Quantum correlation and classical correlation dynamics in the spin-boson model

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Abstract

We study the quantum correlation and classical correlation dynamics in a spin-boson model. For two different forms of spectral density, we obtain analytical results and show that the evolutions of both correlations depend closely on the form of the initial state. At the end of evolution, all correlations initially stored in the spin system transfer to reservoirs. It is found that for a large family of initial states, quantum correlation remains equal to the classical correlation during the course of evolution. In addition, there is no increase in the correlations during the course of evolution.

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It is generally accepted that entanglement plays a crucial role in quantum information processing. However, it is fragile due to the inevitable interaction of the system with its environment. There have been many works on entanglement for all kinds of systems [1–3]. Many extraordinary results have been obtained, such as entanglement sudden death (ESD) [4–6], and sudden birth [7].

However, more and more works show that, even without entanglement, there are quantum tasks [8–10] superior to their classical counterparts. This is due to their nonzero quantum correlation ($Q$). Recently, some special results have been obtained for $Q$ and its counterpart classical correlation ($C$), such as sudden change [11, 12]. Therefore, it is important and interesting to investigate $Q$ and $C$ in more open systems. $Q$, introduced by Henderson and Vedral [13], is identical to quantum discord introduced by Olliver and Zurek [14] in the case of two qubits [11, 15, 16]. $Q$ describes all nonclassical correlations in a state, while entanglement is only a special part of the correlations.

To obtain $Q$, one must acquire $C$ first. As shown in [17], $C(\rho_{s_1s_2}) \equiv \max_{\{\Pi_j\}} [S(\rho_{s_1}) - S_{\{\Pi_j\}}(\rho_{s_1s_2})]$, where the maximum is taken over the set of positive-operator-valued measurements (POVM) $\{\Pi_j\}$ in partition $s_2$. $Q$ is then obtained by subtracting $C$ from the quantum mutual information. In fact, $Q$ is asymmetric [14], it depends on which partition the measurement is carried out in. Only under the condition that $S(\rho_A) = S(\rho_B)$ (where $A$ and $B$ are the two partitions, and $S(\rho)$ denotes the Von Neumann entropy of state $\rho$), does it become symmetric [17]. It is difficult to obtain $Q$ for a general state, owing to the maximal operation. There are only a few works [11, 12, 18] on the dynamics of correlations, and more work is needed to gain a deep understanding of quantum and classical correlations.

In this Brief Report, we investigate the dynamics of $Q$ and $C$ in a spin-boson model, which is conventionally used to study open system. For two different forms of spectral density, we acquire analytical results for both $Q$ and $C$ for different partitions. It is shown that evolutions of $Q$ and $C$ depend closely on the form of the initial state. For a large family of initial states, $Q$ remains equal to $C$ during the course of evolution. We also find oscillation of $Q$ and $C$. Furthermore, we find that all $Q$ and $C$ initially stored in the spin system run into the boson reservoirs continuously without sudden death, which is in sharp contrast with the case for entanglement. In addition, the absence of an increase in correlations is investigated.

The model we are considering is two spins interacting independently with their own boson
reservoirs without interaction between the two spins \[19\]. The Hamiltonian reads as

\[
\hat{H} = \sum_{i=1}^{2} \left( \frac{\omega_i}{2} \hat{\sigma}_i^z + \sum_{k=1}^{N} \omega_k \hat{b}_{i,k}^\dagger \hat{b}_{i,k} \right) + \sum_{i=1}^{2} \sum_{k=1}^{N} g_{i,k} (\hat{\sigma}_i^- \hat{b}_{i,k}^\dagger + \hat{\sigma}_i^+ \hat{b}_{i,k}),
\]

(1)

(with \(N \to \infty\)). Here, \(\hat{b}_{i,k}\) \((i = 1, 2)\) is an annihilation operator of the \(k\)th model in the \(i\)th reservoir with corresponding frequency \(\omega_k\). \(\hat{\sigma}_i^+\), \(\hat{\sigma}_i^-\) \((\hat{\sigma}_i^z)\), and \(\omega\) represent the Pauli operator, raising (lowering) operator, and Zeeman splitting of the \(i\)th spin, and \(g_{i,k}\) denotes the coupling strength between the \(k\)th model in the \(i\)th reservoir and the corresponding spin. As is well known, all information on the coupling between the system and reservoir is included in the spectral density, written as \(J(\omega) = \sum_k |g_k|^2 \delta(\omega - \omega_k)\) (Where we suppose both spins and reservoirs are identical).

