Kinematics of Horizon and Singularity and IR/UV Mixing in AdS

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Abstract

In arXiv:1512.02232 [hep-th] it was argued, based on the construction of a holographic c-function, that the curvature singularity of a black brane can be thought of as a trivial IR fixed point. This is dual to the gapped nature of the thermal state in the IR. So one can say that by taking one kind of low energy limit in the thermal CFT we are probing the near-singularity region. But there is another more conventional low energy limit which corresponds to probing the near-horizon region. Now, instead of one, if we think in terms of these two low-energy limits and take into account the fact that in AdS-CFT the only observables are CFT correlators then we can get a completely different interpretation of the curvature singularity. In a nutshell, the very long wavelength degrees of freedom in the thermal CFT carry information of both the near-horizon and the near-singularity regions, but, the field theory observer cannot, in principle, disentangle the information of the near-horizon region from the information of the near-singularity region using the the thermal CFT correlators. This can be interpreted as a very specific form of holographic "non-locality" in a black hole background which relates the "inside and the outside". We argue in the paper that owing to this "non-locality", the space-like curvature singularity along with its problems, which are all local in nature, completely disappear from the theory or get dissolved. But, the same "non-locality" now tells us that some of the "$e^{-S}$-effects" that one finds, for example, in the late time thermal two-point function, can be thought of as carrying complete information about "Planck-scale effects near the singularity". From the local EFT point of view this may be called "UV-IR-mixing" which is caused by the "non-locality". We also discuss its close connection to black hole complementarity.

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I. A TALE OF TWO INFRAREDS

A. RG-flow in the infalling-frame and Classical Black Hole Singularity

In this paper we consider black brane in AdS formed by collapsing matter which is dual to an approximately thermal state of the boundary CFT living on Minkowski space-time. We are interested in sufficiently late time when the bulk settles down to the static AdS-Schwarzschild black brane to a very good approximation.

In [21] it was suggested that from the holographic RG point of view the curvature singularity can be thought of as a trivial IR fixed point and the high temperature (IR) expansion of the renormalized logarithmic negativity of the dual CFT is a way to probe the region near the classical singularity. The suggestion was based on the following observations:

1) In AdS-CFT correspondence [1][3] radial direction in the bulk is identified with the field theory energy scale [8]. So moving along the radial direction can be thought of as renormalization group (RG) flow in the dual field theory. The irreversibility of the RG flow is encoded in the c-theorem [13] and a holographic c-function can be constructed in certain
FIG. 1: The figure shows a black brane in AdS formed from collapse. The arrow denotes the future bulk light-cone which starts at a boundary point at sufficiently late time and ends at the curvature singularity.

geometries [22]. In [23, 24] it was shown that if we consider a domain-wall geometry then the holographic $c$-function can be given an entropic interpretation in the following way: Consider a point on the boundary of the AdS and draw the future bulk light-cone of that point. Now construct space-like cross sections of the future light-cone and assign Bekenstein-Hawking entropy to each cross section. The cross sections are non-compact in this case and so the right quantity to consider is the entropy density. It turns out that this entropy density decreases monotonically as we move deeper into the bulk along the future directed null geodesic generators of the light-cone. The numerical value of the density coincides with the UV and the IR central charges of the dual field theory when evaluated in the UV region and the IR region of the domain-wall geometry. Hence this entropy density is a holographic $c$-function. The future bulk light-cone of a boundary point is also known as the past causal horizon and the monotonic decrease of the holographic $c$-function also follows from the second law of causal horizon thermodynamics [49]. So the dual of the field theory $c$-theorem may be identified with the second law of causal horizon thermodynamics in the bulk.

This has the virtue of being a covariant prescription for constructing a holographic $c$-function and can be immediately generalized to other geometries. The challenge here is of course to interpret these generalized $c$-functions in the dual field theory.

2) We can now apply this to an $AdS_5$ black brane [21]. Black brane is not a relevant
deformation of the field theory. But due to finite temperature scale invariance is broken and various quantities in field theory show interesting scale variation. We call this RG-flow.

In this case the future bulk light-cone of a boundary point ends in the curvature singularity (See Fig-1). Once again we can construct a holographic \( c \)-function as the Bekenstein-Hawking entropy density on the space-like slices of the light-cone. It turns out that the holographic \( c \)-function has the value \( a_{UV} \) near the boundary of AdS and then it monotonically decreases to zero at the curvature singularity. Therefore curvature singularity can be thought of as a trivial IR fixed point of a gapped system \cite{21}.

