Modelling the drying process in case of combined energy supply

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Abstract. In many chemical, food, biochemical and other industries, drying is the prime process that is characterized by high energy consumption and determines the quality of the finished product. In this regard, the development of mathematical models that allow of optimization and reliable control of the drying process is relevant. A large set of physical and physicochemical phenomena accompanying drying, as well as their mutual influence on each other, gives rise to numerous uncertainties in the mathematical description of this process. The modeling process becomes more complicated under conditions of combined energy supply, for example, convective and microwave energy supply. In this article, based on the laws of chemical kinetics, the authors propose the original approach to modeling the drying process, which, together with experimental studies, allows obtaining a reliable mathematical model that can be used as the basis for optimizing and controlling this process. During the mathematical modeling of drying, it was proposed to apply new process parameters: the degree of drying (the share of removed moisture) and the degree of absorbed energy (the ratio of the amount of energy absorbed by the product at a given moment to the amount of energy absorbed by the product during the entire process). The expediency of characterizing the kinetics of drying by the dependence of the degree of drying on the degree of energy absorbed by the product was demonstrated. The scientific hypothesis was substantiated and confirmed: the generalized characteristic, the kinetics of drying - the dependence of the degree of drying on the degree of energy absorbed by the product, does not depend on the parameters characterizing the drying mode. The proposed model makes it possible to determine the ratio of the components of convective and microwave energy flows when moisture is removed from the material, which can be used as the basis for optimization and reliable control of the process.

1. Introduction

The efficiency of modern industrial production is largely determined by the level of application of computer technologies, which are based on mathematical modeling of the technological processes. The perfection of computer technology significantly depends on the completeness and accuracy of the reflection of all phenomena accompanying the process by the mathematical model. Numerous studies of drying [1, 2, 3, 6] show that this process is described by a whole complex of mutually influencing phenomena - chemical, physicochemical, biochemical, rheological, the kinetics of which, along with...
the kinetics of energy and moisture transfer, determines the drying rate. A large set of these phenomena and the lack of sufficient amount of knowledge about their patterns in the drying process generates numerous uncertainties in the mathematical modeling of this process. Therefore, with the existing theoretical basis, it is not possible to predict with sufficient accuracy the real course of the drying process, which is the reason for the long period of experimental design work in the development of new technologies and drying techniques and determines the relevance of searching for new approaches to mathematical modeling of drying. However, as shown by V.M. Arapov in his thesis [1], most of these phenomena, which have the dominant effect on drying, can be described by the equations of chemical kinetics. In connection with the above, the general goal of this work is to develop theoretical principles of drying based on the laws of chemical kinetics, which make it possible to increase the reliability of mathematical models of drying.

2. Analysis of the performed studies of the issue and setting the current research objectives

According to the current concepts, the drying process includes two stages: the period of constant rate (the first drying period), when

\[ \frac{du}{d\tau} = N_1 = \text{const} \]

\[ U_i \geq U \geq U_{cr} \]

and the period of reducing rate (the second period), when

\[ \frac{du}{d\tau} = N_2(U), \]

\[ U_{cr} > U \geq U_f \]

where \( U_i, U_{cr}, U_f \) – correspondingly, initial, critical, final, and current moisture contents of the product; 
\( \tau \) – drying duration (time).

The scientific foundations of mathematical modeling of drying are physical laws and physicochemical relationships, which the phenomena accompanying drying obey, as well as general methods of research and calculation of production processes and apparatus. Since the emergence of the science of food drying, a large number of mathematical models have been published for various drying methods. Among them, one can single out the models based on the suggestion of T. Sherwood and Lewis on the Fick diffusion law, however, these models are applicable in approximate solutions under isothermal conditions [2]. Another group of mathematical models of drying is based on the thermodynamic approach, in which it is proposed to determine the intensity of any flow of energy or mass as the product of the kinetic coefficient and the corresponding thermodynamic force [3]. This approach to drying is often used for a simplified process flow model. General recognition as a theoretical basis for mathematical modeling of drying was obtained by the system of differential equations for heat and moisture transfer the proposed A.V. Lykov, which describes the unsteady process of heat and mass transfer in wet bodies under any conditions. However, according to A.V. Lykov, the solution of this system of equations in relation to the period of decreasing velocity can be used as a first approximation [3]. Due to the noted difficulties, the empirical approach for the purposes of mathematical modeling of drying [1...3] is widespread, based on the generalization of experimental data using theories of similarity, probability, and mathematical statistics. The experimental data are processed in the form of relationship equations between generalized variables with similarity criteria and simplices. However, as shown in [4], such models are rough and limited.

