A puzzle in $\bar{B}^0_{(s)} \to D^{(*)+}(\pi^-, K^-)$ decays and extraction of the $f_s/f_d$ fragmentation fraction

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Abstract We provide updated predictions for the hadronic decays $\bar{B}^0 \to D^{(*)+}\pi^-$ and $\bar{B}^0 \to D^{(*)+}K^-$. They are based on $O(\alpha_s^2)$ results for the QCD factorization amplitudes at leading power and on recent results for the $\bar{B}_{(s)}^0 \to D^{(*)}_{(s)}$ form factors up to order $O(\Lambda_{QCD}/m_b^2)$ in the heavy-quark expansion. We give quantitative estimates of the matrix elements entering the hadronic decay amplitudes at order $O(\Lambda_{QCD}/m_b)$ for the first time. Our results are very precise, and uncover a substantial discrepancy between the theory predictions and data on the $f_s/f_d$ ratios based on a recent analysis within the heavy-quark expansion. We determine the $f_s/f_d$ fragmentation fraction for the CDF, D0 and LHCb experiments for both scenarios.

1 Introduction

The physics programs of the Large Hadron Collider experiments promise data sets of unprecedented sizes for a variety of $B_s$ decays. Consequently, analyses that emerge from these programs now dominate determinations of absolute branching fractions of $B_s$ decays. The biggest source of uncertainties in these analyses is the poorly known fraction of the $b$ quark fragmentation into $\bar{B}^0_s$ versus $\bar{B}^0$ mesons, denoted as $f_s/f_d$. A promising approach to determine this ratio \cite{1} from data is the measurement of a ratio of branching fractions for hadronic $B$ decays:

\[
\frac{\sigma(pp \to \bar{B}^0_s X) \times B(\bar{B}^0_s \to D^{(*)+}\pi^-)}{\sigma(pp \to \bar{B}^0 X) \times B(\bar{B}^0 \to D^{(*)+}K^-)} \equiv \frac{\sigma(pp \to \bar{B}^0_s X)}{\sigma(pp \to \bar{B}^0 X)} \times R_{s/d}^{P(V)}. \tag{1}
\]

We consider the hadronic decays in this ratio very advantageous from the theory point of view. Since in these decays all valence quarks are distinguishable, we do not have to account for decay topologies involving penguin operators. For the same reason weak annihilation is not an issue either. Moreover, these decays are dominated by the color-allowed tree topology, and the color-suppressed operator enters only through perturbative or power corrections. In Ref. \cite{2} the ratio that we here call $R_{s/d}^{P(V)}$ is given with a relative uncertainty of $\sim 9\%$.

The purpose of the present article is threefold. First, we revisit the QCD factorization framework for the non-leptonic decays $\bar{B}^0 \to D^{(*)+}K^-$ and $\bar{B}^0_s \to D^{(*)+}\pi^-$ beyond leading power. We update the values of the $\bar{B}_q \to D^{(*)}_{q}$ form factors based on a recent analysis within the heavy-quark expansion (HQE) \cite{3}. We also provide, for the first time, conservative numerical estimates for the necessary hadronic matrix elements that enter at next-to-leading-power in $\Lambda_{QCD}/m_b$. Based on these improvements we predict the branching fractions of the four decays and, for the first time, use their theoretical correlation to reduce the uncertainty on their ratios $R_{s/d}^{P(V)}$.

Second, we challenge existing experimental data on two-body non-leptonic decays into a heavy-light final state. We point out the subtleties in the comparison between theory and experiment. Our analysis reveals a puzzling pattern in the comparison of theory predictions and data on the absolute branching fractions of the tree-dominated $\bar{B}^0_{(s)} \to$...
$D_{(s)}^+(\pi^-, K^-)$ decays, whereas the predicted ratios $R_{s/d}^{P(V)}$ are in good agreement with experiment. We subsequently extract the $f_s/f_d$ fragmentation fraction for the CDF, D0 and LHCb experiments in a variety of scenarios.

Last but not least, we critically assess possible origins for the observed puzzle – which amounts to a discrepancy of up to five standard deviations for the individual absolute branching fractions – with one of them being contributions from physics beyond the Standard Model (BSM).

This article is organised as follows. In Sect. 2 we revisit the QCD factorization framework, including a thorough discussion of next-to-leading power hadronic matrix elements. We then discuss the numerical input parameters that enter our expressions, and give results for the non-leptonic branching fractions and the ratios $R_{s/d}^{P(V)}$ at leading power. Our estimate of $R_{s/d}^{P(V)}$ at next-to-leading power shows their robustness against power corrections. In Sect. 3 we compare the theoretical predictions to experimental data, explain in detail our extraction of the $f_s/f_d$ fragmentation fraction for various hadron colliders and uncover the puzzling pattern in non-leptonic $B^0_{(s)} \rightarrow D_{(s)}^+(\pi^-, K^-)$ decays. We determine possible solutions to this puzzle, among them effects from BSM physics. We finally discuss prospects for future extraction of $f_s/f_d$ using the method discussed here and potential alternatives. We conclude in Sect. 4. The article is supplemented by two appendices. In Appendix A we present the light-cone sum rule calculation for the soft-gluon matrix element relegated from Sect. 2, while in Appendix B we give details on the experimental inputs.

2 Theory prediction of $\mathcal{B}(\bar{B}^0 \rightarrow D_{(s)}^{(*)}+K^-)$ and $\mathcal{B}(\bar{B}^0_s \rightarrow D_s^{(*)}+\pi^-)$

We begin by briefly summarizing the framework of collinear factorization [also known as QCD factorization (QCDF)] [4] for the decays $\bar{B}^0 \rightarrow D_{(s)}^{(*)}+K^-$ and $\bar{B}^0_s \rightarrow D_s^{(*)}+\pi^-$ in Sect. 2.1. Numerical results are presented subsequently in Sect. 2.2.

2.1 Framework

Theory predictions for the decays under consideration are both relatively simple and particularly clean [4]. They are relatively simple, since neither penguin nor annihilation topologies contribute; they are particularly clean, again due to the absence of pollution from weak annihilation, but also because no chirally enhanced hard-scattering contributions are present at order $\Lambda_{\text{QCD}}/m_b$. Since the latter two contributions constitute the main limitation of the QCDF approach, the resulting theory predictions are among the most reliable for non-leptonic decays. This is the basis for the phenomenological application of extracting the ratio $f_s/f_d$ from these modes [1]. We emphasize that this statement does not hold for modes where the flavour of the spectator anti-quark is present in the valence content of the light meson. An example is the decay $\bar{B}^0 \rightarrow D^+\pi^-$, which factorizes to leading power, but suffers from larger uncertainties due to endpoint divergences at subleading power.

The effective Lagrangian needed for the description of $\bar{B}^0 \rightarrow D_{(s)}^{(*)}+K^-$ and $\bar{B}^0_s \rightarrow D_s^{(*)}+\pi^-$ decays reads

$$\mathcal{L} = \frac{-4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[ C_1 Q_1^{cb} + C_2 Q_2^{cb} \right],$$

where $q_2 = d, s$, and $V_{cb}, V_{ud}^*$ are CKM matrix elements. The Wilson coefficients $C_{1,2}$ are $q_2$-flavour universal in the Standard Model (SM). We choose these two current-current operators in the CMM basis [5]:

$$Q_{2(1)}^{q_2} \equiv \left[ \bar{c} \gamma^\mu P_L (T^A) b \right] \left[ \bar{q}_2 \gamma_\mu P_L (T^A) u \right].$$

The Wilson coefficients $C_{1,2}$ are known to next-to-next-to-leading logarithmic accuracy [6], and at the scale $\mu = m_b$ we use

$$C_2 = +1.010 \quad C_1 = -0.291.$$ (4)

Their uncertainties are negligible compared to those of the hadronic matrix elements.

