Relating quarks and leptons without grand-unification

S. Morisi,1 E. Peinado,1‡ Yusuke Shimizu,2§ and J. W. F. Valle1

1AHEP Group, Institut de Física Corpuscular – C.S.I.C./Universitat de València
Edifici Institutos de Paterna, Apt 22085, E-46071 Valencia, Spain
2Graduate School of Science and Technology, Niigata University, Niigata 950-2181, Japan

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In combination with supersymmetry, flavor symmetry may relate quarks with leptons, even in the absence of a grand-unification group. We propose an $SU(3) \times SU(2) \times U(1)$ model where both supersymmetry and the assumed $A_4$ flavor symmetries are softly broken, reproducing well the observed fermion mass hierarchies and predicting: (i) a relation between down-type quarks and charged lepton masses, and (ii) a correlation between the Cabibbo angle in the quark sector, and the reactor angle $\theta_{13}$ characterizing CP violation in neutrino oscillations.

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INTRODUCTION

Understanding the observed pattern of quark and lepton masses and mixing [1, 2] constitutes one of the deepest challenges in particle physics. Flavor symmetries provide a very useful approach towards reducing the number of free parameters describing the fermion sector [3]. It has long been advocated that grand unification offers a suitable framework to describe flavor. In what follows we will adopt the alternative approach, assuming that flavor is implemented directly at the $SU(3) \times SU(2) \times U(1)$ level. Typically this requires several $SU(2)$ doublet scalars in order to break spontaneously the flavor symmetry so as to obtain an acceptable structure for the masses and mixing matrices. (One may alternatively introduce “flavons” instead of additional Higgs doublets, but in this case one would have to give up renormalizability).

In order to construct a “realistic” extension of the Standard Model (SM) with flavor symmetry one needs a suitable alignment of the scalar vacuum expectation values (vevs) in the theory [4–7]. There are several multi-doublet extensions of the SM with flavor in the market, but renormalizable supersymmetric extensions of the SM with a flavor symmetry are only a few [8], usually because the existence of additional Higgs doublets spoils the unification of the coupling constants.

Here we choose to renounce to this theoretical argument, noting that gauge coupling unification may happen in multi-doublet schemes due to other effects. What we now present is a supersymmetric extension of the SM based on the $A_4$ group where all the matter fields as well as the Higgs doublets belong to the same $A_4$ representation, namely, the triplet. This leads us to two theoretical predictions. The first a mass relation

$$\frac{m_\tau}{\sqrt{m_\tau m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}},$$

involving down-type quarks and charged lepton mass ratios. Such relation can be obtained by a suitable combination of the three Georgi-Jarlskog (GJ) mass relations [9],

$$m_b = m_\tau, \quad m_s = 1/3 m_\mu, \quad m_d = 3 m_s,$$

which arise within a particular ansatz for the $SU(5)$ model and hold at the unification scale. In contrast to eq. (2), our relation requires no unification group and holds at the electroweak scale. It would, in any case, be rather robust against renormalization effects as it involves only mass ratios.

The second prediction obtained in our flavor model is a correlation between the Cabibbo angle for the quarks and the so-called “reactor angle” $\theta_{13}$ characterizing the strength of CP violation in neutrino oscillations [10, 11]. Within a reasonable approximation we find

$$\lambda_C \approx \frac{1}{\sqrt{2}} \frac{m_\mu m_b}{m_\tau m_s} \sqrt{\sin^2 2\theta_{13}} - \frac{m_\mu}{m_c},$$

which arises mainly from the down-type quark sector [12] with a correction coming from the up isospin diagonalization matrix. This is a very interesting relation, discussed below in more detail.

THE MODEL

Here we propose a supersymmetric model based on an $A_4$ flavor symmetry realized in an $SU(3) \times SU(2) \times U(1)$ gauge framework. The field representation content is given in Table I. Note that all quarks and leptons transform as $A_4$ triplets. Similarly the Higgs superfields with opposite hypercharge characteristic of the MSSM are now upgraded into two sets, also transforming as $A_4$ triplets. Note that since all matter fields transform in the same way under the flavor symmetry one may in principle embed the model into a grand-unified scheme. However, given the large number of scalar doublets, gauge coupling unification must proceed differently, see, for example, Ref. [13].
The most general renormalizable Yukawa Lagrangian for the charged fermions in the model is

\[ L_{\text{Yuk}} = y_{ij}^u \bar{L}_i H_u^d E_k + y_{ij}^d \bar{Q}_i H_d^d \bar{D}_k + y_{ij}^d \bar{Q}_i H_d^d \bar{U}_k , \tag{4} \]

where \( y_{ij}^{u,d} \) are \( A_4 \)-tensors, assumed real at this stage.

