Probing the Phase Boundary between Hadronic Matter and the Quark-Gluon-Plasma in Relativistic Heavy Ion Collisions

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(February 7, 2018)

Abstract

We discuss recent data on particle production with emphasis on the degree of thermal and chemical equilibration achieved. The data are interpreted in terms of a resonance gas model. The phase boundary constructed between the resonance gas and the quark-gluon plasma is shown to be very close to the deduced parameters characterizing the hadronic fireball at freeze-out.

Collisions between ultra-relativistic heavy nuclei create matter of high initial energy and particle density. The experimental program currently underway at the fixed target machines probes at the same time the region of very high initial baryon density. Under such conditions it is expected that a phase of deconfined quarks and gluons, where also chiral symmetry is restored, is more stable than hadronic matter. An experimental heavy ion program has been underway for nearly ten years at the Brookhaven AGS and the CERN SPS. In a number of experiments many hadronic observables have been measured, often over a large fraction of the full solid angle. With these experimental data the final freeze-out stage has been well characterized in terms of the hadronic observables at least for the relatively light Silicon and Sulfur beams and similar data for the heaviest beams, Gold and Lead, are rapidly becoming available. We want to address in this article the question to what extent the final hadronic stage can be characterized in terms of the concepts of thermal
and hadrochemical equilibrium, i.e. to what extent we can talk about a thermodynamic phase. This will lead to the next question, namely which regions of the phase diagram are probed by present experiments. We would like to point out that such studies of freeze-out configurations do not shed direct light on the high density/high temperature phase of the collision. On the other hand, if equilibrium is not reached at freeze-out, it is difficult to see how one can describe the fireball formed in the collision in terms of thermodynamical quantities at earlier stages.

As a starting point for the present discussion we use results from recent numerical solutions of QCD on the lattice. For zero net baryon density the critical temperature for the transition between the hadronic and the quark-gluon phase has been obtained by several groups: $T_c = 145 \pm 10$ MeV. Lattice QCD calculations do not, up to now, shed light on the properties of the phase transition at finite baryon density. In order to continue the lattice results into the region of finite baryon density relevant for present fixed target experiments we construct the phase boundary by equating chemical potential $\mu_B$ and pressure $P$ of a hadron resonance gas with the equivalent quantities of a non-interacting quark-gluon plasma. The hadron resonance gas contains all known baryons and mesons up to a mass of 2 and 1.5 GeV, respectively. Interactions among the baryons are approximately taken into account by an excluded volume correction. The plasma phase contains massless gluons, u and d quarks and strange quarks with $m_s = 150$ MeV. A bag constant of $B = 262$ MeV/fm$^3$ is used to insure that the transition for $\mu_B = 0$ takes place at $T_c = 145$ MeV, consistent with the lattice QCD results. We note that, because of the relatively low transition temperature, the hadron gas is never very dense near $T_c$ especially for $\mu_B < 0.8$ GeV, i.e. the values relevant for fireballs formed in ultra-relativistic nucleus-nucleus collisions (see below). The excluded volume corrections are consequently not very large. Also the neglect of very heavy hadrons (with masses $> 2$ GeV) in the resonance gas is of no consequence for the calculation of the phase boundary in the relevant region.

In order to illustrate our construction of the phase transition we present, in Figure 1, the dependence of pressure on temperature for two values of the baryon chemical potential,
\(\mu_B = 0.17\) and 0.54 GeV, respectively. The pressure in the hadronic phase (starting at \(P = 0\) for \(T = 0\)) rises approximately \(\propto T^6\) because of the resonances included in the hadron gas description, while for the quark-gluon phase \(P \propto T^4\) and, of course, \(P = -B\) at \(T = 0\).

