Averaging Spacetime: Where do we go from here?

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The Idealized Universe

- Universe is assumed to be homogeneous and isotropic on very large scales.
- Some observational data to support these assumptions [CMB, Galaxy Surveys, etc].
- GR results in a Universe described by a single function of time $R(t)$.
- Mathematically elegant.

but ...
There is structure on smaller scales.

The smaller the scale, the larger the inhomogeneity.

What are the effects of these inhomogeneities on our smoothed out idealized model?

Can we ignore these effects?

How do we model these effects?
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\[ G_{\alpha \beta}(g) = \kappa T_{\alpha \beta} \]

considered a success on solar system scales

for larger scales, not quite so

for cosmology, RHS commonly modeled as a fluid on very large scales.

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**The Big Problem:** an averaging/smoothing procedure has been employed without a corresponding averaging/smoothing procedure on the LHS.
Shirokov and Fisher (63): early recognition of the problem

Ellis (84): Detailed description of the issues related to the problem

Both suggested modified gravitational equations for cosmology,

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\bar{G}_{\alpha\beta}[g] = \kappa \bar{T}_{\alpha\beta} = \kappa T^{\text{fluid}}_{\alpha\beta}
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The Big Solution?

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Further Issues: Problem A

How does one average tensor fields on a manifold?
Further Issues

- How can one relate $\overline{G}_{\alpha\beta}[g]$ with $G_{\alpha\beta}[\overline{g}]$?
- Can we simply assume $\overline{G}_{\alpha\beta}[g] = G_{\alpha\beta}[\overline{g}]$?
- **NO**, due to non-linearity of the EFEs.
- **Solution** Both S+F and E, introduce a Gravitational Correlation Tensor $C_{\alpha\beta}$

\[
G_{\alpha\beta}[\overline{g}] + C_{\alpha\beta} = \kappa T_{\alpha\beta}^{\text{fluid}}
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Further Issues: Problem C

What is the nature of the gravitational correlation $C_{\alpha\beta}$?
Shirokov and Fisher (63)

- Appears to be the first to propose a solution
- Employed spacetime averaging procedure

\[
\overline{T}_\beta^\alpha(x) = \frac{\int_{\xi \in \Sigma_x} T_\beta^\alpha(x + \xi) \sqrt{-g(x + \xi)} d^4\xi}{\int_{\xi \in \Sigma_x} \sqrt{-g(x + \xi)} d^4\xi}
\]

- \(x\) is the location of the macro-observer (center of averaging region)
- \(\xi\) is the location of the micro-observer with respect to \(x\)
- Perturbatively determined the nature of the gravitational correlation
- Weakness: non-covariant and perturbative
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Isaacson(68)

- Main interest in Gravitational Radiation, not cosmology
- Appears to be the first to use a covariant averaging procedure

\[ \overline{T}_\beta^\alpha(x) = \int_{\text{all space}} g_{\alpha'}^\alpha(x, x') g_{\beta'}^\beta(x, x') T_{\beta'}^\alpha(x') f(x, x') \, d^4x' \]

- \( f(x, x') \) is a weighting function
- \( \int_{\text{all space}} f(x, x') \, d^4x' = 1 \)
- \( g_{\alpha'}^\alpha(x, x') \) is the parallel propagator along geodesics.
- Perturbatively determined the nature of the gravitational correlation

- Weakness(?): \( \overline{g}_{\alpha \beta} = g_{\alpha \beta} \) and perturbative
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Noonan (84)

- Introduces micro and macro observers and the idea of duality
- Claims to extends Isaacson’s result by averaging the RHS of EFEs, but
- Employed a non-covariant spacetime averaging procedure
- Perturbatively determined the nature of the gravitational correlation
- Also determined corrections to EFE’s due to averaging LHS
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Gasperini, Marozzi, and Veneziano (09)

- Gauge invariant proposal for averaging
- Uses a **Window Function**: Similar to Isaacsons $f(x, x')$ function
- Argues that 3D spatial averaging can be calculated from the 4D with appropriate choice of Window Function
- Has not applied averaging procedure to EFE’s
- Weakness: No gravitational correlation determined
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Zalaletdinov (92)

- Similar to Isaacson
- Assumes bi-local transport operators $A_{\alpha}^\prime(x, x')$
- Defines the spacetime averaging operation

$$
\bar{T}_{\alpha}^\beta(x) = \frac{\int_{x' \in \Sigma_x} A_{\alpha}^\prime(x, x') A_{\beta}^\prime(x, x') T_{\beta}^\prime(x') \sqrt{-g(x')} \, d^4 x'}{\int_{x' \in \Sigma_x} \sqrt{-g(x')} \, d^4 x'}
$$

- Apply averaging procedure to Cartan structure equations,
- Determines an averaged spacetime by defining
  $$
  \Gamma_{\beta\gamma}^\alpha = \left< F_{\beta\gamma}^\alpha \right>
  $$
  to be LC connection for this space
- $F_{\beta\gamma}^\alpha$ a bi-local extension of the LC connection of the original manifold.
Zalaletdinov (92) cont.

