Abstract—We present\textsuperscript{1} a novel approach to maximize the transmission rate in a MIMO relay system, where all nodes are equipped with multiple antennas and the relay is self-sustained by harvesting energy using a non-uniform power splitting method. By deriving a closed-form solution for the optimal power splitting ratios for harvesting at the relay and the optimal power allocation for precoding at both source and relay, we are able to design an efficient primal-dual algorithm to jointly optimize for these variables, instead of solving for them sequentially as traditionally done. We demonstrate the algorithm efficiency at reducing the run-time by several orders of magnitude compared to standard solvers and existing sequential algorithms, making our algorithm highly amenable to massive MIMO settings. Numerical results further reveal the significant rate gain of non-uniform power splitting over traditional uniform splitting.

I. INTRODUCTION

With the rapid evolution of wireless networks, energy efficiency is now deemed as a figure of merit for the design of next generation communication systems. We can envision future communication networks to employ relays capable of providing cooperation in terms of both information and energy. Prior works in joint information and energy transfer have considered systems with multiple antennas at the transmitting and single antennas at the receiving nodes [1] [2], or with multiple antennas only at the relay node [3]. While such systems can provide throughput gains over single-antenna systems, a MIMO system with multiple antennas at all nodes can further enhance the system’s performance not only in terms of the achievable rate, but also the harvested energy. New research efforts also explore energy harvesting and communication using massive MIMO systems [4].

Two practical designs for MIMO channels with energy harvesting and information decoding exist in literature - Time Switching (TS) and Power Splitting (PS) [5] [6]. In general, power splitting has reduced transmission delay and increased spectral efficiency compared to time switching. To simplify the system model, a common assumption in literature is uniform power splitting among all antennas at the relay node [7] [8]. While this can achieve the same rate as non-uniform splitting for systems with multiple antennas at a single node [6], however, it is non-optimal for a MIMO system.

In this letter, we present a novel optimization problem to characterize a MIMO relay system with simultaneous wireless information and power transfer using non-uniform power splitting. The non-uniform power splitting complicates the considered rate optimization problem and renders the traditional sequential approach of fixing the power splitting ratio and optimizing only for the precoding inapplicable [7]. We propose a new approach by explicitly solving in closed-form for the primal variables in terms of the dual variables and use that to design a specialized primal-dual algorithm. Performance comparison between our algorithm and existing methods, carried out for an increasing number of antennas in the MIMO system, reveals extremely low convergence time and significant rate gain from the proposed non-uniform power splitting scheme. These results further corroborate the applicability of our efficient algorithm to massive MIMO systems.

II. SYSTEM MODEL

A. Channel Model

A half-duplex, decode-and-forward two-hop MIMO relay system is considered. We assume that the direct transmission link suffers from significant path loss and fading, such that the relay channel is always used for data transmission from source to destination. The source S, relay R and destination D are equipped with \( N_s \), \( N_r \) and \( N_d \) antennas respectively and the S-R and R-D Rayleigh fading channels are modeled by matrices \( H \in \mathbb{C}^{N_r \times N_s} \) and \( G \in \mathbb{C}^{N_d \times N_r} \) respectively. We assume a quasi-static fading channel with local channel state information (CSI) available to both the transmit and receive nodes to reveal theoretical bounds of the problem considered.

B. Relay Model

The relay is a self-sustained node, employing a harvest-use policy. It is equipped with two rechargeable batteries which store harvested energy and supply power using a TS scheme which caters for the half-duplicity of energy transfer. The power of the received signal at each antenna of the relay is divided for Energy Harvesting (EH) and Information Decoding (ID), according to power splitting ratios \( \rho_i \in (0, 1) \) \( \forall i \in [1, N_r] \), and we define \( \Lambda_{\rho} = \text{diag}(\rho_1 \ldots \rho_{N_r}) \).

Different from existing models, we introduce an analog beamforming/combining matrix at the relay to rotate the received signal and direct the received energy in beams to the power splitter, which acts on each received beam (the traditional power splitting per antenna is included as a special case). This received beamforming matrix creates more flexibility in

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optimizing for both the transmission rate and harvested energy. Accordingly, the received signal at the relay is given by $y_{ID}$ for information decoding and $y_{EH}$ for energy harvesting as

$$y_{ID} = (I - A_p)^{1/2}Hx_s + z_r; y_{EH} = \Lambda_p^{1/2}Q_r x_s$$

where $x_s$ is the source transmitted signal and $z_r \sim \mathcal{CN}(0, \sigma_r^2I)$ denotes the traditional receiver noise. We adopt the standard assumption of perfect power splitting [6], and hence include no noise term for $y_{EH}$. The matrix $Q_r \in \mathbb{C}^{N_r \times N_r}$ is the introduced receiver combining matrix at the relay, which is a unitary matrix representing analog beamforming in the RF domain.

