On the Trace Anomaly of the Chaudhuri–Choi–Rabinovici Model

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Abstract: Recently a non-supersymmetric conformal field theory with an exactly marginal deformation in the large $N$ limit was constructed by Chaudhuri–Choi–Rabinovici. On a non-supersymmetric conformal manifold, the $c$ coefficient of the trace anomaly in four dimensions would generically change. In this model, we, however, find that it does not change in the first non-trivial order given by three-loop diagrams.

Keywords: conformal field theory; trace anomaly

Conformal field theories are no longer conformally invariant in curved space-time due to the trace anomaly in even space-time dimensions. They do, however, play a fundamental role in understanding the structure of the energy–momentum tensor and the renormalization group flow.

In four-dimensional conformal field theories, the trace anomaly has the form

$$T^\mu_\mu = c \text{Weyl} l^2 - a \text{Euler}$$

and it is known that coefficient $a$ cannot change under exactly marginal deformations, but coefficient $c$ may [1–7]. However, there has been no explicit field theory example where $c$ changes (except for the effective holographic constructions in [2]). The main obstruction has been that we have no good examples of non-supersymmetric conformal field theories with exactly marginal deformations; in superconformal field theories, while it is easier to realize exactly marginal deformations, $c$ does not change [8].

Recently, Chaudhuri, Choi and Rabinovici have constructed a non-supersymmetric conformal field theory with an exactly marginal deformation in the large $N$ limit [9] (see also [10,11] for other recently constructed examples of non-supersymmetric field theories with exactly marginal deformations in different dimensions than four). This theory may serve as a first non-trivial check if $c$ can really change under exactly marginal deformations. In this short note, we, however, show that it does not change at the first non-trivial order given by three-loop diagrams.

The model (called complex bifundamental model in [9]) is given by four $SU(N_c)$ gauge theories with names $1, 1', 2$ and $2'$, each of which has $N_f$ Dirac fermions in the fundamental representation. We have two complex scalars in the bifundamental representations $\Phi_1$ (under gauge group 1 and $1'$) and $\Phi_2$ (under gauge group 2 and $2'$). The gauge coupling constant for each gauge group is $g_i$. It has no Yukawa interaction, the absence of which is protected by chiral symmetry, but it has a scalar potential

$$V = \tilde{h}_2 \text{Tr} \left[ \Phi_1^2 \Phi_2 \Phi_1^2 \Phi_2 \right] + \tilde{h}_2 \text{Tr} \left[ \Phi_2^2 \Phi_1 \Phi_2^2 \Phi_1 \right] + \tilde{f}_2 \text{Tr} \left[ \Phi_1^2 \Phi_2 \Phi_1 \Phi_2 \right] + 2 \tilde{c} \text{Tr} \left[ \Phi_1^2 \Phi_1 \right] \text{Tr} \left[ \Phi_2^2 \Phi_2 \right].$$

We take the Veneziano limit of $N_c, N_f \to \infty$ with fixed $x = N_f / N_c$ and consider the limit $x \to \frac{21}{4}$ to make the theory weakly coupled.
In terms of rescaled coupling constants \((i = 1, 2)\)

\[
\lambda_i = \frac{N_c \lambda_i^2}{16\pi^2}
\]

the renormalization group \(\beta\) functions in the Veneziano limit are expressed as (no sum over \(i\) unless explicitly shown)

\[
\begin{align*}
\beta_{\lambda_i} &= -\frac{21}{3} - \frac{4x}{3}\lambda_i^2 + \frac{-54 + 26x}{3}\lambda_i^3 \\
\beta_{h_i} &= 8h_i^2 - 12\lambda_i h_i + \frac{3}{2}\lambda_i^2 \\
\beta_{f_i} &= 4f_i^2 + 16f_i h_i + 12h_i^2 + 4\zeta^2 - 12\lambda_i f_i + \frac{9}{2}\lambda_i^2 \\
\beta_{\zeta} &= \zeta \sum_{i=1}^{2} (4f_i + 8h_i - 6\lambda_i).
\end{align*}
\]

(4)

