QCD corrections to FCNC decays mediated by $Z$-penguins and $W$-boxes.

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Abstract

QCD corrections are evaluated to FCNC processes like $B \to X_s \nu \bar{\nu}$, $K \to \pi \nu \bar{\nu}$, $B \to l^+ l^-$ or $K_L \to \mu^+ \mu^-$, i.e. to processes mediated by effective operators containing neutrino currents or axial leptonic currents. Such operators originate from $W$-box and $Z$-penguin diagrams in the Standard Model. QCD corrections to them are given by two-loop diagrams. We confirm results for those diagrams which are already present in the literature. However, our analytical expressions for the Wilson coefficients disagree, due to a subtlety in regulating spurious IR divergences. The numerical effect of the disagreement is rather small. The size of the perturbative QCD corrections compared to the leading terms depends on the renormalization scheme used at the leading order. It varies from 0 to around 15% for a reasonable class of schemes. The uncertainty originating from uncalculated higher-order (three-loop) QCD corrections is expected to be around 1%.

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1. Introduction

Flavour Changing Neutral Current (FCNC) processes are known to be an important source of information concerning the Standard Model (SM) parameters, as well as a window towards new physics. However, predictions for their amplitudes are often plagued with uncertainties due to low-energy QCD dynamics. Processes dominated by $W$-box or $Z$-penguin diagrams which involve both quarks and leptons form an important class of exceptions from this rule. Low-energy dynamics can be then efficiently factorized, and the theoretical predictions can be made very precise for a given set of fundamental SM parameters.

Phenomenologically, the most interesting processes belonging to this class are $B \rightarrow X_s \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $B \rightarrow l^+ l^-$. FCNC decays involving neutrinos in the final state are obviously receiving contributions only from $W$-box or $Z$-penguin diagrams, because the heavy bosons mediate all the neutrino interactions. In the $B \rightarrow l^+ l^-$ case, photonic penguin contributions vanish due to the electromagnetic current conservation for the lepton pair, while double-photon intermediate states are numerically irrelevant. The analogous $K \rightarrow l^+ l^-$ decays are, on the contrary, dominated by the CKM-favoured double-photon contributions and, in consequence, much less theoretically clean.

![Figure 1: One-loop diagrams which give leading contributions to the considered processes in the Standard Model. The charged would-be Goldstone boson is denoted by $G$.](image.png)

The $W$-box and $Z$-penguin diagrams which give rise to all those processes are presented in fig. 1. Since external momenta in these diagrams are of order $m_B$ or $m_K$, i.e. much smaller than $M_W$, $M_Z$ or $m_t$, one can describe the processes in question by introducing an effective theory which is obtained from the SM by decoupling the heavy electroweak bosons and the
top quark

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}} \text{(leptons and light quarks)} + \frac{4G_F}{\sqrt{2}} \sum_n C_n Q_n. \quad (1) \]

Here, \( Q_n \) stand for dimension > 4 effective interactions like

\[ Q_1 = \frac{g_2^2}{8\pi^2} V_{ts}^* V_{tb}(s \gamma_\alpha P_L b)(l \gamma_\alpha P_L l), \quad (2) \]

\[ Q_2 = -\frac{g_2^2}{8\pi^2} V_{ts}^* V_{tb}(s \gamma_\alpha P_L b)(\bar{\nu} \gamma_\alpha P_L \nu), \quad (3) \]

where \( g_2 \) is the \( SU(2) \) gauge coupling constant, \( V_{ij} \) are the CKM matrix elements, and \( P_L = (1 - \gamma_5)/2 \). Numerical values of their Wilson coefficients

\[ C_n = C_n^{(0)} + \frac{\alpha_s}{4\pi} C_n^{(1)} + ... \quad (4) \]

are found by matching the full and the effective theory Green’s functions. The matching is performed perturbatively in gauge couplings and in (external momenta)/\( M_W \).

