Larmor precession and barrier tunneling time of a neutral spinning particle

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The Larmor precession of a neutral spinning particle in a magnetic field confined to the region of a one dimensional-rectangular barrier is investigated for both a nonrelativistic and a relativistic incoming particle. The spin precession serves as a clock to measure the time spent by a quantum particle traversing a potential barrier. With the help of general spin coherent state it is explicitly shown that the precession time is equal to the dwell time in both the nonrelativistic and relativistic cases. We also present a numerical estimation of the precession time showing an apparent superluminal tunneling.

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I. INTRODUCTION

The time spent by a quantum particle passing through a potential barrier has been one of the most controversial question since the founding of quantum physics and has attracted considerable attention from both a theoretical perspective and a experimental view\textsuperscript{1 2}. There are various approaches dealing with tunneling time\textsuperscript{3}, but there has been no clear-cut answer to this old question\textsuperscript{4 5}. Recently a number of experiments\textsuperscript{6 7 8} indicating superluminal transmission of photons has renewed interest in this subject.

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In quantum mechanics, time enters as a parameter rather than an observable. Thus, there is no direct way to calculate tunneling time. For particles with given energy there exist at least three different notions of the traversing time in the literature [9] [10] [11], i.e., the Wigner-time, the Büttiker-Landauer time and the Larmor-time, corresponding to different criteria. The Wigner time accounts for how long it takes for the peak of a wave packet to emerge from the exit face of the tunnel barrier relative to the time that the peak of the incident wave packet arriving at the entrance face. The calculation of the Wigner time is based on an asymptotic treatment of tunneling as a scattering problem utilizing the method of stationary phase to calculate the position of the peak of a wave packet. This tunneling time is simply the derivative of the phase of the tunneling amplitude with respect to the energy of the particle. Büttiker and Landauer consider the case that the height of the barrier or the amplitude of the incident wave is modulated sinusoidally in time. They have found that if the frequency of the modulation is very low, the tunneling particle will see the instantaneous height of the barrier and the transmitted waves adiabatically follow the modulation. However, as the frequency of the modulation increases, the transmitted waves will no longer be able to follow adiabatically the rapidly varying modulation. The Büttiker-Landauer time is the modulation period such that the transmitted wave begins to depart from an adiabatic following of the modulation.

Larmor precession was first introduced long ago as a thought experiment designed to measure the time associated with scattering events [12]. Subsequently the method was applied to measure the tunneling time of particles penetrating barrier with a magnetic field confined to the barrier region, causing the spin of particle to precess [13]. The original scheme [13] considered only the rotation of the spin in the plane that is perpendicular to the magnetic field. Later it was recognized that a particle tunneling through a barrier in the magnetic field does not actually perform a simple Larmor precession in a plane [11]. The main effect of the magnetic field is to align the spin with the field since the particle with spin parallel to the magnetic field has lower energy and less decay rate in barrier region than that of particle with spin antiparallel to the magnetic field. The total angular change of
the tunneling particle divided by the Larmor precession frequency is the Larmor time \[\tau_L\]. The literature also invokes a dwell time \[\tau_d\] defined as the ratio of integrated probability density over the barrier region to the incident flux. The dwell time measures how long the matter wave in the barrier regardless of whether the particle is reflected or transmitted.

In the present paper we revisit the Larmor precession of a neutral spinning particle with a general spin coherent state as a clock to measure the tunneling time through a barrier and extend for the first time the study of quantum tunneling to the relativistic regime. The advantage of using the spin coherent state is that an equation of motion for the expectation value of spin operator in the magnetic field within a barrier is obtained and identified with the equation of spin precession. With the help of the equation of motion we find that both nonrelativistic and relativistic neutral spin 1/2 particles perform a simple Larmor precession in three-dimensional space. The Lamor precession time of spin in a magnetic field confined to the potential barrier is compared with the time in the absence of barrier in order to show the apparent superluminal tunneling.

