1. Introduction

Development is an integral part of the life cycle of organizations, enterprises, companies, regardless of their type of activity and scale. For enterprises and companies, new qualities that are eventually transformed into an increase in financial performance, are formed as a result of development.

The problem of the development of enterprises (organizations, companies) from the scientific point of view has not lost its relevance for quite a long time. New management concepts are emerging and developing, methodology and international standards for project and program management are designed. These standards structure the knowledge and world experience of the preparation and implementation of projects and programs, through which development is actually carried out in practice. All this leads to the emergence of new approaches, methods and models for the development of enterprises, companies, organizations.

Development is a multifaceted problem [1] and involves a wide range of problems of varying levels and nature. Development can be considered to be a specific stage or a whole sequence of stages that establish a long-term prospect for a developing object. The latter is related to the need to develop a so-called road map that structures the step-by-step achievement of development goals. Paper [2] contains the thesis that the creation of a road map should be a strategic
process aimed at achieving the necessary characteristics with the costs that are real by magnitude, using the right methods and at the right time.

We believe that the use of a road map as a strategic management tool has broad prospects that go beyond those that are currently covered by existing research.

2. Literature review and problem statement

The category of a “road map” has been a popular strategic management tool since the early 2000s. Gradually, the use of this tool goes beyond the strategic development of enterprises and finds its application in the processes of development of new technologies, project implementation, creation and marketing of new products. Today, the largest number of publications in this direction is devoted to the formation of road maps in the field of development of innovation and new information technologies ([3–7] are the examples of such publications). In particular, [3–5] show the results of structuring the road map areas and objectives in the system of “technology – products – market”, as well as the variants of graphic representation of the road map components. In [6], the specified system was considered at the level of a multi-product company level and the approach to harmonizing road maps for different products, markets and technologies was presented. Paper [7] focuses on the visual representation of a road map. What unites papers [3–7] is that the main result is structuring and visualization of road maps for development of various technologies that are aimed at creation of new products designed to be brought out to competitive markets. Therefore, these results cannot be used in their current form to substantiate the road maps of enterprise development in a broad sense in the context of their strategic development. The reason for this attention to road maps for the creation of new products based on new technologies is that the classical methods and approaches of strategic management are not fully applicable to this process. The need to harmonize “technological” and “marketing” processes causes the demand for the appropriate tools.

The main results in the publications on the issue of construction of road maps are generally formalization and visualization of the relationships between different aspects and participants of road map. Specifically, article [8] proposes the procedures for solving these problems, and the author believes that road maps come in different configurations with different objectives, such as forecasting, planning and administration. As a generalization, article [8] states the following: the most typical road map is based on the layers such as the market, product and technology that cover the horizontal timescale. These layers reflect the evolution of competition, markets, products, technologies, and the relationship between these factors. At the same time, the author focuses on the construction of networks of relationships between subjects and objects of road maps. The same path is followed by the authors in studies [3–7], which gives a specificity of a road map from the standpoint of responsible executors.

Combination of different approaches to structuring road maps is used in paper [9], the effect of such combination is the multifaceted structuring of the road map stages, which gives a functionally full coverage of the enterprise activity.

It should be noted that some studies take into consideration the area specifics and offer the appropriate structures of road maps, in particular, as in [10–12].

We should note as an interesting approach to the construction of a universal road map the study [13], which provides a formalized approach to construct a targeted road map based on the multidimensional cluster analysis. In this study, a formalized approach is used to structure the system of development goals. At the same time, special attention is paid to the “dimensionality of a road map”, that is, the space in which the map is constructed. These results can serve as the basis for the space-time structure of a road map.

Thus, the problem of “how to represent and structure a road map” prevails in modern publications over the problems “how to determine the essence of the stages of a road map” in the context of the classic multi-stage development of enterprises.

It should also be noted that there is little or no research focusing on how to substantiate the parameters and indicators that need to be achieved at the stages of a road map. At the same time, resources – something, without which any road map can be implemented – generally remains beyond the attention of the authors and are not taken into consideration in the development of road maps. However, it is the resources that ultimately determine the possibility of the practical implementation of any road map.

The exceptions are studies [14–17]. For example, paper [14] presents the mathematical model of harmonization of target indicators, criteria and separate indicators. This study is based on the decomposition of the set target indicators at different periods of time, which gives for each structural unit of a road map the appropriate parameters, the achievement of which will ensure the achievement of the set global targets.

Instead of the concept of a road map, research [15] uses the “planned development trajectory”, which is in its essence a road map. According to this approach, the trajectory reflects the “spatial” characteristic of the development results. In [15], it was proposed to use time and a set of indicators that characterize the state of an enterprise (financial, production, marketing) as the dimensions of such space. This, in fact, corresponds to the approaches of modern authors to understanding the essence of a road map for the development of enterprises. Since this trajectory is following “a road map”, the ideas presented in [15] can be used to form a road map for the development of enterprises.

