Consolidation analysis and effect analysis in the treatment of soft foundation by vacuum preloading

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Abstract: Vacuum preloading is a common method in the treatment of soft soil foundation. Combined with field test engineering cases, the plastic drainage board is equivalently processed into sand well units. The analytical solution of the sand well consolidation equation under the linear attenuation of vacuum preloading is derived to evaluate the consolidation degree of the site. The results showed that the vacuum negative pressure reached \(-85\) kPa at 20 d, and the pore water pressure in the silt soil continued to dissipate to produce negative pore pressure, and finally reached a stable value at 60 d. Considering that the vacuum negative pressure affects the change of pore pressure, it is concluded that the vacuum negative pressure attenuates 3.4 kPa/m along the depth of the soil. Based on the data obtained on site, the solution in this paper is compared with the Hansbo solution, which is closer to the actual project.

1. Introduction
The vacuum preloading method is based on the theory of sand well consolidation, draining pore water from the soil through the drainage body, thereby accelerating the consolidation of the foundation and meeting the settlement requirements. Kjellmann (1952) \cite{1} first proposed the vacuum preloading method. Hiroshi et al. (1974) \cite{2} considered the effect of well resistance and obtained an analytical solution for its consolidation. Xie Kanghe (1989) \cite{3} believed that radial and vertical seepage effects should be considered. Shen Zhujiang et al. (1986) \cite{4}, Dong Zhiliang (1992) \cite{5}, Zhang Gongxin et al. (2005) \cite{6}, Mei Guoxiong et al. (2006) \cite{7}, Peng Jie et al. (2010) \cite{8}, Wu Yuedong et al. (2016) \cite{9}all believed that there was negative pressure in the process of drainage consolidation. Indraratna et al. (2004) \cite{10} considered that the vacuum negative pressure was distributed in a trapezoidal shape along the depth. Tang et al. (2004) \cite{11} considered that the vacuum negative pressure transmission effect in the drainage plate is better, and the attenuation is small. Chu et al. (2000, 2005) \cite{12,13} obtained a linear distribution of vacuum negative pressure along the depth of the soil through actual engineering measurements.

Based on the analytical solution of the consolidation model of the linear attenuation of vacuum negative pressure in sand wells, this paper relies on a field test of vacuum preloading in Guangdong Province to analyze the effect of vacuum preloading on soil deformation and pore water pressure in soft foundation treatment.
2. Consolidation model establishment

Figure 1 shows the theoretical calculation diagram of sand well consolidation. \( r_w \) and \( d_w \) are the equivalent radius and diameter of the plastic drainage plate, respectively. \( r_s \) and \( d_s \) are the radius and diameter of the smear area, respectively. \( r_e \) and \( d_e \) are the radius and diameter of the affected zone of the plastic drainage board, respectively. \( H \) is the depth of the plastic drainage board inserted into the soil. \( r \) and \( z \) are the radial and vertical coordinates of the soil, respectively. The bottom of the foundation and its surroundings are impervious boundaries, and only the top is permeable.

The basic assumptions are as follows: (1) Barron's equal strain condition is established, and the vertical deformation of each point at any depth within the influence range of the sand well is equal. (2) There is no lateral deformation in the foundation soil. (3) The vertical seepage of the foundation soil is ignored, and only the radial seepage is considered. The seepage in the soil under negative pressure conforms to Darcy's law. (4) The negative pressure (excess pore pressure) in the sand well shows a linear attenuation distribution along the depth, that is \(-p_0 \left[1 - \left(1 - k_i \right) \frac{z}{H} \right]\), and the amount of water flowing into the soil at any depth is equal to the amount of water flowing out. (5) Except for seepage in the sand well and smear area, other properties are the same as the natural foundation soil.

Its boundary conditions: \( \overline{u_r} \big|_{r=0} = 0 \), where \( \overline{u_r} \) is the average excess pore pressure at any depth in the soil in the radial seepage area, that is \( \overline{u_r} = \frac{1}{\pi} \int_{r_e}^{r_s} \int_{r_e}^{r_s} \frac{2\pi r_u dr}{r \gamma_w} \bigg|_{r=r_e} = 0 \).