According to the authors, increase in reliability of mathematical models of drying can be obtained on the basis of the development of the hypothesis proposed by V.M. Arapov, on the possible consideration of drying as a quasi-chemical heterogeneous reaction with the application of the laws of chemical kinetics to its modeling.
The existing theoretical basis does not make it possible to obtain a mathematical model common for both drying periods in an explicit form. Therefore, it is necessary to develop mathematical models separately for the first and second drying periods.

In connection with the above, in order to achieve this goal, the following tasks are considered in this work.

1. Determination of the type of the single mathematical model for the first and second periods of drying, which characterizes the kinetics of drying and is invariant with respect to any mode of this process.

2. Determination of the type of generalized process parameters included in the mathematical model.

3. The research results

3.1. Substantiation of the scientific hypothesis

The idea of applying the laws of chemical kinetics to modeling convective drying processes was successfully implemented by V.M. Arapov in his dissertation work [1], as well as in publication [7]. Subsequently, in [5], the methodology for modeling drying based on the laws of chemical kinetics is presented. When solving the set tasks, we paid special attention to the established and experimentally verified by V.M. Arapov functional dependence for the second drying period:

\[
\frac{T_d - T_w}{T_d - T_w} = \frac{1}{m} \left[ \left( \frac{U - U_p}{U_{cr} - U_p} \right)^{m+1} - 1 \right] + \frac{m+1}{m} \left( \frac{U_{cr} - U}{U_{cr} - U_p} \right),
\]

where \(T_d, T_w\) – absolute temperature of the drying agent measured by dry and wet thermometers; \(T\) – absolute temperature of the product being dried; \(m\) – empiric coefficient constant for this product and independent of the drying mode.

The left side of equation (3) characterizes the ratio of the amount of heat absorbed by the dry part of the product at a given time to the amount of heat absorbed by the dry part of the product during the entire second period. In this regard this value is suggested to be called the heating degree of the dry matter in the second drying period, and the value \(\alpha_U = \frac{U_{cr} - U}{U_{cr} - U_p}\) – the drying degree of the product in the second drying period. Thus, for the second drying period, the functional dependence of the degree of drying on the degree of heating of the dry matter is valid, which is invariant with respect to any drying mode. This fact gives grounds to formulate the following scientific hypothesis. For the whole drying period the drying degree \(\alpha_U\) is a function of the energy absorption degree \(\alpha_E\) for both first and second periods of drying, i.e. value \(\alpha_U\) unambiguously corresponds value \(\alpha_E\). Moreover, this function is invariant to any drying mode. The degree of drying is determined by the following ratio

\[
\alpha_U = \frac{U_i - U(\tau)}{U_i - U_f},
\]

energy absorption degree

\[
\alpha_E = \frac{E(\tau)}{E_{full}},
\]

where \(U_i, U_f, U(\tau)\) – correspondingly, initial, final, and current moisture contents of the product; \(E(\tau), E_{full}\) – the energy amount absorbed by the product at moment \(\tau\), and during the whole drying process, correspondingly.

We suggest to call the said dependence \(\alpha_U = f(\alpha_E)\) general drying kinetics equation for the first and second periods.

3.2. Energy conversion coefficient determination

The amount of energy absorbed by the product during the drying process is proposed to be determined based on the energy balance. All the energy \(E_{full}\) absorbed by the product is spent for heating the dry
part of the product $E_{d.m}$ and moisture $E_m$ up to temperature $T(\tau)$, for disruption of the binding energy between the moisture and the product $E_b$, and for moisture evaporation $E_{ev}$ under temperature $T$. Thus,

$$E_{\text{full}} = E_{d.m} + E_m + E_{ev} + E_b,$$

(6)

In differential form $E_{\text{full}}$ per 1 kg of dry product can be written as follows:

$$dE_{\text{full}} = c_{d.m} \cdot dT + c_m \cdot dT[U(\tau) - dU] + dU \cdot i''(T) - RT(\tau)dU \cdot \ln[\varphi(U,T)].$$

(7)

where $c_{d.m}$ – dry matter heating capacitance;
$c_m$ – water heating capacitance;
$i''$ – enthalpy of dry saturated water vapor under temperature $T$;
$R$ – absolute gas constant;
$\varphi(U,T)$ – the value of relative humidity of the air at equilibrium with the product under temperature $T$ and its moisture content $U$.

The last term of equation (7) determines the value of the binding energy of moisture with matter. In many cases, the final values of moisture content are significantly higher than the moisture content corresponding to monoadsorbed bound moisture, therefore this value can be neglected.

Integration of equation (7) presents great difficulties. Therefore, to determine the value of $E$, it is proposed to apply the zone calculation method. For its application one needs to obtain the experimental drying curves for the given product: $U = f_1(\tau)$ and $T = f_2(\tau)$. Then we divide the whole drying process into $n$ zones. In each zone we define the interval $\Delta U$ and matching $\Delta T$. In this case, it does not matter at all in what mode the experimental curves are obtained; this will not affect the form of the graphical dependence $\alpha_U = f(\alpha_T)$.

