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Minimal-Approximation-Based Adaptive Event-Triggered Control of Switched Nonlinear Systems with Unknown Control Direction

Yumeng Cao 1,*, Ning Zhao 1, Ning Xu 2, Xudong Zhao 3 and Fawaz E. Alsaadi 4

1 College of Control Science and Engineering, Bohai University, Jinzhou 121013, China
2 College of Information Science and Technology, Bohai University, Jinzhou 121013, China
3 Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian 116024, China
4 Department of Information Technology, Faculty of Computing and Information Technology, King Abdulaziz University, Jeddah 22254, Saudi Arabia
* Correspondence: ycao0998@gmail.com

Abstract: In this paper, the adaptive neural network event-triggered tracking problem is investigated for a class of uncertain switched nonlinear systems with unknown control direction and average dwell time switching. To reduce the communication network traffic, an event-triggering mechanism based on the tracking error is explored in the controller-to-actuator channel. Additionally, the minimal approximation technology, which designs virtual control laws as the unavailable intermediate signals, is introduced to reduce the difficulty of the controller design process. Compared with the existing adaptive backstepping designs using the filtering technology, the virtual controllers are recurred into a lumped nonlinear function to settle the explosion of complexity, and one neural network is employed in the recursive process. Meanwhile, a boundedness lemma on Nussbaum function is given to address the unknown control direction under the minimal approximation design framework. The stability of the overall closed-loop system is rigorously proved by the Lyapunov stability theory, and the rationality of the proposed strategy is verified by a simulation example. According to the proposed event-triggered mechanism, 81.25% of the communication resources are saved in the simulation example.

Keywords: switched nonlinear systems; minimal approximation; neural network; event-triggered control; unknown control direction

1. Introduction

In modern decades, switched systems were extensively applied in autonomous underwater vehicle [1,2], power network [3,4], dual drive [5,6], and networked control system [7,8]. Switched systems are a special class of hybrid systems that are composed of a series of discrete-time/continuous-time subsystems and a switching signal that decides the current active subsystem. Recently, a lot of adaptive control algorithms for switched linear systems [9–11] were proposed and extended to switched nonlinear systems [12–14]. In [9], the authors developed an adaptive asymptotic control protocol for a class of switched linear systems by the common Lyapunov function approach. Additionally, the adaptive tracking problem of second-order switched nonlinear systems with arbitrary switchings was investigated by using the multiple Lyapunov functions method [12]. At the same time, neural networks [13,15] or fuzzy logic systems [16,17] were used as function approximators in the adaptive backstepping designs, then the adaptive control protocol for uncertain switched nonlinear systems was designed. However, the above-mentioned backstepping designed methodologies are unable to solve the explosion of complexity introduced by the repeat differentiations of virtual controllers, which makes a great challenge in calculating the actual control input.
Facing this challenge, some scholars in the area employed the filtering technologies [18–21] to remove the repeat differentiation processes. As stated in [19], an adaptive dynamic surface control protocol using first-order filters to approximate the virtual control laws was applied for uncertain switched nonlinear systems. On the basis of dynamic surface control, an adaptive tracking control protocol based on command filters [22] was proposed for uncertain nonlinear systems to address this problem, in which an error compensation method was established to eliminate a part of the filtering errors. Note that the filtering errors were not completely eliminated in the above designs. With this in mind, a minimum approximation method was proposed to address the explosion of complexity and remove the filtering errors [23]. The method modifies the coordinate transformation to design a group of un-implementable virtual control signals that are recursed into a lumped nonlinearity in the last step, then one can eliminate the repeat differentiations for virtual controllers. Recently, the authors proposed several minimal-approximation-based adaptive control algorithms for nonlinear time-delay systems [24], interconnected strict-feedback nonlinear systems [25], and interconnected time-delay systems [26]. In addition, the authors in [27] developed a distributed tracking controller for uncertain nonlinear multi-agent systems under the minimal function approximation (MFA) framework. Nevertheless, the above-mentioned MFA methods only consider the time-triggered manner [28], which may incur the unnecessary waste of resources.

To reduce the depletion of communication resources, an event-triggered control (ETC) strategy has been proposed and widely used in the resource-limited networked systems [29,30]. Under the ETC framework, the updates of control signals are determined by the designed triggering conditions. Therefore, such updates are aperiodic, which can effectively save the communication resources. In [31,32], event-triggered adaptive control strategies were developed for linear systems, and were actively introduced into nonlinear systems by scholars in [33,34]. In addition, the authors in [35] designed an adaptive tracking control protocol for uncertain switched nonlinear systems under the ETC framework. Meanwhile, a neural-networked adaptive event-triggered controller was developed for uncertain switched nonlinear systems subjected to unknown control directions [36]. Nevertheless, the existing ETC schemes can not be extended to the MFA-based adaptive ETC design of uncertain switched nonlinear systems due to the following two difficulties:

1. When the MFA technology is taken into account, the design methodologies in [22,36] can not be used to address the issue of unknown control direction. To be specific, the un-implementable virtual control laws suffer from a difficulty on exploiting the traditional Nussbaum function methods [22,36] which are considered as a celebrated remedy for dealing subjected to unknown control gains;
2. The existing ETC approaches considering the controller-to-actuator channel often adopt the relative threshold strategy [36,37]. However, since the coordinate transformation errors $z_i, i = 2, 3, \ldots, n$, are unknown under the MFA framework, it is not feasible to use the above threshold strategy.

