New constraints on $H_0$ and $\Omega_m$ from SZE/X-RAY data and Baryon Acoustic Oscillations

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Abstract

The Hubble constant, $H_0$, sets the scale of the size and age of the Universe and its determination from independent methods is still worthwhile to be investigated. In this article, by using the Sunyaev-Zel’dovich effect and X-ray surface brightness data from 38 galaxy clusters observed by Bonamente et al. (2006), we obtain a new estimate of $H_0$ in the context of a flat $\Lambda$CDM model. There is a degeneracy on the mass density parameter ($\Omega_m$) which is broken by applying a joint analysis involving the baryon acoustic oscillations (BAO) as given by Sloan Digital Sky Survey (SDSS). This happens because the BAO signature does not depend on $H_0$. Our basic finding is that a joint analysis involving these tests yield $H_0 = 0.765^{+0.035}_{-0.033}$ km s$^{-1}$ Mpc$^{-1}$ and $\Omega_m = 0.27^{+0.03}_{-0.02}$. Since the hypothesis of spherical geometry assumed by Bonamente et al. is questionable, we have also compared the above results to a recent work where a sample of triaxial galaxy clusters has been considered.
1 Introduction

Ten years ago, two different teams using SNe type Ia as standard candles announced that the expansion of the Universe is speeding up and not slowing as believed by the cosmology community [1]. The authors interpreted their results as implying that the cosmological constant \( \Lambda \) is greater than zero, an assumption that is not only surviving but has been strengthened in the last decade by many independent astronomical observations [2, 3, 4, 5, 6, 7] (see also Peebles and Ratra [8], and Padmanabhan [9] for reviews).

On the other hand, until now there is no consensus about the present value of the Hubble parameter even when the estimates of \( H_0 \) are limited to the Cepheid period-luminosity relation and the correlation of the luminosity at maximum light for SNe type Ia. For instance, Riess et al. [10] using a hybrid sample composed by Cepheids in galaxies and recent type Ia SNe concluded that \( H_0 = 73 \pm 6 \text{ km.s}^{-1}.\text{Mpc}^{-1} \) while Sandage [11] and coworkers based on data from the Hubble Space Telescope (HST) key project are claiming that \( H_0 = 62 \pm 5 \text{ km.s}^{-1}.\text{Mpc}^{-1} \). Therefore, is still worthwhile to improve the direct local estimates of \( H_0 \), and, probably, more important, to investigate their concordance with independent methods. An interesting one is provided by the combination of Sunyaev-Zel’dovich Effect (SZE) and measurements of X-ray surface brightness from galaxy clusters.

The so-called SZE is a small distortion on the Cosmic Microwave Background (CMB) spectrum provoked by the inverse Compton scattering of the CMB photons passing through a population of hot electrons [12, 13]. Observing the temperature decrement of the CMB in the direction of galaxy clusters and the X-ray spectrum emitted by the hot electron gas pervading the cluster, it is possible to break the degeneracy between concentration and temperature [14], and, therefore, to calculate the angular diameter distance (ADD) [15, 16, 17]. This technique for measuring distances is completely independent of other methods (as the one provided by the luminosity distance), and it can be used to measure distances at high redshifts directly. Review
papers on this subject have been published by Birkinshaw[18] and Carlstrom, Holder and Reese[19]. More recently, such a technique has been applied for a fairly large number of clusters[20, 21, 22, 23, 24].

In 2006, Bonamente and collaborators[23] determined the ADD distance to 38 clusters of galaxies in the redshift range $0.14 \leq z \leq 0.89$ using X-ray data from Chandra and Sunyaev-Zeldovich effect data from the Owens Valley Radio Observatory and the Berkeley-Illinois-Maryland Association interferometric arrays. Assuming spherical symmetry, the cluster plasma and dark matter distributions were analyzed by using a hydrostatic equilibrium model accounting for radial variations in density, temperature and abundances. In addition, by fixing the earlier cosmic concordance ΛCDM model ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$), they obtained for the Hubble constant $H_0 = 76.9^{+3.5 +10.0}_{-3.4 -8.0}\: km.s^{-1}.Mpc^{-1}$ (statistical followed by systematic uncertainty at 1σ c. 1.). The main advantage of this method for estimating $H_0$ is that it does not rely on the extragalactic distance ladder being fully independent of any local calibrator. In a point of fact, once the angular diameter distance has been measured a large amount of astrophysical and cosmological tests can be performed in order to constrain the relevant free parameters[25, 26, 27].