3.3. Experimental verification of the general equation of drying kinetics

For experimental verification of the general equation of the kinetics of drying, we used the results of the study of drying coriander seeds under conditions of a combined (convective together with microwave energy supply) energy supply, borrowed from a published source [6], which are shown in Figures 1, 2, 3, 4. The results of the generalization of these curves are shown in Figure 5, and they confirm the validity of our hypothesis.

**Figure 1.** The curves for drying and drying rate for coriander seeds under $U_i=0.16 \text{ kg}_\text{moisture}/\text{kg}_\text{dry matter}$:

1 – $T_{dr,agent}=303 \text{ K}$, $Q_{dr,agent}=141.3 \text{ m}^3/\text{h}$, $P_{sp}=180 \text{ W/kg};$
2 – $T_{dr,agent}=333 \text{ K}$, $Q_{dr,agent}=141.3 \text{ m}^3/\text{h}$, $P_{sp}=180 \text{ W/kg};$
3 – $T_{dr,agent}=363 \text{ K}$, $Q_{dr,agent}=141.3 \text{ m}^3/\text{h}$, $P_{sp}=180 \text{ W/kg}.$

**Figure 2.** Temperature curves for coriander seeds under $U_i=0.16 \text{ kg}_\text{moisture}/\text{kg}_\text{dry matter}$:

1 – $T_{dr,agent}=303 \text{ K}$, $Q_{dr,agent}=141.3 \text{ m}^3/\text{h}$, $P_{sp}=180 \text{ W/kg};$
2 – $T_{dr,agent}=333 \text{ K}$, $Q_{dr,agent}=141.3 \text{ m}^3/\text{h}$, $P_{sp}=180 \text{ W/kg};$
3 – $T_{dr,agent}=363 \text{ K}$, $Q_{dr,agent}=141.3 \text{ m}^3/\text{h}$, $P_{sp}=180 \text{ W/kg}.$
To define the drying kinetics equation generalized for the first and second periods, it is sufficient to carry out one experimental study for a given product and equipment design.

4. Conclusion
Drying under conditions of combined energy supply can be considered from the standpoint of chemical kinetics. To describe the kinetics of drying, the form of a unified mathematical model for the first and second periods of drying is determined, invariant with respect to any mode of this process. It has been detected, that the kinetics of drying under conditions of combined energy supply should be characterized by the dependence of the degree of drying on the degree of absorbed energy. It is proposed to determine the amount of energy absorbed by the product during drying using the energy balance.

To define the drying kinetics equation generalized for the first and second periods, it is sufficient to carry out one experimental study for a given product and equipment design.

Figure 3. The curves for drying and drying rate for coriander seeds under $U_i=0.16 \, \text{kg}_{\text{moisture}}/\text{kg}_{\text{dry matter}}$:
1. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=60 \, \text{W/kg}$;
2. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=180 \, \text{W/kg}$;
3. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=300 \, \text{W/kg}$.

Figure 4. Temperature curves for coriander seeds under $U_i=0.16 \, \text{kg}_{\text{moisture}}/\text{kg}_{\text{dry matter}}$:
1. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=60 \, \text{W/kg}$;
2. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=180 \, \text{W/kg}$;
3. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=300 \, \text{W/kg}$.

Figure 5. Dependence of the drying degree on the energy absorption degree during drying of coriander seeds with the experimental setup at $U_i=0.16 \, \text{kg}_{\text{moisture}}/\text{kg}_{\text{dry matter}}$
1. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=180 \, \text{W/kg}$;
2. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=60 \, \text{W/kg}$;
3. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=180 \, \text{W/kg}$;
4. $T_{\text{dr.agent}}=333 \, \text{K}, \, Q_{\text{dr.agent}}=141.3 \, \text{m}^3/\text{h}, \, P_{\text{sp}}=300 \, \text{W/kg}$.
The original approach to modeling the drying process under conditions of numerous uncertainties will provide a reliable mathematical model that can be used as the basis for optimization and control of this process.

5. Theoretical value of the research and the prospects of the following studies of this field
For the practical implementation and development of the idea of applying a generalized mathematical model based on the laws of chemical kinetics to combined drying processes, the drying process of various classification groups of products with various combinations of energy supply (convective and microwave, vacuum and microwave, convective and infrared, as well as other options for combined power supply) should be studied using the indicated methodology.

To establish explicitly for each group of products and various combinations of power supply the dependence of the degree of drying on the degree of absorbed energy and, on this basis, develop algorithms for computer technologies for controlling production processes.

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