Reflective Array with Controlled Focusing for Radiotomographic Application

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Abstract. It’s considered the principle possibility of creation the managed reflectors for formulation of given field distribution in the focus area. Reflectors change the reflect ratio in dependence of the external control. The proposed theoretical modeling of such controlled focused device which provides focus to a specific point in a given distribution of the reflectors. On the basis of numerical simulation it’s considered the application of this approach for the solution of the problem of radiotomography.

1. Introduction

Due to the development of computing technologies enabled one to apply the radiotomography techniques for remote non-destructive testing and the diagnostics of the internal structure of radio-semi-transparent media and the shape recovery of radio opaque objects. The task of radiotomography involves the transformation of the data obtained from the multiangle scanning of target objects into the 3D visualization [1]. The electronically and electromechanically scanned arrays are developed to obtain the multiangle projections of the wave field. However, the design and development of such arrays are complicated and expensive task, as it requires the development and control of numerous microwave transceiver channels.

Nowadays, the ways for simplification and cheapening the arrays with steerable pattern are looked for. It is known that two-dimensional lenses in form of the Fresnel zone plates focus radiation on a certain point. The electronic scanning can be performed in a certain angle sector with the change in geometrical arrangement of zone rings [2–6]. In these conditions, the positions of the radiator and receiver remains constant. In this paper, the solution of the problem by using the reflective array model where the reflection coefficient of every element changed by the external control is proposed.

2. Calculation of Reflection Coefficient Distribution

Consider that the externally controlled array element could have the reflection coefficient 0 and 1. Then, the recovery of the object shape using the 2D array stacked of such reflectors is split into two tasks. The first task is to calculate the reflectors distribution with the coefficient 0 and 1 for the focusing the reflected radiation on a certain point provided that the transmitter position is defined. The
second task consists in calculating the field sequentially scattered by the test object and predefined distribution of reflectors. This field is detected by the receiver.

Consider the task scheme presented in figure 1, in detail. Let a transmitted spherical wave be propagated in a free space defined in the Cartesian coordinate system. The transmitter (1) is placed at \( Z \) axis and at the \( h \) height from the reflective array (2). The test object (3) is placed above the array.

The solution of the first task reduces to the calculating the shape of the Fresnel zones for the predefined position of the transmitter and the focusing point. When the focusing point lies on the \( Z \) axis at a certain height, the Fresnel zones have the shape of a circle. Focusing points with the random positions are the ellipse-shaped which centers don't coincide with the coordinate origin.

\[ \Phi(r_{\alpha}) \]

\[ \sqrt{z_0^2 + (x-x_0)^2 + (y-y_0)^2} + \sqrt{h^2 + x^2 + y^2} = n\lambda \left( 1 + \frac{1}{4} \right) \]

where \( n \) is the number of the Fresnel zone boundary, \( \lambda \) is the wavelength of the sounding signal.

Consider the example when \( y_0 = 0 \). Then, the equation for the Fresnel zones boundaries takes the following form:

\[ \frac{(x-x_0)^2}{A^2} + \frac{y^2}{B^2} = 1 \]

where \( A \) and \( B \) are a major and a minor semiaxis of an ellipse, respectively. The semiaxis are calculated as follows:
\begin{align*}
A^2 &= \left(1 - \frac{x_0^2}{d^2}\right)^2; \quad B^2 = \frac{1 - x_0^2}{d^2};
\end{align*}

where

\[ d = n \lambda \left(1 + \frac{1}{4}\right), \quad q = \frac{z_0^2 + x_0^2 - d^2 - h^2}{2d}, \quad x_c = \frac{q x_0}{x_0^2 - d^2}. \]

The value 1 is assigned to the even zones and 0 is assigned to the odd zones. Then, the reflection coefficient distribution \( \Phi(r) \) is obtained (figure 1). This reflection coefficient distribution provides the radiation focusing on the point \((x_0, 0, z_0)\).

