Signatures of Supersymmetry at B-Factories

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Abstract

We discuss $CP$ asymmetries in $B^0$-meson decays and $B^0_s$-$\bar{B}^0_s$ mixing within the framework of the supersymmetric standard model (SSM). It is shown that $B^0_s$-$\bar{B}^0_s$ and $K^0$-$\bar{K}^0$ mixings could receive sizable new contributions through box diagrams mediated by the charginos and charged Higgs bosons. This implies that the $CP$-violating phase of the Cabibbo-Kobayashi-Maskawa matrix may have a value which is different from the one predicted by the standard model (SM). The value of the $CP$-violating phase affects the amounts of the $CP$ asymmetries and $B^0_s$-$\bar{B}^0_s$ mixing. We examine predictions for these quantities in both the SSM and the SM, exploring the difference between the two models.

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§1. Introduction

In the supersymmetric standard model (SSM) there exist several new sources for flavor-changing neutral current (FCNC) processes, such as $B^0$-$\bar{B}^0$ mixing and radiative $B$-meson decay. The new sources are the interactions in which a quark $q$ couples to a squark $\tilde{q}$ of a different generation and a chargino $\tilde{\omega}$, a neutralino $\tilde{\chi}$, or a gluino $\tilde{g}$. Since the SSM contains two doublets of Higgs bosons, charged Higgs bosons $H^\pm$ also mediate FCNC processes.

In this article we discuss SSM contributions to $B^0_d$-$\bar{B}^0_d$ and $K^0$-$\bar{K}^0$ mixings and their effects on $CP$ asymmetries in neutral $B$-meson decays and $B^0$-$\bar{B}^0$ mixing, searching for signatures of supersymmetry. The measurements of these quantities will be performed at B-factories in the near future. Possible observation of discrepancies within standard model (SM) predictions can give hints for physics beyond the SM. Since the SSM is one of the most plausible extensions of the SM, studying the predictions of the SSM would be important. It will be shown that SSM predictions are deviated from SM predictions in sizable ranges of SSM parameters. For definiteness, we assume the framework of grand unification theories coupled to $N = 1$ supergravity.

Among the new sources for FCNC processes in the SSM, sizable contributions can be expected from the interactions mediated by the charginos and the charged Higgs bosons. The reasons are as follows: Since the $t$-quark has a large mass, the coupling strengths of the related Yukawa interactions for the chargino, the $t$-squark, and the down-type quark and for the charged Higgs boson, the $t$-quark, and the down-type quark are comparable to that of the SU(2) gauge interaction. Consequently, the chargino interaction strengths for the $t$-squarks are made different from those for the $u$- or $c$-squarks which are determined by the gauge interaction alone. Besides, if the squark masses of the first two generations are not much different from the $t$-quark mass, one of the $t$-squarks can be lighter than the other squarks. These effects soften the cancellation among different squark contributions for the chargino box diagrams, which otherwise is rather severe. In the box diagrams exchanging charged Higgs bosons, the charged Higgs boson interactions for the $t$-quark are no longer weak, compared to the standard $W$-boson interactions, while those for the $u$- or $c$-quark are much weaker. Therefore, the chargino and the charged Higgs boson contributions could naturally be the same order of magnitude as the $W$-boson contributions. On the other hand, the gluino and the neutralino contributions are mediated by the down-type squarks, whose masses are quite degenerated. Although the interaction strength for the gluinos is stronger than for the charginos, in the grand unification scheme the gluino mass is proportionally larger than the chargino masses. Therefore, the gluino and the neutralino contributions become smaller than the chargino contributions.
In sect. 2 we briefly review the model. In sect. 3 we discuss the SSM contributions to $B^0_d$-$\bar{B}^0_d$ and $K^0$-$\bar{K}^0$ mixings. In sect. 4 we evaluate the Cabibbo-Kobayashi-Maskawa (CKM) matrix through these mixings and discuss the resultant implications for $CP$ asymmetries in $B^0$-meson decays and $B^0_s$-$\bar{B}^0_s$ mixing. Summary is given in sect. 5.

