Total cross sections for neutron-nucleus scattering

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Systematics of neutron scattering cross sections on various materials for neutron energies up to several hundred MeV are important for ADS applications. Ramsauer model is well known and widely applied to understand systematics of neutron nucleus total cross sections. In this work, we examined the role of nuclear effective radius parameter \( r_0 \) on Ramsauer model fits of neutron total cross sections. We performed Ramsauer model global analysis of the experimental neutron total cross sections reported by W. P. Abfalterer, F. B. Bateman, et. al., from 20MeV to 550MeV for nuclei ranging from Be to U. The global fit functions which can fit total cross section data over periodic table are provided along with the required global set of parameters. The global fits predict within \( \pm 8\% \) deviation to data, showing the scope for improvement. It has been observed that a finer adjustment of \( r_0 \) parameter alone can give very good Ramsauer model description of neutron total scattering data within \( \pm 4\% \) deviation. The required \( r_0 \) values for Ramsauer model fits are shown as a function of nuclear mass number and an empirical formula is suggested for \( r_0 \) values as a function of mass number. In optical model approach for neutron scattering, we have modified the real part of Koning-Deleroche potentials to fit the neutron total cross sections using SCAT2 code. The modified potentials have a different energy dependence beyond 200MeV of neutron energy and fit the total cross sections from Al to Pb.

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The concept of an accelerator driven sub-critical (ADS) system is drawing worldwide attention \[1\] \[2\]. In this ADS system, neutrons are produced by bombarding a heavy element target with a high energy proton beam of typically above 1.0GeV with a current of \( > 10mA \[1\] \[2\]. Such a system serves a dual purpose of energy multiplication and waste incineration. In this context it is important to study the systematics of neutron scattering cross sections on various nuclei for neutron energies up to several hundred MeV. In this paper we propose an empirical approach to reproduce the energy and mass dependence of total cross sections \( (\sigma_{tot}) \). The nuclear Ramsauer model was first proposed by Lawson \[3\] in the year 1953 as a simple means to understand the energy dependence of total cross sections of neutron nucleus scattering. In order to appreciate this model, it is necessary to discuss briefly the optical model (OM) description of neutron scattering. In the OM approach, the scattering amplitudes are obtained by solving the Schrodinger’s equation with complex optical model potentials (OMP), such as in SCAT2 optical model code \[4\] with Koning and Deleroche potentials \[5\]. The absorption arising from imaginary part of OMP gives the reaction cross section. The calculations are usually performed using partial wave expansion method and the phase shifts \( \delta_\ell (\eta_l=e^{2i\delta_\ell} = \alpha_\ell e^{i\beta_\ell}) \) are determined. For a given set of potentials, these complex phase shifts are strongly angular momentum and energy dependent. In terms of the phase shifts and the wave number \( (\lambda = \hbar/\sqrt{2mE}) \) and neglecting spins, various cross sections are given below.

\[
\sigma_{tot} = 2\pi \lambda^2 \sum_\ell (2\ell + 1) |1 - \Re \eta_\ell| \tag{1}
\]

\[
\sigma_{reac} = \pi \lambda^2 \sum_\ell (2\ell + 1) (1 - |\eta_\ell|^2) \tag{2}
\]

\[
\frac{d\sigma}{d\Omega}(\theta) = \frac{\lambda^2}{4} \left| \sum_\ell (2\ell + 1)(1 - |\eta_\ell|)P_\ell(\cos \theta) \right|^2 \tag{3}
\]

In Figure 1, we show the transmission as a function of angular momentum \( \ell \) for n+\(^{208}\)Pb system at high and low energies. The results in this figure are generated using SCAT2 code \[4\] with Koning and Deleroche potentials \[5\]. The transmission, which is related to absolute value of the complex phase shift \( (1 - |\eta_\ell|^2) \), is approximately constant up to some \( \ell_{max} \) for each energy and falls sharply after that cut off angular momentum. We have plotted the SCAT2 transmissions and seen that this angular momentum dependence is very similar in all the systems considered in this work. These results are well known in the field of heavy ion fusion around the Coulomb barrier energies where, a sharp cut off or diffuse cut-off models are often used. When multiplied by factor, The transmission functions give the well known triangular distribution shape of partial wave cross sections \( (\sigma_\ell = (\pi/k^2)(2\ell+1)) \). Ramsauer model in the following discussion is essentially based on this behaviour of transmission function. An excellent experimental data base of neutron total cross sections is presently available in the energy range up

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The most recent work by Koning and Delaroche presents a very exhaustive search for OMP parameters that fit the data very well up to 200 MeV. We made a phenomenological Ramsauer model analysis of the experimental total cross sections. The nuclear Ramsauer model provides a simple method to parameterize the energy dependence of neutron nucleus total scattering cross sections. This model assumes that the scattering phase shifts are independent of angular momentum ($\ell$) as reflected in Eq. (4) ($\eta = \alpha e^{i\beta}$), in contrast to the predictions of the optical model given in Eq. (1). Further, it was proposed that the $\ell$-independent phase shift varies slowly with energy. Peterson applied this model for the study of neutron scattering from various nuclei. There were some attempts (see references therein) to put this Ramsauer model on a sound theoretical basis. The neutron total cross sections have been well studied using this model, over a wide range of nuclear masses and neutron energies up to 500 MeV.

