PeV Scale Right Handed Neutrino Dark Matter in $S_4$ Flavor Symmetric extra U(1) model

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Abstract

Recent observation of high energy neutrino in IceCube experiment suggests existence of superheavy dark matter beyond PeV. We identify the parent particles of neutrino as two degenerated right handed neutrinos, assuming the dark matter is the heaviest right handed neutrino. The $O(V_{cb}) \sim O(10^{-2})$ flavor symmetry breaking accounts for the $O(10^{-4})$ mass degeneracy of right handed neutrinos which is a sizable scale to explain the successful resonant leptogenesis at the PeV scale. At the same time, non-thermal production of the heaviest right handed neutrino gives the right amount of dark matter for $T_{RH} \sim 10$PeV. The footprint of flavor symmetry is left in degenerated mass spectra of extra Higgs multiplet and colored Higgs multiplet which may be testable for LHC or future colliders.
1 Introduction

The standard model (SM) is a successful theory of gauge interactions; however, there are many unsolved issues such as how to generate Yukawa hierarchy, meaning of small neutrino mass, how to generate baryon asymmetry and what dark matter is. Recently, IceCube reported their observation of high energy neutrinos which might be a hint to solve some of those problems [1]. The most important problem of SM is how to stabilize large hierarchy between the electroweak scale, $M_W \sim 10^2\text{GeV}$, and the Planck scale, $M_P \sim 10^{18}\text{GeV}$ against quantum corrections. The elegant mechanism of stabilizing hierarchy is supersymmetry (SUSY) [2], which is the main target of Large Hadron Collider (LHC). The existence of light Higgs boson such as 125-126GeV [3] supports the idea of SUSY.

In the minimal supersymmetric standard model (MSSM), as the Higgs superfields $H^U$ and $H^D$ are vector-like under the SM gauge symmetry $G_{SM} = SU(3)_c \times SU(2)_W \times U(1)_Y$, we can introduce $\mu$-term;

$$\mu H^U H^D,$$

(1)

in superpotential. The natural size of parameter $\mu$ is $O(M_P)$, however $\mu$ must be $O(M_W)$ in order to allow electroweak gauge symmetry to break. This is so-called $\mu$-problem, which is solved by making Higgs superfields chiral under a new $U(1)_X$ gauge symmetry. Such a model is achieved based on $E_6$-inspired extra U(1) model [4]. The new gauge symmetry replaces the $\mu$-term by trilinear term;

$$\lambda SH^U H^D,$$

(2)

which is converted into effective $\mu$-term when singlet $S$ develops $O(1\text{TeV})$ vacuum expectation value (VEV) [5]. At the same time, the baryon and lepton number violating terms in MSSM are replaced by single G-interactions;

$$GQQ + G^c U^c D^c + GU^c E^c + G^c QL,$$

(3)

where $G$ and $G^c$ are new colored superfields which must be introduced to cancel gauge anomaly. Although these terms induce very fast proton decay, it can be suppressed by a $S_4$ flavor symmetry [6][7]. Therefore flavor symmetry plays an important role in stabilizing protons in supersymmetric model.

The $S_4$ flavor symmetry can also solve the hierarchy problem of Yukawa couplings and the problem of flavor violating processes originated from the SUSY contribution simultaneously. The former can be achieved by not assigning the $S_4$-triplet, on the other hand the latter can be done by assigning the $S_4$ doublet for the first and second generations of left handed quarks. Therefore the hierarchies $m_u, m_c < m_t$ and $m_d, m_s < m_b$ are generated by the same manner as $SU(2)_W$, as the mismatch of the sizes of representations among the fields in operator suppresses the coefficient of the operator. We should keep in mind that the hierarchy $m(\text{quark}), m(\text{lepton}) \ll M_P$ is generated by the discrepancy of the sizes of representations of $SU(2)_W$ between left handed fermion and right handed fermion. This manner is also adopted for suppressing single G-interactions when $G$ and $G^c$ are assigned to be $S_4$-triplets. From the sizes of the elements of Cabibbo-Kobayashi-Maskawa (CKM) matrix, the $S_3$ subgroup of $S_4$ should be broken by $O(10^{-4})$.

It is well known that heavy right handed neutrino (RHN) not only realizes small neutrino mass by see-saw mechanism but also generates baryon asymmetry of the universe through leptogenesis. If the first and second generations of RHNs are assigned to be $S_4$-doublet, then the mass degeneracy is solved to be achieved by $O(10^{-4})$. Therefore successful resonant thermal-leptogenesis requires that the mass of the lightest RHN is $O(\text{PeV})$, which is in good agreement with the energy scale of high-energy neutrinos observed in IceCube. Furthermore, if we impose constraint on the reheating temperature, $T_R < 10^7\text{GeV}$, for avoiding gravitino over production [8], the right amount of dark matter density is obtained through non-thermal production of the heaviest RHN. Although the flavor symmetry is broken, its remnant may be observed in the spectrum of extra particles such as $G, G^c$ or extra Higgs.

2 Symmetry Breaking

2.1 Gauge Symmetry

We extend the gauge symmetry from $G_{SM}$ to $G_{42111} = G_{SM} \times U(1)_X \times U(1)_Z$, and add new superfields $N^c, S, G, G^c$ which are embedded in 27 representation of $E_6$ with quark, lepton superfields $Q, U^c, D^c, L, E^c$ and Higgs superfields $H^U, H^D$. Where $N^c$ is RHN, $S$ is $G_{SM}$ singlet and $G, G^c$ are colored Higgs. The two $U(1)$s
The constraint for remains unbroken, the lightest SUSY particle (LSP) is stable.

Table 1: $G_{32111}$ assignment of superfields. Where the $x$, $y$ and $z$ are charges of $U(1)_{X}$, $U(1)_{Y}$ and $U(1)_{Z}$, and $Y$ is hypercharge. The charges of $U(1)_{ψ}$ and $U(1)_{χ}$ which are defined in Eq.(4) are also given.

are linear combinations of $U(1)_{ψ}$ and $U(1)_{χ}$ where $E_{6} \supset SO(10) \times U(1)_{ψ} \supset SU(5) \times U(1)_{χ} \times U(1)_{ψ}$, and their charges $X$ and $Z$ are given as follows

\[ X = \frac{\sqrt{15}}{4} Q_{ψ} + \frac{1}{4} Q_{χ}, \quad Z = -\frac{1}{4} Q_{ψ} + \frac{\sqrt{15}}{4} Q_{χ}. \]  

