Light-Front QCD:
Role of Longitudinal Boundary Integrals

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Abstract

In the canonical light-front QCD, the elimination of unphysical gauge degrees of freedom leads to a set of boundary integrals which are associated with the light-front infrared singularity. We find that a consistent treatment of the boundary integrals leads to the cancellation of the light-front linear infrared divergences. For physical states, the requirement of finite energy density in the light-front gauge ($A_+ = 0$) results in equations which determine the asymptotic behavior of the transverse (physical) gauge degrees of freedom at longitudinal infinity. These asymptotic fields are generated by the boundary integrals and they are responsible for the topological winding number. They also involve non-local behavior in the transverse direction that leads to non-local forces.
1 Introduction

Quantum chromodynamics (QCD) was initially proposed as a strong interaction field theory in light-front coordinates, motivated by light-front current algebra [1]. In recent years, the search for nonperturbative solutions of QCD has led to an extensive exploration of light-front field theory (LFFT). The main attractions for studying nonperturbative QCD in light-front coordinates, called light-front form by Dirac [2], are that [3]: (1) boost invariance in LFFT is a kinematical symmetry, which is important in the study of composite systems, particularly the hadrons in QCD; (2) LFFT is a relativistic field theory with nonrelativistic structure so that the relativistic bound state equations are reduced to Schrödinger-type equations from which the nonrelativistic quark model may find its justification in QCD in light-front form; (3) the positivity of the longitudinal momentum \( k^+ \geq 0 \) in light-front Hamiltonian field theory implies that the light-front vacuum consists only of particles with longitudinal momentum \( k^+ = 0 \), which may simplify the QCD vacuum structure. These properties provide a hope to solve QCD in light-front form for hadrons.

The earliest systematic formulation of the light-front QCD (QCD in light-front form with light-front gauge, referred to simply as LFQCD hereafter) was given about sixteen years ago [4]. For applications of perturbative LFQCD see ref. [5]. In order to understand basic nonperturbative relativistic bound state problems, in the last few years many works on LFFT have mainly focused on various 1 + 1 field theory models, and some on the 3+1 Yukawa model and QED [6, 7]. One main obstacle in extending the study to nonperturbative LFQCD is that a formalism to address simultaneously the major difficulties of QCD in light-front form is still not in place. These difficulties include the renormalization problem (even in perturbation theory), the confinement problem, and the problem of the QCD vacuum and dynamical chiral symmetry breaking.

The renormalization problem in the study of relativistic bound states in LFFT has several aspects. Since power counting is different on the light-front [8], there are additional ultraviolet divergences in LFFT, compared to the instant form [2]. The additional ultraviolet divergences have received some attention recently in the context of the relativistic bound state problem in the 3+1 light-front Yukawa model [9]. LFQCD also contains severe light-front infrared divergences. The resolution of the light-front infrared divergence problems is not complete even in perturbative LFQCD. Issues arising from the possible mixing of the ultraviolet and infrared divergences in the relativistic bound state problems in LFFT have not been addressed so far.

Understanding confinement is crucial for building hadronic bound states in QCD. In the present canonical LFQCD, the associated Hamiltonian contains a linear potential between color charges only in the longitudinal direction, which does not provide a confinement mechanism for quarks and gluons in 3 + 1 dimensions. Therefore, it may not be suitable for describing low-energy hadronic structure. Based on light-front power counting, Wilson recently proposed a formalism to construct a confining light-front quark-gluon Hamiltonian for LFQCD [8]. Wilson suggested that a starting point for analyzing the full QCD with
confinement in light-front form is the linear infrared divergence (i.e., $1/k^+^2$ singularity in momentum space). The counterterms for the linear divergence, which may be constructed from light-front power counting rules, can involve the color charge densities and involve unknown non-local behavior in transverse direction. It is tempting to identify these terms as the source of transverse confinement. However, the analysis is not yet complete and a scheme for practical calculation has yet to be developed.

Dynamical chiral symmetry breaking is another important issue in the study of QCD for hadrons. In instant form, dynamical chiral symmetry breaking is associated with a nontrivial vacuum through the Goldstone mechanism. In LFQCD, the vacuum is trivial when the $k^+ = 0$ sector is ignored. Therefore, it seems to be natural to argue that in order to obtain a nontrivial vacuum, one has to solve the $k^+ = 0$ modes. Solving the $k^+ = 0$ modes may provide us with mechanism for spontaneous chiral symmetry breaking. Yet, the $k^+ = 0$ sector is singular and is very ambiguous. This singularity may exist even in free field theory. Thus, it is not clear whether the nontrivial structure of LFQCD must be associated with the $k^+ = 0$ modes. Furthermore, by involving the $k^+ = 0$ sector, the main advantage of LFQCD that simplifies nonperturbative bound states is lost, and therefore there is no strong reason why we should study nonperturbative QCD in light-front form. In fact, dynamical symmetry breaking can be manifested in different ways in different frames. It may be more attractive if we could formulate LFQCD with a trivial vacuum such that the dynamical breaking of chiral symmetry is manifested explicitly via effective interactions. However, the attempt in this direction has not yet started.

All these problems mentioned above are essential and should involve non-abelian gauge degrees of freedom in QCD. We are still unable to solve QCD at the moment. As a starting point, we shall address in this paper the problems of light-front linear infrared divergences and the associated nontrivial aspects, based on a canonical quantization approach to LFQCD. We hope that these discussions will provide some insight for solving QCD in light-front form in the future.

We apply the conventional canonical procedure to QCD in light-front form. It turns out naturally that QCD is a generalized Hamiltonian system where the first-class gauge and quark constraints emerge explicitly in the Lagrangian. As is known, in the light-front gauge these first-class constraints become solvable first-order differential equations, and are used to eliminate unphysical degrees of freedom to all orders of the coupling constant. However, the gauge constraint equations contains a set of boundary integrals at longitudinal infinity for the longitudinal color electric fields [see eq.(15) and the following discussions]. These longitudinal boundary integrals are the color charge density integrated over the longitudinal space ($x^-$) and are associated with the light-front infrared singularity. The resulting LFQCD Hamiltonian contains a boundary term proportional to these boundary integrals which are overlooked in previous investigations of light-front gauge theory.

We find that in perturbation theory the boundary integrals serve to remove linear infrared divergences in loop integrals. Removing the linear infrared divergences in LFQCD is a serious problem that has not been solved completely. In usual Feynman theory of per-
turbative LFQCD, by use of the gauge fixing term, one can derive the gauge propagator involving $1/k^+$ singularity. Beyond the leading order calculation, this singularity leads to linear infrared divergence in the principal value prescription. In $x^+$-ordering Hamiltonian perturbation theory, the linear infrared divergences emerge even in tree-level and one-loop diagrams. By including the boundary term in the Hamiltonian, we obtain a consistent distribution function for the product of two principal value prescriptions, from which the linear divergences in loop integrals are removed by the same divergences in the instantaneous interaction. This finding is useful for perturbative LFQCD calculations in high-energy processes.

The relevant boundary integrals in the Hamiltonian formulation of axial gauge were indeed pointed out first by Schwinger in 1962 [12]. Due to the different structure between LFFT and the field theory in instant form, the consequences from the boundary integrals we study in this paper have not yet been realized explicitly in axial gauge. One of the important differences is related to the QCD vacuum. In instant form with axial gauge, the QCD vacuum cannot be simple. In LFQCD, generally the vacuum should also be nontrivial because of the $k^+ = 0$ modes. However, the choice of antisymmetric boundary conditions for field variables at longitudinal infinity excludes the $k^+ = 0$ modes. In this case, the LFQCD vacuum remains trivial as the bare vacuum, and thus the nontrivial QCD structure must be carried purely by the boundary behavior of gauge fields.

For physical states, the requirement of finite energy density results in asymptotic equations for transverse (physical) gauge fields at longitudinal infinity. These asymptotic gauge fields are generated by the boundary integrals. It is these asymptotic fields that determine the gauge field configurations for the non-vanishing winding number associated with the topological solutions of QCD. Therefore, although the LFQCD vacuum is trivial with our choice of the boundary conditions, the nontrivial behavior of gauge theory is manifested in field operators. To the best of our knowledge, this is the first time the nontrivial behavior of gauge fields is explored in light-front form with light-front gauge $A^+_a = 0$. It is also the first attempt to address the nontrivial structure with trivial vacuum in QCD, which seems to be possible only in light-front form.

Furthermore, the asymptotic gauge fields at longitudinal infinity which are generated by the boundary integrals not only involve the color charge densities in transverse space but also involve non-local behavior in the transverse direction. We find that by replacing the nontrivial boundary condition with a trivial one for the transverse gluon fields, many transverse non-local potentials are induced by the boundary integrals. These potentials are responsible for the nontrivial QCD behavior and therefore may lead to quark and gluon confinement. This possibility will be explored in further investigations.

The paper is organized as follows. In section 2, a canonical procedure for LFQCD is studied where we focus on the problem of the boundary conditions in solving the light-front constraints. In section 3, the roles of boundary integrals are explored in detail. Some remarks are made for relevant problems in section 4. Finally, the paper includes two appendices. In appendix A, we discuss canonical quantization of LFQCD by use of the rigorous
where the fermion spinor in light-front form is divided into $\psi$ conjugate momenta of field variables $\partial T$. We start from the QCD Lagrangian in one-loop diagrams for the quark mass correction.

We demonstrate the cancellation of linear infrared divergences at the tree level in $q\bar{q}$ scattering and in one-loop diagrams for the quark mass correction.

## 2 Canonical formulation and boundary condition

We start from the QCD Lagrangian

$$
\mathcal{L} = -\frac{1}{2} \text{Tr}(F^{\mu\nu} F_{\mu\nu}) + \bar{\psi}(i\gamma_{\mu}D^\mu - m)\psi,
$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$, $A^\mu = \sum_a A^a_\mu T^a$ is a $3 \times 3$ gluon field color matrix and the $T^a$ are the generators of the SU(3) color group: $[T^a, T^b] = if^{abc}T^c$ and $\text{Tr}(T^a T^b) = \frac{1}{2}\delta_{ab}$.

