Homogeneous cosmologies in scalar tensor theory

Melis Ulu Doğru\textsuperscript{a} and Derya Baykal\textsuperscript{b}

\textsuperscript{a} Department of Physics, Art and Science Faculty, Çanakkale Onsekiz Mart University
Çanakkale, 17020, Turkey

\textsuperscript{b} Institute for Natural and Applied Sciences, Çanakkale Onsekiz Mart University
Çanakkale, 17020, Turkey

melisulu@comu.edu.tr
Abstract

In this study, FRW-cosmologies with some matter groups such as monopole-domain wall, monopole-Chaplygin gas and monopole-strange quark matter in the scalar theory of gravitation based on Lyra geometry are investigated. We expand two exact models as static case and time-depended case for each matter groups in order to solve field equations in the scalar theory. For each matter groups, the solutions are introduced as the models of expanding universe, exponentially. Hubble parameters in the case of $k = 0, -1, 1$ are obtained for these models. Furthermore, we realize interesting result which the well-known relation between scalar theory based on Lyra geometry and Einstein’s theory is an incomplete idea. In opposition to the well accepted idea in the literature, we suggest that Einstein’s theory with no cosmological constant is equivalent of scalar theory based on Lyra geometry with zero displacement vector, completely. If the components of displacement vector in the scalar theory are any constant functions, the scalar theory couldn’t correspond to Einstein’s theory, identically. Even if the components of displacement vector in the scalar theory are the constant, the field equations and their solutions contain the Einstein’s field equations and their solutions, but they are variously more general than Einstein’s theory of gravitation. So, coefficients of constant displacement vector don’t play the role of cosmological constant in keeping with the Einstein’s theory. Finally, the results have been discussed.

1 Introduction

The fundamentals of gravitation theories have been predicated on the Newtonian Theory, early on. But, at the end of 19th century, it was found that the velocity of light has finite value by well-known experiment carried out by Michelson and Morley. Consequently, the Lorentzian transformations invalidate the Galilean transformations. For the many reason, the new gravitation theories with Lorentz invariance was needed to arise. Suggested theories with Lorentz invariance can be classified as Poincare-type, scalar-type, vectorial-type and tensorial-type gravitation theories. In addition to the available theories at present, adopted gravitation theories are called as Einstein’s theory, Teleparallel gravity, Brans-Dicke theory, modified gravity and Lyra geometry. Both of them, Lyra [1] and Brans-Dicke [2], are the alternative scalar-tensor theories [3]. Indeed, an alternative scalar-tensor theory of gravitation was propounded by Weyl [4] in 1918 via associated the electro-dynamical and gravitational states of a space-time, firstly. But geometry of Weyl’s theory was not valid and it was not useful since it was depended on non-integrability of length transfer [5]. Lyra evolved a new theory, composed by modification of Riemannian geometry and based on Weyl’s geometry [6]. In Lyra
geometry, the concept of scalar curvature is defined as opposite of Weyl’s theory [7].

In Lyra geometry, \( n \)-dimensional space-time \((M, \phi, g_{ik})\) is contained a smooth manifold, a smooth scalar field and connection, on condition that \( M, \phi \) and \( \Gamma \) are introduced manifold, the gauge function and Lyra connection, respectively [8]. The coefficients of Lyra connection are given by

\[
\Gamma^c_{ab} = \frac{1}{\phi} \{ \Gamma^c_{ab} \} + \frac{s+1}{\phi^2} g^{cd} (g_{bd} \partial_a \phi - g_{ab} \partial_d \phi)
\]

where \( \{ \Gamma^c_{ab} \} \) is the second kind of Christoffel symbols, \( s \) is a constant. Also, torsion is introduced by

\[
T^c_{ab} = \frac{s}{\phi^2} (\delta^c_b \nabla_a \phi - \delta^c_a \nabla_b \phi).
\]