The charge assignments of the superfields are given in Table 1. To break $U(1)_{Z}$, we add new vector-like superfields $Φ, Φ^{c}$ which are originated in $351^{r} + 351^{r}$ of $E_{6}$. The invariant superpotential under these symmetries is given by

\[
\begin{align*}
W_{32111} & = W_{0} + W_{S} + W_{G} + W_{Φ}, \\
W_{0} & = Y^{U} H^{U} Q^{c} + Y^{D} H^{D} Q^{c} + Y^{L} H^{D} L^{c} + Y^{N} H^{U} L^{c} + Y^{M} Φ N^{c} N^{c}, \\
W_{S} & = kSGG^{c} + λSHU^{D}, \\
W_{G} & = Y^{QQ} GQQ + Y^{UD} G^{c} U^{c} D^{c} + Y^{UF} G^{c} U^{c} E^{c} + Y^{QL} G^{c} QL + Y^{DN} GD^{c} N^{c}, \\
W_{Φ} & = M_{Φ} Φ Φ^{c} + \frac{1}{M_{P}} Y^{Φ} (Φ^{c})^{2},
\end{align*}
\]

where $M_{P} = 2.4353 \times 10^{18}$GeV is reduced Planck scale and unimportant higher dimensional terms are omitted. Since the interactions $W_{S}$ drive squared mass of $S$ to be negative through renormalization group equations (RGEs), spontaneous $U(1)_{X}$ symmetry breaking is realized and $U(1)_{X}$ gauge boson $Z'$ acquires the mass

\[ m(Z') \sim \frac{5}{2\sqrt{3}} g_{X} \langle S \rangle. \]  

where $\langle H^{U,D} \rangle \ll \langle S \rangle$ is assumed based on the experimental constraint for $Z'$. as follow

\[ m(Z') > \left\{ \begin{array}{l}
2.51 \text{TeV (ATLAS[9])} \\
2.26 \text{TeV (CMS[10])}
\end{array} \right. \]  

The constraint for $Z'$ mass is not far from this bound. Note that the mass bound for $Z'$ depends on sparticle mass spectrum [11]. In this paper we assume $\langle H^{U,D} \rangle / \langle S \rangle \sim O(10^{-1})$.

If $M_{Φ} = 0$ in $W_{Φ}$ and the origin of the potential $V(Φ, Φ^{c})$ is unstable, then $Φ, Φ^{c}$ develop large VEVs along the D-flat direction of $⟨Φ⟩ = ⟨Φ^{c}⟩ = V$, $U(1)_{Z}$ is broken and $U(1)_{Z}$ gauge boson $Z''$ acquires the mass

\[ m(Z'') = \frac{8}{3} \sqrt{\frac{5}{2}} g_{Z} V. \]  

After the gauge symmetry breaking, since the R-parity symmetry defined by

\[ R = \exp \left[ iπ \frac{3x - 8y + 15z}{20} \right], \]  

remains unbroken, the lightest SUSY particle (LSP) is stable.
Table 2: $S_4 \times Z_3^W \times Z_3^P \times Z_2 \times Z_2$ assignment of superfields (Where the index $i$ of the $S_4$ doublets runs $i = 1, 2,$ and the index $a$ of the $S_4$ triplets runs $a = 1, 2, 3$. The details of $S_4$ are given in Ref. [13].)

### 2.2 Flavor symmetry

The superpotential defined in Eq.(5)-Eq.(9) has following problems. As the interaction $W_G$ induces too fast proton decay, they must be strongly suppressed. The mass parameter $M_\Phi$ in $W_\Phi$ must be forbidden in order to allow $U(1)_X$ symmetry breaking. In $W_0$, the contributions to flavor changing processes from the extra Higgs bosons must be suppressed [12]. These problems should be solved by flavor symmetry.

If we introduce $S_4$ flavor symmetry and assign $G, G^c$ to be triplets, then $W_G$ defined in Eq.(8) is forbidden. This is because any products of doublets and singlets of $S_4$ do not contain triplets. Note that we assume full $S_U$ symmetry does not realize at Planck scale, therefore there is no need to assign all superfields to the same flavor representations. In this model the generation number three is imprinted in $G, G^c$. Therefore they may be called "G Higgs" (generation number imprinted colored Higgs).

As the existence of G Higgs which has life time longer than 0.1 second spoils the success of Big Ban nucleosynthesis (BBN)[8], $S_4$ symmetry must be broken. Therefore we assign $\Phi^c$ to be triplet and $\Phi$ to be doublet and singlet to forbid $M_\Phi \Phi \Phi^c$. With this assignment, $S_4$ symmetry is broken due to the VEV of $\Phi$ and the effective trilinear terms are induced by non-renormalizable terms

$$W_{N RG} = \frac{1}{M_P} \Phi \Phi^c (GQQ + G^c U^c D^c + GU^c E^c + G^cQL + GD^cN^c).$$

The size of effective coupling constants of these terms should be

$$\frac{\langle \Phi \rangle \langle \Phi^c \rangle}{M_P} > 10^{-14},$$

(15)

to satisfy the BBN constraint [14]. The superpotential of gauge non-singlets $\Phi, \Phi^c$ is given by

$$W_{\Phi} = \frac{Y_\Phi}{M_P} \Phi^c (\Phi^c_1)^2 + (\Phi_2)^2 + (\Phi_3)^2 + \frac{c^2 Y_{\Phi}}{M_P} [\Phi_1^2 + \Phi_2^2] (\Phi_1^c)^2 + (\Phi_2^c)^2 + (\Phi_3^c)^2]
+ \frac{c^2 Y_{\Phi}}{M_P} [2\sqrt{3} \Phi_1 \Phi_2 (\Phi_3^c)^2 - (\Phi_3^c)^2 + (\Phi_1^c)^2 - (\Phi_2^c)^2 - (\Phi_3^c)^2 - 2(\Phi_1^c)^2]
+ \frac{c^4 Y_{\Phi}}{M_P} \Phi_3 [(\Phi_3^c)^2 - (\Phi_3^c)^2 + \Phi_2 ((\Phi_3^c)^2 + (\Phi_3^c)^2 - 2(\Phi_1^c)^2)],$$

(16)
We assume the global minimum of the potential \( V(\Phi, \Phi^c) \) is at \( S_3 \)-symmetric vacuum as follow

\[
\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = 0, \quad \langle \Phi_1^c \rangle = \langle \Phi_2^c \rangle = \langle \Phi_3 \rangle = \frac{V}{\sqrt{3}},
\]  
\( (17) \)

The assignments of the other superfields are determined based on following criterion. (1) The quark and lepton mass matrices reproduce observed mass hierarchies and CKM and Mak-Nakagawa-Sakata (MNS) matrices. (2) The third generation Higgs \( H_3^U, H_3^D \) are specified as MSSM Higgs and the first and second generation Higgs superfields \( H_{1,2}^{U, D} \) are almost inert Higgs (AIH). The representation of all superfields under the flavor symmetry is given in Table 2.