Motivated by the above considerations, the purpose of this paper is to study an MFA-based adaptive ETC strategy for uncertain switched nonlinear systems with unknown control gain. The primary contributions and innovations of the presented scheme in this paper are concluded as follows:

1. Considering that filtering errors are commonly presented in the backstepping designs including the explosion of complexity [20,21], the MFA technique is thus introduced to the proposed scheme, where the explosion of complexity is solved by designing a group of un-implementable virtual control laws, rather than using the filtering technique. In addition, the scheme no longer needs to employ multiple function approximators in the recursive process, which improves the computational efficiency for the actual control inputs effectively;
2. Since the relative threshold strategy [36,37] can not be extended to the MFA framework, an adaptive threshold strategy is considered for the designed ETC problem.
The designed ETM not only enhances the communication efficiency in the controller-sensor channel, but also eliminates the parameter limitation problem existing in the relative threshold strategy;

3. Different from the Nussbaum function methods using global coordinate transformation information [22,36], a tracking-error-based Nussbaum function is designed to handle the unknown control direction. Then, a boundedness lemma on the Nussbaum function is given to show the boundedness of the Nussbaum function.

2. Preliminaries

In this section, we consider the following uncertain nonlinear systems with unknown control direction:

\[
\begin{align*}
\dot{x}_1 &= x_{i+1} + f_{i\sigma(t)}(x_i), i = 1, \ldots, n - 1 \\
\dot{x}_n &= b_n u(t) + f_{n\sigma(t)}(x_n) \\
y &= x_1
\end{align*}
\]

where \( x_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i \) \((i = 1, \ldots, n)\) are the state vectors, \( y \) is the output of the system, \( u(t) \) is the control input, \( f_{i\sigma(t)}(x_i) \) is the unknown smooth nonlinear function, \( f_{n\sigma(t)}(x_n) \) is the unknown smooth nonlinear function, \( \sigma(t) : [0, +\infty) \to M = \{1, \ldots, m\} \) is a switching signal defined as a piecewise function, and \( b_n \) is the unknown control coefficient.

Defining the event error \( e_k(t) \) as

\[
e_k(t) = u(t) - v_k(t), k = 1, \ldots, m
\]

where \( v_k(t) \) and \( u(t) \) are the control signal and event-triggered control input which will be defined later. To simplify the notations, the time variable \( t \) will be omitted later.

Based on the (2), the considered system can be transformed into

\[
\begin{align*}
\dot{x}_1 &= x_{i+1} + f_{i\sigma(t)}(x_i), i = 1, \ldots, n - 1 \\
\dot{x}_n &= b_n(e_k + v_k) + f_{n\sigma(t)}(x_n) \\
y &= x_1.
\end{align*}
\]

Since radial basis function neural networks (RBFNNs) [38] can approximate any continuous nonlinear network with arbitrary accuracy, they are commonly used to approximate unknown nonlinear functions. Therefore, the RBFNNs used in the this paper are as follows:

\[
f(\mu) = W^T S(\mu) + \varepsilon_k(\mu)
\]

where \( \mu \in \mathbb{R}^l \) is the input vector, \( W = [w_1, w_2, \ldots, w_l]^T \in \mathbb{R}^l \) is an optimal weight vector, \( \varepsilon_k(\mu) \in \mathbb{R} \) is the bounded reconstruction error, the number of NN nodes is \( l > 1 \), and a radial basis function vector \( S(\mu) = [s_1(\mu), s_2(\mu), \ldots, s_l(\mu)]^T \in \mathbb{R}^l \). Since the unknown weight vector cannot be used in designing the actual control input, we define \( \hat{W} \) as its estimation and define \( \hat{W} = W - \hat{W} \) as the weight estimation error.

Assumption 1. There exists a positive constant \( \bar{\varepsilon} \) such that \( \varepsilon_k(\mu) \) satisfies \( |\varepsilon_k(\mu)| \leq \bar{\varepsilon} \).

Definition 1 ([39]). Any continuous function \( Q(\varsigma) : \mathbb{R} \to \mathbb{R} \) is a Nussbaum type function with the following properties:

\[
\begin{align*}
\lim_{s \to \infty} \sup_{s} \frac{1}{s} \int_{0}^{s} Q(\varsigma) d\varsigma &= +\infty \\
\lim_{s \to -\infty} \inf_{s} \frac{1}{s} \int_{0}^{s} Q(\varsigma) d\varsigma &= -\infty
\end{align*}
\]

[35x779]Electronics 2022, 11, 3386

3 of 17
where \( z \) is the Nussbaum variate. In fact, many continuous functions can be called the Nussbaum function, such as \( \zeta^2 \sin(\zeta) \), \( \zeta^2 \cos(\zeta) \) and \( \varepsilon \zeta^2 \cos(\zeta) \).

**Definition 2** ([40]). The average dwell time (ADT) is the maximal value \( \tau_a \) that for a given \( \Lambda_0 \geq 0 \) satisfies:

\[
\Lambda_\nu(\tau, t) \leq \Lambda_0 + \frac{t - \tau}{\tau_0}
\]

where \( \tau_a > 0, t > \tau > 0, \) and \( \Lambda_\nu(\tau, t) \) is the switching number that occurs in the interval \([\tau, t)\).

### 3. Main Results

#### 3.1. Controller Design

In what follows, an event-triggered adaptive tracking control strategy in the MFA framework is studied to address the complexity explosion.

Now, we introduce the following notations to simplify the calculation process:

\[
G_i = \prod_{m=1}^{i} g_m
\]

where \( i = 1, 2, \ldots, n, G_0 = 1, \) and \( g_i > 0 \) denote positive design parameters.

