Distributed Wake-Up Scheduling for Energy Saving in Wireless Networks

Francesco De Pellegrini, Karina Gomez, Daniele Miorandi and Imrich Chlamtac

Abstract—A customary solution to reduce the energy consumption of wireless communication devices is to periodically put the radio into low-power sleep mode. A relevant problem is to schedule the wake-up of nodes in such a way as to ensure proper coordination among devices, respecting delay constraints while still saving energy. In this paper, we introduce a simple algebraic characterisation of the problem of periodic wake-up scheduling under both energy consumption and delay constraints. We demonstrate that the general problem of wake-up times coordination is equivalent to integer factorization and discuss the implications on the design of efficient scheduling algorithms. We then propose simple polynomial time heuristic algorithms that can be implemented in a distributed fashion and present a message complexity of the order of the number of links in the network.

Numerical results are provided in order to assess the performance of the proposed techniques when applied to wireless sensor networks.

Index Terms—Wireless Networks, Energy Saving, Wake-up Scheduling, Chinese Remainder Theorem

I. INTRODUCTION

In wireless networks with battery-operated devices, energy saving mechanisms are of paramount importance in order to maximize network lifetime. Measurements have shown that the power consumption associated to the reception of packets is of the same order of magnitude as that involved in the packets transmission [1], [2], [3]. Even worse, due to the power drainage operated by the radio frequency (RF) amplifier, the energy expenditure for passive communication operations – e.g., overhearing the channel for collision avoidance mechanisms – has a very high energy cost as well. As a consequence, power saving mechanisms typically force low-power consumption modes for the RF interface, which in turn inhibits the communications capabilities (transmission, reception and channel sensing) of wireless devices for a certain fraction of time. The typical statement to this respect is that nodes are put into sleep mode.

A customary solution to increase network lifetime is therefore to periodically put nodes into sleep mode. This corresponds to introducing a duty cycle, intertwining sleep and active periods at the device level. Such approach is already in use in a variety of wireless technologies, including power-save modes and beaconing techniques in IEEE802.11 [4] and wake-up cycling methods in wireless sensor networks [5].

The introduction of wake-up scheduling has been identified in the past as a major source of performance degradation in wireless networks. Nodes going in sleep mode increase indeed the latency associated to the delivery of messages. This has been subject to a number of research studies, in particular showing that duty cycles may lead to a data forwarding interruption problem in wireless sensor networks [2], [6].

The problem of scheduling wake-up times may appear at first sight a standard optimization problem: one tries to maximize the wake-up periods while satisfying a given delay bound [7], [8] in order to minimize the associated energy consumption. However, in this trade-off there exists a further element of complexity since the scheduling of activity periods requires coordination at the network level. Indeed, nodes in sleep mode are not able to receive incoming messages, so that it is necessary that both transmitting and receiving nodes become active at the same time. This problem is often referred to as scheduling of rendezvous points among neighboring nodes. While it can be regarded as a synchronization issue, it is inherently different from clock synchronization [9], [10], where one typically aims at synchronizing devices’ clocks network-wide using minimal local message exchange.

A recent research line has addressed the design of quorum-based protocols for the power saving duty cycles [4], [11], [12], [13], [14]. In general, wake-up schedules should be robust with respect to loss of synchronization due to clock skew and should account for possibly different energy/delay constraints on various nodes (due to, e.g., different types of battery installed in different devices etc.). Furthermore, protocols for determining the wake-up schedules should be able to adapt to changes in network topology, due to, e.g., node failures or addition of new nodes to the network. In the aforementioned works, the target is the definition of algorithms and protocols able to ensure that two neighboring nodes are able to communicate (i.e., both are active at the same time) at least once within a given (bounded) time-frame. A different approach to reduce energy consumption is adaptive listening [8], [15], [16], whereby nodes enter a low-power mode if no activity is detected on the channel for a given time interval. However, in this approach, no guarantees can be given in terms of delay and/or energy constraints.

In this work we take a different perspective and aim to answer to the following questions: Assume that energy and delay constraints have been assigned network-wide: do feasible duty-cycle schedules exist? If so, what is the related energy-delay trade-off? Furthermore, do algorithms exist that achieve network-wide distributed scheduling of duty cycles, and scale in the number of required messages?

Our main contributions can be summarized as follows:
• a simple algebraic characterization of the wake-up scheduling problem;
• the analysis of the complexity of the wake-up scheduling problem: it is proved equivalent to that of integer factorization;
• heuristics and algorithms that solve the wake-up scheduling problem with message complexity linear in the number of links in the network;
• fully distributed protocols implementing the aforementioned algorithms and test them in a system-level simulator to evaluate their performance.

The remainder of the paper is organized as follows. Sec. II reports on related works in the literature. Sec. III introduces the system model and formally define the problem of wake-up scheduling problem. Then, we analyze in Sec. IV the complexity of the aforementioned problem by relating it to integer factorization while Sec. V introduces algorithms solving the problem with low messages overhead. In Sec. VI we evaluate the performance of the proposed protocols, obtained by means of system-level simulations. Finally, Sec. VII is devoted to final a conclusions and pointers to promising research directions.

II. RELATED WORK

In wireless networks, periodic wake-up is a convenient mean to avoid idle listening to the channel and to prolong nodes’ lifetime [5], [3], [2], [16]. Typically, the duty cycle (defined as the fraction of time a node is in the active state) is set to values of the order of some percent [2], [16]. This is feasible when the data rate is low, i.e., on the order few packets per minute, as in low-rate wireless sensor networks. In order to schedule periodic wake-ups, a simple solution, adopted by BMAC and other WSN protocols, is to employ preamble sampling [5], [3]. Basically, the preamble is set long enough to guarantee node discovery at each wake-up.

In order to avoid inefficiencies, it has been proposed to match the RF interface activation, i.e., the sleep period, to the traffic pattern. This is the case of SMAC or TMAC protocols [2], [17], [16]. In particular SMAC synchronizes the duty cycle by coordinating neighbouring sensors to the same time slot to reduce the time spent listening to the channel. Adaptive change of the duty cycle is supported in TMAC [17], whereas [16] matches duty cycles to the dynamics of the routing protocol. Tuning such protocols network-wide is typically not considered; scalability issues do indeed arise, due to the need of a potentially large amount of signaling messages. This is the core issue that we address in this paper: we design distributed algorithms for wake-up scheduling that present linear message complexity in terms of the network size and can operate local adaptation in case of nodes joining/leaving in a fully distributed fashion.

In ad hoc networks literature, centralized planning of wake-up schedules was addressed in several works. Solutions that are close to the ones proposed in this paper can be found in [4], [11], [12], [13], [14]. In [4] the target is the design of power saving protocols for WiFi networks, using quorums design and leveraging the power-save mode provided by the IEEE 802.11 standard. In [11] methods for designing asynchronous and heterogeneous wake-up schedules are presented, where the heterogeneity may come from either different applications running or by different node-level features. Along the same line, [12] and [18] provide solutions for creating wake-up schedules whereby the ability of neighbouring nodes to communicate (i.e., to be active in the same time interval) within a wake-up cycle is ensured by combinatorial means. The complexity of quorum design limits the applicability of such combinatorial techniques to regular or simple topologies; further insight into the design of quorums is found in [19].

