Effects of Bionic Bone Flexibility on the Hydrodynamics of Pectoral Fins

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Abstract: Compared with traditional underwater equipment powered by propeller, the manta-ray-inspired vehicle with MPF mode (Median fin/paired fin) has the advantages of stable swimming attitude, high maneuverability, and low noise, etc. As one of the sources of advancing power when the manta-ray-inspired vehicle swims, the flexible deformation of the pectoral fin is an important factor affecting the hydrodynamic performance. In this paper, a mechanical analysis of the two-dimensional flexible pectoral fin using thin wing theory shows that the main factor affecting the hydrodynamic force of the two-dimensional flexible pectoral fin is the level of curvature of the pectoral fin chordal section. By designing a two-stage bionic skeleton at the leading and rear edges of the manta-ray-inspired vehicle, the root–tip section width of the bionic skeleton is used to characterize the level of the bionic pectoral fin’s flexibility, and a tensiometer is used to quantitatively measure the level of flexibility. The root-to-tip ratio of the cross-section was varied to obtain different levels of pectoral fin flexibility, and the hydrodynamic properties of the pectoral fins during flapping were measured using a force sensor and normalized for analysis. The experimental results show that the reduction of the flexibility of the leading edge and the increase of the flexibility of the rear edge are beneficial to the improvement of the thrust performance, and the experimental results are the same as the distribution of the skeletal flexibility in real organisms. Fitting curves of the pectoral fins’ relative flexibility and the normalized thrust/lift show that the flexibility of the pectoral fins has a significant effect on its hydrodynamic force, and a stiffer leading edge and a softer rear edge can improve the hydrodynamic characteristics of the manta-ray-inspired vehicle. Phase differences interacting with flexibility can also enhance bionic pectoral fins’ dynamic properties within 10~30 degree.

Keywords: bioinspired underwater vehicles; pectoral fin; flexibility; dynamic characteristics

1. Introduction

As a kind of underwater equipment with a wide range of applications, UUVs (Unmanned Underwater Vehicles) need to have capabilities of long endurance, high maneuverability, and low noise to cope with multiple tasks such as wide area cruising, narrow water maneuvering, and covert close reconnaissance. Nature has enabled many marine organisms to possess these capabilities at the same time. Among them, manta rays with MPF (median fin/paired fin) swimming mode have received attention from many research teams for their ability of wide range migration, fast maneuvering, and bio-affinity. Manta rays have an excellent streamlined shape and outstanding hydrodynamic characteristics when swimming. Scholars have conducted biological and bionic studies and developed several bionic prototypes, as well as conducted theoretical and experimental research on bionic flapping wings [1–4].
Mark et al. [5,6] proposed a biomechanical model of the pectoral fins and studied the deformation of the pectoral fins of spiny lobster in motion accordingly and found that the pectoral fins did not move synchronously with the movement of the pectoral fins during the swimming of the fish, but there was a phase difference, that is, the movement of the pectoral fins was a compound movement of twisting and flapping. Schaefer et al. [7] found through the study of the muscle types and pectoral fin skeletons of several typical MPF propulsion mode fishes that the pectoral fin skeleton of pectoral fin propulsion mode fishes have certain similarities, with fin bones radiating along the spreading direction and sequentially arranged along the chordal direction, with more severe calcification and lower flexibility in the parts near the head and wing roots, i.e., non-uniform distribution of biological pectoral fin flexibility. The fin bones are connected by joints to form a mesh-like connection, which facilitates the flexible deformation and wave motion of the pectoral fins while forming support. Alben [8] derived power-law scaling by analyzing the fin as a damped resonant system and found input and output power proportional to stiffness (for small-to-moderate rigidity) and pitch frequency (for moderate-to-large frequency). Russo et al. [9] studied the distribution of pectoral fin flexibility in bullnose rays and Atlantic rays by stretching tests on the connective tissue of the pectoral fin skeleton. A biomechanical model was proposed that simulates how bat-like skeletal structures achieve the swimming motion observed in nature, and the model also suggests that changes in skeletal structure will affect pectoral fin deformation properties. Hang H et al. [10] used a self-propelled two-link model to study the effect of phase differences between the posterior and anterior sections of fishes’ body. They found that actively bending fishes’ body can improve speed and efficiency. Lauder et al. [11,12] used DPIV (digital particle image velocimetry) to visualize water flow in the wake of the pectoral fins of bluegill sunfish, and the results show that vorticity is shed by each fin during the downstroke and stroke reversal to generate discrete, roughly symmetrical, vortex rings of near-uniform circulation with a central jet of high-velocity flow. Riggs [13] studied oscillating flexible finned Pumpkinseed Sunfish with a standard NACA-0012 shape. He found that the biomimetic fin produced 26% greater thrust more consistently over each oscillation cycle. Dewey and Moored [14] emphasized the importance of wave motion for manta rays to generate excellent thrust performance and showed that there is a strong interplay between propulsion performance and the flexibility of the drive surface. Dabiri [15] reviewed the concept of optimal vortex formation and studied the relevance to propulsion in biological and bio-inspired systems. He pointed out that optimal vortex formation can potentially be studied as a unifying principle to understand propulsion in nature.

Triantafyllou [16] found that an oscillating foil at the frequencies of maximum spatial amplification could achieve optimal propulsion for St (Strouhal number) values in the range of 0.25 to 0.35. Bi and Cai [17–20] conducted several hydrodynamic experiments and showed that both the thrust coefficient and efficiency reached the maximum at St = 0.4, the most flexible tailplane had the highest propulsive efficiency, and the Reynolds number had a significant effect on the propulsive efficiency. Michelin [21] investigated the effect of the bending stiffness of a flexible heave wing on its propulsion performance in two-dimensional imposed parallel flow in the inviscid limit, and they found that flapping efficiency is greatly enhanced by flexibility. Arastehfar et al. [22–25] measured the thrust generated by the pectoral fins of 16 different designs under free flow and found that the generation of forward thrust increased with the decrease of the fin’s spanwise flexibility. R.G. Bottom [26] draws conclusions from simulation that, when swimming at a slow speed, the thrust contribution of the vortex distribution in the rear half of the stingray body is larger. During fast swimming, the enhancement of the attached vortex on the leading edge is beneficial to the enhancement of thrust. Fish et al. [27] figures out that the eddy current mode and high-efficiency propulsion of the manta ray are obtained through simulation calculations, which are related to the Strouhal number at normal swimming speed. Dong Zhang [28,29] reveals that moderate chordal deformation enhances thrust and improves efficiency, and the tip vortex is the main source of thrust at a high Strouhal number, while
a leading-edge vortex and rear-edge vortex will weaken the thrust generation; at a lower Strouhal number, the leading-edge vortex enhances thrust. Heathcote [30] studied the effect of spanwise flexibility on the thrust, lift, and propulsive efficiency using PIV (particle image velocimetry) through water tunnel experiments. A degree of flexibility was found to a moderately stronger trailing-edge vortex system. Fernández-Gutiérrez [31] calculates thrust and power coefficients, as well as hydrodynamic efficiency by simulating passive elastic deformation and changing the leading and rear edge curvature parameters. The results show that thrust and efficiency are primarily affected by chord-wise curvature, with spanwise curvature having some small additional benefit to efficiency. Menzer [32] found that, by rotating the manta ray model joints to form bending, the pitching ratio of the pectoral fins has an important influence on the generation of thrust through computational fluid dynamics.