The new coordinate transformations are defined as

\[
z_1 = y - y_d
\]

\[
z_i = \frac{1}{g_{i-1}} \left( \frac{1}{G_{i-2}} x_{i,k} - a_{i-1,k} \right), i = 2, \ldots, n
\]

where \( a_{i,k} \) denotes the un-implementable virtual control signal, and \( y_d \) is the reference signal.

**Step 1:** According to (3) and (8), the derivative of \( z_1 \) is expressed as

\[
\dot{z}_1 = g_1 z_2 + a_{1,k} + f_{1,k} (\bar{x}_1) - \dot{y}_d.
\]

Constructing a Lyapunov function \( V_{1,k} = z_1^2 / 2 \), its derivative can be calculated as

\[
\dot{V}_{1,k} = z_1 (g_1 z_2 + a_{1,k} + f_{1,k} (\bar{x}_1,k) - \dot{y}_d).
\]

Choosing the virtual control signal \( a_{1,k} \) with \( F_{1,k}(\bar{x}_1) = f_{1,k}(\bar{x}_1) \) as

\[
a_{1,k} = -g_1 z_1 - F_{1,k}(\bar{x}_1) + \dot{y}_d.
\]

Substituting (11) into (10), it follows:

\[
\dot{V}_{1,k} \leq -g_1 z_1^2 + g_1 |z_1||z_2|.
\]

**Step 2:** Invoking (11) into (8) and using (7), we have

\[
z_2 = \frac{1}{g_1} x_2 + F_{1,k}^* - y_d
\]

where \( F_{1,k}^* = (F_{1,k}(x_1) - \dot{y}_d) / g_1 + x_1 \).

Differentiating \( z_2 \) results in

\[
\dot{z}_2 = \frac{1}{g_1} \dot{x}_2 + \frac{\partial F_{1,k}^*}{\partial x_1} (x_2 + f_{1,k}(\bar{x}_1)) + \frac{\partial F_{1,k}^*}{\partial y_d} \dot{y}_d - \dot{y}_d
\]

\[
= g_2 z_3 + a_{2,k} + F_{2,k}(x_2, \dot{y}_d) - \dot{y}_d
\]

where \( F_{2,k}(x_2, \dot{y}_d) = f_{2,k}(x_2) / g_1 + \left( \frac{\partial F_{1,k}^*}{\partial x_1} \right) (x_2 + f_{1,k}(\bar{x}_1)) + \left( \frac{\partial F_{1,k}^*}{\partial y_d} \right) \dot{y}_d.\)
Constructing $V_{2,k} = V_{1,k} + z_k^2/2$, the derivative of $V_{2,k}$ is computed as

$$V_{2,k} = V_{1,k} + z_2(g_2z_3 + F_{2,k}(g_2, y_d) + \alpha_{2,k} - \dot{y}_d).$$  \hspace{1cm} (15)

The virtual control signal $\alpha_{2,k}$ is designed as

$$\alpha_{2,k} = -g_2z_2 - F_{2,k}(g_2, y_d) + \dot{y}_d.$$  \hspace{1cm} (16)

Taking (16) into (15) yields

$$\dot{V}_{2,k} \leq -g_1z_1^2 + g_1|z_1||z_2| - g_2z_2^2 + g_2|z_2||z_3|$$

$$= -\sum_{i=1}^{2} g_i z_i^2 + \sum_{i=1}^{2} g_i |z_i||z_{i+1}|.$$  \hspace{1cm} (17)

**Step $m$ ($3 \leq m \leq n - 1$):** According to the recursive manner, $z_m$ is given as:

$$z_m = \frac{1}{G_{m-1}} x_m - z_{m-1} + \frac{F_{m-1,k}(x_{m-1}, y_d^{(m-1)}) - \dot{y}_d}{g_{m-1}}$$

$$= \frac{1}{G_{m-1}} x_m + F_{m-1,k}^* - \dot{y}_d$$  \hspace{1cm} (18)

where $F_{m-1,k}^* = F_{m-2,k}^* + x_{m-1}/G_{m-2} + \left( F_{m-1,k}(x_{m-1}, y_d^{(m-1)}) - \dot{y}_d \right)/g_{m-1}$.

From (3) and (8), the time derivative of $z_m$ is given by

$$\dot{z}_m = \frac{1}{G_{m-1}} (x_{m+1} + f_{m,k}(x_m)) + \sum_{i=1}^{m-1} \frac{\partial F_{m-1,k}^*}{\partial y_d^{(i)}} y_d^{(i+1)} + \sum_{i=1}^{m-1} \frac{\partial F_{m-1,k}}{\partial x_i} (f_{i,k}(x_i) + x_{i+1}) - \dot{y}_d$$

$$= g_m z_{m+1} + \alpha_{m,k} + F_{m,k}(x_m, y_d^{(m)}) - \dot{y}_d$$  \hspace{1cm} (19)

where $F_{m,k}(x_m, y_d^{(m)}) = f_{m,k}(x_m)/G_{m-1} + \sum_{i=1}^{m-1} \left( \frac{\partial F_{m-1,k}^*}{\partial y_d^{(i)}} \right) (x_{i+1} + f_{i,k}(x_i)) + \sum_{i=1}^{m-1} \left( \frac{\partial F_{m-1,k}}{\partial x_i} \right) y_d^{(i+1)}$.

