Quantum Description of Electromagnetic Waves in Time-Dependent Linear Media

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Abstract. In this work, we present a quantum description of electromagnetic waves propagating through time-dependent homogeneous nondispersive linear media without sources. By using the Coulomb gauge, we show that this description can be performed in terms of a time-dependent quantum harmonic oscillator. In addition, we construct coherent states for the quantized electromagnetic waves and evaluate the quantum fluctuations in coordinate and momentum as well as the uncertainty product for each mode of the electromagnetic field.

1. Introduction
The quantum description of the electromagnetic field is traditionally performed in empty cavities or in free space by associating a quantum-mechanical harmonic oscillator with each mode of the radiation field [1, 2]. The behavior of electromagnetic waves is well understood in the case of electromagnetic fields in empty cavities or in free space. However, in the presence of external currents and others sources, the behavior of these waves is not so clear. Recently, the study of the quantum properties of electromagnetic waves propagating through material media has drawn special attention of physicists [3, 4, 5, 6, 7, 8, 9], motivated partly by the advent of modern optical materials such as optical fibers and photonic crystal and partly by the growth of experiments on quantum optics processes taking place inside material [6, 8, 9]. Several approaches have been employed to tackle the problem of the quantization of the electromagnetic field inside material media [3, 9], which have mainly considered materials with electric permittivity [5, 6, 10, 11, 12, 13, 14]: (i) real (nondispersive) and either homogeneous or inhomogeneous media; (ii) real and dependent on time and position (time-dependent nonuniform media); or (iii) position and frequency dependent (dispersive inhomogeneous media). It is worth remarking here that, in dispersive media, the inclusion of losses into the system is, in general, introduced by a reservoir (a continuum of harmonic oscillators) which leads to an energy flow from the medium to the reservoir [8, 9, 15]. On the other hand, the problem of conducting media has also been considered [16, 17, 18, 19, 20]. In this case, the losses were introduced phenomenologically as a set of time-dependent parameters.

In this work we present a simple and comprehensive quantum description of electromagnetic waves propagating through homogeneous nondispersive linear media, in the absence of sources, with time-dependent electric permittivity and conductivity. Similarly to the case of empty cavities, we show that this quantization can be performed by associating a damped or attenuated quantum-mechanical time-dependent harmonic oscillator with each mode of the electromagnetic field.
field. Afterwards, we construct coherent states for the quantized electromagnetic waves and evaluate the quantum fluctuations in coordinate and momentum as well as the uncertainty product for each mode of the electromagnetic field.

We organize this paper as follows. In section 2, we investigate classical and quantum properties of electromagnetic waves propagating through time-dependent conducting and nonconducting linear media without charge density. In section 3, we construct coherent states for the quantized electromagnetic waves and evaluate the quantum fluctuations in coordinate and momentum as well as the uncertainty product for each mode of the electromagnetic field. In section 4, we conclude the paper with a short summary.

2. Light propagation in time-dependent linear media

2.1. Classical description

In the absence of charge sources, the electromagnetic field dynamics in time-dependent homogeneous conducting linear media is governed by the phenomenological Maxwell’s equations. Further, the relations between the fields and current are given by $\vec{D} = \varepsilon(t) \vec{E}$, $\vec{B} = \mu_0 \vec{H}$ and $\vec{J} = \sigma(t) \vec{E}$. Here, $\varepsilon(t)$ and $\sigma(t)$ are heuristically introduced as the time-dependent electric permittivity and conductivity, respectively, while $\mu_0$ is the magnetic permeability. In general, the electric permittivity and the magnetic permeability are complex. However, we will restrict our discussion to materials where they are real. This is the case [21, 22], for instance, of poor conductors and other materials for frequency below the resonant frequency. Since Maxwell’s equations are gauge invariant, we are free to choose the most appropriate gauge for our problem. A convenient gauge choice is the Coulomb gauge [1, 2] for which the divergence of the vector potential $\vec{A}$ is zero and the scalar potential is null in the absence of sources. Consequently, both the electric $\vec{E}$ and magnetic $\vec{B}$ fields are determined from the vector potential as

$$\vec{B} = \nabla \times \vec{A} \quad \text{and} \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t}. \quad (1)$$

