On the cosmological gravitational waves and cosmological distances

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Abstract

We show that solitonic cosmological gravitational waves propagated through the Friedmann universe and generated by the inhomogeneities of the gravitational field near the Big Bang can be responsible for increase of cosmological distances.

INTRODUCTION

At present the most popular cosmological model — ΛCDM model successfully explains a wealth of cosmological observations. However, it involves a hypothetical substance, “dark energy”, having unusual physical properties. According to interpretation of various data, there is a need for accelerated expansion in the recent history of the universe, and hence dark energy is required to dominate the energy budget of the universe.

So far the main cause of introduction of dark energy was the discrepancy between observations of distant type Ia supernovae [1, 2] and Friedmann cosmological models with ordinary matter. Additional arguments include: tension between age estimates of globular clusters [3] and the age of the Universe, determined thanks to the measurement of the Hubble parameter [4–6], measurement of the baryon acoustic oscillations signature [7] providing the ratio of absolute distances in different cosmological epochs, and the inference from X-ray observation of massive galaxy clusters of the evolution of their number density [8], all consistent with recent accelerated expansion. Moreover, measurements of anisotropy spectrum of the cosmic microwave background [9, 10] suggest that observed universe is nearly spatially flat, while accounting for matter usually gives about one third of the critical density [10], requiring additional unknown component.

Alternatives to dark energy, discussed in the literature, include modification of gravity, for review see e.g. [11], and back-reaction of density perturbations on the Friedmann background [12, 13].

However, this standard theory does not take into account traces of the strong gravitational waves of cosmological origin left in space. In the present paper we propose the point of view that such traces can be a cause for the aforementioned discrepancy (if so there is no need to search for any enigmatic substance filling the universe). The sources of the long-lived cosmological waves are the solitonic type inhomogeneities of the gravitational field near the Big Bang. In general the inhomogeneities are unavoidable near the initial cosmological singularity and some subset of them has the solitonic structure. Due to expansion of the universe these inhomogeneities decay but expel solitonic gravitational waves which also decay in course of propagation through the expanding space transferring, however, their energies to the Friedmann background deforming it and making the distances different compared with those which would be observed without such waves [17]. The gravitational solitons are most important among all possible types of waves because such disturbances are more stable and survive longer time than others in the course of expansion. This effect has been described in the paper [15] by example of single-soliton cylindrical wave propagating on the Friedmann background where the background is supported by the matter in the form of massless scalar field \( \varphi \). The particular kind of matter is of a little importance since constructed solutions contain only gravitational waves and matter field remains unperturbed, it serves only to support the background. In the case of flat model the background solution of the Einstein equations in cylindrical coordinates is:

\[
- ds^2 = t (-dt^2 + dz^2 + z^2 dx^2 + dy^2) , \quad \varphi = (3/2)^{1/2} \ln t , \tag{1}
\]

and exact solution [15] containing one gravitational soliton propagating on this background has the form:

\[
- ds^2 = f(t, z) (-dt^2 + dz^2) + g_{11}(t, z) dx^2 + g_{22}(t, z) dy^2 + 2 g_{12}(t, z) dx dy , \tag{2}
\]

with the same unperturbed scalar field \( \varphi \) as in (1). Here \((x^0, x^1, x^2, x^3) = (t, x, y, z)\) and metric has the following structure:

\[
g_{11} = \frac{t}{s^2 t^2 + (t^2 + \mu)^2} \times \left( s^2 t^2 z^2 + z^2 (t^2 + \mu)^2 + q z^2 (t^2 + \mu) - q^2 \mu \right) , \tag{3}
\]
Then in case of the background (1) we have:

\[ g_{22} = \frac{t}{s^2 t^2 + (t^2 + \mu)^2} \left[ s^2 t^2 + (t^2 + \mu)^2 - q (t^2 + \mu) \right] , \]  

\[ g_{12} = \frac{t q s \mu}{s^2 t^2 + (t^2 + \mu)^2} , \]  

\[ f = \frac{t l^2 s^2 t^2 + (t^2 + \mu)^2}{s^2 [l^2 t^2 + (t^2 + \mu)^2]} \]  

In these formulas \( l \) and \( s \) are two arbitrary real constants and
\[ q = s^2 - l^2 . \]  

The function \( \mu \) (representing the pole trajectory in the inverse scattering method by which this solution was found) is:
\[ \mu = -\frac{1}{2} (l^2 + t^2 + z^2) + \frac{1}{2} \left( (l^2 + t^2 + z^2)^2 - 4t^2 z^2 \right)^{1/2} , \]  