### I. ANALYSIS OF NEUTRON TOTAL CROSS SECTIONS

We performed the Ramsauer model fits to the experimental neutron total cross sections using Eq. (3). The quantities $R(fm), \alpha$ and $\beta$ are functions of atomic mass number ($A$) and the center of mass energy ($E$).

\[
\sigma_{tot} = 2\pi (R + x)^2 (1 - \alpha \cos \beta) \quad (4)
\]

\[
\beta = \beta_0 A^\frac{1}{2} (\sqrt{E} + V - \sqrt{E}) \quad (5)
\]

\[
V = V_A + V_E \sqrt{E}
\]

\[
V_A = V_0 + V_1(N - Z)/A + V_2/A
\]

\[
\alpha = \alpha_0 + \alpha_A \sqrt{E} \quad (6)
\]

\[
\alpha_A = \alpha_1 \ln(A) + \alpha_2 / \ln(A)
\]

\[
R = r_0 A^\frac{1}{2} + r_A \sqrt{E} + r_2 \quad (7)
\]

\[
r_0 = 1.42988, \quad r_10 = -0.02298, \quad r_11 = 0.10268 \quad (8)
\]

\[
r_2 = 0.23216, \quad V_0 = 46.51099, \quad V_1 = 6.73833
\]

\[
V_2 = -117.52082, \quad V_E = -3.21817, \quad \beta_x = 0.5928
\]

\[
\alpha_0 = 0.02868, \quad \alpha_1 = -0.00274, \quad \alpha_2 = 0.13211
\]

Figures (a,b) show the experimental data represented by symbols and the Ramsauer model fits (solid lines) for $\sigma_{tot}$ cross sections using Eqs. (1)-(8). The fits are obtained with twelve optimised parameters as given in Eq. (8) over wide mass range of $^{24}\text{Mg}$ to $^{208}\text{Pb}$ (Fig. (a)) and $^7\text{Be}$ to $^{24}\text{Mg}$ (Fig. (b)). These fits cover the neutron energy region ($E_{lab}$) of 20-550 MeV. Our functional dependence on energy and mass given in Eq. (5)-(7) with twelve global parameters (Eq. (8)) reproduced the experimental data well. The percentage deviations of these fits from data shown in Figs. (a,b) have been calculated. We observed that the fits are within ±6% for heavy nuclei and within ±8% for light nuclei for energies between 20MeV-550MeV. These deviation functions are shown for all systems in Figs. (a,b). As seen in Figures 3, overall fits are within 8% error from the data. Note that the radius parameters for light systems are different as mentioned on the Figure (b). In order to test these parameterization, we predicted $\sigma_{tot}$ using Eqs. (1)-(8) for six nuclei which were not included in global fits of Figure 2. These predictions and the respective experimental data are shown in Figure 4. We have seen that the predictions are within 8% deviation from data. Model fits to total cross sections were already shown by various groups (see the references therein). Some works reproduced very well the Ramsauer peak, superposed on their smoothly varying parts of the cross sections.

The Ramsauer model global fits of experimental neutron total cross sections are shown in figures (a) and tested for systems which are not used in fitting process as shown in Fig. 4. The fit functions and global set of parameters are provided in Eqs. (1)-(8). These functions and parameters are a convenient starting point for Ramsauer model.
FIG. 2: Ramsauer model fits (solid lines) to the experimental data (symbols) of total cross sections versus $E_{\text{lab}}$ using Eq. (4). The twelve parameters optimised and the functional forms used are mentioned in the text in Eqs. (5)-(8). Fig. (a) is for heavy nuclei and Fig. (b) is for light nuclei. The radius parameters used for light systems are mentioned on the Figure (b).

Analysis of the experimental data and further improved fits can be achieved by local finer variation of these parameters. In this context, we observed that by adjusting only a single parameter $r_0$, it is possible to achieve better fits to experimental data, as has been earlier shown by us for neutron total scattering on targets $^{238}$U and $^{232}$Th [25]. We extended this study for all the systems, light and heavy considered in figures 2, 4. It is well known that the nuclear radius is given by $r_0A^{1/3}$. For Ramsauer model analysis, this is a good approximation only for global fits. However, local minor deviations may occur for nuclear radii from this formula $r_0A^{1/3}$. It is important to note that this radius, $r_0A^{1/3}$, should not be taken to mean the actual radial size of the nucleus. It represents the energy independent part of the radial size required in the Ramsauer model fits, see Eq. (7). Therefore, it represents the size required by neutron total cross sections, very similar to the strong absorption radius in heavy ion elastic scattering. Hence, this radial size is usually much larger than nuclear size. Therefore we performed an elaborate study by varying the $r_0$ parameter and fixing all other parameters as given.

