Moduli stabilization with F-term uplifting in heterotic string theory

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Abstract

We discuss the role of F-term uplifting in stabilizing moduli within the framework of heterotic string theory. It turns out that the uplifting sector plays an important role in fixing the volume modulus at one of the self-dual points of a modular invariant potential. For the volume modulus stabilized at a self-dual point, the F-term uplifting leads to the dilation stabilization which can naturally yield the mirage mediation pattern of soft supersymmetry breaking terms. Generalizing to the case with anomalous $U(1)$ gauge symmetry, we also find that the $U(1)$ sector generically gives a contribution to sfermion masses comparable to the dilaton mediated one while maintaining the mirage mediation pattern.

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I. INTRODUCTION

In supergravity theories derived from string compactification, effective couplings of gauge and matter superfields are determined by the vacuum values of string moduli including the dilaton. To fix moduli vacuum values, one can incorporate various non-perturbative effects and background fluxes which generate a non-trivial potential for the moduli. However, it is usually argued that such sources of the moduli potential, though responsible for stabilizing the moduli, are alone incapable of rendering the vacuum energy adjustable to a small positive value. Typically there arises a moduli vacuum solution with negative vacuum energy. This has led to consider the construction of a realistic de-Sitter vacuum via an uplifting mechanism \[1\] that is implemented by adding extra supersymmetry breaking effects. Since the uplifting effects generally shift the moduli vacuum configuration, how the moduli break supersymmetry would depend crucially on the uplifting procedure \[2\]. The uplifting mechanism has been studied mostly within the framework of type IIB string theory where the use of fluxes is rather flexible \[1, 2, 3\]. Such moduli stabilization has been noticed to provide a natural scheme realizing the mirage mediation of supersymmetry breaking \[4\], in which anomaly mediation is equally important as moduli mediation.

Recently Löwen and Nilles have discussed the possibility of F-term uplifting scenario within the framework of heterotic string theory \[5\]. They considered a moduli potential which is non-perturbatively generated by gaugino condensation in a modular invariant effective supergravity. Then, introducing F-term uplifting sector, they found that the heterotic dilaton can be successfully stabilized with a single gaugino condensation in a controllable approximation while achieving the desired value of the vacuum energy. This is because the uplifting sector plays an important role in fixing the dilaton as well as in adjusting the vacuum energy. In their discussion, the volume modulus in heterotic compactifications is assumed to be stabilized at one of the self-dual points of a modular invariant potential, giving no contributions to the supersymmetry breakdown.

Since the uplifting matter field generically has non-trivial modular transformation, uplifting effects might shift the volume modulus from the self-dual point. Furthermore, it has been noticed that the self-dual points typically correspond to local maxima or saddle points in the absence of uplifting sector. Motivated by these issues, we wish to examine in more detail the role of F-term uplifting sector and the stabilization of moduli in heterotic com-
pactifications while explicitly including the volume modulus. It turns out that the inclusion of uplifting sector is indeed crucial for stabilizing the volume modulus at one of the self-dual points. To see this, we derive the condition for a self-dual point to be a local minimum of modular invariant potential, from which the modular weight of uplifting field is found to be somewhat constrained.

Fixed at a self-dual point, the volume modulus does not participate in supersymmetry breaking. The F-term uplifting scheme then leads to the dilation stabilization that can naturally realize the mirage mediation scenario where anomaly and dilaton mediations give comparable contributions to soft supersymmetry breaking terms. Meanwhile, heterotic compactifications often involve anomalous $U(1)$ gauge symmetries, under which the dilaton transforms non-linearly to implement the Green-Schwarz mechanism of anomaly cancellation [6]. Generalizing the F-term uplifting scheme to a situation involving anomalous $U(1)$ gauge symmetry, we will also examine its implications for supersymmetry breaking. It is found that the $U(1)$ sector gives a comparable contribution to sfermion soft terms as the dilaton mediation [7] while maintaining the mirage pattern.

This paper is organized as following. In section 2, we shall discuss how the potential for heterotic moduli is affected by F-term uplifting sector, and then examine the condition for the volume modulus to be stabilized at one of the self-dual points. Section 3 contains a brief analysis of the pattern of soft terms. Extending to a more general case with anomalous $U(1)$ gauge symmetry, the implications of $U(1)$ sector on soft terms will be examined in section 4. Section 5 is the conclusion.

