Non-relativistic M-Theory solutions based on Kähler-Einstein spaces

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Abstract: We present new families of non-supersymmetric solutions of $D = 11$ supergravity with non-relativistic symmetry, based on six-dimensional Kähler-Einstein manifolds. In constructing these solutions, we make use of a consistent reduction to a five-dimensional gravity theory coupled to a massive scalar and vector field. This theory admits a non-relativistic CFT dual with dynamical exponent $z = 4$, which may be uplifted to $D = 11$ supergravity. Finally, we generalise this solution and find new solutions with various $z$, including $z = 2$.

Keywords: AdS-CFT Correspondence, Supergravity Models, M-Theory
1 Introduction

Over the past year, non-relativistic conformal (NRC) field theories have attracted a lot of attention, primarily driven by the prospect of tailoring the AdS/CFT correspondence so that it may be used as a tool to describe condensed matter systems in a laboratory environment. These systems are described by Schrödinger symmetry, which is a non-relativistic version of conformal symmetry. The corresponding algebra is generated by Galilean transformations, an anisotropic scaling of space, $x \rightarrow \lambda x$, and time, $x^+ \rightarrow \lambda^z x^+$, where $z > 0$ is a real number usually referred to as the dynamical exponent, and an additional special conformal transformation when $z = 2$. For NRC field theories with one time and $d$ spatial dimensions, the corresponding symmetry algebra will be denoted $\text{Sch}_z(1, d)$.

Gravity duals for NRC field theories were initially proposed in [1, 2] and were subsequently embedded in type IIB in [3–5] and $D = 11$ supergravity in [6]. The IIB solutions of [3–5] with $z = 2$ are obtained by coordinate transformations which deform the three-form flux, but in the process break supersymmetry. Other techniques that have been employed in the construction of NRC gravity duals in type IIB and $D = 11$ supergravity include metric deformations [7] and uplift of suitable solutions of the lower dimensional theories to which the $D = 10, 11$ supergravities on Sasaki-Einstein manifolds consistently truncate [5, 6]. Some solutions obtained by these two methods do preserve supersymmetry [7, 8]. Solutions pursued via uplift turn out to permit only set dynamical exponents, whereas more general constructions, still based on Sasaki-Einstein spaces [8–10], allow for richer classes of solutions with many different values of $z$, including $z = 2$. For a selection of other works on gravity duals of NRC field theories in various dimensions, both supersymmetric and non-supersymmetric, see [11].

In all these cases, the $D = 10$ or $D = 11$ metric dual to an NRC field theory in spatial dimension $d$ corresponds to a deformation of a given $D$-dimensional solution containing...
(d + 3)-dimensional Anti-de Sitter space, that breaks the original \( AdS_{d+3} \) isometry \( so(2, d + 2) \) down to its \( \text{Sch}_2(1, d) \) subalgebra. The purpose of this paper is to obtain \( D = 11 \) supergravity solutions with \( \text{Sch}_2(1, 2) \) symmetry, associated to the \( AdS_5 \times KE_6 \) class of \( D = 11 \) supergravity solutions with \( KE_6 \) a six-dimensional Kähler-Einstein space of positive curvature \[12, 13\]. Interestingly enough, despite the lack of supersymmetry of the general \( AdS_5 \times KE_6 \) solution\(^1\) for arbitrary \( KE_6 \), the special case when \( KE_6 \) is \( CP^3 \) has recently been shown to be classically stable \[15\]. We expect our \( \text{Sch}_2(1, 2) \)-invariant solutions, dual to NRC field theories in spatial dimension \( d = 2 \), to inherit the non-supersymmetric character of the original \( AdS_5 \times KE_6 \) solutions.

