Implications for the formation of star clusters from extragalactic star formation rates

C. Weidner,1,2⋆ P. Kroupa1,2† and S. S. Larsen3⋆

1Institut für Theoretische Physik und Astrophysik, Universität Kiel, 24098 Kiel, Germany
2Sternwarte der Universität Bonn, 53121 Bonn, Germany
3European Southern Observatory, 85748 Garching, Germany

Accepted 2004 February 19. Received 2004 January 21; in original form 2003 October 20

ABSTRACT
Observations indicate that young massive star clusters in spiral and dwarf galaxies follow a relation between luminosity of the brightest young cluster and the star formation rate (SFR) of the host galaxy, in the sense that higher SFRs lead to the formation of brighter clusters. Assuming that the empirical relation between maximum cluster luminosity and SFR reflects an underlying similar relation between maximum cluster mass (\(M_{\text{ecl, max}}\)) and SFR, we compare the resulting SFR\((M_{\text{ecl, max}})\) relation with different theoretical models. The empirical correlation is found to suggest that individual star clusters form on a free-fall time-scale with their pre-cluster molecular-cloud-core radii typically being a few parsecs independent of mass. The cloud cores contract by factors of 5–10 while building up the embedded cluster. A theoretical SFR\((M_{\text{ecl, max}})\) relation in very good agreement with the empirical correlation is obtained if the CMF of a young population has a Salpeter exponent of \(\beta \approx 2.35\) and if this cluster population forms within a characteristic time-scale of a 1–10 Myr. This short time-scale can be understood if the interstellar medium is pressurized, thus precipitating rapid local fragmentation and collapse on a galactic scale. Such triggered star formation on a galactic scale is observed to occur in interacting galaxies. With a global SFR of 3–5 M\(_{\odot}\) yr\(^{-1}\), the Milky Way appears to lie on the empirical SFR\((M_{\text{ecl, max}})\) relation, given the recent detections of very young clusters with masses near 10\(^5\) M\(_{\odot}\) in the Galactic disc. The observed properties of the stellar population of very massive young clusters suggests that there may exist a fundamental maximum cluster mass, \(10^6 < M_{\text{ecl, max}}/M_{\odot} < 10^7\).

Key words: stars: formation – open clusters and associations: general – galaxies: evolution – galaxies: interactions – galaxies: starbursts – galaxies: star clusters.

1 INTRODUCTION
In a series of publications Larsen (2000, 2001, 2002) and Larsen & Richtler (2000) examined star-cluster populations of 37 spiral and dwarf galaxies and compared the derived properties with overall attributes of the host galaxy. For this work they used archive HST data, own observations and literature data. They showed that cluster luminosity functions (LFs) are very similar for a variety of galaxies. They also found that the V-band luminosity of the brightest cluster, \(M_V\), correlates with the global star formation rate (SFR), but it is unclear if this correlation is physical or statistical in nature. According to the statistical explanation there is a larger probability of sampling more luminous clusters from a universal cluster LF when the SFR is higher (Larsen 2002; Billett, Hunter & Elmegreen 2002).

Larsen (2001) concluded that all types of star clusters form according to a similar formation process which operates with different masses. Smaller clusters dissolve fast through dynamical effects (gas expulsion, stellar-dynamical heating, galactic tidal field) and only massive clusters survive for a significant fraction of Hubble time (Vesperini 1998; Fall & Zhang 2001; Baumgardt & Makino 2003). The notion is that virtually all stars form in clusters (Kroupa & Boily 2002; Lada & Lada 2003), and that a star formation ‘epoch’ produces a population of clusters ranging from about 5 M\(_{\odot}\) (Taurus–Aurigalike pre-main-sequence stellar groups) up to the heaviest star cluster which may have a mass approaching 10\(^6\) M\(_{\odot}\). The time-scale over which such a cluster population emerges within a galaxy defines its momentary SFR.

The aim of this contribution is to investigate if the empirical \(M_v(SFR)\) relation may be understood to be a result of physical processes. In Section 2 the observational data concerning the correlation between the SFR and \(M_v\) of the brightest star cluster are presented, and the empirical and physical models describing this correlation
2 THE OBSERVATIONAL DATA

Based on various observational results Larsen (2002) concludes that star clusters form under the same basic physical processes, and that the so-called superclusters are just the young and massive upper end of the distribution. We first derive from this observational material a correlation between the absolute magnitude of the brightest cluster and the SFR of the host galaxy.

