The one-loop renormalization of the MSSM Higgs sector and its application to the neutral scalar Higgs masses

A. Dabelstein

Institut für Theoretische Physik
Universität Karlsruhe
Kaiserstr. 12
D-76128 Karlsruhe, Germany

Abstract

The structure of the Higgs sector in the minimal supersymmetric standard model is reviewed at the one-loop level. An on-shell renormalization scheme of the MSSM Higgs sector is presented in detail together with the complete list of formulae for the neutral Higgs masses at the one-loop level. The results of a complete one-loop calculation for the mass spectrum of the neutral MSSM Higgs bosons and the quality of simpler Born-like approximations are discussed for sfermion and gaugino masses in the range of the electroweak scale.

* Supported in part by the European Union under contract CHRX-CT92-0004
1 E-Mail: ADD@DMUMIWH.BITNET
1 Introduction

The Higgs boson of the standard model is the last unobserved particle since the recent results from FERMILAB indicate that the top quark mass is rather heavy [1]. In spite of the success of the standard model, there are also good reasons to consider "new physics" beyond the standard model. One of these theoretical motivations is the appearance of quadratically divergent contributions to the mass of the scalar Higgs particle, so that the Higgs couplings become strong at the TeV scale.

This problem of naturalness is solved in supersymmetric theories. The minimal supersymmetric standard model (MSSM) is the supersymmetric extension of the standard model, with a 2-Higgs doublet sector, where the coefficients of the Higgs potential are restricted by supersymmetry [2].

For the experimental search it is crucial to have precise predictions for the properties of the Higgs bosons under inclusion of radiative corrections, i.e. the characteristic production and decay channels of the MSSM Higgs particles at $e^+e^-$ and $pp$ colliders. These signatures may allow to distinguish between a Higgs sector of different origins.

As a result of the supersymmetric Higgs potential, a light Higgs boson exists with a tree level upper mass bound given by the $Z_0$ mass. Radiative corrections to the Higgs mass spectrum, however, predict an upper limit of the light Higgs mass $\mathcal{O}(130 \text{ GeV})$ [3, 4]. Calculations were performed at the one-loop level using renormalization group technique [4], effective potential approximation [3] and one-loop calculations with top and stop contributions [7, 8]. Two-loop effects to the upper limit of the lightest Higgs boson mass are discussed in [9].

This article contains a complete on-shell renormalization scheme for the MSSM Higgs sector and points out the different treatments of the renormalization conditions of the vacuum expectation values $v_1, v_2$ in the on-shell scheme [10]. The complete MSSM expressions for the 2-point functions of the MSSM Higgs sector are calculated and formulae are listed in the appendix.

In chapter 4 the calculation of the full one-loop contribution to the physical neutral scalar Higgs masses is discussed. The numerical analysis of the one-loop contribution includes the full parameter space of the MSSM, where the mass range of light sfermions ($> 100 \text{ GeV}$) and gauginos ($> 50 \text{ GeV}$) was analyzed in detail. These one-loop Higgs mass predictions are finally compared with simpler approximate formulae. Deviations are within $\approx 2 - 10 \text{ GeV}$ between the full one-loop calculation and approximation formulae.
2 The Higgs sector of the MSSM

2.1 Tree level structure

The Higgs sector of the MSSM consists of two scalar doublets

\[
H_1 = \begin{pmatrix} H^1_1 \\ H^2_1 \end{pmatrix} = \begin{pmatrix} (v_1 + \phi_1^0 - i\chi_1^0)/\sqrt{2} \\ -\phi_1^0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^1_2 \\ H^2_2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ (v_2 + \phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix},
\]

with opposite hypercharge \( Y_1 = -Y_2 = -1 \) and vacuum expectation values \( v_1, v_2 \). The quadratic part of the potential

\[
V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 + m_{12}^2 (\epsilon_{ab} H^a_1 \bar{H}^b_2 + h.c.) + \frac{1}{8} (g_1^2 + g_2^2) (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 - \frac{g_2^2}{2} |H_1 \bar{H}_2|^2,
\]

with soft breaking parameters \( m_1^2, m_2^2, m_{12}^2 \) and the gauge couplings \( g_1, g_2 \) is diagonalized by the rotations

\[
\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix},
\]

\[
\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix},
\]

\[
\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}.
\]

\( G^0, G^\pm \) describe the unphysical Goldstone modes. The spectrum of physical states consists of

- 2 neutral bosons with CP = 1: \( h^0, H^0 \) ("scalars")
- 1 neutral boson with CP = -1: \( A^0 \) ("pseudoscalar")
- 2 charged bosons: \( H^\pm \).

The masses of the gauge bosons and the electromagnetic charge are determined by

\[
M_Z^2 = \frac{1}{4} (g_1^2 + g_2^2) (v_1^2 + v_2^2), \quad M_W^2 = \frac{1}{4} g_2^2 (v_1^2 + v_2^2),
\]

\[
e^2 = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2}.
\]
Thus, the potential (2.2) contains two independent free parameters, which can con-
veniently be chosen as
\[
\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta),
\]
where $M_A$ is the mass of the $A^0$ boson.

Expressed in terms of (2.3), the masses of the other physical states read:
\[
m_{H^0,H^0}^2 = \frac{1}{2} \left[ M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2M_A^2 \cos^2 2\beta} \right]
\]
\[
m_{H^+}^2 = M_A^2 + M_W^2,
\]
and the mixing angle $\alpha$ in the $(H^0,h^0)$-system is derived from
\[
\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad -\frac{\pi}{2} < \alpha \leq 0.
\]
Hence, masses and couplings are determined by only a single parameter more than in the
standard model.

The dependence on $M_A$ is symmetric under $\tan \beta \leftrightarrow 1/\tan \beta$, and $m_{h^0}$ is constrained by:
\[
m_{h^0} < M_Z \cos 2\beta < M_Z.
\]
This simple scenario, however, is changed when radiative corrections are taken into
account.

### 2.2 Renormalization of the MSSM Higgs sector

It is well known since quite some time that radiative corrections modify the tree level
relations (2.6, 2.7) substantially, with the main effect from loops involving the top quark
and its scalar partner $\tilde{t}$ [3, 6]. Various approaches have been applied:

(i) The effective potential method [3, 7]:

The tree level mass matrix $M_0$ of the neutral scalar system is diagonalized by
(2.3). Loop contributions to the quadratic part of the potential (neglecting the
$q^2$-dependence of the diagrams) modify the mass matrix
\[
\mathcal{M}_0 \rightarrow \mathcal{M}_0 + \delta \mathcal{M} = \mathcal{M}.
\]
Re-diagonalizing the one-loop Matrix $\mathcal{M}$ yields the corrected mass eigenvalues
$M_{H^0,h^0}$, replacing (2.6), and an effective mixing angle $\alpha_{eff}$ instead of (2.7).
(ii) The renormalization group method [5]
Solving the renormalization group equations for the parameters of a general 2-doublet model and imposing the SUSY constraints at the scale $\mu = M_{\text{SUSY}}$ yields the effective parameters of the Higgs potential at the electroweak scale. Large log terms are resummed, but effects from realistic mass spectra are not covered by this approximation.

(iii) Complete one-loop calculation [10, 12]:
A complete one-loop calculation to masses and couplings accommodates all SUSY particles and mass parameters (or soft breaking parameters, respectively) in the radiatively corrected version of (2.6, 2.7) and, in addition, provides the 3-point functions required for Higgs boson production and decay processes. They are necessary to check the quality of the approximations (i), (ii) and allow a detailed study of the full parameter dependence of production cross sections and decay rates. Complete one-loop calculations are available for:
- 2-point functions (mass relations) [10, 12]
- 3-point functions $ZZh(H)$, $ZAh(H)$, $h(H)\gamma\gamma$, ... for production and bosonic decay processes [11].
- 3-point functions $Aff$, $h(H)ff$ for fermionic decay processes [12].
Other one-loop calculations with restrictions to the dominating fermion-sfermion loops have been performed in [8].

Loop calculations in the Higgs sector require an extension of the renormalization procedure applied in the minimal version of the standard model, e.g. the on-shell scheme [13].

At the one-loop level, the free parameters and the fields of the Lagrangian are replaced by renormalized parameters and fields, and a set of counter terms:

\[
\begin{align*}
B_\mu &\to (Z_2^B)^{1/2}B_\mu \\
W_\mu^a &\to (Z_2^W)^{1/2}W_\mu^a \\
\psi_j^L &\to (Z_L^j)^{1/2}\psi_j^L \\
\psi_j^R &\to (Z_R^j)^{1/2}\psi_j^R \\
H_i &\to Z_{H_i}^{1/2}H_i \\
g_2 &\to Z_1^W (Z_2^W)^{-3/2}g_2 \\
g_1 &\to Z_1^B (Z_2^B)^{-3/2}g_1 \\
v_i &\to Z_{H_i}^{1/2}(v_i - \delta v_i)
\end{align*}
\]
\[ m_i^2 \rightarrow Z_{H_i}^{-1}(m_i^2 + \delta m_i^2) \]
\[ m_{12}^2 \rightarrow Z_{H_1}^{-1/2}Z_{H_2}^{-1/2}(m_{12}^2 + \delta m_{12}^2) . \] (2.9)

The renormalization constants \( Z_i \) are expanded by \( Z_i \rightarrow 1 + \delta Z_i \). This transforms the potential \( V \) into a renormalized potential \( V_{ren} \) and a counter term part \( \delta V \):
\[ V \rightarrow V_{ren}(m_i^2, g_1, g_2, ...) + \delta V . \] (2.10)

\( V_{ren} \) is formally identical to (2.2). Expressed in terms of the rotated fields (2.3) the coefficients of the quadratic part
\[ V_{ren} = \frac{1}{2}(m_{h_0}^2 h^2 + m_{H_0}^2 H^2 + m_A^2 A^2 + ...) , \] (2.11)
are those of (2.5, 2.6). The coefficients in the counter term potential
\[ \delta V = \delta t_{h_0} h + \delta t_{H_0} H + \frac{1}{2}(\delta m_{h_0}^2 h^2 + \delta m_{H_0}^2 H^2 + \delta m_A^2 A^2 + ...) \] (2.12)
are linear combinations of the counter terms in (2.9).

