A REVISIT OF THE PHASE-RESOLVED X-RAY AND GAMMA-RAY SPECTRA OF THE CRAB PULSAR

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ABSTRACT

We use a modified outer-gap model to study the multifrequency phase-resolved spectra of the Crab pulsar. The emissions from both poles contribute to the light curve and the phase-resolved spectra. Using the synchrotron self-Compton mechanism and by considering the incomplete conversion of curvature photons into secondary pairs, the observed phase-averaged spectrum from 100 eV to 10 GeV can be explained very well. The predicted phase-resolved spectra can match the observed data reasonably well, too. We find that the emission from the north pole mainly contributes to leading wing 1. The emissions in the remaining phases are mainly dominated by the south pole. The widening of the azimuthal extension of the outer gap explains trailing wing 2. The complicated phase-resolved spectra for the phases between the two peaks, namely, trailing wing 1, the bridge, and leading wing 2, strongly suggest that there are at least two well-separated emission regions with multiple emission mechanisms—synchrotron radiation, inverse Compton scattering, and curvature radiation. Our best-fit results indicate that there may exist some asymmetry between the south and north poles. Our model predictions can be examined with GLAST.

Subject headings: gamma rays: theory — pulsars: individual (Crab) — radiation mechanisms: nonthermal — stars: neutron — X-rays: individual (Crab)

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1. INTRODUCTION

It is generally believed that phase-resolved spectra can provide the most detailed information about the structure of pulsar magnetospheres, the acceleration mechanism, and pair creation and radiation processes in the outer magnetosphere. Romani (1996) and Daugherty & Harding (1996) first calculated the phase-resolved spectra of the Vela pulsar. Then, Cheng et al. (2000) and Zhang & Cheng (2002) calculated the phase-resolved spectra of the Crab pulsar in the γ-ray and X-ray regimes separately. We are interested in finding a general scenario that can produce both the γ-ray and the X-ray regimes in the phase-resolved spectra of the Crab pulsar. Our preliminary results were reported at the 2006 COSPAR conference (Jia et al. 2007). We have since continued to study these spectra in more detail. Here we present our recent results with a different parametric fitting (Tang 2007).

According to Cheng et al. (1986a, 1986b, hereafter collectively CHR), the outer gap starts at the null charge surface, ends at the light cylinder, is bounded below by the last closed field line, and is bounded above by a layer of electric current that replenishes charges to the open field lines outside the gap to maintain a steady charge density, the Goldreich-Julian charge density:

\[ \rho_{GJ} \sim -\frac{B \cdot \Omega}{2\pi c} \]  

(Goldreich & Julian 1969). The charge depletion within the outer gap due to global flows of charged particles causes a large electric field along the magnetic field lines, so that \( E \cdot B \neq 0 \) inside the gap. This gap thus acts as an accelerator to boost the charged particles to relativistic speeds. Through a cascade process, high-energy γ-ray photons and \( e^\pm \) pairs are produced. However, the classical outer-gap model has been challenged recently by Hirotani et al. (2003). By solving the set of Maxwell’s and Boltzmann’s equations, they found that a current at nearly the Goldreich-Julian rate can shift the position of the inner boundary of the outer gap. Therefore, we adopt a modified version of the CHR outer-gap model such that the inner boundary of the outer gap is shifted inward.

The photon emission mechanism starts with curvature radiation from the accelerated charged particles in the gap. The emission direction is tangent to the local magnetic field lines. As a photon escapes, it may encounter another, low-energy photon. This may be a thermal photon from the stellar surface or a magnetospheric soft synchrotron photon emitted by the secondary \( e^\pm \) pairs that are created by the curvature photons from the inner field lines. The primary curvature photons will then be converted into the secondary \( e^\pm \) pairs by means of photon-photon pair production, that is, \( \gamma + \gamma \rightarrow e^+ + e^- \) or \( \gamma + X \rightarrow e^+ + e^- \), inside and outside the gap. As pointed out in Cheng et al. (2000), although pair production inside an outer gap is limited to a small region, pair production outside the outer gap can cover a much wider area, because the synchrotron photons produced by the secondary pairs are more abundant than the thermal photons from the stellar surface. Although the secondary synchrotron photons...
cannot get into the outer gap, because of the curvature of the field lines, they can convert most of the primary curvature photons from the outer gap into secondary pairs. Since the synchrotron radiation is concentrated into a small angle by the relativistic beaming effect (Rybicki & Lightman 1979), synchrotron photons will also be seen as more or less tangent to the field lines if observed.

Besides curvature and synchrotron radiation, inverse Compton scattering is another important radiation mechanism in the neutron star magnetosphere. It occurs when a fast-moving electron or positron collides with a photon and net energy is transferred from the particle to the photon. In the far region of the magnetosphere, the relativistic particles collide with the soft synchrotron photons through the inverse Compton scattering process.

Chiang & Romani (1992, 1994) and Romani & Yadigaroglu (1995) calculated pulsar light curves by considering a single outer gap with the photon emission beamed to the outside alone, in order to avoid producing multiple peaks that do not occur in true observational data of the Crab pulsar. Cheng et al. (2000) pointed out the lack of a reason to explain ignoring the incoming true observational data of the Crab pulsar. Cheng et al. (2000) showed that the photons from only one gap; that is, a single-pole outer gap is considered in the canonical model. However, if the gap extends below the null charge surface, photons originating from another gap become measurable by the observer as well. In this paper, therefore, although we follow Chiang & Romani (1992, 1994) and Romani & Yadigaroglu (1995) to calculate the light curve, we consider both gaps; that is, a two-pole model is examined.

We calculate the pulse profile and the phase-resolved spectra for the Crab pulsar with an outer-gap accelerator model. Because this pulsar is one of the brightest γ-ray sources in the sky, detailed observations for the pulse profile and the phase-resolved spectra have been obtained. These provide useful information to study the nonthermal processes in the pulsar magnetosphere. Since the Crab pulsar is still young and is believed to have a thin gap, calculating the contribution from one layer suffices. By fixing an inclination angle and an observing angle, the photon emission locations that produce the light curve are used to calculate the phase-resolved photon spectra, which are compared with the observational data. For the Crab pulsar, we adopt $R = 10$ km for the stellar radius and $B_p = 3.8 \times 10^{12}$ G for the stellar magnetic field strength.

Another major feature of this paper is the relaxing of the assumption that all curvature radiation has been converted into $\gamma$-rays pairs as in previous studies. In fact, by letting a trace amount of photons through the inverse Compton scattering process.

