Resource Allocation for URLLC and eMBB Traffic in Uplink Wireless Networks

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Abstract—In this paper we consider two resource allocation problems of URLLC traffic and eMBB traffic in uplink 5G networks. We propose to divide frequencies into a common region and a grant-based region. Frequencies in the grant-based region can only be used by eMBB traffic, while frequencies in the common region can be used by eMBB traffic as well as URLLC traffic. In the first resource allocation problem we propose a two-player game to address the size of the grant-based region and the size of the common region. We show that this game has specific pure Nash equilibria. In the second resource allocation problem we determine the number of packets that each eMBB user can transmit in a request-grant cycle. We propose a constrained optimization problem to minimize the variance of the number of packets granted to the eMBB users. We show that a water-filling algorithm solves this constrained optimization problem. From simulation, we show that our scheme, consisting of resource allocation according to Nash equilibria of a game, persistent random retransmission of URLLC packets and allocation of eMBB packets by a water-filling algorithm, works better than four other heuristic methods.

keywords: wireless networks, resource allocation, game theory, constrained optimization, water-filling algorithm

1 INTRODUCTION

The fifth-generation networks (5G) and beyond aim to cover three generic connectivity types: (i) enhanced mobile broadband (eMBB), (ii) ultra-reliable low-latency communications (URLLC), and (iii) massive machine-type communications (mMTC) (see, e.g., [1], [2], [3] and references therein). The reliability defined in 3GPP for supporting URLLC services, such as autonomous driving, drones, and augmented/virtual reality, requires a 1 − 10^{-5} success probability of transmitting a layer 2 packet of length 32 bytes within 1 millisecond. mMTC services are characterized by a large number of simple devices. Motivated by these emerging needs in 5G, research communities in wireless networking commonly believe that grant-based communications continue to be feasible for eMBB services, but multiple access schemes in a grant-free manner are more suitable for URLLC and mMTC services. Many multiple access schemes have been proposed in the literature recently. We refer the readers to [4], [5] for references. In this paper we propose a method to deliver URLLC packets reliably with their latency bounds, and to schedule eMBB packets in a fair manner.

Providing satisfactory services to ultra-reliable low-latency communications (URLLC) traffic and enhanced mobile broadband (eMBB) traffic simultaneously is an interesting and challenging research problem. These two traffic types are characterized by very different service requirements. URLLC traffic demands a latency as low as 1 millisecond and a packet loss probability as low as 10^{-5}. On the other hand, an eMBB user cares very much about its throughput and also the fairness in the throughput among all eMBB users. eMBB services can be efficiently supported by a request and grant paradigm. However, a grant-based service is not likely to meet the stringent latency requirement of URLLC services. A grant-free paradigm is more likely to satisfy the latency requirement of URLLC users. Non-Orthogonal Multiple Access (NOMA) offers a good solution to the problem above [7]. The whole frequency band can be used for both grant-based eMBB services and grant-free URLLC services [8], [9], [10], [11], [12]. Using NOMA techniques, one can divide the frequency band into a grant-based region and a common region in uplink 5G networks. Using wireless resource blocks in the grant-based region, eMBB users make requests, receive grants, and transmit packets according to the grants. Properly designed resource allocation algorithms can ensure that eMBB users receive their services in a fair manner. Using wireless resource blocks in the common region, URLLC users sporadically transmit packets in a grant-free manner. eMBB users also can be granted packets in the common region. If so, eMBB users transmit their granted packets at a smaller power in the common region. If eMBB packets collide with URLLC packets in the common region, the eMBB packets hopefully can be recovered by the interference cancellation technique [5].

In this paper we propose to divide wireless resource blocks into a common region and a grant-based region. URLLC packets are transmitted sporadically in a grant-free manner in the common region. We propose a persistent retransmission scheme for the URLLC packets to cope with their stringent latency requirements. One of the contributions of this paper is that we analyze the probability that an URLLC packet fails to meet its latency requirement. In this paper we study two resource allocation problems. In the first
problem, we determine the size of the common region and the size of the grant-based region. We propose to solve this problem by formulating a two-player game. The two players of the game agent URLLC and agent eMBB, who negotiate resource blocks on behalf of all URLLC users and eMBB users, respectively. The payoff of the URLLC agent depends on the probability that an URLLC packet fails to meet its latency requirement. The second contribution of this paper is that we analyze the Nash equilibria of this game. eMBB users can use resource blocks in the grant-based region as well as in the common region to transmit their packets. The transmissions are carried out in a request and grant manner. In the second resource allocation problem, we study the allocation of the overall bits that can be transmitted in a request-grant cycle to individual eMBB users. We solve this problem by formulating a constraint optimization problem to minimize the variance of the number of bits transmitted by the eMBB users. The third contribution of this paper is that we show a water-filling algorithm, solves the nonlinear program. From simulation, we show that our scheme, consisting of resource allocation according to Nash equilibria of a game, persistent random retransmission of URLLC packets and allocation of eMBB packets by a water-filling algorithm, works better than four other heuristic methods.

The outline of this paper is as follows. In Section 2 we present the frame structure of the wireless uplink transmission system. In Section 3 we present a random persistent retransmission scheme for the URLLC packets. We present an analysis of the probability that an URLLC packet fails to be transmitted successfully in this section. In Section 4 we describe a resource block assignment game to allocate the number of resource blocks in the common region and that in the grant-based region. In Section 5 we describe a constrained optimization problem of the eMBB users. We present numerical and simulation results in Section 6. Finally, we conclude this paper in Section 7.

2 RESOURCE BLOCKS AND FRAME STRUCTURE

In this paper we propose that the frequency band is divided into a common region and a grant-based (GB) region in Fig. 1. We assume that URLLC packets can only be transmitted in the common region in a grant-free manner. The eMBB users make transmission requests. The eMBB users can use resource blocks in both the grant-based region and the common region to transmit their granted packets. Since URLLC packets are transmitted in the common region in a grant-free manner, it is possible that URLLC packets and eMBB packets are collided in the common region. We assume that eMBB packets are transmitted at a smaller power, so that URLLC packets are likely to be received successfully. Then, successive interference cancellation techniques are applied to recover the collided eMBB packets.

In this paper we assume that time is discrete and is divided into time slots. Multiple time slots are grouped into a time frame. A time frame is divided into two periods, including a control period and a transmission period. In control periods, eMBB users make transmission requests. In transmission periods, eMBB users transmit their data packets that are granted by a base station. In the control period, the base station also announces how resource blocks are divided into a common region and a grant-based region. In each time slot, one eMBB packet can be transmitted. It has been widely recognized that conventional automatic repeat request (ARQ) mechanisms such as acknowledgments or timeouts cannot solve the reliability issue of URLLC traffic due to its stringent delay requirement. To cope with the ultra stringent latency requirement, it has been proposed that time slots are divided into mini time slots [13], [14], [15]. An URLLC packet can be transmitted in one mini time slot. We assume that a time slot is divided into τ mini time slots. A typical length of a time slot is 1 millisecond (ms), and a typical value for τ is 8. We show time frames, time slots and mini time slots in Fig. 2.

We summarize in Table 1 some symbols used in this paper.

3 RETRANSMISSION SCHEME FOR URLLC TRAFFIC

URLLC traffic is characterized by its ultra stringent delay requirement and extremely high reliability requirement. An URLLC packet needs to be delivered to its destination within 1 millisecond with a success probability higher than
1 − 10⁻⁵. The ultra low latency requirement makes grant-based transmission mode infeasible for URLLC services. It is commonly believed that grant-free transmission is suitable for URLLC services [16].

The traditional semi-persistent scheduling (SPS) scheme [4] pre-schedules resources before sessions begin. SPS would be suitable for URLLC traffic if the traffic were periodic and predictable. However, URLLC traffic is expected to be sporadic. SPS could be very inefficient in resource utilization. It has been widely recognized that conventional automatic repeat request (ARQ) mechanisms such as acknowledgments or timeouts cannot solve the reliability issue of URLLC traffic due to its stringent delay requirement [13], [14], [15]. Packets must be retransmitted persistently without waiting for their negative acknowledgments or expiration of timeout clocks. That is, each URLLC packet will be transmitted multiple numbers of times. An URLLC packet is successfully received if at least one copy is received successfully. This approach has been adopted by the 3GPP standard [14]. However, the multiple access nature of the uplink communication makes it possible that multiple URLLC users attempt to transmit their packets using the same resource block in the same mini time slot. A collision thus can happen. A persistent transmission scheme would imply that all subsequent retransmissions are also collided. In this paper we propose a randomized persistent transmission scheme for URLLC traffic. Suppose that up to 𝜏 copies of each URLLC packet are transmitted. The first copy is transmitted at the time when it arrives. Instead of repeating the transmission of the remaining 𝜏 − 1 copies, one in each of the 𝜏 − 1 subsequent mini time slots, each copy is transmitted with probability 𝑝. With probability 1 − 𝑝, the URLLC user does not transmit a copy. This mechanism can reduce the number of subsequent collisions and let the collided URLLC users have a chance to successfully transmit their packets. Fig. 2 contains a graphical illustration. A time slot is divided into several mini time slots. On the upper side of Fig. 2 the two URLLC users attempt to transmit packets. Original packet transmissions are shown by colored shapes, and retransmissions are shown by uncolored shapes. Both of the two URLLC users transmit in the first mini time slot of a specific time slot using the first resource block. They persistently and randomly retransmit. The first user (shown in rectangle) successfully retransmits its packet in the second retransmission attempt. The second user (shown in circle) successfully retransmits its packet in the third retransmission attempt.