The on-shell renormalization conditions can be formulated in terms of self energies, tadpoles and counter terms. In the following the 't Hooft-Feynman gauge is used and only the \( g_{\mu\nu} \) components of the vector boson propagators are considered. The vector boson self energies \( \Sigma_{\gamma,Z,W,\gamma Z} \) are the transversal components of the vector boson propagators \( \Delta_{\mu\nu}^V (V = \gamma, Z, W) \) [13] :
\[ \Delta_{\mu\nu}^V (k) = -ig_{\mu\nu} \left( \frac{1}{k^2 - M_V^2} - \frac{1}{k^2 - M_V^2} \Sigma_{\gamma Z}^V (k^2) \frac{1}{k^2 - M_V^2} \right) \]
\[ \Delta_{\mu\nu}^{\gamma Z} (k) = ig_{\mu\nu} \frac{1}{k^2 - M_Z^2} \Sigma_{\gamma Z} (k^2) \frac{1}{k^2} . \] (2.13)

The scalar boson self energies \( \Sigma_S \) are related to the scalar boson propagators \( \Delta^S \) through:
\[ \Delta^S (k) = i \left( \frac{1}{k^2 - M_S^2} - \frac{1}{k^2 - M_S^2} \Sigma_S (k^2) \frac{1}{k^2 - M_S^2} \right) . \] (2.14)

The fermion self energies \( \Sigma_f \) are defined by the fermion propagator \( S_f \) :
\[ S_f (k) = \frac{i}{k - m_f} - \frac{i}{k - m_f} \Sigma_f (k) \frac{1}{k - m_f} . \] (2.15)
The $A^0-Z^0$ and $A^0-G^0$ mixing $\Sigma_{AZ}$, $\Sigma_{AG}$ are defined by the mixing propagator $\Delta^{A^0Z^0}$, $\Delta^{A^0G^0}$:

$$
\Delta^{A^0Z^0}(k^2) = \frac{1}{k^2 - M_A^2} \Sigma_{AZ}(k^2) \frac{1}{k^2 - M_Z^2}
$$

$$
\Delta^{A^0G^0}(k^2) = \frac{i}{k^2 - M_A^2} \Sigma_{AG}(k^2) \frac{1}{k^2 - M_Z^2}.
$$

(2.16)

In the following renormalized self energies are denoted by $\hat{\Sigma}$. The vector boson self energies, the $A^0$-boson self energy and the $A^0Z^0$ mixing read:

$$
\hat{\Sigma}_Z(k^2) = \Sigma_Z(k^2) - \delta M_Z^2 + \delta Z^2_Z(k^2 - M_Z^2)
$$

$$
\hat{\Sigma}_W(k^2) = \Sigma_W(k^2) - \delta M_W^2 + \delta Z^2_W(k^2 - M_W^2)
$$

$$
\hat{\Sigma}_\gamma(k^2) = \Sigma_\gamma(k^2) + k^2\delta Z^2_2
$$

$$
\hat{\Sigma}_\gamma Z(k^2) = \Sigma_\gamma Z(k^2) + M_Z^2 \frac{c_W}{s_W} (\delta Z_1^2 - \delta Z_2^2) - \left( k^2 \frac{c_W s_W}{c_W^2 - s_W^2} (\delta Z_2^2 - \delta Z_1^2) \right)
$$

$$
\hat{\Sigma}_A(k^2) = \Sigma_A(k^2) - \delta m_A^2 + k^2(\sin^2 \beta \delta Z_{H_1} + \cos^2 \beta \delta Z_{H_2})
$$

$$
\hat{\Sigma}_{AZ}(k^2) = \Sigma_{AZ}(k^2) + M_Z \sin \beta \cos \beta (\delta Z_{H_1} - \delta Z_{H_2})
$$

(2.17)

The mass counter terms $\delta M_Z^2$, $\delta M_W^2$, $\delta m_A^2$ are defined in (2.28) through the fundamental renormalization constants in (2.9). They correspond to the substitution:

$$
\begin{align*}
M_Z^2 &\rightarrow M_Z^2 + \delta M_Z^2 \\
M_W^2 &\rightarrow M_W^2 + \delta M_W^2 \\
M_A^2 &\rightarrow M_A^2 + \delta m_A^2,
\end{align*}
$$

(2.18)

where the unrenormalized masses are replaced by renormalized masses and a mass counter term. The quantities

$$
s_W = \sin \theta_W , \quad c_W = \cos \theta_W
$$

are the short-hand notations for the electroweak mixing angle in the convention $c_W^2 = M_W^2/M_Z^2$ [14].

The renormalized fermion self energy is given as follows:

$$
\hat{\Sigma}_f(k^2) = \kappa \left( \Sigma_f(V,k^2) + \frac{\delta Z_L + \delta Z_R^f}{2} \right) + \kappa \gamma_5 \left( \Sigma_A(k^2) - \frac{\delta Z_L - \delta Z_R^f}{2} \right)
$$

$$
+ m_f \left( \Sigma_f(S,k^2) - \frac{\delta Z_L + \delta Z_R^f}{2} - \frac{\delta m_f}{m_f} \right)
$$

(2.19)
The renormalization conditions for fixing the counter terms consist of:

1) the on-shell conditions in the gauge sector for $M_W$, $M_Z$, the fermion masses $m_f$ and the electromagnetic charge $e$, as in the minimal standard model. They determine the quantities $\delta g_1$, $\delta g_2$ and $\delta v^2 = \delta(v_1^2 + v_2^2)$ due to (2.4). The on-shell conditions for the physical masses $M_Z$, $M_W$, $m_f$ are defined as:

$$
\text{Re } \hat{\Sigma}_Z(M_Z^2) = 0 \\
\text{Re } \hat{\Sigma}_W(M_W^2) = 0 \\
\text{Re } \hat{\Sigma}_f(m_f^2) = 0.
$$

The QED charge renormalization and residue conditions:

$$
\hat{\Gamma}^{\gamma\mu}(k^2 = 0, \not{p} = \not{q} = m_e) = i e \gamma_\mu \\
\text{Re } \frac{\partial}{\partial k^2} \hat{\Sigma}_\gamma(k^2)|_{k^2=0} = 0 \\
\text{Re } \hat{\Sigma}_\gamma(0) = 0 \\
\text{Re } \frac{1}{k^2 - m_f^2} \hat{\Sigma}_f(k^2)|_{k^2=m_f^2} = 0.
$$

The residue condition for $f$ in (2.23) is considered for a fixed isospin component. It can not be set for the upper and lower component of a fermion doublet simultaneously.

2) the tadpole conditions for vanishing renormalized tadpoles:

$$
T_{H^0(h^0)} + \delta t_{H^0(h^0)} = 0
$$

3) the on-shell and residue condition for the $A^0$-boson:

$$
\text{Re } \hat{\Sigma}_A(M_A^2) = 0
$$
\[ \Re \frac{\partial}{\partial k^2} \tilde{\Sigma}_{A^0}(k^2) \big|_{k^2=M^2_A} = 0. \]  \hspace{1cm} (2.25)

By (2.24) \( \delta m_A^2 \) is determined in terms of the on-shell self energy of \( A^0 \).

4) renormalization of \( \tan \beta \):

The relation \( \tan \beta = v_2/v_1 \) in terms of the "true vacua" is maintained by the condition

\[ \frac{\delta v_1}{v_1} = \frac{\delta v_2}{v_2}. \] \hspace{1cm} (2.26)

5) the vanishing of the \( A-Z \) mixing on-shell:

\[ \Re \tilde{\Sigma}_{A^0Z^0}(M_A^2) = 0. \] \hspace{1cm} (2.27)

By this set of conditions, the extra input for the Higgs sector (besides the parameters already used for the gauge sector) is given by \( \tan \beta \) and the physical mass of the \( A^0 \)-boson.

The mass counter terms of the \( W^\pm, Z^0 \) and \( A^0 \) bosons in the on-shell condition (2.21) are

\[ \Re \Sigma_Z(M_Z^2) = \delta M_Z^2 \]
\[ = M_Z^2 \left( 2\delta Z_1^Z - 3\delta Z_2^Z + \cos^2 \beta \delta Z_{H_1} + \sin^2 \beta \delta Z_{H_2} - 2\cos^2 \beta \frac{\delta v_1}{v_1} \right. \]
\[ \left. - 2\sin^2 \beta \frac{\delta v_2}{v_2} \right) \]

\[ \Re \Sigma_W(M_W^2) = \delta M_W^2 \]
\[ = M_W^2 \left( -\frac{c_W^2}{c_W^2 - s_W^2} (3\delta Z_2^Z - 2\delta Z_1^Z) + \frac{s_W^2}{c_W^2 - s_W^2} (3\delta Z_2^Z - 2\delta Z_1^Z) \right. \]
\[ \left. + \cos^2 \beta \delta Z_{H_1} + \sin^2 \beta \delta Z_{H_2} - 2\cos^2 \beta \frac{\delta v_1}{v_1} - 2\sin^2 \beta \frac{\delta v_2}{v_2} \right) \]

\[ \Re \Sigma_A(M_A^2) = \delta m_A^2 - M_A^2 \left( \sin^2 \beta \delta Z_{H_1} + \cos^2 \beta \delta Z_{H_2} \right) \]
\[ = \frac{1}{2} \left( \sin^2 \beta \delta m_1^2 + \cos^2 \beta \delta m_2^2 - \sin 2\beta \delta m_{12} \right) - \frac{M_Z^2}{4} \cos^2 2\beta \]
\[ \left( \delta Z_1 - 2\frac{\delta v}{v} + \delta Z_H \right) - M_A^2 \left( \sin^2 \beta \delta Z_{H_1} + \cos^2 \beta \delta Z_{H_2} \right), \] \hspace{1cm} (2.28)
where $\delta v/v = \delta v_i/v_i$, $\delta Z = 2\delta Z^*_1 - 3\delta Z^*_2$ and $\delta Z_h = \delta Z_{H_1} + \delta Z_{H_2}$. The photon and $Z^0$-boson vertex and field renormalization constants are as in the minimal standard model:

$$
\begin{align*}
\delta Z^*_1 &= c_w^2 \delta Z^*_1 + s_w^2 \delta Z^*_B \\
&= -\Sigma'_\gamma(0) - \frac{3c_w^2 - 2s_w^2}{s_w c_w} \Sigma_\gamma(0) + \frac{c_w^2 - s_w^2}{s_w^2} \left( \frac{\Sigma_Z(M^2_Z)}{M^2_Z} - \frac{\Sigma_W(M^2_W)}{M^2_W} \right), \\
\delta Z^*_2 &= c_w^2 \delta Z^*_2 + s_w^2 \delta Z^*_B \\
&= -\Sigma'_\gamma(0) - \frac{2c_w^2 - s_w^2}{s_w c_w} \Sigma_\gamma(0) + \frac{c_w^2 - s_w^2}{s_w^2} \left( \frac{\Sigma_Z(M^2_Z)}{M^2_Z} - \frac{\Sigma_W(M^2_W)}{M^2_W} \right), \\
\delta Z_1 &= -\Sigma'_\gamma(0) - \frac{s_w^2 \Sigma_\gamma(0)}{c_w M^2_Z}, \\
\delta Z_2 &= -\Sigma'_\gamma(0). \\
\end{align*}
$$

(2.20)

The Higgs field renormalization constants $\delta Z_{H_i}$ are obtained from (2.25, 2.27):

$$
\begin{align*}
\delta Z_{H_1} &= -\Sigma'_A(M^2_A) - \frac{\cot \beta}{M_Z} \Sigma_{AZ}(M^2_A), \\
\delta Z_{H_2} &= -\Sigma'_A(M^2_A) + \frac{\tan \beta}{M_Z} \Sigma_{AZ}(M^2_A). \\
\end{align*}
$$

(2.30)

From the Higgs potential counter term $\delta V$ (2.12) one obtains the tadpole counter terms expressed by:

$$
\begin{align*}
\delta t_{h^0} &= -\sqrt{2} v_1 \sin \alpha \, \delta m_1^2 + \sqrt{2} v_2 \cos \alpha \, \delta m_2^2 + \sqrt{2} (v_1 \cos \alpha - v_2 \sin \alpha) \, \delta m_{12}^2 + R_{h^0}, \\
\delta t_{H^0} &= \sqrt{2} v_1 \cos \alpha \, \delta m_1^2 + \sqrt{2} v_2 \sin \alpha \, \delta m_2^2 + \sqrt{2} (v_1 \sin \alpha + v_2 \cos \alpha) \, \delta m_{12}^2 + R_{H^0}. \\
\end{align*}
$$

