A Barren Landscape?

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Abstract

We consider the generation of a non-perturbative superpotential in F-theory compactifications with flux. We derive a necessary condition for the generation of such a superpotential in F-theory.

For models with a single volume modulus, we show that the volume modulus is never stabilized by either abelian instantons or gaugino condensation. We then comment on how our analysis extends to a larger class of compactifications. From our results, it appears that among large volume string compactifications, metastable de Sitter vacua (should any exist) are non-generic.

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1 Introduction

The space of four-dimensional N=1 string compactifications is a mysterious, rich, but still largely unexplored region. These compactifications constitute our current best hope of connecting string theory with observed phenomenology. However, there are a number of basic issues that need to be overcome. The first issue is the construction of string vacua with, ideally, no massless scalar fields. Unfortunately, most string vacua have many massless scalar fields, or moduli, that parametrize the ways in which the compactification space can be deformed while preserving supersymmetry. The second issue is breaking supersymmetry without generating an enormous cosmological constant. It is worth stressing that these issues are not special to string theory, but are present in any model of low-energy phenomenology that involves compactifying a higher dimensional theory.

It is very possible that these problems are simply telling us that we are still missing critical ingredients in trying to connect string theory with observation; perhaps, the physics of the initial cosmological singularity. However, there is another alternative first studied in field theory [1, 2]. Namely, that there exist many metastable de Sitter vacua with a distribution of cosmological constants. By membrane nucleation, we tunnel between vacua until we arrive at a metastable vacuum that describes our universe. In string theory, this approach has been studied in [3, 4]. The typical assumption in this kind of analysis is that metastable string solutions exist in regions beyond our current computational control; for example, in small volume compactifications where supergravity cannot be trusted.

A large class of N=1 compactifications can be described in the framework of F-theory [5]. These purely geometric compactifications can be studied in a large volume limit, but they tend to suffer from large numbers of moduli. A more interesting class of vacua are found by considering F-theory compactifications with flux described in [6] based on the M-theory compactifications of [7]. As shown in [6, 8], these warped compactifications typically have far fewer moduli than conventional geometric compactifications. The underlying reason for the existence of these warped, reduced moduli compactifications is, however, a purely stringy one: the existence of a tadpole for D3-brane charge in F-theory [9].

What concerns us in this letter is the structure of non-perturbative contributions to the space-time superpotential. These corrections have been studied in [10] where a criterion for non-vanishing instanton contributions was derived. Our goal is to extend this analysis to warped compactifications with flux. We are motivated, in part, by an interesting but yet unrealized proposal to fix all the moduli of a flux compactification in a regime where supergravity is valid [11]. Recently, there has been a summary of potential problems in...
scenarios of this kind [12, 13]. However, these issues are secondary to the more basic question of whether any metastable vacua actually exist.

The proposal of [11] involves a set of reasonable ingredients. Fluxes can, in principle, freeze all the geometric moduli except for the volume modulus [6, 14]. Freezing the volume modulus, however, requires a non-perturbative stabilization mechanism beyond anything visible in supergravity. Usually, this mechanism involves abelian instantons, or gaugino condensation. From the perspective of low-energy supergravity, we might expect this to be a common occurrence in the space of N=1 compactifications. However, string theory is not supergravity. Generic superpotentials are not necessarily realized in string compactifications. There are many cases that illustrate this point; for example, heterotic string vacua are generically believed to be destabilized by world-sheet instantons [15]. However, recently it has been shown that a large class of these vacua are actually stable [16–18].

In this letter, we will present a kind of F-theory analogue of this result. We will derive a condition for the generation of a non-perturbative superpotential in F-theory (this includes certain type IIB orientifolds [19]). We then consider the simplest class of M-theory compactifications with only two volume moduli, but which admit an F-theory lift. This class of models has been the subject of recent investigations; see, for example, [20,21]. In this case, we show that neither abelian instantons nor gaugino condensation stabilize the volume. This kind of analysis can also be performed in a much wider class of F-theory models with more than one volume modulus, and we present an example. Indeed, it should be possible to analyze most F-theory models that admit non-abelian gauge symmetry. However, that is a subject to be explored elsewhere [22].

Note Added: After submitting this paper, we became aware of some interesting work, [33] and [34], with partial overlap.

