Learning Holistic Geometric Representations for Monocular 3D Object Detection

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Abstract

As a crucial task of autonomous driving, 3D object detection has made significant progress in recent years. However, monocular 3D object detection remains a challenging problem due to the unsatisfactory performance in depth estimation. Most existing monocular methods typically directly regress the depth, while ignoring essential relationships between the depth and various geometric elements (e.g. bounding box sizes, 3D object dimensions, and object poses). In this paper, we propose to learn geometry-guided depth estimation with projective modeling to advance monocular 3D object detection. Specifically, a principled geometry formula with projective modeling of 2D and 3D depth predictions in the monocular 3D object detection network is devised. We further implement and embed the proposed formula to enable geometry-aware deep representation learning, allowing effective 2D and 3D interactions for boosting the depth estimation. Moreover, we provide a strong baseline through addressing substantial misalignment between 2D annotation and projected boxes to ensure robust learning with the proposed holistic geometric formula. Experiments on the KITTI dataset show that our method remarkably improves the detection performance of the state-of-the-art monocular-based method without extra data by 2.80\% on the moderate test setting. The model and code will be released upon acceptance.

1. Introduction

As an important and challenging problem, 3D object detection plays a fundamental role in various computer vision applications, such as autonomous driving, robotics, and augmented/virtual reality. In recent years monocular 3D object detection has received great attention, because it simply uses a monocular camera instead of requiring extra sensing devices as in LiDAR-based [8, 17, 37, 38] and stereo-based [7, 20, 30, 44] methods. However, the performance gap between LiDAR-based and monocular image-based approaches remains significant, mainly because of the lack of reliable depth information. A quantitative investigation is conducted by replacing the depth predictions with the ground-truth depth values on a baseline model. The detection performance can be remarkably improved from 11.84\% to 70.91\% in terms of the AP\textsubscript{40} under the moderate setting of car category on the KITTI val set (see Table 1), which suggests that the depth estimation is a critical performance bottleneck in the monocular 3D object detection.

The depth information has also been successfully ap-
plied as an important 3D geometry element to facilitate the learning in other problems, such as 2D object detection [12, 46, 47], human pose estimation [34], and camera localization [2, 36, 45]. However, how to jointly model the geometry relationships between the depth and different 2D/3D network predictions, such as 2D box sizes, 3D dimensions, and poses, and enable joint learning with the modeled geometry constraints for geometry-aware monocular 3D detection is rarely explored in the literature. An intuitive way to introduce the geometric relationships is to leverage perspective projection between the 3D scene space and the 2D image plane. Prior works [1, 5, 19, 21] either weakly use the geometry considering the projection consistency between 2D and 3D for post-processing or employ perspective projection regardless of the object poses and positions. However, object poses and positions can provide considerably stronger geometric constraints and are extremely important for accurate depth estimation. As can be observed in Fig. 1, the depth values differ by more than 5 meters due to the distinct poses and positions of the cars with the same height of 2D/3D boxes.

In this paper, we propose an effective holistic geometric formula by principled modeling of the relationships between the depth and different geometry elements predicted from the deep network for the task of monocular 3D object detection, including 2D bounding boxes, 3D object dimensions, object poses, and object positions. We further implement the proposed formula to develop a geometry-based network module, which can be flexibly embedded into the deep learning framework, allowing effective geometry-aware learning on the representation level for guiding the depth estimation and advancing the monocular 3D object detection. Besides, the geometry module can be utilized during both the training and inference phases without additional complex post-processing. Moreover, we provide a simple yet strong baseline for ensuring robust learning with the proposed geometry module, which is achieved through addressing the severe misalignment between the annotated 2D box and the projected 2D box from the 3D annotations. This effective baseline achieves an AP of 13.37% under the moderate setting of car category on the KITTI val set.

To summarize, the contribution of this paper is threefold:

- We propose a simple yet strong baseline through dealing with the misalignment between 2D projected boxes and 2D annotation boxes, which achieves 13.37% on the moderate of the KITTI val set. We expect our baseline will be beneficial for the community in future research on monocular 3D object detection.

Extensive experiments conducted on the challenging KITTI [11] dataset clearly demonstrate the effectiveness of the proposed approach and show that our method achieves 13.81% in terms of the AP40 metric, which is 2.80% absolute AP40 improvement compared with the state-of-the-art monocular 3D object detection method on the moderate setting of the KITTI test set for the car category.

2. Related Work

There are two groups of works closely related to ours, i.e., monocular 3D object detection and geometry-guided 3D object detection.

Monocular 3D Object Detection. Compared with the methods with LiDAR and stereo sensors, 3D object detection with monocular images is challenging due to the absence of reliable depth information. Existing works [5, 6, 10, 25, 26, 28] have considered using external pretrained networks, extra training data, and prior knowledge to improve the performance of monocular 3D object detection. Particularly, DeepMANTA [6] utilizes extra 3D shape and template in learning 2D/3D vehicle models, and performs 2D/3D matching for the detection. Inspired by the importance of accurate depth for 3D object detection, many works [10, 25, 26, 32, 48] develop monocular 3D object detection by introducing pretrained external network for depth estimation. In contrast to these methods, we only use the monocular image as input without any extra burden.

