Dynamic Characteristics Analysis of Cracked Magnetic Rotor-Bearing System

Yan Yang1,2, Lingyun Zhang1,2 and Yanxia Zhang1,2
1 School of Mechatronic Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China
2 Key Laboratory of System Dynamics and Reliability of Rail Transport Equipment of Gansu Province, Lanzhou, 730070, China
E-mail: yany@mail.lzjtu.cn

Abstract. A dynamic model of the rotor system supported by active magnetic bearings at both sides is established, the fourth-order Runge-Kutta method is used to simulate. The periodic motion transition of the system and its evolution to chaotic motion are discussed from bifurcation diagrams and phase diagrams. It emphatically analyses the change of crack influencing factor and eccentricity on the system response and stability. The results show that the system presents periodic motion with appropriate parameter conditions. Large crack fatigue damage and value of eccentricity increases the amplitude and chaotic motion window in the low speed region, decrease the system stability. When the rotor crosses the critical speed range and reaches the high rotational speed region, the stable period-1 motion is dominant.

1. Introduction
Active magnetic bearing has been recognized by the industry because of its high speed, low energy consumption, environmental protection and other advantages. Many scholars have studied the dynamic behaviour of active magnetic rotor-bearing in recent years, Xiang[1] considered the time-varying crack stiffness, rub-impact force and nonlinear oil-film force, analysed the nonlinear behaviour of the rotor bearing system. Tiwari[2] established a finite element model of the double-disk rotor-bearing system, and discussed the effect of supporting conditions on the rotor's dynamic behaviour. Ho[3] used Floquet theory to verify the stability and bifurcation of the magnetic force rotor model, and explained the importance of transforming the magnetic rotor model with rub-impact force into a mathematical model. Mohamed[4] found that in a rigidly supported magnetic rotor system, the coupled Hopf bifurcation occurs when the speed reaches a certain value. Jang, Jawaid[5-6] numerical studied the bifurcation and sub-harmonic vibration such as period-2, period-4, quasi-periodic and chaotic motion of the magnetic bearing rotor system. Luo, Babu, Khorrami[7-9] studied the influence of crack characteristics and crack number on the nonlinear dynamic characteristics of different rotor bearing systems. Wang, Subramanya, Bonello[10-12] developed the rotor system dynamic response test rig respectively, the theory of nonlinear vibration characteristics of the rotor bearing system was experimentally verified through data acquisition, influence parameters and other directions. Cracked magnetic rotor-bearing system exhibits stable basic periodic motion under certain parameter conditions. There are few reports on the transition law and bifurcation of such periodic motions at home and abroad. The nonlinear dynamic model of a magnetic bearing with cracks is established, its dynamic characteristics are studied. The influence of the system under the change of crack influencing factor and mass ratio were discussed in detail. The bifurcation characteristics and the evolution of
periodic motion, quasi-periodic motion, and chaotic motion under different rotational speeds are analysed.

2. Mechanical Model

The system dynamics model is shown in Fig.1. The rotor disc with the mass $2M_D$ is crossed on the flexible shaft supported by active magnetic bearings at both sides. There are fatigue cracks on the shaft, a fixed stator outside the rotor. The clearance between the rotor and stator is $\delta$. The assumptions are as follows: (1) The rotor is fixed at the midpoint of the shaft, and its mass is geometrically symmetric; (2) The rotor speed is invariable; (3) The rotor brace stiffness are radial symmetric; (4) The damping forces are regarded as acting on the mid-span disc of rotor due to air viscosity; (5) The unbalance of the rotor refers to the unbalance in a single plane where the rotor disc is located; (6) The axial motion, gyroscopic effect of the rotor are ignored. Only half of the system is considered when modelling the differential equations, because of the active magnetic rotor system is completely symmetrical.

