Flat World of Dilatonic Domain Walls

Mirjam Cvetić

Department of Physics
University of Pennsylvania
Philadelphia, PA 19104–6396

ABSTRACT

We study dilatonic domain walls specific to superstring theory. Along with the matter fields and metric the dilaton also changes its value in the wall background. We found supersymmetric (extreme) solutions which in general interpolate between isolated superstring vacua with non-equal value of the matter potential; they correspond to the static, planar domain walls with flat metric in the string (sigma model) frame. We point out similarities between the space-time of dilatonic walls and that of charged dilatonic black holes. We also comment on non-extreme solutions corresponding to expanding bubbles.

PACS #: 11.17+y, 04.20.-q, 04.65.+e, 11.30.Pb
We address dilatonic domain wall solutions which are specific to superstring theory. In the domain wall background along with the matter fields and metric the dilaton field also changes its value. Such walls are of particular interest because they correspond to configurations which interpolate between isolated superstring vacua and may thus shed light on the nature and connectedness of the superstring vacua. The primary goal is to present supersymmetric (extreme) solutions which correspond to static, planar domain walls in general interpolating between isolated four dimensional (4d) superstring vacua with non-equal value of the matter potential. We also comment on non-extreme walls.

Dilatonic domain walls are a generalization of the “ordinary” domain walls in an analogous way as dilatonic charged black holes are a generalization of “ordinary” black holes. Ordinary domain walls between vacua of non-equal cosmological constant fall into three classes: (i) extreme (supersymmetric) static, planar domain walls, (ii) non-extreme domain walls (expanding bubbles with an inertial observer inside the bubble for each side of the wall) and (iii) ultra-extreme walls (expanding bubbles of false vacuum decay). The energy density \( \sigma_{ultra} \) of the non- [or ultra-]extreme walls is bound from below [or above] by the one \( \sigma_{ext} \) of the extreme ones. Walls are thus an example of configurations for which supersymmetry provides a lower bound for the energy of stable wall configurations. The space-times induced by the walls are non-singular with non-trivial global structure and horizons closely related to the ones of certain black holes: on the anti-deSitter [or Minkowski] side of the wall the induced non-singular space-time is closely related to the ones of the Reissner-Nordström [or Schwarzschild] black holes. The intriguing similarity between the space-time of the walls and the one of the corresponding black-holes reappears in the case of dilatonic walls as well.
In the domain wall case the role of the mass \((M)\) and the charge \((Q)\) of the black hole is played by the energy density \((\sigma)\) of the wall and the cosmological constant \((\Lambda)\) outside the wall, respectively.

Potentially phenomenologically viable superstring vacua are described by an effective 4d \(N = 1\) supergravity theory. The scalar part of the effective Lagrangian involves the metric \(g_{E\mu \nu}\), the dilaton \(S = e^{-2\phi} + ia\) (written in this form as a scalar part of the chiral superfield),\(^19\) matter fields and gauge fields. In this note we do not include gauge fields; however, since the dilaton does couple to gauge fields the study of charged dilatonic walls is interesting and will be examined elsewhere. For the sake of simplicity we take only one (complex) matter field \(T\), a scalar component of a chiral superfield interpolating between isolated minima of the matter potential.

To all orders in string loops the superpotential \(W = W_0(T)\) of the effective Lagrangian of superstring vacua does not depend on the dilaton,\(^16\) i.e., it is only a function of the matter fields. In the Kähler potential \(K\) the dilaton couples\(^17, 18\) in a special way: \(K = -\kappa^{-1} \log(S + S^*) + K_0(T, T^*)\). We put the imaginary part (axion) of the dilaton field to zero \((a = 0)\) which turns out to be the solution of field equations for the dilatonic domain walls anyway. The scalar part of the Lagrangian (in the Einstein frame) is then of the form:

\[
L_E = \sqrt{-g_E} \left( -\frac{1}{2\kappa} R_E + \kappa^{-1} g_E^{\mu \nu} \partial_\mu \phi \partial_\nu \phi + T_0 - \frac{e^{2\phi}}{2} \tilde{V}_0 \right)
\]  

