Non-monotonous Nonparametric Variogram to Model the Land Price of Manado City with Hole Effect Periodicity Structure

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Abstract. In spatial analysis, the experimental variogram fitting generally uses a mathematical variogram model such as monotonous spherical, exponential Gaussian along with increasing distance lag. However, there are certain cases, in this case the land price in Manado which raises experimental variogram in the form of non-monotonous (hole effect) as well as sinusoidal waves. This means that experimental variogram matching must be done as well as possible based on the positive definite function or at least following the hole effect pattern at the first peak. This study offers a variogram multiplicative-additive operation to combine monotonous and non-monotonous models in a nonparametric manner. The monotonous model used is a Gaussian-type for the amplitude of the hole effect, while the non-monotonous model is the first order zero-form Bessel function to indicate the wavelength. Based on measurements of the accuracy of RMSE (Root Mean Square Error) and MAE (Mean Absolute Error), combining the two models is relatively good and successful in fitting experimental variograms of land prices that have indicators of hole effect with periodicity. The hybrid composition of these models (non monotonous and monotonous models) provides a better approach to the experimental variogram compared to the previous mathematical monoton variogram models. To model Manado's land price which has a hole effect, it would be more appropriate to use the Bessel composition model on the basis of $p = 1$ and the Gaussian-type model at $m = 1$ (or Exponential).

Keywords: Non monotonous variogram, nonparametric, Besel, Gaussian-type, hole effect, land price.

1. Introduction
Geographers, planners, decision makers, engineering experts and other parties with an interest in population issues and urban development, desperately need information about the land price to decide about the development of urban structures. Based on Tobler’s law, the price of adjacent land has a relatively similar tendency so that the principle of spatial dependence has a very important role. Several studies have been conducted to model land prices, for example research on the modeling and mapping of urban land prices in the city of Osztyn, Poland [1] and analysis of spatial autocorrelation of land prices in Hubei province, China [9], also [13] modeled land prices using monotonous variogram in the city of Bandung as well as [4] also conducted monotonous variogram analysis of land prices in BoDeTaBek. These shows that the research that has been carried out on land prices has ignored the information available on the hole effect structure available in the experimental variogram. Therefore, it is deemed necessary to develop a study of how to model land prices using point data by...
taking into account the phenomena of spatial structures that occur at observation points. The limited research on land prices with experimental variograms using non-monotonous models was felt to be very influential for the development of variogram modeling. Analysis with spatial statistics, in this case the variogram can provide a strong contribution in finding solutions to land price modeling problems.

The form of the experimental variogram will usually increase continuously when the distance also increases. So that the variogram model that can be used is a monotone model that increase, for example, the power, exponential, gaussian, spherical model, and so on[15]. However, unfortunately there are shortcomings in the Gaussian model where this model will approach the origin with zero gradients [14]. This may cause the kriging equation to be unstable and bring up to a strange effect when estimations as stated by Wackernagel. He introduced a shaped Gaussian-type model ($\gamma(h) = \sigma^2(1 - \exp(-(ah)^m))$ where $1 \leq m \leq 2$ replaces the traditional Gaussian model. The value of $m$ itself is ultimately very important to determine because it determines the suitability of a variogram model in curve fitting and interpolation. Determining the value of $m$ may vary because each data set have different optimal $m$ values.

In addition, sometimes experimental variogram forms that rise and fall (non-monotonous), so the variogram structure will form a down-hole or hole effect. The basic problem is when a researcher performs experimental variogram fitting that is non-monotonous but uses a monotonous variogram model, then results are not optimal. It will give a greater error rate if the monotonous variogram model as commonly used. When ignoring this structure it may cause a lack of important or a variogram model will not realistic in making predictions later.

Actually, the variogram model with a hole effect has been developed previously by Ma and Jones using a base sinus model and is known as the initial hole effect model [11]. However, for some experimental variogram fitting this sine model is not very suitable (there is a shift) when applied. For continuous variables, Hohn and Yao match the experimental variogram by adding cosine functions to the spherical model. The model offered provides little improvement to the sinus model[11].