§2. Model

In the SSM, down-type quarks interact with charginos and up-type squarks. The charginos are the mass eigenstates of the SU(2) charged gauginos and the charged higgsinos. Their mass matrix is given by

$$ M^{-} = \begin{pmatrix} \tilde{m}_2 & -\frac{1}{\sqrt{2}}g v_1 \\ -\frac{1}{\sqrt{2}}g v_2 & m_H \end{pmatrix}, \tag{2.1} $$

where $v_1$ and $v_2$ are the vacuum expectation values of the Higgs bosons, and $\tilde{m}_2$ and $m_H$ respectively denote the SU(2) gaugino mass and the higgsino mass parameter. In the ordinary scheme for generating the gaugino masses, $\tilde{m}_2$ is smaller than or around the gravitino mass $m_3/2$. If the SU(2)×U(1) symmetry is broken through radiative corrections, a relation $\tan \beta (\equiv v_2/v_1) \gtrsim 1$ holds and the magnitude of $m_H$ is at most of order of $m_3/2$. The chargino mass eigenstates are obtained by diagonalizing the matrix $M^{-}$ as

$$ C_R^\dagger M^{-} C_L = \text{diag}(\tilde{m}_{\omega_1}, \tilde{m}_{\omega_2}) \quad (\tilde{m}_{\omega_1} < \tilde{m}_{\omega_2}), \tag{2.2} $$

$C_R$ and $C_L$ being unitary matrices.

The squark fields, as well as the quark fields, are mixed in generation space. Since the left-handed squarks and the right-handed ones are also mixed, the mass-squared matrix for the up-type squarks $M_U^2$ is expressed by a $6 \times 6$ matrix:

$$ M_U^2 = \begin{pmatrix} M_{U,11}^2 & M_{U,12}^2 \\ M_{U,21}^2 & M_{U,22}^2 \end{pmatrix}, \tag{2.3} $$

where $m_U$ denotes the mass matrix of the up-type quarks. The mass parameters $m_{3/2}$, $\tilde{m}_Q$, $\tilde{m}_U$ and the dimensionless constants $a$, $c$ come from the terms in the SSM Lagrangian which break supersymmetry softly: $\tilde{m}_Q$ and $\tilde{m}_U$ are determined by the gravitino and gaugino masses and $\tilde{m}_Q \simeq \tilde{m}_U \sim m_{3/2}$; $a$ is related to the breaking of local supersymmetry; and $c$
represents the magnitude of radiative corrections to the squark masses. At the electroweak energy scale, $a$ is of order unity and $c = -1 - (-0.1)$.

The squark mixings among different generations in Eq. (2.3) are removed by a unitary matrix which consists of the same $3 \times 3$ matrices that diagonalize the up-type quark mass matrix. As a result, the generation mixings in the Lagrangian between the down-type quarks and the up-type squarks in mass eigenstates are described by the CKM matrix of the quarks. The mixings between the left-handed and right-handed squarks can be neglected for the first two generations because of the smallness of the corresponding quark masses. The masses of the left-handed squarks $\tilde{u}_L, \tilde{c}_L$ and the right-handed squarks $\tilde{u}_R, \tilde{c}_R$ are given by

$$
\tilde{M}^2_{uL} = \tilde{M}^2_{cL} = \tilde{m}^2_Q + \cos 2\beta \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) M^2_Z,
$$

$$
\tilde{M}^2_{uR} = \tilde{M}^2_{cR} = \tilde{m}^2_U + \frac{2}{3} \cos 2\beta \sin^2 \theta_W M^2_Z.
$$

