Portfolio Effect on End User Spare parts based on Demand Patterns

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Abstract. This paper use 1577 items of end-user spare parts data from two authorized service-center for a global brand motor cycle product that become the main means of public transportation in Indonesia to study the Portfolio Effect on end-user spare parts based on their demand patterns categorized as smooth, erratic, intermittent and lumpy. It found that items in the lumpy category got larger benefits from Portfolio Effect compared to other categories, with an average of 25.13% and standard deviation 9.70%. Despite their high fluctuations, they have relatively low demand (ρ) and Magnitude (M) coefficients so they can have more benefit from consolidation with relatively low risk.

1. Introduction

The availability of spare parts for end-consumer products is important for after-sales services, spare parts management is special, as it involves large quantities of items, fluctuates, and low demand rates with intermittent tendencies, it often written on the literature as sporadic demand or erratic. Intermittent demand patterns are often found in engineering-spare [1] and end-user spare parts [2].

This conditions bring difficulties for authorized service-centers to provide overall items to anticipate the needs arising from the repair process, as it is difficult to know the customer usage of end product, and their maintenance concept are not known [3]. It makes sense that the service-centres in an area to take benefit from spare parts pooling to realize the economies of scale [4].

Theoretically, inventory-pooling can bring the benefits of a safety stock reduction called the Portfolio Effect [5], since statistically aggregating demand to a centralized location will reduce the demand variability, where high demand at one location will be offset by demand which is low in other locations, and can also fill up on intermittent demand, so that reduced variability also reduces safety- stock [6][7]. Safety-stock stored in a single centralized site is equal to the aggregate safety-stock stored in n locations divided by the roots of the number of locations (\(\sqrt{n}\)) known as square root law (SRL) of location[8], and is evidenced by mathematical analysis by Maister [9]. When the correlation coefficient (\(\rho\)) of past demand from two stocking location is zero, then the benefits from inventory centralization are the same as SRL[6]. Ballou studied empirical data from 16 various industry firms and concluded that SRL is the maximum limit that can be achieved through consolidation [10]. The benefits of inventory consolidation will increase in line with the increase in the number of consolidated location [11].
Zinn et al used the term Portfolio Effect as a percentage reduce of safety-stock made possible by consolidated [5]. They studied consolidation with different demand variability between locations expressed as magnitude. Magnitude (M) is defined as relative values of standard deviation between location, and concluded that the Portfolio Effect maximum when \( \rho = -1 \) and \( M = 1 \), and demonstrated that SRL is the special case of Portfolio Effect when \( \rho = 0 \) and \( M = 1 \). Consolidation is more valuable when demand is less positively correlated [12], and inventory consolidation provides no benefit at all when demand across locations is fully correlated [7]. It is preferred for consolidated inventories between locations that have the same demand variance so the fluctuate can balance each other, and some sub-consolidations can be as attractive as a single large consolidation which is depend on \( \rho \) and \( M \), because when \( \rho = 1 \) or \( M \) is large, then no further benefit can take from consolidation[13][14][15].

When consolidation cannot be done physically, the use of lateral-transhipment can also provide the same benefits when the transhipment costs are zero [16], and Portfolio Effect model can be used to evaluate consolidation by lateral-transhipment approach [17]. Pooling between locations always improves service-level in both locations [18]. Consolidated benefits are greater in safety-stock compared to cycle-stock [19], the determination of replenishment by EOQ can take benefit from consolidation, but there is no benefit from replacement approach [20]. That pooling always improves Type-I service level [21].

This study will be conduct by using empirical data from two authorized service-centres for motorcycle products served by one main dealer who has the same level of sales and is located in an area to study the Portfolio Effect on items in category smooth, erratic, intermittent lumpy as Syntetos et al[22].

The purpose of this study is to see in general how items in each category meet the demand correlation (\( \rho \)) and Magnitude (\( M \)) which is the factors determining the level of the Portfolio Effect that can be obtained from consolidation, so that it can be a reference for managers to determine which items should be focus on safety-stock consolidation decision.