(1)

where \(T_0 = g_E^{\mu \nu} K_{0 \tau \tau^*} \partial_\mu T \partial_\nu T^*\) is the kinetic energy of the matter field, and \(\tilde{V}_0 = e^{\kappa K_0}\times (K_0^{TT^*} |D_T W_0|^2 - 2\kappa |W_0|^2)\) is the part of the potential that depends on the matter fields, only. Here \(K_{0 \tau \tau^*} \equiv \partial_T \partial_T^* K_0\) and \(D_T W_0 \equiv e^{-\kappa K_0} \partial_T (e^{\kappa K_0} W_0)\). We
use the space-time signature (+ − −−) and \( \kappa = 8\pi G \). For supersymmetric minima \( D_T W_0 = 0 \) and thus \( \tilde{V}_0 = -2\kappa e^K_0|W_0|^2 \leq 0 \), i.e., supersymmetric minima have non-negative cosmological constant. The value of the potential \( \tilde{V}_0 = -2\kappa e^K_0|W_0|^2 \leq 0 \) at a supersymmetric minimum is different from the corresponding "ordinary" \( N = 1 \) supergravity one \( V_0 = -3\kappa e^K_0|W_0|^2 \). The additional factor \( \kappa e^K_0|W_0|^2 \) is due to an additional dilaton contribution \( e^{2\phi}|DSW|^{2KSS^*} \equiv \frac{e^{2\phi}}{2} \times \kappa e^K_0|W_0|^2 \) to the total potential in Eq.(1).

A natural frame to which strings couple is the string frame, i.e., the frame of the sigma model expansion of the string effective action. In this case \( (g_s)_{\mu\nu} = e^{2\phi} g_{\mu\nu} \) the scalar part of the action is of the form:

\[
L_s = \sqrt{-g_s e^{-2\phi}} \left[ -\frac{1}{2\kappa} R_s - 2\kappa^{-1} g_s^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + T_0 - \frac{\tilde{V}_0}{2} \right] \quad (2)
\]

We are searching for planar (in \( x,y \) plane), static, supersymmetric dilatonic domain walls interpolating between isolated supersymmetric minima with \( \tilde{V}_{0,1,2} = -2\kappa e^K_0|W_0|^2 \). The metric Ansatz is of the form:

\[
ds^2_E = A_E(z)(dt^2 - dz^2 - dx^2 - dy^2). \quad (3)
\]

and the scalar field \( T(z) \) and the dilaton \( \phi(z) \) depend on \( z \), only. Using a technique of the generalized Nester’s form, as developed for the study of ordinary domain wall configurations in Ref.5, one obtains the relation between supersymmetry transformations and a Bogomol’nyi bound for the ADM energy density of the planar domain wall configuration:

\[
\sigma - |C| = \int_{-\infty}^{\infty} \left[ -\delta_{i\psi}^\dagger g^{ij} \delta_{\psi} \psi_j + K_{TT^*} \delta_{\psi} \chi^\dagger \delta_{\psi} \chi + K_{SS^*} \delta_{\chi} \eta^\dagger \delta_{\chi} \eta \right] dz \geq 0 \quad (4)
\]

where \( \sigma \) is the energy per unit area, \( C \) is the topological charge, \( g_{ij} \) is the metric of
the space coordinates. \( \delta_\epsilon \psi_\mu, \delta_\epsilon \chi \) and \( \delta_\epsilon \eta \) are the supersymmetry variations of the gravitino, supersymmetric partner of the matter field \( T \) and the dilaton \( S \), respectively. For supersymmetric bosonic backgrounds, one has \( \delta_\epsilon \psi_\mu = \delta_\epsilon \chi = \delta_\epsilon \eta = 0 \) which yields coupled first order differential equations (Bogomol’nyi equations) for the metric (3), the complex matter field \( T(z) \) and the dilaton \( \phi(z) \) Ansätze: \(^{21,22}\)