Curvature tensor in Lyra geometry is given by [8]

\[
K^c_{dab} = \frac{1}{\phi^2} [\partial_a (\phi \Gamma^c_{db}) - \partial_d (\phi \Gamma^c_{ab}) + \Gamma^e_{db} \Gamma^c_{ea} - \Gamma^e_{ab} \Gamma^c_{ed}]
\]

and also, contractions of curvature tensor are formed \( K_{ac} = K^b_{abc} \), \( K = g^{ab} K_{ab} \), similar to Riemannian geometry. Finally, scalar curvature is defined by the following form:

\[
K = \frac{R}{\phi^2} + \frac{2(s+1)}{\phi^3} (1-n) \Box \phi + \frac{1}{\phi^4} [(s+1)^2 (3n-n^2-2) - 2(s+1)(2-n)] \nabla^c \phi \nabla_c \phi
\]

where \( \Box \) signs D’alambertian operator [8]. The action in scalar theory based on Lyra geometry is given by

\[
S = \int d^4x \sqrt{-g} (\phi^2 R - 4\omega g^{cd} \nabla_c \phi \nabla_d \phi)
\]

where \( \omega = \frac{3(s^2-1)}{2} \).

\[
R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi \phi_j - \frac{3}{4} g_{ij} \phi^k \phi_k = -8G T_{ij}
\]
where $\phi_i$ is the displacement vector.

Sen [9], Sen and Dunn [10] reproduced the Lyra geometry. Halford [11], suggested that cosmological constant in the Einstein’s theory corresponds the constant displacement vector field $\phi$ in scalar theory of gravitation based on Lyra’s geometry. Some of the studies about Lyra theory displayed the investigation of cosmological models with constant displacement field vector [12] [13] [14] [15] [16]. Pradhan and Pandey [17] obtained the exact solutions of bulk viscosity in LRS Bianchi type-I models with constant deceleration parameter besides Pradhan and Chauhan [17] obtained the exact solutions of perfect fluid in LRS Bianchi type-I models with variable deceleration parameter. Rahaman et. al. [18] proposed two models according to thin domain walls in Lyra geometry and pointed out that thin domain walls have no particle horizons in addition to have gravitational force, effectively. Rahaman [19] studied global texture with time dependent displacement vector using weak field approximation on Lyra geometry. On the other hand, there are some investigations in homogeneous Friedmann-Robertson-Walker (FRW) universe with the framework of Lyra geometry in the literature. For example, Pradhan et. al. [20] studied bulk viscosity in FRW-universe and suggested the solutions of energy density and displacement field vector for power-law or exponential expansion of the universe in the cases of $k = 0$ and $k = -1$. Also, Singh and Desikan [21] displayed the solution of FRW-universe with time dependent displacement field vector and constant deceleration parameter, based on Lyra geometry using the equation of state. Rahaman et. al. [22] propounded the field equations and solutions of higher dimensional spherically symmetric space-time associated with mass-less scalar field with constant potential for the flat region.

In this study, we have investigated the models of FRW-universe with monopoles and domain walls, Chaplygin gas or strange quark matter in the frame work of the scalar theory of gravitation based on Lyra geometry. In Section.2, we have obtained the field equations of scalar theory based on Lyra geometry for given matter groups and universe. We have examined the exact solutions of the field equations for all values of $k = -1, 0, 1$. Finally, our results have been discussed.
2 Field equations of homogeneneous cosmologies in scalar theory based on Lyra geometry

It is known that FRW-models have worked in varied gravity theories in order to understand nature of universe. A spatially homogeneous and isotropic FRW space-time is

\[ ds^2 = dt^2 - R(t)^2 \left[ \frac{1}{1-kr^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\psi^2 \right] \]

where curvature parameter, \( k \), has values of \( k = -1, 0, 1 \) according to open, flat or closed geometry of universe. Also, \( R(t) \) is the cosmic scale factor \([23]\).

