Top mass effects in Higgs production at next-to-next-to-leading order QCD: virtual corrections

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Abstract

Top quark mass suppressed terms are calculated for the virtual amplitude for Higgs production in gluon fusion at three-loop level, i.e. \( \mathcal{O}(\alpha_s^3) \). The method of asymptotic expansions in its automated form is used to evaluate the first three non-vanishing orders in terms of \( \frac{M_H^2}{M_t^2} \), where the first order corresponds to the known results of the effective Lagrangian approach.

1 Introduction

Radiative corrections to Higgs production through gluon fusion are known to be unusually large \[1, 2, 3, 4\]. The inclusive next-to-next-to-leading order (NNLO) cross section \( \sigma(pp/\bar{p}p \rightarrow H + X) \) exceeds the LO prediction by roughly a factor of two at LHC energies, and even up to a factor of three at the Tevatron \[8, 9\]. Recent compilations of the currently available contributions to the production cross section can be found in Refs. \[8, 9\].

The current NNLO prediction is based on the assumption that the top mass dependence is largely determined by the LO expression, while the higher order terms can be evaluated in the limit of infinitely heavy top mass \[10, 3, 4, 11\]. At NLO, where a comparison with the full mass dependence of the cross section is possible, the heavy-top approximation is valid at the 2-3% level for Higgs masses \( M_H < 2M_t \) (see, e.g., Ref. \[12\]). Even at \( M_H \approx 1 \text{ TeV} \), the deviation from the full NLO result amounts to only about 10%.

The fact that the heavy top limit works so well is at first sight surprising, because it assumes that \( M_t \) is larger than any other scale in the process. This is certainly not the case at the LHC with a prospected hadronic center-of-mass energy of \( \sqrt{s} = 14 \text{ TeV} \). However, one can argue that since the cross section is dominated by soft gluon radiation parton scatterings with energies \( \sqrt{s} \) much larger than \( 2M_t \) are strongly suppressed.

It is indeed observed that an expansion of the partonic cross section \( \hat{\sigma} \) in powers of \( (1-z) \), where \( z = \frac{M_H^2}{\hat{s}} \), converges rather quickly to the exact result \[5\]. On the other hand, resummation of the soft terms does not lead to a big effect at any of the three lowest orders in perturbation theory \[13\].

Recently it has been suggested that the size of the radiative corrections is due to the transition from space- to time-like momenta, and in fact, numerical studies show that the
bulk of the radiative corrections can be obtained by resumming the leading \( \pi^2 \)-terms that arise from this transition \[14\].

These unresolved issues leave one with a certain amount of doubt as to the use of the heavy-top limit at NNLO. There is however surprisingly little activity in the field that addresses the validity of this approximation. Besides the NLO calculations for the inclusive cross section mentioned before \[10, 3, 4, 11\], there are studies concerning the mass effects on differential distributions \[15, 16, 17\] which allow one to derive validity ranges on the kinematical variables. Furthermore, in Ref. \[18\], the effects of the partonic high-energy region on the total cross section have been studied by deriving the leading behaviour in this limit.

A rather direct way to check the heavy-top limit is to evaluate formally subleading terms. In this paper, we consider them for the purely virtual corrections at NNLO. While they do not correspond to a physical quantity, they constitute an important gauge-invariant ingredient to the full inclusive cross section. Note that at NLO, the virtual corrections are known in closed analytical form for arbitrary values of \( M_t \) \[19, 20, 21\].