The active and passive deformation of pectoral fins is an important factor affecting bionic propulsion. In terms of the influence of flexibility on pectoral fins, good qualitative research has been obtained, but how to quantitatively characterize the relationship between flexibility and mechanical properties remains to be studied. Dimensional analysis was used to study the physics of the forces acting on the flexible deformation caused by pectoral fin flapping, and the results showed that three dimensionless numbers—the geometry of the pectoral fins, the kinematic properties, and the flexibility of the pectoral fins—are the key factors in such physical scenarios. In this paper, two-dimensional flexible flapping wings are considered, bionic skeletons with different flexibility are designed, and the influence of the flexibility of flapping wings on their force is analyzed through thin-wing theory. Finally, the numerical curve between flexibility and mechanical properties is obtained via the method of polynomial fitting.

2. Dynamics of 2D Flexible Pectoral Fin

For pectoral fin dynamics in three dimensions, the commonly used analyzing method is the blade-element theory. In general, the geometric parameters of the pectoral fin cross-section are similar. Find a pectoral fin cross-section of a certain length, and the forces and motions of this pectoral fin’s microelements can be analyzed, after which the forces on the pectoral fin’s microelements are integrated in the spanwise direction to obtain the dynamic expression of the whole pectoral fin. The thickness of the pectoral fin used in this paper is thin and can be approximated as a two-dimensional plane, which no longer has three-dimensional geometry at the spanwise cross-section. This highlights the role of bionic bones.

When considering the fluid dynamics calculation of a wing with a relatively small thickness, the wing thickness is usually assumed to be zero, and the airfoil shape at this point is replaced by a mid-arc for mechanical calculation, which is called thin-wing theory [33,34]. The two-dimensional flexible flapping pectoral fin can be considered to be a flexible thin wing, and the flapping wings’ curve after the flexible deformation is used instead of the mid-arc line so as to carry out the analysis. The two-dimensional pectoral fin used in this paper is designed as two equilateral triangles considering that the real manta ray shape is approximated as a diamond shape [35].

At a certain moment of the pectoral fin’s upwards movement, the water below the fin’s surface flows faster than water above the fin’s surface, creating a high-pressure area above and a low-pressure area below on both sides of the fin surface based on Bernoulli’s principle. Due to the pressure difference, the surface of the pectoral fin is subjected to a thrust perpendicular to the surface. During the downbeat, the pectoral fins are subjected to thrust from the pressure difference between the low pressure above and the high pressure below. Instantaneous positive and negative thrust is generated during the motion cycle. The actual motion direction of the flapping wing is the result of the combined effect of the positive and negative thrust. The component of the force generated by the upbeat and downbeat in the forward direction is one of the sources of thrust.
To simplify the consideration of the 2D flapping pectoral fin’s motion, the deformation of the flexible pectoral fins in the spanwise direction during flapping is ignored, so the projection of the pectoral fins in the top view is still represented by triangles. The situation can be seen as a thin wing with a fixed angle of attack advancing in the current and creating a certain bend. Projecting the spanwise pectoral fin microelement in Figure 1a onto the yOz surface obtains the plane chord line in Figure 1b.

Figure 1. Force analysis of two-dimensional pectoral fins.

Let the pectoral fin span be $s$. Consider the pectoral fin microelements at the cross-sectional span $x_0$, leading edge point $f_{x_0}$, rear edge point $b_{x_0}$, chord line function $l(x,y)$, the curved surface formed by the perimeter of this pectoral fin microelement $A$, thus velocity circulation of this microelements at span $x_0$ can be defined as

$$\Gamma(x_0) = \int_{f_{x_0}}^{b_{x_0}} \gamma(x_0,y)dy,$$

where $\gamma(x_0,y)$ is velocity circulation distribution at point $(x_0,y)$.

Then, the distribution of the velocity circulation at any span $y$ for the cross-section is

$$\Gamma(x) = \int_{f_x}^{b_x} \gamma(x,y)dy.$$  

(2)

The amount of velocity circulation over the entire fin surface is

$$\Gamma = \int_0^s \Gamma(x)dy = \int_0^s \int_{f_x}^{b_x} \gamma(x,y)dxdy,$$

(3)

where $\gamma(x,y)$ is surface vortex distribution on the foil. According to the Kutta–Zhukovsky equation, the hydrodynamic force on the pectoral fin is proportional to the incoming flow velocity, the angle of attack and the velocity circulation around the pectoral fin, so for flapping wings

$$F = \rho V_\infty \Gamma$$

where $\rho$ is fluid density, $V_\infty$ is the velocity of incoming flow or swimming, and $\Gamma$ is velocity circulation.

The hydrodynamic force on the pectoral fin can be found, so the key to solve the dynamic problem is the information of the surface vortex $\gamma(x,y)$ attached to the pectoral fin.
Consider the chord at the span \( y \). Assuming that the incoming flow during the pectoral fin motion is a uniform linear flow, according to thin-wing theory, the induced velocity at a point of the flapping wing section is

\[
v(x, y) = \int_0^{l(x, y)} \frac{\gamma(x, \xi)}{2\pi(\xi - y)} d\xi,
\]

where \( \xi \) is an integral variable between 0 and \( y \).

