Radiative transitions of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$

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Abstract

We study radiative decays of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ using light-cone QCD sum rules. In particular, we consider the decay modes $D_{sJ}^*(2317) \rightarrow D_s^* \gamma$ and $D_{sJ}(2460) \rightarrow D_s^{(*)} \gamma, D_{sJ}^*(2317) \gamma$ and evaluate the hadronic parameters in the transition amplitudes analyzing correlation functions of scalar, pseudoscalar, vector and axial-vector quark currents. In the case of $D_{sJ}^*(2317) \rightarrow D_s^* \gamma$ we also consider determinations based on two different correlation functions in HQET. The decay widths turn out to be different than previous estimates obtained by other methods; the results favour the interpretation of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ as ordinary $\bar{c}s$ mesons.

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I. INTRODUCTION

The observation of two narrow resonances with charm and strangeness, $D_{sJ}^*(2317)$ in the $D_s\pi^0$ invariant mass distribution and $D_{sJ}(2460)$ in the $D_s^*\pi^0$ and $D_s\gamma$ mass distributions, has raised discussions about the nature of these states and their quark content. The natural identification consists in considering these states as the scalar and axial vector $\bar{c}s$ mesons respectively denoted as $D_{s0}$ and $D_{s1}'$. In the heavy quark limit $m_c \to \infty$ such states are expected to be degenerate in mass and to form a doublet having $s_\ell^P = \frac{1}{2}^+$, with $s_\ell$ the angular momentum of the light degrees of freedom. In that interpretation the two mesons complete, together with $D_{s1}(2536)$ and $D_{s2}(2573)$, the set of four states corresponding to the lowest lying P-wave $\bar{c}s$ states of the constituent quark model. A chiral symmetry between the negative and positive parity doublets $D_{s} - D_{s}^*$ vs $D_{s0} - D_{s1}'$, suggested in ref. [11, 12], would account for the equality of the hyperfine splitting in the two doublets.

However, estimates of the masses of these mesons based on potential quark models generally produce larger values than the measured ones, implying that the two scalar and axial-vector $\bar{c}s$ $D_{s0}$ and $D_{s1}'$ states should be heavy enough to decay to $DK$ and $D^*K$ and should have a broad width. On this basis, other interpretations for $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ have been proposed, for example that of molecular states [13]. Unitarity effects in the scalar $DK$ channel have also been considered [14].

Radiative transitions probe the structure of hadrons, and therefore they are suitable to understand the nature of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ distinguishing among different interpretations [15, 16]. Their rates can be predicted by various methods and the predictions can be compared to the experimental measurements. In particular, it has been suggested that, in the molecular picture, the $D_{sJ}(2460) \to D_{sJ}^*(2317)\gamma$ decay should be driven by the $D^* \to D\gamma$ transition and should occur at a different rate with respect to the rate for a quark-antiquark meson decay [13]; such a suggestion has to be supported by explicit calculations in view of the experimental observations.

The radiative decay widths $D_{sJ}^*(2317) \to D_{s}^*\gamma$ and $D_{sJ}(2460) \to D_{s}^{(*)}\gamma, D_{sJ}^*(2317)\gamma$ have been evaluated using the constituent quark model [11, 15] and the Vector Meson Dominance (VMD) ansatz in the heavy quark limit [10, 16]. In this paper we use a different method, light-cone QCD sum rules, an approach exploited to analyze many aspects of the heavy and
light quark system phenomenology \cite{17,18}, including radiative decays \cite{19,20} (for a review and references see \cite{21}). We apply the method starting from the identification of $D_{sJ}^∗(2317)$ with $D_{s0}$ and $D_{sJ}(2460)$ with $D_{s1}$ and we discuss results and related uncertainties. In particular, in Section II we consider the decay mode $D_{sJ}^∗(2317) \to D_{s}^∗ \gamma$ and describe in detail the calculation of the transition amplitude, the input quantities used in the analysis, the numerical results and the sources of uncertainties. In Section III we carry out a calculation of the same transition amplitude in the infinite heavy quark limit, discussing the deviation from the case of finite mass which is sizeable in the case of charm. The radiative modes of $D_{sJ}(2460)$ are analyzed in Sections IV-VI, where we find different results with respect to those obtained by other methods. In Section VII we discuss the differences; in spite of them, considering the available experimental measurements, we conclude that the description of $D_{sJ}^∗(2317)$ and $D_{sJ}(2460)$ as $q\bar{q}$ states is favoured.

II. $D_{sJ}^∗(2317) \to D_{s}^∗ \gamma$

The amplitude of the E1 transition $D_{s0} \to D_{s}^∗ \gamma$:

$$\langle \gamma(q,\lambda)D_{s}^∗(p,\lambda')|D_{s0}(p+q)\rangle = ed\left[(\varepsilon^∗ \cdot \bar{\eta}^∗)(p \cdot q) - (\varepsilon^∗ \cdot p)(\bar{\eta}^∗ \cdot q)\right], \quad (2.1)$$

with $\varepsilon(\lambda)$ and $\bar{\eta}(\lambda')$ the photon and $D_{s}^∗$ polarization vectors, respectively, and $e$ the electric charge, involves the hadronic parameter $d$ which has dimension mass$^{-1}$. According to the strategy of QCD sum rules, the calculation of this parameter starts from considering the QCD and the hadronic expressions of a suitable correlation function of quark currents.

We consider the correlation function

$$F_{\mu}(p,q) = i \int d^4x \ e^{ipx} \langle \gamma(q,\lambda)|T[J_{\mu}^I(x)J_0(0)]|0\rangle \quad (2.2)$$

of the scalar $J_0 = \bar{c}s$ and the vector $J_{\mu} = \bar{c}\gamma_{\mu}s$ quark currents, and an external photon state of momentum $q$ and helicity $\lambda$. The correlation function can be expressed in terms of Lorentz invariant structures:

$$F_{\mu}(p,q) = F_0 \left[(p \cdot \varepsilon^*)q_\mu - (p \cdot q)\varepsilon^*_\mu\right] + \ldots \quad (2.3)$$

In order to compute $F_0$ (or $F_{\mu}$) in QCD, we carry out the light-cone expansion $x^2 \to 0$ of the product of the two currents in (2.2). This involves non-local matrix elements of quark
operators between the vacuum and the photon state which can be expressed in terms of operator matrix elements of increasing twist. For example, contracting the charm-quark fields in eq. (2.2) we obtain

\[
F_\mu (p, q) = \int \frac{d^4 k}{(2 \pi)^4} \int d^4 x \frac{e^{i(p-k)\cdot x}}{m_c^2 - k^2} \langle \gamma(q, \lambda) | \bar{s}(x) \gamma_\mu (k + m_c) s(0) | 0 \rangle
\]

\[
= \int \frac{d^4 k}{(2 \pi)^4} \int d^4 x \frac{e^{i(p-k)\cdot x}}{m_c^2 - k^2} \left[ k_\mu \langle \gamma(q, \lambda) | \bar{s}(x) s(0) | 0 \rangle - i k^\alpha \langle \gamma(q, \lambda) | \bar{s}(x) \sigma_{\mu\alpha} s(0) | 0 \rangle + m_c \langle \gamma(q, \lambda) | \bar{s}(x) \gamma_\mu s(0) | 0 \rangle \right]; \quad (2.4)
\]

the expressions of the photon matrix elements in terms of distribution amplitudes are collected in the Appendix. This kind of contributions is depicted in fig. (a). Moreover, the light-cone expansion involves higher-twist contributions related to three-particle quark-gluon matrix elements, as depicted in fig. (b); the expressions of the relevant quark-gluon matrix elements can also be found in the Appendix.