In order to realize Yukawa hierarchies, we introduce gauge singlet flavon superfields \( V_i, X_W, W_i, P_W, X_P, P, X \) and fix the VEV of them by

\[
\begin{align*}
\langle X \rangle &= \left( \frac{m_{\text{SUSY}}}{M_P} \right)^{\frac{1}{16}} = 0.1 = \epsilon, \\
\langle X_W \rangle &= \langle X_P \rangle = \left( \frac{m_{\text{SUSY}}}{M_P} \right)^{\frac{1}{2}} = 0.01 = \epsilon^2, \\
\langle W_1 \rangle &= \epsilon^2 a c_W, \\
\langle W_2 \rangle &= \epsilon^2 \beta s_W, \\
\langle P \rangle &= \epsilon^2 \gamma, \\
\langle P_W \rangle &= \epsilon^2 \gamma_W, \\
\langle V_1 \rangle &= \epsilon^2 c_V, \\
\langle V_2 \rangle &= \epsilon^2 s_V,
\end{align*}
\]  
\( (18) \)

where \( c_V = \cos \theta_V, s_V = \sin \theta_V \) and Greek letters are complex except for \( \epsilon \). In this paper, we assume original Lagrangian has CP symmetry and all parameters in it are real. Therefore the complex VEVs given in Eq.(18) induce spontaneous CP violation. We illustrate how to get these VEVs as follows. Without loss of generality, we can define \( \langle X \rangle, \langle X_W \rangle, \langle X_P \rangle, \langle V_1 \rangle \) to be real by the redefinition of superfields. The superpotential of \( V_i \) is given by

\[
W = \frac{a}{M_P} (V_1^2 + V_2^2)^3 (3V_1^2 V_2 - V_2^3) + \frac{b}{M_P} (3V_1^2 V_2 - V_2^3)^3.
\]  
\( (19) \)

This superpotential and soft SUSY breaking terms give polynomial potential as

\[
V(s_3) = C_0 + C_1 s_3 + C_2 s_3^2 + C_4 s_3^3, \quad s_3 = \sin 3\theta_V,
\]  
\( (20) \)

where all coefficients are real. The minimum condition of this potential with \( \theta_V \),

\[
c_3[1 + 2C_2 s_3 + 4C_4 s_3^3] = 0,
\]  
\( (21) \)

has trivial solution \( c_3 = 0 \) and non-trivial solutions which satisfy the equation

\[
C_1 + 2C_2 s_3 + 4C_4 s_3^3 = 0.
\]  
\( (22) \)

Which of these solutions is selected for the global minimum depends on the parameters in the potential. Since the solution \( c_3 = 0 \) gives wrong prediction such as \( V_{ud} = 0 \), we assume the solution of Eq.(22) corresponds to the global minimum. Whether the solution of Eq.(22) is real or complex also depends on the parameters. As we choose real solution, \( \langle V_2 \rangle \) has no relative phase. On the other hand for \( V(X_P, P) \) and \( V(X_W, W_i, P_W) \), we assume \( \langle W_1 \rangle, \langle W_2 \rangle, \langle P \rangle, \langle P_W \rangle \) have relative phases which break CP symmetry. The scale of VEV is fixed as follows. If the superpotential of superfield \( \Psi \) is given by

\[
W = \frac{\Psi^n}{n M_P^{n-3}},
\]  
\( (23) \)

then the potential of \( \Psi \) is given by

\[
V(\Psi) = m_\Psi^2 |\Psi|^2 - \frac{A \Psi^n}{M_P^{n-3}} + \frac{|\Psi^{n-1}|^2}{M_P^{2n-6}},
\]  
\( (24) \)

where \( m_\Psi \sim A \sim m_{\text{SUSY}} \) is assumed. At the global minimum \( \langle \Psi \rangle \neq 0 \), as the each terms in the potential should be balanced, the scale of VEV is fixed by

\[
\langle \Psi \rangle = \frac{m_{\text{SUSY}}}{M_P} \left( \frac{m_{\text{SUSY}}}{M_P} \right)^{\frac{1}{n-2}}.
\]  
\( (25) \)
2.3 Soft SUSY breaking terms

We assume that the effect of SUSY breaking in hidden sector is mediated by gravity and induces soft SUSY breaking terms in observable sector. Since these terms are non-universal in general, large flavor-changing processes are induced by the sfermion exchange. From the experimental constraints on them, the assignment of quarks and leptons under the flavor symmetry are restrictive.

After the flavor violation, soft breaking scalar squared mass matrices become non-diagonal. For the Higgs scalars, this gives the mixing mass terms as follow

\[ V = m^2 \epsilon^4 (H_i^U)^* H_i^U + m^2 \epsilon^5 (H_i^D)^* H_i^D + m_3^2 S_i^a S_i^b, \]  

(26)

which compel extra scalars to develop VEVs as

\[ \langle H_i^U \rangle = O(\epsilon^4) v_u, \quad \langle H_i^D \rangle = O(\epsilon^5) v_d, \quad \langle S_i \rangle = O(\epsilon^6) v_s, \]  

(27)

where we put

\[ \langle H_3^U \rangle = v_u = 150.7 \text{GeV}, \quad \langle H_3^D \rangle = v_d = 87.0 \text{GeV}, \quad \langle S_3 \rangle = v_s \gg \sqrt{v_u^2 + v_d^2} = 174.0 \text{GeV}. \]  

(28)

The VEVs of the extra scalar are very small, we call \( H_i^U \) and \( H_i^D \) ”almost inert-Higgs” (AIH) and \( S_i \) as ”almost inert-singlet” (IS). As \( S_1 \) does not develop VEV, we call it ”inert-singlet” (IS). As the same effects affect the flavons, the VEV directions given in Eq.(17) and (18) are modified as follows,

\[ \langle \Phi_1 \rangle \sim \langle \Phi_2 \rangle \sim O(\epsilon^4) V, \quad \langle \Phi_3 \rangle = \frac{V}{\sqrt{3}} + O(\epsilon^4) V, \quad \frac{V_1}{M_P} = \epsilon^2 c_V + O(\epsilon^6), \quad \cdots, \]  

(29)

and so on. Note that dominant parts of scalar squared mass matrices of AIH and G Higgs are diagonal and degenerated. Due to the smallness of VEVs of AIH and AIS, the superpartners of AIH and G Higgs also have diagonal and degenerated mass matrices. Therefore the trace of \( S_4 \) symmetry is imprinted in their mass spectra which may be testable for LHC or future collider.

2.4 The size of \( V \)

The size of \( V \) given in Eq.(17) should satisfy following condition. The interactions of G Higgs are given by

\[ W = \frac{1}{M_P^2} \Phi_3 \Phi_5 G_a (U_3^c E_3^c + Q_3 Q_3) = \frac{V^2}{\sqrt{3} M_P^2} (G_1 + G_2 + G_3) (U_3^c E_3^c + 2 U_3^c D_3), \]  

(30)

from which each G Higgs can decay to top and tau, where we assume G Higgs is lighter than G higgsino and G higgsino can decay to G Higgs. The decay width of G Higgs is given by

\[ \Gamma(G \to t + \tau) + \Gamma(G \to t + b) \simeq \frac{m_G}{16 \pi} \left( \frac{V^2}{\sqrt{3} M_P^2} \right)^2 (5^2 + 8 \times 13^2), \]  

(31)

where \( m_G \) is mass of G Higgs and the factor 5 and 13 are the approximate value of renormalization factors [7]. As the the life time of G Higgs

\[ \tau(G) \simeq 7.2 \times 10^{-29} \left( \frac{\text{TeV}}{m_G} \right) \left( \frac{M_P}{V} \right)^4 \text{sec}, \]  

(32)

should be shorter than 0.1 second in order not to spoil the success of BBN, we have to require

\[ m_G > 0.072 \left( \frac{10^{-6.5} M_P}{V} \right)^4 \text{TeV}. \]  

(33)

For \( m_G = 1 \text{TeV}, V/M_P > 0.52 \times 10^{-6.5} \) should be satisfied. Assuming the A-term dominant potential as

\[ V(\Phi, \Phi^c) = \left\{ -A \Phi_3^2 (\Phi_3^c)^2 + (\Phi_5^c)^2 + (\Phi_3^c)^2 \right\} + h.c. \right) + (D - \text{term}) + (F - \text{term}), \]  