![Figure 1. The model of rotor supported by magnetic bearings](image)

The magnetic levitation force generated by the active magnetic bearing is strongly nonlinear, affected by AC current, magnetic flux, etc.. The occurrence and propagation of cracks on the rotating shaft reduces the stiffness of the rotor. At the same time, the magnitude and direction of the stress on the crack surface are constantly changing. According to Newton second law, the differential equation of system motion can be established, which has comprehensively consider the impact of the rub-impact force between the rotor and the stator and the effect of the air flow vortex on the system.

$$\begin{align}
M_D \ddot{X}_D + C \dot{X}_D + K_J (X_D - X_J) = & M_D U \Omega^2 \cos \Omega t + F_{EX} - F_{CX} \\
M_D \ddot{Y}_D + C \dot{Y}_D + K_J (Y_D - Y_J) = & M_D U \Omega^2 \sin \Omega t + F_{BY} - M_D g - F_{CY} \\
M_J \ddot{X}_J + K_J (X_D - X_J) = & F_X \\
M_J \ddot{Y}_J + K_J (Y_D - Y_J) = & F_Y - M_J g 
\end{align}$$

In Eq.(1), $M_D$ is half mass of the rotor disk, $M_J$ is mass of the shaft journal, $X_D, Y_D$ are the displacement of disk centre in the x-axis, y-axis direction, $X_J, Y_J$ are the displacement of shaft journal centre in the x-axis, y-axis direction, $C$ is damping coefficient of the rotor shaft, $K_J$ is stiffness coefficient of the shaft, $U$ is the rotor eccentricity, $\Omega$ is the rotor rotational velocity, $\delta$ is acceleration of gravity.

The expression of the magnetic levitation force generated by the active magnetic bearing at the shaft journal in the x-y direction:

$$\begin{align}
F_X = F_{X+} - F_{X-} + \alpha \left( \frac{X}{g_0} \right) (F_{Y+} + F_{Y-}) \\
F_Y = F_{Y+} - F_{Y-} + \alpha \left( \frac{Y}{g_0} \right) (F_{X+} + F_{X-})
\end{align}$$
Where:

\[ F_{X+} = \frac{\mu_b N^2 A_x}{4} \left[ (i_k - \bar{P}X_j - \bar{D}X_j)^2 \right], \quad F_{X-} = \frac{\mu_b N^2 A_x}{4} \left[ (i_k + \bar{P}X_j + \bar{D}X_j)^2 \right], \]

\[ F_{Y+} = \frac{\mu_b N^2 A_y}{4} \left[ (i_k - \bar{P}Y_j - \bar{D}Y_j)^2 \right], \quad F_{Y-} = \frac{\mu_b N^2 A_y}{4} \left[ (i_k + \bar{P}Y_j + \bar{D}Y_j)^2 \right]. \]  

In Eq.(3), \( \mu_b \) is magnetic permeability, \( \bar{D} \) represents the feedback gain ratio in the PD controller, which control the rotational speed, \( \bar{D} \) is differential feedback gain, which control the acceleration, \( \alpha \) is geometric contact parameter, \( \epsilon_0 \) is air gap of magnetic bearing.

The rotor and the stator will rub against each other, when the radial displacement exceeds the clearance \( \delta \), as shown in Fig.2(a).

**Figure 2.** Diagram of rotor-bearing rub-impact and section of crack: (a) Rub-impact ; (b) Crack shaft.

Assume the impact is elastic, the friction meets Coulomb friction law. Express the radial displacement of shaft journal with \( e = \sqrt{(X_D - \varepsilon_x)^2 + (Y_D - \varepsilon_y)^2} \). When impact occurs, the normal friction force \( F_{RN} \) and tangential friction force \( F_{RT} \) can be expressed as:

\[ \begin{aligned}
F_{RN} &= 0 \quad e < \delta \\
F_{RN} &= \mu_e (e - \delta) \quad e \geq \delta \\
F_{RT} &= \mu F_{RN}
\end{aligned} \]  

Decomposed \( F_{RN} \), \( F_{RT} \) into the X and Y axes:

\[ \begin{aligned}
F_{RX} &= -F_{RN} \cos \phi + F_{RT} \sin \phi \\
F_{RY} &= -F_{RN} \sin \phi - F_{RT} \cos \phi
\end{aligned} \]  

\( \phi \) is displacement angle, \( \sin \phi = \frac{Y_D - \varepsilon_y}{e}, \cos \phi = \frac{X_D - \varepsilon_x}{e} \), the rub-impact forces is described as:

\[ \begin{aligned}
F_{RX} &= -\frac{K_e (e - \delta)}{e} \left[ (X_D - \varepsilon_x) - \mu (Y_D - \varepsilon_y) \right] \\
F_{RY} &= -\frac{K_e (e - \delta)}{e} \left[ (Y_D - \varepsilon_y) + \mu (X_D - \varepsilon_x) \right]
\end{aligned} \]  

