Strings with axionic content and baryogenesis

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Abstract

We describe different electroweak strings with axionic content, including non-topological configurations calculated numerically, and show their possible influence on baryogenesis indicating that they may constitute a mechanism competitive to that of bubble nucleation with two Higgs-doublets.
1 Introduction

There is much interest on the possibility that baryogenesis in the universe had occurred during the electroweak (EW) phase transition \[1\], even though particular GUT models with \( B - L \) violation are not excluded \[2\]. Due to the weakly first-order feature of this transition \[3\] it may be important to have contributions from strings \[4\] in addition or as alternative to the one coming from the bubble nucleation. It seems also necessary to find sources of \( CP \) violation stronger than that of minimal Standard Model (SM).

In the present work we estimate the influence on baryogenesis of electroweak strings which include axions. The relevance of this particle, possible candidate for the dark matter, is twofold. On one side it may increase the stability of the otherwise unstable electroweak strings. On the other hand the \( CP \) violation caused by the non-vanishing axion field in the phase where baryogenesis is produced may be an alternative to the two Higgs-doublets model or the supersymmetric extensions \[5\]. We analyze the cases of the electroweak string which attracts axions inside its core because of the smaller mass of these particles in the high temperature phase \[6\], the global axionic string formed at a higher scale which with the addition of electroweak components assures the flux constancy of the \( Z \)-magnetic field \[7\] \[8\], and the non-topological configuration \[9\] due to the interaction of the axion with the neutral gauge boson \( Z \). The existence of these last strings is shown numerically in the thin-wall approximation, and it is found that in the case in which the EW transition is of first order they become unstable below the critical temperature competing with bubbles to transform the metastable symmetric phase into the broken symmetry stable one.

In section 2 we describe baryogenesis in the EW transition with the possible contribution of general strings. In section 3 the strings with axionic content are considered. The existence and relevance of non-topological strings are numerically shown in section 4 and brief conclusions are included in section 5.

2 Baryogenesis at the electroweak transition and strings.

The rate of generation of baryonic number per unit volume \( n_B \) is \[1\]

\[
\frac{dn_B}{dt} \simeq -\frac{\Gamma}{T}\mu_B ,
\]

where \( \Gamma \) is the probability per unit volume and time of anomalous events, \( T \) is the temperature and \( \mu_B \) is the chemical potential of baryonic number, which is related to the densities of all left-handed fermions.

Whereas the unsuppressed anomalous events rate in the symmetric phase is

\[
\Gamma \sim (\alpha_W T)^4 ,
\]

where \( \alpha_W \) is the weak mixing angle.

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with $\alpha_W$ electroweak coupling, $\mu_B$ depends on the possibility that baryogenesis is local or that the bias produced by the $CP$ violation arrives by diffusion to a larger region according to details of the transition. In the former case, if the $CP$ violation is given by the relative phase $\theta$ between two Higgs doublets, the baryon number density ideally produced by its change through the wall of an expanding broken symmetry region which covers the whole space is \[10\]

$$n_B \sim -\frac{\Gamma}{T} \left(\frac{m_t}{T}\right)^2 \Delta \theta,$$  \hspace{1cm} (3)$$

where the top quark is the most relevant of the species because of its high mass which gives larger coupling. In the latter case an effective $1/\alpha_W^2$ factor appears \[11\] and the lepton $\tau$ becomes competitive to the top quark because of its larger diffusion property.

Being the entropy density

$$s \sim g_* T^3,$$  \hspace{1cm} (4)$$

with $g_*$ number of massless particle modes at the EW transition, we may express the known present ratio of baryon to photon densities $\eta$ as

$$\frac{\eta}{7} = \frac{n_B}{s} \sim a\frac{\alpha_W}{g_*} \theta_{CP} \frac{V_{BG}}{V}.$$  \hspace{1cm} (5)$$

In Eq.(3) $n = 4$ (2) for local (non local) baryogenesis and $\theta_{CP}$ is the effective parameter, including dynamical features, due to the change of the $CP$-violating angle through the wall of the defect responsible for the universe non-equilibrium period necessary to produce the matter-antimatter asymmetry. The suppression factor $SF = V_{BG}/V$ is the ratio of the volume active for baryogenesis to the total one.

