A vibration-based structural damage detection method and its applications to engineering structures

K. He and W.D. Zhu*

Department of Mechanical Engineering, University of Maryland, Baltimore County, 1000 Hilltop Circle, Baltimore, MD 21250, USA

(Received 24 May 2011; accepted 1 June 2011)

Two major challenges associated with a vibration-based damage detection method using changes in natural frequencies are addressed: accurate modeling of structures and the development of a robust inverse algorithm to detect damage, which are defined as the forward and inverse problems, respectively. To resolve the forward problem, new physics-based finite element modeling techniques are developed for fillets in thin-walled beams and for bolted joints, so that complex structures can be accurately modeled with a reasonable model size. To resolve the inverse problem, a logistic function transformation is introduced to convert the constrained optimization problem to an unconstrained one, and a robust iterative algorithm using the Levenberg–Marquardt method is developed to accurately detect the locations and extent of damage. The new methodology can ensure global convergence of the iterative algorithm in solving under-determined system equations and deal with damage detection problems with relatively large modeling error and measurement noise. It is applied to various engineering structures including lightning masts, a space frame structure and one of its components, and a pipeline. The exact locations and extent of damage can be detected in the numerical simulation, and the locations and extent of damage can be successfully detected in experimental damage detection.

Keywords: natural frequency; forward problem; inverse problem; Levenberg–Marquardt method; logistic function transformation

1. Introduction

Most damage in a structure, such as corrosion, cracks, fatigue, and loosening of bolted joints, manifests itself as a stiffness reduction. Assuming that mass changes due to damage are negligible, the linear vibration characteristics of a structure, such as natural frequencies and mode shapes, will change due to stiffness reduction. Damage at different locations with different extent can lead to different patterns in the changes in the natural frequencies and mode shapes. Hence one can use the changes in the natural frequencies and mode shapes to detect the locations and extent of damage by solving an inverse problem. Because measurements of mode shapes can usually be difficult and incur a relatively large error (over 10%) [1], and the mode shapes are not very sensitive to small damage in a structure, it is difficult to use changes in the mode shapes to detect damage in the structure. Because natural frequencies of a structure can be remotely measured at a few locations of the structure

*Corresponding author. Email: wzhu@umbc.edu

ISSN 1947-5411 print/ISSN 1947-542X online
© 2011 Taylor & Francis
DOI: 10.1080/19475411.2011.594105
http://www.informaworld.com
and measurements of the natural frequencies can be very accurate (the measurement error is usually less than 1%) [2], it would be better to use changes in the natural frequencies to detect damage in a structure. The authors’ group developed an iterative damage detection method [3] that can detect the locations and extent of damage in simple structures, such as beams [4], using changes in the natural frequencies of the first several modes of the structures. However, to apply the method to complex engineering structures such as space frame structures, there are two major challenges that need to be addressed.

The first challenge is accurate modeling of structures. Because the damage detection method is model-based, an accurate model of a test structure is required to establish the relationship between the natural frequencies of the structure and its stiffness. The model must be physics-based so that it can simulate real changes in the natural frequencies caused by particular damage in the structure. Developing accurate models for complex structures, especially those with bolted joints, is more challenging than that for simple beams. In addition, the model size and computation efficiency must be at an acceptable level so that the models can be used with an iterative damage detection algorithm.

The second challenge is from the inverse analysis, which is the development of a robust algorithm to detect the locations and extent of damage. The changes in the natural frequencies due to damage include the effects of modeling error and measurement noise. Since it is difficult to develop an accurate model for a complex structure, the effect of modeling error can be relatively large in damage detection of complex structures. The presence of large modeling error and measurement noise can significantly affect the accuracy of the damage detection algorithm, and the level of modeling error and measurement noise determines the size of the minimum detectable damage [4]. Hence the damage detection algorithm should have sufficient robustness to deal with relatively large modeling error and measurement noise [5].

With the development of modeling techniques for two common features in complex structures: fillets in thin-walled beams [6] and bolted connections [7], the main challenge associated with the modeling of complex structures can be resolved. By applying the modeling technique for fillets in thin-walled beams [6], thin-walled beams can be accurately modeled using shell and beam elements. The higher mode vibrations of a filleted thin-walled beam, which usually cannot be captured by beam elements due to the plate-like deformations of the thin walls, can be accurately predicted by a shell and beam element model. The bolted connection modeling technique developed in [7] provides a simple, yet efficient, solution for the modeling of bolted joints. A bolted connection is modeled by a solid cylinder; the radius and elastic modulus of the cylinder can be determined by solving a contact problem using an intensive solid element model for the bolted connection. It was found that the radius of the cylinder, which represents the contact area between the two clamped components, is the most critical parameter in the modeling of the bolted connection. The model developed for a bolted joint is physics-based; it can accurately predict the natural frequencies and mode shapes of a structure with tightened or loosened bolted connections as long as the vibrations of the structure are within a linear range. Because the sizes of the models for fillets and bolted joints are relatively small and only regular elements are used in the models, the modeling techniques can be easily applied to an assembled structure with thin-walled beams and bolted joints.

The inverse analysis of the vibration-based damage detection method is essentially solving a nonlinear least-square problem [1,5,8]. Because the number of the unknown stiffness parameters to be determined is usually greater than that of the natural frequencies used in damage detection, the nonlinear least-square problem is under-determined. In [4], an iterative line-search method, which is essentially the Gauss–Newton method [8], was
used to solve the under-determined nonlinear least-square problem [5]. To obtain a unique solution in each iteration and ensure convergence of the iterations for an under-determined system, the Moore–Penrose inverse, which provides the minimum-norm solution [1,8], was used to solve for the search step in each iteration [4]. Using the Gauss–Newton method along with the Moore–Penrose inverse to solve an under-determined nonlinear least-square problem cannot guarantee that the iterations will converge to a stationary point of the objective function [5,9], and the Moore–Penrose inverse will amplify the effects of modeling error and measurement noise when the condition number of the Jacobian matrix is large [1,5]. Hence it is difficult to apply the Gauss-Newton method to detect damage in a complex structure.

To improve the robustness of the damage detection algorithm, a trust-region search method, called the Levenberg–Marquardt (LM) method [8], is used to solve the nonlinear least-square problem [5]. By combing the trust-region search strategy and a regularization method that can reduce the condition number of the Jacobian matrix [8], the LM method can restrict the effects of modeling error and measurement noise to an acceptable level in each iteration. In addition, the LM method can ensure global convergence of the iterations, i.e. convergence to a stationary point of the objective function [5,8].

With the challenges associated with the modeling and inverse analysis resolved, the vibration-based damage detection method was applied to detect various types of damage, including loosening of bolted joints, in slender and three-dimensional structures, including lightning masts [10], a space frame structure [5] and one of its components [10], and a pipeline [11]. The exact locations and extent of damage can be detected in the numerical simulation where there are no modeling error and measurement noise, and the locations and extent of damage were successfully detected in experimental damage detection.