In recent years, some works also only use RGB data as the input for the task [3, 9, 23, 40, 41]. For instance, MonoDIS [40] proposes to leverage a disentangling transformation between different 2D and 3D tasks to optimize the parameters at the loss level. M3D-RPN [3] focuses on the design of depth-aware convolution layers to improve 3D parameter estimation and post-optimization of the orientation by exploring the consistency between projected and annotated bounding boxes. To address the common occlusion issue in monocular object detection, MonoPair [9] proposes to model spatial relationships of objects in paired adjacent RGB images via introducing an uncertainty-based prediction for improving the detection. MoVi-3D [41] builds virtual views where the object appearance is normalized depending on the distance to reduce the visual appearance variability. RAR-Net [23] builds a post-processing method by introducing reinforcement learning to improve the 3D object detection performance. Although these existing methods achieved very promising results, the bene-
We leverage a network to extract features from the monocular image. Then base detection branch is used for generating 2D/3D predictions with depth excluded from image features. The 2D/3D predictions are utilized by the holistic geometric representation learning branch to generate geometric features via the proposed holistic geometric formula implemented in a network module. The geometric features are concatenated with the image features from the backbone for depth estimation. Based on the depth and other 3D predictions from the base detection branch, the detectors outputs the 3D object detection results. The symbol \(\odot\) indicates a concatenation operation.

**3. The Proposed Approach**

**3.1. Framework Overview**

An overview of our end-to-end network is shown in Fig. 2. We model an object as a single point followed by the other 3D predictions from the base detection branch, the detectors outputs the 3D object detection results.
3.2. Base Detection Structure

Our base network structure for 2D detection, 3D dimension, and orientation prediction is derived from the anchor-free 2D object detection [42, 50] with six output branches. Each branch takes the backbone features as input and uses 3x3 convolution, ReLU, and 1x1 convolution for prediction. In the base detection branch, the heatmap branch is used to locate 2D object center; the 2D/3D offset branch is applied for estimating 2D/3D center in 2D image coordinate system; the 2D box size and the 3D dimension branch predicts the size of 2D bounding box and the 3D dimension of the 3D object, respectively. Similar to [9, 29, 50], the orientation branch predicts observation angle \( \alpha \) of the object via encoding it into scalars.

3.3. Holistic Geometric Representation Learning

In this section, we introduce the proposed geometric formula via modeling the relationships between the depth and 2D/3D predictions and present how it can be applied to learn holistic geometric representations for depth estimation.

Formulation and notation. We adopt the 3D object definition described by the KITTI dataset. The coordinate system is constructed in meters with the camera center as the origin of the coordinate. A 3D bounding box is represented as a 7-tuple \((W, H, L, x, y, z, r_y)\), where \(W, H\) and \(L\) are the dimensions of the 3D bounding box, \(i.e.\) width, height, and length, respectively, and \((x, y, z)\) is the bottom center coordinate of the 3D bounding box. As shown in Fig. 4, \(r_y\) denotes the rotation around the Y-axis in the camera coordinate system, in a range of \([-\pi, \pi]\). Moreover, to facilitate the introduction of the proposed geometric formula, we define the 2D bounding box with a 4-tuple \((w, h, u, v)\), where \((w, h)\) and \((u, v)\) represent the size and the center of 2D bounding box, respectively.

3.3.1 Holistic Geometric Formula

The holistic geometric formula models the object depth according to the geometric relationships between the depth and multiple 2D/3D network predictions. In particular, the object depth \(z\) can be formulated as:

\[
z = \frac{1}{2}b + \frac{1}{2} \sqrt{b^2 + 4\left(-\frac{Hf_v}{h} \Delta z_{max} + \Delta z_{max}^2\right)},
\]

where \(b = \frac{L}{h}(2 \tan \beta \Delta z_{max} + H)\). \(\Delta z_{max}\) involves the 3D pose of objects to represent the maximum difference between the eight corners of the objects in the \(z\)-axis, and \(\beta\) denotes the angle between the bottom center of the object and the horizontal plane.

In the following, we first elaborate the relationship between the formula in Eq. 1 and existing works, to better understand the differences between this work and existing ones. Then we show how to derive the holistic geometric formula in Eq. 1 based on a set of 2D and 3D elements.

Relationship to existing works. As we will show later, Eq. 1 is derived from the perspective projection principle that far-away objects tend to be smaller than the near objects. If the object is far away from the camera, we can make two assumptions: (i) the object depth will be considerably larger than the object size, and the term \(\tan(\beta)\) in Eq. 1 will be close to zero in this case; (ii) if \(\Delta z_{max} << z\), then we can ignore the influence from \(\Delta z_{max}\) by setting \(\Delta z_{max} = 0\). Based on the two assumptions, the holistic formula in Eq. 1 can be derived as a simplified version \((i.e. v2)\) of the proposed holistic geometric formula:

\[
z = k \cdot \frac{H}{h},
\]

where \(k\) denotes the factor for the depth scale conversion. The formula in Eq. 2 is widely used in previous works [5, 19]. The formulation from the previous works [5, 19] in Eq. 2 is clearly different from our formula in Eq. 1 in two aspects. First, our formula builds a non-linear relationship between the depth \(z\) and \(H/h\), due to the joint modeling with extra the 3D object pose and the 3D box dimensions, while the formula in Eq. 2 is a linear relationship; Second, the influence of the factors, \(i.e.\) \(\Delta z_{max}\) and \(\beta\), on depth calculation is directly ignored by Eq. 2.

Another possible setting for reducing the computational cost (also investigated in the experimental results) is to consider adopting the first item in Eq. 1 as another simplified version \((i.e. v1)\) of the proposed holistic geometric formula:

\[
z = \frac{f_v}{h}(2 \tan \beta \Delta z_{max} + H).
\]
We report detailed comparison between the proposed holistic geometric formulation in Eq. 1 and the simplified versions (i.e. v1 and v2) in the experiments (see Sec. C). In the following, we derive the formulation in Eq. 1.