In this work, the light pseudoscalar ($\bar{u}q_2$) bound state is denoted as $L$ when the particular quark flavour $q_2$ is not relevant to the discussion. For the discussion below we adopt the power-counting $\varepsilon \sim \Lambda_{\text{QCD}}/E_L \sim \Lambda_{\text{QCD}}/m_b$. Following this power counting, the matrix elements of the current-current operators factorize to leading power [4]:

$$\langle D_{(s)}^{(*)}+L^- | Q_i | \bar{B}^0 \rangle = \sum_j F_j^{\bar{B}_q \rightarrow D_{(s)}^{(*)}} (M_L^2)$$

$$\times \int_0^1 du T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right).$$

The amplitudes to leading power in $\varepsilon$ read

$$A(\bar{B}^0_q \rightarrow D_{q}^+L^-) = i \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} a_1 (D_q^+L^-) f_L$$

$$\times \bar{B}_q \rightarrow D_q^+ (M_{\bar{B}_q}^2 - M_{D_q}^2),$$

$$A(\bar{B}^0_q \rightarrow D_{q}^{*+}L^-) = -i \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} a_1 (D_{q}^{*+}L^-) f_L$$

$$\times \bar{B}_q \rightarrow D_q^{*+} (M_{\bar{B}_q}^2 - M_{D_q}^2) \times 2 M_{D_q} f_L (\lambda = 0) \cdot q$$

$$+ \mathcal{O}(1).$$ (6)
for either a pseudoscalar $D_q$ meson, or a longitudinal ($\lambda = 0$) $D_q^*$ vector meson. The structure of these amplitudes holds to all orders in $\alpha_s$. The effective Wilson coefficients $a_1(D_q^{(*)} + L^-)$ have been computed to next-to-next-to-leading order in $\alpha_s$ in Ref. [2]. We emphasize that the $SU(3)_F$ breaking of the amplitudes is numerically driven by the decay constants of the light-meson. The deviation from the symmetry limit in heavy-to-heavy form factors is additionally suppressed by the heavy-quark masses and therefore expected to be very small [7–9]. A recent analysis of the symmetry breaking in heavy-to-heavy form factors yields results that are compatible with this expectation [3]. To ascertain the stability of QCDF predictions for $R_{s/d}$, control of the symmetry-breaking effects is essential, even if they only arise within power corrections.

The formulas for the $SU(3)_F$ ratios with either a pseudoscalar (P) or vector (V) meson $D_q^{(*)}$ read

$$R_{s/d}^P = \frac{\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{(*)+}\pi^-)}{\mathcal{B}(\bar{B}_s^0 \rightarrow D^{(*)+}\pi^-)},$$

$$R_{s/d}^V = \frac{\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{(*)+}\pi^-)}{\mathcal{B}(\bar{B}_s^0 \rightarrow D^{(*)+}\pi^-)}.$$

Here $a(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ is the Källén function, and the formulas for the individual branching ratios can be found in [4]. To test the consistency of the QCDF predictions, we also study the fixed-flavor ratios of branching fractions to vector $D_q^{(*)}$ mesons over pseudoscalar $D_q$ mesons:

$$R_s^{V/P} = \frac{\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{(*)+}\pi^-)}{\mathcal{B}(\bar{B}_s^0 \rightarrow D^{(*)+}\pi^-)},$$

$$R_d^{V/P} = \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow D_d^{(*)+}\pi^-)}{\mathcal{B}(\bar{B}_s^0 \rightarrow D^{(*)+}\pi^-)}.$$

which have analogous expressions.

Power corrections to the amplitudes in Eq. (6) arise from a variety of effects. To order $\varepsilon$, these can potentially include higher-twist corrections to the light-meson light-cone distribution amplitude (LCDA); emission of a hard-collinear gluon from the spectator $q$, or from the heavy bottom and charm quarks; and exchange of a soft gluon between the $\bar{B}_q^0 \rightarrow D_q$ system and the light meson. We briefly discuss each of these effects and provide updated numerical estimates for their magnitudes. Importantly, using form factors in terms of QCD quark fields in Eq. (6) avoids explicit $A/m_c$ power corrections. We note that implicit corrections due to application of the heavy-quark expansion to the form factors are part of our uncertainty budget as discussed below.

The contributions of the two-particle twist-three light-meson LCDA are suppressed with respect to the leading-twist contribution by one power of $\varepsilon$. As discussed in the literature, see e.g. Ref. [10], twist-three corrections for a pseudoscalar scale like $\mu_L/E_L$, which can be numerically large due to the normalization in terms of the factor

$$\mu_L = \frac{M^2_{L^-}}{m_u + m_q^2}.$$  \hspace{1cm} (11)

However, the twist-three corrections to the hadronic amplitude vanish algebraically at leading-order in $\alpha_s$ [11]. We find that higher-twist corrections only enter at order $\alpha_s \varepsilon^2$ or higher.

Up to $\mathcal{O}(\varepsilon)$, the exchange of a gluon between the heavy-to-heavy transition and the light meson can possibly enter in one of three ways [11].

(i) The exchange of a hard gluon from the $b$ or $c$ quark is a purely perturbative effect, and accounted for by the $\mathcal{O}(\alpha_s)$ correction to the effective Wilson coefficients $a_1(D_q^{(*)}L)$. (ii) The exchange of a hard-collinear gluon from the spectator quark is incompatible with the physical picture of the spectator having a soft momentum inside both the $B_q^0$ and the $D_q$ meson. The exchange of a hard-collinear gluon from the $b$ or $c$ quark is possible, and contributes through a quark–antiquark–gluon Fock state of the light meson through three-particle LCDA. Due to the $V - A$ structure of the weak interaction the twist-three three-particle contribution is absent, and the first contribution emerges at the twist-four level [4].

Within our power counting, the twist-four contribution enters at the $\mathcal{O}(\varepsilon^2)$ level. This power correction breaks $SU(3)_F$ symmetry maximally, since it is absent for the $L = \pi^-$ case by virtue of G parity, but has a finite contribution for the $L = K^-$ case. Using the expression for these corrections in Ref. [4], a model based on the conformal expansion of the LCDA and the numerical results of Ref. [10] we obtain

$$\frac{\mathcal{A}(\bar{B}_q^0 \rightarrow D^{(*)+}\pi^-)_{NNLP(he)}}{\mathcal{A}(\bar{B}_q^0 \rightarrow D^{(*)+}\pi^-)_{LP}} \simeq \frac{C_1}{\alpha_1} \times -0.64\%.$$  \hspace{1cm} (12)

This term has been computed in the limit $m_c \ll m_b$. Analysis of eq. (56) of Ref. [4] reveals that corrections due to a
non-zero charm mass scale like $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}} m_c}{m_b}\right)$, which are negligible to the accuracy for which we aim.

(iii) The exchange of a single soft gluon between the heavy-to-heavy transition and the light meson causes one of the light quarks in the light meson to become hard-collinear [4]. This configuration can be expressed through the heavy-to-heavy matrix elements of a non-local $\bar{c}G b$ current. We estimate the relevant matrix elements using Light-Cone Sum Rules (LCSRs) in Appendix A. Using these estimates, we obtain ranges for the $\mathcal{O}(\varepsilon)$ soft-gluon correction:

$$\frac{\mathcal{A}(\bar{B}_q^0 \to D_q^+ L^-)}{\mathcal{A}(\bar{B}_q^0 \to D_q^0 L^-)} = \frac{C_1}{4a_1} \times [0.76, 7.6] \%,$$

$$\frac{\mathcal{A}(\bar{B}_q^0 \to D_{q+}^+ L^-)}{\mathcal{A}(\bar{B}_q^0 \to D_{q+}^+ L^-)} = \frac{C_1}{4a_1} \times [0.46, 4.6] \%.$$ (13, 14)

The lower value of these ranges corresponds to the sum rule results for our nominal inputs as discussed in Appendix A. To obtain a more conservative estimate, we increase the values of the scale-setting parameters $\lambda^2_E$ and $\lambda^2_H$ by one order of magnitude, yielding the higher of the values. We emphasize that the ratio of the hadronic matrix elements is matching the size expected from naive dimensional analysis. However, the prefactor of $C_1/a_1 \sim -1/3$ renders all estimates for the power corrections small, and supports the picture of these decays being very clean.

