On the reliability of the so far performed tests for measuring the Lense-Thirring effect with the LAGEOS satellites

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Abstract

In this paper we critically discuss the so far performed attempts aimed at the detection of the general relativistic gravitomagnetic Lense-Thirring effect in the gravitational field of the Earth with the existing LAGEOS satellites. In the latest reported measurement of the gravitomagnetic shift with the nodes of the LAGEOS satellites and the 2nd generation GRACE-only EIGEN-GRACE02S Earth gravity model over an observational time span of 11 years a 5-10% total accuracy is claimed at 1-3σ, respectively. We will show that, instead, it might be 15-45% (1-3σ) if the impact of the secular variations of the even zonal harmonics is considered. Possible strategies in order both to make more robust and reliable the tests with the node-only LAGEOS-LAGEOS II combination used and to overcome the problems affecting it with other alternative combinations are presented.
1 Introduction

Recent years have seen increasing efforts aimed to directly\(^1\) detecting various phenomena connected to the general relativistic gravitomagnetic field \([2, 3, 4, 5]\) of the rotating Earth.

The extraordinarily sophisticated and expensive Gravity Probe B (GP-B) mission \([6, 7]\) has been launched in April 2004; it is aimed at the detection of the gravitomagnetic precession of the spins \([8]\) of four superconducting gyroscopes carried onboard at a claimed accuracy of 1% or better.

The Lense-Thirring effect on the orbital motion of a test particle \([9]\) could be measured by analyzing the orbital data of certain Earth artificial satellites with the Satellite Laser Ranging (SLR) technique \([10]\). Up to now, the only performed tests are due to Ciufolini and coworkers \([11, 12, 13, 14]\). In such papers a confirmation of the existence of the Lense-Thirring effect as predicted by the Einstein’s General Theory of Relativity is claimed with a total 20-25\% (node-node-perigeel LAGEOS-LAGEOS II linear combination) and 5−10\% (node-node LAGEOS-LAGEOS II linear combination) accuracy, respectively, according to the adopted observable.

In this paper we will analyze the latest results presented in \([12, 13, 14]\) from a critical point of view in order to show that the total error budgets might have been underestimated.

2 The Lense-Thirring effect on the orbit of a test particle and the strategy to measure it

The gravitomagnetic field of a spinning mass of proper angular momentum \(J\) induces tiny secular precessions on the longitude of the ascending node \(\Omega\) and the argument of pericentre\(^2\) \(\omega\) of a test particle \([9, 15, 3, 16]\)

\[
\dot{\Omega}_{LT} = \frac{2GJ}{c^2a^3(1-e^2)^{3/2}}, \quad \dot{\omega}_{LT} = -\frac{6GJ \cos i}{c^2a^3(1-e^2)^{3/2}},
\]  

where \(G\) is the Newtonian constant of gravitation, \(c\) is the speed of light in vacuum, \(a, e\) and \(i\) are the semimajor axis, the eccentricity and the inclination, respectively, of the test particle’s orbit. See Figure 1 for the orbital

\(^1\) According to K. Nordtvedt \([1]\), the multidecadal analysis of the Moon’s orbit by means of the Lunar Laser Ranging (LLR) technique yields a comprehensive test of the various parts of order \(O(c^{-2})\) of the post-Newtonian equation of motion. The existence of gravitomagnetism as predicted by the Einstein’s General Theory of Relativity would, then, be indirectly inferred from the high accuracy of the lunar orbital reconstruction.

\(^2\) In their original paper Lense and Thirring use the longitude of pericentre \(\varpi = \Omega + \omega\).
geometry of a Keplerian ellipse. In the terrestrial space environment the gravitomagnetic precessions are very small: for the spherically symmetric geodetic SLR LAGEOS satellites, whose orbital parameters are listed in Table 1, they amount to a few tens of milliarcseconds per year (mas yr$^{-1}$ in the following).

The extraction of the Lense–Thirring precessions from the orbit data analysis is very difficult due to a host of competing classical orbital perturbations of gravitational [17, 18, 19, 20] and non-gravitational [21, 22, 23, 24, 25, 26] origin which have various temporal signatures and are often quite larger than the relativistic signal of interest. The most insidious ones are the

Table 1: Orbital parameters of the existing LAGEOS and LAGEOS II and of the proposed LARES and their Lense-Thirring node precessions.

| Satellite  | $a$ (km) | $e$   | $i^\circ$ | $\Omega_{LT}$ (mas yr$^{-1}$) |
|------------|---------|-------|-----------|-----------------------------|
| LAGEOS     | 12270   | 0.0045| 110       | 31                          |
| LAGEOS II  | 12163   | 0.0135| 52.64     | 31.5                        |
| LARES      | 12270   | 0.04  | 70        | 31                          |
perturbations which have the same temporal signature of the Lense-Thirring precessions\(^3\), i.e. secular trends. Indeed, whatever the length of the adopted observational time span \(T_{\text{obs}}\) is, they cannot be fitted and removed from the time series without removing the relativistic signal as well. Then, it is of the utmost importance to assess as more accurately and reliably as possible their aliasing impact on the measurement of the Lense-Thirring effect.

