1. Editor’s note

This issue of the SPM Bulletin announces two conferences which are of interest to anyone working in SPM or general topology. In the conference announced in [§3.2 below] it is planned to have a significant part devoted to SPM. Those who are interested in participating should contact Ljubiša D. R. Kočinac at lkocinac@ptt.yu. Kočinac is a very active mathematician in the field of SPM. We announce here one of his most recent works.

The first issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online:

(1) First issue: http://arxiv.org/abs/math.GN/0301011
(2) Second issue: http://arxiv.org/abs/math.GN/0302062
(3) Third issue: http://arxiv.org/abs/math.GN/0303057

We are looking forward to receive more announcements from the recipients of this bulletin.

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2. Research announcements

2.1. Selection principles in uniform spaces. We begin the investigation of selection principles in uniform spaces in a manner as it was done with selection principles theory for topological spaces. We introduced and characterized uniform versions of classical topological notions of the Menger, Hurewicz and Rothberger properties. The uniform $\gamma$-sets are also considered.

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3. Conferences

3.1. General and Set-Theoretic Topology. (dedicated to the 60th birthday of Istvan Juhasz) Budapest, Hungary, from 8 to 13 August, 2003.

The János Bolyai Mathematical Society, the Alfréd Rényi Institute of Mathematics and the Paul Erdős Summer Research Center of Mathematics are organizing a Colloquium on General and Set-Theoretic Topology in the period of August 8-13, 2003 in Budapest. You are cordially invited to attend this conference. The aim of the Colloquium is to provide ground for the exchange of information on new achievements and on the recent problems of General and Set-Theoretic Topology.

Organizing Committee: A. Csaszar (cochairman), J. Gerlits, A. Hajnal (chairman), E. Makai, G. Sagi, L. Soukup(secretary), Z. Szentmiklosy

The following people will give invited addresses:

- A. V. Archangelskii (Athens, OH)
- A. Dow (Charlton, NC)
- K. Kunen (Madison)
- J. van Mill (Amsterdam)
- S. Romaguera (Valencia)
- S. Shelah (Jerusalem)
- W. A. R. Weiss (Toronto)

Besides these lectures there will be 20-minutes contributed talks.

The conference will take place at the Alfred Renyi Institute of Mathematics, Budapest (V. Realtanoda u. 13–15, Budapest).

E-mail: top2003@renyi.hu
URL: http://www.renyi.hu/~top2003

3.2. Interplay between Topology and Analysis. (accompanying the Congress of MASSEE) September 15–21, 2003, Bulgaria. Organizing and Programme Committee:

- A. V. Arhangel’skii, University of Ohaio, USA
- M. M. Choban, Tiraspol University in Kishineu, Moldova
- D. Dimovski, University of Skopje, Macedonia
- P. S. Kenderov (Co-Chair), Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
Lj. Kocinac, University of Nis, Serbia
S. J. Nedev (Co-Chair), Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
J. Orihuela, University of Murcia, Spain
J. Pelant, Institute of Mathematics, Czech Academy of Sciences
I. Shishkov (Secretary), Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
St. Troyanski, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

The main purpose of this Mini Symposium is to bring together, within the Congress of MASSEE, people interested in the indicated areas, including Ph.D. and university students. Topology and the Analysis occupy central place in Mathematics. The interplay between Topology and Analysis is evident in such topics like Selections of set-valued mappings, Geometry of Banach spaces and, in particular, equivalent renorming of Banach spaces. Recent results in the area show that the renorming of Banach spaces is closely related to $s$-fragmentability of the weak topologies, and thus to the topological theory of metrizability. Useful tools when attacking problems in this area are Topological games and quasi-continuity. Any other related topics are welcome.

