Mathematical Modelling of a Three Tube Heat Exchanger for Hot Two Pass and Cold one Pass Flow (Both from Front end) Arrangement

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Abstract— A three tube heat exchanger is a special type of H.E having three concentric tubes with multi-pass arrangements for one of the fluid. This is a mixed type of heat exchanger which is neither pure counter flow nor pure parallel flow. Simultaneously it gives both parallel and counter flow arrangements for any flow directions of two fluids. This is also a type of multi-pass and a compact type of heat exchanger.

The work involved in this investigation is to analyze the performance of 3-tube heat exchanger by formulating differential equations using energy balance in differential form. These differential equations are solved by using a numerical technique viz., by 4th order Runge-Kutta method with boundary conditions. Figure numbers 5 to 8 shows the temperature plots along the length of the three tube heat exchanger in dimensionless form. The plots are obtained for four different values of F, which is a capacity rate ratio. The temperature profile nature is as expected for different arrangements resulting from entry of hot and cold fluids. By using this temperature plots effectiveness and NTU are determined for different capacity rate ratios. The results are concluded at the end.

Keywords- Effectiveness, Heat exchanger, Heat Transfer Coefficient, Heat capacity ratio, Logarithmic Mean Temperature Difference (LMTD), Mass Flow rates, NTU, Overall heat transfer coefficient.

Notations:
A- outer most tube
B- Inner most tube
C- Center tube
T_h→ Temperature of hot fluid at inlet
T_ho- Temperature of hot fluid at outlet
T_c→ Temperature of cold fluid at inlet
T_co- Temperature of cold fluid at outlet
U-Overall heat transfer coefficient
A-Surface area
F- Flow rate ratio

1. INTRODUCTION

It is necessary to remove the heat in various engineering applications such as in Space, Automobile and Aeronautical Applications where size and weight play a major role. This heat produced in various processes can be removed by an efficient heat exchanger.

One way of increasing the rate of heat transfer is by increasing the surface area or by increasing the temperature difference. The commonly used heat exchangers are either the parallel flow type or the counter flow type. The counter flow arrangements will give the maximum heat transfer rate than the parallel flow arrangements.

There are other types of heat exchangers also namely cross flow and multi-pass heat exchangers, which gives high heat transfer rate, but they are suffered from high operating cost and pressure drop. Hence, counter flow heat exchangers are preferred.

A three tube heat exchanger is a mixed type of heat exchanger which is neither a pure counter flow nor a pure parallel flow. Simultaneously it gives parallel and counter flow arrangements for any flow directions of two fluids. This is also a type of multi-pass and a compact type of heat exchanger. The schematic diagram of a three tube heat exchanger is presented in the below Figure 1.

![Figure 1. Schematic diagram of 3-tube heat exchanger](image)

Figure 1. Schematic diagram of 3-tube heat exchanger

Working of 3-tube heat exchanger:

The schematic diagram shown in figure 1. In this case, the hot fluid enters at port 1 travels to the rear end also through the outer annulus and returns to the front end through the central tube and leaves by port 2. The cold fluid enters at port 3, travels through the middle annulus and leaves by port 4.

![Figure 2: 3D diagram of 3-tube heat exchanger](image)

Figure 2: 3D diagram of 3-tube heat exchanger

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II. METHODOLOGY

Mathematical formulation:
In this investigation there are different flow arrangements are possible. For one of the flow arrangement we analyzed the work in detailed as explained below.

Flow arrangement: Hot two pass cold one pass (Both from front end):
For the above case the simplified energy balance equation for three tubes are given below in differential form.
Consider a differential control volume of width $dx$ at a distance of $x$ from front end as shown in figure 2 and 3. It consists of three sections corresponding to the three tubes.

$$\frac{dT_{ha}}{dx} = -A_1(T_{ha} - T_{cc})$$ \hspace{1cm} (1)

$$\frac{dT_{hb}}{dx} = B_1(T_{hb} - T_{cc})$$ \hspace{1cm} (2)

$$\frac{dT_{cc}}{dx} = C_1(T_{ha} - T_{cc}) + D_1(T_{hb} - T_{cc})$$ \hspace{1cm} (3)

With boundary conditions:

at $x=0$ : $T_{ha}=1, T_{cc}=0$

at $x=1$ : $T_{ha}=T_{hb}$

III. SOLUTION FOR METHODOLOGY

The solution is based on the 4th order Runge-Kutta method. The main steps involved in the computational procedure are shown in the flow chart. The differential equations are in their dimensionless form. The solutions are obtained for four flow rate ratios ($F$) of fluid.

IV. RESULTS AND DISCUSSION

As explained in the above methodology the equations formulated by energy balance equation are solved by using the boundary conditions. Figure numbers 5 to 8 shows the temperature plots along the length of the three tube heat exchanger in dimensionless form. The plots are obtained for four different values of $F$, which is a flow rate ratio. The temperature profile nature is as expected for different arrangements resulting from entry of hot and cold fluids.
The effectiveness of three tube heat exchanger is calculated by using inlet and outlet temperatures.

V. CONCLUSIONS AND SCOPE FOR FURTHER WORK

1. Three tube heat exchanger is a special type of heat exchanger which gives parallel and counter flow arrangements simultaneously.
2. Three tube heat exchanger is a compact type of heat exchanger and the cost requires is also less when compared to pure parallel and pure counter flow heat exchanger.
3. The length required for three tube heat exchanger is less as compared to two tube heat exchanger for same amount of heat transfer area. Hence, it is advantages to have three tube heat exchanger where space is restricted.
4. The length of three tube heat exchanger is less as compared to two tube heat exchanger, hence pressure drop is less.
5. The work is developed for different flow arrangements and different flow rate ratios of both hot and cold fluids.
6. The work is also further developed by providing finned arrangements for higher heat transfer rates.

REFERENCES

1. Raja. H Krishna Chaitanya, ‘Online Mechanical Characterization of a Shell and Tube Heat Exchanger’, International Journal of Pure and Applied Mathematics Volume 114 No. 11 2017, 289-299.
2. Vikas Kannojiya, ‘Performance Investigation Of A Double Pipe Heat Exchanger Under Different Flow Configuration by using Experimental And Computational Technique’, Journal published in Archive of Mechanical Engineering, 2018.
3. Patil Sateesha, ‘Design & Development of Tube in Tube Helical Coil Heat Exchanger’, journal published in Int. Res. J. of Science &amp; Engineering, 2018.
4. Naef A.A. Qasem, ‘Compact and Microchannel Heat Exchangers: A Comprehensive Review of Airside Friction Factor and Heat Transfer Correlations’, published in Science Direct -Energy Conversion and Management.
5. Muhammad Awa, ‘Heat and mass transfer for compact heat exchanger (CHXs) design’, journal published in Science Direct - International Journal of Heat and Mass Transfer.
6. Ammar Ali Abd, ‘Performance analysis of shell and tube heat exchanger: Parametric study’, journal Published in Science Direct - Case Studies in Thermal Engineering.