By including in Fig. 1 all the data points presented by Larsen (2001, 2002) the following equation (1) emerges from a two-dimensional linear least-squares fit:

\[ M_V = -1.93(\pm 0.06) \times \log SFR - 12.55(\pm 0.07) \]  

(1)

with a reduced \( \chi^2 \) of about 17. Excluding four points (A, B, C and D, see Larsen 2002) that lie far above this first fitting leads to

\[ M_V = -1.87(\pm 0.06) \times \log SFR - 12.14(\pm 0.07) \]  

(2)

with a reduced \( \chi^2 \) of about 6. Both fits are shown in Fig. 1. For \( M_V \) the formal error is based on photon statistics, and is always very small (especially as these are the brightest clusters in the galaxies); usually 0.01 mag or less. Most of the errors are systematic, caused by uncertain aperture corrections, contamination within the photometric aperture by other objects and are typically 0.1 mag. The SFRs are derived from infrared fluxes published in the IRAS catalogue which lists typical errors of 15 per cent. However, a major source of uncertainty in the derived SFRs lies in the far-infrared luminosity versus SFR calibration, for which Biau & Xu (1996) quote a typical error of \pm 100 per cent/\pm 40 per cent.

Inverting equation (2) reveals

\[ \log SFR = -0.54(\pm 0.02) \times M_V - 6.51(\pm 0.26), \]  

(3)

while a fit to the inverted data (SFR versus \( M_V \)) gives

\[ \log SFR = -0.54(\pm 0.02) \times M_V - 6.51(\pm 0.19), \]  

(4)

with a reduced \( \chi^2 \) of about 6. Both equations (3) and (4) lead to essentially the same result thus nicely demonstrating a level of robustness.

The exclusion of A, B, C and D is motivated by three of these being clusters in very sparse cluster systems in dwarf galaxies (DDO 165, NGC 1705 and NGC 1569) dominated by a single brightest member. Therefore the present SFR does not describe the rate during the birth of these clusters. It has dropped to the values shown as no further (massive) clusters are seen to be forming. This can be understood as a general trend of aging after a star formation epoch. The underlying (observed) SFR has dropped while the clusters retain an approximately constant luminosity for the first few million years (Table 1). The clusters therefore appear on the left in this diagram (Fig. 1) in dwarf galaxies in which star formation proceeds in bursts. The cluster in NGC 7252 was excluded because this galaxy is merger that is several 10^9 yr old in which the SFR was presumably much higher when most of its clusters formed and in which the brightest ‘cluster’ is probably an unresolved or already merged star-cluster complex (Fallhauer & Kroupa 2002a), and thus not a true single cluster.

3 THE MODELS

3.1 Empirical model

From the second fitting to the observations (equation 2) we derive an empirical model for the dependence of the mass of the heaviest cluster on the underlying SFR of the host galaxy. Using the mass-to-light-ratio, \( k_{ML} \), the magnitude (\( M_V \)) can be converted into a mass (\( M_{\text{cl},\text{max}} \)),

\[ M_V = 4.79 - 2.5 \log \frac{M_{\text{cl},\text{max}}}{k_{ML}}. \]  

(5)

where \( M_{\text{cl},\text{max}} \) is the stellar mass in the cluster. The mass-to-light ratios in Table 1 are derived from (Smith & Gallagher 2001, their fig. 7). The age spread of between 6.0 and 8.0 (in logarithmic units) is used to estimate the mass errors for the individual clusters in the Larsen data set plotted in Fig. 3.

Substituting equation (2) in equation (5) gives

\[ M_{\text{cl},\text{max}} = k_{ML} SFR^{0.79(\pm 0.03)} \times 10^{6.77(\pm 0.02)} \]  

(6)

and equation (5) in equation (4)

\[ \text{SFR} = \left( \frac{M_{\text{cl},\text{max}}}{k_{ML}} \right)^{1.34(\pm 0.04)} \times 10^{-9.07(\pm 0.28)}. \]  

(7)

