A STATISTICAL STRATEGY FOR THE SUNYAEV-ZELDOVICH EFFECT’S CLUSTER DATA

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ABSTRACT

We present a statistical strategy for the efficient determination of the cluster luminosity function from the interferometric Sunyaev-Zeldovich (SZ) effect’s cluster data. To determine the cluster luminosity function from the noise contaminated SZ map, we first define the zeroth-order cluster luminosity function as the difference between the measured peak number density of the SZ map and the mean number density of noise. Then we demonstrate that the noise contamination effects can be removed by the stabilized deconvolution of the zeroth-order cluster luminosity function with the one-dimensional Gaussian distribution. We test this analysis technique against Monte Carlo simulations and find that it works quite well, especially in the medium-amplitude range where the conventional cluster selection method based on the threshold cutoff usually fails.

Subject headings: galaxies: clusters: general — methods: statistical

1. INTRODUCTION

Galaxy clusters are the biggest bound objects of the universe. Being rare and formed relatively late, the abundance of galaxy clusters depends sensitively on the background cosmology (Barbosa et al. 1996; Henry 1997; Bahcall & Fan 1998; Viana & Liddle 1999; Borgani et al. 1999; Fan & Chuieh 2001; Grego et al. 2001; Molnar, Birkinshaw, & Mushotzky 2002). To find as many galaxy clusters as possible and to investigate the evolution of their abundance have thus become two of the most challenging tasks of the current observational cosmology. The cluster survey is categorized by the observing wave band as the X-ray, optical, and Sunyaev-Zeldovich (SZ) effect. In the past, the X-ray or optical surveys were favored because of their high rate of detecting clusters and their relatively low costs. However, thanks to recent developments in technology, the SZ effect is currently considered a powerful cosmological probe (Carlstrom, Joy, & Grego 1996; White et al. 1999; Holder et al. 2000; Lo et al. 2000) that can provide a statistically unbiased sample of clusters, unlike the X-ray and optical surveys.

The SZ effect represents the change in brightness of the cosmic microwave background (CMB) radiation caused by the interaction of the CMB photons with the ionized intracluster gas (Sunyaev & Zeldovich 1972). The underlying physics of the SZ effect is the inverse Compton scattering: free electrons of the hot intracluster gas scatter off the CMB photons as they pass through the galaxy clusters, which results in a shift in the CMB photon frequency and a corresponding change in its radiation energy. Depending on whether the scattering of the CMB photons was by the random or systematic motions of the electrons, the SZ effect is called either thermal or kinetic, respectively. Here we focus on the thermal SZ effect since its contribution is an order of magnitude stronger than the kinetic counterpart (Springel, White, & Hernquist 2001; Zhang, Pen, & Wang 2002).

The SZ effects (\(\Delta I\)) observed at a given frequency (\(f\)) at a given position \(\theta\) on the sky depend on the electron number density (\(n_e\)) and temperature (\(T_e\)) within the galaxy clusters that the CMB photons have encountered all along the path:

\[
\Delta I(\theta) = -2g_y \frac{a_y k_B}{m_e c^2} \int n_e(\hat{\theta}) T_e(\hat{\theta}) dl \equiv -2g_y N(\theta), \tag{1}
\]

with the integral taken along the CMB path in the line-of-sight direction (\(\theta = \theta/|\theta|\)). Here \(y\) is the cluster Comptonization parameter, and \(g_y\) is the frequency-dependent spectral shape factor. Note that the SZ effects are spatially localized with the associated clusters along the line of sight, unlike the primordial CMB fluctuations.

Equation (1) implies that the amplitude of the SZ effects observed in a narrow-frequency band with almost fixed \(g_y\) can be quantified by the \(y\)-parameter. Provided that noise was absent in the SZ map, clusters could be counted as local maxima of \(y\). In practice, however, noise always dominates the SZ map. A critical issue is how to eliminate the dominant noise contamination effects. A common observational practice for identifying true signals from the noise-contaminated field is to select only those peaks with amplitudes above some threshold (usually several times the noise standard deviation). Using this technique, however, one can select only high-amplitude signals for a given observational integration time. In order to increase the signal-to-noise ratio, one has to decrease the noise level by increasing the integration time, which is proportional only to the square of the noise level.

Considering the usual high costs of SZ experiments, this conventional technique of signal selection is too inefficient to apply to the SZ cluster data. One may wish to find a more efficient statistical strategy that can allow one to determine the cluster luminosity function as quickly and accurately as possible. Powerful new generations of SZ instruments such as AMiBA (Array for Microwave Background Anisotropy; see Lo et al. 2000) are already in the pipeline. In fact, with the expectation of plenty of cluster data coming out in a few years, it is quite urgent that we start developing such statistical strategies now. In this Letter, we develop a statistical strategy for the efficient determination of the cluster luminosity function, especially from interferometric SZ surveys using AMiBA as our model experiment.

