Local Axion Cosmic Strings from Superstrings

Stephen C. Davis\textsuperscript{a}, Pierre Binétruy\textsuperscript{b,c}\textsuperscript{†} and Anne-Christine Davis\textsuperscript{d}\textsuperscript{‡}

\textsuperscript{a}Institute of Theoretical Physics, École Polytechnique Fédérale de Lausanne, CH–1015 Lausanne, Switzerland
\textsuperscript{b}LPT, Université de Paris-Sud, Bât. 210, 91405 Orsay Cedex, France
\textsuperscript{c}APC, Université Paris VII, 2 place Jussieu, 75251 Cedex 05, France
\textsuperscript{d}Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Cambridge, CB3 0WA, UK

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Abstract

Axionic cosmic string solutions are investigated in a superstring motivated model with a pseudo-anomalous $U(1)$ gauge symmetry. The inclusion of a gauge field and spatially varying dilaton allow local defect solutions with finite energy per unit length to be found. Fermion zero modes (whose presence is implied by supersymmetry) are also analysed. The corresponding fermion currents suggest strong cosmological bounds on the model. It is shown that the unusual form of the axion strings weakens these bounds. Other cosmological constraints on the underlying theory are also discussed.

1 Introduction

Spontaneous breaking of a pseudo-anomalous $U(1)$ gauge symmetry occurs in a large class of superstring compactifications \cite{1}. The $U(1)$ symmetry arises generically as a remnant of the Green-Schwarz \cite{2} mechanism of anomaly cancellation in the underlying ten-dimensional supergravity.
It is well known that line-like topological defects form during an ordinary $U(1)$ symmetry breaking. Such defects are called cosmic strings, and have many interesting cosmological properties [3]. Similar defects can also form if the symmetry is pseudo-anomalous. These strings are expected to be global, because of their coupling to the axion [4]. Furthermore, it seems that a cutoff must be introduced inside the string core in order to give a finite energy per unit length. It has been shown that by coupling the axion to a gauge field, local axion strings can be obtained (although the cutoff inside the core is still needed) [5]. The coupling of the dilaton-axion field to the gauge fields plays a central role in the Green-Schwarz mechanism, so local axion strings are natural in this context.

Recent work on the formation of topological defects in superstring theory has led to a renewed interest in cosmic strings [6]. In contrast to other work, we will just be considering a four-dimensional model, although we expect our results to be of general relevance.

In this paper we discuss axion string solutions which do not require a cutoff in the core. In a supersymmetric theory, the dilaton forms part of the same supermultiplet as the axion. By allowing the dilaton to vary inside the string, finite energy solutions can be found. The axion field varies (in a topologically non-trivial way) around the string, and so plays the role of the phase of the Higgs field in a conventional $U(1)$ cosmic string model. In a similar way the dilaton plays the role of the magnitude of the Higgs field. This type of solution relies on particular couplings between the axion and dilaton fields, although they are completely natural in a supersymmetric (or superstring) context.

Although we have referred to the dilaton in the above discussion, the same ideas apply for pseudo-anomalous $U(1)$ breaking with an axion-moduli superfield. Similarly the analysis and results of this paper will also apply to moduli, although the corresponding couplings and mass scales coming from the underlying theory will be different.

We will investigate local, axion strings in a superstring motivated, supersymmetric model. A simplified version of a ‘racetrack’ potential [7] will be used to stabilise the dilaton. Such a potential could arise from gaugino condensation. It is quite natural for its energy scale to be far lower than that associated with supergravity, perhaps even as low as the QCD scale. For simplicity we model these lower energy effects with two effective scalar fields. The model is outlined in section 2 and the cosmic string solutions themselves are discussed in section 3.

A common feature of supersymmetric cosmic string models is the existence of conserved, massless fermion currents which flow along the strings [8]. Like the strings themselves, these have interesting cosmological implications [9]. The massless fermion bound states of our solutions are examined in section 4. The cosmological evolution of a network of such strings is discussed in section 5. The bounds on the underlying theory coming from the cosmological properties of the axion strings are derived in section 6. The possibility of cosmic string loops which are stabilised by the fermion currents generally gives rise to particularly strong bounds. However, as we will show, these bounds are significantly weakened for our string solutions.

In section 7 we discuss an apparent analytic simplification of the field equations, and in section 8 we summarise our results.
2 Axion cosmic string model

We will consider a supersymmetric model with a pseudo-anomalous $U(1)$ gauge symmetry. Working in units with $m_{\text{pl}} = 1$, the Lagrangian is

$$\hat{\mathcal{L}} = \left(\Phi_i^* e^{2\eta_i V} \Phi_i + \mathcal{K}\right) + \left(\frac{1}{4} SW^a W_a + W(\Phi_i, S)\right) \delta(\bar{\theta}^2) + (\text{h. c.}) \ . \quad (1)$$

$S(s, 2s_s\chi^a, F_s)$ is the axion-dilaton superfield, $V(A_\mu, s_s^{-1/2} \lambda_\alpha, D)$ is the gauge vector superfield, and $\Phi_i(\phi_i, \psi_{i\alpha}, F_i)$ are chiral superfields with charges $q_i$. The kinetic term for the superfield $S$ is described by the modified Kähler function $\mathcal{K} = -\log(S + \bar{S} - 4\delta_{ GS} V)$, where $\delta_{ GS}$ is the Green-Schwarz parameter $[1]$. If the dilaton is to have a finite vacuum expectation value, a non-trivial superpotential ($W$) is required. We will consider a racetrack [7] style model with the effective superpotential

$$W = \Phi_0 \left\{ h_1(\Phi/\eta)^{n_1} e^{-3S/(2b_1)} - h_2(\Phi/\eta)^{n_2} e^{-3S/(2b_2)} \right\} \quad (2)$$

where $b_i = 3N_i/(16\pi^2)$. The integers $N_i$ are determined by the symmetry breaking of the original theory. The above superpotential resembles that arising from an $E_8 \to SU(N_1) \times SU(N_2)$ breaking, in which case $N_1 + N_2 \leq 10$.