The question of whether the brightest cluster observed is always the heaviest is important because a less massive but younger cluster may appear brighter than an older but more massive cluster as the stellar population fades with age. This does not always hold true for the very youngest phases, where the clusters may briefly brighten owing to the appearance of red supergiant stars (Table 1). We therefore explore this problem with a simple model. Using three different cluster formation rates (CFRs; linearly decreasing, linearly increasing and constant) a number of clusters is formed per
time-step (1 Myr). Taking a power-law mass function (CMF) with an exponent of $\beta = 2$ (equation 11) cluster masses are allocated randomly by a Monte Carlo method. These clusters are then evolved using time-dependent mass-to-light ratios derived from a STARBURST99 simulation (Leitherer et al. 1999) for a Salpeter initial mass function (IMF; $\alpha = 2.35$) from 0.18 to 120 $M_\odot$ for a $M_{\text{cl}} = 10^6 M_\odot$ cluster over 1 Gyr. The lower mass boundary is chosen to have the same mass in stars in the cluster with the Salpeter IMF as in a universal Kroupa IMF (Kroupa 2001). The evolution in $M_V$ of a $M_{\text{cl}} = 10^6 M_\odot$ cluster and a $M_{\text{cl}} = 5 \times 10^5 M_\odot$ cluster is shown in Fig. 2.

For the whole Monte Carlo simulation the heaviest cluster is also the brightest for about 95 per cent of the time and for all three cases of the CFR over the first 500 Myr. We can, therefore, estimate an uncertainty of about 5 per cent for our assertion that the brightest cluster in a population is also the most massive one. This uncertainty can be neglected relative to the larger uncertainties in the cluster ages and therefore in the mass-to-light ratios.

Larsen (2002) points out that a relation between the luminosity of the brightest cluster, $M_V$, and the total SFR arrived at by random sampling using a power-law LF, given an area-normalized SFR $\Sigma_{\text{SFR}}$ and total galaxy size, reproduces the observed correlation. The aim of this contribution is to investigate if the correlation may be the result of physical processes. In essence, the observed correlation is expected because for a massive cluster to form over a similar timescale, a higher SFR is needed than for a low-mass cluster. To probe the physical background of the empirical relation (equations 6 and 7) we calculate a number of different models in Sections 3.2–3.4.

### 3.2 Local data model

In Fig. 3 the data for individual clusters in the Milky Way (MW; Taurus–Auriga, Orion Nebula cluster) and in the Large Magellanic Cloud (LMC: R136, the core of the 30 Doradus region) are compared with the extragalactic cluster-system data. The data points (crosses) were calculated by dividing a mass estimate for each cluster by a formation time of 1 Myr. This is a typical formation timescale as deduced from the ages of the stars in Taurus–Auriga, the Orion Nebula cluster and in R136. The error for the mass scale is constructed using different assumptions from the literature about the number of stars in the cluster and different mean masses (as they vary in dependence of the used IMF) and the maximal possible stellar mass for the particular cluster. We thus have upper and lower bounds on the cluster masses. By dividing the upper mass over a formation time of 0.5 Myr and the lower mass over a formation time of 2 Myr the corresponding errors for the SFR are obtained. The data for these assumptions are taken from Massey & Hunter (1998), Selman et al. (1999), Kroupa (2001), Hartmann (2002), Palla & Stahler (2002) and Briceño et al. (2002). Fig. 3 demonstrates that this simple description leads to reasonable agreement with the observational data. That the local individual cluster data are offset to lower SFRs from the extragalactic data can be understood as being due to the observations measuring the SFR for entire star-cluster populations rather than for individual clusters and/or the formation timescale to vary with cluster mass. In Section 3.3 the star-cluster formation timescale (set here as 1 Myr) is allowed to be the mass-dependent free-fall timescale.

### 3.3 Free-fall model

Here the timescale for the formation of an individual star cluster is the free-fall time, $t_{\text{ff}}$, for a pre-cluster molecular cloud core with radius $R$. This model is motivated by the insight shown by Elmegreen...
appears to be very compact with radii of 0.5–1 pc, and the results from Fig. 4 can be taken to mean that they form in a free-fall period if the pre-cluster cloud cores have radii of about 5 pc at the onset of collapse. The buildup of the stellar population would proceed while the density of the cloud core increases by a factor of about $5^3$ to $10^3$, when the SFR in the embedded cluster probably peaks and declines rapidly thereafter as a result of gas evacuation from accumulated outflows and/or the formation of the massive stars that photo-ionize the cloud core (Matzner & McKee 2000; Tan & McKee 2002).

### 3.4 Total-mass model

The above free-fall model quantifies the theoretical relation for the case that the measurements only capture star formation in the most massive clusters in a galaxy. This can be considered to be a lower bound on the SFR. An upper bound is given by the rate with which all stars are formed, which means the total mass being converted to stars over a given time interval. This total mass is the mass in the star-cluster population and is the subject of this subsection, which begins by assuming that there exists no fundamental maximum star-cluster mass, followed by an analysis in which a physical maximum cluster mass, $M_{\text{max}}$, is incorporated.

