Joint power control and user grouping for uplink power domain non-orthogonal multiple access

Bilal Ur Rehman1, Mohammad Inayatullah Babar1, Arbab Waheed Ahmad2, Hesham Alhumyani3, Gamil Abdel Azim4, Rashid A. Saeed3 and Sayed Abdel Khalek5

Abstract
Orthogonal multiple access schemes based on assignment of communication resource blocks among multiple contenders, although widely available, still necessitate an upper limit on the number of concurrent users for minimization of multiple-user interference. The feature thwarts efforts to cater for pressing connectivity demands posed by modern-day cellular communication networks. Non-orthogonal multiple access, regarded as a key advancement towards realization of high-speed 5G wireless communication networks, enables multiple users to access the same set of resource blocks non-orthogonally in terms of power with controllable interference, thereby allowing for overall performance enhancement. Owing to the combinatorial nature of the underlying optimization problem involving user pairing/grouping scheme, power control and decoding order, the computational complexity in determining optimal and sub-optimal solutions remains considerably high. This work proposes three novel alternative approaches (Randomly, 2-Opt and Hybrid) for arriving at a near-optimal solution for the problem of user pairing/grouping. The algorithms not only offer reduced computational complexity but also outperform orthogonal multiple access and existing schemes reported in the literature for uplink non-orthogonal multiple access systems.

Keywords
Uplink, 5G wireless communication, non-orthogonal multiple access

Date received: 22 March 2021; accepted: 23 August 2021

Handling Editor: Peio Lopez Iturri

Introduction
The exponential increase in traffic originating from wireless communication networks has sparked interest at the end of several researchers. Spurring from both academic and industrial contributors, the efforts have primarily been focused towards developing next-generation (5G) Wireless Communication Networks, aimed at substantial enhancements in user-performance as well as coverage.1 The above-mentioned enhancements are set to overcome the constraints posed by existing communication networks. The transition to 5G networks is posited to address the challenges related to

1Department of Electrical Engineering, Faculty of Electrical and Computer Engineering, University of Engineering and Technology, Peshawar, Pakistan.
2Department of Electrical and Computer Engineering, PAF-IAST, Haripur, Pakistan
3Department of Computer Engineering, College of Computers and Information Technology, Taif University, Taif, Saudi Arabia
4Department of Computer Science, Faculty of Computer Science, Suez Canal University, Ismailia, Egypt
5Mathematics Department, Faculty of Sciences, Taif University, Taif, Saudi Arabia

Corresponding author:
Bilal Ur Rehman, Department of Electrical Engineering, Faculty of Electrical and Computer Engineering, University of Engineering and Technology, Peshawar, KPK 25000, Pakistan.
Email: bur@uetpeshawar.edu.pk
spectral efficiency, massive connectivity and superior coverage.\textsuperscript{2,3} Broadly speaking, multiple access schemes are categorized into two main categories: non-orthogonal multiple access (NOMA) and orthogonal multiple access (OMA), with the choice to select either depending upon the resource allocation strategy assigned to multiple users.\textsuperscript{4} In the OMA scheme, resources are allocated to each user in a cell exclusively, without inter-cell interference. In this arrangement, however, low-complexity decoding techniques can be readily employed at the receiver for recovery of user information. In current conditions, a hefty majority of existing mobile communication standards including Long-Term Evolution (LTE) and LTE Advanced\textsuperscript{5} draw heavily from OMA. More recently, NOMA has managed to grab significant attention as a potential candidate for deployment in 5G technology for wireless communication networks. Although NOMA allocates identical resources in the power domain to all users, inter-user interference remains inevitable. Consequently, a more complex technique known as multi-user detection (MUD) has been recently introduced for recovering user information at the receiver, even in the presence of inter-user interference.

Spectral efficiency obtained using the NOMA scheme has been demonstrated to be greater than that for OMA.\textsuperscript{6–8} In essence, differentiating user signals as distinct entities requires adoption of an approach based on power control at the transmitter. Furthermore, a multitude of approaches aimed at power allocation\textsuperscript{9} has been introduced for a typical single-cell NOMA system. Owing to co-channel interference from different cells, the method for power allocation corresponding to a single cell remains much simpler than that encountered in a multi-cell configuration. With the intent to overcome co-channel interference, a distributed power allocation method for downlink has been proposed for mitigation of total transmitted power in the multi-cell scenario.\textsuperscript{10}

