Predicting Value at Risk for Cryptocurrencies Using Generalized Random Forests

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Abstract

We study the estimation and prediction of the risk measure Value at Risk for cryptocurrencies. Using Generalized Random Forests (GRF) [Athey et al., 2019] that can be adapted to specifically fit the framework of quantile prediction, we show their superior performance over other established methods such as quantile regression and CAViaR, particularly in unstable times. We investigate the small-sample prediction properties in comparison to standard techniques in a Monte Carlo simulation study. In a comprehensive empirical assessment, we study the performance not only for the major cryptocurrencies but also in the stock market. Generally, we find that GRF outperforms established methods especially in crisis situations. We further identify important predictors during such times and show their influence on forecasting over time.

Keywords: Generalized Random Forests, Value at Risk, Quantile Prediction, Backtesting, Cryptocurrencies.

JEL Classification: C58, G17, C22
1 Introduction

Cryptocurrencies are an important and rising part of today’s digital economy. Currently, the market capitalisation of the top 10 cryptocurrencies in the world is close to $2 trillion and growing. The use of cryptocurrencies in terms of daily volume exploded from 2016 to 2018, which not only attracts individuals but also business users such as hedge funds, merchants and long-term investors such as crypto-focused as well as traditional investment funds. However, the crypto asset market remains highly volatile. An investment in bitcoin in 2013 would have seen a return of roughly 20000% in 2017, but an investment in 2017 would have led to a performance of -75% in 2019. Consequently, there is a need to monitor the inherent volatility to manage the risks associated with cryptocurrencies. To address this, we find that classic approaches such as the historical simulation or CaViaR methods are too restrictive. More general non-linear methods provide more flexibility to account for such behaviour which could be caused in part by speculators.

In this paper we propose a novel way for out-of-sample prediction of the Value at Risk, one of the standard and mostly used risk measures in practice. We use a quantile version of Generalized Random Forests (GRF, see Athey et al., 2019), which builds upon standard random forests and extends them to fit quantiles as opposed to the mean in standard ones. This framework shows to be especially promising when dealing with more volatile classes of assets such as cryptocurrencies due to the non-linear structure of their returns. GRF outperforms other established methods such as CAViaR, quantile regression or simple GARCH-models over a rolling window, particularly in unstable times. This can be attributed to the non-parametric approach of random forests that is flexible and adaptable considering important factors and non-linearity. Previous studies have confirmed that there exist speculative bubbles and we find that our approach assesses risks especially well during such times (e.g. in 2018). Moreover, it can be seen that variable importance differs substantially depending on time, where long-term measures of standard deviation, that are an important predictor in stable times, are not good in predicting VaR in unstable, volatile times. As a robustness-check, we repeat the analysis for classic assets such as the S&P 500, the Dow Jones or the Apple Inc. stock and find that with these less volatile assets, CAViaR or quantile regression perform similar to GRF.

See e.g. https://coinmarketcap.com/charts/ accessed at 6th May 2021.
Our paper contributes to the growing literature on cryptocurrencies. Analyses performed in the past include GARCH models (Chu et al., 2017) as well as ARMA-GARCH models (Platanakis and Urquhart, 2019), approaches using RiskMetrics (Pafka and Kondor, 2001) and GAS-models (Liu et al., 2020), application of extreme value theory (Gkillas and Katsiampa, 2018), vine copula-based approaches (Trucios et al., 2020), Markov-Switching GARCH models (Maciel, 2020), non-causal autoregressive models (Hencic and Gouriéroux, 2015) and also some machine learning based approaches (see e.g. Takeda and Sugiyama, 2008). Additionally, cryptocurrencies can be used for diversification in investment strategies with other, traditional assets (see e.g. Trimborn et al. (2020); Petukhina et al. (2021)), as the correlation between them and more established assets tends to be low (Elendner et al., 2018; Platanakis and Urquhart, 2019). This again poses the question of assessing the risks of cryptocurrencies, where new methods of addressing the above mentioned challenges need to be explored.

The paper is structured as follows. Section 2 presents the underlying data and cryptocurrencies we use in our analysis. In Section 3 we introduce main methods used to analyze the data, specify VaR and present the evaluation tests and framework. Section 4 demonstrates the performance of the different methods under various data generating processes in a thorough simulation study. Results for the data are shown in Section 5 before we conclude in Section 6.

2 Data

We use daily log-returns of the four of the largest (by volume) and most popular cryptocurrencies with their value compared to US-Dollar in the period from 01/2015 to 11/2020 where all were highly liquid. Thus we study Bitcoin (BTC), Ethereum (ETH), Tether (USDT) and XRP (XRP, also known as Ripple), and the cryptocurrency index CRIX (Trimborn and Härdle, 2018) as obtained from the CRIX website. Note that each of these currencies also represents a different technical class of cryptocurrencies as described in Härdle et al. (2020). For comparison, we also use daily log-returns of large blue chip stocks and stock indices from the beginning of 01/1996 up to 04/2020 taken from Yahoo finance comprising the S&P 500 (SPX), the Dow Jones (DJIA), the German DAX index (GDAXI) and the stocks of Apple Inc. (APPL) and Volkswagen AG (VOW.DE). Thus for the crypto assets, we have a total of 1911 observations available for a rolling window analysis, while for the classic assets we can use a longer total period of 6100

http://data.thecrix.de
https://finance.yahoo.com
observations since liquidity is not an issue in their case. An overview of descriptive statistics can be found in Table 1. All assets have a large excess-kurtosis indicating that they have large outliers, while we have both positive and negative skewness in the data, suggesting that returns for CRIX, XRP, Apple and Volkswagen are not symmetric, which is important modelling VaR later.

We do not detect any stochastic non-stationarities in the data, which is supported by Augmented Dickey-Fuller (ADF) tests and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests (Kwiatkowski et al., 1992). With the CRIX and the ETH data, KPSS tests against trend stationarity seem slightly significant, while ADF tests suggest stationarity. All results of the stationarity tests can be found in Table 9 in Appendix A.

Table 1: Descriptive Statistics of Data

| BTC | CRIX | ETH | USDT | XRP | Apple | DAX | Dow Jones | S&P 500 | Volkswagen |
|-----|------|-----|------|-----|-------|-----|-----------|---------|------------|
| Min | -0.465 | -0.447 | -0.551 | -0.053 | -0.616 | -0.731 | -0.131 | -0.138 | -0.128 | -0.257 |
| 1%  | -0.111 | -0.116 | -0.165 | -0.016 | -0.142 | -0.072 | -0.044 | -0.033 | -0.034 | -0.064 |
| 5%  | -0.062 | -0.059 | -0.085 | -0.007 | -0.073 | -0.041 | -0.024 | -0.018 | -0.019 | -0.033 |
| Median | 0.002 | 0.003 | 0.000 | 0.000 | -0.002 | 0.001 | 0.001 | 0.001 | 0.001 | 0.000 |
| 95% | 0.062 | 0.061 | 0.101 | 0.007 | 0.088 | 0.043 | 0.022 | 0.017 | 0.017 | 0.034 |
| 99% | 0.111 | 0.107 | 0.185 | 0.017 | 0.228 | 0.079 | 0.037 | 0.031 | 0.034 | 0.061 |
| Max | 0.225 | 0.199 | 0.304 | 0.057 | 1.027 | 0.287 | 0.108 | 0.108 | 0.110 | 0.903 |
| Skewness | -0.914 | -1.207 | -0.190 | 0.465 | 2.959 | -2.546 | -0.200 | -0.383 | -0.389 | 6.404 |
| Excess-Kurtosis | 14.206 | 12.964 | 7.352 | 26.047 | 45.762 | 75.866 | 5.491 | 12.669 | 10.701 | 253.134 |
| Standard Deviation | 0.039 | 0.039 | 0.060 | 0.006 | 0.066 | 0.029 | 0.015 | 0.012 | 0.012 | 0.026 |
| Observations | 1911 | 1910 | 1911 | 1911 | 1911 | 6110 | 6147 | 6110 | 6110 | 6187 |

For the cryptocurrencies, Figure 1 illustrates that in all cases the rolling window empirical standard deviation as a measure for unconditional volatility varies strongly in time depending on the specific currency. Moreover, the volatility increases especially in 2017 and 2018 and decreases again after the so-called crypto bubble bursts (see e.g. Chaim and Laurini (2019)) suggesting different unconditional volatility levels. Furthermore, we can see that USDT’s volatility is much lower as it acts as a so-called stablecoin and it is backed almost fully by US dollar. Finally, we can also see that the CRIX has similar volatility patterns to Bitcoin as it replicates the largest cryptocurrencies by volume and is thus dominated by Bitcoin.

4Since the amount of trading days per year differ for different stocks depending e.g. on the market where they are traded, the exact amount of observations is not equal for each of the classic assets.
Figure 1: 250 days Rolling Window Standard Deviation of the Cryptocurrencies, also see Härdle et al. (2020), Figure 4

3 Methodology

For the prediction of cryptocurrencies, we advocate the use of nonlinear machine learning based techniques. In this way, we intend to accommodate the documented large share of speculation (Ghysels and Nguyen, 2019; Baur et al., 2018; Selmi et al., 2018; Glaser et al., 2014) and resulting frequent changes in unconditional volatility which make predictions in this market peculiar. In particular, we focus on generalized random forest methods that are tailored for conditional quantiles of returns and thus allow to forecast the VaR. The flexible but interpretable nonlinearity of the approach allows for a direct comparison to standard linear and (G)ARCH type models. We also argue that the difference in forecasting performance can moreover be employed to detect periods of bubbles and extensive speculation.

Recall that for daily log returns $r_t$ the $VaR_t$ at level $\alpha \in (0,1)$ conditional on some covariates $x_{t-1}$ is defined as

$$VaR_t^\alpha(x_{t-1}) = \sup_{r_t} (F(r_t|x_{t-1}) < \alpha) ,$$

where $F$ marks the distribution of $r_t$ conditional on $x_{t-1}$. Generally, the conditioning variables
could consist of past lagged returns, standard deviations but also external (market) information or other assets. We focus on a purely time series based approach here to highlight the advantages of interpretable nonlinearities rather than gains from the use of other type of information (see e.g. Chen and Hafner (2019); Zhang et al. (2016)). But such specific information could easily be added if data was available and we would expect additional gains from doing so.