FIG. 1: Diagrams involving photon distribution amplitudes. The dashed lines represent the two currents in the correlation function (2.2). In (a) two-particle contributions and in (b) three-particle quark-gluon contributions are shown.

In addition to the contributions of the photon emission from the soft s quark, we must consider the perturbative photon coupling to the strange and charm quarks, fig. 2 (a) and (b). It produces an expression for \( F_0 \) of the form:

\[
F_0 = \int_{(m_s + m_c)^2}^{+\infty} ds \frac{\rho^P(s)}{(s - p^2)(s - (p + q)^2)}
\]

(2.5)
with

$$\rho^p(s) = -\frac{3e_s}{4\pi^2} \left\{ -m_s \ln \left( \frac{s - m_c^2 + m_s^2 - \lambda^2(s, m_c^2, m_s^2)}{s - m_c^2 + m_s^2 + \lambda^2(s, m_c^2, m_s^2)} \right) + \frac{m_c - m_s}{s} \lambda^2(s, m_c^2, m_s^2) \right\}$$

$$+ \frac{3e_s}{4\pi^2} \frac{m_c + m_s}{2} \lambda^\frac{1}{2}(s, m_c^2, m_s^2) \left( 1 - \frac{m_s^2 - m_c^2}{s} \right) + (s \leftrightarrow c) \quad (2.6)$$

($\lambda$ the triangular function). Furthermore, nonperturbative effects when the photon is emitted from the heavy quark give rise to contributions proportional to the strange quark condensate, corresponding to the diagram in fig.2 (c).

FIG. 2: Perturbative photon emission by the strange (a) and charm (b) quark. In (c) the strange
quark condensate contribution is represented.

The result is an expression of the correlation function (2.2) and of the function $F_0$ in
terms of quantities such as quark masses, condensates and photon distribution amplitudes.
The sum rule for $d$ is obtained by the equality of this QCD expression with a hadronic
expression obtained by a complete insertion of physical states. The two quark currents in
(2.2) have non-vanishing matrix elements between the vacuum and $D_s^*$ and $D_{s0}$:

$$\langle 0| J_\mu^\dagger | D_s^* \rangle = f_{D_s} m_{D_s} \tilde{\eta}_\mu$$

$$\langle D_{s0} | J_0 | 0 \rangle = f_{D_{s0}} m_{D_{s0}}$$

so that $F_\mu$ can be written as

$$F_\mu = \frac{\langle D_{s0} | J_0 | 0 \rangle \langle \gamma D_s^* | D_{s0} \rangle \langle 0 | J_\mu^\dagger | D_s^* \rangle}{(m_{D_{s0}}^2 - (p + q)^2)(m_{D_s}^2 - p^2)} + \text{other resonances + continuum} \quad (2.7)$$

neglecting the widths of $D_s^*$ and $D_{s0}$. The sum rule follows after a double Borel trans-
formation in $-p^2$ and $-(p + q)^2$ of both the QCD and the hadronic representation of the
correlation function, that involves two Borel parameters, $M_1^2$ and $M_2^2$. The transformation
allows to suppress the contribution of the continuum of states and of higher resonances, to
suppress the higher twist terms in the QCD expression of the correlation function and to remove all terms that are either independent of one of the two variables $-p^2$ or $-(p + q)^2$ or depend on it only polynomially. The Borel parameters $M^2_1$ and $M^2_2$ are independent; we choose $M^2_1 = M^2_2$ since this allows, invoking global quark-hadron duality between the hadronic and the OPE expression of the correlation function above some threshold $s_0$, to subtract the continuum in the QCD side through the substitution $e^{-\frac{m^2_0}{M^2_2}} \rightarrow e^{-\frac{m^2_0}{M^2_2}} e^{-\frac{s_0}{M^2_2}}$ in the leading twist term \[18\]. The masses of the charmed mesons involved in the transitions are close to each other, therefore the choice of equal Borel parameters is reasonable. The final expression of the sum rule for $d$ is:

\[
d = \frac{e^{-\frac{m^2_0}{M^2_2}}}{m_{D_{s0}} f_{D_{s0}} m_{D_5} f_{D_5}} \left\{ \int_{(m_c + m_s)^2}^{s_0} ds \ e^{-\frac{s}{M^2_2}} \rho^P(s) \right. \\
+ e_c e^{-\frac{m^2}{M^2}} \langle \bar{s}s \rangle \left( 1 + \frac{m_s^2}{4M^2} + \frac{m_c^2 m_c^2}{2M^4} \right) + e_s \langle \bar{s}s \rangle (e^{-\frac{m^2}{M^2}} - e^{-\frac{s_0}{M^2}}) M^2 \chi \phi(u_0) \\
+ e_s \langle \bar{s}s \rangle e^{-\frac{m^2}{M^2}} \left[ -\frac{1}{4} \left( \Lambda(u_0) - 8\bar{H}_\gamma(u_0) \right) \left( 1 + \frac{m_c^2}{M^2} \right) \right. \\
+ \int_0^{1-u_0} dv \int_0^{m_s} d\alpha_g \mathcal{F}(u_0 - (1 - v)\alpha_g, 1 - u_0 - v\alpha_g, \alpha_g) \\
+ \int_{1-u_0}^1 dv \int_0^{1-u_0} d\alpha_g \mathcal{F}(u_0 - (1 - v)\alpha_g, 1 - u_0 - v\alpha_g, \alpha_g) \\
- 2e_s f_{3\gamma} m_c e^{-\frac{m^2}{M^2}} \Psi^\nu(u_0) \right\} \\
(2.9)
\]

where $\mathcal{F} = S - \bar{S} - T_1 + T_4 - T_3 + T_2 + 2v(-S + T_3 - T_2)$, $\hat{H}_\gamma(u) = \int_0^u du' H_\gamma(u')$, $H_\gamma(u) = \int_0^u du' h_\gamma(u')$ and $\Psi^\nu(u) = \int_0^u du' \psi^\nu(u')$. All the distribution amplitudes are defined in the Appendix; $u_0 = \frac{M^2}{M^2_1 + M^2_2} = \frac{1}{2}$.