It is worth mentioning that in the Coulomb gauge the vector potential is purely transverse [1, 2]. Therefore, it is easy to verify that it obeys the damped wave equation

$$\nabla^2 \vec{A} - \mu_0 (\dot{\varepsilon} + \sigma) \frac{\partial \vec{A}}{\partial t} - \mu_0 \varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = 0. \quad (2)$$

where the dot represents a time-derivative. From Eq.(2) we notice the appearance of the unusual term $\dot{\varepsilon}$. Consequently, the time dependence of the electric permittivity, which, in principle, may be associated with the internal response of the localized charges to an external perturbation, causes an additional attenuation of the electromagnetic field (for $\dot{\varepsilon} > 0$). Furthermore, for nondispersive nonconducting dielectric media, $\sigma(t) = 0$, the media become absorbing because of the time dependence, just as if it were in contact with a reservoir (time-dependent background medium) or if it were a conductor.

Now, we move our attention to the solutions of Eq.(2). By using the well-known separation of variables method, we write the vector potential in terms of the mode $u_l(\vec{r})$ and amplitude $q_l(t)$ functions of each cavity mode [1, 2, 16, 20]

$$\vec{A}(\vec{r}, t) = \sum_l \vec{u}_l(\vec{r}) q_l(t). \quad (3)$$

If we substitute Eq.(3) into the damped wave Eq.(2) we get that

$$\nabla^2 \vec{u}_l(\vec{r}) + \frac{\omega_l^2}{c_0^2} \vec{u}_l(\vec{r}) = 0, \quad (4)$$

$$\frac{\partial^2 q_l}{\partial t^2} + \frac{\dot{\varepsilon} + \sigma}{\varepsilon} \frac{\partial q_l}{\partial t} + \Omega_l^2(t) q_l = 0, \quad (5)$$

where $\omega_l$ is the angular frequency of the $l$th mode.
where $\omega_l$ is the natural frequency of the mode $l$, $c_0 = 1/\sqrt{\mu_0 \varepsilon(0)}$ is the velocity of light inside the medium at $t = 0$ and $\Omega_l(t)$ is a modified frequency defined as

$$\Omega_l = c(t)\omega_l/c_0,$$

with $c(t) = 1/\sqrt{\mu_0 \varepsilon(t)}$ being the velocity of the electromagnetic wave in the time-dependent medium. Here, we notice that the equations of motion for the amplitudes $q_l(t)$ given by Eq.(5) can be directly obtained from the classical Hamiltonian

$$H_l(t) = e^{-\lambda(t)} \frac{p_l^2}{2\varepsilon_0} + \frac{1}{2} e^{\lambda(t)} \varepsilon_0 \Omega_l^2(t) q_l^2,$$

where $q_l$ and $p_l$ are canonical conjugated variables, and $\lambda(t)$ is given by

$$\lambda(t) = \int_0^t \frac{\dot{\varepsilon}(\tau) + \sigma(\tau)}{\varepsilon(\tau)} d\tau.$$

Hence, the total Hamiltonian of the electromagnetic field is a sum of individual Hamiltonians corresponding to each mode, that is, $\sum_l H_l$.

Let us now concentrate on Eq.(4). Considering the electromagnetic field to be contained in a certain cubic volume $V$ of nonrefracting media, the mode functions are required to satisfy the transversality condition, $\nabla \cdot \vec{u}_l(\vec{r}) = 0$ and to form a complete orthonormal set [1, 2]. Further, assuming periodic boundary conditions on the surface, the mode function may be written in terms of plane waves as [1, 2, 16, 20]

$$\vec{u}_l(\vec{r}) = L^{-3/2} e^{\pm i\vec{k}_l \cdot \vec{r}} \vec{e}_{l\nu},$$

