NONEXISTENCE OF NONCONSTANT POSITIVE STEADY STATES OF A DIFFUSIVE PREDATOR-PREY MODEL

SHANSHAN CHEN

Department of Mathematics, Harbin Institute of Technology
Weihai, Shandong, 264209, China

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Abstract. In this paper, we investigate a diffusive predator-prey model with a general predator functional response. We show that there exist no nonconstant positive steady states when the interaction between the predator and prey is strong. This result implies that the global bifurcating branches of steady state solutions are bounded loops for a predator-prey model with Holling type III functional response.

1. Introduction. Since the pioneering work of Holling [9, 10], the predator functional responses, especially the Holling type II functional response, have been investigated extensively. For example, Yi et al. [33] studied the following predator-prey model

\[
\begin{align*}
\frac{\partial u}{\partial t} &= d_1 \Delta u + ru \left(1 - \frac{u}{k}\right) - \frac{buv}{1 + au}, \quad x \in \Omega, \ t > 0, \\
\frac{\partial v}{\partial t} &= d_2 \Delta v - dv + \frac{cuv}{1 + au}, \quad x \in \Omega, \ t > 0,
\end{align*}
\]

(1.1)

and the induced complex spatiotemporal patterns through bifurcations, such as spatially inhomogeneous periodic orbits and steady state solutions, were addressed. Here \(u(x,t)\) and \(v(x,t)\) are the densities of the prey and predator at time \(t\) and location \(x\) respectively, and \(d_1, d_2, r, d, k, b, c\) and \(\alpha\) are all positive constants. Then, Peng and Shi [19] showed that the global bifurcating branches of steady state solutions of system (1.1) are bounded loops. Du and Lou [5, 6] studied a slightly different model,

\[
\begin{align*}
\frac{\partial u}{\partial t} &= d_1 \Delta u + u(a - u) - \frac{buv}{1 + mu}, \quad x \in \Omega, \ t > 0, \\
\frac{\partial v}{\partial t} &= d_2 \Delta v + v(d - v) + \frac{cuv}{1 + mu}, \quad x \in \Omega, \ t > 0,
\end{align*}
\]

(1.2)

where the growth rate of the predator is logistic type, and the existence and nonexistence of the nonconstant steady states were investigated for large \(m\). We refer to [7, 8] for the bifurcations and pattern formations of model (1.2) with the spatial heterogeneity. Moreover, we point out that the corresponding ODE systems of models (1.1) and (1.2) are classical in mathematical biology, and they are usually

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attributed to Rosenzweig and MacArthur [22], see e.g. [25]. The dynamics of the ODE systems can be found in [4, 11, 12] and references therein.

Recently, Wang [27] studied the following nondimensionalized diffusive predator-prey model with Holling type III functional response and no flux boundary conditions,

\[
\begin{align*}
\frac{\partial u}{\partial t} &= d_1 \Delta u + u (1 - u) - \frac{mu^2 v}{a^2 + u^2}, & x \in \Omega, t > 0, \\
\frac{\partial v}{\partial t} &= d_2 \Delta v - dv + \frac{mu^2 v}{a^2 + u^2}, & x \in \Omega, t > 0, \\
\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} &= 0, & x \in \partial \Omega, t > 0, \\
u(x, 0) = u_0(x) \geq (\neq) 0, & v(x, 0) = v_0(x) \geq (\neq) 0, & x \in \Omega,
\end{align*}
\]

(1.3)

and the existence of spatially inhomogeneous periodic orbits and nonconstant steady state solutions were obtained by the bifurcation method and Leray-Schauder degree theory. The Hopf bifurcation of model (1.3) was also investigated in [26, 32]. We remark that there are also many results on the pattern formations for other diffusive predator-prey models and chemical reaction-diffusion models, see [1, 13, 17, 18, 20, 21, 23, 28, 29, 30, 31, 34] and references therein.

