We propose a contagion model to describe the evolution of the political voting trends in Brazil after the dictatorship from 1985 to nowadays. We consider a fully-connected population divided in two voting groups, left and right. Each group includes three kinds of agents, sensitives, inflexibles and radicals. While sensitives may shift their left or right voting, inflexibles and radicals do not. Excluding political interactions with radicals, we found sets of pair interaction parameters which reproduce all the voting outcomes of past presidential elections from 1989 till 2014 excluding 2018. At this stage, we found that adding an empty third voting group overlapping left and right, the populist voting group, can yield Bolsonaro 2018 victory. The initial filling of the populist group is triggered by the breaking of the interaction barrier between sensitives and radicals, which had prevailed for decades. Within each voting group, some of the interacting pairs sensitive/radical recast the sensitive into a populist. This status change from sensitive to populist allows the populist to interact with sensitives of both left and right, thus creating an additional source of filling of the populist group. The process is shown to yield the 2018 55% for Bolsonaro. From the 2018 distribution of votes, we evaluate the parameter changes which recover the recent 2021 poll yielding a victory of Lula against Bolsonaro at a score of 64/36. The magnitude of the parameter changes are very small highlighting the high volatility of what would be a vote under the current conditions. We also present an alternative explanation for the Bolsonaro victory in 2018 election, based on a single varying parameter without populists. The aim was to reproduce all elections outcomes (1989 - 2018) and the 2021 poll. For 2018 election and 2021 poll, we compare the results of the two scenarios, with and without populists. Regarding the two possible scenarios for the 2018 Bolsonaro victory, we discuss about a hypothetic Lula/Bolsonaro 2022
second round voting and the requirements to yield either Lula or Bolsonaro victory in 2022. Both scenarios are feasible.

Keywords: Dynamics of social systems, Collective phenomena, Opinion Dynamics

I. INTRODUCTION

The rise of populism has been observed in many democratic countries in the last years [1–3]. Two most emblematic cases are the elections of Donald Trump in the 2016 US election [4] and Jair Bolsonaro in the 2018 Brazilian election [5]. In parallel to those successful victories of populist leaders, far-right has grown in a series of Western European countries including France, Germany, Italy and Spain [6].

Most of these drastic changes in electoral landscapes were not anticipated by polls and pandits, taken them by surprise. The underlying mechanisms driving the opinion dynamics towards populism and its connection to extreme political views on both the right and the left are still not well understood. In particular the various amplitudes from one country to another are distributed over quite a large spectrum of values.

A recent study investigated the issue in Germany analyzing the changes in political narratives [7]. Here we tackle the problem from the viewpoint of opinion dynamics studied within the frame of sociophysics. A great deal of works have been published along this approach [8–9].

Several studies related to politics and elections were published in the last years [10–13]. They use discrete variables or continuous ones. Among them, simple contagion models were considered to study the evolution of distinct social processes, like tax evasion [14], cooperation [15], rumor spreading [16], epidemic spreading [17], radicalization phenomena [18], ideological conflicts [19], corruption [20], juvenile crimes [21], obesity [22].

In this work we consider a fully-connected population composed of $N$ individuals. Agents are labelled in terms of political affiliations, either left or right. Each group includes three kinds of agents, sensitives, inflexibles and radicals adding to a total of six subgroups. These subgroups are denoted sensitive, inflexible, radical. Their respective proportions are $l$, $ll$, $L$ for the left, and $r$, $rr$, $R$ for the right.

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While inflexibles and radicals do not shift their political affiliations, sensitives are liable to shift to the opposite side via some specific pair interactions. Accordingly, while proportions \( ll, L, rr, R \) are given and constants, proportions \( l \) and \( r \) are functions of time. What makes an individual to become radical or inflexible is out the scope of the present work. We thus have the conservation condition,

\[
l(t) + r(t) + L + ll + R + rr = 1. \tag{1}
\]

We first apply the model to explain the voting outcome for 1989 to 2014 Brazilian elections. After that, we introduce the populists in the model, in order to give an explanation for the Bolsonaro victory in 2018 election. We also consider the recent 2021 poll regarding a possible Lula vs Bolsonaro in a second round in the next year presidential election. We also present an alternative explanation for the Bolsonaro victory in 2018 election, based on a single varying parameter without populists. This alternative scenario can reproduce all election outcomes (1989 - 2018) and the 2021 poll with a single parameter. For 2018 election and 2021 poll, we compare the results of the two scenarios, with and without populists. Regarding the two possible scenarios for the 2018 Bolsonaro victory, we discuss about a hypothetic Lula/Bolsonaro 2022 second round voting and the requirements to yield either Lula or Bolsonaro victory in 2022.

