Extracting the depolarization coefficient $D_{NN}$ from data measured with a full acceptance detector

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Abstract

The spin transfer from vertically polarized beam protons to $\Lambda$ or $\Sigma$ hyperons of the associated strangeness production $\bar{p}p \to pK^+\Lambda$ and $\bar{p}p \to pK^0\Sigma^+$ is described with the depolarization coefficient $D_{NN}$. As the polarization of the hyperons is determined by their weak decays, detectors, which have a large acceptance for the decay particles, are needed. In this paper a formula is derived, which describes the depolarization coefficient $D_{NN}$ by count rates of a 4$\pi$ detector. It is shown, that formulas, which are given in publications for detectors with restricted acceptance, are specific cases of this formula for a 4$\pi$ detector.

1. Introduction

The determination of the depolarization coefficient of the associated strangeness production reaction is an important tool to gain information about the quark configuration of the initial proton and the produced hyperon. (see e.g. [1] and references therein). While analyzing the polarization data measured with the time of flight spectrometer (COSY-TOF) (see e.g. [2]) it was noticed that formulas given in literature for the depolarization coefficient $D_{NN}$ are tailored to the acceptances of the respective detectors [3, 4, 5, 6, 7]. Therefore, they are not valid for the COSY-TOF experiment, which is a 4$\pi$ acceptance detector. In
this paper a formula for a $4\pi$ acceptance detector is derived. This formula can be applied for the reactions $p\bar{p} \rightarrow pK^+\Lambda$ and $p\bar{p} \rightarrow pK^0\Sigma^+$. For the intermediate $\Sigma^0$ production in $p\bar{p} \rightarrow pK^+\Sigma^0 \rightarrow pK^+\Lambda\gamma$ the $D_{NN}$ formula cannot be applied, as in this reaction the $\Lambda$ polarization is diluted by the $\Sigma^0$ polarization and by the additional degree of freedom introduced by the photon momentum [8].

Measurements of $D_{NN}$ with limited acceptance detectors can be described by modifying this formula for the respective acceptance. This is demonstrated with the formula given for the Brookhaven Multiparticle Spectrometer [3].

The analyzing power and the hyperon polarization are discussed first, in order to introduce the “Integral Method” and to define some of the variables. Here and in the following we apply the notations of E. B. Bonner et al. [3], which are in agreement with the Ann Arbor Conventions [9].

2. Analyzing power

The hyperon analyzing power $A_N$ describes how strong the (transversely) polarized beam generates a left right asymmetry of the hyperon. It can be measured by applying the formula:

$$I(\vartheta^*, \Phi) = I_0(\vartheta^*) \cdot (1 + A_N(\vartheta^*) P_B \cos(\Phi)) \quad (1)$$

$I$ is the intensity distribution, $P_B$ the absolute value of the beam polarization, and $\Phi$ is the angle between the direction of the beam polarization ($\pm \vec{e}_y$) and the normal vector $\vec{N}$ to the production plane ($\vec{e}_{beam} \times \vec{e}_{hyperon}$) (see fig. 1). $\vartheta^*$ is the polar scattering angle in the cm system. Alternatively, the polarization variables are often evaluated as a function of the transverse momentum, Feynman $x$, or the rapidity of the hyperon.

For the beam polarization pointing upwards ($+\vec{e}_y$) the azimuthal angle of the hyperon $\varphi$ is connected to the angle $\Phi$:
$$\varphi = \Phi$$

For the beam polarization pointing downwards ($-\vec{e}_y$):
$$\varphi = \Phi - \pi$$

In the following the “Integral Method” of the evaluation of the analyzing power is given. Depending on the position in the left or right hemisphere of the hyperon azimuthal angle $\varphi$ and on the direction of the beam polarization, four count rates can be distinguished:

$\varphi$ in the left hemisphere and beam polarization upwards

$$N_L^\uparrow(\vartheta^*) = \int_0^1 I(\vartheta^*, \Phi) \, d\cos(\Phi) \quad (2)$$

