Sidelobe level reduction in ACF of NLFM waveform

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Abstract: In this study, an iterative method is proposed for non-linear frequency modulation (NLFM) waveform design based on a constrained optimisation problem using Lagrangian method. To date, NLFM waveform design methods have been performed based on the stationary phase concept which has been already used by the authors in a previous work. The proposed method has been implemented for six windows of Raised-Cosine, Taylor, Chebyshev, Gaussian, Poisson, and Kaiser. The results reveal that the peak sidelobe level of autocorrelation function (ACF) reduces about an average of 5 dB in the proposed method compared with the stationary phase method, and an optimum peak sidelobe level is achieved. The minimum error of the proposed method decreases in each iteration which is demonstrated using mathematical relations and simulation. The trend decrement of minimum error guarantees convergence of the proposed method.

1 Introduction

Using a short simple pulse for sending in a radar system needs to employ a high-power transmitter which is expensive and easy to intercept. If the pulse width increases, the required range resolution to detect the several targets is severely degraded; therefore, the best solution is to modulate the pulse [1, 2]. By performing modulation also known as pulse compression, bandwidth and signal-to-noise ratio are increased and range resolution improves [3–8], but the existence of high sidelobe level in the autocorrelation function (ACF) is annoying [6, 9]. Pulse compression is done using various methods such as phase coding (Barker code), amplitude weighing, linear frequency modulation (LFM), and non-linear LFM (NLFM) [10–12].

In the phase coding and amplitude weighing methods, due to the phase discontinuity and variable amplitude, the mismatch loss increases in the receiver [13]; therefore, the use of LFM signals increases significantly due to continuous phase and constant amplitude. Although, the LFM method shows clear advantages over the phase coding and amplitude weighing methods, but the high sidelobe level is still annoying because of masking the smaller targets by sidelobes of bigger targets; thus, the NLFM method is used thanks to its significant decrease in peak sidelobe level (PSL) [14].

In the NLFM method similar to LFM method, the signal amplitude is constant, and an optimal phase is intended to find. The stationary phase concept (SPC) is often used for NLFM method. The SPC expresses that the power spectral density (PSD) in a specific frequency is relatively large if frequency variations are small with respect to time [11]. The stationary phase method used in [15] resulted in significant reduction of PSL while the mainlobe width widens which can be neglected.

In this paper, the proposed method improves the PSL of the stationary phase method.

The remainder of the paper is organised as follows. Section 2 reviews NLFM waveform design based on the stationary phase method. NLFM waveform design based on the proposed method is explained in Section 3 in which the optimal phase is calculated initially and then the convergence of the proposed method is demonstrated using mathematical analysis. Section 4 contains simulation results. Finally, Section 5 concludes the paper.

2 NLFM signal design with stationary phase method

In stationary phase method, the desired signal is defined as

\[ x(t) = a(t)\exp(j\psi(t)), \quad -\frac{T}{2} \leq t \leq \frac{T}{2} \]  

(1)

where \( a(t) \), \( \psi(t) \), and \( T \) are amplitude, phase, and pulse width of \( x(t) \), respectively. We consider \( a(t) = A \) (\( A \) = constant). The \( f_s \) is instantaneous frequency of \( x(t) \) at time \( t_0 \) which is determined as follows:

\[ f_s = \frac{1}{2\pi} \psi'(t_0) \]  

(2)

In this method, the relation between the desired PSD and the frequency variation in desired signal is obtained by using the stationary phase concept. Since the amplitude is constant in NLFM signals, the PSD only depends on the second-order derivative of the signal phase, which is the frequency variation [11].

By solving the equations in the Fourier domain, the group time delay function is obtained by taking the first derivative of the phase in Fourier domain. Inverse group time delay function is used to calculate frequency function of time [16] and the signal phase can be addressed with integrating the frequency function. Finding the inverse group delay function is not always easy, and in some cases it should be carried out numerically.

This method was applied in [15]. In Section 4, the results are compared against the proposed method.