2.2 Numerical inputs and results

**Form factors** For the form factors and form-factor ratios entering our predictions, we use the posterior samples for the HQE parametrization from Ref. [3]. This parametrization includes contributions to orders $1/m_b$, $1/m_c^2$, $\alpha_s$, thereby providing a consistent treatment of the form factors and specifically also their uncertainties including $1/m_c^2$ contributions, see Refs. [3, 12] for details. In Table 1 the values of the form factors used in this work are collected together with the ones used in Ref. [2]. We notice that with respect to Ref. [2] these form factor predictions have reduced uncertainties, especially for the $\bar{B}_q^0 \to D_{q+}^+$ cases. This is dominantly due to improved lattice QCD determinations of the corresponding form factors presented in Refs. [13, 14], which are the most constraining inputs for the analysis in Ref. [3]. The correlation coefficients between these form factor values are

$$\begin{pmatrix}
1.0000 & 0.0814 & 0.1702 & -0.0255 \\
0.0814 & 1.0000 & 0.0059 & 0.4787 \\
0.1702 & 0.0059 & 1.0000 & 0.0134 \\
-0.0255 & 0.4787 & 0.0134 & 1.0000
\end{pmatrix}, \quad (15)$$

with rows and columns ordered as $F_0^{\bar{B} \to D}(M_K^2)$, $A_0^{\bar{B} \to D^*}(M_K^2)$, $F_0^{\bar{B} \to D_{L}^+}(M_K^2)$, and $A_0^{\bar{B} \to D_{L}^+}(M_K^2)$. Accounting for these correlations, we obtain the form factor ratios:

$$\frac{F_0^{\bar{B} \to D_i}(M_K^2)}{F_0^{\bar{B} \to D}(M_K^2)} = 1.001 \pm 0.021, \quad (16)$$

$$\frac{A_0^{\bar{B} \to D^*_{L}i}(M_K^2)}{A_0^{\bar{B} \to D^*}(M_K^2)} = 0.9729 \pm 0.080, \quad (17)$$

$$\frac{F_0^{\bar{B} \to D_{L}i}(M_K^2)}{A_0^{\bar{B} \to D_{L}i}(M_K^2)} = 0.9773 \pm 0.095, \quad (18)$$

$$\frac{F_0^{\bar{B} \to D}(M_K^2)}{A_0^{\bar{B} \to D}(M_K^2)} = 0.9507 \pm 0.095. \quad (19)$$

**Effective Wilson coefficients** Based on the analytic results for the effective Wilson coefficients to NNLO in $\alpha_s$ from Ref. [2], we produce numbers for their absolute values. The numbers used in the present work contain more significant digits, but are otherwise identical to those in Ref. [2], see Table 1. In addition, we require the following three ratios of effective Wilson coefficients:

$$\frac{|V_{cb}|}{|V_{ub}|} \quad (20)$$

$$\frac{|V_{us}|}{f_\pi} \quad (21)$$

$$\frac{|V_{us}|}{f_K} \quad (22)$$

For $|V_{cb}|$, we use the value obtained in Refs. [3, 12], which constitutes an average of the values extracted from inclusive and exclusive semileptonic decays. This value is larger than in Ref. [2] where the value for $|V_{cb}|$ from exclusive decays as of 2016 was used.

| $V_{ub}$ | $V_{us}$ | $f_\pi$ | $f_K$ |
|---|---|---|---|
| $1.0024 \pm 0.0023$ | $1.0013 \pm 0.0006$ | $1.0013 \pm 0.0005$ |

$|V_{us}|/f_\pi$ and light-meson decay constants Throughout this work, we use directly the products of the CKM matrix elements $|V_{ub}|$ or $|V_{us}|$ with the respective light-meson decay constants $f_\pi$ or $f_K$ as obtained by the PDG from leptonic $\pi^-, K^-$ decays [15]. Our current values together with those from Ref. [2] are again collected in Table 1. Due to the partial cancellation of radiative corrections their ratio is even more precisely known than the individual quantities,

$$\frac{|V_{us}|^2}{|V_{us}|^2 \times f_\pi^2} = 13.128 \pm 0.038, \quad (23)$$

leaving a negligible relative uncertainty of $1.5/\%$.

**Leading-power predictions and comparison** Using the above inputs we obtain for the QCDF results at leading
power (LP) in $\epsilon$ the values in Table 1, again listed together with the results of Ref. [2]. The most significant changes arise due to the shifts in $|V_{cb}|$ (+4%), and the $\bar{B}_s^0 \rightarrow D_s^{(*)}$ form factors. The former shift is mostly due to our inclusion of the $|V_{cb}|$ value obtained from inclusive decays [16], while the latter (−4%) for $F_0$ and +33% for $A_0$ is caused by improved calculations of the $\bar{B}_s^0 \rightarrow D_s^{(*)}$ form factors [13,17,18] which have become available since the analysis of Ref. [2], where the only available calculation [19] as of 2016 was used. The resulting shifts in the absolute branching fractions follow correspondingly, with +8% for the $\bar{B}_s^0$ branching fractions, no significant shift in $\bar{B}_s^0 \rightarrow D_s^+ \pi^-$, and +90% in $\bar{B}_s^0 \rightarrow D_s^{(*)}$. Note, that the current form factor results shift the QCDF predictions from 2.24±0.56 (in units of 10$^{-3}$) to the new value 4.30±0.9 (which is now 2.3\sigma away from the experimental value 2.1 ± 0.5 from the third column of Table 2. The uncertainties of all our predictions are dominated by the form factor uncertainties, which will be reduced in the future. In the case of the absolute branching fractions of modes with $D_s$ final states, the uncertainties from $|V_{cb}|^2 \sim 2.4\%$ and $|\alpha_1|^2 \sim 2.4\%$ are comparable to the form factor ones (≈3.3%), while for absolute branching fractions for $D_s^+$ final states and ratios of branching fractions the form factor uncertainties dominate by far.

In addition we obtain, still to leading power, the following ratios of branching fractions:

$$\mathcal{R}_{s/d}^P |_{LP} = 13.5^{+0.6}_{-0.5},$$

$$\mathcal{R}_{s/d}^V |_{LP} = 13.1^{+2.3}_{-2.0},$$

$$\mathcal{R}_{s/d}^{V/P} |_{LP} = 0.97^{+0.20}_{-0.17},$$

$$\mathcal{R}_{d}^{V/P} |_{LP} = 1.01 \pm 0.11.$$ (27)

Two of these values can be compared to the ones in Ref. [2] where

$$\mathcal{R}_{s/d}^P |_{LP} = 14.67^{+1.34}_{-1.28},$$

$$\mathcal{R}_{d}^{V/P} |_{LP} = 0.863^{+0.158}_{-0.147}$$ (29)

are obtained. We observe that the central values are compatible within uncertainties, and that in our new analysis the latter are 55% and 30% smaller, respectively.

**Next-to-leading power predictions** When including the hadronic matrix elements at next-to-leading power (NLP) in $\Lambda_{QCD}/m_b$, we obtain:

$$\mathcal{R}_{s/d}^P |_{NLP}/\mathcal{R}_{s/d}^P |_{LP} - 1 \approx -1.7\%,$$

$$\mathcal{R}_{s/d}^V |_{NLP}/\mathcal{R}_{s/d}^V |_{LP} - 1 \approx -1.7\%,$$

$$\mathcal{R}_{s/d}^{V/P} |_{NLP}/\mathcal{R}_{s/d}^{V/P} |_{LP} - 1 = (0.4 \ldots 8.2)\%,$$

$$\mathcal{R}_{d}^{V/P} |_{NLP}/\mathcal{R}_{d}^{V/P} |_{LP} - 1 = (0.4 \ldots 8.2)\%.$$ (33)

The shift from LP to NLP is negligible, as anticipated in Sect. 2.1. This underscores why the QCDF predictions for these decays are considered to be among the most reliable for two-body $B$ decays.

### 3 Challenging present measurements

We compare our predictions obtained in the previous section with existing data, and aim for a determination of $f_s/f_d$ at the LHCb experiment and the Tevatron experiments CDF and D0. This is not trivial, for the following reasons:

1. We face a circular dependence between some measurement of branching fractions and $f_s/f_d$: for the $B_s^0$ decays in question, the values entering the world average [15] are at least partially using $f_s/f_d$, which we are aiming to extract. In Table 2 we make this dependence explicit, and introduce two independent quantities for the LHCb and CDF experiments, to highlight a potential dependence on the transverse momentum and hence the experiment.

2. Besides correlations due to $f_s/f_d$, the measurements of the decays in question exhibit additional sizeable correlations. The largest ones are introduced because LHCb only measures ratios of branching fractions, albeit with very high precision. Furthermore, in almost all measurements the same few $D_{(s)}$ decay modes are used. Finally, we include explicitly the production fraction of neutral $B$ mesons entering absolute branching fraction measurements by the $B$ factories, since in this case isospin symmetry can be violated sizeably.