**Geometric relationship of 2D and 3D corners.** First, we represent an object in the object coordinate system, in which the origin is the bottom center of the object via the translation transformation from the camera coordinate system. As shown in Fig. 4, the coordinate of the c-th \((c = 1, ..., 8)\) corner in the 3D object bounding box, denoted as \(P_{\text{cor}}^c\), can be given as follows:

\[
P_{\text{cor}}^c = \left[ \pm \Delta x_i^c, \pm \Delta y_i^c, \pm \Delta z_i^c \right]^T \quad \text{s.t.}
\]

\[
\begin{bmatrix}
\Delta x_i^c \\
\Delta y_i^c \\
\Delta z_i^c
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{2} L \cos(r_y) \pm \frac{1}{2} W \sin(r_y) \\
\frac{1}{2} H \pm \frac{1}{2} H \\
\frac{1}{2} L \sin(r_y) \mp \frac{1}{2} W \cos(r_y)
\end{bmatrix},
\tag{4}
\]

where \(\Delta x_i^c, \Delta y_i^c\), and \(\Delta z_i^c\) represent the coordinate difference between the corner and the center of the object in X, Y, and Z direction, respectively; \(i \in \{1, 2\}\) denotes the index of different \(\Delta\) values as shown in Fig. 4. With the position of the object in the camera coordinate system, we can represent the corner in the same coordinate system as:

\[
P_{\text{cam}}^c = P_{\text{obj}} + P_{\text{cor}}^c = \begin{bmatrix} x + \Delta x_i^c \\ y + \Delta y_i^c \\ z + \Delta z_i^c \end{bmatrix},
\tag{5}
\]

where \(P_{\text{obj}}\) and \(P_{\text{cam}}^c\) respectively represent the bottom center coordinate and the corner coordinate of the 3D object bounding box in the camera coordinate system; \(x, y,\) and \(z\) denote the coordinate value along the X, Y, and Z dimension in the camera plane. \(z\) also represents the distance from the bottom center of object to the camera plane, i.e. the depth of the object in the camera coordinate system;

Given the intrinsic matrix of the camera provided by the official KITTI dataset, \(K_{\text{inc}}\), we can project the corner in the camera coordinate system to the pixel coordinate system as:

\[
P_{\text{pix}}^c = \left[ u^c, v^c, 1 \right]^T = \frac{K_{\text{inc}} \cdot P_{\text{cam}}^c}{z^c},
\tag{6}
\]

where \(P_{\text{pix}}^c\) denotes the projected corner coordinate in the pixel coordinate system; \(z^c\) indicates the depth of the \(c\)-th corner; \(u^c\) and \(v^c\) respectively denote the horizontal and vertical coordinate of the corner in the pixel coordinate system.

**Relationship between 2D height and 3D corners.** Given the eight corners of the 3D object box in the pixel plane, the height of the projected 2D bounding box \(h\) can be estimated from the difference between the vertical coordinate of the uppermost corner (i.e. \(\max_c \{v^c\}\)) and that of the lowermost corner (i.e. \(\min_c \{v^c\}\)) in the pixel coordinate system as:

\[
h = \max_c \{v^c\} - \min_c \{v^c\} = \frac{(y + \Delta y_{\text{max}}) f_w}{z - \Delta z_{\text{max}}} - \frac{(y + \Delta y_{\text{min}}) f_w}{z + \Delta z_{\text{max}}},
\tag{7}
\]

where \(v^c\) is derived from Eq. 6: \(\Delta z_{\text{max}} = \max_c \{\Delta z_i^c\}\) represents the maximum of \(\Delta z_i^c\) of the eight corners, analogically for \(\Delta y_{\text{max}}; f_w\) denotes the focal length in the vertical direction of the pixel plane.

**Relationship between depth and other 2D/3D parameters.** Similar to the definition of the bird’s-eye view angle \(\theta\) (see Fig. 4a), we define the angle between the bottom center of the object and the horizontal plane as \(\beta\) (see Fig. 4b). Given the projected coordinate \((u_o, v_o)\) of the object bottom center in the pixel plane based on Eq. 6, we can obtain the following geometric relationship:

\[
y = z \cdot \tan(\beta) = z \cdot \frac{v_o - c_v}{f_w},
\tag{8}
\]

where \(c_v\) is the location of the principal point relative to the origin in the pixel plane. Combining Eq. 7 and Eq. 8, the depth of the object center, \(z\), can be obtained as in Eq. 1.

### 3.3.2 Learning Enhanced Depth with Holistic Geometric Representations

Following the proposed geometric formula, we devise and implement a network module for the geometry-guided deep representation learning for accurate depth prediction, as shown in the holistic geometric representation learning branch of Fig. 2. The module aims to refine depth estimation through removing input noise introduced by using the 2D/3D geometry-related network predictions as input. Specifically, in both the training and inference stage, the module first produces a calculated one-channel depth map with the proposed geometric formula as described in Eq. 1, and a detailed description of the computing flow of the geometric formula is shown in Fig. 3. The depth map is then transformed into 3D maps of 3 channels representing a 3D data point \([x, y, z]\) by introducing camera parameters as the initial geometric input. Then, the 3D map goes through three non-linear transformation blocks, with each block consisting of convolution, BN, and ReLU layer, to learn a robust geometric representation map with \(C\) channels. These geometric representations learned by
Table 1. **Error analysis.** Similar to the error analysis in [50], we replace the predicted depth and 3D dimensions with their corresponding ground-truth values. Using the ground-truth depth remarkably improves the AP from 11.84% to 70.91% on the moderate, suggesting that the depth is a significantly important factor that affects the accuracy the monocular 3D object detection.

| Method         | 3D Detection | BEV          |
|----------------|--------------|--------------|
|                | Easy | Mod. | Hard | Easy | Mod. | Hard |
| Baseline       | 16.42| 11.84| 10.06| 24.47| 17.17| 15.40|
| w/ gt Dim      | 19.85| 14.06| 12.02| 25.06| 18.29| 15.85|
| w/ gt Depth    | 79.82| 70.91| 62.41| 88.60| 82.66| 75.41|

To alleviate overfitting, we adopt data augmentation techniques including random scaling, random horizontal flipping, and random cropping for the 2D detection, and random horizontal flipping for the 3D detection, respectively. We use the Adam optimizer with 1e-5 weight decay to optimize the full training loss as described in [9]. The initial learning rate is 1.25e-4, which is dropped by multiplying 0.1 after the 90-th and the 120-th epoch. To make train stable, we apply the linear warm-up strategy for learning with the geometric network module in the first 5 epochs.

