ANALYSIS OF THE OKISHIO THEOREM

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Abstract: The Okishio theorem is a classic theory of Japanese Marxism that denies the law of the decline of the general rate of profit proposed by Marx in the third volume of Capital. The theorem seriously affects the correct understanding of the mode of production and the economic crisis in capitalist countries today. Analyzing the original text of the Okishio theorem, this article finds that the theorem does not distinguish between paid and unpaid labor, and that its mathematical model is based on an incorrect understanding of Marx’s views and of the capitalist mode of production. The theorem’s mathematical proof process is itself untenable and even contradicts the numerical examples that Okishio gives. In fact, Okishio negates Marx’s view by effectively denying the existence of the capitalist mode of production. Unexpectedly, this article also finds that a decline in the general rate of profit will result even if all enterprises do not increase their organic composition of capital, but if only enterprises with a higher organic composition of capital expand their proportions in the economy.

Key words: Okishio theorem; general rate of profit; Western Marxism; mathematical model

1. Introduction

In the third volume of Capital, Marx proposed the law that the general rate of profit tends to decline. The Japanese scholar Nobuo Okishio has questioned this law, arguing that technological innovations introduced by capitalists will not reduce the general rate of profit unless the real wage rate rises sufficiently (Okishio 1961). This conjecture is known as the Okishio theorem due to its formulation of a mathematical model that has come to represent a classic theory of Japanese Marxism. However, the law of the decline of the general rate of profit is a
scientific theory that was abstracted and deduced by Marx from reality according to the internal logic of economic science, and it cannot be overturned simply by using a mathematical model. The question thus becomes one of identifying errors in the mathematical formulations concerned. It is necessary to make a detailed analysis of the Okishio theorem and to point out its fallacies, because the law of the decline of the general rate of profit has an impact on the correct understanding of the mode of production and on the economic crisis in capitalist countries today.

Before this analysis proceeds further, two brief points should be made. First, Marx in the third volume of *Capital* lists various factors that may prevent the general rate of profit from declining. These factors do not negate Marx’s law of the decline of the general rate of profit, but Marx was well able to see that their effects might for a time prevent the tendency he identified from manifesting itself in practice. Second, and whether the organic composition of capital increases or not, the profit rate of all capital is bound ultimately to decline. This is because, in historical terms, the only condition under which the accumulation rate of capital can be a series that does not tend to zero in proportion to the rate of profit is if the rate of profit cannot decline. According to Okishio’s mathematical theorem, meanwhile, capital accumulation will tend to infinity. The fallacy involved here is exactly what Marx in the third volume of *Capital* criticized as Dr. Price’s wishful thinking. “One penny, put out at our Saviour’s birth to 5 percent compound interest, would, before this time, have increased to a greater sum, than would be contained in a hundred and fifty millions of earths, all solid gold” (Marx and Engels 2010c, 392). More technically, Marx’s critique of price is as follows: “Price entirely forgets that the interest of 5% presupposes a rate of profit of 15%, and assumes it to continue with the accumulation of capital” (393).

2. An Overview of the Okishio Theorem

We will first provide an overview of Okishio’s questioning of Marx’s reasoning, before reviewing it in the next section.

According to Okishio, the criterion that capitalists apply before introducing a new technology is whether it reduces production costs, not whether the technology can increase labor productivity.

The productivity of labor in the creation of Commodity \( i \) is recorded as \( 1/t_i \), where \( t_i \) denotes the amount of direct or indirect labor necessary to produce 1 unit of Commodity \( i \). The quantity \( t_i \) is determined by the following equation:

\[
t_i = \sum a_{ij} t_j + \tau_i, \quad i = 1, 2, \ldots, n,
\]  

(1)

where \( a_{ij} \) denotes the direct quantity of Commodity \( j \) necessary to produce 1 unit of Commodity \( i \), and \( \tau_i \) denotes the amount of direct labor necessary to produce 1 unit of Commodity \( i \).
The above stipulation by Okishio indicates that the first term on the right of Equation (1) is the amount of dead labor (the amount of constant capital) contained in 1 unit of Commodity $i$, which is marked as $c$ by Marx, while the second term is the amount of living labor contained therein (both paid labor $v$ and unpaid labor $m$), which means that $\tau_i$ is the $v + m$ contained in 1 unit of Commodity $i$.

Okishio goes on to stipulate that the condition under which a new production technology can increase the labor productivity of Commodity $k$ in Industry $k$ is

$$\Sigma a_k t_j + \tau_k > \Sigma a'_k t_j + \tau'_k,$$

where $(a_{k1}, a_{k2}, \ldots, a_{kn}, \tau_k)$ denotes a new technology in Industry $k$. Condition (2) is the “productivity criterion.”