![Fig. 1 Theoretical calculation diagram of sand well consolidation](image)

3. Governing equation and solutions

According to Darcy's law, the flow rate through the cylindrical surface of radius \( r \) in unit time \( dt \):

\[
\frac{\partial Q}{\partial t} = k \frac{\partial u_r}{\partial r} A
\]

Where \( A = 2\pi r dz \), \( k \) is the permeability coefficient of the foundation soil, \( \gamma_w \) is the weight of water, and the approximate value is generally equal to 10.0 kN/m³.

At the same time, the vertical volume change rate of the unit soil can be expressed as:

\[
\frac{\partial V}{\partial t} = \frac{\partial \varepsilon}{\partial t} \pi \left( r_e^2 - r^2 \right) dz
\]

where \( V \) and \( \varepsilon \) are the volume and vertical strain of the element soil, respectively. According to the principle of equal strain, the two equations are equal, then:
\[
\frac{k}{\gamma_w} \frac{\partial u_r}{\partial r} 2\pi r dr dz = \frac{\partial E}{\partial t} \pi (r_e^2 - r^2) dz
\]
(3)

From the above formula, the governing equations of the smeared area and the undisturbed area can be expressed as:

\[
\left\{ \begin{array}{l}
\frac{\partial u_r}{\partial r} = \frac{\gamma_w}{2k_s} \frac{\partial E}{\partial t} \left( \frac{r_e^2 - r^2}{r} \right) & r_w \leq r \leq r_s \\
\frac{\partial u_r}{\partial r} = \frac{\gamma_w}{2k_h} \frac{\partial E}{\partial t} \left( \frac{r_e^2 - r^2}{r} \right) & r_s \leq r \leq r_e
\end{array} \right.
\]
(4)

where \( k_s \) and \( k_h \) are the permeability coefficients of soil in smeared area and undisturbed area, respectively.

Integrate the two side pairs of the above formula and introduce boundary conditions, the excess pore water pressure of the soil around the sand well can be obtained as:

\[
u_r = \frac{\gamma_w}{2k_s} \frac{\partial E}{\partial t} \left( r_e^2 \ln \frac{r}{r_w} - \frac{r_e^2 - r_w^2}{2} \right) - p_0 \left[ 1 - (1-k_i) \frac{z}{H} \right] \quad r_w \leq r \leq r_s
\]
(5-1)

\[
u_r = \frac{\gamma_w}{2k_h} \frac{\partial E}{\partial t} \left( r_e^2 \ln \frac{r}{r_w} - \frac{r_e^2 - r_w^2}{2} \right) + \frac{\gamma_w}{2k_h} \frac{\partial E}{\partial t} \left( r_e^2 \ln \frac{r}{r_w} - \frac{r_e^2 - r_s^2}{2} \right) - p_0 \left[ 1 - (1-k_i) \frac{z}{H} \right] \quad r_s \leq r \leq r_e
\]
(5-2)

Incorporate formula (5) into \( \overline{u_r} = \frac{1}{\pi (r_e^2 - r^2)} \int_{r_e}^{r} 2\pi ru_r dr \), follow as:

\[
\overline{u_r} = \frac{\gamma_w r_e^2}{2k_h} F(n) \frac{\partial E}{\partial t} - p_0 \left[ 1 - (1-k_i) \frac{z}{H} \right]
\]
(6)

According to the overall balance equation \( \frac{\partial E}{\partial t} = -m_r \frac{\partial u_r}{\partial t} \), we can get

\[
\frac{\partial u_r}{\partial t} = -\frac{2k_h}{m_r \gamma_w r_e^2 F(n)} \left[ \overline{u_r} + p_0 \left[ 1 - (1-k_i) \frac{z}{H} \right] \right]
\]
(7)

where \( F(n) = \frac{n^2}{n^2 - 1} \left( \ln \frac{n}{s} + \frac{k_h}{k_s} \ln s - \frac{3}{4} \right) + \frac{s^2}{n^2} \left[ 1 - \frac{k_h}{k_s} \right] \left( 1 - \frac{s^2}{4n^2} \right) + \frac{k_h}{k_s} \frac{1}{n^2 - 1} \left( 1 - \frac{1}{4n^2} \right) \),

\( s = \frac{r_s}{r_w}, \quad n = \frac{r_e}{r_w} \).