3 NLFM signal design with the proposed method

Our proposed method is based on a constrained optimisation problem. First, a window as the initial window is considered, and then by solving the constrained optimisation problem, the desired signal is aimed to be found. This method is performed iteratively and the phase obtained from the stationary phase method [15] is used to start. To guarantee the convergence of the proposed method, triangle inequality and mathematical analysis are used to demonstrate the minimum error decrement in each iteration. Additionally, the minimum error value is positive expressing convergence due to the fact that a positive non-increasing sequence certainly converges.
3.1 Optimal phase

In the proposed method, to obtain the phase of the desired signal, first, an initial window is considered where |\( Y(f) \)| is its root. Difference between |\( Y(f) \)| and the amplitude of the Fourier transform of the desired signal (|\( X(f) \)|) is defined as the error. Since in NLFM signals, amplitude is constant, so our goal is to minimise the error with the constraint of being unit the amplitude of \( x(t) \) for \(|t| \leq T/2\), therefore we try to minimise the following equation:

\[
\begin{align*}
\min_{x(f)} & \quad E = \int_{-B/2}^{B/2} |Y(f)|^2 - |X(f)|^2 \, df \\
\text{s.t.} & \quad |x(t)|^2 = 1, \ |t| \leq T/2 \\
& \quad x(t) = 0, \ |t| > T/2
\end{align*}
\]

(3)

If two complex numbers come close to each other, then it can be concluded that the values of their amplitudes also close together, so if the following equation is reduced, then the error can be reduced, which is expressed in the phase matching problems [17–19]

\[
\begin{align*}
\min_{\theta(f), X(f)} & \quad E = \int_{-B/2}^{B/2} |Y(f)| \exp(j\theta(f)) - X(f)|^2 \, df \\
\text{s.t.} & \quad |x(t)|^2 = 1, \ |t| \leq T/2 \\
& \quad x(t) = 0, \ |t| > T/2
\end{align*}
\]

(4)

where \( \theta(f) \) is defined as the phase of \( X(f) \). Assume \( Y_\theta(f) = |Y(f)| \exp(j\theta(f)) \) and \( f = kb(K-1) \), then the error equation can be written in the discrete form as follows:

\[
E = \sum_{k=0}^{K-1} |Y_\theta(k) - X(k)|^2
\]

(5)

where \( X(k) = \sum_{n=0}^{N-1} n_\theta(n) \exp(-j2\pi kn/(K)) \). Consider equations in vector space and assume \( x = \begin{bmatrix} x(0), x(1), \ldots , x(N-1) \end{bmatrix}^T \) and \( W \), matrix as follows:

\[
\begin{bmatrix}
X(0) \\
\vdots \\
X(N) - 1
\end{bmatrix} = Wx
\]

(6)

If we assume \( Y_\theta = [Y_\theta(0), Y_\theta(1), \ldots , Y_\theta(K-1)]^T \), (4) is rewritten as follows:

\[
\begin{align*}
\min_{\theta, x} & \quad (Y_\theta - Wx)^H(Y_\theta - Wx) \\
\text{s.t.} & \quad |x(n)|^2 = 1, \ n = 1, \ldots , N - 1
\end{align*}
\]

(7)

Using Lagrangian method, we solve the obtained constrained optimisation problem [20]

\[
J = (Y_\theta - Wx)^H(Y_\theta - Wx) + \sum_{n=0}^{N-1} \lambda_n |x(n)|^2
\]

(8)

where \( \lambda_n \) is Lagrangian multiplier and \( \Lambda = \text{diag}(\lambda_0, \lambda_1, \ldots , \lambda_{N-1}) \). The symbol \( \text{diag}(.) \) is the diagonal matrix where the entries outside the main diagonal are all zero. Due to the orthogonality of the columns of the matrix \( W \), the value of \( W^H W \) is equal to \( KI_N \) where \( I_N \) is identity matrix of size \( N \). We take \( J \) derivative with respect to \( x \)

\[
\frac{\partial J}{\partial x} = -(W^H Y_\theta)^* + KI_N x^* + \Lambda x^* = 0
\]

\[
\Rightarrow x = (KI_N + \Lambda)^{-1} W^H Y_\theta
\]

(9)

where \( * \) denotes the complex conjugate, \((KI_N + \Lambda)^{-1}\) is the inverse matrix of \( KI_N + \Lambda \) and calculated as follows:

\[
(KI_N + \Lambda)^{-1} = \text{diag}\left(\frac{1}{K + \lambda_0}, \ldots , \frac{1}{K + \lambda_{N-1}}\right)
\]

(10)