It is easy to see that everywhere in space-time the square root in this expression is real and without loss of generality we can define it as positive. In the limit when parameter \( q \) tends to zero \( (s^2 \to l^2) \) the solution gives the background metric (1). It is this fact that permits to interpret the solution as an exact solitonic gravitational perturbation on the Friedmann background. It is convenient to characterize the solitonic field as exact deviation of the metric (2) from its background, which we designate by the upper index \( (0) \). For the \( g \)-components of the metric (2) this field is represented by the symmetric matrix \( H_{ab} \) \( (a, b = 1, 2) \):

\[ H_{11} = \left( g_{11} - g_{11}^{(0)} \right) \left( g_{11}^{(0)} \right)^{-1} , \]  

\[ H_{22} = \left( g_{22} - g_{22}^{(0)} \right) \left( g_{22}^{(0)} \right)^{-1} , \]  

\[ H_{12} = H_{21} = g_{12} \left( g_{11}^{(0)} g_{22}^{(0)} \right)^{-1/2} , \]  

and in the same way can be defined the perturbation \( F \) for the \( f \)-component of the metric (2):

\[ F = \left( f - f^{(0)} \right) \left( f^{(0)} \right)^{-1} \]  

Then in case of the background (1) we have:
\[ H_{11} = \frac{g_{11} - t z^2}{t z^2} , \quad H_{22} = \frac{g_{22} - t}{t} \]  
\[ H_{12} = \frac{g_{12}}{t z} , \quad F = \frac{f - t}{t} . \]  

The solution (3)-(8) contains no singularities other than the usual cosmological singularity already present in the background (1). The axis of cylindrical symmetry \( z = 0 \) is regular. All fields and its derivatives on the axis take some finite values. The solitonic excitations at the initial time \( t = 0 \) are concentrated around the axis \( z = 0 \) with the width \( \delta z \sim l \) and disappear at infinity \( z \to \infty \).

Hence, observers at the initial stage of expansion located far enough from the axis see no difference from the usual Friedmann universe with metric (1). During expansion the disturbance \( H_{ab} \) around the axis vanish but produces a solitonic gravitational wave moving away from the axis to infinity, with amplitude decreasing in time. This wave propagates along light-like line (in two-dimensional section) \( t = z \) inside the strip with the same width \( \delta z \sim l \) as initial disturbances have (see Fig. 1). It turns out that for the same observers located far from the axis but at the final stages of the expansion, that is after the wave passed the region around them, the solution describes again the flat Friedmann model, however, with different measure for the time intervals and space distances in the longitudinal \( z \)-direction (measures of the distances in the transversal directions \( x \) and \( y \) do not change). From (8) it follows that at \( t \to 0 \) the function \( \mu \) tends to zero as \( t^2 \) and in this limit for the metric coefficient \( f \) \( (6) \) we have \( f = t \). In the limit \( t \to \infty \) (and \( t \gg z \) ) the function \( \mu \) in the main approximation does not depend on time and at the late phases of expansion we have \( f = t l^2 s^{-2} \). Consequently, passing of a cosmological gravitational wave through the universe makes the scale factor in the measure of the longitudinal distances for the initial and final Friedmann backgrounds different because perturbation \( F \) of this factor makes the jump (see Fig. 1):
\[ \lim_{t \to 0} F = 0 , \quad \lim_{t \to \infty, t \gg z} F = \frac{t^2 - s^2}{s^2} . \]  

This jump disappears if \( l = s \) that is when the constant defined in \( q \) \( (7) \) vanish, but in such case, as we already mentioned, the solution (2)-(6) coincide identically with the final Friedmann model, however, with different measure for the time intervals and space distances in the longitudinal \( z \)-direction (measures of the distances in the transversal directions \( x \) and \( y \) do not change). From (8) it follows that at \( t \to 0 \) the function \( \mu \) tends to zero as \( t^2 \) and in this limit for the metric coefficient \( f \) \( (6) \) we have \( f = t \). In the limit \( t \to \infty \) (and \( t \gg z \) ) the function \( \mu \) in the main approximation does not depend on time and at the late phases of expansion we have \( f = t l^2 s^{-2} \). Consequently, passing of a cosmological gravitational wave through the universe makes the scale factor in the measure of the longitudinal distances for the initial and final Friedmann backgrounds different because perturbation \( F \) of this factor makes the jump (see Fig. 1):
\[ \lim_{t \to 0} F = 0 , \quad \lim_{t \to \infty, t \gg z} F = \frac{t^2 - s^2}{s^2} . \]  

Now the reasonable question to ask is whether such longitudinal memory effect is only due the cylindrical symmetry and single-solitonic structure of the chosen solution but can be absent in the more general cases. Let’s show that the same effect arises in the solutions containing double-solitonic waves and no matter under which symmetry, cylindrical or planar. These facts suggest a hint that the longitudinal memory effect is the general phenomenon of solitonic waves propagating on an isotropic homogeneous cosmological background. Some general consequences of this phenomenon we will discuss in the section “Summary”. 

