Entropy optimization in MHD nanofluid flow over a curved exponentially stretching surface with binary chemical reaction and Arrhenius activation energy

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Abstract

In this examination article, the production of entropy is investigated through the steady state nanofluid flow with MHD past a curved exponentially stretched surface. By keeping the knowledge of curved surfaces in mind the existing model has been constructed. Put on the limit layer assesses on the Navier-Stokes equation to get the nonlinear coupled PDEs to portray the current nanofluid flow. Then some suitable similarity transformation has been selected to transform these coupled PDEs to non-dimensional coupled ODEs. Coupled ODEs were solved numerically through HAM. The impacts of parameters which are contained in ODEs are displayed by graphs while tables of $C_{r}$, $Nu$ and $Re_{s}$ are showed. The physical features of relevant parameters have been deliberated by plotting the graphs of velocity, temperature, concentration profile, entropy optimization, and Bejan number. Numerical conclusions of gradients of velocity and heat are deliberated through tables by various physical variables. The objective of our study was to examine the rate of warmth transmission through nanofluid. Astonishingly, for thermal border level thickness, solutal border level thickness and momentum border layer thickness became greater when $\lambda > 0$, as associated with the situation when $\lambda < 0$. Entropy generation is a growing function of Brinkman number and local Reynolds number. Moreover, Bejan number reduces in closed vicinity of the stretching sheet and grows far away from it with augmenting values of the magnetic parameter.

Nomenclature

- $r$: Normal to the tangent at any point of the surface
- $s$: Arc length coordinates along the surface
- $R$: Curvature of the curves belt
- $\beta_{0}$: Applied magnetic field
- $c_{p}$: Specific heat at constant pressure $(m^{2} s^{-2} K^{-1})$
- $k$: Thermal conductivity $(W m^{-1} K^{-1})$
- $T_{w}$, $T_{\infty}$: Temperatures near and far away the surface respectively

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\( R_0 \)  
Dimensionless parameter

\( u, v \)  
Velocity components in \( s, r \) directions respectively

\( C_w, C_\infty \)  
Concentration near and far away the surface respectively

\( p \)  
Pressure (dimensional)

\( C, T \)  
Concentration and temperature of the fluid respectively

\( M \)  
Magnetic field parameter

\( P \)  
Pressure (dimensionless)

\( Sh_t \)  
Sherwood number

\( Sc \)  
Schmidt number

\( Cf \)  
Skin friction coefficient

\( Ec \)  
Eckert number

\( Pr \)  
Prandtl number

\( u_w \)  
Suction/injection parameter

\( q_w \)  
Wall heat flux

\( Be \)  
Bejan number

\( N_t \)  
Thermophoresis parameter

\( q_w \)  
Wall heat flux

\( E_1 \)  
Activation energy

\( Nu \)  
Nusselt number

\( h_m \)  
Mass flux

\( Ns \)  
Entropy generation

\( N_b \)  
Brownian parameter

\( k_0 \)  
Curvature parameter

\( Re \)  
Reynolds number

\( Br \)  
Brinkman number

**Greek symbols**

\( \rho \)  
Density (kg m\(^{-3}\))

\( \mu \)  
Dynamic viscosity (kg m\(^{-1}\) s\(^{-1}\))

\( \nu \)  
Kinematic viscosity (m\(^2\) s\(^{-1}\))

\( \alpha \)  
Thermal conductivity (m\(^2\) s\(^{-1}\))

\( \phi \)  
Fluid concentration (dimensionless)

\( \alpha \)  
Dimensionless temperature

\( \beta \)  
Thermal slip parameter

\( \theta \)  
Fluid temperature (dimensionless)

\( \delta \)  
Temperature difference parameter

\( \zeta \)  
Dimensionless concentration

\( \tau_{rs} \)  
Shear stress in rs-plane (kgs\(^{-2}\) k\(^{-1}\))

