Influence of inertial links on the distribution of vibration amplitudes of points of the working body of a technological vibrating

A V Eliseev¹, N K Kuznetsov¹, S V Eliseev², Q T Vuong²

¹ Irkutsk National Research Technical University, 83, Lermontov St., Irkutsk, 664074, Russia
² Irkutsk State Transport University, 15, Chernyshevskiy St., Irkutsk, 664074, Russia

E-mail: eavsh@ya.ru

Abstract. Features of problems of dynamics and formation of dynamic states of working bodies of vibrating technological machines are considered. The purpose of the work is to develop a method for constructing mathematical models of vibration machines that provide the ability to evaluate, control and adjust the forms of movement of working bodies by introducing additional links into the structure of the system, forming a closed internal contour. Methods of structural mathematical modeling are used in the construction of original structural formations of their typical elements. The direction of constructive and technical solutions in the implementation of approaches to configure and adjust the structure of the vibration field of the working body is proposed. The method is implemented using the analytical apparatus of the automatic control theory. The features of dynamic states formation based on the application of transfer functions of structural mathematical models of mechanical oscillatory systems are studied. The use of transfer functions of interpartial connections for the formation of vibration fields of working bodies is proposed. Analytical relations are obtained that determine the possibility of implementing specific dynamic modes, including the decoupling of movements by individual coordinates, taking into account the features of the connectivity of movements by characteristic points of the working body, the manifestation of oscillation nodes or modes of dynamic damping. Features of using structural mathematical modeling methods in problems of dynamics of mechanical oscillatory systems with additional connections are discussed. The work is aimed at specialists in the field of theoretical and applied mechanics, machine science and mechanical engineering.

1. Introduction

The use of additional connections in the structures of mechanical oscillatory systems, considered as calculation schemes of technical objects for various purposes, is a fairly developed direction in solving problems of machine dynamics. In many cases, structural and technical forms of additional connections are implemented as an introduction to the structure of mechanical oscillating systems of various mechanisms: gear, lever, including screw non-self-braking devices. The presence of mechanisms as specific mechanical chains in the structure of oscillatory systems allows us to solve a fairly wide range of dynamics problems, in which changes in the dynamic properties of the system, as a whole, are achieved due to dynamic effects that occur in the interactions of the system elements[1-3]. The article develops a method of structural mathematical modeling in applications to solving...
problems of evaluating the features of dynamic interactions when connecting devices for conversion of movement (DCM) with mass-inertia and elastic typical elements of a mechanical oscillatory system, which is a design scheme of a technological machine[4].

2. The features of technical object. Problem statement.

The technical object whose design scheme is shown in figure 1 is a mechanical oscillatory system with two degrees of freedom. A special feature of the approach is to take into account the connection features of typical elements of a mechanical system [5,6].

![Figure 1. Schematic diagram of a suspension with a device for converting movement (DCM).](image)

An inertial element of mass $m_2$ has a support spring of stiffness $k_3$ and rests in p.(A) on the support surface II (figure 1). in addition, the element $m_2$ rests on an elastic complex element (or a structural formation – a quasi-spring) consisting of a spring of stiffness $k_1$ with contact points $(B)$ and $(B_1)$, respectively, with a pair of elements $k_2$ and L in pp.$(B_1)$, $(B_2)$. At the point $(B_1)$, the mass $m_1$ is concentrated. The second part of the structural block is a parallel connection of elastic elements with stiffness $k_2$ and DCM with an intermediate mass $m_1$. Contact with the inertia element $m_2$ occurs in p.$(B_2)$. The position of the inertial element $m_2$ is determined by the $y_2$ coordinate, and the intermediate mass $m_1$ is determined by the $y_1$ coordinate, respectively. It is assumed that the system has linear properties and makes small fluctuations relative to the static equilibrium position. The movement of the reference surfaces I and II (figure 1) is determined by the known harmonic functions $z_1(t)$ and $z_2(t)$ of the same frequency; there is a functional relationship between the external perturbations

$$\bar{z}_2 = \alpha \bar{z}_1,$$

where $\alpha$ is the constant coupling coefficient of external disturbances.

