Implementation of Multivariate Exponentially Weighted Mean Square (MEWMS) control chart for quality control of wing parts of Airbus aircraft at PT Dirgantara Indonesia

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Abstract. Quality control must be carried out very tightly for the production of single wing multivariate dispersion aircraft manufacturing. This research was conducted at PT. Dirgantara Indonesia in producing components of Airbus aircraft wings centered in Germany and France. The method used includes three stages, namely determining the single observation MEWMS diagram, determining the algorithm for determining L, and implementing MEWMS. The data used are L simulation data for p = 2 and p = 3, ARL = 370, and various values, key characteristic data from 31 wing component components of the aircraft fly with. The results showed that the more variables, the L value as the determinant of the width of the MEWMS control limit was smaller, and the greater the value of ω, the value of L as the width of the MEWMS control border width is wider.

1. Introduction

Multivariate Exponentially Weighted Moving Average (MEWMA), Multivariate Cumulative Sum (MCUSUM) and Minimax Multivariate Quality Control Chart (MMQC) [1-3]. Whereas multivariate dispersion control includes control diagrams based on testing of covariance matrix, generalized variance (GV), Multiple EWMA, Multiple CUSUM and vector variance (VV) [4,5].

In the process of manufacturing, industry or manufacturers need to measure several mutually correlated characteristics that aim to control the quality of the production process by applying multivariate statistical methods. Some methods that have been developed include Chi-squared, Hotelling Statistical control diagram, Multivariate Exponentially Weighted Moving Average Process quality control in general is done by taking n-sized observations within a certain period of time known as rational subgroup concept [1,6]. In the production process with a large scale each period is usually n> 1, while in the production process with a small scale or destructive checks are usually n = 1. PT. Dirgantara Indonesia currently produces aircraft wing components ordered by Airbus aircraft companies based in Germany and France.

The control diagram for controlling multivariate averages with a single sample size is a special form, whereas for controlling multivariate dispersions with a single sample it cannot be developed from the control chart above. The control chart for multivariate dispersion with a single sample size, one of which
is the multivariate control chart, exponentially weighted mean square (MEWMS) with multivariate dispersion size, is total variance (TV) [6-8].

Control limits are made for phase II control (known parameters) depending on the number of parameters, the weight of the observation of the ongoing period and the value of the Average Run Length in control (L). Control limits were determined for the number of variables $p = 2$ and $p = 3$ through Monte-Carlo simulation studies. For this reason, the main focus in this study is to determine the L value as the determinant of the width of the MEWMS control limits for the number of variables $p = 5$ also through the Monte-Carlo simulation method. MEWMS Control Chart is implemented in controlling aircraft wing component products at PT. Indonesian Aerospace with mean vector parameters and covariance matrix calculated based on historical data. MEWMS performance is better than Multiple CUSUM and Multiple EWMA in terms of detecting changes in variance. Based on the explanation above, this research specializes in the formation of MEWMS control diagrams at PT Dirgantara by determining the L value as the determinant of the width of the control boundaries.

2. Method

This research was conducted at PT. Dirgantara Indonesia is producing aircraft wing components ordered by Airbus aircraft companies based in Germany and France. The data used are L simulation data for $p = 2$ and $p = 3$, $ARL = 370$, and various values, key characteristic data from 31 flying wing component products with mean vector parameters and covariance matrix are calculated based on historical data. The research methodology generally consists of 3 (three) important stages in this research, namely the determination of a single observation MEWMS diagram, the determination of algorithms to determine L, and MEWMS Implementation.

2.1. Determine the MEWMS diagram

This stage is a procedure for determining control limits starting with determining a new variable through a transformation with an average vector and variance. Through the algebraic process the size of the multivariate dispersion was determined by using the total variance, statistical covariance expectations, and MEWMS diagram control limits [6,7].

2.2. Determine the value of L

To determine L used the simulation algorithm for the number of variables is $p = 5$ which includes:

- Determine the Average Run Length In Control ($L$) value, the omega value = 0.1 (0.1) 0.9, and the control boundary width $L = (2.5 (0.01) 2.8)$
- Set four quality characteristics, written according to the concept notation $X = (X1, X2, X3, X4, X5) t$.
- Generate a $Z$ random vector from a normal multivariate distribution with an average vector $X$ and a covariance matrix $\Sigma = 1$ sample size one (say this is the sample $j, j = 1,2 ...$).
- Determine the Total Variance ,
- Evaluate whether the data is in control or out of control.
- Replicate 100 times. Calculate the average RL (ARL), if if ARL (returns to step 2), if ARL is $\neq$ (the L is not returned to number 3).

