Research of Active Disturbance Rejection Controller Design for PMSM Servo System

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Abstract. In the permanent magnet synchronous motor (PMSM) speed control system, an improved active disturbance rejection control strategy with higher tracking accuracy for time-varying input is proposed. The traditional active disturbance rejection controller is mainly used for fast and static-free tracking of step signals, but there is a large tracking error for time-varying signals, which limits the application of active disturbance rejection controller. In this paper, the theoretical analysis of the existence of steady-state error is carried out, and then an improved speed active disturbance rejection controller (ADRC) with derivative feedforward and parallel linear extended state observer (P-LESO) is designed to reduce the tracking error of the system. In order to observe and compensate the reverse electromotive force in real time and reduce the current following error, a current loop linear active disturbance rejection controller is designed. By constructing Simulink simulation model for verification, the control system not only improves the tracking accuracy of PMSM for time-varying input, but also has a good dynamic performance for step input.

1. Introduction

In recent years, permanent magnet synchronous motors (PMSM) have been widely used in control systems in various fields due to their small size, small torque ripple, and simple structure. Active disturbance rejection control\cite{1-2} (active disturbance rejection control, ADRC) as a new type of controller has been widely used in permanent magnet synchronous motor servo systems. However, the nonlinear active disturbance rejection controller has certain difficulties in parameter tuning and theoretical analysis of control performance. Compared with this, the linear active disturbance rejection controller (LADRC) has simple parameter tuning and is easier to apply in engineering\cite{3-4}. Literature\cite{5-6} uses active disturbance rejection controllers on industrial sewing machines and bridge power converters. Simulations show that the design effectively compensates for the effects of uncertain disturbances and greatly enhances the dynamic performance of the system. Literature\cite{7} used linear and non-linear ADRC to replace the traditional PI controller in the speed regulation system of permanent magnet synchronous motor, and got good anti-interference performance. Literature\cite{8-9} proposed a compensation control strategy based on the ADRC model, which compensates the known part of the model to the ADRC, improves the observer's estimation accuracy, and makes the system more resistant to load disturbances. From the current research, the application of active disturbance rejection controller is mainly for step input, and for time-varying input, the traditional active disturbance rejection controller
has a large tracking error and poor control accuracy, thus limiting the active disturbance rejection. The scope of application of the controller.

In response to the above problems, the ADRC of the speed loop in this paper adopts the idea of input differential feedforward to eliminate the tracking error of the traditional ADRC, and on this basis, the ADRC of the speed is further improved, and the parallel expansion state observer (LESO) is introduced, which effectively improves the traditional The observation accuracy of the single-channel LESO reduces the steady-state error.

2. The mathematical model of PMSM

This paper selects the surface-mounted permanent magnet synchronous motor model as the research object. The mathematical model under the vector control strategy is:

\[
L_d \frac{di_d}{dt} = -Ri_d + u_d + \omega_l L_c i_q \\
L_q \frac{di_q}{dt} = -Ri_q + u_q - \omega (\varphi_f + L_d i_d) \\
T_e - T_i - B\omega = J \frac{d\omega}{dt} \\
T_e = \frac{3}{2} n_p \varphi_f i_q
\]

In the formula, \(i_d, i_q\) is the motor stator current; \(R\) is the stator resistance; \(u_d, u_q, L_d, L_q\) is d-axis and q-axis voltage and inductance respectively; \(\omega\) is the electrical angular velocity; \(\varphi_f\) is a permanent magnet flux linkage; \(\omega\) is Mechanical angular velocity \(T_e\) is electromagnetic torque; \(n_p\) is extremely logarithmic; \(J\) is the system moment of inertia; \(T_i\) is the load torque.

3. Traditional speed first-order active disturbance rejection controller

In this paper, vector control method is adopted, and the speed equation is expressed as:

\[
\dot{\omega} = \frac{3n_p \varphi_f i_q}{2J} \frac{T_e}{J} - \frac{B\omega}{J}
\]

Rewrite equation (2) as:

\[
\dot{\omega} = f + bi_q
\]

In the formula, \(f = \frac{T_e}{J} - \frac{B\omega}{J}\) is the sum of internal and external disturbances; \(b = \frac{3n_p \varphi_f}{2J}\) is the current gain.

