A Double Feature Extraction Method for Rolling Bearing Fault Diagnosis Based on Slope Entropy and Fuzzy Entropy

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Abstract

Rolling bearings are the key components for the safe operation of mechanical equipment. It plays an irreplaceable role in the normal operation of mechanical equipment. Higher load makes higher failure rate of rolling bearing. Accurate identification of the fault location is an important step in the diagnosis of the rolling bearing fault. In recent years, the entropy features of rolling bearing vibration signals are usually extracted to identify fault. In this paper, a double feature extraction method based on slope entropy (SlE) and fuzzy entropy (FE) is proposed to recognize the fault state of rolling bearing through rolling bearing signals. In the single feature extraction experiment, the recognition rate of these two kinds of entropy is not high. Through the improvement of the single feature extraction experiment, SlE and FE are selected as two feature combinations. After combining approximate entropy (AE), SlE, FE, permutation entropy (PE), and sample entropy (SE), the identification rate of these combinations was calculated using $k$ nearest neighbor (KNN). The result shows that the recognition rate of this combination is 98% and 3.3% higher than other combinations.

1. Introduction

Rolling bearing is an important mechanical element that can effectively reduce friction during mechanical operation and is an indispensable part of many rotating machines. Once the failure of the rolling bearing, the operation of the whole mechanical equipment will become difficult, but as the supporting part of the rotating parts, the rolling bearing will inevitably suffer from wear and tear. Therefore, it is necessary to diagnose the failure status of the rolling bearing in time [1, 2]. Although the extraction of frequency signals, sound signals, and other signals of rolling bearings can well reflect the fault, the extraction of these signals has higher requirements for the equipment and professionalism of personnel [3, 4]. Due to the structural characteristics of rolling bearings, natural vibration [5] is unavoidable when they are working, and this vibration signal is easy to extract and analyze, so there is more research on the features of the vibration signal. However, in the actual work of rolling bearing, the vibration signal collected often has obvious interference and presents nonstationary, nonlinear characteristics. In order to better identify the signal, entropy, as an important technology that can accurately analyze the nonlinear dynamic changes of time series signals, has been gradually used in the field of fault diagnosis [6].

Fault types of rolling bearings are generally divided into three types: outer ring, inner ring, and rolling body. The vibration signals of rolling bearings are usually composed of their own vibration, vibration, and noise of other parts of the machine [7]. At present, several methods of time-frequency analysis have been proposed for the diagnosis of rolling bearing fault signals. Wavelet transform (WT) is one of the most classic ones, but the decomposition of WT for the high-frequency part of signals is not precise enough. And, the existence of the wavelet function means that it is not an adaptive method itself [8]. Some mode decomposition algorithms, such as empirical mode decomposition (EMD) [9] and variational mode decomposition (VMD) [10], are also gradually applied to fault diagnosis [11, 12]. The signal is decomposed into multiple modes and processed to reduce the effect of the noise therein. However, during this process, it is found that the decomposition results of EMD have a
certain decomposition overlap and final effect, which directly affects the decomposition results [13]. While VMD can restrain mode aliasing to some extent, its parameter setting and mode selection affect the decomposition performance.

To solve the above-given problems and obtain better feature information, entropy theory is used to characterize the features of signals [14]. AE [15], FE [16], PE [17], and SE [18] are successively used to extract the characteristics of rolling bearing signals. AE is an important parameter to quantify the regularity and unpredictability of time series fluctuations. The greater the probability of generating new patterns in time series, the more complex the information given and the greater the AE value. AE has a certain noise resistance, if the amplitude of the noise is lower than the similar tolerance parameter $r$, the noise will be suppressed [18–20]. PE differs from other entropy in that it introduces permutation in calculating the complexity of reconstructed subsequences [14, 17]. PE also believes that the more complex the time series, the higher the entropy value is. SE has made some improvements based on AE and is more consistent in parameter selection [18, 21]. FE is improved based on SE, which can describe the degree of ambiguity of a fuzzy set [16]. When using the fuzzy membership function instead of the self-similar function, the greater the probability of time series generating a new mode, the greater the FE value [22]. In 2019, Cuesta-Frau proposed SLE, which was based on continuous differences between input samples and improved on PE in time series magnitude [23, 24], first applied in medicine. SIE is applied in the field of fault diagnosis in this paper.

In agricultural applications [25], hydroacoustic signal recognition [26], and even star recognition [27], the double feature extraction method has been well applied, which can strengthen the classification process, combine the different features extracted, and improve the accuracy of classification results. Based on the above reasons, this paper proposes a double feature extraction method combining SIE and FE and applies it to fault diagnosis.

