Invited Review Paper

Theoretical analysis for CSMA/CA-based wireless full duplex networks

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Received January 10, 2021; Revised March 10, 2021; Published July 1, 2021

Abstract: Full Duplex (FD) communication can potentially double the throughput of a point-to-point link in wireless communication. Additionally, FD communication can mitigate collisions induced by hidden nodes. Though exchanging control frames also mitigate hidden node collisions, FD communication has a smaller overhead for one data transmission than the control-frame scheme. Although FD communication is expected to provide system-performance improvement, the enhancement highly depends on its network topology or system parameters. Besides, it is not easy work to comprehend the relationships between system parameters and network performance. Theoretical analysis is useful for evaluating network performance and the optimal design for the system. This paper introduces theoretical analysis for Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA)-based wireless FD networks, mainly author’s researches, and discusses their performance. In this paper, star-topology single-hop networks and string topology multi-hop networks are considered as analysis subjects. This paper evaluates FD communication potential for each network through the analytical model by comparing network performances with exchanging control frames and FD communication.

Key Words: full-duplex communication, networks, CSMA/CA, MAC protocols, theoretical analysis, Markov-chain model, WLAN, multi-hop networks

1. Introduction

In-band Full-Duplex (FD) communication has attracted significant attention because this technique increases the spectral efficiency of wireless communications [1–3]. This technique enables users to transmit and receive data signals simultaneously in the same frequency band. FD communication can potentially double the throughput of a point-to-point link. This technique not only increases the network capacity but also solves the various problems in wireless networks. One benefit is that FD communication can mitigate hidden node collisions [1–3]. Though exchanging control frames such as Request To Send and Clear To Send (RTS/CTS) also mitigates hidden node collisions, FD communication has a smaller overhead for one data transmission than the control-frame scheme. Therefore, applying FD communication to Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) based wireless networks, where the hidden-nodes collisions cause severe network-throughput degra-
Fig. 1. Full duplex communication with bidirectional transmission mode. (a) bi-directional FD, (b) uni-directional FD.

dation, is expected. Although FD communication is expected to provide various merits to wireless networks, the enhancement highly depends on network topology or system parameters. Finding the parameters which provide more benefit to the networks is not easy work. From the viewpoint of network designer, it is necessary to comprehend the relationships between system parameters and network performance. Theoretical analysis becomes a powerful tool for network performance comprehensions [4].

This paper introduces analytical models for CSMA/CA-based wireless FD networks and discusses the networks’ performance. The analysis subjects considered in this paper are star-topology single-hop networks [5, 6] and string topology multi-hop networks [7, 8]. This paper evaluates FD communication potential for each network through the analytical model by comparing network performances with the control-frame scheme and FD communication.

The rest of this paper is organized as follows. Sec. 2 provides related works for wireless FD networks. Sec. 3 explains theoretical analysis for CSMA/CA-based wireless FD networks, which is the main part of this paper. Sec. 3.1 explains the fundamental part of the analysis based on two Markov-chain models. An expression of throughput for single-hop networks is provided in Sec. 3.2. For throughput analysis of FD multi-hop networks, integration of airtime expression [4] and the Markov-chain models are introduced in Sec. 3.3. Sec. 4 provides performance evaluations for FD single-hop networks and FD multi-hop networks through the analytical model, and conclusions are given in Sec 5.

2. Wireless FD communication networks

2.1 FD transmission in networks

In wireless FD communication, two transmission modes are considered: “Bidirectional Full-Duplex (BFD) transmission mode” and “Unidirectional Full-Duplex (UFD) transmission mode.” In the BFD mode, a pair of an access point (AP) and a station (STA) transmits frames to each other simultaneously (i.e. Fig. 1(a)). On the other hand, in the UFD transmission mode, only a node performs transmission and reception simultaneously. (i.e. Fig. 1(b)).

As shown in Fig. 1, STA 0 cannot sense transmissions of STA 1 because STA 1 is located outside of the carrier-sensing range of STA 0, and vice versa. This relationship is called “hidden nodes.” In CSMA/CA networks, data transmission collisions induced by hidden nodes cause severe network throughput degradation. To overcome the hidden node collisions, IEEE 802.11 prepares RTS/CTS-handshaking method where each node exchanges some control frames before a data transmission [9]. However, overhead due to the handshaking may cause throughput degradation. FD communication has the potential to mitigate hidden node collisions. Using FD communication, AP transmits some signals while STA 0 transmits a data frame, as shown in Fig. 1. Although STA 1 cannot sense the STA-0 transmission, STA 1 overhears the signals transmitted by AP. Because STA 1 detects that the channel is busy, STA 1 refrains from initiating transmission until the ongoing STA-0 transmission is completed. This means that STA 1 recognizes the transmission of STA 0 through FD communication between STA 0 and the AP. FD communication prevents hidden nodes from transmitting data frames. This function is one of the important functions of wireless FD communication in networks. Medium Access Control (MAC) protocols for wireless FD networks have been designed, taking into account
2.2 CSMA/CA-based FD MAC protocol

The FD MAC protocols are classified into two types; RTS/CTS-based FD MAC protocol and header snooping-based FD MAC [1]. The difference between the two protocols comes from how to adjust the FD communication timing, and how to mitigate the hidden-node collisions.

