The decays $\Lambda_{b,c} \to N^* l \nu$ in QCD

M. Emmerich, N. Offen, A. Schäfer
Institut für theoretische Physik,
Universität Regensburg, 93040 Regensburg, Germany

Abstract

We present an exploratory study of the $\Lambda_{c,b} \to N^*$-form factors and the semileptonic decay width within the framework of light-cone sum rules. We use two different methods and two different interpolating currents for the $\Lambda_{c,b}$.

1. We follow [1] and eliminate negative parity partners of the $\Lambda_{c,b}$ by taking linear combinations of different Lorentz-structures.
2. We extract the form factors by choosing the Lorentz-structures with the highest possible powers of $p_+$. As interpolating currents we choose an axial-vector like and a pseudoscalar like current.

Our results show that the procedure of eliminating negative parity partners is not well suited for the case at hand and that the second approach with an axial-vector like interpolating current gives the most reliable results. Our predictions are based on the models obtained in [2, 3]. The largest uncertainty comes from the uncertainty of the twist 4 parameters $\eta_{10}, \eta_{11}$ and we take the spread between the two models in [2] as a measure for this. We get

\begin{align*}
\Gamma(\Lambda_b \to N^*(1535)l\nu) &= (0.0055^{+0.0010}_{-0.0009}) \cdot \left( \frac{V_{ub}}{3.5 \cdot 10^{-3}} \right)^2, \quad \text{LCSR(1)} \\
\Gamma(\Lambda_b \to N^*(1535)l\nu) &= (0.00070^{+0.00012}_{-0.00011}) \cdot \left( \frac{V_{ub}}{3.5 \cdot 10^{-3}} \right)^2, \quad \text{LCSR(2)} \\
\Gamma(\Lambda_c \to N^*(1535)l\nu) &= (0.0064^{+0.0012}_{-0.0011}) \cdot \left( \frac{V_{cd}}{0.225} \right)^2, \quad \text{LCSR(1)} \\
\Gamma(\Lambda_c \to N^*(1535)l\nu) &= (0.00077^{+0.00016}_{-0.00014}) \cdot \left( \frac{V_{cd}}{0.225} \right)^2, \quad \text{LCSR(2)}
\end{align*}

as predictions for the respective decay widths, where LCSR(1) and LCSR(2) refer to the two different models of the distribution amplitudes of the $N^*$. It is seen that even a rough measurement of these decays will greatly help to discriminate different models.

1 Introduction

Measuring the properties of the nucleon resonances and interpreting them in terms of the fundamental degrees of freedom of QCD is one of the goals of the Hall B CLAS 12 detector at Jefferson Lab. [4] We propose here a complementary approach to nucleon resonances using the decays of $\Lambda_b$ and $\Lambda_c$ baryons produced abundantly at LHCb or at the planned PANDA experiment. Our approach is based on light cone sum rules [5] a hybrid of classical SVZ-sum rules [6].
and methods from hard exclusive decays. It allows to relate the \( \Lambda_{b,c} \to N^* \) decay form factors to the distribution amplitudes of the \( N^* \) that is roughly speaking to the momentum distribution of the quarks in different Fock states inside the \( N^* \).

Sufficient experimental data will allow to compare the extracted distribution amplitudes to constraints coming from the lattice \([7,3]\) or from electromagnetic \( N^* \) form factors \([2]\), see also \([8]\) for the general framework. This will provide an alternative approach to obtain information on the structure of low lying nucleon resonances, see also \([9]\) for a review of hadronic decays of heavy mesons and baryons to investigate hadronic resonances. Similar calculations for the \( \Lambda_b \to N \) form factors have been done in \([10,11,1,12]\) and we will refer to \([11]\) for more details on the calculation.

The paper is organized as follows: In section 2 we will give the definitions of the relevant form factors and a short introduction to the method of light cone sum rules. Section 3 is devoted to the numerical analysis of the sum rules and section 4 will give some concluding remarks. The relevant formulas are given in the appendix. In addition we refer to \([2,8,1]\), see also \([13,14]\), for the basic definitions of the distribution amplitudes in order not to overload this publication.

## 2 Form Factors and light cone sum rules

We give the definitions and derivations for the case of the \( \Lambda_c \to N^* \) decay. The transition to \( \Lambda_b \to N^* \) is simply done by replacing \( c \to b \) everywhere and \( d \to u \) in the transition current. The relevant form factors of the vector and axial-vector current are defined as

\[
\langle \Lambda_c(P') | j_{\nu} | N^*(P) \rangle = \bar{u}_{\Lambda_c}(P') \left( f_1(q^2) \gamma_\nu + i \frac{f_2(q^2)}{m_{\Lambda_c}} \sigma_{\nu\mu} q^\mu + \frac{f_3(q^2)}{m_{\Lambda_c}} q_\nu \right) \gamma_5 u_{N^*}(P),
\]

\[
\langle \Lambda_c(P') | j_{\nu5} | N^*(P) \rangle = \bar{u}_{\Lambda_c}(P') \left( g_1(q^2) \gamma_\nu + i \frac{g_2(q^2)}{m_{\Lambda_c}} \sigma_{\nu\mu} q^\mu + \frac{g_3(q^2)}{m_{\Lambda_c}} q_\nu \right) u_{N^*}(P),
\]

where \( P' = P - q \). We don’t consider \( f_3 \) and \( g_3 \) in the following since in semileptonic decays they will contribute with coefficients proportional to the lepton mass.

