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Obscuration elimination in three-dimensional nonsymmetrical optical systems

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Abstract

In order to avoid central obscuration in reflective optical systems, off-axis designs with mirrors tilting and decentering about optical axis are popular. With the help of freeform surfaces, the field of view and resolution of the off-axis mirror systems have been greatly improved. However, it is important to take care of mechanical vignetting during the design process and recalculate the unobscured transmitted field after the design is completed. In this paper, one method to compute obscuration-free fields in a fixed off-axis plane symmetrical system is developed. Then, one algorithm that automatically eliminates obscuration in optical systems which have no symmetry in three-dimensional space is put forward. In the end, two examples are shown to prove the feasibility of this idea.

1. Introduction

The idea of off-axis telescopes like Schiefspiegler, Yolo telescopes [1] was first proposed in 1970s to avoid central obscuration in the usual common-axis telescopes, which degrades the brightness and contrast of the image. An off-axis optical system can be either zigzag arranged or folded to achieve compact size. The tilt angles of the mirrors are specially adjusted to avoid obscuration, and correct third-order astigmatism if possible, which makes the system a good initial system for further optimization, e.g. substituting one spherical surface to freeform surface, to get diffraction limited image for a large field of view [2–4]. During the whole design process, the optical structure should be checked from time to time to ensure that the system is free of obscuration. Therefore, it is necessary to add obscuration as boundary condition when designing off-axis mirror systems with freeform surfaces [5]. As early as in 1987, Rodgers has put forward one method to design unobscured plane symmetrical reflective system by solving the linear inequalities between boundary rays and mirror edges [6]. Normally this work is done by setting the global operands of boundary ray heights on the mirrors in the merit function or writing the extensional codes to solve inequalities for the optical design software e.g. OpticStudio, CodeV, etc. The targets of operands or inequalities have to be updated manually according to the current structure. Then in 2017, 30 years later, Xu et al developed a new algorithm to automatically search for unobscured optical structure with properly defined variables [7] which efficiently reduces the engagement of human being in optical design. The image quality is able to be optimized as well in their algorithm. However, their methods are focusing on mirror movement in plane symmetrical systems with known field of view. On one hand, the question of how large the field can go through one optical system freely without vignetting is also of interest. For example, typical extreme ultraviolet (EUV) lithographic projection systems are consisting of off-axis segment of the mirrors that are centered on common axis [8]. After finishing the design, the size of the unobscured transmitted field needs to be computed. On the other hand, although three-dimensional (3D) nonsymmetrical optical systems have not been broadly studied due to alignment problem, the prospective cannot be neglected [9–11]. In 1987, a general obscuration removing theory for 3D nonsymmetrical systems was published by Tatian [12]. This method was ray-based and had difficulties in defining the aperture points. Therefore, it is desirable to put forward one algorithm, which is easier to handle, to describe the degree of obscuration in 3D nonsymmetrical optical systems and go one more step to automatically find an unobscured initial 3D nonsymmetrical system with given mirror moving constraints.

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In this paper, we first shortly review Xu’s work about obscuration automatic elimination program and continue to extend the ability of his algorithm to search for obscuration-free field area in optical systems. Then we will mainly focus on the obscuration elimination method for optical systems that have no symmetry in 3D space. This method is based on the geometrical structure of the optical systems that solves the problem to find extreme rays in the ray-based method. In the end, two examples by using the described algorithm to obtain two-mirror and three-mirror initial 3D Yolo telescope systems without obscuration will be shown and the combination of obscuration elimination and aberration correction will be discussed.

2. Obscuration elimination in plane symmetrical optical systems

2.1. Review of Xu’s method

For multi-mirror plane symmetrical optical systems, only the optical structure in tangential or sagittal plane is considered depending on the symmetric plane. With ray trace feature in the optical design software (in our case, OpticStudio is used), the global coordinates of upper and lower boundary ray intersecting points on every optical surface can be fast calculated and recorded. The key concept in Xu’s method is to build ray-quadrangles by the intersecting points of lower and upper boundary rays on the neighboring two mirrors as shown in figure 1. The two sides of a ray-quadrangle which lie on the mirrors are called ‘hard’ sides and are denoted by solid lines. The remaining two sides represented by dashed lines are called ‘soft’ sides. One dummy surface is inserted in front of the first mirror to indicate the width of incoming beams and build the first ray-quadrangle with the first mirror. The last ray-quadrangle is constructed by the image plane and the last mirror. Therefore $n+1$ ray-quadrangles are built to describe one system which contains $n$ optical surfaces. In order to cover the mirrors and leave some space for mechanical mounting, each side of ray-quadrangles is extended outwards by certain clearance. The vertices of the extended area are calculated by the cross points between the diagonals of the ray-quadrangle and the extended boundaries. The extended area is therefore cut into four triangles. By studying the relative positions of the non-neighboring ray quadrangles, three types of error functions are established to describe all obscuration cases in off-axis plane symmetrical system.