Our approach is very similar to the calculation of the top mass suppressed terms to the Higgs decay rate into gluons, described in Ref. \[22\]. One might be tempted to use this result obtained for the decay rate as an estimate of the effects for the gluon fusion process. However, one should recall that the kinematics of the two processes are very different. In particular, the top quark mass is indeed the largest scale for the decay, so that the expansion in \( M_H/M_t \) remains within the radius of convergence. This is not the case for the higher order corrections to the gluon fusion production process involving real radiation of gluons and quarks. The partonic center-of-mass energy \( \sqrt{s} \) can well exceed the threshold value of \( 2M_t \), and a series expansion in the limit of large top mass becomes questionable. For the purely virtual effects though, which are the subject of this paper, the partonic center-of-mass energy is fixed to \( M_H \) which, according to the limits derived from electroweak precision fits, can safely be assumed to be lighter than twice the top mass. They will therefore be a useful ingredient for any possible treatment of the full hadronic cross section, be it inclusive or exclusive.

2 Method

Sample diagrams that contribute to the virtual corrections to gluon fusion at LO, NLO, and NNLO are shown in Fig. 1. An efficient and algorithmic procedure for evaluating them in terms of a consistent expansion in \( M_H/M_t \) is the well-known method of asymptotic expansions (see, e.g., Ref. \[23\]). In our case, it expresses the original diagrams as a sum of convolutions of massive vacuum with massless vertex integrals. The diagrammatic representation of this procedure is shown for two particular diagrams in Figs. 2 and 3.

We generate the diagrams with the help of qgraf \[24\] and pass them to q2e/exp \[25, 26\], which automatically carries out the expansion. The resulting 1-, 2-, and 3-loop vacuum integrals are evaluated by MATAD \[27\]. For the 1- and 2-loop vertex integrals we use the method of Ref. \[28\] by applying the relevant modifications \[29\] to MINCER \[30\].

The colour and Lorentz structure of the physical amplitude is given by

\[
P_{\mu\nu}^{ab}(q_1, q_2) \equiv g^{ab} (q_1 \cdot q_2 g_{\mu\nu} - q_1,\mu q_2,\nu),
\]

(1)
Figure 1: Examples of Feynman diagrams contributing to the virtual corrections at NNLO in the gluon fusion process. The solid lines denote top quarks, the springy lines are gluons, and the dashed line is the Higgs boson. In diagram (i), the bubble insertion can be a top quark or any other quark.

Figure 2: Diagrammatic representation for the asymptotic expansion of a particular Feynman diagram in the limit $M_H^2 \ll 4M_t^2$. The diagrams left of $\otimes$ represent subdiagrams of the original diagram that are to be expanded in the momenta corresponding to the dotted external lines before the loop integration. In this way, it is apparent that the original integral, depending on $M_H^2$ and $M_t^2$, is decomposed into products of “tadpole” integrals with vanishing external momenta and massless vertex integrals. The shaded blob in the diagrams right of $\otimes$ represents an effective vertex given by the result of the diagram left of $\otimes$ (for details of asymptotic expansions, see Ref. [23], for example). The three terms right of “$\rightarrow$” are proportional to $N_t^3$, $N_t^2N_h$, and $N_tN_h^2$, respectively (cf. Eq. (3) below). Subdiagrams without external mass scales are not shown.
Figure 3: Diagrammatic representation for the asymptotic expansion of the Feynman diagram in Fig. 1 (i) in the limit $M^2_H \ll 4M^2_t$, when the bubble insertion is a top quark. The second term on the r.h.s. is a source of the term $\sim \zeta^{(1),B}_9$ in Eq. (3) below.

where $q^\mu_1$ and $q^\nu_2$ are the external gluon momenta, and $a$ and $b$ are the corresponding colour indices. We contract the amplitude with $P^\mu_\nu(q_1, q_2)$ in order to arrive at a scalar expression in Lorentz and colour space.

Before the massive two- and three-loop integrals are passed to MATAD, we need to eliminate any external momenta in their numerators by appropriate decompositions into invariants, e.g.

$$\int d^Dl \cdots \frac{(p_1 \cdot l)(p_2 \cdot l)}{(l^2 - m^2)\cdots} = \frac{1}{D} (p_1 \cdot p_2) \int d^Dl \cdots \frac{l^2}{(l^2 - m^2)\cdots},$$

where the dots represent factors that are independent of $l$.

