AT2: Asynchronous Trustworthy Transfers

RACHID GUERRAOUI, EPFL
PETR KUZNETSOV, LTCI, Télécom ParisTech, University Paris-Saclay
MATTEO MONTI, EPFL
MATEJ PAVLOVIC, EPFL
DRAGOS-ADRIAN SEREDINSCHI, EPFL

ABSTRACT Many blockchain-based protocols, such as Bitcoin, implement a decentralized asset transfer system. As clearly stated in the original paper by Nakamoto, the crux of this problem lies in prohibiting any participant from engaging in double-spending. There seems to be a common belief that consensus is necessary for solving the double-spending problem. Indeed, whether it is for a permissionless or a permissioned environment, the typical solution uses consensus to build a totally ordered ledger of submitted transfers.

In this paper we show that this common belief is false: consensus is not needed to implement a decentralized asset transfer system. We do so by introducing AT2 (Asynchronous Trustworthy Transfers), a class of consensusless algorithms.

To show formally that consensus is unnecessary for asset transfers, we first consider this problem in the shared-memory context. We introduce AT2_{SM}, a wait-free algorithm that asynchronously implements asset transfer in the read-write shared-memory model. In other words, we show that the consensus number of an asset-transfer object is one.

In the message passing model with Byzantine faults, we introduce a generic asynchronous algorithm called AT2_{MP} and discuss two instantiations of this solution. First, AT2_{D} ensures deterministic guarantees and consequently targets a small scale deployment (tens to hundreds of nodes), typically for a private, i.e. permissioned, environment. Second, AT2_{P} provides probabilistic guarantees and scales well to a very large system size (tens of thousands of nodes), ensuring logarithmic latency and communication complexity. Instead of consensus, we construct AT2_{D} and AT2_{P} on top of a broadcast primitive with causal ordering guarantees offering deterministic and probabilistic properties, respectively.

Whether for the deterministic or probabilistic model, our AT2 algorithms are both simpler and faster than solutions based on consensus. In systems of up to 100 replicas, regardless of system size, AT2_{D} outperforms consensus-based solutions offering a throughput improvement ranging from 1.5x to 6x, while achieving a decrease in latency of up to 2x. (Not shown in this version of the document.) AT2_{P} obtains sub-second transfer execution on a global scale deployment of thousands of nodes.
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1 INTRODUCTION

In 2008, Satoshi Nakamoto introduced the Bitcoin protocol, implementing an electronic asset transfer system (often called a cryptocurrency) without any central authority [51]. Since then, many alternatives to Bitcoin came to prominence, designed for either the permissionless (public) or permissioned (private) setting. These include major cryptocurrencies such as Ethereum [63] or Ripple [53], as well as systems sparked from research or industry efforts such as Bitcoin-NG [27], Algorand [30], ByzCoin [41], Stellar [48], Hyperledger [7], Corda [34], or Solida [2]. Each of these alternatives brings novel approaches to implementing decentralized transfers, and may offer a more general interface in the form of smart contracts [59]. They improve over Bitcoin in various aspects, such as performance, scalability, energy-efficiency, or security.

A common theme in these protocols, whether they are for transfers [42] or smart contracts [63], is that they seek to implement a blockchain. This is a distributed ledger where all the transfers in the system are totally ordered. Achieving total order among multiple inputs (e.g., transfers) is fundamentally a hard task, equivalent to solving consensus [28, 33].

Consensus is a central problem in distributed computing, known for its notorious difficulty. Consensus has no deterministic solution in asynchronous systems if just a single participant can fail [28]. Algorithms for solving consensus are tricky to implement correctly [1, 21, 23], and they face tough trade-offs between performance, security, and energy-efficiency [8, 13, 32, 62].

As stated in the original paper by Nakamoto, the main problem of a decentralized cryptocurrency is preventing a malicious participant from spending the same money more than once [51]. This is known as the double-spending attack. Bitcoin and follow-up systems typically assume that total order—and thus consensus—is vital to preventing double-spending [29]. Indeed, there seems to be a common belief that solving consensus is necessary for implementing a decentralized asset transfer system [15, 32, 39, 51].

Our main contribution in this paper is showing that this common belief is false. We show that total order is not required to avoid double-spending in decentralized transfer systems. We do so by introducing AT2 (Asynchronous Trustworthy Transfers), a class of consensusless algorithms.

As a starting point, we consider the shared memory model with benign failures. In this model, we give a precise definition of a transfer system as a sequential object type. It is for pedagogical purposes that we start from this model, as it allows us to study the relation between the transfer object and consensus, and later extend our result to the message passing model. We introduce $AT2_{SM}$, an asset transfer algorithm that has consensus number one [35]. In other words, decentralized asset transfer does not need consensus in its implementation, and we can avoid maintaining a total order across transfers.
To get an intuition why total order is not necessary, consider a set of users who transfer money between their accounts in a decentralized manner. For simplicity, assume the full replication model, where every user maintains a copy of the state of every account. We observe that most operations in a transfer system commute, i.e., different users can apply the operations in arbitrary order, resulting in the same final state. For instance, a transfer $T_1$ from Alice to Bob commutes with a transfer $T_2$ from Carol to Drake. This is because the two transfers involve different accounts. In the absence of other transfers, $T_1$ and $T_2$ can be applied in different orders by different users, without affecting correctness.

Consider now a more interesting case when two transfers involve the same account. For example, let us throw into the mix a transfer $T_0$ from Alice to Carol. Assume that Alice issues $T_0$ before she issues $T_1$. Note that $T_0$ and $T_1$ do not commute, because they involve the same account—that of Alice—and it is possible that she cannot fulfill both $T_0$ and $T_1$ (due to insufficient balance). We say that $T_1$ depends on $T_0$, and so $T_0$ should be applied before $T_1$.

Furthermore, suppose that Carol does not have enough money in her account to fulfill $T_2$ before she receives transfer $T_0$ from Alice. In this case, transfer $T_2$ depends on $T_0$. Thus, all users should apply $T_0$ before applying $T_2$, while transfers $T_1$ and $T_2$ still commute. The partial ordering among these three transfers is in fact given by a causality relationship.

In Figure 1 we show the scenario with these three transfers and the causality relation between them. Intuitively, the general ordering constraint we seek to enforce is that every outgoing transfer for an account causally depends on all preceding transfers involving that—and only that—account.

![Fig. 1. A simple scenario illustrating, on the left side, three transfers among four participants. On the right side, we depict the dependencies among these three transfers. The dependencies are established by a causality-based ordering relationship.](image)

We then extend our result to the model of Byzantine fault-prone processes that communicate via passing messages. In this model, we show how to sidestep consensus by presenting $\text{AT2}_{MP}$, a generic algorithm that implements decentralized asset transfer atop a variant of secure broadcast. We describe two variants of this generic solution. The first, called $\text{AT2}_D$, has deterministic guarantees that targets a smaller scale deployment (tens to hundreds of nodes) typically for a private environment. The second, called $\text{AT2}_P$, ensures probabilistic guarantees and scales well to very large system sizes, typically for a public setting.
It is well-known that the main bottleneck in blockchain-based systems is their consensus module [34, 57, 62]. Numerous solutions have emerged to alleviate this problem [31, 34]. Typical techniques seek to employ a form of sharding [42], for instance, or to use a committee-based optimization [27, 30]. With AT2, we circumvent this main bottleneck, yielding new solutions that bypass consensus altogether.

Whether for the deterministic or probabilistic model, our AT2\textsubscript{MP} algorithms are both simpler and faster than solutions based on consensus. In systems of up to 100 replicas, regardless of system size, AT2\textsubscript{D} outperforms consensus-based solutions offering a throughput improvement ranging from 1.5x to 6x, while achieving a decrease in latency of up to 2x. AT2\textsubscript{P} obtains \textit{sub-second} transfer execution on a global scale deployment of thousands of nodes.

The rest of this paper is organized as follows. We first define the asset transfer object type in the crash-stop shared memory model and show that it has consensus number one for accounts with a single owner (§2). We then move to the message passing model with Byzantine failures and present AT2\textsubscript{MP}, a generic algorithm for implementing distributed asset transfer on top of a secure broadcast primitive (§3). Next, we focus on one deterministic and one probabilistic version of the secure broadcast primitive, yielding, respectively, AT2\textsubscript{D} (§4) and AT2\textsubscript{P} (§5), our deterministic and probabilistic flavors of AT2 for the Byzantine message passing model. At the end, we revisit the crash-stop shared memory model to prove that a general asset transfer object has consensus number $k$ if an account is shared by $k$ (but not more) processes (§6). Finally, we discuss related work (§7) and conclude (§8). Insights behind the analyses of our algorithms can be found in the appendices.

2 CONSENSUSLESS ASSET TRANSFER WITH AT2\textsubscript{SM}

In this section we formally define the asset transfer problem and discuss its consensus number. We begin with presenting the shared memory model we use in this section (§2.1) and then precisely define the problem of asset-transfer as a sequential object type (§2.2).

Intuitively, an asset-transfer object consists of accounts whose balances can be read by all processes. Processes are also allowed to transfer assets between accounts, where each account is associated with a subset of processes (owners) that are allowed to issue transfers debiting this account.

Finally, we prove that an object of this type where each account has a single owner has consensus number \textit{one}. We do so by proposing a wait-free implementation of this object called AT2\textsubscript{SM} (§2.3). The interested reader is referred to §6 that generalizes this result, proving that an object containing an account with $k$ owners has consensus number $k$. 
2.1 Shared Memory Model Definitions

Processes. As our basic system model, we assume a set $\Pi$ of $N$ asynchronous processes that communicate by invoking atomic operations on shared memory objects. Processes are sequential—we assume that a process never invokes a new operation before obtaining a response from a previous one.

Object types. A sequential object type is defined as a tuple $T = (Q, q_0, O, R, \Delta)$, where $Q$ is a set of states, $q_0 \in Q$ is an initial state, $O$ is a set of operations, $R$ is a set responses and $\Delta \subseteq Q \times \Pi \times O \times Q \times R$ is a relation that associates a state, a process identifier and an operation to a set of possible new states and corresponding responses. Here we assume that $\Delta$ is total on the first three elements, i.e., for each state $q \in Q$, each process $p \in \Pi$ and each operation in $o \in O$, some transition to a new state is defined, i.e., $\exists q' \in Q, r \in R: (q, p, o, q', r) \in \Delta$.

A history is a sequence of invocations and responses, each invocation or response associated with a process identifier, and a sequential history is a history that starts with an invocation and in which every invocation is immediately followed with a response associated with the same process. A sequential history $(j_1, o_1), (j_1, r_1), (j_2, o_2), (j_2, r_2), \ldots$, where $\forall i \geq 1, j_i \in \Pi, o_i \in O, r_i \in R$, is legal with respect to type $T = (Q, q_0, O, R, \Delta)$ if there exists a sequence $q_1, q_2, \ldots$ of states in $Q$ such that $\forall i \geq 1, (q_{i-1}, j_i, o_i, q_i, r_i) \in \Delta$.

Implementations. An implementation of an object type $T$ is a distributed algorithm that, for each process and invoked operation, prescribes the actions that the process needs to take to perform it. An execution of an implementation is a sequence of events: invocations and responses of operations, send and receive events, or atomic accesses to shared abstractions. The sequence of events at every process must respect the algorithm assigned to it.

Failures. Processes are subject to crash failures. A process may halt prematurely, in which case we say that the process is crashed. A process is called faulty if it crashes during the execution. A process is correct if it is not faulty. All algorithms we present in the shared memory model are wait-free—every correct process eventually returns from each operation it invokes, regardless of an arbitrary number of other processes crashing.

Linearizability and sequential consistency. For each pattern of operation invocations, the execution produces a history, i.e., the sequence of distinct invocations and responses, labelled with process identifiers and unique sequence numbers.

A projection of a history $H$ to process $p$, denoted $H|p$ is the subsequence of elements of $H$ labelled with $p$. An invocation $o$ by a process $p$ is incomplete in $H$ if it is not followed by a response in $H|p$. A history is complete if it has no incomplete invocations. A completion of $H$ is a history $\bar{H}$
that is identical to \( H \) except that every incomplete invocation in \( H \) is either removed or completed by inserting a matching response somewhere after it.

A **sequentially consistent** implementation of \( T \) ensures that for every history \( H \) it produces, there exists a completion \( \bar{H} \) and a legal sequential history \( S \) such that for all processes \( p \), \( H[p] = S[p] \).

A **linearizable** implementation, additionally, preserves the real-time order between operations. Formally, an invocation \( o_1, r_1 \) precedes an invocation \( o_2 \) in \( H \), denoted \( o_1 <_H o_2 \), if \( o_1 \) is complete and the corresponding response \( r_1 \) precedes \( o_2 \) in \( H \). Note that \( <_H \) stipulates a partial order on invocations in \( H \). A linearizable implementation of \( T \) ensures that for every history \( H \) it produces, there exists a completion \( \bar{H} \) and a legal sequential history \( S \) such that (1) for all processes \( p \), \( \bar{H}[p] = \bar{S}[p] \) and (2) \( <_H \subseteq <_S \).

A (sequentially consistent or linearizable) implementation is \( t \)-**resilient** if, under the assumption that at most \( t \) processes crash, it ensures that every invocation performed by a correct process is eventually followed by a response. In the special case when \( t = n-1 \), we say that the implementation is \( \text{wait-free} \).