\[
\text{Im}(\partial_z T \frac{DTW_0}{W_0}) = 0, \\
\partial_z T = -\zeta \left( \frac{Ae^{2\phi}}{2} \right)^{\frac{1}{2}} e^{\frac{\kappa_0}{2}|W_0|} |K_{0T}^T| \frac{D_TW_0}{W_0}, \\
\partial_z \log AE = 2\zeta \kappa \left( \frac{Ae^{2\phi}}{2} \right)^{\frac{1}{4}} e^{\frac{\kappa_0}{2}|W_0|}, \\
\partial_z \phi = -\zeta \kappa \left( \frac{Ae^{2\phi}}{2} \right)^{\frac{1}{4}} e^{\frac{\kappa_0}{2}|W_0|}.
\]

\( \zeta \) is either +1 or -1 and can change sign when and only when \( W \) vanishes.\(^{5,6}\)

When Eqs.(5) are satisfied the (Bogomol’nyi) bound for the energy per area \( \sigma \) is saturated by the absolute value of the topological charge \( |C| \). The charge can be unambiguously determined in the thin wall approximation. Then in the wall region \( (z \sim z_0) \) the matter field \( T \) varies rapidly while the metric and the dilaton are slowly varying. We normalize \( AE(z_0) = 1 \) and chose the boundary condition \( e^{2\phi(z_0)} = 1 \): \(^{23}\)

\[
\sigma = |C| \equiv \kappa^{-1} \sqrt{2}(\alpha_1 \pm \alpha_2)
\]

Here, \( \alpha_{1,2} = \kappa e^{\kappa K_0/2} |W_0|_{1,2} = (-\kappa \tilde{V}_0/2)^{1/2} \) where subscript 1 [or 2] refers to the side of the wall with a more [or less] negative value for \( \tilde{V}_0 \). The signs \( \pm \) correspond to the two classes of the solutions with \( W_0 \) crossing zero and \( W \neq 0 \) everywhere, respectively. Note, there are no static walls with \( \tilde{V}_0 \) on both sides of the wall.
The energy density of ordinary supersymmetric domain walls is of a similar form:
\[ \sigma_{\text{ext}} = 2\kappa^{-1}(\alpha_1 \pm \alpha_2) \]
where \( \alpha_{1,2} \equiv \kappa e^{\frac{\kappa K_0}{2}}|W_0|_{1,2} = (-\kappa \tilde{V}_0/3)^{1/2} \) is defined in terms of \( W_0 \) and \( K_0 \) in the same way as above. An additional factor \( 1/\sqrt{2} \) in the case of dilatonic walls is associated with the dilaton contribution to the quantity \( e^{\frac{\kappa K}{2}}|W_0|_{1,2} = 1/\sqrt{2} \times e^{\frac{\kappa K_0}{2}}|W_0|_{1,2} \). Namely, the boundary condition \( e^{2\phi(z_0)} = 1 \) ensures that the effective cosmological constant on each side of the wall is by a factor of \( 1/2 \) less negative, thus decreasing the energy density of the wall by a factor of \( 1/\sqrt{2} \). There is a parallel relation between the mass \( M \) and the charge \( Q \) for extreme charged (Reissner-Nordstrom) black holes \( (M = Q) \) and extreme charged dilatonic black holes \( (M = Q/\sqrt{2}) \). In the domain wall case the role of the charge is played by the parameters \( \alpha_{1,2} \) associated with the value of the matter potential at each minimum.

The first two equations in (5) govern the evolution of the matter field \( T(z) \); the first one corresponds to the “geodesic” equation for the complex \( T \) field and is identical to the one of ordinary supersymmetric domain walls. In the limit, \( \kappa \to 0 \), it reduces to the constraint that the geodesic path of \( T \) corresponds to \( W \) which is a straight line through the origin.

