Analytical Modelling of Power Swing and Validation Using Real Time Digital Simulator

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Abstract. Grid disturbance is one of the largest areas of concern in the protection of power systems after high penetration of distributive generation. They are responsible for causing mass blackouts. This paper analytically derives the equations for resistances and reactances as a function of time. The derived expressions were validated using real time digital simulator (RTDS) for variation in power factor of system and frequency of the power swing. Results signify that the derived equations were able to provide the locus of instantaneous values of the real time power swing occurring in the system for changes in power factor and swinging frequency. Two dimensional R-X plane and three dimensional R-X-t space plots were also used to study the static and dynamic variations of impedances in time domain. The expressions \( r(t) \) and \( x(t) \) will be useful in determining the instantaneous short circuit ratios and also for tuning the impedance relays.

1. Introduction

With the higher penetration of renewable resources, the large inertia of a synchronous machine is constantly getting replaced by smaller or zero inertia renewable generators like solar, tidal or wind. Decentralisation of large synchronous generators with smaller units of distributed induction machines or solar panels has challenged the grids synchronicity and stability. In view of power system dynamics, stability studies play a vital role for the renewable energy integration. Power swing occurs when a disturbance occurring in one part of a system due to absence or limited inertia leads to propagation of disturbance along the whole grid. These swings cause mis-operation of relays especially the distance relays in transmission lines causing regional or country wide blackout [1].

Power swing is an un-attenuated periodic disturbance caused due to oscillations in voltages and currents thus forcing the impedances to swing around the normal operating point which distorts the locus of tripping and restraining region of relays. Thus it is vital to understand power swing in order to maintain grid stability and for integrating large scale renewable generation with conventional centralised generators [2].

The parameters which are observed by measuring meters at any point in time are voltages and currents. Variation of voltages and currents lead to oscillation of active and reactive power which leads to rise of stability studies during power swing. Eigen value analysis [3] helped in finding the swinging frequencies, yet required numerous computations. Transformation of domain from time to frequency [4] provided the magnitude and frequency of swinging frequencies, yet the continuous online transformations and calculations for small time steps seemed to be memory demanding and


laborious. Thus, it is essential to have a generalised analytical model which could describe the power swing for any frequencies and power factor which eliminates the technical difficulties and solely depend on time varying few sets of non iterative equations [5].

Here, voltages and currents in time domain will be used for analytically deriving the resistances and reactances as a function of time – viz. r(t) and x(t) during power swing. Evaluation of instantaneous resistances and reactances may have the following functions.

1.1 This will shed light on the amount of distributive generations which could be penetrated into the grid based on voltages, currents and power factor before the system loses stability.

1.2 System strength or stiffness is the ability of the power system to maintain voltage stability, frequency stability and quality at the Renewable Energy Source interconnection points. The stiffness of any grid is measured using short circuit ratio which is a function of resistance and reactance. Instantaneous short circuit ratio x(t)/r(t) will help in accessing the impact of renewable energy integration on the systems strength [6].

1.3 The instantaneous generalised locus of impedances during power swing can also be used for tuning of relays depending on the intensity of the disturbance and swinging frequency.

2. Analytical derivation for the impedances

Table 1 below provides the nomenclature for all symbols used in derivation.

| Symbols | Nomenclature | Symbols | Nomenclature |
|---------|--------------|---------|--------------|
| V_p     | Voltage at power frequency | I_p     | Current at power frequency |
| V_s     | Voltage at swinging frequency | I_s     | Current at swinging frequency |
| V_max   | Maximum peak voltage during swinging | I_max   | Maximum peak current during swinging |
| V_min   | Minimum peak voltage during swinging | I_min   | Minimum peak current during swinging |
| v(t)    | Voltage observed during power swing | i(t)    | Current observed during power swing |
| ω_p     | Power angular frequency | ω_s     | Swinging angular frequency |
| m       | Amplitude index | Φ       | Phase shift at power frequency |
| k       | Frequency index | Z_p     | Impedance at power frequency |
| r(t)    | Resistance during power swing | x(t)    | Reactance during power swing |

The voltages at power frequency and swinging frequency can be written in time domain as:

\[ v_p(t) = V_p \cos(\omega_p t) \]  
\[ v_s(t) = V_s \cos(\omega_s t) \]  

Power swing resembles an amplitude modulated waveform for both voltages and currents in time domain as shown in figure 1 with variations in amplitude and frequency [7]. The modulated wave in a AM signal is same as swinging wave during power swing and carrier wave in a AM signal is same as power frequency wave at 60Hz. The variation in differences of peak amplitudes depends on the intensity of the disturbance. Thus, higher the disturbance, greater is the differences between higher and lower peaks. The variation in swinging frequency depends on the system parameters like resistance, reactance, admittance and susceptance.

