1. Introduction

Weak decays of heavy hadrons, of B mesons in particular, provide us with essential information on the quark flavor sector. Since the underlying flavor dynamics of the quarks is masked by strong interactions, a sufficiently precise understanding of QCD effects is crucial to extract from weak decays involving hadrons the basic parameters of flavor physics. Much interest in this respect is being devoted to rare B decay modes such as $B \to \pi\pi$, $\pi K$, $\pi\rho$, $\phi K_S$, $K^{*}\gamma$, $\rho\gamma$ or $K^{*}l^+l^-$. These decays are a rich source of information on CKM parameters and flavor-changing neutral currents. Many new results are now being obtained from the B meson factories and hadron colliders. Both exclusive and inclusive decays can be studied. Roughly speaking, the former are more difficult for theory, the latter for experiment.

In dealing with the presence of strong interactions in these processes the challenge for theory is in general to achieve a systematic separation of long-distance and short-distance contributions in QCD. This separation typically takes the form of representing an amplitude or a cross section as a sum of products of long and short distance quantities and is commonly referred to as factorization. The concept of factorization requires the existence of at least one hard scale, which is large in comparison with the intrinsic scale of QCD. For B decays this scale is given by the $b$-quark mass, $m_b \gg \Lambda_{QCD}$. The asymptotic freedom of QCD allows one to compute the short-distance parts using perturbation theory. Even though the long-distance quantities still need to be dealt with by other means, the procedure usually entails a substantial simplification of the problem.

Various methods, according to the specific nature of the application, have been developed to implement the idea of factorization in the theoretical description of heavy hadron decays. These include heavy-quark effective theory (HQET), heavy-quark expansion (HQE), factorization in exclusive nonleptonic decays and soft-collinear effective theory (SCET). In particular the latter two topics are more recent developments and are still under active investigation and further study. They play an important role for the exclusive rare B decays listed above. Dynamical calculations based on these tools hold the promise to improve our understanding of QCD in heavy-hadron decays significantly and to facilitate the determination of fundamental weak interaction parameters. A different line of approach is the use of the approximate $SU(2)$ or $SU(3)$ flavor symmetries of QCD in order to isolate the weak couplings in a model-independent way. Both strategies, flavor symmetries and dynamical calculations, are complementary to each other and enhance our ability to test quark flavor physics. While the flavor symmetry approach gives constraints free of hadronic input in the symmetry limit, dynamical methods allow us to compute corrections from flavor symmetry breaking.

The following section gives a brief overview of theoretical frameworks for B decays based on the heavy-quark limit. The remainder of this talk then concentrates on the subject of exclusive rare or hadronic decays of B mesons.

2. Tools and Applications

The application of perturbative QCD to hadronic reactions at high energy requires a proper factorization of short-distance and long-distance contributions. One example is given by the operator product expansion (OPE) used to construct effective Hamiltonians for hadronic B decay. This is shown schematically in Fig. 1 for a generic B decay amplitude. The
OPE approximates the nonlocal product of two weak currents, which are connected by $W$ exchange in the full standard model, by local 4-quark operators, multiplied by Wilson coefficients $C(M_W/\mu, \alpha_s)$. In this way the short-distance physics from scales of order $M_W$ (or $m_t$ appearing in penguin loop diagrams) down to a factorization scale $\mu \sim m_b$ is isolated into the coefficient. Determined by high energy scales, the coefficient can be computed perturbatively, supplemented by renormalization-group improvement to resum large logarithms $\sim \alpha_s \ln M_W/m_b$. The QCD dynamics from scales below $\mu$ is contained within the matrix elements of the local operators. These matrix elements depend on the particular process under consideration, whereas the coefficients are universal. The approximation is valid up to power corrections of order $m_b^2/M_W^2$.

In the case of $B$ decay amplitudes, the hadronic matrix elements themselves still contain a hard scale $m_b \gg \Lambda_{QCD}$. Contributions of order $m_b$ can be further factorized from the intrinsic long-distance dynamics of QCD. This is implemented by a systematic expansion in $\Lambda_{QCD}/m_b$ and $\alpha_s(m_b)$ and leads to important simplifications. The detailed formulation of this class of factorization depends on the specific application and can take the form of HQET, HQE, QCD factorization for exclusive hadronic $B$ decays or SCET.

- **HQET** describes the static approximation for a heavy quark, formulated in a covariant way as an effective field theory.\textsuperscript{11,12} It allows for a systematic inclusion of power corrections. Its usefulness is based on two important features: The spin-flavor symmetry of HQET relates form factors in the heavy-quark limit and thus reduces the number of unknown hadronic quantities. Second, the dependence on the heavy-quark mass is made explicit. Typical applications are (semi)leptonic form factors involving hadrons containing a single heavy quark, such as $B \to D^{(*)}$ form factors in semileptonic $b \to c$ transitions or the decay constant $f_B$.