Moreover, monopoles appeared as point-like defects due to global symmetry breaking in the evolution of universe according to the standard cosmology. So, the defects are principally called ”global monopoles”. Although the required geometrical conditions for the formation of all defects are the same, monopoles have more different properties, physically. Global monopoles hold Lagrangian density as

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \Phi^i \partial^{\mu} \Phi^i - \frac{1}{4} \lambda (\Phi^i \Phi^i - \eta_m^2)^2 \]

where \( \Phi^i = \eta_m f(r)(\frac{x}{r}) \) is a scalar field of monopoles and \( i = 1, 2, 3 \). Furthermore, \( h(r) \) is equal to zero at \( r = 0 \) and approaches \( h(r) \rightarrow 1 \) at \( r \gg \delta \), where the size of monopole core \( \delta \) can be described as \( \delta \sim (\sqrt{\lambda} \eta_m)^{-1} \) \([24]\).

The energy-momentum tensor of any matter field is widely known as the form by

\[ T^{\mu}_{\nu} = \partial_\nu \Phi^i \partial^\mu \Phi^i - \mathcal{L} \delta^{\mu}_{\nu} \].

From Eqs. (7)-(8) and Eq. (9), the required components of energy-momentum tensor for global monopoles can be obtained as \([24]\)

\[ T^t_t = T^r_r = \frac{\eta_m^2}{r^2} \].

In terms of standart cosmology, domain walls formed due to the degeneration of discrete symmetry of early universe. In the Goldstone model, a relation can be identified between domain wall surface density \( \sigma_w \) and symmetry breaking scale \( \eta_w \) such as \( \sigma_w \sim \sqrt{\lambda} \eta_w^3 \). If the symmetry breaking scale is not very small, it
is seeing that domain wall surface density must be dominant from the relation. It is concluded that domain walls have an enormous role on the homogeneity of universe \[24\]. The energy-momentum tensor of domain walls in the perfect fluid form is given by

\[ T^i_k = (p_m - \sigma_w)(u^iu_k - \delta^i_k) + (\rho_m + \sigma_w)u^iu_k \]

where \( \rho_m \) and \( p_m \) are density and pressure of the matter, \( \sigma_w \) is the tension of domain wall, respectively \[25\].

Recent observations of Type-Ia Supernovae have pointed out that our universe is spatially flat and has expanded, accelerately \[26\] [27] [28] [29] [30] [31]. It is believed that the source of the expansion is the dark energy, which constitutes %70 of the universe \[32\]. In the unified model of dark energy, one of the dark energy candidates is Chaplygin gas, also called as quartessence \[33\]. The Chaplygin gas has the similar role with cosmological constant at small or large values of scale factor in relation with expansion of universe \[34\] [35] [36]. In the gravitation theories, energy-momentum tensor of Chaplygin gas is given by

\[ T^i_k = \left( \frac{A}{\rho_C} \right)(u^iu_k - \delta^i_k) + \rho_C u^iu_k \]

where \( \rho_C \) is density of the Chaplygin gas and \( A \) is a negative constant \[33\] [36].

In the early universe, there are in some other important stages as well as symmetry breaking. One of them is named Quark-Hadron phase. The phase in which cosmic temperature had the values of 200 MeV, passed away from Quark-Gluon Plasma to Hadron gas \[37\]. Due to Quark-Hadron phase transition, the quark matter occurred. According to bag model, using the proportion between the density and pressure of quark matter such as \( p_q = \frac{\rho_q}{T} \), total energy density and total pressure are respectively given by

\[ \rho_m = \rho_q + B_c, \]

and

\[ p_m = p_q - B_c \]

where \( B_c \) is the bag constant \[37\]. On the other hand, Equation of State (EoS) for strange quark matter is also given by
\begin{equation}
    \rho_m = \frac{1}{3}(\rho_m - 4B_c) .
\end{equation}

Recently, in the Brookhaven National Laboratory, quark-gluon plasma has been achieved in the form of perfect fluid, experimentally \cite{34, 35, 38}. According to the development, it can be considered Quark-Gluon plasma in the form of perfect fluid and thus energy-momentum tensor of strange quark matter can be given by the following form

\begin{equation}
    T^i_k = (p_q - B_c)(u^i u_k - \delta^i_k) + (\rho_q + B_c)u^i u_k .
\end{equation}

In this study, it can be noted that the matter of space-times has been classified in three different groups like as (i) monopoles and domain walls, (ii) monopoles and Chaplygin gas, (iii) monopoles and strange quark matter. Also, it can be chosen the comoving coordinates as \( u^i = \delta^i_0 \), \( u^i \) stands for the four-velocity. The displacement vector is used like as \( \phi_i = (0, 0, 0, \beta) \) and \( \beta \) is the constant.