Figure 2 shows the channel access examples of (a) header snooping-based FD MAC and (b) RTS/CTS-based FD MAC. In Fig. 2, STA 0 and STA 1 are in a hidden node relationship. In the header snooping-based FD MAC protocols, the information regarding FD communication is included in the header part of a data frame [10]. In Fig. 2(a), STA 0 starts to transmit a data with FD communication by decoding the information from the header part of primary data transmission. For preventing hidden nodes from transmitting data frame, the header snooping-based FD MAC uses FD communication [10, 11]. On the other hand, in RTS/CTS-based FD MAC protocols, the information for FD communication is included in RTS and CTS frames. By exchanging the control frames before a DATA-frame transmission with FD communication, network nodes adjusts their FD communication timings. The RTS/CTS handshaking brings not only the timing adjustment for FD communications but also mitigation of hidden node collisions.

In CSMA/CA-based FD MAC protocols, when a node wins the channel contention and starts to transmit a frame (called “primary transmission”), another node starts a “secondary transmission” triggered by the primary transmission. Following Distributed Coordination Function (DCF) in IEEE 802.11, both the AP and STAs set backoff timers from the range \([0, 2^{CW_{\text{min}}}-1]\) prior to frame transmission. Here, \(CW_{\text{min}}\) is the value of minimum CW, and \(s\) is the number of retransmissions (backoff stage [12]). In Fig. 2, \(s = 0\) and \(CW_{\text{min}} = 16\). A STA transmits a frame when its backoff timer is zero. In FD MAC, a primary-transmitter whose backoff timer is zero gives a frame-transmission opportunity to the secondary transmitter, as shown in Fig. 2. Note that the secondary transmitter starts to transmit a frame even if its backoff timer is not zero. It is stated that this operation is a characteristic backoff process in CSMA/CA-based FD MAC protocols.

As shown in Fig. 2(a), through the FD communication between STA 0 and the AP, STA 1 recognizes the transmission of STA 0. Namely, the FD communication prevents hidden nodes from transmitting data frames. The primary transmitter may finish transmitting a data frame before the end of the secondary transmission. The AP transmits a busy tone signal to prevent the hidden node from such characteristic function in FD communication.
transmitting data frames as shown in Fig. 2(a).

Note that wireless FD transmission cannot avoid hidden node collisions completely. Figure 3 shows the example of the transmission failure due to hidden-node collisions. In the header snooping-based FD MAC, when STA 1 starts to transmit a data frame during STA 0 transmitting the header part of the data frame, STA 0 fails its transmission due to the hidden node collision. Such duration is called “vulnerable duration” [13]. Generally, the vulnerable duration of the header snooping-based FD MAC is shorter than that of the RTS/CTS-based FD MAC. As shown in Figs. 2 and 3, the overhead durations of one data frame for the RTS/CTS-based FD MAC are longer than those for header snooping-based FD MAC. However, the wasted duration due to frame transmission collisions for RTS/CTS-based FD MAC is shorter than that for the header snooping-based FD MAC. From the above explanations, the comparison with RTS/CTS-based FD MAC protocol and header snooping-based one can be summarized as shown in Table I.

### 2.3 Theoretical analysis

Some analytical models of FD networks have been proposed by modeling the characteristic operation in FD MAC. To the best of my knowledge, the model of [13] is the first paper that gives the analytical model for FD MAC protocols including the characteristic operation in CSMA/CA-based FD MAC protocols. Analysis subject of [13] is star-topology single-hop networks such as WLAN. In the model, Binary Exponential Backoff (BEB) operation, which is one of the fundamental functions of DCF in CSMA/CA, is not considered. Although the effects of hidden node collisions are considered in [13], this model assumes the network has extremely large $CW_{\text{min}}$. In real networks, $CW_{\text{min}}$ is set as not so large. Therefore, the model of [13] cannot be applied to the network with a large collision probability due to lack of consideration for BEB. To solve this problem, the model of [14] considers BEB operations by extending the model of [13]. However, the hidden node collisions are not considered in the model. Additionally, models of [13] and [14] assume only header snooping-based FD MAC operation. For establishing a general analytical model, the model of [5, 6] proposed throughput analysis with two Markov-chain modes regarding backoff operation in FD MAC and BEB. This model can be applied to both FD WLANs with header snooping-based FD MAC and RTS/CTS-based FD MAC under the consideration of hidden node collisions. Reference [15] proposed the improved backoff scheme for FD WLAN, and those performances are evaluated by using the model of [5, 6]. Additionally, Ref. [16] proposes optimization of contention-window value based on the FD WLANs’ theoretical analysis.

The analytical models for the FD multi-hop networks have been proposed in [7, 8]. In this model, the airtime expressions and the Markov-chain model for the FD MAC’s backoff operation are integrated. The analytical model based on airtime expressions is an effective approach for multi-hop networks analysis using the airtime expression [4, 17–20]. This integration has enabled to divide the transmission airtime into three parts: the primary, secondary transmissions in the FD communication, and the transmissions in the half-duplex (HD) communications. Using the divided transmission airtimes, each node’s operation can be considered in the FD multi-hop network. Because the model of [7, 8] assumes only header snooping-based FD MAC operation in multi-hop networks, the model of [21] has proposed throughput analysis for multi-hop networks with RTS/CTS-based FD MAC by extending the model of [7, 8].

In the next section, detailed explanations of analytical models from [5, 6] and [7, 8], which are the fundamental models of theoretical analysis for CSMA/CA-based wireless full-duplex networks, are provided. The analysis subjects are single-hop networks with hidden nodes and string-topology multi-hop networks. Through the analytical models, the potential of FD communication for each network is evaluated.
3. Theoretical analysis for wireless CSMA/CA-based wireless FD networks

This section provides detailed explanations of analytical models of CSMA/CA-based wireless FD networks. First, Sec. 3.1 explains the derivation of primary-transmission probability for CSMA/CA-based FD MAC protocols from two Markov-chain models. Next, using derived the primary transmission probability, an expression of throughput for single-hop networks is provided in Sec. 3.2. For throughput analysis of FD multi-hop networks, finally, integration of airtime expression and the Markov-chain models are introduced in Sec. 3.3.

