AoI-Constrained Bandit: Information Gathering over Unreliable Channels with Age Guarantees

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Abstract—Age-of-Information (AoI) is an application layer metric that has been widely adopted to quantify the information freshness of each information source. However, few works address the impact of probabilistic transmission failures on satisfying the harsh AoI requirement of each source, which is of critical importance in a great number of wireless-powered real-time applications. In this paper, we investigate the transmission scheduling problem of maximizing throughput over wireless channels under different time-average AoI requirements for heterogeneous information sources. When the channel reliability for each source is known as prior, the global optimal transmission scheduling policy is proposed. Moreover, when channel reliabilities are unknown, it is modeled as an AoI-constrained Multi-Armed Bandit (MAB) problem. Then a learning algorithm that meets the AoI requirement with probability 1 and incurs up to $O(K \sqrt{T \log T})$ accumulated regret is proposed, where $K$ is the number of arms/information sources, and $T$ is the time horizon. Numerical results show that the accumulated regret of our learning algorithm is strictly upper-bounded by $K \sqrt{T \log T}$ and outperforms the AoI-constraint-aware baseline, and the AoI requirement of every source is robustly satisfied.

I. INTRODUCTION

Advances of wireless communications have motivated a wide range of real-time networked applications, e.g., disaster management [1], [2], aerial surveillance [3], [4], smart grid [5], [6], and building automation [7], [8] etc. In these applications, information are gathered from heterogeneous sources to an agent/monitor over unreliable wireless channels whose reliability or state information might be priorly unknown.

To support the real-time decision-making/monitoring requirement in these applications, the gathered information should be fresh enough to avoid outdated decisions or missing anomalies. Moreover, due to the heterogeneity of information sources, the channel reliability and the strictness of information freshness requirement associated with each source varies. For example, sources equipped with lower power transmitter suffers higher path loss [9], and in surveillance, the urgency of data needs in different area monitoring can be prioritized [4]. Overall, in these applications, the information freshness of each source should be bounded, and the bound is normally heterogeneous.

The application-layer information freshness requirement can be well characterized by the Age-of-Information (AoI) metric [10], which is defined as the time elapsed since the last successful data transmission for each information source. Ideally, we may hope that the worst-case AoI of each source is bounded with a threshold. However, data packets might drop during transmissions with a certain probability due to the channel unreliability, indicating that the worst-case AoI of a source is always unbounded when none of the transmission successes. Therefore, to deal with the randomness brought by channel unreliability, we can instead expect to bound the expectation of time-average AoI for each source.

Although extensive AoI-related works have been proposed, the issue of guaranteeing AoI for each of the heterogeneous sources under channels with unknown reliability remains unaddressed. Specifically, most of the existing works aim at minimizing the overall AoI subject to certain constraints (see the complete survey [11]), but obviously these works cannot promise to meet the hard AoI requirement for each source.

There is also another line of research which explores AoI-as-a-constraint setting [10], [12], [13], [14], [15], [16]. Among these works, [12] only addresses the single-source case, and [10], [16] assumes the channel is flawless (i.e., no packet loss), while [13], [14], [15] consider the channel state information as a prior knowledge and aims at minimizing energy consumption or operational cost.

Also, we note that [17] offers the source-average AoI upper bound for his scheduling policy without knowing channel reliabilities, but obviously the scheduling policy does not promise to meet all feasible AoI requirements for each source since the upper bound is independent of reliability of most channels except for the most unreliable one[4]. Overall, none of these works successfully addresses the challenge of transmission failures with unknown probability in guaranteeing the stringent per-source AoI constraint.

In this paper, we address the transmission scheduling problem of gathering information from multiple sources over unreliable channels under the expected time-average AoI thresholds limit for each information source. To be specific, at each time slot, a scheduler selects one source to immediately sample information and transmit data packet over its associated wireless channel and feed it to an agent/monitor, but the transmission might fail with a certain probability (both priorly known and unknown situations are discussed). The throughput (num-

1The author also proposes the AoI upper bound for each source, but the author himself thinks it is "extremely large" and thus impractical.
number of collected data packets) is expected to be maximized to enrich the decision-making/analyzing bases. Meanwhile, to meet the ongoing real-time decision-making/monitoring requirement, the expected (long-term) time-average AoI of each source cannot exceed its given threshold.

Even when the channel reliability is known as a prior, the transmission scheduling problem is challenging since 1) the awareness of channel unreliability imports randomness on both AoI and throughput, and 2) the source heterogeneity asks the scheduler to balance the transmission opportunities assigned to each source to meet different AoI requirements.

When the channel reliability is priorly unknown, the transmission scheduling problem can be viewed as a MAB (Multi-Armed Bandit) problem [18] subject to the AoI constraints. This problem turns to be much more challenging since it requires the scheduler to extra-learn from probabilistic transmission results while maximizing throughput. Specifically, maximizing gathered information from heterogeneous sources over channels with unknown reliability can be modeled as maximizing accumulated reward by pulling different arms, where each arm is associated with a Bernoulli distribution and the reward of pulling an arm is sampled from it. The reward distribution is unknown and the scheduler can only learn by observing the sampling results.

Recently, a few AoI-related bandit problems [19], [20], [21] have been proposed, but all these works focus on AoI minimization instead of guaranteeing AoI performance on each source. Although massive bandit problems aiming at maximizing accumulated reward has been studied in the literature of bandit theory (see the book [22]), none of them explores the AoI-as-a-constraint and can be casted to be bandit literature of bandit theory (see the book [22]), none of them maximizing accumulated reward has been studied in the literature that successfully addresses unknown reliability of wireless channels, it is assumed that packets generated at source $i$ can be successfully received by BS with probability $p_i \in (0, 1)$, where $p_i$ is the channel reliability and $\bar{p} = (p_1, \cdots, p_K)$ is the channel reliability vector. To denote the transmission result at each round $t$, a binary variable $r(t)$ is introduced, i.e.,

$$r(t) = \begin{cases} 1, & \text{if transmission at round } t \text{ succeeds.} \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

To enrich the decision-making/analyzing bases, the information is expected to be gathered as much as possible. Hence the objective of the scheduler is to maximize the network throughput, i.e., the number of successfully transmitted data packets within $T$ time slots. Formally, the network throughput $Q(T)$ is defined as

$$Q(T) = \sum_{t=1}^{T} r(t) \quad (2)$$

A. Age-of-Information Constraint

The Age-of-Information concept is defined from the view of BS, and it indeed captures how unfresh the information is. Literally, the AoI of an information source will keep growing (from 1) if no data packet/update of the source is successfully sent to BS. Formally, let $h_i(t)$ be the AoI of source $i$ at the beginning of round $t$ ($h_i(1)$ is assumed to be 1 for simplicity), then it can be defined as

$$h_i(t + 1) = \begin{cases} 1, & \text{if } i(t) = i \text{ and } r(t) = 1. \\ h_i(t) + 1, & \text{otherwise.} \end{cases} \quad (3)$$

Confidence Bound is designed. We prove that MOSS-CB can guarantee to satisfy the AoI requirements with probability 1, and its accumulated regret can be upper-bounded by $O(K \sqrt{T \log T})$. In MOSS-CB, the upper confidence bound (UCB) of the channel reliability is used to determine the optimal source/arm as well as encouraging exploration. Meanwhile, lower confidence bound (LCB) of channel reliability is used to assign sampling probability to sources in order that sources/arms are sampled frequently enough to meet the AoI requirement.