As mentioned earlier, the first examples of gravitational solutions dual to NRC field theories were found in lower-dimensional theories of gravity coupled to a massive vector field \[1\]. One benefit of much recent work on consistent Kaluza-Klein (KK) truncations \[16–18\] is that these solutions may be uplifted to type IIB \[5\] and \( D = 11 \) supergravity settings \[6\]. In a similar fashion, we will first show, in section 2, that there exists a consistent KK truncation of \( D = 11 \) supergravity on \( KE_6 \) to a \( D = 5 \) theory involving a massive vector and a massive scalar. We subsequently uplift, in section 3, a solution to the \( D = 5 \) theory to eleven-dimensions to find a new M-Theory solution with dynamical exponent \( z = 4 \). In section 4 we perform a generalisation to a class of NRC solutions obtained as deformations of the original \( AdS_5 \times KE_6 \) solution that, in general, cannot be obtained from uplift. In this class, we will find new \( \text{Sch}_z(1, 2) \)-invariant M-Theory solutions with different dynamical exponents \( z \), including \( z = 2 \). Like the analog constructions in \[7–10\], the metric of all these solutions will maintain the \( KE_6 \) part of the original \( AdS_5 \times KE_6 \). Further generalisations should be possible allowing for more general internal geometries \[19\].

The \( AdS_5 \times KE_6 \) geometries that we take as starting point for our analysis are solutions to the equations of motion of \( D = 11 \) supergravity,

\[
\begin{align*}
    dG_4 &= 0 , \\
    d *_{11} G_4 &+ \frac{1}{2} G_4 \wedge G_4 = 0 , \\
    R_{AB} &= \frac{1}{12} G_{AC_1C_2C_3}G_B^{C_1C_2C_3} - \frac{1}{144} g_{AB} G_{C_1C_2C_3C_4} G^{C_1C_2C_3C_4} = 0 ,
\end{align*}
\]

with metric and four-form given, respectively, by

\[
\begin{align*}
    ds_{11}^2 &= ds^2(AdS_5) + ds^2(KE_6) , \\
    G_4 &= cJ \wedge J .
\end{align*}
\]

Here, \( c \) is a constant, \( J \) is the Kähler form on \( KE_6 \), and the metrics \( g_{\mu\nu} \) and \( g_{mn} \) for \( AdS_5 \) and \( KE_6 \), respectively, are normalised so that their with Ricci tensors are

\[
R_{\mu\nu} = -2c^2 g_{\mu\nu}, \quad R_{mn} = 2c^2 g_{mn} .
\]

**Note.** While we were in the process of completing this paper, \[20\] appeared which, although supersymmetric in the main, section 5 therein has some overlap with our analysis.

\(^1\)See \[14\] for the classification of the supersymmetric M-Theory solutions containing \( AdS_5 \).
2 Consistent truncation of $D = 11$ supergravity on $KE_6$

For every general supersymmetric solution $AdS_n \times_w M_{D-n}$, where $\times_w$ denotes warped product, of a $D$-dimensional supergravity theory, there exists a consistent truncation of the $D$-dimensional theory down to a suitable $n$-dimensional pure, massless gauged supergravity [16–18]. For supersymmetric Freund-Rubin backgrounds, the massive supermultiplet containing the breathing mode of the internal space $M_{D-n}$ can also be retained consistently, together with the supergravity multiplet [6]. In all these cases, the $G$-structure on $M_{D-n}$ specified by supersymmetry plays a crucial role in constructing the KK ansatz which describes the embedding of the retained $n$-dimensional fields into the $D$-dimensional ones. In the case at hand here, despite the lack of supersymmetry of the $AdS_5 \times KE_6$ background (1.4), (1.5), the Kähler form $J$ of $KE_6$ will still allow us to build a KK ansatz that consistently includes massive modes, along the lines of [6].