The two operators \( Q_1 \) and \( Q_2 \) written explicitly above are the ones which give dominant contributions to \( B \to l^+l^- \) and \( B \to X_\nu \nu \bar{\nu} \), respectively. Operators relevant for \( K^+ \to \pi^+ \nu \bar{\nu} \), \( K_L \to \pi^0 \nu \bar{\nu} \) and \( K_L \to \mu^+ \mu^- \) are obtained from \( Q_1 \) and \( Q_2 \) by simply replacing \( b \)-quarks by \( d \)-quarks in the operators themselves and in indices of the CKM factors.

There are two main reasons for introducing the effective theory description. One of them is summing up large QCD logarithms like \( [\alpha_s \ln(M_W^2/m_b^2)]^k \) from all orders of the SM perturbation series. This is achieved by applying renormalization group equations to the Wilson coefficients \( C_n \). The other reason is the fact, that different-looking Feynman diagrams can give rise to very similar effective operators whose nonperturbative matrix elements can be related to each other. Such relations are used e.g. for extracting nonperturbative matrix elements in \( K \to \pi \nu \bar{\nu} \) from the measurement of \( K^+ \to \pi^0 \nu_e e^+ \) [7].

In the processes under consideration here, renormalization group evolution of the Wilson coefficients plays only a minor role. The dominant contributions originate from operators with only a single (V–A) quark current, like those in eqns. (2) and (3). Such a current is not renormalized in a mass-independent scheme, because it is conserved in the limit of vanishing quark masses. Consequently, anomalous dimensions of these operators vanish, and RGE running of their Wilson coefficients may arise only at higher orders in \( (\text{light masses})^2/M_W^2 \). Such higher-order running is numerically relevant (though subdominant)
for $K^+ \to \pi^+ \nu \bar{\nu}$ and $(K_L \to l^+ l^-)$ Short Distance only.

So long as the RGE running is negligible, and the low-energy matrix elements can be found reliably, the main calculational effort is required at the point of perturbative matching of the full SM and the effective theory. Since masses of the light particles can be set to zero when matching is performed, Wilson coefficients of $Q_1$ and $Q_2$ are the same as the Wilson coefficients of their $d$-quark analogs. If the Wilson coefficients of $Q_1$ and $Q_2$ were found only at the leading order in QCD, i.e. from the purely electroweak one-loop diagrams in fig. 1, they would contain an inherent uncertainty of order 10% due to two-loop gluonic corrections. If those two-loop corrections were not calculated, this uncertainty would often become the main theoretical uncertainty in the final predictions for branching ratios, for given values of the SM parameters.

A two-loop calculation of the next-to-leading matching conditions for $Q_1$ and $Q_2$ was completed several years ago [1]. It has not been checked by any other group so far. In the present paper, we recalculate all the next-to-leading contributions to the Wilson coefficients of $Q_1$ and $Q_2$. We confirm the results of ref. [1] for all the necessary two-loop Feynman diagrams. However, our final expressions for the Wilson coefficients disagree, due to a subtlety in regulating spurious IR divergences.

Our paper is organized as follows: In the next two sections, we present a detailed description of evaluating Wilson coefficients of $Q_1$ and $Q_2$. Section 2 is devoted to the leading-order (one-loop) matching. The QCD corrections which arise at two loops are taken into account in section 3. Section 4 contains a discussion of numerical significance of the QCD corrections.

2. Leading-order matching for $Q_1$ and $Q_2$.

Leading-order contributions to the Wilson coefficients $C_1$ and $C_2$ can be found by requiring equality of perturbative off-shell amplitudes generated by the full Standard Model and the effective theory. We need to consider $b \to s l^+ l^-$ and $b \to s \nu \bar{\nu}$ transitions in the cases of $Q_1$ and $Q_2$, respectively. On the Standard Model side, we have to calculate the one-loop diagrams from fig. 1 for vanishing external momenta. On the effective theory side, one needs to include only tree-level diagrams with four-fermion vertices corresponding to either $Q_1$ or $Q_2$.