The Higgs scalar potential invariant under \( A_4 \) is

\[
V = (|\mu|^2 + m^2_{H_u})(|H_u^d|^2 + |H_d|^2 + |H_d^d|^2) \\
+ (|\mu|^2 + m^2_{H_d})(|H_d^d|^2 + |H_u|^2 + |H_u^d|^2) \\
- [b(H_u^d H_d^d + H_d^d H_u^d + H_u H_u^d) + \text{c.c.}] \\
+ \frac{1}{8} (g^2 + g'^2)(|H_u|^4 + |H_d|^4 + |H_u^d|^4 + |H_d^d|^4) \] \\
+ |H_d^d|^2 - |H_d|^2 - |H_u|^2 \tag{5} .
\]

Assuming that the Higgs doublets take real vevs given as possible local minima the alignments

\[
\langle H_u^d \rangle = v_u^d \quad \text{and} \quad \langle H_d^d \rangle = v_1^d = \epsilon^d_1 , \tag{6} \]

where \( v_u^d \ll v_u \) and \( \epsilon^d_1 \ll v^d \).

### Charged fermions

By using \( A_4 \) product rules it is straightforward to show that the charged fermion mass matrix takes the following universal structure

\[
M_f = \begin{pmatrix}
0 & y_f^1 & y_f^2 \\
y_f^1 & H_f^u H_f^d & 0 \\
y_f^2 & 0 & H_f^d \end{pmatrix} , \tag{7}
\]

where \( f \) denotes any charged lepton, up- or down-type quarks. Note that, in addition to the "texture" zeros in the diagonal, one has additional relations among the parameters. This may be seen explicitly by rewriting eq. (7) as

\[
M_f = \begin{pmatrix}
0 & a_f \alpha_f & b_f \\
b_f \alpha_f & 0 & a_f \rho_f \\
a_f & b_f & 0 \end{pmatrix} , \tag{8}
\]

where \( a_f = y_f^1 \epsilon^d_1 , b_f = y_f^1 \epsilon^d_1 , \) with \( y_f^{1,2} \) denoting the only two couplings arising from the \( A_4 \)-tensor in eq. (4), \( \rho_f = v_f / \epsilon^d_1 \) and \( \alpha_f = \epsilon^d_1 / \epsilon^d_1 \). Thanks to the fact that the same Higgs doublet \( H_f \) couples to the lepton and to the down-type quarks one has, in addition, the following relations

\[
r_f = r_f , \quad \rho_f = \alpha_f , \quad \tag{9}
\]

involving down-type quarks and charged leptons.

It is straightforward to obtain analytical expressions for \( a_f , b_f \) and \( r_f \) from eq. (8) in terms of the charged fermion masses and \( \alpha_f \),

\[
r_f \approx \frac{m_f}{\sqrt{m_f m_2}} , \quad a_f \approx \frac{m_f}{m_3} \sqrt{m_f m_2} , \quad b_f \approx \frac{\sqrt{m_f m_2}}{m_2} . \tag{10}
\]

From eq. (9) and eq. (10) it follows that

\[ m_d \approx m_3 \sqrt{m_3 m_2} \],

a formula relating quark and lepton mass ratios (to a very good approximation this formula also holds for complex Yukawa couplings). This relation is a strict prediction of our model, and appears in a way similar to the celebrated SU(5) mass relation, despite the fact that we have not assumed any unified group, but just the \( SU(3) \times SU(2) \times U(1) \) gauge structure. It allows us to compute the down quark mass in terms of the charged fermion masses and the \( s \) and \( b \) quarks, as

\[
m_d \approx m_3 \frac{m_d}{m_s} \left( \frac{m_b}{m_s} \right)^2 . \tag{11}
\]

This mass formula predicts the down quark mass at the scale of the Z boson mass, to lie in the region

\[
1.71 \text{ MeV} < m^h_d < 3.35 \text{ MeV} \quad \text{at } 1 \sigma \quad \text{and the best fit point and the GJ prediction.}
\]