II. COMPETITION BETWEEN LEFT AND RIGHT OVER SENSITIVES

During last decades since last World War II, with few exceptions radicals have been excluded from social discussions about voting choices among people from left and right. On both sides, left and right, non radicals refused to discuss with their side-extremists often labelled “fascists” for the right and “communists” for the left. Discussions were restricted between inflexibles and sensitives. During these exchanges, sensitives could eventually shift
side. The various interactive configurations with their respective output are,

\begin{align*}
l + r \xrightarrow{k} l + l, \quad (2) \\
l + r \xrightarrow{1-k} r + r, \quad (3) \\
ll + r \xrightarrow{m} ll + l, \quad (4) \\
ll + r \xrightarrow{1-m} ll + r, \quad (5) \\
l + rr \xrightarrow{n} r + rr, \quad (6) \\
l + rr \xrightarrow{n} l + rr \quad (7)
\end{align*}

where \( k, m, n \) are the various shift probabilities under the constraints \( 0 \leq k \leq 1, 0 \leq m \leq 1, 0 \leq n \leq 1 \). For each case, sensitives either shift side or keep on their current stance.

- The rate \( k \) measures the propensity for a left sensitive to convince a right sensitive and \((1 - k)\) the reverse.
- The rate \( m \) measures the propensity for a left inflexible to convince a right sensitive while \((1 - m)\) is the capacity of a sensitive right to resist this outcome.
- The rate \( n \) measures the propensity for a left sensitive to resist a right inflexible and \((1 - n)\) the reverse.

The corresponding time evolution of \( l(t) \) and \( r(t) \) are given by,

\begin{align*}
\frac{dl(t)}{dt} &= -(1 - 2k) l(t) r(t) + ml r(t) - (1 - n) rr l(t), \quad (8) \\
\frac{dr(t)}{dt} &= (1 - 2k) l(t) r(t) - ml r(t) + (1 - n) rr l(t), \quad (9)
\end{align*}

where \( \frac{dl(t)}{dt} = -\frac{dr(t)}{dt} \) as expected from the fact that the dynamics of affiliation occurs only within the group of sensitives with \( l(t) + r(t) = 1 - ll - L - rr - R = constant \).

Then, solving the equation \( \frac{dl}{dt} = 0 \) yields the steady state,

\begin{equation}
l = \frac{ml r}{(1 - n) rr - (2k - 1) r} \quad (10)
\end{equation}

where \( l = l(t \to \infty) \) and \( r = r(t \to \infty) \) are the stationary values of \( l(t) \) and \( r(t) \), respectively. From Eq. (11), substituting \( r = 1 - l - ll - L - rr - R \) into Eq. (10) yields a second order polynomial for \( l \),

\begin{equation}
Al^2 + Bl + C = 0, \quad (11)
\end{equation}
where

\[ A \equiv -(1 - 2k), \quad (12) \]
\[ B \equiv (1 - n) \, rr + (1 - 2k) \, (1 - D) + m \, ll, \quad (13) \]
\[ C \equiv -m \, (1 - D) \, ll, \quad (14) \]

with

\[ D \equiv ll + rr + L + R. \quad (15) \]

From Eq. (11) we obtain 2 solutions,

\[ l = \frac{-[(1 - n) \, rr + (1 - 2k) \, (1 - D) + m \, ll] \pm \sqrt{\Delta}}{2 \,(2k - 1)}. \quad (16) \]

where \( \Delta = B^2 - 4AC \), with \( A, B \) and \( C \) given by Eqs. (12) - (14). Numerically, we found that the solution with the plus signal is the relevant one giving \( l > 0 \). The stationary value for the sensitive right fraction \( r \) can be obtained from the normalization condition (Eq. (1)), i.e.,

\[ r = 1 - l - L - ll - R - rr. \quad (17) \]

