On the Interaction of a Bonnor–Ebert Sphere with a Stellar Wind

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Abstract

The structure of protostellar cores can often be approximated by isothermal Bonnor–Ebert spheres (BES), which are stabilized by an external pressure. For the typical pressure of $10^4 k_B T$ in low molecular clouds, cores with masses below $1.5 M_\odot$ are stable against gravitational collapse. In this paper, we analyze the efficiency of triggering gravitational collapse with a nearby stellar wind, which represents an interesting scenario for triggering low-mass star formation. We analytically derive a new stability criterion for a BES compressed by a stellar wind, which depends on its initial nondimensional radius $\zeta_{\text{max}}$. If the stability limit is violated the wind triggers a core collapse. Otherwise, the core is destroyed by the wind. We estimate its validity range to $2.5 < \zeta_{\text{max}} < 4.2$ and confirm this in simulations with the SPH-Code GADGET-3. The efficiency of triggering a gravitational collapse strongly decreases for $\zeta_{\text{max}} < 2.5$ since in this case destruction and acceleration of the whole sphere begins to dominate. We were unable to trigger a collapse for $\zeta_{\text{max}} < 2$, which leads to the conclusion that a stellar wind can move the smallest unstable stellar mass to $0.5 M_\odot$ and that destabilizing even smaller cores would require external pressure larger than $10^4 k_B T$. For $\zeta_{\text{max}} > 4.2$ the expected wind strength according to our criterion is small enough that the compression is slower than the sound speed of the BES and sound waves can be triggered. In this case our criterion somewhat underestimates the onset of collapse and detailed numerical analyses are required.

Unified Astronomy Thesaurus concepts: Star formation (1569)

1. Introduction

A Bonnor–Ebert sphere (BES; Bonnor 1956; Ebert 1955) is an often-used theoretical model in simulations for protostellar cores, which is defined as an isothermal, spherically symmetric gas distribution with density $\rho(r)$ that is self-gravitating, in hydrostatic equilibrium and supported by the pressure of the ambient medium. Especially attractive is its well-defined density profile (see Figure 1) as well as its stability behavior under a uniform external pressure. Despite the turbulent nature of the interstellar medium observations show that some molecular cloud cores are in hydrostatic equilibrium and that their density distributions follow a Bonnor–Ebert profile (Alves et al. 2001).

Using the equation of hydrostatic equilibrium one can derive a density distribution $\rho(r)$, which can be nondimensionalized such that it depends on the nondimensional radius $\zeta$. In theoretical studies, one then assumes that there exists an external pressure $P_{\text{ext}}$ that stabilizes the BES and therefore the density profile is cut off at a chosen $\zeta_{\text{max}}$, where the internal pressure is $\rho(\zeta_{\text{max}}) c_s^2 = P_{\text{ext}}$. Here $c_s$ is the isothermal sound speed of the core. Bonnor (1956) and Ebert (1955) showed that the BES is unstable for $\zeta_{\text{max}} > 6.45$, which means that perturbations can grow and the BES finally collapses, while for smaller radii it stays stable. Previous studies can be divided into two categories. On the one hand studies analyzing isolated BES and on the other hand studies analyzing BES that interact with other objects. The main focus of the first type is a better understanding of the evolution of the collapse of BES (Hunter 1977; Ogino et al. 1999; Banerjee et al. 2004; Keto & Caselli 2010) or the derivation of new stability criteria for modified BES such as nonspherical BES (Sipilä et al. 2011, 2015, 2017; Nejad-Asghar 2016). The second type focuses on BES growth and induced star formation, e.g., by mergers (Burkert & Alves 2009).