For the third generation, the large $t$-quark mass leads to an appreciable mixing between $\tilde{t}_L$ and $\tilde{t}_R$. The mass-squared matrix for the $t$-squarks becomes

$$
M^2_t = \begin{pmatrix}
\tilde{M}^2_{uL} + (1 - |c|)m_t^2 & (\cot \beta m_H + a^* m_3/2)m_t \\
(\cot \beta m_H + a m_3/2)m_t & \tilde{M}^2_{uR} + (1 - 2|c|)m_t^2
\end{pmatrix}.
$$

The mass eigenstates of the $t$-squarks are obtained by diagonalizing the matrix $M^2_t$ as

$$
S_t M^2_t S_t^\dagger = \text{diag}(\tilde{M}^2_{t_1}, \tilde{M}^2_{t_2}) \quad (\tilde{M}^2_{t_1} < \tilde{M}^2_{t_2}),
$$

where $S_t$ is a unitary matrix.

Down-type quarks interact also with charged Higgs bosons and up-type quarks. The generation mixings in these interactions are described by the CKM matrix. The parameters which determine FCNC processes, other than the SM parameters, are only the charged Higgs boson mass $M_{H^\pm}$ and $\tan \beta$.

In general, the SSM parameters have complex values. If the magnitudes of their physical complex phases are of order unity, the electric dipole moment of the neutron is predicted to have a large value. Its experimental limits then lead to a lower bound of about 1 TeV on the squark masses. In this case, the SSM does not give any sizable new contributions to FCNC processes. We assume hereafter real values for the parameters other than the SM parameters, so that the constraints from the electric dipole moment of the neutron can be ignored.

§3. $B^0 - \bar{B}^0$ and $K^0 - \bar{K}^0$ mixings

The SSM gives new contributions to $B^0 - \bar{B}^0$ and $K^0 - \bar{K}^0$ mixings through box diagrams mediated by the charginos or the charged Higgs bosons. For the chargino contribution,
exchanged bosons are up-type squarks. For the charged Higgs boson contribution, exchanged fermions are up-type quarks and exchanged bosons are either only charged Higgs bosons or charged Higgs bosons and $W$-bosons.

An observable quantity for $B_d^0$-$\bar{B}_d^0$ mixing is the mixing parameter $x_d = \Delta M_{B_d}/\Gamma_{B_d}$, where $\Delta M_{B_d}$ and $\Gamma_{B_d}$ denote the mass difference and the average width for the $B_d^0$-meson mass eigenstates. The mass difference is induced dominantly by the short distance contributions of box diagrams. The mixing parameter is given by

$$x_d = \frac{G_F^2}{3\pi^2} \frac{m_W^2}{f_{B_d}^2} \frac{M_{B_d}}{f_{B_d}^2} \beta \eta_{B_d} |V_{t\bar{d}}^* V_{t\bar{d}}| A_W^d + A_C + A_H^d,$$  \hspace{1cm} (3.1)$$

where $G_F$, $f_{B_d}$, $\beta$, and $\eta_{B_d}$ represent the Fermi constant, the $B_d^0$-meson decay constant, the bag factor for $B_d^0$-$\bar{B}_d^0$ mixing, and the QCD correction factor. The CKM matrix is denoted by $V$. The contributions of the $W$-boson, chargino, and charged Higgs boson box diagrams are respectively expressed as $A_W^d$, $A_C$, and $A_H^d$, which are explicitly given in Refs. [4, 10]. The box diagrams with $t$-quarks give $A_W^d$ and $A_H^d$.

