Abstract

In this paper we calculate the turbulent heating rates in the solar wind using the Kolmogorov-like MHD turbulence phenomenology with Kolmogorov’s constants calculated by Verma and Bhattacharjee [1995b,c]. We find that the turbulent heating can not account for the total heating of the non-Alfvénic streams in the solar wind. We show that dissipation due to thermal conduction is also a potential heating source. Regarding the Alfvénic streams, the predicted turbulent heating rates using the constants of Verma and Bhattacharjee [1995c] are higher than the observed heating rates; the predicted dissipation rates are probably overestimates because Alfvénic streams have not reached steady-state. We also compare the predicted turbulent heating rates in the solar corona with the observations; the Kolmogorov-like phenomenology predicts dissipation rates comparable to the observed heating rates in the corona [Hollweg, 1984], but Dobrowoly et al.’s generalized Kraichnan model yields heating rates much less than that required.
1 Introduction

The proton temperature $T$ of the solar wind, regarded here as a single magnetofluid parameter, is observed to decrease slower than what adiabatic cooling would predict [Schwenn, 1983; Marsch et al., 1983; Schwartz and Marsch, 1983; Gazis, 1984; Freeman and Lopez, 1985; Lopez and Freeman, 1986; Freeman et al., 1992]. These observations indicate that the protons in the solar wind is heated in its transit. Note that in this paper we will not discuss the temperature evolution of electrons and alpha particles. Also, since the solar wind has complicated flow behaviour, the simplistic arguments presented in this brief report are of a qualitative nature.

Various attempts have been made to explain the observed heating. The proposed sources of heating are shocks [Whang et al., 1990], turbulence [Tu et al., 1984; Tu, 1987; Tu, 1988; Verma et al., 1995a], heat conduction [Gazis, 1984], interactions with neutral particles [Isenberg et al., 1985] etc. Whang et al. [1990] performed observational and simulational studies on the corotating shocks in the solar wind between 1 and 15 AU, and showed that in this region, shocks are major heating sources in the solar wind, with most of the heating confined to 1-5 AU. They found that the increase in the entropy per shock is approximately $0.8 \times 10^{-23}$ J/K/proton. Note that entropy is defined as $k_B \ln(T^{1.5}/n)$, where $T$ is the temperature, $n$ is the particle density, and $k_B$ is the Boltzmann constant. They also derived that the temperature variation due to the observed entropy increase yields $T(r) \propto r^{-0.53}$, where $r$ is the distance from the sun. Note that without any heating, the temperature of the solar wind would be proportional to $r^{-4/3}$.

In a recent paper, henceforth referred to as paper I, Verma et al. [1995a] calculated turbulent heating in the solar wind using the existing MHD turbulence phenomenologies, Kolmogorov-like MHD turbulence phenomenology [Marsch, 1990; Mattheaus and Zhou, 1989; Zhou and Mattheaus, 1990] and Dobrowolny et al.’s generalized Kraichnan phenomenology [Dobrowolny et al., 1980; Kraichnan, 1965]. The basic idea paper I is similar to that of Tu et al. [1984] and Tu [1988] which is that the dissipation rates can be calculated from the inertial range energy spectra without any knowledge of the details of dissipation mechanism. There are some differences between our model and Tu’s [1988] model; they are compared in paper I. In paper I the constants of the phenomenologies were treated as free parameters. Their results were in general agreement with the observations for certain values of the free param-
eters. Recently Verma and Bhattacharjee [1995b,c] have calculated the constants of the Kolmogorov-like phenomenology theoretically using the direct interaction approximation (DIA) technique of Kraichnan [Kraichnan, 1959]. The constants calculated theoretically are in general agreement with the results of numerical simulations by Verma [1994] and Verma et al. [1995d]. However, the constants calculated by DIA and numerical simulations are different from the ones used in paper I. In this paper we calculate the dissipation rates using the constants calculated theoretically, contrary to the procedure of paper I in which the constants were basically free parameters. Section 2 of this paper contains these calculations. Note that the dissipation rates in the solar wind varies with distances. In this paper we estimate typical dissipation rates in the solar wind (at 1 AU).

