Extraction of the $\pi NN$ coupling constant
from NN scattering data

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Abstract

We reexamine Chew’s method for extracting the $\pi NN$ coupling constant from np differential cross section measurements. Values for this coupling are extracted below 350 MeV, in the potential model region, and up to 1 GeV. The analyses to 1 GeV have utilized 55 data sets. We compare these results to those obtained via $\chi^2$ mapping techniques. We find that these two methods give consistent results which are in agreement with previous Nijmegen determinations.

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I. INTRODUCTION

While the $\pi NN$ coupling constant has been known within a few percent for decades, there have been numerous attempts to more precisely determine its value, and to search for possible charge-independence breaking. Recent results from the Nijmegen $NN$ and $\bar{N}N$ analyses [1], and the Virginia Tech $\pi N$ and $NN$ analyses [2], [3] have found values of $g^2/4\pi$ near 13.7 with little evidence for charge dependence. However, the importance of this coupling continues to motivate independent studies [4] utilizing different methods of extraction.

In 1958, Chew suggested [5] that the influence of the pion-pole might be measurable in $NN$ elastic scattering, if appropriate kinematic regions were examined. This technique has recently been applied to $\bar{p}p \rightarrow \bar{n}n$ [6] and np elastic [7] differential cross section measurements. In these studies, the extracted values for $g^2/4\pi$ ($g^2/4\pi = 12.83 \pm 0.36$ [6] and $g^2/4\pi = 12.47 \pm 0.36$ [7]) were somewhat below those found in Refs. [1], [3].

In the present work, selected np differential cross section data [7]−[15] below 1 GeV have been analyzed using a modified form of the Chew method [5], [11]. In the region below 350 MeV, the “difference method” [11] was used in conjunction with the Nijmegen, Bonn and Paris potentials, and various partial-wave solutions. We considered differences between “model” and “experimental” values of the dimensionless quantity $y = x^2 s (d\sigma/d\Omega)/(hc)^2$ expanded in terms of a function of $x = 1 + MT(1 + \cos \theta)/\mu^2$,

$$y_{\text{fit}} = y_{\text{model}} + \sum_{j=0}^{N} a_j b_j(x)$$

where $s$ is the usual Mandelstam variable, $T$ is the laboratory kinetic energy, and $\mu$ ($M$) is the pion (nucleon) mass. While it is conventional to expand ($y_{\text{fit}} - y_{\text{model}}$) as a power series in $x$, we chose to expand in terms of Legendre polynomials, $P_j(\rho)$,

$$b_0 = 1, \quad b_j(j > 0) = P_j(\rho) - 1,$$

with $\rho = 1 - 2(x/x_{\text{max}})$, which resulted in improved numerics but had no effect on the final results (several different expansions were made for comparison purposes). The difference
between the model coupling constant and the experimental value is then given in terms of the first coefficient

\[ g_{\text{exp}}^4 = g_{\text{model}}^4 + a_0, \tag{3} \]

in the above expansion.

The results of our analyses are given in Tables I and II. In Table I, we list values of \( g^2/4\pi \) found via the difference method applied to data below 1 GeV, using our partial-wave solution SM95 as input. In order to treat the different datasets as consistently as possible, we put a number of constraints on the fits.

- Extractions were performed on data backward of 110 degrees.
- Only those datasets with more than 15 qualifying points were accepted.
- An upper limit for the variable \( x \) was taken to be 10.
- The number of fitted parameters was given a lower limit of 4.

The search was terminated either at a minimum \( \chi^2/(\text{degree of freedom}) \) or when the error on \( g^2/4\pi \) exceeded 1.

In order to gauge the effect of renormalizing datasets, two independent determinations of \( g^2/4\pi \) were made for each experiment. In the first case, data were fitted without any renormalization. In the second case, data were first renormalized according to the input solution SM95. The average values of \( g^2/4\pi \), and their associated uncertainties, are listed in Table II for a number of input solutions and energy ranges. The uncertainties were calculated without accounting for systematic errors. We estimate that more realistic uncertainties are larger by at least a factor of 2.