If the EW phase transition is clearly of first order, its traditional mechanism corresponds to the nucleation of bubbles of the broken-symmetry phase in the metastable sea of the symmetric phase. In this case baryogenesis is produced in the non-equilibrium region outside the wall of the expanding bubble so that almost every point in space becomes active and $SF \sim 1$. Since $\alpha_W^2 \sim 10^{-3}$ and $g_* \sim 10^2$, to obtain from Eq.(3) the experimental result $n_B/s \sim 10^{-10}$ a relevant contribution of $\theta_{CP} \sim 10^{-2} - 10^{-5}$ is necessary. This cannot come from the Kobayashi-Maskawa phase which gives $\theta_{CP} \sim 10^{-20}$. But the spontaneous CP breaking due to the two Higgs-doublets extension of the Standard Model allows easily the desired result especially if non-local baryogenesis is envisaged.

Unfortunately for not too low Higgs mass the EW transition is perturbatively calculated to be weakly of first order \[3\] or even of second order with the minimal SM and the bubble mechanism might not be straightforward \[12\] even though non perturbative effects seem to maintain the discontinuous feature of the transition \[13\] which is also enhanced with the two Higgs-doublets models. It is in any case useful to think about the contribution of cosmic strings \[4\]. If we consider strings
formed at a scale $\Lambda$ where a symmetry is broken maintaining the EW one down to the lower scale $v$, its length and average separation from one another will be the correlation length $\xi$ according to the Kibble mechanism $[14]$. After the EW transition inside its width $\delta \sim 1/\sqrt{\lambda v}$, where $\lambda$ is the Higgs potential coupling constant, there will be symmetric phase and the outer region will correspond to the spontaneously broken vacuum so that, if the strings collapse to decrease their energy, the active volume will be

$$V_{BG} \sim \delta^2 \xi \frac{V}{\xi^3}. \quad (6)$$

The suppression factor to be inserted into Eq.(5) would not seem too severe, but one has to take into account that $\xi$ may be estimated as coming from the correlation length at the formation time $t_f$ of a transition different from the EW one. The length that one must introduce into Eq.(6) corresponds on the other hand to the EW time $t_{EW}$. If the string dynamics is still in the friction stage due to the dominance of the universe expansion term, the ratio will be $[15]$

$$\frac{\xi(t_{EW})}{\xi(t_f)} \sim \left( \frac{t_{EW}}{t_f} \right)^{5/4}, \quad (7)$$

with $\xi(t_f) \sim 1/\lambda' \Lambda$ and $\lambda'$ the Higgs potential coupling constant responsible for the string formation. Using the scale expansion for the radiation epoch

$$\frac{\Lambda}{v} \sim \left( \frac{t_{EW}}{t_f} \right)^{1/2}, \quad (8)$$

the suppression factor

$$SF = \left( \frac{\delta}{\xi(t_{EW})} \right)^2 \sim \frac{\lambda^2}{\Lambda} \left( \frac{v}{\Lambda} \right)^3 \quad (9)$$

is therefore very small unless $\Lambda$ is close to $v$.

### 3 Strings with axionic content

One may consider then $Z$-strings produced at the EW transition $[16]$ which would seem to give a relevant baryogenesis. But these strings are unstable for not too low Higgs mass $[17]$ so that their contribution to Eq.(3) would be in fact negligible unless new ingredients succeed in stabilizing them.