In section 3 we estimate the contributions of turbulent thermal conduction to the heating in the solar wind. In section 4 of this paper we have estimated turbulent dissipation rates in the solar corona using Dobrowolny et al.’s generalized Kraichnan phenomenology. Here we also restate Hollweg’s [1986] arguments in terms of the Kolmogorov-like MHD turbulence phenomenology. Section 5 contains the concluding remarks.

2 Turbulent Heating in the Solar Wind

In paper I we have derived a formula for the temperature evolution of the solar wind protons due to turbulent heating. Other source of heating were not included in that paper. However, one can easily generalize the equation for the temperature evolution when heating due to shocks and heat conduction are also included [see Priest, 1982]. The generalized equation is

$$\frac{dT}{dr} + \frac{2RT}{CVr} = \frac{\Sigma}{UCV} \left( \epsilon_{turb} + \epsilon_{shock} + \frac{\nabla \cdot q}{\rho} \right),$$

where $T$ is the temperature, $r$ is the radial distance from the sun, $R$ is the Rydberg’s constant, $CV$ is the specific heat per unit mass at constant volume, $\Sigma$ is the mass per unit mole ($\approx 1$ gm for the solar wind plasma), $U$ is the solar wind mean speed, $\epsilon_{turb}$ is the turbulent dissipation rate per unit mass, $\epsilon_{shock}$ is the dissipation rate per unit mass due to shocks, $q$ is the heat flux, and $\rho$ is the density. Here as well as in paper I we have assumed constant speed for the solar wind, hence $\rho \propto r^{-2}$. We make this assumption to simplify the
analysis even though the density is observed to vary somewhat differently from $r^{-2}$. The Eq. (1) has the immediate solution

$$T(r,U) = \left(\frac{r}{r_0}\right)^{-4/3} T_0(U) + \frac{\Sigma}{UC_V} \int_{r_0}^{r} \left(\frac{s}{r_0}\right)^{4/3} \left(\epsilon_{\text{turb}} + \epsilon_{\text{shock}} + \frac{\nabla \cdot q}{\rho}\right) ds$$

where $T_0(U)$ is the temperature at $r_0$ for a stream with velocity $U$.

In paper I we have calculated the turbulent heating in the solar wind using the Kolmogorov-like MHD turbulence phenomenology and Dobrowolny et al.’s modified Kraichnan’s phenomenology. Here we state the energy spectra in these phenomenologies. In Kolmogorov-like phenomenology, they are

$$E^\pm(k) = C^\pm \left(\epsilon^\pm\right)^{4/3} \left(\epsilon^\mp\right)^{-2/3} k^{-5/3}$$

where $E^\pm(k)$ are the energy spectra of fluctuations $z^\pm = u \pm b$ ($u$ is the velocity fluctuation, and $b$ is the magnetic fluctuation in velocity units), $\epsilon^\pm$ are the dissipation rates of $z^\pm$, and $C^\pm$ are Kolmogorov’s constants for MHD turbulence. By inverting the above equation we can obtain the total dissipation rate which is

$$\epsilon = \frac{\epsilon^+ + \epsilon^-}{2} = \frac{1}{2} \left(\frac{\alpha_1}{C^-\sqrt{C^+}} + \frac{\sqrt{\alpha_1}}{C^+\sqrt{C^-}}\right) \left[E^+(k)\right]^{3/2} k^{5/2}$$

where $\alpha_1 = E^-/E^+$. Note that using this scheme we can obtain the dissipation rates using the energy spectra of the wind and the constants $C^\pm$ without any knowledge of the details of dissipation mechanism.

In this paper, without any loss of generality we take $\alpha_1 \leq 1$. The normalized cross helicity $\sigma_c$, a measure of velocity and magnetic field correlation, is defined as

$$\sigma_c = \frac{2v \cdot b}{v^2 + b^2} = \frac{1 - \alpha_1}{1 + \alpha_1}.$$ 

The normalized cross helicity plays an important role in determining dissipation in the solar wind [Verma et al., 1995a].

