Effect of Spatial Variability of the Elastic Modulus in Composite Ground on the Structural Performance of Large-diameter Tunnels

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Abstract. The spatial variability of soil properties is often neglected in design and construction, resulting in underestimating or overestimating the tunnel structural performance. Furthermore, the risks of ignoring the spatial variability of ground properties increase significantly as the tunnel diameter increases. This study investigates the effects of ground spatial variability on the convergence of shield tunnels in the composite ground, based on a real case of a cross-sea tunnel typically embedded in a ground mixture of soft clay and hard rock in Shenzhen, China. The influence of scale of fluctuation (SOF) for the soil and rock elastic modulus on tunnel convergence is investigated. It is found that the mean value of the tunnel convergence is less affected. In contrast, the tunnel convergence coefficient of variation is strongly affected by changing the elastic modulus SOF of the soil or rock layer. Moreover, the results indicate that as the elastic modulus follows the lognormal distribution, the vertical and horizontal convergence of the tunnel agrees well with the lognormal distribution.

1. Introduction

The tunnel convergence can reflect the safety of the structure directly. Hence, the appropriate prediction and control of the tunnel convergence are essential to avoid accidents during the construction and operation of the shield tunnel. For the design period, the geotechnical parameters of the same layer of soil are often regarded as deterministic or purely random variables. If the spatial variability of the soil is ignored, the extreme case can be easily overlooked, which is not conducive for the safety evaluation of tunnel convergence[1]. As the tunnel diameter increases, the ground conditions faced by tunnel engineering become more complicated. Therefore, for large-diameter shield tunnels, the influence of spatial variability in the composite ground on the performance of the tunnel structure needs to be considered more.

Considering the spatial variability of ground, Vanmarcke[2] proposed a random field model of soil profile based on antecedents and described the spatial variability of geotechnical parameters using the digital characteristics of a random field, which established the calculation method of transition from...
point characteristics of data to average spatial characteristics. The spatial variability is often modeled by random field theory. Random field theory has many applications in tunnels and geotechnical engineering. Mollon et al. [3] analyzed the face stability of a tunnel driven in anisotropic and nonhomogeneous soils by considering the spatial variability of the soil’s shear strength. Huang et al. [4] studied the influence of the foundation reaction coefficient’s spatial variability on the tunnel’s longitudinal differential settlement and affirmed the significant correlation. Cheng et al. [5] explored the impact of the spatial variability of soil elastic modulus on the ground surface deformation.

Among all soil properties, it is widely accepted that the elastic modulus and Poisson’s ratio are the dominant parameters that significantly affect deformations of soil and embedded geo-structures. The Poisson’s ratio is believed to have less spatial variability and only second-order importance to deformational analysis. Therefore, the elastic modulus of the ground is always specifically simulated by the random field for deformational analysis of geotechnical systems. The rock-soil stationary random field is generated by means, the coefficient of variation (COV), distribution type, scale of fluctuation (SOF), and correlation structure. The distribution types of geotechnical parameter random variables, including normal and lognormal distribution, are commonly used. The mean and COV can usually be obtained from geotechnical engineering survey reports, while SOF selection has no clear basis.

This study focuses on the SOF impact of elastic modulus on the deformation of large-diameter tunnels in the composite ground. Firstly, the three-dimensional ground-shell-spring model of the tunnel single ring is established using FLAC3D finite difference software. Then, the horizontal SOF($\delta_x$) and vertical SOF($\delta_y$) of the elastic modulus of the ground are varied to consider the influence of the spatial variability. The mean value and COV of the convergence of the corresponding tunnel are finally obtained. The outcome of this study can provide theoretical guidance for tunnel design, construction, and subsequent operation.

2. Stochastic finite difference model of shield tunnel based on random field

2.1. Engineering background

The Mawan Cross-sea Passage project starts at the intersection of Mawan Avenue and Yueliangwan Avenue in the Mawan Port Area of Nanshan. It passes through Qianhai Bay and ends at Dachan Bay Port Area, Baoan District. The maximum diameter of this tunnel is 15 m, and the stratum conditions of this project are complex. In the geologic section of the project, the specific line location, and direction are shown in Figure 1.