The equation for the conformal factor \( A_E(z) \) and the dilaton field (see Eqs.(5)) imply that \( A_E(z)e^{2\phi(z)} = \text{const.} \) which with the boundary conditions \( A_E(z_0) = e^{2\phi(z_0)} = 1 \) imply:

\[ A_s(z) \equiv A_E(z)e^{2\phi(z)} = 1 \]

Therefore, the metric factor \( A_s(z) \) in the string frame is flat, i.e., independent of the value of the matter potential everywhere in the domain wall background. Although there is a nontrivial matter potential, the dilaton field adjusts itself in the domain
wall background in such a way as to leave the string metric flat; strings do not “feel” the wall. In addition, the second equation for the matter field decouples from the metric and dilaton equations and the metric factor $A_E(z)$ can be expressed in terms of the matter field as $A_E(z) = \exp(\int_{z_0}^z dz \sqrt{2\zeta _K e^\kappa K_0/2}|W_0|)$.

There are two types of $\text{AdS} (\alpha_1 \neq 0) - \text{AdS} (\alpha_2 \neq 0)$ walls corresponding to the two signs in Eq.(6). Here $\text{AdS}$ refers to the space-time with the dilaton modulated negative cosmological constant. The $+$ sign in Eq.(6) corresponds to solution with $W_0$ traversing zero; in this case the form of the metric outside the wall is $A_E(z)_{1,2} = e^{-\sqrt{2}\alpha_1 |z|}$. The $-$ sign in Eq.(6) corresponds to the case with $W_0 \neq 0$ everywhere and the form of the metric outside the wall is: $A_E(z)_1 = e^{-\sqrt{2}\alpha_1 |z|}$ and $A_E(z)_2 = e^{\sqrt{2}\alpha_2 |z|}$. As $z \to \pm \infty$, $A_E(z) = e^{-\sqrt{2}\alpha_1 |z|} \to 0$ and thus both the dilaton field and the curvature blow up in this region. However, this singularity is an infinite geodesic distance away. On the other hand, as $|z| \to \infty$, $A_E(z)_2 = e^{\sqrt{2}\alpha_2 |z|} \to \infty$, which corresponds to the zero curvature space-time and is geodesically complete. Note, that in this region $e^{2\phi} = e^{-\sqrt{2}\alpha_2 |z|} \to 0$ (see Eq.(7)) and thus the effective cosmological constant $\Lambda = \kappa^{1/2} e^{2\phi} \tilde{V}_0/2 \to 0$.

In the following we discuss a special case: $\text{AdS} (\alpha_1 \neq 0) - M(\alpha_2 = 0)$ walls. $M$ refers to the Minkowski space with zero cosmological constant. In this case the thin wall solution (located at $z_0 = 0$) has the explicit form:

$$\sigma = \sqrt{2}\kappa^{-1}\alpha_1; \quad A_E(z)_1 = e^{-\sqrt{2}\alpha_1 z}, \quad z < 0; \quad A_E(z)_2 = 1, \quad z > 0. \quad (8)$$

where $\alpha_1$ is defined after Eq.(6). For illustrative purposes we also present in Figure 1 an explicit finite size wall solution for $T(z) \in \mathcal{R}$ (solid line) and $A_E(z)$ (dashed line). We chose an example with $W_0 = \sqrt{\kappa T^2(T^2 - 2\alpha^2/\kappa)}$, $K_0 = TT^*$ and
\[ a^2 = 0.1. \] The wall interpolates between \( T = 0 \) and \( T \sim a/\sqrt{\kappa} \) and has a thickness \( \mathcal{O}(\sqrt{\kappa}/a^2) \).\(^{24}\)

The Penrose diagram for such walls in the \((z, t)\) plane is given on Figure 2. The \( M \) side \((\alpha_2 = 0)\) corresponds to Minkowski space-time while the \( AdS \) side \((\alpha_1 < 0)\) exhibits singularity an infinite geodesic distance away. Note a formal similarity with the Penrose diagram\(^{10,11}\) for the \((r, t)\) plane of the extreme charged dilatonic black hole.