(34)
the size of $V$ is given by

$$\frac{V}{M_P} \sim \frac{1}{Y_1^\Phi} \left( \frac{A}{M_P} \right)^{\frac{1}{2}} \sim (Y_1^\Phi)^{-1} \times 10^{-7.5},$$

(35)

which requires $Y_1^\Phi \sim 0.1$ to give $V/M_P = 10^{-6.5}$. Note that we assume $Y = O(1)$ means $0.1 \leq Y \leq 1$ in this paper. Therefore the natural size of $V$ is given by $V/M_P \leq 10^{-6.5}$. In order to satisfy Eq.(33) for TeV scale G Higgs, the size of $V$ should be in the region as follow

$$0.52 \times 10^{-6.5} \leq \frac{V}{M_P} \leq 10^{-6.5}.$$

(36)

3 Quark Sector

The superpotential of quark sector is given by

$$W = H_3^U Q Y^U U^c + H_3^D Q Y^D D^c + H_1^U Q Y_{AIH}^U U^c + H_1^U Q Y_{AIH} D^c,$$

(37)

where Yukawa matrices are calculated as follows,

$$Y^U = \begin{pmatrix}
Y_U^{c \ell_6} & -Y_U^{s \ell_3} & Y_U^{c \ell_2} \\
Y_U^{s \ell_6} & Y_{2}^{c \ell_3} & Y_{2}^{s \ell_2} \\
Y_{5}^{c \ell_4} & 0 & Y_{3}^{u \ell_4}
\end{pmatrix},$$

(38)

$$Y^D = \begin{pmatrix}
-Y_{D}^{c \ell_5} & Y_{D}^{s \ell_4} & Y_{4}^{D (a_s c - a_p \gamma s) \ell_4} \\
Y_{1}^{D 2} & Y_{2}^{D 2} \ell_4 & Y_{4}^{D (a_s s + a_p \gamma c) \ell_4} \\
0 & 0 & Y_{3}^{D 2} \ell_4
\end{pmatrix},$$

(39)

where $c_{2V} = \cos 2\theta_V$ and $s_{2V} = \sin 2\theta_V$. As the Kähler potential receives the effect of flavor violation, superfields must be redefined as

$$U^c \rightarrow V_K(U) U^c, \quad D^c \rightarrow V_K(D) D^c, \quad Q \rightarrow V_K(Q) Q,$$

$$V_K(U) = \begin{pmatrix}
1 & a_1 \ell_7 & u_2 \ell_4 \\
a_1 \ell_7 & 1 & a_3 \ell_6 \\
u_2 \ell_4 & a_3 \ell_6 & 1
\end{pmatrix}, \quad V_K(D) = \begin{pmatrix}
1 & \beta_1 \ell_5 & \beta_2 \ell_9 \\
\beta_1 \ell_5 & 1 & \beta_3 \ell_8 \\
\beta_2 \ell_9 & \beta_3 \ell_8 & 1
\end{pmatrix},$$

(40)

in order to get canonical kinetic terms [15]. As the result, quark mass matrices are given by

$$M'_U = \begin{pmatrix}
Y_U^{c \ell_6} & -Y_U^{s \ell_3} & Y_U^{c \ell_2} \\
Y_U^{s \ell_6} & Y_{2}^{c \ell_3} & Y_{2}^{s \ell_2} \\
Y_{5}^{c \ell_4} & 0 & Y_{3}^{u \ell_4}
\end{pmatrix},$$

(41)

$$M'_D = \begin{pmatrix}
-Y_{D}^{c \ell_5} & Y_{D}^{s \ell_4} & Y_{4}^{D (a_s c - a_p \gamma s) \ell_4} \\
Y_{1}^{D 2} & Y_{2}^{D 2} \ell_4 & Y_{4}^{D (a_s s + a_p \gamma c) \ell_4} \\
0 & 0 & Y_{3}^{D 2} \ell_4
\end{pmatrix},$$

(42)

$$\alpha_D = a_s c - a_p \gamma s, \quad \beta_D = a_s s + a_p \gamma c, \quad |\alpha_D|^2 + |\beta_D|^2 = 1,$$

(43)

where some parameters are redefined for simplicity, such as $Y_5^{U} + u_2 Y_3^{U} \rightarrow Y_5^{U}$. Note that the each elements in these matrices include only leading terms and the contributions to mass matrices from $Y_{AIH}$ are negligible with this approximation. These matrices are diagonalized by the superfield redefinitions

$$U \rightarrow L_U U, \quad D \rightarrow L_D D, \quad U^c \rightarrow R_U U^c, \quad D^c \rightarrow R_D D^c,$$

(44)

$$L_U^T = \begin{pmatrix}
cv & -sv & -(Y_3^{U} / Y_5^{U}) \ell_2 \\
-sv & cv & 0 \\
(Y_4^{U} / Y_5^{U}) \ell_2 & (Y_4^{U} / Y_5^{U}) s \ell_2 & 1
\end{pmatrix},$$

(45)
$$L_D^T = \begin{pmatrix} c_{2V} & -s_{2V} & -(Y_{4D}^P/Y_{3D}^P)(\alpha_D c_{2V} - \beta_D s_{2V})\epsilon^2 \\ s_{2V} & c_{2V} & -(Y_{4D}^P/Y_{3D}^P)\beta_D \epsilon^2 \\ (Y_{4D}^P/Y_{3D}^P)\alpha_D \epsilon^2 & (Y_{4D}^P/Y_{3D}^P)\beta_D \epsilon^2 & 1 \end{pmatrix},$$

$$R_U = \begin{pmatrix} 1 & N_U \alpha_2 \epsilon^7 & (Y_{5D}^U/Y_{3D}^U)\epsilon^4 \\ -N_U \alpha_2 \epsilon^7 & 1 & -\alpha_3 \epsilon^5 \\ -(Y_{5D}^U/Y_{3D}^U)\epsilon^4 & -\alpha_3 \epsilon^5 & 1 \end{pmatrix},$$

$$R_D = \begin{pmatrix} 1 & \xi_1 \epsilon^5 & (\xi_1/Y_{3D}^P)\epsilon^3 \\ -\xi_1 \epsilon^5 & 1 & (\xi_2/Y_{3D}^P)\epsilon^4 \\ -(\xi_1/Y_{3D}^P)\epsilon^3 & -(\xi_2/Y_{3D}^P)\epsilon^4 & 1 \end{pmatrix},$$

from which we get

$$L_D^T M_U R_U = \text{diag}(m_u, m_c, m_t) = \text{diag} \left( (Y_{1D}^U - Y_{4D}^U/Y_{3D}^U)\epsilon^6 v_u, Y_{2D}^U \epsilon^a v_u, Y_{3D}^U v_u \right),$$

$$L_D^T M_D^T R_D = \text{diag}(m_d, m_s, m_b) = \text{diag} \left( -Y_{1D}^D \epsilon^5 v_d, Y_{2D}^D \epsilon^a v_d, Y_{3D}^D \epsilon^2 v_d \right),$$

$$V_{CKM} = L_U^T L_D = \begin{pmatrix} c_{3V} & s_{3V} & V_{ub} \\ s_{3V} & c_{3V} & V_{cb} \\ V_{td} & V_{ts} & 1 \end{pmatrix},$$