### 2.2 Asset transfer type

Let \( \mathcal{A} \) be a set of accounts and \( \mu : \mathcal{A} \rightarrow 2^\Pi \) be an “owner” map that associates each account with a set of processes that are, intuitively, allowed to debit the account. The asset-transfer object type associated with \( \mathcal{A} \) and \( \mu \) is then defined as a tuple \((Q, q_0, O, R, \Delta)\), where:

- The set of states \( Q \) is the set of all possible maps \( q : \mathcal{A} \rightarrow \mathbb{N} \). Intuitively, each state of the objects assigns each account with its balance.
- The initialization map \( q_0 : \mathcal{A} \rightarrow \mathbb{N} \) assigns the initial balance to each account.
- Operations and responses of the type are defined as \( O = \{ \text{transfer}(a, b, x) : a, b \in \mathcal{A}, x \in \mathbb{N} \} \) and \( R = \{ \text{true}, \text{false} \} \cup \mathbb{N} \).
- For a state \( q \in Q \), a proces \( p \in \Pi \), an operation \( o \in O \), a response \( r \in R \) and a new state \( q' \in Q \), the tuple \((q, p, o, q', r)\) is \( \Delta \) if and only if one of the following conditions is satisfied:
  - \( o = \text{transfer}(a, b, x) \land p \in \mu(a) \land q(a) \geq x \land q'(a) = q(a) - x \land q'(b) = q(b) + x \land \forall c \in \mathcal{A} \setminus \{a, b\} : q'(c) = q(c) \) (all other accounts unchanged) \( \land r = \text{true} \);
  - \( o = \text{transfer}(a, b, x) \land (p \notin \mu(a) \lor q(a) < x) \land q' = q \land r = \text{false} \);
  - \( o = \text{read}(a) \land q = q' \land r = q(a) \).

In other words, operation \( \text{transfer}(a, b, x) \) invoked by process \( p \) succeeds if and only if \( p \) is the owner of the source account \( a \) and account \( a \) has enough balance, and if it does, \( x \) is transferred from \( a \) to the destination account \( b \). A \( \text{transfer}(a, b, x) \) operation is called outgoing for \( a \) and incoming for \( b \); respectively, the \( x \) units are called outgoing for \( a \) and incoming for \( b \). A transfer is successful
if its corresponding response is \textit{true} and \textit{failed} if its corresponding response is \textit{false}. Operation \textit{read}(a) simply returns the balance of \(a\) and leaves the accounts untouched.

\subsection*{2.3 Asset Transfer Is Easier than Consensus}

In this section, we discuss the “synchronization power” of the asset-transfer type in what we believe to be the most typical use case: each account being associated with a single owner process. We show that such a “1-owned” asset-transfer object type can be implemented in the \textit{wait-free} manner using only read-write registers. Thus, the type has consensus number 1. In §6 we generalize our result and show that a “\(k\)-owned” asset-transfer object has consensus number \(k\).

For now, consider an asset-transfer object associated with a set of accounts \(\mathcal{A}\) and an ownership map \(\mu\) such that \(\forall a \in \mathcal{A}, |\mu(a)| \leq 1\). We now present \(\text{AT2}_{SM}\), our algorithm that implements this object in the read-write shared-memory model.

The implementation is described in Figure 2. The idea is very simple. The \(n\) processes share an atomic snapshot object \([4]\) of size \(N\). Every process \(p\) is associated with a distinct location in the atomic snapshot object storing the sequence of all successful transfer operations executed by \(p\) so far. Since each account is owned by at most one process, all outgoing transfers for an account appear in a single location of the atomic snapshot (associated with the owner process).

Recall that the atomic snapshot (AS) memory is represented as a vector of \(N\) shared variables. The memory can be accessed with two atomic operations: \textit{update} and \textit{snapshot}. An \textit{update} operation modifies the value at a given position of the vector and a \textit{snapshot} returns the state of the vector.

To read the balance of an account \(a\), the process simply takes a snapshot \(S\) and computes the initial balance plus the sum of incoming units minus the sum of all outgoing units. We denote this number by \(\text{balance}(a, S)\). As we argue below, the result is guaranteed to be non-negative, i.e., the operation is correct with respect to the type specification.

To perform \(\text{transfer}(a, b, x)\), a process \(p\), the owner of \(a\), takes a snapshot \(S\) and computes \(\text{balance}(a, S)\), i.e., the balance of \(a\) based on all operations in \(S\) that concern \(a\) (transfer assets from or to \(a\)). If the amount to be transferred does not exceed \(\text{balance}(a, S)\), the operation is appended to the list of \(p\)’s operations in the snapshot object via an \textit{update} operation and \textit{true} is returned. Otherwise, the operation returns \textit{false}.

\textbf{Theorem 1}. A single-owned asset-transfer object type has a \textit{wait-free} implementation in the read-write shared memory model.

\textbf{Proof}. Fix an execution \(E\) of the algorithm in Figure 2. Atomic snapshots can be \textit{wait-free} implemented in the read-write shared memory model \([4]\). As every operation only involves a finite number of atomic snapshot accesses, every process completes each of the operations it invokes in a finite number of its own steps.
Shared variables:
   \( AS, \) atomic snapshot, initially \( \{ \bot \}^N \)

Local variables:
\[ ops_p \in (A \times A \times \mathbb{N^*})^* \]

Upon \( transfer(a, b, x) \)
1. \( S := AS.snapshot() \)
2. \( \text{if } p \notin \mu(a) \lor \text{balance}(a, S) < x \text{ then} \)
3. \( \text{return } \text{false} \)
4. \( ops_p := ops_p \cdot (a, b, x) \)
5. \( AS.update(ops_p) \)
6. \( \text{return } \text{true} \)

Upon \( read(a) \)
7. \( S := AS.snapshot() \)
8. \( \text{return } \text{balance}(a, S) \)

Fig. 2. \( AT2_{SM} \): Wait-free single-owned asset-transfer: code for process \( p \)

Let \( Ops \) be the set of:

- all invocations of \( transfer \) or \( read \) in \( E \) that returned, and
- all invocations of \( transfer \) in \( E \) that completed the \textit{update} operation (line 5) (the atomic snapshot operation has been linearized).

Let \( H \) be the history of \( E \). We define a completion of \( H \) and, for each \( o \in Ops \), we define a linearization point as follows:

- If \( o \) is a \textit{read} operation, it linearizes at the linearization point of the \textit{snapshot} operation in line 7.
- If \( o \) is a \textit{transfer} operation that returns \textit{false}, it linearizes at the linearization point of the \textit{snapshot} operation in line 1.
- If \( o \) is a \textit{transfer} operation that completed the \textit{update} operation, it linearizes at the linearization point of the \textit{update} operation in line 5. If \( o \) is incomplete in \( H \), we complete it with response \textit{true}.

Let \( \bar{H} \) be the resulting complete history and let \( L \) be the sequence of complete invocations of \( \bar{H} \) in the order of their linearization points in \( E \). Note that, by the way we linearize invocations, the linearization of a prefix of \( E \) is a prefix of \( L \).
Now we show that $L$ is legal and, thus, $H$ is linearizable. We proceed by induction, starting with the empty (trivially legal) prefix of $L$. Let $L_\ell$ be the legal prefix of the first $\ell$ invocations and $op$ be the $(\ell + 1)$st operation of $L$. Let $op$ be invoked by process $p$. The following cases are possible:

- $op$ is a read$(a)$: the snapshot taken at the linearization point of $op$ contains all successful transfers concerning $a$ in $L_\ell$. By the induction hypothesis, the resulting balance is non-negative.
- $op$ is a failed transfer$(a, b, x)$: the snapshot taken at the linearization point of $op$ contains all successful transfers concerning $a$ in $L_\ell$. By the induction hypothesis, the resulting balance is non-negative.
- $op$ is a successful transfer$(a, b, x)$: by the algorithm, before the linearization point of $op$, process $p$ took a snapshot. Let $L_k$, $k \leq \ell$, be the prefix of $L_\ell$ that only contain operations linearized before the moment of time when the snapshot was taken by $p$.

We observe that $L_k$ includes a subset of all incoming transfers on $a$ and all outgoing transfers on $a$ in $L_\ell$. Indeed, as $p$ is the owner of $a$ and only the owner of $a$ can perform outgoing transfers on $a$, all outgoing transfers in $L_\ell$ were linearized before the moment $p$ took the snapshot within $op$. Thus, $\text{balance}(a, L_k) \leq \text{balance}(a, L_\ell)$.

By the algorithm, as $op = \text{transfer}(a, b, x)$ succeeds, we have $\text{balance}(a, L_k) \geq x$. Thus, $\text{balance}(a, L_\ell) \geq x$ and the resulting balance in $L_{\ell+1}$ is non-negative.

Thus, $H$ linearizable. □

**Corollary 2.** The asset-transfer object type has consensus number 1.

### 3 AT2 IN THE MESSAGE PASSING MODEL

In this section we abandon the crash-stop shared memory model that we only used for reasoning about the consensus number of the asset-transfer data type. Instead, we consider a distributed system where processes communicate by sending messages through authenticated channels and where a limited fraction of processes may behave in an arbitrary (Byzantine) manner. In this setting, Byzantine processes may attempt to double-spend, i.e., initiate transfers which cannot be justified by the balance in their accounts.

We first refine the specification of our asset transfer abstraction to adapt it to the Byzantine message passing model. We then present present AT2$_{MP}$, an algorithm that implements this abstraction.

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$^{1}$ Analogously to $\text{balance}(a, S)$ that computes the balance for account $a$ based on the transfers contained in snapshot $S$, $\text{balance}(a, L)$, if $L$ is a sequence of operations, computes the balance of account $a$ based on all transfers in $L$. 
3.1 Asset Transfer in Message Passing with Byzantine Faults

We now refine our asset transfer specification for the Byzantine environment. We only require that the transfer system behaves correctly towards benign processes, regardless of the behavior of Byzantine ones. We say that a process is benign if it respects the algorithm and can only fail by crashing.

Informally, we require that every system execution appears to benign processes as a correct sequential execution. As a result, no benign process can be a victim of a double-spending attack.

Moreover, we require that this execution respects the real-time order among operations performed by benign processes [35]: when such an operation completes it should be visible to every future operation invoked by a benign process.

For the sake of efficiency, in our algorithm (§3), we slightly relax the last requirement—while still preventing double-spending. We require that successful transfer operations invoked by benign processes constitute a legal sequential history that preserves the real-time order. A read or a failed transfer operation invoked by a benign process \( p \) can be “outdated”—it can be based on a stale state of \( p \)’s balance. Informally, one can view the system requirements as linearizability [36] for successful transfers and sequential consistency [9] for failed transfers and reads. As progress (liveness) guarantees, we require that every operation invoked by a correct process eventually completes. One can argue that this relaxation incurs little impact on the system’s utility, as long as all incoming transfers are eventually applied.

3.2 AT2\( _{MP} \) Overview

We now present AT2\( _{MP} \), a decentralized algorithm that implements secure transfers in a message passing system subject to Byzantine faults. Instead of consensus, AT2\( _{MP} \) relies on a causally ordered broadcast which precisely captures the semantics of the transfer applications.

Figure 3 depicts the high-level modules of AT2\( _{MP} \). There are two main modules: one for tracking dependencies (i.e., the applied incoming transfers), plus an underlying secure broadcast protocol. For simplicity, in this section we present only the basic transfer algorithm and assume an existing secure broadcast protocol as a black box (which we will describe later). Depending on the exact system model and the properties of this broadcast protocol, we obtain two variants of AT2\( _{MP} \).

Concretely, we consider a deterministic secure broadcast algorithm, which we use to obtain AT2\( _D \) (§4), and a probabilistic secure broadcast protocol, which underlies AT2\( _P \) (§5).

Our algorithm works as follows. To perform a transfer, a process \( p \) broadcasts a message with the transfer details. These details match the arguments of the transfer operation (see §2.2), and consist of the outgoing account (in this case, the account of \( p \)), the incoming account, and the transferred amount.
To ensure the authenticity of operations—so that no process is able to debit another process’s account—we assume that processes sign all their messages before broadcasting them. In practice, similar to Bitcoin and other transfer systems, every process possesses a public-private key pair that allows only $p$ to securely initiate transfers from its corresponding account.

Recall that in a secure transfer system, a correct process should not accept a transfer $P$ before it accepts all transfers $P$ depends on. In particular, transfers originating at the same process $p$ do not commute, and thus must be delivered by all correct processes in the same order. This property is known as the source order, and we ensure it by relying on secure broadcast \[46\].

Secure broadcast, by itself, is not sufficient to ensure that causality is preserved, because there is an additional scenario in which two transfers do not commute. Suppose that process $p$ has initially zero balance and receives money in a transfer $P_1$. Thereafter, process $p$ immediately sends money to process $q$ in a transfer $P_2$. If process $q$ tries to apply $P_2$ before being aware of $P_1$, then $q$ would consider the former transfer invalid due to insufficient funds. Thus, $P_2$ depends on $P_1$ and all processes in the system should apply $P_1$ before $P_2$. We enforce this ordering by attaching a dependency set—called a history—to every transfer. Before delivering a transfer, we require every process to deliver the history attached to that transfer. Note that in practice, this history need not contain the full details of other transfers, but merely their identifiers (in a similar vein as vector clocks \[37\]).