In this paper, we consider the following nondimensionalized diffusive predator-prey model [2, 27],

\[
\begin{align*}
\frac{\partial u}{\partial t} &= d_1 \Delta u + u (1 - u) - \frac{mu^r v}{a^r + u^r}, & x \in \Omega, t > 0, \\
\frac{\partial v}{\partial t} &= d_2 \Delta v - dv + \frac{mu^r v}{a^r + u^r}, & x \in \Omega, t > 0, \\
\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} &= 0, & x \in \partial \Omega, t > 0, \\
u(x, 0) = u_0(x) \geq (\neq) 0, & v(x, 0) = v_0(x) \geq (\neq) 0, & x \in \Omega,
\end{align*}
\]

(2.1)

where \(u(x, t)\) and \(v(x, t)\) are the densities of the prey and predator at time \(t\) and location \(x\) respectively, \(d_1, d_2, m, d, r\) and \(a\) are all positive constants, \(\Omega\) is a bounded domain in \(\mathbb{R}^N (N \leq 3)\) with a smooth boundary \(\partial \Omega\), and \(m\) measures the interaction between the predator and prey. If \(r = 2\), then model (1.4) is reduced to (1.3). In Section 2, we prove that, for \(r > 1\), system (1.4) has no nonconstant positive steady states when \(m\) is sufficiently large. The method used here is motivated by [19], and we find that the result for the case of \(r = 1\) in [19] can be extended to the case of \(r > 1\) or a more general predator functional response, which satisfies Eq. (3.2). Moreover, our result supplements the results in [27] and implies that each global bifurcating branch of steady state solutions of model (1.3) obtained in [27] is a bounded loop, which connects at least two different bifurcation points, (see Section 3).

2. Main results. In this section, we consider the steady states of model (1.4) for large \(m\), which satisfy

\[
\begin{align*}
-d_1 \Delta u &= u (1 - u) - \frac{mu^r v}{a^r + u^r}, & x \in \Omega, \\
-d_2 \Delta v &= -dv + \frac{mu^r v}{a^r + u^r}, & x \in \Omega, \\
\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} &= 0, & x \in \partial \Omega.
\end{align*}
\]

(2.1)
Let \( w = m^{\frac{1}{2}} u, z = m^{\frac{1}{2}} v \) and \( \rho = 1/m^{\frac{1}{2}} \). Then \( w \) and \( z \) satisfy

\[
\begin{aligned}
&-d_1 \Delta w = w (1 - \rho w) - \frac{w^r z}{a^r + \rho^r w^r}, \quad x \in \Omega, \\
&-d_2 \Delta z = - dz + \frac{w^r z}{a^r + \rho^r w^r}, \quad x \in \Omega, \\
&\partial_n w = \partial_n z = 0, \quad x \in \partial \Omega.
\end{aligned}
\]  

(2.2)

To derive a priori estimates for the positive solutions of system (2.2), we first recall the following three well-known results. The first is a maximum principle from \([16]\).

**Lemma 2.1** ([16]). Suppose that \( \Omega \) is a bounded smooth domain in \( \mathbb{R}^N \), \( g \in C(\overline{\Omega} \times \mathbb{R}) \), and \( z \in C^2(\Omega) \cap C^1(\overline{\Omega}) \) satisfies

\[
\begin{aligned}
&\Delta z + g(x, z) \geq 0, \quad x \in \Omega, \\
&\partial_n z \leq 0, \quad x \in \partial \Omega.
\end{aligned}
\]

If \( z(x_0) = \max_{x \in \Omega} z \), then \( g(x_0, z(x_0)) \geq 0 \).

The second is from \([14]\).

**Lemma 2.2** ([14]). If \( \Omega \) is a bounded smooth domain in \( \mathbb{R}^N \), \( d \) is a nonnegative constant, and \( z \in W^{1,2}(\Omega) \) is a non-negative weak solution of the following inequalities

\[
\begin{aligned}
&-\Delta z + dz \geq 0, \quad x \in \Omega, \\
&\partial_n z \leq 0, \quad x \in \partial \Omega,
\end{aligned}
\]

then, for any \( q \in \left[1, \frac{N}{N-2}\right) \), there exists a positive constant \( C \), determined only by \( q, d \) and \( \Omega \), such that

\[
\|z\|_q \leq C \inf_{x \in \Omega} z.
\]

Finally, we cite a Harnack inequality from \([15, 20]\).

**Lemma 2.3** ([15]). If \( \Omega \) is a bounded smooth domain in \( \mathbb{R}^N \), \( c(x) \in L^q(\Omega) \) for some \( q > N/2 \), and \( z \in W^{1,2}(\Omega) \) is a non-negative weak solution of the following problem

\[
\begin{aligned}
&\Delta z + c(x)z = 0, \quad x \in \Omega, \\
&\partial_n z = 0, \quad x \in \partial \Omega,
\end{aligned}
\]

then, there exists a positive constant \( C \), determined only by \( \|c(x)\|_q \), \( q \), and \( \Omega \), such that

\[
\sup_{x \in \Omega} z \leq C \inf_{x \in \Omega} z.
\]

Then, we have a priori estimate for the positive solutions of system (2.2).

