Abstract

The problem of indirect violation of discrete symmetries CP, T and CPT in a neutral meson system can be described using two complex parameters $\varepsilon$ and $\delta$, which are invariant under rephasing of meson and quark fields. For the $B_d$ system, where the width difference between the physical states is negligible, only $\text{Re}(\delta)$ and $\text{Im}(\varepsilon)$ survive. As a consequence, the traditional observables constructed for kaons, which are based on flavour tag, are not useful for the analogous study in this system. We describe how using a CP tag and studying CP-to-flavour transitions of the $B$ mesons, we may build asymmetries, alternative to those used for the kaon, which enable us to test T and CPT invariances of the effective hamiltonian for the $B_d$ system.
Indirect Violation of CP, T and CPT in the $B_d$-system

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1. Introduction

The time evolution of a neutral meson system is governed by an effective hamiltonian \[ H \]. The problem of indirect violation of discrete symmetries refers to the non-invariance of this hamiltonian under the corresponding operations.

For the kaon system, this study has been performed by the CP-LEAR experiment \[2\] from the preparation of definite flavour states $K^0 - \bar{K}^0$. These tagged mesons evolve in time and their later decay to a semileptonic final state projects them again on a definite flavour state. The study of this flavour-to-flavour evolution allows the construction of observables which violate CP and T, or CP and CPT.

Contrary to what happens in the kaon case, for the $B_d$-system the width difference $\Delta \Gamma$ between the physical states is expected to be negligible. In this system the T- and CPT-odd observables proposed for kaons, which are based on flavour tag, vanish. but, making use of CP tag, the $B_d$ entangled states can be used to construct alternative observables which are sensitive to T and CPT independently of the value of $\Delta \Gamma$ \[3\].

2. The parameters

In the neutral $B$-meson system the physical states are a linear combination of $B^0$ and $\bar{B}^0$. If they are written in terms of CP eigenstates, one has to introduce two complex parameters, $\epsilon_{1,2}$, to describe the CP mixing.

\begin{equation}
|B_{\pm}\rangle = \frac{1}{\sqrt{1 + |\epsilon_{1,2}|^2}} \left[ |B_+\rangle + \epsilon_{1,2} |B_-\rangle \right],
\end{equation}

where $|B_{\pm}\rangle \equiv \frac{1}{\sqrt{2}}((I \pm CP)|B^0\rangle)$. Then $\epsilon_{1,2}$ are invariant under rephasing of the meson states, and physical when the CP operator is well defined \[4\].

Alternatively, one may use the parameters $\epsilon \equiv (\epsilon_1 + \epsilon_2)/2$ and $\delta \equiv \epsilon_1 - \epsilon_2$, whose interpretation in terms of symmetries is simpler.

Discrete symmetries impose different restrictions on the effective mass matrix, $H = M - \frac{i}{2} \Gamma$:

- CPT invariance requires $H_{11} = H_{22}$,
- T invariance imposes $\text{Im}(M_{12} \text{CP}_{12}^*) = \text{Im}(\Gamma_{12} \text{CP}_{12}^*) = 0$, and
- CP conservation requires both conditions to be simultaneously satisfied. Furthermore, in the exact limit $\Delta \Gamma = 0$, customary for the $B_d$-system, both $\text{Re}(\epsilon)$ and $\text{Im}(\delta)$ vanish. Therefore we have four real parameters which carry information on the symmetries of the effective mass matrix

- $\text{Re}(\epsilon) \Rightarrow$ CP and T violation, with $\Delta \Gamma \neq 0$;
- $\text{Im}(\epsilon) \Rightarrow$ CP and T violation;
- $\text{Re}(\delta) \Rightarrow$ CP and CPT violation;
- $\text{Im}(\delta) \Rightarrow$ CP and CPT violation, $\Delta \Gamma \neq 0$.

3. The entangled state: CP tag

In a $B$ factory operating at the $\Upsilon(4S)$ peak, correlated pairs of neutral $B$-mesons are pro-
duced through $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$. The special features of this system can be used to study CP [5] and CPT [6] violation in $B$ mesons.

In the CM frame, the resulting $B$-mesons travel in opposite directions, each one evolving with the effective hamiltonian. The $B\bar{B}$ state has definite $L = 1$, $C = -$ and $\mathcal{P} = -$, being $\mathcal{P}$ the operator which permutes the spatial coordinates, so that the initial state may be written as

$$|i> = \frac{1}{\sqrt{2}} \left( |B^0, \Upsilon^0\rangle > - |\bar{B}^0, B^0\rangle \right)$$

(2)

The correlation between both sides of the entangled state holds at any time after the production. As a consequence, one can never simultaneously have two identical mesons at both sides of the detector. This permits the performance of a flavour tag: if at $t = 0$ one of the mesons decays through a channel, such as a semileptonic one, which is only allowed for one flavour of the neutral $B$, the other meson in the pair must have the opposite flavour at $t = 0$.