### 2.1 FRW-cosmologies with monopoles and domain walls in Lyra geometry

Using the energy momentum tensors of monopoles and domain walls in Eqs.(10)-(11) and the line element of FRW space-time in Eq.(7) together with Eq.(6), the field equations in scalar tensor Lyra theory are obtained the following form

\begin{equation}
    \frac{k}{R^2} + \frac{R'^2}{R^2} + \frac{2R''}{R} - \frac{3}{4}\beta^2 = -\chi (p_m - \sigma - \eta \frac{\rho_m}{r^2} ) ,
\end{equation}

\begin{equation}
    \frac{k}{R^2} + \frac{R'^2}{R^2} + \frac{2R''}{R} - \frac{3}{4}\beta^2 = -\chi (p_m - \sigma_w ) ,
\end{equation}

\begin{equation}
    \frac{3k}{R^2} + \frac{3R'^2}{R^2} + \frac{3}{4}\beta^2 = \chi (\rho_m + \sigma_w + \frac{\eta m^2}{r^2} ) .
\end{equation}

Domain wall density and pressure depend on each other with \textit{EoS} given by \( p_m = \gamma \rho_m \), where \( \gamma \) is the constant. With reference to the \textit{EoS} and Eqs.(17)-(19), we have obtained two different exact solutions of FRW-cosmologies with monopoles and domain walls in Lyra scalar theory.
case (i) First solution of FRW-cosmologies with monopoles and domain walls
First set of the exact solutions of FRW-cosmologies with monopoles and domain walls in Lyra scalar theory has been obtained by

\[ R(t) = c_1, \]

\[ \rho_m = \frac{1}{\gamma+1} \left[ \frac{2k}{\chi c_1^2} + \frac{3}{2\chi} \beta^2 \right], \]

\[ p_m = \frac{\gamma}{\gamma+1} \left[ \frac{2k}{\chi c_1^2} + \frac{3}{2\chi} \beta^2 \right] \]

and

\[ \sigma_w = \left( \frac{3\gamma+1}{\gamma+1} \right) \frac{k}{\chi c_1^2} + \left( \frac{\gamma-1}{\gamma+1} \right) \frac{3}{4\chi} \beta^2 - \frac{\eta_m^2}{r^2}. \]

case (ii) Second solution of FRW-cosmologies with monopoles and domain walls
Second set of the exact solutions of FRW-cosmologies with monopoles and domain walls in Lyra scalar theory has been obtained by

\[ R(t) = \frac{c_2}{2} \left[ e^{-\frac{\eta_m}{\sqrt{2}}} + ke^{\frac{\eta_m}{\sqrt{2}}} \right], \]

\[ \rho_m = \left( \frac{1}{\gamma+1} \right) \frac{3}{2\chi} \beta^2, \]

\[ p_m = \left( \frac{\gamma}{\gamma+1} \right) \frac{3}{2\chi} \beta^2 \]

and

\[ \sigma_w = \frac{3}{\chi c_2} + \left( \frac{\gamma-1}{\gamma+1} \right) \frac{3}{4\chi} \beta^2 - \frac{\eta_m^2}{r^2}. \]
2.2 FRW-cosmologies with monopoles and Chaplygin gas in Lyra geometry

Using the energy momentum tensor of monopoles and Chaplygin gas in Eqs. (10), (12) and the line element of FRW space-time in Eq. (7) together with Eq. (6), the field equations in scalar tensor Lyra theory are obtained by the following form

\begin{equation}
\frac{k}{R^2} + \frac{R''}{R} - \frac{3}{4} \beta^2 = -\chi\left(\frac{A}{\rho_C} - \frac{\eta_m^2}{r^2}\right), \tag{28}
\end{equation}

\begin{equation}
\frac{k}{R^2} + \frac{R''}{R} + \frac{3}{4} \beta^2 = -\chi\left(\frac{A}{\rho_C}\right), \tag{29}
\end{equation}

\begin{equation}
\frac{3k}{R^2} + \frac{3R''}{R^2} + \frac{3}{4} \beta^2 = \chi(\rho_C + \frac{\eta_m^2}{r^2}). \tag{30}
\end{equation}

Chaplygin gas density and pressure depend on the each other with EoS given by \( p_C = \frac{A}{\rho_C} \). Starting with the EoS and Eqs. (28)-(30), we have obtained two different exact solutions of FRW-cosmologies with monopoles and Chaplygin gas in Lyra scalar theory.