We propose the use of two different types of random forest based techniques which directly model the conditional VaR in (1). Both build on the classic random forest (Breiman, 2001) which is an ensemble of (decorrelated) decision trees (see e.g. Hastie et al. (2009)) for the mean of $r_t$. In a decision tree, each outcome $r_t$ is sorted into leafs of the tree by binary splits. These splits are performed based on different $x_{t-1}$ components falling above or below specific adaptive threshold values that are computed e.g. by the Gini Impurity or MSE-splitting in Classification and Regression Trees (CART) (Breiman et al., 1984) or other criteria. Finally, the prediction for a new $r_t$ is a weighted version of each tree prediction.

In our first method, the generalized random forest (GRF) based on Athey et al. (2019), the random forest split criterion is adapted to mimic the task of quantile regression rather than minimizing a standard mean squared loss criterion for mean regression tasks. Please see Algorithm 1 for the detailed description. In particular, motivated by moment conditions and gradient approximations, we first transform the response variable $r_t$ in each split to obtain pseudo-outcomes $\rho_t = \sum_{k=1}^{K} 1\{r_t > \theta_k\}$, where $\Theta = (\theta_{q_1}, \ldots, \theta_{q_K})$ describe a set of $K$ pilot-quantiles of $r_t$ with levels $\tau = q_1, \ldots, q_K$ in the parent node that are used to calibrate the split (Athey et al., 2019). In other words, $r_t$ is relabeled to a nominal scale depending on the largest quantile it does not exceed. In a final step, the optimal split on a variable component $p$ of $x_{t-1}$ and $j = 1, \ldots, J$ observations in the parent node is based on minimizing the classic Gini impurity criterion for classification. For a separation into two possible leaf sets $v = l, r$, the Gini impurity for one leaf $v$ is $G^v_p = 1 - \sum_{k=1}^{K} p^2_{k,v}$, where $p_{k,v} = \sum_{j=1}^{J} 1\{\rho_j = k \text{ and } \rho_j \in v\}/|v|$ is the proportion of $\rho_j$ in group $v$ with value $k$. The full loss is then an average weighted by leaf size, yielding

$$G_p = (|l|G^l_p + |r|G^r_p)/(|l| + |r|) .$$

Such a splitting regime can be particularly helpful when dealing with changing variance (and thus time-varying quantiles) of returns over time since it accounts for this specifically in the relabeling step. Algorithm 1 briefly summarizes the tree building algorithm from Athey et al. (2019) for the quantile version of GRF, where the main differences with the splitting regime occur in every step the tree is grown.

\footnote{We use their default tuning parameters $K = 3$ and $\tau = (0.1, 0.5, 0.9)$ in this classification pre-step.}
In addition to that, the outcome that is predicted is not the mean but the α-quantile (i.e. VaR), which is done in a way that you do not calculate a weighted average of \( r_t \) but a weighted average of the empirical CDF \( \hat{F}(r_t|x_{t-1}) = E[1 \{ r_t \} | x_{t-1}] \). Intuitively, log returns \( r_t \) that have similar \( x_{t-1} \) in comparison to a new observation \( x_v \) receive higher weight in the empirical CDF. Similarity weights \( w_t(x_v) \) are measured as the relative frequency on how often \( x_v \) falls in the same terminal leaf as \( x_{t-1} \), for \( t = 1, \ldots, T \), and averaged over all trees for each \( x_{t-1} \). This last step was originally introduced by Meinshausen (2006) for random forests.

Our second technique, the quantile regression forest (QRF) based on Meinshausen (2006), however, uses the same splitting regime as the original CART random forest and therefore does not account explicitly for situations where the variance and therefore the quantile changes, as splits are conducted based on a mean-squared error criterion. Since such volatility changes are to be expected for cryptocurrencies in our data, we expect GRF to perform better than QRF, but still include both in the analysis to see potential differences in predictions (and report the full QRF results in the Appendix). Furthermore, GRF uses so-called ‘honest’ trees, meaning that different data (usually the subsampled data for each tree is split again in half) is used for building and ‘filling’ each of the trees with values.

As standard benchmarks we further include two types of standard time series methods. We use the CAViaR methodology by Engle and Manganelli (2004) and standard quantile regression (Koenker, 2005). Both make use of quantile regression techniques (Koenker and Bassett, 1978) that do not minimize the squared error as in ordinary regression, but use the check function \( \rho_\alpha(u) = u(\alpha - I\{u \leq 0\}) \) to minimize \( L_\alpha(f_\alpha(\cdot), x_t) = \sum_{t=1}^{T} \rho_\alpha(r_t - f_\alpha(x_t)) \). For CAViaR, we use a symmetric absolute value (SAV) component for \( f_\alpha(\cdot) \), i.e. \( f_\alpha(x_t, r_t) = \beta_1 + \beta_2 f_\alpha(x_{t-1}, r_{t-1}) + \beta_3|r_{t-1}| \), and \( f_\alpha(r_{t-1}, x_t) = \beta_4 x_t + \beta_5 r_{t-1} \) for the quantile regression. In contrast to the former methods, they can only capture parametric (non-) linear effects which limits their flexibility.

For comparison we use three simple baseline methods, a GARCH(1,1) (Bollerslev, 1986) model, a simple historical simulation (Hist) meaning we predict \( VaR_{t+1}^\alpha \) at level \( \alpha \) as the sample \( \alpha \)-quantile of the preceeding returns in a window of length \( K \), i.e. \( (r_{t-K+1}, \ldots, r_t) \), and and one that fits a normal distribution to the sample data and uses the theoretical fitted \( \alpha \)-quantile as the prediction for \( VaR_{t+1}^\alpha \) (NormFit).

For the proposed random forest type and all benchmark procedures that allow for additional covariates, we include lagged standard deviations (SD) in addition to the lagged level \( r_{t-1} \) in the model in order to capture the strongly varying levels of unconditional volatility in particular
**Algorithm 1** Generalized Random Forest - Tree Building

**Input:** Set of ‘honest’, subsampled observations $X_T$ and $R_T$; minimum node size $n_m$; quantile probabilities $\tau_K = (\tau_1, \ldots, \tau_K)$

1. **Growing the tree** Create root node $P_0$
2. Initialize queue $Q$ with $P_0$
3. **while** Queue is not empty **do**
   4. Take oldest element from $Q$ (Parent Node $P$) and remove it from $Q$
   5. Take a random subsample of $p$ variables index by set $P_{sub} = \{\hat{1}, \ldots, \hat{p}\}$ from $X_T$ on which to potentially split and take observations $x^{(P_{sub})}_i = (x^{(\hat{1})}_i, \ldots, x^{(\hat{P})}_i)$ from $P$.
   6. Set $\text{loss} = \infty$
   7. **for** $h$ in 1 to $p$ **do**
      8. Compute quantiles $\theta_k$ of $r_t$ from parent node $P$ at $\tau_1, \ldots, \tau_K$ and compute pseudo outcomes $\rho_t = \sum_{k=1}^{K} 1\{r_t > \theta_k\}$ for each $r_t \in P$.
      9. For each possible split point in $x^{(h)}$, compute the criterion from Equation (2)
      10. Save loss $s_h$ that minimizes this splitting criterion
      11. Save optimal split point $\text{split}_h$
      12. **if** $s_h < \text{loss}$ **then**
          13. $\text{loss} \leftarrow s_h$
          14. $\text{ind}_h \leftarrow h$
      15. **end if**
   16. **end for**
   17. **if** Split on variable $h$ with $\text{split}_{\text{ind}_h}$ succeeded (based on hyperparameters) **then**
      18. Determine children $C_1, C_2$ according to optimal split
      19. Add both children $C_1, C_2$ to a new daughter node each with corresponding observations left and add these to $Q$
   20. **end if**
21. **end while**

**Output:** One Tree of the forest
for the cryptocurrencies in the non-linear structure (see Figure 1). To establish a fair common ground in used model complexity for the QR, QRF and GRF, we select a common set of different SD lags as covariates from an additional Monte-Carlo study. In this, we use the simple SAV-model from Section 4 as a baseline of which the respective specification is tailored to regime changes in the unconditional volatilities as observed form the cryptocurrencies in Figure 1. The model is essentially linear autoregressive of order 1 in the VaR and thus directly yields the VaRs as outputs. With this, it is possible to use a MSE-minimizing criterion and select the MSE-minimizing variables for the subsequent simulations and data analysis. Table 2 summarizes the results of this short simulation study, which is why for the forest-based methods, we use 3, 7, 30, and 60 day lagged SD as covariates, while for QR, we only use 7 and 30 SD. One explanation why only taking two out of the 4 additional covariates is preferable for QR is that the linear, additive structure of QR is rather restrictive and two lagged variables are already sufficient to capture the behavior, while adding more covariates might overfit. We further include \( r_{t-1} \) as a covariate for the QR, QRF, and GRF to make them more comparable to the time series based methods.

Table 2: MSE Prediction Error for Different Covariate Combinations

| Lagged SD (in days) | 3   | 7   | 30  | 3 and 7 | 3 and 30 | 7 and 30 | 3, 7 and 30 | 3, 7, 30, and 60 |
|---------------------|-----|-----|-----|---------|----------|----------|-------------|------------------|
| GRF-MSE             | 0.124 | 0.095 | 0.089 | 0.075 | 0.070 | 0.069 | 0.059 | **0.057** |
| QR-MSE              | 0.078 | 0.060 | 0.053 | 0.063 | 0.053 | **0.052** | 0.057 | 0.058 |

MSE prediction error for a simulated SAV-model as in Section 4 for GRF (QRF) and QR. The minimum MSE is marked in bold.

