Filtered Inner Product Projection for Multilingual Embedding Alignment

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Abstract

Due to widespread interest in machine translation and transfer learning, there are numerous algorithms for mapping multiple embeddings to a shared representation space. Recently, these algorithms have been studied in the setting of bilingual dictionary induction where one seeks to align the embeddings of a source and a target language such that translated word pairs lie close to one another in a common representation space. In this paper, we propose a method, Filtered Inner Product Projection (FIPP), for mapping embeddings to a common representation space and evaluate FIPP in the context of bilingual dictionary induction. As semantic shifts are pervasive across languages and domains, FIPP first identifies the common geometric structure in both embeddings and then, only on the common structure, aligns the Gram matrices of these embeddings. Unlike previous approaches, FIPP is applicable even when the source and target embeddings are of differing dimensionalities. We show that our approach outperforms existing methods on the MUSE dataset for various language pairs. Furthermore, FIPP provides computational benefits both in ease of implementation and scalability.

1 Introduction

The problem of aligning sets of embeddings, or high dimensional real valued vectors, is of great interest in natural language processing, with applications in machine translation and transfer learning, and shares connections to graph matching and assignment problems. In machine translation, aligning embeddings trained on corpora from different languages has led to improved performance of supervised and unsupervised word and sentence translation [Zou et al. 2013]. Additionally, aligned source and target input embeddings have been shown to improve the transfer of models learned on a source domain to a target domain [Artetxe et al. 2018a, Wang et al. 2018, Mogadala and Rettinger 2016].

In the bilingual dictionary induction task, one seeks to learn a transformation on the embeddings of a source and a target language so that translated word pairs lie close to one another in the shared representation space. Specifically, one is given embeddings for a source and a target language, $X_s \in \mathbb{R}^{n \times d}$ and $X_t \in \mathbb{R}^{n \times d}$, and a small seed dictionary $D$ containing pairs of translated words. Using this seed dictionary, a transformation is learned on $X_s$ and $X_t$ with the objective that unseen translation pairs can be induced, often through nearest neighbors search.

Previous literature on this topic has focused on aligning embeddings by minimizing matrix or distributional distances [Grave et al. 2018, Jawanpuria et al. 2018, Joulin et al. 2018a]. For instance, [Mikolov et al. 2013a] proposed to using the Orthogonal Procrustes solution restricted to pairs of words in the seed dictionary, $\Omega^* = \arg \min_{\Omega \in O(d)} \|X_s^T \Omega - X_t^T\|_F$, which achieves high word translation accuracy for similar languages. However, these methods usually require the dimensions of the source and target language embeddings to be the same, which often may not hold. Furthermore, due to semantic shifts across languages, it’s often the case that a word and its translation may not co-occur with the same sets of words [Gulordava and Baroni 2011]. Therefore, seeking an
alignment which minimizes all pairwise distances among translated pairs results in using information not common to both the source and target embeddings.

To address these problems, we propose Filtered Inner Product Projection (FIPP) for mapping embeddings from different languages to a shared representation space. FIPP aligns a source embedding $X_s \in \mathbb{R}^{n \times d_s}$ to a target embedding $X_t \in \mathbb{R}^{m \times d_t}$ and maps vectors in $X_s$ to the $\mathbb{R}^{d_t}$ space of $X_t$. Instead of word-level information, FIPP focuses on pairwise distance information, specified by the Gram matrices $X_sX_s^T$ and $X_tX_t^T$. During alignment, FIPP tries to achieve the following two goals. First, it is desired that the aligned source embedding $\text{FIPP}(X_s) = X_s \in \mathbb{R}^{n \times d_t}$ be structurally close to the original source embedding to ensure that semantic information is retained and prevent against overfitting on the seed dictionary. This goal is reflected in the minimization of the reconstruction loss: $\|X_sX_s^T - X_sX_s^T\|_F^2$.