The reduced mass of the control unit, designated as L in Figure B.1, is determined by the expression

$$L = \frac{J}{r_{av}^2 \gamma^2},$$

where $J$ is the moment of inertia of the flywheel nut, $r_{av}$ is the average radius of the screw, and $\gamma$ is the angle of elevation of the helix.

The position of the system is considered in coordinates $y_1$ and $y_2$ in a fixed basis. To build a mathematical model, we will make expressions for the kinetic and potential energies:
\[ T = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} L (\dot{y}_2 - \dot{y}_1)^2 + \frac{1}{2} m_2 \dot{y}_2^2, \]  
\[ \Pi = \frac{1}{2} k_1 (y_1 - z_1)^2 + \frac{1}{2} k_2 (y_2 - y_1)^2 + \frac{1}{2} k_3 (y_2 - z_2)^2. \]

Using the Lagrange equation of the second kind, and then the Laplace transform under zero initial conditions, we write a system of differential equations in operator form [6]:

\[ \ddot{y}_1 (m_1 + L)p^2 + \ddot{y}_1 (k_1 + k_2) - \ddot{y}_2 (Lp^2 + k_2) = k_1 z_1, \]
\[ \ddot{y}_2 (m_2 + L)p^2 + \ddot{y}_2 (k_2 + k_3) - \ddot{y}_1 (Lp^2 + k_2) = k_3 z_2. \]

The structural mathematical model or block diagram of the automatic control system is shown in figure 2. This structure consists of two partial systems that are connected by inertia-elastic, which makes it possible to "zeroing" the connection at the frequency

\[ a_0^2 = \frac{k_2}{L}. \]

The transfer functions of the system under kinematic perturbation from the base are defined by the expressions

\[ W_1(p) = \frac{\ddot{y}_1}{\ddot{z}_1} = \frac{k_1[(m_2 + L)p^2 + k_2 + k_3]]}{A(p)}, \]
\[ W_2(p) = \frac{\ddot{y}_2}{\ddot{z}_2} = \frac{k_1(k_2 + Lp^2)}{A(p)}, \]
\[ W_3(p) = \frac{\ddot{y}_1}{\ddot{z}_2} = \frac{k_2(k_2 + Lp^2)}{A(p)}, \]
\[ W_4(p) = \frac{\ddot{y}_2}{\ddot{z}_2} = \frac{k_1[(m_1 + L)p^2 + k_1 + k_2]]}{A(p)} \]

where \( A(p) = \frac{[m_1 + Lp^2 + k_1 + k_2]](m_2 + Lp^2 + k_2 + k_3)] - (Lp^2 + k_2)^2}{A(p)} \)

– is the characteristic frequency equation of the system.

The diagram (figure 1) shows that \( m_1 \) is a low-inertia element through which the cascade \((Lp^2 + k_2)\) is connected to the elastic element \( k_1 \) (in this case, the point \( B_1 \) is introduced into consideration).

The task of the study is to evaluate the dynamic features introduced into the mechanical oscillatory system by a device for converting motion, as well as by introducing an intermediate mass-inertia element \( m_1 \), which creates two cascades in the suspension system, sequentially connected to each other.

3. Assessment of the dynamic properties of the system upon addition DCM

Usually, the point \( B_1 \), in which three typical elements are simultaneously in contact with the transfer
functions \( k_1, k_2 \) and \( Lp^2 \), is not considered as a characteristic point. The authors interpret such a nodal point; its position is determined by the coordinate \( \tilde{y}_1 \). The limiting case is the situation when \( m_1 \) tends to 0, which predetermines the approach to assessing the dynamic stiffness of a two-stage series connection of system elements, the value of \( m_1 \) affects the distribution of dynamic bond reactions in pp.\((B)\), \((B_1)\) and \((B_2)\).