To compare the performance of the above methods, we use two types of evaluation approaches. First, we test how well each model predicts the conditional \( \alpha \)-VaR over the entire out-of-sample horizon using three different sets of evaluation techniques. The simplest way of checking whether a model predicts \( VaR^\alpha \) correctly over a time horizon is to look at its calibration meaning the number of times \( r_t \) is smaller than the predicted \( VaR^\alpha_t \). In a well calibrated model, this should be exactly \( \alpha T \) times. This measure is called Actual over Expected Exceedances (AoE) and is computed as \( AoE_{\alpha} = 1/(\alpha T) \sum_{t=1}^{T} 1\{r_t < VaR^\alpha_t\} \). To test this intuition formally, we employ three tests, the DQ-test\(^6\) from Engle and Manganelli (2004), the Christoffersen-test (Christoffersen, 1998) and the Kupiec-test (Kupiec, 1995). All three tests assume that under the null hypothesis the forecasts are well calibrated. The Kupiec-test is

\(^6\)We use the implementation from the GAS-package in R (Ardia et al., 2019) including 4 lagged Hit-values, a constant, the VaR-forecast, and the squared lagged (log-)return.
simply the formalization of the above intuition, the Christoffersen-test is robust against serial correlation by assuming that \( g_t = 1 \{ r_t < \text{VaR}^\alpha_t \} \sim \text{Bern}(\alpha) \), and the DQ-test additionally accounts for problems with conditional coverage due to clustering of the hits exceedance sequences \( g_t \) with a regression-based approach. Secondly, for comparing the forecast performance of two models 1 and 2 directly, we implement the one-step ahead test for conditional predictive ability (CPA) from Giacomini and White (2006), Theorem 1, that assumes under the null hypothesis that forecasts of model 1 and model 2 have on average equal predictive ability conditional on previous information. As suggested by Giacomini and Komunjer (2005), we use the quantile loss function \( L^\alpha \) for the test. This tests assumes under the null hypothesis that \[ H_0 : \mathbb{E}[\Delta L_t | \mathcal{F}_{t-1}] = \mathbb{E}[L^\alpha(f_\alpha^{(1)}, r_t) - L^\alpha(f_\alpha^{(2)}, r_t) | \mathcal{F}_{t-1}] = 0 \] is a martingale difference sequence, where \( \mathcal{F}_{t-1} \) contains all information up to time \( t-1 \) and \( f_\alpha^{(1)} \) and \( f_\alpha^{(2)} \) are two competing forecasts. The test statistic is computed using a Wald-type test with a set of factors \( h_{t-1} \) that can possibly predict the loss difference \( \Delta L_t \) and is \( \chi^2_q \)-distributed under \( H_0 \). More specifically, we choose \( h_{t-1} = (1, \Delta L_{t-1}) \) (i.e. \( q = 2 \)), i.e. using the lagged loss difference and an intercept as predictors in a linear regression with parameter \( \beta_0 \) for the simulation and application.

4 Simulation

As both the GRF and the QRF have so far only been studied in sample for cross-section data, we provide simulation results on their out-of-sample performance for VaR\(^\alpha\)-forecasts in a financial time series set-up. Overall, we find that as expected, GRF performs best in all settings, in particular in settings which are designed to mimic the cryptocurrency behaviour over time but also in those similar to stock index behaviour. For QRF the performance is entirely different at least for the chosen parsimonious model specification and the relatively short estimation intervals.

We study two different types of DGPs. The first one is a standard GARCH(1,1) process, i.e

\[
\begin{align*}
  r_t &= z_t \sigma_t \\
  \sigma_t^2 &= \omega + \beta_0 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2,
\end{align*}
\]

where the parameters are estimated on the full Bitcoin data to mimic the behavior of cryptocurrencies (with \( z_t \sim \mathcal{N}(0,1) \)). We denote this setting as \textit{sim GARCH Bitcoin fit}. Moreover, we also consider the specification with \( \beta_0 = 0.1, \beta_1 = 0.8, \omega = 10^{-4} \) and \( z_t \sim \mathcal{N}(0,1) \) (\textit{sim GARCH}) and with \( t_5 \)-distributed (\textit{sim GARCH t}) which corresponds to standard stock index data. Secondly, for the \textit{Sim SAV-Model} setting, we fit a symmetric absolute value (SAV) model
to normal returns, i.e.

\[ \text{VaR}_{t+1} = \gamma_0 + \gamma_1 \text{VaR}_t + \gamma_2 |r_t^{(\text{init})} - \gamma_3|, \]

(5)

with \( r_t^{(\text{init})} \sim N(0, \sigma_t^2) \) and \( \sigma_t \sim \chi^2_2 \) where new draws of \( \sigma_t \) are only taken every 100 observations keeping it constant meanwhile. We then generate the final return as \( r_t \sim N(0, \frac{\text{VaR}_t}{\Phi(\alpha)^{-1}}) \) from the fitted SAV-model, where \( \Phi(\alpha)^{-1} \) is the quantile function of a standard normal variable. We do this to obtain returns that have exactly the \( \text{VaR} \) that we obtained from the SAV model before.

For all settings, we generate 2000 return observations and forecast the one-step ahead VaR over the different rolling window lengths \( l = 500, 1000 \). We repeat this generation process 200 times for \( \alpha = 0.01 \) and \( \alpha = 0.05 \). For comparison of the different methods described in Section 3, we use the DQ-test, the Kupiec test, the Christoffersen-test, and the AoE. Note that for all tests we present aggregate results from two-sided t-tests of the empirical versus the nominal coverage. The results are therefore rejection rates of t-tests against the nominal level of 5%. Therefore, a lower rejection rate and higher mean p-values (in parentheses) indicate better performance.

For GRF, QRF, and QR, we use a common set of lagged covariates as described at the end of Section 3.

Table 3 summarizes the results of the simulation for the 5% VaR. According to the more advanced DQ- and Christoffersen-tests for evaluation, GRF consistently outperforms the other methods in almost all cases, indicating superior predictive quality. This is very much in contrast to the QRF which is consistently dominated by the other models. For the Sim GARCH and Sim GARCH t cases which mimic standard stock indices, the performances of GRF, CAViaR and QR appear in a similar range with mostly advantages for GRF in particular for smaller sample sizes. This holds generally for normally distributed as well as heavy tailed innovations which lead to similar results also in magnitude of the rejection rates. As expected, forecasting performance increases throughout all models with larger estimation windows, though with CAViaR often profiting the most from the larger sample sizes. For these settings with GARCH as true DGP, the performance of the GARCH model serves as a close to oracle reference. In the t-innovation case, it has calibration problems and GRF is even able to outperform it in Christoffersen-test.

For the Sim SAV Model and the Sim GARCH Bitcoin fit the situation, however, differs substantially. In these cryptocurrency-like cases, the GRF clearly dominates the QR and CAViAR particularly strongly in the small 500 observations setting. Considering our application where only a relatively small time span is available, this seems crucial. Moreover, CAViAR runs into calibration problems according to the AoE results which even deteriorate for larger sample sizes.
| Rolling Window | $l = 500$ | $l = 1000$ |
|----------------|----------|----------|
|                | DQ       | Kupiec   | Christoffersen | AoE |
| Sim GARCH Normal |          |          |                |     |
| QRF            | 0.940 (0.008) | 0.325 (0.200) | 0.200 (0.275) | 1.185 |
| GRF            | 0.495 (0.163) | 0.000 (0.583) | 0.015 (0.549) | 1.040 |
| QR             | 0.760 (0.048) | 0.085 (0.441) | 0.090 (0.438) | 1.095 |
| Hist           | 0.835 (0.045) | 0.015 (0.554) | 0.195 (0.365) | 1.047 |
| NormFit        | 0.740 (0.071) | 0.040 (0.551) | 0.205 (0.329) | 1.006 |
| CAVaR          | 0.785 (0.051) | 0.025 (0.514) | 0.010 (0.545) | 1.073 |
| GARCH          | 0.445 (0.209) | 0.030 (0.511) | 0.155 (0.394) | 1.046 |
| Sim GARCH t    |          |          |                |     |
| QRF            | 0.920 (0.018) | 0.315 (0.203) | 0.210 (0.277) | 1.188 |
| GRF            | 0.435 (0.188) | 0.010 (0.579) | 0.025 (0.537) | 1.029 |
| QR             | 0.770 (0.060) | 0.130 (0.407) | 0.105 (0.407) | 1.109 |
| Hist           | 0.725 (0.077) | 0.020 (0.573) | 0.200 (0.360) | 1.036 |
| NormFit        | 0.695 (0.090) | 0.310 (0.271) | 0.370 (0.220) | 0.839 |
| CAVaR          | 0.525 (0.135) | 0.010 (0.523) | 0.010 (0.542) | 1.059 |
| GARCH          | 0.405 (0.232) | 0.135 (0.381) | 0.210 (0.316) | 0.893 |
| Sim SAV-Model  |          |          |                |     |
| QRF            | 0.960 (0.010) | 0.315 (0.188) | 0.180 (0.265) | 1.187 |
| GRF            | 0.495 (0.148) | 0.005 (0.571) | 0.025 (0.570) | 1.047 |
| QR             | 0.895 (0.024) | 0.050 (0.435) | 0.045 (0.452) | 1.102 |
| Hist           | 0.290 (0.220) | 0.035 (0.595) | 0.070 (0.523) | 1.047 |
| NormFit        | 0.215 (0.360) | 0.040 (0.529) | 0.110 (0.502) | 0.994 |
| CAVaR          | 0.775 (0.053) | 0.030 (0.525) | 0.030 (0.563) | 1.067 |
| GARCH          | 0.135 (0.338) | 0.030 (0.567) | 0.065 (0.528) | 1.021 |
| Sim GARCH Bitcoin fit |          |          |                |     |
| QRF            | 0.875 (0.018) | 0.325 (0.200) | 0.200 (0.275) | 1.185 |
| GRF            | 0.255 (0.303) | 0.000 (0.583) | 0.015 (0.549) | 1.040 |
| QR             | 0.635 (0.095) | 0.085 (0.441) | 0.090 (0.438) | 1.095 |
| Hist           | 0.600 (0.102) | 0.015 (0.554) | 0.195 (0.365) | 1.047 |
| NormFit        | 0.540 (0.159) | 0.040 (0.551) | 0.205 (0.329) | 1.006 |
| CAVaR          | 0.970 (0.007) | 0.375 (0.178) | 0.280 (0.224) | 0.814 |
| GARCH          | 0.230 (0.319) | 0.030 (0.536) | 0.180 (0.399) | 1.054 |

The table displays rejection rates of t-tests of empirical quantile levels against the nominal level of 5% for DQ-, Kupiec- and Christoffersen-tests and mean p-values in parentheses. Thus higher p-values and lower rejection rates indicate better model performance.
for Sim GARCH Bitcoin fit. In the latter case, the GRF rejection rates are close to the GARCH benchmark while the strong conditional dependence structure in the tails of the Sim SAV Model setting shows that for such extreme cases, the unconditional coverage of GRF is still excellent, but the conditional coverage measured by the DQ-test is only average among all models for the larger estimation samples and even below for the smaller ones. These relative findings generally prevail for the 1% VaR forecasts, but absolute performance is generally worse for all methods, especially with a smaller rolling window of 500 (see Table 10 in the Appendix.). This is clear since relevant observations for the 1% level should in theory only occur in 5 of the 500 observations, making it harder for the data-driven methods to predict such unlikely events.