The rub-impact forces in the radial and tangential directions can be expressed as:
\[
\begin{aligned}
\left\{ F_{xx} \right\} &= \frac{K_r(e-\delta)}{e} \left[ \begin{array}{cc} 1 & -\mu \\ \mu & 1 \end{array} \right] \left\{ X_d \right\} \\
\left\{ F_{yy} \right\} &= \frac{K_r(e-\delta)}{e} \left[ \begin{array}{cc} 1 & -\mu \\ \mu & 1 \end{array} \right] \left\{ Y_d \right\} \\
F_{xx} &= F_{yy} = 0
\end{aligned}
\]

(7)

After the rotor works for a long time, it is easy to produce fatigue cracks on the rotating shaft and reduce the rigidity. Fig 2(b) shows that the periodic function of cracks opening and closing is in the following form:

\[
f(\varphi) = \frac{1}{2} + \left( \frac{2}{\pi} \right) \cos \varphi - \left( \frac{2}{3\pi} \right) \cos 3\varphi + \left( \frac{2}{5\pi} \right) \cos 5\varphi
\]

(8)

Where \( \varphi \) is difference between rotation angle and vortex angle, which can be expressed as \( \varphi = \theta - \arctan(X_d/Y_d) \), rotation angle: \( \theta = \omega t \). Taking \( \Delta k \) represents the crack influence factor, let \( \Phi = \theta + \beta \), where \( \beta \) is the angle between the direction of unbalance and crack propagation. The force of the cracked rotor influences the shaft stiffness in the horizontal and vertical direction:

\[
\begin{aligned}
F_{cx} &= -\frac{\Delta K f(X)}{2} \left[ (1 - \cos 2\Phi)X_d + (\sin 2\Phi)Y_d \right] \\
F_{cy} &= -\frac{\Delta K f(Y)}{2} \left[ (\sin 2\Phi)X_d + (1 + \cos 2\Phi)Y_d \right]
\end{aligned}
\]

(9)

The non-dimensional quantities are given by:

\[
\begin{aligned}
x_p = \frac{X_p}{g_0}, y_p = \frac{Y_p}{g_0}, x_c = \frac{X_c}{g_0}, y_c = \frac{Y_c}{g_0}, u = \frac{U}{g_0}, p = \frac{P}{g_0}, D = \frac{D}{g_o}, D_i = \frac{D_i}{g_o}, W = \frac{W}{g_o} g_1, \gamma = \frac{M_j}{M_d},
\end{aligned}
\]

\[
\begin{aligned}
\tau = \omega t, \epsilon = \frac{e}{g_0}, \lambda = \frac{\delta}{g_0}, \omega = \frac{\Omega}{2M_d \omega_s}, \zeta = \frac{C}{2M_d \omega_s}, \omega_n = \sqrt{\frac{\mu_s N^2 A_i^2 (P-1)}{M_d g_0}}, \omega_s = \frac{K_s}{\sqrt{M_d}}, \Delta k = \frac{\Delta K}{K_s}, K = \frac{K_s}{K_y}, Q = \frac{\omega_s}{\omega_n}
\end{aligned}
\]

The non-dimensional magnetic force in Eq. (2) can be expressed as follows:

\[
\begin{aligned}
F_x &= F_{x+} - F_{x-} + \alpha \frac{X_p}{g_0} (F_{y+} + F_{y-}) \\
F_y &= F_{y+} - F_{y-} + \alpha \frac{Y_p}{g_0} (F_{x+} + F_{x-})
\end{aligned}
\]

(10)

Where the magnetic force in all directions can be expressed as:

\[
\begin{aligned}
F_{x+} &= \frac{1}{4(P-1)} \left[ (1 - P x_j - D y_j)^2 \right] \\
F_{x-} &= \frac{1}{4(P-1)} \left[ (1 + P x_j + D y_j)^2 \right] \\
F_{y+} &= \frac{1}{4(P-1)} \left[ (1 - P y_j - D x_j)^2 \right] \\
F_{y-} &= \frac{1}{4(P-1)} \left[ (1 + P y_j + D x_j)^2 \right]
\end{aligned}
\]

(11)