We now present an analysis of the probability that a tagged URLLC packet cannot be successfully transmitted. Suppose that there are 𝑛₁ resource blocks in the common region. Assume that there are 𝑛 URLLC users. Assume that URLLC packets arrive according to a Poisson process with rate 𝜌 packets per mini time slot. Assume that each URLLC user randomly chooses a resource block to transmit its packet. It follows that the URLLC packet arrivals to a specific resource block, say block 𝑗, are Poisson with rate 𝜌/𝑛₁ packets per mini time slot. Let 𝜀 = 𝜌/𝑛₁ be the traffic intensity per resource block. We consider a specific packet called tagged packet. We label the mini time slot that the tagged URLLC packet arrives as mini time slot 0. The sender of this tagged packet continuously attempts to retransmit the packet with probability 𝑝 in each of the next 𝜏 − 1 mini time slots. The sender does not retransmit the packet with probability 1 − 𝑝 in each of the next 𝜏 − 1 mini time slots. Let 𝐸_{𝑛₁} denote the event that the tagged URLLC packet cannot be successfully transmitted given that the common region has 𝑛₁ resource blocks.

We now analyze 𝑃(𝐸_{𝑛₁}). Recall that we denote the URLLC packet arrival rate to a resource block by 𝜀 = 𝜌/𝑛₁. We also simply denote the event that the tagged packet fails to be successfully transmitted by 𝐸, rather than 𝐸_{𝑛₁}. Let 𝐺_{𝑖} be the event that the tagged packet is not successfully transmitted in mini time slot 𝑖 for 𝑖 = 0, 1, ..., 𝜏 − 1. Then,

\[ E = G_0 \cap G_1 \cap \cdots \cap G_{\tau - 1}. \]

Let \( X_j \) be the number of arrivals in mini time slot \( j \) not including the tagged packet for \( j = -(\tau - 1), -(\tau - 1) + 1, \ldots, 0, 1, \ldots, \tau - 1 \). Let \( A \) denote the event that \( X_j = x_j \) for \( -(\tau - 1) \leq j \leq \tau - 1 \). That is,

\[ A = \{ X_j = x_j, -(\tau - 1) \leq j \leq \tau - 1 \}. \]

Conditioning on event \( A \), events \( G_0, G_1, \ldots, G_{\tau - 1} \) are independent. By the law of total probability and conditional independence, one has

\[
P(E) = \sum_{x_j=0}^{\infty} P \left( \bigcap_{i=0}^{\tau-1} G_i \middle| A \right) \cdot P(A)
\]

\[
= \sum_{x_j=0}^{\infty} \prod_{i=0}^{\tau-1} P(G_i \mid A) \cdot P(A),
\]

where

\[
P(A) = P(X_j = x_j, -(\tau - 1) \leq j \leq \tau - 1)
\]

\[
= \prod_{j=-(\tau-1)}^{\tau-1} \frac{e^{-\epsilon} \epsilon^{x_j}}{x_j!}.
\]

Now we derive the conditional probabilities in (1). If there are no new arrivals in mini time slot 0, the tagged packet cannot be successfully transmitted. Assume that there are no new arrivals in mini time slot 0, the tagged packet still cannot be successfully transmitted if there is at least

| Symbol | Definition |
|--------|------------|
| 𝜌     | URLLC packet arrival rate |
| 𝜏     | Delay requirement of an URLLC packet |
| 𝑝     | URLLC packet retransmission probability |
| 𝑁     | Total number of resource blocks |
| 𝑒     | Upper bound of the URLLC packet loss probability |
| \( E_k \) | Event that a tagged URLLC packet cannot be successfully transmitted given that the common region has \( k \) resource blocks |
| 𝑏     | Cost of resource blocks |
| 𝑚     | Number of eMBB users |
| 𝑐     | Number of bits that the eMBB users can transmit using one resource block in the grant-based region |

TABLE 1 Definition of parameters used in this paper.
one arrival in mini time slots $-(\tau - 1),-(\tau - 1) + 1,\ldots ,-1$ that is retransmitted in mini time slot 0. Combining these possibilities, we have

$$P(G_0 | A) = 1_{[x_0=1]} + 1_{[x_0=0]} \left( 1 - (1 - p)\sum_{i=-(\tau - 1)}^{1} x_i \right)$$

$$= 1 - 1_{[x_0=0]}(1 - p)\sum_{i=-(\tau - 1)}^{1} x_i ,$$

where notation $1_{[A]}$ is an indicator function of event $A$. The value of $1_{[A]}$ is equal to 1 if event $A$ is true, and is equal to 0 otherwise. For $i = 1, 2, \ldots , \tau - 1$, we have

$$P(G_i | A) = (1 - p) + p \left[ 1_{[x_i=1]} + 1_{[x_i=0]} \left( 1 - (1 - p)\sum_{j=-i-1}^{1} x_j \right) \right]$$

$$= 1 - p \cdot 1_{[x_i=0]}(1 - p)\sum_{j=-i-1}^{1} x_j .$$

(3)

In (3), we consider a specific case, in which $\tau = 3$. That is, each arrival has two opportunities to be retransmitted. Substituting Eqs. (3) and (4) into (1), we obtain an expression for $P(E)$. For general values of $\tau$, we are not able to obtain a closed form expression for $P(E)$. However, we show in the next proposition that $P(E)$ is an increasing function of $\bar{\rho}$. This result is used to study the Nash equilibria of a resource allocation game in Section 3. The proof is presented in Appendix A.

**Proposition 1.** For any $p$ and $\tau$, $P(E)$ is increasing with respect to $\bar{\rho}$.

To proceed further, we consider a specific case, in which $\tau = 3$. That is, each arrival has two opportunities to be retransmitted. Substituting Eqs. (3) and (4) into (1), we have

$$P(G_0 | A)P(G_1 | A)P(G_2 | A)$$

$$= \left( 1 - 1_{[x_0=0]}(1 - p)^{2+\tau x_1} \right)$$

$$\cdot \left( 1 - p \cdot 1_{[x_1=0]}(1 - p)^{2+\tau x_2} \right)$$

$$\cdot \left( 1 - p \cdot 1_{[x_2=0]}(1 - p)^{2+\tau x_3} \right).$$

(5)

In addition, from the independent increment property of Poisson process, we have

$$P(A) = \sum_{j=-2}^{2} e^{-\bar{\rho}}\frac{\bar{\rho}^{|j|}}{|j|!}.$$  

(6)

Substituting (5) and (6) into (1) and simplifying, we obtain

$$P(E) = 1 - (1 + 2p) \cdot \exp(-3\bar{\rho} + 2(1 - p)\bar{\rho})$$

$$+ p(1 + \bar{\rho}) \cdot \exp(-4\bar{\rho} + (1 - p)\bar{\rho} + (1 - p)^2\bar{\rho})$$

$$+ p^2 \cdot \exp(-5\bar{\rho} + (1 - p)\bar{\rho} + (1 - p)^2\bar{\rho}).$$

(7)

A Taylor expansion of (7) around $\bar{\rho} = 0$ is

$$P(E) \approx (1 - p^2 + p^3)\bar{\rho} + o(\bar{\rho}^2).$$

(8)

For general values of $\tau$, it is difficult to evaluate (1). Since URLLC traffic requires a very low packet loss probability of the order $10^{-5}$, the system must operate in a light traffic condition. We now propose a light traffic approximation for $P(E)$ with general values of $\tau$. It is interesting to point out that the order of this light traffic approximation with $\tau = 3$ agrees with the Taylor expansion in (8).

We consider a light traffic model, in which excluding the tagged packet in mini time slot 0, in mini time slots in range $[-(\tau - 1), \tau - 1]$, there is either exactly one packet arrival, or there are no packet arrivals. With probability $1 - (2\tau - 1)\bar{\rho}$ there are no additional arrivals. With probability $(2\tau - 1)\bar{\rho}$, there is exactly one arrival. If there is exactly one arrival, this packet arrives in any one of the $2\tau - 1$ mini time slots in range $[-(\tau - 1), \tau - 1]$ with an equal probability. We now derive $P(E)$ for this light traffic model. Let $H_i$ be the event that a packet arrives in mini time slot $i$, where $-(\tau - 1) \leq i \leq \tau - 1$.

$$P(E | H_i) = 0.$$  

(9)

If $-(\tau - 1) \leq i \leq -1$,

$$P(E | H_i) = p(1-p+\rho \cdot p)\tau+i-1 \cdot (1-p)^i.$$  

(10)

If $i = 0$,

$$P(E | H_0) = (1 - p + p \cdot \bar{\rho})\tau - 1.$$  

(11)

For $i$ in range $[-(\tau - 1), \tau - 1]$,

$$P(H_i) = \frac{(\tau - 1)\bar{\rho}}{2\tau - 1} = \hat{\rho}.$$  

(12)

Let $H^c$ denote the event that there are no additional packet arrivals. Then,

$$P(E | H^c) = 0$$

$$P(H^c) = 1 - (2\tau - 1)\bar{\rho}.$$  

(13)

By the law of total probability and Eqs. (9) - (13), we have

$$P(E) = \sum_{i=-(\tau - 1)}^{\tau - 1} P(E | H_i)P(H_i)$$

$$+ P(E | H_0)P(H_0) + P(E | H^c)P(H^c)$$

$$= \sum_{i=-(\tau - 1)}^{\tau - 1} p(1-p+p^2)\tau+i-1(1-p)^i$$

$$+ (1-p+p^2)\tau - 1\hat{\rho}.$$  

(14)

In a special case where $\tau = 3$, (14) agrees with (8).