Many studies showed that radiation from nearby stars can ionize the outer layers of a BES, which leads to an additional pressure force that can trigger the collapse of the BES or disperse it (Kessel-Deynet & Burkert 2003; Gritschneder et al. 2009; Bisbas et al. 2011; Krumholz et al. 2014; Ngoumou et al. 2014). Flow driven triggered star formation is another mechanism that can induce the collapse of a BES (Frank et al. 2015). Here gas flows from distant supernovae or stellar winds collide with the cloud core and act as an additional ram pressure source. The likelihood of collapse and star formation depends on the Mach number of the flow since higher Mach numbers can lead to instabilities in the BES that can even disperse the original cloud (Boss et al. 2008; Boss & Keiser 2013; Dugan et al. 2017). Ngoumou et al. (2014) analyzed the combined effect of stellar winds and ionizing radiation on a BES. They concentrated on the regime of low Mach number stellar winds, which means that the flow can be approximated by an additional external analytic pressure and the flow itself does not have to be simulated directly. In their simulations, which did not include a stabilizing pressure of the ambient medium, the wind itself was not able to trigger a collapse of the BES and they argued that ionization is the main force behind triggering star formation.

In this paper, we use the stellar wind model of Ngoumou et al. (2014) and show that the wind alone can in fact trigger a collapse if we add the stabilizing external pressure from the ambient medium. We assume that the wind is generated by a star at a distance $d$ with a mass-loss rate $\dot{M}$ and a wind velocity $v_w$. In Section 2 we derive analytically a stability criterion for the BES that only depends on these parameters as well as on the radius $R$ of the BES, its isothermal sound speed $c_s$, and its...
initial nondimensional radius $\xi_{\text{max}}$. We assume here $d \gg R$, which means that the wind can be approximated to be parallel. We also estimate the range of $\xi_{\text{max}}$ for which this criterion should work. In Section 3 we present simulations with the SPH-Code GADGET-3 (Springel 2005) in which we analyze the stability of BES for different $\xi_{\text{max}}$, and stellar properties. We find here a good agreement with the analytical criterion derived in the previous section. In Section 4 we discuss the implications of our results on the stability of protostellar cores in molecular clouds.

2. Analytic Derivation of the Stability Criterion

2.1. Definition of the Bonnor–Ebert Sphere

To fulfill the properties described above, the BES has to fulfill the equation of hydrostatic equilibrium, Poisson's equation and an isothermal equation of state:

$$\frac{1}{\rho} \nabla P = -\nabla \Phi, \quad (1a)$$

$$\nabla^2 \Phi = 4\pi G \rho, \quad (1b)$$

$$P = \rho c_s^2. \quad (1c)$$

In the case of spherical symmetry the first and third equation lead to

$$\rho(r) = \rho_c \exp\left( -\frac{\Phi(r)}{c_s^2} \right) \quad (2)$$

with the central density $\rho_c$ and the definition $\Phi(0) = 0$ for the gravitational potential. By plugging this into Equation (1b) we find:

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d \Phi}{dr} \right) = 4\pi G \rho_c \exp\left( -\frac{\Phi(r)}{c_s^2} \right). \quad (3)$$

By introducing the dimensionless variables $\psi = \Phi/c_s^2$ and $\xi = (4\pi G \rho_c/c_s^2)^{1/2} r$ we can simplify Equation (3) to a special form of the Lane–Emden equation

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\psi}{d\xi} \right) = \exp(-\psi). \quad (4)$$

which can be solved numerically with the boundary conditions $\psi(\xi = 0) = 0$ and $(d\psi/d\xi)_{\xi=0} = 0$. In Figure 1 we show the result $\rho(r)/\rho_c = \exp(-\psi(r))$ that can be well approximated within the stable region by:

$$\frac{\rho}{\rho_c}(\xi) = e^{-0.8444\xi}(1 + 0.8477\xi + 0.1961\xi^2 - 0.07308\xi^3$$

$$+ 0.01252\xi^4). \quad (5)$$

The BES is typically confined by an external pressure $P_{\text{ext}}$ that determines the radius $R$ of the sphere by the condition $\rho(R)c_s^2 = P_{\text{ext}}$:

$$R = \left( \frac{c_s^4}{4\pi G \rho_c} \right)^{1/2} \xi_{\text{max}} \quad (6)$$