The sum rule \[2.9\] involves the meson masses, for which we use the experimental data, and the leptonic constants $f_{D_5}$ and $f_{D_{s0}}$. For the former one, we put $f_{D_5} = f_{D_s}$ and use the central value of the experimental result $f_{D_s} = 266 \pm 32$ MeV \[22\]. As for $f_{D_{s0}}$, a sum rule obtained from the analysis of the correlation function

\[
\Pi(p^2) = i \int d^4x \ e^{ip\cdot x} \langle 0 | T [ J_0(0) J_0^\dagger(x) ] | 0 \rangle ,
\]

(2.10)
allows to obtain $f_{D_s^0} = 225 \pm 25$ MeV, using the parameters in the Appendix.

From eq. (2.9) we can compute $d$ varying the threshold $s_0$ and considering the range of the external variable $M^2$ where the result is independent on it (stability region). In this region a hierarchy in the terms with increasing twist is observed, so that we can presume that the neglect of higher-twist contributions induces a small error. On the other hand, the perturbative term, which depends on both the light and the heavy quark charges, represents a sizeable contribution to the sum rule.

In fig. 3 we plot the curves corresponding to different values of $s_0$. Considering the range $5 \text{ GeV}^2 \leq M^2 \leq 7 \text{ GeV}^2$, where the best stability in $M^2$ is found, together with the variation of the threshold $s_0$, we get: $-0.35 \text{ GeV}^{-1} \leq d \leq -0.28 \text{ GeV}^{-1}$, corresponding to the radiative decay width

$$\Gamma(D_{s0} \to D_{s}^*\gamma) = (4 - 6) \text{ keV.}$$ (2.12)

In (2.12) we have only considered the uncertainty in $s_0$ and $M^2$, and we have used the central values of the QCD parameters collected in the Appendix. Actually, such parameters represent another source of uncertainty. In particular, an important input parameter is the
magnetic susceptibility of the quark condensate, $\chi$, for which we use the value determined in ref. [28]: $\chi = -(3.15 \pm 0.3) \text{GeV}^{-2}$. A different value $\chi = -4.4 \text{GeV}^{-2}$, previously used in other sum rule analyses, would produce a 40% larger value of $|d|$.

The result (2.12) shows that the radiative decay occurs at a typical rate for this kind of transitions (a few keV’s). However, the rate is larger by a factor of 4-5 than that obtained using VMD and the infinite heavy quark limit, and by a factor of 2-3 larger than the estimates based on the constituent quark model. It is interesting to investigate the reason of the numerical differences; aiming at that, we estimate $d$ by light-cone QCD sum rules in the heavy quark limit, using an approach based on the heavy quark effective theory. We discuss such a calculation in the next Section.

III. $D_{sJ}(2317) \rightarrow D_s^* \gamma$ IN THE HEAVY QUARK LIMIT

In order to determine the hadronic parameter $d$ in eq. (2.1) when $m_c \rightarrow \infty$, we consider two different correlation functions:

$$F^{(S)}_\mu(\omega, q \cdot v) = i \int d^4x \, e^{i(\omega v - q \cdot x)} \langle \gamma(q, \lambda) | T[\hat{J}^\mu(x) \hat{J}_0(0)] | 0 \rangle$$ (3.1)

and

$$F^{(D)}_\mu(\omega, q \cdot v) = i \int d^4x \, e^{i(\omega v - q \cdot x)} \langle \gamma(q, \lambda) | T[\hat{J}^\mu_d(x) \hat{J}_0(0)] | 0 \rangle$$ (3.2)

The currents in (3.1) and (3.2) are effective currents constructed in terms of the strange quark field and of the effective field $h_v$ of the heavy quark (in our case the charm quark) with four-velocity $v$. The effective field $h_v$ is related to the heavy quark field $Q$ in QCD through the relation $h_v = e^{imQx} \frac{1 + \gamma \cdot v}{2} Q$ (for a review see [23]). The current $\hat{J}^\mu_0 = h_v \gamma^\mu s$ has the quantum numbers of a vector meson. On the other hand, the currents $\hat{J}_0 = h_v s$ and $\hat{J}_d = h_v (-i) \gamma^\mu \bar{D} \mu s$ have both the quantum numbers of a scalar meson, since $D_t^\mu \equiv g_{t\mu\alpha} D^\alpha \equiv (g_{\mu\alpha} - v_\mu v_\alpha) D^\alpha$, $D$ being the covariant derivative. The latter current has been proposed as better suited for describing scalar heavy-light quark mesons in the heavy quark limit [24], therefore it is interesting to investigate how it behaves in sum rules for radiative decays.

The sum rules for $d$ are obtained from (3.1) and (3.2) using the same procedure followed in Sec III, namely considering the light-cone expansion and the hadronic representation of the correlation functions, making a double Borel transform in the variables $\omega$ and $\omega' = \omega - q \cdot v$.
that involve two Borel parameters $E_{1,2}$, choosing $E_1 = E_2 = 2E$ and invoking quark-hadron duality above some threshold $\nu_0$. From (3.1) we obtain

$$d^{(S)} = \frac{4}{F \bar{F}^+} e^{\frac{2}{E \nu}} \left\{ \frac{3m_s e_s}{8\pi^2} \int_{m_s}^{\nu_0} d\nu \ e^{-\frac{\nu}{E}} \ln \left[ \frac{\nu - (\nu^2 - m_s^2)^{\frac{1}{2}}}{\nu + (\nu^2 - m_s^2)^{\frac{1}{2}}} \right] \right\}$$

$$+ e_s \langle \bar{s}s \rangle \frac{2}{2} E \chi_f \gamma(u_0) \left( 1 - e^{-\frac{\nu_0}{E}} \right) - e_s \langle \bar{s}s \rangle \frac{4}{8E} \left( \frac{\hat{A}(u_0)}{3} - \hat{H}_\gamma(u_0) \right) - e_s \frac{f_{3g}}{2} \Psi^e(u_0) \right\}.$$  \hspace{1cm} (3.3)

On the other hand, from the correlation function (3.2) we get:

$$d^{(D)} = \frac{4}{F \bar{F}^+} e^{\frac{2}{E \nu}} \left\{ \frac{3m_s e_s}{8\pi^2} \int_{m_s}^{\nu_0} d\nu \ e^{-\frac{\nu}{E}} \ln \left[ \frac{\nu - (\nu^2 - m_s^2)^{\frac{1}{2}}}{\nu + (\nu^2 - m_s^2)^{\frac{1}{2}}} \right] (m_s + \nu) \right\}$$

$$+ e_s \langle \bar{s}s \rangle \frac{1}{2} \left[ \Psi^v(u_0) + \frac{1}{4} \psi^s(u_0) - u_0 \frac{\psi^v(u_0)}{4} \right] \left( 1 - e^{-\frac{\nu_0}{E}} \right)$$

$$+ e_s \langle \bar{s}s \rangle \frac{1}{2} \left[ - E^2 \left( \chi_f \gamma(u_0) + u_0 \chi_f \gamma(u_0) \right) \right] \left( 1 - e^{-\frac{\nu_0}{E}} \right)$$

$$+ e_s \langle \bar{s}s \rangle \frac{1}{2} \left[ \frac{1}{16} \left( \hat{A}(u_0) + u_0 \hat{A}'(u_0) \right) - \hat{H}_\gamma(u_0) \right] \right\}.$$  \hspace{1cm} (3.4)