The gapped system is not difficult to identify in the dual field theory. The thermal state which is dual to the black brane behaves like a gapped system in many respects. For example, equal time two point correlation function at finite temperature decays exponentially if the separation between the insertion points is much larger than the inverse temperature. So inverse temperature acts as finite correlation length. Another manifestation of the gapped nature is the absence of any long-range quantum entanglement at finite temperature. This property turns out to be very useful for constructing potential candidate for the \( c \)-function in thermal CFT. The field theory \( c \)-function should have all the properties of the holographic \( c \)-function that we have constructed. So the UV value should be given by the central charge of the CFT and then it should decrease monotonically to zero in the IR. In \cite{21} it was pointed out, based on the calculations of \cite{44}, that in a two dimensional thermal CFT, the UV value of renormalized logarithmic negativity is given by the central charge of the CFT and the IR value is zero. So in two dimensions the renormalized logarithmic entanglement negativity is a potential candidate for the \( c \)-function. The vanishing of logarithmic negativity in the IR is just a reflection of the fact that it is an entanglement measure for mixed states \cite{50} and so can detect the absence quantum entanglement in the IR. In higher dimensions also we expect the renormalized logarithmic negativity to show the same asymptotic behavior based on this physical consideration. What is not clear is whether the decrease from the UV to the IR is monotonic or not although there are some numerical evidence that this may indeed decrease \cite{44}.

Another way to think about this is the following. Suppose we define an ”effective central charge” \( c_{eff} \) in the thermal CFT using say the renormalized logarithmic negativity. In the UV the value of \( c_{eff} \) is given by the central charge \( c \) of the CFT whereas in the IR it decreases to zero. In the gravity approximation in AdS\(_{d+1} \), \( c \sim \left( \frac{L_{AdS}}{L_{pT}} \right)^{d-1} \sim O(N^2) \). This is the UV
value of $c_{\text{eff}}$. Now as we go to longer distances in the CFT, $c_{\text{eff}}$ decreases and at some point in the deep IR, $c_{\text{eff}} \sim \left( \frac{k_{\text{eff}}}{L_{p}} \right)^{d-1} \sim O(1)$. So the IR of the thermal state, as probed by this $c$-function, is dual to a strongly coupled bulk region. In the black brane geometry it is natural to identify this as the region near the curvature singularity.

Another evidence for the gapped nature of the black hole interior comes from the tensor network construction of the thermofield double state [25] which is dual to an eternal black hole in AdS.

Now the above description, based on the construction of holographic $c$-function or its potential field theory candidate at finite temperature, suggests that the IR or very long wavelength degrees of freedom of the CFT at finite temperature encodes information about the region near the classical singularity. One can probe this by studying, for example, the RG-flow of quantum entanglement in the thermal state. Lack of long range quantum entanglement in the thermal state is manifested in the presence of the curvature singularity in the bulk. This is consistent with the well-known scale-radius duality in AdS-CFT [8] and we can think of this as scale-radius duality in the infalling frame where the IR of the field theory is identified with the curvature singularity.

The goal of this paper is to argue that this description of the classical singularity is essentially "classical". In quantum gravity, interpretation of curvature singularity seems to be drastically different and the surprising fact is that the presence of the horizon as a region of high redshift forces this interpretation upon us.

**B. RG-flow in the asymptotic frame and Black Hole Horizon**

From the point of view of the asymptotic observer space-time geometry ends at the horizon of the black hole. The asymptotic observer’s time or the Schwarschild time can be identified with the global (Minkowski) time of the field theory and no bulk object can cross the horizon in finite Schwarschild time. Therefore the standard time evolution in the field theory does not describe horizon crossing.

Scale-radius duality is an important component of AdS/CFT. It is well understood that in a black brane geometry, the near-horizon region corresponds to the IR of the dual field theory and the Schwarschild radial coordinate can be identified with the energy (RG) scale. This can be thought of as the scale-radius duality in the asymptotic frame. The most
prominent reason for this is that the near-horizon region is a region of high redshift. So a bulk object placed very close to the horizon has very little field theory energy. Moreover a boundary excitation with size of the order of the thermal scale or bigger can be thought of as residing in the near horizon region in the bulk as has been argued for example in [45, 46]. There is also strong indication coming from the holographic Wilsonian RG flow approach to various low energy phenomena in the dual field theory [47]. Now, this description of the scale-radius duality where the near horizon region appears in the IR is the one appropriate for an asymptotic observer. This should be contrasted with the scale-radius duality in the infalling frame in which the curvature singularity appears in the IR.

