Electromechanical coupling in suspended nanomechanical resonators with
a two-dimensional electron gas

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Abstract. A physical model describing the piezoelectric-effect-mediated influence of bending of a thin suspended cantilever with a two-dimensional electron gas on the conductivity is proposed. The model shows that the conductivity change is almost entirely caused by the rapid change in mechanical stress near the boundary of suspended and non-suspended areas, rather than by the stress itself. An experiment confirming that the electromechanical coupling is associated with the piezoelectric effect is performed. The experimentally measured conductance sensitivity to the cantilever’s vibrations agree with the developed physical model.

1. Introduction
The typically studied low-dimensional electron systems are created from a two-dimensional electron gas (2DEG) contained in a semiconductor bulk (such as a GaAs/AlGaAs heterostructure). However, a heterostructure with a sacrificial layer can be used as a base for creation of thin suspended membranes with a 2DEG by means of selective etching \cite{1-3}. The nanostructures fabricated from these membranes are mechanically moveable and can be used as nanomechanical resonators. Experience shows that vibration of the nanostructures with electron gas affects electron transport and locally changes 2DEG conductivity. These suspended nanostructures can be considered as nanoelectromechanical systems, whose mechanical motion can be detected by means of electrical measurements. A series of papers describe non-trivial hybrid electromechanical devices combining a nanoscale mechanical resonator and such mesoscopic objects as single-electron transistors \cite{1} and quantum point contacts \cite{2}. However, the physical mechanism of electromechanical coupling in these systems remains essentially unclear, although it is often mentioned that the coupling is possibly associated with the piezoelectric effect. Moreover, there is no common physical model predicting the magnitude of the effect. In the present paper we propose a model describing the coupling and show that its predictions agree with the experimental data.
2. Physical model
Consider a nanomechanical resonator having the form of a thin cantilever with dimensions $L \times W \times t$ performing small flexural vibrations at the fundamental mode frequency $\Omega_0/2\pi$. Introduce a coordinate system with $x$-axis directed along the cantilever and $z$-axis coinciding with the vibrations direction. Let the cantilever contain a 2DEG at the neutral surface (in the middle of the cantilever). The cantilever's shape $U(x, t)$ is known from the Euler-Bernoulli theory [4]. The mechanical stress tensor can be written using the Voigt notations [5] as

$$\sigma_\alpha = \left( -Ez \times \partial^2U/\partial x^2 \ 0 \ 0 \ 0 \ 0 \ 0 \right)^T,$$

where $E$ is the Young’s modulus corresponding to the direction along the cantilever, and $z$ is the distance from the neutral surface. Due to the piezoelectric effect, the stress leads to electrical polarization $P_\alpha = d_{\alpha\beta}\sigma_\beta$, where $d_{\alpha\beta}$ is the piezoelectric matrix. Consider the case of $z$-axis coinciding with [001]-direction in an Al$_{0.33}$Ga$_{0.67}$As crystal and denote the angle of rotation from [100] direction to $x$-axis as $\theta$. Then the piezoelectric matrix can be written in the following form:

$$d_{\alpha\beta} = \frac{d_{14}}{2} \begin{pmatrix} 0 & 0 & 0 & 2 \cos 2\theta & 2 \sin 2\theta & 0 \\ 0 & 0 & -2 \sin 2\theta & 2 \cos 2\theta & 0 & 0 \\ \sin 2\theta & -\sin 2\theta & 0 & 0 & 2 \cos 2\theta \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

where $d_{14}$ is the piezoelectric constant equal to $-3.04 \text{ pm/V}$ for the considered crystal. Electrical polarization $P_\alpha$ for the given stress tensor has only one non-zero component $P_z = -d_{14}/2 \times Ez \sin 2\theta \times \partial^2U/\partial x^2$. This non-uniform polarization can be equivalently represented by the volume bound charge $\rho_b = -\text{div}P = -dP/\text{d}z$ which is compensated by the surface bound changes $\sigma_b = -\rho_b t/2$ on the top and the bottom surfaces of the cantilever.

![Figure 1](image-url)  
Figure 1. (a), (b) The bent cantilever and its orientation with respect to the crystallographic axes. (c) The calculated electron density change $\delta n$ per unit displacement of the cantilever’s free end $u(L)$. The results are shown for the cantilever with length $L = 3 \mu$m and width $t = 166 \text{ nm}$.  

The 2DEG reacts on appearance of these bound charges by changing the electron density distribution to maintain constant electrochemical potential:

$$-e(\delta\phi_b + \delta\phi_{2DEG}) + \delta\mu_{ch} = 0.$$  

Here $e$ is the elementary charge, $\delta\phi_b$ and $\delta\phi_{2DEG}$ are the electrical potentials created by the piezoelectric bound charges and the changed 2DEG density $\delta n$, respectively and $\delta\mu_{ch} = \delta n \times \pih^2/m^*$ is the chemical potential change ($m^*$ is the effective electron mass). To estimate the electron density change, we neglect the term $\delta\mu_{ch}$ and use the model of pure electrostatic screening [6]. Considering the influence of a point charge located at the distance $r$ from the 2DEG plane, it can be shown that such approach is reasonable when $r \gg a_B$, where $a_B \approx 13 \text{ nm}$ is the Bohr radius in GaAs. Since $t \gg a_B$ in our case, the model of pure electrostatic screening takes
into account the most of the induced bound charges. The simplified equation $\delta \phi_b + \delta \phi_{2DEG} = 0$ can be solved using the method of images. To simplify the estimates we neglect the small bending of the cantilever and consider the 2DEG as an infinite zero-potential plane sandwiched between two $t/2$-thick insulating layers with dielectric constant $\varepsilon = 13$. However, we save the calculated bound charges $\rho_b$ and $\sigma_b$ at $0 < x < L$. The calculated electron density change $\delta n$ per unit displacement of the cantilever free end $u(L)$ is shown in Fig. 1 (c). It can be seen that $|\delta n|$ decreases with the distance from the clamped end much more rapidly than the mechanical stress. Moreover, almost the same density distribution can be obtained for the mechanical stress in the form of the Heaviside step function $\sigma_1(x) = \sigma_1(0)\Theta(x)$. This fact shows that the change in the electron density is determined mainly not by the stress itself, but rather by the rapid change in mechanical stress occurring near the clamping point. It is clear that actual characteristic distance at which the stress becomes negligible at $x < 0$ is the cantilever thickness $t$.

