Study on Slip Effects of Simply-supported Composite Beams

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Abstract. The condition of the rigid shear connector of composite beams hardly exists in practice and interface slip is common due to the elastic shear connector. However, the rigid connector is often assumed in the design. What relationships for slip-induced effects? Is it small enough to be neglected? The classical differential equation is used to calculate slip effects exactly. Results show that the slip-induced deflection is above 20% and the risk of crack must be considered for the bottom of concrete slab in midspan.

1. Introduction

Early in 1951, Newmark derived the differential equation on the interface slip of composite beams [1], and plenty of research has been carried out in the analytical method [2-10]. Because many influencing factors are involved in the calculation, it is difficult to obtain a general and simple formula for an arbitrary load. In order to reveal the relationships among slip effects, this paper uses the differential equation and its exact solutions to analyse a simply-supported composite beam subjected to a uniformly distributed load.

2. Derivation of the differential equation

The benefit of composite beam lies in the composite action between the two individual parts (concrete slab and steel beam). Large ratio of the bending moment ($M$) is shared by the internal composite axial force ($N_i$), illustrated by Equation 1.

$$M = (E_c I_c + E_s I_s) \kappa + N_i d \quad \Rightarrow \quad \kappa = \frac{M - N_i d}{E_c I_c + E_s I_s}$$

(1)

where $E$ and $I$ are elastic modular and the second moment of area respectively, the subscribe $c$ and $s$ denote concrete slab and steel beam respectively, $\kappa$ is curvature and $d$ is the distance between the center of gravity of the two parts.

For the rigid shear connector, the composite action reaches its maximum. It will decrease as the rigid shear stiffness becomes elastic. The elastic shear connector normally exists in practical structures. The reduction of the composite action is related with many factors like load type, shear stiffness of interface ($k$), material and geometrical properties.

In order to reveal the influence, the axial force $N_i$ must be known. As shown in Figure 1 $N_i$, shear flow ($p$) and interface slip ($s$) have the following equilibrium and constitutive relationship.

$$\begin{cases}
    p dx = -dN_i \\
    p = ks
\end{cases}$$

(2)
which shows the first order differential relation for the shear flow and the axial force. For sign convention the axial force and strain are positive in tension, bending moment and curvature are positive in a concave beam. The two self-equilibrium internal axial force are expressed by $N_i$ and $-N_i$.

$$\varepsilon_{\text{slip}} = s' = \kappa d - \varepsilon_c + \varepsilon_s \tag{3}$$

where $\varepsilon_c$ and $\varepsilon_s$ are strains of the center of gravity of the two parts.

With Equation 1, 2 and 3, two equivalent differential equations can be obtained (Equation 4), in which slip and axial force are independent variables respectively.

$$\begin{align*}
N_i'' - \omega^2 k N_i &= -\frac{kd}{E_c I_c + E_s I_s} M \\
s'' - \omega^2 s &= \frac{d^2}{E_c I_c + E_s I_s} M', \\
\omega^2 &= \frac{d^2}{E_c I_c + E_s I_s} + \frac{1}{E_c A_c} + \frac{1}{E_s A_s}
\end{align*} \tag{4}$$

To be noticed, $\omega$ has no relationship with the load and only relates with material, size and shape of cross section, and the dimensionless form of $\omega$ is $\omega l$. Bending moment $M$ is dependent on load type, therefore the analytical solutions of Equation 4 are different under different loading conditions. Giving a specific load, the exact solutions of Equation 4 can be obtained without mathematic difficulties.

**3. Example**

Condition of uniformly distributed load is common in practice. An example (Table 1) is considered to analyse its slip effects including changes of deflection, axial strain, axial force and shear flow.

| Concrete slab | Steel beam | Shear connector |
|---------------|------------|----------------|
| $E_c=28\text{Gpa}$ | $E_s=210\text{Gpa}$ | Single row |
| Size: $133\text{cm}\times 10\text{cm}$ | I shape, height: $30\text{cm}$ | $k=67000/193$ |
| $A_c=1330\text{cm}^2$ | $A_s=67.25\text{cm}^2$ | $d=20\text{cm}$ |
| $I_c=11080\text{cm}^4$ | $I_s=9400\text{cm}^4$ | $\omega=0.973\text{m}^{-1}$ |

In the case of no shear connector, the whole flexural stiffness is the direct superposition of the two parts and reaches its minimum, the deflection and interface slip are maximum and internal axial force is zero (Figure 2). In this case the beam can not be named as composite beam.

With rigid shear connector, the whole stiffness increases significantly, and therefore the deflection decreases to 36% of the case of no connector in this example (Figure 2). Great economy can be obtained by means of composite action. However rigid shear connector is the case of idea condition. In practice, the shear connector will produce shear deformation and slip will produce on the interface, which is the case of the elastic shear connector.
To be noticed, here the critical section of crack is midspan rather than support section. Therefore, the effective flexural stiffness is not constant along the beam, shown in Figure 3.

Figure 2 shows that the deflection of the elastic connector increases to 1.25 times of the rigid case, which means the stiffness reduction is about 0.8. In the midspan, the strain of bottom slab tends to tension because of slip although the strain of slip is small (0.18‰), which increase the risk of crack.

Figure 3. Slip effects

The investigation of the composite action is carried out to be seen the magnitude of composite action along the beam, shown in Figure 2. The biggest slip strain occurs at the midspan, which means the stiffness reduction is (−0.35,−0.27,−0.26) and the slip strain is (−1.05,−0.14,−0.01). The internal axial force in midspan (0.74,0.78,1.05) although the strain of slip is small (0.18‰).

Throughout the beam, the shear flow (−0.35,−0.27,−0.26). The slip strain of midspan section is 0.18‰.
4. Conclusions
Based on the differential equation of cross-section, slip effects are obtained exactly in this paper. The influence of interface slip on the deflection of beam is above 20%. It should be considered for beams that is susceptible to the deflection, like large span bridge, while generally interface slip is neglected in the design and rigid shear connector is assumed.

In addition, the slip-induced crack risk lies in the bottom slab of midspan section that is contrary to the action of bending moment. The slip-induced reduction of internal axial force is about 15%.

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