#### 3.4.1 Without a physical maximum cluster mass

The aim is to estimate the SFR required to build a complete young star-cluster population in one star formation epoch such that it is populated fully with masses ranging up to $M_{\text{max}}$. Observational surveys suggest the embedded CMF is a power law, i.e.

$$
\xi_{\text{ecl}}(M_{\text{ecl}}) = k_{\text{ecl}} \left( \frac{M_{\text{ecl}}}{M_{\text{ecl,max}}} \right)^{-\beta}
$$

with $1.5 \lesssim \beta \lesssim 2.5$ (Elmegreen & Efremov 1997; Kroupa 2002; Kroupa & Boily 2002; Kroupa & Lada 2003; Kroupa & Weidner 2003, and references therein). The total mass of a population of young stellar clusters is given by

$$
M_{\text{ecl}} = \int_{M_{\text{ecl,min}}}^{M_{\text{ecl,max}}} M_{\text{ecl}} \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}
$$

where $M_{\text{ecl,max}}$ is the mass of the heaviest cluster in the population. The normalization constant, $k_{\text{ecl}}$, is determined by stating that $M_{\text{ecl,max}}$ is the single most massive cluster, i.e.

$$
1 = \int_{M_{\text{ecl,min}}}^{M_{\text{ecl,max}}} \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}} = k_{\text{ecl}} M_{\text{ecl,max}}^{\beta} \int_{M_{\text{ecl,min}}}^{M_{\text{ecl,max}}} M_{\text{ecl}}^{1-\beta} dM_{\text{ecl}}.
$$

With a CMF power-law index of $\beta = 2$ we get the following from equation (13):

$$
k_{\text{ecl}} = \frac{1}{M_{\text{ecl,max}}^{\beta}}.
$$

Inserting this into equation (12) (again with $\beta = 2$) gives

$$
M_{\text{ecl}} = M_{\text{ecl,max}} \ln \frac{M_{\text{ecl,max}}}{M_{\text{ecl,min}}} - \ln M_{\text{ecl,min}}.
$$
Figure 5. The (logarithmic) total mass of a cluster system, \( M_{\text{tot}} \), in dependence on the (logarithmic) maximal cluster mass, \( M_{\text{ecl,max}} \), for different CMF power-law indices \( \beta \) (2.0–2.7, from bottom to top).

Equations (14) and (15) change into

\[
 k_{\text{ecl}} = \frac{\beta - 1}{M_{\text{ecl,max}}} \tag{16}
\]

and

\[
 M_{\text{tot}} = (\beta - 1) M_{\text{ecl,max}}^{\beta - 1} \left( \frac{M_{\text{ecl,max}}^{2 - \beta} - M_{\text{ecl,min}}^{2 - \beta}}{2 - \beta} \right). \tag{17}
\]

The resulting total mass, \( M_{\text{tot}} \), as a function of the maximal cluster mass, \( M_{\text{ecl,max}} \), is shown in Fig. 5 for different values of \( \beta \).

Given a SFR, a fully populated CMF with total mass, \( M_{\text{tot}} \), is constructed over time, \( \delta t \), where

\[
 M_{\text{tot}} = \text{SFR} \delta t. \tag{18}
\]

Thus, dividing \( M_{\text{tot}} \) by different ad hoc formation times, \( \delta t \), and using different maximal masses, \( M_{\text{ecl,max}} \), results in a series of theoretical \( M_{\text{ecl,max}}(\text{SFR}) \) relations which are shown in Fig. 6. It thus appears that star formation epochs with duration \( \delta t \approx 10 \) Myr suffice for populating complete cluster systems.

