EFFECTS OF LARGE-SCALE NON-AXISYMMETRIC PERTURBATIONS IN THE MEAN-FIELD SOLAR DYNAMO

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ABSTRACT

We explore the response of a nonlinear non-axisymmetric mean-field solar dynamo model to shallow non-axisymmetric perturbations. After a relaxation period, the amplitude of the non-axisymmetric field depends on the initial condition, helicity conservation, and the depth of perturbation. It is found that a perturbation that is anchored at 0.9 Rs has a profound effect on the dynamo process, producing a transient magnetic cycle of the axisymmetric magnetic field, if it is initiated at the growing phase of the cycle. The non-symmetric, with respect to the equator, perturbation results in a hemispheric asymmetry of the magnetic activity. The evolution of the axisymmetric and non-axisymmetric fields depends on the turbulent magnetic Reynolds number Rm. In the range of Rm = 10^2–10^6 the evolution returns to the normal course in the next cycle, in which the non-axisymmetric field is generated due to a nonlinear α-effect and magnetic buoyancy. In the stationary state, the large-scale magnetic field demonstrates a phenomenon of “active longitudes” with cyclic 180° “flip-flop” changes of the large-scale magnetic field orientation. The flip-flop effect is known from observations of solar and stellar magnetic cycles. However, this effect disappears in the model, which includes the meridional circulation pattern determined by helioseismology. The rotation rate of the non-axisymmetric field components varies during the relaxation period and carries important information about the dynamo process.

Key words: dynamo – Sun: activity – Sun: magnetic fields

1. INTRODUCTION

Dynamo theories commonly assume that the magnetic activity of the Sun is approximately axisymmetric on large spatial (size of the Sun) and temporal (period of solar cycle) scales. These models provide a quantitative self-consistent description of the 22-year solar magnetic cycles, and allow us to investigate the basic mechanisms of the solar dynamo. However, deviations from the axisymmetry are rather strong at any particular moment of observations. Intermittent patterns of magnetic fields on the solar surface are formed because the magnetic field emerges on the surface like separated magnetic patches, (e.g., in the form of sunspot groups). Such phenomena make a significant contribution to the large-scale non-axisymmetric magnetic field of the Sun.

Raedler (1986) discussed dynamo generation of a large-scale non-axisymmetric (NA) magnetic field on the Sun. It was found that the differential rotation suppresses the generation of the NA magnetic field. Nonlinear dynamo processes can maintain a weak non-axisymmetric field in expense of the axisymmetric (AS) magnetic field (see, Raedler et al. 1990; Moss 1999; Elstner & Korhonen 2005). It was also found that the dynamo-generated NA magnetic field rotates rigidly. Further theoretical developments (e.g., Moss et al. 1991; Bigazzi & Ruzmaikin 2004; Moss 2004; Berdyugina et al. 2006) showed that some of the properties of the nonlinear non-axisymmetric mean-field dynamo can be invoked for interpretation of the origin and evolution of the so-called active longitudes of solar magnetic activity. The phenomenon of active longitudes (AL) is probably one of the most interesting manifestations of the solar non-axisymmetric magnetic field. It appears when the solar activity persists within a fixed interval of longitudes for a long period of time (see, e.g., Vitinskii 1966; Bumba & Howard 1969; Vitinsky et al. 1986).

We have to mention that the question about the persistence of AL on a century time interval remains a highly controversial issue both from the observational and theoretical points of view. For example, Berdyugina (2004) and Berdyugina et al. (2006; also, e.g., Plyusnina 2010; Zhang et al. 2011) reported on AL that are persistent over a century-long time interval. However, Pelt et al. (2010) found that AL have the maximal lifetime of about one solar cycle. Another phenomenon that is related to AL is the so-called hot spots of the solar flare activity (Bai 1987, 2003). The reason for the different name is because, in general, the longitudinal position of the activity nests is different in the northern and southern hemispheres. Bai (2003) found that the “hot spot could persist rigidly rotating with period about 27 days up to three solar cycles.” The nonlinear dynamo models predict that the energy of the non-axisymmetric modes is only about 10^-3 of the energy of the axisymmetric (AS) component (Berdyugina et al. 2006). It is not clear how such a weak magnetic field can modulate the nests of the sunspots activity.

The mean-field dynamo theory explains the basic features of solar cycle (e.g., the migratory toroidal field belts that are transformed into sunspots via an instability that is not understood yet). The same kind of instability, together with the large-scale flow of the Sun, can induce the large-scale NA magnetic field. The solar activity produces the large-scale NA modes, which are well seen in the coronal hole configurations (Stix 1977). A very recent example of such events was
observed by the SDO/HMI during the last decade of 2015 May. Observations at the Wilcox Solar Observatory (Duvall et al. 1979; Hoeksema 1995) found that the strength of the NA modes of the radial magnetic field (e.g., the mode with the azimuthal number \( m = 1 \)) can be about 1 G during epoch of the solar maxima. The axisymmetric dipole has the same magnitude during the solar minima. It is likely that the origin of this NA field is related to the decay of the solar active regions. However, it is unclear how the evolution of such an NA field may impact the solar dynamo process. The effect of the NA field on the global dynamo has not been studied before.

In this paper we explore a nonlinear response of a mean-field dynamo model to a shallow NA \( m = 1 \) perturbation with the strength of 1 G. It is assumed that these perturbations result from abrupt instability-type events, which can be described by injecting NA perturbations into the system over a very short period of time. We consider a fairly complete theoretical description of the mean turbulent electromagnetic force, taking into account the known properties of the solar convection zone and including the anisotropic turbulent effects due to the global rotation. The model includes a nonlinear magnetic buoyancy effect, and two types of nonlinearity in the \( \alpha \)-effect, which are described as “algebraic” and “dynamical” quenching (Covas et al. 1998). The algebraic quenching is due to the back-reaction of the dynamo-generated magnetic field on helical turbulence. Dynamical quenching results from a magnetic helicity conservation condition (Kleeroin & Ruzmaikin 1982). We will show that the nonlinear non-axisymmetric effects are sufficiently strong to reproduce the “flip-flop” phenomenon and explain the rotation rate of AL on the Sun (Tuominen et al. 2002; Berdyugina et al. 2006; Gulyege et al. 2012).