The effective superfields, $\Phi$ and $\Phi_0$, represent superstring and gaugino condensation effects. We take $q_0 = -1$ and $q_0 < 0$. Gauge invariance implies that $q_0 = n_i - 3\delta_{ GS}/b_i$, and so for our model we require

$$\delta_{ GS} = \frac{N_1 N_2 (n_1 - n_2)}{16\pi^2 (N_2 - N_1)} \ . \quad (3)$$

Typically we expect $\delta_{ GS} \sim 1/10$ [1], but we will consider more general values.

The Lagrangian [1] can be expanded in terms of the component fields. The auxiliary fields can then be eliminated using their equations of motion. Working in Wess-Zumino gauge, the bosonic part of the Lagrangian is

$$\mathcal{L}_B = \frac{1}{4s^2} \partial_\mu s \partial^\mu s + \frac{1}{4s^2} (\partial_\mu a - 2\delta_{ GS} A_\mu)^2 - \frac{s}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a}{4} F^{\mu\nu} \tilde{F}_{\mu\nu} + |D_\mu \phi|^2 + |D_\mu \phi_0|^2 - |F_0|^2 - |F|^2 - \frac{1}{4s^2} |F_s|^2 - \frac{s}{2} D^2 \quad (4)$$

where $s = s_s + ia$, $D_\mu \phi = (\partial_\mu - iA_\mu)\phi$, and $D_\mu \phi_0 = (\partial_\mu + iq_0 A_\mu)\phi_0$. The real part of the dilaton defines the gauge coupling $1/g^2 = s_r$. The auxiliary fields are given by

$$F_0^\dagger = -h_1(\phi/\eta)^{n_1} e^{-3S/(2b_1)} + h_2(\phi/\eta)^{n_2} e^{-3S/(2b_2)} \quad (5)$$

$$D = -\frac{1}{s_r} \left( q_0 |\phi_0|^2 - |\phi|^2 + \delta_{ GS} \right) \ . \quad (6)$$

$F$ and $F_s$ are both proportional to $\phi_0$. The scalar potential is minimised by $s_r = \infty$ or

$$\phi_0 = 0 \ , \ |\phi| = \eta \ , \ s_r = \frac{1}{g_0^2} = \frac{\delta_{ GS}}{\eta^2} \ , \quad (7)$$
where \( g_0 \) is the vacuum value of the coupling \( g \). The superpotential is related to \( g_0 \) by

\[
\frac{2\delta_{\text{GS}}}{n_1 - n_2} \ln \frac{h_1}{h_2} = \frac{1}{g_0^2}.
\]

We see that the model has two distinct mass scales. One,

\[
m_D = \frac{\sqrt{2\eta^2}}{\sqrt{\delta_{\text{GS}}}} \sqrt{1 + \eta^2} = \sqrt{2\delta_{\text{GS}} g_0^2} \sqrt{1 + \eta^2},
\]

arises from the \( D \) term, and because of supersymmetry is also the mass of the gauge field \( A_\mu \). If \( \delta_{\text{GS}} \sim 1/10 \) and \( g_0 \sim 1 \), as suggested by string theory, we will have \( m_D \approx 1/\sqrt{5} \). The other mass scale,

\[
m_F = |n_2 - n_1| \frac{|h_1|}{\eta^2} \exp \left( -\frac{3\delta_{\text{GS}}}{4b_1 \eta^2} \right) \sqrt{1 + \eta^2}
\]

\[
= 16\pi^2 \frac{|h_1|}{g_0^2} \frac{|N_1 - N_2|}{N_1 N_2} \exp \left( -\frac{8\pi^2}{N_1 g_0^2} \right) \sqrt{1 + \eta^2},
\]

arises from the \( F_0 \) term, and will generally be far smaller. If \( \eta \) is small, the fields \( s \) and \( \phi \) are approximate mass eigenstates. Their masses are respectively \( m_F \) and \( m_D \).

### 3 Cosmic Strings

We will look for cosmic string solutions of the form

\[
\phi = \eta f(r) e^{i\varphi}
\]

\[
A_\varphi = n \frac{v(r)}{r}
\]

\[
s = \frac{\delta_{\text{GS}}}{\eta^2 \gamma(r)^2} + 2i\eta \delta_{\text{GS}} \varphi
\]

\[
\phi_0 = 0.
\]

Note that \( g_0 n \) must be an integer so that \( F_0 \) is single valued. As \( r \) approaches infinity \( f, \gamma \) and \( v \) all tend to 1. The boundary conditions at \( r = 0 \) are \( \gamma = f = v = 0 \). In contrast to previous work [4, 5], we allow spatial variations of the dilaton.

The equations of motion for the various fields are

\[
\partial^\mu \left( \frac{1}{s_R} \partial_\mu s_R \right) + \frac{1}{s_R^2} (\partial_\mu a - 2\delta_{\text{GS}} A_\mu)^2 + \frac{s_R}{2} F^{\mu\nu} F_{\mu\nu} + 4s_R F_0 \frac{\partial F_0^*}{\partial s^*} \frac{1}{s_R} \left( |\phi|^2 \delta_{\text{GS}} s_R \right) \left( |\phi|^2 - 3 \delta_{\text{GS}} s_R \right) = 0
\]

\[
\mathcal{D}^\mu \mathcal{D}_\nu \phi + F_0 \frac{\partial F_0^*}{\partial \phi^*} + \frac{1}{s_R} \left( |\phi|^2 - \frac{\delta_{\text{GS}} s_R}{\delta_{\text{GS}} s_R} \right) \phi = 0
\]
The equations for the string profile function are found by substituting the string ansatz (11–14) into the above field equations. It is useful to introduce \( \rho = r \eta^2 \sqrt{2/\delta_{GS}} \). The equations (15–17) for the string solution are then

