How perturbative is quantum gravity?

Eichhorn, Astrid; Lippoldt, Stefan; Pawlowski, Jan M.; Reichert, Manuel; Schiffer, Marc

Published in:
Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics

DOI:
10.1016/j.physletb.2019.01.071

Publication date:
2019

Document version
Final published version

Document license
CC BY

Citation for published version (APA):
Eichhorn, A., Lippoldt, S., Pawlowski, J. M., Reichert, M., & Schiffer, M. (2019). How perturbative is quantum gravity? Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics, 792, 310-314. https://doi.org/10.1016/j.physletb.2019.01.071

Terms of use
This work is brought to you by the University of Southern Denmark through the SDU Research Portal. Unless otherwise specified it has been shared according to the terms for self-archiving. If no other license is stated, these terms apply:

- You may download this work for personal use only.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying this open access version

If you believe that this document breaches copyright please contact us providing details and we will investigate your claim. Please direct all enquiries to puresupport@bib.sdu.dk

Download date: 06. May. 2020
How perturbative is quantum gravity?

Astrid Eichhorn\textsuperscript{a,}\textsuperscript{*}, Stefan Lippoldt\textsuperscript{a,}\textsuperscript{*}, Jan M. Pawlowski\textsuperscript{a,}\textsuperscript{*}, Manuel Reichert\textsuperscript{a,b,}\textsuperscript{*}, 
Marc Schiffer\textsuperscript{a,}\textsuperscript{*}

\textsuperscript{a} Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany
\textsuperscript{b} CP³-Origins, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark

\textbf{A R T I C L E   I N F O}

\textbf{Article history:}
Received 29 November 2018
Received in revised form 18 January 2019
Accepted 18 January 2019
Available online 29 March 2019
Editor: A. Ringwald

\textbf{A B S T R A C T}

We explore asymptotic safety of gravity-matter systems, discovering indications for a near-perturbative nature of these systems in the ultraviolet. Our results are based on the dynamical emergence of effective universality at the asymptotically safe fixed point. Our findings support the conjecture that an asymptotically safe completion of the Standard Model with gravity could be realized in a near-perturbative setting.

© 2019 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP\textsuperscript{3}.

1. Introduction

How nonperturbative is quantum gravity? What appears to be a technical question at the first glance, could actually be critical for the conceptual understanding of quantum spacetime. The nature of quantum gravity in the very early universe, its impact on matter, and the prospects of potential observational tests depend on the extent to which quantum spacetime is nonperturbative.

In a quantum field theoretic setting, the ultraviolet (UV) completion of particle physics including gravity may be asymptotically safe [1], being governed by the interacting Reuter fixed point in quantum gravity [2]. In the past two decades substantial nontrivial evidence has been collected for the existence of this fixed point in gravity-matter systems. In parallel, the search for a universal emergence of spacetime encoded in an interacting fixed point is also ongoing in other quantum gravity approaches. Despite the impressive plethora of results some pressing questions concerning the physical nature of the fixed point have not been answered yet.

In [3], the concept of effective universality of gravitational couplings has been put forward. Within a scalar-gravity system, the gravitational self-coupling and the scalar-gravity coupling are in semi-quantitative agreement. These are avatars of the Newton coupling that agree on the classical level. Their near-agreement in the quantum theory suggests a near-perturbative realization of diffeomorphism invariance in the asymptotically safe UV regime. Simply put, if the fixed point lies in a near-classical regime with small quantum fluctuations, these avatars are nearly equal. This result complements indications for a near-perturbative fixed-point structure based on near-canonical scaling dimensions [4]. This highly intriguing scenario would allow for the application of perturbative methods in the transplanckian regime, which indeed provide indications for the fixed point [5].

In this work we find indications for this scenario in gravity-matter systems, including all types of Standard-Model fields. We discover that effective universality [3] holds for the avatars of the Newton coupling including the gravitational self-couplings and all minimal gravity-matter couplings, see [3,6–11]. This suggests that the UV fixed point lies in a near-perturbative regime.