There is a relationship between induced velocity and angle of attack (AOA) and incoming flow velocity in thin wing theory

\[
v(x, y) = V_\infty \frac{dl(x, y)}{dy} - \alpha,
\]

where \( \frac{dl(x, y)}{dy} \) is chord \( l(x, y) \)'s derivative, and \( \alpha \) is AOA.

Then,

\[
2\pi V_\infty \frac{dl(x, y)}{dy} - \alpha = \int_0^{l(x, y)} \frac{\gamma(x, \xi)}{(\xi - y)} d\xi.
\]

Therefore,

\[
\gamma(x, y) = f(V_\infty, \alpha, \frac{dl(x, y)}{dy}),
\]

where \( \gamma(x, y) \) is the attached surface vortex distribution on airfoil surfaces.

According to Stokes' theorem, the velocity circulation along a closed curve is equal to the eddy flux through any surface bounded by the curve, which is

\[
\Gamma = \oint \nu \cdot dl = \iint_A \nabla \times \nu \cdot dA = \iint_A \omega \cdot dA = J,
\]

where \( A \) is the surface surrounded by the microelement’s boundary, \( \nu \) is the velocity along the curve which bounds surface \( A \), \( \omega \) is the strength of the vortex line through points of surface \( A \), and \( J \) is eddy flux through surface \( A \).

Therefore, the distribution of surface vortices on the fin surface is related to the incoming velocity, the angle of attack, and the chordal curvature of the spanwise section; the distribution of the surface vortices can be obtained to find the distribution of the circulation in this section and then solve the hydrodynamic force on the pectoral fin.

Analysis of \( \Gamma(x, y) \) leads to the conclusion that the magnitude of the attachment surface vortex is closely related to the deformation of the pectoral fin’s chordal curve \( l(x, y) \) in the spanwise. In the flexible pectoral fin scenario, the curvature of the chord \( dl/dx \) changes during the motion due to the flexible deformation, which drastically affects the attachment vortex \( \Gamma(x, y) \) of the wing section. So, we can obtain the information of velocity circulation around the circumferential surface as well as the hydrodynamic force to which the pectoral fin is subjected. Thus, the level of pectoral fin flexibility can greatly affect the hydrodynamic performance of the two-dimensional pectoral fin. This paper will study the corresponding relationship between the flexibility and the force of two-dimensional flexible flapping wings.

The physical phenomena that occur during the movement of the flexible pectoral fins are related to the force \( F \), the incoming flow velocity or the swimming velocity \( \nu \), the chord length \( l \), the upstream area \( S \), the movement frequency \( f \), and the pectoral fin flexibility \( \lambda \) (deformation of tip at given force when fixing pectoral fins’ root).

The respective dimensions are \( F(MLT^{-2}) \), \( \nu(LT^{-1}) \), \( l(L) \), \( S(L^2) \), \( f(T^{-1}) \), \( \lambda(M^{-1}T^2) \). Select \( \nu, l, \lambda \) as loop parameter, so as to traverse all the dimensions involved.

Then define dimensionless number

\[
\Pi_1 = \frac{F}{\nu^{a_1} l^{a_2} \lambda^{a_3}} = M^{1-a_3} L^{1+a_1+a_2} T^{-2-a_1+2a_3}
\]
Then define dimensionless number
\[ \Pi_2 = S v^{b_1} l^{b_2} \lambda^{b_3} = M^{-b_1} L^{2+b_1} T^{-b_1+2b_3} \]  
\[ \Pi_3 = f v^{c_1} l^{c_2} \lambda^{c_3} = M^{-c_1} L^{c_1+c_2} T^{-1-c_1+2c_3} \]  

(11)

(12)

All dimension of these parameter has to be zero, then we can obtain
\[ \Pi_1 = F \lambda / l \]  
\[ \Pi_2 = S / l^2 \]  
\[ \Pi_3 = f l / v. \]  

(13)

(14)

(15)

\( F \lambda / l \) can be regarded as the force related to flexibility, \( S / l^2 \) reflects the geometric factor of the flapping wing, \( f l / v \) is St. The equation \( f(F \lambda / \lambda, S / l^2, f l / v) = 0 \) can summarize what this paper aims towards. When the latter two are constants, there is a strong connection between force and flexibility, which can be described as \( F = f(\lambda) \). The purpose of this paper is to obtain the numerical relationship of force and flexibility.

3. Design of 2D Flexible Pectoral Fins

According to the biological anatomy of real manta rays, the flexibility of the pectoral fin skeleton shows a gradual increase in flexibility from head to tail and from the center of the carapace to the wing tips at both ends [7]. Based on this feature of the skeleton of manta rays, the skeletons of the anterior and posterior edges were designed separately to characterize the overall pectoral fin flexibility distribution.

The prototype has two drive sources on each side to, respectively, drive the motion of the leading-edge skeletons and rear-edge skeletons. The two-stage bionic skeleton is covered with an equilateral triangular silicone film to form a soft-body flapping wing.

The widths of the tips and roots of the leading edge’s bionic skeleton are defined as \( L_{F1} \) and \( L_{F2} \), and those of the rear edge’s bionic skeleton are defined as \( L_{B1} \) and \( L_{B2} \). By means of control variates, the overall flexibility characteristics of the pectoral fins can be changed by changing the width of the cross-section from root to tip of the leading and rear-edge skeleton. According to the flexibility distribution of the real manta ray skeleton, the root is less flexible than the tip, and the fin design is as shown in Figure 2. In addition, the parametric design is given in Figure 3.

![Figure 2. Bionic flapping wing design.](image)

Following the rule that the flexibility of a real manta ray’s pectoral fins tends to be larger from head to tail, the flexibility of the rear-edge fins is larger than that of leading-edge fins, and the root-tip section of leading-edge fins was chosen to be fixed at 6 mm-6 mm (meaning that both the root section’s and tip section’s widths are 6 mm). According to the characterization method of the bionic skeleton’s flexibility distribution defined above, the change of the rear-edge bionic skeleton’s flexibility was characterized by varying the ratio of the tips’ and roots’ widths of the rear-edge bionic skeleton.
Define the rear-edge tip-to-root ratio as $K_B = L_{B1}/L_{B2}$. The flexibility of rear-edge skeletons is gradually reduced from 2 mm/6 mm to 6 mm/6 mm, and the design is provided in Table 1. When changing the rear-edge skeletons’ width, keep the width of the leading-edge skeleton constant which is 6 mm-6 mm. In addition, all the skeletons’ thicknesses are 1.5 mm. The most flexible design is $K_B = 2/6$, while the least flexible design is $K_B = 6/6$.