Due to the stark difference in the sequence following which user information is encoded and decoded at the transmitter and receiver for encoding and decoding information, the mechanism for uplink and downlink NOMA remains considerably distinct. Decoding at the receiver is based upon the application of successive interference cancellation (SIC) and remains a function of power allocation strategy.\textsuperscript{11} To compensate for the path loss propagation, a scheme known as fractional power control (FPC) is introduced for uplink LTE-A.\textsuperscript{12} That said, decoding different user signals at the receiver by exploiting the existing FPC is a difficult ordeal. Addressal of this constraint via an advanced power control scheme that draws heavily from game theory for reduction of interfering-cells thereby lowering the total consumption of power.\textsuperscript{13}

To enhance the network capacity, the concept of Small Cell Networks (SCNs) has been introduced for future-centric mobile communication networks.\textsuperscript{14,15} For uplink NOMA, a novel detection method is proposed for small cell\textsuperscript{16} wherein both near- and far-signal users are facilitated while leveraging dual path loss compensation factors (PLCFs). An analytical framework for power allocation accompanied by the concept of user clustering for hybrid downlink NOMA\textsuperscript{17} networks is presented. The basics of uplink NOMA system and downlink NOMA system, their significant dissimilarities, detection, decoding, implementation complexity, and inter and intra cell interferences are discussed while illustrating the concept of two-user clustering strategy\textsuperscript{18} for uplink/downlink NOMA systems. A power control method\textsuperscript{19} is proposed for uplink multi-cell with two-user clustering strategy based on stochastic geometry along with a discussion on a novel model for inter-cell interference based on the Laplacian transformation. User-pairing and power allocation are the key factors for increasing the capacity of uplink NOMA. A detailed lookup table\textsuperscript{20} is provided for efficient user-pairing and power allocation. In addition, a joint interference cancellation method\textsuperscript{21} is suggested for uplink NOMA to solve the issue of error-propagation and time delay in SIC utilizing a combination of parallel-interface cancellation (PIC) and SIC.

For uplink NOMA systems, a framework elucidating joint user association, power control and user grouping\textsuperscript{22} is introduced for multi-cell environment for reduction of inter-user interference. A technique for the allocation of resource block (RB) and clustering for uplink multi-carrier NOMA communication networks has also been proposed to provide time-based proportional fairness.\textsuperscript{23} Yet another power allocation scheme is proposed to increase the sum rate and reduce interference using a water-filling technique. The approach is a composite of two techniques, namely, interference filling and conventional power filling.\textsuperscript{24} The key differences in the operation of uplink/downlink\textsuperscript{25} for cellular communication networks have been discussed. Moreover, a power allocation scheme for user clustering in NOMA has also been introduced to increase the overall system performance. With the intent to increase spectral efficiency of a network, an iterative technique for uplink NOMA based on shifted-gamma strategy\textsuperscript{26} in the existence of inter-user has also been put forward. Maintaining fairness among users in terms of data rate is a long-standing challenge, especially in scenarios where the inequality is exhibited in data rates over a huge scale, for which a thorough discussion has been carried out concluding that NOMA service, at times, can be crucial when it comes to avoiding strict fairness\textsuperscript{27} in certain conditions. A power allocation scheme is proposed for uplink multi-cell networks to increase the spectral efficiency in the presence of co-channel interference.\textsuperscript{28} Moreover, an optimum and sub-optimum algorithm for efficient user-pairing and power
allocation scheme for uplink NOMA is formulated to enhance the overall system performance.\(^{29}\)

Recent research studies focused on NOMA transmission have unearthed two major limitations encountered during implementation of uplink NOMA systems. First and foremost being interference mitigation among small cells which has not been taken into consideration.\(^{30}\) Severe instances of inter-cell interference leads to pronounced degradation of system level efficiency. The second limitation, centred around a power allocation scheme\(^{31}\) with full channel-inversion, is also discussed. The optimal performance of a network due to fact that PLCFs of different signal users are not jointly formulated to obtain the highest spectral efficiency. A novel approach based on game theory\(^{32}\) is deployed to increase the capacity of the network for uplink NOMA clustering. In addition, an iterative algorithm has also been proposed by assuming Karush Khun Tucker (KKT) conditions for effective addressal of challenges related to power allocation, system throughput and fairness in the context of user clustering in NOMA. Yet another technique is proposed for user clustering and power consumption in multi-cluster multi-input single-output (MISO) NOMA network.\(^{33}\) A combinatorial joint problem is formulated and solved to obtain the closed form global optimal solution for user pairing/grouping in order to achieve sum rate.\(^{34}\) Both OMA and NOMA techniques are discussed for downlink scenario,\(^{35}\) by considering the order of decoding, sum rates, and power allocation techniques and are used to determine the throughput and outage analysis for two users. A framework\(^{36}\) has also been presented for wireless communication networks including the fractions of energy harvesting and evaluates the ergodic rate and outage performance for attaining optimal solution. For each user, efficient energy resource allocation\(^{37}\) with quality of service (QoS) constraint for uplink NOMA is considered.