**Inference:** We first predict 2D bounding boxes, 3D dimensions, and orientations via a shared backbone and several separate task branches. Then, we use the proposed geometric module leveraging the above 2D/3D predictions to predict depth. Finally, similar to [50], we use a simple post-processing algorithm through $3 \times 3$ maxpooling and back-projection to recover 3D bounding boxes from 2D boxes, 3D dimensions, orientations, and the depth.

## 4. Experiments

**Setup.** The KITTI dataset [11] provides widely used benchmarks for various visual tasks in the autonomous driving, including 2D Object detection, Average Orientation Similarity (AOS), Bird’s Eye View (BEV), and 3D Object Detection. The official data set contains 7481 training and 7518 test images with 2D and 3D bounding box annotations for cars, pedestrians, and cyclists. We report the average accuracy (AP) for each task under three different settings: easy, moderate, and hard, as defined in [11]. Moreover, we use 40 recall positions instead of 11 recall positions proposed in the original Pascal VOC benchmark, following [40]. This results in a more fair comparison of the results. Each class uses different IoU standards for further evaluations. We report our results on the official settings of IoU $\geq 0.7$ for cars.

**4.1. Overall Performance Comparison and Analysis**

Table 2 and 3 show the overall performance of the proposed approach on the KITTI 3D test and val sets for cars from the official online leaderboard as of Mar. 12th, 2021. Existing state-of-the-art monocular 3D object detectors, including methods using extra data and only using monocular image are listed in the tables. There are some methods [3, 9, 31] results on the KITTI val quoted from [9].

**Build a simple yet strong baseline for monocular 3D object detection.** We report the enhanced baseline results of 3D monocular object detection in Table 4. Overall, the baseline significantly increases the AP$_{40}$ performance upon the original one by 3.76%, 3.54%, 2.88% on easy, moderate and hard difficulty levels, respectively. This is achieved by introducing three methods to the original baseline. **First,** we adopt the $L_1$ loss with the aleatoric uncertainty in [9, 14], which makes training stage more robust to noise input. **Second,** we use the projected 3D center as the ground-truth for...
Table 2. State-of-the-art comparison on the KITTI test set for the car category in terms of the metric of AP_{40}. Extra data denotes the methods with extra data or external networks used in the training or inference or not. ‘-’ denotes the methods have not been published yet without specific details. The bold black/blue color indicates the best/the second best performing method under the same ‘No’ setting. ‘Improvement’ denotes the increasing in performance compared to methods without extra data.

| Method           | 3D Detection | BEV | AOS | Runtime |
|------------------|--------------|-----|-----|---------|
|                 | Easy Mod. Hard | Easy Mod. Hard | Easy Mod. Hard | Easy Mod. Hard | Easy Mod. Hard | Easy Mod. Hard |
| MonoDLE [27]     | 17.23 12.26 10.29 | 24.79 18.89 16.00 | 93.46 90.23 80.11 | - |
| GrooMeD-NMS [16] | 18.10 12.32 9.65 | 26.19 18.27 14.05 | 90.05 79.93 63.43 | - |
| DDMP-3D [43]     | 19.71 12.78 9.80 | 28.08 17.89 13.44 | 90.73 80.20 61.82 | - |
| Decoupled-3D [5] | Yes 11.08 7.02 5.63 | 23.16 14.82 11.25 | 87.34 67.23 53.84 | - |
| UR3D [39]        | Yes 15.58 8.61 6.00 | 21.8 12.51 9.20 | - - - | 120ms |
| AM3D [26]        | Yes 16.50 10.74 9.52 | 25.03 17.32 14.91 | - - - | ~400ms |
| PatchNet [25]    | Yes 15.68 11.12 10.17 | 22.97 16.86 14.97 | - - - | ~400ms |
| DA-3DNet [48]    | Yes 16.80 11.50 8.9 | - - - | - - - | - |
| D4LCN [10]       | Yes 16.65 11.72 9.51 | 22.51 16.02 12.55 | 90.01 82.08 63.98 | - |
| Kinematic3D [4]  | Yes 19.07 12.72 9.17 | 26.69 17.52 13.10 | 58.33 45.50 34.81 | ~120ms |
| CaDDN [32]       | Yes 19.17 13.41 11.46 | 27.94 18.91 17.19 | 78.28 67.31 59.52 | - |
| GS3D [19]        | No 4.47 2.90 2.47 | 8.41 6.08 4.94 | 85.79 75.63 61.85 | ~2000ms |
| MonoGRNet [31]   | No 9.61 5.74 4.25 | 18.19 11.17 8.73 | - - - | ~60ms |
| MonoDIS [40]     | No 10.37 7.94 6.40 | 17.23 13.19 11.12 | - - - | - |
| M3D-RPN [3]      | No 14.76 9.71 7.42 | 21.02 13.67 10.23 | 88.38 82.81 67.08 | 161ms |
| MonoPair [9]     | No 13.04 9.99 8.65 | 19.28 14.83 12.89 | 91.65 86.11 76.45 | 57ms |
| RTM3D [21]       | No 14.41 10.34 8.77 | 19.17 14.20 11.99 | 91.75 86.73 77.18 | 55ms |
| MoVi-3D [41]     | No 15.19 10.90 9.26 | 22.76 17.03 14.85 | - - - | 45ms |
| RAR-Net [33]     | No 16.77 11.01 4.52 | 22.45 15.02 12.93 | 88.48 83.29 67.54 | - |
| Our method       | No 18.85 13.81 11.52 | 25.86 18.99 16.19 | 94.67 89.44 79.27 | 50ms |
| Improvement      | +2.48 +2.80 +2.00 | +3.10 +1.96 +1.34 | +2.92 +2.71 +2.09 | - |