**Table 1.** Details of rear-edge flexibility design.

| Fin Number | 1       | 2       | 3       | 4       | 5       |
|------------|---------|---------|---------|---------|---------|
| thickness  |         |         | 1.5 mm  |         |         |
| $L_{F1}/L_{F2}$ | 2 mm/6 mm | 3 mm/6 mm | 4 mm/6 mm | 5 mm/6 mm | 6 mm/6 mm |
| $L_{B1}/L_{B2}$ | 2/6 | 3/6 | 4/6 | 5/6 | 6/6 |
| $K_B$       |         |         |         |         |         |

Define the leading-edge tip-to-root ratio as $K_F = L_{F1}/L_{F2}$. The results of the initial experiments as well as the flexibility distribution of real manta rays show that the hydrodynamic performance of the leading-edge skeleton is better when it is less flexible. Therefore, the leading-edge skeleton’s flexibility level is designed from 2 mm/8 mm to 8 mm/8 mm with an overall reduction, and the best hydrodynamic performance for the rear edge’s flexibility distribution is taken as the rear-edge tip-to-root ratio which is 3 mm/6 mm. The design is provided in Table 2. When changing the leading-edge skeletons’ width, keep the width of the rear-edge skeleton constant which is 3 mm/6 mm. In addition, all the skeletons’ thickness are 1.5 mm. The most flexible design is $K_B = 2/6$, while the least flexible design is $K_B = 6/6$.

**Table 2.** Details of leading-edge flexibility design.

| Fin Number | 6       | 7       | 8       | 9       | 10      |
|------------|---------|---------|---------|---------|---------|
| thickness  |         |         | 1.5 mm  |         |         |
| $L_{F1}/L_{F2}$ | 2 mm/8 mm | 3.5 mm/8 mm | 5 mm/8 mm | 6.5 mm/8 mm | 8 mm/8 mm |
| $L_{B1}/L_{B2}$ | 2/8 | 3.5/8 | 5/8 | 6.5/8 | 8/8 |
| $K_F$       |         |         |         |         |         |

Considering the equation of the attachment vortex $\gamma(x, y)$, it is clear that the chordwise curve at any section of the pectoral fin is indispensable for gaining the whole dynamical results, while the curvature parameters of the pectoral fin section during motion cannot easily be obtained. Therefore, the flexibility of the pectoral fin itself at rest is considered, and the relationship between the flexibility of different designs and the dynamics is explored in this paper.

As per Figure 4, we fixed the maximum chordal section of the pectoral fin, applied force $F$ perpendicular to the surface from the wing tip, and measured the increment of the vertical distance $z$ of the wing tip from the initial position when pulling. Define the static
pectoral fin flexibility as $\lambda = z/F$. The static flexibility of the pectoral fins was measured for each working condition using a tensiometer. Normalize the flexibility of pectoral fins under all conditions with the least flexibility as the benchmark, which is $\lambda = \lambda_{\text{min}}$; we named this relative flexibility. The results are shown in the Table 3.

![Figure 4. Static flexibility metric.](image)

**Table 3.** Relative flexibility.

| Fin Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------------|---|---|---|---|---|---|---|---|---|----|
| Relative flexibility | 7.7 | 5.0 | 3.0 | 1.8 | 1.0 | 6.0 | 5.5 | 5.1 | 4.6 | 4.0 |

It is shown that the relative flexibility of the rear-edge group ranges wider than that of the leading-edge group, while the flexibility of the leading-edge group as a whole is larger than that of the rear-edge group.

4. Experiment System

The experimental system is divided into the pool and installation components, the six-dimensional force measurement system, the bionic robot fish with flexible flapping wings, and the control system.

The pool size is 0.8 m * 0.8 m * 1.5 m, as a hydrostatic environment, built by aluminum profiles, surrounded by acrylic panels with a base below. A flat plate is installed above the pool, and the end of the long rod extending below the plate is connected to the mechanical data-acquisition equipment and the experimental prototype by an adapter, and the prototype is installed in a suitable position to avoid the interference caused by the wall and water surface reflux.

The force data-acquisition equipment is ATI company’s Nano17 IP68 force/torque sensor with a data-sampling frequency of 100 Hz. The sensor’s supporting Net Box is connected to a dedicated PC host computer via the Net Box, and the data-acquisition software can output the collected data as data files for subsequent processing and analysis. The sensor’s force resolution is 1/80 N.

The shape of the bionic motion device is based on the 3D shape of a real manta ray and is made of high-performance black nylon7500 material using 3D-printing technology. The maximum chord length of the pectoral fin is 0.12 m, and a DG995 servo is installed on both sides to drive the bionic skeleton’s movement. The skeleton is made of 3D-printing PLA material, with a thickness of 1.5 mm, and the surface is covered with silicone film bonded to form a flexible wing. Nano17 IP68 is installed above the centroid of the fish.

The schematic diagram of the experimental process is shown in Figure 5. CPG (Central Pattern Generator) is a bionic motion modality that can simulate the motion parameters...
of real creatures and output a sine-like waveform. The MCU (Microcontroller Unit) is the core of control system, fixed at the flat plate above the water, and in this paper it is STM32F103ZET6. The PWM (Pulse Width Modulation) is the electrical signal that drives the motion of the servo.

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In this paper, in order to lessen the influence of the interference caused by motion parameters and explore the effect of the flexibility distribution of the bionic skeleton on the propulsion performance of the flexible flapping wing, hydrodynamic experiments of flapping wings are carried out at different frequencies, amplitudes, and phase differences. Considering the results of the initial experiments, the working condition of amplitude 80° and phase difference 30° are used as the reference condition. Frequency ranges are [0.6 Hz, 1.0 Hz], amplitude ranges are [30°, 80°], and phase difference ranges are [−50°, 50°].