For the \( A_2^c \)-form factors one defines \( f_i \) and \( g_i \) analogously to \([1]\) by replacing \( \Lambda_c \) with \( \Lambda_c^* \) and adding a \( \gamma_5 \) after the \( \Lambda_c^* \)-spinor. To get access to these form factors we use the correlation function

\[
\Pi_a(P,q) = i \int d^4 x \ e^{iq \cdot x} \langle 0 | T \{ \eta_{\Lambda_c}(0), j_a(x) \} | N^*(P) \rangle,
\]

where

\[
j_a(x) = \bar{c} \Gamma_a d(x), \quad \Gamma_a = \gamma_\nu, \gamma_\nu \gamma_5,
\]

are the weak transition currents and

\[
\eta_{\Lambda_c} = e^{ijk}(u_i C \Gamma_1 d_j) \Gamma_2 c_k
\]

is the interpolating current with the correct quantum numbers for the \( \Lambda_c \). We will use two different choices for \( \eta_{\Lambda_c} \) namely

\[
\eta_{\Lambda_c}^{(P)} = (u C \gamma_5 d) c \quad \text{and} \quad \eta_{\Lambda_c}^{(A)} = (u C \gamma_5 \gamma_3 d) \gamma_5 c.
\]

The standard procedure of light cone sum rules is to calculate the correlation function \([2]\) in two different ways. On the one hand, one inserts a complete set of states with \( \Lambda_c \) quantum numbers between the two currents and extracts the lowest lying state. On the other hand, one uses the operator product expansion (OPE) around the light cone for space like
moments $(P - q)^2$, $q^2 \ll 0$. This results in two different representations of the correlation function which can be equated using dispersion relations and quark-hadron duality. Taking the Borel-transform to eliminate possible subtraction terms from the dispersion relations and to suppress higher states in the hadronic sum gives the final sum rule.

2.1 Eliminating the $\Lambda_c^*$-pole

We will give a few details on the procedure but refer the reader to [1] for all details of the calculations. To eliminate the $\Lambda_c^*$ pole from the sum rules, we explicitly keep both the $\Lambda_c$ and $\Lambda_c^*$ in the hadronic sum and represent higher states by a dispersion integral. The residue of the poles of the two states is given by a product of the form factors [1] and their decay constants:

$$\langle 0| \eta^{(i)}_c | \Lambda_c(P') \rangle = \lambda^{(i)}_c m_{\Lambda_c} u_{\Lambda_c} (P'),$$

$$\langle 0| \rho^{(i)}_c | \Lambda_c^*(P') \rangle = \lambda^{(i)}_{\Lambda_c^*} m_{\Lambda_c^*} \gamma_5 u_{\Lambda_c^*} (P').$$

(3)

Two main observations: First, using the equation of motion $(P - m_{\Lambda_c^*}) u_{\Lambda_c^*} (P)$, one can decompose the correlation function into six independent, invariant functions:

$$\Pi^{(i)}(P, q) = \Pi^{(i)}_{\mu} P_{\mu} = \Pi^{(i)}_{\mu} P_{\mu} \gamma_\mu + \Pi^{(i)}_{\mu} \gamma_\mu + \Pi^{(i)}_{\mu} \gamma_\mu \gamma_5 u_{\Lambda_c^*} (P),$$

(4)

where $i$ labels the pseudoscalar $P$ and axial-vector $A$ interpolating current.

Second, the form factors enter in front of more than one of the Lorentz-structures in (4). This allows, after one equates the hadronic sum and the OPE result, to construct linear combinations where the contribution of the $\Lambda_c^*$ is eliminated. Taking e.g. the hadronic sum for the vector transition

$$\Pi^{(i)}_{\mu}(P', q) = \frac{\lambda^{(i)}_c m_{\Lambda_c}}{m_{\Lambda_c}^2 - P^2} \left[ 2 f_1(q^2) P_{\mu} - 2 f_2(q^2) P_{\mu} \right]$$

$$+(m_{\Lambda_c^*} - m_{\Lambda_c}) \left( f_1(q^2) + \frac{m_{\Lambda_c^*} - m_{\Lambda_c}}{m_{\Lambda_c}} f_2(q^2) \right) \gamma_\mu + \left( f_1(q^2) + \frac{m_{\Lambda_c^*} - m_{\Lambda_c}}{m_{\Lambda_c}} f_2(q^2) \right) \gamma_\mu \gamma_5 u_{\Lambda_c^*} (P)$$

$$+ \frac{\lambda^{(i)}_{\Lambda_c^*} m_{\Lambda_c^*}}{m_{\Lambda_c^*}^2 - P^2} \left[ -2 f_1(q^2) P_{\mu} + 2 f_2(q^2) P_{\mu} \right]$$

$$+(m_{\Lambda_c^*} - m_{\Lambda_c}) \left( f_1(q^2) + \frac{m_{\Lambda_c^*} + m_{\Lambda_c}}{m_{\Lambda_c}} f_2(q^2) \right) \gamma_\mu - \left( f_1(q^2) + \frac{m_{\Lambda_c^*} + m_{\Lambda_c}}{m_{\Lambda_c}} f_2(q^2) \right) \gamma_\mu \gamma_5 u_{\Lambda_c^*} (P)$$

$$+ \frac{2 f_2(q^2)}{m_{\Lambda_c}^2 - P^2} \left( f_1(q^2) + \frac{m_{\Lambda_c^*} + m_{\Lambda_c}}{m_{\Lambda_c}} f_2(q^2) \right) \gamma_\mu - \left( f_1(q^2) + \frac{m_{\Lambda_c^*} + m_{\Lambda_c}}{m_{\Lambda_c}} f_2(q^2) \right) \gamma_\mu \gamma_5 u_{\Lambda_c^*} (P)$$

$$+ \int \frac{ds}{s - P^2} \left( \rho^{(i)}_1 (s, q^2) P_{\mu} + \rho^{(i)}_2 (s, q^2) P_{\mu} \right)$$

$$+ \rho^{(i)}_3 (s, q^2) \gamma_\mu + \rho^{(i)}_4 (s, q^2) \gamma_\mu \gamma_5 u_{\Lambda_c^*} (P),$$

(5)
where higher states are described by the spectral densities $\rho^{(i)}_j$ with $j = 1, \ldots, 6$. It can be seen that taking a linear combination of the first four Lorentz-structures one can get rid of \( \tilde{f}_1(q^2) \), \( \tilde{f}_2(q^2) \) and get expressions for \( f_1(q^2) \), \( f_2(q^2) \) in terms of $\Pi_1$ to $\Pi_4$.