• Numerical experiments are conducted and results show that the proposed MOSS-CB algorithm robustly meets the AoI constraint for every source and outperforms other baseline algorithms.

II. System Model and Problem Statement

As illustrated in Fig. 1, consider a wireless network where a Base Station (BS) as a scheduler gathers time-sensitive information from $K$ information sources to make real-time decisions. Assume that time is slotted with index $t \in \{1, 2, \cdots, T\}$, and each time slot is equivalently called a round. In each round $t$, to avoid interference, the BS selects only one information source $i(t) \in \{1, 2, \cdots, K\}$ for transmission, then $i(t)$ senses fresh information and transmits the data packet/update to BS over the wireless channel. To model the unreliability of wireless channels, it is assumed that packets generated at source $i$ can be successfully received by BS with probability $p_i \in (0, 1)$, where $p_i$ is the channel reliability and $\bar{p} = (p_1, \cdots, p_K)$ is the channel reliability vector. To denote the transmission result at each round $t$, a binary variable $r(t)$ is introduced, i.e.,

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Consider that in real applications, as data updates from heterogeneous sources have different stringency on AoI requirements, the AoI of each source is expected to be individually upper-bounded. As bounding the worst-case Aois is impossible with the randomness/unreliability of the wireless channel, we consider to upper-bound the expectation of time-average Aois. Formally, let $\bar{\lambda} = (\lambda_1, \cdots, \lambda_K)$ be the AoI threshold vector, where $\lambda_i > 0$ is the AoI threshold for source $i$. The time-average AoI $H_i(t)$ for source $i$ at first $t$ rounds is

$$H_i(t) = \frac{1}{t} \sum_{\tau=1}^{t} h_i(\tau), \forall i$$

Then the expected long-term time-average AoI constraint for each source is formulated as

$$\lim_{T \to \infty} E[H_i(T)] \leq \lambda_i, \forall i$$

Note that if $\lambda_i$ is arbitrarily large, the AoI constraint on source $i$ might be meaningless. For example, if $\lambda_i \geq (T+1)/2$, then constraint (5) is satisfied even if the scheduling algorithm never selects source $i$. Therefore, if $\lambda_i \geq (T+1)/2$, then the source $i$ can be directly removed from the input for simplicity. Hence, without loss of generality, we assume that

$$\lambda_i < \frac{T+1}{2}, \forall i$$

By now, a problem instance can be depicted by $(K, T, \bar{\lambda}, \bar{\rho})$, i.e., the problem input. However, not all inputs are feasible since limited scheduling chances cannot satisfy infinitely stringent AoI requirement. For example, let $K = 2$, $\lambda_1 = \lambda_2 = 1$, then none of the scheduling algorithms can meet the AoI requirement even if all channels are reliable. Here we propose that under assumption (6), a problem input is feasible if and only if it satisfies $\sum_{i=1}^{K} \frac{1}{\lambda_i p_i} \leq 1$, which follows Corollary (to be proved in Section III-A). Therefore, we make the following assumption to guarantee the feasibility of input for simplicity, where only the equality is dropped:

$$\sum_{i=1}^{K} \frac{1}{\lambda_i p_i} < 1$$

B. AC-TMS Problem: Known Channel Reliability

When the channel reliability $\bar{\rho}$ is known as a prior, using the definitions of the throughput and AoI, the AoI-Constrained Throughput Maximization Scheduling (AC-TMS) problem can be defined as follows,

**Problem 1 (AC-TMS Problem).** Given a network setup $(K, T, \bar{\lambda}, \bar{\rho})$ satisfying (4) and (6), select information source $i(t)$ to transmit at each time slot $t$ so that the expectation of network throughput $E[R(T)]$ maximized subject to expected long-term time-average AoI constraint for each source $\bar{\lambda}$.

C. AC-MAB Problem: Unknown Channel Reliability

When channel reliability vector $\bar{\rho}$ is unknown, the AC-TMS problem can be viewed as an AoI-Constrained Multi-Armed Bandit (AC-MAB) Problem. More concretely, at each round $t$, the BS determines which arm (source) to pull (transmit). After pulling an arm $i$, the BS can observe the reward (transmission result) which is sampled from a Bernoulli distribution with mean value $p_i$. Due to the lack of knowledge of $\bar{\rho}$, the BS needs to learn to balance the estimation/exploration of $\bar{\rho}$ (to learn the uncertainty of the channel) and its objective of maximizing accumulated reward (throughput) within $T$ rounds. In fact, the objective of the BS is equivalent to minimizing accumulated regret, which is a commonly used metric in bandit problems. Accumulated regret of a policy is defined as the gap to the optimal solution that knows reward distributions in prior. Formally, let $Q^*(T) = \sum_{t=1}^{T} r^*(t)$ be the network throughput in the optimal solution of AC-TMS problem, where $r^*(t)$ is the transmission result in the optimal solution at time slot $t$. The accumulated regret $E[R(T)]$ under AC-MAB algorithm is defined as $E[R(T)] = E[Q^*(T)] - E[Q(T)]$.

Then the AC-MAB problem can be defined as follows,

**Problem 2 (AC-MAB Problem).** Given a bandit setup $(K, T, \bar{\lambda})$ satisfying (4) and (6), select arm $i(t)$ at each round $t$ so that the accumulated regret $E[R(T)]$ is minimized subject to the expected long-term time-average AoI constraint for each arm $\bar{\lambda}$.

III. OPTIMAL SCHEDULING POLICY FOR AC-TMS

In this section, an optimal method for the AC-TMS problem (global optimal) is proposed. Firstly, in Subsection III-A, a class of Stationary Randomized Sampling (SRS) policies is considered. Moreover, the global-optimum reachability of SRS policy, i.e., a SRS policy that is global optimal always exists, is further guaranteed. Secondly, in Subsection III-B, we propose a SRS policy and prove its global optimality.

A. Stationary Randomized Sampling: Global-Optimum Reachability

For ease of description, we slightly update the previous notations. We use policy $\pi$ as the superscript to denote values under this policy. For example, the network throughput under policy $\pi$ is denoted by $Q^\pi(T)$. 