In § 2, we briefly review the emission geometry in the magnetosphere. Then we discuss the location of the inner boundary of the outer gap and produce the light curve for the Crab pulsar. In § 3, we discuss the electric field component along the magnetic field lines and the three major emission mechanisms, namely, synchrotron radiation, inverse Compton scattering, and curvature radiation. We argue that although most of the primary curvature photons are converted into secondary pairs to produce synchrotron photons, some curvature photons emitted far from the star can survive the cascade process and escape. They contribute to the peak in the high-energy regime at several GeV in the trailing wing 1, bridge, and leading wing 2 phases. In the final section, we summarize our results and discuss briefly the justification behind our assumptions.

2. THEORETICAL LIGHT CURVES OF NEUTRON STARS WITH CRAB PARAMETERS

2.1. Emission Geometry

To calculate the light curves and the spectra, we adopt a rotating dipole field in the magnetosphere. For a rotating dipole, the local magnetic field $B(r)$ is given by

$$B = \hat{r} \left( \frac{3\mu}{c^2 r} + \frac{3\mu}{c^2 r} \hat{r} \right) - \left( \frac{\mu}{c^2 r} + \frac{\mu}{c^2 r} \right)$$

(Cheng et al. 2000), where $\mu = \mu (\hat{r} \sin \alpha \cos \Omega t + \hat{y} \sin \alpha \sin \Omega t + \hat{z} \cos \alpha)$ is the magnetic moment vector, $\hat{r}$ is the radial unit vector, and $\alpha$ is the inclination angle.

We calculate the polar cap’s edge using $(x_0, y_0, z_0) = (R_p \cos \phi_p, R_p \sin \phi_p, (R^2 - R_p^2)^{1/2})$ as initial trials for the computer program. $R_p = R(R/R_p)^{1/2}$ is the polar cap radius of an aligned static dipole, where $R$ is the stellar radius and $R_p = c/\Omega$ is the radius of the light cylinder. The quantity $\phi_p$ is the azimuthal angle about the magnetic axis, which we call the polar cap angle. Then we employ the Runge-Kutta method to trace out the field lines. In subsequent iterations, we try to find the scaling factors $a_0$ such that $(x_0, y_0, z_0) = (a_0 x_0, a_0 y_0, (R^2 - a_0^2 R_p^2)^{1/2})$ correspond to the footpoints of the last closed field lines, that is, the boundary of the polar cap. In Figure 2 of Cheng et al. (2000) it is shown that $a_0 = \phi_p$-dependent. The polar cap of a rotating dipole is not circular in shape, especially for those having large inclination angles. A three-dimensional view of the last closed field lines is shown in Figure 1. Next we define another scaling factor $a_1$ to denote the footpoints of the open field lines, as $(x, y, z) = (a_1 x_0, a_1 y_0, (R^2 - (x^2 + y^2)^{1/2})$. Here $a_1 = 0$ represents the magnetic pole and $a_1 = 1$ represents the last closed field lines.
Since the star is rotating, aberration occurs along the line of sight. With $\beta = |\mathbf{r} \times \Omega|/c$, we have

$$u_0' = u_0 + \beta c / (1 + \beta u_0/c), \quad u_0' = u_0 \sqrt{1 - \beta^2} / (1 + \beta u_0/c),$$

$$u' = \sqrt{1 - \beta^2} / (1 + \beta u_0/c), \quad \Phi = -\phi' - \frac{r \cdot \hat{u}'}{R_L},$$

where $u_0$ and $u_0'$ for $i = \{r, \theta, \phi\}$ are the emission directions in the corotating and observer frames, respectively. Choosing the rotational axis as the $z$-axis, in the observer’s frame the polar angle from the rotational axis is given by

$$\cos \zeta = u_0'/u'$$

(Yadigaroglu 1997), where $\zeta$ is the viewing angle, with $\zeta = 0^\circ$ when the star is viewed directly above its rotational axis and $\zeta = 90^\circ$ when it is viewed over the stellar equator.

Compared with the photons emitted from the center of the star, a photon emitted at any particular location $\mathbf{r}$ will take less time to travel to the light cylinder. The phase difference due to the travel time is given by $\Delta \Phi = -r \cdot \hat{u}'/R_L$. Therefore, the phase angle $\Phi$ in the observer’s frame (Yadigaroglu 1997) is given by

$$\Phi = -\phi' - \frac{r \cdot \hat{u}'}{R_L},$$

where $-\phi' = -\cos^{-1} (u_0'/u')$ is the azimuthal angle in the observer’s frame. Choosing the $\Omega$-$\mu$ plane to be the $x$-$z$ plane, $u_0'$ is the length of the projection of $u'$ onto the $x$-$y$ plane.

We would like to remark that we have assumed a dipolar field in the corotating frame. On the other hand, Takata et al. (2007) assumed a rotating dipole field in the observer’s frame. With this difference in magnetic field configuration, in the observer’s frame the emission direction near the light cylinder is azimuthal for the present case and nearly radial for the case in Takata et al. (2007). Essentially, this difference appears because the poloidal magnetic field dominates the toroidal field near the light cylinder in the observer’s frame for the present case, and the poloidal and the toroidal fields are comparable to each other for the case in Takata et al. (2007). Although there is a large difference in emission direction near the light cylinder, the pulse profiles do not change very much (Takata et al. 2007), because the radiation very close to the light cylinder mainly contributes to the bridge phase. As we will see below, however, the calculated phase-resolved spectra in the present case explain the observations better than the results of Takata et al. (2007). Therefore, the current results suggest that the radiation direction near the light cylinder seems to be in the azimuthal rather than the radial direction. The recent particle simulation for the global structure of the charge-separated magnetosphere by Wada & Shibata (2007) also indicates such behavior of the emission. On the other hand, the time-dependent force-free relativistic MHD solution obtained by Spitkovsky (2006) indicated radial motion of the particles near the light cylinder. Furthermore, Bucciantini et al. (2006) have considered a more general relativistic MHD approach for rotating pulsars, and their solutions asymptotically approach the force-free ones, similar to that obtained by Spitkovsky (2006) in the case of a highly magnetized wind, which is close to our case. Therefore, the global structure of the magnetic field is still an open question.