2. BASIC EQUATIONS

The evolution of the large-scale magnetic field in perfectly conductive media is described by the mean-field induction equation (Moffatt 1978; Parker 1979; Krause & Rädler 1980):

\[
\partial_t \langle \mathbf{B} \rangle = \nabla \times (\mathbf{E} + \langle \mathbf{U} \rangle \times \langle \mathbf{B} \rangle),
\]

where \( \mathbf{E} = \langle \mathbf{u} \times \mathbf{b} \rangle \) is the mean electromotive force; \( \mathbf{u} \) and \( \mathbf{b} \) are the turbulent fluctuating velocity and magnetic field, respectively; and \( \langle \mathbf{U} \rangle \) and \( \langle \mathbf{B} \rangle \) are the mean velocity and magnetic field. Our solution of the dynamo equation will follow the outline given earlier by Moss et al. (1991) and Moss (1999). For convenience, we decompose the magnetic field into the axisymmetric (AS; hereafter \( \mathbf{B} \)-field), and non-axisymmetric (NA) parts (hereafter \( \tilde{\mathbf{B}} \)-field): \( \langle \mathbf{B} \rangle = \mathbf{B} + \tilde{\mathbf{B}} \). We assume that the mean flow is axisymmetric \( \langle \mathbf{U} \rangle = \mathbf{U} \). We let \( \tilde{\mathbf{E}} = \mathbf{e}_\theta \) and \( \mathbf{r} = r \mathbf{e}_r \) be vectors in the azimuthal and radial directions, respectively, and then represent the mean magnetic field vectors as follows:

\[
\langle \mathbf{B} \rangle = \mathbf{B} + \tilde{\mathbf{B}}
\]

\[
\mathbf{B} = \hat{\phi} \mathbf{B} + \nabla \times (A \hat{\phi})
\]

\[
\tilde{\mathbf{B}} = \nabla \times (\hat{r} \mathbf{T}) + \nabla \times \nabla \times (\hat{r} \mathbf{S}),
\]

where \( A, \mathbf{T}, \) and \( S \) are scalar functions representing the AS and NA parts, respectively. Assuming that \( A \) and \( B \) do not depend on longitude, Equations (3) and (4) ensure that the field \( \langle \mathbf{B} \rangle \) is divergence-free. Taking the scalar product of Equation (1) with vector \( \hat{\phi} \) we get equations for the AS magnetic field components,

\[
\partial_t B = \hat{\phi} \cdot \nabla \times (\mathbf{E} + \mathbf{U} \times \mathbf{B}),
\]

\[
\partial_t A = \hat{\phi} \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}).
\]

To get the equation for \( T \), we take the curl of Equation (1) and then calculate the scalar product with vector \( \hat{r} \). Similarly, the equation for \( S \) is obtained by taking twice the curl of Equation (1), and then the scalar product with vector \( \hat{r} \).

Equations for the NA field are

\[
\partial_t \Delta T = \Delta T V^{(U)} + \Delta T V^{(S)},
\]

\[
\partial_t \Delta S = \Delta T U^{(U)} + \Delta T U^{(S)},
\]

where \( \Delta T = \frac{\partial}{\partial \mu} \sin^2 \theta \frac{\partial}{\partial \mu} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \), \( \mu = \cos \theta \), and \( \theta \) is a polar angle, and

\[
\Delta T V^{(U)} = -\hat{r} \cdot \nabla \times \nabla \times (\mathbf{U} \times \tilde{\mathbf{B}}),
\]

\[
\Delta T V^{(S)} = -\hat{r} \cdot \nabla \times \nabla \times \mathbf{E},
\]

\[
\Delta T U^{(U)} = -\hat{r} \cdot \nabla \times (\mathbf{U} \times \tilde{\mathbf{B}}),
\]

\[
\Delta T U^{(S)} = -\hat{r} \cdot \nabla \times \mathbf{E}.
\]

The scalar functions with superscript \( ^{(U)} \) contain contributions from the large-scale AS flows like the differential rotation or meridional circulation. The integration domain includes the solar convection zone from 0.71 to 0.99 \( R_\odot \). The distribution of the mean flows is given by helioseismology (Howe et al. 2011 and Zhao et al. 2013). Profiles of the angular velocity and meridional circulation are illustrated in Figure 1.

We use the formulation for the mean electromotive force obtained by Pipin (2008). The calculations of the mean electromotive force are done using the mean-field magnetohydrodynamics framework and the so-called minimal tau-approximation (see, e.g., Blackman & Field 2002; Rädler et al. 2003; Brandenburg & Subramanian 2005). The tau-approximation suggests that the second-order correlations do not vary significantly on timescale \( \tau_e \), which corresponds to a typical turnover time of the convective flows. The theoretical calculations are performed for anelastic turbulent flows. They take into account the effects of the density stratification, spatial inhomogeneity of the intensity of turbulent flows, and inhomogeneity of the large-scale magnetic fields. The effects of large-scale inhomogeneity of the turbulent flows and magnetic fields are computed in the first order of the Taylor expansion in terms of ratio \( \ell / L \), where \( \ell \) is a typical spatial scale of the turbulence and \( L \) is a spatial scale of the mean quantities. The mean electromotive force, \( \mathbf{E}_T \), is expressed as follows (Pipin 2008):

\[
\mathbf{E}_T = \left( \alpha_{ij} + \gamma_{ij} \right) B_{ij} - \eta_{jk} \nabla \times B_{jk},
\]

where symmetric tensor \( \alpha_{ij} \) models the generation of magnetic field by the \( \alpha \)-effect, antisymmetric tensor \( \gamma_{ij} \) controls the mean drift of the large-scale magnetic fields in turbulent medium, and tensor \( \eta_{jk} \) governs the turbulent diffusion. We take into account the effect of rotation and magnetic field on the mean electromotive force (see, e.g., Pipin 2015 for details). To determine the unique solution of Equations (5)–(8), we
apply the following gauge (see, e.g., Krause & Rädler 1980; Bigazzi & Ruzmaikin 2004):
\[
\int_0^{2\pi} \int_{-1}^1 Sd\mu d\phi = 0, \quad \int_0^{2\pi} \int_{-1}^1 Td\mu d\phi = 0.
\]
(14)
The same gauge is used in Equations (9)–(12).