In particular, the case \( k = 1/2 \) of symmetry balance between shift from sensitive interactions yields,

\[ l = \frac{m \,(1 - D) \, ll}{(1 - n) \, rr + m \, ll} \quad (18) \]
\[ r = \frac{(1 - n) \,(1 - D) \, rr}{(1 - n) \, rr + m \, ll}. \quad (19) \]

Extending the symmetry balance for all interactions leads to,

\[ l = \frac{k \,(1 - D) \, ll}{(ll - rr) \, k + rr} \quad (20) \]
\[ r = \frac{(1 - k) \,(1 - D) \, rr}{(ll - rr) \, k + rr}. \quad (21) \]

III. APPLYING THE MODEL TO 1989, 1994, 1998, 2002, 2006, 2010 AND 2014 ELECTIONS

After a long dictatorship (1964 - 1985) a democratic system has been installed based on a two rounds’ presidential elections. Voting is mandatory although penalty fee for not voting
is moderate. In 1989, the first president to be elected has been the center-right candidate Fernando Collor de Mello. Following elections in 1994 and 1998 were won by the right-wing candidate Fernando Henrique Cardoso at first rounds. The left-wing candidate Luis Inácio Lula da Silva was elected twice in a row at second rounds in 2002 and 2006. His successor, Dilma Roussef, was also elected twice in a row at second rounds in 2010 and 2014. The populist Jair Bolsonaro was elected at second round in the 2018 election.

Limiting the elections before 2018, five had a second round (1989, 2002, 2006, 2010, 2014) and two (1994, 1998) had the president elected at the first round. Respective voting outcomes are:

- 1989: 0.53 (right-wing candidate) and 0.47 (left-wing candidate)
- 1994: 0.61 (right-wing candidate) and 0.39 (left-wing candidate)
- 1998: 0.57 (right-wing candidate) and 0.43 (left-wing candidate)
- 2002: 0.39 (right-wing candidate) and 0.61 (left-wing candidate)
- 2006: 0.39 (right-wing candidate) and 0.61 (left-wing candidate)
- 2010: 0.44 (right-wing candidate) and 0.56 (left-wing candidate)
- 2014: 0.48 (right-wing candidate) and 0.52 (left-wing candidate)

Above results consider only the valid votes, i.e., they do not take into account null votes or abstentions. Moreover, for the first round 1994 election, the value 0.61 for the right-wing candidate is obtained summing outcomes of the actual right-wing winner (≈ 0.54) and the third candidate also right-wing (≈ 0.07). Similarly for the first round 1998 election, we added the outcomes of second (≈ 0.32) and third (≈ 0.11) candidates both being left-wing to yield 0.43 for the left-wing candidate.

For all those seven elections, using Eqs. (16) and (17) we are able to find appropriate sets of parameters $L, ll, R, rr$ and $k, m, n$ producing steady-state values for $l$ and $r$, which are identical to the actual voting outcomes. The parameter identification being performed

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1 The value vary between R$1.05 and R$3.51 in Brazilian currency (Reals, where R$1 corresponds to 0.18 US$).
FIG. 1. (Color online) Time evolution of the densities of sensitives ($l$ and $r$), total right ($r + rr + R$), total left ($l + ll + L$), and the differences $total left - total right$ and $l - r$. (a: upper left) Simulation of 1989 election. (b: upper right) Simulation of 2002 election. (c: lower) Simulation of 2006 election. We are presenting 2 distinct initial conditions, one of them determined by polls (see the black squares) [24]. The initial conditions are: (a) $l(0) = 0.40, r(0) = 0.47$ (given by polls) and $l(0) = 0.30, r(0) = 0.57$; (b) $l(0) = 0.50, r(0) = 0.31$ (given by polls) and $l(0) = 0.60, r(0) = 0.21$; (c) $l(0) = 0.39, r(0) = 0.42$ (given by polls) and $l(0) = 0.49, r(0) = 0.32$. The remaining parameters are described in the text.

