Special relativistic effects on the strength of the fluorescent Kα iron line from black hole accretion disks.

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ABSTRACT
The broad iron Kα emission line, commonly seen in the X-ray spectrum of Seyfert nuclei, is thought to originate when the inner accretion disk is illuminated by an active disk-corona. We show that relative motion between the disk and the X-ray emitting material can have an important influence on the observed equivalent width (EW) of this line via special relativistic aberration and Doppler effects. We suggest this may be relevant to understanding why the observed EW often exceeds the prediction of the standard X-ray reflection model. Several observational tests are suggested that could disentangle these special relativistic effects from iron abundance effects.

Key words: accretion, accretion disks – line:formation – galaxies: Seyfert – X-rays: galaxies

1 INTRODUCTION
Recent X-ray observations of many active galactic nuclei (AGN) have found the Kα emission line of cold iron (at 6.40 keV), and show this line to be broadened and skewed towards low-energies (Mushotzky et al. 1995; Tanaka et al. 1995; Nandra et al. 1997). The line profile is usually found to be in good agreement with that expected if it were to originate from the inner regions of an accretion disk around a black hole, with gravitational redshifts and (relativistic) Doppler shifts being the processes relevant to shaping this profile. In the cases with the best data, many alternative mechanisms for forming such a line can be examined and rejected (Fabian et al. 1995).

The iron line is believed to be a fluorescent line which results when Thomson-thick material, with a relatively low ionization state, is externally illuminated by a hard X-ray source (Basko 1978; George & Fabian 1991; Matt, Perola & Piro 1991). In the case of accreting black hole systems, the optically-thick material can be identified with a thin accretion disk whereas the external X-ray source is probably a disk-corona. The equivalent width (EW), $W$, of the iron line can be predicted given the source geometry, illuminating X-ray spectrum, and chemical abundances. In the ‘standard’ case where the cold material possesses Morrison & McCammon (1983) cosmic abundances and subtends $2\pi$ sr at the X-ray source (which is assumed to have an AGN-like spectrum), the EW is $W_{\text{mm}} = 150$ eV. For a more recent set of cosmic abundances (Anders & Grevesse 1993), this EW increases slightly to $W_{\text{ag}} = 190$ eV (Reynolds, Fabian & Inoue 1995; Matt, Fabian & Reynolds 1997), primarily due to the increased abundance of iron.

Observationally, many iron lines are significantly stronger than the predictions of the previous paragraph. Although the mean EW in the sample of Nandra et al. (1997) is only slightly in excess of $W_{\text{ag}}$, $\langle W \rangle = 230$ eV, there are many objects in this sample with very strong lines, $W \sim 300 – 600$ eV. Several possible explanations for the strength of these lines have been previously discussed. First, the iron may be over-abundant (George & Fabian 1991; Reynolds, Fabian & Inoue 1995). However, the EW grows only logarithmically with iron abundance due to the fact that iron itself contributes to its own Kα line opacity through L-shell photoelectric absorption. Thus, extreme iron overabundances ($\sim 10 Z_\odot$ or more) are required to explain the strongest lines. Secondly, the geometry might be such that the cold material subtends more than $2\pi$ sr at the X-ray source. This geometry is difficult to reconcile with a disk/corona model. Thirdly, gravitational focusing of X-ray flux from a source which is at some height above the disk plane can enhance the EW of the line (Martocchia & Matt 1996; Reynolds & Begelman 1997). However, these General Relativistic (GR) effects are only important for enhancing the emission from the innermost regions of the disk (radii $r < 3 R_{\text{Sch}}$, where $R_{\text{Sch}}$ is the Schwarzschild radius of the central black hole), which produce very redshifted line emission (with observed photon energies of $\sim 4$ keV or less). Thus, whilst GR enhancement effects might be important for understanding the broadest line known, they cannot be relevant to strong iron lines from typical objects.
In this letter, we note that any relative motion between the X-ray source and the accretion disk will also affect (and usually enhance) the EW of the iron line through the effects of special relativistic (SR) aberration and Doppler shifts. Such relative disk/corona motion will naturally occur if the accretion disk and the corona are not rigidly coupled together. For example, some authors treat the disk-corona as an independent, slim accretion disk. Due to the subsequent sub-Keplerian motion of the corona, the disk and corona will be in relative motion at any given radius. However, the corona may well be tightly coupled to the accretion disk by magnetic fields (which will force the disk and corona to accrete together). Even in this circumstance, the SR effects discussed here may be important. Field & Rogers (1993; hereafter FR93) have argued that magnetic instabilities and reconnection events in a disk corona could produce shock waves and/or the streaming of relativistic particles along the magnetic field lines. Thus, the plasma which is instantaneously responsible for the X-ray emission might well be in bulk motion relative to the disk. Such arguments gain qualitative support by drawing an analogy with the solar corona and solar flares.

