A New Experiment for the Measurement of the $g$-Factors of $^{3}\text{He}^{+}$ and $^{3}\text{He}^{2+}$

A Mooser$^{1}$, A Rischka$^{1}$, A Schneider$^{1,2}$, K Blaum$^{1}$, S Ulmer$^{3}$ and J Walz$^{4,5}$

$^{1}$ Max-Planck-Institute for Nuclear Physics, Heidelberg, Germany
$^{2}$ University of Heidelberg, Heidelberg, Germany
$^{3}$ RIKEN, Ulmer Fundamental Symmetries Laboratory, Wako, Japan
$^{4}$ Institute for Physics, Johannes Gutenberg-University Mainz, Mainz, Germany
$^{5}$ Helmholtz-Institute Mainz, Mainz, Germany

E-mail: andreas.mooser@mpi-hd.mpg.de

Abstract. We describe a new experiment that aims at a parts per billion measurement of the nuclear magnetic moment of $^{3}\text{He}^{2+}$ and a 100 parts per trillion measurement of the Zeeman effect of the ground-state hyperfine splitting of $^{3}\text{He}^{+}$. To enable ultrafast and efficient experiment cycles the experiment relies on new technologies such as sympathetic laser cooling of single $^{3}$He-ions coupled to a cloud of Doppler-cooled $^{9}$Be-ions in a Penning trap or a novel spin-state detection scheme.

1. Introduction

Recent advances in quantum-jump spectroscopy of single isolated protons and antiprotons in advanced Penning-trap systems led to the most precise measurements of their nuclear magnetic moments $[1, 2, 3]$. Based on these successes we set up a new experiment dedicated to the measurements of the electronic and nuclear magnetic moments of $^{3}\text{He}^{+}$ and $^{3}\text{He}^{2+}$. The project aims at the first direct measurement of the nuclear magnetic moment of $^{3}\text{He}^{2+}$ with a relative precision on the parts per billion level (ppb) or better and an improved value of the ground-state hyperfine splitting in $^{3}\text{He}^{+}$ by at least a factor of 10. The measurement will not only provide a high-precision determination of electronic and nuclear magnetic moments in a light hydrogen-like atom but will also give access to the zero-field ground-state hyperfine splitting of $^{3}\text{He}^{+}$. Currently, the most precise measurement of the latter comes from hyperfine spectroscopy using a cloud of $^{3}\text{He}^{+}$ ions in a radio-frequency quadrupole ion trap $[4]$. However, the measurement was limited to relative precision of 1 ppb by resonance line broadening due to the second-order Doppler effect in the used room temperature apparatus. We aim to improve the precision of the the ground-state hyperfine splitting by making use of recent developments on single ion spectroscopy in a cryogenic Penning-trap system. The measurement can be used to characterize nuclear effects in $^{3}\text{He}$. For hydrogen-like systems the hyperfine splitting is given to leading order by the Fermi contact term $[5]$, which accounts for...
the interaction of the nuclear and electronic magnetic moments,

\[ E^F = -\frac{8\pi \alpha}{3} \frac{\hbar^3}{m_e^2 c} \frac{\mu_e \cdot \mu_{He}}{\mu_B} |\Psi_{1s}(r = 0)|^2, \] with (1)

|\Psi_{1s}(r = 0)|^2 = \frac{1}{\pi} \left( \frac{Z \alpha m_e}{\hbar} \right)^3 \] (2)

being the square of the wave function at the origin. Here \( \alpha \) is the fine structure constant, \( \hbar \) the reduced Planck-constant, \( m_e \) the mass of the electron, \( \mu_e \) and \( \mu_{He} \) the electron likewise nuclear magnetic moment, \( \mu_B \) the Bohr magneton, \( Z = 2 \) the charge of the nucleus, \( \mu_R = (m_e m_{He})/(m_e + m_{He}) \) the reduced mass and \( c \) the speed of light. Please note that the direction of \( \mu_{He} \) is opposite to the nuclear spin, thus \( \mu_{He} < 0 \), which results in \( E^F < 0 \). Corrections contributing to the energy splitting arise from QED effects \( \delta_{QED} \), the strong interaction \( \delta_{hvp} \) and recoil effects \( \delta_{rec} \). In addition the nuclear polarizability as well as the spatial distribution of the charge and the magnetic moment within the nucleus \( \delta_{nucl} \)- the latter known as Zeemach radius - contribute