One possibility is the inclusion of the axion, hypothetical candidate for dark matter proposed as a dynamical field solution of the strong $CP$ problem $[18]$, which couples to gauge fields in a $CP$ violating way such that $\theta_{CP}$ may be taken as coming from $\Delta a/f_{PQ}$, where $\Delta a$ is the change of the axion field across the defect.
wall and \( f_{PQ} \approx 10^{12} \text{GeV} \) is the scale at which the Peccei-Quinn symmetry was broken \([19]\). Axions have been already considered \([20]\) thinking that for \( T > 1 \text{GeV} \) they are massless so that its field may fluctuate and give an interference with sphalerons which produces however a very small \( \theta_{CP} \sim 10^{-20} \).

We instead consider the possibility that the coherent change of the axion field given by a global string may be the seed of formation of electroweak strings. We remind that a global axionic string is a configuration where along the \( z \)-axis there is a filament of unbroken Peccei-Quinn phase defined by a chiral field \( \Psi = 0 \) whereas out of it in the \( x-y \) plane \( \Psi = f_{PQ} \exp (ia/f_{PQ}) \) with a dependence \( a/f_{PQ} = \alpha \) on the angle \( \alpha \) around the \( z \)-axis. When temperature goes below the critical EW one, two kinds of strings may be formed. One possibility is that a region in the \( x-y \) plane, at a distance \( r \) from the axis of the global string where \( \Psi = f_{PQ} \exp (i\alpha) \) and the Higgs field \( \varphi = 0 \), takes the form of a torus surrounded by the broken EW phase \( |\varphi| = |v|/\sqrt{2} \) and \( \Psi = f_{PQ} \) because a small axion mass forces \( a \) to the potential minimum \( a = 0 \). An alternative is that the \( z \)-axis filament of unbroken Peccei-Quinn phase \( \Psi = 0 \) and \( \varphi = 0 \) enhances to a finite width outside which, if there are two Higgs-doublets, the angle \( a/f_{PQ} \) compensates the difference of the phases of \( \varphi_1 \) and \( \varphi_2 \) giving a topological reason of stability to the EW string \([7]\).

Apart from these possibilities, when the temperature approaches the critical value from above, a torus in the \( x-y \) plane may appear inside which the EW broken phase \( \varphi = v/\sqrt{2} \) is formed and together with \( \Psi = f_{PQ} \exp (ia/f_{PQ}) \) simulates a Chern-Simons model through its coupling with gauge fields \([4]\) which stabilizes the resulting string, whereas outside it we have the symmetric phase \( \varphi = 0 \) and \( \Psi = f_{PQ} \exp (ia_0/f_{PQ}) \) with the approximation of an average value \( a_0 \) for the axion field since the axion mass is practically zero there.

We will consider these three strings in the above order. The fact that the axion must be lighter in the higher temperature phase may be simulated by an effective potential \([6]\) \([8]\) of interaction of the Higgs \( \varphi \) and axion \( a \) fields

\[
U = \lambda \left( \varphi^\dagger \varphi - \frac{v^2}{2} \right)^2 + m_a^2 f_{PQ}^2 + \kappa \left( \varphi^\dagger \varphi - \frac{v^2}{2} \right) \left( 1 - \cos \frac{a}{f_{PQ}} \right),
\]

where \( m_a \) is the axion mass in the phase of EW broken symmetry and \( \kappa \) fixes its extremely small mass \( m_a' \) in the symmetric phase according to

\[
m_a'^2 = \frac{\kappa}{2} \frac{v^2}{f_{PQ}^2} - m_a^2.
\]

The only important features of the effective potential \( U \) are the phase nature of the axion field and the fact that for the broken symmetry vacuum \( a = 0 \) whereas in the symmetric one the axion field prefers a non-vanishing value, due to the fact that there it needs not compensate the argument of the determinant of the mass matrix \([19]\).
The first proposed string has magnetic flux of the neutral $Z$ field through its core but it is unstable considering only EW fields for realistic parameters of the minimal SM $[17]$. The increase of stability due to the addition of axions is twofold $[8]$. On one side the fact that, analogously to bags, the disappearance of the string with the light axions concentrated in its core would require the energy for building the higher mass of axions in the broken symmetry phase. On the other hand, and this is of more relevance, the oscillation of the phase $a/f_{PQ}$ along the core between $0$ and $2\pi$ gives a quasitopological reason for the stability of these closed strings in analogy with the superconducting ones $[21]$. It is important to remark that whereas outside the string $|\varphi| = v/\sqrt{2}$, $a = 0$, inside the core $\varphi \simeq 0$, $a/f_{PQ} \sim \pi$, so that the variation of the $CP$ violating phase across the wall is $\Delta (a/f_{PQ}) \sim 1$, i.e. large without need of two Higgs-doublets.