Studies [16, 17] focus on the impact of the macro and micro environment on the road map construction. In addition, [17] emphasizes aligning the parameters of a road map with fuzziness of the environment parameters.

To sum up, most modern publications on the area under consideration offer conceptual consideration of “road maps” and their structuring without supporting the processes of their development through formalized methods. The use of the latter is necessary because the development of a road map involves:

- operation the significant arrays of information about the current state of the external environment and its dynamics;
- taking into consideration alternative variants to achieve certain indicators;
- possibility of varying resource volumes by changing the time and quality parameters of projects corresponding to the stages of a road map (this, in particular, was explored at the project level in [15]).

All this necessitates the development of the formalized tools that will make it possible to determine the optimal
structure and parameters of a road map for the development of enterprises. In contrast to existing results providing the conceptual representation, the economic and mathematical model will ensure well-grounded and specific parameters of development.

3. The aim and objectives of the study

The aim of this study is to develop the economic and mathematical model to substantiate the optimal structure and parameters of a road map, which ensures the efficiency of business development processes.

To achieve the aim, the following tasks were set:
- to identify the structure and parameters of the strategic network as the basis for the formation of a road map;
- to take into consideration the dynamics of the environment, resource constraints, and the possibility of varying the time parameters of a road map when developing the model;
- to conduct experimental research into the formation of a road map based on the developed model.

4. Representation of the problem on forming a road map for enterprise development as a two-stage problem

As a tool of strategic management, a road map corresponds to the long-term prospect ensuring achievement of the set prospect target. In fact, each stage of a road map in the context in question is the project, the totality of which forms a long-term program of the step-by-step development of an enterprise. Implementation of each step ensures that a specific set of indicators describing the state of an enterprise is achieved. Thus, the long-term strategy is divided into stages, and the state of an enterprise after the implementation of each stage is a certain point of the corresponding phase space, which is presented in papers [12, 15]. This approach is universal in the development of road maps in any field of activity.

The concept of “a strategic network” was introduced and its formalized description was implemented in paper [15]. At the same time, the development of an enterprise was presented as a choice of a route on the given network (similar to the route in the transport network), for which the appropriate optimization model was proposed. These results are proposed to be developed in the context of the formation of a road map for the enterprise development. In particular, a more detailed consideration of alternative variants of road map projects will make it possible to ensure a greater variation in the options of development in the face of limited resources and alternative variants of their use.

Thus, the problem of the formation of a road map for the enterprise development is a two-stage problem. A strategic network is formed at the first stage, an optimal route is chosen and the optimal parameters of the projects that form a path, in fact, a road map, are set at the second stage. The formation of the strategic network should be carried out in a “manual” mode and does not imply the use of special methods. However, a network must be characterized in a certain way. The formation of a road map, in turn, involves the use of the economic and mathematical model, which will make it possible to take into consideration the possible variation of the time parameters of a network.

Existing mathematical approaches to the formation of the optimal route on the transport network, presented particularly in [18, 19], are used to form the model.

According to the accepted essence of “a road map”, it is supposed to form it in the space of “time – indicators of achievement of goals”. With this approach, a road map is clearly determined in the accepted coordinate system. For example, the target indicators may include the capacity of an enterprise, production volumes, sales volumes, competitiveness, financial results, etc. Subsequently, the two-dimensional space will be used for illustration with the view to clear visual representation of the ideas and provisions expressed. However, let us assume that the state of an enterprise is determined in more than a two-dimensional space.

5. Strategic network as the basis for the formation of a road map for development

Thus, point 1 at the initial moment of time (Fig. 1) corresponds to the current state of an enterprise, and point 10 is the desired state of an enterprise after a certain period. We will note that similarly to the transport route, several possible (alternative) paths, that is, variants of road maps, can lead to the set goal. For example, in the case of two-dimensional space, the possible trajectories are formed based on key points of the phase space, which can be achieved through the implementation of a project and can look as follows (Fig. 1).

![Fig. 1. Strategic network as the basis for the formation of a road map for enterprise development](image)

Thus, a network is formed similarly to the transport network. Each node of this network is the state of an enterprise at a certain point of time. The “transition” to a new state is carried out through projects. For example, the financial result is accepted as the ordinate axis in Fig. 1.

Note an important feature of this strategic network: each transition from state to state can be carried out by different variants, that is, through the implementation of different projects. For example, an increase in the capacity of an enterprise or an increase in sales volumes can be achieved by upgrading or renovating production facilities based on a variety of technological solutions.

Moreover, each project can be implemented in different time frames, which causes a certain “mobility” of the points of a strategic network relative to the time axis (Fig. 2).

