Probability prediction of the appearance of overload cycles of variable load case in parts of the aircraft chassis

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Abstract. Distribution function of the maximum values of voltage overload cycles in some aircraft parts is proposed. Based on the data from the flight experiment, more than 100 records of random load case processes were schematized and processed statistically. The proposed model of the distribution of maximum load cycles and processed experimental data has been verified.

1. Introduction
During operation, a large number of power parts of machines operating under conditions of variable loading has rare, but rather high overload cycles, which can significantly affect their bearing capacity. High peak loads can lead both to unacceptable local deformations of the structure and to the initiation of fatigue microcracks, which refers to the parts made of high-strength materials and having zones of high stress concentration as holes, grooves, dumbbell transitions, etc. It is very important to assess the possibility of these high loads when designing technological processes of surface hardening, including surface plastic deformation and technology for repair and restoration of the resource of parts.

The methods of surface plastic deformation, which are widely used in various engineering industry, increase the fatigue life of parts by creating material hardening in the surface layer, characterized by increase in its hardness and strength, favorable field of compressive residual stresses and changes in surface microrelief. Under the influence of cyclic loads, the technological residual stresses decrease and the more intensively, the greater the magnitude and duration of these loads [1]. Under the action of large peak load case, the residual compressive stresses can decrease by so much that the hardening effect becomes completely negligible or even zero. It is especially true for power parts made of high-strength materials, for which the effect of surface hardening in increasing fatigue strength is largely, and sometimes completely, determined precisely by the technological residual compressive stresses.

The studies are based on the results of processing and analysis of experimental data on the variable load case of the chassis parts of Russian transport aircraft. These data are obtained from a flight experiment based on strain gauging of typical parts of the main landing gears under various conditions and airplane movement regimes on the airfield. In total, more than 100 realizations of operational loading processes of individual parts of the chassis of 4 types of aircraft were recorded and processed.

2. Methodology
At the beginning of the processing, a randomization of loading processes of individual recorded implementations of parts was carried out according to the method of “maximizing” the stress
amplitudes. Their relative repetition frequencies were determined. The stress amplitudes are less than 0.2 of the fatigue strength limit, which obviously do not affect fatigue damage, were excluded from the variation series.

To describe the probability prediction of schematized process of variable load case of the power components of the aircraft landing gear, two-parameter exponential distribution law was proposed in [2]:

\[ F(\sigma_a) = 0 \text{ if } \sigma_a < b \gamma \]
\[ F(\sigma_a) = 1 - e^{-\frac{\sigma_a}{\gamma}} \text{ if } \sigma_a > b \gamma \]

with mathematical expectation \( M(\sigma_a) = \gamma (1 + b) \) and the variance \( S_{\sigma_a} = \gamma ; (\gamma \text{ and } b > 0) \) are the parameters of the distribution function.

As an example, in the figure 1 there are the histograms of the distribution of the schematized maximum stress amplitudes that are adequate for the actual implementation of the load case processes, and their theoretical distribution density, corresponding to formula (1), for the slot-joint shelf and the brake rod of the main landing gear of the aircraft.

![Figure 1](image)

**Figure 1.** Histograms of the distribution of stress amplitudes: a – slot-joint shelf (\( \gamma = 13.16; b = 19.72 \)); b – brake rod (\( \gamma = 32.29; b = 176.37 \)); 1 – theoretical distribution density; \( \gamma \) and b are the parameters of function (1).

Conformity assessment of the empirical distribution to the theoretical one was carried out according to the Kolmogorov criterion, selectively, for 25 realizations of the load case process. The calculations showed that for most implementations with a significance level of 0.2, the hypothesis that the empirical distribution of stress amplitudes corresponds to the two-parameter exponential distribution law is not inconsistent.

In forecasting problems of estimating the probability of occurrence of extreme values of random variables, it is most expedient, from the point of view of accuracy, to use the limiting distribution laws of the extreme members of a sequence of independent variables based on the statistical theory of the “weakest link” [3].

According to the theory of the “weakest link”, the maximum sample member \( \sigma_{max} \) will have a distribution defined by the following equality:

\[ F(\sigma_{max} < \sigma) = F^n(\sigma), \]
where \( F(\sigma) \) is the distribution function of the current voltage amplitudes; \( n \) is the volume (parameter) of the sample.