2. Effective universality

In the absence of a cosmological constant, General Relativity with matter is parameterized by one single coupling, the Newton coupling $G_N$. It governs both, the gravitational self-interaction as well as the gravity-matter interactions. In the presence of quantum fluctuations the single classical Newton coupling is promoted to potentially different running couplings, related to the interaction vertices of gravitons with each other, with their Faddeev-Popov ghosts and with matter. The scale-dependence of these avatars of the Newton coupling is encoded in their $\beta$-functions. This structure is familiar from gauge theories such as QED or QCD. For marginal, i.e., dimensionless couplings the corresponding avatars exhibit two-loop universality due to gauge symmetry. Hence in the perturbative regime all vertices can be described by a single gauge coupling. The Newton coupling is not marginal. Therefore universality of the distinct $\beta$-functions is not automatic. Nevertheless, the
underlying symmetry manifests itself in relations between the various Newton couplings, the Slavnov-Taylor identities (STIs). These can lead to significant differences between the various avatars in a nonperturbative regime of the theory. An example is given by Landau gauge Yang-Mills theory in the infrared. There the three-gluon coupling even becomes negative while the other couplings remain positive, see e.g. [12–16].

It is a key result of this work that the \( \beta \)-functions of the different avatars of the Newton coupling agree semi-quantitatively in the asymptotically safe UV regime in gravity-matter systems. This result is summarized in Fig. 1, showing that a region of effective universality exists in the space of couplings. The location of the fixed point falls into this region. We interpret the non-trivial emergence of effective universality as a manifestation of the near-perturbative nature of safely gravity. This supports a rather appealing scenario: the residual interactions in the UV are just strong enough to induce asymptotic safety, while allowing for near-canonical scaling of higher-order operators. The near-canonical scaling is indeed observed, see e.g. [4,17] as is the existence of the fixed point in perturbative studies [5]. Additionally, the asymptotically safe Standard Model [18–23] favors a perturbative nature of the fixed point.

3. Gravity-matter systems

We start from the gauge-fixed Einstein-Hilbert action with minimally coupled scalars and fermions as well as gauge-fixed gauge theory with \( N_e \) gauge fields. The classical Euclidean action reads

\[
S = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \left( 2\Lambda - R \right) + S_{g\phi\text{gh},\text{gravity}}
+ \frac{1}{2} \sum_{i=1}^{N_f} \int d^4x \sqrt{g} g^{\mu\nu} \partial_{\mu} \psi^i \partial_{\nu} \psi^i + \sum_j \int d^4x \sqrt{g} \bar{\psi}^j \gamma^i \gamma^j
+ \frac{1}{2} \int d^4x \sqrt{g} g^{\mu\nu} g^{\rho\sigma} \text{tr} F_{\mu\rho} F_{\nu\sigma} + S_{g\phi\text{gh},\text{gauge}} ,
\]

(1)

where \( F_{\mu\nu} \) is the field-strength tensor of the gauge field \( A_\mu \). The gravity gauge fixing is of the linear de-Donder type in the Landau gauge limit. We also use the Landau gauge in the Yang-Mills sector. For the covariant Dirac operator \( \bar{\psi} \) we use the spin-base invariant formulation [24–26]. Under the impact of quantum gravity, Abelian and non-Abelian gauge theories can approach a free fixed point [10,19,21,27–29], such that the corresponding gauge couplings vanish, and the non-Abelian ghost sector decouples. Hence for our computation only the total number of gauge fields, \( N_g \), is relevant. We focus on \( N_s = 2N_i = 1 \).