3.1 Markov-chain model for CSMA/CA based FD MAC

The characteristic operation of backoff process in FD MAC are considered in Markov-chain model. Figure 4 shows Markov-chain model of backoff process of FD MAC protocol. In Fig. 4, \( t \) denotes the state that a node has backoff timer \( t \). \( \alpha \) is the probability that a node freezes its backoff timer due to sensing the transmissions of neighbor nodes. \( \beta \) is the probability that a node transmits a data frame with secondary transmission in FD mode. \( W \) is the value of contention window. Unlike DCF, a node starts to transmit a frame as secondary transmission in FD MAC protocols even if its backoff timer is not zero. In Fig. 4, therefore, the transition probability including such operation is expressed as

\[
\begin{align*}
P[t|t+1] &= 1 - \alpha - \beta, & t \in (0,W-2), \\
P[t|t] &= \alpha + \frac{\beta}{W}, & t \in (1,W-1), \\
P[t|u] &= \frac{\beta}{W}, & t \in (0,W-1), \quad u \in (1,W-1), \quad t \neq u. \\
P[t|0] &= \frac{1}{W}, & t \in (1,W-1).
\end{align*}
\]

(1)

Because the Markov-chain model of Fig. 4 has recurrent characteristic, the stationary distribution can be derived. The stationary distribution of the Markov-chain model in Fig. 4 is obtained as

\[
b_t = \frac{(\alpha + \beta) \left[ 1 - (1 - \alpha - \beta)^{W-t} \right]}{W(\alpha + \beta) - (1 - \alpha - \beta) \left( 1 - (1 - \alpha - \beta)^W \right)}, \quad \text{for } t = 0,1, \cdots, W-1.
\]

(2)

Because [6] gives the detailed explanations for the derivation of Eq. (2), this paper omits the derivations. The average staying time is obtained as the expected recurrence time of the Markov-chain model, which is the inverse number of \( b_0 \), namely

\[
\omega = \frac{1}{b_0} = \frac{W(\alpha + \beta) - (1 - \alpha - \beta) \left( 1 - (1 - \alpha - \beta)^W \right)}{(\alpha + \beta) \left[ 1 - (1 - \alpha - \beta)^W \right]}.
\]

(3)

For obtaining primary transmission probability, backoff-stage transitions due to collisions are considered. Figure 5 shows Markov-chain model for backoff-stage number transitions. \( \gamma \) is transmission-failure probability. \( m \) is the value of retransmission limit. The transition probabilities between each state in Fig. 5 are expressed as follows:

\[
\begin{align*}
P[s+1|s] &= \gamma, & s \in (1,m-1), \\
P[m|0] &= 1, \\
P[0|s] &= 1 - \gamma, & s \in (0,m)
\end{align*}
\]

(4)
Here, \( \pi_s \), for \( s \in \{0, 1, \cdots, m\} \) denotes the stationary distribution of the Markov-chain model as shown in Fig. 5. From Eq. (4), we obtain the following:

\[
\pi_s = \frac{\gamma^s (1 - \gamma)}{1 - \gamma^{m+1}}. \tag{5}
\]

Here, the value of the contention window of the \( i \)-th backoff stage is expressed as follows:

\[
W_s = \min(2^s CW_{\text{min}}, CW_{\text{max}}), \tag{6}
\]

where \( CW_{\text{max}} \) are the maximum sizes of \( CW \). By replacing \( W = W_s \) in Eq. (6), the average staying time in backoff stage \( s \) is obtained as follows:

\[
\omega_s = \frac{W_s (\alpha + \beta) - (1 - \alpha - \beta) \left( 1 - (1 - \alpha - \beta)^{W_s} \right)}{(\alpha + \beta) \left( 1 - (1 - \alpha - \beta)^{W_s} \right)} \tag{7}
\]

Here, by calculating the limit as \( \alpha \) and \( \beta \) tend to zero of \( \omega_s \), \( \lim_{\alpha, \beta \to 0} \omega_s = \frac{W_s + 1}{2} \). This means \( \omega_s \) is the same as the average number of backoff timer counting for backoff stage \( s \).

When a node’s backoff timer is zero, the node starts to transmit a frame as primary transmission. Namely, a node transmits only one frame in each backoff stage. By considering the average number of backoff timer counting for backoff stage \( s \) is \( \omega_s \), primary transmission attempt probability is expressed as

\[
\tau = \frac{\sum_{s=0}^{m} \pi_s \times \omega_s}{\sum_{s=0}^{m} \pi_s \times \omega_s} = \frac{1}{\sum_{s=0}^{m} \omega_s \pi_s}
= \frac{1}{\sum_{s=0}^{m} (1 - \gamma) \left\{ W_s (\alpha + \beta) - (1 - \alpha - \beta) \left( 1 - (1 - \alpha - \beta)^{W_s} \right) \right\}}
= \frac{1}{\sum_{s=0}^{m} \gamma^s \left( W_s (\alpha + \beta) - (1 - \alpha - \beta) \left( 1 - (1 - \alpha - \beta)^{W_s} \right) \right)}
= \frac{1 - \gamma^{m+1}}{1 - \gamma}
= \sum_{s=0}^{m} \frac{\gamma^s W_s + 1}{2} \tag{8}
\]

By calculating the limit as \( \alpha \) and \( \beta \) tend to zero of \( \tau \), we obtain

\[
\lim_{\alpha \to 0, \beta \to 0} \tau = \frac{1 - \gamma^{m+1}}{1 - \gamma}
= \frac{1}{2} \sum_{s=0}^{m} \frac{W_s + 1}{2} \gamma^s \tag{9}
= \frac{1}{2} \sum_{s=0}^{m} \frac{W_s + 1}{2} \gamma^s + \cdots + \frac{W_m + 1}{2} \gamma^m.
\]