Numerically, for simplicity we took $k = m = n$ turning Eqs. (16) and (17) to,

$$l = \frac{-(1 - k) rr + (1 - 2k) (1 - D) + k ll + \sqrt{\delta}}{2 (2k - 1)}$$

(22)

$$r = 1 - D - l$$

(23)

with $D = ll + rr + L + R$ and $\delta = b^2 - 4ac$, with $a = -(1 - 2k), b = (1 - k) rr + (1 - 2k) (1 -$
FIG. 2. (Color online) Time evolution of the densities of sensitives ($l$ and $r$), total right ($r + rr + R$), total left ($l + ll + L$), and the differences $total\ left - total\ right$ and $l - r$. (a: left panel) Simulation of 2010 election. (b: right panel) Simulation of 2014 election. We are presenting 2 distinct initial conditions, one of them determined by polls (see the black squares) [24]. The initial conditions are: (a) $l(0) = 0.40, r(0) = 0.35$ (given by polls) and $l(0) = 0.59, r(0) = 0.16$; (b) $l(0) = 0.37, r(0) = 0.35$ (given by polls) and $l(0) = 0.55, r(0) = 0.17$. The remaining parameters are described in the text.

The respective sets of parameters compatible with available data [23] are given in Table I. They are determined under the constraint of minimum change of values when going from one election to the following one. Figures 1 and 2 exhibit the various dynamics obtained from the numerical integration of Eqs. (8) and (9) using the sets of parameters given in Table I.

In each figure we plot the densities of sensitives $l$ and $r$, the total densities of left ($l + ll + L$) and right ($r + rr + R$) supporters, the difference $total\ left - total\ right$ and the difference $l - r$. Two distinct initial conditions are shown leading to the same steady states given by Eqs. (16) and (17) with $k = m = n$. One of the initial conditions (represented by the black squares) was obtained by polls [24]. The corresponding values $l(0)$ and $r(0)$ for the initial conditions are given in the caption. The model reproduces both victories of ring-wing and left-wing, depending on the set of parameters. From the set of parameters, even taking into account that the radicals do not interact with the other two classes (sensitives and inflexibles), the presence of such radicals is important. The growing of radicalism observed in Brazil [25] is reflected in the increasing values of $L$ and $R$ over the years.

At this stage, we want to stress that the actual choice of values for the various interaction
TABLE I. Set of parameters used to match the elections’ results [23].

| Election | L   | ll  | R   | rr  | k   |
|----------|-----|-----|-----|-----|-----|
| 1989     | 0.03| 0.05| 0.03| 0.02| 0.48|
| 1994     | 0.04| 0.05| 0.03| 0.02| 0.47|
| 1998     | 0.04| 0.07| 0.03| 0.02| 0.46|
| 2002     | 0.07| 0.07| 0.03| 0.02| 0.48|
| 2006     | 0.07| 0.07| 0.03| 0.02| 0.48|
| 2010     | 0.07| 0.08| 0.05| 0.05| 0.49|
| 2014     | 0.07| 0.08| 0.07| 0.06| 0.49|

parameters is arbitrary in the sense that different sets could reproduce each election outcome. Our aim is to identify a set of sound values which both, can reproduce an actual voting outcome and allow to get another one with very little changes of the values. What matters is to obtain a full and coherent description of all the voting outcomes.

IV. THE EMERGENCE OF POPULISM

At this stage we make a basic hypothesis to include populists among agents. We assume that populists are former sensitive agents from both left and right, who had interacted with their respective radicals. Populists form a seventh subgroup of agents denoted $p(t)$. In addition, once they are created, populists do interact with sensitives from both left and right. The proportion of populists is thus a function of time and sensitives. The normalization condition given by Eq. (1) becomes,

\[
l(t) + r(t) + p(t) + L + ll + R + rr = 1.
\]  

(24)