The non-dimensional rub-impact force is:

\[
\begin{aligned}
\left\{ F_{x} \right\} &= \frac{(\epsilon - \lambda) K}{\epsilon} \left[ \begin{array}{cc} 1 & -\mu \\ \mu & 1 \end{array} \right] \left\{ x_d \right\} \\
\left\{ F_{y} \right\} &= \frac{(\epsilon - \lambda) K}{\epsilon} \left[ \begin{array}{cc} 1 & -\mu \\ \mu & 1 \end{array} \right] \left\{ y_d \right\}
\end{aligned}
\]

(12)

\[
\begin{aligned}
F_{xx} &= F_{yy} = 0
\end{aligned}
\]

The non-dimensional crack influence force is:
\[
\begin{aligned}
F_{cs} &= -\frac{\Delta k f(\phi)}{2} [(1-\cos 2\phi)x_d + (\sin 2\phi)y_d] \\
F_{cy} &= -\frac{\Delta k f(\phi)}{2} [(\sin 2\phi)x_n + (1+\cos 2\phi)y_n]
\end{aligned}
\] 

(13)

The non-dimensional equations of the system are shown as follow.

\[
\begin{aligned}
x_d &= \frac{\omega_n^2}{\omega_h^2} F_{cs} - \frac{\omega_n^2}{\omega_h^2} F_{cy} - 2\zeta \frac{\omega_n}{\omega_h} x_d - \frac{\omega_n^2}{\omega_h^2} (x_d - x) + u\omega^2 \cos \omega t \\
\dot{y}_d &= \frac{\omega_n^2}{\omega_h^2} F_{cs} - \frac{\omega_n^2}{\omega_h^2} F_{cy} - 2\zeta \frac{\omega_n}{\omega_h} \dot{y}_d - \frac{\omega_n^2}{\omega_h^2} (y_d - y) - \frac{\omega_n^2}{\omega_h^2} W + u\omega^2 \sin \omega t \\
x_j &= F_i - \frac{\omega_n^2}{\omega_h^2} (x_j - x) \\
\dot{y}_j &= F_i - \frac{\omega_n^2}{\omega_h^2} (y_j - y) - \frac{\omega_n^2}{\omega_h^2} W
\end{aligned}
\] 

(14)

3. System Numerical Simulation and Analysis

There are many nonlinear factors in the rotor system of cracked magnetic bearings: segmentation of rub-impact force, complexity of magnetic force and crack force, etc., calculate Eq.(14) with the fourth-order Runge-Kutta numerical integration method, construct the Poincaré map of the system: \(\sigma_T = [(x_0, x_d, y_0, y_d, t)] \in \mathbb{R}^4 \times S, \theta = n T\), where \(T = 2\pi / \omega\) denotes period of motivation. Taking the non-dimension parameters: \(W = 0.025, P = 1.1, D = 0.03, \zeta = 0.001, \alpha = 0.24, \lambda = 0.85\), etc. as the criterion parameters, we mainly study the effect of different crack development \(\Delta k\) and eccentricity \(u\) on the system response.