It turns out that the perigees of the LAGEOS-like satellites are severely affected by the non-gravitational perturbations, contrary to the nodes. Moreover, since the non–conservative forces depend on the structure, the shape and the rotational status of the satellite their correct modelling is not a trivial task and, as we will see later, introduces large uncertainties in the correct assessment of the error budget in some of the performed gravitomagnetic tests.

2.1 The gravitational error

The even \((\ell = 2, 4, 6,...)\) zonal \((m = 0)\) harmonic coefficients \(J_\ell\) of the multipolar expansion of the Earth’s gravitational potential, called geopotential, induce secular precessions\(^4\) on the node, the perigee and the mean anomaly of any near-Earth artificial satellite \(^{27}\) which, of course, depend only on its orbital configuration and are independent of its physical structure. Such aliasing effects are many orders of magnitude larger than the Lense-Thirring precessions; the precision with which the even zonal harmonics are known in the currently available Earth gravity models \(^{28, 29, 30, 31, 32, 33, 34, 35, 36}\) would yield errors amounting to a significant fraction of the Lense-Thirring precessions or even larger.

Even more dangerous are the perturbations induced by the secular variations of the low degree even zonal harmonics \(\dot{J}_\ell, \ell = 2, 4, 6\) \(^{37, 38}\). Indeed, such perturbations grow quadratically in time if the shifts in mas are considered and linearly in time if the rates in mas yr\(^{-1}\) are considered. Their impact on the orbital elements of the LAGEOS satellites have been worked out in \(^{39}\). It turns out that, by using the results of \(^{37}\), the errors induced by \(\dot{J}_2\) would amount to 8\%, 14\% and 5.4\% for the nodes of LAGEOS and

\(^3\)Also the perturbations which grow quadratically in time are, of course, very dangerous. Those induced by the secular variations of the even zonal harmonics of the Earth’s geopotential fall in this category, as we will see in detail in Section 3.2.1. Harmonic time-dependent perturbations with periods longer than the observational time span may also be insidious because they would resemble superimposed linear trends \(^{17}\).

\(^4\)Also the subtle non–gravitational Yarkovsky-Rubincam force, which is due to the interaction of the Earth’s electromagnetic IR radiation with the physical structure of the LAGEOS satellites, induces secular effects on their nodes and perigees \(^{24}\).
LAGEOS II and the perigee of LAGEOS II, respectively, over an observational time span $T_{\text{obs}}$ of just one year at $1-\sigma$ level. This clearly shows that it would be impossible to analyze single orbital elements.

The time-dependent harmonic perturbations \cite{17, 19, 20} are less dangerous because if their periods are shorter than the adopted observational time span they can be fitted and removed from the time series. The most insidious tidal perturbation is that induced by the even zonal (055.565) constituent which has a period of 18.6 years and whose nominal impact on the orbital elements of the LAGEOS satellites amounts to thousands of mas \cite{17}. However, it turns out that it does not affect the observables which have been adopted for the performed Lense-Thirring tests because its main component is of degree $\ell = 2$ and order $m = 0$ (see Section 2.1.2).

2.1.1 The butterfly configuration

A possible way to cope with the even zonal gravitational perturbations is represented by the use of a pair of satellites in the so-called butterfly configuration. In it the orbital planes of the two satellites are shifted $180^\circ$ apart, all other orbital parameters being equal. In this case the sum of the nodes \cite{10, 11} $\dot{\Omega}^{(i)} + \Omega^{(i+180^\circ)}$ and the difference of the perigees \cite{12, 13, 44} $\dot{\omega}^{(i)} - \dot{\omega}^{(i+180^\circ)}$ would allow to cancel exactly out the bias due to all the even zonal harmonics of the geopotential\footnote{If totally passive satellites are considered, the difference of the perigees would yield less accurate results than the sum of the nodes \cite{44}.}; the gravitomagnetic precessions would, instead, add up. Indeed, the Lense-Thirring node precessions are independent of $i$, while the classical geopotential node precessions depend on a multiplicative factor $\cos i$ common to all degrees $\ell$ and on sums of even powers of $\sin i$ which are different for the various degrees \ell \cite{18}; the Lense-Thirring perigee precessions depend on $\cos i$ while the classical geopotential perigee precessions depend on even powers of $\cos i$ and $\sin i$ \cite{18}. The butterfly configuration cannot be realized with the present-day existing Earth artificial satellites. In \cite{40} it was proposed to launch a LAGEOS-like satellite-the former LAGEOS III which later became LARES\footnote{The eccentricity of LARES would amount to $e_{\text{LARES}} = 0.04$ in order to perform other tests of post-Newtonian gravity with the perigee.} \cite{45}-with the same orbital parameters of LAGEOS apart from the inclination which should be supplementary. That idea is still alive, although it has not yet been approved by any national space agency or institution. Recently, it has been proposed to adopt the LARES orbital configuration for the OPTIS relativity mission \cite{46, 47, 48} currently under examination by the German Space
Agency (DLR). It should be noted that with a LAGEOS-LARES/OPTIS butterfly configuration the difference of the perigees would not be a good observable because of the small eccentricity of the LAGEOS orbit. The exact cancellation of the classical precessions due to all the even zonal harmonics of geopotential could be achieved only if the LARES/OPTIS orbital parameters were exactly equal to those of LAGEOS. This would pose severe restrictions in term of the quality and, consequently, the cost of the rocket launcher to be used. Indeed, the realistically obtainable precision with the originally proposed sum of the nodes would be affected by the unavoidable departures of the LARES/OPTIS orbital parameters from their nominal values, at least to a certain extent. Also the fact that the eccentricities of LAGEOS and LARES/OPTIS would differ by one order of magnitude should be accounted for. Indeed, in these cases a residual aliasing effect due to all the even zonal harmonics $J_2, J_4, J_6...$ would still occur. These topics have been investigated in [49]. A different observable involving also the nodes of LAGEOS and LAGEOS II has been proposed in [50] for the LARES mission. It would dramatically reduce the dependence of the systematic error due to the even zonal harmonics of the geopotential on the unavoidable orbital injection errors allowing also for a reduction of the costs of the mission. Moreover, as it will become clear later, also the impact of the $\dot{\psi}$ would be greatly reduced. On the contrary, the simple sum of the nodes would be affected by $\dot{\psi}$, $\dot{\psi}_2$, $\dot{\psi}_4$, $\dot{\psi}_6$... which might not be negligible over a time span many years long.

