High Order Inverse Radial Lens Distortion

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Abstract. Lens distortion is one of the highest elements affecting camera calibration. In this paper, a new improved algorithm for calculating the inverse of the best radial distortion model is presented. The presented method provides a model of reverse radial distortion. The method involves lens of focal length (190mm) with a Sony G camera, and calibrating them using Zhang method which entails a chessboard pattern to be detected from several positions, and an exact formula to calculate the inverse radial distortion.

The results show that the highest order radial distortion is the better model to be reverse, it was shown that the 8th order radial distortion is the best model which contain a lowest value of distortion can be acquired after a number of iterations about 1.009842932 x 10⁻²⁴, and this is the best value can be acquired without losing the accuracy, which produces low distortion performs as pincushion distortion. The new improved algorithm displays a good performance in obtaining least value for distortion coefficient.

Keywords: Lens distortion, Distortion correction, Model Camera calibration, Image correction

Introduction: Distortion is a known marvel that in specific circumstances may extraordinarily affect a picture’s geometry without impeding quality or decreasing the data current in the picture. Applying the projective pinhole camera is as often as possible unrealistic without considering the bending brought about by the camera lens. This wonder can be exhibited by a radial distortion, the greatest unmistakable segment, and furthermore with a minor effect, a decentering twisting which possess both tangential and radial fragment. Radial distortion is occurring due to the circular state of the focal point, though tangential distortion occurs due to the decentering and non-symmetry of the focal point parts as for the optical pivot ([1,2]).

Note that radial distortion is greatly associated with focal length. The study on camera lens distortion can be followed back to 1919, when a Conrady originally presented the decentering distortion model. In view of Conrady’s work, in 1966 Brown announced the renowned Brown–Conrady model [1,8]. In this model, Brown characterized lens distortion into radial distortion and tangential distortion, and proposed the well-known plumb line technique to calibrate these distortions. Since then Brown–Conrady model has been generally utilized [3,4,5 and 6].

Three several sort of lens distortion; radial, decentering and thin prism distortion. In this paper radial distortion is considered. Although the radial component of lens distortion is dominating, it is combined with tangential one, as demonstrated in figure 1 radial distortion types the barrel distortion which appears in lenses of short focal lengths and the pincushion distortion in lenses of longer focal lengths, as the other types normally have less significant impacts than the radial. The following polynomial equation symbolizes the 8th order radial distortion, used in this paper:

\[ \text{rd} = rf (r) = r (1 + k_1 r^2 + k_2 r^4 + k_3 r^6 + k_4 r^8 + \ldots) \] (1)
1. METHODOLOGY

Zhang's method is utilized as motivation errand for its flexible technique forwardness and known accuracy. In this paper, the method is utilized for lens distortion model evaluation. It requires several images of a chessboard pattern to be captured from a different position with a known dimension. The algorithm figures the camera alignment parameters utilizing the connection between the checkerboard corners in a camera arrange framework and a world coordinate system joined to the checkerboard plane [5, 6].

A lot of experience after Conrady and Brown is made to treat the eliminating of distortion from images [7, 8]. Considering, inverse distortion problematic is obvious in various applications, and it's kind of the poor connection of the distortion challenges. A few plans to the back-projection issues can be found in writing, as uncovered by Silvén and Heikkilä [9]. Figure 2 demonstrates the calculation used to compute the inverse of the best radial distortion.

Figure 1. Radial Distortion Types

Figure 2. Flow Chart of the New Proposed Algorithm
Experimental work and Results
In this section, we work on one experiment to select the best radial distortion model using automatic selection criteria. And second experiment to examine the inverse formula for radial distortion by applying the forward-converse recipe iteratively inside a loop. Therefore, the inverse of the inverse radial distortion is calculated.

2.1. RADIAL DISTORTION MODEL

A calibration is done using Zhang methods. The used camera is a Sony G camera of 25 mm zoom lens with 10x optical zoom and Chessboard Pattern with resolution 1830×1330 and point’s corner 8×11=88. The results for the 6th order and 8th order radial distortion model are shown in Table 1:

| Radial Distortion Model | k<sub>1</sub> | k<sub>2</sub> | k<sub>3</sub> | k<sub>4</sub> |
|------------------------|-------------|-------------|-------------|-------------|
| 6th order              | 1.410 x 10<sup>-3</sup> | -3.583 x 10<sup>-3</sup> | 9.205 x 10<sup>-8</sup> | -          |
| 8th order              | 1.532 x 10<sup>-3</sup> | -9.656 x 10<sup>-7</sup> | 7.245 x 10<sup>-11</sup> | 0          |