The values in Table I show considerable scatter, as is evident from Fig. 1. Given this variability, the calculated averages are remarkably consistent. Coupling constants determined using potential model and partial-wave inputs below 400 MeV are in good agreement those
found using our partial-wave solutions up to 1 GeV. These averages are also quite consistent with the results of Refs. [1], [3].

As a check, we have repeated the coupling constant extractions of Refs. [7], [11] and agree with their results. In Ref. [11] it is claimed that the 96 MeV and 162 MeV Uppsala cross sections [9], [11] imply a consistent value for $g^2/4\pi$. This is not evident from the values quoted in Table I. We can, however, explain the discrepancy. While order 4 and order 6 fits to the data of Ref. [9] gave values of $g^2/4\pi$ consistent with those found in Ref. [11], the order 5 fit gave a somewhat larger value. In choosing the results listed in Table I, the order 6 fit was rejected, as the associated error was too large, according to our criteria for terminating a search. In contrast, the coupling derived from the data of Ref. [11] was much less sensitive to the chosen order.

As a further consistency check we mapped $g^2/4\pi$ against $\chi^2$ over different energy regions, analyzing $pp$ and $np$ data both separately and in combined fits. This extended a study reported earlier [2]. In analyzing the combined dataset ($pp$ and $np$) below 400 MeV, we found a value of coupling near 13.8. Results were obtained using the Nijmegen Coulomb distorted-wave Born approximation [21] for the one-pion exchange (OPE) contribution. A significantly higher value of $g^2/4\pi$ (near 14.4) was found if we used a plane-wave Born approximation [22] for the OPE contribution. While this appears to confirm our Chew-extrapolation results, less consistency was found when the extraction was broken into separate $pp$ and $np$ contributions, or when the energy range was increased to 1.6 GeV. In fact the sign of $(g_{pp}^2 - g_{np}^2)$ changed when the analysis was extended from 400 MeV to 1.6 GeV. These results are listed in Table III. Results of our analyses to 1.6 GeV are displayed in Fig. 2. The $\chi^2$ map from our analyses of elastic pion-nucleon scattering data is included for comparison.

In summary, we have found that the value of $g^2/4\pi$ extracted from NN scattering data is quite consistent with our previous determinations. The Nijmegen group has also generated $\chi^2$ maps [1] of the Bonner et al. [10] data below 350 MeV, obtaining results in qualitative agreement with ours. However, given the variation of $g^2/4\pi$ values displayed in Fig. 1, we cannot claim to have improved upon the results found from our application of dispersion
relations to elastic pion-nucleon scattering data.

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Figure captions

Figure 1. Plot of $g^2/4\pi$ determinations from np differential cross sections versus the laboratory kinetic energy. The input solution was SM95. The solid line gives the average value. Results for unnormalized data are displayed.

Figure 2. $\chi^2$ maps from fits to all NN data below 1.6 GeV (dot-dashed line), pp data only (dashed line), and np data only (dotted line). The result from fits to elastic pion-nucleon scattering data (solid line) is display for comparison. The variation of $\chi^2$ from minimal values ($\delta \chi^2$) is plotted against $g^2/4\pi$. 
Table I. Values of $g^2/4\pi$ extracted using the difference method applied to data below 1 GeV at backward angles (see text). The laboratory kinetic energy ($T$), the angular range ($\theta$) and number (N) of data used in the analysis, the order (M) of the fit, and the corresponding $\chi^2$/degree of freedom are given for each dataset. In the first column of $g^2/4\pi$ values, datasets were fit without renormalization. In the next column, a normalization factor (Norm), determined from solution SM95, was applied prior to fitting.