II. TUNNELING TIME FOR A NONRELATIVISTIC PARTICLE

Consider the one-dimensional rectangular potential penetration for a neutral particle of spin \(\frac{1}{2}\) with momentum \(p\) and mass \(m\). The Hamiltonian is

\[
H = \frac{p^2}{2m} + V_0 - \frac{\hbar \omega_L}{2} \sigma_3 \quad |x| < d
\]

\[
H = \frac{p^2}{2m} \quad |x| > d
\]

where \(V_0\) is the height of the barrier situated between -d and d, \(\omega_L = \frac{2\mu B}{\hbar}\) is the Larmor frequency and \(\mu, \hbar\) denote the magnetic moment and Planck constant, respectively. Here the time-independent magnetic field \(B\) is assumed in the z-direction and is confined to the barrier. \(\sigma_1, \sigma_2, \sigma_3\) are the Pauli spin matrices. The incoming wave is in the x-direction

\[
\psi_i = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} e^{i\kappa_0 x}
\]
where $\kappa_0 = \sqrt{\frac{2mE}{\hbar^2}}$ denotes the wave number and $E$ is the energy of the particle. The component $u_1$ ($u_2$) of the incoming wavefunction corresponds to spin up (down). We assume the incoming spinor is a normalized spin coherent state, which is the eigenstate of spin operator $\sigma \cdot n$ with unit eigenvalue [15].

$$\sigma \cdot n \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

(3)

where $n = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ is an arbitrary unit vector with a polar angle $\theta$ and azimuthal angle $\varphi$. The two components of the spinor are found to be

$$u_1 = \cos \frac{\theta}{2} e^{-i \frac{\varphi}{2}}$$

$$u_2 = \sin \frac{\theta}{2} e^{i \frac{\varphi}{2}}$$

(4)

We only consider the case of $E < V_0$ for quantum tunneling. The wave function to the left of the barrier ($x < -d$) is

$$\psi_1 = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} e^{i \kappa_0 x} + \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} e^{-i \kappa_0 x}$$

(5)

and the transmitted wave function to the right of the barrier ($x > d$) is

$$\psi_3 = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} e^{i \kappa_0 x}$$

(6)

In the barrier the wave function is

$$\psi_2 = \begin{pmatrix} B_1 e^{\kappa_1 x} \\ B_2 e^{\kappa_2 x} \end{pmatrix} + \begin{pmatrix} C_1 e^{-\kappa_1 x} \\ C_2 e^{-\kappa_2 x} \end{pmatrix}$$

(7)

where $\kappa_1, \kappa_2$ are given by

$$\kappa_1 = \sqrt{\frac{2m}{\hbar^2} (V_0 - E - \frac{\hbar \omega_L}{2})}$$

$$\kappa_2 = \sqrt{\frac{2m}{\hbar^2} (V_0 - E + \frac{\hbar \omega_L}{2})}$$

(8)
The coefficients $A_i$, $B_i$, $C_i$ and $D_i$ ($i = 1, 2$) in the wavefunction are obtained from boundary conditions $\psi_1(-d) = \psi_2(-d)$, $\psi_2(d) = \psi_3(d)$ and $\frac{d}{dx}\psi_1(x)|_{x=-d} = \frac{d}{dx}\psi_2(x)|_{x=-d}$, $\frac{d}{dx}\psi_2(x)|_{x=d} = \frac{d}{dx}\psi_3(x)|_{x=d}$,

\[
D_i = \sqrt{T_i} e^{i \phi_i} e^{-i2d\kappa_0 u_i} \\
A_i = \sqrt{R_i} e^{-i \frac{\kappa}{2} e^{i d\kappa_0 - d\kappa_i} D_i} \\
B_i = \frac{i\kappa_0 + \kappa_i}{2\kappa_i} e^{i d\kappa_0 + d\kappa_i} D_i \\
C_i = \frac{-i\kappa_0 + \kappa_i}{2\kappa_i} e^{i d\kappa_0 + d\kappa_i} D_i
\] (9)

where

\[
T_i = \frac{4\kappa_0^2 \kappa_i^2}{(\kappa_0^2 + \kappa_i^2)^2 \sinh^2(2d\kappa_i) + 4\kappa_0^2 \kappa_i^2} \\
R_i = \frac{(\kappa_0^2 + \kappa_i^2)^2 \sinh^2(2d\kappa_i)}{(\kappa_0^2 + \kappa_i^2)^2 \sinh^2(2d\kappa_i) + 4\kappa_0^2 \kappa_i^2} \\
\phi_i = \arctan\left(\frac{\kappa_0^2 - \kappa_i^2}{2\kappa_0 \kappa_i} \tanh(2d\kappa_i)\right)
\] (10)

For our purposes we shall consider the case of infinitesimal field limit, such that

\[
\kappa_1 \simeq \kappa - \frac{m\omega_L}{2\hbar\kappa}, \quad \kappa_2 \simeq \kappa + \frac{m\omega_L}{2\hbar\kappa}, \quad \kappa = \sqrt{\frac{2m}{\hbar^2} (V_0 - E)}
\] (11)