For convenience, we set the speed of light to unity, \( c = 1 \), throughout this work.

2 A STATIONARY SLAB ILLUMINATED BY A MOVING SOURCE

Most of the SR effects relevant to the EW of the iron line can be studied within a scenario in which the iron line originates from a stationary slab of cold matter. Suppose we have a semi-infinite slab of cold gas filling the half-space \( z < 0 \). Let this cold slab be stationary with respect to a distant observer who views it at an inclination \( i \) with respect to the upward normal to the slab. Furthermore, suppose this cold slab is illuminated by an X-ray source which is at some distance \( h \) above the face of the slab. We make the following assumptions about the X-ray source:

(i) the source is pointlike.

(ii) the X-ray emission is isotropic in the rest-frame of the source.

(iii) the spectrum of the emission, when viewed in the rest frame, is a power-law with photon index \( \Gamma = 2 \) for all energies relevant to this discussion. This choice of photon index is observationally motivated (e.g. Nandra & Pounds 1994; Reynolds 1997).

We further assume that the source is moving at velocity \( v \) relative to the slab. We define \( \alpha \) to be the angle this velocity vector makes with the downward normal (thus \( \alpha = 0 \) corresponds to motion directly towards the slab). We also define \( \beta \) to be the azimuthal direction of the source in the slab plane relative to some reference line on the slab. We choose the projection of the observers line of sight to be this reference line. Finally, we construct a standard 2-d polar coordinate system \( (r, \phi) \) on the slab taking the point that is (instantaneous) below the source to be the origin, and using same reference direction used to define \( \beta \).

From simple vector geometry, we can calculate two other important angles. First, for a given point on the slab \( (r, \phi) \), the angle that the source velocity makes with the line joining the source with that point, \( \theta \), is given by

\[
\cos \theta = \frac{r \sin \alpha \cos \beta \cos \phi + r \sin \alpha \sin \beta \sin \phi + h \cos \alpha}{(r^2 + h^2)^{1/2}}.
\]

Secondly, the angle between the observers line of sight and the source velocity, \( \eta \), is

\[
\cos \eta = \sin \alpha \cos \beta \sin i - \cos \alpha \cos i.
\]

The question we wish to address is this: how does the relative slab-source motion influence the strength of the iron fluorescence line that will result from the illumination. There will be two relevant effects. First, the Doppler shifts and aberration of the source emission will influence both the number and average direction of primary photons that fall above the iron photoelectric threshold. This will change the absolute line photon emission rate as compared with the static source case. Secondly, the fact that the primary emission suffers relativistic aberration whereas the line emission does not will affect the observed ratio of these two emissions and, hence, the EW of the iron line. We will now discuss these two effects in turn.

2.1 Influence of the motion on the absolute line photon emission rate

There are two distinct ingredients involved in determining the absolute emission rate of fluorescent iron line photons. The most obvious one is the number of illuminating photons with energy \( E > E_{th} \), where \( E_{th} = 7.1 \text{ keV} \) is the photoelectric threshold for neutral iron. Only these incident photons can eject one of the K-shell electrons from iron and, thus, initiate the radiative cascade within the atom that results in a K\( \alpha \) line photon being emitted. For a fixed illuminating spectrum, the iron line strength is simply proportional to the normalization of that spectrum.