FIG. 3: The percentage deviations of the empirical fits in Figs. ??(a,b) from the experimental data are shown for heavy and light nuclei respectively in Figs.(a,b).
FIG. 4: The Ramsauer model predictions (solid lines) to the experimental data (symbols) of total cross sections versus $E_{\text{lab}}$ using Eq. (4), using the functions and parameters that were optimised for Figure. ??(a) as in Eqs. (5)-(8).

FIG. 5: The $r_0$ parameter values that result in best Ramsauer model description to experimental data have been shown versus mass number.

FIG. 6: Ramsauer model fits (solid lines) to the experimental data (symbols) of total cross sections versus $E_{\text{lab}}$ using Eq. (4). The global fits are same as in Fig. 2. The red curve shows the fits by varying the $r_0$ parameter. Left panel shows the fits to cross sections for $^{238}\text{U}$ and $^{232}\text{Th}$ systems and right panel shows the percentage deviation of respective fits from the data. The red symbols show less deviation compared to black symbols (global fits) as expected.

As discussed earlier, the $r_0$ parameter values that result in best Ramsauer model description to experimental data are shown in Fig. 5. We obtained $r_0$ adjusted fits and their percentage deviations for all the systems considered in this study and compared these with the global fits mentioned earlier. The required $r_0$ values are shown in Fig. 5 and will be discussed later. These detailed fits and comparisons are shown in Figs. 6-16. In these figures, we show cross section fits in left panel and percentage deviation of fit from data in right panels. In these left panel figures, symbols represent data, black curves for global fits and red curves for $r_0$ adjusted fits. In the right panels, we show % deviations of global fits (black circles) and $r_0$ adjusted fits (red circles). As can be seen in all these figures, the systematic trend in % deviations were removed by $r_0$ adjustment. The % deviations are in most cases within ±4%. It is owing to the requirement of a finer adjustment of $r_0$ parameter values, the global fits produced large deviations up to ±8%.

As discussed earlier, the $r_0$ parameter values that result in best Ramsauer model description to experimental data are shown in Fig. 5. The constant value of $r_0 = 1.42988$ of global fits is shown in the figure by black straight line for reference. The smooth red solid curve in Fig. 5 shows the required variation of mass dependence of the $r_0$. This is given by a formula $r_0(A) = 1.0005 + 0.38045A^{0.02581}$. This functional form and parameters show that $r_0A^{1/3}$ is an approximation, however, the finer deviation of nuclear mass dependence is shown by second term. The
fine adjusted $r_0$ values using Ramsauer model are shown in Table.1. in column 2 and the from the formula given in column 3. These results in Table are to be compared with global fixed value of $r_0=1.42990$. Using this dependence of $r_0$ values would result in improved Ramsauer model fits of the experimental data.

II. OPTICAL POTENTIALS FOR NEUTRON TOTAL CROSS SECTIONS

As mentioned in the introduction, the SCAT2 code [4] with Koning Deleroche (KD) [5] potentials, explains consistently all the measured quantities of neutron nucleus cross sections, such as total cross sections, angular distributions, shape elastic cross sections etc. However, as well known its use is limited to 1KeV to 200 MeV energy region only. After achieving good fits using Ram-
sauer model, we have tried to modify the KD potentials in SCAT2 program such that they explain the total cross sections up to 550 MeV. We use KD potentials up to energy $E_c$ and a modification function $f(E)$ beyond $E_c$ for real potentials. Effectively, $f(E)=1$ below $E_c$ and the $f(E)$ function normalizes the volume term of real part of optical potentials, i.e., $\text{pot}(1)$ array in SCAT2 code. We have chosen exponential energy dependence factor given by $f(E) = \exp((E - E_0)/a_e)$, with $E_0$ and $a_e$ and $E_c$ being adjusted for various nuclei and these values are shown in Table II. In the following figures, we show these details with and without modification of KD potentials. In figures 11-14 symbols show the experimental total cross sections of W. P. Abfalterer, F. B. Bateman, et. al.,[8]. The black lines are for normal SCAT2 code results with the prescribed KD potentials with option $\gamma = 0$. This option refers to no relativistic corrections in potentials. The blue curves represent results with KD potentials with option $\gamma = 1$. As well known, this option refers to relativistic corrections implemented to optical potentials. The red curves show the total cross sections from our modified KD potentials using option $\gamma = 1$. As seen

FIG. 11: As shown in Figs[6] for $^{55}$Mn and $^{59}$Co.

