A plug-and-play, scalable control method for AC island microgrid

Yiwei Feng · Xin Wang

Abstract In this paper, a scalable, plug-and-play (PnP) and system-stable synthesis control method is proposed for the AC island microgrid consisting of a distributed generator units (DGUs) and loads connected by power lines. The proposed method only requires a limited global parameter design controller, so the design process of the controller is decentralized, so that the addition and removal of DGU only need to adjust the controller of the DGU with its physical connection, without affecting the overall stability of the microgrid. In addition, we used the Lyapunov method to proved the stability of the close-loop system. Finally, the simulation verification was carried out with the MATLAB/Simulink and specific cases.

Keywords scalable; controller; stability; plug-and-play; AC island microgrid.

1 Introduction

With the gradual depletion of fossil energy and the increasing environmental pollution, countries all over the world have begun to pay attention to environmental protection, efficient and flexible way of power generation microgrid. Green power generation technologies based on wind, solar, hydrogen and hydropower have proven to be technically and economically feasible, integrating these technologies into the advantages of microgrids [1]. In modern microgrid power systems, renewable energy and energy storage units are used extensively to reduce energy consumption and improve the reliability, power quality and energy efficiency of existing power systems. Due to its scalability, competitive investment costs and flexible operation, the penetration of the microgrid is becoming more and more high. Inevitably, the stable operation of microgrid system has become the most important concern, however, the stable operation of microgrid mainly involves the stability of microgrid voltage and frequency. There are two modes of operation of the microgrid, one is...
grid-connected mode, and its voltage and frequency are determined by the main grid. For another off-grid (island) operating mode, voltage and frequency stability is a big challenge, because in this operating mode, the microgrid voltage and frequency are given by its internal DGUs.

Microgrids are expected to be more robust and economical than traditional centralized grids. However, generators or micro-sources used in microgrids are usually renewable or unconventional distributed energy sources, and some environmentally directly affected interferences, such as wind or solar, are also introduced into the power system, greatly increasing the difficulty and complexity of frequency and voltage control [2]. Because of the composition of the microgrid and the power generation characteristics, it is widely distributed in a certain area, and for the control of voltage and frequency, the [3] proposes a control method for the parallel operation of an island power grid or an infinite bus inverter. The control method is based on the droop control method, controls the distributed power output of the microgrid, controls the reactive power, does not need the communication coordination between the units, can achieve the goal of micro-source PnP and peer control, and ensures the unity of power balance and frequency in the mode of island of the microgrid, and has simple and reliable characteristics.

As the number of parallel inverters in the microgrid increases, the signal lines between inverters become more complex. However, the traditional droop control method of the parallel-running inverter is premised on the line resistance which is much smaller than the line’s resistance, and does not consider the condition that the line impedance value will vary depending on the system voltage level. Moreover, the basic control equation of the traditional droop control method run in parallel is mathematically calculated by the voltage of the common node Point of Common Coupling (PCC) of each parallel inverter, but the power supply is relatively far apart in the power system, resulting in the feedback signal not being able to measure the voltage value at the PCC, so that the mean slower control algorithm in the power system cannot be accurately realized. Based on the electronic power processor, [4] proposed a non-droop, low-voltage residential micro-grid plug-and-play control method, in making full use of energy, distribution at the same time, so that each electronic power processors can run steadily and independently, therefore, the microgrid can be efficient. However, stable operation, adapt to power and load changes, and automatic mode-to-mode switching makes it requires some digital control capabilities, high demands on power converters, and no communication capacity, for the state of adjacent power generation units and the overall supply and demand relationship can not have a good sense, not suitable for the microgrid system with multiple power generation units. Based on some concepts in communication theory, the distributed algorithm of sharing power generation tasks in an optimized way between multiple distributed energy resources in the microgrid is proposed in [5], and the proposed algorithm is as effective in practical application as the centralized full-communication algorithm. But this approach, which is close to the full centralization of communication, requires a central controller to communicate extensively with the controlled unit and collect a large amount of global information, and some unnecessary information is a heavy burden on the processor. A decentralized control scheme is proposed in [6] to ensure the stable operation of the island microgrid, which is made of the interconnection of distributed power generation units. The local controller adjusts the voltage and frequency at the co-coupling point of each DGU, ensuring the stability of the overall island microgrid. The control design process is decentralized because, in addition to the two global scalars, the synthesis of local controllers uses only the corresponding DGU and the information of the lines connected to it. This method supports plug-and-play operations, but when DGUs are plugged in or removed, the controller of the adjacent DGUs that is physically
connected to the DGU needs to be adjusted, because this method is dependent on the information of the power line. The [7] proposes a fully dispersed and scalable voltage regulation method for the DC microgrid, which supports plug-and-play operation without the need to adjust the controller of the DGU with which it is physically connected. The main features of the proposed controller are: 1) DGs controller is completely dispersed, local controller only uses local measurements; 2) controller ensures the stability of the entire system; 3) the controller allows the PnP function of DGUs in the microgrid, which is fully in line with the idea of plug-and-play control methods. Under PnP operation, different DGUs randomly access or disconnect the microgrid without affecting the overall stability of the microgrid.