Additionally, we conduct direct pairwise comparison tests between the superior random forest type method GRF against the best performing non-oracle other parametric methods via CPA-tests for each scenario. The respective results are reported in Table 4. In the majority of cases, GRF outperforms its competitors on average, however, mean p-values are mostly not significant. For example, GRF has a smaller loss (i.e. quantile loss) than QR in about 90% of the forecasts (aggregated over all runs) but a mean p-value of 0.298, which would not qualify as a significant out-performance (on average). This of course does not mean that the test never rejects but is likely be caused by high variances in the p-values over different simulation runs. Furthermore, the competing methods are also slightly improving with window length, which further implies that GRF can deal better with a smaller training time frame.
Table 4: CPA-tests on Predictions of 5% VaR for Different Window Lengths and Different Models

| Rolling Window | $l = 500$ | $l = 1000$ |
|----------------|-----------|------------|
|                | QR  | Hist | CAViaR | QR  | Hist | CAViaR |
| **Sim GARCH Normal** |     |      |        |     |      |        |
| Mean P-Value    | 0.429 | 0.351 | 0.334  | 0.491 | 0.388 | 0.463  |
| No. P-Values < 0.1 | 34  | 55   | 50     | 24   | 44   | 27     |
| GRF-Performance | 0.780 | 0.755 | 0.847  | 0.501 | 0.744 | 0.659  |

| **Sim GARCH t** |     |      |        |     |      |        |
| Mean P-Value    | 0.392 | 0.382 | 0.396  | 0.431 | 0.397 | 0.446  |
| No. P-Values < 0.1 | 43  | 40   | 34     | 30   | 37   | 15     |
| GRF-Performance | 0.732 | 0.707 | 0.692  | 0.439 | 0.717 | 0.529  |

| **Sim SAV-Model** |     |      |        |     |      |        |
| Mean P-Value    | 0.298 | 0.311 | 0.374  | 0.471 | 0.381 | 0.448  |
| No. P-Values < 0.1 | 53  | 63   | 42     | 26   | 50   | 22     |
| GRF-Performance | 0.903 | 0.279 | 0.775  | 0.657 | 0.321 | 0.658  |

| **Sim GARCH Bitcoin fit** |     |      |        |     |      |        |
| Mean P-Value    | 0.429 | 0.351 | 0.001  | 0.491 | 0.388 | 0.003  |
| No. P-Values < 0.1 | 34  | 55   | 200    | 24   | 44   | 199    |
| GRF-Performance | 0.780 | 0.755 | 0.995  | 0.501 | 0.745 | 0.994  |

The table displays CPA-tests for 5% one-day ahead VaR-forecasts of the best performing random forest type techniques GRF in Table 3 versus the best parametric time series models. We report mean p-values, the number of significant p-values over 200 iterations and the rate at which GRF outperforms the competing method (i.e. a value of 0.8 means that GRF has a smaller error loss than the competing method in 80% of the rolling window forecasts over all runs). Low p-values paired with performance rates larger than 0.5 indicate that GRF outperforms the competing methods.
5 Results

We report on the 5% - VaR out-of-sample performance of the proposed random forest technique versus standard time-series methods for cryptocurrencies. Our particular interest is in the risk prediction for cryptocurrencies, and the performance for classic stocks and indices is regarded as a benchmark case. Our results show that the flexible GRF method has its strengths in forecasting especially in highly volatile periods that occur frequently with cryptocurrencies, highlighted by direct comparisons with CPA-tests, while retaining a good overall performance in the classic backtests. We investigate the performance of GRF over time and can therefore identify when and how it obtained an advantage over its standard counterparts (e.g. CAViaR, GARCH).

5.1 Direct Pairwise Forecast Comparisons

We compare the out-of-sample forecast performance of GRF versus QR, Hist, CAViaR, and GARCH, introduced in Section 3. For this, we conduct pairwise CPA-tests and study the time evolution of differences in predicted losses. The results of the full tests are contained in Table 5. In general, the out-of-sample size for cryptocurrencies is rather low for the CPA-tests to have high power. Therefore we additionally look at the direct comparisons of predicted losses (as suggested by Giacomini and White (2006), Section 4), where we compare in the loss series how often GRF is better, i.e. has a smaller loss, than its competitors (GRF-Performance). Note that a value of one thus indicates that GRF has a smaller predicted loss over the full loss series.

GRF clearly outperforms all methods for almost all cryptocurrencies over the full out-of-sample time period according to Table 5, as expected from the simulations. For Bitcoin, by far the cryptocurrency with the highest volume and market capitalization, all tests show significance on a 10% level and performance is higher than 95%. The same holds true for the CRIX-index, although tests are not significant for all instances over the full time period. For the other, smaller and less frequently traded assets, we take a more detailed look at performance over time. In particular, we study the specific events that cause p-values over the full time period to be rather high even though the performance indicates clearly that GRF losses are lower.

Starting with CRIX, where some of the p-values are rather high, we conduct an additional

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8 The CPA-tests require the rolling window length to be smaller than the out-of-sample forecast window to produce valid results, which is not given for the cryptocurrencies and \( l = 1000 \). For completeness, we report the full results including \( l = 1000 \) in the Appendix in Table 15.

9 We use the lagged loss difference and an intercept for loss prediction in an auto-regressive setup since these are the main drivers of the test statistic in the CPA test.
Table 5: CPA-tests on Predictions of 5% VaR for Cryptocurrencies

| Rolling Window | l = 500 |
|----------------|---------|
| GRF vs:        | QR      | Hist | CAViaR | GARCH   |

| CRIX           |         |      |        |        |
|----------------|---------|------|--------|--------|
| P-Value        | 0.170   | 0.025⁺ | 0.265  | 0.174  |
| GRF-Performance| 0.976   | 0.964 | 0.985  | 0.999  |

| Bitcoin        |         |      |        |        |
|----------------|---------|------|--------|--------|
| P-Value        | 0.063⁺  | 0.002⁺ | 0.098⁺ | 0.086⁺ |
| GRF-Performance| 1.000   | 0.987 | 0.991  | 0.985  |

| Ethereum       |         |      |        |        |
|----------------|---------|------|--------|--------|
| P-Value        | 0.955   | 0.946 | 0.559  | 0.982  |
| GRF-Performance| 0.188   | 0.907 | 0.022  | 0.931  |

| USDT           |         |      |        |        |
|----------------|---------|------|--------|--------|
| P-Value        | 0.331   | 0.000⁺ | 0.011⁺ | 0.012⁺ |
| GRF-Performance| 0.034   | 1.000 | 0.993  | 0.973  |

| XRP            |         |      |        |        |
|----------------|---------|------|--------|--------|
| P-Value        | 0.169   | 0.000⁺ | 0.491  | 0.004⁺ |
| GRF-Performance| 0.978   | 0.996 | 0.171  | 0.973  |

Results of CPA-tests are for 5% one-day ahead VaR-forecasts of GRF vs. other methods. We report p-values and the rate at which GRF outperforms the competing method (i.e. a value of 0.8 means that GRF has a smaller error loss than the competing method in 80% of the rolling window forecasts). Low p-values paired with performance rates larger than 0.5 indicate that GRF outperforms the competing methods and are marked with a +.
analysis where CPA tests are run over a rolling window of 800 predictions to see how the performance (i.e. the percentage over the 800 predictions where GRF has smaller loss than other methods) varies over time. Figure 2 summarizes these results. As expected, the power of the rolling window tests is even lower meaning that p-values are generally higher in comparison to Table 5 since we have less observations. For all competing methods except for Hist, which is dominated over the whole period, there are some performance kinks in the beginning of the out-of-sample period. These disappear when the Covid-19 drop in March 2021 enters in the model, meaning that the loss of all methods increased compared to GRF, which was able to handle these large outliers better. The insignificance can therefore be attributed to the rather stable environment in the beginning of the out-of-sample period, where the competing methods were not fully dominated by GRF (See Figure 6 in the Appendix for an overview of out-of-sample log-returns for the cryptocurrencies).

For USDT, only QR is insignificant, which is due to the small difference in losses that is
practically zero (see Figure 7 in the appendix). For XRP, the variance in losses is extremely high for CAViaR and centered around zero, while the CPA-tests over time have kinks mostly around the Covid-19 crash in March 2020 (see Figure 8 and 9 in the appendix). Contrary to the full sample, CAViaR is dominated most of time over the rolling window. The same can be said about QR, where the insufficiency of the test is probably caused by the high variance in returns in the beginning of the out-of-sample period that translates to lower power in the CPA-tests (see Figure 8 and 9 in the appendix). Finally, with Ethereum, there is a mix of both phenomena, where with CAViaR, loss differences have extremely high variance, while with the other methods, the difference is basically zero (see Figure 10 and 11 in the Appendix). This behavior might be caused by the out-of-sample period that starts directly when a big shift of volumes and market capitalization in the whole crypto market takes place. This affects ETH especially since its market share measured by total market capitalization rises strongly (to up to 30%) in the period from 02/2017 reaching its peak approximately three months later and only stabilizing at similar levels as before in the summer of 2018\footnote{see e.g. https://coinmarketcap.com/charts/ for an overview.}. This large instability therefore affects more or less the whole out-of-sample period, where no method could obtain a clear advantage in pairwise tests.

To highlight the properties and the adaptability of GRF during unstable times, Figure 3 shows the predicted loss differences by the CPA-tests for Bitcoin. There, GRF losses are compared to both a simple historical simulation and a more complex CAViaR\footnote{A full set of figures for each possible method combination is available upon request from the authors.}. Unstable times where high volatility occurs seems to favor GRF against simple, non-adaptive methods such as historical simulation especially, while more complex non-linear methods such as CAViaR can better adapt in such situations (right in Figure 3). This pronounced spike on the left side Figure 3 occured right in the beginning of 2018 when the Bitcoin crashed. This suggests that such empirical comparisons between complex and simple methods could work as a detection device for speculative bubbles. In general, however, GRF also performs better against more complex parametric methods as can be seen at the level of predicted losses that is below zero for both methods. This shows that using such non-parametric methods can be beneficial in predicting risk in both speculative and stable times.

