Effects of Geometric Parameters and Axial Tension on Vibration Characteristics

Bohan Wang, Li Cheng, Junliang Ding, Xuechun Bao, Changkai Wang and Dongchun Li
Aeronautic Engineering College, Airforce Engineering University, Xi’an 710038, China
Email: 2215114116@qq.com, cheng_qiaochu@foxmail.com

Abstract. The vibration frequency of the aero-engine blade is affected by its own mass characteristics and steady-state centrifugal force, which can be used to adjust the blade resonance frequency to avoid the exciting source and prolong the fatigue life of the blade, and can also be used to approach the exciting source frequency and guide the design of ultrasonic fatigue specimens. Through the combination of mechanical analysis and finite element simulation, the influence mechanism of geometric parameters and axial tension on modal frequency is analyzed from the point of view of vibration and energy, the influence law is obtained, and the optimal design of two kinds of three-point bending specimens is completed. The resonant frequency error calculated by finite element method is controlled within ±5Hz, and the frequency is 20.03 kHz verified by ultrasonic fatigue test, which meets the accuracy requirements, which provides a reliable theoretical support for the design of simulated blade ultrasonic fatigue specimens.

1. Introduction
Aero-engine blades often produce high-frequency resonance or forced vibration because of their wide aerodynamic excitation frequency, changeable working conditions and harsh working environment, which eventually lead to blade fatigue failure. Among the compressor blade failure cases from 1960s to December 2010 in China, the fatigue failure cases caused by resonance account for 53% [1]. In reality, it is often used to change the cross-sectional area of the blade, remove the local material of the blade body or tip, design reinforcement ribs [2] and other methods to adjust the resonant frequency of the blade to avoid the source of vibration, so as to reduce the vibration damage of the blade and prolong the fatigue life. But even so, the fatigue fracture problem of engine blade with ultra-high cycle (more than 10^7 cycles) is still very acute.

At present, the study of ultra-high cycle fatigue mainly includes traditional fatigue methods (rotary bending, electro-hydraulic servo, electromagnetic resonance and electromagnetic shaking table, etc.) and ultrasonic fatigue test methods. Ultrasonic fatigue test is an accelerated fatigue test method based on resonance principle, and the typical test frequency is 20kHz. The advantage of this method is that it is fast and can greatly shorten the test time, but the disadvantage is that the high frequency of the test leads to the serious temperature rise of the sample and the complexity of the sample design. In the more than 30 years since the mid-1980s, ultrasonic fatigue testing technology has become more and more mature, it has developed from symmetrical axial tension-compression loading in a single room temperature air environment to a variety of loading forms such as variable stress ratio, three-point bending, vibration bending, torsion and fretting fatigue in a variety of complex environments [3-12].
As the ultra-high cycle fatigue mechanism of the material is not completely clear, the ultrasonic fatigue test has not yet formed a unified test standard [13-15].

Aiming at the problem of ultra-high cycle fatigue of aero-engine blade bending vibration and the current situation that titanium alloy is widely used in engine blade manufacturing, some scholars have carried out related research. In 2012, Li and Gao [16][17] set up a cantilever beam bending vibration ultrasonic fatigue test system with excitation frequency of 20kHz. The simulated blade specimens were designed and the ultra-high cycle bending vibration fatigue properties of TC17 titanium alloy were studied. In 2019, based on the axial tension-compression ultrasonic fatigue machine, Lu [18] built a three-point bending ultrasonic fatigue test system for aeronautical TC4 titanium alloy, and successfully carried out ultrasonic bending fatigue test at room temperature. However, from the currently published research results, few scholars have studied the ultra-high cycle fatigue performance of engine blades under real load mode (bending vibration with steady-state centrifugal force). This aspect is subject to the single function of the experimental equipment. On the other hand, it is due to the lack of theoretical support and form innovation in the design of bending vibration samples. The necessary and important part of the ultrasonic fatigue test system is the design of the ultrasonic fatigue test piece. The three-point bending test system provides the lateral vibration load to the test piece. The lateral distribution of vibration displacement and stress along the test piece is affected by the geometric parameters and axial tension. The design of ultrasonic fatigue specimens must meet the resonance conditions of the system [19][20].