where $L = V^{1/3}$ is the size of the cube, $|\vec{k}_l| = \omega_l/c_0$ is the wave vector, and $\vec{e}_{l\nu}$ are unit vectors in the directions of polarization ($\nu = 1, 2$), which must be perpendicular to the wave vector because of the transversality condition. With the spatial mode functions $\vec{u}_l$ completely determined, we only need the canonical variable $q_l(t)$ in order to obtain the vector potential and, consequently, a complete classical description of the electromagnetic field. Then, the electric field confined in the cubic volume of side $L$ can be written as

$$\vec{E}(\vec{r}, t) = \frac{e^{-\lambda(t)}}{\varepsilon_0 L^{3/2}} \sum_l \sum_{\nu} \dot{e}_{l\nu} e^{\pm i\vec{k}_l \cdot \vec{r}} p_l(t).$$

Here we observe that we have quoted some results of this subsection in a recent paper [23].

2.2. Quantum description

In order to obtain a quantum description of electromagnetic waves propagating in a conducting linear media with time-dependent electric permittivity and conductivity we need to quantize the electromagnetic field. Now as the spatial mode functions $\vec{u}_l(\vec{r})$ are completely determined, the amplitude of each normal mode in Eq.(3) needed to specify a particular field configuration is $q_l(t)$ [1]. Further, for each canonical operator $q_l$ the electromagnetic field is completely specified since $\vec{E}$ and $\vec{B}$ fields operators may be derived by inserting the operator $\vec{A}$ given by Eq.(3) into Eq.(1). So, we move our attention to the canonical operator $q_l(t)$ in order to obtain the vector potential. For this purpose, we must solve the Schrödinger equation associated with the Hamiltonian (7)

$$H_l(t)\Psi(q(t), t) = i\hbar \frac{\partial}{\partial t} \Psi(q(t), t),$$

where $\psi_l$ is the natural frequency of the mode $l$, $c_0 = 1/\sqrt{\mu_0 \varepsilon(0)}$ is the velocity of light inside the medium at $t = 0$ and $\Omega_l(t)$ is a modified frequency defined as

$$\Omega_l = c(t)\omega_l/c_0,$$

with $c(t) = 1/\sqrt{\mu_0 \varepsilon(t)}$ being the velocity of the electromagnetic wave in the time-dependent medium. Here, we notice that the equations of motion for the amplitudes $q_l(t)$ given by Eq.(5) can be directly obtained from the classical Hamiltonian

$$H_l(t) = e^{-\lambda(t)} \frac{p_l^2}{2\varepsilon_0} + \frac{1}{2} e^{\lambda(t)} \varepsilon_0 \Omega_l^2(t) q_l^2,$$

where $q_l$ and $p_l$ are canonical conjugated variables, and $\lambda(t)$ is given by

$$\lambda(t) = \int_0^t \frac{\dot{\varepsilon}(\tau) + \sigma(\tau)}{\varepsilon(\tau)} d\tau.$$
where \( p_l \) is now the moment operator \( p_l = -i\hbar \partial / \partial q \) with \( [q_l, p_l] = i\hbar \). The solutions of Eq. (11) can be obtained with the aid of the dynamical invariant method devised by Lewis and Riesenfeld [24, 25]. Following this method, we look for a nontrivial Hermitian operator \( I_l(t) \) which satisfies the equation

\[
\frac{dI_l}{dt} = \frac{1}{i\hbar} [I_l, H_l] + \frac{\partial I_l}{\partial t} = 0. \tag{12}
\]

If the exact invariant \( I(t) \) (constant of motion), does not contain any time-dependent operator, the Schrödinger equation solutions are straightforwardly written in terms of the orthonormalized eigenfunctions \( \phi_n(q_l, t) \) of \( I_l(t) \),

\[
I_l(t)\phi_n(q_l, t) = \lambda_n \phi_n(q_l, t), \tag{13}
\]

and of the phase functions \( \beta_n(t) \) as

\[
\Psi_n(q_l, t) = e^{i\beta_n(t)} \phi_n(q_l, t). \tag{14}
\]

Here, the \( \lambda_n \) are time-independent eigenvalues and the phase functions \( \beta_n(t) \) are determined from the equation

\[
\hbar \frac{d\beta_n(t)}{dt} = \left\langle \phi_n \left| i\hbar \frac{\partial}{\partial t} - H_l \right| \phi_n \right\rangle, \tag{15}
\]

with the orthonormality condition \( \langle \phi_{n'}|\phi_n \rangle = \delta_{n'n} \).

