Non-invasive Measurements of Cavity Parameters by Use of Squeezed Vacuum

Eugeniy E. Mikhailov, Keisuke Goda, and Nergis Mavalvala
LIGO Laboratory, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

We propose and experimentally demonstrate a method for non-invasive measurements of cavity parameters by injection of squeezed vacuum into an optical cavity. The principle behind this technique is the destruction of the correlation between upper and lower quantum sidebands with respect to the carrier frequency when the squeezed field is incident on the cavity. This method is especially useful for ultrahigh Q cavities, such as whispering gallery mode (WGM) cavities, in which absorption and scattering by light-induced nonlinear processes inhibit precise measurements of the cavity parameters. We show that the linewidth of a test cavity is measured to be $\gamma = 844 \pm 40$ kHz, which agrees with the classically measured linewidth of the cavity within the uncertainty ($\gamma = 856 \pm 34$ kHz).

I. INTRODUCTION

High Q cavities such as whispering gallery mode (WGM) cavities have recently demonstrated quality factors ($Q$) as high as $2 \times 10^{10}$ and have shown the potential to reach even higher Q values $\Gamma, \Delta, \Omega$. However, there are difficulties in measurement of the linewidth and $Q$ of such high Q cavities. While in theory, the Q factor could be as high as $10^{12}$ and is limited only by Rayleigh scattering $\Gamma$, in practice, it is limited by other losses in the cavity. They include absorption and scattering losses due to impurities in the cavity material, and light-induced losses due to nonlinear processes. Due to the extremely small mode volume and high Q-factor of the cavity, the cavity build-up intensity is extremely high, even in the case of an input with small power (as small as several mW). Such a high resonator intensity leads to very efficient nonlinear processes inside WGM cavities, such as Raman scattering, second harmonic generation, and four-wave mixing $\Delta$. Whereas this is beneficial in many applications, it causes additional losses in the cavity and thus makes the Q factor measurement unreliable (at least, making it power-dependent) $\Omega$.

Squeezed states of vacuum or light have been used in many applications such as improvement in interferometric $\Gamma, \Delta, \Omega$ and absorption $\Gamma$ measurements, for quantum teleportation $\Delta$ and quantum cryptography $\Gamma$, and for quantum imaging $\Delta$. However, to the best of our knowledge, no experiment for measurements of cavity parameters by use of squeezing has yet been reported. In this paper we propose and demonstrate an alternative method of measuring Q factors by use of a squeezed vacuum field which is equivalent to a field with correlated quantum sidebands $\Gamma, \Omega$. This technique is advantageous over traditional optical methods in that it utilizes the injection of squeezed vacuum into a test cavity not to excite any nonlinear processes in the cavity. When the input field is detuned from the cavity resonance frequency, it transmits only the upper or lower quantum sidebands within the cavity linewidth while reflecting the counterparts (associated upper or lower sidebands) and all the other sidebands. The linewidth of the cavity can then be measured by observing the destruction of the correlation between the upper and lower quantum sidebands with respect to the carrier frequency. We show that the linewidth and Q factor of a test cavity using the method agrees with those measured by traditional optical methods.

This paper is organized as follows: In Sec. II A we describe the theoretical framework for the measurement method. In Sec. II B we explain the validity of the use of squeezed vacuum as a probe for non-invasive measurements and compare the technique to using a classical state. In Sec. III we demonstrate the method using a test cavity with known cavity parameters and compare the parameter values obtained by the new method and the traditional optical methods. The conclusions of the paper are summarized in Sec. IV.