2 Instantons and the Volume Modulus

2.1 Compactifications with flux

We want to describe 4-dimensional string vacua with N=1 supersymmetry. We take space-time to be flat Minkowski space. The class of vacua that we will consider are termed F-theory compactifications [5]. Although string theory is 10-dimensional, strangely enough, F-theory employs an 8-dimensional compactification space.

The way this comes about is as follows: an F-theory compactification is simply type IIB string theory compactified on a 6-dimensional space, \( B \), with positive Ricci curvature. To
compensate the non-vanishing curvature, the type IIB string coupling, $\tau$, must vary over $B$. This variation is nicely captured in the geometry of an 8-dimensional Calabi-Yau space, $\mathcal{M}$, with a torus fibration. The base of this fibration is $B$, while $\tau$ of the torus determines the type IIB string coupling. This structure can be generalized to include integral NS-NS and RR 3-form fluxes. These fluxes, denoted $H_3$ and $F_3$ respectively, combine into the usual IIB complex flux, $G_3$, which to preserve supersymmetry satisfies an imaginary anti-self-duality condition [6]

$$\ast G_3 = -iG_3. \quad (1)$$

As noted in the introduction, these compactifications have a stringy deficit of D3-brane charge. To satisfy this Gauss constraint, we must add a combination of D3-branes and fluxes chosen to satisfy tadpole cancellation [6, 9],

$$\frac{1}{2} \int H_3 \wedge F_3 + N_{D3} = \frac{\chi(\mathcal{M})}{24}, \quad (2)$$

where $N_{D3}$ is the number of D3-branes, while $\chi(\mathcal{M})$ is the Euler character of the 8-dimensional space, $\mathcal{M}$. Since adding D3-branes introduces additional moduli, we will restrict to models with only flux. For special choices of $\mathcal{M}$, F-theory compactifications can be described as type IIB orientifolds [19]. The same is true when fluxes are present [6] although it is worth noting that the resulting IIB orientifolds are not necessarily perturbative string compactifications.

It is often useful to think about these 4-dimensional compactifications as limits of M-theory compactified to 3 dimensions on $\mathcal{M}$. To return to 4 dimensions, we take the area of the torus fiber to zero. In this limit, M-theory goes over to type IIB string theory. Since all F-theory compactifications arise as limits of M-theory, we can freely use either picture. This will be useful later. The last point we should stress is that these flux compactifications are always non-generic. Finding supersymmetric flux vacua requires fine-tuning both complex structure and Kähler structure moduli (see [6] for details). This is the reason that many scalars are frozen. Indeed, in some orbifold examples of [6], and some $K3 \times K3$ examples of [6, 23], all the complex structure moduli are fixed. However, the overall volume modulus is never fixed this way.

2.2 Abelian instantons

Now let us turn to the generation of a non-perturbative superpotential. It is simplest to begin with smooth spaces $\mathcal{M}$ that only give rise to abelian symmetry. We will explain later that this is actually the only case that ever needs to be studied.
Since all F-theory compactifications are limits of M-theory compactifications, we begin in the more general M-theory setting. In this setting, the background flux, denoted $G_4$, is a (half) integral self-dual primitive 4-form. The integrality of $G_4$ is correlated with $\chi/24$ [24]. From the M-theory perspective, instantons are constructed by wrapping Euclidean M2 and M5-branes on cycles of $\mathcal{M}$. To stabilize volume moduli, we are interested in M5-brane instantons which, in the absence of $G_4$ flux, wrap complex divisors, $D$, of $\mathcal{M}$. To generate a term in the superpotential, an instanton must produce exactly the right number of fermion zero-modes. In [10], Witten derived a necessary condition on $D$ for this to be the case,

$$\chi_D = 1,$$  \hspace{1cm} (3)

where $\chi_D$ is the arithmetic genus of the divisor. This condition is necessary but not sufficient. It can be satisfied by divisors which have moduli. Whether those divisors contribute is hard to determine. The only divisors that contribute for sure are rigid (in the absence of D3-branes [25]). We will actually rule out any divisor satisfying (3).

To account for the background flux, we will take the following approach. At the level of the supergravity solution, we are free to ignore flux quantization, and tune the flux to zero. The metric on $\mathcal{M}$, which is conformally Calabi-Yau [7], becomes simply Calabi-Yau and we arrive at a standard M-theory compactification. The warp factor becomes constant. Under this smooth deformation, a BPS instanton should remain BPS, and so must satisfy (3).