FIG. 12: As shown in Figs[6] for $^{31}$P and $^{40}$Ca.

FIG. 13: As shown in Figs[6] for $^{24}$Mg and $^{27}$Al.

FIG. 14: As shown in Figs[6] for $^{13}$C and $^{19}$F.
in figures, the red curves simulate the neutron total cross sections well up to 550 MEV.

### III. CONCLUSION

In conclusion, we performed the Ramsauer model parameterization of experimental neutron total cross sections and derived the systematics of the Ramsauer model parameters. The percentage deviation of fits from data were plotted and displayed to be within ±8%. It was observed that the Ramsauer model fits are sensitive to nuclear radius parameter. By fine adjustment of $r_0$ parameter locally one can obtain very good Ramsauer model fits to neutron total cross sections. The $r_0$ adjusted fits have been compared with the global fits for various systems. These percentage deviations have been presented in de-

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**TABLE I**: Table shows $r_0$ values for various nuclei required to achieve best fits using Ramsauer model for neutron total cross sections. First column shows nucleus, second column shows the $r_0$ value. This is to be compared with a fixed value of 1.43fm used in global analyses.

| Nucleus | $r_0$ (fm) | formula |
|---------|------------|---------|
| 229U    | 1.4442     | 1.43866 |
| 232Th   | 1.44853    | 1.43838 |
| 209Bi   | 1.43457    | 1.43720 |
| 208Pb   | 1.42689    | 1.43714 |
| 197Au   | 1.43534    | 1.43558 |
| 181Ta   | 1.43204    | 1.43653 |
| 120Sn   | 1.41503    | 1.43135 |
| 93Nb    | 1.43409    | 1.42817 |
| 90Zr    | 1.4285     | 1.42780 |
| 89Y     | 1.43287    | 1.42768 |
| 59Co    | 1.43109    | 1.42317 |
| 55Mn    | 1.41083    | 1.42241 |
| 40Ca    | 1.43506    | 1.41895 |
| 31P     | 1.43062    | 1.41621 |
| 27Al    | 1.39221    | 1.41473 |
| 24Mg    | 1.42302    | 1.41347 |
| 19F     | 1.42892    | 1.41099 |
| 13C     | 1.38432    | 1.40999 |
| 11B     | 1.36467    | 1.40524 |
| 10B     | 1.44432    | 1.40425 |
| 9Be     | 1.40193    | 1.40315 |

**TABLE II**: Table shows best adjusted $E_0$, $a_0$ values of the $f(E)$ function that fit the neutron total cross sections for various nuclei using SCAT2 program. First column shows nucleus, second and third columns show the $E_0$, $E_0$, and $a_0$ values.

| Nucleus | $E_0$ (MeV) | $E_0$ (MeV) | $a_0$ (MeV) |
|---------|-------------|-------------|-------------|
| 209Bi   | 350         | 350         | 260         |
| 208Pb   | 325         | 325         | 275         |
| 197Au   | 350         | 350         | 245         |
| 181Ta   | 300         | 300         | 280         |
| 120Sn   | 250         | 250         | 270         |
| 93Nb    | 225         | 225         | 280         |
| 90Zr    | 225         | 225         | 280         |
| 89Y     | 225         | 225         | 285         |
| 59Co    | 225         | 225         | 275         |
| 55Mn    | 210         | 210         | 280         |
| 40Ca    | 225         | 225         | 275         |
| 31P     | 250         | 250         | 270         |
| 27Al    | 225         | 225         | 285         |
| 24Mg    | 250         | 250         | 275         |
| 19F     | 250         | 250         | 275         |
| 13C     | 250         | 250         | 300         |
FIG. 17: Total cross sections for n scattering on $^{13}$C, $^{19}$F, $^{24}$Mg, $^{27}$Al. Symbols show experimental data. Black curve is SCAT2 results with KD potentials with $\gamma = 0$ option. The blue curve represents with $\gamma = 1$ option and red curves represent modified KD potentials together with $\gamma = 1$.

FIG. 18: Total cross sections for n scattering on $^{31}$P, $^{45}$Ca, $^{55}$Mn, $^{60}$Co. The curves are as explained in previous figure.

tail. The $r_0$ parameter values that fit best have been shown as a function of nuclear mass. We modified the KD potentials incorporating new energy dependence. With these new potentials, the SCAT2 code was able to predict the total cross sections well up to 550 MeV.

FIG. 19: Total cross sections for n scattering on $^{89}$Y, $^{90}$Zr, $^{93}$Nb, $^{120}$Sn. The curves are as explained in previous figure.

FIG. 20: Total cross sections for n scattering on $^{181}$Ta, $^{197}$Au, $^{208}$Pb, $^{209}$Bi. The curves are as explained in previous figure.

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