A model for forming the degree of influence of the counterparty when making managerial decisions in mechanical engineering

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Abstract. We consider an arbitrary set of counterparties, united either in a social network or in the field of joint activities, for example, top management of an engineering company. It is required to build a rating for each participant based on the results of mutual evaluation of the participants of the association in question, when the significance of the counterparty’s assessment will depend on how it was evaluated by other participants in the association. To solve this problem, the interaction between counterparties was modeled in the form of a graph. For each vertex, its potential was determined, and for each edge of the graph, the flow along it. Based on these data, ratings were obtained from participants in the scientific and technical council of a machine-building enterprise. It is shown that using a similar algorithm, it is possible to obtain estimates of the significance of agents in social networks when conducting marketing research.

1. Introduction

The machine-building complex of Russia is one of the key sectors of the national economy, which, in general, is the locomotive of the economy, which leads all other sectors of Russian industry. According to data for 2018, engineering accounts for approximately 12% of Russia’s GDP, which in absolute terms is above 8 trillion rubles, and the number of people employed in the manufacturing industries is 14.2% of the total employed population. But, nevertheless, for this sector of the economy, some negative trends are characteristic, namely, the volume of production for the first half of 2019 decreased for the first time in the last three years.

Thus, the country's leading industry, which should ensure the innovative development of the economy, is drawn into crisis phenomena. The way out of this situation is to improve the methods of making managerial decisions in order to increase their objectivity and reduce the subjective component. One of the possibilities in this case is the model support of the management decision-making process. In this case, the capabilities of social networks can be used to model the processes for evaluating the generated management decisions. The possibility of such modeling is represented by graph theory and, in this case, by stream algorithms.

For a long time, the main practical representation of flow problems were pipeline systems and electrical circuits [1, 2]. But with the development of telecommunication technologies, these ideas have expanded significantly. Currently, the network is increasingly gaining independent significance, simultaneously creating its own subculture, which is expressed in the fact that the network is already
forming its own customs, standards of communication and behavior, role models. That is, a kind of “network community” is formed, with an almost unlimited audience.

And so, and another strategy is a kind of extreme embodiment of the capabilities of society and the state, but if the second strategy still occurs in its pure form, then the first strategy exists only hypothetically. Neither society, nor the state, nor the business community can pass by the opportunities presented by networks. Therefore, their development is quite rapid. But development should be accompanied by study, and study by development of methods for managing the network community.

2. Materials and methods
Consider the processes occurring in the network. But for this we somehow need to introduce this network. From the point of view of such modeling, graph theory presents the best opportunities. In this case, the vertices of the graph will represent the network participants or, in terms of the theory of active systems, agents. The edges of the graph will describe the relationship of a particular agent with other partners on the network [3, 4].

The problem arises: how can one evaluate the importance of each agent on the basis of the ratings it gives and the ratings it receives. And here, first of all, the simplest solution opens: to build an assessment based on the ratings received by a specific agent. Moreover, all agents initially have the same weight.

But this idea will not be completely objective, since it does not take into account what kind of assessment each of the survey participants has. It would be logical to link the ratings given by the agent with the rating that he himself has in this procedure. It is possible to build an estimation algorithm taking into account the above circumstances using stream algorithms [5, 6].

For this purpose, we will present the assessment procedure as a certain flow passing through a graph characterizing the interaction of agents with each other. This makes it possible to apply existing flow algorithms to such a graph.

Moreover, the main property of the stream [1, 2] is the conservation property, which in general form can be written in the following form

\[ \sum \phi_{ij} - \sum \phi_{ji} = \Delta \phi_i. \]

where \( \Delta \phi_i \) – change in balance at the top \( i \).

Considering the features of the problem under consideration, when it is necessary to construct an estimation system, the remainder of the flow at an arbitrary vertex must be set equal to zero, i.e. \( \Delta \phi_i = 0 \). Then we obtain the relation of the following form

\[ \sum \phi_{ij} = \sum \phi_{ji}. \]

that is, a stream entering a vertex \( i \) from other vertices is equal to a stream exiting from this vertex to other vertices.