Note that in the collapse of these strings the change $\Delta \theta$ has the same sign as in the expansion of bubbles because in the bias for baryogenesis the explicit $CP$ violation angle and the axion field appear with opposite signs. Regarding the suppression factor, it must be analyzed more carefully when strings are formed close to the scale of a possibly second order EW transition. In this case for the temperature $T_s$, smaller than the critical one $T_c$, when the string can be identified, i.e., when its length becomes larger than the Ginzburg length $[22]$, the width will be

$$\delta \sim \frac{1}{m_H(T_s)} = \frac{1}{\lambda^{4/7}} \left( \frac{m_P}{T_c} \right)^{3/14} \frac{1}{T_c},$$

where $m_P$ is the Planck mass, and the length is

$$\xi \sim \frac{1}{\lambda^{3/7}} \left( \frac{m_P}{T_c} \right)^{2/7} \frac{1}{T_c}.$$  

Therefore the suppression factor turns out to be

$$SF = \left( \frac{\delta}{\xi} \right)^2 \sim \frac{1}{\lambda^{2/7}} \left( \frac{T_c}{m_P} \right)^{1/7} \lesssim 10^{-2}$$

for $T_c \sim v$. As a result $n_B/s$ would be of the correct order of magnitude, if kinematical factors are similar to those of bubbles so that $\theta_{CP} \sim 10^{-3}$, and non-local effects enhance baryogenesis.

For the formation of the string it is favourable that the axion is heavier in the broken symmetry phase. This might occur in a more definite way at a lower scale $\sim 1GeV$ where $QCD$ effects are particularly relevant. We must note that the very strong true magnetic fields present at the electroweak scale may cause the instability of the broken symmetry vacuum $[23]$ delaying the phase transition to a lower scale closer to the $QCD$ one where, due to the universe expansion, the magnetic field is weaker. If this occurs we may trust more in our string, paying however the price of a smaller $T_c$ in Eq.(12) with a slight decrease of $SF$. 


The second proposal for formation is one to one related to the original axionic string, at variance from the previous case where many EW strings may appear from the same global one. Therefore the suppression factor \( f_{PQ} \) will be small due to the formation scale \( f_{PQ} \) much larger than the EW one \( v \).

These strings base their stability on a topological argument which requires two Higgs-doublets with the potential \( V(\varphi_i, \Psi) = \sum_{i=1}^{2} \frac{\lambda_i}{4} (\varphi_i^\dagger \varphi_i - v_i^2)^2 + \frac{\lambda}{2} (\varphi_1^\dagger \varphi_1)(\varphi_2^\dagger \varphi_2) + \frac{\lambda'}{2} (\varphi_1^\dagger \varphi_2)(\varphi_2^\dagger \varphi_1) + f_{PQ} v_1 v_2 - \frac{1}{2} [([\varphi_1^\dagger \varphi_2] \Psi + h.c.) \] \( \text{(15)} \)

which forces the axion field to compensate the difference of Higgs angles, i.e. \( \frac{f_{PQ}}{f_{PQ}} = \theta_1 - \theta_2 \), outside their core.