The entangled $B-\bar{B}$ state can also be expressed in terms of the CP eigenstates $|B_\pm\rangle$ as

$$|i> = \frac{1}{\sqrt{2}} \left( |B_-, B_+\rangle > - |B_+, B_-\rangle \right)$$

(3)

Thus it is also possible to carry out a CP tag, once we have a CP-conserving decay into a definite CP final state, so that its detection allows us to identify the decaying meson as a $B_+$ or a $B_-$. In Ref. [7] we described how this determination is possible and unambiguous to $O(\lambda^3)$, which is sufficient to discuss both CP-conserving and CP-violating amplitudes in the effective hamiltonian for $B_d$ mesons. Here $\lambda$ is the flavour-mixing parameter of the CKM matrix [8]. The determination is based on the requirement of CP conservation, to $O(\lambda^0)$, in the $(sd)$ and $(bs)$ sectors. To this order, however, CP-violation exists in the $(bd)$ sector, and it can be classified by referring it to the CP-conserving direction. A $B_d$ decay that is governed by the couplings of the $(sd)$ or $(bs)$ unitarity triangles, or by the $V_{td}V_{tb}^*$ side of the $(bd)$ triangle, will not show any CP violation to $O(\lambda^3)$. We may say that such a channel is free from direct CP violation. Examples are $J/\Psi K_S$, with CP $= -$, and $J/\Psi K_L$, with CP $= +$.

To extract information on the symmetry parameters we may study the time evolution of the entangled state [4] and its decay into a final configuration $(X, Y)$. In our notation, $X$ is the decay product observed on one side of the detector at a certain time, and $Y$ the product detected on the opposite side after a $\Delta t$.

We will only consider here decay channels $X, Y$ which are either flavour or CP conserving. Then the final configuration $(X, Y)$ corresponds to a certain transition at the mesonic level, i.e. the $B$ state tagged by the $X$ decay evolves for a period $\Delta t$ and is then projected into a flavour or CP eigenstate by means of the $Y$ decay.

4. The asymmetries

By comparing the probabilities corresponding to different processes we build time-dependent asymmetries that allow the extraction of the relevant parameters. The observables can be classified into three types.

4.1. Flavour-to-flavour genuine asymmetries

If one detects semileptonic decays on both sides of the detector, then the transition at the mesonic level is of the kind flavour-to-flavour. The mesonic transitions for such a final configuration appear in Table 4, where $\ell^\pm$ represents the final decay product of a semiinclusive decay $B \rightarrow \ell^\pm X^\mp$. From these processes we can construct

| Table 1 |
| --- |
| Flavour-to-flavour transitions |
| (X, Y) | Transition |
| $(\ell^+, \ell^+)\quad B^0 \rightarrow B^0$ |
| $(\ell^-, \ell^-)\quad B^0 \rightarrow \bar{B}^0$ |
| $(\ell^+, \ell^-)\quad \bar{B}^0 \rightarrow B^0$ |
| $(\ell^-, \ell^+)\quad B^0 \rightarrow \bar{B}^0$ |

two non-trivial asymmetries, which are the analogous, in the $B$-system, to the traditional observables used for kaons. The first two processes in Table 4 are conjugated under CP and also under
T, then we may construct a genuine asymmetry by comparing the corresponding intensities

\[ A(\ell^+, \ell^+) \approx \frac{\text{Re}(\varepsilon)}{1 + |\varepsilon|^2}. \]  

(4)

On the other hand, the last two processes in Table 2 are related by a CP or a CPT transformation. Therefore, the corresponding asymmetry,

\[ A(\ell^+, \ell^-) \approx -2 \left[ \text{Ch} \frac{\Delta m}{2\pi} + \cos(\Delta m \Delta t) \right]^{-1} \left[ \text{Re} \left( \frac{\varepsilon}{\Delta m} \right) \text{Sh} \frac{\Delta m}{2\pi} - \text{Im} \left( \frac{\varepsilon}{\Delta m} \right) \sin(\Delta m \Delta t) \right], \]  

(5)

is also a genuine CP and CPT observable.

In both cases, the resulting asymmetry vanishes unless \( \Delta \Gamma \neq 0 \). Thus measuring a small value for these observables does not impose a straightforward bound on the size of symmetry violation, because the vanishingly small \( \Delta \Gamma \) of B-mesons would hide any symmetry breaking effect.

### 4.2. CP-to-flavour genuine asymmetries

We may construct alternative asymmetries making use of the CP eigenstates, which can be identified in this system by means of a CP tag. If the first decay product, \( X \), is a CP eigenstate produced along the CP-conserving direction, and \( Y \) is a semileptonic channel, then the mesonic transition corresponding to the configuration \((X, Y)\) is of the type CP-to-flavour. The order of appearance of both final states matters, because for the reverted configuration, \((Y, X)\), we have a flavour-to-CP transition. In Table 2 we show the mesonic transitions, with their related final configurations, connected by genuine symmetry transformations to \( B_+ \rightarrow B^0 \), i.e. \((J/\Psi K_S, \ell^+)\). Comparing the intensity of \((J/\Psi K_S, \ell^+)\) with each of them we construct three genuine asymmetries. Next, we show the results to linear order in \( \delta \) and in the limit \( \Delta \Gamma = 0 \).

\[
A_{CP} = -2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) \\
+ \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \left( \frac{2 \text{Re}(\varepsilon)}{1 + |\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \right),
\]

