Initial Conditions for Supersymmetric Inflation

G. Lazarides

Physics Division
School of Technology
University of Thessaloniki
Thessaloniki 540 06, Greece

N.D.Vlachos

Dept. of Theoretical Physics
University of Thessaloniki
Thessaloniki 540 06, Greece

Abstract

We perform a numerical investigation of the fields evolution in the supersymmetric inflationary model based on radiative corrections. Supergravity corrections are also included. We find that, out of all the examined initial data, only about 10% give an adequate amount of inflation and can be considered as "natural". Moreover, these successful initial conditions appear scattered and more or less isolated.
A couple of years ago Linde [1] has proposed, in the context of non-supersymmetric (SUSY) grand unified theories (GUTs), a clever inflationary scenario based on a coupled system of two real scalar fields one of which is a gauge non-singlet. This field remains displaced from its vacuum value during inflation and provides the vacuum energy which drives inflation while the other, which is a gauge singlet, is the slowly varying field during inflation. The main advantage of this hybrid inflationary scenario is that it can reproduce, in contrast to previous realizations of inflation, the observed temperature fluctuations in the cosmic background radiation (CBR) with natural values of the coupling constants. However, inflation terminates abruptly and is followed by a ”waterfall” regime during which topological defects are easily produced if they are predicted by the symmetry breaking associated with inflation.

Recently [2,3], two variants of the above scenario, both sharing its naturalness, were proposed in the context of SUSY GUTs. One of them [2] relies on the lowest order non-renormalizable contribution to the superpotential. This scenario has been called smooth hybrid inflation since the termination of the inflationary period is not as abrupt as in Linde’s scheme. Rather, the entrance of the system into the oscillatory phase is quite smooth. Moreover, the system, already from the beginning of inflation, follows a particular valley of minima that leads to a particular point of the vacuum manifold and, thus, there is no production of topological defects in this case. This is certainly an advantage of this scenario since it makes it applicable even to GUT models where the symmetry breaking associated with inflation does imply the existence of (possibly) cosmologically disastrous topological defects like domain walls or magnetic monopoles. The main advantage though of smooth hybrid inflation is that it can be associated to the SUSY GUT scale $M_{SG} \simeq 2 \times 10^{16}$ GeV consistent with the unification of the minimal supersymmetric standard model (MSSM) gauge couplings. A possible shortcoming of this scenario is that the relevant part of inflation takes place at relatively high values $[(1-3) \times 10^{17}$ GeV] of the inflaton field and, thus, supergravity and superstring corrections may not be so easily tamable.

The second SUSY variant [3] of the hybrid inflationary scenario utilizes the one loop
radiative corrections to the inflationary potential. These corrections are non-trivial since SUSY is broken at the inflationary trajectory in field space. The termination of inflation is as in the non-SUSY Linde’s scenario and topological defects can be copiously produced. It is then important to make sure that no cosmologically dangerous topological defects come into existence as a result of the symmetry breaking associated to inflation. Moreover, as it turns out, the scale at which this symmetry breaking takes place is about five times smaller than $M_{SG}$. This can be considered as a disadvantage of the scheme since it forces us to abandon the successful unification of the MSSM gauge coupling constants. In this case, inflation may be chosen to take place during an intermediate symmetry breaking with no magnetic monopoles or domain walls associated with it. The important advantage of this scenario is that the relevant part of inflation takes place at values of the inflaton field of order $10^{16}$ GeV which are low enough for supergravity and superstring corrections to be under relatively good control [4].

For an inflationary scenario to be considered fully successful, one has to show that it is obtainable for a wide class of ”natural” initial values of the fields and their time derivatives. In ref. [5] this issue was investigated for smooth hybrid inflation. The result was striking and unexpected. It was found that, for almost all the examined initial conditions (except for a narrow transition region), we get inflation with an adequate number of e-foldings. This is certainly an important advantage of this inflationary scenario. The purpose of this work is to discuss the question of initial conditions for the second SUSY hybrid inflationary scenario which is based on radiative corrections. In particular, we will try to see if there exists a wide class of ”natural” initial conditions, i.e., a class of comparable initial values of the fields with vanishing time derivatives for which the system falls at the bottom of the valley of minima so that its subsequent evolution along this valley produces an adequate amount of inflation. To this end, we solve numerically the evolution equations of the system for various sets of initial conditions. Our analysis also includes cases with all initial field values being much smaller than the Planck scale, where the results are expected to be less affected by replacing global by local SUSY. At the end, non-vanishing initial time derivatives of the fields as well
as supergravity corrections were also taken into account.