Based on these considerations, we summarize all relevant experimental results in Table 4 in the Appendix, explicitly highlighting all cross dependence. The individual branching fractions, their ratios, and the different production fractions presented in Table 2 are produced from a series of fits to the data listed in Table 4. In these fits, we allow for independent production fractions $f_s/f_d$ at Tevatron, LHCb (7 TeV) and LHCb (13 TeV). The latter does not enter any measurement here and is only given for comparison. Since only one measurement by CDF is available for the considered modes, this effectively determines a value for $f_s/f_d$ at the Tevatron experiments, which can then be compared to other determinations. We consider therefore the following fit scenarios:

1. We first perform a fit without any input on $f_s/f_d$. The absolute scale of all $\bar{B}_s^0$ branching fractions is in this case provided by the measurement of $\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \pi^-)$ at Belle [21], which yields a very low value for $f_s/f_d$ with large uncertainties.

2. We next include the $f_s/f_d$ value from LHCb measured at 7 TeV in semileptonic decays [22,23], which uses
a method independent from the one investigated here. We make the dependence on the $D_s$ branching fraction explicit also in this case, since it is an important uncertainty and correlated with the other measurements in the fit. The dependence on $B(\bar{D} \to D^+ K^-)$ is not as easily included and its contribution to the uncertainty not as large as for the $D_s$ case.

The fit results for these two scenarios are collected in Table 2 under “our fits (w/o QCDF)”. Both fits describe the available data perfectly, meaning there are no obvious inconsistencies among the measurements. We observe significant shifts compared to the PDG fit results for $\bar{B}^0 \to D^+ K^-$ and $\bar{B}^0 \to D^+ \pi^-$ modes, but overall our results are well compatible with the PDG fit. More importantly, we obtain the full correlation matrix for these branching fractions, which allows to calculate their ratios with reduced uncertainties. Our improvements significantly sharpen the pattern that was apparent already in Refs. [2,4]: the ratios of branching fractions are well reproduced, the largest difference between measurement and prediction is $1.3\sigma$ for $R^{V/P}$. On the other hand, what was a tendency to overestimate the individual branching fractions in the past, is now a clear discrepancy: naively we observe a 4$\sigma$ difference between prediction and measurement in $\bar{B}_s^0 \to D_s^+ \pi^-$, over 5$\sigma$ difference in $\bar{B}_d^0 \to \bar{D}_u^+ K^-$, about 2$\sigma$ in $\bar{B}_u^0 \to D_s^+ \pi^-$ and 3$\sigma$ in $\bar{B}_d^0 \to D_s^{(*)+} K^-$.

A fit to the same data as above, but expressing all branching fractions by their QCDF expressions without allowing for corrections results in $\chi^2_{\text{min}} = 38.7$ for 9 degrees of freedom. We see the following possibilities to resolve this discrepancy:

1. One obvious option is the presence of large non-factorizable contributions of $O(15−20\%)$ at amplitude level in each of the modes. This was already discussed in Ref. [2], where the discrepancy has a smaller statistical significance. When taking our new estimates in Eqs. (12)–(14), which allow already for an enhancement by a factor of 10 in the hadronic matrix elements, at face value, this scenario is clearly and significantly disfavoured at the 4.4$\sigma$ level. We emphasize that we do not only see no enhancement in our calculation of next-to-leading power contributions, but instead a systematic suppression by $C_1/a_1 \sim -1/3$, which renders our result particularly small. Therefore even the generic expectation of $\Lambda_{\text{QCD}}/m_b \sim 10\%$ seems already on the high side. We pursue this scenario nevertheless, which still allows us to extract $f_s/f_D$, albeit with increased uncertainties.

2. We entertain also the possibility that this is an experimental issue. For that it is interesting to note that the fit to the QCDF predictions becomes excellent as soon as the measurements of the absolute branching fractions $\bar{B}^0 \to D^{(*)+}\pi^-$ are excluded from the fit. Both val-

Table 1: Numerical inputs and results for the QCDF expressions for the branching fractions at leading power. We compare the results of Ref. [2] with our results. The predictions for the $\bar{B}_d^0$ branching fractions are not time-integrated, and therefore differ from the measured branching fractions by a factor of $(1 − \gamma_s^2)$ [20]
ues are dominated by the BaBar analysis [24], however, even the less precise CLEO data are already in conflict with the QCDF predictions: the fit without these normalization modes and without input on $f_s/f_d$ yields $B(B^0 \to D^\pm \pi^-) = (3.94^{+0.24}_{-0.20}) \times 10^{-3}$ and $B(B^0 \to D^{++} \pi^-) = (3.98^{+0.38}_{-0.37}) \times 10^{-3}$. This option would therefore imply a serious experimental problem in more than one experiment. We do not consider this option further. Nevertheless, we would like to encourage additional measurements of absolute branching fractions, which is possible with existing data from the Belle experiment and upcoming data at Belle II.

3. The parametric inputs in Eq. (6) are in principle also potential sources of systematic shifts. However, all of them are very well known from several measurements. The quantity $|V_{cb}|$ has the largest uncertainty, but a 20% shift would be in direct contradiction with all existing analyses and the global picture of the CKM unitarity fit.

4. If we assume the experimental results to be correct and assume our estimates in Eqs. (12)–(14) to have the correct order of magnitude, only beyond the Standard Model (BSM) effects can explain the data. The fact that all QCDF predictions are above the corresponding measurements requires this BSM physics to interfere with the SM. Since the ratios are predicted well, an approximately universal factor in $b \to c\bar{d}d$ and $b \to c\bar{s}s$ transitions is preferred.

Also a combination of the above effects is possible. In the following we discuss options I and 4 in detail.

3.1 Extracting $f_s/f_d$

Allowing for non-factorizable contributions beyond the size indicated by the QCDF results in all decays at hand, we parametrize

$$\frac{\mathcal{A}(B^0 \to D^+ K^-)}{\mathcal{A}(B^0 \to D^+ K^-)_{\text{QCDF,LP}}} = 1 + \Delta_p,$$  
(34)

$$\frac{\mathcal{A}(B^0 \to D^{++} K^-)}{\mathcal{A}(B^0 \to D^{++} K^-)_{\text{QCDF,LP}}} = 1 + r_{SU(3)}^P \Delta_p,$$  
(35)

$$\frac{\mathcal{A}(B^0 \to D^{++} \pi^-)}{\mathcal{A}(B^0 \to D^{++} \pi^-)_{\text{QCDF,LP}}} = 1 + \Delta_V,$$  
(36)

$$\frac{\mathcal{A}(B^0 \to D^{++} \pi^-)}{\mathcal{A}(B^0 \to D^{++} \pi^-)_{\text{QCDF,LP}}} = 1 + r_{SU(3)}^V \Delta_V.$$  
(37)

Here we use the leading-power QCDF amplitudes as in Eq. (6), $\Delta_{P,V}$ parametrize non-factorizable contributions to the amplitude (including the parts estimated above) and $r_{SU(3)}^{P,V}$ parametrizes $SU(3)$ breaking beyond that in the leading amplitude. We expect $r_{SU(3)}^{P,V} \approx 1$, i.e., the non-factorizable parts to still scale as the decay constants, which is justified by the analytic structure of the NLP results.
We choose all four parameters $\Delta_P, V, P_{SU(3)}$ real without phenomenological consequences, since we only consider branching fractions here.

Leaving all four parameters in Eqs. (34)–(37) arbitrary is equivalent to not using the QCDF calculation at all. This reproduces the previous fit without QCDF and no input for $f_s/f_d$ in Table 2. Setting $P_{SU(3)} \equiv 1$, but leaving $\Delta_P, V$ arbitrary corresponds to the assumption that the ratios $R_{f_s/d}^{P, V}$ are perfectly predicted by QCDF, while the individual branching fractions receive large non-factorizable contributions. This mimics one of the assumptions employed in Refs. [1, 25] and in particular allows for a comparison with the result in Ref. [25]. We list the results of this scenario in Table 2 under “ratios only”. However, given the significant reduction of the parametric theory uncertainty in this work, the assumption of a negligible uncertainty from $SU(3)$ breaking in the non-factorizable part does not seem appropriate anymore. While we do not use the quantitative results for the NLP contributions here, we still make observations from their analytical structure:

1. In general there is no justification to identify $\Delta_P$ and $\Delta_V$.
2. The deviation of $P_{SU(3)}$ from unity is expected to be small: the sizable breaking from the light-meson decay constants is identical to that of the leading amplitude, and the heavy-to-heavy matrix elements for a single soft gluon at NLP exhibit a similar structure to that of the form factors, for which the results in Ref. [3] show explicitly that the breaking is small, in accordance with theoretical expectations from the HQE [7–9]. We therefore consider $P_{SU(3)} \in [0.9, 1.1]$ to be a conservative estimate of this breaking, leaving the two parameters independent.

