Observation of shot-noise-induced asymmetry in the Coulomb blockaded Josephson junction

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We have investigated the influence of shot noise on the IV-curves of a single mesoscopic Josephson junction. We observe a linear enhancement of zero-bias conductance of the Josephson junction with increasing shot noise power. Moreover, the IV-curves become increasingly asymmetric. Our analysis on the asymmetry shows that the Coulomb blockade of Cooper pairs is strongly influenced by the non-Gaussian character of the shot noise.

Phase coherence of electronic motion is one of the central issues of mesoscopic physics. This characteristic is destroyed by environmental noise sources, which are not necessarily Gaussian. The desire to understand the dephasing mechanisms has given rise to a need of mesoscopic noise detectors that can measure the asymmetry of non-equilibrium noise. Recently, the realization of quantum noise detectors has been studied theoretically by Agudo and Kouwenhoven in double quantum dots \cite{1} and by Schoelkopf et al. in Cooper pair boxes \cite{2}. The possibility of using a Coulomb blockaded Josephson junction (JJ) as a probe of noise was shown experimentally \cite{3}. In addition, the applicability of a SIS mixer for noise detection has been demonstrated by Deblock et al \cite{4}.

We have investigated the potential of a single mesoscopic JJ as a detector of shot noise focusing on the case of a strongly resistive environment, in which Coulomb blockade (CB) of Cooper pair current \cite{5} takes place owing to the delocalization of the phase variable \cite{6}. In this case the current enhances distinctly with added non-equilibrium noise \cite{7}.

The study of higher moments of current fluctuations has recently gained a lot of theoretical attention starting to the theory of Levitov and Reznikov \cite{8} (see other references in \cite{9,10}). Experimentally, the influence of higher moments has proven to be difficult to study with standard spectrum analyzer methods. The third moment and asymmetry of shot noise was, however, measured in this way by Reulet et al \cite{9}. Tobiska and Nazarov discussed a method for the measurement of higher moments by using the asymmetry of escape rates in a current biased, large JJ \cite{11}. In our experiments, we generate shot noise by a separately biased superconductor-insulator-normal metal tunnel junction (SIN TJ). The quasiparticle current of the TJ is found to strongly reduce the CB current of Cooper pairs: its influence can be resolved down to currents of 0.1 pA. We observe a linear enhancement of zero-bias conductance of the Josephson junction with increasing shot noise power. Moreover, the IV-curves of the JJ become increasingly asymmetric due to the non-Gaussian nature of shot noise. Our experiment shows, for the first time, that the Coulomb blockade of Cooper pairs is strongly influenced by the non-Gaussian nature of phase fluctuations generated by shot noise \cite{10}.

When the supercurrent channel is blocked off owing to a high impedance environment, the current in a mesoscopic JJ is carried by incoherent tunneling of Cooper pairs \cite{12,13,14}. This is controlled by the exchange of energy with the environment. As a result, the junction conductance strongly depends on temperature: $G_0 \propto T^{2\rho-2}$, where the parameter $\rho = R/R_Q$ is the ratio between the resistance $R$ of the dissipative ohmic environment and the quantum resistance $R_Q = h/4e^2$.

The simplest way to account for shot noise is to equate the available noise power with a noise temperature $T_N$: $2eI_S = 4k_BT_N/R$. Consequently, the zero-bias conductance $G_0$ under the influence of a quasiparticle current $I_S$ would correspond to the conductance at an elevated effective temperature of $T + T_N$. Hence, a comparison of the current-induced conductance change with the temperature dependence allows one to determine $T_N$.

The effective noise temperature analysis has been proven to be rigorous in the case of small $\rho \ll 1$ \cite{10}. The theoretical paper of Ref. \cite{10} shows that such an approach is not valid in the opposite limit of $\rho \gg 1$. Instead of non-analytic power law dependence, a linear dependence of $G$ vs $I_S$ is obtained in the limit $T \rightarrow 0$.

