Holographic interacting tachyon model

Alberto Rozas-Fernández, David Brizuela and Norman Cruz

1 Instituto de Física Fundamental, CSIC, Serrano 121, 28006 Madrid, Spain
2 Instituto de Estructura de la Materia, CSIC, Serrano 121, 28006 Madrid, Spain
3 Departamento de Física, Facultad de Ciencia, Universidad de Santiago de Chile, Casilla 307, Santiago, Chile
E-mail: a.rozas@cfmac.csic.es

Abstract. We propose an interacting holographic tachyon model of dark energy. A correspondence between the tachyon energy density and the interacting holographic dark energy is established. As a result, we reconstruct the potential of the interacting holographic tachyon field and the dynamics of the tachyon field, in a flat Friedmann-Robertson-Walker background. We show that the evolution of the universe for $-1 < w < 0$ can be completely described by the resulting interacting tachyon model.

1. Introduction

There is a mounting observational evidence in favour of a present accelerating universe. Within the framework of the standard Friedmann-Robertson-Walker (FRW) cosmology, this present acceleration requires the existence of a negative pressure fluid, dubbed dark energy (DE), whose pressure $p_\Lambda$ and density $\rho_\Lambda$ satisfy $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1/3$. Nevertheless, the underlying physical mechanism behind this phenomenon remains unknown.

On the other hand, based on the validity of effective local quantum field theory in a box of size $L$, Cohen et al [1] suggested a relationship between the ultraviolet (UV) and the infrared (IR) cutoffs due to the limit set by the formation of a black hole. The $UV-IR$ relationship gives an upper bound on the zero point energy density

$$\rho_\Lambda \leq L^{-2}M_p^2, \quad (1)$$

where $L$ acts as an IR cutoff and $M_p$ is the reduced Planck mass in natural units. The largest $L$ is chosen by saturating the bound in Eq.(1) so that we obtain the holographic dark energy density

$$\rho_\Lambda = 3c^2M_p^2L^{-2}, \quad (2)$$

where $c$ is a dimensionless $O(1)$ parameter. Interestingly, this $\rho_\Lambda$ is comparable to the observed dark energy density $\sim 10^{-10}eV^4$ for the Hubble parameter at the present epoch $H = H_0 \sim 10^{-33}eV$.

However, $L$ should be taken as the size of the future event horizon of the universe in order to have an accelerated universe [2]

$$R_{ch}(a) = a \int_0^\infty \frac{dt'}{a(t')} = a \int_a^{\infty} \frac{da'}{Ha'^2}, \quad (3)$$

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where \( a \) is the scale factor of the universe. This allows to construct a satisfactory holographic dark energy (HDE) model.

A further development was to consider a possible interaction between dark matter (DM) and the HDE \([3]\).

2. Interacting holographic tachyon dark energy model

The tachyon can act as a source of dark energy and be described by the following effective action \([4]\)

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - V(\phi)\sqrt{1 + g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi} \right],
\]

where \( V(\phi) \) is the tachyon potential. In the flat FRW background the energy density \( \rho_t \) and the pressure \( p_t \) are given by

\[
\rho_t = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad p_t = -V(\phi)\sqrt{1 - \dot{\phi}^2}.
\]

From Eqs. (5) and (6) we obtain the tachyon equation of state parameter

\[
w_t = \frac{p_t}{\rho_t} = \dot{\phi}^2 - 1.
\]

In order to have a real energy density for the tachyon we require \( 0 < \dot{\phi}^2 < 1 \) which implies, from Eq. (7), that the equation of state parameter is constrained to \( -1 < w_t < 0 \).

In order to impose the holographic nature to the tachyon, we should identify \( \rho_t \) with \( \rho_\Lambda \). We consider a spatially flat FRW universe filled with DM and HDE. The Friedmann equation reads

\[
3M_p^2H^2 = \rho_m + \rho_t. \tag{8}
\]

In the case of an interaction between HDE and DM, their energy densities obey

\[
\dot{\rho}_m + 3H\rho_m = Q, \tag{9}
\]

\[
\dot{\rho}_t + 3H(1 + \omega_t)\rho_t = -Q, \tag{10}
\]

where \( Q \) is an interaction term. Here we consider \( Q = 3b^2H(\rho_m + \rho_t) \), where \( b^2 \) is a coupling constant which also measures to what extent the different evolution of the DM due to its interaction with the DE gives rise to a different expansion history of the Universe. A positive \( b^2 \) corresponds to a decay of DE into DM.