### 2.2 Extracting highest powers of $p_+$

We define a light-like vector $n_\mu$ by the condition
\[
q \cdot n = 0, \quad n^2 = 0
\]
and introduce the second light-like vector as
\[
p_\mu = P_\mu - \frac{1}{2} n_\mu \frac{m_\Lambda^2}{P \cdot n}, \quad p^2 = 0,
\]
so that $P \to p$ in the infinite momentum frame, $P \cdot n \to \infty$, or if the nucleon mass can be neglected, $m_N \to 0$. The projector onto the directions orthogonal to $p$ and $n$ is then defined as
\[
g_\mu^\perp = g_{\mu\nu} - \frac{1}{p \cdot n} (p_\mu n_\nu + p_\nu n_\mu).
\]
Taking a look at equations (4) and (5) it is seen that contracting the correlation function $\Pi_{\nu}(P', q)$ with $n_\nu$ and multiplying by the projector $g_\mu^\perp$ the result can be written as
\[
\frac{p_\mu}{2p \cdot n} n^\nu \Pi_{\nu}(P', q) = p \cdot n \left( A(P', q) + \frac{\gamma_5}{m_\Lambda} B(P', q) \right).
\]

The form factors $f_1(Q^2)$ and $f_2(Q^2)$ are then extracted from the sum rules for the functions $A(P', q)$ and $B(P', q)$ respectively. $g_1(Q^2)$ and $g_2(Q^2)$ are extracted in the same way with the only difference being one additional $\gamma_5$.

### 2.3 Deriving the light cone sum rules

For $P^2, q^2 \ll 0$ the product of two currents in (2) can be expanded around the light-cone $x^2 \sim 0$. At leading order the two $c$-quarks in (2) are contracted giving the free propagator and the resulting matrix element is decomposed according to (A.21) in $\ref{2}$. This results in a sum over distribution amplitudes of different twists multiplied by their respective coefficient functions. Using equation of motions the contributions to the invariant functions $\tilde{\Pi}_j$ in $\ref{4}$ can be identified. Neglecting terms which will vanish after Borel-transformation they can in general be written as
\[
\tilde{\Pi}^{(i)}_j(P^2, q^2) = \frac{1}{4} \sum_{n=1,2,3} \int_0^1 dx \frac{w_j^{(i)}(x, q^2)}{D^n},
\]
with the denominator
\[
D = m_\Lambda^2 - (xP - q)^2 = m_\Lambda^2 - xP^2 - xq^2 + x\bar{x}m_N^2,
\]
where $\bar{x} = 1 - x$. The different functions $w_j^{(i)}$ are distinguished by their indizies, where $i$ denotes either $A$ or $P$ for axial- or pseudoscalar interpolating current, $j = 1, \ldots, 6$ parametrizes the invariant amplitude it contributes to and $n = 1, \ldots, 3$ is the power of the denominator.
The functions \( w_{jm}^{(i)} \) are given in appendix A.

We have to write (11) as a dispersion integral in \( P^2 \):

\[
\tilde{\Pi}_j^{(i)}(P^2, q^2) = \frac{1}{\pi} \int_{m_c^2}^{\infty} \frac{ds}{s - P^2} \Im \tilde{\Pi}_j^{(i)}(s, q^2). \tag{12}
\]

Therefore we substitute

\[
s(x) = \frac{1}{x} (m_c^2 - \bar{x} q^2 + \bar{x} \bar{x} m_{N*}^2),
\]

\[
x(x) = \frac{1}{2m_{N*}^2} \left[ m_{N*}^2 + q^2 - s + \sqrt{(s - q^2 - m_{N*}^2)^2 + 4m_{N*}^2(m_c^2 - q^2)} \right] \tag{13}
\]

in the denominator (11) and do a partial integration if the power of the denominator is larger than one. Using quark-hadron duality to approximate the contributions of the hadronic states

\[
\int_{s_0}^{\infty} \frac{ds}{s - P^2} \rho_j^{(i)}(q^2) \approx \frac{1}{\pi} \int_{s_0}^{\infty} \frac{ds}{s - P^2} \Im \tilde{\Pi}_j^{(i)}(q^2), \tag{14}
\]

where \( s_0 \) is the duality threshold, and performing a Borel-transformation that results in

\[
\frac{1}{s - P^2} \rightarrow e^{-s/M^2}, \quad (P^2)^n \rightarrow 0,
\]

with \( M^2 \) being the Borel-parameter leads to the final sum rules

\[
f_1(q^2) = \frac{e^{m_{\Lambda_c}^2/M^2}}{2m_{\Lambda_c}(m_{\Lambda_c} + m_{\Lambda_c}^*)} \frac{1}{\lambda_{\Lambda_c}^{(i)}} \int_{m_c^2}^{s_0} ds e^{-s/M^2} \left[ (m_{\Lambda_c} - m_{N*}) \Im \tilde{\Pi}_1^{(i)}(s, q^2) - (m_{\Lambda_c} + m_{N*}) \Im \tilde{\Pi}_1^{(i)}(s, q^2) \right],
\]