If a vertex $P$ of a ray-quadrangle $b$ is located in any triangle of a non-neighboring extended area which belongs to ray-quadrangle $a$, as shown in figure 2(a), the squared distance from the vertex to the external side of the triangle is calculated as the first type of error function $Erf_{2D1}$, which can be written as [7]

$$
Erf_{2D1}(a, b) = \sum_{j=1}^{4} \begin{cases} 
|P_jE|^2 + \text{Bias}, & \text{if } P_j \in \Lambda \\n0, & \text{if } P_j \not\in \Lambda.
\end{cases}
$$

Since the ray-quadrangles $a$ and $b$ are non-neighboring, $a$ and $b$ have the relation that $a, b \leq n + 1$, $|a - b| > 1$. $j$ is the sequence of vertices of the ray-quadrangle $b$. $\Lambda$ is the extended area of ray-quadrangle $a$. The line segment $|P_jE|$ is perpendicular to the corresponding external side and intersects the side at the point $E$. A positive bias is added to the non-zero value of error function to clearly distinguish zero and non-zero part.
In figure 2(b), one hard side of a ray-quadrangle \( b \) is crossing one side of a non-neighboring ray-quadrangle \( a \) at the point \( Q \) and cutting the side into two parts \( |AQ| \) and \( |BQ| \). \( A \) and \( B \) are two endpoints of the crossed side of the ray-quadrangle \( a \). To indicate this case, the second type of error function \( \text{Erf}_{2D2} \) is introduced as the squared value of the shortest part [7]

\[
\text{Erf}_{2D2}(a, b) = \sum_{i=1}^{4} \sum_{j=1}^{2} \min(|A_i Q_j|^2, |B_i Q_j|^2) + \text{Bias}, \quad \text{if crossed}
\]

\[
= 0, \quad \text{if not crossed}
\]

where \( a \) and \( b \) have the same relation as in \( \text{Erf}_{2D1} \), \( i = 1 - 4 \) are the indices of the four sides of the ray-quadrangle \( a \), and \( j = 1 - 2 \) are the indices of the two hard sides of the ray-quadrangle \( b \). From the indices \( i \) and \( j \) in \( A_i B_i \) and \( Q_j \), it can be known which two sides belonging to the ray-quadrangles \( a \) and \( b \) respectively are crossed.

Until now, only the non-neighboring ray-quadrangles are discussed. However, the image side which belongs to the \((n + 1)\text{th} \) ray-quadrangle may fall into the \( n\text{th} \) ray-quadrangle. To check this circumstance, the last error function \( \text{Erf}_{2D3} \) has the same form as \( \text{Erf}_{2D2} \). By substituting \( a \) and \( b \) to \( n \) and \( n + 1 \) respectively, the intersection relation between the last two ray-quadrangles is investigated by [7]

\[
\text{Erf}_{2D3} = \text{Erf}_{2D2}(n, n + 1) = \sum_{j=3}^{4} \min(|P_j E|^2, |Bi|^2) + \text{Bias}, \quad \text{if } P_j \in \Lambda
\]

\[
= 0, \quad \text{if } P_j \notin \Lambda
\]

where \( j = 3, 4 \) represents the two vertices on the image plane.

The total error function \( \text{Erf}_{2D} \) to evaluate the degree of obscuration is equal to

\[
\text{Erf}_{2D} = \sum_{a, b} (\text{Erf}_{2D1}(a, b) + \text{Erf}_{2D2}(a, b)) + \text{Erf}_{2D3}
\]

In the automatic obscuration elimination process, the upper and lower boundary rays in the plane, which the system symmetric about, are traced and the global coordinates of the ray intersecting points on each mirror are transported from OpticStudio to MATLAB. Then the ray-quadrangles and the value of error function \( \text{Erf}_{2D} \) of the current structure are computed in MATLAB. The optimization variables can be tilt or decenter of the mirrors. The aim is to optimize the value of error function \( \text{Erf}_{2D} \) until zero by utilizing the proper optimization toolbox in MATLAB. Since \( \text{Erf}_{2D} \) is not monotonic decreasing function, global optimization toolbox, for example simulated annealing (SA) chosen by Xu, other than local optimization toolbox should be selected. In each iteration, the variables are assigned back to OpticStudio to trace rays information for the updated system structure and the value of error function \( \text{Erf}_{2D} \) is recalculated.