2.1. Nonlinear Interaction of the Axisymmetric and Non-axisymmetric Modes

The interaction between the AS and NA modes in the mean-field dynamo models can be due to nonlinear dynamo effects, for example, the \(\alpha\)-effect (Krause & Rädler 1980; Moss 1999). In our model, the \(\alpha\) effect takes into account the kinetic and magnetic helicities in the following form:
\[
\alpha_{ij} = C_\alpha \sin^2 \theta \psi_{(\beta)} \alpha_{ij}^{(H)} \eta_\tau + \alpha_{ij}^{(M)} \chi \frac{\tau_c}{4\pi \ell^2},
\]
(15)
where \(C_\alpha\) is a free parameter that controls the strength of the \(\alpha\)-effect due to turbulent kinetic helicity; \(\alpha_{ij}^{(H)}\) and \(\alpha_{ij}^{(M)}\) express the kinetic and magnetic helicity parts of the \(\alpha\)-effect, respectively; \(\eta_\tau\) is the magnetic diffusion coefficient; and \(\chi\) is the fluctuating parts of magnetic field vector-potential and magnetic field vector. Both the \(\alpha_{ij}^{(H)}\) and \(\alpha_{ij}^{(M)}\) depend on the Coriolis number \(\Omega^2 = 4\pi \frac{\tau_c}{P_{rot}}\), where \(P_{rot}\) is the rotational period, \(\tau_c\) is the convective turnover time, and \(\ell\) is a typical length of the convective flows (the mixing length). A theoretical justification for the latitudinal factor, \(\sin^2 \theta\), in Equation (15) was given by Kleeorin & Rogachevskii (2003). Function \(\psi_{(\beta)}\) controls the so-called algebraic quenching of the \(\alpha\)-effect where \(\beta = \frac{|B|}{\sqrt{4\pi \mu_0 u'/2}}\), \(u'\) is the rms of the convective velocity. For the case of the strong magnetic field, \(\beta \gg 1\), \(\psi_{(\beta)} \sim \beta^{-2}\). The interaction between the axisymmetric and non-axisymmetric dynamo modes via \(\psi_{(\beta)}\) is because both modes contribute to parameter \(\beta\). Also, for the case \(\beta > 1\), the latitudinal profile of the \(\alpha\) effect changes. This can affect the dynamo conditions for excitation of the NA modes. Raedler et al. (1990) and Moss (1999) discussed the evolution of the NA magnetic field in a simple dynamo model with such “algebraic” quenching.

The dynamical quenching is caused by the magnetic helicity conservation (see, Kleeorin & Ruzmaikin 1982). This effect was discovered by Frisch et al. (1975) and Pouquet et al. (1975). Contribution of the magnetic helicity to the \(\alpha\)-effect is expressed by the second term in Equation (15). The magnetic helicity density of turbulent field, \(\langle \chi \rangle = \langle a \cdot b \rangle\), is governed by the conservation law (Hubbard & Brandenburg 2012; Pipin et al. 2013):
\[
\frac{\partial \langle \chi \rangle}{\partial t} = - \langle \chi \rangle R_m \tau_c - 2\eta_\chi \langle J \rangle - \nabla \cdot \mathbf{F}_\chi,
\]
(16)
where \(\langle \chi \rangle = \langle a \cdot B \rangle\) is the total magnetic helicity density of the mean and turbulent fields, \(\mathbf{F}_\chi = -\eta_\chi \nabla \langle \chi \rangle\) is the diffusive flux of the turbulent magnetic helicity, and \(R_m\) is the magnetic Reynolds number. The coefficient of the turbulent helicity diffusivity, \(\eta_\chi\), is chosen to be 10 times smaller than the isotropic part of the magnetic diffusivity (Mitra et al. 2010): \(\eta_\chi = \frac{1}{10} \eta_\tau\). Similarly to the magnetic field, the mean magnetic helicity density can be formally decomposed into the axisymmetric and non-axisymmetric parts: \(\langle \chi \rangle = \langle \chi \rangle^{(AS)} + \langle \chi \rangle^{(NA)}\). The same can be done for the magnetic helicity density of the turbulent field, \(\langle \chi \rangle = \bar{\chi} + \tilde{\chi}\), where \(\bar{\chi} = \langle a \cdot b \rangle\) and \(\tilde{\chi} = \langle a \cdot \tilde{b} \rangle\). Then, we have
\[
\langle \chi \rangle^{(AS)} = \bar{\chi} + \bar{\mathbf{A}} \cdot \bar{\mathbf{B}} + \bar{\mathbf{A}} \cdot \tilde{\mathbf{B}},
\]
(17)
\[
\langle \chi \rangle^{(NA)} = \tilde{\chi} + \bar{\mathbf{A}} \cdot \tilde{\mathbf{B}} + \tilde{\mathbf{A}} \cdot \bar{\mathbf{B}} + \tilde{\mathbf{A}} \cdot \tilde{\mathbf{B}},
\]
(18)
The evolution of \(\bar{\chi}\) and \(\tilde{\chi}\) is governed by the corresponding parts of Equation (16). Thus, the model takes into account contributions of the AS and NA fields in the whole magnetic helicity density balance, providing a nonlinear coupling. We see that the \(\alpha\)-effect is dynamically linked to the longitudinally averaged magnetic helicity of the NA \(\bar{\mathbf{B}}\)-field, which is the last term in Equation (17). Thus, the nonlinear \(\alpha\)-effect is non-axisymmetric, and it results in coupling between the AS and NA modes. The coupling works in both directions. For instance, the azimuthal \(\alpha\)-effect results in \(\mathcal{E}_\phi = \alpha_{\phi \phi} \langle B_\phi \rangle\). If we denote the NA part of the \(\alpha_{\phi \phi}\) by \(\tilde{\alpha}_{\phi \phi}\), then the mean electromotive force is \(\tilde{E}_\phi = \tilde{\alpha}_{\phi \phi} \langle B_\phi \rangle\). This introduces a new source in Equation (6), which is usually ignored in the axisymmetric dynamo models.
Magnetic buoyancy is another nonlinear effect that is important in the large-scale dynamo. The part of the mean electromotive force that is responsible for magnetic buoyancy is (Kichatinov & Pipin 1993):

\[
\mathcal{E}^{(\beta)} = V_\beta \hat{\mathbf{r}} \times \mathbf{B},
\]

where \( V_\beta = C_3 \frac{\alpha_{\text{MLT}} U'}{\gamma} \beta K(\beta) \)

\( u' \) is the rms convection velocity, \( K(\beta) \) can be found in (Kichatinov & Pipin 1993; Pipin 2015), \( \gamma \) is the adiabatic exponent, \( \alpha_{\text{MLT}} \) is the mixing-length theory parameter, and \( C_3 \) is a free parameter that switches this effect on or off in the model. For the case \( \beta \ll 1 \), \( K \sim 1 \) and the up-flow velocity, \( V_\beta \), are proportional to the pressure of large-scale magnetic field. Similar to the \( \alpha \)-effect, the \( V_\beta \) is non-axisymmetric and contributes to the source terms in Equations (5) and (6). Note that advection of the large-scale magnetic field by the magnetic buoyancy reduces the concentration of the magnetic field near the bottom of the convection zone, and increases the field strength near the top.

### 2.2. Parameters of the Convection Zone and Numerical Procedure

The distribution of the turbulent parameters, such as the typical convective turnover time, \( \tau_c \), the mixing length, \( \ell \), and the rms convection velocity, \( u' \), are taken from the solar interior model of Stix (2002). We define the mixing length \( \ell = \alpha_{\text{MLT}} |A^{(p)}| \), where \( A^{(p)} = \nabla \log P \) is the inverse pressure scale and the mixing-length parameter \( \alpha_{\text{MLT}} = 2 \).