**case(i) First solution of FRW-cosmologies with monopoles and Chaplygin gas**

First set of the exact solutions of FRW-cosmologies with monopoles and Chaplygin gas in Lyra scalar theory has been obtained by

\begin{equation}
R(t) = c_4, \tag{31}
\end{equation}

\begin{equation}
\rho_C = -\left(\frac{k}{\chi c_4^2} + \frac{3}{4} \beta^2\right) \pm \frac{1}{2} \sqrt{\left(\frac{2k}{\chi c_4^2} + \frac{3}{2} \chi \beta^2\right)^2 - 4A} \tag{32}
\end{equation}

and

\begin{equation}
p_C = A\left[-\left(\frac{k}{\chi c_4^2} + \frac{3}{4} \beta^2\right) \pm \frac{1}{2} \sqrt{\left(\frac{2k}{\chi c_4^2} + \frac{3}{2} \chi \beta^2\right)^2 - 4A}\right]^{-1}. \tag{33}
\end{equation}
case(ii) Second solution of FRW-cosmologies with monopoles and Chaplygin gas

Second set of the exact solutions of FRW-cosmologies with monopoles and Chaplygin gas in Lyra scalar theory has been obtained by

\begin{equation}
R(t) = \frac{c_5}{2} e^{\frac{-\epsilon t}{c_5}} + k e^{\frac{\epsilon t}{c_5}},
\end{equation}

(34)

\begin{equation}
\rho_C = -\frac{3}{4\chi} \beta^2 + \sqrt{\frac{9}{16\chi}} \beta^4 - A
\end{equation}

(35)

and

\begin{equation}
p_C = A \left[-\frac{3}{4\chi} \beta^2 \mp \sqrt{\frac{9}{16\chi}} \beta^4 - A\right]^{-1}.
\end{equation}

(36)

2.3 FRW-cosmologies with monopoles and strange quark matter in Lyra geometry

Using the energy momentum tensors of monopoles and strange quark matter in Eqs. (10), (16) and the line element of FRW space-time in Eq. (7) with together Eq. (6), the field equations in scalar tensor Lyra theory are obtained by the following form

\begin{equation}
\frac{k}{R^2} + \frac{R'^2}{R^2} + \frac{2R''}{R} - \frac{3}{4} \beta^2 = -\chi (p_q - B_c - \frac{n_m^2}{r^2}),
\end{equation}

(37)

\begin{equation}
\frac{k}{R^2} + \frac{R'^2}{R^2} + \frac{2R''}{R} - \frac{3}{4} \beta^2 = -\chi (p_q - B_c)
\end{equation}

(38)

and

\begin{equation}
\frac{3k}{R^2} + \frac{3R'^2}{R^2} + \frac{3}{4} \beta^2 = \chi (p_q + B_c + \frac{n_m^2}{r^2}).
\end{equation}

(39)
Since the strange quark matter has been perfect fluid form, it’s density and pressure depend on each other with EoS given by Eqs. (13)-(15). From the Eqs. (13)-(15) and Eqs. (37)-(39), we have two different exact solutions of FRW-cosmologies with monopoles and strange quark matter in Lyra scalar theory.