It must be noted that with the potential \( \text{(15)} \) there are not \( CP \) violating terms associated with Higgs doublets so that the bias for baryogenesis is entirely due to the axion coupling with gauge fields where, because of the global origin of the string, \( \Delta a/f_{PQ} = 2\pi \) around its axis. If \( CP \) violating terms of the type \( \text{Re}(\varphi_1^\dagger \varphi_2) - v_1 v_2 \cos(\theta_1 - \theta_2) \) and \( \text{Im}(\varphi_1^\dagger \varphi_2) - v_1 v_2 \sin(\theta_1 - \theta_2) \) were added to Eq. \( \text{(15)} \), the strings would become unstable and the bias due to \( \theta_1 - \theta_2 \) would tend to compensate that caused by the axion field.

The stability of the axionic strings with the potential \( \text{(15)} \) should be assured until the \( QCD \) scale, giving time to a delayed EW breaking, when domain walls attach to them making maximum the decay by radiation of axions \( \text{[24]} \). However, even though the dynamics of global strings is difficult because of long range interactions, one may expect a small contribution to baryogenesis due to the suppression factor.

The last string we will consider is the non-topological configuration, regarding a single Higgs doublet, which is made possible by the interaction of the axion with the gauge field \( Z \). The existence of this configuration without a Maxwell kinetic term was numerically proved in 2+1 dimensions where the previous interaction was replaced by a Chern-Simons term \( \text{[9]} \) and evidences were also given including the Maxwell term \( \text{[25]} \). Adding the third spatial dimension, the role of the Chern-Simons constant is played by the constant gradient of the axion field along the \( z \)-axis inside the string, remnant of the axionic string which existed when the configuration was formed. The numerical computation of this case is described in the next section.

4 Non-topological strings

The energy of a static cylindrically symmetric configuration including the coupling of the axion field only with the gauge field \( Z \) for small enough electric field
turns out to be
\[ E = \int d\vec{r} \left[ \frac{1}{2} \vec{B}_Z^2 + \frac{1}{4} \left( \frac{\alpha}{f_{PQ} g} \vec{\nabla} a \cdot \vec{B}_Z \right)^2 + (g \varphi Z \theta)^2 + \left( \vec{\nabla} \varphi \right)^2 + \left( \vec{\nabla} a \right)^2 + U \right] \]  
(16)
where the potential \( U \) is shown in Eq.\((\text{10})\) and we look for the behaviour in the \( x - y \) plane
\[
\varphi \to 0 \quad a \to a_o \quad r \to \infty ,
\]
(17)
\[
\varphi \to \frac{\psi}{\sqrt{2}} \quad a \propto z \quad r \to 0 .
\]
The reason of the possible stability for \( T > T_c \) is that, at variance from the purely Maxwell kinetic term, the Chern-Simons-like contribution allows \( \vec{B}_Z \neq 0 \) only in the region where \( \varphi \neq 0 \), i.e. inside the string core. Moreover, even though \( U \) increases the energy for \( a \) increasing monotonically along the string, for closed loops the change \( \Delta a = N 2 \pi f_{PQ} \) produces again a quasitopological stabilization.

Taking the thin wall approximation for the change of fields \( a \) and \( \varphi \) in a common width \( \varepsilon \ll R \), with \( R \) radius of the string, the equations of motion give the increase of magnetic field as an expansion in powers of \( r \)
\[
B_Z(r) \simeq B_Z(0) \left( 1 + \frac{1}{2} g^2 v^2 r^2 \right) .
\]
According to how many times the radius is larger than the width, the magnetic field will tend to concentrate in the wall. We will take this extreme case which is the most disfavourable for the stability of the string and denote the flux as \( 2 \pi h_0 \).

