The Body Center Cubic Quark Lattice Model

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Abstract

The Standard Model while successful in many ways is incomplete [1]; many questions remain. The origin of quark masses and hadronization of quarks are awaiting an answer. From the Dirac sea concept, we infer that two kinds of elementary quarks (u(0) and d(0)) constitute a body center cubic (BCC) quark lattice with a lattice constant $a \leq 10^{-18}$ m in the vacuum. Using energy band theory and the BCC quark lattice, we can deduce the rest masses and the intrinsic quantum numbers (I, S, C, b and Q) of quarks. With the quark spectrum, we deduce a baryon spectrum. The theoretical spectrum is in agreement well with the experimental results. Not only will this paper provide a physical basis for the Quark Model, but also it will open a door to study the more fundamental nature at distance scales $\leq 10^{-18}$ m. This paper predicts some new quarks $u_c(6490)$ and $d_b(9950)$, and new baryons $\Lambda_c^+(6500)$, $\Lambda_b^0(9960)$.

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I Introduction

The Standard Model of particle physics has been enormously successful in explaining and predicting a wide range of phenomena. For example, the discovery of weak vector
bosons $W^\pm, Z^0$ at the predicted masses and so forth. In spite of the successes, the Standard Model is incomplete\(^1\), and many questions remain. One is how many quarks are there? QCD does not throw any light on how many quark flavors there should be. The origin of quark masses is unknown. The rigorous basis of confinement and hadronization of quarks are other questions which await answers\(^2\), \(^3\). The minimal version of the Standard Model has 19 arbitrary parameters. This high degree of arbitrariness suggests that a more fundamental theory underlies the Standard Model. As M. K. Gaillard, P. D. Grannis and F. J. Sciulli have pointed out\(^1\):

“We do not expect the Standard Model to be valid at arbitrarily short distances. However, its remarkable success strongly suggests that the Standard Model will remain an excellent approximation to nature at distance scales as small as $10^{-18}$ m”.

Thus, from the Dirac sea concept\(^4\), we have proposed a accompanying excitation concept. This concept can give a rigorous basis for the confinement of quarks\(^5\). Then, from the Dirac sea concept, we have inferred (see Appendix A) that two kinds elementary quarks ($u(0)$ and $d(0)$) constitute a body center cubic quark lattice\(^6\) with a lattice constant $a \leq 10^{-18} m$\(^1\) in the vacuum (the BCC Quark Lattice Model). Using only two kinds of elementary quarks ($u(0)$ and $d(0)$), we have deduced the rest masses and the intrinsic quantum numbers (including $I, S, C, b$ and $Q$) of the ground quarks ($u, d, s, c$ and $b$)\(^7\). With the sum law, we found all important baryons that agree well with the experimental results\(^8\).

In terms of the BCC Quark Lattice Model, using only two kinds elementary quarks ($u(0)$ and $d(0)$), we will deduce the rest masses and the intrinsic quantum numbers ($I, S, C, b$ and $Q$) of the low energy quark spectrum first. Then, using the sum laws, we deduce the baryon spectrum that agrees well with the experimental results in this paper. Not only will this paper answer “the origin of quark masses” and “hadronization...
of quarks” (baryon part, we can deduce the meson spectrum also-see our next paper),
two important problems not answered by the Standard Model, but also it will provide a
door to study the nature at distance scales \( \leq 10^{-18} \text{ m} \) and to look for a more fundamental
theory.

II Fundamental Hypotheses

Hypothesis I : There are only two kinds of elementary quarks, \( u(0) \) and \( d(0) \), with
\( S=C=b=0 \) in the vacuum state. There are super strong attractive interactions among
the quarks (colors). These forces make and hold an infinite body center cubic (BCC)
quark lattice with a periodic constant \( a \leq 10^{-18} \text{ m} \) in the vacuum. (see Appendix A)

Hypothesis II : Due to the effect of the vacuum quark lattice, fluctuations of energy
\( \varepsilon \) and intrinsic quantum numbers (such as the Strange number \( S \)) of an excited quark
exist. The fluctuation of the Strange number is always \( \Delta S = \pm 1 \) \[9\].

III The Spectrum of the Quarks

A The Motion Equation

According to the Fundamental Hypothesis I, there is a body center cubic quark
lattice in the vacuum. When an excited quark (\( q \)) is moving inside the vacuum quark
lattice. It obeys the special quark Dirac equation \[7\]:

\[
(i\hbar \frac{\partial}{\partial t} + i\hbar c \vec{\alpha} \cdot \vec{\nabla} - \beta m_q c^2 - V_0) \psi(\vec{r},t) = 0,
\]

where \( m_q \) is the bare mass \[10\] of the elementary quark \( u(0) \) (or \( d(0) \)), \( V_0 \) is a constant in
any time, any location and any reference frame. Since \( (i\hbar \frac{\partial}{\partial t} + i\hbar c \vec{\alpha} \cdot \vec{\nabla} - \beta m_q c^2) \psi(\vec{r},t) \)
= 0 is a free particle Dirac equation and $V_0$ is only a constant, the above equation is Lorentz-invariant.

The purpose of this paper is to deduce the rest masses and the intrinsic quantum numbers of the quarks. These results are independent from the reference frame. Thus we can use the low energy approximation to deduce them.

**B The Low Energy Approximation**

We use a classic approximation of the special quark Dirac equation—the Schrödinger equation (our results will show that this is a very good approximation) to deduce the rest masses and the intrinsic quantum numbers of the quarks:

$$\frac{\hbar^2}{2m_q} \nabla^2 \Psi + (\varepsilon - V_0) \Psi = 0,$$

(2)

where $V_0$ is an average field approximation of the strong interaction periodic field of the vacuum quark lattice with body center cubic periodic symmetries and $m_q$ is the bare mass of the elementary quark $u(0)$ (or $d(0)$) in the pure vacuum. This Schrödinger equation is a strong interactions motion equation of an elementary quark $u(0)$ (or $d(0)$) with body center cubic (BCC) periodic symmetries.

**C The Free Particle Approximation**

The solution of the above equation is a free particle plane wave function of the $u(0)$-quark or the $d(0)$-quark:

$$\varepsilon_\mathbf{k} = \left(\frac{\hbar^2}{2m_q}\right)^2 (k_x^2 + k_y^2 + k_z^2) + V_0,$$

(3)

$$\Psi_\mathbf{k} = e^{i \mathbf{k} \cdot \mathbf{r}}, \quad \mathbf{k} = (k_x, k_y, k_z).$$

(4)
They represent running waves and carry momentum $\vec{P} = \hbar \vec{k}$ of the excited quarks (u or d) with $S = C = b = 0$ since $u(0)$ and $d(0)$ have $S = C = b = 0$. When $\vec{k} = (k_x, k_y, k_z) = 0$, from (3), we get the rest masses of excited state $u$ (or $d$) of the elementary quark $u(0)$ (or $d(0)$)

$$m_u = m_d = \frac{V_0}{c^2}.$$ 

Using the accompanying excited concept [5] and the masses of p and n, we get

$$M_p = m_u' + m_u + m_d' 
\approx M_n = m_d' + m_u' + m_d' 
= 940 \text{ Mev/c}. \quad (5)$$

From [5] and [12]

$$m_u' = 3 \text{ Mev and } m_d' = 7 \text{ Mev}, \quad (6)$$

we have

$$m_u C^2 = m_d C^2 = V_0 = 930 \text{ Mev}. \quad (7)$$

Thus the u-quark has

$$S = C = b = 0, I = I_z = s = \frac{1}{2}, Q = \frac{2}{3}, m = 930 \text{ Mev}, \quad (8)$$

the d-quark has

$$S = C = b = 0, I = -I_z = s = \frac{1}{2}, Q = -\frac{1}{3}, m = 930 \text{ Mev}. \quad (9)$$

The flavored particles are the tracer particles of the BCC quark lattice [7]. In order to deduce the rest masses of the flavored quarks, we have to consider the BCC symmetries.
D The Energy Bands

According to the energy band theory, considering the BCC symmetries, the wave functions of the Schrödinger equation have to satisfy the body center cubic periodic symmetries. The solution will divide into energy bands. If we take

$$\vec{k} = (2\pi/a)(\xi, \eta, \zeta),$$

the energy of the Schrödinger equation is

$$\varepsilon(\vec{k}, \vec{n}) = V_0 + \alpha E(\vec{k}, \vec{n}),$$

(10)

$$\alpha = h^2/2m_qa^2,$$

(11)

$$E(\vec{k}, \vec{n}) = (n_1-\xi)^2 + (n_2-\eta)^2 + (n_3-\zeta)^2.$$  

(12)

and the plane wave function is

$$\Psi_\xi(\vec{r}) = \exp\{(-i2\pi/a)[(n_1-\xi)x + (n_2-\eta)y + (n_3-\zeta)z]\},$$

(13)

where $a$ is the periodic constant of the quark lattice, $a \leq 10^{-18}$ m; and $n_1$, $n_2$ and $n_3$ are integers $n_1 = l_2 + l_3$, $n_2 = l_3 + l_1$, $n_3 = l_1 + l_2$,

$$l_1 = 1/2(-n_1+n_2+n_3),$$

$$l_2 = 1/2(+n_1-n_2+n_3),$$

$$l_3 = 1/2(+n_1+n_2-n_3),$$

(14)

satisfying the condition that only those values of $\vec{n} = (n_1, n_2, n_3)$ are allowed, which make $\vec{l} = (l_1, l_2, l_3)$ an integral vector. Condition (14) implies that the vector $\vec{n} = (n_1, n_2, n_3)$ can only take certain values. For example, $\vec{n}$ cannot take $(0,0,1)$ or $(1,1,-1)$, but can take $(0,0,2)$ and $(1,-1,2)$. This is a result of BCC symmetries.
E   How to find the energy bands

Now we will demonstrate how to find the energy bands.

The first Brillouin zone [15] of the body center cubic lattice is shown in Fig.1. (depicted from [13] (Fig.1)) and [15] (Fig. 8. 10).

![Figure 1](image)

**Figure 1**

Figure 1: The first Brillouin zone of the body center cubic lattice. The symmetry points and axes are indicated. The $\Delta$ -axis is a four-fold rotation axis, $S = 0$, the quark family $q_\Delta$ will be born on the axis. The axes $\Lambda$ and $F$ are three-fold rotation axes, $S = -1$, the quark family $q_\Sigma$ will be born on the axes. The axes $\Sigma$ and $G$ are two-fold rotation axes, $S = -2$, the quark family $q_\Xi$ will be born on the axes. The $D$-axis is parallel to the axis $\Delta$, $S = 0$. The axis is a two-fold rotation axis, the quark family $q_N$ will be born on the axis.
In Fig.1, the \((\xi, \eta, \varsigma)\) coordinates of the symmetry points are:

\[
\Gamma = (0, 0, 0), \quad H = (0, 0, 1), \quad P = (1/2, 1/2, 1/2),
\]
\[
N = (1/2, 1/2, 0), \quad M = (1, 0, 0). \quad (15)
\]

and the \((\xi, \eta, \zeta)\) coordinates of the symmetry axes are:

\[
\Delta = (0, 0, \zeta), \quad 0 < \zeta < 1; \quad \Lambda = (\xi, \xi, \xi), \quad 0 < \xi < 1/2;
\]
\[
\Sigma = (\xi, \xi, 0), \quad 0 < \xi < 1/2; \quad D = (1/2, 1/2, \xi), \quad 0 < \xi < 1/2;
\]
\[
G = (\xi, 1-\xi, 0), \quad 1/2 < \xi < 1; \quad F = (\xi, \xi, 1-\xi), \quad 0 < \xi < 1/2. \quad (16)
\]

For any valid value of the vector \(\vec{n}\), substituting the \((\xi, \eta, \zeta)\) coordinates of the symmetry points (15) or the symmetry axes (16) into Eq. (12) and Eq. (13), we can get the \(E(\vec{k}, \vec{n})\) values and the wave functions at the symmetry points and on the symmetry axes. In order to show how to calculate the energy bands, we give the calculation of some low energy bands in the symmetry axis \(\Delta\) as an example (the results are illustrated in Fig. 2(a)).

First, from (12) and (13) we find the formulae for the \(E(\vec{k}, \vec{n})\) values and the wave functions at the end points \(\Gamma\) and \(H\) of the symmetry axis \(\Delta\), as well as on the symmetry axis \(\Delta\) itself:

\[
E_{\Gamma} = n_1^2 + n_2^2 + n_3^2, \quad (17)
\]
\[
\Psi_{\Gamma} = \exp \{-i(2\pi/a_x)[n_1 x + n_2 y + n_3 z]\}. \quad (18)
\]
\[
E_{H} = n_1^2 + n_2^2 + (n_3 - 1)^2, \quad (19)
\]
\[
\Psi_{H} = \exp \{-i(2\pi/a_x)[n_1 x + n_2 y + (n_3 - 1) z]\}. \quad (20)
\]
\[ E_\Delta = n_1^2 + n_2^2 + (n_3 \cdot \zeta)^2, \quad (21) \]

\[ \Psi_\Delta = \exp \{ -i(2\pi/a_x)[n_1x + n_2y + (n_3 \cdot \zeta)z] \}. \quad (22) \]

Then, using (17)–(22) and beginning from the lowest possible energy, we can obtain the corresponding integer vectors \( \vec{n} = (n_1, n_2, n_3) \) (satisfying (14)) and the wave functions:

1. The lowest \( E(\vec{k}, \vec{n}) \) is at \((\xi, \eta, \zeta) = 0\) (the point \( \Gamma \)) and with only one value of \( \vec{n} = (0,0,0) \) \( \text{ (see (17) and (18)):} \)

\[ \vec{n} = (0,0,0), \quad E_\Gamma = 0, \quad \Psi_\Gamma = 1. \quad (23) \]

2. Starting from \( E_\Gamma = 0 \), along the axis \( \Delta \), there is one energy band (the lowest energy band) \( E_\Delta = \zeta^2 \), with \( n_1 = n_2 = n_3 = 0 \) \( \text{ (see (21) and (22)) ended at the point } E_H = 1: \)

\[ \vec{n} = (0,0,0), \quad E_H = 1, \quad \Psi_\Delta = \exp [i(2\pi/a_x)(\zeta z)]. \quad (24) \]

3. At the end point \( H \) of the energy band \( E_H = 0 \rightarrow E_\Delta = \zeta^2 \rightarrow E_H = 1 \), the energy \( E_H = 1 \). Also at point \( H \), \( E_H = 1 \) when \( n = (\pm 1, 0, 1) \), \((0, \pm 1, 1)\), and \((0, 0, 2)\) \( \text{ (see (19) and (20)):} \)

\[ E_H = 1, \quad \Psi_H = [e^{i(2\pi/a_x)(\pm x)}, e^{i(2\pi/a_y)(\pm y)}, e^{i(2\pi/a_z)(\pm z)}]. \quad (25) \]

4. Starting from \( E_H = 1 \), along the axis \( \Delta \), there are three energy bands ended at the points \( E_\Gamma = 0, E_\Gamma = 2 \text{ and } E_\Gamma = 4 \), respectively:

\[ \vec{n} = (0,0,0), \quad E_H = 1 \rightarrow E_\Delta = \zeta^2 \rightarrow E_\Gamma = 0, \quad \Psi_\Delta = \exp[i(2\pi/a_x)(\zeta z)]. \quad (26) \]
\( \vec{n} = (0,0,2), \)

\[ E_H = 1 \rightarrow E_\Delta = (2-\zeta)^2 \rightarrow E_\Gamma = 4, \]

\[ \Psi_\Delta = \exp \left[ i(2\pi/a_x)(2 - \zeta)z \right]. \]  \hspace{1cm} (27)

\[ \vec{n} = (\pm1,0,1)(0, \pm 1,1), \]

\[ E_H = 1 \rightarrow E_\Delta = 1+(1-\zeta)^2 \rightarrow E_\Gamma = 2, \]

\[ \Psi_\Delta = e^{-i(2\pi/a_x)[\pm x+(1-\zeta)z]}, e^{-i(2\pi/a_x)[\pm y+(1-\zeta)z]}. \]  \hspace{1cm} (28)

