arising due to internal heat generation has been studied theoretically by Sparrow et. al [21] and Roberts [22]. The effect of quadratic basic state temperature gradient caused by uniform internal heat generation was first addressed by Char and Chiang [23] for Benard-Marangoni convection. Wilson [24] used a combination of analytical and numerical techniques to analyze the effect of internal heat generation on the onset of Marangoni convection.

Or et al. [25] studied theoretically the use of feedback control strategies to stabilize long wavelength instabilities in the Marangoni-Benard convection. Bau [26] has shown independently how such a feedback control can delay the onset of Marangoni-Benard convection on a linear basis with no-slip boundary conditions at the bottom. Arifin et al. [27] have shown that a feedback control also can delay the onset of Marangoni-Benard convection with free-slip boundary conditions at the bottom. In the presence of strong electric field, the electric conductivity is affected by the magnetic field. Bachok and Arifin [28] studied the feedback control of the Marangoni-Benard convection in the presence of internal heat generation.

To the best of our knowledge, there has been no work on Benard-Marangoni convection in the presence of internal heat generation in micropolar fluids with feedback control. In an attempt to rectify this situation, we seek to investigate the effect of uniform internal heat generation on the onset of Benard-Marangoni convection in micropolar fluids with feedback control. In this analysis, the stability theory is based on the linear stability theory and the resulting eigenvalue problem is solved using the Galerkin method.

2. Mathematical formulation

Consider an infinite horizontal layer of Boussinesquian micropolar fluids layer of depth $d$, heated from below with the internal heat generation exists within the fluid system. The stability of a horizontal layer of micropolar fluids in the presence of thermal feedback control is examined. The no-spin boundary condition is assumed for micro rotation. Let $\Delta T$ be the temperature difference between the lower and upper surfaces with the lower boundary at a higher temperature than the upper boundary and these boundaries are maintained at constant temperatures. A Cartesian coordinate system ($x$, $y$, $z$) is used with the origin at the bottom of the surface and the $z$-axis vertically upward. We assumed that the upper free is non-deformable and the surface tension $\sigma$ at the free surface is assumed to vary linearly with temperature in the form

$$\sigma^* = \sigma_0^* - \sigma_T^* \nabla T,$$

where $\sigma_0^*$ is the unperturbed value and $-\sigma_T^*$ is the rate of change of surface tension with temperature $T$. The fluid density $\rho$ is assumed to vary linearly with temperature in the form

$$\rho = \rho_0 [1 - \alpha_t (T - T_0)],$$

where $\alpha_t$ is the thermal expansion coefficient and $\rho_0$ is the density at $T = T_0$. The governing equations for the Benard-Marangoni situation in a Boussinesq micropolar fluids are

$$\nabla \cdot q = 0,$$

$$\rho_0 \left[ \frac{\partial q}{\partial t} + (q \cdot \nabla)q \right] = -\nabla p - \rho g k + (2\zeta + \eta)\nabla^2 q + \zeta (\nabla \times \omega).$$
\[
\frac{\partial T}{\partial t} + (q \cdot \nabla) T = \frac{\beta}{\rho C_v} (\nabla \times \omega) \cdot \nabla T + \kappa \nabla^2 T + h_y, \quad (5)
\]

\[
\rho_0 J \left( \frac{\partial \omega}{\partial t} + (q \cdot \nabla) \omega \right) = (\lambda' + \eta') \nabla (\nabla \cdot \omega) + \eta' \nabla^2 \omega + \zeta (\nabla \times q - 2 \omega), \quad (6)
\]

where \( q \) is the velocity, \( \omega \) is the microrotation, \( p \) is the pressure, \( \rho_0 \) is the density, \( T \) is the temperature, \( g \) is the acceleration due to gravity, \( \hat{k} \) is the unit vector in the \( z \)-direction, \( \zeta \) is the coupling viscosity coefficient or vortex viscosity, \( \beta \) is the micropolar heat conduction coefficient, \( C_v \) is the specific heat, \( \lambda \) and \( \eta \) are the bulk and shear kinematic viscosity coefficients, \( \lambda' \) and \( \eta' \) are the bulk and shear spin-viscosity coefficients, \( I \) is the moment of inertia, \( \kappa \) is the thermal conductivity and \( h_g \) is the overall uniformly distributed volumetric internal heat generation within the micropolar fluid layer. The basic state of the fluid is quiescent and is described by

\[
q_b = (0,0,0), \omega_b = (0,0,0), p = p_b(z) \text{ and } T = T_b(z),
\]

where the subscript \( b \) denotes the basic state. Substituting equation (7) into equations (4) and (5), we get the basic state governing equations as

\[
\frac{dp_b}{dz} = -\rho_b g, \quad (8)
\]

\[
\frac{d^2 T_b}{dz^2} = \frac{h_y}{\kappa}, \quad (9)
\]

with

\[
\rho_b = \rho_0 \left[ 1 - \alpha_i (T_b - T_0) \right]. \quad (10)
\]