- **HQE** is a theory for inclusive $B$ decays.\textsuperscript{13,14} It is based on the optical theorem for inclusive decays and an operator product expansion in $\Lambda_{QCD}/m_b$ of the transition operator. The heavy-quark expansion justifies the parton model for inclusive decays of heavy hadrons, which it contains as its first approximation. Beyond that it allows us to study nonperturbative power corrections to the partonic picture. The main applications of the HQE method is for processes as $B \to X_u, c\ell\nu$, $B \to X_s\gamma$, $B \to X_s l^+l^-$, and for the lifetimes of $b$-flavored hadrons.

- **QCD factorization** refers to a framework for analysing exclusive hadronic $B$ decays with a fast light meson as for instance $B \to D\pi, B \to \pi\pi, B \to \pi K$ and $B \to V\gamma$. This approach is conceptually similar to the theory of hard exclusive reactions, described for instance by the pion electromagnetic form factor at large momentum transfer.\textsuperscript{15,16} The application to $B$ decays requires new elements due to the presence of heavy-light mesons.\textsuperscript{17}

- **SCET** is an effective field theory formulation for transitions of a heavy quark into an energetic light quark.\textsuperscript{18} The basic idea is reminiscent of HQET. However, the structure of SCET is more complex because the relevant long-distance physics that needs to be factorized includes both soft and collinear degrees of freedom. Only soft contributions have to be accounted for in HQET. Important applications of SCET are the study of $B \to P, V$ transition form factors at large recoil energy of the light pseudoscalar ($P$) or vector ($V$) meson, and formal proofs of QCD factorization in exclusive heavy hadron decays.

There are further methods, which have been useful to obtain information on hadronic quantities relevant to $B$ decays. Of basic importance are computations based on lattice QCD, which can access many quantities needed for $B$ meson phenomenology (see \textsuperscript{19} for a recent review). On the other hand, exclusive processes with fast light particles are very difficult to treat within this framework. An important tool to calculate in particular heavy-to-light form factors ($B \to \pi$) at large recoil are QCD sum rules on the light cone.\textsuperscript{20,21} We will not discuss those methods.
3. Exclusive Hadronic B Decays in QCD

3.1. Factorization

The calculation of B-decay amplitudes, such as $B \to D\pi$, $B \to \pi\pi$ or $B \to \pi K$, starts from an effective Hamiltonian, which has, schematically, the form

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \lambda_{CKM} C_i Q_i.$$  \hspace{1cm} (1)

Here $C_i$ are the Wilson coefficients at a scale $\mu \sim m_b$, which are known at next-to-leading order in QCD. $Q_i$ are local, dimension-6 operators and $\lambda_{CKM}$ represents the appropriate CKM matrix elements. The main theoretical problem is to evaluate the matrix elements of the operators $\langle Q_i \rangle$ between the initial and final hadronic states. A typical matrix element reads $\langle \pi\pi | (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} | B \rangle$.

These matrix elements simplify in the heavy-quark limit, where they can in general be written as the sum of two terms, each of which is factorized into hard scattering functions $T^I$ and $T^{II}$, respectively, and the nonperturbative, but simpler, form factors $F_j$ and meson light-cone distribution amplitudes $\Phi_M$ (Fig. 2).

Important elements of this approach are: i) The expansion in $\Lambda_{QCD}/m_b \ll 1$, consistent power counting, and the identification of the leading power contribution, for which the factorized picture can be expected to hold. ii) Light-cone dynamics, which determines for instance the properties of the fast light mesons. The latter are described by light-cone distribution amplitudes $\Phi_x$ of their valence quarks defined as

$$\langle \pi(p)|u(0)\bar{d}(z)|0 \rangle = \frac{i f_\pi}{4} \gamma_5 \not{p} \int_0^1 dx \, e^{izp} \Phi_x(x)$$  \hspace{1cm} (2)

with $z$ on the light cone, $z^2 = 0$. iii) The collinear quark-antiquark pair dominating the interactions of the highly energetic pion decouples from soft gluons (colour transparency). This is the intuitive reason behind factorization. iv) The factorized reason amplitude consists of hard, short-distance components, and soft, as well as collinear, long-distance contributions. More details on the factorization formalism can be found elsewhere.

An alternative approach to exclusive two-body decays of B mesons, referred to as pQCD, has been proposed in $^{23}$. The main hypothesis in this method is that the $B \to \pi$ form factor is not dominated by soft physics, but by hard gluon exchange that can be computed perturbatively. The hypothesis rests on the idea that Sudakov effects will suppress soft endpoint divergences in the convolution integrals. A critical discussion of this framework has been given in $^{24}$.

