Metal-insulator transition and glassy behavior in
two-dimensional electron systems

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ABSTRACT

Studies of low-frequency resistance noise demonstrate that glassy freezing occurs in a two-dimensional electron system in silicon in the vicinity of the metal-insulator transition (MIT). The width of the metallic glass phase, which separates the 2D metal and the (glassy) insulator, depends strongly on disorder, becoming extremely small in high-mobility (low-disorder) samples. The glass transition is manifested by a sudden and dramatic slowing down of the electron dynamics, and by a very abrupt change to the sort of statistics characteristic of complicated multistate systems. In particular, the behavior of the second spectrum, an important fourth-order noise statistic, indicates the presence of long-range correlations between fluctuators in the glassy phase, consistent with the hierarchical picture of glassy dynamics.

Keywords: metal-insulator transition, glass transition, noise, two-dimensional systems

1. INTRODUCTION

Since the development of the scaling theory of localization for noninteracting electrons, it was widely believed that all electronic states are localized in a two-dimensional (2D) disordered system. Strictly speaking, however, the possibility of a 2D metallic state and the metal-insulator transition (MIT) in case of strong electron-electron interactions has been an open issue from the theoretical point of view. Recent experiments on dilute 2D electron and hole systems in semiconductor heterostructures have provided considerable evidence for the existence of a true, zero-temperature MIT, and revived interest in the problem of the interplay of disorder and electron-electron interactions in 2D. However, the physics behind the apparent MIT is still not understood.

It is now well established that the MIT occurs in the regime where both Coulomb (electron-electron) interactions and disorder are strong. Theoretically, it is well known that, in the strongly localized limit, the competition between electron-electron interactions and disorder leads to glassy dynamics (electron or Coulomb glass). Some glassy properties, such as slow relaxation phenomena, have been indeed observed in various insulating thin films. Furthermore, recent work has suggested that the critical behavior near the 2D MIT may be dominated by the physics of the insulator, leading to the proposals that the 2D MIT can be described alternatively as the melting of the Wigner, Coulomb, or spin glass. It is clear that understanding the nature of the insulator represents a major open issue in this field.

While glassy systems exhibit a variety of phenomena, studies of metallic spin glasses have demonstrated that mesoscopic, i.e. transport noise measurements are required in order to provide definitive information on the details of glassy ordering and dynamics. We have employed a combination of transport and low-frequency resistance noise measurements to probe the glassy behavior and the MIT in a 2D electron system to form in Si metal-oxide-semiconductor field-effect transistors (MOSFETs). We find that glassy freezing occurs near the MIT.
in the regime of very low conductivity $\sigma$, apparently as a precursor to the MIT. The glass transition is manifested by a sudden and dramatic slowing down of the electron dynamics and by an abrupt change to the sort of statistics characteristic of complicated multistate systems. The properties of the entire glass phase are consistent with the hierarchical picture of glassy dynamics, similar to spin glasses with long-range correlations. We also show that the width of the metallic glass phase, which separates the 2D metal and the glassy insulator, depends strongly on disorder, becoming very small in samples with relatively low disorder. These results are consistent with the model that describes the 2D MIT as the melting of a Coulomb glass.\textsuperscript{17, 24, 25}

2. SAMPLES AND EXPERIMENTAL TECHNIQUE

Measurements were carried out on a 2DES in MOSFETs that were fabricated on the (100) surface of Si. In such a device, the disorder is due to the oxide charge scattering (scattering by ionized impurities randomly distributed in the oxide within a few Å of the interface) and to the roughness of the Si-SiO$_2$ interface.\textsuperscript{26} While the former dominates at low carrier densities $n_s$, the increasing $n_s$ improves screening and reduces the effective disorder due to oxide charges. However, at the same time, the electrons are pushed closer to the interface, and surface roughness scattering becomes more important, becoming dominant at high $n_s$. As a result of this competition, the mobility of the 2DES exhibits a peak as a function of $n_s$. The value of the peak mobility at 4.2 K is usually taken as a rough measure of the disorder.\textsuperscript{26} We have studied two sets of devices with the peak mobilities that differ by about a factor of 40, which, together with the substantial differences in their geometry, size, and many fabrication details, means that these samples span essentially the entire range of Si technology.