The [8] designs a controller for the AC island microgrid, and proposes a microgrid control method based on plug-and-play, which thinks that the system is completely observable and the system state is measurable, but because of the complexity of the system or other factors, the state of the system is not easily measured directly. In this paper, a PnP control method is developed for the island AC microgrid, which is independent of the power line, where in which the local controller of DGU only requires limited global scalar parameters and information about adjacent DGU. Because the system is complex and the state variables are difficult to measure, we use distributed state estimators to estimate the state of the system and design the controller. Whenever a new DGUs are inserted or removed, adjacent DGUs with physical connections do not need to readjust their controllers, because this method is independent of electrical or power line information.

This paper is organized as follows. In section II, we propose an approximate model based on QSL and estimate the state of the system using the distributed state estimator (DSE). In section III, a completely decentralized PnP controller is designed using the estimated system state. In section IV, we simulate the specific example and verify the performance of PnP controller.

Notation.

The identity and null matrices of size $N$ are represented by $I_N$ and $0_N$, respectively, while we use $A > 0$ to indicate a null matrix of appropriate size. The inequality $A \in \mathbb{R}^{n \times n}$ means that the matrix is positive definite.

2 Microgrid model

In this section, we describe the electrical model of a AC island considered in [8]. We assume three-phase electrical signals without zero-sequence components and balanced network parameters.

It includes the following parts: a DC voltage source (modeling a generic renewable resource), a controlled VSC and a local load, connected to the Point of Common Coupling (PCC) through series an RLC filters. We further assume that loads $I_{Li}$ are unknown and act as current disturbances for DGU. The Island microgrid is composed of $N$ DGUs indexed by the set $\mathcal{D} = \{1, \ldots, N\}$. Then, we call two DGUs neighbors if there is a power line connecting their PCCs and denote with $\mathcal{N}_i \subset \mathcal{D}$ the subset of neighbors of DGU $i$. We indicate with $G_i$ is the undirected electric graph induced by the neighboring relation over the node set $\mathcal{D}$, whose internal elements are composed of $G_{el}$. As shown in the right dashed frame of Fig. 1, $R_{ij}$ and $L_{ij}$ are the resistance and inductance of the power line connecting DGUs $i$ and $j$, respectively. In the PCC of each area, we use shunt capacitor $C_{ti}$ to attenuate the effects of the load voltage high frequency harmonics. Let $\omega_0$ be the reference network frequency. Next, we provide a model of DGU $i$ where all electric variables are represented in the $dq$ reference frame rotating with speed $\omega_0$. The state of DG $i$ is $x_i = \left[\begin{array}{c} v_i^d \\ v_i^q \\ i_{di}^t \\ i_{qi}^t \\ p_i^k \end{array} \right]$. 
and collects the d and q components of the PCC voltage and the filter current. The control input is $d_i = [I_{di}, I_{qi}]^T$ and the load current, which acts as an exogenous disturbance, is $d_i = [I_{di}, I_{qi}]^T$. The measured output and controlled variables are $y_i = x_i$ and $z_i = [V_{di}, V_{qi}]^T$ respectively.