Some other works address the issue of network performance in the presence of duty cycles [3], [7]. The goal of such works is to maximize the throughput and minimize the delay, while meeting a given average power consumption constraint. It is shown that the problem is, in general, NP-complete; an analysis is provided for regular topologies (line, grid and tree networks in [7], tree and ring networks in [8]). The distributed heuristic provided in [8] does not provide guarantees in terms of delay bounds. Centralized approaches only are considered in [7].

In [20] the authors address the joint design of routing and wake-up scheduling protocols. The idea is that routing should be aware of the wake-up schedules and vice versa. A distributed implementation (based on the use of distributed versions of Bellman-Ford algorithm) is also possible. The need to coordinate routing and wake-up scheduling may create issues related to the amount of signaling messages to be exchanged in rapidly changing topologies.

Main contributions

In this work we expose the equivalence of the wake-up scheduling problem to integer factorization. We observe that this follows naturally as soon as one imposes constraints on both delay and energy consumptions. Using basic algebraic tools [21] we provide a framework that relies on a very limited set of input parameters, providing means to map energy consumption constraints and end-to-end delay requirements to wake-up scheduling.

We further provide fully distributed heuristic algorithms that solve the wake-up scheduling problems with a message complexity $O(M)$ messages, where $M$ is the number of links in the network. The proposed schemes are validated numerically using system-level simulations.

III. SYSTEM MODEL

We consider a multi-hop wireless network represented as a graph $G(V, E)$ where $G$ is a set of $N$ vertices corresponding to nodes $j = 1,\ldots, N$, and $E$ is a set of $M$ edges. Edge $ij$ connects vertices $j$ and $i$ if the two nodes are within mutual communication range. We assume a discrete time model and denote by $T_0$ the time slot duration.

In the following we will assume $T_0 = 1$ for notation’s sake. We consider a common time axis, and denote the origin by 0.
Nodes do not need to be aware of such common time axis: we will use it only for presentation purposes (see Sec. IV-A). \(N_g(i)\) is the set of neighbors of node \(i\); we further assume that nodes are synchronized on time slot boundaries.

We now define a periodic wake-up schedule. To illustrate the concept and notation, we refer to the simple example reported in Fig. 1. Two neighboring nodes, namely node 1 and node 2, wake-up with period \(n_1 = 5\) and \(n_2 = 3\), respectively. This means that they wake every \(n_1\) (respectively: \(n_2\)) slots. However, their wake-up cycles can start from a different “origin”. In the example, node 1 period starts at slot 1 and node 2 period at slot 2, respectively. In particular, let \(\alpha_i\) be the local time origin for node \(i\): in the example, the two nodes wake up at time 0 with respect to their local time, but at time 1 and 2 with respect to the common time.

We want to characterize the conditions under which for every pair of nodes in the network, a wake-up schedule provides a set of times when nodes are both awake and can communicate; those times are called also rendezvous times.

Nodes may have multiple active slots within a wake-up period. We denote by \(A_i\) the set of such active slots for node \(i\). In other words, node \(i\) follows an activation cycle with period \(n_1\), whereby the active slots are characterized by the set \(A_i\). We call an element of \(A_i\) a phase.

Formally, we define a periodic wake-up schedule for node \(i\) as a function \(S(i) = \{A_i, n_1\}\), where \(\{A_i, n_1\}\) is a pair given by the wake-up epoch set \(A_i \subseteq \{0, 1, \ldots, n_1 - 1\}\) and the wake-up period \(n_1 \in \mathbb{Z}\). Under a wake-up schedule, node \(i\) will wake up at times

\[
A_i + n_1 \mathbb{Z} := \{a + k \cdot n_1 \mid k \in \mathbb{Z}, a \in A_i\}
\]

We say that node \(i\) performs a periodic wake-up with period \(n_1\) and epochs in \(A_i\). With a slight abuse of language, we will call a wake-up schedule the set of functions \(S = \{S_i\}_{i=1}^{N}\); also, for ease of reading, we will denote \(a_{ij} \in A_i\) the phase that node \(i\) uses to communicate with node \(j\).

Denote \(X = \{x_{ij}\}\) the set of the meeting times (or: rendezvous points) for nodes \(i\) and \(j\), i.e., the set of time slots when node \(i\) and \(j\) are both active: \(X = \{A_i + n_1 \mathbb{Z}\} \cap \{A_j + n_2 \mathbb{Z}\}\). As it will be clear in the following, only two cases are possible: \(X = \emptyset\) or \(|X| = \infty\), i.e., either there are no meeting times or there is an infinite number thereof. Quest for periodicity is the reason for such dichotomy.

We consider first energy constraints. Assuming that the energy expenditure is proportional to the fraction of time a node is in the active state, one wants to prolong the lifetime of node \(i\) by letting the duty cycle \(|A_i|/n_i\) satisfy constraints of the form:

\[
n_i \geq L_i \cdot |A_i|, \quad i = 1, \ldots, N
\]

We consider the possibility of nodes having different energy constraints (i.e., different values for \(L_i\)). This may reflect, e.g., heterogeneity in the battery capacity.

Further, depending on the specific applications running on top of the network, specific requirements for the end-to-end delay traffic may have to be satisfied. For instance, the delay bound may represent the maximum latency that is allowed for a certain sensor reading.

In the remainder of the paper, we assume that constraints of such a type are given in terms of the maximum delay between two subsequent rendezvous between node \(i\) and any of its neighbors \(j\). We denote such constraint by \(U_i\); the constraint can then be written as:

\[
x_{ij}^{n_{ij} + 1} - x_{ij}^{n_{ij}} \leq U_i, \quad \forall j \in N_g(i).
\]

It is important to notice here local constraints \(L_i\) and \(U_i\) are input to the problem. In App.E, we show how end-to-end delay constraints can be turned into constraints of the form \(\ref{eq:energy_constraints}\), while accounting at the same time for the presence of energy constraints as in \(\ref{eq:energy_constraints}\).

**Definition 1** A wake-up schedule \(S\) is said to be feasible if for every node \(i\), the number of rendezvous points with node \(j\), where \(j \in N_g(i)\) is infinite and it respects the energy constraints \(\ref{eq:energy_constraints}\).

**Definition 2** A wake-up schedule is said to be tight if it is feasible and if it further respects the constraints \(\ref{eq:energy_constraints}\).

**Definition 3** A wake-up schedule is said to be strictly feasible (respectively: tight) if it is feasible (respectively: tight) and \(|A_i| = 1\) for all \(i = 1, \ldots, N\).

The wake-up scheduling problem can be formalized as:

**Problem 1** (Strong WAKE-UP) Given graph \(G = (V, E)\) and constraints \(\ref{eq:energy_constraints}\) and \(\ref{eq:delay_constraints}\), determine a (strictly) tight wake-up schedule.
A trivial necessary condition is that $U_i \geq L + 1 = 1 + \max\{L_j, \ldots, L_k\}$, $\forall i,j,k \in E$, which we assume holds in the remainder of the paper.