(2.31)

The quantities $R_{h^0}$, $R_{H^0}$, $R_A$ contain the gauge sector counter terms in the Higgs self interaction part:

$$
\begin{align*}
R_{h^0} &= m_1^2 \sin \alpha \, \delta v_1 - m_2^2 \cos \alpha \, \delta v_2 + m_{12}^2 (\sin \alpha \, \delta v_2 - \cos \alpha \, \delta v_1) \\
&+ \frac{M_W M^2_Z}{g_2} \left( - \cos 2 \beta \sin (\beta + \alpha) (\delta Z_{Z} - 3\delta v/v) \right) \\
&- \frac{1}{2} \delta Z_H \sin 2 \beta \cos (\beta + \alpha) - 2 \delta Z_{H_1} \cos^3 \beta \sin \alpha + 2 \delta Z_{H_2} \sin^3 \beta \cos \alpha, \\
R_{H^0} &= -m_1^2 \cos \alpha \, \delta v_1 - m_2^2 \sin \alpha \, \delta v_2 - m_{12}^2 (\cos \alpha \, \delta v_2 + \sin \alpha \, \delta v_1) \\
&+ \frac{M_W M^2_Z}{g_2} \left( \cos 2 \beta \cos (\beta + \alpha) (\delta Z_{Z} - 3\delta v/v) \right).
\end{align*}
$$
\[-\frac{1}{2} \delta Z_H \sin 2\beta \sin(\beta + \alpha) + 2\delta Z_H \cos^3 \beta \cos \alpha + 2\delta Z_H \sin^3 \beta \sin \alpha \]

\[R_A = -\frac{M_Z^2}{4} \cos^2 2\beta (\delta Z_Z - 2\frac{\delta v}{v} + \delta Z_H) .\]

Introducing the linear combinations:

\[\delta X = \frac{1}{v} \left( \cos(\beta - \alpha) (\delta t_{H^0} - R_{H^0}) + \sin(\beta - \alpha) (\delta t_{h^0} - R_{h^0}) \right) ,\]

\[\delta Y = \frac{1}{v} \left( \sin(\beta - \alpha) (\delta t_{H^0} - R_{H^0}) - \cos(\beta - \alpha) (\delta t_{h^0} - R_{h^0}) \right) ,\] (2.32)

where \(v^2 = v_1^2 + v_2^2\), yields the mass counter terms \(\delta m_i^2, \delta m_{i2}\):

\[\delta m_1^2 = \delta X \cos^2 \beta + \delta Y \sin 2\beta + 2 \sin^2 \beta (\delta m_A^2 - R_A)\]
\[\delta m_2^2 = \delta X \sin^2 \beta - \delta Y \sin 2\beta + 2 \cos^2 \beta (\delta m_A^2 - R_A)\]
\[\delta m_{i2}^2 = \frac{1}{2} \sin 2\beta \delta X - \cos 2\beta \delta Y - \sin 2\beta (\delta m_A^2 - R_A) .\] (2.33)

The complete set of gauge and Higgs counter terms is now fixed by the self energies and tadpoles. This subset is required for the calculation of the \(h^0, H^0\), and \(h^0 H^0\) propagators at the one-loop level in the next chapter.

The fermion mass counter term and field renormalization constant follow from (2.21, 2.23):

\[\Re \Sigma_f(m_f^2) = \delta m_f^2 + \frac{m_f^2}{2} (\delta Z_L^f + \delta Z_R^f) - k (\delta Z_V^f - \gamma_5 \delta Z_A^f)|_{\psi = m_f},\]

\[\frac{\delta m_f^2}{m_f^2} = \Sigma_V^f(m_f^2) + \Sigma_S^f(m_f^2)\]

\[\delta Z_V^f = \frac{1}{2} (\delta Z_L^f + \delta Z_R^f)\]

\[\delta Z_A^f = \frac{1}{2} (\delta Z_L^f - \delta Z_R^f)\]

\[\Sigma_f(k) = k \Sigma_V^f(k^2) + k \gamma_5 \Sigma_A^f(k^2) + m_f \Sigma_S^f(k^2) .\] (2.34)

where the decomposition of \(\Sigma_f\) into the invariant functions \(\Sigma_{V,A,S}^f\) is applied:

\[\Sigma_f(k) = k \Sigma_V^f(k^2) + k \gamma_5 \Sigma_A^f(k^2) + m_f \Sigma_S^f(k^2) .\] (2.35)
3 Physical neutral Higgs masses in the MSSM

Radiative corrections to the neutral scalar Higgs particles require the calculation of the renormalized Higgs self energies and the $h^0-H^0$ mixing. They are obtained by summing the loop diagrams and the counter terms:

\[ \hat{\Sigma}_{h^0}(k^2) = \Sigma_{h^0}(k^2) + k^2 (\delta Z_{H_1} \sin^2 \alpha + \delta Z_{H_2} \cos^2 \alpha) - \delta m_{h^0}^2 \]
\[ \hat{\Sigma}_{H^0}(k^2) = \Sigma_{H^0}(k^2) + k^2 (\delta Z_{H_1} \cos^2 \alpha + \delta Z_{H_2} \sin^2 \alpha) - \delta m_{H^0}^2 \]
\[ \hat{\Sigma}_{H^0 h^0}(k^2) = \Sigma_{H^0 h^0}(k^2) + k^2 \sin \alpha \cos \alpha (\delta Z_{H_2} - \delta Z_{H_1}) - \delta m_{H^0 h^0}^2 . \]  (3.1)

The mass counter terms $\delta m_{h^0}^2$, $\delta m_{H^0}^2$ and $\delta m_{H^0 h^0}^2$ are completely fixed in the Higgs potential counter terms:

\[
\begin{align*}
\delta m_{h^0}^2 &= \cos^2(\beta - \alpha) \delta m_A^2 + \frac{g_2 \sin^2(\beta - \alpha) \cos(\beta - \alpha)}{2M_W} T_{H^0} + \sin^2(\beta + \alpha) \Sigma_Z(M_Z^2) \\
& \quad - \frac{g_2 \sin(\beta - \alpha)(1 + \cos^2(\beta - \alpha))}{2M_W} T_{h^0} - M_Z^2 \sin(\beta + \alpha) \sin(\beta - \alpha) (\delta Z_{H_1} - \delta Z_{H_2}) \\
& \quad + M_Z^2 \sin^2(\beta + \alpha) \sin(\beta \delta Z_{H_1} + \cos^2 \beta \delta Z_{H_2}) \\
\delta m_{H^0}^2 &= \sin^2(\beta - \alpha) \delta m_A^2 - \frac{g_2 \cos(\beta - \alpha)(1 + \sin^2(\beta - \alpha))}{2M_W} T_{H^0} \\
& \quad + \cos^2(\beta + \alpha) \Sigma_Z(M_Z^2) + \frac{g_2 \cos^2(\beta - \alpha) \sin(\beta - \alpha)}{2M_W} T_{h^0} \\
& \quad + M_Z^2 \cos(\beta + \alpha) \cos(\beta - \alpha)(\delta Z_{H_1} - \delta Z_{H_2}) \\
& \quad + M_Z^2 \cos^2(\beta + \alpha)(\sin^2 \beta \delta Z_{H_1} + \cos^2 \beta \delta Z_{H_2}) \\
\delta m_{H^0 h^0}^2 &= -\sin(\beta - \alpha) \cos(\beta - \alpha) \delta m_A^2 + \frac{g_2 \sin^3(\beta - \alpha)}{2M_W} T_{H^0} + \frac{g_2 \cos^3(\beta - \alpha)}{2M_W} T_{h^0} \\
& \quad - \cos(\beta + \alpha) \sin(\beta + \alpha) \Sigma_Z(M_Z^2) - M_Z^2 \sin \alpha \cos \alpha (\delta Z_{H_1} - \delta Z_{H_2}) \\
& \quad - M_Z^2 \cos(\beta + \alpha) \sin(\beta + \alpha)(\sin^2 \beta \delta Z_{H_1} + \cos^2 \beta \delta Z_{H_2}) .
\end{align*}
\]

The propagator matrix of the scalar neutral Higgs particles is diagonal in lowest order perturbation theory. Quantum effects give rise to mixing between the light and heavy
Higgs boson states. Therefore the \((h^0, H^0)\) propagator matrix in one loop order is a \(2 \times 2\) matrix. The inverse of the propagator matrix is

\[
\Delta^{-1} = -i \begin{pmatrix}
    k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}(k^2) & \hat{\Sigma}_{h^0H^0}(k^2) \\
    \hat{\Sigma}_{h^0H^0}(k^2) & k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}(k^2)
\end{pmatrix},
\]

(3.2)

where \(\hat{\Sigma}_{h^0}, \hat{\Sigma}_{H^0}, \hat{\Sigma}_{h^0H^0}\) are the renormalized self energies and mixing as indicated in (3.1). The entries in the propagator matrix

\[
\Delta = i \begin{pmatrix}
    \Delta_{h^0} & \Delta_{h^0H^0} \\
    \Delta_{h^0H^0} & \Delta_{H^0}
\end{pmatrix},
\]

(3.3)

are the diagonal and non-diagonal propagators:

\[
\Delta_{h^0} = \frac{1}{k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}(k^2) - \frac{\Sigma_{h^0H^0}(k^2)}{k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}(k^2)}}
\]

\[
\Delta_{H^0} = \frac{1}{k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}(k^2) - \frac{\Sigma_{h^0H^0}(k^2)}{k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}(k^2)}}
\]

\[
\Delta_{h^0H^0} = -\frac{\hat{\Sigma}_{h^0H^0}(k^2)}{(k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}(k^2))(k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}(k^2)) - \Sigma_{h^0H^0}(k^2)}.
\]

(3.4)

The physical one-loop masses of the scalar neutral Higgs particles are the real parts of the one-loop propagator matrix poles. The poles of the propagator system are the solutions of the equation

\[
(k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}(k^2))(k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}(k^2)) - (\hat{\Sigma}_{h^0H^0}(k^2))^2 = 0.
\]

(3.5)

The physical Higgs masses are denoted by capital \(M_{h^0}, M_{H^0}\) in order to distinguish them from the formal tree level Higgs masses \(m_{h^0}, m_{H^0}\) in Eq. (2.6).

In the numerical analysis the Fermi constant \(G_F\) is taken as the input parameter from \(\mu^-\) decay. \(G_F\) is related to \(M_W\) by:

\[
G_F = \frac{\pi \alpha}{\sqrt{2} \cdot s_W \cdot c_W} \cdot \frac{1}{1 - \Delta r},
\]

(3.6)

where \(\Delta r\) is the full MSSM radiative correction to the \(\mu^-\) decay amplitude [15].