\[
\begin{align*}
\frac{1}{\rho} \left( \frac{\rho \gamma'}{\gamma} \right)' - 2 n^2 \eta^4 \left( 1 - v \right)^2 \gamma^4 + n^2 \eta^2 \gamma^2 \left( \frac{v'}{\gamma^2 \rho} \right)^2 - \frac{\eta^2}{4} (f^2 - \gamma^2)(f^2 - 3\gamma^2) \gamma^2 \\
- \frac{2\eta^4}{\gamma^2} \left[ \tilde{h}_1 f^{n_1} e^{-1/\tilde{b}_1 \gamma^2} - \tilde{h}_2 f^{n_2} e^{-1/\tilde{b}_2 \gamma^2} \right] \left[ \tilde{h}_1 \tilde{b}_1^{-1} f^{n_1} e^{-1/\tilde{b}_1 \gamma^2} - \tilde{h}_2 \tilde{b}_2^{-1} f^{n_2} e^{-1/\tilde{b}_2 \gamma^2} \right] = 0
\end{align*}
\] (18)

\[
\begin{align*}
\frac{1}{\rho} \left( \rho f' \right)' - n^2 \left( 1 - v \right)^2 f - \frac{\gamma^2}{2} (f^2 - \gamma^2) f \\
- \eta^2 \left[ \tilde{h}_1 f^{n_1} e^{-1/\tilde{b}_1 \gamma^2} - \tilde{h}_2 f^{n_2} e^{-1/\tilde{b}_2 \gamma^2} \right] \left[ \tilde{h}_1 n_1 f^{n_1 - 1} e^{-1/\tilde{b}_1 \gamma^2} - \tilde{h}_2 n_2 f^{n_2 - 1} e^{-1/\tilde{b}_2 \gamma^2} \right] = 0
\end{align*}
\] (19)

\[
\left( \frac{v'}{\gamma^2 \rho} \right)' + \frac{1 - v}{\rho} \left( f^2 + \eta^2 \gamma^4 \right) = 0
\] (20)

where \( \tilde{h}_i = h_i \eta^{-4} \sqrt{\delta_{GS}/2} \), \( \tilde{b}_i = 2 b_i \eta^2 / (3 \delta_{GS}) \). Hence \( m_F/m_D = (n_1 - n_2) \tilde{h}_i e^{-1/\tilde{b}_i} \).

There are two distinct length scales for this type of string solution. The string has an inner core of radius \( r_D \sim m_D^{-1} (\rho_D \sim 1) \) in which \( v < 1 \) and \( f \neq \gamma \), so \( D \neq 0 \). Around that region there is an outer core in which \( v \approx 1 \) and \( f \approx \gamma \) but \( f, \gamma < 1 \) so \( F_0 \neq 0 \). This region

![Figure 1: Plot of string profile functions](image-url)
is of radius \( r_F \sim m_F^{-1} \), which can be far greater then \( r_D \) even for moderate values of the parameters.

A numerical solution of (18–20) for \( n = 1 \) is shown in figure 1. The parameters used were \( n_1 = 1, n_2 = 0, g_0 = 1, m_F/m_D = 10^{-5} \) and \( b_i = 3N_i/(16\pi^2) \) with \( N_1 = 4, N_2 = 6 \) (as suggested by the symmetry breaking discussed in section 2). The remaining parameters are then \( g_0 = -2, \delta_{GS} = \eta^2 = 3/(4\pi^2), h_1 \approx 111 \) and \( h_2 \approx 0.154 \).

It is useful to have an approximate analytic solution to the field equations (18–20). Inside the inner core (\( \rho < 1 \)) of the string the kinetic terms dominate the field equations. At \( \rho = 1 \) we take \( f = \gamma \) and \( v = 1 \). Assuming \(|n|\eta^2\) is small, the approximate solution for \( \rho < 1 \) is

\[
\gamma \approx \frac{A}{\sqrt{1 - 2|n|\eta^2A^2\log \rho}} \tag{21}
\]

\[
f \approx A\rho^n \tag{22}
\]

\[
v \approx \frac{\rho^2}{1 - 2|n|\eta^2A^2\log \rho}. \tag{23}
\]

If \( m_D \gg m_F \) then an approximate solution for the outer core is also needed. Here \( D \approx 0 \) so \( f \approx \gamma \). The potential terms in the field equations can still be neglected, and \( v \approx 1 \) so some kinetic terms can also be dropped. Solving eq. (18) and imposing continuity of \( \gamma \) and \( \gamma' \) at \( \rho = 1 \) implies

\[
f \approx \gamma \approx A\rho^{2|n|\eta^2} \tag{24}
\]

and \( v = 1 \) for \( 1 < \rho < m_D/m_F \). For \( \rho > \rho_F \) we take \( f = \gamma = v = 1 \). The constant \( A \) is determined by continuity at \( \rho = m_D/m_F \). It satisfies \((- \log A)/A^2 = |n|\eta^2\log(m_D/m_F)\). For small \(|n|\eta^2\log(m_D/m_F)\) this implies \( A \approx 1 - |n|\eta^2\log(m_D/m_F)\).

If \( m_D \sim m_F \), there is no outer core. \( |\phi| \) and \( s_R \) take their vacuum expectation values for \( \rho > 1 \), and \( A = 1 \).

In all cases the dilaton is thus \( s_R \approx -2|n|\delta_{GS} \log \rho + \text{const.} \) for small \( \rho \). Substituting this into the Lagrangian (4) we see that \( \mathcal{L} \sim 1/(r \log r)^2 \) inside the string core. Thus the string energy, which is proportional to \( \int \mathcal{L} r \, dr \), is finite for this type of solution. If we had taken \( s_R = \text{const.} \) then \( \mathcal{L} \sim 1/r^2 \) and the energy would have diverged logarithmically at the string core. We see that our solution is a local string with finite energy, in contrast the usual axion strings which are global.