In the presented fragment, the transition from state $i$ to state $j$ can be implemented by three alternative projects. The time to implement each $k$-th project $\Delta t_k$, $k = 1, 2, 3$ can be minimally admissible $\Delta t_k^{\text{min}}$, $k = 1, 2, 3$ or maximally ac-
ceptable $\Delta t_{ijtk}^\text{max}, k = 1, 2, 3$, which naturally entails changing the need for resources on a project. At the same time, certain possible time limits correspond to each project.

![Fig. 2. Transition between the points of the strategic network](image)

Thus, the variation in project implementation time makes it possible to manage the time of implementation of not only each stage separately, but also a road map in general. This flexibility makes it possible to vary, first of all, the necessary resources.

In addition, the important feature of any project was noted in paper [15]: dependence of the result (for example, financial) on the time of the beginning of a project and obtaining the project product. This is determined by the dynamics of the cost of the resources used in a project and the dynamics of the market of the project product market. Thus, the use of a favorable moment for the beginning or the end of a project can lead to an increase in its result, even on the condition of completion of the project works within a shorter time at a higher price. Moreover, if the project is related to the sale or purchase of production means, the price of them is also subject to a change in dynamics, which, in particular, was demonstrated in [18] on the example of the project of purchasing a vessel.

Thus, the specifics of the projects that determine the “motion” on the road map is:
- dependence of project costs on the intensity of its implementation (it is possible to implement a project faster, but at a higher costs), thus the project money and time are interconnected (Fig. 2);
- project costs depend on the interval of time related to the beginning of the implementation, which is caused by the dynamics of the cost of resources used in a project.

Each node of the network in question (subsequently, a road map) is characterized by the following set of parameters:
- the set indicators that characterize the state of an enterprise (for example sales volumes, financial results, market share, etc.);
- the minimally possible and maximally acceptable moment of time (determined from $\Delta t_{ijtk}^\text{max}, \Delta t_{ijtk}^\text{min} 1/2$ – the boundary terms of implementation of the sequence of projects leading to this node);
- minimal and maximal costs, which are determined by alternatives variants of the costs for reaching the given node (based on possible sequences of projects leading to the given node, as well as their duration).

Note that the moment of time (minimal, maximal) together with the indicators of state form the coordinates of a point (the enterprise location) in the phase space.

For certain points of time within the studied period of the road map, it is possible to establish the “control” values of indicators of the state of an enterprise, which, in fact, determine the admissibility of certain stages (projects) of a road map.

Thus, there arises the challenge to determine the optimal road map for the enterprise development, that is, to choose the sequence of projects and their parameters, which will ensure the achievement of the goal taking into consideration the basic requirements and constraints.

The constraints of development are conditioned by the external and internal environment of an enterprise: the external environment, first of all, determines the possibilities for the implementation of certain projects with specified characteristics; the internal environment of an enterprise sets constraints on resources and key project indicators. The main requirement is to achieve certain indicators that characterize the state of an enterprise by certain points in time and, in fact, the achievement of the ultimate goal (multiple goals) against the deadline.

Thus, the problem of determining the optimal road map for the enterprise development is solved on an oriented network, which is determined as a “strategic network”, which is characterized by the following:

1. Transition from one state to another takes place over a certain period, the duration of which, generally speaking, can vary within the specified boundaries, that is the network is flexible in terms of time.
2. Transition from one state to another requires the use of resources, the amount of which may also have alternatives, depending on the terms of projects implementation.

This network is similar to transport networks (the main provisions of which are outlined, for example, in papers [19, 20]). Unlike transport ones, a strategic network is considered over time and is exposed to outside influence, which is also a dynamic process (demand, prices, level of competition, etc.). For transport networks, the intermediate points are in the usual topological space, for the strategic network under consideration, they are in the phase space of the enterprise state, in this case, the dimensionality of this space can vary significantly in every specific case. Each network link is the process of transition from one stage of development to another through the projects, the implementation of which provides a new state of an enterprise in the phase space.

6. Formalized description of the elements and parameters of the strategic network

According to the above-mentioned judgments, the main elements of a strategic network are “nodes” – the states of an enterprise and the projects that correspond to network transitions from one state to another; and the main parameter is time.

A more detailed description of a strategic network is provided below (formulated based on the results outlined in [15]):

1. A strategic network is assigned in the form of a matrix of relations of separate stages $\{A_{ij}, i, j = 1, n\}$, the elements of which are 0, 1, and $n$ is the number of stages under consideration.

2. Transition from one stage to another is carried out within time $\Delta t_k$, where $k = 1, K$ is alternative projects), which is limited taking into consideration the specific features of the projects and wishes of the managers $\Delta t_{ijtk}^\text{min} \leq \Delta t_{ijtk}^\text{max} \leq \Delta t_{ijtk}^\text{max}$

3. Beginning of the transition from one strategic stage to another $\Delta t_k$ (beginning of the project) can be imple-
We will note that at the stage of the network is the beginning of the project implementation (transformation established based on the previous and the opportunities of borrowed resources). The project costs intensity \( R_i^k(t, \Delta t_i^k, \Delta T_i^{k'}) \) is formed from two components \( \Delta t_i^k \) and \( \Delta T_i^{k'} \) (variables). Let us accept the following:

- \( \Delta t_i^k \) is the duration of the project funding process; \( \Delta T_i^{k'} \) is the beginning of the project implementation (transition to the new stage).