Let us now discuss a simple example which is helpful for the purpose of visualisation.

Consider RG flow of the thermal state in the CFT. A well known property of a thermal state is the absence of long range correlation or quantum entanglement. By long range we mean length scale of the order of inverse temperature or bigger. So if we integrate out UV degrees of freedom and come down to the thermal scale, the effective quantum state in the IR will be separable or unentangled. This state is classical from the point of view of quantum information theory. Entanglement or Von-Neumann entropy in this state reduces to thermal entropy and is a measure of classical or thermal correlations. Along the RG-flow the effective temperature also grows because temperature is a relevant parameter. This effective temperature can be thought of as the local temperature measured by a stationary observer in the bulk at a certain radial distance from the boundary. The temperature diverges as we move closer to the horizon. So these high temperature IR degrees of freedom of the field theory can be thought of as "located" in the near horizon region in the bulk. One way to visualize this is to compute the (renormalized) entanglement entropy in the IR. In the large-N limit one can use Ryu-Takayanagi prescription [48] to perform this computation. One has to take a subsystem of size much bigger than the thermal scale. For such a subsystem the dominant contribution to the entanglement entropy comes from the portion of the minimal surface in the bulk which touches the horizon. This is clear from the fact that for subsystem size much bigger than the thermal scale entanglement entropy crosses over to the thermal entropy. The important point is that the minimal surface for the stationary thermal state does not cross the horizon. An interesting description of this RG-flow appears in [41] from the point of view of entanglement renormalization.
II. INFORMATION THEORETIC DISTINCTION BETWEEN HORIZON AND SINGULARITY

FIG. 2: This is a cartoon of the Schwarzschild black brane geometry from the point of view of holographic-RG when we treat the classical and quantum correlations as "observables". There is a single UV boundary but two distinct IR regions in the interior corresponding to RG-flow in asymptotic (black) and infalling (red) frames. So "locality" in the "radial direction" or "energy-scale" breaks down in a very specific manner. We will argue that this is essentially a "classical".

Fig-2 summarizes our previous discussion. RG flow in a thermal state has two different bulk descriptions which are apparently contradictory. In one description where we consider the flow of classical correlations (e.g, entanglement or Von-Neumann entropy) the IR of the field theory can be identified with the near-horizon region whereas in a different description where we consider the flow of quantum correlations (e.g, some entanglement measure for mixed states like negativity) the IR can be identified with the region near the curvature singularity. It is natural to associate the first description with the asymptotic observer and the second one with the infalling observer.

The upshot of this whole discussion is that the very long wave-length degrees of freedom in a thermal state carry information about the near-horizon as well as the near-singularity regions. In other words, near-horizon and near-singularity regions can be thought of as manifestations of two different aspects (e.g, classical and quantum correlations, respectively) of the same IR degrees of freedom in the field theory. Classical and quantum correlation
is a pair of information theoretic quantities which can distinguish between near-horizon and near-singularity regions in the thermal CFT but there can be many more such pairs. But, although classical and quantum correlations can be quantified, there are no standard quantum mechanical observables which can measure them.

III. TOWARDS RESOLUTION OF THE CLASSICAL SINGULARITY

Since string theory is a consistent theory of quantum gravity, it should be able to make sense of classical black hole singularity. This does not include, for example, the singularity of negative mass Schwarzschild solution [20] but curvature singularity hidden behind a horizon should be resolved in string theory. What we mean by resolved is that if we ask a physical question about the singularity in string theory then we should get a meaningful finite answer [18, 19]. In general the set of such "physical questions" is difficult to determine. One of the potential difficulties is that a question which is physical within the framework of effective field theory may not remain so when embedded in non-perturbative string theory. We will now give some heuristic arguments which suggest that this is indeed the case, at least for Schwarzschild black brane (hole) in AdS.

AdS-CFT duality tell us that the observables of quantum gravity in asymptotically AdS space-times are the correlation functions of the boundary CFT (or string scattering amplitudes in the bulk). Therefore the answer to a physical question in bulk quantum gravity can be obtained by computing some set of correlation functions in the CFT. So if the CFT correlation functions do not contain answer to some bulk question then we will say that the question is not physical.

