ON THE NEWTONIAN AND SPIN-INDUCED PERTURBATIONS FELT BY THE STARS ORBITING AROUND THE MASSIVE BLACK HOLE IN THE GALACTIC CENTER

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ABSTRACT

The S-stars discovered in the Galactic center are expected to provide unique dynamical tests of the Kerr metric of the massive black hole (MBH) that they orbit. In order to obtain unbiased measurements of its spin and the related relativistic effects, a comprehensive understanding of the gravitational perturbations of the stars and stellar remnants around the MBH is quite essential. Here, we study the perturbations on the observables of a typical target star, i.e., the apparent orbital motion and the redshift, due to both the spin-induced relativistic effects and the Newtonian attractions of a single object or a cluster of disturbing objects. We find that, in most cases, the Newtonian perturbations on the observables are mainly attributed to the perturbed orbital period of the target star rather than the Newtonian orbital precessions. Looking at the currently detected star S2/S0-2, we find that its spin-induced effects are very likely obscured by the gravitational perturbations from the star S0-102 alone. We also investigate and discuss the Newtonian perturbations on a hypothetical S-star located inside the orbits of those currently detected. By considering a number of possible stellar distributions near the central MBH, we find that the spin-induced effects on the apparent position and redshift dominate over the stellar perturbations for target stars with orbital semimajor axis smaller than 100–400 au if the MBH is maximally spinning. Our results suggest that, in principle, the stellar perturbations can be removed because they have morphologies distinct from those of the relativistic Kerr-type signatures.

Key words: black hole physics – Galaxy: center – Galaxy: nucleus – gravitation – relativistic processes – stars: kinematics and dynamics

1. INTRODUCTION

It is now widely accepted that a massive black hole (MBH) exists in the center of our galaxy, with the most prominent evidence provided by the so-far Keplerian motion of dozens of the surrounding S-stars (Ghez et al. 2008; Gillessen et al. 2009). They are found to be exclusively B-type dwarfs, and closely orbiting the central MBH within a distance of ~0.04 pc ≈ 8250 au. The continuous monitoring of their orbital motion provides precise measurements of the mass of the central MBH (≥4 × 10^6 M_☉) and, simultaneously, of our distance from the Galactic center (≈8 kpc) (Ghez et al. 2008; Gillessen et al. 2009; Meyer et al. 2012). Theoretical studies suggest that some of the hidden S-stars exist within the orbits of those currently detected, which may be revealed by future telescopes, e.g., the Thirty Meter Telescope (TMT) or European Extremely Large Telescope (E-ELT) (e.g., Zhang et al. 2013). Due to the proximity of these S-stars to the MBH, the strength of the gravitational field around them is orders of magnitude larger than that in the solar system or in pulsar binaries (Angélil et al. 2010; Iorio 2011a). Thus, their trajectories contain various general relativistic (GR) effects, including Lense-Thirring precession and frame-dragging (e.g., Jaroszynski 1998; Fragile & Mathews 2000; Rubilar & Eckart 2001; Weinberg et al. 2005; Will 2008; Preto & Saha 2009; Angélil et al. 2010; Angélil & Saha 2010, 2011; Merritt et al. 2010; Iorio 2011a, 2011b; Zhang et al. 2015), which should be measured accurately by powerful facilities in the near future. Continuous tracking of the orbital motion of S-stars by future telescopes is expected to provide unique dynamical tests of the Kerr metric of the MBH and also the no-hair theorem in the Galactic center (GC) (e.g., Psaltis et al. 2016; Zhang et al. 2015; Johannsen 2016; Yu et al. 2016).

However, in order to make accurate measurements of the spin and its induced GR effects actually feasible, careful handling of the perturbations induced by other stars on the motion of the target ones (the so-called “Newtonian perturbation” or “stellar perturbation”) is required. Indeed, the perturbations are likely contributed by a number of different gravitational sources located in the vicinity of the target star in the GC, e.g., late-type and early-type stars (e.g., Paumard et al. 2006; Gillessen et al. 2009; Bartko et al. 2010), stellar-mass black holes, neutron stars, white dwarfs (e.g., Freitag et al. 2006; Morris 1993), and dark matter (e.g., Iorio 2013). An intermediate-mass black hole (IMBH) possibly exists, with some allowed parameter space for its orbit and mass according to current observations (e.g., Hansen & Milosavljević 2003; Yu & Tremaine 2003; Gualandris & Merritt 2009; Gillessen et al. 2009; Genzel et al. 2010; Gualandris et al. 2010). Other dynamical processes, e.g., gravitational waves and tidal dissipation, are important but only for those S-stars in highly eccentric and/or extremely tight orbits (Psaltis 2012; Psaltis et al. 2013, 2016).

The stellar perturbation can induce additional orbital precessions of the target star and submerge those due to spin-induced effects. N-body post-Newtonian (PN) numerical simulations (Merritt et al. 2010) and analytical estimations based on theories of orbital perturbation (Sadeghian & Will 2011) have both found that orbital precessions of the target star caused by stellar perturbations can obscure those due to the frame-dragging effects (or quadrupole effects) if the target star itself is located more than ~0.5 mpc (or ~0.2 mpc) from the MBH. However, these previous studies have not included other complexities. For example, precessions of the
ascending node, periapsis, and the orbital inclination can only be determined indirectly by fitting the predictions of models that incorporate the various GR effects and also the complexities due to the MBH parameters (e.g., mass, spin, and GC distance) to the directly observable properties of the target star, i.e., its apparent trajectory in the plane of the sky and its redshift. Thus, from a practical point of view, it is more meaningful to compare the predicted perturbations on the apparent trajectory and redshift of the target star due to spin effects with those due to the stellar perturbations.

In our previous study (Zhang et al. 2015, hereafter ZLY15), we developed a fast full GR method to obtain the observables of the target star by considering both its orbital motion around the MBH and the propagation of photons from the target star to a distant observer. We investigated the constraints of the spin parameters by fitting to the observables of the target star without considering stellar perturbations. Relying upon the framework of ZLY15, here we further include the gravitational perturbations due to a single object or a cluster of disturbing objects on the orbital motion of the target stars orbiting the MBH. By performing a large number of numerical simulations, we investigate the Newtonian perturbations on the apparent orbital motion and the redshift of the target star and their dependence on the model parameters. The differences between the spin-induced relativistic effects and the Newtonian perturbations revealed by our study can provide useful clues to methods for separating them, which is essential for accurate measurements of the spin parameters and also for tests of the Kerr metric.

This paper is organized as follows. Section 2 describes the details of the numerical methods. The gravitational attractions of the background perturbers are included as an additional perturbed Hamiltonian term in the equations of motion of the target star. To obtain the projected sky position and the redshift of the star at a given moment, we adopt the light-tracing technique described in ZLY15 to solve the light trajectories propagating from the star to the observer. In Section 3, we describe the details of the methods used to estimate the perturbations on the apparent position and redshift of the target star due to the GR spin effects, Newtonian perturbations, and their combined effects from the numerical simulations. In Section 4, we investigate the stellar perturbations caused by a single perturber, in the specific case that S2/S0-2 is perturbed by the gravitational force of S0-102 (Section 4.1) or in the general case that a hypothetical S-star located inside the orbits of those currently detected is perturbed by a single perturber (Section 4.2). In Section 5, we consider the stellar perturbations of a cluster of disturbing objects. By performing a large number of numerical simulations, we investigate their resulting perturbations on the observables and compare them to those of the spin-induced signals. Discussion and conclusions are provided in Sections 6 and 7, respectively.

2. PERTURBED MOTION OF THE TARGET STAR

The geodesic motion of a star orbiting a Kerr black hole (Kerr 1963) can be described by a Hamiltonian $H_K$. In the Boyer–Lindquist coordinates $(r, \theta, \phi, t)$ (Boyer & Lindquist 1967), it is given by

$$H_K = - \frac{r^2 + a^2}{2\Sigma} - 2r^2 a^{2} \Delta \sin^2 \theta \, \frac{P_r^2}{2\Sigma \Delta} - \frac{2ar}{\Sigma \Delta} P_r P_\phi + \Delta \frac{2\Sigma P_r^2}{2\Sigma \Delta}$$

$$+ \frac{1}{2\Sigma} P_\theta^2 + \Delta \frac{\Delta - a^2 \sin^2 \theta}{2\Sigma \Delta} P_\phi^2$$

where

$$\Sigma = r^2 + a^2 \cos^2 \theta,$$

$$\Delta = r^2 - 2r + a^2.$$  \hspace{1cm} (1)

Here $p_r$, $p_\theta$, and $p_\phi$ are the components of the tetrad-momentum in the Boyer–Lindquist coordinate. Moreover, $a = J c / (M^2 G)$ is the dimensionless spin parameter of the MBH and $J$ is its spin angular momentum. For simplicity, we set $G = c = M = r_g = G M_\odot / c^2 = 1$ above, where $G$, $c$, $M$, and $r_g$ are the gravitational constant, the speed of light, the MBH mass, and the gravitational radius, respectively. Throughout this paper, we assume that the MBH in the GC has a mass of $M = 4 \times 10^6 M_\odot$ and is at a distance of $R_{GC} = 8$ kpc. The corresponding gravitational radius is then given by $r_g \approx 0.04 \text{ au} \approx 5 \mu \text{as} \approx 2 \times 10^{-4} \text{mpc}$.

The equations of motion described by $H = H_K$ can be further reduced to Equations (19)–(22) in ZLY15, from which we integrate numerically the orbital trajectories of a star without any stellar perturbation. As the Hamiltonian relies upon the full Kerr metric, all the various GR effects in the orbital motion of the target star around the Kerr MBH, including the advancements in the periastron and the orbital plane caused by the spin-induced effects, e.g., the frame-dragging and the quadrupole effects, are thus simultaneously included in the simulations.

