Abstract: Phase-shifting in an achromatic configuration of moiré interferometry is explained. The rigorous diffraction theory defines the phase of diffracted beams in terms of the pitch and the relative position of a compensator grating. A numerical analysis proceeds to determine the total phase change between two diffracted beams that produce moiré fringes.

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OCIS codes: (050.1950) Diffraction grating; (050.1960) Diffraction theory; (050.5080) Phase shift; (120.3180) Interferometry.

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1. Introduction
Moiré interferometry is a full-field technique to measure in-plane displacements. It has been used extensively for deformation analyses in the various fields of mechanics [1]. In this method, a high frequency crossed-line phase grating is replicated on the surface of a specimen. A pair of collimated beams strikes the specimen grating and is diffracted to an imaging system.
nominally perpendicular to the specimen. Since the specimen grating deforms together with the specimen, the diffracted beams have warped wave fronts representing the deformation. The two diffracted beams interfere and produce a pattern of fringe. In practice, two pairs of beams interact with each set of crossed-line grating lines to produce two orthogonal displacement fields, i.e., $U$ and $V$.

A pair of beams to produce the $U$ field is illustrated in Fig. 1a where the incident beam is separated into two beams (Beam 1 and Beam 2) by a beam splitter; analogous pair of beams to produce the $V$ field is in the vertical plane (not shown). In routine practice of moiré interferometry, the frequency of specimen grating is 1200 lines/mm, which produces a contour interval of 0.417 $\mu$m displacement per fringe order.

The measurement sensitivity can be enhanced further by using the phase-shifting technique, which utilizes a series of phase-shifted patterns to determine fractional fringe orders. The phase-shifting technique has been adopted for moiré interferometry to increase the measurement sensitivity since early 1990 [2-8]. The phase of the interferogram of moiré interferometry can be shifted by changing the phases of the two beams relative to each other. However, the optical/mechanical schemes, implemented for classical two-beam interferometry, to alter the phase of the reference beam are not most suitable for moiré interferometry inasmuch as it requires two separate shifting mechanisms for the $U$ and $V$ fields.

A simple mechanical system using an analogy to geometric moiré was proposed for simultaneous shift of $U$ and $V$ field moiré fringes [9]. The concept of the mechanical phase-shifting was explained by the concept of the optical path length (OPL). As illustrated in Fig. 1a, the two beams separated by the beam splitter meet again at a point $P$ on the specimen. The OPL difference determines the state of constructive or destructive interference at $P$. If all the optical elements are translated with respect to the specimen grating by the same distance $\Delta$, Beam 2 reflected by the beam-splitter hits the same point $P$ on the specimen without any change of the OPL. However, Beam 1 transmitted through the beam-splitter reaches point $P$ with an OPL change of

$$\delta = 2\Delta \sin \alpha$$

The corresponding phase shift is $2\pi \frac{\delta}{\lambda}$.

![Fig. 1. Illustration of phase-shifting in moiré interferometry; (a) mechanical phase-shifting in a conventional configuration and (b) phase-shifting in an achromatic configuration](image)

Another scheme to produce the four beams required in moiré interferometry is called achromatic system, where a crossed-line phase grating, called compensator grating, is used to create the four beams [10,11]; the most significant advantages of achromatic system is relaxed temporal coherence requirement of a light source. Figure 1b illustrates such a system where
Beams 1 and 2 are the ±n diffraction orders of the incident beam. The phase-shifting can be accomplished by translating the compensator grating by a fraction of its pitch as illustrated in Fig. 1b [11].

The phase-shifting mechanism of the achromatic system cannot be explained by the concept of OPL; the translation of the compensator grating does not change the OPL of the diffracted beams. The phase information of the diffracted beams should be used to explain the phase-shifting in the achromatic system. In this paper, a rigorous diffraction theory is utilized to analyze the phase of diffracted beams. The analysis provides the theoretical and physical explanation for the phase-shifting in the achromatic system.