Naturally, the flow will be transmitted only along the edges of the graph defining the interaction procedure between network participants, and the stronger the connection between specific agents, the greater the flow. It is quite clear that another characteristic of the vertex: its potential, can be taken by the proportionality coefficient in such a dependence. Neither the potentials of the peaks, nor the flows along the arcs, are yet known to us. To successfully solve the problem, you need to calculate them.

3. Results
Information about the interaction of agents with each other is specified in the form of an adjacency matrix. In this case, at the intersection of the \( i \)-th row and the \( j \)-th column, a mark is put, which the \( i \)-th agent rated the \( j \)-th. A zero value in the adjacency matrix means that there is no connection between the agents. The main diagonal of the matrix is filled with zeros, since the agent cannot evaluate itself [7, 8].
Now, using the concepts of flow along the edge, peak potential and throughput of the edge, it is possible, by analogy with electric circuits, to introduce a relation similar to Ohm's law [7, 8]. To do this, suppose that in the network under consideration the flow along the edge will be proportional to the potential difference between the vertices incident to this edge, i.e. \( \phi_{ij} = c_{ij} q_j \).

If we use the property of saving the flow along the vertices of the network and substituting the resulting ratio into it, we obtain the following expression

\[
\sum_j c_{ij} q_j = \sum_j c_{ji} q_i = c_i q_i.
\]

In this ratio, we used an electrical analogy of the bandwidth of a rib with conductivity. And since in this case all edges coming out of the considered vertex are considered, that is, an analog of parallel connection of conductors takes place, the total conductivity of such a section will be equal to the sum of the conductivities of its edges [8]. Thus, we obtain \( c_i = \sum_j c_{ij} q_j \).

In the obtained system of equations, the dimension of which will be equal to the number of vertices of the graph, the potential values of each vertex \( q_i \), are used as unknowns, and the throughput capacities of the edges \( c_{ij} \) are known.

These fundamental properties of the flows in the graph served as the basis for the currently quite common task of evaluating participants in various kinds of surveys, summing up the results of competitions, including sports, etc. In this case, the rating of each of the participants consists of the ratings that other participants put to him, but the significance of each of the participants will depend on the rating received by this participant during this survey.

In this case, such a system for calculating the results of many competitions will be more objective that those methods that are currently adopted when the win against a favorite of the season and an outsider are estimated by the same number of points (just remember football or chess). To apply graph theory algorithms to a similar task, it is necessary to identify the potential of the graph vertex with the participant rating, and the graph itself will be a diagram of the interaction of participants. Therefore, in order to obtain the ratings of participants in a survey, voting or competition, it is necessary to solve a system of equations regarding unknown potential \( q_i \).

The solution to this problem can be approached purely from the standpoint of linear algebra. To do this, it is enough to recall that for any graph it is possible to determine the degree of each vertex and the adjacency matrix \( A \). The degrees of the vertices can also be defined as a matrix \( D \), with the degrees of vertices along the main diagonal and all other elements equal 0. Thus, for any of the graph, one can simply find the Kirchhoff matrix \( K \), calculated as the difference of two matrices (the Kirchhoff matrix is often called the discrete Laplace operator or Laplacian)

\[
K = D - A.
\]

The solution of the system of equations (1) using the Kirchhoff matrix consists in finding an additional minor of the Kirchhoff matrix. If we denote such a minor by \( A^i_j(K) \), then the solution of system (1) can be written in the form

\[
q_i = A^i_j(K).
\]

And in order to get the rating of the \( i \)-th participant, it is enough to multiply the obtained potential by the value \( c_i \), which can be interpreted as the degree of the vertex \( i \) or the total throughput of the edges coming from this vertex, i.e.

\[
\phi_i = A^i_j(K)c_i.
\]

Thus, in order to calculate the potentials of the vertices and flows along incident edges, it is necessary to calculate additional minors of the Kirchhoff matrix. The number of minors to be
calculated will be equal to the number of vertices. That is, the task from a computational point of view is quite laborious. But if we use the properties of the Kirchhoff matrix, we can simplify the problem by reducing it to the inverse of the transformed Kirchhoff matrix.

One of the main properties of the Kirchhoff matrix is the fact that its determinant is always equal to zero, i.e., a relation of the form \( \det(A) = 0 \).