The energy per unit length of the string is equal to

\[
\mu = 2\pi\eta^2 \int \rho d\rho \left\{ \frac{\gamma^2}{\eta^2\gamma^2} + f^2 + n^2 \left( 1 - \frac{v}{\rho^2} \right) (f^2 + \eta^2\gamma^4) + \frac{v^2}{\gamma^2}\rho^2 + \frac{1}{4} (f^2 - \gamma^2)^2 \gamma^2 + \eta^2 \left( \tilde{h}_1 f_{n_1} e^{-1/(\tilde{b}_1 \gamma^2)} - \tilde{h}_2 f_{n_2} e^{-1/(\tilde{b}_2 \gamma^2)} \right) \right\}. \tag{25}
\]

Substituting in the above approximate solution, we find that for small \( \eta \),

\[
\mu = 4\pi n^2 \eta^2 \left( 2a_n + |n|\eta^2 b_n \log \frac{m_D}{m_F} \right) + O(\eta^4) \tag{26}
\]

with \( a_n, b_n \sim 1 \).
4 Fermion Zero Modes

We will now consider the fermionic sector of the theory. The non-zero terms in the fermionic part of the Lagrangian are

\[
\mathcal{L}_F = -i\bar{\chi}\sigma^\mu \left[ \partial_\mu - i\frac{\delta_{GS}}{s_R} a - 2\delta_{GS} A_\mu \right] \chi - i\bar{\lambda}\sigma^\mu \left[ \partial_\mu + i\frac{\delta_{GS}}{2s_R} a \right] \lambda - i\bar{\psi}\sigma^\mu D_\mu \psi
\]

\[
- i\bar{\psi}_0\sigma^\mu D_\mu \psi_0 + \frac{i}{\sqrt{2}s_R} \left( 3\frac{\delta_{GS}}{s_R} - |\phi|^2 \right) (\bar{\lambda}\chi - \lambda\bar{\chi}) + \frac{s_R}{2\sqrt{2}} (\bar{\chi}\sigma^\nu \sigma^\mu \bar{\lambda} + \lambda\sigma^\mu \sigma^\nu \chi) F_{\mu\nu}
\]

\[
- i\frac{\sqrt{2}}{\sqrt{s_R}} (\bar{\phi}\lambda\psi - \phi\lambda\bar{\psi}) + \frac{3}{2}\chi^2 \bar{\chi}^2 - \left[ \frac{\partial^2 W}{\partial\phi_0 \partial\phi} \psi\psi_0 + 2s_R \frac{\partial^2 W}{\partial\phi_0 \partial s} \chi\psi_0 + (\text{h. c.}) \right].
\]

A common feature of cosmic string models is the existence of fermion bound states which are confined to the string core. These occur in models in which fermion fields gain their masses from the cosmic string Higgs bosons. This is the case for all supersymmetric cosmic string models, including the one considered in this paper.

Of particular interest are zero energy fermion bound states, or ‘zero modes’. Excitations of these are massless particles, with the unusual property that all excitations of a given zero mode move in the same direction along the string. The resulting currents can radically alter the model’s cosmology.

In supersymmetric models some of the zero modes can be found by applying a supersymmetry transformation to the string solution [8]. Applying the transformation (52–54), with parameter \( \xi_\alpha = i\sqrt{\delta_{GS}}/(2\eta^2)\xi_\alpha \), to the string solution (11–14), and using a gauge...
transformation to return to Wess-Zumino gauge gives

\[ \delta \psi_\alpha = i \eta \left( \frac{\tilde{h}_2 f_{n2}}{e^{1/(b_2 \gamma^2)}} - \frac{\tilde{h}_1 f_{n1}}{e^{1/(b_1 \gamma^2)}} \right) e^{-i n q_0 \varphi} \epsilon_\alpha \]  

\[ \delta \psi_\alpha = - \left( f' \pm n \frac{1 - v}{\rho} \right) e^{i (n \pm 1) \varphi} \epsilon_\alpha^* \]  

\[ \delta \lambda_\alpha = \left[ \frac{\gamma}{2} (\gamma^2 - f^2) \pm n \frac{v'}{\gamma \rho} \right] \epsilon_\alpha \]  

\[ \delta \chi_\alpha = \left( \frac{\gamma'}{\gamma \eta} \mp n \eta \gamma^2 \frac{1 - v}{\rho} \right) e^{\mp i \varphi} \epsilon_\alpha^* \]  

where the upper (lower) signs corresponds to the index \( \alpha = 1 \) (2). Since (28–31) are non-zero, supersymmetry is completely broken inside the string core. The above expressions must be normalisable if they are to correspond to physical states. At large \( \rho \) they all decay exponentially, so there is no problem there. Using (22–23) we see that (28–30) are all well behaved as \( \rho \to 0 \). The small \( \rho \) behaviour of \( \chi_\alpha \) is less obvious. Solving (18) for small \( \rho \) we find

\[ \left( \frac{\gamma'}{\gamma \eta} \mp n \eta \gamma^2 \frac{1}{\rho} \right) = \frac{1 \mp \text{sgn} n}{-\eta \rho \log \rho} + O(\rho) \]  

so \( \chi_1 (\chi_2) \) is well-behaved as \( \rho \to 0 \) if \( n > 0 \) (\( n < 0 \)). Thus only one of the transformations (\( \xi_1 \neq 0 \) if \( n > 0 \)) gives a normalisable fermion zero mode solution. A plot of this solution is shown in figure 2. Excitations of it are massless currents, which only flow in one direction along the string. The direction is determined by the sign of \( n \). While the other transformation also gives a solution of the fermion field equations, it is unphysical since it is not single valued as \( \rho \to 0 \).