- Defines a 2nd order Connection Correlation tensor,
- Few more assumptions to obtain splitting rules for products of Riemann and metric
- Apply averaging procedure to EFE’s
- Complete set of field equations including a new field, with its own set of equations
- Gravitational Correlation determined exactly (non-perturbative)
- Weakness: existence of bi-local transport operators
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Futamase(88,89,96)

- Futamase(88,89),
  - noncovariant averaging procedure,
  - perturbative determination of the gravitational correlation
- Futamase(96),
  - used geodesic parallel propagator on 3-surface
  - perturbative determination of the gravitational correlation
Foliates spacetime by flow orthogonal hypersurfaces with 3-metric $g_{ij}$

Assumes inhomogeneous dust model,

Averages the energy density only

Defines 3 correlations [Extrinsic Curvature, Ricci 3-Curvature, Density Contrast]

Determines conditions for the EFE’s of inhomogeneous models to have the form of a dust FRW on average.

If met, some of the correlations are determined

Weakness: Part of gravitational correlation is assumed zero
Buchert(00,01)

- Foliate spacetime by flow orthogonal hypersurfaces with 3-metric $g_{ij}$
- Define a spatial averaging operation suitable for scalars

$$
\overline{T}(X^i, t) = \frac{1}{V_D} \int_D T(X^i, t) \sqrt{\det(g_{ij})} d^3 X
$$

- No fixed background
- Define volume scale factor $a_D(t) = \left( \frac{V_D(t)}{V_{D_0}} \right)^{1/3}$
- Apply averaging procedure to scalar parts of the EFEs
- Yields 2 scalar equations for three unknowns
- Weakness: Not Closed: Ignores the tensorial parts of the EFE’s
Boersma (98)
- Defines a general averaging operator \( \hat{A} \)
- Assumes FRW is a stable fixed point of the averaging operator
- Shows that linearized averaging operation for metric perturbations, can be defined as a spatial averaging operation for scalars applied to \( \delta g_{00} \) and \( \delta g_i^i \) in synchronous coordinates

Paranjape and Singh (07)
- Spatial averaging limit of Zalaletdinov averaging
- More general but agrees with Buchert averaging
Other Promising Approaches

- **Debasch (04)**, ensemble averaging, no gravitational correlation
- **Sussman (08)**, defines quasi-local scalars, and averages scalar EFE’s, similar to Buchert
- **Behrend (08)**, spacetime averaging of maximally smooth tetrad field to determine average metric
- **Hehl and Mashoon (08)**, Non-local gravity, GR\(_{\parallel}\), a causal spacetime averaged theory of gravity
- **Khosravi, Mansouri and Kourkchi (08)**, Preliminary ideas of “on” and “in” Light Cone Averaging
- **Coley (09)** Discusses the need for Lightcone Averaging: Averages the Raychaudhuri Equation on the Null Cone
Averaging and parallel transport

- Averaging involves integration/summation of tensor fields
- Not straightforward on an arbitrary affinely connected and curved manifold.
- How to add tensor fields which are located at finitely separated points?
- Requires a notion of parallel transport of a tensor at a point $x'$ along some curve $C$ to a base point $x$ in a unique and well-defined manner.
A well defined a unique transportation will require either

1. transportation along well defined curves: e.g. geodesics or
2. the transportation should be independent of the path
Selection of Unique Curve: Geodesic

- select unique curve, in this case, the geodesic,
- appears “natural”, as there are no other “natural” curves that connect $x'$ and $x$.
- in Riemannian space, the geodesic is the shortest and straightest path connecting points $x'$ and $x$.
- (we assume a unique geodesic exists connecting $x'$ and $x$)
- the **elementary parallel propagators** no longer depend on an arbitrary curve and are functions of the endpoints $x'$ and $x$.
- these special parallel propagators are denoted with a lower case $g$, i.e, $g_w(x, x')$, $g_{\alpha'}(x, x')$, $g_{\alpha'}(x, x')$
Path Independent Parallel Transport

- Parallel Transport is independent of path iff curvature of the connection is zero.
- cannot use the Levi-Cevita connection
- employ a different connection, one in particular that has zero curvature

Let $e_i^\alpha (i = 1, \ldots, n)$ be $n$ linearly independent vector fields
Assume covariantly constant with respect to some unknown connection
This requirement uniquely defines an affine connection $W^{\alpha}_{\beta\gamma} = e_i^\alpha e_i^\beta,\gamma$
The result is a Weitzenbock connection.
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$$ W^\alpha_{\beta\gamma} = e_i^\alpha e_i^\beta,\gamma $$