2. Mechanical Analysis of Effects of Geometric Parameters and Axial Tension on Vibration Characteristics

2.1. Effects of Geometric Parameters on Vibration Characteristics

The design of the three-point bending specimen is carried out by using the undamped vibration theory of the Euler beam with equal cross-section. At this time, the specimen can be regarded as a simply supported beam with a moving fulcrum, and the difference lies in the different boundary conditions between the two. The ultrasonic fatigue specimen model of three-point bending is established, as shown in Fig.1(a), the x-axis direction is recorded as the axial direction and the y-axis direction as the transverse direction, and the corners are chamfered at the two corners to eliminate the stress concentration. The force analysis of the \( dx \) micro-element is shown in Fig.1(b), where \( Q(x,t) \) is the shear force, \( M(x,t) \) is the bending moment, \( w(x,t) \) is the displacement, \( q(x,t) \) is the external force acting on the unit length of the specimen.

![Specimen shape](image1)

![dx micro-element stress analysis](image2)

**Figure 1.** Specimen shape and micro-element stress analysis

By establishing the equilibrium equation of the force and moment of the microelement in the transverse direction and ignoring the higher power term of \( dx \), the following results can be obtained:

\[
\begin{align*}
\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + \frac{\partial Q(x,t)}{\partial x} - q(x,t) &= 0 \\
\frac{\partial M(x,t)}{\partial x} - Q(x,t) &= 0
\end{align*}
\]

(1.1)
Where \( A \) is the cross-sectional area, \( \rho \) is the material density, and \( \rho A \frac{\partial^2 w(x,t)}{\partial t^2} \) is the inertia force acting on the microelement.

The differential equation of the deflection curve can be obtained from the "simple beam theory", and for the free vibration, \( q(x,t) = 0 \). The differential equation of the free vibration of the homogeneous beam with equal cross section can be obtained as follows:

\[
EI_z \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = 0
\]

(1.2)

Where \( I_z \) is the moment of inertia and \( E \) is the modulus of elasticity. From equation 1.2, it is found that the mass characteristic will affect the mode shape of a certain mode, that is, the deflection function \( w \), and its corresponding modal frequency.

The free vibration of the specimen is analyzed by the energy method, and the corresponding elastic potential energy and kinetic energy are expressed as follows:

\[
\begin{align*}
U &= \frac{1}{2} \int EI_z \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dx \\
E_k &= \frac{1}{2} m \dot{w}^2
\end{align*}
\]

(1.3)

It can be seen from the above formula that the kinetic energy and elastic potential energy of the specimen are transformed into each other during free vibration. If the mass characteristic \( m \) changes, taking the decrease of kinetic energy as an example, the degree of decrease of elastic potential energy and kinetic energy is different, and only the change of motion velocity \( \dot{w} \) can maintain the conservation of energy. this means that the frequency of free vibration varies with the characteristics of mass. Unfortunately, it is difficult to obtain universal guidance directly from equations 1.2 and 1.3.

2.2. Effect of Axial Tension on Vibration Characteristics

When there is no axial tension, due to the simple harmonic motion of the specimen, the \( w(x,t) = W(x) \sin \omega t \) carries out variable separation, then the formula 1.2 can be written as follows:

\[
\frac{d^4 W(x)}{dx^4} - k^4 W(x) = 0
\]

(1.4)

Where \( k^4 = \frac{\rho A \omega^2}{EI_z} \), \( \omega = 2\pi f \). Because there is no analytical solution to the frequency equation of the three-point bending specimen, taking a simply supported beam as an example [21-23], the displacement and bending moment at both ends are zero, and it is necessary to ensure that \( W(x) \) has a non-zero solution, from which the characteristic root of the frequency equation can be obtained as follows:

\[
\omega_n = k_n^2 \sqrt{\frac{EI_z}{\rho A}} = \left( \frac{n\pi}{l} \right)^2 \sqrt{\frac{EI_z}{\rho A}} \quad (n = 1, 2, \ldots)
\]

(1.5)

When the beam is subjected to axial tension, the axial force and lateral displacement form a moment, the moment balance equation in formula 1.1 changes, and the transverse vibration equation under axial tensile load becomes:

\[
EI_z \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} - 4F \frac{\partial^2 w(x,t)}{\partial x^2} = 0
\]