Recently Verma and Bhattacharjee [1995b,c] have calculated the values of the constants $C^\pm$ using DIA. They find that the $C^\pm$ are not universal constants as is the case in fluid turbulence, but they depend on the Alfvén ratio $r_A$ (ratio of kinetic and magnetic energy) and the normalized cross helicity $\sigma_c$. In Table 1 we list some of the values of these constants. In these
calculations they take $E^+ > E^-$. Unfortunately, in paper I it was assumed that $C^+ = C^- = C$, an approximation valid only for nonAlfvénic streams. In this paper we will use constants $C^+$ and $C^-$ obtained by DIA.

In Dobrowolny et al.’s generalized Kraichnan’s phenomenology, the dissipation rates are given by

$$\epsilon^+ = \epsilon^- = A^{-2} B_0^{-1} E^+(k) E^-(k) k^3$$

(5)

where $A$ is Kraichnan’s constant, and $B_0$ is the mean magnetic field or the magnetic field of the largest eddies. Verma and Bhattacharjee [1995c] have also calculated the Kraichnan’s constant for a variation of Dobrowolny et al.’s model and found that the Kraichnan’s constant $A$ is approximately 2.0 for nonAlfvénic streams. In this paper we emphasize the Kolmogorov-like phenomenology because its predictions appear most consistent with the observed energy spectra of $-5/3$ for both $z^\pm$ in the solar wind [Matthaeus and Goldstein, 1982; Marsch and Tu, 1989].

From the energy spectra and the constants $C^\pm$ we can estimate the dissipation rate $\epsilon_{\text{turb}}$ at reference position $r_0$ using Eq. (4). Then we can obtain the temperature at any position $r$ using Eq. (2). In paper I the best fit to the observed temperature evolution of the solar wind was found when $C = 1.0$ for nonAlfvénic wind and $C = 8.0$ for Alfvénic wind, and the dissipation rates at 1 AU was found to be of the order of $10^{-3}$ km$^2$/s$^3$. The calculation of the temperature evolution using Dobrowolny et al.’s model with neither $A = 1$ nor $A = 0.5$ provided a good fit to the observed temperature evolution, however here too for $A = 1$, the dissipation rates at 1 AU were of the order of $0.5 \times 10^{-3}$ km$^2$/s$^3$ [Verma et al., 1995a]. See paper I for details.

The turbulent dissipation rate depends on the constant $C$, and it is crucial to use the correct constants $C$ for proper estimation of turbulent heating. In the following subsections we use the constants calculated by Verma and Bhattacharjee [1995b,c] to estimate the turbulent dissipation rates in the solar wind.

### 2.1 NonAlfvénic Streams

In the outer heliosphere most of the solar wind streams have been observed to be nonAlfvénic ($\sigma_c \sim 0$) with $r_A \sim 0.5$. From Table 1 $C = 4.04$ [Verma and Bhattacharjee, 1995b,c] for these streams. In paper I Verma et al. found
that when \( C = 1.0 \), turbulent heating can account for all the heating in the solar wind. They estimated turbulent dissipation rate at 1 AU when \( C = 1 \) to be approximately \( 10^{-3} \) km\(^2\)s\(^{-3}\). However, since theoretical value of \( C \) is approximately 4 in the outer heliosphere [Verma and Bhattacharjee, 1995b,c], the correct turbulent dissipation rate is approximately (refer to Eq. (4))

\[
\epsilon = \epsilon_{\text{paper} I} \left( \frac{C_{\text{paper} I}}{C} \right)^{3/2} \sim \frac{10^{-3}}{4^{3/2}} \sim 1.25 \times 10^{-4} \text{ km}^2\text{s}^{-3}.
\]

Hence, in the outer heliosphere, turbulent heating is not sufficient to heat the plasma to the observed temperature. The rest of the heating can be provided by shocks, thermal diffusion, and other mechanisms. Whang et al. [1990] showed that the shock heating is significant, while Gazis [1984] showed that heating due to thermal conduction is significant (for thermal conduction, also refer to Marsch et al. [1983]; Marsch and Richter [1984]). We briefly sketch the results of their work in the section 3.