Table 3. Monocular 3D object detection results on the KITTI val set for the car category with the evaluation metric of AP_{40}. The results of the previous works are from [9]. Our approach significantly outperforms the previous state-of-the-arts on almost all the different evaluation protocols and settings. The bold black/blue color indicates the best/the second best performing method.

| Method           | 3D Detection IoU≥0.7 | BEV IoU≥0.7 | 3D Detection IoU≥0.5 | BEV IoU≥0.5 |
|------------------|---------------------|-------------|----------------------|-------------|
|                 | Easy Mod. Hard      | Easy Mod. Hard | Easy Mod. Hard      | Easy Mod. Hard |
| CenterNet [50]   | 0.60 0.66 0.77      | 3.46 3.31 3.21 | 20.00 17.50 15.57   | 34.36 27.91 24.65 |
| MonoGRNet [31]   | 11.90 7.56 5.76     | 19.72 12.81 10.15 | 47.59 32.28 25.50   | 52.13 35.99 28.72 |
| MonoDIS [40]     | 11.06 7.60 6.37     | 18.45 12.58 10.66 | - - -               | - - -        |
| M3D-RPN [3]      | 14.53 11.07 8.65    | 20.85 15.62 11.88 | 48.53 35.94 28.59   | 53.35 39.60 31.76 |
| MoVi-3D [41]     | 14.28 11.13 9.68    | 22.36 17.87 15.73 | - - -               | - - -        |
| MonoPair [9]     | 16.28 12.30 10.42   | 24.12 18.17 15.76 | 55.38 42.39 37.99   | 61.06 47.63 41.92 |
| Baseline         | 16.54 13.37 11.15   | 23.62 19.19 16.70 | 53.93 40.97 36.67   | 58.72 45.48 40.02 |
| Our method       | 18.45 14.48 12.87   | 27.15 21.17 18.35 | 56.59 43.70 39.37   | 61.96 47.84 43.10 |

Table 2. State-of-the-art comparison on the KITTI test set for the car category in terms of the metric of AP_{40}. Extra data denotes the methods with extra data or external networks used in the training or inference or not. ‘-’ denotes the methods have not been published yet without specific details. The bold black/blue color indicates the best/the second best performing method under the same ‘No’ setting. ‘Improvement’ denotes the increasing in performance compared to methods without extra data.

Table 3. Monocular 3D object detection results on the KITTI val set for the car category with the evaluation metric of AP_{40}. The results of the previous works are from [9]. Our approach significantly outperforms the previous state-of-the-arts on almost all the different evaluation protocols and settings. The bold black/blue color indicates the best/the second best performing method.

2D heatmap prediction similar to SMOKE [24]. Third, we address the misalignment between 2D ground-truth bounding boxes and the 2D projection bounding boxes by using 2D projected box as the ground-truth. This guarantees the consistency between 2D and 3D boxes from the projection relationships in the proposed geometric formula, and ensure the robust learning with the formula. The enhanced baseline achieves 16.54%, 13.37%, 11.15% on easy, moderate and hard difficulty levels, respectively.

Comparison with monocular image based methods. Our approach achieves a notable improvement over the state-of-the-art monocular image-based detectors [3, 9, 31, 40] on both the val and test sets. As shown in Table 2, the performance of our approach on the KITTI test set, for the detection on the car category, an indispensable part of the 3D object detection task for the autonomous driving scenario, our method achieves 18.85% (2.48% improvement) on the easy, 13.81% (2.80% improvement) on the moderate, and 11.52% (2.00% improvement) on the hard compared with the previous state-of-the-art image-only method.
Figure 5. Qualitative results of our method for multi-class 3D object detection. We use orange box for cars, purple box for pedestrians, and green box for cyclists. All illustrated images are from the KITTI test set. Zoom in the image for more details.

### Table 4. Results of the enhanced baseline on KITTI val set for the car category with the evaluation metric of AP\(_{40}\). Each row adds an extra component to the above row.

| Description            | Easy Detection | BEV  | Easy Detection | BEV  |
|------------------------|----------------|------|----------------|------|
| Original baseline      | 12.78          | 9.83 | 8.27           | 18.32| 14.18| 12.11|
| + Uncertainty          | 15.40          | 11.10| 9.58           | 22.33| 16.53| 14.18|
| + Center3d             | 16.22          | 12.88| 10.94          | 22.61| 17.89| 16.17|
| + Projected box        | 16.54          | 13.37| 11.15          | 23.62| 19.19| 16.70|
| Enhanced baseline      | 16.54          | 13.37| 11.15          | 23.62| 19.19| 16.70|

Table 5. Quantitative comparison on different variants of the proposed approach. The experiments are conducted on the KITTI val set for the car category with the evaluation metric of AP\(_{40}\), to investigate the effect of the proposed geometric formula and geometry-guided representation learning. ‘3D-CAT’, ‘Geo-SV1’ and ‘Geo-SV2’ represents transformation blocks combined with 3D dimension, simplified geometric formula v1, and v2.

| Method                  | Easy Detection | BEV  | Easy Detection | BEV  |
|-------------------------|----------------|------|----------------|------|
| Baseline                | 16.54          | 13.37| 11.15          | 23.62| 19.19| 16.70|
| + 3D-CAT                | 15.87          | 11.80| 10.33          | 21.85| 16.90| 14.51|
| + Geo-SV1               | 17.25          | 13.38| 11.29          | 24.33| 18.57| 16.06|
| + Geo-SV2               | 17.10          | 13.22| 11.13          | 25.02| 18.62| 16.48|
| Ours (full model)       | 18.45          | 14.48| 12.87          | 27.15| 21.17| 18.35|

### 4.2. Ablation Experiments

We conduct extensive ablation studies on the KITTI val set, to demonstrate the effectiveness of the proposed approach for geometry-guided depth learning in advancing the monocular 3D object detection. For all the evaluation, the AP\(_{40}\) metric is employed. We mainly investigate from two aspects, including the effect of the proposed geometric formula and module, and the effect of the geometry-guided representation learning for depth estimation.