Optimal values of these management parameters are set for those projects that will be selected for a road map. These parameters determine the following characteristics of projects (which form the characteristics of the stages of a road map):

- project costs;
- root mean square deviation of project costs;
- financial result after project implementation;
- root mean square deviation of the financial result after the project implementation.

In addition to the above mentioned, the management parameters also include the choice of a project. That is why we will introduce into consideration the variables that ensure, in fact, the formation of a road map for development: \( x_i^k \in \{0, 1\}, k = 1, K, i, j = 1, n \). These variables correspond to the non-zero elements of matrix \( A_{ij} \), \( i, j = 1, n \).

First of all, we will formulate the constraints of the model, which reflect the formation of the sequence of projects – a road map (formed by analogy with the choice of a route on a transport network):

\[
\sum_{j=1}^{n} x_i^k - \sum_{j=1}^{n} x_i^j = 0, \quad (j = \frac{n}{2}, \frac{n}{2} - 1).
\]

\[
\sum_{j=1}^{n} x_i^k = 1,
\]

\[
\sum_{j=1}^{n} x_i^j = 1.
\]

The main constraint of the developed economic and mathematical model is the available resources. Let us assume that the opportunities to attract financial resources for the implementation of strategic development are limited by magnitude \( R(t) \), that characterizes the maximum possible intensity of funding. Since in practice, most often, there is a mixed form of funding for enterprise development projects, \( R(t) \) is formed from two components \( R'(t) = R'(t) + R''(t) \), which, respectively, characterize their own investment opportunities \( R'(t) \) and the opportunities of borrowed funds \( R''(t) \). \( R''(t) \) are established based on the previous experience (for example the use of a bank loan). The sources of formation of \( R'(t) \) may be the profit from the previous periods of operation of the studied or the company, in the structure of which the enterprise is included. In the second situation, in particular, \( R'(t) \) can be established as a certain share \( \alpha \) of the intensity of gaining the profit from various businesses by the company \( F(t) \), in this case \( R'(t) = \alpha \cdot F(t) \).

Note: this study does not address the technology of the construction of a strategic network, but identifies its essence, structure and specificity, that is, designs the requirements for it, based on the specifics of the structure of a road map of an enterprise development. Fig. 1, 2 provide a graphic representation of the model of the strategic network model and its parameters.
bined businesses, and the optimization of the distribution of corporate finance of the company, which goes beyond this work. Therefore, we believe that $R'(t)$ and $R''(t)$ are set beyond the limits of the model and are assigned as source data.

Split the entire time interval $[0, T]$ ($T$ is the time of the road map implementation, which is assigned taking into account covering all upper time boundaries of different project variants $\max \{t_{\text{fin}} + T_{\text{proj}} \} \leq T$) into periods $[t_{t_i}, t_{t_i+1}]$, $i = 1, L$ (some years; a year, half year).

Introduce the function in consideration:

$$R_{x^i} = \begin{cases} R_{x^i}^c(t, \Delta t, T^c) & \text{if } _{-}(t^c + \Delta t^c) \in [0; t] \wedge (t^c + T^c) \in [t, t_i], \\ 0, & \text{if } _{-}(t^c + T^c) \leq t, \end{cases}$$

and $R_{x^i}^c = \int R_{x^i}^c(t, \Delta t, T^c) - k(\alpha)\sigma_{\alpha}^c(t)dt$.

where $c = \begin{cases} t^c, & \text{if } _{-}t^c \in [t_{t_i}, t_l], \\ t_{l_i}, & \text{if } _{-}t^c \leq t_{l_i}, \end{cases}$

d = \begin{cases} \Delta t^c + T^c, & \text{if } _{-}(t^c + \Delta t^c) \in [t_{l_i}, t_t], \\ t_t, & \text{if } _{-}(t^c + \Delta t^c) \geq t_t. \end{cases}$

Thus, the limitation by resources throughout the duration of the road map is reasonable:

$$\sum_{i=1}^{L} \sum_{j=1}^{J} \sum_{k=1}^{K} R_{x^i} \cdot x^i \leq \int R(t)dt, i = 1, L.$$  \hspace{1cm} (6)

As mentioned above, the whole set can act as target indicators of development. However, only one target indicator should actually be used as an optimization criterion, while the rest can serve as additional constraints.