For $K^0$-$\bar{K}^0$ mixing, the $CP$ violation parameter $\epsilon$ is an observable quantity for the short distance contributions of box diagrams. This parameter can be written as

$$\epsilon = -e^{i\pi/4} \frac{G_F^2}{12\sqrt{2}\pi^2} m_W^2 \frac{M_K}{\Delta M_K} f_K^2 B_K \text{Im}[(V_{31}^* V_{32})^2 \eta_{K33} (A_{tt}^W + A_C + A_{tt}^H)] + (V_{21}^* V_{22})^2 \eta_{K22} A_{cc}^W + 2V_{31}^* V_{32} V_{21}^* V_{22} \eta_{K32} A_{tc}^W,$$  \hspace{1cm} (3.2)$$

where $f_K$ and $B_K$ represent the decay constant and the bag factor. The QCD correction factors are denoted by $\eta_{Kab}$. The standard $W$-boson box diagram with $c$-quarks and that with $t$- and $c$-quarks give $A_{cc}^W$ and $A_{tc}^W$, respectively.

The SSM prediction for $x_d$ is different from the SM prediction by $A_C + A_{tt}^H$. The difference between the SSM and the SM predictions for $\epsilon$ is in the term proportional to $(V_{31}^* V_{32})^2$, which is the same amount as for $x_d$. Thus, we can measure the amount of the SSM contributions to $B_d^0$-$\bar{B}_d^0$ and $K^0$-$\bar{K}^0$ mixings by the ratio

$$R = \frac{A_{tt}^W + A_C + A_{tt}^H}{A_{tt}^W}.$$  \hspace{1cm} (3.3)$$

If new contributions are negligible, $R$ becomes unity.

We now examine the amounts of SSM contributions to the mixings. In order to see the chargino and the charged Higgs boson contributions separately, we evaluate, instead of $R$, the ratios $R_C = (A_{tt}^W + A_C)/A_{tt}^W$ and $R_H = (A_{tt}^W + A_{tt}^H)/A_{tt}^W$. In Table I the value of $R_C$ is given for $\tan \beta = 1.2$ (i), 2 (ii) and several values of the higgsino mass parameter $m_H$. The other parameters are set, as typical values, for $\tilde{m}_2 = 200$ GeV, $\tilde{m}_Q = 200$ GeV,
The ratio $R_C$ for $\tilde{m}_2 = 200$ GeV and $\tilde{m}_Q = 200$ GeV. The other parameters are set for $\tilde{m}_U = am_{3/2} = \tilde{m}_Q$ and $|c| = 0.3$: (i) $\tan \beta = 1.2$, (ii) $\tan \beta = 2$.

| $m_H$ (GeV) | −200 | −100 | 100 | 200 |
|-------------|------|------|-----|-----|
| (i)         | 1.26 | 1.43 | 2.25|
| (ii)        | 1.14 | 1.25 | 1.50| 1.35|

$\tilde{m}_Q = \tilde{m}_U = am_{3/2}$, and $|c| = 0.3$. For the $t$-quark mass we use $m_t = 170$ GeV. In case (i) with $m_H = 200$ GeV, the lighter $t$-squark mass-squared becomes negative. The sign of the chargino contribution is the same as that of the $W$-boson contribution, and these contributions interfere constructively. If $\tan \beta$ is not much larger than unity, the ratio $R_C$ can have a value larger than 1.5 in sizable ranges of other SSM parameters. The dependence of $R_C$ on $\tan \beta$ arises from the chargino Yukawa interactions: $R_C$ increases as $\tan \beta$ decreases, since a smaller value for $v_2$ enhances the Yukawa couplings of the charginos to the $t$-squarks.