We have seen that in the thermal CFT the information about the near-singularity region is encoded in the very long wave-length (IR) degrees of freedom. But it is also true that the same long wave-length degrees of freedom carry information about the near-horizon region. So let us consider a thermal correlator in the CFT and focus on a kinematic regime where the correlator is able to probe the very long-wavelength degrees of freedom in the thermal state. To extract information about the near-singularity region from the correlator one has to subtract the contribution of the near-horizon region. In other words, if we want to extract information about the near-singularity region we need to have a unique decomposition of the value of the correlator into two parts, one of which has information about the near-horizon
FIG. 3: This is a cartoon of the Schwarzschild black brane geometry from the point of view of holographic-RG when we use the exact observables of the theory which are the correlation functions of the CFT. Due to non-separability of correlators, two distinct IR regions (classical horizon and classical singularity) in Fig-2 have been replaced by a single IR region which may either be called quantum horizon (in the asymptotic frame) or quantum singularity (in the infalling frame). This suggests that classical singularity by itself is not a physical problem.

region and the other one about the near-singularity region. But such a unique decomposition of correlation functions is not meaningful or rather not observable. The simplest reason being that CFT (or any QFT) gives us a prescription for computing the value of a correlation function only. In this particular context, we have to further decompose the value into two parts based on some criterion determined by say the properties of (thermal) classical (→ horizon) and quantum (→ singularity) correlations (entanglement) of the IR degrees of freedom. But, due to the principle of linear superposition of quantum states, there are no standard quantum mechanical observables that can measure such correlations.¹ This suggests that a field theory observer cannot, in principle, disentangle the information of the near-singularity region from the information of the near-horizon region, that is contained in the CFT correlation functions.² As a result of this, from the CFT point of view, both the

¹ See also [42] for an interesting discussion on some related issues.
² This is somewhat similar to the following situation. Suppose we are given the cross-section for 2 → 2 scattering of electrons in QED. Now can we determine that how much of the cross-section is due to the particle nature and how much is due to the wave nature of the electrons? The answer is obviously no as
near-horizon and the near-singularity regions completely lose their separate identities. This can also be stated as: from the CFT point of view near-horizon and near-singularity regions are non-separable. In the rest of the paper we will refer to this as "non-separability of CFT correlation functions". Therefore in the non-perturbative CFT description of the black hole, the concept of the near-singularity region, by itself, completely disappears from the theory and the same is true for the near-horizon region. Instead, they are replaced by a single concept (Fig-3) which may be called either the quantum horizon or the quantum singularity depending on whether we choose the asymptotic or the infalling frame for the bulk interpretation of the CFT correlation functions. This should be contrasted with the description of the black hole in the framework of local effective field theory where the near-horizon and the near-singularity regions are clearly distinct space-time regions with very different physical properties.

We would like to emphasize that this does not mean that the horizon is replaced by singularity or vice-versa. Rather this should be interpreted as a very specific type of "non-locality" in the bulk which "relates the observations of the asymptotic and the infalling observers". This is actually a statement about the nature of observables in AdS quantum gravity (or perhaps in any holographic \cite{1-7} or asymptotic description) and so it concerns only the kinematics of the theory. Also the fact that the quantum gravity in the bulk is dual to a standard linear quantum mechanical system is crucial for this relation to exist.

Although we are interpreting the non-separability of CFT correlators as a kind of "non-locality" in the bulk, this is actually a misnomer. In particular, we would like to emphasize that this is not the type of non-locality which can be described as a "non-local theory living on a background space-time and the result of non-locality is signal propagation outside the light-cone." We hope to clarify this more in later sections.

Let us now discuss the implications for the classical singularity behind the horizon which is the main focus of this paper.

For the time being let us think about the black hole from a global point of view. This may also be called the "nice-slice point of view". This description is refereed to an imaginary observer who can see all of the Penrose diagram. Now from effective field theory point of view singularity is a UV problem. Since curvature in the near-singularity region can be of

\footnote{long as we use the mathematical framework of quantum mechanics to calculate the cross-section.}
the order of the Planck scale, large stringy and quantum effects are expected in this region. On the contrary, the near-horizon region of a sufficiently large black hole is almost flat space and so the physics in this region is expected to be well-described by the standard low energy theory. Now we have already described in the previous sections that from the holographic RG point of view singularity can be thought of as a trivial IR fixed-point of a gapped system. So a natural guess could be that the new stringy degrees of freedom, which live near the singularity, "smooth it out" to a kind of Planck scale fuzz. These new degrees of freedom are precisely those missed by pure gravity.