We consequently perform another fit, leaving $\Delta_P, V$ independent and arbitrary while constraining the $SU(3)$ breaking beyond that in the leading amplitude to be below 10%. In this way we account for the possibility of relevant additional $SU(3)$ breaking, while still exploiting the information from QCDF. The results of this fit are reported again in Table 2.

Since this scenario interpolates in a way between the fit without QCDF and without $f_s/f_d$ input and the one without $SU(3)$ breaking, it is not surprising that also the results for $f_s/f_d$ lie between the corresponding values. As mentioned before, we observe large non-factorizable contributions of 15–20%, independently of the allowed magnitude of $SU(3)$ breaking. Although the minimal $\chi^2$ decreases by 0.9 when allowing for $SU(3)$ breaking, such a fit has two fewer degrees of freedom, so there is no indication for sizable $SU(3)$ breaking in the data. We consider this scenario nevertheless preferable over the one without breaking, since the two parameters $P_{SU(3)}$ account for the related uncertainty and render our results for $f_s/f_d$ conservative. While this is not the only way to estimate this uncertainty, the sizable shift between the central values and uncertainties in the scenarios with and without breaking, despite the relatively small breaking beyond that of the leading amplitude of $\leq 10\%$, indicates that this source of uncertainty must be taken into account for a meaningful extraction of $f_s/f_d$.

Our results are compatible with previous extractions of $(f_s/f_d)^{\gamma_{Tev}}_{LHCb}$ from $R_s/d$ [23, 25–28], but have significantly reduced uncertainties, despite allowing for larger $SU(3)$ breaking in the non-factorizable part. Both our values are also in excellent agreement with the value from semileptonic decays. The uncertainties are comparable, and can be reduced with more data in the future. However, the problem of the large apparent corrections of the QCDF results remains. Furthermore, while we consider our estimate for the range of $P_{SU(3)}$ to be conservative, it is clear from the data in Table 2 that the result for $f_s/f_d$ sensitively depends on our assumption. An effort should therefore be made to even further improve the understanding of these modes, in order to either understand the source for these large non-factorizable contributions, or to establish the presence of BSM physics in these modes, as discussed in the next subsection.

Finally, let us comment on the value for $(f_s/f_d)^{\gamma_{Tev}}$ extracted in Table 2: the value quoted in Ref. [29] reads $(f_s/f_d)^{\gamma_{Tev}} = 0.334 \pm 0.040$, which differs by $\sim 2\sigma$ from the value obtained in both fits using QCDF in Table 2. It is worth emphasizing that our value is independent from the values entering that average, and more precise. On the other hand it relies on LHCb-data and is hence not a pure CDF measurement. It is not clear from the information provided in Ref. [29] how the average is obtained. Updating the external inputs for the analyses [30, 31] and performing a correlated average, we obtain

$$(f_s/f_d)^{\gamma_{Tev, 123}} = 0.263 \pm 0.031,$$

which is also more precise than the average quoted in Ref. [29] and perfectly compatible with both the LHCb results (at different transverse momentum) and our result from non-leptonic decays.

We therefore conclude that while we do not see whence the required large non-factorizable contributions could originate, assuming their presence yields a consistent picture for all available data and an improved determination of $f_s/f_d$, which can be improved further in the future.

3.2 New physics in $b \rightarrow c\bar{s}(d)$ transitions

We now explore the possibility that the discrepancy between data and the SM calculation is caused by BSM physics. This is in part motivated by the observation that our fits including the QCDF constraints allow for $\Delta_P = \Delta_V$, although this would not be expected for NLP contributions. It would, however,
be expected for BSM contributions to the Wilson coefficients $C_{1,2}^{a_2}$. The ratios $R_{d,s}^{V/P}$ therefore provide tests for our BSM hypothesis. We do not aim at a detailed BSM analysis, only at establishing whether this option is already ruled out by existing data.

The observation that the new contributions should similarly reduce branching fractions with a pseudoscalar meson $D_q$ and a vector meson $D^*_q$ has rather strong consequences, implying specifically a minimal-flavour-violation-like scenario where the BSM contributions scale with the CKM factors of the SM ones. Scalar BSM operators generate predominantly reduce branching fractions with a pseudoscalar meson $D_q$ and a vector meson $D^*_q$ has rather strong consequences, implying specifically a minimal-flavour-violation-like scenario where the BSM contributions scale with the CKM factors of the SM ones. Scalar BSM operators generate...

As suggested by the previous fits, our BSM scenario works very well: we obtain $\chi^2_{\text{min}} = 4.9$ for 7 degrees of freedom. We also obtain $(f_s/f_d)_{\text{LHCb}} = 0.263^{+0.018}_{-0.016}$, which has the same uncertainty as in the “ratios only” scenario; however, clearly this value depends sensitively on our assumption of $\Delta a_1^d = \Delta a_1^s$. Most importantly, we obtain

$$\Delta a_1 = -0.17 \pm 0.03, \quad \text{or} \quad \Delta a_1 = -0.18 \pm 0.03, \quad (39)$$

ignoring potential imaginary parts for now using the same justification as given above.

We check if our BSM physics hypothesis is already excluded by other observables. In the following we list these observables and discuss the impact of their measurements, see Table 3:

- $\Gamma_q$: since we modify the leading contribution to the total decay rates of the $B_q$ mesons, we expect a strong constraint from $\Gamma_q$, $q = d, s$. However, each individual total decay rate is not as precisely predicted as their ratio.
- $\tau_{B_d}/\tau_{B_s}$: this observable is both predicted and measured to very high precision. The main contributions to the individual lifetimes cancel in the ratio, therefore the dominant contribution in our scenario is via dimension-6 operators in the OPE to $\tau_{B_d}$. We calculate the BSM shift at leading order, using the results from Ref. [32] with updated bag factors [33].
- $a_4^d$: the flavour-specific CP-asymmetry in the $B_d \to \bar{B}_d$ system receives also a leading contribution from $b \to c\bar{u}d$ operators, which is linear in the corresponding coefficient [34]. This constraint is therefore complementary to the others.

We do not consider further observables that are not as cleanly predicted as the ones above. We note, however, that other $B$ meson decays like $\bar{B}^0 \to D^{(*)+}\pi^-$ and $\bar{B}_s \to D^{(*)+}K^-$ (and their analogues with $\rho, K^*$) are likewise affected by our BSM physics hypothesis. We emphasize that all corresponding measurements [15] are lower than their (updated) SM predictions [2], thereby strengthening the case for the BSM hypothesis.

We find that surprisingly none of the inclusive observables in Table 3 excludes the possibility of having a shift of $-15$ to $-20\%$ in $a_1$ from BSM physics, in line with Refs. [34,36]. We conclude that BSM physics is a viable possibility that could explain the observed puzzle. Despite requiring a large contribution of $\sim -17\%$ of a CKM-leading tree-amplitude, available constraints from the above inclusive observables do not exclude such a scenario, which therefore has to be taken seriously. A detailed BSM analysis of these constraints is, however, beyond the scope of this article and left for future work.

3.3 Prospects

Our results reduce both main uncertainties in the predictions for $\bar{B}^0(\bar{B}_s) \to D^{(*)}\pi^-$ decays, thereby enabling future precision analyses of these modes. Measurements of absolute branching fractions can and should be carried out at Belle II, if possible also for the $\bar{B}_s$ decays modes. High-precision measurements of ratios of branching fractions can be obtained by the LHC experiments. With further reduced uncertainties, it would also be helpful to have smaller uncertainties on the $D^{(*)}_s$ branching fractions.