Extreme dilatonic domain walls are solutions of the 4\(d\) effective superstring action (evaluated to all orders in string loops) with isolated minima of the matter potential preserving supersymmetry. Eventually, supersymmetry should be spontaneously broken. Current proposals rely on non-perturbatively induced gaugino condensates (of the hidden gauge groups)\(^{25}\) which introduce new terms \( \propto e^{-cS} \) in the superpotential. Here \( c \) is a positive constant proportional to the beta functions of the hidden gauge groups. The analysis for this case has to be redone; there is a wealth of new wall solutions which need not be planar and static anymore and will be addressed elsewhere. Within dilatonic black holes analogous solutions with different dilaton potentials has been studied in Refs. 26,27.

There is an alternative possibility where supersymmetry is broken spontaneously by the matter part of the potential \((\tilde{V}_0)\). This case is similar to the case of non-extreme of charged dilatonic black holes with \( M \neq Q/\sqrt{2} \). Now, the wall need not be static any more. A convenient way is to write the metric in the wall’s rest frame and assuming that the \((2 + 1)d\) space-time internal to the wall is homogeneous, isotropic and geodesically complete. The general form of metric (compatible with the constraint that \( \sigma > 0 \)) is then of the form:\(^4\)

\[
ds^2 = A_{E}(z)[dt^2 - dz^2 - (\cosh \beta t)^2/\beta^2 d\Omega^2_2].
\]

It corresponds to time dependent
bubbles where $\beta$ parametrizes a deviation of this solution from the supersymmetric one. As $\beta \to 0$, the metric reduces to the extreme one (see Eq.(3)).

We address solutions corresponding to the $AdS - M$ expanding bubbles with an inertial observer inside the expanding bubble on each side of the wall. On the $M$ side there is a unique solution: $A_E(z)_2 = e^{-2\beta|z|}$, $\phi_2 = 0$ which is identical to the ordinary Minkowski non-extreme wall solution. In the rest frame of the wall the metric solution exhibits a cosmological horizon identical to the one of the Schwarzschild black hole horizon. In the inertial Minkowski coordinates the wall is expanding for $t > 0$. On the $AdS$ side the equations can be solved perturbatively for $\beta \gg \alpha$: $A_E(z)_1 = \exp \{-2\beta|z| - \alpha_1^2/(6\beta^2)[1 - \exp (-2\beta|z|)]\}$, $\phi_1 = \alpha_1^2/(8\beta^2)[1 - \exp (-2\beta|z|)]$. The energy density of the wall is: $\sigma = \kappa^{-1}[4\beta + \alpha_1^2/(3\beta)]$. On $AdS$ side the metric also exhibits cosmological horizons whose nature is subject to further investigation. In particular one is interested in the global space-time structure on the $AdS$ side of the wall.

We found extreme (supersymmetric) dilatonic domain walls specific to isolated 4d superstring vacua. Such walls are static configurations with matter fields in general interpolating between non-equal minima of the matter potential. Everywhere in the domain wall background the dilaton field adjusts itself in a way as to leave metric in the string frame flat; strings do not “feel” the wall. Intriguing similarities between extreme dilatonic walls and extreme charged dilatonic black holes are pointed out.

I would like to thank H. Soleng for collaboration at the initial stages of the work and for useful discussions. I also benefited from discussions with R. Davis, S. Griffies, and J. Horne. The work was supported by U. S. DOE Grant No. DOE-EY-76-C-02-3071, and NATO Research Grant No. 900-700.
Figure Captions

Figure 1. Solutions for $T(z)\sqrt{\kappa}/a$ (solid line) and $A_E(z)$ as a function of $\tilde{z} = a^2 z/\sqrt{\kappa}$ for the example given in the text and $a^2 = 0.1$.