If a wake-up schedule is feasible but not tight, some of the constraints (2) are not satisfied. This origins a weaker problem that is

**Problem 2 (Weak wake-up)** Given graph $G = (V,E)$ and constraints (2) and (7), determine a (strictly) feasible wake-up schedule.

We denote as violation a condition such that constraints (3) are not satisfied and yet a wake-up schedule is feasible.

### A. Key Assumptions

We summarize here the main assumptions that we will use in the remainder of the paper:

- Nodes do not have global knowledge of the common time axis;
- Nodes do not have global knowledge of the $L_i$ and $U_i$ values assigned to other nodes in the network;
- Energy consumption is directly proportional to the duty cycle. This implies that we neglect non-linear effects and energy costs associated to wake-up operations;
- Neighboring nodes are synchronized on common time slot boundaries [2].

We remark that our results apply to general networks; however, due to the customary need to prolong battery lifetime, wireless sensor networks will be used as our reference case throughout the paper.

### IV. Characterization of the Wake-up Problem

In this section we provide an algebraic characterization of the WAKE-UP problem. Using standard notation, we denote by $(r,s)$ and by $[r,s]$ the greatest common divider (gcd) and the least common multiplier (lcm) of $r,s \in \mathbb{Z}$, respectively [1]. Integer $r$ divides $s$, which we write as $r | s$, if $s = r \cdot v$ for some integer $v$.

We first recall a fundamental result:

**Theorem 1 (Chinese Remainder theorem [21])** Let $n_1, n_2, a_1, a_2 \in \mathbb{Z}$, consider the system

\[
\begin{aligned}
    x & \equiv a_1 \pmod{n_1} \\
    x & \equiv a_2 \pmod{n_2}
\end{aligned}
\]

Then,

(i) system (3) has a solution if and only if $(n_1, n_2)|a_1 - a_2$;
(ii) the general solution of (3) is given by $x = x_0 \mod [n_1, n_2]$, where $x_0$ is a particular solution of (3).

In order to derive a particular solution $x_0$ at step (ii), the extended Euclidean algorithm [21] guarantees that integer $u_1, u_2$ exist that solve the following Bézout’s identity

$$n_1 u_1 - n_2 u_2 = (n_1, n_2)$$

from which

$$x_0 = a_1 - n_1 \frac{u_1(a_1 - a_2)}{(n_1, n_2)} = a_2 - n_2 \frac{u_2(a_1 - a_2)}{(n_1, n_2)}$$

The extended Euclidean algorithm has complexity $O(\log^3 \max\{n_1, n_2\}) = O(1)$. It is clear that when system (3) consists of $k > 2$ congruences, it can be solved iteratively; the solution can be determined repeating the operation for a single pair $k - 1$ times. The time complexity for one step is $O(1)$, which in turn becomes on the order of $O(k)$.

From the model introduced in Sec. III the solution of the WAKE-UP problem has to satisfy the following relation:

$$a_{ij} + h \cdot n_i = a_{ji} + k \cdot n_j,$$  \hspace{1cm} (4)

for $ij \in E$ and $a_{ij} \in A_i$ and $a_{ji} \in A_j$, where $h, k \in \mathbb{Z}$. The sequence of rendezvous times $\{x^t_i\}_{t \in \mathbb{Z}}$ can therefore be seen as solution of (3), with $a_1 = a_{ij}, n_1 = n_i, a_2 = a_{ji}$ and $n_2 = n_j$.

Existence of feasible schedules follows as a consequence of the Chinese Remainder theorem.

**Theorem 2** A strictly feasible wake-up schedule always exists, and can be constructed in time $O(N)$ with a fully distributed asynchronous algorithm requiring $O(M)$ messages.

**Proof:** A feasible wake up schedule is constructed in $N$ steps choosing for all nodes $n_i \geq L_i$ such that $(n_i, n_j) = 1$ for all pairs neighboring nodes $i$ and $j$. For each edge we can apply the Chinese Reminder Theorem to its end nodes. Irrespective of $a_i$ and $a_j$, it holds $1 = (n_i, n_j)(a_i - a_j)$ at each step. The algorithm attains by construction a feasible wake-up schedule.

**Remark 1** The above construction shows a basic effect: the request of synchronization of duty cycles introduces a delay drift (DD). In the worst case since $[n_i, n_j] = n_i \cdot n_j$ products become exponentially large with respect to the network diameter. In order to minimize such drift, one would request $[n_i, n_j] = \min\{n_i, n_j\}$ at each step, i.e., either $n_i | n_j$ or vice versa.

We get therefore:

**Corollary 1** The Weak WAKE-UP problem can be solved in polynomial time.

In the general case, when constraints on the maximum wake-up period are imposed, the problem becomes difficult due to the simultaneous requirements on the phases of nodes. Given a certificate wake-up schedule, we can verify with 2E congruences if the constraints (2) and (1) are satisfied. Also, congruences appearing in (3) have the cost of computing $(n_i, n_j)$ for each pair of nodes $i, j$, i.e., a solution of Strong WAKE-UP problem can be verified in polynomial time with respect to the input size, i.e., $M$.

In the general case, when constraints on the maximum wake-up period are imposed, the given problem is made
Algorithm 1 PERIOD($L_i,U_i$)

Require: $L_i \leq U_i \rightarrow Set : x = L_i, n_i = L_i$

1: while $x \leq U_i$ do
2:     if $B|x$ then
3:         $n_i \leftarrow x$; BREAK
4:     end if
5:     $x \leftarrow x + 1$
6: end while

Theorem 3 WAKE-UP is as hard as INTEGER FACTORIZATION.

Proof: Consider a polynomial time reduction to the INTEGER FACTORIZATION problem. The input is a integer $R > 0$. The reduced instance of the WAKE-UP is a graph composed of two nodes, 1 and 2, and one edge that joins them; furthermore $U_2 = L_2 = R$, $U_1 = R - 1$ and $L_1 = 2$. Assume that a solution of the INTEGER FACTORIZATION problem is given, i.e., $R$ has proper factors $m$ and $n$, i.e., $R = m \cdot n$, $m \not | R$ for some $n_1$ and $n_2$ are rather tight wake-up schedule. Vice versa, assume that a feasible schedule exists. Indeed $n_2 = L_2 = U_2 = R$ by construction. Observe that the $n_2 > 1$ since $L_1 \geq 2$. However, the Chinese Reminder Theorem ensures that $[n_1,R] \leq R$, which in turn implies that $n_1$ and $R$ cannot be co-prime, otherwise $[n_1,R] = n_1R \geq 2R > R$, a contradiction. Hence $n_1 | R$, and it is a proper factor of $R$ since $1 < 2$. Hence, by solving the WAKE-UP problem, we can solve the problem of factorizing $R$ as $R = n_1 \cdot q$ for some pair of proper factors $n_1,q > 1$, which concludes the proof.