4 A TWO-PLAYER GAME

In this section we propose a two-step procedure to the resource block allocation to URLLC users and eMBB users. We first apply the Nash equilibria of a two-player game to determine the number of resource blocks in the common region and that in the grant-based region. We then apply the solution of a constrained nonlinear optimization problem to allocate the number of bits granted to the eMBB users. We explain the two-player game in this section. We present the constrained nonlinear optimization problem in Section 5.

We now explain a two-player resource allocation game. The two players are an URLLC agent and an eMBB agent, negotiating resource blocks on behalf of URLLC users and eMBB users, respectively. The URLLC agent negotiates the number of resource blocks in the common region. The eMBB agent negotiates the number of resource blocks in the grant-based region. Assume that there are $n$ URLLC users and $m$
eMBB users. Assume that there are totally $N$ resource blocks to be allocated. The set of actions for the URLLC agent is $\{0,1,\ldots,N\}$. The eMBB agent has the same set of actions. When the URLLC agent takes action $n_1$ and the eMBB agent takes action $n_2$, where $0 \leq n_1,n_2 \leq N$, the payoff to the URLLC agent is denoted by $P_{\text{URLLC}}(n_1,n_2)$ and the payoff to the eMBB agent is denoted by $P_{\text{eMBB}}(n_1,n_2)$. Recall that $E_k$ denotes the event that a tagged URLLC packet cannot be successfully transmitted given that the common region has $k$ resource blocks. From the light traffic approximation of [14], we have

$$P(E_k) = \begin{cases} \sum_{i=-(k-1)}^{-1} p(1-p+p^2)^{i-1}(1-p)^{-i} & \text{if } k \geq 1 \\ +(1-p+p^2)^{k-1} & \text{if } k = 0. \end{cases}$$ (15)

We assume that the URLLC agent’s payoff is

$$P_{\text{URLLC}}(n_1,n_2) = 1[p(E_{\min(n_1,N-n_2)} \leq \epsilon) \cdot 1[n_1+n_2 \leq N] - \frac{n_1}{N} \cdot b],$$ (16)

where $\epsilon$ is an upper bound of the URLLC packet loss probability, and $b$ is a cost of using $N$ resource blocks. We assume that

$$0 < b < 1.$$ (17)

Suppose that an eMBB user can transmit $c$ bits using one resource block in the grant-based region in a time frame. If action profile $(n_1,n_2)$ is taken by the two players, the grant-based region has $\min(n_2,N-n_1)$ resource blocks that are not overlapped with resource blocks in the common region. Thus, the eMBB users can transmit $\min(n_2,N-n_1) \cdot c$ bits in this region. On the other hand, the common region has $\min(n_1,N-n_2)$ resource blocks that are not overlapped with resource blocks in the grant-based region. We assume that the eMBB users transmit at a smaller power, and their packets are recovered by interference cancellation if they are collided with URLLC packets. Thus, the number of bits that the eMBB users can transmit in this region is $a \cdot \min(n_1,N-n_2) \cdot c$, where $0 < a < 1$. Parameter $a$ reflects the fact that an eMBB user can transmit less data using resource blocks in the common region. We refer the readers to Fig. 1 for an illustration of these regions. The total number of bits that eMBB users can potentially transmit in a time frame given that action profile $(n_1,n_2)$ is taken is $T_{n_1,n_2}$, where

$$T_{n_1,n_2} = [\min(n_2,N-n_1) + a \cdot \min(n_1,N-n_2)] \cdot c.$$ (18)

Define

$$n_2^*(n_1) = \begin{cases} \min\{n_2 : T_{n_1,n_2} \geq r\} & \text{if } \max_{0 \leq n_2 \leq N} T_{n_1,n_2} \geq r \\ N & \text{otherwise}, \end{cases}$$ (19)

where $r$ is the number of bits requested by eMBB users collectively in a time frame. Quantity $n_2^*(n_1)$ is the minimum number of resource blocks for the common region in order for eMBB users to transmit $r$ bits. We assume that the eMBB agent’s payoff is

$$P_{\text{eMBB}}(n_1,n_2) = \begin{cases} \frac{n_2^*(n_1)}{N} \cdot 1[n_1+n_2 \leq N] - \frac{n_2}{N} \cdot b & \text{if } n_2 > n_2^*(n_1) \\ \frac{n_2}{N} \cdot 1[n_1+n_2 \leq N] - \frac{n_2}{N} \cdot b & \text{if } n_2 \leq n_2^*(n_1). \end{cases}$$ (20)

If $\min_{0 \leq n_1 \leq N} P(E_1) \leq \epsilon$, define

$$n_1^* = \min\{n_1 : P(E_{n_1}) \leq \epsilon\}.$$ (21)

Clearly, $1 \leq n_1^* \leq N$.

The following theorem characterizes the Nash equilibria of the two-player resource allocation game. Its proof is presented in Appendix B.

**Theorem 2.** If $\min_{0 \leq i \leq N} P(E_i) \leq \epsilon$, there are three cases as follows.

1) If

$$n_2^*(0) \leq N - n_1^*$$

and $n_2^*(n_1^*) \leq N - n_1^*$.

action profile $(n_1^*,n_2^*(n_1^*))$ is the only one pure Nash equilibrium of the two-player resource allocation game.

2) If

$$n_2^*(0) > N - n_1^*$$

and $n_2^*(n_1^*) \leq N - n_1^*$.

holds, action profiles $(n_1^*,n_2^*(n_1^*))$ and $(0,n_2^*(0))$ are the only pure Nash equilibria of the two-player resource allocation game. In addition,

a) if $n_1^* < N$ or $n_2^*(0) - n_2^*(n_1^*) < N$, $(n_1^*,n_2^*(n_1^*))$ is socially better than the other equilibrium.

b) If $n_1^* = N$ and $n_2^*(0) - n_2^*(n_1^*) = N$, both $(n_1^*,n_2^*(n_1^*))$ and $(0,n_2^*(0))$ are socially optimal.

3) If

$$n_2^*(0) > N - n_1^*$$

and $n_2^*(n_1^*) > N - n_1^*$.

holds, action profiles $(n_1^*,N-n_1^*)$ and $(0,n_2^*(0))$ are the only pure Nash equilibria of the two-player resource allocation game. In addition,

a) if $n_1^* < N$ or $n_2^*(0) < N$, $(n_1^*,N-n_1^*)$ is socially better than the other equilibrium.

b) If $n_1^* = N$ and $n_2^*(0) = N$, both $(n_1^*,N-n_1^*)$ and $(0,n_2^*(0))$ are socially optimal.

If $\min_{0 \leq i \leq N} P(E_i) > \epsilon$, action profile $(0,n_2^*(0))$ is the only one pure Nash equilibrium of the two-player resource allocation game.

**Remark.** We remark that Theorem 2 contains only three cases (i.e. (22), (23) and (24)) when $\min_{0 \leq i \leq N} P(E_i) \leq \epsilon$. We now show that $n_2^*(0) \leq N - n_1^*$ and $n_2^*(n_1^*) \leq N - n_1^*$ cannot occur at the same time. To see this, note from the definition of $n_1^*$ in (21) that

$$T_{n_1^*,N-n_1^*} = [\min(N-n_1^*,N-n_1^*) + a \cdot \min(n_1^*,n_1^*)] \cdot c$$

$$= (N-n_1^* + a \cdot n_1^*) \cdot c$$

$$> [\min(N-n_1^*,N) + a \cdot \min(0,n_1^*)] \cdot c$$

$$= T_{0,N-n_1^*}.$$ (25)

Suppose that $n_2^*(0) \leq N - n_1^*$. From (25), we have $T_{n_1^*,N-n_1^*} > T_{0,N-n_1^*}$, thus $n_2^*(n_1^*) \leq N - n_1^*$. 


We have reached the following separable convex nonlinear program with linear constraints.

\[
\min_{x_1, x_2, \ldots, x_m} \sum_{j=1}^{m} F_j(x_j) \\
\text{subject to} \\
\sum_{j=1}^{m} x_j = L \\
0 \leq x_j \leq r_j \quad \text{for all } j.
\]

We use bold face letters to denote vectors. Let \( x = (x_1, x_2, \ldots, x_m) \), \( r = (r_1, r_2, \ldots, r_m) \), \( e = (1, 1, \ldots, 1) \) and \( 0 = (0, 0, \ldots, 0) \).