Figure 2. Penrose diagram in the $(z,t)$ plane for the finite size extreme dilatonic domain wall. The matter potential $\tilde{V}_0 = 0$ for $z > 0$ ($M$ region) and $\tilde{V}_0 < 0$ for $z < 0$ ($AdS$ region). We use the standard compactified null coordinates $-\pi \leq (u',v') \equiv 2 \tan^{-1}(t \pm z) \leq \pi$. Note the null singularity on the $AdS$ side.
REFERENCES

1. For an attempt to study domain walls within the Jordan-Brans-Dicke theory see H.-S. La, CTP-TAMU-52/92 hep-ph/9212041, CTP-TAMU-78/92 hep-th/9207202.

2. A. Vilenkin, Phys. Lett. 133B, 177 (1983).

3. J. Ipser and P. Sikivie, Phys. Rev. D 30, 712 (1984).

4. M. Cvetič, S. Griffies and H. Soleng, Non- and Ultra- Domain Walls and their Global Space-Time, UPR-546-T/Rev (March 1993).

5. M. Cvetič, S. Griffies, and S.-J. Rey, Nucl. Phys. B381, 301 (1992).

6. M. Cvetič and S. Griffies, Phys. Lett. B 285, 27 (1992).

7. For a review, see: M. Cvetič and S. Griffies, in Proc. Int. Symp. on Black Holes, Membranes and Wormholes, The Woodlands, Texas, January 1992, edited by S. Kalara and D. Nanopoulos (World Scientific, Singapore, in press).

8. M. Cvetič, R. Davis, S. Griffies and H. H. Soleng, Phys. Rev. Lett. 70, 1191 (1993).

9. G. W. Gibbons, DAMTP preprint R-92/38, October 1992, Nucl. Phys. B (in press).

10. G. Gibbons, Nucl. Phys. B207, 337 (1992); G. Gibbons and K. Maeda, Nucl. Phys. B298, 741 (1988); D. Garfinkle, G. Horowitz and A. Strominger, Phys. Rev. D43, 3140 (1991), D45, 3888 (1992)E.

11. For a review see G. Horowitz, The Dark Side of String Theory: Black Holes and Black Strings, USCB-92-32 (October 1992) and references therein.

12. V. A. Berezin, V. A. Kuzmin, and I. I. Tkachev, Phys. Lett. 120B, 91 (1983).
13. H. Sato, Progr. Theor. Phys. 76, 1250 (1986); S. K. Blau, E. I. Guendelman, and A. H. Guth, Phys. Rev. D 35, 1747 (1987); V. A. Berezin, V. A. Kuzmin, and I. I. Tkachev, Phys. Rev. D 36, 2919 (1987).

14. S. Coleman and F. De Luccia, Phys. Rev. D 21, 3305 (1980).

15. In the black hole context, see G. W. Gibbons in: Supersymmetry, Supergravity and Related Topics, edited by F. del Aguila et al. (World Scientific, Singapore 1985), p. 147; R. E. Kallosh, A. D. Linde, T. M. Ortín, A. W. Peet, and A. van Proeyen, Phys. Rev. D 46, 5278 (1992).

16. E. Witten, Nucl. Phys. B268, 373 (1986).

17. E. Witten, Phys. Lett. B155, 151 (1985).

18. A superpotential for the dilaton can, however, be induced non-perturbatively. Also, the Kähler potential receives one-loop corrections due to mixed Yang-Mills-σ model anomalies. See, G. Cardoso and B. Ovrut, Nucl. Phys. B369, 351 (1992); J.P. Derendinger, S Ferrara, C Kounnas and F. Zwirner, Nucl. Phys. B372, 145 (1992). The above effects should eventually be included in the full treatment of the theory.