In the inner heliosphere, the fluctuations are somewhat fluid dominated. Therefore, \( C \) in the inner heliosphere is smaller than \( C \) of the outer heliosphere (see Table 1), hence resulting in a larger turbulent heating. If we take \( r_A \) in the range of 1 to 2, \( C \) is in the range of 3.38 to 2.58 (see Table 1). With these constants too the turbulent heating is utmost \( C^{-3/2} \sim 25 \sim 30\% \) (approximately one-quarter) of the of observed heating (heating rate of paper I) in the solar wind. The corotating shocks are absent in the inner heliosphere. Therefore, shock heating is presumably not significant for non-Alfvénic streams in the inner heliosphere. Hence, neither shocks nor turbulence can provide enough heating in the inner heliosphere. In section 3 we will estimate the contributions of thermal conduction to the overall heating in the inner heliosphere.

Regarding Dobrowolny et al.’s model, for non-Alfvénic streams, the Kraichnan’s constant \( A \) is approximately 2.0. With \( A = 2.0 \), the dissipation rates at 1 AU will be of the order of \( \epsilon_{\text{paper} I}/A^2 \sim 0.5 \times 10^{-3}/A^2 \sim 10^{-4} \text{ km}^2\text{s}^{-3} \) (see Eq. [5]), which again is lower than the observed heating.

### 2.2 Alfvénic streams

In paper I Verma et al. find that the observed temperature evolution of the Alfvénic streams are in good agreement with the predictions of Kolmogorov-like phenomenology if Kolmogorov’s constant for MHD turbulence \( C \) is equal
to 8.0. In paper I $C^+$ and $C^-$ were taken to be equal which is inconsistent with the DIA calculations of Verma and Bhattacharjee [1995b,c]. In the following discussion we modify the dissipation rates of paper I using the theoretical results of Verma and Bhattacharjee [1995b,c].

For large $\sigma_c$, since $\alpha_1$ is small, from Eq. (4)

$$\epsilon_{turb} \sim \frac{1}{2} \sqrt{\alpha_1} C^+ \sqrt{C^-} \left[ E^+(k) \right]^{3/2} \left( k^{5/2} \right).$$

(7)

If we take $\sigma_c = 0.90$ ($\alpha_1 = 0.05$) and $r_A = 2.0$, Verma and Bhattacharjee [1995c] predict $C^+ = 1.54$ and $C^- = 2.08$. With these constants $\epsilon_{turb}$ predicted by the Kolmogorov-like phenomenology is larger by a factor of $8^{3/2}/(C^+ \sqrt{C^-}) \sim 10$ as compared to what was calculated in paper I, i.e., the predicted $\epsilon_{turb}$ with $C^\pm$ of DIA is $10^{-2}$ km$^2$s$^{-3}$. When we take $r_A = 1.0$, unfortunately, the maximum $\sigma_c$ we can get in DIA scheme is 0.5 (after $\sigma_c = 0.5$ there is no consistent solution). This limitation indicates that some of the assumptions of Verma and Bhattacharjee [1995b,c] are not fully consistent. However, from the general trend of $C^\pm$ we believe that the Kolmogorov-like MHD turbulence phenomenology predicts higher dissipation rate as compared to what is observed in the solar wind.

The above study appears to justify the conjecture of Marsch [1991] and Grappin et al., [1991] which states that in the solar wind the Alfvénic streams have not reached steady-state, and the turbulence in these streams is not yet fully developed. The nonlinear interactions may be distributing energy among large and intermediate wavenumbers, with only a small amount of energy flowing into the dissipation range. Therefore, the real dissipation rate is lower than that predicted by the phenomenology, which assumes that the system is in steady-state. Any modelling of “approach towards steady-state turbulence” will shed light to the evolution of energy spectra and help us in correct estimation of turbulent heating in the Alfvénic streams in the solar wind.

