Baryon Acoustic Oscillations in 2D: Modeling Redshift-space Power Spectrum from Perturbation Theory

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We present an improved prescription for matter power spectrum in redshift space taking a proper account of both the non-linear gravitational clustering and redshift distortion, which are of particular importance for accurately modeling baryon acoustic oscillations (BAOs). Contrary to the models of redshift distortion phenomenologically introduced but frequently used in the literature, the new model includes the corrections arising from the non-linear coupling between the density and velocity fields associated with two competitive effects of redshift distortion, i.e., Kaiser and Finger-of-God effects. Based on the improved treatment of perturbation theory for gravitational clustering, we compare our model predictions with monopole and quadrupole power spectra of N-body simulations, and an excellent agreement is achieved over the scales of BAOs. Potential impacts on constraining dark energy and modified gravity from the redshift-space power spectrum are also investigated based on the Fisher-matrix formalism. We find that the existing phenomenological models of redshift distortion produce a systematic error on measurements of the angular diameter distance and Hubble parameter by $1\sim 2\%$, and the growth rate parameter by $\sim 5\%$, which would become non-negligible for future galaxy surveys. Correctly modeling redshift distortion is thus essential, and the new prescription of redshift-space power spectrum including the non-linear corrections can be used as an accurate theoretical template for anisotropic BAOs.

I. INTRODUCTION

Galaxy redshift surveys via the spectroscopic measurements of individual galaxies provide a three-dimensional map of galaxy distribution, which includes valuable cosmological information on structure formation of the Universe. The observed galaxy distribution is, however, apparently distorted due to the peculiar velocity of galaxies that systematically affects the redshift determination of each galaxy. The anisotropy caused by peculiar velocities is referred to as the redshift distortion, which complicates the interpretation of the galaxy clustering data (e.g., \textsuperscript{1,2}).

Nevertheless, redshift distortion provides a unique way to measure the growth rate of structure formation, which has been previously used for determining the density parameters of the Universe (e.g., \textsuperscript{3,4}), and is now recognized with great interest as a powerful tool for testing gravity on cosmological scales (e.g., \textsuperscript{2,9}). Redshift distortion also provides a helpful information on the dark-sector interactions \textsuperscript{10}, where the dark energy is dynamically coupled with dark matter (e.g., \textsuperscript{11,12}). Note that the distortion of the galaxy clustering pattern also arises from the apparent mismatch of the underlying cosmology when we convert the redshift and angular position of each galaxy to the comoving radial and transverse distances. This is known as Alcock-Paczynski effect \textsuperscript{13}, and with the baryon acoustic oscillations (BAOs) as a robust standard ruler, it can be utilized for a simultaneous measurement of the Hubble parameter $H(z)$ and angular diameter distance $D_A(z)$ of distant galaxies at redshift $z$ (e.g., \textsuperscript{14–18}).

In these respects, anisotropic clustering data from galaxy redshift surveys serve as a dual cosmological probe of the cosmic expansion and the gravity on cosmological scales, from which we can address properties of both the dark energy and modification of gravity responsible for the late-time cosmic acceleration. Although current data are not yet sensitive enough to separately measure $H(z)$, $D_A(z)$ and growth rate (see \textsuperscript{7,19–21} for current status), planned and ongoing galaxy redshift surveys aim at precisely measuring the anisotropic power spectrum and/or two-point correlation function in redshift space. Thus, the accurate theoretical modeling of anisotropic power spectrum is crucial and needs to be developed toward future observations.

The purpose of this paper is to address these issues based on the analytical treatment of non-linear gravitational clustering. In the single-stream limit, cosmological evolution of the mass distribution consisting of the cold dark matter (CDM) and baryon is described by the coupled equations for irrotational and pressureless fluid \textsuperscript{22}. Recently, a detailed study on the standard treatments of perturbation theory has been made \textsuperscript{23,25}, and several improved treatments have been proposed \textsuperscript{26,38}, showing that a percent-level accuracy can be achieved for the predictions of power spectrum or two-point correlation function in real space. With a help of these, in this paper, we will develop a model of redshift
distortion, and compute the matter power spectrum in redshift space, with a particular attention to the BAOs. Also, we discuss the impact of model uncertainty of the redshift distortion on the acoustic-scale measurement of BAOs and the estimation of growth rate parameter.

This paper is organized as follows: In Sec. II we start with writing down the relation between real space and redshift space, and derive an exact expression for matter power spectrum in redshift space. We then consider the existing theoretical models of redshift distortion, and compare those with N-body simulations in Sec. III showing that non-negligible discrepancy appears at the scales of BAOs. In Sec. IV, non-linear corrections relevant to redshift space becomes important for the accurate modeling of BAOs. Before addressing the existing models of redshift distortion, and examine distortion, and compute the matter power spectrum in redshift space, with a particular attention to the BAOs. Also, we discuss the impact of model uncertainty of the redshift distortion on the acoustic-scale measurement of BAOs and the estimation of growth rate parameter.

Let us first recall that the redshift distortion arises from the apparent mismatch of galaxy position between real and redshift spaces caused by the contamination of peculiar velocities in the redshift measurement. For nearby galaxies, the position in real space, $\mathbf{r}$, is mapped to the one in redshift space, $\mathbf{s}$, as

$$\mathbf{s} = \mathbf{r} + \mathbf{v}_z(\mathbf{r}) \frac{1}{a H(z)} \hat{\mathbf{z}},$$  

where the unit vector $\hat{\mathbf{z}}$ indicates the line-of-sight direction, and quantity $v_z$ is the line-of-sight component of the velocity field, i.e., $\mathbf{v}_z = \mathbf{v} \cdot \hat{\mathbf{z}}$. The quantities $a$ and $H$ are the scale factor of the Universe and the Hubble parameter, respectively. Then, the density field in redshift space, $\delta^{(S)}(\mathbf{s})$, is related to the one in real space, $\delta(\mathbf{r})$ through the relation

$$\{1 + \delta^{(S)}(\mathbf{s})\} d^3s = \{1 + \delta(\mathbf{r})\} d^3r,$$

which leads to

$$\delta^{(S)}(\mathbf{s}) = \left| \frac{\partial s}{\partial r} \right|^{-1} \{1 + \delta(\mathbf{r})\} - 1. \quad (2)$$

The Fourier transform of this is given by

$$\delta^{(S)}(\mathbf{k}) = \int d^3r \left\{ \delta(\mathbf{r}) - \frac{\nabla_z v_z(\mathbf{r})}{a H(z)} \right\} e^{i(k\mu v_z/H+\mathbf{k} \cdot \mathbf{r})}, \quad (3)$$

where the quantity $\mu$ is the cosine of the angle between $\hat{\mathbf{z}}$ and $\mathbf{k}$. Here, we used the fact that the Jacobian $|\partial s/\partial r|$ is written as $1 + \nabla_z v_z/(aH)$.

From this, the power spectrum of density in redshift space becomes

$$P^{(S)}(\mathbf{k}) = \int d^3x e^{-i\mathbf{k} \cdot \mathbf{x}} \left\{ e^{-ik\mu f \Delta u_z} \times \{ \delta(\mathbf{r}) + f \nabla_z u_z(\mathbf{r}) \} \{ \delta(\mathbf{r}') + f \nabla_z u_z(\mathbf{r}') \} \right\}, \quad (4)$$

where $x = \mathbf{r} - \mathbf{r}'$ and $\langle \cdots \rangle$ is the ensemble average. We defined $u_z(\mathbf{r}) = -v_z(\mathbf{r})/(aHf)$ and $\Delta u_z = u_z(\mathbf{r}) - u_z(\mathbf{r}')$. The function $f$ is the logarithmic derivative of linear growth function $D(z)$ given by $f = d \ln D(z)/d \ln a$. This is the exact expression for power spectrum in redshift space, and no dynamical information for velocity and density fields, i.e., Euler equation and/or continuity equation, is invoked in deriving this equation.

In the expression (4), the power spectrum is written as function of $k$ and $\mu$, and is related to the statistical average of real-space quantities in a complicated manner, but qualitative effects on clustering amplitude of power spectrum are rather clear, i.e., enhancement and damping, well known as Kaiser and Finger-of-God effects. The Kaiser effect basically comes from the braces in the right hand side of the expression (4), which represents the coherent distortion by the peculiar velocity along the line-of-sight direction. In linear theory, the relation $u = \delta$ holds and the strength of clustering anisotropies is controlled by the growth rate parameter $f$. This is the basic reason why the redshift distortion attracts much attention as a powerful indicator for growth of structure. On the other hand, Finger-of-God effect roughly comes from the factor $e^{-ik\mu f \Delta u_z}$ in Eq. (4). Due to the randomness of peculiar velocities, de-phasing arises and it leads to the suppression of clustering amplitude. The apparent reduction of amplitude becomes especially significant around the halo forming regions.

Of course, these two effects cannot be separately treated in principle, and a mixture of Kaiser and Finger-of-God effects is expected to be significant on trans-linear regime, where a tight correlation between velocity and density fields still remains. This is of particular importance for the accurate modeling of BAOs. Before addressing detailed modeling, however, we will first consider currently existing models of redshift distortion, and examine how these models fail to reproduce the major trends of BAO features in redshift space.
III. EXISTING MODELS OF REDSHIFT DISTORTION

A. Perturbation theory description

Let us first examine the perturbation theory (PT) based model of redshift distortion. We here specifically deal with the two representative models: one-loop PT calculations for redshift-space power spectrum from standard PT and Lagrangian PT.

