Prediction of the fatigue life distribution for aluminum through its mechanical characteristics

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Abstract. A novel and reliable theoretical model based on the Birnbaum-Saunders (BISA) distribution is presented from which the fatigue life can be determined. Experimental verification of the model is in progress and will be published in due course.

1. Introduction
According to a poll conducted in 1985 by the American Society for Quality Control, reliability was the second most important attribute among the ten most important products attributes [1]. This result is expected as the impact of product failure can range from minor injuries and/or loss to severe injuries/death and/or loss. Therefore, the precise prediction of failures can save lives and money. Usually, the reliability of the products is determined through one of several types of life testing. The primary objective of these tests is to quantify the reliability of the product, which can be used to determine whether a set of goals for the product are met or not. Typically the result of the reliability test is a set of failure times that is analyzed statistically to predict the reliability distribution through curve fitting. One of the disadvantages of this method is the high cost and time in some circumstances especially when regular life tests are used. Another disadvantage is the low accuracy of the test when accelerated life tests are used due to extrapolation. Moreover, this approach pays no attention to the strong relationship between the mechanical properties and the reliability distribution of the product as it uses parametric or nonparametric statistical methods to predict the reliability from the failure data and not from the mechanical properties of the product.

Little work has been done to predict the life of products through methods other than life testing. Good references for fatigue life prediction methods can be found in [2]. Ref [3] predicted the fatigue life of the tibial component of a polymeric PMMA space rutilizing multiaxial fatigue coupled with computational simulations and material properties. Ref [4] used the S-N curve and the crack growth rate curve to predict the fatigue life. Ref [5] proposed a method to predict the fatigue-life based on linear elastic analysis and used it to predict the fatigue for different materials subjected to constant amplitude multiaxial proportional loading. Ref [6] Used ΔK–N curve utilizing finite element analysis to predict the fatigue life of spot welds starting from coarse finite element meshes and ending at one unique ΔK–N curve.

Among the little available non life-testing prediction methods, prediction through fitting the mechanical properties to the life distribution parameters' was absent. The primary objective of this paper is to propose a method for fatigue failure mode life prediction of carbon steel shafts through their mechanical properties. The primary advantage of this method is in saving time and money. Once the relationship between the mechanical properties and the reliability distribution parameters for the
carbon steel shafts is known, the reliability distribution for any carbon steel shaft can be induced from the mechanical properties of that shaft without conducting reliability tests.

The Birnbaum-Saunders (BISA) distribution was used as the life distribution for the fatigue mode failure for these shafts. Different carbon steel specimens were prepared through heat treatment to generate different micro-structures. The mechanical properties of each type of micro-structure were determined through standard compression test. A number of standard specimens for each micro-structure were prepared and subjected to a simple stress accelerated life test based on standard fatigue test to generate failure times. Bayesian inference was conducted on these failure times to estimate the BISA parameters. The parameters of the BISA were related to the mechanical properties of the specimens such that the shape parameter and the scale parameter of the BISA were expressed as a function of the mechanical properties of the carbon steel.

2. Fatigue mechanism

Fatigue is the progressive and localized structure damage for the part when it is subjected to cyclic loading. The fatigue failure stress values are well below the ultimate tensile strength or even the yield strength of the material. Fatigue failure process can be divided into periods: the crack initiation period and crack growth period. The crack initiation period starts by initiation of micro-cracks due to local accumulation of dislocations, high stresses at local points, or plastic deformation around inhomogeneous inclusions or other imperfections in or under the contact surface [7]. The crack initiation period usually covers a large portion of the fatigue life under stress amplitudes just above the fatigue limit. The microstructure of the specimen, along with the applied stress and the geometry of the specimen, play a vital role in determining the position and mode of the fatigue crack initiation [7]. Consequently, at the same applied stress and geometry, the microstructure of the specimen determines the mode and position of the fatigue crack. The crack growth period follows the crack initiation period in which the cracks propagate until they cause a permanent damage to the part. The fatigue life of the member is the time (cycles) corresponding to those two periods.

