The Hierarchy of Hyperlogics: A Knowledge Reasoning Perspective

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Abstract
We discuss the hierarchy of hyperlogics from a knowledge reasoning perspective. Hyperproperties generalize trace properties by relating multiple traces. Recently, logics for hyperproperties have been obtained from standard logics by adding variables for traces or paths to temporal logics like LTL and CTL*, and by adding the equal-level predicate to first-order and second-order logics, like monadic first-order logic of order and MSO. The resulting hierarchy of hyperlogics provides interesting opportunities for knowledge reasoning research: many epistemic properties and system properties in multi-agent systems, like distributivity, are hyperproperties. At the same time, first-order and second-order reasoning methods become applicable to hyperproperties.

Hyperproperties (Clarkson and Schneider 2010) are system properties that relate multiple execution traces to each other. Hyperproperties play an important role in security policies, such as information flow control (Goguen and Meseguer 1982), call integrity in smart contracts (Grishchenko, Maifei, and Schneidewind 2018), and secrecy in web-based workflows (Finkbeiner et al. 2017), as well as, beyond security, in areas such as fault-tolerance (Nguyen et al. 2017). As demonstrated by high-profile side-channel attacks like Meltdown and Spectre, hyperproperties have significant practical relevance. There is a lot of recent research on model checking (Finkbeiner, Hahn, and Torfah 2018; Coenen et al. 2019b), synthesis (Finkbeiner et al. 2020a; Finkbeiner et al. 2020b), monitoring (Hahn 2019; Finkbeiner et al. 2019), and program repair (Bonakdarpour and Finkbeiner 2019) from hyperproperties.

The standard logics used in the areas of verification, knowledge representation, and reasoning, like linear-time temporal logic (LTL), computation tree logic (CTL), and monadic first-order logic, cannot express hyperproperties. However, these logics have recently been extended to hyperproperties with two principal extensions: the addition of variables for traces or paths to a temporal logic, and the extension of monadic first-order and second-order logics with a relational predicate.

While the standard temporal logics only refer to a single trace or path at a time, the temporal hyperlogics HyperLTL and HyperCTL* quantify over multiple traces (or paths) and relate them using a temporal formula. For example, the HyperLTL formula

$$\forall \pi, \forall \pi'. \Box \bigwedge_{a \in AP} a_\pi \leftrightarrow a_{\pi'}$$

expresses that all pairs of traces must at all times agree on the values of the atomic propositions given as the set $AP$.

The second extension adds the equal-level predicate $E$ (Thomas 2009) to monadic first-order logic of order (FO[$<$]). The equal-level predicate relates points that happen at the same time. The HyperLTL formula (1) is equivalent to the FO[$<$, $E$] formula

$$\forall x, \forall y. E(x, y) \rightarrow \bigwedge_{a \in AP} (P_a(x) \leftrightarrow P_a(y))$$

A natural question is how the two extensions compare in terms of expressiveness. Kamp’s theorem (Kamp 1968) states (in the formulation of Gabbay et al. (Gabbay et al. 1980)) that LTL is expressively equivalent to FO[$<$]. However, the potential analogue of Kamp’s theorem for hyperlogics, that HyperLTL might be equivalent to FO[$<$, $E$], is not true (Finkbeiner and Zimmermann 2017).

In the paper “The Hierarchy of Hyperlogics” (Coenen et al. 2019a), we initiated an extensive study of the expressiveness of temporal and first-order/second-order hyperlogics. The standard hierarchy of linear-time and branching-time logics is shown in Figures 1a and 1b, respectively. Each temporal logic corresponds to an equally expressive FO/SO logic, e.g., LTL to FO[$<$], and QPTL to S1S, where the latter two are both able to express all $\omega$-regular languages.

The hierarchy of hyperlogics is shown in Figures 1c and 1d. The picture differs significantly from the standard hierarchy: for almost every pair of a temporal logic and a FO/SO logic that are expressively equivalent in the standard hierarchy, the corresponding FO/SO logic is more expressive in the hierarchy of hyperlogics. The only exception is MSO[$E$] and HyperQCTL*, which are also equivalent in the hierarchy of hyperlogics. The reason is that HyperQCTL* allows for quantification over

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The discovery of new decidable fragments of the hyperlogics. Procedures for interesting classes of hyperproperties and to techniques to FO hyperlogics could lead to new decision yet been explored for hyperproperties. Generalizing these ample, have been studied exhaustively for FO logics, but not hyperproperties. Proof systems and tableau methods, for ex-


tional observation and fault tolerance (Finkbeiner et al. 2020b). Hyperlogics can thus not only express functional correctness and information flow policies, but also describe other. Which outputs may depend on which inputs can be from the environment and also share information with each security models. In Int. Conf. on Security and Trust.

Another research opportunity afforded by the hierarchy of hyperlogics is that first-order and second-order reasoning methods become applicable to hyperproperties. There is a rich body of FO/\(SO\) reasoning methods, many of which developed for knowledge reasoning, which could be adapted to hyperproperties. Proof systems and tableau methods, for example, have been studied exhaustively for FO logics, but not yet been explored for hyperproperties. Generalizing these techniques to FO hyperlogics could lead to new decision procedures for interesting classes of hyperproperties and to the discovery of new decidable fragments of the hyperlogics. Atomic propositions, which, on trees, corresponds to full second-order quantification.

The hierarchy of hyperlogics provides interesting opportunities for knowledge reasoning research. The hierarchy can provide a frame of reference for the study of epistemic logics, which reason about the knowledge of agents in multi-agent systems. Consider for example the property “Agent A, who can observe the value of variable \(a\), never knows the value of variable \(b\)". To express such a property, one needs to compare computation traces that are equivalent for agent A, it is, thus, a hyperproperty. A common epistemic specification language is LTL extended with the knowledge modality under perfect recall semantics (LTL\(_K\)). In terms of expressiveness, LTL\(_K\) is incomparable to HyperLTL (Bozzelli, Maubert, and Pinchinat 2015). LTL\(_K\) is, however, strictly subsumed by HyperQCTL (Rabe 2016), which can express all \(\omega\)-regular properties. The exact position of LTL\(_K\) in the hierarchy of hyperlogics is an open question.

Reasoning about knowledge is especially important in settings where information is distributed over multiple components of a system. In distributed architectures, for example, it is assumed that several processes receive different inputs from the environment and also share information with each other. Which outputs may depend on which inputs can be formalized as a hyperproperty in HyperLTL (Finkbeiner et al. 2020b). Hyperlogics can thus not only express functional correctness and information flow policies, but also describe the structure of the system itself. Similar properties are partial observation and fault tolerance (Finkbeiner et al. 2020b).

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Figure 1: The linear-time hierarchy of standard logics (a): LTL and its extension with quantification over atomic proposition, QPTL, and their counterparts FO[<] and the monadic second-order logic of one successor (S1S). The branching-time hierarchy of standard logics (b): CTL* and its extension with quantification over atomic propositions, QCTL*, and monadic path logic (MPL) and monadic second-order logic (MSO). The hierarchies of the corresponding hyperlogics (c and d).

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