The argumentation can now be inverted to better quantify the time-scale required to build an entire young cluster population in a star formation epoch with a given SFR. For this purpose we employ the empirical \( \text{SFR}(M_{\text{ecl,max}}) \) relation. For \( \beta = 2 \)

\[
 \delta t = \frac{M_{\text{tot}}}{\text{SFR}} = \frac{M_{\text{ecl,max}}}{\text{SFR}} \ln \left( \frac{M_{\text{ecl,max}}}{M_{\text{ecl,min}}} \right) \tag{19}
\]

and equation (7) can be rewritten as follows:

\[
 \text{SFR} = \left( \frac{M_{\text{ecl,max}}}{k_{\text{ML}}} \right)^s \times 10^{-9.07}, \tag{20}
\]

with \( s = 1.34 \) being the exponent of this empirical \( \text{SFR}(M_{\text{ecl,max}}) \) relation. By combining equations (19) and (20) we finally arrive at

\[
 \delta t = M_{\text{ecl,max}}^{s - 1} \ln \left( \frac{M_{\text{ecl,max}}}{M_{\text{ecl,min}}} \right) k_{\text{ML}}^s \times 10^{9.07} \text{[yr]}. \tag{21}
\]

If \( \beta \neq 2 \) we obtain

\[
 \delta t = (\beta - 1) M_{\text{ecl,max}}^{\beta - 1 - s} \left( \frac{M_{\text{ecl,max}}^{2 - \beta} - M_{\text{ecl,min}}^{2 - \beta}}{2 - \beta} \right) \times k_{\text{ML}}^s \times 10^{9.07} \text{[yr]}, \tag{22}
\]

The cluster-system formation time-scale, or the duration of the star formation ‘epoch’, \( \delta t \), is plotted in Fig. 7 for different \( M_{\text{ecl,max}} \) values – and therefore different \( M_{\text{tot}} \) values – and different CMF slopes (\( \beta \)). For \( \beta \lesssim 2.4 \) a decreasing value of \( \delta t \) for almost all masses is found which indicates that the formation of the whole cluster system can be very rapid (\( \lesssim 10 \) Myr).
We have thus found that the empirical SFR($M_{\text{ecl,max}}$) relation implies that more massive cluster populations need a shorter time to assemble than do less massive populations, unless the embedded CMF is a power law with an index of $\beta \approx 2.35$, strikingly similar to the Salpeter index for stars ($\alpha = 2.35$). In this case the formation time is $\approx 10$ Myr independently of the maximum cluster mass in the population.

Young populations of star clusters extend to superstar clusters mostly in galaxies that are being perturbed or are colliding. The physics responsible for this are connected to the higher pressures in the interstellar medium resulting from squeezed or colliding galactic atmospheres (Elmegreen & Efremov 1997; Bekki & Couch 2003). When this occurs, massive molecular clouds rapidly build up and collapse locally but are distributed throughout the galaxy. If two disc galaxies collide face-on, star formation occurs in a synchronized fashion throughout the discs, while edge-on encounters would lead to the star formation activity propagating through the discs with a velocity of a few hundreds of parsecs per Megayear (the relative encounter velocity) which amounts to synchronization of the star formation activity throughout 10 kpc radii discs to within a few Megayears. Recently Engargiola et al. (2003) found that for M33 the typical lifetimes of giant molecular clouds (GMCs) with masses ranging up to $7 \times 10^4 M_{\odot}$ are $10-20 \times 10^4$ yr, indicating a similar formation time for star clusters born from these clouds. Hartmann, Ballesteros-Paredes & Bergin (2001) deduce from solar-neighbourhood clouds that their lifetimes are also comparable to the ages of the pre-main-sequence stars found within them, again suggesting that molecular clouds form rapidly and are immediately dispersed through the instant onset of star formation.

It is notable that $\beta \approx 2.4$ gives a theoretical $M_{\text{ecl,max}}$ (SFR) relation with virtually the same slope as the empirical relation (Fig. 6). This would imply that the embedded CMF is essentially a Salpeter power law. In addition, in our analysis we neglected to take into account that once an embedded cluster expels its residual gas it expands and loses typically half to two-thirds of its stars (Kroupa 2002; Kroupa & Boily 2002). The observed clusters with ages of $>10$ Myr thus have masses $(0.3-0.5)\times M_{\text{ecl}}$. Taking this into account would shift the theoretical relations downwards by at most 0.5 in log mass which would lead to an increase in $\delta t$ by a factor of a few.

3.4.2 With a physical maximal cluster mass

The most massive ‘clusters’ known, e.g. o Cen (a few $10^6 M_{\odot}$; Gnedin et al. 2002) or G1 ($\approx 15 \times 10^4 M_{\odot}$; Meylan et al. 2001) consist of complex stellar populations with different metallicities and ages (Hilker & Richtler 2000). They are therefore not single-metallicity, single-age populations that formed from a compact population of clusters and with sufficient mass to retain their interstellar medium for substantial times and/or capture field-stellar populations and/or possibly re-accrete gas at a later time to form additional stars (Kroupa 1998; Fellhauer & Kroupa 2002b). A fundamental or physical maximal star-cluster mass may therefore be postulated to exist on empirical grounds in the range $10^6 \lesssim M_{\text{ecl,max}}/M_{\odot} \lesssim 10^7$. In the following we explore the implications of such a fundamental maximum cluster mass on the analysis presented in Section 3.4.1.

