Superconducting gap ratio in a SYK-like model

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Abstract

A lattice model for strongly interacting electrons motivated by a rank-3 tensor model provides a tool for understanding the pairing mechanism of high-temperature superconductivity. Within the framework of this SYK-like model, our calculation indicates that the superconducting gap ratio in this model is higher than the ratio in the BCS theory due to the coupling term and the spin operator. Under certain condition, the ratio also agrees with the BCS theory. Our results may pave the way to gaining insight into the cuprate high temperature superconductors.

1 Introduction

The Sachdev-Ye-Kitaev (SYK) model is a disordered and strongly-coupled quantum system composed by N Majorana fermions with Gauss distribution random [1,2,3]. The connection between the SYK model and the gravity theory with a near-horizon AdS$_2$ geometry could be obtained in [4,5,6]. Various applications of SYK model have been presented, such as topological SYK model [7], SYK-like models [8,9,10,11,12,13,14], transport [15,16,17,18], SYK spectral density [19,20,21,22,23,24], supersymmetric SYK model [25,26,27,28,29], quantum chaos [30], the higher dimensional generalization [31,32,33,34] and the bulk gravity dual of SYK models [35,36,37,38,39].

Recently, it was found that the SYK model could be a powerful method to study superconductivity. Actually, the conventional superconductors can be described by the BCS theory. However, the BCS theory is not capable of explaining the high temperature superconductor. There are some progresses on high-temperature superconductivity within the framework of SYK dots [40,41,43,44]. The single particle phase has been investigated [55]. There is a finite-temperature crossover to an incoherent metallic (IM) and non-fermi liquid...
(NFL) in some lattice model [40, 45]. The SYK model realizes a gapless non-fermi liquid, and it violate the ratio between the zero temperature gap and the critical temperature which predicted by BCS mean-field theory [41].

Unlike the BCS superconductivity theory, there is no quasiparticle description in the SYK model. However, we can still imagine a pairing mechanism existing on a “small” Fermi surface. Although the SYK model describes a non-Fermi liquid state, it actually has marginally relevant paring instability just like the ordinary Fermi liquid state in some previous works [46, 47]. Naturally, we could try to investigate a SYK-like theory which is analogous to the condensation of Cooper pair of the ordinary Fermi liquid below the Debye energy. The Debye frequency or the Debye energy is a cut-off frequency or energy for a crystal lattice. As a candidate theory, in [48], the authors propose a lattice model for strongly interacting electrons motivated by the recently developed “tetrahedron” rank-3 tensor model that mimics much of the physics of the SYK model (See more details in [49, 50, 51]). The single particle Green’s function of this lattice model in large N limit is identical to the disorder-averaged Green’s function of the SYK model. Moreover, the scaling behavior of this model in the large N limit may still apply for finite N [48]. The lattice model leads to a fermion pairing instability just like the BCS instability. It is natural to study the gap equation and the superconducting gap ratio.

Motivated by these facts, we intend to explore the pairing mechanism of high-temperature superconductivity via this 2 + 1-dimensional lattice model for strongly interacting electrons. The paper is organized as follows, in section 2, we construct symplectic group singlet pairs between fermions in the transverse momentum space and the corresponding microscopic model. Then, we derive equations for the correlation functions. In section 3, we investigate the gap function, the transition temperature and the ratio. We also evaluate the influence of the attractive K term and spin $\vec{S}$ term, and compare our results with the BCS theory. The section 4 is the summary and discussion.

2 SP(N) singlet pairs and the microscopic model

In this section, we construct singlet pairs between only two sites and briefly review the microscopic lattice model. We first introduce a 2M-component fermion basis on site 1 and site 2,

$$\Psi = (c_{1,\alpha}, c_{2,\alpha})^T.$$  \hfill (1)

The $2M \times 2M$ Green’s function matrix is given by

$$- <T_\tau \Psi(\tau)\Psi^\dagger(0)> = \begin{pmatrix} - <T_\tau c_{1,\alpha}(\tau)c_{1,\beta}^\dagger(0)> & - <T_\tau c_{1,\alpha}(\tau)c_{2,\beta}^\dagger(0)> \\ - <T_\tau c_{2,\alpha}(\tau)c_{1,\beta}^\dagger(0)> & - <T_\tau c_{2,\alpha}(\tau)c_{2,\beta}^\dagger(0)> \end{pmatrix}.$$  

Then we consider a general dimer of site $(i,j)$. Here $\Delta_{i,j} = J_{\alpha\beta}c_{i,\alpha}c_{j,\beta}$ is a Sp(M) spin singlet fermion pairings on nearest neighbor links $<i,j>$. Motivated by the observation
that the symplectic group $\text{SP}(N)$ allows fermions to form singlet pairs \cite{52,53}, we define
\begin{equation}
G_{i,j}(\tau) = - \langle \tau \delta_{\alpha,\beta} c_{i,\alpha}(\tau)c_{i,\beta}^\dagger(0) \rangle ,
\end{equation}
\begin{equation}
F_{i,j}(\tau) = \langle \tau J_{\alpha,\beta} c_{i,\alpha}(\tau)c_{j,\beta}(0) \rangle ,
\end{equation}
\begin{equation}
F_{i,j}^\dagger(\tau) = \langle \tau J_{\alpha,\beta} c_{i,\alpha}(\tau)c_{j,\beta}(0) \rangle .
\end{equation}