Assuming that on average the energy consumed is equal to the energy harvested to prevent energy-outages in the data transmission phase [9], the transmission power for the relay, $P_r$, is then equal to the harvested power, $P_h$. The relay transmission power can be written as

$$P_r = P_h = \frac{E_h}{T/2} = \frac{\eta w(\Lambda_p Q_r H W_s H^{*} Q_r^{*})}{T/2}$$

(2)

For this paper we assume the energy conversion efficiency $\eta = 1$ [6] and also $T_s/2 = 1$ which makes $P_h = E_h$ in (2).

The relay employs a decode-forward multi-hop relaying scheme [10]. The signal received at the destination is then

$$y_d = G x_r + z_d,$$

(3)

where $z_d \sim \mathcal{CN}(0, \sigma_d^2I)$ is the additive noise at the destination. The receiver then decodes this signal to recover the information transmitted from the source.

Based on the achievable rate for the multi-hop relay channel [10, p. 387], using the signal model in (1) and (3) and assuming optimal Gaussian transmit signals, the achievable rate in bps/Hz, is given as

$$R(\Lambda_p, W_s, W_r, Q_r) = \min \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{(1 - \Lambda_p)Q_r H W_s H^{*} Q_r^{*}}{\sigma_r^2} \right), \frac{1}{2} \log_2 \left( 1 + \frac{G W_r G^{*}}{\sigma_d^2} \right) \right\}$$

(4)

III. THROUGHPUT OPTIMIZATION PROBLEM

The maximization of the achievable end-to-end transmission rate is formulated as the optimization problem given below, where (5b) and (5c) are the transmit power constraints at the source and relay, respectively.

(P) : \[
\max_{\Lambda_p, W_s, W_r, Q_r} R(\Lambda_p, W_s, W_r),
\]

s.t. \(\text{tr}(W_s) \leq P_s\),

\(\text{tr}(W_r) \leq \text{tr}(\Lambda_p Q_r H W_s H^{*} Q_r^{*})\),

\(W_s \succeq 0, \ W_r \succeq 0\).

(5a) (5b) (5c) (5d)

The variables to be optimized here are splitting ratio $\Lambda_p$, precoding matrices $W_s$ and $W_r$, and combining matrix $Q_r$. Objective function $R(\Lambda_p, W_s, W_r)$ is the end to end transmission rate, as defined in (4).

Problem (P) can be simplified by solving for the transmit and receive beamforming matrices analytically, leaving only the precoding power allocation and power splitting as variables. Considering the term $Q_r H W_s H^{*} Q_r^{*}$ which appears in both (5) and (5c), it can be easily seen that without affecting constraint (5b), since $\Lambda_p$ is a diagonal matrix, the S-R rate in (4) and harvested power in (5c) are simultaneously maximized if $Q_r = U_r^{*}$ and $W_s = V_H A_s V_r^{*}$, where $V_H$ is obtained from the Singular Value Decomposition (SVD) of the channel matrix $H$, and $\Lambda_s = \text{diag} (p_1 \cdots p_{K_1})$ represents source precoding power allocation [11]. For matrix $W_r$ which appears in both (5a) and (5c), we note that both expressions in (5c) are independent of the eigenvectors of $W_r$. We can therefore choose these eigenvectors to maximize the R-D rate in (5a) alone, which is then just waterfilling with the optimal solution as $W_r = V_G A_r V_G^{*}$ and $\Lambda_r = \text{diag} (q_1 \cdots q_{K_2})$, where $V_G$ is obtained from the SVD of $G$. Here $K_1$ and $K_2$ are the number of active channels corresponding to the non-zero singular values of $H$ and $G$, respectively.

We can now rewrite the optimization problem (P), as the equivalent problem (P-eq) given below.

(P-eq) : \[
\max_{R, p, p, q} R
\]

s.t. \(R \leq \frac{1}{2} \sum_{i=1}^{K_1} \log_2(1 + (1 - \rho_i)\lambda_{H,i})\)

(6a)

\(R \leq \frac{1}{2} \sum_{i=1}^{K_2} \log_2(1 + q_i \lambda_{G,i})\)

(6b)

\(\sum_{i=1}^{K_1} p_i \leq P_s\)

(6c)

\(\sum_{i=1}^{K_2} q_i \leq \rho_i \lambda_{H,i}\)

(6d)

Here $\lambda_{H,i}$ and $\lambda_{G,i}$ are the eigenvalues for the S-R and R-D channels respectively. Implicit constraints not mentioned here are $p_i, q_i \geq 0$ and $0 \leq p_i \leq 1$ which are later imposed as boundary conditions in Algorithm 1. (P-eq) is equivalent to (P) but is much simpler as the variables are only the power splitting ratios and the power allocation factors. These variables are coupled via the power harvesting constraint (6d), which complicates the optimization problem and is not immediately obvious as a convex constraint. We show in the next lemma that the equivalent problem is convex.

