Additional broader I=0 states in the KN channel near Θ+(1540) are expected in many models, making the absence of any signature in the K⁺-deuteron scattering data even more puzzling.

In an ideal “three-body” picture the Θ is viewed as two compact ud(1)ud(2) ¯s color diquarks and an ¯s quark. A “QCD-type” inequality involving \( m(\Theta^+)\), \( m(\Lambda)\), the mass of the \( \Lambda(1/2^-) \) L=1 excitation and that of a new I=0 tetraquark vector meson then follows. The inequality suggests a very light new vector meson, and is violated.

We note that “associated production” of the pentaquark with another quadriquark or anti-pentaquark may be favored. This along with some estimates of the actual production cross sections suggest that the Θ can be found in BaBar or Belle e⁺-e⁻ colliders.

PACS numbers:

I. Introduction and first comment

The low mass and narrow width of the recently discovered[1]-[8] Θ(1540) pentaquark were anticipated to a certain extent in some chiral soliton models.[9],[10] Yet, despite the extensive research using more conventional QCD quark models,[11]-[17] these remain puzzling features.

This is particularly the case for the narrow width. The evidence comes from two sources: (i) directly from the measured width of the Θ peak in the K⁺n or K⁰p invariant mass distribution, and (ii) indirectly from the lack of evidence for resonances in K⁺d scattering data. The bounds on the width from (i) cannot be lower than the resolution which in most experiments to date is no better than O(10MeV);

\[
\Gamma(\Theta)|\text{(direct observations)}\leq 10\text{MeV}
\]

The indirect bounds[13],[18]-[21] use the fact that KN elastic and charge exchange resonant cross sections have a Breit-Wigner shape with fixed normalization. Hence, in older, low energy K-deuteron data, a K⁺N resonance due to the Θ should manifest as an enhancement of the cross section \( \delta(\sigma) \). As long as the intrinsic \( \Gamma(\Theta) \) is smaller than the broadening due to the Fermi motion of the neutron inside the deuteron, which is clearly the case here, \( \delta(\sigma) \) is proportional to \( \Gamma(\Theta) \).

The analyses vary in sophistication and the scope of experimental information used suggesting:

\[
\Gamma(\Theta)|\text{(indirect bounds)} < 1 - 3\text{MeV}
\]

To explain the small widths we need quark (or other) models where the Θ has a small overlap with the KN decay channel.

It was recently suggested[22] that mixing of (1/2)⁺ states via the KN continuum yields physical states of which one—the Θ(1540)—is anomalously narrow while the other states have “normal”, say, 50-200 MeV widths. Additional states in the \( I = 0 \) KN channel in the 300 MeV range above the Θ(1540) are generic. Indeed the spectrum in the more complex five-quark system is expected to be denser than in the corresponding non-exotic three quark baryonic system. It is instructive to see how these general expectations are realized in specific models. In the model[14],[15] with one (ud) diquark and an (¯sud) compound, the ¯s tunnels between the two ud diquarks. This yields a lowest symmetric (1/2)⁺ state and a slightly higher antisymmetric (1/2)⁻ state. In the two-diquark model, the two I=S=0 diquarks are in a relative orbital angular momentum, L(12)=1[11]-[13]. Coupling this orbital angular momentum with the spin-half of the ¯s antiquark yields the J=1/2 Θ and a slightly higher J=3/2 state. The \( \vec{L} \cdot \vec{S} \) (spin orbit) splittings in ordinary hadrons suggest a splitting of less than 250 Mev between these two states.[16] We assumed all along that
the Θ is a positive parity spin-half particle. However, lattice and some early quark model calculations[16] suggest that we have all quarks in an S wave yielding a (1/2)− state. Thus we could also have a splitting smaller than 300 MeV between the two states of opposite parity.