### 3.3 The AT2$_{MP}$ Algorithm

Figure 4 describes our AT2$_{MP}$ consensusless algorithm that implements a transfer system (as defined in §3.1). Each process $p$ maintains, for each process $q$, an integer $seq[q]$ reflecting the number of transfers which process $q$ has initiated and which process $p$ has applied. Process $p$ also keeps, for every process $q$, an integer $rec[q]$ which reflects the number of transfers which process $q$ has initiated and process $p$ has observed (but not necessarily applied).
Local variables:
rec, initially \( \text{rec}[q] = 0, \forall q \) [received transfers from \( q \)]
seq, initially \( \text{seq}[q] = 0, \forall q \) [validated transfers from \( q \)]
hist, initially \( \text{hist}[q] = \emptyset, \forall q \) [Validated transfers involving \( q \)]
deps, initially \( \emptyset \) [Latest incoming transfers]
toValidate, initially \( \emptyset \) [Received but not validated transfers]

\[
\text{operation} \quad \text{transfer}(a, b, x) \quad \text{where} \quad a = p \quad \\
\quad \text{return} \quad \text{false} \quad \text{if} \quad \text{balance}(a, \text{hist}(p) \cup \text{deps}) < x \quad \text{then} \quad \text{broadcast}([(a, b, x, \text{seq}(p) + 1), \text{deps}]) \quad \\
\quad \text{deps} := \emptyset \quad \\
\text{operation} \quad \text{read}(a) \quad \\
\quad \text{return} \quad \text{balance}(a, \text{hist}(a) \cup \text{deps}) \quad \\
\text{upon} \quad \text{deliver}(q, m) \quad \text{Executed when p delivers message m from process q} \quad \text{upon} \quad (q, [t, h]) \in \text{toValidate} \land \text{Valid}(q, t, h) \quad \\
\quad \text{hist}(q) := \text{hist}(q) \cup h \cup t \quad \\
\quad \text{seq}(q) := s \quad \\
\quad \text{if} \quad d = p \quad \text{then} \quad \text{deps} := \text{deps} \cup (c, d, y, s) \quad \text{Transfer incoming to account of local process p} \quad \\
\quad \text{if} \quad c = p \quad \text{return} \quad \text{true} \quad \text{Transfer outgoing from account of local process p} \quad \\
\text{function} \quad \text{Valid}(q, t, h) \quad \text{function} \quad \text{balance}(a, h) \quad \\
\quad \text{return} \quad (q = c) \quad \\
\quad \text{and} \quad (s = \text{seq}(q) + 1) \quad \\
\quad \text{and} \quad (\text{balance}(c, \text{hist}(q) \cup h) \geq y) \quad \\
\quad \text{and} \quad (h \subseteq \text{hist}(q)) \quad \\
\quad \text{sum of incoming transfers minus outgoing transfers for account a in h} \quad \\
\]

Fig. 4. AT2\(_{MP}\): an algorithm for a consensusless transfer system based on secure broadcast. Code for every process \( p \).

Additionally, there is also a list \( \text{hist}[q] \) of transfers which involve process \( q \). We say that a transfer operation involves a process \( q \) if that transfer is either outgoing or incoming on the account of \( q \).
Finally, process $p$ maintains as well a local variable $deps$ which is a set of incoming transfers for $p$, and which $p$ has applied since it made the last successful transfer.

To perform a transfer operation, process $p$ first checks the balance of its own account, and if there is not enough funding, i.e., the balance is insufficient, returns false (line 11). Otherwise, process $p$ broadcasts a message with this operation via the secure broadcast primitive (line 12). This message includes the three basic arguments of a transfer operation as well as $seq[p] + 1$ and $deps$. Each correct process in the system eventually delivers this message via a callback from the secure broadcast primitive (line 16). Upon delivery, this message is checked for well-formedness (lines 17 and 18), and then added to the set of messages to be validated. We explain the validation procedure later.

Once a transfer from a process $q$ is validated (the predicate in line 29 is satisfied), it is added to the corresponding $hist[q]$ (line 22). We consider that a transfer is applied when it is added to $hist[q]$. If the transfer is incoming for $p$, it is also added to $deps$, the set of $p$’s current dependencies (line 26). If the transfer is outgoing for $p$, i.e., it is the currently pending transfer operation invoked by $p$, the response true is returned (line 28).

To perform a read($a$) operation for account $a$, process $p$ simply computes the balance of this account based on the local history $hist[a]$.

Before appending an operation $op$ from some process $q$ to $hist[q]$ (line 22), process $p$ validates $op$ (line 21) via the Valid function. To be valid, $op$ must satisfy four conditions. The first is condition is that $q$ (the issuer of the transfer) is the owner of the corresponding account (line 31). Second, the preceding transfer from $q$ must have been validated (line 32). Third, that balance of account $q$ does not drop below zero (line 33). Finally, the dependencies of $op$ (encoded in $h$ of line 34) have been validated and put in $hist[q]$.

## 4 AT2$_D$: THE DETERMINISTIC CASE

AT2$_D$ implements a transfer algorithm for the deterministic system model. It builds on a secure broadcast algorithm which assumes an asynchronous network of $N$ processes, where less than $N/3$ of processes can be Byzantine. We briefly discuss this broadcast algorithm here, while the rest of the transfer algorithm is identical to the one we described earlier (Figure 4).

### 4.1 Deterministic Secure Broadcast

Secure broadcast for the deterministic model has its roots in the Asynchronous Byzantine Agreement (ABA) problem, defined by Bracha and Toueg [18]. ABA is a single-shot abstraction for agreeing on the content of a single message broadcast from a designated sender. Informally, secure broadcast is a multi-shot version of ABA, and guarantees that messages broadcast by a given (correct) sender are delivered by all correct processes in the same order.
There are multiple algorithms for implementing secure broadcast in deterministic system models, e.g., the double-echo algorithm, initially described by Bracha [17] as a single-shot version (which appears in several other works, including practical systems [19, 20, 26]), as well as the secure reliable multicast of Reiter [54], which relies on digital signatures and which was optimized in several aspects [46, 47].

Below we sketch a protocol for secure broadcast which we use in AT2\textsubscript{D}. This protocol is not novel (unlike the probabilistic version we use for AT2\textsubscript{P} in §5), and it draws directly from the signature-based algorithm due to Malkhi and Reiter [47]. We chose to use this protocol for its simplicity. This description uses an underlying reliable broadcast primitive [19], and we use the terms reliable-broadcast and reliable-deliver to denote the invocation and callback of this primitive.

To broadcast a message $m$ securely, a process $s$ attaches a sequence number $i$ to that message and then disseminates the tuple $(m, i)$ using reliable-broadcast. Upon reliable-delivery of $(m, i)$, every process $p$ checks whether this is the first time it sees sequence number $i$ from process $s$. If yes, process $p$ replies directly to $s$ with a signed acknowledgment of the tuple $(m, i)$. When process $s$ receives acknowledgments from a quorum of more than two thirds of the processes, $s$ broadcasts the set of obtained acknowledgments using reliable-broadcast. Any process can deliver $m$ after obtaining the correct set of acknowledgments for $(m, i)$, and after having delivered all messages from $s$ that have sequence numbers smaller than $i$.

Intuitively, this algorithm ensures that any two delivered messages are “witnessed” by at least one correct process. This way we ensure that messages from the same source, whether benign or Byzantine, are delivered by benign processes in the same order.

5 AT2\textsubscript{P}: THE PROBABILISTIC CASE

AT2\textsubscript{P} implements an asset transfer algorithm in a probabilistic system model, by building on a novel secure broadcast algorithm with probabilistic guarantees. In the rest of this section we first discuss our system model more precisely (§5.1). We then present the secure broadcast algorithm in three steps. First, we introduce an abstraction for broadcasting a single message with probabilistic delivery guarantees (§5.2). Second, we refine this abstraction to make it consistent, ensuring that processes deliver the same single message (§5.3). Third, we obtain probabilistic secure broadcast as a multi-shot abstraction over consistent broadcast (§5.4).

5.1 Preliminaries

We discuss the system model, including assumptions we make on the network and on the information available both to correct and Byzantine processes. We also introduce notation that will be valid throughout the rest of this section.

For all our probabilistic algorithms, we have the following assumptions:
The set $\Pi$ of processes partaking in the algorithm is fixed. Unless stated otherwise, we let $N = |\Pi|$ denote the number of processes, and refer to the $i$-th process as $\pi_i \in \Pi$.

Any two processes can communicate via reliable, authenticated, point-to-point links [19].

At most a fraction $f$ of the processes is Byzantine, i.e., subject to arbitrary failures. Byzantine processes are under the control of the same adversary, and can take coordinated action. We also assume that the adversary does not have access to the output of local randomness sources of correct processes. Unless stated otherwise, we let $\Pi_C \subseteq \Pi$ denote the set of correct processes and $C = |\Pi_C| = \lceil (1 - f)N \rceil$ denote the number of correct processes.

Byzantine processes can cause arbitrary but finite delays on any link, including links between pairs of correct processes.

Byzantine processes cannot determine which correct processes another correct process is communicating with.

Every process has direct access to an oracle $\Omega$ that, provided with an integer $n \leq N$, yields the public keys of $n$ distinct processes, chosen uniformly at random from $\Pi$.

We later weaken assumption (1) into an inequality, as we generalize our results to systems with slow churn. Assumption (4) represents one of the main strengths of this work: messages can be delayed arbitrarily and maliciously without compromising the security property of any of the algorithms presented in this work. Assumption (5) represents the strongest constraint we put on the knowledge of the adversary. We later show that, without this assumption, an adversary could easily poison the view of the system of a targeted correct process without having to interfere with any local randomness source. Even against ISP-grade adversaries, assumption (5) can be implemented in practice by means of, e.g., onion routing [58] or private messaging [61] algorithms. Assumption (6) reduces, in the permissioned case, to randomly sampling an exhaustive list of processes. We later discuss how a membership sampling algorithm can be used, in conjunction with Sybil resistance strategies, to implement the sampling oracle in the permissionless case (see §5.5).

### 5.2 Probabilistic broadcast

In this section, we introduce the probabilistic broadcast abstraction and discuss its properties. This abstraction serves the purpose of reliably broadcasting a single message from a designated (correct) sender to all correct processes. We then present Erdős-Rényi Gossip, a probabilistic algorithm that implements probabilistic broadcast, and evaluate its security and complexity as a function of its parameters. We use probabilistic broadcast in the implementation of Probabilistic
Double-Echo (see §5.3) to initially distribute a message from the designated sender to all correct processes.

5.2.1 Definition. The probabilistic broadcast interface (instance $pb$, sender $\sigma$) exposes the following two events:

- **Request** $(pb.\text{Broadcast} \mid m)$: Broadcasts a message $m$ to all processes. This is only used by $\sigma$.
- **Indication** $(pb.\text{Deliver} \mid m)$: Delivers a message $m$ broadcast by process $\sigma$.

For any $\epsilon \in [0, 1]$, Probabilistic broadcast is $\epsilon$-secure if:

1. **No duplication**: No correct process delivers more than one message.
2. **Integrity**: If a correct process delivers a message $m$, and $\sigma$ is correct, then $m$ was previously broadcast by $\sigma$.
3. **$\epsilon$-Validity**: If $\sigma$ is correct, and $\sigma$ broadcasts a message $m$, then $\sigma$ eventually delivers $m$ with probability at least $(1 - \epsilon)$.
4. **$\epsilon$-Totality**: If a correct process delivers a message, then every correct process eventually delivers a message with probability at least $(1 - \epsilon)$.

5.2.2 Algorithm. Algorithm 1 lists the implementation of Erdős-Rényi Gossip. This algorithm distributes a single message\footnote{Note that one instance of probabilistic broadcast only distributes a single message. To disseminate multiple messages, we use multiple instances of probabilistic broadcast.} across the system by means of gossip: upon reception, a correct process relays the message to a set of randomly selected neighbors. The algorithm depends on one integer parameter, $G$ (expected gossip sample size), whose value we study in §A.

**Initialization.** Upon initialization, (line 11) a correct process randomly samples a value $\tilde{G}$ from a Poisson distribution with expected value $G$, and uses the sampling oracle $\Omega$ to select $\tilde{G}$ distinct processes that it will use to initialize its gossip sample $G$.

**Link reciprocation.** Once its gossip sample is initialized, a correct process sends a GossipSubscribe message to all the processes in $G$ (line 13). Upon receiving a GossipSubscribe message from a process $\pi$ (line 17), a correct process adds $\pi$ to its own gossip sample $G$ (line 22), and sends back the gossiped message if it has already received it (line 20).

**Gossip.** When broadcasting the message (line 34), a correct designated sender $\sigma$ signs the message and sends it to every process in its sample $G$ (line 28). Upon receiving a correctly signed message from $\sigma$ (line 37) for the first time (this is enforced by updating the value of $\text{delivered}$, line 25), a correct process delivers it (line 30) and forwards it to every process in its gossip sample (line 28).
Algorithm 1 Erdös-Rényi Gossip

1: Implements:
2: ProbabilisticBroadcast, instance pb
3:
4: Uses:
5: AuthenticatedPointToPointLinks, instance al
6:
7: Parameters:
8: $G$: expected gossip sample size
9:
10: upon event (pb.Init) do
11: $\mathcal{G} = \Omega(\text{poisson}(G))$;
12: for all $\pi \in \mathcal{G}$ do
13: trigger (al.Send | $\pi$, [GossipSubscribe]);
14: end for
15: delivered = ⊥;
16:
17: upon event (al.Deliver | $\pi$, [GossipSubscribe]) do
18: if delivered $\neq$ ⊥ then
19: (message, signature) $\leftarrow$ delivered;
20: trigger (al.Send | $\pi$, [Gossip, message, signature]);
21: end if
22: $\mathcal{G} \leftarrow \mathcal{G} \cup \{\pi\}$;
23:
24: procedure dispatch(message, signature) is
25: if delivered $= \bot$ then
26: delivered $\leftarrow$ (message, signature);
27: for all $\pi \in \mathcal{G}$ do
28: trigger (al.Send | $\pi$, [Gossip, message, signature]);
29: end for
30: trigger (pb.Deliver | message)
31: end if
32:
33: upon event (pb.Broadcast | message) do
34: dispatch(message, sign(message));
35: $\triangleright$ only process $\sigma$
36: upon event (al.Deliver | $\pi$, [Gossip, message, signature]) do
37: if verify($\sigma$, message, signature) then
38: dispatch(message, signature);
39: end if
40:
For a discussion on the correctness of Erdős-Rényi Gossip, we refer the interested reader to §A.

5.3 Probabilistic consistent broadcast

In this section, we introduce the probabilistic consistent broadcast abstraction and discuss its properties. We then present Probabilistic Double-Echo, a probabilistic algorithm that implements probabilistic consistent broadcast, and evaluate its security and complexity as a function of its parameters.

The probabilistic consistent broadcast abstraction allows the set of correct processes to agree on a message from a designated sender, potentially Byzantine. Probabilistic consistent broadcast is a strictly stronger abstraction than probabilistic broadcast. Probabilistic broadcast only guarantees that, if the sender is correct, all correct processes deliver its message and, if any correct process delivers a message, every correct process delivers a message. Probabilistic consistent broadcast also guarantees that, even if the sender is Byzantine, no two correct processes will deliver different messages.