Considering $V_{m,k} = V_{m-1,k} + z_m^2/2$, we obtain

$$\dot{V}_{m,k} = \dot{V}_{m-1,k} + z_m \left( g_m z_{m+1} + \alpha_{m,k} + F_{m,k}(x_m, y_d^{(m)}) - \dot{y}_d \right).$$  \hspace{1cm} (20)

Then, we construct the virtual controller as follows:

$$\alpha_{m,k} = -g_m z_m - F_{m,k}(x_m, y_d^{(m)}) + \dot{y}_d.$$  \hspace{1cm} (21)

Based on (20) and (21), we have

$$\dot{V}_{m,k} \leq -\sum_{i=1}^{m} g_i z_i^2 + \sum_{i=1}^{m} g_i |z_i||z_{i+1}| - g_m z_m^2 + g_m |z_m||z_{m+1}|$$

$$= -\sum_{i=1}^{m} g_i z_i^2 + \sum_{i=1}^{m} g_i |z_i||z_{i+1}|.$$  \hspace{1cm} (22)
Step n: Following the previous steps, $z_n$ is given as follows:

$$z_n = \frac{1}{G_{n-1}} x_n - z_{n-1} + \frac{F'_{n-1,k}(x_{n-1}, y^{(n-1)}_d) - y_d}{\tilde{\gamma}_{n-1}}$$

$$= \frac{1}{G_{n-1}} x_n + F'_{n-1,k} - y_d$$

where $F'_{n-1,k} = x_{n-1}/G_{n-2} + \left(F_{n-1,k}(x_{n-1}, y^{(n-1)}_d) - y_d\right)/\tilde{\gamma}_{n-1} + F'_{n-2,k}$.

The derivative of $z_n$ becomes

$$\dot{z}_n = \frac{1}{G_{n-1}} \left(b_n u + f_{n,k}(x_n)\right) + \sum_{i=1}^{n-1} \frac{\partial F'_{n-1,k}}{\partial y^{(i)}_d} y^{(i+1)}_d + \sum_{i=1}^{n-1} \frac{\partial F'_{n-1,k}}{\partial x_i} (f_{i,k}(x_i) + x_{i+1}) - y_d$$

$$= \frac{1}{G_{n-1}} b_n u + F_{n,k}(\tilde{x}_n, y^{(n)}_d) - \dot{y}_d$$

where $F_{n,k}(\tilde{x}_n, y^{(n)}_d) = (1/G_{n-1}) f_{n,k}(x_n) + \sum_{i=1}^{n-1} \left(\frac{\partial F'_{n-1,k}}{\partial y^{(i)}_d} y^{(i+1)}_d + \sum_{i=1}^{n-1} \frac{\partial F'_{n-1,k}}{\partial x_i} (f_{i,k}(x_i) + x_{i+1})\right)$.

Considering Lyapunov function $V_{n,k} = V_{n-1,k} + z_n^2/2, \forall k \in M$. The time derivative of $V_{n,k}$ is

$$V_{n,k} = V_{n-1,k} + z_n \left(\frac{b_n u + F'_{n,k}(x_n, y^{(n)}_d)}{G_{n-1}} - \dot{y}_d\right)$$

$$\leq - \sum_{i=1}^{n-1} \tilde{\gamma}_i \tilde{z}_i^2 + \sum_{i=1}^{n-1} \tilde{\gamma}_i |z_i| |z_{i+1}| + z_n \frac{b_n}{G_{n-1}} \left(u + F'_{n,k}\right) - c_n \tilde{z}_n^2 - \dot{y}_d \tilde{z}_n - \tilde{y}_d \tilde{\gamma}_n z_n$$

where $F'_{n,k} = (G_{n-1}/b_n) \left(F'_{n,k}(x_n, y^{(n)}_d) + A_{n} F'_{n-1,k} + \tilde{\gamma}_{n,x_n}/G_{n-1}\right)$.

To deal with the lumped nonlinearity derived from (25), we employ the following RBFNN:

$$F_{n,k}(\mu) = W_k^T S(\mu) + \epsilon_k(\mu)$$

where $F_{n,k} = F_{n,k} - q$, the variable $q$ will be defined in the following; $\mu = [x_n, y^{(n)}_d]$, $W_k$ and $S(\mu)$ satisfy $||S(\mu)|| \leq \tilde{S}$ and $||W_k|| \leq \tilde{W}_k$. Additionally, $\tilde{W}_k$ denotes the estimation of $W_k$, and $\tilde{W}_k = W_k - \tilde{W}_k$ represents the weight estimation error.

Considering the proposed scheme, we design the following ETM as:

$$u(t) = v_k(t_l), t_l \leq t < t_{l+1}$$

$$t_{l+1} = \inf\{t > t_l | |e_k| > a|z_l| + \gamma, 0 < a < 1, \gamma > 0\}$$

(27)

where $u(t)$ is the control input signal, $a$ and $\gamma$ are user designed constants. It can be shown that the proposed trigger condition depends only on a coordinate transformation error $z_l$, and the control signal remains constant between two adjacent trigger instants.
By adding and subtracting the term \( q = Q(\varsigma) \left( g_1 z_1 + \sum_{i=2}^{n} c_i x_i \right) \) into (25), we have

\[
\dot{V}_{n,k} \leq - \sum_{i=1}^{n-1} g_i z_i^2 + \sum_{i=1}^{n-1} g_i |z_i| |z_{i+1}| + z_n \frac{b_n}{C_{n-1}} \left( u + F_{n,k}^* + q \right) - c_n z_n^2 - \dot{y}_d z_n - y_d \ddot{y}_d z_n
\]

\[
= - \sum_{i=1}^{n-1} g_i z_i^2 + \sum_{i=1}^{n-1} g_i |z_i| |z_{i+1}| + z_n \frac{b_n}{C_{n-1}} \left( e_k + v_k + F_{n,k}^* + q \right) - c_n z_n^2 - \dot{y}_d z_n - y_d \ddot{y}_d z_n. 
\]  

(28)

The MFA-based adaptive ETC framework is depicted in Figure 1, where the input vector of RFBNN is defined as \( \mu = [\bar{x}_n, y_d^{[n]}] \). Under the proposed closed-loop control framework, the system variables \( \bar{x}_i \) and \( y_d \) are transmitted through the sensor to the adaptive law, NN weight update law and Function approximator. Then, the Nussbaum signal \( Q(\varsigma) \) and the estimated nonlinear function \( \hat{W}_k^T S(\mu) \) can be obtained to construct the actual control signal \( v_k(t) \). Subsequently, the proposed ETM transmits the control signal to the actuator in an aperiodic manner, which can effectively save the communication resources.