The fatigue life of the specimen under fatigue failure mode can be modeled using variety of distributions like Weibull, Gamma, Lognormal and BISA (known as fatigue life distribution). The strength of BISA distribution is that it is based on the renewal theory and a set of physical properties of the fatigue process [9]. The main theory behind the BISA distribution is that:

1- The fatigue failure is generated by the cycles of the stress that causes degradation or crack growth until a critical crack size is reached, which causes a point fatigue failure.
2- In each cycle, the crack will extend and these extensions are statistically independent. The central limit theorem guarantees that the total crack extension is approximately normally distributed.

The BISA distribution defines a random variable $T$ as the life time in cycles with the following probability density function (pdf) and cumulative density function (CDF):

$$f_T(t; \alpha, \beta) = \frac{t - \beta}{\alpha \sqrt{2\pi \beta t}} \exp \left( \frac{\beta - t}{\alpha \sqrt{\beta t}} \right)$$

$$F_T(t; \alpha, \beta) = \frac{1}{\sqrt{2\pi}} \exp \left( \frac{\beta - t}{\alpha \sqrt{\beta t}} \right)$$

Where $\alpha$ and $\beta$ are the shape and scale parameters respectively such that $\alpha, \beta > 0$. According to Ref. [10] the scale parameter $\beta$ is the ratio of the critical crack extension to the average incremental
crack extension per cycle. Therefore, $\beta$ represents the median number of cycles to failure as the time to the $p^{th}$ quantile of the BISA distribution is:

$$t_p = \frac{\beta}{4} \left( \alpha z_p + \sqrt{\alpha^2 z_p^2 + 4} \right)^2,$$

where $z_p$ is the $p^{th}$ quantile of the standard normal distribution [11].

3. The effect of the mechanical characteristics on the fatigue life

In general, the metallurgical and/or the mechanical characteristics of an engineering material can be changed either by heat treatment if the material is liable for treatment such as the carbon steels, or more recently refining its structure by the addition of some rare elements. For example, the addition of Ti or Ti+B to Al and Al alloys to their melts prior to solidification to grain refine their metallurgical structure as they solidify in columnar structure with large grain size which tends to reduce their mechanical strength and surface quality [13]. It is worth mentioning in this respect that the addition of other alloying elements in the presence of the main grain refining element may affect its refining efficiency. For example, V, Mo, and C improves grain refining efficiency, whereas other elements like Cr, Zr, and Ta poison the grain refinement, i.e make the grains larger in size.

Figure 1 shows the effect of adding Ti-B at a rate of 0.1% which corresponds to the peritectic point at the Al-Ti-B phase diagram which is the ration normally used in industry.

![Figure 1. Effect of adding Ti-B at a rate of 0.1% Al-Ti-B](image)

The equation that governs the strength of most of the materials is given by:

$$\sigma = k \varepsilon^n$$

where $k$ is the strength factor of the material, $\varepsilon$ is the true strain, and $n$ is the work or strain hardening index. It is easy to notice that as $k$ increases, the strength of the material increases as well. This matter increases the material's ability to resist fatigue, hence increases the fatigue life. On the other hand, increases $n$ decreases the strength of the material as $n$ and $\varepsilon$ both are less than 1. Thus, increasing $n$, reduces the material's resistant to fatigue, consequently, decreases its fatigue life.

Changing the microstructure of the material with refiners changes its $k$ and $n$ values and consequently, changes its fatigue life. Changing the fatigue life changes the shape and scale parameters of the fatigue life distribution of the material.