The low-mobility (i. e. low peak mobility or high disorder) devices were fabricated using standard 0.25 μm Si technology\textsuperscript{27} with poly-Si gates, self-aligned ion-implanted contacts, substrate doping $N_a \sim 2 \times 10^{17}$cm$^{-3}$, oxide charge $N_{ox} = 1.5 \times 10^{13}$cm$^{-2}$, and oxide thickness $d_{ox} = 50$ nm. Their peak mobility at 4.2 K was only 0.06 m$^2$/Vs. Most of the measurements were performed on a 1 μm long, 90 μm wide rectangular sample. The fluctuations of current $I$ (i. e. $\sigma$) were measured as a function of time $t$ in a two-probe configuration using an ITHACO 1211 current preamplifier and a PAR124A lock-in amplifier at ~ 13 Hz. The excitation voltage $V_{exc}$ was kept constant and low enough (typically, a few $\mu$V) to ensure that the conduction was Ohmic. A precision DC voltage standard (EDC MV116J) was used to apply the gate voltage $V_g$, which controls the carrier density $n_s$. The current fluctuations as low as $10^{-13}$ A were measured at $0.13 \leq T \leq 0.80$ K in a dilution refrigerator with heavily filtered wiring. Relatively small fluctuations of temperature $T$, $V_g$, and $V_{exc}$ were ruled out as possible sources of the measured noise, since no correlation was found between them and the current fluctuations. In addition, a Hall bar sample from the same wafer was measured at $T = 0.25$ K in both two- and four-probe configurations, and it was determined that the contact resistances and the contact noise were negligible.

High-mobility (low-disorder) samples had the peak mobility of $\approx 2.5$ m$^2$/Vs at 4.2 K. They were fabricated in a Hall bar geometry with Al gates, $N_a \sim 10^{14}$cm$^{-3}$, and oxide thickness $d_{ox} = 147$ nm.\textsuperscript{28, 29} The resistance was measured down to $T = 0.24$ K using a standard four-probe ac technique (typically 2.7 Hz) in the Ohmic regime. The DC voltage standard was used to apply $V_g$. Contact resistances and their influence on noise measurements were minimized by using a split-gate geometry, which allows one to maintain high $n_s$ ($\approx 10^{12}$ cm$^{-2}$) in the contact region while allowing an independent control of $n_s$ of the 2D system under investigation in the central part of the sample ($120 \times 50$ μm$^2$) (Fig. 3 inset). Nevertheless, care was taken to ensure that the observed noise did not come from either the current contacts or the regions of gaps in the gate. For example, since the noise measured across a resistor connected in series with the sample and having a similar resistance was at least three times lower than the noise from the central part of the sample, the effect of the contact noise on the excitation current $I_{exc}$ could be easily ruled out. Similarly, the resistance and the noise measured between the voltage contact in the region of high $n_s$ (e. g. #5 in Fig. 3 inset) and the one in the central part (#6) were much smaller than those measured between contacts in the central part (e. g. #6 and #7). In fact, they were in agreement with what is expected based on the geometry of the sample, which proves that the gap regions did not contribute to either the measured resistance or noise. In order to minimize the influence of fluctuations of both $I_{exc}$ and $T$, some of the noise measurements were carried out with a bridge configuration.\textsuperscript{30} The difference voltage was detected using two PAR124A lock-in amplifiers, and a cross-spectrum measurement was performed with an HP35665A spectrum analyzer in order to reduce the background noise even further.\textsuperscript{31} The output filters of the lock-in amplifiers and/or spectrum analyzer served as an antialiasing device. Most of the noise spectra
Relative fluctuations of $\sigma$ vs. time for different $n_s$ in a sample with high disorder at $T = 0.13$ K. $\langle \sigma \rangle$ is the time-averaged conductivity. Different traces have been shifted for clarity; the lowest $n_s$ is at the bottom and the highest $n_s$ at the top. The noise decreases dramatically, from $\sim 100\%$ to less than $1\%$, with the increasing $n_s$. The character of the noise also changes dramatically as $n_s$ is varied.

were obtained in the $f = (10^{-4} - 10^{-1})$ Hz bandwidth, where the upper bound was set by the low frequency of $I_{exc}$, limited by the low cut-off frequency of RC filters used to reduce external electromagnetic noise as well as by high resistance of the sample.