2. Methods

2.1. Inventory Pooling

If each location handles their customer demand using their own inventory, then the optimal safety - stock for each location is:

\[
ss_i = z\sigma_i
\]

Which \( z \) is the safety factor from the cumulative table Normal distribution in accordance with the targeted service level, so the safety-stock for the overall location is:

\[
ss = z\sum_{i=1}^{n} \sigma_i
\]

If all inventories are placed in one location to meet the aggregated demand across location are:

\[
x = \sum_{i=1}^{n} x_i
\]

Then the safety-stock on the centralized system is:

\[
ss_c = z\sigma_a
\]

Where

\[
\sigma_a = \left( \sum_{i=1}^{n} (\sigma_i)^2 + 2 \sum_{i=1}^{n} \sum_{i<j}^{n} \sigma_i \rho_{ij} \right)^{1/2}
\]

Where \( \sigma_i \) is the standard deviation of \( x \) and \( \rho_{ij} \) (Pearson product-moment correlation coefficient) is the demand correlation coefficient between location \( i \) and \( j \) of random variable for location \( i \) and \( j \) which can be calculated by the formula:
\[ \rho_{xy} = \frac{\text{Cov}(x, y)}{o_x o_y} \]  

(6)

And COV \((x, y)\) is the covariance to measure how the direction of change of two variables is formulated as follows:

\[ \text{COV}(x, y) = \sum_{n=1}^{\infty} (x_i - \bar{x})(y_i - \bar{y}) \]

(7)

### 2.2. Portfolio Effect

Zinn et al used the term Portfolio Effect (PE) as a percentage of aggregate safety-stock reductions made possible by inventory centralization measures. PE is a function that has two variables, namely sales correlation \((\rho)\) between location as the formulation of Eppen and Magnitude \((M)\) is relative values of standard deviation between location. In formula, the Portfolio Effect for two locations is defined as follows:

\[ \text{PE} = 1 - \frac{ss_a}{ss_1 + ss_2} \]

(8)

Where \(ss_a\) is consolidated safety stock, and \(ss_1\) and \(ss_2\) are safety-stock for location 1 and location 2, and substitution of equation (5) to equation (8) yields:

\[ \text{PE} = 1 - \frac{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho_{12}}{o_1^2 + o_2^2} \]

(9)

In formula, Magnitude \((M)\) is defined as follows:

\[ M = \frac{\sigma_1}{\sigma_2} \quad \text{for} \quad o_1 = o_2 \quad \text{and} \quad o_2 > 0 \]

(10)

And the substitution of equation (10) to equation (9) yields:

\[ \text{PE} = 1 - \frac{(M^2\sigma_2^2 + 2M\sigma_2\sigma_1\rho_{12})^{1/2}}{o_1 + o_2} \]

(11)

### 2.3. Demand Pattern

Syntetos et al categorizes the demand pattern for forecasting purpose into four smooth, erratic, intermittent and lumpy queries that are based on the cut-off value of \(p\) and \(v\), where \(p\) is the average inter-demand interval (ADI) and \(v\) is the square of coefficient variation (CV2). The cut-off value for \(p\) is 1.32 and \(v\) is 0.49.

#### Figure 1

Category of demand patterns by Syntetos et al (2005)

Items with smooth demand patterns have \(\text{ADI} \leq 1.32\) and \(\text{CV2} \leq 0.49\), which means that demand arises with regular intervals and not too much variation on demand. Items with erratic demand patterns have \(\text{ADI} \leq 1.32\) and \(\text{CV2} > 0.49\), which means that demand arises with regular intervals and there are
variations in demand. Items with intermittent demand patterns have ADI> 1.32 and CV2 ≤ 0.49, which means that demand, arise at irregular intervals with no demand in some periods. Items with lumpy demand patterns have ADI> 1.32 and CV2> 0.49, which means that the fluctuating demand with no demand in some periods.

2.3.1 Average Inter-Demand Interval
The average inter-demand interval (ADI) is defined as the average interval between two non-zero requests that are usually expressed in the bucket of the reference time on the business to make a purchase. ADI is used to measure the regularity of demand by calculating the average time intervals between the two demands illustrated in Figure 2.

Where \( E_i \) is non-zero demand in a given period, and it is the time interval between two non-zero requests and ADI is formulated as follows:

\[
ADI = \frac{\sum_{i=1}^{n} E_i}{n} \tag{12}
\]

Where \( n \) is the number of with non-zero demand.