(1.6)
By separating the variables according to the simple harmonic motion and substituting the boundary conditions, the characteristic roots of the frequency equation under axial tension can be obtained as follows:

\[ \omega_n = \frac{\left(\frac{n\pi}{l}\right)^2}{\rho A} \sqrt{\frac{EI}{\rho A} \left(1 - \frac{F_l^2}{EI(n\pi)^2}\right)} \quad (n = 1, 2, \ldots) \] (1.7)

According to equation 1.5 and 1.7, when the axial force is positive (the beam is under compression), the natural frequency decreases, which is equivalent to the stiffness of the beam, and the degree of frequency reduction increases with the increase of pressure. When the pressure increases to \( \pi^2 EI / L^2 \), the first-order vibration frequency of the beam is zero, reaching the unstable state of the simply supported beam; when the axial force is negative (the beam is under tension), the natural frequency will increase, which is equivalent to increasing the stiffness of the beam. From the point of view of energy, the increase of stiffness increases the elastic potential energy in formula 1.3. when the mass characteristic \( m \) is constant, only the increase of motion velocity \( \dot{w} \) can maintain the conservation of energy, which means the increase of free vibration frequency.

From the above analysis, it is known that the steady-state centrifugal force on the engine blade will increase its own vibration frequency, so it is necessary to study the influence of tension on the frequency to ensure that the specimen can meet the resonance requirements.

3. Simulation Analysis of Effects of Geometric Parameters and Axial Tension on Vibration Characteristics

3.1. Simulation Analysis of the Influence Law of Geometric Parameters

Under the condition of 20kHz, the natural vibration mode of the three-point bending specimen is one-dimensional transverse bending vibration [24], and the positions of the two fulcrums are the vibration displacement stationary points of the natural modes of the specimen, then the boundary conditions are:

\[
\begin{align*}
(W''(x))_{x=l} &= 0, (W''(x))_{x=0} = 0 \\
W(-x) &= W(x), (W(x))_{x=l_0} = 0
\end{align*}
\] (2.1)

From the differential equation 1.4 and boundary condition 2.1, the analytical solutions of specimen length \( L \) and span \( L_0 \) can be obtained as follows:

\[
\begin{align*}
L &= 2l = \frac{4.73004}{k} \\
L_0 &= 2l_0 = \frac{2.60948}{k}
\end{align*}
\] (2.2)

The bending moment at the middle section of the specimen is the largest. From the bending normal stress formula \( \sigma = My/I \) of the beam, the maximum vibration bending normal stress at the bottom of the specimen is obtained:

\[ \sigma_{u,\text{max}} = A_yEk^2y \left( \frac{\cos(\theta) - \cosh(\theta)}{\cos(\theta) + \cosh(\theta)} \right) \approx A_0 \cdot C_S \] (2.3)

Where \( y \) is the coordinate of the middle bottom surface and \( C_S \) is the ratio of the vibration bending normal stress to the vibration displacement at the bottom of the specimen, which is called the stress-displacement coefficient in this paper, which is used to calculate the dynamic load in the test.

TC4 titanium alloy is widely used in the manufacture of aero-engine blades and blades, accounting for up to 60% of titanium alloy products [25]. Therefore, the material used in this experiment is \( \phi16 \text{mm} \) aviation grade TC4 bar, and the heat treatment process is 920°C×1h+AC+530°C×4h+AC, and its mechanical properties are shown in Table 1.
Table 1. Mechanical properties parameters of TC4 titanium alloy.

| Property          | Value          |
|-------------------|----------------|
| Density           | 4.38g/cm³      |
| Elastic Modulus   | 112GPa         |
| Poisson's ratio   | 0.33           |
| Tensile strength  | 986MPa         |
| Yield Strength    | 909MPa         |

Figure 2. Stress and displacement cloud diagram of first-order bending vibration under theoretically calculated dimensions