5. The energy bands with four sets of values \( \vec{n} \) \((\vec{n} = (\pm1,0,1), (0,\pm1,1)) \) ended at \( E_\Gamma = 2 \). From (17), \( E_\Gamma = 2 \) also when \( \vec{n} \) takes other eight sets of values: \( \vec{n} = (1,\pm1,0), (-1,\pm1,0), (\pm1,0,-1) \) and \((0,\pm1,-1)\). Putting the 12 sets of \( \vec{n} \) values into Eq. (18), we can obtain 12 plane wave functions:

\[ E_\Gamma = 2, \Psi_\Gamma = [e^{i(2\pi/a_x)(\pm x \pm y)}, e^{i(2\pi/a_x)(\pm y \pm z)}, e^{i(2\pi/a_x)(\pm z \pm x)}]. \]  \hspace{1cm} (29)

6. Starting from \( E_\Gamma = 2 \), along the axis \( \Delta \), there are three energy bands ended at the points \( E_H = 1, \ E_H = 3, \) and \( E_H = 5 \), respectively:

\[ \vec{n} = (\pm 1,0,1)(0, \pm 1,1), \ E_\Gamma = 2 \rightarrow E_\Delta = 1+(1-\zeta)^2 \rightarrow E_H = 1, \]  \hspace{1cm} (30)

\[ \vec{n} = (1,\pm 1,0)(-1, \pm 1,0), \ E_\Gamma = 2 \rightarrow E_\Delta = 2+\zeta^2 \rightarrow E_H = 3, \]  \hspace{1cm} (31)

\[ \vec{n} = (\pm 1,0,-1)(0, \pm 1, -1), \ E_\Gamma = 2 \rightarrow E_\Delta = 1+(\zeta+1)^2 \rightarrow E_H = 5. \]  \hspace{1cm} (32)

Continuing the process, we can find the low energy bands and the corresponding wave functions. The wave functions are not needed for this paper, so we only show the energy bands in Fig. 2-4. There are six small figures in Fig. 2-4. Each of them shows the energy
bands in one of the six axes in Fig. 1. Each small figure is a schematic one where the straight lines that show the energy bands should be parabolic curves. The numbers above the lines are the values of $\vec{n} = (n_1, n_2, n_3)$. The numbers (deg) under the lines are the fold numbers of the energy bands with the same energy (the free particle approximation). The numbers beside both ends of an energy band (the intersection of the energy band line and the vertical lines) represent the highest and lowest $E(\vec{k}, \vec{n})$ values (see Eq. (12)) of the band.
Figure 2

Figure 2: (a) The energy bands on the $\Delta$-axis. $E_\Gamma$ is the value of $E(\vec{k}, \vec{n})$ (see Eq. (12)) at the end point $\Gamma$, while $E_H$ is the value of $E(\vec{k}, \vec{n})$ at other end point $H$. (b) The energy bands on the $\Lambda$-axis. $E_\Gamma$ is the value of $E(\vec{k}, \vec{n})$ (see Eq. (12)) at the end point $\Gamma$, while $E_P$ is the value of $E(\vec{k}, \vec{n})$ at other end point $P$. 
Figure 3: (a) The energy bands on the Σ-axis. \( E_\Gamma \) is the value of \( E(\vec{k}, \vec{n}) \) (see Eq. (12)) at the end point \( \Gamma \), while \( E_N \) is the value of \( E(\vec{k}, \vec{n}) \) at other end point \( N \). (b) The energy bands on the axis \( D \). \( E_P \) is the value of \( E(\vec{k}, \vec{n}) \) (see Eq. (12)) at the end point \( P \), while \( E_N \) is the value of \( E(\vec{k}, \vec{n}) \) at other end point \( N \).
Figure 4

Figure 4: (a) The energy bands on the $F$-axis. $E_P$ is the value of $E(\vec{k}, \vec{n})$ (see Eq. [12]) at the end point $P$, while $E_H$ is the value of $E(\vec{k}, \vec{n})$ at other end point $H$. (b) The energy bands on the axis $G$. $E_M$ is the value of $E(\vec{k}, \vec{n})$ (see Eq. [12]) at the end point $M$, while $E_N$ is the value of $E(\vec{k}, \vec{n})$ at other end point $N$. 
Figure 5: (a) The four-fold degenerate energy bands (selected from Fig. 2(a)) on the \( \Delta \)-axis. (b) The single energy bands (selected from Fig. 2(a)) on the \( \Delta \)-axis. The numbers above the lines are the values of \( \vec{n} \) \((n_1, n_2, n_3)\). (c) The single energy band (selected from Fig. 3(a)) on the \( \Sigma \)-axis.

Putting the values of the \( E(\vec{k},\vec{n}) \) into Eq. (10), we get the energy values (in Mev) in Table 7-12 of the Appendix B.

F The Recognition of the Quarks

We have already found the energy bands which are shown in Fig. 2—Fig. 5. These energy bands represent the excited states of the elementary quarks \( u(0) \) or \( d(0) \). They
are the ground quarks and excited quarks. For the first Brillouin zone, \( \mathbf{n} = (0, 0, 0) \), it is a part of the free quark solution (4) and it has the lowest energy (mass). Thus it represents the lowest mass u-quark (8) and the d-quark (9):

\[
\mathbf{n} = (0, 0, 0), \text{ u}(930) \text{ and d}(930).
\] (33)

Now we deduce the quantum numbers and the rest masses of the energy bands. Using the numbers and the rest masses, we can recognize the quarks.

**F- 1 The Quantum Numbers and Energies of the Energy Bands**

Now we find the formulae that we can use to deduce the quantum numbers and the rest masses of the energy bands, as shown in the following:

1. An excited (from the vacuum) quark, \( q \), has a baryon number

\[
B = \frac{1}{3}
\] (34)

2. The isospin \( I \) is determined by the energy band degeneracy \( \text{deg} \), where

\[
\text{deg} = 2I + 1.
\] (35)

If the \( \text{deg} > \text{the rotary fold} R \) of the symmetry axis

\[
\text{deg} > R,
\] (36)

the \( \text{deg} \) will be divided \( \gamma \)-subdegeneracies first

\[
\gamma = \text{deg}/R;
\] (37)

then, using (35), we can find the isospin values for each subdegeneracy.

The isospin \( I \) from (35), not only is the isospin of the \( q \)-quark that the energy bands represent, but also is the isospin of the baryon \( (qq'q'') \) that the energy bands represent. The components of the isospin \( I \):

\[
I, I-1, I-2, ..., -I
\] (38)
are the components of the baryon’s isospin. The components of the quark are deduced by (50).

3. Strange number $S$ is determined by the rotary fold $R$ of the symmetry axis \[13\] with

$$S = R - 4,$$ \hfill (39)

where the number 4 is the highest possible rotary fold number of the BCC lattice.

For the $\Delta$-axis, $R = 4, S = 0$, \hfill (40)

for the $\Lambda$-axis, $R = 3, S = -1$; \hfill (41)

for the $\Sigma$-axis, $R = 2, S = -2$. \hfill (42)

The three axes (the axes D, F and G) on the surface of the first Brillouin zone have that the D-axis parallel the $\Delta$-axis, the F-axis parallel the $\Lambda$-axis and the G-axis parallel the $\Sigma$-axis. Thus

$$S_D = S_\Delta = 0,$$ \hfill (43)

$$S_F = S_\Lambda = -1,$$ \hfill (44)

$$S_G = S_\Sigma = -2.$$ \hfill (45)
4. Electric charge $Q$ of the exited quark, $q$, is determined completely by the elementary quark $u(0)$ (or $d(0)$) that is excited to produce the exited quark $q$. Since the $u(0)$-quark has $I_z = 1/2 > 0$ and the $d(0)$-quark has $I_z = -1/2 < 0$, for the exited quark $q$ with $I_z > 0$, it is an excited state of the elementary quark $u(0)$,

$$Q_q = Q_u = +2/3; \quad (46)$$

for the exited quark $q$ with $I_z < 0$, it is an excited state of the elementary quark $d(0)$,

$$Q_q = Q_d = -1/3. \quad (47)$$

For the exited quark $q$ with $I_z = 0$, considering the generalized Gell-Mann-Nishijima relation [16], if $q$ with $S_G (S+C+b) > 0$, it is an excited state of the $u(0)$-quark

$$Q_q = Q_u = +2/3; \quad (48)$$

if $q$ with $S_G (S+C+b) < 0$, it is an excited state of the $d$-quark

$$Q_q = Q_d = -1/3. \quad (49)$$

There is not any quark with $I_z = 0$ and $S_G (S+C+b) = 0$.

5. After we find the the electric charge $Q$ and the strange number $S$ (C, b) of the excited quark $q$, using the generalized Gell-Mann-Nishijima relation [16], we can find $I_z$ of the excited quark $q$

$$I_z = Q - \frac{1}{2}(B+S+C+b). \quad (50)$$

It is natural that the $I_z$ of an energy band excited state of the elementary quark $u(0)$ (or $d(0)$) might be different from the $I_z$ of the elementary quark $u(0)$ (or $d(0)$) in vacuum state, but their electric charges will be the same.

6. If a degeneracy (or subdegeneracy) of a group of energy bands is smaller than the rotary fold $R$

$$\text{deg} < R \quad \text{and} \quad R - \text{deg} \neq 2, \quad (51)$$

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then formula (39) will substituted by
\[ \bar{S} = R - 4. \] (52)

The real value of \( S \) is
\[ S = \bar{S} + \Delta S = S_{Axis} \pm 1, \] (53)

from Hypothesis II, \( \Delta S = \pm 1 \). We have a formula to find \( \Delta S \),
\[ \Delta S = [1-2\delta(S)]\text{Sign}(\vec{n}), \] (54)

where
\[ \text{Sign}(\vec{n}) = \frac{n_1 + n_2 + n_3}{|n_1| + |n_2| + |n_3|}. \] (55)

For the \( \Delta \)-axis and the D-axis, \( \delta(S) = 1 \), from (54), we get
\[ \Delta S = - \text{Sign}(\vec{n}). \] (56)

For the \( \Lambda \)-axis, the \( \Sigma \)-axis, the F-axis and the G axis, \( \delta(S) = 0 \), from (54), we have
\[ \Delta S = \text{Sign}(\vec{n}). \] (57)

7. The fluctuation of the strange number will be accompanied by an energy change (Hypothesis II). We assume that the change of the energy (perturbation energy) is proportional to \( -\Delta S \) and a number, \( J \), representing the energy order, as a phenomenological formula:
\[ \Delta \varepsilon = (S+1)100(J+S)(-\Delta S), \] (58)

for a single energy band, \( J \) will take 1, 2, ... from the lowest energy band to higher ones for each of the two end points of the symmetry axes respectively.

8. Charmed number \( C \) and Bottom number \( b \): The “Strange number”, \( S \), in (53) is not completely the same as the strange number in (39). In order to compare it with the
experimental results, we would like to give it a new name under certain circumstances. Based on Hypothesis II, the new names will be the Charmed number and the Bottom number: if $S = +1$, which originates from the fluctuation $\Delta S = +1$, we call it the Charmed number $C$

$$C = +1; \quad (59)$$

if $S = -1$, which originates from the fluctuation $\Delta S = +1$ on a single energy band, and there is an energy fluctuation, we call it the Bottom number $b$

$$b = -1. \quad (60)$$

Similarly, we can obtain charmed strange quarks $q_{\Xi_C}$ and $q_{\Omega_C}$ (Appendix C).

9. We assume that the excited quark’s rest mass is the minimum energy of the energy band that represents the excited quark:

$$m(q) = \text{Minimum}[V_0 + \alpha E(\vec{k}, \vec{n})] + \Delta \varepsilon, \quad (61)$$

where $V_0 = 930$ Mev $\text{(7)}$; $\Delta \varepsilon$ is from $\text{(58)}$; fitting the energy band excited states to the experimental results (the baryon spectrum and the meson spectrum), we find the $\alpha$ in $\text{(61)}$

$$\alpha = \frac{\hbar^2}{2m_qa^2} = 360 \text{ Mev.} \quad (62)$$

This formula $\text{(61)}$ is the united mass formula that can give the masses of all quarks.

Using the above formulae, we can find the quark spectrum. We will start from the $\Delta$-axis.

**F- 2 The Quarks on the $\Delta$-Axis (the $\Gamma$-$H$ axis)**

Since the $\Delta$-axis is a four-fold rotatory axis (see Fig. 1), $R = 4$, from $\text{(39)}$, we have $S = 0$. Because the axis has $R = 4$, from $\text{(36)}$ and $\text{(37)}$, the energy bands of degeneracy 8 will be divided into two four-fold degenerate bands.
1. The four fold degenerate bands on the $\Delta$-axis ($\Gamma$-H)

For four-fold degenerate bands, using (35), we get $I = 3/2$, and $I_z, B = 3/2, 1/2, -1/2, -3/2$ (38). Thus, from (46) and (47), each four-fold degenerate band represents a four-fold quark family $q_\Delta(q_{\Delta^3}, q_{\Delta^1}, q_{\Delta^{-1}}, q_{\Delta^{-3}})$ with

$$B = \frac{1}{3}, S = 0, I = \frac{3}{2}, I_z, B = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}. \quad (63)$$

Using Fig.2(a) and Fig.5(a), we can get $E_\Gamma, E_H, \text{ and } \bar{n}$ values. Then, putting the values of $E_\Gamma$ and $E_H$ into the energy formula (12) and (10), we can find $m_{q_\Delta}$. Thus, we have $[q_\Delta = (q_{\Delta^3}, q_{\Delta^1}, q_{\Delta^{-1}}, q_{\Delta^{-3}})]:

| E     | (n_1n_2n_3, ... ) | $\varepsilon$ | $I$ | qName(m) |
|-------|-------------------|---------------|-----|-----------|
| $E_H = 1$ | (101, -101, 011, 0-11) | 1290 | 3/2 | $q_\Delta$(1290) |
| $E_H = 2$ | (110, 1-10, -110, -1-10) | 1650 | 3/2 | $q_\Delta$(1650) |
| $E_H = 2$ | (10-1, -10-1, 01-1, 0-1-1) | 1650 | 3/2 | $q_\Delta$(1650) |
| $E_H = 3$ | (112, 1-12, -112, -1-12) | 2010 | 3/2 | $q_\Delta$(2010) |
| $E_H = 4$ | (200, -200, 020, 0-20) | 2370 | 3/2 | $q_\Delta$(2370) |
| $E_H = 5$ | (121, 1-21, -121, -1-21) | 2730 | 3/2 | $q_\Delta$(2730) |
| $E_H = 5$ | (211, 2-11, -211, -2-11) | 2730 | 3/2 | $q_\Delta$(2730) |

| $E_H = 5$ | (121, 1-21, -211, -1-21) | 2730 | 3/2 | $q_\Delta$(2730) |
| $E_H = 5$ | (202, -202, 022, 0-22) | 2730 | 3/2 | $q_\Delta$(2730) |
| $E_H = 5$ | (013, 0-13, 103, -103) | 2730 | 3/2 | $q_\Delta$(2730) |
| $E_H = 6$ | (121, 121, 121, 121) | 3090 | 3/2 | $q_\Delta$(3090) |
| $E_H = 6$ | (211, 211, 211, 211) | 3090 | 3/2 | $q_\Delta$(3090) |

2. The single bands on the axis $\Delta(\Gamma$-H) [7]. For the single bands on the $\Delta$-axis, $R = 1, S_\Delta = 0$ from (39); $d = 1, I = 0$ from (39). Since $d = 1 < R = 4$ and $R-d = 3 \neq 2$, ...
According to (51), we will use (53) instead of (39). Using (59) and (58), we have

| E   | n₁, n₂, n₃ | ΔS | J   | Δε | I  | S  | C  | qₙₐₙₖ(m) |
|-----|------------|----|-----|----|----|----|----|-----------|
| E₀  | 0, 0, 0    | 0  | 0   | 0  | 0  | 0  | 0  | qᴺ(930)   |
| E₁  | 0, 0, 2    | -1 | J₁ = 0 | 100 | 1  | 0  | 0  | dₛ(1390)  |
| E₄  | 0, 0, -2   | +1 | J₁ = 0 | -100| 0  | 0  | 1  | u₅(2270)  |
| E₉  | 0, 0, 4    | -1 | J₂ = 2 | 200 | 0  | 0  | 0  | u₅(4370)  |
| E₁₆ | 0, 0, -4   | +1 | J₂ = 2 | -200| 0  | 0  | 1  | u₅(6490)  |
| E₂₅ | 0, 0, 6    | -1 | J₃ = 3 | 300 | 0  | 0  | 0  | u₅(10230)|
| E₃₆ | 0, 0, -6   | +1 | J₃ = 3 | -300| 0  | 0  | 1  | u₅(13590)|
| E₄₉ | 0, 0, 8    | -1 | J₄ = 4 | 400 | 0  | 0  | 0  | u₅(18970)|
| E₆₄ | 0, 0, -8   | +1 | J₄ = 4 | -400| 0  | 0  | 1  | u₅(23570)|

F- 3 The Quarks on the Axis Λ(Γ-P)

Since the Λ-axis is a three-fold rotatory axis (see Fig. 1), R = 3, from (39), we have S = -1. From Fig. 2(b), we see that there is a single energy band with \( \vec{n} = (0, 0, 0) \), and all other bands are three-fold degenerate energy bands (\( d = 3 \)) and six-fold degenerate bands (\( d = 6 \)). From (36) and (37), the six-fold degenerate energy bands will be divided into two three-fold energy bands.

