Development of Method in Precise Multibeam Acoustic Bathymetry

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1 Introduction

Multibeam echo sounder (MES) possesses all characters of side scan sonar system and single beam sounding system. It can fulfill swath, large area and whole covering surveying. In the first approximation, the position of beam footprint can be estimated by the triangular method.

\[ z = z_0 + \frac{c_0 T}{2} \cos \theta, \quad x = x_0 + \frac{c_0 T}{2} \sin \theta \]  (1)

where \((z_0, x_0)\) is the coordinate of transducer in the vessel coordinate system; \(c_0, \theta\) are the sound velocity and the incident angle at the emission time, respectively; \(T\) is the two-way travel time of beam between the transducer and the beam footprint.

In Eq. (1), the sound velocity is thought to be a constant in the whole travel. But, in fact, it changes with temperature, salinity and pressure of seawater in travel. So, generally, the sound ray tracing method is used for the determination of beam footprint position.

\[ X = X_0 - \sum_{i=1}^{n} x_i, \quad T = \sum_{i=1}^{n} t_i \]  (2)

where \(p\) is Snell parameter; \(g_i, c_i, \Delta z_i, x_i, t_i\) are gradient of sound velocity, sound velocity, thickness of layer, the displacement and travel time in the \(i\)th layer, respectively; \(X, T\) are the displacement and travel time in all layers, respectively.

For Eq. (2), the division layer method must be adopted.

KEY WORDS multi-beam echo sounder (MES); position correction method; relative depth error; area difference

ABSTRACT The sound ray tracing method can achieve higher accuracy in determining depths and plan positions with multibeam echo sounding system. In data processing, actual sound speed profile must be used in the method. However, the method is too complicated. In order to overcome the shortcoming, this paper presents a new method, the position correction method. Two situations are considered in the new method, namely, change of sound velocity keeps constant gradient in whole water column (including \(N\) layers) or in different water layer.
The strict layer division should be based on the known depth. However, the depth is an unknown parameter. In order to resolve the problem, the method of appending layer should be used.

2 Method of position correction

If the beam passes through N layers, among which the nth layer is the layer to be studied. On the surface of the nth layer, the sound velocity and the incident angle are \( C_{i-1}, \theta_{i-1} \), respectively; the beam traverses \( \Delta Z_i \) (propagation time \( t_i \)), and goes out from the bottom of the nth layer at sound velocity \( C_i \) and incident angle \( \theta_i \). If the change of sound velocity keeps zero gradient in the travel, according to the triangle relation, the thickness \( \Delta z_i \) (\( \Delta z_i = z_i - z_{i-1} \)) and displacement \( \Delta x_i \) (\( \Delta x_i = x_i - x_{i-1} \)) can be expressed by the following formula.

\[
\Delta z_i = C_{i-1}t_i \cos \theta_{i-1} \\
\Delta x_i = C_{i-1}t_i \sin \theta_{i-1}
\]  

The depth \( z'_i \) and displacement \( x'_i \) can be obtained after the beam traverses the nth layer. If the change of sound velocity is constant gradient (Fig. 1), the thickness \( \Delta z_i \), displacement \( \Delta x_i \), and travel time \( t_i \) of the nth layer can be expressed as in Eq. (4). The depth \( z_i \) and displacement \( x_i \) can be obtained after the beam goes through the nth layer.

\[
\Delta z'_i = C_{i-1}t_i \cos \theta_{i-1} \\
\Delta x'_i = C_{i-1}t_i \sin \theta_{i-1}
\]  

Comparing above two methods with each other, although the triangular method is simple, the estimation accuracy of beam footprint position is lower than by the latter and the latter can meet the requirement of the International Hydrographic Organization (IHO). However, its operation procedure is more complicated. In order to overcome these difficulty, two concepts are introduced, which are the relative position error (including the relative depth error \( f_z \) and the relative displacement error \( f_x \)) and the area difference.

Then, \( f_{z_i}, f_{x_i} \) in the nth layer is defined as:

\[
f_{z_i} = \frac{x_i - x_{i-1} - x_i - x_{i-1}}{x_i - x_{i-1}} = 1 \\
f_{x_i} = \frac{z_i - z_{i-1} - z_i - z_{i-1}}{z_i - z_{i-1}} = 1
\]

The area difference \( \Delta S_i \) in the nth layer is defined as shown in Fig. 2. In Fig. 2, a constant sound velocity profile in Eq. (3) is defined as \( C_{i-1} \); the actual sound velocity profile is a constant gradient \( g_i \) profile; the area difference \( \Delta S_i \), between the two sound velocity profiles and the relative area differences \( \varepsilon_i \), are defined as Eq. (6) shows.