Given the design width $b = 10mm$, thickness $h = 4mm$ and chamfer $R = 1mm$, the length $L$ and span $L_0$ of the specimen are determined according to formula 2.2. According to the mechanical property parameters in Table 1, the material properties are set up, the finite element model of the specimen is established and the modal analysis is carried out. As shown in Fig. 2, the resonant frequency of the bending vibration of the specimen is 19074Hz, which is lower than the theoretical calculation frequency. This is because the aspect ratio of the specimen designed in this paper is less than 10, which can not be regarded as a slender beam, and there is an error between the actual calculation results and the theoretical values. In order to complete the preparation of the specimen, it is also necessary to consider the influence of the machining error, so this section obtains the influence law of the geometric parameters of the specimen on the vibration characteristics through simulation calculation, modifies the theoretical value and determines the appropriate machining accuracy, as shown in Fig. 3.

Fig. 3(a) shows that the resonant frequency and stress-displacement coefficient vary linearly with the length of the specimen, and the calculated value of the finite element method is lower than the theoretical value. Using the least square method for linear fitting, it is found that when the length decreases by 0.1mm, the resonant frequency increases by about 121.67Hz and the stress displacement coefficient increases by about 0.04MPa/μm. In order to ensure the test accuracy, the resonant frequency is controlled in the range of 20 ±0.1kHz, so the machining error of determining the length direction is ±0.05mm.

As shown in Fig. 3(b), the width change has little effect on the resonant frequency and stress-displacement coefficient. For every 1mm increase in the width of the specimen, the resonant frequency only increases by more than 20 hertz, so the machining error in the width direction can be ignored.

Fig. 3(c) shows that the thickness change has a significant effect on the resonant frequency and stress displacement coefficient. After linear fitting, it is found that when the specimen thickness increases by 0.1mm, the resonant frequency increases by 453.54Hz, and the error between the finite element calculation value and the theoretical value tends to increase; the corresponding stress displacement coefficient increment is about 0.16MPa 4.3~4.4mm, and the thickness is the zero error between the actual calculated value and the theoretical value. In order to control the resonant frequency in the range of 20 ±0.1kHz, the machining error in the thickness direction is ±0.02mm.

The finite element calculation values of resonant frequency and stress-displacement coefficient decrease with the increase of chamfer radius, while the theoretical values show the opposite trend. When the chamfer radius is 0, the error between the two is minimized, as shown in Fig. 3(d).
According to Fig.3, the average error between the theoretical calculation value and the finite element calculation value is used to guide the size correction of the specimen, and according to the difference of the influence degree of the four geometric parameters, the appropriate machining accuracy is determined (to ensure the test accuracy, the maximum machining error is less than 0.1mm). The results are shown in Table 2.

**Table 2.** Machining accuracy of four geometric parameters.

| Parameter name       | Length  | Width   | Thickness | Chamfer radius |
|----------------------|---------|---------|-----------|----------------|
| Error (%)            | 5.7455  | 5.7546  | 5.8858    | 6.8135         |
| Precision (mm)       | ±0.05   | ±0.10   | ±0.02     | ±0.10          |

### 3.2. Simulation Analysis of the Influence Law of Axial Tension

The vibration mode function and frequency equation of the equal section beam with fixed ends are the same as those with free ends [26], so the approximate size of the specimen under tensile condition can be obtained according to the calculation formula of ordinary three-point bending specimen in Section 3.1, as shown in Fig.4(a). Based on the modal analysis of the boundary conditions and tensile load on the specimen, it is found that there is stress concentration at the junction of the clamping section and the test section, which can easily lead to fatigue fracture at the junction of the specimen. For this reason, the specimen is set to the form of variable cross-section as shown in Fig.4(b), so that the stress distribution of the test section is in a plane stress state, and the fatigue fracture is easy to occur from the intermediate test section.
Fig. 5 is the first-order bending vibration mode of the variable cross-section beam specimen. It can be seen from the diagram that the resonant frequency is 20079Hz, the stress concentration at the junction of the clamping section and the test section is effectively reduced, and the stress distribution on the lower surface of the middle section of the specimen accords with the plane stress state. It is known from Section 2.2 that the axial tension will increase the natural frequency of the specimen, which may cause the resonant frequency of the specimen to exceed the working range of the equipment. Therefore, the influence of axial tension on resonant frequency is obtained by finite element method.