There are two diagrams at one-loop level, 23 at two-loop level, and 657 at three-loop level, and the calculation of the $1/M^2_t$-suppressed terms takes about $5 \cdot 10^4$s, with the computationally most expensive one shown in Fig. 3.

3 Results

Before we present the results, let us introduce some useful notation. The renormalization scale $\mu$ appears in our calculation only through the factors

$$N_h = e^{i\pi\epsilon} \left(\frac{\mu}{M_H}\right)^{2\epsilon} N, \quad N_t = \left(\frac{\mu}{M_B}\right)^{2\epsilon} N, \quad N = \exp[\epsilon(-\gamma_E + \ln 4\pi)],$$

with Euler’s constant $\gamma_E \approx 0.577216$. These expressions are understood as their Laurent series in $\epsilon = (4 - D)/2$, where $D$ is the number of space-time dimensions used in the calculation.

The perturbative coefficients typically contain the transcendental numbers $\zeta_n \equiv \zeta(n)$, where $\zeta$ is Riemann’s zeta function. The particular values occurring here are

$$\zeta_2 = \frac{\pi^2}{6} \approx 1.64493, \quad \zeta_3 \approx 1.20206, \quad \zeta_4 = \frac{\pi^4}{90} \approx 1.08232.$$


Throughout this paper, bare quantities are labeled by a superscript “B”. Note that since the diagrams are evaluated with a spectrum of six quark flavours, renormalization has to be performed accordingly. To perform on-shell mass renormalization or conversion of $\alpha_s$ from the six- to the five-flavour scheme one must keep the proper number of higher order terms in $\epsilon$ due to the presence of infra-red poles. Furthermore, in order to arrive at a physical result, the external gluons must be renormalized on-shell.

For convenience, the number of light flavours $n_l$ is kept as a free parameter; the physical case corresponds to $n_l = 5$. The virtual cross section for the process $gg \rightarrow H$ can be written as

$$\sigma_{\text{virt}} = \frac{\pi}{576 v^2} \frac{1}{(1 - \epsilon)} \left( \frac{\alpha_s B}{\pi} \right)^2 \delta(1 - z) |N_t H(\epsilon) h(\alpha_s B)|^2,$$

where

$$H(\epsilon) = \Gamma(1 + \epsilon) \left[ 1 + \frac{7}{120} \left( \frac{M_H}{M_t^B} \right)^2 (1 + \epsilon) \right. + \left. \frac{1}{168} \left( \frac{M_H}{M_t^B} \right)^2 (1 + \frac{3}{2} \epsilon + \frac{1}{2} \epsilon^2) \right] + O\left( \frac{M_6}{M_t^B} \right).$$

The amplitude is expanded in terms of a perturbative series:

$$h(\alpha_s B) = 1 + \left( \frac{\alpha_s B}{\pi} \right)^2 h^{(2)} + \ldots,$$

where the coefficients $h^{(n)}$ are functions of $M_H$, $M_t^B$, and the renormalization scale $\mu$. In our approach, they take the form

$$h^{(n)} = h_0^{(n)} + \left( \frac{M_H}{M_t^B} \right)^2 h_2^{(n)} + \left( \frac{M_H}{M_t^B} \right)^4 h_4^{(n)} + \ldots.$$  

The leading terms have been calculated in the framework of an effective Lagrangian. However, for consistency, we present them here in a form that is directly compatible with the mass suppressed terms to be presented below:

$$h_0^{(1)} = a^{(1)} + c^{(1)},$$

$$h_0^{(2)} = a^{(2)} + c^{(2)} + a^{(1)} c^{(1)} - \zeta_g^{(1)} B a^{(1)},$$

where

$$a^{(1)} = N_k \left\{ - \frac{3}{2 \epsilon^2} + \frac{3}{4} \zeta_2 + \epsilon \left( - \frac{3}{2} + \frac{7}{2} \zeta_3 \right) + \epsilon^2 \left( - \frac{9}{2} + \frac{141}{32} \zeta_4 \right) \right\} + O(\epsilon^3),$$