For the three-fold degenerate energy bands, using (35), (38), (46), (47), and (49), we have I = 1, and I₂,₃ = 1, 0, -1. Thus, we get a three-member quark family \( q_Σ(q_Σ, q_Σ, q_Σ) \)
$q_{\Sigma}^{-1}$) with $B = 1/3$, $S = -1$, $I = 1$. Similar to (64), using Fig. 2(b), we get

\[
E = \begin{array}{cccc}
E_{\nu} = 3/4 & (101,011,110) & 1200 & q_{\Sigma}(1200) \\
E_{\nu} = 2 & (1-10,-110,01-1,0-110,10-110) & 1650 & q_{\Sigma}(1650) \\
E_{\nu} = 2 & (-10-1,0-1-1,-1-10) & 1650 & q_{\Sigma}(1650) \\
E_{\nu} = 11/4 & (020,002,200) & 1920 & q_{\Sigma}(1920) \\
E_{\nu} = 11/4 & (121,211,112) & 1920 & q_{\Sigma}(1920) \\
E_{\nu} = 4 & (0-20,-200,00-2) & 2370 & q_{\Sigma}(2370) \\
E_{\nu} = 19/4 & (1-12,-112,21-1,2-11,12-1,-121)) & 2640 & q_{\Sigma}(2640) \\
E_{\nu} = 19/4 & (202,022,220) & 2640 & q_{\Sigma}(2640) \\
E_{\nu} = 6 & (-211,2-1-1,-1-12) & 3090 & q_{\Sigma}(3090) \\
E_{\nu} = 6 & (11-2,-12-1,1-21) & 3091 & q_{\Sigma}(3090) \\
E_{\nu} = 6 & (-1-21,1-2-1,-11-2) & 3091 & q_{\Sigma}(3090) \\
E_{\nu} = 6 & (1-1-2,-21-1,-2-11)) & 3091 & q_{\Sigma}(3090) \\
E_{\nu} = 6 & (-1-2-1,-1-1-2,-2-1-1) & 3091 & q_{\Sigma}(3090) \\
\end{array}
\]

\[
\text{(66)}
\]

. . . .

**F- 4 The Quarks on the Axis }$\Sigma$($\Gamma$-$N$)

The $\Sigma$-axis is a two-fold rotation axis, $R = 2$. From (89), $S = -2$ (see Fig. 3(a)).

1. The two-fold degenerate energy bands on the $\Sigma$-axis ($\Gamma$-$N$)

For the two-fold degenerate energy bands, each of them represents a quark family $q_{\Xi}$ ($q_{\Xi^+}, q_{\Xi^-}$) with $B = 1/3$, $S = -2$, $I = 1/2$ from (65), $I_{z,B} = 1/2, -1/2$. Similar to (64), we
have \([q_{\Xi}(q_{\Xi}^\frac{1}{2}, q_{\Xi}^{-\frac{1}{2}})]\):

\[
\begin{array}{ccccccc}
\text{E} & \text{n}_1\text{n}_2\text{n}_3 & S_{\Xi} & I & m & q_{\Xi}(m) \\
E_{\Gamma} = 2 & (1-10,-110) & -2 & 1/2 & 1650 & q_{\Xi}(1650) \\
E_{N} = 5/2 & (200,020) & -2 & 1/2 & 1830 & q_{\Xi}(1830) \\
E_{\Gamma} = 4 & (002,00-2) & -2 & 1/2 & 2370 & q_{\Xi}(2370) \\
& & & & & & (67) \\
& & & & & & (\cdot \cdot \cdot) \\
E_{N} = 9/2 & (112,11-2) & -2 & 1/2 & 2550 & q_{\Xi}(2550) \\
\end{array}
\]

2. The four-fold degenerate energy bands on the axis \(\Sigma(\Gamma-N)\)

According to (67), each four-degenerate energy band on the symmetry axis \(\Sigma\) will be divided into two two-fold degenerate bands. From (67), each of them represents a quark family \(q_{\Xi}(q_{\Xi}^\frac{1}{2}, q_{\Xi}^{-\frac{1}{2}})\) with \(B = 1/3, S = -2, I = 1/2, I_{z,B} = \frac{1}{2}, -\frac{1}{2}\). Thus, we have

\[
\begin{array}{ccccccc}
\text{E} & \text{n}_1\text{n}_2\text{n}_3, \ldots & I & q_{\Xi}(m) \\
E_{N} = 3/2 & \bar{n} = (101,10-1,011,01-1) & 1/2 & 2 \times q_{\Xi}(1470) \\
E_{\Gamma} = 2 & \bar{n} = (-101,-10-1,0-11,0-1-1) & 1/2 & 2 \times q_{\Xi}(1650) \\
E_{N} = 7/2 & \bar{n} = (121,12-1,211,21-1) & 1/2 & 2 \times q_{\Xi}(2190) \\
\end{array}
\]

\[
(67)
\]

\[
(\cdot \cdot \cdot)
\]

3. The single energy bands on the axis \(\Sigma(\Gamma-N)\), similarly to (65) [7], we have:

\[
\begin{array}{ccccccccc}
\text{E} & \text{n}_1\text{n}_2\text{n}_3 & S & J & I & \Delta \epsilon & q_{Name}(m) \\
E_{N}=1/2 & (1,1,0) & -1 & J_N=1 & 0 & 0 & d_S(1110) \\
E_{\Gamma}=2 & (-1,-1,0) & -3 & J_{\Gamma}=1 & 0 & 0 & d_\Omega(1650) \\
E_{N}=9/2 & (2,2,0) & -1 & J_N=2 & 0 & 0 & d_S(2550) \\
E_{\Gamma}=8 & (-2,-2,0) & -3 & J_{\Gamma}=2 & 0 & 0 & d_\Omega(3810) \\
E_{N}=25/2 & (3,3,0) & -1 & J_N=3 & 0 & +100 & d_b(5530) \\
E_{\Gamma}=18 & (-3,-3,0) & -3 & J_{\Gamma}=3 & 0 & -100 & d_\Omega(7310) \\
E_{N}=49/2 & (4,4,0) & -1 & J_N=4 & 0 & +200 & d_b(9950) \\
E_{\Gamma}=32 & (-4,-4,0) & -3 & J_{\Gamma}=4 & 0 & -200 & d_\Omega(12250) \\
E_{N}=81/2 & (5,5,0) & -1 & J_N=5 & 0 & +300 & d_b(15810) \\
\end{array}
\]

\[
(69)
\]

\[
(\cdot \cdot \cdot)
\]
Continuing the above procedure (see Appendix C), we can use Fig. 2-5 to find the excited states of the quark (the quark spectrum) of lower energies as shown in the following lists. The \( I_z^{Baryon} \) of \( Q^{I_z,Bareon}_{\text{Name}} \) is the z-component of the isospin of the baryon \( (Q^{I_z,q_{\text{Name}}}q_1q_2) \) from \( (70) \). the \( I_z \) of \( Q^{I_z,\text{quark}}_{\text{Ele. quark}} \) is the z-component of the isospin of \( Q^{I_z,\text{quark}}_{\text{quark}} \) from \( I_z = Q \cdot \frac{1}{2}(B+S+C+b) \); the \( Q^{I_z,\text{quark}}_{\text{Ele. quark}} \) is the excited quark of the elementary quark (u(0) or d(0)).

| \( I_z^{\text{Baryon}} \), \( q^{\text{Name}} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |
|---|---|---|---|---|---|---|---|---|---|---|
| S | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | -1 |
| C | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| b | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| I | 1/2 | 1/2 | 3/2 | 3/2 | 3/2 | 3/2 | 0 | 1 | 1 | 1 |
| \( I_z^{\text{baryon}} \) | 1/2 | -1/2 | 3/2 | 1/2 | -1/2 | -3/2 | 0 | 1 | 0 | -1 |
| \( Q^{I_z,\text{quark}}_{\text{Ele. quark}} \) | \( u_N^+ \) | \( d_N^+ \) | \( u_\Delta^+ \) | \( d_\Delta^+ \) | \( d_\Delta^- \) | \( d_\Delta^0 \) | \( u_\Sigma^1 \) | \( d_\Sigma^0 \) | \( d_\Sigma^0 \) | \( d_\Sigma^0 \) |
| \( Q_q \) | \( \frac{2}{3} \) | \( -\frac{1}{3} \) | \( \frac{2}{3} \) | \( \frac{1}{3} \) | \( -\frac{1}{3} \) | \( \frac{2}{3} \) | \( \frac{1}{3} \) | \( -\frac{1}{3} \) | \( \frac{1}{3} \) | \( -\frac{1}{3} \) |

| \( I_z^{\text{Baryon}} \), \( q^{\text{Name}} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |
|---|---|---|---|---|---|---|---|---|---|---|
| S | -2 | -2 | 0 | 0 | -2 | -1 | -1 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| b | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 |
| I | 1/2 | 1/2 | 0 | 0 | 0 | 1/2 | 1/2 | 1 | 0 | -1 |
| \( I_z^{\text{baryon}} \) | 1/2 | -1/2 | 0 | 0 | 0 | 1/2 | -1/2 | 1 | 0 | -1 |
| \( Q^{I_z,\text{quark}}_{\text{Ele. quark}} \) | \( u_\Xi^+ \) | \( d_\Xi^+ \) | \( u_\Sigma^0 \) | \( d_\Omega^0 \) | \( u_\Xi^0 \) | \( d_\Xi^0 \) | \( u_\Sigma^0 \) | \( u_\Sigma^0 \) | \( u_\Sigma^0 \) | \( d_\Omega^1 \) |
| \( Q_q \) | \( \frac{2}{3} \) | \( -\frac{1}{3} \) | \( \frac{2}{3} \) | \( -\frac{1}{3} \) | \( \frac{2}{3} \) | \( -\frac{1}{3} \) | \( \frac{2}{3} \) | \( -\frac{1}{3} \) | \( \frac{2}{3} \) | \( -\frac{1}{3} \) |

where \( \text{Name of } Q^{I_z,\text{quark}}_{\text{quark}} \) is the name of the quark, \( I_z, \text{quark} \) is the z-component of the elementary quark (u(0) or d(0)).
isospin of the baryon \( q_{\text{Name}}^I q_{1q_2} \);

| The | Quark | Spectrum |
|-----|-------|----------|
| Elementary | quarks, \( u(0) \) and \( d(0) \), in vacuum, | \( m_u = Q_u = 0 \), \( m_d = Q_d = 0 \). |
| The two | accompany excited quarks |
| \( u' \)-quark | \( S = C = b = 0 \), \( I = s = \frac{1}{2} \), \( I_z = \frac{1}{2} \), \( Q = \frac{2}{3} \), \( m_u = 3 \) |
| \( d' \)-quark | \( S = C = b = 0 \), \( I = s = \frac{1}{2} \), \( I_z = -\frac{1}{2} \), \( Q = -\frac{1}{3} \), \( m_d = 7 \) |
| The energy | band excited quarks |
| \( q_N(930) \) | \( d_S(1110) \) | \( u_C(2270) \) | \( d_b(5530) \) |
| \( 1q_N(1200) \) | \( 1d_S^0(1390) \) | \( 1q_{\Sigma}(1200) \) | \( 2q_{\Xi}(1290) \) | \( u_C(2550) \) | \( d_b(9950) \) |
| \( 1q_N(1470) \) | \( 1d_S^0(1490) \) | \( 3q_{\Sigma}(1650) \) | \( 3q_{\Xi}(1470) \) | \( 2u_C(2750) \) | \( d_b(15810) \) |
| \( 2q_N(1830) \) | \( 2d_S^0(1830) \) | \( 2q_{\Sigma}(1920) \) | \( 3q_{\Xi}(1650) \) | \( u_C(4140) \) | \( d_b(23010) \) |
| \( 3q_N(1920) \) | \( 3d_S^0(1920) \) | \( 1q_{\Sigma}(2370) \) | \( 1q_{\Xi}(1830) \) | \( u_C(6490) \) | \( d_b(31850) \) |
| \( 1q_N(2010) \) | \( 2d_S^0(2010) \) | \( 3q_{\Xi}(2640) \) | \( 2q_{\Xi}(1920) \) | \( u_C(13590) \) |
| \( 2q_N(2190) \) | \( 2d_S^0(2120) \) | \( 5q_{\Xi}(3090) \) | \( 3q_{\Xi}(2010) \) | \( u_C(23570) \) |
| \( 2q_N(2640) \) | \( 2d_S(2550) \) | \( 4q_{\Xi}(2190) \) | \( u_C(36430) \) |
| \( 1q_N(2910) \) | \( 1d_S^0(2640) \) | \( 2q_{\Xi}(2370) \) |
| \( 1q_N(3360) \) | \( 4d_S^0(2730) \) | \( 3q_{\Xi}(2550) \) |
| \( 1q_{\Delta}(1290) \) | \( 1d_S(4370) \) | \( d_{\Omega}(1650) \) | \( d_{\Omega_{c}}(2730) \) | \( 1q_{\Xi}(2440) \) |
| \( 2q_{\Delta}(1650) \) | \( d_S(10230) \) | \( d_{\Omega}(2350) \) | \( d_{\Omega_{c}}(2750) \) | \( 1q_{\Xi}(2530) \) |
| \( 1q_{\Delta}(2010) \) | \( d_S(18970) \) | \( d_{\Omega}(2730) \) | \( d_{\Omega_{c}}(3670) \) | \( 1q_{\Xi}(2640) \) |
| \( 1q_{\Delta}(2370) \) | \( d_S(30590) \) | \( d_{\Omega}(2870) \) | \( 1q_{\Xi}(2730) \) |
| \( 4q_{\Delta}(2730) \) | \( d_S(45090) \) | \( d_{\Omega}(3810) \) | \( 1q_{\Xi}(2960) \) |
| \( 3q_{\Delta}(3090) \) | \( d_{\Omega}(7310) \) | (71)
IV The Spectrum of the Baryons

We have already found the quantum numbers (70) and the rest masses (71) of the excited quarks and the accompanying excited quarks u' and d' (5). Now we will recognize the three-quark systems (qq'q''). Using the sum laws, we can find the quantum numbers and masses of the three quark (qq'q'') systems first. Then, from the quantum numbers and the masses of the systems, we recognize the baryons.

A The Three Quark (qq'q'') Systems Are the Baryons

The baryon number of the three-quark system (qq'q''), from (71),

\[ B = B_q + B_{q^1} + B_{q^2} = 1. \]  

Thus the three-quark system is a baryon.