The standard PT usually implies a straightforward expansion of the the cosmic fluid equations around their linear solution, assuming that the amplitudes of density and velocity fields are small. This treatment is also applied to the evaluation of redshift-space power spectrum [3], and the resultant expressions for one-loop power spectrum is schematically summarized as (see [29, 42] for complete expressions)

\[ P_{\text{SPT}}^{(S)}(k, \mu) = (1 + f \mu^2)^2 P_{\text{lin}}(k) + P_{\text{1-loop}}^{(S)}(k, \mu), \]  

The first term in the right-hand side is the linear-order result of the redshift-space power spectrum, and the factor \((1 + f \mu^2)^2\) multiplied by the linear power spectrum \(P_{\text{lin}}\) indicates the enhancement due to the Kaiser effect. The second term \(P_{\text{1-loop}}^{(S)}\) represents a collection of the leading-order mode-coupling terms called one-loop correction, arising both from the gravitational clustering and the redshift distortion. This term is basically of the forth order in linear-order density or velocity fields, and is roughly proportional to \(P_{\text{lin}} \Delta^2\) with \(\Delta^2 = k^3 P_{\text{lin}}/(2\pi^2)\).

On the other hand, the Lagrangian PT description of the redshift-space power spectrum is obtained in somewhat different way. Intuitively, we rewrite the exact expression \([1]\) in terms of the displacement vector, and the perturbative expansion is applied to the displacement vector. Although a naive perturbative treatment merely reproduces the standard PT result \([5]\), Ref. [29] has applied a partial expansion, and some of the terms has been kept in some non-perturbative ways. The resultant expressions for power spectrum in redshift space becomes

\[ P_{\text{LPT}}^{(S)}(k, \mu) = e^{-k^2(1+f(j+2)\mu^2)} \sigma_{v,\text{lin}}^2, \]

\[ \times \left[ P_{\text{SPT}}^{(S)}(k, \mu) + (1 + f \mu^2)^2 \left( 1 + f (j+2) \mu^2 \right) k^2 \sigma_{v,\text{lin}}^2 \right], \]

where the quantity \(\sigma_{v,\text{lin}}^2\) is the linear-order estimate of the one-dimensional velocity dispersion given by

\[ \sigma_{v,\text{lin}}^2 = \frac{1}{3} \int \frac{d^3q}{(2\pi)^3} \frac{P_{\text{lin}}(q, z)}{q^2}. \]

The exponential prefactor in Eq. \([1]\) can be regarded as the result of non-perturbative treatment, and in redshift space, this term accounts for the non-linear damping of the BAOs arising both from the gravitational clustering and Finger-of-God effect of redshift distortion.

Fig. \([1]\) compares the PT based models of redshift distortion with N-body simulations of Ref. [34]. Left and right panels respectively show the monopole \((\ell = 0)\) and quadrupole \((\ell = 2)\) moments of power spectrum divided by the smooth reference spectrum at different redshifts, \(z = 3, 1\) and 0.5 (from top to bottom). The reference spectrum \(P_{\ell, \text{no-wiggle}}^{(S)}(k)\) is calculated from the no-wiggle approximation of the linear transfer function in Ref. [43], taking account of the linear-order result of Kaiser effect. The multipole moment of two-dimensional power spectrum is defined by

\[ P_{\ell}^{(S)}(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu \, P^{(S)}(k, \mu) \mathcal{P}_{\ell}(\mu), \]

with \(\mathcal{P}_{\ell}(\mu)\) being the Legendre polynomials.

As it has been repeatedly stated in the literature [23, 25, 34, 44], the standard PT treatment is not sufficiently accurate to describe the BAOs. Fig. \([1]\) confirms that this is indeed true not only in real space, but also in redshift space. While the power spectrum amplitude of N-body simulations tends to be smaller than that of the linear theory prediction (dotted), the predicted amplitude of standard PT generally overestimates the N-body results, and it exceeds the linear prediction on small scales. Compared to the results in real space, the discrepancy between prediction and simulation seems a bit large. Contrastingly, in the Lagrangian PT calculation, the amplitude of power spectrum is rather suppressed, and a better agreement between prediction and simulation is achieved at low-\(k\). This is due to the exponential prefactor in Eq. \([1]\). As a trade-off, however, the predicted amplitude at higher \(k\) modes largely underestimates the result of N-body simulations. Further, a closer look at first peak of BAOs around \(k \sim 0.05-0.1\ \text{hMpc}^{-1}\) reveals a small discrepancy, which becomes significant as decreasing the redshift and can produce few \% errors in power spectrum amplitude.

These results indicate that the existing PT based approaches fail to describe the two competitive effects of redshift distortion in the power spectrum [55]. A proper account of these is thus essential in accurately modeling BAOs.

B. Phenomenological model description

Next consider the phenomenological models of redshift distortion, which have been originally introduced to explain the observed power spectrum on small scales. Although the relation between the model and exact expression \([1]\) is less clear, for most of the models frequently used in the literature, the redshift-space power spectrum is expressed in the form as (e.g., [11, 45, 54])

\[ P^{(S)}(k, \mu) = D_{\text{FoG}}[k \mu f \sigma_v] P_{\text{Kaiser}}(k, \mu), \]

where the term \(P_{\text{Kaiser}}(k, \mu)\) represents the Kaiser effect, and the term \(D_{\text{FoG}}[k \mu f \sigma_v]\) indicates the damping func-
FIG. 1: Ratio of power spectra to smoothed reference spectra in redshift space, \( P^{(s)}_\ell(k) / P^{(s)}_{\ell, \text{no-wiggle}}(k) \). N-body results are taken from the \textit{wmap5} simulations of Ref. [34]. The reference spectrum \( P^{(s)}_{\ell, \text{no-wiggle}} \) is calculated from the no-wiggle approximation of the linear transfer function, and the linear theory of the Kaiser effect is taken into account. Short dashed and dot-dashed lines respectively indicate the results of one-loop PT and Lagrangian PT calculations for redshift-space power spectrum (Eqs. (5) and (6)).

FIG. 2: Same as in Fig. 1 but we here plot the results of phenomenological model predictions. The three different predictions depicted as solid, dashed, dot-dashed lines are based on the phenomenological model of redshift distortion [1] with various choices of Kaiser and Finger-of-God terms (Eqs. (10) and (11)). Left panel shows the monopole power spectra (\( \ell = 0 \)), and the right panel shows the quadrupole spectra (\( \ell = 2 \)). In all cases, one-dimensional velocity dispersion \( \sigma_v \) was determined by fitting the predictions to the N-body simulations. In each panel, vertical arrow indicates the maximum wavenumber \( k_{1\%} \) for improved PT prediction including up to the second-order Born approximation (see Eq. (12) for definition).
tion which mimics the Finger-of-God effect. The quantity $\sigma_\nu$ is the one-dimensional velocity dispersion defined by $\sigma_\nu^2 = \langle \nu^2(0) \rangle$. The variety of the functional form for $P_{\text{Kaiser}}(k, \mu)$ and $D_{\text{FoG}}[k \mu f \sigma_\nu]$ are summarized as follows.

The Kaiser effect has been first recognized from the linear-order calculations [51], from which the enhancement factor $(1 + f \mu^2)^2$ is obtained (see Eq. [53]). As a simple description for the Kaiser effect, one may naively multiply the non-linear matter power spectrum by this factor, just by hand. Recently, proper account of the non-linear effect has been discussed [41, 45], and non-linear model of Kaiser effect has been proposed using the real-space power spectra. Thus, we have

$$P_{\text{Kaiser}}(k, \mu) = \begin{cases} (1 + f \mu^2)^2 P_{\delta \delta}(k) & \text{linear} \\ P_{\delta \delta}(k) + 2f \mu^2 P_{\delta \theta}(k) + f^2 \mu^4 P_{\theta \theta}(k) & \text{non-linear} \end{cases}$$

(10)

Here, the spectra $P_{\delta \delta}$, $P_{\delta \theta}$, and $P_{\theta \theta}$ denote the auto power spectra of density and velocity divergence, and their cross power spectrum, respectively. The velocity divergence $\theta$ is defined by $\theta \equiv \nabla u = -\nabla v/(aHf)$ [60].

On the other hand, the functional form of the damping term can be basically modeled from the distribution function of one-dimensional velocity. Historically, it is characterized by Gaussian or exponential function (e.g., $\exp(-x^2)$; Gaussian and $1/(1 + x^2)$; Lorentzian

(11)

Note that there is analogous expression for exponential distribution, i.e., $D_{\text{FoG}}[x] = 1/(1 + x^2)^2$ [40], but the resultant power spectrum is quite similar to the one adopting the Lorentzian form for the range of our interest, $x \lesssim 1$. Since the Finger-of-God effect is thought to be a fully non-linear effect, which mostly comes from the virialized random motion of the mass (or galaxy) residing at a halo, the prediction of $\sigma_\nu$ seems rather difficult. Our primary purpose is to model the shape and structure of acoustic feature in the power spectrum, and the precise form of the damping is basically irrelevant. We thus regard $\sigma_\nu$ as a free parameter, and determine it by fitting the predictions to the simulations or observations.