The profile of the turbulent diffusivity is taken in the form

\[
\eta_T = C_{\eta} u'^2 r_e, \quad \text{where} \quad f_{ov}(r) = 1 + \exp (50 (r_{ov} - r)),
\]

\( r_{ov} = 0.725 R_{\odot} \) controls quenching of the turbulent effects near the bottom of the convection zone, which is \( r_b = 0.715 R_{\odot} \). The free parameter \( C_{\eta} \) controls the mixing efficiency of the large-scale magnetic field. It is usually employed to tune the period of the dynamo cycle.

The numerical scheme employs the spherical harmonics decomposition for the non-axisymmetric part of the problem, that is, the scalar functions \( T \) and \( S \) in Equations (7) and (8) are represented in the form:

\[
T(r, \mu, \phi, t) = \sum l m \hat{S}_{l,m}(r, t) P_l^m \exp (\text{im} \phi),
\]

\[
S(r, \mu, \phi, t) = \sum l m \hat{S}_{l,m}(r, t) P_l^m \exp (\text{im} \phi),
\]

where \( P_l^m \) is the normalized associated Legendre function of degree \( l \geq 1 \) and order \( m \geq 1 \). The simulations that we will discuss include 600 spherical harmonics \( (\ell_{\text{max}} = 28) \). Note that \( \hat{T}_{l,m} = \hat{S}_{l,m} \) and the same for \( \hat{T} \). We employ the pseudo-spectral approach for integration along latitude. The second-order finite differences are used for discretization in the radial direction. The numerical integration is carried out in latitude from pole to pole, and in radius from \( r_b = 0.715 R_{\odot} \) to \( r_e = 0.99 R_{\odot} \). All the nonlinear terms are calculated in the real space. The transformation between the spectral spherical harmonics and the real 3D space was done using the Intel Fortran FFT library. We implement algorithms of Muciaccia et al. (1997) to speed-up the transform calculations. At the bottom of the convection zone we set up a perfectly conducting boundary condition for the axisymmetric magnetic field, and for the non-axisymmetric field we set the functions \( S \) and \( T \) to zero. At the top of the convection zone the poloidal field is smoothly matched to the external potential field. The boundary conditions for toroidal field allow the field to penetrate the surface (Moss et al. 1991; Pipin & Kosovichev 2011):

\[
\frac{\delta}{r_e} B + (1 - \delta) \mathcal{E}_0 = 0, \quad (22)
\]

\[
\frac{\delta}{R} T - (1 - \delta) \frac{\partial T}{\partial r} = 0, \quad (23)
\]

where parameter \( \delta = 0.99 \).

The particular choice of parameters was discussed in our previous papers (see, e.g., Pipin & Kosovichev 2014). The free parameters are \( C_\alpha = 0.04, C_0 = \frac{1}{3} C_\alpha, \) and \( C_\beta = \frac{1}{15} \), and the anisotropy parameter is \( a = 3 \) (see Pipin & Kosovichev 2014). The \( \alpha \)-effect parameter \( C_\alpha \) is about 30\% above the dynamo generation threshold. For the chosen values of \( C_\eta \) and \( a \), the turbulent diffusion coefficient in the near-surface shear layer at \( r = 0.9 R_{\odot} \) is about \( 10^9 \text{ m}^2 \text{ s}^{-1} \), which is in agreement with surface observations (see, Abramenko et al. 2011). The magnetic helicity conservation is determined by the magnetic Reynolds number, \( R_m \), for which we considered values \( 10^4-10^6 \).

To investigate the influence of meridional circulation, we consider two models: M1 without meridional circulation and M2 with the double-cell meridional circulation with a characteristic velocity of \( 10 \text{ m s}^{-1} \). For model M2, we employ the larger parameter \( C_\alpha \) because this model has a larger critical dynamo threshold (see, Pipin & Kosovichev 2013, 2014). For this value of \( C_\alpha \), model M2 has the same magnitude of the generated AS toroidal magnetic field inside the convection zone as model M1. The set of parameters in the models is summarized in Table 1. The part of the mean electromotive force that is responsible for magnetic buoyancy is (Kichatinov & Pipin 1993):
magnetic field,
\[
P = \frac{E_S - E_A}{E},
\]
\[
\tilde{E}_{SA} = \int \left( \hat{B}(r, \theta, \phi) \pm \hat{B}(r_0, \theta, \phi) \right)^2 \sin \theta d\theta d\phi,
\]
where \( \tilde{E} = \tilde{E}_S + \tilde{E}_A \). The parity of the total magnetic field is
\[
P = (\tilde{P}E + \tilde{P}\tilde{E})/(\tilde{E} + \tilde{E}).
\]
We will see that perturbation can affect the cycle amplitude. Similarly to Raedler et al. (1990), we introduce a parameter to measure the deviation of the toroidal magnetic field from symmetry about the axis of rotation
\[
M = 1 - \frac{E}{E + \tilde{E}}.
\]

We simulate the sunspot number \( W \) using the anzatz (Pipin et al. 2012)
\[
W(t) = \langle B_{\text{max}} \rangle \exp \left( -\frac{B_0}{\langle B_{\text{max}} \rangle} \right).
\]
where in model M1, \( \langle B_{\text{max}} \rangle \) is the maximum strength of the toroidal magnetic field averaged in the subsurface layers over radius in the range of 0.9–0.99 \( R_\odot \), and \( B_0 \) is a characteristic strength of the toroidal magnetic field, \( B_0 = 800 \) G. In model M2 we measure \( B_{\text{max}} \) in the layer of convergence of the two meridional circulation cells, which is in the range of 0.85–0.9 \( R_\odot \). This layer corresponds to the maximum of the toroidal magnetic field strength in the convection zone for model M2. The AS model with the double-cell meridional circulation was discussed in details in our previous papers (Pipin & Kosovichev 2013, 2014).

In our preliminary runs, we tried the weak initial field, which consisted of a superposition sum of polar and equatorial dipoles with the magnetic field strength of 0.01 G. The field evolved to a state in which the axisymmetric dynamo regime dominates. In this regime, the typical strength of the axisymmetric toroidal field in the convection zone is about 1 kG. However, the non-axisymmetric field is rather weak with the strength about \( 10^{-5} \) G. Moreover the preliminary simulations shows that the NA field exponentially decays to zero if the axisymmetric dynamo is switched off. This means that in our model the non-axisymmetric magnetic field is linearly stable, unless the dynamo governing parameters are forced to be non-axisymmetric (e.g., like in the paper by Bigazzi & Ruzmaikin 2004).

2.3. Initial Conditions

Exploring the nonlinear solutions, we found that the evolution of the non-axisymmetric field depends on the initial conditions that include the strength and geometry of both the axisymmetric \( \hat{B} \)-field and non-axisymmetric \( \hat{B} \)-field. The evolution of the large-scale magnetic field also depends on the presence of the meridional circulation.