To reveal which of the covariates strongly influences the VaR predictions for the GRF, we obtain variable importance measures that measure the frequency of inclusion in splits of the forest\footnote{We use a maximum depth of $d_{max} = 5$ corresponding to the number of covariates and a weight decay of 2,} In Figure 4, the importance difference of certain covariates over time for Bitcoin is
Figure 3: Rolling 10-day mean of predicted loss difference series (red) of $h_t \hat{\beta}_0$ of CPA tests on Bitcoin predicted 5% VaR with $l = 500$ for GRF vs. historical simulation (left) and GRF vs. CAViaR (right) with lagged 250 day standard deviation in dashed lines (blue). A negative predicted loss difference indicates that the prediction error of GRF is smaller than of the compared method.

clearly visible. Interestingly, the lagged 60-day standard deviation is more important in the rather stable times of mid 2018 to mid 2019, while in more unstable times, lagged return and 7-day rolling standard deviation have more influence in the predictions of the GRF. Intuitively, this finding seems reasonable as in times of bubbles, when the volatility is driven by some short, bubble-like events and returns are highly variable, volatility lagged over a longer time horizon is less predictive for VaR and predictions are driven by events happening shortly before the prediction. This again could serve as an indicator for more risk in Bitcoin returns, where in more stable times, longer measures of standard deviation are already informative enough for prediction of VaR. Detailed figures on all methods can be found in Figure 12 in Appendix A.

meaning a split further down in each tree receives less weight $w_l$ in the final frequency as it is less important. Specifically, for layer $l = 1, \ldots, 5$, $w_l = \frac{l^{-2}}{\sum_{l=1}^{5} l^{-2}}$. 
Figure 4: Rolling 30-day mean of the GRF variable importance in the out-of-sample period for Bitcoin predicted one-day ahead 5% VaR with $l = 500$. Variable importance of covariate $x_p$ is measured as the proportion of splits on $x_p$ relative to all splits in a respective layer $l$ (over all trees in a trained forest), weighted by layer $l$. Variable description can be seen in Section 2.

5.2 Backtesting and Robustness Checks

Apart from directly comparing the methods against each other, we can also check their ability of predicting VaR in general using the classic backtests. The main results of the 5% VaR-predictions in Table 6 show that for the majority of cryptocurrencies GRF does best in terms of test rejections, while for the other methods, the tests reject more often. As expected, a simple historical simulation fails completely in all test scenarios, while more advanced methods such as QR and CAViaR, are better than in the direct comparisons, mainly when they are fitted with a larger rolling window. It makes sense that these non-adaptive methods perform better when more data is available, while GRF is better with a smaller rolling window. The GARCH-model is also better using a smaller window which seems reasonable given we employed a rather simple GARCH(1,1) model here. With all the backtests, it has to be noted that the power of the tests is probably rather low given the short time frame available for cryptocurrencies in general. Using a larger rolling window reduced the sample available for out-of-sample forecasting significantly making it smaller than the rolling window itself. Therefore, direct comparisons between the different methods are rather weak here compared to the direct CPA-tests, particularly when
methods have rather high p-values.

Table 6: Backtests of 5% VaR Forecasts for Cryptocurrencies

| Rolling Window | $l = 500$ | $l = 1000$ |
|----------------|-----------|------------|
|                | GRF | QR | Hist | CAViaR | GARCH | GRF | QR | Hist | CAViaR | GARCH |
| **CRIX**       |     |    |      |        |       |     |    |      |        |       |
| DQ             | 0.985 | 0.430 | 0.007 | 0.188 | 0.380 | 0.037 | 0.457 | 0.017 | 0.310 | 0.369 |
| Christoffersen | 0.707 | 0.968 | 0.472 | 0.452 | 0.374 | 0.123 | 0.333 | 0.003 | 0.052 | 0.164 |
| AoE            | 0.933 | 1.007 | 1.081 | 0.859 | 0.919 | 0.729 | 0.800 | 0.565 | 0.659 | 0.729 |
| **Bitcoin**    |     |    |      |        |       |     |    |      |        |       |
| DQ             | 0.460 | 0.413 | 0.000 | 0.710 | 0.239 | 0.662 | 0.197 | 0.004 | 0.378 | 0.068 |
| Christoffersen | 0.775 | 0.701 | 0.024 | 0.662 | 0.253 | 0.115 | 0.176 | 0.002 | 0.078 | 0.023 |
| AoE            | 0.947 | 0.933 | 1.170 | 0.992 | 0.947 | 0.705 | 0.752 | 0.517 | 0.682 | 0.752 |
| **ETH**        |     |    |      |        |       |     |    |      |        |       |
| DQ             | 0.413 | 0.025 | 0.019 | 0.285 | 0.249 | 0.151 | 0.055 | 0.070 | 0.262 | 0.004 |
| Christoffersen | 0.839 | 0.798 | 0.559 | 0.724 | 0.086 | 0.167 | 0.436 | 0.051 | 0.581 | 0.003 |
| AoE            | 0.962 | 1.081 | 0.962 | 1.021 | 0.873 | 0.799 | 0.823 | 0.682 | 0.870 | 0.729 |
| **USDT**       |     |    |      |        |       |     |    |      |        |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.010 | 0.023 | 0.000 | 0.010 | 0.000 |
| Christoffersen | 0.016 | 0.249 | 0.000 | 0.294 | 0.036 | 0.050 | 0.119 | 0.006 | 0.704 | 0.041 |
| AoE            | 0.799 | 1.051 | 1.421 | 0.962 | 1.303 | 0.729 | 1.105 | 0.564 | 0.964 | 1.387 |
| **XRP**        |     |    |      |        |       |     |    |      |        |       |
| DQ             | 0.005 | 0.000 | 0.000 | 0.257 | 0.000 | 0.096 | 0.159 | 0.065 | 0.132 | 0.048 |
| Christoffersen | 0.234 | 0.057 | 0.008 | 0.490 | 0.002 | 0.049 | 0.214 | 0.001 | 0.057 | 0.001 |
| AoE            | 1.036 | 1.170 | 1.140 | 1.140 | 0.770 | 0.870 | 0.893 | 0.494 | 0.799 | 0.588 |

Results for the DQ- and Christoffersen-test depict p-values and AoE measures the calibration of the forecast. AoE values close to one and large p-values depict good performance.

When distinguishing the performance between the different cryptocurrencies, we can generally observe that GRF and CAViaR work rather well for the currencies traded in large volume such as ETH and Bitcoin and the CRIX-index which is largely driven by those two currencies. For USDT and XRP, however, results especially for DQ-tests show weak performance over all
methods.

To investigate this behavior further, we repeat the DQ-tests over rolling windows\textsuperscript{13} to uncover which time periods are problematic for the DQ-tests. Looking at Figure 5, we see that for XRP, we have large heterogeneities in p-values depending on the rolling window. The (non-)rejection in the full tests (i.e. over the full out-of-sample period) is therefore likely caused by larges outliers in the beginning of the out-of-sample period. It also highlights that GRF performs better than the other methods for most periods. Notably, all methods except GRF have long periods with small (significant) p-values, indicating problems when dealing with extreme events, while GRF is more robust in that case. For USDT (see Figure 13 in the Appendix), all methods perform poorly in the beginning and end of the out-of-sample period, which is due to extremely high outliers mid 2017 and in the beginning of 2020 (Covid-crisis). Although unconditional coverage seems to be good for some methods in Table 6, forecasts are generally not conservative enough in the beginning and end of the considered time frame, which makes the hits (i.e. the exceedances of VaR) predictive as uncovered by the DQ-tests in Figure 13 in the Appendix.

As expected, the DQ-test is the strictest measure as usually the p-values for Christoffersen test are higher for each respective method. This makes sense since the test of Christoffersen does not take the quality of the forecast into account but only its coverage and serial dependence, while the DQ-test enforces more plausible forecasts. Thus, in our data an economically sound forecast by the DQ-test is usually also serially uncorrelated and gives correct coverage as also tested by the Christoffersen test and AoE.

As a robustness check\textsuperscript{14} we compare the performance of all methods in terms of backtesting to the classic stocks in Table 7. In general, GRF cannot outperform the other methods as much with regard to Christoffersen tests and AoE-values apart from historical simulation and GARCH. Interestingly, more established methods such as quantile regression and CAViaR perform better than with cryptocurrencies, which could be attributed to the longer time horizon available and the less volatile, more predictable assets.

In general it has to be noted that with classic stocks, the sample size and therefore the out-of-sample period of the rolling window is much larger, including the dotcom bubble, the financial crises, the following European sovereign debt crisis, and the corona crisis. This could be an indicator that most methods struggle with the size of losses and predictions, which is clearly

\textsuperscript{13}We use a rolling window of 500 (rather than 800 as for the CPA-tests) to better quantify periods where large changes in the return structure occurred. This is possible here since the DQ-test does not crucially depend on the test sample to be larger than the training sample.