We use probabilistic consistent broadcast in the implementation of Sequenced Probabilistic Double-Echo (see §5.4) as a way to consistently broadcast sequenced messages.

5.3.1 Definition. The probabilistic consistent broadcast interface (instance pcb, sender σ) exposes the following two events:

• **Request**: ⟨pcb.Broadcast | m⟩: Broadcasts a message m to all processes. This is only used by σ.

• **Indication**: ⟨pcb.Deliver | m⟩: Delivers a message m broadcast by process σ.

For any $\epsilon \in [0, 1]$, Probabilistic consistent broadcast is $\epsilon$-secure if:

1. **No duplication**: No correct process delivers more than one message.
2. **Integrity**: If a correct process delivers a message m, and σ is correct, then m was previously broadcast by σ.
3. **$\epsilon$-Validity**: If σ is correct, and σ broadcasts a message m, then σ eventually delivers m with probability at least $(1 - \epsilon)$.
4. **$\epsilon$-Totality**: If a correct process delivers a message, then every correct process eventually delivers a message with probability at least $(1 - \epsilon)$.
5. **$\epsilon$-Consistency**: Every correct process that delivers a message delivers the same message with probability at least $(1 - \epsilon)$.

5.3.2 Algorithm. Algorithm 2 lists the implementation of Probabilistic Double-Echo.
Algorithm 2 Probabilistic Double-Echo

1: Implements:
2: ProbabilisticConsistentBroadcast, instance pcb
3: Uses:
4: AuthenticatedPointToPointLinks, instance al
5: ProbabilisticBroadcast, instance pb
6: Parameters:
7: \( E \): echo sample size \( \hat{E} \): ready threshold
8: \( R \): ready sample size \( \hat{R} \): feedback threshold
9: \( D \): delivery sample size \( \hat{D} \): delivery threshold
10: procedure sample(message, size) is
11: \( \psi = \emptyset \);
12: for size times do
13: \( \psi \leftarrow \psi \cup \Omega(1) \);
14: end for
15: for all \( \pi \in \psi \) do
16: trigger \((al.Send \mid \pi, [message])\);
17: end for
18: return \( \psi \);
19: upon event \((pcb.Init)\) do
20: \( \text{echo} = \perp; \quad \text{ready} = \perp; \quad \text{delivered} = \text{false}; \)
21: \( \mathcal{E} = \text{sample}(EchoSubscribe, E); \quad \text{replies.echo} = \{\perp\}^E; \)
22: \( \mathcal{R} = \text{sample}(ReadySubscribe, R); \quad \text{replies.ready} = \{\perp\}^R; \)
23: \( \mathcal{D} = \text{sample}(ReadySubscribe, D); \quad \text{replies.delivery} = \{\perp\}^D; \)
24: \( \tilde{\mathcal{E}} = \emptyset; \quad \tilde{\mathcal{R}} = \emptyset; \)
25: upon event \((al.Deliver \mid \pi, [EchoSubscribe])\) do
26: if \( \text{echo} \neq \perp \) then
27: \( (message, signature) = \text{echo}; \)
28: trigger \((al.Send \mid \pi, [Echo, message, signature])\);
29: end if
30: \( \tilde{\mathcal{E}} \leftarrow \tilde{\mathcal{E}} \cup \{\pi\} \);
upon event \( \langle al.\text{Deliver} \mid \pi, [\text{ReadySubscribe}] \rangle \) do

\begin{enumerate}
\item \( \text{if } \text{ready} \neq \bot \text{ then} \)
\item \( (\text{message, signature}) = \text{ready}; \)
\item \( \text{trigger} \langle al.\text{Send} \mid \pi, [\text{Ready, message, signature}] \rangle; \)
\end{enumerate}

end if

\( \mathring{\mathcal{R}} \leftarrow \mathring{\mathcal{R}} \cup \{ \pi \}; \)

upon event \( \langle pcb.\text{Broadcast} \mid \text{message} \rangle \) do

\( \triangleright \text{only process } \sigma \)

\( \text{trigger} \langle pb.\text{Broadcast} \mid [\text{Send, message, sign(message)}] \rangle; \)

upon event \( \langle pb.\text{Deliver} \mid [\text{Send, message, signature}] \rangle \) do

\( \text{if } \text{verify}(\sigma, \text{message, signature}) \text{ then} \)

\( \text{echo} \leftarrow (\text{message, signature}); \)

\( \text{for all } \rho \in \mathring{\mathcal{E}} \text{ do} \)

\( \text{trigger} \langle al.\text{Send} \mid \rho, [\text{Echo, message, signature}] \rangle; \)

end for

end if

upon event \( \langle al.\text{Deliver} \mid \pi, [\text{Echo, message, signature}] \rangle \) do

\( \text{if } \pi \in \mathcal{E} \text{ and } \text{verify}(\sigma, \text{message, signature}) \text{ and } \text{replies.echo}[\pi] = \bot \text{ then} \)

\( \text{replies.echo}[\pi] \leftarrow (\text{message, signature}); \)

end if

upon exists \( \text{message} \text{ such that } |\{ \rho \in \mathcal{E} \mid \text{replies.echo}[\rho] = (\text{message, signature}) \}| \geq \hat{\mathcal{E}} \text{ and ready} = \bot \) do

\( \text{ready} \leftarrow (\text{message, signature}); \)

\( \text{for all } \rho \in \mathring{\mathcal{R}} \text{ do} \)

\( \text{trigger} \langle al.\text{Send} \mid \rho, [\text{Ready, } \sigma, \text{message, signature}] \rangle; \)

end for

upon event \( \langle al.\text{Deliver} \mid \pi, [\text{Ready, message, signature}] \rangle \) do

\( \text{if } \text{verify}(\sigma, \text{message, signature}) \text{ then} \)

\( \text{if } \pi \in \mathcal{R} \text{ and } \text{replies.ready}[\pi] = \bot \text{ then} \)

\( \text{replies.ready}[\pi] \leftarrow (\text{message, signature}); \)

end if

\( \text{if } \pi \in \mathcal{D} \text{ and } \text{replies.delivery}[\pi] = \bot \text{ then} \)

\( \text{replies.delivery}[\pi] \leftarrow (\text{message, signature}); \)

end if

end if
This algorithm consistently distributes a single message\(^3\) across the system, as follows:

- Initially, probabilistic broadcast distributes potentially conflicting copies of the message to every correct process.
- Upon receiving a message \(m\) from probabilistic broadcast, a correct process issues an Echo message for \(m\).
- When enough Echo or Ready messages have been collected for the same message \(m\), a correct process issues a Ready message for \(m\).
- When enough Ready messages have been collected for the same message \(m\), a correct process delivers \(m\).

A correct process collects Echo and Ready messages from three randomly selected samples (echo sample, ready sample and delivery sample). The sizes of these samples are determined by three integer parameters (\(E\), \(R\) and \(D\), respectively). Three additional integer parameters (\(\hat{E} \leq E\), \(\hat{R} \leq R\), \(\hat{D} \leq D\)) represent thresholds to trigger the issue of Ready messages and the delivery of the message. We discuss the values of the six parameters of Probabilistic Double-Echo in §B.

Sampling. Upon initialization (line 23), a correct process randomly selects three samples (an echo sample \(\mathcal{E}\) of size \(E\), a ready sample \(\mathcal{R}\) of size \(R\), and a delivery sample \(\mathcal{D}\) of size \(D\)). Samples are selected with replacement by repeatedly calling \(\Omega\) (line 16). A correct process sends an EchoSubscribe message to all the processes in its echo sample, and a ReadySubscribe message to all the processes in its ready and delivery samples (line 19).

\(^3\)Note that one instance of probabilistic consistent broadcast only distributes a single message. To disseminate multiple messages, we use multiple instances of probabilistic consistent broadcast.
**Publish-subscribe.** Unlike in the deterministic version of Authenticated Double-Echo, where a correct process broadcasts its Echo and Ready messages to the whole system, here each process only listens for messages coming from its samples (lines 58, 70, 73).

A correct process maintains an *echo subscription set* $\tilde{E}$ and a *ready subscription set* $\tilde{R}$. Upon receiving a Subscribe message from a process $\pi$, a correct process adds $\pi$ to $\tilde{E}$ (in case of EchoSubscribe, line 37) or to $\tilde{R}$ (in case of ReadySubscribe, line 44). If a correct process receives a Subscribe message after publishing an Echo or a Ready message, it also sends back the previously published message (line 35, 42). A correct process will only send its Echo and Ready messages (lines 53, 65, 81) to its echo and ready subscription sets respectively.

**Echo.** The designated sender $\sigma$ initially broadcasts its message using probabilistic broadcast (line 47). Upon pb.Deliver of a message $m$ (correctly signed by $\sigma$) (line 49), a correct process sends an Echo message for $m$ to all the nodes in its echo subscription set (line 53).

**Ready.** A correct process sends a Ready message for a message $m$ (correctly signed by $\sigma$) to all the processes in its ready subscription sample (lines 65, 81) upon collecting either of:

- At least $\hat{E}$ Echo messages for $m$ from its echo sample (line 62).
- At least $\hat{R}$ Ready messages for $m$ from its ready sample (line 78)

**Delivery.** Upon collecting at least $\hat{D}$ Ready messages for the same message $m$ (correctly signed by $\sigma$) from its delivery sample (line 84), a correct process delivers $m$ (line 86).

For an analysis of Probabilistic Double-Echo, we refer the interested reader to §B.

### 5.4 Probabilistic secure broadcast

In this section, we introduce the probabilistic secure broadcast abstraction. This abstraction allows correct processes to agree on a sequence of messages sent from a designated sender, potentially Byzantine. Probabilistic secure broadcast is a strictly stronger abstraction than probabilistic consistent broadcast, because the former allows for an arbitrary sequence of messages to be delivered in a consistent order. We then present Sequenced Probabilistic Double-Echo, a probabilistic algorithm that implements Probabilistic secure broadcast.

#### 5.4.1 Definition

The probabilistic secure broadcast interface (instance $psb$, sender $\sigma$) exposes the following two events:

- **Request:** $\langle psb.\text{Broadcast} \mid m \rangle$: Broadcasts a message $m$ to all processes. This is only used by process $\sigma$.
- **Indication:** $\langle psb.\text{Deliver} \mid m \rangle$: Delivers a message $m$ broadcast by process $\sigma$. 

Definition 1. Let \( \pi \) be a correct process. Then \( \pi \) \emph{initially broadcasts (or delivers)} a sequence of messages \( m_1, \ldots, m_n \) if the sequence of messages it broadcasts (or delivers) begins with \( m_1, \ldots, m_n \).

For any \( \epsilon \in [0, 1] \), we say that probabilistic secure broadcast is \( \epsilon \)-secure if:

1. **No creation**: If \( \sigma \) is correct, and \( \sigma \) never broadcasts more than \( n \) messages, then no correct process delivers more than \( n \) messages.
2. **Integrity**: If a correct process delivers a message \( m \), and \( \sigma \) is correct, then \( m \) was previously broadcast by \( \sigma \).
3. \( \epsilon \)-**Multi-validity**: If \( \sigma \) is correct, and \( \sigma \) initially broadcasts \( m_1, \ldots, m_n \), then \( \sigma \) eventually initially delivers \( m_1, \ldots, m_n \) with probability at least \( (1 - \epsilon)^n \).
4. \( \epsilon \)-**Multi-totality**: If a correct process delivers \( n \) messages, then every correct process eventually delivers \( n \) messages with probability at least \( (1 - \epsilon)^n \).
5. \( \epsilon \)-**Multi-consistency**: Every correct process that delivers \( n \) messages initially delivers the same sequence of \( n \) messages with probability at least \( (1 - \epsilon)^n \).

5.4.2 Algorithm. Algorithm 3 presents the implementation of Sequenced Probabilistic Double-Echo.

Sequenced Probabilistic Double-Echo consistently distributes across the system a sequence of messages in consistent order. It does so by using one distinct instance of ProbabilisticConsistent-Broadcast (§5.3) for each message in the sequence. Instances are incrementally numbered, which allows for reordering on the receiver’s end.

Initialization. Upon initialization (line 7), all correct processes initialize one instance of ProbabilisticConsistentBroadcast (line 12), that will be used to consistently broadcast the first message. Each process also initializes an array of all the messages received through probabilistic consistent broadcast (line 11).

Broadcast. When broadcasting (line 19), the sender simply triggers \( pcb.Broadcast \) on the latest instance of ProbabilisticConsistentBroadcast (line 20). It then initializes new instance of ProbabilisticConsistentBroadcast. The index of the new instance is maintained in the variable \( next \), and incremented in \( expand() \) (line 14).

Delivery. Upon \( pcb.Deliver \) (line 23), a correct process adds the message to the array of messages that have been received. When the next expected message (i.e., the message with the lowest sequence number which has not yet been \( psb.Delivered \)) is \( pcb.Delivered \) (line 26), it is also \( psb.Delivered \) (line 27).
Algorithm 3 Sequenced Probabilistic Double-Echo

1: **Implements:**
2:   ProbabilisticSecureBroadcast, **instance** psb
3:   
4: **Uses:**
5:   ProbabilisticConsistentBroadcast, **instance** pcb  ➤ multiple instances
6:   
7: **upon event** (psb.Init) **do**
8:   next = 0;
9:   expected = 0;
10: 
11:   messages[next] = ⊥;
12:   Initialize a new instance pcb.next of ProbabilisticConsistentBroadcast;
13: 
14: **procedure** expand() **is**
15:   next ← next + 1;
16:   messages[next] = ⊥;
17:   Initialize a new instance pcb.next of ProbabilisticConsistentBroadcast;
18: 
19: **upon event** (psb.Broadcast | message) **do**  ➤ only process σ
20:   trigger (pcb.next.Broadcast | message);
21:   expand();
22: 
23: **upon event** (pcb.index.Deliver | message) **do**
24:   messages[index] ← message;
25: 
26: **upon messages[expected] ≠ ⊥ do**
27:   trigger (psb.Deliver | messages[expected]);
28:   expected ← expected + 1;
29:   **if** self ≠ σ **then**
30:     expand();
31:   **end if**
32: 

5.5 Permissionless Environment

In this section we discuss the deployment of AT2 (in particular AT2$p$) in a permissionless environment.