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**Fig. 1:** The system model: the base station (BS) gathers information from multiple heterogeneous sources with channel unreliability. The AoI of a source decreases only when its data update is transmitted and received (white dot), and otherwise grows by 1 (blue dot) at each time slot.
The Stationary Randomized Sampling policies is defined as a class of policies that sample a source from a stationary distribution to transmit at each time slot, and we denote the set of SRS policies as \( \Pi_S \). Specifically, for any BS/scheduler using SRS policy \( \pi_s \in \Pi_S \), it selects the information source \( i^{\pi_s}(t) \) at each time slot \( t \) for transmission by sampling from a fixed probability distribution where \( \Pr\{i^{\pi_s}(t) = i\} = \mu_i, \forall i \). Obviously, according to this definition the sampling probability \( \mu_i \) for each source \( i \) in a SRS policy should satisfy
\[
\sum_{i=1}^{K} \mu_i \leq 1 \quad (8)
\]

Next, we show the global-optimum reachability of SRS, that is, there always exists a SRS policy that is optimal for AC-TMS problem.

Let \( \Pi_F \) be the set of policies that are feasible to AC-TMS problem, i.e., policies can meet the expected time-average AoI constraint (5). We first show that for any policy that is feasible to AC-TMS, we can construct an equivalent SRS policy which is also feasible and incurs the same network throughput in expectation. Due to space limit, some proofs will be omitted, and we will instead show the proof sketch.

**Lemma 1.** For any feasible policy \( \pi_f \in \Pi_F \), there exists an equivalent feasible SRS policy \( \pi_s \in \Pi_S \cap \Pi_F \) such that \( \mathbb{E}[Q^\pi_f(T)] = \mathbb{E}[Q^\pi_s(T)] \).

**Proof (Sketch).** Let \( N_i(t) \) be the selected times of source \( i \) till round \( t \). The main idea of the proof is to take the scheduling frequency of sources \( \mathbb{E}[N_i^{\pi_s}(T)]/T \) in the original policy \( \pi_f \) as the sampling probability of sources \( \mu_i^{\pi_s} \) in constructed SRS policy \( \pi_s \). Firstly, we show the expected throughput \( \mathbb{E}[Q(T)] \) depends only on the expected selected times \( \mathbb{E}[N_i(T)] \) of each source \( i \) within \( T \) rounds since
\[
\mathbb{E}[Q(T)] = \sum_{t=1}^{T} \mathbb{E}[r(t)] = \sum_{t=1}^{T} \mathbb{E}[p_i(t)] = \sum_{i=1}^{K} \mathbb{E}[N_i(T)] p_i. \quad (9)
\]
Obviously the \( \mathbb{E}[N_i(T)] \) in \( \pi_f \) and \( \pi_s \) is the same, hence \( \mathbb{E}[Q^\pi_f(T)] = \mathbb{E}[Q^\pi_s(T)] \). Secondly, we prove the feasibility of \( \pi_s \) by contradiction. Specifically, assume that there exists a source \( i' \) of AoI constraint under policy \( \pi_s \) is violated. Then according to the existing result of Proposition 2 in [24]
\[
\lim_{T \to \infty} \mathbb{E}[H_i(T)] = \frac{1}{\mu_i}, \forall i. \quad (10)
\]
there exists a time slot \( T' > 0 \) such that when \( T > T' \) the expected successful transmission times \( p_i \mathbb{E}[N_i^{\pi_s}(T)] \) of \( i' \) under policy \( \pi_s \) is proved to be upper-bounded by \( T'/\lambda_{i'} \). We then prove that the expected long-term average AoI of \( i' \) under policy \( \pi_s \) is greater than \( \lambda_{i'} \), which contradicts with the feasibility of \( \pi_s \). \( \square \)

Based on Lemma [1], the global-optimum reachability of SRS policy set can be easily derived, that is,

**Theorem 1 (Global-Optimum Reachability of SRS).** A SRS policy that achieves the global optimum always exists, i.e.,
\[
\max_{\pi \in \Pi_S \cap \Pi_F} \mathbb{E}[Q^{\pi}(T)] = \max_{\pi \in \Pi_F} \mathbb{E}[Q^{\pi}(T)] \quad (11)
\]
Besides, Lemma [1] indicates that any input feasible for AC-TMS should satisfy the feasibility requirement of SRS’s input and vice versa. Therefore, combining (8) and (10), a feasible input for AC-TMS problem should satisfy the following constraint.

**Corollary 1.** An input of AC-TMS problem is feasible if and only if
\[
\sum_{i=1}^{K} \frac{1}{\lambda_i p_i} \leq 1 \quad (12)
\]

**B. MOSS: Maximum Optimal-Source Sampling Probability**

In this subsection, we propose a SRS policy called Maximum Optimal-Source Sampling probability (MOSS), which will be shown optimal for the AC-TMS problem.

The main idea of developing MOSS is as follows. Specifically, using preceding results (8), (9) and (10), the AC-TMS problem under SRS policy is reformulated as follows,
\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{K} (T \mu_i) \cdot p_i \\
\text{subject to} & \quad \frac{1}{\mu_i p_i} \leq \lambda_i, \forall i; \\
& \quad \sum_{i=1}^{K} \mu_i \leq 1.
\end{align*}
\]
To maximize the objective function in (13a), the highest sampling probability should be assigned to the optimal-source \( i^* = \arg \max_i p_i \) (which is the one whose associated channel is the most reliable). Also, constraint (13b) gives the minimum requirement on sampling probability \( \mu_i \) assigned to each source \( i \). Since the overall sampling probability is upper-bounded by (13c), in the optimal SRS policy the BS/scheduler should firstly follow (13b) to assign the least sampling probability to each sub-optimal source \( i \neq i^* \) so that the AoI requirement is met. Then the remaining sampling probability ought to be assigned to the optimal-source to maximize the network throughput.

**Algorithm 1: MOSS**

```python
for t = 1, 2, · · · , T do
    i(t) ← sample from LS3;
    Select i(t) to transmit.
end for
```

Based on the analysis above, we propose the global optimal SRS policy MOSS as shown in Algorithm 1. As MOSS is a stationary randomized sampling policy, it samples an arm \( i(t) \) from a stationary distribution to transmit at each time slot, where the distribution is named Least Sub-optimal Source Sampling probability (LS3) and formulated as follows.

**Definition 1 (LS3 Probability Distribution).** Given feasible \((K, \lambda, p)\), assume random variable \( I \) follows this distribution, and let \( c = \sum_{i=1}^{K} 1/(\lambda_i p_i) \). Set \( i^* = \arg \max_i p_i \), then for each source \( i \)
\[
\mu_i = \Pr\{I = i\} = \begin{cases} \frac{\lambda_i}{\lambda_i p_i} & \text{if } i = i^* \\ 1 - c + \frac{c}{\lambda_i p_i} & \text{otherwise}. \end{cases}
\]
Next, we show the global optimality of MOSS.