Using Fourier series, the excess pore water pressure in the foundation soil can be obtained as:

\[
u_r = -p_0 \sum_{m=0}^{\infty} \frac{1}{F(n) + D} \left[ \frac{k_h}{k_s} \left( \ln \frac{r}{r_w} - \frac{r_e^2 - r_w^2}{2r_e^2} \right) + D \right] \frac{2}{M} \left[ 1 - (1-k_i) \frac{z}{H} \right] \sin \frac{M}{H} z \cdot e^{-b_r t}
\]

\[+ p_0 \left[ 1 - (1-k_i) \frac{z}{H} \right] \quad r_w \leq r \leq r_s
\]
(8-1)
where \( \beta_r = \frac{8k_s E_s}{\gamma_w d^2 e \left( F(n) + D \right)^2} \), \( M = \frac{2m + 1}{2} \pi \), \( D = \frac{8(n^2 - 1)}{M^2 n^2}. \)

Then the degree of soil consolidation at any depth is expressed as:

\[
U_r = \frac{u_0 - u_r}{u_0} = 1 - \frac{u_r}{u_0}
\]

where \( u_0 \) is the ultimate excess pore water pressure in the soil.

4. Site condition

The soft soils along the southeast coast of China are distributed regionally, especially in the east, south and west of Guangdong Province. They are mainly soft soil layers such as silt and silty soil. They are characterized by low strength, high water content, easy compression, and high sensitivity. The proposed site is in the Pearl River Delta region, with thick layered silt clay widely distributed, shallow buried, and belongs to a highly compressible soil layer. The soil layer information of the site is shown in Table 1, and the vacuum preloading test site distribution is shown in Figure 2.

The SPB-B type 100 mm × 4 mm plastic drainage board was used in the field test site, which was arranged in a plum blossom shape with a pitch of 1.2 m, and the buried depth was 22 m. A layer of 50 cm sand cushion is laid on the upper part of the ground, and horizontal drainage pipes are laid. The distribution of pore water pressure monitoring points is shown in Figure 1. Among them, the monitoring of pore water pressure is realized by vibrating wire pore pressure gauge. The soil is arranged at 3.0 m intervals from top to bottom, and the monitoring period is 83 d. The vacuum value of pump in the site is 85 kPa, and it takes 20 d from the beginning to full load. The loading time curve is shown in Figure 3.
Based on the principle of equal drainage [14], the vertical drainage plate is equivalently converted into sand wells according to the following method:

\[ d_w = \alpha \frac{2(b + \delta)}{\pi} \]

where \( d_w \) is the equivalent diameter of the plastic drainage plate, \( \alpha \) is the conversion factor, between 0.75 and 1.00, generally 1.00, \( b \) and \( \delta \) are the width and thickness of the plastic drainage board, respectively.

### Table 1 Soil layer parameter information table

| Soil layer | \( \gamma \) kN/m³ | \( E_s \) Mpa | \( C \) kPa | \( k_h \) cm/s | \( \phi \) ° | \( e \) | \( \nu \) |
|------------|----------------|-------------|----------|------------|--------|--|---|
| Plain fill | 18.0 | 3.89 | 5.0 | 8.11×10⁻⁸ | 5.00 | 0.769 | 0.30 |
| Silt       | 15.2 | 1.76 | 2.9 | 5.96×10⁻⁸ | 2.30 | 2.013 | 0.41 |
| Sand       | 19.0 | 11.66 | 4.0 | 1.88×10⁻⁴ | 30.6 | 0.658 | 0.34 |
| Silty soil | 17.3 | 3.40 | 8.3 | 2.31×10⁻⁵ | 9.10 | 1.406 | 0.36 |

where \( \gamma \), \( E_s \), \( C \), \( k_h \), \( \phi \), \( e \) and \( \nu \) are weight, elastic modulus, cohesion, permeability coefficient, internal friction angle, void ratio and poisson's ratio of the soil, respectively.