![Figure 1. Minimal-function-approximation-based adaptive ETC framework.](image-url)

**Remark 1.** Unlike the traditional backstepping algorithms for switched nonlinear systems [13,15], the designed virtual control laws \( \alpha_i, i = 1, 2, \ldots, n \), are un-implementable signals. Thus, \( z_i, i = 2, 3, \ldots, n \), can not be calculated under the MFA framework.

Now, the control input, the adaptive law and the weight update law are proposed by using one RBFNN approximator, as follows:

\[
v_k = -Q(\varsigma) \left( \hat{W}_k^T S(\mu) + g_1 z_1 + \sum_{i=2}^{n} g_i x_i \right) \quad (29)
\]

\[
\varsigma = \Gamma_\varsigma \left( z_1^2 - \sigma_\varsigma \varsigma z_1^2 \right) \quad (30)
\]

\[
\hat{W}_k = \Gamma_w (z_1 S(\mu) - \sigma_w |z_1| \hat{W}_k) \quad (31)
\]
where $\Gamma_\varsigma$ is a positive constant, $\Gamma_{w}$ is a positive definite $l$-order matrix, and $\sigma_\varsigma > 0$ and $\sigma_{w} > 0$ are constants.

**Remark 2.** In the case of switched nonlinear systems with unknown control direction [22,41,42], the existing works have designed the tuning law $\varsigma$ of the Nussbaum function by using $z_n$, then the stability of the system has been investigated by invoking the lemma on the Nussbaum function (see [22], Lemma 1). However, since $z_n$ in the MFA framework is unattainable signal, the above methodologies cannot be used in the design of the control input. In this paper, a Nussbaum function based on the tracking error is designed to handle the control gain of unknown sign.

**Lemma 1.** For the uncertain switched nonlinear system (3), consider the virtual control laws and actual control input designed in (11), (16), (21) and (29). Under the the adaptive law (30), the parameter adaptation function $\varsigma$ is bounded.

**Proof of Lemma 1.** Selecting the Lyapunov function $V_\varsigma = (1/2\Gamma_\varsigma)\varsigma^2$, then its derivative can be calculated as

$$
\dot{V}_\varsigma = \varsigma \left( z_1^2 - \sigma_\varsigma \varsigma z_1^2 \right)
\leq -|\varsigma|z_1^2(\sigma_\varsigma|\varsigma| - 1).
$$

(32)

This inequality shows that the negativity of $\dot{V}_\varsigma$ is guaranteed when $|\varsigma| > 1/\sigma_\varsigma$. Then, the boundedness of $\varsigma$ can be expressed as $\varsigma(t) \in \mathbb{R}_\varsigma = \{\varsigma, |\varsigma| \leq 1/\sigma_\varsigma\}$. The proof is completed. $\square$

**Lemma 2.** For the uncertain switched nonlinear system (3), consider the virtual control laws and actual control input designed in (11), (16), (21) and (29). Under the NN weight update law (31) for the uncertain switched nonlinear system, NN weight estimation error $\dot{W}_k$ is bounded.

**Proof of Lemma 2.** Considering the Lyapunov function $V_{w,k} = (1/2)\dot{\bar{w}}_k^T\Gamma_{w}^{-1}\dot{\bar{w}}_k$, then its time derivative along (31) is derived as

$$
\dot{V}_{w,k} = -\dot{\bar{w}}_k^T\Gamma_{w}^{-1}\dot{\bar{w}}_k
= -\bar{w}_k^T \left( z_1 S(\mu) - \sigma_{w} |z_1| (W_k - \dot{\bar{w}}_k) \right).
$$

(33)

Then, according to the condition $\|S(\mu)\| \leq \bar{S}$ and Frobenius norm, we have

$$
V_{w,k} \leq -|z_1| \|\bar{w}_k\| (\sigma_{w} \|\bar{w}_k\| - \bar{S} - \sigma_{w} \bar{w}_k).
$$

(34)

Therefore, the derivative $\dot{V}_{w,k}$ is negative definite only if

$$
\|\bar{w}_k\| > (\bar{S} + \sigma_{w} \bar{w}_k)/\sigma_{w}.
$$

(35)

Then, the boundedness of $\dot{W}_k(t)$ can be expressed as $\dot{W}_k(t) \in \mathbb{R}_w = \{\bar{W}_k, \|\bar{w}_k\| \leq (\bar{S} + \sigma_{w} \bar{w}_k)/\sigma_{w}\}$. The proof is completed. $\square$

**Remark 3.** In the existing function approximation-based adaptive control schemes [19,43], function approximators are included in each virtual controller and the actual controller. Therefore, the computational burden increases with the order of the system. The proposed method reduces the usage frequency. More specifically, the virtual controllers are designed as the unprocuarable intermediate transition signals, and the actual control input is established based on one approximator. In such a scenario, the calculation costs to achieve the control objective no longer increase with the increase in the system order.