2.3.2 Coefficient of Variance\(^2\)
Coefficient of variation (CV) is defined as the ratio of the standard-deviation to mean, CV is commonly used to measure the variability on demand, and CV2 is the squared Coefficient of variation with the following formula:

\[
CV^2 = \left( \frac{\sigma}{\mu} \right)^2 \tag{13}
\]

3. Result and Discussion
This study retrieves 24 months of sales data extracted from the POS system of two official service centres and has equivalent sales rates to study Portfolio Effect if inventory consolidation is made for both of them. We found that not all items of spare parts sold on service-centres 1 (sc1) were also sold on service-centres 2 (sc2), from 2650 items sold on sc1, and 2642 items sold on sc2, only 1557 items sold on both of them, and this 1557 items will be used as data in this research to get descriptive about coefficient correlation of demand(\( \rho \)), Magnitude (M), and Portfolio Effect on smooth, erratic, intermittent and lumpy demand categories.

3.1. Descriptive on Demand Category
Categorization of demand patterns is use the rules created by Syntetos et al (2005) based on ADI and CV2 of each item as presented in Table 1.

| Demand Category | CV2 | ADI   | SC1     | SC2     | Consolidate |
|-----------------|-----|-------|---------|---------|-------------|
|                 |     |       | Items   | %       | Items       | %       |
| Smooth <= 4.9   | <= 1.32 | 59    | 3.79%   | 48      | 3.08%       | 115     | 7.39% |
| Erratic > 4.9   | <= 1.32 | 74    | 4.75%   | 50      | 3.21%       | 122     | 7.84% |
| Intermittent <= 4.9 > 1.32 | 0.06% | 1      | 0.00%   | 1       | 0.06%       | 1319    | 84.71% |
| Lumpy > 4.9 > 1.32 | 91.39% | 1423   | 93.71%  | 1459    | 93.71%      | 1319    | 84.71% |
| Total Item      |      | 1557   | 1557    | 1557    |             |         |        |

Table 1. Number of items in each demand category.
It was shown that above 90% of items in both of location have lumpy demand patterns and only 1% of items have intermittent demand pattern, and if inventory consolidation is done, there is a shift in the demand pattern from lumpy demand to erratic and smooth as evidenced by the increase in smooth to 7.39%, and erratic percentages to 7.84%, and a decrease in the percentage of lumpy to 84.71%.

3.2. Descriptive on Correlation of Demand and Magnitude

As the formula of The Portfolio Effect, benefits are influenced by the correlation of demand ($\rho$) and Magnitude (M). It is preferred to consolidate two sites that have low demand correlation and Magnitude. Based on the processing of 1557 items to be consolidated using formulas (5) and (10), it can be shown the distribution of demand correlation, magnitude, and their combination in Table 2, Table 3 and Table 4.

| Range of $\rho$ | Smooth (items) | Erratic (Items) | Intermittent (Items) | Lumpy (Items) | Total item | Percentage |
|-----------------|----------------|-----------------|----------------------|--------------|------------|------------|
| -1 to -0.9      | 0              | 0               | 0                    | 0            | 0          | 0.00%      |
| -0.9 to -0.8    | 0              | 0               | 0                    | 0            | 0          | 0.00%      |
| -0.8 to -0.7    | 0              | 0               | 0                    | 0            | 0          | 0.00%      |
| -0.7 to -0.6    | 0              | 0               | 0                    | 0            | 0          | 0.00%      |
| -0.6 to -0.5    | 0              | 0               | 1                    | 0            | 1          | 0.06%      |
| -0.5 to -0.4    | 4              | 1               | 0                    | 3            | 8          | 0.51%      |
| -0.4 to -0.3    | 5              | 2               | 0                    | 6            | 13         | 0.83%      |
| -0.3 to -0.2    | 13             | 6               | 0                    | 42           | 61         | 3.92%      |
| -0.2 to -0.1    | 8              | 13              | 0                    | 211          | 232        | 14.90%     |
| -0.1 to 0       | 11             | 34              | 0                    | 687          | 732        | 47.01%     |
| 0 to 0.1        | 18             | 10              | 0                    | 63           | 91         | 5.84%      |
| 0.1 to 0.2      | 11             | 19              | 0                    | 49           | 79         | 5.07%      |
| 0.2 to 0.3      | 20             | 7               | 0                    | 54           | 81         | 5.20%      |
| 0.3 to 0.4      | 9              | 10              | 0                    | 39           | 58         | 3.73%      |
| 0.4 to 0.5      | 6              | 6               | 0                    | 31           | 43         | 2.76%      |
| 0.5 to 0.6      | 4              | 5               | 0                    | 25           | 34         | 2.18%      |
| 0.6 to 0.7      | 1              | 1               | 0                    | 9            | 11         | 0.71%      |
| 0.7 to 0.8      | 2              | 2               | 0                    | 21           | 25         | 1.61%      |
| 0.8 to 0.9      | 2              | 2               | 0                    | 7            | 11         | 0.71%      |
| 0.9 to 1        | 1              | 4               | 0                    | 72           | 77         | 4.95%      |