The SUSY inflationary scenario based on radiative corrections can be realized in the context of a SUSY GUT model with a (semisimple) gauge group $G$ of rank $\geq 5$ which breaks spontaneously directly to the standard model (SM) group $G_S$ at a scale $M$. The symmetry breaking of $G$ to $G_S$ is obtained through a superpotential which includes the terms

$$W = \kappa S (-M^2 + \bar{\phi}\phi).$$

(1)

Here, $\bar{\phi}, \phi$ is a conjugate pair of left-handed SM singlet superfields which belong to non-trivial representations of the gauge group $G$ and reduce its rank through their vacuum expectation values (vevs) and $S$ is a gauge singlet left-handed superfield. The superpotential in eq.(1) is in fact the only renormalizable superpotential consistent with a continuous $U(1)$ R-symmetry under which $W \rightarrow e^{i\theta}W$, $S \rightarrow e^{i\theta}S$, $\bar{\phi}\phi \rightarrow \bar{\phi}\phi$. The potential obtained from $W$ in eq.(1), in the SUSY limit, is

$$V = \kappa^2 |M^2 - \bar{\phi}\phi|^2 + \kappa^2 |S|^2 (|\bar{\phi}|^2 + |\phi|^2) + D - terms .$$

(2)

Vanishing of the D-terms is achieved along the D-flat directions where $|\bar{\phi}| = |\phi|$. The SUSY vacuum

$$< S > = 0, \quad < \bar{\phi} > < \phi >= M^2, \quad | < \bar{\phi} > | = | < \phi > |$$

(3)

lies on the particular D-flat direction $\bar{\phi}^* = \phi$. Restricting ourselves to this direction and performing appropriate gauge and R-transformations we can bring the complex $S,\bar{\phi},\phi$ fields on the real axis, i.e., $S \equiv \frac{\sigma}{\sqrt{2}}$, $\bar{\phi} = \phi \equiv \frac{1}{2} \chi$, where $\sigma$ and $\chi$ are real scalar fields. The potential in eq.(2) then takes the form

$$V(\chi, \sigma) = \kappa^2 (M^2 - \frac{\chi^2}{2})^2 + \frac{\kappa^2 \chi^2 \sigma^2}{4} .$$

(4)

The vacua lie at $< \chi >= \pm 2M$, $< \sigma >= 0$. We observe that the potential possesses an exact flat direction at $\chi = 0$ with $V(\chi = 0, \sigma) = \kappa^2 M^4$. The mass squared of the field $\chi$
along this flat direction is given by $m_{\chi}^2 = -\kappa^2 M^2 + \frac{1}{2} \kappa^2 \sigma^2$ and remains non-negative for $\sigma \geq \sigma_c = \sqrt{2} M$. This implies that, at $\chi = 0$ and $\sigma \geq \sigma_c$, we obtain a flat-bottomed valley of minima which, as it stands, is not suitable for inflation. However, due to SUSY breaking along the bottom of this valley resulting from the non-zero value of $V(\chi = 0, \sigma)$, there exist non-trivial radiative corrections to the effective potential so that this valley acquires the negative slope which is necessary for inflation. The one loop corrected effective potential (along the direction $\sigma \geq \sigma_c$, $\chi = 0$) is given by [3]

$$V_{\text{eff}}(\sigma) = \kappa^2 M^4 + \frac{\kappa^4}{32\pi^2} [2M^4 \ln \frac{\kappa^2 \sigma^2}{2\Lambda^2} + \left(\frac{1}{2} \sigma^2 - M^2\right)^2 \ln \left(1 - \frac{2M^2}{\sigma^2}\right) + \left(\frac{1}{2} \sigma^2 + M^2\right) \ln \left(1 + \frac{2M^2}{\sigma^2}\right)] .$$