\[ \Delta E_{HFS} = E^F \left( 1 + \delta_{QED} + \delta_{rec} + \delta_{nucl} + \delta_{hvp} \right) \] (3)

Since \( \delta_{nucl} \) contributes at a relative precision of 213 parts per million (ppm) any measurement of \( \Delta E_{HFS} \) at dramatically better resolution is highly sensitive to nuclear structure effects. Furthermore, since in leading order \( E^F \) scales with the square of the lepton mass any measurement with a relative precision of 20 parts per trillion (ppt) in this electronic system would complement a ppm measurement of \( \Delta E_{HFS} \) in a muonic system.

In addition a direct high-precision measurement of the nuclear magnetic moment of \(^3\)He has the potential to establish hyper-polarized \(^3\)He NMR probes as an independent standard for precision-magnetometry. Compared to protons in water, hyper-polarized xenon or optical magnetometers \(^3\)He exhibits numerous advantages. These advantages are smaller systematic uncertainties concerning sample shape, impurities and environmental dependencies [9, 10, 11] if compared to water, ten times larger polarization and thus higher signal strengths [12] if compared to xenon, and no lack of a poorly defined blend of hyperfine lines if compared to optical magnetometers [13]. Eventually, owing to the fact that \(^3\)He is an atom rather than a molecule, the diamagnetic
shielding of $^3$He does not have to be calibrated by a set of dependent measurements but is known most precisely by theory for any atom [14]. Using hyper-polarized $^3$He probes relative precisions of ppt within seconds have been already achieved [15]. However, so far, only indirect determinations of the magnetic moment exist, which compared the nuclear magnetic resonance frequency of $^3$He to the nuclear magnetic resonance frequency of protons in water or in molecular hydrogen (compare Fig. 1). These allowed to determine the magnetic moment with relative precisions of 12 ppb only. Thus, we aim for a direct ppb measurement of the nuclear magnetic moment of $^3$He, which will cancel the correlation to proton NMR probes and improve the precision. However, as is known for the ground-state hyperfine splitting, the nuclear-spin dependent part of the atomic $g$-factor is suppressed by 3 orders of magnitude compared to the bound-electron $g$-factor, which limits the determination of the nuclear magnetic moment to the ppm level. Thus in addition to the hyperfine structure measurement we aim for a direct measurement of the nuclear magnetic moment of the helion nucleus $^3$He$^{2+}$. Here we plan to use a similar setup as the one used in the recent 300 ppt measurement of the $g$-factor of the proton [3, 16].

2. Basic measurement principle

The basic measurement principles and experimental setup will be based on methods and techniques that have been developed and improved in the course of proton and antiproton magnetic moment measurements [17]. For this experiment a single isolated $^3$He$^{2+}$ or $^3$He$^+$ will be stored in an advanced cryogenic Penning trap. A Penning trap is a superposition of a homogeneous magnetic field $B$ and an electrostatic quadrupole potential [18]. In such a field configuration a single ion has three eigenmotions, the axial motion with frequency $\nu_z$, and two radial motions: the magnetron and the modified cyclotron motions, at frequencies $\nu_-$ and $\nu_+$, respectively. In addition, an invariance theorem $\nu^2 = \nu^2_+ + \nu^2_+ + \nu^2_z$ [18] relates the three eigenmotions to the free cyclotron frequency $\nu_c = 1/(2\pi)(q/m)B$, where $q/m$ is the charge-to-mass ratio of the ion. Using nondestructive detection systems [19] and phase-sensitive detection schemes [20] cyclotron frequency measurement with relative precisions on the order of 30 ppt have been demonstrated [21]. In addition to the center of mass motion described above, the coupling of the spin magnetic moment(s) and $B$ results in energy splittings between different spin-eigenstates. In the case of $^3$He$^{2+}$ the energy splitting between the two nuclear spin-states is characterized by the Larmor-frequency $\nu_L = 1/(2\pi)(g/2)(q/m)B$. Therefore, a measurement of the frequency ratio

