Unitary decoupling treatment of a quadratic bimodal cavity quantum electrodynamics model

M Guccione¹, M A Jivulescu² and A Messina¹

¹ Dipartimento di Fisica, Università di Palermo, Via Archirafi 36, I-90123 Palermo, Italy
² Department of Mathematics, University ‘Politehnica’ of Timișoara, P-ta Victoriei Nr. 2, 300006 Timișoara, Romania

E-mail: marina.guccione@unipa.it, antonino.messina@unipa.it and maria.jivulescu@mat.upt.ro

Received 1 October 2012
Accepted for publication 22 November 2012
Published 28 March 2013
Online at stacks.iop.org/PhysScr/T153/014032

Abstract
We consider a two-photon quantum model of radiation–matter interaction between a single two-level atom and a degenerate bimodal high-\(Q\) cavity field. Within this tripartite system, the explicit construction of two collective radiation modes, one of which is freely evolving and the other one quadratically coupled to the matter subsystem, is reported. The meaning and advantages of such a decoupling treatment are carefully discussed.

PACS numbers: 42.50.Pq, 32.80.–t

1. Introduction
Investigating the physical properties of confined quantum matter–radiation systems has been, is and shall always be in fashion. This topic indeed meets the increasing demand of new more and more miniaturized devices for industrial applications which, in turn, spurs research activities on fundamental issues of quantum mechanics. Understanding the nano-world physical behavior speeds up the development of current technologies and provides radically new ideas for applicative advances. Many theoretically envisaged and experimentally realized physical situations wherein a matter–radiation system lives in a micro or nano confinement have originated successful new branches of research such as for example cavity quantum electrodynamics (CQED) [1–8], circuit quantum electrodynamics (circuit QED) [9–12], trapped atoms [13–15] and quantum dots [9, 16]. Of course, what plays the role of ‘matter’ and ‘radiation’ depends indeed on the specific scenario. In the simplest case the ‘matter’ subsystem is generally represented by few-level atoms, which might be either true flying or trapped atoms or artificial ones such as two-level quantum dots or charge or flux qubit. The radiation subsystem instead is generally represented by one or two quantized bosonic modes which belong either to a high-\(Q\) electromagnetic cavity in a typical CQED situation [1], to on-chip resonators acting as quantized cavities in the circuit QED realm [18–20] or to a photonic nanostructure embedding a single quantum dot [21]. A common aspect characterizing these exemplary four different scenarios is the achievement of effective interactions on demand resulting from the possibility of a high experimental control of relevant physical parameters. This fact paves the way for theoretical investigations and experimental realizations of appropriately tailored new regimes of matter–radiation interaction. On the basis of the previous considerations, it is not surprising that the dynamical evolution of a confined system is often based on appropriate generalizations of the seminal Jaynes–Cummings model (JCM) [22]. In such extended models the bosonic variables couple in an effective way, sometimes nonlinearly, to fermionic variables describing the internal degrees of freedom of the matter subsystem [23–25]. In this paper, we are interested in a model describing the two-photon interaction between an effective two-level atom and two quantized modes of a bimodal high-\(Q\) degenerate cavity [26] (degenerate means that its two modes involved in the processes possess the same frequency). Compared with the typical CQED setup including a single mode in a high-\(Q\) superconductive cavity, the utilization of bimodal cavities [27] enables experimental investigations on tripartite systems. The aim is to drive and reveal the appearance of quantum correlations [17, 28, 29], which might be useful also in implementing quantum information protocols [28, 30]. This explains the growing theoretical and experimental interest in such systems [20, 28, 31–38]. It is of relevance that quite recently a maximally...
entangled state of two modes of a high-\( Q \) cavity has been experimentally demonstrated \cite{27, 32}. The scope of this paper is to show that the Hamiltonian model of bimodal degenerate cavity resonantly coupled by two-photon processes to a single two-level atom is unitarily equivalent to a generalized exactly solvable quadratic JCM and a further collective quantized free evolving modes. Besides being interesting on its own, such a transformation puts at our disposal a simple conceptual tool to improve the physical interpretation of the well-known rich dynamical behavior exhibited by this tripartite CQED system \cite{39}. It has been demonstrated that the time evolution is indeed dominated by a peculiar sensitivity to the parity of the initial population assigned to the Fock state of one of the two modes, the other one and the atom starting from their respective ground states \cite{26}. Such a behavior is a genuine quantum phenomenon that has been called the parity effect \cite{39-42}. In the next sections, we report our original unitary decoupling treatment leading to a transformed Hamiltonian wherein the two-level atom is effectively (quadratically) coupled to only one of the two collective cavity modes. The analysis of dynamical properties of the original physical model associatable with the existence of such a decoupling will be reported elsewhere.