3.2. CP Violation in $B \to \pi^+\pi^-$

A framework for systematic computations of heavy-hadron decay amplitudes in a well-defined limit clearly has many applications for quark flavor physics with two-body nonleptonic B decays. An important example may serve to illustrate this point. Consider the time-dependent, mixing-induced CP asymmetry in $B \to \pi^+\pi^-$

$$A_{CP}(t) = \frac{\Gamma(B(t) \to \pi^+\pi^-) - \Gamma(B(t) \to \pi^+\pi^-)}{\Gamma(B(t) \to \pi^+\pi^-) + \Gamma(B(t) \to \pi^+\pi^-)} = -S \sin(\Delta M_d t) + C \cos(\Delta M_d t)$$  \hspace{1cm} (3)

Using CKM-matrix unitarity, the decay amplitude consists of two components with different CKM factors and different hadronic parts, schematically

$$A(B \to \pi^+\pi^-) = V_{ub}^\ast V_{ud}(\text{up} - \text{top}) + V_{cb}^\ast V_{cd}(\text{charm} - \text{top})$$  \hspace{1cm} (4)

If the penguin contribution $\sim V_{ub}^\ast V_{cd}$ could be neglected, one would have $C = 0$ and $S = \sin 2\alpha$, hence a direct relation of $A_{CP}$ to the CKM angle $\alpha$. In reality the penguin contribution is not negligible compared to the dominant tree contribution $\sim V_{ub}^\ast V_{ud}$. The ratio of penguin and tree amplitude, which enters the CP asymmetry, depends on hadronic physics. This complicates the relation of observables $S$ and $C$ to CKM parameters. QCD factorization of $B$-decay matrix elements allows us to compute the required hadronic input and to determine the constraint in the $(\rho, \eta)$ plane implied by measurements of the CP asymmetry. This is illustrated for $S$ in Fig. 3. The widths of the bands indicate the theoretical uncertainty $^{25}$. Note that the constraints from $S$ are relatively insensitive to theoretical or experimental uncertainties. The analysis of direct CP violation measured by $C$ is more complicated due to the importance of strong phases. Recent phenomenological analyses were performed in...
3.3. Current Status

QCD factorization to leading power in $\Lambda/m_b$ has been demonstrated at $O(\alpha_s)$ for the important class of decays $B \to \pi\pi, \pi K$. For $B \to D\pi$ (class I), where hard spectator interactions are absent, a proof has been given explicitly at two loops and to all orders in the framework of soft-collinear effective theory (SCET). Complete matrix elements are available at $O(\alpha_s)$ (NLO) for $B \to \pi\pi, \pi K$, including electroweak penguins. Comprehensive treatments have also been given for $B \to PV$ modes and for $B$ decays into light flavor-singlet mesons. A discussion of two-body $B$ decays into light mesons within SCET has been presented in.

Power corrections are presently not calculable in general. Their impact has to be estimated and included into the error analysis. Critical issues here are annihilation contributions and certain corrections proportional to $m_2^2/(m_u + m_d)m_b$, which is numerically sizable, even if it is power suppressed. However, the large variety of channels available will provide us with important cross checks and arguments based on SU(2) or SU(3) flavor symmetries can also be of use in further controlling uncertainties.

3.4. Phenomenology of $B \to PP, PV$

Two-body $B$ decays into light mesons have been widely discussed in the literature. Two-body $B$ decays into light mesons have been widely discussed in the literature.

In general, a phenomenological analysis of these modes faces the problem of disentangling three very different aspects, which simultaneously affect the observable decay rates and asymmetries: First, there are the CKM couplings that one would like to extract in order to test the standard model. Second,
it is possible that some observables could be significantly modified by new physics contributions, which would complicate the determination of CKM phases. Third, the short distance physics, CKM quantities and potential new interactions, that one is aiming for, is dressed by the effects of QCD. A priori any discrepancy between data and expectations has to be examined with these points in mind. Fortunately, the large number of different channels with different QCD dynamics and CKM dependence will be very helpful to clarify the phenomenological interpretation. The following examples illustrate how various aspects of the QCD dynamics may be tested independently.

1. Penguin-to-tree ratio. To test predictions of this ratio a useful observable can be built from the mode $B^- \to \pi^- K^0$, which is entirely dominated by a penguin contribution, and from the pure tree-type process $B^- \to \pi^- \pi^0$:

$$ \left| \frac{\text{penguin}}{\text{tree}} \right| = \left| \frac{V_{ub} f_\pi}{f_K} \sqrt{\frac{B(B^- \to \pi^- K^0)}{2B(B^- \to \pi^- \pi^0)}} \right| $$

This amplitude ratio is not identical to the $P/T$ ratio required for $B \to \pi^+ \pi^-$, but still rather similar to be interesting as a test. Small differences come from $SU(3)$ breaking effects (the dominant ones due to $f_\pi/f_K$ are already corrected for in (12)), and weak annihilation corrections in $B \to \pi K$, and from the color-suppressed contribution to $B^- \to \pi^- \pi^0$. Because the $\pi^- K^0$ and $\pi^- \pi^0$ channels have only a single amplitude (penguin or tree), no interference is possible and the ratio in (12) is independent of the CKM phase $\gamma$. This is useful for distinguishing QCD effects from CKM issues. A comparison of factorization predictions for the left-hand side of (12) with data used to compute the right-hand side in (12) is shown in Fig. 4. The agreement is satisfactory within uncertainties.