2.1.2 The linear combination approach

The problem of reducing the impact of the mismodeling in the even zonal harmonics of the geopotential with the currently existing satellites can be coped in the following way.

Let us suppose we have at our disposal $N$ ($N > 1$) time series of the residuals of those Keplerian orbital elements which are affected by the geopotential with secular precessions, i.e. the node and the perigee: let them be $\psi^A$, $A=$LAGEOS, LAGEOS II, etc. Let us write explicitly down the expressions of the observed residuals of the rates of those elements $\delta\dot{\psi}^A_{\text{obs}}$ in terms of the Lense-Thirring effect $\dot{\psi}^A_{\text{LT}}$, of $N-1$ mismodelled classical secular precessions $\dot{\psi}^A_{\text{c}}\delta J_\ell$ induced by those even zonal harmonics whose impact on the measurement of the gravitomagnetic effect is to be reduced and of the remaining mismodelled phenomena $\Delta$ which affect the chosen orbital element.
\[ \delta \dot{\psi}_\text{obs}^A = \dot{\psi}_{LT}^A \mu + \sum_{N-1 \text{ terms}} \psi_{\ell}^A \delta J_\ell + \Delta^A, \quad A = \text{LAGEOS, LAGEOS II, ...} \quad (2) \]

The parameter\(^7\) \(\mu\) is equal to 1 in the General Theory of Relativity and 0 in Newtonian mechanics. The coefficients \(\dot{\psi}_{\ell}^A\) are defined as

\[ \dot{\psi}_{\ell} = \frac{\partial \dot{\psi}_{\text{class}}}{\partial J_\ell} \quad (3) \]

and have been explicitly worked out for the node and the perigee up to degree \(\ell = 20\) in [10, 18]; they depend on some physical parameters of the central mass \((GM\) and the mean equatorial radius \(R)) and on the satellite’s semi-major axis \(a\), the eccentricity \(e\) and the inclination \(i\). We can think about eq. (2) as an algebraic nonhomogeneous linear system of \(N\) equations in \(N\) unknowns which are \(\mu\) and the \(N-1\) \(\delta J_\ell\): solving it with respect to \(\mu\) allows to obtain a linear combination of orbital residuals which is independent of the chosen \(N-1\) even zonal harmonics. In general, the orbital elements employed are the nodes and the perigees and the even zonal harmonics cancelled are the first \(N-1\) low-degree ones.

This approach is, in principle, very efficient in reducing the impact of the systematic error of gravitational origin because all the classical precessions induced by the static and time-dependent parts of the chosen \(N-1\) \(J_\ell\) do not affect the combination for the Lense-Thirring effect. Moreover, it is flexible because it can be applied to all satellites independently of their orbital configuration, contrary to the butterfly configuration in which the cancellation of the even zonal harmonics can be achieved only for supplementary orbital planes and identical orbital parameters. This approach has been adopted, among other things, in [50, 47] in order to make the requirements on the LARES/OPTIS orbital configuration less stringent. Apart from the first orbital element which enters the combination with 1, the other elements are weighted by multiplicative coefficients \(c_i(a, e, i) \neq 1\) which are built up with \(\dot{\psi}_{\ell}\) and, then, depend on the orbital elements of the considered satellites. Their magnitude is very important with respect to the non-gravitational perturbations, which in general are not cancelled out by the outlined method, and to the other time-dependent perturbations of gravitational origin which are neither even nor zonal so that they affect the obtained combination as well. Values smaller than 1 for the \(c_i\) coefficients are, in general, preferable because they reduce the impact of such uncancelled perturbations. It

\[^7\text{It can be expressed in terms of the PPN } \gamma \text{ parameter [51] as } \mu = (1 + \gamma)/2.\]
is important to note that the order with which the orbital elements enter the combination is important: indeed, while the systematic error due to the even zonal harmonics of the geopotential remains unchanged if the orbital elements of a combination are exchanged, the coefficients \( c_i \) do change and, consequently, also the non-gravitational error. The best results are obtained by choosing the highest altitude satellite as first one and by inserting the other satellites in order of decreasing altitudes.