As reported in App. [D] there exists a further argument for the equivalence proved above: a connection exists between WAKE-UP and INTEGER FACTORIZATION through a standard isomorphism. Also, WAKE-UP is not NP-complete unless $NP = co-NP$ since it is known that INTEGER FACTORIZATION is NP-complete if and only if $NP = co-NP$, which (at present) is believed a convincing argument for ensuring that no polynomial time complexity algorithms exists [24].

A question naturally arising is whether similar issues hold if we formulate the problem in continuous time, as opposed to the discrete-time framework. In App. [D] we show that, under a rationality assumptions, the two formulations are equivalent.

Therefore, no efficient algorithm for exactly solving Strong WAKE-UP can be designed. In the next section we will introduce two heuristics for the Strong WAKE-UP which have message complexity linear in the number of links in the graph.

A. Local time versus global time

From the implementation standpoint, a remark is needed: according to the framework proposed, the elements of $A_i$ are defined with respect to the common time. But, one might question that nodes operations are likely based on local time only. However, from the Chinese Reminder theorem, phase differences $\mod (n_i,n_j)$ matter. In practice, nodes will know their relative phase, i.e., the time when a wake-up occurs for a neighbor with respect to their own time axis. Thus, node $i$ will store the phase corresponding to node $j$ as $\Delta_{ij} = a_{ij} - a_i$ and node $j$ will do the same for $\Delta_{ji} = a_{ji} - a_j$. We can now see that phase differences with respect to the global time axis can be handled using values measured with respect to local time axes. In fact,

$$a_{ij} - a_{ji} \mod (n_i,n_j)$$

$$= (\Delta_{ij} - \Delta_{ji}) \mod (n_i,n_j) - (a_i - a_j) \mod (n_i,n_j)$$

$$= (\Delta_{ij} - \Delta_{ji}) \mod (n_i,n_j) + (a_i - a_i) \mod (n_i,n_j)$$

Where last term corresponds to the offset of the two local clocks and can be easily measured time-stamping the local index of a certain time slot.

In the rest of the paper we will refer to the common time axis for the sake of clarity.

V. ALGORITHMS DESIGN

We assume that each node $i$ maintains a sorted list of neighbors $N_g(i)$ according to their degree $d_i$.

The algorithms that we adopt in the following use a certain factor basis $B = \{p_1,p_2,\ldots,p_n\}$ where $p_8$ are prime numbers [21]. For the sake of notation, let us say that $B|n$ if $n$ factorizes in $B$. Observe that the choice of the factors of the wake-up periods at each node is critical; the Chinese Remainder Theorem suggests that period $n_i$ should be chosen free of large factors.

Nodes choose period $n_i \in [L_i,U_i]$ such that it factorizes according to $B$ using the procedure described in Alg. [I] Alg. [I]
Algorithm 2  

**BFS WAKE-UP**

**Require:** \( L_i \leq U_i \rightarrow Default : A_i = \{ \alpha_i \} \)

1. for all \( i \in V \) do
2. \( n_i = \text{PERIOD}(L_i, U_i) \)
3. \( f(i) \leftarrow 0 \)
4. end for

**Ensure:** Select \( i \in V \) with largest degree
5. \( f(i) = 1 \)
6. \( Q \leftarrow \{ i \} \{ \text{Enqueue the first node} \}
7. while \( Q \neq \emptyset \) \{ Visit all the nodes in BFS \} do
8. \( i \leftarrow Q[1] \{ \text{Get the node on top of the queue} \}
9. for all \( j \in N_q(i) \) do
10. if \( f(j) = 0 \) \{ If it was not explored before \} then
11. \( Q \leftarrow \{ j \} \{ \text{Enqueue this node} \}
12. \( \alpha_j \leftarrow \alpha_i \{ \text{Align time axis of node } i \text{ and node } j \} \)
13. \( A_j \leftarrow \{ \alpha_j \} \{ \text{Update the phase if } j \}
14. \( x_{ij} = x_{ji} = \alpha_i + k \cdot [n_i, n_j], \quad k \in \mathbb{Z} \{ \text{Rendezvous times} \} \)
15. \( f(j) \leftarrow 1 \{ \text{Visited} \}
16. \) end if
17. end for
18. \( Q = Q \setminus \{ i \} \{ \text{Dequeue } i \}
19. n_i \leftarrow [n_i, \{ n_{i_1}, \ldots, n_{i_d} \}] \{ \text{Use largest possible period at } i \}
20. f(i) = 2 \{ \text{Explorred} \}
21. end while

returns the smallest integer in \([L_i, U_i]\) that can be written as \( p_1^{q_1} \cdots p_d^{q_d}, \) \( 0 \leq q_i \in \mathbb{Z}, \) provided that such a number exists. If it does not exist, \( L_i \) is returned. We now introduce two heuristics for Strong WAKE-UP.

A. BFS WAKE-UP Algorithm

Once nodes have chosen their periods according to Alg. 1, the simplest rationale is to align their phases pairwise. Alg. 2 is based on a breadth first search (BFS) visit of the network rooted at a given node; as customary for BFS search a queue \( Q \) is employed.

Each node \( i \) maintains information about nodes in its neighborhood, i.e., the node ID, the phases set \( A_i \) and a special flag \( f \) that is used for the BFS procedure, where (i) \( f(j) = 0 \) means \( j \) unexplored (ii) \( f(j) = 1 \) means node \( j \) discovered but not all neighbors of \( j \) examined (iii) \( f(j) = 2 \) node \( j \) explored. By default, node \( i \) wakes up at the origin of its own period, i.e., at \( \alpha_i \).

The algorithm selects all unexplored nodes and enqueues them at most once, so that it terminates in \( N \) steps.

Alg. 2 reports on the pseudocode of the centralized version of the BFS WAKE-UP algorithm; however, the BFS WAKE-UP algorithm can be easily implemented in a distributed fashion [25]. Observe that BFS WAKE-UP attains phase synchronization and a strictly feasible schedule.

Below, we formalize the properties of the algorithm; the proof is reported in App. A.

**Theorem 4** BFS WAKE-UP has the following properties:

i. It produces a strictly feasible wake-up schedule;
ii. It has complexity \( O(E) \);
iii. A distributed implementation requires \( O(M) \) messages.

**Remark 2** For implementation purposes, the above algorithm can be very easily coupled to the construction of shortest path trees [25]; in such a way customary algorithms such as Dijkstra can conveniently provide joint routing and wake-up scheduling with no need for separate message exchange; in turn, the weight of link \( ij \) is represented naturally by \([n_i, n_j]\).

The convergence time of the algorithm is proportional to the diameter of the network [25]; the BFS WAKE-UP requires a preliminary leader election procedure; further its convergence may be slow. Accordingly, we devised an alternative heuristic algorithm, described below, which effectively addresses such shortcomings by parallelising the computation of the scheduling.

B. PARALLEL WAKE-UP Algorithm

The design of a parallel algorithm for wake-up synchronization requires careful handling of nodes phases; in particular, in order to avoid the increase of the size of the \( A_i \)s, it is possible to operate pairwise adaptation of nodes phases (phase adaptation).