2. Damage detection algorithm

2.1. Nonlinear least-square problem

To illustrate the damage detection methodology, consider the cantilever beam shown in Figure 1. The beam is divided into \( n \) groups, and the stiffness of the \( i \)th group can be represented by a dimensionless stiffness parameter \( 0 < G_i < 1 \), where \( i = 1, 2, \ldots, n \), with \( G_i = 1 \) and \( G_i = 0 \) indicating that the \( i \)th group is undamaged and fully damaged, respectively. When all the stiffness parameters are identified, the locations and extent of damage are known. While smaller groups can provide a more accurate representation of damage, the size of a group cannot be too small since the minimum size of detectable damage depends on the damage information contained in the natural frequencies used in damage detection and decreasing the size of a group will increase the number of stiffness parameters to be identified.

![Figure 1](image.png)

Figure 1. A cantilever beam whose stiffness of which is represented by a set of dimensionless stiffness parameters \( G_i (i = 1, 2, \ldots, n) \).
A damage detection algorithm is to identify what changes in \( G_i \) can cause the changes in the natural frequencies due to damage. The \( m \) natural frequencies of a structure that are used in damage detection are denoted by \( F = [f_1, f_2, \cdots, f_m]^T \), where \( f_j (j = 1, 2, \ldots, m) \) are functions of the set of dimensionless stiffness parameters \( \mathbf{G} = [G_1, G_2, \cdots, G_n]^T \). Note that it is difficult to obtain the analytical expressions of \( f_j \) in terms of \( \mathbf{G} \) unless the structure is relatively simple; the nonlinear relationships between \( f_j \) and \( \mathbf{G} \) are usually numerically represented by a finite element (FE) model of the test structure. The measured natural frequencies of the damaged structure are denoted by \( \mathbf{F}_d = [f_{d1}, f_{d2}, \cdots, f_{dm}]^T \); the changes in the natural frequencies caused by damage are \( \mathbf{r} = [f_1 - f_{d1}, f_2 - f_{d2}, \cdots, f_m - f_{dm}]^T \). To identify what changes in \( \mathbf{G} \) can cause the changes in the natural frequencies indicated by \( \mathbf{r} \), a nonlinear least-square problem that minimizes the objective function,

\[
Q(\mathbf{G}) = \frac{1}{2} \sum_{j=1}^{m} (f_j(\mathbf{G}) - f_{dj})^2 = \frac{1}{2} \mathbf{r}^T \mathbf{r},
\]

in the domain of \( \mathbf{G} \) can be formulated. For a solution of the nonlinear least-square problem, \( \mathbf{G}^* \), each entry in \( \mathbf{G}^* \) is the remaining stiffness of the structure at the corresponding location; if the value is less than one, there is damage at the corresponding location.

### 2.2. Gauss–Newton method

Because the nonlinear least-square problem is a small-residual problem, i.e. \( \mathbf{r} \) is small and will theoretically approach zero, it can be solved using the classical Gauss–Newton method [8]. The second-order derivative of the objective function is approximated by \( \mathbf{J}^T \mathbf{J} \), where \( \mathbf{J} \) is the Jacobian matrix, whose entries are the first-order derivatives of \( \mathbf{r} \) with respect to \( \mathbf{G} \). Since usually \( m < n \), the nonlinear least-square problem is under-determined. Hence there are an infinite number of solutions satisfying the following system equations in each iteration [5]:

\[
(\mathbf{J}^T \mathbf{J}) \delta \mathbf{G} = -\mathbf{J}^T \mathbf{r},
\]

where \( \delta \mathbf{G} = \mathbf{G}^{k+1} - \mathbf{G}^k \), in which \( k \) is the iteration number. To ensure that the iterations will converge, a unique solution needs to be carefully selected in each iteration [8]. The Moore–Penrose inverse is usually used to solve Equation (2) [8], and a minimum-norm solution can be obtained [1]. Whereas using the Moore–Penrose inverse can make the iterations converge for under-determined systems, it cannot guarantee that the first-order derivative of the objective function will approach zero as the number of iterations approaches infinity; a weaker convergence condition is achieved using the Moore–Penrose inverse for under-determined systems [9]. In addition, if the Moore–Penrose inverse is used, the effects of modeling error and measurement noise will be amplified by the large condition number of \( \mathbf{J} \) [1]. If relatively large modeling error and measurement noise are associated with damage detection, the algorithm using the Moore–Penrose inverse may not converge. Regularization methods usually need to be imposed to reduce the condition number of the system matrix [1,4,5].

### 2.3. LM method with a logistic function transformation

To overcome the drawback of the Gauss–Newton method for under-determined systems, the LM method [8] is used to improve the robustness of the damage detection algorithm.
Instead of searching along the steps that are calculated by directly solving the under-determined system equations (Equation (2)), the LM method only accepts those steps whose lengths are less than prescribed radii of trust regions in the search domain [8]. If the length of a search step calculated from Equation (2) is greater than the radius of the trust region, the search step is rejected, and a new search step $\delta G$ is calculated by solving

$$
\begin{align*}
-J^T r &= (J^T J + \lambda I)\delta G \\
\|\delta G\| &= \Delta,
\end{align*}
$$

(3)

where $\lambda$ is a regularization parameter and $\Delta$ is the radius of the trust region. For a given-radius $\Delta$, $\lambda$ and $\delta G$ are simultaneously calculated from Equation (3). The radius of the trust region is selected to ensure that approximating the nonlinear objective function in Equation (1) by its second-order Taylor series expansion is acceptable within the trust region in the search domain. The radius of the trust region will be updated before each iteration based on the descent made by the previous iteration [8]. Since introducing the regularization parameter $\lambda$ can reduce the condition number of $J^T J + \lambda I$ in Equation (3), the error amplification effect of the Moore–Penrose inverse [1] can be significantly reduced in the LM method. More importantly, the LM method can ensure that the algorithm will converge to a stationary point of the objective function [8].

The LM method mentioned above is for unconstrained nonlinear least-square problems. Because the dimensionless stiffness parameters $G$ are bounded between 0 and 1, the nonlinear least-square problem associated with vibration-based damage detection is a constrained problem. In order to apply the LM method to damage detection, a logistic function transformation is used to convert the constrained problem to an unconstrained one:

$$
G_i = \frac{1}{1 + e^{-a S_i}},
$$

(4)

where $S_i (i = 1, 2, \ldots, n)$ are the transformed dimensionless stiffness parameters, and $a > 0$ is an index of the transformation, which can be adjusted to change the convergence speed of the iterations [5,10]. With the logistic function transformation, the damage detection algorithm will search in the domain of $S = [S_1 \, S_2 \, \cdots \, S_n]^T$, whose entries vary from $-\infty$ to $\infty$.

3. Applications and results

3.1. Detecting damage in lightning masts

The vibration-based damage detection method was first used to detect damage in lightning masts. Lightning masts are commonly used structures in electric substations, and are subject to such environmental effects as rain, wind, snow, and temperature changes, which can cause various types of damage, such as corrosion and loosening of bolted joints. If damage can be detected at an early stage, premature failure of lightning masts can be avoided.