In the following section we present results for the non-axisymmetric dynamo, which was perturbed by a finite-amplitude non-axisymmetric \( \hat{B} \)-field in the developed axisymmetric dynamo regime. Such non-axisymmetric perturbations can be developed either due to the evolution of active regions or due to instabilities not described by the mean-field theory (e.g., Dikpati & Gilman 2001). For the seed field we consider a non-symmetric relative to the equator perturbation represented by a sum of the equatorial dipole \((l = 1, m = \pm 1)\) and quadrupole \((l = 2, m = \pm 1)\) components. In Equation (21) we define
\[
S_{1,1} = \frac{1}{2} \left( 1 - \text{erf} \left( \frac{r_c - r}{d} \right) \right) \left( \frac{r_c}{r} \right),
\]
\[
S_{1,2} = \frac{1}{2} \left( 1 - \text{erf} \left( \frac{r_c - r}{d} \right) \right) \left( \frac{r_c^2}{r^2} \right),
\]
where, \( r_c = 0.99 R_\odot \), \( r_s = 0.9 R_\odot \) is the bottom of the subsurface shear layer, \( d = 0.02 R_\odot \). The other \( S_{1,m} \) and \( T_{1,m} \) coefficients are zero in the perturbation. The initial non-axisymmetric perturbation is concentrated in the near-surface shear layer. The depth of the non-axisymmetric perturbation can influence the evolution of the axisymmetric dynamo. At the moment of the initialization of perturbation, the strength of the axisymmetric toroidal field is by two orders of magnitude greater than of the non-axisymmetric one. At the same time, the maximum strength of the NA magnetic field components at the surface is about 10 G, see Figure 2(c). Also, at the moment of initialization, the parity parameters of the AS and NA parts are \( P = -1, \tilde{P} = -0.3 \), respectively, and the index of axisymmetry is \( M \approx 10^{-3} \).

3. RESULTS

Figure 2 illustrates the geometry of the axisymmetric and non-axisymmetric fields just before the initialization of the non-axisymmetric perturbation. The perturbation is initialized at \( t \approx 13.2 \) years for model M1 and \( t \approx 5.7 \) for model M2. Those times correspond approximately to the same phase of the AS toroidal magnetic field in the upper half of the convection zone in the models. Figure 3 illustrates the evolution of the axisymmetric magnetic field before and after the perturbation for two models, with and without the meridional circulation. We show the time–latitude diagrams for the toroidal magnetic field in the subsurface shear layer and the radial magnetic field at the surface. In the model with the meridional circulation, the toroidal magnetic field is shown for the middle of the convection zone (see Pipin & Kosovichev 2013). The radial evolution is shown for \( 30^\circ \) latitude in the northern hemisphere.

The models show that the imposed non-axisymmetric perturbation produces a transient cycle in both models M1 and M2. In the northern hemisphere, where the initiated perturbation is greater, the simulated sunspot number cycle is stronger than in the southern hemisphere. The perturbation affects the reversal of the polar magnetic fields. In the northern hemisphere, the polar field reversal occurs earlier than in the southern hemisphere where we see multiple reversals. Model M1 shows a time shift of about two years for the polar reversals in the northern and southern hemispheres. We find that these phenomena depend on the depth of perturbation (parameters, \( r_s \) and \( d \) in Equation (28)). For instance, the polar field reversal happens earlier in the southern hemisphere than in the northern one if the imposed perturbation is shallower, \( r_s = 0.95 R_\odot \). We also see that in both models, the cycle returns quickly to the previously established axisymmetric state. Additional runs, which are not illustrated here, show that this restoration could take a longer time interval (more than 1 cycle) if the
The axisymmetric field has a mixed parity at the initialization moment.

The evolution of the magnetic helicity density (Equation (16)) depends on the magnetic Reynolds number, \( R_m \), which is a free parameter of the model. For higher \( R_m \) the relaxation in both models is similar. Figure 4 illustrates the results for model M1 for \( R_m = 10^4 \) and \( R_m = 10^6 \) and also for model M1 with a reduced magnetic buoyancy effect (\( C_\beta = C_\eta \)). The developed axisymmetric dynamo regime, which is employed at the beginning of evolution series shown in Figure 4, is related to the case \( R_m = 10^4 \) and \( C_\beta = 1 \). This explains why the magnetic cycles in model M1 with \( C_\beta = C_\eta \) and \( R_m = 10^6 \) do not relax to the original cycle amplitude. The model with a reduced magnetic buoyancy shows a smaller (by a factor of 2) magnitude of the \( T_{1,1} \) mode after the relaxation. The mode \( T_{1,1} \) shows a larger variation of amplitude in the case of \( R_m = 10^6 \) than in the case of \( R_m = 10^4 \) after the relaxation.

The growth and decay of the NA magnetic field is accompanied by oscillations with a period about of 1.5 year. These oscillations are likely related to the NA dynamo eigenmodes. It was reported previously (see, e.g., Raedler 1986; Raedler et al. 1990; Moss 1999; Bigazzi & Ruzmaikin 2004) that the NA \( m = 1 \) mode that is antisymmetric about the equator (in common notation A1) is oscillatory. To shed some light on the issue, we made additional runs for the NA dynamo model where we exclude the axisymmetric magnetic field. The same initial NA magnetic field is used in these simulations. It was found that the initial NA perturbation decays exponentially to zero, oscillating with a period of about one year. The decay rate in this case is faster than the decay rate during the transient phase of the full dynamo model, which we discussed above. Also, it is found that the \( m = 1 \) NA dynamo mode is rigidly rotating.

The restoration of the initial axisymmetric magnetic field evolution after the perturbation does not mean that the non-axisymmetric field completely dissipates. Figures 5 and 6(a), (c), (d) show that a low strength non-axisymmetric field is maintained in the model. We find that the strength of the \( B_\phi \) field decays slowly after the relaxation. Both M1 and M2 models show a deviation of the parity of the AS magnetic field, \( \bar{P} \) from \(-1\) (corresponding to antisymmetric about the equator magnetic field) after the perturbation. The NA magnetic field shows a mixed parity solution during and after the relaxation phase. The important finding is that the NA perturbation evolves through a growing phase for about half a year in model M1, and for about one year in model M2. Thus
the perturbation is not just decaying after the initialization. Variations of the axial symmetry index, $M$, and the parity, $P$, are shown in Figure 7(a). We see that $M$ first grows, and then after about a year the perturbation is reflected in the parity of the large-scale magnetic field. Model M1 shows greater variations of the parities than model M2. We find that in M1, relaxation of the parity takes more than one cycle. After the relaxation, the dynamo model returns to axial symmetry, with $M \approx 10^{-6}$. Even such a low strength non-axisymmetric magnetic field can produce some interesting phenomena, which may be related to solar observations.