Fig. 2 compares the phenomenological models of redshift distortion with combination of Eqs. (10) and (11) with N-body simulations. In computing the redshift-space power spectrum from the phenomenological models, we adopt the improved PT treatment by Refs. [33, 34], and the analytic results including the corrections up to the second-order Born approximation are used to obtain the three different power spectra $P_{\delta \delta}$, $P_{\theta \theta}$ and $P_{\theta \theta}$. The accuracy of the improved PT treatment has been checked in details by Refs. [34], and it has been shown that the predictions of $P_{\delta \delta}$ reproduce the N-body results quite well within 1% accuracy below the wavenumber $k_{1\%}$, indicated by the vertical arrows in Fig. 2. This has been calibrated from a proper comparison between N-body and PT results and is empirically characterized by solving the following equation [27, 34]:

$$\frac{k_{1\%}^2}{6\pi^2} \int_0^{k_{1\%}} dq P_{\text{lin}}(q; z) = C$$

(12)

with $C = 0.7$ and $P_{\text{lin}}$ being linear matter spectrum. Note that the 1% accuracy of the improved PT prediction at $z = 3$ has reached at $k \sim 0.47 h\,\text{Mpc}^{-1}$, outside the plot range. We basically use this criterion to determine $\sigma_\nu$, and fit the predictions of both monopole and quadrupole spectra to the N-body results in the range $0 \leq k \leq k_{1\%}$.

Since we allow $\sigma_\nu$ to vary as a free parameter, the overall behaviors of the model predictions reproduce with N-body results, and the differences between model predictions are basically small compared to the results in the PT description. However, there still exist small but non-negligible discrepancies between N-body results and model predictions, which are statistically significant, and are comparable or exceed the expected errors in upcoming BAO measurements [34]. Although the agreement is somehow improved when we adopt the non-linear model of $P_{\text{Kaiser}}$, there still remains discrepancies of few % in monopole and 5 % in quadrupole moments of power spectrum amplitudes. These are irrespective of the choice of damping function $D_{\text{FoG}}$.  

FIG. 3: Redshift evolution of velocity dispersion $\sigma_\nu$ determined by fitting the predictions of monopole and quadrupole power spectra to the N-body results. While the solid lines represent the linear theory prediction, the symbols indicate the results obtained by fitting models of redshift distortion with various choices of Kaiser and damping terms (see Fig. 2).
Furthermore, the fitted results of $\sigma_v$ show somewhat peculiar behavior. Fig. 3 plots the fitted values of $\sigma_v$ as function of redshift (symbols), which significantly deviate from linear theory prediction (solid line) as increasing the redshifts. This is in contrast with a naive expectation, and indicates that the model based on the expression (9) misses something important, and needs to be reconsidered.

IV. IMPROVED MODEL PREDICTION

A. Derivation

Comparison in previous section reveals that even in the models with fitting parameter, a small but non-negligible discrepancy appears at the scales of BAOs, where the choice of the damping function $D_{\text{FoG}}[x]$ does not sensitively affect the predictions. This implies that there exists missing terms arising from the non-linear mode coupling between density and velocity fields, and those corrections alter the acoustic feature in redshift-space power spectrum. In this section, starting with the exact expression (11), we derive non-linear corrections, which are relevant to explain the modulation of acoustic features in redshift space.

First recall that the expression (11) is written in the form as

$$P^{(s)}(k, \mu) = \int d^3x e^{i k \cdot x} e^{j A_1 A_2 A_3},$$

(13)

where the quantities $j_1$, $A_i$ ($i = 1, 2, 3$) are respectively given by

$$j_1 = -i k \mu f,$$

$$A_1 = u_z(r) - u_z(r'),$$

$$A_2 = \delta(r) + f \nabla u_z(r),$$

$$A_3 = \delta(r') + f \nabla u_z(r').$$

We shall rewrite the ensemble average $\langle e^{j A_1 A_2 A_3} \rangle$ in terms of the cumulants. To do this, we use the relation between the cumulant and moment generating functions. For the stochastic vector field $A = \{A_1, A_2, A_3\}$, we have (e.g., [22, 41]):

$$\langle e^{j A} \rangle = \exp \{\langle e^{j A} \rangle_c \}$$

(14)

with $j$ being arbitrary constant vector, $j = \{j_1, j_2, j_3\}$. Taking the derivative twice with respect to $j_2$ and $j_3$, and we then set $j_2 = j_3 = 0$. We obtain [41]

$$\langle e^{j_1 A_1 A_2 A_3} \rangle = \exp \{\langle e^{j_1 A_1} \rangle_c \} \times [\langle e^{j_1 A_1 A_2 A_3} \rangle_c + \langle e^{j_1 A_1 A_2} \rangle_c \langle e^{j_1 A_1 A_3} \rangle_c].$$

(15)

Substituting this into Eq. (13), we arrive at

$$P^{(s)}(k, \mu) = \int d^3x e^{i k \cdot x} \exp \{\langle e^{j_1 A_1} \rangle_c \} \times [\langle e^{j_1 A_1 A_2 A_3} \rangle_c + \langle e^{j_1 A_1 A_2} \rangle_c \langle e^{j_1 A_1 A_3} \rangle_c].$$

(16)

This expression clearly reveals the coupling between density and velocity fields associated with Kaiser and Finger-of-God effects. In addition to the prefactor $\exp \{\langle e^{j_1 A_1} \rangle_c \}$, the ensemble averages over the quantities $A_2$ and $A_3$ responsible for the Kaiser effect all includes the exponential factor $e^{j_1 A_1}$, which can produce a non-negligible correlation between density and velocity.

Comparing Eq. (16) with the expression (9) with (10) and (11), we deduce that the phenomenological models discussed in Sec. IV.B miss something important, and are derived based on several assumptions or treatments:

- In the integrand of Eq. (10), while taking the limit $j_1 \to 0$ in the bracket, we keep $j_1 \neq 0$ in the exponent of the prefactor.

- For cumulants $\langle A_i^n \rangle_c = \langle [u_z(r) - u_z(r')]^n \rangle_c$ of any integer value $n$, the spatial correlations between different positions are ignored, and the non-vanishing cumulants are assumed to be expressed as $\langle A_i^n \rangle_c = 2^n n! c_n \sigma_v^n$ for even number of $n$, with $c_n$ being constants.

- To further obtain the Gaussian or Lorentzian forms of the damping function $D_{\text{FoG}}[\xi]$, we assume that the conditions, $c_{2n} = 0$ except for $c_2 = 1$, or, $c_{2n} = (2n - 1)!$ and $c_{2n-1} = 0$, are fulfilled.

In the above, the last two conditions play a role for specifying the damping function, and they mainly affect the broadband shape of the power spectrum. On the other hand, the first condition leads to the expression of $P_{\text{Kaiser}}(k)$, which can add the most dominant contribution to the acoustic feature in power spectrum. Since the choice of the damping function (11) is presumably a minor source for discrepancies between the model predictions and simulations, taking the limit $j_1 \to 0$ in the bracket would be the main reason for discrepancy. In this respect, the terms involving the exponential factor can produce additional contributions to the spectrum $P_{\text{Kaiser}}(k)$, which are responsible for explaining the modulated acoustic peak and trough structure in redshift space.

Let us now derive the corrections to $P_{\text{Kaiser}}(k)$. To do this, we keep the last two conditions, and perturbatively treat the terms inside the bracket of Eq. (16). This treatment is reasonable, because the modification of acoustic features should be small for relevant scales of BAOs. On the other hand, the factor $\exp \{\langle e^{j_1 A_1} \rangle_c \}$ is most likely affected by the virialized random motion of the mass around halos, and seems difficult to treat it perturbatively. Here, regarding the quantity $j_1$ as a small expansion parameter, we perturbatively expand the terms in the bracket of the integrand. Up to the second order in...
to order when we explicitly compute it employing the perturbation theory calculation, and is roughly proportional to $O(P_{10}^3)$. We thus drop the higher-order contribution, and collect the leading and next-to-leading order corrections (18), (19) and (20) still have some non-perturbative properties, although the new corrections $A$ and $B$ neglected in the previous phenomenological models are expected to be small, and can be treated perturbatively. In Appendix A based on the standard PT treatment, we summarize the perturbative expressions for the corrections (19) and (20), in which the three-dimensional integrals are reduced to the sum of the one- and two-dimensional integrals.

\[
\langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c \\
\simeq \langle A_2 A_3 \rangle + j_1 \langle A_1 A_2 A_3 \rangle_c \\
+ j_1^2 \{ \frac{1}{2} \langle A_2^2 A_3 \rangle_c + \langle A_1 A_2 \rangle_c \langle A_1 A_3 \rangle_c \} + O(j_1^3). \tag{17}
\]

In the above, the term $\langle A_2^2 A_3 \rangle_c$ turns out to be higher order when we explicitly compute it employing the perturbation theory calculation, and is roughly proportional to $O(P_{10}^3)$. We thus drop the higher-order contribution, and collect the leading and next-to-leading order corrections. Then, Eq. (10) can be recast as

\[
P^{(S)}(k, \mu) = D_{FoG}[k \mu f \sigma_v] \left\{ P_{\delta \delta}(k) + 2 f \mu^2 P_{\delta \theta}(k) + f^2 \mu^4 P_{\theta \theta}(k) + A(k, \mu) + B(k, \mu) \right\}. \tag{18}
\]