2. Phase change of diffracted beam

A rigorous grating theory based on the laws of electromagnetism [12] is used to define the phase of the diffracted beam with respect to the relative position to the compensator grating. The complex field of an incident beam of unit amplitude at an angle of incidence, α, is defined as

$$E_i = \exp \left( i \frac{2\pi}{\lambda} (x \sin \alpha - z \cos \alpha) \right)$$

(2)

Fig. 2. Compensator grating with a pitch of $g_c$ and a profile of $f(x)$ is translated linearly to change the phase of diffracted beams.

Based on the rigorous grating theory, the governing equation of the complex field of the beams diffracted from a grating with a pitch of $g_c$ can be expressed as [12]

$$\nabla^2 E + k^2 E = 0$$

(3)

and the solution of Eq. (3) can be expressed as [12]

$$E(x, z) = \sum_n E_n(x, z) = \sum_n B_n \Phi_n(x, z) = \sum_n B_n \exp \left( i \frac{2\pi}{\lambda} (x \sin \theta_n - z \cos \theta_n) \right)$$

(4)

where $\sin \theta_n = \sin \alpha + \lambda n / g_c$ and $n$ denotes the diffraction order. In Eq. (4) each term of $E_n$ represents a diffraction order, and the 2-D function, $\Phi_n(x, z)$, and the coefficient, $B_n$, define the phase and the amplitude of each diffracted beam, respectively.

As the coefficient $B_n$ in Eq. (4) includes the complex field, the explicit form of solution for the coefficient cannot be obtained analytically. In this study, the Point Matching Method (PMM) is employed for a numerical analysis.

The PMM method is one of the Rayleigh methods and it is relatively simple to use. In the method, the boundary condition is written as [12]
\[ \sum_{n} B_n \Phi_n(x, f(x)) + E_i(x, f(x)) = 0 \]  \hspace{1cm} (5)

where \( f(x) \) is the grating profile. The only unknown in Eq. (5) is a set of complex coefficient \( B_n \). Equation (5) can be changed into a system of \( 2N+1 \) linear equation with \( 2N+1 \) unknowns as

\[ \sum_{n=-N}^{N} B_n^{(N)} \Phi_n(x, f(x)) + E_i(x, f(x)) = 0 \]  \hspace{1cm} (6)

which can be solved by selecting \( 2N+1 \) points in the profile [12].

The sinusoidal profile illustrated in Fig. 2 can be defined as

\[ f(x) = \frac{h}{2} \cos\left(2\pi f_c(x - \Delta)\right) - \frac{h}{2} \]  \hspace{1cm} (7)

where \( h \) is the depth of groove, \( f_c \) is the frequency of the compensator grating, and \( \Delta \) is the amount of translation of the compensator grating. The change of phase in a diffracted beam at point \( P(0, 0) \) then can be calculated as a function of \( \Delta \). The parameters used in the calculation were \( f_c = 1200 \) lines/mm and \( h = 0.1 \) \( \mu \)m. The wavelength of the incident beam was 350 nm; this short wavelength was used to investigate a higher diffraction order.

![Fig. 3. Phase change as a function of linear translation of the compensator grating: (a) the first order and (b) the second order diffraction beams.](image)

The results obtained for the first and second diffraction orders are shown in Fig. 3a and b, respectively. As the grating translates linearly, the phase at \( P(0, 0) \) also changes linearly. The linear relationship between the translation of the compensator grating, \( \Delta \), and the phase change in the diffracted beam, \( \phi \), can be expressed as

\[ \phi = 2\pi n f_c \Delta \]  \hspace{1cm} (8)

The phase change of Eq. (8) can be explained physically using the wave fronts of the diffracted beams. For the normal incidence, the diffraction angle is governed by the scalar diffraction equation

\[ \sin \theta_n = n\lambda f_c \]  \hspace{1cm} (9)

where \( \theta_n \) is the diffraction angle. Equation 9 defines the amount of phase change within one pitch of the compensator grating and it is illustrated graphically in Fig. 4(a) and (b) for the first and second order diffraction orders. The phase change within one pitch is \( 2\pi \) and \( 4\pi \) for the first and second orders, respectively.
The phase change of the wave fronts caused by translating the compensator grating are illustrated in (c) and (d), where the solid lines show the new position of the compensator grating after translation. The diffracted wave fronts advance by $\lambda$ as the compensator grating translates by $\Delta$. Using Eq. (9), the relationship can be expressed as

$$\sin \theta_n = -\frac{n\lambda + \Delta}{\Delta + 1/f_c} = n\lambda f_c \Rightarrow \Delta = n\lambda f_c \Delta$$

