PAPER

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External models of frictional interaction dynamics

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Abstract This investigation suggests a method used to determine the evolution of metallic wear and friction by sliding. The friction of steel moving over brass was taken as an example. The problem of external dynamics friction is investigated through the definition of the dynamic characteristics such as damping factor and natural frequency. Some certain automatic control methods were applied for sliding friction contact, including parametric identification, ARX simulation and Newton’s dynamic equation. The suggested approach allows using amplitude-frequency characteristics to assess the dynamic factors (coefficients) under friction interaction. The research findings indicate that the proposed method allows monitoring the evolution of metallic wear and friction.

1. Introduction
The article investigates some issues referred to the friction study based on input-output models. The main aspect of this approach is to identify damping and vibration properties of a tribological system. The input-output model is widely used to solve a range of experimental problems in automatic control theory [1]. The use of well-known approaches to triboprocess stimulation and the application of multi-order equations allow analyzing friction processes with subject to their surface and intermediate layers [2, 3]. It is important to predict some variations in the phase of a relative face slip in the majority of practical systems where metals, fiber, composite alloys are exposed to sliding with metallic materials. These measurements can be implemented differently. Some of them are vibration, noise monitoring, and temperature control of the friction surface. At present, however, the problem of accurate forecast based on the friction model has not been solved yet, due to the complicacy of the processes during friction interaction. The aim of this work is to study internal dynamic processes, friction evolution and determine the adaptive sliding model [4, 5].

2. Fundamental principles
It is convenient to consider the tribological system with regard to its input-output. The input parameters of this system are dynamic characteristics like sliding velocity, rolling speed, force and pressure. Vibration and power dissipation (heating) belong to its output parameters. Under some external actions on this system it is possible to keep track of its reaction on this impact. The main
types of the system response are its resistance to friction, surface deterioration, temperature rise on the friction surface and adherence of surfaces. [6].

Upon the results of the process identification we can create the mathematical model of the object. This model is represented in time and frequency domains. Moreover, this model reproduces and imitates the adequate object behavior. The operation mode of the Tribal-T device is shown in Figure 1(a). This device is intended to determine the friction factor (coefficient) and perform a real-time monitoring of the process. The main distinction of this device from similar friction and wear test machines is in the free movement of the upper sample relative to a lower one.

![Figure 1](image)

**Figure 1.** a – Schematic diagram of the device and its components: P - load, c - variable rigidity, v(t), a(t) - velocity (speed) and acceleration of the given law of motion b – The basic mode of displacement curves, where $\frac{dt}{dt}$ is the phase shift, Ax, Ay are the amplitudes of oscillations.

The phase shift curve of displacement curves is shown in Figure 1(b). The lower sample $x(t)$ moves forward, while the upper one $y(t)$ is delayed due to spring elasticity and friction forces affecting its movement. There is some relative motion of the samples with a typical sliding friction when the device operates in this mode.

3. **Experimental unit**

The experimental tribological unit Tribal-T (Fig. 2) was developed on that basis. This unit is intended to determine the tribological and mechanical properties of materials. The device for wear testing, which description is given in [2], was used as an analog. The idea [7, 8] of reciprocating motion of test samples relative to each other was implemented in this unit [3].

![Figure 2](image)

**Figure 2:** Tribometer Tribal-T appearance and its main components: 1 - samples; 2 - displacement sensor; 3,4 - load sensors in normal and radial directions; 5 - precision linear sliding bearings; 6.7-
load and speed control board; 8-cyclic linear drive; 9-loading drive.

Lower samples execute cyclically reciprocating movements driven by a cyclic linear drive (8). Upper samples are in motion due to the impact of the friction force. Displacement and pressure sensors allow tracking the absolute displacement of samples and generate a digital signal in a real time. The measured signals are inputs and outputs of the investigated system. The proposed approach provides for using the Tribal-T tribometer to study sliding (rubbing) friction.

4. Mathematical representation of system identification
In the general case, system "input" and "output" can be described by the n-th order differential equation, where the right side is "input", the parameters $a_0, a_1, \ldots, a_n$ are selected for a given input signal, and $b_0, b_1, \ldots, b_m$ are considered being specified.

$$a_0x^{(n)} + a_1x^{(n-1)} + \cdots + a_n x = b_0y^{(m)} + b_1 y^{(m-1)} + \cdots + b_m y$$

where $x^{(n)} = \frac{d^n x}{dt^n}$ under the assumption that $n > m$.