The IV curve of a Josephson junction at small voltages can be represented as a power series \cite{10}:

\begin{equation}
I(V) = I_0 + GV + aV^2 + bV^3.
\end{equation}

Here the terms $I_0$ and $aV^2$ are related to the odd moments of shot noise \cite{10} and, therefore, they are absent without shot noise. They characterize the asymmetry of the IV curves and give rise to the ratchet effect ($I_0$) and a local conductance maximum. In contrast, the terms $GV$ and $bV^3$ may be present even without shot noise. In fact, one should write $b = b_0(T) + b_1$, where $b_0$ is a temperature dependent term which is present without shot noise and $b_1$ is due to shot noise. According to the theory \cite{10}, the shot noise contribution to the zero bias conductance

\begin{equation}
G(T) = G_0 + aV^2 + bV^3
\end{equation}
A scanning electron microscope picture of sample 1 and a schematic view of the circuit. The chrome resistor is denoted by Cr, the superconductor-normal junction by SIN, and the Josephson junction in a SQUID-loop configuration by JJ1 and JJ2.

\[ G = G_0 + G_S \]

\[ G_S = \frac{\pi^{5/2}}{32\sqrt{2}\ln \rho} \left( \frac{E_j}{E_C} \right)^2 \frac{1}{V_C} \rho^{3/2} |I_S|, \]

where \( V_C = E_C/e = e/2C \). The ratchet current is given by \( I_0 = \beta_0 I_S \), where

\[ \beta_0 = \frac{\pi^2}{32} \left( \frac{E_j}{E_C} \right)^2 \rho. \]

From Eq. 1 we also obtain a local conductance extremum (a maximum or minimum – depending on the sign of \( b \))

\[ V_{max} = -\frac{a}{3b} = \text{sign}(I_S) \frac{2V_C \ln \rho}{\pi^2 \rho}, \]

where we assumed \( b_0 = 0 \). The ratchet effect and the location of the extremum are thus odd functions of \( I_S \).

Our experiments were performed using the circuit layout displayed in Fig. 1. The on-chip structure consists of three basic elements: 1) an Al-AlO<sub>x</sub>-Al Josephson junction (JJ), 2) a superconducting-normal Al-AlO<sub>x</sub>-Cu tunnel junction (SIN) with, (3) a thin film, on-chip, Cr resistor located within a few \( \mu \)m from the Josephson junction. The resistor \( R_{bias} \) is located outside the cryostat, and it is taken as large as 10 G\( \Omega \) for good current biasing of the SIN TJ (which has a subgap resistance \( \approx 1 \) G\( \Omega \) around zero bias but 10 – 100 M\( \Omega \) near the superconductive gap, which is our typical operating point). In practice, the large capacitance \( C_0 \) (see Fig. 2 of the measurement leads makes the TJ voltage biased – a necessary condition for shot noise generation in the junction. In sample 1, an additional on-chip resistor of 53 k\( \Omega \), was manufactured before the SIN junction. Two samples (see Table I) with comparable results were investigated. The circuits were fabricated using electron beam lithography and four-angle evaporation. Linearity of the Cr-resistors was found to be better than 5 %, when \( B \) and \( T \) swept over 0 ... 0.2 T and 0.1 ... 4 K, respectively.

| \( R_{bias}^J \) (k\( \Omega \)) | \( R_{bias}^{JN} \) (k\( \Omega \)) | \( R \) (k\( \Omega \)) | \( E_C \) | \( E_j^{min} \) / \( E_j^{max} \) |
|---|---|---|---|---|
| 8.1 | 27.3 | 22.6 | 65 | 22 / 78 |
| 24 | 73 | 179 | 80 | 2.4/24 |

TABLE I: Sample parameters for two samples numbered by the first column. The next two columns give the normal state tunneling resistance of the Josephson junction \( R_J^J \) and the SIN junction \( R_{bias}^{JN} \). \( R \) denotes the impedance of the Cr resistors in the immediate vicinity of the Josephson junction. The last two columns indicate the Coulomb energy, \( E_j \), and the minimum \( E_j^{min} \) and maximum \( E_j^{max} \) values of the Josephson energy. The energies are given in \( \mu \)eV.