19. Since we study the part of the effective Lagrangian which depends on $S + S^*$ only, the Lagrangian in terms of the linear dilaton supermultiplet is equivalent to the one in terms of the chiral dilaton supermultiplet.

20. W. Israel, Nuovo Cimento 44B, 1 (1966).

21. Outside the wall region the same solutions can be obtained (M. Cvetič and H. Soleng, unpublished) in the thin wall approximation. Outside the wall $|z| > z_0$ the metric Ansatz (Eq.(3)) satisfies Einstein’s equations: $\partial_z H - H^2/2 = -2(\partial_z \phi)^2$, $\partial_z H + H^2 = 2\alpha_{1,2}^2 e^{2\phi} A_E(z)$, where $H \equiv \partial_z \ln A_E(z)$, and the
dilaton satisfies the Euler-Lagrange equation: \( H \partial_z \phi + \partial_z^2 \phi + \alpha_{1,2}^2 e^{2\phi} A_E = 0 \). Here \( \alpha_{1,2} = (-\kappa \tilde{V}_0/2)^{1/2}_{1,2} \). Using matching conditions across the wall surface one obtains \( \sigma = H_{\tilde{z}0}^+ - H_{\tilde{z}0}^- \) which is the same as the saturated Bogomol’nyi bound (6).

22. One can obtain related supersymmetric (“no-scale”) domain wall solutions also in the so called no-scale supergravity models with \( K = -\kappa^{-1} C \log(M + M^*) + K_0(T, T^*) \) (\( C = 1, 2, 3 \)) and \( W = W_0(T) \). Along with the matter fields \( T(z) \) and metric also the field \( M(z) \) changes in the wall’s background. Theories with such a form of scalar interactions are realized in certain superstring vacua with \( M \)'s corresponding to untwisted moduli.

23. A more general boundary condition \( e^{2\phi(z_0)} = e^{2\phi_0} \) allows for a family of one parameter solutions with the energy density (6) determined in terms of \( \alpha_{1,2} \equiv \kappa e^{(\phi_0 + \kappa K_0/2)}|W_0|_{1,2} \). For \( \phi_0 > 0 \), \( \sigma \) is increased due to an additional dilaton contribution. Also, Eq.(7) is rewritten as \( A_s(z) = A_E(z) e^{2\phi(z)} = e^{2\phi_0} \) and \( A_E(z) = \exp\left[\int_{z_0}^z dz \sqrt{2} \kappa e^{(\phi_0 + \kappa K_0/2)}|W_0|\right] \).

24. In the case \( a^2 \ll 1 \), equations (5) can be solved explicitly: \( T(z) = a e^{\tilde{z}}/(1 + e^{2\tilde{z}})^{1/2} \) and \( A_E(z) = (1 + e^{2\tilde{z}})^{-a^2/\sqrt{2}} \exp\{-a^2/[\sqrt{2}(e^{-2\tilde{z}} + 1)]\} \) where \( \tilde{z} = a^2 z/\sqrt{\kappa} \).

25. L. Ibáñez and P. Nilles, Phys. Lett. 155 65 (1985); M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B156, 55 (1985).

26. R. Gregory and J. Harvey, Enrico Fermi Preprint EFI-92-49 and J. Horn and G. Horowitz, Santa Barbara Preprint UCSBTH-92-17.

27. 2d black holes with the massive dilaton were studied by M. McGuigan, C.Nappi, and S. Yost, Nucl. Phys. B375, 421 (1992).
28. Note that other solutions $A_E(z)_2 = e^{2\beta |z|}$, $\phi_2 = 0$ and $A_E(z)_2 = \cosh 2\beta |z|$, $\exp 2\phi_2 = \text{tanh} (\beta |z|)^{\sqrt{3}}$ correspond to expanding bubbles with an inertial observer inevitably being hit by the bubble, i.e., those are bubble of false vacuum decay.