With reference to the calculation results of the size of the ordinary three-point bending specimen, it is preliminarily selected that the length of the middle section of the specimen under the tensile condition is 9.2mm, and the relationship between the resonant frequency and the tension is linearly increased, as shown in Fig.6(a). The resonant frequency increases linearly with the axial tension, which is consistent with the theoretical analysis of Section 1.2. When the tension is 280MPa, the resonant frequency is nearly 400Hz higher than that without tension, which reaches the stop condition of the ultrasonic fatigue testing machine (frequency fluctuation range >300Hz), which indicates that ignoring the influence of axial tension will lead to the failure of the test. The stress-displacement coefficient decreases linearly with the axial tensile force, when the tensile force increases by 50 MPa, the stress-displacement coefficient decreases by about 0.05MPa/um. For the variable cross-section specimens which are lack of theoretical calculation, the accurate stress-displacement coefficient can be obtained according to the law shown in Fig.6(a) curve, and the loading accuracy of the test can be improved.
Combined with the content of Section 2.2, it can be seen that the resonant frequency and the stress-displacement coefficient are affected by both geometric parameters and axial tension. According to the results shown in Fig.6(a), the axial tension is selected to be equal to 200MPa, and the influence of the length of the middle section of the specimen on the vibration characteristics is obtained, as shown in Fig.6(b). The resonant frequency and the stress-displacement coefficient decrease linearly with the increase of the length of the middle section. When the length increases by 0.1mm, the frequency decreases by 131.23Hz and the stress-displacement coefficient decreases by 0.0214MPa/um. That is, through the coupling influence of the axial tension and the length of the intermediate section, the optimal design of the tensile specimen with variable cross-section can be realized.

4. Design Scheme and Experimental Verification of Specimen

4.1. Design Schemes of Two kinds of Three-point Bending Specimens

It is known from Section 3.2 that the change of the thickness of the non-tensile specimen with equal cross section has a significant effect on the resonant frequency, while the change of width has no effect on it. In order to control the test accuracy and improve the design efficiency, the optimization parameters are selected as length and chamfer radius. The four alternative schemes shown in Table 3 are drawn up. By comparison, it is found that when the parameter combination is 31.3/10/4/0.7, the resonant frequency deviation is only 4Hz, and the stress-displacement coefficient is relatively large, which is determined as the optimal scheme. The alternative design scheme of tension specimen with variable cross-section is shown in Table 4. When the length of the intermediate section (Record as L) is 9.2mm and the axial tension is 240~250MPa, the frequency deviation can be kept within ±10Hz. When the length of the intermediate section is 9.1mm and the axial tension is 150MPa, the frequency deviation is only 1Hz, and the stress-displacement coefficient is more than 3MPa/um, so the combination is determined to be the optimal scheme.

Table 3. Design alternatives without tensile specimens.

| Parameter combination (mm) | Frequency (Hz) | Stress displacement coefficient (MPa/um) |
|----------------------------|----------------|----------------------------------------|
| 31.2/10/4/1.0              | 20047          | 6.2330                                 |
| 31.2/10/4/1.2              | 20003          | 6.3117                                 |
| 31.3/10/4/0.5              | 20040          | 6.4395                                 |
| 31.3/10/4/0.7              | 19996          | 6.3364                                 |
Table 4. Design alternatives with tensile specimens.

| L (mm) | F(MPa) | Frequency (Hz) | Stress displacement coefficient (MPa/um) |
|--------|--------|----------------|----------------------------------------|
| 9.1    | 140    | 20047          | 6.2330                                 |
| 9.1    | 150    | 20003          | 6.3117                                 |
| 9.2    | 240    | 20040          | 6.4395                                 |
| 9.2    | 250    | 19996          | 6.3364                                 |

According to the selected optimal scheme, combined with the machining errors of the geometric parameters given in Table 2, the final design schemes of two kinds of three-point bending specimens are determined, as shown in Fig.7.

