ABSTRACT

This work reviews the available experimental information on the $\pi-\pi$ scattering lengths, especially the recent near-threshold $\pi N \rightarrow \pi \pi N$ data from several laboratories and the related application of the Chew–Low–Goebel (CLG) technique well below 1 GeV/c momentum. At this time uncertainties stemming from non-pion-exchange backgrounds in near-threshold CLG studies appear to preclude a determination of the $\pi-\pi$ scattering lengths with the desired accuracy of 10% or better.

1. Motivation

Pion-pion scattering at threshold is uniquely sensitive to the explicit chiral symmetry breaking (ChSB) part of the strong interaction. For this reason it has been the subject of much theoretical and experimental study for over thirty years. While QCD has removed the early controversies and established the Weinberg picture of ChSB as valid at the tree level, pion-pion scattering lengths, $a(\pi\pi)$, remain of interest in terms of improving the precision of several basic parameters of the effective chiral lagrangian.

Currently the most accurate predictions of $a(\pi\pi)$ come from chiral perturbation (ChPT) calculations including terms up to two loops. However, a complementary approach, the generalized chiral perturbation theory (GChPT) dispenses with the standard assumption of a strong scalar quark condensate $\langle 0|\bar{q}q|0 \rangle$, allowing it to vary widely, with the conclusion that available experimental evidence favors a fairly weak condensate. The only observables capable of resolving this discrepancy are the $\pi-\pi$ s-wave scattering lengths, with a required precision of $\sim 10\%$ or better.

The result generally regarded as most reliable among the available evaluations of $a^f(\pi\pi)$ was obtained in 1979 in a comprehensive phase shift analysis of peripheral $\pi N \rightarrow \pi \pi N$ reactions and $K_{e4}$ decays:

$$a^0_0 = 0.26 \pm 0.05 \mu^{-1} \quad \text{and} \quad a^2_0 = -0.028 \pm 0.012 \mu^{-1},$$

where $\mu$ is the pion mass, clearly not precise enough to resolve the above quark condensate controversy. We therefore examine the more recent experiments and attempts at extraction of new, more precise values of $a(\pi\pi)$.

2. Experiments on Threshold $\pi-\pi$ Scattering

Since free pion targets cannot be fabricated, experimental evaluation of $\pi\pi$ scattering observables is restricted to the study of a dipion system in a final state of more
complicated reactions. While several reactions have been proposed and/or studied, only
\( \pi N \rightarrow \pi \pi N \) and \( K e^4 \) decays have so far proven useful in studying threshold \( \pi \pi \) scattering, although there are ambitious plans to study \( \pi^+ \pi^- \) atoms (pionium) in the near future.

\( K e^4 \) Decays

By most measures, the \( K^+ \rightarrow \pi^+ \pi^- e^+ \nu \) decay (called \( K e^4 \)) provides the most suitable tool for the study of threshold \( \pi \pi \) interactions. The interacting pions are real and on the mass shell, the only hadrons in the final state. The dipion invariant mass distribution peaks close to the \( \pi \pi \) threshold, and only two states, \( l_{\pi \pi} = I_{\pi \pi} = 0 \) and \( l_{\pi \pi} = I_{\pi \pi} = 1 \), contribute appreciably. These factors, plus the well understood \( V-A \) weak lagrangian giving rise to the decay, favor the \( K e^4 \) process among all others in terms of theoretical uncertainties. Measurements are, however, impeded by the low branching ratio of the decay, \( 3.9 \times 10^{-5} \).

\( K e^4 \) decay data provide information on the \( \pi-\pi \) phase difference \( \delta_0^0 - \delta_1^1 \) near threshold. The most recent published \( K e^4 \) experimental result was obtained by a Geneva–Saclay collaboration in the mid-1970’s.\(^5\) Taken alone these data provide a \( \sim 35 \% \) constraint on \( a_{0}^0 \). Only after being combined with \( \pi \pi \) phase shifts extracted from peripheral \( \pi N \rightarrow \pi \pi N \) reactions (see below) is it possible to reduce the uncertainties to the level of about 20\%, as quoted in Eq. (1).