**Baseline and variant models.** To conduct an extensive evaluation, we consider the following baseline and variant models: (i) Baseline, which is a base model achieving a strong 3D detection performance with an AP\(_{40}\) of 11.8% on the moderate; (ii) 3D-CAT., which directly inputs the concatenation of the 3D network predictions to the non-linear transformation blocks while bypassing the depth calculation with geometric formula; (iii) Geo-SV1, which uses our simplified geometric formula v1 as in Eq. 3; (iv) Geo-SV2, which uses our simplified geometric formula v2 as in Eq. 2.

**Effects of the geometric formula and module.** A detailed ablation study is shown in Table 5. As we can observe, ours (full model) achieves a large gain (2.68% on the moderate) over 3D-CAT, meaning that directly using the network predictions are not effective enough for learning the geometric representations, thus verifying the importance of the proposed geometric formula.

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**Latency.** We test our model on Nvidia GTX 1080 Ti, Pytorch 1.1, CUDA 9.0. As shown in Table 2, the proposed method achieves 20 fps and runs similar to other real-time state-of-the-arts [21, 41]. This clearly demonstrates the efficiency of our method when compared with other competitive methods under the similar experimental environment.
Figure 6. Qualitative results of our method for Bird’s-Eye-View. We use black box for ground-truth, red box for baseline results, and blue box for our results. All the illustrated images are from the KITTI val set. Zoom in on the circles for more detailed comparison.

Figure 7. Depth prediction performance w.r.t. SILog (Scale invariant logarithmic error) and sqRel (Relative squared error) metrics on KITTI val set for all the car samples. Different depth ranges are considered in the performance evaluation.

5. Conclusion

We proposed an effective holistic geometric formula principally modeled from multiple 2D/3D network predictions, to guide the depth estimation and advance the monocular 3D object detection. We design and implement this formula as a neural network module to have geometry-aware feature learning with the image representations to boost the learning of the depth. Extensive experiments demonstrate the effectiveness of the proposed approach, and results also achieve state-of-the-art performance with a large margin.

Effect of the geometry-guided representation learning for depth estimation. Fig. 7 shows a performance comparison between baseline and our approach on the depth estimation. Specifically, we evaluate the predicted depth of all car samples in different depth ranges under two primary metrics (i.e. SILog and sqRel) widely used in depth estimation field. On the KITTI train&val dataset, 87% of the cars are within 40m, while only 5.0% of those are 45m away. Fig. 7 shows that our approach outperforms the baseline consistently in all the depth ranges, especially in the 40m range with most samples, which further validates our idea of using geometry-guided representation learning to boost depth estimation to advance the monocular 3D object detection.
In this Supplementary Material, we provide more elaboration on the implementation details, experiment results, and qualitative results. Specifically, we present the implementation details of the model training in Section A, additional quantitative results, analysis, and limitations in Section B, additional ablation study in Section C, and additional qualitative results in Section D.

A. Additional Implementation Details

The overall network optimization loss of the proposed approach consists of three parts, i.e., a classification loss \( L_c \), a 2D regression loss \( L_{2D} \), and a 3D regression loss \( L_{3D} \). We present the details of these losses one by one: (i) Regarding to the classification loss, similar to \([18, 50]\), we employ the heteroscedastic aleatoric uncertainty term for regressing the depth. Specifically, \( L_c \) is the sum of the three losses written as:

\[
L_c = \begin{cases} 
-(1-p)^\alpha \log(p) & \text{if } y = 1 \\
-(1-p)^\beta p^\alpha \log(1-p) & \text{otherwise},
\end{cases}
\]

where \( y \) and \( p \) represent the ground-truth class probability given by an unnormalized 2D Gaussian and the model’s predicted probability for the class, respectively. And \( \alpha \) and \( \beta \) are hyperparameters that control the importance of each sample. We set \( \alpha \) to 2 and \( \beta \) to 4 as a default setting in our experiments. (ii) For the 2D regression loss \( L_{2D} \), it is defined upon a 6-tuple of ground-truth bounding-box targets and a predicted 6-tuple. Specifically, the 6-tuple consists of two 2D offsets, two 3D offsets, and two 2D box sizes. 2D/3D offsets are used to adjust the 2D/3D center locations before remapping them to the input resolution following \([18, 50]\). We use an \( L_1 \) loss to optimize each 6-tuple parameters. (iii) For the 3D regression loss \( L_{3D} \), it consists of an \( L_1 \) loss for regressing the dimension of the 3D bounding box (i.e., width, height, and length), and an \( L_1 \) loss with an uncertainty term for regressing the depth. Specifically, we follow \([9, 14]\) and employ the heteroscedastic aleatoric uncertainty in the \( L_1 \) depth estimation loss as:

\[
d^*, \sigma^* = f^\theta(x),
\]

\[
L(\theta) = \frac{\sqrt{2}}{\sigma} ||d - d^*|| + \log \sigma^*.
\]

Where \( d^* \) and \( d \) represent the predicted depth and the ground-truth depth, respectively. \( \sigma^* \) is the noisy observation parameter of the model. Hence, the overall optimization loss is the sum of the three losses written as:

\[
L = L_c + \lambda_1 L_{2D} + \lambda_2 L_{3D},
\]

where \( \lambda_1 \) and \( \lambda_2 \) are loss weights controlling the balance between the different losses. We consider \( L_{2D} \) and \( L_{3D} \) equally important and use \( \lambda_1 = \lambda_2 = 1 \) in all experiments.

