Nuclear forces from chiral EFT: The unfinished business

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Abstract. In recent years, there has been substantial progress in the derivation of nuclear forces from chiral effective field theory (EFT). Accurate two-nucleon forces have been constructed at next-to-next-to-next-to-leading order (N^3LO) and applied (in some cases together with three-nucleon forces at NNLO) to nuclear few and many-body systems—with a fair deal of success. This may suggest that the 80-year old nuclear force problem has finally been cracked. Not so! Some pretty basic issues are still unresolved, like, the proper renormalization of the two-nucleon potential and subleading many-body forces. In this contribution, I will focus on the latter issue.

1. Introduction
The fundamental goal of nuclear structure physics is to understand the properties of atomic nuclei and their reactions in terms of the basic forces between the constituents. During the past half century, a large variety of phenomenological forces has been developed and applied in microscopic nuclear structure calculations with some success. But in the long run phenomenology is not good enough and, ultimately, we need nuclear interactions that are based upon proper theory. Since the nuclear force is a manifestation of strong interactions, any serious derivation has to start from quantum chromodynamics (QCD). However, the well-known problem with QCD is that it is non-perturbative in the low-energy regime characteristic for nuclear physics. For many years this fact was perceived as the great obstacle for a derivation of nuclear forces from QCD—impossible to overcome except by lattice QCD. The effective field theory (EFT) concept has shown the way out of this dilemma. One has to realize that the scenario of low-energy QCD is characterized by pions and nucleons interacting via a force governed by spontaneously broken approximate chiral symmetry. This chiral EFT allows for a systematic low-momentum expansion known as chiral perturbation theory (ChPT) [1]. Contributions are analyzed in terms of powers of small external momenta over the large scale: \((Q/\Lambda_{\chi})^\nu\), where \(Q\) is generic for an external momentum (nucleon three-momentum or pion four-momentum) or pion mass and \(\Lambda_{\chi} \approx 1\) GeV is the chiral symmetry breaking scale (‘hard scale’). The early applications of ChPT focused on systems like \(\pi\pi\) [2] and \(\pi N\) [3], where the Goldstone-boson character of the pion guarantees that the expansion converges.

The past 15 years have also seen great progress in applying ChPT to nuclear forces [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. As a result, nucleon-nucleon (NN) potentials of high precision have been constructed, which are based upon ChPT carried to next-to-next-to-next-to-leading order (N^3LO) [15, 17, 18], and applied in nuclear structure calculations with...
great success [19, 20, 21, 22, 23]. However, in spite of this progress, we are not done. Due to the complexity of the nuclear force issue, there are still many subtle and not so subtle open problems, like

- the proper renormalization of chiral nuclear potentials and
- subleading chiral few-nucleon forces.

This contribution is devoted to the latter issue. The renormalization problem is discussed in Refs. [24, 25].

2. Few-nucleon forces and what is missing

Nuclear three-body forces in ChPT were initially discussed by Weinberg [5]. The 3NF at NNLO, was derived by van Kolck [7] and applied, for the first time, in nucleon-deuteron scattering by Epelbaum et al [26]. The leading 4NF (at N³LO) was recently constructed by Epelbaum [27] and found to contribute in the order of 0.1 MeV to the $^4$He binding energy (total $^4$He binding energy: 28.3 MeV) in a preliminary calculation [28], confirming the traditional assumption that 4NF are essentially negligible. Therefore, the focus is on 3NF.

The power of a 3NF diagram is given by

$$\nu = 2 + 2L + \sum \Delta_i,$$

where $L$ denotes the number of loops in the diagram and

$$\Delta_i = d_i + \frac{n_i}{2} - 2$$

with $d_i$ the number of derivatives or pion-mass insertions and $n_i$ the number of nucleon fields (nucleon legs) involved in vertex $i$. The sum in (1) runs over all vertices contained in the diagram under consideration. We will use (1) to analyze 3NF contributions order by order. The lowest possible power is obviously $\nu = 2$ (NLO), which is obtained for no loops ($L = 0$) and only leading vertices ($\sum \Delta_i = 0$). This 3NF happens to vanish [5, 29, 30]. The first non-vanishing 3NF occurs at NNLO.

2.1. The 3NF at NNLO

The power $\nu = 3$ (NNLO) is obtained when there are no loops ($L = 0$) and $\sum \Delta_i = 1$, i.e., $\Delta_i = 1$ for one vertex while $\Delta_i = 0$ for all other vertices. There are three topologies which fulfill this condition, known as the two-pion exchange (2PE), 1PE, and contact graphs (figure 1).