The simulational studies show that the system reaches steady-state after around 1 eddy turnover time [Verma, 1994; Verma et al., 1995d]. For solar wind it corresponds to the distance of approximately 2 AU ($1(l/u_{rms})V_{SW} = 1*(0.1AU/20)*400 = 2AU$). Hence, it is possible that the Alfvénic streams have not reached steady-state during its transit in the inner heliosphere, and the estimated turbulent heating by the turbulence phenomenologies is
higher than realistic heating in the solar wind. However, it is hard to justify the above mentioned conjecture in the outer heliosphere. Also note the non-Alfvénic streams appear to have reached steady-state by the time they reach 1 AU. This could be due to the initial conditions of the non-Alfvénic streams near the sun. We need to perform careful analysis of the approach to steady-state and effects of cross helicity in MHD turbulence to understand this behaviour.

Considering that the turbulent heating is not sufficient to heat the non-Alfvénic streams to the observed temperature, in the following section we consider the heating due to shocks and thermal diffusion.

3 Heating due to Shocks and Thermal Diffusion

Whang et al. [1990] studied the average entropy increase across a shock for heliocentric distances of 1-15 AU. They showed that an average entropy increase per shock, $\Delta S$, is approximately $0.8 \times 10^{-23}$ J/K/proton. The strong shocks were in the region of 1-5 AU. Whang et al.’s [1990] found approximately 400 shocks in 1-15 AU region. Therefore, the dissipation rate $\epsilon_{\text{shock}} = N T \Delta S / \Delta t$, where $N$ is the number of shocks in 1-15 AU, $T$ is the temperature and $\Delta t$ is the time taken to travel 1 AU to 15 AU, will be

$$\epsilon_{\text{shock}} \sim N T \Delta S / \Delta t$$

$$\sim 400 \times 10^5 \times 0.8 \times 10^{-23} / (14 \times 1.5 \times 10^8 / 500) \text{J/proton}$$

$$\sim 400 \times 10^5 \times 0.8 \times 10^{-23} \times 10^{-3} / (m_p \times 14 \times 1.5 \times 10^8 / 500) \text{km}^2 / \text{s}^3$$

$$\sim 10^{-3} \text{km}^2 / \text{s}^3$$

Here $m_p$ is the mass of a proton. From the above calculation we see that shock heating is comparable to the turbulent heating, and they are one of the major sources of heating in the outer heliosphere. However, note that Whang et al.’s [1990] observational study of entropy increase in the solar wind with the radial distance

$$S - S_E = 3.81 \times \log (r/r_E) 10^{-23} \text{J/K/proton}$$

where the subscript $E$ denotes the conditions at $r_0 = 1$ AU, is the total entropy increase due to all the heating sources present in the solar wind, not just due to shocks, as inferred in the Whang et al.’s [1990] paper.
Regarding heat conduction, it is widely believed that the coupling between electrons and protons is weak because the protons-electron collision frequency is much smaller as compared to electron-electron or proton-proton collision frequencies. Due to this reason, for the following discussion we assume that the temperature of the protons in the solar wind are affected by the heat flux of protons alone, not by the heat flux of electrons. Marsch and Richter [1984] performed observational studies of the solar wind at 0.95 AU and reported that the heat conduction by protons \( q_i \) is speed dependent and is approximately \( 10^{-4} \) ergs/cm\(^2\)s (also refer to Hundhausen [1972], Marsch et al. [1983], and Gazis [1984]). At 0.95 AU, using particle density as 5/cc, the proton heating rate \( \nabla \cdot q_i / \rho \) due to this heat flux is \( 10^{-4} \) km\(^2\)s\(^{-3}\), which is comparable to the heating rates due to turbulence. Hence, the dissipation due to heat flux appears to be a likely candidate for the solar wind heating.