Using the structural diagram in Figure 2, we write down the transfer functions, taking into account the connectedness of external influences, defined by the expression (B.1)

\[
W_i^1(p) = \frac{\tilde{y}_i}{z_i} = \frac{k_i[(m_z + L)p^2 + k_z + k_1] + \alpha k_z(Lp^2 + k_z)}{A(p)},
\]

\[
W_i^2(p) = \frac{\tilde{y}_i}{z_i} = \frac{\alpha k_z[(m_1 + L)p^2 + k_z + k_1] + k_z(Lp^2 + k_z)}{A(p)}.
\]

Figures B. 3, a, and b show converted block diagrams with the allocation of elements \( m_1 \) and \( m_2 \) covered by the corresponding negative feedbacks. In the physical sense, the transfer functions of such connections correspond to (or determine) the dynamic rigidity of structural formations.

**Figure 3.** Structural diagrams of the system, expanded relative to the mass-inertia elements \( m_1 \) and \( m_2 \): a-\( m_1 \)-coordinates \( y_1 \); b-\( m_2 \)-coordinates \( y_2 \)

Dynamic reactions at points of contact with the reference surface are defined by the expressions

\[
\left| \bar{R}_{\sup} \right| = \left| \bar{R}_A \right| + \left| \bar{R}_B \right| = k_1 \cdot \tilde{z}_1 \cdot W_2^* (p) + k_1 \cdot \tilde{z}_1 \cdot W_2^* (p) =
\]

\[
= \frac{k_1 \cdot \tilde{z}_1 \cdot \alpha k_z[(m_1 + L)p^2 + k_1 + k_z] + k_z(Lp^2 + k_z) + k_1 \cdot \tilde{z}_1 \cdot k_z[(m_1 + L)p^2 + k_1 + k_z] + \alpha k_z(Lp^2 + k_z)}{A(p)},
\]

or

\[
\tilde{z}_1 \cdot \left[ \alpha k_z^2 (m_1 + L) + k_1^2 (m_z + L) + k_1 k_z (1 + \alpha) \right] p^2 +
\]

\[
\left| \bar{R}_{\sup} \right| = \frac{\alpha k_z^2 (k_1 + k_z) + k_z^2 (k_2 + k_z) + k_1 k_z k_3 (1 + \alpha)}{A(p)}.
\]

The dynamic response \( \left| \bar{R}_{\sup} \right| \) is “zeroing” at the frequency of external influence.
The dynamic response on the $m_2$ «object» is defined through dynamic stiffness, which can be found using structural transformation rules

$$k_{up} = \frac{(Lp^2 + k_2)[(m_1 + L)p^2 + k_1 + k_2] - (Lp^2 + k_2)^2}{(m_1 + L)p^2 + k_1 + k_2}. \quad (18)$$

Thus

$$z_i = \frac{\{\alpha k_3[(m_1 + L)p^2 + k_1 + k_2] + k_1(Lp^2 + k_2)\} \times k_1[(m_1 + L)p^2 + k_1 + k_2] + (Lp^2 + k_2)(m_1p^2 + k_1)}{(m_1 + L)p^2 + k_1 + k_2} A(p). \quad (19)$$

We find the relation of dynamic reactions $|\mathbf{R}_m|$ and $|\mathbf{R}_{sup}|$, using the techniques described in [6]

$$N(\omega) = \frac{|\mathbf{R}_m|}{|\mathbf{R}_{sup}|} = \frac{\{\alpha k_3[(m_1 + L)p^2 + k_1 + k_2] + k_1(Lp^2 + k_2)\} \times k_1[(m_1 + L)p^2 + k_1 + k_2] + (Lp^2 + k_2)(m_1p^2 + k_1)}{\{\alpha k_3[(m_1 + L)p^2 + k_1 + k_2] + k_1(Lp^2 + k_2)\} \times k_1[(m_1 + L)p^2 + k_1 + k_2] + (Lp^2 + k_2)(m_1p^2 + k_1)} A(p). \quad (20)$$

4. Discussion

The ratio of reactions of dynamic connections at characteristic points of the source system can be considered as a parameter that reflects the dynamic state of the system, taking into account the features defined by a set of standard elements, external disturbance conditions, and the choice of parameter values.