\( \lambda_1 \)  
Diffusive constant

\( \xi \)  
Similarity variable

\( \delta_1 \)  
Heat absorption/generation

**Subscripts**

\( \infty \)  
Ambient condition

\( w \)  
Condition on surface

**Superscripts**

\( \prime \)  
Differentiate with respect to \( \xi \)
1. Introduction

The stream provoked by a moving limit is essential in the investigation of expulsion forms and is a subject of the incredible enthusiasm for present-day writing. The limit layer, characterized as the thickness impacts, is significant-close by the surface. The wearisome rundown of its engineering applications incorporates paper creation, assembling of elastic sheets and metallic cables, glass blowing metal turning, the expulsion of polymer pieces, and the refrigerating string going between a liberated roll. However, a closed-form exact solution of an incompressible viscous fluid for a 2-dimensional laminar flow over a linearly stretched sheet has been studied by Crane [1]. Gupta [2] has been observed stretched flow through mass and warmth transmission in the occurrence of injection/suction. Banks [3] has been described as correspondence solutions for the boundary layer equalities, while Magyari and Keller [4] have proposed similar solutions for impermeable stretched surfaces. Further, Wang [3] has examined the viscous flow owed to the suction and surface slip of the stretched sheet. Wang [6] has explained flow analysis to 3D axisymmetric tensile surfaces. Mahapatra and Gupta [7] have found numerical solutions on stretched surfaces below the stagnation point. Ali [8] has emphasized the flow of the boundary layer with buoyancy impact in a continuously stretching sheet. Wang [9] has studied the flow to the shrinking tablet below the stagnation point. He found that there was no solution with a sufficiently high shrinkage rate, and when it existed, it might not be unique in two dimensions. Alblawi et al [10] have been Explored a model of nanofluidic Buongiorno flowing through an exponential curve plate through suction. A force law liquid model over an extending pane under the inaction fact was concentrated by Mahapatra et al [11]. Tie-Gang et al [12] have deliberated the unsteady flow of fluid on a stretching cylinder. Wang [13] have deliberated the common deformation over an upright chamber. Salahuddin et al [14] have considered the progression of nanofluids on an extending chamber close to the stagnation region. Hayat et al [15] have studied the characteristics of variable thermal conductivity and thermal relaxation in stagnation flow over a variable thickness stretched surface with chemical reaction. Khan et al [16] have deliberated the magnetohydrodynamic (MHD) stagnation point flow of Casson fluid towards a stretching sheet. Muhammad et al [17] have considered the magnetohydrodynamic fluid flow in the presence of magnetic characteristics and the impact of electrically conducting liquids. Muhammad et al [18] have deliberated entropy generation minimization (EGM) combines the essential ideologies of heat transfer, thermodynamics, and fluid mechanics. Ibrahim et al [19] have considered the mixed convection flow of SWCNT-Water and MWCNT-Water over a stretchable permeable sheet Nayak et al [20] have analyzed three-dimensional unsteady magnetohydrodynamic streams and entropy generation of micropolar Casson Cross nanofluid subject to nonlinear warm radiation and chemical reaction. Gireesha et al [21] have contemplated nanoparticles inserted water-based hybrid nanofluid stream over permeable longitudinal balance moving with consistent velocity is completed along with warm radiation and common convection condition. Wang et al [22] have deliberated the irreversibility aspects in the MHD flow of viscous nanofluid. Abbas et al [23] have considered the Characteristics of von Kármán swirling flow for Oldroyd-B nanofluid in the presence of heat absorption/generation. Khan et al [24] have considered magnetohydrodynamic mixed convective hybrid nanofluid flow by a rotating disk. Visit the paper from [25–33] for the same study. Lahmar et al [34] have deliberated the variable thermal conductivity and the existence of inclined magnetic field impacts on heat transfer and fluid flow of squeezing unsteady nanofluid flow. Eid et al [35] have considered a numerical solution for the steady convective heat and mass transfer of Carreau nanofluid over non-linearly stretched sheet saturated porous medium with chemical reaction. Eid et al [36] have deliberated the issue of heat transfer in bio-nanofluid flow on a permeable exponentially stretching surface with convective boundary conditions. Visit the paper from [36–38] for the same study. Conventional warmth move liquids like propylene glycol, water, and ethylene glycol, and so forth comprise poor warmth move qualities. The expansion of nanoscale solid particles in the ordinary warmth move liquids incredibly expands the warm characteristics of such liquids. Expression nanofluid was first utilized by Choi [39] and he demonstrated that the expansion of nano particles in the typical liquids expands the warm qualities of these liquids. Eastman et al [40] have deliberated the characteristics of copper nanoparticles in ethylene glycol. In direction to convey nanofluids, Boungiorno reported a mathematical model [41]. Based on his investigations, Boungiorno calculated that the Brownian wave and heat swimming are the two utmost significant factors in warmth transmission. Tiwari and Das [42] have studied the enhancement of heat transfer of nanofluids in heated square cavities driven by double-sided lids. Abu-Nada and Ozturk [43] discussed the effect of inclination on the flow of Cu-water nanofluids. Hassan et al [44] have been established a measured model of the flow of nanofluids on a movable sheet. Ibrahim et al [45] have been discussed the effect of stagnation point flow with a normal magnetic field. Other studies have reported nanofluidic flows with diverse types of flat boundaries. Nadeem and Lee [46] have been investigated the boundary layer of nanofluids flow on exponentially moving surface. Similarly, Rana and Bhargava [47] have been proposed nanofluidic boundary layer flow with nonlinear stretched speed. Haq et al [48] have been explained the effects of nanoparticles, micro spins, and radiation on vertical moving surfaces.
In the previously mentioned article, the surface is viewed as smooth and level while the stream is created because of the movement of the surface. Dynamics of the stream could be very unique on account of bent surface extending as it needs the concern of curvature impacts. Also, the arrangement of the limit layer nearby the outward would be not quite the same as the one for plane surface extending. This wonder has likely applications in the polymer business where the expulsion procedure is being finished because of the development of the bent surface. This marvel has likely applications in the polymer business where the expulsion procedure is being ended because of the development of the bent surface. Saleh et al [49] have inspected the flow of marginally polar liquids through contracted surfaces. Pop et al [50] have been probed the effect of surface curvature with the magnetic field, which indicated a huge change in the skin friction value. What’s more, they showed that there are four answers for little pull and attractive boundary esteems. Arifin et al [51] have considered stream on porous surfaces with curvature and suction effects. Rosca and Pop [52] have been proposed a few arrangements on account of surface shrinkage and reverse. They likewise analyzed the solidity of the solution and introduced the scope of qualities for which a solitary solution; numerous solutions and no solution occur. The instance of contracting is not quite the same as the extending as it will make an opposite stream circumstance. Scientifically on account of contracting more than one arrangement will exist. The progression of Newtonian or non-Newtonian liquids over an extending exterior with warmth and mass exchange is a significant part of different mechanical and engineering firms. The submissions can be found in the refrigerating of sheets, liquefy turning forms, turning of filaments, throwing, and so on. In each procedure, the greatness of the completed items is subject to a warm exchange close to the extending sheet. Crane [53] has been published the primary paper on stream on stretchable surfaces. Mabood et al [54] have studied the border layer gesture of NFs due to the non-linear elongating of the pane, numerically. They explored that the fluid motion shrunk as the value of nonlinear stretching parameters increased. Rangi and Ahmad [55] have been reported heat transfer in flows caused by cylinder stretching. Postelnicu and Pop [56] have been analyzed the motion of power-law fluids on tensile wedges. Imtiaz et al [57] have examined the flow of Fe3O4– along a stretchable bend geometry states that resistance closes the outward increases with increasing curving parameters. Hayat et al [38] have continued this work. Novelty the effect of thermal relaxation time on Darcy–Fochheimer flow. Khan et al [59] have examined flows of fluids along curved surfaces in the occurrence of nanoparticles. Afridi et al [60] have been discussed the heat dissipation and entropy production through hybrid nanofluids on curved surfaces have been studied. Asma et al [61] have been pointed out the facts through MHD flow of nanofluids with Arrhenius activation energy and binary chemical reactions on a turntable have been studied.

Entropy advancement is utilized to exhibit the presence of various frameworks in engineering and mechanical wonders. In this manner, different researchers have concentrated their battle on the entropy generation issue. The measure of any type of vitality that produces in any irreversible procedures in a framework or encompassing is recognized as entropy age. This vitality can’t be utilized to accomplish the proficient effort. Entropy generation originates through thermodynamics 2nd law. Thermodynamics’ subsequent law is increasingly proficient when contrasted with the 1st law. The irreversible procedure encloses the progression of fluids because of resistance stream, dispersion, Joule warming, friction between viscous fluids, and warm radiation, and so on. For improved proficiency of the framework, we measured the entropy age rate. Thermodynamics’ 2nd law expresses that the entropy generation rate inside a framework needs to be zero or more noteworthy than zero. The numerical expression for the entropy generation because of warmth move and viscous dispersal was inferred by Bejan [62]. Later the considerable effort of Bejan [62], the 2nd law investigation of the liquid stream in various conditions turned into an exceptionally alluring examination theme for theoretical thermal researchers. As of late, Butt et al [63] announced the 2nd law examination of the liquid stream over a tending extending tube. Rashidi et al [64] have investigated the production of entropy in the stream of third-grade fluid. Sheikholeslami et al [65] have investigated the impacts of the non-uniform magnetic field on the creation of entropy during the progression of non-Newtonian fluid. Jawad et al [66] have examined entropy creation because of warm radiation and viscous dissipation impact in the progression of CNTs and water-based nanofluid under the effect of MHD between rotating surfaces. Khan et al [67] have investigated the impact of MHD on the flow of viscous fluid past an elongating surface. The readers can see an extensive study about the entropy generation in ref [68–71]. Eid et al [72] have studied the numerical investigation of an incompressible flow comprising MHD non-Newtonian CNTs-based kerosene with entropy generation over a permeable stretching sheet past. Some recent related work can be read in [73–76].