Using the Minimum Description Length criteria (MDL) which is presented by rissanen in 1978, the simplest model that depicts the information adequately will be chosen in MDL, where MDL choose the complexity equal to or less than that of the different criteria without losing an essentially lower error. [10]. It has the following form:

\[
\text{MDL} = N \ln \left( \frac{\text{SSE}}{N} \right) + 2k \log N
\]  

In the following: N refers to the number of samples, k is the number of parameters in the model, and SSE refers to the sum-square-error (SSE) computed as: \( \text{SSE} = \sum_i r_i^2 \) Where \( r_i = \|m_i - m'_i\| \) is the difference between the measured and estimated image points.

As shown in Table 2 shows that the 8th order is the best model with low complexity.

| Radial Distortion Model | MDL   |
|------------------------|-------|
| 6th order              | 0.376 |
| 8th order              | 0.190 |

2.2 Inverse Distortion Loop

The following inverse formulas for radial distortion are applied iteratively within a loop to test the accuracy of the inverse formula. Therefore, the inverse of the inverse radial distortion is calculated 10000 times and contrasted with the first distortion coefficients [11]. Therefore, according to [11], the formulas for the first four coefficients of 8th order radial distortion model is provided according to the best model.

\[
b_1 = -k_4
\]
\[
b_2 = 3k_1^2 - k_2
\]
\[
b_3 = -12k_1^3 + 8k_1k_2 - k_3
\]
\[
b_4 = 55k_1^4 - 55k_1^2k_2 + 5k_2^2 + 10k_1k_3 - k_4
\]
Table 3 Radial distortion calibration and inverse of radial distortion with coefficient from $k_1$... $k_4$.

| Coefficient | Original Distortion Coefficient | Inverse 1 |
|-------------|---------------------------------|-----------|
| $k_1$       | $1.532 \times 10^{-3}$         | - $1.532 \times 10^{-3}$ |
| $k_2$       | - $9.656 \times 10^{-7}$       | 1.7697072 x $10^{-3}$ |
| $k_3$       | 7.245 x $10^{-11}$             | - 3.339416252 x $10^{-11}$ |
| $k_4$       | 0                               | 4.1255518770316804 x $10^{-13}$ |

As the equations give a reverse formula for radial distortion it was repeated 2 time and a comparison between the final results and the original distortion is made. Then, we repeated the process 10000 times and also comparison between the final results and the original distortion is made. Table 4 clarified the loop (L=2) which represent the second inverse compared to the original distortion coefficients. loop (L=10000) represent the 10001 inverse compared to the original distortion coefficient. We can realize that the coefficients $k_1$ and $k_2$ did not change while the delta on $k_3$ is small with respect to the corresponding coefficient.

Table 4 shows the original radial distortion and the computed inverse parameters after n inversions L = 2 and =10,000).

| Coefficient | Original Distortion Coefficient | Inverse 1 | Loop 1 | Loop 10000 |
|-------------|---------------------------------|-----------|--------|-------------|
| $k_1$       | $1.532 \times 10^{-3}$         | - $1.532 \times 10^{-3}$ | 0      | 0           |
| $k_2$       | - $9.656 \times 10^{-7}$       | 1.7697072 x $10^{-3}$ | 0      | 0           |
| $k_3$       | 7.245 x $10^{-11}$             | - 3.339416252 x $10^{-11}$ | 1.292469707114 x $10^{-26}$ | 1.292469707 x $10^{-26}$ |
| $k_4$       | 0                               | 4.1255518770316804 x $10^{-13}$ | 1.009741958682 x $10^{-28}$ | 1.009842932 x $10^{-28}$ |

The main involvement of this work is to minimizing the distortion as possible. this could help in manufacturing of lenses to be closer to ideal.
3. Conclusions

A complete selection of the lowest distortion model is done and the inverse evaluation of 8th order radial lens distortion using Zhang method is presented. The results showed that the lowest value of the distortion can be attained after numerous iterations is $1.009842932 \times 10^{-24}$, and this is the finest value can be acquired without losing the accuracy. This improved method can be used in the lenses assembly to be closer to ideal corresponding on the inverse values.

The test indicates the great stability of the inversion procedure. Nevertheless, also if parameters $k_1...k_4$ are sufficient in order to reward distortion, utilizing high order term of polynomial expression is important for the inversion stability.

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