If $\sigma \gg \sigma_c$, $V_{\text{eff}}(\sigma)$ in eq.(3) reduces to the simpler form

$$V_{\text{eff}}(\sigma \gg \sigma_c) \approx \kappa^2 M^4 \left[1 + \frac{\kappa^2}{16\pi^2} \left(\ln \frac{\kappa^2 \sigma^2}{2\Lambda^2} + \frac{3}{2}\right)\right] .$$

A region of the universe, where $\chi$ and $\sigma$ happen to be almost uniform with negligible kinetic energies and with values close to the bottom of the valley of minima, follows this valley in its subsequent evolution and undergoes inflation. The temperature fluctuations of CBR produced during this inflation can be estimated to be [3]

$$\frac{\delta T}{T} \simeq \sqrt{\frac{N_q}{45}} \left(\frac{M}{M_c}\right)^2 ,$$

where $M_c = M_p/\sqrt{8\pi}$, $M_p = 1.22 \times 10^{19}$ GeV is the Planck mass, and $N_q \simeq 60$ is the number of e-foldings of our present horizon scale during inflation. The Cosmic Background Explorer (COBE) result, $\delta T/T \simeq 6.6 \times 10^{-6}$, can then be reproduced with $M = 5.82 \times 10^{15}$ GeV. Using eq.(3) one finds

$$\kappa = \pi \left(\frac{8}{N_q}\right)^{1/2} \frac{\sigma_Q M}{\sigma_c M_c} ,$$

where $\sigma_Q$ is the value of $\sigma$ when the scale which evolved to our present horizon size crossed outside the de Sitter horizon during inflation. Inflation continues until $\sigma = \sigma_c$ where it terminates abruptly and is followed by a "waterfall", i.e., a sudden entrance into an
oscillatory phase about a global minimum. This is checked by noting the magnitude of the quantities $\epsilon = (V'/V)^2 M_c^2/2, \eta = M_c^2 V''/V$, where the prime refers to derivatives with respect to $\sigma$. Since the system can fall into either of the two available global minima with equal probability, topological defects can be easily produced if they are predicted by the relevant symmetry breaking of the particular particle physics model one is employing.

We will now try to specify the initial conditions for the $\sigma$ and $\chi$ fields which lead to the above described inflationary scenario. In other words, we will try to identify initial conditions for which the system falls at the bottom of the valley of minima of the effective potential so that its subsequent evolution along this valley produces an adequate amount of inflation. We assume that, after "compactification" at some initial cosmic time, a region emerges in the universe where the scalar fields $\sigma$ and $\chi$ and their time derivatives happen to be almost uniform and the energy density $\lesssim M_c^4$. (The initial values of $\sigma$ and $\chi$ together with their time derivatives can always be transformed by appropriate gauge and R-transformations to become positive). The radiative corrections in eqs. (5) and (6), though crucial for inflation, play no essential role in the issue of initial conditions and, thus, they will be ignored. In order to keep the formalism as clear as possible, we shall from now on use dimensionless variables for all relevant quantities as follows:

$$\hat{\chi} = \frac{\chi}{M_c}, \hat{\sigma} = \frac{\sigma}{M_c}, \hat{t} = \kappa M_c \hat{t}, \hat{M} = \frac{M}{M_c}, \hat{H} = \frac{H}{\kappa M_c}, \hat{\theta} = \frac{\theta}{\kappa^2 M_c^4}, \hat{V}(\hat{\chi}, \hat{\sigma}) = \frac{V(\chi, \sigma)}{\kappa^2 M_c^4}, \tag{9}$$

where $H$ is the Hubble parameter and $\theta$ the energy density. The evolution of the system in this region is governed by the following equations of motion

$$\ddot{\chi} + 3\hat{H}\dot{\chi} - \hat{\chi} (\hat{M}^2 - \frac{\hat{\chi}^2}{4}) + \frac{1}{2}\hat{\sigma}^2 \hat{\chi} = 0, \tag{10}$$

$$\ddot{\sigma} + 3\hat{H}\dot{\sigma} + \frac{1}{2}\hat{\chi}^2 \hat{\sigma} = 0, \tag{11}$$

where overdots denote derivatives with respect to the dimensionless time $\hat{t}$, the rescaled Hubble parameter $\hat{H}$ is given by

$$\hat{H} = \frac{\hat{\theta}^{1/2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \left(\frac{1}{2} \hat{\chi}^2 + \frac{1}{2} \hat{\sigma}^2 + \hat{V}(\hat{\chi}, \hat{\sigma})\right)^{1/2}, \quad \hat{V}(\hat{\chi}, \hat{\sigma}) = (\hat{M}^2 - \frac{\hat{\chi}^2}{4})^2 + \frac{\hat{\chi}^2 \hat{\sigma}^2}{4}. \tag{12}$$
and \( \dot{M} = 2.39 \times 10^{-3} \).