A lightning mast in an electrical substation is shown in Figure 2a, which consists of two pipes connected by bolted flanges and a rod-type spike. The lightning mast is made of steel with an elastic modulus of 200 GPa, a Poisson’s ratio of 0.29, and a mass density of 7800 kg/m$^3$. The length, radius, and thickness of the lower pipe are 6.845 m, 0.1055 m, and 0.0081 m, respectively, and those of the upper pipe are 6.8358 m, 0.0806 m, and 0.0071 m, respectively. The length of the spike is 2.1336 m and its radius is 0.0127 m. The radius and thickness of the upper and lower flanges are 0.1762 m and 0.0222 m, respectively.
Figure 2. (a) A lightning mast in an electrical substation, along with expanded views of the two joints; and (b) its FE model with 40,122 degrees of freedom (DOFs).

An impact hammer was used to excite the lighting mast at the ground level, and the natural frequencies of the mast were measured by two accelerometers attached to the lower pipe of the mast in two perpendicular radial directions of the cross-section. Because the shell-like deformations of the upper pipe wall caused by the vibration of the spike cannot be neglected [10], the upper and lower pipes in the lightning mast are modeled by shell elements using SDTools [12] (Figure 2b). The spike of the lightning mast is modeled by beam elements. The steel cap at the top of the upper pipe and the mass of the brackets at the upper joint are included in the FE model (Figure 2). The model developed can accurately predict the natural frequencies of the lightning mast; the maximum error between the first eight measured natural frequencies and the corresponding calculated ones is 1.69% (Table 1).

| Mode | Measured (Hz) | Calculated (Hz) | Error  |
|------|---------------|----------------|--------|
| 1    | 1.17          | 1.15           | −1.30% |
| 2    | 5.09          | 5.08           | −0.23% |
| 3    | 7.81          | 7.87           | 0.71%  |
| 4    | 16.56         | 16.33          | −1.41% |
| 5    | 29.31         | 29.10          | −0.71% |
| 6    | 45.22         | 45.98          | 1.69%  |
| 7    | 47.65         | 46.85          | −1.69% |
| 8    | 53.98         | 53.81          | −0.31% |
Figure 3. Groups in the FE model of the lightning mast in Figure 2.

To apply the damage detection algorithm developed, the lightning mast in Figure 2a, including the spike, is evenly divided into 20 groups with a dimensionless stiffness parameter assigned to each group (Figure 3). The first six measured natural frequencies were used in damage detection. The iterations were terminated in experimental damage detection when \( \Delta \) was less than \( 10^{-3} \) times the initial magnitude of \( G \) or \( \| \nabla Q \| \) was less than \( 10^{-2} \). Note that the selection of the termination criterion is based on the accuracy of the modeling in the forward problem. The criterion can be smaller when the modeling and measurement errors are relatively small and vice versa. The lightning mast was essentially undamaged since there was no significant stiffness reduction in the damage detection result in Figure 4.

The damage detection algorithm was also applied to detect introduced damage in a scale model of the lightning mast in Figure 2a (Figure 5a). The length, radius, and thickness of the lower pipe are 0.776 m, 0.0124 m, and 0.000889 m, respectively, and those of the upper pipe are 0.776 m, 0.0093 m, and 0.000889 m, respectively. The length and radius of the spike are 0.2530 m and 0.0016 m, respectively. The radius and thickness of the flanges
are 0.0254 m and 0.0051 m, respectively. The length, width, and thickness of the base plate are 0.0762 m, 0.0762 m, and 0.0062 m, respectively. The elastic modulus of the scale mast is 200 GPa, the Poisson’s ratio is 0.29, and the mass density is 7,833 kg/m³. A section from 0.0923 m to 0.1348 m from the fixed end and 0.000285 m deep was machined from the surface of the lower pipe, and another section from 0.8959 m to 0.9471 m from the fixed end and 0.000355 m deep was machined from the surface of the upper pipe; they represent a 30% and a 55% stiffness reduction, respectively. An impact hammer was used to excite the lower pipe of the scale mast in two perpendicular radial directions of the cross-section, and the natural frequencies of the mast were measured by a laser vibrometer to avoid mass loading.

Since the shell-like deformation of the upper pipe is negligible for the scale mast [9], the scale mast is modeled mainly using beam elements (Figure 5b); only the flanges and the base plate are modeled by shell elements (Figure 5b). The stiffness reductions caused by the machined sections are taken into account in the FE model by reducing the elastic modulus of the beam elements at the corresponding locations. The mainly beam element model can accurately predict the natural frequencies of the scale mast. The maximum error between the first 15 calculated and measured natural frequencies is 1.88%; the machined sections introduce a maximum change of 3.57% in the first 15 calculated natural frequencies (Table 2).
Figure 5. (a) A scale mast with two machined sections, along with the expanded views of the two joints and the base plate; and (b) its mainly beam element model with 5,892 DOFs.

Table 2. The first 15 measured and calculated natural frequencies of the scale mast in Figure 5a and the differences between the calculated natural frequencies of the undamaged and damaged masts

| Mode | Measured (Hz) (damaged) | Calculated (Hz) (damaged) | Error | Calculated (Hz) (undamaged) | Difference |
|------|-------------------------|---------------------------|-------|-----------------------------|------------|
| 1    | 9.780                   | 9.698                     | −0.85%| 10.026                      | −3.27%     |
| 2    | 9.812                   | 9.817                     | 0.05% | 10.074                      | −2.55%     |
| 3    | 42.341                  | 42.785                    | 1.05% | 44.371                      | −3.57%     |
| 4    | 43.151                  | 43.152                    | 0.00% | 44.427                      | −2.87%     |
| 5    | 74.019                  | 75.152                    | 1.53% | 75.433                      | −0.37%     |
| 6    | 76.714                  | 75.271                    | −1.88%| 75.535                      | −0.35%     |
| 7    | 147.976                 | 146.212                   | −1.19%| 148.146                     | −1.31%     |
| 8    | 149.430                 | 147.555                   | −1.25%| 148.851                     | −0.87%     |
| 9    | 249.081                 | 252.208                   | 1.26% | 257.696                     | −2.13%     |
| 10   | 254.010                 | 254.395                   | 0.15% | 258.158                     | −1.46%     |
| 11   | 425.630                 | 428.982                   | 0.79% | 429.910                     | −0.22%     |
| 12   | 435.639                 | 431.942                   | −0.85%| 432.578                     | −0.15%     |
| 13   | 495.239                 | 498.128                   | 0.58% | 500.235                     | −0.42%     |
| 14   | 505.554                 | 500.641                   | −0.97%| 501.165                     | −0.10%     |
| 15   | 543.332                 | 549.123                   | 1.07% | 557.286                     | −1.46%     |

To apply the damage detection algorithm, the lower pipe, the upper pipe, and the spike are evenly divided into 20, 14, and 4 groups, respectively (Figure 6). The elastic modulus of each element group is represented by a dimensionless stiffness parameter. The first six
measured natural frequencies were used to detect the introduced damage. The iterations were terminated in experimental damage detection when $\Delta$ was less than $10^{-3}$ times the initial magnitude of $G$ or $\|\nabla Q\|$ was less than $10^{-2}$. The locations and extent of damage were successfully detected (Figure 7); the maximum error between the first six calculated and measured natural frequencies of the damaged mast is 1.73%. In the numerical simulation where there are no modeling error and measurement noise, the exact locations and extent of damage were detected (Figure 7). Note that the use of the logistic function transformation can significantly increase the convergence speed of the iterative algorithm; it reduces the number of iterations from 1300 to 13 in detecting the locations and extent of damage [10].