$\varphi$ in the right hemisphere and beam polarization upwards

$$N_R^\uparrow(\vartheta^*) = \int_0^1 I(\vartheta^*, \Phi) \, d\cos(\Phi) \quad (3)$$
\( \varphi \) in the left hemisphere and beam polarization downwards

\[
N_1^L(\varphi^\ast) = \int_{-1}^{0} I(\varphi^\ast, \Phi) d\cos(\Phi) \tag{4}
\]

\( \varphi \) in the right hemisphere and beam polarization downwards

\[
N_1^R(\varphi^\ast) = \int_{0}^{1} I(\varphi^\ast, \Phi) d\cos(\Phi) \tag{5}
\]

Assuming that the absolute value of the beam polarization is the same for both directions and that the detector is fully symmetric in \( \varphi \), the count rates \( N_1^L(\varphi^\ast), N_1^R(\varphi^\ast) \) and \( N_2^L(\varphi^\ast), N_2^R(\varphi^\ast) \) are pairwise the same.

The integration of equation (1) yields:

\[
A_N(\varphi^\ast) = \frac{2}{P_B} \epsilon_A(\varphi^\ast) \tag{6}
\]

with the averaged asymmetry

\[
\epsilon_A(\varphi^\ast) = \frac{(N_1^L(\varphi^\ast) + N_1^R(\varphi^\ast)) - (N_2^R(\varphi^\ast) + N_2^L(\varphi^\ast))}{N_1^L(\varphi^\ast) + N_1^R(\varphi^\ast) + N_2^R(\varphi^\ast) + N_2^L(\varphi^\ast)} \tag{7}
\]

3. Hyperon polarization

It has been discovered that the hyperons are produced polarized even with an unpolarized beam \[10\]. The polarization axis is normal to the production plane. It is determined by measuring the angle \( \theta^\ast \) between the direction of the daughter baryon (in the hyperon rest frame) and the normal vector to the production plane \( \vec{N} \) (s. fig. \[1\]). The hyperon polarization \( P_N \) can be determined by the equation:

\[
I(\varphi^\ast, \theta^\ast) = I_0(\varphi^\ast) \cdot (1 + P_N(\varphi^\ast) \alpha \cos(\theta^\ast)) \tag{8}
\]

with the hyperon decay asymmetry parameter \( \alpha \)

\( \alpha(\Lambda \rightarrow p\pi^-) = 0.642 \pm 0.013, \alpha(\Sigma^+ \rightarrow p\pi^0) = -0.980 \pm 0.017 - 0.052 \[11\] \).

Several methods of evaluating \( P_N(\varphi^\ast) \) from the data can be applied: The count rate distribution versus \( \cos(\theta^\ast) \) can be fitted with a first order polynomial, the inclination represents \( P_N(\varphi^\ast) \) \( \alpha \). For this method the measured distribution has to be normalized to the distribution derived from Monte Carlo simulations. Another method is the calculation of the weighted sum: \( P_N(\varphi^\ast) \alpha = \sum \cos(\theta^\ast)/\sum \cos^2(\theta^\ast) \), which was developed in \[12\].

Here we apply the integral method, in order to prepare the variables for the derivation of the spin transfer equation. Depending on whether the hyperon decay particle is emitted above (same hemisphere as \( \vec{N} \)) or below (opposite hemisphere as \( \vec{N} \)) the production plane and on the azimuthal angle of the hyperon \( \varphi \) (left or right hemisphere) four count rates can be defined:
\(\varphi\) in the left hemisphere and the decay particle above the production plane:

\[
N_A^L(\varphi^*) = \int_0^1 I(\varphi^*, \theta^*) \, d\cos(\theta^*)
\]  

(9)

\(\varphi\) in the right hemisphere and the decay particle above the production plane:

\[
N_A^R(\varphi^*) = \int_0^1 I(\varphi^*, \theta^*) \, d\cos(\theta^*)
\]  

(10)