**Theorem 2** (Global Optimality of MOSS). MOSS is a global optimal solution for AC-TMS problem, that is,

$$
\mathbb{E}[Q^{\text{MOSS}}(T)] = \max_{\pi \in \Pi_F} \mathbb{E}[Q^\pi(T)]
$$

(15)

**Proof (Sketch).** The feasibility of MOSS is MOSS is easy to be validated. That is, we can get $1 - c > 0$ by (7), hence in MOSS policy $\forall i, \mu_i \geq 1/(\lambda_i p_i)$ according to Definition 1. This yields that feasibility constraint (13b) is satisfied, i.e., MOSS is a feasible policy for AC-TMS. Moreover, MOSS assigns the largest assignable sampling probability to the optimal source $i^*$ whose associated channel is most reliable. Therefore, we prove that MOSS achieves the optimum that a SRS policy can do, i.e., $\mathbb{E}[Q^{\text{MOSS}}(T)] = \max_{\pi \in \Pi_F} \mathbb{E}[Q^\pi(T)]$. Combining it with Theorem 1, the theorem is proved. □

IV. LEARNING ALGORITHM FOR AC-MAB

In this section, we tackle the AC-MAB problem where $\bar{p}$ is unknown. Specifically, in Subsection IV-A we will first overview our solution and raise three questions that should be addressed. Then the answers are shown in Subsection IV-B and consequently a learning algorithm named MOSS-CB is proposed in Subsection IV-C. Finally the feasibility and regret bound of MOSS-CB are respectively proved in Subsection IV-D and IV-E.

A. Overview of Solution: Questions to Be Addressed

The difference between AC-TMS and AC-MAB is that $\bar{p}$ is unknown in the latter, therefore a natural idea to solve AC-MAB is to use estimated $\bar{p}$ to run MOSS. Even though the idea is simple, a series of challenging questions should be addressed. The first one is,

**Q1** What kind of $\bar{p}$ estimation should the learning algorithm use to replace the unknown $\bar{p}$ to run MOSS?

Moreover, this idea might fail because estimated $\bar{p}$ is not promised to meet the feasibility requirement for AC-TMS problem input, i.e., estimated $\bar{p}$ might be too rough to satisfy the constraint (7). Especially in earlier rounds, the estimation of $\bar{p}$ can be extremely rough since arms are rarely explored.

Although this problem cannot be avoided, intuitively when each arm has been explored enough times, it is expected that in most cases the estimated $\bar{p}$ is accurate enough and hence feasible to be used to run MOSS. But we still need to address the following two questions,

**Q2** How many times of pullings for each arm would make estimated $\bar{p}$ be accurate enough to run MOSS?

**Q3** What should the learning algorithm do when it is infeasible to run MOSS with the current estimated $\bar{p}$?

To adapt MOSS to this $\bar{p}$-unknown setting, we will address the above three questions and accordingly propose a learning algorithm in the following two subsections.

B. Answers to Q1-Q3

Firstly, we introduce some definitions and useful alternative estimations for $\bar{p}$. For each arm $i$, let $N_i(t)$ be the number of times it is pulled till round $t$, and $\bar{p}_i(t)$ be the average reward received from it till round $t$. In multi-armed bandit problems, Upper Confidence Bound (UCB) is a classical estimation technique which overestimates the unknown random variable to engage exploration while exploitation. In contrast, Lower Confidence Bound (LCB) is the one that underestimates the random variable. The formal definition of UCB and LCB are given as follows.

**Definition 2** (Confidence Bound). For each arm $i$, its UCB is $p_i^U(t) = \bar{p}_i(t) + \epsilon_i(t)/2$ and its LCB is $p_i^L(t) = \bar{p}_i(t) - \epsilon_i(t)/2$, where $\epsilon_i(t)$ is called confidence diameter and

$$
\epsilon_i(t) = 2 \sqrt{\frac{2 \log T}{N_i(t)}}
$$

The benefit of using confidence bound to estimate unknown $\bar{p}$ is that the actual value is bounded by LCB and UCB with high probability (Lemma 1.5, [18]). This result can be formulated as the clean event below.

**Proposition 1.** Let “clean event” be the event that $|p_i - \bar{p}_i(t)| \leq \epsilon_i(t)/2$ for every arm $i$, then $\Pr\{\text{clean event}\} > 1 - 1/T^2$.

Now Q1 is considered as follows. While conducting MOSS without knowing $\bar{p}$, to maximize the accumulated reward as well as to engage exploration, for each arm $i$, we will use its UCB $p_i^U(t)$ as $p_i$’s estimation to determine the round-optimal arm $i^*(t)$, i.e., $i^*(t) = \arg \max_i p_i^U(t)$. The benefit of using UCB here is that the regret contributed by $i^*(t)$ is bounded. Specifically, following $i^*(t)$’s definition and Proposition 1 it is held under clean event that

$$
p_{i^*(t)} - p_{i^*(t)} \leq p_i^U(t) - p_{i^*(t)} + \epsilon_{i^*(t)}/2 \quad (17a)
$$

$$
= p_i^U(t) - p_{i^*(t)} + \epsilon_{i^*(t)}(t) \leq \epsilon_{i^*(t)}(t) \quad (17b)
$$

$$
\leq \epsilon_{i^*(t)}(t) \quad (17c)
$$

Furthermore, to meet the AoI constraint, it is preferable to underestimate the channel reliability during the learning process such that higher sampling probability are assigned to sources. Therefore, for each arm $i$, we will use its LCB $p_i^L(t)$ as $p_i$’s estimation to determine the sampling probability.

Overall, similar to the LS3 probability distribution introduced in the global optimal SRS policy MOSS, we also construct a probability distribution to describe the process above. The distribution is named $SPO_i$, meaning that $SamPle$ an arm like the Optimal algorithm for pulling at round $t$, where the UCB of arms is used to determine the round-optimal arm first, and then LCB of arms is used to determine specific sampling probability. Formally, it can be defined as follows,

**Definition 3** ($SPO_i$, Probability Distribution). Given feasible $(K, \lambda_i, p_i^U(t), p_i^L(t))$, assume random variable $I$ follows this
distribution, and let \( c(t) = \sum_{i=1}^{K} [1/(\lambda_i p_i^t(t))] \). First, set 
\[ i^*(t) = \arg \max_i p_i^t(t), \] for each \( i \)
\[
\Pr\{I = i\} = \begin{cases} \frac{1}{\lambda_i p_i^t(t)}, & \text{if } i \neq i^*(t) \\ 1 - c(t) + \frac{1}{\lambda_i p_i^t(t)}, & \text{otherwise.} \end{cases} \tag{18}
\]

As aforementioned, the \( SPO_t \) distribution cannot be constructed for all rounds as the estimation of \( \bar{p} \) might be too rough to satisfy the constraint \( \tilde{c} \). It is viable only when the following event occurs (or condition is satisfied):

**Definition 4** (\( SPO_t \) Event/Condition). This event occurs, or equivalently this condition is satisfied, if and only if for each arm \( i \) it is held that \( N_i(t) > 0 \) and \( p_i^t(t) > 0 \) and \( c(t) \leq 1 \).