\[
f_2(q^2) = \frac{e^{m_{\Lambda_c}^2/M^2}}{2(m_{\Lambda_c} + m_{\Lambda_c}^*)} \frac{1}{\lambda_{\Lambda_c}^{(i)}} \int_{m_c^2}^{s_0} ds e^{-s/M^2} \left[ \Im \tilde{\Pi}_2^{(i)}(s, q^2) - 2\Im \tilde{\Pi}_1^{(i)}(s, q^2) \right], \tag{15}
\]

with subtraction of the \( \Lambda_{c*}^\ast \)-pole and

\[
f_1(q^2) = \frac{e^{m_{\Lambda_c}^2/M^2}}{2m_{\Lambda_c} \lambda_{\Lambda_c}^{(i)}} \frac{1}{\pi} \int_{m_c^2}^{s_0} ds e^{-s/M^2} \Im \tilde{\Pi}_1^{(i)}(s, q^2),
\]

\[
f_2(q^2) = \frac{e^{m_{\Lambda_c}^2/M^2}}{2\lambda_{\Lambda_c}^{(i)}} \frac{1}{\pi} \int_{m_c^2}^{s_0} ds e^{-s/M^2} \Im \tilde{\Pi}_1^{(i)}(s, q^2), \tag{16}
\]

for those without subtraction. Again the procedure for \( g_1 \) and \( g_2 \) is the same with some trivial sign differences due to an additional \( \gamma_5 \).
| Method       | $|\lambda^N/\lambda^N_1|$ | $f_{N^*}/\lambda^N_1$ | $\varphi_{10}$ | $\varphi_{11}$ | $\varphi_{20}$ | $\varphi_{21}$ | $\varphi_{22}$ | $\eta_{10}$ | $\eta_{11}$ | Ref. |
|--------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|-------------|-------------|------|
| LCSR (1)     | 0.633           | 0.027           | 0.36           | -0.95         | 0              | 0              | 0              | 0.00        | 0.94        | [2]  |
| LCSR (2)     | 0.633           | 0.027           | 0.37           | -0.96         | 0              | 0              | 0              | -0.29       | 0.23        | [2]  |
| LATTICE      | 0.633(43)       | 0.027(2)        | 0.28(12)       | -0.86(10)     | 1.7(14)        | -2.0(18)       | 1.7(26)        | -           | -           | [3]  |

Table 1: Parameters of the $N^*(1535)$ distribution amplitudes at the scale $\mu^2 = 2\text{ GeV}^2$. For the lattice results [3] only statistical errors are shown. The set of parameters indicated as LCSR (1) corresponds to the fit to the form factors $G_1(Q^2)$ and $G_2(Q^2)$ extracted from the measurements of helicity amplitudes in Ref. [24] adding the errors in quadrature. The set of parameters indicated as LCSR (2) is obtained from the fit to helicity amplitudes including all available data at $Q^2 \geq 1.7\text{ GeV}^2$ [25, 26, 27, 24]. $\lambda^N_1$ is given in [3] as $10^2 m_N \lambda^N_1 = -3.88(2)(19)\text{ GeV}^3$. Note the typo in the first column in [2].

3 Numerical Analysis

3.1 Prerequisites

We start this section by specifying the input parameters we use for our numerical analysis. The masses of the involved Baryons are taken from [20] and for the $\Lambda^*_b$ from [19]

$$m_{\Lambda_c} = 2.286\text{ GeV}, \quad m_{\Lambda^*_c} = 2.595\text{ GeV},$$

$$m_{\Lambda_b} = 5.620\text{ GeV}, \quad m_{\Lambda^*_b} = 5.85\text{ GeV}. \quad (17)$$

The values for the shape parameters of the $N^*$-distribution amplitude are given in table 1. For the twist 4 normalization factors $\lambda^{(A,P)}_1$ we use the corresponding leading order two point sum rules instead of inserting fixed values. We take the same two approaches to the two point sum rules as for the light-cone sum rules. The correlation functions for $\lambda^{(A,P)}_1$ and $\lambda^{(A,P)}_1 \cdot f^{(A,P)}_i$ have a very similar $\mu$-dependence which effectively cancels in the quotient. This has the advantage of considerably reducing the $\mu$-dependence of the result. We take the sum rules from [21], see also [1] and have checked their results. A thorough analysis of the sum rules shows that those derived by eliminating the $\Lambda^*_c,b$-pole are plagued by several problems:

1. Except for the form factors $f_1(Q^2)$ and $g_1(Q^2)$ using the pseudoscalar interpolating current there are large numerical cancellations between different Lorentz-structures
2. For the pseudoscalar current there is generally very little hierarchy between contributions of different twists, there are numerical cancellations between different twists and they are very sensitive to the variation of the higher twist parameter $\xi_{10}$
3. There is no set of parameters $M^2, s_0$ so that the sum rules for all four form factors fulfill the basic criteria used to check their viability

Finally we have chosen the sum rules for the functions $A$ and $B$, see [4], with the axial vector interpolating current and the respective two-point sum rule with the highest powers of $p_+$ as our default. The two models from table 1 give a measure for the uncertainty coming from the variation of twist 4 parameters $\eta_{10}$ and $\eta_{11}$. We quote the results for the two models