**Remark 4.** Considering that the relative threshold strategy [37] mainly aim at the controller-to-actuator communication, where the triggering condition is designed as $\epsilon(t) \geq m_1 u(t) + m_2$ (i.e.,
0 < m_1 < 1, m_2 > 0 \). However, due to z_n and a_i are un-implementable signals, the above design methods cannot be extended to the minimal-approximation-based ETC design. Therefore, a tracking error-based adaptive threshold (27) is designed in this paper. It can be seen that the ETM (27) can adaptively adjust the triggering interval according to the tracking error. In addition, the range of threshold coefficients no longer need to satisfy the restrictive condition in [36,37].

3.2. Stability Analysis

In this part, we will certify that all signals in the derived closed-loop system are bounded.

Theorem 1. Considering the switched nonlinear systems with unknown control gain (3), the event-triggering mechanism (27), the control input (29), the adaptive law (30), the NN weight update law (31). The closed-loop system is bounded for any bounded initial condition. Meanwhile, can be as small as possible to adjust.

Proof of Theorem 1. Considering the multiple Lyapunov function as

\[ V_k = V_{n,k} + \frac{1}{2} q^2 + \frac{1}{2} W_k^T W_k. \]  

(36)

Based on (26), (28), Lemmas 1 and 2, we have

\[ \dot{V}_k \leq - \sum_{i=1}^{n-1} g_i z_i^2 + z_n b_n \left( \epsilon_k + Q(\varsigma) W_k^T S(\mu) + (1 - Q(\varsigma)) W_k^T S(\mu) + \epsilon_k(\mu) \right) \]

\[ - c_n z_n^2 - y_d z_n - y_d g_n z_n + \sum_{i=1}^{n-1} g_i |z_i||z_{i+1}|. \]

(37)

From Lemmas 1 and 2 and Q(\varsigma)=e^2 cos(\varsigma), we have

\[ -y_d z_n^2 \leq \frac{1}{4} z_n^2 + y_d^2 \]

(38)

\[ -y_d g_n z_n \leq \frac{1}{4} z_n^2 + (y_d g_n)^2 \]

(39)

\[ z_n b_n \frac{g_n}{G_n-1} \gamma \leq \frac{1}{2} z_n^2 + \frac{1}{2} \gamma^2 \Delta^2 \]

(40)

\[ z_n b_n \frac{a|z_1|}{G_n-1} \leq \frac{1}{2} a^2 z_1^2 + \frac{1}{2} \Delta^2 z_n^2 \]

(41)

\[ \sum_{i=1}^{n-1} g_i |z_i||z_{i+1}| \leq \sum_{i=1}^{n-1} \frac{g_i}{2} z_i^2 + \sum_{i=1}^{n-1} \frac{g_i}{2} z_{i+1}^2 \]

(42)

\[ z_n \frac{b_n}{G_n-1} \left( Q(\varsigma) W_k^T S(\mu) + (1 - Q(\varsigma)) W_k^T S(\mu) + \epsilon_k(\mu) \right) \leq \frac{1}{2} \Delta^2 z_n^2 + \theta_k(\mu) \]

(43)

where |b_n/G_n-1| ≤ Δ, |Q(\varsigma)| ≤ Q, \( \theta = \frac{1}{2} (Q(S + \varsigma w W_k) S / \varsigma w + (1 + Q) W_k^T S + \epsilon)^2 \). Using the inequalities (37)–(41), \( \dot{V}_k \) becomes

\[ \dot{V}_k \leq - \sum_{i=1}^{n} \frac{g_i}{2} z_i^2 + \frac{1}{2} \sum_{i=1}^{n} \frac{g_i}{2} z_{i+1}^2 - \left( \frac{g_n}{2} - \Delta^2 - 1 \right) z_n^2 + \frac{1}{2} \Delta^2 z_1^2 + \rho \]

(44)

where \( y_d^2 + (y_d g_n)^2 + \frac{1}{4} \Delta^2 \gamma^2 + \theta \leq \rho \).
Choosing appropriate design parameters, there exist
\[ p_1 = \frac{g_1}{2} - \frac{1}{2}a^2 \]
\[ p_i = \frac{g_i}{2} - \frac{g_{i-1}}{2} > 0, \quad i = 2, \ldots, n - 1 \]
\[ p_n = g_n - \frac{g_{n-1}}{2} - \Delta^2 - 1 > 0. \]  
(45)

Let \( p = \min\{ p_i, i = 1, 2, \ldots, n \} \), then (45) can be rewritten as
\[ V_k \leq -p V_k + \rho \]  
(46)
\[ V_k \leq \eta V_k \]  
(47)
where \( \eta = \max\{ p_k / p_r \} \) for \( \forall r \in M \).

Furthermore, we can find two functions \( g, \bar{r} \in \kappa_{\infty} \), such as \( g(\|\mu\|) \leq V_r(\mu) \leq \bar{r}(\|\mu\|) \).