For the following $M_{\text{ecl,max}} = 10^7 M_{\odot}$ is adopted. Equation (12) remains unchanged while equation (13) changes to

$$1 = k_{\text{ecl}} M_{\text{ecl, max}}^{\beta} \int_{M_{\text{ecl, max}}}^{M_{\text{ecl}}} M_{\text{ecl}}^{-\beta} dM_{\text{ecl}}. \quad (23)$$

Figure 8. As Fig. 6 but for the case that there exists a fundamental maximum cluster mass $M_{\text{ecl,max}} = 10^7 M_{\odot}$.

This can be evaluated for $\beta \neq 1$

$$k_{\text{ecl}} = \frac{M_{\text{ecl, max}}^{-\beta} \left(1 - \frac{1}{\beta}\right)}{M_{\text{ecl, max}}^{-\beta} - M_{\text{ecl}}^{-\beta}}. \quad (24)$$

If $\beta = 2$,

$$M_{\text{tot}} = -\frac{\ln M_{\text{ecl, max}} - \ln M_{\text{ecl, min}}}{\ln M_{\text{ecl, max}} - \ln M_{\text{ecl}}}, \quad (25)$$

or for $\beta \neq 2$

$$M_{\text{tot}} = \frac{1 - \frac{1}{\beta} M_{\text{ecl, max}}^{2-\beta} - M_{\text{ecl, min}}^{2-\beta}}{2 - \frac{1}{\beta} M_{\text{ecl, max}}^{2-\beta} - M_{\text{ecl, min}}^{2-\beta}}. \quad (26)$$

For a fixed $M_{\text{ecl, max}}$ and a changing $M_{\text{tot}}$ the upper mass $M_{\text{ecl,max}}$ for each cluster system can now be evaluated. The resulting SFR($M_{\text{ecl,max}}$) models are plotted in Fig. 8 for different formation times for the entire cluster population. The conclusions of Section 3.4.1 do not change.

Given the empirical SFR($M_{\text{ecl,max}}$) relation, the time-scale, $\delta t$, needed to build up a fully populated young star-cluster population can be determined as in Section 3.4.1. The result is shown in Fig. 9. Note that in both the limited ($M_{\text{ecl,max}} = 10^7 M_{\odot}$, Fig. 9) and the unlimited ($M_{\text{ecl,max}} = \infty$, Fig. 7) cases it takes an arbitrarily long time to sample the CMF arbitrarily close to $M_{\text{ecl,max}}$.

4 DISCUSSION AND CONCLUSIONS

Observations of young star-cluster systems in disc galaxies show that there exists a correlation between the total SFR and the luminosity of the brightest star cluster in the young cluster population. This can be transformed into a SFR–heaviest-cluster mass relation [SFR($M_{\text{ecl,max}}$), equation 7].

Very young star clusters in the MW and the LMC that are deduced to have formed within a few Megayears follow a similar SFR($M_{\text{ecl,max}}$) relation, although this ‘local’ relation is steeper if it is assumed that the formation time-scale of individual clusters is the same in all cases ($\approx 1$ Myr, Fig. 3). Taking instead the formation time-scale to be the free-fall time of the pre-cluster molecular cloud core the correct slope is obtained if the pre-cluster cloud core radius is independent of the cluster mass by a few parsecs (Fig. 4).
implies that the cores of the cluster-forming molecular cloud may contract by a factor of 5–10 as the clusters form. That the pre-cluster radii appear to vary not much with the cluster mass implies that the pre-cluster cores have increasing density with increasing mass. Indeed, Larsen (2003) finds young extragalactic clusters to have only a mild increase in effective radius with mass, and embedded clusters from the local MW also suggest the cluster radii to be approximately independent of cluster mass (Kroupa 2002; Kroupa & Boily 2002).