The total number of zero modes can be determined by considering solutions to the fermion field equations which are normalisable at large and small \( \rho \). By trying to match up these solutions it is possible to find a lower bound for the total number of physical zero mode solutions. This was done for a general cosmic string solution in ref. [10]. There it was assumed that all fermion fields had power law behaviour near the centre of the string. The \( s_r^{-1} \partial_\mu a \sim 1/(r \log r) \) terms in (27) produce extra logarithmic factors in the small \( r \) solutions, but these do not affect their normalisability, so the expression derived in ref. [10] still applies. We find (for \( q_0 < 0 \)) there are \( |n|(1 - q_0) \) zero modes, all of which are left (right) movers if \( n > 0 \) (\( n < 0 \)).

### 5 String Network Evolution

By considering the cosmological properties of the strings, we can obtain constraints on the underlying theory. Before doing this, we need to consider the evolution of a network of our cosmic strings, which may be non-standard. For simplicity we will assume that \( g_0 \sim 1 \). This implies that the energy per unit length of the string \( \mu \sim m_D^2 \), and that the only other mass scales in the model are \( m_{\text{Pl}} \) and \( m_F \).
When discussing the evolution of a cosmic string network, it is useful to introduce the correlation length of the network, $\xi$. This gives the typical length scale of the network. For example, in a volume $\xi^3$ there will typically be length $\xi$ of string.

At the phase transition ($T = T_c$) where the strings form, $\xi \sim 1/T_c$. Our strings are essentially formed when the $F$-terms appear in the potential, at $T = T_c \sim m_F$. This can be far less than the energy scale of the strings, which is an unusual feature of our model.

After a brief period of acceleration, the network’s evolution is dominated by the frictional force coming from its interaction with the plasma. During this friction domination

$$\xi \sim \frac{m_{pl}^{1/2} m_D}{g_s^{1/4} T_5^{5/2}} \sim g_s^{1/4} t \left( \frac{t}{t_*} \right)^{1/4}$$

(33)

(we have used $t \sim m_{pl}/(\sqrt{g_s} T^2)$). This lasts until temperature

$$T_* \sim \frac{m_D^2}{m_{pl}}$$

(34)
after which the expansion of the universe (which simply stretches the strings) will be more significant than the effects of friction. For our strings, if

$$\frac{m_F}{m_D} \lesssim \frac{m_D}{m_{pl}}$$

(35)

then the friction domination regime will be absent.

For $T < T_*$, the stretching of the strings would soon result in them dominating the energy density of the universe. This is clearly not true for our universe, and so some mechanism is needed for the network to lose energy. In fact, as the strings evolve, loops of string break off from the network. Typically these decay by radiating gravitons. As a result of this energy loss the strings do not dominate the universe and instead the network approaches a scaling solution, where its energy density scales like matter. The strings move with relativistic velocities and $\xi = \gamma t$ with $\gamma \sim 1$. The behaviour of the network in this scaling regime is practically independent of the network’s earlier evolution. We therefore expect this part of the evolution to be standard for our strings.

If the friction regime is absent or very short, then the string density at early times will be much higher than that of the scaling regime. In order for the network to reach a scaling solution, it must undergo a period of intense loop production. Once the strings have reached relativistic speeds (which we assume happens rapidly) we find that during this period the correlation length is given by

$$\xi \approx \gamma t + \xi_c \left( 1 - \frac{t_c}{\xi_c} \right) \left( \frac{t}{t_c} \right)^{3/4}$$

(36)

which approaches the scaling solution as $t$ increases.

Most of the cosmological constraints on our model arise from loop production by the string network. We will take the typical size of a loop produced by an evolving network to
be $\ell \sim \alpha \xi$. The rate at which loops are produced is [3] [15]

$$
\frac{dn_{\text{loop}}}{dt} \sim \frac{1}{\xi^3 \ell} \sim \frac{1}{\alpha \xi^4} \frac{d\xi}{dt}.
$$

(37)

The parameter $\alpha$ depends on the behaviour of the network. In the friction domination regime $\alpha \sim 1$. For strings moving with relativistic velocities (as in the scaling regime), the value of $\alpha$ is unclear. It is bounded below by $\alpha \gtrsim \Gamma G\mu$, where the factor $\Gamma$ is related to how length scales in the network evolve with time. Numerical simulations suggest $\Gamma \sim 65$. Simulations of conventional strings indicate $\alpha \sim 10^{-3}$, so we expect large numbers of small loops will be produced during the scaling regime.

Clearly the scaling regime will not be achieved if the string loops do not decay. In the previous section we have seen that there are fermion zero modes in the core of our strings. Including the $z$ and $t$ dependence of the zero modes results in the string carrying a conserved current [9]. Sufficiently large currents will allow stable string loops, called ‘vortons’, to exist. If these do not decay the string energy density will come to dominate the universe after all. This is generically a potential problem for all current carrying strings. If the zero modes move in both directions the net current will be smaller than the total number of particles, weakening the bounds on the model. However in our model we have either left or right movers, but not both. Thus the net current is maximal. Vorton bounds can sometimes be avoided by using supersymmetry breaking to change the fermion spectrum. However, since our zero modes are chiral, they will survive this [11], as they are unable to mix with other modes and become bound states.

Not all loops will go on to become vortons. Only those with a sufficiently large number of fermion charge carriers will survive. In the friction dominated regime, the majority of the loops produced when $T \sim m_F$ satisfy this requirement. Calculating the typical loop length and corresponding current allows the vorton density to be estimated [12]

$$
\frac{\rho_v}{g_\ast T^3} \sim \epsilon \frac{m_F^3}{\sqrt{g_\ast F m_p m_D}}
$$

(38)

where $g_\ast F = g_\ast (T = m_F)$. For our model, anything in the range $10 \lesssim g_\ast F \lesssim 10^3$ is possible. The parameter $\epsilon$ measures the efficiency with which a network produces potential vortons. It will be in the range $0 < \epsilon \lesssim 1$.