Select the mechanical angular velocity \(\omega\) as the state variable \(x_1\), the total disturbance action \(f\) is the state variable \(x_2\), and the q-axis given control variable \(i_q\) is written as \(u\), then the state equation of equation (3):

\[
\dot{x}_1 = x_2 + bu
\]

3.1. Design of Extended State Observer

Establish an expanded state observer for the state equation (4):

\[
\dot{e} = z_1 - \omega \\
\dot{z}_1 = z_2 - \beta e + bu \\
\dot{z}_2 = -\beta e
\]
In the formula $\omega$ is the actual speed of the motor; $z_i$ is the observed value of the speed; $z_i$ is the observed value of the disturbance; $e$ is the observation error; and $\beta$, $\beta_i$ is the parameter of the expanded state observer.

The controllers designed in this paper use the first-order active disturbance rejection control technology, omitting the TD units.

### 3.2. Error feedback control rate
First-order linear error feedback control rate:

$$
\begin{align*}
    e &= \omega^* - z_i \\
    u_e &= ke \\
    u &= \frac{u_e - z_i}{b} = z_i \rightarrow \omega \\
    z_i \rightarrow f
\end{align*}
$$

(6)

In the formula, $\omega^*$ is the given input signal for ADRC; $k$ is the scale factor.

Bring equation (6) into equation (4), the system is equivalent to a linear system:

$$
\dot{\omega} = u_e
$$

(7)

The tracking error is defined as, according to equation (4), the state equation can be expressed as:

$$
\begin{align*}
    \dot{e} &= \omega^* - \omega = \omega^* - bi_i - x_z
\end{align*}
$$

(8)

Simultaneous (6), (7), (8) can get:

$$
\begin{align*}
    \dot{e} &= \omega^* - \omega = \omega^* - u_e = \omega^* - ke = \omega^* - bi_i - x_z
\end{align*}
$$

(9)

Substituting the observations of ESO and combining equation (9), the control quantity is:

$$
i_v = \frac{ke - z_i}{b}
$$

(10)

### 3.3. Problems with traditional first-order ADRC
Substitute equation (8) gives:

$$
\dot{e} = \omega^* - ke + z_i - x_z
$$

(11)

Laplace transformation of equation (11) can be obtained:

$$
e(s) = \frac{s\omega^* + z_i(s) - x_z(s)}{s + k}
$$

(12)

It can be obtained from equation (12) that the tracking error is mainly affected by the input first derivative element and the observation error and $k$ value of the disturbance. The traditional ADRC given input signal is generally a step signal because the first derivative of the step signal is zero, so the tracking error of the system is small. However, when the given input signal is a time-varying signal, its first derivative is not zero, and the system generates a large modeling error. Although the tracking error can be reduced by increasing $k$, according to equation (7), the speed loop after disturbance compensation is equivalent to a first-order integration element, and the $k$ value at this time is the closed-loop bandwidth $\omega_c$. Therefore, by increasing the system bandwidth $\omega_c$ to reduce the tracking error, the system noise will too, which will affect the performance of the system.

### 4. Improved speed loop first-order active disturbance rejection controller
In this section, the ADRC with input differential feedforward is designed, and LESO is redesigned on this basis. Reduce the tracking error of the system.

#### 4.1. Introduce differential feedforward
For the tracking error caused by time-varying input, the disturbance compensation link is redesigned, and the input differential feedforward is introduced on the basis of (10). The new control quantity is:
Bring equation (13) into equation (8) and perform Laplace transform:

\[ e(s) = \frac{z_s(s) - x_s(s)}{s + k} \]  

(14)

According to the tracking error equations (12) and (14), the introduction of differential feedforward eliminates modeling errors \( s\omega' \). When the input is a time-varying signal (the first derivative is not zero), such as a sine wave \( \omega' = A \sin(\alpha x) \), \( A > 0, \alpha > 0 \)

The modeling error is:

\[ s\omega' = A \cdot a \cos(\alpha x) \leq |A \cdot a| \]  

(15)

It can be known from equation (15) that the magnitude of the modeling error eliminated is related to the amplitude and frequency of the time-varying signal, and is limited by the absolute value of the product of amplitude and frequency.

4.2. Design of parallel expansion state observer

On the basis of equation (14), the tracking error is further eliminated, and the system's tracking accuracy of the input signal is improved.

For the systems (4) and (7), the purpose of the extended state observer for real-time estimation and compensation of disturbance is to compensate the original system as a first-order series integration system. According to this idea, to construct an ideal first-order integration system that assumes input, thus simulating the PMSM control system. Thus, the extended state observer fails to accurately estimate the compensation disturbance (residual disturbance: the difference between the output of the ideal integral system and the actual system). On this basis, assume the original expanded state observer is LESO1, and according to the design principle of LESO1, construct LESO2 shown in the dashed frame of Figure 1 to observe the remaining disturbance and compensate it.