(10)

Recalling the relationship between the path length and the phase, $\phi = 2\pi \frac{\Delta}{\lambda}$, Eq. (10) becomes identical to Eq. (8). Physically, the phase of the wave front change cyclically from 0 to $2\pi$ as the gratings moves to the positive $x$-direction from 0 to $1/nf_c$.

3. Phase-shifting in achromatic system

Figure 5 shows the two diffracted beams used in the achromatic moiré interferometry; the first (Beam 1) and the second beam (Beam 2) are $-n_0$ and is $+n_0$ order, respectively. Translating the compensator grating in the positive $x$-direction causes a phase shift in the two beams with the equal amount but opposite sign, as illustrated in the figure. From Eq. (8), the phase shift of each beam can be expressed as

$$\phi_{1,n} = -2\pi nf_c \Delta \quad \text{and} \quad \phi_{2,n} = 2\pi nf_c \Delta$$

(11)
The intensity of fringe resulting from the interference of the two beams in moiré interferometry can be expressed as [1]

\[
I(x, z) = I_1(x, z) + I_2(x, z) + 2\sqrt{I_1(x, z)I_2(x, z)} \cos \left( 2\pi \cdot 2nf_{g}U(x, z) + \phi_1 - \phi_2 \right)
\]

\[
= I_1(x, z) + I_2(x, z) + 2\sqrt{I_1(x, z)I_2(x, z)} \cos 2\pi \left( 2nf_{g}U(x, z) + 2nf_{g}\Delta \right)
\]

(12)

where \(I_1(x, z)\) and \(I_2(x, z)\) are the intensity of Beams 1 and 2, respectively, \(m\) is the diffraction order from specimen grating, and \(f_{g}\) is the frequency of specimen grating. It is clear from Eq. (12) that the phase shift of \(2\pi\) at every point in the field can be achieved by translating the compensator grating by \(\Delta_{2\pi}\), which is defined as

\[
\Delta_{2\pi} = \frac{1}{2nf_{g}}
\]

(13)

Fig. 5. Two diffracted beams in an achromatic system; the directions of phase shift are opposite.

The \(\pm 1\) diffraction orders are typically used in the achromatic systems for optimum light utilization. Then the amount of translation for \(2\pi\) phase shift is half the pitch of the compensator grating. It is worth noting that the phase-shifting is governed only by the compensator grating regardless of the frequency of the specimen grating (or the virtual reference grating [1]). The configuration shown in Fig. 5 can be used for a specimen grating with a much higher frequency. In Refs. [13,14], a specimen grating of 2400 lines per mm was used in an achromatic configuration utilizing the concept of an immersion interferometer. The compensator grating of the system was 1200 lines per mm and the amount of compensator grating translation remained half the pitch of the compensator grating, 0.417 \(\mu\)m, in spite of the doubled specimen grating frequency. In practice, the compensator grating can be translated at a 45° angle to the \(x\) and \(y\) axes so that phase-shifting for the \(U\) and \(V\) fields can be achieved simultaneously.

4. Conclusion

We have explained how phase-shifting occurs in the achromatic configuration of moiré interferometry using a rigorous diffraction theory. In the analysis, the theory defined the phase change in a diffracted beam as a linear function of compensator grating translation. The results showed that the translation of the compensator grating by a fraction of its pitch produces a phase change in diffracted beams by the same fraction of \(2\pi\) multiplied by the...
diffraction order. Consequently, the phase shift of $2\pi$ at every point in moiré patterns can be achieved by translating the compensator grating by half the pitch divided by the diffraction order.