Studying Inelastic Electron Scattering of Magnetic Form Factors M1 for ⁴⁸Ca Nucleus Using Skyrme Interaction

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Abstract. The M1 inelastic scattering form factors from ⁴⁸Ca nucleus have been investigated through nuclear shell theory. The nuclear effective two body interaction fpbm of Richter has been utilized to generate the two body wave vectors with Harmonic oscillator wave function as a single particle wave function in Fp shell model space. Core polarization effect has been used to include the discarded space (core+ higher configuration) via model space with realistic interaction of Skyrme Sly5 to couple the model space active particles with the pair (particle-hole) excited in energy of 2ћω. Calculated results has been compared to available experimental data.

1. Introduction

The form factors of the elastic and inelastic electron scattering, especially their parities, spins, the intensity and the operators of both excited and ground stated transition, have proven to be an effective technique for investigating excited nuclei. Centered on practical Born approximation calculations of shell model wave functions, for certain light elements that are available as targets for experiments, this should be achieved. Efforts to establish practical hypotheses for higher mass number nuclei have made considerable progress and more should continue [1], modern inelastic electron scattering experiments have supplies valuable data as multipole transitions in a number of doubly even (FP) shell nuclei within a transfer momentum range up to [q=3.0 fm⁻¹]. As well as single particle wave function, the two-body matrix element plays a very important participation in the calculation of the nuclear properties mentioned above, so that a variety shell model space effective NN interaction had been introduced depending on two body interaction strength, adjustable parameter obtained from real nuclear interactions and fitting process [1]. Magnetic form-factors by 180° electron scattering for wide range of nuclei including ⁴¹,⁴²,⁴⁴,⁴⁸Ca was studied [2]. In order to study the electrical and magnetic structure of nuclei, elastic and inelastic electron scattering experiments are explained [3], definitions are limited to primary beam energies of up to 100 MeV where the structure of individual nucleons does not have a direct effect on the findings. New knowledge on nuclear states presents an overview of the experimental cross-sections. Elastic magnetic electron scattering in a first Born approximation, was investigated and indicated the corrections expected to arise from a static magnetic moment [4]. The facility of 180° electron scattering was designed [5] and claimed that the research of elastic electron scattering from nuclei, at angles near 180 °, as well as evaluating some aspects of quantum electrodynamics, an analysis of nuclear magnetic multipole distributions is allowed. Electro-excitation experiments at low momentum transfer may be corrected in independent model way for static Coulomb effects [6]; in the case of magnetic transitions this is demonstrated for various
phenomenological current and magnetization distributions. Efficient interaction For the definition of the core Random Phase Approximation (RPA), core-renormalization between nucleons in the fp-shell is greater than for the definition of the lowest-order perturbation theory [7]. Inelastic electron scattering with backward-angle high-resolution on $^{40,42,44,48}$Ca and assessment of a really efficient ground state magnetic dipole transition in $^{48}$Ca had been carried out [9]. The magnetic dipole transitions from the ground state for even Ca isotopes to high lying $J^\pi = 1^+$ states by application of high resolution and low momentum transfer inelastic electron scattering was explained. The excitation of dipole and quadrupole magnetic transitions in nuclei were investigated [10], clearly shows that a number of selected isotopes ($^{16,18}$O, $^{28}$Si, $^{40,48}$Ca, $^{46,50}$Ti) how long-range correlations have been researched by distributions of the magnetic strength and magnetic giant resonances in the nuclear ground state and the structure of the efficient spin-spin and spin-isospin particle hole in nuclei. The strength distributions of the M1 inelastic magnetic electron scattering and the nuclear structure on the N = 28 isotones $^{40}$Ca, $^{50}$Ti and $^{56}$Fe and the structure and magnitude of the M1 were studied [11]. The M1 strength identified in the observed excitation energy area Ex 7-12 MeV is very fragmented and significantly quenched in comparison to predictions of shell model calculations in a model space that includes excitations of up to two particles-two holes. The strength of magnetic dipole distribution in the fp-shell nuclei $^{40,42,48}$Ca and in the N = 28 isotones $^{50}$Ti, $^{51}$V, $^{52}$Cr and $^{54}$Fe was investigated [12]. Following the M1 strength distribution research's in the even-even N = 28 isotones $^{48}$Ca, $^{50}$Ti, $^{52}$Cr and $^{54}$Fe by inelastic electron scattering, in comparison to a recent (p, p') experiment, it was found that no significant M1 excitation was observed.