When the total disturbance is too large, observe it through LESO1 and LESO2:

\[ f = z_i + f' = z_i(s) + z_i' \]  

(16)

In the formula, \( z_i \) is the observation value of LESO1 to the disturbance \( f \), the observation \( z_i' \) value of LESO2 to the remaining disturbance \( f' \), and the latest observation value of the total disturbance is \( z_i(s) \).

Therefore, the improved tracking error is:

\[ e(s) = \frac{z_i'(s) - x_s(s)}{s + k} \]  

(17)
It can be seen from Sections 2.3 and 3.1 that the tracking error expression (14) is mainly affected by the observation error, and the degree of the tracking error is only determined by the observation performance of the observer. Only by improving the observation performance of the observer and reducing the observation error can it be reduce tracking error. Therefore, through the design of (P-LESO), the estimation burden of LESO1 is reduced, the observation error is reduced, and the tracking error is further reduced.

It can be deduced from equations (16) and (17):
\[
\|z'(s) - z(s)\| \leq \|z(s) - x(s)\| \quad (18)
\]
\[
\epsilon(s) = \|x(s) - x(s)\| \leq \|z(s) - x(s)\| \quad (19)
\]

It can be seen from equations (16) and (18) that \(z'(s)\) compared with \(z\), and from (19) it can be concluded that the tracking error \(z\) is smaller than that of equation (14), thereby improving The tracking accuracy of the system to the time-varying signal is discussed.

5. **Current loop first order active disturbance rejection controller**

In the PMSM control system, the current inner loop is mainly affected by changes in stator resistance and back electromotive force. Based on the idea of auto-interference rejection, LADRC is designed to replace the traditional PI in the current loop. The vector control adopted \(i_q = 0\) in this paper, therefore, rewrite equation (1) as:

\[
\frac{di_d}{dt} = \frac{1}{L_d} (-Ri_d + u_d + \omega_L i_q) = f_d + b_d u_d
\]
\[
\frac{di_q}{dt} = \frac{1}{L_q} (u_q - Ri_q - \omega_L \psi) = f_q + b_q u_q
\]

In the formula, \(f_d = \frac{1}{L_d} (-Ri_d + \omega_L i_q)\) is the total disturbance of the \(d\) shaft, \(b_d = \frac{1}{L_d}\) is the shaft voltage gain; \(f_q = \frac{1}{L_q} (Ri_q + \omega \psi)\) is the total disturbance of the \(q\) axis, \(b_q = \frac{1}{L_q}\) is the \(q\) axis voltage gain;

6. **Simulation results and analysis**

In order to verify the control effect of the auto disturbance rejection controller designed in this paper on the PMSM servo system, a general block diagram of the control system was built on the Matlab/Simulink platform and a simulation study was conducted, as shown in Figure 2. Parameters of permanent magnet synchronous motor used: \(R_s = 2.875 \Omega, L_s = L_q = 0.0085 \text{ mH}, \psi_s = 0.175 \text{ Wb}, J = 0.0008 \text{ kg.m}, n_p = 4\). Both the speed loop and the current loop are first-order linear ADRC, so the tuning method based on the system bandwidth \(k = \omega, \beta = 2\omega, \beta_2 = \alpha^2\) is used to set the parameters of the controller.

![Figure 2 Scheme of servo system with double closed-Loop based on ADRC](image-url)
6.1. Simulation analysis
Both the sine and triangle given have an amplitude of 1000 r/min, a frequency of 80 Hz, and the step setting uses an amplitude of 1000 r/min.

From the analysis of equations (12) and (14), it can be seen that the tracking error of the traditional ADRC controller is mainly composed of the first derivative of the time-varying input and the observation error of the disturbance. Figure 3 shows that when the sine is given, the traditional LADRC+PI control method has the largest tracking error without IDF; the introduction of IDF eliminates the modeling error caused by the first derivative, making ADRC+(IDF)+PI tracking error is significantly reduced by 364.9 r/min, but there is still a large tracking error; considering the influence of the current inner loop stator resistance and back electromotive force changes, the traditional PI controller with poor calibration parameters has poor control effect. In order to further improve the tracking accuracy, the current loop is changed to LADRC, so that the ADRC (IDF)+LADRC tracking error is reduced by 27.1 r/min again, and the tracking effect is improved again; then, through the parallel LESO idea, the traditional linear expansion state observer pair is improved. For the observation accuracy of disturbance, equation (16) obtained from theoretical analysis is reduced compared to equation (14). The tracking error caused by the disturbance observation error is reduced again, making the final design of ADRC (IDF+P-LESO)+LADRC have a higher Tracking accuracy, the error is reduced by 5.9 r/min again, and the tracking effect reaches the best.