II. THEORY

A. Destruction of Quantum Sideband Correlation as a Probe for Cavity Parameter Measurements

Consider a squeezed vacuum field with carrier and sideband frequencies, $\omega_0$ and $\omega_0 \pm \Omega$ respectively. As shown in Fig. 1, when the upper sideband of the squeezed vacuum field $a(\omega_0 + \Omega)$ is injected into an optical cavity with resonance frequency $\omega_c$ and mirror reflectivities $R_1, R_2,$ and $R_3$, the reflected field $b(\omega_0 + \Omega)$ and its adjoint $b^{\dagger}(\omega_0 - \Omega)$ are given in terms of $a(\omega_0 + \Omega)$ and its adjoint $a^{\dagger}(\omega_0 - \Omega)$ by

\begin{align}
b(\omega_0 + \Omega) &= r(\omega_0 + \Omega) a(\omega_0 + \Omega) + l(\omega_0 + \Omega) r^{\ast}(\omega_0 + \Omega), \\
b^{\dagger}(\omega_0 - \Omega) &= r^{\ast}(\omega_0 - \Omega) a^{\dagger}(\omega_0 - \Omega) + l^{\ast}(\omega_0 - \Omega) r^{\ast}(\omega_0 - \Omega),
\end{align}

where $r(\omega_0 \pm \Omega)$ is the frequency-dependent cavity reflection coefficient and $l(\omega_0 \pm \Omega)$ is the vacuum noise coupling coefficient associated with transmission and intra-cavity losses. When the cavity is not perfectly mode-matched, the reflected field contains the cavity-coupled reflection $a_c^{\dagger}$ and the promptly reflected field $a_m$ that does not
couple to the cavity due to mode mismatch such that
\[
\begin{align*}
    r(\omega_0 + \Omega)a(\omega_0 + \Omega) &= r_c(\omega_0 + \Omega)a_c(\omega_0 + \Omega) + r_m a_m(\omega_0 + \Omega), \\
    r^*(\omega_0 - \Omega)a^\dagger(\omega_0 - \Omega) &= r^*_c(\omega_0 - \Omega)a^\dagger_c(\omega_0 - \Omega) + r^*_m a^\dagger_m(\omega_0 - \Omega), \\
    l(\omega_0 + \Omega)v(\omega_0 + \Omega) &= l_c(\omega_0 + \Omega)v_c(\omega_0 + \Omega) + l_m v_m(\omega_0 + \Omega), \\
    l^*(\omega_0 - \Omega)v^\dagger(\omega_0 - \Omega) &= l^*_c(\omega_0 - \Omega)v^\dagger_c(\omega_0 - \Omega) + l^*_m v^\dagger_m(\omega_0 - \Omega),
\end{align*}
\]
where \(a_c\) and \(a_m\) are spatially orthogonal and
\[
    r_c(\omega_0 \pm \Omega) = r_c(\omega_d \pm \Omega) = \frac{\sqrt{R_1}}{1 - \sqrt{R_1 R_2 R_3} e^{-i[\phi_c(\omega_d) \pm \phi_c(\Omega)]}}.
\]
Here, \(\omega_d\) is the detuning from the cavity resonance given by \(\omega_d = \omega_0 - \omega_c\) and we have assumed that the resonance frequency of \(a_m\) is far from that of \(a_c\) such that the reflection coefficient \(r_m\) can be treated as a frequency-independent constant at frequencies around the resonance frequency of \(a_m\). The vacuum noise coupling coefficients are then given by
\[
    l_c(\omega_0 \pm \Omega) = l_c(\omega_d \pm \Omega) = \sqrt{1 - |r_c(\omega_d \pm \Omega)|^2},
\]
\[
    l_m(\omega_0 \pm \Omega) = l_m(\omega_d \pm \Omega) = \sqrt{1 - |r_m|^2}.
\]
The cavity mirror reflectivity and transmission of each mirror satisfies
\[
    R_i + T_i + L_i = 1, \quad \text{for } i = 1, 2, 3,
\]
where \(L_i\) is the loss of each mirror. The intra-cavity losses can be absorbed into \(R_3\).