| $T$ (MeV) | $\theta$ (°) | N  | Ref | M  | $\chi^2$/dof | (unnormalized) $g^2/4\pi$ | Norm  | (normalized) $g^2/4\pi$ |
|-----------|--------------|----|-----|----|-------------|---------------------------|-------|-------------------------|
| 91.0      | 119 – 177    | 16 | 8   | 4  | 0.81        | 12.59 (1.41)              | 1.056 | 10.96 (2.36)            |
| 96.0      | 117 – 179    | 32 | 9   | 4  | 0.61        | 16.50 (0.57)              | 0.982 | 16.37 (0.56)            |
| 162.0     | 122 – 178    | 43 | 10  | 4  | 1.51        | 12.96 (0.75)              | 1.050 | 13.19 (0.77)            |
| 162.0     | 119 – 179    | 31 | 11  | 4  | 1.05        | 14.65 (0.36)              | 0.938 | 14.32 (0.34)            |
| 177.9     | 122 – 179    | 44 | 10  | 4  | 1.08        | 13.06 (0.67)              | 1.042 | 13.25 (0.69)            |
| 194.5     | 121 – 179    | 42 | 10  | 4  | 1.54        | 12.15 (0.63)              | 1.040 | 12.29 (0.65)            |
| 211.5     | 120 – 178    | 43 | 10  | 4  | 0.76        | 12.75 (0.54)              | 1.029 | 12.86 (0.55)            |
| 212.0     | 111 – 177    | 30 | 12  | 6  | 0.69        | 14.67 (0.68)              | 0.970 | 13.88 (0.49)            |
| 229.1     | 120 – 178    | 49 | 10  | 4  | 1.36        | 12.80 (0.47)              | 1.027 | 12.90 (0.48)            |
| 247.2     | 119 – 178    | 53 | 10  | 4  | 0.73        | 13.21 (0.42)              | 1.014 | 13.26 (0.42)            |
| 265.8     | 118 – 179    | 63 | 10  | 4  | 0.84        | 12.97 (0.37)              | 1.002 | 12.98 (0.37)            |
| 284.8     | 117 – 179    | 73 | 10  | 4  | 1.14        | 13.32 (0.23)              | 1.024 | 13.41 (0.24)            |
| 304.2     | 116 – 179    | 79 | 10  | 6  | 0.97        | 13.16 (0.76)              | 0.975 | 12.72 (0.88)            |
| 319.2     | 117 – 177    | 40 | 12  | 6  | 0.85        | 11.85 (0.69)              | 0.989 | 11.75 (0.69)            |
| 324.1     | 115 – 179    | 80 | 10  | 4  | 1.10        | 13.57 (0.18)              | 1.028 | 13.67 (0.19)            |
| 344.3     | 118 – 179    | 74 | 10  | 4  | 1.12        | 13.14 (0.21)              | 1.003 | 13.15 (0.21)            |
| 365.0     | 120 – 179    | 62 | 10  | 5  | 1.33        | 12.72 (0.46)              | 1.046 | 12.89 (0.50)            |
| 386.0     | 121 – 179    | 55 | 10  | 4  | 1.44        | 13.51 (0.20)              | 1.066 | 13.73 (0.21)            |
Table I (continued).