Our implementation of AT2$p$ relies on Erdős-Rényi Gossip and Probabilistic Double-Echo, both of which use an oracle $Ω$ for obtaining random samples of processes. The size of a sample is optimized for representativeness: this ensures, e.g., that with high probability only a bounded fraction of a sample is Byzantine.
In private / permissioned systems with full membership view, \( \Omega \) can be obtained from a simple local randomness generator by picking a sample from a local list of processes.

In a public / permissionless system, however, sampling a very large, dynamically changing set of processes is non-trivial, especially in a Byzantine environment.

Moreover, in a permissionless setting, the assumption of a limited fraction of faulty processes might be broken by Sybil attacks.

To implement \( \Omega \), we use an existing Byzantine-tolerant sampling protocol (Brahms [16]). However, Brahms requires, but does not provide, a Sybil resistance mechanism.

As in all public / permissionless solutions, the problem of Sybil attacks needs to be addressed in our permissionless variant as well. An effective solution that does not rely on any trusted admission control scheme inevitably requires linking at least some critical parts of the protocol to some real or virtual resource that is by its nature limited. In Bitcoin, this is achieved through the (in)famous proof-of-work scheme, where computation power is used as this limited resource. Algorand leverages the virtual currency itself to attribute "voting power" to participants (proof-of-stake). Such proof-of-* mechanisms rely on a participant having a proof of her “right to vote” that any other participant can easily verify. Moreover, these mechanisms are usually an integral part of the protocol that uses them. The way AT2 prevents Sybil attacks has two advantages over traditional methods.

First, our solution decouples the Sybil resistance mechanism from the protocol itself. Any sub-protocol can be used to prove that participants are genuine (as opposed to Sybil identities). Proof-of-work is just an example of a mechanism that such a sub-protocol can use. Other mechanisms, even ones yet to be invented, are easily pluggable into our design.

Second, we relax the properties of the Sybil resistance mechanism that are necessary for it to be used with AT2. In particular, while most current strategies are based on a globally verifiable proof of the ownership (or at least control) of some resource, we only require local verifiability: only a small subset of participants needs to be able to verify a proof. Specifically, a participant only presents proofs to those participants she is directly communicating with, and it is sufficient if these participants are able to verify these proofs.

Removing the need for global verifiability opens new doors to much more efficient Sybil resistance techniques, as we show below.

For the Sybil resistance sub-protocol, we propose a novel proof-of-bandwidth scheme. As the name suggests, the voting power of participants is bound to the (generally limited) resource of bandwidth. A participant proves herself to the peers by sending them data over the network. Participant \( A \) considers another participant \( B \) to be genuine (and takes into account \( B \)’s messages when taking decisions) only if \( B \) periodically sends data to \( A \). Nodes that fail to send enough data
to A are disregarded by A. Note that in general B cannot prove to anyone else than A that data has been transmitted, but in our case this is sufficient (no need for global verifiability).

The advantage of this approach is that protocol data itself can serve as such proof, and thus no additional resources need to be wasted, as is the case for proof-of-work or proof-of-storage. In the case where the protocol itself does not generate enough data for a proof, even other completely unrelated data exchange between two participants (e.g. BitTorrent traffic) can be used as proof. Since we only require local verifiability, it is only up to the two communicating parties to agree on what traffic constitutes a proof-of-bandwidth. In the absolutely worst case, where no otherwise useful data can be exchanged between two participants, they can resort to exchanging garbage data.

6 SHARING ACCOUNTS AMONG k PROCESSES

We now return to the crash-stop shared memory model defined in §2 and consider the general case with an arbitrary owner map μ. We show that an asset-transfer object’s consensus number is the maximal number of processes sharing an account. Formally, the consensus number of an asset-transfer object is the maximal k such that there exists \( a \in A \) with \( |\mu(a)| = k \).

Let Ø be an asset-transfer object defined on a set of accounts \( A \) with an ownership map \( \mu \). We say that Ø is k-owned iff \( \max_{a \in A} |\mu(a)| = k \). In other words, if \( \mu \) allows at least one account to be owned by \( k \) processes, and no account is owned by more than \( k \) processes.

We show that the consensus number of any k-owned asset transfer is \( k \), which generalizes the result of §2. We first show that k-owned asset transfer has consensus number at least \( k \) by implementing consensus for \( k \) processes using only registers and an instance of k-owned asset transfer. We then show that k-owned asset transfer has consensus number at most \( k \) by reducing k-owned asset transfer to k-consensus, an object that enables up to \( k \) processes to solve consensus [38]. (Recall that k-consensus object has consensus number \( k \).)

6.1 k-owned asset transfer has consensus number at least \( k \)

To prove that k-owned asset transfer has consensus number at least \( k \), we provide a wait-free algorithm that solves consensus among \( k \) processes using only instances of k-owned asset transfer and registers. The algorithm is described in Figure 5. Intuitively, \( k \) processes use one shared account \( a \) to elect one of them whose input value will be decided. Before a process \( p \) accesses the shared account, \( p \) announces its input in a register (line 1). \( p \) then tries to perform a transfer from account \( a \) to another account. The amount of money withdrawn this way from account \( a \) is chosen specifically such that:

1. only one transfer operation can ever succeed, and
Shared variables:
- \( R[i], i \in 1, \ldots, k \) registers, initially \( R[i] = \perp, \forall i \)
- \( B \), asset-transfer object containing:
  - an account \( a \) with initial balance \( 2k \) owned by processes \( 1, \ldots, k \)
  - some account \( s \)

Upon \( \text{propose}(v) \):
1. \( R[p].\text{write}(v) \)
2. \( B.\text{transfer}(a, s, 2k - p) \)
3. \text{return} \( R[B.\text{read}(a)].\text{read()} \)

Fig. 5. Solving \( k \)-process consensus using a \( k \)-owned asset transfer. Code for process \( p \in \{1, \ldots, k\} \).

(2) if the transfer succeeds, the remaining balance on \( a \) will uniquely identify process \( p \).

To satisfy the above conditions, we initialize the balance of account \( a \) to \( 2k \) and have each process \( p \in \{1, \ldots, k\} \) transfer \( 2k - p \) (line 2). Note that transfer operations invoked by distinct processes \( p, q \in \{1, \ldots, k\} \) have arguments \( 2k - p \) and \( 2k - q \) such that \( 2k - p + 2k - q \geq 2k - k + 2k - (k - 1) = 2k + 1 \). Therefore, the first transfer operation to be applied to the object succeeds and the remaining operation will have to fail.

When \( p \) reaches line 3, at least one transfer must have succeeded:

(1) Either \( p \)'s transfer succeeded, or
(2) \( p \)'s transfer failed due to insufficient balance, in which case some other process must have previously succeeded.

Let \( q \) be the process whose transfer succeeded and thus the balance of account \( a \) be \( q \). Since \( q \) performed a transfer operation, by the algorithm, \( q \) must have previously written its proposal to the register \( R[q] \). Regardless of whether \( p = q \) or \( p \neq q \), reading the balance of account \( a \) returns \( q \) and \( p \) decides the value of \( R[q] \).

6.2 \( k \)-owned asset transfer has consensus number at most \( k \)

To prove that \( k \)-owned asset transfer has consensus number at most \( k \), we reduce \( k \)-owned asset transfer to \( k \)-consensus. A \( k \)-consensus object exports a single operation \( \text{propose} \) that, the first \( k \) times it is invoked, returns the argument of the first invocation. All subsequent invocations return \( \perp \). Given that \( k \)-consensus is known for having consensus number exactly \( k \) [38], a wait-free algorithm implementing \( k \)-owned asset transfer using only registers and \( k \)-consensus objects implies that the consensus number of \( k \)-owned asset transfer is not more than \( k \).
Shared variables:
- \( AS \), atomic snapshot object
- for each \( a \in A \):
  - \( R_a[i], i \in \Pi \), registers, initially \([\bot, \ldots, \bot]\)
  - \( kC_a[i], i \geq 0 \), list of instances of \( k \)-consensus objects

Local variables:
- \( \text{hist} \): a set of completed transfers, initially empty
- for each \( a \in A \):
  - \( \text{committed}_a \), initially \( \emptyset \)
  - \( \text{round}_a \), initially 0

Upon \text{transfer}(a, b, x):
1. if \( p \notin \mu(a) \) then
2. return false
3. \( tx = (a, b, x, p, \text{round}_a) \)
4. \( R_a[p].\text{write}(tx) \)
5. \( \text{collected} = \text{collect}(a) \setminus \text{committed}_a \)
6. while \( tx \in \text{collected} \) do
7. \( \text{req} = \text{the oldest transfer in collected} \)
8. \( \text{prop} = \text{propose}(\text{req}, AS.\text{snapshot}()) \)
9. \( \text{decision} = kC_a[\text{round}_a].\text{propose}(\text{prop}) \)
10. \( \text{hist} = \text{hist} \cup \{ \text{decision} \} \)
11. \( \text{AS.}\text{update}($ \text{hist} $) \)
12. \( \text{committed}_a = \text{committed}_a \cup \{ t : \text{decision} = (t, \ast) \} \)
13. \( \text{collected} = \text{collected} \setminus \text{committed}_a \)
14. \( \text{round}_a = \text{round}_a + 1 \)
15. if \( (tx, \text{success}) \in \text{hist} \) then
16. return true
17. else
18. return false

Upon \text{read}(a):
19. return \( \text{balance}(a, AS.\text{snapshot}()) \)

collect(a):
20. \( \text{collected} = \emptyset \)
21. for all \( i = \Pi \) do
22. if \( R_a[i].\text{read}() \neq \bot \) then
23. \( \text{collected} = \text{collected} \cup \{ R_a[i].\text{read}() \} \)
24. return \( \text{collected} \)

propose\(_a\)((a, b, p, x), \text{snapshot})\):
25. if \( \text{balance}(a, \text{snapshot}) \geq x \) then
26. \( \text{prop} = ((a, b, x), \text{success}) \)
27. else
28. \( \text{prop} = ((a, b, p, x), \text{failure}) \)
29. return \( \text{prop} \)

balance\(_a\), \(\text{snapshot}\)\):
30. \( \text{incoming} = \{ tx : tx = (\ast, a, \ast, \ast, \ast) \land (tx, \text{success}) \in \text{snapshot} \} \)
31. \( \text{outgoing} = \{ tx : tx = (a, \ast, \ast, \ast, \ast) \land (tx, \text{success}) \in \text{snapshot} \} \)
32. return \( \sum_{(\ast, a, x, \ast, \ast) \in \text{incoming}} x - \sum_{(a, \ast, x, \ast, \ast) \in \text{outgoing}} x \)

Fig. 6. Wait-free implementation of a \( k \)-owned asset transfer object using \( k \)-consensus objects. Code for process \( p \).
We present the algorithm reducing $k$-owned asset transfer to $k$-consensus in Figure 6. In the reduction, we associate a series of $k$-consensus objects with every account $a$. Up to $k$ owner of $a$ use the $k$-consensus objects to agree on the order of outgoing transfers for $a$.

The state of the implemented $k$-owned asset transfer object is maintained using an atomic snapshot object $AS$. Every process $p$ uses a distinct entry of $AS$ to store a set $\text{hist}$ of all transfers completed so far. Each element in the set is represented as $((a, b, x, p, r), \text{result})$, where $a$, $b$, and $x$ are the respective source account, destination account, and the amount transferred, $p$ is the originator of the transfer, and $r$ is the round in which the transfer was invoked by the originator. The value of $\text{result} \in \{\text{success}, \text{failure}\}$ indicates whether the transfer succeeds or fails. Each process $p$, for every account $a$ that $p$ is allowed to debit (i.e., such that $p \in \mu(a)$), stores this set in its entry of $AS$. A transfer becomes “visible” when any process appends it to its copy of the list in $AS$. To read the balance of account $a$, a process takes a snapshot of $AS$, and then sums the amounts of all successful incoming transfers and subtracts the amounts of successful outgoing transfers represented in $AS$.

To execute an outgoing transfer $o$, a process $p$ first announces $o$ in a register $R_a[p]$ that can be written by $p$ and read by any other process. This enables the “helping” mechanism needed to ensure wait-freedom to the owners of $a$.

Next, $p$ collects the transfers proposed by other owners and tries to agree on the order of the collected transfers and their results using the series of $k$-consensus objects. A transfer-result pair as a proposal for the next instance of $k$-consensus is created as follows. Process $p$ picks up the “oldest” collected but not yet committed operation (based on the round number $\text{round}_a$ attached to the transfer operation when $p$ registers it). Then $p$ takes a snapshot $S$ of $AS$ and simulates the application of the chosen transfer as if it happened in isolation on the state represented by $S$. The transfer is equipped with a success / failure flag, based on the result in the simulated execution. The resulting ordered list of transfer-result pairs constitutes $p$’s proposal for the ordering of these transfers. The currently executed transfer by process $p$ returns as soon as it is decided by a $k$-consensus object and the flag of the decided value (success/failure) implies the transfer’s response (true/false).

**Theorem 3.** Any $k$-owned asset transfer asset-transfer object type has a wait-free implementation in the read-write shared memory model equipped with $k$-consensus objects.

**Proof.** We essentially follow the footpath of the proof of Theorem 1. Fix an execution $E$ of the algorithm in Figure 6. Let $H$ be the history of $E$.

To perform a transfer $o$ on an account $a$, $p$ registers it in $R_a[p]$ (line 9) and then proceeds through a series of $k$-consensus objects, each time collecting $R_a$ to learn about the transfers concurrently proposed by other owners of $a$. Recall that each $k$-consensus object is wait-free. Suppose, by
contradiction, that \( o \) is registered in \( R_a \) but is never decided by any instance of \( k \)-consensus. Eventually, however, \( o \) becomes the request with the lowest round number in \( R_a \) and, thus, some instance of \( k \)-consensus will be only accessed with \( o \) as a proposed value (line 9). By validity of \( k \)-consensus, this instance will return \( o \) and, thus, \( p \) will be able to complete \( o \).