Second, as the usage of words and their translations vary across languages, instead of requiring $\tilde{X}_s$ to use all of the distance information from $X_t$, FIPP selects a filtered set $K$ of word pairs that have similar distances in both the source and target languages: $K = \{(i,j) \in D : |X_s[i]X_t[j] - X_t[i]X_t[j]| \leq \epsilon\}$. FIPP then minimizes a transfer loss on this set $K$, the squared difference in distances between the aligned source embeddings and the target embeddings: $\sum_{(i,j) \in K} (X_s[i]X_s[j]^T - X_t[i]X_t[j]^T)^2$.

We show FIPP can be efficiently solved using either low-rank semidefinite approximations or stochastic gradient descent. Also, we formulate a least squares projection to infer aligned representations for words outside the seed dictionary. The method is illustrated in Figure 1.

![Figure 1: FIPP alignment of source and target embeddings, $X_s$ and $X_t$, to a common representation space. Note $X_s$ is modified using information from $X_t$ and mapped to $\mathbb{R}^{d_t}$ while $X_t$ is unchanged.](image)

Compared to previous approaches, FIPP has improved generality and efficiency. First, since FIPP’s alignment between the source and target embeddings is performed on Gram matrices, i.e. $X_sX_s^T$ and $X_tX_t^T \in \mathbb{R}^{|D| \times |D|}$, embeddings are not required to be of the same dimensions. This is particularly helpful for aligning embeddings trained on smaller corpuses, such as in low resource domains, or compute-intensive settings where embeddings may have been compressed to lower dimensions. Secondly, alignment modifications made on filtered Gram matrices can incorporate constraints on alignment at the most granular level, pairwise distances. Lastly, FIPP is easy to implement as it involves only matrix operations and scales well to large seed dictionaries.

We conduct a thorough evaluation of FIPP bilingual dictionary induction on the MUSE dataset [Conneau et al. 2017]. Among 22 language pairs between English and another language, FIPP outperforms previous baselines on 13 pairs, most of which involve a low-resource language dissimilar to English.

2 Related Work

2.1 Distributional Methods for Quantifying Semantic Shifts

Prior work has shown monolingual text corpora from different communities or time periods exhibit variations in semantics and syntax [Hamilton et al. 2016b]. Word embeddings [Mikolov et al. 2013b], Pennington et al. [2014], Bojanowski et al. [2017] map words to representations in a continuous space such that the inner product between any two words is approximately proportional to its co-occurrence frequency. By comparing pairwise distances in monolingual embeddings trained on separate corpuses, one can quantify semantic shifts associated with biases, cultural norms, and temporal differences. [Gulordava and Baroni 2011], [Sagi et al. 2014], [Kim et al. 2014]. Recently proposed metrics on embeddings compare all pairwise inner products of two embeddings, $E$ and $F$, of the form $\|EE^T - FF^T\|_F$ [Yin et al. 2018]. While these metrics have been applied in quantifying
monolingual semantic variation, they have not been explored in context of mapping embeddings to a common representation space or in multilingual settings.

2.2 Multilingual Embedding Alignment

The first work in this field by Mikolov et al. [2013a]. It proposed the problem of bilingual embedding alignment with the presence of an incomplete matching between words of two languages or a bilingual seed dictionary. The authors proposed using the alignment recovered by the Orthogonal Procrustes problem, \( \Omega^* = \arg\min_{\Omega \in O(d)} \| X^D \Omega - X^T \|_F \), and found that the approach was able to induce a matching between similar languages with > 75% accuracy. Dinu and Baroni [2014] proposed corrections to the "hubness" problem in embedding alignment, where certain word vectors may be close to many other word vectors, arising due to nonuniform density of vectors in the \( R^d \) space. Smith et al. [2017] propose the inverted softmax metric for inducing matchings between words in embeddings of different languages. Artetxe et al. [2016] studied the impact of normalization, centering and orthogonality constraints in linear alignment functions. Jawanpuria et al. [2018] proposed a composition of orthogonal operators and a Mahalanobis metric of the form \( UBV^T, U, V \in O(d), B = 0 \) to account for observed correlations and moment differences between embedding dimensions. Søgaard et al. [2018]. Joulin et al. [2018a] proposed an alignment based on neighborhood information to account for differences in density and shape of embeddings in their respective \( R^d \) spaces. Artetxe et al. [2018c] proposed a framework which unifies many existing alignment approaches as compositions of matrix operations such as Orthogonal mappings, Whitening, and Dimensionality Reduction. In the unsupervised setting, where a bilingual seed dictionary is not provided, approaches using adversarial learning, distributional matching, and noisy self-supervision have been used to concurrently learn a matching and an alignment between embeddings (Cao et al. [2016], Zhang et al. [2017], Hoshen and Wolf [2018], Grave et al. [2018], Artetxe et al. [2017, 2018b], Alvarez-Melis and Jaakkola [2018]). Further discussion on unsupervised approaches has been deferred to the Appendix. Recently, further interest in the bilingual dictionary induction task has been spurred by the release of various large-scale datasets and measurement tools (Joulin et al. [2018a]).