With the inclusion of the realistic interaction code M3Y[29], a computer framework written in FORTRAN 90 language to measure electron scattering form factors[28] was used, the code[29] was updated to measure the realistic effective interaction Skyrme type (SLy5) as a residual interaction to measure the contribution of discarded space through the so-called core polarization effect

1.1. Aim of the Present Work

The aim of the current effort is to use the realistic efficient nucleon-nucleon (NN) interaction of Skyrme (Sly5) as a residual interaction to measure the effects of core polarization (CP) through a microscopic theory, with the choice of efficient model space interaction that generates model space wave (shell model wave functions) functions and strongly excited states. Harmonic oscillator wave function will be adopted as a single feature of particle wave. For the 2p1f-shell Ca48 nucleus of the low lying states, we will respond to the core-polarization effects on the form factors of elastic and inelastic electron scattering. Interaction of Skyrme potential (SLy5) of B. A. Brown, (2000)[32] and H., respectively, For the core polarization matrix components, Nakada (2003)[31], Elliotte suitable sets of parameters are to be introduced as a residual interaction.

2. General theory

2.1. Single, Effective and Residual Interactions

The shell-model calculations expected a $^{40}$Ca core and valence nucleons spread over the full fp space. In order to remove stalker states from the data set designated [30]. The simple NN interaction resulted in the so-called Skyrme (Sly5) interaction, by balancing the interaction of Skyrme (Sly5) with G-matrix. The Skyrme (Sly5) relationship, defined by the sum of the Yukawa functions, will be controllable in many models. The Skyrme (Sly5) interaction has been shown to bounce matrix elements as well as stable shell model interactions. Additionally, Skyrme (Sly5) interaction has been positively related to nuclear reactions, including nuclear densities, with a certain modification. Efficient interaction is a core component of the nuclear shell model's triumph [31].
2.2. Electron scattering

The differential scattering cross-section for the scattering of an electron into a solid angle \(d\Omega\) from a nucleus of charge \(Z\) and mass \(M\) in the plane wave Born approximation (PWBA) is given by \[32\]:

\[
\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} f_{\text{rec}} \sum |F_j(q, \theta)|^2
\]  

The Mott cross-section \(\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}\) for high-energy scattering electron from a point spinless nucleus, is given by:

\[
\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left[\frac{Za \cos \left(\frac{\theta}{2}\right)}{2E_i \sin^2 \left(\frac{\theta}{2}\right)}\right]^2
\]  

Where \(a = \frac{\hbar^2}{2m_c} = \left(\frac{1}{137}\right)\) is the constant of the fine structure which is representing the order of interaction \[32\], \(Z\) is the atomic number of the target nucleus, \(\theta\) is the angle of the scattering and \(E_i\) is the energy of incident electron.

The recoil factor of the nucleus is given by:

\[
f_{\text{rec}} = \left[1 + \frac{2E_i \sin^2 \left(\frac{\theta}{2}\right)}{M}\right]^{-1}
\]  

where \(M\) is the mass of the target.

\[
|F_j(q)|^2 = \left(\frac{qa}{q}\right)^4 |F_j^E(q)|^2 + \left[\frac{a^2}{2q^2} + \tan^2 \left(\frac{\theta}{2}\right)\right] |F_j^T(q)|^2
\]  

The four-momentum transfer \(q_\mu\) is given by, (with \(\hbar=c=1\) for abbreviation):

\[
q_\mu^2 = q^2 - (E_i - E_f)^2
\]  

squared transverse form factor is given \[13\] as the sum of the squared electric form factor and squared magnetic form factor as follows:

\[
|F_j^T(q)|^2 = |F_j^E(q)|^2 + |F_j^\text{mag}(q)|^2
\]  