\footnote{Note a typo in the dimension six part of $\text{Im} \tilde{F}_1(s)$, eq. (91), in [1]. The correct expression is [22] \[
\text{Im} \tilde{F}_1^{\text{dim}6}(s) = \frac{4(\bar{q}q)^2}{72} \delta(s - m_N^2)(11 + 26 - 13s^2) \]

6
separately to demonstrate the ability to discern different models by measuring these decays. The pseudoscalar interpolating current has similar problems as mentioned above. In most channels there is no clear hierarchy between different twists and there occur large numerical cancellations. In addition the sum rules for the pseudoscalar current are very sensitive to the basically unknown parameter $\xi$. Varying this parameter in the range of $-0.2 \leq \xi \leq 0.2$ changes the result by up to a factor of 6. In contrast the axial-vector sum rules depend only very mildly on this parameter. The Borel-parameter and duality threshold are chosen in a way that the usual sum rule criteria, that is the suppression of continuum states and of higher twist contributions, are fullfilled. We observe that for the range of

\begin{align}
M_b^2 &= 15 - 25 \text{ GeV}^2 \\
M_c^2 &= 5 - 10 \text{ GeV}^2
\end{align}

these criteria are fullfilled and that the sum rules are reasonably stable with respect to the variation of the Borel-parameter, see figure 1.

### 3.2 Decay-widths

Since the light cone sum rules are only valid up to $q^2 \leq q_{\text{max}}^2 = (m_{\Lambda_c,b} - m_{N^*})^2$ we need to extrapolate the sum rule results to the whole physical region to calculate the decay-widths. We do this by making a two-parameter fit to our numerical values using the fit-function proposed in [23]

\begin{align}
f_i(q^2) &= \frac{f_i(0)}{1 - \frac{q^2}{m_{B^*}^2(z, \ldots)}} \left\{ 1 + b_i \left( z(q^2, t_0) - z(0, t_0) \right) \right\} \\
g_i(q^2) &= \frac{g_i(0)}{1 - \frac{q^2}{m_{B^*}^2(1, \ldots)}} \left\{ 1 + \tilde{b}_i \left( z(q^2, t_0) - z(0, t_0) \right) \right\}
\end{align}

with

\begin{align}
z(q^2, t_0) &= \sqrt{t_+ - q^2} - \sqrt{t_+ - t_0} \\
&= \sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}
\end{align}

Figure 1: The Borel- and $\mu$-dependence of the $\Lambda_b \to N^*$-form factors at $Q^2 = 0$
and

\[ t_\pm = (m_{\Lambda_b} \pm m_{N^*})^2, \]
\[ t_0 = t_+ - \sqrt{t_+ - t_-} = t_- - \sqrt{t_+ - t_-}. \]  \hspace{1cm} (20)

\( t_{\min} \) is the lowest value of \( q^2 \) mapped to \( z(q^2, t_0) \), so that \( t_{\min} = q_{\min}^2 \leq q^2 \leq t_- \). We extend the fit region by calculating the form factor starting from \( q_{\min}^2 = -6 \text{ GeV}^2 \). We perform a weighed fit using as weights the uncertainties coming from the input parameters of the sum rules added in quadrature. For asymmetric errors we take the mean value and shift the central value by the difference of the mean value and the asymmetric error to get symmetric errors. The fits compared to our sum rule values can be found in figure 2 and the results in table 2.

As can be seen the largest uncertainty comes from the twist 4 parameters which are taken from table 1. This means these decays are very sensitive to the shape of the distribution amplitudes of the \( N^* \) that parametrize relative orbital angular momentum of the quarks.

Figure 2: The four different form factors for the decay \( \Lambda_b \rightarrow N^* \). LCSR(1) and LCSR(2) refer to the two models from table 1. The thick lines give the central value of the fits to the sum rule results (large dots), the dashed lines give the 2\( \sigma \) uncertainties coming from all parameters except twist 4.
Table 2: The fit parameters according to equation (18) for the \( \Lambda_b \to N^* \) form factors and the two models LCSR(1) and LCSR(2).

We calculate the decay width by the following expression:

\[
\frac{d\Gamma}{dq^2}(\Lambda_b \to pln) = \frac{G_F^2 m_{\Lambda_b}^2}{192\pi^3} |V_{ub}|^2 \lambda^{1/2} (1, r^2, t) \left\{ [(1 - r)^2 - t][|(1 + r)^2 + 2t]|g_1(q^2)|^2 \\
+ [(1 + r)^2 - t][|(1 - r)^2 + 2t]|f_1(q^2)|^2 - 6t[(1 - r)^2 - t]|g_1(q^2)g_2(q^2)| \\
- 6t[(1 + r)^2 - t]|(1 - r)f_1(q^2)f_2(q^2) + t|(1 - r)^2 - t][2(1 + r)^2 + t]|g_2(q^2)|^2 \\
+ t|(1 + r)^2 - t][2(1 - r)^2 + t]|f_2(q^2)|^2 \right\}.
\] (21)
Compared to [1, 12] we have to exchange $f_1 \leftrightarrow g_1$ and $f_2 \leftrightarrow g_2$ and we neglect $f_3$ and $g_3$ since their coefficient is always proportional to the lepton mass. Our results are

\[
\begin{align*}
\Gamma(\Lambda_b \to N^*(1535)l\nu) &= \left(0.0058^{+0.0010}_{-0.0009}\right) \cdot \left(\frac{V_{ub}}{3.5 \cdot 10^{-3}}\right)^2, \quad \text{LCR}(1) \\
\Gamma(\Lambda_b \to N^*(1535)l\nu) &= \left(0.0070^{+0.0012}_{-0.0011}\right) \cdot \left(\frac{V_{ub}}{3.5 \cdot 10^{-3}}\right)^2, \quad \text{LCR}(2) \\
\Gamma(\Lambda_c \to N^*(1535)l\nu) &= \left(0.0064^{+0.0012}_{-0.0011}\right) \cdot \left(\frac{V_{cd}}{0.225}\right)^2, \quad \text{LCR}(1) \\
\Gamma(\Lambda_c \to N^*(1535)l\nu) &= \left(0.0077^{+0.0016}_{-0.0014}\right) \cdot \left(\frac{V_{cd}}{0.225}\right)^2, \quad \text{LCR}(2)
\end{align*}
\]

where we have given the results for the two different models separately to support our claim that the measurement of these decay widths will help to understand the structure of the twist 4 distribution amplitudes of the $N^*$.