Let \( Ops \) be the set of all complete operations and all transfer operations \( o \) such that some process completed the update operation (line 11) in \( E \) with an argument including \( o \) (the atomic snapshot and \( k \)-consensus operation has been linearized). Intuitively, we include in \( Ops \) all operations that took effect, either by returning a response to the user or by affecting other operations. Recall that every such transfer operation (on an account \( c \)) was agreed upon an instance of \( k \)-consensus, let it be \( kC^o \). Therefore, for every such transfer operation \( o \), we can identify the process \( q^o \) whose proposal has been decided in that instance.

We now determine a completion of \( H \) and, for each \( o \in Ops \), we define a linearization point as follows:

- If \( o \) is a read operation, it linearizes at the linearization point of the snapshot operation in line 19.
- If \( o \) is a transfer operation that returns false, it linearizes at the linearization point of the snapshot operation (line 8) performed by \( q^o \) just before it invoked \( kC^o.propose() \).
- If \( o \) is a transfer operation that some process completed the update operation (line 11), it linearizes at the linearization point of the first update operation in \( H \) (line 11) that includes \( o \).

Furthermore, if \( o \) is incomplete in \( H \), we complete it with response true.

Let \( \hat{H} \) be the resulting complete history and let \( L \) be the sequence of complete operations of \( \hat{H} \) in the order of their linearization points in \( E \). If multiple successful transfer operations share the linearization point (they are all linearized at the linearization point of an update operation), we choose them to be linearized contiguously in an arbitrary order. Note that, by the way we linearize operations, the linearization of a prefix of \( E \) is a prefix of \( L \). Also, by construction, the linearization point of an operation belongs to its interval.

Now we show that \( L \) is legal and, thus, \( H \) is linearizable. We proceed by induction, starting with the empty (trivially legal) prefix of \( L \). Let \( L_\ell \) be the legal prefix of the first \( \ell \) operation and \( op \) be the \((\ell + 1)\)st operation of \( L \). Let \( op \) be invoked by process \( p \). The following cases are possible:

- \( op \) is a read\((a)\): the snapshot taken at the linearization point of \( op \) contains all successful transfers concerning \( a \) in \( L_\ell \). By the induction hypothesis, the resulting balance is non-negative.
- \( op \) is a failed transfer\((a, b, x)\): the snapshot taken at the linearization point of \( op \) contains all successful transfers concerning \( a \) in \( L_\ell \). By the induction hypothesis, the balance corresponding to this snapshot non-negative. By the algorithm, the balance is less than \( x \).
• op is a successful transfer\((a, b, x)\). Let \(L_s, s \leq \ell\), be the prefix of \(L_\ell\) that only contains operations linearized before the moment of time when \(q^o\) has taken the snapshot just before accessing \(kC^o\).

As before accessing \(kC^o\), \(q\) went through all preceding \(k\)-consensus objects associated with \(a\) and put the decided values in \(AS\), \(L_s\) must include all outgoing transfer operations for \(a\). Furthermore, \(L_s\) includes a subset of all incoming transfers on \(a\). Thus, \(balance(a, L_k) \leq balance(a, L_\ell)\).

By the algorithm, as \(op = transfer(a, b, x)\) succeeds, we have \(balance(a, L_k) \geq x\). Thus, \(balance(a, L_\ell) \geq x\) and the resulting balance in \(L_{\ell+1}\) is non-negative.

Thus, \(H\) linearizable. \(\Box\)

Corollary 4. Any \(k\)-owned asset transfer object has consensus power \(k\).

7 RELATED WORK
In this section we discuss related work with regards to various parts of AT2. We start by considering other asset transfer systems, both for the private (permissioned) and public (permissionless) setting. We then elaborate on ordering of inputs, and finally we discuss work related to secure broadcast, which is an important building block of AT2.

7.1 Asset Transfer Systems
The different flavors of AT2 are suitable for both a private (permissioned) and public (permissionless) setting. A private setting implies the assumption of an access control mechanism, specifying who is allowed to participate in the system. In this case, we assume that this mechanism is external to the system itself. Private protocols, such as Corda [34], Hyperledger Fabric [7], or Vegvisir [39] rely on such a mechanism.

Importantly, the access control mechanism rules out the possibility of Sibyl attacks [25], where a malicious party can take control over a system by using many identities, toppling the one third assumption on the fraction of Byzantine participants. Once this is done, the malicious party can engage in a double-spending attack.

Decentralized systems for the public, i.e., permissionless, setting are open to the world. They do not have an explicit access control mechanism and allow anyone to join. Systems which fall into this category include Bitcoin [51], Ethereum [63], Avalanche [60], ByzCoin [41], Algorand [30], Hybrid consensus [52], PeerCensus [24], or Solida [2]. To prevent malicious parties from overtaking the system, these systems rely on Sybil-proof techniques, e.g., proof-of-work [51], or proof-of-stake [12].
These systems, whether they address the permissionless or the permissioned environment, seek to solve consensus in their implementation. It is worth noting that many of these solutions can allow for more than just transfers, and enable access to smart contracts. Our focus is, however, on decentralized transfer systems, and the surprising result of this paper is that we can implement such a system without resorting to consensus.

To deploy AT2 in a public setting where anybody can participate, Sibyl attacks need to be addressed and the system needs to be scalable to accommodate many participants. This is possible by using a Sybil-resistant and scalable implementation of secure broadcast, while keeping the transfer algorithm (see Figure 4) unchanged. To the best of our knowledge, there is no such implementation of secure broadcast in the literature so far. Our probabilistic secure broadcast implementation is scalable; transfers can be accepted within $O(\log(N))$ message delays ensuring security with overwhelming probability. We also show how to make our protocol Sybil-resistant (see §5.5) and thus deployable in an open (permissionless) environment.

7.2 Ordering Constraints

In the blockchain ecosystem, there exist several efforts to avoid building a totally ordered chain of transfers. The idea is to replace the totally ordered linear structure of a blockchain with that of a directed acyclic graph (DAG) for structuring the transfers in the system. Notable systems in this spirit include Byteball [22], Vegvisir [39], the GHOST protocol [56], Corda [34], or Nano [44]. Even if these systems use a DAG to replace the classic blockchain, their algorithms still employ consensus. As we show here, total order (obtained via consensus) is not necessary for implementing decentralized transfers.

We can also use a DAG to characterize the relation between transfers in AT2, but we do not resort to solving consensus to build the DAG, nor do we use the DAG in order to solve consensus. More precisely, we can regard each account as having an individual history. Each such history is managed by the corresponding account owner without depending on a global view of the system state. Another way to characterize AT2 is that we do not build a totally ordered sequence of all transfers in the system (like a classic blockchain [51]). Instead we maintain a per-owner sequence of transfers. Each sequence is ordered individually (by its respective owner), and is loosely coupled with other sequences (through dependencies established by causality).

Another parallel we can draw is between the accounts in our system (as we defined them in §2.2) and conflict-free replicated data types (CRDTs) [55]. Specifically, similar to a CRDT, in AT2 we support concurrent updates on different accounts while preserving consistency. Since each account has a unique owner, this rules out the possibility of conflicting operations on each (correct) account. In turn, this ensures that the state at correct nodes always converges to a consistent version. In the terminology of [55], we provide strong eventual consistency.
As we explained earlier, the ordering among transfers in AT2 is based on causality, as defined through the happened-before relationship of Lamport [43]. Various causally-consistent algorithms exist [19, 45]. One problematic aspect in these algorithms is that the metadata associated with tracking dependencies can be a burden [5, 10, 49]. This happens because such algorithms track all potential causal dependencies. In our AT2 algorithm for the message passing model (Figure 4) we track dependencies explicitly [10], permitting a more efficient implementation with a smaller set of dependencies. More concretely, we specify that each transfer outgoing from an account only depends on previous transfers outgoing from and incoming to that—and only that—account, ignoring the transfers that affect other (irrelevant) accounts.

7.3 Asynchronous Agreement Protocols

An important strength of the whole AT2 class of algorithms is that they are asynchronous, meaning that they do not have to rely inherently on timeouts to ensure liveness guarantees. This is in contrast to deterministic consensus-based solutions. Such solutions have to resort to fine-tuned timeout parameters which affect their performance [14, 50].

Asynchronous protocols for consensus exist, but they typically employ heavy cryptography relying on randomization to overcome the FLP impossibility [28, 40]. A few recent efforts are trying to make these protocols more efficient [6, 11, 26, 50]. These asynchronous protocol are designed for a relatively small- to medium-scale, similar to our deterministic algorithm AT2D. Compared to AT2D, asynchronous consensus protocols have higher complexity.

8 CONCLUSIONS

In this paper we revisited the problem of implementing a decentralized asset transfer system. Since the rise of the Bitcoin cryptocurrency, this problem has garnered significant innovation. Most of the innovation, however, has focused on improving the original solution which Bitcoin proposed, namely, that of using a consensus mechanism to build a blockchain where transactions across the whole system are totally ordered.

We did not aim to investigate current consensus-based solutions and push the envelope on performance or other metrics. Instead, we showed that we can implement a decentralized transfer system without resorting to consensus. To this end, we first precisely defined the transfer system object type and proved that it has consensus number one if a single account is not shared by multiple processes. That is, it occupies the lowest rank of the consensus hierarchy and can be implemented without the need for solving consensus. We proved this surprising result by borrowing from the theory of concurrent objects, and then leveraged the result to build AT2, the first consensusless transfer system.
A consensusless solution for decentralized transfers has multiple advantages. Concretely, the transfer algorithm in AT2 is not subject to the FLP impossibility [28]. Additionally, AT2 is significantly simpler than consensus-based solutions (because it relies on a simpler secure broadcast primitive) and exhibits higher performance.

We compared the performance of AT2 to that of a transfer system based on BFT-Smart, a state-of-the-art consensus-based state machine replication system. AT2 provides performance superior to that of the consensus-based solution. In systems of up to 100 replicas, regardless of system size, we observed a throughput improvement ranging from 1.5x to 6x, while achieving a decrease in latency of up to 2x.

We implemented AT2 for a private (permissioned) environment that is prone to Byzantine failures and where participants do not need to trust each other. This implementation is easily extensible to the large-scale permissionless setting, and this is the focus of our concurrent work. Our preliminary results are highly encouraging. Most notably, we can obtain sub-second transfer execution on a global scale deployment of thousands of nodes.

REFERENCES

[1] Abraham, I., Gueta, G., Malkhi, D., Alvisi, L., Kotla, R., and Martin, J.-P. Revisiting fast practical byzantine fault tolerance. arXiv preprint arXiv:1712.01367 (2017).
[2] Abraham, I., Malkhi, D., Nayak, K., Ren, L., and Spiegelman, A. Solida: A blockchain protocol based on reconfigurable byzantine consensus. CoRR abs/1612.02916 (2016).
[3] Acemoglu, D., and Ozdaglar, A. 6.207/14.15: Networks - lecture 4: Erdős–rényi graphs and phase transitions. https://economics.mit.edu/files/4622, 2009.
[4] Afek, Y., Attiya, H., Dolev, D., Gafni, E., Merritt, M., and Shavit, N. Atomic snapshots of shared memory. J. ACM 40, 4 (1993), 873–890.
[5] Akkoorath, D. D., Tomsc, A. Z., Bravo, M., Li, Z., Crain, T., Bieniusa, A., Preguica, N., and Shapiro, M. Cure: Strong Semantics Meets High Availability and Low Latency. In NSDI (2016).
[6] Alistarh, D., Aspnes, J., King, V., and Saia, J. Communication-efficient randomized consensus. Distributed Computing 31, 6 (2018), 489–501.
[7] Androulaki, E., Barger, A., Bortnikov, V., Cachin, C., Christidis, K., Caro, A. D., Enyeart, D., Ferris, C., Laventman, G., Manevich, Y., Muralidharan, S., Murthy, C., Nguyen, B., Sethi, M., Singh, G., Smith, K., Sorniotti, A., Stathakopoulou, C., Vukolic, M., Cocco, S. W., and Yellick, J. Hyperledger fabric: a distributed operating system for permissioned blockchains. In EuroSys (2018).
[8] Antoniadis, K., Guerraoui, R., Malkhi, D., and Seredinschi, D.-A. State Machine Replication is More Expensive Than Consensus. In DISC (2018).
[9] Attiya, H., and Welch, J. L. Sequential consistency versus linearityability. ACM TOCS 12, 2 (1994), 91–122.
[10] Bailis, P., Fekete, A., Ghodsi, A., Hellerstein, J. M., and Stoica, I. The potential dangers of causal consistency and an explicit solution. In ACM SoCC (2012).
[11] Bazzi, R., and Herlihy, M. Clairvoyant state machine replications. In International Symposium on Stabilizing, Safety, and Security of Distributed Systems (2018), Springer, pp. 254–268.
[12] Bentov, I., Gabizon, A., and Mizrahi, A. Cryptocurrencies without proof of work. In *International Conference on Financial Cryptography and Data Security* (2016), Springer, pp. 142–157.

[13] Berman, P., Garay, J. A., and Perry, K. J. Towards Optimal Distributed Consensus. In *FOCS* (1989).

[14] Bessani, A., Sousa, J., and Alchieri, E. E. State Machine Replication for the Masses with BFT-SMaRt. In *DSN* (2014).

[15] Bonneau, J., Miller, A., Clark, J., Narayanan, A., Kroll, J. A., and Felten, E. W. SoK: Research Perspectives and Challenges for Bitcoin and Cryptocurrencies. In *IEEE S&P* (2015).

[16] Bortnikov, E., Gurevich, M., Keidar, I., Kliot, G., and Shraer, A. Brahms: Byzantine resilient random membership sampling. *Computer Networks* 53, 13 (2009), 2340–2359.

[17] Bracha, G. Asynchronous Byzantine agreement protocols. *Information and Computation* 75, 2 (1987), 130–143.

[18] Bracha, G., and Toueg, S. Asynchronous Consensus and Broadcast Protocols. *JACM* 32, 4 (1985).