Lemma 1. The optimization problem (P-eq) is jointly convex in the optimizing variables.

Proof. It can be easily seen that the objective function and constraints (6b), (6c) are affine. To establish the joint convexity for (6a) and (6d) respectively, we define $g_1(p_1, p_i) = 1 + (1 - \rho_i)\lambda_{H,i}p_i$ and $g_2(p_1, p_i) = \rho_i\lambda_{H,i}p_i$. The Hessians for $g_1$ and $g_2$ are indefinite, however, applying the implicit constraints; $\rho_i, p_i \geq 0 \implies \text{dom} \equiv \mathbb{R}_+^2$, and the super-level sets are convex which makes $g_1$ and $g_2$ quasi-concave [12]. The composition function, $f = h \circ g_1$ in constraint (6a), of the
Algorithm 1 Solution for Rate Optimization Problem

**Given:** Channel Matrices, $H, G$, Precision, $\epsilon_0$

**Initialize:** Dual variables, $\alpha, \nu, \mu$. Primal variables, $p, \rho$.

**Begin Algorithm**

- Use closed form expression in (9) to find $\rho_i^*$, if $\rho_i^*$ is $\notin (0,1)$, use boundary conditions,
  - If $\rho_i < 0 \implies$ set $\rho_i^* = \epsilon$
  - Elseif $\rho_i > 1 \implies$ set $\rho_i^* = 1 - \epsilon$, where $\epsilon \to 0, \epsilon \neq 0$
- Waterfill to find $p_i^*$ from (8a) and $q_i^*$ from (8b)
- Check $g(\alpha, \nu, \mu) = \mathcal{L}(p^*, q^*, \alpha, \nu, \mu)$
  - If dual variables converge with required precision, stop
  - Else, use (11) to update dual-variables and continue

**End Algorithm**

non-decreasing function $h(x) = \log(x)$ and $g_1$ preserves quasiconcavity. The problem then is a maximization of a convex function, over convex sets and convex sub-level sets, and is therefore a convex optimization problem. □

IV. **Efficient primal-dual algorithm**

While problem (P-eq) is convex, standard convex solvers can be inefficient due to their inability to exploit specific problem structure. Existing algorithms which fix the power splitting ratio and solve for the precoding power allocation only [7] are no longer applicable since the power splitting ratio in this non-uniform case is a vector rather than a single variable. Thus we proceed with our analysis to design a specialized primal-dual algorithm for jointly solving for the power splitting ratios and the precoding power allocation.

A. **Dual Problem Formulation**

Let $\alpha, \beta, \nu, \mu$ the dual variables associated with constraints (6a)-(6d) respectively. By forming the Lagrangian and taking the derivatives with respect to $R$, we can easily show that $\beta = 1 - \alpha$. Thus the Lagrangian for (P-eq) can be written as

$$
\mathcal{L}(\rho, p, q, \alpha, \nu, \mu) = \alpha R_1 + (1 - \alpha) R_2 - \nu \left( \sum_{i=1}^{K_1} p_i - P_s \right) - \mu \left( \sum_{i=1}^{K_2} q_i - \sum_{i=1}^{K_1} \rho_i p_i \lambda_{H,i} \right)
$$

(7)

Here $R_1$ and $R_2$ are the rates of the first and second hop as given on the right hand side in (6a) and (6b) respectively. The Lagrange dual function of (P-eq) can be defined as $g(\alpha, \nu, \mu) = \max \mathcal{L}(\rho, p, q, \alpha, \nu, \mu)$ with the dual problem defined as P-Dual $= \min_{\alpha, \nu, \mu \geq 0} g(\alpha, \nu, \mu)$. Since the problem is convex, and Slater’s condition is satisfied, the duality gap is zero, which implies that the primal and dual variables can be jointly solved for as a primal-dual pair, $(p_i^*, q_i^*, \rho_i^*, \alpha^*, \nu^*, \mu^*)$.

B. **Optimal Solution**

Next, we solve the KKT conditions based on the Lagrangian in (7) to obtain closed form expressions for the primal variables in terms of the dual variables and then design an efficient primal-dual algorithm utilizing these expressions.