Expanding the metric about a flat background

\[
g_{\mu\nu} = \delta_{\mu\nu} + \sqrt{G_N} h_{\mu\nu} ,
\]

(2)

schematically leads to interactions of the form

\[
\Gamma \sim \int d^4x \left( \sqrt{G_{N,h}} (\partial h)(\partial h) + \sqrt{G_{N,c}} (\partial c)(\partial c) \right) + \sqrt{G_{N,\psi}} (\partial \psi)(\partial \psi) + \sqrt{G_{N,\bar{\psi}} \gamma^j \partial \bar{\psi} \psi^j}
+ \sqrt{G_{N,A}} (\partial (\partial A)) + \ldots
\]

(3)

where we replaced \( G_N \) by avatars of the Newton coupling \( G_{N,i} \) corresponding to the interactions, \( i \in \{ h, c, \psi, \bar{\psi}, A \} \). In addition to the Newton couplings, the expansion of the cosmological constant term results in a two-graviton coupling \( \mu \) and a three-graviton coupling \( \lambda_3 \) [6,30].

4. \( \beta \)-functions

We compute the \( \beta \)-functions with the functional renormalization group (FRG), for general reviews see [31–36]. For gravity it was pioneered in the seminal paper of [2], for reviews see [37–42]. Here we follow the setup in [3,6–11,30,43]. The involved algebra is handled using the symbolic manipulation system form [44,45] and the FormTracer [46] as well as the Mathematica package xAct [47–50]. We provide a Mathematica notebook containing the final \( \beta \)-functions in numerical form [51].

We work with dimensionless running couplings, e.g., \( G_i = G_{N,i}k^2 \) for all avatars of the Newton coupling. It is already instructive to examine a simplified form of the \( \beta \)-functions for the different avatars of the Newton coupling,

\[
\beta_{G_i} = 2G - a_i G^2 + O(G^3) ,
\]

(4)

where all avatars of the Newton coupling are identified, \( G_i = G \). The coefficients \( a_i \) of the quadratic terms read

\[
(a_h, a_c, a_\psi, a_{\bar{\psi}} , a_A) = (3.7, 3.8, 2.9, 2.9, 2.6) ,
\]

(5)

when evaluated at \( \mu = -0.58 \) and \( \lambda_3 = 0.096 \). These are precisely the fixed-point values that we will present later, see (8). Already in the present simple approximation, the coefficients differ by no more than 32%. For quantitative studies, we use a measure for the relative deviation of the \( \beta \)-functions introduced in [3],

\[
e_{ij}(G, \mu, \lambda_3) = \left| \frac{\Delta \beta_{G_i} - \Delta \beta_{G_j}}{\Delta \beta_{G_i} + \Delta \beta_{G_j} \bigg| G_i = G_j = G \right| ,
\]

(6)

where \( i, j \in \{ h, c, \psi, \bar{\psi}, A \} \). \( \Delta \beta_{G_i} \) is the anomalous part of the \( \beta \)-function \( \beta_{G_i} \), obtained by subtracting the canonical running.

\[
\Delta \beta_{G_i} = \beta_{G_i} - 2G_i .
\]

(7)

In the case of effective universality \( e_{ij} \) is close to zero. Larger values of \( e_{ij} \) signal a stronger deviation from effective universality.
This measure can be applied pairwise to the 10 distinct pairs of \( \beta \)-functions. Due to a rather mild \( G \) dependence around the fixed point with \( G_h^* = 0.58 \), cf. (8), we focus the discussion on the \( \mu \)-\( \lambda_3 \)-plane.

A first nontrivial result concerns the existence of distinct lines where the individual components of \( \epsilon_{ij} \) vanish in the \( \mu \)-\( \lambda_3 \)-plane, cf. Fig. 2. These lines cross pairwise in a bounded region. Moreover, the different crossing points lie near each other. This is crucial and highly nontrivial, as the distinct \( \epsilon_{ij} = 0 \) lines have quite different slopes. Therefore, there is a priori no reason to intersect pairwise in a relatively small region in the \( \mu \)-\( \lambda_3 \)-plane. As we tentatively correlate the emergence of effective universality, i.e., an agreement of the \( \beta \)-functions, with a near-perturbative nature of the system, the vicinity of the \( \epsilon_{ij} = 0 \) lines is a preferred region for the couplings.