**Theorem 2.4.** Assume that \( r > 1, d_1, d_2, d, a \) and \( \rho \) are all positive constants, \( \Omega \) is a bounded domain in \( \mathbb{R}^N \) (\( N \leq 3 \)) with a smooth boundary \( \partial \Omega \), and \( (w_\rho, z_\rho) \) is a positive solution of system (2.2). Then the following two statements are true.

(i) There exists \( \overline{C} > 0 \) such that

\[
\sup_{\rho > 0} \sup_{x \in \Omega} w_\rho, \sup_{\rho > 0} \sup_{x \in \Omega} z_\rho \leq \overline{C}.
\]  

(2.3)
(ii) There exists $M > 0$ such that
\[
\inf_{0 < \rho < M} w_{\rho} > 0 \quad \text{and} \quad \inf_{0 < \rho < M} z_{\rho} > 0.
\]
Equation (2.4)

That is, there exists $C > 0$, which depends on $M$, such that $w_{\rho}, z_{\rho} \geq C$ for all $0 < \rho < M$ and $x \in \Omega$.

**Proof.** We first derive the existence of the upper bounds for $\{w_{\rho}\}_{\rho > 0}$ and $\{z_{\rho}\}_{\rho > 0}$. Since
\[
d_1 \Delta \rho w \leq \rho (1 - \rho w),
\]
it follows from Lemma 2.1 that $0 \leq \rho w_{\rho} \leq 1$ for all $\rho > 0$. Dividing the second equation of (2.2) by $z_{\rho}$ and integrating the result over $\Omega$, we get
\[
\int_{\Omega} \left( \frac{w_{\rho}}{\rho^n + \rho^r w_{\rho}^r} - d \right) dx = -d_2 \int_{\Omega} \Delta z_{\rho} dx = -d_2 \int_{\Omega} \frac{|\nabla z_{\rho}|^2}{z_{\rho}^2} \leq 0,
\]
which yields
\[
\int_{\Omega} w_{\rho} dx \leq d(a^n + 1)|\Omega| \quad \text{for all} \quad \rho > 0.
\]
Equation (2.5)

Then, integrating each equation of system (2.2) over $\Omega$, we have
\[
d \int_{\Omega} z_{\rho} dx \leq \int_{\Omega} w_{\rho} dx \leq \left( \int_{\Omega} w_{\rho}^r dx \right)^{\frac{1}{r}} |\Omega|^{1 - \frac{1}{r}},
\]
which leads to
\[
\inf_{\Omega} z_{\rho} \leq (a^n + 1)^{\frac{1}{r}} d^{\frac{1}{r} - 1} \quad \text{for all} \quad \rho > 0.
\]
Equation (2.6)

By virtue of Lemma 2.2, we see that, for any $p \in [1, p^*)$, there exists a positive constant $C_0$, depending on $p$, such that
\[
\|z_{\rho}\|_{p} \leq C_0 \inf_{\Omega} z_{\rho} \quad \text{for all} \quad \rho > 0.
\]
Equation (2.7)

Here $p^* = \infty$ for $N = 1$ or 2, and $p^* = 3$ for $N = 3$. In the following, we denote $C_0$ by $C_0(p)$ to avoid confusion. Then, we claim that there exist $q > \frac{N}{2}$ and a positive constant $C_1$, depending on $q$, such that
\[
\|w_{\rho}^{r - 1} z_{\rho}\|_{q} \leq C_1 \quad \text{for all} \quad \rho > 0.
\]
Equation (2.8)

We first consider the case of $N = 1$ or 2. Due to Eqs. (2.5)-(2.7) and the Hölder inequality, we see that for any given $q \in \left(1, \frac{1}{r - 1}\right)$,
\[
\int_{\Omega} w_{\rho}^{(r - 1)q} z_{\rho} dx \leq \left( \int_{\Omega} w_{\rho}^{r} dx \right)^{\frac{1}{r - 1}q} \left( \int_{\Omega} \frac{rq}{\rho^n + \rho^r w_{\rho}^r} dx \right)^{1 - \frac{1}{r - 1}q}
\]
\[
\leq \left( \int_{\Omega} w_{\rho}^{r} dx \right)^{\frac{1}{r - 1}q} \left[ C_0 \left( \frac{rq}{r - (r - 1)q} \right) \inf_{\Omega} z_{\rho} \right]^q
\]
\[
\leq \left( d(a^n + 1)|\Omega| \right)^{\frac{1}{r - 1}q} \left[ C_0 \left( \frac{rq}{r - (r - 1)q} \right) (a^n + 1)^{\frac{1}{r}} d^{\frac{1}{r} - 1} \right]^q.
\]