\(\varphi\) in the left hemisphere and the decay particle below the production plane:

\[
N_B^L(\varphi^*) = \int_{-1}^0 I(\varphi^*, \theta^*) \, d\cos(\theta^*)
\]  

(11)

\(\varphi\) in the right hemisphere and the decay particle below the production plane:

\[
N_B^R(\varphi^*) = \int_{-1}^0 I(\varphi^*, \theta^*) \, d\cos(\theta^*)
\]  

(12)

\(N_A^L(\varphi^*)\) and \(N_B^R(\varphi^*)\) correspond to the hyperon polarization in \((+\varepsilon_y)\) direction. They are identical because the cross section is symmetric in \(\varphi\), an unpolarized beam is assumed and detector asymmetries are ignored. The same holds for \(N_B^L(\varphi^*)\) and \(N_A^R(\varphi^*)\) for the hyperon polarization in the \((-\varepsilon_y)\) direction.

The integration of equation 8 yields:

\[
P_N(\varphi^*) = \frac{2}{\alpha} \epsilon_P(\varphi^*)
\]  

(13)

with the asymmetry

\[
\epsilon_P(\varphi^*) = \frac{(N_A^L(\varphi^*) + N_B^R(\varphi^*)) - (N_B^L(\varphi^*) + N_A^R(\varphi^*))}{N_A^L(\varphi^*) + N_B^R(\varphi^*) + N_B^L(\varphi^*) + N_A^R(\varphi^*)}
\]  

(14)

4. Depolarization coefficient

The depolarization coefficient \(D_{NN}\) describes the transfer of the beam-proton polarization to the hyperon. It is positive if the polarization of the hyperon follows the beam polarization, negative if the hyperon polarization is opposite to the beam polarization and zero, if the hyperon polarization is independent of the beam polarization. \(D_{NN}\) is given by the equation:

\[
I(\varphi^*, \Phi, \theta^*) = 
I_0(\varphi^*) \cdot (1 + A_N(\varphi^*)P_B \cos(\Phi) 
+ P_N(\varphi^*) \alpha \cos(\theta^*) 
+ D_{NN}(\varphi^*) \alpha P_B \cos(\Phi) \cos(\theta^*))
\]  

(15)
A formula of the intensity distribution dependent on the angles $\theta^*$ and $\Phi$ is given for the reaction $\bar{p}p \rightarrow \Lambda \bar{\Lambda}$. Formula 15 is derived from this formula by omitting the terms, which are related to the $\Lambda$ and exchanging the notation for the target polarization with the notation for the beam polarization. The term $D_{NN}(\theta^*) \alpha P_B \cos(\Phi) \cos(\theta^*)$ describes the polarization transfer from the beam-proton to the hyperon. This has to be nearly zero, if the normal of the production plane is almost perpendicular to the beam-proton polarization direction. The transfer is maximal if the normal to the production plane has the same direction as the beam-proton polarization. This dependency is described with the term $\cos(\Phi)$.

Eight count rates can be defined in order to determine $D_{NN}$. These are extensions to the count rates defined for the hyperon polarization by adding the two possible beam polarization states:

- $\varphi$ in the left hemisphere and the decay particle above the production plane and the beam polarization upwards:

$$N_L^{A\uparrow}(\theta^*) = \int_{0}^{1} \left[ \int_{0}^{1} I(\vartheta^*, \Phi, \theta^*) \ d\cos(\theta^*) \right] \ d\cos(\Phi) \quad (16)$$

- $\varphi$ in the left hemisphere and the decay particle above the production plane and the beam polarization downwards:

$$N_L^{A\downarrow}(\theta^*) = \int_{-1}^{0} \left[ \int_{0}^{1} I(\vartheta^*, \Phi, \theta^*) \ d\cos(\theta^*) \right] \ d\cos(\Phi) \quad (17)$$