Expressing (31), (32) and (33) in terms of vectors, we have

\[
\min_{x} F(x) \\
\text{subject to} \\
h(x) = 0 \\
g_1(x) \leq 0 \\
g_2(x) \leq 0,
\]

where

\[
F(x) \overset{\text{def}}{=} \sum_{j=1}^{m} F_j(x_j) \\
h(x) \overset{\text{def}}{=} \sum_{j=1}^{m} x_j - L \\
g_1(x) \overset{\text{def}}{=} -x \\
g_2(x) \overset{\text{def}}{=} x - r.
\]

Suppose that \( x^* \) is a regular point that satisfies constraints (35), (36) and (37). Then by the Karush-Kuhn-Tucker theorem [17, p. 352], a necessary condition for \( x^* \) to be a relative minimum point for the problem in (34) is that there exist a real number \( \lambda \) and column vectors \( \mu \) and \( \omega \) in \( \mathbb{R}^m \) with \( \mu \geq 0 \) and \( \omega \geq 0 \) such that

\[
\nabla F(x^*) + \lambda \nabla h(x^*) + \mu^T \nabla g_1(x^*) + \omega^T \nabla g_2(x^*) = 0 \\
\mu^T g_1(x^*) = 0 \\
\omega^T g_2(x^*) = 0,
\]

where \( \mu^T \) denotes the transpose of \( \mu \). We remark that \( \nabla F(x) \) and \( \nabla h(x) \) are \( 1 \times m \) row vectors. \( \nabla g_i(x^*) \), \( i = 1, 2 \), are \( m \times m \) matrices. An inequality constraint such as (36) is said to be active at a feasible point \( x \) if \( (g_i)(x) = 0 \) for some \( 1 \leq i \leq m \). Otherwise, the inequality constraint is said to be inactive. An entry of \( \mu \) or \( \omega \) may be non-zero only if the corresponding constraint is active. If one knew in advance which constraints were active, the solution of (42) can easily be solved. Specifically, let sets \( S, S_1 \) and \( S_2 \) be defined as

\[
S = \{ j : (36) \text{ is both inactive} \} \\
S_1 = \{ j : (36) \text{ is active and (37) is inactive} \} \\
S_2 = \{ j : (36) \text{ is inactive and (37) is active} \}.
\]

Note that it is impossible that both inequality constraints are active at the same time. It follows that

\[
S \cup S_1 \cup S_2 = \{1, 2, \ldots, m\}
\]
and the three sets are mutually exclusive. From the definition in (38), \( \nabla F(x) \) is a \( 1 \times m \) row vector whose \( i \)-th entry is
\[
(\nabla F(x))_i = \frac{2(x_i - \eta_i)}{m - 1}.
\]
(45)

From the definition in (39), \( \nabla h(x) \) is a \( 1 \times m \) row vector whose entries are all one, i.e.
\[
(\nabla h(x))_i = 1.
\]
(46)

From the definitions in (40) and (41), both \( \nabla g_1(x) \) and \( \nabla g_2(x) \) are \( m \times m \) matrices. It is easy to verify that
\[
\nabla g_1(x) = -I, \quad \nabla g_2(x) = I,
\]
where \( I \) denotes the \( m \times m \) identity matrix. From (45), (46) and (47), one can simplify (42) to obtain
\[
x_i^* = \eta_i - \frac{m - 1}{2} (\lambda - \mu_i + \omega_i),
\]
(48)
where \( \mu_i \) and \( \omega_i \) are the \( i \)-th entry of \( \mu \) and \( \omega \), respectively.

Now we derive \( \lambda, \mu \) and \( \omega \). One can rewrite constraint (32) as follows.
\[
L = \sum_{j=1}^{m} x_j^* = \sum_{j \in S} x_j^* + \sum_{j \in S_1} x_j^* + \sum_{j \in S_2} x_j^*
\]
\[
= \sum_{j \in S} x_j^* + 0 + \sum_{j \in S_2} r_j
\]
\[
= \sum_{j \in S} \eta_j - \frac{m - 1}{2} \lambda \cdot |S| + \sum_{j \in S_2} r_j,
\]
(49)
where the last equality is due to (48) and \( |S| \) denotes the number of elements in set \( S \). From (49), we have
\[
\lambda = \frac{\sum_{j \in S} \eta_j + \sum_{j \in S_2} r_j - L}{(m - 1)|S|/2}.
\]
(50)

Now we determine \( \mu \) and \( \omega \). For index \( i \), if
\[
0 < \eta_i - (m - 1)\lambda/2 < r_i,
\]
both inequality constraints are satisfied and are inactive. In this case, \( \mu_i = 0 \) and \( \omega_i = 0 \). If \( \eta_i - (m - 1)\lambda/2 \leq 0 \), constraint (56) is violated and constraint (57) is satisfied. In this case,
\[
\omega_i = 0
\]
\[
\mu_i = -\frac{2\eta_i}{m - 1} + \lambda.
\]
(51)

Clearly, \( \mu_i > 0 \). Finally, if \( \eta_i - (m - 1)\lambda/2 \geq r_i \), constraint (56) is satisfied and constraint (57) is violated. In this case,
\[
\mu_i = 0
\]
\[
\omega_i = \frac{2(\eta_i - r_i)}{m - 1} - \lambda.
\]
(52)

Clearly, \( \omega_i > 0 \).

In the following proposition, we present some properties of the optimal point given in (48). The proof of the proposition is presented in Appendix C. This proposition suggests a method to find the optimal solution in (48). This method is called water-filling algorithm and will be presented in the next section.

**Proposition 3.** Let \( x^* \) be the optimal solution of (34).

1) Suppose that \( z_i + r_i \geq z_j + r_j \), then
\[
x_i^* = r_i \text{ implies that } x_j^* = r_j.
\]
(53)
2) Suppose that \( z_i \geq z_j \), then
\[
x_j^* = 0 \text{ implies that } x_i^* = 0.
\]
(54)
3) Suppose that \( 0 < x_i^* < r_i \) and \( 0 < x_j^* < r_j \), then
\[
x_i^* + z_i = x_j^* + z_j.
\]
(55)

### 5.1 A water-filling algorithm

In this section we propose an algorithm to find the optimal point \( x^* \) and sets \( S, S_1 \) and \( S_2 \). Proposition 3 at the end of the last section provides a view to the optimization problem (34). One can view bits as water, requests as buckets, the number of cumulatively granted bits as the height of bits on the top of which buckets sit, and the total number of bits that can be transmitted as the amount of water. In this view, the solution of problem (34) is a method of filling water into buckets of finite sizes, sitting on tops of bins of various heights. Specifically, let there be \( m \) buckets. The sizes of the buckets are \( r_1, r_2, \ldots, r_m \). The \( j \)-th bucket sits on the top of a bin of height \( z_j \). There are \( L \) units of water. The goal is to fill water into the buckets such that the height of water surfaces in buckets are as even as possible. See Fig. 3 for a graphical illustration. Statements (1) and (2) of Proposition 3 imply that water starts to fill buckets on lower bins first, and buckets with higher tops are full only after buckets with lower tops are full. Statement (3) says that partially filled buckets have the same water level. We refer the reader to Fig. 3 for an example. Finding the optimal solution of (34) is equivalent to filling water into buckets simultaneously starting from buckets on lowest bins. Once a bucket is full, it stops receiving water, and water is filled into other buckets until it is exhausted. This water-filling problem can easily be solved by a recursive algorithm shown in Algorithm 1. This algorithm works by raising the water level from one possible level to the next higher possible level. There are three possible water levels in the algorithm. The possible water levels are

1) bottoms of one of the buckets;
2) tops of one of the buckets;
3) water level at a partially filled bucket.

Let \( P(z, r, m, L) \) denote the recursive function in Algorithm 1. The output of this function is the optimal point of the optimization problem (34), i.e.
\[
x^* = P(z, r, m, L).
\]

Let
\[
\hat{j} = \arg\min_{i} [z_i : r_i > 0, 1 \leq i \leq m].
\]

User \( j \) is said to be least granted.

Let \( \mathcal{N} \) be the set of users whose requests are non-zero, i.e.
\[
\mathcal{N} = \{ i : r_i > 0, 1 \leq i \leq m \}.
\]

Then, sort \( z_i, i \in \mathcal{N} \), into an ascending sequence. Label user’s identities such that
\[
z_1 = z_2 = \ldots = z_k < z_{k+1} \leq \ldots \leq z_{|\mathcal{N}|}
\]
(56)
The following theorem states that the water-filling algorithm solves the constrained optimization problem in (31). Its proof is presented in Appendix C.

**Theorem 4.** If \( L \leq \sum_{j=1}^{m} f_j \), the water-filling algorithm finds the optimal point \( x' \) that satisfies (48), (50), (51) and (52).