In the gravity-matter system investigated in this work the interacting Reuter fixed point lies at

\[
(G_h^*, G_V^*, G_{\phi^*}, G_{\theta^*}, \mu^*, \lambda_3^*)
\]

(8)

The values of the different vertex couplings are related by the STIs but they are not necessarily identical. This potential difference is ignored in the ensuing qualitative discussion where we assume full universality for the sake of simplicity.

The flow in the vicinity of the fixed point is governed by the critical exponents at linearized order of the \( \beta \)-functions. The STIs entail that only two of our couplings are independent. They are related to the classical Newton coupling and the cosmological constant. This renders only a subset of critical exponents physical. In our setting, the critical exponents turn out to be quantitatively similar, and close to the values obtained by setting \( G_i = G \) before calculating the stability matrix by taking derivatives of the \( \beta \)-function. This constitutes another strong indication for effective universality, and provides an estimate of \( \Re \theta \approx 1.3 - 1.7 \) for the physical critical exponent of the Newton coupling.

Crucially, our result requires nontrivial cancellations between the diagrams that contribute to the distinct \( \beta \)-functions. It is therefore not an automatic consequence of our choice of truncation that is inspired by classical diffeomorphism invariance. As a specific example we highlight \( \beta_{G_h} \) and \( \beta_{G_{\theta^*}} \) in Fig. 3. There we find an overall similarity of the \( \beta \)-functions while individual diagrams do not agree. Such cancellations appear unlikely to be generated by chance and we interpret them as a hint that the existence of the \( \epsilon_{ij} = 0 \) lines and a fixed point in their vicinity is indeed a nontrivial result.

5. Sources of deviations from effective universality

For a stringent assessment of the deviation of the fixed point from \( \epsilon_{ij} = 0 \), we have to evaluate its possible origins. In the present work we explore the Einstein-Hilbert truncation with minimally coupled matter. Yet, at an asymptotically safe fixed point, higher-curvature couplings have not been considered. Nonminimal gravity-matter couplings \([9,11]\) and matter-self interactions \([29,56,57]\) are also present. Accordingly, our fixed-point values as well as the location of the \( \epsilon_{ij} = 0 \) lines are subject to a systematic error \( \delta \epsilon \). A comparison of the fixed point to the \( \epsilon_{ij} = 0 \) lines is only meaningful within \( \delta \epsilon \). A rough estimate for the systematic error – strictly speaking an estimate for a lower bound on it – can be obtained by comparing changes in fixed-point values under extensions of the truncation. Specifically, we compare a state-of-the-art study \([58]\) with a previous work in the same scheme \([7]\) to obtain differences in fixed-point values \( \delta G_h \), \( \delta \mu, \delta \lambda_3 \). The relative variation of fixed-point values is similar in other extensions of truncations, see, e.g., \([9,59]\). The average \( \delta \epsilon \) of the \( \delta \epsilon_{ij} \) is given by

\[
\delta \epsilon = \frac{1}{10} \sum_{ij} \left[ \frac{\partial \epsilon_{ij}}{\partial G_h} \delta G_h + \frac{\partial \epsilon_{ij}}{\partial \mu} \delta \mu + \frac{\partial \epsilon_{ij}}{\partial \lambda_3} \delta \lambda_3 \right] = \epsilon_{ij} \approx 0.2
\]

resulting in \( \delta \epsilon \approx 0.2 \). As a key result, we stress that \( \epsilon_{ij} \approx 0.2 \) is compatible with effective universality within this estimate of the error \( \delta \epsilon \), cf. contours in Fig. 2. Accordingly, at the UV fixed point, the avatars of the Newton coupling are compatible with effective universality.

The presence of higher-order operators results in a second source of deviations of a more involved nature: It is rooted in the challenge of projecting correlation functions onto specific operators and the related couplings, e.g., the avatars of the Newton coupling. For instance, at the level of the graviton three-point function, the terms contributing to our result for \( \beta_{G_h} \) include \( \sqrt{\mathcal{R}} \) and \( \sqrt{\mathcal{R}_{\mu\nu} R^{\mu\nu}} \), both expanded to third order in \( \hbar \). In the present gauge-fixed and regularized setting, one faces the additional challenge to account for nondiffeomorphical propagators.

