Designing a Spin Channel for Perfect Quantum Communication

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We propose a scheme for using a spin chain with fixed symmetry interaction as a channel for perfect quantum communication. The perfect quantum communication is determined by the eigenvalues that form a special arithmetical progression, the concrete interaction parameter \( J_i \) is obtained as a function of integer and perfect transmission time \( t_p \). There are infinite choices of \( J_i \) for perfect transmission and one can design the channel according to the requirement of \( t_p \). This scheme will provide more choices of spin chain for future experiment in quantum communication.

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Introduction. In quantum information processing, it is needed to transfer a quantum state from one place to another, such as quantum key distribution [1], teleportation [2] and quantum computation (communication between quantum processors) [3]. Thus it is very important to find physical systems to serve as channels for quantum communication. A very successful area is quantum optics, in which the implementation of quantum state transmission has been realized. The carriers of information can be addressed and transmitted with high control and with a low level of decoherence, while this kind of state transport needs interfacing between quantum processor and quantum channels, which make the quantum computer complexity and lower speed.

So a great attention is focused on the problem of transferring quantum information in a solid-state environment, in which one can either first use the channel to share entanglement with a separated party and then use this entanglement for teleportation, or can directly transmit a state through the channel. The key step of using teleportation to transport a state is looking for a quantum channel with ideal entanglement, such as a spin chain with ideal boundary entanglement [4, 5, 6].

Bose first propose a scheme to use a spin chain as a channel for short distance quantum communication [7]. The communication is achieved by state dynamical evolution in spin chain, which does not require the ability to switch “on” and “off” the interactions between the spins, it also does not require any modulation by external fields either. This approach is restricted to zero temperature and single spin-wave states. Basing on Bose’s scheme, many works focus on the perfect transmission in spin systems [8, 9, 10, 11, 12, 13, 14]. Christandl et al. [9] found the simplest scheme to make a proper choice of the modulation of the coupling strengths in spin chain then realize a perfect transmission in arbitrary long distance, Zhang et al. [15] realize this scheme in three qubit case by using liquid nuclear magnetic resonance. Seeking for the ideal channel is important, as well as its transmitted time, the shorter the better. Christandl’s scheme [9] is masterly, but the interaction \( J_i = J_{N-i} = \sqrt{i}(N-i) \) for perfect transmission is not unique.

In this paper, the shortest time of perfect transmission will be studied in spin chain, as well as the corresponding properties of interaction by constructing the eigenvectors with the ways different from Ref. [7, 9, 10].

The eigenvectors and the transmit amplitude that described the transmission. The Hamiltonian of N qubit Heisenberg XXX open chain is \( H_{XX}^N = \sum_{j=1}^{N-1} J_i (\sigma_i^+ \sigma_{i+1}^- + \sigma_i^- \sigma_{i+1}^+) + \frac{1}{2} \sigma_i^z \sigma_{i+1}^z \), where \( \sigma^x, \sigma^y, \sigma^z \) are the Pauli matrices, \( J_i \) the exchange hopping. If the \( \sigma^z \) term vanishes, this system degenerates into XX model note as \( H_{XX}^N = \sum_{j=1}^{N-1} J_i (\sigma_i^+ \sigma_{i+1}^- + \sigma_i^- \sigma_{i+1}^+) \).

The initial state of the system is prepared as the first qubit to be spin up and the others spin down. Since the Hamiltonian commutes with the total spin component along the z direction, the relevant sector of the Hilbert space must be spanned by the states \( |j> = |0_1, 0_2, \ldots, 0_{j-1}, 1_j, 0_{j+1}, \ldots, 0_N \rangle > \) with \( j = 1, \ldots, N \).

The open spin chain is symmetrical about the middle point. Let \( J_i = J_{N-i} \), \( |j> > \) and \( |N+1-j> > \) must have the same probability in the eigenvectors, so the eigenvectors of the system can be written as

\[
|\psi_m> = \sum_{j=1}^{k} C_{mj} e^{i \alpha_m j}|j> + |N+1-j>, N = 2k;
\]

\[
|\psi_m> = \sum_{j=1}^{k} C_{mj} e^{i \alpha_m j}|j> + |N+1-j>,
\]

\[
+ C_{m,k+1}|k+1>, N = 2k + 1
\]

(1)

where \( C_{mj}, \alpha_m (0 or \pi) \), \( j = 1, 2, \ldots, k, m = 1, 2, \ldots, N \) are the parameters determined by \( H|\psi> = E|\psi> \) and the normalization condition.