B The Rest Masses and the Quantum Numbers of the Three Quark (qq'q'') Systems

Now, we give some phenomenological formulas with which we can find the quantum numbers and rest masses of the three quark (qq'q'') systems, as shown in the following:

1. The quantum numbers of the system (qq'q'') are the sums of the constituent quarks (q, q', and q''). Since \( S_{q_1} = C_{q_1} = b_{q_1} = S_{q_2} = C_{q_2} = b_{q_2} = 0 \) from Hypothesis I,

\[ S_{qu'd'} = S(q), \quad C_{qu'd'} = C(q), \quad b_{qu'd'} = b(q), \quad \text{and} \quad Q_{qu'd'} = Q_q + Q_{q_1} + Q_{q_2}. \]  

2. The Isospin \( I_B \) of the system (qq'q'') is found by

\[ \vec{I}_B = \vec{I}_q + \vec{I}_{q_1} + \vec{I}_{q_2}. \]
Then, the z-component of the isospin of the baryon \((qq'1q'2)\):

| \(I_B\)     | \(I_{z,B}\)          |
|------------|----------------------|
| 3/2        | 3/2, 1/2, -1/2, -3/2 |
| 1          | 1, 0, -1             |
| 1/2        | 1/2, -1/2            |
| 0          | 0                    |

(75)

3. The top limit \((I_{max})\) of the isospin of the baryons on the symmetry axis is determined by

\[2I_{max}(axis) + 1 = 4 + S.\] (76)

From (39), we have:

for the axis \(\Delta\) and the axis \(D\), \(S = 0, I_{max} = 3/2\); (77)

for the axis \(\Lambda\) and the axis \(F\), \(S = -1, I_{max} = 1\); (78)

for the axis \(\Sigma\) and the axis \(G\), \(S = -2, I_{max} = 1/2\). (79)

4. The top limit \((I_{max})\) of the isospin of the baryons at a symmetry point is determined by the highest dimension \((D_{high})\) of the irreducible representations of the double point group \([17]\) at the point

\[2I_{max} + 1 = D_{high}.\] (80)

Since the highest dimension \(D_{high} = 4\) for the \(\Gamma\)-group, the \(H\)-group, the \(M\)-group, and the \(P\)-group, we get

\[I_{max} = 3/2, \text{ at } \Gamma, H, M, \text{ and } P.\] (81)
and from that the highest dimension $D_{\text{high}} = 2$ for the double point group $N$ [17], we know

$$I_{\text{max}} = 1/2, \text{ at the point } N. \tag{82}$$

5. The isospin $I_B$ and $I_{B,Z}$ of the baryon $(qq'_1q'_2)$ has already been given by the name of the excited quark $q_{\text{Name}}^{I_Z}$ in (70). The electric charge $Q_q$ and the $I_Z$ of the excited quark $Q_{\text{quark}}^{I_Z}$ have already been given by (70) also. Thus, the two accompanying excited quarks $q'_1$ and $q'_2$ are selected by the excited quark $q$ from $u'u'$, $u'd'$, and $d'd'$, according to the quantum numbers $I_Z$ and $Q$ of the baryon, using (83) and (84)

$$I_{Z,B} = I_{Z,q} + I_{Z,q'_1} + I_{Z,q'_2}, \tag{83}$$

$$Q_B = Q_q + Q_{q'_1} + Q_{q'_2}. \tag{84}$$
We show the results in the following list

| \( q^i \) | \( q^0_N \) | \( q^3_N \) | \( q^1_N \) | \( q^0_\Delta \) | \( q^3_\Delta \) | \( q^1_\Delta \) | \( q^0_\Omega \) | \( q^3_\Omega \) | \( q^0_\Xi \) | \( q^3_\Xi \) |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| S        | 0        | 0        | 0        | 0        | 0        | 0        | -1       | -1       | -1       | -1       |
| C        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| b        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| I        | 1/2      | 1/2      | 3/2      | 3/2      | 3/2      | 3/2      | 0        | 1        | 1        | 1        |
| \( u^{1z} (d^{1z}) \) | \( u^T_N \) | \( d^T_N \) | \( u^T_\Delta \) | \( d^T_\Delta \) | \( d^T_\Delta \) | \( d^T_\Delta \) | \( u^T_\Xi \) | \( d^T_\Xi \) | \( d^T_\Xi \) | \( d^T_\Xi \) |
| \( q_1 \) | \( u' \) | \( u' \) | \( u' \) | \( u' \) | \( u' \) | \( u' \) | \( u' \) | \( u' \) | \( u' \) | \( d' \) |
| \( q_2 \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) |
| \( I_{Z,B} \) | \( 1/2 \) | \( -1 \) | \( 3/2 \) | \( 1/2 \) | \( -1 \) | \( -3/2 \) | 0        | 1        | 0        | -1       |
| \( Q_B \) | 1        | 0        | 2        | 1        | 0        | -1       | 0        | 1        | 0        | -1       |
| \( B_{aryon} \) | \( N^+ \) | \( N^0 \) | \( \Delta^{++} \) | \( \Delta^+ \) | \( \Delta^0 \) | \( \Delta^- \) | \( \Lambda \) | \( \Sigma^+ \) | \( \Sigma^0 \) | \( \Sigma^- \) | \( \Omega^- \) |

| \( q^{i'} \) | \( q^{i'}_\Xi \) | \( q^{i'}_c \) | \( q^{0'}_c \) | \( q^{0'}_\Omega_c \) | \( q^{i'}_\Xi_c \) | \( q^{i'}_\Omega_c \) | \( q^{i'}_\Xi_c \) | \( q^{i'}_\Omega_c \) | \( q^{i'}_\Xi_c \) | \( q^{i'}_\Omega_c \) |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| S        | -2       | -2       | 0        | 0        | -2       | -1       | -1       | 0        | 0        | 0        |
| C        | 0        | 0        | 1        | 0        | 1        | 1        | 1        | 1        | 1        | 1        |
| b        | 0        | 0        | 0        | -1       | 0        | 0        | 0        | 0        | 0        | 0        |
| I        | 1/2      | 1/2      | 0        | 0        | 0        | 1/2      | 1/2      | 0        | -1       |
| \( I_Z \) | 1/2      | -1/2     | 0        | 0        | 0        | 1/2      | -1/2     | 1        | 0        | -1       |
| \( u^{1z} (d^{1z}) \) | \( u^T_c \) | \( d^T_c \) | \( u^T_b \) | \( d^T_b \) | \( d^T_b \) | \( u^T_b \) | \( d^T_b \) | \( u^T_b \) | \( d^T_b \) | \( d^T_b \) |
| \( q_1 \) | \( d' \) | \( d' \) | \( u \) | \( u \) | \( u \) | \( u \) | \( u \) | \( u \) | \( u \) | \( u \) |
| \( q_2 \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) | \( d' \) |
| \( I_{Z,B} \) | \( 1/2 \) | \( -1 \) | \( 0 \) | \( 0 \) | \( 1/2 \) | \( -1 \) | \( 0 \) | \( 1 \) | \( -1 \) |
| \( Q_B \) | 0        | -1       | 1        | 0        | 0        | 1        | 0        | 2        | 1        | 0        |
| \( B_{ar} \) | \( \Xi^0 \) | \( \Xi^- \) | \( \Lambda_c \) | \( \Lambda_b \) | \( \Omega^0_c \) | \( \Xi^+_c \) | \( \Xi^0_c \) | \( \Sigma^+_c \) | \( \Sigma^0_c \) |

(85)

6. The energy of the system \((qq'q_1)\) equals the sum of the energies of the excited quarks \(q, q', \) and \(q_1\):

\[
M_{(qq'q_1)} = m_q + m_{q'} + m_{q_1}.
\]  

(86)
For a quark \( q \) with isospin \( I_q \), adding two accompanying excited quarks \( q'_1 \) and \( q'_2 \) (one of \( u'u', u'd' \), and \( d'd' \)), the three-quark system \( qq'_1q'_2 \) may have \( I_{\text{baryon}} = I_q \pm 1 \) from \[74\]. This baryon \( (qq'_1q'_2) \) may have added energy. For simplicity, we assume

\[
\delta \varepsilon = 100S_G(q)[S_G(q)+\Delta I],
\] (87)

where \( S_G(q) = S + C + b \) of \( q \), \( \Delta I = |I_{\text{baryon}}-I_q|= + 1 \).

Using the formulae \[70\] - \[87\], we can find the baryons on the symmetry axes.

### B- 1 The Baryons On the \( \Delta \)-Axis (\( \Gamma-H \))

#### 1. The four-fold degenerate bands on the \( \Delta \)-axis (\( \Gamma-H \))

\[
\begin{align*}
E_H = 1 & \quad \vec{n} = (101,-101,011,0-11) \quad q_{\Delta}(1290) \quad \Delta(1300) \quad N(1300) \\
E_H = 2 & \quad \vec{n} = (110,1-10,-110,-1-10) \quad q_{\Delta}(1650) \quad \Delta(1660) \quad N(1660) \\
E_H = 2 & \quad \vec{n} = (10-1,-10-1,1-0,1-1) \quad q_{\Delta}(1650) \quad \Delta(1660) \quad N(1660) \\
E_H = 3 & \quad \vec{n} = (112,1-12,-112,-1-12) \quad q_{\Delta}(2010) \quad \Delta(2020) \quad N(2020) \\
E_H = 4 & \quad \vec{n} = (200,-200,020,0-20) \quad q_{\Delta}(2370) \quad \Delta(2380) \quad N(2380) \\
E_H = 5 & \quad \vec{n} = (121,1-21,-121,–1-21) \quad q_{\Delta}(2730) \quad \Delta(2740) \quad N(2740) \\
& \quad \vec{n} = (121,2-11,-121,-2-11) \quad q_{\Delta}(2730) \quad \Delta(2740) \quad N(2740) \\
E_H = 5 & \quad \vec{n} = (202,-202,022,0-22) \quad q_{\Delta}(2730) \quad \Delta(2740) \quad N(2740) \\
E_H = 5 & \quad \vec{n} = (013,0-13,103,-103) \quad q_{\Delta}(2730) \quad \Delta(2740) \quad N(2740) \\
\end{align*}
\] (88)

#### 2. The single bands on the axis \( \Delta(\Gamma-H) \) [7], (see Appendix B)

\[
\begin{align*}
E_H = 1 & \quad \Delta S = -1 \quad J_H = 1 \quad q_S(1390) \quad \Lambda(1400) \quad \Sigma(1400) \\
E_H = 4 & \quad \Delta S = +1 \quad J_H = 1 \quad q_C(2270) \quad \Lambda_C^+(2280) \quad \Sigma_C(2480) \\
E_H = 9 & \quad \Delta S = -1 \quad J_H = 2 \quad q_S(4370) \quad \Lambda(4380) \quad \Sigma(4380) \\
E_H = 16 & \quad \Delta S = +1 \quad J_H = 2 \quad q_C(6490) \quad \Lambda_C^+(6500) \quad \Sigma_C(6700) \\
E_H = 25 & \quad \Delta S = -1 \quad J_H = 3 \quad q_S(10230) \quad \Lambda(10240) \quad \Sigma(10240) \\
E_H = 36 & \quad \Delta S = +1 \quad J_H = 3 \quad q_C(13590) \quad \Lambda_C^+(13600) \quad \Sigma_C(13800) \\
\end{align*}
\] (89)
B- 2 The Baryons On the Axis $\Lambda$ (Γ-P)

\[ E_P = \frac{3}{4} \quad \vec{n} = (101,011,110) \quad q_\Sigma(1200) \quad \Sigma(1210) \quad \Lambda(1210) \]
\[ E_\Gamma = 2 \quad \vec{n} = (1-10,-110,01-1,\ldots) \quad q_\Sigma(1650) \quad \Sigma(1660) \quad \Lambda(1660) \]
\[ E_\Gamma = 2 \quad \vec{n} = (-10-1,0-1-1,1-10) \quad q_\Sigma(1650) \quad \Sigma(1660) \quad \Lambda(1660) \]
\[ E_P = \frac{11}{4} \quad \vec{n} = (020,002,200) \quad q_\Sigma(1920) \quad \Sigma(1930) \quad \Lambda(1930) \]
\[ E_P = \frac{11}{4} \quad \vec{n} = (121,211,112) \quad q_\Sigma(1920) \quad \Sigma(1930) \quad \Lambda(1930) \]
\[ E_\Gamma = 4 \quad \vec{n} = (0-20,-200,00-2) \quad q_\Sigma(2370) \quad \Sigma(2380) \quad \Lambda(2380) \]
\[ E_P = \frac{19}{4} \quad \vec{n} = (1-12,-112,21-1,\ldots) \quad q_\Sigma(2640) \quad \Sigma(2650) \quad \Lambda(2650) \]
\[ E_P = \frac{19}{4} \quad \vec{n} = (202,022,220) \quad q_\Sigma(2640) \quad \Sigma(2650) \quad \Lambda(2650) \]
\[ E_\Gamma = 6 \quad \vec{n} = (-211,2-1-1,2-1-1) \quad q_\Sigma(3090) \quad \Sigma(3100) \quad \Lambda(3100) \]
\[ E_\Gamma = 6 \quad \vec{n} = (-1-21,1-2-1,11-2) \quad q_\Sigma(3090) \quad \Sigma(3100) \quad \Lambda(3100) \]
\[ E_\Gamma = 6 \quad \vec{n} = (-1-2-1,-1-1-2,2-1-1) \quad q_\Sigma(3090) \quad \Sigma(3100) \quad \Lambda(3100) \]
\[ E_\Gamma = 6 \quad \vec{n} = (-211,2-1-1,2-1-1) \quad q_\Sigma(3090) \quad \Sigma(3100) \quad \Lambda(3100) \]

B- 3 The baryons On the Axis $\Sigma(\Gamma-N)$

1. The two-fold energy bands on the axis $\Sigma(\Gamma - N)$

\[ E_\Gamma = 2 \quad \vec{n} = (1-10,-110) \quad q_\Xi(1650) \quad \Xi(1660) \]
\[ E_N = \frac{5}{2} \quad \vec{n} = (200,020) \quad q_\Xi(1830) \quad \Xi(1840) \]
\[ E_\Gamma = 4 \quad \vec{n} = (002,00-2) \quad q_\Xi(2370) \quad \Xi(2380) \quad (91) \]
\[ \vec{n} = (-200,0-20) \quad q_\Xi(2370) \quad \Xi(2380) \]

2. The four-fold degenerate energy bands on the axis $\Sigma(\Gamma - N)$

\[ E_N = \frac{3}{2} \quad \vec{n} = (101,10-1,011,01-1) \quad 2 \times q_\Xi(1470) \quad 2 \Xi(1480) \]
\[ E_\Gamma = 2 \quad \vec{n} = (-101,-10-1,0-11,0-1-1) \quad 2 \times q_\Xi(1650) \quad 2 \Xi(1660) \quad (92) \]
\[ E_N = \frac{7}{2} \quad \vec{n} = (121,12-1,211,21-1) \quad 2 \times q_\Xi(2190) \quad 2 \Xi(2200) \]
3. The single energy bands on the axis $\Sigma(\Gamma-N)$ [7], (see Appendix B)

\[
\begin{align*}
E_N &= 1/2 \quad \vec{n} = (110) \quad q_s(1110) \quad \Lambda(1120) \\
E_\Gamma &= 2 \quad \vec{n} = (-1-10) \quad q_\Omega(1650) \quad \Omega^-(1660) \\
E_N &= 9/2 \quad \vec{n} = (220) \quad q_s(2550) \quad \Lambda(2560) \\
E_\Gamma &= 8 \quad \vec{n} = (-2-20) \quad q_\Omega(3810) \quad \Omega^-(3820) \\
E_N &= 25/2 \quad \vec{n} = (330) \quad q_b(5530) \quad \Lambda_b(5540) \\
E_\Gamma &= 18 \quad \vec{n} = (-3-30) \quad q_\Omega(7310) \quad \Omega^-(7320) \\
E_N &= 49/2 \quad \vec{n} = (440) \quad q_b(9950) \quad \Lambda^0_b(9960)
\end{align*}
\]

The baryons of the D-axis, the F-axis and the G-axis are shown in Appendix C.

C Comparing the Results

Using Tables 1-6, we compare the theoretical baryon spectrum of the BCC Quark Lattice Model with the experimental results [8]. In the comparison, we do not take into account the angular momenta of the experimental results. We assume that the small differences of the masses in the same group of baryons with the same quantum numbers and similar masses are from their different angular momenta. If we ignore this effect, their masses would be essentially the same. In the comparison, we use the baryon name to represent the intrinsic quantum numbers as shown in the second column of Table 1.

Table 1. The Ground States of the Baryons.
In the fourth column, \( R = \left( \frac{\Delta M}{M} \right) \% \).