2. Factorization test for $B^- \to \pi^- \pi^0$. It is of interest to test predictions for the tree-amplitude alone using a classical factorization test of the form

$$ B(B^+ \to \pi^+ \pi^0) = 3\pi^2 f_\pi^2 |V_{ud}|^2 \times \frac{\tau(B^+)}{\tau(B_d)} |a_1 + a_2|^2 $$

where $a_1$, $a_2$ are QCD coefficients. The advantage of this test is that $B^- \to \pi^- \pi^0$ receives neither penguin nor annihilation contributions. It thus gives information on the other aspects of the QCD dynamics in $B \to \pi \pi$. This test was discussed recently in (37,32).

3. Direct CP asymmetries. From the heavy-quark limit one generally expects strong phases to be suppressed, except for a few special cases. This circumstance should suppress direct CP asymmetries. Of course those also depend sensitively on weak phases and a detailed analysis has to consider individual channels. At present, qualitatively, one may at least say that the non-observation of direct CP violation in $B$ decays until today, with experimental bounds typically at the 10% level, are not in contradiction with the theoretical expectation.

4. Weak annihilation. Amplitudes from weak annihilation represent power suppressed corrections, which are uncalculable in QCD factorization and so far need to be estimated relying on models. At present there are no indications that annihilation terms would be anomalously large, but they do contribute to the theoretical uncertainty. Effectively, annihilation corrections may be considered as part of the penguin amplitudes. To some extent, therefore, they are tested with the help of the penguin-to-tree ra-
tio discussed above. Nevertheless, in order to disentangle their impact from other effects it is of great interest to test annihilation separately. This can be done with decay modes that proceed through annihilation or at least have a dominant annihilation component.

An example is the pure annihilation channel $B_d \to D_s^- K^+$. Even though this case is somewhat different from the reactions of primary interest here, because of the charmed meson in the final state, it is still useful to cross-check the typical size of annihilation expected in model calculations. Treating the $D$ meson in the model estimate for annihilation as suggested in, one finds a central (CP-averaged) branching ratio of $B(B_d \to D_s^- K^+) = 1.2 \times 10^{-5}$. Allowing for a 100% uncertainty of the central annihilation estimate, which in the case of the penguin-to-tree ratio shown in Fig. 4 corresponds to the inner (solid) error region around the theoretical value (marked by the cross), gives an upper limit of $5 \times 10^{-5}$. This is in agreement with the current experimental result $(3.8 \pm 1.1) \times 10^{-5}$ (see refs. in).

Additional tests should come from annihilation decays into two light mesons, such as $B \to K K$ modes. These, however, are CKM suppressed and only upper limits are known at present. The $K^+K^0$ and $K^0\bar{K}^0$ channels have both annihilation and penguin contributions. On the other hand $B \to K^+K^-$ is a pure weak annihilation process and therefore especially important. Further discussions can be found in.

At present, within current experimental and theoretical uncertainties, there are no clear signals of significant discrepancies between measurements and SM expectations in hadronic $B$ decays, neither with respect to QCD calculations nor suggesting the need for new physics. However, a few experimental results have central values deviating from standard predictions, which attracted some attention in the literature. Even though the discrepancies are not significant at the moment, it will be interesting to follow future developments. We comment on some of those possible hints here, with a view on QCD predictions within the SM.

- As seen in (11) the measurement of $C = -0.38 \pm 0.16$ suggests the possibility of large direct CP violation in $B \to \pi^+\pi^-$ decays. On the other hand, this is largely due to the result from Belle, whereas BaBar gives a smaller effect. In the SM one expects $C \approx 0.1$ with an error of about the same size. It is interesting to note that the perturbative strong interaction phase predicted to lowest order in QCD factorization gives a positive value for $C$ while the measurements seem to prefer negative values. Since the strong phase is a small effect in the heavy-quark limit, uncalculable power corrections could possibly compete with the perturbative contribution. A small negative $C$ is therefore not excluded, but the reliability of a lowest order perturbative calculation of the strong phase would then be in doubt. (A logical possibility for $C < 0$ would be that the positive sign of the strong phase is correct, but the weak phase is negative, which would require new physics in $\varepsilon_K$.) In any case, a clarification of the experimental situation will be important. It may also be noted that the central numbers from Belle, which are large for both $S$ and $C$, would violate the absolute bound $S^2 + C^2 \leq 1$ when taken at face value.