The significant point of the current correspondence is to explore the effect of nanoparticles in the two dimensional limit layer stream of thick fluid with entropy generation because of a nonlinear bent extendible pane. Convective warmth and mass settings are used. Warmth and mass exchange possessions are examined through Brownian dissemination and thermophoresis. All these circumstances give us a mathematical model in the form of coupled PDEs; by applying suitable similarity transformation these PDEs are changed to ODEs which are solved by applying HAM. Momentum, energy, concentration profiles, and the engineer’s interest quantities like Skin fraction, Nusselt Number, Sherwood Number are discussed via graphs and numerical tables.
also. It is a fact that the production of entropy is the function of Brinkman number and local Reynolds number. Moreover, Bejan number reduces in closed vicinity of stretched sheet and grows far away from it with augmenting values of magnetic parameters.

2. Mathematical formulations

Consider steady 2D viscous nano-liquid flow through a curved stretching surface of radius $R$ with thermal generation/absorption. Normal to tangent vector is $r$ and arc length is $s$ along the flow direction figure 1(a).

The extending velocity of the curved belt is $u_w = ce^z$ in the path of $s$ and $c$ is the real constant, whereas, $u_w$ is suction/injection constraint which signifies two cases. Case: I when $u_w > 0$ it indicates suction, Case: II if $u_w < 0$ it indicates the injection. But we only analyze the Case: I $u_w > 0$ in our work (i.e. Suction). The mathematical representation of our considered problem with the boundary layer estimates are [10]:

\[
\frac{1}{r + R}(r + R)u_r + \frac{R}{r + R}u_r = 0, \quad (1)
\]

\[
\frac{u^2}{r + R} = \frac{1}{\rho_f} \quad (2)
\]

\[
v u_r + \frac{R}{r + R}uu_r + \frac{vu}{r + R} + \frac{1}{\rho_f(r + R)}p_f = \frac{\beta_f}{\rho_f(r + R)}\left(u_r + \frac{1}{r + R}u_r + \frac{1}{(r + R)^2}u\right)
\]

\[
= \frac{1}{\rho_f^2}(\sigma \beta_f^2 u + \frac{v}{k}u), \quad (3)
\]

\[
v T_r + \frac{R u}{r + R} = \alpha_f \left(T_r + \frac{1}{r + R}T_r\right) + \frac{\rho_f}{\rho_f(T_r)} \left(D_b T_r C_r + \frac{D_i}{T_{\infty}}(T_r) \right) + \frac{Q}{(\rho c_p f)}(T - T_{\infty}), \quad (4)
\]

\[
v C_r + \frac{R u}{r + R} = D_b \left(C_r + \frac{1}{r + R}C_r\right) + \frac{D_i}{T_{\infty}} \left(T_r + \frac{1}{r + R}T_r\right)
\]

\[-K_r (C_r - C_{\infty}) \left(\frac{T}{T_{\infty}}\right)^n \exp \left(-\frac{E_n}{kT}\right), \quad (5)
\]

With respected boundary conditions

\[
at \quad r \to 0 \quad u = ce^z, \quad v = v_w, \quad T = T_w + \lambda(T_r), \quad D_b C_r + \frac{D_i}{T_{\infty}}(T_r) = 0,
\]

\[
at \quad r \to \infty \quad u \to u_w, \quad T \to T_{\infty}, \quad C \to C_{\infty}. \quad (6)
\]
Similarity transformations are follows as [10]

$$
T = T_\infty + \varepsilon^2 (T_w - T_\infty) \theta (\xi),
\xi = \frac{a}{\nu' f'} r, \ u = a \varepsilon^2 \eta' (\xi), \ v = \frac{R}{(r + R) \sqrt{\nu'/a}} e^2 f' (\xi),
$$

$$
C = C_\infty + \varepsilon^2 (C_w - C_\infty) \phi (\xi), \ P = \rho \sigma^2 (e^{2\xi}) \rho (\xi).
$$

(7)

On the equations (1)–(6) an appropriate similarity transformations are applied. The, viscosity, thermal diffusivity and density are represented by $\mu_f$, $\alpha_f$ and $\rho_f$ respectively; $T_\infty$ is the fence hotness, pressure represented by $p$, curvature of the curves belt is represented by $R$, and free stream temperature is labeled by $T$. Apply the preselected similarity transformation on the modeled PDEs; we got the coupled ODEs as below:

$$
P' = \frac{f'^2}{(\xi + k_0)}, \quad (8)
$$

$$
\frac{2K}{\xi + k_0} p (\xi) = \left( f'' + \frac{1}{(\xi + k_0)} f'' + \frac{1}{(\xi + k_0)^2} f' \right) + \frac{k_0 R_0}{(\xi + k_0)^2} f'' = \frac{k_0 R_0}{(\xi + k_0)} f'' + \frac{R_0}{(\xi + k_0)} f' = 0, \quad (9)
$$

$$
\frac{1}{\Pr} \left( \theta'' + \frac{1}{(\xi + k_0)} \theta \right) - \frac{k_0 R_0}{(\xi + k_0)} (f' \theta' + f'' \theta) + N_6 R_0 \phi' \theta' + N_6 R_0 \theta'' + 2 \delta \theta = 0, \quad (10)
$$

$$
\left( \phi'' + \frac{1}{(\xi + k_0)} \phi \right) - \frac{k_0 R_0}{(\xi + k_0)} (f' \phi' + f'' \phi) + \frac{N_7}{N_6} \theta'' + \frac{1}{(\xi + k_0)} \theta = 0, \quad (11)
$$

Solving equations (8) and (9), by eliminating the pressure term

$$
\left( f'' + \frac{1}{(\xi + k_0)} f'' - \frac{1}{(\xi + k_0)^2} f' \right) + \frac{k_0 R_0}{(\xi + k_0)} (f' f'' - f'' f') + \frac{k_0 R_0}{(\xi + k_0)} f' f'' = \frac{k_0 R_0}{(\xi + k_0)} f' f'' + \frac{R_0 f''}{(\xi + k_0)} f' = 0, \quad (12)
$$

$$
\left( \theta'' + \frac{1}{(\xi + k_0)} \theta \right) - \frac{k_0 R_0}{(\xi + k_0)} (f' \theta' + f'' \theta) + N_6 R_0 \phi' \theta' + N_6 R_0 \theta'' + 2 \delta \theta = 0, \quad (13)
$$

$$
\left( \phi'' + \frac{1}{(\xi + k_0)} \phi \right) - \frac{k_0 R_0}{(\xi + k_0)} (f' \phi' + f'' \phi) + \frac{N_7}{N_6} \theta'' + \frac{1}{(\xi + k_0)} \theta = 0, \quad (14)
$$

Boundary conditions are

$$
f (0) = \gamma, \ f' (0) = \lambda + \xi (f'' (0) - f' (0) / k_0), \ f' (\infty) = 1
$$

$$
f'' (\infty) = 0, \ N_6 \phi' (0) + N_7 \theta' (0) = 0,
$$

$$
M \theta' (0) + 1 = \theta (0), \ \theta (\infty) = 0, \ \phi (\infty) = 0 \quad (15)
$$