Subject to the boundary conditions \( T_b = T_0 \) at \( z = 0 \) and \( T_b = T_0 - \Delta T \) at \( z = d \), Eq. (9) is solved and we obtained

\[
T_b(z) = -\frac{h_y}{2\kappa} z^2 + \left( \frac{h_y d}{2\kappa} - \frac{\Delta T}{d} \right) z + T_0. \quad (11)
\]

Take note that equation (11) is a parabolic distribution with the liquid layer height due to the existence of the internal heat generation. Without the internal heat generation; \( Q = 0 \), the basic state temperature distribution in the fluid layer is linear. Let the basic state be disturbed by an infinitesimal thermal perturbation and we now have

\[
q = q_b + q', \omega = \omega_b + \omega', p = p_b + p' \text{ and } T = T_b + T', \quad (12)
\]

where the primes indicate that the quantities are infinitesimal perturbations. Substituting equation (12) into equations (3) – (6) and linearized in the usual manner, we obtained the linearised equations in the form

\[
\nabla \cdot q' = 0, \quad (13)
\]

\[
0 = -\nabla p' - \rho' \hat{g} k + (2\zeta + \eta) \nabla^2 q' + \zeta (\nabla \times \omega'), \quad (14)
\]

\[
-W \frac{\Delta T}{d} = \frac{\beta}{\rho_0 C_v} \left( \nabla \times \omega' \right) \cdot \left[ \frac{-\nabla T}{d} \right] k + (\nabla T' + \kappa \nabla^2 T') + \left( \frac{zh_y}{\kappa} + \frac{h_y d}{2\kappa} + \frac{\nabla T}{d} \right) W. \quad (15)
\]
\[0 = (\lambda' + \eta') \nabla (\nabla \cdot \omega') + \eta' \nabla^2 \omega' + \zeta (\nabla \times \mathbf{q}' - 2 \omega') \quad (16)\]

In the present problem, we assume the principle of exchange of stability is valid and deal only with stationary convection. Hence, the time derivatives have been dropped in equations (13) – (16). The perturbation equations are non-dimensionalized using the following definitions

\[ (x', y', z') = \left( \frac{x}{d}, \frac{y}{d}, \frac{z}{d} \right), \quad W' = \frac{W}{x/d}, \quad \Omega' = \frac{(\nabla \times \omega')z}{x/d}, \quad \text{and} \quad T' = \frac{T - T_0}{\Delta T}. \quad (17)\]

Substituting equation (17) into equations (14), (15) and (16), eliminating the pressure term by operating curl twice on the resulting equation of (14), operating curl once on equation (16) and non-dimensionalising we get

\[(1 + N_i) \nabla^4 W + N_i \nabla^2 \omega + R_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0, \quad (18)\]

\[\nabla^2 T + \left[ 1 - Q(1 - 2z) \right] W - N_s \Omega = 0, \quad (19)\]

\[N_s \nabla^2 \Omega - 2N_s \Omega - N_s \nabla^2 W = 0, \quad (20)\]

where the asterisks have been dropped for simplicity. Here,

\[N_i = \frac{\zeta}{\eta + \zeta}, \quad \text{is the coupling parameter,}\]

\[N_s = \frac{\eta'}{(\eta + \zeta)d^2}, \quad \text{is the couple stress parameter,}\]

\[N_s = \frac{\beta}{\rho_0 C_d d^2}, \quad \text{is the micropolar heat conduction parameter,}\]

\[R_s = \frac{\alpha g \Delta T \rho_a d^3}{(\eta + \zeta) \chi}, \quad \text{is the Rayleigh number,}\]

and \(Q = \frac{h_i d^3}{2k \Delta T}\) is the heat source strength.