Total item: 115 122 1 1319 1557

It was shown that 83.34% of the total number of items that have demand-correlation below 0.3, so that the potential of PE benefits is high if both locations consolidate their safety-stock, because the smaller the $\rho$ the greater the PE benefits, under the condition $\rho = -1$ means the fluctuations in both locations are mutually exclusive so no safety-stocks are to be provided, otherwise $\rho = 1$ means there is a perfect correlation where demand increases at one location is also followed by an increase in demand at the second location, so that consolidation will not provide any benefit at all.

After demand-correlation factor, it is preferred to consolidate location with the same fluctuation rate, so that fluctuations in one location can be balanced by fluctuations in the second location. The comparison of fluctuations between locations is expressed as Magnitude (M) is shown in table 3.
Table 3. Distribution of Magnitude (M)

| Range of M | Smooth (Items) | Erratic (Items) | Intermittent (Items) | Lumpy (Items) | Total item | Percentage |
|------------|----------------|----------------|----------------------|---------------|------------|------------|
| 1 to 1.5   | 56             | 53             | 1                    | 765           | 875        | 56.20%     |
| 1.5 to 2   | 19             | 16             | 0                    | 223           | 258        | 16.57%     |
| 2 to 2.5   | 11             | 11             | 0                    | 155           | 177        | 11.37%     |
| 2.5 to 3   | 16             | 10             | 0                    | 51            | 77         | 4.95%      |
| 3 to 3.5   | 4              | 8              | 0                    | 42            | 54         | 3.47%      |
| 3.5 to 4   | 2              | 5              | 0                    | 27            | 34         | 2.18%      |
| 4 to 5     | 3              | 6              | 0                    | 17            | 26         | 1.67%      |
| 5 to 6     | 1              | 4              | 0                    | 16            | 21         | 1.35%      |
| >= 6       | 3              | 9              | 0                    | 23            | 35         | 2.25%      |
| Total item | 115            | 122            | 1                    | 1319          | 1557       |            |

It was shown that 56.20% items have M below 1.5, but it need to be confirmed by a combination of ρ to M as shown in Table 4, because consolidation of demand will give better benefit if ρ is small and the value of M is approaching one

Table 4. Descriptive distribution of combinations ρ to M

| Range of Magnitude (M) | <1.5 | 1.5 to 2 | 2 to 2.5 | 2.5 to 3 | 3 to 3.5 | 3.5 to 4 | 4 to 5 | 5 to 6 | >= 6 | Total Item |
|------------------------|------|----------|----------|----------|----------|----------|--------|--------|------|------------|
| -0.6 to -0.5           | 1    | 1        |          |          |          |          |        |        |      | 1          |
| -0.5 to -0.4           | 3    | 5        | 1        |          |          |          |        |        |      | 5          |
| -0.4 to -0.3           | 6    | 5        | 2        | 1        |          |          |        |        |      | 16         |
| -0.3 to -0.2           | 21   | 9        | 9        | 2        |          |          |        |        |      | 49         |
| -0.2 to -0.1           | 90   | 38       | 46       | 25       | 14       | 11       | 7      | 2      | 3    | 236        |
| -0.1 to 0.0            | 529  | 104      | 68       | 11       | 7        | 7        | 5      | 5      | 9    | 745        |
| 0.0 to 0.1             | 33   | 17       | 14       | 8        | 5        | 3        | 2      | 5      | 4    | 91         |
| 0.1 to 0.2             | 31   | 19       | 7        | 7        | 6        | 6        | 2      | 1      | 3    | 82         |
| 0.2 to 0.3             | 31   | 16       | 8        | 11       | 4        | 1        | 2      | 4      |      | 77         |
| 0.3 to 0.4             | 26   | 7        | 10       | 3        | 4        |          | 2      | 3      | 3    | 58         |
| 0.4 to 0.5             | 10   | 15       | 2        | 3        | 5        | 6        | 2      |        |      | 43         |
| 0.5 to 0.6             | 7    | 19       | 4        | 2        |          | 1        | 1      | 2      |      | 36         |
| 0.6 to 0.7             | 21   | 2        |          |          |          | 1        |        | 2      |      | 26         |
| 0.7 to 0.8             | 2    | 2        |          |          |          | 1        | 1      |        |      | 7          |
| 0.8 to 0.9             | 6    | 4        |          |          |          |          | 1      |        | 1    | 14         |
| >= 0.9                | 56   | 2        | 9        |          | 2        | 1        |        | 1      |      | 71         |
| Total item            | 873  | 259      | 178      | 77       | 53       | 36       | 25     | 21     | 35   | 1557       |