Thus the amplitude modulated voltage [8] or a swinging voltage as a function of time can be represented as:

\[ v(t) = \left[ V_p + V_s \cos(\omega_s t) \right] \cos(\omega_p t) \]
Amplitude modulation index is defined as:
\[
m = \frac{V_{\text{max}} - V_{\text{min}}}{V_{\text{max}} + V_{\text{min}}} = \frac{V_s}{V_p}
\]  
(4)

\[
v(t) = V_p \cos(\omega_p t) [1 + m\cos(\omega_s t)]
\]  
(5)

Similarly, currents can be written with inclusion of delay angle in both power and swing frequency as:
\[
i(t) = I_p \cos(\omega_p t - \phi) [1 + m\cos(\omega_s t - k\phi)]
\]  
(6)

\(m\) also indicates the ratio of maximum peak voltage to the minimum peak voltage or ratio of maximum peak current to the minimum peak current. It is same for both voltages and currents due to ohms law. If \(V_s/I_s = V_p/I_p\) due to impedance; then \(V_s/V_p = I_s/I_p\).

Finding impedance as a ratio of voltages and currents yields
\[
z(t) = \frac{v(t)}{i(t)} = \frac{V_p \cos(\omega_p t) [1 + m\cos(\omega_s t)]}{I_p \cos(\omega_p t - \phi) [1 + m\cos(\omega_s t - k\phi)]} = \frac{Z_p \cos(\omega_p t)[1 + m\cos(\omega_s t)]}{Z_p \cos(\omega_p t - \phi)[1 + m\cos(\omega_s t - k\phi)]}
\]  
(7)

Rewriting the above equation in exponential format
\[
z(t) = Z_p e^{i[\omega_p t + m\omega_s t]} = Z_p e^{i(\omega_p t + m\omega_s t)}[1 + m e^{-j\omega_s t}]^{-1}
\]  
(10)

Binomially expanding the \((1 + xe^{-j\omega_s t})^{-1}\) and rearranging the terms in ascending powers of \(x\),
\[
z(t) = Z_p e^{i\omega_p t} \sum_{n=0}^{\infty} \left( -m \right)^n \sin(\omega_s t + \phi - \frac{nk\pi}{2}) \sin(\omega_p t + \phi - \frac{(n-1)k\pi}{2})
\]  
(11)

Eulers identity: \(Z e^{i\phi} = Z\cos(\phi) + jZ\sin(\phi) = r(t) + jx(t)\)

Separating the real and imaginary parts as resistance and reactance using Eulers Identity,
\[
r(t) = Z_p \left[ \cos(\omega_s t + \phi) - \cos(\omega_p t + \phi - k\phi) - m^2 \cos(2\omega_s t + \phi - 2k\phi) + m^3 \cos(3\omega_s t + \phi - 3k\phi) - m^4 \cos(4\omega_s t + \phi - 4k\phi) + m^5 \cos(5\omega_s t + \phi - 5k\phi) - m^6 \cos(6\omega_s t + \phi - 6k\phi) + \ldots \right]
\]  
(12)

Using cosine identities in ascending powers of \(m\),
\[
r(t) = Z_p \left[ \cos(\omega_s t + \phi) + 2 \sin\left(\frac{k\phi}{2}\right) \sum_{n=1}^{\infty} \left(-m\right)^n \sin(\omega_s t + \phi - \frac{(n-1)k\pi}{2}) \right]
\]  
(13)

Generalising the expression yields
\[
r(t) = Z_p \left[ \cos(\omega_s t + \phi) + 2 \sin\left(\frac{k\phi}{2}\right) \sum_{n=1}^{\infty} \left(-m\right)^n \cos(\omega_s t + \phi - \frac{(n-1)k\pi}{2}) \right]
\]  
(14)

Similarly, \(x(t)\) can be derived as
\[
x(t) = Z_p \left[ \sin(\omega_s t + \phi - \frac{k\phi}{2}) \sum_{n=1}^{\infty} \left(-m\right)^n \cos(\omega_s t + \phi - \frac{(n-1)k\pi}{2}) \right]
\]  
(15)
3. Test Bench using Real Time Digital Simulator

Real time digital simulator (RTDS) was used for testing the validity of the derived analytical equations in time domain which are r(t) and x(t). Figure 2 shows the test setup of the RTDS, RSCAD and computer workstation.