Here, we replaced the exponential prefactor $\exp\{\langle e^{j_1 A_1} \rangle_c \}$ with the damping function $D_{FoG}$. The corrections $A$ and $B$ are respectively given by

\[
A(k, \mu) = j_1 \int d^3x \ e^{i k \cdot x} \langle A_1 A_2 A_3 \rangle_c,
\]

\[
B(k, \mu) = j_1^2 \int d^3x \ e^{i k \cdot x} \langle A_1 A_2 \rangle_c \langle A_1 A_3 \rangle_c.
\]

In terms of the basic quantities of density $\delta$ and velocity divergence $\theta = -\nabla v/(aHf)$, they are rewritten as

\[
A(k, \mu) = (k \mu f) \int \frac{d^3p}{(2\pi)^3} \frac{p_z}{p^2} \times \left\{ B_\sigma(p, k - p, -k) - B_\sigma(p, k, -k - p) \right\}, \tag{19}
\]

\[
B(k, \mu) = (k \mu f)^2 \int \frac{d^3p}{(2\pi)^3} F(p) F(k - p); \tag{20}
\]

\[
F(p) = \frac{p_z}{p^2} \left\{ P_{\theta \theta}(p) + f \frac{p_z^2}{p^2} P_{\theta \theta}(p) \right\},
\]

where the function $B_\sigma$ is the cross bispectra defined by

\[
\langle \theta(k_1) \delta(k_2) + f k_2^2 \theta(k_2) \rangle \left\{ \delta(k_3) + f k_3^2 \delta(k_3) \right\}
= (2\pi)^3 \delta_D(k_1 + k_2 + k_3) B_\sigma(k_1, k_2, k_3). \tag{21}
\]

In deriving the expression (18), while we employed the low-$k$ expansion, we do not assume that the terms $A_1$ themselves are entirely small. In this sense, the expressions (18), (19) and (20) still have some non-perturbative properties, although the new corrections $A$ and $B$ neglected in the previous phenomenological models are expected to be small, and can be treated perturbatively. In Appendix A based on the standard PT treatment, we summarize the perturbative expressions for the corrections (19) and (20), in which the three-dimensional integrals are reduced to the sum of the one- and two-dimensional integrals.

To see the significance of the newly derived terms $A$ and $B$, we evaluate the monopole and quadrupole contributions to the functions defined by

\[
P_{\ell, corr}^{(S)}(k) \equiv \frac{2\ell + 1}{2} \int_{-1}^1 d\mu \ D_{FoG}(k \mu f \sigma_v) \left\{ \begin{array}{l} A(k, \mu) \\ B(k, \mu) \end{array} \right\}. \tag{22}
\]

The results are then plotted in Fig. 4 divided by the smooth reference power spectrum, $P_{\ell, corr}^{(S)}(k)/P_{\ell, no-wiggle}^{(S)}(k)$ (Eq. (22)). We adopt the Gaussian form of the damping function $D_{FoG}$, and adopt the linear theory to estimate $\sigma_v$ (see Eq. (17)).

The corrections coming from the $A$ term show oscillatory behaviors, and tend to have a larger amplitude than those from the $B$ term. While the corrections from the $B$ term are basically smooth and small, they still yield a non-negligible contribution, especially for quadrupole power spectrum. Although the actual contributions of these corrections to the total power spectrum are determined by the fitting parameter $\sigma_v$, and thus the resultant amplitudes shown in Fig. 4 do not simply reflect the correct amplitudes, the new corrections $A$ and $B$ can definitely give an important contribution to the acoustic feature in power spectrum.

Finally, it is interesting to note that while the new formula for redshift-space power spectrum (18) would be applicable to the non-linear regime where the standard PT calculation breaks down, the resultant expression itself is similar to the one for redshift-space power spectrum in the one-loop standard PT. The one-loop power spectrum in redshift space, $P_{SPT}^{(S)}(k, \mu)$ given at Eq. 13,
The function $C$ to the third order in $\delta$ is defined by

$$C(k, \mu) = (k \mu)^2 \int \frac{d^3 p d^3 q}{(2\pi)^6} \delta_D(k-p-q) \frac{\mu^2_\varphi}{p^2} P_{\theta\theta}(p) \times \{P_{\delta\delta}(q) + 2 f \mu^4_\varphi P_{\delta\theta}(q) + f^2 \mu^2_\varphi P_{\theta\theta}(q)\} \simeq (k \mu)^2 \int \frac{d^3 p d^3 q}{(2\pi)^6} \delta_D(k-p-q) \frac{\mu^2_\varphi}{p^2} (1 + f \mu^2_\varphi)^2 \times P_{\text{lin}}(p) P_{\text{lin}}(q)$$

with $\mu_\varphi = p_z/|p|$ and $\mu_q = q_z/|q|$. The second equality is valid for the one-loop PT calculation. Hence, if we adopt either of Lorentzian or Gaussian form in Eq. (14) and just expand it in powers of its argument, the new formula (18) reduces to the one-loop result (23) just dropping the term $C$.

The $C$ term is originated from the spatial correlation of the velocity field, and is obtained through the low-$k$ expansion of the exponential prefactor $\exp\{e^{i A_1}\}$ in Eq. (10). For the scales of BAOs, the $C$ term monotonically increases the amplitude of power spectrum, and it does not alter the acoustic structure drastically. Indeed, our several examinations reveal that the effect of this can be effectively absorbed into the damping function $D[k\mu f \sigma_\nu]$ with varying the velocity dispersion $\sigma_\nu$. Rather, the main drawback of the standard PT expression (23) comes from a naive expansion of all the terms in the exact formula (4), which fails to describe the delicate balance between the Finger-of-God damping and the enhancement from Kaiser effect and non-linear gravitational growth. As we will see in next subsection, both keeping the damping term $D_{\text{FoG}}$ and including the corrections $A$ and $B$ seem essential, and with this treatment, even the standard PT calculation of the power spectrum can give a excellent result which reproduces the N-body simulations fairly well.

B. Comparison with N-body simulations

We now compare the new prediction of redshift-space power spectra with the result of N-body simulations. Fig. 5 shows the monopole (left) and quadrupole (right) power spectra divided by their smooth reference spectra. The analytical predictions based on the model (18) are plotted adopting the Gaussian form of the Finger-of-God term $D_{\text{FoG}}[k\mu \sigma_\nu]$, and the velocity dispersion $\sigma_\nu$ is de-
For monopole (FIG. 6) contributions of the corrections \( A \) \( (S) \) and \( B \) \( (S) \) and each contribution is separately plotted dividing by smoothed reference spectra, \( P^{(S)} \) \( \text{corr} \). Here, the spectrum \( P^{(S)} \) \( \text{corr} \) (dotted) is the contribution of non-linear Kaiser term \( (10) \) convolved with the Finger-of-God damping \( D_{\text{FoG}} \), and the corrections \( P^{(S)} \) \( \text{corr,A} \) and \( P^{(S)} \) \( \text{corr,B} \) are those given by Eq. \( (22) \).

The spectra \( P^{(S)} \) \( \text{corr,A} \) and \( P^{(S)} \) \( \text{corr,B} \) represent the actual contributions of the corrections \( A \) and \( B \) defined by Eq. \( (22) \), with fitted value of \( \sigma_v \). The corrections \( A \) and \( B \) give different contributions in the amplitude of monopole and quadrupole spectra, and their total contribution can reach \( \sim 10\% \) and \( \sim 40\% \) for monopole and quadrupole spectra at \( k \lesssim 0.2h\text{Mpc}^{-1} \), respectively. Thus, even though the resultant shape of the total spectrum \( P^{(S)}(k) \) apparently resembles the one obtained from phenomenological model, the actual contribution of the corrections \( A \) and \( B \) would be large and cannot be neglected.

Note, however, that a closer look at low-\( z \) behavior reveals a slight discrepancy around \( k \sim 0.15h\text{Mpc}^{-1} \) and \( 0.22h\text{Mpc}^{-1} \) in the monopole spectrum. Also, discrepancies in the quadrupole spectrum seems bit large, and eventually reach \( \sim 5\% \) error in some wavenumbers at \( z = 0.5 \). This is partially ascribed to our heterogeneous treatment on the corrections \( A \) and \( B \) using the standard PT calculations. It is known that the standard PT result generically gives rise to a strong damping in the BAOs, and it incorrectly leads to a phase reversal of the BAOs. Thus, beyond the validity regime of the standard PT, the predictions including the small corrections tend to oversmear the acoustic feature, leading to a small discrepancy shown in Fig. \( 5 \).