It is similar to the Cooper pair in the neighbor site $\langle \tau c_{-\vec{r},\uparrow}(\tau)c_{\vec{r},\downarrow}(0) \rangle$. The creation operator in Fourier space is $c_{j,\alpha}^\dagger = \sum_p e^{ij\cdot p} c_{\alpha,p}^\dagger$. Thus, the Fourier transformations of the pair are
\begin{equation}
c_{j,\alpha}(\tau)c_{j,\beta}^\dagger(0) = \sum_{p,p'} e^{-ij\cdot(p-p')} c_{p,\alpha}(\tau)c_{p',\beta}^\dagger(0) ,
\end{equation}
\begin{equation}
c_{i,\alpha}(\tau)c_{j,\beta}(0) = \sum_{p,p'} e^{-i(p+j\cdot p')} c_{p,\alpha}(\tau)c_{p',\beta}(0) ,
\end{equation}
\begin{equation}
c_{i,\alpha}(\tau)c_{j,\beta}(0) = \sum_{p,p'} e^{i(p+j\cdot p')} c_{p,\alpha}(\tau)c_{p',\beta}^\dagger(0) .
\end{equation}

Here $i = (i_x, i_y)$ and $p = (p_x, p_y)$. By introducing the momentum and the hopping term, we modify the interacting electron Hamiltonian in \cite{48} as follows,
\begin{equation}
H = \sum_{\vec{q},\vec{p}} \bar{\vec{U}}(\vec{p}) c_{\sigma,\vec{q}}^\dagger c_{\sigma,\vec{q}+\vec{p}}^\dagger \epsilon_{\gamma,\vec{p}} c_{\gamma,\vec{q}}^\dagger + \sum_{\vec{p}} \xi_{\rho,\sigma} c_{\rho,\sigma}^\dagger c_{\rho,\sigma} + \frac{1}{4} \frac{J}{\sum_{\vec{p},\vec{q}}} \sum_{\sigma,\sigma'} \sum_{\vec{r},\vec{s}} c_{\rho,\sigma}^\dagger c_{\rho,\sigma'} c_{\sigma,\gamma}^\dagger c_{\gamma,\sigma'} + \frac{1}{2} \frac{J}{\sum_{\vec{p},\vec{q}}} \sum_{\sigma,\gamma} \sum_{\vec{r},\vec{s}} c_{\rho,\sigma}^\dagger c_{\rho,\gamma} c_{\sigma,\gamma}^\dagger c_{\gamma,\sigma} + K \sum_{\vec{q},\vec{p}} \left( \epsilon_{\alpha,\beta} \epsilon_{\gamma,\tau} c_{\alpha,\vec{q}}^\dagger c_{\beta,\vec{q}+\vec{p}}^\dagger c_{\gamma,\vec{q}} + H.C. \right) .
\end{equation}
We have set the volume $v = 1$ for simplicity. $\hat{n}_i = \hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow}$ is the total electron number on site $i$. $\vec{S}_i = \frac{1}{2} c_{1,\alpha}^\dagger \sigma_{\alpha,\beta} c_{i,\beta} = \frac{1}{2} c_{i,\alpha}^\dagger \sigma_{\alpha,\beta} c_{i,\beta}$ is the spin operator, and $\vec{S}_i \cdot \vec{S}_j = \frac{1}{2} \vec{S}_{\alpha,\beta,i} \vec{S}_{\alpha,\beta,j}$. $\xi_q$ is the energy of the single particle which hoppings between the two sublattices as perturbations. $K$ satisfies

\begin{equation}
K \begin{cases}
< 0, & |\xi_q| < \omega_D \\
= 0, & |\xi_q| > \omega_D
\end{cases}
\end{equation}

Here $\omega_D$ is the Debye energy. The term with the coupling $K$ takes a spin singlet pair of electrons on two diagonal sites $j, j + \hat{x} + \hat{y}$ of a plaquette to the two opposite diagonal sites $j + \hat{x}, j + \hat{y}$ of the same plaquette. When $\bar{U} = K = \pm J/2$, the interacting electron model in \cite{48} is equivalent to a tetrahedron model with three indices: the $\text{Sp}(M)$ spin, the $x$ coordinate, and $y$ coordinate.

\begin{equation}
\frac{g}{N_a N_b N_c} J_{c1c1'} J_{c2c2'} c_{1,1b1c1'} ^\dagger c_{1b2c1'} ^\dagger c_{0b1c2} ^\dagger c_{0b2c2} ^\dagger c_{0a2b1c2} ^\dagger c_{0a2b1c2} ^\dagger
\end{equation}

\begin{equation}
\frac{g}{N\sqrt{M}} J_{a,\beta} J_{a,\gamma} c_{jx,jy,\alpha} ^\dagger c_{jx,r+jy+r,\beta} c_{jx,jy,r,\gamma} ^\dagger c_{jx+r,jy,\sigma} ,
\end{equation}

where $g$ is the same order as the coupling $J$. The total symmetry of this model is $U(N_a) \times U(N_b) \times \text{Sp}(N_c)$.
3 The gap function and the transition temperature