In addition to the social interactions given by Eqs. (2)-(7), we have,
\[ L + l \xrightarrow{\alpha} L + p, \quad (25) \]
\[ L + l \xrightarrow{1-\gamma} L + l, \quad (26) \]
\[ R + r \xrightarrow{\beta} R + p, \quad (27) \]
\[ R + r \xrightarrow{1-\beta} R + r, \quad (28) \]
\[ l + p \xrightarrow{\gamma} p + p, \quad (29) \]
\[ l + l \xrightarrow{1-\gamma} l + l, \quad (30) \]
\[ r + p \xrightarrow{\delta} p + p, \quad (31) \]
\[ r + r \xrightarrow{1-\delta} r + r, \quad (32) \]

which turn yield the evolution equations,

\[
\frac{dl(t)}{dt} = -(1 - 2k) l(t) r(t) + m ll r(t) - (1 - n) rr l(t) - \alpha L l(t) \\
+ (1 - 2\gamma) l(t) p(t), \quad (33)
\]
\[
\frac{dr(t)}{dt} = (1 - 2k) l(t) r(t) - m ll r(t) + (1 - n) rr l(t) - \beta R r(t) \\
+ (1 - 2\delta) r(t) p(t), \quad (34)
\]
\[
\frac{dp(t)}{dt} = \alpha L l(t) + \beta R r(t) - (1 - 2\gamma) l(t) p(t) - (1 - 2\delta) r(t) p(t). \quad (35)
\]

We performed a numerical integration of the evolution equations \((33) - (35)\) in order to obtain the 2018 election results, namely Bolsonaro received about 55% of the votes and the left-wing candidate Haddad received about 45% of the votes \([23]\). In Figure \(3\) we exhibit two graphics showing the time evolution of the densities of interest. For both graphics, we consider the same initial condition given by polls \([24]\), namely \(l(0) = 0.26\) and \(r(0) = 0.40\). In addition, for the case with populism coupling (Figure \(3\) (a)), we also consider \(p(0) = 0\) since we did not consider populists in the previous elections. In addition, the same set of parameters \(L, ll, R, rr\) and \(k\) were considered for the two graphics of Figure \(3\) namely \(L = 0.08, ll = 0.08, R = 0.12, rr = 0.06\) and \(k = 0.50\). In the left side of Figure \(3\) we consider the model with populism coupling discussed in this section. The set of parameters were chosen to match the election results 0.55 and 0.45 for Bolsonaro and Haddad, respectively. For such a case, we considered \(\alpha = 0.02, \beta = 0.25, \gamma = 0.52\) and \(\delta = 0.33\) (see the discussion
FIG. 3. (Color online) Time evolution of the densities. These results simulate the evolution of votes in the 2018 election. For both figures we considered $L = 0.08$, $ll = 0.08$, $R = 0.12$, $rr = 0.06$ and $k = 0.50$. (a, left panel:) considering the populism coupling, we observe the Bolsonaro victory considering the total right ($r + rr + R$) plus the populists $p$, i.e., for $r + rr + R + p$. The remaining parameters are $\alpha = 0.02$, $\beta = 0.25$, $\gamma = 0.52$ and $\delta = 0.33$. (b, right panel:) for the same parameters $L, ll, R, rr$ and $k$, in the absence of the populism coupling the final victory is of the left candidate. The black squares in both panels denote the initial conditions, obtained by polls \cite{24}, i.e., $l(0) = 0.26$ and $r(0) = 0.40$.

below). Considering the total right plus populists ($r + rr + R + p$), Bolsonaro wins. On the other hand, in the absence of populists, i.e., considering the same set of parameters $L, ll, R, rr$ and $k$, the model discussed in section 2 leads to the left-wing victory (see Figure 3, right side).

As mentioned above, it is not the values of the parameters themselves that matter, but the respective change of their values from one election to another. Here, our choice of values for the new parameters $\alpha, \beta, \gamma$ and $\delta$ is based on the following considerations:

- Comparing with the 2014 election, the left-wing candidate support decreased from 0.52 in 2014 to 0.45 in 2018. In addition, the right-wing candidate support increased from 0.48 in 2014 to 0.55 in 2018.

- Based on the model, we obtained the equilibrium values total left $= 0.45$, total right $= 0.44$ and $p = 0.11$, leading to total right plus populists $= 0.55$. In other words, the
fraction of right individuals decreased from 0.48 in 2014 to 0.44 in 2018.

- However, the fraction of populists increased from zero in 2014 to 0.11 in 2018. Those choices are based on the assumption that right sensitives $r$ turned in populists $p$ from interacting with few right radicals $R$, and once a seed of populists $p$ is formed it sucks directly a part of right sensitives $r$ and also a small part of left sensitives $l$.