The Josephson junction was, in fact, made of two 100 \times 100 nm\(^2\) junctions in a SQUID geometry in order to enable tuning of its Josephson energy with an external flux \( \Phi \). The Josephson energy \( E_j^{max} \) at zero magnetic flux was calculated from the tunneling resistance using the Ambegaokar-Baratoff relation. The minimum Josephson coupling energy, \( E_j^{min} \), was obtained from the smallest achieved inelastic current peak in the JJ IV curve. This peak should, according to the \( P(E) \)-theory [13], go down as \( \propto E_j^2 \), which is also experimentally verified when \( E_j < E_C \). Intermediate values of \( E_j \) can then be determined using the theoretical flux dependence \( E_j = [E_j^2 + E_j^2 + 2E_j E_j \cos(2\pi \Phi / \Phi_0)]^{1/2} \) for the case of asymmetric Josephson energies \( E_j^1 \) and \( E_j^2 \) for a SQUID. The Coulomb energy \( E_C \) was estimated from the IV curves in the normal state: the sum of junction capacitances, \( C = C_{SIN} + C_{JJ} \), was obtained from the voltage offset at large bias voltages using the formula \( V_{offset} = \frac{\Phi}{\pi} \).

Our samples were cooled with a Leiden Cryogenics dilution refrigerator. The samples were mounted inside a tight copper enclosure and the measurement leads were filtered using powder rf-filters (Leiden) and 1 m of Thermocox cable at mixing chamber temperature. The JJ current was measured using a DL 1211 current amplifier.

From the measurements of the temperature dependence of \( G_0(T) \), we found that the effective temperature of the sample was between 45 – 80 mK, somewhat higher than the 25 – 35 mK base temperature of the cryostat. Nevertheless, as Johnson-Nyquist noise and shot noise are uncorrelated in our case, this enhanced thermal noise means only a minor shift in the initial point (\( G_0 \)) of the quasiparticle current \( I_j \) induced conductance changes.

The parameters of sample 2 were most suitable for quantitatively testing the theory of Ref. 14. Therefore, in the following paragraphs, all the results and figures refer to sample 2.

First, we look at the effect of shot-noise power on the conductance of the JJ, a symmetric effect with respect to sign change of \( I_S \). The zero bias conductance changes substantially under a small applied current through the TJ. This effect is summarized in Fig. 2 for different values of the ratio \( E_j/E_C \). The linear dependence on \( I_S \)
is clear (for $I_S > 5$ pA when $G_S$ becomes comparable to $G_0$), as well as the increase of the effect with growing $E_J$. Changing the sign of $I_S$ showed indeed that $G_S = G_S(|I_S|)$. Since the temperature dependence of $G_0$ becomes weaker with growing $E_J$, the data of Fig. 2 also indicate that rising electronic temperature is not able to explain the change as a function of $I_S$. In the inset of Fig. 2 the slopes $G/I_S$ are plotted together with the predicted $\propto (E_J/E_C)^2$ behavior from Eq. (2). The agreement with the theory is within the expected error margin due to the large sensitivity of $G_S$ to errors in the experimental determination of $E_J$ and $C$.

Next, we consider the asymmetric effects, which are related to the non-Gaussian character of the phase fluctuations caused by quasiparticles tunneling through the SIN TJ [10]. In Fig. 3, we show conductance curves when a small TJ current is applied. One can clearly observe the breaking of the symmetry in the shift of the conductance minimum as well as through the appearance of a local maximum. The symmetry with respect to sign change of $I_S$ is also verified by comparing curves with opposite sign of the TJ current. However, the sign of the shift in the minimum conductance indicates that $b_0 > b_1$, which renders the curvature term $b$ in Eq. (2) positive, and makes the extremum a minimum. The magnitude of the shift is quite small ($\approx 2\mu$eV) as expected from theory. Also, the locations of the extrema are independent of $I_S$.