$$a^{(2)} = N_k^2 \left\{ \frac{9}{8 \epsilon^4} + \frac{1}{\epsilon^3} \left[ - \frac{33}{32} + \frac{1}{16} n_l \right] + \frac{1}{\epsilon^2} \left[ - \frac{67}{32} - \frac{9}{16} \zeta_2 + \frac{5}{48} n_l \right] \right. \left. + \frac{1}{\epsilon} \left[ \frac{17}{12} + \frac{99}{32} \zeta_2 - \frac{75}{16} \zeta_3 + n_l \left( - \frac{19}{72} - \frac{3}{16} \zeta_2 \right) \right] \right. \left. + \frac{5861}{288} + \frac{201}{32} \zeta_2 + \frac{11}{16} \zeta_3 - \frac{189}{32} \zeta_4 + n_l \left[ - \frac{605}{216} - \frac{5}{16} \zeta_2 - \frac{7}{8} \zeta_3 \right] \right\} + O(\epsilon),$$

and

$$c^{(1)} = N_k \left\{ - \frac{3}{2 \epsilon^2} + \frac{3}{4} \zeta_2 + \epsilon \left( - \frac{3}{2} + \frac{7}{2} \zeta_3 \right) + \epsilon^2 \left( - \frac{9}{2} + \frac{141}{32} \zeta_4 \right) \right\} + O(\epsilon^3),$$

$$c^{(2)} = N_k^2 \left\{ \frac{9}{8 \epsilon^4} + \frac{1}{\epsilon^3} \left[ - \frac{33}{32} + \frac{1}{16} n_l \right] + \frac{1}{\epsilon^2} \left[ - \frac{67}{32} - \frac{9}{16} \zeta_2 + \frac{5}{48} n_l \right] \right. \left. + \frac{1}{\epsilon} \left[ \frac{17}{12} + \frac{99}{32} \zeta_2 - \frac{75}{16} \zeta_3 + n_l \left( - \frac{19}{72} - \frac{3}{16} \zeta_2 \right) \right] \right. \left. + \frac{5861}{288} + \frac{201}{32} \zeta_2 + \frac{11}{16} \zeta_3 - \frac{189}{32} \zeta_4 + n_l \left[ - \frac{605}{216} - \frac{5}{16} \zeta_2 - \frac{7}{8} \zeta_3 \right] \right\} + O(\epsilon),$$

and

$$\zeta_g^{(1)} = N_k \left\{ \frac{3}{2 \epsilon^2} - \frac{3}{4} \zeta_2 + \epsilon \left( \frac{3}{2} - \frac{7}{2} \zeta_3 \right) + \epsilon^2 \left( \frac{9}{2} + \frac{141}{32} \zeta_4 \right) \right\}.$$
are the perturbative coefficients of the effective Higgs-gluon vertex as presented in Ref. [29] which we quote here for the sake of completeness. Furthermore, we find
\[ c_1^{(1)} = N_l \left[ \frac{3}{4} - \frac{11}{6} \epsilon + \left( \frac{17}{4} + \frac{3}{8} \zeta_2 \right) \right] + O(\epsilon^3), \]
\[ c_2^{(2)} = N_l^2 \left[ \frac{3}{32 \epsilon^2} + \frac{1}{\epsilon} \left( \frac{61}{152} \right) - \frac{4529}{1152} + \frac{33}{32} \zeta_2 + \frac{73}{288} n_l \right] + O(\epsilon). \]

It may be worth noting that \( c_1^{(1)} \) and \( c_2^{(2)} \) correspond to the one- and two-loop results for the bare coefficient function of the effective Lagrangian:
\[ C_1^B = -\frac{1}{3} \frac{\alpha_s^B}{\pi} \Gamma(1 + \epsilon) \left[ 1 + \frac{\alpha_s^B}{\pi} c_1^{(1)} + \left( \frac{\alpha_s^B}{\pi} \right)^2 c_2^{(2)} + O(\alpha_s^3) \right]. \]

