Neutrino Oscillations at JUNO, the Born Rule, and Sorkin’s Triple Path Interference

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We argue that neutrino oscillations at JUNO offer a unique opportunity to study Sorkin’s triple-path interference, which is predicted to be zero in canonical quantum mechanics by virtue of the Born rule. In particular, we compute the expected bounds on triple-path interference at JUNO and demonstrate that they are comparable to those already available from electromagnetic probes. Furthermore, the neutrino probe of the Born rule is much more direct due to an intrinsic independence from any boundary conditions, whereas such dependence on boundary conditions is always present in the case of electromagnetic probes. Thus, neutrino oscillations present an ideal probe of this aspect of the foundations of quantum mechanics.

INTRODUCTION

Obtaining a deeper understanding of Quantum Mechanics (QM) is homework leftover from the 20th century. The question is becoming more acute with the development of QM based technologies already impacting our everyday lives (semi-conductors, super-conductors, etc.) as well as the promise of various quantum information technologies that may be realized in the not too distant future [1]. In this letter, we emphasize the relevance of neutrino physics to address various foundational questions in QM. In particular, we consider the potential of neutrino oscillations to probe the triple-path interference of Sorkin [2] as a direct test of the Born rule and compare the expected bound to those currently available from electromagnetic experiments.

There are many features that distinguish QM from classical mechanics (CM). Though the statistical nature of QM as opposed to the deterministic nature of CM is often emphasized in textbooks, many other differences exist as well; for instance, in correlations [3, 4], and in the presence or absence of interference. However, it has been noted that not only do QM correlations go beyond those of CM but are themselves restricted [5, 6], and are not as large as that allowed by logic and relativity [7]. QM interference is also restricted in that the Born rule only allows for pair-wise interference between paths, but not for triple-path interference or higher [2].

The fact that canonical QM itself is restricted, with no apparent physical or logical reason, points to the possible existence of consistent theories that go beyond QM boundaries, perhaps at the expense of some of our cherished principles which we currently hold to be fundamental. It is up to experiments to determine whether Nature always stays within those QM boundaries, or occasionally ventures outside, and under what conditions. Indeed, it has been argued that it would (or should) in the context of quantum gravity and cosmology [9, 10], or in the domain of macroscopic quantum systems [11], or in the realm of quantum measurement [12], or in the domain of macroscopic quantum systems [12].

The celebrated Bell’s inequality [3, 4] (1964) and its generalization by Clauser, Horne, Shimony, and Holt, the CHSH inequality [5] (1969), provide bounds that CM correlations cannot violate. That experimental correlations violate these bounds has been confirmed by various groups using photons [13] (1981), ions and atoms, Josephson junctions, and NV centers in diamond (See FIG. 1 of [14]). However, Tsirelson has shown that the CHSH correlator is bounded from above for QM as well [6, 7] (1980) and while this bound is naturally larger than that of CM, it does not saturate the logically allowed maximum, despite the fact that such a saturation does not contradict relativistic causality as demonstrated by Popescu and Rohrlich [8] (1994). Whether experimental correlations violate the CM Tsirelson bound or not is being checked using photons [14, 15].

The Bell/CHSH inequalities involve correlations between entangled pairs of spatially separated observables. In contrast, the Leggett-Garg inequalities (LGI’s) [9] (1985) involve correlations between temporary separated measurements of a single observable and are expected to hold for “macroscopic” systems. Since QM satisfies neither of the two assumptions underlying the LGI’s, namely macroscopic realism and non-invasive measurability, QM correlations can be expected to violate them, though there is some subtlety in how those correlations should be defined in QM to reflect the setup of each experiment. Nevertheless, experimental checks that demonstrate the violation of some form of the LGI have been performed using a variety of systems including superconducting qubits, nuclear spins, photons (see Table 1 of Ref. [17]), and more recently using the neutrino oscillation data from Minos [19] (2016) and Daya-Bay [20] (2017).

