1. Introduction

Quantum chromodynamics (QCD) is the fundamental theory of strong interaction. The study of the QCD vacuum is a very nontrivial issue in high energy and nuclear physics. The usual vacuum can be viewed as a kind of substance filled with condensations of quarks and gluons due to the non-perturbative nature of QCD. If there are valence quarks in the vacuum, those quarks will be confined in hadron states [1]. This is the usual confinement which can be viewed as an emergent phenomenon of quarks in the normal vacuum. However, according to the view of T. D. Lee, the vacuum is a real substance whose properties could be changed. In 1970s T. D. Lee had proposed “vacuum engineering” [2]. The central idea was to change the vacuum by heavy ion collisions and produce deconfined quarks and gluons. By heating the vacuum, which means by high energy collisions of heavy nuclei, the tremendous kinetic energy is converted to the heat energy of the fireball in the center region of the collision, where the vacuum condensations are melted and the quarks and gluons are deconfined. In the experiments of relativistic heavy ion collisions (RHIC) in Brookhaven National Lab (BNL), a new emergent state of quarks and gluons has finally formed in a small region of deconfined vacuum at high temperatures, which is called strongly interacting quark gluon plasma (sQGP) [3-6]. The collective flows in heavy ion collisions known as the radial and elliptic flow could be well described by ideal hydrodynamics which means the sQGP at RHIC is the most perfect liquid [7].

Though sQGP has been produced by the RHIC, we still lack understanding about the non-perturbative vacuum of QCD. In the early years, like in 1970s, T. D. Lee used a phenomenological scalar field to describe the complicated nonperturbative features of QCD vacuum [2]. He has further introduced a phenomenological model which is called Friedberg-Lee (FL) model to study the confinement and hadron properties in the vacuum [1]. The model consists of quark fields interacting with a phenomenological scalar field $\sigma$. There is a non-topological soliton solution in this model which represents a hadron. Phenomenologically it was successful in describing the static properties of
hadrons and their behaviors at low energies [8,9]. The model has been also extended to finite temperatures and densities [10-16]. However at high temperatures the sQGP indicates that the nonperturbative thermal vacuum is even more complicated than we thought before. In recent years, the nonperturbative structure of QCD vacuum at finite temperatures has been seriously considered by Shuryak and Zahed[17-19], by whom the topological solitons in QCD, like instantons and dyons, have been thoroughly investigated, which reveals a topological structure of thermal QCD vacuum. The topology and vacuum is closely related to the confinement and chiral symmetry breaking. However this whole theory of topological solitons based on QCD is quite involved and it is difficult to extrapolate the theory to the zero temperature case.

It is very challenging to find a consistent theoretical scheme directly based on QCD to describe the whole physical story from low energy hadrons which are confined quark states to high temperature liquid sQGP which is deconfined but strongly coupled quark states. However in this paper we will use the FL model which is the phenomenological model to present a simple description on some of the emergent properties of quarks in a confined or deconfined vacuum in a self-consistent scheme, from which one could have a holistic and apparent physical picture about the varying emergent phenomenon of quarks in a vacuum with temperatures varying from low temperatures to very high temperatures.

2. The FL model and the confinement
The Lagrangian of the FL model is defined as

$$L = \bar{\psi} (i\gamma_{\mu} \partial^{\mu} - g\sigma) \psi + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - U(\sigma),$$

where $\psi$ is the quark field, $\sigma$ is the phenomenological scalar field which can be identified with the gluon condensate. $U(\sigma)$ is the potential as

$$U(\sigma) = \frac{a}{2!} \sigma^2 + \frac{b}{3!} \sigma^3 + \frac{c}{4!} \sigma^4 + B,$$

Where $a$, $b$, $c$ and $g$ are the constants which are generally fitted in producing the properties of hadrons. $B$ is the bag constant. Presumably the classical potential $U(\sigma)$ has two minima which are at

$$\sigma_0 = 0,$$

$$\sigma_v = \frac{3|b|}{2c} \left[ 1 + \left( 1 - \frac{8ac}{3b^2} \right)^{1/2} \right].$$