In this study, we proposed a periodicity model approach to describe spatial correlations from land price data. In contrast to the approach taken by previous researchers in modeling land prices, the advantages and contributions of this study are the use of nonparametric nonmonoton variogram models to approach the characteristics of spatial structures with hole effects because of the complexity of land price data. We arrange the writing structure here: in part 2 discusses the importance of the basic concept of variogram, we give part 3 the proposed model and method, followed by the results and discussion in section 4 and ends with a conclusion in section 5.

2. Variogram

Spatial data obtained from the measurement results of a location. Spatial data comes from different spatial locations that indicate dependencies between measurement values and location. Expressed that $\{Z(s): s \in D\}$ is a spatial process for $D$ of a particular nature and $D \subset \mathbb{R}^2$, Euclidean space of the two dimensions and $s$ is the position of the location [3].

**Definition.** Stationary Process of Second Order [5]

$Z$ is a second-order stationary process in $D$ if it has a constant mean and translational-invariant covariance, $c: \forall s, t \in D: E(Z(s)) = \mu$ and $c(s, t) = Cov(Z(s), Z(t)) = C(t - s)$.

For second-order stationary processes the variogram and covariance are equivalent, applicable:

$$2\gamma(h) = \text{var}\left[\{Z(s + h)\} - Z(s)\right]$$

$$= \text{cov}\left[Z(s) - (Z(s + h)), Z(s) - (Z(s + h))\right]$$

$$= C(s, s) + C(s + h, s + h) - 2C(s, s + h)$$

(1)

when there is a weak stationary process it can be simplified as follows [8]:

$$\gamma(h) = C(0) - C(h) \text{ or } C(h) = C(0) - \gamma(h)$$

(2)

$$\gamma(h) = C(0)[1 - \rho(h)]$$

(3)
The classic estimator for variogram proposed by Matheron using the moment method as follows [10][11]:

\[ 2\hat{\gamma}(h) = \frac{1}{|N(h)|} \sum_{(i,j) \in N(h)} (Z(s_i) - Z(s_j))^2 \]  
\[ (4) \]

where \( N(h) \equiv \{(i,j): s_i - s_j = h\} \) and \(|N(h)|\) is the number of finite elements of \( N(h) \).

2.1. Variofilm with Hole Effect

Sometimes, the variogram can behave non-monotonically compared to monotonous. When the variogram reaches the maximum point for the first time, the variogram can decrease to the minimum (local) then rise again to the maximum. This phenomenon in variogram is called “hole effect” [11][16]. But unfortunately, this phenomenon is ignored and considered as a nuisance, because in the mining industry the variogram is expected to behave continuously. Variofilms with hole effects that regularly periodic converging on sill are called decreases (dampened) and variofilms that are not convergent but are undampened continuously. Experimental variogram modeling is useful because the results in the form of a smooth curve fitting produce estimators at certain lags. At present geostatistics practitioners are very subjective in choosing a variogram model using empirical guidelines [7].

2.2. Parametric Variofilm

Parametric variofilms are monotonically up. Wackernagel defines a Gaussian type model in the form of a variogram [14]:

\[ \gamma(h) = \sigma^2 \left( 1 - \exp\left(- (ah)^m \right) \right) \]  
\[ (5) \]

where a is a parameter to determine the weights of neighbors and \( 1 \leq m \leq 2 \). When \( m \to 1 \) (exponential), the model will be more linear when small lags and spatial correlations between adjacent points will decrease. When \( m \to 2 \) (gaussian), the model will approach the center with a more quadratic shape and more towards the sigmoidal, and spatial correlation between adjacent points is higher.

2.3. Nonparametric Variofilm

Valid variofilms must fulfill definite positive assumptions, and the idea behind the emergence of nonparametric estimators for covariofilms is Bochner’s theorem which gives a spectral representation of positive definite function/pdf. In particular, a covariance function \( C(h) \) is pdf if and only if it fulfills the form [6]:

\[ C(h) = \int_{R^d} \cos(h^T x) dF(x) \]  
\[ (6) \]

where \( F \) is symmetrical and is measured positively on \( \mathbb{R}^d \).