The higher-order contributions in the three-graviton and the graviton-matter vertices are not related to each other, as they are linked to distinct classically diffeomorphism invariant operators such as, e.g., \( \sqrt{\mathcal{R}} R_{\mu\nu} R^{\mu\nu} \) vs \( \sqrt{\mathcal{R}} R_{\mu\nu} \partial_\mu \phi \partial_\nu \phi \). From the observed small value of the \( \epsilon_{ij} \), we conclude that such higher-order operators, which could spoil effective universality completely, have a subleading impact.

Let us elucidate this point with an explicit example: Assume for the moment, that our evaluation of \( \beta_{G_h} \) would lead to \( 2G + \Delta \beta_{G_h} + \delta \beta_{G_h} \). Here, \( \delta \beta_{G_h} \) is an additional part of similar magnitude as \( \Delta \beta_{G_h} \), that originates from the running of the \( \sqrt{\mathcal{R}} R_{\mu\nu} R^{\mu\nu} \) coupling and contributes to \( \beta_{G_h} \) due to our (non-diagonal) projection procedure. Even assuming perfect agreement between all actual \( \beta_{G_h} \), our result for \( \delta \epsilon_{ij} \) would be greater than \( 0.3 \). The observation that all \( \epsilon_{ij} \) satisfy \( \epsilon_{ij} \lesssim 0.2 \) can tentatively be interpreted as
a hint for the subleading nature of higher-curvature couplings. Accordingly, our projection prescription isolates the various Newton couplings without a large ‘contamination’ from higher-order terms. At the same time, this suggests that the ‘backreaction’ of these specific higher-order terms, once included in a truncation, should be small, as indeed observed in several approximations, e.g., [4,17].

6. Modified Slavnov-Taylor identities

In a gauge-fixed setting, the fluctuation couplings are related by nontrivial Slavnov-Taylor identities (STIs). In the flow-equation setup, the regulator function is quadratic in the fluctuation fields and further breaks the diffeomorphism invariance. This turns the STIs into modified STIs (mSTIs) that now contain explicit regulator contributions, see e.g. [2,32,60–63]. The mSTIs imply that couplings that derive from the same classical structure differ at the quantum level. Thus, \( \epsilon_{ij} = 0 \) is not to be expected in a quantum setting, even in the absence of the systematic effects discussed above. Since the mSTIs arise as a consequence of quantum effects, the perturbative limit with vanishing couplings features trivial mSTIs. In the non-perturbative regime of gauge theories, the mSTIs become nontrivial with QCD being an excellent example, see e.g. [12,64]. If we ascribed the full difference in the fixed-point values of the different avatars of the Newton coupling to nontrivial mSTIs then \( \epsilon_{ij} \approx 0.2 \) would translate into a factor of roughly 0.7 between the fixed-point values. Taking our cue from QCD, where different avatars of the gauge coupling even feature distinct signs in the nonperturbative regime and thus much larger relative differences, we tentatively conclude that our results imply a near-perturbative nature of the asymptotically safe fixed point. More specifically, the analogue of mSTIs in QCD, see [64], suggests a grouping of fixed-point values into the pair \( \{ G_b, G_c \} \) and the triple \( \{ G_b, G_c, G_A \} \), with a nontrivial contribution from the mSTI differentiating between the former and the latter. This grouping is indeed apparent in the fixed-point values in [8]. In contrast, the ‘matter-like’ behavior of \( \epsilon_{\text{ghat}} \) and \( \epsilon_{\text{hc}} \) away from the fixed point, cf. Fig. 1, is a direct consequence of the diagrammatic structure underlying the \( \beta \)-functions.

7. Implications

We observe that the fixed point yields \( \epsilon_{ij} \approx 0.2 \), which is compatible with zero within our estimate for the systematic error. This entails a compatibility of the fixed point with effective universality within our present setup. The result has several important implications.