Another source for the discrepancies may come from the effect of finite-mode sampling caused by the finite boxsize of the N-body simulations. As advocated by Refs. \( 23, 52 \), due to the finite number of Fourier modes, the matter power spectrum measured from N-body simulations may not agree well with the predictions of linear theory nor standard PT even at very large scales, and tends to systematically deviate from them. While we follow and extend the procedure of Ref. \( 23 \) to correct this effect in redshift space, it relies on the leading-order calculations of standard PT, and the correction for finite-

In Fig. \( 6 \) we divide the total power spectrum \( P^{(S)} \) \( \text{total} \) into the three pieces as \( P^{(S)} \) \( \text{total} = P^{(S)} \) \( \text{Kaiser} \) + \( P^{(S)} \) \( \text{corr,A} \) + \( P^{(S)} \) \( \text{corr,B} \), and each contribution is separately plotted dividing by smoothed reference spectra, \( P^{(S)} \) \( \text{corr} \). Note, however, that a closer look at low-\( z \) behavior reveals a slight discrepancy around \( k \sim 0.15h\text{Mpc}^{-1} \) and \( 0.22h\text{Mpc}^{-1} \) in the monopole spectrum. Also, discrepancies in the quadrupole spectrum seems bit large, and eventually reach \( \sim 5\% \) error in some wavenumbers at \( z = 0.5 \). This is partially ascribed to our heterogeneous treatment on the corrections \( A \) and \( B \) using the standard PT calculations. It is known that the standard PT result generically gives rise to a strong damping in the BAOs, and it incorrectly leads to a phase reversal of the BAOs. Thus, beyond the validity regime of the standard PT, the predictions including the small corrections tend to oversmear the acoustic feature, leading to a small discrepancy shown in Fig. \( 5 \).

In Fig. \( 6 \) to see the significance of the contributions from corrections \( A \) and \( B \), we divide the improved PT prediction of power spectra \( P^{(S)}(k) \) at \( z = 1 \) into the three pieces as \( P^{(S)} \) \( \text{Kaiser} \), \( P^{(S)} \) \( \text{corr,A} \), and \( P^{(S)} \) \( \text{corr,B} \), which are separately plotted as dotted, long-dashed, and short dashed lines, respectively. The power spectrum \( P^{(S)} \) \( \text{Kaiser} \) is the contribution of the non-linear Kaiser term given in Eq. \( (10) \), convolved with the damping function \( D_{\text{FoG}} \).
mode sampling has been restricted to the low-\(k\) modes, \(k \lesssim 0.1 h \text{Mpc}^{-1}\) \cite{34}. Hence, the high-\(k\) modes of the power spectrum plotted here may be affected by the effect of finite-mode sampling, and it would be significant for higher-multipole spectrum because of its small amplitude. This might be still serious even with the 30 independent data of N-body simulations.

Perhaps, the best way to remedy these discrepancies at low-\(z\) is both to apply the improved PT treatment to the corrections \(A\) and \(B\), and to consider the higher-order contributions for correcting the effect of finite-mode sampling over the relevant range of BAOs. The complete analysis along the line of this need some progress and is beyond the scope of this paper. Nevertheless, it should be stressed that the model given by Eq. \ref{eq:18} captures several important aspects of redshift distortion, and even the present treatment with standard PT calculations of the corrections \(A\) and \(B\) can provide a better description for power spectra. In Fig. \ref{fig:7}, we plot the fitted values of the velocity dispersion obtained from the new predictions shown in Fig. \ref{fig:5}. The redshift dependence of the fitted results roughly matches physical intuition, and is rather consistent with the linear theory prediction. This is contrasted to the cases neglecting the corrections (see Fig. \ref{fig:5}).

As another significance, we plot in Fig. \ref{fig:8} the quadrupole-to-monopole ratios for redshift-space power spectra. The new model predictions using standard and improved PT calculations (solid and dashed) are compared with those neglecting the corrections \(A\) and \(B\) (dot-dashed). The amplitude of the ratio \(P_{2}^{(S)}/P_{0}^{(S)}\) basically reflects the strength of the clustering anisotropies, and is proportional to \((4f/3+4f^2/7)/(1+2f/3+f^2/5)\) in the limit \(k \to 0\) (e.g., \cite{1,3,51}). One noticeable point is that the N-body results of quadrupole-to-monopole ratio do exhibit an oscillatory behavior, and the model including the corrections \ref{eq:18} reproduces the N-body trends fairly well. On the other hand, the phenomenological model neglecting the corrections generally predicts the smooth scale-dependence of the ratio \(P_{2}^{(S)}/P_{0}^{(S)}\), and thus it fails to reproduce the oscillatory feature. Since this oscillation is originated from the acoustic feature in BAOs, Fig. \ref{fig:8} implies that the quadrupole-to-monopole ratio possesses helpful information not only to constrain the growth-rate parameter \(f\), but also to determine the acoustic scales. In other words, any theoretical template for redshift-space power spectrum neglecting the corrections \(A\) and \(B\) may produce a systematic bias in determining the growth-rate parameter \(f(z)\), Hubble parameter \(H(z)\) and angular diameter distance \(D_A(z)\), which we will discuss in details in next section.

V. IMPLICATIONS

The primary science goal of future galaxy surveys is to clarify the nature of late-time cosmic acceleration, and thereby constraining the parameters \(D_A(z), H(z)\) and \(f(z)\) through a precise measurement of BAOs in redshift space would be the most important task. However, these constraints may be biased if we use the incorrect model of redshift distortion as theoretical template fitting to observations. In this section, we explore the potential impact on the uncertainty and bias in the parameter estimation for \(D_A(z), H(z)\) and \(f(z)\).

\section{A. Recovery of parameters \(D_A, H\) and \(f\)}

Let us first examine the parameter estimation using the new model of redshift distortion. Fitting the theoretical template of power spectrum to the N-body data, we will check if the best-fit parameters for \(D_A(z), H(z)\) and \(f(z)\) can be correctly recovered from the monopole and quadrupole moments of anisotropic BAOs.

We model the power spectrum of N-body simulations by

\[
P^{(S)}_{\text{model}}(k, \mu) = \frac{H(z)}{H_{\text{fid}}(z)} \left( \frac{D_{A,\text{fid}}(z)}{D_A(z)} \right)^2 P^{(S)}(q, \nu), \tag{25}
\]

where the comoving wavenumber \(k\) and the directional cosine \(\mu\) for the underlying cosmological model are related to the true ones \(q\) and \(\nu\) by the Alcock-Paczynski

\[k = (1+z) \frac{d_A(z)}{d_{\text{fid}}(z)} \mu, \quad \mu = \frac{\cos \theta}{1+z},
\]

and \(d_A(z)\) and \(d_{\text{fid}}(z)\) are the angular diameter distance for the fiducial cosmology and the underlying cosmological model, respectively.
effect through (e.g., 18 40 50)

\[ q = k \left[ \left( \frac{D_{A,\text{fid}}}{D_A} \right)^2 + \left\{ \left( \frac{H}{H_{\text{fid}}} \right) - \left( \frac{D_{A,\text{fid}}}{D_A} \right)^2 \right\} \mu^2 \right]^{1/2}, \tag{26} \]

\[ \nu = \left( \frac{H}{H_{\text{fid}}} \right) \mu \times \left[ \left( \frac{D_{A,\text{fid}}}{D_A} \right)^2 + \left\{ \left( \frac{H}{H_{\text{fid}}} \right) - \left( \frac{D_{A,\text{fid}}}{D_A} \right)^2 \right\} \mu^2 \right]^{-1/2}, \tag{27} \]

The quantities \( D_{A,\text{fid}} \) and \( H_{\text{fid}} \) are the fiducial values of the angular diameter distance and Hubble parameter adopted in the N-body simulations. For a given set of cosmological parameters, the redshift-space power spectrum \( P^{(S)} \) is calculated from Eq. (18), but we here treat the quantity \( f \) as free parameter in addition to the velocity dispersion \( \sigma_v \). Further, to mimic a practical data analysis using galaxy power spectrum, we introduce the bias parameter \( b \), assuming the linear deterministic relation, i.e., \( \delta_{\text{sim}} = b \delta_m \) 67. Then, fitting the monopole and quadrupole power spectra of Eq. (25) to those of the N-body simulation at \( z = 1 \), we determine the best-fit values of \( D_A, H \) and \( f \), just marginalized over the parameters \( \sigma_v \) and \( b \). To do this, we use the Markov chain Monte Carlo (MCMC) technique described by Ref. 58, and adopt the likelihood function given by

\[ -2 \ln L = \sum_n \sum_{\ell,\ell'=0,2} \left\{ P_{\ell,\text{sim}}^{(S)}(k_n) - P_{\ell,\text{model}}^{(S)}(k_n) \right\} \times \text{Cov}_{\ell,\ell'}^{-1}(k_n) \left\{ P_{\ell',\text{sim}}^{(S)}(k_n) - P_{\ell',\text{model}}^{(S)}(k_n) \right\}, \tag{28} \]

where the quantity \( \text{Cov}_{\ell,\ell'} \) represents the covariance matrix between different multipoles. The range of wavenumber used in the likelihood analysis was chosen as \( k \leq k_{\text{max}} = 0.205h\text{Mpc}^{-1} \), so as to satisfy \( k_{\text{max}} \leq k_{\text{vec}} \). As for the covariance, we simply ignore the non-Gaussian contribution (see Ref. 54 for validity of this treatment), and use the linear theory to estimate the diagonal components of the covariance, \( \text{Cov}_{\ell,\ell'} \), including the effect of shot-noise contribution assuming the galaxy number density \( \pi_g = 5 \times 10^{-4} h^3 \text{Mpc}^{-3} \). The explicit expression for the covariance is presented in Appendix C. We checked that the linear theory estimate reasonably reproduces the N-body results of the covariance matrix for the range of our interest \( k \leq 0.3 h \text{Mpc}^{-1} \) at \( z = 1 \).