However, to solve this equation it is necessary, first of all to take into account the order of differentiation, which provides physical interpretation. When $n = 2$ we will get second-order differential motion equation:

$$x - \text{the input value, } n - \text{damping factor (depends on loading rate); } f(t) - \text{external action on the system; } w_0 - \text{natural-vibration frequency. Second-order differential motion equation, where } n=2, \complies with the model of the system investigated on the basis of the Tribal-T tribometric system, where the relative displacement of two surfaces takes place. The dynamics of mechanical oscillating system can be represented by Newton equation [9, 10].

Disturbing forces are located on the right side of the equation, while the system response to the external action is on the left side of this equation. To integrate the equation we have created the performance (characteristic) equation and evaluated its roots:

$$z^2 + 2nz + w_0^2 = 0$$

$$z_1 = -n + \sqrt{n^2 - w_0^2}, \quad z_2 = -n - \sqrt{n^2 - w_0^2}$$

Damped frequency:

$$\omega_* = \sqrt{\omega_0^2 - n^2}$$

The period of damped oscillations is the time interval between two successive passes through the point "0" in the propagation (traveling) direction:

$$T^* = \frac{\tau}{\sqrt{1 - \left(\frac{n}{\omega_*}\right)^2}}$$

Where: $T = 2\pi/w_0$ is the natural period of vibration. Hence, oscillation damping period lasts much longer than its natural period of vibration.

The period of damped oscillations exposed to low resistance can be equal to natural oscillation. Damping is very fast, even at low resistance. Thus, low frequency components have the main influence on resistance during free oscillations. The laws of movements, described by the second-order differential equation, are the most understandable from the viewpoint of classical mechanical mechanics [11].

5. Computational solution of the system in a state space
Let us consider the solution of simultaneous equations (1) for the second-order dynamic characteristics. The solution of this equations is based on identification of a nonlinear autoregression model (ARX). The vector-matrix form of the first-order differential equation system, also called the
equation of state, is the basis of the mathematical model of a multidimensional system in the time domain. The equation of state fully describes the control object, taking into account the change of its location at a time.

The model of friction (rubbing) state of the tribometer Tribal-T is as follows:

\[
\begin{align*}
\dot{x}(t + Ts) &= A \cdot x(t) + B \cdot u(t) + K \cdot e(t) \\
y(t) &= C \cdot x(t) + D \cdot u(t) + e(t)
\end{align*}
\]

(7)

\( Ke(t) \) and \( e(t) \) are correlated stochastic processes. The effective recursive Kalman filter is chosen on condition of \( Ke(t) \) and \( e(t) \) stationarity.

Applying the Matlab application package for the solution of the system we create a discrete model (for the second-order differential equation), where \( th = \text{canstart}(z, 2,1, [0,1,0]) \);

where: \( z \) is the matrix of input and output discrete signals;

\( «2» \) is the order of the differential equation, which is selected depending on the type of triboprocess;

\( «1» \) is the number of inputs per one channel;

\( «0» \) means that the delay is left out of account;

\( «0» \) is zero-initial condition.

Further solution of this equation allows determining the transfer function and characteristic parameters of the system. These parameters have oscillating nature such as damping coefficient and self-resonant (natural) frequency [12, 13].

The obtained parameters such as system eigenvalue (characteristic constant), damping coefficient and natural (undamped) frequency allow describing the system features and predicting their further changes [6]. Besides, the dynamic friction coefficient can be determined with regard to the interaction rate:

\[
L(\omega) = 20 \log A(\omega)
\]

(8)

where: \( A(\omega) = A(\text{out})/ A(\text{in}) \) is the ratio of the amplitude of input and output signals.

6. Practical application of the method and discussions

This part presents the testing of the above mentioned mathematical apparatus. Sliding friction of the samples made from the constructional bearing steel (CBS15) and L90 brass (C22000, CuZn10 analogs) has been investigated. The input data were the displacements (movements) of upper and lower samples Fig. (1). In the course of the experiment we studied 10 samples with the same initial roughness (Ra 0.25).

