Dynamics of magnetic islands and confinement transitions in TJ-II

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Abstract. Bolometric observations over a large number of discharges under very different conditions in the TJ-II Heliac, together with magnetic and line-emission diagnostics have indicated that the plasma behavior near rational magnetic surfaces is rather peculiar, which involves the formation and destruction of internal transport barriers associated with magnetohydordynamic (MHD) activity \cite{1}. The interpretation of the results is that around the magnetic islands that appear at resonant low-order rational surfaces there is a sheared flow that gives rise to the formation of a transport barrier, as the anomalous transport is decreased. This is evidenced by low recycling at the edge shown by low H\textalpha emission. Then an MHD event arises, possibly due to magnetic reconnection, which destroys the barrier and releases high-energy particles. This sequence of events is observed as sawtooth-like oscillations in the bolometric emission with a well defined repetition time. In this work a model is presented where this ingredients are incorporated in a transport simulation that includes a turbulence model based on resistive ballooning modes \cite{2}. The sheared flow is described by a neoclassical modeling of the ambipolar electric field given in \cite{3}. A transition model \cite{4} is included in the transport system of equations to verify that the appropriate signals are caused by the mechanisms described above. It is found that the model reproduces in an acceptable way the experimental results, supporting the assumed phenomenology: confinement regulated by rapid particle expulsions related to the break of magnetic islands locally and intermittently. The repetition rates are on the ms time-scale. The breaking of magnetic islands is described in terms of a magnetic reconnection model which is related to the presence of sheared flows just outside the islands. The unstable tearing mode includes the effect of the polarization drift in the outer region.

1. Introduction

Plasma confinement in stellarators has been found to be improved under certain circumstances leading to a transition in the confinement time with some features similar to the L-H transition in tokamaks \cite{5}. The nature of the transport barriers in stellarators, however, has its own properties. For instance, in high-power electron cyclotron heating (ECH) an internal transport barrier (ITB) is created, characterized by peaked electron temperature profiles and a large positive radial electric field in the core plasma region, the so-called core electron root confinement (CERC) \cite{6}. For neutral beam injection (NBI) heated plasmas, edge transport barriers (ETB) can be produced although the confinement improvement is more modest than in tokamaks \cite{7}. The origin of the transport barriers is somewhat complex: it has been argued that the sheared flows associated with the radially varying radial electric field can suppress the turbulence-driven transport, following the usual paradigm for tokamaks; but there is evidence that MHD...
phenomena play an important role by modifying the confining magnetic field. A clear piece of evidence is the relationship that has been found between the presence of low-order rational magnetic surfaces and the formation of transport barriers [1, 6].

In the TJ-II flexible heliac stellarator [8] it has been possible to perform many studies about rational surfaces and related phenomena. Some facts emerged already in the first campaigns [9] and were confirmed over the years. A brief overview of the important results includes: close to the predicted locations of resonant layers in the bulk plasma the generation of enhanced electron confinement regimes has been observed [10], in addition to the de-trapping of barely trapped particles [11] and unexpected enhanced line impurity emissions [12]. Near the expected position of singular surfaces, radial electric field shearing at the time of L–H transitions and suprathermal electron accumulation have been detected [13]. These results are complemented with the knowledge of a systematic modification of plasma gradients that resembles the existence of (weak) transport barriers accompanying the main magnetic resonances throughout the plasma [14].

The specific characteristics of TJ-II, i.e. low magnetic shear and high magnetic configuration flexibility, allow controlling the position of low-order rational values within the rotational transform profile, which allows the study of how the magnetic topology affects the transport. It is known that rational surfaces are resonant with magnetic perturbations and thus tend to form magnetic islands. The presence of magnetic islands, in turn, contributes to the outward electron flux that creates a locally strong positive radial electric field [10]. This is due to the fact that the diffusion coefficient in magnetic islands depends on the parallel velocity of the species; consequently, the presence of the island will enhance the electron flux more than the ion flux, modifying the equilibrium ambipolar radial electric field. On the other hand, the transport reduction resulting from the strong electric field radial gradient may modify the behavior of the island and produce MHD activity. This interplay of phenomena is reflected in the various measurements using different diagnostics which has been described in [11], trying to identify a process that links MHD activity with transport barriers. Based on these data, we have started to develop a model that puts together all the relevant processes in order to reproduce the experimental observations. Here we present the basis of the model which describes transport through a turbulent process which is modulated by the presence of magnetic islands. To this end, we first give in Section 2 a description of the different experimental results on which we base our model. Then, in Section 3 we describe the transport model which assumes a resistive ballooning mode as the drive for turbulence, which incorporates the presence of magnetic islands in a phenomenological way. It will be shown that the main features of the time evolution of emission profiles are reproduced quite well. A physical study of the dynamics of the magnetic islands is presented in Section 4, where we incorporate the presence of sheared flows on the stability of tearing modes. Finally, in Section 5 preliminary conclusions of this work are given.