We remark that the idea of viewing resources as water and allocating resources as if filling water in buckets has been used in several resource allocation problems [18, 19]. In fact, several algorithms that solve resource allocation problems are also called water-filling algorithms. There are two types of water-filling problems in the literature. We refer the reader to Fig. 1 in [19]. The first type, shown in Fig. 1(a) of the reference, arises in a power control problem of Gaussian interference channels [20, 21]. However, the corresponding optimization problem has a logarithmic objective function. In addition, the assigned power has no upper limit. The corresponding water-filling problem can be viewed as a special case of our problem, in which the buckets have infinite capacities. The water-filling problem in Fig. 1(b) of [19] arises in a max-min fairness allocation problem. The water-filling problem is also a special case of ours, in which the heights of bins \( \{ z_j : 1 \leq j \leq m \} \) are all zero. We refer the reader to [18] for more information on max-min fairness allocation.

**Algorithm 1 Water filling algorithm**

**Function:** \( x = P(z, r, m, L) \)

**Inputs:** \( z, r, m, \) and \( L \)

**Outputs:** \( x \)

1: Let \( N' = \{ i : r_i > 0 \} \).
2: Label the users in set \( N' \) and let \( k \) be such that \( z_1 = z_2 = \ldots = z_k < z_{k+1} \leq \ldots \leq z_{|N'|} \).
3: In addition, label users 1, 2, ..., \( k \) such that \( r_1 \leq r_2 \leq \ldots \leq r_k \).
4: Let \( \bar{m} = \min \{ r_1, z_{k+1} - z_k \} \).
5: if \( L \leq \bar{m}k \) then
6: \( x_1 = x_2 = \ldots = x_k = L/k \).
7: \( x_j = 0, j = k + 1, k + 2, \ldots, |N'| \).
8: return \( x \)
9: end if
10: Evaluate \( z' \) according to (60).
11: Evaluate \( r' \) according to (61).
12: Evaluate \( L' \) according to (62).
13: Call \( y = P(z', r', m, L') \).
14: if \( r_1 \leq z_{k+1} - z_k \) then
15: Compute \( x \) according to (63).
16: else if \( z_{k+1} - z_k \leq r_1 \) then
17: Compute \( x \) according to (64).
18: end if
19: return \( x \)

### 6 Numerical and Simulation Results

In this section we present numerical and simulation results. We present the results in the following two sub-sections. In Section 6.1 we examine the light traffic analysis of \( P(E_i) \) by comparing numerical results with simulation results. In
Section 6.2, we study the Nash equilibria of the two-player resource allocation games and the performance of the water-filling algorithm.

6.1 Light traffic approximation

In this section we first examine the light traffic analysis of \( P(E_i) \) by comparing numerical results with simulation results. Then, we study the effect of \( p \) and \( \tau \) to the packet loss probability \( P(E_i) \).

In Fig. 4, we compare the light traffic approximation with \( \tau = 3 \) in (15) and its exact result in (7). We assume that the retransmission probability \( p \) for URLLC packets is 0.3. We consider three sizes for the common region: \( i = 24 \) for a small common region, \( i = 48 \) for a medium sized common region and \( i = 100 \) for a large common region. From Fig. 4, we see that the light traffic approximation and the exact result are very close to each other. Next we show \( P(E_i) \) versus \( \rho \) in Fig. 5. As expected, \( P(E_i) \) decreases with \( \rho \) initially. Later it increases with \( \rho \). There is an optimal value of \( \rho \) at which \( P(E_i) \) achieves a minimal value.

In Fig. 6, we compare the light traffic approximation in (15) and its exact result in (7). We assume that the number of bits to each eMBB buffer in a time frame is an independent discrete uniform random variable over \([0, 3 \times 10^5]\). Values of other parameters are shown in Table 2. We compare the performance with that of a few other resource allocation strategies. Specifically, we compare the performance of socially optimal Nash equilibrium \((n_1', n_2' (n_1'))\) with the non-optimal Nash equilibrium \((0, n_2' (0))\), profiles \((n_1' + 1, N - n_1' - 1)\) and \((N - 1, 1)\), and a random profile. In time slot \( t \), the random profile assigns \( N_{1,t} \) resource blocks to the common region, and assigns \( N_{2,t} \) resource blocks to the grant-based region. Discrete random variable \( N_{1,t} \) is uniform over the interval \([0, N]\). Conditioning on \( N_{1,t} \), discrete random variable \( N_{2,t} \) is uniform over the interval \([0, N - N_{1,t}]\). The packet loss probability of eMBB traffic is shown in Fig. 7. The packet loss probability of URLLC traffic is shown in Fig. 8. From these two figures, we see that the resource allocation based on socially optimal profile \((n_1', n_2' (n_1'))\) achieves a good balance between the two loss probabilities. We have also simulated the social payoff, which is defined as the sum of payoffs of two game players. The result is shown in Fig. 9. As expected, the socially optimal Nash equilibrium has the largest social payoffs.

6.2 Resource allocation

In this section we study the two resource allocation algorithms presented in Section 4 and Section 5, respectively. We simulate the system shown in Fig. 6. The sizes of common region and grant-based region are determined by a resource allocation algorithm. Each eMBB user has a buffer which can store \( B \) eMBB bits. eMBB packet arrivals in frame \( t \) are stored in the corresponding buffer. If the residual space of a buffer is not enough to store new arrivals, eMBB packets can be lost. At the beginning of frame \( t + 1 \), eMBB users make a request to transmit all bits in their buffers. A resource allocation algorithm determines the number of bits that each user can transmit. Packets that cannot be transmitted stay in the buffers. We will compare the water-filling algorithm with a few other resource allocation algorithms.

First, we study the packet loss probability of eMBB traffic when wireless resources are assigned according to the socially optimal Nash equilibrium. We assume that eMBB bits are scheduled using the water-filling algorithm in Algorithm 1. We assume that the number of bits to each eMBB buffer in a time frame is an independent discrete uniform random variable over \([0, 3 \times 10^5]\). Values of other parameters are shown in Table 2. We compare the performance with that of a few other resource allocation strategies. Specifically, we compare the performance of socially optimal Nash equilibrium \((n_1', n_2' (n_1'))\) with the non-optimal Nash equilibrium \((0, n_2' (0))\), profiles \((n_1' + 1, N - n_1' - 1)\) and \((N - 1, 1)\), and a random profile. In time slot \( t \), the random profile assigns \( N_{1,t} \) resource blocks to the common region, and assigns \( N_{2,t} \) resource blocks to the grant-based region. Discrete random variable \( N_{1,t} \) is uniform over the interval \([0, N]\). Conditioning on \( N_{1,t} \), discrete random variable \( N_{2,t} \) is uniform over the interval \([0, N - N_{1,t}]\). The packet loss probability of eMBB traffic is shown in Fig. 7. The packet loss probability of URLLC traffic is shown in Fig. 8. From these two figures, we see that the resource allocation based on socially optimal profile \((n_1', n_2' (n_1'))\) achieves a good balance between the two loss probabilities. We have also simulated the social payoff, which is defined as the sum of payoffs of two game players. The result is shown in Fig. 9. As expected, the socially optimal Nash equilibrium has the largest social payoffs.

Fig. 4. \( P(E_m) \) versus \( \rho \).

Fig. 5. \( P(E_m) \) versus \( p \).

Fig. 6. System architecture.
TABLE 2
Parameter values.

| Parameter | Value                          | Parameter | Value |
|-----------|-------------------------------|-----------|-------|
| $\rho$    | $6.3 \times 10^{-3}$ packets per mini-slot | $\tau$    | 8     |
| $p$       | 0.3                           | $n$       | 60    |
| $e$       | $10^{-5}$                     | $n_i^*$   | 30    |
| $b$       | 0.8                           | $c$       | $3.2 \times 10^4$ bits |
| $B$       | $3.8 \times 10^4$ bits       | $m$       | 8     |

Fig. 7. Packet loss probability of eMBB traffic versus $a$.

Fig. 8. Packet loss probability of URLLC traffic versus $a$.

water-filling algorithm, other resource allocation algorithms that we consider are smallest request first, largest request first, random order allocation, two-step averages, and max-min fairness allocation. We now briefly explain these allocation methods. The smallest request first and the largest request first methods are straightforward. Sort the requests {$r_j : 1 \leq j \leq m$} into an ascending list. In the case of smallest request first, grant requests in the ascending order. On the other hand, in the case of largest request first, grant requests in the descending order. In either case, grant requests until the allowance $L$ is exhausted or all requests have been satisfied. The random order allocation is similar. Arrange requests according to a randomly selected permutation and grant requests according to the order of this permutation until $L$ bits are exhausted or all requests are satisfied. The two-step average method works as follows. There are two steps in this method. In the fist step, compute and grant requests according to the average $v = L/m$. The number of bits granted to user $i$ in step 1 is $x^{(1)}_i = \min(r_i, v)$. The residual request size of user $i$ is $r^{(1)}_i = r_i - x^{(1)}_i$. The original grant allowance is $L$ and the residual allowance after the first step is $L^{(1)} = L - \sum_{i=1}^{m} x^{(1)}_i$. In the second step, sort {$r^{(1)}_i : 1 \leq i \leq m$} into a descending list. Grant requests in the order of this list until $L^{(1)}$ is exhausted or all requests have been satisfied. Finally, we explain the max-min fairness method. This allocation method was proposed to allocate bandwidth [18]. The max-min fairness allocation can be solved by a water-filling problem. This problem is a special case of $P(z, r, m, L)$, in which $z$ is set to a zero vector, i.e. $z = \mathbf{0}$. Although the max-min allocation can be solved by a water-filling algorithm, there is a simpler algorithm to solve it. First, sort {$r_i : 1 \leq i \leq m$} into an ascending sequence. Relabeling indexes if necessary, we can assume that the sequence {$r_i : 1 \leq i \leq m$} is ascending. For convenience, let $r^{(0)}_i = r_i$ for $i = 1, 2, \ldots, m$, and let $L^{(0)} = L$. In step $j$, compute the average number of bits that can be granted to $j$ users, i.e. $v^{(j)} = L^{(j-1)}/(m-j+1)$.