with $\psi(\xi_{\text{max}}) = \psi_{\text{max}} = -\ln(P_{\text{ext}}/(\rho_c c_s^2))$. The total mass of the sphere is

$$M(\xi_{\text{max}}) = 4\pi \int_0^R r^2 \rho(r) dr' = 4\rho_c^{-1/2} \left( \frac{c_s^2}{4\pi G} \right)^{3/2} \xi_{\text{max}} \exp\left( -\frac{\psi(\xi_{\text{max}})}{c_s^2} \right) \quad (7)$$

and can be made dimensionless by the definition

$$m(\xi_{\text{max}}) = \frac{1}{\rho_c c_s^4} \frac{G^2 3^2 M}{c_s^4} = \left( 4\pi \frac{\rho_c}{\rho(R)} \right)^{-1/2} \left( \frac{c_s^2 \exp(-\psi)}{d\xi/d\xi} \right)_{\xi=\xi_{\text{max}}}. \quad (8)$$

$m(\xi)$ is shown in the right panel in Figure 1 and can be approximated by:

$$m(\xi) = \begin{cases} 0.00131\xi^2 + 0.109\xi^3 - 0.0291\xi^4 & 0 \leq \xi \leq 2 \\ e^{0.1536(\xi-2)}(0.4012 + 0.3678(\xi - 2) - 0.1178(\xi - 2)^2 + 0.01015(\xi-2)^3) & 2 \leq \xi \leq 5.5 \\ 1.16503 + 0.04056(\xi - 5.5) - 0.02852(\xi - 5.5)^2 + 0.005152(\xi - 5.5)^3 & 5.5 \leq \xi \leq 6.45 \end{cases}. \quad (9)$$
It reaches a maximum \( m_{\text{max}} = 1.18 \) for \( \xi_{\text{max}} = 6.45 \), which corresponds to a density contrast \( \rho_c/\rho(R) = 14 \), and all configurations with \( \xi_{\text{max}} > 6.45 \) lead to a gravitational collapse.

2.2. Derivation of the Stability Criterion

The starting point for our analysis is a BES in equilibrium that is stabilized by an external pressure

\[
P_{\text{ext}} = \frac{C^8 m^2}{G^3 M^2}. \tag{10}
\]

To first approximation one could add a uniform pressure \( P_W \) representing the effect of the wind. In this case, \( m \) can be increased until the BES becomes unstable for \( m > m_{\text{max}} = 1.18 \).

The critical uniform \( P_W \) for a BES with initially \( m = m_0 \) is therefore given by:

\[
P_{W,\text{crit}} = P_{\text{ext}} \left( \frac{m_{\text{max}}^2}{m_0^2} - 1 \right), \tag{11}
\]

which we also show in Figure 2. However, the parallel wind impacts the clump from one side. It therefore does not lead to an isotropic pressure and the pressure \( P_W \) that is required to trigger a collapse cannot directly be calculated but has to be approximated. We use the ansatz:

\[
P_W = C \frac{M v_w}{4 \pi d^2}, \tag{12}
\]

where \( C \) is a nondimensional parameter that should typically be \( O(1) \). \( C = 1 \) corresponds to the maximum wind pressure at any point of the BES, \( C = 1/3 \) corresponds to the wind pressure averaged over the half of the surface of the BES that is directly affected by the wind, and \( C = 1/6 \) corresponds to the wind pressure averaged over the whole surface. We now can rewrite the original external pressure term using Equation (6) and the density \( \rho_h \) at the surface of the BES:

\[
P_{\text{ext}} = C^2 \rho_B = C^2 \frac{\rho_B}{\rho_c} = C^4 \frac{\xi_{\text{max}}}{\rho_c} \frac{\rho_B}{4 \pi G R^2 \rho_c}. \tag{13}
\]

Plugging this into Equation (11) we finally find for the condition that the BES is unstable:

\[
\frac{GMv_w R^2}{d^2 \xi_s^4} > \frac{\xi_{\text{max}}^2}{C^2} \frac{\rho_B}{\rho_c} \left( \frac{m_{\text{max}}^2}{m_0^2} - 1 \right). \tag{14}
\]

The right-hand side of this inequality is only a function of \( \xi_{\text{max}} \) and \( C \) and can easily be calculated. Figure 4 shows the dependence of the left side of \( \xi_{\text{max}} \).