$$V_{ub} = (r_D(\alpha_D^c c_{2V} + \beta_D s_{2V}) - r_U)\epsilon^2 = (a_s r_D - r_U)\epsilon^2, \quad V_{cb} = r_D(\beta_D^c c_{2V} - \alpha_D s_{2V})\epsilon^2 = a_p r_D \gamma^*\epsilon^2, \quad V_{td} = (r_D(\alpha_D s_{2V} - \beta_D c_{2V}) - s_D r_U)\epsilon^2 = [s_D (r_U - a_s r_D) - a_p r_D s_D \gamma]\epsilon^2, \quad V_{ts} = (r_D s_{3V} - s_D(\alpha_D^c s_{2V} + \beta_D^c c_{2V}))\epsilon^2 = [s_D (r_U - a_s r_D) - a_p r_D s_D \gamma]\epsilon^2, \quad r_U = Y_{1D}^U/Y_{3D}^U, \quad r_D = Y_{4D}^U/Y_{3D}^U.$$
where these matrices are defined for canonically normalized superfields. After the diagonalization of Yukawa matrices, the squared mass matrices are given by

\[
(m^2_U)^{SCKM} = R^\dagger U m^2 U R_U = m^2 \begin{pmatrix}
O(1) & e^7 & e^4 \\
\ell^4 & O(1) & e^5 \\
e^5 & e^4 & O(1)
\end{pmatrix},
\]

(60)

\[
(m^2_D)^{SCKM} = R^\dagger D m^2 D R_D = m^2 \begin{pmatrix}
O(1) & e^5 & e^5 \\
e^5 & O(1) & e^4 \\
e^5 & e^4 & O(1)
\end{pmatrix},
\]

(61)

\[
(m^2_Q)^{SCKM} = L^\dagger U_D m^2 U_D L_U,D = m^2 \begin{pmatrix}
1 & e^4 & e^2 \\
\ell^4 & 1 & e^2 \\
e^2 & e^2 & O(1)
\end{pmatrix},
\]

(62)

and the (1,1) elements of \(A_U\) and \(A_D\) remain real except for the next leading terms which are at most \(O(\ell^{10})\) and \(O(\ell^9)\), therefore the SUSY contributions to electric dipole moment of the neutron are negligible. Note that the (1,2) and (2,1) elements of \((m^2_Q)^{SCKM}\) are complex. The off-diagonal elements of squark mass matrices contribute to flavor and CP violation through the squark exchange, on which are imposed severe constraints. With the mass insertion approximation, the most stringent bound for the squark mass \(M_Q\) is given by \(\epsilon K\) as

\[
\sqrt{\text{Im}[(m^2_Q)_{12}(m^2_D)_{12}]/M_Q} = \epsilon^{4.5} < 4.4 \times 10^{-4} \left(\frac{M_Q}{\text{TeV}}\right) \rightarrow M_Q > 72\text{GeV},
\]

(63)

where \(M_Q = M(\text{gluino}) = M(\text{squark})\) is assumed [18]. This bound is very weak and the SUSY FCNC problem is solved. Note that if CP symmetry was not imposed, \(A^D_1\) would be complex and the constraint on neutron EDM would give stronger bound, \(M_Q > 625\text{GeV}\).

The contribution to FCNC from AH\(I\)H exchange is also suppressed due to the hierarchical structure of \(Y^{U,D}_{A\bar{I}H}\). Assuming the mass degeneracy of CF-even AH\(I\)H and CP-odd AH\(I\)H, the strongest mass bound for AH\(I\)H is given by \(D^0 - D^0\) as \(m_{A\bar{I}H} > 79\text{GeV}\) [12].

### 4 Lepton Sector

The superpotential of lepton sector is given by

\[
W = H^D_3 L Y^E E^c + H^U_3 L Y^N N^c + \frac{1}{2} \Phi_3 N^c Y^M N^c + H^D_1 L Y^E_{A\bar{I}H} E^c + H^U_1 L Y^N_{A\bar{I}H} N^c + \frac{1}{2} \Phi_1 N^c Y^M_{A\bar{I}H} N^c,
\]

(64)

where

\[
Y^E = \begin{pmatrix}
Y^E_{1\ell^5} \\
Y^E_{4\alpha E \ell^3} & -Y^E_{4\beta S W \ell^3} & Y^E_{3 \alpha C W \ell^2} \\
Y^E_{4\beta E \ell^5} & Y^E_{2 \alpha C W \ell^3} & Y^E_{3 \beta S W \ell^2}
\end{pmatrix},
\]

(65)

\[
Y^N = \begin{pmatrix}
\ell^5 & 0 & 0 \\
Y^N_{1 C V} & Y^N_{2 N} + Y^N_{5 S W} & Y^N_{1 C V} \\
Y^N_{5 \alpha C W} + Y^N_{2 N} & Y^N_{2 N} - Y^N_{5 S W}
\end{pmatrix},
\]

(66)

\[
Y^M = \begin{pmatrix}
Y^M_{1 \ell^6} & 0 & 0 \\
Y^M_{1 \ell^6} & 0 & 0 \\
0 & 0 & Y^M_{3 \ell^4}
\end{pmatrix},
\]

(67)

\[
\alpha_E = b_s C V + b_w (\beta C V S W + \alpha S C W) - b_p \gamma W S V, \\
\beta_E = b_s S V + b_w (\alpha C V C W - \beta S V S W) + b_p \gamma W C V, \quad |\alpha_E|^2 + |\beta_E|^2 = 1.
\]

(68)

Because the all elements of MNS matrix are \(O(1)\), the contributions of flavor violation in Kähler potential and \(Y_{A\bar{I}H}\) and \(Y^M_{A\bar{I}H}\) are negligible. Therefore the lepton mass matrices are given by

\[
M_E = Y^E_{v d} v_d \begin{pmatrix}
Y^E_{4 \ell^5} & 0 & 0 \\
Y^E_{4 \alpha E \ell^3} & -Y^E_{4 \beta S W \ell^3} & Y^E_{3 \alpha C W \ell^2} \\
Y^E_{4 \beta E \ell^5} & Y^E_{2 \alpha C W \ell^3} & Y^E_{3 \beta S W \ell^2}
\end{pmatrix},
\]