$$\frac{\nu_L}{\nu_c} = \frac{\mu_{He}}{\mu_N} \frac{m_{He}}{2m_p} = \frac{g}{2} \frac{m_{He}}{m_p}$$

(4)

determines the $g$-factor, which is a measure of the nuclear magnetic moment $\mu_{He}$ in units of the nuclear magneton $\mu_N$. Here $m_{He}/m_p$ is the helion-to-proton mass ratio.

In the case of $^3$He$^+$ the Zeeman effect of the hyperfine-splitting can be described by the well-known Breit-Rabi equation [22],

$$E_F = I_{\pm1/2} = -\frac{1}{4}\Delta E^{HFS} + gj\mu_K m_F B \pm \frac{1}{2}\Delta E^{HFS} \sqrt{1 - 2m_F x + x^2}, \text{ with}$$

$$x = \frac{(g_j\mu_B - gj\mu_K) B}{\Delta E^{HFS}}.$$  

(5)

(6)

Here $\mu_K$ is the nuclear magneton, $I$ the nuclear spin, and $g_j$ as well as $g_I$ the electronic likewise nuclear $g$-factors. $F$ and $m_F$ are the low-field quantum numbers. Please note that $\Delta E^{HFS} < 0$. The scaling of the level splitting is shown in Fig. 2. Experimentally the energy splitting of
the two nuclear spin transitions, $\nu p_1 = E_1 (F = 0, m_F = 0) \nu h - E_2 (F = 1, m_F = 1) \nu h$ and $\nu p_2 = E_3 (F = 1, m_F = 0) \nu h - E_4 (F = 1, m_F = -1) \nu h$, as well as the energy splitting of the electronic spin transition $\nu_e = E_2 (F = 1, m_F = 1) \nu h - E_3 (F = 1, m_F = 0) \nu h$ are accessible. In addition the magnetic field in units of the ion mass can be determined via a measurement of $\nu_c$. The measurement of this set of frequencies thus enables a magnetic field independent determination of the electronic $g_j$ and nuclear $g_I$ g-factor as well as the Fermi-energy $E_F$. Since the magnetic field is measured in units of the ion’s mass, the electron-to-helion mass ratio, the proton-to-helion mass ratio and additional contributions due to the binding energy of the remaining electron enter the determination. Both mass ratios were determined with relative precisions in the order 30 ppt [21, 23, 24, 25], however, the results for the helion mass reported in [24] and in [25] disagree by 250 ppt and require further experimental studies. By comparison, the contributions due to the binding energy of the electron pose no limitation.

3. Measurement of the energy splitting between spin-states - continuous Stern-Gerlach effect

In both cases, $^3\text{He}^+$ as well as $^3\text{He}^{2+}$, the energy splitting between the spin states will be probed with the continuous Stern-Gerlach effect [26]. Here the spin magnetic moment is coupled to the particle’s oscillation frequency by superimposing an inhomogeneous magnetic field to the trap, a so-called magnetic bottle, $B_z = B_0 + B_2 z^2$. The coefficient $B_2$ characterizes the strength of the superimposed inhomogeneity. A spin quantum jump shifts the axial frequency by