On the other hand, the “cost criterion” is

$$\Sigma a_k q_j + \tau_k > \Sigma a'_k q_j + \tau'_k,$$

where $q_j = p_j / w$, and where $p_j$ and $w$ denote the price and money wage rate of Commodity $j$, respectively.

The quantity $q_j$ as set by Okishio is meaningless. It will be analyzed later.

According to Okishio, the “productivity criterion” (2) and the “cost criterion” (3) are consistent only if $q_j = t_j$ holds for all of $i$. However, $q_j > t_j$ applies to all of $i$ in a capitalist economy, because there must be positive profit in each industry. Therefore, the following inequality must be true:

$$q_i > \Sigma a_i q_j + \tau_i, \quad i = 1, 2, \ldots, n.$$  

Comparing Inequality (4) with Equation (1), it can be seen that $q_j > t_j$ holds for all of $i$.

Here, Okishio stipulates that $q_j > t_j$ represents the positive profit, and thus the part by which $q_j$ exceeds $t_j$ is the surplus value part $m$, or what is converted from $m$. Therefore, $t_j$ can only represent the cost $(c + v)$ at this point, not the full value of the commodity $(c + v + m)$ as in Equation (1). As a result, $\tau_i$ on the right of Inequality (4) can only represent the amount of paid labor $v$, not $m$. The meaning is not the same as $\tau_i$ in Equation (1). For the same reason, $\tau_k$ in Inequality (2) and $\tau_k$ in Inequality (3) are also different. The former contains $(v + m)$, while the latter contains only $v$.

Okishio believes that the “productivity criterion” is different from the “cost criterion.” Capitalists apply the cost criterion rather than the productivity criterion, and therefore the introduction by the capitalists of new production technology does not necessarily improve labor productivity, although it will inevitably reduce
the cost of production. Marx pointed this out long ago, writing in the first volume of *Capital* that

the use of machinery for the exclusive purpose of cheapening the product, is limited in this way, that less labor must be expended in producing the machinery than is displaced by the employment of that machinery. For the capitalist, however, this use is still more limited. Instead of paying for the labor, he only pays the value of the labor power employed; therefore, the limit to his using a machine is fixed by the difference between the value of the machine and the value of the labor power replaced by it. (Marx and Engels 2010b, 396)

What Marx was saying here is that capitalists will introduce new production technologies, namely, machines, only when these machines reduce the cost of production.

Okishio continues to investigate the improving of labor productivity, noting that

\[ c_i = \sum a_{ij} t_j, \]
\[ v_i = \tau_i \sum b_{jtj}, \]

where \((b_1, b_2, \ldots, b_n)\) refers to the consumer goods package that a worker receives in return for 1 unit of labor, that is, the real wage rate. The organic composition of capital in an industry thus depends on two factors: the production technology that determines \(a_{ij}, \tau_i,\) and \(t_i,\) as well as the real wage rate that determines \(b_j.\) The change in the real wage rate will also alter the organic composition of capital even if the production technology remains constant.

Okishio refers to \(c_i/(v_i + m_i)\) as the “organic composition of production” of Industry \(i.\)

The surplus value rate \(m/v\) is then expressed symbolically as follows:

\[
\frac{m}{v} = \frac{1 - \sum b_j t_j}{\sum b_j t_j}. \tag{5}
\]

If \(b_j > 0,\) then Industry \(j\) is a wage-goods industry. Okishio refers to all wage-goods industries and the industries inseparable from wage-goods industries as “basic industries.” Thus, given the real wage rate, the rate of surplus value depends solely on the production technologies of the basic industries.

If a new production technology is introduced into a basic industry and the labor productivity of some wage products increases—if, in other words, for \(i\) with \(b_i > 0,\) \(t_i\) decreases—then given the real wage rate, the rate of surplus value must increase.
Nevertheless, the change in production technology in non-basic industries does not affect the rate of surplus value.

According to Okishio, the rate of profit, \( m/(c+v) \), cannot exceed the reciprocal of the organic composition of production, i.e.,

\[
m/(c+v) \leq (v+m)/c. \tag{6}
\]

Okishio believes that it is incorrect for Marx to calculate the general rate of profit in the form of value by dividing total surplus value by total capital in the form of value, i.e., \( m/(c+v) \). The general rate of profit, \( r \), in his view, is determined by the following equation:

\[
q_i = (1 + r)(\Sigma a_j q_j + \tau_j), \quad i = 1, 2, \ldots, n,
\]

\[
1 = \Sigma b_j q_j. \tag{7}
\]

However, Equation (7) merely calculates the rate of profit by dividing the profit contained in the commodity by the cost of the commodity itself, which greatly overestimates the rate of profit of enterprises with large fixed asset investment. In addition, \( \tau_i \) in Equation (7) is actually equal to \( v_i \) of \( \tau_j b_j f_j \) since it is included in the cost. Moreover, \( q_i \) does not represent the correctly calculated amount of labor. As a result, Equation (7) is not valid in any way.