The transmission and reflection probabilities can be expanded as the power series of the small quantity $\frac{m\omega_L}{2\hbar\kappa}$. The first order approximation is

\[
T_1 = T(\kappa_1) \simeq T(\kappa) - \frac{\partial T}{\partial \kappa} \frac{m\omega_L}{2\hbar\kappa}, \quad T_2 = T(\kappa_2) \simeq T(\kappa) + \frac{\partial T}{\partial \kappa} \frac{m\omega_L}{2\hbar\kappa} \\
R_1 = R(\kappa_1) \simeq R(\kappa) - \frac{\partial R}{\partial \kappa} \frac{m\omega_L}{2\hbar\kappa}, \quad R_2 = R(\kappa_2) \simeq R(\kappa) + \frac{\partial R}{\partial \kappa} \frac{m\omega_L}{2\hbar\kappa}
\] (12)

and we have

\[
\sqrt{T_1 T_2} \simeq T(\kappa), \quad \sqrt{R_1 R_2} \simeq R(\kappa)
\] (13)

If we denote the transmission and reflection probabilities of spin up (down) by $T_+$ ($T_-$) and $R_+$ ($R_-$) respectively, then it can be easily shown that
\[ T_+ \equiv |D_1|^2 = T_1|u_1|^2, \quad T_- \equiv |D_2|^2 = T_2|u_2|^2 \]
\[ R_+ \equiv |A_1|^2 = R_1|u_1|^2, \quad R_- \equiv |A_2|^2 = R_2|u_2|^2 \]

\[ T_+ + R_+ = |u_1|^2, \quad T_- + R_- = |u_2|^2 \] (14)

\[ T_+ + R_+ + T_- + R_- = 1 \] (15)

which indicates the conservation of probability. From the viewpoint of scattering, the outgoing wave packet consists of both a reflected and a transmitted wave packets, which are separated from each other. The outgoing wave packet must be normalized to unity, since the incoming wave packet is normalized to unity.

The expectation values of spin for the transmitted wave \( \psi_t = (D_1) \) in the infinitesimal field limit are

\[ \langle S_1 \rangle_t = \frac{\hbar}{2} \frac{T(\kappa) \sin \theta \cos(\phi_2 - \phi_1 + \varphi)}{T(\kappa) - \frac{\partial T(\kappa)}{\partial \kappa} \frac{m \omega l}{2 \hbar} \cos \theta} \]
\[ \langle S_2 \rangle_t = \frac{\hbar}{2} \frac{T(\kappa) \sin \theta \sin(\phi_2 - \phi_1 + \varphi)}{T(\kappa) - \frac{\partial T(\kappa)}{\partial \kappa} \frac{m \omega l}{2 \hbar} \cos \theta} \]
\[ \langle S_3 \rangle_t = \frac{\hbar}{2} \frac{T(\kappa) \cos \theta - \frac{\partial T(\kappa)}{\partial \kappa} \frac{m \omega l}{2 \hbar} \cos \theta}{T(\kappa) - \frac{\partial T(\kappa)}{\partial \kappa} \frac{m \omega l}{2 \hbar} \cos \theta} \] (16)

The expectation values of spin for the reflected wave \( \psi_r = (A_1) \) in the infinitesimal field limit are

\[ \langle S_1 \rangle_r = \frac{\hbar}{2} \frac{R(\kappa) \sin \theta \cos(\phi_2 - \phi_1 + \varphi)}{R(\kappa) - \frac{\partial R(\kappa)}{\partial \kappa} \frac{m \omega l}{2 \hbar} \cos \theta} \]
\[ \langle S_2 \rangle_r = \frac{\hbar}{2} \frac{R(\kappa) \sin \theta \sin(\phi_2 - \phi_1 + \varphi)}{R(\kappa) - \frac{\partial R(\kappa)}{\partial \kappa} \frac{m \omega l}{2 \hbar} \cos \theta} \]
\[ \langle S_3 \rangle_r = \frac{\hbar}{2} \frac{R(\kappa) \cos \theta - \frac{\partial R(\kappa)}{\partial \kappa} \frac{m \omega l}{2 \hbar} \cos \theta}{R(\kappa) - \frac{\partial R(\kappa)}{\partial \kappa} \frac{m \omega l}{2 \hbar} \cos \theta} \] (17)

The second terms in the expectation values of the z-component of spin for both reflected and transmitted waves have an obvious interpretation as given in Ref.[11] that the particle with spin parallel to the magnetic field has lower energy and less decay rate in barrier region.
than that of particle with spin antiparallel to the magnetic field. Equations (16) and (17) show that the spin still performs a Larmor precession around the z-axis which cannot be realized using the special spin-polarization of an incoming particle with polar angle $\theta = \frac{\pi}{2}$ and azimuthal angle $\varphi = 0$. To see the spin precession explicitly we may take the sum of expectation values of spin components for the reflected and transmitted waves with an infinitesimal magnetic field, i.e. $\langle S_i \rangle = \langle \psi_t | S_i | \psi_t \rangle + \langle \psi_r | S_i | \psi_r \rangle$.