These two minima are corresponding to two vacuums in the system: $\sigma_0$ is the local minimum which represents the meta-stable or perturbative vacuum, while $\sigma_v$ is a global minimum which represents the physical or non-perturbative vacuum. If one sets the potential energy at the physical vacuum to be zero, the bag constant is $B = -\frac{a}{2!} \sigma_v^2 + \frac{b}{3!} \sigma_v^3 + \frac{c}{4!} \sigma_v^4$ which can be identified as the potential energy difference between the two vacuums.

In the mean field approximation the sigma field is treated as the time-independent classical field and only the N valence quarks of the lowest energy are considered, which means

$$\sigma(\vec{r}, t) = \sigma(\vec{r}), \psi(\vec{r}, t) = e^{-iEr} \sum_{i=1}^{N} \psi_i(\vec{r}).$$

The value of N is usually taken as 3 for a baryon or 2 for a meson. Then the static classical Euler-Lagrange equations could be obtained from the above Lagrangian,
\[
(-i\gamma_0 \vec{r} \cdot \nabla + \gamma_0 g \sigma)\psi_i = E \psi_i ,
\]
\[
-\nabla^2 \psi + \frac{dU(\sigma)}{d\sigma} + g \sum_{i=1}^{N} \vec{\psi}_i \psi_i = 0 .
\]  

If the spherical symmetrical configurations of the fields are considered, which means
\[
\sigma(\vec{r}) = \sigma(r) , \quad \psi_i(\vec{r}) = \begin{pmatrix} u_i(r) \\ i\sigma \cdot r v_i(r) \end{pmatrix} \chi ,
\]
where \(\sigma\) is the Pauli matrice, \(\chi\) is the spinor as \(\chi = (1,0)^T\) or \(\chi = (0,1)^T\) and \(\vec{r}\) is the spatial unit vector, the equations (6) and (7) can be further divided into the radial forms as
\[
\frac{du(r)}{dr} = -(E + g \sigma(r))v(r) ,
\]
\[
\frac{dv(r)}{dr} = -\frac{2}{r} v(r) + (E - g \sigma(r))u(r) ,
\]
\[
\frac{d^2 \sigma(r)}{dr^2} + \frac{2}{r} \frac{d\sigma(r)}{dr} - \frac{dU}{d\sigma} = Ng(u^2(r) - v^2(r)) .
\]

The quark wave functions \(u\) and \(v\) should satisfy the normalization condition
\[
4\pi \int r^2 (u^2(r) + v^2(r))dr = 1 .
\]

The subscript “i” has been omitted as the wave functions of the N valence quarks are identical.

In the physical vacuum with the condensation \(\sigma = \sigma_v\) the valence quark acquires a mass \(m = g \sigma_v\) and would be forced to rest at the bottom of the continuum spectrum. However it is possible that for a small region in the physical vacuum the vacuum condensation is depleted by the valence quarks with some nontrivial spatial configuration of \(\sigma\) field created in that small region. Now let us presume the valence quark mass in the physical vacuum is very large, then the valence quarks will automatically stay away from the physical vacuum and be settled around the perturbative vacuum at \(\sigma = \sigma_0\). Apparently speaking the N valence quarks create a bag in the physical vacuum which leads to the confinement in the FL model. Inside the bag the quark is nearly massless with an eigenenergy \(0 < E < m\). The vacuum inside the bag is different from the physical vacuum outside. This is the nontopological soliton solution in the FL model.

3. The emergent properties of quarks in different vacuums
From the above discussions it is shown that the valence quarks are confined in a bag in the physical vacuum at zero temperature in FL model. In this case the physical vacuum can be also viewed as a confined vacuum. In this section we will discuss the the FL model at finite temperatures. Finite temperature FL model have been discussed by many authors [10-15]. However the emergent properties of quarks in both confined case and deconfined case have not yet been fully explored in the FL model.

