Spectral and Energy Efficiency for Massive MIMO Two-way Hybrid Relaying with Multi-Pair Users

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Abstract: In this paper, we derive a general non-vanishing expression of signal to interference and noise ratio (SINR) considering hybrid relaying processing (HRP) in a massive multiple-input multiple-output (MIMO) half-duplex amplify-and-forward (AF) two-way relaying networks with multi-pair users. Then the general asymptotic values of system spectral efficiency (SSE) and system energy efficiency (SEE) can be compared under different power scaling schemes, when the relaying antenna number $M$ increases sufficiently large. Then we achieve the optimal power scaling scheme of SSE and SEE with the considerably large $M$. Finally, all comparison analyses of SSE and SEE are confirmed by simulation results.

Keywords: Massive MIMO, Two-way relaying, Hybrid relaying processing, Energy efficiency, Spectral efficiency

Classification: Wireless communication technologies

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1 Introduction

In conventional relaying with massive MIMO, relatively high energy consumption, implementation cost and computational complexity are problems because each radio-frequency (RF) chain at the relay station (RS) is dedicated to one antenna. To alleviate this challenge, analog and digital hybrid relaying architecture, where the RF chain number is less than the antenna number, has been investigated in massive MIMO networks [1]. The system is composed of an analog high-dimensional RF beamformer/combiner to adjust only the signal phase by analog phase shifters and a digital low-dimensional baseband processor to adjust both amplitude and phase of the signal.

Recently, some researches on hybrid processing have been investigated. Sharma et al. [2] derived asymptotic SSE and SEE expressions for three power scaling schemes with zero-forcing reception/zero-forcing transmission processing. The spectral efficiency of each user-pair through an algebraic norm was maximized in [3]. Oh et al. [4] optimized the sum-rate maximization problem based on a hybrid filter design including the RF filter design and the baseband filters. Sharma et al. [5] optimized for the proposed generalized singular value decomposition transceiver for digital relay precoder and the precoder/decoders of the user. However, the general expressions of SSE and SEE with considerably large $M$ for the massive MIMO AF two-way hybrid relaying networks under power scaling schemes based on the signal processing of the maximum-ratio combining/maximum-ratio transmission (MRC/MRT) has not been deduced yet, which motivates our research.

In this paper, we determine the general expression of SINR analytically, from which the asymptotic SSE and SEE can be deduced, as the number of relaying antenna increases sufficiently large. Then the general asymptotic expressions of the SSE and SEE are deduced in the symmetric hybrid massive MIMO AF two-way relaying networks. For a non-vanishing SINR, three specific power scaling scheme cases are achieved by modifying the power scaling factors of transmit power of both the RS and user terminals. By comparing those power scaling cases, we can achieve the optimal power scaling case of SSE and SEE, respectively. Finally, we employ Monte-Carlo simulations to verify our analytical results.
2 Large $M$ Analysis with Power Scaling

We consider a cooperative wireless system, where $2K$ users make up $K$ communication pairs for exchanging information within a communication pair via a half-duplex hybrid AF RS deployed with $M$ ($M \gg 2K$). Each user is placed with a single antenna. Two users in the $i$th communication pair denoted by $(2i-1, 2i)$, $i = 1, \ldots, K$, exchange information within each other. For user $k'$, the signal from user $k$ is the effective signal. For simple analysis, assuming that there is no direct link between all communication pairs according to severe shadow fading or path loss attenuation. At the RS side, to keep low system cost, we assume an analog combiner $F_1 \in \mathbb{C}^{K_r \times M}$ with $K_r$ receive RF chains and an analog precoder $F_2 \in \mathbb{C}^{K_t \times M}$ with $K_t$ transmit RF chains, satisfying $2K \leq K_r, K_t \ll M$. This analog processing is relatively inflexible and high-dimensional. Meanwhile, a low-dimensional and complicated signal processing is implemented in the digital baseband region, which is expressed as a digital combiner $W_1 \in \mathbb{C}^{K_t \times 2K}$ and a digital precoder $W_2 \in \mathbb{C}^{K_t \times 2K}$.