Without limiting the commonality, the financial result of an enterprise in the new state as the difference between the financial inflows of an enterprise and the costs of implementation of road map projects will be considered as objective function of the model:

$$\sum_{i=1}^{L} \sum_{j=1}^{J} \sum_{k=1}^{K} \left( F_{a_t^i}^c(t) - R_{x^i}^c(t, \Delta t^c, T^c)^i \right) \left( t^c + \Delta t^c + T^c \right) x^i = \max. \hspace{1cm} (7)$$

It had previously been accepted that project costs and financial inflows were random magnitudes. That is why for each transition, there arises a risk of increased costs and decreased inflows, which can be assessed, for example, with the help of the procedure [21], based on the VAR approach and be considered as:

$$k(\alpha) = \int (F_{a_t^i}^c(t) - k(\alpha)\sigma_{\alpha}^c(t)dt).$$  \hspace{1cm} (8)

where $k(\alpha)$ is the coefficient that is determined based on the Laplace function depending on assigned probability $\alpha$. Thus, (8) is the magnitude of possible losses, and the probability that these losses will be higher is $\alpha$ (as a rule, $\alpha$ is accepted as 0.05, and corresponding $k(\alpha) = 1.65$); similarly, the project investment costs can be exceeded by magnitude

$$k(\alpha) \int (F_{a_t^i}^c(t, \Delta t^c, T^c)^i)dt.$$

Given both types of risk in the objective function, expression (7) can be corrected as follows:

$$\int (F_{a_t^i}^c(t) - \sigma_{\alpha}^c(t))dt - \sum_{i=1}^{L} \sum_{j=1}^{J} \sum_{k=1}^{K} \left( R_{x^i}^c(t, \Delta t^c, T^c)^i \right) \left( t^c + \Delta t^c + T^c \right) x^i \rightarrow \max. \hspace{1cm} (9)$$

This expression is the financial result of the implementation of a road map taking into consideration possible losses.

Maximizing the financial outcome of development may not provide the necessary efficiency, both ultimate and current. That is why, we should introduce the constraint, the satisfaction of which would ensure the necessary final effectiveness of activities, taking into consideration the possible additional costs and a decrease in cash inflows:

$$\sum_{i=1}^{L} \sum_{j=1}^{J} \sum_{k=1}^{K} \left( F_{a_t^i}^c(t) - \sigma_{\alpha}^c(t) \right) \left( t^c + \Delta t^c + T^c \right) x^i \geq \int \left( R_{x^i}^c(t, \Delta t^c, T^c)^i \right) \left( t^c + \Delta t^c + T^c \right) dt \rightarrow \max. \hspace{1cm} (10)$$

It is also necessary in the development process to ensure the assigned level of intensity of cash flows $CF(t)$ during the entire time period under consideration. To form a constraint for the intensity of cash flows, we will introduce the following function in consideration:

$$F_{a_t^i}^c = \left\{ \begin{array}{ll} F_{a_t^i}^c(t, \Delta t^c, T^c)^i, & \text{if } _{-}(t^c + \Delta t^c) \in [0; t_t] \wedge (t^c + T^c) \in [t_{l_i}, T], \\ 0, & \text{if } _{-}(t^c + \Delta t^c) \leq t_{l_i}; \end{array} \right.$$  \hspace{1cm} (12)

where

$$a = \begin{cases} t^c + \Delta t^c, & \text{if } _{-}(t^c + \Delta t^c) \in [t_{l_i}, t_t], \\ t_{l_i}, & \text{if } _{-}(t^c + \Delta t^c) \leq t_{l_i}; \end{cases}$$

$$b = \begin{cases} t^c + \Delta t^c + T^c, & \text{if } _{-}(t^c + \Delta t^c + T^c) \in [t_{l_i}, t_t], \\ t_t, & \text{if } _{-}(t^c + \Delta t^c + T^c) \geq t_t. \end{cases}$$
Satisfaction of condition
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} \left( F^{ij}_{t} - F^{ij}_{s} \right) x_{ij} \geq \int_{t_o}^{t_f} CF(t) dt, i = 1, \ldots, l, \] (13)
ensures a certain level of cash flow throughout the entire road map of development.

Note that the conditions formulated in (10), (11) take into consideration the worst version of the terms of implementation of the road map, for example, the impact of unfavorable market conditions and the deliberate excess of project costs, since the relevant financial indicators are adjusted for the magnitude of risk. Practically, the road map should be optimized for different variants of accounting risk, for example, not taking into consideration possible losses at all, or taking into consideration only one type of risk. Further analysis for comparison of the obtained options and additional analysis of possible risks will eventually make it possible to choose the best variant of a road map with regard to the most likely conditions for its implementation.

Since not only the road map in general, but also each of its projects must ensure certain efficiency, the model should be subject to appropriate restrictions:

For \( x_{ij} = 1, i, j = 1, n, k = 1, K \)
\[ \int_{t_o}^{t_f} F^{ij}_{t} dt x_{ij}^{ij} \geq E_{k}, \] (14)
\[ \int_{t_o}^{t_f} R^{ij}_{t} (t, \Delta_{ij}^{t}, T_{ij}^{t}) dt x_{ij}^{ij} \geq E, \] (15)

where \( E_{k} \), \( E > 0 \) are the assigned minimally admissible levels of effectiveness.