II. MODULI POTENTIAL AND F-TERM UPLIFTING

To describe the low energy dynamics of heterotic string theory, we consider the effective four-dimensional supergravity under the assumption that the moduli dependence of Kähler potential is well approximated by the string tree-level expression:

$$K = -3 \ln(T + \bar{T}) - \ln(S + \bar{S}),$$

(1)

where $T$ is the volume modulus and $S$ is the dilaton. The supergravity action is specified by the Kähler invariant function $G = K + \ln |W|^2$, where $W$ is the holomorphic superpotential. For the above tree-level Kähler potential, the effective superpotential is required to have
explicit dependence on both $T$ and $S$ in order to generate a non-trivial potential for their stabilization. Since the tree-level gauge kinetic function is given by $f_a = S$ for all gauge groups $G_a$, gaugino condensation \[8\] can naturally generate non-perturbative superpotential for the dilaton. As for the volume modulus, a dependence on $T$ is introduced in the superpotential if one requires the modular invariance of effective supergravity action that arises in orbifold compactifications \[9\]. Considering such sources of moduli dependence, it follows that the superpotential generically has a moduli dependence of the form

$$W = \frac{1}{\eta(T)^6} \omega(S),$$

(2)

which can be inferred from the modular invariance of $G$. Here $\eta(T) = e^{-\pi T/12} \prod_n (1 - e^{-2\pi n T})$ is the Dedekind eta function. With this superpotential involving $T$ and $S$, it is now possible to provide a potential for the moduli.

Since there is no mixing between the dilaton and the volume modulus in the tree-level Kähler potential, the scalar potential reads

$$V = e^G \left( G_{1\bar{1}}^1 G_I G_J - 3 \right) = e^G \left( \frac{1}{G_{TT}} |G_T|^2 + \frac{1}{G_{SS}} |G_S|^2 - 3 \right),$$

(3)

where $G_I \equiv \partial_I G$, and $G_{1\bar{1}}^{-1} = K_{1\bar{1}}^{-1}$ denotes the inverse Kähler metric. This potential should have a minimum at an acceptable value for the dilaton since the gauge coupling constant is given by $1/g^2 = \text{Re}(S)$. The vacuum value of $S$ is determined by solving the extremum condition $\partial_S V = 0$:  

$$\frac{G_{SS}}{G_{SS}^2} |G_S|^2 - (1 + e^{-G} V) G_S - \frac{G_{SS}}{G_{SS}} G_S = \frac{G_{TS}}{G_{TT}} G_T,$$  

(4)

in which $G_{TS} = 0$ for the modular invariant action derived from (1) and (2). Hence there is a possible solution at $G_S = 0$ regardless of the value of $G_T$ which is proportional to the F-term of $T$ \[10\]. Multiple gaugino condensations\(^*\) indeed lead to the potential which can develop a minimum along the $S$-direction at $G_S = 0$ through the racetrack mechanism \[11\]. On the other hand, it is quite difficult to achieve a solution with non-vanishing $G_S$ for $\text{Re}(S) = O(1)$ because the typical size of $G_{SS}$ is $|G_{SS}| \gg 1$ if the dilaton superpotential arises non-perturbatively from gaugino condensation.

Meanwhile, the modular invariance of supergravity action ensures that the self-dual points $T = 1, e^{i\pi/6}$ always correspond to extrema of the potential where the F-term of volume modulus

\(^*\text{For a single gaugino condensation } \omega(S) = Ae^{-aS}, \text{ the potential runs away as } S \text{ goes to infinity.}\)
modulus is vanishing. To stabilize the volume modulus at one of the self-dual points, the condition for a local minimum requires
\[ \partial_T \partial_T V > |\partial_T^2 V|, \]
where we have used \( \partial_T \partial_S V = \partial_T \partial_S V = 0 \) at the self-dual points. However, for the moduli potential derived from (1) and (2), this requirement cannot be fulfilled as long as the dilaton is stabilized at \( G_S = 0 \). Actually the potential develops a minimum in \( T \) close to the point \( T = 1.2 \) [12], which corresponds to a deep anti de-Sitter with non-vanishing F-term for \( T \).