It was shown on table 4 and Figure 1 that there is a large concentration of ρ between -0.1 to 0.0 and Magnitude below 1.5.
3.3. **Descriptive on Portfolio Effect**

Based on the calculation of PE by using formula (9) from 1557 items, it can be presented Portfolio Effect benefits according to each category of demand patterns in Table 5.

| Range PE | smooth | erratic | intermittent | lumpy | Total |
|----------|--------|---------|--------------|-------|-------|
|          | Items  | %       | Items        | %     | Items | %     | Items  | %     | Items | %     |
| = 0%     | 0      | 0.00%   | 0            | 0.00% | 68    | 5.16% | 68     | 4.37% |
| 0% to 5% | 5      | 4.35%   | 9            | 7.38% | 14    | 1.06% | 28     | 1.80% |
| 5 to 10% | 6      | 5.22%   | 10           | 8.20% | 53    | 4.02% | 69     | 4.43% |
| 10% to 15%| 14    | 12.17%  | 17           | 13.93%| 70    | 5.31% | 101    | 6.49% |
| 15% to 20%| 19    | 16.52%  | 18           | 14.75%| 95    | 7.20% | 132    | 8.48% |
| 20% to 25%| 19    | 16.52%  | 16           | 13.11%| 113   | 8.57% | 148    | 9.51% |
| 25% to 30%| 23    | 20.00%  | 18           | 14.75%| 252   | 19.11%| 293    | 18.82%|
| 30% to 35%| 14    | 12.17%  | 29           | 23.77%| 613   | 46.47%| 656    | 42.13%|
| 35% to 40%| 11    | 9.57%   | 5            | 4.10% | 36    | 2.73% | 52     | 3.34% |
| 40% to 45%| 2     | 1.74%   | 0            | 0.00% | 3     | 0.23% | 5      | 0.32% |
| 45% to 50%| 2     | 1.74%   | 0            | 0.00% | 2     | 0.15% | 4      | 0.26% |
| 50% to 55%| 0     | 0.00%   | 1            | 100.00%| 0    | 0.00% | 1      | 0.06% |
| >= 60%    | 0     | 0.00%   | 0            | 0.00% | 0     | 0.00% | 0      | 0.00% |

| Mean(μ)   | 23.29% | 21.12% | 51.36%       | 25.13% | 25.22% |
| SD(σ)     | 9.91%  | 9.80%  | 0.00%        | 9.70%   | 9.70%  |

*Special conditions see notes

Since the number of items in the intermittent category is too small, the result cannot be used for conclusion, so we conclude that items with lumpy categories get a larger Portfolio Effect (PE) benefits with an average of 25.13% deviation 9.70%, and there are 68.69% items that get PE above 25%.

Note:
Then there is also a special thing that can be seen is the existence of 5.16% items in the lumpy category that get $PE = 0$ because of special cases associated with recall-product in certain periods, resulting in perfect demand correlation between locations for these items.

4. Conclusion

The management of end-user parts is special, as it involves large number of items, fluctuate and intermittent, based on 1557 items spare-part that have sales in two official service centres of a global brand motor cycle product, 84.7% of items have lumpy demand. The potential benefits that can be gained from the consolidation of safety-stock between service centres can be high because in general the spare parts demand have low correlation-coefficient ($\rho$) and Magnitude (M), this is indicated from the data that 83.34% of items has below $\rho < 0.3$, and 56.2% of items have $M < 1.5$. The safety-stock consolidation of 1557 items overall is expected to reduce the average safety-stock by 25.22% with standard deviation 9.70%.

Especially for lumpy items there are 84.3% items that have $\rho < 0.3$, and 57.99% items have $M < 1.5$, can benefits from Portfolio Effect with average 25.7% and deviation 9.70%, so it can be concluded that in general the items in the lumpy category can get a greater Portfolio Effect compared to the other categories.

Practically this research can be a reference for managers who involve in end-user spare parts management to focus to items in lumpy categories which have large fluctuation but has a relatively low value of $\rho$ and $M$ in order to get high benefits from safety-stock consolidation with relatively low risk.

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