At any rate, there is an argument about which modes one should expect to be able to keep in the truncation of $D = 11$ supergravity on $KE_6$. Consider first the particular case when the internal $KE_6$ is $CP^3$, which has isometry group SU(4), and for which the KK spectrum is explicitly known [15]. Following [21], one should be able to truncate consistently the KK tower of $D = 11$ supergravity on $CP^3$ to its SU(4) singlet sector. This contains the massless graviton, one massive real scalar and one massive real vector [15], both with mass $12c^2$. Now, it is precisely the singlet character of these modes under the relevant SU(4) symmetry of the particular $KE_6 = CP^3$ that makes them expected to be universal for all $KE_6$ spaces. We can thus predict a consistent truncation of $D = 11$ supergravity on any $KE_6$ to a $D = 5$ theory with the field content quoted above. In particular, no massless vector that could enter the $D = 5 N = 2$ supergravity multiplet along with the metric should be expected to survive the truncation, so the resulting $D = 5$ theory should not correspond to a supergravity.\footnote{This is to be constrained with the analog situation for skew-whiffed Freund-Rubin backgrounds: in spite of also breaking all supersymmetry, they do allow for a consistent truncation to a supergravity theory [6].}

Without much further ado, consider the following KK ansatz

$$ds^2_{11} = ds^2_5 + e^{2U} ds^2(KE_6),$$
$$G_4 = H_4 + H_2 \wedge J + cJ \wedge J,$$  \(2.1, 2.2\)

where $U$, $H_4$ and $H_2$ are, respectively, a scalar (the breathing mode of the internal $KE_6$), a four-form and a two-form on the external five-dimensional spacetime, with line element $ds^2_5$, and $J$ is again the Kähler form on $KE_6$. By choosing the coefficient in the $J \wedge J$ term to be the same constant $c$ that appears in the background flux (1.5) we are anticipating that this coefficient cannot be turned into a dynamical $D = 5$ field without violating the $D = 11$ Bianchi identity for $G_4$. Also, one could have tried to add to the KK ansatz (2.2) terms involving the holomorphic $(3,0)$-form $\Omega$ defining the complex structure on $KE_6$, but it is unclear how to deal with those terms when plugging the ansatz into the $D = 11$ equations of motion.

The KK ansatz (2.1), (2.2) reduces to the background solution (1.4), (1.5) for $U = H_4 = H_2 = 0$, $ds^2_5 = ds^2(AdS_5)$. More generally, direct substitution of (2.1), (2.2) into (1.1)–(1.3)
shows that the KK ansatz also solves the $D = 11$ supergravity field equations provided the $D = 5$ fields satisfy

\begin{align}
    dH_4 &= 0, \quad (2.3) \\
    dH_2 &= 0, \quad (2.4) \\
    d(e^{6U} \ast H_4) + 6cH_2 &= 0, \quad (2.5) \\
    d(e^{2U} \ast H_2) + 2cH_4 + H_2 \wedge H_2 &= 0, \quad (2.6) \\
    d(e^{6U} \ast dU) + 2c^2(e^{-2U} - e^{4U})\text{vol}_5 - \frac{1}{6}e^{6U}H_4 \wedge \ast H_4 &= 0, \quad (2.7)
\end{align}

\[
R_{\alpha\beta} = -2c^2e^{-8U}\eta_{\alpha\beta} + 6(\nabla_\beta \nabla_\alpha U + \partial_\alpha U \partial_\beta U) + \frac{3}{2}e^{-4U}\left(H_{\alpha\lambda}H_{\beta}^{\lambda} - \frac{1}{6}\eta_{\alpha\beta}H_{\lambda\mu\rho}H^{\lambda\mu\rho}\right).
\]

All the dependence on the internal $KE_6$ drops out, leaving fully-fledged $D = 5$ equations of motion for the $D = 5$ fields. This shows the consistency of the truncation.