This does not mean that the $G_4$ flux is irrelevant! In fact, it is harder to generate a superpotential in flux compactifications because not all divisors contribute. The M5-brane world-volume contains a coupling between $G_4$ and the anti-self-dual world-volume 2-form, $b_2$. This coupling is proportional to

$$\int_D b_2 \wedge G_4.$$  \hspace{1cm} (4)

Wrapping a Euclidean M5-brane on a divisor through which $G_4$ flux threads generates a source for $b_2$. For a compact divisor, these sources must sum to zero so the total flux through $D$ must vanish.

### 2.3 Reformulating the arithmetic genus condition

So now we are in the situation of trying to find divisors, $D$, in $\mathcal{M}$ with $\chi_D = 1$. Our task is aided by the integral expression for the arithmetic genus,

$$\chi_D = \int_D \text{Td}(D),$$  \hspace{1cm} (5)
where \( \text{Td}(D) \) is the Todd class of \( D \). In terms of Chern classes, the Todd class is given by,

\[
\text{Td}(D) = 1 + \frac{1}{2}c_1 + \frac{1}{12}[c_2 + c_1^2] + \frac{1}{24}c_2c_1. \tag{6}
\]

We denote the \((1,1)\) cohomology class Poincaré dual to \( D \) by \([D]\). This form acts like a delta function restricting us to \( D \) so

\[
\int_D \eta = \int_M \eta \wedge [D]
\]

for all forms \( \eta \) on \( M \). Now by the adjunction formula, the total Chern class of \( D \) is given in terms of the Chern classes of \( M \) by

\[
c(D) = \frac{c(M)}{1+[D]} = 1 - [D] + (c_2(M) + [D]^2) + (c_3(M) - c_2(M)[D] - [D]^3), \tag{7}
\]

where all forms are understood to be pulled back to \( D \). In particular, all of the Chern classes of \( D \) are pull-backs of forms on \( M \). Then\(^3\)

\[
\chi_D = \frac{1}{24} \int_D c_2(D)c_1(D) = -\frac{1}{24} \int_M \left(c_2(M)[D]^2 + [D]^4\right), \tag{9}
\]

and the arithmetic genus condition becomes

\[
\int_M \left(c_2(M)[D]^2 + [D]^4\right) = -24. \tag{10}
\]

### 2.4 Divisors dual to the Kähler cone of \( M \)

An interesting case to consider is the case when \([D]\) lies inside the Kähler cone of \( M \), i.e., when \([D]\) would be an acceptable choice for the Kähler form of \( M \). In this case, we immediately have that

\[
\int_M [D]^4 = 4! \times \text{Volume}(M) \geq 0. \tag{11}
\]

In order for such a divisor to have \( \chi_D > 0 \) it follows from \([\text{9}]\) that

\[
\int_M c_2(M)[D]^2 < 0. \tag{12}
\]

\(^3\)Alternatively, this follows directly from the Hirzebruch-Riemann-Roch formula for this case,

\[
\chi_D = \int_M \left(1 - e^{-[D]}\right) \text{Td}(M). \tag{8}
\]
However, this is impossible by the following inequality on a Calabi-Yau $n$-fold $X$ with Kähler class $[\omega]$ \[^{4}\]
\[
\int_X c_2(X)[\omega]^{n-2} \geq 0.
\]  
(14)

So if $D$ is dual to a Kähler class then instantons wrapping $D$ cannot contribute to the superpotential.

As an immediate application of this result, consider an $\mathcal{M}$ that has only one Kähler modulus which parametrizes the overall volume of $\mathcal{M}$. In this case every effective divisor must lie in the Kähler cone since the Kähler class necessarily spans $H^2(\mathcal{M})$ (an effective divisor in this case will be a positive multiple of the Kähler class which is also integral, but we do not need this here). So in this case, the volume cannot be stabilized by abelian instantons. Models with one Kähler modulus cannot be elliptic fibrations, so these models only exist as 3-dimensional M-theory compactifications. To rule out F-theory models with one Kähler modulus, we need to do more work.