The second ingredient in determining the absolute iron line emission is the geometry of the illumination (Basko 1978; George & Fabian 1991). Consider normally incident photons with energy \( E > E_{th} \). On average, such photons traverse a distance corresponding to unity optical depth prior to being photoelectrically absorbed. The resulting iron fluorescence photons have to travel through at least the same optical depth of material in order to escape the slab. Some fraction of these iron line photons will be absorbed in this process (either by K-shell photoionization of low-Z metals or L-shell photoionization of iron). Now, consider photons that are incident on the slab with a large inclination (i.e. grazing collision). Again, these photons are absorbed after a unity optical depth, but this now corresponds to a significantly smaller vertical depth in the slab. Thus, the resultant iron line photons can escape significantly less impeded by subsequent absorption. The net result is that the effective fluorescent yield increases with increasing inclination (see Fig. 1 of George & Fabian 1991).

This geometrical effect only becomes important when most of the incident flux strikes the disk at a high inclination. In the scenario under discussion here, that corresponds to irradiation by sources which are rapidly moving in a direction parallel to the disk plane. Given this fact, and that analytical descriptions of the geometrical dependence are somewhat cumbersome (and approximate; e.g. Basko 1978),
we shall ignore this effect and study just the Doppler-shifting phenomenon.

With this restriction, and the assumption of a power-law primary spectrum, the problem simply amounts to determining how the normalization of the illuminating spectrum at some given energy \( E = E_{th} \), say), integrated over the surface of the slab, is affected by the source motion. Making use of the phase space invariant \( I_r / \nu^3 \), we see that the SR enhancement in absolute iron line production is

\[
\Upsilon_{\text{abs}} = \frac{\int \int F(r) \delta(\theta)^{(2+\Gamma)} r \, dr \, d\phi}{\int \int F(r) \, r \, dr \, d\phi},
\]

where \( F(r) \) is the illuminating flux striking a unit area of the slab in the absence of any relativistic effects,

\[
F(r) \propto \frac{h}{r^2 + h^2}^{3/2},
\]

and \( \delta(\theta) \) is the beaming parameter,

\[
\delta(\theta) = \frac{(1-v^2)^{1/2}}{(1-v \cos \theta)}.
\]

Simple expressions can be obtained for \( \Upsilon_{\text{abs}} \) in three cases:

(i) source motion directly towards the slab \( (\alpha = 0) \):

\[
\Upsilon_{\text{abs}} = \frac{(1-v^2)(1+v)^2}{1-v^2}.
\]

(ii) source motion parallel to the slab \( (\alpha = \pi/2) \):

\[
\Upsilon_{\text{abs}} = \frac{1 + \frac{1}{3}v^2}{1-v^2}.
\]

(iii) source motion directly away from the slab \( (\alpha = \pi) \):

\[
\Upsilon_{\text{abs}} = \frac{(1+v)(1-v^2)^2}{1-v^2}.
\]

In Fig. 1, we plot \( \Upsilon_{\text{abs}}(v) \) for these cases. As intuitively expected, the former two cases enhance the absolute iron emission. For motion along the slab normal \( (\alpha = 0, \pi) \), the beaming can influence the absolute line production by a factor of two for velocities that are only mildly relativistic \( (v \sim 0.4) \).

2.2 Differential beaming and the equivalent width of the line

The fact that the primary emission is beamed whereas the fluorescent emission is not beamed has direct consequences for the observed EW of the line. The relevant quantity is the ratio of the iron line flux to the normalization of the observed primary continuum at the iron line energy. Due to the effects of relativistic beaming, this primary flux normalization is proportional to \( \delta(\eta)^{2+\Gamma} \) where, to recap, \( \eta \) is the angle between the source motion and the observers line of sight. Noting that \( \Gamma = 2 \), the equivalent width of the iron line is given by

\[
\frac{W(v)}{W(0)} = \frac{\Upsilon_{\text{abs}} [1 - v (\sin \alpha \cos \beta \sin i - \cos \alpha \cos i)]^4}{(1 - v^2)^2}.
\]