### 5. Results and discussion

The duration curve of the excess pore water pressure of the soil at FC03 point in the site area is shown in Figure 4. At the beginning, the excess pore water pressure of each layer of soil is zero, and then the excess pore water pressure slowly accumulates under the action of vacuum negative pressure in the plastic drainage plate. At 20 days, the negative pressure in the vacuum pump reached -85 kPa, and the excess pore pressure in the soil at different depths continued to increase. Because the silt soil layer in the site area is thick, covering almost the entire treatment range of the plastic drainage board, the accumulation of excess pore pressure requires a time process. Now select the pore pressure monitoring data to get the distribution map of the excess pore pressure along the depth, as shown in Figure 5. The excess pore water pressure values in each layer of soil tend to be stable, which the super pore pressures of the soil at depths of 3 m, 9 m, 15 m and 21 m are -73 kPa, -54 kPa, -35 kPa and -14 kPa. It can be seen that the bottom layer of the plastic drainage board (that is the bottom of the silt soil layer) is affected by the vacuum negative pressure at the upper part, and the corresponding excess pore pressure is generated, thereby generating reinforcement settlement. It can be obtained that the excess pore pressure in the soil shows a linear attenuation law along the soil, and the fitting linear equation is calculated, and the linear attenuation coefficient is 3.4 kPa/m. The excess pore pressure in the soil is caused by vacuum negative pressure. According to the linear equation of the distribution of excess pore pressure, the distribution of vacuum negative pressure in the plastic drainage plate in the field can
be considered.

\[ p = -3.41h + 84.81 \]

Fig. 4 Excess pore water pressure duration curve

Fig. 5 Distribution of excess pore water pressure along depth

Based on the analytical solution of consolidation under linear attenuation, using the field test data obtained from the above monitoring, the consolidation problem of soft soil foundation treated by vacuum preloading is analyzed and studied. Because the solution in the article is suitable for single-layer homogeneous foundation soil, the main treatment layer on site is silt soil layer, and the buried depth is large. The weighted sum of the soil parameters of the artificial fill and the silt layer can be used to obtain the soil parameters of the entire foundation.

From the field engineering data, take the horizontal permeability coefficient \( k_h \) of the undisturbed soil as \( 6.26 \times 10^{-8} \) m/s, compression modulus \( E_s \) is 2.05 Mpa, \( P_0 \) is 85 kPa. Cen Yangrun (2003) [15] obtained a linear attenuation of vacuum negative pressure in the soil based on field test data, and the attenuation coefficient is between 0.3 and 0.9, which \( k_1 \) is chosen as 0.88 in this paper. The equivalent transformation radius \( r_w \) of the plastic drainage board is 0.033 m. According to the specification [16], the horizontal permeability coefficient \( k_s \) of the smeared area is 1/3 to 1/5 of \( k_h \), which is \( 1.90 \times 10^{-8} \) m/s. The radius value \( r_s \) of the smearing area is 2 to 3 times of \( r_w \), which is 0.066 m, that is, \( s \) is 2. The effective drainage radius value \( r_e \) is 15 to 30 times that of \( r_w \), which is 0.63 m, that is, \( n \) is 19.

Figure 6 shows the comparison of the degree of consolidation under the Hansbo solution, the solution in this paper, and the monitoring value calculation. It can be seen that, because the Hansbo solution considers the consolidation of the ideal underground soil, the conditions are too ideal, and the calculation results in practical applications are quite different. In this paper, the linear attenuation of
vacuum negative pressure is considered, and a more accurate value is obtained.