The whole point of the above discussion is to emphasize the important point that in the framework of effective field theory the usual questions about the classical singularity and their expected answers are all local in nature. Therefore one can use these local bulk questions and answers to single out the near-singularity region in the black hole geometry. But we can easily convince ourselves that this is in sharp contradiction with the non-separability of the CFT correlation functions. Due to the same reason, local bulk statements like "space-time curvature blows up at the singularity" or "quantum fluctuation of the metric becomes large near the singularity" are also unphysical from the CFT point of view. It is important to note that "unphysical" does not mean wrong. What it means is that within the framework of holography it is not possible to decide, in principle, whether the above statements are true or false. From the bulk point of view we would like to say that, classical black hole singularity, by itself, is not a physical problem in quantum gravity. Very roughly speaking the "non-locality" of the holographic description in the presence of a black hole does not allow us to zoom in near the singularity and as a result the classical singularity, which is essentially local in nature, is completely dissolved. We have used the term "dissolved", rather than "resolved", to emphasize the fact that the classical singularity does not appear to be removed or smoothed out by any dynamical mechanism. But what is happening is that the singularity as a local space-time concept has completely disappeared from the theory due to "mixing" with the horizon. The reason for this seems to be the linearity of the dual description and the fact that quantum gravity is holographic in nature.

Now from the bulk point of view, the reason that such local or EFT questions about the classical singularity cannot be posed in quantum gravity, seems to be related to a fundamental limitation on the type of local measurements that an infalling observer can perform. This limitation has nothing to do with quantum mechanical uncertainty in the bulk, but
appears to be closely related to the holographic bound on the number of degrees of freedom in quantum gravity. Non-separability of CFT correlation functions is a reflection of this limitation. The standard picture of local space-time physics can be trusted in the regime where this limitation can be ignored in the same way that classical description is trustworthy in the regime where quantum mechanical uncertainty can be ignored. Although we do not know what this limitation on bulk measurements is but the duality is powerful enough to let us guess some of its consequences and do precise calculations.

IV. DIAGNOSTICS FOR THE "CLASSICAL-SINGULARITY"

From the previous discussions it is clear that for an unambiguous space-time interpretation of CFT correlation functions we have to refer to a particular bulk observer because the global description is not consistent with the non-separability of correlators. This is an important point. But, as we will discuss, even with the choice of a particular observer space-time interpretation is only approximate if our arguments are correct. In other words, exact observables do not have exact space-time interpretation. The reason is that there are always some effects, that an observer can detect, which have no conventional space-time interpretation. So let us discuss the asymptotic observer.

Although our arguments in the previous sections involving (holographic)RG-flow is very well-suited for black branes, we will assume that the non-separability of CFT correlation functions is also true for an AdS-Schwarschild black hole with compact horizon. Another way to think about this will be that the CFT lives on a sphere of radius $R$ and the temperature $T$ is $>> R^{-1}$. Now we will give a heuristic argument to suggest that at late time the thermal correlation functions contain contributions that can be thought of as "coming from the region near the classical singularity". In particular we will consider the thermal two-point function which is also a diagnostic for information loss in AdS-CFT [9].

Let us consider the thermal two-point function $< O(t)O(0) >_{th}$ where $O$ is some operator in the CFT. The operator $O(0)$ creates an excitation in the bulk at Schwarschild time $t = 0$ which then falls towards the black hole. With respect to the asymptotic observer the excitation does not cross the horizon in finite Schwarschild time, but after a sufficiently long time we can safely assume that the excitation starts probing the near-horizon region of the black hole. Now according to our previous discussions, if we use the exact observables of the
theory which are the CFT correlation functions or string scattering amplitude in the bulk, then the near-horizon and the near-singularity regions are non-separable. Now owing to this non-separability of correlators, an excitation which is in the near-horizon region can also be thought of as ”probing” the region near the singularity. So the late time limit of a thermal correlation function, in particular the two-point function, has contributions ”coming from the region near the classical singularity”.