The field variable $\psi$ describes quarks with three colors and $N_f$ flavors, $D^\mu = \frac{1}{2}\partial^\mu - igA^\mu$ is the symmetric covariant derivative, and $m$ is an $N_f \times N_f$ diagonal quark mass matrix. The Lagrange equations of motion are

$$
\partial_\mu F_{a\mu}^{\mu} + g f^{abc} A_{\mu b} F_{c\mu}^{\mu} + g \bar{\psi} \gamma^\nu T^a \psi = 0,
$$

$$
(i\gamma_{\mu}\partial^\mu - m + g\gamma_{\mu}A^\mu)\psi = 0.
$$

The light-front coordinates are defined as: $x^\pm \equiv x^0 \pm x^3$, $x^i_\perp \equiv x^i(i = 1, 2)$, where $x^+$ is chosen as the “time” direction along which the states are evolved, and $x^-$ and $x^\perp$ become naturally the longitudinal and transverse coordinates. The inner product of any two four-vectors is then $a_\mu b^\mu = \frac{1}{2}(a^+ b^+ + a^- b^-) - a_\perp b_\perp$, and the and space derivatives $(\partial^\mu = \frac{\partial}{\partial x^\mu})$ and the 4-dimensional volume element are given by $\partial^+ = 2\frac{\partial}{\partial x^+}$, $\partial^- = 2\frac{\partial}{\partial x^-}$, $\partial^i = -\frac{\partial}{\partial x^i}$, and $d^4x = \frac{1}{2}dx^+dx^-d^2x_\perp$, respectively.

Naively, the canonical theory of QCD in light-front form is constructed by defining the conjugate momenta of field variables \{\$A^a_\mu(x), \psi(x), \bar{\psi}(x)\} as

$$
E^\mu_a(x) = \frac{\partial \mathcal{L}}{\partial (\partial^\mu - A_\mu a)} = -\frac{1}{2}F_{a\mu}(x),
$$

$$
\pi_\psi(x) = \frac{\partial \mathcal{L}}{\partial (\partial^\psi - \bar{\psi})} = i\frac{1}{4}\bar{\psi}\gamma^+ = i\frac{1}{2}\psi^\dagger(x),
$$

$$
\pi_{\bar{\psi}}(x) = \frac{\partial \mathcal{L}}{\partial (\partial^\bar{\psi} - \psi)} = -i\frac{1}{4}\gamma^0\gamma^+\psi = -i\frac{1}{2}\psi(x),
$$

where the fermion spinor in light-front form is divided into $\psi = \psi_+ + \psi_-$, $\psi_\pm = \Lambda_\pm \psi$ with $\Lambda_\pm \equiv \frac{1}{2}\gamma^0 \gamma^\pm$. Following a similar procedure in instant form described by Faddeev and Slavnov\[10\] for gauge theory, we separate the time derivative terms from the Lagrangian,

$$
\mathcal{L} = \left\{ \frac{1}{2} F_{a\mu} \bar{\psi} (\partial^- A^a_\mu) + \frac{i}{2} \bar{\psi}^\dagger \psi_+ (\partial^- \psi_+) - \frac{i}{2} \bar{\psi}_+ (\partial^- \psi^\dagger) \psi_+ \right\} - \mathcal{H}
$$

$$
- \left\{ A^a_c C_a + \frac{1}{2} (\psi^\dagger_+ C + C^\dagger \psi_+ ) \right\},
$$

5
where

\[ H = \frac{1}{2}(E_a^{-2} + B_a^{-2}) + \frac{1}{2} \left\{ \psi^+_1 \{\alpha_\perp \cdot (i\partial_\perp + gA_\perp) + \beta m\}\psi_- + h.c. \right\} \\
+ \left\{ \frac{1}{2} \partial^+(E_a^- A_a^-) - \partial^i(E_a^i A_a^i) \right\} \] (8)

and

\[ C_a = \frac{1}{2}(\partial^+ E_a^- + gf^{abc} A_b^+ E_c^-) - (\partial^i E_a^i + gf^{abc} A_b^i E_c^i) + g\psi^\dagger_1 T^a \psi_+ \] (9)

\[ C = (i\partial^+ + gA_\perp)\psi_- - (i\alpha_\perp \cdot \partial_\perp + g\alpha_\perp \cdot A_\perp + \beta m)\psi_+ . \] (10)

In eq.(8), we have defined \( B_a^- = F_a^{12} \) as the longitudinal component of the light-front color magnetic field.

The reason for writing the Lagrangian in the above form is to make the Hamiltonian density and also the dynamical variables and constraints manifest. In eq.(7), the first term contains all the light-front time derivative terms. From the definition of eqs.(4-6), it immediately follows that only the transverse gauge fields \( A_a^\perp \) and the up-component quark fields \( \psi_+ \) and \( \psi^\dagger_1 \) are dynamical variables. The second term in eq.(7), \( H \), is a Hamiltonian density. It contains three parts, the first part involves the light-front color electric and magnetic fields; the second, the usual quark Hamiltonian with coupling to the gauge field, and the last a surface term. Besides the kinetic term and the Hamiltonian density, eq.(7) also contains an additional term. This is a constraint term which indicates that the longitudinal gauge field \( A_a^- \) and the down-component quark fields \( \psi_- \) (\( \psi^\dagger_1 \)) are only the Lagrange multipliers for the constraints \( C_a, C \ (C^\dagger) = 0 \). These constraints arise from the definition of canonical momenta in the light-front coordinates and are consistent with the Lagrangian equations of motion. The gauge field constraint, \( C_a = 0 \), is in fact the light-front Gauss law which is an intrinsic property of gauge theory. The fermion constraint, \( C \ (C^\dagger) = 0 \) is purely a consequence of using the light-front form.

The existence of constraint terms simply implies that QCD in the light-front form is a generalized Hamiltonian system\(^{[11]}\). These constraints are all secondary, first-class constraints\(^{[1]}\) in the Dirac procedure of quantization\(^{[13]}\). To obtain a canonical formulation of LFQCD for non-perturbative calculations, we need to explicitly solve the constraints, namely to determine the Lagrange multipliers, to all orders of the coupling constant. Generally, it is very difficult to analytically determine the Lagrange multipliers from the constraints \( C_a, C = 0 \) since they are coupled by \( A_+^a \). Only in the light-front gauge\(^{[13, 14]}\),

\[ A_+^a(x) \equiv A_0^a(x) + A_3^a(x) = 0, \] (11)

\(^{1}\)In light-front field theory, there always exist the so-called primary, second-class constraints due to the fact that the light-front Lagrangian is linear in the first-order \( x^+ \)-derivative and therefore the canonical momenta are functions of field variables. These constraints are not real constraints in the generalized phase space quantization procedure and are easily handled. We will put these discussions in the Appendix A.
are these two constraints reduced to solvable one-dimensional differential equations:

\[
\begin{cases}
\frac{1}{2} \partial^+ E_a^- = \partial^i E_a^i + g(f^{abc} A_b^i T^a \psi_+) = G_a \\
i \partial^+ \psi_- = (i \alpha_\perp \cdot \partial_\perp + g \alpha_\perp \cdot A_\perp + \beta m) \psi_+
\end{cases}
\]

(12)

In order to solve eq.(12), we have to define the operator \(1/\partial^+\). In general,

\[
\left( \frac{1}{\partial^+} \right) f(x^-, x^+, x_\perp) = \int_{-\infty}^{\infty} dx_1^- \varepsilon(x^- - x_1^-) f(x_1^-, x^+, x_\perp) + C(x^+, x_\perp)
\]

(13)

where \(\varepsilon(x) = 1, 0, -1\) for \(x > 0, = 0, < 0\), respectively, and \(C(x^+, x_\perp)\) is a \(x^-\) independent constant. However, since the canonical conjugate of transverse gauge field in LFQCD is a dependent variable \([E_a^i = -\frac{1}{2} \partial^+ A_a^i, \text{see eq.}(4)\] with \(A_a^+ = 0\), one has to impose a \(\text{a priori}\) a boundary condition for \(A_a^i\) in order to derive the canonical commutation relations for the physical field variables. It has been shown that the suitable definition of \(1/\partial^+\) which uniquely determines the initial value problem at \(x^+ = 0\) for independent field variables is \(C(x^+, x_\perp) = 0\). This corresponds to choosing an antisymmetric boundary condition for field variables in the longitudinal direction.

Using eq.(13), we can explicitly express \(E_a^-\) in terms of transverse gauge fields \(A_a^i\) and the independent light-front quark field \(\psi_+\) from the gauge constraint in eq.(12),

\[
E_a^-(x) + \partial_\perp A_a^i(x) = -\frac{g}{4} \int_{-\infty}^{\infty} dx^- \varepsilon(x^- - x^-) \left(f^{abc} A_b^i \partial^+ A_c^i + 2 \psi_1^a T^a \psi_+ \right) + C_a(x^+, x_\perp),
\]

(14)

here, \(E_a^i = -\frac{1}{2} \partial^+ A_a^i\) has been used. To uniquely determine their initial values at \(x^+ = 0\), we require that the \(E_a^-\) and \(A_a^i\) satisfy antisymmetric boundary conditions at longitudinal infinity, namely, \(C_a(x^+, x_\perp) = 0\). As a result,

\[
E_a^-(x) = -\partial_\perp A_a^i(x) - \frac{g}{4} \int_{-\infty}^{\infty} dx^- \varepsilon(x^- - x^-) \left(f^{abc} A_b^i \partial^+ A_c^i + 2 \psi_1^a T^a \psi_+ \right)
\]

\[
x^- = \pm \infty \rightarrow \partial_\perp A_a^i \big|_{x^- = \pm \infty} = + \frac{g}{4} \int_{-\infty}^{\infty} dx^- \left(f^{abc} A_b^i \partial^+ A_c^i + 2 \psi_1^a T^a \psi_+ \right).
\]

(15)

Since \(E_a^-\) satisfies now an antisymmetric boundary condition, its boundary values at longitudinal infinity are completely determined by the second equality in eq.(15), where the second term is boundary integrals over \(x^-\) for the color charge densities. These integrals are the source of light-front infrared singularity. We call them the longitudinal boundary integrals, or simply the boundary integrals.