$$\Delta \nu_z = \frac{1}{2 \pi^2 m_{\text{He}} \nu z} B_2 \mu_s,$$

where $m_{\text{He}}$ is the mass of the nucleus and $\mu_s$ represents the electronic or nuclear magnetic moment. The successful implementation of this method will allow for the measurement of the spin flip probability as a function of the spin-flip drive frequency from which the energy splitting between the spin states can be determined. This method has been applied with great success in the measurements of the magnetic moment of the free electron [27], free positron [28] as well as electrons bound in highly charged ions [23, 29, 30]. However, subject of this project is the nuclear magnetic moment of $^3\text{He}$ which is
smaller by $\mu_{\text{He}}/\mu_e \approx 1/880$. Thus, in order to have a detectable frequency jump caused by a spin transition, a much larger magnetic field inhomogeneity is needed. Using the material with the highest saturation magnetization available a magnetic field inhomogeneity of $B_2 = 300000 \text{T/m}^2$ is superimposed on the Penning trap. This leads to an axial frequency jump of $170 \text{mHz}$ in the case of the proton/antiproton, which has to be resolved on top of a typical axial frequency of $700 \text{kHz}$ resulting from the electrostatic quadrupole potential. For the case of the nuclear magnetic moment of $^3\text{He}$ the frequency jump will further decrease to $40 \text{mHz}$ due to the three times higher mass and the slightly smaller nuclear magnetic moment.

To overcome this limitation in the case of $^3\text{He}^+$, a novel method will be implemented, where the nuclear spin-state detection is transferred to the detection of an electronic spin transition. The method makes use of the nuclear and electronic spin configuration of the hyperfine structure in a magnetic field. Here, an electronic spin-transition from $E_2 (m_j = 1/2, m_I = 1/2)$ to $E_3 (m_j = -1/2, m_I = 1/2)$ is only possible if the nucleon is in the spin-up state, compare Fig. 2. Thus the possibility to excite an electron transition can be used to detect the nuclear spin-state and thus nuclear spin-transitions. Eventually, an electronic spin-transition, which scales with the Bohr magneton, results in $\Delta \nu_z \approx 100 \text{Hz}$ and thus is much easier to detect.

This method, however, cannot be applied to a measurement of the magnetic moment of $^3\text{He}^{2+}$ and a frequency jump due to a spin-transition of $40 \text{mHz}$ has still to be detected. Here the challenge is, that superimposed electronic noise of order of a few $10 \text{pV/Hz}^{1/2}$ randomly excites cyclotron quantum transitions $\partial n_+ / \partial t \propto E_+ [31]$, which increase with the energy of the cyclotron mode $E_+$. Eventually, due to the strong coupling to the axial caused by to the huge magnetic field inhomogeneity these transitions induce axial frequency fluctuations, which make the identification of spin transitions difficult, see Fig. 3 a).

Calculations of the spin-flip detection fidelity, which is defined as the fraction of correctly assigned spin flips in a series of frequency measurements [33], are shown in Fig. 3 b). To achieve single spin flip resolution with $100\%$ detection fidelity, the cyclotron motion of the $^3\text{He}$ ion needs to be cooled to energies of $E_+/k_B < 30 \text{mK}$. This is about a factor of 7 lower.

Figure 3. : a) Frequency fluctuations as a function of the cyclotron energy $E = k_B T$ measured with a single proton [32]. b) Comparison of conventional resistive particle cooling and laser cooling and its implications on the spin state detection. Spin state detection fidelity as a function of cyclotron energy for a single proton (black) and a single $^3\text{He}^{2+}$ ion (red). In green the probability of preparing a particle below a given cyclotron energy using the conventional resistive cooling method. Please note the logarithmic scale. The vertical blue line indicates the energies achieved by sympathetic laser cooling and achieved with resistive cooling. In case of sympathetic laser-cooling a quasi-deterministic preparation of ions with much lower cyclotron energy is possible.
compared to the proton/antiproton, because the larger mass reduces $\Delta \nu_z$ and cyclotron quantum fluctuations are more likely to be excited by the electrical noise field due to the higher charge state. Typically these low energies are achieved by sub-thermal resistive cooling, using thermal baths with temperatures of $T = 4$ K and correlation time constants of 180 s [32]. Since this technique is a statistical process, under typical operation conditions several cooling attempts are conducted to achieve detection fidelities of 90 %, see Fig. 3 b). Thus the method is insufficient if applied to $^3$He and more advanced cooling techniques allowing for lower energies and quasi-deterministic energy preparations are mandatory in the case of $^3$He$^{2+}$.