We can now introduce the Lagrangian of the $D = 5$ theory and work out the masses of the various fields. First of all, the Bianchi identities (2.3), (2.4) for $H_4$ and $H_2$ can be trivially solved by introducing a three-form and a one-form potential such that

\begin{align}
    H_4 &= dB_3, \quad (2.9) \\
    H_2 &= dB_1. \quad (2.10)
\end{align}

The Lagrangian that gives rise to the $D = 5$ equations of motion (2.5)-(2.8) upon variation of $B_3, B_1, U$ and the $D = 5$ metric $g_{\mu\nu}$ can then be worked out. It reads

\[
\mathcal{L} = e^{6U}R\text{vol}_5 + 30e^{6U}dU \wedge \ast dU - \frac{1}{2}e^{6U}H_4 \wedge \ast H_4 - \frac{3}{2}e^{2U}H_2 \wedge \ast H_2 \\
+ 6c^2(2e^{4U} - e^{-2U})\text{vol}_5 - B_1 \wedge (6cH_4 + H_2 \wedge H_2), \quad (2.11)
\]

or, in terms of the Einstein frame metric $\bar{g}_{\mu\nu} = e^{4U}g_{\mu\nu},$

\[
\mathcal{L}_{\text{Einstein}} = \bar{R}\text{vol}_5 - 18dU \wedge \bar{\ast} dU - \frac{1}{2}e^{12U}H_4 \wedge \bar{\ast} H_4 - \frac{3}{2}H_2 \wedge \bar{\ast} H_2 \\
+ 6c^2(2e^{-6U} - e^{-12U})\text{vol}_5 - B_1 \wedge (6cH_4 + H_2 \wedge H_2), \quad (2.12)
\]

with barred quantities referring to the Einstein frame metric.

It is useful to dualise $B_3$ into a scalar $B$. In order to do this, define $H_5 = dH_4$ and add the piece

\[
\mathcal{L}' = -BH_5 \quad (2.13)
\]

to the Lagrangian (2.12). Integrating out $H_4$ we find that it is now given as

\[
H_4 = -e^{-12U}\ast H_1, \quad (2.14)
\]
where we have found it convenient to define
\[ H_1 = dB - 6cB_1 . \]  

Substituting this back into \( \mathcal{L}_{\text{Einstein}} + \mathcal{L}' \) we find the dual Lagrangian
\[ \mathcal{L}_{\text{dual}} = \bar{R} \text{vol}_5 - 18dU \wedge \bar{d}U - \frac{1}{2} e^{-12U} H_1 \wedge \bar{\star}H_1 + \frac{3}{2} H_2 \wedge \bar{\star}H_2 \]

\[ + 6c^2 \left( 2e^{-6U} - e^{-12U} \right) \text{vol}_5 - B_1 \wedge H_2 \wedge H_2 . \]  

The masses of the \( D = 5 \) fields can now be computed by expanding the Lagrangian (2.16) about the \( \text{AdS}_5 \) vacuum, keeping up to quadratic terms. Doing this, for \( U \) and \( B_1 \) we find
\[ m_U^2 = m_{B_1}^2 = 12c^2 , \]  

while \( B \) (the scalar dual to \( B_3 \)) is just a St"uckelberg field that can be gauged away to give \( B_1 \) its mass. As anticipated, the \( D = 5 \) theory obtained upon consistent KK truncation of \( D = 11 \) supergravity on \( KE_6 \), and described by the Lagrangian (2.12) or (2.16), contains the \( D = 5 \) metric, one massive scalar and one massive vector with mass (2.17). When \( KE_6 = CP^3 \), the SU(4)-neutrality (table 2 of [15]) and the masses (tables 3 and 4 of [15]) of \( U \) and \( B_1 \) show that these are the modes in the \( k = 0 \) level of the \( (k+3)(k+4)c^2 \) towers of real scalars and real one-forms, respectively.

We are interested in solutions to the \( D = 5 \) field equations (2.3)–(2.8) displaying NRC symmetry. Rather than working with the full theory, we will consider a suitable further truncation. There are three further consistent truncations, apparently no longer explained by a group theory argument as the one above. The first is obtained by setting \( H_4 = H_2 = 0 \), leaving only the five-dimensional metric and the breathing mode \( U \). The second, leading to five-dimensional General Relativity with a cosmological constant, is trivially obtained by insisting on \( H_4 = H_2 = 0 \) and further setting \( U = 0 \). The third, which is the one we are interested in, will be described in the next section.

