Phantom Energy Accretion onto Black Holes in Cyclic Universe

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Abstract

Black holes pose a serious problem in the cyclic or oscillating cosmology. It is speculated that, in the cyclic universe with phantom turnarounds, black holes will be torn apart by the phantom energy before turnaround before they can create any problems. In this paper, using the mechanism of the phantom accretion onto black holes, we find that black holes do not disappear before the phantom turnaround. But the remanent black holes will not cause any problems due to the Hawking evaporation.

1 Introduction

The scenario of cyclic or oscillating universes is an attractive idea in the theoretical cosmology since it is expected to avoid the initial singularity by providing an infinitely oscillating universe. This idea has a long history [1, 2]. In recent years, there have been many discussions on such a topic [3, 4, 5]. Generally, however, cyclic models of the universe confront a serious problem: black holes. If black holes formed during expansion of the universe survive into the next cycle, they will grow even larger from cycle to cycle and act as

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defects in an otherwise nearly uniform universe. Eventually, the black holes will occupy the entire horizon volume, and then the cyclic models break away.

In \cite{4}, by assuming the existence of the phantom dark energy and using the modified Friedmann equation, the authors suggested an oscillating cosmology. It is argued that the any black holes produced in an expanding phase in the universe are torn apart before they can create problems during contraction. A rough calculation has been given. In general relativity, the source for a gravitational potential is the volume integral of $\rho + p$, where $\rho$ is the energy density in the universe and $p$ is the pressure. So an object of radius $R$ and mass $M$ is pulled apart when $-(4\pi/3)(\rho + p)R^3 \sim M$. Then a black hole of mass $M$ and horizon radius $R = 2GM$ is pulled apart when the energy density of the universe has climbed up to a value $\rho_{BH} \sim (3/32\pi)(M^2G^3|1+w|)^{-1}$, where $p = w \rho$ is the equation of state (for phantom dark energy $w < -1$). Then black holes will be torn apart before turnaround, if $\rho_{BH} < \rho_c$, where $\rho_c$ is the critical energy density in the cyclic model, namely the energy density corresponding to the turnaround (and bounce).

However, it is obvious that the destruction of black holes is not an instantaneous event just happened at $\rho \sim \rho_{BH}$, but a process. At the same time, the qualitative analysis in \cite{4} is too rough and can not be taken as a mechanism of tearing up black holes. We needs such a mechanism in order to know whether the analysis in \cite{4} does work or not. In \cite{5}, the authors have suggested a mechanism in which, by accreting the phantom energy, the mass of a black hole decreases at the rate $\dot{M} = 4\pi AM^2(\rho + p)$, where $A$ is a positive dimensionless constant. Replacing $\rho$ and $p$ by the effective energy density $\rho_{eff} = \rho(1 - \rho/\rho_c)$ and the effective pressure $p_{eff} = p(1 - 2\rho/\rho_c) - \rho^2/\rho_c$ respectively, the author of \cite{7} used this mechanism, $\dot{M} = 4\pi AM^2(\rho_{eff} + p_{eff})$, to study the destruction of black holes in cyclic models. The conclusion in \cite{7} is that, in the expanding stage of the universe, through the phantom accretion, the masses of black holes first decrease and then increase. And at the turnaround, black hole masses restore their initial values. So it is claimed that black hole cannot be torn up in the cyclic model of \cite{4}.

But in \cite{8}, the author argued that $\rho_{eff}$ and $p_{eff}$ are not proper physical quantities. Taking into account this view, in this paper, we will study the destruction of black holes in the phantom cyclic universe by using $\dot{M} = 4\pi AM^2(\rho + p)$. Similar application in the brane cosmology has been discussed in \cite{9}.

The paper is organized as follows. In section 2, we analyze the phantom energy accretion onto black holes in the cyclic model of \cite{4}. Section 3 contains
conclusion and discussion.

2 Accretion of Phantom Fluid in Cyclic Models

Here, we consider that the dark energy fluid, covers the whole space in the homogeneous and isotropic form with the dark energy density $\rho$ and pressure $p$. For an asymptotic observer, the black hole mass $M$ changes at the rate

$$\dot{M} = 4\pi AM^2(\rho + p).$$

(1)