2. SZ CLUSTER LUMINOSITY FUNCTION

The SZ cluster luminosity function, \(n_y(y)\), is defined as the number density of galaxy clusters with the associated cluster Compton parameter in the range of \([y, y + dy]\). Its cumulative function, \(N_y(y) \equiv \int n_y(y')dy'\), is connected to the cluster mass function (see eq. [7] in Barbosa et al. 1996) and thus can be used in principle as a cosmological discriminator (Holder...
et al. 2000; Fan & Chiuhe 2001; Diego et al. 2002; Grego et al. 2001; Molnar et al. 2002; Benson, Reichardt, & Kamionkowski 2002). In practice, the direct conversion of the cluster luminosity function into the cluster mass function is fraught with difficulties related to the rather large scatter in the correlation between the cluster mass and the SZ effect strength (e.g., see Metzler 2002).

Anyway, our goal here is to discover an efficient way to find $n_{cl}(y)$ from the observed SZ map that is expected to be significantly contaminated by noise. There are two different sources of noise: instrumental noise and primordial CMB fluctuations. The primordial fluctuations, however, turned out to be negligible in interferometric SZ cluster surveys (Zhang et al. 2002). In the following analysis, we concentrate on instrumental noise only.

Our model experiment, AMiBA, as an interferometric SZ survey, will employ the drift-scan method to optimize the observations. For a detailed description of AMiBA and the drift-scan method, see Lo et al. (2000) and Pen et al. (2002), respectively. Among the many advantages of the drift-scan method, it makes noise analysis tractable: in an SZ map measured from the drift-scanned CMB sky, noise is Gaussian white. Therefore, a total SZ map measured by AMiBA will be a combination of non-Gaussian cluster sources with Gaussian noise. In the following two subsections, we simulate a total SZ map by means of a Monte Carlo method, and we reconstruct the cluster luminosity function by eliminating the noise contamination effects from the SZ map.

2.1. Monte Carlo Simulations of Drift-scan SZ Maps

We have constructed a random field on a 2048^2 mesh in a periodic box of linear size 1° using the Monte Carlo method in such a way that the random field possesses the main statistical properties of a cleaned SZ map expected from an AMiBA drift-scan survey over a unit area per a unit hour, assuming the flat-sky approximation. By a cleaned SZ map, we mean an SZ map smoothed by an optimal filter: in Fourier $u$-space ($u = |u|$), the optimal filter for AMiBA drift-scan observations is given (Pen et al. 2002) as $W(u) = W_k(u)W_c(u)$, where $W_k(u)$ and $W_c(u)$ are the cluster intrinsic shape and the natural beam, respectively. We use $W_k(u) = 1/|u|$ and approximate $W_c(u)$ as $W_c(u) \approx \exp(-u^2\theta_n^2/2) - \exp(-u^2\theta_p^2/2)$, where the two angular scales, $\theta_n$ and $\theta_p$, represent the size of the natural and primary beam, respectively, related to each other by $\theta_p/\theta_n = 1/\beta$ (U.-L. Pen 2002, private communications). We first constructed a Gaussian random field with the white-noise power spectrum and convolved it by $W_k(u)$, and we rescaled the field by its rms fluctuations, $\sigma_\nu^2 = [1/(2\pi)^2] \int |W_k(u)|^2 d^2 u$.

Second, we simulated the cluster sources by generating a sparse set of two-dimensional Gaussian functions of which peak locations and amplitudes were chosen randomly. Zhang et al. (2002) showed that the optimal scan rate for the purpose of the AMiBA cluster search is around 150 hr deg^-2, which could find one cluster every 8 hr. Thus, the total number of the cluster sources was set to be 20, with the expectation that the number of clusters per square degree from the AMiBA cluster search would be around the same number. We locate the cluster sources in the two-dimensional map deliberately so that they are not overlapping one another. The size of each cluster source, i.e., the length scale of each Gaussian function, was chosen to be twice the pixel size, which is consistent with the expected AMiBA drift-scan map. The randomly chosen amplitudes of the cluster sources were in the range of $[0, 5\sigma_\nu]$, which is where we would like to reconstruct the cluster luminosity function from the noise-contaminated map. The cluster amplitudes were chosen to be distributed exponentially, mimicking the real cluster distribution in this range (Zhang et al. 2002).