3 NRC solutions from uplift

It is consistent with the \( D = 5 \) equations of motion to set \( H_4 = 6c e^{-6U} \ast B_1 \), where the Hodge dual here refers again to the metric appearing in the Lagrangian (2.11), and \( B_1 \) is defined in (2.10). Rather than a further truncation, this just corresponds to gauging away \( B_3 \) or, alternatively, the St"uckelberg scalar \( B \), as can be seen from equations (2.14), (2.15). The third possible further truncation referred to above is obtained (having gauged away \( B_3 \)) by further setting \( U = 0 \) (and, thus, \( H_4 = 6c \ast B_1 \)) while restricting \( B_1 \) to light-like configurations,
\[ B_1 \wedge \ast B_1 = 0 , \quad H_2 \wedge H_2 = 0 . \]  

In this case, the equations of motion (2.5)–(2.8) reduce to (3.1) together with
\[ d \ast H_2 + 12c^2 \ast B_1 = 0 , \]  

\[ R_{\alpha\beta} = -2c^2 \eta_{\alpha\beta} + \frac{3}{2} H_\alpha H_\beta + 18c^2 B_\alpha B_\beta \]  

(3.3)
(with $H_2 = dB_1$). Indeed, setting $U = 0$ and $H_4 = 6c \ast B_1$, equation (2.5) is identically satisfied; equations (2.6) and (2.7) reduce, respectively, to the second and first conditions in (3.1); equation (2.3) is obtained by differentiating (3.2); and, finally, the Einstein equation (2.8) reduces to (3.3).

The equations of motion (3.2), (3.3) can be derived from the Lagrangian\(^3\)

$$\mathcal{L} = R \text{vol}_5 + 6c^2 \text{vol}_5 - \frac{3}{2} H_2 \wedge \ast H_2 - 18c^2 B_1 \wedge \ast B_1 ,$$

which was argued in [1] to allow for solutions with metric displaying Schrödinger symmetry. These solutions should be supported by a light-like massive vector of the form $B_1 \propto r^z dx^+$ (see [5]), where $z$ is the dynamical exponent, thus immediately satisfying (3.1). Specifically, we look for solutions to (3.1), (3.2), (3.3) of the form

$$ds_5^2 = -\alpha^2 r^{2z} (dx^+)^2 + \frac{2}{c^2 r^2} dr^2 + \frac{2}{c^2} r^2 (-dx^+ dx^- + dx_1^2 + dx_2^2) ,$$

$$B_1 = \beta r^z dx^+ .$$

(3.5)

where $\alpha$, $\beta$ and the dynamical exponent $z$ are constants to be determined. The configuration (3.5) does satisfy the conditions (3.1) and turns out to also solve the equations (3.2), (3.3) provided that

$$z(z + 2) = 24 ,$$

$$\alpha^2 (z^2 - 1) = \beta^2 \left( \frac{3}{4} z^2 + 18 \right) .$$

(3.6)

(3.7)

Thus, as in [5], we indeed find solutions for $z = 4$ (and $\beta = \frac{\alpha}{\sqrt{2}}$) and $z = -6$ (and $\beta = \frac{\alpha \sqrt{2}}{3}$). By convention $z > 0$, so we ignore the latter possibility.

The $z = 4$ solution can now be uplifted to $D = 11$ with the help of the KK ansatz (2.1), (2.2). We find

$$ds_{11}^2 = -\alpha^2 r^8 (dx^+)^2 + \frac{2}{c^2} \frac{dr^2}{r^2} + \frac{2}{c^2} r^2 (-dx^+ dx^- + dx_1^2 + dx_2^2) + ds^2(KE_6) ,$$

$$G_4 = 12 \frac{\alpha}{c^2} r^5 dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - 2\sqrt{2} \alpha r^3 dx^+ \wedge dr \wedge J + cJ \wedge J .$$

(3.8)

This is a new (non-supersymmetric) M-Theory solution dual to a NRC field theory in spatial dimension $d = 2$ with dynamical exponent $z = 4$. One can generalise this solution and consider more general ansatze for $D = 11$ supergravity solutions dual to $d = 2$ non-relativistic conformal field theories with dynamical exponent $z$, where the internal directions still correspond to a $KE_6$ space. We now turn to this point.