2.3. Transverse Magnetic Operator (\(\eta=m\))

The transverse form factor arises from the interaction of the electron with the current \(\tilde{J}(\vec{r}, t_z)\), and the magnetization distributions \(\tilde{\mu}(\vec{r}, t_z)\) of the nucleus. The magnetic operator is given by \[32\]:

\[
\tilde{M}_{jLM}(q, \vec{r}) = \int d\vec{r} \tilde{M}_{jLM}(q, \vec{r}) \cdot \tilde{J}(\vec{r}, t_z)
\]  

with

\[
\tilde{M}_{jLM}(q, \vec{r}) = j_L(qr) \tilde{Y}_{LM}(\Omega_r)
\]  

where \(\tilde{Y}_{LM}(\Omega_r)\) is the vector spherical harmonics and \(j_L(qr)\) is the spherical Bessel function with order \(L\), defined as \[18\]:

\[\text{....} \]
2.4. Core polarization effects

Microscopic theory will include the discarded space as a first order perturbation that is particle hole state (p-h), and using mixing interaction in order to calculate these effects as a residual interaction, Skyrme-type Hamiltonian (SLy5) [31]. The fp model space is the correct space for nuclei with mass number \( A > 40 \), atomic number \( Z \), and neutron number \( N \geq 20 \) [32], the core of \(^{40}\text{Ca}\) is required.

There are two parts of the reduced electron scatterer operator matrix elements, one for the matrix elements of 'Model space' and the second for the matrix elements of 'Core polarization'.

\[
\langle \gamma_f || \hat{T}_A^n || \gamma_i \rangle = \langle \gamma_f || \hat{T}_A^n || \gamma_i \rangle_{MS} + \langle \gamma_f || \delta \hat{T}_A^n || \gamma_i \rangle_{CP} \tag{9}
\]

Where, \( \langle \gamma_f || \hat{T}_A^n || \gamma_i \rangle_{MS} \) these are the matrix components of the model-space, and \( \langle \gamma_f || \delta \hat{T}_A^n || \gamma_i \rangle_{CP} \) are the core-polarization matrix elements. \( \langle \gamma_i \rangle \) and \( \langle \gamma_f \rangle \) are labelled by the model vectors.

The model-space in the light fp-shell nuclei is distinct by the ensuing configuration: \((1f7/2 1f5/2 2p3/2 2p1/2)\).

\[
\langle \gamma_f || \delta \hat{T}_A^n || \gamma_i \rangle_{CP} = \sum_{\alpha, \beta} \text{OBDM}(\alpha, \beta) \langle a || \delta \hat{T}_A^n || \beta \rangle \tag{10}
\]

\[
\langle a || \delta \hat{T}_A^n || \beta \rangle = \left( a | V_{res} \frac{q}{E-H(\alpha)} \hat{T}_A^n | \beta \right) + \left( a | \hat{T}_A^n \frac{q}{E-H(\alpha)} V_{res} | \beta \right)
\]

\[
\langle a | \hat{T}_A^n | p \rangle = \sum_{\rho, \nu} \Delta_\rho \Delta_\nu \langle c \rho \nu | \hat{T}_A^n | p \rangle \tag{11}
\]

The three potentials are expressed as:

\[
V_{12}^{(C)} + V_{12}^{(LS)} + V_{12}^{(TN)} \tag{12}
\]

3. Results and Discussion

Experimental data must be present in order to make sure that the carried theoretical calculations meet the experimental result with an acceptable errors or differences and the experimental data are reliably enough with respect to the circumstances that’s affecting the calculation and comparison. Results must be clear and carefully viewed in order to reveals the details. The model space effective interaction FPmb of Richter [30] has been used to give the \((1f7/2 1f5/2 2p3/2 2p1/2)\) wave functions for \(^{40}\text{Ca}\).