If $m_F < T_*$ then our strings will form in the scaling regime, and the above expression [38] will not apply. During the scaling regime, far fewer loops will possess enough charge carriers for them to form vortons. The number density of vortons will therefore be smaller, as will the typical current per vorton. Taking these issues into account gives a smaller expression for the vorton density [12]

$$
\frac{\rho_v}{g_\ast T^3} \sim \epsilon \alpha^{3/2} \frac{m_F^2 m_{Pl}^2}{\Gamma^2 m_D^3} \left( \frac{\Gamma m_D}{\sqrt{g_\ast F m_p m_D}} \right)^{\epsilon/(3-\epsilon)}
$$

(39)

The smaller value of $\rho_v$ will weaken any vorton-related constraints on our model.
The unusual form of the axion strings in this model can reduce the vorton density even further. Typically the radius of a stable loop will be of order $10^2 m_D^{-1}$. If $m_F/m_D \lesssim 10^{-2}$ the outer cores of each side of a potential vorton will overlap. We expect there to be significant interaction between the charge carriers on the opposite sides of the loop, allowing the current to decay. It is then energetically favourable for the loop to decay, releasing the parent particles as radiation. This will avoid any constraints on the model coming from vortons. However other constraints relating to particle production will apply instead.

6 Cosmological Constraints

6.1 Dark matter bounds

The existence of conserved currents on cosmic strings has profound effects on their cosmology. As we discussed in the previous section, loops of string can be stabilised by the angular momentum of the charge carriers, producing vortons. If large numbers of such vortons are produced they can, for example, overclose the Universe, or interfere with nucleosynthesis. Thus, current-carrying strings have a vorton problem, similar to the monopole problem of grand unified theories. In ref. [13] it was shown that vortons place very stringent constraints on the underlying particle physics theory. Indeed, the most stringent constraints arise for chiral strings (like those in our model) since the current is maximal in this case [12].

For our model to be consistent with the observed light element abundances, the universe must be radiation dominated at nucleosynthesis. This implies $\rho_v \ll \rho_{\text{DM}}$ when $T = T_N \sim 10^{-4}\text{GeV}$.

If vortons form during a period of friction domination then $\rho_v$ is given by eq. (38) and (taking $\epsilon \sim 1$) $\rho_v \ll g_* T_N^4$ implies

$$g_*^{-1/5} \left( \frac{m_F}{T_*} \right)^{6/5} G \mu \ll \left( \frac{T_N}{m_{Pl}} \right)^{2/5} \sim 10^{-9} .$$

(40)

Since the above expression is only valid for $m_F > T_*$, we see that (assuming the vortons do not decay) the above nucleosynthesis constraint implies $\delta_{GS} \sim G \mu \ll 10^{-9}$. This is a very small value for $\delta_{GS}$, and it is hard to reconcile it with the string theory motivation for our model. On the other hand, if $m_F/m_D \lesssim 10^{-2}$ and the vortons are unstable when their cores overlap, there is no problem. For $m_F \gtrsim 10^{-2} m_D$, eq. (40) implies that we must have $\delta_{GS} \sim G \mu \ll 10^{-17}$.

If the vortons form during the scaling regime, $\rho_v$ will instead be given by eq. (39). If vortons are unstable for small $m_F$, then this expression only applies in the limited parameter range $m_D/m_{Pl} \gtrsim m_F/m_D \gtrsim 10^{-2}$. For $\epsilon = 1$ and $\alpha \sim \Gamma G \mu$, this implies $\rho_v/(g_* T^3) \gtrsim 10^{-9}$. Hence this region of parameter space is ruled out. If however the vortons do not decay when their outer cores overlap, the constraint $\rho_v \ll g_* T_N^4$ implies

$$g_*^{-1/8} (G \mu)^{5/8} \frac{m_F}{m_D} \ll \left( \frac{T_N}{m_{Pl}} \right)^{1/2} .$$

(41)
for $\epsilon = 1$ and $\alpha \sim \Gamma G\mu$. By taking $m_F$ sufficiently small this can be satisfied for any $m_D$ (even $m_D = m_{pl}$, in which case $m_F \ll 10^8$ GeV).

Of course, if $m_F \lesssim 10^{-2} m_D$ and the fermion currents do decay due to core overlap, the vortons will not be stable. Then the nucleosynthesis constraint does not apply in the first place, irrespective of whether the strings form in the scaling regime or the friction regime.

A similar bound comes from the considering the effects of the dark matter density on galaxies. An excessive amount of dark matter (this includes vortons) will lead to problems with galaxy formation. To avoid this, the vorton density needs to be less than the critical closure density $\rho_c \approx g_* m_c T^3$, with $m_c \sim 1$ eV \[12\]. We find that similar constraints to eqs. (40) and (41) apply, with $T_N$ replaced by $m_c$. If the vortons do not decay, we need $G\mu \sim \delta_{GS} \lesssim 10^{-10}$ if the strings form during friction domination. If the strings form in the scaling regime, then $\delta_{GS}$ is not bounded if $m_F \lesssim 10^7$ GeV. Of course, if the vortons are not stable, the dark matter bound is evaded.

### 6.2 Particle production

If $m_F \lesssim 10^{-2} m_D$ vortons are unlikely to form. Instead the loops will decay, although in contrast to more conventional models, the main decay mechanism will be particle production rather than gravitational radiation.

The dilaton couples to our axions strings with a similar strength to gravity, and so decaying string loops will radiate not only gravitons but also dilatons \[14\]. Furthermore, when the radius of a loop is comparable to its thickness, microphysical forces will become important, and all the energy of the loop will be released as a burst of radiation \[15\].