2. Experimental observations

The TJ-II device has a major radius of \( R = 1.5 \) m with bean-shaped cross-sectional magnetic surfaces of average minor radius \( a \approx 0.2 \) m and four toroidal periods. It operates at a mean magnetic field of \( \approx 1 \) T with a low magnetic shear and high rotational transform \( \iota(a)/2\pi \sim 3/2 \). The plasmas considered are NBI heated with line-average densities \( 1.5 \times 10^{19} m^{-3} \leq \pi \leq 4.0 \times 10^{19} m^{-3} \). However, features similar to those described next are also seen in high power ECH plasmas. The NBI power is around 0.5 MW, which produces a rather moderate plasma pressure (volume average \( \langle \beta \rangle \leq 0.5\% \)) and small net electric currents (\( |I_p| \leq 2 \) kA). In TJ-II plasmas the typical transition to the improved confinement manifests in a drop of electromagnetic activity and fluxes towards the wall, accompanied by a widening of the density profiles and a change in the radiation profiles from bell-shaped to dome-like. The formation of a sheared rotation layer is generally found at around \( 0.7 \leq \rho \leq 0.8 \) depending on the magnetic configuration, particularly
Figure 1. (a) Time evolution of discharge #24014 line-averaged electron density, diamagnetic energy, Hα emission and line-integrated emissivity, $P$, from two bolometer chords tangent to flux surfaces with $\rho \approx 0.29$ and $\rho \approx 0.57$. (b) Enlargement of Mirnov coil signal, Hα emission and multi-chord bolometry, showing sawtooothing activity, transition to H-mode and an internal crash. (c) Unfiltered traces of the outer bolometry chords and Hα. (d) Coherent modes in the spectrogram of a bolometer signal whose line of sight crosses the whole plasma.

due to the placement of different low-order rational values of the rotational transform. In Fig. 1 we show the time evolution of a discharge that presents three different phenomena: first sawtooothing activity, then a transition to high confinement and later on an internal crash.

After $t=1135$ ms bursts of activity develop and continue to $t=1145$ ms when an H-mode is established. This period is detailed in Figs. 1(b)-(d). A radius where the bolometry time traces invert growth (inversion point) is manifested during the pre-transition phase: there is a trace with no or little modification during the sawteeth, while the signals have opposite jumps inside and outside. During the bursty period of the Hα signal, the MHD events (Mirnov coil signal, $\delta B^2$) coincide with perturbations in all the viewing chords. Figure 1(c), shows details of unfiltered outer bolometry traces, $\rho \geq 0.77$, where the profile change near the plasma edge is apparent according to the different evolution of the $\rho = 0.77$ chord with respect to $\rho > 0.77$ chords. Figure 1(d) is the spectrogram of a bolometry chord passing very close to the magnetic axis, thus collecting information from practically all radii. A clear mode at 17 kHz becomes prominent just before H-mode onset. It is seen that the H-mode remains quiescent until another MHD event shows up in the core of the cooler, more collisional plasma at $t \approx 1155$ ms. However, the apparent avalanche gets stopped at a radial position $0.67 < \rho < 0.77$ without leaving a detectable trace further out.

The bolometric measurements of the three types of events that are usually observed are compared in Figure 2. These are the evolving profiles as a function of the effective minor radius. The off-axis sawteeth event (same discharge as in Fig. 1), is presented in Fig. 2 showing intermittent rapid radial redistributions of radiation with a pivot or inversion point near the plasma periphery and it is appreciated that the events cover the entire plasma despite their relative small magnitude. Coincident with sawtooth crashes there is the onset of a quasi-coherent mode of about 30 kHz detected in the edge bolometer and in the magnetic signals,
which is resonant with the 8/5 magnetic rational surface. There is also an intensification of Hα activity, features that are shown in Fig. 3. This points to a process started by an edge MHD mode that triggers the rise in edge radiation and the quasi-coherent mode, followed by a burst of particles reaching the wall. In Fig. 2(b) we present the profile evolution for an internal crash like the one in the later stage of Fig. 1 characterized by initial growth and subsequent damping of radiation oscillations suggesting the presence of an island with growing width. For this discharge (#24018) there are two internal crashes involving a small central volume accompanied by a modest increase beyond the inversion radius at around \( \rho = 0.3 \), but leaving the edge region unperturbed. Consequently, no edge magnetic signals are detected but the possible mechanism is that the pressure profiles increase until an internal MHD mode is triggered causing a momentary loss of confinement and a consequently rapid profile readjustment. The behavior of these two events can be compared with the profile evolution of the L-H transition presented in Fig. 2(c). Here the rise in emissivity produces a narrowing of the profiles indicating a cooler edge as a result of the presence of a transport barrier further in. It can be said that the same process involving transport barriers in Figs. 2(a) and (b) is present when an L-H transition happens, with the particularity that the resonant region in this case is more effective in creating a transport barrier.