### 3. EXPERIMENTAL RESULTS

In both types of samples, we have observed strong fluctuations of $\sigma$ with time at low $n_s$ and $T$. Figure 1 shows the fluctuations of $(\sigma - \langle \sigma \rangle)/\langle \sigma \rangle$ ($\langle \ldots \rangle$ represents averaging over time intervals of, typically, several hours) in a low-mobility sample for several $n_s$ at $T = 0.13$ K. It is quite striking that, for the lowest $n_s$, the fluctuation amplitude is of the order of $100\%$. In addition to rapid, high-frequency fluctuations, both abrupt jumps and slow changes over periods of several hours are also evident. The amplitude of the fluctuations decreases dramatically with increasing either $n_s$ (Fig. 1) or $T$, as discussed in more detail below. Perhaps even more interesting is the change in the character of the noise: at high enough $n_s$, the slow modulations and the discrete events are no longer apparent in the raw data and, in fact, the variance of the noise no longer varies with time. Figure 2 shows that similar behavior is observed in the resistance noise of high-mobility samples. Here we choose to plot $(\rho - \langle \rho \rangle)/\delta \rho$, where $\delta \rho = \langle (\rho - \langle \rho \rangle)^2 \rangle^{1/2}$ and resistivity $\rho = 1/\sigma$, in order to make the changes in the character of the noise with the variation of $n_s$ more apparent. In order to try to understand the origin of the observed noise, it is important to study the transport characteristics first, because they provide information on the mechanism of conduction in the regime of interest.

#### 3.1. Transport

Transport properties of high-mobility samples almost identical to ours have been studied extensively by several groups.\textsuperscript{2, 28, 29} Naturally, we find similar results for the behavior of the time-averaged resistivity $\langle \rho \rangle$ as a function of $T$ for different $n_s$, as shown in Fig. 3. For the lowest $n_s$ and $T$, the data are described by an activated form $\langle \rho \rangle \propto \exp(T_0/T)$, corresponding to transport in the insulating regime. The vanishing of $T_0$ is often used as a criterion to determine $n_s$,\textsuperscript{32, 33} the critical density for the MIT. Using this method, we find that $n_c \approx 9.7 \times 10^{10}\text{cm}^{-2}$ (Fig. 3 inset). In this sample, $d\langle \rho \rangle/dT$ changes sign at $n_s^* \approx 9.7 \times 10^{10}\text{cm}^{-2}$, so that $n_c \approx n_s^*$.
Figure 2. Resistance noise in a high-mobility (low disorder) sample for several $n_s$ shown on the plot. $(\rho - \langle \rho \rangle)/\delta\rho$ is plotted ($\delta\rho^2$ = variance, $\rho$ = resistivity) in order to make the change in the character of the noise with $n_s$ more apparent. Different traces have been shifted vertically for clarity.