Equation (9) is the same as Eq. (1) in [23]. This means that the primary transmission probability includes both IEEE 802.11 DCF and the FD MAC operations.
3.2 Throughput analysis for single hop FD networks

In this section, an expression of throughput for a single hop network with FD MAC are derived. The analysis subject is a star-topology single-hop network such as WLAN. The network has \( n+1 \) FD-enable nodes (\( n \) STAs and one AP). All the STAs and an AP have full-duplex capabilities. We assume that each STA has hidden nodes whose average number for a STA is \( n_h \). In addition, we assume average number of nodes within carrier-sensing range of a STA is \( n_c \) and \( n_c + n_h = n \) is satisfied [13]. The AP has no hidden nodes. The other assumptions of this section are following:

**Assumptions 1**

i. FD transmission is assumed to have perfect self-interference cancellation. Additionally, only BFD transmission mode is considered. Therefore, inter-user interference [1–3] is not considered. Transmission failures only occur due to frame collisions [13, 14].

ii. All of the nodes have at least one data frame in their buffer. Namely, networks in saturation condition is considered.

iii. All nodes transmit fixed-sized UDP data packets whose size is \( L \) bytes.

iv. The AP attempts to transmit a data packet to each STA with equal probability \( \frac{1}{n} \).

v. When STA successfully transmits a data packet to AP, AP starts secondary transmission a data frame whose destination is the STA.

As shown in Eq. (8), fixing the value of \( \tau \) requests the expressions of \( \beta \) and \( \gamma \) for STA and AP. In the following discussions at this subsection, we apply the superscript per-STA parameters as \( \beta_{sta} \), \( \tau_{sta} \), and \( \gamma_{sta} \) for a STA, and \( \beta_{ap} \), \( \tau_{ap} \), and \( \gamma_{ap} \) for the AP.

When no node transmits any frames, STA counts its backoff timer. Considering STA recognizes the transmission of hidden nodes through FD communication or RTS/CTS handshaking, the probability that a STA freezes its backoff timer due to sensing neighbor nodes’ data transmissions is expressed as

\[
\alpha_{sta} = 1 - (1 - \tau_{ap})(1 - \tau_{sta})^{n_c - 1}(1 - \tau_{sta})^{n_h - 1} = 1 - (1 - \tau_{ap})(1 - \tau_{sta})^{n - 1} \quad (10)
\]

Similarly, the probability that AP freezes its backoff timer is expressed as

\[
\alpha_{ap} = 1 - (1 - \tau_{sta})^n. \quad (11)
\]

A STA starts secondary transmission when the AP succeeds transmitting a frame as the primary transmission. From the assumption 1-v, therefore, the secondary transmission probability of a STA is expressed as follows:

\[
\beta_{sta} = \tau_{ap}(1 - \tau_{sta})^{n - 1}. \quad (12)
\]

Similarly, AP starts secondary transmission when a STA succeeds transmitting a data packet to AP. The secondary transmission probability of AP is expressed as follows:

\[
\beta_{ap} = n\tau_{sta}(1 - \tau_{sta})^{n - 1}. \quad (13)
\]

For obtaining the transmission-failure probability expression, collisions induced by hidden nodes and the nodes within the carrier-sensing range are considered. As shown in Fig. 3, the hidden nodes start to transmit a data frame if their backoff timer is smaller than \( d \), where \( d \) is vulnerable duration whose unit is slot. Under the use of the header snooping-based FD MAC, \( d \) is calculated as \( T_{HEADER}/\sigma \), where \( T_{HEADER} \) is the times required to send the header part of a DATA frame and \( \sigma \) is a system slot time. In the RTS/CTS-based FD MAC case, \( d \) is calculated as \( T_{RTS}/\sigma \), where \( T_{RTS} \) is duration of a RTS-frame transmission. Here, \( f_{jk} \) is transition rate to state \( j \) from \( k \) in the Markov-chain model, and \( f_{kj} = \frac{b_j}{b_k} \) [22]. From Fig. 4, the states within the vulnerable area are state \( j \in \{0, 1, \cdots, d\} \). Therefore, sum of the transition rate to state 0 from state \( j \in \{0, 1, \cdots, d\} \) is calculated as
\[ \eta_s = \sum_{j=0}^{d} f_{0j} \]
\[ = \sum_{j=0}^{d} \frac{1 - (1 - \alpha - \beta)^{W_s-j}}{1 - (1 - \alpha - \beta)^{W_s}} \]
\[ = \frac{(\alpha + \beta)(d+1) - (1 - \alpha - \beta)^{W_s+1} + (1 - \alpha - \beta)^{W_s-d}}{\beta [1 - (1 - \alpha - \beta)^{W_s}]} . \]  

(14)

Therefore, the probability that a hidden node starts to transmit a data frame is obtained as
\[ \tau_h = \sum_{s=0}^{m} \pi_s^{m} \sum_{i=0}^{m} \gamma^i \eta_s / \sum_{s=0}^{m} \gamma^s \omega_s . \]  

(15)

A STA has \( n_h \) hidden nodes and \( n_c \) contention nodes. Therefore, transmission failure probability of STA is obtained as
\[ \gamma_{sta} = 1 - (1 - \tau_{ap})(1 - \tau_{sta})^{n_c-1}(1 - \tau_{h,sta})^{n_h-1} . \]  

(16)