The most important baryons are shown in Table 1. These baryons have relatively long lifetimes. They are the most important experimental results of the baryons. From Table 1, we can see that all theoretical intrinsic quantum numbers (\( I, S, C, b \) and \( Q \)) are the same as those in the experimental results. Also the theoretical mass values agree well with the experimental values.

Table 2. Two Kinds of Strange Baryons \( \Lambda \) and \( \Sigma \) (\( S = -1 \))

| Theory | Quantum. No | Experiment | R  | Life Time |
|--------|-------------|------------|----|-----------|
| Name(M) | S, C, b, I, Q | Name(M) |    | 10^{25} \text{years} |
| \( N^+(940) \) | 0, 0, 0, 1/2, 1 | p(938) | 0.2 | \( >10^{25} \text{years} \) |
| \( N^0(940) \) | 0, 0, 0, 1/2, 0 | n(940) | 0.0 | 885.7 s |
| \( \Lambda^0(1120) \) | -1, 0, 0, 0, 0 | \( \Lambda^0(1116) \) | 0.4 | \( 2.6 \times 10^{-10} \text{s} \) |
| \( \Sigma^+(1210) \) | -1, 0, 0, 1, 1 | \( \Sigma^+(1189) \) | 1.8 | \( 8.0 \times 10^{-10} \text{s} \) |
| \( \Sigma^0(1210) \) | -1, 0, 0, 1, 0 | \( \Sigma^0(1193) \) | 1.4 | \( 7.4 \times 10^{-20} \text{s} \) |
| \( \Sigma^-(1210) \) | -1, 0, 0, 1, -1 | \( \Sigma^-(1197) \) | 1.1 | \( 1.5 \times 10^{-10} \text{s} \) |
| \( \Xi^0(1300) \) | -2, 0, 0, 1/2, 0 | \( \Xi^0(1315) \) | 1.2 | \( 2.9 \times 10^{-10} \text{s} \) |
| \( \Xi^-(1300) \) | -2, 0, 0, 1/2, -1 | \( \Xi^-(1321) \) | 1.6 | \( 1.6 \times 10^{-10} \text{s} \) |
| \( \Omega^-(1660) \) | -3, 0, 0, 0, -1 | \( \Omega^-(1672) \) | 0.7 | \( 8.2 \times 10^{-10} \text{s} \) |
| \( \Lambda_c^+(2280) \) | 0, 1, 0, 0, 1 | \( \Lambda_c^+(2285) \) | 0.2 | \( 200 \times 10^{-15} \text{s} \) |
| \( \Xi_c^+(2450) \) | -1, 1, 0, 1/2, 1 | \( \Xi_c^+(2466) \) | 0.6 | \( 442 \times 10^{-15} \text{s} \) |
| \( \Xi_b^0(2450) \) | -1, 1, 0, 1/2, 0 | \( \Xi_b^0(2470) \) | 0.8 | \( 98 \times 10^{-15} \text{s} \) |
| \( \Omega_b^0(2750) \) | 0, 0, -1, 0, 0 | \( \Omega_b(2698) \) | 1.9 | \( 64 \times 10^{-15} \text{s} \) |
| \( \Lambda_b^0(5540) \) | 0, 0, -1, 0, 0 | \( \Lambda_b^0(5641) \) | 1.8 | \( 1.23 \times 10^{-12} \text{s} \) |
| \( \Sigma_{c+}^+(2480) \) | -1, 1, 0, 1, 2 | \( \Sigma_{c+}^+(2453) \) | 1.1 | \( \Gamma = 2.0 \text{ Mev} \) |
| \( \Sigma_{c-}^+(2480) \) | -1, 1, 0, 1, 1 | \( \Sigma_{c+}^+(2451) \) | 1.2 | \( \Gamma < 4.6 \text{ Mev} \) |
| \( \Sigma_b^0(2480) \) | -1, 1, 0, 1, 0 | \( \Sigma_{b+}^+(2452) \) | 1.2 | \( \Gamma = 1.6 \text{ Mev} \) |
| \( \Delta^{++}(1240) \) | 0, 0, 0, 3/2, 2 | \( \Delta^{++}(1232) \) | 0.7 | \( \Gamma = 120 \text{ Mev} \) |
| \( \Delta^{++}(1240) \) | 0, 0, 0, 3/2, 1 | \( \Delta^{+}(1232) \) | 0.7 | \( \Gamma = 120 \text{ Mev} \) |
| \( \Delta^{0}(1240) \) | 0, 0, 0, 3/2, 0 | \( \Delta^{0}(1232) \) | 0.7 | \( \Gamma = 120 \text{ Mev} \) |
| \( \Delta^{-}(1240) \) | 0, 0, 0, 3/2, -1 | \( \Delta^{-}(1232) \) | 0.7 | \( \Gamma = 120 \text{ Mev} \) |
| Theory | Experiment | $\Delta M/\%$ | Theory | Experiment | $\Delta M/\%$ |
|--------|------------|--------------|--------|------------|--------------|
| Λ(1120) | Λ(1116)    | 0.36         | Σ(1210) | Σ(1193)    | 1.4          |
| Λ(1400) | Λ(1405)    |              | Σ(1400) | Σ(1385)    |              |
| Λ(1500) | Λ(1520)    |              |        |            |              |
| Λ(1450) | Λ(1463)    | 0.9          | Σ(1400) | Σ(1385)    | 1.1          |
| Λ(1660) | Λ(1600)    |              | Σ(1660) | Σ(1670)    |              |
| Λ(1660) | Λ(1670)    |              | Σ(1660) | Σ(1750)    |              |
| Λ(1660) | Λ(1690)    |              | Σ(1660) | Σ(1775)    |              |
| Λ(1660) | Λ(1653)    | 0.4          | Σ(1660) | Σ(1714)    | 3.2          |
| Λ(1840) | Λ(1800)    |              | Σ(1930) | Σ(1915)    |              |
| Λ(1840) | Λ(1810)    |              | Σ(1930) | Σ(1940)    |              |
| Λ(1930) | Λ(1820)    |              |        |            |              |
| Λ(1930) | Λ(1830)    |              |        |            |              |
| Λ(1930) | Λ(1890)    |              |        |            |              |
| Λ(1894) | Λ(1830)    | 3.5          | Σ(1930) | Σ(1928)    | .10          |
| Λ(2020) | Λ(2100)    |              | Σ(2020) | Σ(2030)    | .50          |
| Λ(2020) | Λ(2110)    |              |        |            |              |
| Λ(2130) | Λ(2130)    |              |        |            |              |
| Λ(2071) | Λ(2105)    | 1.4          |        |            |              |
| Λ(2380) | Λ(2350)    |              | Σ(2130) | Σ(2080)*   |              |
| Λ(2380) | Λ(2350)    |              | Σ(2130) | Σ(2250)    |              |
| Λ(2380) | Λ(2350)    | 1.3          | Σ(2130) | Σ(2165)    | 1.6          |
| 2Λ(2560) | Λ(2585)*   | 1.0          | Σ(2380) | Σ(2455)*   | 3.0          |
| 6Λ(2650) | Λ(2585)*   |              | 6Σ(2650) | Σ(2620)*   | 1.1          |

*Evidences of existence for these baryons are only fair, they are not listed in the Baryon Summary Table [8].

Two kinds of the strange baryons Λ and Σ are compared in Table 2. Their theoretical and experimental intrinsic quantum numbers are the same. The theoretical masses of the baryons Λ and Σ agree well with the experimental results.
| Theory | Experiment | $\Delta M$% | Theory | Experiment | $\Delta M$% |
|--------|------------|-------------|--------|------------|-------------|
| $2N(1210)$ | $N(1300)$ | $\Delta(1210)$ | $1\Delta(1300)$ | $\Delta(1232)$ |
| $\bar{\bar{N}}(1240)$ | $\bar{\bar{N}}(1498)$ | $1.2$ | $\bar{\bar{N}}(1689)$ | $\bar{\bar{N}}(1640)$ | $1.2$ |
| $N(1480)$ | $N(1440)$ | $\Delta(1660)$ | $\Delta(1600)$ | $\Delta(1620)$ | $\Delta(1700)$ |
| $N(1660)$ | $N(1650)$ | $\Delta(1930)$ | $\Delta(2420)$ | $\Delta(2750)^*$ | $0.4$ |
| $\bar{\bar{N}}(1930)$ | $\bar{\bar{N}}(1923)$ | $0.4$ | $\bar{\bar{N}}(2220)$ | $\bar{\bar{N}}(2240)$ | $0.9$ |
| $N(2200)$ | $N(2190)$ | $\Delta(2380)$ | $\Delta(2420)$ | $1.7$ |
| $N(2380)$ | $N(2600)$ | $\Delta(2740)^*$ | $\Delta(2750)^*$ | $0.4$ |
| $3N(2650)$ | $N(2700)^*$ | $4\Delta(2740)$ | $\Delta(2950)^*$ | $5.1$ |

*Evidences are fair, they are not listed in the Baryon Summary Table [8].

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A comparison of the theoretical results with the experimental results of the unflavored baryons $N$ and $\Delta$ is made in Table 3. From Table 3, we can see that the intrinsic quantum numbers of the theoretical results are exactly the same as those of the experimental results. Also the theoretical masses of the baryons $N$ and $\Delta$ agree well with the experimental results. The theoretical results $N(1210)$ and $N(1300)$ are not found in the experiment. We believe that they are covered up by the experimental baryon $\Delta(1232)$ because of the following reasons: (1) they are unflavored baryons with the same $S$, $C$ and $b$; (2) the width (120 Mev) of $\Delta(1232)$ is very large, and the baryons $N(1210)$ and $N(1300)$ both fall within the width region of $\Delta(1232)$; and (3) the average mass (1255 Mev) of $N(1210)$ and $N(1300)$ is essentially the same as the mass (1232 Mev) of $\Delta(1232)$ ($\Gamma = 120$ Mev).

Table 4. The Baryons $\Xi$ and the Baryons $\Omega$

| Theory       | Experiment | $\frac{\Delta M}{M}$% | Theory       | Experiment | $\frac{\Delta M}{M}$% |
|--------------|------------|------------------------|--------------|------------|------------------------|
| $2\Xi(1300)$ | $\Xi(1318)$| 1.4                    | $\Omega(1660)$| $\Omega(1672)$| 0.7                    |
| $3\Xi(1480)$ | $\Xi(1530)$| 3.3                    | $\Omega(2360)$| $\Omega(2250)$|                        |
| $3\Xi(1660)$ | $\Xi(1690)$| 1.5                    | $\Omega(2360)$| $\Omega(2380)^*$|                        |
| $1\Xi(1840)$ | $\Xi(1820)$| 1.1                    | $\Omega(2360)$| $\Omega(2470)^*$|                        |
| $2\Xi(1930)$ | $\Xi(1950)$| 1.0                    | $\Omega(2360)$| $\Omega(2367)$| 0.3                    |
| $3\Xi(2020)$ | $\Xi(2030)$| 0.5                    | $\Omega(2740)$|              |                        |
| $4\Xi(2200)$ | $\Xi(2250)^*$| 2.2                  | $\Omega(2880)$|              |                        |
| $2\Xi(2380)$ | $\Xi(2370)^*$| 0.4                  | $\Omega(3820)$|              |                        |
| $3\Xi(2560)$ | $\Xi(2500)^*$| 0.3                  |              |              |                        |

*Evidences of existence for these baryons are only fair, they are not listed in the Baryon Summary Table [8].

The theoretical intrinsic quantum numbers of the baryons $\Xi$ and $\Omega$ are the same as the experimental results (see Table 4). The theoretical masses of the baryons $\Xi$ and $\Omega$ are compatible with the experimental results.
Table 5. Charmed $\Lambda^+_c$ and Bottom $\Lambda^0_b$ Baryons

| Theory      | Experiment | $\Delta M/\%$ | Theory      | Experiment | $\Delta M/\%$ |
|-------------|------------|--------------|-------------|------------|--------------|
| $1\Lambda^+_c(2280)$ | $\Lambda^+_c(2285)$ | 0.22 | $1\Lambda^0_b(5540)$ | $\Lambda^0_b(5641)$ | 1.8 |
| $1\Lambda^+_c(2560)$ | $\Lambda^+_c(2593)$ | 1.2 | $1\Lambda^0_b(9960)$ |              |              |
| $1\Lambda^+_c(2760)$ | $\Lambda^+_c(2625)$ |              | $1\Lambda^0_b(15820)$ |              |              |
| $1\Lambda^+_c(2760)$ | $\Lambda^+_c(2880)^*$ |              |              |              |              |
| $1\Lambda^+_c(2760)$ | $\Lambda^+_c(2723)$ | 1.4 | $1\Lambda^0_b(15820)$ |              |              |
| $1\Lambda^+_c(2800)$ |              |              |              |              |              |

The charmed and bottom baryons $\Lambda^+_c$ and $\Lambda^0_b$ can be found in Table 5. The experimental masses of the charmed baryons ($\Lambda^+_c$) and bottom baryons ($\Lambda^0_b$) coincide with the theoretical results.

Table 6. Charmed Strange Baryon $\Xi_c$, $\Sigma_c$ and $\Omega_c$

| Theory      | Experiment | $\Delta M/\%$ | Theory      | Experiment | $\Delta M/\%$ |
|-------------|------------|--------------|-------------|------------|--------------|
| $1\Xi_c(2450)$ | $\Xi_c(2469)$ | 0.8 | $1\Sigma_c(2480)$ | $\Sigma_c(2500)$ |              |
| $1\Xi_c(2540)$ | $\Xi_c(2577)$ | 1.4 | $1\Sigma_c(2480)$ | $\Sigma_c(2488)$ | 0.3 |
| $1\Xi_c(2650)$ | $\Xi_c(2645)$ | 0.2 | $1\Omega_c(2740)$ | $\Omega_c(2760)$ |              |
| $1\Xi_c(2740)$ | $\Xi_c(2790)$ |              | $1\Omega_c(2750)$ | $\Omega_c(2698)$ | 1.9 |
| $1\Xi_c(2970)$ | $\Xi_c(2815)$ |              |              |              |              |
| $1\Omega_c(2750)$ | $\Omega_c(2880)$ |              |              |              |              |

*Evidences of existence for these baryons are only fair, they are not listed in the Baryon Summary Table [8].

Finally, we compare the theoretical results with the experimental results for the charmed strange baryons $\Omega_c$, $\Xi_c$ and $\Sigma_c$ in Table 6. Their intrinsic quantum numbers are all matched completely, and their masses also agree well.

In summary, the BCC Quark Lattice Model explains all baryon experimental intrinsic quantum numbers and masses. Virtually no experimentally confirmed baryon is not included in the model. The angular momenta and the parities of the baryons, however, are not included in this paper. They depend on the wave functions of the energy bands.
We will discuss them in some later papers.

V Predictions

A New Quarks

1. There is an energy band excited quark spectrum \( \overline{\Pi} \). Some of the quarks have already been discovered: \( q_N(930), d_S(1110), u_C(2270), d_b(5530), q_\Sigma(1200), q_\Xi(1290), \Omega(1650), d_S(1390), q_\Sigma(1650) \) and so forth.

2. There is always an excited \( q \) in a baryon. It has more than 98% of the mass of the baryon. We hope experimental physasts to find them.

3. The new quarks \( d_S(4370), u_C(6490) \) and \( d_b(9950) \) still need to be found.

B New Baryons

\[
\begin{align*}
I=0 & \quad C=1 & \quad Q=1 & \quad \Lambda_C^+(6500) \\
I=0 & \quad S=-1 & \quad Q=0 & \quad \Lambda^0(4380) \\
I=0 & \quad b=-1 & \quad Q=0 & \quad \Lambda_b^0(9960)
\end{align*}
\]

VI Discussion

1. The constant \( \alpha = \frac{\hbar^2}{2m_q a^2} = 360 \text{ Mev} \),

with a lattice constant \( a \leq 10^{-18} \text{ m} \). Thus the bare mass \( (m_q) \) of the elementary quarks

\[
\begin{align*}
m_q &= \frac{\hbar^2}{2 \times 720 \text{ Mev} \times a^2} \\
&\geq \frac{43.90 \times 10^{-68} (\text{J s})^2}{720 \times 10^9 \times 1.602 \times 10^{-19} \text{J} \times 10^{-36} \text{m}^2} \\
m_q &\geq 3.8 \times 10^{-21} \text{kg} = 2.27 \times 10^6 m_p,
\end{align*}
\]

39
is much larger than the excited quark masses. This ensures that the Schrödinger equation (2) is a good approximation of the special quark Dirac equation (1).