A simple 3 bus, 735kV, 2200MVA system was used consisting of generator, two transmission lines and a load. The measurements for voltages and currents were taken from the bus which is situated at the middle of the transmission lines. The generator model used can generate both the power frequency and swinging frequency. Transmission lines were modelled as long lines using toolbox from RSCAD. The load is a variable load which can be varied during run time. The r(t) and x(t) obtained will be plotted in two dimensional static impedance plot (R-X plane) and three dimensional spatial dynamic plot (R-X-t space) [9]. The test system used in RTDS is shown in the figure 3. The equations used for obtaining the impedances are shown below [10]:

\[ Z(t) \angle \theta(t) = \frac{V(t)}{I(t)+dI_0(t)} = \frac{V(t)}{I(t)+d\left(\frac{R(t)+jX(t)+jI_0(t)}{3}\right)} \]  

(16)

where,

- \( V \) and \( I \) are phase voltages and currents
- \( d \) is the sensitivity factor for zero sequence current
- \( I_0 \) is the zero sequence current.

\( Z(t) \angle \theta(t) \) will be transformed to \( r(t)+jx(t) \) in order to compare the graphs with the derived equations.

4. Analytical Model vs RTDS

Validation of the analytical model using equations will be verified using RTDS. The validation will be done by varying power factor and swinging frequency for the analytically derived equations (14) and (15) and in RTDS test bench.

4.1. Variation of Power factor

Power factor will be varied from -60° to +60° for both the analytical model and RTDS. The power factors, reactive power and real power used are as shown in the Table 2.

| Load   | Power Factor Angle | Real Power | Reactive Power | Power Factor  |
|--------|--------------------|------------|----------------|--------------|
| 2200 MVA | -60 °              | 1100 MW   | -1905 MVAR    | 0.5 leading  |
| 2200 MVA | -30 °              | 1905 MW   | -1100 MVAR    | \( \sqrt{3}/2 \) leading |
| 2200 MVA | 0 °               | 2200 MW   | 0 MVAR        | 1            |
| 2200 MVA | 30 °              | 1905 MW   | -1100 MVAR    | \( \sqrt{3}/2 \) lagging |
| 2200 MVA | 60 °              | 1100 MW   | -1905 MVAR    | 0.5 lagging  |

All the simulations were carried out at the swinging frequency of 3Hz for different power factors.
4.1.1. Analytical Model
Power factor was varied for the analytical model according to the Table 2 using the equations (14) and (15). The R-X and R-X-t plots for the same are shown in the figures 4 and 5 respectively.

4.1.2. Real Time Digital Simulator
Power factor was varied for the RTDS variable load according to the Table 2. The two dimensional R-X and three dimensional R-X-t plots for the same are shown in the figure 6 and 7 respectively.

4.2. Variation of Swinging frequency
Swinging frequency will be varied as 10/3Hz, 20/3Hz and 10Hz for both the analytical model and RTDS. This considers both slow swing which is 3.33Hz and fast swing which is 10Hz [11]. All the simulations were investigated at a power factor of 0.95 lead for different swinging frequencies.

4.2.1. Analytical Model
Swinging frequency was varied for the analytical model using the equations (14) and (15). The static R-X plot and dynamic R-X-t plots for the same are shown in the figures 8 and 9 respectively.

4.2.2. Real Time Digital Simulator
Swinging frequency was varied for the RTDS variable generator. The two dimensional R-X planar plot and three dimensional R-X-t spatial plots are shown in the figures 10 and 11 respectively.
5. Observations
The analytically derived equations and the plots taken from RTDS possess similarities and differences.