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This jump disappears if \( l = s \) that is when the constant defined in \( q \) \( (7) \) vanish, but in such case, as we already mentioned, the solution (2)-(6) coincide identically with the background (1), that is the case \( l = s \) means the absence of any waves in the course of evolution. 

Now the reasonable question to ask is whether such longitudinal memory effect is only due the cylindrical symmetry and single-solitonic structure of the chosen solution but can be absent in the more general cases. Let’s show that the same effect arises in the solutions containing double-solitonic waves and no matter under which symmetry, cylindrical or planar. These facts suggest a hint that the longitudinal memory effect is the general phenomenon of solitonic waves propagating on an isotropic homogeneous cosmological background. Some general consequences of this phenomenon we will discuss in the section "Summary".
DOUBLE-SOLITON GRAVITATIONAL WAVES ON THE FRIEDMANN BACKGROUND

Cylindrical symmetry

Using the same method we can add to Friedmann background (1) two solitonic gravitational waves of the same type as in the previous case. The matter field \( \varphi \) is again unperturbed. The corresponding exact solution is described in Appendix I and here we outline only its properties relevant to the effect we are interested in. Qualitatively these properties are the same as in the previous single-soliton case. Again, the inhomogeneous solitonic perturbations at the initial time \( t = 0 \) are concentrated around the axis of cylindrical symmetry and during expansion decay but creates a gravitational waves moving along the light-like line \( z = t \) away from the axis to infinity with amplitudes decreasing in time (see Fig. 2). For an region located far from the axis and at the initial stage of expansion (that is before the waves reached this region) the metric is as in Friedmann background (1) but at the final stages of the expansion (after the waves passed this region) the metric again take the Friedmann form, however, with different measure for the time intervals and distances in the longitudinal \( z \)-direction (measures of the distances in the transversal directions \( x \) and \( y \) again do not change). Calculations show that in the limit \( t \to 0 \) we have \( f = t \) but when \( t \to \infty \) (and \( t \gg z \)) the scale factor \( f \) becomes \( f = t(l_1^2 \lambda_1 - l_2^2 \lambda_2)^2 [l_1 l_2 (\lambda_1 - \lambda_2)]^{-2} \), where \( l_1, l_2, \lambda_1, \lambda_2 \) are four arbitrary constants the solution contains (see Appendix I). Again, the perturbation \( F \) of this factor makes the jump (see Fig. 2):

\[
\lim_{t \to 0} F = 0, \quad \lim_{t \to \infty, t \gg z} F = \frac{(l_1^2 - l_2^2)}{l_1 l_2 (\lambda_1 - \lambda_2)^2} .
\]

(15)

This jump vanish only in the limit \( l_1^2 \to l_2^2 \) but, as shown in Appendix I, in this limit any waves disappear and solution coincide identically with the background Friedmann solution (1). Then we have the same phenomenon as in the previous single-soliton case.

Plane wave symmetry

The previous two cases describe cylindrical gravitational waves in the Friedmann universe. To construct the plane waves on the Friedmann background we need to represent it by Cartesian 3-dimensional space coordinates:

\[
- ds^2 = (dt^2 + dz^2 + dx^2 + dy^2) , \quad \varphi = (3/2)^{1/2} \ln t .
\]

(16)

However, for the real pole trajectories (real functions \( \mu \) as in the previous two cases) the inverse scattering method for this form of background produce solutions with discontinuities of the first derivatives of the metric tensor on the light cone \( t^2 = z^2 \). This phenomenon is related to the shock waves and need an appropriate physical interpretation. Nevertheless, the wide set of fully analytical solutions also exists. This set represents the gravitational solitons which correspond to an arbitrary number of complex-conjugated pairs of the poles in the inverse scattering machinery (that is an arbitrary number of complex-conjugated pairs of functions \( \mu \)). In the present paper we consider the simplest case when there is only one such pair \( \mu_1 = \rho e^{i\psi}, \mu_2 = \rho e^{-i\psi} \). The exact form of solution is given in Appendix II and omit details of its derivation (for guide see [13], subsection 2.2).