In this paper, we neglect the contribution from the inward emission for the following reasons: In general, since the charged particles accelerate only within the gap and they lose energy during the cascade process, the incoming charged particles, and hence the incoming photons, cannot have an energy exceeding $\gamma_e m_e c^2$, which is the energy of a charged particle immediately after it leaves the gap; $\gamma_e \sim 10^4$ is the local Lorentz factor of a charged particle upon leaving the outer gap (see eq. [8]). On the other hand, charged particles within the gap are being accelerated continuously and will gain an energy of $eV_{\text{gap}}$ with $V_{\text{gap}} \approx 6.6 \times 10^{13} / eB_0 R_L^2 P^{-2} V \sim 10^{15} V$ (Romani & Yadigaroglu 1995), where $f_0 \approx 0.2$ is the average value of the local gap size at $R_L$. Therefore, we can roughly estimate the ratio of the intensity of the radiation due to the incoming photons to the radiation due to the outgoing photons as $N_{\text{gap}} \gamma_e m_e c^2 / N_{\text{gap}} eV_{\text{gap}} \sim 0.5\%$. As a result, when we compute the light curve we neglect the contribution from the incoming photons.

By considering the radiation to be emitted tangent to the magnetic field lines in the corotating frame, we project the radiating points onto the $\zeta-\Phi$ plane. Figure 2 shows the photon emission pattern for an inclination angle $\alpha = 50^\circ$ and $a_1 = 0.97$. Here we assume the emission region to extend from the stellar surface to the light cylinder and assume a symmetry between the north and south poles. In other words, when there is a photon emitted with $(\zeta, \Phi)$ from the north pole, there is another photon emitted with $(180^\circ - \zeta, 180^\circ + \Phi)$ from the south pole as well. In Figure 2, the gray lines correspond to the outgoing photons emitted from the north pole, the pole that makes an acute angle with the rotational axis. The black lines correspond to the outgoing photons emitted from the south pole. For example, for an observer at a viewing angle smaller than $90^\circ$, the emission region near the north pole (gray) corresponds to the radiation emitted in the region between the inner boundary (stellar surface) and the null charge surface, and the emission region near the south pole (black) corresponds to the radiation emitted in the region beyond the null charge surface.

The viewing angle, which is the angle between the observer and the rotational axis, can be set to a certain value $\zeta_0$. Then we can measure the number of photons traveling in the $\zeta_0$-direction and produce a theoretical light curve. However, before we move on to produce the theoretical curve, we need to mention that the radiation is, in fact, emitted within a finite cone of half-angle $\phi(r)$ instead of simply tangent to the field lines. The criterion for counting a photon becomes $\zeta - \phi(r) \leq \zeta_0 \leq \zeta + \phi(r)$. This effect can be understood from a geometric point of view. Figure 3 shows a
close-up of two field lines, which approximate two concentric circles; \( h(r) \) is the local thickness of the outer gap, \( s(r) \) is the local radius of curvature of the field line, and \( \lambda(r) \) is the pair creation mean free path. According to Figure 3, since the secondary pairs are produced just above the outer boundary of the outer-gap accelerator, we can estimate the pitch angle of the newborn pairs as

\[
\sin^2 \varphi(r) = \frac{2s(r)R_L}{r(r)} ,
\]

where \( f(r) \) is the fractional gap thickness, defined by \( f(r) = h(r)/R_L \). The self-sustained outer-gap model of Zhang & Cheng (1997) estimates the fractional gap thickness as \( f(R_L/2) \sim 5.5 P_{12} / B_{-2}^{3/7} \), which is \( \sim 0.04 \) for the Crab pulsar and \( \sim 0.13 \) for the Vela pulsar. Since \( r(s) = (r(R_L))^1/2 \) in the static dipole approximation, \( \sin \varphi(r) = (r/R_L)^{1/2} \sin \varphi(R_L) \).

2.2. Inner and Outer Boundaries of the Outer Gap

As the electrodynamical studies have shown (Hirotani et al. 2003; Takata et al. 2004, 2007; Hirotani 2006a), the inner boundary of the outer-gap accelerator is shifted inward from the null charge surface with an increase in the current through the gap. In fact, if there is no current injection from the inner and the outer boundaries, the inner boundary will be located at a position at which \( B_z/B = j_B \) is satisfied, where \( j_B \) is the current density in units of \( \Omega B/2\pi \) carried by the pairs created in the gap and is constant along the field line for the steady state (Takata et al. 2004). For example, if no current is created in the gap (\( j_B = 0 \)), the inner boundary is located at the position where \( B_z/B = 0 \) is satisfied, that is, on the null charge surface. On the other hand, if \( j_B \sim \cos \alpha \) on a particular magnetic field line, the inner boundary on the field line is located at the stellar surface, where \( B_z/B \sim \cos \alpha \) is satisfied. We expect that the created current density is proportional to the pair creation rate, which depends on the radial distance as \( r^{-3/8} \) (Cheng et al. 2000). Since most of the pairs are created around the null charge surface in the outer gap, we may be able to relate the created current density to the radial distance to the null charge surface of each last closed field line as \( j_B(\phi_p) = j_B(0)[r_B(0)/r_B(\phi_p)]^{-3/8} \), where \( j_B(0) \) and \( r_B(0) \) are respectively the created current density on and the radial distance to the null charge surface on the last closed field line, with the polar cap angle \( \phi_p = 0^\circ \). We would know the location of the inner boundary given the azimuthal angle if we could estimate the created current density \( j_B(0) \). As demonstrated by the electrodynamical studies, however, the current structure in the gap is very sensitive to the gap geometry, such as the transfield thickness and the longitudinal width. Since the gap geometry in the pulsar magnetosphere should be determined by global conditions (Wada & Shibata 2007) and since there is no study of the three-dimensional magnetosphere of an inclined rotator, we deal with the created current \( j_B(0) \) by using some model parameters. Figure 4 summarizes the variation of the radial distances to the inner boundary and the null charge surface on the last closed field lines with the polar cap angle of the field lines around the magnetic axis. From Figure 4, one can see that only a small current is created around \( \phi_p \sim 180^\circ \), so the outer gap must be less active there. In this paper, we constrain the width of the polar cap angle with the field lines on which the pair creation mean free path, \( \lambda(r) \sim 2s(r)f(r/R_L)^{1/2} \sim 2f^{1/2}(R_L/2)r \), at the null charge surface of the active field lines is estimated to be shorter than \( R_L \). This condition produces an azimuthal extension of the outer gap of \( \Delta \phi_p \sim 250^\circ \).