Substituting the value of the function \( F(\sigma) \) from (1) into (2), we obtain:

\[
F(\sigma_{max} < \sigma) = 1 - e^{-\frac{\sigma}{\gamma + b}}
\]  

(3)

It follows from expression (3) that for any constant value \( \sigma > \gamma b \) and for \( n \to \infty \),

\( F(\sigma_{max} < \sigma) \to 0 \) and \( F(\sigma_{max} \geq \sigma) \to 1 \).

It can be proved that the mathematical expectation of distribution (3) is approximately determined by the following expression:

\[
M \sigma_{max} \approx \gamma (1 + b) \ln n.
\]

Assuming that \( \sigma = \gamma (1 + b) \ln n + z \) (\( z \) can have any sign), from (3) we get the following:

\[
F[\sigma_{max} < \gamma (1 + b) \ln n + z] = \left[ 1 - e^{-\frac{(1+b) \ln n \cdot z}{\gamma + b}} \right]^n = \left[ 1 - \frac{e^{\frac{z}{\gamma + b}}}{n^{(1+b)}} \right]^n
\]

(4)

It is known that \( (1 - u/n)n \to e \to u \) for \( n \to \infty \), then under the same condition from expression (4), if \( \sigma > 0 \) and \( b > 0 \), we may have the following:

\[
F[\sigma_{max} < \gamma (1 + b) \ln n + z] \to e^{-\frac{z}{\gamma + b}}
\]

(5)

Introducing new notation, from the expression (5) we obtain the limiting distribution law of the maximum member of the variation series, expressed by the following formula

\[
F(\sigma_{max}) = e^{-e^{-\alpha(\sigma - \beta)}}
\]

(6)

where \( \alpha > 0 \) and \( \beta \) are the distribution parameters.

In order to determine the distribution law of the maximum values of the stress amplitudes in each implementation of the schematized load case process, one single maximum amplitude was selected, from the totality of them the variation series were formed separately for each part [4].

To estimate the distribution parameters (6) of \( \alpha \) and \( \beta \), the method proposed by Gumbel was used. This method consists in determining the parameters \( \alpha \) and \( \beta \) from a linear equation:

\[
Y[F_{em}(\sigma_{max})] = \alpha(\sigma - \beta)
\]

The values of \( Y \), called the normalized mode deviation, are determined by the double potentiating of function (6):

\[
Y[F_{em}(\sigma_{max})] = -\ln[-\ln F_{em}(\sigma_{max})]
\]

(7)

where \( F_{em}(\sigma_{max}) = (m - 0.5)/n \) is the empirical distribution frequency (\( m \) is the accumulated frequency; \( n \) is the total number of realizations under consideration for one part, that is, the volume of the variation series of values of maximum amplitudes voltage) [5].

Having determined by formula (7) a series of \( Y \), values corresponding to certain values of the maximum amplitudes \( \sigma_{max} \), the parameters \( \alpha \) and \( \beta \) are determined by the least squares method according to the following formulas:

\[
\alpha = \frac{n \sum_{i=1}^{n} \sigma_{max} Y_i - \sum_{i=1}^{n} \sigma_{max} \sum_{i=1}^{n} Y_i}{n \sum_{i=1}^{n} \sigma_{max}^2 - (\sum_{i=1}^{n} \sigma_{max})^2}
\]

\[
\beta = \frac{1}{\alpha} \left( \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} \sigma_{max} \right)
\]

(8)

For a preliminary assessment of the convergence of empirical and theoretical distribution functions, a correlation coefficient is determined, for its convenience of calculating the expression is used:
\[ R_{Y_i\sigma_{\text{max}}} = \alpha \sqrt{\frac{n \sum_{i=1}^{n} \sigma_{\text{max}_i}^2 - (\sum_{i=1}^{n} \sigma_{\text{max}_i})^2}{n \sum_{i=1}^{n} Y_i^2 (\sum_{i=1}^{n} Y_i)^2}} \tag{9} \]

3. Results

There are the values of the parameters \( \alpha \) and \( \beta \) and the correlation coefficient \( R_{Y_i\sigma_{\text{max}_i}} \) obtained by formulas (8) and (9) in Table 1.

| Chassis detail            | Number of implementations | Parameters of equation 2.9 | Correlation coefficient | Pearson criterion \( \chi^2 \) | \( f \) | \( \chi^2_{0.95} \) |
|---------------------------|---------------------------|-----------------------------|--------------------------|-------------------------------|-----|-------------------|
| Brake rod                 | 31                        | 0.02165                     | 297.0                    | 0.97                          | 6.7 | 11                | 12.6 |
| Beam of the cart          | 19                        | 0.06765                     | 382.6                    | 0.97                          | 10.8| 10                | 18.3 |
| Slot-join shelf           | 23                        | 0.01350                     | 102.5                    | 0.96                          | 19.5| 12                | 21.0 |
| Front strut               | 68                        | 0.05630                     | 130.0                    | 0.99                          | 5.9 | 3                 | 7.8  |

In figure 2, on the probability paper corresponding to the distribution law (6), there are the empirical and theoretical distributions of the maximum stress amplitudes on smooth parts.