The close correlation of rational surfaces with the inversion points in the emissivity profiles is supported by the fact that increasing the radial position of the vacuum rational over different discharges also increases the radial position of the inversion point. This link indicates that the resonances have tampered with transport in some way. Regardless of the position or the strength of the events, the plasma behavior is systematic in a way: particle flows are released and move radially leaving a magnetic signature detectable in Mirnov coils and radiation signals. The effect is clearly electromagnetic. In some cases, if the location of the resonances is appropriate the effectiveness of the transport barriers increases to the extent that they seem to be at the origin of the L–H transition, which occurs associated with low-order rationals located at about \( \rho \sim 0.8 \).

**Figure 2.** Time evolution of reconstructed emissivity profiles for (a) off-axis sawtoothing event in discharge #27222 (b) internal crash in discharge #24018 and (c) an improved confinement mode (H-mode) in discharge #21319.

The sequence of events just described has to be interpreted in terms of how the magnetic structures associated with rational surfaces (i.e. magnetic islands) can affect transport in the way observed.

3. Transport model in presence of magnetic islands

It is well known that neoclassical transport is strongly affected by the radial electric field which is important in a stellarator due to the non-ambipolar nature of fluxes. Therefore, any process that can potentially modify the electric field should affect plasma transport. Such is the case...
of a magnetic island since the presence of closed magnetic surfaces allows the particles to travel along the field lines efficiently producing fast radial transport, but more so for electrons. It is expected then that a radial electric field across the island be present, modifying in turn the confinement. In order to incorporate this effect in the overall plasma transport we used the Astra transport code [15] introducing a model for turbulent transport and the presence of the islands. We chose to adopt a model of transport due to resistive ballooning modes [2] since it seems to be a likely candidate for stellarators. In the model we have to include the effect of sheared-flow suppression of turbulence [4] in order to allow for the formation and destruction of transport barriers. In the Astra code the electric field is computed from the electron and ion
fluxes $\Gamma_j(E_r)$ by solving the ambipolarity condition in the form

$$\frac{\partial E_r}{\partial t} = K[\Gamma_e(E_r) - Z_i \Gamma_i(E_r)]. \quad (1)$$

until the stationary state is established. In this relation, only the neoclassical fluxes are taken since the anomalous transport is usually intrinsically ambipolar. For the neoclassical coefficients we use the analytical expressions given by Kovrizhnykh [3]. In order to better model the TJ-II plasmas we have also included the presence of impurities which contribute to the radiation emission; three charge states of carbon are assumed. The equations considered in the model can be written as

\begin{align*}
\frac{\partial n_e}{\partial t} &= -\nabla \cdot \Gamma_e + S_e \\
\frac{\partial}{\partial t} \left( \frac{3}{2} n_e T_{e,i} \right) &= -\nabla \cdot \mathbf{q}_{e,i} + Q_{e,i} \\
\frac{\partial n_{C2}}{\partial t} &= n_e(n_{C3}R_{C3} - n_{C2}(R_{C2} + I_{C2})) - \nabla \cdot \Gamma_{C2} \\
\frac{\partial n_{C3}}{\partial t} &= n_e(R_{C2} - n_{C3}(R_{C3} + I_{C3}) + n_{C4}R_{C4}) - \nabla \cdot \Gamma_{C3} \\
\frac{\partial n_{C4}}{\partial t} &= n_e(R_{C3} - n_{C4}(R_{C4} + I_{C4})) - \nabla \cdot \Gamma_{C4} \\
\Gamma_j &= -(D_{NC,j} + D_{tb,j})\nabla n_j \\
\mathbf{q}_j &= -n_j(\chi_{NC,j} + \chi_{tb,j})\nabla T_j + \frac{5}{2} \Gamma_j T_j \\
S &= S_{rec} + S_{gb} + S_{NBI} \\
Q_e &= -P_{ei} + P_{NBI} - P_{rad} \\
Q_i &= P_{ei} + P_{NBI} - P_{cx} \\
P_{rad} &= n_e(C_{eff} n_I + C_{C2} n_{C2} + C_{C3} n_{C3} + C_{C4} n_{C4}) \\
\frac{\partial \epsilon}{\partial t} &= (\gamma_{RB} - \alpha_1 \epsilon + \alpha_2 |\omega_s|)\epsilon - \nabla \cdot \Gamma_e
\end{align*}