Figure 7. Design schemes of two kinds of three-point bending specimens

4.2. Experimental Verification of Specimen Design Scheme
The ultra-high cycle fatigue test of three-point bending was carried out on HC-DF2030GD ultrasonic test system. The temperature of the specimen will rise obviously under the influence of the thermal effect of high-frequency vibration, especially during the test, the cyclic bending deformation of the specimen will lead to a small reciprocating relative slip between the specimen and the fulcrum, and this kind of contact friction heat generation is more intense than vibration heat production. In order to eliminate the effect of temperature on fatigue performance, it is necessary to carry out necessary forced cooling to ensure that ablation does not occur at the fulcrum.

In the experiment, it is found that the resonant frequency is 20.03 kHz at the beginning, and the resonant frequency decreases gradually with the test, but it can still maintain the resonant state. According to the analysis, along with the test, there are cracks on the surface or inside of the specimen, and gradually expand with the increase of the load cycle, resulting in the gradual degradation of the stiffness of the specimen and the gradual decrease of the resonant frequency. When the frequency decreases by more than 300Hz, the fatigue fracture of the specimen occurs and the test system stops working immediately.

Fig. 8 shows the macroscopic fracture morphology of two kinds of three-point bending specimens, and the fracture positions of the two specimens are at the middle section, which is consistent with the maximum stress distribution of the specimen. From Fig.8(b), it can be seen that the fatigue fracture surface is divided into three regions from the bottom to the upper surface of the specimen: fatigue source zone, fatigue crack propagation zone and instantaneous fracture zone. The difference between the three regions is obvious, which can be clearly distinguished by the naked eye. The test results are consistent with the expectation, and the rationality and accuracy of the specimen design are verified.
5. Summary
In this paper, the influence of geometric parameters and axial tension on vibration characteristics is studied, and the undamped vibration theory of equal cross-section Euler beam is used for mechanical analysis. The influence mechanism of geometric parameters and axial tension on vibration characteristics is analyzed from the point of view of vibration and energy, and the feasibility of specimen design method is verified.

The influence rules of geometric parameters and axial tension are obtained. The geometric parameters simulate the quality characteristics, in which the influence of the thickness change is the greatest, the influence of the width change can be ignored, and the influence of the length and chamfer change on the resonant frequency is moderate. According to the influence law of four kinds of geometric parameters, the average error and machining accuracy of finite element calculation and theoretical calculation are determined to complete the design and preparation of the specimen. The axial tension simulates the steady-state centrifugal force, which is equivalent to increasing the stiffness of the beam, and the resonant frequency increases with the increase of the axial tension. When the tension is 280MPa, the resonant frequency increases by nearly 400Hz compared with that without tension. The coupling effect of the axial tension and the length of the middle section can realize the optimal design of the tensile specimen with variable cross-section.

The optimal design and experimental verification of two kinds of three-point bending specimens are completed. Through the combination of mechanical analysis and finite element calculation, the resonant frequency error of the two specimens is controlled at ±5Hz, and the frequency is 20.03kHz, which fully meets the precision requirements. The two kinds of specimens all break from the middle section, which is consistent with the expected results. The design method lays a foundation for the ultra-high cycle fatigue test of engine blade materials in the future.

6. References
[1] LIU Q X. Manufacturing technology and failure analysis of titanium alloy blade for aeroengine [M]. Beijing: Aviation Industry Press, 2018:972.
[2] SONG Z H. Aeroengine Reliability and Fault Suppression Engineering [M]. Beijing: Beijing University of Aeronautics and Astronautics Press, 2002: 114.
[3] Geathers J, Torbet C J, Jones J W, et al. Investigating environmental effects on small fatigue crack growth in Ti6242S using combined ultrasonic fatigue and scanning electron microscopy[J]. International Journal of Fatigue, 2015, 70:154-162.
[4] Reo Kasahara, Masato Nishikawa, Yoshinobu Shimamura, et al. Evaluation of Very High Cycle Fatigue Properties of β-Titanium Alloy by Using an Ultrasonic Tensile-Compressive Fatigue Testing Machine. Key Engineering Materials, 2016, 725:366-371.

[5] Xiaolong Liu, Chengqi Sun, Youshi Hong. Faceted crack initiation characteristics for high-cycle and very-high-cycle fatigue of a titanium alloy under different stress ratios [J]. International Journal of Fatigue, 2016, 92:434-441.