In order to make the experimental data real and effective, repeatability and reliability tests need to be carried out during the experiment. Through multiple independent repeated tests (Bernoulli experiments) of each working condition, the Pearson correlation coefficient matrix is listed, and the correlation coefficient and p value are analyzed. The data with small correlation coefficients and large p values are discarded. Since the systematic error of the installation and the random error in the measurement process cannot be avoided, the data with the standard deviation (SD) within the range of 5% are regarded as confident. We remove erroneous data in mechanical parameters within a cycle and filter the original data to make it a smooth curve. Due to the focus on the effect of flexibility, the mean values of thrust and lift over the period are calculated. In order to compare and reflect on the changing trend of different experiments, the results of the experiments were normalized.

\[
F_{x,z} = \frac{F_{x,z}}{(F_{x,z})_{\text{max}}} \tag{16}
\]

\[
RTL = \frac{F_x}{F_z} \tag{17}
\]
\[ \eta = \frac{F_x}{F_x + F_z} = \frac{RTL}{RTL + 1}, \]  

(18)

where (16) is the normalized thrust/lift coefficient, (17) is the ratio of the normalized thrust coefficient and the normalized lift coefficient, and (18) is the decomposition efficiency of the force in the thrust direction. Among them, \( F_{x,z} \) is the average thrust/lift over several periods, and \( (F_{x,z})_{\text{max}} \) is the maximum value of thrust/lift among all experiments. In this paper, \( \eta \) stands for the proportion of the thrust component in joint force instead of work efficiency.

The number of independent repeated experiments for each working condition was 5, the data with a standard deviation over 5% were discarded as wrong data, and three groups of data with the smallest standard deviation were selected for data processing. Each experiment was carried out for at least fifteen cycles, the first three cycles and the last three cycles were discarded, and the middle nine cycles were selected for data processing.

For the curve fitting between the relative flexibility and the normalized thrust coefficient, a one-way ANOVA (analysis of variance) was used to analyze the correlation between the two parameters. The curve of the experimental data is numerically fitted by means of polynomial interpolation.

5. Results and Discussion

The dynamic characteristics of the pectoral fin can be considered in terms of the normalized thrust coefficient, the normalized lift coefficient, the thrust–lift ratio, and the thrust decomposition efficiency. In the application of a manta ray-inspired robot fish, the thrust is the main indicator, which reflects the propulsion capability of the vehicle. The lift not only reflects the ability of the vehicle to pitch quickly, but also affects the attitude stability during the cruise phase. The thrust–lift ratio and thrust decomposition efficiency reflect the force characteristics in the direction of thrust when the vehicle’s pectoral fins flap to generate force. In order to highlight the overall influence of flexibility on the mechanical properties, the mechanical properties of different flexible designs at different frequencies are analyzed together.

The data curve in Figure 6 shows that the flexibility of the leading edge has a great influence on the thrust force, which continues to increase with the decrease of the flexibility level to the maximum value and then slowly decreases or remains unchanged. For the 1.0 Hz condition, where the change is most obvious, the normalized thrust force continuously increases from a maximum flexibility of 2 mm/8 mm to 6.5 mm/8 mm with an increase of 12%, and then shows a decreasing trend. It shows that, in the conditions of this paper, a suitably small leading-edge flexibility allows more leading-edge vortices to attach to the pectoral fin’s surface, delaying the dislodgement of vortices. The aggregation of attached vortices leads to a greater force generated by the pectoral fin. Due to the angular phase difference between the outputs of the two servo units, the fin surface will have a certain angle of attack when moving, and, in combination with its own flexible deformation, the force generated by the fin will have the component of the forward direction of the bionic fish. Therefore, the harder leading edge can gather the attachment surface vortex, thus enhancing the propulsive performance of the pectoral fin. It is also noted that, the higher the frequency of the flap beat, the greater the change in normalized lift. This is because, as the flapping frequency increases, the interaction between the flexible pectoral fins and the water flow becomes more pronounced, and the flexibility plays a more important role. As the flapping frequency increases, the amount of velocity circulation around the pectoral fin circumference increases, thus generating greater hydrodynamic forces.
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Figure 6. Results of leading edge.

The leading-edge flexibility shows a monotonic increasing trend for normalized lift in general, but the increase is small compared to the normalized lift. Therefore, leading-edge flexibility affects the thrust performance of the vehicle more. Since the normalized lift tends to increase slowly and monotonically with decreasing flexibility, and the thrust decomposition efficiency is positively correlated with the thrust–lift ratio, the trends of thrust ratio and thrust decomposition efficiency are similar to those of normalized thrust. It shows that the leading edge of the vehicle should be designed with less flexibility, which can optimize the thrust performance as well as the propulsion efficiency of the vehicle at the same time.

As seen in Figure 7, the influence of the rear-edge flexibility on the normalized thrust differs significantly from that of the leading edge. When the rear edge is highly flexible, the normalized thrust of the vehicle is at a high level, and, as the flexibility decreases, the normalized thrust shows a tendency to decrease, and the decrease is significant, with a maximum decrease of 15%. When the rear-edge flexibility is large, the passive deformation generated by the interaction with the current and the active deformation of the fin surface provided by the phase difference of the input source will increase the wave number during the pectoral fin’s motion. According to Zhang Dong’s [28] conclusion of the manta ray’s fluid simulation calculation, the larger the wave number of the pectoral fin motion in a certain range, the higher the thrust generated will increase. Therefore, for the rear edge, the flexibility level should be designed to be high, so that the pectoral fin can obtain better thrust performance. The effect of flapping frequency on the normalized thrust has a highly similar trend for the leading and rear edges, indicating that a higher flapping frequency can result in a higher thrust level for the pectoral fins as a whole.

The rear-edge flexibility has a more pronounced effect on the normalized lift compared to the leading-edge flexibility. The lift continues to increase with a decrease in the level of flexibility, with a 10% increase. It shows that the rear-edge flexibility has a greater
effect on both thrust and lift at the same time. A higher rear-edge flexibility level with higher thrust and lower lift is the ideal flexibility design in the application of a manta ray-inspired vehicle.

![Figure 7. Results of rear edge.](image)

From the sample interpolation curves of the data points, it can be observed that the rear-edge flexibility reaches the maximum normalized thrust at close to 3 mm/6 mm design, and the thrust–lift ratio and thrust decomposition efficiency have the same characteristics. So, when designing the rear edge of the pectoral fin, the flexibility of the rear edge should be reasonably increased to obtain better hydrodynamic performance.