By using the identity[12],

\[
\frac{1}{2} \int_{-\lambda/2}^{\lambda/2} dx^- \varepsilon(x^- - x^-) \varepsilon(x^- - x^-) = -|x^- - x^-| + \frac{1}{2} \lambda,
\]

(16)
where the parameter $\lambda$ denotes the distance between two boundary points in the longitudinal direction, the color electric field energy in the Hamiltonian becomes

$$
H_E = \frac{1}{2} \int_{-\infty}^{\infty} dx^- d^2x_\perp (E_a^-)^2 = \frac{1}{2} \int_{-\infty}^{\infty} dx^- d^2x_\perp \left\{ (\partial^i A_a^i)^2 + \frac{1}{2} \int_{-\infty}^{\infty} dx'^- \left[ g \partial^i A_a^i \varepsilon(x^- - x'^-) (f^{abc} A_b^j \partial^+ A_c^i + 2 \psi_+^i T^a \psi_+) \right] \right\} + \frac{g^2}{8} \int_{-\infty}^{\infty} dx'^- \left( f^{abc} A_b^i \partial^+ A_c^i + 2 \psi_+^i T^a \psi_+ \right) |x^- - x'^-| (f^{ade} A_d^j \partial^+ A_e^j + 2 \psi_+^j T^a \psi_+)}
$$

(17)

In the above equation, the last term (a boundary term) involves the boundary integrals and is associated with the infrared divergence in the light-front instantaneous interactions. As we will discuss later, in perturbation theory, this term is regularized by the distribution function of the product of two principal value prescriptions and leads to the cancellation of the light-front linear infrared divergences. For physical states, the requirement of finite energy density results in the asymptotic equations for the transverse gauge fields which show that the asymptotic transverse gauge fields do not vanish at longitudinal infinity and are generated by the boundary integrals. The non-vanishing asymptotic transverse gauge fields determine the nontrivial QCD structure in light-front form. Thus, the boundary integrals can inherently affect QCD dynamics.

The Lagrange multipliers in eq.(7) can be determined easily now. The Lagrange multiplier $\psi_-$ is the solution of the quark constraint in eq.(12),

$$
\psi_-(x) = -\frac{i}{4} \int_{-\infty}^{\infty} dx'^- d^2x'_\perp \varepsilon(x^- - x'^-) \delta^2(x_\perp - x'_\perp) \times [a_\perp \cdot (i\partial_\perp + gA_\perp(x')) + \beta m] \psi_+(x').
$$

(18)

The Lagrange multiplier $A_a^-$ is obtained from the definition $E_a^- = -\frac{1}{2} \partial^+ A_a^-$ and eq.(15),

$$
A_a^-(x) = -\frac{1}{2} \int_{-\infty}^{\infty} dx'^- \varepsilon(x^- - x'^-) E_a^- (x^+, x'^-, x_\perp)
$$

(19)

For this solution, the first surface term in eq.(8) vanishes. Moreover, it is reasonable to assume that the transverse color electric fields $E_a^i$ as well as $A_a^i$ vanish as $O(r^{-2})$ and $O(r^{-1})$ at $r = |x_\perp| \rightarrow \infty$ because the gauge freedom is totally fixed at the transverse infinity. Hence the other surface term in eq.(8) vanishes as well.

After the determination of the Lagrange multipliers, the LFQCD Hamiltonian is given simply by

$$
H = \int dx^- d^2x_\perp \left\{ \frac{1}{2} (E_a^-)^2 + B_a^- \right\} + \psi_+^i \left\{ a_\perp^i (i\partial_\perp + gA^i) + \beta m \right\} \psi_-
$$
\[
\begin{align*}
&= \int dx^- d^2 x_\perp \left\{ \frac{1}{2} (\partial^i A^i_\perp)^2 + g f^{abc} A^i_a A^j_b \partial^i A^j_c + \frac{g^2}{4} f^{abc} f^{ade} A^i_a A^j_b A^k_c A^l_d \right. \\
&\quad + \frac{1}{4} \int_{-\infty}^{\infty} dx^- \left[ g \partial^i A^i_\perp \varepsilon(x^- - x'^-) (f^{abc} A^i_\perp \partial^+ A^i_a + 2 \psi^\dagger_+ T^a \psi_+) \\
&\quad - i \psi^\dagger_+ \{ \alpha^i_\perp (i \partial^-_\perp + g A^i) + \beta m \} \varepsilon(x^- - x'^-) \{ \alpha^i_\perp (i \partial^-_\perp + g A^i) + \beta m \} \psi_+ \right] \\
&\quad - \frac{g^2}{16} \int_{-\infty}^{\infty} dx^- (f^{abc} A^i_a \partial^+ A^i_c + 2 \psi^\dagger_+ T^a \psi_+) \\
&\quad \left. \left| x^- - x'^- \right| (f^{ade} A^j_d \partial^+ A^j_e + 2 \psi^\dagger_+ T^a \psi_+) \right\} \\
&\quad + \left( \lim_{\lambda \to \infty} \lambda \right) \frac{g^2}{16} \int d^2 x_\perp \left\{ \int_{-\infty}^{\infty} dx^- (f^{abc} A^i_a \partial^+ A^i_c + 2 \psi^\dagger_+ T^a \psi_+) \right\}^2 .
\end{align*}
\]  

A detailed procedure to quantize the above formulation is presented in appendix A. This Hamiltonian contains the naive canonical LFQCD Hamiltonian plus a boundary term which is the square of the boundary integrals, as a result of eq. (15). Choosing antisymmetric boundary conditions in the longitudinal direction in LFQCD has the following advantages:

1). With any other boundary condition, eq. (14) contains an arbitrary \( x^- \)-independent function. Such an arbitrary function leads to ambiguities in formulating LFQCD. Only with an antisymmetric boundary conditions, is this arbitrary term zero and formally LFQCD can be completely defined.

2). For the definition eq. (13) with \( C(x^+, x^-) = 0 \), all field variables in LFQCD satisfy antisymmetric boundary conditions at longitudinal infinity except \( \psi_+(x) \), whose boundary condition is not specified. However, the equation of motion for \( \psi_+(x) \) contains \( \frac{1}{\pi^*} \psi_+ \) [see Appendix A] which forces it to satisfy the antisymmetric boundary condition. As a result, the \( k^+ = 0 \) modes are completely excluded in the momentum expansion of field variables. Since the LFFFT vacuum is occupied only by the \( k^+ = 0 \) particles, with the help of antisymmetric boundary condition the LFQCD vacuum is ensured to be trivial as the bare vacuum. The important consequence is, as we shall discuss in the next section, that in this case the nontrivial structure of QCD is carried purely by field operators.

3). By the choice of antisymmetric boundary conditions, the residual gauge freedom in \( A^+_a = 0 \) is completely fixed \[18\] (also see the discussion later).

It is worth pointing out that there is a disadvantage for using eq. (13) in perturbative light-front Feynman loop integrals. In perturbative theory, eq. (13) with \( C = 0 \) leads to the principal value prescription for the light-front longitudinal infrared singularity. Although this prescription removes severe linear infrared divergences as we will show next, severe logarithmic infrared divergences are still present. These logarithmic infrared singularities correspond to the “spurious” poles in Feynman integrals, which prohibit any continuation to euclidean space (Wick rotation) and hence the use of standard power counting arguments for Feynman loop integrals \[13\]. This causes difficulties in addressing renormalization of QCD in Feynman perturbation theory with light-front gauge. In the last decade there are many investigations in attempting to solve this problem. One excellent solution is given by Mandelstam and Leibbrandt, i.e., Mandelstam-Leibbrandt (ML) prescription \[20\], which
allow continuation to euclidean space and hence power counting. This prescription has also been derived in the equal-time quantization with light-front gauge by Bassetto et al. [21]. It has been also shown that with ML prescription, the multiplicative renormalization in the two-component LFQCD Feynman formulation is restored [22].

In the present paper, we study QCD in light-front equal-\(x^+\) quantization. Unfortunately, ML prescription cannot be applied to equal-\(x^+\) quantization because ML prescription is defined on the boundary condition which involves \(x^+\) itself [23] and are not allowed in equal-\(x^+\) canonical theory. Yet, as is pointed out recently by Wilson [8], light-front power counting differs completely from the power counting in equal-time quantization. Furthermore, the current attempts to understand nonperturbative QCD in light-front form is based on the \(x^+\)-ordered (old-fashioned) diagrams in which no Feynman integral is involved [5, 6]. Thus the problem with the power counting criterion in Feynman loop integrals does not affect our discussions in this paper. The renormalization of light-front Hamiltonian is an entirely new subject where investigations are still in their preliminary stage [8, 9].

The main aim of this paper is to show that the boundary integrals play an important role in understanding the nontrivial features of LFQCD. The logarithmic infrared divergences are completely cancelled in the complete loop diagrams of dynamical processes, as was previously shown in the calculation of QCD correction to the scale evolution of hadronic structure function up to two-loop [24]. A simple example of such a cancellation in \(x^+\)-ordered perturbative theory is also given for quark mass renormalization in Appendix B-2.

In our forthcoming papers [25] we will present a detailed discussion on \(x^+\)-ordered perturbative loop calculations and light-front renormalization in LFQCD Hamiltonian theory, where the logarithmic infrared divergences are again completely cancelled in coupling constant renormalization. However, based on light-front power counting, the linear infrared divergences only involve color charge density and involve non-local behavior in the transverse direction, which may be the source of transverse confinement in LFQCD [8]. The severe linear infrared divergences in LFQCD has not been explored in light-front Hamiltonian. In the present paper, we shall focus on linear infrared singularity associated with the boundary integrals, which may relate to the nonperturbative aspects of LFQCD in physical states, as we will see below.