By using QSL approximations of power lines one obtains the LTI system

$$\Sigma_{DCU}: \begin{cases} \dot{x}_i(t) = A_{ii}x_i(t) + B_iu_i(t) + M_id_i(t) + \xi_i(t) \\ y_i(t) = C_i x_i \\ z_i(t) = H_i y_i(t) \end{cases}$$

(1)

where $\xi_i = \sum_{j \in A_i} A_{ij}(x_j - x_i)$ accounts for the coupling with neighboring DGUs through PCC voltages and

$$A_{ii} = \begin{bmatrix} 0 & \frac{1}{C_{ti}} & 0 \\ -\omega_0 & 0 & \frac{1}{C_{ti}} \\ 0 & \frac{1}{L_{ti}} & -\omega_0 \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} \frac{R_{ij}}{Z_{ij}} & \frac{x_i}{Z_{ij}} & 0 \\ \frac{-x_i}{Z_{ij}} & \frac{R_{ij}}{Z_{ij}} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 & 0 \\ \frac{1}{C_{ti}} & 0 \\ 0 & \frac{1}{C_{ti}} \end{bmatrix}, \quad M_i = \begin{bmatrix} -\frac{1}{C_{ti}} & 0 & 0 \\ 0 & -\frac{1}{L_{ti}} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C_i = I_{si}, \quad H_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(2)

where $X_{ij} = \omega_0 L_{ij}$ and $Z_{ii} = |R_{ij} + iX_{ij}|$. 

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[Image of the electrical model of DGU i, power line ij, and local PnP Voltage and Frequency controller.]
The overall Island microgrid model is given by

$$\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Md(t) \\
y(t) &= Cx(t) \\
z(t) &= Hy(t)
\end{align*}$$

(4)

where \( x = [x_1^T, \ldots, x_N^T]^T \in \mathbb{R}^{nN} \), and vectors \( u, d, y, \) and \( z \) are similarly defined. Matrices \( A, B, M, C \) and \( H \), which can be easily derived from (2) – (3).

The model (4) describes the interactions of the DGU with the rest of the microgrid over a wide range of possible operating conditions. It should be noted that variable \( d(t) \), in addition to modeling the effects of the microgrid topological changes and parametric mismatches, also embeds other sources of uncertainties such as load current demand changes. The disturbance is assumed to be unknown but the rate of disturbance is bounded. Due to the complexity of system dynamics and difficulty in measuring the state, we use the DSE to estimate the state of the system. We use a DSE that supports PnP operations for state estimation. We design an observer for each DGU \( i \) to calculate the control input \( u \) based on the information transmitted by the measured output \( y \) and its adjacent DGUs, and need to guarantee the closed-loop stability of the system. Here, we use Luenberger observer to estimate the local state of the system. First, we rewrite equation (4) as:

$$\Sigma_i : \begin{align*}
\dot{x}_i' &= A_{ii}x_i + B_{ii}u_i + \sum_{j\in\mathcal{N}_i} A_{ij}x_j + Md_i \\
y_i &= C_{ii}x_i
\end{align*}$$

(5)

where \( x_i' \) stand for \( x \) at time \( t+1 \), \( A_{ii} \in \mathbb{R}^{n_i\times n_i}, \) \( j \in \mathcal{N}_i, i \neq j \). As we can see, the measurement output \( y_i \) depends only on the local state variable \( x_i \), hence, the system is output-decoupled. For the system (5), we define a local Luenberger observer as follows

$$\Sigma_i : \begin{align*}
\dot{x}_i &= A_{ii}\tilde{x}_i + B_{ii}u_i - L_{ii}(y_i - C_{ii}\tilde{x}_i) + \\
\sum_{j\in\mathcal{N}_i} A_{ij}x_j - \sum_{j\in\mathcal{N}_i} L_{ij}(y_j - C_{ij}\tilde{x}_j)
\end{align*}$$

(6)

We add a binary entry \( \bar{\eta}_{ij} \in \{0, 1\} \), such as the Equation (7)

$$\Sigma_i : \begin{align*}
\dot{x}_i &= A_{ii}\tilde{x}_i + B_{ii}u_i - L_{ii}(y_i - C_{ii}\tilde{x}_i) + \\
\sum_{j\in\mathcal{N}_i} A_{ij}x_j - \sum_{j\in\mathcal{N}_i} \bar{\eta}_{ij}L_{ij}(y_j - C_{ij}\tilde{x}_j)
\end{align*}$$