We have

$$\langle S_1 \rangle = \frac{\hbar}{2} \sin \theta \cos (\phi_2 - \phi_1 + \varphi)$$

$$\langle S_2 \rangle = \frac{\hbar}{2} \sin \theta \sin (\phi_2 - \phi_1 + \varphi)$$

$$\langle S_3 \rangle = \frac{\hbar}{2} \cos \theta$$

which are of the same form as for the Larmor precession of a spin in a uniform magnetic field. To see this let us consider a neutral particle in a uniform constant magnetic field $B$ along the z-direction in the absence of potential barrier. The Larmor precession is obtained by solving the Heisenberg equation

$$\frac{d}{dt} S(t) = \frac{1}{i\hbar} [S(t), H_s]$$

with the spin Hamiltonian

$$H_s = -\frac{1}{2} \hbar \omega_L \sigma_3$$

If the initial wave function is given by the spin coherent state i.e.

$$\psi_i = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

the expectation values of the spin components at time $t$ are

$$\langle S_1(t) \rangle = \frac{\hbar}{2} \sin \theta \cos (-\omega_L t + \varphi)$$

$$\langle S_2(t) \rangle = \frac{\hbar}{2} \sin \theta \sin (-\omega_L t + \varphi)$$

$$\langle S_3(t) \rangle = \frac{\hbar}{2} \cos \theta$$

(22)
Comparing Eqs.(18) and Eqs.(22) the Larmor tunneling time \( \tau_L \) is obviously obtained as

\[ \tau_L = \omega_L^{-1}(\phi_1 - \phi_2) \]  

(23)

Using Eq.(11) we expand \( \kappa_{1,2} \) in Eq.(10) up to the first order of the small quantity \( \frac{m\omega_L}{2\hbar c} \). the Larmor tunneling time is found to be

\[ \tau_L = \frac{m\kappa_0}{\hbar\kappa} \left( \frac{4d\kappa(\kappa^2 - \kappa_0^2) + (\kappa^2 + \kappa_0^2)\sinh(4d\kappa)}{4\kappa_0^2\kappa^2 + (\kappa^2 + \kappa_0^2)^2\sinh^2(2d\kappa)} \right) \]  

(24)

We assume that the incoming particle is a neutron with energy \( E \) and the width and height of the rectangular barrier are \( 2d=8\,\text{Å} \), \( V_0=470\text{MeV} \) respectively. The Larmor tunneling time as a function of the particle energy \( E \) is shown in Fig. 1(a). The peculiar feature, however, characteristic of Larmor tunneling time is that it increases with the energy of incoming particle monotonically in agreement with the observation in Ref. [16]. It is interesting to compare the Larmor tunneling time with the Larmor time of a neutron traversing a constant magnetic field \( B \) confined in region \(-d<x<d\), but without a barrier. With the same procedure as that for the case with a barrier we find that the transmission probability tends to one in the small field limit. The Larmor time of passage through the magnetic field region in the absence of a barrier is

\[ \tau_L^0 = \frac{2md}{\hbar\kappa_0} \]  

(25)

which is just the ratio of the traveling distance \( 2d \) of the spinning particle to its speed \( v = \frac{\hbar\kappa_0}{m} \). Using the same parameter as in Fig. 1(a), the plot of \( \tau_L^0 \) as a function of energy \( E \) is shown in Fig. 1(b). The ratio \( r = \frac{\tau_L}{\tau_L^0} \) can be smaller than one. In other word the speed of a neutron in a barrier is larger than that in the free space [17]. For the parameters chosen here, if the speed of incoming particle is one tenth of the speed of the light in the vacuum, the speed of particle though the barrier would be \( 5.7 \times 10^{15} \text{m/s} \).