As we are going to evaluate the gap ratio, let us first consider the time development

\[
\frac{d}{d\tau} c_{\alpha,\vec{p}}(\tau) = [H, c_{\alpha,\vec{p}}] = \sum_{\vec{q},\vec{p}'} \tilde{U} c_{\sigma,\vec{q}+\vec{p}'} c_{\alpha,\vec{p}'} + \frac{J}{2} \epsilon_{\gamma} c_{\beta,\vec{p}'} c_{\sigma,\vec{q}} - \tilde{\Sigma} c_{\gamma,\vec{p}'} c_{\sigma,\vec{q}}. \tag{9}
\]

\[
\frac{d}{d\tau} c_{\alpha,\vec{p}}(\tau) = [H, c_{\alpha,\vec{p}}] = \sum_{\vec{q},\vec{p}'} \tilde{U} c_{\sigma,\vec{q}+\vec{p}'} c_{\alpha,\vec{p}'} - \frac{J}{2} c_{\alpha,\vec{p}'} c_{\sigma,\vec{q}} + \frac{J}{2} \epsilon_{\gamma} c_{\beta,\vec{p}'} c_{\sigma,\vec{q}} - \tilde{\Sigma} c_{\gamma,\vec{p}'} c_{\sigma,\vec{q}}. \tag{10}
\]

Here \( \tilde{\Sigma} = \frac{1}{2} \sum_{\vec{q}} \epsilon_{\gamma} c_{\alpha,\vec{q}} c_{\beta,\vec{q}} \) and \( \vec{q} = (0, p_y - p_y) \). The equations for the correlation functions

\[
G(\vec{p},\tau) = -<T_{\tau} \delta_{\alpha\beta} c_{\vec{p},\alpha}(\tau) c_{\vec{p},\beta}(0)>, \\
F(\vec{p},\tau) = <T_{\tau} J_{\alpha\beta} c_{\vec{p},\alpha}(\tau) c_{\vec{p},\beta}(0)>,
\]

are determined by

\[
\frac{\partial}{\partial \tau} G(\vec{p},\tau) = -\delta(\tau) - <T_{\tau} \delta_{\alpha\beta}\frac{\partial}{\partial \tau} c_{\vec{p},\alpha}(\tau) c_{\vec{p},\beta}(0)>, \tag{12}
\]

\[
\frac{\partial}{\partial \tau} F(\vec{p},\tau) = <T_{\tau} J_{\alpha\beta}\frac{\partial}{\partial \tau} c_{\vec{p},\alpha}(\tau) c_{\vec{p},\beta}(0)>. \tag{13}
\]

Combined with the results [9][10] and the gap function

\[
\Delta(\vec{p}) = 4K \sum_{\vec{q}} <F(\vec{p} - \vec{q},\tau = 0)>, \tag{14}
\]

the derivative of the equation for the correlation function after Fourier transforming is given as,

\[
(ip_n - \xi_p)G(\vec{p},ip_n) + \Delta(\vec{p})F(\vec{p},ip_n) + J <T_{\tau} \sigma_{\alpha\beta} c_{\vec{p},\alpha} \tilde{\Sigma} c_{\vec{p},\beta} > \\
+ 2 \sum_{\vec{q},\vec{p}'} \tilde{U} c_{\sigma,\vec{q}+\vec{p}'} c_{\alpha,\vec{p}'} c_{\sigma,\vec{p}'} - \frac{J}{2} \sum_{\vec{q}} c_{\alpha,\vec{p}'} c_{\sigma,\vec{q}+\vec{p}'} c_{\sigma,\vec{q}+\vec{p}'} = 1, \tag{15}
\]

\[
(ip_n + \xi_p)F(\vec{p},ip_n) + J_{\alpha\beta} c_{\vec{p},\alpha} \Delta(\vec{p})G(\vec{p},ip_n) + J <T_{\tau} J_{\alpha\beta} c_{\vec{p},\alpha} \tilde{\Sigma} c_{\vec{p},\beta} > \\
+ 2 \sum_{\vec{q},\vec{p}} J_{\alpha\beta} \tilde{U} c_{\sigma,\vec{q}+\vec{p}'} c_{\alpha,\vec{p}'} c_{\sigma,\vec{p}'} - \frac{J}{2} \sum_{\vec{q}} J_{\alpha\beta} c_{\sigma,\vec{q}+\vec{p}'} c_{\sigma,\vec{q}+\vec{p}'} = 0. \tag{16}
\]
The summation over \(n\), we simplify the final results as follows,

\[
\mathcal{G}(\tilde{p}, \tilde{q}, n) = \left[ \tilde{p}^2_n + \xi^2_p + \Delta(\tilde{p})J_{ab}\epsilon_{ab}\Delta^\dagger(\tilde{p}) \right]^{-1}
\]

\[
\Delta(\tilde{p}) \left( J < T_r J_{ab}\sigma_\beta\epsilon_{a\tilde{q}, \tilde{p}}\tilde{S}c_{-\tilde{p}, b}\tilde{b} > +2 \sum_{\tilde{q}, \tilde{p}} J_{ab}\tilde{U}c^\dagger_{a\tilde{q}, \tilde{p}}c_{a\tilde{q}, \tilde{p}}+2 \right)
\]

\[
- \frac{J}{2} \sum_{\tilde{q}} J_{ab}\epsilon_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger \right) \right]
\]