(7)

where \( \tilde{x}_i \in \mathbb{R}^{n_i} \) is the state estimate, \( x_i' \) is the state of the next moment, \( L_{ij} \) is the gain matrix. As mentioned above, the adjacent DGUs distributed observer will exchange information, and we will add a binary entry, and when \( \bar{\eta}_{ij} = 1 \), the estimator will accept the information from the adjacent DGUs to estimate the current state. When \( \bar{\eta}_{ij} = 0 \), the estimator will not accept the information from adjacent DGUs, reducing the number of information transmitted. So, the matrix consisting of \( \bar{\eta}_{ij} \) is a sparse matrix, which greatly reduces the time and complexity of computing while guarantee accuracy.

Designing the distributed state estimator, the main problem is the need to guarantee the following conditions. For a given \( \bar{\eta}_{ij} \), calculate \( L_{ii} \) and \( L_{ij}, j \in \mathcal{N}_i \) such that \( \delta_i \leq 1 \).

$$\delta_i = \sum_{j\in\mathcal{N}_i} \sum_{k=0}^{\infty} \left\| (A_{ii} + L_{ii}C_i)^k (A_{ij} + \bar{\eta}_{ij}L_{ij}C_i) \right\|_\infty < 1$$

(8)

For the constraint (8), it is always satisfied [9]. If the subsystem is decoupled, then the constraint (8) is automatically satisfied, and if the subsystem is not decoupled and has additional coupling effects, then an additional condition is required to guarantee the existence of a Robust Positive Invariance (RPI) to guarantee that the DSE can be designed.
Because the variables of the system are complex and not easily measured, we use the state of DSE estimation to design the controller. The goal of local controllers is to guarantee offset-free tracking of constant references $z_{\text{ref}}(t) = \bar{z}_{\text{ref}}$, when $d(t)$ is constant. Note that, as show in Fig.1, local references $z_{\text{ref}} = V_{\text{ref}i}$ are $dq$ signals and therefore provide complete information about the desired amplitude and frequency for the voltage $V_i$. Typically, $Z_{\text{ref}i}$ is provided by higher control layers devoted to power flow regulation. For offset-free tracking, We argue the model with integrators. The dynamic model of the integrator is as follows

$$
\dot{v}_i(t) = e_i(t) = z_{\text{ref}i}(t) - z_i(t)
$$

$$
= z_{\text{ref}i}(t) - H_i C_i \ddot{x}_i(t)
$$

The augmented DGU model is

$$
\Sigma_{\text{DGU}} : \begin{cases}
\dot{x}_i(t) = A_{ii} x_i(t) + B_i u_i(t) + M_i d_i(t) + \xi_i(t) \\
\dot{y}_i(t) = C_i x_i(t) \\
z_i(t) = H_i \ddot{y}_i(t)
\end{cases}
$$

(9)

where $\dot{x}_i = [\dot{x}_i^T, \dot{v}_i^T]^T \in \mathbb{R}^6$ is the state, $\ddot{x}_i$ is the state of DSE estimation, $\ddot{y}_i = \ddot{x}_i$ the measurable output, $\dot{d}_i = [\dot{d}_i^T, \dot{z}_{\text{ref}i}^T]^T \in \mathbb{R}^4$ collects the exogenous signals and $\ddot{\xi}_i = \sum_{j \in N_i} A_{ij} (\ddot{x}_j - \ddot{x}_i)$ Moreover, matrices in (9) have the form

$$
\tilde{A}_{ii} = \begin{bmatrix}
A_{ii} & 0 \\
-H_i C_i & 0
\end{bmatrix},
$$

\begin{align}
\tilde{\mathcal{A}}_{11,i} &= \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix}, \\
\tilde{\mathcal{A}}_{22,i} &= \begin{bmatrix}
\frac{L_i}{R_{iz}} & \omega_0 \\
-\omega_0 & \frac{L_i}{R_{iz}}
\end{bmatrix}, \\
A_{ij} &= \begin{bmatrix}
A_{ij} & 0 \\
0 & 0
\end{bmatrix},
\end{align}