Let us first consider the evolution of the initial state with two excitations in the spin system,

\[
|\Phi_0\rangle = (\alpha|0\rangle_{s_1}|0\rangle_{s_2} + \beta|1\rangle_{s_1}|1\rangle_{s_2})|0\rangle_{r_1}|0\rangle_{r_2},
\]

(2)

with \(|\alpha|^2 + |\beta|^2 = 1\), where the collective state \(|0\rangle_{r_i} = \prod_{k=1}^{N} |0_k\rangle_{r_i}\) indicates there is no excitation in reservoir \(r_i\). Evolution of the whole system is given by

\[
|\Psi_t\rangle = \beta(\xi(t)|1\rangle_{s_1}|0\rangle_{r_1} + \sum_{k=1}^{N} \lambda_k(t)|0\rangle_{s_1}|1_k\rangle_{r_1})
\]

\[
\times (\xi(t)|1\rangle_{s_2}|0\rangle_{r_2} + \sum_{l=1}^{N} \lambda_l(t)|0\rangle_{s_2}|1_l\rangle_{r_2})
\]

\[
+ \alpha|0\rangle_{s_1}|0\rangle_{s_2}|0\rangle_{r_1}|0\rangle_{r_2}.
\]

(3)

After introducing the collective state \(|1\rangle_{r_i} = \frac{1}{\chi} \sum_{k=1}^{N} \lambda_k(t)|1_k\rangle_{r_i}\) with one excitation in the reservoir, the reduced density matrix of the spin system is

\[
\rho_{s_1 s_2}(t) = \begin{pmatrix}
|\alpha|^2 + |\beta\chi|^2 & 0 & 0 & \alpha\beta^*\chi^2 \\
0 & |\beta\chi|^2 & 0 & 0 \\
0 & 0 & |\beta\chi|^2 & 0 \\
\alpha^*\beta\chi^2 & 0 & 0 & |\beta\chi|^2
\end{pmatrix}.
\]

(4)

In our case, we find that the maximum can be obtained at \(\theta = \pi/4\) \[11\]; therefore, the
analytical results of $C$ and $Q$ are

$$C(\rho_{s_1s_2}(t)) = Q(\rho_{s_1s_2}(t)) = H(|\beta\xi|^2)$$

$$- H\left(\frac{1}{2}(1 + \sqrt{1 - 4|\beta\xi|^2})\right),$$

(5)

where we have introduced the Shannon entropy $H(x) = -x\log_2 x - (1 - x)\log_2(1 - x)$. For the reservoirs, $C$ and $Q$ read as

$$C(\rho_{r_1r_2}(t)) = Q(\rho_{r_1r_2}(t)) = H(|\beta\chi|^2)$$

$$- H\left(\frac{1}{2}(1 + \sqrt{1 - 4|\beta\xi|^2})\right).$$

(6)

First, when the spectral density is flat,

$$J(\omega) = \gamma,$$

(7)

where $\gamma$ is a constant that is commonly used as the Markov approximation with the interval of the spectral density much broader than the corresponding energy scale of the system. We obtain parameters

$$\chi = \sqrt{1 - e^{-\gamma t}}, \quad \xi = e^{-\gamma t/2}.$$  

(8)

The concurrence of the system is max\{0, $e^{-\gamma t}(|\alpha\beta| - |\beta|^2(1 - e^{-\gamma t}))$\}. If $|\alpha| < |\beta|$, there will be ESD [19]. As time approaches infinity, the asymptotic behavior of $C$ for the spin system $\rho_{s_1s_2}$ becomes

$$\lim_{t \to \infty} C(\rho_{s_1s_2}(t)) \sim (|\beta|^2 - |\beta|^4)\gamma t e^{-2\gamma t}/\ln 2,$$

(9)

which does not depend on the initial value $C(\rho_{s_1s_2}(0)) = -(1 - |\beta|^2)\log_2(1 - |\beta|^2) - |\beta|^2\log_2|\beta|^2$. It is shown that $C$ of the spin system is depleted gradually without sudden death.