![Figure 1. Route diagram of the Mawan Cross-sea Passage project.](image)

2.2. Finite difference model

This study establishes a three-dimensional finite difference model of large-diameter shield tunnel (as shown in Figure 2) using FLAC3D software, taking Shenzhen’s Mawan cross-sea passage as the research object. The outer diameter (D) of the tunnel is 7.5 m. The inner diameter, the ring width, and the lining thickness of the tunnel are 6.85 m, 2 m, and 0.65 m, respectively. A block-by-block elastic shell unit simulates the lining structure and the bolts between the segments are simulated by a spring
unit. The density, elastic modulus, and Poisson’s ratio of the concrete segment are 25 kN/m³, 36 GPa, and 0.167, respectively. Considering the model’s boundary effect, the model’s length is 105 m (52.5 m on the left and right of the tunnel centerline, 3.5D), and the height is 75 m (the buried depth of the tunnel is 30 m). Considering that the variation of the soil layer often does not change much within the same tunnel section, only one ring is considered in the longitudinal direction of the model (the longitudinal length is 2 m), which is a common method in the stochastic finite difference model (FDM)[6]. The yield criterion meets the Mohr–Coulomb ideal elastoplastic criterion, and the parameter values are shown in Table 1. The Mohr–Coulomb failure criterion is widely used in rock mechanics, rock engineering, and rock material mechanics for two advantages. First, its parameters have direct physical meaning. Second, conventional experimental methods can measure all parameters, so the Mohr–Coulomb yield criterion makes the process simple[7]. Since the underlying ground profiles for the Mawan cross-sea passage includes an upper soil layer and a lower rock layer, the interface between the soil layer and the rock layer is assumed to pass through the center of the shield tunnel for convenience of and calculation.

![Figure 2. Finite difference model of the large-diameter tunnel in the composite ground (sectional view).](image)

| Ground | Density/(kg/m³) | E/MPa | Poisson’s ratio | Cohesion/kPa | Internal friction angle/° |
|--------|----------------|-------|-----------------|--------------|---------------------------|
| Soil   | 1860           | 22.67 | 0.32            | 28.21        | 21.33                     |
| Rock   | 2590           | 109.69| 0.26            | 40.66        | 33.80                     |

When analyzing the influence of the spatial variability of the soil elastic modulus of the two-layer composite layer on the mechanical properties of the shield tunnel, a research framework combining the random field theory, the finite difference simulation, and the method Monte–Carlo is used in this study, the random field is imported into the FDM mentioned above to calculate the tunnel excavation. This calculation step is repeated N times to obtain the probability distribution and statistical eigenvalues (mean and variance) of the convergence deformation of the excavated tunnel. Thus, the research steps are as follows:

1. Create a FDM of the tunnel and its surroundings. The model’s grid position information is then derived.
2. Using the appropriate random field discrete method, obtain the rock and soil parameter random field corresponding to the FDM based on the model’s grid position information. This step also determines the random field’s statistical characteristics (correlation structure, SOF, mean and variability, and so on) as well as the number of Monte–Carlo simulations N.
3. Import the discrete random field into the FDM established in the first step and perform engineering simulation analysis according to research needs.
4. Repeat the third step for N times and get the engineering results’ probability distribution and statistical characteristics. This study focused on the probability distribution and statistical eigenvalues (mean and variance) of the convergence deformation of the excavated tunnel.
The flow chart of this research is shown in Figure 3. It is noted that the rationality of the method has been verified by Zhang et al.[8], and the calculation results of this method agree well with the actual tunnel convergence.

**Figure 3. The framework of the research**

### 2.3. Random field model of elastic modulus

Generally, we can obtain the mean $\mu_E$ and the COV$_E$ of the ground elastic modulus according to the engineering geological survey report. To ensure the nonnegativity of the elastic modulus, it is assumed that the elastic modulus $E$ obeys the lognormal distribution; that is, $\ln E$ follows the normal distribution. Its standard deviation $\zeta_{\ln E}$ and mean $\lambda_{\ln E}$ can be calculated based on equation (1) and equation (2):

$$
\zeta_{\ln E} = \sqrt{\ln(1 + \text{COV}_E^2)} 
$$

$$
\lambda_{\ln E} = \ln \mu_E - \frac{1}{2} \sigma_{\ln E}^2
$$

In this study, the random field is discrete using the method proposed by Huang et al.[9]. The parameter assignment of the numerical ground model adopts the center method; that is, the parameter value of each unit is equal to the value of the random field variable at the geometric center point of the unit. In addition, the spatial variability of the ground in the longitudinal direction of the tunnel is not considered.