(69)
The charged lepton mass matrix is diagonalized by superfield redefinitions

\[ E \rightarrow L_E E, \quad E^c \rightarrow R_E E^c, \quad N \rightarrow L_E N \]

\[ L_E^T = \begin{pmatrix} 1 & \epsilon^4 & \epsilon^6 \\ \epsilon^4 & 1 & \epsilon^2 \\ \epsilon^6 & \epsilon^2 & 1 \end{pmatrix}, \]

\[ R_E = \begin{pmatrix} 1 & \epsilon^2 & \epsilon \\ \epsilon^2 & 1 & \epsilon \end{pmatrix}, \]

from which we get

\[ L_E^T M_E R_E = \text{diag}(m_e, m_\mu, m_\tau) = \text{diag}(Y_1^E \epsilon^5 v_d, -Y_2^E (\beta^2 s_W^2 + \alpha^2 c_W^2) \epsilon^3 v_d, Y_3^E \epsilon^2 v_d), \]

\[ M'_E = L_E^T \sqrt{M_\nu} L_E \]

\[ = \begin{pmatrix} x_1^2 & x_1 (M_{21} c_Y + M_{22} s_Y) & x_1 (M_{31} c_Y + M_{32} s_Y) \\ x_1 (M_{21} c_Y + M_{22} s_Y) & M_{21}^2 + M_{22}^2 & M_{21} M_{31} + M_{22} M_{32} \\ x_1 (M_{31} c_Y + M_{32} s_Y) & M_{21} M_{31} + M_{22} M_{32} & M_{31}^2 + M_{32}^2 \end{pmatrix}, \]

\[ M_{21} = \beta s_W x_2 + (\beta^2 s_W^2 - \alpha^2 c_W^2) x_5 - \alpha c_W c_W x_4, \]

\[ M_{31} = \alpha^* c_W x_2 + (\alpha^* \beta + \alpha^* \beta^*) c_W s_W x_5 + \beta^* s_W s_W x_4, \]

\[ M_{22} = -\alpha c_W x_2 + 2 \alpha^* c_W s_W x_3 + \beta^* s_W s_W x_4, \]

\[ M_{32} = \beta^* s_W x_2 + (\beta^2 s_W^2 - \alpha^2 c_W^2) x_5 - \alpha^* c_W c_W x_4. \]

The experimental values of charged lepton running masses at 1TeV

\[ m_e = 4.895 \times 10^{-4}, \quad m_\mu = 0.1033, \quad m_\tau = 1.757, \quad (\text{GeV}) \quad [17], \]

are realized by putting the parameters at \( \mu = M_\nu \) as follows

\[ |Y_1^E| = 0.30, \quad |Y_2^E (\alpha^2 c_W^2 + \beta^2 s_W^2)| = 0.62, \quad |Y_3^E| = 1.1, \]

where we use the renormalization factors given in [7]. Charged lepton mass hierarchy is realized without fine tuning which is the same as for quark sector.

In order to realize neutrino mass scale \( m_\nu \sim O(0.01) \text{eV} \), the relations

\[ x_1^2 = \frac{(Y_i^N \epsilon^2 v_u)^2}{Y_i^{M V}} \sim O(0.01) \text{eV} \quad \rightarrow \quad Y_1^{M V} \sim 10^{11} \text{GeV}, \quad M_1 = \epsilon^6 Y_1^{M V} \sim 10^5 \text{GeV} \]

are required. After the diagonalization of charged lepton Yukawa matrix, the squared mass matrix and A-term matrix are given by

\[ (m_L^2)_{SMNS} = (L_E^T m_L^2 L_E^*) = m^2 \begin{pmatrix} O(1) & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & 1 & \epsilon^4 \\ \epsilon^4 & \epsilon^4 & 1 \end{pmatrix}, \]

\[ (A'_E)_{SMNS} = L_E^T A_E R_E = v_d \begin{pmatrix} A_1 \epsilon^5 & O(\epsilon^7) & O(\epsilon^6) \\ O(\epsilon^5) & O(\epsilon^3) & O(\epsilon^4) \\ O(\epsilon^5) & O(\epsilon^3) & O(\epsilon^2) \end{pmatrix}, \]

where the (1,1) element of \( A'_E \) is real at leading order and the SUSY contribution to electric dipole moment of the electron is negligible. Based on consideration of the lepton flavor violations, the most stringent bound for slepton mass \( M_L \) is given by \( \mu \rightarrow e + \gamma \) as

\[ \frac{v_d \epsilon^5}{M_L} < 3.4 \times 10^{-6} \left( \frac{M_L}{300 \text{GeV}} \right) \quad \rightarrow \quad M_L > 300 \text{GeV}, \]
where $M_L = M(\text{slepton}) = M(\text{photino})$ is assumed [18]. Without CP symmetry, the constraint on electron EDM would give stronger bound, $M_L > 1765\text{GeV}$.

The contribution to lepton flavor violation from AIH exchange is also suppressed due to the hierarchical structure of $Y^L_{AIH}$ as same as quark sector. Assuming the mass degeneracy of CP-even AIH and CP-odd AIH, the strongest mass bound for AIH is given by $\mu \to e + \gamma$ as $m_{AIH} > 38\text{GeV}$ [12].

For the canonically normalized superfields, RHN mass matrix is given by

$$M_N = V_K^T(N)Y^MVV_K(N) = \begin{pmatrix} M_1 & M\epsilon^4 & 0 \\ M\epsilon^4 & M_1(1 + \epsilon^4) & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad M \sim M_1, \quad M_3 = Y^N_N V \epsilon^4,$$

(87)

$$V_K(N) = \begin{pmatrix} 1 & \epsilon^4 & 0 \\ \epsilon^4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(88)

which gives degenerated mass spectrum of RHNS as follow

$$M_1 \simeq M_2 = M_1(1 + \epsilon^4) \ll M_3 \rightarrow \delta_N = \frac{M_2 - M_1}{M_1} \sim \epsilon^4.$$

(89)

The right handed sneutrinos have same spectrum. In the early universe, the out-of-equilibrium decay of $\eta_i^c$ and $N_i^c$ generates B-L asymmetry which is transferred to a baryon asymmetry by EW sphaleron processes. Following Ref. [19], the baryon asymmetry is given by

$$B_f \sim \frac{\kappa\epsilon_{CP}}{3g_s},$$

(90)

where $g_s = 340$ is the degree of freedom of radiation, $\kappa$ is dilution factor which is given by

$$\kappa \sim \frac{1}{K \ln K},$$

(91)

$$K = \frac{\Gamma(M_1)}{2H(M_1)}, \quad \Gamma(M_1) = \frac{K_{11}M_1}{8\pi}, \quad H(M_1) = \sqrt{\frac{\pi^2 g_s M_1^4}{90 M_P^2}}, \quad K_{ij} = \sum_{l=1}^3 (Y_{i_i}^N)^*(Y_{i_j}^N),$$

(92)

and $\epsilon_{CP}$ is given by

$$\epsilon_{CP} = -\frac{1}{2\pi} \frac{\text{Im}(K_{12}^2)}{K_{11}} \left( \frac{2\sqrt{x}}{x - 1} + \sqrt{x} \ln \frac{1 + x}{x} \right) \simeq -\frac{\text{Im}[K_{12}^2]}{2\pi K_{11}\delta_N}, \quad x = \frac{M_2^2}{M_1^2} \simeq 1 + 2\delta_N.$$

(93)

From the order estimations as follows

$$K_{12} \sim K_{11} \sim \epsilon^{10}, \quad K \sim \left( \frac{0.7\text{PeV}}{M_1} \right), \quad \epsilon_{CP} \sim 10^{-6}, \quad M_1 \sim 10^5\text{GeV},$$

(94)

we get the correct amount of baryon asymmetry $B_f \sim 10^{-10}$. Right sign of baryon number corresponds to

$$0^\circ < 2\theta_{CP} < 180^\circ, \quad \theta_{CP} = \arg(K_{12}).$$

(95)