For the outer boundary, the position should be determined with a global model such as in Wada & Shibata (2007) with current. For the present local model, this position is a free parameter, and we place it at the light cylinder because we assume that the emissivity of the curvature radiation declines rapidly near/beyond the light cylinder, that the radiation beyond the light cylinder is beamed out of the line of sight as a result of the magnetic bending, or both. We may place the outer boundary inside the light cylinder. However, the resultant pulse profile and the spectra do not change very much unless the outer boundary is located close to the null charge surface, because the accelerated particles inside the gap contribute to the total radiation emission outside the outer gap (Wada & Shibata 2007).

2.3. Light Curve

Since the general features of the light curve such as the standing phase of the pulse are mainly affected by the geometry of the emission regions, we produce a theoretical light curve by assuming constant emissivity. Figure 5 shows the theoretical light curve for a pulsar with inclination angle \( \alpha = 50^\circ \), \( a_1 = 0.97 \), viewing angle \( \zeta_0 = 76^\circ \), and azimuthal extension of the outer gap \( \Delta \phi_p = 250^\circ \). The pitch angle at \( R_L \) is treated as a fitting parameter and is assumed to be \( \sin \varphi(R_L) = 0.04 \). These parameters are chosen so...
that the modeled light curve explains the general features of the observations, such as two peaks in a single period with a phase separation of ~140° between the two peaks. A breakdown of the light curve to show the contribution from the two poles separately is given in Figure 6. The color scheme is the same as for the emission pattern in Figure 2. As we shall see below, the inclination angle and the viewing angle chosen to explain the observed emission pattern in Figure 2. As we shall see below, the inclination angle and the viewing angle chosen to explain the observed light curve also produce phase-resolved spectra that are consistent with observations.

3. ENERGY SPECTRA OF THE OBSERVED PHOTONS

3.1. Acceleration and Emission in the Gap

We adopt the local electric field equation in the CHR model for the region beyond the null charge surface. By assuming that the local electric field decreases in a quadratic manner in the region between the null charge surface \( r_{\text{null}} \) and the inner boundary of the pair production region \( r_{\text{in}} \), we have

\[
E_{||} = \begin{cases} 
\frac{\Omega B(r)\hbar^2(r)}{cs(r)}, & \text{if } r \geq r_{\text{null}}, \\
E_{\gamma}(r_{\text{null}}) \left( \frac{(r/r_{\text{in}})^2 - 1}{(r_{\text{null}}/r_{\text{in}})^2 - 1} \right), & \text{if } r < r_{\text{null}}.
\end{cases}
\]  

For \( r \geq r_{\text{null}} \), \( E_{||} \) is the vacuum solution given in Cheng et al. (1986a). The vacuum solution is a good approximation for pulsars with thin gaps such as the Crab pulsar.

The radial distance \( r \) to the null charge surface varies with the field lines, and the local curvature radius also depends on the field lines. Therefore, strictly speaking, the electric field component along a magnetic field line is a function of both the radial distance \( r \) and the polar cap angle \( \phi_p \), that is, \( E_{||} = E_{||}(r, \phi_p) \).

When a relativistic charged particle that is accelerated continuously along a magnetic field line by the strong local electric field radiates by means of curvature radiation, the power gained by the particle as it goes through the electric potential, \( eE_{||}(r, \phi_p)c \), is transformed to the power radiated as curvature radiation in order to maintain equilibrium. The total radiated power for each particle is \( I_{\text{cur}}(r) = 2e^2c^4 \gamma_{\text{e}}^4(r)/3s^2(r) \). The local Lorentz factor \( \gamma_{\text{e}} \) of the primary particles can be found by requiring \( eE_{||}(r, \phi_p)c = I_{\text{cur}} \), and hence

\[
\gamma_{\text{e}}(r) = \left[ \frac{3 \sqrt{2}}{2} e E_{||}(r, \phi_p) \right]^{1/4}.
\]  

The characteristic energy of the radiated curvature photons is given by \( E_{\text{cur}}(r) = (3/2)e^2c^4 \gamma_{\text{e}}^4(r)/s(r) \), and Figure 7 shows the variation of \( E_{\text{cur}}(r) \) along the field lines with polar cap angles \( \phi_p \) of 0° (solid line), 90° (dashed line), 180° (dotted line) and 270° (dash-dotted line). From Figure 7, we find that the particles are accelerated up to the ultrarelativistic regime, so that 10 GeV photons are emitted in the outer-gap accelerator of the Crab pulsar by means of the curvature process.

3.2. Synchrotron Radiation and Inverse Compton Scattering from Secondary Pairs

Since the relation \( E_{\text{cur}}(r)dN_e/dt \sim I_{\text{cur}}(r)\gamma_{\text{e}}^4 \) is satisfied, the radiation spectrum of the primary particles in a unit volume is approximately described by

\[
\frac{d}{dV} \left( \frac{d^2N_{\gamma}}{dE_{\gamma} \, dt} \right) \approx \frac{l_{\text{cur}}(r)n}{E_{\text{cur}}(r) E_{\gamma}},
\]

where \( n = dN/dV \) is the number density of the primary particles. Here we use \( n = \Omega B(r)c/2\pi c \), which comes from the local Goldreich-Julian number density, disregarding the angle between
the local magnetic field direction and the rotational axis. Since the energy of each photon comes from curvature radiation, \( E_\gamma \leq E_{\text{cur}}(r) \). These primary curvature photons collide with the soft photons produced by synchrotron radiation of the secondary electrons or positrons to produce even more secondary \( e^+\) pairs. In this way, the synchrotron photons become abundant, and nearly all the curvature photons are converted into secondary \( e^\pm \) pairs. As a result, the energy distribution of the secondary electrons or positrons is

\[
\frac{dN(r)}{dE_e} \approx \frac{1}{E_e} \int_{E_e}^{E_{\text{max}}(r)} \frac{d^2N_\gamma(E_\gamma = 2E_e')}{dE_{e'} dt} (1 - e^{-\tau_{\gamma}(E_e, r)}) dE_e'
\]

\[
\approx \frac{1}{E_e} \frac{\tau_{\gamma}(r) \Omega B(r)}{2\pi\epsilon c E_{\text{cur}}(r)} \Delta V(r) \ln \left[ \frac{E_{\text{cur}}(r)}{E_e} \right] \quad \text{for} \quad \tau_{\gamma} \rightarrow \infty, \quad (10)
\]

where the upper integration limit \( E_{\text{max}}(r) \) is taken to be \( E_{\text{cur}}(r)/2 \), as the energy of each photon is divided between an \( e^\pm \) pair, \( E_e \) represents the energy of each \( e^\pm \), and \( E_{\text{cur}} = 2eB^2(r) \times \sin^2 \beta(r) E_{\text{cur}}^2/3m_e^2c^3 \) is the synchrotron energy-loss rate. The quantity \( \tau_{\gamma}(E_e, r) \) is the attenuation depth for the absorption of the curvature photons. It will be very large in most of the magnetospheric regions except in the trailing wing 1, bridge, and leading wing 2 phases. A detailed discussion of the absorption of curvature photons is given in § 3.3.