2. The Hamiltonian model

The one-photon interaction between a two-level atom and two degenerate radiation modes of a lossless high-\( Q \) cavity is described by the following Hamiltonian \cite{35}:

\[
H = \hbar \omega_0 S_z + \hbar \omega \sum_{\mu=1}^{2} \sigma_{\mu}^{+} \sigma_{\mu}^{-} + \hbar g \sum_{\mu=1}^{2} (\sigma_{\mu}^{+} S_{z\mu} + \hbar g \sigma_{\mu}^{+} S_{z\mu}).
\]  

(1)

The energy separation of the two atomic levels \(|+\rangle\) and \(|-\rangle\) is \(\hbar \omega_0\) and the pseudospin operators \(S_z\) and \(S_{z\mu}\), describing the interval degrees of freedom, are such that

\[
S_z |\pm\rangle = \pm \frac{1}{2} |\pm\rangle, \quad S_{z\mu} |\pm\rangle = \frac{1}{2} |\mp\rangle.
\]  

(2)

The \(\mu\)th radiation mode is described by Bose operators \(\sigma_{\mu}^{+}\) and \(\sigma_{\mu}^{-}\). No atom–cavity resonance condition is assumed. It is well known that the dynamical behavior of this extended JCM exhibits a rich variety of attractive quantum effects stemming from the assumption of cavity degeneracy. In particular, when at \(t = 0\), one of the two modes is coherent and the other one and the atom are in their respective ground states, the field dynamics is indeed dominated by oscillations with exchange of a large number of photons between the two modes, displaying collapses and revivals in the field populations. The physical origin of this quantum interference effect may be traced back to the circumstance that in the cavity–atom system two collective modes emerge. What is peculiar is that one of them is coupled to a two-level atom in accordance with a JCM characterized by an effective coupling constant, whereas the other one evolves freely. In recent years, much attention has been devoted to quadratic atom–high-\( Q \) cavity interaction models since it has been demonstrated that they predict remarkable quantum features in the dynamics of transparent observables easily accessible experimentally \cite{26, 41}. In the trapped ion context \cite{43-46}, the same effect is at the origin of a robust parity-dependent entanglement between the vibrational and the electronic subsystem \cite{47-50}. In this paper, we focus our attention on the Hamiltonian model first introduced in \cite{26}:

\[
H_{\text{eff}} = \hbar \omega_0 S_z + \hbar \omega \sum_{\mu=1}^{2} \alpha_{\mu}^{+} \alpha_{\mu}^{-} + \hbar s \sum_{\mu=1}^{2} \alpha_{\mu}^{+} \alpha_{\mu}^{-} + \hbar (r_1 \alpha_1^{+} \alpha_2^{-} + r_2 \alpha_1^{-} \alpha_2^{+} S_z) + \text{h.c.} \\
+ \left[ \sum_{\mu=1}^{2} \lambda_{\mu} \alpha_{\mu}^{3} + \hbar g \alpha_1 \alpha_2 S_z + \text{h.c.} \right]
\]  

(3)