5.1. Variation in power factor
Figures 4, 5, 6 and 7 provide plots for variation in power factor according to Table 2.

Static two dimensional R-X plots for analytically derived equations and RTDS system are shown in figures 4 and 6 respectively. The major similarities observed in these two plots are the formation of biconical structures. The apex of the biconical structure is the normal operating point of the system in the absence of power swing (Zp = Vp/Ip). The circular formation extending beyond the normal operating point is due to the power swing. These are the swings responsible for unwanted tripping of relays. Both analytical expression and RTDS indicate that the swinging impedance undergoes phase shift depending on power factor and the oscillations being higher for lower power factors. At unity power factor, the plot is not visible in figure 4 but appears as a straight line in figure 6. This could be because the analytical equations did not take into account of any disturbances or noise which could appear in a real time digital power system.

Dynamic three dimensional R-X-t plots for analytically derived equations and RTDS system are shown in figures 5 and 7 respectively. Formations of biconical structures are evident in both plots. Figure 5 has connectivity between the outermost part of one cone to that of other cone apart from apex. This was not observed in figure 7, where the only connection between the two cones is the apex. Thus, apart from the outermost connecting loops, the inner loops are similar in both the analytical and RTDS plots. At unity power factor, figure 5 has a straight line passing along the normal operating impedance Zp indicating the absence of time varying resistance and reactance. In figure 7 there is an oscillating resistance which varies with respect to time which was not observed in figure 5. Thus, finer tuning is needed for the analytical expressions in order to accurately describe the locus of power swing as in real time.

5.2. Variation in swinging frequency
Swinging frequency was varied for both analytical and RTDS in steps of 3.333Hz as: 0Hz, 3.33Hz, 6.66Hz and 10Hz. Figures 8, 9, 10 and 11 provides plots during changes in swinging frequency.

Static two dimensional R-X plots for analytically derived equations and RTDS system are shown in figures 8 and 10 respectively. Though both plots cannot be completely superimposed, there are some major similarities in both the plots. The amplitude of swinging of impedance is proportional to the swinging frequency. It can be noticed that the 3.33Hz plot is inside the 6.66Hz which is inside 10Hz plot. Also, the number of oscillations in a specific duration was found to be low for 3.33Hz and high for 10Hz. In the absence of power swing, figure 10 indicates a small circle around the coordinates (235,-72). This explains the absence of any plot in figure 8 as it depicts a single accurate point due to the usage of analytically derived equation which is devoid of any uncertainty or disturbance.

Dynamic three dimensional R-X-t plots for analytically derived equations and RTDS system are shown in figures 9 and 11 respectively. The plot for absence of power swing was not clear in two dimensional plots, but they are clear in R-X-t plots. The absence of power swing in figure 9 was represented by a straight line parallel to time axis indicating the absence of any changes. In figure 11, absence of power swing was represented by a line which has very small oscillations with respect to R and X. Unlike analytical plots depicted by figure 9, the real time plots possess other minor unknown disturbances which could interfere with the measurements.

6. Conclusion
The analytically derived equation tends to approximately define the behaviour of power swing in time domain. Both the planar and spatial plots observed for the analytically derived equations and RTDS have some agreement with each other. Since measurements are taken at random from the transmission
line, it will cover the disturbances caused by generators and loads. Thus grid disturbances happening within the centralised grid or integrated renewable microgrids will influence the $r(t)$ and $x(t)$ accordingly. The analytically derived equations require further tuning in order to fully track the locus of power swing in real time for all situations. This will later help in setting of relays only based on bus voltage, operating current and power factor without the requirement of sophisticated tooling or software for both centralised grids and integrated microgrids.

7. Future Work
The above derived analytical equations will undergo further refinement in order to completely fit the real time power swing. These parameters possess two applications as follows:
1. $\theta$ is tangent of $X/R$ which is a measure of short circuit ratio. Analysing $\theta(t)$, derived from $x(t)$ and $r(t)$ will indirectly indicate the stiffness and immunity of the grid which is either entirely made of renewables or possessing higher percentage of renewables.
2. $dr(t)/dt$, $dx(t)/dt$ and $d\theta(t)/dt$ will help in developing a robust technique for differentiating a fault, stable swing and unstable swing only based on time domain voltages and currents which can be easily obtained from measuring devices thus eliminating complicated numerical calculations.

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