Fig. 9 summarizes the result of the MCMC analysis assuming an ideally large survey with \( V_s = 20h^{-3}\text{Gpc}^3 \). The two-dimensional contour of the 1-\( \sigma \) marginalized errors are shown for \( D_A/D_{A,\text{fid}} \) vs \( H/H_{\text{fid}} \) (bottom left), \( D_A/D_{A,\text{fid}} \) vs \( f \) (middle left), and \( f \) vs \( D_A/D_{A,\text{fid}} \) (bottom center). Also, the marginalized posterior distribution for each parameter are plotted in the top left, middle center, and bottom right panels. In each panel, blue and red lines respectively represent the results using the model of redshift distortion with and without the terms \( A \) and \( B \).

As it is clear from Fig. 9, the model including the corrections shows a better performance. Within the 1-\( \sigma \) errors, which roughly correspond to the precision of a percent-level, it correctly reproduces the fiducial values of the parameters (indicated by crosses). On the other hand, the two-dimensional errors of the results neglecting the corrections show a clear evidence for the systematic bias on the best-fit parameters. Accordingly, the resultant value of \( \chi^2 \) around the best-fit parameters, given by \( \chi^2 = -2 \ln L \), is larger than that of the case including the corrections: \( \chi^2 = 10.1 \) and 22.2 for the cases with and without corrections, respectively. Although the deviation from the fiducial values seems somewhat small except for the growth-rate parameter \( f \), this is solely due to the fact that we only use the monopole and quadrupole power spectra. It would be generally significant in the analysis using the full shape of redshift-space power spectrum.
for which the statistical errors are greatly reduced, and thereby the systematic biases would be prominent.

**B. Impact of redshift distortion on future measurements of \(D_A, H\) and \(f\)**

Given the fact that the robust measurement of \(D_A, H\) and \(f\) can be made with the new model of redshift distortion, we then move to the discussion on the potential impact on the future measurements using the full shape of the redshift-space power spectrum. Here, for illustrative purpose, we consider the two surveys around \(z = 1\), with volume \(V_s = 4\) and \(20\ h^{-3}\)Gpc\(^3\), and quantitatively estimate how the wrong model of redshift distortion leads to the incorrect measurements of \(D_A, H\) and \(f\).

The fundamental basis to estimate the uncertainties and systematic biases on model parameters is the Fisher matrix formalism. The Fisher matrix for galaxy survey is given by

\[
F_{ij} = \frac{V_s}{(2\pi)^2} \int_{k_\text{min}}^{k_{\text{max}}} dk\ k^2 \int_{-1}^{1} d\mu \times \frac{\partial \ln P^{(S)}_{\text{obs}}(k, \mu)}{\partial p_i} \frac{\partial \ln P^{(S)}_{\text{obs}}(k, \mu)}{\partial p_j} \left\{ \frac{\pi_g P^{(S)}_{\text{obs}}(k, \mu)}{\pi_g P^{(S)}_{\text{true}}(k, \mu)} + 1 \right\}^2
\]

with \(\pi_{\text{gal}}\) being the number density of galaxies, for which we specifically set \(\pi_{\text{gal}} = 5 \times 10^{-4}\ h^3\)Mpc\(^{-3}\). The minimum wavenumber available for a given survey, \(k_{\text{min}}\), is set to \(2\pi/V_s^{1/3}\). Here, the observed power spectrum \(P_{\text{obs}}^{(S)}(k, \mu)\) is given by Eq. (29), and we allow to include the influence of galaxy biasing adopting the deterministic linear relation, \(\delta_{\text{gal}} = b\ \delta_m\). Then, we have five parameters in total, given by \(p_i = \{b, f, \sigma_v, D_A/D_{A,\text{fid}}, H/H_{\text{fid}}\}\). Fiducial values of these parameters are set as \(b = 2, f = 0.858, D_A/D_{A,\text{fid}} = 1\) and \(H/H_{\text{fid}} = 1\). As for the velocity dispersion \(\sigma_v\), we use the fitted result to N-body simulations adopting the new model of redshift distortion in Sec. [LVIII] and set \(\sigma_v = 395\ km\ s^{-1}\).

Based on the Fisher matrix (29), the systematic bias for parameter \(p_i\) caused by incorrectly modeling theoretical power spectrum is estimated from the following formula:

\[
\Delta p_i = - \sum_j (F^{-1})_{ij} \ s_j
\]

where \(F^{-1}\) is the inverse Fisher matrix evaluated at the fiducial parameter set, but is obtained from an incorrect model of redshift distortion as a theoretical template of redshift-space power spectrum. The vector \(s_j\) is given by

\[
s_j = \frac{V_s}{(2\pi)^2} \int dk\ k^2 \int_{-1}^{1} d\mu \frac{\partial \ln P^{(S)}_{\text{obs}}(k, \mu)}{\partial p_j} \left\{ \frac{\pi_g P^{(S)}_{\text{obs}}(k, \mu)}{\pi_g P^{(S)}_{\text{true}}(k, \mu)} + 1 \right\}^2
\]

The function \(P^{\text{true}}(k, \mu)\) is the theoretical template adopting the incorrect model of redshift distortion. The systematic differences in the power spectrum amplitude are quantified as \(P^{\text{obs}}(k, \mu) - P^{\text{true}}(k, \mu)\), where \(P^{\text{true}}(k, \mu)\) is the correct template for redshift-space power spectrum \(P^{(S)}\), for which we assume the new model of redshift distortion including the terms \(A\) and \(B\) (Eq. [13]). Below, we will quantify the magnitude of systematic biases if we incorrectly apply the model of redshift distortion neglecting the corrections \(A\) and \(B\) for the power spectrum template.

Fig. [10] plots the results of the Fisher matrix calculations marginalized over the nuisance parameters \(b\) and \(\sigma_v\). The uncertainties and biases for the best-fit values of \(f\), \(D_A\) and \(H\) are estimated assuming \(k_{\text{max}} = 0.12h\)Mpc\(^{-1}\) (left) and \(0.2h\)Mpc\(^{-1}\) (right), and the results are shown for the surveys with \(V_s = 4 h^{-3}\)Gpc\(^3\) (open) and \(20 h^{-3}\)Gpc\(^3\) (shade). In each panel, two-dimensional contours around the crosses and filled triangles show the expected 1-\(\sigma\) (68\% C.L.) errors around the best-fit values adopting the model of redshift distortion with and without the corrections, respectively. The differences between best-fit values (crosses and filled triangles) represent the systematic biases estimated from Eq. (30), which remain unchanged irrespective of the survey volume \(V_s\). Since the size of marginalized uncertainties is proportional to \(V_s^{-1/2}\), the systematic bias in the best-fit parameters become relatively prominent and is considered to be a serious problem if we increase the survey volume. Note that similar to the result in Fig. [9] there exists a tight correlation of the parameters between the growth rate parameter \(f\) and quantities \(D_A\) and \(H\). This is consistent with the finding by Ref. [55], indicating that the distinguishing dark energy from modified gravity needs another observational constraint.

Fig. [10] implies that phenomenological model of redshift distortion neglecting the corrections can produce a large systematic error. As increasing the maximum wavenumber \(k_{\text{max}}\), the bias on the measurements of angular diameter distance and Hubble parameter reaches \(\sim 1 - 2\%\) error, while the best-fit value for the growth rate parameter \(f\) would be seriously biased with \(\sim 5\%\) error. If we conservatively choose a smaller value of \(k_{\text{max}} \lesssim 0.12h\)Mpc\(^{-1}\), these systematics could be still within the size of statistical error for surveys with typical volume of \(V_s \sim 4 h^{-3}\)Gpc\(^3\). However, for a survey of larger volume with \(V_s \gtrsim 20 h^{-3}\)Gpc\(^3\), the systematic error on the growth rate parameter becomes outside the marginalized uncertainty. If we aggressively choose \(k_{\text{max}} \sim 0.2h\)Mpc\(^{-1}\) in order to reduce statistical uncertainties, the systematic biases become definitely serious issues in all of the parameters \(f, D_A\) and \(H\) for both surveys of volume \(V_s = 4\) and \(20 h^{-3}\)Gpc\(^3\). Hence, correctly modeling redshift distortion would be very crucial for both stage-III and -IV class surveys defined by the Dark Energy Task Force [50].
VI. DISCUSSION AND CONCLUSION

In this paper, we have investigated the power spectrum in redshift space, and presented a new model of redshift distortion, which is particularly suited for modeling anisotropic BAOs around $k = 0 \sim 0.3 \, h \text{Mpc}^{-1}$. Contrary to the previous phenomenological modes in which the effects of Kaiser and Finger-of-God are separately treated in a multiplicative way, the new model includes the corrections coming from the non-linear coupling between velocity and density fields, which give rise to a slight uplift in the amplitude of monopole and quadrupole power spectra. The model predictions can give a good agreement with results of N-body simulations, and a percent level precision is almost achieved.

