Spin-dependent transport through helical Aharonov-Bohm interferometer

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Abstract. We discuss spin-dependent transport via tunneling Aharonov-Bohm interferometer formed by helical edge states tunnel-coupled to helical leads. We focus on the experimentally relevant high-temperature case as compared to the level spacing and obtain the full 4×4 matrix of transmission coefficients in the presence of magnetic impurities. We show that spin conserving and spin-flip transmission coefficients of the setup can be effectively tuned by the magnetic flux. These features are attractive due to possible applications for spintronics, magnetic field detection, and quantum computing.

1. Introduction

The physics of topological insulators is one of the recently emerged, but already very actively studied areas of condensed matter physics [1, 2]. In these materials, as in conventional insulators, the Fermi level lies in the band gap, so that there is no bulk conductivity. At the same time, conducting states exist on the surface of a topological insulator, and the conduction through these states is not suppressed by disorder scattering due to topological reasons. Surface excitations exist due to strong spin-orbit interaction and have chiral symmetry: the direction of the momentum is rigidly related to the spin projection. These are the so-called helical edge states (HES), in which electrons with different spins propagate in opposite directions.

In two-dimensional topological insulators, interference of electron waves propagating at the edge state along the perimeter of the system is possible. Thus, the transport through the edge states of the topological insulator is, in fact, the transport through the interferometer (see figure 1). The ability to control interference by a magnetic field due to Aharonov-Bohm effect [3] provides an additional potential for experimental and theoretical analysis [4].

Therefore, HES are candidates for efficient creation, control, and transfer of electron polarization. It is very promising from the point of view of the application in quantum spin-sensitive interferometry, especially in the modern and rapidly developing field of spintronics, as well as for the creation of quantum informatics devices using spin degrees of freedom, such as spin filters and quantum qubits.

In this paper, we focus on an experimentally relevant case, when temperature exceeds the level spacing, \( T \gg \Delta \). Even for such high \( T \), the Aharonov-Bohm effect is not entirely suppressed and reveals itself in sharp antiresonances in the dependence of conductance, \( G \), on the magnetic flux, \( \Phi \) [5]. Current progress in the research of helical Aharonov-Bohm interferometer is given
in recent review [6]. Here, we generalize previous studies and consider all possible ways of electrons transmission through interferometer including backscattering. Such generalization allows to study case of helical leads instead of metallic leads (see figure 3).

2. Model

We consider a two-dimensional topological insulator and assume that there are HES on its edge (see figure 1). The length of the upper (lower) shoulder is \(L_1\) (\(L_2\)) and the full length of the edge is \(L = L_1 + L_2\). HES are coupled to metallic leads by tunneling point contacts. Since time-reversal symmetry protects HES from backscattering on non-magnetic impurities than scattering matrix, \(S_{pc}\), which describes the contacts, has a block-diagonal form:

\[
S_{pc} = \begin{pmatrix} S_{pc}^{(2)} & 0 \\ 0 & S_{pc}^{(2)} \end{pmatrix}, \quad S_{pc}^{(2)} = \begin{pmatrix} -t & r \\ r & t \end{pmatrix}, \quad r^2 + t^2 = 1, \quad t = e^{-\lambda}.
\]  

(1)

\(S_{pc}\) connects incoming states (\(\psi_\uparrow, h_\uparrow, \psi_\downarrow, h_\downarrow\)) with outgoing states (\(\psi'_\uparrow, h'_\uparrow, \psi'_\downarrow, h'_\downarrow\)) (see figure 2). \(\psi_\alpha\) denotes electron at the lead, \(h_\alpha\) denotes electron at HES. \(\lambda\) is the transparency of the contact: \(\lambda = 0\) (\(\lambda = \infty\)) corresponds to the closed (open) interferometer.