Wherever in the case of, if $\gamma > 0$ is the injection and $\gamma < 0$ is the suction, Brownian motion parameter $N_6$, $M$ is the magnetic field parameter, $\beta$ represents the stretching parameter, the thermophoresis parameter $N_6$, $R_0$ is the dimensionless constraint, the curvature constraint is $k_0$ and $k$ is the thermal conductivity. Along the s direction the physical properties such as Nusselt number and skin friction coefficients are shown as

$$
C_f = \frac{\tau_w}{\rho_1 u_w}, \ Nu = \frac{\sigma q_w}{k_f (T_w - T_\infty)}, \ Sh_s = \frac{sh_m}{D_0 (C_w - C_\infty)} \quad (16)
$$

Here, the shear stress at wall is denoted by $\tau_w$ and heat flux by $q_w$. The mathematical expressions for shear stress at wall and heat flux are:

$$
q_w = \left( \frac{\partial T}{\partial r} \right)_{r=0}, \ \tau_w = \left( \frac{\partial u}{\partial r} - \frac{u}{r + R} \right)_{r=0}, \ h_m = - \left( \frac{\partial C}{\partial r} \right)_{r=0} \quad (17)
$$
We get the dimensionless form by using equation (18) in equation (17)

\[
\text{Re}_\tau \frac{1}{\gamma} C_f = \left( f''(0) - \frac{f'(0)}{k} \right),
\]

(18)

\[
\text{Re}_\tau \frac{1}{\gamma} \text{Nu} = -\theta'(0),
\]

(19)

\[
\text{Re}_\tau \frac{1}{\gamma} \text{Sh} = -\phi'(0).
\]

(20)

Where Reynolds number is \( \text{Re}_\tau = \frac{\omega \bar{d}}{v} \).

3. Entropy analysis

The measurement of unwanted heat energy which is created during any irreversible processes in the nature is recognized as entropy production. The mathematical dimensionless expression for the entropy production is given below [67]:

\[
E_G = \frac{k_{a f}}{T_\infty^2} (T_r)^2 + \frac{\mu_{a f}}{T_\infty} \left( u_r + \frac{1}{r + R} u \right)^2 + \frac{\sigma_{a f} \beta_0^2}{T_\infty} u + \frac{\text{RD}}{T_\infty} (C_r T_r) + \frac{\text{RD}}{C_\infty} (C_r)^2,
\]

(21)

The rate of transfer for the characteristic entropy is explained via a mathematical expression as

\[
E_0 = \frac{ak_f(T_r - T_\infty)}{T_\infty^\gamma}.
\]

The rate of volumetric entropy production \( (N_s) \) is mathematically expressed by the ratio of total entropy production \( (E_G) \) to the rate of transfer of characteristic entropy \( (E_0) \) given as under:

\[
N_s = \frac{E_G}{E_0} = \frac{T_\infty^\gamma}{ak_f(T_r - T_\infty)} \left[ \frac{k_{a f}}{T_\infty^2} (T_r)^2 + \frac{\mu_{a f} \text{H}_{a f}}{T_\infty} \left( u_r + \frac{1}{r + R} u \right)^2 + \frac{\sigma_{a f} \beta_0^2}{T_\infty} u + \frac{\text{RD}}{T_\infty} (C_r T_r) + \frac{\text{RD}}{C_\infty} (C_r)^2 \right] + \frac{\text{RD}}{T_\infty} (C_r T_r) + \frac{\text{RD}}{C_\infty} (C_r)^2.
\]

(22)

Use the non-dimensional transformation in equation (7) the above equation take the following non-dimensional form

\[
N_s = \frac{\alpha \theta'^2 + \frac{Br}{\alpha} \left( f'' + \frac{1}{\xi + k_0} f' \right)^2 + MB_r f'^2 + \frac{\lambda_1 \zeta \phi'}{\alpha} + \frac{\lambda_1 \zeta^2}{\alpha} \phi^2}{\alpha \theta'^2 + \frac{Br}{\alpha} \left( f'' + \frac{1}{\xi + k_0} f' \right)^2 + MB_r f'^2 + \frac{\lambda_1 \zeta \phi'}{\alpha} + \frac{\lambda_1 \zeta^2}{\alpha} \phi^2}.
\]

(23)

Where \( \lambda_1 = \frac{RD \xi}{k_1} \) is diffusive constant parameter, \( Br = \frac{\nu_1 \xi}{\xi (T_r - T_\infty)} \) is Brinkman number, and \( \alpha = \frac{(T_r - T_\infty)}{T_\infty} \) is dimensionless temperature, \( \zeta = \frac{C_r - C_\infty}{C_\infty} \) is dimensionless concentration.

The Bejan number is defined as

\[
\text{Be} = \frac{\alpha \theta'^2 + \frac{Br}{\alpha} \left( f'' + \frac{1}{\xi + k_0} f' \right)^2 + MB_r f'^2 + \frac{\lambda_1 \zeta \phi'}{\alpha} + \frac{\lambda_1 \zeta^2}{\alpha} \phi^2}{\alpha \theta'^2 + \frac{Br}{\alpha} \left( f'' + \frac{1}{\xi + k_0} f' \right)^2 + MB_r f'^2 + \frac{\lambda_1 \zeta \phi'}{\alpha} + \frac{\lambda_1 \zeta^2}{\alpha} \phi^2}.
\]

(24)

4. Solution by HAM

An optimal approach is utilized for the solution procedure. Conditions (12–14) with boundary conditions (15) are settled by HAM. Mathematica programming is utilized for this point. This technique is semi-analytical and presents solutions to the problem in functional form. In order to derive this technique he has taken into consideration one of the elementary idea of topology known as homotopy. We know that two functions are homotopic if one can be continuously distorted in other functions. The fundamental induction of the model equation through HAM is given in detail below.

\[
L_f (\tilde{f}) = \tilde{f}'' \quad L_\phi (\tilde{\phi}) = \tilde{\phi}'' \quad L_\phi (\tilde{\phi}) = \tilde{\phi}''
\]

(25)

\[
L_f, L_\phi, \text{ and } L_\phi \text{ are signified as}
\]

\[
L_f (e_1 + e_2 \xi + e_3 \xi^2 + e_4 \xi^3) = 0, \quad L_\phi (e_5 + e_6 \xi) = 0, \quad L_\phi (e_7 + e_8 \xi) = 0,
\]

(26)
The consistent non-linear operators are reasonably designated as $N_f$, $N_{\phi}$ and $N_{\bar{\phi}}$ and recognize in system:

\[
N_f \left[ \ddot{f}(\xi; \zeta) \right] = \left( \frac{\partial f}{\partial \xi} \right)_{\xi=0} = \gamma, \quad \frac{\partial^2 f}{\partial \xi^2} \left|_{\xi=0} \right. = 0, \quad N_{\phi} \left[ \ddot{\phi}(\xi; \zeta) \right] = \left. \frac{\partial \phi}{\partial \xi} \right|_{\xi=0} = 0, \quad M \left[ \ddot{\bar{\phi}}(\xi; \zeta) \right] = \left. \frac{\partial \bar{\phi}}{\partial \xi} \right|_{\xi=0} = 0
\]