\[
\mathcal{F}(\tilde{p}, \tilde{q}, n) = \left[ \tilde{p}^2_n + \xi^2_p + \Delta(\tilde{p})J_{ab}\epsilon_{ab}\Delta^\dagger(\tilde{p}) \right]^{-1}
\]

\[
\left( J < T_r J_{ab}\sigma_\beta\epsilon_{a\tilde{q}, \tilde{p}}\tilde{S}c_{-\tilde{p}, b}\tilde{b} > +2 \sum_{\tilde{q}, \tilde{p}} J_{ab}\tilde{U}c^\dagger_{a\tilde{q}, \tilde{p}}c_{a\tilde{q}, \tilde{p}}+2 \right)
\]

\[
- \frac{J}{2} \sum_{\tilde{q}} J_{ab}\epsilon_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger \right) \right]
\]

By inserting (18) into

\[
\Delta(\tilde{p}) = \Delta^\dagger(\tilde{p}) = -4 \sum_{\tilde{q}} K\mathcal{F}(\tilde{p}, \tilde{q}, \tau = 0) = -4 \sum_{\tilde{q}, \tilde{p}, \tilde{q}_n} K\mathcal{F}(\tilde{p}, \tilde{q}, \tau = 0)
\]

we obtain the equation for the gap function, which is

\[
\Delta(\tilde{p}) = -4 \sum_{\tilde{q}, \tilde{p}, \tilde{q}_n} K \left[ (p_n - q_n)^2 + \xi^2_p + J_{ab}\epsilon_{ab}\Delta(\tilde{p})\tilde{q}^2 \right]^{-1}
\]

\[
\left( J < T_r J_{ab}\sigma_\beta\epsilon_{a\tilde{q}, \tilde{p}}\tilde{S}c_{-\tilde{p}, b}\tilde{b} > +2 \sum_{\tilde{q}, \tilde{p}} J_{ab}\tilde{U}c^\dagger_{a\tilde{q}, \tilde{p}}c_{a\tilde{q}, \tilde{p}}+2 \right)
\]

\[
- \frac{J}{2} \sum_{\tilde{q}} J_{ab}\epsilon_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger c_{a\tilde{q}, \tilde{q}}^\dagger \right) \right]
\]

We define the excitation energy of the superconductor as

\[
E_{\tilde{p}-\tilde{q}} = \sqrt{\xi^2_p + J_{ab}\epsilon_{ab}\Delta(\tilde{p})\tilde{q}^2}.
\]

The summation over \(i(p_n - q_n)\) is evaluated by the contour integral

\[
\oint \frac{dz}{2\pi i} n_F(z) \frac{\Delta^\dagger(\tilde{p})\tilde{q}}{z^2 - E_{\tilde{p}-\tilde{q}}}, \oint \frac{dz}{2\pi i} n_F(z) \frac{\xi_p}{z^2 - E_{\tilde{p}-\tilde{q}}^2}.
\]
and the poles of Fermi distribution \( n_F(z) = \frac{1}{e^{\beta E_p} - 1} \) gives the summation over \( z = i(p_n - q_n) \).

Since \( \frac{\Delta}{2E_p} = e^{-\beta E_p - \Delta E_p} e^{\beta E_p} = \frac{\Delta}{2E_p} \tan(h(\beta E_p \frac{\Delta}{2})) \), now the gap function

\[
\Delta(p) = \sum_{q} f(q) = 4 \sum_{q} \left[ -K J_{ab \ell ab} \left( 1 - J < T \sigma_\alpha \beta c_{\vec{p} \vec{q}}^{\dagger} \vec{S}_{\vec{p} \vec{q}, \alpha} \right) - 2 \sum_{\vec{q}, \vec{p}} \bar{U} c_{\vec{q}, \vec{q} + \vec{p}} \bar{c}_{\vec{q}, \vec{p}} c_{\vec{q}, \vec{p}, \alpha} \left( \beta \frac{E_{p-q}}{2} \right) \right] \tag{23}
\]

It is convenient to change the summation to an integration

\[
\sum_{q} f(q) = \int \frac{d^3 q}{(2\pi)^3} f(q) = N_F \int_{-\omega_D}^{\omega_D} d\xi f(\xi), \tag{24}
\]

where we have approximately substituted the constant \( N_F \) for density of states near the Fermi surface. Taking the zero temperature limit, \( \beta = 1/T \to \infty \), we obtain

\[
\Delta(p) = -4KN_F \left[ J_{ab \ell ab} \left( 1 - J < T \sigma_\alpha \beta c_{\vec{p} \vec{q}}^{\dagger} \vec{S}_{\vec{p} \vec{q}, \alpha} \right) + \sum_{\vec{q}, \vec{p}} \bar{U} c_{\vec{q}, \vec{q} + \vec{p}} \bar{c}_{\vec{q}, \vec{p}} c_{\vec{q}, \vec{p}, \alpha} \right] \tag{25}
\]