We assume that each shoulders of the interferometer may contain one magnetic impurity (MI) of arbitrary strength. The internal dynamics of MI is neglected. Scattering on such MI is described by the matrix

\[
S_M = \begin{pmatrix} e^{i\xi} \cos \theta & i \sin \theta e^{i\varphi} \\ i \sin \theta e^{-i\varphi} & e^{-i\xi} \cos \theta \end{pmatrix}.
\]  

(2)

Here, \(\theta\) is related to the MI strength: \(\theta\) is small for a weak MI and \(\theta = \pi/2\) for a strong MI – all electrons are fully reflected by it. The forward scattering phase, \(\zeta\), can be removed by appropriate shift of \(\varphi\), and we set \(\zeta = 0\) below. The corresponding transfer matrix reads

\[
M = \frac{1}{\cos \theta} \begin{pmatrix} 1 & i \sin \theta e^{i\xi} \\ -i \sin \theta e^{-i\xi} & 1 \end{pmatrix},
\]  

(3)

where \(\xi = \varphi - 2kx_0\) and \(k\) is the electron momentum.
3. Transmission coefficients

Now we can compute the scattering matrix of the whole setup, $S$, which relates incoming states $(\psi_L^+, \psi_L^-, \psi_R^+, \psi_R^-)$ and outgoing states $(\psi_L'^+, \psi_L'^-, \psi_R'^+, \psi_R'^-, \psi_R'^-, \psi_R'^-)$. The superscript “L” (“R”) stands for the electron in the left (right) contact (see figure 1):

$$
S = \left( \begin{array}{cc}
\kappa^2 & 0 \\
0 & \kappa^{-2}
\end{array} \right) + \frac{t^2}{l} \left( \begin{array}{cc}
\kappa^{-1} & 0 \\
0 & \kappa\Lambda_1
\end{array} \right) \left( \begin{array}{cc}
\kappa & 0 \\
0 & (\Lambda_1\kappa)^{-1}
\end{array} \right),
$$

$$
g = \left( \begin{array}{cc}
\kappa^2 M_2 & 0 \\
0 & \kappa^{-2}
\end{array} \right) \left( \begin{array}{cc}
D & D \\
D & D
\end{array} \right) \left( \begin{array}{cc}
\kappa^2 M_1 & 0 \\
0 & \kappa^2
\end{array} \right) \left( \begin{array}{cc}
\sigma_3 & 0 \\
0 & \sigma_3
\end{array} \right),
$$

$$
D = (1 - \kappa^2 M_1 \kappa^2 M_2)^{-1},
$$

$$
\Lambda_1 = \left( \begin{array}{cc}
e^{iKL_1} & 0 \\
e^{-iKL_1} & e^{2\pi\phi L/\kappa}
\end{array} \right) e^{2\pi\phi L/\kappa},
$$

$$
\sigma_3 = \left( \begin{array}{cc}
1 & 0 \\
0 & -1
\end{array} \right).
$$

Here $\phi = \Phi/\Phi_0 = 1/2 + n$ is the dimensionless flux, with $\Phi_0 = \hbar c/\epsilon$ the flux quantum. $M_1$ ($M_2$) is the transfer matrix for MI, equation (3), in the upper (lower) shoulder.

The matrix of transmission coefficients is defined by $Y_{ij} = \langle |S_{ij}|^2 \rangle$, where the thermal averaging, $\langle \cdots \rangle = \int d\epsilon f_F(\epsilon)$, is performed with the Fermi function $f_F(\epsilon)$. Computation of $Y_{ij}$ in an analytical way is possible in two cases: either for weak impurities in both shoulders or for one strong impurity in one of the shoulders. We focus on strong MI because the spin polarization of outgoing electrons can reach 100% in this case.