Because the AP has no hidden node, on the other hand, transmission failure probability of the AP is obtained as
\[ \gamma_{ap} = 1 - (1 - \tau_{sta})^{n} . \]  

(17)

Network throughput is obtained following the traditional analytical method based on Bianchi’s Markov-chain model [12]. The probability that there is at least a single-frame transmission in the considered slot time is expressed as:
\[ P_{tr} = 1 - (1 - \tau_{ap})(1 - \tau_{sta})^{n} . \]  

(18)

FD transmission is successful when AP or a STA transmits a frame as a primary transmission without frame collisions. Therefore, the probability that the FD transmission will succeed is expressed as follows:
\[ P_{suc} = \tau_{ap} + n_{sta}(1 - \tau_{ap})^{n_c-1}(1 - \tau_{h,sta})^{n_h-1} / P_{tr} . \]  

(19)

Network throughput is obtained as the time fraction wherein the channel is occupied for successful transmission [13, 14]. Therefore, the saturated throughput is expressed as:
\[ E = P_{tr}P_{suc}2L / (1 - P_{tr}) + P_{tr}P_{suc}T_{suc} + P_{tr}(1 - P_{suc})T_{fail} . \]  

(20)

Here, \( T_{suc} \) and \( T_{fail} \) are transmission-success duration and transmission-failure one, respectively. As following explanations of [13], \( T_{suc} \) and \( T_{fail} \) are expressed as
\[ T_{suc} = T_{DIFS} + T_{DATA} + T_{SIFS} + T_{ACK} + T_{HEADER} , \]
\[ T_{fail} = n_c (T_{DIFS} + T_{DATA}) + n_h \frac{3}{2} (T_{HEADER} + T_{DATA} + T_{DIFS}) , \]  

(21)

where \( T_{DIFS}, T_{SIFS}, \) and \( T_{ACK} \) are the durations of the DIFS, SIFS, and ACK-frame transmission times, respectively, and \( T_{DATA} \) are the times required to send the payload, respectively. In the case of RTS/CTS frame, these values are following as:
\[ T_{suc} = T_{DIFS} + T_{RTS} + T_{CTS} + T_{DATA} + 3T_{SIFS} + T_{ACK} , \]
\[ T_{fail} = n_c (T_{DIFS} + T_{RTS}) + n_h \frac{3}{2} (T_{RTS} + T_{DIFS}) , \]  

(22)

where \( T_{CTS} \) is the durations of the CTS-frame transmission duration.
3.3 Throughput analysis for FD multi-hop networks

In this section, an expression of throughput for a multi-hop network with FD MAC are derived. Figure 6 shows string topology $H$-hop networks with one-way flow, which are the analysis subjects in this subsection.

The analysis in this section is based on the following assumptions:

**Assumptions 2**

i. Only Node 0 generates the fixed-sized UDP data frames, the payload size of which is $L$ bytes. The destination of the data frames is Node $H$. Node $i$, for $i = 1, 2, \ldots, H - 1$, generates a data frame by receiving the data frame from Node $i - 1$. Namely, the intermediate nodes never generate the data frames by itself [17–20, 25].

ii. Channel conditions of all the links are ideal. Additionally, the self-interference cancellation is perfect. Namely, the transmission failures occur only due to the frame collisions [17–20, 25].

iii. Each node’s transmission range is the same as its carrier sensing range [25]. Therefore, Node $i + 1$ and Node $i - 1$ are within the transmission range of Node $i$ as shown in Fig. 6. Similarly, Node $i + 1$ and Node $i - 1$ are in the carrier sensing range of Node $i$. In this network, Node $i$ and Node $i + 2$ are in the hidden node relationships.

iv. UFD mode is considered. Node $i$ starts to transmit a data frame as a secondary transmitter when Node $i$ has at least one frame in the buffer at the data frame receiving from Node $i - 1$. Under this assumption, Node 0 never be a secondary transmitter as shown in Fig. 6.

First, the primary transmission airtime and the secondary transmission one are expressed. The two types of airtimes enable to express carrier sensing airtime and hidden node collision probability in FD multi-hop networks. Finally, we obtain the end-to-end throughput of FD multi-hop networks.

3.3.1 Airtime expressions for FD multi-hop networks

To achieve the primary transmission airtime and the secondary transmission one, a ratio of the number of the primary transmissions and that of the total number of the data frame transmissions is derived from two Markov-chain models as shown in Figs. 4 and 5. For the simplification of the analysis in multi-hop networks, it is assumed $\alpha = 0$ in Fig. 4. It is seen from Fig. 4 that all the arrows from each state flows in point A. The arrow from state 0 flows into point A with probability 1. This transition means selecting a next backoff timer after primary transmission. On the other hand, The arrows from state $t \in (1, W - 1)$ flows into point A with probability $\beta$. These transitions mean selecting a next backoff timer after secondary transmissions. Considering the probability flows regarding above transitions in Fig. 4, from characteristics of Markov-chain model [22], the ratio of the number of the primary transmissions and the total number of the data frame transmissions can be derived. The sum of probability flows which flow into point A is $1 \cdot b_0 + \beta \sum_{t=1}^{W-1} b_t$, where the first term means probability flow regarding primary transmission and the second term means probability flow regarding secondary transmission. Therefore, the ratio of the number of the primary transmissions and that of the total number of the data frame transmissions in backoff stage $s$ is expressed as
\[
\delta_s = \frac{1 \cdot b_0}{1 \cdot b_0 + \beta \sum_{t=1}^{W_s-1} b_t} = \frac{1 - (1 - \beta) W_s}{W_s \beta}.
\] (23)