3.2. Detecting damage in a space frame structure and one of its components

To study damage detection of space frame structures, a modular space frame structure with L-shaped beams and bolted joints was fabricated in the authors’ laboratory (Figure 8). The structure is made of 6061-T651 aluminum with an elastic modulus of $E = 68.9$ GPa, a Poisson's ratio of 0.33, and a mass density of 2731.4 kg/m$^3$. The height, width, and depth of the space frame structure are 1.849 m, 0.511 m, and 0.611 m, respectively. It consists of 12 vertical L-shaped beams, 12 horizontal rectangular beams, and two diagonal rectangular beams. The width and thickness of the rectangular beams are 0.0381 m
Figure 7. Experimental (upper bars) and numerical (lower bars) damage detection results of the scale mast in Figure 5 with a 30% and a 55% stiffness reduction at a section of the lower and the upper pipe, respectively, using changes in the first six natural frequencies.

and 0.0127 m, respectively; the lengths of the longer horizontal and diagonal beams are 0.586 m and 0.755 m, respectively, and those of the shorter ones are 0.486 m and 0.650 m, respectively. The bolts (Hex Head M10 × 35-8.8) are made of steel with an elastic modulus of 210 GPa. The four lowest L-shaped beams are bolted to a plate through four cubes, and the length of each side of the cubes is 0.0402 m. The plate, whose thickness, width, and depth are 0.0108 m, 0.5 m, and 0.6 m, respectively, is bolted to the ground through four bolts (Figure 8). Note that the plate is not in contact with the ground surface; there are three washers with a thickness of 0.0024 m and a radius of 0.0135 m, inserted between the plate and the ground on each bolt mounted to the ground.

Twenty seven accelerometers were used to measure the natural frequencies of the structure and to match the calculated and measured mode shapes. Two accelerometers were placed at each of the 12 bolted joints that connect the L-shaped beams and the horizontal beams, to measure the vibrations of the structure in two perpendicular horizontal directions. Two accelerometers were placed in the middle of two perpendicular horizontal beams at the top of the structure, and the other accelerometer was placed on the bottom plate (Figure 8). An impact hammer was used to excite the vibrations of the structure in
two perpendicular horizontal directions. The measured data were collected and analyzed using a 36-channel LMS spectrum analyzer.

To conduct vibration-based damage detection on the space frame structure in Figure 8, an accurate FE model of the structure needs to be developed. Modeling such a complex structure using simple beam elements will lead to large modeling error due to the bending-torsion coupled vibration of a relatively short L-shaped beam and complexity of the bolted joints [7]. A more intensive model of the space frame structure with a reasonable model size needs to be developed.

3.2.1. Modeling of fillets in an L-shaped beam

Whereas the bending-torsion coupled vibration of an L-shaped beam can be captured by a shell element model, it is difficult to model a fillet in an L-shaped beam, which is introduced to reduce stress concentration in the L-shaped beam, using only shell elements [6]. The effect of the fillet on the linear vibration of the L-shaped beam usually cannot be neglected [6].

A simple shell and beam element model was developed to model fillets in thin-walled beams [6]. The equivalent stiffness of a fillet can be decomposed into the in-plane stiffness and the out-of-plane stiffness, which can be assumed to be uncoupled and separately calculated [6]. The in-plane stiffness is associated with the cross-sectional deformation of an L-shaped beam and the out-of-plane stiffness is associated with the bending and torsional stiffness of the L-shaped beam. Shell elements are used to mainly capture the in-plane stiffness though they also provide some out-of-plane stiffness; beam elements are used to compensate for the out-of-plane stiffness that is not fully represented by the shell elements [6]. A shell and beam element model of the L-shaped beam was then developed to capture its bending-torsion coupled vibration. The natural frequencies and mode shapes of the L-shaped beam predicted by the shell and beam element model have essentially the same accuracy as that of an intensive solid element model that has ten times more DOFs [6].
3.2.2. Modeling of bolted connections

Because a bolted joint that connects an L-shaped beam and a horizontal beam in the space frame structure in Figure 8 contains 10 bolted connections and its size is approximately one fifth of that of an L-shaped beam, the bolted joint cannot be simply modeled by a stiffness in the FE model [7]. A simple, yet accurate, linear model of a bolted connection was developed [7]. In the FE model of a bolted joint that connects two L-shaped beams through a bracket (Figure 9), the walls of the L-shaped beams and the bracket are modeled using shell elements, and a bolted connection is modeled by a solid cylinder [7]. The length of the solid cylinder is the sum of the distance between the two center planes of the clamped components (i.e. the walls of an L-shaped beam and the bracket) and the length of the extruded part of the bolt outside the center plane of the wall of the bracket. The radius and the material properties of the solid cylinder are the parameters that need to be determined. It was observed from the numerical simulation that the natural frequencies are more sensitive to the changes in the radius of the solid cylinder than the elastic and shear moduli [7]. The radius of the solid cylinder, which is the radius of the effective area of the bolted connection, can be approximated by the radius of the static contact area of the two clamped components under a sufficiently large clamping force, which can be calculated from an intensive contact model using ABAQUS; the resulting radius is 0.01043 m [7]. From the numerical simulation, it was observed that the contact area is not sensitive to the changes in the contact interface properties and the material properties of the clamped components, and will remain a constant once the clamping force is above a threshold value; hence the FE model will be valid as long as the bolted connection is tightened and the clamping force is sufficiently large compared to potential external loading [7].

The elastic modulus of the solid cylinder can be obtained by matching the axial stiffness of the cylinder with that of the bolted connection calculated using the Rotsher’s pressure cone method [13]; the calculated elastic modulus of the cylinder is 1.17E \[5\]. For a tightened bolted connection, the shear modulus of the solid cylinder is set to be that of the material shear modulus of the clamped components, since the slip between them is neglected in the linear model of the bolted connection [7]. In addition, rigid-link constraints

![Figure 9](image)

Figure 9. (a) A bolted joint that connects two L-shaped beams through a bracket; and (b) its FE model from SDTools [12], with a bolted connection modeled by a solid cylinder.
that can restrict the relative translational motion between the corners of the L-shaped beams and the bracket, and prevent penetration between the clamped components, are included in the FE model (Figure 9b) [7]. With the parameters of the solid cylinder determined, the stiffness effect of a bolted connection can be accurately modeled. The mass effect of a bolted connection is modeled by changing the mass density of the solid cylinder, so that the mass of the cylinder equals that of the bolted connection [7].

