ANOMALOUS PHASE DIAGRAM IN SIMPLEST PLASMA MODEL

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INTRODUCTION

Problem of Phase Transition (PT) in Strongly Coupled Coulomb Systems (SCCS) is of great interest in plasma theory during very long time. Besides the study of hypothetical PT in real plasmas a complementary approach is developing where the main subject of interest is definitely existing PT in simplified plasma models. In our previous study we dealt with a phase transition in the set of plasma models with common feature - combination of (i) absence of individual correlations (coupling) between charges of opposite sign, and (ii) total compressibility of system. The simplest example of such a system is One Component Plasma (OCP) on uniform, but compressible compensating background (following notation - OCP{c}). The well-known prototype model is OCP with a rigid background (notation - OCP{r}). This variant of OCP is studied carefully nowadays. It can not collapse or explode spontaneously. The only phase transition - crystallisation - occurs in OCP{r} without any density change.

Transition to the OCP on uniform and compressible background leads to appearance of a new first-order phase transition of gas-liquid type. New phase diagram combines previous crystallisation, now with a finite density change, with a qualitatively different coexistence curve of the new phase transition. The structure and parameters of this phase diagram strongly depend on exact definition of thermodynamic contribution of background. The simplest variant of OCP{r} is the «Single OCP» - the system of classical point charges with a compressible background of ideal fermi-gas of electrons. This variant of OCP was declared repeatedly but the discussed phase transition was out of consideration. Closely similar structure of global phase diagram was obtained in «Combined OCP» by superposition of two non-coupled OCP-s of mass-non-symmetrical charged particles of opposite sign.

PHASE DIAGRAM OF SINGLE OCP{c}

Three qualitatively different situations should be distinguished for the OCP{c} depending on the value of charge number Z:

1) Low value of charge number \(- Z < Z_1^* \approx 35\)
2) High value of charge number \(- Z > Z_2^* \approx 45\)
3) Intermediate value of charge number \(- Z_1^* < Z < Z_2^*\)
Phase diagram of the Single OCP\{r\} of classical point charges on uniform compressible background of ideal fermi-gas of electrons at intermediate value of charge number (Z_1^* < Z = 40 < Z_2^*). Notations: 1- melting line of prototype OCP\{r\} (\Gamma \sim \Gamma_{melt} \approx 178); 2,2''; 3,3'' global crystal-fluid (gas and liquid) coexistence: 2,2'' - melting, 3,3'' sublimation; 4 - spinodal of metastable liquid-gas coexistence; 5 - its critical point (cp).

Figure 1. Phase diagram of the Single OCP\{c\} of classical point charges on uniform compressible background of ideal fermi-gas of electrons at intermediate value of charge number (Z_1^* < Z = 40 < Z_2^*). Notations: 1- melting line of prototype OCP\{r\} (\Gamma \sim \Gamma_{melt} \approx 178); 2,2''; 3,3'' global crystal-fluid (gas and liquid) coexistence: 2,2'' - melting, 3,3'' sublimation; 4 - spinodal of metastable liquid-gas coexistence; 5 - its critical point (cp).

Critical Point (Fluid-Gas)

\[ \Gamma = \Gamma_{melt} = \text{const} \]

\[ 6.0 \quad 7.0 \quad 8.0 \quad 9.0 \]

\[ T, \text{ a.e.} \]

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \]

\[ n \]

\[ 10^6 \quad 10^7 \quad 10^8 \quad 10^9 \]

\[ Z_1^* < Z < Z_2^* \]

\[ \text{Crystal-Gas Binodal} \]

\[ \text{Fluid-Gas Spinodal} \]

\[ 1 \quad 2 \quad 3 \quad 4 \]

\[ n/nc \]

\[ 0 \quad 1 \quad 2 \quad 3 \]

\[ T/T_c \]

\[ 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0 \]

\[ \text{Critical Point} \]

\[ \Gamma = \Gamma_{melt} = \text{const} \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ n \]

\[ 10^6 \quad 10^7 \quad 10^8 \quad 10^9 \]

\[ Z = Z_1^* \]

\[ \text{Fluid-Gas Spinodal} \]

\[ \text{Fluid-Gas Binodal} \]

\[ \text{Crystal-Gas Binodal} \]

\[ \text{Critical Point} \]

\[ \Gamma = \Gamma_{melt} = \text{const} \]

\[ 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0 \]