**Theorem 1.** The optimal power allocation, $p$ and $q$, and the optimal power splitting ratio, $\rho$, can be obtained in closed-form in terms of the dual variables as

$$
\rho_i^* = \left( \frac{\alpha}{2\nu - 2\mu \rho_i \lambda_{H,i}} - \frac{1}{(1 - \rho_i) \lambda_{H,i}} \right)^+ \tag{8a}
$$

$$
q_i^* = \left( \frac{1 - \alpha}{2\mu} - \frac{1}{\lambda_{G,i}} \right)^+ \tag{8b}
$$

$$
p_i^* = \left[ 1 - \frac{1 - \rho_i \lambda_{H,i}}{\mu(2\mu - 1)} \right]_0^1 \tag{9}
$$

**Proof.** Obtained by setting $\nabla p_i \mathcal{L} = 0$, $\nabla q_i \mathcal{L} = 0$ and $\nabla \mathcal{L}_{p_i} = 0$, respectively. Details omitted due to space limitation. Here $(x)^+ = \max(x,0)$, and $[x]_0^1 = \max(\min(x,1),0)$ to ensure implicit constraints $p_i, q_i \geq 0$ and $0 \leq \rho_i \leq 1$ respectively. □

**Remark 1.** We see varying power levels for $p_i$ in (8a) in terms of the channel eigenmodes and PS ratios, but constant "water-level" for $q_i$ in (8b) in terms of the dual variables.

**Remark 2.** In the expression for $\rho_i^*$ in (9), we see that the term $\frac{1}{2\mu - 1}$ is not constant, therefore the optimal PS ratio is non-uniform. For uniform PS scheme in $V$, we use $\rho_i = \rho \forall i$.

**Remark 3.** For the case of CSI available at the receivers only (CSIR-only), we assume that the relay has knowledge of the R-D eigenvalues, $\lambda_{G,i}$, via feedback or estimation [13]. In this case, power allocation in (8a) and (8b) becomes equal (average) allocation as $p = P_s/N_s$ and $q = P_r/N_r$. [10]. The PS ratio in (9) then changes to the expression below and Algorithm 1 applies using these new update equations.

$$
\rho_i^* = \left[ 1 - \frac{P_s}{P_r \lambda_{H,i}} \left( \frac{\alpha}{2\mu} - 1 \right) \right]_0^1 \tag{10}
$$

C. **Primal Dual Algorithm**

Using the optimal primal solution in Theorem 1, we obtain the dual function, $g(\alpha, \nu, \mu)$. The problem then reduces to minimizing the dual function in terms of the dual variables. We can use a subgradient based method to solve for this minimization problem, with the subgradient terms given as

$$
\Delta \alpha = \frac{1}{2} \sum_{i=1}^{K_1} \log_2(1 + (1 - \rho_i)p_i \lambda_{H,i}) - \frac{1}{2} \sum_{i=1}^{K_2} \log_2(1 + q_i \lambda_{G,i})
$$

$$
\Delta \nu = P_s - \sum_{i=1}^{K_1} p_i \quad \Delta \mu = \sum_{i=1}^{K_1} \rho_i p_i \lambda_{H,i} - \sum_{i=1}^{K_2} q_i \tag{11}
$$

Our algorithm works as follows. At each iteration, we use the shallow cut ellipsoid method [12] to solve for the dual variables based on (11), then use Theorem 1 to update the primal variables using the current dual variables and the primal variable values from the previous iteration. From the expressions for $p_i^*$ and $q_i^*$ in (8a) and (9), we see that they are interdependent, therefore, for the first iteration only we initialize the algorithm with $p_i,0 \geq 0$, and $0 \leq \rho_i,0 \leq 1$. 

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As the dual variables reach their optimal values, the primal variables also converge to their respective optimal values by strong duality. The algorithm is summarized in Algorithm 1.

V. NUMERICAL RESULTS AND ANALYSIS

In this section, we show the performance of our proposed method, with respect to convergence and throughput, for optimal transmission in a MIMO system. For simulations, path loss exponent $\gamma = 3.2$, noise floor $N_0 = -100$dBm; $d_{sr} = 2m$ and $d_{sd} = 10m$ are the distances between S-R and R-D respectively. Numerical results are averaged over 5000 independent channel realizations of $H$ and $G$.