3.2.3. Modeling of the space frame structure

An accurate FE model of the space frame structure in Figure 8 was developed using SDTools [11] (Figure 10). The bottom plate and the rectangular beams are modeled by shell elements. The cubes, which are modeled by hexahedron solid elements, are fully connected to the plate and the lowest L-shaped beams through rigid links in the FE model. There are no brackets at the four bolted joints at the top of the structure; the rectangular beams are directly bolted on the flanges of the L-shaped beams. The radius of the effective area of these bolted connections is the same as that of a bolted connection in Figure 9, since the thickness of the flange of the L-shaped beam is smaller than those of the rectangular beam and the bracket, and the radius of the static contact area is controlled by the thinner clamped component [7]. The radius of the effective area of the bolted connections that are used to mount the bottom plate to the ground is the radius of the washers between

![Figure 10](image-url). (a) The FE model of the space frame structure in Figure 8 with 223,296 DOFs; and (b) an enlarged view of a bolted joint, where eight bolted connections are later loosened to hand-tight.
the plate and the ground, which is 0.01004 m. The elastic moduli of the solid cylinders for the bolted connections at the top of the structure and those at the boundaries are calculated to be 1.22E. There are three clamped components in a bolted connection that connects a horizontal rectangular beam to a bracket and an L-shaped beam; the elastic modulus of the solid cylinder between the center planes of the rectangular beam and the bracket is 1.32E, and that between the center planes of the bracket and the L-shaped beam is 1.27E. The maximum error between the first 30 calculated and measured natural frequencies is 1.86% (Table 3).

When eight bolted connections in a bolted joint are loosened to hand-tight (Figure 10), by reducing the shear moduli of the solid cylinders corresponding to the loosened bolted connections, the FE model developed can still accurately model the space frame structure. When the shear moduli of the solid cylinders are reduced to 3.41% of their original values, the maximum error between the calculated and measured natural frequencies is 1.82% for the first 30 modes (Table 3).

Table 3. The measured (Exp) and calculated (FEM) natural frequencies of the first 30 modes of the space frame structure in Figure 8, with all the bolted connections tightened (undamaged) and eight bolted connections in a bolted joint loosened to hand-tight (Figure 10) (damaged), and the changes in the measured natural frequencies due to damage

| Mode | Exp (Hz) | FEM (Hz) | Error | Exp (Hz) | FEM (Hz) | Error | Change |
|------|----------|----------|-------|----------|----------|-------|--------|
| 1    | 17.847   | 18.137   | 1.62% | 17.656   | 17.794   | 0.78% | -1.07% |
| 2    | 20.690   | 20.414   | -1.33%| 19.146   | 19.402   | 1.34% | -7.46% |
| 3    | 28.031   | 28.088   | 0.20% | 25.689   | 26.156   | 1.82% | -8.36% |
| 4    | 39.031   | 38.367   | -1.70%| 38.133   | 37.470   | -1.74%| -2.30% |
| 5    | 57.695   | 58.102   | 0.71% | 56.489   | 56.871   | 0.68% | -2.09% |
| 6    | 61.215   | 60.892   | -0.53%| 60.114   | 60.215   | 0.17% | -1.80% |
| 7    | 73.809   | 74.863   | 1.43% | 72.481   | 73.337   | 1.18% | -1.80% |
| 8    | 80.398   | 80.232   | -0.21%| 79.737   | 80.017   | 0.35% | -0.82% |
| 9    | 88.476   | 88.548   | 0.08% | 87.250   | 87.551   | 0.34% | -1.39% |
| 10   | 90.649   | 90.863   | 0.24% | 90.054   | 90.521   | 0.52% | -0.66% |
| 11   | 93.839   | 94.233   | 0.42% | 92.413   | 93.114   | 0.76% | -1.52% |
| 12   | 98.522   | 98.394   | -0.13%| 96.288   | 96.949   | 0.69% | -2.27% |
| 13   | 108.559  | 108.361  | -0.18%| 105.234  | 105.619  | 0.37% | -3.06% |
| 14   | 115.100  | 114.841  | -0.24%| 112.425  | 113.210  | 0.70% | -2.32% |
| 15   | 126.888  | 125.744  | -0.90%| 122.437  | 120.203  | -1.82%| -3.51% |
| 16   | 129.707  | 129.071  | -1.26%| 125.722  | 124.638  | -0.86%| -3.07% |
| 17   | 137.472  | 135.328  | -1.56%| 134.254  | 134.559  | 0.23% | -2.34% |
| 18   | 148.130  | 146.295  | -1.24%| 145.327  | 144.804  | -0.36%| -1.89% |
| 19   | 150.431  | 149.123  | -0.87%| 148.733  | 147.108  | -1.09%| -1.13% |
| 20   | 157.488  | 157.504  | 0.01% | 153.045  | 154.230  | 0.77% | -2.82% |
| 21   | 161.676  | 161.293  | -0.24%| 161.925  | 160.016  | -1.18%| 0.15%  |
| 22   | 165.153  | 166.449  | 0.78% | 165.045  | 164.083  | -0.58%| -0.07% |
| 23   | 168.036  | 167.465  | -0.34%| 168.080  | 165.259  | -1.68%| 0.03%  |
| 24   | 181.142  | 180.334  | -0.45%| 181.032  | 179.906  | -0.62%| -0.06% |
| 25   | 187.13   | 183.650  | -1.86%| 182.246  | 182.577  | 0.18% | -2.61% |
| 26   | 192.408  | 189.267  | -1.63%| 186.243  | 184.114  | -1.14%| -3.20% |
| 27   | 198.522  | 197.591  | -0.47%| 197.072  | 196.494  | -0.29%| -0.73% |
| 28   | 209.405  | 207.822  | -0.76%| 209.659  | 207.563  | -1.00%| 0.12%  |
| 29   | 215.103  | 213.014  | -0.97%| 215.315  | 212.893  | -1.12%| 0.10%  |
| 30   | 217.930  | 216.117  | -0.83%| 216.898  | 213.689  | -1.48%| -0.47% |
3.2.4. Detecting damage in the space frame structure

The vibration-based damage detection method was used to detect various types of damage in the space frame structure in Figure 8 [5]. The structure is divided into 39 groups in the FE model (Figure 11). Each solid cylinder that models a bolted connection connecting the bottom plate to the ground is represented by a group (groups 1–4). The bottom plate is represented by a group (group 5). Each of the L-shaped beams and the horizontal and diagonal beams is represented by a group (groups 6–8, 11–13, 16–18, 21–23, 26–39). The eight solid cylinders representing the eight bolted connections that connect the L-shaped beams and the bracket, in each of the eight bolted joints in the middle of the structure, are grouped together and represented by a stiffness parameter (groups 9, 10, 14, 15, 19, 20, 24, 25). The iterations were terminated in experimental damage detection when $\Delta$ was less than $10^{-5}$ times the initial magnitude of $\mathbf{G}$ or $\|\nabla Q\|$ was less than $10^{-2}$.