In the following we discuss the effects from virtual supersymmetric particles in radiative corrections to the physical light and heavy Higgs mass.
4 Discussion

- Higgs mass approximation formulae $O(m_t^4)$

The complete numerical analysis of the physical neutral Higgs masses confirms that virtual fermions and sfermions give the largest contributions to radiative corrections through the Yukawa couplings. The top-stop loops give rise to the dominant term increasing with the top quark mass $\sim m_t^4$. These leading contributions to the renormalized self energies $\hat{\Sigma}_h^0, \hat{\Sigma}_H^0, \hat{\Sigma}_{hH}^0$ are given by

$$
\begin{align*}
\hat{\Sigma}_h^0(k^2) &= -\omega_t \cos^2 \alpha \\
\hat{\Sigma}_H^0(k^2) &= -\omega_t \sin^2 \alpha \\
\hat{\Sigma}_{hH}^0(k^2) &= -\omega_t \sin \alpha \cos \alpha ,
\end{align*}
$$

where

$$\omega_t = \frac{N C G_F m_t^4}{\sqrt{2} \pi^2 \sin^2 \beta} \log \left( \frac{m_{\tilde{t}L} m_{\tilde{t}R}}{m_t^2} \right).$$

(4.1)

Thereby the masses of the left and right stop squark states are $m_{\tilde{t}L}, m_{\tilde{t}R}$ and $N_C$ is the number of colors. In this approximation, the solution of equation (3.5) is the following simple modification of Eq. (2.6):

$$M_{H,h, eff}^2 = \frac{M_A^2 + M_Z^2 + \omega_t}{2}$$

$$\pm \sqrt{\left( \frac{M_A^2 + M_Z^2}{4} \right)^2 + \omega_t^2} - M_A^2 M_Z^2 \cos^2 2\beta + \frac{\omega_t \cos 2\beta}{2} (M_A^2 - M_Z^2).$$

(4.3)

As compared to the lowest order behaviour, the following striking changes occur:

- the upper limit for the light Higgs mass $M_{h^0}$ exceeds the tree level limit $m_{h^0} \leq M_Z$ significantly.
- the spectrum is no longer symmetric under $\tan \beta \leftrightarrow 1/ \tan \beta$.
- $M_{h^0} \neq 0$ for $M_A \rightarrow 0$ in general.

These features are already described by the approximate formulae (4.3). In the next sections the complete one-loop calculation is compared with the approximation formulae (4.3).

- Higgs mass dependence on $M_A, \tan \beta$

Figs. 1a,b show the light and heavy physical Higgs masses $M_h, M_H$ as functions of the input parameters $M_A, \tan \beta$. The values for $M_A, \tan \beta$ are varied in the range

$$0.5 \leq \tan \beta \leq m_t / m_b$$
For the top quark mass we take the value $m_t = 175$ GeV in accordance with the experimental indication $m_t = 174.1 \pm 17$ GeV [1]. The SUSY soft breaking parameters are chosen at $\mu = 100$ GeV, $M = 400$ GeV, $m_{sf} = 500$ GeV. $m_{sf}$ defines the sfermion soft breaking parameter, which is assumed to be equal for all squarks and sleptons (4.4). A diagonal squark and slepton mass matrix is also assumed. The complete one-loop result is displayed in Figs. 1a,b. The differences with the approximation formulae (4.3), are depicted in Tab. 1, for $M_A = 300$ GeV:

| $\tan \beta$ | $M_{h,\text{eff}}$ [GeV] | $M_{h,1\text{-loop}}$ [GeV] | $M_{H,\text{eff}}$ [GeV] | $M_{H,1\text{-loop}}$ [GeV] |
|--------------|------------------|----------------|----------------|----------------|
| 0.5          | 87.4             | 92.4           | 340.0          | 352.0          |
| 2            | 86.5             | 83.0           | 311.5          | 312.2          |
| 5            | 108.9            | 105.8          | 302.7          | 302.9          |
| 10           | 113.5            | 110.7          | 300.7          | 300.8          |
| 30           | 115.0            | 113.7          | 300.1          | 300.1          |

Table 1: Physical neutral Higgs masses in the approximation of Eq. (4.3) $M_{h,\text{eff}}, M_{H,\text{eff}}$ and in the full one-loop result $M_{h,1\text{-loop}}, M_{H,1\text{-loop}}$.

A nearly constant light Higgs mass plateau is obtained for fixed $\tan \beta$ values and pseudoscalar masses $M_A > 200$ GeV (Fig. 1a). In the full one-loop result, lower $\tan \beta$ values yield larger light Higgs masses $M_h$ (5 GeV for $\tan \beta = 0.5$) compared to the approximate result. The exact value can be smaller than the approximate result ($\approx 1 - 3$ GeV for $\tan \beta \geq 2$).

- Higgs mass dependence on $m_t$

Figs. 1c,d illustrate the expected top quark dependence of the light and heavy Higgs masses. SUSY soft breaking parameters and $\tan \beta$ values are taken from Figs. 1a,b and a pseudoscalar mass $M_A = 200$ GeV is selected. The leading $\sim m_t^4$ dependence in (4.2) is shown for a top quark mass in the experimental $2\sigma$ top mass range between 150 GeV $< m_t < 200$ GeV. In case of the light Higgs this $m_t$ dependence change the Higgs mass by $\pm 12$ GeV from its mean value at $m_t = 175$ GeV and $M_A = 200$ GeV. The heavy Higgs mass shows sizeable $m_t$ dependent effects only for low $\tan \beta$ values. For $\tan \beta > 5$ the heavy Higgs mass deviates by $\pm 1$ GeV within the $2\sigma$ top mass range. The quality of the
approximation formulae at $m_t = 150$ GeV compared to the full one-loop result is shown in Tab. 2 where the result of Eq. (4.3) is listed.

The results for $\tan \beta \geq 2$ in the full calculation are $\approx 2 - 4$ GeV smaller than the approximation of Eq. (4.3). A value $\tan \beta = 0.5$ yields a difference of 1 GeV between the full result and the approximation formulae.

**Higgs mass dependence on squarks and sleptons**

The squark and slepton sector of the MSSM is described by a $2 \times 2$ mass matrix:

$$M^2_i = \begin{pmatrix} \tilde{M}_Q^2 + \tilde{M}_Z^2 \cos 2\beta (I_3 - Q_f s^2_W) + m_f^2 & m_f (A_f + \mu \{\cot, \tan\} \beta) \\ m_f (A_f + \mu \{\cot, \tan\} \beta) & \tilde{M}_{\tilde{U}, \tilde{D}}^2 + \tilde{M}_Z^2 \cos 2\beta Q_f s^2_W + m_f^2 \end{pmatrix},$$

with SUSY soft breaking parameters $\tilde{M}_Q, \tilde{M}_{\tilde{U}, \tilde{D}}, A_f,$ and $\mu$. In the following discussion the soft breaking parameters are taken to be equal $m_{s_f} = \tilde{M}_Q = \tilde{M}_{\tilde{U}, \tilde{D}}$. Up and down type sfermions in (4.4) are distinguished by setting $f=u,d$ and the $\{u,d\}$ entries in the parenthesis. The parameter $\mu$ in the off-diagonal matrix elements in (4.4) is also present in the gaugino sector. The sfermion masses, obtained from diagonalizing (4.4) are:

$$m^2_{f_i} = \frac{1}{2} (\text{Tr} M^2_i \pm \sqrt{(\text{Tr} M^2_i)^2 - 4 \text{Det} M^2_i}) , \ i = 1, 2 ,$$

where the corresponding rotation matrices are described by the sfermion mixing angle $\tilde{\theta}_f$:

$$\tan 2\tilde{\theta}_f = \frac{2 m_f (A_f - \mu \{\cot, \tan\} \beta)}{M^2_f - M^2_{\tilde{U}, \tilde{D}} + M^2_Z (I_3 - 2 Q_f s^2_W) \cos 2\beta} .$$

As a first step, a diagonal mass matrix is obtained by setting:

$$A_f + \mu \{\cot, \tan\} \beta = 0 .$$

| $\tan \beta$ | $M_{h, eff}$ [GeV] | $M_{h, 1-loop}$ [GeV] | $M_{H, eff}$ [GeV] | $M_{H, 1-loop}$ [GeV] |
|--------------|------------------|-----------------|-----------------|-----------------|
| 0.5          | 77.1             | 78.4            | 240.8           | 243.6           |
| 2            | 72.6             | 68.4            | 216.4           | 216.7           |
| 5            | 98.4             | 95.2            | 204.3           | 204.5           |
| 10           | 104.3            | 101.3           | 201.2           | 201.3           |
| 30           | 106.2            | 104.1           | 200.1           | 200.2           |

Table 2: Physical neutral Higgs masses in the approximation of Eq. (4.3) $M_{h, eff}, M_{H, eff}$ and in the full one-loop result $M_{h, 1-loop}, M_{H, 1-loop}$.
Together with the present experimental lower bounds on squark masses \cite{16}, Figs. 2a,b show the light and heavy Higgs masses in the range $100\ \text{GeV} < m_{\tilde{s}f} < 1\ \text{TeV}$, for the parameters of Figs. 1a,b and $M_A = 200\ \text{GeV}$. The result of Eq. (4.3) is listed in Tab. 3 for $m_{\tilde{s}f} = 200\ \text{GeV}$ (1 TeV) and the parameters of Figs. 2a,b.

| $\tan\beta$ | $M_{h,\text{eff}}$ [GeV] | $M_{h,1-\text{loop}}$ [GeV] | $M_{H,\text{eff}}$ [GeV] | $M_{H,1-\text{loop}}$ [GeV] |
|-------------|----------------------------|-----------------------------|-----------------------------|-----------------------------|
|             | $(m_{\tilde{s}f} = 200\ \text{GeV})$ | $(m_{\tilde{s}f} = 200\ \text{GeV})$ | $(m_{\tilde{s}f} = 200\ \text{GeV})$ | $(m_{\tilde{s}f} = 200\ \text{GeV})$ |
| 0.5         | 70.1                       | 69.6                        | 230.2                       | 233.4                       |
| 2           | 65.2                       | 61.2                        | 215.3                       | 215.7                       |
| 5           | 92.5                       | 89.9                        | 204.1                       | 204.2                       |
| 10          | 98.5                       | 96.1                        | 201.1                       | 201.2                       |
| 30          | 100.4                      | 106.0                       | 200.1                       | 200.2                       |

|             | $(m_{\tilde{s}f} = 1\ \text{TeV})$ | $(m_{\tilde{s}f} = 1\ \text{TeV})$ | $(m_{\tilde{s}f} = 1\ \text{TeV})$ | $(m_{\tilde{s}f} = 1\ \text{TeV})$ |
| 0.5         | 94.1                       | 96.6                        | 281.0                       | 287.4                       |
| 2           | 96.1                       | 92.5                        | 221.3                       | 221.3                       |
| 5           | 119.3                      | 115.6                       | 205.7                       | 205.7                       |
| 10          | 125.1                      | 121.4                       | 201.6                       | 201.7                       |
| 30          | 127.1                      | 123.6                       | 200.2                       | 200.3                       |

Table 3: Physical neutral Higgs masses in the approximation of Eq. (4.3) $M_{h,\text{eff}}$, $M_{H,\text{eff}}$ and in the full one-loop result $M_{h,1-\text{loop}}$, $M_{H,1-\text{loop}}$.

A squark soft breaking parameter $m_{\tilde{s}f} = 200\ \text{GeV}$ (1 TeV) yields a result in the full one-loop calculation which is by 3-6 (3-4) GeV smaller than the formulae (4.3) with $\tan\beta \geq 2$.