To be specific we assume MI in the upper shoulder, the matrix $g$ can be represented as follows:

$$
g = H_0 + \sum_{\alpha = \pm} \frac{1 + \alpha \hat{H}}{1 - t^2 e^{i(kL + \alpha2\pi\phi_0)}},
$$

$$
H_0 = \left( \begin{array}{cc}
-1 & i \sin \theta e^{i\xi} - \cos \theta & 0 \\
0 & 0 & 0 \\
0 & -\cos \theta & \sin \theta e^{-i\xi}
\end{array} \right),
$$

$$
H = \left( \begin{array}{cc}
a_2 & b_2 e^{i\xi} & a_1 & b_1 e^{i\xi} \\
b_2 e^{-i\xi} & -a_2 & b_1 e^{i\xi} & -a_1 \\
a_1 & b_1 e^{i\xi} & a_2 & b_2 e^{i\xi} \\
b_1 e^{-i\xi} & -a_1 & b_2 e^{-i\xi} & -a_2
\end{array} \right),
$$

$$
a_1 = i(e^{-2\pi\phi_0} - \cos(2\pi\phi_0) \cos \theta) / \sin(2\pi\phi_0),
b_1 = e^{-2\pi\phi_0} \sin \theta / \sin(2\pi\phi_0),
a_2 = i(e^{-2\pi\phi_0} \cos \theta - \cos(2\pi\phi_0)) / \sin(2\pi\phi_0),
b_2 = \sin \theta \cot(2\pi\phi_0),
$$

where $\phi_0$ obeys $\cos(2\pi\phi_0) = \cos \theta \cos(2\pi\phi)$. The elements of matrix $H$ obey $a_1^2 + b_1^2 = \cos^2 \theta$, $a_2^2 + b_2^2 = \cos^2 \theta$ and depend on the strength of the impurity and the magnetic flux only, while the dependence on the energy is encoded in the exponents $e^{\pm i\xi}$. In $Y_{ij}$ these exponents are absent. Thus, thermal averaging acts only on the following combinations:

$$
\left\langle \frac{1}{1 - t^2 e^{i(kL + \alpha2\pi\phi_0)}} \right\rangle = 1,
$$

$$
\left\langle \frac{1}{1 - t^2 e^{\pm i(kL + \alpha2\pi\phi_0)}} \right\rangle = 1.
$$

Now one is in a position to compute the matrix of transmission coefficients.

The metallic lead allows to study only unpolarized beams of incoming electrons. For example, the transmission coefficient for electrons with spin up and down through the interferometer from the left lead to the right are defined as $T_\uparrow = Y_{31} + Y_{41}$ and $T_\downarrow = Y_{32} + Y_{42}$. Then the transmission coefficient for the whole setup is $T = (T_\uparrow + T_\downarrow)/2$. The spin transmission coefficient is defined as $T_s = (T_\uparrow - T_\downarrow)/2$. 


However, the matrix \( Y_{ij} \) contains information about the contributions of different spins separately. Thus, we can consider the cases of ferromagnetic leads or the situation when HES form leads themselves (see figure 3). In the latter case one has to demand that electrons with opposite spin propagate in different directions. For example, lead “1” consist of incoming state \( \psi_{L,\uparrow} \) and outgoing state \( \psi'_{L,\downarrow} \). In such a basis the matrix of transmission coefficients has to be rotated \( \bar{Y} = R_1 Y R_2 \), where

\[
R_1 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{pmatrix}, \quad R_2 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix}.
\]

(7)

**Figure 3.** Helical Aharonov-Bohm interferometer with leads formed by helical states.

Let us discuss electron transport. For weak MIS the transmission of electrons from the left leads, 1 and 4, to the right leads, 2 and 3, depends on the transparency of the contacts, \( \lambda \). For small \( \lambda \) the transmission is small, for large \( \lambda \) the transmission tends to unity. However, for strong MI, \( \theta = \pi/2 \), the transmission in one of the interferometer shoulders is blocked. In this case for MI in the upper (lower) shoulder the transmission from the lead 1 (4) to the right lead is absent. In the intermediate region of parameters \( \lambda \) and \( \theta \), the spin-flip processes have significant contribution into the transmission as shown in figure 4. Incoming electrons from lead 1 have spin “up”, outgoing electrons in lead 2 have the same spin, whereas in lead 3 electrons have spin “down”. Accordingly, \( \bar{Y}_{21} \) describes spin conserving processes, \( \bar{Y}_{31} \) describes spin-flip process. It is seen, that these transmission coefficients may be of the same order. Besides, fine-tuning by magnetic flux is possible to make one of the coefficients larger or lower than the other. Transmission coefficients for electron emitted from lead 4 are shown in figure 5. This electron has spin “down” and \( \bar{Y}_{23} \) describes spin conserving processes, whereas \( \bar{Y}_{24} \) describes the spin-flip process.