The experimental damage detection was first conducted to detect damage in a beam member (Figure 12). The damage was introduced to the structure in Figure 8 by loosening the upper bolted connection of a diagonal beam (group 26 in Figure 11) to hand-tight; the diagonal beam did not provide any stiffness to the space frame structure, but retained its mass effect. The first 14 measured natural frequencies were used to detect the damage. The maximum change in the natural frequencies caused by the damage was 4.18% for the first 14 modes (Table 4). The location and extent of the introduced damage were detected after
Figure 12. Experimental and numerical damage detection results of the space frame structure in Figure 8, where the upper bolted connection of a diagonal beam (group 26 in Figure 11) was loosened to hand-tight.

Table 4. The measured natural frequencies of the first 14 modes of the space frame structure in Figure 8, with the upper bolted connection of a diagonal beam (group 26 in Figure 11) loosened to hand-tight

| Mode | Undamaged (Hz) | Damaged (Hz) | Change |
|------|---------------|--------------|--------|
| 1    | 17.847        | 17.733       | −0.64% |
| 2    | 20.69         | 20.422       | −1.30% |
| 3    | 28.031        | 27.5198      | −1.82% |
| 4    | 39.031        | 38.8875      | −0.37% |
| 5    | 57.695        | 57.4616      | −0.40% |
| 6    | 61.215        | 60.9476      | −0.44% |
| 7    | 73.809        | 72.7762      | −1.40% |
| 8    | 80.398        | 80.0831      | −0.39% |
| 9    | 88.476        | 85.9499      | −2.86% |
| 10   | 90.649        | 89.5057      | −1.26% |
| 11   | 93.839        | 91.6187      | −2.37% |
| 12   | 98.522        | 94.5392      | −4.04% |
| 13   | 108.559       | 104.024      | −4.18% |
| 14   | 115.1         | 111.676      | −2.97% |

14 iterations (Figure 12); the maximum error between the calculated and measured natural frequencies of the first 14 modes of the damaged space frame structure was 0.29%. The resultant stiffness at the damage location was 1.23% of its original value.

The second case was to detect loosening of a bolted joint (Figure 13). The damage was introduced to the structure in Figure 8 by loosening eight bolted connections in a bolted joint (group 9 in Figure 11) to hand-tight (Figure 10); note that the two bolted connections that connect the two horizontal beams to the bracket and the L-shaped beam remained tightened, and loosening them would reduce the stiffness effects of the two horizontal beams.
The first 10 measured natural frequencies were used to detect the damage. The maximum change in the natural frequencies caused by the damage was 8.36% for the first 10 modes (Table 3). The location and extent of the introduced damage were detected after 15 iterations; the maximum error between the calculated and measured natural frequencies of the first 10 modes of the damaged space frame structure was 1.09%. The resultant stiffness at the damage location was 3.37% of its original value.

The third case was to detect damage at a lower boundary of the space frame structure (Figure 14). The damage was introduced to the structure in Figure 8 by loosening one of the four bolted connections that connected the bottom plate to the ground (group 4 in Figure 11) to hand-tight. The first nine measured natural frequencies were used to detect the damage. The maximum change in the natural frequencies caused by the damage was 4.35% for the first nine modes (Table 5). The location and extent of the introduced damage were detected after 12 iterations; the maximum error between the calculated and measured natural frequencies of the first nine modes of the damaged space frame structure was 1.82%. The resultant stiffness at the damage location was 10.91% of its original value. When a similar damage was introduced to the FE model of the space frame structure in the numerical simulation, the exact location and extent of damage was detected in each case (Figures 12–14).

3.2.5. Detecting loosened bolted connections in a component of the space frame structure

The damage detection method was also used to detect loosened bolted connections in a component of the space frame structure in Figure 8, with free boundaries (Figure 15) [10]. Four of the 10 bolted connections in the bolted joint were loosened to hand-tight, which causes a maximum change of 4.31% in the natural frequencies of the first 16 elastic modes [9] (Table 6). The first 16 elastic modes were used to detect the damage. The iterations were terminated in experimental damage detection when \( \frac{\Delta}{G} \) is less than \( 10^{-4} \) times the initial magnitude of \( G \) or \( ||\nabla Q|| \) is less than \( 10^{-2} \). The portion of each L-shape beam that is not in contact with the bracket is evenly divided into 10 groups (groups 1 to 10 for one
Figure 14. Experimental and numerical damage detection results of the space frame structure in Figure 8, where a bolted connection that connects the bottom plate to the ground (group 4 in Figure 11) was loosened to hand-tight.

Table 5. The measured natural frequencies of the first nine modes of the space frame structure in Figure 8, with one of the four bolted connections that connected the bottom plate to the ground (group 4 in Figure 11) loosened to hand-tight

| Mode | Undamaged (Hz) | Damaged (Hz) | Change |
|------|----------------|--------------|--------|
| 1    | 17.847         | 17.305       | −3.04% |
| 2    | 20.69          | 20.59        | −0.48% |
| 3    | 28.031         | 27.817       | −0.76% |
| 4    | 39.031         | 38.817       | −0.55% |
| 5    | 57.695         | 57.22        | −0.82% |
| 6    | 61.215         | 61.008       | −0.34% |
| 7    | 73.809         | 73.761       | −0.07% |
| 8    | 80.398         | 80.341       | −0.07% |
| 9    | 88.476         | 84.625       | −4.35% |

L-shaped beam and groups 18 to 27 for the other L-shaped beam) in the FE model, the elements representing the bracket are grouped together as one group (group 11), and the elements representing the L-shaped beams in contact with the bracket are grouped together as one group (group 17). The elastic modulus of each group is represented by a dimensionless stiffness parameter. Each pair of the bolted connections that are symmetric about the corner of the bracket, which is assumed to have the same tightness, is grouped together as one group (groups 12 to 16), and the corresponding shear modulus of the cylinders is represented by a dimensionless stiffness parameter. The loosened bolted connections were successfully detected, and the maximum error between the calculated and measured natural frequencies of the first 16 elastic modes of the damaged component is 1.50% (Table 6). The exact locations and extent of damage were detected in the numerical simulation (Figure 15).
Figure 15. Experimental (left columns) and numerical (right columns) damage detection results of a component of the space frame structure with four of the 10 bolted connections loosened.

Figure 16. A full-size steel pipeline, consisting of two pipes, each welded to a steel flange through a circumferential weld, and eight bolted connections that bolt the two flanges together, was put on two air beds to simulate the free boundaries. Each pipe is 2.11 m long, the outer radius of the pipe is 0.1091 m, and the thickness of the wall is 0.0082 m. The span of each octagonal flange is 0.3388 m, and the thickness of the flange is 0.0254 m. The flanges and a rubber gasket with a thickness of 0.0056 m are clamped together through eight hex head steel bolts (5/8”, Grade 5).