In agreement with other studies,$^{32,33}$ we point out, however, that a small but systematic difference of a few percent has been reported$^{34,35}$ such that $n^*_s > n_c$. At even higher $n_s$, a pronounced drop of $\langle \rho \rangle$ with decreasing $T$ is observed. This strong metallic temperature dependence of resistivity has been a subject of considerable research effort,$^2$ and some progress has been made recently in understanding its origin.$^{36,37}$ On the other hand, the low-mobility samples used in our work have not been studied before. We find that the behavior of $\langle \sigma(n_s, T) \rangle$ in these samples (Fig. 4) is qualitatively similar to that of high-mobility devices. At the highest $n_s$, for example, low-mobility samples exhibit a metallic-like behavior with $d\langle \sigma \rangle/dT < 0$ (i.e. $d(\rho)/dT > 0$). The change of $\langle \sigma \rangle$ in a given $T$ range, however, is small (only 6% for the highest $n_s = 20.2 \times 10^{11}$ cm$^{-2}$) as observed in other Si MOSFETs with a large amount of disorder.$^{34}$ $d\langle \sigma \rangle/dT$ changes sign when $\langle \sigma(n_s) \rangle = 0.5 e^2/h$ similar to other 2D samples.$^2$ For the lowest $n_s$, the data are again best described by the simply activated form $\langle \sigma \rangle \propto \exp(-T_0/T)$ (Fig. 5(a)). $T_0$ decreases linearly with increasing $n_s$ (Fig. 5(a) inset), and vanishes at $n_c \approx 5.2 \times 10^{11}$ cm$^{-2}$. Surprisingly, we find that, close to $n_c$, the data are best described by the metallic power-law behavior $\langle \sigma(n_s, T) \rangle = a(n_s) + b(n_s)T^x$ with $x \approx 1.5$ (Fig. 5(b)). The fitting parameter $a(n_s)$ is relatively small and, in fact, vanishes for $n_s(10^{11}$ cm$^{-2}) = 4.72$ and 4.92. Such a simple power-law $T$-dependence of $\sigma$, given by $\langle \sigma(n_c, T) \rangle \propto T^x$, is consistent with the one expected in the quantum critical region (QCR) of the MIT based on general arguments,$^{38}$ and with the behavior observed in 3D systems$^{38}$ and other Si MOSFETs$^{39}$ within the QCR. Moreover, the $T^{3/2}$ correction is consistent with the recent theory for the metallic glass phase.$^{24}$ Therefore, based on the analysis of $\langle \sigma(n_s, T) \rangle$ in both insulating regime and QCR, we conclude that the critical density $n_c = (5.0 \pm 0.3) \times 10^{11}$ cm$^{-2}$, which is more than a factor of two smaller than $n^*_s$. Such a large difference between $n_c$ and $n^*_s$ is attributed to a much higher amount of disorder in these samples than in high-mobility Si MOSFETs.$^{23,32-35}$
3.2. Noise

We begin our discussion of noise by considering low-mobility samples first. A simple analysis shows\(^{22}\) that \(\frac{\delta \sigma}{\langle \sigma \rangle} (\delta \sigma^2 = \text{variance})\) does not depend on \(n_s\) and \(T\) at high enough \(n_s\). However, below a certain density \(n_g = (7.5 \pm 0.3) \times 10^{11} \text{cm}^{-2}\), which does not seem to depend on \(T\), an enormous increase of \(\frac{\delta \sigma}{\langle \sigma \rangle}\) is observed with decreasing either \(n_s\) or \(T\). It is interesting that \(\frac{\delta \sigma}{\langle \sigma \rangle}\) does not exhibit any special features near \(n_c\) or \(n_s^*\).

A more detailed study of the noise has been carried out by calculating the normalized power spectra \(S_I(f) = \frac{S(I, f)}{I^2 (f)}\) \((f - \text{frequency})\) of \((\sigma - \langle \sigma \rangle)/\langle \sigma \rangle\) for all \(n_s\) and \(T\). Most of the spectra were obtained in the \(f = (10^{-4} - 10^{-1}) \text{Hz bandwidth}\), where they follow the well-known empirical law \(S_I \propto 1/f^\alpha\).\(^{40, 41}\) The background noise was measured by setting \(I = 0\) for all \(n_s\) and \(T\). It was always white and usually several orders of magnitude smaller than the sample noise. Nevertheless, a subtraction of the background spectra was always performed, and the power spectra of the device noise were averaged over frequency bands \((\leq \text{an octave})\). Some of the resulting \(S_I\) are presented in Fig. 6. At the highest \(n_s\), \(S_I(f)\) does not depend on \(n_s\). However, it is obvious that, by reducing \(n_s\) below \(n_g\), \(S_I\) increases enormously, by up to six orders of magnitude at low \(f\), as shown in Fig. 7. This striking increase of the slow dynamic contribution to the conductivity reflects a sudden and dramatic slowing down of the electron dynamics. This is attributed to the freezing of the electron glass.