If the target star is surrounded by $N_p$ perturbers—for example, a cluster of stars or stellar remnants in the field—the gravitational attractions from these sources can cause the orbital motion of the target star to deviate from the GR prediction. Considering only the leading order perturbations in Newtonian gravity, we ignore the mutual gravitational interactions between the perturbers and take the target star as a test particle, then the perturbations on the target star contributed by these sources can be approximately expressed by a Hamiltonian $H_p$, which is given by (see also Angelini & Saha 2014; Wisdom & Holman 1991)

$$H_p = \sum_{j=1}^{N_p} m_{pj} \left( \frac{r}{r_j} \cos \zeta_j - \frac{1}{d} \right).$$  \hspace{1cm} (3)

Here $(r_j, \theta_j, \phi_j)$ and $m_{pj}$ are the spatial position and the mass of the $j$th perturber, respectively. $\zeta_j = \arccos \left[ \sin \theta \sin \theta_j \cos (\phi - \phi_j) + \cos \theta \cos \theta_j \right]$ is the angle between the position vector of the target star and that of the $j$th perturber, and $d = \sqrt{r^2 + r_j^2 - 2rr_j \cos \zeta_j}$ is the distance between the target star and the $j$th perturber. Then the motion of the perturbed target star can be described by the modified Hamiltonian $H = H_K + H_p$.

Note that as the mutual

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4 We notice that assuming a non-zero mass of the target star could possibly lead to non-negligible back-reaction effects on the motion of the MBH and the perturbers. We defer the exploration of such potentially relevant effects to future works.
perturbations between the perturbers are disregarded, the orbital motion of each perturber rotating around the black hole can be fully described by the Hamiltonian $H = H_k$ and integrated by Equations (19)–(22) in ZLY15.

The orbital evolutions of the target star and the perturbers can be integrated from their Hamiltonian equations of motion once the initial orbital elements of the star and of the perturbers, and also the mass, the spin parameters, and the distance of the MBH, are provided (see details in ZLY15). Here the six orbital elements of the target star (the perturber) are the semimajor axis $a_s$ ($a_p$), eccentricity $e_s$ ($e_p$), inclination $I_s$ ($I_p$), position angle of the ascending node $\Omega_s$ ($\Omega_p$), angle to periapsis $\omega_s$ ($\omega_p$), and the true anomaly $f_s$ ($f_p$) (or the time of pericenter passage $t_{0s}$ ($t_{0p}$)), which are defined with respect to the sky plane. The spin direction of the MBH is defined by two angles: $\iota$ and $\epsilon$. Here $\iota$ is the line-of-sight inclination of the spin, and $\epsilon$ is the angle between the projection of the direction of the spin onto the sky plane and a reference direction.5

The observed sky position and the redshift of the target star can be obtained if the parameters describing the trajectory of the light emitted from the star to the distant observer are determined. The spin-induced position difference of the star in the sky plane is found to be of approximately the same order of magnitude as that caused by light-bending effects, so it is crucial to include the light-tracing technique in the simulations to model accurately the apparent position of the star (see Section 5.1 in ZLY15). As the total enclosed mass of the perturbers distributed around the MBH (typically $\lesssim 10$ mpc in this paper, see also Section 5) is much smaller than the mass of the central MBH, we assume that the light trajectories can be approximately integrated by the equations of motion in the Kerr metrics. In this work, we adopt the backward light-tracing technique described in ZLY15 to calculate the observed right ascension (R.A.) and declination (decl.) of the target star in the sky plane, and also the redshift ($Z$) of the target star as a function of the observational time $t_{obs}$ for an observer located at a distance of $R_{GC}$. Here $t_{obs} = t_s + t_{prop}$, where $t_s$ is the local time of the star in the Boyer–Lindquist coordinates and $t_{prop}$ is the time used for a photon propagating from the star to the observer. All the various GR effects affecting the propagation of photons from the star to the observer, including both the displacement of the image position in the sky plane due to the gravitational bending of light and the gravitational redshift of the target star (e.g., Iorio 2011b; Angélil et al. 2010; ZLY15), are then simultaneously included in the mock observables of the target star.

The full GR effects can be divided into two parts: the spin-zero and the spin-induced effects. The spin-zero effects are those when the black hole is not spinning, e.g., the Schwarzschild precession, the time dilation, and so on. The spin-induced effects include the frame-dragging ($\propto a$), quadrupole momentum ($\propto a^2$), and other high-order spin-related GR effects as well. Both the spin-zero and the spin-induced effects have been automatically included in the orbital motion and the mock observables of the target star as we adopt a full Kerr metric. However, in the following sections, we deal only with the spin-induced effects, and single them out by removing the spin-zero effects from the full GR effects. For a given set of initial conditions, this can be done by examining the differences between the results of the simulation with $\alpha = 0.99$ and those with $\alpha = 0$ (see details in Section 3). We will not discuss about effects of the spin-zero terms because they are not directly connected to the purpose of this study. We defer discussions of the spin-zero effects and their differences from the Newtonian perturbations to future works.

The Keplerian orbital elements of a star can be obtained from its instantaneous position and velocity. In this work, we calculate the orbital elements of the target star at any given moment by using its three-position and three-velocity measured in the local non-rotating rest frame (LNRF) of the target star. Here the three-velocity can be derived according to Equations (9)–(11) of ZLY15. We notice that if alternatively the elements are estimated from the instantaneous position and momentum in the LNRF (see also Pretor & Saha 2009), the results will be slightly different. We also notice that the orbital elements calculated in this work can be different from those obtained in the PN simulations, because the adopted spacetime metrics are different. However, we find that the results of these two methods are generally consistent (see more details in Section 4.1.3).

3. EVALUATING THE PERTURBATIONS FELT BY THE TARGET STAR

The orbital motion and the observables of a target star rotating around a spinning black hole can be affected by both the spin-induced effects and the gravitational attractions from other stars/stellar remnants. In this work, we deal mainly with three types of perturbations experienced by the target star: (1) the spin-induced perturbation; (2) the Newtonian perturbation; (3) a combination of (1) and (2). If $Y$ is any quantity relative to the motion of the target star, then the three types of perturbations listed above can be denoted as $\delta_1 Y$, $\delta_2 Y$, and $\delta_3 Y$, respectively. Here, $Y$ can be any one of the orbital elements of the target star, e.g., $a_s$, $\epsilon_s$, $I_s$, $\Omega_s$, $\omega_s$, $f_s$, ... or the observables, e.g., the coordinates of the apparent position (R.A. and decl.) and the redshift ($Z$) of the star. Note that $Y$ can also be the position vector of the target star as it appears in the sky plane, i.e., $R = (R_{\mathrm{A},}\, \text{decl})$; then, its perturbations are denoted as $\delta R = (\delta R_{\mathrm{A}}, \delta \text{decl})$.

The three types of perturbations can be obtained by comparing between two simulations that have the same set of initial orbital parameters of the target star and the perturber(s); however, the effects of spin and/or Newtonian gravity are turned on in one simulation and off in the other: (1) $\delta_1 Y$ is the difference in $Y$ between the simulation with $\alpha = 0$ and that with $\alpha = 0.99$, both of which ignore the stellar perturbations; (2) $\delta_2 Y$ is the difference in $Y$ between the simulation ignoring and that including the Newtonian gravity of the perturbers, for both of which the spin is set to $\alpha = 0$. (3) $\delta_3 Y$ is the difference in $Y$ between the simulation including both the Newtonian and spin-induced perturbations and that ignoring both of them. We found that $\delta_3 Y \approx \delta_2 Y + \delta_1 Y$.

As shown in the following sections, the signals of all these perturbations usually vary significantly as a function of time. The overall contributions of these perturbations can be estimated from their rms values. Within a period of time $T_{\text{tot}}$.
the rms perturbation in $Y$ is defined by

$$\delta Y = \frac{1}{T_{\text{tot}}} \int_{t_{\text{obs}}}^{T_{\text{tot}}} \delta Y(t_{\text{obs}}) dt_{\text{obs}}.$$  \hspace{1cm} (4)

Note that for the apparent position vector of the target star, i.e., $\mathbf{R} = (\text{R.A.}, \text{decl.})$, the rms magnitude of its perturbation, i.e., $|\delta \mathbf{R}| = \sqrt{\delta \text{decl.}^2 + \delta \text{R.A.}^2}$, is defined by

$$|\delta \mathbf{R}| = \frac{1}{T_{\text{tot}}} \int_{t_{\text{obs}}}^{T_{\text{tot}}} |\delta \mathbf{R}(t_{\text{obs}})| dt_{\text{obs}}.$$  \hspace{1cm} (5)

Similarly, if “$Y$” before $Y$ or $\mathbf{R}$ in Equation (4) or (5) is replaced by “$\delta_r$,” “$\delta_p$,” or “$\delta_c$,” it means the rms spin-induced perturbations, stellar perturbations, or their combination, respectively.

4. THE NEWTONIAN PERTURBATIONS OF A SINGLE PERTURBER

In this section, we study the simple case that the target star orbiting the MBH is perturbed by a single perturber. The target star and the perturber are considered to be the currently detected ones (see Section 4.1) or those undetected but very likely to exist in the vicinity of the MBH in the GC (see Section 4.2). The importance of such analysis is mainly threefold. (1) As we will see in the latter sections, such studies help us to understand the behavior of the Newtonian perturbation and its difference from the spin-induced effects. (2) They help to reveal the Newtonian perturbations contributed by the individual perturber when the target star is embedded in a stellar cluster. (3) As currently there are relatively large uncertainties for the mass profiles in the vicinity of the MBH (see details in Section 5), it remain possible that a target star within a few hundred au of the MBH is attracted by just a few perturbers located inside or nearby (see also Figure 8). In these cases, such a three-body problem of target star, perturber, and MBH may be important if one of the perturbers plays a dominant role in the signals of the Newtonian perturbations.