The program of the Workshop will include invited talks of approximately 45 min. duration, as well as contributed presentations of approximately 20 min. duration (including 5 min. discussion), depending on the total number of contributions. For the accommodation, participation fees, deadlines, abstracts, etc., please refer to

http://www.math.bas.bg/massee2003

Ivailo Shishkov, shishkov@math.bas.bg

4. Modern selection principles: Open problems

By modern selection principles we mean selection principles involving modern types of open covers. These include groupable and weakly groupable covers [4, 5], and $\tau$-covers. In this note we concentrate on this latter type of covers. The following is based on [7].

Fix a zero-dimensional, separable metric space $X$. (Topologically this is simply a zero-dimensional set of reals.) Recall that the symbols $\mathcal{O}$, $\Omega$, and $\Gamma$ denote the by-now-classical collections of (countable open) covers, $\omega$-covers, and $\gamma$-covers of $X$. $U$ is a $\tau$-cover of $X$ if it is a large cover of $X$, and for each $x, y \in X$, either $\{U \in U : x \in U, y \notin U\}$ is finite, or $\{U \in U : y \in U, x \notin U\}$ is finite. As any $\omega$-cover of a set of reals contains a countable $\omega$-cover of that set, and every large subcover of a $\tau$-cover is a $\tau$-cover (and every $\omega$-cover is a large cover), we have that Every open $\tau$-cover of $X$ contains a countable $\tau$-cover of $X$. Let $T$ denote the collection of countable open
τ-covers of $X$. Then

$$\Gamma \subseteq T \subseteq \Omega \subseteq \mathcal{O}.$$ 

The notion of τ-covers introduces seven new pairs—namely, $(T, \mathcal{O})$, $(T, \Omega)$, $(T, T)$, $(T, \Gamma)$, $(\mathcal{O}, T)$, $(\Omega, T)$, and $(\Gamma, T)$—to which any of the selection operators $S_1$, $S_{fin}$, and $U_{fin}$ can be applied. This makes a total of 21 new selection hypotheses. Fortunately, some of them are easily eliminated, and the surviving properties appear in the following diagram.

Below each property in the above diagram appears a “serial number” (to be used below), and its critical cardinality, the minimal cardinality of a set of reals not satisfying the property. The cardinal numbers $p$, $t$, $b$, and $d$ are the well-known pseudo-intersection, tower, (un)bounding, and dominating numbers, and $\text{cov}(\mathcal{M})$ is the covering number for the meager ideal (see, e.g., [2] or [1] for definitions and details). $\tau$ is the excluded-middle number, and is equal to $\max\{s, b\}$, where $s$ is the splitting number [6].

As indicated in the diagram, some of the critical cardinalities were not found yet.

**Problem 4.1.** What are the unknown critical cardinalities in the above diagram?
Recall that Issue 2’s Problem of the month mentioned two unsettled implications in the corresponding diagram for the classical types of open covers. As there are many more properties when \(\tau\)-covers are incorporated into the framework, and since this investigation is new, there remain many unsettled implications in the above diagram.

To be precise, there are exactly 76(!) unsettled implications in this diagram. These appear as question marks in the following implications table. Entry \((i, j)\) in the table \((i\text{th row, } j\text{th column})\) is to be interpreted as follows: It is 1 if property \(i\) implies property \(j\), 0 if property \(i\) does not imply property \(j\) (that is, consistently there exists a counter-example), and ? if the implication is unsettled.