We note that \( K e^4 \) decays provide no information on the \( I = 2 \) \( \pi \pi \) phase shifts, meaning that information from other reactions is required.

Peripheral \( \pi N \rightarrow \pi \pi N \) Reactions at High Momenta

Goebel as well as Chew and Low showed in 1958/59 that particle production in peripheral collisions can be used to extract information on the scattering of two of the particles in the final state.\(^7\) This approach is, of course, useful primarily for the scattering of unstable particles and has been used to great advantage in the study of the \( \pi \pi \) system. The method relies on an accurate extrapolation of the double differential cross section to the pion pole, \( t = \mu^2 \) (\( t \) is the square of the 4-momentum transfer to the nucleon), in order to isolate the one pion exchange (OPE) pole term contribution. Since the exchanged pion is off-shell in the physical region (\( t < 0 \)), this method requires measurements under conditions that maximize the OPE contribution and minimize all background contributions—typically peripheral pion production at values of \( t \) as close to zero as possible, which is practical at incident momenta above \( \sim 3 \) GeV/c. Since the CLG method relies on extrapolation in a two-dimensional space, it requires kinematically complete data of high quality, both in terms of measurement statistics and resolution—main limiting factors in all analyses to date.

The data base for peripheral CLG analyses has not changed essentially since the early 1970’s, and is dominated by two experiments, performed by the Berkeley\(^8\) and CERN-Munich\(^9\) groups. A comprehensive analysis of this data base, with addition of the Geneva–Saclay \( K e^4 \) data, was performed by Nagels et al.\(^5\), with results given in Eq. (1).

A 1982 analysis by the Kurchatov Institute group was based on a set of some 35,000 \( \pi N \rightarrow \pi \pi N \) events recorded in bubble chambers.\(^10\) This analysis was recently updated by including available data on the \( \pi N \rightarrow \pi \pi \Delta \) reaction, as well as the published \( K e^4 \) data.\(^11\) The resulting s-wave \( \pi \pi \) scattering lengths were bounded by

\[ 0.205 \mu^{-1} < a^0_0 < 0.270 \mu^{-1} \quad \text{and} \quad -0.048 \mu^{-1} < a^2_0 < -0.016 \mu^{-1}. \] (2)
Although the limits on $a_0^0$ carry slightly smaller uncertainties than the $a_0^0$ value of Nagels et al. given in Eq. (1), the result of Patarakin et al. still cannot rule out any of the two competing pictures of chiral symmetry breaking (strong vs. weak scalar quark condensate). The central value, though, is lower than in (1), more in line with the conventional, strong condensate picture that leads to the standard ChPT two-loop prediction of $a_0^0 \simeq 0.21 \mu^{-1}$.

New high energy ($E_\pi > 3$ GeV) peripheral $\pi N \to \pi \pi N$ measurements have not been planned for some time, so that much attention has been devoted to the study of the $\pi N \to \pi \pi N$ reaction at lower energies, $p_\pi \leq 500$ MeV, as discussed below.

**Inclusive $\pi N \to \pi \pi N$ Reactions Near Threshold**

Weinberg showed early on that the OPE graph dominates the $\pi N \to \pi \pi N$ reaction at threshold, inspiring vigorous theoretical and experimental study of the $\pi \pi$ and $\pi N \to \pi \pi N$ threshold amplitudes. Results of near-threshold $\pi \pi N$ studies published before 1995 are reviewed in detail in Ref. [12]. That impressive data base has been augmented by the addition of new, more precise $\pi^\pm p \to \pi^\pm \pi^\pm n$ cross sections very near threshold from TRIUMF [13]. The new measurements have confirmed the same group’s earlier published data [14] on the $\pi^+ p \to \pi^+ \pi^0 n$ reaction, thus definitively invalidating older data taken by the OMICRON collaboration at CERN [15].