4.1. Perturbations on S2/S0-2 from S0-102

Among all of the currently detected S-stars, the star S2/S0-2 is of particular interest for testing spin-induced effects. It is in close proximity to the MBH and has a relatively high eccentricity, so continuous monitoring of the orbital motion of S2 by future facilities, i.e., TMT or E-ELT, can be used to provide constraints on the spin parameters of the MBH (see ZLY15). However, this requires a clean separation of the stellar perturbations caused by other surrounding S-stars or still undetected stars/stellar remnants. One of the closest S-stars is the recently discovered S0-102, which has the shortest orbital period known so far (Meyer et al. 2012). It has been found that S0-102 is not particularly suited to probing the spin parameters of the MBH (see ZLY15), but it may introduce gravitational perturbations on the orbits of S2/S0-2. In this section, we study the perturbations on both the orbital elements and the direct observables of S2/S0-2 due to the gravitational pull from S0-102.

We adopt the initial conditions of the target star S2/S0-2 and the perturber S0-102 according to recent observations (Gillessen et al. 2009; Meyer et al. 2012, see Table 1). We simulate the orbital motion of S2/S0-2 over three orbital periods, beginning in the year 2020 (corresponding to $t_{\text{obs}}=0$), to mimic the observational signals probed by the future telescopes. The mass of S0-102 is poorly known at present so we assume it to be $m_p = 0.5M_{\odot}$. As we will see later in this section, such an assumed mass for S0-102 results in Newtonian perturbations almost comparable with those caused by the spin-induced effects on both the position and redshift signals of S2/S0-2. We will discuss the dependence of the results on the mass of S0-102 later in this section.

We assume three different cases for the magnitude and direction of the MBH spin: (1) $a = 0.99$, $i = 45^\circ$, and $e = 200^\circ$, such that both the spin-induced effects on the position and redshift signals of S2/S0-2 are modest; (2) $a = 0.99$, $i = 49^\circ$, and $e = 125^\circ$, such that the spin-induced position displacement of S2/S0-2 is most significant; (3) $a = 0.99$, $i = 28^\circ$, and $e = 127^\circ$, such that the spin-induced redshift differences of S2/S0-2 are most significant (see also Yu et al. 2016).

4.1.1. Perturbations on the Observables of S2/S0-2

The simulated apparent trajectories of these two stars in the sky plane are shown in Figure 1. Figure 2 show the simulation results of the perturbations on the observables of S2/S0-2 due to the spin-induced effects when $a = 0.99$, $i = 45^\circ$, and $e = 200^\circ$ (magenta dashed lines), the Newtonian attractions of S0-102 (green dotted lines), and their combined effects (blue solid lines). Table 2 shows the maximum and the rms values of these perturbations over the three orbits. The details of the results are discussed as follows.

The evolutions of the spin-induced perturbations on the apparent position ($\delta_r\mathbf{R}$), its distance in the sky plane ($\delta_p\mathbf{R}$), and the redshift ($\delta_pZ$) of S2/S0-2 are shown by the magenta dashed lines in the left, middle, and right panel of Figure 2, respectively. We can see that the spin-induced position displacement is spiral-like, increases, and peaks near the apocenter in each orbit, while the spin-induced redshift difference is most variable and peaks near the pericenter in each orbit. Note that these spin-induced effects depend on the assumed spin orientations. The values of the maximum and rms spin-induced perturbations over three orbits in three cases of spin orientations are shown in Table 2.

The Newtonian perturbations on both the position and redshift signals of S2/S0-2 show quite different evolutions from those of the spin-induced effects (see the green dotted lines in Figure 2). The Newtonian perturbations on both position and redshift signals peak around the pericenter in each orbit, with the maximum values given by $|\delta_p\mathbf{R}|_{\text{max}} \sim 17.9 \mu \text{as}$ and $|\delta_pZ|_{\text{max}} \simeq 3.4 \text{km s}^{-1}$ around the third pericenter passage. The corresponding rms values are given by $|\delta_p\mathbf{R}| = 6.7 \mu \text{as}$ and $|\delta_pZ| = 0.8 \text{km s}^{-1}$ for the apparent position and the redshift signals, respectively.

The combined perturbations of the Newtonian and spin-induced effects are complex (see the blue solid lines in each panel of Figure 2), because in these simulations they are of comparable magnitude, i.e., $|\delta_p\mathbf{R}| \simeq |\delta_p\mathbf{R}|$ or $|\delta_pZ| \simeq |\delta_pZ|$. Note that the combined perturbations on the position signal of S2/S0-2 show peaks near both the pericenter and apocenter, which are contributed by the Newtonian and spin-induced effects, respectively.

We find that the Newtonian perturbations are proportional to the mass of S0-102 ($m_p$), as shown by the green dotted lines in Figure 3. The rms Newtonian perturbations are given by
Table 1

Initial Orbital Elements of the Target Stars and Perturbers

| Target star | \(a_\star\) (au) | \(r_{\text{per,}\star}\) (\(r_\odot\)) | \(e_\star\) | \(I_\star\) (deg) | \(\Omega_\star\) (deg) | \(\omega_\star\) (deg) | \(t_{0\star}\) (yr) | \(f_\star\) (deg) |
|-------------|-----------------|-----------------|----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| S2/S0-2\(^a\) | 984 | 2993 | 0.88 | 135 | 225 | 63 | 2.32 | ... |
| T1\(^b\) | 50–800 | 152–2434 | 0.88 | 45 | 0 | 0 | ... | 180 |
| T2\(^b\) | 50–800 | 887–14198 | 0.3 | 45 | 0 | 0 | ... | 180 |

Notes.

\(^a\) Distance at pericenter.
\(^b\) Time of the pericenter passage with respect to the year 2000.
\(^c\) Taken from Gillessen et al. (2009).
\(^d\) Taken from Meyer et al. (2012).

The orbital semimajor axis (\(a_\star\)) takes one of ten values that are logarithmically spaced between 50 and 800 au, which are 50, 68, 93, 126, 171, 233, 317, 432, 588, and 800 au. The distance at pericenter (\(r_{\text{per,}\star}\)) in units of \(r_\odot\) for T1 and T2 can then be obtained correspondingly.

In the MC simulation, \(I_\star\), \(\Omega_\star\), \(\omega_\star\), and \(t_{0\star}\) of the perturber take random values between 0° and 360°, and \(t_{0\star}\) of the perturber takes random values between 0 and \(P(a_\star)\), where \(P(a_\star)\) is the orbital period of the perturber.

\[
[\delta_p \mathbf{R}] \simeq 13.4 (m_\star/1M_\odot) \, \mu\text{as} \quad \text{and} \quad \delta_\tau Z \simeq 1.6 (m_\star/1M_\odot) \, \text{km s}^{-1}
\]

for the position and redshift signals of S2/S0-2, respectively. Thus, there is a critical mass of S0-102, i.e., \(m_{\text{crit}}^\star\) (or \(m_{\text{crit}}^w\)), when the rms value of the Newtonian perturbations on the position (or redshift) signal is equal to those resulting from the spin-induced effects, i.e., \([\delta_p \mathbf{R}] = [\delta_\tau Z] = [\delta_w Z]\). The spin effects in position (or redshift) of S2/S0-2 will be obscured by the Newtonian perturbations if \(m_\star > m_{\text{crit}}^\star\) (or \(m_\star > m_{\text{crit}}^w\)). The critical masses in the cases of different spin orientations are shown in Table 2. When the spin-induced effects become most significant, we get the upper limit of the critical mass, i.e., \(m_{\text{crit}}^\star = 0.7 M_\odot\) and \(m_{\text{crit}}^w = 0.28 M_\odot\) for the position and redshift signals of S2/S0-2, respectively. As the S-stars are exclusively B-type main-sequence stars with masses \(\gtrsim 3 M_\odot\) (Ghez et al. 2008; Gillessen et al. 2009), S0-102 is likely more massive than these critical values. Thus, it is quite plausible that the spin-induced signals of S2/S0-2 are obscured by the perturbations from S0-102 alone.

Observationally, the spin-induced effects can be measured by subtracting the stellar perturbations from the total measured shifts. In the case of S0-102, they may be well removed in the ideal case when it is the only S-star closer to the MBH than S2/ S0-2, and if the orbital parameters and the mass of S0-102 are observationally determined. The difference between the spin-induced effects and the stellar perturbations from S0-102 may provide helpful hints on discerning them from the observational data.

4.1.2. Perturbations on the Orbital Elements of S2/S0-2

The standard osculating orbital elements of stars are instantaneously calculated from the values of their position and velocity vectors from the usual Keplerian formulas. Here we calculate the orbital elements of S2/S0-2 from the three-position and the three-velocity in its LNRF (see Equations (9)–(11) in ZLY15). Both the Newtonian perturbations from S0-102 and the relativistic spin-induced effects induce variations of the orbital elements of S2/S0-2. The perturbations on the observables, i.e., \(\delta \mathbf{R}\) and \(\delta Z\), are approximately related to the variations of the orbital elements by

\[
\delta \mathbf{R}(t_{\text{obs}}) \simeq \sum_k \frac{\partial \mathbf{R}}{\partial k} \delta k(t_{\text{obs}})
\]

and

\[
\delta Z(t_{\text{obs}}) \simeq \sum_k \frac{\partial Z}{\partial k} \delta k(t_{\text{obs}}),
\]

respectively, where \(k = a_\star, e_\star, I_\star, \Omega_\star, \omega_\star\), and \(t_{0\star}\) (or \(f_\star\)). We calculate the perturbations on the orbital elements of S2/S0-2 according to the method described in Section 3. The results in the case that \(a = 0.99, i = 45^\circ\), and \(e = 200^\circ\) are depicted in Figure 4 and discussed as follows (the results for other two spin orientations are similar).