(6)

is the CP odd asymmetry, which has contributions from T-violating and CPT-violating terms. The first term, odd in \( \Delta t \), is governed by the T-violating \( \text{Im}(\varepsilon) \), whereas the second term, \( \Delta t \) even, is sensitive to CPT violation through the parameter \( \text{Re}(\delta) \).

\[
A_{T} = -2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) \\
\left[ 1 - \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \frac{2 \text{Re}(\varepsilon)}{1 + |\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \right],
\]

(7)

the T asymmetry, needs \( \varepsilon \neq 0 \), and includes CPT even and odd terms. Moreover, in the limit we are considering, turns out to be purely odd in \( \Delta t \).

\[
A_{CPT} = \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \frac{\text{sin}^2 \left( \frac{\Delta m \Delta t}{2} \right)}{1 - \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \text{sin}(\Delta m \Delta t)},
\]

(8)

is the CPT asymmetry. It needs \( \delta \neq 0 \), and includes both even and odd time dependences, so that there is no definite symmetry under a change of sign of \( \Delta t \).

Measuring the presented asymmetries with good time resolution, so to separate even and odd \( \Delta t \) dependences, should be enough to determine the parameters \( \frac{2 \text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \) and \( \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \frac{2 \text{Re}(\varepsilon)}{1 + |\varepsilon|^2} \), which govern CP, T violation and CP, CPT violation, respectively, in the \( B_d \) mixing.

Contrary to what happened in the case of flavour tag, the CPT and T asymmetries based on a CP tag do not vanish due to the smallness of \( \Delta \Gamma \). Instead, they provide a set of observables which could separate the parameters \( \delta \) and \( \varepsilon \).

### 4.3. CP-to-flavour non-genuine asymmetries

The asymmetries defined in the previous paragraphs are genuine observables, since each of them compares the original process with its conjugated under a certain symmetry and is thus odd under the corresponding transformation. Nevertheless the measurement of all those quantities requires to tag both \( B_+ \) and \( B_- \) states. The last needs, from the experimental point of view, a good reconstruction of the decay \( B \rightarrow J/\Psi K_L \),

#### Table 2

| (X, Y) | Transition | Transformation |
|--------|------------|----------------|
| \((J/\Psi K_S, \ell^-)\) | \(B_+ \rightarrow B^0\) | CP |
| \((\ell^-, J/\Psi K_L)\) | \(B^0 \rightarrow B_+\) | T |
| \((\ell^+, J/\Psi K_L)\) | \(B^0 \rightarrow B_+\) | CPT |
not so easy to achieve as for the corresponding $J/\Psi K_S$ channel.

But it is also possible to construct useful asymmetries from final configurations $(X, Y)$ with only $J/\Psi K_S$. In Table 3 we show the different transitions we may study from such final states. From the comparison between $(J/\Psi K_S, \ell^+)$ and each process in the table we can construct three asymmetries. The first one will correspond to the genuine CP asymmetry $A(J/\Psi K_S, \ell^-) = A_{CP}$. We find that, in the exact limit $\Delta \Gamma = 0$, $\Delta t$ and T operations become equivalent, so that $A(\ell^+, J/\Psi K_S) = A_T$ and $A(\ell^-, J/\Psi K_S) = A_{CPT}$. But these asymmetries are not genuine. They do not correspond to true T- and CPT-odd observables, for the processes we are comparing are not related by a symmetry transformation. This implies that the presence of $\Delta \Gamma \neq 0$ may induce non-vanishing values for them, even if there is no true T or CPT violation. But even if that is the case, it is possible to separate out the different parameters, if good enough $\Delta t$ is provided.

Table 3
Final configurations with only $J/\Psi K_S$.

| $(X, Y)$         | Transition | Transformation |
|------------------|------------|----------------|
| $(J/\Psi K_S, \ell^-)$ | $B_+ \rightarrow B^0$ | CP             |
| $(\ell^+, J/\Psi K_S)$  | $B^0 \rightarrow B_-$ | $\Delta t$   |
| $(\ell^-, J/\Psi K_S)$  | $B^0 \rightarrow B_-$ | $\Delta t + CP$ |

We classify the observables into three different types:

- Genuine asymmetries for T or CPT violation, based on flavour-to-flavour transitions at the meson level, which need $\Delta \Gamma \neq 0$.
- Genuine observables, based on the combination of flavour and CP tags, which do not need $\Delta \Gamma$.
- Making use of the equivalence between $\Delta t$ and T reversal operations for $\Delta \Gamma = 0$, we have also considered non genuine observables, involving only the hadronic decay $J/\Psi K_S$.

5. Conclusions

We present an overview of the possibilities to explore indirect violation of CP, T and CPT in a neutral meson system from the quantities that $B$-factories can measure. The asymmetries analyzed here exploit their time dependences in order to separate out two different ingredients: on one hand CP and T violation, described by $\varepsilon$, and on the other CP and CPT violation, given by $\delta$. Such a study is possible, even if $\Delta \Gamma = 0$, if one goes beyond flavour-to-flavour transitions and makes use of CP tags.

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