Notwithstanding the abundance and high accuracy of recent near-threshold inclusive pion production data, their interpretation in terms of $\pi \pi$ amplitudes has been plagued by theoretical uncertainties. This shortcoming was addressed in 1995 using the heavy baryon chiral perturbation theory (HBChPT), yielding:

$$a_0^0 \simeq 0.21 \pm 0.07 \mu^{-1} \quad \text{and} \quad a_0^2 = -0.031 \pm 0.007 \mu^{-1} . \quad (3)$$

The above result for $a_0^0$ was subsequently refined by Olsson who used the so-called universal curve, a model-independent relation between $a_0^0$ and $a_0^2$ due to the forward dispersion relation or, equivalently, to the Roy equations [15]. Olsson found

$$a_0^0 = 0.235 \pm 0.03 \mu^{-1} . \quad (4)$$

Any analysis based on HBChPT cannot, however, be expected to result in $\pi \pi$ scattering lengths significantly different from the standard ChPT prediction because the latter is built into the lagrangian used.

**Chew–Low–Goebel Analysis of Low Energy $\pi N \to \pi \pi N$ Data**

Given the theoretical uncertainties in the interpretation of inclusive $\pi N \to \pi \pi N$ data near threshold, it was suggested some time ago to apply the Chew–Low method to low energy $\pi N \to \pi \pi N$ data [18]. Recently several exclusive $\pi N \to \pi \pi N$ data sets suitable for such treatment have become available. These are, in the order in which they were measured:

(a) $\pi^- p \to \pi^0 \pi^0 n$ data from BNL [19] and
(b) $\pi^+ p \to \pi^+ \pi^0 p$ data from LAMPF [20], and
(c) $\pi^- p \to \pi^- \pi^+ n$ data from TRIUMF [21].

We next briefly review the current results of these experiments.

A Virginia–Stanford–LAMPF team studied the $\pi^+ p \to \pi^+ \pi^0 p$ reaction at LAMPF at five energies from 190 to 260 MeV [23]. The LAMPF $\pi^0$ spectrometer and an array of plastic scintillation telescopes were used for $\pi^+$ and $p$ detection. Three classes of exclusive events were recorded simultaneously and independently: $\pi^+ \pi^0$ and $\pi^0 p$ double
coincidences, and $\pi^+\pi^0p$ triple coincidences. The $\pi^p \rightarrow \pi^+\pi^0p$ reaction is sensitive only to the $I = 2$ s-wave $\pi\pi$ scattering length.

The main source of difficulty in this analysis was the relatively broad missing mass resolution: $\sigma_p \simeq 11$ MeV and $\sigma_\pi \simeq 17$ MeV. This energy resolution considerably smears the cross section data bins in a Chew–Low plot of $m_{\pi\pi}$ against $t$. Consequently, in order to obtain a physically interpretable array of double differential cross section bins, a complicated deconvolution procedure had to be implemented first. Limited counting statistics presented an additional difficulty in the analysis, as it increased the uncertainties in both the deconvolution procedure and in the final CLG extrapolation.

![Figure 1: Chew–Low extrapolation to the pion pole from $\pi^p \rightarrow \pi^+\pi^0p$ exclusive cross sections at 260 MeV (preliminary). Full circles: data points included in the fit. Open circles: data points excluded from the fit. The extrapolated value of the $\pi\pi$ total cross section at $m_{\pi\pi} = 2.26 \pm 0.18 \mu$ is indicated.](image)

Preliminary results of this analysis for one bin of $m_{\pi\pi} = 2.26 \pm 0.18 \mu$ are shown in Fig. 1. Open circles in the figure indicate data points excluded from the Chew–Low extrapolation procedure due to large values of $|t| > 6 \mu^2$, where OPE is weak, and the smallest $|t|$ point which has a large normalization uncertainty due to the cross section deconvolution procedure. The resulting $\pi\pi$ cross section is $0.79 \pm 0.56$ mb, which translates to a phase shift of $\delta_0^2 = -8.3^\circ \pm 3.0^\circ$. This does not provide a strong new constraint when compared with existing information.