A joint power allocation and user association scheme is postulated based on the swap-matching algorithm to improve system coverage. Furthermore, an algorithm has been suggested to accentuate the sum rate capacity for uplink multi-carrier NOMA system wherein the number of sub-carriers assigned to each user is not limited.\(^{38}\)

Research on power control and user pairing/grouping, particularly for uplink NOMA systems, remains a crucial aspect improving upon which is of paramount importance. With the said as the core motivation, user pairing/grouping in NOMA for uplink case is investigated by examining specific predetermined power control approaches with near-perfect SIC. In this article, we examine a combinatorial optimization problem for uplink NOMA system and propose three algorithms (Randomly, 2-Opt and Hybrid) for user pairing/grouping problem to enhance the system performance and reduce the computational complexity. The obtained results are compared with those for conventional OMA scheme and existing NOMA approaches reported in the literature for uplink scenario. Simulation results indicate that the proposed algorithms achieve a significant improvement in spectral efficiency.

The rest of the article is organized into the following sections. The uplink NOMA system is described in section ‘System model’. The problem formulation is discussed in section ‘Spectral efficiency maximization’. In section ‘Solution of proposed model’, the solution of the research problem is presented. Results are described in section ‘Simulation results’, and finally, the conclusion of this research is provided in section ‘Conclusion’.

**System model**

Consider a single-cell C with an uplink NOMA system, consists of \(N\) number of users served by a single base station (BS) located at the centre of the cell as shown in Figure 1. The number of physical resource blocks (PRBs) assigned to multiple users is represented by \(K\). To achieve the signal/information of several users, the \(N\) users are allocated to \(K\) PRBs/groups.

To perform NOMA transmission, all users in the same group maintaining same PRB while in OMA transmission, different users group allotted different PRB. Therefore, the received signal \(z_k\) at BS can be expressed as

\[
z_k = \sum_{n=1}^{N} \beta_{k,n} g_n \sqrt{\alpha_n} P s_n + \omega_k
\]

where \(\beta_{k,n} \in \{0,1\}\) is the user \(n\) indicator allocated to the \(k\)th group. The transmission link between user \(n\) and BS is denoted by \(g_n\), which is Gaussian distribution. The power allocation coefficient is indicated by \(\alpha_n\) (\(0 \leq n \leq 1\)). For each user \(n\), the transmission power and the signal are denoted by \(P\) and \(s_n\), respectively.

![Figure 1. NOMA uplink transmission.](image)
where $\mathbb{E}(|s_n|^2) = 1$. The additive white Gaussian noise (AWGN) power is denoted by $\omega_k$ with an average power $\sigma^2$. Thus, the maximum spectral efficiency of user $n$ is represented as

$$S_n = \log_2(1 + \psi_n)$$  \hspace{1cm} (2)

where $\psi_n$ represents the signal to interference noise ratio of user $n$. Hence, the SIC is performed at the BS, to decode the users signal for each PRB. Consider $\Delta_{k,n}$, which indicates the decoding order of user $n$ in a cell, where $\Delta_{k,n} = a > 0$ implies that any user $n$ in a group is the $a$th one in the $k$th PRB is to be decoded. Hence, the maximum spectral efficiency of user $n$ can be expressed as

$$S_n = \log_2 \left(1 + \frac{|g_n|^2 \alpha_n \Gamma}{\sum_{j \neq n}^{N} |g_j|^2 \alpha_j \Gamma_j + \sigma^2} \right)$$  \hspace{1cm} (4)

where $\Gamma$ indicates the transmission power-to-noise ratio, where $\Gamma = P/\sigma^2$, and $\Delta_{k,n} > \Delta_{k,n}$ represents the decoding order in a group. For example, user $n$ and $j$ in the same group, then it implies that user $n$ is decoded first. Consider that the BS has aware of channel state information (CSI) of each user inside the coverage area to effectively decode the desired signals. To achieve efficient user grouping and power control for uplink NOMA, each user $N$ in a cell transmit their power control coefficient $\alpha_n$ along with user indicator $\beta_{k,n}$. Hence, the maximum spectral efficiency of the users in the $k$th PRB/group can be expressed as

$$S_T(k) = \sum_{\beta_{k,n} = 1}^{\alpha_n} S_n$$  \hspace{1cm} (5)

$$S_T(k) = \log_2 \left(1 + \sum_{\beta_{k,n} = 1}^{\alpha_n} |g_n|^2 \alpha_n \Gamma \right)$$  \hspace{1cm} (6)

It is clear from equation (6) that the spectral efficiency in each group has not affected by the order of decoding but the spectral efficiency of each user is not same as a result of different decoding order.