4 Conclusions

We propose to measure the $\Lambda_{b,c} \to N^*$ form factors at PANDA and LHCb as an alternate way to extract information on the distribution amplitudes of the $N^*$. We did a leading order calculation in the framework of light cone sum rules taking into account three particle Fock-states up to twist 6.

We eliminated the $\Lambda_{b,c}^*$ contribution following [1] and investigated two different interpolating currents for the $\Lambda_{b,c}$. We found that all our sum rules using the elimination of the $\Lambda_{b,c}^*$-pole are plagued by large numerical cancellations between different Lorentz-structures except for the form factors $F_1(Q^2)$ and $G_1(Q^2)$ if one uses the pseudoscalar interpolating current. While this might look like a good sign these sum rules have the problem that there are large cancellations between contributions of different twist. Our final verdict is, that the elimination of $\Lambda_{b,c}^*$ is not well suited for the case at hand: The mixing of different Lorentz-structures leads to a larger continuum contribution and to large numerical cancellations. The only two exceptions where the method could work are sum rules which have bad characteristics for each Lorentz-structure separately.

The comparison of the two interpolating currents leads to the conclusion that the axial-vector current is generally better suited for our calculation. It shows a clearer hierarchy of contributions of different twist, lesser cancellations and a lower sensitivity to the only badly known $\lambda_2^{N^*}$ and $\xi_{10}$.

In general it is seen that in contrast to the $\Lambda_{b,c} \to N$ case the sum rules are dominated by twist 4 contribution, i.e. contributions with one unit of relative angular momentum, due to the higher mass of the $N^*$ and the smaller normalization factor of the leading order distribution amplitude $f_{N^*}$. A similar feature although not quite as pronounced was already observed in [2].

This circumstance makes this decay an ideal candidate to constrain the twist 4 distribution
amplitudes. The predicted decay widths for the two models LCSR(1) and LCSR(2)

\[ \Gamma(\Lambda_b \rightarrow N^*(1535)l\nu) = (0.0058^{+0.0010}_{-0.0009}) \cdot \left(\frac{V_{ub}}{3.5 \cdot 10^{-3}}\right)^2, \text{ LCSR}(1), \]

\[ \Gamma(\Lambda_b \rightarrow N^*(1535)l\nu) = (0.00070^{+0.00012}_{-0.00011}) \cdot \left(\frac{V_{ub}}{3.5 \cdot 10^{-3}}\right)^2, \text{ LCSR}(2), \]

\[ \Gamma(\Lambda_c \rightarrow N^*(1535)l\nu) = (0.0064^{+0.0012}_{-0.0011}) \cdot \left(\frac{V_{cd}}{0.225}\right)^2, \text{ LCSR}(1), \]

\[ \Gamma(\Lambda_c \rightarrow N^*(1535)l\nu) = (0.00077^{+0.00016}_{-0.00014}) \cdot \left(\frac{V_{cd}}{0.225}\right)^2, \text{ LCSR}(2), \]

support this claim by showing that even a rough measurement would already be able to discriminate between the two models. With upcoming experimental data and increasing precision a NLO-analysis will be the next task since these corrections are expected to be sizeable. See, e.g. [8, 2] for the case of the electromagnetic form factors. This is a huge calculation since in contrast to [8] there will be an additional mass scale due to the heavy quark and additional structures contributing at the same order of the twist expansion.

On the other hand combined with experimental data of sufficient precision it will give an excellent complementary possibility to make quantitative statements on the low lying nucleon resonances in terms of the fundamental degrees of freedom of QCD.

**Acknowledgements**

We appreciate helpful discussion with V. M. Braun. This work was funded by the BMBF under the contract number 05P12WRFTE.

**A Correlation functions**

The coefficient functions \( w^{(i)}_{jn} \) are listed below for:
pseudoscalar interpolating current

\[
\begin{align*}
  w_{11}^{(P)} &= x_2 m_N \phi_1^{(P)}, & w_{12}^{(P)} &= x_2 m_N^3 \left[ x_2 \phi_2^{(P)} + 2 \phi_3^{(P)} \right], \\
  w_{13}^{(P)} &= 4 x_2 m_N^2 m_c \phi_3^{(P)}, \\
  w_{21}^{(P)} &= w_{23}^{(P)} = 0, & w_{22}^{(P)} &= x_2 m_N \phi_2^{(P)}, \\
  w_{31}^{(P)} &= \frac{m_N^*}{2} (m_c + x_2 m_N^*) \phi_1^{(P)}, \\
  w_{32}^{(P)} &= \frac{m_N^*}{2} \left[ m_c (m_c + x_2 m_N^*) \phi_2^{(P)} + 2 x_2 m_N^2 \phi_3^{(P)} \right], \\
  w_{33}^{(P)} &= 2 m_N^2 m_c^2 (m_c + x_2 m_N^*) \phi_3^{(P)}, \\
  w_{41}^{(P)} &= \frac{m_N^*}{2} \phi_1^{(P)}, & w_{42}^{(P)} &= \frac{m_N^*}{2} \left[ m_c \phi_2^{(P)} + 2 m_N \phi_3^{(P)} \right], \\
  w_{43}^{(P)} &= 2 m_N^3 m_c^2 \phi_3^{(P)}, \\
  w_{51}^{(P)} &= - m_N \phi_1^{(P)}, & w_{52}^{(P)} &= - m_N^3 \left[ x_2 \phi_2^{(P)} + 2 \phi_3^{(P)} \right], \\
  w_{53}^{(P)} &= - 4 m_N^3 m_c^2 \phi_3^{(P)}, & w_{61}^{(P)} &= w_{63}^{(P)} = 0, & w_{62}^{(P)} &= - m_N^2 \phi_2^{(P)},
\end{align*}
\]