The 3NF at NNLO has been derived (without the $1/M_N$ corrections) [7, 26] and applied in calculations of few-nucleon reactions [26, 31, 32, 33], structure of light- and medium-mass nuclei [21, 34, 35], and nuclear and neutron matter [23, 36] with a fair deal of success. However, the famous ‘$A_y$ puzzle’ of nucleon-deuteron scattering is not solved [26, 31], and there exists
a similar problem with the analyzing power in $p^3$He scattering [37, 38, 39]. Furthermore, the spectra of light nuclei leave room for improvement [21].

We note that there are further 3NF contributions at NNLO, namely, the $1/M_N$ corrections of the NLO 3NF diagrams. Part of these corrections have been calculated by Coon and Friar in 1986 [30]. These contributions are believed to be very small.

In summary, because of various unresolved problems in microscopic nuclear structure, the 3NF beyond NNLO is very much in need. In fact, it is no exaggeration to state that the 3NF at sub-leading orders is presently one of the most important outstanding issues in the chiral EFT approach to nuclear forces.

2.2. The 3NF at $N^3$LO

According to (1), the value $\nu = 4$, which corresponds to $N^3$LO, is obtained for the following classes of diagrams.

**3NF loop diagrams at $N^3$LO.** For this group of graphs, we have $L = 1$ and, therefore, all $\Delta_i$ have to be zero to ensure $\nu = 4$. Thus, these one-loop 3NF diagrams can include only leading order vertices, the parameters of which are fixed from $\pi N$ and $NN$ analysis. We show two samples of this very large class of diagrams in figure 2. One sub-group of these diagrams (“$2\pi$ exchange graphs”) has been calculated by Ishikawa and Robilotta [40], and two other topologies ($2\pi-1\pi$ and ring diagrams) have been evaluated by the Bonn-Jülich group [41]. The remaining topologies, which involve a leading order four-nucleon contact term (e.g., second diagram of figure 2), are under construction by the Bonn-Jülich group. The $N^3$LO $2\pi$-exchange 3NF has been applied in the calculation of nucleon-deuteron observables in [40] producing very small effects.

The smallness of the $2\pi$ loop 3NF at $N^3$LO is not unexpected. It is consistent with experience with corresponding 2NF diagrams: the NLO 2PE contribution to the $NN$ potential, which involves one loop and only leading vertices, is also relatively small.

By the same token, one may expect that also all the other $N^3$LO 3NF loop topologies will produce only small effects.

**3NF tree diagrams at $N^3$LO.** The order $\nu = 4$ is also obtained for the combination $L = 0$ (no loops) and $\sum_i \Delta_i = 2$. Thus, either two vertices have to carry $\Delta_i = 1$ or one vertex has to be of the $\Delta_i = 2$ kind, while all other vertices are $\Delta_i = 0$. This is achieved if in the NNLO 3NF graphs of figure 1 the power of one vertex is raised by one. The latter happens if a relativistic $1/M_N$ correction is applied. A closer inspection reveals that all $1/M_N$ corrections of the NNLO 3NF vanish and the first non-vanishing corrections are proportional to $1/M_N^2$ and appear at $N^3$LO. However, there are non-vanishing $1/M_N^2$ corrections of the NLO 3NF and there are so-called drift corrections [42] which contribute at $N^3$LO (some drift corrections are claimed to contribute even at NLO [42]). We do not expect these contributions to be sizable. Moreover, there are contributions from the $\Delta_i = 2$ Lagrangian [43] proportional to the low-energy constants $d_i$. As
it turns out, these terms have at least one time-derivative, which causes them to be $Q/M \supset N^4$ suppressed and demoted to $N^4$LO.

Thus, besides some minor $1/M^2 N$ corrections, there are no tree contributions to the 3NF at $N^3$LO.

**Summarizing the entire $N^3$LO 3NF contribution:** For the reasons discussed, we anticipate that this 3NF is weak and will not solve any of the outstanding problems. In view of this expectation, we have to look for more sizable 3NF contributions elsewhere.