[19] Cachin, C., Guerraoui, R., and Rodrigues, L. *Introduction to reliable and secure distributed programming*. Springer Science & Business Media, 2011.

[20] Cachin, C., and Poritz, J. A. Secure intrusion-tolerant replication on the internet. In *DSN* (2002).

[21] Cachin, C., and Vukolić, M. Blockchains consensus protocols in the wild. *arXiv preprint arXiv:1707.01873* (2017).

[22] Churyumov, A. Byteball: A decentralized system for storage and transfer of value. *https://byteball.org/Byteball.pdf* (2016).

[23] Clement, A., Wong, E. L., Alvisi, L., Dahlin, M., and Marchetti, M. Making Byzantine Fault Tolerant Systems Tolerate Byzantine Faults. In *NSDI* (2009).

[24] Decker, C., Seidel, J., and Wattenhofer, R. Bitcoin meets strong consistency. In *Proceedings of the 17th International Conference on Distributed Computing and Networking* (2016), p. 13.

[25] Douceur, J. R. The Sybil Attack. In *IPTPS* (2002).

[26] Duan, S., Reiter, M. K., and Zhang, H. BEAT: Asynchronous BFT Made Practical. In *CCS* (2018).

[27] Eyal, I., Gencer, A. E., Sirer, E. G., and Van Renesse, R. Bitcoin-NG: A Scalable Blockchain Protocol. In *NSDI* (2016).

[28] Fischer, M. J., Lynch, N. A., and Paterson, M. S. Impossibility of distributed consensus with one faulty process. *J. ACM* 32, 2 (Apr. 1985), 374–382.

[29] Garay, J., Kiayias, A., and Leonidas, N. The bitcoin backbone protocol: Analysis and applications. In *Advances in Cryptology - EUROCRYPT 2015* (2015).

[30] Gilad, Y., Hemo, R., Micali, S., Vlachos, G., and Zeldovich, N. Algorand: Scaling byzantine agreements for cryptocurrencies. In *SOSP* (2017).

[31] Golan-Gueta, G., Abraham, I., Grossman, S., Malkhi, D., Pinkas, B., Reiter, M. K., Seredinschi, D., Tamir, O., and Tomescu, A. SBFT: a scalable decentralized trust infrastructure for blockchains. *CoRR abs/1804.01626* (2018).

[32] Guerraoui, R., Pavlović, M., and Seredinschi, D.-A. Blockchain protocols: The adversary is in the details. In *Symposium on Foundations and Applications of Blockchain* (2018).

[33] Hadzilacos, V., and Toueg, S. Fault-tolerant broadcasts and related problems. In *Distributed Systems*, S. J. Mullender, Ed. Addison-Wesley, 1993, ch. 5, pp. 97–145.

[34] Hearn, M. Corda: A distributed ledger. *Corda Technical White Paper* (2016).

[35] Herlihy, M. Wait-free synchronization. *ACM Trans. Program. Lang. Syst.* 13, 1 (1991), 123–149.

[36] Herlihy, M. P., and Wing, J. M. Linearizability: A correctness condition for concurrent objects. *ACM Transactions on Programming Languages and Systems* (TOPLAS) 12, 3 (1990).

[37] J. Fidge, C. Timestamps in message-passing systems that preserve partial ordering. In *Proceedings of the 11th
[38] Jayanti, P., and Toueg, S. Some results on the impossibility, universability and decidability of consensus. In *International Workshop on Distributed Algorithms* (1992), vol. 647 of LNCS, Springer Verlag.

[39] Karlsson, K., Jiang, W., Wicker, S., Adams, D., Ma, E., Van Renesse, R., and Weatherspoon, H. Vegvisir: A Partition-Tolerant Blockchain for the Internet-of-Things. In *ICDCS* (2018).

[40] King, V., and Saia, J. Scalable byzantine computation. *ACM SIGACT News* 41, 3 (2010), 89–104.

[41] Kogias, E. K., Jovanovic, P., Gailly, N., Khoffi, L., Gasser, L., and Ford, B. Enhancing bitcoin security and performance with strong consistency via collective signing. In *USENIX Security* (2016).

[42] Kokoris-Kogias, E., Jovanovic, P., Gasser, L., Gailly, N., Syta, E., and Ford, B. Omniledger: A secure, scale-out, decentralized ledger via sharding. In *IEEE S&P* (2018).

[43] Lamport, L. Time, clocks, and the ordering of events in a distributed system. *Communications of the ACM* 21, 7 (1978).

[44] LeMahieu, C. Nano: A feeless distributed cryptocurrency network. *Nano* [Online resource]. URL: https://nano.org/en/whitepaper (date of access: 18.01. 2019) (2018).

[45] Lloyd, W., Freedman, M. J., Kaminsky, M., and Andersen, D. G. Don’t settle for eventual. In *SOSP* (2011).

[46] Malkhi, D., Merritt, M., and Rodeh, O. Secure Reliable Multicast Protocols in a WAN. In *ICDCS* (1997).

[47] Malkhi, D., and Reiter, M. K. A high-throughput secure reliable multicast protocol. *Journal of Computer Security* 5, 2 (1997), 113–128.

[48] Mazieres, D. The stellar consensus protocol: A federated model for internet-level consensus. *Stellar Development Foundation* (2015).

[49] Mehdi, S. A., Littley, C., Crooks, N., Alvisi, L., Bronson, N., and Lloyd, W. I Can’t Believe It’s Not Causal! Scalable Causal Consistency with No Slowdown Cascades. In *NSDI* (2017).

[50] Miller, A., Xia, Y., Croman, K., Shi, E., and Song, D. The Honey Badger of BFT Protocols. In *CCS* (2016).

[51] Nakamoto, S. Bitcoin: A peer-to-peer electronic cash system, 2008.

[52] Pass, R., and Shi, E. Hybrid consensus: Efficient consensus in the permissionless model. In *Cryptology ePrint Archive, Report 2016/917* (2016).

[53] Rapoport, P., Leal, R., Griffin, P., and Sculley, W. The Ripple Protocol, 2014.

[54] Reiter, M. Secure Agreement Protocols: Reliable and Atomic Group Multicast in Rampart. In *CCS* (1994).

[55] Shapiro, M., Preguiça, N., Baquero, C., and Zawirski, M. Conflict-free replicated data types. In *Stabilization, Safety, and Security of Distributed Systems*. Springer, 2011.

[56] Sompolinsky, Y., and Zohar, A. Accelerating Bitcoin’s transaction processing: fast money grows on trees, not chains. *IACR Cryptology ePrint Archive, 2013:881* (2013).

[57] Sousa, J., Bessani, A., and Vukolić, M. A byzantine fault-tolerant ordering service for the hyperledger fabric blockchain platform. In *DSN* (2018).

[58] Syverson, P., Dingledine, R., and Mathewson, N. Tor: The second generation onion router. In *Usenix Security* (2004).

[59] Szabo, N. Formalizing and securing relationships on public networks. *First Monday* 2, 9 (1997).

[60] Team-Rocket. Snowflake to Avalanche: A Novel Metastable Consensus Protocol Family for Cryptocurrencies. *White Paper* (2018). Revision: 05/16/2018 21:51:26 UTC.

[61] Van Den Hooff, J., Lazar, D., Zaharia, M., and Zeldovich, N. Vuvuzela: Scalable private messaging resistant to traffic analysis. In *SOSP* (2015).

[62] Vukolić, M. The Quest for Scalable Blockchain Fabric: Proof-of-work vs. BFT Replication. In *International
Workshop on Open Problems in Network Security (2015), Springer, pp. 112–125.

[63] Wood, G. Ethereum: A secure decentralized generalized transaction ledger. White paper, 2015.
Appendices

A ANALYSIS OF ERDŐS-RÉNYI GOSSIP

We now discuss the correctness of Erdős-Rényi Gossip.

No duplication: A correct process maintains a delivered variable that it checks and updates before delivering a message. This prevents any correct process from delivering more than one message.

Integrity: Before broadcasting a message, the sender signs that message with its private key. Before delivering a message $m$, a correct process verifies $m$’s signature. Under the assumption that signatures cannot be forged, this prevents any correct process from delivering a message that was not previously broadcast by the sender.

Validity: Upon broadcasting a message, the sender also immediately delivers it. Since this happens deterministically, and thus Erdős-Rényi Gossip satisfies 0-validity, independently from the parameter $G$.

A.1 Totality

Erdős-Rényi Gossip satisfies $\epsilon_t$-totality with $\epsilon_t$ upper-bounded by a function that decays exponentially with $G$, and polynomially increases with the fraction $f$ of Byzantine faults.

Indeed, the network of connections established among the correct processes is an undirected Erdős–Rényi graph, and totality is satisfied if such graph is connected. This allows us to bound the probability of totality not being satisfied, using a well-known result on the connectivity of Erdős–Rényi graphs.

Upon initialization, a correct process randomly selects a sample of other processes (it uses an oracle to achieve this, see Assumption 6) with which it will exchange messages.

We start by noting that every link is eventually reciprocated by correct processes, i.e., if a correct process $\pi$ is in the sample of $\rho$, then $\rho$ will eventually be in the sample of $\pi$ (this is due to the fact that messages are always eventually delivered, see Assumption 4).

We consider the sub-graph of connections only between correct processes. This network is eventually undirected. We show that, if such graph is connected, then Erdős-Rényi Gossip satisfies totality. This is due to the fact that every message will eventually propagate through all the gossip links, reaching every correct process (again, due to Assumption 4).

We show that any two correct processes have an independent probability of being connected. This is due to the fact that, upon initialization, the number of elements in a correct process’ gossip sample is sampled from a Poisson distribution. Poisson distributions quickly limit to binomial
distributions for large systems, and we show that selecting a binomially distributed number of distinct objects from a set is equivalent to selecting each object with an independent probability. This proves that the sub-graph of connections between correct processes is an Erdős–Rényi graph.

Erdős–Rényi graphs are well known in literature [3] to display a connectivity phase transition: when the expected number of connections each node has exceeds the logarithm of the number of nodes, the probability of the graph being connected steeply increases from 0 to 1 (in the limit of infinitely large systems, this increase becomes a step function). We use that result to compute the probability of the sub-graph of correct processes being connected and, consequently, of Erdős-Rényi Gossip satisfying totality.

Fig. 7. Upper bound for the $\epsilon$-security of Erdős-Rényi Gossip, as a function of the gossip sample size ($G$), the fraction of Byzantine failures ($f$), and the system size ($N$).
Figure 7 shows the $\epsilon$-security of Erdős-Rényi Gossip, as a function of the gossip sample size ($G$), the fraction of Byzantine failures ($f$) and the size of the system ($N$).

**B ANALYSIS OF PROBABILISTIC DOUBLE-ECHO**

We now discuss the correctness of Probabilistic Double-Echo.

*No duplication.* A correct process maintains a *delivered* variable that it checks and updates before delivering a message. This prevents any correct process from delivering more than one message.

*Integrity.* Before broadcasting a message, the sender signs that message with its private key. Before delivering a message $m$, a correct process verifies $m$’s signature. Under the assumption that signatures cannot be forged, this prevents any correct process from delivering a message that was not previously broadcast by the sender.

In order to study validity, totality and consistency, we first establish some auxiliary results.

**B.1 Auxiliary results**

For a correct process, the execution of Probabilistic Double-Echo reduces to three operations: publishing an Echo message, publishing a Ready message, and delivering a message. Each operation is triggered by one or more conditions:

- A correct process publishes an Echo message upon pb.Deliver of a Send message.
- A correct process publishes a Ready message upon collecting either enough Echo messages from its echo sample, or enough Ready messages from its ready sample.
- A correct process delivers a message upon collecting enough Ready messages from its delivery sample.

We study the probability of each condition being fulfilled in steps.

**Definition 2** (Ready, E-ready, R-ready). Let $\pi$ be a correct process, let $m$ be a message. Then:

- $\pi$ is **Ready** for $m$ if it eventually publishes a Ready message for $m$.
- $\pi$ is **E-ready** for $m$ if it is ready for $m$ as a result of having collected enough Echo messages for $m$.
- $\pi$ is **R-ready** for $m$ if it is ready for $m$ as a result of having collected enough Ready messages for $m$.

Let $\pi$ be a correct process, let $m$ be a message.

*E-ready probability.* We compute lower and upper bounds for the probability of $\pi$ being E-ready for $m$, given the number of correct processes that echo (i.e., publish an Echo message for) $m$. 
Echo samples are uniformly picked with replacement from the set of processes (correct processes use an oracle to achieve this, see Assumption 6). Therefore, each element of \( \pi \)'s echo sample has an independent probability of being Byzantine and, if correct, of having echoed \( m \).

Let \( \rho \) be an element of \( \pi \)'s echo sample. We consider two scenarios. In the first scenario, no Byzantine process ever sends an Echo message for \( m \). In this scenario, the probability of \( \pi \) receiving an Echo message for \( m \) from \( \rho \) reduces to the probability of \( \rho \) being correct and having echoed \( m \) (this is due to the fact that messages are always eventually delivered, see Assumption 4).

In the second scenario, all Byzantine processes send an Echo message for \( m \) to all the correct subscribed processes. In this scenario, the probability of \( \pi \) receiving an Echo message for \( m \) from \( \rho \) reduces to the probability of \( \rho \) being Byzantine, or \( \rho \) being correct and having echoed \( m \).

The probability of \( \pi \) being E-ready for \( m \) is minimized in the first scenario, and maximized in the second. Both probabilities can be computed by noting that the number of Echo messages that \( \pi \) receives is binomially distributed.

**Ready feedback.** We study the feedback mechanism produced by correct processes being R-ready for \( m \), i.e., ready as a result of having received enough Ready messages for \( m \).

Ready samples are uniformly picked with replacement from the set of processes. We define a random **ready multigraph** \( g \) allowing multi-edges and loops whose nodes represent the set of correct processes. The predecessors\(^4\) of a node \( \nu \) represent the correct processes in \( \nu \)'s ready sample.