### 2.2. Extension to unobscured field searching

With specified field of view, the structure of an optical system can be optimized by Xu’s method to eliminate obscuration. However, sometimes on the contrary, e.g. for an EUV lithographic projection system, the optical structure is given but the size of the unobscured transmitted field is unclear that needs to be searched. The above method can be extended to this application.

For a single field, ray-quadrangles can be built in the same way as introduced in section 2.1 and the error functions from \( \text{Erf}_{2D1} \) to \( \text{Erf}_{2D3} \) are calculated to evaluate the overlapping situation among the ray bundles without extension of ray-quadrangles. Since the size and location of each optical component is fixed, the obscuration between the ray bundles and the optical elements needs to be considered. For this reason, two more types of error functions are introduced.

The first case is that the aperture of the ray bundle shrinks due to the limiting size of the mirror. As explained in figure 3(a), the mirror \( s \) has upper and lower boundary points at \( H_1 \) and \( H_2 \). The shadowed part of the light reflected from mirror \( s - 1 \) exceeds the margin of mirror \( s \) and is lost. If the whole ray bundle is propagated to mirror \( s \) without energy loss, the edge of mirror \( s \) has to be extended to at least \( H_3 \) which is the boundary ray height on mirror \( s \). The distance \( |H_3 H_1| \) indicates how much light is outside of mirror \( s \). The forth error function \( \text{Erf}_{2D4} \) is written as square of \( |H_3 H_1| \) and a positive bias.

![Figure 2. Two cases that two ray-quadrangles overlap. (a) One vertex P of the ray-quadrangle b inside the extended area of the ray-quadrangle a. (b) One hard side (blue) of the ray-quadrangle b crosses any side of the ray-quadrangle a [7].](image-url)
where $s = 2$ to $n + 1$ denotes that $\text{Erf}_{2D}$ is calculated for every surface except the first dummy surface and image plane; $k = 1$ to 2 denotes the upper and lower boundaries of the ray bundle and corresponding edge of mirror.

The other possibility is that the edge of a mirror falls into a ray-quadrangle so that another error function $\text{Erf}_{2D3}$ is introduced to detect it. When discussing $\text{Erf}_{2D4}$, the clear apertures of mirrors that are responsible for light propagation are used. But in this case, the mirror edges have to be extended by certain clearance to include mounting elements. As can be seen from figure 3(b), the mechanical edge $G_2$ of the mirror $s$ falls into the ray-quadrangle $a$, which is consisting of mirrors $a$ and $a + 1$, although the ray bundles (represented by blue lines) are not truncating each other. $\text{Erf}_{2D3}$ is able to deal with this point-to-ray-quadrangle problem. The difference is that, ray-quadrangle $a$ is not extended here. But it is still divided into four triangles by the two diagonals. If one edge of a mirror $s$ is inside a triangle, the distance $|G_kD|$ from the mechanical edge $G_k$ to the corresponding side on the ray-quadrangle is calculated. $D$ is the intersection point. $k = 1$ to 2 are the indices of the two edges. The error function can be written as

$$\text{Erf}_{2D3}(a, s) = \text{Erf}_{2D3}(a, s) = \sum_{k=1}^{2} \left\{ \begin{array}{ll}
|G_kD|^2 + \text{Bias}, & \text{if } G_k \in \text{ray - quadrangle } a \\
0, & \text{if } G_k \notin \text{ray - quadrangle } a
\end{array} \right.$$

(6)

where $a \leq n + 1, s = 2$ to $n + 1$ and $s = a, a + 1$. For every ray-quadrangle, the edges of mirrors that do not belong to this ray-quadrangle are checked.

Now two more parts are added to the total error function:

$$\text{Erf}_{2D} = \sum_{a,b} (\text{Erf}_{2D1}(a, b) + \text{Erf}_{2D2}(a, b)) + \text{Erf}_{2D3} + \sum_s \text{Erf}_{2D4}(s) + \sum_{a,s} \text{Erf}_{2D5}(a, s).$$

(7)

Before running the program to calculate the obscuration error function for each field in MATLAB, the required input are the semi-diameters of the clear aperture of mirrors, the matrix of field points which are equally sampled in the object plane, clearance and bias. One dummy surface is inserted before the optical system and near the object plane to help build the first ray-quadrangle. The global coordinates of the clear aperture edges and mechanical edges for each mirror can be computed in MATLAB by the input semi-diameters, the radii and rotational matrices of mirrors that are collected in OpticStudio. Then for each field, the global coordinates of the upper and lower boundary ray heights on every surface are recorded by ray trace in OpticStudio and transported to MATLAB to construct ray-quadrangles for calculating the obscuration error function which is based on ray-quadrangles and global coordinates of the mirror edges. In the end, the field area with zero error functions is found as unobscured field area.