describing, in the dipole approximation, the physical behavior of a single effective two-level atom coupled by two-photon processes to a bimodal, degenerate, high-\( Q \) cavity field. The two modes share the same angular frequency \(\omega\) and differ either by the polarization vector or by the direction of propagation. This Hamiltonian is an effective model reflecting all the energy exchange channels among the three constitutive subsystems under the two-photon resonance condition \(\omega_0 = 2\omega\). Thus \(g\) is the complex (in general) coupling constant relative to atomic transitions accompanied by a simultaneous gain or loss of one photon in both modes. The terms controlled by the real parameters \(r_1\) and \(r_2\) are Rayleigh terms, while the ones involving \(s\) (real) are called Stark terms. Finally, the complex, in general, parameter \(\lambda_{\mu}\) with \(\mu = 1, 2\) is associated with two-photon exchange processes between the atom and the \(\mu\)th cavity mode. The time evolution of the tripartite system represented by the nonlinear Hamiltonian \(H_{\text{eff}}\) displays the previously quoted parity effect. This phenomenon consists of an intermode net transfer of photons which turns out to be sensitive to the parity of the Fock population \(n(0)\) initially injected in one of the modes. More in detail, after a rapid reduction to 50% of \(n(0)\) and a relatively longer time interval characterized by a negligible intermode exchange of photons, a sudden inversion of the dynamical role of the two modes occurs if \(n(0)\) is even. In contrast, if \(n(0)\) is odd, the initially empty mode gives back its adsorbed 50% of the initial population to the other mode. The physical origin of this phenomenon can be traced back to quantum interference manifestations. As for the case of the linear Hamiltonian model given by equation (1) where the unitary reduction leads to a deep understanding of the origin of the quantum features dominating the dynamics of the atom–mode–mode system, we wonder whether an analogous treatment of the quadratic Hamiltonian model may contribute to a more profound insight on the parity effect and its peculiar consequences. This approach amounts to looking for the existence of a unitary operator accomplishing the canonical transformation of \(H_{\text{eff}}\) to a new fictitious system describable as a quadratic interaction between the two-level atom and a collective mode (generalized quadratic JCM) and a second free evolving collective mode. The scope of this paper is to investigate this question, providing a conditioned positive reply.
3. The canonical transformation of $H_{\text{eff}}$

To this end, we introduce the following two-parameter transformation:

$$
\tilde{\alpha}_1 = (\cos^2 \frac{\theta}{2} + e^{-i\eta} \sin^2 \frac{\theta}{2}) \alpha_1 + \frac{i}{2} \sin \theta (1 - e^{-i\eta}) \alpha_2 \\
= V^* \alpha_1 V,
$$

$$
\tilde{\alpha}_2 = \frac{1}{2} \sin \theta (e^{-i\eta} - 1) \alpha_1 - (e^{-i\eta} \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}) \alpha_2 \\
= V^* \alpha_2 V,
$$

where $V$ is the unitary operator accomplishing it, certainly existing since it is straightforward to show that whatever the real parameters $\theta$ and $\eta$ are, the given transformation expressed by equation (4) is canonical. Even if equations (4) are enough to transform $H_{\text{eff}}$, we wish however to construct $V$, and to this end we consider the following Hermitian operator:

$$
K = \Omega_1 \alpha_1^+ \alpha_1 + \Omega_2 \alpha_2^+ \alpha_2 + \lambda (\alpha_1^+ \alpha_2 + \alpha_2^+ \alpha_1),
$$

where the adimensional parameters $\Omega_1$, $\Omega_2$ and $\lambda$ are real. It is immediate to demonstrate that the unitary operator

$$
S(\theta) = \exp[\theta (\alpha_1^+ \alpha_2 - \alpha_2^+ \alpha_1)]
$$

transforms $K$ as follows:

$$
\tilde{K} := S^* K S = \Omega_1 \alpha_1^+ \alpha_1 + \Omega_2 \alpha_2^+ \alpha_2 + \lambda (\alpha_1^+ \alpha_2 + \alpha_2^+ \alpha_1),
$$

where

$$
\begin{aligned}
\tilde{\Omega}_1 &= \Omega_1 \cos^2 \theta + \Omega_2 \sin^2 \theta - \lambda \sin 2\theta, \\
\tilde{\Omega}_2 &= \Omega_1 \sin^2 \theta + \Omega_2 \cos^2 \theta + \lambda \sin 2\theta, \\
\tilde{\lambda} &= \frac{1}{2} (\Omega_1 - \Omega_2) \sin 2\theta + \lambda \cos 2\theta.
\end{aligned}
$$

It is legitimate, without loss of generality, to link $\theta$ to $\Omega_1$, $\Omega_2$ and $\lambda$ putting $\tilde{K}$ in diagonal form: $\tilde{\lambda} = 0$, that is,

$$
(\Omega_1 - \Omega_2) \sin 2\theta = -2\lambda \cos 2\theta,
$$

which may be satisfied whatever $\Omega_1$, $\Omega_2$ and $\lambda$ are. To appreciate the role of the operators $K$ and $S$ in our search for $V$, let us transform $\alpha_1$ and $\alpha_2$ using the unitary operator $e^{iK}$ [42]