By averaging Eq. (23) for all the backoff stages, the ratio of the number of the primary transmissions and that of the total number of the data frame transmissions is derived as

\[
\Delta = \sum_{s=0}^{m} \pi_s \delta_s = \frac{1 - \gamma}{1 - \gamma^{m+1}} \sum_{s=0}^{m} \frac{1 - (1 - \beta) W_s}{W_s \beta} \cdot \gamma^s.
\] (24)

Here, \(\Delta_i\) means the ratio of the number of the primary transmissions and that of the total number of the data frame transmissions for Node \(i\). The primary and the secondary transmission airtime of Node \(i\) can be expressed as

\[
X_{PR_i} = \Delta_i X_i,
\] (25)

and

\[
X_{SC_i} = (1 - \Delta_i) X_i,
\] (26)

respectively. It is necessary to consider that a node transmits a data frame with HD transmission due to the empty of the secondary transmitter’s buffer. The primary transmission airtime should also be divided into the HD case and the FD one. As an essential characteristic in string-topology FD multi-hop networks, it is supposed that the number of primary transmissions for Node \(i\) is approximately equal to the number of secondary transmissions for Node \(i + 1\). From this relationship, the HD transmission airtime of Node \(i\) is expressed as

\[
X_{HD_i} = X_{PR_i} - X_{SC_{i+1}} = \Delta_i X_i - X_{SC_{i+1}}.
\] (27)

In the multi-hop network, it is assumed that the primary transmission does not fail because the secondary transmission prevents the data frame transmission by the hidden nodes. Therefore, the throughput of Node \(i\) is expressed as

\[
E_i = [X_{HD_i}(1 - \gamma_i) + (X_{PR_i} - X_{HD_i}) + X_{SC_i}(1 - \gamma_i)] \frac{L}{T_{suc}}
\]

\[= [X_i - \gamma_i(X_{HD_i} + X_{SC_i})] \frac{L}{T_{suc}}. \] (28)

In the case of HD, \(\Delta_i\) for all the node is equal to one, namely \(X_i = X_{HD_i}\) and \(X_{SC_i} = 0\). In HD networks, therefore, Eq. (28) is rewritten as \(X_i(1 - \gamma_i) \frac{L}{T_{suc}}\), which is the same expression as Eq. (15) in [19].

### 3.3.2 Carrier-sensing airtime and idle airtime

The carrier sensing airtime consists of the frame reception durations from the previous node and the carrier sensing durations from the other nodes in the carrier sensing range. Node \(i\) is the secondary transmitter when Node \(i - 1\) transmits a data frame as the primary transmitter. Therefore, the carrier sensing airtime of Node \(i\) is regarded as the sum of the frame transmission durations in all of the nodes in the carrier sensing range excluding “Node \(i - 1\) primary transmission durations in FD case” and “Node \(i + 1\) secondary transmission durations”. Therefore, the carrier sensing airtime of Node \(i\) is expressed as
\[ Y_i = (X_{i-1} - X_{SC_i}) + (X_{i+1} - X_{SC_{i+1}}) - \frac{(X_{i-1} - X_{SC_i})(X_{i+1} - X_{SC_{i+1}})}{1 - X_i} \]

\[ = X_{SC_{i-1}} + X_{HD_{i-1}} + X_{PR_{i+1}} - \frac{(X_{SC_{i-1}} + X_{HD_{i-1}})X_{PR_{i+1}}}{1 - X_i}. \] (29)

In the case of string-topology HD networks, \( X_i = X_{HD_i} = X_{PR_i} \) is satisfied. Namely, the right-hand side of Eq. (29) is \( X_{i-1} + X_{i+1} - X_{i-1}X_{i+1}/(1 - X_i) \), which is the same as Eq. (10) in [18]. This result shows the analytical expressions regarding Eq. (29) includes both FD and HD multi-hop networks.

When a node is in neither the transmission state nor the carrier sensing states, the channel related to the node is idle. Namely, the channel idle airtime is expressed as

\[ Z_i = 1 - X_i - Y_i. \] (30)

### 3.3.3 Frame-existence probability

Frame existence probability defined as the probability that Node \( i \) has at least one data frame when the node is in the channel idle state [19]. From the explanation in [19], this probability is expressed as the rate of total durations for backoff timer decrement and the channel-idle durations. The backoff timer decrement duration of the overall time is the product of the data frame reception rate and the average slot number of the backoff timer decrement for single data frame transmission success. From the assumption 2-i, the frame arrival(reception) rate for Node \( i \) is expressed as

\[ \lambda_i = \begin{cases} 
O & \text{for } i = 0 \\ 
\frac{E_{i-1}}{L} & \text{for } i = 1, 2, \ldots, H - 1
\end{cases}, \] (31)

where \( O \) is the network offered load. The denominator of the right hand side in Eq. (8) means the average number of backoff timer decrement for one frame transmission success. Therefore, frame existence probability for Node \( i \) is expressed as

\[ q_i = \min \left( \lambda_i \sigma \sum_{s=0}^{m} \gamma \frac{W_s \beta_1 - (1 - \beta_1) (1 - (1 - \beta_1)^{W_s})}{\beta_1 [1 - (1 - \beta_1)^{W_s}]}, 1 \right). \] (32)

### 3.3.4 Secondary transmission probability

From the assumption 2-iv, Node \( i \) performs the secondary transmission when Node \( i - 1 \) starts to perform a primary transmission during Node \( i \) is in the backoff timer decrement state. In the FD multi-hop networks as shown in Fig. 6, however, Node \( i \) does not perform the secondary transmission when Node \( i - 1 \) has an empty buffer. Similarly, when Node \( i - 2 \) is in the transmission state, Node \( i \) does not perform the secondary transmission. This is because Node \( i - 1 \) senses the transmission of Node \( i - 2 \). The probability of that Node \( i - 1 \) has an empty buffer when Node \( i \) is in the backoff decrement state is expressed as \( (1 - q_{i-1})Z_{i-1}q_iZ_i \). Additionally, the probability that Node \( i - 2 \) is in the transmission state when Node \( i \) is in the backoff timer decrement state is expressed \( (X_{HD_{i-2}} + X_{SC_{i-2}})q_iZ_i \).