Linear invariant operators satisfying Eq. (12) are innumerable [26, 27], but in this paper we are interested in dealing with a quadratic Hermitian invariant. Now, it is known that one such quadratic invariant is given by [28, 29]

\[
I_l(t) = \frac{1}{2} \left[ \left( \frac{q_l}{\rho_l} \right)^2 + \left( \rho_l p_l - \varepsilon_0 e^{\lambda(t)} \dot{\rho}_l q_l \right)^2 \right], \tag{16}
\]

where \( \rho_l(t) \) is a time-dependent real function satisfying the Milne-Pinney equation [28, 30, 31]

\[
\ddot{\rho}_l(t) + \frac{\dot{\varepsilon} + \sigma}{\varepsilon} \dot{\rho}_l(t) + \Omega_l^2(t) \rho_l = \frac{e^{-2\lambda(t)}}{\varepsilon^2 \rho_l^3}. \tag{17}
\]

Next, we look for the eigenstates \( \phi_n(q, t) \) of \( I_l(t) \). For this purpose, we consider the unitary transformation [28, 29]

\[
\phi_n'(q_l, t) = U \phi_n(q_l, t), \tag{18}
\]

with

\[
U = \exp \left( -\frac{i\varepsilon_0 e^{\lambda(t)} \dot{\rho}_l}{2\hbar \rho_l} \frac{q_l^2}{2} \right). \tag{19}
\]

Making use of this transformation, we can rewrite the eigenvalue equation (13) as

\[
I_l' \phi_n'(q_l, t) = \lambda_n \phi_n'(q_l, t), \tag{20}
\]

where

\[
I_l' = UI_lU^\dagger = -\frac{\hbar^2}{2} \rho_l^2 \frac{\partial^2}{\partial q_l^2} + \frac{1}{2} \frac{q_l^2}{\rho_l^2}. \tag{21}
\]

If we now define a new variable \( z_l = q_l / \rho_l \), we can express Eq. (20) as
\[
\left[-\frac{\hbar^2}{2} \frac{\partial^2}{\partial z_l^2} + \frac{q_l^2}{2}\right] \varphi_n(z_l) = \lambda_n \varphi_n(z_l), \tag{22}
\]

where \(\varphi_n\) is related to \(\phi'_n\) by

\[
\varphi_n(z_l) = \varphi_n(q_l/\rho_l) = \rho_l^{1/2} \phi'_n(q_l, t). \tag{23}
\]

The factor \(\rho_l^{1/2}\) has been introduced to satisfy the normalization condition. Therefore, the solution \(\varphi_n(z_l)\) of Eq. (22) are the eigenfunctions

\[
\varphi_n(z_l) = \left[\frac{1}{\pi^{1/2}\hbar^{1/2} n! 2^n}\right]^{1/2} \exp \left(-\frac{z_l^2}{2\hbar}\right) H_n \left[\left(\frac{1}{\hbar}\right)^{1/2} z_l\right], \tag{24}
\]

with the respective eigenvalues

\[
\lambda_n = \hbar (n + \frac{1}{2}). \tag{25}
\]

Here \(H_n\) is the Hermite polynomial of order \(n\). So, using Eqs. (18), (19), (23) and (24) we find that

\[
\phi_n(q_l, t) = \left[\frac{1}{\pi^{1/2}\hbar^{1/2} n! 2^n \rho_l}\right]^{1/2} \exp \left[\frac{i\varepsilon_0 e^{\lambda(t)}}{2\hbar} \left(\frac{\dot{\rho}_l}{\rho_l} + \frac{ie^{-\lambda(t)}}{\varepsilon_0 \rho_l^2}\right) q_l^2 \right] \times H_n \left[\left(\frac{1}{\hbar}\right)^{1/2} q_l/\rho_l\right]. \tag{26}
\]