The dwell time \( \tau_d \) is defined as the ratio of the probability \( P_b \) of finding a particle within the barrier to the incident probability flux \( J_i \)

\[ \tau_d = \frac{P_b}{J_i} \]  

(26)
The incident probability flux $J_i$ is

\[
J_i = -\frac{ih}{2m} (\psi_i^+ \nabla \psi_i - \psi_i \nabla \psi_i^+) = \frac{h\kappa_0}{m} \tag{27}
\]

and the probability for the particle to be in the barrier is

\[
P_b = \int_{-d}^{d} \psi_m^+ \psi_m dx = \kappa_0^2 \frac{4d\kappa (\kappa^2 - \kappa_0^2) + (\kappa^2 + \kappa_0^2) \sinh(4d\kappa)}{\kappa} \frac{4\kappa_0^2 \kappa^2 + (\kappa^2 + \kappa_0^2)^2 \sinh^2(2d\kappa)}{4d} \tag{28}
\]

The dwell time is found to coincide with the Larmor time exactly, $\tau_d = \tau_L$ in agreement with the result of Ref. \[11\] for the spin polarization perpendicular to the magnetic field. The identity $\tau_d = \tau_L$ has also been demonstrated in Ref. \[18\] for potential barrier of arbitrary shape.

### III. QUANTUM TUNNELING OF RELATIVISTIC PARTICLE

A relativistic neutral particle of spin $\frac{1}{2}$ with mass $m$ and magnetic moment $\mu$, moving in an external electromagnetic field denoted by the field strength tensor $F_{\mu\nu}$, is described by a four-component spinor wave function $\psi$ obeying the Dirac-Pauli equation

\[
[\gamma^\mu \frac{c}{i} \partial_\mu + mc^2 + \frac{1}{2} \mu \sigma_{\mu\nu} F_{\mu\nu}] \psi = 0 \tag{29}
\]

where $c$ is the velocity of light in vacuum, $\gamma^\mu = (\gamma^0, \gamma)$ are Dirac matrices satisfying

\[
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{30}
\]

with $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and

\[
\sigma_{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \tag{31}
\]

It can be shown that

\[
\frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu} = i\alpha \cdot E - \Sigma \cdot B \tag{32}
\]
where \( E \) and \( B \) are the external electric and magnetic fields, \( \alpha = \gamma^0 \gamma, \beta = \gamma^0 \). Here we make use of the Pauli representation

\[
\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \Sigma_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \quad i = 1, 2, 3
\]

and \( S_i = \frac{\hbar}{2} \Sigma_i \) is the spin operator.

We again consider the one-dimensional rectangular potential barrier. The magnetic field \( B \) assumed in the z-direction is confined within the barrier region. The Hamiltonian is seen to be

\[
H_D = c \alpha_1 p_x + \beta mc^2 \quad |x| > d
\]

\[
H_D = c \alpha_1 p_x + \beta [(mc^2 + V_0) - \frac{\hbar}{2} \omega_L \Sigma_3] \quad |x| < d
\]

When \( E < V_0 \), the wave function satisfying the stationary Dirac-Pauli equation

\[
H_D \psi = E \psi
\]

is

\[
\psi_1 = \frac{1}{\sqrt{1 + f_0^2}} \begin{pmatrix} u_1 \\ u_2 \\ f_0 u_2 \\ f_0 u_1 \end{pmatrix} e^{\frac{i k_0 x}{\hbar}} e^{-\frac{i k_1 x}{\hbar}} + \begin{pmatrix} A_1 \\ A_2 \\ -f_0 A_2 \\ -f_0 A_1 \end{pmatrix} e^{-\frac{i k_0 x}{\hbar}} e^{\frac{i k_1 x}{\hbar}} x < -d
\]

\[
\psi_2 = \begin{pmatrix} B_1 e^{\frac{k_1 x}{\hbar}} \\ B_2 e^{\frac{k_2 x}{\hbar}} \\ -if_2 B_2 e^{\frac{k_2 x}{\hbar}} \\ -if_1 B_1 e^{\frac{k_1 x}{\hbar}} \end{pmatrix} e^{-\frac{i k_0 x}{\hbar}} + \begin{pmatrix} C_1 e^{-\frac{k_1 x}{\hbar}} \\ C_2 e^{-\frac{k_2 x}{\hbar}} \\ if_2 C_2 e^{-\frac{k_2 x}{\hbar}} \\ if_1 C_1 e^{-\frac{k_1 x}{\hbar}} \end{pmatrix} e^{-\frac{i k_0 x}{\hbar}} |x| < d
\]

\[
\psi_3 = \begin{pmatrix} D_1 \\ D_2 \\ f_0 D_2 \\ f_0 D_1 \end{pmatrix} e^{\frac{i k_0 x}{\hbar}} e^{-\frac{i k_1 x}{\hbar}} x > d
\]
where