2.5 An F-theory condition on instantons

To study compactifications with a type IIB description, we want $\mathcal{M}$ to be an elliptically-fibered Calabi-Yau 4-fold with base $B$. We can present $\mathcal{M}$ in Weierstrass form as follows: let $W$ be a $\mathbb{P}^2$ bundle over $B$ with homogeneous coordinates $(x, y, z)$ which are sections of $\mathcal{O}(1) \otimes K^{-2}$, $\mathcal{O}(1) \otimes K^{-3}$, and $\mathcal{O}(1)$, respectively. The line-bundle $\mathcal{O}(1)$ is the degree one bundle over the $\mathbb{P}^2$ fibre and $K$ is the canonical bundle of $B$. Then $\mathcal{M}$ is given by
\[
0 = s = -zy^2 + x^3 + axz^2 + bz^3,
\]  
(15)

where $a$ and $b$ are sections of $K^{-4}$ and $K^{-6}$ and $s$ is a section of $\mathcal{O}(3) \otimes K^{-6}$. Now the cohomology ring of $W$ is given by adding the element $\alpha = c_1(\mathcal{O}(1))$ to the cohomology ring of $B$ along with the relation,
\[
0 = \alpha(\alpha + 2c_1)(\alpha + 3c_1),
\]  
(16)

where $c_1 = c_1(B)$. This relation follows because $x$, $y$, and $z$ are not allowed to have any common zeroes. In the cohomology ring of $\mathcal{M}$, there is a further simplification. Since $\mathcal{M}$

\[^{4}\]This is a special case of a more general inequality which holds for Einstein-Kähler manifolds,
\[
\int_X \left( c_1(X)^2 - \frac{2(n+1)}{n} c_2(X) \right) [\omega]^{n-2} \leq 0.
\]  
(13)
is given by \( s = 0 \), we always multiply by \( c_1(\mathcal{O}(3) \otimes K^{-6}) = 3(\alpha + 2c_1) \) before integrating, so the relation above can be simplified to

\[
0 = \alpha(\alpha + 3c_1) \quad \Rightarrow \quad \alpha^2 = -3\alpha c_1
\]

on \( \mathcal{M} \).

Write the total Chern class of \( B \) as \( c(B) = 1 + c_1 + c_2 + c_3 \). Now the total Chern class of \( \mathcal{M} \) is calculated by adjunction

\[
c(\mathcal{M}) = (1 + c_1 + c_2 + c_3) \frac{(1 + \alpha + 2c_1)(1 + \alpha + 3c_1)(1 + \alpha)}{1 + 3\alpha + 6c_1} = 1 + (c_2 + 11c_1^2 + 4\alpha c_1) + (c_3 - c_1c_2 - 60c_1^3 - 20\alpha c_1^2) + 4\alpha(c_1c_2 + 30c_1^3).
\]

In M-theory, superpotentials are generated by M5-branes wrapping divisors with arithmetic genus one, \( \chi(D) = 1 \) given by (9). In terms of the (1,1) cohomology class of the divisor, \([D]\), and the expression derived above for the Chern class of \( \mathcal{M} \), we can rewrite the arithmetic genus constraint as

\[
[D]^4 + (c_2 + 11c_1^2 + 4\alpha c_1) [D]^2 = -24.
\]

Finally, we would like to take the F-theory limit and relate this to a compactification of type IIB string theory on \( B \). Let \( \pi : \mathcal{M} \to B \) be the projection onto the base. If \( \pi(D) = B \) then the contribution to the superpotential vanishes on taking the F-theory limit, so we need only restrict ourselves to the case that \( D = \pi^{-1}(C) \), where \( C \) is a divisor in the base \( B \). In particular, this implies that

\[
[D]_M^4 = \pi^{-1}([C]_B^4) = 0
\]

and also that

\[
(c_2[D]^2)_M = \pi^{-1}((c_2[C]^2)_B) = 0, \quad (c_1^2[D]^2)_M = 0.
\]

So for this situation, the constraint reduces greatly to read

\[
4\alpha c_1[D]^2 = -24.
\]

Finally, we can do the fiber integration by multiplying \( 22 \) by \( 3(\alpha + 2c_1) \) and picking out the \( \alpha^2 \) term, leaving only a condition on the base

\[
c_1[C]^2 = -2.
\]
One immediate application of this formula is to show that in cases with only one Kähler modulus, there can be no superpotential generated by smooth instantons. Indeed, if \( h^{1,1}(B) = 1 \), then the Kähler form \( J \) generates all of \( H^{1,1}(B) \). In particular, we know that \( c_1(B) \) is a positive \((1,1)\)-form, since \( c_1(B) = c_1(K^{-1}(B)) \), and (e.g.) \( K^{-4} \) had at least one nonzero section, so \( c_1 = aJ \) for some \( a > 0 \). We also know that \([C] = bJ\) for some real (in fact positive, but we do not actually require that) \( b \). Then

\[
c_1[C]^2 = ab^2 \int_B J^3 = 3! \times ab^2 \times Vol(B) \geq 0.
\]