Although the moduli \( T \) and \( S \) can both be stabilized reliably, there still remains a problem since it is difficult to construct a realistic de-Sitter vacuum consistent with the cosmological observations. Therefore one has to assume the existence of some contributions compensating the negative vacuum energy.

To get a desired value of the vacuum energy, Löwen and Nilles have considered F-term uplifting mechanism under the assumption that the volume modulus is stabilized at one of the self-dual points with vanishing F-term [5]. They found that the uplifting sector plays an important role not only in adjusting the vacuum energy but also in achieving the stabilization of dilaton. In this work, we examine in detail the role of uplifting sector while explicitly including the volume modulus. Including the uplifting matter field \( Z \), the effective Kähler potential is now written as
\[ K = -3 \ln(T + \bar{T}) - \ln(S + \bar{S}) + \frac{|Z|^2}{(T + \bar{T})^{n_Z}}, \]
where \( n_Z \) denotes the modular weight of \( Z \), which is generically a rational number of order unity. For the supergravity action invariant under modular transformations, the modular invariance of \( G \) leads to the relations
\[ \partial_T V \propto n_Z Z \partial_Z V, \]
\[ F^T \propto G_{ZZ} G_T - G_{TZ} G_Z = 0, \]
at the self-dual points \( T = 1, e^{i \pi/6} \). It is therefore obvious that extrema of the potential still occur along the \( T \)-direction at the self-dual points where its F-term does vanish. Since the uplifting sector gives an additional contribution to the potential for the volume modulus, it is necessary to reexamine the possibility that the self-dual points can correspond to a local minimum of the potential.
If the volume modulus is stabilized at one of self-dual points, it has a vanishing F-term
and thus the scalar potential is written as
\[ V = e^G \left( \frac{1}{G_{SS}} |G_S|^2 + \frac{1}{G_{ZZ}} |G_Z|^2 - 3 \right), \tag{8} \]
and the stationary condition for the dilaton is given by
\[ \frac{G_{SSS}}{G_{SS}^2} |G_S|^2 - (1 + e^{-G} V) G_S - \frac{G_{SS}}{G_{SS}} G_{\bar{S}} G_{\bar{S}} = \frac{G_{SZ}}{G_{ZZ}} G_{\bar{Z}}, \tag{9} \]
from which the dilaton should be fixed in a realistic region with \( \text{Re}(S) = \mathcal{O}(1) \). As discussed already, if \( G_Z = 0 \), a plausible solution would be \( G_S = 0 \) since typically \( |G_{SS}| \gg 1 \) for the dilaton superpotential induced by gaugino condensation. Here it should also be noted that the potential can avoid a run-away behavior of \( S \) with just one gaugino condensation provided that the uplifting sector leaves a constant superpotential. This indicates that the uplifting sector can play a role in stabilizing \( S \) as pointed out in [5]. Now turning on \( G_Z \) which is proportional to \( F^Z \), one can find that the vanishing vacuum energy requires \( G_S = \mathcal{O}(1) \) while \( G_S \) is suppressed compared to \( G_Z \) unless the mixing \( G_{SZ} \) is unnaturally large. Hence \( F^Z \) is the dominant source of supersymmetry breaking and provides the uplifting effects which account for the slight shift of the dilaton vacuum configuration from \( G_S = 0 \). In fact, the situation is qualitatively same as in usual uplifting scenarios that have been considered within the framework of type IIB string theory [1, 2, 3].