Next, Q2 is considered. Based on the selected estimation technique, i.e., the confidence bound, it can be found that Q2 essentially asks how accurate should \( p_i^t(t) \) be to satisfy \( SPO_t \) condition.

To meet the input feasibility constraint \( \tilde{c} \) of MOSS, we first propose that the following bias is allowed when estimating \( \bar{p} \).

**Lemma 2.** Given \( c = \sum_{i=1}^{K} 1/(\lambda_i p_i) < 1 \), for any \( \epsilon = \{\epsilon_i | 0 < \epsilon_i < (1 - c) p_i, \forall i\} \), there exists \( \delta = 1 - c/[1 - \max_i (\epsilon_i/p_i)] \in (0, 1) \) such that
\[
\sum_{i=1}^{K} \frac{1}{\lambda_i (p_i - \epsilon_i)} \leq 1 - \delta \tag{19}
\]

**Proof.** By the range of \( \epsilon_i \), it is easy to be validated that \( \delta \in (0, 1) \). Then we have \( 1 - \delta = 1/(1 - \max_i \epsilon_i) \) and \( \sum_{i=1}^{K} \frac{1}{\lambda_i p_i} \geq \sum_{i=1}^{K} [1/(1 - \epsilon_i) \cdot \frac{1}{\lambda_i p_i}] = \sum_{i=1}^{K} \frac{1}{\lambda_i (p_i - \epsilon_i)}. \)

Now we show how many times of arm-pulling will be enough to meet \( SPO_t \) condition. Assume that \( \xi = 8/(1 - c)^2 p_i^t \), \( N_{\min}(t) = \min_i N_i(t) \), and let \( t_M \) be the latest time slot that there exists at least one arm that is pulled less than \( \xi \log T \) times, i.e., \( t_M = \max \{t | \exists i, N_{\min}(t) < \xi \log T\} \), then

**Theorem 3.** \( SPO_t \) condition occurs with probability 1 under clean event assumption when \( t > t_M \), that is,
\[
\Pr(SPO_t \text{ event}|t > t_M, \text{ clean event}) = 1 \tag{20}
\]

**Proof.** By the definition of \( SPO_t \) event, we will prove that, when \( t > t_M \), the three conditions \( \forall i, N_i(t) > 0, p_i^t(t) > 0 \), and \( c(t) \leq 1 \) are satisfied in turn.

Firstly, by the definition of \( t_M \), when \( t > t_M \), we have \( N_{\min}(t) \geq \xi \log T \). This implies that \( \forall i, N_i(t) \geq \frac{8 \log T}{(1 - c)^2 p_i^t} > 0 \). Then by the definition of \( \epsilon_i(t) \) in \( \tilde{c} \), it can be derived that
\[
\epsilon_i(t) \leq (1 - c/p_i), \forall i. \tag{21}
\]

Secondly, in clean event, using Proposition [1] it is held that
\[
p_i^t(t) = \frac{\epsilon_i(t)}{2} \geq p_i - \epsilon_i(t), \forall i. \tag{22}
\]

By \( \tilde{c} \), it immediately follows that \( \forall i, p_i^t(t) \geq c p_i \).

Thirdly, let \( \delta(t) = 1 - c/[1 - \max_i (\epsilon_i(t)/p_i)] \), then the definition of \( c(t) \) yields that
\[
c(t) = \sum_{i=1}^{K} \frac{1}{\lambda_i p_i^t} \leq \sum_{i=1}^{K} \frac{1}{\lambda_i (p_i - \epsilon_i)} \leq 1 - \delta(t) < 1 \tag{23}
\]

where the first inequality is derived by \( \tilde{c} \), and the second is derived by Lemma 2. Hereby Theorem 3 is proved.

**Theorem 3** tells us that when each arm \( i \) has been pulled at least \( \xi \log T \) times, \( SPO_t \) condition is always satisfied under clean event, where we say that the learning algorithm enters the \( SPO-is-Mostly-Feasible (SPO-MF) \) phase. In other words, in the excellent SPO-MF phase, the estimations of unknown \( \bar{p} \) we use are accurate enough to be used to run optimal policy MOSS, which answers Q2.

Now we address Q3, which seeks the fallback policy when \( SPO_t \) condition is not satisfied. Based on the preceding analysis, a natural idea is to adaptively pull arm that is currently explored the least to help the learning algorithm to enter into the SPO-MF phase. Therefore, when \( SPO_t \) condition is not satisfied, we propose the Least-pull-Arm First (LAF) fallback policy which pulls arm \( i_{\min}(t) = \arg \min_i N_i(t) \). Intuitively, under the LAF policy, \( N_{\min}(t) \) increases rapidly, hence the learning algorithm can soon step into the SPO-MF phase within limited rounds. Moreover, with the LAF fallback policy, the number of rounds that \( SPO_t \) condition is not satisfied (LAF is conducted) can be bounded as follows.

**Corollary 2.** Let \( S_{LAF} = \{t | SPO_t \text{ condition is not satisfied}\} \) be the set of round indices where LAF is conducted, then
\[
\Pr(|S_{LAF}| \leq \xi K \log T | \text{clean event}) = 1 \tag{24}
\]

**Proof.** Assume \( |S_{LAF}| > \xi K \log T \), then there exists \( t' \in S_{LAF} \) such that LAF is conducted at round \( t' \) where \( N_{\min}(t') > \xi \log T \). By Theorem 3 round \( t' \) should be in SPO-MF phase, i.e., \( t' \geq t_M \). However, this also conflicts with Theorem 3 that \( SPO_t \) condition is always feasible in SPO-MF phase under clean event. Thus, the corollary is proved.

**C. MOSS-CB: Adapting MOSS with Confidence Bound**

Based on the answers to Q1-Q3, we propose the learning algorithm named MOSS with Confidence Bound (MOSS-CB) to solve the AC-MAB problem.

As mentioned in Q2 and Q3, at each round \( t \), if the \( SPO_t \) condition (given in Definition 4) is not satisfied, i.e., estimation of \( \bar{p} \) is infeasible to be used to run MOSS, then fallback policy LAF is conducted. With LAF, the arm \( i_{\min}(t) \) that is pulled currently the least is selected. More importantly, such a fallback policy 1) is able to ensure that the learning algorithm can enter into SPO-MF phase by Theorem 3 where \( SPO_t \) condition is satisfied with high probability, 2) will be conducted only \( \Omega(K \log T) \) times with high probability by Corollary 2.

