Light propagation and interaction modelling in ring fibre nonlinear microcavities

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Abstract. Usage of the "Cabaret" explicit-implicit numerical scheme is proposed to model field dynamics both in microcavities and fibre cavities. Two opposite running waves are investigated with a number of the nonlinear effects included in the model, with potential to include more if necessary.

1. Introduction
Modern communications network is a complex system that requires a lot of problems being solved to reach the required levels of the information being transferred per second, with the demand of it and thus the load on the network growing every year. The situation can basically be summarized by the requirement of the data transmission to be within the Tb/sec range, while the devices employed for that has to be as small as possible, preferably microscopic or even nanoscale level. And the biggest potential in the terabit communications seems to belong to the optical frequency combs [1]. Furthermore, generating Kerr combs in the nonlinear cavities [2, 3] could create the combs with tens of GHz in spacing, but that can also create a 'vicious circle' of sorts, because these combs are prone to strong phase noise [4, 5, 6], resulting in a new set of the data transmission problems. Experiments show [7] that, despite the mentioned difficulties, it is the Kerr combs that are the best suited to answer the modern communication demands.

So, optical connections capable of transmitting such data amounts are the most apparent way to get past the bottlenecks in the world communication networks and computational centres. One could use wavelength-division multiplexing (WDM) with tens or hundreds of channels together with the advanced data modulation formats to achieve the required terabit speeds, but with the energy consumption remaining quite optimal.

Distributed feedback lasers are a common method to create optical WDM line. However, it is quite problematic to make them fully silicon based, because they would then require multiple interfaces between various parts, leading to the total size growth. Moreover, spectral efficiency starts to suffer as well. One way to solve such problems would be to use the OFCs as a WDM source, because each of the spectral lines in a comb could be modulated on its own [1, 8, 9].

Hence it is obvious that being able to predict the field propagation inside of the cavities has an immense practical value. The task is made more difficult due to the high nonlinearity of the microcavities, resulting in the numerical only investigations being possible, and in the process it is common to look for the models that both are adequate in the process simulation, but which do
not consume a lot of time during the calculations. The previous trend [10, 11, 12, 13, 14, 15, 16] was to use the modal approach, but the result is a system of up to hundreds of nonlinear bound equations, and solving those numerically is a daunting task. Modal method is one of the spectral ones, but there is an alternative way, which is to use a transport equations difference scheme, what have been already used to simulate Raman and SBS lasers [17, 18, 19]. This article further develops our model [17], and we discuss the results obtained with the approach.

2. Model introduction
The ultra-short pulse theory uses mostly two ways to solve problems of the kind: slowly varying amplitude and slow envelopes. As we have already state in our previous articles [17, 18], the left side of the transport equations is as follows:

$$\frac{\partial E}{\partial t} + v_g \frac{\partial E}{\partial z} - i \omega'' \frac{\partial^2 E}{\partial z^2} + \ldots = 0,$$

(1)

Here \(v_g\) - group velocity, \(\omega'' = -\beta'' v_g^3\), \(\beta'' = -\frac{\lambda^2}{2\pi c D^2}\), where \(D^2\) - is group velocity dispersion parameter [20]. Thus, the pulse equations within a microcavity are given as follows:

$$2i \left( \frac{\partial F}{\partial t} + v \frac{\partial F}{\partial z} \right) + D \frac{\partial^2 F}{\partial z^2} + 2\chi \left( |F|^2 + 2|B|^2 \right) F = 0,$$

(2)

$$2i \left( \frac{\partial B}{\partial t} - v \frac{\partial B}{\partial z} \right) + D \frac{\partial^2 B}{\partial z^2} + 2\chi \left( 2|F|^2 + |B|^2 \right) B = 0.$$

The boundary conditions for the equations are:

$$F(0) = \sqrt{1-R}\sqrt{1-r}F(L) + \sqrt{R}\sqrt{A}\sqrt{1-r} + \sqrt{\tau}B(0),$$

$$B(L) = \sqrt{1-R}\sqrt{1-r}B(0) - \sqrt{\tau}(1-r)F(L) + \sqrt{Rr}\sqrt{1-R}\sqrt{A}$$

(3)

Detailed description of the model was provided by us before [21]. In the current investigation of two opposite running waves, we include in our model the ring cavity itself (1), nonlinear...
and dispersive; we also include the coupler (2), and a regular intracavity mirror (3) to simulate the light scattering. Modulation instability can also be present in the system [17, 20, 22], due to the negative GVD in the medium, resulting in the solution instability, while intensity stays time-constant. Their combined effect leads to the complicated dynamics under the investigation. The effective second order difference scheme "Cabaret" [23] is used to solve the problem, that we have already proven possible [21]. Computational algorithm stability was proven by having less than 1% of numerical losses after more than a 1000 cavity roundtrips.

3. Simulation results

Figures 2 and 3 show the unidirectional simulation results from the experiment start up to the appearing of the opposite wave born of the nonlinearity. It can be noted that a characteristic solitonic-like comb has been formed, with its spikes spaced roughly at the roundtrip time, resulting in a OFC appearing in the signal.

![Figure 2. Pulse propagation in the single direction regime.](image2)

![Figure 3. OFC appearence due to the nonlinear interaction. Phase modulation coefficient is extremely low ($\chi = 0.001$).](image3)

Next we introduce counterpropagating wave and simulate cavity rotation, which would lead to the increase in the nonreciprocal phase shift. Wave interface coefficient $\chi$ in (2) is increased from 0.001 to 0.25. That results in the OFC deformity and transition to the chaotic regime. Modeling results are presented in Figures 4 and 5. In both cases phase coefficient $\chi = 0.25$. Wave interface coefficient $r = 0.001$.

![Figure 4. Chaotic regime after the second wave was introduced.](image4)

![Figure 5. Detailed field profile of the clockwise wave interacting with the counterclockwise one.](image5)

Final step was to add Rayleigh scattering to the system. It is simulated by inserted mirrors in every $\Delta z$ interval with the phase being given a new random value after the scattering occurs.
To deduce how it influences the fields, we increase the wave interface coefficient $r$ from 0.001 to 0.05. The results are demonstrated in Figures 6 and 7. $\chi = 0.25$. Coupling coefficient $r = 0.001$. Rayleigh loss coefficient is always 0.0001.

**Figure 6.** Several overlapping combs interfacing with each other and contributing phase noises into the signal.

**Figure 7.** Detailed view of the field profile.

4. Conclusion

In conclusion we would like to state that our numerical model not just possesses increased stability, but it is very easy to add nearly any kind of a nonlinear effect in the investigated system without a noticeable growth in the computation times. Our further goals are to first make sure that our model fits the existing experimental data and refine it if necessary, and then attempt to find a promising regime that could have a significant practical value.

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