The perturbation quantities in a normal mode form are

\[(W, T, \Omega) = [W(z), \Theta(z), G(z)] \exp \left[ i(a, x + a, y) \right], \quad (21)\]

where \(W(z), \Theta(z), G(z)\) are amplitudes of the perturbations of vertical velocity, temperature and spin, and \(a = \sqrt{a^2 + a^2}\) is the wavenumber of the disturbances at the liquid layer. Substituting equation (21) into equations (18) – (20) we get

\[(1 + N_i) \left( D^2 - a^2 \right) W + N_i \left( D^2 - a^2 \right) G = R_s a^2 \Theta, \quad (22)\]

\[(D^2 - a^2) \Theta = [Q(1 - 2z) - 1] W + N_s G, \quad (23)\]

\[N_s \left[ \left( D^2 - a^2 \right) W + 2G \right] - N_s \left( D^2 - a^2 \right) G = 0, \quad (24)\]
where $D = d/dz$.

Following the proportional feedback control in (1999), the continuously distributed actuators and sensors are arranged in a way that for every sensor, there is an actuator positioned directly beneath it. The determination of a control; $q(t)$ can be accomplished using the proportional-integral-differential (PID) controller of the form

$$q(t) = r + K[\varepsilon(t)]$$

where $r$ is the calibration of the control, $\varepsilon(t)$ an error or deviation from the measurement, $m(t)$ from some desired or reference value, $\dot{m}(t)$ from some desired or reference value, $r$ is the calibration of the control, $e(t)$ an error or deviation from the measurement, $m(t)$ from some desired or reference value, $K_p$, proportional gain, $K_d$, differential gain and $K_i$, integral gain. Based on equation (25), for one sensor plane and proportional feedback control, the actuator modifies the heated surface temperature using a proportional relationship between the upper, $z = 1$ and the lower, $z = 0$ thermal boundaries for perturbation field

$$T'(x, y, 0, t) = -KT'(x, y, 1, t),$$

where $T'$ denotes the deviation of the fluid’s temperature from its conductive state and $K$ is the scalar controller gain in which it will be used to control our system.

Equations (21) - (23) are solved subject to appropriate boundary conditions that are

$$W = DW = G = \Theta(0) + K\Theta(1) = 0 \text{ at } z = 0.$$  \tag{27}

Equations (22) – (24) are solved subject to appropriate boundary conditions that are

$$W = D\Theta = G = D^2W = 0 \text{ at } z = 0, \tag{28}$$

$$W = D\Theta = G = D^2W + a^2\Theta M = 0 \text{ at } z = 1, \tag{29}$$

where $M_o = \sigma_o \Delta Td/\mu \gamma$ is the Marangoni number. Equation (25) indicates the use of rigid and isothermal for lower boundary, and for equation (26) the upper surface is assumed to be stress free and insulating. The condition on $G$ is the no-spin boundary condition for both boundaries.

3. Method of Solution

Equations (22)–(24) together with the boundary conditions (25) and (26) constitute a Sturm-Liouville problem with the Marangoni number $M_o$ or the Rayleigh number $R_a$ as an eigenvalue while keeping other physical parameters fixed. The Galerkin method is used to solve the resulting eigenvalue problem. Accordingly, the variables are written in a series of basis function as

$$W = \sum_{i=1}^{N} A_i W_i(z), \ \Theta = \sum_{i=1}^{N} C_i \Theta_i(z) \text{ and } G = \sum_{i=1}^{N} D_i \Gamma_i(z), \tag{30}$$

where the trial functions $W_i(z)$, $\Theta_i(z)$ and $\Gamma_i(z)$ will be chosen in such a way that they satisfy the respective boundary conditions and $A_i$, $C_i$ and $D_i$ are constants. Substituting equation (27) into equations (22) – (24), multiplying the resulting equation (22) by $W_i(z)$, equation (23) by $\Theta_i(z)$ and equation (24) by $\Gamma_i(z)$; performing the integration by parts with respect to $z$ between $z = 0$ and $1$ and using the boundary conditions (25) and (26), we obtain the following system of linear homogeneous algebraic equations
\[ C_j A_j + D_j C_j + E_j D_j = 0, \]  
\[ F_j A_j + G_j C_j + H_j D_j = 0, \]  
\[ I_j A_j + J_j D_j = 0. \]