To obtain a lognormal distributed random field $E(X)$, a random field $G(X)$ which obeys the normal distribution, should be obtained first. Then, the mean of $G(X)$ is zero, and the mean square error is 1. Its autocorrelation model $\rho(x, y)$ is established by using a separable exponential autocorrelation model[10], which is shown in equation (3):

$$
\rho(x, y) = \exp \left( -\frac{2|x|}{\delta_x} \right) \exp \left( -\frac{2|y|}{\delta_y} \right)
$$

Among them, $\delta_x$ and $\delta_y$ represent the horizontal and vertical SOF, respectively. $x$ and $y$ are the difference between the horizontal and vertical coordinates of two points in the space. After obtaining the normally distributed random field $G(X)$, the required lognormal random field $E(X)$ can be obtained from equation (4), where $X_i$ represents the coordinates of any point in the space.

$$
E(x_i) = \exp \left( \lambda_{\ln E} + \zeta_{\ln E} G(x_i) \right)
$$

### 2.4. Monte–Carlo simulation number

To obtain the stable probability characteristic value of the tunnel structure convergence, it is necessary to determine the number of Monte–Carlo simulations $N$. Figure 4 shows the relationship between the
Monte–Carlo simulation number \(N\) and the characteristic probability values (mean and standard deviation) of the convergence.

![Image](a) Mean of the convergence (b) Standard deviation of the convergence

Figure 4. Tunnel convergence deformation statistics vs. the simulation times \(N\) for the Monte–Carlo simulation.

When the number of simulations \(N\) is greater than 400, the mean and standard deviation of the convergence of the tunnel section has stabilized. Therefore, the number \(N\) of Monte–Carlo simulations is set to 400 in this study to save the calculation time.

2.5. SOF of rock and soil

To describe the random variability of geotechnical parameters, random variable models or random field models are commonly used presently. The random variable model, on the other hand, is unable to adequately describe the spatial variability of geotechnical parameters. To simulate the spatial variability of the elastic modulus of the double-layer composite layer, a stationary random field model is established in this study to characterize the spatial variability of geotechnical parameters. For this random field model, the mean of elastic modulus, COV, probability distribution, \(\delta_x\) and \(\delta_y\) should be known. The existing research results on SOF of rocks and soils at home and abroad have been counted in this study. The variation range of \(\delta_y\) of rocks is 10–80 m, and the variation range of \(\delta_x\) is 0.5–8.0 m. However, the variation range of \(\delta_x\) of soil is 3–80 m, and the variation range of \(\delta_y\) is 0.1–7.14 m, which indicates that \(\delta_x\) of penetration resistance of rock and soil is generally significant than the \(\delta_y\).

SOF of the penetration resistance of different soil types cannot be directly equal to the SOF of the rock and soil elastic modulus \(E\). However, since the penetration resistance of the soil is approximately linear with the elastic modulus, the SOF of the rock and soil elastic modulus is approximately equal to the SOF of the penetration resistance.

2.6. Cases design

To research the influence of the SOF of the upper soil layer and the lower rock layer in the double-layer composite layer on the mechanical properties of the shield tunnel, the \(\delta_x\) and \(\delta_y\) of the rock layer and the soil layer are changed in this study. In Table 2, cases ISO-1–ISO-7 explore the influence of the \(\delta_y\) of the upper soil layer on the mechanical properties of the shield tunnel by changing the \(\delta_x\) of the rock layer. Cases ISO-8–ISO-13 changes the \(\delta_x\) of the rock layer to explore the influence of the \(\delta_y\) of soil on the shield tunnel. The \(\delta_x\) and \(\delta_y\) of soil for ISO-1–ISO-13 are 30 m and 4 m, respectively. Similarly, cases ISO-14–ISO-20 and ISO-21–ISO-26 are used to explore the influence of the \(\delta_x\) and \(\delta_y\) of the upper soil layer on the mechanical properties, respectively. The \(\delta_x\) and \(\delta_y\) of rock for ISO-14–ISO-26 are 30 m and 4 m, respectively.