A non-diagonal mass matrix (4.4) induces a mixing of the left and right sfermion states described by the sfermion mixing angle $\tilde{\theta}_f$. As a consequence, the Higgs couplings to sfermions contain the mixing angles $\tilde{\theta}_f$ and modify the approximation formulae (4.3) sizeably. The leading $\sim m_t^4$ contribution to the physical light Higgs mass is obtained in replacing (4.2) by

$$
\omega_t = \frac{N_C G_F m_t^4}{\sqrt{2\pi^2 \sin^2 \beta}} \left( \log \left( \frac{m_{t_1} m_{t_2}}{m_t^2} \right) + \frac{A_t (A_t + \mu \cot \beta)}{m_{t_1}^2 - m_{t_2}^2} \log \frac{m_{t_1}^2}{m_{t_2}^2} \right)
$$
In Figs. 2c,d the light Higgs mass is plotted as a function of the parameters $\mu$ and $A = A_u = A_d$ for $\tan\beta = 30(2)$. A large $\tan\beta = 30$ in Fig 2c shows a variation of the light Higgs by 2 GeV for $\mu$ in the range $-250$ GeV $< \mu < 250$ GeV. Small $\tan\beta = 2$ values in Fig. 2d give strong deviations of the light Higgs mass with $\mu$. Not all squark and gaugino masses are experimentally allowed in Fig. 2c,d.

Table 4 presents the light Higgs mass in the approximation of Eq. (4.8) for the parameters of Figs. 2c,d.

| $\mu$ [GeV] | $M_{h,\text{eff}}$ [GeV] | $M_{h,1\text{-loop}}$ [GeV] | $M_{h,\text{eff}}$ [GeV] | $M_{h,1\text{-loop}}$ [GeV] |
|-------------|-----------------|-----------------|-----------------|-----------------|
|             | (tan $\beta = 30$) | (tan $\beta = 30$) | (tan $\beta = 30$) | (tan $\beta = 30$) |
|             | (A = 0)          | (A = 0 GeV)     | (A = 300)       | (A = 300 GeV)   |
| -250        | 98.7             | 97.7            | 109.5           | 106.9           |
| -100        | 100.4            | 98.1            | 111.2           | 108.4           |
| 0           | 100.4            | 98.1            | 111.5           | 108.8           |
| 100         | 100.4            | 98.0            | 111.5           | 109.0           |
| 250         | 98.7             | 96.5            | 110.3           | 107.4           |

|             | (tan $\beta = 2$) | (tan $\beta = 2$) | (tan $\beta = 2$) | (tan $\beta = 2$) |
|-------------|-----------------|-----------------|-----------------|-----------------|
|             | (A = 0)         | (A = 0 GeV)     | (A = 300)       | (A = 300 GeV)   |
| -250        | 68.8            | 66.2            | 68.6            | 66.2            |
| -100        | 65.8            | 63.5            | 74.2            | 71.8            |
| 0           | 65.2            | 61.8            | 79.1            | 76.1            |
| 100         | 65.8            | 61.8            | 84.8            | 81.3            |
| 250         | 68.8            | 64.4            | 92.2            | 89.2            |

Table 4: Physical light Higgs mass in the approximation of Eq. (4.3) $M_{h,\text{eff}}$ and in the full one-loop result $M_{h,1\text{-loop}}$.

The data from the approximation formulae are larger (smaller) by $< 1-4$ GeV than the full result of Figs. 2c,d, depending in detail on the chosen parameters.
Slepton masses and slepton mixing angles show numerically small effects in the neutral scalar Higgs mass. The variation of the slepton soft breaking parameter $M_\tilde{l}$ between 100 GeV $< M_\tilde{l} < 500$ GeV while keeping squark masses constant results in shifts of the light and heavy Higgs mass by 0.3 GeV.

- Higgs mass dependence on gauginos

Gauge bosons and gauginos in the virtual states of self energies and tadpole diagrams are described in the MSSM by the SUSY soft breaking parameters $\mu$, $M$, $M'$ and $\tan \beta$. In the numerical analysis $M' = 5/3 \tan \theta_W \ M$. In Figs. 3a,b the light Higgs mass is plotted for $m_t = 175$ GeV, $M_A = 200$ GeV, $\tan \beta = 2(30)$ and $m_{sf} = 500$ GeV and a diagonal sfermion mass matrix. The values for $\mu$ and $M$ are in agreement with the experimentally lower gaugino mass bounds. The variation of the light Higgs mass dependence with $M$ in Figs. 3a,b is below 2 GeV and decreases for larger $M$ values. By the choice of $\mu$, $A_t$ and $A_b$ are fixed in terms of Eq. (4.7). However, in Fig. 3b $A_b$ can reach its maximum value of 7.5 TeV and is substantially larger than all other SUSY soft breaking parameters. Both plots show spikes as an effect of threshold effects of gauginos in the various self energies. These effects can not be described by a perturbation expansion and have to be skipped from the numerical analysis.

The full one-loop calculation of the neutral Higgs mass spectrum is in good agreement with the simpler approximation formulae in most of the parameter space. The deviations between the full one-loop calculation and the approximate result are within $2-10$ GeV in the considered parameter regions.

- Comparison with the complete on-shell renormalization scheme [10]

The full one-loop calculation of [10] makes use of different renormalization conditions. [10] fixes the counter terms $\delta v_i$ by an $\overline{MS}$ subtraction:

$$
\frac{\delta v}{v} = \frac{\delta v_i}{v_i} = - \frac{1}{(4\pi)^2} \frac{3g_2^2 + g_1^2}{4} \Delta,
$$

where the UV singularity $\Delta$ is given in (A.12). Instead of (4.9) the renormalization condition for the residue of the pseudoscalar Higgs propagator is used:

$$
\text{Re} \frac{\partial}{\partial k^2} \hat{\Sigma}_A^0(k^2) \big|_{k^2=m_A^2} = 0,
$$

The numerical results of [10] for the neutral Higgs masses, however, are in agreement within 0.4% with the one-loop calculation described in this article.
A Self energies and Tadpoles

The Feynman rules of the minimal supersymmetric standard model are given in [2]. All analytical formulae are calculated in the 't Hooft-Feynman gauge. The one- and two-point functions \(A, B_0, B_1, B_{22}\) are defined at the end of appendix A. The self energies and tadpoles are split into the 2 Higgs doublet SM-, sfermion- and gaugino contributions.

- \(Z^0\) - self energy:

\[
\Sigma_{Z,\text{Higgs}}(k^2) = \frac{\alpha}{4\pi} \frac{1}{s_W^2 c_W^2} \left( - \sin^2(\beta - \alpha)(B_{22}(k^2, M_A, m_{H^0}) + B_{22}(k^2, M_Z, m_{H^0})) \right. \\
- \cos^2(\beta - \alpha)(B_{22}(k^2, M_A, m_{H^0}) + B_{22}(k^2, M_Z, m_{H^0})) + 2 c_W^4 B_{22}(k^2, M_W, M_W) \\
- \cos^2 2\theta_W (B_{22}(k^2, m_{H^+}, m_{H^+}) + B_{22}(k^2, M_W, M_W)) \left. + \frac{1}{4} (A(m_{H^0}) + A(m_{H^0})) \right. \\
+ A(M_A) + A(M_Z)) + \frac{\cos^2 2\theta_W}{2}(A(M_W) + A(m_{H^0})) + M_Z^2 (2 s_W^4 c_W^2 B_0(k^2, M_W, M_W) \\
+ \sin^2(\beta - \alpha) B_0(k^2, M_Z, m_{H^0}) + \cos^2(\beta - \alpha) B_0(k^2, M_Z, m_{H^0}) ) + c_W^4 (A(M_W) \\
- 4 M_W^2) - c_W^4 F_2(k^2, M_W, M_W) \\
- s_W^2 c_W^2 \sum_f N_C \frac{F_1(k^2, m_f, m_f, \frac{v_f + a_f}{2}, \frac{v_f - a_f}{2}, \frac{v_f + a_f}{2}, \frac{v_f - a_f}{2})}{N_C} \\

\]

\[
\Sigma_{Z,\text{sfermion}}(k^2) = - \frac{\alpha}{4\pi} \sum_f (I_3^f s_{\tilde{\theta}}^2 - Q_f s_W^2) \left( \frac{4(I_3^f s_{\tilde{\theta}}^2 - Q_f s_W^2)^2}{s_W^2 c_W^2} B_{22}(k^2, m_{\tilde{f}_1}, m_{\tilde{f}_1}) + \frac{c_{\tilde{\theta}}^2 s_{\tilde{\theta}}^2}{s_W^2 c_W^2} \right) \\
+ (B_{22}(k^2, m_{\tilde{f}_2}, m_{\tilde{f}_2}) + B_{22}(k^2, m_{\tilde{f}_2}, m_{\tilde{f}_2})) + 2 \frac{(I_3^f s_{\tilde{\theta}}^2 - Q_f s_W^2)^2}{s_W^2 c_W^2} B_{22}(k^2, m_{\tilde{f}_2}, m_{\tilde{f}_2}) \\
- 2 (I_3^f - Q_f s_W^2)^2 c_W^2 + 4 Q_f^2 s_W^2 s_{\tilde{\theta}}^2 A(m_{\tilde{f}_2}) - 2 \frac{(I_3^f - Q_f s_W^2)^2 s_{\tilde{\theta}}^2 + 4 Q_f^2 s_W^2 c_W^2}{s_W^2 c_W^2} A(m_{\tilde{f}_2}) \\

\]

\[
\Sigma_{Z,\text{gaugino}}(k^2) = - \frac{\alpha}{4\pi} \frac{1}{4 s_W^2 c_W^2} \left( \frac{1}{2} \sum_{i,j=1}^{4} F_1(k^2, m_{\tilde{\chi}_i^0}, m_{\tilde{\chi}_j^0}, O_{ij}^{\mu L}, O_{ij}^{\mu R}, O_{ij}^{\nu L}, O_{ij}^{\nu R}) \\
+ \sum_{i,j=1}^{2} F_1(k^2, m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_j^-}, O_{ij}^{L R}) \right) \\

\]
with the vector and axial vector coupling notation:

$$v_f = \frac{I_3^f - 2s_W^2 Q_f}{2c_W s_W}, \quad a_f = \frac{I_3^f}{2c_W s_W}.$$  

(A.1)

$I_3^f = \pm 1/2$ is the weak isospin and $Q_f$ the fermion charge and $c_\theta = \cos \theta$, $s_\theta = \sin \theta$. The chargino and neutralino couplings are:

$$O_{ij}^{\prime L} = -V_{i1}V_{j1}^* - \frac{1}{2} V_{i2} V_{j2}^* + \delta_{ij} s_W^2; \quad O_{ij}^{\prime R} = -U_{i1}^* U_{j1} - \frac{1}{2} U_{i2}^* U_{j2} + \delta_{ij} s_W^2$$

$$O_{ij}''^{L} = -\frac{1}{2} N_{i3} N_{j3}^* + \frac{1}{2} N_{i4} N_{j4}^*; \quad O_{ij}''^{R} = -O_{ij}''^{L*},$$  

(A.2)

where the chargino matrix $U_{ij}$, $V_{ij}$ and neutralino matrix $N_{ij}$ are given in appendix B.