If \( C(h) \) is isotropic, Bochner’s theorem can be written as follows [7][12]:

\[ C(h) = \int_0^\infty \Omega_d(th) F(dt) \]  
\[ (7) \]

where \( \Omega_d(x) = \left( \frac{2}{\pi} \right)^\frac{d-2}{2} \Gamma \left( \frac{d}{2} \right) j_{d/2}(x) \) is the basis for functions in \( \mathbb{R}^d \).

One of the reasons for choosing the Bessel model is to be used to describe the function of connectedness because it can describe the purpose of the structure of the hole effect on the variogram function [12]. The covariance functions of the Bessel model are stated as follows [16]:

\[ C(h) = 2^\nu \Gamma \left( \frac{d}{2} \right) (bh)^{-\nu} j_\nu(bh) \]  
\[ (8) \]

where \( b \) is the number of changes in the sign \( b=2,3,4,5,... \), \( d \) is the dimension of the data, \( \nu = \left( \frac{d}{2} \right) - 1 \), and \( j_\nu(\cdot) \) is the first form of Bessel function from order to \( \nu \). When \( d = l \), then \( C(h) = \cos(bh) \), when \( d = 2 \), then \( C(h) = j_0(bh) \); when \( d = 3 \) then \( C(h) = \sin(bh) / bh \), when \( d \to \infty \), then \( C(h) = \exp\left(- (bh)^2 \right) \) [7][11]. The periodicity function has a weak hole effect structure when \( d \) increases and becomes a Gaussian function when \( d \to \infty \).
For two dimensional random field \((d = 2)\), when \(J_0(\cdot)\) which is the first form Bessel function with order \(\nu\) and \(\nu = \left(\frac{d}{2}\right) - 1\), then the idea becomes \(\nu = \left(\frac{d}{2}\right) - 1 = 0\). Therefore \(C(h) = J_0(\nu h)\) which can be expressed as: \[C(h) = J_0(\nu h) = \sum_{k=0}^{\infty} \left(\frac{(-1)^k}{k!}\right)^2 \nu^{2k} h^{2k}\] \((9)\)

### 3. Proposed Model

#### 3.1. Model Approach.

To increase flexibility when modeling a variogram, we use linear combinations of valid covariant functions. The treatment of valid covariance functions is to consider it as a base function to produce a nonparametric model fitting approach. The covariant nonparametric model is defined:

\[
\gamma(h) = \sigma^2 \left\{1 - \sum_{k=1}^{p} \alpha_k J_0(k \nu h)\right\}
\] \((10)\)

where \(p\) is the number of basic functions, while \(\alpha_k\) is weight. Theoretically the more base functions are used, the fitting gets better. However, in practice the calculations will be very complex for large datasets to increase the number of base functions. Also, when the base function size increases and fitting gets better, the model obtained is less general and difficult to interpret. When the Bessel function \(J_0(\cdot)\) is used as a base function, it is recommended that the size \(p\) should be at most five. As a result the variogram model will be as follows:

\[
\gamma(h) = \sigma^2 + \sigma^2 \left\{1 - \sum_{k=1}^{p} \alpha_k J_0\left(\frac{k \nu h}{\omega}\right)\right\}
\] \((11)\)

where \(\sigma^2\) is the effect nugget and \(\sigma^2\) is sill and the weight \(0 \leq \alpha_1 \leq 1\) and \(\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4 > \alpha_5\), such that \(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 1\) or \(\sum_{k=1}^{p} \alpha_k = 1\). Therefore the order of the \(b\) order that describes the sign of a curve change in the structure of the effect of the hole is very important. The basic composition with the covariance function \(C(h)\) can be carried out as long as the function \(C(h)\) satisfies the symmetrical and measured positively in \(\mathbb{R}^d\).