We compare our proposed primal-dual algorithm with the convex standard solver CVX and existing sequential methods as in [7] for an NxNxN MIMO system(s) where $N_s = N_r = N_d = N$. Figure 1 shows the stark difference in the convergence time of our proposed algorithm and the other two. The gain over CVX comes from the use of ellipsoid algorithm in ours with complexity $O(n^2)$ in contrast to the heuristic successive approximation method of CVX which is several times slower due to its iterative approach, and has a complexity cost of $O(n^3)$ [14]. The gain over the sequential approach in [7] comes from the fact that this approach solves for both hops separately as a split problem, and uses an iterative grid search to find the uniform power splitting ratio.

Figure 2 shows a performance comparison of the optimal non-uniform power splitting scheme as proposed with perfect CSI and an adapted algorithm for the case of CSIR-only as per Remark 3, to the uniform power splitting scheme. We see that for $-5$dBm < $P_s \leq 20$dBm, the CSIR-only scheme using non-uniform power splitting performs better than the uniform power splitting scheme with perfect CSI, while non-uniform PS with perfect CSI outperforms the uniform PS scheme for all values of $P_s$. We see only around a $25\%$ decrease in rate from the perfect CSI case with non-uniform power splitting to the CSIR-only scheme which may justify the practical approach of having CSI at the receiving nodes only.

VI. CONCLUSION

In this letter we formulated a novel optimization problem and designed an efficient algorithm to maximize the achievable rate of a wirelessly powered MIMO relay system. Numerical analysis showed that our proposed algorithm is efficient and runs faster than the CVX solver and existing iterative algorithms by several orders of magnitude. Further comparison with standard uniform power splitting revealed that non-uniform power splitting achieves significantly higher rate especially at low source transmit power. Rate evaluation for the CSIR-only with non-uniform power splitting scheme also shows promising performance with only small rate decrease compared to the perfect CSI case.

REFERENCES

[1] J. Qiao, H. Zhang, F. Zhao, and D. Yuan, “Secure transmission and self-energy recycling with partial eavesdropper CSI,” IEEE Journal on Selected Areas in Communications, pp. 1–1, 2018.
[2] M. T. Mamaghani, A. Kuhestani, and K. Wong, “Secure two-way transmission via wireless-powered untrusted relay and external jammer,” IEEE Transactions on Vehicular Technology, pp. 1–1, 2018.
[3] M. Mohammadi, H. A. Suraweera, G. Zheng, C. Zhong, and I. Krikidis, “Full-duplex mimo relaying powered by wireless energy transfer;” in 2015 IEEE 16th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), June 2015, pp. 296–300.
[4] K. Xu, Z. Shen, X. Xia, Y. Wang, and D. Zhang, “Hybrid time-switching and power splitting swipt for full-duplex massive mimo systems: A beam-domain approach,” IEEE Transactions on Vehicular Technology, pp. 1–1, 2018.
[5] A. Alsharoa, H. Ghazzai, A. E. Kamal, and A. Kadri, “Optimization of a power splitting protocol for two-way multiple energy harvesting relay system,” IEEE Transactions on Green Communications and Networking, vol. 1, no. 4, pp. 444–457, Dec 2017.
[6] R. Zhang and C. K. Ho, “Mimo broadcasting for simultaneous wireless information and power transfer,” IEEE Transactions on Wireless Communications, vol. 12, no. 5, pp. 1989–2001, May 2013.
[7] F. Benkhelifa, A. S. Salem, and M. S. Alouini, “Rate maximization in mimo decode-and-forward communications with an eh relay and possibly imperfect csi,” IEEE Transactions on Communications, vol. 64, no. 11, pp. 4534–4549, Nov 2016.
[8] J. Liao, M. R. A. Khandaker, and K. K. Wong, “Energy harvesting enabled mimo relaying through power splitting,” in 2016 IEEE 17th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), July 2016, pp. 1–5.
[9] S. Ulukus, A. Yener, E. Erkip, O. Simeone, M. Zorzi, P. Grover, and K. Huang, “Energy harvesting wireless communications: A review of recent advances,” IEEE Journal on Selected Areas in Communications, vol. 33, no. 3, pp. 360–381, March 2015.

[10] A. Gamal and Y. Kim, Network Information Theory. Cambridge University Press, 2011.

[11] R. Horn, R. Horn, and C. Johnson, Matrix Analysis. Cambridge University Press, 1990. [Online]. Available: https://books.google.com/books?id=PYQN0ypTwEC

[12] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge University Press, 2004.

[13] S. Srinivasa and S. A. Jafar, “Vector channel capacity with quantized feedback,” in IEEE International Conference on Communications, 2005. ICC 2005. 2005, vol. 4, May 2005, pp. 2674–2678 Vol. 4.

[14] M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming, version 2.1,” http://cvxr.com/cvx, Mar. 2014.