| \( T \) (MeV) | \( \theta \) (°) | \( N \) | Ref | \( M \) | \( \chi^2/dof \) | (unnormalized) \( g^2/4\pi \) | Norm | (normalized) \( g^2/4\pi \) |
|---------------|----------------|------|-----|-----|----------------|----------------|------|----------------|
| 407.3         | 123 – 179     | 52   | [10]| 4   | 0.52           | 13.34 (0.20)   | 1.065| 13.56 (0.20)   |
| 418.0         | 128 – 177     | 20   | [12]| 5   | 0.63           | 12.72 (0.52)   | 1.010| 12.70 (0.52)   |
| 418.0         | 127 – 178     | 28   | [12]| 5   | 0.74           | 12.44 (0.49)   | 0.986| 12.37 (0.49)   |
| 421.4         | 152 – 180     | 42   | [13]| 4   | 1.29           | 15.87 (1.31)   | 0.949| 15.51 (1.27)   |
| 428.9         | 134 – 180     | 51   | [10]| 4   | 0.87           | 13.38 (0.26)   | 1.006| 13.41 (0.26)   |
| 450.9         | 135 – 180     | 50   | [12]| 4   | 0.75           | 12.92 (0.24)   | 1.061| 13.66 (0.56)   |
| 457.2         | 152 – 180     | 42   | [13]| 4   | 1.47           | 14.35 (0.89)   | 1.021| 14.50 (0.90)   |
| 459.3         | 127 – 180     | 76   | [13]| 6   | 1.62           | 12.19 (0.54)   | 1.112| 12.68 (0.59)   |
| 473.2         | 134 – 180     | 54   | [10]| 5   | 0.82           | 12.91 (0.46)   | 1.073| 13.33 (0.48)   |
| 493.0         | 128 – 172     | 20   | [12]| 4   | 1.23           | 13.90 (0.46)   | 1.003| 13.91 (0.47)   |
| 494.6         | 151 – 180     | 43   | [13]| 4   | 2.26           | 13.75 (0.74)   | 1.069| 14.16 (0.77)   |
| 495.7         | 133 – 180     | 62   | [10]| 4   | 1.06           | 13.52 (0.16)   | 1.053| 13.74 (0.16)   |
| 518.5         | 132 – 180     | 67   | [10]| 4   | 0.85           | 13.68 (0.14)   | 1.047| 13.86 (0.14)   |
| 532.0         | 133 – 180     | 70   | [13]| 4   | 1.84           | 12.67 (0.26)   | 1.045| 12.82 (0.27)   |
| 541.6         | 131 – 180     | 71   | [10]| 4   | 1.02           | 13.72 (0.14)   | 1.038| 13.87 (0.14)   |
| 564.9         | 133 – 180     | 69   | [10]| 4   | 1.03           | 13.95 (0.15)   | 1.019| 14.03 (0.15)   |
| 570.9         | 134 – 180     | 69   | [13]| 5   | 1.20           | 12.62 (0.41)   | 1.086| 13.36 (0.20)   |
| 588.4         | 134 – 180     | 67   | [10]| 4   | 0.91           | 13.90 (0.17)   | 1.021| 13.98 (0.17)   |
| 610.3         | 134 – 180     | 68   | [13]| 5   | 1.40           | 12.44 (0.44)   | 1.056| 12.68 (0.46)   |
| 612.2         | 134 – 179     | 67   | [10]| 6   | 0.95           | 15.23 (0.58)   | 1.019| 15.19 (0.56)   |
| 636.2         | 137 – 179     | 65   | [13]| 4   | 0.83           | 14.12 (0.23)   | 0.980| 14.03 (0.23)   |
| 647.5         | 136 – 180     | 47   | [13]| 5   | 0.92           | 13.61 (0.40)   | 0.993| 13.62 (0.38)   |
| 649.6         | 146 – 180     | 50   | [13]| 4   | 2.21           | 12.01 (0.51)   | 1.043| 12.18 (0.53)   |
| 660.4         | 136 – 179     | 66   | [10]| 4   | 1.27           | 14.33 (0.22)   | 1.001| 14.34 (0.22)   |
| 684.8         | 137 – 179     | 65   | [10]| 6   | 0.84           | 15.42 (0.70)   | 1.016| 16.19 (0.76)   |
| 690.2         | 150 – 180     | 42   | [13]| 4   | 1.03           | 12.28 (0.50)   | 1.044| 12.47 (0.51)   |
| 709.3         | 138 – 179     | 63   | [10]| 6   | 0.87           | 13.82 (0.89)   | 0.997| 13.99 (0.22)   |
| 731.3         | 138 – 180     | 57   | [13]| 5   | 1.28           | 11.77 (0.52)   | 1.061| 12.49 (0.48)   |
| 734.1         | 138 – 179     | 62   | [10]| 6   | 1.01           | 13.17 (0.75)   | 1.006| 13.30 (0.73)   |
| 770.6         | 139 – 179     | 63   | [10]| 4   | 1.50           | 13.84 (0.12)   | 1.068| 14.12 (0.12)   |
| 772.9         | 139 – 180     | 58   | [13]| 4   | 1.15           | 13.00 (0.20)   | 1.067| 13.22 (0.21)   |
| 801.9         | 141 – 180     | 50   | [13]| 4   | 0.98           | 13.47 (0.19)   | 1.091| 13.81 (0.20)   |
| 814.9         | 141 – 180     | 56   | [13]| 5   | 1.43           | 11.63 (0.47)   | 1.053| 11.89 (0.48)   |
| 856.8         | 142 – 180     | 54   | [13]| 4   | 0.97           | 12.88 (0.25)   | 1.054| 13.04 (0.26)   |
| 899.3         | 143 – 180     | 52   | [13]| 4   | 1.85           | 13.38 (0.25)   | 1.098| 13.69 (0.27)   |
| 942.5         | 144 – 180     | 50   | [13]| 4   | 1.45           | 13.87 (0.27)   | 1.056| 14.07 (0.28)   |
| 986.0         | 144 – 180     | 49   | [13]| 5   | 1.14           | 14.53 (0.42)   | 1.112| 15.20 (0.45)   |
Table II. Table of average $g^2/4\pi$ values obtained using partial-wave analyses (SM95, VV40, and NI93), and potential models (NY93, BONN, and PARIS) as input. N is the number of datasets involved in the analysis.