According to Okishio, the following inequality can be obtained from Equations (1) and (7):

\[
r < \tau / \Sigma a_j f_j, \text{ for an } i. \tag{8}
\]

Since Equation (7) is itself false, then even if Inequality (8) is true, it is irrelevant to Equation (7). If fixed assets are excluded, then \( r = m/(c+v) \), while \( \tau / \Sigma a_j f_j = (v+m)/c \). Inequality (8) is merely a repetition of Inequality (6).

Okishio continues to assume that the production technology \( (a_{k1}, a_{k2}, \ldots, a_{kn}, \tau_k) \) in Industry \( k \) is replaced by the new technology \( (a'_{k1}, a'_{k2}, \ldots, a'_{kn}, \tau'_k) \), and satisfies Inequality (3). He holds that the following conclusions can be drawn:

1) If the industry in which the new technology is introduced is a non-basic industry, the general rate of profit will not be affected at all.

The proof process is as follows:

According to the definition, the products of non-basic industries are not wage goods, so \( b_l = 0 \) in Equation (7), where \( l \) is the number of non-basic industries. Similarly, by definition, the products of non-basic industries are not the inputs of wage-goods industries, or of those industries required directly or indirectly for the
production of wage goods. Thus, if $l$ to $m$ denotes the wage-goods industry and the industries necessary for the production of wage goods, then

$$a_{ii} = 0, \ i = 1, 2, \ldots, m,$$  \hspace{1cm} (A.1)

where $l$ is the number of non-basic industries. In this way, $m + 1$ equations can be made out:

$$q_i = (1 + r) \left( \sum a_{ij}q_j + \tau_i \right), \ i = 1, 2, \ldots, m,$$

$$1 = \sum b_iq_i.$$  \hspace{1cm} (A.2)

Moreover, these equations are sufficient to determine the general rate of profit. For this reason, technological changes in non-basic industries are unlikely to affect the $r$ thus determined.

Nevertheless, the so-called proof here is meaningless since Okishio directly excludes non-basic industries from the industries that determine the general rate of profit. All that the above equations show is that he has taken his conclusion as a premise. If non-basic industries increase their profit by introducing new technologies, the capital in basic industries will be freely transferred to the non-basic industries in order to maximize profit. This in turn will lead to a decrease in competition and an increase in profit in the basic industries, and to an increase in competition and a decrease in profit in the non-basic industries. The rates of profit in the two categories of industry will eventually be equalized, and the free flow of capital between them will reach an equilibrium. At this point, the general rate of profit will be higher than the original general rate of profit. Okishio’s conclusion merely demonstrates that he confuses the rate of surplus value with the rate of profit. If the labor productivity of non-basic industries increases, this will affect the general rate of profit, not the rate of surplus value.

Another conclusion Okishio comes to is this:

2) If the industry in which the new technology is introduced is one of the basic industries, the general rate of profit is bound to rise.

The proof is obtained through the following process:

Suppose that Industry $k$ is a basic industry, and that the new production technology introduced satisfies:

$$\sum a_{kj}q_j + \tau_k > \sum a'_{kj}q_j + \tau'_k.$$  \hspace{1cm} (A.3)

Let $\beta = 1/(1 + r)$. (A.2) can then be rewritten as:

$$\beta q_i = \sum a_{ij}q_j + \tau_i, \ i = 1, 2, \ldots, m,$$

$$1 = \sum b_iq_i.$$  \hspace{1cm} (A.5)
Under the new technology, the general rate of profit is determined by (A.5) and the following two equations:

\[ \beta q_i = \sum a_i q_j + \tau_i, \quad i = 1, \ldots, k-1, k+1, \ldots, m, \tag{A.6} \]

\[ \beta q_k = \sum a'_k q_j + \tau'_k. \tag{A.7} \]

