Generation of two-mode entanglement via atomic coherence created by non-resonant dressed-state transitions

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Two-mode squeezing and entanglement is obtained in a atom-cavity system consisting a three-level atom and a two-mode cavity with driving laser fields. Here non-resonant dressed-state transitions between the cavity modes and atom are used to get the effective Hamiltonian.

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I. INTRODUCTION

Quantum entanglement is regarded as the magic part of mechanics and has become one central part in quantum information and quantum computation. Recently, continuous-variable (CV) entanglement has attracted much great interest in the application of entanglement-based quantum information processing [1]. Thus generation continuous-variable entanglement have gained much attention since they can provide useful entanglement sources for quantum teleportation [2], quantum dense coding [3] and quantum swapping [4]. There are many interaction system candidates to generate CV entanglement such as cavity QED system [5-9], nondegenerate parametric down-conversion optical crystals [10,11], NV centers system [12,13], and so on.

As for cavity QED system, it has been a good candidate to investigate the quantum interaction in physics and quantum mechanics. Many schemes have been presented to generate entangled states. For example, the scheme for generating CV entangled light via correlated emission laser has been proposed [14]. They obtain a two-mode entangled light with large photon number. The scheme to generate CV entanglement via a single atom laser has also been presented [15,16]. Among the entanglement generating schemes, some schemes use atomic coherence to build the entanglement between various cavity modes [17,18]. The atomic coherence can be input or created by driving related transitions. In these schemes, atomic coherence induces entanglement somewhat. One scheme is proposed in Ref. [19], a two-step operation to get highly entangled two-mode light by using the atoms as a reservoir is proposed. They use the dressed states of the driving field and choose resonant transition for the two cavity modes and the squeeze-transformed modes successively interact with the dressed atoms. One of the limitations in this scheme are as following. First, the two-step procedure and different initial state preparation make the experimental setup complicated. It is useful to investigate the scheme that has one-step procedure and simple initial state preparation.

Motivated by this, we propose a scheme to extend the atomic system to a three-level atom, which has an auxiliary driving transition to generate the two-mode entanglement light. By introducing the auxiliary driving transition and choosing non-resonant atom-cavity interaction, the dressed atomic-cavity interaction can be an effective two-mode squeezing interaction, which can be used to generate two-mode entangled light.

II. MODEL AND EQUATIONS

The system we consider is a three-level Λ atomic system located in a two-mode optical cavity with drivings. Two classical laser fields with Rabi frequency $\Omega_j$ and frequency $\omega_{d,j}$ resonantly drive the two atomic transitions $|j\rangle \leftrightarrow |3\rangle$, respectively for $j=1, 2$. The two cavity modes with coupling constant $g_n$ and frequency $\omega_{c,n}$ coupling transitions $|2\rangle \leftrightarrow |3\rangle$ ( $n = 1, 2$). The whole system can be described by the following Hamiltonian ($\hbar = 1$)

\[
H = H_\Omega + H_c
\]

\[
H_\Omega = - \sum_{j=1,2} \Omega_j (|3\rangle\langle j| + |j\rangle\langle 3|)
\]

\[
H_c = \sum_{n=1,2} \delta_n a_n^\dagger a_n + \sum_{n=1,2} (g_n a_n \sigma_{32} + g_n^\ast a_n^\dagger \sigma_{23}) (1)
\]

where $\Delta_j = \omega_{3j} - \omega_{d,j}$ ( $j = 1, 2$) and $\delta_n = \omega_{c,n} - \omega_{d,2}$ with $\omega_{3j}$ are atomic resonant frequency for transition $|j\rangle \leftrightarrow |3\rangle$ ( $j = 1, 2$). Atomic operator $\sigma_{jk}$ are flip operators when $j \neq k$ and are population operators when $j = k$. $a_n$ ($a_n^\dagger$) is the creation (annihilation) operator for the nth cavity mode.

Assume that the laser fields are much strong, we first calculate the dressed states created by part Hamiltonian $H_\Omega$. The eigenvalues are $\lambda_0 = 0$, $\lambda_{\pm} = \pm \sqrt{\Omega_1^2 + \Omega_2^2} = \pm \Omega_c$, and related eigenstates are
\[ |0\rangle = \cos \theta |2\rangle - \sin \theta |1\rangle \]
\[ |\rangle = \frac{1}{\sqrt{2}} |3\rangle + \frac{\sin \theta}{\sqrt{2}} |2\rangle + \frac{\sin \theta}{\sqrt{2}} |1\rangle \] (2)
\[ |\rangle = \frac{1}{\sqrt{2}} |3\rangle + \frac{\sin \theta}{\sqrt{2}} |2\rangle + \frac{\sin \theta}{\sqrt{2}} |1\rangle \]

in which \( \sin \theta = \frac{E_0}{\mu} \). Thus in the eigenstates basis, the relations hold
\[ H_0 = \Omega (\sigma_+ - \sigma_-) \]
\[ \sigma_3 = \frac{1}{2} \cos \theta (\sigma_+ + \sigma_-) + \frac{\sin \theta}{2} (\sigma_+ - \sigma_- + \sigma_+ - \sigma_-). \]