Notice that in (3.3)-(3.4) only photon emission from the light quark contributes. In the heavy quark limit the current-vacuum matrix elements are defined as follows: $\langle 0|\hat{J}^\mu|D^*_{s0}(v, \lambda)\rangle_H = \hat{F}^H \eta^\mu(\lambda), \langle 0|\hat{J}_d|D_{s0}(v)\rangle_H = \hat{F}_d^H$ (the subscript $H$ indicates that the states are normalized as used in HQET; $\hat{F}^H$ and $\hat{F}_d^H$ have dimension mass$^{3/2}$ and mass$^{5/2}$, respectively). Moreover, $\hat{A}$ and $\hat{A}^+$ are mass parameters defined as $\hat{A} = m_{D^*_s} - m_c$, $\hat{A}^+ = m_{D_{s0}} - m_c$ (in the heavy quark limit). We use the numerical values: $\hat{F} = 0.35$ GeV$^2$, $\hat{F}_d = 0.45$ GeV$^2$, $\hat{F}_d^+ = 0.44$ GeV$^2$ and $\hat{A} = 0.5$ GeV, $\hat{A}^+ = 0.86$ GeV. In fig.4 (left) we depict the result corresponding to eq.(3.3). Considering the region where $d^{(S)}$ is independent of the Borel parameter $E$: $1.2$ GeV $\leq E \leq 1.6$ GeV, and the variation of the threshold $\nu_0$, we obtain $-0.16$ GeV$^{-1} \leq d^{(S)} \leq -0.13$ GeV$^{-1}$. Therefore, we obtain in the heavy quark limit a value compatible with the value obtained by VMD in the same
FIG. 4: The parameters $d^{(S)}$ obtained from eq. (3.3) (left) and $d^{(D)}$ from eq. (3.4) (right) versus the Borel parameter $E$. The continuous, long-dashed and dashed lines correspond to the thresholds $\nu_0 = 1.1, 1.2$ and $1.3$ GeV, respectively.

limit: $d \simeq -0.15$ GeV$^{-1}$; finite quark mass effects are large, and enhance the $D_{s0} \to D_s^*\gamma$ amplitude by at least a factor of two.

From the second sum rule eq. (3.4), taking into account the dependence on the Borel parameter $E$ for the continuum subtraction, we obtain a smaller result, see fig. 4 (right). This is due to a nearly complete cancellation between two different terms, the perturbative and the leading twist term, and therefore it critically depends on the input parameters of the QCD side of the sum rule, making the numerical result less reliable.

IV. $D_{sJ}(2460) \to D_s\gamma$

Coming back to the case of finite charm quark mass, let us consider three radiative decay modes of $D_{s1}'$, the transitions into a pseudoscalar $D_s$, a vector $D_s^*$ and a scalar $D_{s0}$ meson with the emission of a photon. The calculation of the decay amplitudes is analogous to the one carried out in Section III, therefore we present only the relevant formulae.

The decay amplitude of $D_{s1}' \to D_s\gamma$:

$$\langle \gamma(q,\lambda)D_s(p)|D_{s1}'(p + q,\lambda'')\rangle = e g_1 [(\varepsilon^* \cdot \eta)(p \cdot q) - (\varepsilon^* \cdot p)(\eta \cdot q)]$$ (4.1)

with $\eta(\lambda'')$ the $D_{s1}'$ polarization vector, involves the hadronic parameter $g_1$ that can be computed considering the correlation function

$$T^\mu(p,q) = i \int d^4 x \, e^{ip \cdot x} \langle \gamma(q,\lambda)|[J^\mu_5(x), J^A_\mu(0)]|0\rangle.$$ (4.2)

The quark currents are $J_5 = \bar{c}i\gamma_5 s$ and $J^A_\mu = \bar{c}\gamma_\mu\gamma_5 s$; $T^\mu$ can be expanded in Lorentz-invariant
structures:

\[ T_\mu(p, q) = T \left[ (\varepsilon^* \cdot p) q_\mu - (p \cdot q) \varepsilon^*_\mu \right] + \ldots \]  \hspace{1cm} (4.3)

The sum rule for \( g_1 \), obtained from the function \( T \), reads:

\[
g_1 = \frac{e^{2 m_D^2 + m_D^2 s}}{m_{D'_s} f_{D'_s} m_{D_s} f_{D_s}} \left\{ \int_{(m_c + m_s)^2}^{s_0} ds \ e^{-\frac{s}{M^2}} \rho^P(s) \right\}
\]

\[
+ e_c \ e^{-\frac{m_c^2}{M^2}} \langle \bar{s} s \rangle \left[ 1 - \frac{m_c m_s}{M^2} + \frac{m_s^2}{2M^2} \left( 1 + \frac{m_c^2}{M^2} \right) \right]
\]

\[
- e_s \langle \bar{s} s \rangle (e^{-\frac{m_c^2}{M^2}} - e^{-\frac{m_s^2}{M^2}}) M^2 \chi_\gamma(u_0)
\]

\[
- e_s \langle \bar{s} s \rangle e^{-\frac{m_s^2}{M^2}} \left[ -\frac{1}{4} \left( \mathcal{A}(u_0) - 8\mathcal{H}_\gamma(u_0) \right) \left( 1 + \frac{m_c^2}{M^2} \right) \right]
\]

\[
- \int_0^{1-u_0} dv \int_0^{u_0} d\alpha_g \mathcal{F}_1(u_0 - (1 - v)\alpha_g, 1 - u_0 - v\alpha_g, \alpha_g)
\]

\[
- \int_0^{1-u_0} dv \int_0^{1-u_0} d\alpha_g \mathcal{F}_1(u_0 - (1 - v)\alpha_g, 1 - u_0 - v\alpha_g, \alpha_g)
\]

\[
+ 2e_s f_{3\gamma} m_c e^{-\frac{m_c^2}{M^2}} \Psi^s(u_0) \right\}
\]

where \( \mathcal{F}_1 = S + \bar{S} - T_1 - T_2 + T_3 + T_4 + 2v(-S - T_3 + T_2) \) and the spectral function \( \rho^P \) is:

\[
\rho^P(s) = -\frac{3e_s}{8\pi^2} \left\{ 2m_s \ln \left( \frac{s - m_c^2 + m_s^2 - \lambda^2(s, m_c^2, m_s^2)}{s - m_c^2 + m_s^2 + \lambda^2(s, m_c^2, m_s^2)} \right) \right.
\]

\[
+ (m_c - m_s) \frac{(m_c^2 - m_s^2 - s)}{s^2} \lambda^2(s, m_c^2, m_s^2) \left\} - (s \leftrightarrow c) \right. \hspace{1cm} (4.5)
\]