The remaining structure of this paper is as follows: Section 2 describes the basic principles of the two kinds of entropy used in this method in this paper; In Section 3, the proposed methods based on SIE and FE are introduced in detail. Section 4 illustrates the validity of the proposed method through specific extraction and classification experiments. Section 5 summarizes the innovations and conclusions of the whole experiment.

### 2. Basic Theory

#### 2.1. Slope Entropy

The complexity of the time series is expressed by analyzing the difference between two consecutive samples of the time series, that is, the slope. The specific steps to calculate SE are as follows:

1. Given a time series $[x_1, x_2, \ldots, x_N]$. According to the embedded dimension $m$, it is decomposed into $j$ subsequences in the following form:

   $$x''_m = \{x_i, x_{i+1}, \ldots, x_{i+m-1}\},$$

   \hspace{1cm} (1)

   where $i = 1, 2, \ldots, j, j = N - m + 1$.

2. Two soft threshold parameters $\gamma$ and $\delta$ are introduced to calculate the symbolic patterns of these subsequences, where $0 < \delta < \gamma$.

3. For elements in a subsequence, the difference $d$ is taken $d = x_{i+1} - x_i$. These elements can be distinguished into five modes by comparing $d$ with the soft threshold parameters, which are defined as $2, 1, 0, -1, -2$, with the relationships shown as follows:

   $$\text{pattern}=2, \quad d > \gamma,$$

   $$\text{pattern}=1, \quad \delta < d \leq \gamma,$$

   $$\text{pattern}=0, \quad |d| \leq \delta,$$

   $$\text{pattern}=-1, -\gamma \leq d < -\delta,$$

   $$\text{pattern}=-2, \quad d < -\gamma.$$

4. Based on these five modes, we can get $5^{m-1}$ sequence combinations. Record the number of occurrences $f_n$ of each combination, The relative frequency $p_n$ of the combination is calculated as follows:

   $$p_n = \frac{f_n}{f}$$

   where $n = 1, 2, \ldots, 5^{m-1}$.

5. Based on the calculation results of the relative frequency, the formula of SLE can be obtained using the Shannon entropy formula:

   $$\text{SLE}(m, \gamma, \delta) = -\sum_{n=1}^{5^{m-1}} p_n \ln p_n.$$  \hspace{1cm} (4)

#### 2.2. Fuzzy Entropy

FE is a quantitative statistical index of signal complexity. It introduces the theory of fuzzy sets, uses the fuzzy membership function as the definition of the similarity degree of entropy, and is more adaptable to nonlinear and nonstationary fault signals. Its specific calculation process is as follows:

1. For time series $[u(i), i = 1, 2, \ldots, N]$, embedding dimension $m$ is defined to reconstruct the phase space, the reconstructed vector is

   $$x(i) = [u(i), u(i + 1), \ldots, u(i + m - 1)] - u_0(i), i = 1, 2, \ldots, N - m + 1,$$

   \hspace{1cm} (5)

   where $u_0(i)$ is a scalar value:

   $$u_0(i) = \frac{1}{m} \sum_{p=0}^{m-1} u(i + j).$$  \hspace{1cm} (6)

2. Define $d''_{ij}$ as the distance between two vectors $x(i)$, $x(j)$ in the reconstructed matrix:
\[ d_{ij}^m = \max_{k=1,2,\ldots,m} \{ |u(i+k-1) - u_0(i)| - |u(j+k-1) - u_0(j)| \}, \]

(7)

where \( i \geq 1, j \leq N - m + 1 \) and \( i \neq j \).

(3) The fuzzy membership function is introduced according to the distance to calculate the similarity:

\[
A(x) = \begin{cases} 
1, & x = 0, \\
\exp\left[ -\ln\left( \frac{x}{r} \right)^2 \right], & x > 0,
\end{cases}
\]

(8)

where \( r \) is the similarity tolerance parameter, which is usually 0.2 times the standard deviation of the original one-dimensional sequence. On the basis of the function, the similarity is calculated as follows:

\[ A_{ij}^m = \exp\left[ -\ln\left( \frac{d_{ij}}{r} \right)^2 \right]. \]

(9)

(4) For these similarities, we can get the following formula:

\[ C_i^m (r) = \frac{1}{N - m} \sum_{j=1, j \neq i}^{N-m+1} A_{ij}^m. \]

(10)

(5) From this, we can get the relationship under \( m \) dimension:

\[ \Phi^m (r) = \frac{1}{N - m} \sum_{i=1}^{N-m+1} C_i^m (r). \]

(11)