(10)

$$
\tilde{C}_i = \begin{bmatrix}
C_i & 0 \\
0 & M_i
\end{bmatrix},
$$

(11)

$$
\tilde{M}_i = \begin{bmatrix}
M_i & 0 \\
0 & I_2
\end{bmatrix},
$$

(12)

Since the pair $(\tilde{A}_{ii}, \tilde{B}_i)$ is controllable[6], system (9) can be stabilized in absence of coupling. The overall augmented system is obtained from (9) as

$$
\begin{cases}
\tilde{\dot{x}}(t) = \tilde{A} \tilde{x}(t) + \tilde{B} u(t) + \tilde{M} d(t) \\
\ddot{y}(t) = \tilde{C} \tilde{x}(t) \\
z(t) = \tilde{H} \ddot{y}(t)
\end{cases}
$$

(13)

where $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{M}$ and $\tilde{H}$ collect variables $\tilde{x}_i, \tilde{y}_i$ and $\tilde{d}_i$, respectively, and matrices $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{M}$ and $\tilde{H}$ are derived directly from (9).
3 Stability analysis

We equip each DGU $\Sigma^{DGU}_i$ with the following state-feedback controller.

$$u_i(t) = K_i y_i(t) = K_i x_i(t)$$

The local controller $C[i]$ introduces a decentralized control architecture that requires only the state of the system $\Sigma^{DGU}_i$. Moreover, we need to guarantee the closed-loop asymptotic stability of the system to ensure the offset-free tracking of the system. According to Lyapunov’s theory, if there is a symmetric matrix $P_i$ such that

$$\text{(15)}$$

where $P_i > 0$, and close-loop system be a feedback interconnection of (13) with (14). However, due to the existence of coupling terms $\xi_i$, coupling conditions may undermine the stability of the closed-loop system. In order to guarantee the stability of the system, we will prove that the closed-loop system is asymptotically stable under the following conditions so that guarantee equations (15) and (16) are equivalent

$$\text{(16)}$$

where $P = \text{diag} (P_1 \cdots P_N)$, $K = \text{diag} (K_1 \cdots K_N)$ and $\tilde{A}, \tilde{B}, K$ collect matrices $\tilde{A}_i, \tilde{B}_i, K_i$

Next, we will consider the stability of the closed-loop system given below

$$\text{(17)}$$

Assumption 1. For (15), design $P$ has the following structure

$$P_i = \begin{bmatrix} \rho_{i2} & 0 & 0 \\ 0 & \rho_{22,i} & \rho_{23,i} \\ 0 & \rho_{32,i} & \rho_{33,i} \end{bmatrix}$$

where, $\rho_{i,j}$ denotes an arbitrary entry, $P_i > 0, \rho_{i,j} > 0$ and $P_i^T = P_i$.

**Theorem 1**: If Assumption 1 is fulfilled and the matrix

$$\Pi = \begin{bmatrix} \tilde{A}_i - \sum_{j \in N_i} \frac{\rho_{ij}}{\rho_{ij}} \tilde{B}_j \tilde{C}_j & 0 & 0 & 0 \\ 0 & \tilde{A}_{i+1} & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \tilde{A}_{NN} \end{bmatrix}$$

is negative definite, that is $\Pi < 0$, then the closed-loop system (17) is asymptotically stable.

**Proof**: According to Lyapunov theory

$$\dot{V}(\tilde{x}) = \tilde{x}^T \{ (\tilde{A} + \tilde{B}K)^T P + P(\tilde{A} + \tilde{B}K) \} \tilde{x}$$

In the presence of the coupling term, we define $\tilde{A}$ in the following form

$$\tilde{A} = \tilde{A}_i + \tilde{A}_{i+1} + \tilde{A}_N$$
where $\hat{A}_a = \text{diag} (\hat{A}_{a_i} \cdots \hat{A}_{a_N})$ collects the local dynamics only. $\hat{A}_b = \text{diag} (\hat{A}_{b_i} \cdots \hat{A}_{b_N})$, takes into account the dependence of each local state on neighboring DGUs. $\hat{A}_c$ represents the possible coupling effects of other DGUs in the region, and $i \neq j, i,j \in N$.