where the functions \( \phi_i^{(P)} \) are

\[
\begin{align*}
  \phi_1^{(P)} &= 2 \tilde{A}_1 + 4 \tilde{A}_3 + 2 \tilde{A}_{123} + 2 \tilde{B}_1 + 2 \tilde{S}_1 + 6 \tilde{T}_1 - 12 \tilde{T}_7 - \tilde{T}_{123} - 5 \tilde{T}_{127} - 2 \tilde{V}_1 + 4 \tilde{V}_3 + 2 \tilde{V}_{123}, \\
  \phi_2^{(P)} &= 3 \tilde{A}_{34} + 2 \tilde{A}_{123} - \tilde{A}_{1345} + 2 \tilde{P}_{21} + 2 \tilde{S}_{12} - 12 \tilde{T}_{78} - 2 \tilde{T}_{123} - 4 \tilde{T}_{127} - 6 \tilde{T}_{158} + 8 \tilde{T}_{234578} - 3 \tilde{V}_{23} + 2 \tilde{V}_{123} + \tilde{V}_{1345}, \\
  \phi_3^{(P)} &= - \tilde{A}_1^M - 3 \tilde{T}_1^M + \tilde{V}_1^M + \tilde{A}_{123456} - 3 \tilde{T}_{125678} + \tilde{T}_{234578} + \tilde{V}_{123456}.
\end{align*}
\]
axial-vector interpolating current

\[ u_{11}^{(A)} = 2 \left( -2m_c\phi_1^{(A)} - x_{22}m_{N^*} (2\phi_1^{(A)} + \phi_2^{(A)}) + 2m_{N^*}\phi_3^{(A)} \right) , \]
\[ u_{12}^{(A)} = 2m_{N^*} \left[ x_2^2m_{N^*}\phi_4^{(A)} - x_{22}m_{N^*}m_c\phi_5^{(A)} + 2m_c^2\phi_3^{(A)} + 2x_2m_{N^*}\phi_6^{(A)} \right] , \]
\[ u_{13}^{(A)} = 8m_{N^*}^2m_c \left[ -m_c^2\phi_7^{(A)} + x_2m_{N^*}m_c\phi_6^{(A)} - x_2^2m_{N^*}\phi_8^{(A)} \right] , \]
\[ u_{21}^{(A)} = -4\phi_1^{(A)} , \quad u_{23}^{(A)} = 8m_{N^*}m_c \left[ -m_c\phi_7^{(A)} - x_{22}m_{N^*}\phi_8^{(A)} \right] , \]
\[ u_{22}^{(A)} = 2m_{N^*} \left[ 2m_c\phi_3^{(A)} + x_2m_{N^*}\phi_4^{(A)} - 2m_c\phi_7^{(A)} \right] , \]
\[ u_{31}^{(A)} = -2 \frac{m_c^2 - q^2}{x_{22}} \phi_1^{(A)} + m_{N^*}m_c\phi_2^{(A)} - m_c^2\phi_9^{(A)} + 2x_2m_{N^*}\phi_{10}^{(A)} + 2 \frac{m_{N^*}^2\phi_4^{(A)}}{x_{22}} , \]
\[ u_{32}^{(A)} = m_{N^*} \left[ -2(q^2 - x_2^2m_{N^*})\phi_3^{(A)} + 2 \frac{q^2 + m_c^2}{x_{22}} \phi_7^{(A)} + x_2m_{N^*}m_c\phi_{11}^{(A)} - m_c^2\phi_5^{(A)} - 2\phi_3^{(A)} \right] + 2m_{N^*}(m_c - x_{22}m_{N^*})\phi_8^{(A)} , \]
\[ u_{33}^{(A)} = 4 \frac{m_c^2m_{N^*}^2}{x_{22}} \left[ -(m_c^2 - q^2)\phi_7^{(A)} + x_2m_{N^*}m_c\phi_{12}^{(A)} - x_2^2m_{N^*}\phi_8^{(A)} \right] , \]
\[ u_{41}^{(A)} = 2m_{N^*} \left[ (\phi_1^{(A)} + \phi_9^{(A)}) - \frac{\phi_2^{(A)}}{x_{22}} \right] , \quad w_{43}^{(A)} = 4m_{N^*}^2m_c^2\phi_{13}^{(A)} , \]
\[ u_{42}^{(A)} = \frac{m_{N^*}}{x_{22}} \left[ 2(m_c^2 + x_2^2m_{N^*}^2 - q^2)\phi_3^{(A)} + x_2m_{N^*}m_c\phi_{10}^{(A)} + 2x_2m_{N^*}\phi_4^{(A)} \right] , \]
\[ u_{51}^{(A)} = 2m_{N^*}\phi_2^{(A)} , \quad w_{53}^{(A)} = 8m_{N^*}m_c \left[ m_c\phi_{14}^{(A)} + x_2m_{N^*}\phi_8^{(A)} \right] , \]
\[ u_{52}^{(A)} = 2m_{N^*} \left[ -x_{22}m_{N^*}\phi_4^{(A)} + m_c(\phi_5^{(A)} + 2\phi_3^{(A)}) + 2m_{N^*}\phi_{14}^{(A)} \right] , \]
\[ u_{61}^{(A)} = 0 , \quad w_{62}^{(A)} = -2m_{N^*}\phi_4^{(A)} , \quad w_{63}^{(A)} = 8m_{N^*}m_c\phi_8^{(A)} , \]
They agree with those in [1] if one replaces $m_N \to m_N^*$ and $m_c \to -m_c$.