[6] LIU Y J, HE C, YANG S B, et al. Study on an Ultrasonic Bending Fatigue Testing Method for Thin Sheet [J]. Journal of Sichuan University (Engineering Science), 2012, 44(S2):154-157.

[7] Nikitin A, Bathias C, Palin-Luc T. A new piezoelectric fatigue testing machine in pure torsion for ultra-sonic gigacycle fatigue tests: application to forged and extruded titanium alloys [J]. FFEMS 2015, 38(11): 1294-1304.

[8] JIAO S B, CHENG L, CHEN X, et al. On the Very High Cycle Fatigue and Life Prediction Model of Ti-6Al-4V Titanium Alloy Subjected to Vibration Bending [J]. Journal of Experimental Mechanics, 2016, 31(06):730-740.

[9] Baohua Nie, Zihua Zhao, Yongzhong Ouyang, et al. Effect of Low Cycle Fatigue Predamage on Very High Cycle Fatigue Behavior of TC21 Titanium Alloy[J]. materials, 2017, 10(1384).

[10] Xiaojian Cao, Luopeng Xu, Chong Wang, et al. Effect of shot peening on the long life fatigue properties of Ti6Al4V with different heat treatment [J]. Key Engineering Materials, 2015, 664: 81-86.

[11] Xuechun Bao, Li Cheng, Junliang Ding, et al. The effect of microstructure and axial tension on three-point bending fatigue behavior of TC4 in high cycle and very high cycle regimes[J]. Materials 2020, 13(68).

[12] Jiukai Li, Yongjie Liu Qingyuan Wang, et al. Effect of temperature and loading frequency on the fatigue behavior of Ti-17 [J]. Engineering Fracture Mechanics, 2016, 664:131-139.

[13] Hong Y S, Sun C Q, Liu X X. mechanism and mod-el of ultrahigh cycle fatigue of alloy materials [J]. Advances in mechanics, 2018, 48:201801.

[14] H. MAYER. Recent developments in ultrasonic fatigue[J]. Fatigue & Fracture of Engineering Materials & Structures, 2016, 39:3-29.

[15] PENG Wen-jie, XUE Huan, GE rui, et al. The influential factors on very high cycle fatigue testing results[J]. MATEC Web of Conferences, 2018, 165.

[16] Li Q T, LIU Q C, SHEN J S, et al. Fatigue test on ultrahigh cycle bending of titanium alloy [J]. Journal of Aerodynamics, 2012, 27(3):617-622.

[17] Gao C, Cheng L, Peng H, et al. Investigation of ultra-high cycle fatigue behavior of TC17 alloy at a frequency of 20kHz[J]. Journal of Aerospace Power, 2012, 027(004):811-816.

[18] Lu K J, Cheng L, Chen X, et al. Very-High-Cycle-Fatigue Properties of TC4 Titanium Alloy Under Three-point Bending[J]. Rare Metal Materials and Engineering, 2019, 48(10):3175-3182.

[19] Roth L D. Ultrasonic fatigue testing [J]. Metal Handbook, Mechanical testine. ASTM, Ohio USA, 1985, 8(9): 24-25.

[20] Manson. W. P. Piezoelectric crystals and their appli-cation in ultrasonic [M], New York, Van Nostrand, 1950.

[21] LIU J B, DU X L. Structural Dynamics [M]. Beijing: Machinery Industry Press, 2005.1:160-167 (reprinted in 2019.1).

[22] Li D X. Advanced Structural Dynamics (second edition) [M]. Beijing: Science Press, 2010:319-336.

[23] SHENG H Y. Structural Dynamics (second edition) [M]. Hefei: Hefei University of Technology Press, 2005.3:203-212.

[24] XUE H X. Study on ultrahigh cycle fatigue properties of materials under ultrasonic vibration [D]. Northwest University of Technology, 2006, 3:27-32.

[25] HUANG X, LI Z X, GAO F, et al. Research progress of new high-temperature titanium alloy for aeroengine[J]. Aeronautical Manufacturing Technology, 2014, 451(7):70-75.

[26] Singiriesu S Rao. Mechanical vibration[M]. Li X Y, YANG LC, translated. Beijing: Tsinghua University Press, 2016:542-546.