Neutrino experiments are, in many respects, ideal laboratories for foundational quantum research given the long coherence times that neutrino states have, and the fundamental role of interference in neutrino oscillation phenomena. In addition to probing the LGI’s [19, 20], the violation of which is not surprising, the potential of neutrino oscillations to constrain models that generalize and go beyond canonical QM have also been explored. For
In this letter, we explore the potential of the neutrino oscillation experiment JUNO [27] to look for the triple-path interference of Sorkin [2] (1994). Sorkin provides a classification scheme for theories that go beyond CM in terms of the existence of multi-path interferences. Only double-path interferences exist in QM due to the Born rule though, in principle, triple and higher-order interferences are possible. Experimental bounds have been placed on the presence of triple-path interference using photons [28] (2010) and Liquid State NMR [29] (2012). Both experiments report upper bounds on the ratio of triple-path to double-path interferences of order $10^{-2}$. Improving the photon bound requires controlling the sensitivity of multi-slit interference to the change in the boundary condition due to the opening and closing of slits [30–34]. Neutrino oscillation, on the other hand, does not involve any slits and always has three mass eigenstates interfering with each other. However, at different baselines, neutrino energies, and neutrino energy resolutions, it effectively becomes a double-path interference experiment due to the large separation in scale between $\delta m_{21}^2$ and $\delta m_{31}^2$. Indeed, JUNO [27] is expected to be the first experiment in which the interference between the atmospheric and solar oscillation amplitudes is clearly visible [35].

In the following, we first review Sorkin’s definition of multi-path interference and the classification of theories based on their presence or absence [2]. We then review our previous work from Ref. [35], which looked at the potential of JUNO to detect deviations of the neutrino oscillation probabilities from their QM predictions. This is effectively the same problem as we are considering in this letter, albeit imposing a particular normalization for the possible triple-path interference. The expected bounds on triple-path interference at JUNO for several other normalization/parametrization choices with details of the analyses are presented next. The parametrizations are chosen under the caveat that the contribution of the triple-path interference should be invisible to pre-JUNO experiments, and includes the one which facilitates comparison of the bound with the photon/NMR results. We conclude with a discussion on the necessity of a concrete model which predicts triple-path interference to derive a more physically meaningful bound.

**THE BORN RULE AND SORKIN’S MULTI-PATH INTERFERENCE**

We follow the discussion of Sorkin in Ref. [2]. Let

$$P_n(A, B, C, \cdots)$$

(1)

denote the probability of a system to go from an initial state $|\alpha\rangle$ to a final state $|\beta\rangle$ when $n$ pathways $A, B, C, \ldots$ connecting the two are available. Classically, we have

$$P_n(A, B, C, \cdots) = P_1(A) + P_1(B) + P_1(C) + \cdots,$$

(2)

for any number of paths. Quantum mechanically, we have for two paths

$$P_2(A, B) = |\psi_A + \psi_B|^2$$

$$= |\psi_A|^2 + |\psi_B|^2 + (\psi_A^* \psi_B + \psi_B^* \psi_A),$$

(3)

The extra term

$$I_2(A, B) = P_2(A, B) - P_1(A) - P_1(B)$$

(4)

is the “interference” of the two paths $A$ and $B$. The non-vanishing of this double-path interference, $I_2(A, B) \neq 0$, distinguishes QM from CM.

In QM the superposition principle allow us to superimpose an arbitrary number of “paths” on top of each other. Indeed, in the path integral approach we superimpose an infinite number of them [36]. However, the Born Rule dictates that all the superimposed paths only interfere with each other in a pairwise manner. For instance, for three paths we have

$$P_3(A, B, C) = |\psi_A + \psi_B + \psi_C|^2$$

$$= |\psi_A|^2 + |\psi_B|^2 + |\psi_C|^2 + (\psi_A^* \psi_B + \psi_B^* \psi_A + \psi_C^* \psi_B + \psi_B^* \psi_C + \psi_C^* \psi_A + \psi_A^* \psi_C)$$

$$= P_2(A, B) + P_2(B, C) + P_2(C, A)$$

$$- P_1(A) - P_1(B) - P_1(C).$$

(5)