Then, we consider the case of $N = 3$. Multiplying $w_{\rho}^{r - 1}$ to the first equation of (2.2) and integrating the result over $\Omega$, we see from Eq. (2.5) that
\[
\int_{\Omega} w_{\rho}^{2r - 1} z_{\rho} dx \leq (a^n + 1) \int_{\Omega} w_{\rho}^{r} dx \leq d(a^n + 1)^2 |\Omega|.
\]
Equation (2.9)
Note that, for \( q \in \left( \frac{3}{2}, \frac{6r-3}{4r-3} \right) \),

\[
\frac{(r - 1)q}{2r - 1} < 1 \quad \text{and} \quad \frac{rq}{(2r - 1) - (r - 1)q} < 3.
\]

Then, by virtue of the Hölder inequality and Eqs. (2.6), (2.7) and (2.9), we obtain that, for any given \( q \in \left( \frac{3}{2}, \frac{6r-3}{4r-3} \right) \),

\[
\int_{\Omega} w_\rho^{(r-1)q} z_\rho^2 \, dx \leq \left( \int_{\Omega} w_\rho^{2r-1} z_\rho \, dx \right)^{\frac{(r-1)q}{2r-1}} \left( \int_{\Omega} z_\rho^{\frac{rq}{r - 1} - (r - 1)q} \, dx \right)^{1 - \frac{(r-1)q}{2r-1}}
\]

\[
\leq [d(a^r + 1)^{2|\Omega|}]^{\frac{(r-1)q}{2r-1}} \left[ C_0 \left( \frac{rq}{(2r - 1) - (r - 1)q} \right) \inf_{\Omega} z \right]^{\frac{rq}{2r-1}}
\]

\[
\leq [d(a^r + 1)^{2|\Omega|}]^{\frac{(r-1)q}{2r-1}} \left[ C_0 \left( \frac{rq}{(2r - 1) - (r - 1)q} \right) (a^r + 1)^{ \frac{1}{2} - \frac{1}{2} } \right]^{\frac{rq}{2r-1}}.
\]

Therefore, the claim is true and Eq. (2.8) holds. This, combined with Lemma 2.3, implies that there exists a positive constant \( C_2 \) such that

\[
\sup_{\Omega} w_\rho \leq C_2 \inf_{\Omega} w_\rho \quad \text{for all } \rho > 0. \tag{2.10}
\]

It follows from Eq. (2.5) that \( \inf_{\Omega} w_\rho \leq d^{\frac{1}{2}} (a^r + 1)^{ \frac{1}{2} } \) for all \( \rho > 0 \). Consequently, there exists a positive constant \( C_3 \) such that

\[
\sup_{\Omega} w_\rho \leq C_3 \quad \text{for all } \rho > 0. \tag{2.11}
\]

Again, by Lemma 2.3 and Eq. (2.11), we see that there exists a positive constant \( C_4 \) such that

\[
\sup_{\Omega} z_\rho \leq C_4 \inf_{\Omega} z_\rho \quad \text{for all } \rho > 0. \tag{2.12}
\]

Then, it follows from Eq. (2.6) that there exists a positive constant \( C_5 \) such that

\[
\sup_{\Omega} z_\rho \leq C_5 \quad \text{for all } \rho > 0. \tag{2.13}
\]

Choosing \( \mathcal{C} = \max\{C_3, C_5\} \), we see that Eq. (2.3) holds.

Now, we prove part \((ii)\). We first claim that there exists \( M_1 > 0 \) such that \( \inf_{0 < \rho < M_1} \inf_{x \in \Omega} w_\rho > 0 \). If it is not true, then there exists a sequence \( \{ \rho_k \}_{k=1}^\infty \) such that \( \lim_{k \to \infty} \rho_k = 0 \) and \( \lim_{k \to \infty} \inf_{x \in \Omega} w_{\rho_k} = 0 \). By Eq. (2.10), we obtain that \( w_{\rho_k} \to 0 \) uniformly on \( \overline{\Omega} \) as \( k \to \infty \), which yields

\[
\int_{\Omega} \left( d - \frac{w_{\rho_k}}{a^r + \rho_k^r w_{\rho_k}} \right) z_{\rho_k} \, dx > 0
\]