A comprehensive analysis of the hydrodynamic characteristics due to the flexibility of the leading and rear edges shows that a flexible design similar to that of a living creature has better hydrodynamic characteristics and achieves the original purpose of biomimetics, which means better hydrodynamic characteristics can be obtained when the level of flexibility increases from the head to the tail and from the center of the torso to the fin tips on both sides. When the flexibility of the leading edge is small, the leading edge of the pectoral fin can make better use of the velocity circulation around the circumference of the pectoral fin, delaying the shedding of the leading-edge vortex and obtaining greater hydrodynamic force. When the flexibility of the rear edge is large, the flexible deformation of the pectoral fins interacting with the current and the phase difference of the input source itself can increase the wave number of the pectoral fins to obtain greater hydrodynamic force.

A polynomial fit of the relative flexibility to the normalized thrust and normalized lift is shown in Figure 8. Due to systematic errors in installation and measurement methods, as well as random errors that cannot be avoided by independent repetitive experiments, the variation patterns of some of the raw data points deviate from the trends of the fitted curves, but the confidence levels are within acceptable limits. When analyzing the same series of data samples subject to a certain distribution, the joint hypothesis test (F-test) analysis is used. We use the goodness of fit, R-squared, to evaluate the quality of the
fitting results. The null hypothesis of this paper is that flexibility is not related to the hydrodynamic performance of pectoral fins. The fitted curves of the normalized thrust and relative flexibility are roughly negative quadratic polynomials, and the fitted results are as follows.

When flapping frequency is 0.6 Hz, \( \bar{F}_x = 0.3982 + 0.03346\bar{\lambda} - 0.00459\bar{\lambda}^2 \). The goodness of fit R-squared is 0.7901, and the F-value is 12.85.

When frequency is 0.8 Hz, \( \bar{F}_x = 0.5353 + 0.07651\bar{\lambda} - 0.00796\bar{\lambda}^2 \). The goodness of fit R-squared is 0.7044, and the F-value is 8.311.

When frequency is 1.0 Hz, \( \bar{F}_x = 0.6027 + 0.1229\bar{\lambda} - 0.01182\bar{\lambda}^2 \). The goodness of fit R-squared is 0.7851, and the F-value is 6.911.

The above fits were all significantly better at the significant level \( \alpha = 0.05 \) than \( y = \text{constant} \). We analyze the statistical results of the fitted curve of the normalized thrust, all of the above R-squared goodness of fit are close to 0.8, indicating that the fitting results fit well to a certain extent. The \( F \) value reflects the probability that the data sample falls into the hypothesis rejection region, and the results show that all curves above reject the null hypothesis, which means that the relative flexibility has a strong correlation with normalized thrust.

The fitted curves of normalized lift force and relative flexibility are approximately cubic polynomials, and the fitted results are as follows.

When frequency is 0.6 Hz, \( \bar{F}_z = 0.6202 - 0.09272\bar{\lambda} + 0.03316\bar{\lambda}^2 - 0.00303\bar{\lambda}^3 \). The goodness of fit R-squared is 0.8381, and the F-value is 9.473.

When frequency is 0.8 Hz, \( \bar{F}_z = 0.7951 - 0.1086\bar{\lambda} + 0.04665\bar{\lambda}^2 - 0.00449\bar{\lambda}^3 \). The goodness of fit R-squared is 0.8290, and the F-value is 9.302.

When frequency is 0.8 Hz, \( \bar{F}_z = 0.8859 - 0.1276\bar{\lambda} + 0.05715\bar{\lambda}^2 - 0.00559\bar{\lambda}^3 \). The goodness of fit R-squared is 0.8897, and the F-value is 14.58.

![Figure 8. Polynomial fit of thrust/lift and relative flexibility.](image-url)
The above fits were all significantly better at the significant level $\alpha = 0.05$ than $y = \text{constant}$. Compare the above statistics of normalized thrust with lift, all $R$-squared goodness of fit values exceed 0.8, which proves that the fitting results are good. In addition, the high $F$ values indicate that the null hypothesis is rejected, which indicates that the relative flexibility has a strong correlation with the normalized lift. Meanwhile, since the wing with high stiffness also has high lift, it is reasonable that there is an increasing trend for normalized lift in the direction of decreasing flexibility.

The fitted curves show that flexibility has a significantly nonlinear effect on the hydrodynamics of the pectoral fins; therefore, the distribution of their flexibility needs to be considered when designing the pectoral fins.

In the analytical conclusion of the thin-wing theory, the chordal curvature of the spanwise section of the fin surface is an important factor affecting the hydrodynamic force during the pectoral fin’s motion. In addition to the consideration of the passive flexible deformation of the pectoral fin, the angular phase difference of the input source is also an important factor affecting the chordal curvature, so it is also necessary to analyze the effect of the phase difference on the normalized thrust and lift of the pectoral fin.

Take the normalized thrust as an absolute value. As shown in Figure 9, the trend of the effect of the phase difference of the input source is highly similar for different flexible designs of the leading edge of the pectoral fin. In the phase-difference interval of this paper, the trend of normalized thrust is decreasing and then increasing, reaching a maximum value and then slowly decreasing. The thrust decreases to a minimum value at $-20^\circ$ phase difference and increases to a maximum value at $30^\circ$. Due to the flexibility of the pectoral fin itself, the normalized thrust of the pectoral fin does not reach the minimum value when the phase difference of the input source is $0^\circ$ but reaches the minimum value at a phase difference of $-20^\circ$. When the phase difference is $0^\circ$, although the leading and rear edges are flapping at the same time, because the rear edge is more flexible compared with the leading edge, a flexible deformation will occur, thus leading to a certain actual phase difference and generating a positive thrust. When the phase difference is a small negative value, the passive flexible deformation and the deformation caused by the phase difference of the input source cancel each other out, so the pectoral fins produce zero thrust at this time. Regardless of the positive or negative phase difference, within a certain range, as the phase difference increases the wave number of the pectoral fin motion, an increase in the positive and negative phase difference is accompanied by an increase in the thrust force.

![Figure 9](image-url)

**Figure 9.** Effect of phase difference on leading edge.

For different flexible leading edges, the lift decreases slowly from $-50^\circ$ to $10^\circ$ with a phase difference of only 5%. When the phase difference increases from $10^\circ$, the lift shows a tendency to continue decreasing. This is due to the fact that, when the phase difference continues to increase, the component of the combined force in the direction of the lift decreases, and the excessive bending of the fin surface causes an increase in drag, thus decreasing the combined force. Therefore, $30^\circ$ is an ideal phase difference.
As seen in Figure 10a, the phase difference on thrust for different rear-edge flexibility designs is overall similar to that of the leading edge, but differs from the leading edge in a few places. Observing the normalized thrust curves, it is found that the positions of the 4/6, 5/6, and 6/6 minima of the less flexible rear edge are shifted to $-10^\circ$. It can be seen that, when the flexibility of the rear edge decreases, the flexible deformation of the pectoral fin decreases, and the deformation due to the phase difference of the input source can be offset more easily, thus generating positive thrust earlier.