This method was explicitly adopted for the first time in \cite{52} with the nodes of the LAGEOS satellites and the perigee of LAGEOS II. The obtained combination is

\[
\delta \dot{\Omega}^{\text{LAGEOS}}_{\text{obs}} + c_1 \delta \dot{\Omega}^{\text{LAGEOS II}}_{\text{obs}} + c_2 \delta \omega^{\text{LAGEOS II}}_{\text{obs}} \sim \mu 60.2, \tag{4}
\]

where \( c_1 = 0.304 \), \( c_2 = -0.350 \) and 60.2 is the slope, in mas yr\(^{-1}\), of the expected gravitomagnetic linear trend. Eq. \ref{eq:4} is insensitive to the first two even zonal harmonics \( J_2 \) and \( J_4 \). It has been used in \cite{11} when the level of accuracy of the JGM3 \cite{28} and EGM96 \cite{29} Earth gravity models, available at that time, imposed the cancellation of the first two even zonal harmonics, at least.

In view of the great improvements in the Earth gravity field modelling with the CHAMP \cite{53} and, especially, GRACE \cite{54} missions an extensive search for alternative combinations has been subsequently performed \cite{55, 56, 57, 58}. In \cite{39, 56} the following combination has been proposed\footnote{The possibility of using only the nodes of the LAGEOS satellites in view of the improvements in the Earth gravity models from GRACE has been proposed for the first time in \cite{51}, although without quantitative details.}

\[
\delta \dot{\Omega}^{\text{LAGEOS}}_{\text{obs}} + k_1 \delta \dot{\Omega}^{\text{LAGEOS II}}_{\text{obs}} \sim \mu 48.2, \tag{5}
\]

where \( k_1 = 0.546 \) and 48.2 is the slope, in mas yr\(^{-1}\), of the expected gravitomagnetic linear trend. It has been adopted for the test performed in \cite{13} with the 2nd generation GRACE-only EIGEN-GRACE02S Earth gravity model \cite{34}. Eq. \ref{eq:5} allows to cancel out the first even zonal harmonic \( J_2 \).

### 3 The performed Lense-Thirring tests with the LAGEOS satellites

The only performed tests aimed at the detection of the Lense-Thirring precessions of eq. \ref{eq:1} in the gravitational field of the Earth with the existing LAGEOS satellites have been performed, up to now, by Ciufolini and...
coworkers. They have used the node-node-perigee combination of eq. (4)\cite{11,12} and the node-node combination of eq. (5)\cite{13}. In these works it has often been claimed that “[...] the Lense-Thirring effect exists and its experimental value, [...] fully agrees with the prediction of general relativity.”

The main objections to the results presented in these works can be summarized as follows

- The authors have not performed really robust and reliable tests e.g. by varying the length of the adopted observational time span, running backward and forward the initial epoch of the analysis, varying the secular rates of the even zonal harmonics in order to check their impact over different time spans, using different Earth gravity models in order to obtain a scatter plot of the obtained results. Instead, it seems that the authors, for a given data set, have always used from time to time those Earth gravity models which yielded just the closest results to what it is a priori expected from the General Theory of Relativity. Moreover, the impact of a priori ‘imprint’ effects of the Lense-Thirring signature itself on the coefficients of the Earth gravity models used may have driven the outcome of the performed tests just towards the expected result.

- The total error budget has probably been underestimated, especially the systematic error of gravitational origin. E.g., the impact of the secular variations of the even zonal harmonics of the geopotential, which may become a very limiting factor over time spans many years long as those used, has not been addressed in a realistic, reliable and satisfactorily manner. Moreover, in some cases it has been incorrectly calculated by summing in quadrature the various contributions to the gravitational error (static even zonal harmonics, tides, secular variations of the even zonal harmonics) which can, instead, hardly be considered as uncorrelated. Almost always $1–\sigma$ results have been presented without any explicit indication of this fact.

- All the relevant works of other authors, in which many of these issues have been addressed, have always been consistently ignored.

- The node-node combination of eq. 5 has been repeatedly and explicitly presented as a proper own result of the authors with references to their works (ref. 6 of 12 and ref. 19 of 13) which, instead, have nothing to do with eq. 5.
3.1 The node-node-perigee tests

The combination of eq. (4) has been analyzed by using the EGM96 [29] Earth gravity model over 4 years in [11] and over 7.3 years in [12]. The claimed total error budget amounts to 20-25% over 4 years and to 20% over 7.3 years.

3.1.1 The gravitational error

The impact of the remaining uncancelled even zonal harmonics of the geopotential $J_6$, $J_8$, $J_{10}$, ... on eq. (4) has been estimated by Ciufolini and coworkers with the full covariance matrix of EGM96 in a root-sum-square calculation. In [11] and, six years later, in [12] it is claimed to be $\lesssim 13\%$. Apart from the fact that this is a $1-\sigma$ level estimate, in [59], as later acknowledged in a number of papers [55, 49, 39, 56, 58], the use of the full covariance matrix of EGM96 has been questioned. Indeed, it has been noted that in the EGM96 solution the recovered even zonal harmonics are strongly reciprocally correlated; it seems, e.g., that the 13% value for the systematic error due to geopotential is due to a lucky correlation between $J_6$ and $J_8$ which are not cancelled by eq. (4). The point is that, according to [59], nothing would assure that the covariance matrix of EGM96, which is based on a multi-year average that spans the 1970, 1980 and early 1990 decades, would reflect the true correlations between the even zonal harmonics during the particular time intervals of a few years adopted in the analyses by Ciufolini and coworkers. Then, a more conservative, although pessimistic, approach would be to consider the sum of the absolute values of the errors due to the single even zonal as representative of the systematic error induced by our uncertainty in the terrestrial gravitational field according to EGM96 [56, 39]. In this case we would get a conservative upper bound of 83% (1-$\sigma$). If a root-sum-square calculation is performed by neglecting the correlations between the even zonals a 45% 1-$\sigma$ error is obtained [18, 56, 39, 58].