We also find that, for \(n_s < n_g\), \(S_I(f)\) increases exponentially with decreasing \(T\) (Fig. 7 inset), consistent with early studies on Si MOSFETs where \(dS_R/dT < 0\) was observed for \(T = 1.5, 4.2 \text{K}\).\(^{42}\) Such a temperature dependence shows that the noise in our system cannot be explained by the models of thermally activated charge trapping,\(^{41, 43, 44}\) noise generated by fluctuations of \(T\),\(^{45}\) and a model of noise near the Anderson transition.\(^{46, 47}\) Likewise, the models of noise in the Mott and Efros-Shklovskii variable-range hopping (VRH) regimes\(^{48-51}\) predict either \(dS_R/dT > 0\) or a saturation of \(S_R\) below 10-100 Hz, both in clear disagreement with the experiment. Therefore, the observed noise cannot be a result of single electron hops even when Coulomb interactions are included through the Coulomb gap. In principle, it is possible that the VRH models of noise\(^{48-51}\) may not be applicable to our low-mobility samples with their relatively short and wide geometry. In such cases, one may expect\(^{52, 53}\) that the percolation cluster will be reduced to a small number of parallel hopping chains, resulting...
Figure 4. Low-mobility sample: $\langle \sigma \rangle$ vs. $T$ for different $n_s$. The data for many other $n_s$ have been omitted for clarity. The error bars show the size of the fluctuations. $n^*_s$, $n_g$ (glass transition density), and $n_c$ (critical density for the metal-insulator transition) are marked by arrows. They were determined as explained in the main text.

in a weaker temperature dependence of conductivity. Measurements of $1/f$ noise in 2D hopping of electrons in GaAs samples with lengths (0.5 and 1 µm) comparable to ours and performed at higher $T$ (> 4.2 K) and $f$ (1 Hz), have shown a power-law increase of noise with decreasing $T$, in agreement with the VRH models of noise. In the shortest sample, with the length $\sim 0.2$ µm, the cluster was reduced down to a set of well separated linear chains. In that sample, the temperature dependence of noise was found to be even weaker than in longer samples, which is exactly the opposite of what is observed in our samples. These results show clearly that the noise in our low-mobility samples cannot be explained based on the model of hopping chains. We also note that a small number of parallel hopping chains would result in large ($\sim 100\%$) fluctuations of conductivity with the gate voltage, which moves the Fermi energy through different sets of localized states. While such fluctuations have been observed in other experiments, here we find that the fluctuations of $\langle \sigma \rangle$ with $V_g$ are only of the order 0.1% in the range of $n_s$ where noise has been studied. On the other hand, an increase of noise at low $T$ similar to our results has been observed in mesoscopic spin glasses, in wires in the quantum Hall regime for tunneling through localized states, and in Si quantum dots in the Coulomb blockade regime. Perhaps the most striking feature of our data is the sharp jump of the exponent $\alpha$ at $n_s \approx n_g$ (Fig. 8). While $\alpha \approx 1$ ("pure" $1/f$ noise) for $n_s > n_g$, $\alpha \approx 1.8$ below $n_g$, reflecting a sudden shift of the spectral weight towards lower frequencies. This is yet another manifestation of the sudden and dramatic slowing down of the electron dynamics at $n_g$. Similar large values of $\alpha$ have been observed in some spin glasses above the MIT, and in submicron wires. This will be discussed in more detail below. Here we point out that the onset of glassy dynamics occurs on the metallic side of the MIT, at the density $n_g > n_c$ (where VRH models are clearly not applicable!). This implies the existence of the metallic glass phase for $n_c < n_s < n_g$, which is actually consistent
Figure 5. Low-mobility sample: a) $\langle \sigma \rangle$ vs. $T^{-1}$ for several $n_s(10^{11}\text{cm}^{-2})$ in the insulating regime. The error bars show the size of the fluctuations, and the lines are fits to $\langle \sigma \rangle \propto \exp(-T_0/T)$. Inset: $T_0$ vs. $n_s$ with a linear fit, and an arrow showing $n_c$. b) $\langle \sigma \rangle$ vs. $T^{1.5}$ for a few $n_s(10^{11}\text{cm}^{-2})$ near $n_c$. The solid lines are fits; the dashed line is a guide to the eye, clearly showing insulating behavior ($\langle \sigma(T \to 0) \rangle = 0$).
with recent predictions of the model of interacting electrons near a disorder-driven MIT.\(^{25}\) We also note that, in the glassy phase, $\alpha$ decreases with increasing $T$ (Fig. 8 inset). As a result of such $\alpha(T)$: (i) the large values of $\alpha$ are observable only at relatively low $T$, and (ii) the rise in $\alpha$ with decreasing $n_s$ becomes smoother, \textit{i.e.} less sharp at higher $T$.