3 Filtered Inner Product Projection (FIPP)

In this section, we introduce Filtered Inner Product Projection (FIPP), a method for aligning embeddings in a shared representation space. The rest of this section details the method definition and is split into defining the optimization problem and approaches for finding approximate solutions.

3.1 Filtered Inner Product Projection Objective

FIPP aligns a source embedding \( X_s \in \mathbb{R}^{n \times d_1} \) to a target embedding \( X_t \in \mathbb{R}^{m \times d_2} \) and projects vectors in \( X_s \) to \( \tilde{X}_s \in \mathbb{R}^{c \times d_1} \). Let \( X_s \in \mathbb{R}^{c \times d_1} \) and \( X_t \in \mathbb{R}^{c \times d_2} \) be the source and target embeddings for pairs in the seed dictionary \( D, |D| = c \ll \min(n, m) \). FIPP’s objective is to minimize a linear combination of a reconstruction loss, which regularizes changes in the pairwise inner products of the source embedding, and a transfer loss, which aligns the source and target embeddings on common portions of their geometries.

\[
\min_{\tilde{X}_s \in \mathbb{R}^{c \times d_1}} \left( \| \tilde{X}_s X_s^T - X_s X_s^T \|_F^2 + \lambda \| \Delta^c \circ (\tilde{X}_s X_s^T - X_t X_t^T) \|_F^2 \right)
\]

(1)

where \( \lambda, \epsilon \in \mathbb{R}^+ \) are tunable hyperparameters whose effects are discussed in section 6.1.

3.1.1 Reconstruction Loss

Due to the limited, noisy supervision in our problem setting, an alignment should be regularized against overfitting. Specifically, the aligned space needs to retain a similar geometric structure to the original source embeddings; this has been enforced in previous works by ensuring that alignments are close to orthogonal mappings. Mikolov et al. [2013a], Joulin et al. [2018a], Jawanpuria et al. [2018]. As \( \tilde{X}_s \) and \( X_s \) can be of differing dimensionality, we check structural similarity by comparing pairwise inner products, captured by a reconstruction loss known as the PIP distance or Global Anchor Metric: \( \| \tilde{X}_s X_s^T - X_s X_s^T \|_F^2 \). Yin and Shen [2018], Yin et al. [2018].
Suppose \( E \in \mathbb{R}^{n \times d}, F \in \mathbb{R}^{n \times d} \) are two matrices with orthonormal columns and \( \Omega^* = \arg \min_{\Omega \in \mathcal{O}(d)} \| E\Omega - F \|_F \). It follows that \cite{Yin2018}:

\[
\| E\Omega^* - F \|_F \leq \| EE^T - FF^T \|_F \leq \sqrt{2}\| E\Omega^* - F \|_F.
\]  

(2)

This metric has been used in quantifying semantic shifts and has been proven \cite{Yin2018} to be equivalent to the residual of the Orthogonal Procrustes problem up to a small constant factor, as shown in Theorem 1. Note that the PIP distance is invariant to orthogonal operations such as rotations which are known to be present in unaligned embeddings.

### 3.1.2 Transfer Loss

In different languages, it’s often the case that a word and its translation may not be used in all of the same contexts, commonly known as a semantic shift. As an example, consider the translated word pair ("department", "departamento") between English and Spanish. In Spanish, "departamento" is also used to refer to an "apartment" unlike its English translation, information which should not be utilized in alignment of an English embedding to a Spanish embedding. In aligning \( X_s \) to \( X_t \), we seek to only utilize common geometric information between the two embedding spaces. To do so, we first find the pairwise distances similar in both embedding spaces, denoted as inner product filtering and illustrated in Figure 2.