4. Sympathetic laser cooling

To efficiently prepare particles at low cyclotron energy, a new method which will allow for highly efficient cooling to low energies within seconds will be implemented. The scheme, which relies on a variety of techniques proposed in [34], is known as the common end-cap technique. It is based on coupling of the trapped $^3$He$^{2+}$ ion to a cloud of $^9$Be$^+$ ions laser-cooled to the Doppler limit, which sympathetically cools the $^3$He ion. The principle is to store a cloud of $^9$Be$^+$ ions in one Penning trap and a single $^3$He$^{2+}$ ion in a neighboring Penning trap. The coupling will take place by image currents induced into a common electrode shared by both Penning traps, compare Fig. 4.

The coupling strength between both ions, which defines the cooling time, depends on the trap specific length $D$, the oscillation frequency $\nu_z$, the trap capacitance $C_T$ and the number of stored beryllium ions $N_{Be}$ [34],

$$\tau_{exc} = \frac{2\pi^2\nu_z C_T D^2}{q^2\sqrt{m_{He}m_{Be}} \frac{1}{\sqrt{N_{Be}}}}.$$  

Accordingly $D$ and $C_T$ have to be minimized in the design of the Penning trap to reach a short coupling time. Due to the direct impact of the trap diameter on $D$, which decreases with decreasing trap diameter, a small trap diameter is chosen for the coupling traps. To further decrease $D$ the endcap is used together with the two neighbouring electrodes to increase the signal pick-up. To this end the electrodes are capacitively coupled such that the total electrode distance decreases by a factor of three compared to similar Penning traps, but still an adiabatic

---

**Figure 4.** Schematic of the trap layout for the common endcap technique with endcaps (EC), correction electrodes (CE) and ring electrode (RE). The common endcap is a combination of four electrodes (CE+EC+EC+CE) that are connected capacitively to decrease the effective electrode distance and thus increase the coupling strength. On the right side a zoomed view of the region that shows the major contribution to the total capacitance between ring electrode and correction electrode is shown.
transport between the coupling traps by applying individual DC-voltages is possible. The Penning traps are designed such that the capacitance of the combined common endcap (CE+EC+EC+CE) is minimized by reducing the area seen between the ring and correction electrode. To this end the distance between the electrodes is increased and the amount of dielectric material is reduced. In addition sapphire (Al$_2$O$_3$) spacers between ring and correction electrodes were replaced with quartz (SiO$_2$) spacers, the latter exhibiting a lower relative permittivity by a factor of 2.5.

The right side of Fig. 4 details the geometrical steps taken to minimize $C_T$. These are: 1) reduction of the overlap between the ring electrode and quartz spacer and 2) enlargement in thickness of the quartz spacer, mechanically constrained by the inner lengths of the ring and correction electrodes.

For the final geometry, a capacitance of 8.8 pF with sapphire spacers has been calculated. If quartz spacers are used between the ring and the correction electrode the latter can be reduced to 5.6 pF, which is a factor of 3 lower compared to other standard Penning-trap electrodes with typical trap capacitances of up to 15 pF.

To improve the trap capacitance even further the removal of conducting material from the electrodes would allow for smaller electrode surfaces and thus a lower trap capacitance. Alternatively the electrodes could be produced from sapphire and only the inner Penning-trap surfaces gold-plated that would drastically reduce the capacitance.

Eventually, cooling times on the order of seconds for about 100 $^9$Be$^+$ ions are expected. This is much shorter than the thermalisation time of the cyclotron mode of $>100$ h, measured with a single proton [31]. To fulfill the resonance condition, i.e. equal oscillation frequencies for both ion species, it is planned to connect the common endcap to one common detection system. Thus the methods rely on tunable ion-detector interactions that avoid heating due to the detection system that acts as a thermal bath with a temperature of 4 K.