2.3. The 3NF at $N^4$LO of the $\Delta$-less theory

The obvious step to be taken is to proceed to the next order, $N^4$LO or $\nu = 5$, of the $\Delta$-less theory which is the one we have silently assumed so far. (The $\Delta$-full theory will be introduced and discussed below.) Some of the tree diagrams that appear at this order were mentioned already: the $1/M^2 N$ corrections of the NNLO 3NF and the trees with one $d_i$ vertex which are $1/M_N$ suppressed. Because of the suppression factors, we do not expect sizable effects from these graphs. Moreover, there are also tree diagrams with one vertex from the $\Delta_i = 3 \pi N$ Lagrangian [44, 45] proportional to the LECs $e_i$. Because of the high dimension of these vertices and assuming reasonable convergence, we do not anticipate much from these trees either.

However, we believe that the loop contributions that occur at this order are truly important. They are obtained by replacing in the $N^3$LO loops (figure 2) one vertex by a $\Delta_i = 1$ vertex [with LEC $c_i$]. We show one symbolic example of this large group of diagrams in figure 3(a). This 3NF is presumably large and, thus, what we are looking for.

The reasons, why these graphs are large, can be argued as follows. Corresponding 2NF diagrams are the three-pion exchange (3PE) contributions to the $NN$ interaction. In analogy to Figs. 2 and 3(a), there are 3PE 2NF diagrams with only leading vertices and the ones with one (sub-leading) $c_i$ vertex (and the rest leading). These diagrams have been evaluated by Kaiser in Refs. [46] and [47], respectively. Kaiser finds that the 3PE contributions with one sub-leading vertex are about an order magnitude larger than the leading ones.

2.4. $N^5$LO 3NF contributions in the $\Delta$-full theory

The above considerations indicate that the $\Delta$-less theory exhibits, in some cases, a bad convergence pattern. The reason for the unnaturally strong subleading contributions are the large values of the $\Delta_i = 1$ LECs, $c_i$. The large values can be explained in terms of resonance saturation [48]. The $\Delta(1232)$-resonance contributes considerably to $c_3$ and $c_4$. The explicit inclusion of the $\Delta$ takes strength out of these LECs and moves this strength to a lower order, thus improving the convergence [49, 6, 10, 50, 51]. Figure 3 illustrates this fact for the 3NF under consideration: the diagram of the $\Delta$-less theory shown in (a) is (largely) equivalent to

![Figure 3.](image-url)
diagram (b) which includes one $\Delta$ excitation. Note, however, that diagram (a) is $N^4\text{LO}$, while diagram (b) is $N^3\text{LO}$. Moreover, there are further $N^3\text{LO}$ one-loop diagrams with two and three $\Delta$ excitations, which correspond to diagrams of order $N^5\text{LO}$ and $N^6\text{LO}$, respectively, in the $\Delta$-less theory.

This consideration clearly shows that the inclusion of $\Delta$ degrees of freedom in chiral EFT makes the calculation of sizable higher-order 3NF contributions much more efficient.

2.5. Summarizing the open 3NF business
To make a complicated story short, this is the bottom line concerning 3NF:

- The chiral 3NF at NNLO is insufficient. Additional sizable 3NF contributions are needed.
- The chiral 3NF at $N^3\text{LO}$ (in the $\Delta$-less theory) most likely does not produce sizable contributions.
- Sizable contributions are expected from one-loop 3NF diagrams at $N^4\text{LO}$ of the $\Delta$-less or $N^3\text{LO}$ of the $\Delta$-full theory (figure 3). These 3NF contributions may turn out to be the missing pieces in the 3NF puzzle and have the potential to solve the outstanding problems in microscopic nuclear structure.\(^1\)

3. Conclusions and Outlook
The past 15 years have seen great progress in our understanding of nuclear forces in terms of low-energy QCD. Key to this development was the realization that low-energy QCD is equivalent to an effective field theory which allows for a perturbative expansion that has become known as chiral perturbation theory. In this framework, two- and many-body forces emerge on an equal footing and the empirical fact that nuclear many-body forces are substantially weaker than the two-nucleon force is explained automatically.

In spite of the great progress and success of the past 15 years, there are still some unresolved issues that will need our attention in the near future. For this contribution, we picked the issue of the few-nucleon forces beyond NNLO (“sub-leading few-nucleon forces”) which are needed to hopefully resolve some important outstanding nuclear structure problems. We believe that we identified correctly where these forces will emerge within the systematic scheme of ChPT.

If the open issues discussed in this paper will be resolved within the next few years, then, after 80 years of desperate struggle, we may finally claim that the nuclear force problem is essentially solved. The greatest beneficiary of such progress will be the field of ab initio nuclear structure physics.

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\(^1\) Note that the Illinois 3NF [52] includes two one-loop diagrams with one and two $\Delta(1232)$-isobars. The deeper reason for this may be in arguments we are presenting.
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