![Figure 2: Inner product filtering](image)

Specifically, compute a matrix \( \Delta^c \in \{0, 1\}^{c \times c} \) where \( \Delta^c_{ij} \) is an indicator on whether \( |X_{s,i}X_{s,j}^T - X_{t,i}X_{t,j}^T| < \epsilon \). Note that \( \epsilon \) is a hyperparameter which determines how close pairwise distances must be in the source and target embeddings in order to be deemed similar. We then define a transfer loss as being the squared difference between the converted source embedding \( \tilde{X}_s \) and target embedding \( X_t \), but only on pairs of words in \( K \): \( \| \Delta^c \circ (\tilde{X}_sX_s^T - X_tX_t^T) \|_F^2 \), where \( \circ \) is the Hadamard product. The FIPP objective is a linear combination of the reconstruction and transfer losses.

### 3.2 Approximate Solutions to the FIPP Objective

We provide two approaches for obtaining solutions to the FIPP objective using Low-rank Semidefinite Approximations and Stochastic Gradient Descent.

#### 3.2.1 Solutions using Low-rank Semidefinite Approximations

Denote the Gram matrices \( G^s \triangleq X_sX_s^T, G^t \triangleq X_tX_t^T \) and \( \tilde{G}^s \triangleq \tilde{X}_s\tilde{X}_s^T \).

**Lemma 2.** The Gram matrix \( \tilde{G}^s \) which minimizes the FIPP objective for the fixed \( \lambda \) and \( \epsilon \) has entries:

\[
G^*_{ij} = \begin{cases} 
\frac{(X_sX_s^T)_{ij} + \lambda(X_sX_s^T)_{ij}}{1 + \lambda}, & \text{if } (i,j) \in K \\
(X_sX_s^T)_{ij}, & \text{otherwise}
\end{cases}
\]  

(3)

**Proof.** For a fixed \( \lambda \) and \( \epsilon \), \( \mathcal{L}_{FIPP_{\lambda,\epsilon}}(\tilde{X}_s\tilde{X}_s^T) \) can be decomposed as follows:

\[
\mathcal{L}_{FIPP_{\lambda,\epsilon}}(\tilde{X}_s\tilde{X}_s^T) = \| \tilde{X}_s\tilde{X}_s^T - X_sX_s^T \|_F^2 + \lambda\| \Delta^c \circ (\tilde{X}_s\tilde{X}_s^T - X_sX_s^T) \|_F^2 \\
= \sum_{i,j \in K} ((G^*_{ij} - G^s_{ij})^2 + \lambda(G^s_{ij} - G^s_{ij})^2) + \sum_{i,j \notin K} (G^s_{ij} - G^s_{ij})^2
\]  

(4)
By taking derivatives with respect to \( \hat{G}^*_{ij} \), the matrix \( G^* \) which minimizes \( \mathcal{L}_{FIPP, \lambda, \epsilon}(\cdot) \) is:

\[
G^* = \arg \min_{\hat{X}^\perp \in \mathbb{R}^{c \times d}} \mathcal{L}_{FIPP, \lambda, \epsilon}(\hat{X}^\perp \hat{X}^\perp^T), \quad G^*_{ij} = \begin{cases} (X_s X_s^T)_{ij} + \lambda (X_s X_s^T)_{ij}, & \text{if } (i, j) \in K \\ (X_s X_s^T)_{ij}, & \text{otherwise} \end{cases}
\]

We now have the matrix \( G^* \in \mathbb{R}^{c \times c} \) which minimizes the FIPP objective. However, for \( G^* \) to be a Gram matrix, it is required that \( G^* \in \mathcal{S}^+_{c \times c} \), the set of symmetric Positive Semidefinite matrices. Additionally, to recover an aligned embedding \( \hat{X}_s^\perp \), \( \hat{X}_s^\perp \in \mathbb{R}^{c \times d} \), which minimizes \( \|X_s \hat{X}_s^T - G^*\|_F \), we instead perform a rank-constrained semidefinite approximation to find \( \min_{\hat{X}^\perp \in \mathbb{R}^{c \times d}} \|X_s \hat{X}_s^T - G^*\|_F \).