Since \( \Delta \) is constant and \( \ln(\xi + \sqrt{\sum_{\vec{q}, \vec{p}} c_{\vec{q}, \vec{p}} \bar{c}_{\vec{q}, \vec{p}}}) \approx 2 \ln(\frac{2\omega_D}{\sqrt{\sum_{\vec{q}, \vec{p}} c_{\vec{q}, \vec{p}} \bar{c}_{\vec{q}, \vec{p}}}}) \), \( \Delta \) leaves the equation for the energy gap

\[
\Delta = -4KN_F (A \Delta \ln(\frac{2\omega_D}{\sqrt{\sum_{\vec{q}, \vec{p}} c_{\vec{q}, \vec{p}} \bar{c}_{\vec{q}, \vec{p}}}}) + \omega_D B), \tag{26}
\]

\[
A = J_{ab \ell ab} \left( 1 - J < T \sigma_\alpha \beta c_{\vec{p} \vec{q}}^{\dagger} \vec{S}_{\vec{p} \vec{q}, \alpha} \right) + \sum_{\vec{q}, \vec{p}} \bar{U} c_{\vec{q}, \vec{q} + \vec{p}} \bar{c}_{\vec{q}, \vec{p}} c_{\vec{q}, \vec{p}, \alpha} \right) \tag{27}
\]

\[
B = J < T \sigma_\alpha \beta c_{\vec{p} \vec{q}}^{\dagger} \vec{S}_{\vec{p} \vec{q}, \alpha} \right) + \sum_{\vec{q}, \vec{p}} \bar{U} c_{\vec{q}, \vec{q} + \vec{p}} \bar{c}_{\vec{q}, \vec{p}} c_{\vec{q}, \vec{p}, \alpha} \right) - \sum_{\vec{q}, \vec{p}} \bar{U} c_{\vec{q}, \vec{q} + \vec{p}} \bar{c}_{\vec{q}, \vec{p}} c_{\vec{q}, \vec{p}, \alpha} \right) \tag{28}
\]

We set \( \omega_D = 1 \) to fit the gap, and choose a small correction for spin operator. In BCS theory (i.e. \( A = 1, B = 0 \)), the energy gap for \( K = -\frac{1}{2}V_0 \) is \( \Delta = 2\omega_D e^{-1/V_0 N_F} \) at zero temperature \( V_0 > 0 \). \(-V_0\) is the attractive and constant potential in BCS theory. Equation 26 could be numerically calculated and the result is shown in Figure 1 (Here we have neglected the effect of B term due to the following analysis on \( T_c \)). We could conclude that the energy gap in the "tetrahedron" model is higher than the BCS energy gap represented by red line when \( U = K = -J/2 \).
Figure 1: The relation between the gap $\Delta$ and the coupling $-5 < K < 1/2$ with different $S = < T_\tau \sigma_\alpha \beta c^\dagger_{\bar{q} \bar{p}} q^\dagger_{\bar{q} \bar{p}} b^\dagger_{\bar{p}} b_{\bar{q}} > = 0, 0.01, 0.05$ represented by red, blue, green respectively. The figure on the left corresponds to the case of $U = K = J/2$. The figure on the right corresponds to the case of $U = K = -J/2$. The gap changes abruptly when $K$ goes from negative to zero.

Furthermore, we know $\Delta(T = T_c) = 0$ at the transition temperature $T_c$. Then, (25) becomes

$$1 = -4KN_F \int_{-\omega_D}^{\omega_D} d\xi \left[ \frac{A}{2\xi} \tanh \left( \frac{\xi}{2T} \right) + \frac{B}{2\Delta} \right] _{\Delta \to 0} \left( 1 - \tanh \left( \frac{\xi}{2T} \right) \right) .$$

(29)

Using the Euler integral formula, we obtain the transition temperature as approximately as follow

$$T_c = 1.13\omega_D e^{1/(4KN_F A)} .$$

(30)

Since we have required that (29) must be regular, it yields

$$A = J_{ab} \epsilon_{ab} \left( 1 - J < T_\tau \sigma_\alpha \beta c^\dagger_{\bar{q} \bar{p}} q^\dagger_{\bar{q} \bar{p}} b^\dagger_{\bar{p}} b_{\bar{q}} > \right) , B = 0 .$$

(31)

We notice that the critical temperature is $T_c = 1.13\omega_D e^{-1/N_F}$ in the BCS theory. While our solution of $T_c$ is modified by $K$ and $< T_\tau \sigma_\alpha \beta c^\dagger_{\bar{q} \bar{p}} q^\dagger_{\bar{q} \bar{p}} b^\dagger_{\bar{p}} b_{\bar{q}} >$. We plot the transition temperature $T_c$ as a function of the coupling $K$ of the SYK-like term in Figure 2. The transition temperature decrease as $K$ increase. $K$ is the SYK-like coupling. As to the energy gap, the transition temperature diverges as $K$ goes from negative to zero.
Figure 2: The figure shows the relation between the transition temperature $T_c$ and $-5 < K < 1/2$ in the case of $U = K = \pm J/2$. The transition temperature changes abruptly when K goes from negative to positive.

Now we have both the energy gap and the transition temperature. The ratio of these two results is $\frac{2\Delta}{T_c} = 3.5$ in the BCS theory. When $\frac{2\Delta}{T_c} > 3.5$, it is the case of strong coupling. As we know, the energy gap and the critical temperature are dependent on the coupling $V_0$, while $\frac{2\Delta}{T_c}$ is independent on $V_0$ in the BCS theory. Since the ratio $\frac{2\Delta}{T_c}$ is dependent on the coupling $K$ in the “tetrahedron” model. It is interesting to show the numerical evaluation of $\frac{2\Delta}{T_c}$ in Figure 3.