2. The paper has shown that the u-quark and the c-quark are the excited states of the elementary u(0)-quark and that the d-quark, the s-quark and the b-quark are the excited states of the elementary d(0)-quark. The u(0)-quark and the d(0)-quark have a SU(2) symmetry (u(0) and d(0)). Therefore SU(3) (u, d and s), SU(4) (u, d, s and c) and SU(5) (u, d, s, c and b) are correct although there are large differences in masses between the quarks. In fact, the SU(3), SU(4) and SU(5) are natural expansions of the SU(2). Since the bare masses of the elementary quarks u(0) and d(0) (95) are huge

\[ m_{u(0)} (\text{or } m_{d(0)}) \geq 2.27 \times 10^6 m_p = 2.13 \times 10^9 \text{Mev.} \] (96)

Thus the bare masses (taking the absolutely empty space as the zero energy point) of the quarks (u, d, s, c, b, u’ and d’) are huge too:

\[
\begin{align*}
    m_{u}^{\text{bare}} &= m_{u(0)} + 930, \\
    m_{d}^{\text{bare}} &= m_{d(0)} + 930, \\
    m_{c}^{\text{bare}} &= m_{u(0)} + 2270, \\
    m_{s}^{\text{bare}} &= m_{d(0)} + 1110, \\
    m_{b}^{\text{bare}} &= m_{d(0)} + 5530, \\
    m_{u'}^{\text{bare}} &= m_{u(0)} + 3, \\
    m_{d'}^{\text{bare}} &= m_{d(0)} + 7.
\end{align*}
\] (97)

From (96) and (97), we can see that the bare masses of the quarks (u, d, s, c, b, u’ and d’) are essentially the same. This is a rigorous physical basis of the SU(3), SU(4), SU(5) and so forth symmetries. We had long thought that SU(4) and SU(5) do not have a rigorous physical basis since the differences of the quark masses are too large. Considering the bare masses of these quarks, we now believe that SU(4) and SU(5) symmetries really exist.

3. At distance scales \( \leq 10^{-18} \text{ m} \), we can see the BCC quark lattice and can deduce the rest masses (71) and the intrinsic quantum numbers (I, S, C, b and q) (70) of the quarks. The BCC Quark Lattice Model not only provides a rigorous physical basis for
the Quark Model, but also opens a door to study more fundamental structure than the Standard Model.

4. At the distance scales $>10^{-18}m$, although we cannot see the quark lattice, we can see the u(0)-quark Dirac sea and the d(0)-quark Dirac sea (the lattice looks like Dirac sea). Sometimes we can also see the s-quark, the c-quark and the b-quark (inside baryons and mesons); and from the Dirac sea concept, we guess that there will be an s-quark Dirac sea, a c-quark Dirac sea and a b-quark Dirac sea also. Since we cannot see the quark lattice (we can only see the Dirac seas), we cannot deduce the masses and the intrinsic quantum numbers (we can measurement them by experiment). Naturally we think that the quarks (u, d, s, c and b) are all independent elementary particles. The Standard Model is a reasonably excellent approximation to nature at distance scales as small as $10^{-18}m$ [1]. Thus there is no contradiction between the Standard Model and the BCC Quark lattice Model; however the BCC Quark Lattice Model does provide a physical basis for the Standard Model.

5. After the discovery of superconductors, we now understand the vacuum material. In a sense, the vacuum material (skeleton– the BCC quark lattice) works like a superconductor. Since the transition temperature is much higher than the temperature at the center of the sun, all phenomena that we can see are under the transition temperature. Thus there are no electric or mechanical resistances to any particle or to any physical body moving inside the vacuum material. Moving inside it, they look as if they are moving in completely empty space. The vacuum material is a super superconductor.

6. A baryon is composed of three quarks in the Quark Model. The masses of the quarks ($m_u = 1$ to $4.5$ Mev, $m_d = 5$ to $8.5$ Mev, $m_s = 80$ to $155$ Mev, $m_c = 1.0$ to $1.4$ Gev and $m_b = 4.0$ to $4.5$ Gev) [12] are too small to build a stable baryon. Using the sum laws and the quark masses of the Quark Model, we find the theoretical masses of the most important baryons. We find that the theoretical baryon masses of the Quark Model are too small to match the experimental masses. Thus there may be one quark with a large
mass inside the baryon. According to the BCC Quark Lattice Model, this quark with a large mass really exists in a baryon—it is excited quark \((q)\) \((98)\); the quarks with small masses are the accompanying excited quarks \((u'\) and \(d')\) \([5]\). The theoretical masses of the baryons of the BCC Quark Lattice Model agree well with the experimental results. We list the theoretical results and the experimental results \([8]\) for both the Quark Model and the BCC Quark Lattice Model as follows \((98)\):

| Baryon | Quark. | Mass | BCC | Mass | Exper. |
|--------|--------|------|-----|------|--------|
| \(p(938)\) | uud | 13 | uu’d’ | 940 | 938 |
| \(n(940)\) | udd | 16 | du’d’ | 940 | 940 |
| \(\Lambda_{(1116)}\) | uds | 128 | d, u’d’ | 1120 | 1116 |
| \(\Sigma^0_{(1193)}\) | uds | 128 | \(q_\Sigma^0 u'd'\) | 1210 | 1193 |
| \(\Xi^0_{(1315)}\) | uss | 238 | \(q_{\Xi^0} u'd'\) | 1300 | 1315 |
| \(\Omega(1672)\) | sss | 353 | \(q_\Lambda u'd'\) | 1660 | 1672 |
| \(\Lambda_{C(2285)}\) | udc | 1210 | \(q_{\Xi^0} u'd'\) | 2280 | 2285 |
| \(\Lambda_{b(5624)}\) | udb | 4260 | \(q_{\Xi^0} u'd'\) | 5540 | 5624 |

Thus, the experimental results support the baryon model \((qu'd')\) of the BCC Quark Lattice Model.

### VII Conclusions

1. There are only two kinds of the elementary quarks \((u(0)\) and \(d(0)\)) in the vacuum state; other quarks \((u, d, s(d_s), c(u_c), b(d_b), u', d'\) and so forth) are all the excited states (from the vacuum) of the elementary quarks \((u(0)\) or \(d(0)\)).

2. The rest masses of the quarks are the energy minima of the energy bands that represent the quarks. The baryon spectrum agrees well with experimental results. This shows that the theoretical rest masses of the quarks are correct.

3. The strange number, the charmed number and the bottom number are the products of the body center cubic periodic symmetries and the fluctuation of the BCC quark lattice.

4. The BCC Quark Lattice Model not only provide a physical basis for the Quark Model \([18]\), but also opens a door to study the more nature at distance scales \(\leq 10^{-18}\)
m and to look for the more fundamental theory than the Standard Model.

5. Due to the existence of the vacuum material, all observable particles are constantly affected by the vacuum material (the vacuum state quark lattice). Thus some laws of statistics (such as fluctuation) cannot be ignored.

6. The classic approximation (2) is really a good approximation of the special quark Dirac equation (1) for deducing the rest masses and the intrinsic quantum numbers of the quarks, but it cannot deduce the spin angular momentums of the quarks. Thus we need to resolve the special quark Dirac equation (1) to find the angular momentum and to improve the mass spectrum of the quarks that we have deduced using the classic approximation.

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Appendix A: The Body Center Cubic Quark Lattice in the Vacuum

According to Dirac’s sea concept [4], there is an electron-Dirac sea, a $\mu$-lepton Dirac sea, a $\tau$-lepton Dirac sea, a $u$-quark Dirac sea, a $d$-quark Dirac sea, an $s$-quark Dirac sea, a $c$-quark Dirac sea, a $b$-quark Dirac sea and so forth in the vacuum. All of these Dirac seas are in the same space, at any location, that is, at any physical space point. These particles will interact with one another and form the perfect physical vacuum material. Some kinds of particles, however, do not play an important role in forming the physical vacuum material. First, the main force which makes and holds the structure of the physical vacuum material must be the strong interactions, not the electromagnetic interactions. Hence, in considering the structure of the vacuum material, we leave out the Dirac seas of those particles which do not have strong interactions ($e$, $\mu$ and $\tau$). Secondly, the physical vacuum material is super stable, hence we also omit the Dirac seas which can only make unstable baryons (the $s$-quark, the $c$-quark and the $b$-quark). Finally, there are only two kinds of possible particles left: the vacuum state $u(0)$-quarks and the vacuum state $d(0)$-quarks. There are super strong attractive forces between the $u(0)$-quarks and the $d(0)$-quarks (colors) that will make and hold the densest structure of the vacuum material.

According to solid state physics [19], if two kinds of particles (with radius $R_1 < R_2$) satisfy the condition $1 > R_1/R_2 > 0.73$, the densest structure is the body center cubic crystal [6]. We know the following: first, the $u(0)$-quarks and the $d(0)$-quarks are not exactly the same, thus $R_u \neq R_d$; second, they are very close to each other (the same isospin with different $I_z$), thus $R_u \approx R_d$. Hence, if $R_u < R_d$ (or $R_d < R_u$), we have $1 > R_u/R_d > 0.73$ (or $1 > R_d/R_u > 0.73$). Therefore, we conjecture that the vacuum state $u(0)$-quarks and $d(0)$-quarks construct a body center cubic quark lattice in the vacuum.
IX Appendix B: The Energy Bands

In Table 7-12, $E_{\text{Start}}$ ($E_{\text{end}}$) means the starting (end) energy ($\varepsilon$) of the energy band, it is the lowest (the highest) energy of the energy band (bands). $d$ (in the third column) is the number of the energy bands with the same energy. We mark the energy band using $n_1 n_2 n_3$ ($n_1$, $n_2$, $n_3$ of (14)) and $\overline{n}_i = - n_i$.

Table 7. The Energy Bands of the $\Delta$-Axis

| $E_{\text{Start}}$ | $\varepsilon$ | $d$ | Energy Band ($n_1 n_2 n_3$, ...) | $E_{\text{end}}$ |
|---------------------|---------------|-----|----------------------------------|------------------|
| $E_{\Gamma} = 0$    | 930           | 1   | (000)                            | $E_{\Gamma} = 1$ |
| $E_{\Gamma} = 1$    | 1290          | 4   | (101,010,110,011)                | $E_{\Gamma} = 2$ |
| $E_{\Gamma} = 1$    | 1290          | 1   | (002)                            | $E_{\Gamma} = 4$ |
| $E_{\Gamma} = 2$    | 1650          | 4   | (110,010,101,010)                | $E_{\Gamma} = 3$ |
| $E_{\Gamma} = 2$    | 1650          | 4   | (101,001,001,010)                | $E_{\Gamma} = 5$ |
| $E_{\Gamma} = 3$    | 2010          | 4   | (112,012,121,012)                | $E_{\Gamma} = 6$ |
| $E_{\Gamma} = 4$    | 2370          | 4   | (200,020,020,020)                | $E_{\Gamma} = 5$ |
| $E_{\Gamma} = 4$    | 2370          | 1   | (002)                            | $E_{\Gamma} = 9$ |
| $E_{\Gamma} = 5$    | 2730          | 8   | (121,121,121,121,121,211,211,211) | $E_{\Gamma} = 6$ |
| $E_{\Gamma} = 5$    | 2730          | 4   | (202,022,022,022)                | $E_{\Gamma} = 8$ |
| $E_{\Gamma} = 5$    | 2730          | 4   | (013,013,013,013)                | $E_{\Gamma} = 10$|
| $E_{\Gamma} = 6$    | 3090          | 8   | (121,121,121,121,121,211,211,211) | $E_{\Gamma} = 9$ |
| $E_{\Gamma} = 6$    | 3090          | 4   | (112,112,112,112)                | $E_{\Gamma} = 11$|
| $E_{\Gamma} = 9$    | 4170          | 1   | (004)                            | $E_{\Gamma} = 16$|
| $E_{\Gamma} = 16$   | 6690          | 1   | (004)                            | $E_{\Gamma} = 25$|
| $E_{\Gamma} = 25$   | 9930          | 1   | (006)                            | $E_{\Gamma} = 36$|
| $E_{\Gamma} = 36$   | 13890         | 1   | (006)                            | $E_{\Gamma} = 49$|
| $E_{\Gamma} = 49$   | 17640         | 1   | (008)                            | $E_{\Gamma} = 64$|
| $E_{\Gamma} = 64$   | 23970         | 1   | (008)                            | $E_{\Gamma} = 81$|
| $E_{\Gamma} = 81$   | 31020         | 1   | (009)                            | $E_{\Gamma} = 100$|
Table 8. The Energy Bands of the Σ-Axis

| $E_{\text{Start}}$ | $\varepsilon$ | d | Energy Band ($n_1n_2n_3$) | $E_{\text{end}}$ |
|---------------------|---------------|---|---------------------------|-----------------|
| $E_\Gamma = 0$      | 930           | 1 | (000)                     | $E_{N}=1/2$     |
| $E_{N}=1/2$         | 1110          | 2 | (1,1,0)                   | $E_{\Gamma}=2$  |
| $E_{N}=3/2$         | 1470          | 4 | (101,101,011,011)         | $E_{\Gamma}=2$  |
| $E_{\Gamma}=2$      | 1650          | 2 | (101,101,011,011)         | $E_{N}=5/2$     |
| $E_{\Gamma}=2$      | 1650          | 4 | (101,101,011,011)         | $E_{N}=7/2$     |
| $E_{\Gamma}=2$      | 1650          | 2 | (1,1,0)                   | $E_{N}=9/2$     |
| $E_{N}=5/2$         | 1830          | 2 | (200,020)                 | $E_{\Gamma}=4$  |
| $E_{N}=7/2$         | 2190          | 4 | (121,121,211,211)         | $E_{\Gamma}=6$  |
| $E_{\Gamma}=4$      | 2370          | 2 | (002,002)                 | $E_{N}=9/2$     |
| $E_{\Gamma}=4$      | 2370          | 2 | (200,020)                 | $E_{N}=13/2$    |
| $E_{N}=9/2$         | 2550          | 2 | (112,112)                 | $E_{\Gamma}=6$  |
| $E_{N}=9/2$         | 2550          | 1 | (2,2,0)                   | $E_{\Gamma}=8$  |
| $E_{\Gamma}=8$      | 3810          | 1 | (2,2,0)                   | $E_{N}=25/2$    |
| $E_{N}=25/2$        | 5430          | 1 | (3,3,0)                   | $E_{\Gamma}=18$ |
| $E_{\Gamma}=18$     | 7410          | 1 | (3,3,0)                   | $E_{N}=49/2$    |
| $E_{N}=49/2$        | 9750          | 1 | (4,4,0)                   | $E_{\Gamma}=32$ |
| $E_{\Gamma}=32$     | 12450         | 1 | (4,4,0)                   | $E_{N}=81/2$    |
| $E_{N}=81/2$        | 15510         | 1 | (5,5,0)                   | $E_{\Gamma}=50$ |
| $E_{\Gamma}=50$     | 18930         | 1 | (5,5,0)                   | $E_{N}=121/2$   |
| $E_{N}=121/2$       | 22710         | 1 | (6,6,0)                   | $E_{\Gamma}=72$ |
| $E_{\Gamma}=72$     | 26850         | 1 | (6,6,0)                   | $E_{N}=169/2$   |
| ...                 | ...           | ... | ...                      | ...             |
### Table 9. The Energy Bands of the Λ-Axis

| $E_{\text{Start}}$ | $\epsilon$ | d | Energy Band ($n_1n_2n_3$, ...) | $E_{\text{End}}$ |
|---------------------|-------------|---|---------------------------------|-----------------|
| $E_\Gamma$ = 0      | 930         | 1 | (000)                           | $E_p$ = 3/4     |
| $E_p$ = 3/4         | 1200        | 3 | (101,011,110)                   | $E_\Gamma$ = 2  |
| $E_\Gamma$ = 2      | 1650        | 6 | (1\overline{T}0,1\overline{T}0,01\overline{T}, 0\overline{T}1,10\overline{T},\overline{T}01) | $E_p$ = 11/4    |
| $E_\Gamma$ = 2      | 1650        | 3 | (1\overline{T}0,01\overline{T},\overline{T}01) | $E_p$ = 19/4    |
| $E_p$ = 11/4        | 1920        | 3 | (020,002,200)                   | $E_\Gamma$ = 4  |
| $E_p$ = 11/4        | 1920        | 3 | (121,211,112)                   | $E_\Gamma$ = 6  |
| $E_\Gamma$ = 4      | 2370        | 3 | (020,002,002)                   | $E_p$ = 27/4    |
| $E_p$ = 19/4        | 2640        | 6 | (1\overline{T}2,1\overline{T}1,2\overline{T}, 2\overline{T}1,12\overline{T},\overline{T}21) | $E_\Gamma$ = 6  |
| $E_p$ = 19/4        | 2640        | 3 | (202,022,220)                   | $E_\Gamma$ = 8  |
| $E_\Gamma$ = 6      | 3090        | 6 | (211,2\overline{T},1\overline{T}2,11\overline{T},\overline{T}2\overline{T},\overline{T}1) | $E_p$ = 27/4    |
| $E_\Gamma$ = 6      | 3090        | 6 | (1\overline{T}2,1\overline{T}1,1\overline{T}2,1\overline{T}1,2\overline{T},2\overline{T}1) | $E_p$ = 35/4    |
| $E_\Gamma$ = 6      | 3090        | 3 | (1\overline{T}1,1\overline{T}2,2\overline{T}) | $E_p$ = 43/4    |
| $E_p$ = 27/4        | 3360        | 1 | (222)                           | $E_\Gamma$ = 12 |
| ...                 | ...         | ...| ...                             | ...            |