The spin-induced effects cause the variations of all the orbital elements (see the magenta dashed lines in Figure 4). The perturbations on the semimajor axis \([\delta_s a_\star]\) and eccentricity \([\delta_s e_\star]\) due to the spin-induced effects oscillate periodically, with local maxima at both periapsis and apoapsis. The orbit-averaged values of \(\delta_s a_\star\) and \(\delta_s e_\star\) remain constant as a function of time.
Due to the frame-dragging and also other high-order GR effects, $\delta_i$, $\delta_i\Omega_i$, and $\delta_i\omega_i$ increase with time. At the third pericenter, the variations of these orbital elements amount to $\delta_i = 0.16$, $\delta_i\Omega_i = -0.10$, and $\delta_i\omega_i = -0.44$. The time of pericenter passage $\delta_i t_{0p}$ (or similarly, $\delta_i t_p$) also increases slightly for each orbit, because the orbital periods of a star orbiting an MBH with and without spinning are different. One can show that the spin-induced position difference $|\delta_i R|$ and $|\delta_i Z|$ are contributed by both the orbital precession ($\delta_i L$, $\delta_i \Omega_i$, and $\delta_i \omega_i$) and $\delta_i t_{0p}$ (or $\delta_i t_p$) (see also Yu et al. 2016; ZLY15).

The gravitational attractions of S0-102 can also cause changes in all the orbital elements of S2/S0-2 (see the green dotted lines in Figure 4). We find that the stellar perturbations on the orbital elements are complex and quite different from the spin-induced relativistic ones. In particular, we found that the orbit-averaged Newtonian perturbations on the orbital semimajor axis do not remain constant but vary after each orbit. At the third pericenter, the variations of the elements of S2/S0-2 are given by $\delta_{i, a_s} = 0.03\,a_s$, $\delta_{i, e_s} = 10^{-7}$, $\delta_{i, L} = 0.014$, $\delta_{i, \Omega} = 0.041$, $\delta_{i, \omega} = -0.0035$, and $\delta_{i, t_{0p}} = 0.034$ day (or $\delta_{i, f_{p, t}} = -4.3$). As $\delta_{i, L} \ll \delta_{i, \Omega}$, $\delta_{i, \Omega} \ll \delta_{i, \omega}$, and $\delta_{i, \omega} \ll \delta_{i, \omega}$, the Newtonian orbital precessions are negligible compared to the relativistic spin-induced ones. As shown by the analytical calculations below, it turns out that the Newtonian perturbations on the observables, $|\delta_i R|$ and $|\delta_i Z|$, are mainly explained by perturbed orbital periods (or $\delta_{i, f_{p, t}}$) rather than the Newtonian orbital precessions.

The changes in the orbital semimajor axis, i.e., $\delta_{i, a_s}$ due to the Newtonian attractions of S0-102 cause variations in the orbital period and the time of arrival at each point in the orbit of the target star. Suppose that the evolution of the true anomaly of S2/S0-2 with and without perturbations of S0-102 is given by $f_i(t_i)$ and $f_i(t_i)$, respectively, then when the star reaches the same true anomaly, i.e., $f_i(t_i) = f_i(t_i)$, the difference in the time of arrival in these two cases is given by $\delta_i t_{0p} = t_i - t_i$. If we assume that the perturbation is small, i.e., $df_i(t_i)/dt_i \approx df_i(t_i)/dt_i$, then at a given moment $t_i$, the difference in the true anomaly is given by

$$\delta_i f_i(t_i) = f_i'(t_i) - f_i(t_i) \simeq -\frac{df_i}{dt_i} \delta_i t_i \simeq -\frac{\sqrt{M_G a_s(1-e^2)}}{r^2} \delta_i t_i.$$  \(8\)

At the pericenter, we simply have $\delta_i t_i = \delta_i t_{0p}$. Then the variation of the true anomaly is given by

$$\delta_i f_i \simeq -\frac{v_{per}}{a_s(1-e_s)} \delta_i t_{0p}. \quad (9)$$

Here $v_{per} = \sqrt{\frac{MG}{a_s}}\sqrt{\frac{1+e_s}{1-e_s}}$ is the velocity of the star at the pericenter. Note that if we substitute the simulation result $\delta_i t_{0p} = 0.034$ day into Equation (9), we have $\delta_i f_i \simeq -4.3$, which is also consistent with the result of the simulation.

At the third pericenter, the resulting position displacement and difference in the velocity in the line of sight (which can be approximately regarded as the redshift) due to the perturbed period (or the variations of $f_{p, t}$) are given by

$$|\delta_i R|_{f_z=0} = a_s(1-e_s)(1-cos^2\omega_s\sin^2L_s)^{1/2} |\delta_i f_i| \simeq 17.8 \mu as \quad (10)$$

and

$$|\delta_i Z|_{f_z=0} \simeq \sqrt{\frac{GM}{a_s(1-e_s)^3}} \sin \omega_s \sin L_s |\delta_i f_i| \simeq 3.2 \text{ km s}^{-1}, \quad (11)$$

respectively. This analysis is consistent with the results of numerical simulations, i.e., $|\delta_i R| \simeq 17.9 \mu as$ and $|\delta_i Z| \simeq 3.4 \text{ km s}^{-1}$, at the third pericenter of S2/S0-2 (see Table 2).

Similarly, we can obtain the analytical expressions for $|\delta_i R|$ and $|\delta_i Z|$ for arbitrary $f_{p, t}$. Note that, according to Equation (8), $\delta_i f_i$ peaks around the pericenter in each orbit, thus the resulting perturbations on the observables of S2/S0-2 also peak around the pericenter (see Section 4.1.1 and Figure 2).

Meanwhile, we find that the Newtonian orbital precessions induce negligible changes in the observables of S2/S0-2. According to Equations (6) and (7), near the third pericenter passage, the changes in the observables due to the Newtonian orbital precessions are given by (similar to Equations (31) and
Table 2

| (i, ϵ) | |δR|_{max} | |δZ|_{max} | |δ℘|_{max} | |δt|_{max} | |δp|_{max} | |δZ|_{max} | |δ℘|_{max} | |δZ|_{max} | |m_p^{PV} | |m_p^{LC} |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| (45°, 200°) | 17.8 | 17.9 | 22.3 | 6.1 | 6.7 | 10.5 | 3.4 | 4.4 | 0.32 | 0.79 | 1.1 | 0.45 | 0.20 |
| (49°, 125°) | 26.5 | 17.9 | 27.2 | 9.3 | 6.7 | 13.1 | 4.9 | 0.41 | 0.79 | 1.2 | 0.69 | 0.26 |
| (28°, 127°) | 24.8 | 17.9 | 25.7 | 8.5 | 6.7 | 12.5 | 1.6 | 3.4 | 5.0 | 0.44 | 0.79 | 1.2 | 0.64 | 0.28 |

Note. Summaries of the perturbations on position and redshift signals of S2/S0-2 over three orbital periods. Column (1): i and ϵ are the two angles defining the spin orientation; i is the line-of-sight inclination of the spin, ϵ is the angle between the projection of the direction of the spin onto the sky plane and a reference direction (see Figure 1 in ZLY15). Columns (2)–(7): the maximum (columns (2)–(4)) and rms values (columns (5)–(7)) of the perturbations on the position signal of S2/S0-2 over three orbits, in units of μas. Columns (8)–(13): the maximum (columns (8)–(10)) and rms values (columns (11)–(13)) of the perturbations on the redshift signal of S2/S0-2 over three orbits, in units of km s⁻¹. The quantities with “i” “℘,” and “Z” mean the perturbations due to spin-induced effects, the Newtonian attractions of S0-102 with m_p = 0.5M⊙, and their combined effects, respectively. Columns (14) and (15): m_p^{PV} is the mass of SO-102 when |δ℘/Z| = |δR/|Z|, m_p^{LC} is the mass of SO-102 when |δ℘/Z| = |δR/|Z|.

\[
|\delta R|_{\text{Max}} \approx \left( 1 + e_2 \right) \frac{G M_{\odot}}{a_s (1 - e_2)^2} \left[ \cos \omega_s \cos I_s \delta p I_s \right. \\
\left. + \sin \omega_s \sin I_s \delta \Omega_s \delta p I_s \right] \\
\approx 0.03 \ \mu\text{as}
\]  

(36) in ZLY15

for the position and redshift signals, respectively. These values are orders of magnitude lower than those in Equations (10) and (11).

Similarly, for most of the simulations performed in the later sections, i.e., in which a generic target star is perturbed by a single disturbing object (Section 4.2) or multiple such objects (Section 5), we find that the Newtonian perturbations on the observables are mainly caused by the perturbed orbital periods of the target star, rather than the Newtonian orbital precessions. Only in some rare cases, in which the perturber has some particular orbital configurations, are the effects due to the perturbed orbital period smaller than those of the Newtonian orbital precessions.

4.1.3. Comparison with the PN Simulations

The orbital elements of the target star are defined according to its simultaneous position and velocity. Thus, the signals of the Newtonian perturbations on them may show different behaviors if the adopted spacetime metric is different. It would be interesting to compare the Newtonian perturbations on the orbital elements resulting from our simulations with those from the PN numerical simulations. Here, with the same initial conditions, we implement the PN numerical simulations to calculate the orbital evolutions of S2/S0-2 due to the gravitational attraction of S0-102. We adopt the PN formalism in Kidder (1995) up to order 2.0 and ignore the spin-related PN terms. For both the simulations adopting PN formalism and the method in this work, we extract the effects of the Newtonian attraction of S0-102 from the total effects by examining the differences in the orbital elements when we turn the Newtonian gravity of SO-102 on and then off (see the extraction method described in Section 3).