6. Conclusions
This paper is based on the sand drain foundation consolidation model and considers the hypothetical conditions of the vertical attenuation of the vacuum negative pressure along the soil. The Fourier transform is used to obtain the pore water pressure values and soil consolidation degree at different depths in the soil. The conclusions are as follows:

The pore water pressure in the soil is affected by the vacuum negative pressure. At 20 days, the excess pore water pressure continues to accumulate and continues to increase. At 60 days, it tends to stabilize. It can be seen that the accumulation of excess pore pressure in the silt soil requires a time course, which is later than the loading days.

Under the action of vacuum negative pressure, the soil produces excess pore pressure. It can be considered that the distribution of excess pore pressure along the depth reflects vacuum negative pressure. Based on the distribution characteristics of excess pore pressure, it can be considered that the linear attenuation degree of vacuum negative pressure soil is 3.4 kPa/m.

Based on the changes in the over-pore pressure on site, the Fourier transform is used to obtain the consolidation analytical solution. Compared with the Hansbo solution, the solution in this paper is more in line with actual engineering case applications.

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References
[1] Kjellmann, W. Consolidation of clay soil by means of atmospheric pressure [A] // Proceedings on Soil Stabilization Conference, Boston[C]. USA, 1952: 258-263.
[2] HIROSHI Y, HIROFUMI N. Consolidation of soils by vertical drain wells with finite permeability[J]. Soils and foundation, 1974, 14(2):35-46.
[3] Xie Kanghe, Zeng Guoxi. Analytical theory of sand drain foundation consolidation under constant strain conditions[J]. Chinese Journal of Geotechnical Engineering,1989(02):3-17.
[4] SHEN Zhujiang, LU Shunying. Analysis of consolidation deformation of soft ground by vacuum preloading[J]. Chinese Journal of Geotechnical Engineering, 1986, 8(3) : 7-15.
[5] Dong Zhiliang. Analytical Theory of Consolidation of Sand Drain Foundation with Heap Loading and Vacuum Preloading[J]. Water Transportation Engineering,1992(09):1-7.
[6] ZHANG Gongxin, MO Haihong, DONG Zhiliang, et al. Analysis of relationship between vacuity and pore-water pressure in vacuum preloading[J]. Rock and Soil Mechanics, 2005, 26(12) : 1949-1952.
[7] MEI Guoxiong, XU Kai, ZAI Jinming, et al. Deformation mechanism of soft foundation under vacuum preloading[J]. Chinese Journal of Geotechnical Engineering, 2006, 28(9): 1168-1172.

[8] Peng Jie, Dong Jiangping, Song Enrun, Hong Lei. Axisymmetric analytical solution of vacuum preloading considering loading process[J]. Rock and Soil Mechanics, 2010, 31(S1): 79-85.

[9] WU Yuedong, WU Hongsheng, LUO Ruping, ZENG Chuichang. Analytical solutions for vacuum preloading consolidation considering vacuum degree attenuation and change of permeability coefficient in smear zones[J]. Journal of Hohai University(Natural Sciences), 2016, 44(2): 122-128.

[10] INDRARATNA B, BAMUNAWITA C, KHAMBAZ H. Numerical modeling of vacuum preloading and field applications[J]. Canadian Geotechnical Journal, 2004, 41(6): 1098-1110.

[11] TANG M, SHANG J Q. Vacuum preloading consolidation of Yaoqiang Airport runway[J]. Geotechnique, 2004, 50(6): 613-623.

[12] CHU J, YAN S W, YANG H. Soil improvement by the vacuum preloading method for an oil storage station[J]. Geotechnique, 2000, 50(6): 625-632.

[13] CHU J, YAN S W. Estimation of degree of consolidation for vacuum preloading projects[J]. International Journal of Geomechanics, 2005, 5(2): 158-165.

[14] HANSBO S. Consolidation of caly by band-shaped prefabricated drains[J]. Ground Engineering, 1979, 12(5): 16-18.

[15] CEN Yangrun. Vacuum preloading: experiment and theory[Ph. D. Thesis][D]. Hangzhou : Zhejiang University, 2003.

[16] The Professional Standards Compilation Group of People’s Republic of China. JGJ 79—2012 Technical code for ground treatment of buildings[S]. Beijing: China Architecture and Building Press, 2012.(in Chinese)