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 | ? | ? | ? | 0 | 0 | 0 | 0 | 1 | 1 | ? | ? | 0 | 0 | 1 | 1 | 1 |
| 1 | ? | 1 | 1 | 1 | 0 | ? | ? | ? | 0 | 0 | 0 | 0 | 1 | 1 | ? | ? | 0 | 0 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 1 | 0 | 0 | ? | ? | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | ? | 0 | 0 | 0 | 0 | 1 |
| 3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | ? | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | ? | ? | 1 | 1 | 1 | 1 | 0 | ? | 1 | 1 | 1 | 1 | 1 |
| 5 | ? | 1 | 1 | 1 | ? | 1 | 1 | 1 | 0 | ? | ? | 1 | 1 | 1 | 1 | 0 | ? | ? | 1 | 1 | 1 |
| 6 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | ? | ? | 0 | 1 | 0 | 1 | 0 | ? | 0 | 0 | 1 |
| 7 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | ? | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | ? | 1 | 1 | 1 | ? | 1 | 1 | 1 | ? | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | ? | 1 | 1 |
| 10 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 11 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | ? | ? | ? | ? | 0 | ? | ? | ? | 0 | 0 | 0 | 0 | 1 | 1 | ? | ? | 0 | 0 | 1 | 1 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 14 | ? | ? | ? | ? | ? | ? | ? | ? | ? | 0 | 0 | ? | 1 | 1 | 1 | 0 | ? | ? | 1 | 1 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | ? | 0 | 0 |
| 16 | ? | ? | ? | ? | ? | ? | ? | ? | ? | 1 | 1 | 1 | 1 | 1 | 1 | 1 | ? | 1 | 1 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ? | 0 | 0 | 0 | 1 | 1 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ? | 0 | 0 | 0 | 1 | 1 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ? | 0 | 0 | 0 | 0 | 1 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Problem 4.2. Settle any of the unsettled implications in the above table.

Almost any solution logically implies several other solutions. The above implications table, as well as the remarks which follow, were obtained by using a simple computer program.

For example, if the solution to Issue 2’s Problem of the month is negative, that is, \(U_{fin}(\Gamma, \Gamma)\) does not imply \(S_{fin}(\Gamma, \Omega)\) (a plausible assumption), then 6 implications are settled. If, in addition, the solution to Issue 1’s Problem of the month is positive, then only 53 implications (out of the 76 we began with) remain unsettled.

Marion Scheepers asked us which single solution would imply as many other solutions as possible. The answer found by our program is the following: If entry \((12, 5)\) is 1 (that is, \(S_{fin}(\Gamma, T)\) implies \(S_1(T, T)\)), then there remain only 33 (!) open problems.
The best possible negative entry in (16, 3): If $S_{\text{fin}}(\Omega, T)$ does not imply $S_1(\Gamma, \mathcal{O})$, then only 47 implications remain unsettled.

Finally, observe that any solution in Problem 4.1 would imply several new nonimplications.

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5. Problem of the month

Scheepers chose the following problem out of all the problems discussed above as the most interesting.

**Problem 5.1.** Does $S_1(\Omega, T)$ imply the Hurewicz property $U_{\text{fin}}(\Gamma, \Gamma)$?

The reason for this choice is that if the answer is positive, then $S_1(\Omega, T)$ implies the Gerlitz-Nagy (⋆) property [3], which is equivalent to another modern selection property.

Observe that a positive answer to Issue 1’s Problem of the month implies a positive answer to this problem too.

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References

[1] A. R. Blass, *Combinatorial cardinal characteristics of the continuum*, in: **Handbook of Set Theory** (eds. Foreman, Kanamori, and Magidor), to appear.
[2] E. K. van Douwen, *The integers and topology*, in: **Handbook of Set Theoretic Topology** (eds. K. Kunen and J. Vaughan), North-Holland, Amsterdam: 1984, 111–167.
[3] J. Gerlits and Zs. Nagy, *Some properties of $C(X)$, I*, Topology and its Applications 14 (1982), 151–161.
[4] Lj. D. R. Kočinac and M. Scheepers, *Combinatorics of open covers (VII): groupability*, Fundamenta Mathematicae, to appear.
[5] L. Babinkostova, Lj. D. R. Kocinac, and M. Scheepers, *Combinatorics of open covers (VIII)*, preprint.
[6] S. Shelah and B. Tsaban, *Cardinals related to the minimal tower problem*, Journal of Applied Analysis, to appear.
[7] B. Tsaban, *Selection principles and the minimal tower problem*, submitted.