Coupled magnetostatic modes of ferromagnetic / antiferromagnetic cylindrical bilayers

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Abstract. A theory is developed for the bulk and surface magnetostatic modes in bilayer systems with cylindrical geometries, where an antiferromagnetic layer is grown in contact with a ferromagnetic layer. Specifically the dispersion relations and localization properties of the modes are derived for ferromagnetic / antiferromagnetic bilayers as long concentric tubes around a nonmagnetic core. The results differ depending on which type of material is in the outer layer and also contrast with the planar bilayer geometries studied previously. Numerical examples are given for Permalloy and Gadolinium Aluminate as the materials.

1. Introduction
Recently there has also been considerable interest in the dynamical properties of bilayer films grown as ferromagnetic (FM) and antiferromagnetic (AFM) materials in direct contact, e.g., with regard to the coupled magnetostatic modes, exchange bias effects, and interface magnetic anisotropies [1,2]. The magnetostatic modes are the spin excitations in the system at long wavelengths (or small wave vectors) where the dynamic effects of the exchange interactions are neglected compared the magnetic dipole-dipole interactions, and the latter are treated using Maxwell’s equations without retardation (see, e.g., [3] for a review). A motivation for the present work is to extend the above bilayer studies, which were for planar geometries, to the case of curved interfaces. For this we focus on nanotubes, which are essentially hollow cylinders. In the case of a single magnetic material, such structures can be fabricated in dense arrays [4] and their magnetic excitations studied by techniques such as Brillouin light scattering (BLS) [5]. Magnetostatic theory was used to describe the spin excitations in nanotubes [6] for appropriate values of the wave number \( q \).

Here we present magnetostatic calculations for nanotubes where the walls are FM / AFM bilayers. It is found that the coupled magnetic excitations are modified, compared with the planar case, and there is a structural asymmetry depending on whether the FM material forms the inner or outer part of the bilayer.

2. Magnetostatic theory for a cylindrical bilayer
The assumed geometry for the cylindrical bilayer is shown in figure 1, where the double-walled nanotube has inner and outer radii \( R_1 \) and \( R_2 \), respectively and an internal interface at \( R_2 \). An external magnetic field is taken parallel to the cylindrical axis (the \( z \) axis) and a large length-to-diameter aspect ratio is assumed. The magnetostatic modes are characterized by the wave number \( q \) along the cylinder axis of symmetry. Generally we take \( q \sim 10^6 \) or \( 10^7 \) m\(^{-1}\), which is typical of BLS when a 90° scattering...
have a gyromagnetic form with the nonzero frequency-dependent elements applying electromagnetic boundary conditions at all interfaces. The magnetic susceptibility tensors for the dynamic response within each material using Maxwell's equations (without retardation) and medium (assumed here to be vacuum).

The regions internal and external to the nanotube are filled by a nonmagnetic material (e.g., AFM) extending from radius \( R_1 \) to \( R_2 \), while the darker shaded region is the other magnetic material (e.g., FM) from \( R_2 \) to \( R_3 \). The magnetic field \( H_0 \) and wave number \( q \) are along the \( z \) axis, parallel to the FM magnetization and AFM sublattice magnetization.

By extension of the calculations for a nanotube composed of just one material [6], we now solve for the dynamic response within each material using Maxwell’s equations (without retardation) and applying electromagnetic boundary conditions at all interfaces. The magnetic susceptibility tensors have a gyromagnetic form with the nonzero frequency-dependent elements \( \chi_{xx} = \chi_{yy} = \chi_a \) and \( \chi_{xy} = -\chi_{yx} = i\gamma \omega \), which is applicable for the geometry in figure 1. Ignoring damping, we have for a wave of angular frequency \( \omega \) in the FM case [7,8]

\[
\chi_a = \omega_0 \omega_m / \left( \omega^2_0 - \omega^2 \right), \quad \chi_b = \omega \omega_m / \left( \omega^2_0 - \omega^2 \right),
\]