As mentioned earlier, the aim of this paper is to find the equations that relate the mechanical characteristics i.e., $k$ and $n$ with the fatigue life distribution parameters i.e., $\alpha$ and $\beta$ of Al-Ti-B
alloys. For this purpose, different Al-Ti-B alloys were prepared using different weight percentages of Ti-B and their $k$ and $\eta$ values were found using compression test. For each alloy, number of specimens were prepared and a Type I simple step accelerated fatigue life test was conducted on them. The resulting censored data was fitted to BISA distribution using Bayesian inference to determine $\alpha$ and $\beta$ for each alloy. Non-linear exponential-based regression model was used to find the equations relating $\alpha$ and $\beta$ to $k$ and $n$ for Al-Ti-B alloys such that

$$\alpha = \exp(a_0 + a_1 k + a_2 n) \quad (5)$$
$$\beta = \exp(b_0 + b_1 k + b_2 n) \quad (6)$$

4. Life testing

The purpose of life testing is to assess the probability that the product can perform its intended tasks adequately in its useful life under normal operating conditions. Life tests are usually conducted under accelerated conditions to accelerate the failures and hence reduce the testing time. Unfortunately, the data collected under the abnormal conditions is not reliable as the data collected under normal conditions as the accelerated conditions impose abnormal operating circumstances on the specimens; therefore, extrapolation between the abnormal and the normal circumstances must be done to predict the life of the specimen under normal operating conditions.

To relate the parameters of the life distribution under abnormal conditions to the parameters under normal conditions, stress-acceleration models are usually used. The log-linear stress-acceleration model is widely used in literature to relate the mean time to failure of the distribution to the stress level. Under this acceleration model, usually the scale parameter is the only parameter affected.

In this paper a Type-I right censoring simple step accelerated fatigue life test (SSAFLT) is conducted to generate failure times. Figure 2 shows a schematic diagram for the SSALFT under Type-I right censoring.

![Figure 2. Schematic diagram for Type-I right censoring for SSAFLT](image-url)

The SSAFLT under Type I censoring can be described as follows: $n$ samples are stressed under $x_1$ stress level for $\tau$ time (cycles) during which $n_1$ samples fail. A higher stress, $x_2$, is applied on the survived ones until the end of the test time $t_e$ (cycles) such that $x_2 > x_1$. In this paper, the SSAFLTs were carried out on Wholer Testing Machine at the laboratories of the Mechanical and Industrial Engineering Department in the Applied Science University.

Cumulative exposure model is assumed to account for the cumulative damage under $x_j$ stress. The cumulative exposure model is inspired from palmgren-miner rule which states that the fatigue failure is expected when such life fractions sum to unity, that is, when 100% of the life is exhausted such that

$$\sum_{j=1}^{n} \frac{N_j}{N_{fj}} = 1,$$

where $N_j$ is the number of cycles under stress $j$ and $N_{fj}$ is the number of cycles to failure for stress $j$ from the S-N curve.
Figure 3 shows a conceptual graph for Palmgren-Miner rule under SSAFLT. The figure shows that the specimen will start at the first curve with $\beta_1$ for $\tau$ cycles and then will continue on the second curve with $\beta_2$ until the end of the test cycles $t_c$. During $\tau$ cycles, damage (fatigue) will be accumulated in the specimen due to the stress $x_1$ and when the specimen is subjected to the second stress $x_2$, the specimen will not be fresh; hence the accumulated fatigue caused by the stress $x_1$ must be accounted for.

The accumulated damage is manifested as a shift of the value $s$ in the time scale for the second step in the test. The value of $s$ as a function of the BISA parameters and can be derived as:

$$s = \frac{\beta_2 \left[ \xi^2 \left( \frac{1}{\beta_1} \right) + 2 \right] + \sqrt{-\beta_2 \left[ \xi^2 \left( \frac{1}{\beta_1} \right) + 2 \right]^2 - 4 \beta_2^2}}{2}$$

(7)

5. Bayesian Inference
Consider a vector of failure data points $t$, and a life distribution with parameters $\theta$. The posterior distribution of the parameters given the data is:

$$f(\theta; t) = \frac{f(\theta)L(t; \theta)}{f(t)}$$

(8)

Where $L(t; \theta)$ is the likelihood of the data given the model parameters $\theta$, $f(\theta)$ is the joint prior distribution of the parameters, and $f(t) = \int_{\theta} f(\theta)L(t; \theta)$ is the preposterior marginal distribution of $t$. Since the preposterior marginal distribution of $t$ is constant, then the unnormalized posterior distribution of the parameters given the data can be expressed as:

$$f(\theta; t) \propto f(\theta)L(t; \theta)$$

Gibbs sampling can be used to sample from $f(\theta; t)$. Gibbs sampling is built in WINBUGS software which is a stand-alone program to allow practical MCMC methods available to applied statisticians.