For the plane wave case the solitonic fields we define by the same formulas (10)-(12) but these perturbations take a different functional form with respect to the previous two cases since the background (16) is different:

\[
H_{11} = \frac{g_{11} - t}{t} , \quad H_{22} = \frac{g_{22} - t}{t} , \quad H_{12} = \frac{g_{12}}{t} , \quad F = \frac{f - t}{t} .
\]

(17)

From qualitative point of view the regular plane solitonic waves behave similarly as in the preceding cases for the cylindrical solutions. The inhomogeneous solitonic perturbation at the initial time \( t = 0 \) is concentrated around the central plane \( z = 0 \) of the 3-dimensional space and during expansion decays but creates two gravitational waves moving (one to the left and other to the right) inside some strips along the light cone \( z^2 = t^2 \) away from the central plane to infinities \( z \to \pm \infty \) with amplitudes decreasing in time, see Fig. 3. At the initial stage of expansion for the left and right regions located far from the central disturbance (that is before the waves reached these regions) the metric is as in Friedmann background (16) but at the final stages of the expansion (after the waves passed these regions) the metric again takes the Friedmann form, however, with different measure for the time intervals and distances in the longitudinal \( z \)-direction (measures of the distances in the transversal directions \( x \) and \( y \) again do not change). Calculations show that in both left and right far regions the metric coefficient \( f \) near Big Bang \( t = 0 \) is the same as in background, that is \( f = t \), but at the late stages of expansion \( t \to \infty \) (and \( t \gg z \)) it takes the value \( f = t \cosh^2 \sigma_0 (\sin^2 \tau_0)^{-1} \), where \( \sigma_0 \) and \( \tau_0 \) are arbitrary real constants the solution depends on (see Appendix II). Again, the perturbation \( F \) of the metric coefficient \( f \) makes the jump (see Fig. 3):

\[
\lim_{t \to 0} F = 0, \quad \lim_{t \to \infty, t \gg z} F = \frac{\cosh^2 \sigma_0 - \sin^2 \tau_0}{\sin^2 \tau_0} .
\]

(18)

This jump vanishes only in that special case when \( \sigma_0 = 0 \) and \( \tau_0 = \pi/2 \) but, as follows from the exact form of the solution (see Appendix II), in this case the waves disappear and solution gives identically the background Friedmann metric (16).
What is new and intriguing here (with respect to the two preceding cases) is the fact that for the plane waves (corresponding to the complex-conjugated pair of the pole trajectories in the inverse scattering method) the memory effect always increases the time intervals and longitudinal distances (because \( \cosh^2 \sigma_0 \geq \sin^2 \tau_0 \) for any values of the arbitrary constants).

**SUMMARY**

The crucial point in each complicated physical process is to find the basic elementary phenomenon. We hope that by above analysis we identified such basic point in the problem of excess of the value of the scale factor over its usual Friedmann prescription. One may expect in the real universe a huge amount chaotically distributed inhomogeneous solitonic perturbations near the Big Bang and in the course of the expansion all of them will decay sending the gravitational waves to the space, each in different direction. Solitons do not prevent each other to propagate keeping their shapes and directions of propagations. All of them will change their longitudinal distances, however, some direction in space can be transversal for one wave but longitudinal for the other. As a result the distances will be changed equally in any direction (probably with some anisotropy). The examples with real functions \( \mu \) (real poles in the inverse scattering technique) show that distances during expansion can be either increasing or decreasing which depends on the values of the arbitrary constants in the expressions [14] and [15]. But complex \( \mu \) (complex poles) produce effect only with increasing distances: the ratio of the metric coefficients \( f \) after the waves passed to its value before the waves came is always greater than unity. The last property probably is more preferable from the point of view of observations and it looks interesting that it can be given some support of statistical flavour. The main step in construction of solitonic solution is to choose the position of poles (that is the character of the functions \( \mu \), real or complex) of the dressing matrix in the complex plane of the spectral parameter. Of course, we cannot have any information on how many initial cosmological disturbances can correspond to the complex poles and how many to real. In the absence of such information we are forced to evaluate the relative amounts of the number of poles just by the volumes of those parts of the complex plane of the spectral parameter where corresponding poles can be located. It is evident that the number of the complex poles should dominate essentially because a line (real axis of the complex plane) represents a set of measure zero with respect to the whole plane. Then in the mixture of the solitonic cosmological waves the dominant contribution will come from those, which increase the distances and the global average effect also will correspond to the increasing of the time interval and space distances. This new effect may provide alternative explanation for apparent cosmic acceleration inferred from cosmological observations.

