Modification of the Stamp Topological Optimization Taking into Account Cyclic Fatigue based on the Finite Element Approach

Ivan K. Andrianov1,2,
1Federal State-Financed Educational Institution of Higher Learning
"Komsomolsk-na-Amure State University";
Lenina pr., 27, Khabarovsk Krai, Komsomolsk-na-Amure, 681013, Russia
2Khabarovsk Federal Research Center of the Far Eastern Branch of the Russian Academy of Sciences,
51 Turgeneva street, Khabarovsk, 680000, Russia

Received: July 1, 2021. Revised: August 28, 2021. Accepted: September 1, 2021. Published: September 3, 2021.

Abstract—The study is devoted to optimizing the volume of stamping tools used in pressure processing processes. The relevance of the research is due to the active development of additive technologies and the possibility of producing stamping tools from plastic of optimal shape, which has an important practical significance in the manufacture of thin-walled products in the aviation and automotive industries. The purpose of the study was to carry out a mathematical formulation of the problem of topological optimization of a forming die made of a polymer material with restrictions on fatigue durability and minimum volume. The task of topological optimization was to maximize the stiffness of the die under multicyclic loading. The vector description of topological optimization was based on the finite element approach. The optimization model was built on the basis of the solid isotropic material penalization method with the introduction of additional restrictions in the model of searching for pseudo-densities of the material, taking into account the duration of the force action on the stamp under multicyclic loading. In view of the nonlinearity of the resulting system of equations, the solution of the conditional optimization problem is proposed to be carried out by constructing the Lagrange objective function and using the Lagrange multiplier method. The result of the study is the proposed approach to the topological optimization of the stamp, taking into account the multicyclic loading and restrictions on the desired volume.

Keywords—modeling, finite element method, stamp, topology, optimization

I. INTRODUCTION

The modern development of automotive and aircraft manufacturing is inextricably linked with the processes of metal forming for the manufacture of thin-walled products of the shell type. Dies, punches, reverses are the main stamping tools, and they are mostly all-metal. It is proposed to construct a numerical model for calculating the redistribution of the stamp material using the topological optimization method for a given reduction in the stamp volume. The aim of the work was to construct a mathematical model for optimizing the topology of the forming die with restrictions on cyclic fatigue based on the finite element approach. According to the research task, it was necessary to consider a model for optimizing the topology of a die with restrictions on fatigue life using the variational method. The novelty of the work lies in the fact that the SIMP (solid isotropic material penalization) and BESO (bi-directional evolutionary structural optimization) methods of topological optimization do not take into account the duration of loading, but determine the distribution of the material based on the stress level or strain energy. It is proposed to introduce restrictions on the stress state of the polymer due to fatigue life and the thermal state due to self-heating into the numerical model.

Thus, within the framework of the study, the following tasks were set:

- to carry out a mathematical formulation of the problem of topological optimization of the stamp: to determine the fixed areas of the stamp that are free from optimization; loading conditions, optimization areas, the target function and optimization variables.
- to carry out a finite-element formulation of die optimization with the introduction of the fatigue failure conditions, relations describing the development of creep deformation and self-heating of the polymer under multicyclic loading at the topological optimization model.
to determine an approach to solving the nonlinearity of the topological optimization problem using the Lagrange multiplier method.

Optimization of the topology of tools and parts under stress conditions in the last decade has become one of the most relevant areas, since it allows you to reduce the weight of the product while maintaining the strength characteristics, which is one of the components of rational production. However, the issue of product material redistribution under long-term loading conditions remains unexplored despite a large number of optimization approaches. The importance of this issue is due to the accumulation of damage, the development of creep phenomena, which must be taken into account especially for products made of polymer materials.

The optimal distribution of the material in the forming dies largely depends on the type of stress state. Methods of forming with the use of hydroforming, forming with the help of elastic and rigid punches affect the nature of the distribution of the stress-strain state in the sheet billet and stamping tools.