**Spectral efficiency maximization**

In this article, an efficient approach for power control, decoding order and user grouping is proposed to increase the spectral efficiency of each uplink NOMA users in a single cell. Therefore, a joint combinatorial problem of power control, decoding order and user pairing/grouping is formulated to maximize the spectral efficiency. The minimum spectral requirement of each user in the network is $s_n$. Therefore, the spectral efficiency maximization problem$^{29,34}$ can be formulated as

$$\{\beta_{k,n}, \Delta_{k,n}\} \in \theta, \{\alpha_n\} \quad S_T = \sum_{k=1}^{K} S_T(k)$$  \hspace{1cm} (7a)

s.t.  
$$C_1: 0 \leq \alpha_n \leq 1, \forall n$$  \hspace{1cm} (7b)

$$C_2: S_n \geq s_n, \forall n$$  \hspace{1cm} (7c)

$$C_3: \beta_{k,n} \in \{0,1\}, \forall n, \forall k$$  \hspace{1cm} (7d)

$$C_4: \sum_{k=1}^{K} \beta_{k,n} = 1, \forall n$$  \hspace{1cm} (7e)

where $\Delta_{k,n}$ and $\theta$ indicate the decoding order and set of all possible decoding orders for all users in a cell. $C_1$ indicates the upper bound of transmission power. $C_2$ guarantees the minimum rate of a user. $C_3$ represents the user indicator, and $C_4$ ensures that user $n$ assigned to PRB/group.

**Solution of proposed model**

To acquire the global optimal solution of the Problem (7a). Hence, optimization variables $\{\beta_{k,n}, \Delta_{k,n}\}$ and $\{\alpha_n\}$ are strongly correlated, which makes the problem obdurate. In view of the fact that user-pairing variables $\Delta_{k,n}$ are combinatorial integer programming variables. Therefore, first, solve the combinatorial problem of power control and decoding order, and then determine the optimal solution of user-pairing/grouping.

Noted that, for any fixed user-pairing/grouping approach, $\{\beta_{k,n}\}$ are independent among all different groups in case of decoding and power control.

$$\{\Delta_{k,n}\} \in \theta, \{\alpha_n\} \quad S_T(k)$$  \hspace{1cm} (8a)

s.t.  
$$S_n = s_n, n \in \mathcal{N}_k$$  \hspace{1cm} (8b)

$$0 \leq \alpha_n \leq 1, n \in \mathcal{N}_k$$  \hspace{1cm} (8c)

where $\mathcal{N}_k$ indicates the set of users in the $k$th PRB/group.

**Decoder design for user grouping**

In order to apply SIC and decoding order for $n$ users in a cell at the BS, it is necessary to maintain the distinctiveness of the different users signals that are being superposed represented as $z_k$.\textsuperscript{18} For example, consider two users, one user is close and second user far to the
BS in the coverage area. The closest user has the highest channel-gain means with high signal-to-noise ratio (SNR), this signal is to be decoded first user in the same PRB/group and experience interferences from all other users in the same group, where the farther user has weak channel-gain and probably the weakest at the BS is decoded after first. The farther user experiences zero interference from all users in the same PRB/group. For the uplink NOMA system, the efficient decoding is obtained based on $D_n$’s value, where all the users belong to the same PRB/group. Each user in a group has a different decoding order is related to different feasible regions of the power control and $s_n$. Therefore, it can be expressed as

$$D_n = |g_n|^2 \left(1 + \frac{1}{\lambda_n} \right)$$

where

$$\lambda_n = 2^{s_n} - 1$$

It implies that the user in a cell with larger value of $D_n$ is first to decode. It is also clear that the decoding order does not affect the spectral efficiency of each PRB/group.

**Power control**

The proposed power control approach is a mixed integer non-linear programming (MINLP) and the nature of the problem is combinatorial. Therefore, it is required to find all the possible group of combination for user pairing/grouping. Consider a single-cell $C$ with $M$ users. Without loss of generality, it is required to simplify the mathematical procedure corresponding to decoding order $\Delta_{k,n}$.

Assume that, the users in a $C$ is listed according to the decreasing order, for example, 1, 2, 3, $\ldots$, $M$. In this way, equation (8a) is comparable to

$$\max \{ \alpha_m \} \sum_{m=1}^{M} |g_m|^2 \alpha_m \Gamma$$

$$\text{s.t.} |g_m|^2 \alpha_m \Gamma \geqslant \lambda_m \left( \sum_{j=m+1}^{M} |g_j|^2 \alpha_j \Gamma + 1 \right), \forall m$$

$$0 \leqslant \alpha_m \leqslant 1, \forall m$$

where $\{ \alpha_m \}$ represents the power control variables. It is clear that equations (11a) and (11b) are translated to SNR formulations and show linearity.