Table 2. SOF of the ground of cases.

| Case  | SOF of rock/m |   | Case  | SOF of soil/m |
|-------|---------------|---|-------|---------------|
|       | Horizontal    | Vertical |       | Horizontal    | Vertical |
| ISO-1 | 30            | 0.5        | ISO-14| 30            | 0.5        |
| ISO-2 | 30            | 1.5        | ISO-15| 30            | 1          |
| ISO-3 | 30            | 3          | ISO-16| 30            | 2          |
| ISO-4 | 30            | 4          | ISO-17| 30            | 4          |
The influence of the SOF of ground elastic modulus on the convergence of shield tunnel

3.1. The influence of the SOF of rock elastic modulus on the tunnel convergence

This section presents the results for Cases 1–7. Figure 5 shows the schematic of the tunnel diagram. Figure 6 shows the variation of the mean and the COV of the shield tunnel convergence with the $\delta_y$ of the elastic modulus of the lower rock layer. It can be seen that the mean of the horizontal convergence ($\Delta D_x$) and vertical convergence ($\Delta D_y$) of the tunnel decrease with the increase of the $\delta_y$ of the rock layer. The $\delta_y$ of the rock layer has little effect on the mean of the convergence. Figure 6(b) shows that the COV of both the $\Delta D_x$ and $\Delta D_y$ of the tunnel increases with the increase of $\delta_y$ of the rock layer, while the COV of the $\Delta D_x$ is strongly affected by the $\delta_y$ of the rock layer. This is because the increase of the SOF of the rock layer makes the ground more homogeneous while the influence of horizontal spatial variability is more obvious. The increase of the $\delta_y$ of the lower rock layer is not conducive to the $\Delta D_x$ and $\Delta D_y$. This indicates that the $\delta_y$ of the rock layer is an essential factor affecting the COV in tunnel convergence.

| ISO-5  | 30 | 6  | ISO-18 | 30 | 8  |
|--------|----|----|--------|----|----|
| ISO-6  | 30 | 8  | ISO-19 | 30 | 12 |
| ISO-7  | 30 | 12 | ISO-20 | 30 | 16 |
| ISO-8  | 10 | 4  | ISO-21 | 5  | 4  |
| ISO-9  | 15 | 4  | ISO-22 | 10 | 4  |
| ISO-10 | 20 | 4  | ISO-23 | 20 | 4  |
| ISO-11 | 45 | 4  | ISO-24 | 45 | 4  |
| ISO-12 | 60 | 4  | ISO-25 | 60 | 4  |
| ISO-13 | 80 | 4  | ISO-26 | 80 | 4  |

Figure 5. Schematic diagram of the tunnel convergence.

Figure 6. Influence of $\delta_y$ for the rock layer on the $\Delta D_x$ and $\Delta D_y$ statistics.

The results for Cases 8–13 are shown in this section. Figure 7 shows the variation of the mean and the COV of the shield tunnel convergence with $\delta_x$ of the elastic modulus of the lower rock layer. We can find that the means of the $\Delta D_x$ and $\Delta D_y$ increase firstly, then decrease, then increase, and then decrease with the increase of the $\delta_x$ of the rock layer. The mean of the convergence reaches the minimum when the SOF of the two layers is equal (case ISO-4). However, the mean of the convergence is less affected by the change of the $\delta_x$ of the rock layer. This is because the model is symmetrical, and the convergence of the tunnel is a result of stress and strain propagation in the whole space. Figure 7(b) shows that the COVs of the $\Delta D_x$ and $\Delta D_y$ first increase, then decrease, and then...
decrease with the increase of the $\delta_x$ of the rock layer. The COV of the convergence reaches the maximum value when the $\delta_x$ of the rock layer is twice the $\delta_x$ of the soil layer. It means that $\rho_{\text{rock}}=2\rho_{\text{soil}}$ is the most unfavorable condition for convergence.

\[ \Delta D = 40.716 \text{ mm} \quad \text{and} \quad \Delta D = 46.889 \text{ mm} \]

\[ D_n = 0.0324 \quad \text{and} \quad D_n = 0.0257 \]

Thus, the convergence of the tunnel obeys a lognormal distribution with the mean and COV agreeing with the results of the random field calculation when the 95% confidence interval is satisfied.