The analytical expansion in Ref. [11] found by considering the phase fluctuations over the JJ is only valid near zero voltage, and as we move away from that regime additional terms may become significant. Therefore, to explain the appearance and location of the conductance maximum we have to resort to a qualitative argument, which considers the potential dynamics due to quasiparticle tunneling through the SIN junction. When the particle has tunneled, it increases the voltage $V_{JJ}$ over the JJ by $e/C$ before relaxing after the time $\tau = RC$. During this time, the Coulomb blockade of the junction is lifted and current can flow.

For sample 2 the ratchet effect could be observed in the IV-characteristics at zero voltage. In Fig. 4, IV curves of the blockade region are shown for small applied TJ currents. The shift of the IV curves and the non-zero current at zero bias voltage are clearly seen. The fact that the ratchet effect changes sign for negative $I_S$ shows that the observed shift is not simply due to current amplifier offset, which is independent of the sign of $I_S$. The direct measurement was only accurate for small $I_S < 5$ pA, after that the tilting of the conductance curve started to obscure the ratchet effect at zero voltage. The possibility that the non-zero bias gain would be due to a trivial current division between the chromium resistor $R$ and the resistance of the JJ, can be ruled out as the Coulomb blockade of the JJ is quite strong, yielding resistances $> 1\, \Omega$ and a leakage of less than $0.10 I_S$.

In Fig. 5 the dependence of $I_0/I_S$ on $I_S$ and $E_J/E_C$ is shown. The first three points give $I_0/I_S \approx 0.7$, while the theoretical gain given by Eq. (4) is 0.77. For larger values of $I_S$ than 2 pA, $\beta_0$ seems to decline rapidly (roughly as $1/I_S$). In the same figure, we see that the $E_J/E_C$ dependence of the ratchet effect at small currents ($< 2$ pA) is consistent with the theoretical prediction in Eq. (3).

For sample 1 the asymmetric effects were only observed in the shift of the minimum conductance to finite currents $I_{JJ}^{\text{fin}}$, whose sign depends on the sign of $I_S$. From Eq. (11) one gets that for small currents also this shift, although not strictly at zero voltage, is dominated by the ratchet.
FIG. 5: The ratchet current as function of current is shift of IV curve and the resulting ratchet effect when the TJ detector bandwidth, to shot noise. Its sensitivity comes from the large de-  

term. We found that this indirectly observed ratchet effect for sample 1 was around 0.8, while the theoretical prediction is 0.5.  

According to our analysis, a single Josephson junction provides a good detector for the total noise power due to shot noise. Its sensitivity comes from the large detector bandwidth, $\sim E_C/h$, foremost owing to the closeness of the source and the detector. According to our experiments, a Coulomb blocked Josephson junction can resolve noise from a quasiparticle current as small as 0.1 pA, which clearly exceeds the sensitivity of standard high-resolution noise experiments.  

For the low currents used in this experiment, subsequent voltage pulses of tunneling quasiparticles do not overlap and, therefore, correlations between pulses and their statistics are not relevant. For larger currents ($>100$ pA), however, correlations alter the picture and the method could in principal be used to detect the non-Gaussian electron counting statistics.  

In summary, our measurements of $G$ vs. $I$ curves for solitary, resistively confined small Josephson junctions show that the Cooper pair blockade is a very sensitive probe of not only the total noise power but also of the asymmetry of shot noise pulses. In our measurements, in which the shot noise was induced by quasiparticle current in a nearby SIN-junction, we find a linear enhancement in the zero bias conductance with increasing shot noise power as well as asymmetric features that depend on the sign of the applied current. The asymmetric effects are consequences of the non-Gaussian phase fluctuations of shot noise and are in agreement with theoretical predictions.  

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