The transition amplitude between the boundary qubits
to simplify the model, so we change XXX model and calculate its transition amplitude \( |f_{\text{XXX}}^N(J,t)| = \left| -iC_{1}^{t} \sin(E_{1}t) + 2C_{2}^{t} \sin(E_{2}t) \right| \). Here \( C_{1}^{t} = \sqrt{1/(2+2E_{1}^{2})} \), \( C_{2}^{t} = \sqrt{1/(2+2E_{2}^{2})} \), \( E_{1,2} = \pm 7/\sqrt{2(2+E_{2}^{2})} \) Perfect transmission means \(|f_{\text{XXX}}^N(J)| = 1\), as in the XXX model, two conditions must be satisfied in the XXX model: (1) \( 2C_{1}^{2} + 2C_{2}^{2} = 1 \); (2) \( E_{2}^{\pm} = \pm \sqrt{2(2+E_{2}^{2})} \). When \( k_{1} = 1 \) and \( k_{2} = -1 \), \( J = 2/\sqrt{3} \), i.e. the interaction in the middle is stronger than the boundary, at this condition, the time of perfect transmission is \( t_{\text{min}} = \sqrt{3}/2 \), this is coincidence with \( J_{1} = J_{3} = \sqrt{3} \) and \( J_{2} = 2 \) in Ref. \[8\]. When \( k_{1} = 2 \) and \( k_{2} = -2 \), \( J = 2/\sqrt{15} \), i.e. the interaction in the middle is weaker than the boundary, the time of perfect transmission is \( t_{\text{min}} = \sqrt{15}/2 \). If \( k_{1} \) and \( k_{2} \) take other values, one can obtain corresponding interaction for perfect transmission.

### Table 1. The eigenvalues and eigenvectors of four qubit XXX open chain.

| \( m \) | \( E_{m} \) | \( C_{m1}(\alpha_{m1}) \) | \( C_{m2}(\alpha_{m2}) \) |
| --- | --- | --- | --- |
| 1 | 1 + \( \frac{1}{2} \) | \( \frac{\sqrt{3}}{2} \) | \( \frac{\sqrt{3}}{2} \) |
| 2 | \( \frac{1}{2} - 1 \) | \( \frac{\sqrt{3}}{2} \) | \( \frac{\sqrt{3}}{2} \) |
| 3 | \( \frac{1}{2} + \sqrt{2} \) | \( \frac{\sqrt{3} \sqrt{2}}{2} \) | \( \frac{\sqrt{3} \sqrt{2}}{2} \) |
| 4 | \( \frac{1}{2} - \sqrt{2} \) | \( \frac{\sqrt{3} \sqrt{2}}{2} \) | \( \frac{\sqrt{3} \sqrt{2}}{2} \) |

Using Eq. (2) one can get the transition amplitude between the boundary qubits: \( f_{\text{XXX}}^N(J,t) = C_{11}^{t} e^{-iE_{1}t} + C_{2}^{t} e^{-iE_{2}t} - C_{3}^{t} e^{-iE_{3}t} - C_{4}^{t} e^{-iE_{4}t} \). In order to obtain perfect transition (i.e. \( |f_{\text{XXX}}^N(J)| = 1 \)), two conditions must be satisfied at the same time, they are \( \sum_{m=1}^{4} C_{m1}^{2} = 1 \) and \( e^{-iE_{1}t} = e^{-iE_{2}t} = e^{-iE_{3}t} = e^{-iE_{4}t} = 1 \). The first condition is satisfied naturally, but the second one, the reason is that \( E_{2} = E_{3} = 0, E_{4} = \sqrt{2}, E_{1} = \frac{2i}{\sqrt{2} \pi} \). Using this condition, \( J_{1} = J_{2} = J_{3} + 1 \), \( J \), the eigenvalues are \( \{1,2,3,4\} \) are integer. So this model can not give perfect transmission.

In Ref. \[7\], Bose manipulated a uniform open XXX chain with magnetic field which has no effect to the transition amplitude, and claimed that “\( N = 4 \)” gives perfect quantum transmission, his result based on “numerical analysis”. The eigenvectors \( \{1,2,3,4\} \) are the same as Table 1. when \( J = 1 \), while the eigenvalues are just different in a constant which comes from the magnetic field, the corresponding order of eigenvectors are \( |1> = -|\psi_{1}>, |2> = -|\psi_{3}>, |3> = -|\psi_{2}>, |4> = -|\psi_{4}> \). Although four qubit XXX open chain can not realize perfect transmission, proper value \( J \) in Table 1 can make \( |f_{\text{XXX}}^N(J)| \rightarrow 1 \), see Table 2.