As indicated, equation (11a) is increasing for the power control variables $\alpha_m$. Hence, the optimal power control will always be upper bound, which is the optimal solution and to acquire lower bound of power control, solve the following equation accordingly

$$\alpha_m^0 = \frac{\lambda_m \gamma_m}{|g_m|^2 \Gamma}, \quad 1 \leqslant m \leqslant M$$

where

$$\gamma_m = \prod_{u=m+1}^{M} (\lambda_u + 1)$$

which signifies that the spectral efficiency requirements are equal to the sum of spectral efficiencies of all the users. If $\alpha_m^0 \geqslant 1$, exceeds the limit of upper bound, and hence, no feasible solution for equation (11a). If $0 \leqslant \alpha_m^0 \leqslant 1$, equation (11a) has the feasible solution due to bound of $\alpha_m$ variables. Therefore, for all users $N$ in a cell, the optimal solution of the $\alpha_m$ variables can be illustrated as

$$\alpha_m^* = \min \{1, b_m\}$$

where

$$b_m = \min \left\{ \frac{|h_i|^2 \Gamma}{\alpha_m}, \sum_{q=u+1}^{m-1} |h_q|^2 \Gamma - \sum_{j=m+1}^{M} |h_j|^2 \alpha_j^2 \Gamma - 1 (u = 1, 2, 3, \ldots, m - 1) \right\}$$

In reference to equations (14) and (15), the optimal power control variables $\alpha_m^*$ mentioned in problem (11a) is achieved. Specifically, if $\alpha_m^* = b_m$, for other users, the optimal power control variables are $\alpha_j^* = \alpha_j^0$ for $j > m$.

**Optimal user grouping**

A low computational time algorithm for efficient user-pairing/grouping is one of the primary concerns for successful uplink NOMA systems. For uplink NOMA system, a low-complexity algorithm is proposed for user-pairing/grouping as a result to enhance the spectral efficiency of the network and reduce the complexity.

For uplink NOMA scheme, we propose three alternative algorithms for user pairing/grouping that exploits the channel-gain difference among different users and the objective is to increase the spectral efficiency. To determine the optimum user pairing/grouping, a specific approach of solving user pairing/grouping problem is using exhaustive search approach. For fixed user-pairing/grouping scheme, the optimal solution is obtained. Then, list all the users in the decreasing order of $D_n$ accordingly. The proposed three alternative algorithms for user pairing/grouping are illustrated in Algorithms 1, 2 and 3, respectively. The feasible solution of user-pairing/grouping problem has been defined in the beginning of all algorithms. However, it signifies that the computational complexity of algorithm 1 is $O(NK)$, and Algorithms 2 and 3 are $O(N^2)$. The computational complexity of algorithms
Algorithm 1. Randomly algorithm.

**Data:** Set the input control variables \( N, K, \Gamma, \{g_i\}, \{s_u\}, C \)

**Result:** \( \beta_{k,n}( \text{opt solution} = s_u^*, \text{optmax}(f(s_u^*)) \)

**System Initialization;**

**Procedure:**

- List all the users with decreasing order of \( D_u \) accordingly.
- for \( i = 1 \) : total iterations
  - Generate a feasible solution \( s_u \)
  - for \( n = 1 : N \)
    - Measure \( f(s_u) \)
      - (according to equations (5) and (6)) to find, \( s_u^{(k,n)} \) for \( k = 1, 2, 3, \ldots, K \).
      - if \( f(s_u) \geq \text{optmax} \)
        - \( \text{optmax} = f(s_u) \)
        - \( \text{optsolution} = s_u \)
      - end
    - end for
  - end for

**return** \( \beta_{k,n}( \text{optsolution} = s_u^*, \text{optmax}(f(s_u^*)) \)

Algorithm 2. 2-Opt algorithm.

**Data:** Set the input control variables \( N, K, \Gamma, \{g_i\}, \{s_u\}, C \)

**Result:** \( \beta_{k,n}( \text{opt solution}, \text{optmax} \)

**System Initialization;**

**Procedure:**

- Given a random solution \( s_{u1} \), for each user \( i \) and \( j \), where \( i \) and \( j \) are different and belong to different groups \( u \) and \( v \), swap \((i, j)\).
- The result is a new solution for user grouping \( s_{u2} \).
- for \( n = 1 : N \)
  - if \( f(s_{u2}) \geq f(s_{u1}) \) (according to equations (5) and (6))
    - \( \text{optmax} = f(s_{u2}) \)
    - \( \text{optsolution} = s_{u2} \)
  - end
- end for

**return** \( \beta_{k,n}( \text{opt solution}, \text{optmax} \)

Algorithm 3. Hybrid algorithm.