At the first signal transmission phase, the whole $2K$ users send their individual signals to the RS, then the received signal at the RS is

$$y_R = Gx + n_R,$$

where $y_R \in \mathbb{C}^{M \times 1}$, and $x = [x_1, x_2, \ldots, x_{2K}]^T$ is the transmit signal from $2K$ users with independent symbol satisfying $E\{x_k x_k^*\} = p$ for $k = 1, \ldots, 2K$; $n_R \sim \mathcal{CN}(0, \sigma_R^2 I_M)$ is the Additive White Gaussian Noise (AWGN) at the RS. We consider the channel reciprocity. $G = [g_1, \ldots, g_{2K}] = HD^2 \in \mathbb{C}^{M \times 2K}$ is the channel matrix from $2K$ users to the RS with $g_k \sim \mathcal{CN}(0, \eta_k I_M)$, where $H \in \mathbb{C}^{M \times 2K}$ is the small-scale fading channel matrix between $2K$ users and $RS$ with each element satisfying the distribution of $\mathcal{CN}(0, 1)$, and $D \in \mathbb{C}^{2K \times 2K}$ denotes the diagonal large-scale fading channel matrix with $[D]_{kk} = \eta_k$. We assume that all channels between RS and $2K$ users follow the Rayleigh fading and experience independent identically distribution.

Then the processed transmit signal from RS is

$$x_R = Py_R,$$

where $P = \alpha F_2^H W_2 Q W_1^H F_1$ is the hybrid AF relaying matrix. The element amplitude of matrices $F_1$ and $F_2$ should be fixed by $1/\sqrt{M}$, i.e., $\angle[F_1]_{i,j} = -\angle[G]_{j,i}$, $\angle[F_2]_{i,j} = -\angle[G^*]_{j,i}$, and $|[F_1]_{i,j}| = |[F_2]_{i,j}| = \frac{1}{\sqrt{M}}$; $Q = \text{diag}(Q_1, Q_2, \ldots, Q_K), Q_i = [0 \ 1 \ 1 \ 0]$ for $i = 1, \ldots, K$; and $\alpha$ is a factor of normalization constant to assure the transmit power constraint from the RS. Because MRC/MRT guarantees a near-optimal system performance [6], we set $W_1^H = G^H F_1^H$ to combine all received signals at the RS and $W_2 = F_2 G^*$ to broadcast all received signals from the RS to the user terminals, thus we have $P = \alpha F_2^H W_2 G^* Q G^H F_1^H$.

In the next information transmission phase, the RS broadcasts $x_R$ to all $2K$ user terminals with the transmit power $Tr(E\{x_R x_R^*\}) = P_R$, then every user receives this signal packet which is given by

$$y_D = G^T P G x + G^T P n_R + n_D,$$
where $n_D \sim \mathcal{CN}(0, \sigma_D^2 I_M)$ is the AWGN vector of all $2K$ users. Assuming that $2K$ users know their own transmit symbol, and so user $k'$ in pair $(k, k')$ can remove its information. The remaining signal at user $U_{k'}$ is

$$y_{D_{k'}} = g_{k'}^T P g_k x_k + \sum_{i=1,i \neq k,k'}^{2K} g_{k'}^T P g_i x_i + g_{k'}^T P n_R + n_{D_{k'}}. \quad (2.4)$$

Therefore, the SINR of $U_{k'}$ for $k' = 1, \ldots, 2K$ is [2]

$$\text{SINR}_{k'} = \frac{p |g_{k'}^T P g_k|^2}{p \sum_{i=1,i \neq k,k'}^{2K} |g_{k'}^T P g_i|^2 + \sigma_R^2 \|g_{k'}^T P\|^2 + \sigma_D^2}. \quad (2.5)$$

We consider power scaling schemes both at RS and user sides with $p = E_U / M^a$, $P_R = E_R / M^b$, where $E_U$ and $E_R$ are both constants, $a \geq 0$ and $b \geq 0$. Here $a$ and $b$ are the power scaling factor of transmit power of each user and RS, respectively. The asymptotic SINR of user $k'$ under power scaling scheme for $k' = 1, \ldots, 2K$ when $M \to \infty$ is

$$\text{SINR}_{k'} \xrightarrow{a.s.} M^{-a-b} \left(\frac{\pi}{4}\right)^4 \eta_k^2 \eta_k^2 \frac{E_U E_R}{M} \cdot \frac{M^1 - b E_R \left(\frac{\pi}{4}\right)^3 \eta_k^2 \eta_k \sigma_R^2 + M^{1-a} \eta_k^2 \sigma_D^2 E_U \kappa_1 + \sigma_R^2 \sigma_D^2 \kappa_2}{M^1 - b E_R \left(\frac{\pi}{4}\right)^3 \eta_k^2 \eta_k \sigma_R^2 + M^{1-a} \eta_k^2 \sigma_D^2 E_U \kappa_1 + \sigma_R^2 \sigma_D^2 \kappa_2}. \quad (2.6)$$