5. Measurement Scheme

The inhomogeneous magnetic field needed for the detection of the spin-state causes a line broadening of the Larmor and cyclotron resonance lines [35]. This ultimately limits the measurement precision to the ppm level. Thus, in state-of-the-art magnetic moment measurements the so-called double Penning-trap technique is employed [36]. Here a second Penning trap with a $10^6$ times more homogeneous magnetic field is used, in which excitations at the Larmor-frequency and measurements of the cyclotron frequency can be performed with at least a 100 fold improved precision. Spin state detection is carried out in the Penning trap with the inhomogeneous magnetic field called the analysis trap. In the case of $^3$He$^+$ the standard double-Penning trap method, as described in [36], will be used to subsequently obtain magnetic-field independent determinations of $\nu_{p1}$, $\nu_{p2}$ and $\nu_e$. In the case of $^3$He$^{2+}$ laser cooling has to be incorporated and two additional Penning traps dedicated to the common end cap coupling scheme are needed. Thus the double Penning-trap technique will be extended to a four Penning-trap technique. The additional Penning traps are called the coupling trap and the cooling trap. In Fig. 5 a first design of a multi-Penning-trap system capable of sympathetic laser cooling and a high-precision magnetic moment measurement is shown.

The Penning trap will be loaded with $^3$He$^{2+}$ or likewise $^3$He$^+$ using a $^3$He loaded titanium filament, as has been demonstrated in the closed Penning trap setup at the university of Washington [25] and a cloud of $^9$Be$^+$ ions using a laser ablation source. The starting conditions for one cycle of the four Penning-trap technique are a single isolated $^3$He$^{2+}$ ion in the coupling trap and a cloud of $^9$Be$^+$ ions in the cooling trap:

- First Preparation: By adjusting the voltages applied to the electrodes the axial oscillation frequency of the $^3$He$^{2+}$ ion and $^9$Be$^+$ ions are tuned into resonance with each other. Afterwards, the axial mode of the $^9$Be$^+$ cloud is laser cooled to the Doppler limit. Eventually
Figure 5. Penning-trap assembly as planned to be set up for the $^3$He magnetic moment measurements consisting of four individual Penning traps. A cooling trap stores a cloud of $^9$Be$^+$ ions for laser cooling. A coupling trap is used to sympathetically cool the cyclotron mode of the $^3$He$^{2+}$ ion. An analysis trap, where the central electrode is made out of ferromagnetic material, allows for spin state detection using the continuous Stern-Gerlach effect. A precision trap for precise frequency measurements. The bottom graph indicates the magnetic field distribution along the axial direction. The top left graph shows single spin transitions of a single isolated proton. In the top right graph the detection signal is shown. Image currents induced by the ions motion into the trap electrodes are picked up by highly sensitive, superconducting coils. After performing a Fast Fourier Transform the signal is observed. The broad resonance is due to the Johnson Nyquist noise of the parallel tuned circuit formed by the inductance and the trap capacitance. At its resonance frequency the ion, here a single proton, shortens the latter spectrum, which results in the sharp signal drop. From such spectra the cyclotron frequency can be determined [19].

- The laser cooled $^9$Be$^+$ ions act as a thermal bath, that sympathetically cools the axial mode of the $^3$He$^{2+}$ ion. In the last preparation step, the cyclotron mode of the $^3$He$^{2+}$ ion is cooled to 30 mK by coupling the axial mode and cyclotron mode at the respective lower sideband [37].
- Detection of the initial spin state: The $^3$He$^{2+}$ is transported to the analysis trap and its spin state is detected.
- Precision measurement: Then the actual precision measurement will be performed in the precision trap. To this end the cyclotron frequency of the $^3$He$^{2+}$ ion is measured while spin flips at the Larmor frequency are driven.
- Recooling: The ion is transported back to the coupling trap to re-cool the cyclotron mode.
- Detection of final spin state: In the final step the spin state is detected in the analysis trap.
a second time.