Forming with the help of a liquid and elastic medium provides a more uniform loading, which is reflected in the work [1]. The issues of cyclic loading and the development of the stress-strain state during fatigue deformation are studied in [2]. The features of thermal and power loading are reflected in the work [3]. The problems of estimating the stress-strain state of thin-walled products in the processes of metal forming are investigated in [4]. One of the important tasks in the problems of forming, which are mostly covered in scientific research, is the appearance of defects in the stamping processes: corrugation, jamming of the blank sheet. The issues of modeling of corrugation and methods of eliminating these defects were studied in [5, 6]. These problems are primarily caused by the process of manual production of the stamp from metals. The use of three-dimensional printing tools made of plastic will solve the problem of deviations of the stamp shape from the required geometric parameters. To solve these problems, it is necessary to study mathematical models for optimizing the topology of polymers.

Currently, methods of topological optimization using regular and irregular structures are being developed. Modeling of the optimal topology of dies using mesh structures, models of the topology of dies with restrictions on fatigue life, as well as the evaluation of the variable parameters of topological optimization are presented in the works [7 - 12]. The problems of solving optimization problems associated with the evaluation of stress criteria are reflected in [13].

It is important to note the practical implementation of forming dies of optimal topology by 3D prototyping methods, which makes it possible to rationalize the consumption of material resources, as well as to ensure their reuse. The problems of reuse of metalworking industry waste and the issues of production optimization are investigated in the works [14-17]. Methods of 3D printing, applications of additive technologies, as well as systems for automated analysis of shaping processes are studied in [1, 5, 18].

Since the problem of finding the optimal material distribution is described by a complex nonlinear system of differential equations, especially given the geometric nonlinearity of the die forming surface, one of the solution tools is the use of the finite element method. The features of the finite element approach application in the processes of sheet stamping, the choice of finite elements based on tests of mechanical properties, are reflected in the works [19-21]. The issues of topological optimization and methods of finite element modeling in various software complexes are considered in the articles [22-29].

II. METHODS

The application of the topological optimization method in the study of the stamp under conditions of long-term cyclic loading is considered in this paper. The essence of the method is to determine the location of voids in a solid die based on the condition of minimizing the volume (Fig. 1). The location of voids in the optimized object depends on the unevenness of loading, the mechanical properties of the material. The surface loading of the die p(t) is considered as an external load. The problem is solved in a quasi-static formulation, the inertial terms can be neglected due to the small accelerations when moving the points of the medium during the loading of the forming die.

where p(t) is the force loading, which is a periodic function of time, $S_p$ is the loading surface of the stamp, free from topological optimization, $S_u$ is the fixing surface, free from topological optimization, $V_{opt}$ is the optimized volume of the stamp, $V_{sav}$ is the preserved area of the topology, $V_{del}$ is the
The task of topological optimization is to determine the optimal distribution of the die material, which reduces the volume of the material based on minimizing the malleability of the die. With a small number of loading cycles, the main role in assessing the durability is played by the level of the stress state, in this regard, the distribution of the material is determined based on the exhaustion of the strength resource of the stamp, since self-heating of plastics with a small number of cycles is not significant. The construction of the topology optimization model based on the maximization of rigidity is due to the influence of the loading duration on the durability of the die due to the development of fatigue, as well as the phenomenon of self-heating. The type of stress state in the study of the strength of plastic plays a special role. All of them are under compression conditions when using solid dies. However, the use of topological optimization methods can significantly change the type of stress state, in addition to the main compressive stress, the presence of voids due to the removal of material can lead to the appearance of bending stresses.

In this regard, the construction of stamp topological optimization model will be considered on the basis of finding the optimal number and location of the stamp voids under the condition of maximizing rigidity, which is due to the preservation of the stamp shape under cyclic loading, where the phenomenon of self-heating of the polymer and changes in the stress state, in this regard, the distribution of the material is based on minimizing the malleability of the die. The mathematical description of the die topology optimization problem under the conditions of surface force loading is considered. The minimization of the stamp volume is defined as (\(V_0 - \) initial volume of the stamp):

\[
\min_{\sigma \in U_\sigma} V = kV_0, \quad 0 < k < 1.
\]

The problem of die volume optimization is constructed under constraints on the stress-strain state, taking into account the creep and self-heating of the polymer material, described by the relations:

- static equations:
  \[
  \frac{\partial \sigma_{ij}}{\partial x_i} = 0.
  \]
- geometric Cauchy relations:
  \[
  \epsilon_{ij} = \frac{1}{2} (u_{ij} + u_{ji}).
  \]
- Saint-Venant strain compatibility equations:
  \[
  \text{rot} (\text{rot} \epsilon_{ij}) = 0.
  \]
- rheological equation:
  \[
  \sigma_{ij} = \lambda (1 - \dot{\lambda}) \epsilon_{kk} \delta_{ij} + 2\mu (1 - M) \epsilon_{ij},
  \]
- durability condition of the polymer under long-term loading:
  \[
  \int_0^{NT_c} \frac{1}{\tau_0} \frac{\psi(\tau) - \psi_0}{K(T(\tau))} d\tau \leq 1.
  \]

- equation of self-heating under cyclic loading:
  \[
  c \rho V \frac{\partial T}{\partial t} = \frac{V}{t} \int_0^t \sigma d\epsilon - \alpha_g (T - T_g) S_f, \quad (2)
  \]

- boundary and initial conditions:
  \[
  \sigma_{ij} n_i |_{S_p} = p, \quad u |_{S_u} = 0, \quad T |_{t=0} = T_0.
  \]

where \(\sigma_{ij}\) is the stress tensor, \(i, j = 1,3\), \(\epsilon_{ij}\) is the strain tensor, \(u_{ij}\) is the displacement tensor, \(\tau_0 = 10^{-12} \ldots 10^{-13}\) is the material constant, \(U_0\) is the activation energy of the fracture in the absence of stress, \(g\) is the material constant, \(K(t)\) is the absolute temperature of the element as a function of time, \(c\) is the specific heat capacity, \(\rho\) is the density of the material, \(\alpha_g\) is the heat transfer coefficient, \(T_g\) is the temperature, where \(\lambda, \mu\) are the Lame coefficients:

\[
\lambda = \frac{E v}{(1 + \nu)(1 - 2\nu)}, \quad \mu = \frac{E}{2(1 + \nu)}
\]

\(\Lambda, M\) — operators of the form:

\[
\Lambda f(t) = \int_0^t \Lambda(\tau)f(\tau) d\tau, \quad M f(t) = \int_0^t M(\tau)f(\tau) d\tau,
\]

where \(\Lambda(\tau), M(\tau)\) are monotonically decreasing positive functions.

The presented system of equations is nonlinear, respectively, the use of numerical methods is required to solve it. Due to the fact that the forming dies can have a complex shape, it is proposed to optimize the stamps using the finite element method. Therefore, the mathematical description of the problem is carried out in vector form based on minimizing the work of external forces acting on the stamp:

\[
A_p = \int \{p\}^T \{u\} dS \rightarrow \min. \quad (3)
\]

The potential strain energy accumulated by the body is determined by:

\[
U = \frac{1}{2} \int \{\epsilon\}^T [D] \{\epsilon\} dV. \quad (4)
\]

For the \(i\)-th element of the stamp volume, the displacements are determined by the column vector:

\[
\{u\} = [N]\{v^i\}, \quad (5)
\]

where \([N]\) are the position functions, \([v^i]\) are the nodal displacements of the \(i\)-th displacement.
The strains is presented in matrix form through displacements:

\[ \{\varepsilon\} = [B]\{\nu\}, \]

where \([B]\) is the deformation approximation matrix.

The total energy for the element, taking into account the relations (4)-(6):

\[ \mathcal{E}_e = \frac{1}{2} \left\{ v^i \right\}^T \left[ K^i \right] \left\{ v^i \right\} - \left\{ P^i \right\}^T \left\{ v^i \right\}, \]

where \([K^i]\) is the element stiffness matrix:

\[ [K^i] = \int [B]^T [D] [B] \, dV, \]

\(\{P^i\} - \text{вектор узловых сил:}\)

\[ \{P^i\} = \int \{ p \}^T [N] \, dS \]

The equation of the work of external forces according to the relations (3) - (6) has the form:

\[ A_p = \{v\}^T [K] \{v\}. \]

The problem of topological minimization according to (3) has the form:

\[ \sum_{i=1}^{n} v_i^T k_i v_i \rightarrow \min. \]