- $W^\pm$ - self energy:

$$\Sigma_{W,\text{Higgs}}(k^2) = \frac{\alpha}{4\pi} \frac{1}{s_W^2} \left( \frac{-\sin^2(\beta - \alpha)(B_{\bar{2}2}(k^2, m_{H^+}, m_{H^0}) + B_{2\bar{2}}(k^2, M_W, m_{h^0}))}{-\cos^2(\beta - \alpha)(B_{\bar{2}2}(k^2, m_{H^+}, m_{h^0}) + B_{2\bar{2}}(k^2, M_W, m_{H^0})) + 2s_W^2 B_{2\bar{2}}(k^2, 0, M_W) - B_{\bar{2}2}(k^2, M_W, M_Z) + B_{\bar{2}2}(k^2, M_W, M_A) + 2c_W^2 B_{2\bar{2}}(k^2, M_W, M_Z) + \frac{1}{4} \left( A(m_{h^0}) + A(m_{H^0}) + A(M_A) + A(M_Z) + 2A(M_W) + 2A(m_{H^+}) \right) + M_W^2 (\sin^2(\beta - \alpha) B_0(k^2, M_W, m_{h^0}) + \cos^2(\beta - \alpha) B_0(k^2, M_W, m_{H^0}) + s_W^2 B_0(k^2, 0, M_W) + \frac{c_W^2}{s_W^2} B_0(k^2, M_W, M_W) + 3A(M_W^2) - 2M_W^2 + c_W^2 (3A(M_Z^2) - 2M_Z^2) - c_W^2 F_2(k^2, M_W, M_W) - s_W^2 F_2(k^2, 0, M_W) - \sum_f \frac{N_C}{2} F_1(k^2, m_{f^+}, m_{f^-}, \frac{1}{2}, 0, 0, 0) \right)$$

$$\Sigma_{W,\text{sfermion}}(k^2) = -\frac{\alpha}{4\pi} \frac{1}{s_W^2} \sum_f N_C \left(2c_{\tilde{g}}^2 c_{\tilde{g}}^2 B_{\bar{2}2}(k^2, m_{\tilde{t}_1^+}, m_{\tilde{t}_1^-}) + 2c_{\tilde{g}}^2 s_{\tilde{g}}^2 \right.$$  

$$B_{\bar{2}2}(k^2, m_{\tilde{t}_1^+}, m_{\tilde{t}_1^-}) + 2s_{\tilde{g}}^2 c_{\tilde{g}}^2 B_{\bar{2}2}(k^2, m_{\tilde{t}_2^+}, m_{\tilde{t}_2^-}) + 2s_{\tilde{g}}^2 s_{\tilde{g}}^2 B_{\bar{2}2}(k^2, m_{\tilde{t}_2^+}, m_{\tilde{t}_2^-})$$

$$- \frac{c_{\tilde{g}}^2}{2} + \left( A(m_{\tilde{t}_1^+}) + A(m_{\tilde{t}_1^-}) \right) - \frac{s_{\tilde{g}}^2}{2} + \left( A(m_{\tilde{t}_2^+}) + A(m_{\tilde{t}_2^-}) \right)$$

$$\frac{N_C}{2} F_1(k^2, m_{\chi_{i}^+}, m_{\chi_{j}^-}, \chi_{i}^0, O_{ij}^{L}, O_{ij}^{R}, O_{ij}^{L}, O_{ij}^{R}),$$  

with

$$O_{ij}^{L} = -\frac{1}{\sqrt{2}} N_{j4} V_{i2}^* + N_{j2} V_{i1}^*; \quad O_{ij}^{R} = +\frac{1}{\sqrt{2}} N_{j3}^* U_{i2} + N_{j2}^* U_{i1}.$$  

(A.3)

- Photon - self energy:

$$\Sigma_{\gamma,\text{Higgs}}(k^2) = -\frac{\alpha}{4\pi} \left(4B_{22}(k^2, m_{H^+}, m_{H^-}) + 2B_{22}(k^2, M_W, M_W)\right)$$

with

$$O_{ij}^{L} = -\frac{1}{\sqrt{2}} N_{j4} V_{i2}^* + N_{j2} V_{i1}^*; \quad O_{ij}^{R} = +\frac{1}{\sqrt{2}} N_{j3}^* U_{i2} + N_{j2}^* U_{i1}.$$  

(A.3)
\[-2M_W^2B_0(k^2, M_W, M_W) - 2A(M_{H^+}) - 8A(M_W) + 4M_W^2 + F_2(k^2, M_W, M_W) + \sum_f N_C Q_f^2 \, F_1(k^2, m_f, m_f, 1/2, 1/2, 1/2, 1/2) \]

\[\Sigma_{\gamma, sfermion}(k^2) = -\frac{\alpha}{4\pi} 2 \sum_f N_C Q_f^2 \left( 2B_{22}(k^2, m_{\tilde{f}_1}^-, m_{\tilde{f}_1}) - A(m_{\tilde{f}_1}) + 2B_{22}(k^2, m_{\tilde{f}_2}, m_{\tilde{f}_2}) - A(m_{\tilde{f}_2}) \right)\]

\[\Sigma_{\gamma, gaugino}(k^2) = -\frac{\alpha}{4\pi} \sum_{i=1}^2 F_1(k^2, m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_i^-}, 1/2, 1/2, 1/2, 1/2)\]

- $\gamma Z^0$ - mixing:

\[\Sigma_{\gamma, 2-Higgs}(k^2) = -\frac{\alpha}{4\pi} \left( \frac{2 \cos 2\theta_W}{s_W c_W} \left( B_{22}(k^2, m_{H^+}, m_{H^-}) + B_{22}(k^2, M_W, M_W) \right) \right.\]

\[+ \frac{2c_W}{s_W} B_{22}(k^2, M_W, M_W) - 2s_W c_W M_Z^2 B_0(k^2, M_W, M_W)\]

\[-\frac{\cos 2\theta_W}{s_W c_W} \left( A(m_{H^+}) + A(M_W) \right) + \frac{\cos 2\theta_W}{s_W} \left( 6A(M_W) - 4M_W^2 \right) - \frac{c_W}{s_W} \]

\[F_2(k^2, M_W, M_W) - \sum_f N_C Q_f \, F_1(k^2, m_f, m_f, 1/2, 1/2, 1/2) \]

\[\Sigma_{\gamma, sfermion}(k^2) = -\frac{\alpha}{4\pi} 2 \sum_f N_C Q_f \left( \frac{\cos 2\theta_W}{s_W c_W} \left( 2B_{22}(k^2, m_{\tilde{f}_1}^-, m_{\tilde{f}_1}) - A(m_{\tilde{f}_1}) \right) + \frac{\cos 2\theta_W}{s_W c_W} \left( 2B_{22}(k^2, m_{\tilde{f}_2}, m_{\tilde{f}_2}) - A(m_{\tilde{f}_2}) \right) \right)\]

\[\Sigma_{\gamma, gaugino}(k^2) = -\frac{\alpha}{4\pi} \sum_{i=1}^2 F_1(k^2, m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_i^-}, 1/2, 1/2, O_{ii}^H, O_{ii}^H)\]

The fermion loop contributions of vector boson self energies are described by the following functions with masses and couplings in its arguments.

\[F_1(k^2, m_1, m_2, a, b, a', b') = 8 \left[ (aa' + bb')(-2B_{22} + A(m_2) + m_1^2 B_0 + k^2 B_1) - (ab' + a'b) m_1 m_2 B_0 \right] (k^2, m_1, m_2)\]

\[F_2(k^2, m_1, m_2) = \left[ 10B_{22} + (4k^2 + m_1^2 + m_2^2) B_0 + A(m_1) + A(m_2) - 2(m_1^2 + m_2^2 - \frac{k^2}{3}) \right] (k^2, m_1, m_2). \quad (A.4)\]

- Pseudoscalar Higgs self energy:

\[\Sigma_{A, 2-Higgs}(k^2) = \frac{\alpha}{4\pi} \frac{1}{s_W} \left( \frac{1}{2} S_1(k^2, m_{H^+}, M_W) + \frac{\sin^2(\beta - \alpha)}{4c_W^2} S_1(k^2, m_{H^0}, M_Z) \right)\]
The Higgs self energy contains the following functions:

\[
\begin{align*}
\Sigma_{A,sfermion}^{(k^2)} & = -\frac{\alpha}{4\pi} \sum_f N_C \left( -\frac{m_f^2 (\mu - A_t \cot \beta)^2}{2s_W^2 M_W^2} B_0(k^2, m_{f_i}^2, m_{f_j}^2) \\
+ & \left( \frac{Q_3 - Q_2}{2s_W^2} \right) \cos 2\beta - \frac{m_f^2}{2s_W^2 M_W^2} \cot 2\beta \left( c_\theta^2 A(m_{f_i}^2) + s_\theta^2 A(m_{f_j}^2) \right) \\
+ & \left( \frac{Q_1 - Q_2}{2s_W^2} \right) \cos 2\beta - \frac{m_f^2}{2s_W^2 M_W^2} \cot 2\beta \left( c_\theta^2 A(m_{f_i}^2) + s_\theta^2 A(m_{f_j}^2) \right) \\
+ & \left( \frac{Q_2 + Q_1}{2s_W^2} \right) \cos 2\beta - \frac{m_f^2}{2s_W^2 M_W^2} \cot 2\beta \left( c_\theta^2 A(m_{f_i}^2) + s_\theta^2 A(m_{f_j}^2) \right) \right) \\
\Sigma_{A,gauge}(k^2) & = \frac{\alpha}{4\pi} \frac{1}{4s_W^2} \left( \sum_{i,j=1}^{2} S_2(k^2, m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_j^-}, O_{ij}, -O_{ij}^+, O_{ji}, -O_{ji}^+) + \frac{1}{2} \sum_{i,j=1}^{4} S_2(k^2, m_{\tilde{\chi}_i^0}, m_{\tilde{\chi}_j^0}, O_{ij}, -O_{ij}^+, O_{ji}, -O_{ji}^+) \right).
\end{align*}
\]

The Higgs self energy contains the following functions:

\[
\begin{align*}
S_1(k^2, m_1, m_2) & = - \left[ (k^2 + m_1^2)B_0 - 2k^2 B_1 + A(m_2) \right] (k^2, m_1, m_2) \\
S_2(k^2, m_1, m_2, a, b, a', b') & = 8 \left[ (ab + ba') (k^2 B_1 + A(m_1) + m_2^2 B_0) + \right. \]