According to the previous studies of FL model at finite temperatures [12-15], the finite temperature effect could be introduced by replacing the potential \(U(\sigma)\) in the equation (11) by a thermal effective potential \(\Omega(\sigma,T)\). The thermal effective potential \(\Omega(\sigma,T)\) could be obtained from the Lagrangian (1) by calculating a path integral through the finite temperature field theory. In finite temperature field theory the mean field approximation means the effective potential is calculated to the one loop order of the quark fields. The sigma fields are treated as classical fields and the Dirac sea effects of quarks are neglected in the mean field approximation. The temperature dependent part of the one loop effective potential is finite. In this case the final result is
\[ \Omega(\sigma, T) = U(\sigma) - \frac{T \gamma}{\pi^2} \int_0^\infty p^2 \ln \left[ 1 + \exp \left( -\sqrt{p^2 + g^2 \sigma^2} \right) \right] dp , \]  

where \( T \) is the temperature, and \( \gamma \) is a degeneracy factor, \( \gamma = 2(\text{spin}) \times 2(\text{flavor}) \times 3(\text{color}) = 12 \). Substitute (13) into equation (11) and replace the potential \( U(\sigma) \), then the equations (9)-(11) together with the normalization condition (12) could be numerically solved at different temperatures. It is known that there exists a first order deconfinement phase transition in the FL model with a transition temperature \( T_c \). Taking one set of values for the parameters as \( a = 17.70 \text{fm}^{-2} \), \( b = -1457.4 \text{fm}^{-1} \), \( c = 20000 \) and \( g = 12.16 \) [10-12], one obtains the transition temperature \( T_c \approx 120 \text{MeV} \). The thermal effective potentials as functions of \( \sigma \) at different temperatures are plotted in figure 1. The deconfinement phase transition takes place at the temperature \( T_c \) where the two minima of the effective potential are degenerate. When the system is deconfined, the perturbative vacuum at \( \sigma = \sigma_0 \) will be the global minimum of the thermal effective potential, which means the vacuum condensation is melted and the perturbative vacuum becomes the true vacuum. After deconfinement the perturbative vacuum can be viewed as a deconfined vacuum. In the following we will discuss the emergent properties of quarks in confined and deconfined vacuum respectively at finite temperatures.

![Figure 1](image-url)

**Figure 1.** The thermal effective potentials as functions of \( \sigma \) at different temperatures.

### 3.1. Hadron states in confined vacuum for \( T < T_c \)

When \( T < T_c \), from figure 1 one can see that the global minimum of the effective potential keeps staying at \( \sigma = \sigma_v \), which means the nonperturbative vacuum is the true vacuum and the system keeps being confined. The valence quarks are confined in the bag which is generated by the soliton solution. The soliton solutions at different temperatures for \( T < T_c \) are illustrated in figure 2. One can see the radius of the bag is about 1fm. Inside the bag the \( \sigma \) field is nearly zero, while outside the bag the sigma field approaches its vacuum condensation value \( \sigma_v \) which is now dependent on temperatures. The valence quark density \( \psi^\dagger \psi = u^2 + v^2 \) provides a force keeping the \( \sigma \) field close to zero inside the bag. On the other side the physical vacuum exerts an inward pressure on the valence quarks which makes the valence quark density localized in the bag. The bounded states of the valence quarks are corresponding to the hadron states.
Figure 2. The radial functions of the valence quark density $u^2 + v^2$ and the $\sigma$ field at different temperatures for $T < T_c$.

With temperature increasing, from figure 2 one can see the radial functions of the valence quark density and $\sigma$ field become more and more flat, which means the bag radius is increasing and the valence quark density can be distributed to a larger area. As a result the valence quarks though confined could move in a relatively larger region at higher temperatures.