Finally, we obtained a simulated SZ map by combining the Gaussian noise field with the cluster sources. Then we identified the local maxima of the total field by selecting those pixels whose amplitudes exceed the amplitudes of their eight closest neighboring points, and we count the number density of the total SZ map, $n_{cl}(\nu)$, as a function of the rescaled amplitude, $\nu = y/\sigma_\nu$.

2.2. Deconvolution Method

If there were no clusters, the measured SZ map would be just a map of Gaussian noise, whose mean number density of local maxima, $n_0(\nu)$, is analytically derived to be (Longuet-Higgins 1957; Bond & Efstathiou 1987)

$$n_0(\nu) = \frac{1}{(2\pi)^{3/2}} R_{\nu}^2 e^{-\nu^2/2} \int_0^\infty \left( x^2 + e^{-x^2} - 1 \right) \exp \left[ \frac{-(1/2)(x - \nu)^2}{(1 - \gamma^2)} \right] \frac{dx}{2\pi(1 - \gamma^2)^{3/2}}$$

(2)

Here $\sigma_\nu^2 = [1/(2\pi)^2] \int |W_k(u)|^2 u^2 d^2 u$ for our noise spectrum, $R_{\nu} = \sqrt{2}(\sigma_n/\sigma_\nu)$, and $\gamma = \sigma/\sigma_n$. The difference between the peak number density of the total SZ map and equation (2) provides a zeroth-order approximation to the cluster luminosity function: $n_{cl}^{(0)}(\nu) = n_p(\nu) - n_0(\nu)$. At a high-$\nu$ tail ($\nu > 1$) where the mean number density of noise peaks drops practically to zero, $n_{cl}(\nu) \approx n_{cl}^{(0)}(\nu) \approx n_p(\nu)$. At a low-$\nu$ range ($0 \leq \nu \leq 2$) where Gaussian noise strongly dominates, $n_{cl}(\nu) \approx n_0(\nu)$, and $n_{cl}^{(0)}(\nu)$ measures just the Poissonian noise scatter around equation (2).

The intriguing section is at the medium-$\nu$ range ($\nu \sim 3$), where $n_{cl}^{(0)}(\nu)$ includes not only the noise scatter but also the noise-contaminated cluster peaks to a nonnegligible degree. With the conventional cluster selection procedure based on the amplitude cutoff, the number density of cluster peaks in this intermediate range cannot be determined since the peaks in this range are all disregarded as noise. Thus, this is the medium-amplitude section where one needs a better analysis technique to find the number density.

Noise contaminates the cluster number density by changing its amplitude. Let $x$ measure the noise contamination effects on the peak amplitude such that $\nu = \nu_0 + x$, where $\nu_0$ and $\nu$ are the real and contaminated amplitude of a cluster peak, respectively. For a Gaussian noise, $x$ can be assumed to be a Gaussian variable. Now the probability distribution of $\nu$ can be written as a convolution of the probability distributions of $\nu_0$ and $x$. In terms of the number density, one can say

$$n_{cl}^{(0)}(\nu) = \frac{N_{cl}^{tot}}{N_{cl}^{tot} + N_{sys}^{tot}} \int p(x)n_{cl}(\nu - x)dx,$$

(3)

where $p(x) = (1/\sqrt{2\pi}) e^{-x^2/2}$ and $N_{cl}^{tot}$ and $N_{sys}^{tot}$ are the total number of peaks of the SZ map and that of real clusters, respectively.

Equation (3) implies that the cluster luminosity function, $n_{cl}(\nu)$, can be found by the deconvolution of the $n_{cl}^{(0)}(\nu)$ and $p(x)$. In theory, the deconvolution of $n_{cl}^{(0)}(\nu)$ and $p(x)$ could be easily conducted, just by dividing the Fourier transform of
Let and be the real and corrupted power spectrum of , respectively; then the Wiener filter in the Fourier -space () is the Fourier counter part of () can be written as 

\[ W_r(v_i) = P_r(v_i)/P(v_i) \]

One can easily see that \( W_r(v_i) \approx 1 \) in which the error is negligible, and that \( W_r(v_i) \approx 0 \), in which the error is dominant. Unfortunately, we cannot determine the exact functional form of \( W_r(v_i) \) since the only available quantity is \( P_r(v_i) \). Nevertheless, since the Wiener filter works in the least-squares sense (Press et al. 1992), even a fairly reasonable approximation to \( W_r(v_i) \) can make it work quite well. Figure 1 plots \( P_r(v_i) \), showing that there is a sharp boundary between the noise-dominant and negligible sections, where \( P_r(v_i) \) has two distinct behaviors. Finding a turning point \( v_t \), by eye, we approximated the Wiener filter by a step function such that \( W_r(v_t) = \Theta(|v_t| < v_0) \), given the asymptotic behaviors of \( W_r(v_t) \).