**Lemma 1** Let \( ij, jk \in E \) and fix \( n_i, n_j, n_k, a_{kj} \) and \( a_{ij} \). A feasible wake-up schedule for \( i, j, k \) such that \( a_{jk} = a_{ij} \), exists if and only if \( (n_j, (n_i, n_k)) \mid (a_{kj} - a_{ij}) \). If such condition holds, the schedule is obtained for \( a_{jk} = a_{ji} = a_{kj} + N_k \cdot (x_0 \cdot e) \), where \( e, x_0 \in \mathbb{Z} \) are solutions of \( a_{kj} - a_{ij} = (n_j, n_k) \cdot e \cdot x_0 + (n_i, n_j) \cdot e \cdot y_0 \), with \( y_0 \in \mathbb{Z} \).

The proof is reported in App. A.

The algorithm pseudo-code is reported in Alg. 4. Each node \( i \) maintains information about nodes in its neighborhood, i.e., the node ID, the phases set \( A_i \), the wake-up period \( n_i \) and two special flags \( f \) and \( t \). Special flags are used in order to determine \( A_i \) and \( n_i \). In particular (i) \( f(j) = 0 \) means \( j \) unexplored (ii) \( f(j) = 1 \) means node \( j \) discovered but not all neighbors of \( j \) have been fixed yet and all neighbors of \( j \) have been explored (iii) \( f = 2 \) when node \( j \) has been explored and therefore \( a_{ij} \) and \( n_j \) have been fixed. Also, \( t(j) = 0 \) means \( j \) that can adjust \( a_{ij} \) (ii) \( t(j) = 1 \) means \( i \) cannot change \( a_{ij} \) value because some of its neighbors used \( a_{ij} \).

At the start of the algorithm each node \( i \) chooses \( n_i \) using PERIOD and maintains a list of neighbors \( N_g(i) \). Then node \( i \) is marked unexplored and available to change \( a_{ij} \) value \( f(i) = t[i] = 0 \). Hence, each node starts a random timer: when the timer expires node \( i \) freezes neighbors’ timers and determines rendezvous times with all nodes in its neighborhood.

If \( i \) is unexplored, i.e., \( f(i) = 0 \), node \( i \) checks whether it is in range of an already existing node \( j \) explored. If so, \( i \) just aligns \( a_i = a_j ; j \) is then marked not available to change its time axis anymore, i.e., \( t(j) = 1 \). Otherwise, if \( i \) is discovered, i.e., \( f(j) = 1 \), \( i \) checks whether it is in range of two already
Theorem 5 PARALLEL WAKE-UP:

i. produces a wake-up schedule;

ii. it requires $O(M)$ messages for a distributed implementation.

In principle, since multiple phases may be added at one node, it is possible that the output of the algorithm is a schedule that is not feasible. A simple bound on the average duty cycle (DC) is provided by the following

Theorem 6 Under Alg. 3 $DC < \frac{d}{L}$, where $L = \min L_i$ and $d$ is the average node degree.

Proof: The algorithm adds at most $d_i-1$ phases per node:

$$\frac{1}{N} \sum_{i=1}^{N} \frac{|A_i|}{n_i} \leq \frac{1}{NL} \sum_{i=1}^{N} (d_i - 1) = \frac{d-1}{L}$$

Remark 3 Observe that the above algorithm can be used to adapt the wake-up scheduling dynamically in case one or more nodes join the network. In particular, it is possible to combine the two algorithms in order to initialize a wake-up schedule network-wide with BFS WAKE-UP, and use PARALLEL WAKE-UP to add new nodes to existing schedules or to repair locally a schedule, should a node fail or lose synchronization.

C. Example

We reported in Fig. 2 a simple example illustrating the BFS WAKE-UP and of the Parallel WAKE-UP algorithm on a sample graph with 7 nodes. In that instance $U = 20$, $L_i = \{2, 3, 9, 7, 11, 5, 2\}$, $\alpha_i = \{1, 3, 7, 9, 6, 1, 0\}$ and $B = \{2\}$. Several wake-up periods coexist: 4, 8, 16.

The BFS WAKE-UP run generates the list of visited nodes (1, 2, 3, 4, 5, 6, 7); observe that every node aligned to $\alpha_1 = 1$. In the Parallel WAKE-UP node 2 and 7 perform the algorithm operations first, then 1, 5, 6 and finally 4. As we can see from the example, Parallel WAKE-UP assigns heterogeneous phases; at node 4 phase adaptation according to Thm. 1 is not possible so that finally $A_4 = \{1, 3\}$.

D. Particular cases

There exist specific cases when strictly tight wake-up schedules are attained easily.

Lemma 2 Let for any node $i$ be $B|L_i$, $B|U = U_i$ and $L_i \leq U$, then there exists a strictly tight wake-up schedule; it can be attained using BFS WAKE-UP.

Proof: Consider any link $ij$ and let $n_i = L_i$ and $n_j = L_j$. The statement follows immediately from the Chinese Reminder theorem since $[n_i, n_j]|U$ and one can use BFS WAKE-UP to assign the same phase to all nodes in the network.

In particular the following example shows a simple application of the above case. Consider a tree topology $T$ and assume nodes are labeled in order to respect the partial order induced by the distance from the root node 1. Let $L_0$ be associated to the root node, and $L_k = L_0 k$ be associated to nodes at level $k$. A feasible schedule can be constructed as follows: root node 0 wakes at times $x_0 = 0 \mod L_0$. Node $j$ wakes up at $x_j = a_j \mod p^{|r_j|} L_0$ where $r_j$ is the level occupied by node $j$ and $g$ is any positive integer function. In this case $U = L_0 p^{|g(r)|}$.
VI. PERFORMANCE EVALUATION

In this section, we report on the outcomes of experiments for the BFS WAKE-UP and PARALLEL WAKE-UP algorithms. In the first set of experiments, performed using Matlab®, we tested the algorithms on random topologies.

We considered a square playground of side 100 m with 200 nodes randomly deployed according to a uniform distribution. Nodes were assumed to be connected if their mutual distance was below a given value (communication range), varied in the range 10 – 80m. For each setting, 100 runs were performed. For the construction of the schedules, two factor bases were considered, \( B = \{2\} \) and \( B = \{2, 3, 5\} \). The constraints were set by taking, for each node, \( L_i \) uniformly distributed in \([1, 35]\) and \( U_i \) uniformly distributed in \([50, 100]\).

We considered as performance metric the average duty cycle, as introduced in the previous section, and the average normalised delay drift, defined as:

\[
DD = \frac{1}{2M} \sum_{i=1}^{N} \sum_{j \in N_g(i)} \frac{n_i n_j}{U_i} \tag{5}
\]

It is worth remarking that \( DD > 1 \) implies that some constraints on the delay were violated.

Results are reported in Fig. [3]. In terms of duty cycle, BFS WAKE-UP outperforms PARALLEL WAKE-UP, as expected, since the second one may add multiple phases per node. The performance difference reduces in dense networks (i.e., with large transmission range). A closer look at the results showed that, for the parameters chosen, the schedules generated by PARALLEL WAKE-UP were not feasible in less than 1% of the cases. In terms of average normalised delay drift, the two algorithms attain the same performance. BFS WAKE-UP is much slower in converging in sparse networks, but its performance increase (and, actually, outperforms PARALLEL WAKE-UP) in very dense networks. As it may be seen, the use of a richer basis improves significantly the performance in terms of duty cycle, but it also leads to worse delay drift performance.