We shall first examine the case where initially \( \dot{\sigma} \gg \dot{\chi}, 1 \). Under these conditions, the last term in eq. (10) dominates and this equation reduces to

\[
\ddot{\chi} + 3\dot{H}\dot{\chi} + \frac{1}{2}\dot{\sigma}^2 \dot{\chi} \simeq 0. \tag{13}
\]

Furthermore, the potential energy density \( \hat{V}(\hat{\chi}, \hat{\sigma}) \) in eq. (12) is dominated by its last term provided that \( \hat{\chi}\hat{\sigma} \gg 2\hat{M}^2 \). Let us for the moment assume that, during the early stages of evolution, \( \dot{\sigma} \) remains almost constant. Then, for \( |\dot{\chi}| \ll \dot{\chi}\dot{\sigma}/\sqrt{2} \), \( \dot{H} \) is given approximately by

\[
\dot{H} \simeq \frac{1}{2\sqrt{3}} \dot{\chi}\dot{\sigma}, \tag{14}
\]

so that \( \dot{H} \) and \( \dot{\chi} \) satisfy the following differential equations

\[
\ddot{\chi} + \frac{\sqrt{3}}{2}\dot{\sigma}\dot{\chi} + \frac{1}{2}\dot{\sigma}^2 \dot{\chi} \simeq 0, \tag{15}
\]

\[
\ddot{H} + 3\dot{H}\dot{H} + \frac{1}{2}\dot{\sigma}^2 \dot{H} \simeq 0. \tag{16}
\]

Integrating once, with \( \dot{\chi}(\dot{t} = 0) = 0 \) for simplicity, we get

\[
\dot{\chi}(\hat{t})^2 \simeq \dot{\chi}(0)^2 + \frac{4}{3} \ln \left[ 1 + \frac{\sqrt{3}}{\dot{\sigma}} \dot{\chi}(\hat{t}) \right] - \frac{4}{\sqrt{3}\dot{\sigma}} \dot{\chi}(\hat{t}), \tag{17}
\]

\[
\dot{H}(\hat{t})^2 \simeq \dot{H}(0)^2 + \frac{\dot{\sigma}^2}{9} \ln \left[ 1 + \frac{6}{\dot{\sigma}^2} \dot{H}(\hat{t}) \right] - \frac{2}{3} \dot{H}(\hat{t}). \tag{18}
\]

Using these equations one can show that \( \dot{\chi}(\hat{t}), \ddot{\chi}(\hat{t}), \dot{H}(\hat{t}) \) and \( \ddot{H}(\hat{t}) \) are initially (for \( \dot{\chi}(\hat{t}) > 0 \)) monotonically decreasing functions of \( \hat{t} \). At some time \( \hat{t}_0 \), \( \dot{\chi}(\hat{t}) \) becomes small enough and starts performing damped oscillations over the maximum at \( \dot{\chi} = 0 \). For \( \hat{t} \geq \hat{t}_0 \), the continuity equation \( \dot{\theta} = -3\dot{H}(\dot{\theta} + \dot{p}) \), with \( \dot{p} = p/\kappa^2 M^4_c \), \( \dot{p} \) being the pressure, averaged over one oscillation of \( \dot{\chi} \) becomes \( \dot{\theta} = -3\dot{H}\gamma\dot{\theta} \), where \( \gamma = 1 \) for a \( \dot{\chi}^2 \) potential \[7\]. This together with the fact that \( \dot{\theta} \) is proportional to \( \dot{H}^2 \) gives
\[
\dot{H} \simeq \frac{2}{3t}, \quad \dot{t} \equiv \dot{t} - \dot{t}_0 + \frac{2}{3} \ddot{H}(\dot{t}_0)^{-1}, \dot{t} \geq \dot{t}_0 \quad .
\]