### Table 2. The relation between the maximal transition amplitude and \( J \).

| \( J \) | \( |f_{\text{XXX}}^N(J)|_{\text{max}} \) | \( J \) | \( |f_{\text{XXX}}^N(J)|_{\text{max}} \) |
| --- | --- | --- | --- |
| 2 \( \pi \) | 0.99981 | 7 \( \pi \) | 0.99997 |
| 2 \( 2\pi \) | 0.99798 | 8 \( 2\pi \) | 0.99998 |
| 3 \( 2\pi \) | 0.99914 | 9 \( 2\pi \) | 0.99999 |
| 4 \( 2\pi \) | 0.99970 | 10 \( 2\pi \) | 0.99999 |
| 5 \( 2\pi \) | 0.99988 | 6 \( 2\pi \) | 0.99999 |
| 6 \( 2\pi \) | 0.99994 | 7 \( 2\pi \) | 0.99999 |

In order to get perfect transmission, we must simplify the restricted conditions. The natural idea is...
$J_0$, $J_3 = J_5$ and $J_4$, the number of eigenvalue is eight, the eigenvalues satisfy $E_i = -E_{k-i}$, $i = 1, 2, 3, 4$. The transmit amplitude between the boundary qubits is $|f^{Nxx}_{1N}(J_i, t)| = | - i(2C_{2j}^2 \sin(E_{2j} t) + 2C_{2j}^2 \sin(E_{2j} t) + 2C_{2j}^2 \sin(E_{2j} t) + 2C_{2j}^2 \sin(E_{2j} t))|$. As in six qubit case, perfect transmission is assured by the condition $E_1 = (8n-1)\pi$, $E_2 = (4n-1)\pi$, $E_3 = -\frac{2\pi}{3}$ and $E_4 = -\frac{4n+1}{3}\pi$ with $n = 1, 2, \ldots$, one can construct a special arithmetical progression $\pi, 3\pi, 5\pi, \ldots$.

The $J_i$ corresponds to the results in [1] if $n = 1$, except this, $J_i$ has infinite choices for perfect transmission. For example $n = 2$, $t_p = \frac{\pi}{5}$ in Eq. (4) one will obtain $J_1 = J_4 = \sqrt{27}$, $J_2 = J_5 = \sqrt{44}$ and $J_3 = J_6 = \sqrt{35}$ and $J_4 = 12$, these is an order “weak-strong-weak-strong-\ldots”.

In odd qubit open chain, three qubit case is a uniform chain, $J_1 = J_3$, the transition amplitude between the boundary qubits is $|f^{Nxx}_{1N}(t)| = |(\cos(2J_1 t) - 1)|$, perfect state transfer can be obtained at $t = \frac{\pi}{2J_1}$. For five qubit case, $J_1 = J_4$, $J_2 = J_3$, $|E_{15}| = J_2^2 \cos(\frac{\sqrt{J_2 + J_3} t}{2}) + \frac{J_2^2}{2(2J_2 + J_3)} \cos(\sqrt{J_2 + J_3} t) - \frac{1}{2} \cos J_1 t$. When $J_1 = \frac{\pi}{t_p}$, $J_2 = \sqrt{\frac{(2k)^2}{2} - \frac{\pi}{6}}$, $k = 1, 2, \ldots$ or $J_2 \gg J_1$, perfect transmission can be obtained at $t_p = \frac{\pi}{2J_1}$. For seven qubit case, we can get

$$
J_1 = J_6 = \frac{\pi}{t_p} \frac{(2n_1 - 1)(2n_2 - 1)}{\sqrt{A}},
J_2 = J_3 = \frac{\pi}{t_p} \frac{\sqrt{2n_3^2 A - (2n_1 - 1)^2(2n_2 - 1)^2}}{\sqrt{A}},
J_4 = \frac{\pi}{t_p} \sqrt{\frac{A}{2}},
$$

where $A = (2n_1 - 1)^2 + (2n_2 - 1)^2 - (2n_3)^2$, $n_1, n_2, n_3 = 1, 2, \ldots$, and the value of $n_i$, $i = 1, 2, 3$ must keep $J_i = \frac{\pi}{2J_1}$, $i = 1, 2, 3$ real. Similarly as in even qubit case, one has infinite choices for perfect transmission except for $J_4 = J_{N-4} = \sqrt{\frac{\pi}{2}(N-4)}$. The different lies in one need more than one integer to determine $J_i$.