Since the constraints of the optimisation problem are real, then the Lagrange multipliers \( \lambda_n \) are real and the vector \( x \) is expressed as follows:

\[
\begin{bmatrix}
x(0) \\
\vdots \\
x(N-1)
\end{bmatrix} = \sum_{k=0}^{K-1} \frac{1}{K + \lambda_k} \begin{bmatrix} W_{k+1}^H Y_\theta(k) \\
\vdots \\
W_{N-k}^H Y_\theta(k) \end{bmatrix}
\]

(11)

Due to the constraints of the problem, the values \( \lambda_n \) must be such that the square of the amplitude of each coefficient of \( x \) is equal to one, so

\[
\frac{1}{K + \lambda_n} \sum_{k=0}^{K-1} |W_{k+1}^H Y_\theta(k)|^2 = 1
\]

(12)

Therefore, the vector \( x \) is calculated as follows:

\[
\begin{bmatrix}
x(0) \\
\vdots \\
x(N-1)
\end{bmatrix} = \Lambda W^H Y_\theta
\]

(13)

where \( \Lambda_i \) is a \( N \times N \) matrix as follows:

\[
\begin{bmatrix}
[A_{ij}]_n = \sum_{k=0}^{K-1} |W_{k+1}^H Y_\theta(k)|^2, \ i = j \\
0, \ i \neq j
\end{bmatrix}
\]

(14)

To achieve the desired signal, the proposed method is performed as an iterative algorithm; therefore, with respect to (13), in \( r \)th iteration, the desired signal will be as follows:

\[
x^{(r)} = \Lambda_i^{-1} Y_\theta^{(r-1)}
\]

(15)

\( \theta^{(r)} \) is the phase of \( X(f) \) in \( r \)th iteration, which can be calculated as follows:

\[
\theta^{(r)} = \text{phase}(Wx^{(r)})
\]

(16)

By calculating the \( \theta^{(r)} \) value, vector \( Y^{(r)}_\theta \) and then the matrix \( \Lambda_i^{(r)} \) are calculated

\[
Y^{(r)}_\theta = \begin{bmatrix} Y(0) \exp(j\theta^{(r)}(0)) \\
\vdots \\
Y(K - 1) \exp(j\theta^{(r)}(K - 1)) \end{bmatrix}
\]

(17)

\[
[A^{(r)}_{ij}] = \sum_{k=0}^{K-1} |W_{k+1}^H Y^{(r)}_\theta(k)|^2, \ i = j \\
0, \ i \neq j
\]

(18)
In (19), are the projection and orthogonal complement matrices, obtain the desired NLFM signal, and because the amplitude of 3.2 Convergence of the proposed method proposed method is Doppler sensitive. This is due to variations of of the NLFM signal [15]. Thus, by repeating the algorithm, we obtained signal in the constant coefficient method is Doppler robust while the waveform designed by the summary of the proposed method.

To start the algorithm, we set the equal to the obtained phase value on the stationary phase method.

For convergence of the proposed method, the error value must be reduced with increasing of iterations. In other words, for the iteration as

\[ E_{\text{Min}}^{(r+1)} = \int_{B(\Delta)} |Y(f)\exp(j\theta^r(f)) - X^{r+1}(f)|^2 df \]

\[ \leq \int_{B(\Delta)} |Y(f)\exp(j\theta^r(f)) - X^{r}(f)|^2 df \quad (24) \]

From the comparison of (23) and (24), we conclude \( E_{\text{Min}}^{(r+1)} \geq E_{\text{Min}}^{(r)} \).

Since we considered two arbitrary successive iterations, so (21) is satisfied for each two arbitrary successive iterations. Since \( E_{\text{Min}} \) is a positive non-increasing sequence, convergence of the proposed method is guaranteed.

4 Simulation results

The proposed method is performed for six initial windows of Raised-Cosine, Taylor, Chebyshev, Gaussian, Poisson, and Kaiser. Table 1 shows the formula of the selected windows, the group time delay functions, and their constant parameters. As already mentioned, the group time delay function for some windows is numerically calculated. In Table 1, erff() and sgnf() are defined as the error and sign functions, respectively. The design parameters such as bandwidth \( B \), pulse width \( T \), and sampling rate \( SR \) are considered equal to 10–100 MHz, 2.5 \( \mu \)s, and 0.1–1 GHz, respectively. Table 2 compares the results obtained from the stationary phase method and the proposed method for PSL of the ACF. The results indicate that the average of PSL reduction is about 5 dB revealing the maximum PSL reduction associated with the Poisson window.