It should be stressed, however, that in the above quoted determination of the Hubble constant, a specific cosmology was fixed from the very beginning. This is needed because the determination of the Hubble parameter from SZE/X-ray technique is endowed with a strong degeneracy with respect to the matter density parameter which can only be broken by considering additional cosmological tests. An interesting possibility is given by the signature of the baryon acoustic oscillations (BAO) from the last scattering surface (LSS).

In 2005, Eisenstein et al.[28] presented the large scale correlation function from the Sloan Digital Sky Survey (SDSS) showing clear evidence for the baryon acoustic peak at $100h^{-1}\: Mpc$ scale, a result in good agreement with the analyses from CMB data[3]. The breaking on the degeneracy between $H_0$ and $\Omega_m$ by applying this BAO
signature is possible because it does not depend on $H_0$ and is highly sensitive to the matter density parameter. The combination SZE/X-ray with BAO has been recently discussed in the literature by Cunha, Marassi and Lima[29].

In this article, we discuss the determinations of $H_0$ and $\Omega_m$ by considering the same 38 clusters from Bonamente et al. sample where spherical symmetry was assumed. Our basic findings follow from a joint analysis involving the data from SZE and X-ray surface brightness with the recent SDSS measurements of the baryon acoustic peak. The influence of the intrinsic cluster geometry on $H_0$ it will be quantified by comparing the results derived here with a recent determination of both parameters[29] based on De Filippis et al. sample formed by 25 triaxial clusters[22].

2 Basic Equation and Statistical Analysis

Let us now consider that the Universe is described by a flat Friedmann-Robertson-Walker (FRW) geometry

$$ds^2 = dt^2 - a^2(t) \left( dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi) \right),$$  \hspace{1cm} (1)

driven by cold dark matter plus a cosmological constant ($\Lambda$CDM).

In this background, the angular diameter distance, $D_A$, can be written as[29, 30, 31]

$$D_A(z; h, \Omega_m) = \frac{3000h^{-1}}{(1+z)} \int_0^z \frac{dz'}{\mathcal{H}(z'; \Omega_m)} \text{ Mpc},$$  \hspace{1cm} (2)

where $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$ and the dimensionless function $\mathcal{H}(z'; \Omega_m)$ is given by $\mathcal{H} = [\Omega_m(1+z')^3 + (1 - \Omega_m)]^{1/2}$. As it appears, the above expression has only two free parameters ($h, \Omega_m$). In this way, we perform a statistical fit over the $h - \Omega_m$ plane in light of Bonamente et al. data sample[23]. In our analysis we use a maximum likelihood that can be determined by a $\chi^2$ statistics,

$$\chi^2(z|p) = \sum_i \frac{(D_A(z_i; p) - D_{Ao,i})^2}{\sigma_{D_{Ao,i}}^2 + \sigma_{stat}^2},$$  \hspace{1cm} (3)
where $D_{Ao,i}$ is the observational ADD, $\sigma_{D_{Ao,i}}$ is the uncertainty in the individual distance, $\sigma_{\text{stat}}$ is the contribution of the statistical errors (see table 3 in Bonamente et al. [23] (2006)) added in quadrature ($\approx 20\%$) and the complete set of parameters is given by $\mathbf{p} \equiv (h, \Omega_m)$.

In what follows, we first consider the SZE/X-ray distances separately, and, further, we present a joint analysis including the BAO signature from the SDSS catalog. Note that a specific flat $\Lambda$CDM cosmology has not been fixed a priori in the analysis below.