For equation (19), we use the condition (20), which can be obtained

$$\left( \hat{A}_a + \hat{B}K \right)^T P + P \left( \hat{A}_a + \hat{B}K \right) + \hat{A}_a^T P + P \hat{A}_b + \hat{A}_c^T P + P \hat{A}_c < 0$$

(21)

Note that team (a) is a block diagonal matrix that collects on the diagonal all left sides of (15). Hence, team (a) is negative definite matrix. For part (b) of equation (21), it can be obtained by calculation

\[
\hat{A}_{xi} = \frac{1}{c_{xi}} \begin{bmatrix}
\sum_{j \in N_i} \frac{R_i}{c_{xij}} & 0 \\
\sum_{j \in N_i} \frac{X_i}{c_{xij}} & 0
\end{bmatrix}
\]

(22)

\[
\hat{A}_{xi}^T P_i + P_i \hat{A}_{xj} =
\begin{bmatrix}
\sum_{j \in N_i} \frac{R_{ij}}{c_{xij} z_{ij}} & 0 \\
0 & -\sum_{j \in N_i} \frac{R_{ij}}{c_{xij} z_{ij}}
\end{bmatrix}
\]

(23)

We can calculate that the Equation (23), that is, the (b) part of equation (21), is negative semidefinite.

Regarding the (c) part of equation (21), each $(i,j)$ block is equal to

\[
\begin{cases}
\hat{A}_{ji}^T P_j + P_j \hat{A}_{ij} & \text{if } j \in N_i, i \neq j \\
0 & \text{otherwise}
\end{cases}
\]

(24)

by the calculation

\[
P_i \hat{A}_{ij} = \begin{bmatrix}
\frac{R_{ij}}{c_{xij} z_{ij}} & \frac{X_{ij}}{c_{xij} z_{ij}} & 0 \\
\frac{X_{ij}}{c_{xij} z_{ij}} & \frac{K_i}{c_{xij}} & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(25)

\[
\hat{A}_{ji}^T P_j + P_j \hat{A}_{ij} = \begin{bmatrix}
\frac{R_{ij}}{c_{xij} z_{ij}} & \frac{X_{ij}}{c_{xij} z_{ij}} & 0 \\
\frac{X_{ij}}{c_{xij} z_{ij}} & \frac{K_i}{c_{xij}} & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(26)

It turns out that term (c) can be made arbitrarily close to zero by setting coefficients $\rho_i$ small enough. That means that the (c) part of equation (21) is equal to zero. We add up the calculations of the three parts (a) (b) (c) to correspond to the elements, and we got a matrix like equation (18). Because $\hat{A}_a = \text{diag} (\hat{A}_{a_i} \cdots \hat{A}_{a_N})$ is negative definite matrix, we can see
that matrix $\Pi$ is a diagonal matrix with only elements on the diagonal and both negative values, so $\Pi$ is negative definite matrix. Therefore, there is always exist $\rho_i$ to guarantee that (21) is fulfilled. Concluding, we have shown that (21) holds.

**Remark.** The asymptotic stability that we have demonstrated here is different from the[6], and the information of the power line is needed in the literature, for example, line are mainly inductive, so that $R_{ij}/Z_{ij}^2 \approx 0$, in this paper, only need to set the coefficient $\Pi$ appropriately, we can get the system’s asymptotic stability.

We used the Lyapunov method to prove the stability of the system. But for the design of the controller, the gain matrix $K$ is very important to the controller, and it also affects the overall performance of the control strategy. For the design of the controller we need to address the following problem.

**Problem 1.** The solution and calculation of matrix $K$ is an important problem for the design of controller $V_{[i]}$ such that closed-loop subsystem is asymptotically stable. In the case that the matrix $K$ has a solution, controller $V_{[i]}$ is designed so that Assumption 1 and Equation (16) hold.