2.3. Example

As an example, an EUV reflective lithographic projection system with eight aspherical mirrors is shown in figure 4 [13]. Although the system looks like an off-axis system, actually every mirror is a mirror segment which centers on a common axis. Therefore, the mirror segments are not rotated from the optical axis to the current positions but part of the mirrors that are symmetric about optical axis. Also the sag values of the mirror segments are not rotationally symmetric. As a result, the maximum and minimum $y$-coordinates of the mirror segments (listed in table 1) instead of the semi-diameters are given as input to compute the boundaries of mirrors. The input fields are defined by polar coordinates with radius from 105 to 140 mm and azimuthal angle from 40° to

Figure 3. Sketch of the situation in: (a) $\text{Erf}_{2D4}$. The ray bundle border between the two mirrors is indicated by blue solid lines. The shadowed part of the ray bundle cannot reach clear aperture of mirror $s$ which is defined in between $H_k$ and $H_1$. (b) $\text{Erf}_{2D5}$. The mechanical edge $G_2$ of the mirror $s$ falls into the ray-quadrangle $a$ which is consisting of mirrors $a$ and $a + 1$. 

(Equation 5)
140°. Normally the clearance is set as relative 5% of the mirror diameter. For the large mirror segments M1–M4 and M8, the clearance is set to be 5 mm. For the other small mirror segments, the clearance is set to be 2 mm. The bias that on purpose generating a gap between obscured and unobscured fields is set as 5 mm², which has the same unit as error function.

After running the program in MATLAB, the results are shown in figure 5. On the left side, the degree of obscuration for the object field points on the y-axis is shown. Due to the bias, the obscured and unobscured parts are clearly distinguished. The obscuration-free field height in y is between 107.6 mm and 127.2 mm. The degree of obscuration for the 2D-fields are on the right side. The drawn field area is in special shape because the field points are changed back from polar coordinates to Cartesian coordinates. The area between the two white curves is free of obscuration. For the fields that have the same y-height and varying x-heights, the degree of obscuration is nearly the same and symmetric about y-axis. This is the property of plane symmetrical system.

What’s more, the RMS spot radius for each field can be added to the total error function to indicate the image quality. The RMS spot radius is normalized to the Airy radius. When the normalized RMS spot radius (Erf_spot) is smaller than 1, the spot is diffraction limited. The performance criterion in this example is arbitrarily fixed by the request Erf_spot < 2. The error function is updated as Erf_2D + Erf_spot. Because the bias has a value of 5, the error function only contains the component of Erf_spot when its value is smaller than 5. In other words, the two parts of error function—obscuration and image quality does not mix with each other when the system is free of obscuration and aberration is small. The plot of the error function versus fields is shown in figure 6. The value of the error function within the area indicated by the white boundary is smaller than 2 which means that the object fields in this area are free of obscuration and the image quality meets the requirement. It can be seen that this area is kidney-shaped which is typical for a scanning field shape in the EUV reflective lithographic systems [13].

Figure 4. Layout of an EUV reflective lithographic projection system with eight aspherical mirrors (M1–M8). The black curves are the mirror segments being used. With the extended gray dashed curves, it can be seen that they are centered on the optical axis.

Table 1. Input of local minimum and maximum y-heights on the surfaces.

| Surface      | Local min y-height (mm) | Local max y-height (mm) |
|--------------|-------------------------|-------------------------|
| Dummy surface| 0                       | 140                     |
| M1           | 0                       | 120                     |
| M2 (stop)    | −43                     | 43                      |
| M3           | −100                    | 0                       |
| M4           | −180                    | 0                       |
| M5           | −240                    | −140                    |
| M6           | −140                    | −90                     |
| M7           | −50                     | 10                      |
| M8           | −105                    | 150                     |
| Image plane  | 0                       | 50                      |
3. Obscuration elimination in 3D optical systems

3.1. Method

The above discussion is restricted to the range of plane symmetrical systems. In this section, the algorithm to automatically eliminate obscuration in nonsymmetrical reflective systems is introduced. One important assumption in this method is that the mirrors are circular. Therefore, the semi-diameter of a mirror is defined by the outermost ray height on that mirror. The obscuration problem can be investigated by describing the relation among the mirrors.