The cumulative distribution function considering the cumulative exposure model for the BISA is expressed as:
\[ F(t; \alpha, \beta_1) = \begin{cases} 
\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2\alpha^2}\right) & 0 < t < \tau \\
\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(t-\tau)^2}{2\beta_2}\right) & \tau \leq t \leq t_c 
\end{cases} \]  

(9)

and the probability density function is:

\[ f(t; \alpha, \beta_1, \beta_2) = \begin{cases} 
\frac{t - \beta_1}{\alpha \sqrt{2\pi \beta_1} t^3} \exp\left(\frac{\beta_1 - t}{\alpha \sqrt{\beta_1} t}\right) & 0 < t < \tau \\
\frac{(t - (\tau - s)) - \beta_2}{\alpha \sqrt{2\pi \beta_2 (t - (\tau - s))^3}} \exp\left(\frac{\beta_2 - (t - (\tau - s))}{\alpha \sqrt{\beta_2 (t - (\tau - s))}}\right) & \tau \leq t \leq t_c 
\end{cases} \]  

(10)

Moreover, the reliability function for the survived units is:

\[ R(t_c; \alpha, \beta_1, \beta_2) = 1 - \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t_c^2}{2\beta_2}\right) \]  

(11)

Under Type-I right censoring, the likelihood function has three parts: one for the censored data and the other two corresponding to the two steps. In this model, the likelihood function will be written such that the log-linear relation between \( \beta_i \) and the stress level \( x_i \) is assumed as in equation (1):

\[ \ln(\beta_i) = a + bx_i \]  

(12)

Where \( a \) and \( b \) are the model parameters such that their values depend on the product under test and the test method used.

The likelihood function for \( n \) samples utilizing the cumulative exposure model and the log-linear model is:
6. Specimens preparation

For our purposes, Al-Ti-B was used as a refiner for pure Al where 20 micro-alloys of Al-Ti-B were prepared.

6.1. Base metal

99.8% purity commercial aluminum was used in preparing the specimens for this study. The chemical composition of the 99.8% purity commercial aluminum is shown in Table 1.

Table 1. The chemical composition (Wt%) of the 99.8% purity commercial aluminum

| Element | Fe   | Si   | Cu   | Mg   | Ti   | V    | Zn   | Mn   | Na   | Al   |
|---------|------|------|------|------|------|------|------|------|------|------|
| Wt. %   | 0.09 | 0.05 | 0.005| 0.004| 0.004| 0.008| 0.005| 0.001| 0.005| bal. |

The chemical composition of the commercially available ternary Al-Ti-B master alloy is shown in Table 2. The ternary Al-Ti-B master alloy was used to manufacture the Al-%Ti-%B and its micro-alloys.

Table 2. Chemical composition, wt% of the ternary Al-Ti-B alloy

| Element | Ti | B  | Fe | Si | V  | Al   |
|---------|----|----|----|----|----|------|
| Wt%     | 4.6| 0.92| 0.12| 0.09| 0.12| Balance |

High purity aluminum and titanium powder were used in manufacturing the Al-Ti binary alloy. Graphite crucible and stirring rods were used in all of experiments.

6.2. Al base metal preparation

The high purity aluminum wire was pickled in HNO₃ to remove any contamination, and then they melted in a graphite crucible inside an electric furnace at about 800°C. The molten was then poured into a hollow cylindrical brass rod of 10 mm inside diameter and 55 mm external diameter and left to solidify. After the solidification, the rods were rolled into sheets of 2 mm thickness, 10 mm width and 240 mm length.