In step $j$, allocate

$$x^{(j)}_i = \min(v^{(j)}, r^{(j-1)}_i)$$

to user $i$. Calculate the residual request sizes for the next step, i.e.

$$r^{(j)}_i = r_i^{(j-1)} - x^{(j)}_i, \quad 1 \leq i \leq m.$$  

If $r^{(j)}_i = 0$ for all $i$, the algorithm stops. Otherwise, the algorithm continues and we compute the residual allowance

$$L^{(j)} = L^{(j-1)} - \sum_{i=1}^{m} x^{(j)}_i.$$
We simulate the water-filling algorithm and the five other resource allocation algorithms for $T = 10^6$ frames. The sample variance of a sequence $\{z_i(t+1) : 1 \leq i \leq m\}$ is defined in (27). Another well known measure of fairness is the Jain’s fairness index defined as

$$\frac{\left(\sum_{i=1}^{m} z_i(t+1)\right)^2}{m \cdot \sum_{i=1}^{m} z_i(t+1)^2}.$$  

The eMBB packet loss probability, sample variance (defined in (27)) and Jain’s fairness index in Figs. 10, 11 and 12 respectively. From Fig. 10 the smallest request first method has the smallest eMBB packet loss probability. The water-filling algorithm and the max-min fairness method have very similar results. From Figs. 11 and 12, the water-filling algorithm has the best performance in both sample variance and the Jain’s fairness index.

7 CONCLUSIONS

In this paper we studied a wireless resource allocation problem and a packet scheduling problem of the URLLC traffic and eMBB traffic in an uplink wireless network. We proposed to divide frequencies into a grant-based region for eMBB traffic only, and a common region for both URLLC traffic and eMBB traffic. To cope with the ultra stringent latency and reliability requirement of URLLC packets, we proposed a persistent random retransmission scheme for URLLC packets. We derived the probability that a randomly selected URLLC packet fails to meet its latency requirement. We proposed a two-player game to determine the sizes of the common region and the grant-based region. We studied the Nash equilibria of this game. For the request-and-grant allocation of eMBB packets, we proposed a constraint nonlinear program which minimizes the variance of the number of bits granted to the eMBB users. We show that a water-filling algorithm can recursively solve the nonlinear program. From simulation, we show that our scheme, consisting of resource allocation according to Nash equilibria of a game, persistent random transmission of URLLC packets and scheduling eMBB packets by a water-filling algorithm, works better than four other heuristic methods.

Appendix A

**Proof of Proposition 1.** Let $X_j$ denote the number of Poisson arrivals in mini time slot $j$, $-(\tau-1) \leq j \leq \tau-1$, not including the tagged arrival in mini time slot 0. We use bold face letters to denote vectors. Let $x$ be a $1 \times (2\tau-1)$ row vector, i.e.

$$x = (x_{-(\tau-1)}, \ldots, x_{-1}, x_0, x_1, \ldots, x_{\tau-1}).$$

Define function $f_i(x)$ according to (3) and (4)

$$f_i(x) = \begin{cases} 1 - 1_{(x_0=0)}(1-p)^{\sum_{j=(i-1)}^{j=(r-1)} x_j} & \text{if } i = 0 \\ 1 - p \cdot 1_{(x_i=0)}(1-p)^{\sum_{j=(i-1)}^{j=(r-1)} x_j} & \text{if } 1 \leq i \leq \tau-1. \end{cases}$$  

Also define function

$$f(x) = \prod_{i=0}^{\tau-1} f_i(x).$$
It is easy to verify that \( f(x) \) is non-decreasing with respect to \( x_j \) for any \( j = -(\tau - 1), \ldots, \tau - 1 \). Thus, \( f(x) \) is also non-decreasing. Let \( X = (X_{-(\tau - 1)}, \ldots, X_{\tau - 1}, X_0, X_1, \ldots, X_{\tau - 1}) \), where \( X_k \) is the number of Poisson arrivals in mini time slot \( i \). One can rewrite (1) as follows,

\[
E[1_{E_i}|X] = \prod_{i=0}^{\tau-1} f_i(X) = f(X)
\]

\[
P(E) = E[E[1_{E_i}|X]] = E[f(X)].
\]

Let \( Y = \{Y_j : -(\tau - 1) \leq j \leq \tau - 1\} \) be a sequence of independent Poisson random variables with mean \( \rho' \), where \( \rho' \geq \rho \). It is well known that Poisson random variables are stochastically increasing in means (see Example 9.2(B) on page 411 of Ross [22]). Thus,

\[
Y_j \geq \text{st} X_j
\]

for all \( j = -(\tau - 1), \ldots, \tau - 1 \). By Example 9.2(A) of Ross [22] p. 410,

\[
E[f(Y)] \geq E[f(X)].
\]

It follows that \( P(E) \) increases with \( \hat{\rho} \).

**Appendix B**

**Proof of Theorem 2** Suppose that \((n_1, n_2)\) is an action profile of the two-player resource allocation game. Recall that profile \((n_1, n_2)\) is a pure Nash equilibrium if action \(n_1\) and action \(n_2\) are best responses to each other.

Now we consider the case, in which \( \min_{0 \leq i \leq N} P(E_i) \leq \epsilon \). There are three sub-cases. Since their proofs are similar, we only present the proof of the second sub-case. Proofs of the other sub-cases are omitted. We also omit the proof of the case, in which \( \min_{0 \leq i \leq N} P(E_i) > \epsilon \). Suppose that (23) holds. We shall show that profiles \((n_1', n_2'(n_1'))\) and \((0, n_2'(0))\) are pure Nash equilibria. We also need to show that profile \((i, j)\) is not a Nash equilibrium if \((i, j) \neq (n_1', n_2'(n_1'))\) and \((i, j) \neq (0, n_2'(0))\). In addition, we shall discuss the social optimality of these two equilibria.

To begin, we show that profile \((n_1', n_2'(n_1'))\) is a pure Nash equilibrium. We show that action \(n_1'\) of the URLLC agent is a best response to action \(n_2'(n_1')\) of the eMBB agent. For \( i < n_1' \),

\[
P_{\text{URLLC}}(i, n_2'(n_1')) = 1 - \frac{n_1'}{N} b
\]

and

\[
P_{\text{URLLC}}(i, n_2'(n_1')) = - \frac{i}{N} b.
\]

Since \( 0 < b < 1 \), it follows that

\[
P_{\text{URLLC}}(i, n_2'(n_1')) > P_{\text{URLLC}}(i, n_2'(n_1'))
\]

for all \( i < n_1' \). Now consider \( i \), such that \( i > n_1' \). We have

\[
P_{\text{URLLC}}(i, n_2'(n_1')) = 1 \cdot 1_{[i + n_2'(n_1') \leq N]} - \frac{i b}{N}
\]

\[
= \begin{cases} 
- \frac{i b}{N} & \text{if } i > N - n_2'(n_1') \\
1 - \frac{i b}{N} & \text{if } i \leq N - n_2'(n_1')
\end{cases}
\]

Comparing (66) with (68), we obtain

\[
P_{\text{URLLC}}(i, n_2'(n_1')) > P_{\text{URLLC}}(i, n_2'(n_1'))
\]

for all \( i > n_1' \). (67) and (69) establish that action \(n_1'\) is a best response to action \(n_2'(n_1')\). We also need to show that action \(n_2'(n_1')\) is a best response to action \(n_1'\). The payoff of the eMBB agent when profile \((n_1', n_2'(n_1'))\) is taken is

\[
P_{\text{EMBB}}(n_1', n_2'(n_1')) = \frac{n_2'(n_1')}{N} \left[ 1_{[n_2'(n_1') + n_1'] \leq N} - \frac{n_2'(n_1') b}{N} \right]
\]

\[
= \frac{n_2'(n_1')(1 - b)}{N}.
\]

The payoff of the eMBB agent for an arbitrary action \( j \) is

\[
P_{\text{EMBB}}(n_1', j) = \begin{cases} 
\frac{n_2'(n_1')}{N} \left[ 1_{[n_2'(n_1') + j] \leq N} - \frac{j b}{N} \right] & \text{if } j > n_2'(n_1') \\
0 & \text{if } j \leq n_2'(n_1')
\end{cases}
\]

Since \( n_1' + n_2'(n_1') \leq N \), the preceding payoff is equal to

\[
P_{\text{EMBB}}(n_1', j) = \frac{n_2'(n_1')}{N} \left[ 1_{[n_2'(n_1') + j] \leq N} - \frac{j b}{N} \right]
\]

From (70), (71) and assumption \( b < 1 \), we have

\[
P_{\text{EMBB}}(n_1', n_2'(n_1')) \geq P_{\text{EMBB}}(n_1', j).
\]

Hence, action \(n_2'(n_1')\) is a best response to action \(n_1'\).