3. Experiment

The experimental samples are created from the [001]-oriented GaAs/AlGaAs heterostructure with a 2DEG described in detail elsewhere [3]. The 2DEG density is $n \approx 6.7 \times 10^{11}$ cm$^{-2}$ and the mobility is $\mu \approx 1.2 \times 10^6$ cm$^2$V$^{-1}$s$^{-1}$. Lateral geometry of the samples is defined by means of electron beam lithography followed by reactive ion etching. The sacrificial layer is selectively removed from-under the nanostructures by means of wet etching in HF water solution. Each experimental sample represents a suspended conductive cantilever having a length of $L = 3 \mu$m, a width of $W = 2 \mu$m and a thickness of $t = 166$ nm (see Fig. 2 (a)). The cantilever is perforated by a series of through holes arrayed in a line. The regions between the holes are non-conductive due to the edge depletion, except for a single constriction near the cantilever’s base where the inter-hole distance is enlarged up to 600 nm. This is the only conductive channel connecting the source and the drain areas. The measured constriction’s resistance is $1/G_0 \approx 1$ k$\Omega$ (after substraction of the resistance of the contacts and the mesa). Each experimental chip contains two distanced identical cantilevers oriented in the perpendicular directions $[110]$ and $[\overline{1}10]$.

The cantilevers’ flexural vibrations are driven in the direction perpendicular to the surface via

Figure 2. (a) A scanning electron microscope image of an experimental sample. Notations: S - source, D - drain, G1, G2, G3 - gates. Amplitudes (b,e) and phases (c,d) of the conductance change as a function of the driving frequency. (f) Identical stresses applied in the perpendicular directions lead to opposite electrical polarizations.
electrostatic capacitive actuation [3] by means of applying a voltage $V_{dc} + V_{ac} \cos \Omega t$ to the gate G1 surrounding the cantilever’s free end. The force in the considered direction arises because the substrate underlying the cantilever has a high dielectric constant and acts as a bottom gate [3]. Amplitude $\delta G_0$ and phase $\varphi$ of the change in the constriction conductance are obtained using the heterodyne down-mixing method [3]. During the experiment, the samples are placed in a vacuum chamber and cooled down to the liquid helium temperature 4.2 K.

Fig. 2 (b-e) shows $\delta G_0$ and $\varphi$ as a function of the driving frequency for both cantilevers. The measurements are performed at $V_{dc} = 2$ V and $V_{ac} = 50$ mV. These curves (obtained by means of the electrical measurements) have the form characteristic for amplitude-frequency and phase-frequency dependences, which can be obtained for a classical driven linear oscillator. This fact demonstrates linearity of the electromechanical coupling. The measured resonant frequencies $\Omega_0/2\pi$ are 11.455 MHz and 10.889 MHz, while quality factors $Q$ are 18 000 and 23 400.

The measured phase-frequency curves show that identical vibrations of the orthogonal cantilevers lead to the conductance responses differing by 180°. In other words, identical bending of the cantilevers leads to opposite changes in the conductance. This anisotropy is a qualitative evidence of the fact that electromechanical coupling has piezoelectric nature. Indeed, for the piezoelectric matrix determined by equation (2), we can expect opposite electrical responses for the perpendicularly oriented cantilevers (see Fig. 2 (f)).

The vibrations’ amplitude, as shown in [3], can be estimated as $u(L) = C'V_{dc}V_{ac}Q/\left(2m\Omega_0^2\right)$, where $m = 0.24\rho WL$ is the cantilever’s effective mass, $\rho$ is the mass density, $C' = -0.39\varepsilon_0 WL/d_0^2$ and $d_0 = 400$ nm is the distance between the cantilever and the substrate equal to the sacrificial layer’s thickness. These estimates give the amplitudes of 20 and 29 nm and the relative conductance sensitivities to the vibrations $(1/u(L))\delta G_0/G_0$ of $5.6 \times 10^{-6}$ and $-0.86 \times 10^{-6}$ nm$^{-1}$ for the [110]- and [110]-oriented cantilevers, respectively. These values can be compared to the relative change in electron density $(1/u(L))\delta n/n$ predicted by the developed physical model. The value of $\delta n$ should be taken at the distance $x = 1.3$ μm between the constriction’s center and the boundary between suspended and non-suspended areas whose position can be easily determined with a scanning electron microscope. The determined value $(1/u(L))\delta n/n = 0.55 \times 10^{-6}$ nm$^{-1}$ agrees with the experimental data and confirms the fact that the developed physical model adequately describes the experiment.

4. Conclusion
A physical model is developed, which predicts the change in the conductivity of a 2DEG contained in a vibrating nanomechanical resonator. The model shows that the conductivity change is determined mainly by rapid change in mechanical stress near the boundary of suspended and non-suspended areas, rather than by the stress itself. An experiment is described, that shows piezoelectric nature of the electromechanical coupling in nanoelectromechanical systems with a 2DEG. The experimental result agree with the model predictions.

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