Since the carrier is detuned from the cavity resonance frequency, the reflection acquires extra frequency-dependent phase shifts at the detuned carrier frequency and the sideband frequencies, respectively given by
\[
    \phi_c = \frac{p}{c} \frac{\omega_d}{\omega_{FSR}}, \quad \phi_m = \frac{p}{c} \frac{\Omega}{\omega_{FSR}},
\]
where \(p\) and \(\omega_{FSR}\) are the round-trip length and free spectral range of the cavity, and \(c\) is the speed of light in vacuum.

For simplicity, we transform into the rotating frame of the carrier frequency \(\omega_0\) in the frequency domain, such that Eqs. (1) and (2) become
\[
\begin{align*}
    b(\Omega) &= r_c(\omega_d + \Omega)a_c(\Omega) + r_m a_m(\Omega) + l_c(\omega_d + \Omega)v_c(\Omega) + l_m v_m(\Omega), \\
    b^\dagger(-\Omega) &= r^*_c(\omega_d - \Omega)a^\dagger_c(-\Omega) + r^*_m a^\dagger_m(-\Omega) + l^*_c(\omega_d - \Omega)v^\dagger_c(-\Omega) + l^*_m v^\dagger_m(-\Omega),
\end{align*}
\]
where \(a_c(\Omega)\) and \(a^\dagger_c(-\Omega)\) satisfy the commutation relations
\[
    [a_c(\pm \Omega), a^\dagger_c(\pm \Omega')] = 2\pi \delta(\Omega - \Omega'),
\]
and all others vanish (similarly for \(a_m(\Omega), a^\dagger_m(-\Omega), v_c(\Omega), v^\dagger_c(-\Omega), v_m(\Omega),\) and \(v^\dagger_m(-\Omega)\)). In the two-photon representation, the amplitude and phase quadratures of \(a_c\) are defined by
\[
\begin{align*}
    a^z_c(\Omega) &= a_c(\Omega) + a^\dagger_c(-\Omega), \\
    a^z_m(\Omega) &= -i [a_c(\Omega) - a^\dagger_c(-\Omega)]
\end{align*}
\]
respectively (similarly for \(a_m, b, v_c,\) and \(v_m\)). A little algebra yields the amplitude and phase quadrature fields of the reflected light in compact matrix form,
\[
b = M a_c + r_m a_m + H v_c + l_m v_m,
\]
where we use the two-photon matrix representation
\[
a_c \equiv \begin{pmatrix} a^z_c \\ a^y_c \end{pmatrix}
\]
for the operator \(a_c\) (similarly for \(a_m, b, v_c,\) and \(v_m\)),
\[
M = e^{i \varphi_-} \begin{pmatrix} \cos \varphi_+ & -\sin \varphi_+ \\ \sin \varphi_+ & \cos \varphi_+ \end{pmatrix} \begin{pmatrix} A_+ & i A_- \\ -i A_- & A_+ \end{pmatrix}
\]
is a matrix representing propagation through the cavity, and
\[
H = \begin{pmatrix} l^*_+ & il^*_- \\ -il_+ & l^- \end{pmatrix}
\]
\(M\) comprises an overall phase shift \(\varphi_-\), rotation by angle \(\varphi_+\), and attenuation by factor \(A_+\). Here we have defined
\[
\begin{align*}
    \varphi_\pm &= \frac{1}{2} \left[ \arg(r_c(\Omega)) \pm \arg(r_c(-\Omega)) \right], \\
    A_\pm &= \frac{1}{2} \left[ |r_c(\Omega)| \pm |r_c(-\Omega)| \right], \\
    l_\pm &= \frac{1}{2} \left[ l_c(\omega_d + \Omega) \pm l_c(\omega_d - \Omega) \right].
\end{align*}
\]
In the case of no carrier detuning ($\omega_d = 0$), $r_c(\Omega) = r_+^*(-\Omega)$, and $\varphi_+ + \varphi_- + \Delta \varphi$ vanish, giving neither quadrature angle rotation nor asymmetrical amplitude attenuation. In the case of cavity detunings ($\omega_d \neq 0$), nonzero $\varphi_+$ gives quadrature angle rotation.