3 Role of boundary integrals

1. Removing linear infrared divergences. In the past decade, applications of LFQCD are mostly restricted in perturbation theory. Naively, the boundary term in eq.(20) is ignored so that the light-front instantaneous interactions are thought to be linear potentials [13, 20]. However, this negligence leads to severe infrared singularities in the perturbation theory. To see this clearly, we consider the formulation in momentum space. For the prescription of \(1/\partial^+\) expressed in terms of the integral of eq.(13), the standard Fourier transform leads...
to the principal value prescription in momentum space as follows,

\[
\left( \frac{1}{\partial^+} \right) f(x^-) = \frac{1}{4} \int_{-\infty}^{\infty} dx'^- \varepsilon(x^- - x'^-) f(x'^-)
\]

\[
\rightarrow \frac{1}{2} \left( \frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon} \right) f(k^+) \equiv \frac{1}{[k^+]^2} f(k^+), \tag{21}
\]

\[
\left( \frac{1}{\partial^+} \right)^2 f(x^-) = \frac{1}{4^2} \int_{-\infty}^{\infty} dx'^- dx''^- \varepsilon(x^- - x'^-) \varepsilon(x'^- - x''-) f(x''^-)
\]

\[
\rightarrow \left[ \frac{1}{2} \left( \frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon} \right) \right]^2 f(k^+) \equiv \frac{1}{[k^+]^2} f(k^+). \tag{22}
\]

Eq.(22) defines the product of two principal value prescriptions of eq.(21) in terms of the distribution function. In this derivation, it follows (see eq.(16)) that the boundary term in eq.(20) have been regularized. It is known that eq.(22) leads to linear infrared divergences in loop integrals. In order to avoid this divergence, one naively introduces the following prescription \[^5\]

\[
\left( \frac{1}{\partial^+} \right)^2 f(x^-) = \frac{1}{4^2} \int_{-\infty}^{\infty} dx'^- |x^- - x'^-| f(x'^-)
\]

\[
\rightarrow \frac{1}{2} \left( \frac{1}{(k^+ + i\epsilon)^2} + \frac{1}{(k^+ - i\epsilon)^2} \right) f(k^+) \equiv \frac{1}{[k^+]^2} f(k^+). \tag{23}
\]

This corresponds to the case that the longitudinal boundary term in eq.(16) is ignored. Equivalently, the last term in eq.(20) is dropped. Apparently, this prescription removes the linear infrared divergence originated from the instantaneous interactions. Unfortunately in such a prescription, beyond leading order calculations in Feynman perturbation theory or even in leading order calculation in the old-fashion Hamiltonian perturbation theory, the product of two principal value prescriptions appearing from three-point vertex either is not defined or leads to linear infrared divergences. We shall show that it is the prescription of eq.(22) which serves for the cancellation of linear infrared divergences originated from the three-point vertex and from the instantaneous interactions. Here we only discuss the \(x^+\)-ordered (old-fashion) perturbative calculations.

First at the tree level, for example for \(q\bar{q}\) scattering (see appendix B-1), only the linear potential leads to a \(1/\epsilon^2\) divergence as \(k^+ \to 0\). The scattering involving one-gluon-exchange is finite due to the principal value prescription. Thus in the naive prescription \(23\), even the lowest order \(q\bar{q}\) scattering amplitude is \(1/\epsilon^2\) divergent. By including the boundary term, this divergence is cancelled. In loop calculations, for example for the one-loop correction to the quark self-energy (see appendix B-2), the one gluon exchange diagram (which contains an integral of \(1/[k^+]^2\)) leads to a \(1/\epsilon\) divergence; the linear potential is, however, infrared finite in the relevant integral of \(1/[k^+]^2\). Hence, in the naive prescription, loop calculations also contain the severe \(1/\epsilon\) infrared divergence. In prescription \(22\), the instantaneous interactions are the linear potentials accompanied by the boundary term, which produce a \(1/\epsilon\) divergence from the integral of \(1/[k^+]^2\) that cancels precisely the same divergence in
the one gluon exchange diagram. Furthermore, the cancellation in the one-loop correction of quark-gluon vertex has also been verified [25].

The reason that the linear infrared divergences are removed by using prescription (22) can be understood as follows. From eq.(15), the $k^+$ singularity originated from the boundary integrals. The color electric energy in LFQCD Hamiltonian contains two sources for the $k^+$ singularity. One is the explicit boundary term, the last term in eq.(20), which is $1/k^{+2}$-singular. The other belongs to the gluon emission vertex. The resultant gluon emission vertex is the first term in the square bracket in eq.(20), which is $1/k^+$-singular. Therefore, in one gluon exchange diagrams, it produces a $1/k^{+2}$-singularity, namely the product of two principal value prescriptions for the definition of eq.(21). The associated linear divergence in loop integrals is the same as that from the $1/k^{+2}$-singularity of the boundary terms in the prescription of eq.(22), with a different sign from an energy denominator, and therefore the linear infrared divergence is cancelled. Note that in eq.(20) there is another $1/k^+$-singularity [in the second term in the square bracket], which comes from the quark constraint [see eq.(18)]. Yet, in one-gluon exchange diagrams, it leads to a form $1/(p^+_1 p^+_2) \ (p^+_1 = p^+_2 + k^+)$ which does not generate infrared divergences. Thus, all linear infrared divergences originate from the same source, the boundary integrals. Any negligence of boundary term in the Hamiltonian through eq.(23) will lead to unwanted infrared divergences.

However, the cancellation of the linear infrared divergences in higher order loop-integrals (beyond the one-loop diagrams) may also depend on the regularization of ultraviolet divergences. The cancellation beyond leading order should be true for gauge invariant regularization. For gauge variant regularization, such as transverse dimensional regularization [1], boost invariant cutoff regularization [21] and the explicit cutoff regularization [28] used in the $x^+$-ordered perturbative LFQCD, we need to introduce gluon mass counterterms. These counterterms break gauge invariance and thereby may also spoil the cancellation of linear infrared divergences in higher order diagrams. However, if we set the quark mass $m = 0$ in perturbative LFQCD, the transverse dimensional regularization results in a zero gluon mass correction. In this case the cancellation is still satisfied in two-loop diagrams. In deep inelastic scattering, one often sets $m = 0$ in calculating high-order corrections to the scale evolution of hadronic structure functions [24]. A more detailed discussion on perturbative LFQCD will be presented in a separate paper [25]. For low-energy dynamics, the light quark mass is crucial and perturbation theory is no longer useful. Removes infrared divergences needs to be treated in an alternative way, which we shall discuss later.

We may point out that in $1 + 1$ LFQCD [21], the boundary integral is the color charge operator. The corresponding boundary term occurring in the Hamiltonian [30] is then proportional to the square of color charge. It is indeed this term resulting in an infinite quark mass which is regarded as evidence of quark confinement in $1 + 1$ QCD. Explicitly, the linear potential does not provide an infinite mass for the quark, as shown above (also see ref. [31]), but the boundary term adds a $1/k^{+2}$ singularity (the $1/e$ divergence) to the quark propagator. Since there are no transverse gluons in $1 + 1$ QCD to cancel this divergence, the boundary term recovers ’t Hooft’s solution of the infinite quark mass pole [29]. In physical
(zero color charge) states, the boundary term does not contribute to physical observables since it is the square of the color charge operators. Quark confinement in gauge-invariant states arises purely from the linear potential. This implies that ignoring the boundary integral in 1 + 1 QCD may not affect any observable. In 3 + 1 QCD, the existence of transverse gluons changes these consequences.

2. **Winding number.** The winding number is a topological quantity associated with nontrivial structure of U(1) problem in QCD (in Euclidean space, this is the topological charge or Pontryagin index associated with the instanton solution of non-abelian gauge theory) \[32, 33\]. The second important consequence from the boundary integrals is that they determine a non-vanishing winding number.

As it is known, \( A^+ = 0 \) cannot completely fix the gauge degrees of freedom. There are residual gauge transformations under which the theory is invariant. The generators of residual gauge transformations are

\[
R_a = -\frac{1}{2} \int_{-\infty}^{\infty} dx^- \left\{ 2 \partial^i \partial^+ A^{i}_a + g (f^{abc} A^b_i \partial^+ A^c_i + 2 \psi^\dagger T^a \psi) \right\}
\]  

(24)

As we have shown \[18\] the residual gauge transformations generated by \( R_a \) break the antisymmetric boundary condition for \( A^i_a \) at longitudinal infinity and therefore are not allowed with our choice of the boundary conditions. However, by using the antisymmetric boundary conditions, the first term in eq.(24) can be integrated out explicitly over \( x^- \) and we have,

\[
R'_a = \mp 4 \partial^i A^i_a |_{x^- = \pm \infty} - \frac{g}{2} \int_{-\infty}^{\infty} dx^- (f^{abc} A^b_i \partial^+ A^c_i + 2 \psi^\dagger T^a \psi).
\]  

(25)

It has been verified \[18\] that for this definition, \( R'_a \) generate the gauge transformations preserving the antisymmetric boundary conditions. But these operators do not commute with the LFQCD Hamiltonian (20). In other words, there is no longer an additional gauge freedom to choose other \( A^i_a \) such that the resulting Hamiltonian remains invariant. Therefore, with antisymmetric boundary conditions, the residual gauge freedom in \( A^+_a = 0 \) is completely fixed \[18\].

Furthermore, for physical states, finite energy density requires that the longitudinal color electric field strength must vanish in the infinity (A similar requirement was used by Chodos in axial gauge \[12\]):

\[
E^a_{-} |_{x^- = \pm \infty} = 0
\]  

(26)

or explicitly

\[
\partial^- A^a_{-} |_{x^- = \pm \infty} = \mp \frac{g}{4} \int_{-\infty}^{\infty} dx^- (f^{abc} A^b_i \partial^+ A^c_i + 2 \psi^\dagger T^a \psi).
\]  

(27)

Eq.(27) is consistent with our choice of antisymmetric boundary condition. Moreover, this condition explicitly shows that the transverse gauge fields at longitudinal infinity are generated by the boundary integrals.\[4\] Clearly, eq.(27) is satisfied only for physical states. In perturbation theory, we cannot use this condition because in perturbative QCD, we

\[2\]It may be worth pointing out that unlike the 1 + 1 QCD, the boundary integrals in 3+1 LFQCD are
consider not only physical states but also color non-singlet states for which eq. (27) may not be satisfied. Therefore the main effect of eq. (27) should be manifested in nonperturbative dynamics. Now we shall discuss how eq. (27) determines the non-vanishing topological winding number.

To make the discussion clear and without any loss of generality, we consider the case of zero quark mass only \[34\]. It is well-known that the axial current for \(N_f\)-flavor quarks has an anomalous divergence,

\[
\partial_\mu j^\mu_5 = N_f g^2 \frac{8 \pi^2}{\Tr(F_{\mu\nu} \tilde{F}^{\mu\nu})},
\]

where the axial current is \(j^\mu_5 = \bar{\psi} \gamma^\mu \gamma_5 \psi\), and the dual field strength is \(\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}\).