| Solution | (Model) | Ref | Range (MeV) | N | (unnormalized) $g^2/4\pi$ | (normalized) $g^2/4\pi$ |
|----------|---------|-----|-------------|---|---------------------------|--------------------------|
| SM95     | 13.75   | present | 0 – 1000 | 55 | 13.58 (0.04) | 13.75 (0.04) |
| SM95     | 13.75   | present | 0 – 400  | 18 | 13.47 (0.08) | 13.51 (0.08) |
| VV40     | 13.75   | present | 0 – 400  | 18 | 13.55 (0.08) | 13.66 (0.08) |
| NI93     | 13.58   | [17] | 0 – 350  | 16 | 13.51 (0.08) | 13.65 (0.09) |
| NY93     | 13.58   | [18] | 0 – 350  | 16 | 13.04 (0.09) | 13.31 (0.09) |
| BONN     | 14.40   | [19] | 0 – 325  | 15 | 13.72 (0.10) | 13.74 (0.10) |
| PARIS    | 14.43   | [20] | 0 – 350  | 16 | 13.46 (0.08) | 13.76 (0.09) |
Table III. Table of optimal $g^2/4\pi$ and $\chi^2$ values obtained from $\chi^2$ maps. Solution A has only the one-pion exchange (OPE) contribution for $J>6$. Solution B has only the OPE contribution for $J>5$. Solution C is the same as A, with a plane-wave Born approximation for the OPE. The region below 400 MeV contains 2170 pp and 3532 np data. The region below 1600 MeV contains 12838 pp and 11171 np data.

| Solution       | $g^2/4\pi$ (pp data only) | $\chi^2$ | $g^2/4\pi$ (np data only) | $\chi^2$ | $g^2/4\pi$ (pp and np data) | $\chi^2$ |
|----------------|---------------------------|----------|---------------------------|----------|-----------------------------|----------|
| A(0-400 MeV)   | 13.61(0.09)               | 3034     | 14.16(0.12)               | 4533     | 13.80(0.07)                 | 7582     |
| B(0-400 MeV)   | 13.70(0.08)               | 3055     | 14.07(0.11)               | 4555     | 13.83(0.07)                 | 7617     |
| C(0-400 MeV)   | 14.42(0.08)               | 3046     | 14.51(0.11)               | 4554     | 14.38(0.06)                 | 7602     |
| (0-1600 MeV)   | 13.67(0.06)               | 22030    | 13.42(0.04)               | 17625    | 13.51(0.03)                 | 39668    |