Based on the SIMP-method of topological optimization for the exclusion of material elements, we introduce the concept of a fictitious element density:

\[ 0 < \rho_{\text{min}} < \rho_i \leq 1. \]

The density \(\rho_i\) based on the SIMP-method determines the presence or absence of material in the element. The relationship between the elastic modulus and the fictitious density is defined as [22]:

\[ E_i = \rho_i^5 E_0 + (1 - \rho_i^5) E_{\text{min}}. \]

Based on the presented relations (1)-(6), a model of finite element analysis of the stamp topology with a minimum volume is constructed:

\[ \sum_{i=1}^{n} (\rho_i)^5 v_i^T k_i v_i \rightarrow \min, \]

\[ k_i v_i - P_i = 0, \quad i \]

\[ \rho_{\text{min}} V_0 < \sum_{i=1}^{n} \rho_i V_i < V_0, \]

\[ \int_{0}^{N_f} \frac{1}{\tau_0} \frac{\gamma \sigma_{f_p} (t) - u_0}{k_f} \, dt = 1. \]

To solve the nonlinearity system equations, we use the Lagrange multipliers method:

\[ L(\rho_i, v_i) = \sum_{i=1}^{n} (\rho_i)^5 v_i^T k_i v_i + \sum_{i=1}^{n} \lambda_i^{(3)} (k_i v_i - P_i) \]

\[ + \lambda^{(2)} \left( \sum_{i=1}^{n} \rho_i V_i - V_0 \right) + \lambda^{(3)} \left( \rho_{\text{min}} V_0 - \sum_{i=1}^{n} \rho_i V_i \right) + \sum_{i=1}^{n} \lambda_i^{(5)} \omega_i \]

\[ + \sum_{i=1}^{n} \lambda_i^{(4)} \left( \int_{0}^{t} \frac{1}{\tau_0} e^{\int_{0}^{\tau} \frac{-a_g \sigma_f}{cpV_i} \, d\tau} \, dt \right) \quad = 0, \]

where \(\omega_i = T_i - \rho_i \left( \frac{a_g \sigma_f}{cpV_i} \right) \int_{0}^{t} \frac{1}{\tau_0} e^{\int_{0}^{\tau} \frac{-a_g \sigma_f}{cpV_i} \, d\tau} \, dt \)

\[ L_i^{(1)}, L_i^{(2)}, L_i^{(3)}, L_i^{(4)}, L_i^{(5)} - \text{Lagrange multipliers} \]

III. RESULTS AND DISCUSSION

The finite element calculation of the forming stamp under short-term and long-term loads was carried out in the program Ansys. PLA plastic is considered as a material. The external pressure was 8 MPa. The number of loading cycles is \(10^5\). The calculation results are shown in Fig1.- Fig.3.
Fig. 1 – Stress state of the stamp

Fig. 2 - Optimized stamp model for short-term loading

Fig. 3 - Optimized stamp model taking into account the loading time

According to the results of the calculation of a solid stamp, the intensity of stresses according to the Mises criterion is shown in Fig. 1. The initial volume of the forming die is 4793 mm$^3$. The optimized volume was 3104 mm$^3$ with topological optimization under short-term loading conditions. As a result, the volume of the stamp decreased by 35% (Fig. 2). The optimized volume was 3454 mm$^3$, taking into account the duration of the stamp loading as a result of topological optimization. The volume of the stamp has decreased by 28% (Fig. 3).

IV. CONCLUSION

Thus, the finite element formulation of the stamp volume minimizing problem with restrictions on fatigue life is carried out in the work. The accumulation of damage taking into account the creep model is taken into account when constructing a model of material redistribution. The proposed approach is important due to the inadmissibility of changing the shape of the die forming surface, since its geometric accuracy is crucial in the manufacture of thin-walled products in the aircraft industry and automotive industry. The possibility of solving the nonlinear problem of calculating the pseudo-densities of the stamp volume elements and the possibility of introducing the optimal stamp topology into the finite element calculation are described using the Lagrange multiplier method. The advantage of the proposed approach is that it allows to take into account the multi-cycle loading of the stamp in the process of force action. In practice, as a rule, the problem of topological optimization of stamps is solved in a static formulation with short-term loading. Among the limitations of this approach, it should be noted that the method does not take into account the possibility of a "chessboard" problem appearing in the optimization process, which requires further research.