In particular, no divisors \( C \) can satisfy the condition \([23]\). Therefore, the volume is not stabilized in F-theory models with one Kähler modulus.\(^5\)

### 2.6 Gaugino Condensation

Another natural stabilization mechanism contemplated in [11] is a superpotential generated by gaugino condensation. Non-abelian gauge symmetry arises in F-theory via coincident 7-branes which wrap a divisor \( S \) of \( B \). By an \( SL(2, \mathbb{Z}) \) transformation, we can choose these 7-branes to be \( D7\)-branes. In M-theory, this gauge symmetry comes about via an \( ADE \) singularity fibered over \( S \). We want models with pure N=1 Yang-Mills so we need to freeze the scalars on the \( D7\)-branes. This can be achieved in two ways: the first is by choosing an \( S \) so that \([27]\)

\[
h^{1,0}(S) = h^{2,0}(S) = 0.
\]

This condition ensures that there are no moduli from either Wilson lines on \( S \), or from the twisted scalars on the \( D7\)-branes. In this situation, the \( D7\)-branes simply cannot move, and we get pure gauge symmetry. There is one known F-theory compactification to 6 dimensions that has a pure gauge factor of this kind \([28]\).

The other possibility is to consider coincident motile \( D7\)-branes. This notion is really only precise at points where all the branes are mutually local \( D7\)-branes like the orientifold point \([19]\). We can try to freeze the moduli of motile \( D7\)-branes using flux. In this case, there are moduli from either \( h^{1,0}(S) \) or \( h^{2,0}(S) \). The former correspond to moduli from \( h^{2,1}(\mathcal{M}) \) of the four-fold \( \mathcal{M} \), while the latter are complex structure deformations generated by \( h^{3,1}(\mathcal{M}) \). Both classes of moduli can be frozen by flux \([6]\). However, whether there is

\(^5\)We should note that the condition \( h^{1,1}(B) = 1 \) does not imply that \( h^{(1,1})(\mathcal{M}) = 2 \). There can be more than two Kähler moduli on \( \mathcal{M} \) which arise from reducible singularities in the elliptic fibration. None of these models can be stabilized by our argument. We wish to thank P. Aspinwall and the Duke CGTP group for this observation.
any unbroken gauge symmetry depends on the kind of background $G_4$. Some choices of $G_4$ lift to instantons embedded in the gauge group on the D7-branes wrapped on $S$. These instantons can either partially or completely break the gauge symmetry. In addition, there are moduli associated with the embedding of these instantons.

Fortunately, these caveats will not affect our conclusions. Geometrically, we have the following projections, $\mathcal{M} \to B \to S$. We assume that $h^{1,1}(B) = 1$, and that the fibers of $\mathcal{M}$ over $S$ are of $ADE$ type so that we have enhanced gauge symmetry. If $J$ is the Kähler form of $B$ then $\int_B J^3 > 0$. However, by construction, $J$ is necessarily the pull back of some $(1, 1)$ class $H$ on $S$. Since $H^3 = 0$ on the complex surface $S$, this implies that $J^3 = 0$ which is a contradiction. Since there is no space $B$ with only one Kähler modulus that can give gaugino condensation, the volume cannot be stabilized in these models.

Actually, the entire discussion about gaugino condensation is superfluous. As mentioned earlier, we are always free to compute the superpotential in M-theory. Reducing an $N=1$ Yang-Mills vector multiplet to 3 dimensions always results in one adjoint-valued scalar from the Wilson line in the direction of reduction. In 3 dimensions, we therefore always have a Coulomb branch, and we are free to break the gauge symmetry down to abelian. This corresponds to resolving the $ADE$ fibers over $S$ by a resolution parameter that depends on the area of the elliptic fiber. The situation then reduces to the case studied in section 2.5, and we arrive at the same conclusion.