- Mixing-induced CP violation $S$ in $B \to \phi K_S$ and $B \to \eta' K_S$, which proceed through the penguin transition $b \to s\bar{s}s$, could be strongly affected by new physics. In the SM one expects $S_{\phi K_S}$ and $S_{\eta' K_S}$ to be close to the benchmark observable $S_{\psi K_S}$ of mixing-induced CP violation in $B \to \psi K_S$.40 Hints of deviations in the data from Belle, and to a much lesser extent from BaBar, have motivated several analyses in the literature on this issue.41,42,43 Experimentally one finds for the world average

$$S_{\phi K_S} - S_{\phi K_S} = -0.89 \pm 0.33$$
$$S_{\eta' K_S} - S_{\psi K_S} = -0.47 \pm 0.22$$

where the first result combines the BaBar and Belle values ignoring the rather poor agreement between them. This can be compared with the SM expectation based on a recent QCD analysis in

$$S_{\phi K_S} - S_{\psi K_S} = 0.025 \pm 0.016$$
$$S_{\eta' K_S} - S_{\psi K_S} = 0.011 \pm 0.013$$

More information on possible new physics implications can be found in.
• Current data for the ratio of $B \to \pi^+\pi^-$ and $B \to \pi^+\pi^0$ branching fractions appear to be somewhat low in comparison with theoretical calculations for a CKM phase $\gamma < 90^\circ$ as given by standard fits of the CKM unitarity triangle. This feature is often interpreted as a hint for a larger value of $\gamma > 90^\circ$. Such a value could change a constructive interference of tree and penguin amplitudes in the $\pi^+\pi^-$ mode into a destructive one, and thus reduce the ratio of branching fractions. In a different, QCD related possibility was discussed that could account for the suppression of $B \to \pi^+\pi^-$ relative to $B \to \pi^+\pi^0$, even for $\gamma < 90^\circ$. In this scenario, which can be realized without excessive tuning of input parameters, the factorization coefficient $a_2$ (color-suppressed tree) is enlarged, while the $B \to \pi$ form factor is somewhat smaller than commonly assumed. This keeps $B \to \pi^+\pi^0$ roughly constant and suppresses $B \to \pi^+\pi^-$, which is independent of $a_2$. The factorization test mentioned in point 2. above would be very useful to check such a scenario. This could also help to clarify the situation with $B \to \pi^0\pi^0$, which is very sensitive to $a_2$ and for which first measurements from BaBar and Belle indicate a substantial branching fraction. Theoretically $a_2$ is subject to sizable uncertainties, because color suppression strongly reduces the leading order value and makes the prediction sensitive to subleading corrections. 

• The ratio (CP averaged rates are understood) 
\[
R_{00} = \frac{2\Gamma(B^0 \to \pi^0 K^0)}{\Gamma(B^- \to \pi^- K^0)} \tag{18}
\]
appears to be larger than expected theoretically. This is shown in Fig. 5. The ratio $R_{00}$ is almost insensitive to the CKM angle $\gamma$ and it is essentially impossible to enhance the prediction in the SM by QCD effects. The discrepancy of about $2\sigma$ can also be seen in a different way, using the Lipkin-Gronau-Rosner sum rule, which relates all four $\pi K$ modes using isospin symmetry. 

The ratio 
\[
R_L = \frac{2\Gamma(B^0 \to \pi^0 K^0) + 2\Gamma(B^- \to \pi^0 K^-)}{\Gamma(B^- \to \pi^- K^0) + \Gamma(B^0 \to \pi^+ K^+)} \tag{19}
\]
can be shown to be 1 up to corrections of second order in small quantities. Experimentally it is also about $2\sigma$ high. If the discrepancy should become statistically significant, it would be a strong indication of physics beyond the SM. 

The status of QCD calculations for $B \to PV$ modes is presented in and a more general discussion of new physics aspects is given by 44.

4. Rare and Radiative $B$ Decays

4.1. Radiative Decays $B \to V\gamma$

Factorization in the sense of QCD can also be applied to the exclusive radiative decays $B \to V\gamma$ ($V = K^*, \rho$). The factorization formula for the operators in the effective weak Hamiltonian can be written as 
\[
\langle V\gamma(\epsilon)|Q|B\rangle = \left[ F^{B \to V}(0) T_i^l + \int_0^1 d\xi \, dv \, T_i^{lI}(\xi, v) \Phi_B(\xi) \Phi_V(v) \right] \cdot \epsilon
\]
where $\epsilon$ is the photon polarization 4-vector. Here $F^{B \to V}$ is a $B \to V$ transition form factor, and $\Phi_B$, $\Phi_V$ are leading twist light-cone distribution amplitudes (LCDA) of the $B$ meson and the vector meson $V$, respectively. These quantities describe the long-distance dynamics of the matrix elements, which is factorized from the perturbative, short-distance interactions expressed in the hard-scattering kernels $T_i^l$ and $T_i^{lI}$. The QCD factorization formula (20) holds up to corrections of relative order $\Lambda_{QCD}/m_b$. Annihilation topologies are power-suppressed, but
still calculable in some cases. The framework of QCD factorization is necessary to compute exclusive $B \to V\gamma$ decays systematically beyond the leading logarithmic approximation. Results to next-to-leading order in QCD, based on the heavy quark limit $m_b \gg \Lambda_{QCD}$ have been computed \cite{49,50} (see also \cite{51}).