- On this basis, it makes sense to assume the very small value $\alpha = 0.02$, related to the $l \rightarrow p$ transition (interaction $L + l$), and the higher value $\beta = 0.25$, related to the $r \rightarrow p$ transition (interaction $R + r$), used to match the 2018 election results.

- The other parameters, $\gamma$ and $\delta$, related to direct $l$ and $r$ interactions with $p$, should have intermediate values reflecting the fact that we need to have a balance among the sensitive populations $r$ and $l$, and the populism population cannot grow excessively. In such a case, we used $\gamma = 0.52$ and $\delta = 0.33$.

V. COMMENT REGARDING THE 2021 POLL ABOUT A POSSIBLE BOLSONARO VS LULA IN 2022

A recent poll was conducted in Brazil in September 2021 about a possible 2022 second round Bolsonaro / Lula (right / left). The results yielded a substantial advantage for Lula with 0.64 against 0.36 for Bolsonaro, a large shift of 0.19 with respect to the 2018 results.

Within our model, such a shift implies (1) A decrease of the populism support, that was $p = 0.11$ for 2018; (2) A large number of right sensitives $r$ turning into left sensitives $l$.

Indeed, an increase in $k$ with a simultaneous decrease of $\alpha$, $\beta$ and $\gamma$ can produce the poll observed shift of 0.19 from right to left. The increase of $k$ leads to more transitions $r \rightarrow l$. On the other hand, the decrease of $\alpha$ leads to a decrease of the transitions $l \rightarrow p$, which leads to the increase of left sensitives $l$ and the decrease of the populists $p$. For lower values of $\beta$ we have less transitions $r \rightarrow p$, which decreases $p$ and increases $r$. Yet, this increase of $r$ leads to the increase of $l$ triggered by the increase of $k$. Finally, the decrease of $\gamma$ leads to less transitions $l \rightarrow p$, which increases $l$ and decreases $p$.

Looking at the model’s parameters, we can see that an upper limit for the left votes can be obtained if we take $\gamma = 0$. A very low gamma turns most $p$ to $l$ via the interaction $l + p$. Thus, keeping invariant all the parameters considered for the 2018 election and taking $\gamma = 0$,
we obtain the equilibrium values: total left = 0.604, total right = 0.383 and $p = 0.013$, which lead to total right = 0.396 $\approx$ 0.40 and total left $\approx$ 0.60.

Our findings suggest the poll did overestimate the left to 0.64 while it could be only up to 0.60. This possible discrepancy is sound since a 19% shift appears to be huge comparing to the shifts which occurred in precedent elections. Still, a shift of 15% is substantial and could be explained by both Bolsonaro handling of Covid-19 and a persistent support for Lula, specially from people from lower economic classes, despite his fall in popularity due to corruption scandals.

VI. AN ALTERNATIVE EXPLANATION FOR 2018 BOLSONARO VICTORY

To explain 2018 Bolsonaro victory, we have introduced the mechanism of interaction between sensitives and radicals to create populists, who then can spread over interacting with sensitives from both left and right. Then, to recover Bolsonaro outcome of 0.55 we found the equilibrium value $p = 0.11$ for populists, which, contrary to what was expected for a populist leader, is not that high.

This finding prompted us to consider an alternative scenario along the series of previous presidential elections without involving populists. The aim being to reproduce all election outcomes and the 2021 poll, using a single varying parameter without populists.

Indeed, keeping $L = ll = R = rr = 0.04$ unchanged, for each election a value of $k$ is found to yield the actual outcome. It is worth to notice that the variations of $k$ from one election to another are very small contained between $0.50 - 0.012$ and $0.50 + 0.016$, i.e less than 2% variation as seen from Table 2.

For comparison, we have also calculated the values of $k$ obtained for the first scenario when the populist interactions are activated in 2018 and 2021. For $\alpha = \beta = \gamma = \delta = 0.30$ we get respectively $k = 0.496$ and $k = 0.520$ (see Tabel 2) and the same equilibrium value of populists $p = 0.03$. When $\alpha = \beta = \gamma = \delta = 0.40$ we found $k = 0.499$ and $k = 0.528$ and still a unique $p = 0.08$ (see Table 2). All the results are exhibited Figure 4(a, b).