Let us now examine how the potential for \( T \) is affected by the presence of the F-term uplifting sector and then the possibility to develop a minimum in \( T \) at one of its self-dual points. For this, it is convenient to change the field basis as \( Z' = \eta(T)^{2n_Z} Z \) so that the uplifting matter field does not transform under the modular transformations. The effective superpotential is then generically written as
\[ W = \frac{1}{\eta(T)^6} \omega(S, Z'), \tag{10} \]
as can be inferred from the modular invariance of \( G \). In order for the self-dual point to correspond to a minimum of the potential, it is simply required to satisfy \( \partial_T \partial_{\bar{T}} V > |\partial_T V| \)
in the new basis, because \( \partial_T \partial_I V = \partial_T \partial_{\bar{I}} V = 0 \) for \( I = S, Z' \). For \( V = 0 \), the mass matrix components evaluated at self-dual points read
\[ \partial_T \partial_{\bar{T}} V \simeq \frac{3e^G}{(T + T^*)^2} \left( 1 - n_Z + |\lambda(T)|^2 \right), \]
\[ \partial_T^2 V \simeq \frac{3e^G}{(T + T^*)^2} (2 - n_Z) \lambda(T), \tag{11} \]
for $|Z| \ll 1$ and $\text{Re}(S) = \mathcal{O}(1)$, where $\lambda(T) = 3/2 - 2(T + T^*)^2 \partial_T^2 \eta / \eta$, and small corrections suppressed by $ZG_Z$ and $SG_S$ have been neglected since $G_Z = \mathcal{O}(1)$ and $|G_S| \ll 1$. Finally, we find that the condition $\partial_T \partial_T V > |\partial_T^2 V|$ requires the modular weight of uplifting field to satisfy

$$T = 1 : \quad -0.6 \lesssim n_Z \lesssim 2.6,$$

$$T = e^{i\pi/6} : \quad n_Z \lesssim 1,$$

(12)

in order for the corresponding self-dual point to be a local minimum of the potential at $V = 0$. It should be noted that the above constraint on $n_Z$ does not depend on the detailed form of the superpotential of $Z$. The matter modular weight is known to have a rational number of $\mathcal{O}(1)$ in orbifold models [13]. For example, if the uplifting field arises from untwisted sector, the corresponding modular weight is given by $n_Z = 1$ for which the condition for a minimum can be satisfied at both $T = 1$ and $T = e^{i\pi/6}$.

Therefore the uplifting sector plays an important role also in stabilizing the volume modulus. Provided that the uplifting field has a modular weight satisfying the constraint (12), it is indeed possible to get a minimum of the potential along the $T$-direction at its self-dual points. Fixed at a self-dual point, the volume modulus has a vanishing F-term and thus gives contributions neither to the vacuum energy nor soft supersymmetry breaking terms. This is phenomenologically desirable as $T$-mediated soft terms are generically flavor-dependent. It is also found that the volume modulus has a mass comparable to the gravitino mass, whereas the dilaton is much heavier than the gravitino since its non-perturbative superpotential yields a large supersymmetric mass.

III. SUPERSYMMETRY BREAKING

In this section, we briefly discuss the pattern of soft supersymmetry breaking terms arising in the moduli stabilization with F-term uplifting. The matter part of effective supergravity action is given by

$$\mathcal{L}_{\text{matter}} = \int d^4 \theta C \bar{C} Y_i |Q^i|^2 + \left( \int d^2 \phi C^3 \frac{1}{6} \lambda_{ijk} Q^i Q^j Q^k + \text{h.c.} \right),$$

(13)

where $Q^i$ denote matter superfields, and $C$ is the chiral compensator for the super-Weyl invariance. The perturbative shift symmetry associated with $\text{Im}(S)$ ensures that holomorphic Yukawa couplings $\lambda_{ijk}$ are independent of $S$, while the modular invariance constrains
their dependence on $T$. Furthermore, we can simply assume that $\lambda_{ijk}$ are $Z$-independent by imposing an appropriate symmetry. On the other hand, the effective Kähler potential unavoidably involves non-renormalizable cross-couplings between $Q^i$ and $Z$, to which sfermion masses are particularly sensitive, unless they are sequestered from each other. Taking into account such cross-couplings, the matter wave function is written as

$$Y_i = (T + \bar{T})^{1-n_i}(S + \bar{S})^{1/3} \left(1 + \frac{\epsilon_i}{(T + \bar{T})^{n_z}} |Z|^2\right),$$

(14)

for the tree-level Kähler potential, where $n_i$ is the modular weight and the cross-coupling $\epsilon_i$ is a moduli-independent constant. The structure of sfermion soft terms is essentially determined by how $Y_i$ depends on the supersymmetry breaking fields.