To achieve the desired result five test runs with different time domains (intervals) were carried out. These time intervals lasted 30 min., 60 min., 90 min. The experiments were carried out from the time zero till reaching the time domain. After each time interval the profiles of the surface samples were measured, the model of a system was constructed and the dynamic parameters of the system were calculated. At each stage of the experiment we recorded the input \( u(t) \) and output data \( y(t) \) for further dynamic system identification.

Figure 3: Bode diagram (a – amplitude and b – phase characteristics) transfer functions of the system at different time domains (intervals) depending on the frequency: 1- beginning, 2 - 30, 3 - 60, 4 - 90.
Figure 4: Response to step impulse action at different time domains depending on frequency: 1 - beginning, 2 - 30, 3 - 60, 4 - 90.

The variation in surface roughness of samples was not registered. We determined the tendency of the intensification of the oscillating nature of process by the increase of the resonance spike in Bode diagram (Fig. 3). It is confirmed by Impulse and Step Responses illustrated in the diagram (Fig. 4 from right to the left). The system input is exposed to single step impact of impulse characteristic with specified length. Both systems are equivalent if the oscillation phase is not considered.

Figure 5: The diagrams show the variation in damping coefficient and natural vibration frequency (total duration of the experiment is 90 minutes)

In accordance with the specified task of external dynamics the following parameters were determined: n – damping coefficient (rate), w - natural (undamped) frequency (Fig. 5). It can be concluded that for given experimental conditions the damping coefficient and natural-vibration frequency have differently directed competitive dynamics.
7. Conclusion
Based on the results of modeling and experimental findings friction process between the couples steel-brass were studied. Using experimental data the input-output model process was constructed and numerical calculations for Newton’s dynamic equation were made. Thus, we came to the following conclusions:

1. The input-output method can be put into practice if the samples are exposed to percentage slip relative to each other. The output data concerning the sample shift (displacement) include information about the amplitude and phase of the test specimens.
2. Numerical calculations for the Newton’s dynamic equation on experimental data allow determining the model and the variations of the tribological contact at each time domain.
3. Tribological system can be described by two parameters, namely, by damping factor and self-resonant frequency. The experiment showed that the variations of these parameters are mutually independent.
4. Upon the obtained data it was concluded that the method of sliding friction can be used to predict the tribological contact evolution.

References
[1] Koltunova E A, Ikonnikova K V, Lyapushkin S V 2017 MATEC Web of Conferences 91 01041
[2] Tyurin A E, Ismailov G M, Vlasov Y A 2013 Construction of composite materials. 2 58–64
[3] Tyurin A E, Ismailov G M, Beloenko E V, Vard Baranov A V 2017 Monitoring vibrations and microdisplacement for “pin on disc” tribology studies IOP Conf. Series: Journal of Physics: Conf. 803
[4] Pat. 2600080 the Russian Federation. Investigation device for Tribotechnical characteristics of materials [Text] / G. M. Ismailov, A. E. Tyurin, V. M. Musalimov [et al.]. – Published. 15.08.2016, Bulletin № 1.
[5] Ding J, Leen S B, Williams E J, Shipway P H 2009 Proc Inst Mech Eng Part J: J Eng Tribol, 223 1019–1031
[6] Olsson M, Bexell U 2011 Wear 271 1903–1908
[7] Ikonnikova K, Ikonnikova L., Koltunova E 2015 Turkish Online Journal of Educational Technology 2015 488
[8] Ikonnikova K, Ikonnikova L., Koltunova E 2015 Turkish Online Journal of Educational Technology 2015 557-560
[9] Oh J C, Yun E, Golkovski M G and Lee S 2003 Mater. Sci. Eng. A 351 98-108
[10] Bhushan B 2013 Introduction to tribology (John Wiley & Sons, Chichester).
[11] Wen S, Huang P 2012 Principles of tribology (John Wiley & Sons, Singapore)
[12] Lundberg J 1995 Tribol Int, 28 (5) 317–322
[13] Archard J F 1953 J Appl Phys, 24 (8) 981–988