The comparisons between results of the PN simulation and those of this work are shown in Figure 5. We can see that these two methods predict almost the same evolutions of the perturbations, especially for some of the orbital elements, i.e.,
I, \( \Omega', \omega, \) \( \Omega_j, \omega_j, \) \( t_{0k}, \) and \( t_{0p} \). For the other elements defined in the orbital plane, i.e., \( \alpha, e, \) and \( \omega, \), there is a relatively large discrepancy near each pericenter passage. Such differences can be explained by the fact that the spacetime metric in the PN simulations is based on the weak-field approximation, while here we use the pure Kerr metric. Nonetheless, we find that they lead to only a negligible difference in the overall effects of the Newtonian perturbations of S0-102. For example, after three orbital evolution, the relative difference in the Newtonian perturbations on the orbital elements obtained from this work \( \delta_{\rho,\rho_{\text{km}}} \) and those from the PN simulations \( \delta_{\rho,\rho_{\text{km}}} \) is about \( \left| \frac{\delta_{\rho,\rho_{\text{km}}}}{\rho_{\text{km}}} \right| \approx 10^{-2} \), where \( \kappa \) is any orbital element of S2/S0-2. The PN simulations result in \( \delta_{\rho}/f \approx 4/2 \) at the third pericenter. From Equations (10) and (11) we find that the resulting perturbations are given by \( |\delta_{\rho}\rho| \approx 17.5 \text{ m}\text{s}^{-1} \) and \( |\delta_{\rho}Z|/f \approx 3.1 \text{ km s}^{-1} \), which are consistent with those from our method. However, the PN simulations consider only the orbital variations in the local frame of the star. As we additionally include the light-tracing technique, our simulation has the power to predict the stellar perturbations on the direct observables of the target star, which are also the main focus of this study.

4.2. The Perturbations on a Hypothetical S-star

It is plausible that some currently undetected S-stars are located inside the orbit of S2/S0-2 or S0-102 in the GC, which may be revealed by future facilities (e.g., Zhang et al. 2013). These S-stars are better GR probes than S2/S0-2 and may be able to put tight constraints on the spin parameters (e.g., Angéllil et al. 2010; ZLY15). Similarly to S2/S0-2, these S-stars are likely to be disturbed by other surrounding stars or stellar remnants, and such perturbations should be carefully handled to give unbiased measurements of the spin-induced effects. By performing a large number of Monte Carlo (MC) simulations, here we study the case that a hypothetical S-star located inside the orbit of S2/S0-2 or S0-102 (\( \leq 800 \text{ au} \)) is perturbed by a single perturber. As these stars are currently undetected, we consider various initial conditions for both the target star and the perturber. The details of the simulations and the results are shown in the following sections.

4.2.1. The MC Simulations

We explore the problem by performing the MC numerical simulations as follows: we consider a target star where \( \alpha \), takes a value between 50 and 800 au, and \( e_s = 0.88 \) (star T1) or \( e_s = 0.3 \) (star T2). For each target star, we perform \( N_{\text{MC}} = 100 \) MC simulations in which it is perturbed by a perturber with randomly selected values of \( I_p, \Omega_p, \omega_p, \) and \( t_{0p} \) over three orbits. The orbital semimajor axis of the perturber in these MC simulations is \( a_p = 100 \text{ au} \) (perturber P1) or 600 au (perturber P2). The initial conditions of the target stars and the perturbers are listed in Table 1. We estimate the log-average value of the rms position displacement in these MC simulations, i.e., \( \log \left( \langle |\delta R| \rangle \right) \), from

\[
\log \left( \langle |\delta R| \rangle \right) = \frac{\sum_{j=1}^{N_{\text{MC}}} \log \left( |\delta R|_j \right)}{N_{\text{MC}}},
\]

where \( N_{\text{MC}} = 100 \) and \( |\delta R|_j \) is the rms perturbations on the position signal in the \( j \)th MC simulation. Similarly, we can define the log-average value for the redshift signal, i.e., \( \langle \delta z \rangle \), or any other quantities of the target star. The simulation results of ZLY15.
and $d_{\text{an}}$ for the target stars $T_1$ and $T_2$ are shown in Figures 6 and 7, respectively. The associated error bars show the standard deviations (about 1–1.5 dex) due to the randomly selected initial values of $I_p$, $J_p$, and $t_{0p}$. Note that for the spin-induced signals, we simply have $d_{\text{an}} = R_s$ and $d_{\text{an}} = Z_s$.

### 4.2.2. Results

From Figures 6 and 7, it appears that the scaling of $\langle |\delta_p R| \rangle$ with $a_*$ is different from that of $\langle |\delta_p Z| \rangle$. Such a difference can be explained as follows. According to Section 4.1.2, the rms Newtonian perturbations on the observables are related to the perturbations on the true anomaly of the target star by $|\delta_p R| \propto a_* |\delta p \Omega_p|$ and $|\delta_p Z| \propto a_*^{-1/2} |\delta p T_p|$ (see Equations (10) and...
After averaging the different MC runs, we have \( \langle \delta_p \bar{R} \rangle \propto a_s \langle \delta_p f_e \rangle \) and \( \langle \delta_p \bar{Z} \rangle \propto a_s^{-1/2} \langle \delta_p f_e \rangle \). Here \( \langle \delta_p f_e \rangle \) is the log-average value estimated by a method similar to Equation (14). Thus, \( \langle \delta_p \bar{R} \rangle \propto a_s^{3/2} \langle \delta_p \bar{Z} \rangle \), i.e., the scaling relations of these two signals differ by a factor of \( a_s^{3/2} \).

The Newtonian perturbations depend on the location of the perturber relative to the target star. From Figures 6 and 7, a perturber imposes stronger Newtonian perturbations on the observables of a target star located around or outside its orbit \( (a_s \gtrsim a_p) \) than on one located inside its orbit \( (a_s \lesssim \lambda a_p) \). Note that \( \lambda \approx 0.5-0.8 \) is a factor determined by the simulations. For target stars located inside the orbit of the perturber, the Newtonian perturbations are increasing functions of \( a_s \). For example, in Figure 6, for the perturber with \( a_p = 600 \) au (the green dotted lines), \( \langle \delta_p \bar{R} \rangle \) increases from \( 2.5 \times 10^{-6} \) mas to \( 3.1 \mu \) as, and \( \langle \delta_p \bar{Z} \rangle \) increases from 0.002 to 0.9 km s\(^{-1} \), if \( a_s \) of the target star increases from 50 to 432 au.

The Newtonian perturbations depend on other parameters of the target star and the perturber, and we describe them briefly here. (1) Both the Newtonian perturbation and the spin-induced effects depend strongly on the eccentricity of the target star. From Figures 6 and 7, we can see that the target stars with \( e_s = 0.3 \) feel much smaller spin-induced effects and Newtonian perturbations than those target stars with \( e_s = 0.88 \). (2) The Newtonian perturbations are proportional to the mass of the perturber, i.e., \( \langle \delta_p \bar{R} \rangle \propto m_p \) and \( \langle \delta_p \bar{Z} \rangle \propto m_p \) (see also Section 4.1 or Figure 3). We find that such a relation remains true if the mass of the perturber is in the range \( 0.1M_\odot < m_p < 100M_\odot \). (3) The average values of stellar perturbations depend weakly on the eccentricity of the perturber, i.e., \( e_p \). For example, if the eccentricities of the perturbers in the MC simulations of Figures 6 and 7 are replaced by \( e_p = 0.3 \), we find that the results are quite similar. (4) The Newtonian perturbations depend complexly on other parameters related to the orbital configurations of the target star, i.e., \( I_s, \Omega_s, \omega_s, \), and \( f_s \). We found that the Newtonian perturbations may differ by a factor of several to one order of magnitude if different values of these parameters are adopted.

We can see that the Newtonian perturbations by a \( 10 M_\odot \) perturber are already large enough to obscure the spin-induced signals. For the simulations shown in Figures 6 and 7, the spin-induced effects on the apparent position (or the redshift) can be drowned by the Newtonian perturbations if the orbital semimajor axis of the target star is larger than 260–500 au (or 200–430 au).

### 5. THE NEWTONIAN PERTURBATIONS OF A STAR CLUSTER

Currently, the mass distribution on a milliparsec scale in the GC together with its actual composition remain largely uncertain. Infrared imaging and spectroscopic observations in the past two decades have revealed thousands of bright stars within a parsec of the GC (e.g., Genzel et al. 2010; Schödel et al. 2009). Most of these observed stars can be classified into two distinct categories. (1) Early-type stars, which are the young \( (\sim 10\) Myr), massive \( (>7M_\odot) \), and main-sequence O-type or B-type stars. Although these stars are rare (there are several hundred, e.g., Paumard et al. 2006), they dominate the total luminosity within the inner parsec of the GC. (2) Late-type stars, which are old (several Gyr), K-type or M-type giant stars with masses of \( 1–2M_\odot \). These stars dominate the total star counts observed in the inner parsec of the GC. Because of their long lifetime, it is believed that they are the most promising tracers of the stellar distributions in the GC.

Observations of the proper motion of the late-type stars suggest that the extended mass within one parsec of the GC is \( (0.5–1.5) \times 10^5 M_\odot \) (e.g., Schödel et al. 2009). However, the radial distribution of the extended mass is not well constrained. Theoretical works expect that cusp profiles should appear if the stars around the MBH are dynamically relaxed by two-body interactions, i.e., \( n(r) \propto r^{-3} \), with \( \gamma = 3/2-7/4 \) (Bahcall & Wolf 1976, 1977). However, recent observations of the late-type stars suggest a flattened core-like profile with \( \gamma = 0–1 \) toward the inner region (e.g., Buchholz et al. 2009; Do et al. 2009, 2013a, 2013b; Bartko et al. 2010). The deficit of inner stars is currently not well understood. It may suggest that stars/stellar remnants in the inner parsec are not in equilibrium, or that a significant number of the late-type stars in this region are destroyed by stellar collisions (e.g., Alexander 1999; Freitag et al. 2006; Dale et al. 2009).