Here is our recipe for the determination of the cluster luminosity function from the total SZ map: Construct \( n_{cl}^{(0)}(v) \) on a one-dimensional discrete grid by counting the peak number density of the total SZ map and subtracting equation (3) from it. Calculate its power spectrum, \( P_r(v_i) \), by measuring the mean square amplitude of its Fourier transform. Plot \( P_r(v_i) \) to find the turning point \( v_t \). Approximate \( W_r(v_t) \) as a step function such that \( W_r(v_t) = \Theta(|v_t| < v_0) \). Convolve \( n_{cl}^{(0)}(v) \) by \( W_r(v_t) \) and deconvolve the Wiener filter by a step function such that \( W_r(v_t) = \Theta(|v_t| < v_0) \), given the asymptotic behaviors of \( W_r(v_t) \).

In the upper panel of Figure 2, we show the cumulative cluster luminosity function \( N_{cl}(\geq v) \) (solid line) reconstructed from the SZ map with the above recipe and compare it with the real distribution (filled squares). We also plot the cumulative peak number density of the total SZ map (long-dashed line), the cumulative noise mean number density (dashed line), and the cumulative zeroth-order cluster luminosity function (dotted line) for comparison. Figure 2 reveals that the reconstructed cluster luminosity function is indeed in good agreement with the real distribution, especially in the medium range of \( 2 \leq v \leq 4 \). Note also that in the low- \( v \) section \( (v \leq 1) \), noise dominates the SZ peaks, while in the high- \( v \) tail \( (v \geq 4) \), the total SZ peaks are mainly the cluster peaks, as expected.

When determining \( n_{cl}(v) \), the total number of cluster peaks, \( N_{cl}^{tot} \), are assumed to be given as priors. In case \( N_{cl}^{tot} \) is not available, we can still determine the probability density distribution of cluster peaks, \( P_r(\geq v) = n_{cl}(\geq v)/N_{cl}^{tot} \). The lower panel of Figure 2 plots the cumulative probability density distributions that can be determined using our analysis technique without any prior. Again, the real and reconstructed distributions agree with each other quite well. We tested our technique against different realizations of the SZ maps by varying the total number of cluster peaks and distribution shapes, and we found it quite robust.

3. SUMMARY AND DISCUSSIONS

We have developed a useful analysis technique to determine the cluster luminosity function efficiently, using AMiBA, a drift-scan interferometric SZ survey, as a model experiment. We have simulated a total SZ map using the Monte Carlo method, and we have counted the peak number density from it. The total SZ map is constructed by combining a Gaussian noise field with cluster sources. We noted that the peak number density of the SZ map at a medium peak amplitude range has nonnegligible contributions from the cluster peaks, including the noise contamination effects.

To determine the cluster number density, i.e., the cluster luminosity function, we first have to measure the zeroth-order cluster luminosity function by subtracting the available mean noise number density from the peak number density of the total SZ map. We have quantified the noise contamination effects included in the zeroth-order approximation by a single Gaussian
variable, and we have found that the cluster luminosity function can be expressed as the deconvolution of the zeroth-order approximation by the one-dimensional Gaussian distribution.

We have stabilized the deconvolution process by convolving the zeroth-order approximation with a Wiener filter. The approximate functional form of the Wiener filter has been determined from the information on the power spectrum of the zeroth-order approximation. Finally, by deconvolving the Wiener-filtered zeroth-order approximation of the cluster luminosity function, we have determined the cluster luminosity function from the simulated SZ map. We have compared the reconstructed (cumulative) cluster luminosity function with the real one and have found good agreements between them, especially in the medium-amplitude range where the conventional technique fails.

The consequence of the statistical strategy presented here is that it can allow us to find the cumulative distribution of the sources, even when the number of sources occupies only small fraction of the total number of maximum peaks. We also expect this statistical strategy to be applied to the construction of the cluster mass function in weak gravitational lensing analysis.

However, it is worth noting that the accuracy of the measurement of the cluster luminosity function may be improved on by improving the accuracy of the approximation of the Wiener filter, and it is also worth noting that although our technique determines the cluster number density efficiently, it cannot select the cluster peaks from the SZ map. Furthermore, to examine the usefulness of our analysis technique in real practice, testing it against real SZ hydrodynamic simulations will be necessary. Our future work will be in this direction.

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