From eq. (28), the time derivative of the light-front axial charge is given by

\[
\partial^- Q_5 = \int dx^- d^2 x_\perp \partial^- (\bar{\psi}_+^\dagger \gamma_5 \psi_+)
= N_f g^2 \frac{8 \pi^2}{\Tr(F_{\mu\nu} \tilde{F}^{\mu\nu})}.
\]

In obtaining eq. (29), we have used the fact that the other three surface terms of axial currents at longitudinal and transverse infinity do not contribute to \(\partial^- Q_5\). This is clear if we note that \(j^\mu_5(x) \to 0\) at \(x_\perp \to \pm \infty\) and \(j_5^{-}(x) = 2 \bar{\psi}_-(x) \gamma_5 \psi_-(x)\) which leads to \(j^\mu_5|_{x^- = \infty} - j^\mu_5|_{x^- = -\infty} = 0\) due to the antisymmetric boundary behavior of \(\psi_\pm\) at longitudinal infinity (see eq. (19)).

The anomaly alone does not imply that the right-hand side (r.h.s.) of eq. (29) must be nonzero \[32\]. The nonzero contribution of the r.h.s of eq. (29) is given by the gauge field configurations determined by eq. (27). This can be seen from the winding number in LFQCD defined as the net charge between \(x^+ = -\infty\) and \(x^+ = \infty\),

\[
\Delta Q_5 = \frac{1}{2} \int dx^+ \partial^- Q_5 = N_f g^2 \frac{8 \pi^2}{\Tr(F_{\mu\nu} \tilde{F}^{\mu\nu})}.
\]

The integration on the r.h.s. of the above equation is defined in Minkowski space. By using the identity

\[
\Tr(F_{\mu\nu} \tilde{F}^{\mu\nu}) = 4 \delta_\mu K^\mu,
\]

where

\[
K^\mu = e^{\mu\nu\rho\sigma} \Tr \left\{ A_\nu \partial_\sigma A_\rho + \frac{2}{3} A_\nu A_\sigma A_\rho \right\},
\]

color charge densities in the transverse space and not the color charge operators. The color charge operator is defined as follows:

\[
Q_a = \int dx^- d^2 x_\perp (f_{abc} A^a_\nu \partial^\nu A^b_\nu + 2 \bar{\psi}^\dagger T^a \psi_+)
= \int dx^- d^2 x_\perp (\rho^a_\perp (x^- , x_\perp) + \rho^a_q (x^- , x_\perp))
\]

where \(\rho^a_\perp\) is the charge density carried by gluon field and \(\rho^a_q\) that by quark field in light-front form. Therefore, we cannot simply drop the boundary integrals for color singlet states. In other words, removing the boundary integrals in eq. (20) implies ignoring the non-vanishing asymptotic \(A^a_\nu\) fields at longitudinal infinity, and therefore loses the possibility to address the nontrivial properties in LFQCD.
the r.h.s. of eq.(30) is reduced to surface integrals. In the light-front gauge $A^+_a = 0$, the second term in $K^{+,i}$ is zero. Thus, the surface integrals at transverse infinity vanish because $K^i$ falls off as $r^{-3}$ for $r = |x_\perp| \to \infty$. Meanwhile, since $x^+$ and $x^-$ are symmetric, it is reasonable to use the same boundary condition (i.e., antisymmetric boundary condition) for $A^-_a$ at $x^+ = \pm \infty$. Therefore, the contribution from the surface integral at $x^+$-infinity vanishes as well. At $x^- = \pm \infty$, the surface integral contribution for the first term of $K^-$ is also zero due to the antisymmetric boundary condition for the $A^-_a$ and $A^+_a$ fields. Finally, eq.(30) is reduced to

$$\Delta Q_5 = -N_f g^2 \pi \int dx^+ d^2 x_\perp \text{Tr} \left( A^{-} [A^1, A^2] \right) \bigg|_{x^- = -\infty}^{x^- = \infty} \bigg|_{x^- = -\infty}^{x^- = \infty} = -2N_f g^2 \pi \int dx^+ d^2 x_\perp \text{Tr}(A^{-} [A^1, A^2]), \quad \text{at } x^- = \infty,$$

(33)

Here we have used again the antisymmetric boundary condition of $A^i_a$ and $A^{-}_a$ at longitudinal infinity. Eq.(33) shows that a non-vanishing $\Delta Q_5$ is generated from the asymptotic fields of $A^+_a, A^-_a$ at longitudinal infinity, which are generated by the boundary integrals by eq.(27). The non-zero $A^-_a \big|_{x^- = \pm \infty}$ is determined by transverse gauge fields (from Eq.(19)),

$$A^{-}_a(x^+, x_\perp) \big|_{x^- = \pm \infty} = \pm \frac{1}{2} \int_{-\infty}^{\infty} dx^- x^- G_a(x^+, x^-, x_\perp).$$

(34)

Thus, the boundary integrals are essential for the non-vanishing winding number in QCD.

The above derivation shows that for an antisymmetric boundary condition, although the LFQCD vacuum is trivial, the nontrivial QCD structure is switched to the field operators. This structure is manifested in the asymptotic behavior of transverse gauge fields at longitudinal infinity and is explicitly associated with the boundary integrals. The trivial vacuum with nontrivial field variables in the present formulation of LFQCD may provide a practically useful framework for describing hadronic states. We now turn to this discussion.

3. **Non-local potentials in the transverse direction.** One of the nonperturbative approaches to solve bound states in LFFT is the Tamm-Dancoff approach, which truncates the Fock space to be a few-body state space \[35\]. Such an approach becomes practically applicable only when the vacuum is trivial. We have given a realization of a trivial vacuum in this paper. However, to address hadronic bound states, the existence of nontrivial potentials, namely, confinement potentials, is crucial. An explicit construction of such potentials from QCD is still lacking. The LFQCD Hamiltonian contains linear potentials only in the longitudinal direction (see eq.(20)). Quark and color confinement certainly requires similar potentials in the transverse direction as well. We suggest that these nontrivial potentials in LFQCD might hide in the condition of eq.(27).

From eq.(27) we see that the asymptotic $A^i_a$ fields at longitudinal infinity are proportional to the color charge density in transverse space and also that they involve non-local
behavior in the transverse direction (induced by the transverse derivative). Intuitively, we may separate the transverse gauge potentials into a normal part plus a boundary part,

$$A^i_a = A^i_{aN} + A^i_{aB}$$

where

$$A^i_{aN}|_{x^- = \pm \infty} = 0, \quad \partial^i A^i_{aN}|_{x^- = \pm \infty} = 0,$$  \hspace{1cm} (36)

$$\partial^i A^i_{aB}|_{x^- = \pm \infty} = \mp \frac{g}{4}(\rho^i_a(x_\perp) + \rho^i_a(x_\perp)).$$ \hspace{1cm} (37)

In eq.(38), $\rho_a(x_\perp)$ denote the color charge densities integrated over $x^-$, where the definition of the color charge densities is given in footnote 2. The conditions of eqs.(36) and (37) do not uniquely determine the separation of eq.(35). Generally, there are two types of separation for eq.(35). One is to consider $A^i_{aB}$ the long-distance fields generated by the boundary integrals and $A^i_{aN}$ the short-distance fields determined by free theory. Thus, $A^i_{aB}$ correspond to the gauge field configuration for the non-vanishing winding number discussed in the previous subsection. In this case, if we are only interested in the low-energy dynamics, the effect of the $A^i_{aN}$ fields may be ignored. Thus, it is very attractive but is also very difficult to analytically find the $A^i_{aB}$. Another possibility is to choose a simple solution for the $A^i_{aB}$ that satisfy eq.(37). In this case, the $A^i_{aN}$ have the trivial boundary condition eq.(36) but are not determined by free theory. The Hamiltonian is then expressed only in terms of the $A^i_{aN}$, and the boundary behavior of transverse gauge fields are replaced by the effective interactions. A convenient choice for $A^i_{aB}$ which satisfy eq.(37) is

$$A^i_{aB}(x) = -\frac{g}{16} \int dx^- dx'^- \varepsilon(x^- - x'^-)(\rho^i_a(x_\perp) + \rho^i_a(x'_\perp)), \quad i = 1, 2.$$ \hspace{1cm} (38)

Substituting the separation of eq.(35) with (38) into the LFQCD Hamiltonian, we obtain a new Hamiltonian in terms of $A^i_{aN}$ that contains many effective interactions induced by eq.(27). All these effective interactions involve the color charge densities and involve non-local behavior in both the longitudinal and transverse directions. One of the lowest order interactions, for example, is given by

$$H_{b1} \propto \sum_{ij} \int_{-\infty}^{\infty} dx^i dx^- dx'^i dx'^- d\xi^i \eta^{ij} \{\partial^i \rho^j_a(x^-, x^i, x'^j)|x^- - x'^-| \xi^j \rho^i_a(x'^- - x^i, x'^j)|x^j - x'^j|\}$$ \hspace{1cm} (39)

where $\eta^{ij} \equiv 1 (0)$ for $i \neq (=) j$. Hence, eq.(37) leads to numerous many-body non-local color charge interactions which are functions of boundary integrals, and which may lead to confinement.

As we have mentioned in the introduction, Wilson recently proposed a formalism to construct a confining light-front quark-gluon Hamiltonian for LFQCD \[8\]. Wilson suggested that a starting point for analyzing the full QCD with confinement in light-front form is
the linear infrared divergence (i.e., $1/k^2$ singularity in momentum space). Our procedure for constructing effective interactions from the boundary integrals is indeed associated with the linear infrared divergences. However, Wilson’s approach is totally different from what we have discussed here. His analysis based on light-front power counting shows that possible confining potentials may be obtained from the counterterms of the linear infrared divergence. In QED, the counterterms for infrared divergences are forbidden because the divergences arise from integration over the square of the electron scattering amplitude rather than integrals over the amplitude itself and it is only the latter that can be regulated by counterterms. However, in QCD, since quarks and gluons serve only as constituents, there are no scattering cross section for them, and there exist counterterms for the linear infrared divergence. These counterterms involve the color charge densities integrated over $x^-$ and involve non-local behavior in the transverse direction. However, the non-local structure of the counterterms is unknown since the power counting itself cannot determine it. They also violate longitudinal boost invariance so the coefficients of all the counterterms may further be determined by the requirement of boost invariance. Wilson pointed out that it is tempting to identify these terms as the source of transverse confinement.