- $\varphi$ in the right hemisphere and the decay particle above the production plane and the beam polarization upwards:

$$N_R^{A\uparrow}(\theta^*) = \int_{-1}^{0} \left[ \int_{0}^{1} I(\vartheta^*, \Phi, \theta^*) \ d\cos(\theta^*) \right] \ d\cos(\Phi) \quad (18)$$

- $\varphi$ in the right hemisphere and the decay particle above the production plane and the beam polarization downwards:

$$N_R^{A\downarrow}(\theta^*) = \int_{0}^{1} \left[ \int_{0}^{1} I(\vartheta^*, \Phi, \theta^*) \ d\cos(\theta^*) \right] \ d\cos(\Phi) \quad (19)$$

- $\varphi$ in the left hemisphere and the decay particle below the production plane and the beam polarization upwards:

$$N_L^{B\uparrow}(\theta^*) = \int_{0}^{1} \left[ \int_{-1}^{0} I(\vartheta^*, \Phi, \theta^*) \ d\cos(\theta^*) \right] \ d\cos(\Phi) \quad (20)$$

- $\varphi$ in the left hemisphere and the decay particle below the production plane and the beam polarization downwards:

$$N_L^{B\downarrow}(\theta^*) = \int_{-1}^{0} \left[ \int_{-1}^{0} I(\vartheta^*, \Phi, \theta^*) \ d\cos(\theta^*) \right] \ d\cos(\Phi) \quad (21)$$
\( \varphi \) in the right hemisphere and the decay particle below the production plane and the beam polarization upwards:

\[
N_{RB}^{B\uparrow}(\vartheta^*) = \int_{-1}^{0} \left[ \int_{-1}^{0} I(\vartheta^*, \Phi, \theta^*) \, d\cos(\theta^*) \right] \, d\cos(\Phi) \tag{22}
\]

\( \varphi \) in the right hemisphere and the decay particle below the production plane and the beam polarization downwards:

\[
N_{RB}^{B\downarrow}(\vartheta^*) = \int_{0}^{1} \left[ \int_{0}^{-1} I(\vartheta^*, \Phi, \theta^*) \, d\cos(\theta^*) \right] \, d\cos(\Phi) \tag{23}
\]

As the asymmetry produced with an up polarized beam in the left hemisphere is the same as in the right hemisphere produced with a down polarized beam (for a symmetrical acceptance and same beam polarization in both directions) the following count rates are pairwise identically:

\[ N_{LA}^{\uparrow}, N_{RB}^{\downarrow}, N_{LB}^{\uparrow}, N_{RB}^{\downarrow}, N_{RA}^{\uparrow}, N_{LA}^{\downarrow}, N_{RA}^{\downarrow}, N_{LA}^{\downarrow}. \]

For the following terms the hyperon polarization and the beam polarization have the same directions:

\[ \text{same: } N_{LA}^{\uparrow} + N_{RB}^{\downarrow} + N_{RA}^{\uparrow} + N_{LA}^{\downarrow} \]

For the following terms the hyperon polarization and the beam polarization have opposite directions:

\[ \text{opposite: } N_{LA}^{\downarrow} + N_{RB}^{\uparrow} + N_{RA}^{\downarrow} + N_{LA}^{\uparrow} \]

The evaluation of the integrals in equations 19 - 23 yields:

\[
N_{\text{same}}(\vartheta^*) = I_0(\vartheta^*)(4 + D_{NN}(\vartheta^*)\alpha P_B) \tag{24}
\]

\[
N_{\text{opposite}}(\vartheta^*) = I_0(\vartheta^*)(4 - D_{NN}(\vartheta^*)\alpha P_B) \tag{25}
\]

with the definition of the normalized rate differences as

\[
\epsilon_D(\vartheta^*) = \frac{N_{\text{same}}(\vartheta^*) - N_{\text{opposite}}(\vartheta^*)}{N_{\text{same}}(\vartheta^*) + N_{\text{opposite}}(\vartheta^*)} \tag{26}
\]

the depolarization coefficient can be evaluated:

\[
D_{NN}(\vartheta^*) = \frac{4}{\alpha P_B} \epsilon_D(\vartheta^*) \tag{27}
\]