(14) provides a minimum allowable level for projects selected in a road map in the context of traditional economic efficiency (profitability). Satisfaction of (15) determines that the average cash flow intensity of the project corresponds to the minimum allowable level.

Restrictions for the values of control parameters
\[ \Delta_{s}^{\min} \leq \Delta_{ij}^{t} \leq \Delta_{s}^{\max}, \] (16)
\[ t_{ij}^{\min} \leq t_{ij}^{k} \leq t_{ij}^{\max}, \] (17)
\[ T_{ij}^{\min} \leq T_{ij}^{k} \leq T_{ij}^{\max} \] (18)
complete the formation of the economic and mathematical model, the composition of which: (1) to (3), (6), (7), (10), (11), (14) to (18). This model makes it possible to determine the optimal road map for the development of an enterprise, taking into consideration the possibility of varying time parameters. We accept as the main time parameters: beginning of each stage, duration of transition from stage to stage, duration of the funding phase. In addition, the model takes into consideration the probabilistic nature of investment costs and cash inflows of an enterprise after the implementation of development activities.

8. Experimental calculations on the proposed model

In order to prove the operation ability and reliability of the proposed model, the relevant experimental studies were carried out on the example of the following strategic network (Fig. 3), a fragment of the main temporal characteristics of which is shown in Table 1. Without limiting the totality, it was accepted that each transition on the road map could be performed with the help of a single project.

Fig. 3. Strategic network for formation of a road map (calculation example)

| Table 1 | Boundaries of temporal parameters, months |
|---|---|
| Transition | \( \Delta_{s}^{\min} \) | \( \Delta_{s}^{\max} \) | \( t_{ij}^{\min} \) | \( t_{ij}^{\max} \) | \( T_{ij}^{\min} \) | \( T_{ij}^{\max} \) | \( T_{ij} \) |
| 1–2 | 6 | 9 | 5 | 15 | 3 | 13 | 15 |
| 1–4 | 2 | 6 | 5 | 15 | 2 | 10 | 20 |
| 1–3 | 1 | 7 | 5 | 15 | 3 | 8 | 15 |
| 2–4 | 3 | 5 | 8 | 20 | 4 | 12 | 10 |
| 2–5 | 6 | 8 | 8 | 20 | 4 | 12 | 15 |
| 2–6 | 2 | 9 | 8 | 20 | 5 | 10 | 15 |
| 3–6 | 3 | 9 | 15 | 25 | 5 | 10 | 12 |
| 4–5 | 11 | 15 | 16 | 26 | 6 | 10 | 10 |
| 5–7 | 1 | 9 | 18 | 28 | 3 | 12 | 15 |
| 6–7 | 2 | 12 | 22 | 30 | 3 | 12 | 10 |

It was accepted in the experimental calculations that dynamics \( R_{ij}(t, \Delta_{ij}, T_{ij}^{t}) \) is described by the linear dependence on time and by the quadratic dependence on the terms of funding (which was proved by corresponding statistical research). Thus, as the fundamental form \( R_{ij}(t, \Delta_{ij}, T_{ij}^{t}) \) and \( F_{ij}(t) \), in the experimental research it was accepted that
\[ R_{ij} = \Delta_{ij} (a + b \cdot t + c \cdot (T_{ij}^{t})^2 + d \cdot T_{ij}^{t}) \] and
\[ F_{ij} = g + f \cdot t \quad (a, b, c, d, g, f \in R), \] respectively. In particular, for numerical experiments, we used the following dependences (Table 2), the graphical illustration of which for separate projects was shown in Fig. 4, 5.
As a result of the optimization, the following results were obtained for assigned conditions (Table 3) – the values of parameters of model management, which determine structure \( x_0 \) and parameters \( (\Delta t^0, \Delta t^1, T^0) \) of the road map of development of an enterprise which is optimal by the assigned criterion (10).

It should be noted that in the framework of experimental calculations, the investment opportunities (resources) \( R(t) \), were varied, the reduction of which naturally led to a change in the optimal road map (Fig. 6).