In general, we may conclude that PARALLEL WAKE-UP represents a meaningful choice in most situations, presenting performance close to BFS but with a shorter convergence time.

![Example. BFS WAKE-UP and PARALLEL WAKE-UP on a sample topology. The table reports on the final output of the algorithms.](image)

| \( i \) | \( \alpha_i \) | \( L_i \) | \( U_i \) | \( p_i^\alpha \) | \( (\alpha_i, n_i) \) |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 20 | 2 | \((1, 4)\) |
| 2 | 3 | 3 | 20 | 4 | \((1, 4)\) |
| 3 | 7 | 9 | 20 | 16 | \((1, 16)\) |
| 4 | 9 | 7 | 20 | 8 | \((1, 4)\) |
| 5 | 6 | 11 | 20 | 16 | \((1, 16)\) |
| 6 | 1 | 5 | 20 | 8 | \((1, 8)\) |
| 7 | 0 | 2 | 20 | 2 | \((1, 8)\) |

**TABLE II: Performance of BFS WAKE-UP and PARALLEL WAKE-UP**

| Violation | \( B \) | \([75, 100]\) | \([60, 100]\) | \([45, 100]\) |
|---|---|---|---|---|
| Magnitude | \( \{2\} \) | 0 | 2.04 ± 0.07 | 9.32 ± 0.26 |
| Magnitude | \( \{2, 3, 5\} \) | 0 | 88.38 ± 1.84 |
| Percentage | \( \{2\} \) | 0 | 1.25% ± 0.12 | 5.43% ± 0.27 |
| Percentage | \( \{2, 3, 5\} \) | 0 | 9.36% ± 0.43 |

**Discrete events simulation:** In a second set of experiments, we implemented the proposed techniques in an event-based simulator, Omnet++, to assess the advantage of supporting heterogeneous wake-up schedules. We considered 50 sensors deployed over a 100m × 100m area, with communication range of 10m and set the time slot duration to 100ms. We assumed that packets were generated at each node every 30 s and that all packets were routed towards a sink, located in (1, 1). A tree was constructed at the beginning of the simulation to route packets to the sink.

As sensors far from the sink relay less traffic, their bounds on energy consumption can be tightened. We considered a situation in which the constraints \( L_i \) depended on the distance from the sink as follows: \( L_i = 2 \) for nodes at distance \( \leq 2 \) hops, \( L_i = 4 \) for nodes at distance of 3 and 4 hops and so on. As performance metrics we considered the average packet delay (from source to sink) and the lifetime of the nodes. The latency was computed up to the time at which the first node died. We compared the performance attained by BFS (PARALLEL turned out to offer the same performance) with those obtained in the absence of sleep mode (‘No Power Saving’) and those obtained when all nodes go to sleep synchronously for one slot every two (‘Uniform’). The results are reported, plotted against the distance from the sink, in Fig. [4]. As it can be seen, BFS is able to effectively prolong the lifetime of sensors far from the sink, while at the same time limiting the increase in packet latency.

VII. CONCLUSIONS

In this paper we introduced a framework for the synchronization of wake-up schedules, which plays a central role for energy saving in wireless networks, under the joint request to satisfy per node energy and delay constraints. We showed that the related WAKE-UP problem has a strong relation with
integer factorization, which adds a novel algebraic perspective.

This work left out several interesting directions. In particular, for relatively small numbers, factorization can be done in practice [23]. In turn, the optimal choice of the factor basis and of periods is a key problem in order to limit the drift of the rendezvous times.

Furthermore, mapping end to end delay onto bounds on the wake-up period has been only touched briefly in App. E. However, while there exist several works in literature tackling the general problem of bounding the end-to-end delay [26], a network calculus accounting for power saving duty cycles has not been proposed so far. Relating the WAKE-UP problem to those works is part of future work.

References

[1] L. M. Feeney and M. Nilsson, “Investigating the energy consumption of a wireless network interface in an ad hoc networking environment,” in Proc. of IEEE INFOCOM, Apr. 2001, pp. 1548–1557.
[2] W. Ye, J. Heidemann, and D. Estrin, “An energy-efficient MAC protocol for wireless sensor networks,” in Proc. of IEEE INFOCOM, vol. 3, Jun. 2002, pp. 1567–1576.
[3] C. Enz, A. El-Hofydi, J.-D. Decotignie, and V. Peiris, “WiseNET: An ultralow-power wireless sensor network solution,” IEEE Computer, vol. 37, no. 8, pp. 62–70, Aug. 2004.
[4] T. H. Yu-Chee Tseng, C-S Hsu, “Power saving protocols for IEEE802.11-based multi-hop ad hoc networks,” in Proc. of IEEE INFOCOM, 2002, pp. 200–209.
[5] J. Polastre, J. Hill, and D. Culler, “Versatile low power media access for wireless sensor networks,” in Proc. of ACM SenSys, 2004, pp. 95–107.
[6] G. Lu, B. Krishnamachari, and C. Raghavendra, “An adaptive energy-efficient and low-latency MAC for data gathering in sensor networks,” in Proc. of IPDPS, Santa Fe, NM, Apr. 2004.
[7] S. Guha, C.-K. Chau, and P. Basu, “Green wave: Latency and capacity-efficient sleep scheduling for wireless networks,” in Proc. of IEEE INFOCOM, 2010.
[8] G. Lu, N. Sadagopan, B. Krishnamachari, and A. Goel, “Delay efficient sleep scheduling in wireless sensor networks,” in Proc. of IEEE INFOCOM, 2005, pp. 2470–2481.
[9] Q. Li and D. Rus, “Global clock synchronization in sensor networks,” in Proc. of IEEE INFOCOM, Hong Kong, China, 2004, pp. 564–574.
[10] R. E. Mirollo, Steven, and H. Strogatz, “Synchronization of pulse-coupled biological oscillators,” SIAM J. Appl. Math, vol. 50, pp. 1645–1662, 1990.
[11] R. Zheng, J. C. Hou, and L. Sha, “Asynchronous wakeup for ad hoc networks,” in Proc. of ACM Mobihoc, 2003, pp. 35–45.
[12] S. Hu, C.-M. Chen, and M.-S. Chen, “AAA: Asynchronous, Adaptive, and Asymmetric Power Management for Mobile Ad Hoc Networks,” in Proc. of IEEE INFOCOM, 2009, pp. 2541–2545.
[13] J. Wu, F. Dai, M. Gao, and I. Stojmenovic, “On calculating power-aware connected dominating sets for efficient routing in ad hoc wireless networks,” Journal of Communications and Networks, pp. 1–12, 2002.
[14] J.-R. Jiang, Y.-C. Tseng, C.-S. Hsu, and T.-H. Lai, “Quorum-based asynchronous power-saving protocols for IEEE 802.11 ad hoc networks,” Mobile Networks and Applications, vol. 10, no. 1-2, pp. 169–181, 2005.