Next, we show that profile \((0, n_2'(0))\) is a pure Nash equilibrium. First, we show that action 0 of the URLLC agent is a best response to action \(n_2'(0)\) of the eMBB agent. Since

\[
P(E_{\min(0, -n_2'(0))}) = P(E_0) = 1 > \epsilon
\]

and \( 0 + n_2'(0) \leq N \), it follows that

\[
P_{\text{URLLC}}(0, n_2'(0)) = 0
\]

For arbitrary \( i \),

\[
P_{\text{URLLC}}(i, n_2'(0)) = 1 \cdot \left[ P(E_{\min(i, n_2'(0))}) \right] 1_{[i + n_2'(0)] \leq N} - \frac{i b}{N}
\]

We know that \( n_1' + n_2'(0) \geq N \). For \( 1 \leq i < n_1' \), the first indicator function is 0. For \( i \geq n_1' \), the second indicator function is 0. Thus, (73) is reduced to

\[
P_{\text{URLLC}}(i, n_2'(0)) = - \frac{i b}{N} < P_{\text{URLLC}}(0, n_2'(0))
\]

for both cases. Hence, action 0 is a best response to action \(n_2'(0)\). We also need to show that action \(n_2'(0)\) is a best response to action 0. Note that

\[
P_{\text{EMBB}}(0, n_2'(0)) = \frac{n_2'(0)}{N} \left[ 1_{[0 + n_2'(0)] \leq N} - \frac{n_2'(0) b}{N} \right]
\]

\[
= \frac{n_2'(0)(1 - b)}{N}
\]

since \( n_2'(0) \in [0, N] \). On the other hand, the payoff of the eMBB agent for an arbitrary action \( j \) is

\[
P_{\text{EMBB}}(0, j) = \begin{cases} 
\frac{n_2'(0)}{N} \left[ 1_{[0 + j] \leq N} - \frac{j b}{N} \right] & \text{if } j > n_2'(0) \\
0 & \text{if } j \leq n_2'(0)
\end{cases}
\]

Comparing (66) with (68), we obtain

\[
P_{\text{URLLC}}(i, n_2'(n_1')) > P_{\text{URLLC}}(i, n_2'(n_1'))
\]

for all \( i > n_1' \). (67) and (69) establish that action \(n_1'\) is a best response to action \(n_2'(n_1')\). We also need to show that action \(n_2'(n_1')\) is a best response to action \(n_1'\). The payoff of the eMBB agent when profile \((n_1', n_2'(n_1'))\) is taken is

\[
P_{\text{EMBB}}(n_1', n_2'(n_1')) = \frac{n_2'(n_1')}{N} \left[ 1_{[n_2'(n_1') + n_1'] \leq N} - \frac{n_2'(n_1') b}{N} \right]
\]

\[
= \frac{n_2'(n_1')(1 - b)}{N}.
\]

This implies that action \(n_2'(0)\) is a best response to action 0.

Now we show that any profile \((i, j)\) such that \( (i, j) \neq (n_1', n_2'(n_1')) \) and \( (i, j) \neq (0, n_2'(0)) \), cannot be a Nash equilibrium. For each profile \((i, j)\) that is not an equilibrium, we
identify a better response $i′$ to action $j$, or a better response $j′$ to action $i$. In the process of proving that $(n^∗_1, n^∗_2(n^∗_1))$ is a pure Nash equilibrium, for instance, we have shown that actions $n^*_1$ and $n^*_2(n^*_1)$ are best responses to each other. Thus, to show that profile $(i, j)$ is not a Nash equilibrium, it is sufficient to assume that

$$i \neq n^*_1, \quad i = 0 \quad j \neq n^*_2(n^*_1), \quad j = n^*_2(0).$$

(75)

For any profile $(i, j)$ such that (75) holds, we show that

1) if $j \leq N - n^*_1$, then action $n^*_1$ is a better response to action $j$ than action $i$;
2) if $j > N - n^*_1$, then action 0 is a better response to action $j$ than action $i$.

Indeed, suppose that $j \leq N - n^*_1$, we have

$$P_{URLLC}(n^*_1, j) = 1 \cdot 1[i^*_1 + j \leq N] - n^*_1 b - \frac{n^*_1 b}{N}$$

$$P_{URLLC}(i, j) = \begin{cases} 1 & i \neq n^*_1, \quad i = 0 \quad j \neq n^*_2(n^*_1), \quad j = n^*_2(0) \end{cases}$$

The preceding equations imply that

$$P_{URLLC}(n^*_1, j) \geq P_{URLLC}(i, j)$$

for $i \neq n^*_1$ and $j \leq N - n^*_1$. Now assume that $j > N - n^*_1$, we have

$$P_{URLLC}(0, j) = 0 - \frac{0 \cdot b}{N} = 0$$

$$P_{URLLC}(i, j) = \begin{cases} 1 & i \neq n^*_1, \quad i = 0 \quad j \neq n^*_2(n^*_1), \quad j = n^*_2(0) \end{cases}$$

which implies that $x^*_1 = r_j$. We have completed the proof of part (1). The proof of part (2) is similar and we omit the proof. We now consider part (3). Since $0 < x^*_1 < r_i$ and $0 < x^*_1 < r_j$, it follows that $\mu_l = \omega_l = \mu_j = \omega_j = 0$. Thus,

$$x^*_1 + z_i = \frac{1}{m} \left( \sum_{k=1}^{m} z_k + L \right) - \frac{(m-1)\lambda}{2} + z_i$$

$$x^*_1 + z_j = \frac{1}{m} \left( \sum_{k=1}^{m} z_k + L \right) - \frac{(m-1)\lambda}{2} + z_j$$

The proof is completed.

**Proof of Proposition 3** We first prove part (1) of Proposition 3. Since $x^*_1 = r_j$, we have

$$\eta_j - (m-1)\lambda/2 \geq r_j.$$  

From (13), we have

$$\frac{1}{m} \left( \sum_{k=1}^{m} z_k + L \right) - \frac{(m-1)\lambda}{2} + z_i$$

Hence,

$$\eta_j - (m-1)\lambda/2 \geq r_j,$$

which implies that $x^*_1 = r_j$. We have completed the proof of part (1). The proof of part (2) is similar and we omit the proof. We now consider part (3). Since $0 < x^*_1 < r_i$ and $0 < x^*_1 < r_j$, it follows that $\mu_l = \omega_l = \mu_j = \omega_j = 0$. Thus,

$$x^*_1 + z_i = \frac{1}{m} \left( \sum_{k=1}^{m} z_k + L \right) - \frac{(m-1)\lambda}{2} + z_i$$

$$x^*_1 + z_j = \frac{1}{m} \left( \sum_{k=1}^{m} z_k + L \right) - \frac{(m-1)\lambda}{2} + z_j$$

The proof is completed.

**Appendix C**

In this appendix we prove Proposition 3 and then Theorem 4.

Since $x^*_1 = r_j$, we have

$$\eta_j - (m-1)\lambda/2 \geq r_j.$$  

From (13), we have

$$\frac{1}{m} \left( \sum_{k=1}^{m} z_k + L \right) - \frac{(m-1)\lambda}{2} + z_i$$

Hence,

$$\eta_j - (m-1)\lambda/2 \geq r_j,$$

which implies that $x^*_1 = r_j$. We have completed the proof of part (1). The proof of part (2) is similar and we omit the proof. We now consider part (3). Since $0 < x^*_1 < r_i$ and $0 < x^*_1 < r_j$, it follows that $\mu_l = \omega_l = \mu_j = \omega_j = 0$. Thus,

$$x^*_1 + z_i = \frac{1}{m} \left( \sum_{k=1}^{m} z_k + L \right) - \frac{(m-1)\lambda}{2} + z_i$$

$$x^*_1 + z_j = \frac{1}{m} \left( \sum_{k=1}^{m} z_k + L \right) - \frac{(m-1)\lambda}{2} + z_j$$

The proof is completed.

**Proof of Theorem 4** We now prove Theorem 4. We note that if $\sum_{j=1}^{m} r_j < L$, the feasible region of the optimization problem in (31) is empty. If $\sum_{j=1}^{m} r_j = L$, it is clear that the feasible region contains only one point. Thus, the solution is $x_j = r_j$ for all $1 \leq j \leq m$. Thus, in the proof we assume that $\sum_{j=1}^{m} r_j > L$.

Suppose that $L \leq \tilde{m} k$. In this case, the optimal solution of (34) is stated in Claim 1 below.