Only pairwise interferences between the pairs $(A, B)$, $(B, C)$, and $(C, A)$ appear. Therefore, it makes sense to define any deviation from this relation as the triple-path interference:

$$I_3(A, B, C)$$

$$= P_3(A, B, C) - P_2(A, B) - P_2(B, C) - P_2(C, A)$$

$$+ P_1(A) + P_1(B) + P_1(C).$$

(6)

For both CM and QM, this triple-path interference is zero for any triplet of paths.
In a similar fashion, the $n$-path interference for $n \geq 4$ can be defined as
\[
I_n(A_1, A_2, \cdots, A_n) = P_n(A_1, A_2, \cdots, A_n) - \sum P_{n-1}(A_1, A_j, \cdots) + \sum P_{n-2}(A_1, \cdots) \cdots - (-1)^n P_1(A_i),
\] (7)
which are always zero for both CM and QM. Therefore, CM and QM can be characterized by
\[
\text{CM} : I_n = 0 \text{ for } n \geq 2,\\
\text{QM} : I_2 \neq 0, \; I_n = 0 \text{ for } n \geq 3.
\] (8)

Experimental confirmation of $I_3 = 0$ would be a confirmation of the Born rule. In Refs. [28, 29], bounds were placed on the parameter
\[
\kappa = \frac{\varepsilon}{\delta},
\] (9)
where
\[
\varepsilon = I_3(A, B, C),\\
\delta = |I_2(A, B)| + |I_2(B, C)| + |I_2(C, A)|.
\] (10)

Ref. [28] reports $\kappa = 0.0064 \pm 0.0119$ for a multi-slit experiment with a single photon source, while Ref. [29] reports $\kappa = 0.007 \pm 0.003$ based on a liquid state NMR experiment. Thus, the $1\sigma$ deviation of $\kappa$ from zero allowed by these experiments is $|\kappa| < 0.01 \sim 0.02$.

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Here, we review the analysis of Ref. [35]. According to canonical QM, the neutrino oscillation amplitude for $\nu_\beta \rightarrow \nu_\alpha$ at distance $x$ from the source is given by the superposition of the contributions of the three mass eigenstates:
\[
S^{(123)}_{\alpha\beta} = U_{\alpha1}U^{*}_{\beta1} + U_{\alpha2}U^{*}_{\beta2}e^{-i\Delta_{21}x} + U_{\alpha3}U^{*}_{\beta3}e^{-i\Delta_{31}x} = \left(\begin{array}{c}
U_{\alpha1}U^{*}_{\beta1} + U_{\alpha2}U^{*}_{\beta2} + U_{\alpha3}U^{*}_{\beta3} \\
\delta_{\alpha\beta}^\delta + U_{\alpha2}U^{*}_{\beta2} \left(e^{-i\Delta_{21}x} - 1\right) + U_{\alpha3}U^{*}_{\beta3} \left(e^{-i\Delta_{31}x} - 1\right)
\end{array}\right).
\] (11)
where
\[
\Delta_{ij} = \frac{\delta m^2_{ij}}{2E}.
\] (12)
The Born rule gives the QM oscillation probability for $\nu_\beta \rightarrow \nu_\alpha$ as
\[
P_{\text{QM}}(\nu_\beta \rightarrow \nu_\alpha) = \left|S^{(123)}_{\alpha\beta}\right|^2
\]
\[
= \delta_{\alpha\beta}^\delta + \left|S^{\text{sol}}_{\alpha\beta}\right|^2 + \left|S^{\text{atm}}_{\alpha\beta}\right|^2 + 2\delta_{\alpha\beta}^\delta \Re\left(S^{\text{sol}}_{\alpha\beta}S^{\text{atm}}_{\alpha\beta}\right)
+ \left(S^{\text{sol}^*}S^{\text{atm}}_{\alpha\beta} + S^{\text{sol}}S^{\text{atm}^*}_{\alpha\beta}\right)\frac{P_{\text{non-int-fer}}}{P_{\beta\alpha}},
\] (13)
This is the QM prediction. To check this relation, Ref. [35] introduces the parameter $q$ as
\[
P_{\text{exp}}(\nu_\beta \rightarrow \nu_\alpha) = P_{\text{non-int-fer}} + qP_{\text{int-fer}},
\] (14)
and discusses the bounds that can be placed on $q$ by the JUNO experiment [27], that is, the expected $P_{\text{exp}}(\nu_\beta \rightarrow \nu_\alpha)$ at JUNO assuming canonical QM is simulated using GLoBES [37, 38] to which the right-hand side of Eq. (14) is fit to calculate the expected bound on $q$. Since the $P_{\text{non-int-fer}}$ term includes interference effects between the 1-2 and 1-3 mass eigenstates, the parameter $q$ checks for 2-3 interference. However, any deviation of $P_{\text{exp}}(\nu_\beta \rightarrow \nu_\alpha)$ from the QM prediction can also be interpreted as due to Sorkin’s triple-path interference. Indeed, Eq. (14) effectively parametrizes the size of triple-path interference as
\[