From Eq. 18, when we perform homodyne detection of the reflected field with a local oscillator (LO) field, the measured amplitude and phase quadrature variances of the field, defined by $V_i = \langle b_i^* b_i \rangle - \langle b_i \rangle^2$, and $V_{ic} = \langle b_{ic}^* b_{ic} \rangle - \langle b_{ic} \rangle^2$ (similarly for $V_{a}, V_{ac}, V_{am},$ and $V_{ac}$), are found in terms of the mode-matched input amplitude and phase quadrature variances $V_{ic}$ and $V_{ac}$ to be

$$
\begin{align*}
\left( \begin{array}{c}
V_{b}^b \\
V_{b}^c
\end{array} \right) &= \\
&= \eta_c \left( \begin{array}{c}
\cos^2 \varphi + \sin^2 \varphi \\
\sin^2 \varphi + \cos^2 \varphi
\end{array} \right) \left( \begin{array}{cc}
A^2 & A^2 \\
A^2 & A^2
\end{array} \right) \left( \begin{array}{c}
V_{ic}^a \\
V_{ic}^c
\end{array} \right) \\
&+ \eta_m r_m^2 \left( \begin{array}{c}
V_{icm}^a \\
V_{icm}^c
\end{array} \right) + \eta_c \left[ 1 - (A^2 + A^2) \right] \left( \begin{array}{c}
1 \\
1
\end{array} \right) \\
&+ \eta_m (1 - r_m^2) \left( \begin{array}{c}
1 \\
1
\end{array} \right) + \eta_c \left( \begin{array}{c}
1 \\
1
\end{array} \right),
\end{align*}
$$

where $\eta_c$ and $\eta_m$ are the composite efficiencies of detection associated with the cavity-coupled and cavity-mismatched modes respectively, $\eta_c$ is the coupling of detection losses, and $\eta_c + \eta_m + \eta_l = 1$. The detection efficiency is a product of the quantum efficiency of the photodiodes and the mode-overlap efficiency with the LO mode. Eq. 25 can be rewritten in terms of the quadrature variances of the incident field $V_{12}^\alpha$ since the cavity-coupled reflection $V_{12}^\alpha$ and the mode-mismatch reflection $V_{12}^{am}$ originate from the same incident field $V_{12}^\alpha$, such that

$$
\begin{align*}
\left( \begin{array}{c}
V_{12}^\alpha \\
V_{12}^{am}
\end{array} \right) &= \\
&= \left( \begin{array}{c}
V_{1}^a \\
V_{12}^a
\end{array} \right),
\end{align*}
$$

and therefore,

$$
\begin{align*}
\left( \begin{array}{c}
V_{12}^b \\
V_{12}^c
\end{array} \right) &= \\
&= \left[ \eta_c \left( \begin{array}{c}
\cos^2 \varphi + \sin^2 \varphi \\
\sin^2 \varphi + \cos^2 \varphi
\end{array} \right) \right] \left( \begin{array}{cc}
A^2 & A^2 \\
A^2 & A^2
\end{array} \right) + \eta_m r_m^2 \left( \begin{array}{c}
V_{1}^a \\
V_{12}^a
\end{array} \right) \\
&+ \left[ 1 - \eta_c \left( A^2 + A^2 \right) - \eta_m r_m^2 \right] \left( \begin{array}{c}
1 \\
1
\end{array} \right).
\end{align*}
$$

Note that if the input field is in a vacuum or coherent state such that $V_{12}^a = V_{12}^b = 1$, then $V_{12}^b = V_{12}^c = 1$, as expected, and no cavity information is contained in the output state $b$.