2.3. Expected Validity Range of the Stability Criterion

In the derivation of inequality (14) we used several approximations, which we hide in the parameter \( C \). We assumed that we still can use the Equations (10) and (11) for a uniform pressure, which means the external wind pressure is only a perturbation to the hydrostatic equilibrium of the BES: this does not hold anymore for the case \( P_{\text{ext}} \ll P_W \) and simulations show deviations from inequality (14) for \( \xi_{\text{max}} < 2.5 \) (see Section 3.4).

A one-sided pressure component can also accelerate substantially the BES as a whole downstream if the rate of compression of the BES is smaller than its sound speed and sound waves are triggered. To derive the \( \xi_{\text{max}} \) range in which this is the case we will assume a pressure equilibrium between the internal and external pressure and take \( P_W \) as a perturbation. For a BES in equilibrium, the last assumption is at least true at the beginning of the simulation. We assume the wind to be parallel to the \( x \)-axis, the center of the BES to be lying on the \( x \)-axis, and the point of the surface hit first by the wind to be at the origin of the coordinate system, and we concentrate on the evolution of the BES on the \( x \)-axis. We then find from Newton’s law:

\[
\frac{M v_w}{4 \pi d^2} = \frac{P_C}{\rho_0} \frac{d}{dt} (x(t)) = \rho_0 \frac{d^2}{dt^2} \left( \frac{x^2}{2} \right), \tag{15}
\]

where \( \rho_0 \) is the mean clump density and \( x(t) \) is the position of the point upstream to the wind as a function of time \( t \). With the initial conditions \( x(0) = 0 \) and \( x(0) = 0 \) we find the solution for \( t > 0 \) (note there is a discontinuity in \( \dot{x} \) at \( t = 0 \)):

\[
x(t) = \frac{P_C}{\rho_0} t; \quad v_{\text{compr}} = \ddot{x} = \frac{P_C}{\rho_0}. \tag{16}
\]

We expect a nonnegligible fraction of the imposed wind momentum to go directly into acceleration of the whole sphere for \( v_{\text{compr}}/c_s < 1 \) as in this case the whole core can react to the momentum transfer by the wind, impacting from the left. With the definition \( \rho_0 = B(\xi_{\text{max}}) \rho_c \) we find that acceleration effects could matter for:

\[
\frac{GMv_w R^2}{d^2 \xi_s^4} < \xi_{\text{max}}^2 B(\xi_{\text{max}}). \tag{17}
\]

The right-hand side is again a function of \( \xi_{\text{max}} \) and if we choose \( B = \rho_B/\chi_C \) and \( C = 1/3.8 \) (see Section 3.4) we expect for \( \xi_{\text{max}} \geq 4.2 \) deviations from inequality (14). It is important to stress that the inequality (17) should not be interpreted as a stability criterion on its own since we highly simplified the evolution of the compression of the BES and a gravitational collapse can also occur if there are sound waves. Conversely even a collapsing core will have been accelerated as a whole to
some extent. Especially for a BES that is barely stable a small
cmpression is sufficient to deepen the gravitational potential
enough in order to trigger a collapse that is much faster than a
sound wave. In summary we expect inequality (14) to be valid for:

\[ 2.5 < \xi_{\text{max}} < 4.2. \]  

(18)