In this regard, the algorithm for calculating the potentials of the vertices of the graph in question consists in performing the following sequence of actions:

1. **Preliminary step.** In the Kirchhoff matrix replace the first row with a row of the form \((1,0, \ldots,0)\).
2. **1 step.** We handle the matrix obtained at the preliminary step.
3. **2 step.** We calculate the determinant of this matrix. In this case, for verification, we can use the property of the determinants of the direct and inverse matrices \( \det(A^{-1}) = \frac{1}{\det(A)} \).
4. **3 step.** It is necessary to divide the values of the inverse matrix in the first column by the determinant of the inverse matrix. The components of the resulting vector will be the potentials of the vertices of the graph. I.e.

\[
q_i = \frac{a_{i1}}{\det(A^{-1})},
\]

where \(a_{i1}\) - values of the first column of the inverse matrix.

Unfortunately, in the general case, it is completely unclear what to take for the ranking indicator of the objects in question: the peak potential or the flow along an arc. This issue should be resolved based on the specific conditions of the problem being solved.

4. **Discussion**

Consider an example of applying the described algorithm.

It is necessary to determine the rating of each member of the scientific and technical council (STC) of an enterprise, using the data from a survey of members of the STC on the performance of council members. It is possible to organize a direct survey of board members using meeting statistics. In this case, data was accumulated on those proposals that were discussed at the council, who initiated them and information on who voted for which proposal. It is necessary to determine the significance of each member of the STC. The composition of the STC includes 9 people.

To solve this problem, it is necessary to obtain the adjacency matrix \(A\), which will describe the activities of the STC members for the analyzed period. According to the STC secretariat, the following data are available, which are presented in the form of an adjacency matrix \(A\)

\[
A = \begin{pmatrix}
I & II & III & IV & V & VI & VII & VIII & IX \\
I & 0 & 2 & 1 & 3 & 0 & 2 & 4 & 4 & 1 \\
II & 3 & 0 & 2 & 4 & 5 & 7 & 0 & 2 & 4 \\
III & 2 & 1 & 0 & 3 & 2 & 1 & 6 & 0 & 5 \\
IV & 3 & 2 & 1 & 0 & 4 & 0 & 2 & 1 & 1 \\
V & 2 & 5 & 6 & 1 & 0 & 4 & 1 & 3 & 0 \\
VI & 5 & 3 & 1 & 0 & 2 & 0 & 1 & 5 & 2 \\
VII & 3 & 2 & 5 & 7 & 6 & 2 & 0 & 1 & 3 \\
VIII & 7 & 6 & 7 & 4 & 4 & 6 & 7 & 0 & 4 \\
IX & 1 & 2 & 1 & 2 & 5 & 7 & 3 & 6 & 0
\end{pmatrix}
\]

Using the notion of the degree of a vertex, one can construct the Kirchhoff matrix or Laplacians using expression (2).
In order to verify the correctness of the construction, one of the properties of the Laplacian can be used: the sum over each column of the Laplacian should be equal to zero.

In this case, when the Laplacian is known, the flow through each vertex can be determined by solving the system of equations (4), for which it will be necessary to calculate additional minors of the Kirchhoff matrix, which is a rather time-consuming operation. We use the algorithm described above related to the matrix inversion procedure.

**Preliminary step.** In the Kirchhoff matrix replace the first row with a row of the form: (1,0, …,0).

1 step. We handle the matrix obtained at the preliminary step. We get the following inverse matrix.

2 step. We calculate the determinant of the obtained inverse matrix $A^{-1}$ for the Kirchhoff matrix. In this case, for verification, you can use the property of the determinants of the direct and inverse matrices. We finally get $\det(A^{-1}) = 4,258\cdot10^{-11}$.

3 step. It is necessary to divide the values of the inverse matrix in the first column by the determinant of the inverse matrix. The components of the resulting vector will be the potentials of the vertices of the graph. I.e.

$$q_i = \frac{a_{ij}}{\det(A^{-1})},$$

where $a_{ij}$ values of the first column of the inverse matrix.