**Theorem 3.** Let \( G^* = Q \Lambda Q^T \) be the Eigendecomposition of \( G^* \). A matrix \( \hat{X}_s^\perp \in \mathbb{R}^{m \times d_2} \) which minimizes \( \|X_s \hat{X}_s^T - G^*\|_F \) is given by \( \sum_{i=1}^{d_2} \lambda_i^2 q_i \), where \( \lambda_i \) and \( q_i \) are the \( i \)th largest eigenvalue and corresponding eigenvector.

**Proof.** Since \( G^* \in \mathcal{S}^+_{c \times c} \), its Eigendecomposition is \( G^* = Q \Lambda Q^T \) where \( Q \) is orthonormal. Let \( \hat{\lambda}, \hat{\mathbf{q}} \) be the \( d_2 \) largest nonnegative eigenvalues in \( \Lambda \) and their corresponding eigenvectors; additionally, denote the complementary eigenvalues and associated eigenvectors as \( \hat{\lambda}^\perp = \Lambda \setminus \hat{\lambda}, \hat{\mathbf{q}}^\perp = Q \setminus \hat{\mathbf{q}} \). Using the Eckart–Young–Minsky Theorem for the Frobenius norm [Kishore Kumar and Schneider 2017], note that for \( G \in \mathcal{S}^+_{c \times c} \), \( \text{Rank}(G) \leq d_2 \); \( \|G^* - G\|_F \geq \|q^\perp \hat{\lambda}^\perp \hat{\mathbf{q}}^\perp^T\|_F = \sum_{i=1}^{d_2} \lambda_i \) and that \( \|G^* - G\|_F \) is minimized for \( G = \hat{\lambda} \hat{\mathbf{q}}^T \). Using this result, we can recover \( \hat{X}_s^\perp \):

\[
\arg \min_{\hat{G} \in \mathcal{S}^+_{c \times c}, \text{Rank}(\hat{G}) \leq d_2} \|G^* - \hat{G}\|_F = \sum_{\lambda_i \in \hat{\lambda}} (\lambda_i^2 q_i) (\lambda_i^2 q_i)^T = \hat{X}_s \hat{X}_s^T
\]

Using the above matrix approximation, we find our aligned embedding \( \hat{X}_s \) is a minimizer of \( \|X_s \hat{X}_s^T - G^*\|_F \).

Due to the rank constraint on \( G \), we are only interested in the \( d_2^2 \) largest eigenvalues and their corresponding eigenvectors which incur a complexity of \( \mathcal{O}(d_2 c^2) \) using power iteration [Panju 2011].

### 3.2.2 Solutions using Stochastic Gradient Descent

Alternatively, solutions to the FIPP objective can be obtained using Stochastic Gradient Descent (SGD). This requires defining a single variable \( \hat{X}_s \in \mathbb{R}^{c \times d_2} \) over which to optimize. We find that the solutions obtained with SGD are close, with respect to the Frobenius norm, to those obtained with low rank PSD approximations up to a rotation. However, the complexity of solving FIPP using SGD is \( \mathcal{O}(Tc^2) \), where \( T \) is the number of training epochs. Empirically we find \( T \gg d_2 \) for convergence of SGD and, as a result, this approach incurs a complexity orders of magnitude larger than that of low-rank semidefinite approximations.