The next step is to find the phase function given by Eq. (15). After some basic calculations, we get that

\[
\beta_n(t) = -\left(n + \frac{1}{2}\right) \int_0^t \frac{e^{-\lambda(\tau)}}{\varepsilon_0 \rho_l^2(\tau)} d\tau. \tag{27}
\]

Therefore, we can write the solutions of the Schrödinger equation (11) as

\[
\phi_n(q_l, t) = \exp[i\beta_n(t)] \left[\frac{1}{\pi^{1/2}\hbar^{1/2} n! 2^n \rho_l}\right]^{1/2} \times \exp \left[\frac{i\varepsilon_0 e^{\lambda(t)}}{2\hbar} \left(\frac{\dot{\rho}_l}{\rho_l} + \frac{ie^{-\lambda(t)}}{\varepsilon_0 \rho_l^2}\right) q_l^2 \right] H_n \left[\left(\frac{1}{\hbar}\right)^{1/2} q_l/\rho_l\right], \tag{28}
\]

with the phase function \(\beta_n(t)\) given by Eq. (27). The above expression represents the exact wave functions for each mode of the electromagnetic field. Finally, we observe that for \(\varepsilon\) and \(\sigma\) constants, the results of this section coincide with those of Ref. [29].

### 3. Coherent states for the quantized electromagnetic waves

In this section, we construct coherent states for the quantized electromagnetic waves propagating in time-dependent conducting media. In doing so, we introduce the annihilation and creation
operators defined as

\[ b'_l = \left( \frac{1}{2\hbar} \right)^{1/2} \left[ \frac{q_l}{\rho_l} + i\rho_l p_l \right], \quad (29) \]

\[ b'^\dagger_l = \left( \frac{1}{2\hbar} \right)^{1/2} \left[ \frac{q_l}{\rho_l} - i\rho_l p_l \right], \quad (30) \]

with \([b'_l, b'^\dagger_l] = 1\). In terms of these operators, the invariant \(I'_l\) given by Eq.(21) can be rewritten as

\[ I'_l = \hbar (b'^\dagger_l b'_l + \frac{1}{2}), \quad (31) \]

whose coherent states have the form [28, 32, 33]

\[ \varphi_\alpha(z_l, t) = e^{\frac{-|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{(n!)^{1/2}} e^{i\beta_n(t)} \varphi_n(z_l), \quad (32) \]

where \(\alpha\) is an arbitrary complex number. Then, using Eqs.(18),(19),(23) and (32) we find that the coherent states for the system described by the Hamiltonian (7) are given by

\[ \phi_\alpha(q_l, t) = \frac{1}{\rho_l^{1/2}} e^{\frac{i\varepsilon_0 e^{\wedge(t)} \rho_l^2}{2\hbar \rho_l q_l}} \varphi_\alpha(z_l, t). \quad (33) \]

These states satisfy the eigenvalue equation

\[ b_l \phi_\alpha(q_l, t) = \alpha_l(t) \phi_\alpha(q_l, t), \quad (34) \]

with \(b_l\) and \(b'_l\) related by

\[ b_l = U'^\dagger b'_l U = \left( \frac{1}{2\hbar} \right)^{1/2} \left[ \frac{q_l}{\rho_l} + i(\rho_l p_l - \varepsilon_0 e^{\wedge(t)} \rho_l q_l) \right], \quad (35) \]

and

\[ \alpha_l(t) = \alpha_l e^{2i\beta_0(t)}, \quad (36) \]

\[ \beta_0(t) = -\frac{1}{2} \int_0^t \frac{e^{\wedge(\tau)}}{\varepsilon_0 \rho_l^2(\tau)} d\tau. \quad (37) \]

