Fuzzy chromatic number of a wheel graph

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Abstract. Fuzzy graphs have many more applications in modelling real time systems where the level of information inherent in the system varies with different levels of precision. Colouring of graphs is a most important concept in which we partition the vertex (edge) set of any associated graph so that adjacent vertices (edges) belong to different sets of the partition. In this paper we introduce k-Fuzzy total colouring of a wheel graph.

Keywords: Chromatic number, Fuzzy wheel graph, k-Fuzzy total colouring, chromatic number, Round table conference problem

1. Introduction

An introduced the fuzzy set as a class of object with a continuum of grades of membership. Fuzzy relation on a set was first defined by Zadeh in 1965, the first definition of a Fuzzy graph was introduced by Kaufmann in1973and the structure of Fuzzy graphsdeveloped by Azriel Rosenfeld in 1975.

Colouring of graphs is a most important concept in which we partition the vertex or edge set of any associated graph so that adjacent vertices or edges belongs to different sets of the partition. The colouring problem consists of determining the chromatic number of a graph and an associated colouring function.

Let G be a simple graph with n vertices. A colouring of the vertices of G is a mapping f: V(G)→N, such that adjacent vertices are assigned different colours. In 1965, Behzad and Vizing have posed independently a new concept of a graph colouring called total colouring. A k-colouring of graph G is an assignment of k colours to the vertices and edges in such a way that adjacent vertices and incident edges are received different colours. In this paper we determine the chromatic number of Fuzzy Wheel graph. The proper colouring of a graph is the colouring of the vertices with minimal number of colours such that no two adjacent vertices have the same colour. The minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour is the chromatic numberχ(G). Here we find the fuzzy chromatic number of a wheel graph with suitable example. First we give the definitions of basic concept of fuzzy sets and fuzzy graph.
Preliminary Definitions:

Definition 1:
A fuzzy set $A$ defined on a non empty $X$ is the family $A = \{ (x, \mu_A(x)) / x \in X \}$ where $\mu_A: X \rightarrow [0, 1]$

Definition 2:
Let $V$ be a finite non empty set. The triple $\tilde{G} = (V, \sigma, \mu)$ is called a fuzzy graph on $V$ where $\sigma$ and $\mu$ are fuzzy sets on $V$ and $E$ respectively, such that $\mu(uv) \leq \sigma(u) \land \sigma(v)$ for all $u, v \in V$ and $uv \in E$.

For fuzzy graph $\tilde{G} = (v, \sigma, \mu)$, the elements $V$ and $E$ are called set of vertices and set of edges of $G$ respectively.

Definition 3:
Two vertices $u$ and $v$ in $\tilde{G}$ are called adjacent if $\left( \frac{1}{2} \right) [\sigma(u) \land \sigma(v)] \leq \mu(uv)$.

Definition 4:
The least value of $K$ which has a $k$-fuzzy colouring, denoted by $\chi(\tilde{G})$ is called the fuzzy chromatic number of $G$.

Definition 5:
The degree of vertex $v$ in $\tilde{G}$, denoted by $\text{deg}_G(v)$ is the number of adjacent vertices to $v$ and the maximum degree of $\tilde{G}$ is defined by $\Delta(\tilde{G}) = \max \{ \text{deg}_G(v) / v \in V \}$.

Definition 6:
Two edges $v_iv_j$ and $v_iv_k$ are said to be incident if $2\{ \mu(v_iv_j) \land \mu(v_iv_k) \} \leq \sigma(v_i)$ for $i = 1, 2, ..., |v|$ and $1 \leq j, k \leq |v|$.

Definition 7:
A family $\Gamma = \{ \gamma_1, \gamma_2, ..., \gamma_k \}$ of fuzzy sets on $V$ is called a $k$-fuzzy colouring if $\tilde{G} = (v, \sigma, \mu)$ if

a) $\forall \Gamma = \sigma$

b) $\gamma_i \land \gamma_j = 0$

c) For every strong edge $uv$ of $\tilde{G}$, $\gamma_i(u) \land \gamma_i(v) = 0$ for $1 \leq i \leq k$. The above definition of $k$-fuzzy colouring was defined by the authors Eslahchi and Onagh [1] on fuzzy set of vertices. This has been extended to both fuzzy set of vertices and fuzzy set of edges by Lavanya.S and Sattanathan .R [3] as $k$-fuzzy total colouring as follows.

Definition 8:
A family $\Gamma = \{ \gamma_1, \gamma_2, ..., \gamma_k \}$ of fuzzy sets on $V \cup E$ is called a $k$-fuzzy total colouring if $\tilde{G} = (v, \sigma, \mu)$ if

a) $\max_i \{ \gamma_i(v) \} = \sigma(v)$ for all $v \in V$ and $\max_i \{ \gamma_i(uv) \} = \mu(uv)$ for all edges $uv \in E$.

b) $\gamma_i \land \gamma_j = 0$
c) for every adjacent vertices $u,v$ of $\{ \gamma_i(u), \gamma_j(v) \} = 0$ and for every incident edges $\{ \gamma_i(v_j) \}$ are set of incident edges from the vertex $v_j = 0, j = 1,2,...,|v|$. 

**Definition 9:**

A wheel graph with fuzzy colouring is called a fuzzy wheel graph.

A wheel in a fuzzy graph consists of two node sets $v$ and $u$ with $|v| = 1$ and $|u| \geq 1$ such that $\mu(v,u_i) > 0$ where $i = 1$ to $n-1$ and $\mu(u_i,u_{i+1}) > 0$ where $i = 1$ to $n-2$.