According to Eq. (5), $C$ is equal to $Q$ during the evolution. As shown in Figure 1(a), with $\alpha = 1/\sqrt{2}, \beta = 1/\sqrt{2}$, and $\alpha = 1/\sqrt{10}, \beta = 3/\sqrt{10}$, the correlations tend to zero monotonically and continuously.

According to Eq. (6) for reservoirs, $C$ equals $Q$, and their asymptotical behaviors are described by

$$\lim_{t \to \infty} Q(\rho_{r_1r_2}(t)) = C(\rho_{r_1r_2}(t)) \sim Q(\rho_{s_1s_2}(0)).$$

(10)

Figures 1(a) and (b) show that both $Q$ and $C$ initially stored in the spin system run into reservoirs gradually as the whole system achieves equilibrium. Evolutions of $Q$ and
FIG. 1: (Color online) Evolution of $Q$ and $C$ among different partitions with flat spectral density for the initial state $(\alpha|0\rangle_{s1}|0\rangle_{s2} + \beta|1\rangle_{s1}|1\rangle_{s2})|0\rangle_{r1}|0\rangle_{r2}$. Blue diamonds and magenta squares denote $Q$ and $C$ for the Bell state, respectively, while dark triangles and red circles denote $Q$ and $C$ for $\alpha = 1/\sqrt{10}$, $\beta = 3/\sqrt{10}$. (a) Spin system $\rho_{s1s2}$. (b) Reservoirs $\rho_{r1r2}$. (c) Spin $s_1$ with its corresponding reservoir $r_1$, $\rho_{s1r1}$. (d) Spin $s_1$ with the other reservoir $r_2$, $\rho_{s1r2}$.

FIG. 2: (Color online) Dynamics of $Q$ and $C$ for the initial state $(\alpha|0\rangle_{s1}|0\rangle_{s2} + \beta|1\rangle_{s1}|1\rangle_{s2})|0\rangle_{r1}|0\rangle_{r2}$ and the spectral density taking the Lorentz form $W/\lambda = \sqrt{200}$. Blue diamonds and magenta squares denote $Q$ and $C$ for the Bell state, respectively, while dark triangles and red circles denote $Q$ and $C$ for $\alpha = 1/\sqrt{10}$, $\beta = 3/\sqrt{10}$. (a) Spin system $\rho_{s1s2}$. (b) Reservoirs $\rho_{r1r2}$. (c) Spin $s_1$ with its corresponding reservoir $r_1$, $\rho_{s1r1}$. (d) Spin $s_1$ with the other reservoir $r_2$, $\rho_{s1r2}$.

$C$ between the system and reservoirs may reflect the transference processes of $Q$ and $C$ between the system and reservoirs. $s_1$ ($s_2$) becomes correlated with $r_1$ ($r_2$) owing to their interaction as shown in Fig. 1(c), therefore, $r_1$ tangles with $r_2$ through $s_1$ and $s_2$. At the end of evolution there is no entanglement between reservoirs and system; therefore, the system
can be discarded without any effect, and all correlations transfer to reservoirs.

We also find that \( Q, C \), concurrence between \( s_1 \) amd \( r_1 \), and concurrence between \( s_2 \) and \( r_2 \), do not increase during the course of evolution. They are measured by the square of correlations \( Q(t)^2 \equiv Q(\rho_{s_1s_2}(t))^2 + Q(\rho_{s_1r_2}(t))^2 + Q(\rho_{s_2r_1}(t))^2 + Q(\rho_{r_1r_2}(t))^2 \leq Q(0)^2 \), \( C(t)^2 \equiv C(\rho_{s_1s_2}(t))^2 + C(\rho_{s_1r_2}(t))^2 + C(\rho_{s_2r_1}(t))^2 + C(\rho_{r_1r_2}(t))^2 \leq C(0)^2 \), and \( \text{Con}(t)^2 \equiv \text{Con}(\rho_{s_1s_2}(t))^2 + \text{Con}(\rho_{s_1r_2}(t))^2 + \text{Con}(\rho_{s_2r_1}(t))^2 + \text{Con}(\rho_{r_1r_2}(t))^2 \leq \text{Con}(0)^2 \). It can be seen that for partitions \( s_1 \) and \( s_2 \), interaction between \( s_2 \) and \( r_2 \) is simply a local unitary operation; therefore, correlations will not exceed those initially stored between \( s_1 \) and \( s_2 \).