Then, we construct a function \( \Theta(t) = e^{pt} V_{r(t)}(\mu(t)) \) that is piecewise differentiable along the solution of the closed-loop system (3). From (45), we obtain on each interval \([t_j, t_{j+1}]\) that
\[ \Theta(t) = pe^{pt} V_{r(t)}(\mu) + e^{pt} V_{r(t)}(\mu) \]
\[ \leq pe^{pt}, \quad t \in [t_j, t_{j+1}). \]  
(48)

Taking (46) into account, we obtain
\[ \Theta(t_{j+1}) = e^{pt_{j+1}} V_{r(t_{j+1})}(\mu(t_{j+1})) \]
\[ \leq \eta e^{pt_{j+1}} V_{r(t_{j+1})}(\mu(t_j)) \]
\[ = \eta \Theta(t_{j+1}) \]
\[ \leq \eta \left[ \int_{t_{j}}^{t_{j+1}} e^{pt} dt + \Theta(t_j) \right]. \]  
(49)

Furthermore, by selecting an arbitrary \( T > t_0 = 0 \) and integrating the above inequality from \( j = 0 \) to \( j = \Lambda_{\nu}(T, 0) - 1 \), we have
\[ \Theta(T^-) \leq \Theta(t_{\Lambda_{\nu}(T, 0) - 1}) + \int_{t_{\Lambda_{\nu}(T, 0)}}^{T} e^{pt} dt \]
\[ \leq \eta \left[ \int_{t_{\Lambda_{\nu}(T, 0)} - 1}^{t_{\Lambda_{\nu}(T, 0)}} e^{pt} dt + \Theta(t_{\Lambda_{\nu}(T, 0) - 1}) + \eta^{-1} \int_{t_{\Lambda_{\nu}(T, 0)}}^{T} e^{pt} dt \right] \]
\[ \leq \cdots \]
\[ \leq \eta^{\Lambda_{\nu}(T, 0)} \left[ \Theta(0) + \left\{ \sum_{j=0}^{\Lambda_{\nu}(T, 0) - 1} \eta^{-j} \int_{t_{j}}^{t_{j+1}} e^{pt} dt \right\} + \eta^{-\Lambda_{\nu}(T, 0)} \int_{t_{\Lambda_{\nu}(T, 0)}}^{T} e^{pt} dt \right]. \]  
(50)

For \( \forall \delta \in (0, p - (\log \eta / \tau_0)) \) and \( \tau_0 > (\log \eta / p) \), we have
\[ \Lambda_{\nu}(T, t) \leq \Lambda_0 + \frac{(p - \delta)(T - t)}{\log \eta}, \quad \forall T \geq t \geq 0. \]  
(51)

Notice that \( \Lambda_{\nu}(T, 0) - j \leq 1 + \Lambda_{\nu}(T, t_{j+1}), \quad j = 0, 1, \ldots, \Lambda_{\nu}(T, 0), \) thus we can infer that
\[ \eta^{\Lambda_{\nu}(T, 0) - j} \leq \eta^{1 + \Lambda_{\nu}(T, 0) - t_{j+1}}, \]  
(52)
In addition, we obtain from \( \delta < p \) that
\[
\int_{t_j}^{t_{j+1}} pe^{\beta t} dt \leq e^{(p-\delta)t_{j+1}} \int_{t_j}^{t_{j+1}} pe^{\beta t} dt.
\] (53)
Then, it follows from (50)-(53) that
\[
\Theta(T^-) \leq \eta^{1+\Lambda_0} e^{(p-\delta)t_{j+1}} \int_0^T pe^{\beta t} dt + \eta^{\Lambda_0} (T,\theta) \Theta(0).
\] (54)
We obtain
\[
\Theta(T^-) \leq V_{\alpha}(T^-) (\mu(T^-))
\leq e^{\Lambda_0 \log \eta} e^{(\frac{\Lambda_0 \eta}{p} - p)} T_{\alpha} (\mu(0)) + \eta^{1+\Lambda_0} \Theta(0), \forall T > 0, \forall T > 0.
\] (55)
It is clear that all signals of the considered system are bounded in the presence of the switching signals \( k(t) \) satisfying the ADT \( T_{\alpha} > (\log \eta / p - \delta) \).
To avoiding the Zeno behavior, we prove that the inter-event time has lower bounded.
From the definition \( e_k(t) = u(t) - v_k(t) \), we obtain:
\[
\frac{d}{dt} e_k = \frac{e_k e_k}{\| e_k \|} \leq \| \dot{e}_k \| = \| \dot{u} \|
\] (56)
Due to the variables \( y, x_1, \ldots, x_n, \hat{W}_i \) and \( \tilde{z} \) in \( u \) are bounded, there is \( \bar{u} \) such that \( \| \dot{u} \| \leq \bar{u} \). Depending on the triggering condition of (27), the minimum inter-event time satisfies \( t_{\min} \geq \gamma / \bar{u} > 0 \). Therefore, Zeno behavior can be avoided. \( \square \)