The cp effects investigated with the realistic Sly5 interaction [12, 13] as a residual interaction. In this interaction, the Elliot fitting has been used to calculate the radial integral. In this approach one-derived two-body matrix elements in a particular single-particle basis directly from the scattering phase shift. Using the HO potential, the radial component of single-particle vectors is calculated, with the size parameter \( b \) equipped to obtain the root mean square radius \( (r_{\text{rms}}) \) of each nucleus. The nucleus with an abundance of neutrons is \(^{40}\text{Ca}\) and the lightest doubly magic nucleus. It is considered to be a good nucleus of the shell-model and thus presents an outstanding test ground for nuclear
models. In fact, the nucleus $^{48}\text{Ca}$ is more inert than $^{40}\text{Ca}$, $^{48}\text{Ni}$ and $^{56}\text{Ni}$ because of the closed sub shell neutron $1f7/2$ so that it is an motivating one in fp shell nuclei.

3.1. The M1 Transverse magnetic Form Factor For $1^+4$ state

Magnetic dipole transition in $^{48}\text{Ca}$ had been examined in detail by authors because of the mystery inherent with the measurements of the properties like magnetic moments, β-decay and Gamow-Teller (GT) transition and these properties need to be corrected by different approaches including the use of first order configuration mixing through first order perturbation theory where the core is included to the calculations beside the use of well-defined model space, so we shall introduce the most important result obtained for this types of transition.

From Figure (1), (2), (3), (4), and(5) it is clear that the core, model space and the total form factor have a well contribution in this type of transition, someone expects that the total form factors is decreased as the excitation energies increased but this behavior does not happened as a results of giant values of form factors at the fourth excited state (E= 7.405 MeV) and as mentioned before [2, 3, 4, 5, 6, 7, 8, 9, 10, 11] there is an abnormal phenomena inherent experimentally at E=10.23 MeV excitation energy where a giant magnetic dipole moment arise, and according to our approach phenomena resulted at the fourth state. Return to the Figures, the protons and neutrons through their g factors have a contributions to the formation of dipole moment and M1 form factors so the model space (filled by neutrons only) and the core part which has an equal numbers of neutrons and protons have competitive participations, in general the core parts have a destructive interference with model space so that the total form factor is smaller than that of model space and larger than that of core. All of these contributions are distributed between $q=0$ to $q=3$ fm$^{-1}$, and they are rippled in the range of form factors less than $10^{-12}$ for first excited state, less than $2\times10^{-8}$ for the second excited state, less than $2\times10^{-7}$ for the third excited state, less than $3\times10^{-5}$ for the fourth excited state, and finally the fifth excited state is rippled less than $1\times10^{-7}$.

![Figure 1](image_url)

**Figure 1.** The transverse inelastic form factor M1 for the $1^+_1$ (6.333000 MeV) state in $^{48}\text{Ca}$, experimental data for the studied are obtained from reference [25].
Figure 2. The transverse inelastic form factor $M_1$ for the $1^+_2$ (6.374 MeV) state in $^{40}\text{Ca}$, experimental data for the studied are obtained from reference [25].

Figure 3. The transverse inelastic form factor $M_1$ for the $1^+_3$ (6.910 MeV) state in $^{40}\text{Ca}$, experimental data for the studied are obtained from reference [25].
Figure 4. The transverse inelastic form factor $M_1$ for the $1^+_1 (7.405000 \text{ MeV})$ state in $^{40}\text{Ca}$, experimental data for the studied are obtained from reference [25].

Figure 5. The transverse inelastic form factor $M_1$ for the $1^+_2 (7.641000 \text{ MeV})$ state in $^{48}\text{Ca}$, experimental data for the studied are obtained from reference [25].
4. Conclusions
The behavior of the curves are similar to that of M3Y as a residual interaction from the points of interfering and rippling ranges of momentum and form factors but the giant dipole moments in is observed at the third excited state and in our work is seen in the fourth excited state, the model space wave vectors in our work generated by FPMb effective interaction makes the giant dipole moment shifted toward the higher excited states and stronger than that in, the model of model space +discarded space shell model is still good enough to produce the form factors in $^{48}$Ca nucleus.

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