\textsuperscript{14}Additional results for the robustness check can be found in the Appendix.
Figure 5: Top plot: log-returns of XRP for the out-of sample period. Other plots: p-values of DQ tests using 500 out-of-sample predictions (from $l = 500$ training points) over a rolling window (10 days) for different methods. Red dotted lines mark a p-value of 5%. The date on the x-axis marks the start of the testing period and contains approximately two years of daily data (i.e. 500 observations).
Table 7: Robustness Checks for Classic Stocks: Backtests of 5% VaR Forecasts

| Rolling Window | l = 500 | l = 1000 |
|----------------|---------|----------|
|                | GRF     | QR       | Hist    | CAViaR  | GARCH   | GRF     | QR       | Hist    | CAViaR  | GARCH   |
| **SP500**      |         |          |         |         |         |         |          |         |         |         |
| DQ             | 0.000   | 0.000    | 0.000   | 0.000   | 0.000   | 0.000   | 0.000    | 0.000   | 0.000   | 0.000   |
| Christoffersen  | 0.155   | 0.075    | 0.000   | 0.509   | 0.000   | 0.235   | 0.782    | 0.000   | 0.274   | 0.001   |
| AoE            | 1.034   | 1.117    | 1.150   | 1.002   | 1.250   | 0.943   | 0.998    | 1.113   | 0.919   | 1.164   |
| **DowJones**   |         |          |         |         |         |         |          |         |         |         |
| DQ             | 0.000   | 0.000    | 0.000   | 0.000   | 0.000   | 0.000   | 0.000    | 0.002   | 0.000   | 0.000   |
| Christoffersen  | 0.314   | 0.132    | 0.000   | 0.868   | 0.000   | 0.513   | 0.904    | 0.000   | 0.733   | 0.001   |
| AoE            | 1.016   | 1.117    | 1.164   | 1.013   | 1.214   | 0.962   | 0.978    | 1.081   | 0.966   | 1.160   |
| **DAX**        |         |          |         |         |         |         |          |         |         |         |
| DQ             | 0.000   | 0.000    | 0.000   | 0.000   | 0.000   | 0.000   | 0.000    | 0.729   | 0.000   | 0.070   |
| Christoffersen  | 0.002   | 0.004    | 0.000   | 0.729   | 0.000   | 0.297   | 0.608    | 0.000   | 0.013   | 0.000   |
| AoE            | 1.088   | 1.192    | 1.199   | 1.045   | 1.350   | 0.991   | 1.058    | 1.014   | 0.885   | 1.321   |
| **Apple**      |         |          |         |         |         |         |          |         |         |         |
| DQ             | 0.000   | 0.000    | 0.000   | 0.000   | 0.000   | 0.090   | 0.078    | 0.000   | 0.032   | 0.000   |
| Christoffersen  | 0.294   | 0.329    | 0.000   | 0.824   | 0.000   | 0.298   | 0.214    | 0.001   | 0.332   | 0.000   |
| AoE            | 1.009   | 1.052    | 1.009   | 1.031   | 1.002   | 0.927   | 0.927    | 0.911   | 0.887   |
| **VOW**        |         |          |         |         |         |         |          |         |         |         |
| DQ             | 0.001   | 0.000    | 0.000   | 0.000   | 0.000   | 0.002   | 0.048    | 0.000   | 0.148   | 0.000   |
| Christoffersen  | 0.018   | 0.060    | 0.000   | 0.374   | 0.000   | 0.071   | 0.071    | 0.000   | 0.614   | 0.000   |
| AoE            | 1.027   | 1.095    | 1.109   | 1.070   | 0.995   | 0.967   | 1.026    | 0.901   | 0.991   | 0.901   |

Results for Christoffersen-tests depict p-values and AoE measures depict the calibration of the forecast. AoE values close to one and large p-values depict good performance.

visible in the DQ-tests that always reject, but not so much with coverage issues. The forecasts for the Dow Jones, for example, are often too conservative, meaning that the forecast might be well calibrated at the cost of precision. Figure 14 in the Appendix highlights exactly this behavior, where DQ tests reject for all methods during times of crashes. By using an expanding window, such behavior could be alleviated at least for adaptive procedures such as the GRF.

Comparing their performance directly with CPA tests in Table 8, we can confirm that the established methods are better than before, with QR and GARCH sometimes being significantly better than GRF, while CAViaR and especially Historical Simulation are often significantly outperformed by GRF.

All in all, we found that especially for cryptocurrencies, the fully non-parametric GRF tuned for quantile forecasting can significantly outperform established methods such as CAViaR or GARCH and therefore provide a better risk understanding of this asset class. This outperformance could be largely attributed to the fast-changing volatility and structural changes.
Table 8: Robustness Checks for Classic Stocks: CPA-tests on Predictions of 5% VaR

| Rolling Window | 1 = 500 |
|----------------|--------|
| GRF vs:        | QR     | Hist | CAViaR | GARCH |
| SP500          |        |      |        |       |
| P-Value        | 0.049− | 0.000+ | 0.275 | 0.013− |
| GRF-Performance| 0.017  | 0.995 | 0.971  | 0.020 |
| DowJones       |        |      |        |       |
| P-Value        | 0.218  | 0.000+ | 0.347 | 0.014− |
| GRF-Performance| 0.051  | 0.999 | 0.990  | 0.045 |
| DAX            |        |      |        |       |
| P-Value        | 0.153  | 0.000+ | 0.001+ | 0.003− |
| GRF-Performance| 0.017  | 0.995 | 0.937  | 0.015 |
| Apple          |        |      |        |       |
| P-Value        | 0.142  | 0.000+ | 0.161  | 0.446 |
| GRF-Performance| 0.048  | 1.000 | 0.558  | 0.983 |
| VOW            |        |      |        |       |
| P-Value        | 0.493  | 0.000+ | 0.607  | 0.324 |
| GRF-Performance| 0.014  | 0.956 | 0.986  | 0.014 |

Results of CPA-tests are for 5% one-day ahead VaR-forecasts of GRF vs. other methods. We report p-values and the rate at which GRF outperforms the competing method (i.e., a value of 0.8 means that GRF has a smaller error loss than the competing method in 80% of the rolling window forecasts). Low p-values paired with performance rates larger (smaller) than 0.5 indicate that GRF outperforms (is outperformed by) the competing methods and are marked with a + (−).

In these cryptocurrencies, where the flexible random forest methodology is able to adapt faster than methods relying on parametric specifications. As could be expected, this advantage is smaller with more established, better researched, and less extreme assets, where the parametric procedures sometimes outperform the GRF. It has to be noted however, that all methods struggle with very sudden, unpredictable shocks such as the COVID-19 related crash in March 2020. This also affects the more flexible GRF somehow, although more with classic stocks, where such large outliers are less common and not present in the training data.

6 Conclusion

In this paper, we show that random forests can significantly improve the forecasting performance for VaR-predictions when tailored to the task of quantile regression. In both simulations and analyzing return data of the largest cryptocurrencies, the proposed random forest (i.e., GRF) proves to be the most reliable method. This can be attributed to the non-linear form of the
return data with large time-variations of volatility that call for methods that can adapt to changes in a non-parametric way, while other classic methods break down. We further show that the GRF is better in assessing the tail risk of cryptocurrencies in times where speculation and therefore volatility in returns is high, e.g. when there is a speculative bubble. There, more simple procedures perform especially bad and the comparison of predicted losses could thus GRF could serve as an easy, empirical alternative to detect such bubbles.

Our findings are highly relevant for the risk assessment of cryptocurrencies, where high volatility changes and large returns are often found. Classic methods can therefore lead to false security and miss-assessment of risks (and chances) of these assets. Comparing the results for cryptocurrencies to a forecasting scenario using standard stocks and indices, we find that the classic methods such as CAViaR perform much better and are comparable to the performance of GRF. This highlights that cryptocurrencies are structurally different, which can be nicely captured with more flexible, non-parametric methods obtaining proper risk assessments. Furthermore, the GRF-method is readily available in programming languages such as R and Python and can be easily used to compute the VaR.

The random forest methodology allowed us to identify important factors which we show to be time-varying and that are particularly important for forecast performance in unstable times. For future research, an interesting extension would therefore be to even augment this set of covariates with other potentially driving factors, such as for example social media information. The relevance of such factors might also provide additional guidance for relevant exogenous information to be included in standard parametric models such as CAViaR.

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A Appendix

Figure 6: Log-returns of the 4 cryptocurrencies and the CRIX-index over the out-of-sample period.
Figure 7: Rolling 10-day mean of predicted loss difference series (red) of $h_t \hat{\beta}_0$ of CPA tests on USDT predicted 5% VaR with $l = 500$ for GRF vs. QR with lagged 250 day standard deviation in dashed lines (blue). A negative predicted loss difference indicates that the prediction error of GRF is smaller than of the compared method.

Figure 8: Rolling 10-day mean of predicted loss difference series (red) of $h_t \hat{\beta}_0$ of CPA tests on XRP predicted 5% VaR with $l = 500$ for GRF vs. CAViaR (left) and GRF vs. QR (right) with lagged 250 day standard deviation in dashed lines (blue). A negative predicted loss difference indicates that the prediction error of GRF is smaller than of the compared method.
Figure 9: Performance against GRF (red) and p-value (grey) for CPA-tests of GRF against other methods for XRP. The dotted lines are at 0.5 and 0.05 for performance and p-values, respectively. Results are over rolling windows in the out-of-sample period of length 800. Predictions are from $l = 500$ observation rolling windows of the respective method. The bottom dates indicate the start date of the test window and the top dates indicate the end of the test window.

Figure 10: Rolling 10-day mean of predicted loss difference series (red) of $h_t \hat{\beta}_0$ of CPA tests on Ethereum predicted 5% VaR with $l = 500$ for GRF vs. CAViaR (left) and GRF vs. GARCH (right) with lagged 250 day standard deviation in dashed lines (blue). A negative predicted loss difference indicates that the prediction error of GRF is smaller than of the compared method.
Figure 11: Rolling 10-day mean of predicted loss difference series (red) of $h_t \hat{\beta}_0$ of CPA tests on Ethereum predicted 5% VaR with $l = 500$ for GRF vs. historical simulation (left) and GRF vs. QR (right) with lagged 250 day standard deviation in dashed lines (blue). A negative predicted loss difference indicates that the prediction error of GRF is smaller than of the compared method.