In the case that $T$ is stabilized at a self-dual point, the volume modulus does not play a role of supersymmetry breaking messenger since $F^T = 0$. Combined with the condition for vanishing vacuum energy, the stationary condition \[\text{then leads to} \ G_Z = \mathcal{O}(1) \text{ and} \ |G_S| \ll 1 \] since the dilation potential is non-perturbatively generated by gaugino condensation. Hence the F-terms of $S$ and $Z$ are quite different in size from each other

$$\frac{F^Z}{m_{3/2}} = \mathcal{O}(1), \quad \frac{1}{m_{3/2}} \left|\frac{F^S}{S}\right| \ll 1,$$

(15)

in a region with $\text{Re}(S) = \mathcal{O}(1)$, where $m_{3/2} = e^{G/2}$ denotes the gravitino mass. Particularly, this indicates that the dilaton mediation can be comparable in size to the anomaly mediation \[\text{which is always present and arises at loop level by the F-term of chiral compensator,}\]

$F^C/C \simeq m_{3/2}$. In such case, one can find that gaugino masses take a mirage pattern since the gauge coupling functions are determined by the dilaton with the universal form, $f_a = S$ at the gauge coupling unification scale $M_{\text{GUT}}^\dagger$.

The characteristic feature of mirage mediation is the appearance of a mirage messenger scale $M_{\text{mir}}^\dagger$:

$$M_{\text{mir}} = M_{\text{GUT}} \left(\frac{m_{3/2}}{M_{\text{Pl}}}\right)^{\alpha/2}, \quad \alpha = \frac{1}{\ln(M_{\text{Pl}}/m_{3/2})} \frac{F^C/C_0}{M_0},$$

(16)

where $M_0 = F^S/(S + S^*)$ denotes the universal dilaton-mediated gaugino mass at $M_{\text{GUT}}$, and $\alpha$ parameterizes the relative size of anomaly mediation. For the simple case

\[\dagger\text{Due to the cross-couplings in (14), gaugino masses receive a non-universal contribution from $F^Z$ at loop level as a result of the Konishi anomaly [13]. This piece is of order $\epsilon_i Z F^Z/8\pi^2$ and thus negligibly small for $|Z| \ll 1$ compared to the anomaly mediated contribution.}\]
with a single gaugino condensation, the dilaton stabilization typically leads to \( F^{S}/S = \mathcal{O}(m_{3/2}/\ln(M_{Pl}/m_{3/2})) \), thereby yielding \( \alpha = \mathcal{O}(1) \), regardless of the detailed form of uplifting sector\(^{\dagger} \). In addition, one can always make both \( \alpha \) and \( M_0 \) to be real by using \( U(1)_R \) and the perturbative shift symmetry associated with \( \text{Im}(S) \). The mirage mediation scheme shows the mirage unification of gaugino masses at \( M_{\text{mir}} \), \( M_0(M_{\text{mir}}) = M_0 \), while the gauge couplings are still unified at \( M_{\text{GUT}} \). This reflects that anomaly mediated contributions precisely cancel the renormalization group evolved parts of gaugino masses at the mirage messenger scale.

In mirage mediation, low energy A-parameters and sfermion masses are parameterized in terms of

\[
\tilde{A}_{ijk} = -F^I \partial_I \ln \left( \frac{\lambda_{ijk}}{Y_i(M_{\text{GUT}})Y_j(M_{\text{GUT}})Y_k(M_{\text{GUT}})} \right),
\]

\[
\tilde{m}_i^2 = -F^I F^{J*} \partial_I \partial_J \ln Y_i(M_{\text{GUT}}),
\]

(17)

where \( I \) denote the supersymmetry breaking chiral superfields, i.e. \( S \) and \( Z \). As was noticed in \([4]\), a mirage pattern is expected for A-parameters if \( \tilde{A}_{ijk} = M_0 \) for non-negligible Yukawa coupling \( \lambda_{ijk} \). If the condition \( \tilde{m}_i^2 + \tilde{m}_j^2 + \tilde{m}_k^2 = M_0^2 \) is additionally satisfied for \( \lambda_{ijk} \), sfermion masses also take the mirage pattern. Provided that above two conditions are satisfied for non-negligible Yukawa couplings, sfermion soft parameters renormalized at \( M_{\text{mir}} \) read

\[
A_{ijk}(M_{\text{mir}}) = \tilde{A}_{ijk}, \quad m_i^2(M_{\text{mir}}) = \tilde{m}_i^2.
\]

(18)