Figures 5, 6(b) show the evolution of longitude of the NA $m=1$ mode and longitude of the maxima of the NA magnetic field in the southern and northern hemispheres in the subsurface shear layer. The longitude is computed for the coordinate frame rotating with the period of about 25 days. After the perturbation initialization, the longitude of the $m=1$ mode drifts around the Sun. There is no fluid motion associated with this drift, which is an analogue of the latitudinal dynamo waves in the slowly rotating regime. The effect was suggested earlier in the study of the linear dynamo regimes by Raedler (1986). This effect was found in observations (e.g., Tuominen et al. 2002; Lindborg et al. 2013) and in direct numerical simulations (Cole et al. 2014). The speed of the drift changes after the relaxation. Model M1 shows almost fixed positions for the longitude of the $m=1$ mode during epochs of the maximum of the AS toroidal magnetic field. It is found that after relaxation, the position of the maximum of the NA magnetic field in the southern hemisphere resides at the longitude of about 20°. In the northern hemisphere, the position of maximum varies around 140° longitude. The latitude–longitude positions of the maxima of the large-scale toroidal magnetic field strength after relaxation are illustrated in Figure 7(b). Model M1 (without the meridional circulation) shows the periodic changes of the longitude by 180° during the magnetic cycle decay. The change of the longitude is accompanied by a change of the hemispheric position of the field maximum. Thus, the orientation of the global non-axisymmetric field is reversed every cycle during the minima of the toroidal magnetic field. This behavior may correspond to the “flip-flop” phenomenon of the AL suggested for stellar magnetic cycles (Berdyugina 2004). It is seen that the longitude of the field maximum is more chaotic than the longitude of the NA $m=1$ mode. Thus, the flip-flop is likely most evident when we restrict the study to the NA modes with low $m$.

Model M2, which includes the meridional circulation, has no stable positions of the non-axisymmetric magnetic field azimuth. Figures 5(a) and 6(a) clarify the reason for this. The effect disappears because the circulation mixes the magnetic field in the subsurface shear layer, with the magnetic field of the deep interior with a period that approximately corresponds to the period of the magnetic cycle. We see that oscillations of the $S_{1,1}$ and $T_{1,1}$ harmonics have a $\pi/2$ phase shift. Thus, for a persistent appearance of the AL, the phases of the $S_{1,1}$ and $T_{1,1}$ harmonics should be consistent. Model M2 shows a continuous drift of the longitude of the large-scale toroidal magnetic field strength in the course of the magnetic activity evolution. Besides the longitude of the NA $m=1$ mode that continuously rotates around the Sun, we see that the longitudinal position of the maximum of the NA magnetic field in the southern and northern hemispheres is robust and persists during the epochs of the maxima of the AS magnetic field. Additional runs, which are not illustrated here, show that the model with the circulation could produce a sort of AL phenomenon, but for another combination of the circulation cells different from the solar case shown in Figure 1(b). More specifically, the strength of the bottom cell should be reduced compared to model M2. In this case, we find that AL only occupy the northern hemisphere.

Figure 8 shows snapshots of the axisymmetric and non-axisymmetric fields after a half year evolution of the initial perturbation. We also show the configuration of the external
potential magnetic field. This period of time corresponds to a maximum of the toroidal magnetic field in the upper part of the convection zone, and the epoch of the polar field reversal. The non-axisymmetric part of the field is concentrated to the surface (as the initial field). The longitude-latitude diagram shows the

Figure 4. Evolution of the simulated sunspot number (a) and $T_{1,1}$ harmonic (b) in model M1 for the different values of the parameter $R_m$ and $C_\beta$: $R_m = 10^4$, $C_\beta = 1$ (red); $R_m = 10^4$, $C_\beta = C_\eta$ (blue); and $R_m = 10^6$, $C_\beta = 1$ (black).

Figure 5. Non-axisymmetric modes of model M1: (a) the time–radius evolution of the dynamo modes $T_{1,1}$ (background image) and $S_{1,1}$ modes (contours are in the same range of values as the color scale), (b) evolution of longitude of the NA $m = 1$ mode and longitude of the maxima of the NA magnetic field in the southern and northern hemispheres (S- and N- nest, respectively), (c) the mean strength of the near-surface axisymmetric and non-axisymmetric toroidal magnetic field and the strength of the $m = 1$ T-potentials, and (d) evolution of parities, $P$ and $\tilde{P}$, and the index of the axisymmetry $M$.

Figure 6. Same as Figure 5 for model M2.
distributions of the large-scale non-axisymmetric magnetic field. It illustrates how the differential rotation stretches the initial magnetic field configuration (cf., Figure 2(a)). The snapshots in Figures 8 and 9 show the large-scale unipolar regions that extend from the equator to the high-latitude regions the poles. The increasing of the non-axisymmetric magnetic field in the polar regions results in twisted field lines in the polar caps.

Figure 8. Model M1 a half year after the initialization of the non-axisymmetric perturbation, (a) distribution of the axisymmetric toroidal magnetic field (background image) and poloidal field lines in the meridional cross-section; (b) azimuth-latitude distribution of the non-axisymmetric radial magnetic field (background image) and toroidal magnetic field (contours are in the same range as the color scale) on the surface; (c) the same as in panel (a) for the non-axisymmetric components of magnetic field; and (d) the magnetic field lines of the non-axisymmetric magnetic field above the surface.

Figure 10 illustrates snapshots of the magnetic field configuration during the decaying phase of the magnetic cycle in model M1 after six years from the initialization. We see that the non-axisymmetric toroidal field distributed over the convection zone has a maxima at the bottom of the convection zones and in the near equatorial region. The strength of the non-axisymmetric field is much smaller than the strength of the axisymmetric field. Model M2 has long overlaps between the
subsequent cycles. Snapshots for this model are shown in Figure 10(bottom) for the growing phase of the cycle. The snapshots show the situation when the symmetric with respect to the equator $m = 1$ mode dominates at the surface. In the deep layers the general distribution of the non-axisymmetric magnetic field is close to model M1.

The stationary dynamo evolution begins about 15 years after the initialization of the non-axisymmetric magnetic field in model M1 (see Figure 5). The relaxation time of model M2 is about one cycle. In the stationary stage, the non-axisymmetric field is concentrated at the top of the convection zone like in the snapshots shown in Figures 8 and 9 (also see Figures 5(a) and 6(a)). The antisymmetric $m = 1$ mode with mixed parity dominates in both models.

In addition to the models M1 and M2 presented in the paper, we calculated the models for different initial conditions by changing the spatial distribution of the non-axisymmetric perturbation and the initialization time relative to the different epochs of the magnetic cycle. In model M1 the effect of perturbation is the greatest when it is initiated at the growing phase of the cycle. Also, the impact of the perturbation, and the amplitude of the non-axisymmetric field after the relaxation are stronger with the increase of the perturbation depth. However, if the perturbation in the form of Equation (28) is located near the bottom of the convection zone, it produces only a weak effect on the large-scale distributed dynamo.