[15] L. Dai, P. Basu, and J. Redi, “An energy efficient and accurate slot synchronization scheme for wireless sensor networks,” in Proc. of BROADNETS, 2006.

[16] S. Du, A. K. Saha, and D. B. Johnson, “Rmac: A routing-enhanced duty-cycle mac protocol for wireless sensor networks,” in INFOCOM, 2007, pp. 1478–1486.

[17] C. Schurgers, V. Tsiatsis, S. Ganeriwal, and M. Srivastava, “Optimizing sensor networks in the energy-latency-density design space,” IEEE Trans. on Mobile Computing, vol. 1, no. 1, pp. 70–80, Jan. 2002.

[18] Z.-T. Chou, “Optimal adaptive power management protocols for asynchronous wireless ad hoc networks,” in Proc. of IEEE WCNC, 2007, pp. 61–65.

[19] S. Lai, B. Zhang, B. Ravindran, and H. Cho, “Cqs-pair: Cyclic quorum system pair for wakeup scheduling in wireless sensor networks,” in Proc. of OPODIS, 2008, pp. 295–310.

[20] G. Lu and B. Krishnamachari, “Minimum latency joint scheduling and routing in wireless sensor networks,” Ad Hoc Netw., vol. 5, no. 6, pp. 832–843, 2007.

[21] N. Koblitz, A course in number theory and cryptography. Springer, 1994.

[22] M. Agrawal, N. Kayal, and N. Saxena, “PRIMES is in P,” Ann. of Math, vol. 2, pp. 781–793, 2002.

[23] P. Giblin, An Introduction to Number Theory with Computing. Cambridge UP, 1994.

[24] S. Arora and B. Barak, Computational Complexity. A modern approach. Cambridge University Press, 2009.

[25] T. H. Cormen, C. E. Leiserson, and R. L. Rivest, Introduction to Algorithms. Cambridge, Massachusetts, USA: MIT Press, 1990.

[26] I. Chlamtac, H. Zhang, A. Faragó, and A. Fumagalli, “A deterministic approach to the end-to-end analysis of packet flows in connection-oriented networks,” IEEE/ACM Trans. Netw., vol. 6, no. 4, pp. 422–431, 1998.

[27] J.-Y. Le Boudec and G. Hébuterne, “Comments on “a deterministic approach to the end-to-end analysis of packet flows in connection-oriented networks”,” IEEE/ACM Trans. Netw., vol. 8, no. 1, pp. 121–124, 2000.

[28] S. Lang, Undergraduate Algebra. Springer Verlag, 2005.
Appendix

A. Proof of Thm. 2

Proof: i. Consider node \( i \) and assume that the algorithm started at node 1 for simplicity: at the end of the algorithm it holds \( (A_1, n_1) = \{(a_1), (n_1, n_{i_1}), \ldots, (n_1, n_{i_N})\} \). Notice that \( a_i - a_j = 0 \) for all \( j_i \in N_i \) and also
\[
(n_i, n_{i_1}), \ldots, (n_i, n_{i_N}) = (n_i, (n_{i_1}, \ldots, n_{i_N}))
\]
Hence, every pair \( (a_i, n_i) \) solves the modular congruence system according to the Chinese Remainder theorem. ii. For each node \( j \), \( N_g(j) \) operations are performed, including the calculation of \( [n, n_j] \), which requires on \( \mathcal{O}(\log^3 \max(n_i, n_{j})) \) operations. The overall complexity of the algorithm is then bounded as \( \sum_{i=1}^{N} \sum_{j \in N_g(i)} \log^3 \max(n_i, n_{j}) = 2E \log^3 U \). iii. The statement follows immediately by a double counting argument since every node \( i \) needs to exchange at most 2 messages with each neighbor in order to visit neighbors, communicate its phase and receive acknowledgment. ■

B. Proof of Lemma 1

Proof: A novel phase \( a \) can be given to node \( j \) iff the following system solves
\[
x = a_i \mod n_i = a \mod n_j
\]
from which \( (n_k, n_j)[a_k] - a \) and \( (n_i, n_j)[a - a_i] \). Thus, if a novel phase can be assigned then a solution exists for
\[
c = a_k - a_i = x (n_j, n_k) + y (n_i, n_j), \quad x, y \in \mathbb{Z}
\]
From (2), \( d = (n_i, (n_j, n_k))[a_k] - a_i \); let \( de = a_k - a_i \) consider a solution \( x_0, y_0 \)
\[
d = (n_j, n_k)x_0 + (n_i, n_j)y_0, \quad x_0, y_0 \in \mathbb{Z}
\]
then
\[
c = de = (n_j, n_k)c x_0 + (n_i, n_j)c y_0
\]
the novel phase writes
\[
a = a_k - (n_j, n_k)c x_0
\]
Observe that \( a - a_i = a_k - (n_j, n_k)c x_0 - a_i = (n_i, n_j)c y_0 \) which concludes the proof. ■

C. Continuous-Time Analogue

Here we consider wake-up scheduling in continuous time, under the additional requirement that the difference of phase at different nodes is rational (observe that this is a practical requirement since numerical representation is rational). We conclude that the continuous time approach plus the request for a rational difference of phase at two nodes is equivalent to operate wake-up scheduling in the discrete time domain. In fact, consider the general case of a network where wake-up scheduling is operated and nodes are assigned phases \( \alpha_i \) and period \( T_i \). It holds for any node \( i \):
\[
K_{ij}T_i + \alpha_i = K_jT_j + \alpha_j, \quad j \in N_g(i)
\]
Now, assume \( G \) is connected, and consider \( N \) paths \( P_1, P_2, \ldots, P_N \) from node 1 to nodes \( j = 1, \ldots, N \), respectively: it is easy to see that the following set of equalities hold
\[
T_j/T_1 = q_{j1} + \sum_{r \in P_{rj}} p_{rs} \alpha_{rs}
\]
where \( \alpha_{rs} = \alpha_r - \alpha_s \), whereas \( q_{j1} \) and \( p_{rs} \) are rational. Thus, it follows that also \( T_j/T_1 \) is rational. This is equivalent to admit that the overall system time, i.e., the period of each node \( T_i \) can be reduced to a convenient multiple of a certain \( T_0 = T_1/N_0 \).