Figure 3: The dependence of $\frac{2\Delta}{T_c}$ on K and S. The figure on the left shows that the ratio decrease as K decrease and S increase in the case of $U = K = J/2$. The figure on the right shows that the ratio increase as K decrease and S increase in the case of $U = K = -J/2$.

When $< T_{\tau...c^\dagger Sc^\dagger} > = 0.05$, $K = -J/2 = -5$, $\frac{2\Delta}{T_c} \approx 5$. If $< T_{\tau...c^\dagger Sc^\dagger} >$ vanishes, the ratio $\frac{2\Delta}{T_c} = 3.5$ in the “tetrahedron” model ($K < 0$) is always the same as the ratio in the
BCS theory ($K = 1$). In other words, the ratio is independent of the coupling $K$ in such case.

4 Conclusion and discussion

In this paper, we attempt to understand the pairing mechanism of high-temperature superconductivity in a SYK-like model. Sp(N) singlet pairing operator is proposed. Equations for the correlation functions are derived. Our analysis shows how the superconducting gap, the transition temperature and the ratio change with the coupling $K$ and spin $T_{\tau,\sigma}c^\dagger_{Sc}$. Specially, our results could return to the BCS theory if $<T_{\tau,\sigma}c^\dagger_{Sc}> = 0$.

There are still some subtle issues. Firstly we focus on the spinor operator, but its quantum contribution is not considered. Actually, the single particle Green’s function in the large N limit is identical to the disordered averaged Green’s function of the SYK models [48, 54]. The full Green’s function and the current vertex of the translational invariant model with random interaction terms could be solvable in the large N limit [45]. Thus, in the SYK model at large N limit, the quantum contribution to (27)(28) of the rank-3 tensor model can be summed analytically. Secondly our calculation may be not applied in the large N limit, due to the long range interaction between lattices. Although we could not generalize our calculations to large N limit, enhancement of the gap ratio is still seen in the model at large N limit [41]. In some holographic superconductors, the gap ratio increases as well [42]. Two lattice models are proposed with on-site SYK interactions exhibiting a transition from an IM to an s-wave superconductor in [41]. On the other hand, [48] also argue that the correction to the NFL solution in this model is suppressed rapidly with increasing N. Therefore, our results without so large N shows a qualitatively agreement.

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References

[1] S. Sachdev and J. Ye, “Gapless spin fluid ground state in a random, quantum Heisenberg magnet,” Phys. Rev. Lett. 70 (1993) 3339, [cond-mat/9212030].
[2] A. Kitaev, “A simple model of quantum holography,” Talks at KITP, April 7, 2015 and May 27, 2015.
[3] J. Maldacena, D. Stanford and Z. Yang, “Conformal symmetry and its breaking in two dimensional Nearly Anti-de-Sitter space,” PTEP 2016, no. 12, 12C104 (2016) doi:10.1093/ptep/ptw124 [arXiv:1606.01857 [hep-th]].
[4] J. Maldacena and D. Stanford, “Remarks on the Sachdev-Ye-Kitaev model,” Phys. Rev. D 94, no. 10, 106002 (2016) doi:10.1103/PhysRevD.94.106002 [arXiv:1604.07818 [hep-th]].
[5] J. Polchinski and V. Rosenhaus, “The Spectrum in the Sachdev-Ye-Kitaev Model,” JHEP 1604, 001 (2016) doi:10.1007/JHEP04(2016)001 [arXiv:1601.06768 [hep-th]].

[6] K. Jensen, “Chaos in AdS2 Holography,” Phys. Rev. Lett. 117, no. 11, 111601(2016) doi:10.1103/PhysRevLett.117.111601 [arXiv:1605.06098 [hep-th]].

[7] Pengfei Zhang, Hui Zhai, “Topological Sachdev-Ye-Kitaev Model,” arXiv:1803.01411 [cond-mat.str-el].

[8] Xin Dai, Shao-Kai Jian, Hong Yao, “The global phase diagram of the one-dimensional SYK model at finite N,” arXiv:1802.10029 [cond-mat.str-el].

[9] V. Bonzom, L. Lionni and A. Tanasa, “Diagrammatics of a colored SYK model and of an SYK-like tensor model, leading and next-to-leading orders,” J. Math. Phys. 58, no. 5, 052301 (2017) doi:10.1063/1.4983562 [arXiv:1702.06944 [hep-th]].

[10] C. Krishnan, K. V. P. Kumar and S. Sanyal, “Random Matrices and Holographic Tensor Models,” JHEP 1706, 036 (2017) doi:10.1007/JHEP06(2017)036 [arXiv:1703.08155 [hep-th]].

[11] C. Peng, “$\mathcal{N} = (0, 2)$ SYK, Chaos and Higher-Spins,” arXiv:1805.09325 [hep-th].

[12] C. Krishnan and K. V. Pavan Kumar, “Complete Solution of a Gauged Tensor Model,” arXiv:1804.10103

[13] D. J. Gross and V. Rosenhaus, “A Generalization of Sachdev-Ye-Kitaev,” JHEP 1702, 093 (2017) doi:10.1007/JHEP02(2017)093 [arXiv:1610.01569 [hep-th]].