\[ f_0 = \frac{ck_0}{mc^2 + E} \]
\[ f_1 = \frac{ck_1}{mc^2 + V_0 + E + \mu B} \]
\[ f_2 = \frac{ck_2}{mc^2 + V_0 + E - \mu B} \]  \hfill (39)

and

\[ k_0 = \frac{1}{c} \sqrt{E^2 - (mc^2)^2} \]
\[ k_1 = \frac{1}{c} \sqrt{(mc^2 + V_0)^2 - (E + \mu B)^2} \]
\[ k_2 = \frac{1}{c} \sqrt{(mc^2 + V_0)^2 - (E - \mu B)^2} \]  \hfill (40)

The incoming wave, i.e. the first term on the right hand side of Eq.(36), is assumed to be a normalized spin coherent state as in Eq.(4). The coefficients \( A_i, B_i, C_i \) and \( D_i \) \((i = 1, 2)\) in the wavefunction are obtained from boundary conditions \( \psi_1(-d) = \psi_2(-d) \) and \( \psi_2(d) = \psi_3(d) \), namely,

\[ D_i = \sqrt{T_i} e^{i\phi_i} e^{\frac{i2dk_i}{\hbar}} u_i \]
\[ A_i = \sqrt{R_i} e^{-\frac{i\pi}{2}} e^{i\phi_i} e^{\frac{i2dk_i}{\hbar}} u_i \]
\[ B_i = \frac{if_0 + f_i}{2f_i} e^{\frac{i\hbar k_0 - dk_i}{\hbar}} D_i \]
\[ C_i = -\frac{if_0 + f_i}{2f_i} e^{\frac{i\hbar k_0 + dk_i}{\hbar}} D_i \]  \hfill (41)

where

\[ T_i = \frac{4f_0^3 f_i^2}{(1 + f_0^2)(f_0^2 + f_i^2)^2 \sinh^2(\frac{2dk_i}{\hbar}) + 4f_0^2 f_i^2} \]
\[ R_i = \frac{(f_0^2 + f_i^2)^2 \sinh^2(\frac{2dk_i}{\hbar})}{(1 + f_0^2)(f_0^2 + f_i^2)^2 \sinh^2(\frac{2dk_i}{\hbar}) + 4f_0^2 f_i^2} \]
\[ \phi_i = \arctan \left( \frac{f_0^2 - f_i^2}{2f_0 f_i} \tanh \frac{2dk_i}{\hbar} \right) \]  \hfill (42)

To our purpose we again consider the infinitesimal field limit
\[ k_1 \simeq k - \frac{E \hbar \omega_L}{2c^2 k}, \quad f_1 \simeq \frac{k}{\xi} - \frac{\hbar}{2c \xi} \left( \frac{k}{\xi} + \frac{E}{ck} \right) \omega_L \]
\[ k_2 \simeq k + \frac{E \hbar \omega_L}{2c^2 k}, \quad f_2 \simeq \frac{k}{\xi} + \frac{\hbar}{2c \xi} \left( \frac{k}{\xi} + \frac{E}{ck} \right) \omega_L \]  

where

\[ k = \frac{1}{c} \sqrt{(mc^2 + V_0)^2 - E^2}, \quad \xi = \frac{1}{c} (mc^2 + V_0 + E) \]

is the zero order approximation. The transmission and reflection probabilities can be expanded as the power series of the small quantity \( \frac{E \hbar \omega_L}{2c^2 k} \). The first order approximation is

\[ T_1 = T(k_1) \simeq T(k) - \frac{\partial T}{\partial k} \frac{E \hbar \omega_L}{2c^2 k}, \quad T_2 = T(k_2) \simeq T(k) + \frac{\partial T}{\partial k} \frac{E \hbar \omega_L}{2c^2 k} \]
\[ R_1 = R(k_1) \simeq R(k) - \frac{\partial R}{\partial k} \frac{E \hbar \omega_L}{2c^2 k}, \quad R_2 = R(k_2) \simeq R(k) + \frac{\partial R}{\partial k} \frac{E \hbar \omega_L}{2c^2 k} \]  

and we have

\[ \sqrt{T_1 T_2} \simeq T(k), \quad \sqrt{R_1 R_2} \simeq R(k) \]
\[ (1 + f_0^2)(T(k) + R(k)) = 1 \]