B. Additional Results and Analysis

B.1. Additional Results for the Pedestrian/Cyclist Category & Limitations

| Category & Limitations | Method  | 3D Detection/BEV |
|------------------------|---------|------------------|
|                        |         | Easy            | Mod. | Hard |
| Ped.                   | OFTNet  | 0.63/1.28       | 0.36/0.81 | 0.35/0.51 |
|                        | SS3D    | 2.31/2.48       | 1.78/2.09 | 1.48/1.61 |
|                        | M3D-RPN | 4.92/5.65       | 3.48/4.05 | 2.94/3.29 |
|                        | MoVi-3D | 8.99/10.08      | 5.44/6.29 | 4.57/5.37 |
|                        | MonoPair| 10.02/10.99     | 6.68/7.04 | 5.53/6.29 |
|                        | Ours    | 8.00/9.54       | 5.63/6.77 | 4.71/5.83 |
| Cyc.                   | OFTNet  | 0.14/0.36       | 0.06/0.16 | 0.07/0.15 |
|                        | SS3D    | 2.80/3.45       | 1.45/1.89 | 1.35/1.44 |
|                        | M3D-RPN | 0.94/1.25       | 0.65/0.81 | 0.47/0.78 |
|                        | MoVi-3D | 1.08/1.45       | 0.63/0.91 | 0.70/0.93 |
|                        | MonoPair| 3.79/4.76       | 2.12/2.87 | 1.83/2.42 |
|                        | Ours    | 4.73/5.93       | 2.93/3.87 | 2.58/3.42 |

Table 1. Monocular 3D object detection results on the KITTI test set for the Pedestrian and Cyclist categories with the evaluation metric of AP$_{30}$. The IoU threshold is set to 0.5. The bold black/blue color indicates the best/the second best performing method, respectively.

As mentioned in the main paper, the KITTI \([11]\) official data set contains 7,481 training and 7,518 test images with 2D and 3D bounding box annotations for pedestrian and cyclist categories. We report our quantitative results in Table 1, using the official settings with IoU $\geq 0.5$ for pedestrians and cyclists on the KITTI test set. Our method establishes new state-of-the-art performance on all the three detection levels (i.e., easy, medium, and hard) for the cyclist category with only slight drop for the pedestrian category. We investigate the slight performance drop in the pedestrian category by comparing 2D detection results between car and pedestrian. In fact, the advantage of the proposed geometric formula is independent of different classes as 2D images conform with projective camera models, and every object meets the geometric reasoning. However, a performance gap between car detection and pedestrian/cyclist detection commonly exists in ours and many previous works on the KITTI dataset. This is mainly due to insufficient training samples of pedestrian and cyclist categories on KITTI, leading to unstable training, sensitivity to hyperparameters, and inaccurate prediction of 2D/3D information (e.g., 2D boxes, orientation, and the 3D dimensions) with high variance. This imbalance of the category data is however a common issue on the KITTI dataset for the 3D object detection task. Table 2 shows that the 2D detection results on the moderate level are only 50.48% and 44.63% for cyclist and pedestrian respectively, while up to 90.14% for car on the test set. Similarly for orientation estimation, the pedestrian (39.76%) has less than half of the car (89.44%) on the moderate. The two factors above introduce more noise into our geometry formula to affect the
geometry-guided representation learning. However, our results for pedestrians and cyclists are highly competitive with other SOTA methods on the KITTI test set.

| Cat. | Method   | 2D Detection/AOS | Easy | Mod. | Hard |
|------|----------|------------------|------|------|------|
|      |          |                  |      |      |      |
| Car  | SS3D [17] | 92.72/92.57      | 84.92/84.38 | 70.35/69.82 |
|      | M3D-RPN [3] | 89.04/88.38 | 85.08/82.81 | 69.26/67.08 |
|      | Ours      | 95.11/94.67      | 90.14/89.44 | 80.19/79.27 |
| Ped. | SS3D [17] | 61.58/53.72      | 45.79/39.60 | 41.14/35.40 |
|      | M3D-RPN [3] | 56.64/44.33 | 41.46/31.88 | 37.31/28.55 |
|      | Ours      | 58.49/52.87      | 44.63/39.76 | 40.41/35.83 |
| Cyc. | SS3D [17] | 32.97/22.95      | 35.48/25.79 | 31.07/24.26 |
|      | M3D-RPN [3] | 61.54/48.11 | 41.54/31.09 | 35.23/26.10 |
|      | Ours      | 65.42/55.58      | 50.48/42.05 | 42.48/35.48 |

Table 2. Monocular 2D object detection results on the KITTI test set for all categories, with metrics of AP40. The metric AP40 is used for detection evaluation and the IoU threshold is set to 0.5. The bold colors indicate the best and the second best performing method, respectively.