Following the suggestion of [3], we plot, in Figure 2, the equation of state in a plot of pressure divided by energy density vs. energy density, i.e. \(P/\epsilon\) as function of \(\epsilon\) for \(\mu_B = 0.54\) GeV. Similar to the results obtained by [3] our equation of state at finite baryon density also has a “soft point”, i.e. smallest pressure for a given energy density, at \(\epsilon \approx 1.2\) GeV/fm\(^3\), although the detailed shape of the curve, especially in the hadron gas region, is quite different from that of [3]. We furthermore note that, at the softest point, the pressure is about \(P \approx 50\) MeV/fm\(^3\), not negligible but small when compared to \(\epsilon/3\). Also shown in Figure 2 is the dependence of the square of the speed of sound, \(c_s^2\) on \(\epsilon\). The speed of sound, \(c_s = \sqrt{dP/d\epsilon}\) vanishes by construction in the mixed phase. The relatively low pressure and small speed of sound at the “softest” point suggest the possibility of a very long lived fireball.

The full phase diagram resulting from the above described construction is shown in Figure 3 both in terms of the chemical potential and in terms of the baryon density. The fact that the baryon densities on the quark-gluon and on the hadron side differ as soon as they are finite reflects the first order nature of the phase transition constructed in this way. For \(\mu_B < 0.8\) GeV the transition temperature varies only slightly, reaching a value of \(T_c \approx 120\) MeV at \(\mu_B = 0.8\) GeV. For larger \(\mu_B\) values the position of the phase boundary becomes rather uncertain because of the rather crude way in which we incorporate baryon-baryon repulsion through an excluded volume correction [3,4]. However, our analysis will focus on smaller values of the baryon chemical potential, where baryon densities at freeze-out are significantly below normal nuclear matter density. In this region the interaction corrections are small and we consider the phase diagram to be consistent within the framework chosen.

As a next step we demonstrate to what accuracy the hadron abundances from AGS and SPS experiments with Si and S beams can be described in terms of the two thermodynamic variables \(T\) and \(\mu_B\). This is shown in Figure 4 and Figure 5. The details of the analysis are described in [3,4]. The resulting values are \(\mu_B = 0.54\) GeV, \(120 \leq T \leq 140\) MeV at
AGS energy and $\mu_B = 0.17(0.18)$ GeV, $T = 160(170)$ MeV The overall agreement between the measured particle ratios and the thermal model predictions is rather impressive, on the 20% level or better in most cases. Note that, because of the large baryon chemical potential and relatively low temperature, the particle ratios at AGS energy vary over six orders of magnitude, while the ratios measured at the SPS are much more bunched together, consistent with a relatively small $\mu_B$ value and higher temperature.

The baryon density at freeze-out for both the AGS and the SPS data is $\rho_B \approx 0.06/fm^3$, while the corresponding pion densities are $\rho_\pi \approx 0.08/fm^3$ (AGS, $T = 120$ MeV) and $\rho_\pi \approx 0.30/fm^3$ (SPS, $T = 160$ MeV). This may reflect the fact that freeze-out at the AGS, where the pion/nucleon ratio is near unity, is determined by the pion-nucleon and nucleon-nucleon dynamics and the associated large cross sections ($\sim 100$ mb) while the small $\pi\pi$ cross section ($\sim 10$ mb) is the relevant quantity at SPS energies where pions dominate nucleons by about 5 to 1 in the central region. To within the accuracy of present data this analysis shows that the hadronic freeze-out configuration is very close to thermal and chemical equilibrium. In particular, the strangeness suppression which is well known for data from nucleon-nucleon collisions is not observed in nucleus-nucleus collisions. Rather, strangeness degrees of freedom are close to the values expected for a hot hadronic system in chemical equilibrium.

Armed with these results one may predict the hadronic abundances which should be produced in Au-Au collisions at future colliders such as RHIC or LHC. For these predictions we merely set $\mu_B = 0$ and set $T = T_c(\mu_B = 0) = 145$ MeV. The results for spatial densities and rapidity densities of hadrons are shown in Table 1. While the spatial densities are a direct result of the thermal model calculations, predictions of the rapidity densities imply fixing of the fireball volume. We have adjusted this volume to $V = 9524$ fm$^3$, such that the total pion rapidity density is 2000, \textit{i.e.} a value typical for what is expected for experiments at RHIC. The results show that a thermally equilibrated fireball will contain about 170 nucleons and anti-nucleons, and the $K^+/\pi^+$ ratio will be about 19%.