![Figure 1](image.png)

**Figure 1:** a) Angular diameter distance as a function of redshift for $\Omega_m = 0.3$, $\Omega_A = 0.7$ and some selected values of the $h$ parameter. The data points correspond to the the SZE/X-ray distances for 38 clusters from Bonamente et al. sample. b) Confidence regions (68.3%, 95.4% and 99.7%) in the ($\Omega_m, h$) plane provided by the SZE/X-ray data. The best fit values are $h = 0.743$ and $\Omega_m = 0.37$. As remarked in the text, the possible values of $H_0$ are heavily dependent on the allowed values of $\Omega_m$, and, therefore, such a degeneracy need to be broken by adding a new cosmological test.

### 2.1 Limits from SZE/X-ray

To begin with, let us consider the influence of the Hubble parameter on the ADD once the cosmology is fixed.
In Fig. 1(a), we display the residual Hubble diagram and the galaxy cluster data sample by considering a flat cosmic concordance model ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$). As expected, for a given redshift, the distances increase for smaller values of $H_0$.

In Fig. 1(b) we show the contours of constant likelihood (68.3%, 95.4% and 99.7%) in the space parameter $h - \Omega_m$ for the SZ/X-ray data above discussed. Note that a large range for the $h$ parameter is allowed, $(0.65 \leq h \leq 0.86)$, at 1$\sigma$ of confidence level. In particular, we found $h = 0.755^{+0.065}_{-0.065}$ with $\chi^2_{min} = 35.2$ at 68.3% confidence level (c.l.). Naturally, such bounds on $h$ are reasonably dependent on the cosmological model adopted. For example, if we fix $\Omega_m = 0.3$ we have $h = 0.76$, for $\Omega_m = 1.0$ we have $h = 0.65$, and both cases are permitted with high degree of confidence. This means that using only the Bonamente et al. sample (spherical cluster hypothesis) we cannot constrain severely the energetic components of the $\Lambda$CDM model with basis on the SZ/E/X-ray data alone. As one may conclude, one additional cosmological test (fixing $\Omega_m$) is necessary in order to break the degeneracy on the $(\Omega_m, h)$ plane.

2.2 Joint analysis for SZ/E/X-ray and BAO

As remarked in the introduction, more stringent constraints on the space parameter $(h, \Omega_m)$ can be obtained by combining the SZ/E/X-ray with the BAO signature[28]. The peak detected (from a sample of 46748 luminous red galaxies selected from the SDSS Main Sample) is predicted to arise precisely at the measured scale of $100 \, h^{-1}$ Mpc. Let us now consider it as an additional cosmological test over the spherical cluster sample. Such a measurement is characterized by

$$\mathcal{A} \equiv \frac{\Omega_m^{1/2}}{\mathcal{H}(z_*)^{1/3}} \left[ \frac{1}{z_*} \Gamma(z_*) \right]^{2/3} = 0.469 \pm 0.017,$$

(4)

where $z_* = 0.35$ is the redshift at which the acoustic scale has been measured and $\Gamma(z_*)$ is the dimensionless comoving distance to $z_*$. Note that the above quantity is independent of the Hubble constant, and, as such, the BAO signature alone constrains
Figure 2: a) Contours in the $\Omega_m - h$ plane using the SZE/X-ray and BAO joint analysis. The contours correspond to 68.3%, 95.4% and 99.7% confidence levels. The best-fit model converges to $h = 0.765$ and $\Omega_m = 0.27$. b) Likelihood function for the $h$ parameter in a flat $\Lambda$CDM universe from SZE/X-ray emission and BAO. The horizontal lines are cuts in the regions of 68.3% probability and 95.4%.

In Fig. 2(a), we show the confidence regions for the SZE/X-ray cluster distance and BAO joint analysis. By comparing with Fig. 1(b), one may see how the BAO signature breaks the degeneracy in the $(\Omega_m, h)$ plane. As it appears, the BAO test presents a striking orthogonality centered at $\Omega_m = 0.274^{+0.036}_{-0.026}$ with respect to the angular diameter distance data as determined from SZE/X-ray processes. We find $h = 0.765^{+0.035}_{-0.033}$ at 68.3% c.l. and $\Omega_m = 0.273^{+0.03}_{-0.02}$ at 68.3% c.l. for 1 free parameter ($\chi^2_{\min} = 35.3$). In light of these results, the important lesson here is that the combination of SZE/X-ray with BAO provides an interesting approach to constrain the Hubble constant.