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\[ \text{ Crystal-Gas Binodal} \]

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\[ 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0 \]

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\[ n/nc \]

\[ 0 \quad 1 \quad 2 \quad 3 \]

\[ Z = Z_1^* \]

\[ \text{ Fluid-Gas Spinodal} \]

\[ \text{ Fluid-Gas Binodal} \]

\[ \text{ Crystal-Gas Binodal} \]

\[ \text{ Critical Point} \]

\[ \Gamma = \Gamma_{melt} = \text{const} \]

Low Values of Charge Number (Z ~ 1)

Phase diagram of the model was carefully studied in6-8. The ordinary structure of global phase diagram was obtained in this case: the relative position of critical and triple points, melting «stripe», gas-liquid and gas-crystal coexistence, are totally equivalent to those for normal substances.

High Values of Charge Number (Z ~ 100).

Highly anomalous structure of global phase diagram was announced at previous study6-7. The melting «stripe» (\Gamma \approx 178) crosses gaseous part of coexistence curve of the new phase transition.

• Triple point is placed at gaseous part of global phase boundary.
• Critical point is placed at crystalline part of global phase boundary.
• Crystal-crystal coexistence of two dense and expanded crystalline phases of the same structure occurs in OCP\{r\} at such a high values of charge number Z.

Intermediate Values of Charge Number (Z_1^* < Z < Z_2^*)

The most remarkable anomalous phase diagram corresponds to the case when the melting line of prototype OCP\{r\} (\Gamma \sim \Gamma_{melt} \approx 178) crosses coexistence curve of the new gas-liquid phase transition just closely to its critical point. As a result of this coincidence:

• The only phase transition exists in the model. It corresponds to the global crystal - fluid coexistence – continuous superposition of melting and sublimation (see Figure 1).
• There is no true critical point.
• There is no triple point.
• Coexistence curve in $P\leftrightarrow T$ (pressure↔temperature) plane is a continuous, infinite curve. There is no any break at this curve.

Boundary Values of Intermediate Charge Number Interval $(Z_1^* < Z < Z_2^*)$

Remarkable feature of phase diagram of OCP{c} at $Z = Z_1^*$ or $Z = Z_2^*$ is an existence of pseudo-critical point where the well-known standard conditions are fulfilled:

\[
\left(\frac{\partial P}{\partial V}\right)_T = 0 \quad \left(\frac{\partial^2 P}{\partial V^2}\right)_T = 0
\]

$Z = Z_1^* \approx 34.6^*$ – on gaseous part of crystal-fluid binodal (see Figure 2)

$Z = Z_2^* \approx 45.4^*$ – on crystalline part of crystal-fluid binodal.

When we use the same as in 6-8 analytical fits for equation of state of both subsystems, OCP{r} and background, we obtain following parameters of the both pseudo-critical points:

| Table 1. Parameters of pseudo-critical point in OCP of classical point charges on the uniform and compressible background of ideal fermi-gas of electrons ($Z = Z_1^*$ or $Z_2^*$) |
|---|---|---|---|---|---|---|
| $Z$ | $T_C$, a.u | $(n_e)_C$,cc$^{-1}$ | $P_C$, a.u | $\Gamma_C$ | $(r_S)_C$ | $(n_e A^2_\phi)_C$ |
| $Z = Z_1^*$ | 34.6 | 6.38 | 2.24 $10^{25}$ | 11.4 | 140 | 0.416 | 3.30 | 2.91 |
| $Z = Z_2^*$ | 45.4 | 9.29 | 3.96 $10^{25}$ | 28.4 | 181 | 0.344 | 3.26 | 2.89 |