(6) Set the embedding dimension to \( m + 1 \), repeat the above five steps to obtain the following equation:

\[ \Phi^{m+1} (r) = \frac{1}{N - m} \sum_{i=1}^{N-m} C_i^{m+1} (r). \]

(12)

(7) Finally, the expression of FE is as follows:

\[ FE(m, r, N) = \ln \Phi^m (r) - \ln \Phi^{m+1} (r). \]

(13)

3. Implementation of the Proposed Method

In order to reflect the characteristics of signals more comprehensively and obtain more effective experimental results, this paper uses two kinds of entropy: SlE and FE to represent the characteristics of bearing signals. The features are processed by using the double feature extraction method. The distribution of double feature was observed and the identification results were calculated to reflect the good effect of the method. The experimental process of the method is shown in Figure 1. The steps of the experiment are described as follows:

(1) Rolling bearing signal are divided into 100 samples, and each sample contains 1024 data points

(2) Extract SlE and FE features from samples, respectively

(3) Obtain the distribution of double features

(4) Identify and classify signals using KNN

(5) Compute the recognition rate of signal and evaluate whether the recognition is accurate or not

4. Feature Extraction Experiments

4.1. Rolling Bearing Signals. The purpose of this paper is to accurately identify the fault condition of rolling bearing signals, so six different types of fault bearing signals are selected. These bearing signals contain three fault types: ball fault, inner race fault, and outer race fault. In addition, these bearings come in two different sizes, 0.007 feet and 0.021 feet. Based on the above information, these six types of signals are labeled as IR007, B007, OR007, IR021, B021, and OR021, respectively. Six types of rolling bearing signals are shown in Figure 2.
4.2. Feature Extraction and Classification. In this section, the SIE&FE double feature extraction method proposed in this paper is verified by extracting and analyzing the features of the six given rolling bearing fault signals, and then using the classification algorithm to recognize and classify the feature extraction results.

4.2.1. Single Feature Experiments. In this section, we use the five types of entropy mentioned earlier to extract the features of the six types of signals. The original signal is taken as a sample every 1024 sample points, and 100 samples are extracted for each type of signal, $m = 3$. Under the above-given conditions, the entropy values of the five kinds of

![Figure 2: Six types of rolling bearing signals. (a) IR007. (b) B007. (c) OR007. (d) IR021. (e) B021. (f) OR021.](image-url)
Figure 3: The five kinds of entropy feature the distribution of the six types of bearing signals. (a) AE. (b) SIE. (c) FE. (d) PE. (e) SE.
entropy are calculated, and the characteristic distribution maps are drawn. The five kinds of entropy feature distribution maps of the six types of bearing signals are displayed in Figure 3.

As can be seen from Figure 3, under the same kind of entropy, there are more repetitive parts of the entropy values of the six types of signals; the AE, FE, and SE features of B007 and B021 are mixed obviously; in the feature distribution of PE and FE, the difference between the entropy value of IR021 and other signals is large, and the feature distribution range of IR007 and OR021 in AE is less mixed with other signals; OR007 is obviously mixed with other signals in the feature distribution of each type of entropy.

### 4.2.2. Classification and Recognition of Single Features

The above-given feature extraction results are classified to observe the accuracy of recognition, and KNN is selected to classify the features. It can judge the type of sample according to the samples around a sample. The first 50 samples of each kind of signal are selected as training set and the rest as test set. The latest sample number is set to 1. Table 1 illustrates the recognition results under different features of the six types of bearing signals.

It can be seen from the table that the recognition results of these five kinds of entropy are not ideal for these six kinds of signals. The highest AE of recognition rate is only 80.3%; the recognition rates of SI and PE are lower than 50%; moreover, the recognition rate of these entropy varies greatly for different types of signals; the recognition rate of AE for IR007 and OR021, PE for IR021, and SE for IR007 has reached 100%; however, PE achieved only 26% and 24% recognition rates for identifying IR007 and B021, respectively; considering that different entropy reflects different features of signals, in order to improve the accuracy of experiments and make the classification results more precise, we introduce double feature extraction.

### 4.2.3. Double Feature Experiments

Select two of the five entropy to combine and get ten combinations, the parameters for calculating the five entropy values are the same as the previous one. The feature distributions of these combinations are plotted for observation; the transverse and longitudinal coordinates of the figure of feature distribution are the two entropies used for combination. The signals used to extract the features are still the same as the original six. Feature distributions of the ten combinations are shown in Figure 4.