The method defines a systematic, model-independent framework for $B \to V\gamma$. An important conceptual aspect of this analysis is the interpretation of loop contributions with charm and up quarks, which come from leading operators in the effective weak Hamiltonian. These effects are calculable in terms of perturbative hard-scattering functions and universal meson light-cone distribution amplitudes. They are $O(\alpha_s)$ corrections, but are leading power contributions in the framework of QCD factorization. This picture is in contrast to the common notion that considers charm and up-quark loop effects as generic, uncalculable long-distance contributions. Non-factorizable long-distance corrections may still exist, but they are power-suppressed. The improved theoretical understanding of $B \to V\gamma$ decays strengthens the motivation for still more detailed experimental investigations, which will contribute significantly to our knowledge of the flavor sector.

The uncertainty of the branching fractions is currently dominated by the form factors $F_{K\gamma}$, $F_{\rho\gamma}$. A NLO analysis \cite{50} yields (in comparison with the experimental results in brackets) $B(B \to \bar{K}^{(*)}\gamma)/10^{-5} = 7.1\pm 2.5 (4.21\pm 0.29)\,\text{\cite{52}}$ and $B(B \to \rho^{(*)}\gamma)/10^{-6} = 1.6\pm 0.6 (<2.3)\,\text{\cite{53}}$. Taking the sizable uncertainties into account, the results for $B \to K^{(*)}\gamma$ are compatible with the experimental measurements, even though the central theoretical values appear to be somewhat high. $B(B \to \rho\gamma)$ is a sensitive measure of CKM quantities.\cite{50,54,55} This is illustrated in Fig. 6.

\subsection*{4.2. SCET}

In decay processes of $B$ mesons with highly energetic light quarks in the final state, HQET alone is not sufficient to account for the complete long-distance degrees of freedom that need to be represented in an effective theory description. A first step towards implementing the missing ingredients was made in \cite{56}. In this paper a framework, called large-energy effective theory (LEET), was suggested that describes the interactions of energetic light quarks with soft gluons. To correctly reproduce the infrared structure of QCD, also collinear gluons need to be included, which was emphasized in \cite{18}. The authors of \cite{18} constructed an effective theory, the SCET, for soft and collinear gluons, applicable to energetic heavy-to-light transitions. These transitions may be inclusive heavy-to-light processes, such as $b \to u$ decays, but also exclusive $B \to P, V$ form factors at large recoil of the light final state meson. Similarly the SCET is a useful language to investigate factorization properties in hadronic $B$ decays in general terms.

For the construction of the SCET one writes the four-momentum $p$ of an energetic light quark (collinear quark) in light-cone coordinates

\begin{equation}
    p^\mu = \frac{1}{\sqrt{2}} (p_- n^\mu + p_+ \bar{n}^\mu) + p_\perp^\mu
\end{equation}

where $n$ is a light-like four-vector in the direction of the collinear quark and $\bar{n}$ is a similar vector in the opposite direction, that is

\begin{equation}
    n^2 = \bar{n}^2 = 0 \quad n \cdot \bar{n} = 2
\end{equation}

The four-vector $p_\perp$ contains the components of $p$ perpendicular to both $n$ and $\bar{n}$. For $p$ collinear to the light-like direction $n$ the components scale as $p_- \sim M$, $p_\perp \sim M \lambda$, $p_+ \sim M \lambda^2$, where $M$ is the hard scale ($\sim m_b$) and $\lambda$ is a small parameter, such that $p^2 = 2p_+ p_- + p_\perp^2 \sim M^2 \lambda^2$. The dependence

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Impact of the current experimental upper limit on $B(B \to \rho\gamma)/B(B \to K^{(*)}\gamma)$ in the $(\rho, \eta)$ plane. The area to the left of the dark band is excluded. The width of the dark band reflects the variation of $\xi \equiv F_{K\gamma}/F_{\rho\gamma} = 1.33 \pm 0.13$ (second ref. in \cite{20}). The case of $\xi = 1$ is illustrated by the dashed curve. The intersection with the light-shaded band from the measurement of $\sin 2\beta$ defines the apex of the unitarity triangle and the length of $R_l = \sqrt{(1 - \rho)^2 + \eta^2} \sim |V_{ud}|$, once the upper limit will be turned into a measurement. The irregular area represents the standard unitarity triangle fit.}
\end{figure}
on the larger components of \( p, p_\perp \) and \( p_\perp \) is then removed from the light-quark field \( \psi(x) \) in full QCD by writing

\[
\psi(x) = \sum_\hat{p} e^{-i\hat{p} \cdot x} \psi_{n,p} \quad (24)
\]

\[
\hat{p} \equiv \frac{1}{\sqrt{2}} p_\perp - n + p_\perp \quad (25)
\]

