Free vibration analysis of functionally graded plates using four parameter power law in Abaqus

R Wangdus and H D Chalak
Department of Civil Engineering, National Institute of Technology Kurukshetra, India-136119

E-Mail: rigzinwangdus3775@gmail.com

Abstract: Functionally graded material (FGM) are most widely use material with various application because of its properties adaptability to different situations by changing the constituents as per the requirement. In present study, using (ABAQUS) software for free vibration analysis of natural frequency of functionally graded material (FGM) plate is performed. Aluminium and zirconia are the materials considered for the analysis. The effective properties are varying along the thickness with differ with different volume fraction of each material. Four parameter power law is use to accomplish different combination of FGM profile such as classic, symmetrical and asymmetrical. It is assumed that both materials are gradually varying and also homogeneous. Thick and thin FGM plate are being analysis to find natural frequency value of the FGM plate for different boundary condition with different material combination profile. The accuracy of the present model is studied by comparing the present results with those available in literature.

1. INTRODUCTION

Due to huge disadvantage of various laminate structure problem facing in layer interface and stress discontinuity along the thickness [5,6] for which new method of composite of two materials came into exists in mid-980 when Japanese scientists were doing research on ultra-high temperature resistant materials for aerospace application. The first book on functionally graded material was published in London (UK) this book contains comprehensive explanation on fundamental manufacturing processes, design and current application. Eventually many researches have been done on functionally graded materials (FGM) in various field of engineering. Basically, functionally graded materials are microscopically heterogeneous and made from isotropic materials such ceramics and metals. The material properties vary in a regular fashion along the thickness of the FGM structure in a regular fashion.

Power law is the most common method used. Generally, most of researcher has done material properties gradation base on power law. Still there are many more method of gradation such as exponential law, sigmoidal law and mori-tanaka scheme. All this method has their own way to define materials properties distribution in functionally graded materials plate or shell.

2. MATHEMATICAL FORMULATION

In the present study, four parameter power law is used to describe the distribution of materials properties. Where Tornabene [2,3] has recently worked on it and it is given by:

\[ V_c = (1-a(0.5+\frac{z}{h})) + b(0.5+\frac{z}{h})^c \]
Where \(a, b, c\) are three parameters that describe the power law distribution of materials and \(p\) is the exponential variable for which values assume to vary between zero to infinity. When the value of \(p\) is zero it resembles homogenous case of ceramic and when value of \(p\) is infinity is resembling the metal. When this value of \(p\) is varying from zero to infinity distribution of combination of material gets gradually change. The representation of profile is that has chosen in his paper are:

1. Classic profile \([a=1, b=0]\)
   - Ceramic at top and metal at bottom classic
2. Symmetric profile \([a=b=1, c=2]\)
   - Ceramic at both top and bottom
3. Asymmetric profile \([a=1, b=0, c=2.0]\)
   - 50% both ceramic and metal at top and full ceramic at bottom.

Figure 1 (a), (b), (c) shows the variation between volume fraction of ceramic and non-dimensional depth \((z/h)\).

Many researchers have published a scientific literature for analysis of functionally graded material plate with different mathematical formula for volume fraction of distinct material with gradually. A review of article by Jha and Kant [4] briefly discuss on various previous literature exist since it came to exists on thermoelastic, static and vibration. All the analysis done so far is based on FGM structure considers the transverse shear deformation and obtained in 2D results are validated with 3D elasticity solution. However, none of the work shows the application of available finite element software packages for the analysis of sandwich FGM plates. In present work, an attempt has been made to carry out the free vibration analysis of sandwich FGM plates using ABAQUS.
b) Symmetric Profile

c) Asymmetric Profile

**Figure 1.** Different profiles of FGM considered in the present study

### 3. MATERIAL PROPERTIES

The materials properties for the FGM plate are aluminium as metal with elasticity constant $E=70$ GPa, density $=2707$ kg/m$^3$ and zirconia as ceramic with elasticity constant $E=168$ GPa, density $=5700$ kg/m$^3$. Effect of passion ratio for both the material constant value of 0.3.

### 4. MODELLING

A model has been create in ABAQUS to analysis the different mode of natural frequency in thin FGM plate i.e. $(0.01m)$ and moderate thick FGM plate $(0.1m)$ thickness square plate of $(1m*1m)$ as aspect ratio $(a/h)$=10 with different boundary condition (clamped, simply supported-clamped and simply support ). Element with 20-noded quadratic brick has been used as shown in Figure 2.
5. RESULTS AND DISCUSSION

At first convergence study is carried out in order to determine the efficient mesh size that has to be used during the study. The convergence graph is shown in figure 3 for thin FGM simply supported classical plate (p=0). From the graph it is clear that the present results converge at mesh size of 20x20. Hence, in further studies same mesh size is taken.