The statistical error of the coefficient is given by

\[
\Delta D_{NN}(\vartheta^*) = \frac{8}{\alpha P_B} \sqrt{\frac{N_{\text{same}}(\vartheta^*)N_{\text{opposite}}(\vartheta^*)}{(N_{\text{same}}(\vartheta^*) + N_{\text{opposite}}(\vartheta^*))^3}} \tag{28}
\]
4.1. Application to measurements with limited acceptance

For experiments with limited acceptance the relevant angular regions have to be taken into account for the integration limits. In the Brookhaven Multiparticle Spectrometer [3] for example only particles are detected, which are produced on the left side of the proton beam. In addition, the acceptance restricts the production plane to be nearly horizontal, this allows in the derivation of the $D_{NN}$ formula to set $\Phi = 0$ or $\pi$ (in equation 12 of [3]). Regarding only the count rate of equations 16 - 23 in the left hemisphere and setting $\Phi = 0$ for beam polarization upwards and $\Phi = \pi$ for beam polarization downwards the following equations are obtained:

\begin{align}
N_{A}^\uparrow & = 1 + P_B A_N + \frac{1}{2} P_N \alpha + \frac{1}{2} P_B D_{NN} \\
N_{A}^\downarrow & = 1 - P_B A_N - \frac{1}{2} P_N \alpha - \frac{1}{2} P_B D_{NN} \\
N_{B}^\uparrow & = 1 - P_B A_N - \frac{1}{2} P_N \alpha - \frac{1}{2} P_B D_{NN} \\
N_{B}^\downarrow & = 1 - P_B A_N - \frac{1}{2} P_N \alpha + \frac{1}{2} P_B D_{NN}
\end{align}

Inserting these count rates into equations 13 and 14 the following relation is obtained:

\[ P_{\uparrow\downarrow} = \frac{P_N \pm P_B D_{NN}}{1 \pm P_B A_N} \]

which corresponds to equation 13 in [3].
Applying the same conditions of $\Phi$ on the equation 15 yields:

\begin{equation}
I(\vartheta^*, \Phi = 0, \theta^*)^\uparrow = \nonumber
I_0(\vartheta^*) \cdot (1 + A_N(\vartheta^*) P_B + P_N(\vartheta^*) \alpha \cos(\theta^*) + D_{NN}(\vartheta^*) \alpha P_B \cos(\theta^*))
\end{equation}

\begin{equation}
I(\vartheta^*, \Phi = 0, \theta^*)^\downarrow = \nonumber
I_0(\vartheta^*) \cdot (1 - A_N(\vartheta^*) P_B + P_N(\vartheta^*) \alpha \cos(\theta^*) - D_{NN}(\vartheta^*) \alpha P_B \cos(\theta^*))
\end{equation}

The asymmetry of the intensities in equations 33 and 34 is:

\begin{equation}
\frac{I(\theta^*)^\uparrow - I(\theta^*)^\downarrow}{I(\theta^*)^\uparrow + I(\theta^*)^\downarrow} = \frac{P_B A_N + P_B D_{NN} \alpha \cos(\theta^*)}{1 + \alpha P_N \cos(\theta^*)}
\end{equation}

this corresponds to equations 15 and 16 in [3].

5. Conclusion

For the associated strangeness reactions $\overline{p}p \to pK^+\Lambda$ and $\overline{p}p \to pK^0\Sigma^+$ it is shown that the depolarization coefficient according to the “integral method” is given by

\begin{equation}
D_{NN}(\vartheta^*) = \frac{4}{\alpha P_B \epsilon_D(\vartheta^*)}
\end{equation}

The asymmetry $\epsilon_D(\vartheta^*)$ can be expressed by 8 different count rates depending on the beam polarization, the position in the left or right hemisphere, and the position above or below the production plane. Applying restrictions in the acceptance of other measurements as for instance of the Brookhaven Multiparticle Spectrometer onto this formula, the corresponding formulas are derived.

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