This is a general case when \( y_0 \neq 0 \). One can reduce it to the equation (1) with the following substitution

\[ x_0 \rightarrow \text{sgn}(x_0) \sqrt{x_0^2 + y_0^2}, \quad \text{where} \quad \text{sgn}(x_0) = \begin{cases} 
1, & x_0 > 0; \\
0, & x_0 = 0; \\
-1, & x_0 < 0.
\end{cases} \]

and the following recalculation of \( \Phi(r) \) from one coordinate system to another one using the rotation matrix

\[ A_\phi = \begin{pmatrix}
\cos(\phi) & \sin(\phi) & 0 \\
-\sin(\phi) & \cos(\phi) & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad \text{where} \quad \phi = \arctg\left(\frac{y_0}{x_0}\right). \]

Therefore, we obtained the solution for the first task, the distribution of reflectors, which provide the radiation focusing on the predefined point.

Consider the solution of the second task. The field in the test object surface at the height \( z_0 \) after the plate reflection is written as

\[ E_1(r) = \int G(r - r_0) \Phi(r) G(r_0 - r) dr_0. \]

Here, \( G(r) \) is the Green's function of a point-source radiator. When radiation reflected from the object, the field at the reflector plane is written as follows:

\[ E_2(r) = \int E_1(r_0) \gamma(r_0) G(r_0 - r) dr_0, \]

where \( \gamma(r_0) \) is the function describing the geometry of the test object. The function is equal to 1 inside the object boundaries and it is equal to 0 out of the boundaries. The field in the receiver plane at the \( h \) height is

\[ E_3(r) = \int E_2(r_0) \Phi(r_0) G(r_0 - r) dr. \]

Obviously the best focusing is achieved when the receiver point coincides with the transmitter point.

Thus, the solutions of both tasks were achieved. The equation describing the reflection coefficient distribution in the reflective array to provide the radiation focusing on the certain point is obtained. This results will be applied in modeling the reflective array and taking the radio image of a test object.
It should be noted that the data on the field detected in the receiver recorded during the focusing on all points in the scanned volume is the radiotomogram of the test space.

3. Numerical Simulation
A flat array of elementary reflectors 20×20 cm was used in numerical simulation. The size of each reflector was of 0.5 cm. The coordinates of the transmitter and the receiver were \((0, 0, 20)\). The focusing point coordinates were \((0, 0, 30)\). The reflection coefficient distribution \(\Phi(r_e)\) for the frequency 24 GHz was calculated (figure 2). Figure 3 presents the radiation focused by the array on the test object plane. The field distribution in the receiver is presented in figure 4. The maximum value of the scattered field coincides with the coordinates of the transmitter.

![Figure 2. Fresnel zone distribution.](image)

![Figure 3. Field focusing on the object plane.](image)

![Figure 4. Field distribution on the receiver plane.](image)

The reflection coefficient distribution for the focusing point \((5, 2, 30)\) is shown in figure 5. Radiation focused by the array in the test object plane and the field distribution for that focusing point in the receiver plane are shown in figures 4 and 5, respectively. As the figures show, the field in the object plane was focused in the same way as with the focusing point at the \(oz\) axis. The situation with the field distribution at the receiver plane is quite a bit different, as noise increases with the shift of the focusing point from the \(oz\) axis.

![Figure 5. Fresnel zone distribution.](image)

![Figure 6. Field focusing on the object plane.](image)

![Figure 7. Field distribution on the receiver plane.](image)

So, the best focusing is provided when the position of the transmitter coincides with the receiver position.

As a test object, a flat object in a shape of a stepped triangle was used. Each step was of 3 cm. The object was placed at the height of 10 cm above the reflective array. Figure 9 illustrates the radio image
of the test object above the reflective array 10×10 cm in size. The result for the reflective array 20×20 cm in size is presented in figure 10.

![Figure 8. Test object.](image1)

![Figure 9. Radio image of the test object above the 10×10 cm array.](image2)

![Figure 10. Radio image of the test object above the 20×20 cm array.](image3)

The triangle used as the test object is plainly seen in the radio image. Using the reflective array with larger aperture provided the radio image of higher sharpness and with a lower artifact level.

4. Conclusion
The conducted research demonstrated the using the 2D reflective array with controlled reflection coefficient to focus the monochromatic radiation for radiotomographic application. The numerical simulations with a test object in a form of a stepped triangle approved the recovery of the test object's shape.

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