3.1.2 The non-gravitational error

Another important class of systematic errors is given by the non-gravitational perturbations which affect especially the perigee of LAGEOS II. The main problem is that it turned out that their interaction with the structure of LAGEOS II changes in time due to unpredictable modifications in the physical properties of the LAGEOS II surface (orbital perturbations of radiative origin, e.g. the solar radiation pressure and the Earth albedo) and in the evolution of the spin dynamics of LAGEOS II (orbital perturbations of
thermal origin induced by the interaction of the electromagnetic radiation of solar and terrestrial origin with the physical structure of the satellites, in particular with their corner–cube retroreflectors). Moreover, such tiny but insidious effects were not entirely modelled in the GEODYN II software at the time of the analysis of [11, 12], so that it is not easy to correctly and reliably assess their impact on the total error budget of the measurement performed during that particular time span. According to the evaluations in [23], the systematic error due to the non–gravitational perturbations over a time span of 7 years amounts to almost 28%. However, according to [59], their impact on the measurement of the Lense–Thirring effect with the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II is, in general, quite difficult to be reliably assessed.

So, by adding quadratically the gravitational and non–gravitational errors of [23] we obtain for the systematic uncertainty $\delta\mu_{\text{systematic}} \sim 54\%$ if we assume a 45% error due to geopotential. The sum of the absolute values of the errors due to geopotential added quadratically with the non–gravitational perturbations would yield a total systematic error of $\delta\mu_{\text{systematic}} \sim 88\%$. It must be noted that the latter estimate is rather similar to those released in [59]. Note also that they are 1-$\sigma$ evaluations. Moreover, it should be considered that the perigee of LAGEOS II is also sensitive to the eclipses effect on certain non–gravitational perturbations. Such features are, generally, not accounted for in all such estimates. An attempt can be found in [21] in which the impact of the eclipses on the effect of the direct solar radiation pressure on the LAGEOS–LAGEOS II Lense–Thirring measurement has been evaluated: it should amount to almost 10% over an observational time span of 4 years.

3.2 The node-node tests

The situation might turn out to be even worse for the results presented for the node-node combination of eq. (5). Such observable only cancels out the gravitational bias of the first even zonal harmonic $J_2$, but has the great advantage of discarding the perigee of LAGEOS II and its insidious non-gravitational perturbations.

It has explicitly been proposed for the first time in [56, 39], although the possibility of using a LAGEOS-LAGEOS II node-only observable was presented for the first time in [51] without quantitative details. In [12] it seems that the author refers to it as a proper own result with the ref. [6], i.e. [11] of the present paper. In [13] the authors, instead, explicitly attribute it to themselves with ref. 19, i.e. [40] of the present paper. In [14] an unfair
3.2.1 The gravitational error

In [12] the node-node combination of eq. (5) has been analyzed with the EIGEN2 [32] and GGM01 [35] Earth gravity models over a time span of almost 10 years.

In [39] the impact of the static part of the geopotential, according to the CHAMP-only EIGEN2 Earth gravity model, is evaluated as 18% (1-σ root-sum-square covariance calculation), 22% (1-σ root-sum-square calculation) and 37% (1-σ upper bound). Ciufolini only reports 18% obtained with the full covariance matrix of EIGEN2 for which the same remarks as for EGM96 holds. Moreover, he seems to ignore that EIGEN2 is only based on six months of data and that the released sigmas of the even zonal harmonics of low degree, which are the most relevant in this kind of analyses with the LAGEOS satellites, are rather optimistic, as pointed out in [32] and acknowledged in [39]. In regard to the GGM01 model, the covariance matrix was not publicly released. Cufolini correctly presents a 19% which is the 1-σ upper bound obtained in [39]. However, GGM01 is only based on 111 days of data. The author’s claim “We conclude, using the Earth gravity model EIGEN-2S, that the Lense-Thirring effect exists and its experimental value, \( \mu = 0.98 \pm 0.18 \), fully agrees with the prediction of general relativity” seems too optimistic.

In [13] the authors use the 2nd generation GRACE-only EIGEN-GRACE02S Earth gravity model [34]. Also in this case the full covariance matrix was not available and the authors correctly report a systematic error due to the even zonal harmonics of 3% (root-sum-square calculation) and 4% (upper bound) at 1-σ level. The same results have been obtained in [60].