CRITICAL EXONENTS

Remarkable feature of two discussed pseudo-critical points at $Z = Z_1^*$ or $Z = Z_2^*$ is the non-standard values of all critical exponents in comparison with the ordinary (van der Waals like) critical exponents that correspond to the case of OCP{c} with the charge number $Z$ beyond the discussed interval $Z_1^* \div Z_2^*$. For example, at the latter case ($Z < Z_1^*$ or $Z > Z_2^*$), the standard density↔temperature relation is valid

\[
(\rho - \rho_C) \sim |T - T_C|^{1/2}
\]

For the pseudo-critical points ($Z = Z_1^*$ or $Z = Z_2^*$) the following relation may be proved:

\[
(\rho - \rho_C) \sim |T - T_C|^{1/3}
\]

Direct calculation gives:

\[
|\rho/\rho_C - 1| \approx 4.57 \left|T/T_C - 1\right|^{1/2} \quad (Z = 1)
\]

\[
|\rho/\rho_C - 1| \approx 4.07 \left|T/T_C - 1\right|^{1/3} \quad (Z = Z_1^* \approx 34.6)
\]

SATURATION CURVE

Similar violation is observed for saturation ($P_{St} \leftrightarrow T_{St}$) curve. So-called Plank − Gibbs rule (equal slope of saturation curve at $T = T_C - \varepsilon$ and critical isohore at $T = T_C + \varepsilon$) is valid for an ordinary critical point ($Z < Z_1^*$ or $Z > Z_2^*$),

\[
\left(\frac{dP}{dT}\right)_{St} = \left(\frac{\partial P}{\partial T}\right)_{V_C}
\]
It is not evident (see Figure 3), but it can be proved that this rule is not valid for pseudo-critical points ($Z = Z_1^*$ or $Z = Z_2^*$).

Figure 3. Saturation curve and isochors in reduced coordinates for the Single OCP[c] of classical point charges with uniform compressible background of ideal fermi-gas of electrons at boundary value of charge number $Z = Z_1^* \approx 34.6$. Notations: 1 - sublimation; 2 - melting; 3 - pseudo-critical «termination» point; 4 - critical isochore; 5,6 - sub-critical isochors.

Figure 4. The same as on Figure 3 in Log↔Log - coordinates. Notations: 1-4 - as on Figure 1.

This statement is illustrated on Figure 4. Small deviation in position of binodal (curves 1,2) and critical isochore (curve 4) corresponds to the small difference in slope of both the curves at pseudo-critical point (Figure 3).

REFERENCES

1. G.E. Norman and A.N. Starostin, Teplofiz. Vysokih. Temp. (High Temp) 6:410 (1968).
2. W. Ebeling, W.D. Kraeft and D. Kremp. «Theory of Bound states and Ionization Equilibrium in Plasmas and Solids,» Academie-Verlag, Berlin, (1976).
3. D. Saumon, G. Chabrier, Phys. Rev. Let. 62: 2397 (1989).
4. P.N. Vorontsov-Veliaminov, V.P. Chasovskih, Teplofiz. Vysokih. Temp. 13:1153 (1975)
5. M.E. Fisher, J. Stat. Phys. 75:1 (1994)
6. I.L. Iosilevski, High Temperatures. 23:807 (1985)
7. I.L. Iosilevski and A.Yu. Chigvintsev in «Physics of Nonideal Plasmas» W. Ebeling, A.Förster and R. Radtke, ed., Teubner Texte, Stuttgart- Leipzig (1992) p.87.
8. I.L. Iosilevski and A.Yu. Chigvintsev in «Physics of Strongly Coupled Plasmas»
W.D. Kraeft and M. Schlanges, ed., World Scientific, New Jersey-London (1996) p.145.
9. M. Baus and J.P. Hansen, Phys. Reports, 59:1 (1980)
10. S. Ishimaru, H. Yyetomi and S. Tanaka, Phys. Reports, 149:91 (1987).
11. E.L. Pollock and J.P Hansen, Phys. Rev. A-8:3110 (1973).
12. B. Alder, E. Pollock and J.P. Hansen. Proc. Natl. Acad. Sci. USA.,77:6272 (1980).
13. Y. Levin and M.E. Fisher, Physica. A225:164 (1996)