The ratio $R_C$ also depends on $\tilde{m}_Q$, whereas it does not vary so much with $\tilde{m}_2$. In Table II we show $R_H$ for $\tan \beta = 1.2$ (i), 2 (ii) and $M_{H^\pm} = 100, 200, 300$ GeV. The charged Higgs boson box diagrams also contribute constructively. In case (i) $R_H$ is larger than 1.5 for $M_{H^\pm} \lesssim 180$ GeV. Similarly to the chargino contribution, the value of $R_H$ increases as $\tan \beta$ decreases. The net amount of the SSM contribution is given by the sum of all the contributions. The ratio $R$ in Eq. (3.3) is larger than $R_C$ or $R_H$. For example, in case (i) with $M_{H^\pm} = 200$ GeV, the ratio $R$ becomes $R \simeq 1.9$ for $m_H = −100$ GeV ($\tilde{M}_{t1} \simeq 188$ GeV, $\tilde{m}_{\omega 1} \simeq 119$ GeV) and $R \simeq 2.7$ for $m_H = 100$ GeV ($\tilde{M}_{t1} \simeq 85$ GeV, $\tilde{m}_{\omega 1} \simeq 56$ GeV), where $\tilde{M}_{t1}$ and $\tilde{m}_{\omega 1}$ denote the lighter $t$-squark mass and the lighter chargino mass, respectively.

Table II. The ratio $R_H$: (i) $\tan \beta = 1.2$, (ii) $\tan \beta = 2$.

| $M_{H^\pm}$ (GeV) | 100 | 200 | 300 |
|-------------------|-----|-----|-----|
| (i)               | 1.75| 1.45| 1.30|
| (ii)              | 1.24| 1.15| 1.10|

§4. $CP$ asymmetries

We discuss what an enhancement of $R$ implies for observable quantities. The values of $x_d$ and $\epsilon$ have been experimentally measured as $x_d = 0.71 \pm 0.06$ and $|\epsilon| = 2.26 \times 10^{-3}$.

An enhanced value of $R$ is considered to give a prediction, for the CKM matrix, different from the SM prediction through Eqs. (3.1), (3.2). This can be seen easily, if we adopt the
standard parametrization\(\textsuperscript{3}\) for the CKM matrix:

\[
V = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},
\]

where \(c_{ab} = \cos \theta_{ab}\) and \(s_{ab} = \sin \theta_{ab}\). Without loss of generality, the angles \(\theta_{12}, \theta_{23},\) and \(\theta_{13}\) can be taken to lie in the first quadrant, leading to \(\sin \theta_{ab} > 0\) and \(\cos \theta_{ab} > 0\). At present, the experiments give \(|V_{12}| = 0.22, |V_{23}| = 0.04 \pm 0.004,\) and \(|V_{13}/V_{23}| = 0.08 \pm 0.02\textsuperscript{3}\) within the framework of the SM. Since these values have been measured through the processes for which new contributions by the SSM, if any, are negligible, they are also valid in the SSM. Among the four independent parameters of the CKM matrix, the value of \(\sin \theta_{13}\) is determined by \(\sin \theta_{13} = |V_{13}|\). Owing to this smallness of \(\sin \theta_{13}\), the values of \(\sin \theta_{12}\) and \(\sin \theta_{23}\) are given by \(\sin \theta_{12} = |V_{12}|\) and \(\sin \theta_{23} = |V_{23}|\). The remaining undetermined parameter is the \(CP\)-violating phase \(\delta\), which can be measured by \(x_d\) or \(\epsilon\). Therefore, the value of \(\delta\) depends on \(R\) and the SSM with \(R > 1\) and the SM predict different values for it.