![Figure 10. Effect of phase difference on rear edge.](image)

As seen in Figure 10b, the case of lift is more complicated. Overall, the lift value tends to decrease with an increasing phase difference, and the decrease is significant. A comprehensive analysis of different leading and rear-edge flexibility designs shows that the vehicle reaches the optimal thrust performance when the phase difference is 30°. Therefore, the active deformation caused by the phase difference of the input source also reflects the role of pectoral fin flexibility.

In summary, when designing a flexible pectoral fin, the flexibility of the leading edge should be smaller than that of the rear. When it comes to integral flexibility, a middle level of flexibility can enhance propulsion, in this paper it is 3–4 for relative flexibility. Phase difference also has significant influence on pectoral fins’ flexible deformation, and 10–30 degrees is good.

The research conclusions of this paper can provide a reference for flapping wing design. As per Figure 11, if we mark different regions of pectoral fin as 1–4, the flexibility of these regions $flex_i$ should follow the rule $flex_1 < flex_2 < flex_3 < flex_4$ under the condition that the strength requirements are met.

![Figure 11. Regions of pectoral fin.](image)

6. Conclusions

In this paper, through the theoretical analysis of the two-dimensional thin-wing theory, it is concluded that the camber level of the pectoral fin chord determines the surface vortex distribution of the two-dimensional fin surface, that is, the flexibility level of the pectoral fin will affect its hydrodynamic performance. Following the bone flexibility distribution of the
real manta ray, the flexibility of the pectoral fin is defined as the synthesis of the two levels of bionic bone flexibility at the front and rear edges, and different levels of pectoral fin flexibility are obtained by changing the width of the bionic bone cross-section. Quantitative measurements of the flexibility of pectoral fins with different flexibility designs were normalized using a tension meter. The force and torque generated during the movement of the bionic pectoral fins were extracted using the mechanical acquisition equipment, and, after the reliability and repeatability of the experimental data were tested, the data were normalized to analyze the respective effects of flexibility on the thrust and lift force.

For the leading-edge bionic bone, reducing the flexibility level can delay the shedding of the leading-edge vortex and obtain better thrust performance, and, at the same time, the lift performance will also be improved. For the rear edge, increasing the flexibility level can increase the wave number of the pectoral fin motion, which can make the thrust performance better and the relative lift of the highly flexible rear edge small. Therefore, the highly flexible rear edge has a larger push-to-lift ratio, resulting in greater propulsion efficiency. The results of the research on the hydrodynamic performance of the pectoral fin by the flexibility distribution of the bionic skeleton of the vehicle are similar to the skeletal flexibility distribution of the real manta ray, so a reasonable flexibility level needs to be considered when designing the pectoral fin. The phase difference of the input source also has a significant influence on the hydrodynamic characteristics of the two flexible pectoral fins during movement. Within a certain range of the phase difference of zero thrust generated by the pectoral fins, when increasing or decreasing the phase difference, the thrust generated by the pectoral fins continued to increase, reached a maximum value at 30° and then slowly decreasing or remaining unchanged. The fitting curves of the relative flexibility and the normalized thrust/lift force show that the relationship of normalized thrust and the relative flexibility can be explained by a negative quadratic polynomial and that of normalized lift and the relative flexibility by a cubic polynomial relationship. Ultimately, this paper provides a reference scheme when designing flexible pectoral fins.

Our next study will continue to focus on the hydrodynamic performance of the pectoral fins. We will introduce the PIV system to study the flow field structure of the pectoral fin wake and analyze the hydrodynamic performance of the pectoral fin from the perspective of hydrodynamics, so as to establish a more complete pectoral fin design and performance evaluation system.

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References

1. Christianson, C.; Goldberg, N.N.; Deheyn, D.D.; Cai, S.; Tolley, M.T. Translucent soft robots driven by frameless fluid electrode dielectric elastomer actuators. Sci. Robot. 2018, 3, eaat1893. [CrossRef] [PubMed]

2. Li, T.; Li, G.; Liang, Y.; Cheng, T.; Dai, J.; Yang, X.; Liu, B.; Zeng, Z.; Huang, Z.; Luo, Y.; et al. Fast-moving soft electronic fish. Sci. Adv. 2017, 3, e1602045. [CrossRef] [PubMed]

3. Rosenberger, L.J. Pectoral fin locomotion in batoid fishes: Undulation versus oscillation. J. Exp. Biol. 2001, 204, 379–394. [CrossRef] [PubMed]

4. Sfakiotakis, M.; Lane, D.M.; Davies, J.B.C. Review of fish swimming modes for aquatic locomotion. IEEE J. Ocean. Eng. 1999, 24, 237–252. [CrossRef]

5. Drucker, E.G.; Lauder, G.V. Locomotor forces on a swimming fish: Three-dimensional vortex wake dynamics quantified using digital particle image velocimetry. J. Exp. Biol. 1999, 202, 2393–2412. [CrossRef]

6. Liu, G.; Ren, Y.; Dong, H.; Akanyeti, O.; Liu, B.; Zeng, Z.; Huang, Z.; Luo, Y.; et al. Fast-moving soft electronic fish. Sci. Adv. 2017, 3, e1602045. [CrossRef] [PubMed]

7. Schaefer, J.T.; Summers, A.P. Batoid wing skeletal structure: Novel morphologies, mechanical implications, and phylogenetic patterns. J. Morphol. 2005, 264, 298–313. [CrossRef]

8. Alben, S. Optimal flexibility of a flapping appendage in an inviscid fluid. J. Fluid Mech. 2008, 614, 355–380. [CrossRef]

9. Russo, R.S.; Blemker, S.S.; Fish, F.E.; Bart-Smith, H. Biomechanical model of batoid (skates and rays) pectoral fins predicts the influence of skeletal structure on fin kinematics: Implications for bio-inspired design. Bioinspiration Biomim. 2015, 10, 046002. [CrossRef]