This indicates that flavor-independent \( \tilde{m}_i^2 \) would lead to the mirage unification of the squark and slepton masses at \( M_{\text{mir}} \). For the effective action (13), one can easily find

\[
\tilde{A}_{ijk} = M_0 + \frac{\epsilon_i + \epsilon_j + \epsilon_k}{(T + T^*) \text{Re}Z} Z^* F^Z,
\]

\[
\tilde{m}_i^2 = \frac{1}{3} M_0^2 - \frac{\epsilon_i}{(T + T^*) \text{Re}Z} |F^Z|^2,
\]

(19)

for \( \lambda_{ijk} \) having no dependence on \( S \) and \( Z \). This shows that A-parameters can take the mirage pattern for \( |Z| \ll 1/8\pi^2 \). On the other hand, sfermion masses are generically dominated by the contribution from \( F^Z = \mathcal{O}(m_{3/2}) \) unless \( |\epsilon_i| \ll 1/(8\pi^2)^2 \). In the limit of vanishing cross-couplings, which corresponds to the case that \( Z \) is sequestered from the visible matter superfields, sfermion masses can share the mirage pattern since \( \tilde{m}_i^2 + \tilde{m}_j^2 + \tilde{m}_k^2 = M_0^2 \).

\(^{\dagger}\) In \([3]\), they considered a Polony-like superpotential to describe the supersymmetry breaking dynamics in the uplifting sector, and found that the dilaton is stabilized to yield \( F^{S}/S \sim m_{3/2}/\ln(M_{Pl}/m_{3/2}) \).
IV. EFFECTS OF ANOMALOUS $U(1)_A$ GAUGE SYMMETRY

In this section, we examine the effects of anomalous $U(1)_A$ gauge symmetry on soft supersymmetry breaking terms. Anomalous $U(1)_A$ gauge symmetry often appears in effective supergravity derived from heterotic compactifications, where the apparent anomaly is removed by the Green-Schwarz mechanism \[6\]. To implement this mechanism of anomaly cancellation, the dilaton transforms non-linearly under $U(1)_A$ transformations

$$S \rightarrow S - \frac{i}{2} \Lambda(x) \delta_{\text{GS}}, \quad (20)$$

where $\Lambda(x)$ is the transformation function and $\delta_{\text{GS}}$ is the Green-Schwarz coefficient of $O(1/8\pi^2)$. The non-linear transformation of $S$ leads to a field-dependent Fayet-Iliopoulos (FI) term in the effective lagrangian

$$\xi_{\text{FI}} = \frac{1}{2} \delta_{\text{GS}} \partial S K. \quad (21)$$

In order to cancel this large FI D-term, it is necessary to introduce a hidden matter field $X$ that is charged under the $U(1)_A$ gauge group. Then, along the D-flat direction, the field $X$ acquires a large vacuum value of $O(\sqrt{\xi_{\text{FI}}})$ for $\text{Re}(S) = O(1)$, and subsequently $U(1)_A$ is spontaneously broken at a very high energy scale.

For the analysis of $U(1)_A$ breaking, we can simply set the visible matter superfields as $Q^i = 0$. Including the $U(1)_A$ sector, the Kähler potential then takes the form

$$K = -3 \ln(T + \bar{T}) - \ln(S + \bar{S} - \delta_{\text{GS}} V_A) + \frac{|Z|^2}{(T + \bar{T})^{n_Z}} + \frac{X e^{-2V_A} X}{(T + \bar{T})^{n_X}}, \quad (22)$$

where $V_A$ is the $U(1)_A$ vector superfield, $n_X$ is the modular weight of $X$, and the $U(1)_A$ charge of $X$ is normalized as $q_X = -1$. From the Kähler potential, the D-term reads

$$D_A = \xi_{\text{FI}} + \frac{|X|^2}{(T + T^*)^{n_X}} = -\delta_{\text{GS}} \frac{2}{2(S + S^*)} + \frac{|X|^2}{(T + T^*)^{n_X}}, \quad (23)$$

in which a large value of $\xi_{\text{FI}}$ should be cancelled in order to get a vanishing vacuum energy. As a result, $U(1)_A$ is spontaneously broken at $|X|$ and the vector gauge boson acquires a superheavy mass