**case(i) First solution of FRW-cosmologies with monopoles and strange quark matter**

First set of FRW-cosmologies with the exact solutions of monopoles and strange quark matter in Lyra scalar theory has been obtained by

\begin{align}
(40) & \quad R(t) = c_7, \\
(41) & \quad \rho_q = \frac{3}{4} \left[ \frac{2k}{\chi c_7^3} + \frac{3}{4\chi} \beta^2 \right], \\
(42) & \quad p_q = \frac{1}{4} \left[ \frac{2k}{\chi c_7^3} + \frac{3}{4\chi} \beta^2 \right] \\
\text{and} & \quad B_c = \frac{3k}{2\chi c_7^3} + \frac{3}{8\chi} \beta^2 - \frac{\eta_m^2}{r^2}. \\
\end{align}

**case(ii) Second solution of FRW-cosmologies with monopoles and strange quark matter**

Second set of the exact solutions of FRW-cosmologies with monopoles and strange quark matter in Lyra scalar theory has been obtained by

\begin{align}
(44) & \quad R(t) = \frac{c_8}{2} \left[ e^{\frac{(27+q)}{8}} + ke^{\frac{(27+q)}{8}} \right], \\
(45) & \quad \rho_q = \frac{9}{8\chi} \beta^2, \\
(46) & \quad p_q = \frac{3}{8\chi} \beta^2.
\end{align}
3 Conclusion

Some alternative gravitation theories to Einstein’s theory are possible to be seen in the literature. One of them is accepted as the scalar theory based on Lyra geometry, improved from Weyl theory. In this study, we have investigated FRW-cosmologies associated with three matter groups, consisted of monopoles-domain walls, monopoles-Chaplygin gas and monopoles-strange quark matter, in the framework of Lyra scalar theory of gravitation. The components of displacement vector of Lyra theory are chosen a constant. It is noted that two exact solutions for each matter groups are obtained (see in Sec. 2). Our solutions have different features about cosmic scale factor. In the first solutions, the cosmic scale factor is constant as given by Eqs. (20), (31) and (40). So, the solutions with constant $R(t)$, indicate the static universe. In the second solutions, the cosmic scale factor is time-dependent as given by Eqs. (24), (34) and (44). The solutions have acceleration and indicate expanding universe. These solutions can be expounded analogously to the following concept.

In the second solution of FRW-cosmologies with monopoles and domain walls, it is clear that the cosmic scale factor $R(t)$ depends on the time, exponentially. The scale factor $R(t)$ in Eq. (24) transforms the following functions for the values of curvature parameter, $k = 0, -1, 1$, respectively;

\begin{equation}
 k \to 0, \quad R(t) = \frac{c_2}{2} e^{\frac{c_3}{c_2} t},
\end{equation}

\begin{equation}
 k \to -1, \quad R(t) = \frac{c_2}{2} \sinh\left(\frac{c_3}{c_2} t\right),
\end{equation}

and

\begin{equation}
 k \to 1, \quad R(t) = \frac{c_2}{2} \cosh\left(\frac{c_3}{c_2} t\right).
\end{equation}

As the relation with the functions of $R(t)$ in Eqs. (48)-(50), it must be called attention to similar solutions of Friedmann equations for the matter with pressureless
Figure 1. The cosmic scale factor in the non-static solution of FRW models with monopole, Chaplygin gas, domain wall or strange quark matter in Lyra geometry and constant density in Einstein’s theory [39]. If the models have the expansion and the domain walls have the non-zero surface tension, the constant $c_2$ must be non-zero ($c_2 \neq 0$). From Eq. (24), speed of expansion and acceleration of universe are given by

\begin{align*}
\dot{R}(t) &= \frac{1}{2} \left[ e^{\frac{t+c_3}{c_2}} \pm ke^{\frac{-t+c_3}{c_2}} \right] \\
\ddot{R}(t) &= \frac{1}{2c_2} \left[ e^{\frac{t+c_3}{c_2}} \pm ke^{\frac{-t+c_3}{c_2}} \right].
\end{align*}
In Einstein’s theory, speed and acceleration of universe had the similar values to Eqs. (51)-(52) in Lyra theory. From the field equation in Eq. (19) and the solutions in Eqs. (25) and (27), we get

\[ \frac{\dot{R}(t)^2}{R(t)^2} = \frac{1}{c_2} - \frac{k}{R^2}. \]  