Suppose that the solutions of (A.4) and (A.5) are \((\beta, q_1, \ldots, q_n)\), and that the solutions of (A.6), (A.7), and (A.5) are \((\beta', q'_1, \ldots, q'_n)\). From (A.4)–(A.7), it can be seen that:

\[ \beta' \Delta q_i = \sum a_{ij} \Delta q_j - q_i \Delta \beta, \quad i = 1, \ldots, k-1, k+1, \ldots, m, \tag{A.8} \]

\[ \beta' \Delta q_k = \sum a'_{kj} \Delta q_j - q_k \Delta \beta + \{\sum \Delta a_{kj} q_j + \Delta \tau_k\}, \tag{A.9} \]

\[ 0 = \Sigma b_i \Delta q_i, \tag{A.10} \]

where \(\Delta q_i = q'_i - q_i, \Delta \beta = \beta' - \beta, \Delta a_{kj} = a'_{kj} - a_{kj}, \) and \(\Delta \tau_k = \tau'_k - \tau_k\).

Since \(q'_i > 0\) for all \(i\), the coefficients of \(\Delta q\) in (A.8) and (A.9) satisfy the Hawkins-Simon condition. Further, it can be seen from (A.3) that the third term on the right of (A.9) is negative. Thus, if \(\Delta \beta \geq 0\), we find \(\Delta q_k < 0\) and \(\Delta q_i \leq 0\) in (A.8) and (A.9) for all cases in which \(I \neq k\). Industry \(k\) is a basic industry, so there is at least one wage-goods industry that satisfies \(\Delta q_i < 0\). This, however, is in contradiction with (A.10), so it follows that \(\Delta \beta < 0\), or \(r' > r\).

Nevertheless, since Okishio’s first conclusion above is untenable, his second conclusion is untenable as well. The improvement of labor productivity in basic industries will increase the rate of surplus value, but will not necessarily increase the general rate of profit. Marx demonstrated long ago that changes in the rate of surplus value can be in the same or opposite direction compared to changes in the rate of profit. The above mathematical derivation by Okishio will be further analyzed later.

Okishio illustrates the above two propositions through a simple numerical example.

It is assumed that there is a capital-goods industry (I), a wage-goods industry (II), and a luxury-goods industry (III). The production technologies of these industries are shown in the table below:

|     | I    | II   | III  |
|-----|------|------|------|
| I   | 1/2  | 1/4  | 1/5  |
| Labor | 10   | 15   | 16   |

This table shows that in Industry II, for example, 1/4 of a unit of means of production and 15 units of direct labor are necessary to produce 1 unit of wage goods.
Next, suppose that the actual wage rate is $1/45$ of a unit of wage goods.

Thus, the general rate of profit $r$ is determined by the following system of equations:

\[
q_1 = (1 + r) \left( \frac{1}{2} q_1 + 10 \right), \quad (9.1)
\]

\[
q_2 = (1 + r) \left( \frac{1}{4} q_1 + 15 \right), \quad (9.2)
\]

\[
q_3 = (1 + r) \left( \frac{1}{5} q_1 + 16 \right), \quad (9.3)
\]

\[1 = q_2/45. \quad (9.4)
\]

It can readily be seen that for $r = 50\%$, $q_1 = 60$, $q_2 = 45$, $q_3 = 42$, where $q_i = p_i/w$.

Replace (9.3) with

\[
q_3 = (1 + r) \left( \frac{1}{20} q_1 + \frac{3}{4} q_3 + 6 \right) \quad (9.3')
\]

and as before, $r = 50\%$, $q_1 = 60$. The negative $q_3$ can be obtained. This means that there is no general rate of profit between industries. Therefore, although the production technologies of non-basic industries do not affect the general rate of profit, they are related to the existence of the level of the general rate of profit itself.

Leaving aside the $\left( \frac{1}{20} q_1 + 6 \right)$ in (9.3'), however, Okishio requires that 1 unit of $q_3$ is produced from $3/4$ units of $q_3$, and that the value be increased by $50\%$, that is, to reach $9/8$ units of $q_3$, which is a completely arbitrary imposition. In fact, without considering (9.3) and (9.3'), that is, without considering the non-basic industries at all, another calculation result can be obtained from Equations (9.1), (9.2), and (9.4):

\[r = 500\%, \quad q_1 = -30. \]

Okishio cannot discard this result simply because it is unrealistic, because the existence of this result shows precisely that his demonstration of the above is wrong.

In addition, it is wrong to use 1 unit of wage goods to represent 1 unit of labor in Equation (9.4). The former is only equivalent to the necessary labor part $v$, while the latter also includes the surplus labor part, which is equivalent to $v + m$. 