22
\[(aa' + bb')m_1m_2B_0 \mid (k^2, m_1, m_2)\]

\[S_3(k^2, m) = [2m^2B_0 + A(m) + k^2B_1 \mid (k^2, m, m),\]

together with the mixing coefficients of charginos and neutralinos to the pseudoscalar Higgs

\[O_{ij} = Q_{ij} \sin \beta + S_{ij} \cos \beta\]

\[O'_{ij} = Q''_{ij} \sin \beta - S''_{ij} \cos \beta\]

\[Q_{ij} = \sqrt{\frac{1}{2}} V_{i1} U_{j2}, \quad S_{ij} = \sqrt{\frac{1}{2}} V_{i2} U_{j1}\] \hspace{1cm} (A.5)

\[Q''_{ij} = \frac{1}{2} \left[ N_{i3}(N_{j2} - N_{j1} \tan \theta_W) + N_{j3}(N_{i2} - N_{i1} \tan \theta_W) \right]\]

\[S''_{ij} = \frac{1}{2} \left[ N_{i4}(N_{j2} - N_{j1} \tan \theta_W) + N_{j4}(N_{i2} - N_{i1} \tan \theta_W) \right].\]

\[\bullet \Sigma_{h^0}, \Sigma_{H^0} \] self energy and \(h^0H^0\) - mixing \(\Sigma_{hH}\):

\[
\Sigma_{(h^0,H^0,hH),2-Higgs} = \frac{\alpha}{4\pi} \frac{1}{s_W^2} \left( \frac{\cos^2, \sin^2, - \sin \cos}{2} \right) (S_1(k^2, m_{H^+}, M_W)
\]

\[+ \frac{1}{2c_W^2} S_1(k^2, M_A, M_Z) + \frac{\sin^2, \cos^2, \sin \cos}{2} (S_1(k^2, M_W, M_Z))\]

\[+ \frac{1}{2c_W^2} S_1(k^2, M_Z, M_Z) + M_W^2 \{\sin^2, \cos^2, \sin \cos\} (\beta - \alpha) (4B_0(k^2, M_W, M_W) - 2\]

\[+ \frac{1}{2c_W^2} (4B_0(k^2, M_Z, M_Z) - 2)) + M_W^2 \{c_{1,1,1}^2, c_{1,2}^2, c_{1,1}c_{1,2}\} B_0(k^2, m_{H^+}, m_{H^+})\]

\[+ \frac{\{s_{1,1,1}^2, s_{1,2}^2, s_{1,1}s_{1,2}\}}{2} B_0(k^2, m_{H^0}, m_{H^0}) + \{s_{1,2}^2, s_{2,2}^2, s_{1,2}s_{2,2}\} B_0(k^2, m_{H^0}, m_{H^0})\]

\[+ \frac{\{s_{2,2}^2, s_{2,1}^2, s_{2,2}s_{2,1}\}}{2} B_0(k^2, m_{H^0}, m_{H^0}) + \{p_{1,1}^2, p_{1,2}^2, p_{1,1}p_{1,2}\} B_0(k^2, M_A, M_A)\]

\[+ B_0(k^2, M_Z, M_Z) + 2B_0(k^2, M_W, M_W)) + \{p_{2,1}^2, p_{2,2}^2, p_{2,1}p_{2,2}\} B_0(k^2, M_A, M_Z)\]

\[+ \frac{M_W^2 \{c_{2,1}^2, c_{2,2}^2, c_{2,1}c_{2,2}\}}{2} B_0(k^2, M_W, m_{H^+}) - \frac{M_W^2 \{\sin^2, \cos^2, \sin \cos\} (\beta - \alpha)}{2}\]

\[\cdot (B_0(k^2, M_W, M_W) + \frac{1}{2c_W^4} B_0(k^2, M_Z, M_Z) + \{1, 1, 0\} (2A(M_W) - M_W^2)\]

\[+ \frac{1}{2c_W^4} (2A(M_Z) - M_Z^2) \right) + \frac{\{3 \cos^2 2\alpha, 3 \sin^2 2\alpha - 1, 3 \sin 2\alpha \cos 2\alpha\}}{8c_W^2} A(m_{h^0})\]
\[
\Sigma_{\{h, H^0, hH\}, sfermion} = \frac{\alpha}{4\pi} \frac{1}{2s_W^2} \sum_f N_C \left( \left( c_\theta^2 \{u_{1,1}, u_{2,1}, u_{1,1}\} + c_\theta^2 \{u_{1,2}, u_{2,2}, u_{1,2}\} + \sin 2\theta \{u_{1,1}, u_{1,2}, u_{1,1}\}\right) \right.
\]
\[
+ \frac{2}{c_W^2} \left( s_\theta \{u_{1,2}, u_{2,2}, u_{1,1}\} + s_\theta \{u_{1,1}, u_{1,2}, u_{1,1}\}\right) \right) \frac{m_{f+}}{M_W^2 \cos^2 \beta} \left\{ \cos^2 \alpha, \sin^2 \alpha, -\sin \alpha \cos \alpha \right\} S_3 \left( k^2, m_{f+} \right)
\]
\[
+ \frac{m_{f-}}{M_W^2 \cos^2 \beta} \left\{ \sin^2 \alpha, \cos^2 \alpha, -\sin \alpha \cos \alpha \right\} S_3 \left( k^2, m_{f-} \right) \right)
\]
\[
\Sigma_{\{h, H^0, hH\}, gaugino} = \frac{\alpha}{4\pi} \frac{1}{4s_W^2} \left( \sum_{i,j=1}^2 S_2 \left( k^2, m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_j^-}, \{O_{ij}, O_{ij}, O_{ij}\}, \{O_{ij}, O_{ij}, O_{ij}\}, \{O_{ij}, O_{ij}, O_{ij}\}, \{O_{ij}, O_{ij}, O_{ij}\} \right) \right.
\]
\[
+ \frac{4}{2} \sum_{i,j=1}^2 S_2 \left( k^2, m_{\tilde{\chi}_i^0}, m_{\tilde{\chi}_j^0}, \{O_{ij}, O_{ij}, O_{ij}\}, \{O_{ij}, O_{ij}, O_{ij}\}, \{O_{ij}, O_{ij}, O_{ij}\}, \{O_{ij}, O_{ij}, O_{ij}\} \right) \right)
\]
with the mixing coefficients

\[ O_{ij}^h = Q_{ij} \sin \alpha - S_{ij} \cos \alpha \]

\[ O_{ij}^H = Q_{ij} \cos \alpha + S_{ij} \sin \alpha \]

\[ O_{ij}^{''h} = Q_{ij}^{''} \sin \alpha + S_{ij}^{''} \cos \alpha \]

\[ O_{ij}^{''H} = Q_{ij}^{''} \cos \alpha - S_{ij}^{''} \sin \alpha \]

and the notation for Higgs couplings

\[
(c_{i,j}) = \begin{pmatrix}
\sin(\beta - \alpha) + \frac{1}{2c_w} \cos 2\beta \sin(\alpha + \beta), & \cos(\beta - \alpha) - \frac{1}{2c_w} \cos 2\beta \cos(\alpha + \beta) \\
\cos(\beta - \alpha) - \frac{1}{2c_w} \sin 2\beta \sin(\alpha + \beta), & -\sin(\beta - \alpha) + \frac{1}{2c_w} \sin 2\beta \cos(\alpha + \beta)
\end{pmatrix}
\]

\[
(s_{i,j}) = \begin{pmatrix}
\frac{3M_Z}{2c_w} \cos 2\alpha \sin(\alpha + \beta), & \frac{M_Z}{2c_w} (2 \sin 2\alpha \sin(\alpha + \beta) - \cos(\alpha + \beta) \cos 2\alpha) \\
\frac{3M_Z}{2c_w} \cos 2\alpha \cos(\alpha + \beta), & -\frac{M_Z}{2c_w} (2 \sin 2\alpha \cos(\alpha + \beta) + \sin(\alpha + \beta) \cos 2\alpha)
\end{pmatrix}
\]

\[
(p_{i,j}) = \begin{pmatrix}
\frac{M_Z}{2c_w} \cos 2\beta \sin(\alpha + \beta), & -\frac{M_Z}{2c_w} \cos 2\beta \cos(\alpha + \beta) \\
\frac{M_Z}{2c_w} \sin 2\beta \sin(\alpha + \beta), & -\frac{M_Z}{2c_w} \sin 2\beta \cos(\alpha + \beta)
\end{pmatrix}
\]

\[
(v_{i}) = \begin{pmatrix}
\sin 2\alpha \sin 2\beta - \tan^2 \theta_W \cos 2\beta \cos 2\alpha, & \sin 2\beta \cos 2\alpha + \tan^2 \theta_W \cos 2\beta \sin 2\alpha
\end{pmatrix}
\]

\[
(u_{1,j}) = \begin{pmatrix}
\frac{M_Z}{c_w} (\pm \frac{1}{2} - Q_{i,j} s^2_{W}) \sin(\alpha + \beta) - \frac{m_f}{2MW \{\sin \beta, \cos \beta\}} m^2_{\{\cos \alpha, - \sin \alpha\}} \\
\frac{M_Z}{c_w} Q_{i,j} s^2_{W} \sin(\alpha + \beta) - \frac{m_f}{2MW \{\sin \beta, \cos \beta\}} \mu \sin \alpha - A_u \cos \alpha, \mu \cos \alpha - A_d \sin \alpha
\end{pmatrix}
\]

\[
(u_{2,j}) = \begin{pmatrix}
-\frac{M_Z}{c_w} (\pm \frac{1}{2} - Q_{i,j} s^2_{W}) \cos(\alpha + \beta) - \frac{m_f}{2MW \{\sin \beta, \cos \beta\}} m^2_{\{\sin \alpha, \cos \alpha\}} \\
-\frac{M_Z}{c_w} Q_{i,j} s^2_{W} \cos(\alpha + \beta) - \frac{m_f}{2MW \{\sin \beta, \cos \beta\}} \mu \cos \alpha + A_u \sin \alpha, \mu \sin \alpha + A_d \cos \alpha
\end{pmatrix}
\]

(A.6)

(A.7)

- \( A^0 - Z^0 \) - mixing:

\[
\Sigma_{AZ, gauge} = \frac{\alpha}{4\pi} \frac{1}{2s^2_W c^2_W} \left( -M_Z^2 \cos(\beta - \alpha) \sin(\beta - \alpha) (M_1(k^2, m_{h^0}, M_Z) \\
- M_1(k^2, M_{h^0}, M_Z)) - \frac{M_Z^2 \cos 2\beta}{2} (\sin(\beta + \alpha) \cos(\beta - \alpha) M_2(k^2, M_A, m_{h^0}) \\
+ \cos(\beta + \alpha) \sin(\beta - \alpha) M_2(k^2, M_A, m_{h^0}) ) \\
+ \frac{M_Z^2 \sin 2\beta}{2} (\sin(\beta + \alpha) \sin(\beta - \alpha) M_2(k^2, M_Z, m_{h^0})
\right)
\]

25
\[-\cos(\beta + \alpha) \cos(\beta - \alpha) M_2(k^2, M_Z, m_{H^0}) \]
\[-\sum_f N_C \left( \frac{m_f s_W c_W^2 \tan \beta}{M_W} M_3(k^2, m_{f^-}, m_{f^+}, v_f + a_f, v_f - a_f, -\frac{1}{2} \frac{1}{2} \right) \\
+ \frac{m_f s_W c_W^2 \cot \beta}{M_W} M_3(k^2, m_{f^+}, m_{f^-}, v_f + a_f, v_f - a_f, -\frac{1}{2} \frac{1}{2} \right) \bigg) \]
\[\Sigma_{AZ,sfermion} = 0 \]
\[\Sigma_{AZ,gaugino} = -\frac{\alpha}{4\pi} \frac{1}{4 s_W^2 c_W} \left( \sum_{i,j=1}^{2} M_3(k^2, m_{\tilde{\chi}_i^+, m_{\tilde{\chi}_j^-}}, O_{ij}^\prime, -O_{ij}^\prime, O_{ij}^{\prime L}, O_{ij}^{\prime R}) \right) \]
\[+ \frac{1}{2} \sum_{i,j=1}^{4} M_3(k^2, m_{\tilde{\chi}_i^0}, m_{\tilde{\chi}_j^0}, O_{ij}^\prime, -O_{ij}^\prime, O_{ij}^{\prime L}, O_{ij}^{\prime R}) \bigg) \]

where the functions in the $A^0-Z^0$ mixing are:
\[
M_1(k^2, m_1, m_2) = -(B_0 - B_1)(k^2, m_1, m_2) \\
M_2(k^2, m_1, m_2) = (B_0 + 2B_1)(k^2, m_1, m_2) \\
M_3(k^2, m_1, m_2, a, b, c, d) = -8( (ac + bd) m_2 (B_0 + B_1) \\
+ (ad + bc) m_1 B_1 )(k^2, m_1, m_2),
\]