The coefficients \( C_j - J_j \) involve the inner products of the basis functions and are given by

\[ C_j = -N_i \left[ \langle DW \rangle \langle DG \rangle + a^2 \langle WG \rangle \right], \]
\[ C_2 = \langle G_i^2 \rangle \left( 2N_i + N_i a^2 + N_i \langle DG \rangle \right), \]
\[ C_3 = -(1 + N_i) \left[ \langle (D^2 W)^2 \rangle + 2a^2 \langle (D W)^2 \rangle + a^4 \langle W^2 \rangle \right], \]
\[ C_4 = \langle (D \Theta)^2 \rangle + a^2 \langle \Theta^2 \rangle, \]
\[ C_s = -N_s \langle G \Theta \rangle, \]
\[ C_s = (1 + N_i) \langle \Theta \rangle \langle DW \rangle, \]
\[ C_5 = N_i \left[ \langle DG \rangle \langle DW \rangle + a^2 \langle GW \rangle \right], \]
\[ C_6 = \left[ 1 - Q(1 - 2z) \right] \langle W \Theta \rangle, \]
\[ C_6 = \langle \Theta W \rangle, \]
\[ C_{10} = K \Theta(1), \]
\[ C_{11} = C_4 + C_{10}, \]

where the angle bracket \( \langle \cdots \rangle \) denotes the integration with respect to \( z \) from 0 to 1.

The eigenvalue has to be extracted from the characteristic equation (32). Now, we choose the trial functions as

\[ W_i = z^2 (1 - z^2) T_{i-1}, \quad G_i = z^2 (1 - z) T_{i-1}, \quad \text{and} \quad \Theta_i = z (2 - z) T_{i-1} \]  

where \( T_i \) are the Chebyshev polynomials, such that \( W, \Theta, \) and \( G_i \) satisfy the corresponding boundary conditions except \( D^2 W + a^2 M_0 \Theta = 0 \) at \( z = 1 \) but the residuals from the equations are included as residuals from the differential equations.

4. Result and Discussion

The criterion for the onset of Benard-Marangoni convection in micropolar fluids in the presence of feedback control and internal heat generation is investigated theoretically. The sensitiveness of critical Rayleigh number, \( R_{ac} \) and Marangoni number, \( M_{ac} \) to the changes in the micropolar fluids parameters; \( N_i, N_s, \) and \( N \) are also studied.

Table 1 shows the comparison of the critical Marangoni number, \( M_{ac} \) for different values of Rayleigh number, \( R_a \) and internal heat generation effect, \( Q \) when \( K = 0, N_i = 0.5, N_j = 2 \) and \( N_s = 1. \) Our results are compared with Siddheshwar and Pranesh [18] in the absence of internal heat generation and feedback control, and the results are in a good agreement. Thus, this validates our numerical procedure. From this table, we found that the critical Marangoni number decreases as the
value of internal heat generation increase for all $R_a$ values considered. This shows that the internal heat generation parameter has a significant impact on the thermal and surface tensile modes, making the system unstable. Besides that, we also notice that the $M_{ac}$ decreases with the increase in $R_a$.

The variation of $R_a$ and $M_{ac}$ with wavenumber, $a$ for different values of $Q$ are illustrate in Figure 1 and Figure 2 respectively. We choose $N_1 = 0.5$, $N_2 = 2$, $N_3 = 1$ and $K = 2$ for both cases, and we found that the internal heat generation has a rapid impact on the stability of the system. This can be proved by looking at the difference in the $R_a$ and $M_{ac}$ number, where the $R_a$ and $M_{ac}$ number recorded between $Q = 0$ and $Q = 2$ are much larger compare with the value of $Q = 4$ and $Q = 6$. It is evident that the internal heat generation is a destabilizing factor for the system to be more unstable.

Figure 5 and Figure 6 show the plot of $R_a$ and $M_{ac}$ versus the coupling parameter, $N_1$ respectively for values of $Q = 0$ and $Q = 10$ when $N_2 = 2$, $N_3 = 1$ and $K = 2$. In each of these plots, the critical number increases with increasing of $N_1$ for all values of $Q$ considered. $N_1$ indicates the concentration of microelements, and increasing of $N_1$ is to elevate the concentration microelements number. When this happened, a greater part of the energy of the system is consumed by these elements in developing gyrational velocities of the fluid and thus delayed the onset of convection.

Figure 7 and Figure 8 show the plot of $R_a$ and $M_{ac}$ versus the coupling parameter, $N_1$ respectively for values of $K = 0$ and $K = 10$ when $N_3 = 2$, $N_1 = 1$ and $Q = 2$. In each of these plots, the $R_a$ and $M_{ac}$ increases with increasing of $N_1$ for all values of $K$ considered. It is found that $N_1$ can delay the onset of convection.