In the present formulation, as we have seen from eq.(27) or (38), the asymptotic gluon fields which induce effective interactions are proportional to color charge densities integrated over $x^-$ (the boundary integrals) and also involve non-local behavior in the transverse direction. Furthermore, the non-local behavior of effective interactions in the transverse direction is also determined by eq.(27) or (38). Thus, we can explicitly construct many effective interactions from (38). It can be shown that the effective interactions related to fermion part are similar in both QED and QCD but are very different for the gauge part. In QED, there is no any effective interaction generated by the boundary integrals that involves gauge fields. However, in QCD, there are numerous number of effective interactions which are coupled to the non-abelian gauge fields. These effective interactions may be responsible for quark confinement since they originate from the nontrivial behavior of the non-abelian transverse gauge fields. Yet, it is interesting to see that the analyses based on different approaches have the same consequence that non-local potentials in the transverse direction in LFQCD are related to linear infrared divergences which have not been paid attention in the previous investigations.

Still the Hamiltonian contains, in principle, an infinite number of many-body interactions generated by the boundary integrals (or obtained from the counterterms of the linear infrared divergences). This is a consequence of the boundary integrals in a non-abelian gauge theory due to the existence of nonlinear gluon interactions. It is also true in other gauge choices, such as Coulomb gauge [36] or axial gauge [12]. Practically, as the first step, we may only keep two-body interaction terms, such as eq.(41), in the new Hamiltonian. Because of the trivial vacuum in the present formulation of LFQCD, using such an approximate LFQCD Hamiltonian, we can apply the light-front Tamm-Dancoff approach to find hadronic bound states, where the bound states contain only a few particles, such as one quark-antiquark pair, one quark-antiquark pair with one and two gluons. This is certainly
one of the most attractive approaches for low-energy QCD. A numerical investigation along this consideration is in progress.

4 Discussions

In the previous section, we have discussed some primary properties of boundary integrals which we think to be important in understanding LFQCD. In the current investigations of LFFT, one of the most active topics is the problem of the \( k^+ = 0 \) modes and the \( A^+ = 0 \) gauge. The implications of the boundary conditions in determining the nontrivial behavior of gauge theory has, however, been overlooked. In this section, we have some remarks to make about the relation of the \( k^+ = 0 \) modes, the \( A^+_a = 0 \) gauge and boundary conditions at longitudinal infinity.

In previous LFFT investigations, much attention has been paid to how to construct a nontrivial vacuum from the \( k^+ = 0 \) modes. All attempts have focused on 1 + 1 field models \cite{7}. The motivation for these attempts, as we have mentioned in the introduction, is to try to understand spontaneous chiral symmetry breaking in LFFT. In instant form, the vacuum is, of course, crucial for hadronic structure since we believe that axial charges \( Q^a_5 \) create pseudoscalar particles (the lowest bound states in strong interaction region) from the vacuum. However, the role of light-front axial charges in hadronic structure is totally different. The success of light-front current algebra in describing low-energy hadronic structure is based on the properties of light-front \( Q^a_5 \) with a trivial vacuum \cite{37}. In this case, \( Q^a_5 \) annihilate the vacuum so that the vacuum in LFFT itself is not essential in understanding chiral symmetry. The importance of light-front \( Q^a_5 \) lies in their matrix elements between hadronic states. These matrix elements are proportional to hadronic decay constants involving pseudoscalar mesons and therefore carry the basic information of hadronic structure. In instant form, the matrix elements of \( Q^a_5 \) in hadronic states with zero momentum transfer are zero if one does not make use of the infinite momentum limit \cite{38}. In other words, the instant \( Q^a_5 \) itself is practically not useful for hadronic structure except for the Nambu-Goldstone picture of spontaneous symmetry breaking, where the important ingredient is the axial current. These totally opposite properties of axial charges in light-front and instant forms implies that to address dynamical breaking of chiral symmetry in LFQCD, one may need to understand the relation between light-front axial charge operators and the Hamiltonian operator rather than the structure of vacuum in LFFT. The present work is motivated by this consideration.

Second, the canonical quantization of light-front gauge theory is often considered in a box with a periodic boundary condition for gauge fields. In this case, it was argued that one cannot use \( A^+_a = 0 \) for the \( k^+ = 0 \) sector because it is incompatible with the periodic boundary condition in finite volume \cite{39}. This argument is true but incomplete. The \( A^+_a = 0 \) condition does not totally fix the gauge freedom, and the residual gauge fixing is responsible for the non-trivial gauge field configurations which, however, are lacking when one imposes a periodic boundary condition [see eq.(33)]. Thus, if one prefers to use a
periodic boundary conditions to quantize LFQCD in a box, one has to choose other gauges. However, for any gauge fixing other than the $A^+_a = 0$, eqs.(9) and (10) show that the constraint conditions are extremely difficult to solve except for numerical calculations, such as lattice gauge calculations. Our treatment in this paper shows that we can address the non-trivial QCD structure by use of $A^+_a = 0$ gauge and the trivial vacuum if we take into account the residual gauge fixing in antisymmetric boundary conditions for field variables at longitudinal infinity. In principle for $A^+_a = 0$ gauge, other boundary conditions can also be used yet the resulting theory is currently intractable due to the existence of the $k^+ = 0$ modes.

Finally, we discuss briefly the difference between LFQCD and the canonical formulation of QCD in instant form with axial gauge $A^3_a = 0$. The main difference is as follows. In LFQCD, the finite energy density for physical states results explicitly in the asymptotic equation for transverse gauge fields in longitudinal infinity [see eq.(27)] due to the fact that the conjugate momenta of $A^i_a$ are dependent variables in light-front form, $(E^i_a = \frac{1}{2} \partial^+ A^i_a)$ is not a light-front time derivative). In axial gauge, $A^i_a, i = 1, 2$ and their conjugate momenta are all the dynamically independent variables. Thus, a similar condition proposed by Chodos leads to a very complicated formalism which may not be practically useful even for perturbation theory, as noted by himself. The second major difference is the vacuum. In axial gauge, the QCD vacuum is still complicated regardless of the boundary condition chosen. In such a case, it is very difficult (if not impossible) to do non-perturbative calculations before knowing the vacuum structure. In LFQCD, with antisymmetric boundary conditions, the vacuum is trivial and the nontrivial behavior of QCD would be manifested directly in Hamiltonian operators induced by the boundary integrals. Thus it is straightforward to use quantum mechanical non-perturbative approaches to compute bound states. Moreover, in axial gauge, the boost invariance is not manifested kinematically so that it is not a good framework to study low-energy QCD, which deals with composite particles of quarks and gluons. In LFQCD, as we have mentioned in the introduction, boost invariance is a kinematical symmetry which is very useful in addressing hadronic structures.

In summary, the essential point in determining nontrivial behavior of LFQCD seems to be the boundary conditions. A suitable choice of boundary condition for physical fields in LFQCD is crucial because it determines whether the nontrivial behavior of QCD can be decoupled from the vacuum so that the property of the trivial vacuum in LFFT becomes useful for solving hadrons from QCD. We have derived the canonical formulation of LFQCD with great care for boundary integrals, which have not been paid enough attention in previous investigations. We show that the boundary integrals are the source of the light-front linear infrared singularity and involve color charge densities and non-local behavior in the transverse direction that lead to non-local forces generated by the boundary integrals which are also responsible for the non-vanishing topological winding number. Clearly, our understanding of the physics from the boundary integrals in LFQCD is far from complete and much work remains to be done. Particularly, two questions are very interesting for the understanding of hadronic physics. One is which terms among the numerous non-local
interactions are essential for hadronic bound states. The other is how we can find an explicit field configuration for the non-zero winding number in LFQCD that satisfies the asymptotic behavior of eq.(27). These are two of the main problems in nonperturbative LFQCD for the future.

Acknowledgement

We would like to thank R. J. Perry for many critical comments and helpful discussions during this work. We would also like to acknowledge fruitful discussions with R. J. Furnstahl, K. Hornbostel, J. Shigemitsu, T. S. Walhout, and K. G. Wilson. We want to specially thank R. J. Furnstahl for his carefully reading of the manuscript and many useful comments for the paper. This work was supported by National Science Foundation of United States under Grant No. PHY-9102922, PHY-8858250 and PHY-9203145.

Appendix A. Canonical quantization of LFQCD

A self-consistent formulation of LFQCD requires that the resulting Hamiltonian must generate the correct equations of motion for the physical degrees of freedom $(A^i_a, \psi_+, \psi_+^\dagger)$. This appendix is devoted to the derivation of canonical quantization and a check of the consistency.

To see how to correctly reproduce the Lagrangian equations of motion, we need to find consistent commutators for physical field variables.

In the light-front gauge, the Lagrangian of eq.(7) is reduced to

$$\mathcal{L} = \frac{1}{2} \partial^+ A^i_a \partial^- A^i_a + \frac{i}{2} (\psi_+^\dagger \partial^- \psi_+ - \partial^- \psi_+^\dagger \psi_+) - \mathcal{H}. \quad (A.1)$$

The canonical momenta of the physical field variables $A^i_a, \psi_+, \psi_+^\dagger$ are

$$\mathcal{E}^i_a = \frac{1}{2} \partial^+ A^i_a, \quad \pi_{\psi_+} = \frac{i}{2} \psi_+^\dagger, \quad \pi_{\psi_+^\dagger} = -\frac{i}{2} \psi_+. \quad (A.2)$$

However, eq.(A.2) shows that all the canonical momenta are functions of the independent field variables. Thus, after determining all the Lagrange multipliers, the system is still a constrained Hamiltonian system. Usually, in order to quantize such a constrained Hamiltonian system, one has to use the Dirac procedure, by imposing the so-called primary, second-class constraints $E^i_a + \frac{1}{2} \partial^+ A^i_a = 0$ (similarly for $\pi_{\psi_+, \psi_+^\dagger}$) to construct Dirac brackets.

However, for these trivial primary constraints, the mathematically well-defined canonical one-form offers a rigorous phase space structure for canonical quantization [13]. In this appendix, we will use such an approach for light-front quantization.