Eq. (4.4) involves parameters already used in previous Sections and the photon DA collected in the Appendix; it also involves the leptonic constant \( f_{D'_s} \), defined by the matrix element

\[
\langle 0| J^{A}_\mu | D'_s \rangle = f_{D'_s} m_{D'_s} \eta_\mu \hspace{1cm} (4.6)
\]

which can be obtained, starting from the two-point correlation function

\[
\Pi_{\mu\nu}(p^2) = i \int d^4x \ e^{ip \cdot x} \langle 0| T[J^{A}_\mu(0) J^{A\dagger}_\nu(x)] | 0 \rangle \hspace{1cm} (4.7)
\]
from the sum rule:

\[ f_{D'_s1}^2 = \frac{e^{m_{D'_s1}^2}}{m_{D'_s1}^2} \left\{ \frac{1}{8\pi^2} \int_{(m_c+m_s)^2}^{s_0} ds \lambda^2(s, m_c^2, m_s^2) \left[ 2 - \frac{m_c^2 + m_s^2 + 6m_cm_s}{s} - \frac{(m_c^2 - m_s^2)^2}{s^2} \right] e^{-\frac{s}{M^2}} \right\} + e^{-\frac{s_0^2}{M^2}} \left( \bar{s}s \left( m_c - \frac{m_c^2m_s}{2M^2} + \frac{m_s^3m_c^3}{2M^4} \right) - \frac{\langle \bar{s}\sigma g_s G_s \rangle m_c^3}{4M^4} \right) \right\} . \] (4.8)

We get \( f_{D'_s1} \simeq f_{D_{s0}} \).

The calculation of \( g_1 \) produces the curves depicted in fig. 5. Considering the range

\[ 3 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2, \]

together with the variation of the threshold \( s_0 \), we obtain:

\[ -0.37 \text{ GeV}^{-1} \leq g_1 \leq -0.29 \text{ GeV}^{-1}, \]

and therefore

\[ \Gamma(D'_{s1} \rightarrow D_s \gamma) = (19 - 29) \text{ keV}. \] (4.9)

As for \( D_{s0} \rightarrow D_s^* \gamma \), the result of light-cone sum rules for the width of \( D'_{s1} \rightarrow D_s \gamma \) is larger than previous estimates. We shall discuss this point later on.

V. \( D_{sJ}(2460) \rightarrow D_s^* \gamma \)

The calculation of the dimensionless hadronic parameter \( g_2 \) appearing in the \( D'_s1 \rightarrow D_s^* \gamma \) transition amplitude:

\[ \langle \gamma(q, \lambda)D_s^*(p, \lambda')|D'_s1(p + q, \lambda'') \rangle = i e g_2 \varepsilon_{\alpha\beta\sigma\tau} \eta_{\alpha}^{\ast} q_{\beta}^{\ast} \varepsilon_{\sigma} q_{\tau}^{\ast}, \] (5.1)
with \( \tilde{\eta}(\lambda') \) and \( \eta(\lambda'') \) the polarization vectors of \( D_+^* \) and \( D_+^{*1} \), is based on the analysis of the correlation function

\[
T_{\mu\nu}(p, q) = i \int d^4x \ e^{ip\cdot x} \langle \gamma(q, \lambda)|T[J_\mu^+(x)J_\nu^+(0)]|0 \rangle \tag{5.2}
\]

expanded in Lorentz invariant structures

\[
T_{\mu\nu}(p, q) = T_A \varepsilon_{\mu\sigma\tau} \varepsilon^{*\sigma} q^\tau + T_B p_\mu \varepsilon_{\nu\beta\sigma\tau} p^\beta \varepsilon^{*\sigma} q^\tau
\]

\[
+ T_C (p + q)_\nu \varepsilon_{\alpha\mu\sigma\tau} p^\alpha \varepsilon^{*\sigma} q^\tau + \ldots. \tag{5.3}
\]

A sum rule for \( g_2 \) is obtained from \( T_A \):

\[
g_2 = \frac{e^{\frac{m_2 D_1^* + m_2 D_1^*}{2m^2}}}{m_2 D_1^* f_{D_1^*} m_2 D_1^* f_{D_1^*}} \left\{ \int_{(m_c + m_s)^2}^{s_0} ds e^{-\frac{s}{M^2}} \rho^p(s) + e_c m_c e^{\frac{m_2^2}{M^2}} \langle \bar{s}s \rangle \left[ 1 - \frac{m_2^2}{M^2} \right] \right.
\]

\[
+ e_c m_c \langle \bar{s}s \rangle \left( e^{-\frac{m_2^2}{M^2}} - e^{-\frac{s}{M^2}} \right) M^2 \chi_{\gamma}(u_0)
\]

\[
+ e_c m_c \langle \bar{s}s \rangle e^{-\frac{m_2^2}{M^2}} \left[ - \frac{1}{4} \frac{m_c^2}{M^2} A(u_0) - H_\gamma(u_0)(1 - u_0) - H_\gamma(u_0) \left( 1 - \frac{2m_c^2}{M^2} \right) \right]
\]

\[
+ e_s f_3 M^2 \left( e^{-\frac{m_2^2}{M^2}} - e^{-\frac{s}{M^2}} \right) \left[ \frac{1}{4} (1 - u_0) \psi^\alpha(u_0) - \frac{1}{4} \tilde{\psi}^\alpha(u_0) - \Psi^\alpha(u_0) \left( 1 + \frac{2m_c^2}{M^2} \right) \right]
\]

\[
+ (1 - u_0) \psi^\alpha(u_0) \right) \left[ \int_0^{1-u_0} dv \int_0^{m_c} d\alpha_g \mathcal{F}_2(u_0 - (1 - v)\alpha_g, 1 - u_0 - v\alpha_g, \alpha_g)
\]

\[
+ \int_{1-u_0}^{1} dv \int_0^{1-u_0} d\alpha_g \mathcal{F}_2(u_0 - (1 - v)\alpha_g, 1 - u_0 - v\alpha_g, \alpha_g) \right]
\]

\[
- e_s f_3 M^2 \left( e^{-\frac{m_2^2}{M^2}} - e^{-\frac{s}{M^2}} \right) \left[ \int_0^{u_0} d\alpha_q \int_{u_0-\alpha_q}^{1-\alpha_q} d\alpha_g \frac{d\mathcal{F}_3(1 - \alpha_q - \alpha_g, \alpha_q, \alpha_g)}{\alpha_g^2} \right]
\]

\[
- \int_0^{u_0} d\alpha_q \frac{1}{u_0 - \alpha_q} \mathcal{F}_3(1 - u_0, \alpha_q, u_0 - \alpha_q) \right\} \right\}, \tag{5.4}
\]
with $\mathcal{F}_2 = S + \tilde{S} + T_1 - T_2 - T_3 + T_4$ and $\mathcal{F}_3 = \mathcal{A} + \mathcal{V}$. The perturbative spectral function $\rho^P$ reads:

$$\rho^P(s) = \frac{3e_s}{4\pi^2 m_cm_c} \ln \left( \frac{s - m_c^2 + m_s^2 - \lambda_1^2(s, m_c^2, m_s^2)}{s - m_c^2 + m_s^2 + \lambda_1^2(s, m_c^2, m_s^2)} \right) + (s \leftrightarrow c).$$ (5.5)