## 5 Dark Matter

The lightest SUSY particles of this model is singlino $s_2$ which is the superpartner of AIS $S_2$. The superpotential of Higgs sector is given by

$$W = \lambda_1 S_3 (H_1^U H_1^D + H_2^U H_2^D) + \lambda_3 S_3 H_3^U H_3^D + \lambda_2 S_2 [c_V (H_1^U H_2^D + H_2^U H_1^D) + s_V (H_1^U H_1^D - H_2^U H_2^D)] + \lambda_4 S_2 [c_V H_1^U + s_V H_2^U] S_2 H_3^D + \lambda_5 S_2 e^U H_3^D + s_V H_3^D,$$

(96)

which gives

$$M(s_2) \sim \frac{\epsilon^5 \lambda_4 v_d (\epsilon^6 \lambda_5 v_u)}{\epsilon \lambda_1 c_s} \sim 1\text{eV}.$$

(97)
Although $s_2$ is not the dominant component of dark matter, it may help to explain the delay of structure formation [20]. The massless singlino $s_1$ and LSP $s_2$ behave as extra neutrinos and change the effective neutrino generation number to

$$N_{\text{eff}} = 3.194 \quad [21],$$

where $m_{Z'} < 4700\text{GeV}$ is assumed. This extra contributions soften the discrepancy of the expansion rate $H_0$ between the measurements of type Ia supernovas and Cepheid variable and CMB data [20].

The interaction of bino is given by

$$\mathcal{L} = -i\frac{g_Y}{\sqrt{2}}(H^D_3)^*\lambda_Y h^D_3,$$

where higgsino $h^D_3$ has mixing mass term with $s_2$ as follow

$$\mathcal{L} = \lambda_4 e^5 (H^U_3)^* s_2 h^D_3 \sim e^9 v_s s_2 h^D_3.$$  

(100)

Therefore bino life time is calculated as follow

$$\Gamma(\lambda_Y \rightarrow H + s_2) \sim \frac{g^2_{\text{SUSY}}}{4\pi} \left(\frac{e^9 v_s}{m_{\text{SUSY}}}\right)^2 \sim 10^{-10}\text{eV} \rightarrow \tau \sim 10^{-5}\text{sec},$$

which is consistent with standard cosmology.

Five of the six flavon multiplets $\Phi_i$ have lighter masses than $100\text{TeV}$ and are produced non-thermally through the $U(1)_Z$ gauge interaction. As the lightest flavon (LF) is quasistable, therefore it is the candidate of DM. Solving the Boltzmann equation with the boundary condition $n_{\text{LF}}(T_{RH}) = 0$, we get a relic abundance of the LF as [22]

$$\Omega_{\text{LF}} h^2 = 5.0 \times 10^{-3} \left(\frac{T_{RH}}{10^7\text{GeV}}\right)^3 \left(\frac{10^{12}\text{GeV}}{V}\right)^4 \left(\frac{m_{\text{LF}}}{4\text{TeV}}\right).$$

(102)

The heaviest RHN $n_3$ behaves like LF and has a relic abundance given by

$$\Omega_{n_3} h^2 = 0.6 \left(\frac{T_{RH}}{10^7\text{GeV}}\right)^3 \left(\frac{10^{12}\text{GeV}}{V}\right)^4 \left(\frac{M_3}{10^7\text{GeV}}\right);$$

(103)

where $M_3$ is defined by Eq.(87) and we put $M_3 \sim 10^7\text{GeV}$ and $Y^M_3 \sim 0.1$ based on following reason. As $M_3 \gg m_{\text{LF}}$ is satisfied, the contribution of LF to $\Omega_{\text{DM}}$ is negligible. $n_3$ can decay through the interaction in Kähler potential as

$$K = \frac{1}{M_P^2} [X^\dagger N_3(X^\dagger)^* X_W W_i S_1(S_3)^* + (P\nu-\text{contribution})]_{ij} = \frac{e^5}{M_P^2} n_3^5 (\xi_1 n_1^c + \xi_2 n_2^c) s_1 s_3,$$

(104)

and has a life time given by

$$\Gamma(n_3^c \rightarrow n_1^c + s_1 + s_3) \sim \frac{e^{10} M_3^5}{O(100)\pi^3 M_P^4} \sim 10^{-43}\text{eV} \rightarrow \tau \sim 10^{28}\text{sec}.$$  

(105)

The daughter particles $n_{1,2}^c$ decay to $\nu + H$ and give neutrino flux at $E_\nu \sim M_3/6 \sim \text{PeV}$. From the latest results of IceCube [1], the mass and life time of DM are around PeV and $10^{28}\text{sec}$ [23]. If we identify $n_3^c$ as DM, $M_3$ is fixed at $10^7\text{GeV}$ from the IceCube results. Note that reheating temperature should be $T_{RH} < 10^7\text{GeV}$ to avoid gravitino over production. After the three parameters in Eq.(103) are fixed as follows

$$0.52 \times 10^{-6.5} < \frac{V}{M_P} < 10^{-6.5}, \quad M_3 \sim 10^7\text{GeV}, \quad 10^7\text{GeV} > T_{RH} > M_3,$$

(106)

the right amount of DM, $\Omega_{n_3} h^2 \sim O(1) - O(10)$ is realized. This is an interesting prediction of our model. For the reheating temperature $T_{RH} \sim 10^7\text{GeV}$, the thermal mass of flavon $\Phi_3$ is estimated as follow

$$m_T(\Phi_3) \sim e^6 T_{RH} \sim 10\text{GeV},$$

(107)
which is small enough to avoid symmetry restoration. If inflaton $\Psi$ decays through Planck suppressed interaction, for example, such as

$$W = \frac{1}{M_p} \Psi H^c Q_3 U^c_3,$$

(108)

then reheating temperature is given by

$$T_{RH} \sim \sqrt{M_p \Gamma(\Psi)} \sim \left( \frac{M_p m^3(\Psi)}{M_p^2} \right)^{\frac{1}{2}},$$

(109)

which requires inflaton mass $m(\Psi) \sim 10^{11}$GeV for our model.

### 5.1 Numerical analysis

Finally we calculate detection probabilities of neutrino flavors emitted through dark matter decay and argument of $K_{12}$. The neutrino mass matrix is given by

$$M'_{\nu} = U'_{MNS} \text{diag}(0, m_2, m_3) U'_{MNS}^\dagger,$$

(110)

$$U_{MNS} = \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & -c_{23} s_{13} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -s_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{pmatrix} \text{diag}(1, e^{i\phi}, 1),$$

(111)

on the basis that charged lepton mass matrix is diagonalized. From the experimental values as follows [16],

\begin{align*}
\sin^2 \theta_{12} &= 0.308 \pm 0.017, \\
\sin^2 \theta_{23} &= 0.437^{+0.033}_{-0.023}, \\
\sin^2 \theta_{13} &= 0.0234^{+0.0020}_{-0.0019}, \\
\delta / \pi &= 1.39^{+0.38}_{-0.27}, \\
\Delta m^2_{21} &= (0.753 \pm 0.018) \times 10^{-4} \text{eV}^2, \\
\Delta m^2_{32} &= (24.4 \pm 0.6) \times 10^{-4} \text{eV}^2,
\end{align*}