The transition and reflection probabilities of spin up and down are given respectively by

\[ T_+ \equiv (1 + f_0^2)|D_1|^2 = (1 + f_0^2)T_1|u_1|^2 \]
\[ T_- \equiv (1 + f_0^2)|D_2|^2 = (1 + f_0^2)T_2|u_2|^2 \]
\[ R_+ \equiv (1 + f_0^2)|A_1|^2 = (1 + f_0^2)R_1|u_1|^2 \]
\[ R_- \equiv (1 + f_0^2)|A_2|^2 = (1 + f_0^2)R_2|u_2|^2 \]  

We have probability conservation so

\[ T_+ + R_+ = |u_1|^2, \quad T_- + R_- = |u_2|^2 \]
\[ T_+ + R_+ + T_- + R_- = 1 \]

The expectation values of spin for the transmitted wave are obtained in the infinitesimal field limit as
\[ \langle S_1 \rangle_t = \frac{\hbar T(k)}{2} \sin \theta \cos(\phi_2 - \phi_1 + \varphi) \]
\[ \langle S_2 \rangle_t = \frac{\hbar (1 - f_0^2) T(k)}{2} \sin \theta \sin(\phi_2 - \phi_1 + \varphi) \]
\[ \langle S_3 \rangle_t = \frac{\hbar (1 - f_0^2) T(k)}{2} \cos \theta \]

The reflected part reads
\[ \langle S_1 \rangle_r = \frac{\hbar R(k)}{2} \sin \theta \cos(\phi_2 - \phi_1 + \varphi) \]
\[ \langle S_2 \rangle_r = \frac{\hbar (1 - f_0^2) R(k)}{2} \sin \theta \sin(\phi_2 - \phi_1 + \varphi) \]
\[ \langle S_3 \rangle_r = \frac{\hbar (1 - f_0^2) R(k)}{2} \cos \theta \]

The sum of expectation values of spin components for the reflected and transmitted waves with an infinitesimal magnetic field is
\[ \langle S_1 \rangle = \frac{\hbar}{2} \sin \theta \cos(\phi_2 - \phi_1 + \varphi) \]
\[ \langle S_2 \rangle = \frac{\hbar (1 - f_0^2)}{2} \sin \theta \sin(\phi_2 - \phi_1 + \varphi) \]
\[ \langle S_3 \rangle = \frac{\hbar (1 - f_0^2)}{2} \cos \theta \]

which are formally the same as the Larmor precession equation of spin \( \Sigma \) in a magnetic field. To see this we solve the Heisenberg equation (19) with the Hamiltonian
\[ H_s = -\frac{1}{2} \hbar \omega_L \beta \Sigma_3 \]
and the initial wave function
\[ \psi_i = \frac{1}{\sqrt{1 + f_0^2}} \begin{pmatrix} u_1 \\ u_2 \\ f_0 u_1 \\ f_0 u_2 \end{pmatrix} \]
The expectation values of the spin components at time $t$ is

\[
\langle S_1(t) \rangle = \frac{\hbar}{2} \sin \theta \cos(-\omega_L t + \varphi)
\]

\[
\langle S_2(t) \rangle = \frac{\hbar}{2} \frac{1 - f_0^2}{1 + f_0^2} \sin \theta \sin(-\omega_L t + \varphi)
\]

\[
\langle S_3(t) \rangle = \frac{\hbar}{2} \frac{1 - f_0^2}{1 + f_0^2} \cos \theta
\]

(56)

Using the approximation Eq.(43) for $k_1, k_2$ in Eq.(39) and Eq.(42), the Larmor tunneling time which is defined by $\tau_L = \frac{\phi_1 - \phi_2}{\omega_L}$ is obtained as

\[
\tau_L = \frac{f_0}{c^2 k_0} \frac{4dkE(k^2 - f_0^2) + \hbar (ck^2 + E\xi)(k^2 + f_0^2) \sinh\left(\frac{4dk}{\hbar}\right)}{4f_0^2 \xi^2 k^2 + (k^2 + f_0^2)^2 \sinh^2\left(\frac{2dk}{\hbar}\right)}
\]

(57)

For a relativistic neutron and a rectangular potential barrier of width $2d=8\,\text{Å}$ and height $V_0=6000\,\text{MeV}$, the Larmor tunneling time as a function of kinetic energy $E_k$, which in relativistic case is defined as the total energy $E$ minus the static energy $mc^2$, is shown in Fig. 2(a) which is similar to the nonrelativistic case (Fig. 1(a)) except the time scale. The Larmor time through the magnetic field region in the absence of a barrier is

\[
\tau_L^0 = \frac{2dE}{c^2 k_0}
\]

(58)

which also is exactly the ratio of the traveling distance $2d$ to speed $v = c\sqrt{1 - \left( \frac{mc^2}{E} \right)^2}$.