Major problems may arise when the authors show their a priori error analysis for the time-dependent gravitational perturbations (solar and lunar Earth tides, secular trends in the even zonal harmonics of the Earth’s field and other periodic variations in the Earth’s harmonics). Indeed, in [13] they claim that, over an observational time span of 11 years, their impact would be 2%. This evaluation is based on ref. 30 of [13] which refers to the WEBER-SAT/LARES INFN study; it has nothing to do with the present node-only LAGEOS-LAGEOS II combination.
3.2.2 The impact of the secular rates of the uncancelled even zonal harmonics

Moreover, this estimate may turn out to be optimistic because of the secular variations of the even zonal harmonics\(^9\) \(\dot{J}_\ell\). Indeed, eq. (5) allows to cancel out \(\dot{J}_2\), but, in principle, is sensitive to \(\dot{J}_4, \dot{J}_6, \ldots\), as pointed out in [60]. The uncertainties in the \(\dot{J}_\ell\) are still quite large: e.g., according to Table 1 of [38], there is not yet even full consensus on the sign of such rates. On the other hand, their impact on the Lense–Thirring measurement grow linearly in time\(^10\). Indeed, the mismodelled shift, in mas, of eq. (5) due to the secular variations of the uncancelled even zonal harmonics can be written as

\[
\sum_{\ell=2} \left( \dot{\Omega}_{\ell}^{\text{LAGEOS}} + k_1 \dot{\Omega}_{\ell}^{\text{LAGEOS II}} \right) \frac{\delta \dot{J}_\ell}{2} T_{\text{obs}},
\]

where the coefficients \(\dot{\Omega}_{\ell}\) are \(\partial \dot{\Omega}_{\text{class}} / \partial J_\ell\) and have explicitly been calculated up to degree \(\ell = 20\) in [10, 18]. It must be divided by the gravitomagnetic shift, in mas, of eq. (5) over the same observational time span

\[
\left( \dot{\Omega}_{\text{LT}}^{\text{LAGEOS}} + k_1 \dot{\Omega}_{\text{LT}}^{\text{LAGEOS II}} \right) T_{\text{obs}} = 48.2 \text{ mas yr}^{-1} T_{\text{obs}}.
\]

By assuming \(\delta \dot{J}_4 = 0.6 \times 10^{-11} \text{ yr}^{-1}\) and \(\delta \dot{J}_6 = 0.5 \times 10^{-11} \text{ yr}^{-1}\) [38], it turns out that the percent error on the combination eq. (5) grows linearly with \(T_{\text{obs}}\) and would amount to \(1\%\) over one year at \(1 - \sigma\) level. This means that, over 11 years, their impact might range from \(11\%\) (1-\(\sigma\)) to \(33\%\) (3-\(\sigma\)). These evaluations hold on the assumption that \(\dot{J}_4\) and \(\dot{J}_6\) retain their constant sign over the adopted observational time span. At present, there is, in fact, evidence for an inversion only in \(\dot{J}_2\) since 1998 [38], but eq. (5) is insensitive to it. In [14] some explanations about the procedure followed in the analysis of [13] can be found. Basically, they are as follows

- The time series of the combined residuals of the nodes of LAGEOS and LAGEOS II, built up without the even zonal rates in the background reference model for the Earth gravity field, has been fitted with some harmonic signals, a linear trend and a parabolic signal from which a

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\(^9\)The problem of the secular variations of the even zonal harmonics in post-Newtonian tests of gravity with LAGEOS satellites has been quantitatively addressed for the first time in [61]. In regard to the Lense–Thirring measurement with eq. (5), it has been, perhaps, misunderstood in [39].

\(^10\)For a possible alternative combination which would cancel out the first three even zonal harmonics along with their temporal variations see [57, 58].
\[ \dot{J}_4^{\text{eff}} \sim 1.5 \times 10^{-11} \text{ yr}^{-1} \] has been determined. Then, the so obtained \( \dot{J}_4^{\text{eff}} \) would have been inserted in the dynamical force models of the orbital processor in order to re-analyze the same data yielding a certain value of \( \mu \) which we conventionally define as \( \mu_{\dot{J}_4^{\text{eff}}} \).

- The time series of the combined residuals of the nodes of LAGEOS and LAGEOS II, built up with the default value of the even zonal rate \( \dot{J}_4^{\text{eff}} = 1.41 \times 10^{-11} \text{ yr}^{-1} \) in the EIGEN-GRACE02S model, has been fitted, again, with some harmonic signals, a linear trend and a parabolic signal yielding a certain value of \( \mu \) which we conventionally define as \( \mu_{\dot{J}_4^{\text{default}}} \).

- The so obtained values \( \mu_{\dot{J}_4^{\text{eff}}} \) and \( \mu_{\dot{J}_4^{\text{default}}} \) have been compared by finding a 1% variation only. The author of [14] writes “[...] using the value \( \dot{J}_4^{\text{effective}} \equiv 1.5 \times 10^{-11} \) for the LAGEOS satellites that we obtained from fitting the combined residuals [...] resulted in a change of the measured value of frame-dragging by about 1% only with respect to the case of using \( \dot{J}_4 = 1.41 \times 10^{-11} [...] \)”

### 3.2.3 Some possible criticisms to the outlined approach

The value \( \dot{J}_4^{\text{eff}} \sim 1.5 \times 10^{-11} \text{ yr}^{-1} \) measured with the combination of eq. (5) is affected not only by \( \dot{J}_6 \) and the other higher degree even zonal harmonics but, more importantly, by the Lense-Thirring signature itself. Indeed, the combination of eq. (5) is designed in order to only disentangle \( J_2 \) and the Lense-Thirring effect. So, it is not admissible to use the so obtained \( \dot{J}_4^{\text{eff}} \), which is coupled by construction to the Lense-Thirring effect, in order to reliably and correctly measure the Lense-Thirring effect itself. A combination suitably designed in order to measure \( J_4 \) independently of \( J_2, J_6 \) and the General Theory of Relativity has been proposed in [62].