#### 3.2. Variogram Model of Additive-Multiplication Composition

We will construct a general model based on multiplication-addition principle of Bessel functions for \((p \leq 5)\) and Gaussian-type models. The chosen of Gaussian-type model \((1 \leq m \leq 2)\) because it is an improvement from the traditional Gaussian model. The choice of \(m\) value is very influential in the model’s development through fitting experimental variogram curves. In general, equation (5) and (11) can be written to the \(n\)th term as follows:

\[
\gamma_n(h) = \left[1 - \sum_{k=1}^{n} \alpha_k J_0\left(\frac{k \nu h}{\omega}\right)\right] \left[1 - \exp\left(-\frac{h}{\nu}\right)^m\right]
\] \((12)\)

The use of a Gaussian-type base function aims to refine the amplitude of the curve made. Giving weight is very influential in the formation of the variogram model. The weight will decrease to half of the previous weight where \(\sum_{k=2}^{n} \alpha_k \to 1\) and \(\alpha_n = \frac{1}{2^{n-1}}\alpha_1 = \frac{1}{2} \alpha_{n-1}\).

#### 3.3. Variogram Fitting Method

A very challenging problem is to make fitting that really matches the empirical variogram because it is very difficult to match all peaks and valleys using the variogram model. In doing variogram fitting there are curve fitting procedures that involve fitting through visual observation and statistical fitting which are summarized in the following three steps [15]:

i. ignore point-to-point fluctuations and concentrate on trends in general;

ii. variogram estimation will be accurate in short lags and low accuracy in long lags.

iii. match the periodicity pattern of the variogram at least at the first or overall peak.

We then apply equation (12) to do a non-monotonous variogram curve fitting, following these two aspects [11]:

i. Variograms with hole effect describe periodicity. A variogram model must meet at least the first peak where the traditional (monotonous) variogram model cannot fulfill.
ii. Variogram with hole effect must converge to sill. Some cases that rarely occur in the variogram hole are perfect effects which the variogram does not converge to sill.

4. Results and Discussion

In this study, we use processed data obtained from BP2RB (Regional Tax and Retribution Management Agency) of Manado which is listed in the Source of Taxpayers’ Association according to the book category of the Manado municipality for 2018. In this processed data with attributes consisting on behalf of taxpayers, the object tax number, the taxpayer’s address, the object of both land and building tax, each of which is divided into the object area, object class and NJOP (Nilai Jual Objek Pajak), as well as property tax provisions.

![Image: Administrative map of Manado city and 150 observation points. The red color indicates the high price of the land while the bright color states the low land price as stated in the legend.](image)

There are 150 spatial locations were selected to be used in this study (Figure 1). The data has 1 dependent variable such as land price and 4 independent variables such as the closeness distance between the observation points and the location of the office, the location of the college, the location of the police station and the location of the hospital. Legend on the map illustrates the land price in rupiahs, where the red color indicates the high land price (30000-350000) while the bright color states the land price is low (0-500000).

4.1. Experimental Variogram

Determination of sampled data pairs (bin) in calculating experimental variogram can use the combination form C(n, 2) where n is the amount of data. From Table 1 it is shown that observation observation points totaling 150 sample points, then with C(n, 2) or C(150, 2) a number of 22350 sample pairs in the variogram cloud can form an experimental variogram.

In Figure 2(a) it will be very difficult to determine a suitable theoretical variogram model, so that calculations are needed using experimental variogram to obtain a plot of the average value of the varogram, \( \gamma(h) \), to distance \( h \). We get the experimental variogram using equation (4) about 21 classes as the maximum lag distance, where each class has several data pairs and the distance between each data (Table 1). The size of the number of classes is chosen such that the number of sample pairs for each class is not less than 30 sample pairs. The experimental variogram gamma value is plotted against distance, \( h \), as shown in Figure 2(b). The experimental variogram curve for land prices in Manado city shows a periodicity hole effect that needs to be considered so it requires a suitable non-monotone variogram model. There are two minimum locales which are at a distance of 1120.5375 (class 3) and distance 6505.1751 (class 15), and the local maximum at a distance of 3363.3598 (class...
8). This maximum and minimum local presence shows a hole effect, so the curve gradient can change from negative to positive and vice versa.