**APPENDIX I**

Consider the case when we have two real pole trajectories in the inverse scattering machinery and correspondingly two functions \( \mu_1(t, z) \) and \( \mu_2(t, z) \):

\[
\mu_1 = -\frac{1}{2} (l_1^2 + t^2 + z^2) + \frac{1}{2} \left[ (l_1^2 + t^2 + z^2)^2 - 4t^2z^2 \right]^{1/2},
\]

\[
\mu_2 = -\frac{1}{2} (l_2^2 + t^2 + z^2) + \frac{1}{2} \left[ (l_2^2 + t^2 + z^2)^2 - 4t^2z^2 \right]^{1/2},
\]

where \( l_1 \) and \( l_2 \) are two arbitrary real constants. Everywhere in space-time the square roots in these expressions are real and without loss of generality we can define them as positive (taking the different signs in front of these roots does not change the basic results we are interested in). Apart from \( l_1 \) and \( l_2 \) another two arbitrary real constants appear in the solution which constants we denote as \( \lambda_1 \) and \( \lambda_2 \). Now the interval has the same form as in [2] but with more cumbersome metric coefficients:

\[
g_{11} = U + \gamma^2 V + 2\gamma W, \quad g_{22} = V, \quad g_{12} = W + \gamma V,
\]

where by \( \gamma \) we denote the constant

\[
\gamma = -\frac{l_1^2 - l_2^2}{l_1^2\lambda_1 - l_2^2\lambda_2},
\]

and quantities \( U, V, W \) are [18]:

\[
U = \frac{t^2z^2}{2Q} \left[ 2Q + M_1^2 + M_2^2 + M_1^2M_2^2 + 1 + \frac{\mu_1 + \mu_2}{\mu_1 - \mu_2} (M_1^2 - M_2^2) - \frac{t^2z^2}{t^2z^2 - \mu_1\mu_2} (M_1^2M_2^2 - 1) \right],
\]

4
The metric coefficient $f$ such limit procedure corresponding to the pair of complex conjugated poles ($\mu_1, \mu_2$) solution we described already in the text. Gravitational perturbations at initial time $t$ in cylindrical symmetry contains no singularities other than the usual cosmological singularity already presented in the background. The $l$, $l$ of order of unity if constants $\lambda_1, \lambda_2$, scalar matter field $\varphi$ remains the same as in background:

$$f = (3/2)^{1/2} \ln t.$$  

The complexity of this exact solution creates no serious difficulties because we need to know only few basic properties of the solution and its asymptotic behavior for large and small values of coordinates. These properties and asymptotics can be established easily. First of all in the limit $l_1^2 \rightarrow l_2^2$ solution again turns to the Friedmann background [8]. [To see this it is necessary to expand all terms with respect to the difference $l_1^2 - l_2^2$ assuming it infinitesimally small, in such limit $\mu_1 - \mu_2 \sim l_1^2 - l_2^2$ and $Q \sim (l_1^2 - l_2^2)^{-2}$. This means that solution indeed can be interpreted as representing the exact gravitational waves propagating on the flat Friedmann background.

All other properties of the solution (19)-(29) are very similar to those for the single-soliton case. The solution contains no singularities other than the usual cosmological singularity already presented in the background. The axis of cylindrical symmetry $z = 0$ is regular. All fields and its derivatives on the axis take some finite values. The gravitational perturbations at initial time $t = 0$ are concentrated around the axis $z = 0$ with the widths in $z$-direction of order of unity if constants $l_1, l_2, \lambda_1, \lambda_2$ are chosen to be of the order of unity. Other relevant properties of this solution we described already in the text.