We will start by finding an expression for the number of dilatons emitted by a loop as it shrinks. For simplicity we will only consider strings in the scaling regime, where frictional forces can be neglected, and $\ell \sim \alpha t$. We follow the method of ref. \[14\]. There it was assumed that $\ell \ll \ell_{crit} = 4\pi/m_F$ ($m_F$ is the dilaton mass for our model). This will not be the case for our strings, although the analysis is easily extended to general $\ell$. The dilaton number density is then

$$n_F \sim \left(\frac{\alpha m_{pl}}{\sqrt{g_F} m_F}\right)^{1/2} G\mu \times \begin{cases} (\ell m_F)^{1/2} & \ell \ll \ell_{crit} \\ (\ell m_F)^{-5/6} & \ell \gg \ell_{crit} \end{cases}$$

where $s \sim g_* T^3$ is the entropy density. We see that the maximum contribution comes from loops with $\ell \sim \ell_{crit} \sim m_F^{-1}$.

Since the string network does not exist before $t_c$, the minimum value of $\ell$ is $\alpha t_c$, which can be greater than $\ell_{crit}$. Hence

$$n_F \sim \left(\frac{\sqrt{g_F} m_F}{\alpha m_{pl}}\right)^{1/3} G\mu$$

when $m_F < \alpha m_{pl}/\sqrt{g_F}$. This leads to weaker constraints than the $\ell = \ell_{crit}$ results of ref. \[14\].
The derivation of the above expression assumed that the behaviour of the network was given by a scaling solution. This is not the case for our strings, since if \( m_D \sim m_F \) there will also be the friction regime. Even if \( m_F \ll m_D \) and the friction regime is absent, the initial behaviour of the network (which is when most dilatons are produced) will instead be given by eq. (36). The string density was higher at this time, and so we expect more dilatons to be produced. The actual bound for our model will be somewhere between the two expressions (42) and (43).

If the dilatons have a lifetime of less than 0.1s, they will decay well before nucleosynthesis, and there will be no significant bounds on the model from dilaton production. This will be the case if \( m_F \gtrsim 10^2 \text{TeV} \). For a lighter dilaton, with \( m_F \sim 10^4 - 10^5 \text{GeV} \), we need \( n_F/s \lesssim 10^{-14} \). Taking \( \alpha \sim \Gamma G \mu \), the bound (43) implies \( G \mu \lesssim 10^{-11}(\text{TeV}/m_F)^{1/2} \). Alternatively if we use the bound (42) with \( \ell = \ell_{\text{crit}} \), we have instead \( G \mu \lesssim 10^{-15}(m_F/\text{TeV})^{1/3} \). Either way a very small value of \( m_D \) is required to satisfy the constraints for light dilatons.

Since the dilaton is one of the particles which our strings are made up of, our model has an additional dilaton production mechanism. When the loop size is comparable to its width, it will collapse and release its energy as a burst of radiation. This will be consist of dilatons and all the other particles that the strings are made of.

Using eq. (37) and following Brandenberger et al. [15], the total number of quanta produced in loop collapse from a string network is

\[
 n_Q(t) \sim \frac{N_Q}{a(t)^3} \int_{\xi_c}^{\xi} \frac{a(t')^3}{\alpha \xi(t')^4} d\xi'
\]  

(44)

where \( N_Q \) is the number of quanta produced by a single collapsing loop.

At the time of decay, the length of loop will be about one order of magnitude greater than the width of the outer core, i.e. \( \ell \sim \beta m_F^{-1} \), with \( \beta \geq 2\pi \). We expect to get \( N_F = \ell m_F \sim \beta \sim 10 \) particles of mass \( m_F \) from the outer core and \( N_D = \ell m_D / m_F \) from the inner core.

In the friction dominated regime \( \xi \sim t^{5/4} \), and so [15]

\[
 \frac{n_Q}{s} \sim \frac{N_Q}{\alpha g_{sF}}
\]

(45)

(with \( \alpha \sim 1 \)). If \( g_{sF} \sim 10^3 \), as would be the case if \( m_F \) is around the GUT scale, \( n_F/s \sim N_F/g_{sF} \sim 10^{-2} \) and \( n_D \sim (m_D/m_F) n_F \).

An equivalent calculation in the scaling regime would give smaller values of \( n_Q \). However the early evolution of our strings is not actually a scaling solution, and it is the early evolution which produces the largest number of loops. Using eq. (36) we find that the above expression (45) still applies, although this time \( \alpha \) is much smaller, and so the number of particles produced is very high. Unless the lifetime of the dilatons is less than 0.1s, this is completely incompatible with nucleosynthesis constraints [14]. This imposes a lower bound on \( m_F \) of \( 10^2 \text{TeV} \).
6.3 Microwave background constraints

Finally, if the theory evades the vorton bounds then the string network will give rise to anisotropies in the cosmic microwave background. Experimentally these are detected at the level of about $10^{-6}$. In the cosmic string scenario for structure formation they are of order $G\mu$. So unless $g_0^2\delta_{GS} \lesssim 10^{-7}$ they are too large and can be ruled out by the COBE result. This bound comes from considering the properties of the late time scaling solution of the network. Since this is independent of the non-standard early behaviour of our strings, it is hard to see how this bound can be avoided. It is hard to reconcile such a small value of $\delta_{GS}$ with the underlying theory which motivates our model. If $s$ were a moduli field rather than the dilaton, a smaller value of $\delta_{GS}$ could be possible.

It should be noted that, throughout our analysis, we have assumed that our strings intercommute (i.e. when two strings collide, the different halves of the two strings exchange partners). For the usual $U(1)$ strings this is only true if the gauge mass is smaller than the Higgs mass. It is not clear if our strings satisfy an equivalent relation. If they do not intercommute, then a network of our strings will not produce loops, and will evolve in a significantly different way. The microwave background bound will also be changed. However if loops do not break off from the string network, then it is hard to see how it can reach a scaling solution and avoid dominating the universe. Thus this kind of non-standard evolution could actually produce even stronger constraints on the model.