Since it is inconvenient to discuss problem in 3D space, it would be comfortable to derive the 3D problem to 2D by projecting the mirrors onto a reference plane. In total three mirrors are considered at one time—two successive mirrors \( a \) and \( a + 1 \) (the space between the two mirrors represents the ray bundle) and any other arbitrary mirror \( b \). The mirrors are viewed as circles which are constructed by their circular outlines. As explained in figure 7, the projection plane can be built by the midpoints of the three circles. The plane is going through the three circles and cut them into three lines. It is obvious that these three lines are on the same plane. The two lines belonging to the two successive mirrors \( a \) and \( a + 1 \) establish a ray-quadrangle \( a \). By comprehensively studying the combinations between the ray-quadrangle \( a \) and the other line \( b \), all possible cases in 3D space can be described. All the possible combinations are plotted in the figure 8. When the line is crossing or inside the ray-quadrangle, it blocks the ray path. If the line is outside of the ray-quadrangle, it may or may not block the ray path.

Comparing figures 2 and 8, it is obvious that the 2D error functions are able to be used here. In the figures 8(a)–(c), one or two endpoints on the line are inside the ray-quadrangle. This case can be described by the first 2D error function \( \text{Erf}_{2D} \). Referring back to the section 2.1, the ray-quadrangle is extended by certain clearance and the squared distance from the endpoint \( P \) of the line to the corresponding extended side is calculated if \( P \) is located inside one of the triangles of the extended area. Thus the first error function in 3D is

\[
\text{Erf}_{3D} = \text{Erf}_{2D} + \text{Erf}_{\text{spot}}
\]
written as

\[
\text{Erf}_{j \in \{1, 2\}}(a, b) = \sum_{j=1}^{2} \begin{cases} 
|P_j Q|^2 + \text{Bias}, & \text{if } P_j \in A \\
0, & \text{if } P_j \notin A
\end{cases}
\]  

(8)

where \(a\) is the ray-quadrangle, \(b\) is the line which belongs to mirror \(b\). One dummy surface is inserted before the optical system which contains \(n\) mirrors. In total there are \(n + 1\) ray-quadrangles and \(n + 2\) surfaces (\(n\) optical surfaces plus one dummy surface and one image plane) being considered. Therefore, \(a\) and \(b\) have the relation that \(a \leq n + 1\), \(b = 1\) to \(n + 2\) and \(b = a, a + 1\). \(A\) is the extended area of ray-quadrangle \(a\). The line segment \(|P_j E_j|\) is perpendicular to the corresponding external side and intersects the side at the point \(E\). \(j = 1\) to \(2\) denotes the two endpoints on the line.

There is also the possibility that the line crosses with one or two sides of the ray-quadrangles. In the figures 8(a), (b) and (d)–(f), the crossing situation between the line and the ray-quadrangle is considered. The second error function \(\text{Erf}_{j \in \{1, 2\}}\) has the similar form as \(\text{Erf}_{j \in \{1, 2\}}\)

\[
\text{Erf}_{j \in \{1, 2\}}(a, b) = \sum_{i=1}^{4} \begin{cases} 
\min(|A_i Q|^2, |B_i Q|^2) + \text{Bias}, & \text{if crossed} \\
0, & \text{if uncrossed}
\end{cases}
\]  

(9)

where \(a\) and \(b\) have the same range in \(\text{Erf}_{j \in \{1, 2\}}\) and \(i = 1–4\) corresponds to the four sides of the ray-quadrangle. \(A_i\) and \(B_i\) are the two endpoints on the side \(i\) which is crossed by the line \(b\) at the point \(Q\).

From the 2D view, it seems that the third circle does not block the ray path when the line is outside of the ray-quadrangle, as shown in figure 8(g). However, a 3D view is required to get the correct conclusion. The possible conditions are summarized in figure 9. The simplest case is that there is no intersection as shown in figure 9(a). Figure 9(b) shows the case where circle \(b\) intersects with circle \(a\). In the figures 9(c) and (d), the 2D views are exactly the same, but the ray path is truncated in (c) and not truncated in (d) by the circle \(b\).
In order to avoid tremendous calculation, the absolute non-intersection region is calculated at first when the line is outside of the ray-quadrangle. When a circle is rotating around its midpoint, it is inside the sphere which has the same radius. Therefore, when the connecting distance between the midpoints of two circles is longer than the sum of the two radii, the two circles can never touch each other. When the distances from the midpoint of the third circle to the two soft sides are longer than its radius, then the circle can never block the ray path. If the above two cases are fulfilled at the same time, as shown in figure 10, circle \( b \) is in the absolute non-intersection region.