A trivial yet key observation is that the indirect bounds (ii) apply not only to the putative Θ(J=1/2+), but to any resonance in the KN channels. Hence, not only is Γ(Θ) constrained, but also the width of any other I=0 state which couples to the KN channel in the mass range of 1540-18000 MeV. (I spin forbids the decay of such states into Θ(1540) and a pion maintaining a small inelasticity). These states could be the extra, broader (1/2)+ states invoked[22] to explain the narrow width of Θ or any one of the higher states discussed above. Thus, let us consider the Θ(3/2)+. The resonant cross section is: \( \sigma(KN) = (2J+1)/p(cm)^2 \). The higher 3/2 spin makes it roughly the same as the resonant KN cross sections due to Θ J=1/2 at a lower p(cm). Hence, the width of a J=3/2 state in the 300 MeV mass range above the Θ should, like that of the Θ(1540), be bound by

\[
\Gamma(\Theta(J(P) = (3/2)^+) < 1 - 3 MeV
\]

This new indirect bound (NIB) is even more problematic than the original IB on Γ(Θ J=1/2): First, the centrifugal barrier factor, \( \sim p(cm)^3 \), suggests that the higher Θ(3/2+) state is much broader. Second, the Θ(1540) corresponds to P[K](Lab) 440 MeV near the lower limit of all past measurements, whereas the higher states would manifest at higher kaon momenta where more detailed and accurate data from several different K+d experiments are available.

Very broad, say, \( \Gamma > 200 \) MeV, resonances may escape detection in experiments looking for bumps in invariant (missing) K-N mass distributions. However, these I=0 resonances reflect in K+d scattering. The total cross section over the relevant of (p(Lab)= 500-900 MeV) range should increase from \( \sigma(K+d) \sim 2\sigma(K^+p) \sim 25 \) mb by the huge amount of \( \sigma \) (resonance) \( \sim 30 \) mb. Thus the impact of the K+d data on pentaquark modeling is far-reaching: Not only should the Θ(1540) be very narrow, similar bounds apply to the width of any I=0 K-N resonance in the 1540-1800 MeV region.

II. Second comment: A “strict” point-like diquark-diquark \( \bar{s} \) picture of the Θ conflicts with “QCD inequalities”

The simple appealing model of the Θ(1540) as two I=S=0 ud diquarks carrying a \( \bar{3}(c) \) of color each and an \( \bar{s} \)[11–13] maximizes the attractive HF interactions between the lighter, nonstrange quarks. The two diquarks with \( \bar{3}(c) \) couple anti-symmetrically in color to a \( 3(c) \) so as to form an overall color singlet with the remaining \( \bar{s} \) antiquark. Bose statistics for the spinless identical diquarks then implies an odd relative orbital angular momentum, L(1,2), between the two diquarks so as to ensure overall symmetry. The lowest odd angular momentum, L=1, coupled with the spin-1/2 of \( \bar{s} \) to the likely lower energy J=1/2 state, yields a Θ(1540) with spin/parity (1/2)+ just as in the chiral soliton models. The centrifugal barrier due to the orbital angular momentum also helps understanding the small Θ width. (“tensor” diquarks with \( \bar{6}(c) \) of color were considered in Ref. [23]). However, the L=1 relative angular momentum generates extra kinetic energy and the dq(1) dq(2)\( \bar{s} \) configuration may not be as light as 1540 MeV. We next argue that this is indeed the case if we treat the diquarks as elementary scalars carrying 3\( \bar{c} \) color.

The Θ pentaquark is then analogous to an (anti-) baryon with two among the three antiquarks replaced by scalar diquarks, dq(1) and dq(2), with the same 3\( \bar{c} \) color. The pair-wise interaction among the three colored objects in the baryon or in the idealized pentaquark model has two parts with \( \vec{X} \cdot \vec{X} \) (or \( 1^i \cdot 1^i \)) color structure. We will assume that the first class of interactions dominate (as is the case in the large \( N_c \) limit). As indicated in Ref. [24], we can then generate the three-particle wave function—by charge conjugating one particle at a time—trial wave functions for three mesonic two-body states. The variational principle yields inequalities, between \( m(0)[B(i,j,k)] \) and \( m(0)[M(i,j)] \), etc., the masses of the lightest baryon mesons with the specific i,j,k flavors indicated are:

\[
2 \cdot m(0)[B(i,j,k)] \geq m(0)[M(i,j)] + m(0)[M(j,i)] + m(0)[M(j,k)]
\]

Rigorous inequalities \( m(N) > m(\pi) \) [25] and \( m(N) > 3/2m(\pi) \) [26] were obtained using the path integrals for Euclidean qqq(x)qqq(y) (baryonic) and \( q\bar{q}(x)q\bar{q}(y) \) (mesonic) correlators. Table (ii) in the recent review of QCD
inequalities\cite{27} shows that the above more detailed flavor-dependent inequalities hold with a substantial 200-500 MeV margin in all testable cases.