In high-mobility samples, the time series of the relative changes in resistance $\Delta R(t)/\langle R \rangle$ and the corresponding power spectra $S_R(f)$ have been studied in detail, and qualitatively the same behavior has been found.\(^{23}\) At high $n_s$, the low-frequency (\textit{e.g.} 1 mHz) noise power depends rather weakly on both $T$ and $n_s$. In the vicinity of $n_g \approx 10 \times 10^{11}\text{cm}^{-2}$, however, a dramatic change in the behavior of $S_R$ is observed at low $T$. The noise amplitude starts to increase strongly with decreasing $n_s$, and the exponent $\alpha$ jumps from $\approx 1$ to $\approx 1.8$, which is again attributed to the freezing of the electron glass. The temperature dependence of $\alpha$ for these samples is shown in Fig. 9. We point out that these samples are much longer than the low-mobility ones and, in fact, have a completely opposite geometry (long and narrow, as opposed to short and wide). This provides further evidence that the observed behavior of noise cannot be attributed to geometric effects.

Since the amount of disorder in these samples is considerably lower than in the low-mobility ones, the absolute value of $n_g$ is, not surprisingly, almost an order of magnitude lower. However, in addition to affecting the values of $n_c$, $n_g$, and $n_s^*$, the disorder plays another, nontrivial role. In particular, $n_c$ and $n_g$ were found to differ from each other considerably in low-mobility devices ($n_c$, $n_g$, and $n_s^*$ were 5.0, 7.5, and 12.9, respectively, in units of $10^{11}\text{cm}^{-2}$), whereas in high-mobility devices $n_g$ is at most a few percent higher than $n_c$. Therefore, the emergence of glassy dynamics here seems almost to coincide with the MIT. Obviously, the size of the metallic ($n_c < n_s < n_g$) glass phase depends strongly on disorder, in agreement with theoretical predictions.\(^{25}\)

We have established that the exponent $\alpha \approx 1$ in the 2D metallic phase (above $n_g$) in both low- and high-mobility samples. On the other hand, $\alpha \approx 1.8$ in the glassy phase. In general, such high values of $\alpha$ may be also obtained if noise results from a superposition of a small number of independent two-state systems (TSS).\(^{41, 43}\) However, even though some distinguishable discrete events can be seen at low $n_s$ (Figs. 1 and 2), they do not show the characteristic repetitive form of stable TSS. On the contrary, both the shape and the magnitude of noise exhibit random, nonmonotonic (which exclude aging) changes with time. A quantitative measure of such spectral wandering is the so-called second spectrum $S_2(f_2, f)$, which is the power spectrum of the fluctuations.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{The averaged noise power spectra $S_I \propto 1/f^\alpha$ vs. $f$ for several $n_s$ in a low-mobility sample. Solid lines are linear least-squares fits with the slopes equal to $\alpha$.}
\end{figure}
of $S_R(f)$ with time,\textsuperscript{21,64} i.e. the Fourier transform of the autocorrelation function of the time series of $S_R(f)$. If the fluctuators (e.g. TSS) are not correlated, $S_2(f_2, f)$ is white (independent of $f_2$\textsuperscript{21,41,64} and equal to the square of the first spectrum. Such noise is called Gaussian. On the other hand, $S_2$ has a nonwhite character, $S_2 \propto 1/f_2^{1-\beta}$, for interacting fluctuators.\textsuperscript{21,41,64} Therefore, the deviations from Gaussianity provide a direct probe of correlations between fluctuators.
Figure 9. $\alpha$ vs. $T$ for several $n_s$ in a high-mobility sample; $n_g \approx 10 \times 10^{10}$ cm$^{-2}$. Lines are guides to the eye.

Figure 10. (a) Second spectral density $S_2(f_2)$ vs. $f_2$ for $n_s(10^{10}$ cm$^{-2}$) shown on the plot; $f_L = 1$ mHz. Inset: $S_2(f_2)$ for spin glasses Cu$_{0.91}$Mn$_{0.09}$ and Cd$_{0.93}$Mn$_{0.07}$Te. Solid lines are fits. Exponent $1 - \beta$ vs. $n_s$ for (b) high-mobility and (c) low-mobility samples. The low-mobility device is the same as the one studied in Ref. 22.