for sufficiently large \( k \). This is a contradiction, and the claim is proved. Then we claim that there exists \( M_2 > 0 \) such that \( \inf_{0 < \rho < M_2} \inf_{x \in \Omega} z_\rho > 0 \). To the contrary, there exists a sequence \( \{ \rho_j \}_{j=1}^\infty \) such that \( \lim_{j \to \infty} \rho_j = 0 \) and \( \lim_{j \to \infty} \inf_{x \in \Omega} z_{\rho_j} = 0 \), which also implies that \( z_{\rho_j} \to 0 \) uniformly on \( \overline{\Omega} \) as \( j \to \infty \) from Eq. (2.12). Noticing that \( \sup_{\rho > 0} \sup_{x \in \Omega} w_\rho \leq \mathcal{C} \), we have

\[
\int_{\Omega} w_{\rho_j} \left( 1 - \rho_j w_{\rho_j} - \frac{w_{\rho_j}^{-1} z_{\rho_j}}{a^r + \rho_j^r w_{\rho_j}} \right) \, dx > 0
\]

for sufficiently large \( j \), which is a contradiction. Therefore, the claim is proved. Choosing \( M = \min\{M_1, M_2\} \), we see that Eq. (2.4) holds.
For $\rho = 0$, we have the following result on the steady states of system (2.2).

**Theorem 2.5.** Suppose that $r > 1$, $\rho = 0$, and $d_1$, $d_2$, $d$, and $a$ are all positive constants. Then system (2.2) has a unique positive solution $(w_*, z_*) = (ad^\frac{1}{r}, ad^\frac{1}{r-1})$.

**Proof.** One can easily check that $(w_*, z_*)$ is the unique constant positive steady state of system (2.2) for $\rho = 0$. Suppose that $(w, z)$ is a positive steady state of system (2.2) for $\rho = 0$, and set
\[
V(w, z) = \int_\Omega \left\{ \frac{w^r - w_*^r}{w^r} \left[d_1 \Delta w + w \left(1 - \frac{w^{r-1}}{a^r}\right)\right] \right\} dx
+ \int_\Omega \left\{ \frac{z - z_*}{z} \left[d_2 \Delta z + z \left(-d + \frac{w^r}{a^r}\right)\right] \right\} dx.
\]
Clearly, $V(w, z) = 0$, and after a careful calculation, we also have
\[
V(w, z) = -\int_\Omega \left( r d_1 \frac{w_*^r |\nabla w|^2}{w^r+1} + d_2 \frac{z_* |\nabla z|^2}{z^2} \right) dx
+ \int_\Omega \left( w^r - w_*^r \right) \left( \frac{1}{w^{r-1}} - \frac{1}{w_*^{r-1}} \right) dx.
\]
which leads to $(w, z) = (w_*, z_*)$. This completes the proof. \hfill \Box

Now, based on Theorem 2.4 and 2.5, we have the following result on the nonexistence of nonconstant positive solutions of system (2.2) for small $\rho$ (or equivalently, large $m$ for system (2.1)).

**Theorem 2.6.** Assume that $r > 1$, $d_1$, $d_2$, $d$, and $a$ are all positive constants, and $\Omega$ is a bounded domain in $\mathbb{R}^N$ $(N \leq 3)$ with a smooth boundary $\partial \Omega$. Then there exists a positive constant $\rho_* = \rho_*(d_1, d_2, d, a, r)$ such that, for $\rho \in (0, \rho_*)$, system (2.2) has a unique constant positive solution and no nonconstant positive solutions.

**Proof.** We argue indirectly and assume that there exists $\{\rho_i\}_{i=1}^\infty$ such that
\[
\lim_{i \to \infty} \rho_i = 0,
\]
and system (2.2) has a nonconstant positive steady state $(w_i(x), z_i(x))$ for any $\rho = \rho_i$. As in [3, 19], by virtue of Theorems 2.4 and 2.5, the standard regularity theory, and the embedding theorems, we see that there exists a subsequence $\{i_k\}_{k=1}^\infty$ such that $(w_{i_k}(x), z_{i_k}(x)) \to (w_*, z_*)$ in $C^2(\overline{\Omega})$ as $k \to \infty$, where $(w_*, z_*) = (ad^\frac{1}{r}, ad^\frac{1}{r-1})$ is the unique positive solution of system (2.2) for $\rho = 0$. A direct calculation implies that all the eigenvalues of the linearized system at $(w_*, z_*)$ are negative when $\rho = 0$. Then, it follows from the implicit function theorem that there exists $\rho_* > 0$ such that, for $\rho \in (0, \rho_*)$, system (2.2) has a unique positive solution in the neighborhood of $(w_*, z_*)$ in $C^1(\overline{\Omega})$, which is constant and locally asymptotically stable for the corresponding parabolic system. Therefore, $(w_{i_k}(x), z_{i_k}(x))$ is constant for sufficiently large $k$, which is a contradiction. This completes the proof. \hfill \Box