Therefore if the line is outside of the ray-quadrangle, the first step is to test if the circle is in the absolute non-intersection region. If so, the value of total error function \( \text{Erf}_{3D} \) is zero. If not, the third error function \( \text{Erf}_{3D3} \) is established to check if the circles are intersecting with each other. When the circle \( b \) is intersecting with circle \( a \) or \( a + 1 \), the two circles have a common intersection line starting from \( I_1 \) to \( I_2 \). The intersection length is \( |I_1I_2| \) as shown in figure 11(a). Then \( \text{Erf}_{3D3} \) has the form

\[
\text{Erf}_{3D3}(a, b) = \sum_{i=a}^{a+1} \left\{ |I_{i1}I_{i2}|^2 + \text{Bias, if crossed} \right. \\
0, \quad \text{if uncrossed}
\]

where \( i = a \) to \( a + 1 \) are two circle sides of the ray-quadrangle \( a \).
Then for the two possibilities in figures 9(c) and (d), as illustrated in figure 11(b), starting from one endpoint of the line on circle \( b \) to the other, the points on the half part of the circle that close to the ray-quadrangle are checked one-by-one. The way to do is that for each point, it constructs one projection plane with the two centers of the successive circles. This new projection plane cuts the two successive circles into two new lines which establish a new ray-quadrangle. If the point is inside the ray-quadrangle with extended clearance, the squared distance from the point to the corresponding side is calculated and compared with squared distance that computed with next point on the edge of circle to find the maximum squared distance, which is the value of \( \text{Erf}_{3D4} \).

\[
\text{Erf}_{3D4}(a, b) = \max_a \left\{ \begin{array}{ll}
|P_bE|^2 + \text{Bias}, & \text{if } P_b \in A \\
0, & \text{if } P_b \not\in A
\end{array} \right.
\]

(11)

where \( P_b \) are the points on the edge of circle \( b \) from one endpoint of line \( b \) (with azimuthal angle \( \theta = 0^\circ \)) to another (\( \theta = 180^\circ \)), \( \Lambda \) is the extended area. When drawing a line starting from \( P_b \) and perpendicular to the corresponding side of \( \Lambda \), \( E \) is the intersecting point. If from one endpoint of the line to the other, no point on the edge of circle \( b \) is inside the ray-quadrangle, the value of \( \text{Erf}_{3D4} \) is zero, which is the case in figure 9(d).

According to the above discussion, in total four error functions and one test calculation of the absolute non-intersection region are desired to fully describe all the obscuration conditions in nonsymmetrical optical systems.

The workflow to realize the calculation of total error function \( \text{Erf}_{3D} \) is shown in figure 12. In figure 12(a), the steps of choosing surfaces are shown. In figure 12(b), the detailed steps and logic of calculating error function for each surface combination are shown.

The total error function \( \text{Erf}_{3D} \) is able to evaluate the degree of obscuration in a nonsymmetrical system where mirrors can move freely in 3D space. The next step is to establish the error function in MATLAB and optimize the structure of the optical system to eliminate obscuration by using the optimization toolbox in MATLAB. The required input to MATLAB is optimization variables (e.g. tilt angles and decenters of mirrors), their range, clearance and bias. In OpticStudio, field is set properly and one dummy surface is inserted in front of the first mirror. Semi-diameters of the mirrors are determined by outermost ray height on the mirrors if they are set with the type ‘automatic’. The mirror parameters such as semi-diameters, radii, rotational matrices are transferred from OpticStudio to MATLAB to calculate normal vectors of the circles, which are constructed by circular outlines of the mirrors, as well as the global coordinates of the midpoints of the circles. Two perpendicular unit vectors starting from the midpoint of each circle are also calculated. Then the circles and projection planes in 3D space can be analytically represented by equations. The error function is calculated based on these equations and the workflow in figure 12. The optimization algorithm can be chosen as simulated annealing and also the maximum iteration number is fixed. During the optimization, the value of current variables are sent back to OpticStudio to update mirror parameters for MATLAB to calculate new error function. The iteration stops when the value of error function is decreased to zero. If the value of error function is larger than zero after maximum iterations, which could happen for complex optical systems, a new optimization loop can be manually started by user with current error function value.

![Figure 11](image-url)

Figure 11. (a) Circle \( b \) intersects circle \( a \) or \( a + 1 \), the intersection length is \( |l_1l_2| \). (b) When line \( b \) (blue) does not intersects the ray-quadrangle (green), every edge point \( P_b \) on half of the circle \( b \), which is close to ray bundle, constructs a new plane with the midpoints of circle \( a \) and \( a + 1 \). The new plane cuts the ray bundle to a new ray-quadrangle (red).
3.2. Examples

In this section, two examples are shown to demonstrate the method.