Table 3 compares PSL with the integrated sidelobe level (ISL) for the six selected windows in the proposed method \( B = 100 \) MHz and \( SR = 1 \) GHz. Figs. 2 and 3 show the ACFs of the designed signals using the stationary phase method (SPM) and the proposed method (PM) for the six selected windows, respectively. Figs. 4 and 5 show the frequency of the designed signals using the stationary phase method and the proposed method for the six selected windows, respectively. Fig. 6 shows the PSL for the six selected windows in different iterations. Also, Fig. 7 shows the minimum error for the six selected windows in different iterations. The minimum error in Fig. 7 is calculated according to (20). As mentioned in Section 3.2, the minimum error of the proposed method has a decreasing trend. First, this error has a significant value, but it begins to decrease and the trend of its change is almost constant in high iterations.

5 Conclusion

In the proposed method, an NLFM signal by solving a constrained optimisation problem using Lagrangian method is obtained. By this iterative method, the PSL of the ACF reduced about 5 dB compared with the stationary phase method. PSL reduction for the Poisson window compared to other windows is significant. Using mathematical analysis, we showed that the minimum error of the proposed method has a decreasing trend and this guarantees the convergence of the proposed method. The minimum errors of six selected windows also reveal the validity of this statement.

\[ \text{Input} : \theta^{(0)} \text{ where is equal to the obtained phase value from the stationary phase method} \]

\[ \text{Output} : \text{Vector} \ x^{(N)} \]

\[ \text{for } r = 1, 2, \ldots, N \text{ do} \]

1. Generate \( Y_{\theta}^{(r)} \) by (17)
2. Calculate \( A_{\theta}^{(r)} \) using (18)
3. Calculate \( x^{(r)} \) using (15)
4. \( \theta^{(r)} = \text{phase}(Wx^{(r)}) \)

end

\[ \gamma \text{ Note: } N \text{ is the maximum number of the iterations} \]

Fig. 1 Algorithm 1: Summary of the proposed method

To start the algorithm, we set the \( \theta^{(0)} \) equal to the obtained phase value from the stationary phase method for the Fourier transform of the NLFM signal [15]. Thus, by repeating the algorithm, we obtain the desired NLFM signal, and because the amplitude of the NLFM signal is constant, so we can multiply the amplitude of the obtained signal in the constant coefficient. Fig. 1 illustrates the summary of the proposed method.

Note that the NLFM waveform designed by the stationary phase method is Doppler robust while the waveform designed by the proposed method is Doppler sensitive. This is due to variations of the phase of the signal in each iteration.
### Table 1  Selected windows

| Initial windows | Formula | Group time delay function | Constant parameters |
|-----------------|---------|---------------------------|---------------------|
| Raised-Cosine   | \( w(n) = k + (1 - k) \cos(\frac{2\pi n}{M-1}) \) | \( T \left( \frac{1 - k}{2\pi} \right) \sin(\frac{2\pi n}{B}) \) |  \( k = 0.17 \) |
| Taylor          | \( w(n) = 1 + \sum_{m=1}^{N} F_m \cos(\frac{2\pi mn}{M-1}) \) | \( T \left( \frac{1 - k}{2\pi} \right) \sin(\frac{2\pi n}{B}) \) | \( k = 0.17 \) |
| Chebyshev       | \( W(m) = \frac{\cos[M \cos^{-1}(\frac{\beta}{M})]}{\cosh[M \cos^{-1}(\beta)\left(\frac{1}{10^9}\right)\sin(\frac{2\pi n}{B})]} \) |  \( \eta = 88.5 \text{ dB} \) | \( \alpha = 2 \) |
| Gaussian        | \( w(n) = \exp\left(-\frac{k}{\frac{1}{2}(M-1)}\right) \) |  \( \eta = 88.5 \text{ dB} \) | \( \alpha = 2 \) |
| Poisson         | \( w(n) = \exp\left(-\frac{\eta}{2\pi n_f} \right) \) |  \( \eta = 88.5 \text{ dB} \) | \( \alpha = 2 \) |
| Kaiser          | \( w(n) = \begin{cases} T \left( \frac{1 - k}{2\pi} \right) \sin(\frac{2\pi n}{B}) \end{cases} \) |  \( \eta = 88.5 \text{ dB} \) | \( \alpha = 2 \) |