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\(^3\)This $D = 5$ theory, with even the same mass for the vector $B_1$ if we choose $c = \sqrt{2}$, was first discussed in section 4.2 of [5], but the $D = 5$ parent theories with Lagrangian (2.16) above and (4.21) of [5] are very different. As in [5, 6], the Lagrangian (3.4) only reproduces the equations (3.2), (3.3) and not the light-like condition (3.1). Since (3.1), (3.2), (3.3) can be consistently obtained upon truncation of $D = 11$ supergravity on $KE_6$, any choice of five-dimensional metric and lightlike $B_1$ (thus subject to (3.1)) which also solves the equations of motion (3.2), (3.3) that derive from the Lagrangian (3.4), can be safely uplifted to $D = 11$. 

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\(\text{JHEP}07(2009)081\)
Some generalisations

As we have just mentioned, the $D = 11$ solution (3.8) is locally invariant under $S_{ch_4}(1, 2)$. In particular, the scale invariance acts on coordinates as

$$(x^+, x^-, x_i, r) \rightarrow (\lambda^z x^+, \lambda^{2-z} x^-, \lambda x_i, \lambda^{-1} r), \quad i = 1, 2$$

(with $z = 4$ in (3.8)), while leaving the $KE_6$ coordinates unchanged. Following [7, 8], we can generalise the metric in (3.8) as:

$$ds^2_{11} = \frac{2}{c^2} \left[ - f_0 r^{2z}(dx^+)^2 - r^2 dx^+(dx^- + r z^{-2} C_1) + \frac{1}{r^2} dr^2 ight.$$

$$+ r^2 (dx_1^2 + dx_2^2) + ds^2(KE_6), \quad (4.2)$$

where $C_1$ is a one-form and $f_0$ a function, both of them defined on the internal $KE_6$. Both $C_1$ and $r^{2z} f_0$, serve the same role of breaking the $SO(2, 4)$ isometry of the original $AdS_5 \times KE_6$ metric (1.4) down to $S_{ch_2}(1, 2)$.

An ansatz for the accompanying four-form flux may be constructed by considering the forms invariant under $S_{ch_4}(1, 2)$ symmetry (see [22]), though the equations of motion constrain the candidate forms. The specific ansatz we then consider for the four-form flux is:

$$G_4 = - \frac{1}{z + 2} d(\mu_0 r^{z+2} dx^+ \wedge dx^1 \wedge dx^2) - \frac{1}{z} d(\mu_2 \wedge r^z dx^+) + cJ \wedge J,$$

(4.3)

where, in general, $\mu_0$ is a function and $\mu_2$ a two-form, both defined on $KE_6$. The latter can be taken to be proportional to the Kähler form on $KE_6$, as for the uplifted $z = 4$ solution (3.8), but other choices are also possible (see subsection 4.2 below). Indeed, the solution (3.8) is recovered from (4.2), (4.3) by setting $C_1 = 0$, $f_0 = \frac{1}{2} c^2 \alpha^2$, $\mu_0 = \frac{2\alpha}{c^2}$ and $\mu_2 = -2\sqrt{2\alpha} J$, for some constant $\alpha$. More generally, the non-trivial mixing of external and $KE_6$ coordinates in the metric (4.2) will prevent it from being obtainable as the uplift of any $D = 5$ metric. The requirement that (4.2), (4.3) do solve the equations of motion (1.1)–(1.3) of $D = 11$ supergravity leads to restrictions and relations for $f_0$, $C_1$, $\mu_0$ and $\mu_2$. In the following, we will spell out several interesting cases.