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Okishio further assumes that Industry II improves labor productivity and the organic composition of production, and that it reduces costs at the original prices and wages. The technology in Industry II is replaced by the unit in the table below:

|   | I | Labor |
|---|---|-------|
| II| 1/3| 35/24 |

The labor productivity under the old technology is denoted as $t_i$, which is determined by the following formulas:

\[ t_1 = \frac{1}{2} t_i + 10, \]
\[ t_2 = \frac{1}{4} t_i + 15. \tag{10.1} \tag{10.2} \]

Here, $t_1 = 20$, $t_2 = 20$, while under the new technology, (10.2) is replaced by:

\[ t_2 = \frac{1}{3} t_i + \frac{35}{24}. \tag{10.2'} \]

The labor $t_2$ needed to produce 1 unit of wage goods is decreased dramatically, declining from 20 to 8.125. The organic composition of Industry II has been greatly improved.

The effect of this change on the general rate of profit can be obtained by replacing Equation (9.2) with the following one:

\[ q_2 = (1 + r) \left( \frac{1}{3} q_i + \frac{35}{24} \right). \tag{9.2'} \]

The solutions of Equations (9.1), (9.2), and (9.4) are: $q_1 = 80$, $q_2 = 45$, $r = 60\%$. The general rate of profit is increased.

Similarly, Okishio discards another solution to the above system of equations for no reason.

### 3. Further Analysis of the Okishio Theorem

The content of the Okishio theorem has been reviewed in the previous summary, and will be further analyzed here.

First, the $q_j$ in Okishio’s Inequality (3) implies that the amount of labor embodied in other goods is equal to the ratio of the price of the goods to the money wage rate, as he completely ignores the surplus value contained in the commodity. If the price of the commodity is equal to the value, which is equivalent to 2 (working
days), and the money wage rate is equal to 0.5 (working days), i.e., the rate of surplus value is 100%, then \( q_j \) means that the amount of labor in the commodity is \( 2/0.5 = 4 \) (working days), not 2 (working days). Marx once pointed out,

> It would be wrong to assume that if we [say]: £100 capital is laid out in agriculture and if one working day=£l, then 100 working days are laid out. In general, if a capital of £100=100 working days then, in whatever branch of production this capital may be laid out, [the value of the product created by this capital] is never [= to 100 working days]. Supposing that one gold sovereign equals one working day of 12 hours, and that this is the normal working day, then the first question is, what is the rate of exploitation of labor? (Marx and Engels 2010a, 510; square brackets in the original).

Therefore, the expression of Inequality (3) is faulty, and its correct expression should be:

\[
\Sigma a_{kj}p_j + \tau_kw > \Sigma a'_{kj}p_j + \tau'kw. \tag{3'}
\]

Inequality (3) and Inequality (3’) are mathematically identical when \( w \) is not zero, but they have completely different economic implications. Inequality (3’) shows that the production cost of the new technology (the former term is constant capital cost and the latter term is variable capital cost) is lower than the original production cost. \( q_i \) in Inequality (3) simply indicates how many working days it takes for a worker to earn the wage required to purchase 1 unit of commodity \( i \). The two terms on each side of the inequality are of no economic or practical significance.

Second, a closer look at why there is an unequal sign in Inequality (4) while there is an equal sign in Equation (1) reveals that \( \tau_i \) on the right of Inequality (4) is only paid labor, while \( \tau_i \) on the right of Equation (1) includes both paid and unpaid labor. This problem also exists in Inequality (8) because it is derived from Equation (7) and Equation (1), and \( \tau_i \) in Equation (7) has the same meaning as in Inequality (4). This problem also exists in the numerical example given by Okishio later. 10 in Equation (9.1) and 15 in Equation (9.2) are obviously calculated as paid labor, while they are calculated as total living labor, including unpaid labor in Equations (10.1) and (10.2).

As a result, and even though Marx noted the difference between the “productivity criterion” and the “cost criterion,” Okishio fails to express the difference between the two criteria correctly, but misrepresents different entities by using the same symbol. In other words, his challenge to Marx on this point is not valid. Further, what capitalists pursue is profit rather than cost, and thus the ultimate
criterion they consider cannot be cost but only profit. Capitalists who produce luxury goods will never turn luxury goods into ordinary goods in order to reduce their costs. Even if a lessening of costs does not lead to a reduction in the quality of the goods, the value of the goods themselves is still lowered. Therefore, the rate of profit may not be increased when the price of the commodity returns to its value.