From the detected initial spin state and final spin state in the analysis trap one can conclude whether or not spin flips were driven in the homogeneous magnetic field of the precision trap during the actual precision measurement. By repeating this scheme several hundred times and with different spin-flip drive frequencies one obtains a measure of the spin-flip probability as a function of the ratio of the applied spin-flip drive frequency and the measured cyclotron frequency. From this resonance line the magnetic moment can be extracted with a precision on the order of 100 ppt as demonstrated by the measurement of the nuclear magnetic moment of the proton in [3].

6. Summary

We described a new experiment currently being constructed, which aims at a high-precision measurement of the nuclear magnetic moment of $^3$He$^{2+}$. The experiment uses methods, which rely on sympathetic laser cooling of a single isolated ion in a Penning trap and allow for an efficient high-fidelity spin-state detection. This enables a measurement of the nuclear magnetic moment with a relative precision on the parts per billion level or better. In addition, we target a measurement of the hyperfine structure in $^3$He$^+$. Here a novel double resonance method will be implemented. The method maps the readout of the nuclear spin state onto a detection of the electronic spin state using the continuous Stern-Gerlach effect, which greatly facilitates the nuclear spin-state detection. At the expense of a less precise measurement of the nuclear magnetic moment, the measurement will allow for investigations of nuclear structure effects in a light electronic system.

Acknowledgments

This work was supported by the Max-Planck Society and the RIKEN EEE Pioneering Project Funding.