We investigate $S_2$ using digital filtering in a given frequency range $f = (f_L, f_H)$ (usually $f_H = 2f_L$). The normalized second spectra, with the Gaussian background subtracted, are shown in Fig. 10(a) for two $n_s$, just above and just below $n_g$. It is clear that there is a striking difference in the character of the two spectra. Similar differences are observed between various spin glasses (Fig. 10(a) inset), where $S_2$ is white in the absence of long range interactions, and nonwhite when long range RKKY interaction leads to hierarchical glassy dynamics. A detailed dependence of the exponent $(1 - \beta)$ on $n_s$ has been determined for both high- and low-mobility samples (Figs. 10(b) and (c), respectively). In both cases, $S_2$ is white for $n_s > n_g$, indicating that the observed $1/f$ noise results from uncorrelated fluctuators. It is quite remarkable that $S_2$ changes its character in a dramatic way exactly at $n_g$ in both types of samples. For $n_s < n_g$, $S_2$ is strongly non-Gaussian, which demonstrates that the fluctuators are strongly correlated. This, of course, rules out independent TSS (such as traps) as possible sources of noise when $n_s < n_g$. In fact, a sudden change in the nature of the fluctuators (i.e. correlated vs. uncorrelated) as a function of $n_s$ rules out any traps, defects, or a highly unlikely scenario that the observed
glassiness may be due to some other time dependent changes of the disorder potential itself. Instead, it provides an unambiguous evidence for the onset of glassy dynamics in a 2D electron system at $n_g$.

In the studies of spin glasses, the scaling of $S_2$ with respect to $f$ and $f_2$ has been used to unravel the glassy dynamics and, in particular, to distinguish generalized models of interacting droplets or clusters (i.e. TSS) from hierarchical pictures. In the former case, the low-$f$ noise comes from a smaller number of large elements because they are slower, while the higher-$f$ noise comes from a larger number of smaller elements, which are faster. In this picture, which assumes compact droplets and short-range interactions between them, big elements are more likely to interact than small ones and, hence, non-Gaussian effects and $S_2$ will be stronger for lower $f$. $S_2(f_2, f)$, however, need to be compared for fixed $f_2/f$, i.e. on time scales determined by the time scales of the fluctuations being measured, since spectra taken over a fixed time interval average the high-frequency data more than the low-frequency data. Therefore, in the interacting “droplet” model, $S_2(f_2, f)$ should be a decreasing function of $f$ at constant $f_2/f$. In the hierarchical picture, on the other hand, $S_2(f_2, f)$ should be scale invariant: it should depend only on $f_2/f$, not on the scale $f$.\cite{21, 64} Fig. 11 shows that no systematic dependence of $S_2$ on $f$ is seen in our samples. The observed scale invariance of $S_2(f_2, f)$ signals that the system wanders collectively between many metastable states related by a kinetic hierarchy. Metastable states correspond to the local minima or “valleys” in the free energy landscape, separated by barriers with a wide, hierarchical distribution of heights and, thus, relaxation times. Intervallen transitions, which are reconfigurations of a large number of electrons, thus lead to the observed strong, correlated, $1/f$-type noise, remarkably similar to what was observed in spin glasses with a long-range correlation of spin configuration.\cite{21, 64} We note that, unlike droplet models,\cite{68, 69} hierarchical pictures of glassy dynamics\cite{70} do allow for the existence of a finite $T$ (or finite Fermi energy) glass transition in presence of a symmetry-breaking field, such as the random potential in an electron glass.