### 3.2.4 Some suggestions to improve the reliability and the robustness of the discussed test

- The values of \( \dot{J}_j \) to be used in the dynamical force models of the orbital processors should have been determined in a Lense-Thirring-free fashion in order to avoid ‘imprinting’ effects on the outcome of the analysis. Indeed, they could easily drive it just towards the expected result. In regard to this point, suitably designed linear combinations of the residuals of some orbital elements of the existing SLR satel-
lites could be used in order to disentangle, e.g., $J_2$, $J_4$, $J_6$ and the gravitomagnetic force

- For a given observational time span and a given background reference Earth gravity model, two time series of the combined residuals, built up with and without the aforementioned $\dot{J}_\ell$ GTR-free values in the background reference models, should be analyzed. The difference between the so obtained $\mu$ parameters can, then, be evaluated.

- The aforementioned procedure should be repeated for various observational time spans in order to have a more robust assessment of the importance of the secular variations of the even zonal harmonics in the proposed measurement. Indeed, their systematic error on the Lense-Thirring effect grows linearly.

- The aforementioned procedures should be repeated for various Earth gravity models in order to have a scatter plot.

### 3.2.5 On the correct evaluation of the systematic error of gravitational origin

Another controversial point is that it is unlikely that the various errors of gravitational origin can be summed in a root-sum-square way because of the unavoidable correlations between the various phenomena of gravitational origin. Indeed, in the adopted EIGEN-GRACE02S model $\dot{J}_4$ and $\dot{J}_6$ are not solved for: $\dot{J}_2$ and $\dot{J}_4$ have been assumed fixed, while $\dot{J}_6$ is absent. This means that the recovered $J_\ell$ are affected by such rates. It would, then, be more conservative to linearly add the errors induced by them. In this case, the $(J_\ell^{(0)} - \dot{J}_\ell)$ error would range from 15% (4%+11%) at 1-$\sigma$ level to 45% (12%+33%) at 3-$\sigma$ level over a 11-years long observational time span. The so obtained global gravitational error can be added in quadrature to the non-gravitational error. Even by assuming the 2% authors’ estimate of the time-dependent part of the gravitational error, the upper bound errors would be $\sqrt{(4+2)^2 + 2^2} = 6\%$ at 1-$\sigma$, $\sqrt{(8+4)^2 + 4^2} = 13\%$ at 2-$\sigma$ and $\sqrt{(12+6)^2 + 6^2} = 19\%$ at 3-$\sigma$. Instead, in [13] the authors add in quadrature the doubled error due to the static part of the geopotential (the 2×4% value obtained from the sum of the individual error terms), their perhaps optimistic evaluation of the error due to the time dependent part of the geopotential and the non-gravitational error getting $\sqrt{8^2 + 4^2 + 4^2} = 10\%$ at 2-$\sigma$. On the other hand, when they calculate the 3-$\sigma$ upper bound it

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11The evaluations of eq. [5] have been used.
seems that they triple the 3% error due to the static part of the geopotential obtained with a root-sum-square calculation and add it in quadrature to the other (not tripled) errors getting $\sqrt{9\%^2 + 2\%^2 + 2\%^2} \leq 10\%$ at 3-$\sigma$.

### 3.2.6 The a priori ‘memory’ effect of the Lense-Thirring signature on the adopted Earth gravity model

The following point seems also worthy of discussion. The recovered values of the even zonal harmonics in the GRACE models like EIGEN-GRACE02S retain, in principle, a sort of ‘imprint’ or ‘memory’ of the General Theory of Relativity, which, in fact, has not been modelled in the currently released GFZ-GRACE-based models (F. Flechtner, GFZ team, private communication, 2004). This is certainly true for the Earth gravity models of the pre-CHAMP and GRACE era obtained from multidecadal laser ranging to the geodetic satellites of LAGEOS type, as discussed in [52, 59, 58]. This feature is also valid for, e.g. GRACE which recovers the low degree even zonal harmonics from the tracking of both satellites by GPS and the medium-high degree geopotential coefficients from the observed intersatellite distance variations. Indeed, from [64] it can be noted that the variation equations for the Satellite-to-Satellite Tracking (SST) range $\Delta \rho$ and range rate $\Delta \dot{\rho}$ of GRACE can be written in terms of the in-plane radial and, especially, along-track components $R, T, V_R, V_T$ of the position and velocity vectors, respectively. In turns, they can be expressed as functions of the perturbations in all the six Keplerian orbital elements (see (10)-(11), (A4)-(A6), (A14)-(A16) and (A28)-(A30) of [64]). Now, the gravitomagnetic off-diagonal components of the spacetime metric also induce short-periodic 1 cycle per revolution (1 cpr) effects [9, 63, 15] on all the Keplerian orbital elements, apart from the secular trends on the node and the pericentre. This means that there is also a Lense-Thirring signature in all the other typical satellite and intersatellite observables like ranges and range-rates. It is likely that it mainly affects the low-degree even zonal harmonics. In this case, it might happen that the results of [12, 13], obtained with the $J_2$–free node-node LAGEOS-LAGEOS II combination, have been driven close to the expected result, at least to a certain extent, just by this a priori Lense-Thirring ‘imprint’ on the Earth gravity model used.