Therefore, the secondary transmission probability of Node \( i \) is obtained as conditional probability following as:

\[ \beta_i = \frac{q_iZ_i - (1 - q_{i-1})Z_{i-1}q_iZ_i - (X_{HD_{i-2}} + X_{SC_{i-2}})q_iZ_i}{q_iZ_i} \tau_{i-1} \]

\[ = [1 - (1 - q_{i-1})Z_{i-1} - X_{HD_{i-2}} - X_{SC_{i-2}}] \tau_{i-1}. \] (33)

### 3.3.5 Transmission failure probability

In string-topology multi-hop networks, there are two types of the hidden node collision, which are (1) protocol hidden node collision and (2) physical hidden node collision [18]. The protocol hidden
Table II. System Parameters.

| Parameter                      | Value     |
|-------------------------------|-----------|
| Data rate                     | 18 Mbps   |
| ACK rate                      | 12 Mbps   |
| RTS/CTS frame bit rate        | 6 Mbps    |
| \( T_{\text{HEADER}} \)       | 35 \( \mu \)sec |
| \( T_{\text{DATA}} \) (L = 1000 bytes) | 704 \( \mu \)sec |
| \( T_{\text{RTS}} \)          | 48 \( \mu \)sec |
| \( T_{\text{CTS}} \)          | 48 \( \mu \)sec |
| \( T_{\text{ACK}} \)          | 28 \( \mu \)sec |
| \( T_{\text{SIFS}} \)         | 16 \( \mu \)sec |
| \( T_{\text{DIFS}} \)         | 34 \( \mu \)sec |
| Slot time (\( \sigma \))      | 9 \( \mu \)sec |
| \( CW_{\text{min}} \)         | 16        |
| Retransmission limit (\( m \)) | 7         |

Node collision occurs when Node \( i \) starts to transmit a data frame during Node \( i + 2 \) transmitting a data frame. On the other hands, the physical hidden node collision occurs when Node \( i + 2 \) starts to transmit a frame during Node \( i \) transmission. Namely, the frame collision occurs when the backoff timer of Node \( i + 2 \) is smaller than \( d \) at starting the frame transmission. Because both Node \( i \) and Node \( i + 2 \) cannot transmit a data frame when Node \( i + 1 \) transmits a data frame, collision probability of Node \( i \) is expressed as

\[
\gamma_i = \frac{aX_{PR_{i+2}}}{1 - X_{PR_{i+1}}} + \frac{q_i + 2Z_{i+2}T_h_{i+2}}{1 - X_{PR_{i+1}} - X_{PR_{i+2}}},
\]

where \( a = T_{\text{DATA}}/T_{\text{suc}} \).

### 3.4 Flow constraint in multi-hop networks

In string-topology multi-hop networks, each node relays the data frame generated by the source node. Therefore, the throughput of each node satisfies

\[
E_0 = E_1 = \cdots = E_{H-1}.
\]

The relationship in Eq. \( (35) \), the flow-constraint condition, expresses the network layer property, which is the flow-constraint condition [4, 18–20]. From Eqs. \( (31), (33), (34) \) and \( (35) \), 4\( H \) algebraic equations are obtained. These equations contain 4\( H \) unknown parameters, \( X_i, \lambda_i, \beta_i \), and \( \gamma_i \) for \( i = 0, 1, 2, \ldots, H - 1 \). It is possible to fix the 4\( H \) unknown parameters when the offered load \( O \) is given. The end-to-end throughput is obtained as \( E_{H-1} \).

### 4. Verifications and evaluations

This section verifies the theoretical model introduced in this paper and discusses the performances of wireless full-duplex networks for single-hop and multi-hop cases. Table II gives the system parameters, which are based on IEEE 802.11.

#### 4.1 Single-hop networks

First, single-hop networks case is considered. The network topology used for simulations of this scenario is also the star-topology networks with an AP and \( n \) STAs as shown in Fig. 7. The STAs are randomly deployed around the AP within the distance of 50 m. We carry out 100 simulations to take one plot. In the simulations, both transmission and carrier-sensing ranges of each node are also set as 50 m. As the STAs’ density increases, the average number of hidden nodes for each STA also increases. From the “back-of-the-envelope” approximation technique from [24], the average number of hidden nodes for a STA \( n_h \) is calculated as 0.41\( n \) in this network scenario where the carrier-sensing range is set as the same range as the transmission range [13]. Similarly, the average number of nodes within the carrier-sensing range for a STA \( n_c \) is 0.59\( n \).
Figure 8 shows the saturation throughput as a function of the number of STAs. It is seen from Fig. 8 that analytical results agree with simulation ones quantitatively, which demonstrates the validity of this analysis. There remains a little difference between analytical results and simulation results. There are various reasons of the difference. It is stated that one of the reasons for the difference in Fig. 8 comes from the assumption that the average number of nodes within the carrier-sensing range for a STA $n_c$ is set as $0.59n$ [13, 24]. Because of a statistical characteristics, this assumption may not be suitable especialy for a network with small number of STAs. In Fig. 8, the throughput of RTS/CTS-based FD MAC is larger than that of header snooping-based FD MAC when $n$ is larger than 3. In this setting, payload size $L$ is 1000 bytes. In this case, the wasted time due to each frame transmission failure in header snooping-based FD MAC is large compared with that in RTS/CTS-based FD MAC. In the result, throughput with RTS/CTS-based FD MAC is larger than that with header snooping-based FD MAC when the AP handles many STAs.