**Theorem 2.** If Assumption 1 is fulfilled, there are slack matrices $Y_i = P_i^{-1}$ and $G_i = K_i Y_i$, where $G_i$ is the electric undirected graph, then Problem 1 can be solved, that is, the gain matrix $K$ of the local controller has the solution $K_i = G_i Y_i^{-1}$, and existing a positive definite matrix $\Gamma_i$ such that follow conditions hold.

\[
\begin{align*}
\mathcal{O}_i & \quad \min \\
Y_i, G_i, \gamma_i, \gamma_2, \beta_i, \zeta_i & \quad \alpha_i \gamma_i + \alpha_2 \gamma_2 + \alpha_3 \beta_i + \alpha_4 \zeta_i,
\end{align*}
\]

\[
Y_i = \begin{bmatrix} \rho_i^{-1} I_2 & 0_2 & 0_2 \\ 0_2 & y_{22,i} & y_{23,i} \\ 0_2 & y_{31,i} & y_{33,i} \end{bmatrix} > 0 \tag{27}
\]

\[
\begin{bmatrix} Y_i \dot{\lambda}_i^T + G_i^T B_i^T & \dot{\lambda}_i - G_i Y_i & -\Gamma_i \\ -\beta_i I & G_i - I \end{bmatrix} < 0, \quad \begin{bmatrix} Y_i & I \\ I & \zeta_i I \end{bmatrix} > 0 \tag{28}
\]

\[
\gamma_i > 0, \quad \gamma_2 > 0, \quad \beta_i > 0, \quad \zeta_i > 0 \tag{29}
\]

where $\alpha_k, k = 1, \ldots, 4$ are positive weights, $\Gamma_i = \text{diag}(\gamma_i, \gamma_2)$ is a positive definite matric, which can be used as a boundary condition for robust stability when the inequality (28) is fulfilled. The constraint (29) is always feasible, and its purpose is to restrain excessive control behavior, because there is an upper bound $||K_i||_2 = \sqrt{\beta_i \zeta_i}$.

**Proof:** Inequality (16) is equivalent to having a positive matrix $Y_i$ such that inequality (31) hold.

\[
(\bar{A}_i + \bar{B}_i K_i)^T P_i + P_i (\bar{A}_i + \bar{B}_i K_i) + \Gamma_i^{-1} \leq 0 \tag{31}
\]

using Schur complement can be obtained

\[
\begin{bmatrix} (\bar{A}_i + \bar{B}_i K_i)^T P_i & P_i (\bar{A}_i + \bar{B}_i K_i) \\ 0 & I - \Gamma_i \end{bmatrix} \leq 0 \tag{32}
\]

where $\Gamma_i = \text{diag}(\gamma_i, \gamma_2)$ represents robust stability margin, the discussion of it is the focus of future work. $P_i$ is already defined in Assumption 1. since the inequality is nonlinear in $K_i$ and $P_i$, so we introduce two new matrices

\[
Y_i = P_i^{-1}, \quad G_i = K_i Y_i \tag{33}
\]
where \( G \) represents undirected electric graph, containing elements that are \( g_{ij} \). Therefore, we can get the solution of the controller gain matrix \( K_i = G_iY_i^{-1} \), have the same structure as \( P_i \).

\[
Y_i = \begin{bmatrix}
    \rho_i^{-1}I_2 & 0_2 & 0_2 \\
    0_2 & y_{22,i} & y_{23,j} \\
    0_2 & y_{23,j} & y_{33,j}
\end{bmatrix}
\]  

(34)

By pre-and post-multiplying (32) with \( [Y_i 0] \), one can obtain

\[
\begin{bmatrix}
    Y_iA_i^T + G_i^TB_i^T + \hat{A}_iY_i + \hat{B}_iG_i & Y_i \\
    -\Gamma_i & 0
\end{bmatrix} \leq 0
\]  

(35)

The constraint (28) introduces the concept of the undirected electric graph, which makes DGUs more specific and operable in inter-plugging or connectivity/disconnecting, and makes the stability of the closed-loop subsystem guarantee accordingly. We found that the constraints (27)(28)(29)(30) only depend on the local fixed matrix \((\hat{A}_i, \hat{B}_i)\) and the local design parameters \((\alpha_{1i}, \alpha_{21i}, \alpha_{3i}, \alpha_{4i})\), which makes the design of the controller independent of each other in the case of global parameter determining.