In the computer program, we divide the polar cap into \( N_\phi \) equal divisions so that each span an angle of \( \Delta \phi_p = 360^\circ/N_\phi \). Therefore, each division, represented by a magnetic field line, will take up a magnetic flux \( \Phi_{\text{gap}}(r) \). The volume element \( \Delta V(r) \) is represented by \( \Delta A(r) \Delta l(r) \), where \( \Delta A(r) \) and \( \Delta l(r) \) denote the area and the length of the tubelike volume element along a field line where observable photons are produced. Because of magnetic flux conservation, \( B(r) \Delta A(r) = \Phi_{\text{gap}}(r) \), so that equation (10) becomes

\[
\left[ \frac{dN(r)}{dE_e} \right]_i \sim \frac{1}{E_e} \frac{\tau_{\gamma}(r) \Omega B(r)}{2\pi\epsilon c E_{\text{cur}}(r) N_\phi} \ln \left[ \frac{E_{\text{cur}}(r)}{E_e} \right], \quad (11)
\]

where \( \Phi_{\text{gap}} = 2\pi f(R_\star)B(R_\star)^2R_\star^2 \).

The photon spectrum of the synchrotron radiation is given by

\[
F_{\text{syn}}(\gamma, r) = \sum_{i=1}^{N_\phi} \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dN(r)}{dE_e} \left[ \frac{d^2N_\gamma(E_\gamma = 2E_e')}{dE_{e'} dt} \right] \frac{dE_{e'}}{dE_e} \quad \text{syn}
\]

\[
= \sqrt{3}eB(r) \sin \varphi(r) \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dN(r)}{dE_e} \left[ \frac{d^2N_\gamma(E_\gamma = 2E_e')}{dE_{e'} dt} \right] F(x) dE_e, \quad (12)
\]

where \( F(x) = x \int_{x}^{\infty} K_{5/3}(\zeta) d\zeta \) with \( K_{5/3} \) modified Bessel function of order \( 5/3 \), \( x = E_\gamma/E_{\text{syn}}(r) \), and

\[
E_{\text{syn}}(r) = 3\hbar eB(r) \sin \varphi(r) E_e^2/2m_e^2c^5
\]

is the critical synchrotron photon energy. For the integration, the upper integration limit \( E_{\text{max}} = E_{\text{cur}}(r) \) as \( E_{\text{cur}}(r)/2 \), while the lower limit is chosen in a way such that \( E_{\text{min}}/m_e^2c^2 = 20 \) (Takata et al. 2007).

Similarly, the photon spectrum due to inverse Compton scattering is given by

\[
F_{\text{ICS}}(\gamma, r) = \sum_{i=1}^{N_\phi} \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dN(r)}{dE_e} \left[ \frac{d^2N_\gamma(E_\gamma = 2E_e')}{dE_{e'} dt} \right] \frac{dE_{e'}}{dE_e} \quad \text{ICS}
\]

\[
= \int_{\epsilon_1}^{\epsilon_2} \frac{3\pi \gamma c}{4E_e(m_e^2c^2)^{3/2}} n_{\text{syn}}(\epsilon, r) n_\gamma(\epsilon, r) \frac{d\epsilon}{\epsilon} \times \left[ 2q \ln q + (1 + 2q)(1 - q) + \frac{(\Gamma q^2(1 - q)^2}{2(1 + \Gamma q)^2} \right], \quad (15)
\]

where \( n_\gamma(\epsilon, r) = \frac{1}{\pi^2 (h\epsilon)^3} \exp(\epsilon/\Gamma q) - 1 \left( \frac{R}{r} \right)^2 \).

The other source of soft photons arises from synchrotron radiation. The absorption due to this kind of photon is more significant near the light cylinder. The number density of the synchrotron photons, \( n_{\text{syn}}(\epsilon, r) \), is described by

\[
n_{\text{syn}}(\epsilon, r) = \frac{F_{\text{syn}}(\epsilon, r)}{\epsilon^2 \Delta \Omega(r)}, \quad (17)
\]

where \( \Delta \Omega(r) \) is the solid angle of the beam of synchrotron photons and is estimated as

\[
\Delta \Omega(r) = \int_0^{2\pi} \int_0^{\pi/2} d\phi \sin\theta d\theta \approx \pi \varphi^2(r). \quad (18)
\]

The \( E_e \) in \( dN(r)/dE_e \) in equation (14) is the sum of both the synchrotron energy-loss rate and the inverse Compton energy-loss rate due to thermal photons, given by \( E_e = \sigma_T(m_e^2c^2)/16h^3 \times \ln (4\gamma c/kT/m_e^2c^2 - 3) - 0.5772 - 0.5700 \) (Blumenthal & Gould 1970). The surface temperature of the Crab pulsar is taken to be \( 2 \times 10^8 \) K (Tennant et al. 2001).

In order for equation (15) to be valid, \( \epsilon_1 \) must be positive, so \( E_1 < 1 \). Moreover, since the number of photons cannot be negative, we require the quantity in brackets in equation (15) to be greater than or equal to zero. The upper integration limit \( \epsilon_2 \) is chosen in such a way that \( F_{\text{syn}}(\epsilon_2, r) \) is very small for that particular field line, whereas the lower integration limit \( \epsilon_1 \) is taken to be greater than \( 1 \) eV.