$$
\beta_{\mu} = e^{-iK} \alpha_{\mu} e^{iK} = S \left( \frac{\theta}{2} \right) \left[ S^* \left( \frac{\theta}{2} \right) \alpha_{\mu} S \left( \frac{\theta}{2} \right) \right] \\
\times S^* \left( \frac{\theta}{2} \right) \alpha_{\mu} S \left( \frac{\theta}{2} \right) S^* \left( \frac{\theta}{2} \right) \alpha_{\mu} S \left( \frac{\theta}{2} \right)
$$

thus obtaining

$$
\beta_1 = (\cos^2 \frac{\theta}{2} + e^{-i\eta} \sin^2 \frac{\theta}{2}) (e^{i\Omega_1} \alpha_1) \\
+ \frac{1}{2} \sin \theta (e^{-i\eta} - 1) (e^{i\Omega_2} \alpha_1),
$$

$$
\beta_2 = \frac{1}{2} \sin \theta (e^{-i\eta} - 1) (e^{i\Omega_2} \alpha_1) \\
+ (\sin^2 \frac{\theta}{2} + e^{-i\eta} \cos^2 \frac{\theta}{2}) (e^{i\Omega_2} \alpha_2),
$$

where $\eta = \tilde{\Omega}_1 - \tilde{\Omega}_2$. Using finally the unitary operator $T = \exp[-i(\tilde{\Omega}_1 \alpha_1^+ (\tilde{\Omega}_2 + \pi) \alpha_2)]$, it is immediate to verify that

$$
\begin{aligned}
T^* \beta_1 T &= \tilde{\alpha}_1, \\
T^* \beta_2 T &= \tilde{\alpha}_2,
\end{aligned}
$$

so that in conclusion we obtain that the unitary operator $V$ realizing the canonical transformation given by equation (4) may be taken in the form

$$
V = e^{-i\alpha_1^+ \alpha_1} e^{-i\Omega_1 (\alpha_1^+ \alpha_2 + \alpha_2^+ \alpha_1)} e^{iK} \equiv V(\theta, \eta)
$$

under the condition given by equation (9). Now we concentrate on how $V$ transforms $H_{\text{eff}}$. To this end, we limit ourselves to noting that the structure of $V^* H_{\text{eff}} V$ is the same as that of $H_{\text{eff}}$ for arbitrary values of the two parameters $\eta$, $\theta$, except that all the coefficients are now also $\eta$- and $\theta$-dependent. In view of our target, we then search for conditions for putting at zero the coefficients of the operators $\alpha_1^+ \alpha_2$, $\alpha_2^+ \alpha_2$ (and of their h.c.) in the expression of $V^* H_{\text{eff}} V$. In this way we arrive at an algebraic system where we look for a compatible solution after imposing, as a nontrivial possibility, $\eta = \pi$. The resulting conditions become

$$
\begin{aligned}
\lambda_1 \sin^2 \theta + \lambda_2 \cos^2 \theta - \frac{1}{2} g \sin 2\theta &= 0, \\
(\lambda_1 - \lambda_2) \sin 2\theta + g \cos 2\theta &= 0, \\
\cos 2\theta &= 0,
\end{aligned}
$$

then necessarily requiring $\theta = \pi/4$, which in turn determines the model constraints $\lambda_1 = \lambda_2 = \Lambda = -\frac{1}{2} g$. This solution, on the one hand, is compatible with equation (9), which indeed contributes only to establishing the coefficients in $K$ and, on the other hand, reduces the canonical transformation given by equation (4) to the following one:

$$
\begin{aligned}
\tilde{\alpha}_1 &= \cos \frac{\pi}{2} \alpha_1 + \sin \frac{\pi}{2} \alpha_2, \\
\tilde{\alpha}_2 &= -\sin \frac{\pi}{2} \alpha_1 + \cos \frac{\pi}{2} \alpha_2.
\end{aligned}
$$

This transformation may be immediately accomplished by $S(\theta = \frac{\pi}{2})$ deducible from equation (6). The transformed Hamiltonian then acquires the following form:

$$
V^* H_{\text{eff}} V = h \omega_0 S_0 + [h \omega + (S - r_2) S_2 - r_1] \alpha_1^+ \alpha_1 \\
+ [2 \Lambda \alpha_1^+ \alpha_2 + \text{h.c.}] + [h \omega + (S + r_2) S_2 + r_1] \alpha_2^+ \alpha_2.
$$