The zero of the \(\beta\) functions was studied in [9] and they found that there exists a conformal manifold given by

\[
\begin{align*}
\lambda_1 &= \lambda_2 = \lambda = \frac{21 - 4x}{-54 + 26x} \\
h_1 &= h_2 = \frac{3 - \sqrt{6}}{4}\lambda \\
f_p &\equiv \frac{f_1 + f_2}{2} = \sqrt{3}\lambda \\
\zeta^2 + f_m^2 &= \frac{18\sqrt{6} - 39}{16}\lambda^2
\end{align*}
\]

(5)

where \(f_m \equiv \frac{f_1 - f_2}{2}\). From the last line of Equation (??), we see that it has the topology of a circle. As long as \(\lambda\) is small, we may neglect higher order corrections.

We now ask if the coefficient \(c\) in the trace anomaly can change on this conformal manifold. In addition to the coupling constant-independent contributions from the one-loop diagrams (that count a number of fields), the coupling constant-dependent contributions to the trace anomaly that are relevant for us come from the three-loop diagrams shown in Figure 1. The detailed computation for Figure 1A (as well as other two-loop diagrams) can be found in [12–14], but we only need the relative coefficient, so we can simply work on combinatorics.
Figure 1. Three-loop Feynman diagrams that could contribute to $c$. Wavy lines correspond to gauge fields and dotted lines correspond to scalar fields.

The three-loop Figure 1B–D are not evaluated in the literature, but we see that Figure 1B,C do not contribute to $c$. This is because the divergence can be simply removed by adding the “mass counter-term”. Figure 1D may contribute in general, but the contributions to $c$ in our theory do not depend on $\zeta$ or $f_m$ from the symmetry of the diagrams (It cannot be proportional to $\zeta$ because the gauge fields cannot connect $\Phi_1$ and $\Phi_2$. The relevant diagrams are all symmetric with respect to the exchange of $f_1$ and $f_2$).

As for Figure 1A, since the overall contribution to $c$ is known, we can just enumerate diagrams appearing in the Wick contractions of

$$\langle \tilde{f}_1 \text{Tr}[\Phi_1^4 \Phi_1]\text{Tr}[\Phi_1^4 \Phi_1](x)\tilde{f}_1 \text{Tr}[\Phi_1^4 \Phi_1]\text{Tr}[\Phi_1^4 \Phi_1](y)\rangle_{\text{free}}$$

or

$$\langle 2\tilde{c}_1 \text{Tr}[\Phi_1^4 \Phi_1]\text{Tr}[\Phi_2^4 \Phi_2](x)2\tilde{c}_1 \text{Tr}[\Phi_1^4 \Phi_1]\text{Tr}[\Phi_2^4 \Phi_2](y)\rangle_{\text{free}}$$
We see that only planar diagrams will contribute in the Veneziano limit. Up to an overall proportionality factor, the result in the Veneziano limit is summarized as
\[ c_{2,3\text{-loop}} = -4f_m^2 - 4c^2 + c_A\lambda^2 \] (6)
on the conformal manifold, where \( c_A \) is some numerical constant, which is unimportant for our discussions (A typo in the two-loop gauge contribution [14] that could affect \( c_A \) has been corrected in [15]). Since the relative coefficient appearing here coincides with what appears in the last line of Equation (??), we conclude that \( c \) does not change on the conformal manifold, although the value itself is perturbatively corrected. We also note that these two- and three-loop diagrams do not change the value of \( a \) as anticipated [1,16] (rather trivially without cancellation, unlike \( c \)).

The result is surprising in the sense that we generically expect that \( c \) would change on a non-supersymmetric conformal manifold. It is an interesting question to see whether the higher loop corrections modify our conclusion. It may be possible to relate the all-loop argument for the existence of the exactly marginal deformation in [9] with the computation of \( c \) by closing all the external lines in beta functions to make vacuum diagrams.

**Funding:** This work is in part supported by JSPS KAKENHI Grant Number 17K14301.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Acknowledgments:** This work is in part supported by JSPS KAKENHI Grant Number 17K14301. It is motivated from the online talk by Z. Komargodski at YITP workshop on Strings and Fields 2020, which the author watched on Youtube later.

**Conflicts of Interest:** The author declares no conflict of interest.