where all quantities have their usual meaning, with $R_a$, $I_a$ the recombination and ionization rates, $\epsilon$ is the turbulence fluctuation level, whose evolution is determined by the linear growth rate of resistive ballooning modes (RBM)

$$\gamma_{RB} \propto \left( \frac{\eta \beta |p'|}{a \tau_A \rho \epsilon^2} \right)^{2/3}$$

the nonlinear saturation coefficient

$$\alpha_1 = \frac{|p'|^{3/4}_{RB}}{\rho \sqrt{a^2 \eta \sqrt{\gamma_A}}}$$

and the shearing rate $\omega_s = \partial_r (cE \times B/B^2)$. Now, the presence of the island is included as an additional electron flux $\Gamma^R_e$ at the island position in Eq. 1 in addition to the neoclassical contribution $\Gamma^{NC}_e$. Thus, the ambipolar state implies $\Gamma^{NC}_e(\rho) + \Gamma^R_e(\rho) = Z_i \Gamma_i^{NC}(\rho)$, with the island flux localized at the resonant surface $\rho_R$:

$$\Gamma^R_e(\rho) = \Gamma_0 \exp[-((\rho - \rho_R)/\Delta_R)^2] \quad (2)$$
where $\Delta R$ is a measure of the island width. This determines the radial electric field.

The effect of the resonant flux is to increase the radial electric field around $\rho = \rho_R$ and therefore the shear rate, $\omega_s = \partial_r E_r + (\delta - 1)E_r/r$, increases substantially. This in turn, tends to stabilize the RBM turbulence, which reduces the transport, creating a barrier there. The transport reduction should affect the island too but this effect is not included in the present model; we force the feedback manually just to reproduce the cyclic evolution. The results of simulations with the model just described are shown in Figure 4, for parameters taken from discharge #24006. In panel (a) the time evolution of the total emission at different radial positions is presented for the case when an island is placed at $\rho_R = 0.76$ for a time of 1 ms and then removing it. After the barrier is formed the emission increases at radii smaller than $\rho_R$ and decreases for $\rho > \rho_R$ in qualitative agreement with the observations. The different strength peaks are forced when the shearing parameter $\omega_s$ is varied keeping the electron-ion imbalance fixed. In panel (b) we have the radial electric field profile, exhibiting that the effect of the island is to increase the $E_r$ values with the consequent high gradient of both $E_r$ and shearing rate, which is responsible for the mode stabilization. The local perturbations in the simulation have maximum amplitudes consistent with the experimental values, namely, 10% for the density, 20% for the radiation emission, 4% for temperature and less than 2% for the potential.

The set of profiles of several parameters taken from the Astra simulation are shown in Figure 5 for a time when the barrier is not present. It shows that the carbon profiles for CIII and CIV peak near the edge and the $E_r$ field is negative, as in all high density NBI discharges. The lower panel confirms that the instability dynamics, given by $\gamma$ (gamma) is concentrated near the edge, as expected for resistive ballooning modes. Consequently, the electron thermal conductivity $\chi_e$ (Xe) is also quite large close to the edge.

4. Magnetic island evolution

In addition to the qualitative reproduction of the dynamics it is necessary to address the issue of what is the physical mechanism responsible for the flux enhancement assumed in the model. An ingredient from the experiment that has also to be considered is the expulsion of fast particles to the wall, which points to some mechanism that either accelerates particles or releases high-energy particles trapped in the islands when the MHD activity starts. In any case, the dynamics of the islands associated with the rational surfaces seems to be the important process to study. Regardless of whether the island is present in the vacuum magnetic field or it is formed during the
plasma discharge, the dynamics should be determined by tearing mode stability. The magnetic reconnection process resulting from the tearing instability can potentially accelerate the electrons detected by the Hα signal. Reconnection could also open island field lines and release trapped energetic particles.