**Data:** Set the input control variables \( N, K, \Gamma, \{g_i\}, \{s_u\}, C \)

**Result:** \( \beta_{k,n}( \text{opt solution, optmax} \)

**System Initialization;**

**Procedure:**

- Applied Algorithm 2, where the initial solution is the opt solution obtained by Algorithm 1.

**return** \( \beta_{k,n}( \text{optmax, opt solution} \)

\[
\rho = \begin{pmatrix}
1 & 2 & \cdots & n \\
1 & 2 & \cdots & m
\end{pmatrix}
\] (18)

where it can be expressed in mathematical form as

\[
\rho(j) = i, i \in \{1, 2, \ldots, m\}, j \in \{1, 2, \ldots, n\}
\] (19)

### Complexity of Algorithms 1, 2 and 3

The generator of feasible solution is complexity order \( O(NK) \), then the worst case of complexity of Algorithm 1 is order \( O(NK) \).

Consider a quasi permutation matrix \( (\rho) \) define in an equation (18). The total number of operations results in an exchange between two elements \( i \) and \( j \) in two groups \( (m \) groups and \( n \) users) and \( i_1, i_2, \ldots, i_m \) are the cardinality of each group

\[
f(n) = (m - 1) + (m - 2) + \cdots + 2 + 1 - [i_1 - 1] + (i_2 - 1) + \cdots + (i_m - 1)\]
\]

\[
= \sum_{i=1}^{n-1} i - \sum_{j=1}^{m} i_j - m
\] (21)

\[
= \frac{(n-1)n}{2} - [n - m]
\] (22)

\[
= \frac{n^2}{2} - n + m
\] (23)

\[
= \frac{n^2}{2} - \frac{3n}{2} + m
\] (24)

\[
= O(n^2)
\] (25)

Then, the total number of operations executed in Algorithm 3 is given as

\[
f(n) = \frac{n^2}{2} - \frac{3n}{2} + m + mn
\] (26)

\[
= O(n^2)
\] (27)

where \( mn < n^2 \), \( O(mm) \) is a complexity of randomly.

**Definition.** A feasible solution of the user pairing/grouping can be defined by the following matrix

\[
A = a_{ij}, i \in \{1, 2, \ldots, m\}, \quad j \in \{1, 2, \ldots, n\}
\] (16)

where \( m \) and \( n \) represents the number of groups and users

\[
a_{ij} = \begin{cases}
1, & \text{if user } j \text{ } \text{in group } i \\
0, & \text{otherwise}
\end{cases}
\] (17)

The same solution can be represented as a form of quasi permutation matrix as following
Simulation results

This section evaluates the proposed algorithms (namely, Random, 2-Opt, and Hybrid) for their performance considering user-pairing/grouping scheme for uplink NOMA system. Parameters characterizing the simulation are summarized in Table 1.

Both channel of the users and location are allocated randomly in the simulation. Therefore, the range between the user and BS are uniformly distributed and the channel response is assumed to be a Gaussian distribution.34

Figure 2 shown above compares the spectral efficiency of NOMA systems with that for OMA systems under varying transmission power-to-noise ratio for different $N$, where $N = 7, 8, 9$ and $10$. The simulated results validate the superiority of proposed algorithms for NOMA over OMA systems in terms of spectral efficiency. In addition, the formulated scheme has also been found to attain optimality along with performance enhancement while offering significantly low computational complexity than technique presented in

| Parameter                        | Symbol | Value  |
|----------------------------------|--------|--------|
| Number of cells                  | $C$    | 1      |
| Total number of users            | $N$    | 10     |
| Number of PRBs                  | $K$    | 3      |
| Minimum rate requirement         | $s_n$  | 1.1 bits/s/Hz |
| (Figure 2)                       |        |        |
| Transmission power-to-noise ratio| $\Gamma$ | 30 dB  |

Table 1. Parameters for proposed uplink power domain multiple access.

Figure 2. Illustration of spectral efficiency of NOMA with varying $\Gamma$ for different $N$. (a) $N = 7$, (b) $N = 8$ (c) $N = 9$ and (d) $N = 10$. 
Zhang et al. The effect of an increase in total number of users within a cell on the spectral efficiency of both NOMA and OMA systems for different $s_n$, where $s_n = 1.2$, $1.3$, $1.4$ and $1.5$ bits/s/Hz is demonstrated in Figure 3. It can easily be observed that the NOMA scheme clearly outperforms OMA. In addition, NOMA-centric user-pairing/grouping algorithms are explored for their spectral efficiency and are found to be appreciably better than their OMA-based counterparts. It is also evident that the spectral efficiency corresponding to all NOMA-based algorithms for user-paring/grouping surpasses that for the OMA scheme. Moreover, solutions to the proposed random and hybrid algorithms approach optimality. The algorithms also exhaust less computational resources and easier to be implemented.