(A.8)

- Higgs - tadpoles:
\[
T_{\{h^0, H^0\},gauge} = \frac{e}{(4\pi)^2} \frac{1}{s_W} \left( -M_W \{\sin, \cos\}(\beta - \alpha) (4A(M_W) - 2M_W^2) \right) \\
+ \frac{1}{2c_W^2} (4A(M_Z) - 2M_Z^2) \right) M_W \{c_{1,1}, c_{1,2}\} (A(M_{H^0}) - \frac{1}{2}(s_{1,1}, s_{1,2}) A(m_{h^0}) \\
- \frac{1}{2}(s_{2,2}, s_{2,1}) A(m_{h^0}) + \frac{1}{2}(p_{1,1}, p_{1,2}) (A(M_A) - A(M_Z) - 2A(M_W)) \\
+ M_W \{\sin, \cos\}(\beta - \alpha) (A(M_W) + \frac{1}{2c_W^2} A(M_Z)) \\
+ \sum_f N_C \left( \frac{2m_f^2 \{\cos \alpha, \sin \alpha\}}{M_W \sin \beta} A(m_{f^+}) + \frac{2m_f^2 \{-\sin \alpha, \cos \alpha\}}{M_W \cos \beta} A(m_{f^-}) \right) \bigg) \]
\[T_{\{h^0, H^0\},sfermion} = \frac{e}{(4\pi)^2} \frac{1}{s_W} \sum_f N_C \left( c_f^2 \{u_{1,1}, u_{2,1}\} + s_f^2 \{u_{1,2}, u_{2,2}\} + \sin 2\theta \right) \\
\{u_{1,3}, u_{2,3}\} A(m_{f_1}) + ( c_f^2 \{u_{1,1}, u_{2,1}\} + s_f^2 \{u_{1,2}, u_{2,2}\} - \sin 2\theta \{u_{1,3}, u_{2,3}\} \right) \]
\[A(m_{f_2}) \bigg) \]
\[T_{\{h^0, H^0\},gaugino} = -\frac{e}{4\pi^2} \frac{1}{s_W} \left( \sum_{i=1}^{2} \{Q_{ii}\{\sin \alpha, -\cos \alpha\} - S_{ii}\{\cos \alpha, \sin \alpha\}\} m_{\tilde{\chi}_i^+} \right) \]
\[ A(m_{\tilde{\chi}_i^\pm}) + \frac{1}{2} \sum_{i=1}^{4} \left( Q''_{ii} \{ \sin \alpha, -\cos \alpha \} + S''_{ii} \{ \cos \alpha, \sin \alpha \} \right) m_{\tilde{\chi}_i^\pm} A(m_{\tilde{\chi}_i^0}) \].

In the self energies and tapoles the scalar 1-point integral is given by:

\[ A(m) = m^2 \left( \Delta - \log \frac{m^2}{\bar{\mu}^2} + 1 \right) . \] (A.9)

For the scalar 2-point integral we can write

\[ B_0(q^2, m_1, m_2) = \Delta - \int_0^1 dx \log \frac{x^2 q^2 - x(q^2 + m_1^2 - m_2^2) + m_1^2 - i\epsilon}{\bar{\mu}^2} . \] (A.10)

Explicit analytic expression are presented in [13]. In terms of \( B_0 \), \( A \) one obtains for \( B_1 \) and \( B_{22} \):

\[ B_1(q^2, m_1, m_2) = -\frac{q^2 + m_1^2 - m_2^2}{2q^2} B_0(q^2, m_1, m_2) + \frac{m_1^2 - m_2^2}{2q^2} B_0(0, m_1, m_2) \]
\[ B_{22}(k^2, m_1, m_2) = \frac{1}{3} \left( \frac{1}{2} A(m) + m_1^2 B_0(k^2, m_1, m_2) + \frac{1}{2}(k^2 + m_1^2 - m_2^2) B_1(k^2, m_1, m_2) + \frac{m_1^2 + m_2^2}{2} - \frac{k^2}{6} \right) . \] (A.11)

The expression

\[ \Delta = \frac{2}{\varepsilon} - \gamma + \log 4\pi, \quad \varepsilon = 4 - D , \] (A.12)

and the mass scale \( \bar{\mu} \) are the conventional UV parameters in dimensional regularization.

**B Gaugino mass matrix**

The chargino \( 2 \times 2 \) mass matrix is given by

\[ M_{\tilde{\chi}^\pm} = \begin{pmatrix} M & M_W \sqrt{2} \sin \beta \\ M_W \sqrt{2} \cos \beta & \mu \end{pmatrix} , \] (B.1)

with the SUSY soft breaking parameters \( \mu \) and \( M \) in the diagonal matrix elements. The physical chargino mass states \( \tilde{\chi}_i^\pm \) are the rotated wino and charged Higgsino states:

\[ \tilde{\chi}_i^+ = V_{ij} \psi_j^+ \]
\[ \tilde{\chi}_i^- = U_{ij} \psi_j^- ; \quad i, j = 1, 2 . \] (B.2)
$V_{ij}$ and $U_{ij}$ are unitary chargino mixing matrices obtained from the diagonalization of the mass matrix (B.1):

$$U^* \mathcal{M}_{\tilde{\chi}^\pm} V^{-1} = \text{diag}(m^2_{\tilde{\chi}_1^\pm}, m^2_{\tilde{\chi}_2^\pm}). \quad (B.3)$$

The neutralino $4 \times 4$ mass matrix yields:

$$\mathcal{M}_{\tilde{\chi}^0} = \begin{pmatrix}
M' & 0 & -M_Z \sin \theta_W \cos \beta & M_Z \sin \theta_W \sin \beta \\
0 & M & M_Z \cos \theta_W \cos \beta & -M_Z \cos \theta_W \sin \beta \\
-M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & 0 & -\mu \\
M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & -\mu & 0
\end{pmatrix} \quad (B.4)$$

where the diagonalization introduces the unitary matrix $N_{ij}$ by:

$$N^* \mathcal{M}_{\tilde{\chi}^0} N^{-1} = \text{diag}(m^2_{\tilde{\chi}_i^0}). \quad (B.5)$$

References

[1] F. Abe et al., CDF Collaboration, Phys. Rev. Lett. 73 (1994) 225
[2] H.P. Nilles, Phys. Rep. 110 (1984) 1;
H.E. Haber, G. Kane, Phys. Rep. 117 (1985) 75;
J.F. Gunion, H.E. Haber, Nucl. Phys. B 272 (1986) 1;
J.F. Gunion, H.E. Haber, G. Kane, S. Dawson: The Higgs Hunter’s Guide, Addison-Wesley 1990
[3] H.E. Haber, R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815
[4] Y. Okada, M. Yamaguchi, T. Yanagida, Prog. Theor. Phys. 85 (1991) 1
D. Pierce, A. Papadopoulos, S. Johnson, Phys. Rev. Lett. 68 (1992) 3678
[5] R. Barbieri, M. Frigeni, Phys. Lett. 258 B (1991) 395
R. Barbieri, M. Frigeni, F. Caravaglios, Phys. Lett. 258 B (1991) 167
J.R. Espinosa, M. Quiros, Phys. Lett. 266 (1991) 389
K. Sasaki, M. Carena, C.E.M. Wagner, Phys. Rev. Lett. 381 B (1992) 66
H.E. Haber, R. Hempfling, Phys. Rev. D 48 (1993) 4280
[6] J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. 257 B (1991) 83
    J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. 262 B (1991) 477

[7] Y. Okada, M. Yamaguchi, T. Yanagida, Phys. Lett. 262 B (1991) 54
    J.L. Lopez, D.V. Nanopoulos, Phys. Lett. 266 B (1991) 397
    A. Brignole, J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. 271 B (1991) 123

[8] A. Yamada, Phys. Lett. 263 B (1991) 233
    A. Brignole, Phys. Lett. 281 B (1992) 284
    A. Brignole, Phys. Lett. 277 B (1992) 313
    D. Pierce, A. Papadopoulos, Phys. Rev. D 47 (1992) 222

[9] J. Kodaira, Y. Yasui, K. Sasaki, Hiroshima preprint HUPD-9316, YNU-HEPTh-93-102 (Nov. 1993)
    R. Hempfling, A. H. Hoang, Phys. Lett. B 331 (1994) 99
    J.A. Casas, J.R. Espinosa, M. Quiros, A. Riotto, CERN-TH. 7334/94 (July 1994)

[10] P.H. Chankowski, S. Pokorski, J. Rosiek, Phys. Lett. 274 B (1992) 191
    P.H. Chankowski, S. Pokorski, J. Rosiek, Nucl. Phys. 243 B (1994) 437

[11] P.H. Chankowski, S. Pokorski, J. Rosiek, Nucl. Phys. 243 B (1994) 497

[12] A. Dabelstein, Ph.D. Thesis, MPI preprint MPI-Ph 93-64

[13] M. Böhm, W. Hollik, H. Spiesberger, Fortschr. Phys. 34 (1986) 687
    W. Hollik, Fortschr. Phys. 38 (1990) 165

[14] A. Sirlin, Phys. Rev. D 22 (1980) 971
    W.J. Marciano, A. Sirlin, Phys. Rev. D 22 (1980) 2695

[15] D. Garcia, J. Solà, Mod. Phys. Lett. A 9 (1994) 211
    P.H. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek, Nucl. Phys. B 417 (1994) 101

[16] Particle Data Group, Phys. Rev. D 50 (1994) 1173
**FIGURE CAPTIONS**

**Figure 1.** Complete one-loop result of the light and heavy neutral Higgs masses $M_h, M_H$ for the input parameters $M_A$ and $\tan \beta = 0.5, 2, 5, 10, 30$ in Figs. 1a,b with a top quark mass $m_t = 175$ GeV. Figs. 1c,d. show the light and heavy Higgs masses for $150$ GeV $< m_t < 200$ GeV and various choices for $\tan \beta$ together with a fixed pseudoscalar mass $M_A = 200$ GeV. A sfermion soft breaking parameter $m_{s f} = 500$ GeV and $\mu = 100$ GeV is chosen and absence of left-right mixing is assumed in Figs. 1a,b,c,d. The gaugino soft breaking parameter $\mu = 400$ GeV in all figures.

**Figure 2.** Dependence of the light and heavy Higgs masses $M_h, M_H$ on sfermions for $100$ GeV $< m_{s f} < 1$ TeV and various $\tan \beta$ values in Figs. 2a,b. $\mu = 100$ GeV and no left-right mixing. The effects of the left-right mixing is presented in Figs. 2c,d for the light Higgs mass $M_h$ as a function of $\mu$ and $A = A_t = A_b = 0, 100, 200, 300$ GeV. In Fig. 2c(d) $\tan \beta = 30$ (2). $M_A = 200$ GeV, $m_t = 175$ GeV and $M = 400$ GeV.

**Figure 3.** Effects of the gaugino sector in the light Higgs mass $M_h$ in Figs. 3a(b) for $\tan \beta = 2$ (30). $M_A = 200$ GeV, $m_t = 175$ GeV, $m_{s f} = 500$ GeV, no left-right mixing.
Figure 1:
Figure 2:
Figure 3:
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9409375v2
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9409375v2
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9409375v2
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9409375v2