The minimization of the energy described by Eq.\((\text{16})\) with respect to \( R \), which is expected to vary with \( z \), gives
\[
\frac{2 \pi}{\varepsilon} (v^2 + a_0^2) + 2 \pi U_0 R - \left[ \frac{2 \pi^3}{\varepsilon} \left( \frac{\alpha h_0}{g v L} \right)^2 + \frac{\pi \varepsilon}{8} (g v h_0)^2 + \frac{2 \pi h_0^2}{\varepsilon} \right] \frac{1}{R} \]
\[
- \frac{2 \pi}{\varepsilon} (v^2 + a_0^2) R \ddot{R} - \frac{4 \pi}{\varepsilon} (v^2 + a_0^2) R \dot{R} - \frac{(4 \pi)^2 f_{PQ} a_0}{\varepsilon L} R \dot{R}
\]
\[
- \frac{2 \pi h_0^2}{\varepsilon} \dot{R} + \frac{\pi h_0^2}{\varepsilon} \frac{\dot{R}^2}{R^2} = 0 ,
\]
(18)
where \( L \) is the string length, dots represent derivatives with respect to the string axis \( z \) and inside the core the potential is
\[
U_0 = m_a^2 f_{PQ}^2 \left( 1 - \cos \frac{2 \pi}{L} z \right) .
\]
We are considering \( U = 0 \) outside the string to simulate the inclusion of temperature effects such that \( T = T_c \). Eq.\((\text{18})\) has been solved first for the terms
independent of \( z \) and without \( U_0 \), together with the one which gives the minimization of the energy with respect to the wall width \( \varepsilon \), valid if \( L \gg 10^{-4} GeV^{-1} \),

\[
\epsilon^2 = \frac{h_0^2 + 2a_0^2R_0^2}{(gebo)^2} + 2\left(\frac{\pi f_{PQ}}{L}\right)^2 R_0^2,
\]

(19)

giving the constant radius \( R_0 \). With the resulting \( \varepsilon \) and taking \( R_0 \) as input we solved the complete Eq.(18) with the conditions \( R(0) = R(L) \) looking for solutions such that \( \dot{R}(0) = \dot{R}(L) \) in order to have periodic conditions and consider the string as a closed loop. To allow this interpretation we require \( L > 10^1 R \) and look for solutions with only one oscillation of \( a \) along the loop in correspondence to lowest excited states.

In Table 1 we show the parameters of the found solutions with accepted values of the physical constants appearing in Eq.(18). These solutions are very close to the constant ones. Even though the flux is high, it corresponds for many of the indicated solutions to magnetic fields \(|\vec{B}_Z| < 10^{24} G\) which is the critical value \([23]\) for the destabilization of the broken symmetry vacuum. The upper limit for \( L \) corresponds to the fact that a single oscillation \( 0 \leq a/f_{PQ} \leq 2\pi \) does not allow to stabilize too long strings. The lower limit for \( L \) comes from the need that \( \epsilon < R \).

| \( L \) | \( R/L \) | \( \epsilon/L \) | \( h_0 \) |
|------|------|------|------|
| \( 10^4 \) | 0.077 | \( 0.41 \times 10^{-1} \) | \( 10^{14} \) |
| \( 10^5 \) | 0.077 | \( 0.41 \times 10^{-2} \) | \( 10^{14} \) |
| \( 10^7 \) | 0.077 | \( 0.41 \times 10^{-3} \) | \( 10^{15} \) |
| \( 10^9 \) | 0.077 | \( 0.41 \times 10^{-4} \) | \( 10^{16} \) |
| \( 10^{11} \) | 0.077 | \( 0.41 \times 10^{-5} \) | \( 10^{17} \) |
| \( 10^{13} \) | 0.077 | \( 0.41 \times 10^{-6} \) | \( 10^{18} \) |
| \( 10^{15} \) | 0.077 | \( 0.41 \times 10^{-7} \) | \( 10^{19} \) |
| \( 10^{17} \) | 0.077 | \( 0.41 \times 10^{-8} \) | \( 10^{20} \) |
| \( 10^{19} \) | 0.077 | \( 0.41 \times 10^{-9} \) | \( 10^{21} \) |

**Tab. 1.** For the string solutions of Eq.(18) we report the length \( L \) measured in \( GeV^{-1} \); the ratio, with respect to \( L \), of the mean radius \( R \), and the width \( \epsilon \) defined in Eq.(19), together with the Z-magnetic flux \( h_0 \). The following constants have been used: \( \alpha = 1/137 \), \( g = 0.3 \), \( v = 250 GeV \), \( f_{PQ} = 10^{12} GeV \), \( a_0 = 10^{14} GeV \) and \( m_a = 10^{-14} GeV \).