These small values of the proportions of populists are unexpected and puzzling when dealing with a candidate pictured as populist. To check this “anomaly” we look at what happens increasing the populist coupling to $\alpha = \beta = \gamma = \delta = 0.50$. The increase of only 0.10 reveals a sudden sharp jump in the equilibrium value of populists at $p = 0.84$ against
| Election | k    |
|----------|------|
| 1989     | 0.497|
| 1994     | 0.488|
| 1998     | 0.493|
| 2002     | 0.512|
| 2006     | 0.512|
| 2010     | 0.506|
| 2014     | 0.502|
| 2018     | 0.495|
| 2018     | 0.496\(^1\)|
| 2018     | 0.499\(^2\)|
| 2021     | 0.516|
| 2021     | 0.520\(^1\)|
| 2021     | 0.528\(^2\)|

TABLE II. Values of $k$ found to reproduce the elections' results considering the model without populists (model of Section 2) and the model with populists (model of Section 4) for $\alpha = \beta = \gamma = \delta = 0.30$ (values with \(^1\)) and $\alpha = \beta = \gamma = \delta = 0.40$ (values with \(^2\)).

$p = 0.08$ for $\alpha = \beta = \gamma = \delta = 0.40$.

Above jump in the value of populists hints at a singularity at some value of the populist coupling. To identify this singularity we plot in Figure 4 (b, c, d) the variations of stationary fractions of populists $p$, left sensitives $l$ and right sensitives $r$ as a function of $\alpha = \beta = \gamma = \delta$ for three different values $k = 0.10, 0.40, 0.70$. Indeed, abrupt quasi-vertical jump and falls are seen to occur at $\alpha = \beta = \gamma = \delta = 0.50$.

In particular, Figure 4 (b) shows that the increase of $p$ is very slow and stays at low values till the vicinity of $\alpha = \beta = \gamma = \delta = 0.45$, before a sudden jump to each high values.
FIG. 4. (Color online) (a: upper left) Values of $k$ matching the elections results with no populism. The values with populism are added for 2018 and 2021. The x-axis is the time (in years) since the first election: 0 represents the 1989 election, 5 represents the 1994 election, and so on. (b: upper right, c: lower left, d: lower right) The plots show the stationary densities of populists $p$, left sensitives $l$ and right sensitives $r$ as a functions of $\alpha = \beta = \gamma = \delta$ with $L = ll = R = rr = 0.04$. The symbols are numerical results, and the lines are just guides to the eye. In panel (b), the inset exhibits the same data for $p$ only with symbols (circles, squares and triangles), together with the analytical result of Eq. (36) (full black line).

This behavior sheds a new light on the phenomenon of populism, which could explain why it is almost impossible to oppose it when it bursts. Indeed, since the impact of populism is minor on the opinion dynamics during about half the range of the associated parameters, it generates a low proportional reaction, which in turn is overwhelmed by the tidal wave
populism when it reaches its breaking point.

These numerical results regarding the stationary fraction of populists as a function of \( \alpha \) can be understood if we inspect Eq. (35). In the stationary state we have \( dp/dt = 0 \), and Eq. (35) leads to the expression \([(1 - 2\gamma)l + (1 - 2\delta)r]p = \alpha L l + \beta R r \). Considering \( L = ll = R = rr \) and \( \alpha = \beta = \gamma = \delta \) as before, we obtain

\[
p = p(\alpha) = L \frac{\alpha}{1 - 2\alpha}.
\]  

This result is valid for \( \alpha < 0.50, l \neq 0 \) and \( r \neq 0 \). Eq. (36) shows that the stationary values of \( p \) does not depend on \( k \), as we discussed before. Indeed, \( p \) is solely a function of \( \alpha \), and the function \( p(\alpha) \) has a jump at \( \alpha = 0.50 \). Eq. (36) explains our previous observations, and it is exhibited in the inset of Figure 4 (b) together with the numerical results for the stationary \( p \) obtained from the numerical integration of Eqs. (33) - (35). We observe a perfect agreement between the two results. In addition, as we observed in Figure 4 (c,d) that \( l = r = 0 \) for \( \alpha > 0.50 \), from the normalization condition (24) we have for \( L = ll = R = rr \) that \( p = 1 - 4L \). For the case of Figure 4 (b), we have \( L = 0.04 \) and thus \( p = 0.84 \) for \( \alpha > 0.50 \).