ACKNOWLEDGMENT

The reported study was funded by the Russian Fund for Fundamental Research, project «Razrabotka modeli optimizacii form shtampovychnych osnastok metodom effektivnogo pereraspredeleniya materiala», project number 19-38-6002020

References

[1] Ahmetoglu M., Sutter K., Li X.J., Altan T. (2000). Tube hydroforming: current research, applications and need for training. Journal of Materials Processing Technology. V. 98. № 2. p. 224-231.
[2] Andrianov, I.K. (2020). Modeling of Forming Die Under Cyclic Loading Conditions. 2020 International Multi-Conference on Industrial Engineering and Modern Technologies, FarEastCon 2020, 1-4. DOI: 10.1109/FarEastCon50210.2020.9271120.
[3] Bychkov, S. & Tarasov, Fedor & Frizen, Vasily & Smolyanov, Ivan. (2017). Stamp tool induction heating system. 1-4. 10.1109/CPEE.2017.8093084.
[4] Andrianov, I & Stankevich, A. (2019), The stress-strain state simulation of the aircraft fuselage stretch forming in the ANSYS, Journal of Physics: Conference Series, Vol. 1333, DOI: 10.1088/1742-6596/1333/8/082002.
[5] Andrianov, I.K. & Stankevich, A.V. (2019). Finite-Element Model of the Shell-Shaped Half-Pipes Forming for Blanks Behavior Investigating During Corrugating at the Stamping, International science and technology conference "EASTCONF", EASTCONF 2019, Vladivostok, 01-02 march 2019 r. pp.1-3. DOI: 10.1109/EastConf.2019.8725322.
[6] Kawka M., Olejnik L., Rosochowski A., Sunaga H., Makinouchi A. (2001) Simulation of wrinkling in sheet metal forming. Journal of Materials Processing Technology. V. 109. № 3. p. 283-289.
[7] Andrianov, I.K. & Belykh, S.V. (2019). The Finite Element Simulation of the Stamping Die Optimal Topology, International science and technology conference "EASTCONF", EASTCONF 2019, Vladivostok, 01-02 march 2019 , pp. 1-3, DOI: 10.1109/EastConf.2019.8725410.
[8] Andrianov, I.K. (2019). Modeling of Effective Material Distribution of Stamping Equipment in Forming
Processes, International science and technology conference "EASTCONF", EASTCONF 2019, Vladivostok, October 2019 , pp. 1-3. DOI: 10.1109/FarEastCon.2019.8933949.

[9] Andrianov, I.K. (2020). Mathematical Model of Optimal Stamping Topology Based on the Stability Criterion, 2020 International Multi-Conference on Industrial Engineering and Modern Technologies, FarEastCon 2020, 1-4. DOI: 10.1109/FarEastCon50210.2020.9271408

[10] Ballo, Federico & Gobbi, Massimiliano & Mastinu, Giampiero & Previati, Giorgio. (2021). Topology Optimisation of Continuum Structures. 10.1007/978-3-030-60835-0_11.

[11] Chiu, Louis & Stojanov, Daniel & Rolfe, Bernard & Yan, Wenyi. (2016). Effect of Optimisation Parameters in Topology Optimisation. Key Engineering Materials. 725. 529-534. 10.4028/www.scientific.net/KEM.725.529.

[12] Ranjan, Rajit & Ayas, Can & Langelaar, Matthijs & Keulen, Fred. (2020). Topology Optimisation Techniques. 10.1201/9780429436543-2.

[13] Andrianov, I.K. (2019). Optimization Model of Thermal Loading of Multi-layer Shells Based on the Strength Criterion , International science and technology conference "EASTCONF", EASTCONF 2019, Vladivostok, October 2019 , pp. 1-4. DOI: 10.1109/FarEastCon.2019.8934017.

[14] Kolesnikov A.S. Kinetic investigations into the distillation of nonferrous metals during complex processing of waste of metallurgical industry. Russian Journal of Non-Ferrous Metals. 2015. V. 56. № 1. P. 1-5. https://doi.org/10.3103/S1068798X21050013

[15] Jirathearanat, S. & Prapakorn, K.. (2008). Mechanical property testing of sheet steels for accurate stamping finite element simulation. 37. 38-42.