3. Simulations

3.1. Numerical Methods

For all simulations presented in this paper we used an
improved version of the SPH-Code GADGET-3 (Springel
2005), which is presented in detail in Beck et al. (2015). It
supports the density formulation of SPH with entropy as the
thermodynamical variable as in Springel & Hernquist (2002),
a time-dependent artificial viscosity according to Cullen &
Dehnen (2010) as well as adaptive gravitational softening as
presented in Iannuzzi & Dolag (2011), which is important to
achieve hydrostatic equilibrium. We also use the wakeup
scheme presented in Pakmor et al. (2012), which activates
particles if there are other particles in the smoothing kernel with
much smaller timesteps. In all simulations, we use the
Wendland \( C^4 \) kernel (Dehnen & Aly 2012) with 200 neighbors
in the SPH kernel. For the equation of state we use

\[ P = \max \left( \frac{c_s^2}{2} \rho \left[ 1 + \left( \frac{\rho}{\rho_{\text{crit}}} \right)^{-\gamma} \right] - P_{\text{ext}}, 0 \right), \]  

(19)

with \( \rho_{\text{crit}} = 10^{-13} \text{ g cm}^{-3} \) and \( c_s = 200 \text{ m s}^{-1} \) corresponding to
\( T = 10 \text{ K} \) and the external pressure \( P_{\text{ext}} \) to stabilize the BES.
The idea behind Equation (19) is that for \( \rho < \rho_{\text{crit}} \) the core can
cool efficiently and is isothermal. For \( \rho > \rho_{\text{crit}} \) it becomes
optical thick and the equation of state becomes adiabatic.
We create sink particles above the density threshold of
\( 10^{-11} \text{ g cm}^{-3} \) and run all simulations with self-gravity and
advanced SPH but without periodic boundary conditions. In all
simulations we use \( 2 \times 10^6 \) particles, which means the typical
mass of a SPH particle is \( O(2 \times 10^{-6} M_\odot) \). The minimum Jeans
mass for \( \rho = \rho_{\text{crit}} \) and \( T = 10 \text{ K} \) is \( M_j = 4 \times 10^{-3} M_\odot \),
which corresponds to around 2000 particles. This is more than
\( 2N_{\text{neigh}} = 400 \) for our kernel and can therefore always be
resolved according to the criterion from Bate & Burkert (1997).

For the wind we use a similar model as presented in
Ngoumou et al. (2014), which does not sample the wind ejecta
itself but only takes into account the momentum of the wind
ejecta: we assume a constant mass-loss rate \( \dot{M} \) and terminal
wind velocity \( v_w \), which is equivalent to a momentum
production rate of \( d\rho/dt = \dot{M} v_w \). In every time step, the
momentum generated since the last time step is injected
isotropically in the environment of the star using the
HEALPix algorithm (Gorski et al. 2005). If in one of the pixels no
SPH particle is found, the corresponding momentum is
ignored. As in Ngoumou et al. (2014) we also allow the
splitting of rays and smooth the momentum injection over one
smoothing length. The splitting is especially important if \( d \gg R \)
holds since in this case the BES only spans a small solid
angle seen from the star and most of the initial HEALPix rays are empty.

3.2. Initial Conditions

Especially for \( \xi_{\text{max}} \) close to 6.45 it is important to reduce the
noise in the initial conditions since too large noise can already
trigger a collapse. We therefore use the method described in
Pakmor et al. (2012) that uses the HEALPix algorithm to create
spherical symmetric initial conditions. The idea is to build up
the sphere by spherical shells that themselves consist of
approximately cubic cells, all with the same mass. In each cell
one SPH particle is set. For a detailed mathematical description
we refer to Pakmor et al. (2012). To further reduce the initial
noise and also to get rid of preferred directions we let evolve
the initial conditions without the wind until the density
distribution becomes static. As one can see in the upper panel
of Figure 3 the gaps between the different shells disappear
with time.