Although the late time value of the two-point function has contributions that can be thought of as coming from the ”near-singularity region”, the asymptotic observer will naturally interpret them as some effects which originate near the horizon. The point here is that this ”horizon”, at late time, can no longer be completely described by the standard low energy effective field theory in the asymptotic frame because of its ”mixing” with the singularity. So the horizon that the asymptotic observer observes (at late times) is not the ”horizon of the effective field theory” but may be called the ”quantum horizon” as we have discussed in the last section. From the point of view of global effective field theory description of black hole this may be called ”IR/UV mixing” where IR refers to the classical near-horizon region and UV refers to the classical near-singularity region where Planck scale effects are supposed to be dominant. This mixing is visible to a low energy asymptotic observer. Now what is the order of magnitude of such effects. This can be estimated by looking at the deviation of the CFT correlation function from the bulk effective field theory prediction at late time and it is natural to associate some of the $O(e^{-S_{BH}} \sim e^{-N^2})$- effects \cite{9-14} with the ”classical singularity behind the horizon”. So in the large-N limit this can be a very small effect. But the important point is that this effect from ”behind the horizon” is required for the consistency of the holographic description of black hole from the point of view of the asymptotic observers.

The reader may be worried by an apparent violation of causality (and locality) in the above description. But let us emphasize that the ”information” about the singularity is not physically or dynamically transferred from the near-singularity region to the near-horizon region because physical transfer implies that we are able to distinguish between the near-horizon and the near-singularity regions. But our previous discussions suggest that this distinction is unphysical due to the non-separability of CFT correlators. So we should not describe this as causality-violation. This is different from the situation in quantum field theory (QFT) where non-locality generically leads to causality violation. In fact strictly
speaking these effects may not have any space-time interpretation in the sense of canonical QG.

Before we conclude we would like to emphasize that whatever we have said so far is strictly meant for a black hole formed from collapse. For a two-sided black hole the story seems to be quite different \[15\]-\[17\]. We do not completely understand the reason behind this difference. But, it is important to have some understanding of this. A first step in this direction will be to generalize our information-theoretic approach to the two-sided case perhaps using the idea of computational complexity \[35\] and various bulk reconstruction methods \[36\]-\[40\] that have been studied in the literature.

V. FATE OF THE SINGULARITY (?)

If our arguments are correct then the ”classical singularity” is dissolved due to the ”non-locality” of the holographic description.

Our arguments further suggest that in the large-N limit quantum gravity effectively makes the black hole singularity ”slightly naked”. This, in a sense, is consistent with the Cosmic censorship. The asymptotic observer does not see any ”violent” Planck scale effect. Instead, due to the absence of local degrees of freedom or the holographic bound on the number of degrees of freedom in quantum gravity, the ”Planck scale effects in the near-singularity region” are turned into ”soft (\(\sim e^{-N^2}\)) IR effects in the almost flat near-horizon region”. This is then described by the asymptotic observer as deviation from the bulk low-energy effective field theory prediction at late time. And moreover, the completeness of the CFT description tells us that such deviations carry complete information about the ”Planck scale effects near the singularity”. The surprising fact is that this should now be accessible to low energy observers, at least in principle. Calculation of such effects requires knowledge of the non-perturbative theory which in this case is the dual CFT.

Another way of saying this will be that the outside description is a complete description although there are some late-time effects which, in the conventional global space-time description, can be thought of as coming from the ”region near the singularity”. But, as we have already argued, this, in a sense, is ”non-local” but not causality-violating. Hopefully the recent progress in understanding the late-time behaviour of thermal correlators \[10\]-\[13\] will shed more light on the nature of such effects.
VI. SOME COMMENTS ON BLACK HOLE COMPLEMENTARITY

The reader may have noticed that our observations so far are strikingly similar to the Black Hole Complementarity idea [26–28] which says that the space-time location of the information depends on the choice of the observer. In the asymptotic frame the infalling matter ends up in the near-horizon region whereas in the infalling frame the same matter smoothly passes through the horizon and finally hits the singularity. In spite of this, no single observer should be able to know about both the end-points. This leaves open the possibility that an imaginary global observer who can see all of the Penrose diagram can apparently see ”duplication” of the information. Since no such observer exists, the question is can the theory, which we use to calculate say the unitary S-matrix for an evaporating black hole, allow two distinct fates of the infalling matter? If we now apply the arguments of this paper then the answer is clearly no. We can easily convince ourselves that non-separability of correlation functions does not allow the CFT to distinguish between these two fates. This is a schematic argument but we hope to have conveyed the sense in which ”duplication” does not happen in QG and it is very closely related to the fact that the bulk is dual to a standard linear quantum mechanical theory. It will be very interesting to connect the picture that we have tried to produce with the one given in [34].

We would also like to know if the arguments in this paper have anything to say about the firewall paradox [31, 32]. We hope to have something to say about this in future.

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