This is analogous to the construction of the HQET, where the dependence on the large components \( v \) of the heavy-quark velocity is isolated in a similar way. The new fields \( \psi_{n,p} \) are then projected onto the spinors

\[
\xi_{n,p} = \frac{\hat{p}}{4} \psi_{n,p} \quad \xi_{n,p} = \frac{\hat{p}}{4} \psi_{n,p} \quad (26)
\]

The field \( \xi_{n,p} \) represents the collinear quark in the effective theory. The smaller components \( \xi_{n,p} \) are integrated out in the construction of the effective theory Lagrangian \( \mathcal{L}_{\text{SCET}} \) from the Lagrangian of full QCD. \( \mathcal{L}_{\text{SCET}} \) contains collinear quarks \( \xi_{n,p} \), the heavy-quark fields from HQET, \( h_u \), and soft and collinear gluons.

A typical application is the analysis of \( B \to P, V \) form factors at large recoil. Bilinear heavy-to-light currents \( \bar{q}Vb \) have to be matched onto operators of the SCET, schematically

\[
\bar{q}Vb \to C_i \xi_{n,p} \bar{\Gamma}_i h_u \quad (27)
\]

where the \( C_i \) are Wilson coefficient functions. For \( B \to P, V \) transitions in full QCD there is a total of ten different form factors describing the matrix elements of the possible independent bilinear currents. In SCET the equations of motion

\[
\not{\hat{p}} h_u = h_u, \quad \not{\hat{p}} \xi_{n,p} = 0 \quad (28)
\]

imply constraints, which reduce the number of independent form factors to three, to leading order in the heavy-quark limit. An application to \( B \to K^*l^+l^- \) decays will be discussed in the following section. Further developments and applications of the SCET framework to rare, radiative and hadronic \( B \) decays can be found in \( 57,58,59,60,61 \).

### 4.3. Forward-Backward Asymmetry Zero in \( B \to K^*l^+l^- \)

Substantial progress has taken place over the last few years in understanding the QCD dynamics of exclusive \( B \) decays. The example of the forward-backward asymmetry in \( B \to K^*l^+l^- \) nicely illustrates some aspects of these developments.

The forward-backward asymmetry \( A_{FB} \) is the rate difference between forward (\( 0 < \theta < \pi/2 \)) and backward (\( \pi/2 < \theta < \pi \)) going \( l^+ \), normalized by the sum, where \( \theta \) is the angle between the \( l^+ \) and \( B \) momenta in the centre-of-mass frame of the dilepton pair. \( A_{FB} \) is usually considered as a function of the dilepton mass \( q^2 \). In the standard model the spectrum \( dA_{FB}/dq^2 \) (Fig. 7) has a characteristic zero at

\[
\frac{q^2}{m_B^2} = -\alpha_+ \frac{m_B C_7}{m_B C_9^{\alpha f}} \quad (29)
\]

depending on short-distance physics contained in the coefficients \( C_7 \) and \( C_9^{\alpha f} \). The factor \( \alpha_+ \), on the other hand, is a hadronic quantity containing ratios of form factors.

It was first stressed in \( 62 \) that \( \alpha_+ \) is not very much affected by hadronic uncertainties and very similar in different models for form factors with \( \alpha_+ \approx 2 \). After relations were found between different heavy-light form factors (\( B \to P, V \)) in the heavy-quark limit and at large recoil \( 63 \), it was pointed out in \( 64 \) that as a consequence \( \alpha_+ = 2 \) holds exactly in this limit. Subsequently, the results of \( 63 \) were demonstrated to be valid beyond tree level \( 49,18 \).

The use of the \( A_{FB} \)-zero as a clean test of standard model flavor physics was thus put on a firm basis and NLO corrections to (29) could be computed \( 49 \). More recently also the problem of power corrections to heavy-light form factors at large recoil in the heavy-quark limit has been studied \( 57 \). Besides the value of \( q_0^2 \), also the sign of the slope of \( dA_{FB}/dq^2 \) can be used as a probe of new physics. For a \( B \) meson,
The hard process is characterized by a scale $B$ light-cone distribution amplitude of the form factors posed, and shown to one loop in QCD, that the form where $T_{66}$ non-trivial way also depends on the structure of the hard-scattering process, but soft momentum of the spectator quark. The decay where the present discussion applies), and $k$ ing as like momentum of the photon with components scal-
tains a light-quark propagator that is off-shell by an amount $(q - k)^2 \sim q_k$. Here $q$ is the hard, light-like momentum of the photon with components scaling as $m_b$ (this restricts the region of phase-space where the present discussion applies), and $k$ is the soft momentum of the spectator quark. The decay is thus determined by a hard-scattering process, but also depends on the structure of the $B$ meson in a non-trivial way. Recently, in it has been proposed, and shown to one loop in QCD, that the form factors $F$ for this decay factorize as