### 4 Inference and Evaluation

In this section, we detail how to use the aligned embedding obtained by FIPP, \( \hat{X}_s \), for the Bilingual Dictionary Induction task.
4.1 Rotation via Orthogonal Procrustes

Since $\tilde{X}_s \in \mathbb{R}^{c \times d_2}$ has been optimized only with concern for its inner products, $\tilde{X}_s$ must be rotated so that the basis of $\tilde{X}_s$ match those of $X_s$. Here, we use the closed form solution to the Orthogonal Procrustes problem\cite{1966SCHONEMANN}:

$$\text{SVD}(X_i^T \tilde{X}_s) = U\Sigma V^T, \quad \Omega^* = \arg\min_{\Omega \in O(d_2)} \| \tilde{X}_s \Omega - X_i \|_F^2 = UV^T,$$

where $O(d_2)$ is the set of $d_2 \times d_2$ unitary matrices and SVD($A$) is the singular value decomposition of $A$. After obtaining $\Omega^*$, we apply the rotation on $\tilde{X}_s$. Note applying the rotation $\Omega^*$ does not change the Gram matrix $\tilde{G}^*$ and, therefore, $\tilde{X}_s \Omega^*$ retains the same loss on the FIPP objective as $\tilde{X}_s$.

4.1.1 Inference with Least Squares Projection

During inference, FIPP must align source embeddings for the entire vocabulary to obtain $\hat{X}_s \in \mathbb{R}^{n \times d_2}$, not just for those words in the seed dictionary. As we do not have matchings between the unseen source words and their counterparts in the target embedding, we make the assumption that unseen source words should preserve their distances to those in the seed dictionary, i.e., $X_s \hat{X}_t^* \approx \hat{X}_s \tilde{X}_t^*$. Using this assumption, we formulate a least squares projection\cite{2004BOYD} on an overdetermined system of equations:

$$\tilde{X}_s^T \hat{X}_s^i (\tilde{X}_s \hat{X}_s)^{-1} \hat{X}_s^T \tilde{X}_s \hat{X}_s^T \hat{X}_s^i = \begin{pmatrix} \tilde{X}_s \hat{X}_s & \tilde{X}_s \hat{X}_s \end{pmatrix} \in \mathbb{R}^{d_2 \times 2} \quad \begin{pmatrix} \hat{X}_s^T \hat{X}_s \end{pmatrix} \in \mathbb{R}^{2 \times 1}$$

Note that the least squares solution is being calculated independently for each column in $\hat{X}_s$ and therefore can be parallelized across words.

4.2 Nearest Neighbors Search

The evaluation of the bilingual dictionary induction task involves finding a matching between rows of $\hat{X}_s$ and $X_i$. Given a source word, $i$, a traditional nearest neighbor search looks for the vector, $y \in X_i$ with the smallest angle to $\hat{X}_s^i$ : $\arg\min_{y \in X_i} - \cos(\hat{X}_s^i, y)$. In our evaluation, we use a modified version of nearest neighbor search known as Cross-domain Similarity Local Scaling (CSLS)\cite{2018JOULIN} frequently used for the task of bilingual dictionary induction. This search approach scales up distances to target words which are close to many source words and vice-versa for target words which are far away from any source words. This density scaling addresses the “hub problem” with language-based embeddings where some words may be nearest neighbors for a large number of other words while others may be far away from most other words. Specifically, CSLS finds a matching using the below objective:

$$\arg\min_{y \in X_i} -2 \cos(\hat{X}_s^i, y) + \frac{1}{k} \sum_{y' \in \mathcal{N}_{X_s^i}(x)} \cos(\hat{X}_s^i, y') + \frac{1}{k} \sum_{x' \in \mathcal{N}_{X_s^i}(y)} \cos(x', y)$$

where $\mathcal{N}_{X_s^i}(x)$ and $\mathcal{N}_{X_s^i}(y)$ correspond to the $k$ nearest points to the query in the specified embedding space. During experimentation, we use the implementation provided by\cite{2018JOULIN} with default parameters.

5 Experimentation

5.1 Dataset Description

The MUSE dataset\cite{2017CONNEAU} contains bilingual dictionaries for 110 pairs of languages. For each language pair, the training seed dictionaries contain approximately 5000 word pairs while the evaluation sets contain 1500 word pairs. Each of bilingual dictionaries has been manually curated

\footnote{https://github.com/facebookresearch/MUSE}
to control for multiple translations of words and false negatives which may arise from stemming. All embeddings used in our experimentation and previous approaches are FastText vectors trained on Wikipedia dumps\(^2\) [Bojanowski et al. 2017]. The vocabularies of the embeddings are the 200K most frequent words in each language. Performance for each language pair is measured as being the percentage of source words in the evaluation set which are correctly matched to their corresponding translations in the target language, i.e. the precision@1 metric.