Table 6. The measured natural frequencies of the first 16 elastic modes of the component in Figure 15 with all the bolted connections tightened (undamaged) and four of the 10 bolted connections loosened to hand-tight (damaged), and the changes in the natural frequencies due to damage; the calculated natural frequencies of the first 16 elastic modes of the damaged component are also shown.

| Mode | Undamaged (Hz)(measured) | Damaged (Hz)(measured) | Change | Damaged (Hz)(calculated) | Error |
|------|--------------------------|------------------------|--------|--------------------------|-------|
| 1    | 105.83                   | 104.118                | −1.62% | 102.765                  | −1.30%|
| 2    | 182.21                   | 174.351                | −4.31% | 174.9026                 | 0.32% |
| 3    | 254.02                   | 248.216                | −2.28% | 246.6243                 | −0.64%|
| 4    | 313.75                   | 310.17                 | −1.14% | 314.8317                 | 1.50% |
| 5    | 371.75                   | 366.39                 | −1.44% | 367.86                   | 0.40% |
| 6    | 470.45                   | 464.933                | −1.17% | 461.3521                 | −0.77%|
| 7    | 555.46                   | 550.085                | −0.97% | 543.7143                 | −1.16%|
| 8    | 650.46                   | 634.573                | −2.44% | 628.7344                 | −0.92%|
| 9    | 903.25                   | 893.243                | −1.11% | 890.2972                 | −0.33%|
| 10   | 917.8                    | 907.601                | −1.11% | 905.5672                 | −0.22%|
| 11   | 927.06                   | 916.04                 | −1.19% | 922.6876                 | 0.73% |
| 12   | 1057.26                  | 1022.72                | −3.27% | 1024.462                 | 0.17% |
| 13   | 1306.62                  | 1281.12                | −1.95% | 1272.863                 | −0.64%|
| 14   | 1328.52                  | 1319.41                | −0.69% | 1306.332                 | −0.99%|
| 15   | 1381.74                  | 1332.49                | −3.56% | 1338.277                 | 0.43% |
| 16   | 1533.87                  | 1492.72                | −2.68% | 1484.867                 | −0.53%|

3.3. Detecting loosening of bolted connections in a pipeline

The last application of the vibration-based damage detection method was to detect loosening of bolted connections in a pipeline [11]. A full-size steel pipeline, consisting of two pipes, each welded to a steel flange through a circumferential weld, and eight bolted connections that bolt the two flanges together, was put on two air beds to simulate the free boundaries (Figure 16). Each pipe is 2.11 m long, the outer radius of the pipe is 0.1091 m, and the thickness of the wall is 0.0082 m. The span of each octagonal flange is 0.3388 m, and the thickness of the flange is 0.0254 m. The flanges and a rubber gasket with a thickness of 0.0056 m are clamped together through eight hex head steel bolts (5/8”, Grade
The outer and inner radii of the washer are 0.0224 m and 0.0101 m, respectively, and the thickness of the washer is 0.0022 m. The eight bolted connections are evenly distributed along a circle around the center of the octagonal flanges with a radius of 0.1401 m. The radius of each hole in the flanges is 0.0094 m. The elastic modulus, the Poisson’s ratio, and the mass density of the pipes and flanges are 197 GPa, 0.29, and 7800 kg/m³, respectively. An impact hammer was used to excite the pipeline. Three accelerometers were used to measure the bending/breathing and torsional vibrations of the pipeline: two accelerometers were placed inside the pipeline in two perpendicular radial directions of the cross-section, and one accelerometer was attached along the circumferential direction of the cross-section through a steel bracket that is fixed to the outer wall of the pipeline (Figure 16). The measured natural frequency of the highest rigid body mode was 7.74% of that of the first elastic mode, which validated the free boundaries of the pipeline [2].

The FE model of the pipeline was developed using SDTools [12] (Figure 17). The pipes and flanges can be easily modeled by shell elements; the difficulty lies in the modeling of the bolted connections. The modeling technique developed for bolted connections in Section 3.2.2 [7] cannot be directly applied here due to the relatively thick flanges and the gasket between them. It was observed from the numerical simulation [11] that the pressure generated by a large clamping force widely spreads out in the relatively thick clamped component, and due to the presence of the gasket between the flanges, the flanges will be fully in contact with the gasket under a large clamping force. Hence the flanges cannot be assumed to be in contact within an area around a bolted connection, as the case
in Section 3.2.2, and the technique of modeling a bolted connection by determining the radius of the effective area [7] cannot be applied here. However, each bolted connection can still be modeled by a solid cylinder connecting the shell elements that represent the flanges (Figure 17), but unlike modeling a bolted connection in Section 3.2.2, in which a solid cylinder models the equivalent stiffness and mass effects of a bolted connection, a solid cylinder here only represents a bolt. The radius of the solid cylinder is set to be that of the shaft of each bolt, and its material properties are set to be those of the bolt as well. The density of the solid cylinder is calculated by setting the mass of the cylinder to be that of the bolt. The shell elements that represent each flange are put on the surface of the flange that is in contact with the gasket. The gasket is modeled by solid elements that are fully in contact with the shell elements representing the flanges (Figure 17b). Rigid links are used to connect the shell elements representing a flange and those representing the connecting pipe (Figure 17).

The natural frequencies of the pipeline can be accurately modeled; the maximum error between the calculated and measured natural frequencies of the first 21 elastic modes is 1.82% (Table 7). When all the bolted connections in the pipeline were slightly loosened, which causes a maximum change of 16.54% in the natural frequencies of the first 21 elastic modes (Table 7), the model developed can also be used to model the damaged pipeline. When the elastic and shear moduli of the solid elements representing the bolted connections and the rubber gasket in the FE model are reduced to 0.12 times their original values, the maximum error between the calculated and measured natural frequencies of the first 21 elastic modes of the damaged pipeline is 1.75% (Table 7).

The vibration-based damage detection method was used to detect the loosened bolted connections. Each pipe is evenly divided into 20 groups (groups 1–20, 22–41) (Figure 18). The solid elements representing the bolted connections and the rubber gasket are grouped together as one group (group 21). The damage detection algorithm was terminated when
### Table 7. The measured (Exp) and calculated (FEM) natural frequencies of the pipeline in Figure 16 with tightened and loosened bolted connections, and the changes in the measured natural frequencies due to loosening of the bolted connections