In comparison, the BNL $\pi^-p \rightarrow \pi^0\pi^0n$ data, while having much higher event statistics, are characterized by an even broader energy resolution and poorer coverage of the low $|t|$ region critical for the Chew–Low extrapolation. This limitation and/or strong influence of non-OPE backgrounds led to nonphysical results (negative extrapolated cross sections), as shown in Fig. 2.

The most significant development in this field in recent years has been the construction and operation of the Canadian High Acceptance Orbit Spectrometer (CHAOS), a sophisticated new detector at TRIUMF. This impressive device, composed of a number of concentric cylindrical wire chamber tracking detectors and total energy counters mounted between the poles of a large bending magnet, provides nearly 360° of angular
Figure 2: Chew–Low extrapolation to the pion pole $t = +\mu^2$ based on $\pi^- p \rightarrow \pi^0 \pi^0 n$ exclusive cross sections measured at three beam momenta (preliminary). Full circles: data points included in the fit. Open circles: data points excluded from the fit. The unphysical negative extrapolated values of the $\pi\pi$ total cross section are indicated.

coverage for in-plane events, with excellent acceptance for multi-particle events.

The CHAOS $\pi^- p \rightarrow \pi^+ \pi^- n$ data set covers four incident beam energies between 223 and 284 MeV. Unlike the LAMPF and BNL measurements, these data have an excellent energy resolution of $\sigma \simeq 4.8$ MeV, resulting in good linear Chew–Low extrapolations, as shown in Fig. 3.

From the CLG fits the authors extracted $\pi\pi$ cross sections at six $\pi\pi$ energies in the range $m_{\pi\pi}^2 = 4.15–5.65 \mu^2$ with uncertainties ranging from about 16% at the lowest energy to 63% at the highest. These $\pi\pi$ cross section data were then added to the data base of Ref. [11], and a Roy equation constrained phase shift analysis was performed following the same procedure as in Ref. [11], allowing $a_0^0$ to vary freely. Minimizing the $\chi^2$ of the fit, the authors obtained

$$a_0^0 = 0.206 \pm 0.013 \mu^{-1},$$

which would strongly confirm the validity of the standard ChPT and the strong scalar
Figure 3: Chew–Low extrapolation fits produced by the CHAOS group from measured $\pi^- p \to \pi^+ \pi^- n$ data.\textsuperscript{21} The points at $t = +\mu$ are deduced from extrapolation and yield the $\pi\pi$ cross section. Solid circles: data points used in the linear fit; crosses: data points not used in the fit.

quark condensate implied therein, at the same time ruling out the possibility of the weak scalar quark condensate proposed by the Orsay group.\textsuperscript{3}

However, Bolokhov et al. of the Sankt Petersburg State University have recently performed a detailed study of the reliability of the Chew–Low method at low energies using sets of synthetic $\pi N \to \pi\pi N$ “data” between 300 and 500 MeV/c.\textsuperscript{24} In this work the authors constructed data sets with: (a) the OPE contribution only, (b) OPE + other allowed mechanisms, (c) all mechanisms without the OPE. Both linear and quadratic Chew–Low extrapolation were used. The authors found 25–35\% deviations in the reconstructed OPE strength in case (a), 100–300\% deviations under (b), and large “OPE amplitude” without any pion pole in the synthetic data under (c). This led the authors to conclude that the Chew–Low method appears to give completely unreliable results. However, given the complex nature of the issue, it would be premature to write off using the method at low energies altogether. Clearly, a critical examination of the problem is imperative. In the meantime, before the matter is finally resolved, we cannot accept the CHAOS result in Eq. (5) as definitive.
3. Summary of Current Results and Future Prospects

Theoretical predictions and experimental results on the $\pi\pi$ scattering lengths published to date are plotted in Fig. 4 in the $a_0^0$ against $a_0^0$ plane. It is clear that the current analyses of the available $K_{e4}$ and $\pi N \rightarrow \pi\pi N$ data (excluding the not yet fully established low energy application of the Chew–Low method) are not sufficiently accurate to distinguish between the two scenarios of chiral symmetry breaking, i.e., between the standard picture and the one with a weak $\langle 0|\bar{q}q|0 \rangle$.