The same reasoning yields in the present case:

\[ 2 \cdot m(\Theta(1540)) = 2 \cdot m(0)(ud, ud, \bar{s}) \geq a \cdot m(A_4(1/2)^+) + (2 - a) \cdot m(A_4(1/2)^-) + m(0)[(ud)(\bar{u}\bar{d})(J(P) = 1^-)] \]  
(5)

The baryon meson mass inequalities can also be derived in a QCD string/flux tube picture for the baryons and mesons with just one junction point in the baryons\cite{24},\cite{27}. A similar description with two \((JJ)\) and three \((J,J,J)\) junction points and minimal \(3(c)\) (or \(3(c)\)) chromoelectric flux lines has been suggested for a putative exotic tetraquark and the pentaquark. \cite{28},\cite{29}. It is straightforward to verify that the derivation of the new inequality holds in such a scenario as well.

The Lorentz quantum numbers of the states on the right-hand side of Eq. \(5\) are inferred from the pentaquark state. The \(I=0\) vector mesons are the well-known ud(1) and ud(2) diquarks inside the \(\Theta\) have relative orbital angular momentum \(L(1,2) = 1\). Hence, \(\Theta\) is a \(I=S=0\) pentaquark. \cite{28},\cite{29}. It is straightforward to verify that the derivation of the new inequality holds in such a scenario as well.

Using \(m(\Theta(1540))\) and \(m(\Theta(1540))\) instead of \(m(\Lambda(1/2)^+)\) and \(m(\Lambda(1/2)^-)\) would have been detected in \(e^+e^- \rightarrow 3\pi\) reactions. Hence, the last \(m(0)[(ud)(\bar{u}\bar{d})(J(P) = 1^-)] = m(Tq(1^-))\) value in Eq. \(5\) is larger than 1020. Also, \(L[\text{dq}(1),\text{dq}(2)] = 1\) excludes having both \(\text{dq}(1)\) and \(\text{dq}(2)\) in an \(L=0\) state relative to the remaining \(\bar{s}\). We find that the probability of having inside the \(\Theta\) \(\text{dq}(1) - \bar{s}\) and \(\text{dq}(2) - \bar{s}\) in an \(L=1\) state exceeds 50\%, i.e., \(a < 1\) in Eq. \(5\) above. Using \(m(\Lambda) = 1115\) MeV and 1405 MeV for the mass of \(\Lambda(1/2^-)\) — the first negative parity \(L=1\) excitation — we find that the inequality is strongly violated: 2 \(\cdot 1540\) MeV is smaller than \((1115+1405+1020)\) MeV by 450 MeV rather than larger by \(\sim 200-400\) MeV, the margin with which the meson baryon inequalities were satisfied.

Obviously this does not exclude a light pentaquark \(\Theta(1540)\). Rather the extreme version of the two diquark model may be wrong. Treating the diquarks as idealized point-like scalars omits significant hyperfine attractive interactions between the \(\bar{s}\) and each of the four quarks. (The latter were incorporated via constructing the \(u - \bar{s} - d\) aggregate in Refs. \cite{14},\cite{15}).

Still, the above suggests that a light pentaquark is accompanied by light (crypto) exotic \(dq - \bar{d}q\) tetraquarks.

Heavier flavor analogs of \(\Theta(1540)\) with \(\bar{c}/\bar{b}\) replacing \(\bar{s}\) were considered by many authors. The smaller HF interactions with the heavier \(\bar{Q}\) suggest that the idealized \(dq(1)dq(2)\bar{Q}\) picture and the analog of the inequality \(5\) are more applicable here. The latter reads:

\[ 2m(\Theta(c)) > m[\Lambda(c)1/2^+] + m[\Lambda(c)1/2^-] + m(Tq(1^-)) \]  
(6)

with \(Tq(1^-)\) the vector tetraquark state encountered above. Using 2.285 GeV for the mass of \(\Lambda(c)\), 2.593 GeV for its \(L=1\) excitation, and \(m(Tq(1^-)) > 1020\) as above we find that \(m(\Theta(c)) > 2.95 > m_D + m_N = 2790\). Thus \(\Theta(c)\) is likely to be unstable decaying into a D meson + Nucleon.