The quantities \(\dot{\chi}(\dot{t}_0)\), \(\dot{H}(\dot{t}_0)\) as well as their time derivatives can be estimated as follows. When the \(\dot{\chi}\) field starts oscillating, \(\dot{H}(\dot{t})\) satisfies the equation

\[
\dot{H}(\dot{t}_0)^2 \simeq -\frac{2}{3} \ddot{H}(\dot{t}_0) \quad ,
\]

as one deduces from eq.(19). Eqs.(14) and (18) then give

\[
\dot{\chi}(\dot{t}_0) \simeq \frac{2\sqrt{3}}{\dot{\sigma}} \dot{H}(\dot{t}_0) \simeq -\frac{\dot{\sigma}}{\sqrt{3}} \left[ 1 - \exp\left[ -\frac{3}{4} \dot{\chi}(0)^2 \right] \right] \cdot
\]

Substituting into eq.(20) we, furthermore, get

\[
\dot{\chi}(\dot{t}_0) \simeq \frac{2\sqrt{3}}{\dot{\sigma}} \dot{H}(\dot{t}_0) \simeq \frac{2}{\sqrt{3}} \sqrt{1 - \exp\left[ -\frac{3}{4} \dot{\chi}(0)^2 \right]} \cdot
\]

Note that eqs.(21) and (22) imply that |\(\dot{\chi}\)| < \(\dot{\chi}\)\(\dot{\sigma}/\sqrt{2}\), for \(\dot{t} \leq \dot{t}_0\), as was required for the validity of eq.(14). Also, for \(\dot{t} \leq \dot{t}_0\), \(\dot{\sigma} \gg \dot{\chi} \gsim 1\) and in eq.(11) the Hubble parameter dominates over the "frequency" \(\dot{\chi}/\sqrt{2}\). This equation then reduces to \(\dot{\sigma} \approx -\dot{\chi}/\sqrt{3}\) and the variation of \(\dot{\sigma}\) till \(\dot{t}_0\) is of order \(\dot{\chi}(0)\dot{H}(\dot{t}_0)^{-1} \sim \dot{\chi}(0)/\dot{\sigma} \ll 1\). This justifies our assumption that \(\dot{\sigma}\) remains constant for \(\dot{t} \leq \dot{t}_0\).

Upon substituting eq.(19) into eq.(13) we get

\[
\dddot{\chi} + \frac{2}{t} \dddot{\chi} + \frac{3}{2} \dddot{\sigma}^2 \dot{\chi} \simeq 0 \quad .
\]

For \(\dot{\sigma}\) constant, this is a differential equation of the Emden type whose solution satisfying the (approximate) boundary conditions at \(\dot{t}_0\) is given by

\[
\dot{\chi}(\dot{t}) \simeq \frac{4}{\sqrt{3} \dot{\sigma} \dot{t}} \cos \left[ \frac{1}{\sqrt{2}} \dot{\sigma} (\dot{t} - \dot{t}_0) \right], \quad \dot{t} \geq \dot{t}_0 \quad .
\]

Eq.(11), for \(\dot{t} \gg \dot{t}_0\), averaged over one oscillation of \(\dot{\chi}\) then gives

\[
\ddot{\sigma} + \frac{2}{t} \dot{\sigma} + \frac{4}{3\dot{\sigma} \dot{t}^2} \simeq 0 \quad .
\]

This equation implies
\[ \hat{t} - \hat{t}_0 \simeq \int_{x(\hat{t}_0)}^{x(\hat{t})} \frac{dx}{\sqrt{-\frac{4}{3} \ln \frac{x^2}{x(\hat{t}_0)^2} + \dot{x}(\hat{t}_0)^2}}, \tag{26} \]

where \( x(\hat{t}) \equiv \hat{t}\sigma(\hat{t}) \), \( \dot{x}(\hat{t}) > 0 \). We see that \( x(\hat{t}) \) is a periodic function of time corresponding to a bounded motion of \( x \) in a logarithmic potential. The turning point \( x_m(>0) \) is defined through the relation

\[ \frac{4}{3} \ln x(\hat{t}_0)^2 + \dot{x}(\hat{t}_0)^2 = \frac{4}{3} \ln x_m^2 \tag{27} \]

and eq.(26) becomes \((x(\hat{t}) > 0)\)