Combining the definition of HDE (3) and that of the future event horizon (4) we take the derivative with respect to \( x = \ln a \) and obtain

\[
\rho_t' \equiv \frac{d\rho_t}{dx} = -6M_p^2H^2\Omega_t(1 - \frac{\sqrt{\Omega_t}}{c}), \tag{11}
\]

where \( \Omega_t = \rho_t/(3M_p^2H^2) \). Given that \( \dot{\rho}_t \equiv d\rho_t/dt = \rho_t H \) and making use of the Friedmann equation (8), Eq. (10) can be written as

\[
\rho_t' + 3(1 + \omega_t)\rho_t = -9M_p^2b^2H^2. \tag{12}
\]

Combining the last two equations, we are led to the equation of state of this IHDE model,

\[
w_t = -\frac{1}{3} - \frac{2\sqrt{\Omega_t}}{3} - \frac{b^2}{\Omega_t}, \tag{13}
\]
Inserting Eq. (13) into Eq. (12) and using the definition of $\Omega_t$, we arrive at

$$\frac{H'}{H} = -\frac{\Omega_t'}{2\Omega_t} + \sqrt{\frac{\Omega_t}{c}} - 1.$$  \hspace{1cm} (14)

On the other hand, replacing $\dot{H} = H' H$ and $p_t = w_t \rho_t$ into the derivative of the Friedmann equation with respect to cosmic time $\dot{H} = -\frac{1}{2H^2}(\rho + p)$ (where $\rho$ and $p$ are the total energy density and pressure respectively), we have

$$\frac{H'}{H} = \frac{1}{2} \Omega_t + \frac{\Omega_t^{3/2}}{c} + \frac{3b^2}{2} - \frac{3}{2}.$$  \hspace{1cm} (15)

If we combine now the last two equations, we find the evolution equation for $\Omega_t$

$$\frac{d\Omega_t}{dx} = \Omega_t(1 - \Omega_t) \left(1 + \frac{2\sqrt{\Omega_t}}{c} - \frac{3b^2}{1 - \Omega_t}\right).$$  \hspace{1cm} (16)

Since $\frac{d}{dx} = H \frac{d}{dx} = -H(1 + z) \frac{d}{dz}$ we can rewrite the above equation with respect to $z$ as

$$\frac{d\Omega_t}{dz} = -(1 + z)^{-1} \Omega_t(1 - \Omega_t) \left(1 + \frac{2\sqrt{\Omega_t}}{c} - \frac{3b^2}{1 - \Omega_t}\right).$$  \hspace{1cm} (17)

Therefore, the differential equation for the Hubble parameter $H(z)$ can be expressed as

$$\frac{dH}{dz} = -(1 + z)^{-1} H \left(\frac{1}{2} \Omega_t + \frac{\Omega_t^{3/2}}{c} + \frac{3b^2}{2} - \frac{3}{2}\right).$$  \hspace{1cm} (18)

The above equations can be solved numerically to obtain the evolution of $\Omega_t$ and $H$ as a function of the redshift.

Using Eqs. (5), (7) and (18), we derive the interacting holographic tachyon potential

$$\frac{V(\phi)}{\rho_{cr,0}} = H^2 \Omega_t \sqrt{-w_t},$$  \hspace{1cm} (19)

where $\Omega_t$ and $w_t$ are respectively given by Eqs. (17) and (13), being $\rho_{cr,0} = 3M_p^2 H_0^2$ the critical energy density of the universe at the present epoch. Besides, using Eqs. (7) and (18), the derivative of the interacting holographic tachyon scalar field $\phi$ with respect to the redshift $z$ can be expressed as

$$\frac{\phi'}{H_0^{-1}} = \pm \frac{\sqrt{1 + w_t}}{H(1 + z)}.$$  \hspace{1cm} (20)

The sign is in fact arbitrary as it can be changed by a redefinition of the field $\phi \to -\phi$.

The evolutionary form of the interacting holographic tachyon field can be easily obtained integrating it numerically from $z = 0$ to a given value $z$. The field amplitude at the present epoch ($z = 0$) is taken to vanish, $\phi(0) = 0$.

As already discussed in [5] the interaction $Q$ is very weak and positive and the parameters $b^2$ and $c$ are not totally free; they need to satisfy some constraints. Following the latest observational results for the IHDE models [6, 7], we take $0 \leq b^2 \leq 0.03$ and $\sqrt{\Omega_t} < c < 1.255$.

The reconstructed $V(\phi)$ is plotted in Fig. 2. The tachyon scalar field $\phi(z)$ is also shown in Fig. 1. Figs. 1 and 2 display the dynamics of the interacting tachyon scalar field explicitly. All the potentials are more steep in the early epoch, tending to be flat near today. Consequently, $\phi$ rolls down the potential more slowly as the universe expands and the equation of state parameter tends to negative values close to $-1$ as $\phi \to 0$. As a result $dw_t/d\ln a < 0$. Note that $\phi(z)$ decreases as the universe expands.

A similar behaviour has been obtained in [9] for a holographic tachyon model without interaction.
Figure 1. The evolution of $\phi(z)$, where $\phi$ is in units of $H_0^{-1}$, for $c = 1$ and different values of the coupling. We take here $\Omega_{m0} = 0.27$.

Figure 2. $V$ vs $\phi$, where $\phi$ is in units of $H_0^{-1}$ and $V(\phi)$ in \( \phi_{cr,0} \), for $c = 1$ and different values of the coupling. We take here $\Omega_{m0} = 0.27$.

3. Conclusions

We have proposed an interacting holographic tachyon model of dark energy with the future event horizon as infrared cut-off. We have also carried out a throughout analysis of its evolution and deduce its cosmological consequences. We have used the tachyon scalar field model to mimic the evolving behaviour of the IHDE. As a result, we have reconstructed the interacting holographic tachyon model in the region $-1 < w < 0$.

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