The result is reported in fig. 6. Considering the range $4 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2$ and the variation of the threshold $s_0$, we get $-0.18 \leq g_2 \leq -0.13$, i.e.

$$\Gamma(D_{s1}^s \to D_s^* \gamma) = (0.6 - 1.1) \text{ keV}.\quad (5.6)$$

The small value of $g_2$ is due to large cancellations between the various contributions to the sum rule (5.4): perturbative, twist two and higher twist contributions, as shown in fig. 7. In particular, the contribution proportional to $f_3\gamma$ turns out to be 50% of the contribution proportional to the magnetic susceptibility of the quark condensate. In the cancellation the detailed shapes of the distribution amplitudes and the numerical values of the parameters are of critical importance; this sensitivity induces us to consider the result for $g_2$ as less accurate than the results for the other channels.

VI. $D_{sJ}(2460) \to D_{sJ}^*(2317) \gamma$

The last radiative decay mode we consider for $D_{sJ}(2460)$ is the M1 transition $D_{s1}^s \to D_{s0} \gamma$ which is governed by the amplitude

$$\langle \gamma(q, \lambda)|D_{s0}(p)|D_{s1}^s(p + q, \lambda') \rangle = ig_3 \varepsilon_{\alpha\beta\gamma\tau} \varepsilon^{\alpha\beta} p^\sigma q^\tau.\quad (6.1)$$
The parameter $g_3$ can be evaluated starting from the correlation function

$$W_\mu(p,q) = i \int d^4x \; e^{i p \cdot x} \langle \gamma(q, \lambda) | T[J_0^\dagger(x) J_\mu^A(0)] | 0 \rangle$$

(6.2)

written as

$$W_\mu = i \varepsilon_{\mu \alpha \sigma \tau} \varepsilon^* \sigma^\dagger q^\tau W_0.$$

(6.3)

We work out the sum rule for $g_3$:

$$g_3 = \frac{e^{m_{D_0}^2 + m_{D_1}^2}}{m_{D_0} f_{D_0} m_{D_1} f_{D_1}} \left\{ \int_{(m_c + m_s)^2}^{s_0} ds \; e^{-\frac{s}{M^2}} \rho^P(s) + e_c e^{-\frac{m_c^2}{M^2}} \langle \bar{s}s \rangle \left( 1 + \frac{m_c m_s}{2M^2} + \frac{m_c^2 m_s^2}{8M^4} \right) \right.\right.$$

$$+ \left. e_s \langle \bar{s}s \rangle (e^{-\frac{m_s^2}{M^2}} - e^{-\frac{m_c^2}{M^2}}) M^2 \chi_{\gamma\gamma}(u_0) \right.$$

$$+ \left. e^{-\frac{m_c^2}{M^2}} e_s \langle \bar{s}s \rangle \left[ -\frac{1}{4} A(u_0) (1 + \frac{m_c^2}{M^2}) \right] - \frac{m_c}{2} e_s f_{3\gamma} \psi^a(u_0) e^{-\frac{m_c^2}{M^2}} \right.$$

$$+ \left. e^{-\frac{m_c^2}{M^2}} e_s \langle \bar{s}s \rangle \left[ \int_0^{1-u_0} dv \int_0^{u_0-v} d\alpha g F_4(u_0 - (1 - v)\alpha, 1 - u_0 - v\alpha, \alpha) \right.\right.$$

$$+ \left. \int_{1-u_0}^1 dv \int_0^{1-u_0-v} d\alpha g F_4(u_0 - (1 - v)\alpha, 1 - u_0 - v\alpha, \alpha) \right\} \right\}$$

(6.4)
with \( F_4 = \mathcal{S} + \tilde{\mathcal{S}} + T_1 + T_4 - T_2 - T_3 + 2v(-\tilde{\mathcal{S}} + T_3 - T_4) \) and

\[
\rho^P(s) = \frac{3e_s}{4\pi^2} \left\{ \frac{(m_c + m_s)}{s} \lambda^\frac{1}{2}(s, m_s^2, m_s^2) + m_s \ln \left( \frac{s - m_c^2 + m_s^2 - \lambda^\frac{1}{2}(s, m_s^2, m_s^2)}{s - m_c^2 + m_s^2 + \lambda^\frac{1}{2}(s, m_s^2, m_s^2)} \right) \right\}
- \left( s \leftrightarrow c \right).
\] (6.5)

Considering the range \( 4 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2 \) and varying the threshold \( s_0 \) we get:

\[ -0.35 \text{ GeV}^{-1} \leq g_3 \leq -0.27 \text{ GeV}^{-1} \], corresponding to

\[ \Gamma(D_{s1}^* \rightarrow D_{s0}\gamma) = (0.5 - 0.8) \text{ keV}. \] (6.6)

VII. DISCUSSION AND CONCLUSIONS

As seen in the previous Sections, the radiative decay amplitudes of the charmed mesons considered here, when evaluated by light-cone QCD sum rules, are determined by two main contributions, the perturbative photon emission from the heavy and light quarks, and the contribution of the photon emission from the soft light quark. Other terms represent small corrections. In general, these two terms have different signs, and produce large cancellations; this allows to understand the role of QCD parameters like the magnetic susceptibility \( \chi \). The delicate balancing of the two contributions determines the difference between the radiative widths of charged and neutral mesons.

In Table \[ \text{I} \] we collect the LCSR results together with the results of other methods \[ 10, 11, 12, 16 \]. With the exception of \( D_{sJ}(2460) \rightarrow D_{s}^*\gamma \), the rates of all the modes are larger
TABLE I: Radiative decay widths (in keV) of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ obtained by light-cone sum rules (LCSR). Vector Meson Dominance (VMD) and constituent quark model (QM) results are also reported.

| Initial state | Final state | LCSR | VMD [10, 16] | QM [15] | QM [11] |
|---------------|-------------|------|--------------|---------|---------|
| $D_{sJ}^*(2317)$ | $D_s^*\gamma$ | 4-6  | 0.85         | 1.9     | 1.74    |
| $D_{sJ}(2460)$  | $D_s\gamma$  | 19-29| 3.3          | 6.2     | 5.08    |
|                | $D_s^*\gamma$ | 0.6-1.1| 1.5        | 5.5     | 4.66    |
| $D_{sJ}^*(2317)\gamma$ | —       | 0.5-0.8| —        | 0.012   | 2.74    |

than obtained by other approaches. In particular, $\Gamma(D_{sJ}(2460) \to D_s\gamma)$ turns out to be considerably wider; notice that $D_{sJ}(2460) \to D_s\gamma$ is the only radiative mode observed so far, as shown in Table II. The peculiarity in $D_{sJ}(2460) \to D_s\gamma$ is that the perturbative contribution to the sum rule is the largest term, while in the other cases the leading twist term is the largest one in the theoretical side of the sum rules.