3.3. Absorption of Curvature Photons

In our previous studies (Cheng et al. 2000), we anticipated that most of the curvature photons would be converted into secondary pairs through the pair creation process with the magnetospheric X-rays. The typical pair creation mean free path is estimated from \( \Gamma^{-1}(R_L/r_2) \sim (1 - \cos\theta_{\text{pol}}) n_5(R_L/r_2)\sigma_{\gamma\gamma} \), where \( n_5(R_L/r_2) \) is the typical nonthermal X-ray number density, \( \sigma_{\gamma\gamma} \sim \sigma_{\gamma\gamma}/3 \) is the pair creation cross section with \( \sigma_{\gamma\gamma} \) the Thomson cross section, and \( \theta_{\text{pol}}(R_L/r_2) \sim [2f(R_L/r_2)R_L/s(R_L/r_2)]^{1/2} \sim \sqrt{0.2} \) is the typical collision angle.
between the magnetospheric X-rays and the γ-rays emitted in the gap. For the Crab pulsar, the typical number density of X-rays is $n_X \sim \frac{L_X}{\delta \Omega (R_L/2)^2 c E_X} \sim 6 \times 10^{17} \text{ cm}^{-3}$, where we used the typical energy $E_X \sim (2m_e c^2)/(1 - \cos \theta_{\text{eq}})(10 \text{ GeV}) \sim 260 \text{ eV}$, the typical nonthermal X-ray luminosity $L_X \sim 5 \times 10^{34} \text{ ergs s}^{-1}$, and the solid angle $\delta \Omega = 1 \text{ rad}$. As a result, the mean free path becomes $l \sim 2 \times 10^7 \text{ cm}$ ($\sim R_L/5$) for the $10 \text{ GeV}$ photons, and therefore we have believed that most of the $10 \text{ GeV}$ photons emitted are converted into secondary pairs. In this paper, on the other hand, we take into account the fact that some curvature photons emitted far away from the star (or near the light cylinder) could avoid the photon-photon pair creation process as a result of the shorter escape distance and lower photon density near the light cylinder, and we explicitly calculate the optical depth for photons emitted at a given position. It can be shown that indeed some $10 \text{ GeV}$ photons emitted near the light cylinder and almost all photons with sub-GeV energies can escape from the magnetosphere, because their mean free path becomes longer than the light radius. These surviving curvature photons will contribute to the high-energy peaks around $10 \text{ GeV}$ in the trailing wing 1, bridge, and leading wing 2 phases. Following Ding & Cheng (1997), the photon spectrum of the surviving γ-ray photons is

$$F_{\text{cur, sur}} = F_{\text{cur}}e^{-\tau(E, r)},$$

(19)

![Fig. 8.—Phase-averaged spectrum of the Crab pulsar. The observational data are from Kuiper et al. (2001). The fit parameters are $\alpha = 50^\circ$, $a_1 = 0.97$, $\zeta = 76^\circ$, $f(R_L) = 0.2$, and $\sin \varphi(R_L) = 0.06$. [See the electronic edition of the Journal for a color version of this figure.]]

![Fig. 9.—Phase-resolved spectra, with $\alpha = 50^\circ$, $a_1 = 0.97$, $\zeta = 76^\circ$, $f(R_L) = 0.2$, and $\sin \varphi(R_L) = 0.06$, breaking down Fig. 8.]

where $F_{\text{cur}}$ is the curvature spectrum and is described by

$$F_{\text{cur}}(E_\gamma, r) = \frac{dN}{2\pi \hbar s E_\gamma} \frac{\sqrt{3e^2 \gamma_x}}{F(x)}$$  \hspace{1cm} (20)$$

with $dN$ the number of primary $e^\pm$ pairs in the emission region, given by $dN = n e \Delta A \Delta l$, and $x = E_\gamma / E_{\text{cur}}(r)$. The attenuation depth $\tau(E_\gamma, r)$ is calculated from

$$\tau(E_\gamma, r) = l(r) \int_{E_{\text{min}}}^{E_{\text{max}}} [n_{\text{syn}}(\epsilon, r) + n_{\text{e}}(\epsilon, r)] \sigma_{\gamma\gamma}(E_\gamma, \epsilon) d\epsilon$$  \hspace{1cm} (21)$$

(Jauch & Rohrlich 1976) with $l(r)$ being the distance between the emission location and the light cylinder and $\sigma_{\gamma\gamma}(E_\gamma, \epsilon)$ the cross section for photon-photon pair creation, which is given by

$$\sigma_{\gamma\gamma}(E_\gamma, \epsilon) = \frac{3}{16} \sigma_T(1 - v^2) \left[ (3 - v^4) \ln \left( \frac{1 + v}{1 - v} \right) - 2\pi(2 - v^2) \right],$$  \hspace{1cm} (22)$$

where

$$v = \sqrt{1 - m_e c^2 / E_\gamma \epsilon}. \hspace{1cm} (23)$$

The total photon flux received at Earth is

$$F(E_\gamma) = \frac{1}{\Delta\Omega D^2} \sum_r [F_{\text{syn}}(E_\gamma, r) + F_{\text{ICS}}(E_\gamma, r) + F_{\text{cur, cur}}(E_\gamma, r)],$$  \hspace{1cm} (24)$$

where $D = 2$ kpc is the distance to the Crab pulsar from Earth and $\Delta\Omega$ is the solid angle, chosen to be 1 sr for simplicity.

### 3.4 Phase-resolved Spectra

The computation of the photon flux is divided into four steps. First, we calculate the synchrotron radiation photon flux. Then we use this synchrotron flux to calculate the inverse Compton scattering photon flux. Next, we adjust the peak intensities of the synchrotron and the inverse Compton scattering spectra by requiring $\int E_\gamma F(E_\gamma) dE_\gamma$ for the synchrotron spectrum alone, before consideration of the inverse Compton scattering radiation, to be the same as the sum of the synchrotron and the inverse Compton scattering spectra. However, the relative peak intensities of the synchrotron and the inverse Compton scattering spectra are kept unchanged. We employ this third step to comply with energy conservation because all the energy in inverse Compton–scattered photons comes from the synchrotron photons. Finally, we calculate the surviving curvature radiation.

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**Figure 8** shows the observed data for the phase-averaged spectrum of the Crab pulsar and the theoretical fitted spectrum with $f(R_\text{l}) = 0.2$ and $\sin \varphi(R_\text{l}) = 0.06$, calculated using the synchrotron self-Compton mechanism together with the surviving curvature photons from 100 eV to 10 GeV. The medium and thin lines show the emission beyond the null charge surface (i.e., from the south pole) and between the inner boundary and the null charge surface (i.e., from the north pole), respectively. For each region, dashed lines represent the synchrotron spectrum, dotted lines represent the inverse Compton scattering spectrum, dash-dotted lines represent the surviving curvature spectrum, and solid lines are the total. The thick solid line is the sum of the three spectra from both poles. The fitted spectrum reproduces the observations from 100 eV to 10 GeV.