We introduce **Threshold Contagion**, a game played on the nodes of \( g \) where readiness for \( m \) spreads like a disease: an **infected** node represents a correct process that is ready for \( m \), and whenever enough predecessors of a node \( \nu \) are infected, \( \nu \) becomes infected (i.e., R-ready) as well.

Threshold Contagion is played in rounds. At the beginning of each round, a **player** infects an arbitrary set of nodes (this models a set of processes being E-ready for \( m \)). Throughout the rest of the round, the disease automatically propagates to all the nodes that have enough infected predecessors.

Given the number of rounds and the number of nodes infected per round, we use Markov chains to compute the probability distribution underlying the number of nodes that are infected at the end of Threshold Contagion.

We use Threshold Contagion in three settings:

- When evaluating **validity**, a correct sender takes the role of the player in a single-round game of Threshold Contagion.

\( ^{4} \)Node \( \mu \) is a predecessor of node \( \nu \) in the multigraph \( g \) if \((\mu \rightarrow \nu) \in g \). Since multigraphs allow for multiple edges, the set of predecessors of a node is a multiset.
When broadcasting, the correct sender uses probabilistic broadcast to distribute $m$. Unless the totality of probabilistic broadcast is compromised, every correct process eventually publishes an Echo message for $m$. Given the fraction of Byzantine processes, we compute the probability of any correct process being E-ready for $m$. The number of correct processes that are E-ready for $m$ is used as input to (the single round of) Threshold Contagion; its outcome represents the number of correct processes that are ready for $m$. This allows us to compute the probability of a correct process (and, in particular, the sender) eventually delivering $m$.

- When evaluating totality, a Byzantine sender takes the role of the player in a multi-round game of Threshold Contagion. The game starts with no infected nodes. At the beginning of each round, the player infects one healthy node; throughout the round, the infection propagates until either all nodes are infected, or no uninfected node has enough infected predecessors. In order to compromise totality, a Byzantine adversary must cause at least one, but not all correct processes to deliver a message.

  We show that a Byzantine adversary that can cause processes to become E-ready for arbitrary messages can easily perform an attack to compromise totality. We therefore restrict ourselves to the case where the Byzantine adversary can arbitrarily cause any correct process to be E-ready for only one message $m$, and assume that totality is compromised if the adversary can cause two distinct processes to be E-ready for two distinct messages. By doing so, we effectively compute an upper bound for the probability of totality being compromised.

  Under Assumption 5, the adversary has no knowledge of any correct process’ ready sample. As a result of this, we later show (see §B.3) that every action implemented by the adversary can be modeled by a multi-round game of Threshold Contagion.

  Intuitively, lacking any information to meaningfully distinguish correct processes with respect to the topology of the ready multigraph, the minimal action the adversary can perform on the system is causing a single correct process to become E-ready for $m$. As a result, zero or more additional correct processes become R-ready. The adversary can then carry on, causing more correct processes to become E-ready for $m$, or stop.

  In a game of Threshold Contagion, therefore, the adversary’s strategy reduces to playing in rounds, and stopping as soon as any process delivers $m$. At the end of each round, we compute the probability of at least one, but not all correct processes delivering $m$. Totality is compromised if this happens in any round.

- When evaluating consistency, a Byzantine sender takes the role of the player in a single-round game of Threshold Contagion.
Let \(m\) and \(m'\) be two conflicting messages. As we do when evaluating totality, we compute an upper bound for consistency by assuming that if any two correct processes become E-ready for \(m\) and \(m'\), then consistency is compromised.

Under the assumption that correct processes can be E-ready for at most one message, if \(m\) and \(m'\) are delivered by at least one correct process, then either \(m\) or \(m'\) is delivered without any correct process being E-ready for it.

In order to compute the probability of \(m\) being delivered even if no correct process is E-ready for \(m\), we consider a scenario where all Byzantine processes send a Ready message for \(m\) to their correct subscribers. Noting how, in this scenario, the behavior of a Byzantine process is identical to that of a correct process that is E-ready for \(m\), we can compute the probability of \(m\) being delivered by playing a game of Threshold Contagion where Byzantine processes are included as initially infected nodes.

**Delivery probability.** We compute lower and upper bounds for the probability of \(\pi\) delivering \(m\), given the number of correct processes that are ready for \(m\).

This can be achieved using the same technique that we employed when computing lower and upper bounds for the probability of a process being E-ready for \(m\), given the number of correct process that echoed \(m\). For the sake of brevity, we don’t repeat that analysis here.

**Early consistency.** We call **early consistency** the condition where no two correct processes are E-ready for two different messages, \(m_1\) and \(m_2\).

As we see in §§ B.3 and B.4, we compute upper bounds for the probability of compromising totality and consistency by assuming that if early consistency is compromised, then both totality and consistency are compromised.

Under Assumption 5, the adversary has no knowledge of the echo sample of any correct process. Therefore, the adversary has no way of meaningfully distinguishing two correct processes, based on the effect that their Echo messages will have on the system.

We consider a scenario where an adversary releases two different messages \(m_1\) and \(m_2\), and can:

- Cause any correct process to echo any of \(m_1\) and \(m_2\).
- Determine if any correct process is E-ready for \(m_1\) or \(m_2\).

Early consistency is compromised if at least one correct process is E-ready for \(m_1\), and one correct process is E-ready for \(m_2\).

We start by noting that, if the echo threshold is larger than half of the sample size, then the order in which the adversary causes each process to echo either \(m_1\) or \(m_2\) does not affect the probability of compromising early consistency.
Therefore, any adversary that causes \( n_1 \) correct processes to echo \( m_1 \), and \( n_2 \) correct processes to echo \( m_2 \), has the same probability of compromising early consistency as one that first causes \( n_1 \) correct processes to echo \( m_1 \), then \( n_2 \) correct processes to echo \( m_2 \).

Moreover, the probability of any correct process being E-ready for \( m_2 \) is an increasing function of \( n_2 \). Given that at least one correct process is E-ready for \( m_1 \), the probability of compromising early consistency is maximized by the adversary that maximizes \( n_2 \).

Therefore, the probability of compromising early consistency is maximized by an adversary that causes one correct process at a time to echo \( m_1 \), until at least one correct process is E-ready for \( m_1 \), then causes all the other correct processes to echo \( m_2 \).

An adversary could release more than two different messages. We argue, however, that the adversary maximizes the probability of violating early consistency using only two messages. If the adversary needs to make one process E-ready for \( m_1 \) and another process E-ready for \( m_2 \), it needs enough echoes for \( m_1 \) as well as enough echoes for \( m_2 \). The more echoes for each respective message, the higher the chance of a process becoming E-ready for it. Intuitively, introducing a third message would only decrease the chance of a correct process becoming E-ready for either of \( m_1 \) and \( m_2 \).

Feedback creation. We compute an upper bound for the probability of \( m \) being delivered by any correct process, given that no correct process is E-ready for \( m \).

In order to do so, we consider a scenario where every Byzantine process sends a Ready message for \( m \) to every correct subscriber. Noting that, in this scenario, a Byzantine process behaves identically to a correct process that is E-ready for \( m \), we compute the probability underlying the number of correct processes that are ready for \( m \) by playing a game of Threshold Contagion where also the Byzantine processes are included as initially infected nodes.

We then use the upper bound on the delivery probability to bound the probability of \( m \) being delivered.

B.2 Validity

We compute an upper bound for the probability of validity being compromised.

Validity is compromised if a correct sender broadcasts, but does not deliver, a message \( m \). We compute an upper bound for the probability of this happening by assuming that, if the totality of probabilistic broadcast is compromised, then the validity of Probabilistic Double-Echo is compromised as well.

If the totality of probabilistic broadcast is not compromised, then every correct process publishes an Echo message for \( m \). We use our lower bound on the E-ready probability to compute the probability distribution underlying the number of correct processes that are E-ready for \( m \).
We then play a game of Threshold Contagion to compute the probability distribution underlying the number of correct processes that are ready for $m$.

Finally, we use our lower bound for the delivery probability to bound the probability that a correct process (and, specifically, the sender) will not deliver $m$.

### B.3 Totality

We compute an upper bound for the probability of **totality** being compromised.

Totality is compromised if at least one, but not all processes deliver a message. We compute an upper bound for the probability of this happening by assuming that totality is compromised if early consistency is compromised.

If early consistency is not compromised, then a Byzantine sender will cause processes to be E-ready for at most one message $m$.

We consider a scenario where a Byzantine adversary can:

- Cause any correct process to be E-ready for $m$.
- Determine if any correct process delivered $m$.

Under Assumption 5, the adversary has no knowledge of neither the ready nor the delivery sample of any correct process. Therefore, the adversary has no way of meaningfully distinguishing two correct processes, based on the effect that their Ready messages will have on the system.

Let $n$ represent the number of correct processes that are E-ready for $m$. The number of processes that deliver $m$ is a non-decreasing function of $n$. Therefore, the probability of compromising totality, given that at least one correct process delivered $m$, is maximized by the adversary that minimizes $n$.

The adversary that has the highest probability of compromising totality, therefore, will cause one correct process at a time to be E-ready for $m$ until at least one correct process delivers $m$. Totality is compromised if at least one correct process does not deliver $m$.

We model the above using a multi-round game of Threshold Contagion, where the player infects one more uninfected node at the beginning of each round. At the end of each round, we use both our bounds for the delivery probability to determine whether or not totality can be compromised at that round.

### B.4 Consistency

We compute an upper bound for the probability of **consistency** being compromised.

Consistency is compromised if two correct processes deliver two conflicting messages, $m$ and $m'$. We compute an upper bound for the probability of this happening by assuming that consistency is compromised if early consistency is compromised.
If early consistency is not compromised, but both \( m \) and \( m' \) are delivered by at least one correct process, then either \( m \) or \( m' \) is delivered without any correct process being E-ready for it. We bound the probability of this happening using our result on feedback creation.

C ANALYSIS OF SEQUENCED PROBABILISTIC DOUBLE-ECHO

We now discuss the correctness of Sequenced Probabilistic Double-Echo.

No creation. A correct process delivers a message only if it was previously pcb.Delivered. Moreover, pcb.Broadcast is invoked by the sender process only upon psb.Broadcast.

Since probabilistic consistent broadcast satisfies no duplication, each instance of pcb will deliver at most once. Since probabilistic consistent broadcast also satisfies integrity, no correct process will pcb.Deliver without a corresponding invocation of pcb.Broadcast on the side of the sender.

Therefore, if the sender never broadcasts more than \( n \) messages, then no correct process will deliver more than \( n \) messages.

Integrity. A correct process delivers a message only if it was previously pcb.Delivered. Moreover, pcb.Broadcast is invoked by the sender process only upon psb.Broadcast.

Since probabilistic consistent broadcast satisfies integrity, then no correct process will deliver a message \( m \), unless \( m \) was previously broadcast by the sender.

C.1 Multi-validity

Probabilistic secure broadcast satisfies \( \epsilon \)-multi-validity if the underlying abstraction of probabilistic consistent broadcast satisfies \( \epsilon \)-validity.

If the sender \( \sigma \) is correct, and it initially broadcasts \( m_1, \ldots, m_n \), then \( \sigma \) pcb.(\( i - 1 \)).Broadcasts \( m_i \) for every \( i \in \{1, \ldots, m_i\} \)

If probabilistic consistent broadcast satisfies \( \epsilon \)-validity, then each \( m_i \) is eventually pcb.Delivered by \( \sigma \) with probability at least \( (1 - \epsilon) \). Therefore, \( m_1, \ldots, m_n \) are eventually pcb.Delivered with probability at least \( (1 - \epsilon)^n \).

Upon pcb.Deliver of \( m_i \), messages[\( i - 1 \)] is set to \( m_i \). When all \( m_1, \ldots, m_n \) are pcb.Delivered, the first \( n \) entries of messages are updated to a value different than \( \bot \). As a result, \( m_1, \ldots, m_n \) are delivered in sequence.

Consequently, the sender process delivers \( m_1, \ldots, m_n \) with probability at least \( (1 - \epsilon)^n \).

C.2 Multi-totality

Probabilistic secure broadcast satisfies \( \epsilon \)-multi-totality if the underlying abstraction of probabilistic consistent broadcast satisfies \( \epsilon \)-totality.
Let $\pi$ be a correct process. If $\pi$ delivered $n$ messages, then $\text{messages}[j] \neq \bot$ for all $j \in \{0, \ldots, n-1\}$. Moreover, if $\pi$ is correct, then $\text{messages}[j]$ is set to a value other than $\bot$ only upon pcb.$j$.Deliver.

Each instance of pcb satisfies totality with a probability at least $(1-\epsilon)$. Therefore, with probability at least $(1-\epsilon)^n$, all pcb.$j$ satisfy totality, and every correct process eventually sets $\text{messages}[j] \neq \bot$, with $j \in \{0, \ldots, n-1\}$.

Consequently, every correct process delivers $n$ messages with probability at least $(1-\epsilon)^n$.

### C.3 Multi-consistency

Probabilistic secure broadcast satisfies $\epsilon$-**multi-consistency** if the underlying abstraction of probabilistic consistent broadcast satisfies $\epsilon$-**consistency**.

Let $\pi, \rho$ be two correct processes. If $\pi$ and $\rho$ initially delivered $m_1, \ldots, m_n$ and $m'_1, \ldots, m'_n$ respectively, then, for every $i \in \{1, \ldots, n\}$, $\pi \text{ pcb.}(i-1).\text{Delivered } m_i$, and $\rho \text{ pcb.}(i-1).\text{Delivered } m'_i$.

Each instance of probabilistic consistent broadcast satisfies consistency with probability at least $(1-\epsilon)$. Therefore, with probability at least $(1-\epsilon)^n$, pcb.$j$ satisfies consistency for all $j \in \{0, \ldots, n-1\}$, and $m_i = m'_i$ for all $i \in \{1, \ldots, n\}$.

Therefore, with probability at least $(1-\epsilon)^n$, all correct processes that initially deliver $n$ messages deliver $m_1, \ldots, m_n$. 