Experimentally, it might be advantageous to normalize to a different mode, like $\bar{B}^0 \to D^{(*)+}\pi^-$, since it has a larger branching fraction and systematic uncertainties due to the pions cancel. As an example, the prediction for the ratio $B(\bar{B}^0 \to D^{(*)}\pi^-)/B(\bar{B}^0 \to D^{(*)+}\pi^-)$ involves a different ratio of form factors than the ones already provided. Its value obtained from Ref. [3],

$$\left| \frac{F_0^{\bar{B}^0 \to D_s}}{F_0^{\bar{B}^0 \to D_s}} \right| = 1.005 \pm 0.021, \quad (40)$$

is rather precise. We caution that these modes suffer from significantly larger uncertainties due to presently unquan-
ttifiable $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ corrections. Therefore, the theoretically cleanest ratios for the purpose of determining $f_s/f_d$ are $R_{s/d}^{P/V}$, discussed in the previous sections.

As a workaround, we suggest a two-staged approach: the measurement of $B(B \rightarrow X)/B(\bar{B}^0 \rightarrow D^+K^-)$ – which, depending on the choice of $X$, can be determined at a precision of a few percent and potentially further improved at Belle II or LHCb – can convert the more easily measurable ratio $B(\bar{B}_s^0 \rightarrow D_s^+\pi^-)/B(\bar{B} \rightarrow X)$ into one of $R_{s/d}$, i.e.,

$$R_{s/d}^P = \frac{B(\bar{B}_s \rightarrow D_s^+\pi^-)}{B(\bar{B} \rightarrow X)} \frac{B(\bar{B} \rightarrow D^+K^-)}{B(\bar{B}_s^0 \rightarrow D_s^+\pi^-)}$$

For instance, with $X = D^+\pi^-$ the required experimental ratio would already be available at the 3–4% level:

$$R_{d}^{D^+\pi^-} = \frac{B(\bar{B}_s^0 \rightarrow D_s^+\pi^-)}{B(\bar{B}^0 \rightarrow D^+K^-)} \exp(12.17_{-0.37}^{+0.42}).$$

The situation is similar for $X = D^0\pi^-$; the determination of the ratio $R_{ind}^{D^0\pi^-} = B(B^- \rightarrow D^0\pi^-)/B(\bar{B}^0 \rightarrow D^+K^-)$ requires additionally the determination of the production fraction of charged and neutral $B$ meson pairs at the $B$ factories, or the ratio $f_u/f_d$ at LHCb (which is however expected to be close to unity). We are looking forward to see these prospects realised in future analyses by the ATLAS, Belle II, CMS and LHCb experiments.

4 Discussion and conclusion

In this work we revisit, extend and update the theoretical predictions for the branching fractions of the decays $\bar{B}^0 \rightarrow D^{(*)+}K^-$ and $B_s^0 \rightarrow D_s^{(*)+}\pi^-$. We obtain our predictions in the framework of QCD factorization (QCDF), using next-to-next-to leading order results for the effective Wilson coefficients and including next-to-leading power (NLP) corrections for the first time. Moreover, we employ updated values of the form factors with reduced theoretical uncertainties compared to previous estimates. Beyond the prediction of the branching fractions, we provide updated results for the fragmentation fraction $f_s/f_d$ in various scenarios. To obtain a reliable error estimate for the fragmentation fraction, we consider it mandatory to account for potential $SU(3)$ breaking in the non-factorizable corrections. Our results agree with the values extracted by LHCb and CDF from semileptonic decays.

Our comparison of the various experimental measurements of these modes and our theoretical predictions shows a clear and very significant discrepancy at the level of 4.4$\sigma$. This high level is due to drastically reduced parametric uncertainties. We identify the following possible four causes of the discrepancy, none of which is fully satisfactory on its own:

1. The current measurements of the absolute branching fractions for the modes considered could have a systematic bias in form of a downward shift.

This is unlikely to be the case, since the modes we discuss have large branching fractions, and have only charged particles in the final state, rendering them experimentally well accessible. A systematic bias of the order of $\sim -30\%$ would lead to questioning the validity of all measurements of $B$ meson branching fractions.

2. The theoretical results within the framework of QCDF could miss a large contribution of $\sim -20\%$ at the amplitude level in color-allowed tree topologies.

This is unlikely to be the case, since QCDF predictions for color-allowed tree decays into light mesons are in reasonable agreement with the measurements. If such a shift in heavy-light final states is necessary it is hard to understand why a similarly sized shift in light-light final states is absent.

3. The hadronic matrix elements for the non-local soft gluon terms could be underestimated, and cause the observed shift.

To increase the $\bar{B} \rightarrow D^{(*)}$ matrix elements to the size needed for agreement with the measurements, we would need an enhancement by a factor of $\sim 50$. This is also quite unlikely to be the case, since we have already enlarged the possible range of our estimates by multiplying our nominal results by a factor of 10 to obtain conservative estimates.
4. The discrepancy could be caused by contributions from physics beyond the SM.

Also this option seems unlikely, given that the partonic transition is generated by a tree-level $W$ exchange, and it would be surprising if a correction of $\sim -20\%$ had eluded attention so far. On the other hand, the same pattern exists for $b \to c\bar{u}(d/s)$ transitions that are not quantitatively discussed in this work. The interpretation is not excluded either by a number of inclusive observables like the flavor-specific CP asymmetry $a_{d,s}$, the total width of the $B$ meson $\Gamma_d$, or the ratio of lifetimes $\tau_s/\tau_d$.

Given the significance of this puzzle and the potential impact of any of the potential explanations discussed above, each of them should be studied in more detail. Doing so exhaustively requires dedicated efforts on both the experimental and the theoretical side. We look forward to further study these options in detail in a forthcoming publication.

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**Appendix A: Light-cone sum rule for the soft-gluon matrix element**

The next-to-leading power term in the factorization formula for $B_q \to D^{(*)+}_q L^-$ involves an exchange of a soft-gluon between the heavy-to-heavy transition and the light-meson. This exchange can be described in terms of the hadronic matrix elements of the operator [4, 11]

$$
\mathcal{O} \equiv -2 \int_0^{1/\Lambda_{QCD}} ds \bar{c}(0)\gamma^\mu (1-\gamma_5) G_{\mu\nu}(-sn)n^\nu b(0),
$$

(A1)

where $n$ is a light-cone vector and $q_\mu \simeq E_L n_\mu$ in the B-meson rest frame. Note that we absorb a factor of $-1$ into the definition of the operator above. The explicit factor of two accounts for the difference between the definition of $\tilde{G}$ in Refs. [4, 11] and our definition

$$
\tilde{G}_{\mu\nu} \equiv \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta},
$$

(A2)

which agrees with the one in Refs. [37, 38]. Matrix elements of the operator $\mathcal{O}$ can be estimated in a similar fashion as done for the matrix elements at subleading power for the non-local contributions in exclusive $b \to s \ell^+ \ell^-$ decays [37]. We closely follow the procedure to estimate the latter.

Using the translation operator, we can relate the gluon field on the light cone to the gluon field at the origin:

$$
G_{\mu\nu}(-sn) = \exp[-i(-sn) \cdot (iD)] G_{\mu\nu}(0).
$$

(A3)

This step allows us to express $\mathcal{O}$ as follows:

$$
\mathcal{O} = -2 \int_0^{1/\Lambda_{QCD}} \frac{d\omega_2}{\omega_2} \int_0^{1/\Lambda_{QCD}} ds \, e^{i\omega_2 s} \times \bar{c}(0)\gamma^\mu (1-\gamma_5) \delta [\omega_2 - (n \cdot iD)] \tilde{G}_{\mu\nu}(0)n^\nu b(0).
$$

(A4)

To leading-power in $\Lambda_{QCD}/E_L$ the effective integration range for $s$ is $[0, \infty)$. This leads us to the momentum space representation

$$
\mathcal{O} = -2i \int_0^{\infty} \frac{d\omega_2}{\omega_2} \times \bar{c}(0)\gamma^\mu (1-\gamma_5) \delta [\omega_2 - (n \cdot iD)] \tilde{G}_{\mu\nu}(0)n^\nu b(0)
$$

(A5)

$$
= -2i \int_0^{\infty} \frac{d\omega_2}{\omega_2} \bar{\mathcal{O}}(\omega_2) \equiv -2i \bar{\mathcal{O}},
$$