A model according to which the total mass of the young cluster population, $M_{\text{cl, max}}$, is assumed to be assembled in a star formation ‘epoch’ with an a priori unknown duration, $\delta t$, gives the corresponding SFR = $M_{\text{tot}}/\delta t$ and leads to good agreement with the empirical SFR($M_{\text{cl, max}}$) relation for $1 \lesssim \delta t \text{ Myr}^{-1} \lesssim 10$. A particularly good match with the empirical relation results for $\delta t \approx \text{a few} \times 10 \text{ Myr}$ and for a power-law CMF with $\beta \approx 2.35$. It should be noted that the slope of this CMF for stellar clusters is virtually the same as for the Salpeter IMF ($\alpha = 2.35$) which applies for the early-type stars in the LMC (Kroupa 2002).

The same holds true if a fundamental maximum star-cluster mass near $M_{\text{cl, max}} = 10^5 M_\odot$ is introduced. The existence of such a fundamental maximum cluster mass is supported by ‘clusters’ with $M \gtrsim 5 \times 10^6 M_\odot$ having complex stellar populations more reminiscent of dwarf galaxies that cannot be the result of a truly single star formation event.

The short time-span of $\delta t \approx \text{a few} \times 10 \text{ Myr}$ involved for Completely populating a CMF up to the maximum cluster mass of the population, $M_{\text{cl, max}} \lesssim M_{\text{cl, max}}$, can be understood as being owing to the high ambient pressures in the interstellar medium needed to raise the global SFR high enough for populous star clusters to be able to emerge. This short time-scale, which we refer to as a star formation ‘epoch’, does not preclude the star formation activity in a galaxy continuing for many ‘epochs’, whereby each epoch may well be characterized by different total young star-cluster masses, $M_{\text{tot}}$. According to this notion, dwarf galaxies may experience unfinished ‘epochs’, in the sense that during the onset of intense star formation activity, which could be triggered through tidal perturbation, for example, the ensuing feedback (which may include galactic winds) may momentarily squelch further star formation within the dwarf such that the cluster system may not have sufficient time to completely populate the CMF. Squelching would typically occur once the most massive cluster has formed. Dwarf galaxies would therefore deviate notably from the $M_{\text{cl, max}}$(SFR) relation (Section 2).

The conclusion is therefore that the observed SFR($M_{\text{cl, max}}$) data can be understood as being natural outcomes of star formation in clusters and that the SFR at a given epoch dictates the range of star-cluster masses formed given a CMF that appears to be a Salpeter power law. The associated formation time-scales are short being consistent with the conjecture by Elmegreen (2000) that star formation is a very quick process on all scales. Within about $10^7$ yr a complete cluster system is built (Fig. 6, Section 3.3), while individual clusters form on a time-scale of $10^6$ yr and stars in about $10^5$ yr. Correspondingly, molecular-cloud lifetimes are short [\approx \text{a few} \times 10 \text{ Myr}] supporting the assertion by Hartmann et al. (2001).

Applying the empirical SFR($M_{\text{cl, max}}$) relation to the MW which has SFR $\approx 3–5 M_\odot$ yr$^{-1}$ (Prantzos & Aubert 1995) a maximum cluster mass of about $10^5 M_\odot$ is expected from equation (6). It is interesting that only recently have Alves & Homeier (2003) revealed a very massive cluster in our MW with about 100 O stars (similar to R136 in the LMC). Knödlseder (2000) notes that the Cygnus OB2 association contains 2000 $\pm$ 400 OB stars and about 120 O stars with a total mass of $(4 \times 10) \times 10^4 M_\odot$, and that this ‘association’ may be a very young globular-cluster-type object with a core radius of approximately 14 pc within the MW disc at a distance of about 1.6 kpc from the Sun (but see Bica, Bonatto & Dutra 2003). This object may be expanded after violent gas expulsion (Boily & Kroupa 2003a,b). The MW therefore does not appear to be unusual in its star-cluster production behaviour.

**ACKNOWLEDGMENTS**

This work has been funded by DFG grants KR1635/3 and KR1635/4.