### 3.2.7 The LARES mission

Finally, it is hard to understand why the authors of [13] very often refer to the LAGEOS-LARES proposed experiment and to the related simulations and
error budgets. It is rather confusing and misleading. As already explained, the LAGEOS-LAGEOS II combination of eq. [3] is, by construction, designed in order to exactly cancel out the $J_2$ term with an approach which can be applied to any orbital configuration given a pair of satellites in different orbits or a pair of different Keplerian orbital elements of the same satellite (see Section 2.1.2). On the contrary, the observable originally proposed for the LAGEOS-LARES mission is the simple sum of their nodes. As pointed out in Section 2.1.1 if the orbital parameters of LARES, quoted in Table 1, were exactly equal to their nominal values, all the even zonal harmonics would be exactly cancelled out. Instead, the sum of the nodes would be affected, to a certain extent, by the whole range of the even zonal harmonics of the geopotential due to unavoidable departures from the LARES nominal configuration because of the orbital injection errors and mission design (the eccentricity of LARES would be one order of magnitude larger than that of LAGEOS), i.e. the coefficients of the classical nodal precessions would not be exactly equal and opposite $\dot{\Omega}_{\text{LAGEOS}}^{\ell} \neq -\dot{\Omega}_{\text{LARES}}^{\ell}$ for $\ell = 2, 4, 6, 8, ...$. In [13] the combination of the nodes of LAGEOS and LAGEOS II of eq. [5] is presented as if it is only slightly different with respect to the sum of the nodes of the originally proposed LAGEOS-LARES configuration, apart from a $18^\circ$ offset in the inclination of LAGEOS II with respect to LARES. The differences in the eccentricities and the semimajor axes, which do play a role [49], have been neglected.

4 Conclusions

In this paper we have performed a detailed critical analysis of the reliability and robustness of the so far performed tests aimed at the detection of the Lense-Thirring effect in the gravitational field of the Earth with the existing or proposed LAGEOS satellites.

We can summarize our conclusions as follows

- In regard to the node-node-perigee LAGEOS-LAGEOS II combination, still presented in recent works as [12], the claimed $20-25\%$ total accuracy obtained with the EGM96 Earth gravity model seems to be too optimistic mainly because of an underestimation of the systematic error due to the geopotential.

- In regard to the node-node LAGEOS-LAGEOS II combination of eq. [5], extensive and thorough tests are required in order to check the impact of the secular variations of the even zonal harmonics of the
geopotential which may represent a very limiting factor over time spans many years long. Such tests should be conducted by varying the data sets, shifting backward and forward the initial epoch of the analyses, varying the magnitudes of the $\dot{J}_\ell$ in the force models for a given observational time span and the length of the time span for given values of $\dot{J}_\ell$. If it will turn out that $\dot{J}_4, \dot{J}_6,...$ do affect eq. (5), only improvements in our knowledge of the $\dot{J}_\ell$ will allow to use this observable in a truly reliable and confident way; a better knowledge only of the static part of the geopotential would not be sufficient. Moreover, also the problem of the Lense-Thirring ‘memory’ in the adopted gravity model should be accounted for.

- Alternative combinations involving the use of existing SLR targets other than the LAGEOS satellites should be analyzed. The most promising combination involves the nodes of LAGEOS, LAGEOS II, Ajisai and Jason-1. It cancels out the first three even zonal harmonics $J_2, J_4, J_6$, along with their temporal variations, at the price of introducing the relatively huge non-gravitational perturbations on Jason-1 which, however, should have a time-dependent harmonic signature with short periodicities. Moreover, in this case the possible impact of the Lense-Thirring ‘imprint’ would be greatly reduced. According to the recently released combined CHAMP+GRACE+terrestrial gravimetry/altimetry EIGEN-CG01C Earth gravity model, the systematic error due to the remaining even zonal harmonics would amount to 0.7\% (root-sum-square calculation) and 1.6\% (upper bound) at $1-\sigma$. The possibility of getting long time series of the Jason’s node should be seriously investigated with real data tests.

- The launch of a third LAGEOS-like satellite would allow for a really robust and confident measurement of the Lense-Thirring effect. Indeed, it would be possible to combine its node-and, perhaps, also its perigee if it will be built up in such a way to sufficiently reduce the impact of the non-gravitational perturbations-with the nodes of the other currently existing LAGEOS satellites in order to cancel out $J_2, J_4$ and, perhaps, $J_6$ along with their temporal variations. The great improvements in our knowledge of the terrestrial gravitational field would allow to abandon the very tight and stringent requirements of the originally proposed butterfly configuration. However, since the LARES data should be combined with those of the existing LAGEOS satellites, the limit of the total available accuracy is mainly set by the

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impact of the non-gravitational perturbations, i.e. it is of the order of 1%. The systematic error of gravitational origin could be kept well below this value.

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