While the embedding constraint $\zeta \in [0, 1]$, to control for $h_f$, $h_{\phi}$ and $h_{\bar{\phi}}$ are utilized. While $\zeta = 0$ and $\zeta = 1$ we have:

\[
\begin{align*}
f(\xi; 1) &= f(\xi; \zeta = 1) = \bar{\phi}(\xi), \quad \bar{\phi}(\xi; 1) = \phi(\xi), \\
\end{align*}
\]

Expand the $f(\eta; \zeta)$, $\bar{\phi}(\eta; \zeta)$ and $\phi(\eta; \zeta)$ through Taylor's series for $\zeta = 0$

\[
\begin{align*}
f(\xi; \zeta) &= f_0(\xi) + \sum_{n=1}^{\infty} f_n(\xi) \zeta^n \\
\bar{\phi}(\xi; \zeta) &= \bar{\phi}_0(\xi) + \sum_{n=1}^{\infty} \bar{\phi}_n(\xi) \zeta^n \\
\phi(\xi; \zeta) &= \phi_0(\xi) + \sum_{n=1}^{\infty} \phi_n(\xi) \zeta^n
\end{align*}
\]

\[
\begin{align*}
\ddot{f}_n(\xi) &= \frac{1}{n!} \left. \frac{\partial^n f}{\partial \xi^n} \right|_{\xi=0}, \\
\ddot{\phi}_n(\xi) &= \frac{1}{n!} \left. \frac{\partial^n \phi}{\partial \xi^n} \right|_{\xi=0}, \\
\ddot{\bar{\phi}}_n(\xi) &= \frac{1}{n!} \left. \frac{\partial^n \bar{\phi}}{\partial \xi^n} \right|_{\xi=0}
\end{align*}
\]

Whereas BCs are:

\[
\begin{align*}
\ddot{f}(0) &= f'(0) = \lambda + s(f''(0) - f'(0)/k_0), \\
\ddot{f}(\infty) &= 0, \\
M\dddot{\bar{\phi}}(0) &= + 1 = \bar{\theta}(0), \quad \bar{\theta}(\infty) = 0, \quad \bar{\phi}(\infty) = 0.
\end{align*}
\]
Table 1 displays the impact of various physical constraints like $k_0$, $k$, $R_0$, and $M$ on Skin friction $Re$ $\tau^\nu_{ij} = \left( f''(0) - \frac{1}{\chi} f'(0) \right)$, when 
$\delta = 0.35$, $\gamma = 0.12$, $Pr = 0.65$, $Nb = 0.27$, $Nt = 0.33$, $Sc = 0.37$, $\delta_1 = 0.22$, $E_t = 0.32$, $\sigma = 0.19$, $n = 1$.

| $k_0$ | $k$ | $R_0$ | $M$ | $f''(0) - \frac{1}{\chi} f'(0)$ |
|-------|-----|-------|-----|----------------------------------|
| 0.2   | 0.8 | 0.3   | 0.7 | 1.5587488                       |
| 0.3   |     |       |     | 1.4488124                       |
| 0.4   |     |       |     | 1.3237973                       |
| 0.2   |     | 0.3   | 0.7 | 6.3112779                       |
| 0.3   |     | 0.3   | 0.4 | 4.6611815                       |
| 0.4   |     | 0.4   | 0.2 | 3.8434812                       |
| 0.2   |     | 0.3   | 0.3 | 6.6247341                       |
| 0.3   |     | 0.3   | 0.4 | 6.6148066                       |
| 0.4   |     | 0.4   | 0.2 | 6.6048550                       |

Now

$\mathfrak{N}_{n}^\xi(\xi) = \left( f''_{n-1} + 2 f''_{n-1} (\xi + k_0) - \frac{1}{(\xi + k_0)^2} f''_{n-1} + \frac{1}{(\xi + k_0)^3} f''_{n-1} \right) - \frac{k_0 R_0}{(\xi + k_0)^4} \left( \sum_{j=0}^{n-1} f''_{w-1-j} \tilde{j}_{n} - \sum_{j=0}^{n-1} f''_{w-1-j} \tilde{j}_{n} \right)$

$+ \frac{k_0 R_0}{(\xi + k_0)^3} \sum_{j=0}^{n-1} f''_{w-1-j} \tilde{j}_{n} + \frac{k_0 R_0}{(\xi + k_0)^2} \left( f''_{n-1} + \sum_{j=0}^{n-1} f''_{w-1-j} \tilde{j}_{n} \right) + ((Ha)^2 + k)^{\nu}_{n-1}$

(38)

$\mathfrak{N}_{n}^\nu(\xi) = \frac{1}{Pr} \left( \frac{1}{(\xi + k_0)} \tilde{j}_{n-1} \right) + \frac{k_0 R_0}{(\xi + k_0)^2} \sum_{j=0}^{n-1} \tilde{j}_{w-1-j} \tilde{j}_{n} + \sum_{j=0}^{n-1} \tilde{j}_{w-1-j} \tilde{j}_{n} + 2k \tilde{j}_{n-1}$

(39)

$\mathfrak{N}_{n}^\phi(\xi) = \frac{1}{Sc} \left( \frac{1}{(\xi + k_0)} \tilde{j}_{n-1} \right) + \frac{k_0 R_0}{(\xi + k_0)^3} \sum_{j=0}^{n-1} \tilde{j}_{w-1-j} \tilde{j}_{n} + \sum_{j=0}^{n-1} \tilde{j}_{w-1-j} \tilde{j}_{n}$

$+ \frac{Nt}{Nb} \left( \frac{1}{(\xi + k_0)} \tilde{j}_{n-1} \right) + \sigma \phi (\delta \tilde{j}_{n-1} + 1)^{\nu} \exp \left( - \frac{E}{(\delta \tilde{j}_{n-1} + 1)} \right)$

(40)

While

$\chi_n = \begin{cases} 
0, & \text{if } \xi \leq 1 \\
1, & \text{if } \xi > 1.
\end{cases}$

(41)