Here and after, the overhead dot denotes the derivative with respect to the cosmic time and $G = c = 1$. Assuming the universe dominated by the dark energy, the expansion of the universe is governed by the Friedmann equation

$$H^2 = \frac{8\pi}{3}\rho$$

(2)

and the local energy conservation law of the dark energy

$$\dot{\rho} + 3H(\rho + p) = 0,$$

(3)

where $H \equiv \dot{a}/a$ is the Hubble parameter. For the phantom dark energy with the equation of state $w = p/\rho < -1$, $\rho \propto a^{-3(1+w)}$ increases as the expansion of the universe. In order to avoid the big rip [10], we use the modified Friedmann equation

$$H^2 = \frac{8\pi}{3}\rho(1 - \frac{\rho}{\rho_c}),$$

(4)

where $\rho_c$ is the critical energy density about at the order of the Planck density. The modified Friedmann equation (4) has been suggested from different set-ups [11] [12] [13]. Then, in the expanding phase of the universe, the phantom energy density $\rho$ increases. When $\rho = \rho_c$, due to Eq.(4), a turnaround occurs. After the turnaround, the universe begins to contract. In the contracting phase, the energy densities of other non-phantom components in the universe increase, and eventually dominate the evolution of the universe. When the dominant energy density reaches the critical energy density $\rho_c$ again, a bounce occurs. Roughly, this is the scenario of an oscillating cosmology in [4]. Now let us study the evolution of black hole masses as the phantom energy accretion in the scenario.
2.1 Before Turnaround

Using the two equations (1) and (3), we get

$$\frac{dM}{M^2} = -\frac{4\pi A}{3H} d\rho.$$  

(5)

Before the turnaround, we have

$$H = \sqrt{\frac{8\pi}{3}} \sqrt{\rho (1 - \frac{\rho}{\rho_c})}.$$  

(6)

Substituting this equation into Eq.(5), we get

$$\frac{dM}{M^2} = -\frac{D}{\sqrt{\rho (1 - \rho/\rho_c)}} d\rho,$$  

(7)

with $D \equiv \sqrt{\frac{2\pi}{3}} A$. The integration of (7) gives

$$M = \frac{M_i}{1 + 2DM_i \sqrt{\rho_c} (\arcsin \sqrt{\rho/\rho_c} - \arcsin \sqrt{\rho_i/\rho_c})},$$  

(8)

with $\rho_i \leq \rho \leq \rho_c$. Here $M_i$ and $\rho_i$ denote respectively the black hole mass and the phantom energy density at the moment when the phantom energy begins to dominate the evolution of the universe. Generally, $\rho_i \ll \rho_c$. Then we obtain

$$M \simeq \frac{M_i}{1 + 2M_iD\sqrt{\rho_c} \arcsin \sqrt{\rho/\rho_c}}.$$  

(9)

So the black hole mass at the turnaround is

$$M_c \simeq \frac{M_i}{1 + \pi M_i D \sqrt{\rho_c}}.$$  

(10)

This result means that black holes, by accreting the phantom energy, do not disappear before the turnaround. But our result is different from the result of [7]. In [7], it is claimed that, through the phantom accretion, black hole mass will decrease first, and then increase until restoring its initial mass at the turnaround. Here, our result, Eq.(8), indicates, through the phantom accretion, the black hole mass always decreases as the expansion of the universe, and, at turnaround, reaches the minimum $M_c$ in the expanding phase. For $M_i \gg M_p = G^{-1/2}$, $M_c$ becomes independent of $M_i$

$$M_c \simeq \frac{1}{\pi D \sqrt{\rho_c}}$$  

(11)
2.2 After Turnaround

After turnaround, the universe begins to contract and the phantom energy density $\rho$ decreases as the contraction. Eq. (8) cannot be used directly because $H$ is negative in the contracting phase of the universe. Substituting the equation

$$H = -\sqrt{\frac{8\pi}{3}} \sqrt{\rho(1 - \frac{\rho}{\rho_c})}$$

into Eq. (5), we obtain

$$\frac{dM}{M^2} = -\frac{D}{\sqrt{\rho(1 - \rho/\rho_c)}} d\rho.$$  \hspace{1cm} (13)

The integration of the equation gives

$$M = \frac{M_c}{1 + DM_c\sqrt{\rho_c}(\pi - 2 \arcsin \sqrt{\rho/\rho_c})},$$

with $\rho \leq \rho_c$. Then Eq. (14) shows that, after turnaround, as the universe contracting, the phantom energy density decreases and the black hole masses continue to decrease. When $\rho \ll \rho_c$, the black hole mass is

$$M_f \simeq \frac{M_c}{1 + \pi DM_c\sqrt{\rho_c}}.$$  \hspace{1cm} (15)

For $M_i \gg M_p$, using Eq. (11), we find the final mass of black holes is

$$M_f \simeq M_c \frac{1}{2} \simeq \frac{1}{2\pi D\sqrt{\rho_c}}.$$  \hspace{1cm} (16)

2.3 Destruction of Black holes

The analysis above shows that, by accreting the phantom energy, a black hole in the cyclic universe with phantom turnaround does not disappear, but has a remanent mass $M_c$ at the turnaround. This means, through the phantom energy accretion, black holes in the cyclic model of [4] can not be pulled apart before turnaround. Then it seems that the argument of destruction of black holes in [4] is wrong and the problem of black holes still stand.