Now we will obtain theoretical estimates of the heating due to thermal conduction using order of magnitude calculations and compare them with the observational results. The source of heating due to thermal conduction, according to Eq. (1), is

\[
H = -\frac{\nabla \cdot q_i}{\rho} = \frac{\nabla \cdot (K_i \nabla T_i)}{\rho} \sim \frac{K_i T_i}{r^2 \rho},
\]

where \( K_i \) are the thermal conductivity of protons, \( T_i \) are the temperature of protons, and \( r \) is the radial distance from the sun. Here we take \( r = 1 \) AU and \( T_i = 10^5 \)K. Note that the quantities \( H \) decreases with distance. The proton thermal conductivity \( K_i \sim \rho c_V v l \sim \rho c_V \nu \) [Priest, 1982], where \( c_V \) is the specific heat per unit mass of protons, \( v \) is the relevant velocity scale, \( l \) is the length-scale, and \( \nu \) is the kinematic viscosity. Therefore,

\[
H = \frac{c_V \nu T_i}{r^2}.
\]

Note that \( R = 8.31 \) J/K/mole, \( c_V \) for protons is 12.5 Joule/K/gm.

Montgomery [1983] estimated viscosity in the solar wind using the classical transport theory of Braginskii [1965] and obtained typical \( \nu \sim 10^{-6} \) km\(^2\)s\(^{-1}\) [Montgomery, 1983]. We assume that the proton temperature are approximately \( 10^5 \) K. With the above viscosity we find that at 1 AU (\( r = 1 \) AU), \( H \sim 10^{-14} \) km\(^2\)s\(^{-3}\). This \( H \) is negligible as compared to the turbulent dissipation rates and is also inconsistent with the \( H \) obtained from the
observations \cite{Marsch1984, Marsch1983, Hundhausen1972}.

However, since the solar wind plasma is turbulent, thermal conduction coefficient should be determined by large-scale velocity and length scales \cite{Landau1987}. The basic idea is that in a turbulent plasma, the large scale eddies carry heat flux from one region to the other. For this reason, we must use turbulent eddy viscosity due to the large eddies in our calculation for \( K \). Therefore, \( \nu \sim vl \), where \( v \sim 20 \text{ km/s} \) (speed of large eddies) and \( l \sim 10^{12} \text{ cm} \) (size of the large eddies). This yields \( H \sim 10^{-5} \text{ km}^2\text{s}^{-3} \). Since our calculations involve only order of magnitude estimates, we can say that our estimates are in general agreement with the observational results of \cite{Marsch1984, Marsch1983, Hundhausen1972}. Therefore, heating due to thermal conduction, corotating shocks, as well as turbulence appear to be the heating sources of the solar wind in the outer heliosphere.

Gazis \cite{Gazis1984} estimated the thermal energy flux at 1 AU in the solar wind and found it to be equal to \( (2.5 \pm 1.0) \times 10^{-2} \text{ ergs/cm}^2\text{s} \), which corresponds to \( \nabla \cdot \mathbf{q}/\rho \sim 2 \times 10^{-2} \text{ km}^2\text{s}^{-3} \), that is approximately 10 times larger than the dissipation rate estimated in paper I. However, note that in Gazis’ formalism thermal condition was considered as the only source of entropy increase (see Eq. (24) of Gazis \cite{Gazis1984}), therefore, the heating rate calculated by Gazis was an over-estimate. This is not surprising since Gazis \cite{Gazis1984} sets an upper limit on the total heat flux \( \mathbf{q} \).

Regarding heating due to thermal conduction in the inner heliosphere, the substitution of \( T_i = 10^6 \text{K} \), \( r = 0.3 \text{ AU} \), \( v = 20 \text{ km/s} \), and \( l = 10^{12} \text{ cms} \) in Eq. (11) yields \( H_i \sim 10^{-3} \text{ km}^2\text{s}^{-3} \). This quantity is comparable to turbulent heating rate in the solar wind. Thus, ion heating due to thermal conduction is a potentially significant heating source in the inner heliosphere and could provide significant part of the heating in the inner heliosphere. Note that the corotating shocks are absent in the inner heliosphere, therefore, shock heating is presumably not significant for non-Alfvénic streams in the inner heliosphere. Of course, other sources, e.g., stream-stream interactions, neutral ions etc. may also provide significant heating in the inner heliosphere.