The final result can be written in the following form: the potential of each of the vertices will be determined by the values presented in the second column. But taking into account that the obtained values of the potentials are quite significant, it is possible to simplify these expressions by bringing them to the same basis. As such a basis, the smallest potential is selected and all other values are divided by this value. Thus, according to the value of the potential, which in this case will characterize the significance of each member of the STC, we can say that in descending order of authority of the members of the STC can be arranged as follows:

VIII(3.234) – IX(2.496) – VII(1.894) – II(1.889) – III(1.471) – V(1.383) – VI(1.349) – I(1.233) – IV(1.0).

It is clear that if the purpose of the decision was to determine the significance or rating of the members of the council for a certain period, then the decision was received.

If it would be necessary to choose the best member of the STC, then calculating the potential of each of them would not be enough. This would have to be supplemented by calculating flows along all edges of the relationship graph. For this purpose, it is necessary to multiply the absolute value of the potential of the graph vertex by the degree of this vertex, that is, use a formula of the form $\varphi = c_{ij}q_i$.

In this case, the members of the STC were as follows VIII – IX – VII – II – VI – V – III – I – IV.

Comparing rankings with the help of potential, we can conclude that they basically coincide: leaders and outsiders stand out clearly. But this is far from always the case. Computational experiments show that quite often this does not coincide, that is, the most significant agents, that is, those with the highest potential value, do not always receive the highest rating value.

5. Conclusion

Thus, an algorithm was considered that made it possible to obtain a quantitative estimate for each agent of a social network, taking into account the ratings that other members of the same network set to it. This algorithm can be used in various production situations when it is necessary to build estimates for a group of participants in the procedure under consideration, taking into account the opinions of each [8, 9]. The capabilities of the algorithm were shown by the example of determining the rating of members of the scientific and technical council of an enterprise for the purpose of subsequent accounting when summing up the voting results. But this does not exhaust the possibilities of the presented algorithm, which can be used in other procedures.

For example, when conducting various types of surveys using social networks and determining consumer preferences, when it is necessary to pre-calculate the weight of each participant. Or when determining the expert competency coefficient. This may be the task of finding consensus, when a parameter called the influence of the participant should be used as the source data. This algorithm can
be used in personnel incentive systems [10, 11] when it is necessary to take into account the opinion of all members of the production team.

References
[1] Ford L R and Fulkerson D R 1966 Streams in Networks (Moscow: Mir) 276
[2] Hu T 1974 Integer Programming and Network Flows (Moscow: Mir) 520
[3] Gubanov D A, Novikov D A and Chkhartishvili A G 2010 Social Networks: Models of Information Influence, Management and Confrontation (Moscow: Fizmatlit) 228
[4] Novikov D A 2003 Network Structures and Organizational Systems (Moscow: IPU RAN) 102
[5] Barkalov S A, Kurochka P N and Lukmanova I G 2017 Building Integrated Estimates Based on Logistic Regression News of Higher Educational Institutions. Textile Technology 1(367) 37-47
[6] Zhilyakova L Yu and Kuznetsov O P 2017 Resource Network Theory (Moscow: RIOR: INFRA-M) 283
[7] Chebotarev P Yu and Ageev R P 2009 Coordination of characteristics in multi-agent systems and spectra of Laplace matrices of digraphs Automation and telemechanics 3 136-51
[8] Bondarenko Yu V, Sviridova T A and Averina T A 2019 Aggregated multi-criteria model of enterprise management engineering, taking into account the social priorities of the region IOP Conf. Series: Materials Science and Engineering 537 042045
[9] Avdeeva E, Belyantseva O and Smorodina E 2018 Constituents of sustainable development potential of a logistics company Proc. Int. Conf. on MATEC Web of Conferences Siberian Transport Forum - TransSiberia 08004
[10] Barkalov S A, Burkov V N and Kurochka P N 2019 Designing systems of group stimulation in the management of energy complex objects Advances in Intelligent Systems and Computing V(983) 55-68
[11] Burkov V N, Burkova I V, Averina T A and Nasonova T V 2017 Formation of the program to improve the level of competence of the organization's staff Proc. of Tenth International Conference Management of Large-Scale System Development (MLSD) 8109602