3.3. Overview of Simulations

In all simulations we use \( R = 0.2 \text{ pc} \) and \( d = 3 \text{ pc} \) and fix
\( \xi_{\text{max}}, M_{vw}, \) and \( P_{\text{ext}} \) at the beginning of the simulation. Inspired
by inequality (14) we use the dimensionless physical quantity:

\[ C_{\text{sim}} = \frac{\xi_{\text{max}}^2 M_{vw} d^2 c_s^2 \rho_{\text{crit}}}{GM_{vw} R^2 \rho c_s^2 m_0^2 - 1} = \frac{P_{w,\text{crit}}}{P_{C_1}}, \]  

(20)

together with \( \xi_{\text{max}} \) to unambiguously classify the simulations.
We vary \( C_{\text{sim}} \) and \( \xi_{\text{max}} \) to find the boundary between the stable
and unstable regime. All simulations are stopped after the
formation of the first sink particle or when the sphere has
dissolved (i.e., the maximum density is half of the maximum
density at the beginning of the simulation).

3.4. Results

As one can see in Figures 4 and 5 the stability of the BES
can be well described by inequality (14) with:

\[ C = \left\{ \begin{array}{ll}
1/3.8 + (\xi - 2.42)(0.595) & \text{for } 2 \leq \xi \leq 2.42 \\
1/3.8 & \text{for } 2.42 \leq \xi \leq 3.8 \quad (21)
\end{array} \right. \]

In Figures 6 and 7 we show examples of the evolution of
BES with \( \xi_{\text{max}} = 2, \xi_{\text{max}} = 4, \) and \( \xi_{\text{max}} = 6 \) and in Figure 8
the evolution of their maximum density. For \( \xi_{\text{max}} = 2 \) at the edge
of the BES gas gets blown away while the rest gets compressed.
The compression of the cloud is faster than the sound speed,
which means the shielded side of the BES is not
affected by the wind until the compression hits it. Afterwards,
the whole remnant begins to move downstream. For \( C_{\text{sim}} = 0.03 \) at \( t = 290 \text{ kyr} \) a gravitational collapse is triggered.
Due to the low mass of the remnant even a maximum density
of \( n = 10^3 \text{ cm}^{-3} \) is not enough for \( C_{\text{sim}} = 0.042 \) to trigger a
gravitational collapse. Clearly the assumption of spherical
symmetry as made in Section 2 is violated.

For \( \xi_{\text{max}} = 4 \) also gas at the surface of the BES is blown
away and the compression is faster than the sound speed. Now,
the compression is enough to trigger a runaway gravitational
collapse toward a star for \( C_{\text{sim}} = 0.238 \), while for \( C_{\text{sim}} = 0.256 \)
the compression is too weak and the remnant of the BES gets
blown away. For \( \xi_{\text{max}} = 6 \) in both simulations the compression
is slower than the speed of sound, which means a sound
wave can reach the shielded surface of the BES before
the compression front hits it. The sound wave leads to an
expansion of the shielded side of the BES, which prevents the gravitational collapse for $C = 0.042$. For $C = 0.033$ the compression is fast enough to trigger a gravitational collapse.

4. Discussion

In the previous sections we showed that a BES with $2 < \xi < 6.45$ can be efficiently destabilized by a nearby wind source. Equation (8) follows for the mass of the core:

$$M_{\text{BES}} = m(\xi_{\text{max}}) \frac{c_s^4}{G^{3/2} \rho_{\text{ext}}^{1/2}} \approx 3.96 \left(\frac{c_s}{200 \text{ km s}^{-1}}\right)^4 \times \left(\frac{P_{\text{ext}}/k_B}{10^4 \text{ K cm}^{-3}}\right)^{-1/2} m(\xi_{\text{max}}),$$