Otherwise if \( SPO_t \) condition is satisfied, the learning algorithm samples an arm \( i(t) \) from the \( SPO_t \) distribution (given in Definition 3). As mentioned in the answer of Q1, to jointly explore unknown \( \bar{p} \) and maximize accumulated reward, each arm’s UCB that overestimates \( p_i \) is used to determine the round-optimal arm \( i^*(t) \), and LCB that underestimates \( \bar{p} \) is used to determine the specific sampling probability of arms such that each arm is sampled frequently enough to satisfy AoI constraints. The algorithm is illustrated in Algorithm 2.
Algorithm 2: MOSS-CB

1: for \( t = 1, 2, \ldots, T \) do
2: \( i(t) \leftarrow \text{ion}(t) = \arg \min \, N_i(t); \)
3: if SPO condition is satisfied then
4: \( i(t) \leftarrow \text{sample from SPO}_i; \)
5: end if
6: Select \( i(t) \) to transmit.
7: end for

D. Feasibility of MOSS-CB

In this subsection, we will prove the feasibility of MOSS-CB, i.e., AoI constraint (5) under MOSS-CB is satisfied with probability 1. The main idea of the proof is the following: 1) we first construct an algorithm that is Weaker than MOSS-CB (named W-MOSS-CB) but easier to do AoI calculation under the clear event assumption, 2) then we show that for each arm we first construct an algorithm that is Weaker than MOSS-CB required sampling probability 1. The main idea of the proof is the following: 1) based on the preceding idea, we first construct the weaker algorithm W-MOSS-CB as follows,

Definition 5 (W-MOSS-CB Algorithm). If SPO condition is not satisfied, do nothing; Otherwise, select arm \( i(t) \) by sampling from probability distribution where \( \forall i, \Pr \{i(t) = i\} = 1/\lambda_i, \) and do nothing with probability \( 1 - c. \)

Note that W-MOSS-CB assigns the least AoI-constraint-required sampling probability \( 1/\lambda_i \) to each arm \( i \) with the channel reliability \( p_i \) (actually unknown) as depicted in (13b). Comparatively, MOSS-CB underestimates \( p_i \) with LCB \( p^*_i(t) \) and assigns at least \( 1/(\lambda_ip^*_i(t)) \) sampling probability to each arm. Since \( p_i < p^*_i(t) \) under clean event, each arm in MOSS-CB is sampled more frequently than W-MOSS-CB. This makes the expected time-average AoI in W-MOSS-CB larger, i.e.,

\[ \mathbb{E}[H^{\text{W-MOSS-CB}}_{i}(T)] \leq \mathbb{E}[H^{\text{MOSS-CB}}_{i}(T)], \forall i. \tag{25} \]

Next, we prove the feasibility of W-MOSS-CB.

Lemma 3 (Feasibility of W-MOSS-CB). The W-MOSS-CB algorithm is feasible for AC-MAB problem under the clean event, i.e., \( \lim_{T \to \infty} \mathbb{E}[H^{\text{W-MOSS-CB}}_{i}(T)] \leq \lambda_i, \forall i. \)

Proof (Sketch). Firstly, assume that SPO condition is satisfied at round \( t_1, t_2, \ldots, t_r \) (\( V \leq T \)), where \( t_i = t_{i+1} - t_i - 1 \). For ease of denotation, we export \( \tau_0 = 1 \) and \( \tau_{r+1} = T + 1 \). Then \( \sum_{i=1}^{r+1} t_i = T - V = |S_{\text{LAT}}| \leq \xi K \log T \) by Corollary 2 and we further upper-bound \( \sum_{i=0}^{r+1} t_i^2 \) by \( (\xi K \log T)^2/(V + 1) < O((\log T)^2) \) through applying KKT condition here. Secondly, we show that the expected time-average AoI \( \sum_{t=0}^{T} \mathbb{E}[h_i(t)] \) for each arm \( i \) in W-MOSS-CB is \( \frac{1}{2} \sum_{i=0}^{V} l_i^2 + \frac{1}{2} \sum_{i=0}^{V} l_i + \sum_{i=0}^{V} \mathbb{E}[h_i(\tau_i)] + \sum_{i=0}^{V} l_i \mathbb{E}[h_i(\tau_i)] \), where the last two terms remain to be upper-bounded. Thirdly, we prove that the third term satisfies \( \sum_{i=0}^{V} \mathbb{E}[h_i(\tau_i)] \leq \lambda_i \xi K \log T + \lambda_i V \), where the last two terms remain to be upper-bounded. Fourthly, using Jensen’s inequality, we show \( \sum_{i=0}^{V} l_i \mathbb{E}[h_i(\tau_i)] \leq \sqrt{\left( \sum_{i=0}^{V} l_i^2 \right) \left( \sum_{i=0}^{V} \mathbb{E}[h_i(\tau_i)]^2 \right)} \leq \sqrt{\left( (K \xi K \log T)^2 \right) \left( (\lambda_i \xi K \log T + \lambda_i V)^2 \right)} = (K \xi)^2 \lambda_i (\log T)^2 + \lambda_i \xi K \log T \). By applying existing bounds. Finally, combing the results above, we have \( \lim_{T \to \infty} \mathbb{E}[H_{i}(t)] = \lim_{T \to \infty} \sum_{t=1}^{T} \mathbb{E}[h_i(t)]/T \geq \lim_{T \to \infty} O((\log T)^2)/T + \lim_{T \to \infty} \lambda_i V/T \leq 0 + \lim_{T \to \infty} (\lambda_i T/T) = \lambda_i. \) The lemma follows.

Therefore, according to (25), the above lemma implies that MOSS-CB is feasible to the AC-MAB problem under clean event. As the clean event happens with probability 1 as \( T \to \infty \), we conclude the feasibility of MOSS-CB as follows,

Theorem 4 (Feasibility of MOSS-CB). The feasibility of MOSS-CB algorithm is guaranteed with probability 1, i.e.,

\[ \Pr \{ \lim_{T \to \infty} \mathbb{E}[h_i(T)] \leq \lambda_i, \forall i \} = 1 \tag{26} \]

E. Regret Bound of MOSS-CB

In this subsection, the regret under MOSS-CB is proved to be \( O(K \sqrt{T \log T}) \). According to the existing result in bandit theory [18], it can be easily found that the overall regret is dominated by the regret under clean event, i.e., \( \mathbb{E}[R(T)] \leq \mathbb{E}[R(T)|\text{clean event}]+1/T. \) Therefore we focus on analyzing the regret bound under the clean event.

For each round \( t \), let \( \Delta(t) = r^*(t) - r(t) \) be the regret generated at this round, where \( r^*(t) \) is the reward under MOSS optimal policy when knowing \( \tilde{p} \) and \( r(t) \) is the reward under MOSS-CB. Firstly, we show that the regret accumulated before \( t_M \) can be bounded.

Lemma 4. In MOSS-CB, \( \sum_{t=1}^{t_M} \mathbb{E}[\Delta(t)] = O(K \sqrt{T \log T}) \)

Proof. The intuition behind this lemma is that in earlier rounds, \( N_{\text{min}}(t) \) increases rapidly since in LAF the \( \text{ion}(t) \) is selected and in SPO \( \text{ion}(t) \) gets chance to be selected with positive probability. This yields that \( t_M \) is upper-bounded, hence the learning algorithm enters the SPO-MF phase rapidly and incurs upper-bounded regret before stepping into it.