**Figure 3.** Illustration of spectral efficiency of NOMA with increasing $N$ for different $s_n$. (a) $s_n = 1.2$ bits/s/Hz (b) $s_n = 1.3$ bits/s/Hz, (c) $s_n = 1.4$ bits/s/Hz and (d) $s_n = 1.5$ bits/s/Hz.

**Conclusion**

This work jointly investigates decoding order, power control and user grouping to enhance network performance. An optimal solution for determination of appropriate decoding order and power control is formulated. Thereafter, three distinct algorithms for suboptimal user-grouping solutions are proposed which bring about significant improvement in spectral efficiency while offering lowered complexity and ease of implementation. Simulated results validate the efficacy of the proposed computationally efficient approach for uplink NOMA system by demonstrating improvement in performance when stacked against conventional OMA scheme. Moreover, the performance surpasses the existing approaches reported recently in the
literature, making the proposed strategy a good candidate for addressing the demand posed by 5G wireless communication networks.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship and/or publication of this article: This research was supported by Taif University Researchers Supporting Project Number (TURSP-2020/216), Taif University, Taif, Saudi Arabia.

ORCID iD
Bilal Ur Rehman https://orcid.org/0000-0003-2349-7518

References
1. Thompson J, Ge XC, Wu Irmer R, et al. 5G wireless communication systems: prospects and challenges. IEEE Commun Mag 2014; 52(2): 62–64.
2. Agiwal M, Roy A and Saxena A. Next generation 5G wireless networks: a comprehensive survey. IEEE Commun Surv Tut 2016; 18(3): 1617–1655.
3. Zou X, He B and Jafarkhani H. An analysis of two-user uplink asynchronous non-orthogonal multiple access systems. IEEE T Wirel Commun 2019; 18(2): 1404–1418.
4. Wang P, Xiao J and Ping L. Comparison of orthogonal and nonorthogonal approaches to future wireless cellular systems. IEEE Veh Technol Mag 2006; 1(3): 4–11.
5. Ghosh A, Ratasuk R, Mondal B, et al. LTE-advanced: next-generation wireless broadband technology. IEEE Wireless Commun 2010; 17(3): 10–22.
6. Ding Z, Yang Z, Fan P, et al. On the performance of nonorthogonal multiple access in 5G systems with randomly deployed users. IEEE Signal Proc Let 2014; 21(12): 1501–1505.
7. Zeng M, Yadav A, Dobre O, et al. Capacity comparison between MIMO-NOMA and MIMO-OMA with multiple users in a cluster. IEEE J Sel Area Comm 2017; 35(10): 2413–2424.
8. Wang H, Leung S and Song R. Precoding design for two-cell MIMO-NOMA uplink with CoMP reception. IEEE Commun Lett 2018; 22(12): 2607–2610.
9. Ding Z, Fan P and Poor H. Impact of user pairing on 5G nonorthogonal multiple-access downlink transmissions. IEEE T Veh Technol 2016; 65(8): 6010–6023.
10. Fu Y, Chen Y and Sung C. Distributed power control for the downlink of multi-cell NOMA systems. IEEE T Wirel Commun 2017; 16(9): 6207–6220.
11. Liu Y, Qin Z, Elkashlan M, et al. Nonorthogonal multiple access for 5G and beyond. Proc IEEE 2017; 105(12): 2347–2381.
12. Dahlanman E, Parkvall S and Skold J. 4G LTE/LTE-advanced for mobile broadband. New York: Academic, 2013.
13. Sung C and Fu Y. A game-theoretic analysis of uplink power control for a non-orthogonal multiple access system with two interfering cells. In: Proceedings of the IEEE 83rd Vehicular technology conference, Honolulu, HI, 21–26 July 2016, pp.1–5. New York: IEEE.
14. Wang H, Song R and Leung S. Optimal uplink access in cognitive femtocell networks with adaptive modulation. IEEE Commun Lett 2016; 20(5): 1050–1053.
15. Wang H, Leung S and Song R. Uplink area spectral efficiency analysis for multichannel heterogeneous cellular networks with interference coordination. IEEE Access 2018; 6: 14485–14497.
16. Wang HFUY, Shi Z and Song R. Fractional power control for small cell uplinks with opportunistic NOMA transmissions. In: IEEE international conference (ICC), Shanghai, China, 20–24 May 2019, pp.5386–8088. New York: IEEE.
17. Wang K, Liang W and Yuan Y. User clustering and power allocation for hybrid non-orthogonal multiple access systems. IEEE T Veh Technol 2019; 68(12): 12052–12065.
18. Tabassum H, Ali M, Hossain E, et al. Non-orthogonal multiple access (NOMA) in cellular uplink and downlink: challenges and enabling techniques. Networking and internet architecture, arXiv:1608.05783, 2016.
19. Liang Y, Li X and Huanggi M. Non-orthogonal multiple access (NOMA) in uplink Poisson cellular networks with power control. IEEE T Commun 2019; 67(11): 8021–8036.
20. Azam I, Shahab M and Shin S. User pairing and power allocation for capacity maximization in uplink NOMA. In: 42nd international conference on telecommunications and signal processing (TSP), Budapest, 1–3 July 2019, pp.690–694. New York: IEEE.
21. Wang G, Zhao Z and Xu T. A joint interference cancellation method for non-orthogonal multiple access uplink signals. In: 14th IEEE international conference on signal processing (ICSP), Beijing, China, 12–16 August 2018, pp.694–697. New York: IEEE.
22. Guo G, Lu H, Zhu D, et al. Joint user association, grouping and power allocation in uplink NOMA systems with QoS constraints. In: IEEE international conference on communications (ICC), Shanghai, China, 20–24 May 2019, pp.1–6. New York: IEEE.
23. Pischella M and Ruyet D. NOMA-relevant clustering and resource allocation for proportional fair uplink communications. IEEE Wirel Commun Le 2019; 8(3): 873–876.
24. Yukekayya B and Toker C. Power and interference regulated water-filling for multi-tier multi-carrier interference aware uplink. IEEE Wirel Commun Le 2018; 7(4): 494–497.
25. Ali M, Tabassum H and Hossain E. Dynamic user clustering and power allocation for uplink and downlink non-orthogonal multiple access (NOMA) systems. IEEE Access 2016; 4: 6325–6343.
26. Liu Y, Derakhshani M and Lambotharan S. Outage analysis and power allocation in uplink non-orthogonal multiple access systems. IEEE Commun Lett 2017; 22(2): 336–339.
27. Chen L, Ma L and Xu Y. Proportional fairness-based user pairing and power allocation algorithm for
non-orthogonal multiple access system. IEEE Access 2019; 7: 19602–19615.