Figure 2a shows the behaviour of \( W(v) \) for sources moving directly towards the slab \( (\alpha = 0) \) for various inclinations of the observer \( i \). It can be seen that the SR beaming has a major effect on the EW of the iron line for relatively small velocities, especially at low inclinations. The EWs seen in Seyfert 1 nuclei (which have typical inclinations of \( i \sim 30^\circ \)) are often 2–3 times more than predicted by the standard model (see Introduction). If all of this enhancement was due to SR effects (rather than iron overabundance), we would require downwards motion at \( v \sim 0.1 - 0.2 \). By contrast, Fig. 2b shows the behaviour of \( W(v) \) for sources moving parallel to the plane of the slab \( (\alpha = \pi/2) \). In plotting these curves, it is assumed that there is an ensemble of such sources with velocity vectors that are isotropically distributed in that plane (i.e. we average over \( \beta \)). Once this average is performed, the resulting expression for the equivalent width is

\[
\frac{W(v)}{W(0)} = \frac{(1 + \frac{1}{3}v^2)(1 - v^2 \sin^2 i)^{7/2}}{(1 + v)(1-v^3)(1 + \frac{1}{3}v^2 \sin^2 i)}.
\]

It is seen from Fig. 2b that much higher source velocities are required to produce a given enhancement in the iron line. To explain Seyfert 1 spectra would require \( v \sim 0.5 \).

3 DISCUSSION AND CONCLUSIONS

Using a stationary slab model, we have shown that relative disk/source motion can significantly affect (and usually enhance) the EW of the fluorescent iron Kα line. This mechanism might be responsible for producing the strong iron lines seen in many AGN. Viable alternative explanations require a rather large iron overabundance, or invoke some geometry in which the disk subtends more than \( 2\pi \) sr as seen by the X-ray source.

As is intuitively clear, the SR enhancement of the line
Figure 2. The enhancement in the iron line equivalent width $W(v)/W(0)$ for (a) downwards moving sources with velocity $v$, and (b) an ensemble of sources moving with speed $v$ (isotropically) parallel to the slab plane. Four different inclinations are shown: $i = 0^\circ$ (solid line), $i = 30^\circ$ (dashed line), $i = 60^\circ$ (dotted line), and $i = 90^\circ$ (dot-dashed line).

EW is especially important if the X-ray emitting material is moving directly towards the disk. This is exactly the flow pattern envisaged in the magnetized accretion disk model of FR93. By analogy with solar flares, they argue that the corona is magnetically confined by loops of magnetic field which have footpoints in the accretion disk. They suggest two processes that might lead to significant material motion. Firstly, if the footpoints of the loops are forced (by motions in the disk) into a configuration whose topology permits a lower energy magnetic field structure, the coronal loop structure may become unstable, leading to the release of magnetic field energy into bulk kinetic energy of the plasma. Coronal shock waves will result. Secondly, the disk motion may force loops of opposite polarity together, thereby resulting in magnetic reconnection. This would also channel a substantial amount of energy into particle energy. Since these phenomena are likely to occur predominately near the tops of the coronal loops (Field & Rogers 1992), the streaming of accelerated particles along the field lines will result in downwards motion of X-ray emitting material. FR93 use the subsequent beaming of the emission to channel much of the coronal energy back into the accretion disk (which is eventually lost as optical/UV thermal emission from the disk surface).

In the very near future, observations will begin to address the origin of the strong iron lines. Very broad band X-ray observations (with RXTE or Beppo-SAX, for example) will allow the iron line, iron edge and Compton backscattered continuum to be simultaneously constrained. If the iron lines are strong due to an iron-overabundance, this will be revealed as an enhancement of the line relative to the Compton backscattered continuum. On the other hand, if observations show both the iron line and backscattered continuum to be enhanced above the predictions of the ‘standard’ model, then we must either consider a geometry in which the disk subtends more than $2\pi$ sr at the X-ray source, or invoke (special or general) relativistic effects to enhance the overall X-ray reflection. It is interesting to note that preliminary results for the Seyfert 1 galaxy MCG−6-30-15 indicate that the backscattered continuum may somewhat enhanced above the prediction of the standard model (Molendi et al. 1997), although a self-consistent analysis, which includes the effects of iron abundance on the shape of the backscattered continuum, still has to be performed.

If it is confirmed that an overall enhancement of the X-ray reflection is required, the inclination dependence of the iron line properties will be important for disentangling the enhancement mechanisms. As shown in this work, if source motion is responsible for the enhancement, the EW of the line will decrease significantly as one considers sources at higher inclination (note that the inclination of the disk can be measured robustly from the iron line profile). This effect will be much stronger than the inclination dependence of the line just based on limb-darkening (George & Fabian 1991). A careful analysis of existing ASCA datasets might allow such a trend to be addressed.

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