The structure of $V^* H_{\text{eff}} V$ describes a fictitious quadratic one-mode–one–atom generalized JCM and a free evolving collective mode, here labeled by index 2, whose presence influences in an effective way the energy separations of the true (that is, untransformed) two-level system. Indeed, since $[\alpha_2^+ \alpha_2, V^* H_{\text{eff}} V] = 0$, we might study the dynamics fixing the Fock population of the decoupled mode. In so doing, in the particular subspace wherein $\alpha_2^+ \alpha_2$ assumes the integer non-negative value $n_2$ with certainty, the dynamics of the decoupled quadratic Jaynes–Cummings mode is determined by the following one-atom–one-mode Hamiltonian:

$$
(V^* H_{\text{eff}} V)_{\text{red}} = (h \omega_0 + r_2 (S + r_2)) S_0 + [h \omega + r_1] \alpha_2 \\
+ [2 \Lambda \alpha_1^+ \alpha_2 + \text{h.c.}] \\
+ [h \omega + (S - r_2) S_2 - r_1] \alpha_1^+ \alpha_1.
$$
which exhibits an interesting ‘double’ intensity-dependent nature. In fact, the effective atomic energy separation turns out to be $n_2$-dependent, as well as the effective energy coefficient of the coupled collective mode depends on $S_z$ [51, 52].

4. Conclusive remarks

Equations (16) and (17) constitute the results of this short paper. The structure of $(V^+ H_{\text{eff}} V)_{\text{red}}$, in particular, provides on its own an interesting starting point to investigate the physical origin of the parity effect and its consequences, without neglecting other possible as yet unexplored quantum features characterizing the time evolution determined by the Hamiltonian $H_{\text{eff}}$. For example, the property $[\alpha_2^z, V^+ H_{\text{eff}} V] = 0$ means that $H_{\text{eff}}$ has a constant of motion and the physical implications of this fact deserve to be investigated. A systematic study and possible further results will be presented elsewhere.

Acknowledgments

The authors gratefully acknowledge inspiring comments and stimulating discussions with A Napoli and R Messina. AM acknowledges support from the Italian Ministry of Research and Education under project no. PRIN2008, the responsibility of Professor M Cirillo, University Tor Vergata, Rome.