Our system can be characterized by the matrix of conductances, \( G_{ij} \) as \( I_i = G_{ij} V_j \). Here, \( I_i \) is the current flowing in the channel \( i \) and \( V_j \) is the voltage applied to the channel \( j \). The Kubo formula relates \( G_{ij} \) with \( Y_{ij} \), namely \( G_{ij} = (\delta_{ij} - \bar{Y}_{ij})/2 \). Kirchhoff’s rules result in constraints on \( G_{ij} \) such that \( \sum_i G_{ij} = \sum_j G_{ji} = 0 \). Consequently, it is possible to define more convenient combinations of \( I_i \) and \( V_j \):

\[
\begin{align*}
I_R &= (I_1 - I_2 - I_3 + I_4)/2, \quad V_R = (V_1 - V_2 - V_3 + V_4)/2, \\
I_D &= (I_1 + I_2 - I_3 - I_4)/2, \quad V_D = (V_1 + V_2 - V_3 - V_4)/2, \\
I_S &= (I_1 - I_2 + I_3 - I_4)/2, \quad V_S = (V_1 - V_2 + V_3 - V_4)/2, \\
I_{\Sigma} &= \sum_j I_j / 2, \quad V_{\Sigma} = \sum_j V_j / 2.
\end{align*}
\]

(8)

Physically, such combinations have the following meanings: \( I_R \) is the charge current flowing to the right, \( I_D \) is the charge current flowing to the down and \( I_S \) is the spin current. In this new
basis, the matrix of conductances has a simple form

\[
G = R_0^T G R_0 = \begin{pmatrix}
T & 0 & T_s & 0 \\
0 & 1 - \tanh \lambda & 0 & 0 \\
T_s & 0 & 1 + T_s/\tanh \lambda & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad R_0 = \frac{1}{2} \begin{pmatrix}
1 & 1 & 1 & 1 \\
-1 & 1 & -1 & 1 \\
-1 & -1 & 1 & 1 \\
1 & 1 &-1 & 1
\end{pmatrix}, \quad (9)
\]

\[
T = \tanh \lambda \left(1 - \frac{\sin^2 \theta \sin^2 \lambda \cosh(2\lambda)}{\cosh^2(2\lambda) - \cos^2 \theta \cos^2(2\pi \phi)}\right), \quad T_s = -\frac{\sin^2 \theta \sinh^2 \lambda \cosh(2\lambda)}{\cosh^2(2\lambda) - \cos^2 \theta \cos^2(2\pi \phi)}.
\]

4. Conclusion
We have studied Aharonov-Bohm interferometer formed by helical edge states of 2D topological insulators and tunnel coupled to helical leads (see figure 3). We calculated the full $S$-matrix of the interferometer as a whole, including both transmission and reflection processes. In high temperature regime, $T \gg \Delta$, the transmission coefficients, $Y_{ij}$ are oscillatory functions of dimensionless magnetic flux, $\phi$, with the period 1/2. $Y_{ij}$ additionally depend only on the transparency of the contact, $\lambda$, and the strength of magnetic impurity, $\theta$. Other details, such as the geometry of the device, the relation between $L_1$ and $L_2$, the position of the magnetic impurities or the Berry phase, do not affect $Y_{ij}$. Manipulating by the magnetic flux allows one to tune spin conserving and spin-flip transmission coefficients. For example, it is possible to make one of the coefficients dominant. Our study thus indicates possible directions for expansion of quantum spin-sensitive interferometry research.

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