\footnote{It arises from diagrams with double effective operator insertions and charm-quark loops. They correspond to $W$-boxes and $Z$-penguins with charm quarks on the full Standard Model side. Such diagrams are suppressed by $m_c^2/M_W^2$ but enhanced by CKM angles with respect to the leading top-quark contributions.}

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For purposes of the next section, we shall perform the matching in $D = 4 - 2\epsilon$ dimensions, i.e. without using any identity from only 4-dimensional Dirac algebra. In consequence, two other operators need to be introduced explicitly on the effective theory side

$$Q_1^E = \frac{g_2^2}{8\pi^2} V_t^* V_b (\bar{s}\gamma_{\alpha_1} \gamma_{\alpha_2} \gamma_{\alpha_3} P_L b) (\bar{t}\gamma^{\alpha_3} \gamma^{\alpha_2} \gamma^{\alpha_1} P_L l) - 4Q_1,$$  

(5)

$$Q_2^E = -\frac{g_2^2}{8\pi^2} V_t^* V_b (\bar{s}\gamma_{\alpha_1} \gamma_{\alpha_2} \gamma_{\alpha_3} P_L b) (\bar{\nu}\gamma^{\alpha_1} \gamma^{\alpha_2} \gamma^{\alpha_3} P_L \nu) - 16Q_2.$$  

(6)

The superscript "E" stands for the name "evanescent" which originates from the fact that such operators vanish in 4 dimensions due to the identity

$$\gamma_{\alpha_1} \gamma_{\alpha_2} \gamma_{\alpha_3} = g_{\alpha_1 \alpha_2} \gamma_{\alpha_3} - g_{\alpha_1 \alpha_3} \gamma_{\alpha_2} + g_{\alpha_2 \alpha_3} \gamma_{\alpha_1} + i\epsilon_{\beta \alpha_1 \alpha_2 \alpha_3} \gamma^{\beta} \gamma_{5}.$$  

(7)

This identity cannot be analytically extended to D dimensions. Operators $Q_1^E$ and $Q_2^E$ arise in D-dimensional matching, because $W$-box diagrams in fig. 1 contain triple products of Dirac matrices which cannot be reduced to anything simpler in D-dimensions.

Elementary calculation of the one-loop diagrams in fig. 1 with vanishing external momenta in the Feynman-'t Hooft gauge gives us the following results for the leading-order contributions to the considered Wilson coefficients$^3$

$$C_1^{(0)} = C_0(x_t) - B_0(x_t) + O(\epsilon),$$  

(8)

$$C_2^{(0)} = C_0(x_t) - 4B_0(x_t) + O(\epsilon),$$  

(9)

$$C_{i}^{E(0)} = C_{2}^{E(0)} = -\frac{1}{4}B_0(x_t) + O(\epsilon),$$  

(10)

where $x_t = m_t^2/M_W^2$ and $^4$

$$B_0(x) = \frac{x}{4(x-1)^2} \ln x - \frac{x}{4(x-1)},$$  

(11)

$$C_0(x) = \frac{3x^2 + 2x}{8(x-1)^2} \ln x + \frac{x^2 - 6x}{8(x-1)}.$$  

(12)

The functions $B_0$ and $C_0$ originate from $W$-box and $Z$-penguin diagrams, respectively.

3. Next-to-leading order matching for $Q_1$ and $Q_2$.

Before performing the NLO matching, we need to learn the structure of one-loop UV counterterms on the effective theory side. Evaluating the UV QCD counterterms can be done separately in the lepton ($Q_1, Q_1^E$) and neutrino ($Q_2, Q_2^E$) sectors, thanks to their different

$^3$ Here, we ignore contributions suppressed by $m_c^2/M_W^2$. They are relevant in $K^+ \to \pi^+ \nu\bar{\nu}$, i.e. for the $d$-quark analog of $Q_2$, because of their CKM enhancement.
leptonic content. In each of the two sectors, bare quantities are replaced by the QCD-renormalized ones as follows:

\[ CQ + C^E Q^E \rightarrow Z_\psi \left( C Z_{NN} Q + C Z_{NE} Q^E + C^E Z_{EN} Q + C^E Z_{EE} Q^E \right). \]  

(13)

Here, \( Z_\psi \) denotes the usual quark wave-function renormalization constant. The remaining renormalization constants are found in the MS scheme from the following two conditions which the \( n \)-loop effective theory amplitudes have to satisfy [10]:

- Renormalized amplitudes proportional to the coefficient \( C \) of the ”normal” operator \( Q \) have to be finite in the limit \( D \rightarrow 4 \). Counterterms which make them finite can contain nothing but \( 1/\epsilon^k \) poles, with \( 1 \leq k \leq n \).