3.2. The influence of the SOF of soil elastic modulus on the tunnel convergence

This section presents the results for Cases 14–20. Figure 9 shows the variation of the mean and the COV of the shield tunnel convergence with the $\delta_y$ of the elastic modulus of the upper soil layer. It can be found that the mean convergence is significantly reduced when the $\delta_y$ of the soil layer increases from 0.5 to 1.0 m. When the $\delta_y$ of the soil layer exceeds 1.0 m, the mean convergence does not change obviously. This is because the scale of the constraint for the tunnel is proportional to the SOF. A larger scale of constraint means lower convergence. Figure 9(b) suggests that the $\Delta D_x$ and $\Delta D_y$ are consistent with the $\delta_y$ of the soil layer. When $\delta_y = 0.5 \text{ m}$, the COV of the convergence is the smallest. When $\delta_y = 8 \text{ m}$, the COV of the convergence is the largest, which is unfavorable to the convergence of the tunnel.
This section presents the results for Cases 21–26. Figure 10 shows the variation of the mean and the COV of the shield tunnel convergence with the $\delta_x$ of the elastic modulus of the upper soil layer. It can be found that the $\delta_x$ of the soil layer has little effect on the mean of convergence. The mean of convergence is the smallest when $\rho_x$-rock $=\rho_x$-soil $=30$ m. Figure 10(b) indicates that the COV of convergence first decreases, then increases, then decreases, and then increases again as the SOF of the soil layer increases, so this trend is contrary to the case in the SOF of the rock layer. When the $\delta_x$ of the soil layer is smaller ($\rho_x$-soil $=5$ m) or larger ($\rho_x$-rock $=80$ m), the COV of the convergence keeps larger, which is not conducive to the convergence of the tunnel. However, the COV of the convergence is the smallest when the $\delta_x$ of the soil layer is 20 m (about 1.33D).

Figure 11 shows the frequency distribution of tunnel convergence under ISO-14. It can be found that the $\Delta D_x$ and $\Delta D_y$ obtained by deterministic calculation are small compared to the calculation results of the random field. It means that neglecting the spatial variability of the elastic modulus of the soil layer will lead to a lower estimation of the tunnel convergence, which makes the design more unsafe.

According to the K-S test, both the $\Delta D_x$ and $\Delta D_y$ in the case ISO-14 may obey the lognormal distribution with the mean and COV agreeing with the results of the random field calculation when the confidence interval is 95%. Other cases equally agree with the lognormal distribution. Therefore, it can be assumed that the $\Delta D_x$ and $\Delta D_y$ obey the lognormal distribution.
The spatial variability of the elastic modulus may change the stress and strain propagation in the ground, which may influence the response of geo-structural deformation. Ignoring the spatial variability will obtain an unsafe estimation of tunnel deformation. It can be found that the SOF of the rock layer has a significant influence on the mean and the COV of the tunnel convergence compared with the soil layer. The COV of the tunnel convergence increases as SOF of the rock layer increases. In addition, the COV of the tunnel convergence fluctuates up and down with the increase of the SOF of the rock layer.

4. Conclusion
This study aims to reveal the influence of the SOF of the ground elastic modulus on the convergence of large-diameter tunnels. The mean and COV of the $\Delta D_x$ and $\Delta D_y$ of the corresponding tunnel are obtained considering different $\delta_x$ and $\delta_y$ of the elastic modulus. As a result, the following conclusions are obtained:

1. For the rock layer, the SOF of the elastic modulus has little effect on the mean of the tunnel convergence, but it greatly influences the COV of the tunnel convergence. The COV of the tunnel convergence increases with the increase of the $\delta_y$ of the elastic modulus and reaches the maximum value when $\rho_{y\text{-soil}}=2\rho_{y\text{-rock}}$ and $\rho_{x\text{-soil}}=2\rho_{x\text{-rock}}$.

2. For the soft soil layer, the SOF of the elastic modulus shows little influence on the mean of the tunnel convergence, but it greatly influences the COV of the tunnel convergence. When $\rho_{y\text{-soil}}=2\rho_{y\text{-rock}}$ and $\rho_{x\text{-soil}}=2\rho_{x\text{-rock}}$, the COV of convergence reaches its maximum, which is adverse for the tunnel.

3. As the elastic modulus of the ground follows the lognormal distribution, the $\Delta D_x$ and $\Delta D_y$ agree well with the lognormal distribution.

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