At variance from bubbles, non-topological strings (NTS) are produced when \( T > T_c \). If the EW transition is of first order, when the system cools below \( T_c \) the non-topological string survives and becomes first metastable and finally unstable expanding as the bubble.

It is important to compare the contribution of critical bubbles and NTS to the phase transition through their Boltzmann probability factor \( exp(-E/T) \).
For the former ones, their energy is infinite for $T = T_c$ and decreases for $T < T_c$. It is easy to estimate the bubble energy minimizing the sum of the negative volume contribution and positive surface one. We get

$$E_b = \frac{16}{3} \pi \frac{v^6}{\epsilon_b^2 (\Delta U)^2},$$

(20)

where $\Delta U$ is the difference of the energy density for symmetric and broken symmetry vacua and $\epsilon_b \sim 100^{-1} GeV^{-1}$ is the bubble wall width.

Regarding the string energy for $T < T_c$ its most important contributions come from the negative volume term which must be included in $U$ of Eq.(16), the surface and Maxwell terms. The equality of the bubble and string energy occurs when

$$\Delta U_{EQ} \simeq 10^{-4} GeV^4$$

for the shortest strings

and

$$\Delta U_{EQ} \simeq 10^{-16} GeV^4$$

for the longest ones.

When $\Delta U < \Delta U_{EQ}$ one may expect that the NTS are relevant because their energy is smaller than the bubble one. However one must verify that the critical difference of vacua energy densities $\Delta U_c$, which makes the NTS unstable, is smaller than $\Delta U_{EQ}$. One may estimate $\Delta U_c$ taking the most relevant terms for the string energy and putting the condition that the second derivative with respect to the mean radius is not positive in order to avoid the local metastable minimum. Whereas for the shortest strings $\Delta U_c \gg \Delta U_{EQ}$ indicating that they remain metastable while bubbles expand and produce the phase transition, for the longest strings $\Delta U_c \simeq 10^{-21} GeV^4 \ll \Delta U_{EQ}$. Therefore the latter become unstable very soon and contribute to the phase transition more than bubbles in the initial stage. But one has to note that bubbles which equal the energy of the longest strings are extremely large and have $E_b \sim 10^{52} GeV$ i.e. of the order of Earth mass so that most of the phase transition will presumably occur for lower temperatures when bubbles can be easily formed by thermal fluctuations.

Our model for the non-topological string has been oversimplified particularly in having fixed a common wall width for the Higgs and axion fields giving therefore a very high contribution of kinetic energy of the latter, and also in not taking into account in detail temperature effects on the potential to describe the cosmological evolution. It is however interesting to have shown that there is a period during which the longest NTS expand and have energy smaller than the bubble one.

Since the NTS expand as the bubbles they have a suppression factor $SF \sim 1$. On the other hand the change through their wall of the axion field is on the average $\Delta(a/f_{PQ}) \sim \pi$ having the correct sign for the baryogenesis bias. In this sense they might replace the two Higgs-doublets except for the fact that the latter are probably needed to assure the first order feature of the EW transition.
5 Conclusions

We have seen that electroweak strings originated in axionic ones increase their stability and may contribute to the baryogenesis even if the transition is of second order.

The addition of the axion to the standard model represents an extension smaller than the others proposed to explain the observed matter-antimatter asymmetry.

The limitation to the usefulness of these strings, apart from the obvious question regarding the existence of the axion, may be the possibility of other decay channels which decrease their number as well as the necessity of analyzing the details of the probability of their formation.

Through a careful study of the cosmological evolution of axionic strings it may be possible to determine their quantitative influence on baryogenesis.

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