To address the tearing mode stability in this case it is necessary to include the presence of shear flows, since they are always present when a transport barrier is formed. The presence of flows at the island location and their consideration in the nonlinear tearing mode dynamics should shed light on the interplay between barrier formation and island growth. An important effect to take into consideration when the island width is as small as the ion Larmor radius is the polarization drift. This effect is important when the island begins to grow and it has been variously described in the literature with some discrepancies as explained in [16]. The relevant effect that determines the contribution to the growth is the velocity profile across the island. It enters the evolution of the island width $w$ through a contribution of the polarization drift $\Delta_p'$ to the tearing stability parameter $\Delta'$ as,

$$\frac{dw}{dt} \propto \Delta' + \Delta_p' = \Delta' + g(\omega - \omega_{ni})$$

where $g$ depends on the velocity profile and its sign determines the stability. Depending on the frequency range, it can be stabilizing or destabilizing [16].

When there is shear flow the asymmetry in the velocity profiles modifies the stability; as shown in [17] the shear contribution is stabilizing for low and high shear but it destabilizes the mode for intermediate values of the shear. This may lead to oscillations of the island width.

An important issue to consider is the island rotation velocity. This is related to the frequency of the magnetic perturbations measured in the experiment because, in the plasma frame, it is determined by the electron or ion diamagnetic velocities as: $\omega = f\omega_{ce} + (1 - f)\omega_{ni}$, where $f$ is the flattening parameter that measures the degree of temperature profile flattening inside the island. $f = 1$ means there is no flattening. Since in TJ-II NBI plasmas there is no flattening detected during off-axis sawteeth, one would expect $\omega \approx \omega_{ce}$. According to Connor et al. [16],

![Figure 5. Astra profiles for a time when there is no transport barrier. Each panel contains two profiles in red and blue.](image-url)
for \( w \leq \rho_i \), \( g \sim \omega - \omega_{se} \) giving for the polarization drift contribution

\[
\frac{dw}{dt} = \frac{I}{k^2 w} \cdot 10^{-6}(\omega - \omega_{se})(\omega - \omega_{ri}).
\]

(3)

This would give \( \Delta_p' \approx 0 \) for TJ-II when \( \omega \approx \omega_{se} \). Then there should be stable island rotation in the frame of reference where \( E_r = 0 \). Moving to a frame with \( E_r = -3 \text{ kV/m} \), Eq. 3 predicts there can be instability for frequencies outside the range \( 0 < \nu < 20 \text{ kHz} \) for a resonance 8/5 near the edge, for NBI plasma of moderate density. The corresponding growth rate is of order \( 10^5 \text{s}^{-1} \) when \( I/(kW)^2 \sim 1 \). This seems to be consistent with the experimental data.

Another ingredient one should consider is the fact that rational surfaces have magnetic islands even in the vacuum magnetic field. Therefore, the island dynamics analysis should start with these vacuum islands. This problem has been addressed recently in [18] in relation to island healing, considering the self-consistent torque balance. In our case it is relevant since an island can grow or shrink depending on the velocity profiles, so it is important to start with a finite island size. The program to follow is then to study the behavior of the polarization stability parameter from the initial vacuum island, providing the velocity profiles characteristic of the sheared flows observed around TJ-II rational surfaces. This should determine the island evolution.

5. Conclusions

The experimental results obtained over a large range of plasma conditions in TJ-II have evidenced the very important role magnetic rational surfaces play in the formation and destruction of transport barriers. We have presented here a summary of the measurements that associate the onset of MHD activity with the modification of plasma confinement at the position of a low-order rational surface. The picture that emerges is that a transport barrier can form at a rational resonant surface when an enhanced sheared flow is set there, due probably to the increased \( E_r \)-field that appears at the magnetic island produced by large parallel electron transport. The sheared flows can trigger a magnetic instability which eventually destroys the transport barrier releasing outward flows and fast particles that hit the wall.

This interpretation is incorporated in a transport model that includes the effect of the island through a localized radial electron flux centered at the resonant surface. The turbulent transport is produced by resistive ballooning modes that may be stabilized when the \( \mathbf{E} \times \mathbf{B} \) shearing rate is large enough. The preliminary results of this model seem to reproduce in a satisfactory qualitative way the observations. Particularly, the increased radial electric field gradient is of the right size to stabilize the RBM turbulence and create a local small transport barrier, producing sawteeth in the total emission that have an inversion point at the island position. The model is imperfect in that it only describes the effect of the island on transport but the feedback effect of transport on the island is not included.

In order to study the later effect the island evolution is considered from the point of view of the tearing mode stability. The polarization drift contribution to the stability parameter is considered as a factor that modifies the island width evolution and this is critically determined by the velocity profiles across the island. Therefore, it is necessary to study the dynamics of an initial vacuum magnetic island that is subjected to the shear flows around the island typical of a transport barrier. This study is on-going and will be presented elsewhere. When this is completed, an improved transport model will be constructed that incorporates the ingredients for a self-consistent description.

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