However, it has been argued in [4] that for a black hole with mass $M = 10^5M_p$, it Hawking evaporates in a time $\tau \sim (25\pi M^3/M_p^4) \sim 10^{-27}$ sec and
causes no problems. Let us estimate the value of $M_c$. We have taken $G = M_p^{-2} = 1$ in the derivations above. Keeping $M_p$ explicitly, we can rewrite Eq.(10) as

$$M_c \simeq \frac{M_i}{1 + \pi M_i D \sqrt{\rho_c / M_p^3}}.$$  

(17)

$D$ may be taken as a constant of order unity and $\rho_c \sim M_p^{-4}$. Then we have $M_c \sim \frac{M_i}{1 + M_i / M_p}$. This implies, for a black hole with $M_i \gg M_p$, the remanent mass $M_c$ is about at the order of the Planck mass $M_p$ and the remanent black hole Hawking evaporates in a time $\tau \sim 10^{-43}$ sec at the order of the Planck time. Then, fortunately, the remanent black holes do not cause problems. Now, we know, in the cyclic model with phantom energy turnarounds, black hole masses decrease due to the phantom energy accretion. Before a turnaround, black holes can not be torn apart, but the remanent black holes with masses $M_c \sim M_p$ are left at the turnaround. However, the remanent do not cause problems in the cyclic model because of the Hawking evaporation.

3 Conclusion and Discussion

In the cyclic or oscillating cosmology, black holes pose a serious problem. In [4], an oscillating cosmology with phantom energy turnarounds is suggested and it is argued roughly that black holes are torn apart before the turnaround. In [6], a successful mechanism in which the black hole masses decrease due to the phantom energy accretion is obtained. In this paper, by using the result of [6], we have surveyed the destruction of black holes in the cyclic cosmology with phantom energy turnarounds.

Similar work has been done in [7]. Their conclusion is that, through the phantom accretion, black hole mass will decrease first, and then increase until restoring its initial mass at the turnaround. Then the author claimed the problem of black holes still stands in the cyclic model with phantom turnaround. However, this conclusion is obtained by using the effective energy density and pressure which are unphysical variables [8], rather than the energy density and pressure. Using the energy density and pressure, we find that, due to the phantom energy accretion, the mass of a black hole always decreases before turnaround, and at turnaround reaches the remanent mass $M_c$. After the turnaround the remanent black hole mass continues to decrease. For black holes with mass much more massive than the Planck mass,
the black hole masses approach $M_c/2$ asymptotically. Of course, our evaluation after the turnaround is not very rigid, because, as the contraction of the universe, the phantom energy will become subdominant in the universe eventually.

So the remanent mass $M_c$ implies that black holes in the cyclic model cannot be pulled apart by the phantom energy before turnaround. Here, we note, if the Friedmann equation (2) is used, no turnaround occurs, but the big rip. And then we get

$$M = \frac{M_i}{1 + 2DM_i(\sqrt{\rho} - \sqrt{\rho_i})}.$$  

So, in this case, as $\rho \rightarrow \infty$, black holes disappear $M \rightarrow 0$. But, in the cyclic cosmology, the modified Friedmann equation (1) is used. The big rip is avoid and then, through the phantom accretion, black holes can not be eliminated. This result can be obtained in another way. Only using Eq. (1), we can obtain

$$\frac{1}{M} - \frac{1}{M_i} = -\int_{t_i}^{t} 4\pi A (\rho + p) dt$$

The black hole mass $M$ will not be zero unless the integration on the right-hand side is divergent. However, A key property of the cyclic cosmology is that the energy density and pressure are always well defined. So it is impossible for black holes in the cyclic model to be eliminated by accreting the phantom energy.

However, fortunately, we find the remanent masses of black holes at turnaround do not cause problems. The reason is that these remanent black holes Hawking evaporate in a time $\tau \sim 10^{-43}$.

So our analysis indicates that, although, through the phantom energy accretion, black holes do not disappear before turnaround, they do not cause problems in the cyclic models with phantom turnovers.

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