TABLE II: Measurements and 90% CL upper limits of ratios of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ decay widths.

| $\frac{\Gamma(D_{sJ}^*(2317)\to D_s^*\gamma)}{\Gamma(D_{sJ}(2317)\to D_s\pi^0)}$ | Belle | BaBar | CLEO [2] |
|---------------------------------|-------|-------|---------|
| $\Gamma(D_{sJ}(2460)\to D_s\gamma)$ | $0.55 \pm 0.13 \pm 0.08$ [3] | $0.375 \pm 0.054 \pm 0.057$ [9] | $< 0.49$ |
| $\Gamma(D_{sJ}(2460)\to D_s\pi^0)$ | $0.38 \pm 0.11 \pm 0.04$ [4] | $0.274 \pm 0.045 \pm 0.020$ [7] | $< 0.23$ [9] |
| $\frac{\Gamma(D_{sJ}(2460)\to D_s^*\gamma)}{\Gamma(D_{sJ}(2460)\to D_s^*\pi^0)}$ | $< 0.31$ [3] | —     | $< 0.16$ |

In order to quantitatively understand the data in Table II one should precisely know the widths of the isospin violating transitions $D_{s0} \to D_s\pi^0$ and $D_{s1}' \to D_s^*\pi^0$. In the description of these transitions based on the mechanism of $\eta - \pi^0$ mixing [13, 15, 16, 30] one should accurately determine the strong couplings $D_{s0}D_s\eta$ and $D_{s0}'D_s^*\eta$ for finite heavy quark mass. Considering the results in Tables II the couplings should be larger than obtained in the heavy quark and SU(3) limit, an issue which deserves further investigation.
The obtained dominance of $D'_{s1} \to D_s \gamma$ with respect to other modes, in agreement with observation, induces us to consider our results as consistent with the interpretation of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ as ordinary $\bar{c}s$ mesons. The $D_{sJ}(2460) \to D^*_{sJ}(2317) \gamma$ decay turns out to be suppressed; in the molecular interpretation it would be somehow enhanced. The observation of all the radiative decay modes with the predicted rates would of course reinforce our statement; in the meanwhile, we can reasonably conclude that invoking non-standard interpretations is not necessary.

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APPENDIX A: PHOTON DISTRIBUTION AMPLITUDES

For completeness, we collect in this Appendix the light-cone expansions of the photon matrix elements relevant for the calculation of the radiative decays of $D_{s0}$ and $D_{s1}$. We also collect the expressions of the photon distribution amplitudes and the numerical values of the related parameters, as reported in [28]. In all the expressions $\varepsilon(\lambda)$ is the photon polarization vector and $\bar{\varepsilon}_\mu = \varepsilon^*_\mu - q_\mu \varepsilon^* \cdot x$, $\bar{g}_{\mu\nu} = g_{\mu\nu} - \frac{1}{q \cdot x} (q_\mu x_\nu + q_\nu x_\mu)$; the variable $\bar{u}$ is defined as $\bar{u} = 1 - u$; $\bar{G}_{\mu\nu}$ is the dual field $\bar{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\alpha\beta} G^{\alpha\beta}$. We neglect quark mass corrections, that have not been worked out for all matrix elements.

\[
\langle \gamma(q, \lambda) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle = -ie e_q \langle \bar{q} q \rangle (\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu) \int_0^1 du \, e^{i \bar{u} q \cdot x} \left( \chi \phi_{\gamma}(u) + \frac{x^2}{16 A_0(u)} \right)
\]

\[
- ie e_q \langle \bar{q} q \rangle \left( x_\nu \bar{\varepsilon}_\mu - x_\mu \bar{\varepsilon}_\nu \right) \int_0^1 du \, e^{i \bar{u} q \cdot x} h_{\gamma}(u)
\]

\[
\langle \gamma(q, \lambda) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle = e e_q f_{3\gamma} \bar{\varepsilon}_\mu \int_0^1 du \, e^{i \bar{u} q \cdot x} \psi^\nu(u)
\]

\[
\langle \gamma(q, \lambda) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle = -\frac{1}{4} e e_q f_{3\gamma} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^*_{\nu} q_\alpha x_\beta \int_0^1 du \, e^{i \bar{u} q \cdot x} \psi^\alpha(u)
\]

\[
\langle \gamma(q, \lambda) | \bar{q}(x) g_8 G_{\mu\nu}(vx) q(0) | 0 \rangle = -ie e_q \langle \bar{q} q \rangle (\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu) \int D\alpha_i \, e^{i (\alpha_i + \nu \alpha_i) q \cdot x} S(\alpha_i)
\]

\[
\langle \gamma(q, \lambda) | \bar{q}(x) g_8 G_{\mu\nu}(vx)i \gamma_5 q(0) | 0 \rangle = -ie e_q \langle \bar{q} q \rangle (\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu) \int D\alpha_i \, e^{i (\alpha_i + \nu \alpha_i) q \cdot x} S(\alpha_i)
\]

\[
\langle \gamma(q, \lambda) | \bar{q}(x) g_8 \bar{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle = e e_q f_{3\gamma} q_\alpha (\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu) \int D\alpha_i \, e^{i (\alpha_i + \nu \alpha_i) q \cdot x} A(\alpha_i)
\]

\[
\langle \gamma(q, \lambda) | \bar{q}(x) g_8 G_{\mu\nu}(vx)i \gamma_\alpha q(0) | 0 \rangle = e e_q f_{3\gamma} q_\alpha (\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu) \int D\alpha_i \, e^{i (\alpha_i + \nu \alpha_i) q \cdot x} V(\alpha_i)
\]
\[ \langle \gamma(q, \lambda)|q(x)\sigma_{\alpha\beta}g_{\alpha}G_{\mu\nu}(vx)q(0)|0 \rangle = \]

\[
\left[ e_{q}\langle \bar{q}q \rangle \right] e_{\bar{q}}\langle \bar{q}q \rangle \epsilon_{\alpha} \epsilon_{\beta} + \epsilon_{\beta} \epsilon_{\alpha} (\mu \leftrightarrow \nu) \int D\alpha_{i} e^{i(\alpha q + \nu \bar{q})x} \mathcal{T}_{1}(\alpha_{i}) \]

\[
 + \left[ \tilde{\epsilon}_{\alpha} \bar{g}_{\alpha}q_{\beta} - \tilde{\epsilon}_{\beta} \bar{g}_{\beta}q_{\alpha} \right] \int D\alpha_{i} e^{i(\alpha q + \nu \bar{q})x} \mathcal{T}_{2}(\alpha_{i}) \]

\[
 + \left[ \frac{(q_{\mu}x_{\nu} - q_{\nu}x_{\mu})(\epsilon_{\alpha}q_{\beta} - \epsilon_{\beta}q_{\alpha})}{q \cdot x} \right] \int D\alpha_{i} e^{i(\alpha q + \nu \bar{q})x} \mathcal{T}_{3}(\alpha_{i}) \]

\[
 + \left[ \frac{(q_{\alpha}x_{\beta} - q_{\beta}x_{\alpha})(\epsilon_{\mu}q_{\nu} - \epsilon_{\nu}q_{\mu})}{q \cdot x} \right] \int D\alpha_{i} e^{i(\alpha q + \nu \bar{q})x} \mathcal{T}_{4}(\alpha_{i}) \right] \}