(A6)

where $\omega_2$ is the light-cone component of the gluon momentum.
We proceed to estimate the matrix elements
\[
\langle D^{(*)+} (k) | \tilde{O} | \tilde{B}^0 (q + k) \rangle. \tag{A7}
\]
For this task we use light-cone sum rules with an on-shell $B$-meson. The relevant three-particle $B$-meson LCDAs are defined in the heavy-quark limit and therefore only represent the result of the matrix element of $\tilde{O}$ to leading power in $\Lambda_{\text{QCD}}/m_b$. We define the correlation functions
\[
\Pi_{D^{(*)}}(q, k) = i \int d^4 x e^{i k x} \left\{ 0 \right| J_{\text{int}}^{D^{(*)}}(x) \big| \tilde{B}^0(q + k) \right\}, \tag{A8}
\]
where $J_{\text{int}}^{D^{(*)}}(x) \equiv \tilde{d}(x) \Gamma_{D^{(*)},c}(x)$ denotes a current suitable to interpolate the $D$ or $D^*$. We use
\[
\Gamma_D = i \gamma_5, \quad \Gamma_{D^*} = \bar{q} + \bar{k}. \tag{A9}
\]
Inserting a complete set of states between the two currents, we obtain the hadronic representation of the correlation functions. For the pseudoscalar meson $D = D(k)$ we obtain:
\[
\Pi_{D^0}^{\text{had}}(q, k) = \frac{f_D m_D^2}{m_c} \langle D^0 | \tilde{O} | \tilde{B}^0(q + k) \rangle
\]
\[
- i \int_{s_0}^\infty \frac{ds}{\pi} \frac{\bar{\rho}_D(s, q^2)}{s - k^2}, \tag{A10}
\]
For a longitudinal vector meson $D^* \equiv D^*(k, \lambda = 0)$ we obtain:
\[
\Pi_{D^*}^{\text{had}}(q, k) = \frac{f_{D^*} \sqrt{\lambda_{\text{kin}}}}{2 m_{D^*}^2} \langle D^{*+} | \tilde{O} | \tilde{B}^0(q + k) \rangle
\]
\[
+ i \int_{s_0}^\infty \frac{ds}{\pi} \frac{\bar{\rho}_{D^*}(s, q^2)}{s - k^2}, \tag{A11}
\]
where $\lambda_{\text{kin}} \equiv \lambda(m_B^2, m_{D^*}^2, q^2)$ is the Källén function. For both cases above $\bar{\rho}_{D^{(*)}}$ denotes the spectral density of the respective exited and continuum states.

At $q^2 \simeq 0$ the correlators Eq. (A8) are accessible in a light-cone operator product expansion (OPE) if $k^2 < 0$, and $|k^2| \gg \Lambda_{\text{QCD}}^2$. For these kinematics, the hard and soft contributions factorize. The hard contributions are calculated to leading order in $\alpha_s$, while the soft contributions are expressed in terms of the $B$-mesons LCDAs. For both the $D$ and the $D^*$ meson, the OPE result can be conveniently expressed as:
\[
\Pi_{D^{(*)}}^{\text{OPE}}(q, k) = \int d\omega_2 \int d^3 x \int d^4 p' e^{i(k-p')x}
\]
\[
x \left[ \Gamma_{D^{(*)}} \frac{p'_\mu + m_c}{m_c^2 - p'^2} \gamma^\mu (1 - \gamma_5) \right]_{ab}
\]
\[
x \langle 0 | \tilde{d}(x) \delta (\omega_2 - n \cdot D) \tilde{G}_{\mu \nu}(0) n^\nu h^*_0(0) | \tilde{B}^0 \rangle. \tag{A12}
\]
The sum rules are then obtained by matching the OPE results (A12) onto the hadronic representations (A10) and (A11). To remove continuum contributions, the assumption of semiglobal quark-hadron duality approximation is used, as discussed in Ref. [18]. We Borel transform the result to suppress the tail of the OPE calculations and the continuum contributions, thereby reducing the numerical impact of violation of quark-hadron duality. For our results we use the twist classification and modelling of the LCDAs as in Ref. [38], and truncate the twist expansion at the twist-four level.

Using the same inputs as in Ref. [18], our numerical results for $q^2 = 0$ are
\[
\langle D^{(*)+} (k) | \tilde{O} | \tilde{B}^0 (q + k) \rangle = i [0.13, 1.3] \text{ GeV}^2, \tag{A13}
\]
\[
\langle D^{*+} (k, \epsilon(\lambda = 0)) | \tilde{O} | \tilde{B}^0 (q + k) \rangle = i [0.078, 0.78] \text{ GeV}^2. \tag{A14}
\]

The lower bounds in the ranges above are obtained using the nominal values of our input parameters. To have a conservative estimate of these contributions, we compute the upper bounds by increasing the values of $\lambda_{D^*}^2$ and $\lambda_{D^*}^2$ by one order of magnitude. These two parameters enter approximately linearly in the normalization of the $B$-meson LCDAs. An increase in their central values implies an increase in our prediction of the matrix elements (A7). Normalizing these non-local matrix elements with the corresponding local matrix elements, we obtain
\[
\frac{\langle D^{(*)+} (k) | \tilde{O} | \tilde{B}^0 (q + k) \rangle}{i(m_B^2 - m_D^2) F^B_{0 \rightarrow D} (0)} = [0.76, 7.6] \%, \tag{A15}
\]
\[
\frac{\langle D^{*+} (k, \epsilon(\lambda = 0)) | \tilde{O} | \tilde{B}^0 (q + k) \rangle}{i(m_B^2 - m_{D^*}^2) A_0^{B \rightarrow D^*} (0)} = [0.46, 4.6] \%. \tag{A16}
\]
Note that here $A_0$ is defined as in Ref. [4], which differs in the phase convention from Ref. [18].

Appendix B: Experimental inputs

We update the experimental analyses to account for new results regarding, e.g., $D^{*-}$ branching fractions, lifetimes, and production fractions, which is only partially done in the averages available. In Table 4 we summarize the pertinent measurements; we use the information in the articles to extract the quantities listed there, which allow to explicitly account for all correlations due to $f_s/f_d$ as well as the charm branching fractions. A few comments are in order:

- The branching fraction from Ref. [21] depends on the production of $B_s$ in $\Upsilon(5S)$ decays. This is the only mea-
Table 4 Relevant experimental measurements entering our determination of branching fractions and ratios for the considered modes. A * indicates that here $f_1/f_d$ corresponds to the value at Tevatron, see text. The measurements marked by a † from Ref. [25] have a correlation coefficient of $-56\%$. The reference marked with § uses both $D^- \rightarrow K^+ \pi^- \pi^-$ and $D^- \rightarrow K_S \pi^-$ decays, however, the former is dominating.

| Measurement | Source | Reference(s) |
|-------------|--------|--------------|
| $B(B_s^0 \rightarrow D_s^+ \pi^-)$ | (3.6 ± 0.5 ± 0.5) $10^{-3}$ | Belle [15,21] |
| $f_d$ | (6.7 ± 0.5)$%$ | CDF [41]* |
| $f_d$ | 0.174 ± 0.007 | LHCb [42] |
| $f_d$ | 2.08 ± 0.08 | LHCb [25] |
| $B(B^0 \rightarrow D^- K^+)$ | (8.22 ± 0.28)$\%$ | LHCb [25] |
| $B(B^0 \rightarrow D^- \pi^+)$ | (6.8 ± 1.7)$\%$ | Belle [43] |
| $f_{00}B(B^0 \rightarrow D^- (\rightarrow K^+ \pi^- \pi^-)\pi^+)$ | (1.21 ± 0.05) $10^{-4}$ | BaBar/CLEO [24,44] |
| $B(B^0 \rightarrow D^- (\rightarrow K^+ \pi^- \pi^-)\pi^+)$ | (2.88 ± 0.29) $10^{-4}$ | BaBar [45] § |
| $B(B_s^0 \rightarrow D_s^+ \pi^-)$ | 0.66 ± 0.16 | Belle [46] |
| $B(B_s^0 \rightarrow D_s^+ \pi^-)$ | (7.75 ± 0.30)$\%$ | LHCb/BaBar/Belle [43,47,48] |
| $B(B^0 \rightarrow D^- \pi^+)$ | (2.72 ± 0.14) $10^{-3}$ | BaBar/CLEO [24,49] |
| $B(B^0 \rightarrow D^- \pi^+)$ | 0.99 ± 0.14 | BaBar [45] |
| $B(D^- \rightarrow \phi (\rightarrow K^+ K^-)\pi^-)$ | (2.27 ± 0.08)$\%$ | PDG average [15] |
| $B(D^- \rightarrow K^+ K^- \pi^-)$ | (5.45 ± 0.17)$\%$ | PDG average [15] |
| $B(D^- \rightarrow K^+ \pi^- \pi^-)$ | (9.38 ± 0.16)$\%$ | PDG average [15] |
| $B(D^- \rightarrow K^+ K^- \pi^-)(f_{1}/f_{d})^{\text{TeV}}_{\text{LHCb,sl}}$ | 0.0144 ± 0.0010 | LHCb [22,23] |
| $B(D^- \rightarrow K^+ K^- \pi^-)(f_{1}/f_{d})^{13\text{TeV}}_{\text{LHCb,sl}}$ | 0.0133 ± 0.0005 | LHCb [50] |
| $(f_{1}/f_{d})^{\text{TeV}}$ | 0.334 ± 0.040 | HFLAV average [29] |
| $f_{00}$ | 0.488 ± 0.010 | Pheno comb. of BaBar/Belle [40,51,52] |