Until now, we have mainly focused on the weak coupling regime, where dynamics of correlations are monotonic and irreversible. Things will be different in the strong coupling regime. We consider spectral density taking the Lorentz form, which is usually encountered in a cavity [21]. Here we take the spectral density centered at \( \omega_0 \) (resonance with the spin system):

\[
J(\omega) = \left( W^2 \lambda / \pi \right) \frac{1}{(\omega - \omega_0)^2 + \lambda^2},
\]

where \( \lambda \) is used to measure the correlation time of the reservoir. For \( \lambda \approx 0 \), we obtain \( J(\omega) \approx W^2 \delta(\omega - \omega_0) \), which displays a strong non-Markov effect. We have the parameters

\[
\xi = e^{-\lambda t/2}[\sqrt{\lambda^2 - 4W^2} \sinh(\sqrt{\lambda^2 - 4W^2}t/2)] + \cosh(\sqrt{\lambda^2 - 4W^2}t/2),
\]

\[
\chi = \sqrt{1 - \xi^2}.
\]

Concurrence due to the strong non-Markov effect (measured by \( W/\lambda \)) is given by \( \max\{0, |\alpha^* \beta \xi^2| - |\beta^2 \xi^2 \chi^2| \} \), which has oscillation. \( C \) and \( Q \) oscillate too, as shown in Fig. 2(a). According to Eq. (5), for this kind of initial state, although the coupling has changed, \( C \) is equal to \( Q \). As shown in Fig. 2(a), the evolutions of \( Q \) and \( C \) are continuous. Evolutions of \( Q \) and \( C \) for reservoirs also have oscillation, as shown in Fig. 2(b). From the analytical results, we find that both \( Q \) and \( C \) initially stored in the spin system transfer to the reservoirs over time. Figures 2(c) and 2(d) show the evolutions of correlations for \( \rho_{s_1r_1} \) and \( \rho_{s_1r_2} \), respectively. A non increase in correlations and concurrence is noted, but both correlations and concurrence oscillate.

We then consider the initial state with only one excitation in the spin system:

\[
|\Phi_0\rangle = (\alpha|0\rangle_{s_1}|1\rangle_{s_2} + \beta|1\rangle_{s_1}|0\rangle_{s_2})|0\rangle_{r_1}|0\rangle_{r_2}.
\]
FIG. 3: (Color online) Evolution of $Q$ and $C$ for the initial state $(\alpha|0\rangle_{s_1}|1\rangle_{s_2} + \beta|1\rangle_{s_1}|0\rangle_{s_2})|0\rangle_{r_1}|0\rangle_{r_2}$ with both reservoirs having a flat spectrum. Blue diamonds and magenta squares denote $Q$ and $C$ for the Bell state, respectively, while dark triangles and red circles denote $Q$ and $C$ for $\alpha = 1/\sqrt{10}, \beta = 3/\sqrt{10}$. (a) Spin system $\rho_{s_1s_2}$. (b) Reservoirs $\rho_{r_1r_2}$. (c) Spin $s_1$ with reservoir $r_1$, $\rho_{s_1r_1}$. (d) Spin $s_1$ with reservoir $r_2$, $\rho_{s_1r_2}$.