**Proof.** Recalling $\text{Tr} \left( \left[ x_R x_R^H \right] \right) = P_R$, we have $\alpha = \frac{P_R}{p \text{Tr} (\Omega_1) + \sigma_R^2 \text{Tr} (\Omega_2)}$, where $\Omega_1 = F_1^H F_2 G^{*} Q G^{H} F_1^H H F_1^H G G^{H} F_1^H F_2$ and $\Omega_2 = F_2^H F_2 G^{*} Q G^{H} F_1^H H F_1^H F_2 G G^{H} F_1^H F_2$. According to the large number law and lemma 2-3 in [7], we have

$$F_1 G \xrightarrow{a.s.} \sqrt{\frac{M \pi}{4}} D^\frac{1}{2}, \text{ and } G^T F_2^H \xrightarrow{a.s.} \sqrt{\frac{M \pi}{4}} (D^\frac{1}{2})^T. \quad (2.7)$$

Substituting Eq.(2.7) into $\Omega_1$ and $\Omega_2$ above, we have $\text{Tr} (\Omega_1) \xrightarrow{M \to \infty} M^3 \kappa_1$, where $\kappa_1 = \sum_{k=1}^{K} \eta_{k_1} \kappa_2$, and $\text{Tr} (\Omega_2) \xrightarrow{M \to \infty} M^2 \kappa_2$, where $\kappa_2 = \sum_{k=1}^{K} \sum_{k=1}^{K} \eta_{k_1} \eta_{k_2}$. Plugging asymptotic values of $\text{Tr} (\Omega_1)$ and $\text{Tr} (\Omega_2)$ into the equation of $\alpha$ above, we obtain $\alpha \xrightarrow{a.s.} \sqrt{\frac{P_R}{p \text{Tr} (\Omega_1) + \sigma_R^2 \text{Tr} (\Omega_2)}}$.

Substituting Eq.(2.7) and asymptotic $\alpha$ into Eq.(2.4), the received signal from user $k'$ for $M \to \infty$ is

$$y_{D_{k'}} \xrightarrow{M \to \infty} \alpha \left(\frac{M \pi}{4}\right)^2 \sqrt{\eta_k^2 k_{k'}} (D^\frac{1}{2})^* Q (D^\frac{1}{2})^H \sqrt{\eta_k^2 k_{k'}} x_k \quad (2.8)$$

$$\quad + \alpha \sum_{i=1,i \neq k,k'}^{2K} \left(\frac{M \pi}{4}\right)^2 \sqrt{\eta_k^2 k_{k'}} (D^\frac{1}{2})^* Q (D^\frac{1}{2})^H \sqrt{\eta_k^2 k_{k'}} x_i$$

$$\quad + \alpha \left(\frac{M \pi}{4}\right)^2 \sqrt{\eta_k^2 k_{k'}} (D^\frac{1}{2})^* Q (D^\frac{1}{2})^H n_R + n_{D_{k'}}$$

$$\quad = \alpha \left(\frac{M \pi}{4}\right)^2 \eta_k^2 x_k + \alpha \left(\frac{M \pi}{4}\right)^2 \eta_k^2 n_{R_{k'}} + n_{D_{k'}}.$$

From Eq.(2.8), the asymptotic expression of SINR of $U_{k'}$ for $k' = 1, \ldots, 2K$ can be easily obtained. □
The equations of SSE and SEE for two-way hybrid relaying networks are defined in [2] as
\[
R = \frac{1}{2} \sum_{i=1}^{2K} \mathbb{E} \left[ \log_2 \left( 1 + \text{sINR}_i \right) \right],
\]
\[
\rho = \frac{R}{2KP + PR},
\]
where 1/2 in Eq.(2.9) of SSE means two phases of the whole signal transmission process, and \(2KP + PR\) in Eq.(2.10) of SEE is the total system power consumption from RS and all 2K users. The asymptotic SSE and SEE will be deduced when substituting Eq.(2.6) into Eq.(2.9) and Eq.(2.10), respectively.