Third, Okishio claims that Marx’s view was that \((v + m)/c \to 0\), which is a vulgarization of Marx’s real position. In the first place, according to Marx, “It is no contradiction at all on this self-contradictory basis that there should be an excess of capital simultaneously with a growing surplus of population” (Marx and Engels 2010c, 243). This means that a portion of \(c\) will be idle and thus not participate in the calculation of \((v + m)/c\). In addition,

Alongside the fall in the rate of profit the mass of capitals grows, and hand in hand with this there occurs a depreciation of existing capitals which checks the fall and gives an accelerating motion to the accumulation of capital values. (Marx and Engels 2010c, 247)

That is to say, a portion of \(c\) will be destroyed, especially during economic crises, and then accumulated again; accordingly, \((v + m)/c\) does not simply go to zero. Marx’s law of the decline of the general rate of profit can never be vulgarized as the law of the decline of the general rate of profit to zero.

Okishio’s so-called organic composition of production \(c/(v + m)\) is of much weaker theoretical and practical significance compared to Marx’s alleged organic composition of capital \(c/v\).

Fourth, whether \(r\) in Equation (7) is equal to \(m/(c + v)\) depends on which variables in Equation (7) are known quantities and which are unknown, and on how the known quantities are obtained. In general, \(r = m/(c + v)\) appears as a known quantity in Equation (7) so as to provide a solution for the production price \(p\), and thus for the abstract \(q\), etc.

There are two problems in Equation (7). The first is that “the price of the output and the prices of the inputs are determined simultaneously” (Yu 2009, 68), thus the same symbol \(p_i\) or the transformed \(q_i\) is used to express the prices of goods in the same Industry \(i\) in total output and total inputs. This means that the inputs have no price until they emerge from the production process, and that the capitalist then decides at what price to pay for the inputs he has purchased earlier, making this decision according to the output and general rate of profit after the inputs take on their finished form. In other words, when the capitalist buys inputs as means of production from the supplier who sells them, the transaction takes the form of a forced or coercive trade. However, the capitalist does not take advantage of his position as a privileged buyer to pay an arbitrary sum; instead, he compensates the
supplier who has sold inputs to him on the basis of how much he, the capitalist, has made, so that the supplier also earns some money. This is the capitalist mode of production as Okishio learned it from Sraffa. The second problem is that the amount of capital and capital accumulation are not considered at all. Equation (7) is set for one unit of commodity quantity in each industry. It thus applies only to a capitalist economy in which just one unit of each commodity is produced, and in which the amount of capital in the market suffices for this and no more.

The capitalist economy as analyzed by Marx, however, is an economy engaged in capital accumulation. “Essentially, the capitalist production process is simultaneously a process of accumulation” (Marx and Engels 2010c, 216). In this realistic capitalist economy, the accumulated surplus value will be zero if it is not converted into capital. The capitalist will still choose to complete the conversion into capital even if the rate of profit realized in the original capital cannot be reached, so long as the total amount of profit increases; what the capitalist pursues is the volume of profit, not its rate. As Marx pointed out,

In the first case, the product does not have to yield the usual profit, even in capitalist production. It must only yield as much above the usual rate of interest as will make worth while the trouble and risk of the farmer to prefer the industrial employment of his spare capital to its employments as “monied” capital. (Marx and Engels 2010a, 545)

In discussing Ricardo, Marx states that it is necessary to “identify this investment of additional capital with the application of additional capital to new soils” (Marx and Engels 2010a, 545).

In addition, while a high rate of profit will lead to a high volume of profit under certain circumstances, this is not necessarily the case. As Marx pointed out clearly in the third volume of Capital,

The number of laborers employed by capital, hence the absolute mass of the labor set in motion by it, and therefore the absolute mass of surplus labor absorbed by it, the mass of the surplus value produced by it, and therefore the absolute mass of the profit produced by it, can, consequently, increase, and increase progressively, in spite of the progressive drop in the rate of profit. And this not only can be so. (Marx and Engels 2010c, 216)

Of course, this is only true in the case of “a growth of the total capital at a pace more rapid than that at which the rate of profit falls” (Marx and Engels 2010c, 221). All in all, it is absurd and ridiculous for Okishio to attempt to negate Marx’s law of the decline in the general rate of profit by taking as a starting point
Equation (7), which is completely incompatible with the reality of the capitalist mode of production.

Fifth, Okishio believes that technological change in non-basic industries is unlikely to affect the general rate of profit. At the same time, non-basic industries are included in Equation (7), which implies precisely that the production situation in non-basic industries does indeed affect the general rate of profit. In particular, Okishio admits that the production technology in non-basic industries is relevant to the existence of the level of the general rate of profit itself. Does it therefore follow that there is no general rate of profit if the production technology of non-basic industries does not accord with his assumption?