\varepsilon = P_{\text{exp}}(\nu_\beta \rightarrow \nu_\alpha) - P_{\text{QM}}(\nu_\beta \rightarrow \nu_\alpha) = (q - 1)P_{\text{int-fer}}.
\] (15)

Note that this parametrization of the triple-path interference renders it invisible to all pre-JUNO experiments. Thus the bound on $q - 1$ can be reinterpreted as a bound on $\varepsilon$ normalized to $P_{\text{int-fer}}$. Using Wilks’ Theorem [39], we find that the analysis of Ref. [35] imposes a $1\sigma$ allowed range on $q - 1$ given by
\[
-0.17 < (q - 1) < 0.12
\] (16)

Note that in the absence of an actual theory with triple-path interference which predicts the energy and baseline dependence of $\varepsilon$, we must make a somewhat arbitrary choice as in Eq. (15). By fitting the data with constant $\kappa = \varepsilon/\delta$, Refs. [28, 29] are assuming that $\varepsilon \propto \delta$, cf. Eq. (10), which is another arbitrary choice. However, $\delta$ does have the advantage over $P_{\text{int-fer}}$ in that all three paths are treated equally. Due to this, and also for the ease of comparison with the photon and NMR results, we redo the analysis of [35] with this normalization.

**POSSIBLE JUNO BOUND ON $\kappa$**

The parameter $\kappa$ is introduced as
\[
\varepsilon = P_{\text{exp}}(\nu_\beta \rightarrow \nu_\alpha) - P_{\text{QM}}(\nu_\beta \rightarrow \nu_\alpha) = \kappa \left(|I_{\alpha\beta}(1, 2)| + |I_{\alpha\beta}(1, 3)| + |I_{\alpha\beta}(2, 3)|\right),
\] (17)
where
\[ I_{\alpha\beta}(i,j) = 2 \Re \left( U_{\alpha i} U_{\beta i}^* U_{\alpha j} U_{\beta j}^* e^{-i \Delta_{ij} \tau} \right) \] (18)
is the interference between the \(i\)-th and \(j\)-th mass eigenstates. Strictly speaking, one should perform a global fit to all neutrino oscillation experiments to bound \(\kappa\). However, \(|I_{\alpha\beta}(2,3)|\) was invisible to all pre-JUNO experiments and would have averaged to a small constant, while the other terms would have been absorbed into the uncertainties in the mixing angles. This justifies our JUNO-only analysis.

We use GLoBES \cite{37,38} to simulate the JUNO experiment as detailed in Ref. \cite{35}. The simulation is set up with two detectors: a JUNO-like far detector with a fiducial mass of 1 ton and an energy resolution of 1.7%/\(\sqrt{E}\) at a distance of 53 km from a nuclear reactor source with a total power of 36 GWth, and a TAO-like \cite{10} near detector with a fiducial mass of 1 ton and an energy resolution of 3%/\(\sqrt{E}\) at a distance of 30 m from a 4.6 GWth nuclear reactor core; we assume a total data taking time of 6 years.