4.1 A solution with $z = 2$

We can find a $D = 11$ supergravity solution with dynamical exponent $z = 2$ by setting, for some constant $\alpha$, $f_0 = \frac{12\alpha}{c^2}$, choosing $C_1$ such that $dC_1 = \alpha J$, while writing $\mu_0 = \frac{12\alpha}{c^2}$, $\mu_2 = -2\sqrt{2\alpha} J$ so that the flux (4.3) reads

$$G_4 = \frac{12\alpha}{c^2} r^3 dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - \frac{2\alpha}{c^2} r dx^+ \wedge dr \wedge J + cJ \wedge J.$$

(4.4)

A generalisation of this solution appeared previously in [20], where the internal space is a variant of $CP^3$ [13].
4.2 A class of solutions with \( z \geq \sqrt{3} \)

Setting \( C_1 = 0 \) in the metric \((4.2)\) and \( \mu_0 = 0, \mu_2 = 0 \) in \((4.3)\) (which takes the flux back to its background value \((1.5)\)), some calculation reveals that the resulting combination of metric and four-form provides a solution of \( D = 11 \) supergravity if \( f_0 \) is an eigenfunction of the Laplacian \( \Delta_{KE} \equiv *d *d + d*d \) on \( KE_6 \) with eigenvalue \( 2(z^2 - 1)c^2 \):

\[
\Delta_{KE}f_0 = 2(z^2 - 1)c^2 f_0. \tag{4.5}
\]

This class of solutions thus provides a \( D = 11 \) counterpart of the Type IIB solutions first discussed in [7].

For the particular case \( KE_6 = CP^3 \), these eigenvalues are \( k(k + 3)c^2 \), \( k = 0, 1, \ldots \), with the corresponding eigenfunctions transforming in the \((k0k)\) irrep of \( SU(4) \) [15, 23]. Ruling out \( k = 0 \), which just corresponds to a space locally isometric to \( AdS_5 \times KE_6 \), we have a sequence of families of solutions with dynamical exponents \( z_k = \sqrt{1 + 1/2k(k + 3)} \), \( k = 1, 2 \ldots \),

\[
\text{thus obeying the bound} \quad z_k \geq \sqrt{3}. \tag{4.6}
\]

For each \( k = 1, 2, 3 \ldots \), this class contains a family of \( \text{dim}(k0k) = 15, 84, 300, \ldots \) supergravity solutions with the dynamical exponent \( z_k \) in \((4.6)\).

As noted in [7], this class of solutions should be unstable. Stability could be restored in [7] by appropriately turning on fluxes. We can try to do the same here by setting, for simplicity, \( \mu_2 \) to be proportional to the Kähler form \( J \). In this case, only for \( z = 4 \) do we find a solution with metric \((4.2)\) (with \( C_1 = 0 \)), supported by the flux

\[
G_4 = \alpha r^5 dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - \frac{\alpha c^2}{3\sqrt{2}} r^3 dx^+ \wedge dr \wedge J + cJ \wedge J, \tag{4.8}
\]

for any constant \( \alpha \). In this case, \( f_0 \) gets shifted by a positive term proportional to \( \alpha^2 \), which can be tuned to render the solution stable [7]. The shifted \( f_0 \) still fulfils \((4.5)\), now with eigenvalue \( 30c^2 \), corresponding to \( z = 4 \). We are unaware, however, of any \( KE_6 \) space for which this eigenvalue is permissible.