2.3. MEWMS implementation

The steps for determining the MEWMS control chart include:

- Multivariate normal assumption checker
- Perform an assessment and then be seen as and
- for the manufacture of control charts used in controlling the next process,
- Perform process control based on data which is then taken in the period through simulations with 2 scenarios.
The MEWMS control diagram with $p = 5$, will be used $= 0.2$ which corresponds to $L = 3.12$. The analysis procedure was carried out using software assistance, Wolfram Mathematica 7.0, and Ms. Excel 2010.

3. Results and discussion

3.1. Value of $L$ simulation results for $P = 2, 3, \text{ and } 5$

The simulation results $p = 5$, are presented as follows: MEWMS diagram $p = 5$ with $\text{ARL} \simeq 3704.3$.

| $\omega$ | $L$ (p=2) | $L$ (p=3) | $L$ (p=5) |
|----------|------------|------------|------------|
| 0.1      | 2.9013     | 2.8212     | 2.72       |
| 0.2      | 3.487      | 3.3281     | 3.12       |
| 0.3      | 3.8713     | 3.6621     | 3.42       |
| 0.4      | 4.1683     | 3.915      | 3.64       |
| 0.5      | 4.4023     | 4.1133     | 3.85       |
| 0.6      | 4.5859     | 4.2715     | 3.91       |
| 0.7      | 4.7266     | 4.3902     | 4.09       |
| 0.8      | 4.8298     | 4.4766     | 4.17       |
| 0.9      | 4.8945     | 4.5315     | 4.19       |

By using simulations in setting $L$ values for $p = 2$ and $p = 3$, $\text{ARL} = 370$, and various values, for $p = 5$ can be obtained with a clearer trend [6]. All values of the $L$ parameters for $p$ that vary show a more general trend that is clearer so as to facilitate decision making. The value of $L$ continues to increase with increasing $\omega$.

3.2. Calculation of control chart

To plot the quantum Mahalanobis distance for each observation of the average vector with theoretical quantiles, as seen in figure 1.
Showing data comes from a multivariate normal distribution because the points on the graph are almost all in a straight line, there are 1 observation that is not close to a straight line, but can still be tolerated [9]. The mean vector estimates and covariance matrices respectively are:

\[
\mu_0 = [0.15891336 \quad 1.80082993 \quad 0.000016 \quad 0.000021 \quad 0.000053]^T
\]

and

\[
\Sigma_0 = \begin{bmatrix}
0.0121852 & -0.00379 & 0.0028912 & 0.0042337 & 0.005618 \\
-0.00379 & 0.0179384 & 0.0011357 & 0.0020644 & 0.000659 \\
0.0028912 & 0.0011357 & 0.0054434 & 0.005876 & 0.005777 \\
0.0042337 & 0.0020644 & 0.005876 & 0.0077835 & 0.008063 \\
0.0056177 & 0.0006589 & 0.0057769 & 0.0080635 & 0.010261
\end{bmatrix}
\]

By treating \( \mu_0 \) and \( \Sigma_0 \) as a process parameter, the control limit is obtained to control the next process as follows.

\[
BKA = 5 + L \sqrt{10 \sum_{i=1}^{i} c_i^2}
\]

\[
GP = 5
\]

\[
BKB = 5 - L \sqrt{10 \sum_{i=1}^{i} c_i^2}
\]

3.3. MEWMS implementation

Scenario I with, \( \omega = 0.2 \) from table 3 obtained \( L = 3.12 \). The limits of control and plots of each observation of generation results can be seen in figure 2.

**Scenario I**, result :

![Figure 2. MEWMS control chart with scenario I.](image)

In figure 2, the control boundaries appear to be constant starting from observations 10 and 96 giving out of control signals, because indeed the 51st and 100th observations begin, the process average has shifted [9,10].