(22)
which depends on the one hand on the inner properties of the BES (the sound speed) and on the other hand on the environment (the external pressure). Although the pressure within molecular clouds can vary from $10^4 k_B\text{ K cm}^{-3}$ to $10^7 k_B\text{ K cm}^{-3}$ (Sun et al. 2020), typical values from observations and simulations show a pressure between $10^4 k_B\text{ K cm}^{-3}$ and $10^5 k_B\text{ K cm}^{-3}$ (Heigl et al. 2018; Anathpindika & Di Francesco 2021; Sun et al. 2020) in the ISM. As a result, cores with $M < 1.5 M_\odot$ are typically stable. This boundary can be shifted to $0.5 M_\odot$ by a nearby star if we assume that the BES with $\xi = 2$ can be destabilized. To further analyze the wind strength required to destabilize the BES one can rewrite inequality (14) using Equation (6):

$$\frac{\dot{M}_{\text{wind}}}{d^2} > \frac{4\pi}{C} \left( 1.18^2 \frac{c_s^8}{G^3 M_{\text{BES}}^2} - P_{\text{ext}} \right).$$ (23)

In Figures 9 and 10 we show the right side of inequality (23) as a function of $M_{\text{BES}}$ and $P_{\text{ext}}$. Given certain wind properties
difference of the O star is small but, especially for a larger external pressure or smaller mass of the BES, the Wolf–Rayet star is able to influence the stability of the BES. This is important in more extreme environments like stellar clusters ($P_{\text{ext}} \approx 10^6 k_B$ K cm$^{-3}$ for evolved H II regions) in which also distances significantly smaller than 1 pc are common (Olivier et al. 2021). Even larger external pressures can be found in young H II regions ($10^8 k_B$ K cm$^{-3}$ < $P_{\text{ext}} < 10^{10} k_B$ K cm$^{-3}$; Olivier et al. 2021) or in the galactic center ($P_{\text{ext}} > 10^{10} k_B$ K cm$^{-3}$; Burkert et al. 2012). Those regions are also associated with strong radiation fields and interacting stellar outflows of neighboring stars, which makes those systems more complex. In particular, the interplay of evaporation and compression by those effects needs to be understood in greater detail before applying our criterion. Other wind sources are AGB stars with typical velocities of around $v_w \approx 10$ km s$^{-1}$ and mass-loss rates of $M = 10^{-7}$ M$_\odot$ yr$^{-1}$ to $M = 10^{-5}$ M$_\odot$ yr$^{-1}$ (Höfner & Olofsson 2018). Due to their lower wind velocities in comparison to O stars, they only have an influence in low-pressure regions ($P_{\text{ext}} \approx 10^4 k_B$ K cm$^{-3}$) and if their distance to the BES is significantly smaller than 1 pc.

In the derivation of criterion (14) we explicitly assume a constant compression over time, i.e., it cannot be directly applied to time-dependent phenomena like the collision with a supernova remnant or the influence of bipolar outflows from protostars. In a future study, we will analyze how the criterion (14) can be modified to also cover those cases.

\section{5. Summary}

In this paper, we have analyzed the interaction of a Bonnor–Ebert sphere in hydrostatic equilibrium with the stellar wind of a nearby star. We concentrated on the low Mach regime for the wind so that the wind can be approximated by an additional external force and does not have to be modeled separately. In Section 2 we first derived analytically a stability criterion for the BES (see inequality (14)) and predicted that it should be valid for $2.5 < \xi_{\text{max}} < 4.2$. In Section 3 we presented simulations with GADGET-3 and were able to verify the criterion and also its validity range. We showed that the efficiency of the wind to trigger a collapse strongly decreases for $\xi_{\text{max}} < 2.5$ and for $\xi_{\text{max}} < 2$ we were not able to form a sink particle. In Section 4 we discussed the implications of our results for protostellar cores in molecular clouds. We find that winds have an influence in low-pressure regions and on large cores. The smallest core that can be destabilized for a typical external pressure of $10^5 k_B$ K cm$^{-3}$ by a wind has a mass of around $0.5 M_\odot$, while otherwise cores below $1.5 M_\odot$ would be stable.

In the future, we plan to carry out further studies using the moving mesh code Arepo (Springel 2010; Weinberger et al. 2020), which will allow us to directly model the ejecta of the wind. We also plan to continue the simulations presented in this paper until the BES is completely dispersed and to analyze the mass distribution of the formed sink particles.

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