\[
\frac{\Delta S_i}{S_i} = \frac{(C_i - C_{i-1})(z_i - z_{i-1})}{2C_{i-1}}
\]

According to the definition of \( p, \theta_i \), and \( \sin \theta_i \) can be expressed as follows.

\[
\sin \theta_i = \frac{C_i \sin \theta_{i-1}}{C_{i-1}}, \quad \theta_i = \arcsin \frac{C_i \sin \theta_{i-1}}{C_{i-1}}
\]

Integrated Eqs. (3), (4), (6), (7) and (5), \( f_z, f_x \) can be expressed as
Eq. (8) shows that the relative position error is only correlated with the incident angle, the sound velocity on the surface of the \(i\)th layer and the area difference in the \(i\)th layer. So, the compensation model for\( x_i'\) and \( z_i'\) can be written as Eq. (9).

\[
f_i = \frac{(2\varepsilon_i - 1) \tan \theta_i (\theta_i - \arcsin(2\varepsilon_i \sin \theta_i)) \ln(2\varepsilon_i + 1) - 1}{4\varepsilon_i^2}
\]

\[
f_i = \frac{(2\varepsilon_i - 1) \tan \theta_i (\theta_i - \arcsin(2\varepsilon_i \sin \theta_i)) \ln(2\varepsilon_i + 1) \cos \theta_i - 1}{4\varepsilon_i \sin \theta_i^2}
\]

The first situation is just discussed above, in which the water column of the beam level is divided into \(N\) layers, and the change of the sound velocity keeps constant gradient in each layer. For the other situation, namely, \(N\) layers are treated as one layer. In the whole layer, the change of the sound velocity is a constant gradient \(g\), and the initial incident angle, sound velocity, depth and displacement at the surface of the transducer are \(\theta_0\), \(C_0\), \(z_0\) and \(x_0\), respectively. The similar method that is used to estimate the vertical distance and the displacement of each layer in the first situation is adopted. The position of beam footprint \((x_B, z_B)\) can be obtained as:

\[
x_B = \sum_{i=1}^{N} x_i, \quad z_B = \sum_{i=1}^{N} z_i
\]

where \(x_B'\) and \(z_B'\) are the positions of beam footprint and can be estimated by the triangular method.

In Eq. (11), \(\theta_0\), \(C_0\), \(z_0\), \(x_0\) at the surface of the transducer and the travel time \(T\) from the surface of the transducer to the seabed are only used to estimate the position of beam footprint.

### 3 Comparison between the new method and the traditional method

The new method has the following advantages over the sound ray tracing method.

1) Less dependency on the actual sound profile. Although the two methods are all concerned with the actual sound profile, the relevant degrees are different. The sound ray tracing method depends on the actual sound velocity profile completely in each layer. The position correction method only uses the actual sound velocity profile to determine the area difference. Moreover, in the efficient scope of the actual sound velocity profile, the area difference is thought to be a constant. Thus the procedure is simplified.

2) Meeting the accuracy requirement of IHO. IHO requires \(\varepsilon\%\) relative depth error in bathymetry. The accuracy of depth is very high and can achieve about \(1\%\) of \(\varepsilon\). For the new method, only the actual sound velocity profile is used in the determination of the area difference. So the accuracy of depth is lower than that in the sound ray tracing method, but can meet the accuracy requirement of IHO. According to the experiment (shown in Fig. 3(c)), \(4\%\) of relative depth error can be achieved.

Fig. 3(c) shows the result of data processing...
of the new method under the second condition. For the first condition, the method for data processing is similar to that in the sound ray tracing method. So similar result can be obtained.

Through analyzing Figs. 3(b), 3(c) and 3(d), the relative depth errors estimated by the sound ray tracing method and the position correction method are far less than that determined by the triangular method. From Fig. 3, another conclusion can be drawn that the more complicated the actual sound profile is, the bigger the relative depth error is. However, the sound ray tracing method and the position correction method can always meet the requirement of IHO.

3) Simple calculating procedure. The position correction method differs from the sound ray tracing method in data processing. For the former, only one layer is processed, and some required parameters in data processing are obtained easily. The actual sound profile is only used to calculate the area difference that is a constant in the efficient scope of the actual sound profile. The complicated procedure in the sound ray tracing method is simplified.

4) To simplify and improve the sound ray tracing method. According to above analysis, the more dense and representative the actual sound velocity profiles are, the higher the accuracy of the depth is. Otherwise, the effect of representative error of sound velocity profile will be stronger. Without the actual sound velocity profile in some special situations, the sound ray tracing method can not be used in fact. However, once the area difference is obtained, the position correctional method can be applied expediently.

![Graphs showing depth relative error comparisons](image)

**Fig. 3** Comparison of depth relative error by using different methods in depth determining

**References**

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