Firstly, it strongly suggests that the zero of the \( \beta \)-functions observed above is actually a true fixed point, in contrast to a truncation artefact. For the latter, there is no reason why delicate cancellations as observed above should occur. Their presence strongly hints at the impact of a symmetry principle. We view the delicate cancellations that occur in all pairs of \( \beta \)-functions as strong evidence for the physical nature of the asymptotically safe Reuter fixed point.

Secondly, we contrast the observed fixed-point structure with that of a system where \( h_{\mu\nu} \) is a spin-2 field living on a fixed background. Then \( h_{\mu\nu} \) would not be part of the dynamic spacetime geometry. If it was just another ‘matter’ field protected by shift symmetry, it would feature derivative couplings like those that we have examined here. Yet, the absence of a symmetry principle relating the distinct \( G_i \)’s would make a semi-quantitative agreement of the fixed-point values rather unlikely. The presence of this dynamical symmetry linked to the underlying dynamical diffeomorphism invariance of quantum gravity is corroborated further by the previously observed momentum locality of specific propagator and vertex flows, [6,58]. This property entails that the leading momentum dependence of different diagrams cancel non-trivially at large momenta. Our result could thus be interpreted as highlighting the geometric origin of \( h_{\mu\nu} \) with its corresponding spacetime diffeomorphism symmetry and background independence.

Thirdly, we contrast the relative deviation of fixed-point values of different avatars of the Newton coupling with significantly larger deviations in nonperturbative QFTs. The relation between different avatars of the gauge coupling is carried by mSTIs, which allow large deviations of these classically equal avatars in a non-perturbative regime governed by large quantum fluctuations. The significantly smaller differences between the different Newton couplings can accordingly be interpreted as a consequence of a near-perturbative regime, where mSTIs simplify and a more ‘classical’ notion of diffeomorphism symmetry is realized.

8. Conclusions

In summary we find strong indications that quantum gravity, though perturbatively nonrenormalizable, can be described within a quantum field theory in a near-perturbative regime. The corresponding ‘small parameter’ would be related to the deviation from canonical scaling of higher-order couplings. Further evidence is required to back our discovery of potential near-perturbativity of asymptotically safe gravity. The ‘minimally quantum’ nature of spacetime in the sense of small quantum fluctuations provides a strong backing of the robustness and reliability of various approximations commonly used in the literature and could constitute a key cornerstone in the understanding of quantum gravity.

At the physical level, our finding signifies that huge quantum fluctuations of spacetime appear to be subdominant in the ultraviolet. Our result could have important implications for other quantum-field theoretic approaches to quantum gravity in which the search for a continuum limit is an ongoing quest. This includes approaches with causal and Euclidean dynamical triangulations [65–67], the tensor track [68–70], and Loop Quantum Gravity/Spin foams [71–73]. In these approaches not much is known about the underlying universality classes in four dimensions yet. Our findings suggest that there is a universality class for the continuum limit which is near-perturbative in nature, resulting in a near-canonical spectrum of scaling exponents. This observation could provide guidance in the construction of a suitable dynamics in these models.

A scenario with a near-perturbative UV completion also provides an important input for studies of the very early universe, as well as the construction of phenomenological models geared towards observational tests of quantum gravity. Importantly, our discovery fits well to the perturbative nature of the Standard Model at the Planck scale and could provide a compelling picture of Planckian dynamics controlled by a near-perturbative fixed point.

Acknowledgements

This work is supported by an Emmy-Noether-grant of the DFG under AE/1037-1, is supported by EMMI, is part of and supported by the DFG Collaborative Research Centre “SFB 1225 (ISOQUANT)”, and is supported by the Danish National Research Foundation under grant DNRF:90. MR also acknowledges funding from the HGSFP and the IMPRS–PTFS. AE also acknowledges support through a visiting fellowship at Perimeter Institute. Research at Perimeter Institute is supported by the Government of Canada through the Department of Innovation, Science and Economic Development and by the Province of Ontario through the Ministry of Research and Innovation.