\[ \hat{t} - \hat{t}_0 \simeq \frac{\sqrt{3\pi}}{2\sqrt{2}} x_m \left[ \Phi\left( \sqrt{\ln \frac{x_m}{x(\hat{t}_0)}} \right) - \Phi\left( \sqrt{\ln \frac{x_m}{x(\hat{t})}} \right) \right], \tag{28} \]

where \( \Phi(x) \) is the probability integral. The turning point \( x_m \) can be estimated from eq.(27) and the fact that \( \dot{x}(\hat{t}_0) = \dot{\sigma}(\hat{t}_0)\hat{t}_0 + \dot{\sigma}(\hat{t}_0) \approx \dot{\sigma}(0) \) for \( \dot{\sigma}(0) \gg 1 \). This implies that the system, at \( \hat{t} = \hat{t}_0 \), starts from a position far away from the turning point which could subsequently be reached after the lapse of time \( \delta \hat{t} \approx x_m \sqrt{3\pi/2\sqrt{2}} \approx (\sqrt{3\pi/2\sqrt{2}}) \exp(3\dot{\sigma}(0)^2/8) x(\hat{t}_0) \) as one deduces from eq.(28). Using \( x(\hat{t}_0) = \dot{\sigma}(\hat{t}_0)\hat{t}_0 \approx (2/3)\dot{\sigma}(0)\hat{H}(\hat{t}_0)^{-1} \approx 2[1 - \exp[-3\dot{\chi}(0)^2/4]]^{-1/2} \) from eq.(22), \( \delta \hat{t} \) can be related to the initial values of the fields \( \dot{\chi}(0), \dot{\sigma}(0) \). The time needed for the amplitude of \( \dot{\chi} \) to drop to about \( 2\hat{M}^2/\dot{\sigma} \), where the \( \hat{M}^4 \) term starts dominating the potential energy \( \hat{V}(\dot{\chi}, \dot{\sigma}) \) in eq.(12), can be estimated from eq.(24) to be \( \hat{t}_0 \approx 2/\sqrt{3}\hat{M}^2 \approx 2 \cdot 10^5 \ll \delta \hat{t} \) for \( \dot{\sigma}(0) \gg 1 \) and \( \dot{\chi}(0) \) not extremely small. We then deduce that \( \dot{\sigma} \) is an extremely slowly decreasing function of time, for \( \hat{t}_0 \leq \hat{t} \leq \hat{t}_0 + \delta \hat{t} \), and, thus, our assumption that \( \dot{\sigma} \) remains constant is justified in this time interval too.

For cosmic times \( \geq \hat{t}_d \), the \( \hat{M}^4 \) term dominates the potential energy in eq.(12) and the Hubble parameter becomes approximately constant and equal to \( \hat{H} = \hat{M}^2/\sqrt{3} \) and remains so thereafter. Eq.(13) still holds and, assuming that \( \dot{\sigma} \) remains essentially unchanged, \( \dot{\chi} \) performs damped oscillations of frequency \( \dot{\sigma}/\sqrt{2} \gg \hat{H} \). Eq.(11) averaged over one oscillation of \( \dot{\chi} \) then becomes
\[ \ddot{\sigma} + 3\dot{H}\dot{\sigma} + \frac{1}{4}\chi^2_m\dot{\sigma} = 0 \quad , \]  

(29)

where \( \chi_m \) is the amplitude of \( \dot{\chi} \). The decreasing "frequency" of \( \dot{\sigma} \) which is \( \dot{\chi}_m/2 \leq \dot{M}^2/\dot{\sigma} \) soon becomes much smaller than \( \dot{H} \) and eq.(29) reduces to

\[ 3\dot{H}\dot{\sigma} + \frac{1}{4}\chi^2_m\dot{\sigma} \approx 0 \quad . \]  

(30)