where \( \omega_0 = \gamma \omega_0 H_0 \) and \( \omega_m = \gamma \omega_0 M_{FM} \) with \( \gamma \) and \( M_{FM} \) denoting the gyromagnetic ratio and the saturation magnetization, respectively. There are analogous expressions for \( \chi_a \) and \( \chi_b \) in the case of a uniaxial AFM [7,8] but the definition of \( \omega_m \) is now in terms of the sublattice magnetization \( M_{AFM} \) and there are additional frequencies \( \omega_0 \) and \( \omega_{0b} \) relating to the bulk anisotropy and static exchange, respectively. Also the poles for \( \omega \) in the AFM occur at \( \Omega_0 \pm \omega_b \) where \( \Omega_0 = [\omega_0(2\omega_{0b} + \omega_0)]^{1/2} \) is the antiferromagnetic resonance frequency.

Generalizing [6] the appropriate Maxwell’s equations are re-expressed in terms of the magnetostatic scalar potential, which satisfies the Walker equation inside each of the magnetic materials and Laplace’s equation in the vacuum regions outside. In cylindrical polar coordinates the solutions have the form of a radial function multiplied by \( \exp(i m \theta) \) \( \exp(i q z) \), where \( m \) is an integer. The radial function has solutions involving the Bessel functions \( I_m(qr) \) for \( r < R_1 \), combinations of \( I_m(aqr) \) and \( K_m(aqr) \) inside the tube, and \( K_m(aqr) \) for \( r > R_3 \). The frequency-dependent \( \alpha \) is defined for a magnetic material by \( \alpha = (1 + \chi_a)^{-1/2} \), which we denote as \( \alpha_0 \) for \( R_1 < r < R_2 \) and \( \alpha_{0s} \) for \( R_2 < r < R_3 \). The surface-like (localized) modes and bulk-like modes correspond to real and imaginary \( \alpha \), respectively.

As discussed for FM / AFM planar bilayers [2], it is important to take account of the anisotropy due to the two materials in contact (at the \( r = R_2 \) interface in our case). This causes the effective external magnetic field in the FM material and the AFM material to be modified as

\[
H_{FM} = H_0 + (M_{AFM}/\langle M \rangle) H_I, \quad H_{AFM} = H_0 + (M_{FM}/\langle M \rangle) H_I,
\]

where \( H_I \) is the interface anisotropy field and \( \langle M \rangle \) is a volume-weighted average of \( M_{FM} \) and \( M_{AFM} \) for the bilayer. The final step is to apply the magnetostatic boundary conditions [6] at the three interfaces, giving rise to six coupled equations involving the six undetermined coefficients in the scalar potential. A dispersion relation for the magnetostatic modes is then obtained in the form \( \det \mathbf{X} = 0 \), where \( \mathbf{X} \) is a \( 6 \times 6 \) matrix with its nonzero elements depending on frequency through the susceptibility terms.

Figure 1. A bilayer cylindrical nanotube showing the coordinate axes. The light shaded region represents one magnetic material (e.g., AFM) extending from radius \( R_1 \) to \( R_2 \), while the darker shaded region is the other magnetic material (e.g., FM) from \( R_2 \) to \( R_3 \). The magnetic field \( H_0 \) and wave number \( q \) are along the \( z \) axis, parallel to the FM magnetization and AFM sublattice magnetization.

Geometry is employed. The regions internal and external to the nanotube are filled by a nonmagnetic medium (assumed here to be vacuum).
3. Results and discussion
The theory is now applied to bilayer structures in which the FM is Permalloy (or Ni\textsubscript{0.8}Fe\textsubscript{0.2}), for which \(\omega_m/2\pi = 23.9\) GHz (see [6]). The AFM is uniaxial GdAlO\textsubscript{3}, for which \(\omega_m/2\pi = 22.0\) GHz, \(\omega_A/2\pi = 10.2\) GHz, and \(\omega_{Ex}/2\pi = 52.6\) GHz) [6] implying a relatively low AFMR frequency corresponding to \(\Omega_0/2\pi = 34.4\) GHz. The applied magnetic field and the interface anisotropy field are chosen as \(\mu_0H_0 = 0.2\) T and \(\mu_0H_I = 0.05\) T.