Table 1 and 2 shows the value of fundamental and second mode frequency of simply supported thin and thick FGM plates respectively. The classic, symmetric and asymmetric profile are taken for consideration of different exponential values of power law ranging from (0<p<1). When value p=0 it represents pure ceramic plates as value of exponential value of power law increases pure ceramic convert to FGM plate. This common trend is observing in all the profile (classic, symmetric and asymmetric). From Tables 1 & 2, it is seen that as metal content in plates increases results in lowering the stiffness of plate thus also decreasing the value of frequency as exponential value of power law increases.

Due to isotopic properties of plates value of frequency at p=0 shows same for all three profile. In table 1 and 2 it has been observed that classic and asymmetric profile the value of nature frequency is decreasing as the exponential value is increasing from (0<p<1). This indicate that lowering stiffness
of plate. But for symmetric profile values of nature frequencies is increasing which indicate higher stiffness, so it will better plate as compare to other two profile FGM plates.

Tables 3 and 4 shows value of first two frequencies for clamped condition for thin and moderated thick FGM plates respectively. As clamped boundary has high bending the value of frequency observe in table 3 and 4 are higher as compare to results presented in Tables 1, 2, 5 and 6. For case of simply supported clamped (Table 5 for thin FGM plate and Table 6 for thick FGM plate) the value observe are low as compare to clamped condition because of two side are simply supported. Therefore, again symmetric profile shows better than other two profile for both cases (clamped and simply clamped boundary condition).

For all three-boundary conditions, value of natural frequency for thin FGM plate are low than moderated thick FGM plate. Which can be analysis that thick FGM plates are better performance FGM plates. Figures 4- figure 11 shows first two modes of frequency for different conditions of FGM plate. From the figures, it is observed that the behaviour of the plate widely depends upon the boundary condition and exponential power law function (p).

| Table 1. First two frequency for simply supported h=0.1m |
|--------------------------------|
| P | Source | 0 | 0.2 | 0.4 | 1 |
|--------------------------------|
| Classic | Present | 323.1181 | 322.2672 | 320.0005 | 317.8324 |
| | Taj et al. [1] | 324.1771 | 323.3509 | 322.4659 | 319.5509 |
| Symmetric | Present | 323.1181 | 324.5681 | 326.2789 | 328.0075 |
| | Taj et al. [1] | 324.1771 | 324.9004 | 325.5331 | 326.7571 |
| Asymmetric | Present | 323.1181 | 322.9607 | 321.5129 | 319.8126 |
| | Taj et al. [1] | 324.1771 | 323.6979 | 323.145 | 321.0901 |

| Table 2. First two frequency for simply supported h=0.01m |
|--------------------------------|
| P | Source | 0 | 0.2 | 0.4 | 1 |
|--------------------------------|
| Classic | Present | 3129.362 | 3122.987 | 3115.535 | 3087.041 |
| | Taj et al. [1] | 3132.852 | 3125.462 | 3117.48 | 3090.867 |
| Symmetric | Present | 3129.362 | 3136.154 | 3141.268 | 3149.895 |
| | Taj et al. [1] | 3132.852 | 3139.264 | 3144.812 | 3155.066 |
| Asymmetric | Present | 3129.362 | 3125.254 | 3119.246 | 3104.716 |
| | Taj et al. [1] | 3132.852 | 3128.547 | 3123.524 | 3104.584 |
Table 3. First two frequency for clamped $h=0.1m$

| $P$ | Source       | 0     | 0.2   | 0.4   | 1     |
|-----|--------------|-------|-------|-------|-------|
|     | Classic      |       |       |       |       |
|     | Present      | 5354.7| 5205.0| 5174.7| 4987.7|
|     |              | 10228.3| 9912.4| 9876.5| 9797.1|
|     | Taj et al. [1]| 5351.332| 5340.513| 5328.595| 5287.73|
|     |              | 10228.3| 10210.51| 10190.54| 10120.19|
| Symmetric | Present | 5354.7| 5366.9| 5378.5| 5410.0|
|     |              | 10236.0| 10253.0| 10268| 10310|
|     | Taj et al. [1]| 5351.332| 5360.513| 5368.242| 5380.771|
|     |              | 10228.3| 10243.06| 10255.12| 10271.63|
| Asymmetric | Present | 5354.7| 5366.9| 5340.9| 5314.0|
|     |              | 10236.0| 10253.0| 10204.0| 10146|
|     | Taj et al. [1]| 5351.332| 5344.962| 5337.332| 5307.607|
|     |              | 10228.3| 10217.71| 10204.7| 10152.5|