The layout of an extended nonsymmetrical 2-mirror Yolo telescope with f-number 2.16 and field of view $1.5^\circ \times 1.5^\circ$ is shown in figure 13(a) as the first example. The first mirror is tilted around x-axis with an angle of $6^\circ$ and the second mirror is tilted around y-axis with an angle of $3^\circ$. From the layout it can be seen that the second mirror is intersecting with the dummy surface, which means that the second mirror truncates the incoming beam, and the image plane is located inside the ray bundle.

Before optimization, the clearance is set to be 5 mm and bias is 5 mm$^2$. The variables are tilt angles of the two mirrors around x- and y-axis respectively. The range of variables are shown in table 2. The maximum iteration number is 100. After three iterations the error function is optimized to zero, which is shown in figure 14. The error function converges very fast because of the broad variable range. The tilt angles after optimization can be seen from table 2. The layout of the unobscured system is drawn in figure 13(b). The image quality is not considered here. The focal length is automatically fixed because it is not relevant to surface tilt.

The second example is the initial system design of a 3-mirror nonsymmetrical Yolo telescope. In the beginning, three mirrors are placed on the common optical axis with f-number 2.25 and field of view $1^\circ \times 1^\circ$. The layout is shown in figure 15(a). The aim is to rotate the mirrors in 3D space to achieve an unobscured initial system.

Before optimization, the clearance is set to be 5 mm and bias is 5 mm$^2$. The variables are tilt angles of the two mirrors around x- and y-axis respectively. The range of variables are shown in table 2. The maximum iteration number is 100. After four iterations the error function is optimized to zero, which is shown in figure 16. The layout of the optimized system is shown in figure 15(b). It can be utilized as a starting system for further optimization with freeform surfaces.
3.3. Discussion of correction strategy

In the above two examples, the obtained solutions are not unique. The relation between the error function and the tilt angle is binary. Within the allowed range of tilt angles, the error function equals zero in several regions. This means, in finite combination of mirror tilt angles allow to achieve an unobscured optical structure. But the optimization process stops when one combination is found.

However, when aberrations are considered at the same time, not all unobscured structure are suitable as initial system and the structure with minimum aberration is of largest interest. For example, according to Coddington equations [14], the smaller the incident angle of chief ray on the mirror, the smaller the astigmatism, therefore small tilt angles are required. Especially, for some tilt angles, third-order astigmatism can be corrected [15–17].

In the following, a nonsymmetrical 2-mirror Yolo telescope design with minimized 3rd order astigmatism is given as a simple example. In the beginning, as shown in figure 17, the mirrors are placed on a common axis with f-number 2.18 and the system has only one on-axis field. The radii of curvature of the mirrors are \( R_1 = -197.53 \) mm, \( R_2 = 66.43 \) mm. To indicate primary astigmatism in the system, the error function of

![Diagram of a nonsymmetrical 2-mirror Yolo telescope](image)

**Figure 13.** Layout of an extended nonsymmetrical 2-mirror Yolo telescope (a) with obscuration and (b) without obscuration after optimization.

| Parameter | Initial value | Lower boundary | Upper boundary | Final value |
|-----------|---------------|----------------|----------------|-------------|
| M1 tilt X (°) | 6 | 0 | 20 | 17.7839 |
| M2 tilt Y (°) | 3 | 0 | 30 | 29.0227 |

Table 2. Initial values, optimization boundaries and values after optimization of the variables for the first example.
Figure 14. The change of error function $E_{f(3)}$ for the first example during optimization.

Figure 15. Layout of an extended nonsymmetrical 3-mirror Yolo telescope (a) mirrors are centered on axis and (b) obscuration eliminated after optimization.

| Parameter   | Initial value | Lower boundary | Upper boundary | Final value  |
|-------------|---------------|----------------|----------------|--------------|
| M1 tilt X ($^\circ$) | 0             | 0              | 20             | 19.9331      |
| M1 tilt Y ($^\circ$) | 0             | -15            | 0              | -14.6976     |
| M2 tilt Y ($^\circ$) | 0             | -30            | 0              | -25.0453     |
| M3 tilt X ($^\circ$) | 0             | 0              | 25             | 16.0118      |
| M3 tilt Y ($^\circ$) | 0             | -20            | 0              | -1.7787      |
astigmatism is calculated in OpticStudio as

\[ \text{Erf}_{\text{asti}} = \sqrt{c_5^2 + c_6^2}, \tag{12} \]

where \( c_5 \) and \( c_6 \) are Zernike Fringe coefficients. Then the total error function

\[ \text{Erf} = \text{Erf}_{3D} + \text{Erf}_{\text{asti}} \tag{13} \]

contains both obscuration and astigmatism parts.