3.1. Impact of Crack Influence Factor Changes on the System

Due to the long term impact load of the rotor system, fatigue cracks appear on the rotating shaft near the rotor, the existence of cracks mainly affects the stiffness coefficient of the shaft. Taking the eccentricity \(u = 0.2\), other parameters taking criterion values, we mainly study the system dynamic responses infected by different crack influence factors. Fig.3 is bifurcation diagrams of the system under different crack influence factors. The increase of the crack influence factor leads to the rotor displacement amplitude increase in the system response, when \(\Delta k\) increases, the chaotic response window of the system increases in the lower speed range, and the stability is reduced. Conversely, when \(\Delta k\) is small, the system tends to stable. Fig.3(a) & (b) shows that the system emerges period-1 motion during the rotor acceleration stage, but the period-1 motion is unstable at this time, and quickly embed into chaos as the speed increases. With the rotational speed decreases, the chaotic motion degenerates to period-1 motion, and jump near \(\omega = 0.459\), into period-2 motion. As the rotational speed continues to increase, period-2 motion is converted to the period-1 motion by the inverse period-doubling bifurcation (P-D Bif). We can learn from Fig3(c) that when the crack influence factor is large, the system response stays period-1 motion in the high-speed region, and the rotor motion is relatively stable, but when the rotational speed is low, the amplitude of chaotic motion and quasi-periodic motion is increases, the vibration is getting serious.
Different crack influence factors make the system show different motion forms. Fig. 4 is to select charts of axes track during the rotor acceleration process as the research object when the crack influence factor $\Delta k = 0.1$, it is found that the system exhibits rich dynamic characteristics with increasing speed. During the initial start stage of the rotor, the system presents short period-1 motion movement (see Fig.4(a)), the period-1 motion is unstable at this time, and there is eddy phenomenon exists, then the system enters a state of chaotic motion. As the rotational speed increases, the system degenerates from chaotic motion to period-1 motion and period-2 motion, and undergoes inverse period-doubling bifurcation transition to period-1 motion. The system enters chaotic motion when $\omega = 0.735$ as shown in Fig.4(b), the chart of axes track in this parameter domain is messy curves. When the rotational speed reaches 0.74, the system evolves from chaotic motion to quasi-periodic motion, Fig.4(c) is corresponding chart of axes track of quasi-periodic motion. Fig.4(d) shows that the system enters period-4 motion when the rotational speed reaches 0.75. With the rotation speed increases, the system presents period-2 motion (see Fig.4(e)) and period-1 motion (see Fig.4(f)) through the inverse period-doubling sequence. It can be learned from the charts of axes track, the motion trajectory during the high-speed period-1 motion is relatively regular, and the rotor system runs smoothly. Therefore the working speed should be stabilized in the high-speed periodic motion as soon as possible under the actual conditions.

**Figure 3.** Bifurcation diagram of system response at different $\Delta k$ values:
(a) $\Delta k = 0.05$; (b) $\Delta k = 0.1$; (c) $\Delta k = 0.3$.

**Figure 4.** Charts of axes track at different rotational speeds $\omega$, $\Delta k = 0.1$:
(a) Period-2 motion, $\omega = 0.5$; (b) Chaotic motion, $\omega = 0.735$; (c) Quasi-periodic motion, $\omega = 0.74$;
(d) Period-4 motion, $\omega = 0.75$; (e) Period-2 motion, $\omega = 0.76$; (f) Period-1 motion, $\omega = 1.0$. 
3.2. Impact of Eccentricity Changes on the System

The rotor system inevitably has a certain amount of eccentricity. When other parameters are determined, taking crack influence factor $\Delta k = 0.1$, we studied the effect about the change of eccentricity to dynamic characteristics of the system. Fig. 5 is bifurcation diagrams of the system under different eccentricities. It can be observed that when the eccentricity $u$ is small (see Fig. 5(a)), the system basically maintains period-1 motion during the entire acceleration process. When $u$ is further increased (see Fig. 5(b)), the system appears multi-period motion, chaotic and quasi-periodic motion. At this time, the system motion exhibits rich dynamic characteristics at low rotational speeds, but the motion form at high rotational speeds is still mainly period-1. When the eccentricity value exceeds 0.15 (see Fig. 5(c)), the system motion at low rotational speeds is mainly dominated by chaos, quasi-periodic and interspersed with unstable period-2 motion, period-3 motion, etc., but the system still performs period-1 motion at high rotational speeds.

![Figure 5. Bifurcation diagram of system response at different eccentricity values: (a) $u = 0.05$; (b) $u = 0.1$; (c) $u = 0.15$.](image)

4. Conclusions

In this paper we consider the rotor system supported by active magnetic bearing at both sides, the dynamic behaviours of the system affected by the nonlinear crack force on the rotating shaft is considered. The influences of crack influence factor and eccentricity on the dynamic response of entire system are analysed. The research indicates: (1) The cracked magnetic rotor-bearing system shows complex dynamic behaviours include multi-periodic motion, quasi-periodic motion and chaotic motion, etc. in different rotational speeds. (2) The increase of the crack influence factor leads to the expansion of the chaotic response window of the system, and the amplitude of chaotic motion and quasi-periodic motion increases accordingly. As the amount of eccentricity increases, the chaotic motion of the rotor system within the working speed continues to increase, and the periodic motion continuously decreases. The dynamic characteristics of the cracked rotor bearing system can provide a reference for fault diagnosis. (3) The selection of the best parameters in engineering applications should make the system in stable period-1 motion. Here we only consider the influence of crack influence factor and eccentricity on the system dynamic behaviours. In engineering practice, the influence of parameters such as friction coefficient, stiffness coefficient, clearance, etc. should be considered comprehensively.

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