The soliton energy is a sum of the quark eigen energy and the energy of sigma fields [9], which is

$$E = N\epsilon + \int_0^\infty 4\pi r^2 \left[ \frac{1}{2} \left( \frac{d\sigma}{dr} \right)^2 + \Omega(\sigma, T) \right] dr.$$  \hspace{1cm} (14)

In order to see the variation of the soliton more clearly a radial energy function of the soliton at finite temperatures could be defined as [12]

$$\omega(r) = 4\pi r^2 \left[ \frac{1}{2} \left( \frac{d\sigma}{dr} \right)^2 + \Omega(\sigma, T) \right],$$  \hspace{1cm} (15)

Figure 3. The radial functions of the soliton energy at different temperatures for $T < T_c$.

which are plotted at different temperatures as shown in figure 3. One can see the soliton energy is localized in the certain area with the dimension about 1 fm. The position of the peak energy is located
approximately at the bag surface. When the temperature is increasing, the position of the peak is shifted to the larger radius and the energy value of the peak is decreasing. That means the bag is swelling and bag energy becomes smaller at higher temperatures. In other words, the valence quarks in the bag are more likely to be deconfined at higher temperatures.

3.2. Liquid states in deconfined vacuum for $T > T_c$

When $T > T_c$, from figure 1 one can see the global minimum of the effective potential is changed to the $\sigma = \sigma_0$, which means the perturbative vacuum becomes the true vacuum. The condensation of the vacuum is melted and the system is deconfined. As a result, the soliton solution disappears at $T > T_c$. Some researchers regard that the valence quarks are immediately free when the soliton disappears [13, 14]. However, we think that the valence quarks are not necessarily free after the disappearance of the soliton. In our previous studies, we even indicate that there are possibly bound states after deconfinement in FL model though it is in a very different calculation scheme [12, 15]. In the scheme of the present work, there still exists scattering solution in the system after deconfinement which is shown in figure 4. The $\sigma$ field has a shallow potential well in the region where $r \geq 1.5\text{fm}$. This potential well produces certain attractions among the valence quarks. When the distance between the valence quarks gets large ($r \geq 1\text{fm}$), the valence quarks become quasi-free. Thus the valence quark density is not localized and could not be normalized in the whole space as shown in figure 4. However, the nontrivial $\sigma$ field generates some interaction potential between the valence quarks which is attractive in a small distance and oscillatory in a long distance. This means a thermodynamic system composed of the valence quarks with this interaction potential will exhibit short range order and long range disorder.

![Figure 4](image-url)

**Figure 4.** The radial functions of the valence quark density $u^2 + v^2$ and the $\sigma$ field at different temperatures for $T > T_c$.

To make it clearer, one can define a radial distribution function (RDF) through the static interaction potential as

$$g(r) = \exp[-g\sigma(r)/T],$$

which is plotted at different temperatures as shown in figure 5. One can clearly see the damping oscillation of the RDF. When $r$ is small, the RDF oscillates significantly around the value 1, which means the valence quarks are much correlated at short distances. When $r$ is large, the RDF approaches 1, which means the valence quarks are decorrelated and quasi-free at large distances. This
is a typical feature of liquids which indicates the system after deconfinement in FL model is in a liquid state.

![Radial Distribution Functions](image)

Figure 5. The radial distribution functions of the valence quark at different temperatures for $T > T_c$.

4. Summary and conclusions

In this paper we have shown the different behaviors of the quarks in the FL model before and after the deconfinement. Before deconfinement the vacuum is filled with condensation and the quarks are confined in the bag which represents the hadron state. After deconfinement the vacuum condensation is melted and the quarks are not bounded but still correlated, which indicates the system behaves like a liquid. The result after deconfinement is qualitatively consistent with the liquid behavior of sQGP discovered in RHIC experiments. However it should be reminded that the result here is model dependent.

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