D. Standard isomorphism

A basic result in algebra that provides a natural link of the Chinese Remainder theorem with the problem of factorization of integers. With standard notation denote \( \mathbb{Z}/n\mathbb{Z} \) the quotient ring of integers modulus \( n \). Define the standard isomorphism
\[
\Psi : \mathbb{Z}/n\mathbb{Z} \times \mathbb{Z}/m\mathbb{Z} \rightarrow \mathbb{Z}/[m, n] \mathbb{Z}
\]
\[
(a, b) \rightarrow c
\]
where \( c \) solves for
\[
c = a \mod n
\]
\[
c = b \mod m
\]
Assume now that \( m, n \) are two coprime integers, i.e., \( (m, n) = 1 \); it is easy to see that \( \Psi \) is an isomorphism. Translating with the terminology used in wake-up scheduling used throughout this paper, this means that for every wake-up schedule \( \{(a, n), (b, m)\} \), there exists a unique rendezvous time \( c_0 \) such that \( c = c_0 \mod [m, n] \) for any other rendezvous time \( c \) generated by the schedule. Vice versa, given a non-empty set of rendezvous times for two nodes having coprime periods, there exists a unique pair of phases \( a \) and \( b \) that corresponds to such set. This also means that phases are automatically determined by the set of rendezvous times and the coprime periods, i.e., the only information needed is that \( mn = [m, n] \), i.e., \( (m, n) = 1 \). This provides another interpretation of the reason why \( \text{WAKE-UP} \) is as hard as \( \text{INTEGER FACTORIZATION} \), as proved in Sec. [IV]

E. On the Dimensioning of the Upper Bounds

In general, application–level constraints for the application setting we target are expected to be given in terms of:

- Expected lifetime of the network. This can be translated in terms of an upper bound on the duty cycle of single nodes. This is reflected in a constraint of the form like (1).
- End-to-end delay. Bounds will be given in terms of the delay incurred between the generation of a message at a node (which corresponds, in our framework, to the reading of a sensor) and the delivery at the appropriate sink, where such data may be processed or transferred, via some other communications technology, to a remote processing center.
For the latter case, we need to identify means for translating end-to-end delay constraints into node-level delay constraints of the form (4). The problem is then to translate a number of constraints, given on routes, to constraints on the single links constituting the routes.

In order to do so, we start by introducing some notation. We define by $R_i$ the route starting from node $i$ and ending in one of the sink nodes. We say that node $j$ lies along the route $R_i$ if all traffic originating from $i$ passes through $j$ in order to reach the sink, and write $j \in R_i$. The length (in hops) of route $R(i)$ is denoted by $|R(i)|$. Conversely, we denote by $F_i$ the set of routes passing through node $i$. The worst-case delay associated to route $R_i$ is denoted by $D(R_i)$. The delay constraint on route $R_i$ is expressed as $D(R_i) \leq \eta_i$, where $\eta_i$ is a constant.

The difficulty of the problem lies in the fact that the constraint on the energy expenditure (1) impacts the minimal latency experienced by a message passing through a given node.

We assume that (i) routes are static throughout the lifetime of the network (ii) nodes arriving during an active period are sent at the next active period occurrence (iii) the duration of a time slot is sufficient for transmitting all messages queued at any given node.

Now we consider the latency experienced at a given node and imposed by the duty cycle. Under the aforementioned assumptions, the worst-case delay incurred by a message passing through node $i$ can be lower bounded by $n(i) + 1$. The term ‘worst-case’ here refers to two factors. One, to the arrival time of the message, whereby the worst case coincides with a message arriving right after the beginning of an active slot. Second, to the rendez-vous points. In order to be able to move the message one hop further along the route, it is not sufficient for node $i$ to be in the active state, but, given the system model presented in Sec. III, it requires also the next hop node to be in the active state. In general, the delay will be a multiple of $n(i) + 1$ plus the time slot necessary to forward the message. Given condition (1), this implies that such worst-case delay is lower bounded by $L_i + 1$.

Clearly, a set of feasible $U_i$ has to satisfy $U_i \geq L_i + 1$ for all nodes $i$. Furthermore, in order to be compatible with the end-to-end delay constraints it has to be $\sum_{j \in R_i} U_j \leq \eta_i$ for all routes $R_i$.

We thus need to find a solution of the following system of inequalities:

$$
\begin{align*}
\sum_{j \in R_i} U_j & \leq \eta_i, \\
\vdots & \\
\sum_{j \in R_N} U_j & \leq \eta_N, \\
U_1 & \geq 1 + L_1, \\
\vdots & \\
U_N & \geq 1 + L_N.
\end{align*}
$$

The system has a solution if and only if the following condition is satisfied:

$$|R_i| + \sum_{j \in R_i} L_j \leq \eta_i \forall i = 1, \ldots, N. \tag{9}$$

If such a condition is respected, a feasible solution can be found as follows. We associate to route $R_i$ a “delay budget” $\eta_i$. We consider how much is consumed by the duty cycle at the various nodes along the route $R_i$, which, using the bound outlined above, will be taken as $|R_i| + \sum_{j \in R_i} L_j$. We then distribute the remaining “delay budget” in a uniform manner across all nodes of the route $R_i$. We do this for all routes. Then, for each node $i$, we consider as $U_i$ the minimum value of such quantity among all routes passing through it. In symbols:

$$U_i = L_i + 1 + \min_{R_j \in F_i} \left\{ \frac{\eta_j - |R_j| - \sum_{k \in R_j} L_k}{|R_j|} \right\}. \tag{10}$$

It is trivial to prove that under (9) solutions of the form (11) satisfy $U_i \geq L_i + 1 \forall i$. In order to prove that the other conditions in (8) are satisfied we note that:

$$
\begin{align*}
\sum_{i \in R_j} U_i & = \sum_{i \in R_j} \left\{ L_i + 1 + \min_{R_k \in F_i} \left\{ \frac{\eta_k - |R_k| - \sum_{h \in R_k} L_h}{|R_k|} \right\} \right\} \leq \sum_{i \in R_j} L_i + |R_j| + \sum_{i \in R_j} \min_{R_k \in F_i} \frac{\eta_k - |R_k| - \sum_{h \in R_k} L_h}{|R_k|} \\
& \leq \sum_{i \in R_j} L_i + |R_j| + \sum_{i \in R_j} \frac{\eta_j - |R_j| - \sum_{h \in R_j} L_h}{|R_j|} \\
& = \sum_{i \in R_j} L_i + |R_j| + \eta_j - |R_j| - \sum_{h \in R_j} L_h = \eta_j,
\end{align*}
$$

where we used the fact that if $i \in R_j$ then $R_j \in F_i$ and that $\min_{i \in S} \{x_i\} \leq x_j \forall j \in S$.

We recall that the method outlined above provides only one of the possible solutions of the system (8). In particular, there is no reason for which a uniform distribution of the remaining delay budget along a route is optimal, though this seems reasonable in a variety of settings.

For the special case in which $L_i = L \forall i$ and $\eta_i = \eta \forall i$, i.e., all nodes have the same energy constraint and all messages
should be delivered to a sink within the same deadline, regardless of their distance, Eq. (10) simplifies as:

\[ U_i = L + 1 + \min_{R_j \in F_i} \left[ \eta - \left| R_j \right| - \sum_{k \in R_j} L \right] = (14) \]

\[ = L + 1 + \min_{R_j \in F_i} \left[ \frac{\eta - \left| R_j \right| \cdot (L + 1)}{\left| R_j \right|} \right] = (15) \]

\[ = \eta \frac{\eta}{\max_{R_j \in F_i} \left| R_j \right|} (16) \]

In the uniform case, therefore, the bounds \( U_i \) depend only on the maximal length of the routes passing through node \( i \).