5. Results and discussion

5.1. Skin friction, nusselt number and sherwood number

Table 1 displays the impact of various physical constraints like $k_0$, $k$, $R_0$, and $M$ on Skin friction $Re$ $\tau^\nu_{ij} = \left( f''(0) - \frac{1}{\chi} f'(0) \right)$, $\Delta C_i$. The numerical table 1 displays that with the augmentation in curvature parameter $k_0$ the skin friction enhances. By the increase in warm conductivity $k$ of the fluid the viscosity of the fluid reduces so the skin friction of the fluid reduces. When we are stretching (enhancing $R_0$) the belt then the skin fraction of the fluid past the belt becomes low. Also, we observe from the same table 1 that with the augmentation of the thermal slip parameter $M$ the skin friction of the fluid enhances. From table 2 we see various physical parameters impact the heat exchange rate. At the point when we improve the curving parameter of the bent outward, the warmth move rate over the curved surface upgrades, additionally with the extending (upgrading) the trading of warmth becomes quick. As we generate more heat (enhancing the value of $\delta_i$) the exchange rate of heat boosts over the curved surface. By the augmentation in Prandtl number (Pr) the rate of flow of heat reduces. The enhancement in $Nt$ the Nusselt number reduces, but the action for $Nb$ is reverse. Table 3 displays the outcome of different physical parameters over Sherwood parameters. From table 3 we see that the Sherwood number enhances for $k_0$, $\delta$ and $Sc$ while the reverse action is observed for $R_0$, $\sigma$, and activation energy $E_t$. 
5.2. Consequences of physical constraints on velocity profile

We considered a steady state mixed convection flow past a curved surface. We established a logical model that we unraveled arithmetically utilizing the HAM technique. The physical limits which are engaged with the liquid stream conduct have been featured through graphs and tables. The effects of the $k_0$, $R_0$, and $k$ on velocity profile are shown in figures 2–5. Figure 2 displays the effects of the physical constraint on the velocity profile. The explanation is that by the expanding curvature parameter, the twisting of the surface just as the radius diminishes. Henceforth, less friction will be offered to liquid particles by the surface and velocity inspires. Figure 3 indicates the effect of $R_0$ on $f'(\xi)$. The force limits level thickness increments as $R_0$ increments. It is featured that accomplishments of the force limit level thickness is further delicate to the extending constraint than the contracting constraint. Figure 4 signifies the upshot of expanding of $k$, the augmentation in the values of $k$ goes to the reduction in a fluid motion. Physically this can be interpreted as, an increase in values of $k$ increases the porous spaces in a medium which creates opposition in the stream track and ultimately drops the

| $k_0$ | $R_0$ | $\delta_i$ | $Pr$ | $Nr$ | $Nb$ | $-\theta'(0)$ |
|-------|-------|-------------|------|------|------|-------------|
| 0.2   | 0.3   | 0.4         | 2    | 0.7  | 0.5  | 0.9067078   |
| 0.3   | 1.0186833 |
| 0.4   | 1.1409559 |
| 0.2   | 0.8859071 |
| 0.3   | 0.9067078 |
| 0.4   | 0.9277133 |
| 0.2   | 1.1111925 |
| 0.3   | 1.0682112 |
| 0.4   | 1.0244866 |
| 0.2   | 1.0253902 |
| 0.3   | 1.0333111 |
| 0.4   | 1.0380384 |
| 0.2   | 1.0426811 |
| 0.3   | 1.0590229 |
| 0.4   | 1.0353744 |
| 0.2   | 0.8935999 |
| 0.3   | 0.8979632 |
| 0.4   | 0.9023325 |

| $k_0$ | $R_0$ | $\sigma$ | $\delta$ | $Sc$ | $E_i$ | $-\phi'(0)$ |
|-------|-------|----------|-----------|------|------|-------------|
| 0.3   | 0.3   | 0.4      | 1.2       | 0.8  | 0.2  | 0.9731142   |
| 0.4   | 0.9984171 |
| 0.5   | 1.0246052 |
| 0.3   | 0.9519709 |
| 0.4   | 0.9541636 |
| 0.5   | 0.9563667 |
| 0.3   | 0.9621656 |
| 0.4   | 0.9793572 |
| 0.5   | 0.9965097 |
| 1.3   | 0.9823888 |
| 2.3   | 1.0126503 |
| 3.3   | 1.0428117 |
| 1.3   | 0.9552305 |
| 1.5   | 0.9563354 |
| 1.7   | 0.9578408 |
| 0.3   | 0.9458132 |
| 0.5   | 0.9274655 |
| 0.7   | 0.9110749 |

Table 2. Impact of various physical parameters over Nusselt number

Table 3. Impact of various physical parameters over Sherwood number

5.2. Consequences of physical constraints on velocity profile

We considered a steady state mixed convection flow past a curved surface. We established a logical model that we unraveled arithmetically utilizing the HAM technique. The physical limits which are engaged with the liquid stream conduct have been featured through graphs and tables. The effects of the $k_0$, $R_0$, $M$, and $k$ on velocity profile are shown in figures 2–5. Figure 2 displays the effects of the physical constraint on the velocity profile. The explanation is that by the expanding curvature parameter, the twisting of the surface just as the radius diminishes. Henceforth, less friction will be offered to liquid particles by the surface and velocity inspires. Figure 3 indicates the effect of $R_0$ on $f'(\xi)$. The force limits level thickness increments as $R_0$ increments. It is featured that accomplishments of the force limit level thickness is further delicate to the extending constraint than the contracting constraint. Figure 4 signifies the upshot of expanding of $k$, the augmentation in the values of $k$ goes to the reduction in a fluid motion. Physically this can be interpreted as, an increase in values of $k$ increases the porous spaces in a medium which creates opposition in the stream track and ultimately drops the
Figure 2. Influences of $k_0$ on $f'(\xi)$. 

Figure 3. Effect of $R_0$ on $f'(\xi)$. 

Table 4. Impact of various physical parameters over Velocity profile $f'(\xi)$ when, $\delta = 0.35$, $\gamma = 0.12$, $Pr = 0.65$, $Nb = 0.27$, $Nt = 0.33$, $Sc = 0.37$, $\delta_1 = 0.22$, $E_1 = 0.32$, $\sigma = 0.19$, $n = 1$. 

| $k_0$ | $k$ | $R_0$ | $M$ | $f'(\xi)$ |
|-------|-----|-------|-----|-----------|
| 0.2   | 0.8 | 0.3   | 0.7 | 0.7371958 |
| 0.3   |     |       |     | 0.7974685 |
| 0.4   |     |       |     | 0.8661532 |
|       | 0.2 |       |     | 0.2155668 |
|       | 0.3 |       |     | 0.2074967 |
|       | 0.4 |       |     | 0.1999968 |
|       |     | 0.2   |     | 0.0838738 |
|       |     | 0.3   |     | 0.0880568 |
|       |     | 0.4   |     | 0.0922541 |
|       |     |       | 0.2 | 0.1059668 |
|       |     |       | 0.3 | 0.1006621 |
|       |     |       | 0.4 | 0.0964444 |
momentum of nanofluid and thickness of connected boundary layer. Figure 5 displays that the $f'('\xi')$ is a reducing function of $M$. Actually, with augmentation in the magnetic field, the Lorentz force also jumps up that acts in the opposite direction to flow of fluid which ultimately reduces the motion of flow system. Hence increasing values of the magnetic parameter reduces the velocity profile. The numerical table of the velocity profile is given in table 4, from table 4 we see that the velocity profile enhances for curvature parameter $k_0$, stretching parameter $R_0$, while the velocity profile from table 4 decreases with the augmentation in thermal conductivity $k$ thermal slip condition $M$ of the nanofluid.