An intriguing question is raised by the rather surprising result of chemical equilibration at
freeze-out for strange particles. Assuming free hadron-hadron cross sections for strangeness production one would expect that very long lifetimes are needed to bring strange particles into chemical equilibrium. One way to bring strangeness into equilibrium is the possibility that a deconfined phase (quark-gluon plasma) was formed during the course of the collision. If part of the large strangeness content of a thermalized plasma is carried over into the hadronic phase, the large ratios observed for strange/non-strange particle production yields are readily explained.

An alternative but somewhat related option has been demonstrated by Ko, Brown and collaborators. It is that the possible reduction of particle masses in the hot and dense fireball leads to a strong enhancement in strangeness production cross sections, and consequently to much more rapid equilibration. In particular, the dependence on temperature and density of nonstrange hadron masses is parameterized in as:

\[ \frac{m^*}{m} = \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]^{(1/3)} \left[ 1 - 0.2 \frac{\rho_B}{\rho_0} \right], \]

while for kaons,

\[ \frac{m_{K^*}}{m_K} = \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]^{(1/3)} \left[ 1 - 0.2 \frac{\rho_B}{\rho_0} \right]^{1/2} \]

If we, considering the above determined freeze-out parameters, assume, for the case of a fireball created at AGS energy, that \( \langle \rho_B/\rho_0 \rangle \approx 2 \) and \( \langle T/T_c \rangle \approx 0.8 \), where the brackets imply average over the lifetime of the fireball, we get \( \langle m^*/m \rangle = 0.43 \) for nonstrange hadrons and 0.55 for the kaons. Especially for the \( \rho \rho \rightarrow K \bar{K} \) cross section such mass reductions lead to substantial increases and, consequently, much more rapid thermalization. One should note, however, that the question, how hadron and, in particular, kaon masses behave in dense matter, is not settled. In particular, Lutz et al. come to rather different conclusions for the in-medium mass change, with the \( K^+ \) mass actually increasing with increasing baryon density.

Nevertheless, if chemical equilibration of the strangeness degrees of freedom at freeze-out is confirmed for the heavy systems such as Au+Au at the AGS and Pb+Pb at the SPS,
this could be considered evidence, albeit indirect, that either the phase boundary has been crossed or that at least partial restoration of chiral symmetry with a concomitant reduction of hadron masses has been achieved in the hot and dense fireball created in the collision.

Similar conclusions may be drawn from Figure 6, where we have compared the above determined experimental freeze-out parameters with the calculated position of the phase boundary. The surprising result is that, even at freeze-out, the fireball parameters are very close to those expected for a the quark-gluon plasma. Although the system is, by definition, purely hadronic at freeze-out, its proximity to the phase boundary suggests that the boundary was reached or even crossed during the course of the evolution of the fireball towards freeze-out. The arrows indicate where the systems should evolve from if one takes predictions by cascade models such as RQMD as guideline for the system parameters at the time of maximum compression and temperature.
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FIG. 1. Dependence of pressure on temperature for the hadron gas and quark-gluon plasma equations of state described in the text. Solid (dashed) lines are for $\mu_B = 0.17(0.54)$ GeV.
FIG. 2. Top: $P/\epsilon$ vs. $\epsilon$ for $\mu_B = 0.54$ GeV. Bottom: the square of the speed of sound, $c_s^2$ as function of the energy density $\epsilon$. Note that $c_s^2$ vanishes by construction in the mixed phase.
FIG. 3. Phase boundary between hadronic and quark-gluon matter. Top: as function of baryon density. Note the large jump in baryon density from hadronic (dash-dotted line) to quark-gluon matter (dashed line). Bottom: as function of baryon chemical potential.
FIG. 4. Comparison of thermal model prediction to experimental data for hadron abundance ratios at AGS energy. For details see text.
FIG. 5. Comparison of thermal model prediction to experimental data for hadron abundance ratios at SPS energy. The notation (+) or (-) refers to the density of positively or negatively charged hadrons. For more details see text.
FIG. 6. Comparison of experimentally determined freeze-out parameters at AGS and SPS energy with the phase boundary calculated in the present approach.
Table 1: Thermal model predictions for particle and rapidity densities at RHIC energy.

For details see text.