References

[1] Mabuchi H and Doherty A C 2002 Science 298 1372
[2] Haroche S and Raimond J-M 2006 Exploring the Quantum (Oxford: Oxford University Press)
[3] Walther H, Varcoe B T H, Englert B G and Becker T 2006 Rep. Prog. Phys. 69 1325
[4] Koch M, Sames C, Balbach M, Chibani H, Kubanek A, Murr K, Wilk T and Rempe G 2011 Phys. Rev. Lett. 107 023601
[5] Mcke M, Figueroa E, Bochmann J, Hahn C, Murr K, Ritter S, Villas-Bas C J and Rempe G 2010 Nature 465 755
[6] Kampschulte T, Alt W, Brakhane S, Eckstein M, Reimann R, Widera A and Meschede D 2010 Phys. Rev. Lett. 105 153603
[7] Reick S, Alt W, Eckstein M, Kampschulte T, Kong L, Reimann R, Thobe A, Widera A and Meschede D 2010 J. Opt. Soc. Am. B 27 A152
[8] Khadaverdyan M, Alt W, Kampschulte T, Reick S, Thobe A, Widera A and Meschede D 2009 Phys. Rev. Lett. 103 123006
[9] Wallraff A, Schuster D I, Blais A, Frunzio L, Girvin S M and Schmoolkopf R J 2004 Nature 431 162
[10] Chiorescu I, Bertet P, Semba K, Nakamura Y, Harmans C J P M and Mooij J E 2004 Nature 431 159
[11] Johansson J, Saito S, Meno T, Nakano H, Ueda M, Semba K and Takayanagi H 2006 Phys. Rev. Lett. 96 127006
[12] Schmoolkopf R J and Girvin S M 2008 Nature 451 664
[13] Kieplinski D, Monroe C and Wineland D J 2002 Nature 417 709
[14] Leibfried D, Meekhof D M, Monroe C, King B E, Itano W M and Wineland D J 1997 J. Mod. Opt. 44 2485
[15] Maniscalco S 2005 J. Opt. B: Quantum Semiclass. Opt. 7 R1
[16] Khitrova G, Gibbs H M, Kira M, Koch S W and Scherer A 2006 Nature Phys. 2 81
[17] Gonta D, Fritzschke S and Radtke T 2008 Phys. Rev. A 77 062312
[18] Mariantoni M, Deppe F, Marx A, Gross R, Wilhelm F K and Solano E 2008 Phys. Rev. B 78 104508
[19] You J Q and Nori F 2005 Phys. Today 58 42
[20] Sun C P, Wei L F, Liu Y and Nori F 2006 Phys. Rev. A 73 022318
[21] Gerard J M, Sermage B, Gayral B, Legrand B, Costard E and Thierry-Mieg V 1998 Phys. Rev. Lett. 81 1110
[22] Becher C, Kiraz A, Michler P, Imamoglu A, Schoenfeld W V, Petroff P M, Zhang L and Hu E 2001 Phys. Rev. B 63 121312
[23] Kiraz A, Reese C, Gayral B, Zhang L, Schoenfeld W V, Gerardot B D, Petroff P M, Hu E L and Imamoglu A 2003 J. Opt. B: Quantum Semiclass. Opt. 5 129
[24] Jaynes E T and Cummings F W 1963 Proc. IEEE 51 89
[25] Larson J 2007 Phys. Scr. 76 146
[26] Napoli A and Messina A 1996 J. Mod. Opt. 43 649–73
[27] Benivegna G, Messina A and Napoli A 1994 Phys. Lett. A 194 353
[28] Napoli A and Messina A 1997 Quantum Semiclass. Opt. 9 587
[29] Rauschenbeutel A, Bertet P, Osnaghi S, Nogues G, Brune M, Raimond J M and Haroche S 2001 Phys. Rev. A 64 050301
[30] Messina A, Maniscalco S and Napoli A 2003 J. Mod. Opt. 50 1
[31] Gonta D, Radtke T and Fritzschke S 2009 Phys. Rev. A 79 062319
[32] Dong Y, Zou X, Zhang S, Yang S, Li C and Guo G 2009 J. Mod. Opt. 56 1230
[33] Benivegna G and Messina A 1988 Phys. Rev. A 37 4747
[34] Munhoz P P and Semio F L 2010 Eur. Phys. J. D 59 509
[35] Mahmoud A-A 2009 Opt. Commun. 282 4556
[36] Larson J 2008 Phys. Rev. A 78 033833
[37] Benivegna G and Messina A 1994 J. Mod. Opt. 41 907
[38] Prado F O, Luiz F S, Villas-Bas J M, Alcalde A M, Duzzioni E I and Sanz L 2011 Phys. Rev. A 84 053839
[39] Elsayed T and Aljalal A 2011 Phys. Rev. A 83 068333
[40] Mahmoud A-A 2006 Physica A 368 119
[41] Napoli A and Messina A 1997 J. Mod. Opt. 44 2075
[42] Napoli A and Messina A 1996 J. Electron Spectrosc. Relat. Phenom. 79 319
[43] Benivegna G and Messina A 1996 Nucl. Instrum. Methods 116 465
[44] Benivegna G and Messina A 1994 J. Phys. B 27 L453
[45] Monroe C, Meekhof D M, King B E, Jefferts J R, Itano W M and Wineland D J 1995 Phys. Rev. Lett. 75 4011
[46] Wallentowitz S and Vogel W 1997 Phys. Rev. A 55 4438
[47] Steinbach J, Twamley J and Knight P L 1997 Phys. Rev. A 56 4815
[48] Leibfried D, Bratt L, Monroe C and Wineland D 2003 Rev. Mod. Phys. 75 281
[49] Maniscalco S, Messina A, Napoli A and Vitali D 2001 J. Opt. B: Quantum Semiclass. Opt. 3 308
[50] Maniscalco S, Messina A, Napoli A and Vitali D 2001 Quantum Communication, Computing and Measurement vol 3 (New York: Kluwer/Plenum) pp 419–22
[51] Maniscalco S, Messina A and Napoli A 2000 Acta Phys. Slovaca 50 333
[52] Maniscalco S, Messina A and Napoli A 2001 J. Mod. Opt. 48 2065
[53] Shore B W and Knight P 1993 J. Mod. Opt. 40 1195
[54] Naderi M H, Soltanamolkotabi M and Roknizadeh R 2005 Eur. Phys. J. D 32 397