\footnote{If the first-class constraints cannot be solved explicitly, for example for other gauge choices in light-front form, using the Dirac procedure may be necessary to construct the canonical commutation relations for all variables.}
The phase space structure for the physical variables \((E_i^a, A_i^a; \pi \psi^+, \psi^+; \pi \psi^+, \psi^+)\) is determined rigorously by rewriting eq.(A.1) as a Lagrangian one-form \(\mathcal{L}dx^+\) (apart from a total light-front time derivative),

\[
\mathcal{L}dx^+ = \frac{1}{2}dE_i^a + \pi \psi^+ d\psi^+ + d\psi^+_+ \pi \psi^+ + A_i^a dE_i^a - d\pi \psi^+ \psi^+ - \psi^+_+ d\pi \psi^+ - Hdx^+ + \frac{1}{2} \Gamma_{\alpha \beta} dq^\alpha dq^\beta - Hdx^+ \tag{A.3}
\]

where the first term in the right-hand side is called the canonical one-form of the physical phase space, and quark fields are anticommuting \(c\)-numbers (Grassmann variables). Correspondingly, the symplectic structure or the Poisson brackets of the phase space is given by

\[
\omega = \frac{1}{2} \Gamma_{\alpha \beta} dq^\alpha dq^\beta \quad \text{or} \quad [q^\beta, q^\alpha] = \Gamma_{\alpha \beta}^{-1}. \tag{A.4}
\]

Canonical quantization is realized by replacing the Poisson brackets by the equal-\(x^+\) commutation relations

\[
[q^\beta, q^\alpha] = i \Gamma_{\alpha \beta}^{-1}. \tag{A.5}
\]

Explicitly

\[
[A_i^a(x), \psi^+_+(y)]_{x^+ = y^+} = \frac{1}{2} \delta_{ab} \delta^{ij} \delta^3(x - y), \tag{A.6}
\]

\[
\{\psi^+(x), \pi \psi^+(y)\}_{x^+ = y^+} = \frac{1}{2} \Lambda_+ \delta^3(x - y), \tag{A.7}
\]

\[
\{\psi^+_+(x), \psi^+_+(y)\}_{x^+ = y^+} = -\frac{1}{2} \Lambda_+ \delta^3(x - y) \tag{A.8}
\]

or

\[
[A^+_a(x), \delta^+ A^+_b(y)]_{x^+ = y^+} = i \delta_{ab} \delta^{ij} \delta^3(x - y), \tag{A.9}
\]

\[
[A^+_a(x), A^+_b(y)]_{x^+ = y^+} = -i \delta_{ab} \delta^{ij} \frac{1}{4} \varepsilon(x^{-} - y^{-}) \delta^2(x_\perp - y_\perp), \tag{A.10}
\]

\[
\{\psi^+(x), \psi^+_1(y)\}_{x^+ = y^+} = \Lambda_+ \delta^3(x - y) \tag{A.11}
\]

where \(\delta^3(x - y) \equiv \delta(x^+ - y^+) \delta^2(x_\perp - y_\perp)\). All other commutators between the physical degrees of freedom vanish. Note that, unlike in the instant form, the commutator \([A^+_a(x), A^+_b(y)]\) does not vanish since the canonical momentum is a function of the coordinates. Eq.(A.10) is defined consistently with the antisymmetric boundary condition, \(A^+_a(-\infty) = -A^+_a(\infty)\), where it is also required that

\[
\lim_{x^- \to -\infty, y^- \to -\infty} \varepsilon(x^- - y^-) = 0 \tag{A.12}
\]

Topologically, this requirement is identical to \(\varepsilon(0) = 0\). This can easily be understood if we divide the longitudinal line into boxes and define the \(\varepsilon\)-function in each box.
Using the above basic commutation relations, it is straightforward to verify that the equations of motion are consistent with eqs. (2) and (3),

\[ \partial^- \psi^+ = \frac{1}{i} [\psi^+, H] \]

\[ = \left\{ igA^+ - \frac{1}{4} \left\{ \alpha_\perp \cdot (i\partial_\perp + gA_\perp) + \beta m \right\} \times \int_{-\infty}^{\infty} dx^- \varepsilon(x^- - x^-') \{ \alpha_\perp \cdot (i\partial_\perp + gA_\perp) + \beta m \} \right\} \psi^+ \]

\[ \partial^- A_a^i = \frac{1}{i} [A_a^i, H] \]

\[ = \frac{1}{4} \int_{-\infty}^{\infty} dx^- \varepsilon(x^- - x^-')[D_{ab}^i F_{b}^{ii}(x^+, x^-, x_\perp) - D_{ab}^i E_{b}^-(x^+, x^-, x_\perp) - g_{ab}^i(x^+, x^-, x_\perp) - g_{fabc}(A_b^- \partial^+ A_c^i)(x^+, x^-, x_\perp)] \]

(A.13)

where \( D_{ab}^i = \delta_{ab} \partial^i - g_{fabc} A_c^i \), and \( A_a^- \) and \( E_a^- \) are given by eqs. (19) and (15).

Appendix B. Cancellation of linear infrared divergence

In this appendix, we shall use the \( x^+ \)-ordering perturbative rule which we developed recently [25] for the two-component LFQCD to check the cancellation of linear infrared divergence in perturbative LFQCD. For the reader’s convenience, we list some relevant diagrammatic rules in Table 1 for the following calculation. For a complete list of the \( x^+ \)-ordering perturbative rules and Feynman rules for two-component LFQCD, see Ref. [25].

B-1. Tree level (\( q\bar{q} \) scattering)

The lowest-order \( q\bar{q} \) scattering amplitude is given by

\[ M_{fi} = M_{f_i}^q + M_{f_j}^\bar{q} + M_{f_i}^\bar{q}, \]

(B.1)

which corresponds to the diagrams shown in Fig. 1. Using the rules listed above and the \( x^+ \)-ordering perturbative theory, we immediately obtain that

\[ M_{f_i}^q + M_{f_j}^\bar{q} = g^2 T_{21}^\| T_{43}^\| \Gamma_0^q(p_2, p_1) \chi_1 \chi_4^\dagger T_0^q(p_4, p_3) \chi_3 \]

\[ = \left\{ \frac{1}{p_1^- - p_1^- - p_1^- + k^-} + \frac{1}{p_1^- - p_2^- - p_3^- - k^-} \right\}, \]

(B.2)

\[ M_{f_i}^\bar{q} = \left\{ \begin{array}{ll}
2 g^2 T_{21}^\| T_{43}^\| \chi_1^\dagger \chi_4 \chi_3, & \text{NB,} \\
2 g^2 T_{21}^\| T_{43}^\| \chi_1^\dagger \chi_4 \chi_3, & \text{WB}
\end{array} \right\}, \]

(B.3)

where

\[ \Gamma_0^q(p_2, p_1) = \left[ 2 \frac{k^i}{k^+} - \frac{\sigma \cdot p_2\perp - im}{p_2^+}, \sigma^i - \sigma^i \frac{p_1\perp + im}{p_1^+} \right], \]

(B.4)
$p_i^-$ is the total energy of the initial state, $k^\mu = (p_1^+ - p_2^+, p_1^i - p_2^i, \frac{(p_1^+ - p_2^+)^2 + m^2}{p_1^+ - p_2^+})$. NB denotes no boundary term and WB means including the boundary term. It follows that in the principal value prescription, $M_{f_i}^q + M_{f_i}^b$ is free of infrared divergences, while $M_{f_i}^c$ without the boundary term has a $1/\epsilon^2$ divergence when $k^+ \to 0$. When the boundary term is included, the $1/\epsilon^2$ is cancelled [see (B.3)]. Therefore, it is necessary to include the boundary term in order to obtain a finite amplitude for the lowest-order $q\bar{q}$ scattering. A similar discussion for $e^+e^-$ scattering in LFQED is given in ref.[23].

### B-2. Loop corrections (one-loop quark self-energy)

Based on the $x^+$-ordering perturbative theory, the quark on-shell self-energy (mass correction) up to one-loop is determined by

$$\Sigma(p^2 = m^2) = \Sigma_a + \Sigma_b + \Sigma_c. \quad (B.5)$$

The three terms in the right-hand side are denoted by the three diagrams shown in Fig.2. Again, using the rules listed above, we find that

$$\Sigma_a = g^2C_f \int \frac{dk^+ d^2k_\perp}{16\pi^3} \left\{ \frac{\theta(k^+)\theta(p^+ - k^+)}{k^+} \left\{ 2 \frac{k^i}{k^+} \frac{-\sigma \cdot p_{\perp} - im}{p^+} \sigma_i \right\} + \sigma \cdot p_{\perp} - \sigma \cdot k_\perp + im \right\} 2 \frac{k^i}{k^+} \frac{\sigma \cdot p_{\perp} - \sigma \cdot k_\perp - im}{p^+ - k^+} \sigma_i$$

$$\Sigma_b = g^2C_f \int \frac{dk^+ d^2k_\perp}{16\pi^2} \left\{ \frac{1}{k^+ + p^+ - k^+} \right\}$$

$$\Sigma_c = \left\{ \begin{array}{ll} 2g^2C_f \int \frac{dk^+ d^2k_\perp}{16\pi^2} \left\{ \frac{1}{(p^+ + k^+)^2} - \frac{1}{(p^+ + k^+)^2} \right\} & \text{NB} \\ 2g^2C_f \int \frac{dk^+ d^2k_\perp}{16\pi^2} \left\{ \frac{1}{(p^+ - k^+)^2} - \frac{1}{(p^+ - k^+)^2} \right\} & \text{WB} \end{array} \right. \quad (B.7)$$

where $C_f = 4/3$. A direct calculation shows that

$$\Sigma_a = \frac{g^2}{8\pi^2} C_f \int d^2k_\perp \left( \int_0^1 dx \left( \frac{2m^2}{k_\perp^2 + x^2m^2} + 1 - \frac{\pi p^+}{2\epsilon} + \ln \frac{\epsilon}{p^+} \right) \right)$$

$$\Sigma_b = \frac{g^2}{8\pi^2} C_f \left( \ln \frac{\epsilon}{p^+} \right)$$

$$\Sigma_c = \left\{ \begin{array}{ll} \frac{g^2}{8\pi^2} C_f \int d^2k_\perp (-2) & \text{NB} \\ \frac{g^2}{8\pi^2} C_f \int d^2k_\perp (-1 + \frac{\pi p^+}{2\epsilon}) & \text{WB} \end{array} \right. \quad (B.11)$$

which tells us that in the one-loop correction to the quark self-energy, the one-gluon exchange contains both linear and logarithmic infrared divergences. The instantaneous fermion interaction contains only one logarithmic divergence (see $\Sigma_b$ in eq.(B.10)), which
cancels the logarithmic divergence in $\Sigma_a$. The naive instantaneous-gluon interaction (namely the linear potential in the longitudinal direction) is free of infrared divergence. Therefore, without boundary term, the quark mass correction involves a linear infrared divergence, which is an inconsistent solution. By combining the boundary term with the linear potential, we see that $\Sigma_c$ has a linear infrared divergence which precisely cancels the same divergence in $\Sigma_a$. Thus the quark mass correction is now free of infrared divergences,

$$\delta m^2 = p^+ \Sigma = \frac{m^2}{4\pi^2} C_f \ln \frac{\Lambda^2}{m^2} + \text{finite} \quad (B.12)$$

where $\Lambda$ is the transverse momentum cut-off. In eq.(B.9), the coefficient (1/4) in the mass correction is different from the covariant result (3/8) because the regularization scheme is different. This coefficient is the same as that in the light-front calculation with dimensional regularization in the transverse direction and the explicit cutoff in the longitudinal direction $[1]$. Note that in their calculation, the expressions of eqs.(B.9–11) are different but the sum is the same as eq.(B.12) where the linear divergence is also cancelled.