In the previous sections we designed a plug-and-play controller that would ensure the stability of the local microgrid (Consisting of several DGUs subsystems and not connected to the grid), and now we’ll analyze its specific operation. When there is a new \( \Sigma_{DGU}^{N+1} \) that wants to access the local microgrid system, it is equipped with a controller \( C^{N+1} \). If \( \Sigma_{DGU}^{N+1} \) meet the criteria (e.g., can solve the Problem 1 compute \( K_i \)), it can be connected to the local microgrid system. In the previous section we mentioned that the restrictions only depend on \((\hat{A}_i, \hat{B}_i)\) and \((\alpha_{1i}, \alpha_{21i}, \alpha_{3i}, \alpha_{4i})\), then the new access, and the \( \Sigma_{DGU}^{N+1} \) equipped with controller \( \sigma_i \), we can stabilize the system by adjusting only a limited number of local controllers.

When one or a few \( \Sigma_{DGU}^k \) disconnected from the local microgrid operation, the situation is similar to the access, when \( \Sigma_{DGU}^k \) removed, Only a limited number of other DGUs controllers that are physically connected to the DGUs need to be updated or adjusted without any impact or failure on the overall stable operation of the island microgrid.

4 Simulation results

To verify the performance of the proposed control approach, we consider an islanded inverter-interfaced microgrid consisting of 8 DGUs with meshed topology.

![Fig. 2 scheme of the microgrid composed by 8 DGUs and plugging-in of \( \Sigma_{DGU}^{[8]} \)](image)

The simulation case studies are carried out in MATLAB/SimPowerSystems Toolbox. Then, local voltage controllers are designed through the convex optimization problem given in (27)(28)(29)(30), which is solved using YALMIP toolbox to solve.
We used eight DGUs equipped with PNP controller for the case study. Further, we used RL load instead of constant current load to get closer to the real situation. To demonstrate the capability of the proposed control strategy in PnP operation of DGs. We refer to the parameters in [10] to set DGU 1-8. we assume that DGU 6 is plugged in system at $t = 0.2$ s with DGUs 5 and 7. Then, DGU 6 is plugged out of the system at $t = 0.4$ s and due to this operated, all the connections attached to DGU 6 are disconnected. The $dq$ component of the voltages at PCCs 5, 6, 7 are shown in Fig.3, Fig.4, Fig.5 respectively. It can be seen
when DGU 6 is connected at $t = 0.2$ s, the voltage changes of the connected DGU 5 and 7 are shown in Fig. 4 and Fig. 5. When DGU 6 disconnects from DGU5 and 7 and exits the system at $t = 0.4$ s, the system can still operate stably. The results verify that the voltage controllers guarantee stability and provide a desirable performance according to the IEEE standards.

Further, we tested the controller’s performance by changing the local load. The local load exists as a parallel RLC network. The load resistances R at PCC4 in the three phases are equally changed from 152 to 76 Ω at $t = 0.5$ s. As a result of this change, the DG4 and its neighbors DG voltage changes are shown in Fig. 6, Fig. 7, Fig. 8. The results show that under the condition of load change, the microgrid system can still recover quickly and run steadily. The results demonstrate the good performance of the controllers we design.

**Fig. 6** Dynamic responses of DG 4 due to load change at $t = 0.5$s

**Fig. 7** $d$-component of the load voltages at PCC7.
5 Conclusions

In this paper, we have proposed a completely distributed voltage and frequency control method for the island AC microgrid, which is based on the QSL approximation model of the microgrid. In particular, we have proved the closed-loop stability of the system through the Lyapunov theory, and verified through simulation that the proposed control scheme provides the stability and expected performance of the closed-loop system under nominal conditions, and can also resist some Changes, such as random access and removal of DGs, changes in microgrid load.

In the context of today’s cyber-physical system, our next step is to combine the microgrid and network control system, study the influence of the network system on the microgrid control, and realize the remote control and cluster control of the microgrid. Interesting directions for future microgrid research are also include voltage control dynamics, secondary frequency control schemes, extensions to lossy networks, as well as the use of more involved classes of Lyapunov functions that can provide further flexibility in the combinational analysis of networked control microgrid systems.

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