At the same time the available analyses seem to favor slightly higher values of both $a_0^0$ and $a_0^2$ than the values predicted by standard ChPT (strong $\langle 0|\bar{q}q|0 \rangle$).

The threshold $\pi-\pi$ scattering experimental data base will improve significantly in the near future as several new experiments bear fruit. These are: (a) the forthcoming $K_{e4}$ data from BNL E865 (experiment completed, analysis in progress) and the KLOE detector at DAΦNE (experiment to start soon), as well as (b) the planned measurement of the lifetime of the $\pi^+\pi^-$ atom (the DIRAC project at CERN). If all goes as planned, these experiments combined will provide $\sim 5\%$ limits on the scattering lengths.

As noted above, further theoretical work is required to make use of the existing $\pi N \rightarrow \pi\pi N$ data, in particular to clarify the applicability of the Chew–Low–Goebel method at low energies. Additionally, better understanding of the electromagnetic cor-
rections will be necessary in order to take full advantage of the forthcoming $K_{e4}$ and pionium data. Thus, the next few years will be interesting on both the experimental and theoretical fronts.

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References

1. S. Weinberg, Phys. Rev. Lett. 17 (1966) 616; ibid. 18 (1967) 188.
2. J. Bijnens, et al., Phys. Lett. B374 (1996) 210; Nucl. Phys. B508 (1997) 263.
3. J. Stern, H. Sazdjian and N. H. Fuchs, Phys. Rev. D 47 (1993) 3814; M. Knecht, B. Moussallam and J. Stern, Nucl. Phys. B429 (1994) 125.
4. M. Knecht, et al., Nucl. Phys. B457 (1995) 513, ibid., B471 (1996) 445.
5. M. M. Nagels, et al., Nucl. Phys. B147 (1979) 189.
6. L. Rosselet, et al., Phys. Rev. D 15 (1977) 574.
7. C. J. Goebel, Phys. Rev. Lett. 1 (1958) 337; G. F. Chew and F. E. Low, Phys. Rev. 113 (1959) 1640.
8. S. D. Protopopescu et al., Phys. Rev. D 7 (1973) 1279.
9. G. Grayer et al., Nucl. Phys. B75 (1974) 189.
10. E. A. Alekseeva et al., Zh. Eksp. Teor. Fiz. 82 (1982) 1007 [Sov. Phys. JETP 55 (1982) 591].
11. O. O. Patarakin, V. N. Tikhonov and K.N. Mukhin, Nucl. Phys. A598 (1996) 335.
12. D. Počanić, in “Chiral Dynamics, Theory and Experiment”, A. M. Bernstein and B. R. Holstein, eds., Lect. Notes in Phys. Vol. 452, (Springer Verlag, 1995) 95.
13. J. B. Lange, et al., Phys. Rev. Lett. 80 (1998) 1597.
14. M. E. Sevior, et al., Phys. Rev. Lett. 66 (1991) 2569.
15. G. Kernel et al., Z. Phys. C48 (1990) 201.
16. V. Bernard, N. Kaiser and Ulf G. Meissner, Nucl. Phys. B457 (1995) 147.
17. M. G. Olsson, Phys. Lett. B410 (1997) 311.
18. D. Počanić et al., proposal for LAMPF experiment E1179 (1989).
19. J. Lowe, et al., Phys. Rev. C 44 (1991) 956.
20. D. Počanić, et al., Phys. Rev. Lett. 72 (1994) 1156; E. Frlež, Ph.D. Thesis, Univ. of Virginia, 1993 (Los Alamos Report LA-12663-T, 1993).
21. M. Kermani, et al., TRIUMF preprint (1997).
22. S. E. Bruch, M.Sc. Thesis, Univ. of Virginia (1995).
23. G. R. Smith, et al., Nucl. Instrum. Meth. A362 (1995) 349.
24. A. A. Bolokhov, M. V. Polyakov and S. G. Sherman, e-print hep-ph/9707406 (1997).
25. J. Gasser and H. Leutwyler, Phys. Lett. B125 (1983) 325.