- Renormalized amplitudes proportional to the coefficient \( C^E \) of the ”evanescent” operator \( Q^E \) have to vanish in the limit \( D \rightarrow 4 \). Counterterms which make them vanish can contain nothing but \( 1/\epsilon^k \) poles, with \( 0 \leq k \leq n - 1 \) if they are proportional to the ”normal” operator, and with \( 1 \leq k \leq n \) if they are proportional to the ”evanescent” operator. This means that uniquely defined finite counterterms occur in this case, too.

\[ Z_{NN} = 1, \quad Z_{NE} = 0, \quad Z_{EN} = \frac{\alpha_s}{4\pi} r + \mathcal{O}(\alpha_s^2), \quad Z_{EE} = 1 + \mathcal{O}(\alpha_s^2), \]  

(14)

with \( r = 32 \) and \( r = -32 \) in the lepton \( (Q_1, Q^E_1) \) and neutrino \( (Q_2, Q^E_2) \) sectors, respectively.
The above equations mean that the renormalization constant $Z_\psi$ was always enough to remove the $1/\epsilon$ pole from the one-loop diagram in fig. 2. Actually, the first two equations are true to all orders in QCD, because the quark current in $Q$ is conserved for vanishing quark masses, as already mentioned in the introduction. The important part of eqn. (14) is the finite renormalization constant $Z_{EN}$ which is going to affect our final results.

Figure 3: Two-loop QCD corrections to the $W$-box diagrams from fig. 1.

Figure 4: Two-loop QCD corrections to the $Z$-penguin diagrams from fig. 1.

Figure 5: Diagrams with quark mass counterterms.

We are now ready to perform the NLO matching, i.e. to find $C_1^{(1)}$ and $C_2^{(1)}$. On the full Standard Model side, one needs to calculate the two-loop diagrams shown in figs. 3 and 4, as well as the diagrams with quark mass counterterms in fig. 5. We ignore the quark wavefunction renormalization now. This is allowed so long as one does it on the effective theory side of the matching equation, too. As in the previous section, it is enough to calculate all
the diagrams in the limit of vanishing external momenta. The calculation is particularly simple when masses of all the light particles are set to zero. Then, the 1PI parts of the diagrams depend on masses only via the ratio $x_t$, and they are very easy to handle with recurrence relations for two-loop vacuum integrals [9].

Setting light masses to zero may lead to generation of spurious IR divergences. They should cancel with similar divergences on the effective theory side, so that the final results for the Wilson coefficients are finite in the limit $D \to 4$. This cancellation occurs only after comparing the full and effective theory amplitudes. Thus, one needs to calculate both amplitudes in $D$ dimensions, even after their UV renormalization. This was the reason why we needed to perform the matching for $Q^E_k$ in the previous section.

The sum of the diagrams in figs. 3, 4 and 5 turns out to be finite in the limit $D \to 4$. However, as we shall see in the following, it contains finite terms originating from multiplication of $1/\epsilon$ spurious IR divergences with $O(\epsilon)$ terms from the Dirac algebra.

Calculating the effective theory side of the next-to-leading matching condition is particularly simple, because we have set all the light masses to zero on the full theory side. Thus, we now need to do the same on the effective theory side, where only light particles occur. Consequently, all the loop diagrams on the effective theory side vanish in dimensional regularization. What remains are only tree-level diagrams with insertions of either the original operator vertices or the UV counterterms. The matching equation takes then the following form:

\[
\left< \text{1- and 2-loop SM diagrams} \right> = (\text{const.}) \times \left\{ C^{(0)} + \frac{\alpha_s}{4\pi} \left( C^{(1)} + r C^{E(0)} \right) \right\} \langle Q \rangle_{\text{tree}} + \left( \text{something finite in } D=4 \right) \langle Q^E \rangle_{\text{tree}}. 
\]  

(15)

We have now consistently ignored the quark wave-function renormalization on both the full and effective theory sides.