[14] P. Chaturvedi, Y. Gu, W. Song and B. Yu, “A note on the complex SYK model and warped CFTs,” arXiv:1808.08062 [hep-th].

[15] S. K. Jian and H. Yao, “Solvable SYK models in higher dimensions: a new type of many-body localization transition,” arXiv:1703.02051 [cond-mat.str-el].

[16] R. A. Davison, W. Fu, A. Georges, Y. Gu, K. Jensen and S. Sachdev, “Thermoelectric transport in disordered metals without quasiparticles: The Sachdev-Ye-Kitaev models and holography,” Phys. Rev. B 95, no. 15, 155131 (2017) doi:10.1103/PhysRevB.95.155131 [arXiv:1612.00849 [cond-mat.str-el]].

[17] W. Cai, X. H. Ge and G. H. Yang, “Diffusion in higher dimensional SYK model with complex fermions,” JHEP 1801, 076 (2018) doi:10.1007/JHEP01(2018)076

[18] X. H. Ge, S. K. Jian, Y. L. Wang, Z. Xian and H. Yao, “Shear viscosity in SYK islands,” [arXiv:1809.xxxxx]

[19] Y. Jia and J. J. M. Verbaarschot, “Large $N$ expansion of the moments and free energy of Sachdev-Ye-Kitaev model, and the enumeration of intersection graphs,” arXiv:1806.03271 [hep-th].

[20] A. M. Garca-Garca, Y. Jia and J. J. M. Verbaarschot, “Exact moments of the Sachdev-Ye-Kitaev model up to order $1/N^2$,” JHEP 1804, 146 (2018) doi:10.1007/JHEP04(2018)146 [arXiv:1801.02696 [hep-th]].

[21] A. M. Garca-Garca and J. J. M. Verbaarschot, “Spectral and thermodynamic properties of the Sachdev-Ye-Kitaev model,” Phys. Rev. D 94, no. 12, 126010 (2016) doi:10.1103/PhysRevD.94.126010 [arXiv:1610.03816 [hep-th]].
[22] A. M. Garca-Garca and J. J. M. Verbaarschot, “Analytical Spectral Density of the Sachdev-Ye-Kitaev Model at finite N,” Phys. Rev. D 96, no. 6, 066012 (2017) doi:10.1103/PhysRevD.96.066012 [arXiv:1701.06593 [hep-th]].

[23] S. R. Das, A. Ghosh, A. Jevicki and K. Suzuki, “Three Dimensional View of Arbitrary q SYK models,” JHEP 1802, 162 (2018) doi:10.1007/JHEP02(2018)162 [arXiv:1711.09839 [hep-th]].

[24] A. M. Garca-Garca and M. Tezuka, “Many-Body Localization in a finite-range Sachdev-Ye-Kitaev model,” arXiv:1801.03204 [hep-th].

[25] W. Fu, D. Gaiotto, J. Maldacena and S. Sachdev, “Supersymmetric Sachdev-Ye-Kitaev models,” Phys. Rev. D 95, no. 2, 026009 (2017) Addendum: [Phys. Rev. D 95, no. 6, 069904 (2017)] doi:10.1103/PhysRevD.95.069904, 10.1103/PhysRevD.95.026009 [arXiv:1610.08917 [hep-th]].

[26] C. Peng, M. Spradlin and A. Volovich, “A Supersymmetric SYK-like Tensor Model,” JHEP 1705, 062 (2017) doi:10.1007/JHEP05(2017)062 [arXiv:1612.03851 [hep-th]].

[27] T. Li, J. Liu, Y. Xin and Y. Zhou, “Supersymmetric SYK model and random matrix theory,” JHEP 1706, 111 (2017) doi:10.1007/JHEP06(2017)111 [arXiv:1702.01738 [hep-th]].

[28] N. Hunter-Jones, J. Liu and Y. Zhou, “On thermalization in the SYK and supersymmetric SYK models,” arXiv:1710.03012 [hep-th].

[29] P. Narayan and J. Yoon, “Supersymmetric SYK Model with Global Symmetry,” JHEP 1808, 159 (2018) doi:10.1007/JHEP08(2018)159 [arXiv:1712.02647 [hep-th]].

[30] J. Maldacena, S. H. Shenker, and D. Stanford, “A bound on chaos,” JHEP 08,106 (2016).

[31] D. V. Khveshchenko, “Thickening and sickening the SYK model,” SciPost Phys. 5, 012 (2018) doi:10.21468/SciPostPhys.5.1.012 [arXiv:1705.03956 [cond-mat.str-el]].

[32] J. Murugan, D. Stanford and E. Witten, “More on Supersymmetric and 2d Analogs of the SYK Model,” JHEP 1708, 146 (2017) doi:10.1007/JHEP08(2017)146 [arXiv:1706.05362 [hep-th]].

[33] P. Narayan and J. Yoon, “SYK-like Tensor Models on the Lattice,” JHEP 1708, 083 (2017) doi:10.1007/JHEP08(2017)083 [arXiv:1705.01554 [hep-th]].

[34] M. Berkooz, P. Narayan, M. Rozali and J. Simn, “Higher Dimensional Generalizations of the SYK Model,” JHEP 1701, 138 (2017) doi:10.1007/JHEP01(2017)138 [arXiv:1610.02422 [hep-th]].