To investigate the rotation rate of the non-axisymmetric modes, we calculate power spectra of the $m = 1$ mode azimuth for different radii. Figure 11 shows the spatial distributions of the rotational periods of the NA $m = 1$ modes in the solar convection zone. The time series covers a period of about five years after the initialization of the non-axisymmetric field in the models. At the initialization period, the $m = 1$ mode was rotating with a period of 27.26 days, which corresponds to the rotation period at the latitude of the maxima of the initial perturbation. For model M1, it is found that in the subsurface shear layer the $m = 1$ mode rotates with the periods of 25.1–25.5 days. This corresponds to rotation of the subsurface shear layer at 30° latitude. In the bottom part of the convection zone the dominating period is about 25 days, which corresponds to the NA magnetic fields near the solar equator. The origin of these rotational periods has to be studied further. It seems that the periods of rotation follow the rotation profile in the solar convection zone (e.g., see the typical bow of the magnetic field distribution in Figure 10(c)). In model M2, the meridional circulation mixes all layers of the convection zone, producing a unique maximum of the non-axisymmetric magnetic field rotating with period of 25.4 days, which corresponds to the differential rotation period at latitude of 30° and $r = 0.9R$.

Finally, the animated evolution of the large-scale magnetic field is illustrated by two videos (see Figure 12), http://www.youtube.com/watch?v=0i5tSwaxao and http://www.youtube.com/watch?v=buNK91Sb3OA, showing results for the model M1. The first video illustrates the evolution of the total magnetic field for the transient phase of evolution. The second video shows the evolution of the NA magnetic field for the whole run. It illustrates the flip-flop of the NA magnetic field during the stationary phase of evolution.
4. DISCUSSION

In this paper we explored the evolution of non-axisymmetric (NA) magnetic field perturbations in the mean-field solar-type dynamo models. We perturb the dynamo system by a mix of the NA $m = 1$ modes of the strength of about 1 G, according to observations at the Wilcox Solar Observatory (Duvall et al. 1979; Hoeksema 1995). The models are kinematic with respect to the mean flow. The distribution of the mean flow is taken from the recent results of helioseismology, including the subsurface rotational shear layer and the double-cell meridional circulation pattern that was recently suggested by the results of...
Zhao et al. (2013). We studied models with and without meridional circulation. The non-axisymmetric dynamo model takes into account the mean turbulent electromotive force in a fairly complete form. The mean electromotive force, which is employed in the non-axisymmetric part of the model, is the same as for the axisymmetric (AS) part, except that the \( b \)-effect \((\Omega \times J)\) term; Rädler 1969) was omitted in the non-axisymmetric electromotive force. We plan to investigate it separately. We did not discuss the origin of the NA perturbations or their impact on the energy budget in the dynamo system. In the model, the NA magnetic field contributes to the mean-field buoyancy of the AS magnetic field, increasing the loss of magnetic flux from the dynamo region. We postpone discussions of these issues for a separate study.

The dynamo models studied here show two distinct evolution phases. The transient phase starts after the initialization of the NA perturbation and ends approximately after one dynamo cycle period. The results suggest that the non-axisymmetric magnetic field can considerably affect the axisymmetric dynamo. This depends on the amplitude of the perturbation, the depth of perturbation, and the phase of evolution of the axisymmetric magnetic field. We illustrated the effect of a perturbation represented by a mix of the odd and even parities of the \( m = 1 \) mode of the magnetic field with a magnitude of 1 G. The depth of the perturbation is \( r = 0.9 \, R_\odot \).

The models show that during the transient phase the NA magnetic field is amplified from the level \( M \approx 10^{-3} \) of total magnetic energy to \( M = 0.3 \), which is 30% of the total magnetic field energy. This affects the north–south asymmetry of the magnetic field about the equator. The rotation of the \( m = 1 \) mode during the transient phase shows a continuous spectrum of the rotation periods because the evolution of the NA magnetic field is strongly coupled with the differential rotation. The maximum in the spectrum of the rotation periods is about 25.4 days.

The transient phase of the dynamo evolution is followed by a “stationary” phase when the NA magnetic field evolves slowly, varying on a time interval that is longer than the dynamo period. Solutions for the second phase could be compared with the results of previous NA dynamo models (see, Raedler 1986; Raedler et al. 1990; Moss et al. 1991; Moss 1999; Elstner & Korhonen 2005). In the “stationary” phase we found only a weak NA magnetic field \((M \approx 10^{-6})\) with the dominant \( m = 1 \) structure and the antisymmetric relative to the equator magnetic parity. The NA dynamo modes rotate rigidly after relaxation.

Model M1 (without the meridional circulation) shows that the rigid rotation of the \( m = 1 \) dynamo modes is accompanied by changes of the longitude and hemispheric position of the maximum of the \( m = 1 \) mode with a period of one dynamo cycle. This regime resembles the so-called flip-flop phenomenon, which is found in the stellar magnetic activity (Jetsu et al. 1991; Korhonen et al. 2002; Berdyugina 2004; Lindborg et al. 2013). This was also previously suggested by other NA dynamo models (Tuomina et al. 2002; Moss 2004; Elstner & Korhonen 2005; Berdyugina et al. 2006). The flip-flop phenomenon may be related another phenomenon, the so-called active longitudes (AL; e.g., Vitinskii 1966; Vitinsky et al. 1986). In the Introduction, it was mentioned that the persistence of AL on the Sun on the century timescale remains a highly controversial issue. The origin of flip-flop phenomenon in nonlinear non-axisymmetric dynamo models was discussed in detail by Moss (2004; see, also, discussion in Berdyugina et al. 2006). In our calculations, AL is fixed when the \( T_{1,1} \) and \( S_{1,1} \) dynamo modes are in phase. The flip-flop occurs when the orientation of the equatorial dipole changes sign. If this effect is accompanied by the equatorial symmetry variations of the axisymmetric magnetic field, then the orientation of the large-scale magnetic field changes by 180°. The current explanation remains qualitative because the amplitude of the NA magnetic field in the model is rather small, with he axial symmetry index, \( M \approx 10^{-6} \). In the subsurface shear layer, the AS toroidal magnetic field has a typical magnitude of about 500 G and the NA magnetic field has a maximum magnitude of about 100 G during the transient phase and 0.1 G during the stationary phase (see Figure 5). It is not clear how such a weak NA magnetic field could modulate the longitudinal sunspot activity.

Figure 12. Model M1. Panel (a) illustrates the evolution of the total magnetic field for the transient phase of evolution, the background color image shows the strength of the radial magnetic field. Panel (b) shows the evolution of the non-axisymmetric part of magnetic field for the whole run. It illustrates the flip-flop of the NA magnetic field during the stationary phase of evolution.