### 5.2 Self-supervised Augmentation of Seed Dictionary

Exact string matches between words in the source and target language have been used to augment the seed dictionary [Joulin et al. 2018b]. The incidence of these words is due to phenomena such as loan words and proper nouns. However, exact string matches between words of different languages can be potentially be unrelated, known as false cognates, and, therefore, provide noisy supervision signal. Using such word pairs, we augment the seed dictionary and can solve FIPP in semi-supervised (supervised dictionary + string match pairs) and unsupervised (only string match pairs) settings.

### 5.3 Experimental Results

Table 1: Retrieval accuracy (Word Translation Precision@1) on 11 languages to and from English. The best results are in bold; CSLS search is used for all methods.

| Language Pair | Procrustes | MSF | GeoMM | RCSLS | FIPP |
|---------------|------------|-----|-------|-------|------|
| En-Es         | 81.4       | 80.5| 81.9  | 84.1  | 81.7 |
| Es-En         | 82.9       | 83.8| 85.5  | 86.3  | 84.3 |
| En-Fr         | 81.1       | 80.5| 82.1  | 83.3  | 82.0 |
| Fr-En         | 82.4       | 83.1| 84.2  | 84.1  | 83.9 |
| Ru-En         | 51.7       | 50.5| 52.8  | 57.9  | 53.0 |
| En-Zh         | 42.7       | 32.3| 49.1  | 45.9  | 43.7 |
| Zh-En         | 36.7       | 43.4| 45.3  | 46.4  | 40.7 |
| En-Af         | 28.3       | 35.7| 29.2  | 28.7  | 35.7 |
| Af-En         | 34.4       | 42.5| 37.0  | 38.1  | 44.6 |
| En-Bs         | 23.5       | 27.6| 23.6  | 25.4  | 31.5 |
| Bs-En         | 35.6       | 41.4| 38.9  | 39.5  | 42.1 |
| En-Ms         | 45.4       | 55.2| 53.9  | 49.1  | 52.7 |
| Ms-En         | 44.8       | 50.8| 47.7  | 49.1  | 56.9 |
| En-Sl         | 34.3       | 38.0| 2.7   | 36.0  | 42.1 |
| Sl-En         | 49.6       | 55.2| 4.8   | 54.2  | 55.4 |
| En-Th         | 23.3       | 24.1| 24.8  | 23.7  | 25.5 |
| Th-En         | 13.9       | 16.3| 19.3  | 16.6  | 30.2 |
| En-Tl         | 15.8       | 22.6| 0.0   | 17.7  | 26.9 |
| Tl-En         | 19.1       | 23.4| 0.0   | 22.3  | 32.4 |
| En-Vi         | 42.1       | 50.1| 49.5  | 44.3  | 54.9 |
| Vi-En         | 54.9       | 60.0| 57.9  | 56.5  | 68.9 |

We evaluate FIPP along with standard supervised and semi-supervised approaches on 11 languages, to and from English, on the MUSE benchmark. FIPP results reported are semi-supervised, using the supervision of the MUSE seed dictionary and exact string match word pairs. FIPP produces stable results for all language pairs across runs of the same experiment; performance for other methods have been averaged over 5 re-runs. Every approach uses fastText vectors trained on monolingual Wikipedia corpora and a CSLS search criteria. We find that FIPP outperforms previous approaches.

\(^2\)https://fasttext.cc/docs/en/pretrained-vectors.html
on 13 of 22 language pairs, most of which involve languages dissimilar to English. In particular, for language pairs involving Thai (Th) and Tagalog (Tl), which have the lowest retrieval accuracy among the languages evaluated, FIPP increases relative retrieval accuracy by $>25\%$ when compared to the best previous approach.