\textit{N qubit XX open chain as ideal channel.} For general N qubit case, the idea of constructing ideal channel is the same as finite qubit case: at the condition of the spin chain with symmetry interaction $J_i = J_{N-i}$, one can figure out the expression of transmit amplitude $|f^{Nxx}_{1N}|$, then find the perfect transmission is determined by the eigenvalues of the system, at use the proper eigenvalues to solve the corresponding $J_i$. The transmit amplitude expression has great difference for the parity of $N$.

(1) $N$ is even, at this case, one have to divided it into two classes because there still has some difference between $N = 4k$ and $N = 4k - 2$, $k = 1, 2, 3, \ldots$.

When $N = 4k$, the transmit amplitude at this case can be written as $|f^{Nxx}_{1N}| = | - i(\sum_{j=1}^{k} 2C_{2j}^2 \sin(E_{2j} t))|$, the number of $E_j > 0$ and $E_j < 0$ is equal, as long as $E_j t$ can construct a special arithmetical progression $a_i = (1 - 4(k - 1)n)^{\frac{\pi}{2}}$, $a_{i+1} - a_i = (4n)^{\frac{\pi}{2}}$, then we can realize perfect transmission. The concrete expression of $J_i$ is obtained by comparing the coefficients in the following two equations

$$
\begin{align*}
2k & \prod_{j=1}^{k} (x - E_j) = \prod_{i=-k}^{k} [x - (-1 + 4in)^{\frac{\pi}{2t}}] = 0, \\
\text{Det} \begin{pmatrix}
-x & J_1 & 0 & \ldots & 0 \\
J_1 & -x & J_2 & \ldots & 0 \\
0 & J_2 & -x & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & J_{2k-1} & J_{2k} - x \end{pmatrix} & = 0, \quad (7)
\end{align*}
$$

When $N = 4k - 2$, the transmit amplitude is $|f^{Nxx}_{1N}| = | - i(\sum_{j=1}^{k} 2C_{2j}^2 \sin(E_{2j} t))|$, the number of $E_j > 0$ and $E_j < 0$ is $k - 1$, as long as $E_j t$ can construct a special arithmetical progression $a_i = (1 - 4(k - 1)n)^{\frac{\pi}{2}}$, $a_{i+1} - a_i = (4n)^{\frac{\pi}{2}}$, then we can realize perfect transmission. The concrete expression of $J_i$ is obtained by comparing the coefficients in the following two equations

$$
\begin{align*}
2k & \prod_{j=1}^{k} (x - E_j) = \prod_{i=-k}^{k-1} [x - (-1 + 4in)^{\frac{\pi}{2t}}] = 0, \\
\text{Det} \begin{pmatrix}
-x & J_1 & 0 & \ldots & 0 \\
J_1 & -x & J_2 & \ldots & 0 \\
0 & J_2 & -x & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & J_{2k-2} & J_{2k-1} - x \end{pmatrix} & = 0, \quad (9)
\end{align*}
$$

(2) $N$ is odd, we still divided it into two classes because the slightly difference between $N = 4k - 1$ and $N = 4k + 1$, where $k = 1, 2, 3, \ldots$.

When $N = 4k - 1$, the transmit amplitude is $|f^{Nxx}_{1N}| = | \sum_{j=1}^{k} 2C_{2j}^2 \cos(E_{2j} t) - (C_{2k+1}^2 + \sum_{j=2k+1}^{2k} 2C_{2j}^2 \cos(E_{2j} t))|$, the number of $E_j > 0$ and $E_j < 0$ is equal, the number of $E_j = 0$ is one. $J_i$ is obtained by comparing the following
two group equations,
\[ \prod_{j=1}^{k} (x - E_j) = \prod_{j=1}^{k} (x - \frac{2n_j \pi}{t}) = 0, \]  
(10)

\[ \begin{vmatrix}
-x & J_1 & 0 & \ldots & 0 \\
J_1 - x & J_2 & \ldots & 0 \\
0 & J_2 & -x & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & J_{2k-1} & -x \\
\end{vmatrix}_{2k \times 2k} = 0, \]  
(11)

\[ x \prod_{j=2k+1}^{2k+k+1} (x - E_j) = x \prod_{j=2k+1}^{2k+k+1} (x - \frac{(2n_j - 1) \pi}{t}) = 0, \]  
(12)

\[ \begin{vmatrix}
-x & J_1 & 0 & \ldots & 0 \\
J_1 - x & J_2 & \ldots & 0 \\
0 & J_2 & -x & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & J_{2k-2} & -x \\
\end{vmatrix}_{(2k-1) \times (2k-1)} = 0, \]  
(13)

where \( n_j = 1, 2, \ldots \) and their values must satisfy \( J_i \) is real, similarly in \( N = 4k + 1 \) case.