The tilt of mirror 1 around \( x \)-axis and the tilt of mirror 2 around \( y \)-axis are set as variables and they both have the range from \(-30^\circ\) to \(30^\circ\). The clearance is set to be 0 and bias is 100 mm. Due to the positive bias, the value of \( \text{Erf}_{3D} \) jumps from larger than 100 to 0 at the boundary between obscured and unobscured optical structure. But \( \text{Erf}_{\text{asti}} \) is a continuous function. Therefore, if \( \text{Erf} \) is smaller than 100, the contribution only comes from \( \text{Erf}_{\text{asti}} \). The total error function \( \text{Erf} \) as a function of the both two variables are plotted in figure 18(a). The value of \( \text{Erf} \) in between 0 and 100 are mapped with colorbar, all values of \( \text{Erf} \) larger than 100 are displaced by 100. It can be seen that \( \text{Erf} \) is symmetric about \( x \)- and \( y \)-axis. The result is obvious according to the geometry of the optical system.

In the central cross area where the tilt angle of any mirror is small, \( \text{Erf} \) is quite large because of severe obscuration. In the four corners where the tilt angles of both mirrors are large, \( \text{Erf} \) is smaller than 100 which means the structures have no obscuration and the remaining astigmatism is different. Every corner has a dark blue valley where astigmatism is minimized. In figure 18(b), the tilt angle of mirror 2 is \( 20^\circ \), the relation between \( \text{Erf} \) and the tilt angle of mirror 1 is shown. There are two symmetric rotating angles of mirror 1 for correcting primary astigmatism.
In the next step, the global optimization is utilized to search for global minimum. Since for this special 2-mirror setup, the error function is symmetric about the two axes, the range of the tilt angles of the two mirrors are adapted to $0^\circ$–$30^\circ$. The bias is reduced to 5 mm$^2$ for better continuity of the error function. The maximal iteration number is 200. After 200 iterations, as shown in figure 19, the minimal error function is found (marked with red circle) with the tilt angles $\theta_1 = 7.67^\circ$, $\theta_2 = 11.11^\circ$.

From figure 18(a), it can be seen that tilt angle $\theta_1 = 7.67^\circ$ of mirror 1 around x-axis is almost the smallest angle that avoiding the truncation of mirror 2 to ray bundle. As mentioned, the smaller the tilting angle, the smaller the astigmatism. The optimization result in turn demonstrates that the optimization turns to find the minimal rotating angles which eliminate obscuration and primary astigmatism at the same time. The solution is plotted in figure 20(a) and the corresponding spot diagram is shown in figure 20(b). It can be seen that the primary astigmatism is well corrected and the residual aberration is dominated by coma.

At this point, the system is a good initial off-axis mirror system. For the further improvement in particular for larger apertures, freeform surfaces are an intelligent option with huge correction capability. The obscuration error function and image quality merit function should be utilized together to search for unobscured optical system with high performance.
4. Conclusion

In this paper, an algorithm to calculate unobscured field of view in an off-axis plane symmetrical mirror system is successfully established by extending Xu’s method of representing ray bundles between the mirrors by ray-quadrangles. An error function is introduced to evaluate the degree of obscuration by investigating the possible overlapping cases among the ray-quadrangles and relative positions between ray bundles and mirrors. When the values of error function are computed for the interested fields, the area that the values equal zero is free of obscuration.

Then an error function is built to describe the degree of obscuration in nonsymmetrical optical system. The overlapping situation between two successive mirrors and one additional mirror is investigated at one time by projecting them to a common plane. For any nonsymmetrical system with obscuration, unobscured structure can be achieved by optimizing the error function to zero by selecting proper variables, for instance tilt and decenter of mirrors, and optimization algorithm. The number of mirrors in an optical system is not limited by using this method.

For further work, the method can be extended to eliminate obscuration in 3D nonsymmetrical systems with rectangular mirrors by appropriately defining the circumscribed rectangle of the multi-fields footprint on each mirror. Aberration correction can be taken into consideration during the process of eliminating obscuration in nonsymmetrical system by substituting spherical surface to biconic surface or freeform surface and adding aberration criterion to the error function. Also the current error function can be scaled by the length of optical system to avoid extreme large numbers for large telescope system.

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