In Fig. 2(b), we have plotted the likelihood function for the $h$ parameter in a flat $\Lambda$CDM universe for the SZE/X-ray + BAO data set. The dotted lines are cuts in the regions of 68.3% probability and 95.4% (1 free parameter).

At this point, it is interesting to compare our results with others recent works.
In table 1, a list of recent Hubble parameter determinations based on cluster data is displayed. Note that in the first five works the $h$ values were obtained by fixing the cosmology ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$), while in our estimate the SZE/X-ray + BAO technique was used (no one specific cosmology has been fixed).

On the other hand, the importance of the intrinsic geometry of the cluster has been emphasized by many authors\cite{32,33,34,35}. As a consequence, the standard spherical geometry has been severely questioned, since Chandra and XMM-Newton observations have shown that clusters usually exhibit an elliptical surface brightness. In this concern, a previous determination of $H_0$ from SZE/X-ray + BAO by Cunha et al.\cite{29} was based in a smaller sample (25 triaxial clusters) observed by De Fillipis and collaborators\cite{22}. Their results suggested that 15 clusters are in fact more elongated along the line of sight, while the remaining 10 clusters are compressed. Actually, the assumed cluster shape seems to affect considerably the SZE/X-ray distances, and, therefore, the $H_0$ estimates.

As shown in table 1, the central value of $H_0$ in this case (triaxial geometry) is exactly the same determined by Riess et al.\cite{10} by using a hybrid sample (Cepheids + Supernovae). It is also closer to the Hubble Space Telescope (HST) key project determination of the Hubble parameter announced by Friedman et al.\cite{36} based only on Cepheids as distance calibrators.

### 3 Comments and Conclusions

In this paper we have discussed a new determination of the Hubble constant based on the SZE/X-ray distance technique for a sample of 38 clusters as compiled by Bonamente et al. assuming spherical symmetry\cite{23}. The degeneracy on the $\Omega_m$ parameter was broken trough a joint analysis applying the baryon acoustic oscillation signature from the SDSS catalog. The Hubble constant was constrained to be $h = 0.765^{+0.035}_{-0.033}$ for $1\sigma$ and $\Omega_m = 0.273^{+0.03}_{-0.02}$. These limits were derived assuming the flat $\Lambda$CDM scenario.
and a spherical $\beta$-model. The central $h$ value derived here is in perfect agreement with the Bonamente et al. value, $h = 0.769$ (assuming a hydrostatic equilibrium and fixing the concordance model), as well as with others recent estimates coming from WMAP and Hubble Space telescope Key Project, where $h = 0.73$. However, it differs slightly of a recent estimation by Cunha, Marassi & Lima, $h = 0.74$, where the SZE/X-Ray + BAO technique was applied for 24 triaxial galaxies clusters. In general ground, such results are suggesting that the combination of these three independent phenomena (SZE/X-ray and BAO) provides an interesting method to constrain the Hubble constant.

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| Reference (data)                     | $\Omega_m$ | $h \, (1\sigma)$ | $\chi^2$ |
|-------------------------------------|------------|------------------|----------|
| Mason et al. 2001 (7 clusters)      | 0.3        | 0.66$^{+0.14}_{-0.11}$ | $\simeq 2$ |
| Reese et al. 2002 (18 cluster)      | 0.3        | 0.60$^{+0.04}_{-0.04}$ | 16.5     |
| Reese 2004 (41 clusters)            | 0.3        | 0.61$^{+0.03}_{-0.03}$ | -        |
| Jones et al. 2005 (5 clusters)      | 0.3        | 0.66$^{+0.11}_{-0.10}$ | -        |
| Bonamente et al. 2006 (38 clusters) | 0.3        | 0.77$^{+0.04}_{-0.03}$ | 31.6     |
| Cunha et al. 2007 (24 triaxial clusters) + BAO. | 0.273$^{+0.03}_{-0.02}$ | 0.738$^{+0.042}_{-0.033}$ | 24.5     |
| This paper (38 clusters)+BAO.       | 0.273$^{+0.03}_{-0.02}$ | 0.765$^{+0.035}_{-0.033}$ | 35.3     |