### 2.7 More general models

Finally, we will outline how our analysis extends to more general models with many Kähler moduli. As a specific example of another case, consider an elliptically-fibered Calabi-Yau 4-fold, $\mathcal{M}$, over a base $B = \mathbb{P}^2 \times \mathbb{P}^1$. The cohomology ring of the base is generated by the Kähler classes of the two factors, which we denote by $J$ and $\beta$ for $\mathbb{P}^2$ and $\mathbb{P}^1$, respectively. These classes obey the relations, $J^3 = \beta^2 = 0$. We can choose $J$ and $\beta$ to generate the integral cohomology of $B$ so that

$$\int_{\mathbb{P}^2} J^2 = \int_{\mathbb{P}^1} \beta = 1. \quad (26)$$

The class of an effective divisor can then be expressed in terms of $J$ and $\beta$,

$$[C] = nJ + m\beta, \quad (27)$$

where $n$ and $m$ are non-negative integers.

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6We wish to thank Sheldon Katz for explaining the following argument to us.
To find the (possibly) contributing divisors, we use the criterion (23) which means we only have to compute
\[ c_1(B)[C]^2 = \int_B (3J + 2\beta)(nJ + m\beta)^2 = 2n^2 + 6nm \geq 0. \] (28)

So we see immediately that there are no contributing divisors in this case.

Actually, it is a simple matter to extend this analysis to the case of any product \( B = S \times \mathbb{P}^1 \) (this is, for instance, the case considered in [29], where the superpotential is calculated for a specific \( S \)). Here \( S \) must itself be a \( c_1 \) positive surface with arithmetic genus one so that \( B \) also has these properties. Let \( \beta \) again denote the integral class of \( \mathbb{P}^1 \). Then we have \( c_1(B) = c_1(S) + 2\beta \) and we can write any effective divisor as \([C] = [E] + k\beta \) where \([E] \) is a divisor on \( S \) and \( k \) is a non-negative integer. We again compute
\[ c_1(B)[C]^2 = \int_B (c_1(S) + 2\beta)([E] + k\beta)^2 \]
\[ = \int_B (2[E]^2 + 2k\beta c_1(S)[E]) \beta = 2 \int_S [E] ([E] + k\beta c_1(S)). \] (29)

Since \( c_1(S) \) is a positive integral class on \( S \), \([E] \) and \([E] + k\beta c_1(S) \) both represent effective divisors on \( S \). If \([E] \) is a multiple of \( c_1(S) \) (this was the case in the \( S = \mathbb{P}^2 \) example above) then this is proportional to \( \int_S c_1(S)^2 \geq 0 \). If \([E] \) is not proportional to \( c_1 \) and \( k > 0 \) then \([E] \) and \([E] + k\beta c_1(S) \) really represent two different curves in \( S \) and hence intersect non-negatively. So the only remaining case is that \( k = 0 \) and \( E \) has negative self-intersection. For a smooth abelian instanton contribution, we see that the condition is in fact \( \int_S [E]^2 = -1 \).

Moreover, if \([E] \) is irreducible, then by adjunction again we see that
\[ c(E) = \frac{c(S)}{1 + [E]} = 1 + (c_1(S) - [E]), \] (30)
and hence
\[ \chi(E) = \int_E c_1(E) = \int_S (c_1(S) - [E])[E] \]
\[ \Longrightarrow \int_S [E]^2 = \int_S c_1(S)[E] - \chi(E) = \int_S c_1(S)[E] - 2 + 2g, \] (31)
where \( g \) is the genus of the curve \( E \). Since the integrand on the right hand side above is again non-negative, as is \( g \), we have a negative total result only for \( g = 0 \) and \( \int_S c_1(S)[E] < 2 \). To get a contributing divisor we need specifically \( \int_S c_1(S)[E] = 1 \).

It is not difficult to construct such examples. For instance if \( S \) is a del Pezzo surface or the Hirzebruch surface \( \mathbb{F}_1 \). However, since all contributing divisors wrap the \( \mathbb{P}^1 \), their
volumes are invariant if we scale the size of $\mathbb{P}^1$ by a factor $t$ while also scaling the surface $S$ by a factor $t^{-1}$. But the non-perturbative superpotential depends only on the volumes of the contributing divisors, and hence is independent of the Kähler modulus associated with this deformation. So we again conclude that there is at least one Kähler modulus unfixed by these non-perturbative effects.

It is possible to perform a similar analysis in many even more general classes of examples [22]. This will extend prior work on superpotentials in M and F-theory [10, 29–32] to broad classes of non-singular spaces. However, what seems clear is that complete moduli stabilization in string theory is hard to achieve!

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