**Scenario I** : The first 50 data are generated from multivariate normal distributions with mean vectors \( \mu_0 \) and covariance matrices \( \Sigma_0 \). The 51st to 100th observations are generated from the same distribution.
as the mean vector shifts to \( \mu_2 \) and the covariance matrix does not change. 101st observation until 150 is generated with the mean and the first three variances having a 100% increase, namely \( \sigma_{11}^2 = k\sigma_{10}^2 \), \( \sigma_{21}^2 = k\sigma_{20}^2 \) and \( \sigma_{31}^2 = k\sigma_{30}^2 \), with \( k = 1.25; 1.5; 1.75; 2 \), and observations from 151 to 200 are generated with the mean \( \mu_1 \) and covariance matrix \( \Sigma_1 \). For MEWMS control diagram with \( p = 5 \), it will be used \( \omega = 0.2 \).

**Scenario II**, result:

Figure 3. MEWMS control chart with scenario II.

Figure 3 shows that the 56th observation gives an out of control signal, because indeed from the 51st and 100th observations, the average process has shifted with the value \( L = 3.12 \), \( \omega = 0.2 \), \( k = 1, 25 \) [9,10]. The results of the scenario for data processing, changes to variance \( k \) can be seen in the table as follows.

**Scenario II**: The first 50 data are generated from multivariate normal distributions with mean vectors \( \mu_0 \) and covariance matrices \( \Sigma_0 \). The 51st observation to 100 is generated with the mean \( \mu_0 \) and the first three variances having a 100% increase, namely \( \sigma_{11}^2 = k\sigma_{10}^2 \), \( \sigma_{21}^2 = k\sigma_{20}^2 \) and \( \sigma_{31}^2 = k\sigma_{30}^2 \), with \( k = 1.25; 1.5; 1.75; 2 \). 101 to 150 observations are generated from the same distribution with the mean vector shifting to \( \mu_1 \) and the covariance matrix does not change and observations from 151 to 200 are generated with the mean and \( \mu_1 \) covariance matrix \( \Sigma_1 \).

**Table 2.** Shift 3 first variance provides out of control signals.

| Scenario I | Scenario II |
|------------|-------------|
| \( \omega \) | 1.25 | 1.5 | 1.75 | 2 |
| \( \omega \) | 1.25 | 1.5 | 1.75 | 2 |
| 0.1 | 17 | 17 | 17 | 18 |
| 0.2 | 96 | 102 | 101 | 102 |
| 0.3 | 107 | 105 | 104 | 102 |
| 0.4 | 150 | 106 | 103 | 103 |
| 0.5 | 168 | 107 | 106 | 104 |
| 0.6 | 120 | 101 | 105 | 105 |
| 0.7 | 110 | 111 | 102 | 106 |
| 0.8 | 188 | 113 | 126 | 108 |
| 0.9 | 198 | 158 | 112 | 112 |
From the simulation results in table 2, it appears that scenario I gives the first signal out of control greater than scenario II, this strengthens the MEWMS control chart for the dispersion control chart because in the first scenario or from 50 to 100 it changes only the average, and in the second scenario which is shifted is the covariance matrix, then in general the MEWMS control chart gives an out of control signal faster and indeed the process reality is out of control [5,6].

From the results of the calculation of data on 31 products in the form of flight wing components with 5 (five) key characteristics at PT. Indonesian Aerospace can obtain results that by using the MEWMS method the quality control process will run well when using a value = 0.2 with an L value of 3.12 because the first 50 data do not show an out of control signal [5,6].

Visually the wing components made by PT. Indonesian Aerospace for Airbus aircraft can be seen in figure 4.

![Airbus aircraft](image1)

*Figure 4. Airbus aircraft.*

The picture of the Airbus wing section model is shown in figure 5.

![Airbus wing section model](image2)

*Figure 5. Key characteristics of model 5.*
4. Conclusion

Quality control of the manufacturing production process with multivariate dispersion can be fulfilled with a multivariate control chart exponentially weighted mean square (MEWMS). The more variables, the smaller the L value as the determinant of the width of the MEWMS control limit. The greater the value of $\omega$, then the value of L as the determinant of the width of the MEWMS control border is wider.

To control the aircraft wing components can be obtained by using vector parameters averaging $\mu_0$ and covariance matrices $\Sigma_0$. In the simulation data with the average vector $\mu_0$ and covariance matrix $\Sigma_0$, the results obtained are significant at $\omega = 0.2$ (L = 3.12). Changes in covarinas matrix can be done for faster detection.

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