For the purpose of producing the simulated data \(P_{\exp}^{\nu_\beta \rightarrow \nu_\alpha}\), we assume canonical QM with normal ordering to be the true mass ordering and the relevant oscillation parameters to be \(\Delta m^2_{21} = 7.54 \times 10^{-5} \text{eV}^2\), \(\Delta m^2_{31} = 2.43 \times 10^{-3} \text{eV}^2\), \(\theta_{12} = 33.6^\circ\), and \(\theta_{13} = 8.9^\circ\) \cite{11}. The theoretical QM rates \(P_{\text{QM}}^{\nu_\beta \rightarrow \nu_\alpha}\) are calculated with the same inputs and the difference between \(P_{\exp}^{\nu_\beta \rightarrow \nu_\alpha}\) and \(P_{\text{QM}}^{\nu_\beta \rightarrow \nu_\alpha}\) is fit with Eq. (17). The results of our simulation are shown in Fig. 1 for the following three analyses:

(a) (solid line) For each detector, we use a model for non-linear effects in the reconstruction of the positron energy as described in Ref. \cite{12}, which includes terms up to the cubic in the positron energy. To account for the uncertainties in the reactor antineutrino flux prediction, we conservatively introduce a nuisance parameter to each of our 100 energy bins with the spectrum computed before applying the energy resolution function. This is equivalent to the assumption of no prior knowledge of fluxes, as in Ref. \cite{12}.

(b) (dashed line) The same analysis repeated except assuming that the energy calibration error for each detector is linear.

(c) (dotted line) Simulation without a near detector while assuming perfect knowledge of detector and source systematics.

An interesting feature of Fig. 1 is that analysis (c) with no systematic uncertainties provides a weaker constraint for \(\kappa\) than analyses (a) and (b) with systematic uncertainties and a near detector. This demonstrates that the presence of a near detector not only constrains the neutrino flux, but also provides extra constraints on possible deviations of the neutrino oscillation probabilities from canonical QM. Using Wilks' Theorem, the 1σ allowed range of \(\kappa\) for analysis (a) is found to be
\[-0.017 < \kappa < 0.015\] (19)

**CONSTRaining OTHER FORMS OF THE INTERFERENCE TERM**

In addition to Eq. (17), which we will refer to as case (1), we consider two other forms for the triple-path interference \(\varepsilon\): (2) constant \(\varepsilon\), i.e. \(\varepsilon\) independent of \(L/E\), and (3) \(\varepsilon\) proportional \(1 - P_{\text{QM}}^{\nu_\beta \rightarrow \nu_\alpha}\), that is
\[ \varepsilon = P_{\exp}^{\nu_\beta \rightarrow \nu_\alpha} - P_{\text{QM}}^{\nu_\beta \rightarrow \nu_\alpha} = k \left( 1 - P_{\text{QM}}^{\nu_\beta \rightarrow \nu_\alpha} \right), \] (20)
where \(k\) is a constant. In case (2), we are considering the possibility that triple-interference is hidden in the uncertainty of the overall count rate, while in case (3) we are assuming
\[ P_{\exp}^{\nu_\beta \rightarrow \nu_\alpha} = k + (1 - k) P_{\text{QM}}^{\nu_\beta \rightarrow \nu_\alpha}. \] (21)
For each case, we perform the same three analyses as in the previous section and plot the results in Figs. 2 and 3 respectively. For case (2), the 1σ allowed range for \(\varepsilon\) from analysis (a) is
\[-0.065 < \varepsilon < 0.042\] (22)
while for case (3), the bound on $k$ from analysis (a) is

$$-0.040 < k < 0.069$$  \hspace{1cm} (23)$$

In order to compare the constraints for cases (1), (2), and (3), we define

$$q_1 \equiv \kappa \langle \delta(E) \rangle, \quad q_2 \equiv \varepsilon, \quad \text{and} \quad q_3 \equiv k \langle 1 - P_{QM}(\nu_\alpha \rightarrow \nu_\beta) \rangle,$$  \hspace{1cm} (24)$$

where $\langle f(E) \rangle$ is the average of $f(E)$ over the interval $1.8 \text{MeV} \leq E \leq 8.0 \text{MeV}$ using the oscillation parameters that minimized the $\chi^2$ at each point. The results for analysis (a) including all systematic uncertainties is shown in Fig. 4.