Then we have the following result on the nonexistence of the nonconstant positive steady states of system (1.4) when the interaction between the predator and prey is strong.

**Corollary 2.7.** Assume that $r > 1$, $d_1$, $d_2$, $d$, and $a$ are all positive constants, and $\Omega$ is a bounded domain in $\mathbb{R}^N$ $(N \leq 3)$ with a smooth boundary $\partial \Omega$. Then there exists a positive constant $m_* = m_*(d_1, d_2, d, a, r)$ such that, for $m > m_*$, system
Nonexistence of nonconstant steady states

1.4 has a unique constant positive steady state and no nonconstant positive steady states.

3. Global bifurcations and generalization. In this section, we first give some remarks on the global bifurcations of steady state solutions for model (1.3). We recall from [27] that when $m > d$, system (1.3) has a unique constant positive equilibrium $(\lambda(m), v(\lambda(m)))$, where

$$\lambda(m) = \sqrt{\frac{a^2 d}{m - d}}, \quad v(\lambda(m)) = \frac{(1 - \lambda)(a^2 + \lambda)}{m \lambda},$$

and if the assumptions of Theorem 4.12 in [27] are satisfied, then there exists a sequence of steady state bifurcation points $\lambda_i$, $v(\lambda_i)$ such that $\lambda_i$ is equivalent to $m_i$. We see that there also exists a sequence of steady state bifurcation points $m_i$. As in [19], by virtue of Corollary 2.7, Theorem 4.12 in [27] and the global bifurcation theorem in [24], we see that each global branch of steady state solutions bifurcating from $(m_i, \lambda_i, v(\lambda_i))$ is a bounded loop, which contains another $(m_j, \lambda_i, v(\lambda_j))$ for $j \neq i$. This result supplements Theorem 4.12 in [27].

Finally, we remark that the results in Section 2 can be extended to a more general diffusive predator-prey model,

\begin{align*}
\frac{\partial u}{\partial t} &= d_1 \Delta u + ru \left(1 - \frac{u}{k}\right) - mf(u)v, \quad x \in \Omega, \ t > 0, \\
\frac{\partial v}{\partial t} &= d_2 \Delta v - dv + mf(u)v, \quad x \in \Omega, \ t > 0, \\
\partial_\nu u &= \partial_\nu v = 0, \quad x \in \partial \Omega, \ t > 0, \\
u(x, 0) &= u_0(x) \geq (\neq)0, \quad v(x, 0) = v_0(x) \geq (\neq)0, \quad x \in \Omega,
\end{align*}

where $f(u)$ satisfies

$$\lim_{u \to 0^+} \frac{f(u)}{u^r} = \alpha > 0 \quad \text{for some} \quad r \geq 1. \quad (3.2)$$

Actually, let

$$g(u) = \begin{cases} f(u) \quad &\text{if} \quad u > 0, \\ \alpha \quad &\text{if} \quad u = 0, \end{cases} \quad (3.3)$$

and then model (3.1) can be rewritten as follows:

\begin{align*}
\frac{\partial u}{\partial t} &= d_1 \Delta u + ru \left(1 - \frac{u}{k}\right) - mg(u)u^r v, \quad x \in \Omega, \ t > 0, \\
\frac{\partial v}{\partial t} &= d_2 \Delta v - dv + mg(u)u^r v, \quad x \in \Omega, \ t > 0, \\
\partial_\nu u &= \partial_\nu v = 0, \quad x \in \partial \Omega, \ t > 0, \\
u(x, 0) &= u_0(x) \geq (\neq)0, \quad v(x, 0) = v_0(x) \geq (\neq)0, \quad x \in \Omega.
\end{align*}

Therefore, if $g(u)$ is smooth, then we also have the similar result as system (1.4). That is, system (3.1) has no nonconstant positive steady states, when $m$ is sufficiently large. Here we omit the details.

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E-mail address: chens@hit.edu.cn