Vanishing of all the loop diagrams on the effective theory side means, in particular, that all the UV divergences in those diagrams cancel with spurious IR divergences (which arise due to setting all the external momenta and masses to zero). Thus, UV counterterms on the effective theory side actually reproduce the spurious IR divergences. The spurious IR divergences are the same on the full and effective theory sides, and cancel in the matching equation. In our case, there are no $1/\epsilon$ poles involved in this cancellation. We only have
the finite "\(rC^E(0)\)" term. However, \(1/\epsilon\) poles would occur in a generic case.\(^4\) Thus, the "\(rC^E(0)\)" term on the effective theory side can be interpreted as the one which cancels finite terms originating from multiplication of \(1/\epsilon\) spurious IR divergences with \(O(\epsilon)\) terms from the Dirac algebra on the full theory side.

Since we do not need to recover the NLO Wilson coefficients of the evanescent operators, we can take the limit \(D \to 4\) in the matching equation (15). Then \(\langle Q^E\rangle_{\text{tree}}\) vanishes, and we find

\[
C_1^{(1)} = C_1(x_t) - B_1(x_t, -1/2) + O(\epsilon),
\]

\[
C_2^{(1)} = C_1(x_t) - 4B_1(x_t, +1/2) + O(\epsilon),
\]

where

\[
C_1(x) = \frac{x^3 + 4x}{(x-1)^2} \text{Li}_2(1-x) + \frac{x^4 - x^3 + 20x^2}{2(x-1)^3} \ln^2 x + \frac{-3x^4 - 3x^3 - 35x^2 + x}{3(x-1)^3} \ln x
\]

\[
+ \frac{4x^3 + 7x^2 + 29x}{3(x-1)^2} + 8x \frac{\partial C_0(x)}{\partial x} \ln \frac{\mu^2}{M_W^2},
\]

\[
B_1(x, -1/2) = \frac{2x}{(x-1)^2} \text{Li}_2(1-x) + \frac{3x^2 + 6}{(x-1)^3} \ln^2 x + \frac{-11x^2 - 5x}{3(x-1)^3} \ln x
\]

\[
+ \frac{-3x^2 + 19x}{3(x-1)^2} + 8x \frac{\partial B_0(x)}{\partial x} \ln \frac{\mu^2}{M_W^2},
\]

\[
B_1(x, +1/2) = \frac{2x}{(x-1)^2} \text{Li}_2(1-x) + \frac{3x^2 + 6}{(x-1)^3} \ln^2 x + \frac{-31x^2 - x}{6(x-1)^3} \ln x
\]

\[
+ \frac{3x^2 + 29x}{6(x-1)^2} + 8x \frac{\partial B_0(x)}{\partial x} \ln \frac{\mu^2}{M_W^2},
\]

Here, \(\mu\) stands for the renormalization scale at which the top quark mass is \(\overline{\text{MS}}\)-renormalized. The functions \(B_1(x_t, \pm 1/2)\) and \(C_1(x_t)\) originate from gluonic corrections to \(W\)-boxes and \(Z\)-penguins, respectively. The difference between \(B_1(x, -1/2)\) and \(B_1(x, +1/2)\) is proportional to \(B_0(x)\)

\[
B_1(x, -1/2) - B_1(x, +1/2) = 6B_0(x).
\]

Our result for the function \(C_1(x)\) is in perfect agreement with ref. [1]. However, the results for \(B_1(x, \pm 1/2)\) disagree. The disagreement is due to the term "\(rC^E(0)\)" in our matching equation (15).

Since our results differ from the previously published ones due to a subtlety in regulating spurious IR divergences, it is worthwhile to cross-check them using light quark masses as

\(^4\)They do occur e.g. when two-loop off-shell matching for \(b \to s\gamma\) is performed for vanishing light particle masses.
IR regulators. Calculation of the 2-loop SM diagrams is then somewhat more complicated. One-loop diagrams on the effective theory side are nonvanishing in this case, and one needs to calculate their finite parts. However, the evanescent operators are unimportant. The matching equation can be written in 4 dimensions. Both the effective and the full theory amplitudes depend on the light quark masses in the same way. Terms dependent on the light quark masses cancel out in the matching equation. The Wilson coefficients we obtain in the end are exactly the same as the ones already found in eqns. (16)–(20). Thus, our NLO matching results are cross-checked by calculating them twice with use of two different regulators for spurious IR divergences: dimensional regularization and light quark masses.