If the carrier frequency is detuned downward from the cavity resonance frequency, the cavity transmits only the upper sidebands within the cavity linewidth and replaces them by vacuum at those frequencies while reflecting the associated lower sidebands and all the other sidebands. Hence, the cavity-coupled reflected field is composed of the uncorrelated sidebands within the linewidth and the reflected correlated sidebands outside of it. The consequence is the destruction of the correlation within the linewidth between the upper and lower quantum sidebands. This is analogous to the destruction of the correlation between electro-optically modulated coherent sidebands in pairs, in which the beat between the carrier and the upper or lower sideband can be measured only when either sideband is absorbed into the cavity, reflecting the carrier and other sideband. The beat could not be observed if all the fields were reflected. Similar measurements could be done with the transmission of the squeezed vacuum field through the cavity. However, the signal-to-noise ratio would not be as good as in the reflection method because the background of the transmission signal is shot noise.

It is convenient to define the test cavity linewidth $\gamma$, the quality factor $Q$, and the finesse $F$, as

$$
\begin{align*}
\gamma &= \frac{2}{\pi} \omega_{FSR} \sin^{-1} \left[ 1 - \sqrt{R_1 R_2 R_3} \right] \frac{1}{2(R_1 R_2 R_3)^{1/4}} \\
&\approx \frac{1}{\pi(R_1 R_2 R_3)^{1/4} \omega_{FSR}},
\end{align*}
$$

and

$$
F = \frac{\pi(R_1 R_2 R_3)^{1/4} \omega_{FSR}}{1 - \sqrt{R_1 R_2 R_3}} \approx \frac{\omega_{FSR}}{\gamma},
$$

respectively. The approximations made in Eqs. 27 and 28 are valid for high $Q$ cavities. $R_1, R_2, R_3$, and $\omega_{FSR}$ will be treated as free fitting parameters. We also assume the input mirror is lossless such that $T_1 = 1 - R_1$.

### B. Squeezed/Anti-squeezed Vacuum vs. Classically Noisy Light

Since we are interested in having as little light (at the carrier frequency) as possible in the test cavity, it is instructive to calculate the average photon number in the field we use. The average photon number in squeezed light with squeeze factor $r$ and squeeze angle $\theta$ is given by

$$
\langle N \rangle = \langle a^\dagger a \rangle = \langle |a|^2 (\cosh^2 r + \sinh^2 r) - (\alpha^2)^2 e^{i\theta} \sinh r \cosh r - \alpha^2 e^{-i\theta} \sinh r \cosh r + \sinh^2 r, \rangle
$$

where $\alpha$ is the coherent amplitude of the light. As the number of coherent photons becomes zero ($\alpha \rightarrow 0$), resulting in squeezed vacuum, Eq. 31 becomes

$$
\langle N \rangle = \langle a^\dagger a \rangle = \sinh^2 r.
$$

This is the average photon number in squeezed vacuum generated by squeezing. Note that if the field is unsqueezed ($r = 0$), $\langle N \rangle = 0$. For a squeeze factor of 1.5
corresponding to the squeezed or anti-squeezed level of −13 dB which is the current experimental limit \( \langle N \rangle = 4.53 \). Therefore, it is fair to say that the optical influence of ideal squeezed vacuum on cavities is negligible.

Similarly, it is instructive to compare this technique to using a classical state. For simplicity, assuming that the quadrature variance in both quadratures is frequency-independent, we consider the case in which the lower sideband is fully transmitted through an impedance-matched cavity and the upper sideband is fully reflected at the input mirror such that \( r_c(-\Omega) = 0 \) and \( r_c(\Omega) = 1 \) at \( \Omega = \omega_d \), respectively, which gives \( A_+ = A_- = 1/2 \) from Eq. (28). Thus, the amplitude and phase quadrature variances of the reflected field are found to be

\[
V^b(\omega_d) = V^b_{1}(\omega_d) = \frac{1}{4} (V^a_1 + V^a_2) + \frac{1}{2},
\]