(Animations a and b of this figure are available.)
It was also mentioned that detection of AL in the solar data encounters a lot of controversial issues. From our results, as well as results of previous studies (e.g., Moss 1999), this favors small strength regular NA magnetic field of the Sun. On the other hand, the longitudinal activity nests (Brouwer & Zwaan 1990) are well detected in the solar activity (Brouwer & Zwaan 1990; Bai 2003). An interesting finding is that our model M2, which does not show the flip-flop effect, demonstrates stable positions of the maximum of the NA magnetic field in the northern and southern hemisphere during the stationary phase of evolution (see, Figure 6(b)). The stellar activity shows both the AL and the longitudinal activity nests (Jetsu et al. 1991; Korhonen et al. 2002; Berdyugina 2004; Lindborg et al. 2013; Olsert et al. 2015).

One interesting feature of model M1 is that in the stationary evolution phase the antisymmetric parity of the $m = 1$ mode dominates. This is different from previous results of the linear theory by Raedler (1986) and the previous nonlinear models (Raedler et al. 1990; Moss et al. 1991; Moss 1999). In addition, the maximum of $m = 1$ is located on the mid-latitude zone, which is different from the typical poleward concentration of the NA magnetic field suggested by results in the aforementioned papers. The origin of this disagreement is unclear. Note that in our models we employ fairly complete information about the radial profiles of the $\alpha$-effect and the differential rotation suggested by the modern results of the helioseismology, as well as the results of recent theoretical works. We also include the magnetic helicity conservation that was not previously taken into account in the non-axisymmetric dynamo models. The dynamo parameters, such as the $\alpha$-effect and the turbulent diffusion coefficients, were tuned to reproduce the 22-year dynamo cycle by the axisymmetric dynamo. Considering the ratio between the $\alpha$ and $\Omega$-effects in our models, we find $\frac{\alpha_0}{\Omega_0} \sim 10^3$, where $\alpha_0$ is the magnitude of the $\alpha$-effect. Thus, the $\alpha$ effect in our models is about one order of magnitude smaller than that employed by Raedler et al. (1990) and Moss (1999). The finding similar to ours, that is, a concentrated to the equator antisymmetric $m = 1$ magnetic field mode was reported by Nelson et al. (2013) for the global numerical simulation of the convective dynamo on the solar-type star rotating three times faster the modern Sun.

Model M2 with the meridional circulation illustrates another feature predicted by the linear analysis of the kinematic NA dynamo models. In this model, the longitudinal position of the $m = 1$ mode in the stationary phase of evolution drifts around the Sun with a period of about one dynamo cycle. This can be interpreted as an azimuthal dynamo wave. The effect was predicted by the mean-field theory (see, e.g., Krause & Rädler 1980), which was found in the mean-field NA dynamo models (Raedler 1986; Raedler et al. 1990; Moss et al. 1991), and in the direct numerical simulations (Cole et al. 2014). It was also found in observations of magnetic activity on fast-rotating late-type stars (e.g., Tuominen et al. 2002; Lindborg et al. 2013).

### 5. Conclusions

We considered non-axisymmetric mean-field dynamo models including the nonlinear magnetic helicity and magnetic buoyancy effects, which were not studied before. The study confirms the previous findings of Moss (1999) that the non-axisymmetric dynamo component is rather weak if we start from a weak initial (seed) non-axisymmetric field. We notice that our models can be characterized as weakly nonlinear because the parameter of the $\alpha$-effect in the model is only 30% above the dynamo instability threshold. In addition, the magnetic helicity conservation and magnetic buoyancy prevent the generation of a magnetic field of the super-equipartition strength (Brandenburg & Käpylä 2007; Hubbard & Brandenburg 2012). Thus, the low strength non-axisymmetric magnetic field generated from a weak seed field can be explained by the linear stability of the non-axisymmetric field, and by the weak nonlinearity of the dynamo system. However, finite-amplitude non-axisymmetric perturbations, which can be developed in the complex dynamical system may have significant effects on the dynamo process.

The modeling results show that the magnetic helicity conservation (also known as dynamical quenching of the $\alpha$-effect) is an important factor to preserve the non-axisymmetric field from a complete decay. It is found that for the magnetic Reynolds number $R_m = 10^6$ the coupling of the non-axisymmetric magnetic field with the axisymmetric dynamo process is stronger than in case of $R_m = 10^4$. The strong coupling results in a synchronization between oscillations of the non-axisymmetric and axisymmetric magnetic fields. Our models also include nonlinear effects of magnetic buoyancy. It is found that if the magnetic buoyancy effect is switched off, then the strength of the non-axisymmetric field after the relaxation is decreased by a factor of two.

The growth and decay of the NA magnetic field perturbation is accompanied by oscillations with a period of about 1.5 year. This period is a little longer than the period of the NA eigen $m = 1$ dynamo mode, which is oscillating with period of one year. Thus, the origin of these oscillations has to be studied separately. For the final stage of evolution, it is found that the NA perturbation order of $M = 10^{-3}$ of the energy axisymmetric magnetic field is quenched by the system dynamics to the order of $M = 10^{-6}$ of the energy AS magnetic field. This is different from the results of previous studies (e.g., by Moss 1999, who found that initially weak NA $m = 1$ mode is amplified to the order of $M = 10^{-3}$ of the energy AS magnetic field). Thus our model maintains the smaller strength NA magnetic field than the model by Moss (1999). It is found that the dynamo period is not affected by the NA perturbation (i.e., the the strong transient cycle has the same period as the other cycles before and after perturbations).

This paper illustrates the initial results of our nonlinear non-axisymmetric mean-field dynamo model. The axisymmetric part of the model is based on our previous results. The non-axisymmetric perturbations are assumed to be located in the near-surface rotational shear layer. For the first time we demonstrate that nonlinear coupling between the asymmetric and the non-axisymmetric fields can impact the generation of the axisymmetric field in the case of finite-amplitude perturbations. The effect depends strongly on the dynamo mechanisms involved in the problem, the spatial distribution of perturbation, the phase of the dynamo cycle at the time of initialization of the perturbation, and on how well the magnetic helicity is conserved in the system. These factors determine the subsequent evolution of the dynamo system, including the dynamo cycle, evolution, and rotation of the non-axisymmetric modes of large-scale magnetic fields.
We summarize with the following points.

1. The mean-field dynamo model shows an amplification of the non-axisymmetric perturbation at the beginning of transient phase of evolution, producing the magnetic field configuration that deviates considerably from the axial symmetry.

2. In the solar dynamo models, non-axisymmetric magnetic field perturbations developed in the near-surface rotational shear layer affect the strength of the dynamo cycles. The dynamo period remains stable.

3. The stationary phase of evolution without the meridional circulation the non-axisymmetric $m = 1$ dynamo mode shows a flip-flop phenomenon and also has a fixed longitudinal position resembling the AL phenomenon. During this stage, the strength of the non-axisymmetric magnetic field is three orders of magnitude smaller than for the axisymmetric magnetic field. The solar-type (double-cell) meridional circulation destroys the flip-flop effects. Instead, the non-axisymmetric field represents a traveling in the azimuthal direction dynamo wave.

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