**Figure 9** shows the predicted phase-resolved spectra as a breakdown of the phase-averaged spectrum in Figure 8. The blue and red lines show the emission beyond the null charge surface (i.e., from the south pole) and between the inner boundary and the null charge surface (i.e., from the north pole), respectively. The phase intervals are defined in the same way as in Fierro et al. (1998). The pulse of the Crab pulsar is divided into eight phases, namely, leading wing 1 (LW1), peak 1 (P1), trailing wing 1 (TW1), bridge, leading wing 2 (LW2), peak 2 (P2), trailing wing 2 (TW2), and off-pulse. The criteria for the division are listed in Table 1. We employ this third step to comply with energy conservation because all the energy in inverse Compton–scattered photons comes from the synchrotron photons. Finally, we calculate the surviving curvature radiation.

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**Table 1: Definitions of Phase Components for the Crab Pulsar**

| Component          | Abbreviation | Phase Interval (deg) | Width (deg) |
|--------------------|--------------|----------------------|-------------|
| Leading wing 1     | LW1          | 8.8–30.4             | 21.6 ± 0.06 |
| Peak 1             | P1           | 30.4–66.4            | 36 ± 0.10   |
| Trailing wing 1    | TW1          | 66.4–102.4           | 36 ± 0.10   |
| Bridge             | Bridge       | 102.4–142.0          | 39.6 ± 0.11 |
| Leading wing 2     | LW2          | 142–167.2            | 25.2 ± 0.07 |
| Peak 2             | P2           | 167.2–206.8          | 39.6 ± 0.11 |
| Trailing wing 2    | TW2          | 206.8–239.2          | 32.4 ± 0.09 |
| Off-pulse          | OP           | 239.2–8.8            | 129.6 ± 0.36 |

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Figure 10 shows the predicted phase-resolved spectra as a breakdown of the phase-averaged spectrum in Figure 8. The blue and red lines show the emission beyond the null charge surface (i.e., from the south pole) and between the inner boundary and the null charge surface (i.e., from the north pole), respectively. The phase intervals are defined in the same way as in Fierro et al. (1998). The pulse of the Crab pulsar is divided into eight phases, namely, leading wing 1 (LW1), peak 1 (P1), trailing wing 1 (TW1), bridge, leading wing 2 (LW2), peak 2 (P2), trailing wing 2 (TW2), and off-pulse. The criteria for the division are listed in Table 1. We chose an azimuthal angle of 52° from the $\Omega$–$\mu$ plane to be phase zero. The footprints corresponding to each phase are plotted for both poles in Figure 10. Although our prediction is not too good (e.g., in LW1 the peak at ~100 MeV cannot be produced, whereas...
in P2 and TW2, the peaks at ~100 MeV are much too high), most of the spectral features can be explained. However, it has been argued that the north and south poles need not be perfectly symmetric. Since the dipole may not be at the center of the star, just as in the sunspot geometry (Ruderman 1991), it is possible that the two poles could have small differences. Furthermore, $\sin^2(\alpha(R_L))$ is calculated according to a simple static dipole field, which is clearly not a good approximation in the outer magnetosphere of a realistic rotating dipole field with current flow. In the best fit, we thus allow the gap sizes to be different for the two poles and the pitch angles to vary for individual phases.

Figure 11 compares the observed data of the phase-resolved spectra of the Crab pulsar with the theoretical best-fit results. Table 2 summarizes the values of $f(R_L)$ and $\sin^2(\alpha(R_L))$ for different phases for each pole. We find that the fitted pitch angles for

![Graphs and images](https://example.com/graphs)

**Figure 11.**—Phase-resolved spectra of the Crab pulsar from 100 eV to 30 GeV in the seven narrow pulse-phase intervals. Two spectra (TW1 and LW2) are displayed twice for easy comparison. The observational data are from Kuiper et al. (2001), and the spectra are calculated from the theoretical model with $\alpha = 50^\circ$, $a_1 = 0.97$, and $\zeta = 76^\circ$. We take $f(R_L)$ = 0.25 for the north pole and $f(R_L)$ = 0.22 for the south pole. The values of the fit parameter $\sin^2(\alpha(R_L))$ are given in Table 2.

**Table 2.** Values of Fitted Parameter $\sin^2(\alpha(R_L))$

| Phase  | North Pole | South Pole |
|--------|------------|------------|
| LW1    | 0.02       | ...        |
| P1     | 0.03       | 0.08       |
| TW1    | ...        | 0.08       |
| Bridge | ...        | 0.08       |
| LW2    | ...        | 0.06       |
| P2     | 0.08       | 0.08       |
| TW2    | 0.07       | 0.08       |

*Note.*—We have taken $f(R_L)$ to be 0.25 for the north pole and 0.22 for the south pole.

![Graph](https://example.com/graphs)

**Figure 12.**—Emission locations for photons counted in Fig. 6.
phases consisting of field lines at polar angles closer to $\phi_p = 0^\circ$ are smaller, while those for the phases consisting of field lines at polar angles closer to $\phi_p = 180^\circ$ are larger. This agrees with the theory that the outer gap is thinner around $\phi_p = 0^\circ$ and thicker around $\phi_p = 180^\circ$.

As we consider the emission region to extend inward to a boundary inside the null charge surface, we can fit the spectrum of LW1. This could not be done in Cheng et al. (2000), which considered the emission region to be from the null charge surface to the light cylinder alone, that is, one pole only. Figure 11 shows that this phase is mainly contributed by the radiation from the north pole. Another phase that could not be produced in Cheng et al. (2000) is TW2. That work estimated $\Delta \phi_p \sim 160^\circ$, but in § 2.2 above, we estimated that $\Delta \phi_p$ can be $\sim 250^\circ$ for the extended new emission geometry. The widening of the azimuthal extension of the outer gap allows us to obtain a reasonable fit for phase TW2.

From Figure 11, one can see that except for LW1 and the 100 MeV regime of P1, all the phase-resolved spectra are dominated by the radiation from the south pole. In general, the spectra are mainly contributed by the part from the null charge surface to the light cylinder, so the results in Cheng et al. (2000) are good approximations. The peaks at the high-energy regime near several GeV in TW1, the bridge, and LW2 are believed to be the surviving curvature radiation emitted at locations far from the star (cf. Fig. 12). This agrees with the model in which most of the curvature radiation has been converted into $e^\pm$ pairs and then emitted as synchrotron and inverse Compton-scattered photons.