Now we bound the expectation of \( t_M \). We first show that the probability \( N_{\text{min}}(t) \) does not increase in a length-of-\( K \) period (which contains successive \( K \) rounds) is small, formally, \( \Pr\{N_{\text{min}}(t+K) = N_{\text{min}}(t)\} = \Pi_{t=0}^{K-1} \Pr\{\text{SPO event}(1) - \frac{1}{\lambda_{\text{min}}(t)} \leq 1 \} \leq (1 - \frac{1}{\lambda_{\text{min}}(t)})^K \), and we denote the last term as \( \alpha^K < \alpha < 1 \).

Then, we will infer that the probability that \( N_{\text{min}}(t) \) does not increase to \( [\xi \log T] \) till round \( [K \sqrt{T \log T}] \) is small. This is because at round \( t = [K \sqrt{T \log T}] \), if an event \( N_{\text{min}} \left( [K \sqrt{T \log T}] \right) \leq [\xi \log T] \) happens, then at least \([K \sqrt{T \log T}] / K - [\xi \log T] \) length-of-\( K \) periods exist such that in every period the \( N_{\text{min}}(t) \) does not increase. Formally,
\[ \Pr \left\{ N_{\text{min}} \left( \left[ K \sqrt{T \log T} \right] \right) \leq \left\lceil \xi \log T \right\rceil \right\} = \sum_{n=0}^{\left\lceil \xi \log T \right\rceil} \Pr \left\{ N_{\text{min}} \left( \left[ K \sqrt{T \log T} \right] \right) = n \right\} = \sum_{n=0}^{\left\lceil \xi \log T \right\rceil} \alpha^T \left( [K \sqrt{T \log T}] - \xi \log T \right) + n \leq \sum_{n=0}^{\left\lceil \xi \log T \right\rceil} \alpha^n \leq \frac{1}{\alpha(1-\alpha)} \alpha^{\sqrt{T \log T} - \xi \log T} = \frac{1}{\alpha(1-\alpha)} \alpha^{T} \frac{\sqrt{T}}{\log T} \leq O \left( \frac{1}{T^2} \right) \]

where the equality in the last step holds because

\[ \lim_{T \to \infty} \frac{\alpha \sqrt{T \log T} - \xi \log T}{1/T^2} = \lim_{T \to \infty} \exp \left\{ \sqrt{T \log T} \left( 2 + \xi \log \left( \frac{1}{\alpha} \right) \right) \right\} = \exp(\alpha) = 0 \]

The analysis in equation (27) suggests that all arms are pulled over \( \xi \log T \) times in at most \( \left\lceil K \sqrt{T \log T} \right\rceil \) rounds, i.e.,

\[ t_M \leq \left\lceil K \sqrt{T \log T} \right\rceil \]. Let \( A \) denote the event that \( N_{\text{min}} \left( K \sqrt{T \log T} \right) \leq \left\lceil \xi \log T \right\rceil \), and \( A' \) denote its collectively exhaustive event \( N_{\text{min}} \left( K \sqrt{T \log T} \right) > \left\lceil \xi \log T \right\rceil \). Then, the expectation of \( t_M \) can be bounded as follows,

\[ \mathbb{E}[t_M] = \Pr[A] \mathbb{E}[t_M | A] + \Pr[A'] \mathbb{E}[t_M | A'] \leq 1 \star \left\lceil K \sqrt{T \log T} \right\rceil + O(1/T^2) = O(K \sqrt{T \log T}) \]

Therefore, the expectation of accumulated regret \( \mathbb{E}[R(t_M)] \) before round \( t_M \) follows \( \mathbb{E}[R(t_M)] = \sum_{t=1}^{t_M} \mathbb{E}[\Delta(t)] \leq \mathbb{E}[\sum_{t=1}^{t_M} 1] = \mathbb{E}[t_M] \leq O(K \sqrt{T \log T}) \), which completes the proof.

Next, we show that the expectation of the accumulated regret after round \( t_M \) can be bounded.

**Lemma 5.** In MOSS-CB, \( \sum_{t=t_M+1}^{T} \mathbb{E}[\Delta(t)] = O(K \sqrt{T \log T}) \)

**Proof (Sketch).** Recall that the global optimal MOSS algorithm assigns extra \( 1 - \epsilon \) sampling probability to \( i^* \) other than the AoI-constraint-required \( 1/(\lambda_i p_i) \) (see (14)) probability to maximize throughput. Similarly, MOSS-CB assigns extra \( 1 - c(t) \) sampling probability to \( i^* \) (see (18)). Therefore we first upper-bound the expected accumulated regret \( \sum_{t_M+1}^{T} \mathbb{E}[\Delta(t)] \) of MOSS-CB after round \( t_M \) according to the bound of \( c(t) \) at (23), and the bound of \( p_i \star - p_i\star(t) \) at (17). Specifically, we prove that under MOSS-CB, the regret \( \sum_{t=t_M+1}^{T} \mathbb{E}[\Delta(t)] \leq \sum_{t=t_M+1}^{T} 2 \sqrt{T \log T} \min_{p_i} \mathbb{E} \left[ 1/\sqrt{N_{\text{min}}(t)} \right] \).

Secondly, applying the definition of the expectation and \( \Pr \{ i_{\text{min}}(t) = i \} \leq 1 \), we show

\[ \sum_{t=t_M+1}^{T} \mathbb{E} \left[ \frac{1}{\sqrt{N_{\text{min}}(t)}} \right] \leq \sum_{t=t_M+1}^{T} \sum_{i=1}^{K} \mathbb{E} \left[ \frac{1}{\sqrt{N_i(t)}} \right] \]

Thirdly, as each arm is sampled less frequently in W-MOSS-CB under clean event, we can further bound the right-hand side term of (30) in MOSS-CB by that of W-MOSS-CB as follows.

\[ \sum_{t=t_M+1}^{T} \sum_{i=1}^{K} \mathbb{E} \left[ \frac{1}{\sqrt{N_i(t)}} \right] \leq \sum_{t=t_M+1}^{T} \sum_{i=1}^{K} \mathbb{E} \left[ \frac{1}{\sqrt{N_i(t)}} \right] \]

\[ \leq 3 \sum_{i=1}^{K} \mathbb{E} \left[ \frac{1}{\sqrt{N_i(t)}} \right] \]

The first step is derived by the fact that \( 1/\sqrt{x} < 3(\sqrt{x+1} - \sqrt{x}) \) when \( x \geq 1 \), and the second step is derived in W-MOSS-CB each arm \( i \) is pulled with probability \( 1/(\lambda_i p_i) \) which yields \( \mathbb{E} \left[ \frac{1}{\sqrt{N_i(t)}} \right] = \frac{1}{\lambda_i p_i} \mathbb{E} \left[ \frac{1}{\sqrt{N_i(t)}} \right] + (1 - \frac{1}{\lambda_i p_i}) \mathbb{E} \left[ \frac{1}{\sqrt{N_i(t)}} \right] \).