Ternary Al-Ti-B grain refiner is commercially available and where provided by ARAL company in Jordan.

Fatigue test specimens (Al-(Ti-B)ₓ%) of the dimensions shown in Figure 4 were machined into the standard fatigue specimen as per ASTM.
The specimens were machined using a Colchester CNC-1000 lathe machine at a cutting spindle speed of 1030 rpm (30m/min), a depth of cut 0.5 mm and a feed rate of 50 mm/min. All specimens were machined at the same cutting conditions, and thus, have the same surface roughness, which is essential to avoid any discrepancies in the fatigue result which may be caused by different cutting conditions.

7. Experimentation
Twenty different micro-structures will be used in the experimentation. For each micro-structure the mechanical properties need to be determined and 50 samples need to be prepared from it. These 50 samples will be subjected to SSAFLT and the BISA parameters will be estimated using WINBUGS. This will provide 20 sets of parameters corresponding to the 20 different sets of mechanical properties. Since model parameters are independent, we can perform regression analysis between the mechanical properties and the scale and shape parameters of the distribution individually to come up with two functions that relate the mechanical properties to the BISA parameters and thus to the reliability of the specimens.

Due to the limited allowable space for the paper to fulfill the requirements and the lengthy experimentation the verification of this theoretical model will be the subject of another paper.

8. Conclusion
A novel and reliable theoretical model based on the Birnbaum-Saunders (BISA) distribution is presented from which the fatigue life can be determined. Experimental verification of the model is in progress and will be published in due course.

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10. References
[1] Quality progress 1985 18 (Nov.), 12-17
[2] Weicheng Cui 2002 A state-of-the-art review on fatigue life prediction methods for metal structures J Mar SciTechnol7, 43–56
[3] Carnelli D1, Villa T, Gastaldi D, Pennati G 2011 Predicting fatigue life of a PMMA based knee spacer using a multiaxial fatigue criterion J ApplBiomaterBiomech.9(3), 185-92. doi: 10.5301/JABB.2011.8917.
[4] Wang XS et al (1999). "Prediction of fatigue life of carbon steel using only the tensile strength". In: Wu XR, Wang ZG (eds) Proceedings of the 7th International Fatigue Congress (Fatigue’99). Higher Education Press, Beijing, pp 845–850
[5] Guechichi H, Benkabouche S, Amrouche A, Benkhettab M 2011 A high fatigue life prediction methodology under constant amplitude multiaxial proportional loadings *Materials Science And Engineering: A* 528, 4789-4798

[6] Henrysson, H F 2000 Fatigue Life Predictions of Spot Welds Using Coarse FE Meshes *Fatigue and Fracture of Engineering Materials and Structures*, 23, 737-746.

[7] Fadiga G, Sraml M 2009 Fatigue crack initiation and propagation under contact loading *Engineering Fracture Mechanics* 79, 1320-1335

[8] Beden S M, Abdullah S, Arffin A K 2009 Review of fatigue crack initiation and Propagation models of metallic components *European Journal of Scientific Research* 28, 364-397

[9] Birnbaum Z W, Saunders S C 1969 A New Family of Life Distributions *J. Appl. Prob.* 6, 319-327

[10] Engelhardt M, Bain L J, Wright F T 1981 Inference on the parameters of the Birnbaum–Saunders fatigue life distribution based on maximum likelihood estimation *Technometrics* 23(3):251–256

[11] Chang D S, Tang L C 1994 Percentile bounds and tolerance limits for the Birnbaum–Saunders distribution. Communications in Statistics, Part A Theory and Methods 23, 2853–2863

[12] Fatemi A, Yang L 1998 Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials *Int J Fatigue* 20, 9–34

[13] Zaid A I O, Al-Qawabah S M A 2012 Effect of Zirconium Addition on the Grain Size and Mechanical Behavior of Aluminum Grain Refined by Titanium Plus Boron (Ti+B) in the as Cast and Cold Extruded Conditions *Key Engineering Materials*