Above results lead to question the actual narrative which explains Bolsonaro 2018 victory as a populist event. At this stage, our analysis yields two possible scenarios with very different underlying political mechanisms:

1. Bolsonaro 2018 victory was not the result of populism but the outcome of the regular competition between sensitives from left and right. The appropriate value of \( k \) is coherent with all other values for past elections as seen in Figures 1 and 2.

2. Bolsonaro did benefit from populism but at a very low level with only a few percent.

It is important to emphasize that we considered here the simplified hypothesis \( \alpha = \beta = \gamma = \delta \) when we considered the model with populists. This restricts us to a subspace of possibilities but even in this case it gives us important insights about what are the possible different mechanisms that can explain the Bolsonaro 2018 victory.

VII. ANTICIPATING THE 2022 ELECTION

In case above second scenario is valid, to anticipate the 2022 outcome we need to stress that as long as \( \alpha = \beta = \gamma = \delta < 0.50 \), the equilibrium value of \( p \) stays low and what
matters is the value of \( k \). However, as soon as \( \alpha = \beta = \gamma = \delta \geq 0.50 \) the value of \( k \) becomes irrelevant, and in such a case the populism would contribute strongly to Bolsonaro making likely his victory in 2022.

That would make the 2021 poll misleading to forecast the 2022 outcome since it was implemented with low populism couplings putting Bolsonaro in a status of an hidden winner since during the 2022 campaign these couplings could be increased moderately to reach the jump of \( p \) as seen from Figure 3 (b) and then propels Bolsonaro to victory. That would result in a surprising reversing of the 2021 Lula tilde like poll outcome in 2022 at the benefit of Bolsonaro.

\section*{VIII. FINAL REMARKS}

In this work, using a contagious-like model we have studied the dynamics of opinion to describe the evolution of the political voting trends in Brazil after the dictatorship from 1985 to nowadays. The model articulates around a system of coupled ordinary differential equations to account for the interactions of left and right supporters. For each side we have considered three kinds of agents, sensitives, inflexibles and radicals where only sensitives can shift opinions.

First, we have assumed that radicals do not interact. They are kept part from sensitives and inflexibles. The evolution of vote intentions are then ruled by interactions among sensitives and inflexibles, as well as among sensitives themselves. The model was shown to reproduce all elections outcomes after Brazilian dictatorship (1989 - 2014) using one single parameter \( k \) with some fixed set of densities of inflexibles and radicals.

Second, to obtain the winning of Bolsonaro in the 2018 election, we have activated interactions of radicals with sensitives to create populists from sensitives. Once created, populists interact with sensitives to accelerate their spreading.

In parallel, we have explored an alternative path to obtain the Bolsonaro 2018 victory without involving populists. Along the first approach driven solely by \( k \), we showed that we can recover the 2018 outcome as well as the 2021 poll. Our hypothesis would mean that despite Bolsonaro populist narrative, his victory was the result of the same mechanisms, which have been at play during all past presidential elections.

In our study, we found two scenarios of very different political contents, which could
have equally led to Bolsonaro victory. While this finding has no effect on the past 2018 election, it could put at stake the current belief that Bolsonaro will lose the 2022 election in case the second scenario is valid. Moreover, if it is the first scenario, according to our findings, the 2021 poll has overestimated Lula outcome.

To conclude, we are aware that our results are bounded to the actual values chosen for the parameters. However, we emphasize that what counts in our findings are not the numbers themselves but the trends and the connections obtained between the variables within the same framework. Accordingly, although it is too early to select the operative scenario for 2022, the overall framework allows a warning about current certainties of a Bolsonaro failure in 2022.

ACKNOWLEDGMENTS

NC acknowledges financial support from the Brazilian scientific funding agencies CNPq (Grant 310893/2020-8) and FAPERJ (Grant 203.217/2017). We also thank L. Sigaud for a critical reading of the manuscript.

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