Figure 9 shows the saturation throughput for $n = 20$ as a function of the payload size $L$. It is seen from Fig. 9 that analytical results agree with simulation ones quantitatively. In Fig. 9, the point of intersection between two types of lines are obtained when payload size $L$ is 350 bytes. The point of intersection may function as the criteria for using header snooping-based FD MAC or RTS/CTS-based one. The points for various cases can be obtained easily from the analytical model.

Figure 10 shows saturated throughput $E$ as a function of payload size $L$ and the number of STAs.
Fig. 9. Saturated throughput $S$ as a function of payload size $P$ for $n = 20$.

Fig. 10. Saturated throughput $S$ as a function of payload size $L$ and number of STAs $n$. The value of data rate is set as (a) 18 Mbps, (b) 36 Mbps, and (c) 54 Mbps, respectively.

It is seen from Fig. 10 that throughput from header snooping-based FD MAC is larger than that from RTS/CTS-based FD MAC as increasing data rate. The network throughput depends on the length of DATA-transmission durations. The length of DATA-transmission duration becomes short as increasing data rate. This result shows that avoiding hidden node collision in header snooping-based
FD MAC is effective especially when the duration of a data transmission is short. From the analytical model, throughputs for both RTS/CTS-based and header snooping-based FD MAC can be easily obtained for various parameters, such as the number of STAs, payload size, and data rate.

4.2 Multi-hop network

Second, a multi-hop network case is considered. The network topologies used for the simulation are string-topology $H$-hop networks as shown in Fig. 6.

Figure 11 shows the end-to-end throughput as a function of offered load. It is seen from Fig. 11 that the analytical results have good agreements with the simulation results. This result shows the analytical model is valid for multi-hop networks.

Figures 12 and 13 show the frame existence probabilities as a function of offered load in FD and HD three-hop networks, respectively. The results for HD multi-hop networks can be derived by defining the secondary transmission probabilities of all the nodes is zero, namely $\beta_i \to 0$, for $i = 0, 1, \cdots, H-1$.

It is seen from Fig. 12 that Node 0 is the bottleneck in FD three-hop networks because the frame existence probability of Node 0 reaches one at $O = 2.21$ Mbps. Similarly, from Fig. 13, the bottleneck node in HD three-hop networks is also Node 0. This is because Node 0 suffers the effect of hidden node collisions due to the transmission of Node 2 in three-hop networks. Node 0 has much longer the average frame service time for one data frame than the other nodes. In FD three-hop networks, Node 0 cannot perform the secondary transmission while the other nodes can. Therefore, Node 0 becomes the bottleneck node as the same reason as HD three-hop networks. However, it is seen from Figs. 12 and 13 that Node 1 with FD has lower frame existence probability compared with HD cases. This is because the relay nodes in FD networks can achieve more transmission opportunities than HD networks because of secondary transmission. This is the difference of node’s behavior between HD multi-hop networks and FD ones. It is confirmed that the analytical model expresses such difference, and clarify the bottleneck node in FD multi-hop networks.

Figure 14 shows the maximum throughput of three-hop networks as a function payload size. It is seen from Fig. 14 that the analytical results show quantitative agreements with the simulation ones. Additionally, it is seen that the end-to-end throughput of FD multi-hop networks is improved from that of HD multi-hop networks. This is because the FD transmissions enable to decrease durations spending for the carrier sensing as shown in Eq. (29). This result indicates the effectiveness of applying FD to multi-hop networks.

Finally, performance evaluations for RTS/CTS-based and header snooping-based FD MACs in multi-hop networks are given. For obtaining the analytical results for multi-hop networks with RTS/CTS-based FD MAC, this paper adopted the model of [21] and the same system parameters. Figure 15 shows the maximum end-to-end throughput as a function of the number of hops for RTS/CTS-based and header snooping-based FD MACs. Figure 16 shows the maximum end-to-
end throughput as a function of payload size in a five-hop network with the RTS/CTS-based and header snooping-based FD MACs. It is seen from Figs. 15 and 16 the throughput for the header snooping-based FD MAC is much higher than that for the RTS/CTS-based FD MAC. This is because of the mechanism of exchanging control frames in multi-hop networks. In RTS/CTS-based FD
Fig. 15. Maximum end-to-end throughput as a function of number of hops ($L = 500$ bytes, Data rate = 54 Mbps).

Fig. 16. Maximum end-to-end throughput of five-hop network as a function of payload size (Data rate = 54 Mbps).

MAC, when a node transmits an RTS frame in the multi-hop network, the neighboring node of the RTS frame transmitter sets Network Allocation Vector (NAV). The neighboring node is forbidden transmitting any frames due to the NAV setting. As the result, transmission opportunities among network nodes are decreased. For achieving the potential of FD communication in multi-hop networks, it is stated that header snooping-based FD MAC is more effective than RTS/CTS-based FD MAC.

5. Conclusion and future works
This paper introduced analytical models for CSMA/CA-based wireless FD networks and discussed the networks’ performance. The analytical subjects considered in this paper are a star-topology single-hop network and a string topology multi-hop network. FD communication potential for each network is evaluated through the analytical model by comparing network performance with control-frame scheme and FD communication. From the viewpoint of network design, the theoretical analysis provides practical knowledge.

Acknowledgments
This research was supported in part by the Telecommunications Advancement Foundation and Scholarship Foundation and Grant-in-Aid for scientific research (No. 17K14681) from the JSPS.
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