![Dynamic response of tracking sine reference](image)

**Figure.3** Dynamic response of tracking sine reference

Given in the triangle, the following characteristics of the PMSM speed control system corresponding to the four control modes are shown in Figure 4. Similar to the analysis in Figure 3, the traditional LADRC+PI control method has the largest tracking error, and the final improved ADRC (IDF+P-LESO)+LADRC tracking effect is the best. As shown in Figure 4, the tracking errors of the four control modes from left to right are sequentially reduced: 239.2 r/min, 5.8 r/min, 13.8 r/min, indicating that the ADRC designed in this paper can achieve better tracking, the control accuracy of the system is the best.

![Dynamic response of tracking triangular reference](image)

**Figure.4** Dynamic response of tracking triangular reference

When the step is given at 1000 r/min, it starts at no load and suddenly increases 5 N·m in 0.15 seconds and decreases 5 N·m in 0.3 seconds, as shown in Figure 5. As can be seen from Figure 5, in the PMSM speed control system, compared with the traditional ADRC and PI controller control methods, the improved speed ADRC (IDF+P-LESO)+LADRC did not appear overshoot during the motor acceleration process. After load increase or decrease, the motor speed fluctuates. Compared with the other three control methods, the improved ADRC (IDF+P-LESO)+LADRC system has the smallest speed drop (swell) error after disturbance and has better capacity of resisting disturbance. Therefore, it
can be seen that the improved system has better control effect on the permanent magnet synchronous motor.

![Figure.5 Dynamic response of tracking step reference](image)

7. Conclusion
In the PMSM speed control system, in view of the traditional ADRC's large tracking error problem under time-varying signals and the control requirements of the servo control system, this paper proposes a speed active-disturbance rejection control based on differential feedforward and parallel extended state observer. The differential feedforward eliminates the modeling error. On this basis, the parallel extended state observer reduces the observation error, thereby improving the tracking performance of the system. Simulation results show that, compared with traditional ADRC, the system designed in this paper reduces the steady-state error under time-varying input and improves the steady-state performance of the system; at the same time, it also has a better capacity of resisting disturbance for step input, no overshoot, high tracking accuracy, fast dynamic response, etc. In addition, this design parameter is easy to tune, whether it is a time-varying signal or a step signal, it has a better control effect and has more practical application value. This article only proves the feasibility of the design in theory, but it may cause cost increase when the hardware implements the control strategy of this article.

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References
[1] HAN Jingqing. From PID technique to active disturbances rejection control technique[J]. Control Engineering of China, 2002,9(3):13.
[2] Gao Z. Active disturbance rejection control: From an enduring idea to an emerging technology[C]. International Workshop on Robot Motion and Control, 2015(10):269-282. DOI:10.1109/RoMoCo.2015.7219747.
[3] Zheng Q,Gao Z Q. Active disturbance rejection control: between the formulation in time and the understanding in frequency[J]. Control Theory and Technology, 2016,14(3):250-259.
[4] Chu Z,Wu C,Sepehri N. Active Disturbance Rejection Control Applied to High-order Systems with Parametric Uncertainties[J]. International Journal of Control, Automation and Systems, 2019,11(8):1194-1204.
[5] Wang H,Zhao R,Cheng F et al. Active Disturbance Rejection Control for PMSM Servo System Applied in Industrial Sewing Machines[C]. International Conference on Electrical and Control Engineering, 2010:3249-3252.
[6] Sun B,Gao Z Q. A DSP-based active disturbance rejection control design for a 1-kW H-bridge DC-DC power converter[J]. IEEE Transactions on Industrial Electronics,2005,52(5):1271-1277.
[7] LIU Chunqiang,LUO Guangzhao,TU Wencong,et al. Servo systems with double closed-loops based on active disturbance rejection controllers[J]. Proceedings of the CSEE, 2017,37(23):7032-7039.
[8] LIU Zhigang, LI Shihua. Active disturbance rejection controller based on permanent magnetic synchronous motor model identification and compensation[J]. Proceedings of the CSEE, 2008, 28(24):118-123.

[9] GAI Jiangtao, HUANG Shoudao, HUANG Qing, et al. Active disturbance rejection controller for permanent magnet motor drive system control based on load observer[J]. Transactions of China Electrotechnical Society, 2016, 31(18):29-36.