To summarize, turbulence provides only part of the dissipation in the solar wind. Corotating shocks as well as thermal conduction appear to be other heating sources in outer heliosphere and could provide rest of the heating. In the inner heliosphere, heating due to thermal conduction appear to be significant. In absence of corotating shocks in the inner heliosphere, thermal...
conduction may provide significant part of the dissipation here. In this paper we have performed only order of magnitude estimates of these quantities, therefore, we cannot make any definite statements regarding energy budget in the solar wind.

Turbulent heating is considered to be one of the potential mechanism which could explain solar corona heating problem. In the following section we estimate amount of turbulent heating in the solar corona using the above mentioned MHD turbulence phenomenologies.

4 Turbulent Heating in the Solar Corona

The temperature of the sun decreases radially outwards to about 6000 K at photosphere, then it rises abruptly upto $2 - 3 \times 10^6$ K at the corona. Why the coronal temperature rises so steeply remains an open problem. Various heating mechanisms have been proposed, turbulent heating being one of them [Hollweg, 1984; Browning and Priest, 1984; Browning, 1991 and references therein]. In the following discussion we state Hollweg’s [1984] calculation of the dissipation rates in the solar corona using the Kolmogorov-like phenomenology, and we calculate the dissipation rates using Dobrowolny et al.’s generalized phenomenology. We compare these estimates with the observational results. Here too we use dimensional arguments similar to those used for solar wind. The power of these arguments lies in the fact that we can estimate the dissipation rates from the inertial range spectra or from the energy fed in at large-scales, but without any knowledge of the dissipation mechanism which may be active at small-scales.

Here we rephrase the Hollweg’s arguments [1984] on the estimation of turbulent heating in the corona in terms of $E^\pm(k)$. We assume that in the corona $E^+(k) \sim E^-(k) \sim E(k)$. Therefore, the dissipation rate according to the Kolmogorov-like phenomenology will be given by

$$\epsilon \sim C^{-3/2}(E(k)k)^{3/2}k.$$  \hspace{1cm} (12)

Taking the average speed of the large-scale eddies $v_k$ as 30 km/s, $E(k)k \sim v_k^2 \sim 10^3$ km$^2$s$^{-2}$. The size of the largest eddies is roughly $10^5$ km. Therefore, $\epsilon \sim 10$ km$^2$s$^{-3}$. Hollweg [1984] argues that the above dissipation rate is compatible with the heating requirements of coronal active region loops.
In contrast, the dissipation rate in Dobrowolny et al.’s generalized phenomenology [Dobrowolny et al., 1980] will be

\[ \epsilon = A^{-2}B_0^{-1}(E(k)k)^2k. \] \hspace{1cm} (13)

With mean number density as $10^{15}$ particles/m$^3$ and the magnetic field of the order 10-100 mT, $B_0$ in velocity units is $10^3 - 10^4$ km/s. Taking $E(k)k \sim 10^3$ km$^2$s$^{-2}$ as before, we obtain $\epsilon \sim 10^{-2} - 10^{-3}$ km$^2$s$^{-3}$. This dissipation rate is three to four order of magnitude lower than that estimated by the Kolmogorov-like model. Therefore, if turbulent heating is playing a major role in the coronal heating, the Kolmogorov-like phenomenology must be at work in the solar corona. This is surprising because one would expect Dobrowolny et al.’s model to work due to the dominance of the mean magnetic field over the fluctuations. In the solar wind too, even though the Parker field is stronger than the fluctuations, the energy spectra follows roughly $k^{-5/3}$ power law. Also the temperature evolution of the solar wind is better explained by the Kolmogorov-like model rather than Dobrowolny et al.’s phenomenology (see paper I). These observations indicate that Kolmogorov-like phenomenology is applicable under conditions where it is not expected to work.