The illustration of the couple stress parameter, $N_3$ in both thermal buoyancy and surface tension can be seen in Figure 9 and Figure 10 respectively when $N_1 = 0.1$, $N_2 = 10$ and $K = 2$. From these graphs, it can be clearly seen that in the presence of internal heat heating, an increased of $N_3$ decrease the values of $R_a$ and $M_{ac}$ for $Q = 10$ and $Q = 15$ considered. These situations revealed that the system become more unstable much faster when the couple stress parameter increasing.

Figure 11 and Figure 12 show the plot of $R_a$ and $M_{ac}$ versus the couple stress parameter, $N_3$ when $N_1 = 0.1$, $N_2 = 10$ and $Q = 2$. From the graph, it can be clearly seen that in the presence of controller gain, $K$, an increased of $N_3$ decrease the values of $R_a$ and $M_{ac}$ for all $K$ considered. Therefore, the system become more unstable when the couple stress parameter, $N_3$ increasing.

Figure 13 and Figure 14 show the plot of $R_a$ and $M_{ac}$ respectively, versus micropolar heat conduction parameter, $N_1$ when $N_1 = 0.1$, $N_2 = 2$ and $K = 2$. Contrast with the effect of internal heat generation, $Q$, increasing the micropolar heat conduction, $N_1$ increased the critical Rayleigh number and critical Marangoni number as well. The reason behind this is, when $N_1$ increases, the heat induced into the fluid due to the microelements is also increased an thus reducing the heat transfer from the bottom to the top of the system. The decrease in heat transfer is responsible for delaying the onset of instability for both thermal buoyancy and surface tension system. Thus, increasing $N_1$ promotes stability in the micropolar system in the presence of internal heat generation.

Figure 15 and Figure 16 indicate the variation values of $R_a$ and $M_{ac}$ for $K = 0$ and $K = 5$ respectively, when $N_1 = 0.1$, $N_2 = 2$ and $Q = 2$. It is found that as the number of $R_a$ and $M_{ac}$ increases, the value of gain controller, $K$ also increases for both thermal buoyancy and surface tension considered and thus stabilized the system.
Table 1. Comparison of $M_{sc}$ for different values of $R_a$ and $Q$ when $K = 0$

| $R_a$ | Siddheshwar & Pranesh [18] | Present Analysis |
|-------|----------------------------|------------------|
|       | $Q = 0$ | $Q = 0$ | $Q = 2$ | $Q = 3$ |
| 0     | 98.5736 | 98.5736 | 54.146 | 44.188 |
| 100   | 94.9228 | 94.9228 | 50.495 | 40.537 |
| 500   | 80.3196 | 80.3196 | 35.892 | 25.933 |
| 1000  | 62.0657 | 62.0657 | 17.638 | 7.680 |

Figure 1. Variation of $R_a$ with $a$ for different values of $Q$. 
Figure 2. Variation of $M_a$ with $a$ for different values of $Q$.

Figure 3. Variation of $R_a$ with $a$ for different values of $K$. 
**Figure 4.** Variation of $M_a$ with $a$ for different values of $K$.

**Figure 5.** Plot of $R_w$ versus $N_f$ for different values of $Q$. 
Figure 6. Plot of $M_{ac}$ versus $N_f$ for different values of $Q$.

Figure 7. Plot of $R_{ac}$ versus $N_f$ for different values of $K$. 
Figure 8. Plot of $M_{ac}$ versus $N_f$ for different values of $K$.

Figure 9. Plot of $R_{ac}$ versus $N_3$ for different values of $Q$. 
Figure 10. Plot of $M_{ac}$ versus $N_f$ for different values of $Q$.

Figure 11. Plot of $R_{ac}$ versus $N_f$ for different values of $K$. 
**Figure 12.** Plot of $M_{ac}$ versus $N_j$ for different values of $K$.

**Figure 13.** Plot of $R_{ac}$ versus $N_j$ for different values of $Q$. 
Figure 14. Plot of $M_{ac}$ versus $N_5$ for different values of $Q$.

Figure 15. Plot of $R_{ac}$ versus $N_5$ for different values of $K$. 
5. Conclusion
The stability analysis of feedback control of the Benard-Marangoni convection in micropolar fluids with internal heat generation is investigated theoretically. We found that the effect of internal heat generation, \( Q \) in the micropolar fluid has a significant influence on the Benard-Marangoni convection where increasing the internal heating will destabilize the system. As for the effect of controller gain, \( K \) in the micropolar fluid, increasing the values of \( K \), elevate the critical Marangoni number and Rayleigh number, and thus promote stability in the system. Although the effect of \( Q \) is to destabilize the system, the increase of the microelement concentration, \( N_1 \) and \( N_5 \) help to slow down the process of destabilizing. The effect of increasing the \( N_3 \), decrease the couple stress of the fluid and hence promote the destabilizing effect in the system. Therefore, the coupling parameter, \( N_3 \), couple stress parameter, \( N_3 \) and micropolar heat conduction parameter, \( N_5 \) has a significant effect on the onset of Benard-Marangoni convection.