4. Simulation
In this section, we use a numerical model to clarify the validity of the designed control scheme under the condition of setting the sampling time to 0.01s. Considering the below second-order switched nonlinear system
\[
\begin{align*}
\dot{x}_1 &= x_2 + f_{1,k}(x_1) \\
\dot{x}_2 &= b_n u(t) + f_{2,k}(x_2) \\
y &= x_1
\end{align*}
\] (57)
where \( f_{1,1}(x) = 0.1 \exp(\cos(x_1)), f_{1,2}(x) = 0.1 \exp(x_1) + 1.5 \sin(x_2), \) and \( f_{2,1}(x) = 0.15 \exp(x_1) + 0.2 \cos(x_2), f_{2,2}(x) = 0.15 \cos(x_2) + 0.2 \sin(x_1) \) and \( k \in M = \{1, 2\} \). The unknown control gain is defined as \( b_n = -1 \). We construct the NN weight update laws as
\[
\begin{align*}
\dot{W}_1 &= \Gamma_{w1}(z_1 S(\mu) - \sigma_{w1} |z_1| \hat{W}_1) \\
\dot{W}_2 &= \Gamma_{w2}(z_1 S(\mu) - \sigma_{w2} |z_1| \hat{W}_2)
\end{align*}
\] (58)
where \( \sigma_{w1} = 80, \sigma_{w2} = 60, \Gamma_{w1} = 1.5 \) and \( \Gamma_{w2} = 1 \).
Using the designed ETM, the proposed MFA-based controller for (29)–(31) is given by
\[
\begin{align*}
u(t) &= v_k(t), t_j \leq t < t_{j+1} \\
t_{j+1} &= \inf \{ t > t_j \| e_k \| > a |z_1| + \gamma, 0 < a < 1, \gamma > 0 \} \\
v_1 &= Q(\epsilon)(-g_1 z_1 - g_2 x_2 - \hat{W}_1^T S(\mu)) \\
v_2 &= Q(\epsilon)(-g_1 z_1 - g_2 x_2 - \hat{W}_2^T S(\mu)) \\
\zeta &= \Gamma_{\epsilon} \left( z_1^2 - c_{\epsilon} z_1^2 \right)
\end{align*}
\] (59)
where $a = 20$, $\gamma = 0.25$, $Q(\varsigma) = e^{\varsigma^2} \cos(\varsigma)$ and $y_d = 0.1(\cos(2t) + 1.5\cos(t))$. The initial values of variables are given as $x_1(0) = 0.5$, $x_2(0) = 0.1$, $g_1 = 13.1$, $g_2 = 15.1$, $\varsigma = 1.5$, $\sigma_\varsigma = 0.55$, $\Gamma_\varsigma = 6$, $\hat{W}_1(0) = [0.6, \ldots, 0.6]^T \in \mathbb{R}^{25}$ and $\hat{W}_2(0) = [0.9, \ldots, 0.9]^T \in \mathbb{R}^{25}$. Furthermore, it is easy to calculate that $\tau_a = 0.15$.

The simulation results for the above example are shown in Figures 2–9. As is shown in Figure 2, the switching signal changes every 3 s. Additionally, it is observed from Figure 3 that the output of the considered system can effectively track the desired reference trajectory. Then, the tracking error is shown in Figure 4, and it can stabilize within $[0.0059, 0.0306]$ after 0.5 s. From Figures 5 and 6, the boundedness of NN weights can be guaranteed after 0.5 s. Meanwhile, the event-triggered control input is shown in Figure 7. Combining Figures 7 and 8, the control input $u$ changes at the triggering instants shown in Figure 8.

![Figure 2](image-url). The trajectory of the switching signal.

![Figure 3](image-url). The system’s output $y_1$ and reference signal $y_d$.

![Figure 4](image-url). The trajectory of the tracking error $z_1$. 
Figure 5. The trajectory of the norm for the NN weight $\hat{W}_1$.

Figure 6. The trajectory of the norm for the NN weight $\hat{W}_2$.

Figure 7. The red line is the trajectory of the time-triggered controller, and the blue line is the trajectory of the event-triggered controller.

Figure 8. The horizontal axis and vertical axis represent the trigger times and trigger intervals, respectively.
Discussion

According to the results given in Theorem 1, the boundedness of the system can be theoretically guaranteed via the proposed scheme. In addition, the effectiveness of the proposed ETC algorithm can be further proven from the results shown in Figures 2–9. To further demonstrate the advantage of the strategy presented in this paper, the transmission numbers of the conventional time-triggered control method and the proposed ETC method are shown in Figure 9, with the transmission numbers of 2000 and 375, respectively. Compared with the time-triggered scheme [28], the method in this paper is more efficient in saving communication resources. Additionally, it can be seen from the control algorithms given in (58) and (59) that the computational burden is less, and the complexity of the controller design is simpler compared with the existing adaptive ETC methodologies using the filtering technology [18, 21].

5. Conclusions

In this paper, an MFA-based event-triggered control protocol is proposed for uncertain switched nonlinear systems with unknown control direction. With the help of the MFA technology, three excellent results are obtained as follows:

1. The virtual controllers are designed as intermediate signals that do not need to be implemented, which effectively simplifies the complexity of the conventional backstepping design framework.
2. The issue of explosion of complexity existed in the conventional backstepping methods does not need to be considered.
3. There no longer needs to employ multiple function approximators in the recursive process, which actually cuts down the computational costs.

Moreover, the proposed scheme increases the communication efficiency by using the ETM with adaptive threshold. Meanwhile, by using the Lyapunov stability theorem, the boundedness of the considered system is guaranteed, and the simulation example successfully verifies the theoretical results. Finally, the main frame of the study is shown as Figure 10. In our future work, we will pay attention to how to solve the self-triggering control problem in the MFA framework.
Electronics 2022, 11, 3386

Local controller

Weight update law and adaptive law

Switched nonlinear system

\[
\dot{x}_i = x_{i+1} + f_{\text{act}}(\tau_i), i = 1, \ldots, n - 1
\]

\[
\dot{x}_n = h(t) + f_{\text{net}}(\tau_i)
\]

\[
y = x_n
\]

\[
\mathbf{z}_i(t), y(t)
\]

\[
Q(z), S(\mu)
\]

\[
\mathbf{z}_1(t)
\]

Sensor

Actuator

Network

\[
\mathbf{u}(t)
\]

\[
\mathbf{v}_k(t)
\]

\[
u(t) = \mathbf{v}_k(t), t_i < t < t_{i+1}
\]

\[
t_{i+1} = \inf \{t > t_i \mid |\mathbf{v}_k| > a|z| + \gamma\}
\]

Figure 10. The main frame of the article.

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