The value of \(\delta\) is determined as a function of \(R\) or vice versa independently by \(x_d\) and \(\epsilon\). Using consistency in these two evaluations, we can specify the values of \(\delta\) and \(R\). In Table III we show the allowed range of \(\cos \delta\) derived from the experimental values of \(x_d, \epsilon,\) and CKM matrix elements, for several values of the ratio \(R\). The sign of \(\sin \delta\) should be positive in order to give \(\epsilon\) correctly. We have assumed the experimental central value for \(x_d\) and taken three sets of values for the CKM matrix elements: \((|V_{13}/V_{23}|, |V_{23}|) = (0.08, 0.04)\) (a), \((0.08, 0.044)\) (b), \((0.1, 0.04)\) (c). The theoretical uncertainties of \(B_K, B_{B_d}\), and \(f_{B_d}\) are incorporated as \(0.6 < B_K < 0.9\) from a combined result of lattice\textsuperscript{4} and \(1/N\textsuperscript{3}\) calculations and \(180\text{ MeV} < f_{B_d}\sqrt{B_{B_d}} < 260\text{ MeV}\) from a lattice calculation\textsuperscript{4}. For the QCD correction factors we have used \(\eta_{B_d} = 0.55\) in Eq. (3.1) and \(\eta_{K_{33}} = 0.57, \eta_{K_{22}} = 1.1,\) and \(\eta_{K_{32}} = 0.36\) in Eq. (3.2)\textsuperscript{3}. For \(R = 2.5\) in case (a) and \(R = 2, 2.5\) in case (b), there is no allowed range of \(\cos \delta\). If we assume case (a), which corresponds to experimental central values for \(|V_{13}/V_{23}|\) and \(|V_{23}|\), the allowed ranges are \(0.8 \lesssim R \lesssim 2.1\) and \(-0.5 \lesssim \cos \delta \lesssim 0.8\), while \(-0.1 \lesssim \cos \delta \lesssim 0.3\) for the SM of \(R = 1\). The ratio \(R\) cannot be much larger than 2, which rules out some regions in the SSM parameter space. In particular, the existence of a very light \(t\)-squark becomes unlikely\textsuperscript{38}. Within the possible ranges for \(|V_{13}/V_{23}|\) and \(|V_{23}|\) taking into account the experimental uncertainties, the ratio \(R\) can be at most \(R \sim 3\).

The value of \(\cos \delta\) in the SSM could be larger than that allowed in the SM, as seen from Table III. For instance, in case (a) the range \(0.3 \lesssim \cos \delta \lesssim 0.8\) is allowed only in the SSM. In near future experiments, \(CP\) asymmetries in \(B^0\)-meson decays and amount of \(B^0_s-\bar{B}^0_s\) mixing will be measured, which depend on \(\cos \delta\). It is possible that these physical quantities have values outside the ranges predicted by the SM.
Table III. The allowed range of \( \cos \delta \): (a) \(|V_{13}/V_{23}| = 0.08, |V_{23}| = 0.04\), (b) \(|V_{13}/V_{23}| = 0.08, |V_{23}| = 0.044\), (c) \(|V_{13}/V_{23}| = 0.1, |V_{23}| = 0.04\).

| \( R \) | 1       | 1.5     | 2       | 2.5     |
|---------|---------|---------|---------|---------|
| (a)     | (0.09, 0.29) | (0.46, 0.64) | (0.74, 0.77) |         |
| (b)     | (0.20, 0.65) | (0.66, 0.82) |         |         |
| (c)     | (0.01, 0.51) | (0.45, 0.73) | (0.68, 0.82) | (0.81, 0.88) |

The \( CP \) asymmetries enable to measure the angles of the unitarity triangle given by

\[
\phi_1 = \text{arg} \left( -\frac{V_{21}V_{23}^*}{V_{31}V_{33}} \right), \quad \phi_2 = \text{arg} \left( -\frac{V_{31}V_{32}^*}{V_{11}V_{12}} \right), \quad \phi_3 = \text{arg} \left( -\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*} \right).
\]

For instance, the decays \( B_d^0 \to \psi K_s, B_d^0 \to \pi^+\pi^- \), and \( B_s^0 \to \rho K_S \) can be used to determine \( \sin 2\phi_1, \sin 2\phi_2 \), and \( \sin 2\phi_3 \), respectively. These asymmetries are expressed in terms of \( \cos \delta \) and \( r = \cot \theta_{12}(\sin \theta_{13}/\sin \theta_{23}) \), to an excellent accuracy:

\[
\sin 2\phi_1 = \frac{2r \sin \delta (1 - r \cos \delta)}{1 - 2r \cos \delta + r^2}, \quad \sin 2\phi_2 = \frac{2 \sin \delta (r - \cos \delta)}{1 - 2r \cos \delta + r^2}, \quad \sin 2\phi_3 = \sin 2\delta.
\]