**DISCUSSION**

In this letter, we point out the relevance of neutrino physics for addressing foundational questions in QM. In particular, we have examined the potential of the JUNO experiment to probe for the triple-path interference among the three neutrino mass eigenstates and thereby test the Born rule. The potential JUNO 1σ bound of $-0.017 < \kappa < 0.015$ is competitive with those available from electromagnetic probes \cite{28, 29}. Moreover, the prospects of electromagnetic probes to improve their bounds is limited due to the sensitivity of the interference pattern on the change in boundary condition caused by the opening and closing of slits \cite{30–34}, whereas neutrino oscillations are independent of such considerations.

One drawback of our analysis is that we currently lack a theory which can model departures from the Born rule, and predict how triple-path interferences would depend on experimental parameters. This is reflected in the arbitrary choices we must make to normalize the triple-path interference $\varepsilon$ in our fits. We are also assuming...
that triple-path interference will be introduced without any modification to the existing double-path interferences, which may not be the case for a complete theory. Without such a theory, we also cannot disentangle triple-path interference from other effects such as neutrino non-standard interactions (NSI’s), or the presence of sterile neutrinos.

A promising candidate theory that could potentially incorporate triple-path interferences within its framework is Nambu QM [25]. This theory generalizes the time evolution of QM states in a way which is non-canonical yet unitary. In essence, Nambu QM generalizes the space in which the “phase” (in the sense of the “phase” of a complex number) of a state is allowed to evolve, leading to non-canonical time evolution as well as non-canonical double-path interference. Indeed, we have recently discussed how the vanilla version of the theory can be constrained using atmospheric neutrinos by looking at interference effects [24]. Given the larger freedom that the “phase” is allowed in Nambu QM, we envision generalizations (most probably a non-associative one) in which the triple-path interference and the departure from the Born rule could be precisely modeled.

We close this discussion by recalling that neutrinos can also probe the Leggett-Garg inequalities [19, 20], spontaneous collapse models of quantum measurement [24, 22], and, if Mössbauer neutrinos can be realized, the time-energy uncertainty relation [26]. Further studies will indubitably lead to other ways to utilize neutrinos for foundational QM studies.