The variation of \( \dot{\sigma} \) within one expansion time is then estimated to be \( |\Delta\dot{\sigma}/\dot{\sigma}| \sim \dot{\chi}_m^2/12 \leq \dot{M}^4/3\dot{\sigma}^2 \ll 1 \). This justifies the assumption that \( \dot{\sigma} \) does not change much. The overall conclusion is that, at times much larger than \( \tilde{t}_0 \approx 2/\sqrt{3}\dot{M}^2 \), the \( \dot{\chi} \) field falls into the valley of minima of the potential in eq.(12) and relaxes at the bottom of this valley whereas the \( \dot{\sigma} \) field still remains more or less unchanged. After that, the radiative corrections in eq.(3) come into play and the system follows the valley of minima towards the supersymmetric vacuum and, therefore, the hybrid inflationary scenario is realized for initial values of the fields satisfying the inequality \( \dot{\sigma} \gg 1, \dot{\chi} \). These initial conditions, however, cannot be considered as totally satisfying because there is a considerable discrepancy between the initial values of the fields. Also, the inclusion of supergravity is expected to invalidate the above discussion of initial conditions which involve values of the field \( \dot{\sigma} \geq 1 \).

For SUSY inflation based on radiative corrections to be considered as a fully successful inflationary scenario, one must show that it is obtained for a wide class of initial conditions which are more "natural" than the ones just discussed. This can be checked only numerically. To this end, we have conducted numerical integration of eqs.(10) and (11) for the following sets of initial conditions:

1. \( 4 \geq \dot{\chi} \geq 0.5, \quad 6 \geq \dot{\sigma} \geq 0.5 \) with vanishing initial velocities.

2. \( 1 \geq \dot{\chi} \geq 0.1, \quad 1 \geq \dot{\sigma} \geq 0.1 \) with vanishing initial velocities.

3. \( 1 \geq \dot{\chi} \geq 0.1, \quad 1 \geq \dot{\sigma} \geq 0.1 \) at the same points as in (2) introducing equal positive initial velocities so that the initial kinetic energy equals the corresponding potential energy.
The last two cases, although corresponding to pretty low values of the initial energy density, were included since our results there are expected to remain essentially unaffected if we replace global by local SUSY with canonical Kaehler potential. On the contrary, the results of case (1) should be strongly affected by supergravity corrections. To see this we reexamined this case by including supergravity corrections with minimal Kaehler potential. This is done by replacing the potential energy density in eq.(12) and its derivatives with respect to $\hat{\chi}$ and $\hat{\sigma}$ in eqs.(10) and (11) by

$$\hat{V}(\hat{\chi}, \hat{\sigma})_{sc} = \exp \left[ \frac{1}{2} \hat{\chi}^2 + \frac{1}{2} \hat{\sigma}^2 \right] \times \left\{ \left( \frac{1}{4} \hat{\chi}^2 - \hat{M}^2 \right)^2 \left[ 1 - \frac{1}{2} \hat{\sigma}^2 + \frac{1}{4} \hat{\sigma}^2 (\hat{\chi}^2 + \hat{\sigma}^2) \right] + \frac{1}{2} \hat{\sigma}^2 \hat{\chi}^2 \left( \frac{1}{4} \hat{\chi}^2 - \hat{M}^2 \right) + \frac{1}{4} \hat{\sigma}^2 \hat{\chi}^2 \right\} \quad (31)$$

and its respective derivatives.

The integration of the two coupled equations was performed by implementing a variant of the Bulrish-Stoer variable step method in a Fortran program. As a general rule, the initial step for the dimensionless time variable $\hat{t}$ was chosen to be 2 while the sought accuracy was put to $10^{-15}$. This choice was found to ensure reasonable stability in all cases.

The results of our search are shown in Figs.1,2,3, matching the corresponding sets of initial conditions, and 4 where supergravity corrections were included in the set of initial conditions (1). Each point on the $\hat{\sigma}-\hat{\chi}$ plane corresponds to given initial conditions and the way it is depicted corresponds to a definite evolution type for the $\hat{\sigma}-\hat{\chi}$ system. We have used three different kinds of symbols.

(a) Open circles: Both fields oscillate and fall towards the supersymmetric minima without producing any appreciable amount of inflation.

(b) Filled triangles: Here, only $\hat{\chi}$ oscillates. The field $\hat{\sigma}$ starts-off at large values ($\hat{\sigma} > \hat{\chi}$) and remains almost constant for a very large period of time. The system eventually relaxes at the bottom of the valley of minima of $\hat{V}(\hat{\chi}, \hat{\sigma})$ in eq.(12). Its subsequent evolution along this valley produces an adequate amount of inflation.
(c) Filled circles: Both fields start oscillating at the beginning. Then $\hat{\sigma}$ settles down at large values and the system follows an evolution of type (b) with adequate inflation.