[35] Y. H. Qi, Y. Seo, S. J. Sin and G. Song, “Schwarzian correction to quantum correlation in SYK model,” arXiv:1804.06164 [hep-th].

[36] S. R. Das, A. Ghosh, A. Jevicki and K. Suzuki, “Space-Time in the SYK Model,” JHEP 1807, 184 (2018) doi:10.1007/JHEP07(2018)184 [arXiv:1712.02725 [hep-th]].

[37] Y. Z. Li, S. L. Li and H. Lu, “Exact Embeddings of JT Gravity in Strings and M-theory,” arXiv:1804.09742 [hep-th].
[38] S. K. Jian, Z. Y. Xian and H. Yao, “Quantum criticality and duality in the Sachdev-Ye-Kitaev/AdS$_2$ chain,” Phys. Rev. B 97, no. 20, 205141 (2018) doi:10.1103/PhysRevB.97.205141 [arXiv:1709.02810 [hep-th]].

[39] R. G. Cai, S. M. Ruan, R. Q. Yang and Y. L. Zhang, “The String Worldsheet as the Holographic Dual of SYK State,” arXiv:1709.06297 [hep-th].

[40] A. A. Patel, J. McGreevy, D. P. Arovas and S. Sachdev, “Magnetotransport in a model of a disordered strange metal,” Phys. Rev. X 8, no. 2, 021049 (2018) doi:10.1103/PhysRevX.8.021049 [arXiv:1712.05026 [cond-mat.str-el]].

[41] A. A. Patel, M. J. Lawler and E. A. Kim, “Coherent superconductivity with large gap ratio from incoherent metals,” arXiv:1805.11098 [cond-mat.str-el].

[42] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, “Building a Holographic Superconductor,” Phys. Rev. Lett. 101, 031601 (2008) doi:10.1103/PhysRevLett.101.031601 [arXiv:0803.3295 [hep-th]].

[43] A. M. Garca-Garca, Y. Jia and J. J. M. Verbaarschot, “Universality and Thouless energy in the supersymmetric Sachdev-Ye-Kitaev Model,” Phys. Rev. D 97, no. 10, 106003 (2018) doi:10.1103/PhysRevD.97.106003 [arXiv:1801.01071 [hep-th]].

[44] Aaron Chew, Andrew Essin, Jason Alicea, “Approximating the Sachdev-Ye-Kitaev model with Majorana wires,” arXiv:1703.06890 [cond-mat.dia-nn].

[45] D. Chowdhury, Y. Werman, E. Berg and T. Senthil, “Translationally invariant non-Fermi liquid metals with critical Fermi-surfaces: Solvable models,” Phys. Rev. X 8, no. 3, 031024 (2018) doi:10.1103/PhysRevX.8.031024 [arXiv:1801.06178 [cond-mat.str-el]].

[46] Z. Bi, C. M. Jian, Y. Z. You, K. A. Pawlak and C. Xu, “Instability of the non-Fermi liquid state of the Sachdev-Ye-Kitaev Model,” Phys. Rev. B 95, no. 20, 205105 (2017) doi:10.1103/PhysRevB.95.205105 [arXiv:1701.07081 [cond-mat.str-el]].

[47] Z. Luo, Y. Z. You, J. Li, C. M. Jian, D. Lu, C. Xu, B. Zeng and R. Laflamme, “Observing Fermion Pair Instability of the Sachdev-Ye-Kitaev Model on a Quantum Spin Simulator,” arXiv:1712.06458 [quant-ph].

[48] Xiaochuan Wu, Xiao Chen, Chao-Ming Jian, Yi-Zhuang You, Cenke Xu, “A candidate Theory for the “Strange Metal” phase at Finite Energy Window,” arXiv:1802.04293 [cond-mat.str-el].

[49] E. Witten, “An SYK-Like Model Without Disorder,” arXiv:1610.09758 [hep-th].

[50] I. R. Klebanov and G. Tarnopolsky, “Uncolored random tensors, melon diagrams, and the Sachdev-Ye-Kitaev models,” Phys. Rev. D 95, no. 4, 046004 (2017) doi:10.1103/PhysRevD.95.046004 [arXiv:1611.08915 [hep-th]].

[51] R. Gurau, “Colored Group Field Theory,” Commun. Math. Phys. 304, 69 (2011) doi:10.1007/s00220-011-1226-9 [arXiv:0907.2582 [hep-th]].

[52] Rebecca Flint, M. Dzero, P. Coleman, “Heavy electrons and the symplectic symmetry of spin,” Nature Physics 4, 643 - 648 (2008).

[53] N. Read and Subir Sachdev, “Large-N expansion for frustrated quantum antiferromagnets,” Phys. Rev. Lett. 66, 1773 (1991); Subir Sachdev and Ziquiang Wang, Pairing in two dimensions: A systematic approach, Phys. Rev. B 43, 10229 (1991).
[54] S. Sachdev, “Bekenstein-Hawking Entropy and Strange Metals,” Phys. Rev. X 5, no. 4, 041025 (2015) doi:10.1103/PhysRevX.5.041025 [arXiv:1506.05111 [hep-th]].

[55] A. A. Patel and S. Sachdev, “A critical strange metal from fluctuating gauge fields in a solvable random model,” [arXiv:1807.04754 [cond-mat.str-el]].