Considering the symmetric system, i.e., \(\eta_i = 1\) for \(i = 1, \ldots, 2K\) and \(E_U = E_R = E\). From Eq.(2.6) for \(0 \leq a, b \leq 1\), SINR increases as \(M\) rises without bound. Particularly, SINR converges to the constant for three cases: Case I: \(2 - a - b = 1 - b > \max(1 - a, 0)\), Case II: \(2 - a - b = 1 - a > \max(1 - b, 0)\) and Case III: \(2 - a - b = 1 - b = 1 - a = 0\); or equivalently Case I \((a = 1, 0 \leq b < 1)\), Case II \((0 \leq a < 1, b = 1)\) and Case III \((a = b = 1)\), respectively. Substituting the SINR in Eq.(2.6) into Eq.(2.9) and Eq.(2.10), the SSEs and SEEs for three cases in symmetric system become
\[
P_{c1}^{\text{sym}} \overset{\text{a.s.}}{\to} K \log_2 \left(1 + \frac{\pi E}{4\sigma^2_R} \right), \quad \rho_{c1}^{\text{sym}} \overset{\text{a.s.}}{\to} \frac{K}{E} \log_2 \left(1 + \frac{\pi E}{4\sigma^2_R} \right),
\]
\[
P_{c2}^{\text{sym}} \overset{\text{a.s.}}{\to} K \log_2 \left(1 + \frac{E(\frac{x}{y})^4}{2K\sigma^2_D} \right), \quad \rho_{c2}^{\text{sym}} \overset{\text{a.s.}}{\to} \frac{1}{2E} \log_2 \left(1 + \frac{E(\frac{x}{y})^4}{2K\sigma^2_D} \right),
\]
\[
P_{c3}^{\text{sym}} \overset{\text{a.s.}}{\to} K \log_2 \left(1 + \frac{\pi E}{4\sigma^2_R} \times \frac{E(\frac{x}{y})^4}{2K\sigma^2_D + \frac{\pi}{4}} \right),
\]
\[
\rho_{c3}^{\text{sym}} \overset{\text{a.s.}}{\to} \frac{MK}{2KE} \log_2 \left(1 + \frac{\pi E}{4\sigma^2_R} \times \frac{E(\frac{x}{y})^4}{2K\sigma^2_D + \frac{\pi}{4}} \right).
\]

Due to \(\frac{xy}{x+y+z} < \frac{xy}{x+y} \leq \min\{x, y\}\), comparing Eq.(2.11)-(2.14), it can be achieved that Case I \((a = 1, 0 \leq b < 1)\) is the greatest scheme of SSE. But Case III \((a = b = 1)\) is the optimal case for SEE because the asymptotic SEE in Case III increases linearly with \(M\). Meanwhile, the asymptotic SSE and SEE in Case I \((a = 1, 0 \leq b < 1)\) are both proportional to \(K\) but the asymptotic SEE in Case II \((0 \leq a < 1, b = 1)\) decreases with \(K\).

3 Simulations

In this section, numerical and simulated results by the Monte Carlo method are presented to investigate the system performance. In Figures 1 and 2, we denote "Asym" and "Sim" as the asymptotic results and the simulation results. We assume that \(E_U = E_R = 10\), \(K = 2\) and \(K_r = K_t = 2K\), the setting of \(M\) refers to [5], and all noise variance \(\sigma_n^2 = \sigma_r^2 = 1\).

The simulated and asymptotic SSE and SEE versus \(M\) are shown as Fig.1 and Fig.2, respectively. We can observe that with the increase of \(M\), all SSEs and SEEs approach to the corresponding certain constant values for \(K = 2\).
Fig. 1: The SSE versus $M$ where $K = 2, 4$.

Fig. 2: The SEE versus $M$ where $K = 2, 4$.

and $K = 4$, respectively. When $M \to \infty$, the asymptotic SSE in Case I for $K = 4$ is about 12.58 bps/Hz which is twice as 6.29 bps/Hz for $K = 2$. Additionally, the asymptotic SEE in Case I for $K = 4$ is 1.26 bits/J which is about twice as 0.63 bits/J for $K = 2$. For Case II, when $M \to \infty$, the simulated value of SEE for $K = 2$ is greater than that for $K = 4$. Therefore, both the asymptotic SSE and SEE for Case I grow linearly with $K$ but SEE in Case II and $K$ are negatively correlated, which verifies our analyses. When $K = 2$ and $M \to \infty$, the SSE in Case II goes to 3.13 bps/Hz and the SSE in Case III tends to 2.56 bps/Hz. Therefore, we can observe that Case III has the lowest SSE among three cases, which is consistent with our analytical results. Particularly, it can be directly observed that the SEE grows linearly with $M$, which means Case III will achieve the maximal SEE.

4 Conclusions

In this paper, considering HRP with massive MIMO, we investigated the asymptotic SSE and SEE under specific three power scaling cases in the multiple communication pair two-way networks. When the antenna number at RS $M$ tends to considerably large, the best power scaling case of SSE is Case I, where only the transmit power of each user was reduced by $M$. But Case III, with the transmit power of both each user and RS scaled down by $M$, has the best power scaling scheme of SEE for $M \to \infty$. Numerical results in the simulation section are presented to verify our analyses.