When \( N = 4k + 1 \), the transmit amplitude is \( |f_{N,xx}^{\text{opt}}| = \left| -(C_{k+1}^2 + \sum_{j=1}^{k} 2C_{j+1}^2 \cos E_j t + \sum_{j=2k+1}^{2k} 2C_{j+2}^2 \cos E_j t) \right| \), the number of \( E_j > 0 \) and \( E_j < 0 \) is equal, the number of \( E_j = 0 \) is one. \( E_j \) is obtained by comparing the following two group equations,

\[ x \prod_{j=1}^{k} (x - E_j) = \prod_{j=1}^{k} (x - \frac{2n_j \pi}{t}) = 0, \]  
(14)

\[ \begin{vmatrix}
-x & J_1 & 0 & \ldots & 0 \\
J_1 - x & J_2 & \ldots & 0 \\
0 & J_2 & -x & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & J_{2k-2} & -x \\
\end{vmatrix}_{(2k+1) \times (2k+1)} = 0, \]  
(15)

\[ \prod_{j=2k+2}^{2k+k+1} (x - E_j) = \prod_{j=2k+2}^{2k+k+1} (x - \frac{(2n_j - 1) \pi}{t}) = 0, \]  
(16)

\[ \begin{vmatrix}
-x & J_1 & 0 & \ldots & 0 \\
J_1 - x & J_2 & \ldots & 0 \\
0 & J_2 & -x & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & J_{2k-1} & -x \\
\end{vmatrix}_{2k \times 2k} = 0. \]  
(17)

Someone may doubt how can we solve such equations when \( N \) is large enough. The doubt is reasonable but here is unnecessary, because there exist a recursion relation between \( J_i \) and \( J_{i+1} \), the interaction in the middle is easily obtained. Obviously, even qubit chain has some advantage than odd qubit chain, the former only need one parameter \( n \) to determine \( J_i \), while the later need \( (N - 1)/2 \) parameters \( n_1, n_2, \ldots, n_{N/2 - 1} \) to determine \( J_i \), from this point even qubit chain is a good candidate for experiment.

Discussions. Our results offer infinite choices of interaction \( J_i \) for perfect transmission. The perfect transmission time \( t_p \propto 1/J_i \), it will be short enough as long as \( J_i \) can be tuned to arbitrarily large. Our scheme can be generalized to transmit more than one bit information.

The further calculation show that four qubit open chain with symmetry interaction can be used to transmit two bits information perfectly.

It is also interesting to study the boundary entanglement of the perfect transmission channel. For four qubit XXX open chain, the boundary entanglement is

\[ C_{14} = \frac{2(C_{i1}^2 e^{-E_{i1}^2} + C_{i2}^2 e^{-E_{i2}^2} - C_{i3}^2 e^{-E_{i3}^2} - C_{i4}^2 e^{-E_{i4}^2})}{\sqrt{4}} \sum_{m=1}^{2} \sqrt{2m} \],

(18)

where \( C_{i1} \) and \( E_m \) can be seen from Table 1. The concurrence \( C_{14} \) is a half if \( J \to 0 \) and vanishes as \( J \) to be enough large. The boundary entanglement of N qubit XX open chain is \( C_{1N} = \sqrt{\frac{1}{N^2}} \) under the interaction \( J_i = \sqrt{i(N-i)} \). For other perfect interaction, one can also calculate the corresponding \( C_{1N} \), for example, N=4 XX open chain with \( J_1 = J_3 = 1, J_2 = \frac{2}{\sqrt{15}}, C_{14} = \frac{3}{8}; \)
N=5 XX open chain with \( J_1 = J_4 = 1, J_2 = J_3 = \sqrt{30}, C_{15} = \frac{15}{16} \) etc.. Therefore we can conclude that the perfect transmission channel is not the ideal entanglement channel.

In summary, we found a way of constructing an ideal spin channel for quantum communication in study of the finite spin chain with symmetry interaction \( J_i = J_{N-i} \). The equation of \( J_i \) satisfied ideal channel condition and the corresponding solutions are obtained. Besides possessing the advantage of Bose’s and Christandl’s propose, our scheme offers infinite choices of interaction for perfect quantum communication, and one can design an ideal channel according to the requirement of the perfect transmission time \( t_p \). This scheme will provide more choices of spin chain for future experiment in quantum communication and it can also be generalized to transmit more than one bit information.

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