Based on the new model of redshift distortion, we proceeded to the parameter estimation analysis, and checked if the theoretical prediction correctly recovers the cosmological information from the monopole and quadrupole spectra of N-body simulations. MCMC analysis revealed that while the new model of redshift distortion combining the improved PT calculation faithfully reproduces the fiducial parameters $D_A$, $H$ and $f$ and the precision can reach at a percent level, the model neglecting the corrections ($A$ and $B$ terms) exhibits a slight offset of the best-fit values. In order to estimate the potential impact on the future measurement, we have further made the Fisher matrix analysis using the full shape of power spectrum $P^{(3)}(k, \mu)$, and found that the existing phenomenological models of redshift distortion neglecting the corrections produce a systematic error on measurements of the angular diameter distance and Hubble parameter by $1 \sim 2\%$, and the growth rate parameter by $\sim 5\%$. This would become non-negligible for stage-III and -IV class surveys defined by the Dark Energy Task Force. Correctly modeling redshift distortion is thus crucial, and the new prescription of redshift-space power spectrum presented here plays an essential role in constraining the dark energy and/or modified gravity from anisotropic BAOs.

Finally, we note several remaining tasks in practical application to the precision measurement of BAOs. One is the improved treatment for calculation of the corrections, $A$ and $B$ terms, which needs to evaluate the bispectrum of density and velocity fields. In doing this, a systematic treatment using multi-point propagator developed by Ref. [57] would be useful and indispensable. Also, the effects of the new contributions to the redshift-space clustering in the presence of the primordial non-Gaussianity and the dark sector interaction would be presumably important (e.g., [10, 58, 59]), and should deserve further investigation. Of course, the biggest issue is the galaxy biasing. Recent numerical and analytical studies claim that the scale-dependent and stochastic properties of the galaxy bias can change the redshift-space power spectrum, and the potential impact on the determination of the growth-rate parameter would be significant [60, 61]. A realistic modeling of galaxy biasing relevant for the scale of BAOs is thus essential, and a further improvement of the power spectrum template needs to be developed.
Note that the bispectrum (A1) possesses the following energy. In this Appendix, we present the perturbative expressions for the corrections $A$ and $B$ defined in Eqs. (19) and (20), which are originated from the coupling between Kaiser and Finger-of-God effects. Let us first consider the correction $A$, which involves the bispectrum $B_\sigma$ of density and velocity divergence (see Eq. (21)). Using the perturbative solutions up to the second-order, the leading-order result of the bispectrum becomes

\[ B_\sigma(k_1, k_2, k_3) = (\cdot f)^2 \left( 1 + \frac{k_2^2}{k_1^2} \right) \left( 1 + \frac{k_2^2}{k_3^2} \right) G_2(k_2, k_3) P_{\text{lin}}(k_2)P_{\text{lin}}(k_3) \]

\[ + \left( 1 + \frac{k_2^2}{k_1^2} f \right) \left\{ F_2(k_1, k_3) + \frac{k_2^2}{k_3^2} f G_2(k_1, k_3) \right\} P_{\text{lin}}(k_1)P_{\text{lin}}(k_3) \]

\[ + \left( 1 + \frac{k_2^2}{k_1^2} f \right) \left\{ F_2(k_1, k_2) + \frac{k_2^2}{k_1^2} f G_2(k_1, k_2) \right\} P_{\text{lin}}(k_1)P_{\text{lin}}(k_2) \quad (A1) \]

with $F_2$ and $G_2$ being the second-order perturbation kernels given by (e.g., [22, 26, 62])

\[ F_2(k_1, k_2) = \frac{5}{7} + \frac{k_1 \cdot k_2}{2k_1 k_2} \left( \frac{k_1}{k_2} \cdot \frac{k_2}{k_1} \right) + \frac{2}{7} \left( \frac{k_1 \cdot k_2}{k_1 k_2} \right)^2, \]

\[ G_2(k_1, k_2) = \frac{3}{7} + \frac{k_1 \cdot k_2}{2k_1 k_2} \left( \frac{k_1}{k_2} \cdot \frac{k_2}{k_1} \right) + \frac{4}{7} \left( \frac{k_1 \cdot k_2}{k_1 k_2} \right)^2. \]

Let us first consider the correction $A$, which involves the bispectrum $B_\sigma$ of density and velocity divergence (see Eq. (21)). Using the perturbative solutions up to the second-order, the leading-order result of the bispectrum becomes

\[ A(k, \mu) = -k \mu \sum_{m,n} \frac{d^3 p}{(2\pi)^3} f^m p_z^n Q_{mn}(k, p), \quad (A2) \]

with the function $Q_{mn}$ is the scalar function of $k$ and $p$. To further perform the angular integral, we use the formulae presented in Appendix B (Eq. (153)), which can be obtained by utilizing the rotational covariance of the integral. After straightforward but lengthy calculation, the correction $A(k, \mu)$ is finally reduced to the following form:

\[ A(k, \mu; z) = \sum_{m,n=1}^{3} \mu^{2m} f^n \left( \frac{3}{2\pi} \right)^2 \left[ \int_{0}^{\infty} dr \int_{-1}^{+1} dx \left\{ A_{mn}(r, x) P_{\text{lin}}(kr; z) + \bar{A}_{mn}(r, x) P_{\text{lin}}(kr; z) \right\} \right. \]

\[ \times \frac{P_{\text{lin}}(k \sqrt{1 + r^2 - 2rx; z})}{(1 + r^2 - 2rx)^2} + P_{\text{lin}}(k; z) \int_{0}^{\infty} dr a_{mn}(r) P_{\text{lin}}(kr; z) \right], \quad (A3) \]

where we introduce the quantities $r = k/p$ and $x = (k \cdot p)/k$. Note again that $\mu$ is the cosine of the angle between line-of-sight direction $\hat{z}$ and the vector $k$, i.e., $\mu = (k \cdot \hat{z})/k$. The non-vanishing components of $A_{mn}$, $\bar{A}_{mn}$ and $a_{mn}$ are

\[ A_{11} = -\frac{r^3}{7} \left\{ x + 6x^3 + r^2 x(-3 + 10x^2) \right. \]

\[ \left. + r(-3 + x^2 - 12x^4) \right\}, \]

\[ \bar{A}_{11} = \frac{r^3}{7} \left\{ x + 6x^3 + r^2 x(-3 + 10x^2) \right. \]

\[ \left. + r(-3 + x^2 - 12x^4) \right\}, \]

\[ a_{11} = \frac{r^3}{7} \left\{ x + 6x^3 + r^2 x(-3 + 10x^2) \right. \]

\[ \left. + r(-3 + x^2 - 12x^4) \right\}.
\[ A_{12} = \frac{r^4}{14} (x^2 - 1)(-1 + 7rx - 6x^2), \]
\[ A_{22} = \frac{r^3}{14} \left\{ r^2x(13 - 41x^2) - 4(x + 6x^3) + r(5 + 9x^2 + 42x^4) \right\}, \]
\[ A_{23} = A_{12}, \]
\[ A_{33} = \frac{r^3}{14} (1 - 7rx + 6x^2) \left\{ -2x + r(-1 + 3x^2) \right\}, \]

for \( A_{mn} \),
\[ \tilde{A}_{11} = \frac{1}{7} (x + r - 2rx^2)(3r + 7x - 10rx^2), \]
\[ \tilde{A}_{12} = \frac{r}{14} (x^2 - 1)(3r + 7x - 10rx^2), \]
\[ \tilde{A}_{22} = \frac{1}{14} \left\{ 28x^2 + rx(25 - 81x^2) + r^2(1 - 27x^2 + 54x^4) \right\}, \]
\[ \tilde{A}_{23} = \frac{r}{14} (1 - x^2)(r - 7x + 6x^2), \]
\[ \tilde{A}_{33} = \frac{1}{14} (r - 7x + 6x^2)(-2x - r + 3rx^2), \]

for \( \tilde{A}_{mn} \), and
\[ a_{11} = -\frac{1}{84r} \left[ 2r(19 - 24r^2 + 9r^4) - 9(r^2 - 1)^3 \log \left| \frac{r + 1}{r - 1} \right| \right], \]
\[ a_{22} = \frac{1}{336r^3} \left[ 2r(9 - 185r^2 + 159r^4 - 63r^6) + 9(r^2 - 1)^3(2r^2 + 1) \log \left| \frac{r + 1}{r - 1} \right| \right], \]
\[ a_{23} = a_{12}, \]
\[ a_{33} = \frac{1}{336r^3} \left[ 2r(9 - 109r^2 + 63r^4 - 27r^6) + 9(r^2 - 1)^3(3r^2 + 1) \log \left| \frac{r + 1}{r - 1} \right| \right], \]

for \( a_{mn} \).