As can be seen from Figure 4, 100 samples of these six bearing signals show varying degrees of confusion in the double feature distribution; the mixed samples in combination AE&FE, AE&PE, AE&SE, FE&PE, and FE&SE are mainly B007 and B021, while the distribution of these two types of samples in other subgraphs is similar, too; the samples of OR007 and IR021 in combination AE&SE, AE&SE and SE&SE were significantly mixed; mixing of the three samples can be easily observed in combination SE&PE, SE&SE, and PE&SE; mixed portions of these six samples of samples are the least in the combined SI&FE.

### 4.2.4. Classification and Recognition of Double Features

Using KNN for double feature classification and recognition, the selection of training and test sets is the same as that of a single feature. The effectiveness of the proposed method is verified by observing and analyzing the results of classification and recognition. Recognition results of the ten combinations are illustrated in Figure 5, recognition results for ten types of feature combinations can be obtained in Table 2.

It is easy to see from Figure 5 that there are obvious identification errors in all combinations except SI&FE; the combination of AE&FE, AE&PE, AE&SE, FE&PE, FE&SE has a weak recognition ability for B007 and B021; all combinations except SI&PE and FE&PE can fully recognize IR007; AE&SE and SI&SE SE are difficult to identify OR007 and IR021; there are many errors in the combination of SI&PE when identifying IR007 and OR021; OR021 has good recognition effect in nine combinations except SI&PE; the combination of SI&FE has a good recognition effect on these six signals.

As can be seen from Table 2, the feature combination of SI&FE has the highest average recognition rate, which is up to 98%; the lowest average recognition rate is the combination of SI&PE, only 75.3%; the average recognition rate of other combinations is lower than 95%, which has some differences with SI&FE combinations; besides, when recognizing different types of signals, the recognition rate of the same combination varies greatly; for example, SI&SE has a recognition rate of 100% when identifying IR007 and B007, but only 44% when identifying OR007; the recognition rate of AE&FE combination for B007 is also only 44%, but for IR007, IR021 and OR021, the recognition rate is 100%. In general, the SI&FE method proposed in this paper has the best effect.

### Table 1: Recognition rates under different features of the six types of bearing signals.

|        | IR007 (%) | B007 (%) | OR007 (%) | IR021 (%) | B021 (%) | OR021 (%) | Average (%) |
|--------|-----------|----------|-----------|-----------|----------|-----------|-------------|
| AE     | 100       | 62       | 82        | 86        | 52       | 100       | 80.3        |
| SE     | 38        | 86       | 26        | 48        | 34       | 40        | 45.3        |
| FE     | 70        | 44       | 96        | 98        | 72       | 90        | 78.3        |
| PE     | 26        | 38       | 54        | 100       | 24       | 36        | 46.3        |
| SE     | 100       | 54       | 60        | 40        | 62       | 80        | 66          |

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Figure 4: Continued.
Figure 4: Feature distributions of the ten combinations. (a) AE&SlE. (b) AE&FE. (c) AE&PE. (d) AE&SE. (e) SlE&FE. (f) SlE&PE. (g) SlE&SE. (h) FE&PE. (i) FE&SE. (j) PE&SE.

Figure 5: Continued.
Figure 5: Continued.
Conclusions

In this paper, SlE and FE are applied to fault diagnosis through a feature extraction experiment; a single feature extraction method is applied, and then the two entropy are further combined to achieve double feature extraction. The validity of the method is proved by classifying six types of rolling bearing signals according to the feature, and the recognition rate achieves 98%. The main conclusions are demonstrated as follows:

1. A double feature extraction method based on the combination of SlE and FE is proposed and introduced into the field of fault diagnosis.
2. Combining the two kinds of entropy with a poor recognition effect greatly improves the recognition effect, and the recognition rate is 19.7% higher than that of a single feature.
3. Compared with other double feature combinations, the method proposed in this paper has a better effect, and the recognition rate is at least 3.3% higher than that of other combinations.

Abbreviations

| Abbreviation | Description |
|--------------|-------------|
| SlE | Slope entropy |
| FE | Fuzzy entropy |
| AE | Approximate entropy |
| PE | Permutation entropy |
| SE | Sample entropy |
| KNN | $K$ nearest neighbor |
| WT | Wavelet transform |
| EMD | Empirical mode decomposition |
| VMD | Variational mode decomposition |
| IR007 | Inner race fault signal 0.007 feet in size |
| B007 | Ball fault signal 0.007 feet in size |
| OR007 | Outer race fault signal 0.007 feet in size |
| IR021 | Inner race fault signal 0.021 feet in size |
| B021 | Ball fault signal 0.021 feet in size |
| OR021 | Outer race fault signal 0.021 feet in size |

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.
Conflicts of Interest

The authors declare that they have no conflicts of interest.

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