It is expected that a cluster of stellar-mass black holes (with masses of \( \sim 10M_\odot \)) may exist in the vicinity of the MBH, if the two-body relaxation time is less than a few Gyr in the GC (Hopman & Alexander 2006; Alexander & Hopman 2009). These black holes may form from the collapse and explosion of early-type stars at the end of their main-sequence lives, and may later concentrate toward the center through mass segregation (e.g., Freitag et al. 2006). However, so far it remains largely unclear whether they dominate at the milliparsec scale.

Due to the large uncertainties in this region, we explore the stellar perturbations on the observables of a target star surrounded by a star cluster with some possible mass distributions and its composition. The details of the simulations and the results are shown in the following sections.

#### 5.1. The Model Parameters

We adopt six models (M1–M6, see Table 3) with different initial conditions of the star cluster to cope with the large uncertainties in the mass profile within several milliparsecs of the MBH. The details of the model setups are described as follows.

### Table 3

Parameters of the Clusters in Different Models

| Model | \( M_1 \) | \( M_2 \) | \( \gamma \) | \( \beta \) | \( m_p \) | \( N_p \) | \( N_{ac} \) |
|-------|---------|---------|----------|--------|--------|--------|--------|
| M1    | 1780    | 100     | 1.75     | 0.5    | 10     | 178    | 80     |
| M2    | 1780    | 100     | 1.75     | 0.5    | 1      | 1780   | 8      |
| M3    | 530     | 30      | 1.75     | 0.5    | 10     | 53     | 280    |
| M4    | 530     | 30      | 1.75     | 0.5    | 1      | 530    | 28     |
| M5    | 1581    | 5       | 0.5      | –0.5   | 1      | 1581   | 10     |
| M6    | 316     | 1       | 0.5      | –0.5   | 1      | 316    | 48     |

Note. The initial conditions of the clusters in different models. Column (1): the total mass of the cluster in units of \( M_\odot \). Note that the perturbers in the clusters have \( 40 \) au \( < a_p < 2062 \) au (0.2 mpc \( \lesssim a_p \lesssim 10 \) mpc). Column (2): total mass of the perturbers with 0.2 mpc \( \lesssim a_p \lesssim 1 \) mpc, i.e., \( M_1 = M_p(<1 \) mpc), in units of \( M_\odot \). Column (3): slope of the density profile, i.e., \( n(r) \propto r^{-\gamma} \). Column (4): the velocity anisotropy, given by \( \beta = 1 - \sigma_\phi^2/\sigma_\alpha^2 \), where \( \sigma_\phi \) and \( \sigma_\alpha \) are the velocity dispersion in the transverse and line-of-sight directions, respectively. Column (5): the mass of the perturbers in units of \( M_\odot \). Column (6): the total number of perturbers in the cluster, i.e., \( N_p = M_p/m_p \). Column (7): the total number of MC simulations.
We assume that the cluster consists of perturbers with equal mass \( m_p \), and orbital semimajor axis and eccentricity, i.e., \( a_p \) and \( e_p \), following the distribution functions \( g(a_p) \) and \( h(e_p) \), respectively. We assume that the perturbers have randomly selected initial values of the inclination \( \iota_p \), position angle of ascending node \( \Omega_p \), angle to periapsis \( \omega_p \), and time of pericenter passage \( t_{0p} \). We assume that the number density of stars in the cluster is given by \( n(r) \propto r^{-\gamma} \), and their velocity anisotropy is given by \( \beta = 1 - \sigma_t^2/\sigma_{ho}^2 \), where \( \sigma_t \) and \( \sigma_{ho} \) are the velocity dispersion in the transverse and line-of-sight directions, respectively. Then it turns out (Merritt et al. 2010) that \( a_p \) and \( e_p \) of the perturbers in the cluster follow distributions \( g(a_p) \propto a_p^{-\gamma} \) and \( h(e_p) \propto (1 - e_p^2)^{-\beta/2} \), \( \beta \leq \gamma - 1/2 \). Then the total mass and number of perturbers with semimajor axis smaller than \( a_p \), i.e., \( M_p(<a_p) \) and \( N_p(<a_p) \), are given by \( M_p(<a_p) = M_1 \left( a_p / (1 \text{ mpc}) \right)^{3-\gamma} \) and \( N_p(<a_p) = M_p(<a_p)/m_p \), respectively. Here \( M_1 = M_p < 1 \text{ mpc} \) is the mass of perturbers with \( a_p < 1 \text{ mpc} \).

We restrict the model to \( 40 \text{ au} < a_p < 2062 \text{ au} \) (or \( 0.2 \text{ mpc} \leq a_p \leq 10 \text{ mpc} \)), and denote the total mass and number of perturbers in the cluster by \( M_p \) and \( N_p \). The outer boundary (10 mpc) is found to be large enough for the convergence of the simulation results in this work. The inner boundary (0.2 mpc) is set to avoid perturbers being too close to the MBH, which is found to slow down the numerical simulation quite significantly; their removal causes negligible difference to the simulation results. We avoid those stars whose periapsis distance is within their tidal radius, i.e., \( \approx \left( \frac{\eta^2 M_1 / m_p}{\beta_3} \right)^{1/3} (m_p/M_1)^{0.47} R_\text{MBH} \), where \( \eta = 2.21 \) (Mageror & Tremaine 1999). We also avoid the stars with a timescale for orbital gravitational wave radiation \( T_{gw} < 100 \text{ Myr} \), because the total number of these stars may be substantially suppressed due to the rapid orbital decay.

The extent to which the stars and stellar remnants in the vicinity of the MBH are dynamically relaxed by two-body interactions is still unknown. Thus, we consider two extreme cases. (1) They are dynamically relaxed, so the density profile approaches the Bahcall–Wolf cusp profile, i.e., \( \gamma = 1.75 \) (models M1–M4). In this case, it is possible that the stellar–mass black holes may dominate the vicinity of the MBH as a result of mass segregation effects (e.g., Hopkins & Alexander 2006; Alexander & Hopman 2009). We assume that all of the perturbers in the cluster are either low-mass main-sequence stars with \( m_p = 1M_0 \) or stellar–mass black holes with \( m_p = 10M_0 \). We set \( M_1 = 100M_0 \) or \( 30M_0 \) in each case. Then the total mass of the cluster is given by \( M_p = 1780M_0 \) (models M1 and M2) or \( 530M_0 \) (models M3 and M4). (2) The mass distribution in the vicinity of the MBH has not yet reached the equilibrium state via two-body relaxation; then, the density profile is likely flatter than the Bahcall–Wolf cusp profiles. We assume that the mass distribution follows a core-like profile with \( \gamma = 0.5 \) (models M5 and M6). In this case, we assume that the perturbers are all low-mass main-sequence stars with \( m_p = 1M_0 \) and \( M_1 = 5M_0 \) or \( M_1 = 1M_0 \). Then the total stellar mass is given by \( M_p = 1581M_0 \) (model M5) or \( 316M_0 \) (model M6). The total mass and the number of perturbers with orbital semimajor axis smaller than \( a_p \), i.e., \( M_p(<a_p) \) and \( N_p(<a_p) \), for all the models in Table 3 are shown in Figure 8.

We find that the results in this section are insensitive to the eccentricity distribution of the perturber (see also Section 4.2.2), so we assume \( \beta = 0.5 \) for models with \( \gamma = 1.75 \) and \( \beta = -0.5 \) for models with \( \gamma = 0.5 \) in order to fulfill the condition \( \beta \leq \gamma - 1/2 \).

A target star is assumed to be embedded in each of these clusters. Similarly to Section 4.2, we consider the target star T1 or T2 (see Table 1). We integrate the perturbations on the target star over three orbits. Unless specified otherwise, we assume that the spin parameters are given by \( a = 0.99 \), \( i = 45^\circ \), and \( \epsilon = 180^\circ \). For each model in Table 3, \( N_{\text{MC}} \) independent realizations are performed such that the total number of perturbers in the combined set of integrations is \( \sim 15,000 \). Then we estimate the changes in the orbital elements and the observables of the target star due to the Newtonian and spin-induced perturbations. The details of the results are described in the following sections.

5.2. The Perturbed Observables of the Target Star

We use \( \langle \delta \mathbf{R} \rangle \) and \( \langle \delta \mathbf{Z} \rangle \) to denote the log-average values of the perturbations of the clusters (similar to Equation (14)), for the position and redshift signals, respectively. The simulation results of \( \langle \delta \mathbf{R} \rangle \) and \( \langle \delta \mathbf{Z} \rangle \) of the target star T1 in clusters M1–M6 are shown in Figures 9 and 10, respectively. We find that similar results can be obtained for the target star T2. Note that both the Newtonian and spin-induced perturbations felt by the target star T2 are smaller than those felt by T1, simply due to its smaller eccentricity (\( e_\star = 0.3 \)).

According to Figures 9 and 10, both \( \langle \delta \mathbf{R} \rangle \) and \( \langle \delta \mathbf{Z} \rangle \) are increasing functions of \( a_\star \), although the slope index of \( \langle \delta \mathbf{R} \rangle \) is larger than that of \( \langle \delta \mathbf{Z} \rangle \) according to \( \langle \delta \mathbf{R} \rangle / \langle \delta \mathbf{Z} \rangle \propto a_\star^{3/2} \) (see also Section 4.2.2). As the spin-induced effects on both position and redshift are decreasing functions of \( a_\star \), i.e., \( \langle \delta \mathbf{R} \rangle \propto a_\star^{-3/2} \) and \( \langle \delta \mathbf{Z} \rangle \propto a_\star^{-2} \), for each model there is a critical orbital semimajor axis of the target star where the effects of Newtonian attraction and the spin equal to each other (see more details in Section 5.4). Their combined effects (see the blue
solid line in each panel of Figures 9 and 10) are thus dominated by the spin-induced effects in the inner region and by the Newtonian perturbations in the outer region.