From the Eq. (53), we get the Hubble parameter/constant as

\[ k \to 0, \quad H = a, \]  

\[ k \to 1, \quad H(t) = \left[ a^2 - \text{sech}^2(a(t + c_3)) \right]^{\frac{1}{2}} \]  

and

\[ k \to -1, \quad H(t) = \left[ a^2 + \text{cosech}^2(a(t + c_3)) \right]^{\frac{1}{2}} \]

where \( a = \frac{1}{c_2} \). Provided that the field equations of FRW-cosmologies with monopoles and domain walls in Lyra scalar theory are compared with the Friedmann equations in de Sitter cosmology of Einstein’s theory, we get the relation between the Hubble constant \( H \) of the solutions in Lyra scalar theory and cosmological constant \( \Lambda \) of Einstein’s theory as

\[ H = a = \pm \sqrt{\frac{\Lambda}{3}}. \]

Also, it is interesting result that surface tension of domain wall is directly related to the subtraction to matter density and pressure from Eqs. (25)-(27) as

\[ \sigma_w = \frac{3a^2}{\chi} + \frac{(p_m - \rho_m)}{2}. \]

So, in the flat universe, on condition that \( \eta_m \to 0 \) and the matter of domain walls is stiffly, \( p_m = \rho_m \), the surface tension of domain walls directly depends the Hubble constant as...
The result has indicated that domain walls can be responsible for the reason of expansion, in that Hubble parameter are directly depended the domain wall tension.

It is widely-known that the present universe is expanding with positive acceleration from observations of Type-Ia Supernovae, cosmological redshift and Hubble’s law [26, 27, 28, 29, 30, 31]. It is suggested that the exotic matter is accounted for the reason of the expansion. Some candidates of the exotic matter with $p = γρ$ ($γ < 0$), are compiled such as cosmological constant, dark energy, phantom energy, domain walls, Chaplygin gas and tachyon. In our solutions, as given by Eq. (54), FRW-cosmologies with Chaplygin gas and monopoles in the framework of Lyra scalar theory have time-dependent results which grow exponentially and cause an expanding universe, analogously with FRW-cosmologies with domain walls and monopoles. In as much as Chaplygin gas and domain walls are already candidates of exotic matter, these solutions in Lyra scalar theory are suitable results,
predictably. In FRW-cosmologies with Chaplygin gas and monopoles, provided that $\beta^4 = \frac{167^2}{9}a$ in Eqs. (35)-(36), the matter density and pressure of Chaplygin gas are equal to others, as $\rho C = p C$. In this case, the matter of Chaplygin gas is stiff and expanding universe feature of the model is no degeneration as seen from the Eq. (34).

The FRW-cosmologies with monopoles and strange quark matter in Lyra scalar theory have the anticipated equation of state, $p_q = \frac{\rho_q}{3}$, from Eqs. (41)-(42) and (45)-(46). A result of the models is that Bag constant of bag model is directly related the subtraction to matter density and pressure from Eqs. (45)-(47) as similar to FRW-cosmologies with domain wall and monopoles, the bag constant is

$$B_c = \frac{3}{\chi^2 c_8^2} \left( \frac{p_q - \rho_q}{2} \right) - \frac{\eta m^2}{r^2}.$$  

Also, cause of the features of Bag constant, it must be considered the equations near the $r \rightarrow r_0$. In the flat universe, on condition that $\eta m \rightarrow 0$, the Bag constant directly depends the Hubble constant as

$$B_c = \frac{3}{\chi} H^2 + \frac{\rho_q}{3} = \frac{3}{\chi} H^2 + p_q$$

where $H = \frac{1}{c_8}$. Because of the perfect fluid form of strange quark matter, FRW-cosmologies with strange quark matter and monopoles in Lyra geometry have the solutions which include same features of expanding universe.

It is sighted that matter density and pressure of domain walls don’t affect from existence of monopoles according to the Eqs. (21)-(22) and Eqs. (25)-(26). Otherwise from Eqs. (17)-(18), Eqs. (28)-(29) and Eqs. (37)-(38), it is clearly seen that existence of monopole vanishes cause of $\frac{\eta m^2}{r^2} = 0$. So, in this case, it is called that the scalar tensor theory based on Lyra geometry doesn’t allow the solutions of FRW-cosmologies with monopoles. Thus, there are no monopoles in the expanding and accelerated FRW-universe according to the scalar theory based on Lyra geometry.