### Table 1. Number of classes, pairs of data, gamma variogram values and lag distances.

| Class | np  | h     | γ    | h width |
|-------|-----|-------|------|---------|
| 1     | 10  | 396   | 273.3662 | 1.2589 | 0       |
| 2     | 10  | 691   | 679.0334 | 1.1929 | 405.6672 |
| 3     | 10  | 787   | 1120.5375 | 1.1218 | 441.5041 |
| 4     | 10  | 809   | 1577.6771 | 1.2154 | 457.1396 |
| 5     | 10  | 833   | 2023.0232 | 1.3689 | 445.3461 |
| 6     | 10  | 833   | 2479.5024 | 1.3830 | 456.4792 |
| 7     | 10  | 968   | 2920.5116 | 1.6224 | 441.0092 |
| 8     | 10  | 921   | 3363.3598 | 1.7494 | 442.8482 |
| 9     | 10  | 711   | 3821.2396 | 1.5642 | 457.8798 |
| 10    | 10  | 643   | 273.3662 | 1.5515 | 442.7622 |
| 11    | 10  | 569   | 4729.8568 | 1.7496 | 442.7622 |

The selection of the theoretical variogram model is appropriate by fitting the theoretical variogram model under the experimental variogram. Non-monotone variograms with periodicity will converge to a constant when lag increases, or is called sill. The oscillation that occurs can cause the curve to go through sill for several times.

![Cloud Variogram](image1.png)

**Figure 2.** Cloud Variogram and Experimental Variogram for land price with n-pairs which has a hole effect.

### 4.2. Variogram Modelling

The fitting of the experimental variogram curve was carried out based on the model developed. It starts from the Bessel function model with one (base) to five bases which is composed of the Gaussian-type model proposed by Wackernagel (2003) to correct the instability possessed by traditional Gaussian models. The selection of \( m \) values in the Gaussian-type model is very influential in variogram modeling. This Bessel function is then multiplied by the Gaussian-type model with \( 1 \leq m \leq 2 \), to see the value of \( m \) corresponding to the Gaussian-type model. This is intended to see the compatibility of the model curve with the experimental variogram.
First, consider matching using the monotonous variogram model as in Figure 3. Whereas for monotonous mathematical variogram curves it is seen that it cannot follow the movements of experimental variograms in the form of hole effects. As a result there are several experimental variogram points which are far from the exponential and gaussian variogram curves which may cause greater errors.

![Experimental variogram fitting using Monoton Variogram (Exponential and Gaussian).](image)

Figure 3. Experimental variogram fitting using Monoton Variogram (Exponential and Gaussian).

Figure 4 show that the experimental variogram model at 21 lag observation uses a model with 1 base, 2 bases, 3 bases and 4 bases. The four models proposed appear to follow relatively good experimental curves that form hole-effects. The nonparametric model approach provides a curve approach that approaches the experimental curve of the variogram best suited for each base. It can be seen that the proposed periodical model can follow an empirical variogram that has a very well-formed hole effect compared to the two monotonically rising parametric models (Figure 4). Hybrid variogram shows the periodicity when the lag increases, indicating the existence of a functional relationship between each observation point location. This clearly shows that the structure of hole effects can be used to describe data that has periodicity where data like this usually has outliers.

To see the comparison of curve fitting between monotonous variogram models with nonparametric nonmonoton variogram models presented in Figure 5. It is very clear that the model of a combination of non-monotonous and monotonous functions can follow an experimental variogram better than just using a monotonous Gaussian-type function or just a Bessel function.

The appropriate variogram model is obtained by calculating the number of squared errors at each lag. The magnitude of the error is obtained from the difference in experimental variogram values with theoretical variogram values in each lag. The model with the smallest RMSE value is the model that fits and is closest to the experimental variogram. The approach taken is to minimize the distance between true values (experimental variogram) to the value of the approach (hybrid variogram) based on the distance from the binning process.