5.3. Consequences of physical constraints on heat profile

The impacts of $k_0$, $R_0$, $N_a$, $N_r$, Pr and $\delta$ on temperature profile are shown in figures 6–11. The impacts of physical parameters $k_0$ and $R_0$ over temperature profile of the fluid are illustrated in figures 6 and 7. From these two figures 6 and 7, we observed that the augmentation of $k_0$ and $R_0$ the thermal boundary layer thickness reduces. We noted a fantastic outcome that as compared to shrinking in the surface the thermal boundary layer thickness jumps up with stretching in the surface. The relation among $N_a$ and temperature profile $\theta(\xi)$ are portrayed in figure 8. From this figure we see that raise in $N_a$ induces random acceleration in nanoparticles. In this physical phenomenon, extra energy is generated in the flow system that in turn increases the thermal boundary layer, and hence the temperature of fluid rises. The sway of $N_r$ on $\theta(\eta)$ is existing in figure 9. Since fluid particles at a colder level forced back those particles which are at the warmer level in thermophoresis.

Figure 4. Impact of $k$ on $f'('\xi')$.

Figure 5. Effect of $M$ on $f'('\xi')$. 

Figure 6.
phenomenon. This results in the transmission of fluid particles from the warmer level to the colder level and increases the thermal boundary layer for the flow system. Hence increase in $N_t$ augments heat of nanofluid. It is evident from figure 10 that the bigger $Pr$ constrains to diminish the thin film section the liquid heat. Expanding $Pr$, diminishes the thickness of the warm limit layer. Then $Pr$ is the extent of the comparative consequence of warm diffusivities and momentum, and subsequently bigger the $Pr$ lesser the warm dissemination. Thusly the consequences of the physical significance of $Pr$ are in incredible congruity. Figure 11 shows the impression of the heat generation/absorption parameter $d_1$ on the heat profile $\theta(\xi)$ of the active fluid. There are two cases; Case 1 when $d_1 > 0$ it shows the generation of heat while when $d_1 < 0$ it leads to heat absorption. The temperature profile $\theta(\xi)$ and thermal layer both are boosted for the augmentation in $d_1$. Table 5 displays numerically the effect of various bodily limits over the heat profile $\theta(\xi)$ of the nanofluid.

5.4. Belongings of physical parameters to concentration profile

The impacts of $k_0$, $R_0$, $N_t$, $N_p$, $\delta$, $Sc$, $E$, and $\sigma$ on concentration profile are revealed in figures 12–18. The effect of the physical limits $k_0$ and $R_0$ on the absorption profile $\phi(\xi)$ is portrayed in figures 12 and 13. From figure 12 we reveal that the $\phi(\xi)$ of the active fluid reduces with the augmentation in $k_0$. The solutal layer thickness of the fluid boosted with stretching of the sheet as compared to the shrinking of the sheet. The impact of the physical parameter $R_0$ is demonstrated in figure 13. We observed from figure 13 that when we increase the value of $R_0$ the...
Figure 8. Bearing of $N_b$ on $\theta(\xi)$.

Figure 9. Effect of $N_t$ on $\theta(\xi)$.

Table 5. Influence of various physical parameters over Temperature profile $\theta(\xi)$ When $Ha = 0.7$, $M = 0.6$, $\delta = 0.34$, $\gamma = 0.12$, $Sc = 0.37$, $E_t = 0.32$, $\sigma = 0.19$, $n = 1$.

| $k_e$ | $R_d$ | $\delta_l$ | $Pr$ | $N_t$ | $N_b$ | $\theta(\xi)$ |
|-------|-------|------------|------|-------|-------|----------------|
| 0.2   | 0.3   | 0.4        | 2    | 0.7   | 0.5   | 0.2845811      |
| 0.3   |       |            |      |       |       | 0.2561686      |
| 0.4   |       |            |      |       |       | 0.2266090      |
|       | 0.2   |            |      |       |       | 0.2911684      |
|       | 0.3   |            |      |       |       | 0.2845811      |
|       | 0.4   |            |      |       |       | 0.2779911      |
|       |       | 0.2        |      |       |       | 0.4138982      |
|       |       | 0.3        |      |       |       | 0.4311034      |
|       |       | 0.4        |      |       |       | 0.4488238      |
|       |       |            | 0.2  |       |       | 0.3467673      |
|       |       |            | 0.3  |       |       | 0.3466320      |
|       |       |            | 0.4  |       |       | 0.3465538      |
|       |       |            |      | 0.2   |       | 0.4440804      |
|       |       |            |      | 0.3   |       | 0.4450293      |
|       |       |            |      | 0.4   |       | 0.4459781      |
|       |       |            |      |       | 0.2   | 0.2884831      |
|       |       |            |      |       | 0.3   | 0.2871825      |
|       |       |            |      |       | 0.4   | 0.2858819      |
Figure 10. Sway of Pr on $\theta(\xi)$.

Figure 11. Impression of $\delta_1$ on $\theta(\xi)$.

Figure 12. Consequences of $k_0$ on $\phi(\xi)$.
concentration profile $\phi(\xi)$ of the fluid increased the solutal layer thickness. The estimation of solutal layer thickness boosted in extending when contrasted with the instance of contracting. We can see the impact of $N_s$ and $N_b$ over the concentration profile $\phi(\xi)$ of the active fluid in figures 14 and 15. From figure 14 we observed that the concentration profile $\phi(\xi)$ of the fluid decreases as the value of $N_b$ rises. Since a rise in $N_b$ causes an increase in the random motion of nanofluid, which diminishes the thickness of the boundary layer, as a result of which concentration profile of flow system reduces. Figure 15 exhibit the effect of the $N_s$ on $\phi(\xi)$ of the fluid. It is realized that the $\phi(\xi)$ expanded for more prominent estimations of $N_s$ whereas on account of the extending exterior, the estimations of solutal layer thickness expanded when contrasted with the contracting exterior. Figure 16 shows the decrease in concentration profile $\phi(\xi)$ for the bigger estimations of the Schmidt number $Sc$. According to physical explanation, the Schmidt number $Sc$ illustrates the Brownian diffusivity.