[16] Kolesnikov, A.S. Chemical and Phase Transitions in Oxidized Manganese Ore in the Presence of Carbon / A.S. Kolesnikov, I. V. Sergeeva, N. E. [et al.] // Steel in Translation. 2017. - Vol. 47. - No.9. - P. 605-609. https://doi.org/10.3103/S0967091217090078

[17] Nadirov K.S., Zhantazov M.K., Bimbetova G.Z., Kolesnikov A.S., Sadyrbaeva A.S., Orynbasarov A.K., Kutzanova A.N., Turemuratov R.S., Botabaev N.E., Zhantazova D. Examination of optimal parameters of oxy-ethylating of fatty acids with a view to obtaining demulsifiers for deliquefaction in the system of skimming and treatment of oil: a method to obtain demulsifier from fatty acids. Chemistry today. 2016. V. 34. № 1. P. 72-77.

[18] Asnafi, Nader & Rajalampi, Jukka & Aspenberg, David. (2019). Design and Validation of 3D-Printed Tools for Stamping of DP600. IOP Conference Series: Materials Science and Engineering. 651. 012010. 10.1088/1757-899X/651/1/012010.

[19] Kolesnikov, A.S. Chemical and Phase Transitions in Oxidized Manganese Ore in the Presence of Carbon / A.S. Kolesnikov, I. V. Sergeeva, N. E. [et al.] // Steel in Translation. 2017. - Vol. 47. - No.9. - P. 605-609. https://doi.org/10.3103/S0967091217090078

[20] Livatyali H., Ergeldi M.M. (2006) Design of over-crown in sheet metal stamping using finite element method. Journal of Materials Processing Technology. 2006. V. 173. № 1. p. 14-20.

[21] Livatyali H., Ergeldi M.M. (2006) Design of over-crown in sheet metal stamping using finite element method. Journal of Materials Processing Technology. 2006. V. 173. № 1. p. 14-20.

[22] Ranjan, Rajit & Ayas, Can & Langelaar, Matthijs & Keulen, Fred. (2020). Topology Optimisation Techniques. 10.1201/9780429436543-2.

[23] Ribeiro, Tiago & Bernardo, Luis & Andrade, Jorge. (2021). Topology Optimisation in Structural Steel Design for Additive Manufacturing. 11.2112.10.3390/app11052112.

[24] Sosnowski W., Marczewska I., Marczewski A. (2002) Sensitivity based optimization of sheet metal forming tools. Journal of Materials Processing Technology. V. 124. № 3. p. 319-328.

[25] Sun J.S., Lee K.H., Lee H.P. (2000). Effects of geometry and fillet radius on die stresses in stamping processes. Journal of Materials Processing Technology. 2000. V. 104. № 3. p. 254-264.

[26] Tomasz M. (2016) The effect of equivalent strain on punch load for cold backward extrusion of copper cans. Key Engineering Materials. V. 716. p. 68-75.

[27] Yershov A.A., Kotov V.V., Loginov Yu.N. (2012) Optimization of the initial form of a semifinished product in PAM-STAMP 2G. Metallurgist. 2012. V. 56. № 3-4. p. 231-235.

[28] Andrianov, Ivan. (2021). The Mathematical Model of the Stamp Material Redistribution Using Uniform Bar Structures. Lecture Notes in Networks and Systems, 2021, 200, pp. 47–54 10.1007/978-3-030-69421-0_6.

[29] Andrianov, I.. (2021). Rod-Based Model for Optimization of a Polymer Die with Self-Heating in Cyclic Loading. Russian Engineering Research. 41. 403-406. 10.3103/S1068798X21050038.

Ivan K. Andrianov, Ph.D. (Eng.), associate professor of the department «General Physics», Federal State-Financed Educational Institution of Higher Learning “Komsomolsk-na-Amure State University”, pr. Lenina, 27, Komsomolsk-na-Amure, 681013, Russian Federation, e-mail: ivan_andrianov_90@mail.ru

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0
https://creativecommons.org/licenses/by/4.0/deed.en_US