![Figure 2](image)

Figure 2. Distribution of maximum stress amplitudes in chassis parts: 1) ○ - shelf of slot-join shelf; 2) □ - brake rod; 3) ● - beam of the cart; - - - theoretical distributions according to the law (6); ---- - confidence intervals if \( P_{\text{conf}}=0.95 \).

![Figure 3](image)

Figure 3. Histograms of the distribution of maximum stress amplitudes - 1 and theoretical distribution density - 2: a) for slot-join link; b) for brake rod.
A good agreement between empirical and theoretical distributions is evidenced by a close to unity value of the linear correlation coefficient $R_{Y_i, \sigma_{\max i}}$ and the proximity of location of empirical points and theoretical distributions in the figures 2 and 3.

Confidence intervals for the distribution functions from the Figure 2 are built on the basis of the method for assessing the agreement of empirical and theoretical dependences proposed in [6]:

$$S(\sigma_{\max i}) \approx \frac{\sqrt{n} \cdot S(Y_i)}{\sqrt{n} \cdot \alpha}$$

where the value $\sqrt{n} \cdot S(Y_i)$ is taken from the tables depending on the value of $Y_i$.

From the graph in the Figure 2, it follows that almost all empirical values (points) lie within confidence intervals corresponding to a significance level of 0.05, at which the deviation of the boundary of the confidence interval from the center is $\sim 1.96\sigma(Y_i)$.

More rigorous assessment of the adoption of hypothesis on correspondence of empirical distributions of maximum stress amplitudes of distribution function (6) was carried out according to the Pearson criterion [7]. In accordance with this criterion, the following value was calculated:

$$\chi^2 = \sum_{i=1}^{n} \frac{[n_i - n f_i(\sigma_{\max})]^2}{n \cdot f_i(\sigma_{\max})}$$

(10)

where $n_i$ is the empirical frequency of the maximum voltage amplitude, $\sigma_{\max i}$; $f_i(\sigma_{\max})$ is the distribution density determined by the following expression:

$$f(\sigma_{\max}) = \alpha e^{-\alpha(\sigma-\beta)} * e^{-e^{-\alpha(\sigma-\beta)}}$$

The values of $\chi^2$ obtained by formula (10), for a given confidence probability $p$, are compared with the tabulated value $\chi^2_{1-p}$. If $\chi^2 < \chi^2_{1-p}$ then the hypothesis that the empirical distributions correspond to theoretical functions can be accepted [8]. The results of calculations of the convergence estimation by the Pearson criterion given in the Table 1 indicate that, at a significance level of 0.05, the empirical distributions of the maximum values of voltage amplitudes can be described by function (6).

By the formula (11), it is possible to estimate the probability of voltage amplitude exceeding any given $\sigma_K$ and possible number of voltage overload cycles in details for a certain operation period [9].

$$F(\sigma_{\max} > \sigma_K) = 1 - e^{-e^{-\alpha(\sigma_K-\beta)}}$$

(11)

As an example, the brake draft of the chassis, made of 30HGSN2A steel with a tensile strength of 1650 MPa, with a fork end with two eyes, is considered. The values of the theoretical stress concentration coefficient of these eyes under tensile load are 3.2. Calculations by formula (11) showed that the probability of the appearance of peak stresses above the boundary value of 0.8 of the ultimate strength (i.e., more than 1320 MPa) is $\sim 0.0005$. At higher boundary stress values, the likelihood of overloads will naturally decrease [10]. Given the resource of modern transport aircraft should be at least 50 thousand flight hours or about 10 thousand flights, the structural and technological design should consider the possibility of the effect of rare reloading cycles on their bearing capacity.

4. Conclusions

Based on the studies, there are the following conclusions:

Proposed function (6) describes quite accurately the distribution of maximum over case voltage cycles in the power components of the aircraft chassis during operation.

In the structural and technological design of highly loaded parts operating under variable loading conditions and having significant stress concentrators, it is desirable to consider the likelihood of high overload cycles and the possibility of their influence on the bearing capacity, since in operation local stresses in high concentration zones can even reach slightly exceed the value of the conditional yield strength of the material.
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