Note also that, thanks to supersymmetry, we obtain a relation only among the charged lepton and down-type quark mass ratios, avoiding the unwanted relation found by Wilczek and Zee in Ref. [10].
The charged lepton mass matrix is mainly diagonalized by a rotation in the 12 plane. The first of these we have already seen, namely the mass relation in eq. [1] and Fig. [1]. The second is a quark-lepton mixing angle relation concerning the Cabibbo angle $\lambda_C$ and the “reactor angle” $\theta_{13}$ describing neutrino oscillations. To derive it note first that the matrix in eq. (8) is diagonalized on the left by a rotation in the 12 plane, namely

$$\sin \theta_{12} \approx \sqrt{\frac{m_1^2}{m_2^2 - m_{13}^2}}.$$

In order to give an analytical expression for the relation between Cabibbo and reactor angles, we neglect mixing of the third family of quarks and go in the limit where our neutrino mass matrix, eq. (15) is $\mu - \tau$ invariant, that is $\alpha^u = 1$ and $y = z$. In this approximation, the reactor mixing angle is given by

$$\sin \theta_{13} = \frac{1}{\sqrt{2}} \sin \theta_{12} = \frac{1}{\sqrt{2}} \sqrt{\frac{m_{13}^2}{m_\mu \sqrt{\alpha^l}}}, \quad \lambda_C = \frac{m_\mu}{m_s} \frac{m_\mu}{m_\tau} \frac{1}{\sqrt{\alpha^d}} - \sqrt{\frac{m_{13}^2}{m_\mu \sqrt{\alpha^l}}}. \quad (16)$$

Comparing eq. (16) with eq. (17) leads immediately to equation (3). In order to display this prediction graphically we take the quark masses at 1 $\sigma$, obtaining the curved band shown in fig. 2. The narrow horizontal band indicates current determination of the Cabibbo angle, while the two vertical dashed lines represent the expected sensitivities of the Double-Chooz [18] and Daya-Bay [19] experiments on the “reactor mixing angle” $\theta_{13}$. The curved line corresponds to the analytical approximation for the best fit value of the quark masses in eq. (3). Clearly the width of the curved band characterizing our prediction is dominated by quark mass determination uncertainties.

1 Specific realizations of $L_{cd}$ within various seesaw schemes can, of course, be envisaged.
2 The charged lepton mass matrix is mainly diagonalized by a rotation in the 12 plane.
Finally note that mixing parameters of the third family of quarks \( U_{D3}^{q} \approx \frac{m_{q}^{3}}{m_{q}^{2}} \sqrt{\frac{m_{q}^{2} m_{u}^{2}}{m_{u}^{2}}} \frac{1}{\sqrt{\alpha}} \) and \( U_{D3}^{q} \approx \frac{m_{q}^{3}(m_{q}^{2})^{2}}{(m_{q}^{2})^{2}} \frac{1}{\alpha} \) \((q = u, d)\) are negligible, and cannot account for the measured values of \( V_{ub} \) and \( V_{cb} \). The predicted values obtained for these are too small so that in its simplest presentation described above our model can not describe the CP violation found in the decays of neutral kaons. However there is a simple solution which maintains the good predictions described above, namely, adding colored vector-like \( SU(2)_{L} \) singlet states. In this case acceptable values for \( V_{ub} \) and \( V_{cb} \), leading to adequate CP violation can arise solely from non-unitarity effects of the quark mixing matrix.

**OUTLOOK**

We proposed a supersymmetric extension of the standard model with an \( A_{4} \) flavor symmetry, where all matter fields in the model transform as triplets of the flavor group. Charged fermion masses arise from renormalizable Yukawa couplings while neutrino masses are treated in an effective way. The scheme illustrates how, in combination with supersymmetry, flavor symmetry may relate quarks with leptons, even in the absence of a grand-unification group. Two good predictions emerge: (i) a relation between down-type quarks and charged lepton masses, and (ii) a correlation between the Cabibbo angle in the quark sector, and the reactor angle \( \theta_{13} \) characterizing CP violation in neutrino oscillations, which lies within the sensitivities of upcoming experiments.

Although the predicted values for the other mixing parameters \( V_{ud} \) and \( V_{cb} \) of the Cabibbo-Kobayashi-Maskawa matrix are too small, we mentioned a simple way to circumvent this, making the scheme fully realistic.

Finally note that, with few exceptions such as those in Refs. [20, 21], grand-unified flavor models are not more predictive than the novel idea proposed here and illustrate through this simple scheme. As it stands the model fits well with the idea that gauge coupling unification may be an effect of the presence of extra dimensions rather than of grand-unified interactions [13]. Notwithstanding, we wish to stress that our model is manifestly embeddable into a standard Grand-Unified scenario, which would result in further predictive power. A detailed study of this particular model lies outside the scope of this letter and will be taken up elsewhere.

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\* Electronic address: morisi@ific.uv.es
1 Electronic address: epeinado@ific.uv.es
2 Electronic address: shimizu@muse.sc.nigata-u.ac.jp
3 Electronic address: valle@ific.uv.es
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