The eddy turnover time-scale in the solar corona is $l/v_l \sim 10^5/30 \sim 300$ sec. Since the plasma reaches steady-state in around 1 eddy-turnover time [Verma, 1994; Verma et al., 1995d], the question arises whether it is possible for the plasma to reach steady-state and be able to supply enough energy in the dissipation range. It is suggested that on the solar surface and in the coronal loops plasma could stay for around 300 seconds, and therefore, turbulent heating could be one of the possible candidate for coronal heating (see Gómez and Fontán [1988]; Browning [1991] and references therein). To resolve the above issues one needs to resort to model calculations; for some of the recent calculations refer to Gómez and Fontán [1988], Heyvaert and Priest [1992], Zirker [1993], Vekstein et al. [1993].

5 Conclusions

In this paper we have estimated the contributions of turbulent dissipation in the heating of the solar wind using Kolmogorov’s constants for MHD turbulence calculated theoretically by Verma and Bhattacharjee [1995b,c]. We
find that for the non-Alfvénic streams, turbulence contributes only partly to the total heating in the solar wind. The remaining part of the heating should be provided by corotating shocks, thermal conduction, stream-stream interactions, interactions with the neutral ions, and by other sources. The corotating shocks are present in the outer heliosphere. As shown by Whang et al. [1990], they could be one of the major heating sources in the outer heliosphere, at least in 1-15 AU where they have been studied, and could provide a major fraction of the heating. The order of magnitude calculations show that the heating due to thermal conduction is significant both in the inner as well as outer heliosphere. In absence of corotating shocks in the inner heliosphere, thermal conduction probably plays an important role in the solar wind heating and could possible provide significant fraction of the heating here. Our results are based on order of magnitude estimates, therefore, we cannot make definite statements. Models incorporating these results may help us in making definite predictions about the contributions by these sources to the solar wind heating.

For the Alfvénic streams, the Kolmogorov-like MHD turbulence phenomenology with Kolmogorov’s constants calculated by Verma and Bhat-tacharjee [1995b,c] predicts higher dissipation rate as compared to what is observed in the wind. The resolution of the paradox could be that the Alfvénic streams have not reached steady-state [Marsch, 1991 and references therein; Grappin et al., 1991], and the energy is just being distributed among various modes, with only a small amount of energy flowing into the dissipation range and heating the plasma. Hence the dissipation rates predicted by the Kolmogorov-like MHD turbulence phenomenology are over-estimates. The studies on “approach to steady-state” will help us in proper estimation of turbulent heating in the Alfvénic streams and in understanding of other related problems.

Lastly, we have discussed the contributions of turbulent dissipation in the coronal heating. Hollweg [1984] has calculated the heating rate using Kolmogorov’s fluid turbulence phenomenology and showed that the results are consistent with the required energy flux in the coronal active loops. In this paper we calculate the heating rates using Dobrowolny et al.’s generalized model and show that the prediction from this model is four orders of magnitude smaller than the required heating rates. This calculation shows that if turbulent heating is major source of heating in the corona, then the Kolmogorov-like phenomenology rather than Dobrowolny et al.’s phe-
nomenclature is valid in the corona. In the corona, the mean magnetic field in velocity units is much larger than the fluctuations, hence, if we assume that the heating in the solar corona is due to turbulence, the this result is contrary to the assumptions of the phenomenologies. However, the result is in agreement with the solar wind results, where the Kolmogorov-like model works even though the Parker’s field is larger than the fluctuation. This is an important puzzle in MHD turbulence, and future theoretical and numerical work will help in clarifying this issue.

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Table 1: Kolmogorov’s constants for MHD turbulence for various values of Alfvén ratio and normalized cross helicity [Verma and Bhattacharjee, 1995b,c]

| $r_A$ | $\sigma_c$ | $C^+$ | $C^-$ |
|-------|-------------|-------|-------|
| 0.5   | 0.0         | 4.04  | 4.04  |
| 1.0   | 0.0         | 3.38  | 3.38  |
| 2.0   | 0.0         | 2.58  | 2.58  |
| 5.0   | 0.0         | 1.92  | 1.92  |
| 1.0   | 0.43        | 0.54  | 53.80 |
| 1.5   | 0.67        | 1.44  | 6.47  |
| 2.0   | 0.90        | 1.54  | 2.08  |