In the absence of coherent light, the signal contrast can be defined as the quadrature variance at detuning frequency \( \omega_d \) compared to the cavity-uncoupled quadrature variance at off-resonance frequencies (\( |\Omega - \omega_d| \gg \gamma \)), in which case \( V^b_1 = V^a_1 \) and \( V^b_2 = V^a_2 \), and the signal contrasts at the two orthogonal quadratures are respectively given by

\[
S_1(\omega_d) = \frac{V^a(\omega_d)}{V^a_1} = \frac{\frac{1}{2}(V^a_1 + V^a_2) + \frac{1}{2}}{V^a_1} \]

\[
S_2(\omega_d) = \frac{V^a}{V^a_{2}(\omega_d)} = \frac{V^a_2}{\frac{1}{2}(V^a_1 + V^a_2) + \frac{1}{2}}.
\]

We see that \( S_2 \) has about the same limiting level as in the classical case, while \( S_1 \) grows if \( V^a_1 \) gets smaller. Classically, \( V^a_1 \geq 1 \) (the shot noise limit), but using squeezed vacuum we can obtain \( V^a_1 < 1 \), or improved signal contrast for a measurement in the squeezed quadrature. This is illustrated in Fig. 2 where we compare the signal contrast for measurement of the cavity linewidth using a classical field with the signal contrast for squeezed field injection. The cavity-coupled responses of the classical and anti-squeezed quadrature variances behave almost identically in the case of the impedance-matched cavity, whereas squeezing improves the signal contrast of the measurement.

C. Fundamental Limit on Measurement Uncertainty

It is important to note that even in the absence of technical noise, quadrature variance measurements are intrinsically contaminated by quantum noise itself. The standard deviation of the quadrature variances is given by

\[
\Delta V^b_j = \sqrt{2} V^b_j \quad \text{for } j = 1, 2.
\]

Thus, the noise of the measurement is proportional to the measured value itself, and many averages can be performed to achieve smaller uncertainty levels.

This is different from the classical case where the parameters of a cavity are measured by measuring the transmission of a probe optical field incident on the cavity as a function of cavity detuning. In this case, the measurements are fundamentally limited by shot noise: the number of measured photons (\( N \)) has uncertainty proportional to \( \sqrt{N} \). Therefore, the signal-to-noise ratio grows as the number of the transmitted photons increases.

III. EXPERIMENT

The experiment is schematically shown in Fig. 3. The Nd:YAG laser (Lightwave Model 126) gives an output of cw 700 mW at 1064 nm, which is injected into the squeezed vacuum generator (squeezer). The squeezer is composed of a second harmonic generator (SHG) and an optical parametric oscillator (OPO), both using 5% MgO:LiNbO\(_3\) nonlinear crystals placed within optical cavities (hemilith for the SHG and monolith for the OPO) in the Type I phase-matching configuration. The SHG pumped by the Nd:YAG laser generates 250 mW at 532 nm, which then pumps the OPO below threshold.
Servo: PDH-locking servo. The oscillators (OC1, OC2) are dynel photo-detectors, BS: 50/50 beamsplitter, S: subtractor, transducers, PD: photo-detector, HD1 and HD2: homo-acousto-optic modulator, EOM: electro-optic modulator, squeezed vacuum generator, FI: Faraday isolator, AOM: the cavity linewidth, destroying the correlation between this frequency shift, only the upper sidebands are within 00 MHz from the TEM00 mode by 11 MHz and 13 MHz respectively. The squeezed vacuum generator is composed of an optical parametric oscillator (OPO) and a second harmonic generator (SHG) that pumps the OPO. The cavity length is locked to the laser frequency by the PDH-locking servo and PZT (PZT2). The homodyne angle is locked by the noise-locking servo and PZT (PZT1).

with a vacuum seed. The resultant field generated by the OPO is a squeezed vacuum field with a squeezing bandwidth of 66.2 MHz defined by the OPO cavity linewidth. A sub-carrier field, frequency-shifted by an acousto-optic modulator (AOM) to a frequency that is coincident with the cavity TEM01 mode, is injected into the other end of the OPO cavity. The cavity is thus locked to the TEM01 mode, offset by 220 MHz from the carrier frequency, using the Pound-Drever-Hall (PDH) locking technique. The frequency-shift is necessary to ensure that no cavity transmitted light at the fundamental frequency is injected into the OPO cavity since it acts as a seed and degrades broadband squeezing due to the imperfect isolation of the Faraday isolator. This is especially important for high Q cavities with linewidths as narrow as kHz because low-frequency squeezing is difficult to achieve.