Then make a unitary transformation \( e^{iH_0 t} H e^{-iH_0 t} \) with
\[ H_0 = H_\Omega + \sum_{n=1,2} \delta_n a_n^\dagger a_n \]
and assume that the detunings \( \delta_1 = 2\Omega e - d \) and \( \delta_2 = 2\Omega e + d \), where \( d > 0 \) is a small quantity compared with \( 2\Omega e \) but is still large when compared with the effective coupling strength (which will show later). Because of the strong driving condition, the unitary transformation will contain fast oscillating terms \( e^{\pm i\Omega \epsilon t} \) and \( e^{\pm 2i\Omega \epsilon t} \), by neglecting these terms, i.e., when secular approximation is made, the cavity related Hamiltonian now has the form

\[ H_{c1} = (G_1 a_1 - G_2 a_1^\dagger) \sigma_{++} + e^{i\epsilon_{0} t} \]
\[ + (G_1 a_1^\dagger a_1 + G_2 a_2 a_2^\dagger) (\sigma_{+-} - \sigma_{-+}), \] (3)

in which \( G_1 = \frac{a_0}{\sqrt{2}} \sin \theta \), and \( G_2 = \frac{a_0}{\sqrt{2}} \sin \theta \). If \( d \gg |G_1|, |G_2| \) is fulfilled, an effective Hamiltonian \( H_e \) can be obtained as

\[ H_e = \lambda (\sigma_+ \sigma_- - \sigma_- \sigma_+) \]
\[ + |\frac{G_1}{d} a_1^\dagger a_1 + \frac{G_2}{d} a_2 a_2^\dagger| (\sigma_{+-} - \sigma_{-+}) \] (4)

where \( \lambda = \frac{G_1 G_2}{d} \) is the effective coupling between the two cavity modes, and \( \lambda_{++} = \frac{G_1^2}{d^2}, \lambda_{--} = \frac{G_2^2}{d^2} \). Assume that the two cavity modes are initially in their vacuum states \( |0\rangle_1 |0\rangle_2 \), then the terms related to \( a_j^\dagger a_j \) \( (j = 1, 2) \) have no contribution to the evolution and can be omitted. Thus the final effective Hamiltonian is

\[ H_{eff} = \lambda (\sigma_+ \sigma_- - \sigma_- \sigma_+) \]
\[ + \lambda_{++} \sigma_{+-} - \lambda_{--} \sigma_{-+} \] (5)

Next we will show how to obtain squeezing and entanglement from this effective Hamiltonian for the two cavity modes. If the initial state of the whole system is \( |\psi(0)\rangle = |+\rangle |0_1\rangle |0_2\rangle \), then the state will evolve as
\[ |\psi(t)\rangle = e^{-iH_e t} |\psi(0)\rangle \]
to the following state

\[ |\psi(t)\rangle = e^{-i\lambda_{++} t} |+\rangle e^{-i\lambda_{a_1 a_2} a_1 a_2^\dagger t} |0_1\rangle |0_2\rangle \] (6)

Thus if the whole atom is initially in state \( |+\rangle |0_1\rangle |0_2\rangle \), then after time \( t \), the atom will still be in its state \( |+\rangle \), but the two cavity modes will evolve into the following state

\[ |\varphi(t)\rangle_c = e^{-i\lambda_{a_1 a_2} a_1 a_2^\dagger} |0_1\rangle |0_2\rangle \] (7)

It is obvious that \( |\varphi(t)\rangle_c \) is a two-mode squeezed vacuum state. Let us define the quadratic operators of the two cavity modes which will be used to judge squeezing and entanglement as \( x_j = \frac{a_j + a_j^\dagger}{\sqrt{2}}, p_j = -i \frac{a_j - a_j^\dagger}{\sqrt{2}} \), and \( u = x_1 + x_2, v = p_1 + p_2 \). The quantities \( \langle \Delta u \rangle^2 = \langle u^2 \rangle - \langle u \rangle^2 \) and \( \langle \Delta v \rangle^2 = \langle v^2 \rangle - \langle v \rangle^2 \) are needed to be calculated. According to the entanglement criterion \( 20 \), if \( M = \langle \langle \Delta u \rangle^2 + \langle \Delta v \rangle^2 \rangle < 2 \), then the two cavity modes are entangled. Using the state \( |\varphi(t)\rangle_c \) in Eq. (7), the following relations hold \( \langle u \rangle = \langle v \rangle = 0, \langle a_1 a_1^\dagger \rangle = \langle a_2 a_2^\dagger \rangle = \sinh(\lambda t) \) and \( \langle a_1 a_2^\dagger \rangle = -i \cosh(\lambda t) \sinh(\lambda t) \). Thus \( M = 2 + (e^{2\lambda t} - e^{-2\lambda t})^2 = 2 (e^{2\lambda t} + e^{-2\lambda t}) \). If \( \lambda = i|\lambda| \) \( (|\lambda| \neq 0) \), then \( M = 2\cos(2\lambda t) < 2 \), that is to say if we choose \( g_1 = |g|, g_2 = i|g|, \) then \( \lambda = \frac{G_1 G_2}{d} = i|g|^2 \), and entanglement criterion \( M < 2 \) always holds. Thus two cavity modes are entangled states. Actually the state \( |\varphi(t)\rangle_c \) is a two-mode entangled state and is an ideally entangled state when \( \lambda = i|\lambda| \) \( (|\lambda| \neq 0) \) holds.

### III. CONCLUSION

In conclusion, we have presented a scheme to generate two-mode entangled state in a cavity QED system by using the atomic coherence created by the two strong classical driving fields. The result shows that when we appropriately adjusting the coupling strengths of the two cavity modes, a two-mode squeezed vacuum state which is a two-mode entangled state is generated.

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