In a conventional mechanical oscillatory system with two degrees of freedom ($m_1 \neq 0$, $L = 0$, $\alpha = 0$), the amplitude-frequency response in the coordinate has a mode of dynamic damping of vibrations at the frequency

$$\omega^2_{idyn} = \frac{k_2 + k_3}{m_2}, \quad (21)$$

what follows from (8) for $L = 0$. If $m_1 = 0$, $L = 0$, then the original system is reduced to a system with one degree of freedom; in this case, the coordinate is canceled; the frequency of natural vibrations of the system with one degree of freedom in this case is determined by the expression

$$\omega^2_{nat} = \frac{k_1k_2 + k_2k_3 + k_1k_3}{m_2(k_1 + k_2)}. \quad (22)$$

To assess the influence of parameters of typical elements, a model problem is used, which considers changes in $\alpha$ (0, 0.5, 2); $m_1 = 100$ kg; $m_2 = 1000$ kg; $k_1 = 100$ kN/m; $k_2 = 200$ kN/m; $k_3 = 300$ kN/m; $L$ (0, 100 kg).

In general, the system can display resonant modes at the natural oscillation frequencies determined from the frequency characteristic equation (12). In the transition to the amplitude-frequency characteristics (AFC) of coupling reactions, which is defined by the expression (20), it should be noted
that the following features will be observed in the form of AFC: at $\omega \to \infty$, the ratio $\frac{\bar{R}_{m_1}}{\bar{R}_{sup}}$ will tend to a certain limit.

Frequency characteristics of the bond reaction are shown in figure 4.

**Figure 4.** Frequency characteristics of the bond reaction ratio at $m_1 = 100$ kg, $L = 0$: a - $\alpha = 0, 0.5, 1$; b - $\alpha = 0, -0.5, -1$
At the frequency $\omega_{dyn2R}$ the graph, the relations of bond reactions as previously mentioned intersect with the abscissus axis (p. (2) in figure 4,a and p. (2') in figure 4, b) regardless of the coefficient $\alpha$, as determined from (b. 24'). In figure 4, and all graphs of the relation of bond reactions with positive values of $\alpha$ still pass through common points (p. (1) and p. (3)), also respectively p. (1') and p. (3') in figure 4, b for negative values of $\alpha$.

5. Conclusion
A method for constructing mathematical models of a system of transport suspensions is proposed, which allows evaluating their dynamic capabilities under the action of various external perturbations, including the simultaneous action of several factors. The idea of the method of analysis and dynamic systems is to use a structural mathematical model in the form of a block diagram of an automatic control system. It is shown that a structural diagram of a suspension with a selected object whose dynamic state is evaluated can be considered as a basis for determining the dynamic stiffness of the system at characteristic points. This allows, in turn, to determine the dynamic reactions of connections as the product of dynamic stiffness on the dynamic displacement of the point in question, determined by the transfer function of the system. The paper introduces the concept of amplitude-frequency characteristics, in which the «output signal» is the dynamic response of connections. The analysis shows that changes in the dynamic reactions of connections can differ significantly from the ideas about the dynamic properties of systems obtained in the usual way. Setting up dynamic suspensions and evaluating their dynamic properties can be performed using such dynamic characteristics as the ratio of bond reactions occurring on the support surface and on the protected object. In this case, the dynamic state can be controlled by changing the system's stiffness ratio.

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