B.2. Further Analysis on Depth Estimation from Geometry Modeling

We conduct a further depth statistic analysis on the train+val set. Table 3 shows that for two cars with the same height in both the 2D bounding box and the 3D bounding box, the depth values of their centers may differ by more than 5 meters due to their distinct poses and locations. This confirms the critical importance of considering 3D pose and locations simultaneously in the geometric modeling for depth estimation, which is however not investigated by previous works.

| h | depth | The height of 3D bounding boxes |
|---|-------|--------------------------------|
|   |       | avg. | 1.49m | 1.50m | 1.51m | 1.52m |
| 30 | max  | 39.51 | 40.23 | 40.39 | 42.23 | 39.47 |
|    | min  | 37.69 | 36.53 | 36.53 | 37.21 | 37.25 |
|    | diff | 1.82  | 3.70  | 3.86  | 5.02  | 2.22  |
| 35 | max  | 34.04 | 34.68 | 35.69 | 34.12 | 36.40 |
|    | min  | 32.99 | 31.72 | 31.77 | 32.05 | 31.75 |
|    | diff | 1.05  | 2.96  | 3.92  | 2.07  | 4.65  |

Table 3. Depths values on the training set (in meter). We show the maximum (max) and minimum (min) depth values of cars with the same height of 2D bounding boxes h and the same height of 3D bounding boxes, and the difference (diff.) between the maximum and minimum depth values.

B.3. Additional Results at Different Distances

We provide additional results on depth estimation and monocular 3D object detection at different distances. Table 4 shows more depth estimation results on KITTI val set via comparing the enhanced baseline and our method. Specifically, we evaluate the depth estimation by computing Scale Invariant Logarithmic (SILog) error, squared Relative (sqRel) error, absolute Relative (absRel) error, and Root Mean Squared Error of the inverse depth (iRMSE). Our method outperforms the enhanced baseline by large margins on all these evaluation metrics. The depth estimation results clearly demonstrate the effectiveness of our proposed idea of using geometry-guided representation learning to boost depth estimation from monocular images for advancing the monocular 3D object detection.

| Depth Range | Num. | SILog↓ | absRel↓ | sqRel↓ | iRMSE↓ |
|-------------|------|--------|---------|--------|--------|
| 0-10m       | 867  | 16.49  | 8.65    | 67.55  | 16.53  |
| 0-20m       | 4236 | 12.12  | 6.02    | 31.42  | 9.98   |
| 0-30m       | 7379 | 11.00  | 5.70    | 25.25  | 8.16   |
| 0-40m       | 9797 | 10.49  | 5.65    | 24.68  | 7.23   |

Table 4. Depth prediction results on the KITTI val set for all car samples. We show first the baseline and then ours (bold) for each row (i.e., each depth range). ‘Num.’ denotes the number of car samples on val set, which has in total 11,178 car samples. Moreover, we conduct experiments about the 3D monocular object detection improvement at different distances. Table 5 reports performance on AP40 at different object distance ranges following [35]. It is clear that our method consistently outperforms the baseline at different ranges.

| Description | 3D Detection | BEV |
|-------------|--------------|-----|
|             | 15m | 30m | all | 15m | 30m | all |
| Baseline    | 18.85 | 15.42 | 11.32 | 26.95 | 21.94 | 16.82 |
| Ours        | 22.29 | 17.38 | 12.87 | 31.37 | 24.82 | 18.35 |

Table 5. Performance on KITTI val at different ranges.

C. Additional Ablation Study for Uncertainty and Equation

We investigate the effect of uncertainty with our geometric module as requested on the KITTI val set in Table 7. It can be seen that the uncertainty is helpful for learning the geometry, but the main improvement is from the proposed principled geometric modeling. To further validate the effectiveness of Eq. (6), we compare all predictions followed by pointwise MLP as the reviewer described with our geometric module in Table 6. Ours is significantly better than the pointwise MLP.

D. Additional Qualitative Results

Fig. 8 also show the comparison results between the enhanced baseline and the proposed method from the Bird-Eye-View. Figure 9 also present additional qualitative 3D detection results on the images with a comparison between those two on the KITTI val set.
Figure 8. Qualitative results of our method for Bird’s-Eye-View. We use black box for ground-truth, red box for baseline results, and blue box for our results. All the illustrated images are from the KITTI val set. Zoom in on the circles for more detailed comparison.

| Description | 3D Detection | BEV |
|-------------|--------------|-----|
|             | Easy | Mod. | Hard | Easy | Mod. | Hard |
| Baseline    | 16.54 | 13.37 | 11.15 | 23.62 | 19.19 | 16.70 |
| Pointwise MLP | 17.09 | 13.12 | 11.05 | 23.79 | 18.20 | 16.26 |
| Ours        | 18.79 | 14.53 | 12.77 | 26.48 | 20.75 | 18.04 |

Table 6. Results of different modules on KITTI val with AP_{40}.

| All Other Enhancements | Uncertainty | Geometric Module | 3D Detection | BEV |
|------------------------|--------------|-----------------|--------------|-----|
| ✓                      | ✓            | ✓               | 11.81        | 17.51 |
| ✓                      | ✓            | ✓               | 14.44        | 19.77 |
| ✓                      | ✓            | ✓               | 14.48        | 21.17 |

Table 7. Ablation study on KITTI val set for uncertainty and geometric modeling on the moderate setting of cars.

We could observe from the figures that the proposed geometry-guided learning approach can achieve significantly better 3D detection and localization performance than the enhanced baseline.

Figure 10 and 11 show additional visualization of the prediction results on KITTI 3D raw data in both the image plane and the LiDAR coordinate system, respectively. We use orange box, purple box, and green box for car, pedestrian, and cyclist, respectively. Our approach is able to accurately localize the different-depth 3D objects.
Figure 9. **Qualitative Results.** The predictions on the KITTI val set. Results are from the enhanced baseline (left column) and ours (right column).
Figure 10. Qualitative results of our method for multi-class 3D object detection. We use orange box for cars, purple box for pedestrians, and green box for cyclists. All illustrated images are from the KITTI test set. Zoom in on the images for more details.
Figure 11. Qualitative results of our method for multi-class 3D object detection. We use orange box for cars, purple box for pedestrians, and green box for cyclists. All illustrated images are from the KITTI test set. Zoom in on the images for more details.
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