Table 4. First two frequency for clamped $h=0.01m$

| $P$ | Source       | 0     | 0.2   | 0.4   | 1     |
|-----|--------------|-------|-------|-------|-------|
|     | Classic      |       |       |       |       |
|     | Present      | 592.4| 590.3| 587.6| 584|
|     |              | 1203.6| 1199.4| 1198.2| 1188.7|
|     | Taj et al. [1]| 590.5254| 589.0234| 587.4141| 582.1116|
|     |              | 1203.612| 1200.557| 1197.284| 1186.493|
| Symmetric | Present | 592.4| 593.1| 594.4| 595.5|
|     |              | 1203.6| 1205.2| 1207.6| 1210.4|
|     | Taj et al. [1]| 590.5254| 591.84| 592.9895| 595.2106|
|     |              | 1203.612| 1206.285| 1208.622| 1213.131|
| Asymmetric | Present | 592.4| 591.1| 588.4| 586.3|
|     |              | 1203.6| 1202.3| 1198.4| 1196.1|
|     | Taj et al. [1]| 590.5254| 589.6541| 588.6485| 584.9001|
|     |              | 1203.612| 1201.84| 1199.794| 1192.183|

Table 5. First two frequency for simply-clamped $h=0.1m$

| $P$ | Source       | 0     | 0.2   | 0.4   | 1     |
|-----|--------------|-------|-------|-------|-------|
|     | Classic      |       |       |       |       |
|     | Present      | 5354.7| 5190.4| 5170.2| 5112.0|
|     |              | 10236.0| 9894.4| 9873.2| 9803.4|
|     | Taj et al. [1]| 4156.973| 4147.89| 4137.986| 4104.511|
|     |              | 8792.393| 8775.471| 8756.735| 8692.041|
| Symmetric | Present | 5354.7| 5365.7| 5378.5| 5392.4|
|     |              | 10236.0| 10251.6| 10268.0| 10283.5|
|     | Taj et al. [1]| 4156.973| 4164.77| 4164.77| 4183.034|
|     |              | 8792.393| 8806.671| 8806.671| 8837.217|
| Asymmetric | Present | 5354.7| 5348.5| 5340.9| 5314.0|
|     |              | 10236.0| 10221| 10204.0| 10146|
|     | Taj et al. [1]| 4156.973| 4151.655| 4145.37| 4121.291|
|     |              | 8847.72| 8837.613| 8825.438| 8777.686|
Table 6. First two frequency for simply-clamped h=0.01

| P  | Source     | 0  | 0.2 | 0.4 | 1    |
|----|------------|----|-----|-----|------|
| Classic | Present | 447.8 | 445.9 | 442.4 | 438.7 |
|       | Taj et al. [1] | 444.1443 | 443.0135 | 441.802 | 437.8109 |
|       | Present | 447.8 | 449.3 | 451.2 | 452.4 |
|       | Taj et al. [1] | 444.1443 | 445.1342 | 446.00 | 447.6739 |
| Symmetric | Present | 447.8 | 446.4 | 444.3 | 441.6 |
|       | Taj et al. [1] | 444.1443 | 443.4884 | 442.7314 | 439.9176 |
| Asymmetric | Present | 447.8 | 1010.2 | 1008.3 | 1005.7 | 1003.2 |

Figure 4. First two mode shape on classic clamped when p=0.4 at (a/h) =10

Figure 5. First two mode shape on classic simply clamped when p=0.4 at (a/h)=10
Figure 6. First two mode shape on symmetric clamped when $p=0.4$ at $(a/h)=10$

Figure 7. First two mode shape on symmetric simply clamped when $p=0.4$ at $(a/h)=10$

Figure 8. First two mode shape on symmetric simply supported when $p=0.4$ at $(a/h)=10$
6. CONCLUSION

From the present study it is seen that the present model is able to predict free vibration behaviour of sandwich FGM plates more efficiently without any great computational efforts. Also, the present model is able to predict the behaviour of thick sandwich FGM plates with great accuracy. The behaviour of the plate largely depends upon the ceramic and metal content in the plate along the thickness along with boundary conditions. Therefore, the present model can be used for the industrial or research applications on the similar topic.
7. SCOPE OF FUTURE STUDY

The present study can be extended to following:
1. Analysis of sandwich FGM beams and shells.
2. Analysis of sandwich FGM structures under hygro-thermal conditions.
3. Effects of dampers can also be studied.

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