**REFERENCES**

Alves J., Homeier N., 2003, ApJL, 589, L45
Baumgardt H., Makino J., 2003, MNRAS, 340, 227
Bekki K., Couch W. J., 2003, ApJL, 596, L13

Bica E., Bonatto Ch., Dutra C. M., 2003, A&A, 405, 991
Billett O. H., Hunter D. A., Elmegreen B. G., 2002, AJ, 123, 1454
Boily C. M., Kroupa P., 2003a, MNRAS, 338, 665
Boily C. M., Kroupa P., 2003b, MNRAS, 338, 673

Brandl B., Sams B. J., Bertoldi F., Eckart A., Genzel R. et al., 1996, ApJ, 466, 254

Briceno C., Luhman K. L., Hartmann L., Stauffer J. R., Kirkpatrick J. D., 2002, ApJ, 580, 317

Buat V., Xu C., 1996, A&A, 306, 61

Elmegreen B. G., 2000, ApJ, 530, 277
Elmegreen B. G., Efremov Y. N., 1997, ApJ, 480, 235

Engargiola G., Plambeck R. L., Rosolowsky E., Blitz L., 2003, ApJS, 149, 343

Fall S. M., Zhang Q., 2001, ApJ, 561, 751
Fellhauer M., Kroupa P., 2002a, MNRAS, 320, 642

Fellhauer M., Kroupa P., 2002b, AJ, 124, 2006
Gnedin O. Y., Zhao H., Pringle J. E., Fall S. M., Livio M., Meylan G., 2002, ApJL, 568, L23

---

**Figure 9.** As Fig. 7 but assuming the fundamental cluster mass limit is $M_{\text{cl, max}} = 10^5 M_\odot$. 

---

© 2004 RAS, MNRAS 350, 1503–1510

Downloaded from https://academic.oup.com/mnras/article-abstract/350/4/1503/986580 by guest on 29 July 2018
Gomez M., Hartmann L., Kenyon S. J., Hewett R., 1993, AJ, 105, 1927
Hartmann L., 2002, ApJ, 578, 914
Hartmann L., Ballesteros-Paredes J., Bergin E. A., 2001, ApJ, 562, 852
Hillenbrand L. A., Hartmann L. W., 1998, ApJ, 492, 540
Hilker M., Richtler T., 2000, A&A, 362, 895
Hunter D. A., Elmegreen B. G., Dupuy T. J., Mortonson M., 2003, AJ, 126, 1836
Knödlseder J., 2000, A&A, 360, 539
Kroupa P., 1998, MNRAS, 300, 200
Kroupa P., 2001, MNRAS, 322, 231
Kroupa P., 2002, MNRAS, 330, 707
Kroupa P., Boily C. M., 2002, MNRAS, 336, 1188
Kroupa P., Weidner C., 2003, ApJ, 598, 1076
Kroupa P., Aarseth S., Hurley J., 2001, MNRAS, 321, 699
Lada C. J., Lada E. A., 2003, ARAA, 41, 57
Larsen S. S., 2000, A&A, 360, 539
Kroupa P., 2001, MNRAS, 322, 231
Kroupa P., Boily C. M., 2002, MNRAS, 336, 1188
Kroupa P., Weidner C., 2003, ApJ, 598, 1076
Kroupa P., Aarseth S., Hurley J., 2001, MNRAS, 321, 699
Lada C. J., Lada E. A., 2003, ARAA, 41, 57
Larsen S. S., 2001, IAU Symp. Series, 207
Larsen S. S., 2002, AJ, 124, 1393
Larsen S. S., 2003, IAU Joint Discussion, 6, 21
Larsen S. S., Richtler T., 2000, A&A, 354, 836
Leitherer C., Schaerer D., Goldader J. D., González Delgado R. M., Robert
C., Foo Kune D., De Mello D. F., Devost D., 1999, ApJ, 123, 3
Matz-Apellániz J., 2001, ApJ, 563, 151
Massey P., Hunter D. A., 1998, ApJ, 493, 180
Matzner C. D., McKee C. F., 2000, ApJ, 545, 364
Meylan G., Sarajedini A., Jablonka P., Djorgovski S. G., Bridges T., Rich R.
M., 2001, AJ, 122, 830
Palla F., Stahler S. W., 2002, ApJ, 581, 1194
Prantzos N., Aubert O., 1995, A&A, 302, 69
Selman F., Melnick J., Bosch G., Terlevich R., 1999, A&A, 347, 532
Smith L. J., Gallagher J. S. III, 2001, MNRAS, 326, 1027
Tan J. C., McKee C. F., 2002, in Crowther P. A., ed., ASP Conf. Ser. Vol.
267, Hot Star Workshop III: The Earliest Stages of Massive Star Birth.
Astron. Soc. Pac., San Francisco, p. 267
Vesperini E., 1998, MNRAS, 299, 1019
Wuchterl G., Tscharnuter W. M., 2003, A&A, 398, 1081

This paper has been typeset from a TeX/LaTeX file prepared by the author.