6 Discussion

6.1 Effect of $\epsilon, \lambda$ on translation performance

In our experiments, we tune the hyperparameters $\epsilon$ and $\lambda$ which signify the level of discrimination in the inner product filtering step and the weight of the transfer loss respectively. To account for the sparsity of $\Delta^\epsilon$, we scale $\lambda$ in the transfer loss by $\gamma = c^2_{NNZ(\Delta^\epsilon)}$ where $NNZ(\Delta^\epsilon)$ is the number of nonzeros in $\Delta^\epsilon$. Empirically, we find that among the language pairs we evaluate, $\epsilon \in [0.025, 0.10]$ and $\lambda \in [0.5, 1.5]$ provided the best performance. Values of $\epsilon, \lambda$ which are close result in similar performance and, when using the matrix approximation in Section 3.2.1, performance is stable across reruns of the same experiment. As no validation set is provided in MUSE, hyperparameters are tuned by holding out 20\% of the training set.

6.2 Word Translation Performance for Embeddings of Different Dimensionality

| Language Pair | $d_1$  | $d_2$  | Linear | FIPP |
|---------------|--------|--------|--------|------|
| En-Vi         | 300    | 300    | 42.1   | 54.9 |
| En-Vi         | 200    | 300    | 41.9   | 51.3 |
| En-Vi         | 100    | 300    | 35.7   | 42.2 |

Table 2: Retrieval accuracy (Word Translation Precision@1) for Linear and FIPP alignment methods on embeddings of differing dimensionality.

One advantage of FIPP when compared to existing approaches is the ability to align embeddings of different dimensionalities to a common representation space. In this section, we evaluate bilingual dictionary induction performance on embeddings of different dimensionalities for the (English, Vietnamese) language pair. We align English embeddings with dimensions $d_2 \in \{100, 200, 300\}$ to a Vietnamese embedding of dimension 300. We compare the performance of FIPP with the best linear transform with orthonormal rows, $\Omega^* = \arg \min_{\Omega \in R^{d_1 \times d_2}} \| X_s \Omega - X_t \|_F$, equivalent to the Orthogonal Procrustes solution when $d_1 = d_2$. The performance of both methods decreases as $d_2 - d_1$ increases. However, the performance improvement of FIPP over the best linear alignment is maintained.

6.3 Monolingual Task Performance of Aligned Embeddings

As FIPP does not perform an orthogonal transform, it modifies the inner products of word vectors in the source embedding which can impact performance on monolingual task accuracy. We evaluate the aligned embedding learned using FIPP, $\tilde{X}_s$, on monolingual word analogy tasks and compare these results to the original fastText embeddings $X_s$. We set our source language to be English and compute an aligned embedding $\tilde{X}_s$ for various target languages. In Table 5, we compare monolingual English word analogy results for $\tilde{X}_s$ which have been aligned to a Chinese target embedding using FIPP. Evaluation of the aligned and original source embeddings on multiple English word analogy experiments show that aligned FIPP embeddings retain performance on monolingual tasks.

Table 3: Monolingual Analogy Task Performance for English embedding before/after alignment to Chinese embedding.
7 Conclusion

In this paper, we introduced Filtered Inner Product Projection (FIPP) a method for aligning multiple embeddings to a common representation space using pairwise inner product information. FIPP accounts for semantic shifts and aligns embeddings only on common portions of their geometries. Additionally, unlike previous approaches, FIPP can be applied to pairs of embedding spaces regardless of their dimensionalities. We provide two approaches for finding approximate solutions to the FIPP objective and show that it can be solved efficiently even in the case of large seed dictionaries. We evaluate FIPP on the task of bilingual dictionary induction using the MUSE dataset, on which it achieves state-of-the-art performance on dissimilar language pairs. While we focus on the method exposition and experimental results for supervised word translation in this paper, embedding alignment has a wide range of applicability in natural language processing tasks and our method provides a novel approach to the problem of shared representation learning.

Broader Impact

Our paper provides an easy-to-compute alignment method between embeddings of multiple languages that is shown to improve the induction of word translations, especially in the case of dissimilar languages. This approach facilitates work in machine translation, transfer learning, and representation learning and can help with model building for languages dissimilar to English, which are often under-represented in machine learning. However, various biases have been shown to exist in embedding models which our approach does not remedy. Additional bias-removal techniques should be used along with our method.

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