Next consider the corrections \( B \). This term is already of the order \( \mathcal{O}(\{P_{\alpha\beta}(k)\}^2) \), and the non-vanishing contribution can be estimated without employing the perturbative calculations. Just applying the formulae (B3) in Appendix B to Eq. (20), we obtain

\[ B(k, \mu) = \sum_{n=1}^{4} \sum_{a,b=1}^{2} \mu^{2n}(-f)^{a+b} k^3 \int_{0}^{\infty} dr \int_{-1}^{+1} dx B_{ab}(r, x) \frac{P_{a2}(kr \sqrt{1 + r^2 - 2rx})}{(1 + r^2 - 2rx)^{a}}, \]  

(A4)

where \( P_{12}(k) = P_{\alpha\theta}(k) \) and \( P_{22}(k) = P_{\theta\theta}(k) \). The non-vanishing coefficients \( B_{ab}^n \) are
\[ B_{11}^1 = \frac{r^2}{2} (x^2 - 1)^2, \]
\[ B_{12}^1 = \frac{3r^2}{8} (x^2 - 1)^2, \]
\[ B_{21}^1 = \frac{3r^4}{8} (x^2 - 1)^2, \]
\[ B_{22}^1 = \frac{5r^4}{16} (x^2 - 1)^3, \]
\[ B_{11}^2 = \frac{r}{2} (r + 2x - 3rx^2), \]
\[ B_{12}^2 = -\frac{3r}{4}(x^2 - 1)(-r - 2x + 5rx^2), \]
\[ B_{21}^2 = \frac{3r^2}{4}(x^2 - 1)(-2 + r^2 + 6rx - 5r^2x^2), \]
\[ B_{22}^2 = -\frac{3r^2}{16}(x^2 - 1)^2(6 - 30rx - 5r^2 + 35r^2x^2), \]
\[ B_{12}^3 = \frac{r}{8} \{ 4x(3 - 5x^2) + r(3 - 30x^2 + 35x^4) \}, \]
\[ B_{21}^3 = \frac{r}{8} \{ -8x + r \{ -12 + 36x^2 + 12rx(3 - 5x^2) + r^2(3 - 30x^2 + 35x^4) \} \}, \]
\[ B_{22}^3 = \frac{3r}{16}(x^2 - 1) \{ -8x + r \{ -12 + 60x^2 + 20rx(3 - 7x^2) + 5r^2(1 - 14x^2 + 21x^4) \} \}, \]
\[ B_{22}^3 = \frac{r}{16} \{ 8x(-3 + 5x^2) - 6r(3 - 30x^2 + 35x^4) + 6r^2x(15 - 70x^2 + 63x^4) + r^3 \{ 5 - 21x^2(5 - 15x^2 + 11x^4) \} \}. \]

The expression (A1) is still non-perturbative in the sense that we do not perturbatively treat the power spectra \( P_{56} \) and \( P_{66} \) in the integrand. For the leading-order calculation, we simply apply the linear-theory calculation to these quantities, and replace both \( P_{56} \) and \( P_{66} \) with the linear spectrum \( P_{\text{lin}} \).

**Appendix B: Some useful formulae for integrals**

In this Appendix, we give the integral formulae used in Appendix A to derive the perturbative expressions for the correction \( A \) and \( B \).

Let us first consider the integral of an arbitrary scalar function \( f(k, p) \) times some vectors over \( p \). A simple example of the integrand are \( p_i p_j f(k, p) \), where subscript \( i, j \) selects \( x-, y- \) or \( z- \) direction. The rotationally invariant properties of the integral implies that the resultant form of the integral is given by

\[ \int \frac{d^3p}{(2\pi)^3} p_i p_j f(k, p) = P \delta_{ij} + Q k_i k_j, \quad (B1) \]

irrespective of the functional form of \( f(k, p) \). The coefficients \( P \) and \( Q \) are obtained by contracting the above integral with \( \delta_{ij} \) and \( k_i k_j \), and are the functions of \( k = |k| \).

We have

\[ P = \frac{k^5}{(2\pi)^5} \int dr r^2 \int_{-1}^{1} dx x^2 \frac{1}{2} \left( 1 - x^2 \right) f(k, r, x), \]
\[ Q = \frac{k^3}{(2\pi)^3} \int dr r^2 \int_{-1}^{1} dx x^2 \frac{3}{2} \left( 3x^2 - 1 \right) f(k, r, x), \]

where we write \( p = kr \) and \( k \cdot p = k^2rx \). Thus, as a special case with \( i = j = z \), we get the following formula:

\[ \int \frac{d^3p}{(2\pi)^3} p_z^2 f(k, p) = P + (k\mu)^2 Q \quad (B2) \]

with \( k_z = k\mu \).

The above procedure can be generalized to the cases of integrals involving arbitrary numbers of multiplicative vectors. As a useful formula, we here explicitly write down the result summing up the integrals of arbitrary scalar functions \( f_n \) times the power \( p_z^n \) up to the sixth order:

\[ \sum_{n=0}^{6} \int \frac{d^3p}{(2\pi)^3} p_z^n f_n(k, p) = \frac{k^3}{(2\pi)^3} \sum_{m,n=0}^{6} \mu^n \times \int_{0}^{\infty} dr r^2 \int_{-1}^{1} dx (kr)^m G_{nm}(x) f_m(k, r, x), \quad (B3) \]

The non-vanishing coefficients \( G_{nm} \) as functions of \( k, r \) and \( x \) are summarized as follows:

\[ G_{00} = 1, \]
\[ G_{02} = -\frac{1}{2}(x^2 - 1), \]
\[ G_{04} = \frac{3}{8}(x^2 - 1)^2, \]
\[ G_{06} = -\frac{5}{16}(x^2 - 1)^3, \]
\[ G_{11} = x, \]
\[ G_{13} = -\frac{3}{2}x(x^2 - 1), \]
\[ G_{15} = \frac{15}{8}x(x^2 - 1)^2, \]
\[ G_{22} = \frac{1}{2}(3x^2 - 1), \]
\[ G_{24} = -\frac{3}{4}(5x^4 - 6x^2 + 1), \]
\[ G_{26} = \frac{15}{16}(7x^2 - 1)(x^2 - 1)^2, \]
\[ G_{33} = \frac{1}{2}x(5x^2 - 3), \]
\[
G_{35} = -\frac{5}{4} \pi (7x^4 - 10x^2 + 3), \\
G_{44} = \frac{1}{8} (35x^4 - 30x^2 + 3), \\
G_{46} = \frac{15}{16} (-21x^6 + 35x^4 - 15x^2 + 1), \\
G_{55} = \frac{1}{8} (63x^4 - 70x^2 + 15), \\
G_{66} = \frac{1}{16} (231x^6 - 315x^4 + 105x^2 - 5).
\]

Appendix C: Covariance between multipole power spectra

Here, we give the explicit expression for covariance between multipole power spectra used in the MCMC analysis in Sec. V A.

Neglecting the non-Gaussian contribution, the nonvanishing part of the covariance only appears at the diagonal components (i.e., correlation between the same Fourier modes), which are expressed as (e.g., Ref. [64])

\[
\text{Cov}_{\ell, \ell'}(k) = \frac{2}{N_k} \frac{(2\ell + 1)(2\ell' + 1)}{2} \int_{-1}^{1} d\mu P_{\ell}(\mu) P_{\ell'}(\mu) \left\{ P^{(S)}(k, \mu) + \frac{1}{n_g} \right\}^2, \quad (C1)
\]

where \(N_k\) is the number of Fourier modes within a given bin at \(k\), given by \(N_k = 4\pi k^3 \Delta k/(2\pi/\ell_s)^3\) with \(\Delta k\) and \(\ell_s\) being the bin width and survey volume, respectively.

For an analytic estimate of the covariance, we assume that the power spectrum is simply written as \(P^{(S)}(k, \mu) = (1 + \beta \mu^2) b^2 P_{\text{lin}}(k)\), where \(b\) is the linear bias parameter and \(\beta\) is defined by \(\beta \equiv f/b\). Substituting this into Eq. (C1), we obtain

\[
\text{Cov}_{0,0}(k) = \frac{2}{N_k} \times \left[ \left(1 + \frac{4}{3} f + \frac{6}{5} \beta^2 + \frac{4}{7} \beta^3 + \frac{1}{9} \beta^4 \right) \{b^2 P_{\text{lin}}(k)\}^2 \\
+ \frac{2}{n_g} \left(1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2 \right) b^2 P_{\text{lin}}(k) + \frac{1}{n_g^2} \right] \quad (C2)
\]

for \((\ell, \ell') = (0, 0),\)

\[
\text{Cov}_{0,2}(k) = \frac{2}{N_k} \times \left[ \left(\frac{8}{3} \beta + \frac{24}{7} \beta^2 + \frac{40}{21} \beta^3 + \frac{40}{99} \beta^4 \right) \{b^2 P_{\text{lin}}(k)\}^2 \\
+ \frac{2}{n_g} \left(\frac{4}{3} \beta + \frac{4}{7} \beta^2 \right) b^2 P_{\text{lin}}(k) \right] \quad (C3)
\]

for \((\ell, \ell') = (0, 2)\) or \((2, 0),\)

\[
\text{Cov}_{2,2}(k) = \frac{2}{N_k} \times \left[ \left(5 + \frac{220}{21} \beta + \frac{90}{7} \beta^2 + \frac{1700}{231} \beta^3 + \frac{2075}{1287} \beta^4 \right) \{b^2 P_{\text{lin}}(k)\}^2 \\
+ \frac{2}{n_g} \left(5 + \frac{220}{21} \beta + \frac{30}{7} \beta^2 \right) b^2 P_{\text{lin}}(k) + \frac{5}{n_g^2} \right] \quad (C4)
\]

for \((\ell, \ell') = (2, 2).\)
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