References

[1] A. Khodjamirian, C. Klein, T. Mannel and Y. -M. Wang, JHEP 1109, 106 (2011).
[2] I. V. Anikin, V. M. Braun and N. Offen, Phys. Rev. D 92 (2015) 1, 014018 [arXiv:1505.05759 [hep-ph]].
[3] V. M. Braun et al., Phys. Rev. D 89 (2014) 9, 094511 [arXiv:1403.4189 [hep-lat]].
[4] I. G. Aznauryan et al., Int. J. Mod. Phys. E 22, 1330015 (2013). R.W.Gothe et al., Nucleon Resonance Studies with CLAS12, Experiment E12-09-003.
[5] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Sov. J. Nucl. Phys. 44, 1028 (1986); Nucl. Phys. B 312, 509 (1989); V. L. Chernyak and I. R. Zhiltzitsky, Nucl. Phys. B 345, 137 (1990).
[6] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147 (1979) 385. doi:10.1016/0550-3213(79)90022-1
[7] V. M. Braun et al., Phys. Rev. Lett. 103, 072001 (2009).
[8] I. V. Anikin, V. M. Braun and N. Offen, Phys. Rev. D 88 (2013) 114021 doi:10.1103/PhysRevD.88.114021 [arXiv:1310.1375 [hep-ph]].
[9] E. Oset et al., Int. J. Mod. Phys. E 25 (2016) no.01, 1630001 doi:10.1142/S0218301316300010 [arXiv:1601.03972 [hep-ph]].

[10] M. -q. Huang and D. -W. Wang, Phys. Rev. D 69, 094003 (2004).

[11] Y. -M. Wang, Y. -L. Shen and C. -D. Lu, Phys. Rev. D 80, 074012 (2009).

[12] K. Azizi, M. Bayar, Y. Sarac and H. Sundu, Phys. Rev. D 80 (2009) 096007 doi:10.1103/PhysRevD.80.096007 [arXiv:0908.1758 [hep-ph]].

[13] I. V. Anikin and A. N. Manashov, Phys. Rev. D 89 (2014) no.1, 014011 doi:10.1103/PhysRevD.89.014011 [arXiv:1311.3584 [hep-ph]].

[14] I. V. Anikin and A. N. Manashov, Phys. Rev. D 93 (2016) no.3, 034024 doi:10.1103/PhysRevD.93.034024 [arXiv:1512.07141 [hep-ph]].

[15] V. Braun, R. J. Fries, N. Mahnke and E. Stein, Nucl. Phys. B 589 (2000) 381 [Nucl. Phys. B 607 (2001) 433] doi:10.1016/S0550-3213(00)00516-2 [hep-ph/0007279].

[16] V. M. Braun, A. Lenz and M. Wittmann, Phys. Rev. D 73 (2006) 094019 doi:10.1103/PhysRevD.73.094019 [hep-ph/0604050].

[17] V. M. Braun, A. N. Manashov and J. Rohrwild, Nucl. Phys. B 807 (2009) 89 doi:10.1016/j.nuclphysb.2008.08.012 [arXiv:0806.2531 [hep-ph]].

[18] K. G. Chetyrkin, J. H. Kuhn, A. Maier, P. Maierhofer, P. Marquard, M. Steinhauser and C. Sturm, Phys. Rev. D 80 (2009) 074010 doi:10.1103/PhysRevD.80.074010 [arXiv:0907.2110 [hep-ph]].

[19] Z. G. Wang, Eur. Phys. J. C 68 (2010) 479 doi:10.1140/epjc/s10052-010-1365-8 [arXiv:1001.1652 [hep-ph]].

[20] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38 (2014) 090001 doi:10.1088/1674-1137/38/9/090001

[21] E. Bagan, M. Chabab, H. G. Dosch and S. Narison, Phys. Lett. B 301 (1993) 243. doi:10.1016/0370-2693(93)90696-F

[22] Private communication with the authors of [1].

[23] C. Bourrely, I. Caprini and L. Lellouch, Phys. Rev. D 79 (2009) 013008 [Phys. Rev. D 82 (2010) 099902] doi:10.1103/PhysRevD.79.013008 [arXiv:0807.2722 [hep-ph]].

[24] I. G. Aznauyran et al. [CLAS Collaboration], Phys. Rev. C 80, 055203 (2009).

[25] H. Denizli et al. [CLAS Collaboration], Phys. Rev. C 76, 015204 (2007).

[26] M. M. Dalton, G. S. Adams, A. Ahmidouch, T. Angelescu, J. Arrington, R. Asaturyan, O. K. Baker and N. Bennmouna et al., Phys. Rev. C 80, 015205 (2009).

[27] C. S. Armstrong et al. [Jefferson Lab E94014 Collaboration], Phys. Rev. D 60, 052004 (1999).