The evolution pattern (b) includes the limiting area $\hat{\sigma} \gg 1$, $\hat{\chi}$ already described and analyzed by means of semianalytic arguments. In case (c), although $\hat{\sigma}$ starts mostly at moderate or small values $\hat{\sigma} \sim \hat{\chi}$, it appears to increase in amplitude absorbing energy from the fast oscillating field $\hat{\chi}$, creating eventually conditions favoring evolution of type (b). Here, we have an example of large energy transfer between two strongly-coupled non-linear oscillators. A closer look at Figs. 1-4 reveals that, in contrast to the case of the smooth hybrid inflationary model, connected regions corresponding to a definite evolution pattern do not appear to emerge except for the filled triangles area in Fig.1. This area, however, although leading to successful inflation cannot be considered as being "natural" since it requires relatively large differences between the initial values of the fields and is destroyed by supergravity as we shall soon see. Points following evolution type (c) are significantly better since, for many of them, the initial values of the fields are more or less of the same order of magnitude. These points, however, are scattered and do not appear to constitute an extended and well bounded continuous area as in the case of the smooth hybrid inflationary model. We have checked in more detail the area around most of the filled circle points and found out that almost all the points in their immediate neighborhood correspond to the evolution type (a) suggesting that evolution of type (c) occurs only for more or less isolated points. We conclude then that, in the model analyzed here, only about 10% of the examined points lead to successful inflation and lie in the area which can be considered "natural". Finally, the results with supergravity corrections included (Fig.4) show that the filled triangle area disappears and we are left with evolution of the types (a) and (b). This is to be expected, since the very steep walls of the potential in the $\hat{\sigma}$ direction do not allow the $\hat{\sigma}$ field to linger at large values for long. It is remarkable to realize that here also only about 10% of the examined points lead to adequate inflation. The initial energy density $\rho$ can be calculated from eqs. (9),(12) for global SUSY or (9),(31) in the supergravity case with $\kappa$ given by eq.(8) where $\sigma_Q/\sigma_c$ is taken close to 2.
get adequately large initial energy densities. In summary, only a scattered set of "natural" but more or less isolated initial conditions, constituting about 10% of the examined cases, do lead to the SUSY inflationary model based on radiative corrections. It appears that this scenario is less satisfactory than smooth hybrid inflation which is obtainable for almost all initial conditions.

ACKNOWLEDGMENTS

This work was supported in part by E.U. grant ERBFMRXCT 960090
REFERENCES

[1] A. D. Linde, Phys. Rev. D49 (1994) 748.

[2] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D52 (1995) R559.

[3] G. Dvali, Q. Shafi and R. Schaefer, Phys. Rev. Lett. 73 (1994) 1886

[4] G. Lazarides, R.K. Schaefer and Q. Shafi, Trieste preprint IC/96/115 (1996)
(hep-ph 9608256)

[5] G. Lazarides, C. Panagiotakopoulos and N.D. Vlachos, Phys. Rev. D54 (1996) 1369.

[6] For an earlier attempt at inflation with this superpotential, see E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart and D. Wands, Phys. Rev. D, 49, 6410 (1994). Supersymmetric inflation has a long history and there are several reviews available, see, e.g., A.D. Linde, “Particle Physics and Inflationary Cosmology”, Harwood Academic, Switzerland (1990).

[7] M. Turner, Phys. Rev. D28 (1983) 1243.

[8] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, Numerical Recipes in Fortran: The Art of Scientific Computing, 2nd Ed. (Cambridge University Press, Cambridge, England, 1992)
FIGURE CAPTIONS

**Fig. 1**: Evolution patterns for the $\hat{\sigma} - \hat{\chi}$ system with vanishing initial velocities. Open circles represent points that do not lead to inflation. Filled triangles give adequate inflation having only the $\hat{\chi}$ field oscillating. Filled circles give adequate inflation with both fields oscillating initially.

**Fig. 2**: Same as in Fig.1. The initial values are now restricted to lie near the beginning of the axes.

**Fig. 3**: Same as in Fig.2. with positive initial velocities and equality of potential and kinetic energies.

**Fig. 4**: Same as in Fig.1. Supergravity corrections are now being taken into account.
Fig. 2
Fig. 3