Alternatively, following [8–10], rather than setting \( \mu_2 \) to be proportional to the Kähler form, one may take it to be primitive and transverse.\(^4\) Setting, for convenience, \( \mu_0 = C_1 = 0 \), a calculation shows that the configuration \((4.2), (4.3)\) is a solution to \( D = 11 \) supergravity provided

\[
\Delta_{KE}f_0 + 2(z^2 - 1)c^2 f_0 = \frac{c^4}{4}|\mu_2|^2 + \frac{c^2}{2z^2}|d\mu_2|^2, \]

\[
\Delta_{KE}\mu_2 = \frac{1}{2} z(z + 2)c^2 \mu_2, \tag{4.9}
\]

\(^4\) A \((p, q)\)-form \( Y^{p,q} \) on a Kähler space is said to be primitive if its contraction with the Kähler form vanishes, \( J^{mn}Y_{mn}^{p,q} = 0 \), and transverse if \(*d *Y^{p,q} = 0 \).
where $|\mu_2|^2 = \frac{1}{2}\mu_{2\,ab}\mu_{2b}^a$, etc. Now, $f_0$ has devolved the Laplacian eigenvector character upon $\mu_2$, which corresponds to a two-form eigenfunction with eigenvalue $\frac{1}{2}z(z+2)c^2$. In the special case $KE_6 = CP^3$, the eigenvalues of the Laplacian acting on transverse, primitive $(1,1)$-forms (respectively, $(2,0)$-forms) are $(k+2)(k+3)c^2$ (respectively, $(k+3)(k+4)c^2$), for $k = 0, 1, \ldots$ [15, 23]. We thus see that solutions to (4.9) correspond to NRC gravity duals with dynamical exponents bounded below by $z \geq -1 + \sqrt{13}$ (respectively, $z \geq 4$), if $\mu_2$ is a chosen to be (the real part of) a $(1,1)$-form (respectively, $(2,0)$-form). See [10] for a discussion of a solving technique for systems of equations like (4.9). It would be interesting to study the stability of this class of solutions.

5 Final comments

We have constructed solutions of $D = 11$ supergravity dual to NRC field theories in 2 spatial dimensions and with different values of the dynamical exponent $z$. They correspond to suitable deformations of the class of solutions $AdS_5 \times KE_6$, that break the SO(2,4) symmetry down to its Schrödinger subalgebra $Sch_z(1,2)$. Important insight was obtained by first dealing with a simpler, particular solution with $z = 4$. Specifically, $D = 11$ supergravity reduced on the internal $KE_6$ truncates consistently to a $D = 5$ gravity theory involving a massive vector. A suitable solution of this theory, with $z = 4$, was found and subsequently uplifted to eleven-dimensions. We also discussed a more general class of $D = 11$ supergravity solutions, locally invariant under $Sch_z(1,2)$, that contains this solution, along with other examples that can no longer be obtained upon uplift. We are able to find explicitly a solution with $z = 2$, a class of solutions with dynamical exponents $z \geq \sqrt{3}$, and implicitly, solutions with $z \geq -1 + \sqrt{13}$ and $z \geq 4$.

The Schrödinger algebra $Sch_z(1, d)$ is not the only NRC symmetry one may consider. In fact, there also exists a conformal version of the Galilean algebra that, unlike $Sch_z(1, d)$, can be obtained as an Inönü-Wigner contraction of the relativistic conformal algebra $so(2, d+2)$. Some issues regarding the Galilean conformal algebra have been recently discussed, including its supersymmetrisation [24–26] and its implementation, both in the dual field theories and the gravity bulk [27, 28]. As pointed out in [28], a drawback of backgrounds with this conformal Galilean symmetry is that, in contrast to $Sch_z(1, d)$-invariant ones, their metrics exhibit a non-Lorentzian signature. While this would require better understanding, progress on the way NRC symmetries are implemented in gravity duals may be achieved by a systematic characterisation [19] of Type IIB and M-Theory backgrounds with $Sch_z(1, d)$ symmetry, for generic values of $z$ and $d$.

Acknowledgments

We would like to thank Ido Adam, Dumitru Astefanesei, José A. de Azcárraga, Jerome Gauntlett, Sean Hartnoll, Hironobu Kihara, Dario Martelli, Carlos Núñez, Ioannis Papadimitriou, María J. Rodríguez and Stefan Theisen for helpful discussions. OV is supported by an Alexander von Humboldt postdoctoral fellowship and, partially, through the Spanish Government research grant FIS2008-01980.
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