$$M_A^2 = 2g_A^2 \left( \frac{\delta_{\text{GS}}^2}{4(S + S^*)^2} + \frac{|X|^2}{(T + T^*)^{n_X}} \right), \quad (24)$$

where $g_A$ is the $U(1)_A$ gauge coupling. From the D-flat condition, one can easily find that the $U(1)_A$ gauge boson has a mass dominated by the contribution from $|X|^2$, and thus its
longitudinal component comes mostly from $X$ rather than from the dilaton. In fact, the mass eigenstate vector superfield $\tilde{V}_A$ is given by

$$\tilde{V}_A \simeq V_A - \ln |X|,$$

while $S$ remains as a flat direction of the D-term potential. Below the $U(1)_A$ breaking scale, $\tilde{V}_A$ can be integrated out to construct an effective supergravity. The effects of $U(1)_A$ sector are then encoded in effective couplings of the low energy lagrangian.

Meanwhile, it is possible to estimate the contribution of the $U(1)_A$ sector to supersymmetry breaking by combining the gauge invariance with the stationary conditions for $U(1)_A$ charged superfields [7, 17]. Requiring the gauge invariance of the theory, the stationary conditions $\partial S/X V = 0$ lead to the relations

$$F_X/X \simeq - F_S/S + S^*, \quad D_A \simeq - \frac{1}{g_A} \left| \frac{F_S}{S + S^*} \right|^2,$$

for $V = 0$, where small corrections suppressed by $\delta_{GS} = \mathcal{O}(1/8\pi^2)$ have been neglected. This indicates that $X$-mediation can be as important as the dilaton mediation. Moreover, if charged under $U(1)_A$, sfermions receive a mass contribution from the D-term as $\Delta m^2 \sim D_A$, which are also comparable in size to the dilaton mediated one. The D-term potential however gives negligible contribution to the vacuum energy $\Delta V \sim D_A^2$ compared to F-term contributions. Integrating out the heavy vector superfield, effective couplings of visible superfields will have a moduli dependence that reflects these features.

To derive an effective theory below the $U(1)_A$ breaking scale, we need to integrate out heavy $\tilde{V}_A$ by solving its equation of motion $\partial K/\partial \tilde{V}_A = 0$ [7]. This should be done after including the visible matter superfields. Requiring $U(1)_A$ gauge invariance, the matter part of supergravity action is written as

$$\mathcal{L}_{\text{matter}} = \int d^4\theta C \bar{C} Y_i \bar{Q}^i e^{2q_i V_A} Q^i + \left( \int d^2\theta C^3 \frac{1}{6} \lambda_{ijk} X^{q_i+q_j+q_k} Q^i Q^j Q^k + \text{h.c.} \right),$$

where $q_i$ denotes the $U(1)_A$ charge of $Q^i$, the matter wave function $Y_i$ is given by [14], and $\lambda_{ijk}$ have no dependence on $S$ and $Z$. Integrating out the heavy $U(1)_A$ vector superfield using its equation of motion, the matter part of effective supergravity action is written as

$$\mathcal{L}_{\text{matter}} = \int d^4\theta C \bar{C} Y_i^{\text{eff}} |Q^i|^2 + \left( \int d^2\theta C^3 \frac{1}{6} \lambda_{ijk}^{\text{eff}} Q^i Q^j Q^k + \text{h.c.} \right),$$

There are in general non-renormalizable cross-couplings between $X$ and $Q^i$, which are however irrelevant for our discussion on soft terms since $F^X = \mathcal{O}(X F^S)$ and $|F_S| \ll |F_Z|$.
where, under appropriate field redefinition, the effective couplings are given by

\[ Y_{i}^{\text{eff}} = (T + \bar{T})^{1-n_{i}+q_{i}n_{X}}(S + \bar{S})^{1/3+q_{i}} \left( 1 + \frac{\epsilon_{i}}{(T + T^{*})^{n_{Z}}} |Z|^{2} \right) (1 + \mathcal{O}(\delta_{GS})) , \]

\[ \chi_{ij}^{\text{eff}} = |\delta_{GS}/2|^{(q_{i}+q_{j}+q_{k})/2} \lambda_{ijk} , \]  

(29)

in which \( \delta_{GS} = \mathcal{O}(1/8\pi^{2}) \), and \( \epsilon_{i} \) remains unchanged since we are considering the case that the uplifting field is not charged under \( U(1)_{A} \) gauge group\(^{\ddagger} \). Note that the moduli dependence of matter wave function is modified in a way that depends on the \( U(1)_{A} \) charge. Such change of moduli dependence reflects the effects of \( U(1)_{A} \) sector, showing that the \( U(1)_{A} \) mediation generates sfermion soft terms which are comparable in size to the moduli mediated ones.