| Mode | Exp (Hz) | FEM (Hz) | Error | Exp (Hz) | FEM (Hz) | Error | Change |
|------|----------|----------|-------|----------|----------|-------|--------|
| 1    | 62.011   | 62.491   | 0.77% | 52.153   | 52.435   | 0.54% | -15.90% |
| 2    | 63.454   | 62.491   | -1.52%| 52.957   | 52.435   | -0.98%| -16.54% |
| 3    | 194.658  | 198.202  | 1.82% | 188.442  | 187.583  | -0.46%| -3.19%  |
| 4    | 195.024  | 198.202  | 1.63% | 188.647  | 187.583  | -0.56%| -3.27%  |
| 5    | 309.831  | 311.883  | 0.66% | 281.344  | 282.294  | 0.34% | -9.19%  |
| 6    | 315.935  | 311.883  | -1.28%| 282.194  | 282.294  | 0.04% | -10.68% |
| 7    | 442.947  | 448.452  | 1.24% | 442.847  | 448.446  | 1.26% | -0.02%  |
| 8    | 446.767  | 444.452  | 0.85% | 444.614  | 448.446  | 0.86% | -0.01%  |
| 9    | 453.997  | 448.452  | -1.21%| 453.695  | 448.468  | -1.15%| -0.07%  |
| 10   | 455.345  | 448.482  | -1.51%| 455.202  | 448.468  | -1.48%| -0.03%  |
| 11   | 458.334  | 455.729  | -0.57%| 457.543  | 455.391  | -0.47%| -0.17%  |
| 12   | 460.035  | 455.729  | -0.94%| 459.264  | 455.391  | -0.84%| -0.17%  |
| 13   | 509.456  | 504.653  | -0.94%| 509.218  | 503.478  | -1.13%| -0.05%  |
| 14   | 509.849  | 504.653  | -1.02%| 509.614  | 503.478  | -1.20%| -0.05%  |
| 15   | 519.823  | 515.210  | -0.89%| 515.900  | 509.242  | -1.29%| -0.75%  |
| 16   | 520.514  | 515.210  | -1.02%| 518.302  | 509.242  | -1.75%| -0.42%  |
| 17   | 578.728  | 578.491  | -0.04%| 578.464  | 584.846  | 1.10% | -0.05%  |
| 18   | 641.373  | 635.871  | -0.86%| 640.890  | 633.994  | -1.08%| -0.08%  |
| 19   | 643.988  | 635.871  | -1.26%| 643.480  | 633.994  | -1.47%| -0.08%  |
| 20   | 663.754  | 656.428  | -1.10%| 650.444  | 644.370  | -0.93%| -2.01%  |
| 21   | 664.980  | 656.428  | -1.29%| 655.256  | 644.370  | -1.66%| -1.46%  |

\( \Delta \) is less than \( 10^{-6} \) or \( \| \nabla Q \| \) is less than \( 10^{-5} \). The measured natural frequencies of the first five even elastic modes were first used in experimental damage detection. Whereas the approximate location of the loosened bolted connections can be detected (Figure 19a), the extent of damage detected is not accurate. The result indicates that there are small stiffness reductions (less than 10%) all over the pipeline and relatively large stiffness reductions near the flanges. The maximum error between the calculated and measured natural frequencies of the first four even elastic modes is 0.87%, but the error is 3.32% for the fifth even elastic mode. The measured natural frequencies of the first six even elastic modes were then used in damage detection. The location and extent of damage can be successfully detected in this case (Figure 19b), but the result indicates that there is a relatively large stiffness reduction (over 15%) in the 31st group. The maximum error between the calculated and measured natural frequencies is 1.48% for the first four even elastic modes and the sixth even elastic mode, but the error is 3.08% for the fifth even elastic mode. The error in damage extent was mainly caused by the relatively large modeling error of the fifth even elastic mode. When the measured natural frequencies of the first four even elastic modes and the sixth even elastic mode were used in damage detection, excellent damage
Figure 19. (a–c) Experimental and (d) numerical damage detection results of the pipeline in Figure 17 with loosened bolted connections.

detection result was obtained (Figure 19c). The maximum error between the calculated and measured natural frequencies of the first six even elastic modes of the damaged pipeline is 1.99%. The exact location and extent of damage were detected in the numerical simulation (Figure 19d).

4. Conclusions

A vibration-based damage detection method that uses changes in natural frequencies to detect damage is a global method that uses minimum measurement data and can minimize local inspections of a test structure. The creation of an accurate physics-based model for both the undamaged and damaged states of a structure and the development of a robust iterative algorithm for an under-determined system are the keys to implementing the vibration-based damage detection method. The real lightning mast and the scale mast can be accurately modeled using mainly shell and beam elements, respectively. The new modeling techniques for fillets in thin-walled beams and for bolted connections can be used to accurately model the space frame structure with tightened or loosened bolted connections. The bolted connections in the pipeline are modeled using a slightly modified approach due to the relatively thick flanges and the gasket between them. The LM method can resolve the error amplification problem associated with the Gauss–Newton method in solving an under-determined nonlinear least-square problem, and improve the robustness of the iterative damage detection algorithm. With the logistic function transformation, which converts the constrained optimization problem to an unconstrained one, the damage detection algorithm is ensured to converge to a stationary point of the objective function. The logistic function transformation can also significantly increase the convergence speed of the iterative algorithm. The new methodology can successfully detect various types of damage in the scale mast, the space frame structure and one of its components, and the pipeline, including loosening of bolted joints or connections, where the maximum modeling and measurement errors are less than 2%, and the maximum change in the natural frequencies
due to damage can be less than 4%. The exact locations and extent of damage can be detected in the numerical simulation where there are no modeling error and measurement noise.

Acknowledgments
This work was supported by the National Science Foundation through Grant No. CMS-0600559 and the American Society for Nondestructive Testing (ASNT) through the 2007 ASNT Fellowship Award. The authors would also like to thank the Baltimore Gas and Electric Company and the Maryland Technology Development Corporation for their previous support, and Benjamin Emory for measuring the natural frequencies of the lightning mast in an electric substation and for building the scale mast and the pipeline in the laboratory.

References
[1] M.I. Friswell and J.E. Mottershead, *Finite Element Model Updating in Structure Dynamics*, Kluwer Academic, Netherlands, 1995.
[2] D.J. Ewins, *Modal Testing: Theory, Practice and Application*, 2nd ed., Research Studies Press, Baldock, UK, 2000.
[3] C.N. Wong, W.D. Zhu, and G.Y. Xu, *On an iterative general-order perturbation method for multiple structural damage detection*, J. Sound Vib. 273 (2004), pp. 363–386.
[4] G.Y. Xu, W.D. Zhu, and B.H. Emory, *Experimental and numerical investigation of structural damage detection using changes in natural frequencies*, J. Vib. Acoust. 129 (2007), pp. 686–700.
[5] K. He and W.D. Zhu, *Detection of damage in space frame structures with L-shaped beams and bolted joints using changes in natural frequencies*, IMAC – XXIX, Jacksonville, FL, January 2011.
[6] K. He and W.D. Zhu, *Modeling of fillets in thin-walled beams using shell/plate and beam finite elements*, J. Vib. Acoust. 131 (2009), 051002(1–16).
[7] K. He and W.D. Zhu, *Finite element modeling of structures with L-shaped beams and bolted joints*, J. Vib. Acoust. 133 (2011), 011011(1–13).
[8] J. Nocedal and S.J. Wright, *Numerical Optimization*, Springer-Verlag, New York, 1999, pp. 252–270.
[9] A. Galantai, *The theory of Newton’s method*, J. Comput. Appl. Math. 124 (2000), pp. 25–44.
[10] K. He and W.D. Zhu, *Detection of damage in lightning masts and loosening of bolted connections in structures using changes in natural frequencies*, Mater. Eval. 68 (2010), pp. 721–732.
[11] K. He and W.D. Zhu, *Detecting loosening of bolted connections in a pipeline using changes in natural frequencies*, ASME Biennial Conference on Mechanical Vibration and Noise, Washington DC, August 2011.
[12] E. Bálmes, J.P. Bianchi, and J.M. Leclère, *SDTools, Structural Dynamics Toolbox, Users Guide, Version 6.1*, Scientific Software Group, Paris, 2009.
[13] J.E. Shigley and C.R. Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill, New York, 1989.