7 Bogomolnyi equations

It is actually possible to reduce the field equations of the model (15–17) to first order differential equations by using the symmetries of the model. Using the $z$ and $t$ independence of the fields, $\phi_0 = 0$, and integration by parts, the integral of the Lagrangian (4) can be rewritten as

$$\mu_1 = \int d^2x \left\{ |(\partial_x - iA_x)\phi \pm i(\partial_y - iA_y)\phi|^2 
+ \frac{1}{4s_R^2} |(\partial_x s - 2i\delta_{GS}A_x) \pm i(\partial_y s - 2i\delta_{GS}A_y)|^2 + \frac{s_R}{2} (F_{xy} \pm D)^2 + |F_0|^2 \right\}.$$  \hspace{1cm} (46)

We use the upper (lower) sign for $n > 0$ ($n < 0$). If $F_0 = 0$ and

$$f' = |n| \frac{1 - v}{r} f$$ \hspace{1cm} (47)

$$s'_R = 2\delta_{GS} |n| \frac{1 - v}{r}$$ \hspace{1cm} (48)

$$|n| \frac{v'}{r} = \frac{1}{s_R} \left( \frac{\delta_{GS}}{s_R} - f^2 \right) \hspace{1cm} (49)$$

then $\mu_1 = 0$. Since $F_0 = 0$ implies $s = 2\delta_{GS} \ln(\phi/\eta) + \delta_{GS}/\eta^2$, eq. (17) is consistent with eq. (18). Although the above equations give a solution of the full field equations, the string
solution has $s_r$ passing through zero. This implies the coupling $g \to \infty$ there, and so the solution is unphysical.

8 Conclusions

In this paper we have investigated a non-standard class of axionic cosmic string solutions. The axion model used was motivated by an effective superstring action. In this type of model the axion and dilaton form part of the same complex field. By allowing the dilaton to vary, and by including a gauge field in the model, it was possible to find finite energy cosmic string solutions without having to introduce cutoffs. This contrasts with the usual axion string models, in which the dilaton is absent or constant.

Another unusual feature of our strings is the existence of two length scales for the size of the string core. The string’s magnetic field is confined to an inner core, while the dilaton (and a Higgs field) can vary over a far greater distance. The two length scales arise from the two mass scales in the theory’s potential. One ($m_D$) comes from high energy supergravity contributions, and the other ($m_F$) from a lower energy gaugino condensate, which is used to stabilise the dilaton.

Being supersymmetric, our model also contains fermion fields. As is common with topological defects in supersymmetric models, our strings possess zero energy fermion bound states. Excitations of these form conserved chiral currents on the strings. These currents can allow stable string loops (vortons) to exist. The existence of vortons suggests extremely strong cosmological constraints on the model.

However, the fact that the strings have two mass scales alters their evolution. By taking $m_F$ to be small, the strings form at a later time with a smaller density. This leads to far weaker vorton constraints than those that would be expected for the string’s large energy per unit length ($m_D^2$). If $m_F/m_D$ is sufficiently small the outer cores of the opposite sides of the vorton will overlap. Since the fermion bound state wavefunctions do reach to the outer core, this suggests the currents will decay, destabilising the vortons and evading the corresponding constraints on the strings.

If string loops are unstable there will be other constraints on the model from particle production by the strings. They will radiate dilatons as they shrink (in addition to gravitational radiation). Furthermore, when the loops collapse, they will decay into yet more dilatons (as well as heavy Higgs and gauge particles). The number of dilatons produced by these processes is very high, and will rule out the model unless the dilaton lifetime is short. This imposes a lower bound of $10^2$ TeV on the dilaton mass, which is $m_F$ for our model. This rules out the gluino condensation as the source of the potential which stabilises the dilaton. Combining the above bounds gives $10^{-2}m_D \gtrsim m_F \gtrsim 10^2$ TeV.

If $m_F \ll m_D$ the strongest constraints on the model will come from cosmic microwave background measurements. These suggest $m^2_D/m^2_{Pl} \sim \delta_{GS} \lesssim 10^{-7}$. Although weaker than the vorton bounds, this is still difficult to reconcile with the string theory motivation for the model. This would be a less serious problem if we took the superfield $S$ to be a moduli field rather than the dilaton. Thus this type of cosmic string could still be
both theoretically and observationally viable. To satisfy all the above bounds, we require
$10^{15} \text{ GeV} \gtrsim m_D \gtrsim 10^7 \text{ GeV}$.

An important assumption in the microwave background constraints is that the strings intercommute. This may not be the case for our strings, in which case a string network will not produce loops or approach a scaling solution in the usual way. This would significantly alter the bounds on the model, although unfortunately we expect it to make them much stronger.

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A Conventions

We use the metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

Under the U(1) gauge symmetry the fields transform as follows

$$\Phi \to e^{2i\Lambda} \Phi, \quad S \to S + 4i\delta_{GS} \Lambda,$$

$$V \to V + i(\Lambda - \Lambda^\dagger), \quad \Phi_0 \to e^{-2i\eta_0 \Lambda} \Phi_0$$

so $q_\phi = -1$.

Under the R symmetry

$$S \to S + 2b_0 i\alpha, \quad \Phi \to e^{-ir_\phi^\alpha \Phi}, \quad \Phi_0 \to e^{-ir_0^\alpha \Phi_0}$$

Under a supersymmetry transformation of the bosonic fields, the change in the fermion fields (in Wess-Zumino gauge) is

$$\delta \psi = \sqrt{2} F_\xi + i\sqrt{2} \sigma^\mu \bar{\xi} D_\mu \phi$$

$$s_R^{-1/2} \delta \lambda = iD\xi + \frac{1}{2} \sigma^\mu \sigma^\nu \xi F_{\mu\nu}$$

$$2s_R \delta \chi = \sqrt{2} F_\xi \xi + i\sqrt{2} \sigma^\mu \bar{\xi} (\partial_\mu s - 2i\delta_{GS} A_\mu)$$

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