Possible further difficulties encountered in explaining \(N^*\) widths in an SU(3) flavor extended version of the \(dq\bar{d}q\bar{q}\) picture have also been noted.\cite{30} This is hardly surprising as the \(dq(1)\)\(dq(2)\) picture is even more questionable if we replace the \(\bar{s}\) by even lighter \(\bar{u}/\bar{d}\).

III. Third comment: “Associated Production of Exotics” and some crude estimates of \(\Theta(1540)\) production in \(e^+e^-\) colliders

The \(\Theta(1540)\) pentaquark has been discovered in many experiments. The early searches in \(K^+\)-deuteron mentioned in Sec. I and \(e^+e^-\) machines are two notable exceptions.

Recently new charmed states spectroscopy and a crypto-exotic \(cc\bar{q}\bar{q}\) mesons have been discovered in BaBar and Belle. These \(e^+e^-\) accelerators primarily investigate CP violation and b-quark physics. However, with 4\(\pi\) coverage, precise momentum measurements and good particle identification, the O(10\(^9\)) events involving lighter primary quarks collected there are a unique “hadronic treasure trove”.
For $\Theta(\bar{s}uudd)$ production the $e^+e^-$ initial state is the opposite of K+n collisions where all the requisite four quarks and $\bar{s}$ are initially present. To produce $\Theta$ (or $\bar{\Theta}$) in

$$e^+e^- \rightarrow \Theta + X \quad (7)$$

we need to create $udud$ and $\bar{s}$ and their anti-particles. At the BaBar/Belle cm energy of $W=\sim11$ GeV many quark pairs are eventually produced. However, $\Theta$ is an aggregate of specific five quarks within $O$(Fermi) spatial proximity and with small relative momenta. Thus the production rate can be strongly suppressed reducing the sensitivity of searches for $\Theta$ in $e^+e^-$ collisions.

This seems to be even more the case for “double”, i.e., $\Theta\Theta$ pair production:

$$e^+e^- \rightarrow \Theta + \bar{\Theta} + X \quad (8)$$

Our third comment is that this need not be the case. The $\Theta(1540)$ is the lightest member of a novel family of exotic hadrons. Once being produced, the latter hadrons will often decay into the ground state $\Theta$. We next argue that joint “associated” production of the new exotics is likely and the fraction of all hadronic events with double $\Theta$ production (DTP) far exceeds the square of the fraction of single $\Theta$ production events (STP).

Building the complicated pentaquark structure with all the specific quarks and their correlations starting with the hadronic vacuum and using the local flavor and color conserving QCD interactions, we simultaneously produce a “mirror” set of antiparticles with the same color, spin and space, etc., correlations. Both sets will, in general, yield very excited pentaquark configurations, say, of the above $dqdq\bar{s}$ form. If de-excitation occurs via pion or the $\eta$ (but not via kaon emission!)—which can happen in a substantial fraction of the cases—the ground state $\Theta(1540)$ stays in the final state. Thus while, as we show below, single $\Theta$ production (STP) is heavily suppressed, the even more striking events where we have two $\Theta$’s (DTP) produced in the same $e^+e^-$ collision may be only moderately further suppressed relative to (STP).

We next present some rough estimates of these rates. At $W \sim 10$ GeV in $e^+e^-$ collisions, the initial hard short-distance process yields mainly just a quark and anti-quark ($u\bar{u}$ and $d\bar{d}$ in about half the cases). We will use the chromoelectric flux tube model developed for the soft hadronization process in this case.[31] In this model a constant chromoelectric field is generated inside a tube of fixed diameter between the the separating initial quarks. Tunneling within this field yields extra $q\bar{q}$ pairs and eventually multiple mesons. The tunneling rate is proportional to $E^2\exp(-(\pi m^2)/g_s E)$ with $m$ the quark mass and $E$ the constant chromoelectric field. To produce a $\Theta$ pentaquark specific deviations from the generic scenario for multiple meson production are required: Focusing on the quark side we need first to produce a $d$ or $u$ quark which couples with the primary hard quark to a $\bar{s}$, so as to make a $ud\bar{s}$ cluster with overall $\bar{3}$ color. The production of such an $\bar{s}$ is suppressed not only by $\epsilon$ due to the reduced field, but due to the higher $s$ mass the overall suppression factor is $\epsilon/9 \sim 1/100$. The chromoelectric field driving the tunneling here has half the value as in multi-meson production. The suppression due to $m_s^2 > m_q^2$ is here 9; the square of the factor 3 suppression for $s$ quark production relative to $u/d$ in the usual multi-meson production case,