Using the same parameters as Fig. 2(a), $\tau_L^0$ the function of the particle kinetic energy $E_k$ is plotted in Fig. 2(b). For the parameters chosen here, if the speed of incoming particle is $2.9 \times 10^8 \,\text{m/s}$, the speed of particle tunneling through the barrier would be $6.4 \times 10^{15} \,\text{m/s}$. It is interesting to see the difference numerically between the non-relativistic expression (24) and the relativistic formula (61). To this end, we plot the results from both expressions in Fig.(3) with the same kinetic energy of the incoming particles.

In the Dirac theory, the incident probability flux $J_i$ is

\[
J_i = \psi_i^+ c \alpha_1 \psi_i = \frac{2cf_0}{1 + f_0^2}
\]

(59)

and the probability for the particle to be in the barrier is
\[
P_b = \int_{-d}^{d} \psi_m^+ \psi_m dx
\]
\[
= \frac{f_0^2}{4} \left[ -4dk(k^2 - \xi^2)(k^2 - f_0^2 \xi^2) + h(k^2 + \xi^2)(k^2 + f_0^2 \xi^2) \sinh\left(\frac{4dk}{\hbar}\right) \right]
\frac{(1 + f_0^2)k[4f_0^2 k^2 \xi^2 + (k^2 + f_0^2 \xi^2)^2 \sinh^2\left(\frac{2dk}{\hbar}\right)]}{(1 + f_0^2)k[4f_0^2 k^2 \xi^2 + (k^2 + f_0^2 \xi^2)^2 \sinh^2\left(\frac{2dk}{\hbar}\right)]}
\]
(60)

The dwell time \( \tau_d \) is the ratio of Eq. (60) to Eq.(59),
\[
\tau_d = \frac{f_0}{2ck} \frac{-4dk(k^2 - \xi^2)(k^2 - f_0^2 \xi^2) + h(k^2 + \xi^2)(k^2 + f_0^2 \xi^2) \sinh\left(\frac{4dk}{\hbar}\right)}{4f_0^2 k^2 \xi^2 + (k^2 + f_0^2 \xi^2)^2 \sinh^2\left(\frac{2dk}{\hbar}\right)}
\]
(61)

Using the relation
\[
E = \frac{c\xi^2 - ck^2}{2\xi}
\]
(62)

it is obvious that the dwell time Eq.(61) equals exactly the Larmor time Eq.(57).

**IV. CONCLUSION**

Using spin coherent state of an incoming particle we show that a neutral spinning particle penetrating the potential barrier with a constant magnetic field gives rise to a Larmor precession from which the barrier interaction time i.e. a time length for particle to remain in the barrier is determined. The Larmor time coincides with the dwell time in both the nonrelativistic and relativistic cases. The numerical calculation shows that the Larmor tunneling time can be much smaller than the time that the particle penetrates a constant magnetic field without a barrier, which implies the apparent superluminal tunneling.

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**Figure Caption:**

Fig.1(a) Larmor time calculated from the non-relativistic expression Eq.(24) in a barrier as a function of the particle energy with $2d = 8\,\text{Å}$, $m = 1.67 \times 10^{-27}\,\text{kg}$ and $V_0 = 470\,\text{MeV}$.

Fig.1(b) Larmor time of Eq.(25) without a barrier as a function of the particle energy with $2d=8\,\text{Å}$ and $m = 1.67 \times 10^{-27}\,\text{kg}$.

Fig.2(a) Larmor time for the relativistic case Eq.(57) in a barrier as a function of the particle kinetic energy with $2d = 8\,\text{Å}$, $m = 1.67 \times 10^{-27}\,\text{kg}$ and $V_0 = 6000\,\text{MeV}$.

Fig.2(b) Larmor time of Eq.(58) without a barrier as a function of the particle kinetic energy with $2d=8\,\text{Å}$ and $m = 1.67 \times 10^{-27}\,\text{kg}$.
Fig.3 Larmor times as a function of the kinetic energy for both the non-relativistic (dotted line) and relativistic (solid line) particles with the same barrier width $2d=8\text{Å}$ but various heights: (a) $V_0=1000\text{MeV}$, (b) $V_0=3000\text{MeV}$, (c) $V_0=8000\text{MeV}$.
FIG. 1
FIG. 2
FIG. 2
(a)

FIG. 3
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{(b)}
\end{figure}

\textbf{FIG. 3}
FIG. 3