With the enhancing of Schmidt number $Sc$ the particles of the nanofluid start more Brownian diffusion. Due to the more Brownian diffusion of the particle concentration field declines. From figure 17 we observed the impact of the non-dimensional energy $E$ on the concentration profile $\phi(\xi)$. The enhancement in the values of activation energy $E$ the Arrhenius work done $\left( \frac{E_v}{TK} \right)$ decreases. This unavoidably develops the generatively manufactured reaction on account of which concentration profile upgrades. From figure 18 we observed that the concentration profile $\phi(\xi)$ and the corresponding layers of the fluid reduces with the augmentation in the

Table 6. Impact of various physical parameters over concentration profile $\phi(\xi)$ When $Ha = 0.7$, $M = 0.6$, $\delta = 0.47$, $\gamma = 0.12$, $Pr = 0.65$, $Nb = 0.5$, $Nt = 0.7$, $n = 1$.

| $k_0$ | $R_0$ | $\sigma$ | $\delta$ | $Sc$ | $E_1$ | $\phi(\xi)$ |
|------|------|------|------|------|------|---------|
| 0.3  | 0.3  | 0.4  | 1.2  | 0.8  | 0.2  | 0.3518281 |
| 0.4  |      |      |      |      |      | 0.3456313 |
| 0.5  |      |      |      |      |      | 0.3393285 |
|      | 0.3  |      |      |      |      | 0.3501816 |
|      | 0.4  |      |      |      |      | 0.3498006 |
|      | 0.5  |      |      |      |      | 0.3494186 |
|      | 0.3  |      |      |      |      | 0.3534884 |
|      | 0.4  |      |      |      |      | 0.3501816 |
|      | 0.5  |      |      |      |      | 0.3468895 |
|      | 1.3  |      |      |      |      | 0.3497059 |
|      | 2.3  |      |      |      |      | 0.3449645 |
|      | 3.3  |      |      |      |      | 0.3402527 |
|      |      | 1.3  |      |      |      | 0.5903515 |
|      |      | 1.5  |      |      |      | 0.5899128 |
|      |      | 1.7  |      |      |      | 0.5894740 |
|      | 0.3  |      |      |      |      | 0.3393409 |
|      | 0.5  |      |      |      |      | 0.3973285 |
|      | 0.7  |      |      |      |      | 0.6012696 |
Figure 14. Bearing of $N_b$ on $\phi(\xi)$.

Figure 15. Consequences of $N_f$ on $\phi(\xi)$.

Figure 16. Concerns of $Sc$ on $\phi(\xi)$. 
Figure 17. Influence of $E$ on $\phi(\xi)$.

Figure 18. Outcome of $\sigma$ on $\phi(\xi)$.

Figure 19. Impression of $\delta$ on $\phi(\xi)$ $w$. 

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Figure 20. Sway of $M$ on $N_s$.

Figure 21. Control of $Br$ on $N_s$.

Figure 22. Impression of $\alpha$ on $N_s$. 
Figure 23. Impact of $\lambda_1$ on $N_s$.

Figure 24. Consequences of $M$ on $Be$.

Figure 25. Effect of $\alpha$ on $Be$. 
compound response parameter $\sigma$. Figure 19 shows the influences of $\delta$ over the concentration profile $\phi(\zeta)$. Here we see that the concentration profile $\phi(\zeta)$ of the fluid reduces with augmentation in $\delta$. Table 6 displays numerically the effect of various physical constraints over the concentration profile $\phi(\zeta)$ of the nanofluid.

### 6. The production in entropy and Bejan number

The impact of the magnetic parameter $M$, diffusive constant parameter $\lambda$, parameter of the temperature difference $\alpha$, and Brinkman number $Br$ upon entropy generation rate $Ns$ is shown in figures 20–23. Figure 20 we see that with increase in magnetic there is a reduction in $Ns$. Figure 21 indicates the impression of Brinkman number $Br$ upon entropy generation rate. Since $Br$ establishes effects of viscosity in nanofluid which reduces $Ns$ as shown in figure 21. It is also obvious that viscous forces resist the motion of fluid and augments fluid friction that ultimately results in the augmentation of entropy generation. Impact of temperature difference parameter $\alpha$ upon $Ns$ portrays in figure 22. It is perceived that $Ns$ decreases due to increase in temperature difference parameter. Figure 23 shows that entropy age ascends with expanding estimations of diffusive consistent parameter $M$, Magnetic parameter $M$, Brinkman number $Br$ and temperature difference parameter $\alpha$ have some impacts upon Bejan number $Be$ as given in figures 24. Figure 24 indicates that for dissimilar values of Brinkman number, there are some variations in Bejan number. From this figure, we see that $Be$ diminishes because of a comparing increment in Brinkman number. Figure 25 indicates that for increasing the values of the magnetic parameter the Bejan number reduces in the closed vicinity of the stretching sheet. It is observed that the Bejan number increases with the corresponding augmentation in temperature difference parameter respectively as shown in these figures. Table 7 displays numerically the impact of various physical parameters over the production of unwanted energy ($Ns$) during the motion of the fluid.

### 7. Conclusions

In the current work, the steady stream of nanofluid past an exponentially stretched belt is analyzed. The curved is made up of a spongy medium. The influence of Brownian diffusion and thermophoresis diffusion on the flow of the nanofluid is also analyzed in this work. The major points which are noted during the research work are organized as:

- The Larger Magnetic field parameter shows a diminishing pattern for the two velocities.
- The momentum boundary layer thickness increases as $Re$ increases.
- An increase in values of $k_1$ decreases the momentum of the nanofluid and thickness boundary layer.
- Higher $Pr$ relates to more vulnerable temperature while the opposite conduct is realized for $\delta_i$.
- Durable temperature dissemination is grasped for $N_x$ and $N_t$.
- Advanced enactment energy $E$ shows more concentration profile.
- Upgrading in the compound reaction constraint $\sigma$ shows the decay in the concentration profile and its related layer.

| $\alpha$ | $\lambda$ | $M$ | $Br$ | $Ns(\zeta)$ |
|----------|-----------|-----|------|-------------|
| 0.3      | 0.8       | 0.1 | 0.4  | 0.2780552   |
| 0.5      | 0.2       | 0.4 |      | 0.1245551   |
| 0.7      | 0.3       | 0.5 |      | 0.0763490   |
| 0.3      | 0.5       | 0.7 |      | 0.1459788   |
| 0.5      | 0.6       | 0.7 |      | 0.1988094   |
| 0.7      | 0.7       | 0.3 |      | 0.2516399   |
| 0.3      | 0.8       | 0.5 |      | 0.2756749   |
| 0.5      | 0.9       | 0.7 |      | 0.2726079   |
| 0.7      | 0.3       | 0.1 |      | 0.2689921   |
| 0.3      | 0.5       | 0.5 |      | 0.2618669   |
| 0.7      | 0.7       | 0.7 |      | 0.2942434   |
| 0.3      | 0.7       | 0.9 |      | 0.3266198   |
The entropy age ascends with expanding estimations of the diffusive constant parameter.

Both \( \phi (\xi) \) and its related layer are a lessening component of higher \( Sc \).

Concentration \( \phi (\xi) \) shows turn around behavior for \( N_t \) and \( N_s \).

Both concentration \( \phi (\xi) \) and its related layer thickness are diminishing elements of \( Sc \).

With the increase in the magnetic, there is a reduction in \( N_s \).

It is seen that \( N_s \) diminishes because of increment in temperature difference parameter.

The entropy age ascends with expanding estimations of the diffusive constant parameter.

The \( Be \) decreases due to a corresponding increase in Brinkman number.

Bejan number increases with the corresponding augmentation in temperature difference parameter.

Conflicts of interest

The authors declare that they have no competing interests.

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