- **Claim 1** If $L \leq \tilde{m} k$, the optimal solution is $x_j = L/k$ for $1 \leq j \leq k$, and $x_j = 0$ for $k+1 \leq j \leq |N|$.

On the other hand, if $\tilde{m} k < L < \sum_{j=1}^{m} r_j$, the proof needs the following claims.

- **Claim 2** The objective function of $P(z, r, m, L)$ and that of $P(z', r', m', L')$ are the same.

- **Claim 3** The objective function is convex. There is a unique optimal point.

- **Claim 4** Assume that $L \geq \tilde{m} k$. The feasible region $\Omega'$ of $P(z', r', m', L')$ is a subset of the feasible region $\Omega$ of $P(z, r, m, L)$. In addition, the optimal point of $P(z, r, m, L)$ is in $\Omega'$.

- **Claim 5** Since $L < \sum_{j=1}^{m} r_j$ and each call to $P$ strictly decreases $L$, the recursive function terminates in a finite number of calls.

If $L > \tilde{m} k$, the water-filling algorithm in Algorithm 1 raises the water level by $\tilde{m}$ and call $P(z', r', m', L')$. From Claim 2,
the original objective function and the new objective function are the same. From Claim 3, the constrained nonlinear program has a unique solution. From Claim 4, this unique solution is in the feasible region of \( P(z', r', m, L') \). The water-filling algorithm successively reduces the feasible region. In this process, \( L \) is strictly reduced. In a finite number of steps, the condition \( L \leq mk \) will hold and the algorithm will stop.

Now we prove Claims 1, 2, 4. Claims 3 and 5 are obvious. We omit their proofs.

**Proof of Claim 1.**
First, if \( L = 0 \), it is obvious that the feasible region has only one point and \( x_i = 0 \) for all \( i \) gives the optimal solution. We assume that \( 0 < L \leq mk \). Let \( x \) denote the optimal solution. If \( x_i = 0 \), by the second statement in Proposition 3 we have \( x_i = 0 \) for all \( 2 \leq i \leq |\Omega| \). It follows that \( \sum_{i=1}^{m} x_i = 0 \), which contradicts the assumption that \( L > 0 \). If \( x_i = r_i \), then

\[
\frac{1}{m} \left( \sum_{i=1}^{m} z_i + L \right) - z_i = \frac{m-1}{2} \lambda > r_i.
\]

Since \( z_j = z_1 \) for all \( j = 2, 3, \ldots, k \), it follows that

\[
\frac{1}{m} \left( \sum_{i=1}^{m} z_i + L \right) - z_j = \frac{m-1}{2} \lambda > r_i.
\]

Thus, \( x_j > r_1 \). We have

\[
L = \sum_{i=1}^{m} x_i \geq \sum_{i=1}^{k} x_i > k \lambda \geq km,
\]

which is a contradiction. Thus,

\[
0 < x_i < r_i. \tag{77}
\]

Since \( z_1 = z_2 = \ldots = z_k \) and \( r_1 \leq r_2 \leq \ldots \leq r_k \), it follows from the definition of \( \eta_j \) in (60) that \( x_1 = x_2 = \ldots = x_k \). We now show that \( x_j = 0 \) for \( k + 1 \leq j \leq |\Omega| \). If \( z_{k+1} = z_k \geq r_1 \), i.e. \( \tilde{m} = r_1 \), we have

\[
\eta_{k+1} = \frac{m-1}{2} \lambda = \frac{1}{m} \left( \sum_{i=1}^{m} x_i + L \right) - z_i = \frac{m-1}{2} \lambda = x_1 + z_1 - z_{k+1} < r_1 - (z_{k+1} - z_k) \leq 0.
\]

Thus, \( x_j = 0 \) for \( k + 1 \leq j \leq |\Omega| \). On the other hand, consider the case where \( z_{k+1} = z_k < r_1 \), i.e. \( \tilde{m} = z_{k+1} - z_k \). Assume that \( x_j > 0 \) for some \( j = k + 1, \ldots, |\Omega| \). We look for a contradiction. Since \( x_j > 0 \), we have

\[
\frac{1}{m} \left( \sum_{i=1}^{m} z_i + L \right) - z_j = \frac{m-1}{2} \lambda > 0.
\]

From the preceding inequality and the definition of \( \eta_1 \) in (60), we have

\[
\eta_1 - \frac{m-1}{2} \lambda > z_j - z_1.
\]

This implies that \( x_1 > z_j - z_1 \). Thus,

\[
L = \sum_{i=1}^{m} x_i \geq \sum_{i=1}^{k} x_i > k(z_j - z_1) \geq km.
\]

This is a contradiction.

**Proof of Claim 2.**
We assume that \( L > mk \). We use notation \( \eta_j(z, r, L) \) to explicitly show the dependency with \( z, r, \) and \( L \). From (30) and \( z', r', \) and \( L' \) defined in (60), (61) and (62), we have

\[
\eta_j(z', r', L') = \frac{1}{m} \left( \sum_{i=1}^{m} z_i(t) + L' \right) - z_j(t) = \frac{1}{m} \left( \sum_{i=1}^{m} z_i(t) + \sum_{k+1}^{m} z_i(t) + L' \right) - z_j(t) = \frac{1}{m} \left( \sum_{i=1}^{k} z_i(t) + \tilde{m} + \sum_{k+1}^{m} z_i(t) + L - \tilde{m}k \right) - z_j(t) = \eta_j(z, r, L) - \tilde{m}1_{[1 \leq j \leq k]}.
\]

where \( 1_{[A]} \) is equal to 1 if \( A \) is true, and is equal to 0 otherwise. It follows from (79) that one can rewrite

\[
x_j - \eta_j(z, r, L) = y_j - \eta_j(z', r', L')
\]

through a change of variables

\[
y_j = x_j - \tilde{m}1_{[1 \leq j \leq k]}.
\]

Thus, the objective functions in \( P(z, r, m, L) \) and \( P(z', r', m, L') \) are identical.

**Proof of Claim 4.**
Let \( x \) and \( y \) denote the solution of problems \( P(x, r, m, L) \) and \( P(z', r', m, L') \), respectively, where \( x \) and \( y \) are related by (63) or (64) depending on whether \( r_1 < z_{k+1} - z_k \) or \( r_1 \geq z_{k+1} - z_k \). We assume that indexes are labeled such that (56) and (57) hold. Recall that \( z', r' \) and \( L' \) are defined in (60), (61) and (62), respectively. The feasible region of problem \( P(z, r, m, L) \) is

\[
\Omega = \left\{ x : \sum_{i=1}^{m} x_i = L, 0 \leq x_i \leq r_i, 1 \leq i \leq m \right\}. \tag{80}
\]

In terms of \( y \), feasible region \( \Omega' \) is

\[
\Omega' = \left\{ y : \sum_{i=1}^{m} y_i = L', 0 \leq y_i \leq r'_i, 1 \leq i \leq m \right\}.
\]

If \( r_1 < z_{k+1} - z_k \), one can apply a change of variables using (63) to express \( \Omega' \) as

\[
\Omega' = \left\{ x : \sum_{i=1}^{j} x_i = L, x_i = \tilde{m}1 \leq i \leq j, 0 \leq x_{i+j} - \tilde{m} - r_i - \tilde{m}j + 1 \leq \ell \leq k, 0 \leq x_{i+j} - r_i, j + 1 \leq \ell \leq m \right\}.
\]

With a little manipulation, we obtain from the preceding

\[
\Omega' = \left\{ x : \sum_{i=1}^{m} x_i = L, x_i = r_i, 1 \leq i \leq j, 0 \leq x_{i+j} - r_i, j + 1 \leq \ell \leq m \right\}. \tag{81}
\]
From (80) and (81), it is clear that $\Omega' \subseteq \Omega$. The proof for the case, where $r_i \geq z_{k+1} - z_k$ is similar. We omit the details.

We now show that the optimal point of problem $P(z, r, m, L)$ actually lies in region $\Omega'$. We prove the case where $r_1 < z_{k+1} - z_k$. (82)

From (81), we need to show that the optimal solution satisfies $x_i = r_i$ for $i = 1, 2, \ldots, j$. Assume that $x_1 < r_1$. From (48), since $x_1 < r_1$, it follows that $\mu_1 = \omega_1 = 0$ and $1/m \left( \sum_{k=1}^{m} z_k + L \right) - z_1 - m - 1/2 \lambda < r_1$. (83)

Since $z_1 = z_i$ for $i = 2, 3, \ldots, k$, it follows that $x_i = x_1 < r_1$.

In addition, for $i = k + 1, \ldots, m$, we have

$$1/m \left( \sum_{k=1}^{m} z_k + L \right) - z_i - m - 1/2 \lambda < r_1 + z_1 - z_2 \leq 0,$$

where the last inequality is due to the assumption (82).

Thus, $x_i = 0$ for $i = k + 1, \ldots, m$. It follows that $L = \sum_{k=1}^{m} x_\ell = \sum_{k=1}^{m} x_\ell + m \cdot \ell < m k$, which contradicts the assumption that $L \geq m k$. The proof for the other case where $r_1 \geq z_{k+1} - z_k$ is similar. We omit the details. The proof is complete.

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