Finally, combining the results above, the lemma is proved.

Combining Lemma 4 and 5, we conclude that the accumulated regret under MOSS-CB is \( O(K \sqrt{T \log T}) \).

**Theorem 5.** (Regret under MOSS-CB). Regret \( \mathbb{E}[R^{\text{MOSS-CB}}(T)] \) under MOSS-CB algorithm is \( O(K \sqrt{T \log T}) \).

**V. Numerical Results.**

In this section, numerical experiments are conducted to evaluate the proposed MOSS-CB learning algorithm.

We notice that, to the best knowledge of the authors, none of existing algorithms are developed to meet the expected time-average AoI constraints and maximize the throughput simultaneously over the unreliable channels. Therefore we will compare the proposed algorithm MOSS-CB with a classic bandit algorithm UCB1 [25] that is AoI-constraint-oblivious and a natural AoI-constraint-aware baseline named MAGF (Maximal AoI-Gap First). Specifically, at each round \( t \), UCB1 selects the arm with the highest UCB \( p_i^t(t) \) to jointly explore and exploit, and MAGF selects the arm with largest AoI gap \( H_i(t) - \lambda_i \), i.e., the arm whose gap between its current time-average AoI \( H_i(t) \) and its AoI threshold \( \lambda_i \) is the largest. In the experiments, the cases of \( K = 3 \) and \( K = 10 \) are considered, respectively. To be specific, the reliability vector \( \tilde{p} \) is respectively set to \( (0.40, 0.60, 0.90) \), and \( (0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.90) \) when \( K = 3 \) and \( K = 10 \). Furthermore, three types of AoI requirement \( \lambda \) are examined, respectively, i.e., 1) **Type L** where a source with Looser AoI requirement is associated with a more reliable channel, 2) **Type H** where a source with Harsher AoI requirement is associated with more reliable channel, and 3) **Type S** where all sources own Similar AoI requirement. The complete setting of AoI requirement \( \lambda \) is detailed in Table I. Each algorithm under each parameter setting is simulated 1000 times for \( T = 2 \times 10^4 \) rounds.
TABLE I: Setting of $\vec{\lambda}$

| Type | $\vec{\lambda}$ ($K = 3$) | $\vec{\lambda}$ ($K = 10$) |
|------|------------------------|--------------------------|
| L    | (5.88, 9.83, 17.87)   | (26.85, 27.24, 28.35, 29.61, 33.86, 35.26, 39.88, 46.15, 63.03, 92.59) |
| H    | (12.50, 8.33, 5.56)   | (55.56, 47.62, 41.67, 37.04, 33.33, 30.3, 27.78, 25.64, 18.52) |
| S    | (8.40, 8.60, 8.90)    | (35.25, 37.61, 35.74, 36.23, 33.81, 36.34, 34.0, 37.79, 34.87, 35.42) |

A. Accumulated Regret

To visualize the increase rate of accumulated regret of our algorithm, the $\sqrt{T \log T}$–divided accumulated regret of MOSS-CB, UCB, and MAGF under all parameter settings is shown in Fig. 2. Results below are observed in experiments.

Firstly, the UCB1 algorithm incurs negative accumulated reward (blue dotted lines) due to its infeasibility for AoI constraints (which will be discussed in next subsection). Secondly, the $\sqrt{T \log T}$–divided accumulated regret of MOSS-CB (green solid lines) is strictly bounded by $K$, i.e., its accumulated regret is always strictly less than $K \sqrt{T \log T}$, while MAGF (orange dashed lines) exceeds $K \sqrt{T \log T}$ in Fig. 2(a). Thirdly, as time grows, the $\sqrt{T \log T}$–divided accumulated regret in MOSS-CB tends to be stable under all parameter settings, but in MAGF it still shows an increasing trend at round $T$ under parameter type L and S in both Fig. 2(a) and 2(b). Fourthly, in most cases (type L and S parameter setting), MOSS-CB achieves lower accumulated regret compared to MAGF. Although under type H parameter setting, MAGF shows a slightly better but still close regret, the overall average accumulated regret over all parameter types of MOSS-CB is only about 50.11% and 80.78% of MAGF when $K$ is 3 and 10, respectively.

B. Time-Average AoI

To check the feasibility of MOSS-CB, UCB1, and MAGF, the time-average AoI gap $H_i(t) - \lambda_i$ of each arm $i$ is examined. If the time-average AoI gap at round $T$ is non-positive, we consider the AoI requirement of this arm to be satisfied. As shown in Fig. 3, the following results are observed in both $K = 3$ and $K = 10$ cases, where the figure of case $K = 10$ is omitted due to space limit.

Firstly, under our MOSS-CB algorithm, for every arm $i$ at each parameter setting, the time-average AoI gap $H_i(T) - \lambda_i$ at round $T$ is limited to less than 0. In other words, the AoI constraint for each source/arm is successfully met.

Secondly, the AoI requirement of arms under MAGF algorithm is also satisfied, which is not surprising as it always pulls the arm whose time-average AoI gap is currently maximum. Although AoI constraints are satisfied, the MAGF tends to not schedule sources whose associated channel is more reliable more frequently and hence fails to maximize the throughput (as depicted in Fig. 2).

Thirdly, under any of the parameter setting, UCB1 fails to meet the AoI requirement for most arms except the one whose estimated channel reliability is the highest. For instance, the time-average AoI gap of arm 1 and 2 (blue dotted line) in Fig. 3 increases rapidly to above 5 in very earlier rounds and never falls back to below 0 again. This is due to that, to maximize the accumulated reward, the UCB1 essentially aims at identifying the arm with maximal expected reward and keep pulling it. Such obliviousness of AoI in UCB1 makes it violate the AoI requirement on most of the arms.

In summary, the proposed MOSS-CB algorithm can robustly meet the AoI requirement of each source, while the AoI-constraint-oblivious algorithm UCB1 cannot. Although MAGF also satisfies the AoI requirement of arms in the experiments, it incurs a regret that exceeds $K \sqrt{T \log T}$. Moreover, our proposed algorithm outperforms MAGF in the accumulated regret or equivalently throughput on average. These together validate the efficiency of the proposed algorithm.

VI. CONCLUSION

In this paper, we investigate the problem of maximizing the throughput with channel unreliability under AoI constraints.
of heterogeneous sources. Both known and unknown channel reliability situations are studied. When channel reliability is priorly known, we propose the global optimal stationary randomized sampling policy MOSS. When channel reliability is priorly unknown, we propose the bandit learning algorithm MOSS-CB, then prove its feasibility and accumulated regret upper bound $O(K \sqrt{T \log T})$. Numerical experiments demonstrate that MOSS-CB is strictly upper-bounded by $K \sqrt{T \log T}$ and outperforms other baselines while satisfying the AoI constraint for every source. In the future work, there are many interesting and possible extensions to be studied, which include, for example, the consideration of combinatorial-arm scenarios with channel availability or sleeping-arm scenarios with source availability.
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