We find that, for a given total mass \( M_p \), the Newtonian perturbations of a cluster of low-mass stars are smaller than those of stellar-mass black holes (see also Merritt et al. 2010). For example, the stellar perturbations of cluster M1 (with \( m_p = 10 M_\odot \) and \( N_p = 178 \)) are larger than those of cluster M2 (with \( m_p = 1 M_\odot \) and \( N_p = 1780 \)) by a factor of \( \sim 2 \). The reason is probably that M1 consists of more perturbers than M3, such that its potential is more isotropic and the induced stellar perturbations are less significant. Similar results can be obtained if we compare model M3 with M4.

As shown in Section 4, the Newtonian perturbations felt by a target star are mainly attributed to perturbers located around or inside its orbit (\( a_p \ll a_* \)). For a given total mass \( M_p \), stellar perturbations of the clusters with a cusp profile are larger than those with a core profile, because the former contain more perturbers in the inner region than the latter. For example, the mass of cluster M2 is quite similar to that of M5, but the stellar perturbations of cluster M2 are about 4–5 times larger than those of cluster M5 (see Figures 9, 10, and Table 5).

The observables of the target stars with \( a_* \sim 100–400 \) au may be dominated by either Newtonian or spin-induced perturbations, depending on the details of the cluster. Figure 11 shows the perturbations on the observables of a target star T1 with \( a_* = 126 \) au for three models of the cluster (M1, M2, and M3). The target star in each model is selected from \( N_{MC} \) MC runs, with \( \langle \delta_p R \rangle \sim \langle \delta_p \omega \rangle \) and \( \delta_p Z \sim \delta_p \Omega \) (these values are shown in Table 5). From top to bottom panel of Figure 11, the dominating factor of the signal gradually changes from stellar to spin-induced perturbations. Figure 11 suggests that the following are true for a target star in a cluster. (i) The stellar perturbations on the apparent position and the redshift always peak around the periapsis passage, and are caused mainly by the perturbed orbital period (see Section 4.1.2 and the following Section). (2) The combined perturbations are not similar to either Newtonian or spin-induced ones if those are comparable to each other, i.e., \( \langle \delta_p \kappa \rangle \sim \langle \delta_p \Omega \rangle \) or \( \delta_p Z \sim \delta_p \Omega \). Thus, a complex morphology of the signal strongly suggests that contamination by stellar perturbations occurs. (3) The details of the morphologies/evolutions of the Newtonian perturbations are quite different from those of the spin-induced effects. In principle, they can be separated from each other according to their distinctive features.

5.3. The Perturbed Orbital Elements of the Target Star

We describe the results of the stellar perturbations on the orbital elements in this section. We find that \( (\delta_p \kappa) \sim a_* \), where \( \kappa \) is any orbital element and \( \epsilon \) is a slope index depending on the details of the cluster. For models in Table 3, we found \( \epsilon \sim 1.9–2.5 \) if \( \kappa = a_* \), and \( \epsilon \sim 0.7–1.5 \) if \( \kappa = e, I, \Omega, \omega, \) or \( f_* \). The dependence of \( (\delta_p \kappa) \) on the parameters of the clusters is quite similar to that of the observables (see Table 5).

Table 5 shows the log-average stellar perturbations on the orbital elements and observables of the target star T1 with \( a_* = 126 \) au. From Table 5, the stellar perturbations cause the precessions of both the argument of periapsis \( (\delta_p \omega) \) and the orientation of the orbital plane (described by \( (\delta_p \Omega) \)) and \( (\delta_p \Omega_\perp) \)). We found that these results are roughly consistent with the following analytical arguments. The Newtonian precession of the argument of pericenter in each revolution is
given by (Madigan et al. 2011)
\[ \delta_p \omega_* = 1.72 \times \mathcal{F}(e_*, \gamma) \frac{4 \times 10^6 M_\odot}{2 - \gamma} \times \frac{M_p(r < a_*)}{10^3 M_\odot}. \]  
(15)

Here \( \mathcal{F}(e_*, \gamma) \) is a factor depending on the eccentricity of the target star and the density profile. In the case \( \gamma = 1.75 \), it is given by (Madigan et al. 2011)
\[ \mathcal{F}^{-1}(e_*, 1.75) = 0.681 + \frac{0.975}{\sqrt{1 - e_*}} + 0.373(1 - e_*). \]  
(16)

\( M_p(r < a_*) \) is the mass enclosed within a radius \( r = a_* \). For a target star with \( a_* = 126 \) au in cluster M1 or M2, \( M_p(r < 126 \) au) \( \sim 40 M_\odot \). After three orbits, its rms Newtonian precession is \( \delta_p \omega_* \approx 3 \times 2 \delta_p \omega_* \approx 0.16 \), which is roughly consistent with the numerical results in model M1 or M2. Note that Equation (15) assumes that the potential of the cluster is smooth and isotropic, which may not be fully satisfied in the simulation. The simulated clusters consist of a finite number of perturbers and thus the potential is somewhat anisotropic. This may explain the discrepancies between the results of simulations and that predicted by Equation (15).

The anisotropy of the potential can also lead to effects from resonant relaxations, which cause the precession of the orbital plane of the target star (e.g., Rauch & Tremaine 1996; Hopman & Alexander 2006). According to Merritt et al. (2010),

\[ \phi_{\text{per,}r} = a_p^{\text{rc}}(1 - e_*) \text{ and } \phi_{\text{per,}z} = a_p^{\text{zc}}(1 - e_*) \]  
in different

Equations (26) and (27)), in each revolution,
\[ \frac{|\delta L|}{L_c} \approx \beta_v \frac{m_p}{M_c} \sqrt{\mathcal{N}_p(r < a_*)}. \]  
(17)

where \( \beta_v = 1.8 \) and \( \frac{|\delta L|}{L_c} \approx (1 - e_*^2)^{1/2}(\delta_p l_2^2 + \sin l_2^2) \approx 0.11 \) (or 0.04). These results are consistent with the predictions of Equation (17), which are 0.1 for cluster M1 and 0.03 and for M2.

Similar to Section 4.1.2, we found that, in most of the MC simulations performed, the Newtonian perturbations on observables of the target star are caused mainly by the perturbed orbital period rather than the Newtonian orbital precessions. Take the target star T1 as an example. From Table 5 we can see that \( \langle |\delta_p f_1| \rangle \gg \langle |\delta_p l_2| \rangle, \langle |\delta_p l_2| \rangle, \langle |\delta_p f_1| \rangle \rangle \) or \( \langle \delta_p \omega_* \rangle \). By the rough estimations according to Equations (10)–(13), it is simple to show that the perturbations on observables due to \( \langle |\delta_p f_1| \rangle \) are orders of magnitude larger than those due to \( \langle |\delta_p f_1| \rangle, \langle |\delta_p l_2| \rangle, \langle |\delta_p \omega_* \rangle \rangle \).

5.4. The Critical Semimajor Axis

From Figures 9 and 10, we can see that the total perturbation is dominated by the spin-induced effects in the inner region of the cluster and by the stellar perturbations in the outer region. For each model there is a critical orbital semimajor axis of the target star where \( \langle |\delta_p R| \rangle = 0 \) or \( \langle |\delta_p Z| \rangle = 0 \). Denote \( a_p^{\text{rc}} \) and \( a_p^{\text{zc}} \) as the critical orbital semimajor axis for the position and redshift signals, respectively, then these values of the target star T1 or T2 and the corresponding pericenter distance \( r_{\text{per,}r} = a_p^{\text{rc}}(1 - e_*) \) and \( r_{\text{per,}z} = a_p^{\text{zc}}(1 - e_*) \) in different
models are shown in Table 4. If target stars have $a_*$ larger than the critical value, the detection of the spin-induced effects from the observables is likely not feasible as they are quite significantly submerged by the stellar perturbations.

As both the Newtonian and spin-induced perturbations are less significant for target stars with low orbital eccentricities, the resulting critical orbital semimajor axis of the target star T1 is found to be similar to those of the target star T2. For the clusters explored here, $a_*^{\text{cr}} \approx 120$–390 au and $a_*^{\text{cr}} \approx 100$–330 au for the star T1, or $a_*^{\text{cr}} \approx 100$–300 au and $a_*^{\text{cr}} \approx 110$–350 au for the star T2. As the different MC realizations lead to scatters of about $\sim$1 dex on the stellar perturbations, the critical orbital semimajor axis given above may vary by up to 25%.

Note that the critical orbital semimajor axis depends also on the spin parameters (its magnitude and orientation). The spin orientation assumed here ($i = 45^\circ$, $\epsilon = 180^\circ$) exhibits modest spin-induced effects. If alternatively we choose some other spin magnitude or orientation, then the orbital semimajor axis will be changed accordingly. Take target star T1 as an example. If the spin magnitude of the MBH is smaller, i.e., $a_* = 0.3$, then for the position signal (or redshift signal), the critical semimajor axis in models M1–M6 is given by $a_*^{\text{cr}} \approx 100$–330 au (or $a_*^{\text{cr}} \approx 60$–210 au). If we assume $a_* = 0.99$, $i = 72^\circ$, and $\epsilon = 91^\circ$, such that the spin-induced position displacement is most significant, then $a_*^{\text{cr}} \approx 200$–600 au. If we assume $a_* = 0.99$, $i = 38^\circ$, and $\epsilon = 46^\circ$, such that the spin-induced redshift difference is most significant, then $a_*^{\text{cr}} \approx 110$–340 au.