Table 9: P-Values of Non-Stationarity Tests

|        | Level KPSS | Trend KPSS | ADF  |
|--------|------------|------------|------|
| BTC    | >0.100     | >0.100     | <0.010 |
| CRIX   | >0.100     | 0.089      | <0.010 |
| ETH    | 0.099      | 0.081      | <0.010 |
| USDT   | >0.100     | >0.100     | <0.010 |
| XRP    | >0.100     | >0.100     | <0.010 |
| Apple  | >0.100     | >0.100     | <0.010 |
| DAX    | >0.100     | >0.100     | <0.010 |
| Dow Jones | >0.100   | >0.100     | <0.010 |
| S&P 500 | >0.100     | >0.100     | <0.010 |
| Volkswagen | >0.100     | >0.100     | <0.010 |

Note: Tests are performed on the data set as described in Section 2.
Figure 12: Rolling 30-day mean of the GRF variable importance in the out-of-sample period for predicted 5% VaR of different cryptocurrencies with \( l = 500 \). Variable importance of covariate \( x_p \) is measured as the proportion of splits on \( x_p \) relative to all splits in a respective layer \( l \) (over all trees in a trained forest), weighted by layer \( l \). Notice that the USDT y-axis scale is different.
Figure 13: Top plot: log-returns of USDT for the out-of sample period. Other plots: p-values of DQ tests using 500 out-of-sample predictions (from l=500 training points) over a rolling window (10 days) for different methods. Red dotted lines mark a p-value of 5%. The date on the x-axis marks the start of the testing period and contains approximately two years of daily data (i.e. 500 observations).
Figure 14: Top plot: log-returns of the DowJones index for the out-of-sample period. Other plots: p-values of DQ tests using 500 out-of-sample predictions (from $l = 500$ training points) over a rolling window (10 days) for different methods. Red dotted lines mark a p-value of 5%. The date on the x-axis marks the start of the testing period and contains approximately two years of daily data (i.e. 500 observations).
Results for 1% VaR show rejection rates of t-tests of empirical levels against the nominal level of 1% for DQ-, Kupiec- and Christoffersen-tests and mean p-values in parentheses. Higher p-values and lower rejection rates indicate better model performance. GARCH depicts an oracle GARCH-model that fits an GARCH(1,1) process with normally distributed errors.
Table 11: Backtests of 5% VaR Forecasts for Cryptocurrencies

| Rolling Window | QRF | GRF | QR | Hist | NormFit | CAViaR | GARCH |
|----------------|-----|-----|----|------|---------|--------|-------|
| CRIX           |     |     |    |      |         |        |       |
| DQ             | 0.255 | 0.985 | 0.430 | 0.007 | 0.000 | 0.188 | 0.380 |
| Christoffersen  | 0.767 | 0.707 | 0.968 | 0.472 | 0.195 | 0.452 | 0.374 |
| Kupiec         | 0.578 | 0.570 | 0.950 | 0.498 | 0.950 | 0.225 | 0.486 |
| AoE            | 1.067 | 0.933 | 1.007 | 1.081 | 1.007 | 0.859 | 0.919 |
|                |     |     |    |      |         |        |       |
| Bitcoin        |     |     |    |      |         |        |       |
| DQ             | 0.007 | 0.460 | 0.413 | 0.000 | 0.000 | 0.710 | 0.239 |
| Christoffersen  | 0.603 | 0.775 | 0.701 | 0.024 | 0.119 | 0.662 | 0.253 |
| Kupiec         | 0.360 | 0.655 | 0.566 | 0.163 | 0.582 | 0.945 | 0.655 |
| AoE            | 1.110 | 0.947 | 0.933 | 1.170 | 1.066 | 0.992 | 0.947 |
|                |     |     |    |      |         |        |       |
| ETH            |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.413 | 0.025 | 0.019 | 0.257 | 0.285 | 0.249 |
| Christoffersen  | 0.412 | 0.839 | 0.798 | 0.559 | 0.103 | 0.724 | 0.086 |
| Kupiec         | 0.301 | 0.749 | 0.502 | 0.749 | 0.043 | 0.857 | 0.276 |
| AoE            | 1.125 | 0.962 | 1.081 | 0.962 | 0.770 | 1.021 | 0.873 |
|                |     |     |    |      |         |        |       |
| USDT           |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Christoffersen  | 0.294 | 0.016 | 0.249 | 0.000 | 0.024 | 0.294 | 0.036 |
| Kupiec         | 0.749 | 0.080 | 0.669 | 0.001 | 0.106 | 0.749 | 0.015 |
| AoE            | 0.962 | 0.799 | 1.051 | 1.421 | 0.814 | 0.962 | 1.303 |
|                |     |     |    |      |         |        |       |
| XRP            |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.005 | 0.000 | 0.000 | 0.000 | 0.257 | 0.000 |
| Christoffersen  | 0.124 | 0.234 | 0.057 | 0.008 | 0.000 | 0.490 | 0.002 |
| Kupiec         | 0.202 | 0.761 | 0.163 | 0.248 | 0.000 | 0.248 | 0.043 |
| AoE            | 1.155 | 1.036 | 1.170 | 1.140 | 0.489 | 1.140 | 0.770 |

Results for the DQ-, Kupiec, and Christoffersen-test depict p-values for the t-tests of empirical quantile levels against the nominal level of 5% and AoE measures the calibration of the forecast. AoE values close to one and large p-values depict good performance.
Table 12: Backtests of 5% VaR Forecasts for Classic Stocks

| Rolling Window | $l = 500$ | $l = 1000$ |
|----------------|-----------|-----------|
|                | QRF | GRF | QR | Hist | NormFit | CAViaR | GARCH | QRF | GRF | QR | Hist | NormFit | CAViaR | GARCH |
| **SP500**      |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Kupiec         | Christoffersen | 0.010 | 0.155 | 0.075 | 0.000 | 0.000 | 0.509 | 0.000 | 0.000 | 0.274 | 0.000 | 0.000 | 0.000 | 0.274 | 0.001 |
| AoE            | 1.142 | 1.034 | 1.117 | 1.150 | 1.171 | 1.062 | 1.250 | 1.077 | 0.943 | 0.998 | 1.113 | 1.089 | 0.919 | 1.164 |
| **DowJones**   |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Kupiec         | Christoffersen | 0.021 | 0.314 | 0.132 | 0.000 | 0.000 | 0.868 | 0.000 | 0.000 | 0.733 | 0.001 | 0.000 | 0.000 | 0.733 | 0.001 |
| AoE            | 1.164 | 1.016 | 1.117 | 1.164 | 1.160 | 1.013 | 1.214 | 1.061 | 0.962 | 0.978 | 1.081 | 1.073 | 0.966 | 1.160 |
| **DAX**        |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Kupiec         | Christoffersen | 0.006 | 0.782 | 0.049 | 0.000 | 0.000 | 0.830 | 0.000 | 0.000 | 0.581 | 0.011 | 0.000 | 0.000 | 0.581 | 0.011 |
| AoE            | 1.264 | 1.088 | 1.192 | 1.199 | 1.253 | 1.045 | 1.350 | 1.160 | 0.991 | 1.058 | 1.014 | 1.014 | 0.885 | 1.321 |
| **Apple**      |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Kupiec         | Christoffersen | 0.002 | 0.294 | 0.129 | 0.000 | 0.000 | 0.824 | 0.000 | 0.000 | 0.332 | 0.000 | 0.000 | 0.000 | 0.332 | 0.000 |
| AoE            | 1.211 | 1.009 | 1.052 | 1.009 | 0.861 | 1.031 | 1.002 | 1.125 | 0.927 | 0.927 | 0.976 | 1.014 | 0.911 | 0.887 |
| **VOW**        |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Kupiec         | Christoffersen | 0.008 | 0.641 | 0.108 | 0.065 | 0.186 | 0.234 | 0.934 | 0.035 | 0.591 | 0.671 | 0.099 | 0.000 | 0.880 | 0.099 |
| AoE            | 1.173 | 1.027 | 1.095 | 1.109 | 0.924 | 1.070 | 0.995 | 1.131 | 0.967 | 1.026 | 0.901 | 0.628 | 0.991 | 0.901 |

Results for the DQ-, Kupiec, and Christoffersen-test depict p-values for the t-tests of empirical quantile levels against the nominal level of 5% and AoE measures the calibration of the forecast. AoE values close to one and large p-values depict good performance.
Table 13: Backtests of 1% VaR Forecasts for Cryptocurrencies

| Rolling Window | QRF | GRF | QR | Hist | NormFit | CAViaR | GARCH | QRF | GRF | QR | Hist | NormFit | CAViaR | GARCH |
|----------------|-----|-----|----|------|---------|--------|-------|-----|-----|----|------|---------|--------|-------|
| **CRYX**       |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.015 | 0.046 | 0.003 | 0.036 | 0.000 | 0.265 | 0.000 | 0.015 | 0.046 | 0.003 | 0.036 | 0.000 | 0.265 | 0.000 |
| Christoffersen  | 0.119 | 0.873 | 0.279 | 0.779 | 0.000 | 0.823 | 0.000 | 0.119 | 0.873 | 0.279 | 0.779 | 0.000 | 0.823 | 0.000 |
| Kupiec         | 0.058 | 0.891 | 0.156 | 0.687 | 0.000 | 0.676 | 0.000 | 0.058 | 0.891 | 0.156 | 0.687 | 0.000 | 0.676 | 0.000 |
| AoE            | 1.556 | 0.963 | 1.407 | 1.111 | 2.741 | 0.889 | 2.296 | 1.556 | 0.963 | 1.407 | 1.111 | 2.741 | 0.889 | 2.296 |
| **Bitcoin**    |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.066 | 0.250 | 0.001 | 0.384 | 0.000 | 0.062 | 0.000 | 0.066 | 0.250 | 0.001 | 0.384 | 0.000 | 0.062 | 0.000 |
| Christoffersen  | 0.119 | 0.822 | 0.072 | 0.780 | 0.000 | 0.780 | 0.000 | 0.119 | 0.822 | 0.072 | 0.780 | 0.000 | 0.780 | 0.000 |
| Kupiec         | 0.058 | 0.674 | 0.033 | 0.689 | 0.000 | 0.689 | 0.000 | 0.058 | 0.674 | 0.033 | 0.689 | 0.000 | 0.689 | 0.000 |
| AoE            | 1.554 | 0.888 | 1.628 | 1.110 | 2.295 | 1.110 | 2.147 | 1.554 | 0.888 | 1.628 | 1.110 | 2.295 | 1.110 | 2.147 |
| **ETH**        |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.000 | 0.015 | 0.000 | 0.000 | 0.013 | 0.000 | 0.000 | 0.000 | 0.015 | 0.000 | 0.015 | 0.000 | 0.013 |
| Christoffersen  | 0.026 | 0.873 | 0.663 | 0.780 | 0.072 | 0.663 | 0.044 | 0.026 | 0.873 | 0.663 | 0.780 | 0.072 | 0.663 | 0.044 |
| Kupiec         | 0.010 | 0.888 | 0.508 | 0.689 | 0.033 | 0.508 | 0.018 | 0.010 | 0.888 | 0.508 | 0.689 | 0.033 | 0.508 | 0.018 |
| AoE            | 1.776 | 0.962 | 1.184 | 1.110 | 1.628 | 1.184 | 1.702 | 1.776 | 0.962 | 1.184 | 1.110 | 1.628 | 1.184 | 1.702 |
| **USDT**       |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Christoffersen  | 0.007 | 0.337 | 0.001 | 0.000 | 0.000 | 0.015 | 0.000 | 0.007 | 0.337 | 0.001 | 0.000 | 0.015 | 0.000 | 0.000 |
| Kupiec         | 0.010 | 0.689 | 0.005 | 0.005 | 0.000 | 0.033 | 0.000 | 0.010 | 0.689 | 0.005 | 0.005 | 0.033 | 0.000 | 0.000 |
| AoE            | 1.776 | 1.110 | 1.850 | 1.850 | 2.665 | 1.628 | 3.331 | 1.776 | 1.110 | 1.850 | 1.850 | 2.665 | 1.628 | 3.331 |
| **XRP**        |     |     |    |      |         |        |       |     |     |    |      |         |        |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Christoffersen  | 0.001 | 0.856 | 0.002 | 0.188 | 0.529 | 0.015 | 0.015 | 0.001 | 0.856 | 0.002 | 0.188 | 0.529 | 0.015 | 0.015 |
| Kupiec         | 0.001 | 0.894 | 0.033 | 0.098 | 0.359 | 0.005 | 0.005 | 0.001 | 0.894 | 0.033 | 0.098 | 0.359 | 0.005 | 0.005 |
| AoE            | 1.999 | 1.036 | 1.628 | 1.480 | 1.258 | 1.850 | 1.850 | 1.999 | 1.036 | 1.628 | 1.480 | 1.258 | 1.850 | 1.850 |