**Table 2**

| Transition (project) | \( R_i(t, \Delta t^i, T^i) \) | \( F_i(t) \) |
|----------------------|----------------------------|--------------|
| 1–2                  | \( \Delta t^0 \) \((1.5 + 0.09 \cdot t) \left(-0.2T^0_{ij} + 0.18T^1_{ij}\right) \) | \( F_i = 2 + 4 \cdot t \) |
| 1–4                  | \( \Delta t^1 \) \((5.3 - 0.3 \cdot t) \left(T^1_{ij} + 1.2T^2_{ij}\right) \) | \( F_i = 3 - 0.02 \cdot t \) |
| 1–3                  | \( \Delta t^2 \) \((15 - 0.4 \cdot t) \left(-T^0_{ij} + 4.2T^2_{ij}\right) \) | \( F_i = 1 - 0.04 \cdot t \) |
| 2–4                  | \( \Delta t^3 \) \((16 - 0.6 \cdot t) \left[12T^1_{ij} - 0.4T^2_{ij}\right] \) | \( F_i = 12 - 0.4 \cdot t \) |
| 2–5                  | \( \Delta t^4 \) \((2 - 0.06 \cdot t) \left[5T^1_{ij} - 0.09T^2_{ij}\right] \) | \( F_i = -0.2 + 0.5 \cdot t \) |
| 2–6                  | \( \Delta t^5 \) \((22 - 0.3 \cdot t) \left[9T^1_{ij} - 0.4T^2_{ij}\right] \) | \( F_i = 1 + 0.04 \cdot t \) |
| 3–6                  | \( \Delta t^6 \) \((14 - 0.04 \cdot t) \left[-8T^1_{ij} + 1.4T^2_{ij}\right] \) | \( F_i = 1.2 + 0.4 \cdot t \) |
| 4–5                  | \( \Delta t^7 \) \((12 - 0.3 \cdot t) \left[6T^1_{ij} - 0.12T^2_{ij}\right] \) | \( F_i = 2 - 0.1 \cdot t \) |
| 5–7                  | \( \Delta t^8 \) \((28 - 0.7 \cdot t) \left[13T^1_{ij} - 0.09T^2_{ij}\right] \) | \( F_i = -1 + 0.1 \cdot t \) |
| 6–7                  | \( \Delta t^9 \) \((20 - 0.4 \cdot t) \left[4T^1_{ij} + 0.11T^2_{ij}\right] \) | \( F_i = 1.2 - 0.02 \cdot t \) |

**Fig. 4.** Dynamics in the intensity of investment costs \( R_i(t, \Delta t^i, T^i) \) in calculation example for separate projects at minimal admissible values of \( \Delta t^i, T^i \)

**Fig. 5.** Dynamics in the intensity of profit by an enterprise \( F_i(t) \) after project implementation

**Table 3**

| Transition (project) | \( \Delta t^0 \) | \( \Delta t^1 \) | \( T^0 \) | \( x_0 \) |
|----------------------|----------------|----------------|---------|---------|
| 1–2                  | 9              | 8              | 3       | 1       |
| 1–4                  | 2              | 14             | 2       | 0       |
| 1–3                  | 1              | 5              | 4       | 0       |
| 2–4                  | 3              | 8              | 4       | 0       |
| 2–5                  | 6              | 21             | 4       | 1       |
| 2–6                  | 9              | 8              | 12      | 0       |
| 3–6                  | 3              | 25             | 5       | 0       |
| 4–5                  | 15             | 16             | 12      | 0       |
| 5–7                  | 1              | 27             | 3       | 1       |
| 6–7                  | 2              | 29             | 3       | 0       |

**Fig. 6.** The optimal structure of the road map under various resource constraints:

\( a \) – at \( 150 \leq R(t) \leq 250 \), \( b \) – \( 250 \leq R(t) \leq 350 \)

In addition, in the framework of the experimental calculations, there was an increase in the possible risk (root mean square deviation of the intensity of obtaining profit from one of the stages according to (8)), which also led to a change in the optimal road map (Table 4).

**Table 4**

| Value of \( \sigma \), share | «road map» |
|-----------------------------|------------|
| \( \sigma_{x^0} = 0.3 \)    | 1-3-6-7    |
| \( \sigma_{x^1} = 0.2 \)    | 1-2-5-7    |
| \( \sigma_{x^2} = 0.1 \)    | 1-2-5-7    |

To study the impact of time, we explored several variants of the boundaries for the beginning of projects...
Based on the numerical experiments for the assigned source data, the following conclusions were made: two variants of the road map 1-2-5-7 and 1-3-6-7 are preferable; in this case, path 1-2-5-7 ensures the best financial results – objective function (10) for this path takes value 900, for 1-3-6-7 – 400. In addition, the second path has the higher profitability (according to (11)).

Thus, the availability of financial resources, profit dynamics, possible time limits and the magnitude of the possible risk determine the choice of the road map structure. Note that the restrictions on the lower boundary of cash flows $CF(t)$, are of great importance in the optimization of the process of the enterprise development. Thus, high efficiency in terms of profitability of the variant of road map 1-3-6-7 ensures much smaller inflows of cash than the road map variant 1-2-5-7. Variation $R(t)$, $CF(t)$ and the ranges of possible values of time parameters make it possible to determine the preferable variant of the road map of the enterprise development.

Experimental studies of the model demonstrated a clear correlation between the results of optimization and input data, as well as the existence of a certain level of “sensitivity” of results. This is reflected, in particular, in the fact that only a certain level of change, for example, in profits, measure of risk and availability of resources determines the fundamental change in the structure and parameters of a road map. In addition, the “reaction” of each constraint and the objective function at a change in the source data in totality with the above conclusion substantiates the performance of the proposed model.