Finally,
\[ \zeta_2^{(1),B} = \frac{N_l}{6} \left( \frac{1}{\epsilon} + \frac{\epsilon}{2} \zeta_2 - \frac{\epsilon^2}{3} \zeta_3 \right) + O(\epsilon^3) \]
is the 1-loop term of the bare decoupling constant for \( \alpha_s \) for the transition from \( n_f = 6 \) to \( n_f = 5 \) flavour QCD (see, e.g., Ref. [32]). The origin of the term involving \( \zeta_2^{(1),B} \) in Eq. (12) is the fact that the coefficients \( a_1^{(n)} \) in Eq. (29) were evaluated in 5-flavour QCD, while the \( h_2^{(n)} \) of Eq. (3) are based on 6-flavour QCD. Therefore, diagrams like the right-most one in Fig. 3 do not have a correspondence in the effective theory calculation of Ref. [29].

The expressions presented so far correspond to known results and have been included in this paper only for the sake of the reader’s convenience. They should facilitate any implementation of the newly calculated terms to be presented below. Besides that, they serve as a useful check of our setup. It should be noted that in our approach, we directly calculate the coefficients \( h_2^{(n)} \), and the decomposition into \( a_1^{(n)} \) and \( c_2^{(n)} \) is just for comparison to the literature.

The new results of this paper are the contributions to the virtual 3-loop amplitude that are formally suppressed by powers of \( M_H/M_t \). In the notation of Eqs. (7) and (8), the first two subleading orders read:\[^2\]
\[ h_2^{(1)} = N_l \left[ -\frac{7}{60} + \frac{79}{540} \epsilon + \frac{37}{2400} - \frac{7}{120} \zeta_2 \right] + O(\epsilon^3), \]
\[ h_2^{(2)} = N_l N_h \left[ \frac{7}{40 \epsilon^3} + \frac{79}{360 \epsilon^2} - \frac{47}{1600} + \frac{58863}{1296000} - \frac{7}{15} \zeta_3 \right] + O(\epsilon^3), \]
\[ h_2^{(3)} = N_l^2 \left[ \frac{7}{1244160} n_l - \frac{49}{1440} \zeta_2 + \frac{955667}{552960} \zeta_3 + n_l \left( \frac{49729}{518400} + \frac{7}{720} \zeta_2 \right) \right] + O(\epsilon^3), \]
\[ + N_h^2 \left[ \frac{7}{320} + \frac{1441}{103680} n_l - \frac{559}{2560} \zeta_2 + \frac{36377}{311040} n_l \right] + O(\epsilon). \]

\(^1\)Renormalizing \( C_1^B \) according to Ref. [31] (where \( C_1^B \) is called \( C_1^B \)), one may derive the coefficient function \( C_1 \) quoted in Eq. (3) of Ref. [29].

\(^2\)For the sake of brevity we insert SU(3) colour factors. The result for general colour factors can be obtained upon request from the authors.
\[ h^{(1)}_4 = \mathcal{N}_t \left[ -\frac{857}{50400} \frac{1}{\epsilon} - \frac{3301}{113400} + \epsilon \left( -\frac{1064509}{76204800} - \frac{857}{100800} \zeta_2 \right) 
+ \epsilon^2 \left( -\frac{506399}{21952000} - \frac{3301}{226800} \zeta_2 + \frac{857}{151200} \zeta_3 \right) \right] + \mathcal{O}(\epsilon^3), \]