### 3.3. Slow relaxations and history dependence

The 2D electron glass in Si MOSFETs also exhibits other phenomena characteristic of glassy systems. In particular, for $n_s < n_g$, we have observed history dependent behavior, and long relaxation times following a large change in $V_g$. While further careful investigation is required in order to study these effects in detail, here we present an example of the results obtained following two different cooling protocols in a low-mobility sample. In the first one, $n_s(10^{11}\text{cm}^{-2})$ was changed slowly ($\approx 4.5$ hours) from 15.93 ($> n_g$) to 6.01 ($< n_g$) at $T = 0.8$ K (from point 1 to point 2 in Fig. 12), and then the system was allowed to relax (top trace in Fig. 12 inset). The conductivity reached a stationary value after about 70 hours. The system was then cooled to $T = 0.13$ K (point 3 in Fig. 12), where no further relaxation (i.e. monotonic decrease) of $\sigma$ was observed (the height of “point” 3 in Fig. 12 reflects fluctuations of $\sigma$ with time). The system was then warmed back up to $T = 0.8$ K (point 4 in Fig. 12), and $n_s$ changed back to its starting value (point 5 in Fig. 12, identical to 1). In the next, second cooling protocol, the system was first cooled down to 0.13 K (point 6 in Fig. 12), and $n_s(10^{11}\text{cm}^{-2})$ was then
changed from 15.93 to 6.01 (point 7 in Fig. 12) at about the same rate as before. This time, the relaxation (7 → 8 in Fig. 12, and bottom trace in the inset) was nonmonotonic and it took about 80-90 hours for $\sigma$ to reach a stationary value (point 8 in Fig. 12). Most importantly, the final values of $\sigma$ (points 3 and 8) obtained for the same $n_s$ and $T$ following two different cooling protocols differ by a factor of 2, clearly demonstrating history-dependent, i.e. nonergodic behavior.

The history dependence of $\sigma$ at low $n_s$ and $T$ was actually the first “unusual” property that we observed in this system. In order to study its transport properties and obtain reproducible values of $\langle \sigma(n_s, T) \rangle$ shown in Figs. 3 and 4, it was necessary to vary $n_s$ at a relatively high $T$ (we used 0.8 K and $\approx 2$ K for low- and high-mobility samples, respectively).

4. CONCLUSIONS

We have presented evidence for the freezing of the electron glass at a well-defined density $n_g$ in a 2DES in Si. By studying the statistics of low-frequency resistance noise in Si MOSFETs with a wide range of characteristics, including a vast difference in the amount of disorder, we have established that the glassy ordering of a 2DES near the metal-insulator transition is a universal phenomenon in Si inversion layers. Such ordering is observable only at sufficiently low $T$, and becomes more pronounced with decreasing $T$. Glassy freezing occurs in the regime of very low conductivities but on the metallic side of the MIT. The size of the metallic glass phase, which separates the 2D metal and the (glassy) insulator, depends strongly on disorder, becoming extremely small in high-mobility samples. The existence of such a metallic glass phase and its dependence on disorder are consistent with theoretical predictions. The glass transition is manifested by a sudden and dramatic slowing

**Figure 12.** Two different cooling protocols. I $n_s(10^{11} \text{cm}^{-2})$ was changed slowly ($\approx 4.5$ hours) from 15.93 to 6.01 at $T = 0.8$ K ($1 \rightarrow 2$, open squares), and the system was allowed to relax (top trace in the inset). After $\sigma$ reached a stationary value, the sample was cooled down to 0.13 K (point 3 on the plot), where no further relaxation occurred. The system was then warmed up (4), and $n_s$ returned to its initial value ($4 \rightarrow 5$, open triangles). This was followed by the second protocol: II The system was first cooled down to 0.13 K (point 6, solid circle), and $n_s$ was then changed to $6.01 \times 10^{11} \text{cm}^{-2}$ ($6 \rightarrow 7$, solid circles) at about the same rate as before. The system was allowed to relax ($7 \rightarrow 8$, and bottom trace in the inset). $\sigma$ reached a stationary value (point 8) that was, by a factor of 2, different from the value (point 3) obtained in the first protocol for the same $n_s$ and $T$, clearly demonstrating history-dependent behavior.
down of the electron dynamics and by an abrupt change to the sort of statistics characteristic of complicated multistate systems, consistent with the hierarchical picture of glassy dynamics and very similar to spin glasses with long-range correlations. Most recent studies of noise in parallel magnetic fields have shown that the glass transition persists even in fields such that the 2DES is fully spin polarized. Therefore, our results provide strong support to the theoretical proposals that attempt to describe the 2D metal-insulator transition as the melting of a Coulomb glass.\textsuperscript{17, 24, 25}

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