The values of \( \sin 2\phi_1 \) and \( \sin 2\phi_2 \) only depend on \( \cos \delta \) and \( r \), while \( \phi_3 = \delta \). In most of the plausible ranges for \( \cos \delta \) inferred from the analyses of \( x_d \) and \( \epsilon \), the values of \( \sin 2\phi_2 \) and \( \sin 2\phi_3 \) monotonously change between \(-1\) and \( 1 \), while \( \sin 2\phi_1 \) does not vary much with \( \cos \delta \). The dependence on \(|V_{13}/V_{23}|\) is weak for both \( \sin 2\phi_1 \) and \( \sin 2\phi_2 \). These show the reasons why, in the SM, the prediction of \( \sin 2\phi_1 \) has been made more specifically than those of \( \sin 2\phi_2 \) and \( \sin 2\phi_3 \). Taking into account the present constraints on \( \cos \delta \) and \(|V_{13}/V_{23}|\), the value of \( \sin 2\phi_1 \) should satisfy \( 0.4 \lesssim \sin 2\phi_1 \lesssim 0.8 \), whereas \( \sin 2\phi_2 \) and \( \sin 2\phi_3 \) can have any values from \(-1\) to \( 1 \).

The mixing parameter \( x_s \) for \( B_s^0/\bar{B}_s^0 \) mixing is given by an equation analogous to Eq. (3.1). The ratio of \( x_s \) to \( x_d \) becomes

\[
\frac{x_s}{x_d} = \frac{|V_{32}|^2}{|V_{31}|^2},
\]

where we have neglected the small differences between \( B_d^0 \) and \( B_s^0 \) caused by the SU(3)\text{\textsubscript{flavor}} breaking. This ratio is expressed in terms of \( \cos \delta \) and \( r \) as

\[
\frac{x_s}{x_d} = \frac{\cot^2 \theta_{12} + 2r \cos \delta}{1 - 2r \cos \delta + r^2}.
\]

As \( \cos \delta \) increases, \( x_s/x_d \) monotonously increases. The present constraints on \( \cos \delta \) and \(|V_{13}/V_{23}|\) give \( 10 \lesssim x_s/x_d \lesssim 56 \). It is worth emphasizing that the three \( CP \) asymmetries and the ratio \( x_s/x_d \) only depend on \( \cos \delta \) and \(|V_{13}/V_{23}|\).
Table IV. The ranges of $\sin 2\phi_1$, $\sin 2\phi_2$, $\sin 2\phi_3$, and $x_s/x_d$ predicted by the SSM and the SM:

(a) $|V_{13}/V_{23}| = 0.08$, $|V_{23}| = 0.04$, (b) $|V_{13}/V_{23}| = 0.08$, $|V_{23}| = 0.044$, (c) $|V_{13}/V_{23}| = 0.1$, $|V_{23}| = 0.04$.

|       | $\sin 2\phi_1$   | $\sin 2\phi_2$   | $\sin 2\phi_3$   | $x_s/x_d$   |
|-------|-------------------|-------------------|-------------------|--------------|
| (a) SSM | (0.55, 0.66)       | (-0.96, 0.74)     | (-0.18, 1.00)     | (17, 36)     |
|       | (0.61, 0.66)       | (0.15, 0.74)      | (-0.18, 0.54)     | (17, 21)     |
| (b) SSM | (0.46, 0.66)       | (-1.00, 0.31)     | (0.39, 1.00)      | (20, 40)     |
|       | (0.63, 0.66)       | (-0.65, 0.31)     | (0.39, 0.98)      | (20, 30)     |
| (c) SSM | (0.56, 0.80)       | (-1.00, 0.73)     | (0.02, 1.00)      | (17, 53)     |
|       | (0.74, 0.80)       | (-0.15, 0.73)     | (0.02, 0.88)      | (17, 27)     |
| SM    |                   |                   |                   |              |