The squeezed vacuum is injected into a triangular test cavity with the FSR of 713 MHz and FWHM of 856 ± 34 kHz, both measured by traditional methods using light. The frequency shift, of the subcarrier is 231 ± 0.1 MHz so that the carrier frequency is detuned from the TEM00 mode by 11.0 ± 0.1 MHz. As a result of this frequency shift, only the upper sidebands are within the cavity linewidth, destroying the correlation between the upper and lower sidebands and, therefore, destroying the squeezing or anti-squeezing. This cavity-coupled squeezed vacuum reflection is measured by balanced homodyne detection, where the field to be measured interferes with a local oscillator (LO) field and is detected by two (nearly) identical photodetectors. The difference of the two photodetector signals is sent to an HP4195A spectrum analyzer (SA) to measure the noise variance of the squeezed or anti-squeezed quadrature. The results are shown in Fig. 4. The experimental data are exponentially averaged 100 times. The resolution bandwidth of the spectrum analyzer is 100 kHz. Since the squeezed vacuum does not carry any coherent amplitude, the noise-locking technique is employed to lock the homodyne angle to either the squeezed or anti-squeezed quadrature.

Before fitting the experimental data points, the homodyne efficiencies \(\epsilon_h\) and \(\epsilon_{hm}\), and the quantum efficiency of the photo-detectors \(\eta_{QE}\) need to be taken into account. The sum of the homodyne efficiencies and the quantum efficiency were independently measured to be 90% and 85% respectively. The sum of the efficiencies \(\eta_c + \eta_m\) in Eq. (24) is given by \(\eta_c + \eta_m = (\epsilon_h + \epsilon_{hm})\eta_{QE}\). We ignore \(\epsilon_{hm}\) since the cavity mode-matching efficiency is 82% and hence \(\epsilon_{hm} \ll \epsilon_h\), which yields \(\eta \approx 1 - \eta_c\). Moreover, we have assumed that the input mirror \(M_1\) is lossless. This assumption is valid since it is a single-pass loss and does not influence the linewidth of the cavity. We then fit Eq. (24) to the measured data points with free parameters \(R_1, R_2, R_3\), and \(\omega_d\); both the data and the fits are shown in Fig. 4. The resulting fitting values are \(\sqrt{R_1 R_2 R_3} = 0.99628 \pm 0.00016\), \(\sqrt{R_1} = 0.99783 \pm 0.00005\), and \(\omega_d/(2\pi) = 11.098 \pm 0.017\) MHz.
Therefore, the FWHM linewidth of the cavity is found to be $\gamma = 844 \pm 40$ kHz, which agrees with the classically measured linewidth of the cavity within the uncertainty ($\gamma = 856 \pm 34$ kHz). We note that $\omega_{\text{FSR}}$ can be determined from the fit, but here we have used the optically measured value to estimate the linewidth. This is valid because any loss in the cavity does not change the FSR.

IV. CONCLUSION

We have proposed and experimentally demonstrated a method for non-invasive measurements of optical cavity parameters by use of squeezed vacuum. The technique has the advantage over traditional optical methods that the injection of a squeezed vacuum field as a probe for cavity parameters does not excite any nonlinear processes in cavities, and is, therefore, useful for ultrahigh Q cavities such as whispering gallery mode (WGM) cavities. We have shown that when a squeezed vacuum field is injected into a detuned cavity, the linewidth and $Q$ factor of a test cavity can be determined by measuring the destruction of upper and lower quantum sidebands with respect to the carrier frequency. The linewidth of a test cavity is measured to be $\gamma = 844 \pm 40$ kHz, which agrees with the classically measured linewidth of the cavity within the uncertainty ($\gamma = 856 \pm 34$ kHz). We have also show that the use of squeezed fields leads to better signal contrast, as expected.