(A1)

\[ \alpha_{i} = \{\alpha_{q}, \alpha_{\bar{q}}, \alpha_{g}\} \] and \[ \int D(\alpha_{i}) \equiv \int_{0}^{1} d\alpha_{q} \int_{0}^{1} d\alpha_{\bar{q}} \int_{0}^{1} d\alpha_{g} \delta(1 - \alpha_{q} - \alpha_{\bar{q}} - \alpha_{g}) . \]

The photon distribution amplitudes (DA) have the following expressions:

\[ \phi_{\gamma}(u) = 6u\bar{u} \left(1 + \varphi_{2}C_{2}^{2} \right) \]

\[ A(u) = 40u^{2}\bar{u}^{2}(3k - k^{+} + 1) + 8(\zeta_{2}^{+} - 3\zeta_{2}) \left[ u\bar{u}(2 + 13u\bar{u}) \right. \]

\[ + 2u^{3}(10 - 15u + 6u^{2}) \ln u + 2\bar{u}^{3}(10 - 15\bar{u} + 6\bar{u}^{2}) \ln \bar{u} \]

\[ h_{\gamma}(u) = -10(1 + 2k^{+})C_{2}^{2} \]

\[ \psi_{v}(u) = 5 \left(3(2u - 1)^{2} - 1\right) + \frac{3}{64} \left(15\omega_{\gamma}^{v} - 5\omega_{\gamma}^{A}\right) \left(3 - 30(2u - 1)^{2} + 35(2u - 1)^{4}\right) \]

\[ \psi_{a}(u) = 5 \left( (2u - 1)^{2} - 1 \right) \left(5(2u - 1)^{2} - 1\right) \frac{5}{2} \left(1 + \frac{9}{16}\omega_{\gamma}^{v} - \frac{3}{16}\omega_{\gamma}^{A}\right) \]

\[ \mathcal{V}(\alpha_{q}, \alpha_{\bar{q}}, \alpha_{g}) = 540\omega_{\gamma}^{v}(\alpha_{q} - \alpha_{\bar{q}})\alpha_{q}\alpha_{\bar{q}}\alpha_{g}^{2} \]

\[ \mathcal{A}(\alpha_{q}, \alpha_{\bar{q}}, \alpha_{g}) = 360\alpha_{q}\alpha_{\bar{q}}\alpha_{g} \left[1 + \omega_{\gamma}^{A}\frac{1}{2}(7\alpha_{g} - 3) \right] \]

\[ 20 \]
\[ S(\alpha_q, \alpha_{\bar{q}}, \alpha_g) = 30\alpha_g^2 \left[ (k + k^+) (1 - \alpha_g) + (\zeta_1 + \zeta_1^+) (1 - \alpha_g) (1 - 2\alpha_g) \right. \\
+ \left. \zeta_2 [3(\alpha_q - \alpha_g)^2 - \alpha_g (1 - \alpha_g)] \right] \]

\[ \hat{S}(\alpha_q, \alpha_{\bar{q}}, \alpha_g) = -30\alpha_g^2 \left[ (k - k^+) (1 - \alpha_g) + (\zeta_1 - \zeta_1^+) (1 - \alpha_g) (1 - 2\alpha_g) \right. \\
+ \left. \zeta_2 [3(\alpha_q - \alpha_g)^2 - \alpha_g (1 - \alpha_g)] \right] \]

\[ T_1(\alpha_q, \alpha_{\bar{q}}, \alpha_g) = -120(3\zeta_2 + \zeta_2^+) (\alpha_q - \alpha_{\bar{q}}) \alpha_q \alpha_g \alpha_g \]

\[ T_2(\alpha_q, \alpha_{\bar{q}}, \alpha_g) = 30\alpha_g^2 (\alpha_q - \alpha_{\bar{q}}) \left[ (k - k^+) + (\zeta_1 - \zeta_1^+) (1 - 2\alpha_g) + \zeta_2 (3 - 4\alpha_g) \right] \]

\[ T_3(\alpha_q, \alpha_{\bar{q}}, \alpha_g) = -120(3\zeta_2 - \zeta_2^+) (\alpha_q - \alpha_{\bar{q}}) \alpha_q \alpha_g \alpha_g \]

\[ T_4(\alpha_q, \alpha_{\bar{q}}, \alpha_g) = 30\alpha_g^2 (\alpha_q - \alpha_{\bar{q}}) \left[ (k + k^+) + (\zeta_1 + \zeta_1^+) (1 - 2\alpha_g) + \zeta_2 (3 - 4\alpha_g) \right] . \]

(A2)

The parameters in the distribution amplitudes are: \( f_{3\gamma} = -(0.0039 \pm 0.0020) \text{ GeV}^2 \), \( \omega_\gamma^V = 3.8 \pm 1.8 \), \( \omega_\gamma^A = -2.1 \pm 1.0 \) [28]; \( k = 0.2 \), \( \zeta_1 = 0.4 \), \( \zeta_2 = 0.3 \), \( \varphi_2 = k^+ = \zeta_1^+ = \zeta_2^+ = 0 \) (at the renormalization scale \( \mu = 1 \text{ GeV} \)) [19]. The other parameters in the QCD sides of the sum rules, at the same renormalization scale, are: \( m_c = 1.35 \text{ GeV} \), \( m_s = 0.125 \text{ GeV} \) [29], \( \langle \bar{s} s \rangle = 0.8 \langle \bar{q} q \rangle \) \( (q = u, d) \), \( \langle \bar{q} q \rangle = (-0.245 \text{ GeV})^3 \) and \( \langle \bar{q} g \sigma G q \rangle = m_0^2 \langle \bar{q} q \rangle \) with \( m_0^2 = 0.8 \) GeV\(^2\). Finally, for the magnetic susceptibility of the quark condensate \( \chi \) we use the value \( \chi = -(3.15 \pm 0.3) \text{ GeV}^{-2} \) obtained in [28].

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