10. Hang, H.; Heydari, S.; Costello, J.H.; Kanso, E. Active tail flexion in concert with passive hydrodynamic forces improves swimming speed and efficiency. J. Fluid Mech. 2021, 932, A35. [CrossRef]

11. Drucker, E.G.; Lauder, G.V. Locomotor forces on a swimming fish: Three-dimensional vortex wake dynamics quantified using digital particle image velocimetry. J. Exp. Biol. 1999, 202, 2393–2412. [CrossRef]

12. Liu, G.; Ren, Y.; Dong, H.; Akanyeti, O.; Liu, J.C.; Lauder, G.V. Computational analysis of vortex dynamics and performance enhancement due to body–fin and fin–fin interactions in fish-like locomotion. J. Fluid Mech. 2017, 829, 65–88. [CrossRef]

13. Riggs, P.; Bowyer, A.; Vincent, J.F.V. Advantages of a Biomimetic Stiffness Profile in Pitching Flexible Fin Propulsion. J. Bionic Eng. 2010, 7, 113–119. [CrossRef]

14. Moore, K.W.; Dewey, P.A.; Leftwich, M.C.; Bart-Smith, H.; Smits, A.J. Bioinspired Propulsion Mechanisms Based on Manta Ray Locomotion. Mar. Technol. Soc. J. 2011, 45, 110–118. [CrossRef]

15. Dabiri, J.O. Optimal Vortex Formation as a Unifying Principle in Biological Propulsion. Annu. Rev. Fluid Mech. 2009, 41, 17–33. [CrossRef]

16. Triantafyllou, G.; Triantafyllou, M.S.; Greschner, M.A. Optimal Thrust Development in Oscillating Foils with Application to Fish Propulsion. J. Fluids Struct. 1993, 7, 205–224. [CrossRef]

17. Cai, Y.; Bi, S.; Ma, H. Research on Robotic Fish Propelled by Oscillating Pectoral Fins; Springer: Berlin/Heidelberg, Germany, 2015.

18. Cai, Y.; Bi, S.; Zhang, L.; Gao, J. Design of a robotic fish propelled by oscillating flexible pectoral fins. In Proceedings of the 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems, St. Louis, MO, USA, 10–15 October 2009; pp. 2138–2143. [CrossRef]

19. Cai, Y.; Bi, S.; Zhang, L. Design Optimization of a Bionic Fish with Multi-Joint Fin Rays. Adv. Robot. 2012, 26, 177–196. [CrossRef]

20. Cai, Y.; Chen, L.; Bi, S.; Li, G.; Zhang, H. Bionic Flapping Pectoral Fin with Controllable Spatial Deformation. J. Bionic Eng. 2019, 16, 916–930. [CrossRef]

21. Michelin, S.; Smith, S.G.L. Resonance and propulsion performance of a heaving flexible wing. Phys. Fluids 2009, 21, 071902. [CrossRef]

22. Arastehfar, S.; Chew, C.-M. Effects of root chord movement on thrust generation of oscillatory pectoral fins. Bioinspiration Biomim. 2020, 16, 036009. [CrossRef]

23. Arastehfar, S.; Chew, C.-M.; Jalalian, A.; Gunawan, G.; Yeo, K.S. A Relationship Between Sweep Angle of Flapping Pectoral Fins and Thrust Generation. J. Mech. Robot. 2018, 11, 011014. [CrossRef]

24. Chew, C.-M.; Arastehfar, S.; Gunawan Yeo, K.S. Study of sweep angle effect on thrust generation of oscillatory pectoral fins. In Proceedings of the 2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Vancouver, BC, Canada, 24–28 September 2017; pp. 6271–6276. [CrossRef]

25. Chew, C.-M.; Lim, Q.-Y.; Yeo, K.S. Development of propulsion mechanism for Robot Manta Ray. In Proceedings of the 2015 IEEE International Conference on Robotics and Biomimetics (ROBIO), Zhuhai, China, 6–9 December 2015; pp. 1918–1923. [CrossRef]

26. Bottom, L.I.R.G.; Borazjani, I.; Blevins, E.L.; Lauder, G.V. Hydrodynamics of swimming in stingrays: Numerical simulations and the role of the leading-edge vortex. J. Fluid Mech. 2016, 788, 407–443. [CrossRef]

27. Fish, F.E.; Schreiber, C.; Moore, K.W.; Liu, G.; Dong, H.; Bart-Smith, H. Hydrodynamic Performance of Aquatic Flapping: Efficiency of Underwater Flight in the Manta. Aerospace 2016, 3, 20. [CrossRef]

28. Zhang, D.; Huang, Q.; Pan, G.; Yang, L.-M.; Huang, W. Vortex dynamics and hydrodynamic performance enhancement mechanism in batoid fish oscillatory swimming. J. Fluid Mech. 2021, 930, A28. [CrossRef]

29. Zhang, D.; Zhang, J.-D.; Huang, W. Physical models and vortex dynamics of swimming and flying: A review. Acta Mech. 2022, 233, 1249–1288. [CrossRef]
30. Heathcote, S.; Wang, Z.; Gursul, I. Effect of Spanwise Flexibility on Flapping Wing Propulsion. *J. Fluids Struct.* 2006, **24**, 183–199. [CrossRef]

31. Fernández-Gutiérrez, D.; van Rees, W.M. Effect of leading-edge curvature actuation on flapping fin performance. *J. Fluid Mech.* 2021, **921**, A22. [CrossRef]

32. Menzer, A.; Gong, Y.; Fish, F.E.; Dong, H. Bio-Inspired Propulsion: Towards Understanding the Role of Pectoral Fin Kinematics in Manta-like Swimming. *Biomimetics* 2022, **7**, 45. [CrossRef]

33. Lan, C.E. A Quasi-Vortex-Lattice Method in Thin Wing Theory. *J. Aircr.* 1974, **11**, 518–527. [CrossRef]

34. Liu, T.; Montefort, J. Thin-Airfoil Theoretical Interpretation for Gurney Flap Lift Enhancement. *J. Aircr.* 2007, **44**, 667–671. [CrossRef]

35. Xing, C.; Cao, Y.; Cao, Y.; Pan, G.; Huang, Q. Asymmetrical Oscillating Morphology Hydrodynamic Performance of a Novel Bionic Pectoral Fin. *J. Mar. Sci. Eng.* 2022, **10**, 289. [CrossRef]