References

[1] H. Fritzsch, and M. Gell-Mann, *Proc. XVI int. Conf. on High Energy Phys.* eds. J. D. Jackson and A. Roberts, (Fermilab, 1972) Vol.2, p.135; H. Fritzsch, M. Gell-Mann, and H. Leutwyler, *Phys. Lett.* 47B (1973) 365.

[2] P. A. M. Dirac, *Rev. Mod. Phys.* 21 (1949) 392.

[3] For an extensive list of light-front references see: *Sources for light-front physics*, available via anonymous FTP from pacific.mps.ohio-state.edu in the subdirectory pub/infolight.

[4] A. Casher, *Phys. Rev.* D14 (1976) 452. Also see W. A. Bardeen and R. B. Pearson, *Phys. Rev.* D14 (1976) 547; W. A. Bardeen, R. B. Pearson and E. Rabinovici, *Phys. Rev.* D21 (1980) 1037 for LFQCD lattice formulation. For the earliest discussion of light-front Yang-Mills theory see E. Tomboulis, *Phys. Rev.* D8 (1973) 2736.

[5] S. J. Brodsky and G. P. Lepage, in *Perturbative quantum chromodynamics*, edited by A. H. Mueller (World Scientific, Singapore, 1989) p.93 and references therein.

[6] H. C. Pauli, and S. Brodsky, *Phys. Rev.* D32 (1985) 1993; S. Brodsky and H. C. Pauli, in “Recent aspect of quantum field theory”, *Lect. Notes in Phys.* Vol.396, ed. by H. Mitter and H. Gausterer, (Springer Verlag, Berlin, 1991) and references therein; R. J. Perry, A. Harindranath and K. G. Wilson, *Phys. Rev. Lett.* 65 (1990) 2959; R. J. Perry, and A. Harindranath, *Phys. Rev.* D43 (1991) 4051.
[7] For a summary see X. Ji, Quantum field theory in light-cone coordinates, MIT preprint CTP-2152 (1992), (submitted to Comments on Nucl. Part. Phys.);
S. Brodsky, G. McCartor, H. C. Pauli, and S. Pinsky, The challenge of light-cone quantization of gauge field theory, SLAC-PUB-5811 (1992).

[8] K. G. Wilson, “Light-Front QCD”, OSU Internal Report, 1991.

[9] P. M. Wort, Phys. Rev. D47 (1993) 608;
St. Glazek, A. Harindranath, J. Shigemitsu, S. Pinsky and K. G. Wilson, Phys. Rev. D47 (1993) 1599;
St. Glazek, and K. G. Wilson, Phys. Rev. D (in press).

[10] L. D. Faddeev and A. A. Slavnov, Gauge Fields, (Benjamin/Cummings, Reading, Massachusetts, 1980), ch.3.

[11] P. A. M. Dirac, Can. J. Math. 2 (1950) 129.

[12] J. Schwinger, Phys. Rev. 130 (1963) 402;
Y.-P. Yao, J. Math. Phys. 5 (1964) 1319;
A. Chodos, Phys. Rev. D17 (1978) 2624;
I. Bars, and F. Green, Nucl. Phys. B142 (1978) 157.

[13] L. D. Faddeev and R. Jackiw, Phys. Rev. Lett. 60 (1988) 1692.

[14] P. A. M. Dirac, Lectures on Quantum Mechanics, (Yeshiva University, New York, 1964).

[15] J. B. Kogut, and D. E. Soper, Phys. Rev. D1 (1970) 2901.

[16] F. Rohrlich, Acta Phys. Austr., 32 (1970) 87;
R. A. Neville, and F. Rohrlich, Nuovo Cimento, A1 (1971) 625.

[17] R. Jackiw, Springer Tracts in Mod. Phys. 62 (1972) 1.

[18] W. M. Zhang, and A. Harindranath, Residual Gauge Fixing in Light-Front QCD, OSU preprint (submitted to Phys. Lett. B, 1993).

[19] D. M. Capper, J. J. Dulwich and M. J. Litvak, Nucl. Phys. B241 (1984) 463.

[20] S. Mandelstam, Nucl. Phys. B213 (1983) 149;
G. Leibbrandt, Phys. Rev. D29 (1984) 1699; Rev. Mod. Phys. 59 (1987) 1067.

[21] A. Bassetto, G. Nardelli and R. Soldati, Phys. Rev. D31 (1985) 2012; Yang-Mills theories in algebraic non-covariant gauge, (World Scientific, 1991).

[22] D. M. Capper, and D. R. T. Jones, Nucl. phys. B252 (1985) 718;
H. C. Lee, and M. S. Milgram, Nucl. Phys. B268 (1986) 543.

[23] A. C. Tang, Phys. Rev. D37 (1988) 3014.
For an example, see G. Curci, W. Furmanski, and R. Petronzio, *Nucl. Phys.* B175 (1980) 27.

W. M. Zhang, and A. Harindranath, *Two-component theory of LFQCD (I)*, (in preparation);  
A. Harindranath, and W. M. Zhang, *Two-component theory of LFQCD (II)*, (in preparation).

T. M. Yan, *Phys. Rev.* D7 (1973) 1760.

S. J. Brodsky, G. P. Lepage, T. Huang, and P. B. Mackenzie in *Particles and Fields 2*, edited by A. Z. Capri and A. N. Kamal, (Plenum Press, New York, 1983).

C. B. Thorn, *Phys. Rev.* D20 (1979) 1934.

G. ’t Hooft, *Nucl. Phys.* B75 (1975) 461;  
For more detailed discussion see K. Hornbostel, SLAC-333, Ph. D. Thesis (1988).

Such a term has been noted by Bergknoff in 1+1 Schwinger model, see H. Bergknoff,  
*Nucl. Phys.* B122 (1977) 215.

C. G. Callan, N. Coote, and D. Gross, *Phys. Rev.* D13 (1976) 1649;  
M. B. Einhorn, *Phys. Rev.* D14 (1976) 3451.

For example, see W. J. Marciano and H. Pagels, *Phys. Rep.* 36 (1978) 137, Sec.6 and references therein.

S. Coleman, *Aspects of Symmetry* (Cambridge Univ., Cambridge, 1985) ch.6.

Recently, D. Mustaki found in the covariant form of free fermion theory that the axial currents in the light-front form are different than in instant form. Only in the limit of zero quark mass, do they become identical (private communication). We have further found that this result can be derived consistently in two-component light-front Lagrangian formalism, and the result is generally true for interaction theory (QCD). For details see W. M. Zhang and A. Harindranath, (in preparation).

I. Tamm, *J. Phys.* (USSR) 9 (1949) 449;  
S. M. Dancoff, *Phys. Rev.* 78 (1950) 382;  
For the light-front Tamm-Dancoff approach see R. J. Perry, A. Harindranath, and K. G. Wilson, in ref.[3]

N. Christ, and T. D. Lee, *Phys. Rev.* D22 (1980) 939.

F. Feinberg, *Phys. Rev.* D7 (1973) 540; E. Eichten, F. Feinberg and J. F. Willemsem,  
*Phys. Rev.* D8 (1973) 1204.

F. J. Gilman and H. Harari, *Phys. Rev.*, 165 (1968) 1803.
[39] V. A. Franke, Y. V. Novozhilov, and E. V. Prokhvatilov, *Lett. Math. Phys.* **5** (1981) 239.

[40] P. J. Steinhardt, *Ann. Phys.* **128** (1980) 425.

[41] D. Mustaki, S. Pinsky, J. Shigemitsu and K. G. Wilson, *Phys. Rev.*, **D43** (1991) 3411.
Table 1: Some $x^+$-ordering diagrammatic rules for the two-component LFQCD:

\[
-g(T^a)\chi_\alpha \left\{ 2 \frac{k^i}{[k^+]} - \sigma^i \frac{p_{+1} - im}{[p_1^+]} - \sigma_i \frac{p_{+1} + im}{[p_1^+]} \right\} \chi_\epsilon \epsilon^i
\]

\[
g^2(T^a T^b)\chi_\beta \frac{\sigma^i \epsilon^j}{[p_1^+ + k_1^+]} \chi_\alpha \epsilon^i \epsilon^j
\]

\[
\begin{cases}
2g^2(T^a T^a)\chi_\beta \chi_\alpha \frac{1}{[p_1^+ - p_2^+]} \chi_\gamma & \text{NB} \\
2g^2(T^a T^a)\chi_\beta \chi_\alpha \frac{1}{[p_1^+ - p_2^+]^2} \chi_\delta \chi_\gamma & \text{WB}
\end{cases}
\]

Remarks. The $\chi_\alpha$ are two-component light-front quark spinors, $\chi_\uparrow = (1,0)$ and $\chi_\downarrow = (0,1)$, and the $\epsilon^i$ are two-component gluon polarization vectors. We have also used the Majorana representation of the $\gamma$ matrices in the above realization of the two-component formulation. In additional, the internal integral for quark momentum is given by $\int \frac{dp^+ dp^+ dp^- \theta(p^+)}{16\pi^2}$, while for gluon it is $\int \frac{dk^+ dk^+ dk^- \theta(k^+)}{16\pi^2}$. The light-front free quark and gluon energies are $p^- = \frac{p^2 + m^2}{[p^+]}$ and $k^- = \frac{k^2}{[k^+]}$.
Figures:

Fig.1. The $x^+$-ordering graphs for the lowest-order $q\bar{q}$ scattering in perturbative LFQCD.

Fig.2. The $x^+$-ordering graphs for the one-loop correction of quark self-energy in perturbative LFQCD.