As discussed in the previous section, the dilaton stabilization naturally leads to a mirage mediation scheme for \( T \) fixed at a self-dual point. In the presence of the \( U(1)_{A} \) sector, the gaugino masses still show the mirage unification at \( M_{\text{mir}} \) since they are not affected by the sector. For the sfermion soft terms, the effective matter wave function (29) now gives

\[ \tilde{A}_{ijk} = (1 + q_{i} + q_{j} + q_{k}) M_{0} + \frac{\epsilon_{i} + \epsilon_{j} + \epsilon_{k}}{(T + T^{*})^{n_{Z}}} Z^{*} F^{Z} , \]

\[ \tilde{m}_{i}^{2} = \left( \frac{1}{3} + q_{i} \right) M_{0}^{2} - \frac{\epsilon_{i}}{(T + T^{*})^{n_{Z}}} |F^{Z}|^{2} , \]  

(30)

where \( M_{0} = F^{S}/(S + S^{*}) \). Since the Yukawa coupling is given by

\[ \lambda_{ijk}^{\text{eff}} \sim \frac{\lambda_{ijk}}{(4\pi)^{q_{i}+q_{j}+q_{k}}} , \]  

(31)

it is quite plausible to assume \( q_{i} + q_{j} + q_{k} = 0 \) for a large Yukawa coupling of order unity. Provided that \( q_{i} + q_{j} + q_{k} = 0 \) for non-negligible Yukawa couplings \( \lambda_{ijk}^{\text{eff}} \), the inclusion of \( U(1)_{A} \) sector does not induce any shift in both \( \tilde{A}_{ijk} \) and \( \tilde{m}_{i}^{2} + \tilde{m}_{j}^{2} + \tilde{m}_{k}^{2} \) while providing \( \tilde{m}_{i}^{2} \) with an additional piece proportional to \( q_{i} \). This indicates that the \( U(1)_{A} \) sector can allow more freedom for the values of sfermion masses while maintaining the mirage pattern of soft parameters. Further, one can arrange the \( U(1)_{A} \) charges to be flavor-universal in order to avoid flavor violations. Such flavor-universal charge assignment would then lead to the mirage unification of the squark and slepton masses at \( M_{\text{mir}} \), if the associated cross-couplings \( \epsilon_{i} \) are sufficiently small.

\(^{\ddagger} \) If the uplifting field is charged under \( U(1)_{A} \), one can find \( D_{A} = \mathcal{O}(|F^{Z}|^{2}/\delta_{GS}) \) by combining the stationary conditions \( \partial_{S,X,Z} V = 0 \) with the gauge invariance. In the effective lagrangian, this results in a large cross-coupling between \( Z \) and \( Q^{i} \), \( \epsilon_{i} \sim q_{i} q_{Z} / \delta_{GS} \).
V. CONCLUSION

In this paper, we have examined the role of F-term uplifting in stabilizing moduli within the framework of heterotic string theory. It is noted that the inclusion of an uplifting sector is crucial for fixing the volume modulus at one of the self-dual points of a modular invariant potential while achieving a vanishing vacuum energy. Since the volume modulus does not participate in supersymmetry breaking at the self-dual points, the F-term uplifting leads to the dilaton stabilization which can naturally realize the mirage mediation scenario for a moduli potential generated by gaugino condensation. Extending to a more general case involving anomalous $U(1)$ gauge symmetry, we have also examined its implications for supersymmetry breaking. The supersymmetry breaking in the $U(1)$ sector is essentially related to how the dilaton is stabilized because the dilaton is responsible for the Green-Schwarz mechanism of anomaly cancellation. It turns out that the $U(1)$ sector generically gives a contribution to sfermion masses comparable to the dilaton mediated one while maintaining the mirage mediation pattern of soft terms.

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