\[ h^{(2)}_4 = \mathcal{N}_t \mathcal{N}_h \left[ \frac{857}{33600} \frac{1}{\epsilon^3} + \frac{3301}{75600} \frac{1}{\epsilon^2} + \frac{1175389}{50803200} \frac{1}{\epsilon} + \frac{240009257}{3556224000} - \frac{857}{12600} \zeta_3 \right]
+ \mathcal{N}_h^2 \left[ \frac{1}{\epsilon^2} \left( \frac{22801}{1209600} + \frac{857}{604800} n_t \right) + \frac{1}{\epsilon} \left( \frac{28471}{1088640} + \frac{94907}{29030400} n_t \right) + \frac{5277060458353}{1170505728000} + \frac{22801}{1209600} \zeta_2 + \frac{9358312739}{2477260800} \zeta_3 
+ n_t \left( \frac{13897721}{1143072000} + \frac{857}{604800} \zeta_2 \right) \right]
+ \mathcal{N}_h^2 \left[ \frac{1}{\epsilon} \left( \frac{87}{89600} + \frac{80231}{87091200} n_t \right) + \frac{1481}{153600} + \frac{781}{102060} n_t \right] + \mathcal{O}(\epsilon), \]

where \( h^{(1)}_2 \) is the two-loop result for which the \( \epsilon^{-1} \) and \( \epsilon^0 \) terms can be compared to Ref. [10], with full agreement, of course. Note that since the leading order amplitude has been factored out in terms of \( H(\epsilon) \), the NLO and NNLO expressions for \( h \) start only at \( \mathcal{O}(1/\epsilon) \) and \( \mathcal{O}(1/\epsilon^3) \) respectively. The leading poles are thus fully determined by the leading terms in \( 1/M_t \).

There are a number of checks that we can perform on our result: (i) calculating the amplitude in an arbitrary covariant gauge, we find it to be independent of the gauge parameter; (ii) replacing the projector of Eq. (1) by \( \delta_{ab} q_1,\mu q_2,\nu \) leads to a vanishing result, which also checks gauge invariance; (iii) the poles of order \( \alpha_s^{n+1}/\epsilon^k \), \( k = 1, \ldots, 2n \) were checked against the general formula of Ref. [33] (with additional input from Ref. [34] for the \( \alpha_s^3/\epsilon \) terms; see also Ref. [35]) and found to be in full agreement.

Another observation is that in the ratio of the amplitudes taken at time-like and space-like momenta, all mass effects cancel and the result is the one given by the general formula of Ref. [36, 37]. Specifically,

\[ \left| \frac{\hat{h}(M^2_H)}{\hat{h}(-M^2_H)} \right|^2 = \left| \frac{a(M^2_H)}{a(-M^2_H)} \right|^2, \]

where

\[ \hat{h}(q^2) = Z_{\alpha_s}^{OS} H(\epsilon) h(\alpha_s^B) \bigg|_{M^2_H=q^2} \]

with the on-shell gluon renormalization constant \( Z_{\alpha_s}^{OS} \) [32] (see also Ref. [38]). The ratio on the left-hand side of Eq. (16) can also be found in Ref. [29]. It is understood that the bare quantities \( M_H^B \) and \( \alpha_s^B \) in Eq. (16) are expressed in terms of their renormalized values [39].

### 4 Conclusions

The top mass suppressed terms for the virtual corrections to Standard Model Higgs production in gluon fusion were presented through three-loop order. They are an ingredient
for the inclusive NNLO QCD cross section. We have subjected the result to various checks and found full confirmation.

The next steps towards the top mass effects in the Higgs production cross section will be the evaluation of the mass suppressed terms in the real radiation amplitudes. We defer this problem to a forthcoming publication.

Upon completion of this paper, we became aware of a similar calculation \[40\]. We have compared our results and found full agreement.

Acknowledgments. This work has been supported by Deutsche Forschungsgemeinschaft, contract HA 2990/3-1 and the Helmholtz Alliance Physics at the Terascale. We would like to thank A. Pak, M. Rogal, and M. Steinhauser for useful comments, as well as M. Czakon for enlightening discussions concerning the renormalization of the amplitude.

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