FIG. 4: (Color online) Dynamics of $Q$ and $C$ for the initial state $(\alpha|0\rangle_{s_1}|1\rangle_{s_2} + \beta|1\rangle_{s_1}|0\rangle_{s_2})|0\rangle_{r_1}|0\rangle_{r_2}$ with the spectral density of reservoirs taking the Lorentz form $W/\lambda = \sqrt{200}$. Blue diamonds and magenta squares denote $Q$ and $C$ for the Bell state, respectively, while dark triangles and red circles denote $Q$ and $C$ for $\alpha = 1/\sqrt{10}, \beta = 3/\sqrt{10}$. (a) Evolution of spin system $\rho_{s_1s_2}$. (b) Reservoirs $\rho_{r_1r_2}$. (c) Spin $s_1$ with reservoir $r_1$, $\rho_{s_1r_1}$. (d) Spin $s_1$ with reservoir $r_2$, $\rho_{s_1r_2}$.
Evolution of the spin system is given by the reduced density matrix:

$$\rho_{s_1s_2}(t) = \begin{pmatrix} |\chi|^2 & 0 & 0 & 0 \\ 0 & |\alpha\xi|^2 & \alpha^*\beta|\xi|^2 & 0 \\ 0 & \alpha^*\beta|\xi|^2 & |\beta\xi|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{14}$$

Interestingly, in this situation, $C$ has the same value as that in the first situation given by Eq. (5), while for $Q$, we have

$$Q(\rho_{s_1s_2}(t)) = -H(|\xi|^2) + H(|\alpha\xi|^2) + H\left(\frac{1}{2}(1 + \sqrt{1 - 4|\beta\xi\chi|^2})\right). \tag{15}$$

For reservoirs $\rho_{r_1r_2}$, we obtain

$$C(\rho_{r_1r_2}(t)) = H(|\beta\chi|^2) - H\left(\frac{1}{2}(1 - \sqrt{1 - 4|\beta\xi\chi|^2})\right),$$

$$Q(\rho_{r_1r_2}(t)) = H\left(\frac{1}{2}(1 + \sqrt{1 - 4|\beta\xi\chi|^2})\right) - H(|\chi|^2) + H(|\alpha\chi|^2). \tag{16}$$

With flat spectral density, we obtain the parameters in Eq. (8). Concurrence of the spin system reads as $|\alpha^*\beta|e^{-\gamma t}$, and therefore, there is no ESD. As time approaches infinity, $Q$ can be represented asymptotically by

$$\lim_{t \to \infty} Q(\rho_{s_1s_2}(t)) \sim Q(0)e^{-\gamma t}, \tag{17}$$

with $Q(0) = H(|\alpha|^2)$.

For this kind of initial state, $Q$ is no longer equal to $C$ as in Fig. 3(a), but the correlations fall to zero monotonically and continuously. For reservoirs, both correlations approach a definite value monotonically as in Fig. 3(b). The transference processes of quantum and classical correlations are similar to those of the initial state with two excitations, as shown in Figs. 3(c) and 3(d). As time tends to infinity, both $Q$ and $C$ initially stored in the spin system run into reservoirs. In addition, there are no increases in the correlations.

For the Lorentz form in Eq. (11), we have the same parameters as in Eq. (12). Concurrence of the spin system is given by $|\alpha^*\beta\xi|^2$. $Q$ is not equal to $C$. As shown in Fig. 4(a), correlations of the spin system also oscillate. Figure 4(b) shows evolutions of correlations in
reservoirs. Figures 4(c) and 4(d) show evolutions for $s_1 r_1$ and $s_1 r_2$. Although there is large oscillation due to the non-Markov effect, there are no increases in correlations.

In conclusion, we have investigated the dynamics of both $Q$ and $C$ in the spin-boson model with two spins independently coupled to their own reservoirs. From the analytical results obtained for $Q$ and $C$, we studied the evolutions of $Q$ and $C$ among different partitions in detail. We found that the dynamics of $Q$ and $C$ depend closely on the form of the initial state. At the end of evolution, all $Q$ and $C$ initially stored in the spin system transfer to reservoirs. We found that $Q$ is more robust than entanglement, for there is no sudden death with $Q$. At the same time, during the evolution process, all partitions had nonzero $Q$, which is not the case for entanglement. We also found that for a large family of initial states, $Q$ remains equal to $C$ during the course of evolution. In addition, there was no increase in either correlation. There are many differences between this paper and \cite{11,12}; for example, we focused on a more concrete case of the spin-boson model, which is usually encountered, and we concentrated on the transference processes of both $Q$ and $C$.

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