$$F = \int d\tilde{k}_+ \Phi_B(\tilde{k}_+) T(\tilde{k}_+)$$

(30)

where $T$ is the hard-scattering kernel and $\Phi_B$ the light-cone distribution amplitude of the $B$ meson defined as

$$\Phi_B(\tilde{k}_+) = \int dz e^{i\tilde{k}_+ z -} \langle 0|b(0)\bar{u}(z)|B\rangle|_{z_+=z_-=0}$$

(31)

The hard process is characterized by a scale $\mu_F \sim \sqrt{m_b}\Lambda$. At lowest order the form factors are proportional to $\int d\tilde{k}_+ \Phi_B(\tilde{k}_+)/\tilde{k}_+ \equiv 1/\Lambda_B$, a parameter that enters hard-spectator processes in many other applications. The analysis at NLO requires resummation of large logarithms $\ln(m_b/\tilde{k}_+)$. An extension of the proof of factorization to all orders was subsequently given by within the SCET.

Progress has also been made recently towards a better understanding of the $B$ meson light-cone distribution amplitude itself.

5. Conclusions

QCD has been very successful as a theory of the strong interaction at high energies, based on expansions in inverse powers of the high-energy scale and perturbation theory in $\alpha_s$. This general framework of QCD has recently found new applications in the treatment of exclusive decays of heavy hadrons. It is particularly exciting that these developments come at a time where a large amount of precision data is being collected at the experimental $B$ physics facilities.

Factorization formulas in the heavy-quark limit have been proposed for a large variety of exclusive $B$ decays. They justify in many cases the phenomenological factorization ansatz that has been employed in many applications. In addition they enable consistent and systematic calculations of corrections in powers of $\alpha_s$. Non-factorizable long-distance effects are not calculable in general but they are suppressed by powers of $\Lambda_{QCD}/m_b$. So far, $B \to D^\pm \pi^-$ decays are probably understood best. Decays with only light hadrons in the final state such as $B \to \pi\pi$, $K^{\ast}\gamma$, $\rho\gamma$, or $K^{\ast}l^+l^-$ include hard spectator interactions at leading power and are therefore more complicated. An important new tool that has been developed is the soft-collinear effective theory (SCET), which is of use for proofs of factorization and for the theory of heavy-to-light form factors at large recoil. Studies of the process $B \to l\nu\gamma$ have also led to a better understanding of QCD dynamics in exclusive hadronic $B$ decays. These are promising steps towards controlling the QCD dynamics in exclusive hadronic or rare $B$ decays in a reliable way. In many cases the required theoretical accuracy is not extremely high and even moderately precise, but robust predictions will be very helpful. Using all the available tools we can hope to successfully probe CP violation, weak interaction parameters and new phenomena in the quark-flavor sector.
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DISCUSSION

Brendan Casey (Brown University): Does the range in predictions for $\bar{B}^0 \rightarrow D^+_s K^-$ of $(1 \div 5) \times 10^{-5}$ correspond to the $1\sigma$ contours or to the $5\sigma$ contours in the $P/T$ predictions?

Gerhard Buchalla: The default model estimate for the annihilation term gives $1.2 \times 10^{-5}$ for the branching ratio of $\bar{B}^0 \rightarrow D^+_s K^-$. Allowing for a 100% uncertainty of the default value gives the upper limit of $5 \times 10^{-5}$. This corresponds to the inner (solid line) of the three error contours shown in the plot of the $P/T$ prediction (see Fig. 4).

Harry Lipkin (Weizmann Institute): Do you have anything to say about the $B$ decays to the new charmed-strange axial and scalar mesons that have been observed? When I predicted last year a large $B$ decay to the $D^*_s$ axial vector, I was told by HQET experts that this decay would be small.

Gerhard Buchalla: The $D^*_s$ emitted in $B$ decay is a heavy-light meson and therefore represents an extended hadronic object, in contrast to a pion or a similar energetic light meson. The usual factorization formulas do not apply to this situation and it is thus difficult to control QCD uncertainties in the predictions.

Ikaros Bigi (Notre Dame University): When you consider $B \rightarrow VV$, like $B \rightarrow \rho\rho$, and calculate the polarization of $V$, there are corrections of order $1/m_b$. Those are sensitive to long-distance dynamics, right?

Gerhard Buchalla: That is correct.