### Table 10. The Energy Bands of the D-Axis

| $E_{\text{Start}}$ | $\epsilon$ | d | Energy Band ($n_1n_2n_3$, ...) | $E_{\text{End}}$ |
|---------------------|-------------|---|---------------------------------|-----------------|
| $E_N$ = 1/2         | 1110        | 2 | (000,110)                       | $E_p$ = 3/4     |
| $E_p$ = 3/4         | 1200        | 2 | (101,011)                       | $E_N$ = 3/2     |
| $E_N$ = 3/2         | 1470        | 2 | (1\overline{T}0, 01\overline{T}) | $E_p$ = 11/4    |
| $E_N$ = 5/2         | 1830        | 4 | (1\overline{T}0,1\overline{T}0,020,200) | $E_p$ = 11/4    |
| $E_p$ = 11/4        | 1920        | 4 | (1\overline{T}0,1\overline{T}1,211,121) | $E_N$ = 7/2     |
| $E_p$ = 11/4        | 1920        | 2 | (002,112)                       | $E_N$ = 9/2     |
| $E_N$ = 7/2         | 2190        | 4 | (1\overline{T}2,2\overline{T}1,1\overline{T}0,0\overline{T}1) | $E_p$ = 19/4    |
| $E_N$ = 9/2         | 2550        | 2 | (220,1\overline{T}0)           | $E_p$ = 19/4    |
| $E_N$ = 9/2         | 2550        | 2 | (1\overline{T}2,002)           | $E_p$ = 19/4    |
| $E_p$ = 19/4        | 2640        | 2 | (1\overline{T}1,2\overline{T}1) | $E_N$ = 11/2    |
| $E_p$ = 19/4        | 2640        | 4 | (1\overline{T}2,1\overline{T}2,202,022) | $E_N$ = 13/2    |
| $E_N$ = 11/2        | 2910        | 2 | (2\overline{T}1,\overline{T}2\overline{T}) | $E_p$ = 27/4    |
| ...                 | ...         | ...| ...                             | ...            |
Table 11. The Energy Bands of the F-Axis (the P-H axis)

| $E_{Start}$ | $\varepsilon$ | d | Energy Band (n_1n_2n_3...) | $E_{end}$ |
|-------------|--------------|---|-----------------------------|----------|
| $E_{P}=3/4$ | 1200         | 3 | (000,011,101)               | $E_{H}=1$|
| $E_{P}=3/4$ | 1200         | 1 | (110)                       | $E_{H}=3$|
| $E_{H}=1$   | 1290         | 3 | (002,101,01T)               | $E_{P}=11/4$|
| $E_{P}=11/4$| 1920         | 3 | (112,1T0,1T0)               | $E_{H}=3$|
| $E_{H}=1$   | 1920         | 6 | (01T,10T,121,211,020,200)   | $E_{H}=5$|
| $E_{H}=3$   | 2010         | 3 | (1T0,1T2,1T2)               | $E_{P}=19/4$|
| $E_{H}=3$   | 2010         | 1 | (1T2)                       | $E_{P}=27/4$|
| $E_{P}=19/4$| 2640         | 6 | (202,022,1T21,2T1,0T1,1T0)  | $E_{H}=5$|
| $E_{P}=19/4$| 2640         | 3 | (220,2T1,12T)               | $E_{H}=9$|
| $E_{H}=5$   | 2730         | 6 | (020,300,311,1T1,013,103)   | $E_{P}=27/4$|
| $E_{H}=5$   | 2730         | 6 | (022,302,2T1,2T2,1T3,0T3)   | $E_{P}=35/4$|
| ...         | ...          | ...| ...                         | ...      |

Table 12. The Energy Bands of the G-Axis

| $E_{Start}$ | $\varepsilon$ | d | Energy Band (n_1n_2n_3,...) | $E_{end}$ |
|-------------|--------------|---|-----------------------------|----------|
| $E_{N}=1/2$ | 1110         | 2 | (000, 110)                  | $E_{M}=1$|
| $E_{M}=1$   | 1290         | 2 | (101, 10T)                  | $E_{N}=3/2$|
| $E_{M}=1$   | 1290         | 2 | (200, 1T0)                  | $E_{N}=5/2$|
| $E_{N}=3/2$ | 1470         | 2 | (011, 01T)                  | $E_{M}=3$|
| $E_{N}=5/2$ | 1830         | 2 | (020, 1T0)                  | $E_{M}=5$|
| $E_{M}=3$   | 2010         | 4 | (0T1, 0T1, 211, 21T)        | $E_{N}=7/2$|
| $E_{M}=3$   | 2010         | 2 | (2T1, 2T1)                  | $E_{N}=11/2$|
| $E_{N}=7/2$ | 2190         | 4 | (1T01,1T01, 121, 12T)       | $E_{M}=5$|
| $E_{N}=9/2$ | 2550         | 6 | (112, 112, 002, 002, 220,1T0) | $E_{M}=5$|
| $E_{M}=5$   | 2730         | 6 | (202, 202, 1T2, 1T2, 310, 020) | $E_{N}=13/2$|
| $E_{M}=5$   | 2730         | 3 | (301,30T,121,12T)           | $E_{N}=15/2$|
| $E_{M}=5$   | 2730         | 2 | (310,2T0)                   | $E_{N}=17/2$|
| $E_{N}=11/2$| 2910         | 2 | (1T21,1T2)                  | $E_{M}=9$|
| ...         | ...          | ...| ...                         | ...      |

50
Appendix C: The Quarks and the Baryons on the D-axis, the F-axis and the G-axis

For the three symmetry axes (the D-axis (P-N), the F-axis (P-H) and the G-axis (M-N)) that are on the surfaces of the first Brillouin zone (see Fig. 1), the energy bands with the same energy may have asymmetric $\vec{n}$ values (see Fig. (3b), (4a) and (4b)). For symmetric $\vec{n}$, we give a definition: a group of $\vec{n} = (n_1, n_2, n_3)$ values is said to be symmetric if any two $\vec{n}$ values in the group can transform into each other by various permutations (change component order) and by changing the sign “±” (multiplied by “−1”) of the components (one, two or three). Otherwise they are asymmetric. For example, $(-2, -1, 3)$ and $(-3, 2, 1)$ are symmetric; $(-3, 0, 2)$ and $(-3, 0, 1)$ are asymmetric. For these energy bands (‘degeneracy’) with the same energy but asymmetric $\vec{n}$ values, if the ‘deg’ > the rotary fold ($R$) of the symmetry axis,

$$\text{‘deg’} > R,$$

the ‘degeneracy’ will be divided to $\gamma$-subdegeneracies first (the first kind of division, $K = 0$),

$$\gamma = \text{‘deg’} / R.$$  \hspace{1cm} (100)

There is not a change of the strange number and energy for the first kind of division. Each subgroup of the $\gamma$-subgroups has $R$ energy bands and the same strange number with the symmetrery axis. If the $\vec{n}$ values are symmetric, using (35), we can find the isospin values. If the $\vec{n}$ values are asymmetric, the subgroup will be divided into two sub-subgroups with symmetric (or single) $\vec{n}$ values (the second kind of division, $K = 1$) again. Then, using (35), we can find the isospin values of the sub-subgroups. For the each sub-subgroup, using (52)

$$S = S_{axis} + \Delta S,$$  \hspace{1cm} (101)
we can find the strange numbers. The $\Delta S$ can be found using (54) and (55). If $\text{Sign}(\vec{n}) = 0$

$$\Delta S = (-1)^{S_{\text{Axis}}}.$$  \hspace{1cm} (102)

The fluctuation of the strange number will be accompanied by an energy change (Hypothesis II). We assume that the change of the energy (perturbation energy) is proportional to $\Delta S$ and a number, $J$, representing the energy level with an asymmetric $\vec{n}$ values, as a phenomenological formula:

$$\Delta \varepsilon=(-1)^S200(J-2-K)\Delta S, \hspace{0.5cm} J= (R-SK-2)+(1,2,3,...) \hspace{1cm} (103)$$

where $K$ is the division number of the energy bands and $R$ is the symmetric rotation number of the symmetry axis. For a single energy band, $J$ will take 1, 2, and so forth from the lowest energy band to higher ones for each of the two end points of the axes respectively.

There are three energy bands ($\vec{n} = (000)$, $\vec{n} = (100)$ and $\vec{n} = (200)$) that have already been recognized inside the Brillouin zones. The bands ($\vec{n} = (000)$, $\vec{n} = (100)$ and $\vec{n} = (200)$) on the surfaces of the Brillouin zones are the same quarks:

| $\vec{n}$   | Bands (Inside Brillouin zone) | Bands (on Surface ) | Quark           |
|-------------|-------------------------------|---------------------|-----------------|
| (000)       | $E_{\Gamma}(0) \rightarrow E_{N}(\frac{3}{2})$, $E_{p}(\frac{3}{2})$, $E_{H}(1)$ | $E_{M}(1) \leftarrow E_{N}(\frac{3}{2}) \rightarrow E_{p}(\frac{3}{2}) \rightarrow E_{H}(1)$ | $q_{N}(930)$ |
| (100)       | $E_{N}(\frac{3}{2}) \rightarrow E_{\Gamma}(2)$ | $E_{M}(1) \leftarrow E_{N}(\frac{3}{2}) \rightarrow E_{p}(\frac{3}{2}) \rightarrow E_{H}(3)$ | $d_{S}(1110)$ |
| (200)       | $E_{H}(1) \rightarrow E_{\Gamma}(2)$ | $E_{M}(1) \rightarrow E_{N}(\frac{5}{2})$, $E_{H}(1) \rightarrow E_{p}(\frac{3}{2})$ | $d_{S}(1390)$ |

(104)
A  The Axis D(P-N)

From (43), the D-axis has $S = 0$. For low energy level, there are four-fold degenerate energy bands and two-fold degenerate energy bands on the axis (see Fig. (3b)).

A- 1  The Quarks and the Baryons on the Four-fold Energy Bands

We can see that each four-fold degenerate energy band has four symmetric $\pi^\pm$ values. They can be divided into two groups (37). Each of them has two symmetric $\pi^\pm$ values. Using (35), for the two-fold degenerate energy bands, we get $I = 1/2$; $I_{Z,Baryon} = 1/2$, $-1/2$; $Q = 2/3$, $-1/3$ from (16) and (17). Thus, for the four-fold degenerate energy bands, we have $q_N(m) = [u^\pm_N(m), d^\pm_N(m)]$. Using (35), for a $q_N(m)$, we have a $N(m+m_{q_1}+m_{q_2})$ and a $\Delta(m+m_{q_1}+m_{q_2})$ from (33) at the point P; we have a $N(m+m_{q_1}+m_{q_2})$ only from (32) at the point N:

$$
\begin{align*}
\text{E}_N &= 5/2 \quad (1\overline{1}0,\overline{1}0,020,200) \quad 1830 \quad J_N = 2 \quad K = 0 \\
\Delta S &= 0 \quad (1\overline{1}0,\overline{1}0) \quad q_N(1830) \quad N(1840) \\
\Delta S &= 0 \quad (020,200) \quad q_N(1830) \quad N(1840) \\
\text{E}_P &= 11/4 \quad (1\overline{1}01,01,211,121) \quad 1920 \quad J_P = 1 \quad K = 0 \\
\Delta S &= 0 \quad (1\overline{1}01,01) \quad q_N(1920) \quad N(1930) \quad \Delta(1930) \\
\Delta S &= 0 \quad (211,121) \quad q_N(1920) \quad N(1930) \quad \Delta(1930) \\
\text{E}_N &= 7/2 \quad (12\overline{2}1,21\overline{2},0\overline{1}0,0\overline{1}0) \quad 2190 \quad J_N = 3 \quad K = 0 \quad (105) \\
\Delta S &= 0 \quad (12\overline{2}1,21\overline{2}) \quad q_N(2190) \quad N(2200) \\
\Delta S &= 0 \quad (0\overline{1}01,0\overline{1}0) \quad q_N(2190) \quad N(2200) \\
\text{E}_P &= 19/4 \quad (1\overline{2}12,1\overline{2}1,220,022) \quad 2640 \quad J_P = 2 \quad K = 0 \\
\Delta S &= 0 \quad (1\overline{2}12,1\overline{2}1) \quad q_N(2640) \quad N(2650) \quad \Delta(2650) \\
\Delta S &= 0 \quad (202,022) \quad q_N(2640) \quad N(2650) \quad \Delta(2650) \\
\end{align*}
$$

...  

...  

...  

...  