6. DISCUSSION

The high-order relativistic effects, e.g., frame-dragging and the quadrupole effects, can be used in verifying the theory of general relativity, testing the quasi-Kerr metrics and also the no-hair theory (e.g., Johannsen 2016). However, they should be measured from orbital motion of a target star after a clean removal of the competing perturbations caused by other perturbers. The contaminations of stellar perturbations cause
Other dynamical processes can also cause similar shifts in the apparent position and redshift of the target star: for example, gravitational waves and tidal dissipation. These effects are negligible for the S-stars considered in this paper, because they are important only for those S-stars in highly eccentric and/or extremely tight orbits, e.g., $a_{\text{per}} \lesssim 10^4 R_{\text{T}} \approx 40$ au (Psaltis 2012; Psaltis et al. 2013, 2016). It is still possible that there is an IMBH in the GC, with a mass around $10^{-4} M_\odot$. As the stellar perturbations are approximately proportional to the mass of the IMBH (see Figure 3), they can create perturbations on the target star that completely drown the spin effects if the mass of the IMBH is large enough.

Dynamical simulations suggest that pulsars possibly exist in the intermediate vicinity around the MBH in the GC (e.g., Zhang et al. 2014). If they are close enough to the MBH, the high-order GR effects, including the frame-dragging and quadrupole effects, can be probed by their precise timing signals. Detections of such pulsars and the measurements of their signal can possibly be made by the Square Kilometer Array (SKA) (e.g., Pfahl & Loeb 2004; Liu et al. 2012; Psaltis et al. 2016). However, the problems of stellar perturbations and also their removal from the timing signals should also be considered in these studies.

On the other hand, the measurements of stellar perturbations constrain simultaneously the mass profiles in the vicinity of the MBH in the GC, which are essential for studies of various important dynamical problems in the GC, e.g., the formation and dynamical evolution histories of the S-stars and stellar remnants (e.g., Perets et al. 2009; Madigan et al. 2009, 2011; Merritt et al. 2011; Zhang et al. 2013), the tidal disruptions of stars by black holes (e.g., Madigan et al. 2011; Bromley et al. 2012), and the rates of gravitational wave in-spirals (e.g., Hopman & Alexander 2006; Merritt et al. 2011).

### Table 4

| Model | $a_{\text{per}}$ (1) | $r_{\text{per}}$ (2) | $a_{\text{red}}$ (3) | $r_{\text{red}}$ (4) | $a_{\text{spin}}$ (5) | $r_{\text{spin}}$ (6) | $a_{\text{tot}}$ (7) | $r_{\text{tot}}$ (8) |
|-------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| M1    | 120                 | 360                 | 100                 | 300                 | 100                 | 1770                | 110                 | 1950                |
| M2    | 170                 | 520                 | 130                 | 400                 | 130                 | 2310                | 150                 | 2660                |
| M3    | 170                 | 520                 | 130                 | 400                 | 130                 | 2310                | 150                 | 2660                |
| M4    | 230                 | 700                 | 170                 | 520                 | 170                 | 3020                | 190                 | 3370                |
| M5    | 270                 | 820                 | 200                 | 610                 | 200                 | 3550                | 210                 | 3730                |
| M6    | 390                 | 1190                | 330                 | 1000                | 300                 | 5320                | 350                 | 6210                |

Note. The critical orbital semimajor axis or the distance at the pericenter of the target stars where the effects of Newtonian attraction are equal to the spin-induced ones. Columns (1)-(4): $a_{\text{per}}$ (or $a_{\text{red}}$) is the orbital semimajor axis when $\langle \delta R \rangle \approx \langle R \rangle$ (or $\langle \delta Z \rangle \approx \langle Z \rangle$) for the target star T1 (with $e_\text{s} = 0.88$), in units of au. $r_{\text{per}, \text{red}} = a_{\text{per}, \text{red}} (1 - e_\text{s})$ (or $r_{\text{per}, \text{red}} = a_{\text{per}, \text{red}} (1 - e_\text{s})$) is the corresponding distance of the pericenter, in units of $r_\text{c} \approx 0.04$ au. Columns (5)-(8): similar to columns (1)-(4) but for the target star T2 (with $e_\text{s} = 0.3$). See Table 1 for the parameters of these target stars.

### Table 5

| Name | $<\delta R_{\text{f3}}>$ (1) | $<\delta R_{\text{f1}}>$ (2) | $<\delta R_{\text{f2}}>$ (3) | $<\delta R_{\text{f1}}>$ (4) | $<\delta R_{\text{f3}}>$ (5) | $<\delta R_{\text{f2}}>$ (6) |
|------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| M1   | 14.7                          | 0.23                          | 0.14                          | 0.23                          | 10.9                          | 22.8                          |
| M2   | 5.5                           | 0.08                          | 0.06                          | 0.09                          | 4.1                           | 8.5                           |
| M3   | 5.5                           | 0.1                           | 0.06                          | 0.1                           | 4.1                           | 8.5                           |
| M4   | 2.1                           | 0.04                          | 0.03                          | 0.05                          | 1.6                           | 3.2                           |
| M5   | 1.0                           | 0.03                          | 0.01                          | 0.02                          | 0.8                           | 1.6                           |
| M6   | 0.5                           | 0.01                          | 0.004                         | 0.008                         | 0.4                           | 0.8                           |

Note. The stellar perturbations on the orbital motion and observables of a target star T1 with $a_{\text{s}} = 126$ au. The values for the orbital elements of the target star ($\omega$, $i$, $\Omega$, and $\omega_\star$) are shown in columns (1)-(4) in units of arcmin. The values for the position and redshift signals of the target star are shown in column (5) in units of $\mu$as and in column (6) in units of km s^{-1}, respectively. The standard deviations is $0.5-1$ dex for each values in the table. As a comparison, the rms spin position and redshift signals of this target star are given by $\langle \delta R \rangle = 9.6 \mu$as and $\langle \delta Z \rangle = 10$ km s^{-1}, respectively.

The S-stars discovered in the close vicinity of the MBH in the GC are anticipated to provide tight constraints on the MBH spin and metric from continuous monitoring of their orbits. However, the gravitational attractions of other stars and stellar remnants in this region may cause the orbit of a target S-star to deviate from GR predictions; thus, adequately modeling and removing them is quite essential. To understand the stellar perturbations comprehensively, here we consider both the spin-related relativistic effects and the Newtonian perturbations felt by the target stars, and their resulting perturbations on the observables of the target star, i.e., the apparent position in the sky plane and the redshift.

The relativistic numerical methods adopted here rely upon the framework of ZLY15. The gravitational attractions of the background stars/stellar remnants are considered by adding an additional perturbation term to the Hamiltonian equations of motion of the target star. The apparent orbital motion and the observed redshift of a target star are obtained by the ray-tracing techniques. We find that the simulated variations of the orbital elements of the target star due to the Newtonian perturbations resulting from the method adopted in this work are generally consistent with those from the PN method with corrections of order 2.0.

Investigations of the gravitational perturbations by a single perturber can provide helpful hints in understanding the properties and the nature of the stellar perturbations. In this case, the Newtonian perturbations on S2/S0-2 caused by S0-
102 are of particular interest. We find that the spin-induced effects on image position (or redshift) can be blurred by the gravitational perturbations from S0-102 alone if the mass of the latter is $\lesssim 0.6M_\odot$ (or $\gtrsim 0.2M_\odot$), which is very likely because the S-stars are found to be exclusively B-type stars with masses $\gtrsim 3M_\odot$.

The changes in the observables of S2/S0-2 result from the changes in its orbital elements. We find that the perturbed observables of S2/S0-2, which are caused by the Newtonian attractions of S0-102, are mainly ascribed to the change in its orbital period. Meanwhile, the Newtonian orbital precessions due to S0-102 induce a negligible difference in the observables of S2/S0-2. As a result, the Newtonian perturbations on the observables of S2/S0-2 peak around the time of the pericenter passage in each orbit and evolve quite differently from those of the spin-induced effects. We find that these conclusions also remain true for the general cases in which a target star is perturbed by a single or multiple disturbing object(s).

By performing a large number of MC simulations, we study the case of a hypothetical S-star inside the orbit of S2/S0-2 or S0-102 perturbed by a single perturber with various initial conditions. We find that the Newtonian perturbations on the observables of the target star are proportional to the mass of the perturber, and depend complexly on the orbital configurations of both the perturber and the target star. The Newtonian perturbations of a single perturber located inside the orbit of the target star are found to be much more significant than those caused by a perturber located outside. It is found that in some cases the Newtonian perturbations on the observables due to a single perturber with a mass of $10M_\odot$ are large enough to overwhelm the spin-induced effects (see Figures 6 and 7).

At present the mass distribution and its composition in the vicinity of the MBH in the GC are rather uncertain. By performing a large number of numerical simulations that consider a number of possible initial conditions, we investigate the stellar perturbations of a cluster of disturbing objects. We find that, for a given total mass of the cluster, the stellar perturbations due to a cluster of stellar-mass black holes (with masses of $1M_\odot$) are larger than those due to a cluster of low-mass main-sequence stars (with masses of $1M_\odot$). The stellar perturbations of a cluster with the cusp profile are generally larger than those of a cluster with the core profile, because the former contains more stars in the inner region. When the central MBH is maximally spinning, the Newtonian perturbation of a cluster can drown the spin-induced signals if the target star has an orbital semimajor axis larger than 100–400 au.

As shown in the numerical simulations performed in this study, the morphologies of the stellar perturbations seem quite different from the GR spin effects in the evolution of both the orbital elements and the observables, i.e., the signals of the apparent position and the redshift. Their different features and also the dependences on the model parameters suggest that, in principle, the stellar and the spin-induced perturbations are separable. We defer the separations of these two effects and accurate measurements of the spin parameters in the presence of Newtonian perturbations to future studies.

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