- The production fraction $f_{00}$ of $B/\bar{B}$ pairs in $\Upsilon(4S)$ decays is assumed to be 1/2 in most measurements (and for all results in Ref. [15]), but can differ sizeably from that value, see Ref. [40] for a recent discussion.

References

1. R. Fleischer, N. Serra, N. Tuning, Phys. Rev. D 83, 014017 (2011). https://doi.org/10.1103/PhysRevD.83.014017. arXiv:1012.2784 [hep-ph]
2. T. Huber, S. Kränsel, X.-Q. Li, JHEP 09, 112 (2016). https://doi.org/10.1007/JHEP09(2016)112. arXiv:1606.02888 [hep-ph]
3. M. Bordone, N. Gubernari, M. Jung, D. van Dyk, Eur. Phys. J. C 80, 347 (2020). https://doi.org/10.1140/epjc/s10052-020-7850-9. arXiv:1912.09335 [hep-ph]
4. M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 591, 313 (2000). https://doi.org/10.1016/S0550-3213(00)00559-9. arXiv:hep-ph/0006124
22. R. Aaij et al. (LHCb), Nucl. Phys. B 520, 279 (1998). [https://doi.org/10.1016/S0550-3213(98)00131-X. arXiv:hep-ph/971280]

23. P. Gambino, K.J. Healey, S. Turczyk, Phys. Lett. B 520, 93 (2001). [https://doi.org/10.1016/S0370-2693(01)01071-9. arXiv:hep-ph/0101071]

24. E.E. Jenkins, M.J. Savage, Phys. Lett. B 281, 331 (1992). [https://doi.org/10.1016/0370-2693(92)91150-8. arXiv:hep-ph/9202276]

25. C. Boyd, B. Grinstein, Nucl. Phys. B 451, 177 (1995). [https://doi.org/10.1016/0550-3213(95)00339-T. arXiv:hep-ph/9502311]

26. A. Kobach, Continuity and semileptonic $B_{(s)} \to D_{(s)}^* f$ form factors. Phys. Lett. B 809, 135708 (2020). [https://doi.org/10.1016/j.physletb.2020.135708]

27. P. Ball, V. Braun, A. Lenz, JHEP 05, 004 (2006). [https://doi.org/10.1088/1126-6708/2006/05/004. arXiv:hep-ph/0603063]

28. M. Benecke, J. Phys. G 27, 1069 (2001). [https://doi.org/10.1088/0954-3899/27/6/303. arXiv:hep-ph/0009328]

29. M. Bordone, M. Jung, D. van Dyk, Eur. Phys. J. C 80, 74 (2020). [https://doi.org/10.1140/epjc/s10052-020-7616-4. arXiv:1908.09398 [hep-ph]]

30. E. McLean, C. Davies, J. Koponen, A. Lytle, Phys. Rev. D 101, 074513 (2020). [https://doi.org/10.1103/PhysRevD.101.074513. arXiv:1906.00701 [hep-lat]]

31. J.A. Bailey et al. (MILC), Phys. Rev. D 92, 034506 (2015). [https://doi.org/10.1103/PhysRevD.92.034506. arXiv:1503.07237 [hep-lat]]

32. M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018). [https://doi.org/10.1103/PhysRevD.98.030001]

33. P. Gambino, K.J. Healey, S. Turczyk, Phys. Lett. B 763, 60 (2016). [https://doi.org/10.1016/j.physletb.2016.07.023. arXiv:1606.06174 [hep-ph]]

34. E. McLean, C. Davies, A. Lytle, J. Koponen, Phys. Rev. D 99, 114512 (2019). [https://doi.org/10.1103/PhysRevD.99.114512. arXiv:1904.02046 [hep-lat]]

35. N. Gubernari, A. Kokulu, D. van Dyk, JHEP 01, 150 (2019). [https://doi.org/10.1007/JHEP01(2019)150. arXiv:1811.00983 [hep-ph]]

36. P. Blasi, P. Colangelo, G. Nardulli, N. Paver, Phys. Rev. D 49, 238 (1994). [https://doi.org/10.1103/PhysRevD.49.238. arXiv:hep-ph/9307290]

37. K. DeBruyn, R. Fleischer, R. Knegjens, P. Koppenburg, M. Merk, N. Tuning, Phys. Rev. D 86, 014027 (2012). [https://doi.org/10.1103/PhysRevD.86.014027. arXiv:1204.1735 [hep-ph]]

38. R. Louvot et al. (Belle), Phys. Rev. Lett. 102, 021801 (2009). [https://doi.org/10.1103/PhysRevLett.102.021801. arXiv:0809.2526 [hep-ex]]

39. R. Aaij et al. (LHCb), Phys. Rev. D 85, 032008 (2012). [https://doi.org/10.1103/PhysRevD.85.032008. arXiv:1111.2357 [hep-ex]]

40. B. Storaci, Updated average $f_{K^*}/f_K$ b-hadron production fraction ratio for 7 TeV pp collisions (2013)

41. R. Aaij et al. (LHCb), JHEP 04, 001 (2013). [https://doi.org/10.1007/JHEP04(2013)001. arXiv:1301.5286 [hep-ex]]

42. R. Aaij et al. (LHCb), Phys. Rev. Lett. 107, 211801 (2011). [https://doi.org/10.1103/PhysRevLett.107.211801. arXiv:1106.4445 [hep-ex]]

43. J.A. Bailey et al., Phys. Rev. D 85, 114502 (2012). [https://doi.org/10.1103/PhysRevD.85.114502. arXiv:1202.6346 [hep-lat] [Erratum: Phys. Rev. D 86, 039904 (2012)]]

44. C.J. Monahan, H. Na, C.M. Bouchard, G.P. Lepage, J. Shigemitsu, Phys. Rev. D 95, 114506 (2017). [https://doi.org/10.1103/PhysRevD.95.114506. arXiv:1703.09728 [hep-lat]]

45. Y.S. Amhis et al. (HFLAV) (2019), arXiv:1909.12524 [hep-ex]

46. T. Affolder et al. (CDF), Phys. Rev. Lett. 84, 1663 (2000). [https://doi.org/10.1103/PhysRevLett.84.1663. arXiv:hep-ex/9909011]

47. T. Aaltonen et al. (CDF), Phys. Rev. D 77, 072003 (2008). [https://doi.org/10.1103/PhysRevD.77.072003. arXiv:0801.4375 [hep-ex]]

48. M. Neubert, C.T. Sachrajda, Nucl. Phys. B 483, 339 (1997). [https://doi.org/10.1016/S0550-3213(96)00559-7. arXiv:hep-ph/9603202]

49. M. Kirk, A. Lenz, T. Rauh, JHEP 12, 068 (2017). [https://doi.org/10.1007/JHEP12(2017)068. arXiv:1711.02100 [hep-ph] [Erratum: JHEP 06, 162 (2020)]

50. C. Bobeth, U. Haisch, A. Lenz, B. Pecjak, G. Tetlalmatzi-Xolocotzi, JHEP 06, 040 (2014). [https://doi.org/10.1007/JHEP06(2014)040. arXiv:1404.2531 [hep-ph]]

51. F. Krimmer, A. Lenz, T. Rauh, Nucl. Phys. B 876, 31 (2013). [https://doi.org/10.1016/j.nuclphysb.2013.07.028. arXiv:1305.5390 [hep-ph]]