Note that, in terms of the operator \(b_l\), the invariant in Eq.(16) can be expressed as \(I_l = \hbar \left( b'^\dagger_l b'_l + \frac{1}{2} \right)\). Hence, the expectation values of \(q_l\) and \(p_l\) in the coherent states \(\phi_\alpha(q_l, t)\) are

\[ \langle q_l \rangle = (2\hbar |\alpha|^2 \rho_l^2)^{1/2} \sin(-2\beta_0(t) + \delta_l), \quad (38) \]

\[ \langle p_l \rangle = (2\hbar |\alpha|^2 \rho_l^2)^{1/2} [\varepsilon_0 e^{\wedge(t)} \rho_l \sin(-2\beta_0(t) + \delta_l) - \frac{1}{\rho_l} \cos(-2\beta_0(t) + \delta_l)], \quad (39) \]

where \(\delta_l\) is the argument of the complex number \(\alpha\). Here we note that the expression (38) is a solution of Eq. (5). Hence, the center of the coherent state wave packet follows the motion of a
classical particle. Therefore, the above result agrees with the original idea of Schrödinger about the coherent states, who was interested in finding quantum mechanical states which followed the motion of a classical particle in a given potential [34]. Also, we can use the result (39) to show that when the electric field (10) is in a state $|\phi,\alpha(q_l, t)\rangle$, its expectation value looks like a classical field, as it should be. In what follows we evaluate the quantum fluctuations in $q_l$ and $p_l$ in the state $|\phi,\alpha(q_l, t)\rangle$. After some algebra we find that

$$<\Delta q>_2 = <q_l^2> - <q_l^2> = \frac{\hbar}{2} \rho_l^2,$$  

$$<\Delta p>_2 = <p_l^2> - <p_l^2> = \frac{\hbar}{2} \left[ \frac{1}{\rho_l^2} + (\varepsilon_0 e^{\land(t)} \dot{\rho}_l)^2 \right].$$  

Thus, the uncertainty product is expressed as

$$(\Delta q_l)(\Delta p_l) = \frac{\hbar}{2} \left[ 1 + (\varepsilon e^{\land(t)} \rho_l \dot{\rho}_l)^2 \right]^{1/2}.$$  

Here, we observe that the uncertainty relation (42), in general, does not attain its minimum value. This occurs because the states $|\phi,\alpha(q_l, t)\rangle$ correspond to the well-known squeezed states [29, 35, 36, 37, 38, 39]. It is also worth mentioning that if the medium is a dielectric material [$\sigma(t) = 0$] with constant permittivity [$\varepsilon(t) = \varepsilon_0 = \text{cte}$] and if we take the particular solution $p_l = (1/\varepsilon_0 \omega_l)^{1/2}$ of the Milne-Pinney equation (17), the uncertainty product attains its minimum value. This occurs because, in this case, both the Hamiltonian (7) and the coherent states $|\phi,\alpha(q_l, t)\rangle$ reduce to the ordinary harmonic oscillator model. Finally, we remark that we have quoted some of the results of this section and of the previous subsection 2.2 in [40].

4. Summary

In this work, we have presented a direct and simple quantum description of electromagnetic waves propagating in time-dependent conducting and nonconducting media. We have shown that this description can be performed by associating a damped quantum-mechanical oscillator with each mode of the electromagnetic field. As a consequence, we have established an unification of the procedure to obtain the quantum behavior of electromagnetic waves in empty cavities (or free space) and cavities filled with a (time-dependent) material medium. In the former case, it is usually performed by the association of an ordinary oscillator with each mode of the quantized field, and in the latter one it can be performed by associating a time-dependent harmonic oscillator. We have also constructed coherent states for the quantized electromagnetic waves and have calculated the quantum fluctuations in coordinate and momentum as well as the uncertainty product for each mode of the electromagnetic field. We have seen that this uncertainty product, in general, does not attain its minimum value. Finally, we expected that the approach developed in this work can be useful to investigate subjects related to the quantization of electromagnetic waves propagating in conducting and nonconducting media with material properties varying in time.

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