According to Okishio, if there is a higher rate of profit in the non-basic industries, the capitalists in the basic industries will limit themselves to expressions of envy, to avoid a situation in which a general rate of profit comes into being. Conversely, the capitalists in non-basic industries will merely be forced to lower their rates of profit and bring them onto a par with the rates in the basic industries, so as to remedy a situation that is abnormal for a capitalist economy. As this demonstrates, Okishio has no idea how the general rate of profit is formed. This is why he denies that the general rate of profit exists, and with his Equation (9.3′) implicitly denies the existence of the capitalist mode of production.

By contrast, Marx’s ideas can be applied to explain the existence of a general rate of profit under the production technology corresponding to Equation (9.3′). For this reason the rate of surplus value, which represents the core of Marx’s ideas and is never addressed by Okishio in his essay, should be established first. To get closer to the example Okishio uses, the non-basic industries should first be set aside. The results of Equations (9.1), (9.2), and (9.4) can then be used to estimate the rate of surplus value. The sum of paid labor for Industries I and II amounts to $10 + 15 = 25$, and the sum of profit, i.e., the sum of surplus labor, comes to $20 + 15 = 35$. Thus, the rate of surplus value may be estimated as $35/25 = 140\%$.

Next, we can calculate the general rate of profit for the three industries in the case of Equation (9.3′). Let us suppose that 1 unit of value is equal to 1 unit of labor wage. If the price of inputs is based on Okishio’s previous calculation results of $q_i$ in the three industries, and if the total paid labor in these three industries is $10 + 15 + 6 = 31$, the total surplus value, i.e., the total profit, is $31 \times 140\% = 43.4$. The production costs of these three industries are 40, 30, and 40.5, a total of 110.5, and thus the general rate of profit is $43.4/110.5 = 39.3\%$. Substituting this result into Equations (9.1), (9.2), and (9.3′), the output prices of the three industries after transformation come to 55.7, 41.8 and 56.4, respectively, and the gaps with their value quantity are 1.7, –9.2, and 7.5, respectively.

According to Okishio, one of the reasons why Marx failed to draw a correct conclusion was because of a lack of thoroughness in his analysis of the so-called
transformation problem. However, it is Okishio who allows the price of a commodity to remain constant before the transformation (input) and after it (output), so that there is no transformation at all.

Sixth, and according to Equation (7), $q_i$ of Commodity $i$ as output is the same as that of input, and $q_i$ as input after the adoption of the new technology should also be the same as that of the product $q'_i$ after the use of the new technology. Hence (A.3) should be:

$$\sum a_{kj}q_j + \tau_k > \sum a'_{kj}q'_j + \tau'_k.$$  

(A.3')

There is thus no longer a $\Delta q$ in (A.9), and consequently, no coefficient of $\Delta q$ satisfying the Hawkins-Simon condition. The proof of Okishio’s second proposition is not valid even if the same commodity in output and input is not distinguished on the basis of Equation (7). The reason is that Okishio fails to prove that at least one wage-goods industry satisfies $\Delta q_i < 0$ since Industry $k$ is a basic industry. On the contrary, his numerical example has Equation (9.4) keeping $q$ constant in wage-goods Industry II regardless of the introduction of new technology in basic Industry I. As a result, not only does the entire wage-goods industry fail to satisfy $\Delta q < 0$, but $\Delta q = 0$. Okishio’s alleged proof is disproved by his own example.

Seventh, the examples of Equations (9.1), (9.2), and (9.4) fail to establish the conclusion—in fact, just the incorrect result of a miscalculation—that the general rate of profit has risen. First of all, the capitalists of Industry II are shown here as holding idle capital. According to Okishio’s calculation, the original capital invested by the capitalists of Industry II is 30, but after the new technology has been adopted the investment is only 28.125, which means that capital of 1.875 has been left, generating a zero profit rate. If the capitalists of the Industry II are not to leave their capital idle, they should expand the scale of production and improve labor productivity. However, Okishio’s whole calculation is based on the premise that each industry produces only 1 unit of goods.

Meanwhile, the status of $\Sigma a_{ij}q_j$ and $\tau_i$ is symmetrical in Okishio’s Equation (7). In other words, all means of production create surplus value, as with living labor. Not only that, but these means of production create surplus value according to their respective value or price quantities at the same rate of surplus value as living labor. This is no longer a Marxist perspective at all, nor is it in line with Smith’s or Ricardo’s viewpoints, but rather it is something that Okishio has appropriated from Sraffa, who claims that commodities produce commodities.