References

[1] Mooser A, Ulmer S, Blaum K, Franke K, Kracke H, Leiteritz C, Quint W, Rodegheri C C, Smorra C and Walz J 2014 Nature 509 596.
[2] Smorra C, Sellner S, Borchert M J, Harrington J A, Higicji T, Nagahama H, Tanaka T, Mooser A, Schneider G, Bohman M, Blaum K, Matsuda Y, Ospelkaus C, Quint W, Walz J, Yamazaki Y and Ulmer S 2017 Nature 550 371.
[3] Schneider G, Mooser A, Bohman M, Schön N, Harrington J, Higuchi Y, Nagahama H, Sellner S, Smorra C, Blaum K, Matsuda Y, Quint W, Walz J and Ulmer S 2017 Science 358 1081.
[4] Schuessler H A, Fortson E N and Dehmelt H G 1968 Phys. Rev. 187 5.
[5] Fermi E 1930 Z. Phys. 60 320.
[6] Neronov Y I and Barzakh A E 1987 Zh. Eksp. Teor. Fiz 75 1521.
[7] Neronov Y I and Seregin N N 2014 Metrologia 51 54.
[8] Flowers J L, Petley B W and Richards M G 1993 Metrologia 30 75.
[9] Muo g-2 Collaboration (Grange et al.) 2015 arXiv:150106858.
[10] Roderick V. R. 1975 Phys. Rev A 11 403.
[11] Fei X, Hughes V W and Prigl 1997 Nucl. Instr. and Meth. A 394 349.
[12] Chupp T and Swanson S 2001 Adv. At. Mol. Opt. Phy. 45 41.
[13] Budker D 2013 Optical Magnetometry (Cambridge University Press).
[14] Rudzinski A, Puchalski M and Pachucki K 2009 J.Chem. Phys. 130 244102.
[15] Nikiel A, Blümer P, Heil W, Hohn M, Karpuk S, Maul A, Otten E, Schreiber L M and Terekhov M 2014 Eur. Phys. J. D 68 330.
[16] Bohman M, Mooser A, Schneider G, Schön N, Wiesinger M, Harrington J, Higuchi T, Nagahama H, Smorra C, Sellner S, Blaum K, Matsuda Y, Quint W, Walz J and Ulmer S 2017 J. Mod. Opt. 65 568.
[17] Smorra C, Blaum K, Bojtár L, Borchert M, Franke K, Higuchi T, Leefer N, Nagahama H, Matsuda Y, Mooser A, Niemann M, Ospelkaus C, Schneider G, Sellner S, Tanaka T, Van Gorp S, Walz J, Yamazaki Y and Ulmer S 2015 Eur. Phys. J. Spec. Top. 224 3055.
[18] Brown L S and Gabrielse G 1982 Phys. Rev. A 25 2423.
[19] Nagahama H, Schneider S, Mooser A, Smorra C, Sellner S, Harrington J, Higucji T, Borchert M, Tanaka T, Besrill M, Blaum K, Matsuda Y, Ospelkaus C, Quint W, Walz J, Yamazaki Y and Ulmer S 2016 Rev. Sci. Instr. 87 113305.
[20] Sturm S, Wagner A, Schabinger B and Blaum K 2011 Phys. Rev. Lett. 107 143003.
[21] Heifle F, Köhler-Langes F, Rau S, Hou J, Junck S, Kracke A, Mooser A, Quint W, Ulmer S, Werth G, Blaum K and Sturm S 2017 Phys. Rev. Lett. 119 033001.
[22] Breit G and Rabi I I 1931 Phys. Rev. 38 2082.
[23] Sturm S, Köhler F, Zatorski J, Wagner A, Harman Z, Werth G, Quint W, Keitel C H and Blaum K 2014 Nature 506 467.
[24] Myers E G, Wagner A, Kracke H and Wesson B A 2015 Phys. Rev. Lett. 114 013003.
[25] Zafonte S L and Van Dyck R S 2015 Metrologia 52 280.
[26] Dehmelt H 1986 Proc. Natl. Acad. Sci. 83 2291.
[27] Hanneke D, Fogwell S and Gabrielse G 2008 Phys. Rev. Lett. 100 120801.
[28] Van Dyck R S Jr., Schwingberg P B Jr. and Dehmelt H G 1987 Phys. Rev. Lett. 59 26.
[29] Wagner A, Sturm S, Köhler F, Glazov D A, Volotka A V, Plunien G, Quint W, Werth G, Shabaev V M and Blaum K 2013 Phys. Rev. Lett. 110 033003.
[30] Köhler F, Blaum K, Block M, Chenmarev S, Eliseev S, Glazov D, Goncharov M, Hou J, Kracke W, Nesterenko A, Novikov Y, Quint W, Ramirez E M, Chabaev V, Sturm S, Volotka V and Werth G 2015 Nature Comm. 7 10246.
[31] Mooser A, Kracke F, Blaum K, Bräuninger S A, Franke F, Leiteritz C, Quint W, Rodegheri C C, Ulmer S and Walz J 2013 Phys. Rev. Lett. 110 140405.
[32] Mooser A, Bräuninger, Franke K, Kracke H, Leiteritz C, Rodegheri C C, Nagahama H, Schniedr G, Smorra C, Blaum K, Matsuda Y, Quint W, Walz J, Yamazaki Y and Ulmer S 2013 Phys. Lett. B 723 78.
[33] Smorra C, Mooser A, Besirli M, Bohman M, Borchert M, Harrington J, Higuchi T, Nagahama H, Schneider G, Sellner S, Tanaka T, Blaum K, Matsuda Y, Ospelkaus C, Quint W, Walz J, Yamazaki Y and Ulmer S 2017 Phys. Rev. B 769 1.
[34] Heinzen D J and Wineland D J 1990 Phys. Rev. A 42 2977.
[35] Nagahama H, Smorra C, Sellner S, Harrington J, Higuchi T, Borchert M J, Tanaka T, Besirli M, Mooser A, Schneider G, Blaum K, Matsuda Y, Ospelkaus C, Quint W, Walz J, Yamazaki Y and Ulmer S 2017 Nat. Comm. 8 14084.
[36] Häffner H, Beier T, Djekic S, Hermanspahn N, Kluge H-J, Quint W, Stahl S, Verdu J, Valenzuela T and Werth G 2003 Eur. Phys. J. D 22 163.
[37] Cornell E, Weisskoff R, Boyce K and Pritchard D 1990 Phys. Rev. A 41 312.