(112-117)

we fix the parameters as follows

\begin{align*}
\theta_{12} &= 33.709^\circ, \\
\theta_{23} &= 41.381^\circ, \\
\theta_{13} &= 8.799^\circ, \\
m_1 &= 0 \text{eV}, \\
m_2 &= 0.867756 \times 10^{-2} \text{eV}, \\
m_3 &= 5.015277 \times 10^{-2} \text{eV}, \\
\delta &= 250.0^\circ.
\end{align*}

(118-124)

By the phase rotations of lepton doublets $L_a$, we define the diagonal elements of $(M'_\nu)_{\exp}$ to be real and non-negative and the real parts of $(1,2)$ and $(1,3)$ elements of $(M'_\nu)_{\exp}$ to be non-negative. The same procedure is performed for $M'_\nu$ which is given in Eq.(76). The matching condition $M'_\nu = (M'_\nu)_{\exp}$ gives seven equations with nine free parameters as follows

$$\phi, \quad \theta_a = \arg(\alpha), \quad \theta_b = \arg(\beta), \quad \theta_c = \arg(\gamma_W), \quad \theta_W, \quad x_1, \quad x_2, \quad x_4, \quad x_5.$$ 

(125)

Note that two of nine equations in matching condition are automatically solved due to the condition $\det M'_\nu = \det (M'_\nu)_{\exp} = 0$. In order to solve the equations, two constraints should be added by hand. We fix $(\phi, x_4)$ to solve the equations numerically and calculate branching ratios and argument of $K_{12}$. The former is given by

$$B_e = \frac{\Gamma(n_1 \rightarrow \nu_e H) + \Gamma(n_2 \rightarrow \nu_e H)}{\Gamma(n_1 \rightarrow \nu_e H)} = \frac{x_1^2}{x_1^2 + 2(x_2^2 + x_4^2 + x_5^2)},$$

(126)

$$B_\mu = \frac{\Gamma(n_1 \rightarrow \nu_\mu H) + \Gamma(n_2 \rightarrow \nu_\mu H)}{\Gamma(n_1 \rightarrow \nu_\mu H)} = \frac{x_2^2 + x_3^2 + x_4^2 + 2x_2x_5s_W \cos \theta_b - 2x_4x_5c_W \cos(\theta_a - \theta_c)}{x_1^2 + 2(x_2^2 + x_3^2 + x_4^2)},$$

(127)
\[ B_\tau = \frac{\Gamma(n_1 \rightarrow \nu_\tau H) + \Gamma(n_2 \rightarrow \nu_\tau H)}{\Gamma(\nu H)} = \frac{x_2^2 + x_3^2 + x_4^2 - 2x_2x_3 s_W \cos \theta_b + 2x_4x_5 c_W \cos(\theta_a - \theta_c)}{x_1^2 + 2(x_2^2 + x_3^2 + x_4^2)}, \tag{128} \]

\[ \Gamma(\nu H) = \Gamma(n_1 \rightarrow \nu_\tau H) + \Gamma(n_2 \rightarrow \nu_\tau H) + \Gamma(n_1 \rightarrow \nu_\mu H) + \Gamma(n_2 \rightarrow \nu_\mu H), \tag{129} \]

where we put \( \Gamma(n_3 \rightarrow n_1 + s_1 + s_3) = \Gamma(n_3 \rightarrow n_2 + s_1 + s_3) \) by hand. The latter is given by

\[ \theta_{CP} = \arg(K_{12}) = \arg \left[ (x_1^2 c_V s_V + 2x_2x_5 c_W \cos \theta_a - 2x_4x_5 s_W \cos(\theta_b - \theta_c)) \right. \]

\[ + \left. i(2x_5^2 c_W s_W \sin(\theta_a - \theta_b) - 2x_2x_4 \sin \theta_c) \right], \tag{130} \]

where we assume the contributions from AIH are negligible for simplicity. Due to the neutrino oscillation, the neutrinos emitted by RHN-decay change the flavor, therefore the detection probabilities of neutrino at detector are given by

\[ P(\nu_e) = B_c |U_{e1}|^4 + |U_{e2}|^4 + |U_{e3}|^4 \]

\[ + B_\mu |U_{\mu1}|^2 |U_{\mu2}|^2 + |U_{\mu3}|^4 \]

\[ + B_\tau |U_{\tau1}|^2 |U_{\tau2}|^2 + |U_{\tau3}|^2 \], \tag{131} \]

\[ P(\nu_\mu) = B_c |U_{\mu1}|^2 |U_{\mu2}|^2 + |U_{\mu3}|^4 \]

\[ + B_\mu |U_{\mu1}|^4 + |U_{\mu2}|^4 + |U_{\mu3}|^4 \]

\[ + B_\tau |U_{\tau1}|^2 |U_{\tau2}|^2 + |U_{\tau3}|^2 \], \tag{132} \]

\[ P(\nu_\tau) = B_c |U_{\tau1}|^2 |U_{\tau2}|^2 + |U_{\tau3}|^2 \]

\[ + B_\mu |U_{\mu1}|^2 |U_{\mu2}|^2 + |U_{\mu3}|^2 \]

\[ + B_\tau |U_{\tau1}|^4 + |U_{\tau2}|^4 + |U_{\tau3}|^4 \], \tag{133} \]

where we assume the baseline is longer than PeV/m_c^2 \sim 10^{-4}pc. As the equation \( 3\theta_V = 13.003^\circ \) has three solutions,

\[ \theta_V = 4.334^\circ, \quad 124.334^\circ, \quad 244.334^\circ, \tag{134} \]

we calculate \( \theta_{CP}, P(\nu_{e,\mu,\tau}) \) for each cases. The results are given in Table 3. The dependences of detection probability \( P(\nu_{e,\mu,\tau}) \) on parameters are weak. Depending on \( \phi \), both sign of \( \sin(2\theta_{CP}) \) are possible.

### Table 3: The detection probabilities of neutrino flavors and argument of \( K_{12} \)

Without loss of generality, we can define \( (x_1, x_2, x_3, x_5) \geq 0 \) and \( 180^\circ \geq (\theta_a, \theta_W) \geq 0^\circ \). All angles are given in the unit of degree and \( x_i \) are given in the unit of 0.1\sqrt{m_e^2}.

### 6 Conclusion

In this paper we consider \( S_4 \) flavor-symmetric extra \( U(1) \) model with taking account of high energy neutrino flux observed by IceCube and obtain following results. If we specify dark matter is the heaviest RHN, the mass
scale PeV is understood as follow. The structure of CKM matrix requires $O(10^{-2})$ flavor symmetry breaking and the symmetry should be non-abelian to suppress flavor changing processes induced by sfermion exchange. Due to this symmetry, two lighter RHNs form $S_4$-doublet and have same mass. As the $O(10^{-2})$ flavor symmetry breaking solves the mass degeneracy of $S_4$-doublet RHN by $O(10^{-4})$, the mass of RHN should be 100TeV for a successful resonant leptogenesis. The 3-body decay of heaviest RHN with 10PeV mass generates PeV energy neutrino through the following decay of lighter RHN. Non-thermal production of the heaviest RHN gives right amount of dark matter. The information about leptogenesis may be extracted from neutrino flux.

Although the flavor symmetry is broken, as the footprint of it is left in the degenerated mass spectra of almost inert-Higgs and G Higgs and their superpartners, the existence of flavor symmetry may be testable for LHC or future colliders.

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