V. ACKNOWLEDGMENTS

We would like to thank our colleagues at the LIGO Laboratory, especially Thomas Corbitt and Christopher Wipf, and Stan Whitcomb for his valuable comments on the manuscript. We gratefully acknowledge support from National Science Foundation Grant Nos. PHY-0107417 and PHY-0457264.

[1] V. S. Ilchenko, A. A. Savchenkov, A. B. Matsko, and L. Maleki, Phys. Rev. Lett. 92, 043903 (pages 4) (2004).
[2] K. J. Vahala, Nature 424, 839 (2003).
[3] A. A. Savchenkov, V. S. Ilchenko, A. B. Matsko, and L. Maleki, Phys. Rev. A 70, 051804 (2004).
[4] M. L. Gorodetsky, A. D. Pryamikov, and V. S. Ilchenko, J. Opt. Soc. Am. B 17, 1051 (2000).
[5] A. A. Savchenkov, A. B. Matsko, D. Strekalov, M. Mohageg, V. S. Ilchenko, and L. Maleki, Phys. Rev. Lett. 93, 243905 (2004).
[6] A. A. Savchenkov, Private communication (2006).
[7] C. M. Caves, Phys. Rev. D 23, 1693 (1981).
[8] K. McKenzie, D. A. Shaddock, D. E. McClelland, B. C. Buchler, and P. K. Lam, Phys. Rev. Lett. 88, 231102 (2002).
[9] M. Xiao, L.-A. Wu, and H. J. Kimble, Phys. Rev. Lett. 59, 278 (1987).
[10] P. Grangier, R. E. Slusher, B. Yurke, and A. LaPorta, Phys. Rev. Lett. 59, 2153 (1987).
[11] F. Marin, A. Bramati, V. Jost, and E. Giacobino, Opt. Comm. 140, 146 (1997).
[12] A. Furusawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science 282, 706 (1998).
[13] C. H. Bennett, F. Bessette, G. Brassard, L. Salvail, and J. Smolin, J. Crypto. 5, 3 (1992).
[14] M. I. Kolobov, Rev. Mod. Phys. 71, 1539 (1999).
[15] C. M. Caves and B. L. Schumaker, Phys. Rev. A 31, 3068 (1985).
[16] B. L. Schumaker and C. M. Caves, Phys. Rev. A 31, 3093 (1985).
[17] A. E. Siegman, Lasers (University Science Books, 1986).
[18] M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge University Press, 1997).
[19] P. K. Lam, T. C. Ralph, B. C. Buchler, D. E. McClelland, H.-A. Bachor, and J. Gao, J. Opt. B: Quantum Semiclass. Opt. 1, 469 (1999).
[20] T. Aoki, G. Takahashi, and A. Furusawa, quant-ph/0511239 (2005).
[21] K. McKenzie, E. E. Mikhailov, K. Goda, P. K. Lam, N. Grosse, M. B. Gray, N. Mavalvala, and D. E. McClelland, J. Opt. B: Quantum Semiclass. Opt. 7, S421 (2005).
[22] R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, Appl. Phys. B 31, 97 (1983).
[23] K. McKenzie, N. Grosse, W. P. Bowen, S. E. Whitcomb, M. B. Gray, D. E. McClelland, and P. K. Lam, Phys. Rev. Lett. 93, 161105 (2004).
[24] W. P. Bowen, R. Schnabel, N. Treps, H.-A. Bachor, and P. K. Lam, J. Opt. B: Quantum Semiclass. Opt. 4, 421 (2002).