In Table IV we give the predicted values of $\sin 2\phi_i$ and $x_s/x_d$ in the SSM and in the SM for $(|V_{13}/V_{23}|, |V_{23}|) = (0.08, 0.04)$ (a), (0.08, 0.044) (b), (0.1, 0.04) (c). In each case there are wide ranges of $\sin 2\phi_2$, $\sin 2\phi_3$, and/or $x_s/x_d$ which are possible only in the SSM. If the experimental values are found in these ranges, then this would indicate that $R > 1$, with significant new contributions to $B^0-\bar{B}^0$ and $K^0-\bar{K}^0$ mixings arising from the SSM. For case (a) the possible experimental results $\sin 2\phi_2 < 0$, $\sin 2\phi_3 \sim 1$, and $x_s/x_d \sim 30$ suggest the SSM effects. Note, however, that the predicted ranges vary with the values of $|V_{13}/V_{23}|$ and $|V_{23}|$. More precise measurements for these quantities are necessary to make predictions definitely.

§5. Summary

In the SSM there exist several new interactions which can induce FCNC processes. We have discussed their effects on $B^0_d-\bar{B}^0_d$ and $K^0-\bar{K}^0$ mixings. These mixings receive contributions from box diagrams in which charginos and up-type squarks, or charged Higgs bosons and up-type quarks are exchanged. We have calculated the ratio $R$ of the SSM contribution to the contribution in the SM. The new SSM contributions interfere constructively with the standard $W$-boson contribution. The ratio $R$ is sizably larger than unity, if $\tan \beta$ has a value around unity and a chargino, a $t$-squark, and/or a charged Higgs boson are not much heavier than 100 GeV.

The enhanced SSM contributions to $B^0_d-\bar{B}^0_d$ and $K^0-\bar{K}^0$ mixings make the $CP$-violating phase $\delta$ of the CKM matrix have a value different from the SM prediction. We have discussed the ranges of $\cos \delta$ and $R$ which are derived from $x_d$ and $\epsilon$. The present uncertainties in $|V_{13}/V_{23}|$, $|V_{23}|$, $f_{B_d}\sqrt{B_{B_d}}$, and $B_K$ are still large, and the allowed ranges for $\cos \delta$ and
$R$ vary with the values of those quantities. However, if the present experimental central values for $|V_{13}/V_{23}|$ and $|V_{23}|$ are in the vicinities of actual values, $\cos\delta$ and $R$ should lie in the ranges $(-0.1, 0.8)$ and $(1.0, 2.1)$, respectively, while a constraint $-0.1 \lesssim \cos\delta \lesssim 0.3$ is obtained for the SM, which corresponds to $R = 1$. This $CP$-violating phase $\delta$ can be probed by $CP$ asymmetries in $B^0$-meson decays and amount of $B_s^0-\bar{B}_s^0$ mixing. We have shown the possibility that the measurements of $\sin 2\phi_1$, $\sin 2\phi_2$, $\sin 2\phi_3$, and $x_s/x_d$ disclose values of $\cos\delta$ and $R$ which are not allowed in the SM, thereby implicating the existence of supersymmetry.

The SSM interactions mediated by the charginos and the charged Higgs bosons could also sizably contribute to radiative $B$-meson decay\[.\] The chargino contribution interferes with the standard $W$-boson contribution either constructively or destructively depending on the parameter values, while the charged Higgs boson contribution interferes constructively. Compared with the SM prediction, the branching ratio of $B \to X_s\gamma$ becomes either enhanced or reduced, if $\tan\beta$ is much larger than unity or the relevant particles are not heavy. Radiative $B$-meson decay therefore provides information on the SSM complimentarily to $B^0-\bar{B}^0$ and $K^0-\bar{K}^0$ mixings.

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