Appendix II

To construct the solitonic plane waves on the Friedmann background we apply the standard inverse scattering procedure corresponding to the pair of complex conjugated poles ($\mu_1 = \rho e^{i\psi}$, $\mu_2 = \rho e^{-i\psi}$) which results in the solution of the following form:

$$g_{11} = \frac{t}{D} \left[ D + \cos 2\tau_0 + \cosh 2\sigma_0 + \frac{z (t^2 - \rho^2)}{w_0 (t^2 + \rho^2)} \sin 2\tau_0 - \frac{t^2 + \rho^2}{t^2 - \rho^2} \sinh 2\sigma_0 \right],$$ (30)

$$g_{22} = \frac{t}{D} \left[ D + \cos 2\tau_0 + \cosh 2\sigma_0 - \frac{z (t^2 - \rho^2)}{w_0 (t^2 + \rho^2)} \sin 2\tau_0 + \frac{t^2 + \rho^2}{t^2 - \rho^2} \sinh 2\sigma_0 \right],$$ (31)

$$g_{12} = \frac{2t}{D} \left[ \frac{z (t^2 - \rho^2)}{w_0 (t^2 + \rho^2)} \sinh \sigma_0 \sin \tau_0 + \frac{t^2 + \rho^2}{t^2 - \rho^2} \cosh \sigma_0 \cos \tau_0 \right],$$ (32)

$$D = \frac{(t^2 - \rho^2)^4 \sin^2 \tau_0 + 16 w_0^2 t^4 \rho^4 \cosh^2 \sigma_0}{4 w_0^2 \rho^2 (t^2 - \rho^2)^2},$$ (33)
\[ f = \frac{t \left[ (t^2 - \rho^2)^2 \sin^2 \tau_0 + 16w_0^2 t^2 \rho^4 \cosh^2 \sigma_0 \right]}{\left[ (t^2 - \rho^2)^2 + 16w_0^2 t^2 \rho^4 \right] \sin^2 \tau_0}, \] (34)

where \( \sigma_0, \tau_0 \) and \( w_0 \) are three real arbitrary constants. The function \( \rho(t, z) \) is real and positive and follows from the equation:

\[ \frac{4z^2 \rho^2}{(t^2 + \rho^2)^2} + \frac{4w_0^2 \rho^2}{(t^2 - \rho^2)^2} = 1. \] (35)

The matter field, as in the previous cases, is \( \varphi = (3/2)^{1/2} \ln t \) that is remains unperturbed. The algebraic equation (35) has two real and positive solutions for function \( \rho(t, z) \). We chose that one which in the limit \( t \to 0 \) has asymptotes \( \rho = t^2 \left[ 4 \left( w_0^2 + z^2 \right) \right]^{-1/2} \) and in the limit \( t \to \infty \) tends to \( \rho = t - w_0 \) (second root for \( \rho \) gives the same physical results).

It is easy to see that there are two different ways in the parameter space to get the background limit. First is to take \( \sigma_0 = 0 \) and \( \tau_0 = \pi/2 \) in which case solution gives the seed metric [16]. The second and independent possibility is to take limit \( w_0 \to 0 \) under which solution also tends to coincide with this background. Then solution indeed represents the two exact solitonic plane gravitational wave propagating on the Friedmann universe.

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[17] The phenomenon we describe here is due to the strong non-linear gravitational waves of cosmological origin, however, it is akin to the Zeldovich-Polnarev [14] memory effect produced by the weak linear gravitational waves generated by the colliding astrophysical objects.
[18] It is worth mentioning that \( g_{11} = U, g_{22} = V, g_{12} = W \) with the same \( f \) and \( \varphi \) from (28) also is a solution of the Einstein equations but the regular behavior of the solution on the axis \( z = 0 \) demands transformation of the dummy coordinates \( x, y \) as \( x \to x, y \to y + \gamma x \). This produces the metric coefficients in the form (21) as linear combinations of quantities \( U, V, W \).
FIG. 1: Single-soliton solution with parameters $l = 1, s = 1/2$ as a function of time $t$ and spatial coordinate $z$. From left to right and from top to bottom the following functions are shown: $F$, $H_{22}$, $H_{11}$ and $H_{12}$. 
FIG. 2: Double-soliton solution with parameters $l_1 = -3$, $l_2 = -2.8$, $\lambda_1 = 1.1$, $\lambda_2 = 1.2$ as a function of time $t$ and spatial coordinate $z$. From left to right and from top to bottom the following functions are shown: $F$, $H_{22}$, $H_{11}$ and $H_{12}$. 
FIG. 3: Double-soliton solution with plane wave symmetry having parameters $\omega_0 = 1$, $\sigma_0 = 2$, $\tau_0 = -1$ as a function of time $t$ and spatial coordinate $z$. From left to right and from top to bottom the following functions are shown: $F$, $H_{22}$, $H_{11}$ and $H_{12}$. Function $H_{22}$ is finite everywhere, but its peak is not shown for better illustration of wave propagation.