...
A- 2 The Quarks and Baryons on the Two-fold Energy Bands

From (33), (56), (102) and (103), we have ($\Delta \varepsilon = 200(J-2)\Delta S$, $J = 1, 2, 3, ...$):

\[
E_N = \frac{1}{2} \begin{array}{c}
(000,110) \\
(000) \text{ from (104)}
\end{array}, \quad J_N = 1, \quad K = 0 \\
\Delta S = -1 \begin{array}{c}
(110) \text{ from (104)} \\
1110 q(930) \quad N(940)
\end{array}
\]
\[
E_p = \frac{3}{4} \begin{array}{c}
(101,011) \\
\end{array}, \quad q(1200) \quad N(1210) \quad \Delta(1210)
\]
\[
E_N = \frac{3}{2} \begin{array}{c}
(10\bar{1}, 01\bar{1}) \\
\end{array}, \quad q(1470) \quad N(1480)
\]
\[
E_p = \frac{11}{4} \begin{array}{c}
(002,112) \\
\end{array}, \quad J_p = 1, \quad K = 0 \\
\Delta S = -1 \begin{array}{c}
(002) \quad \Lambda(1230) \quad \Sigma(2130)
\end{array}
\]
\[
E_N = \frac{9}{2} \begin{array}{c}
(220,1\bar{1}0) \\
\end{array}, \quad J_N = 2, \quad K = 0 \\
\Delta S = +1 \begin{array}{c}
(1\bar{1}0) \quad q(2550) \quad \Lambda_c(2560)
\end{array}
\]
\[
E_p = \frac{19}{4} \begin{array}{c}
(1\bar{2}1,2\bar{1}1) \\
\end{array}, \quad q(2640) \quad N(2650) \quad \Delta(2650)
\]
\[
E_N = \frac{11}{2} \begin{array}{c}
(2\bar{1}1,1\bar{2}1) \\
\end{array}, \quad q(2910) \quad N(2920)
\]
\[
E_p = \frac{27}{4} \begin{array}{c}
(103,013) \\
\end{array}, \quad q(3360) \quad N(3370) \quad \Delta(3370)
\]

B The Axis F(P-H)

The axis is a three-fold symmetric axis, $S = -1$ from (14). There are six-fold energy bands, three-fold energy bands and single energy bands on the axis (see Fig. 4(a)). Using Fig. 4(a), we get:

B- 1 The Quarks and Baryons on the Single Energy Bands

For the single bands, the strange number $S = -1$, the isospin $I = 0$ from (35), and electric charge $Q = -1/3$ from (47). Each single energy band represents an excited quark
qs (d^0_S) with S = -1, I = 0, and Q = -1/3:

\[
\begin{align*}
E_P &= 3/4 & \pi &= (110) \text{ (104)} & E &= 1110 & d^0_S(1110) & \Lambda(1120) \\
E_H &= 3 & \pi &= (\bar{T}T2) & E &= 2010 & d^0_S(2010) & \Lambda(2020)
\end{align*}
\]

B- 2 The Quarks and Baryons on the Three-fold Energy Bands

From [33], [57], [102] and [103], we have \( \Delta \varepsilon = -200(J-2)\Delta S \) \( J = 2, 3, 4, \ldots \):

\[
\begin{align*}
E_P &= 3/4 & (000, 011, 101) & 1200 & J_P = 1 & K = 0 & \text{from (33)} \\
& & (000) & q_N(930) & N(940) \\
\Delta S &= +1 & (011, 101) & 1400 & q_N(1200) & N(1210) & \Delta(1210) \\
E_H &= 1 & (002, \bar{T}01, 0\bar{T}1) & 1290 & J_H = 1 & K = 0 \\
\Delta S &= 0 & (002) & q_s(1390) & \Lambda(1400) & \Sigma(1400) \\
\Delta S &= -1 & (\bar{T}01, 0\bar{T}1) & 1290 & q_\Xi(1290) & \Xi(1300) \\
E_P &= 11/4 & (112, 1\bar{T}0, \bar{T}10) & 1920 & J_P = 2 & K = 0 \\
\Delta S &= 0 & (112) & q_s(1920) & \Lambda(1930) & \Sigma(1930) \\
\Delta S &= -1 & (1\bar{T}0, \bar{T}10) & 1920 & q_\Xi(1920) & \Xi(1930) \\
E_H &= 3 & (\bar{T}10, \bar{T}12, 1\bar{T}2) & 2010 & J_H = 2 & K = 0 \\
\Delta S &= 0 & (\bar{T}10) & q_s(2010) & \Lambda(2020) & \Sigma(2020) \\
\Delta S &= +1 & (21\bar{T}, 12\bar{T}) & 2010 & q_N(2010) & N(2020) & \Delta(2020) \\
E_P &= 19/4 & (220, 21\bar{T}, 12\bar{T}) & 2640 & J_P = 3 & K = 0 \\
\Delta S &= 0 & (220) & q_s(2640) & \Lambda(2650) & \Sigma(2650) \\
\Delta S &= +1 & (21\bar{T}, 12\bar{T}) & 2440 & q_\Xi_c(2440) & \Xi_c(2450) \\
E_P &= 27/4 & (130, 310, 11\bar{Z}) & 3360 & J_P = 4 & K = 0 \\
\Delta S &= 0 & (11\bar{Z}) & q_s(3360) & \Lambda(3370) & \Sigma(2650) \\
\Delta S &= +1 & (130, 310) & 2960 & q_\Xi_c(2960) & \Xi_c(2970) \\
\end{align*}
\]

\ldots
The Quarks and Baryons on the Six-fold Energy Bands

From (100), (57), (102) and (103), we have \( \Delta \varepsilon = -200(J-3)\Delta S \quad J = 3,4,5,... \)

\[
E_{P}=11/4 \quad (01\mathrm{T},10\mathrm{T},121, \\
211, 020, 200) \quad 1920 \quad J_{P}= 2 \quad K=0
\]

\[
\Delta S= 0 \quad (01\mathrm{T},10\mathrm{T},121) \quad 1920 \quad J_{P}= 2 \quad K=1
\]

\[
\Delta S= 0 \quad (121) \quad 1920 \quad q_{S}(1920) \quad \Lambda(1930) \quad \Sigma(1930)
\]

\[
\Delta S= -1 \quad (01\mathrm{T},10\mathrm{T}) \quad 1920 \quad q_{\Xi}(1920) \quad \Xi(1930)
\]

\[
\Delta S= 0 \quad (211, 020, 200) \quad 1920 \quad J_{P}= 2 \quad K=1
\]

\[
\Delta S= 0 \quad (211) \quad 1920 \quad q_{S}(1920) \quad \Lambda(1930) \quad \Sigma(1930)
\]

\[
\Delta S= 1 \quad (020, 200) \quad 1920 \quad q_{N}(1920) \quad N(1930) \quad \Delta(1930)
\]

\[
E_{P}=19/4 \quad (202,022,\overline{1}21, \\
2\overline{1}1,0\overline{1}1,0\overline{1}1) \quad 2640 \quad J_{P}= 3 \quad K=0
\]

\[
\Delta S= 0 \quad (202,022,2\overline{1}1) \quad 2640 \quad J_{P}= 3 \quad K=1
\]

\[
\Delta S= +1 \quad (202,022) \quad 2640 \quad q_{\Xi_{C}}(2640) \quad q_{\Xi_{C}}(2650)
\]

\[
\Delta S= 0 \quad (\overline{1}21) \quad 2640 \quad q_{S}(2640) \quad \Lambda(2650) \quad \Sigma(2650)
\]

\[
\Delta S= 0 \quad (2\overline{1}1,0\overline{1}1,0\overline{1}1) \quad 2640 \quad J_{P}= 3 \quad K=1
\]

\[
\Delta S= -1 \quad (01\overline{1},10\overline{1}) \quad 2640 \quad q_{\Xi}(2640) \quad \Xi(2650)
\]

\[
\Delta S= 0 \quad (\overline{2}1) \quad 2640 \quad q_{S}(2640) \quad \Lambda(2650) \quad \Sigma(2650)
\]

\[
E_{H}= 5 \quad (020,\overline{2}00,\overline{2}11, \\
1\overline{2}1,013,103) \quad 2730 \quad J_{H}= 3 \quad K=0
\]

\[
\Delta S= 0 \quad (020,2\overline{2}0,2\overline{1}1) \quad 2730 \quad J_{H}= 3 \quad K=1
\]

\[
\Delta S= 0 \quad (\overline{2}11) \quad 2730 \quad q_{S}(2730) \quad \Lambda(2740) \quad \Sigma(2740)
\]

\[
\Delta S= -1 \quad (0\overline{2}0,\overline{2}00) \quad 2730 \quad q_{\Xi}(2730) \quad \Xi(2740)
\]

\[
\Delta S= 0 \quad (1\overline{2}1,013,103) \quad 2730 \quad J_{H}= 3 \quad K=1
\]

\[
\Delta S= +1 \quad (1\overline{2}1) \quad 2730 \quad q_{S}(2730) \quad \Lambda(2740) \quad \Sigma(2740)
\]

\[
\Delta S= 0 \quad (013,103) \quad 2730 \quad q_{\Xi_{C}}(2730) \quad \Xi_{C}(2730)
\]

\[
E_{H}= 5^2 \quad (022,\overline{2}02,2\overline{1}1, \\
1\overline{2}1,013,103) \quad 2730 \quad J_{H}= 4 \quad K=0
\]

\[
\Delta S= 0 \quad (0\overline{2}2,202,2\overline{1}1) \quad 2730 \quad J_{H}= 4 \quad K=1
\]

\[
\Delta S= -1 \quad (0\overline{2}2,\overline{2}02) \quad 2930 \quad q_{\Xi}(2930) \quad \Xi(2940)
\]

\[
\Delta S= 0 \quad (\overline{2}11) \quad 2730 \quad q_{S}(2730) \quad \Lambda(2740) \quad \Sigma(2740)
\]

\[
\Delta S= 0 \quad (\overline{1}21,0\overline{1}3,103) \quad 2730 \quad J_{H}= 3 \quad K=1
\]

\[
\Delta S= +1 \quad (0\overline{1}3,103) \quad 2530 \quad q_{\Xi_{C}}(2530) \quad \Xi_{C}(2540)
\]

\[
\Delta S= 0 \quad (\overline{1}21) \quad 2730 \quad q_{S}(2730) \quad \Lambda(2740) \quad \Sigma(2740)
\]
C  The Axis G(M-N)

The axis G(M-N) is a two-fold symmetric axis, S = -2 from \( \text{[100]} \). There are two-fold, four-fold and six-fold energy bands on the axis (see Fig. 4(b)). Using Fig. 4(b), we get:

C-1  The Quarks and Baryons on the Two-fold Energy Bands

From \( \text{[100]}, \text{[102]}, \text{[110]} \) and \( \text{[110]} \), we have \( \Delta \varepsilon = 200(J-2)\Delta S, \ J = 1, 2, 3, 4, \ldots \):

| \( E_N \) | \( J_N \) | \( K \) | \( \Delta S \) | \( \Delta S \) |
|----------|------|-----|-------|-------|
| \( 1/2 \) | \( (000,110) \) | \( 1 \) | \( 0 \) | \( +1 \) | \( \text{[100]} \) 930 | \( q_N(930) \) | \( N(940) \) |
| \( 1 \) | \( (00) \) | \( 1 \) | \( 0 \) | \( +1 \) | \( \text{[100]} \) 1110 | \( q_S(1110) \) | \( \Lambda(1120) \) |
| \( 1 \) | \( (101,10\bar{1}) \) | \( 1290 \) | \( q_\Xi(1290) \) | \( \Xi(1300) \) |
| \( 1 \) | \( (200,1\bar{1}0) \) | \( 1290 \) | \( J_M = 1 \) | \( K = 0 \) | \( \text{[102]} \) 1390 | \( q_S(1390) \) | \( \Lambda(1400) \) |
| \( 1 \) | \( (1\bar{1}0) \) | \( 1490 \) | \( J_M = 1 \) | \( \text{[102]} \) 1470 | \( q_\Xi(1470) \) | \( \Xi(1480) \) |
| \( 3/2 \) | \( (011,01\bar{1}) \) | \( 1470 \) | \( J_N = 2 \) | \( K = 0 \) | \( \text{[110]} \) 1830 | \( q_\Xi(1830) \) | \( \Lambda(1840) \) |
| \( 5/2 \) | \( (020,1\bar{1}0) \) | \( 1830 \) | \( J_N = 2 \) | \( \text{[110]} \) 2010 | \( q_\Xi(2010) \) | \( \Xi(2020) \) |
| \( 3 \) | \( (2\bar{1}1,2\bar{1}0) \) | \( 2010 \) | \( J_M = 3 \) | \( K = 0 \) | \( \text{[110]} \) 2730 | \( q_\Xi(2730) \) | \( \Lambda(2740) \) |
| \( 5 \) | \( (310,2\bar{1}0) \) | \( 2730 \) | \( J_M = 3 \) | \( \text{[110]} \) 2910 | \( q_\Xi(2910) \) | \( \Xi(2920) \) |

\[
\begin{align*}
E_N &= 1/2 \quad (000,110) \\
J_N &= 1 \quad K = 0 \\
\Delta S &= +1 \\
\Delta S &= +1
\end{align*}
\]

\[
\begin{align*}
E_M &= 1 \quad (101,10\bar{1}) \\
E_M &= 1 \quad (200,1\bar{1}0) \\
\Delta S &= +1 \quad (00) \\
\Delta S &= +1 \quad (1\bar{1}0) \\
E_N &= 3/2 \quad (011,01\bar{1}) \\
E_N &= 5/2 \quad (020,1\bar{1}0) \\
\Delta S &= +1 \quad (020) \\
\Delta S &= +1 \quad (\bar{1}, 1, 0) \\
E_M &= 3 \quad (2\bar{1}1,2\bar{1}0) \\
E_M &= 5 \quad (310,2\bar{1}0) \\
\Delta S &= +1 \quad (310) \\
\Delta S &= +1 \quad (2\bar{1}0) \\
E_N &= 11/2 \quad (\bar{1}21,\bar{1}2\bar{1}) \\
\end{align*}
\]
C- 2 The Quarks and Baryons on the Four-fold Energy Bands

From (100), (57), (102) and (103), we have:

\[
\begin{align*}
E_M &= 3 \quad (0\bar{T}1,0\bar{T}1,211,21\bar{T}) \quad 2010 \quad J_M = 2 \quad K = 0 \\
\Delta S &= 0 \quad (0\bar{T}1,0\bar{T}1) \quad 2010 \quad q(2010) \quad \Xi(2020) \\
\Delta S &= 0 \quad (211,21\bar{T}) \quad 2010 \quad q(2010) \quad \Xi(2020) \\
E_N &= 7/2 \quad (\bar{T}01,\bar{T}0\bar{T}, 121, 12\bar{T}) \quad 2190 \quad J_N = 3 \quad K = 0 \\
\Delta S &= 0 \quad (\bar{T}01,\bar{T}0\bar{T}) \quad 2190 \quad q(2190) \quad \Xi(2200) \\
\Delta S &= 0 \quad (121, 12\bar{T}) \quad 2190 \quad q(2190) \quad \Xi(2200) \\
E_M &= 5 \quad (301,30\bar{T},121,12\bar{T}) \quad 2730 \quad J_M = 3 \quad K = 0 \\
\Delta S &= 0 \quad (301,30\bar{T}) \quad 2730 \quad q(2730) \quad \Xi(2740) \\
\Delta S &= 0 \quad (121,12\bar{T}) \quad 2730 \quad q(2730) \quad \Xi(2740) \\
\end{align*}
\]
The Quarks and Baryons on the Six-fold Energy Bands

From (100), (57), (102) and (103), we have:

| $E_N = 9/2$ | $\begin{bmatrix} 112,112,002,002, \\ 220, \bar{T}10 \end{bmatrix}$ | 2550 | $J_N = 4$ | $K = 0$ |
| $\Delta S = 0$ | (112,112) | 2550 | $q_\Xi(2550)$ | $\Xi(2560)$ |
| $\Delta S = 0$ | (002,002) | 2550 | $q_\Xi(2550)$ | $\Xi(2560)$ |
| $\Delta S = 0$ | (220, \bar{T}10) | 2550 | $J_N = 4$ | $K = 1$ |
| $\Delta S = +1$ | (220) | 2750 | $q_{\Omega}(2750)$ | $\Omega_C(2760)$ |
| $\Delta S = -1$ | (\bar{T}10) | 2350 | $q_\Omega(2350)$ | $\Omega(2360)$ |

| $E_M = 5$ | $\begin{bmatrix} 202,202,1T2,1T2, \\ 310, 020 \end{bmatrix}$ | 2730 | $J_M = 3$ | $K = 0$ |
| $\Delta S = 0$ | (1T2,1T2) | 2730 | $q_\Xi(2730)$ | $\Xi(2740)$ |
| $\Delta S = 0$ | (202,202) | 2730 | $q_\Xi(2730)$ | $\Xi(2740)$ |
| $\Delta S = 0$ | (310,020) | 2730 | $J_M = 3$ | $K = 1$ |
| $\Delta S = +1$ | (310) | 2730 | $q_{\Omega}(2730)$ | $\Omega_C(2740)$ |
| $\Delta S = -1$ | (020) | 2730 | $q_\Omega(2730)$ | $\Omega(2740)$ |

| $E_N = 13/2$ | $\begin{bmatrix} 112,112,022,022, \\ 130, \bar{2}00 \end{bmatrix}$ | 3270 | $J_N = 5$ | $K = 0$ |
| $\Delta S = 0$ | (112,112) | 3270 | $q_\Xi(3270)$ | $\Xi(3280)$ |
| $\Delta S = 0$ | (022,022) | 3270 | $q_\Xi(3270)$ | $q_\bar{\Xi}(3280)$ |
| $\Delta S = 0$ | (130,\bar{2}00) | 3270 | $J_N = 5$ | $K = 1$ |
| $\Delta S = +1$ | (130) | 3670 | $q_{\Omega}(3670)$ | $\Omega_C(3680)$ |
| $\Delta S = -1$ | (\bar{2}00) | 2870 | $q_\Omega(2870)$ | $\Omega(2880)$ |

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