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References
[1] Eringen A C 1966 Theory of micropolar fluids J. Math. Mech. 16 1-18
[2] Eringen A C 1972 Theory of thermo-microfluids J. Math. Anal Appl. 38 480-496
[3] Eringen A C 2001 Microcontinuum Field Theories II: Fluent Media Springer-Verlag, New York 71 1-10
[4] Eringen A C 1964 Simple microfluids Int. J. Engng Sci. 2 205-217
[5] Eringen A C 1969 Micropolar fluids with stretch Int. J. Engng Sci. 7 115-127
[6] Eringen A C 1980 Theory of anisotropic micropolar fluids Int. J. Engng Sci. 18 15-17
[7] Eringen A C 1990b Theory of thermo-microstretch fluids and bubbly liquids Springer-Verlag 28 133-143
[8] Eringen A C 1991 Memory-dependent orientable nonlocal micropolar fluids Int. J. Engng. Sci. 29 1515-1529
[9] Benard H 1900 Les tourbillons cellulaires dans une nappe liquid Revue Generale des entries Sciences Pures et Appliquées 11 1261-1271
[10] Benard H 1901 Les tourbillons cellulaires dans une nappe liquide transportant de la chaleur par convection en regime permanent Ann. Chem. Phys. 23 62-144
[11] Rayleigh L 1984 On convection currents in a horizontal layer of fluid when higher temperature is on the under side Philos. Mag. 32 529-546
[12] Rajee V 1984 Onset of instability in a heat conducting micropolar fluid layer Acta Mech. 32 79 – 84
[13] Sastry V U K and Ramamohan R V 1983 Numerical study of thermal instability of a rotating micropolar fluid layer Int. J. Engng. Sci. 21 449 – 461
[14] Qin Y and Kaloni P N 1992 A thermal instability problem in a rotating micropolar fluid Int. J. Engng Sci. 30 1117 – 1126
[15] Ahmadi G and Shahinpoor M 1974 Universal stability of magneto micropolar fluid motion Int. J. Engng Sci. 12 657 – 663
[16] Murty Y N and Ramana R V V 1999 Effect of throughflow on Marangoni convection in micropolar fluids Acta Mech. 138 211 – 217
[17] Siddheshwar P G and Pranesh S 2002 Magnetoconvection in fluids with suspended particles under 1g and μg Aero. Sci. Tech. 6 105 – 114
[18] Tang J and Bau H H 1993 Stabilization of the no-motion state in Rayleigh-Benard convection through the use feedback control Phys. Rev. Lett. 70 1795-1798
[19] Howle L E 1997 Linear stability analysis of controlled Rayleigh-Benard convection using shadowgraphic measurement Phys. Fluids 9 3111-3113
[20] Sparrow A, Goldstein R J and Jonsson V K 1964 Thermal stability in a horizontal fluid layer: Effect of boundary conditions and non-linear temperature J. Fluid Mech. 18 513-528
[21] Roberts P H 1967 Convection in horizontal layers with internal heat generation: Theory J. Fluid Mech. 30 33-49
[22] Char M I and Chiang K T 1994 Stability analysis of Benard-Marangoni convection in fluids with internal heat generation Biometrika 71 1-10
[23] Wilson S K 1997 The effect of uniform internal heat generation on the onset of steady Marangoni convection in a horizontal layer of fluid Acta Mech 124. 63-78
[24] Or A C, Kelly R E, Cortelezzi L and Speyer J L 1999 Control of long wavelength Benard-Marangoni convection J. Fluid Mech. 387 321-341
[25] Bau H H 1999 Control of Marangoni-Benard convection Int. J. Heat Mass Transfer 42 1327-1341
[26] Arifin N M, Nazar R and Senu N 1999 Feedback control of the Marangoni-Benard instability in a fluid layer with free-slip bottom J. Phys. Soc. Jpn. 76 1-4
[27] Bachok N and Arifin N M 2010 Feedback control of the Marangoni-Benard convection in a horizontal fluid layer with internal heat generation Micro. Sci. Technol. 22 97-105