To validate which model is suitable for modeling land prices presented in Table 2. The RMSE value is always greater than the MAE value. This causes the selection of the model to use basis will be different, where the smallest RMSE value is in the model with base 1 for all values of m (1; 1.5; 2) while the smallest MAE value is on the third base for all values of m (1; 1.5; 2), respectively. For reasons of sensitivity to outliers, an error assessment to compare this model is used RMSE assessment rather than MAE.
Figure 4. Experimental variogram fitting models using Non-Monotone Variogram on the basis of $p = 1$ (a), $p = 2$ (b), $p = 3$ (c) and $p = 4$ (d) for $m = 1$.

The RMSE value given by the Bessel base 1 model which is composed with Gaussian $m = 1$, is smaller than the mathematical variogram model $\gamma(h) = \sigma^2\{1 - \exp(-(ah)^m)\}$ that is Exponential (at $m = 1$) of 0.2522 and Gaussian (at $m = 2$) of 0.2988 or natural Bessel which about 0.3446, indicating that the two Bessel Gaussian-type composition models are still better. This means that the composition model (RMSE = 0.2170) given is relatively good in approaching the empirical variogram by considering the RMSE value.

These results contradict what is theoretically illustrated about the use of a large number of Bessel function terms to estimate experimental variograms. The use of 1 term based on the RMSE measurement indicator is considered sufficient to model the land price. Even though theoretically the more terms forms are used in Bessel functions, the better the model is used to estimate.
Figure 5. Experimental variogram models fitting using 5 non-parametric variogram models with Bessel-Gaussian-type $m=1$ (a) and $m=1.5$ (b).

Table 2. Accuracy measurement of Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) for Hybrid model for $p < 5$ and Gaussian-type $1 \leq m \leq 2$.

| model         | RMSE  | MAE  |
|---------------|-------|------|
| basis 1       | 0.2170| 0.1625|
| basis 2       | 0.2276| 0.1730|
| $m=1$         | basis 3  | 0.2234| 0.1693|
|               | basis 4  | 0.2236| 0.1702|
|               | basis 5  | 0.2235| 0.1707|
|               | basis 1  | 0.2375| 0.1776|
|               | basis 2  | 0.2469| 0.1822|
| $m=1.5$       | basis 3  | 0.2423| 0.1793|
|               | basis 4  | 0.2424| 0.1806|
|               | basis 5  | 0.2423| 0.1807|
|               | basis 1  | 0.2501| 0.1895|
|               | basis 2  | 0.2588| 0.1916|
| $m=2$         | basis 3  | 0.2543| 0.1881|
|               | basis 4  | 0.2543| 0.1893|
|               | basis 5  | 0.2543| 0.1895|
| Exponential   | 0.2522| 0.1957|
| Gaussian      | 0.2988| 0.2424|
| Natural Bessel (p=5) | 0.3446| 0.3037|

5. Conclusion
Sometimes experimental variogram fitting, usually using a monotonous variogram model. However, in the experimental variogram that has the characteristics of the hole effect periodicity, the monotonous variogram model is not suitable to be used so it is necessary to use a non-monoton variogram. Therefore, in this study, which has the characteristics of the hole effect, it is used to combine non-parametric and non-monotonous variogram models. There are two functions that are used, such as the
Bessel function represents a nonmonotonous and a Gaussian-type function that represents a monotonous nature. Both of these functions are definitively positive. The alternative offered in this study is to develop a multiplication-composition model, so that the first peak of the experimental variogram can be followed properly.

Most variogram models have a hole effect structure that is rapidly dampened from strong to weak periodicity with increasing lag \((h)\). However, in the case of Manado city land prices, the variogram model looks slow to decline so it tends to be undampened. So that the use of Bessel and Gaussian-type functions separately is very difficult to use to follow the hole effect pattern. Information on spatial variability that has been neglected, can finally be explored relatively well when using non-parametric variograms in a nonparametric manner in fitting the model curve.

Based on the MAE and RMSE error assessment, modeling of Manado city land prices that have a hole effect is more appropriate using the Bessel composition model on the basis of \(p = 1\) and the Gaussian-type model at \(m = 1\) (exponential). This form is also related to computational effective and efficiency. The composition of these models (nonmonotonous and monotonous models) provides a better approach to the experimental variogram rather than the other mathematical variogram models.

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