4. Numerical significance of the NLO corrections

Branching ratios of the decays $B \to X_s \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ are proportional to the squared Wilson coefficient $|C_2|^2$, while $B \to l^+l^-$ is proportional to $|C_1|^2$. Thus, numerical significance of the NLO matching in these decays can be seen in figs. 6 and 7 where $|C_k|^2$ are plotted as functions of the matching scale $\mu$. Dependence on this scale enters via $x_t = [m_t^{\overline{MS}}(\mu)/M_W]^2$, $\alpha_s(\mu)$ and explicitly in $C_k^{(1)}$. We use $m_t^{\text{pole}} = 175$ GeV and $\alpha_s(M_Z) = 0.118$. Solid lines present the full NLO predictions, while dashed lines correspond to the LO results. Dotted lines indicate what the NLO results would be if the "$rC^{E(0)n}$" term was not included in the matching equation.

![Figure 6: $|C_1(\mu)|^2$ at the leading and next-to-leading order.](image)

One can see that the NLO QCD corrections to $|C_k|^2$ range from 0 to around 15% for a reasonable class of renormalization schemes, i.e. for $\mu \in [M_W, m_t]$. The remaining $\mu$-
dependence at NLO is an estimate of uncalculated higher order (three-loop) QCD corrections. It is around 1%. The numerical effect of the "$rC^{E(0)}$" term is seen to be rather small (around 2%). To some extent, it is due to the fact that it originates from $W$-boxes. Contributions from $W$-boxes tend to a constant at large $x_t$, while $Z$-penguin contributions grow with $x_t$, and are dominant for the physical value of $x_t \simeq 5$. Thus, from the numerical point of view, our results are a confirmation of what has been found in ref. [1] and in the following phenomenological analyses [2].

The decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \mu^+ \mu^-$ are somewhat more complicated because their amplitudes receive other significant contributions which are not proportional to the Wilson coefficients discussed in sections 2 and 3. Both of them are affected by $O(m_c^2/M_W^2)$ terms which are CKM-enhanced with respect to the leading top-quark contributions. Those charm-quark loops are numerically as large as 20%-40% of the top-quark ones [4]. In addition, the $K_L \rightarrow \mu^+ \mu^-$ mode is actually dominated by contributions from double-photon intermediate states which arise at higher order in $\alpha_{em}$, but involve $u$-quark loops and, in consequence, are strongly CKM enhanced with respect to the $W$-box and $Z$-penguin diagrams. Thus, QCD corrections to the top-quark loops are of mild importance for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, and totally unimportant in the $K_L \rightarrow \mu^+ \mu^-$ case.

5. Summary

We have evaluated QCD corrections to processes mediated by $Z$-penguin and $W$-box diagrams. Two-loop matching of the full Standard Model and the effective theory was per-
formed off-shell, for vanishing external momenta and light particle masses. In consequence, calculation of the necessary two-loop Feynman diagrams was very easy, but the matching had to be performed in D dimensions, due to dimensionally regulated spurious IR divergences. In effect, Wilson coefficients of evanescent operators had to be found, and they affected the final physical results. Such a phenomenon is expected to take place in a generic process of heavy particle decoupling, when light particle masses are set to zero in the matching procedure.

We have confirmed the results of ref. [1] for the two-loop diagrams, but our final expressions for the NLO Wilson coefficients disagree due to the very contribution from the evanescent operators. We have cross-checked our results using light quark masses as IR regulators, in which case there was no need to consider evanescent operators.

The effect of the NLO QCD corrections on the branching ratios varies between 0 and around 15% in a reasonable class of renormalization schemes. Higher order effects are expected to be around 1%. The effect of the evanescent operator contribution in the matching turns out to be small. Thus, from the numerical point of view, our results are a confirmation of what has been found in ref. [1] and in the following phenomenological analyses [12].

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