In this regard, no matter how the production scale of each industry changes, it will not change Equation (7), nor will it change the general rate of profit. For instance, suppose that in the case of Equations (9.1), (9.2), and (9.4), Industry I
produces 3 units of goods, while Industry II still only produces 1 unit of goods. According to Okishio’s calculation, Equation (9.1) is replaced by:

\[ 3q_i = (1+r) \left[ 3 \times \left( \frac{1}{2} q_i + 10 \right) \right]. \tag{9.1'} \]

If 3 is divided out from both sides of (9.1’) simultaneously, there is no difference between Equations (9.1’) and (9.1).

This may be contrasted with the perspective offered by the labor theory of value elaborated by Marx. From this perspective, leaving aside Industry III, the production cost of Industry I is constant capital \(3 \times 30 = 90\), variable capital \(3 \times 10 = 30\), and surplus value \(30 \times 140\% = 42\) when Industry I produces 3 units of goods. Industry II, meanwhile, only produces 1 unit of goods; its production cost is constant capital 15, variable capital 15, and surplus value 15 \(\times\) 140\% = 21. Therefore, the total surplus value (i.e., the total profit) is 63, the total cost is 150, and the general rate of profit is \(63/150 = 42\%\). When both Industry I and Industry II produce only 1 unit of goods, the general rate of profit is 50\%. It is obvious that a decrease in the general rate of profit will be facilitated even if no enterprise increases its organic composition of capital, but only those with a higher organic composition of capital expand their share in the economy. Conversely, the decrease in the general rate of profit will be hindered if enterprises with a low organic composition of capital expand their share in the economy.

It is particularly significant that Okishio still follows (9.4), and does not acknowledge a decrease in variable capital \(v\) and an increase in the rate of surplus value when Industry II increases labor productivity and the production technology changes from Equation (9.2) to (9.2’). If the rate of surplus value remains unchanged, and the 140\% rate estimated earlier continues to apply, it can be seen that the production cost of Industry I is constant capital 30, variable capital 10, and surplus value 10 \(\times\) 140\% = 14. The new production cost of Industry II is constant capital 20, variable capital 1.5, and surplus value 1.5 \(\times\) 140\% = 2.1. Therefore the total surplus value, i.e., the total profit is 16.1, the total cost is 61.5, and the general rate of profit is 26.2\%, lower than the original rate of general profit of 50\%. This outcome reflects the increase in the organic composition of total capital, and a decrease in the general rate of profit will be facilitated.

Finally, and more fundamentally, the transformation of surplus value into average profit, and hence of value into production price, is realized in the field of circulation rather than in the field of production. As a result, it is impossible to calculate the general rate of profit directly using a system of input–output equations, not to mention that this system of equations fails to take into account the existence of fixed capital and the differences in the turnover period of floating capital.

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4. Summary

To sum up, Okishio “negates” Marx’s law of the decline of the general rate of profit with his own inaccurate presuppositions and inaccurate calculations and derivations. The so-called Okishio theorem is not tenable at all.

Okishio reveals a failure to really understand what Marx’s analysis was when he claims that Marx was unable to reflect on an analysis of his own thinking. The truth is exactly as Okishio acknowledges: one of Marx’s basic doctrines is that profit is an expression of surplus value. However, Okishio’s argument that the rate of surplus value is only relevant to basic industries, and has nothing to do with non-basic industries, is insufficient. The reason is that the basic industries mainly determine relative surplus value, while the main determinant of absolute surplus value, i.e., the struggle to set the boundaries of the working day, is “a struggle between collective capital, i.e., the class of capitalists, and collective labor, i.e., the working class” (Marx and Engels 2010b, 243). It is still insufficient for Okishio to argue that basic and non-basic industries play different roles in the general rate of profit, i.e., that non-basic industries play only a passive role. This is inadequate first of all because the conversion of surplus value into profit does not occur solely in basic industries, while at a more fundamental level, Okishio completely ignores the capitalists’ pursuit of profit.

Okishio claims that the law of the decline of the general rate of profit is not the cornerstone on which all the edifices of Marx’s system are built, and that attempts to deduce economic crises from the law of the decline of the general rate of profit are doomed to failure. The only way to respond to this is to note Marx’s comment at one point on the misinterpretation of his arguments; if this was Marxism, he observed, “All I know is that I’m not a Marxist” (Marx and Engels 2010d, 22). If Okishio and his followers are Marxists, they are surely “Marxists” in the sense to which Marx was alluding.

Note

1. Equation (A.1) is Equation (25) in the appendix to Okishio’s original text. For the sake of convenience, the so-called proof in the appendix to the original text is taken out of order, and thus Equations (25)–(34) in the original text are changed to Equations (A.1)–(A.10).

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