There are two possible bilayer structures to consider, depending on which material forms the inner layer, and we now show that they give rise to contrasting behaviour for the magnetostatic modes. In figure 2 we show an example where the FM forms the inner layer of the tube. The frequencies of the coupled surface magnetostatic modes for \(|m| = 1\) and \(2\) are plotted versus \(q\) for the above parameters and assuming values for \(R_1, R_2\) and \(R_3\) as indicated. Qualitatively the modes have some features that are similar to those for cylindrical nanotubes with one magnetic material [6], except that there are two bands of frequencies, labelled as the AFM Region and the FM Region, which are characteristic of the component materials. However, due to the coupling, the surface modes within each region are perturbed in frequency and their localization properties are modified. As in [6], the frequency of each branch decreases monotonically as \(q\) increases until there is a cut-off value above which no localized modes occur. Also the modes exist only within specific ranges of frequency, as indicated by the horizontal lines. Quantitatively there are important differences that include the existence of the two bands of frequencies, as well as two branches for each \(lmn\) in both bands. Other new features, which are a consequence of coupling across the cylindrical interface leading to restrictive conditions for localization, are the cut-off values when \(q\) is decreased seen for the uppermost surface branch in the AFM region.

![Figure 2](image1.png)

**Figure 2.** Frequencies of surface modes in a Permalloy / GdAlO\textsubscript{3} nanotube plotted versus wave number \(q\), where the FM forms the inner layer. Solid and broken lines refer to \(lmn = 1\) and \(2\) (see text for other parameters).

![Figure 3](image2.png)

**Figure 3.** As in figure 2 but for the inverse bilayer nanotube structure where the AFM now forms the inner layer, using the same values of the radii.

In figure 3 we present results for the inverse structure to that just described, i.e., the same values of the radii are employed for the bilayer but the AFM is now the inner layer. It can be seen that the results are quite different in terms of the mode frequencies, cut-off values for \(q\), hybridization (mode-mixing), etc.

A final numerical example is given in figure 4, where we again take the case of a bilayer with the AFM as the inner layer (as in figure 3), but we illustrate the effects of varying the radii. The mode dispersion curves are shown for \(lmn = 1\) only, but we consider two different sets of values for the radii (see the solid and broken lines) in addition to those quoted in figure 3 for an analogous bilayer. In each
case the four surface modes can be approximately associated with localization near the inner and outer radii of the AFM and near the inner and outer radii of the FM, although there is also some degree of mode mixing. The modes in figure 4 that are the least affected by the size variations are those represented by the almost degenerate curves starting at ~ 30.3 GHz in the AFM Region, and we can identify these as having their maximum amplitude near the vacuum / AFM interface (at $r = R_1$) and mainly localized within the AFM layer. The other mode (localized near $R_2$) in the AFM Region is shifted in frequency due to the role of the interface anisotropy, which gives different $H_{FM}$ and $H_{AFM}$ values for the two structures in accordance with equation (2) due to a modified $\langle M \rangle$. Likewise the mode localized near $R_2$, but mainly in the FM layer, is represented by the branches starting at ~ 16.2 and 16.5 GHz for the two structures in Figure 4.

![Figure 4. Frequencies of surface modes with $|lm| = 1$ in a Permalloy / GdAlO$_3$ nanotube plotted versus $q$, where the AFM forms the inner layer. Solid and broken lines refer to structures with different radii.](image)

4. Conclusions
We have presented calculations for AFM / FM double structures with a cylindrical geometry, generalizing the previous studies which were for planar systems. Taking Permalloy and GdAlO$_3$ as examples we showed how the dispersion relations for surface magnetostatic modes are modified and how the inverse structure has different properties. Analogous conclusions follow regarding the coupled bulk modes of these magnetic bilayers. Calculations using other choices of materials, such as Ni for the FM and MnF$_2$ for the AFM, have been carried out and lead to qualitatively similar results. However, the applications using GdAlO$_3$ are likely to be of greater interest since this AFM has a much weaker uniaxial anisotropy (implying a smaller AFM resonance frequency) than is typically the case. Inelastic light scattering (which can be either Brillouin or Raman scattering, depending on the frequency range of the modes) provides a convenient experimental technique to study the coupled magnetostatic modes described here.

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