For the three phases between the peaks (Figure 11, middle row), namely, TW1, bridge, and LW2, the synchrotron peaks from the south pole are much wider than their counterparts from the north pole. In fact, they show a kind of double-peaked feature with a plateau around 1 MeV. According to Figure 12, the radiation of each of these three phases is actually emitted from two well-separated regions. For TW1, the radiation is emitted from an outer region around $1.15R_L < r < 1.72R_L$ and an inner region around $0.46R_L < r < 0.67R_L$. For the bridge, these are $1.28R_L < r < 1.72R_L$ and $0.31R_L < r < 0.45R_L$. For LW2, they are $1.15R_L < r < 1.27R_L$ and $0.29R_L < r < 0.31R_L$. In order to understand this feature more clearly, we plot further-decomposed phase-resolved spectra for TW1, the bridge, and LW2 in Figure 13. In this figure, we decompose each blue line in Figure 11 into two components. The thin lines represent the radiation coming from the outer emission region, and the thick lines represent that from the inner region.

From Figure 13, one may note that the radiation emitted at a larger $r$, that is, from the outer region, produces a synchrotron peak at a lower energy. This can be explained by the typical synchrotron photon energy given in equation (13) with $E_{\text{syn}} \propto B(r) \sin \varphi(r) \propto r^{-2.5}$. Let us take the bridge as an example. We choose the representative $r$ to be the midpoint of the emission region from Figure 12, that is, $1.25R_L$ for the outer region and $0.35R_L$ for the inner region. In this case, the ratio of $r$-values is about 3.57 and hence the characteristic $E_{\text{syn}}$ of the inner region is about 24 times that of the outer region. From the modeled fit, the ratio of the peaks is about 27. Therefore, the theory matches quite well with the observation in this case.

4. CONCLUSIONS AND DISCUSSION

We have used a modified three-dimensional outer magnetospheric gap model in which the inner boundary of the outer gap is extended from the null charge surface to near the stellar surface. The exact location of the inner boundary does not affect the fitting results. The “inwardly extended” part of the outer gap contributes to phases LW1 and TW2 of the light curve with a slight modification of P1 and P2. Such a modified outer-gap geometry also plays a vital role in explaining the optical polarization properties of the Crab pulsar (Takata et al. 2007). Together with the results of Takata et al. (2007) on the gap accelerator, this model explains the pulse profile, phase-resolved spectra, and polarization of the Crab pulsar. Also, the outer-gap model can explain the observed complex morphological changes in light curve that occur as a function of photon energy (Takata & Chang 2007).

Four adjustable parameters are used to simulate the light curve: $\alpha$, the inclination angle of the magnetic axis to the rotational axis; the scaling factor $a_1$; the viewing angle $\xi$ to the rotational axis; and the emission-cone pitch angle due to the geometry of the
emission location, sin $\varphi(R_e)$. As constrained by the phase separation of the double peaks, we choose the values $\alpha = 50^\circ$ and $\zeta = 76^\circ$. From radio observations, Rankin (1993) estimated $\alpha \approx 84^\circ$ with $\zeta$ unknown. By using polarimetric observations at frequencies between 1.4 and 8.4 GHz, Moffett & Hankins (1999) calculated $\alpha \approx 56^\circ$ and $\zeta = 117^\circ$. Therefore, further observations are required in order to determine these two parameters more accurately.

For the photon spectra, our model fitting requires $f(R_e) \approx 0.2$, which is larger than the theoretical estimate $[f(R_e) \approx 0.12]$ by a factor of $\sim 2$. However, the theory (Zhang & Cheng 1997; Cheng et al. 2000) assumes a vacuum dipole potential. It has been pointed out that in order to explain the observed radiation power in the high-energy region, current must flow in the accelerator, and hence the potential at the gap will be reduced (Hirotani 2006b, 2007; Takata et al. 2006). Consequently, the gap size with current flow should be larger than in the vacuum case.

Moreover, we have taken the stellar radius of a neutron star to be $10^6$ cm. Because of the lack of direct methods to determine the size of a neutron star (Lattimer & Prakash 2004), and given that the equations of state inside a neutron star provided by different theoretical models do not provide a unanimous value for the neutron star size, we can only determine the magnetic moment, that is, $B_{p} R^3$, of the pulsar from the energy-loss rate. However, although the dipole radiation comes from the spin-down power of the star, it is not all of it. For example, the vacuum formula for the dipole radiation of an aligned rotator is usually used to infer the dipole moment of a pulsar. On the other hand, the MHD solution of Spitkovsky (2006) suggests that the effect of the current increases the spin-down luminosity from the vacuum formula. Therefore, we may have overestimated the value of the magnetic moment and, therefore, the strength of the magnetic field. Since $E_{\text{syn}} \propto B(r)$ and hence $B_{p} R^3$, the synchrotron peak may have been overestimated. Similarly, the peak synchrotron flux $F_{\text{syn}} \propto B(r)$ may also be overestimated. The power of synchrotron radiation and inverse Compton scattering can be compared by means of the ratio of the local magnetic energy density and the photon energy density: $P_{\text{syn}}/P_{\text{ICS}} \approx U_{\text{B}}/U_{\text{E}} \propto B^2(r)/2\pi \left[E_{\text{syn}}(r) m_{\text{syn}}(E_{\text{syn}}, r)\right]$, which is independent of $B_{p} R^3$, showing that $P_{\text{ICS}}$ may not be affected. On the other hand, since $F_{\text{syn}}$ may have been overestimated, $n_{\text{syn}}$ should be smaller. Hence, fewer curvature photons should be absorbed.

The Crab pulsar is one of the most important pulsars because of its proximity and strong radiation signals at all frequencies. Although observational data on it have been gathered for about 40 years, the underlying physics involved in this pulsar is not completely understood. For example, the structure of the charged-particle accelerator in the magnetosphere can be different in the polar cap model, the outer-gap model, and the newly proposed caustic model (Dyks & Rudak 2003), which envisions a two-pole configuration with thin gap regions running from each polar cap to the light cylinder. We hope that further observation by the more sensitive instruments on future missions, for example, the Gamma-Ray Large Area Space Telescope (GLAST), will bring crucial new information so that we can determine the mechanisms that are truly at work. For example, as shown in Figure 11, we predict that there is a clear component (30 MeV–30 GeV) that is mainly dominated by the surviving curvature photons in the TW1, bridge, and LW2 phases. Our model also suggests that the high-energy turnover in TW1, the bridge, and LW2 is at $\sim 10$ GeV, whereas the high-energy turnover of LW1 is only $\sim 100$ MeV (much lower than the other phases). This difference emerges as a consequence of the radiation in LW1 being emitted from a different pole in the region between the inner boundary and the null charge surface, where the local electric field is much weaker (cf. eq. [7]). With an increase in sensitivity and a widening of the energy range over EGRET, GLAST may be able to confirm our predictions.

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