- The result is a Weitzenbock connection.
In this case, the elementary parallel propagators are factorable and can be shown to have the form:

\[
P_w(x, x') = \left( \frac{e(x)}{e(x')} \right)^w \quad e = \det(e^\alpha_i) \quad (1)
\]

\[
P_\alpha^{\prime}(x, x') = e_i^\alpha(x)e_i^{\alpha'}(x') \quad (2)
\]

\[
P_{\alpha'}^{\alpha}(x, x') = e_i^\alpha(x)e_i^{\alpha'}(x') \quad (3)
\]

Basically the frame components of any tensor are invariant under parallel transport with respect to \( W^{\alpha}_{\beta\gamma} \).
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**Basically** the frame components of any tensor are invariant under parallel transport with respect to \( W_{\beta\gamma}^{\alpha} \).
Our Options

Two options in developing a well defined covariant averaging procedure. Parallel transport along geodesic,

- Curve uniquely chosen
- Parallel transported along geodesic with respect to Levi-Cevita connection
- Use $g^{\alpha'}_{\alpha}$ as the transporter
- Approach used by Isaacson

or ...
Our Options

Path Independent transportation

- Parallel transported with respect to the Weitzenbock connection
- Use $P^\alpha_{\alpha'}$ as the transporter
- closely resembles the approach of Zalaletdinov

One can now integrate vector and/or tensor fields over compact regions of the manifold, and consequently, can define an averaging procedure.
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Path Independent transportation

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One can now integrate vector and/or tensor fields over compact regions of the manifold, and consequently, can define an averaging procedure.
Definition (Averaging/Smoothing Procedure)

Let $\mathcal{M}$ be a simply connected metric manifold. Let $T^\alpha_\beta(x)$ be a continuous tensor field defined on some simply connected region $\mathcal{R} \subset \mathcal{M}$. Let $\Sigma_x$ be a compact subset of $\mathcal{R}$ at supporting point $x$. We define the average of the tensor field $T^\alpha_\beta(x)$, denoted $\overline{T}^\alpha_\beta(x)$, as the definite integral at supporting point $x$,

$$
\overline{T}^\alpha_\beta(x) \equiv \frac{1}{V_{\Sigma_x}} \int_{x' \in \Sigma_x} P^\alpha_{\alpha'}(x, x') P^\beta_{\beta'}(x, x') T^\alpha_{\beta'}(x') \sqrt{-g(x')} d^4x'
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In either procedure, $\overline{g}_{\alpha\beta} = g_{\alpha\beta}$

Bonus: Constant Curvature spacetimes are fixed points of either procedure

Does it make sense to average the metric?

Which geometrical object of the micro geometry when averaged, yields information about the macro geometry?

Levi-Cevita connection? Possibly.

Perhaps it is $R^\alpha_{\beta\gamma\delta}(g)$? Better possibility?

Perhaps it is the Kontosion tensor?

Illustrated an averaging procedure for tensor fields (Problem A), have not averaged the EFE’s (Problem C), so more work to do
INTRODUCTION
AVERAGING AND GRAVITATIONAL CORRELATIONS
PROPOSED SOLUTION TO PROBLEM A
CONCLUDING REMARKS

The Transport Problem
Parallel Transport along Geodesic
Path Independent Parallel Transport
Covariant Averaging Procedure

Comments

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Presented a fresh look at a fully covariant approach to averaging.
Made some arguments and constructions to possibly elucidate the Zalaletdinov averaging procedure.
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- On what length scale is GR the appropriate Gravitational Theory?
- If GR is appropriate for the solar system, then what is the effective change to the Einstein Field Equations upon averaging?
- Should result be GR plus bits, or a new theory?
- Should we average $GR\parallel$ equivalent to GR?
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The gravitational correlation (polarization) should it be determined

- through perturbative techniques, or
- assigned via geometrical assumptions a la Zalaletdinov? or
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- Symmetries and Averaging?
- Is the average of a bundle of null geodesics, a null geodesic? What about Causality?
- Should we use a fully covariant spacetime averaging procedure, or one better suited to cosmology (1+3 split).
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- Is the average of a bundle of null geodesics, a null geodesic? What about Causality?
- Should we use a fully covariant spacetime averaging procedure, or one better suited to cosmology (1+3 split).
Can the inhomogeneities in the un-averaged geometry manifest an effective acceleration in the averaged geometry?

Cosmology is tested with observations, and observations take place down the Null Cone: Should we not be averaging down the Null Cone?

The Fitting Problem.
Unanswered Questions and Other Issues

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INTRODUCTION
AVERAGING AND GRAVITATIONAL CORRELATIONS
PROPOSED SOLUTION TO PROBLEM A
CONCLUDING REMARKS

Overview
Questions to Stimulate Further Discussion

DISCUSSION