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REFERENCES

[1] I. H. Deutsch, PRX Quantum 1, 020101 (2020).
[2] R. D. Sorkin, Mod. Phys. Lett. A9, 3119 (1994), arXiv:gr-qc/9401003 [gr-qc].
[3] J. Bell, Physics Physique Fizika 1, 195 (1964).
[4] J. Bell, Speakable and Unspeakable in Quantum Mechanics (Cambridge University Press, 2004).
[5] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).
[6] B. Cirelson, Lett. Math. Phys. 4, 93 (1980).
[7] L. J. Landau, Phys. Lett. A 120, 54 (1987).
[8] S. Popescu and D. Rohrlich, Found. Phys. 24, 379 (1994).
[9] R. Penrose, Found. Phys. 44, 557 (2014).
[10] M. Gell-Mann and J. B. Hartle, Phys. Rev. A 89, 052125 (2014), arXiv:1312.7454 [quant-ph].
[11] S. Weinberg, Phys. Rev. A 93, 032124 (2016), arXiv:1603.06008 [quant-ph].
[12] A. Leggett, In: Quantum Theory: Reassessment of Foundations, J. yöru, ed. (World Scientific, 2009).
[13] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 47, 460 (1981).
[14] H. S. Poh, S. K. Joshi, A. Cerè, A. Cabello, and C. Kurtsiefer, Phys. Rev. Lett. 115, 180408 (2015), arXiv:1506.01865 [quant-ph].
[15] Z. Tian, Y.-Y. Zhao, H. Wu, Z. Wang, and L. Luo, Sci. China Inf. Sci. 63, 180506 (2020).
[16] A. Leggett and A. Garg, Phys. Rev. Lett. 54, 857 (1985).
[17] C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. 77, 016001 (2014), arXiv:1304.5133 [quant-ph].
[18] C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. 77, 039501 (2014).
[19] J. Formaggio, D. Kaiser, M. Murskyj, and T. Weiss, Phys. Rev. Lett. 117, 050402 (2016), arXiv:1602.00041 [quant-ph].
[20] Q. Fu and X. Chen, Eur. Phys. J. C 77, 775 (2017), arXiv:1705.08601 [hep-ph].
[21] S. Donadi, A. Bassi, L. Ferrialdi, and C. Cuccureano, Found. Phys. 43, 1066 (2013), arXiv:1207.5997 [quant-ph].
[22] M. Bahrami, S. Donadi, L. Ferrialdi, A. Bassi, C. Cuccureano, A. Di Domenico, and B. Hiesmayr, Sci. Rep. 3, 1952 (2013), arXiv:1305.6168 [quant-ph].
[23] G. C. Ghirardi, P. M. Pearle, and A. Rimini, Phys. Rev. A 42, 78 (1990).
[24] D. Minic, T. Takeuchi, and C. H. Tze, arXiv:2006.00583 [hep-ph] (2020).
[25] D. Minic and C. H. Tze, Phys. Lett. B 536, 305 (2002), arXiv:hep-th/0202173.
[26] R. S. Raghavan, D. Minic, T. Takeuchi, and C. H. Tze, arXiv:1210.5639 [quant-ph] (2012).
[27] F. An et al. (JUNO), J. Phys. G 43, 030401 (2016), arXiv:1507.05613 [physics.ins-det].
[28] U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, and G. Weihs, Science 329, 418 (2010), arXiv:1007.4193 [quant-ph].
[29] D. K. Park, O. Moussa, and R. Laflamme, New J. Phys. 14, 113025 (2012), arXiv:1207.2321 [quant-ph].
[30] H. Yabuki, Int. J. Theor. Phys. 25, 159 (1986).
[31] H. De Raedt, K. Michielsen, and K. Hess, Phys. Rev. A 85, 012101 (2012), arXiv:1112.2665 [quant-ph].
[32] R. Sawant, J. Samuel, A. Sinha, S. Sinha, and U. Sinha, Phys. Rev. Lett. 113, 120406 (2014), arXiv:1308.2022 [quant-ph].
[33] A. Sinha, A. H. Vijay, and U. Sinha, Sci. Rep. 5, 10304 (2015), arXiv:1412.2198 [quant-ph].
[34] G. Rengaraj, U. Prathwiraj, S. N. Sahoo, R. Somasekhar, and U. Sinha, New J. Phys. 20, 063049 (2018), arXiv:1610.09143 [quant-ph].
[35] P. Huber, H. Minakata, and R. Pestes, Phys. Rev. D 101, 093002 (2020), arXiv:1912.02426 [hep-ph].
[36] R. P. Feynman and A. R. Hibbs, Quantum Mechanics and Path Integrals: Emended Edition (Dover Publications, 2010) (1st Edition published by McGraw-Hill, 1965).
[37] P. Huber, M. Lindner, and W. Winter, Comput. Phys. Commun. 167, 195 (2005), arXiv:hep-ph/0407333.
[38] P. Huber, J. Kopp, M. Lindner, M. Rolinec, and W. Winter, Comput. Phys. Commun. 177, 432 (2007), arXiv:hep-ph/0701187.

[39] S. S. Wilks, The Annals of Mathematical Statistics 9, 60 (1938).

[40] M. Sisti (JUNO), J. Phys. Conf. Ser. 1468, 012150 (2020).

[41] P. F. de Salas, D. V. Forero, S. Gariazzo, P. Martínez-Miravé, O. Mena, C. A. Ternes, M. Tórtola, and J. W. F. Valle, JHEP 02, 071, arXiv:2006.11237 [hep-ph].

[42] D. V. Forero, R. Hawkins, and P. Huber, arXiv:1710.07378 [hep-ph] (2017).