The last two steps of generating an excited $\Theta$-like state by creating another ud and du quark are analogous to those in baryon production with an $\epsilon/10$ suppression. Collecting all factors we expect one “Primary” pentaquark production event in $2 \cdot (e^{-3.9 \times 2 \cdot 10^4 e^+e^-}$ collisions. The primary excited pentaquark may decay via kaon emission to some excited $N^*$, in which case we will not find the $\Theta(1540)$ in the final state. However, we expect that for an appreciable fraction, $f \sim 0.5$, of all primary pentaquarks this will not be the case and:

$$\text{Probability of } \Theta(1540) \text{ production } \sim 5 \cdot 10^{-5} \cdot f \sim 2.5 \cdot 10^{-5}.$$ 

A key observation is that the probability of producing two $\Theta(1540)$ particles in the same event is not the square of the last small probability. Because our production mechanism naturally yields two excited pentaquarks, all we need is for both to decay into the ground state pentaquark and therefore:

$$\text{Probability of } \Theta(1540) \text{ production } \sim 5 \cdot 10^{-5} \cdot f \sim 2.5 \cdot 10^{-5}.$$ 

A key observation is that the probability of producing two $\Theta(1540)$ particles in the same event is not the square of the last small probability. Because our production mechanism naturally yields two excited pentaquarks, all we need is for both to decay into the ground state pentaquark and therefore:
Probability of $\Theta(1540) + \bar{\Theta}(1540)$ production $\sim 5 \cdot 10^{-5} \cdot f^2 \sim 10^{-5}$.

Since the $\Theta(1540)$ decays into $K^0 p$ in half the cases, and we further have a 1/3 reduction from the demand that the $K^0$ will decay as $K_S$ into charged (rather than neutral pion pair) the requirement of detectability reduces $f$ to an “effective $f'' \sim f/6$. This suggests a probability of $\sim 2 \cdot 10^{-6}$ for detecting a $\Theta(1540)$ per collision.

While these probabilities are pretty small, the large number of collisions may still yield a significant signal. We also note that if the $\Theta(1540)$ signal at Zeus and, in particular, the appearance of approximately one hundred $\Theta(1540)$ (which cannot be proton fragments anyway) in four million e-proton collisions are real, then the last probability is, in fact, $2.5 \cdot 10^{-5}$—consistent with our estimate!

The CFT model can be applied to hadron production and to pentaquark production, in particular, also off hadronic targets.

Thus in $\gamma$ proton collision we can consider the struck $u$, say, quark separating from the ud diquark instead of the initial state of, say, $\bar{u} - u$ in $e^+ e^-$ collisions.

Following similar steps as above we see that the production of the pentaquark requires one less step and, hence, we gain a $1/\epsilon \sim 10$ in production strength so that we expect $\Theta(1540)$ to be produced in $10^{-4}$ of all cases. Also, the “mirror” exotic that will be generated in this case with the pentaquark will be a strange tetraquark $(\bar{s} \bar{u})(ud)$.

We note that neither expectation seems to be born out: first the accompanying hadrons were the ordinary non-exotic kaon, or $K^*$. Finally, the pentaquark production rate is considerably larger in this case than our estimates.

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I would like to thank Marek Karliner for asking me some time ago if QCD inequalities can be applied to the pentaquark, and to Ralf Gothe for helpful discussions.

I enjoyed discussing aspects of pentaquark physics with Tom Cohen. I did not, however, take his (sound!) advice and write a special purpose paper with only one single motif—running the risk that many readers will note, as in the first case[13], only one of the three comments...

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