In this study, for the each of matter groups, FRW-cosmologies in Lyra scalar theory have two solutions such as the model of static universe and the model of time-depended expanding universe. It is possible to correlate between both of the models. It is emerged that given the first solutions in case(i) of statical universe model is the particular circumstance of the model of time-depended expanding universe which is given second solutions in case(ii), provided that $k = 0$ at any time ($t = t_0$). The case can be pointed out that a part of time-depended model
of universe is similar to take a photograph of the universe at \( t = t_0 \), instantaneously. In contrast, first solutions completely wide apart from second solutions, provided \( k \neq 0 \). The obtained solutions of time-depended expanding model for FRW-cosmologies with monopoles and domain walls, Chaplygin gas or strange quark matter in Lyra scalar theory agree with the solutions for FRW-cosmologies with perfect fluid in Einstein’s theory. But, it is obtained the completely different solutions called static model in the Lyra scalar theory. Halford [11] proposed that constant displacement vector \( \Phi_i \), therefore number \( \beta^2 \) in Lyra scalar theory of gravitation, play the role of the cosmological constant \( \Lambda \) in Einstein’s theory. Halford’s suggestion has been extensively informed in many studies, in the literature \([3, 21, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56]\). Even if, Halford can be right in his conjecture, his suggestion must be modified due to contain inadequate information. In the case of constant displacement vector like as \( \beta = \text{constant} \), it can be obtained solutions of field equations in Lyra scalar theory. These solutions can agree with the solutions which is obtained solutions of field equations in Einstein’s theory. But, Lyra scalar theory has the more general solutions than Einstein’s theory. Because, in field equations, there are two terms of \( \Phi_i \Phi^k \) and \( \Phi^m g_{ik} \) in Lyra scalar theory besides the term with cosmological constant of \( \Lambda_{ik} \) in Einstein’s theory. Provided to \( \beta = \text{constant} \), the term of \( \Phi^m g_{ik} \) in Lyra scalar theory can be equivalent with the term of \( \Lambda_{ik} \) in Einstein’s theory. But it is clearly that the term of \( \Phi_i \Phi^k \) in Lyra scalar theory can add a constant term to field equations unlike in Einstein’s theory. Because the term of \( \Phi_i \Phi^k \) in Lyra scalar theory does not multiply the metric potential \( g_{ik} \), the field equations have the extra term in comparison with Einstein’s theory. Thus, the field equations in Lyra theory must be different from fields equations in Einstein’s theory. This means that obtained solutions from field equations in Lyra theory must be more general from Einstein’s, even though displacement vector has constant components.

Also, it can be seen from the solutions in this study, in addition to this result which can be directly emerged with compare field equations of both theories. Firstly, we get the model of static universe for the FRW-cosologies in Lyra scalar theory. In the Einstein’s theory, there is no similar to the solution for Friedmann equations. On the other hand, the Hubble parameter for FRW-cosmologies in the Einstein’s theory depends on the cosmological constant as Eq. (57). Already, the cosmological constant is considered to be responsible for the expansion of universe. If widely-known consider in literature could be correct completely, the reason of the expansion of universe would need to be the constant of \( \beta \) which called to play the role of cosmological constant, in Lyra scalar theory. So, Hubble parameter in Lyra scalar theory would need to depend on the constant of \( \beta \), but it is seen that the Hubble parameter in Lyra scalar theory doesn’t depend on the constant of \( \beta \) in view of Eqs. (54)-(56). For example, the Hubble parameter in flat universe is the constant which is different from \( \beta \), getting arbitrarily from obtained the solutions.
Consequently, the suggestions of many authors must be generalized: "field equations in Einstein theory could be the particular situation of field equations in Lyra scalar theory, provided that $\beta = constant$. Lyra scalar theory is the equivalent theory of Einstein with no cosmological constant provided that $\beta = 0$, completely."

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