On the mechanics of a fusiform cerebral aneurysm: Mooney-Rivlin mathematical model for the experimental data

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Abstract. In this work, mathematical modelling of the process of rupturing the wall of a human fusiform aneurysm is carried out. We consider 3-parameter and 5-parameter Mooney-Rivlin models. The comparison between the results of the modelling and the experimental data is made. Analyzing the relation of parameters of 3-parameters and 5-parameters Mooney-Rivlin models, the relation between the greatest (least) value and Cauchy strain was found. The field of application of these models for hemodynamics was shown.

1. Introduction
An intracranial aneurysm (IA) is a cerebrovascular disease, which occurs approximately in 1 out of 50 people [1, 2], but nevertheless is rarely diagnosed. To know the risk of rupture of given aneurysm is one of the main concerns of the neurosurgery. According to statistics approximately only 1-2% of unruptured aneurysms will rupture at some point. To calculate the risks of rupture the numerous factors are to be considered and it presents a demanding problem to examine all of them at the same time. However doing so provides the better understanding of the aneurysm structure and properties which increases the accuracy of prediction whether or not the aneurysm is going to rupture. The mechanism of the aneurysm formation is not entirely known yet, hence the studies of unruptured IAs, such as [3, 4] and [5], all use results from various experimental approaches.

Mathematical modelling of aneurysm tissue provides more profound understanding of its mechanics [6]. The model that sufficiently corresponds experimental and clinical data can be used in FSI calculations during the estimation of critical parameters of aneurysm rupture. Besides each model of that kind has its own field of application. Mooney-Rivlin kind of models are widely used in mathematical physics for modelling the mechanics of elastic materials and particularly in hemodynamics related field [7].

2. Methods
The experiments on investigating the mechanic properties of cerebral aneurysm tissue were first performed at studies [8]. And the technology was significantly developed by the group under the leadership of A.M. Robertson [9, 10].

After the neurosurgeries were performed, the obtained IA tissues were taken to laboratory, preserved with sodium 0.9% at +2°C-+5°C. According to [11] refrigerated aneurysm show slightly different results with relation to used temperature. At the laboratory the rupture machine (machine Zwick/Roell Z010, Germany) was used to perform a series of experiments on tissues. As the tissue is
quickly deteriorating becoming unfit for the further investigation, only one series of experiments is possible for every sample. Speed of pulling equals 1mm per minute and was the same for all experiments.

Finding Mooney-Rivlin constants

As the conducted tension of aneurysm tissue in mechanic tests was uniaxial we will consider the mathematical model under that condition.

In that case:

\[
S_{3p} = 2C_{10} \left( \lambda - \frac{1}{\lambda} \right) + 2C_{01} \left( 1 - \frac{1}{\lambda^2} \right) + 6C_{11} \left( \lambda^2 - \lambda - 1 + \frac{1}{\lambda^2} + \frac{1}{\lambda^3} - \frac{1}{\lambda^4} \right) 
\]

\[
S_{5p} = 2C_{10} \left( \lambda - \frac{1}{\lambda} \right) + 2C_{01} \left( 1 - \frac{1}{\lambda^3} \right) + 6C_{11} \left( \lambda^2 - \lambda - 1 + \frac{1}{\lambda^2} + \frac{1}{\lambda^3} - \frac{1}{\lambda^4} \right) 
+ 4C_{20} \lambda \left( 1 - \frac{1}{\lambda^3} \right) \left( \lambda^2 + \frac{2}{\lambda} - 3 \right) + 4C_{02} \left( 2\lambda + \frac{1}{\lambda^2} - 3 \right) \left( 1 - \frac{1}{\lambda^3} \right) 
\]

Where \( S_{3p} \) and \( S_{5p} \) are corresponding uniaxial stress for 3-parameters and 5-parameters models, \( \lambda \) is a stretch ratio and \( C_{10}, C_{01}, C_{11} \) are parameters for 3-parameters model while \( C_{10}, C_{01}, C_{11}, C_{02}, C_{20} \) are parameters for 5-parameters model.

To find the Mooney-Rivlin constants that correspond best to the data of each experiment we used Wolfram Mathematica (commercial software, License of NSU). The approximate function for strain-stress curve was built, using least squares method. After that, based on obtained function, the Mooney-Rivlin constants were found, independently for each experiment.

![Figure 1. Reconstructed 3D model of vessel with aneurism](image)
3. Results and discussion
We have examined the effect that growth of the relative elongations of the sample has on the changes in the parameters of the Mooney-Rivlin models under cyclic loading. In Figures 4-11 plot points correspond to largest Cauchy strain of previously described experiment. As it can be noted from the plots, in area of ε from 1.6 to 2.6 every one of parameters for each model reaches its extremum. From the Figure 3 it is clear that after given Cauchy stress value the error increases significantly (up to 13%) to in relation to experimental data.

Figure 2. Results of the experiment №2 (blue) and mathematical modelling with 3-parameter (red) and 5-parameter (yellow) Mooney-Rivlin models

Figure 3. Results of the experiment №4 (blue) and mathematical modelling with 3-parameter (red) and 5-parameter (yellow) Mooney-Rivlin models

Figure 4. Coefficient C01 value for 3-parameters model (1) in relation to Cauchy strain

Figure 5. Coefficient C10 value for 3-parameters model (1) in relation to Cauchy strain
Figure 6. Coefficient $C_{11}$ value for 3-parameters model (1) in relation to Cauchy strain

Figure 7. Coefficient $C_{01}$ value for 5-parameters model (1) in relation to Cauchy strain

Figure 8. Coefficient $C_{10}$ value for 3-parameters model (1) in relation to Cauchy strain

Figure 9. Coefficient $C_{11}$ value for 5-parameters model (2) in relation to Cauchy strain

Figure 10. Coefficient $C_{20}$ value for 5-parameters model (2) in relation to Cauchy strain

Figure 11. Coefficient $C_{02}$ value for 5-parameters model (2) in relation to Cauchy strain

4. Conclusion
The satisfying correspondence of stress-strain curves were obtained during the mathematical modelling for both 3-parameters and 5-parameters Mooney-Rivlin models. It is shown that for the relative elongations in the range from 1.6 to 2.6 the coefficients of the above models reach their extremum. With further increase in the relative elongations of the aneurysm material, the error in the approximation of the models of experimental data of technologies increases (up to 13%).
This analysis suggests that the usage of 3-parameters and even 5-parameters Mooney-Rivlin models is justified for relatively small deformation of the material and particularly it is adequate to use such models in FSI calculations in cases where the area of aneurysm does not contain diverticula (protrusions) which are usually are characterized by relatively large displacements relative to the dome of the aneurysm.

Acknowledgements
This work was supported by a grant from Russian Science Foundation, No. 17-11-01156.

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