### Table 2  Comparison of PSL for the stationary phase method (SPM) and proposed method (PM) through different bandwidths

| Peak sidelobe level, dB | Raised-cosine | Taylor | Chebyshev | Gaussian | Poisson | Kaiser |
|-------------------------|---------------|--------|-----------|----------|---------|--------|
| B, MHz                  | SPM/PM        | SPM/PM | SPM/PM    | SPM/PM   | SPM/PM  | SPM/PM |
| 10                      | -25.05/-28.95 | -25.08/-28.52 | -23.18/-28.90 | -24.84/-29.86 | -21.57/-28.71 | -22.79/-29.22 |
| 20                      | -29.42/-31.03 | -29.43/-30.95 | -26.12/-30.85 | -28.33/-32.00 | -20.83/-30.20 | -25.62/-29.00 |
| 30                      | -30.94/-34.28 | -30.94/-31.64 | -27.93/-33.21 | -29.67/-32.18 | -20.65/-33.13 | -27.16/-32.51 |
| 40                      | -31.80/-34.92 | -31.81/-33.64 | -29.09/-33.11 | -30.56/-34.18 | -20.54/-33.88 | -28.25/-35.43 |
| 50                      | -32.29/-34.75 | -32.29/-35.28 | -28.85/-34.15 | -31.10/-33.52 | -20.51/-33.24 | -29.03/-35.55 |
| 60                      | -32.72/-35.86 | -32.72/-35.77 | -30.44/-35.29 | -31.59/-35.61 | -20.51/-35.95 | -29.65/-35.25 |
| 70                      | -32.92/-36.53 | -32.91/-35.44 | -30.88/-35.10 | -31.83/-35.84 | -20.46/-35.98 | -30.11/-34.26 |
| 80                      | -33.07/-36.04 | -33.05/-35.41 | -31.24/-37.33 | -32.03/-36.85 | -20.43/-36.61 | -30.49/-35.20 |
| 90                      | -33.22/-37.34 | -33.20/-36.28 | -31.55/-36.30 | -32.23/-37.23 | -20.41/-36.80 | -30.77/-36.76 |
| 100                     | -33.34/-37.89 | -33.34/-37.73 | -31.77/-37.37 | -32.38/-37.67 | -20.39/-37.67 | -30.98/-36.82 |

### Table 3  Comparison between PSL and ISL for the six selected windows in the proposed method

| B, MHz | SR, GHz | Raised-cosine | Taylor | Chebyshev | Gaussian | Poisson | Kaiser |
|--------|---------|---------------|--------|-----------|----------|---------|--------|
|        |         | PSL/ISL       | PSL/ISL| PSL/ISL   | PSL/ISL  | PSL/ISL | PSL/ISL |
| 100    | 1       | -37.89/-48.60 | -37.73/-47.98 | -37.37/-47.54 | -37.67/-48.99 | -37.67/-47.84 | -36.82/-47.18 |

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**Image**

![Raised-Cosine ACF](a)

**Image**

![Taylor ACF](b)
Fig. 2 ACFs of the designed signals by using the stationary phase method for the six windows of Raised-Cosine, Taylor, Chebyshev, Gaussian, Poisson, and Kaiser

(a) ACF of the designed signal by using SPM for Raised-Cosine window, (b) ACF of the designed signal by using SPM for Taylor window, (c) ACF of the designed signal by using SPM for Chebyshev window, (d) ACF of the designed signal by using SPM for Gaussian window, (e) ACF of the designed signal by using SPM for Poisson window, (f) ACF of the designed signal by using SPM for Kaiser window
Fig. 3 ACFs of the designed signals by using the proposed method for the six windows of Raised-Cosine, Taylor, Chebyshev, Gaussian, Poisson, and Kaiser
(a) ACF of the designed signal by using PM for Raised-Cosine window, (b) ACF of the designed signal by using PM for Taylor window, (c) ACF of the designed signal by using PM for Chebyshev window, (d) ACF of the designed signal by using PM for Gaussian window, (e) ACF of the designed signal by using PM for Poisson window, (f) ACF of the designed signal by using PM for Kaiser window

Fig. 4 Frequency of the designed signals by using the stationary phase method for the six windows of Raised-Cosine, Taylor, Chebyshev, Gaussian, Poisson, and Kaiser

Fig. 5 Frequency of the designed signals by using the proposed method for the six windows of Raised-Cosine, Taylor, Chebyshev, Gaussian, Poisson, and Kaiser
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