Thus, the use of the presented model in actual situations of strategic planning will make it possible to find the variant of development of a road map that will meet all set conditions.

9. Discussion of results of studying the model for determining the optimal structure and parameters of the road map for enterprise development

The main results of the study are the economic and mathematical model of the formation of the road map for the enterprise development (1) to (3), (6), (7), (10), (11), (14) to (18) and the model for strategic network (graphic representation in Fig. 1, 2), which serves as its basis.

Unlike the existing approaches to the development of road maps, based on the condition that the development “trajectory” has already been set, the totality of the proposed results makes it possible to structure, formalize and optimize the road map at the level of its initiation and fundamental determining. Thus, the proposed results make it possible to form the structure and parameters of the road map, substantiated by calculation. This provides a theoretical basis for decision making at a higher level of consideration of a road map – at the level of its development in the form of the structured totality of projects.

The strategic network is oriented in the space “time – indicators of the state of an enterprise” and is a sequence of alternative projects, characterized by the totality of time parameters that allow the variation within the set limits. This approach is fundamentally new to road maps and universal in terms of the essence of development. The proposed results apply not only to the development through new products and the implementation of innovations (which is most often explored in publications), but also to the development by increasing, for example, the capacity of an enterprise.

Unlike the existing informal approaches to the development of road maps, the model takes into consideration the market dynamics (external environment), indirectly in profits $F_i^O(t)$ of an enterprise after the implementation of projects, which forms the objective function (10) and the constraint on efficiency (11); the possibility to vary resources to provide certain boundaries not only of final efficiency, but also by separate time intervals in (13). Moreover, the model makes it possible to carry out the experiments “what happens if…” some external conditions or resource availability (6) change, for example. The specificity of the model takes into consideration the possibility of temporary shifts of the stages of the road map within admissible limits, which is taken into account in constraints (16) to (18), and also provides flexibility in the use and distribution of resources.

The experimental studies of the formation of a road map based of the developed model were conducted, which proved its operation ability. We believe that the model can find its wide practical application, for example, in solving the issues on the construction of road maps of a step-by-step increase in production volumes, or a step-by-step equipment replacement (technical development).

The constraint of the use of this model in practice is the need to have the information on the dynamics of market conditions for the formation of $F_i^O(t)$ and $G_{ij}^d(t)$, which can be difficult during a sufficiently long period of a road map. That is why the development of the proposed result for sorting out this problem may be to take into consideration different scenarios for the development of external environment. In addition, one of the directions of further development of the obtained results is the construction of a road map of corporate structures – multidisciplinary companies. At this level, it becomes necessary, on the one hand, to harmonize the distribution of general corporate resources in the areas of activity, on the other hand, to take into consideration the specific conditions of implementation of development for each business direction.

10. Conclusions

1. The formation of a strategic network implies positioning the states of an enterprise in the course of its development in the space “time – indicators of the state of an enterprise”. This network is based on the principle of formation of transport networks that make it possible to form alternative paths from the starting point to the final point. Accordingly, the strategic network makes it possible to form different variants of the road map for the enterprise development and is the basis for the optimization of its structure and parameters.

The main elements of the strategic network are “nodes” – the states of an enterprise and “projects” that correspond to network transitions from one state to another. The main parameters of the strategic network are time parameters. The following parameters were determined as time parameters:
- duration of transition from one state to another, determined by the intensity of project activity;
- duration of the project funding process;
- beginning of project implementation (transition to a new stage).
The specified parameters determine the following characteristics of projects (which form the characteristics of the road map):

- project costs;
- root mean square deviation of project costs;
- financial result after the project implementation;
- root mean square deviation of the financial result after the project implementation.

2. The structure of the economic and mathematical model for establishing the optimal road map for the enterprise development is the following: the objective function is the financial outcome of the road map implementation, taking into consideration the possible losses; the constraints reflect the enterprise’s capabilities to fund the development projects, the possible duration of projects, and establish continuity of road map transitions. The model takes into account the possibility of varying the time parameters of the strategic network, which serves as the basis for the optimization. In addition, the model takes into consideration the probabilistic nature of investment costs and inflows of the funds of an enterprise after the implementation of development activities.

3. The experimental studies of the formation of a road map based on the developed model proved its operation capacity and the compliance of the results with the logic of an actual choice under different conditions. In particular, an increase in available resources determines the variant of a road map, in which an enterprise can achieve higher target results at the earlier stages of development. In the example, this is variant 1-2-5-7, which, unlike variant 1-3-6-7 at a lower level of resource capacity provides not only a higher total result, but also higher indicators at each stage of development. In addition, an increase in resources makes it possible to implement projects within a shorter time frame, which reduces the time for achieving the end goal.

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