28. Khan W, Jameel F, Ristaniemi T, et al. Efficient power allocation for multi-cell uplink NOMA network. In: IEEE 89th vehicular technology conference (VTC 2019-Spring), Kuala Lumpur, Malaysia, 28 April–1 May 2019, pp.1–5. New York: IEEE.

29. Sedaghat M and Miller R. On user pairing in uplink NOMA. IEEE T Wirel Commun 2018; 17(5): 3474–3486.

30. Zhang Z and Hu R. Uplink non-orthogonal multiple access with fractional power control. In: Proceedings of the IEEE wireless communication network conference, San Francisco, CA, 19–22 March 2017, pp.1–6. New York: IEEE.

31. Zhang Z, Sun H and Hu R. Downlink and uplink non-orthogonal multiple access in a dense wireless network. IEEE J Sel Area Comm 2017; 35(12): 2771–2784.

32. Zheng H, Hou S, Li H, et al. Power allocation and user clustering for uplink MC-NOMA in D2D underlaid cellular networks. IEEE Wirel Commun Le 2018; 7(6): 2162–2337.

33. Sun Z and Jing Y. Average power analysis and user clustering design for MISO-NOMA systems. In: IEEE 21st international workshop on signal processing advances in wireless communications (SPAWC), Atlanta, GA, 26–29 May 2020, pp.26–29. New York: IEEE.

34. Zhang J, Zhu L, Xiao Z, et al. Optimal and sub-optimal uplink NOMA: joint user grouping, decoding order and power control. IEEE Wirel Commun Le 2019; 9: 254–257.

35. Do D, Le A and Lee B. On performance analysis of underlay cognitive radio-aware hybrid OMA/NOMA networks with imperfect CSI. Electronics 2019; 8: 819.

36. Do D and Le C. Application of NOMA in wireless system with wireless power transfer scheme: outage and ergodic capacity performance analysis. Sensors 2018; 18: 3501.

37. Zeng M, Yadav A, Dobre O, et al. Energy-efficient joint user-RB association and power allocation for uplink hybrid NOMA-OMA. IEEE Internet Things 2019; 6: 5119–5131.

38. Zeng M, Nguyen N, Dobre O, et al. Spectral- and energy-efficient resource allocation for multi-carrier uplink NOMA systems. IEEE T Veh Technol 2019; 68(9): 9293–9296.