Results for the DQ-, Kupiec, and Christoffersen-test depict p-values for the t-tests of empirical quantile levels against the nominal level of 1% and AoE measures the calibration of the forecast. AoE values close to one and large p-values depict good performance.
### Table 14: Backtests of 1% VaR Forecasts for Classic Stocks

| Rolling Window |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|----------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|                | QRF | GRF | QR | Hist | NormFit | CAViaR | GARCH | QRF | GRF | QR | Hist | NormFit | CAViaR | GARCH |
| **SP500**      |     |     |    |      |         |         |       |     |     |    |      |         |         |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Christoffersen | 0.000 | 0.005 | 0.000 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.004 | 0.003 | 0.000 | 0.000 | 0.022 | 0.000 |
| Kupiec         | 0.000 | 0.024 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.026 | 0.003 | 0.000 | 0.000 | 0.009 | 0.000 |
| AoE            | 1.982 | 1.315 | 1.640 | 1.658 | 2.595 | 1.514 | 2.198 |     |     |    |      |         |         |       |
| **DowJones**   |     |     |    |      |         |         |       |     |     |    |      |         |         |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Christoffersen | 0.000 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.005 | 0.050 | 0.000 | 0.000 | 0.002 | 0.000 |
| Kupiec         | 0.000 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.026 | 0.000 | 0.000 | 0.001 | 0.000 |
| AoE            | 2.018 | 1.297 | 1.658 | 1.694 | 2.793 | 1.658 | 2.198 |     |     |    |      |         |         |       |
| **DAX**        |     |     |    |      |         |         |       |     |     |    |      |         |         |       |
| DQ             | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.001 | 0.000 | 0.000 | 0.098 | 0.000 |
| Christoffersen | 0.000 | 0.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.082 | 0.088 | 0.006 | 0.000 | 0.147 | 0.000 |
| Kupiec         | 0.000 | 0.010 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.085 | 0.056 | 0.056 | 0.000 | 0.129 | 0.000 |
| AoE            | 2.076 | 1.360 | 1.718 | 1.772 | 2.595 | 1.790 | 2.076 |     |     |    |      |         |         |       |
| **Apple**      |     |     |    |      |         |         |       |     |     |    |      |         |         |       |
| DQ             | 0.000 | 0.008 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.087 | 0.132 | 0.000 | 0.000 | 0.018 | 0.002 |
| Christoffersen | 0.000 | 0.039 | 0.000 | 0.019 | 0.000 | 0.004 | 0.000 | 0.000 | 0.135 | 0.135 | 0.215 | 0.000 | 0.495 | 0.016 |
| Kupiec         | 0.000 | 0.018 | 0.000 | 0.133 | 0.001 | 0.003 | 0.000 | 0.000 | 0.116 | 0.116 | 0.242 | 0.002 | 0.624 | 0.006 |
| AoE            | 1.946 | 1.333 | 1.586 | 1.225 | 1.694 | 1.423 | 1.640 |     |     |    |      |         |         |       |
| **VOW**        |     |     |    |      |         |         |       |     |     |    |      |         |         |       |
| DQ             | 0.000 | 0.393 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.022 | 0.350 | 0.000 | 0.000 | 0.316 | 0.000 |
| Christoffersen | 0.000 | 0.331 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.332 | 0.261 | 0.000 | 0.000 | 0.253 | 0.000 |
| Kupiec         | 0.000 | 0.376 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.323 | 0.355 | 0.112 | 0.064 | 0.858 | 0.013 |
| AoE            | 1.848 | 1.120 | 1.671 | 1.528 | 1.830 | 1.564 | 1.671 |     |     |    |      |         |         |       |

Results for the DQ-, Kupiec, and Christoffersen-test depict p-values for the t-tests of empirical quantile levels against the nominal level of 1% and AoE measures the calibration of the forecast. AoE values close to one and large p-values depict good performance.
Table 15: Additional CPA-tests on Predictions of 5% VaR for Cryptocurrencies

| Rolling Window | $l = 500$ | $l = 1000$ |
|----------------|-----------|------------|
|                | GRF vs.   | QR | Hist | CAViaR | GARCH | QR | Hist | CAViaR | GARCH |
| **CRIX**       |           |    |      |        |       |    |      |        |       |
| P-Value        | 0.170     | 0.025+ | 0.265 | 0.174  | 0.571 | 0.153 | 0.799 | 0.153  |
| GRF-Performance| 0.976     | 0.964 | 0.985 | 0.999  | 0.980 | 0.985 | 0.709 | 0.835  |
| **Bitcoin**    |           |    |      |        |       |    |      |        |       |
| P-Value        | 0.063+    | 0.002+ | 0.098+ | 0.086+ | 0.461 | 0.032+ | 0.482 | 0.537  |
| GRF-Performance| 1.000     | 0.987 | 0.991 | 0.985  | 0.987 | 0.998 | 0.968 | 0.979  |
| **Ethereum**   |           |    |      |        |       |    |      |        |       |
| P-Value        | 0.955     | 0.946 | 0.559 | 0.982  | 0.800 | 0.267 | 0.501 | 0.696  |
| GRF-Performance| 0.188     | 0.907 | 0.022 | 0.931  | 0.076 | 0.989 | 0.012 | 0.969  |
| **USDT**       |           |    |      |        |       |    |      |        |       |
| P-Value        | 0.331     | 0.000+ | 0.011+ | 0.012+ | 0.772 | 0.000+ | 0.019+ | 0.027+ |
| GRF-Performance| 0.034     | 1.000 | 0.993 | 0.973  | 0.012 | 0.995 | 1.000 | 1.000  |
| **XRP**        |           |    |      |        |       |    |      |        |       |
| P-Value        | 0.169     | 0.000+ | 0.491 | 0.004+ | 0.047+ | 0.000+ | 0.623 | 0.000+ |
| GRF-Performance| 0.978     | 0.996 | 0.171 | 0.973  | 0.982 | 0.976 | 0.992 | 0.982  |

Results of CPA-tests are for 5% one-day ahead VaR-forecasts of GRF vs. other methods. We report p-values and the rate at which GRF outperforms the competing method (i.e. a value of 0.8 means that GRF has a smaller error loss than the competing method in 80% of the rolling window forecasts). Low p-values paired with performance rates larger than 0.5 indicate that GRF outperforms the competing methods and are marked with a +.
Table 16: Additional CPA-tests on Predictions of 5% VaR for Classic Stocks

| Rolling Window | \( l = 500 \) | \( l = 1000 \) |
|---------------|---------------|---------------|
|                | QR    | Hist | CAViaR | GARCH | QR    | Hist | CAViaR | GARCH |
| \textit{SP500} |       |      |        |       |       |      |        |       |
| P-Value        | 0.049⁻ | 0.000⁺ | 0.275  | 0.013⁻ | 0.035⁻ | 0.000⁺ | 0.009⁺ | 0.004⁻ |
| GRF-Performance| 0.017  | 0.995 | 0.971  | 0.020  | 0.000  | 0.998 | 0.923  | 0.025  |
| \textit{DowJones} |       |      |        |       |       |      |        |       |
| P-Value        | 0.218  | 0.000⁺ | 0.347  | 0.014⁻ | 0.040⁻ | 0.000⁺ | 0.023⁺ | 0.021⁻ |
| GRF-Performance| 0.051  | 0.999 | 0.990  | 0.045  | 0.028  | 1.000 | 0.994  | 0.020  |
| \textit{DAX}   |       |      |        |       |       |      |        |       |
| P-Value        | 0.153  | 0.000⁺ | 0.001⁺ | 0.003⁻ | 0.077⁻ | 0.000⁺ | 0.012⁺ | 0.121  |
| GRF-Performance| 0.017  | 0.995 | 0.937  | 0.015  | 0.087  | 0.996 | 0.948  | 0.005  |
| \textit{Apple} |       |      |        |       |       |      |        |       |
| P-Value        | 0.142  | 0.000⁺ | 0.161  | 0.446  | 0.561  | 0.000⁺ | 0.003⁺ | 0.124  |
| GRF-Performance| 0.048  | 1.000 | 0.558  | 0.983  | 0.114  | 0.994 | 0.986  | 0.944  |
| \textit{VOW}   |       |      |        |       |       |      |        |       |
| P-Value        | 0.493  | 0.000⁺ | 0.607  | 0.324  | 0.357  | 0.000⁺ | 0.326  | 0.597  |
| GRF-Performance| 0.014  | 0.956 | 0.986  | 0.014  | 0.015  | 0.965 | 0.995  | 0.032  |

Results of CPA-tests are for 5% one-day ahead VaR-forecasts of GRF vs. other methods. We report p-values and the rate at which GRF outperforms the competing method (i.e. a value of 0.8 means that GRF has a smaller error loss than the competing method in 80% of the rolling window forecasts). Low p-values paired with performance rates larger (smaller) than 0.5 indicate that GRF outperforms (is outperformed by) the competing methods and are marked with a + (-).