**1.1 k-Fuzzy total colouring on Fuzzy Wheel Graph:**

We extend the definition of $k$-fuzzy colouring on Fuzzy wheel graph in the definition given below. Since we deal with fuzzy wheel graph for which $\mu(v,u_i) > 0$ where $i = 1$ to $n-1$ and $\mu(u_i,u_{i+1}) > 0$ where $i = 1$ to $n-2$, the definition can be stated as follows.

**Definition 1.1.1**

A family $\Gamma = \{ \gamma_1, \gamma_2, ..., \gamma_k \}$ of fuzzy sets on $V \cup E$ is called a $k$-fuzzy total colouring of $\Gamma$ if

a) $\bigvee \gamma_i(v) = \sigma(v)$ and $\bigvee \gamma_i(uv) = \mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u,v \in V, uv \in E$ and $1 \leq i \leq k$.

b) $\gamma_i \wedge \gamma_j = 0$ for $1 \leq i, j \leq k$.

c) For every strong edge $uv$ of $\Gamma$, $\gamma_i(u) \wedge \gamma_i(v) = 0, (1 \leq i \leq k)$ and for any set of incident edges $uv$ on vertex $u \in V$ of $\Gamma$, $\gamma_i(uv) = 0 (1 \leq i \leq k)$.

**Example 1.1.1**

The round table conference problem:

In order to analyze the concept of total colouring of fuzzy wheel graph and its associated chromatic number, a Round table conference problem is presented. The problem has been modelled as an assignment problem and has been studied as fuzzy wheel graph. The Round table conference is held in the department of Mathematics on behalf of conducting the National level seminar. The Department has seven staff members Kavitha, Sreesha, Raeshma, Mazhila, Vini, Chitra and Jenitha. There are five subcommittees with following membership.

- Fund raising (FR): K,R,V,M
- Food committee (FC): K,R,V,M
- Stage decoration (SD): J,V,C,M
- Seating arrangement (SA): J,V,C,M
- Co-ordinating the programme (C): M

Each time the committee has a meeting, at first each of the subcommittees will meet with appropriate college officers and then the committee gets together as a whole to go over subcommittee recommendations and make decisions.
We draw a graph [figure 1] in which the vertices are the subcommittees. The subcommittees are adjacent to the neighbourhood committees and all the members of the subcommittees are adjacent to the co-ordinator of the programme who takes the final decisions.

The graph [figure 1] is a wheel graph with two node sets u and v. In figure 1 the wheel graph has five vertices each vertex has degree 3 except the centre vertex whose degree is 4. Each pair of vertices is adjacent and they require two different colours, but the same members in the subcommittees have the same colour. Therefore the minimum number of colours needed to properly colour the graph is 3. Hence the chromatic number of the graph is 3. Obviously the subcommittee meetings depend in the wheel graph which helps to make clear decisions and arrangements for the seminar. This concept could be fuzzy and it could be associated with some numerical values and the problem in example 1.1.1 could be modelled by means of a fuzzy wheel graph $G_k = (v, \sigma, \mu)$ and vertex set $V = \{u_1, u_2, u_3, u_4, v\}$ and edge set $E = \{v_iu_j | ij = 12, 15, 23, 25, 34, 35, 41, 45\}$ and the corresponding membership functions which has been dealt in detail in following section 1.2

1.2 The total chromatic number of a fuzzy wheel graph

Consider the fuzzy wheel graph $G_k = (v, \sigma, \mu)$ and vertex set $V = \{u_1, u_2, u_3, u_4, v\}$ and edge set $E = \{v_iu_j | ij = 12, 15, 23, 25, 34, 35, 41, 45\}$ the membership functions are defined as follows
\[ \sigma(u_i, v) = \begin{cases} 0.12, & i = 1,3 \\ 0.19, & i = 2,4 \\ 0.2, & i = 5 \end{cases} \]

\[ \mu(v_i, u_j) = \begin{cases} 0.04, & ij = 12,25 \\ 0.02, & ij = 23,35 \\ 0.01, & ij = 34,45 \\ 0.07, & ij = 41 \\ 0.08, & ij = 15 \end{cases} \]

Let \( \Gamma = \{ \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 \} \) be a family of fuzzy sets defined on \( V \cup E \) as follows

\[ \gamma_1(u_i) = \begin{cases} 0.12, & i = 1 \\ 0, & \text{otherwise} \end{cases} \]
\[ \gamma_2(u_i) = \begin{cases} 0.19, & i = 2 \\ 0, & \text{otherwise} \end{cases} \]
\[ \gamma_3(u_i) = \begin{cases} 0.12, & i = 3 \\ 0, & \text{otherwise} \end{cases} \]
\[ \gamma_4(u_i) = \begin{cases} 0.19, & i = 4 \\ 0, & \text{otherwise} \end{cases} \]
\[ \gamma_5(v) = \begin{cases} 0.2, & i = 5 \\ 0, & \text{otherwise} \end{cases} \]
Table for set of vertices:

|   | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\gamma_5$ | max   |
|---|------------|------------|------------|------------|------------|-------|
| 1 | 0.12       | 0          | 0          | 0          | 0          | 0.12  |
| 2 | 0          | 0.19       | 0          | 0          | 0          | 0.19  |
| 3 | 0          | 0          | 0.12       | 0          | 0          | 0.12  |
| 4 | 0          | 0          | 0          | 0.19       | 0          | 0.19  |
| 5 | 0          | 0          | 0          | 0          | 0.2        | 0.2   |

$\gamma_1(v_{ij}) = \begin{cases} 
0.04 & , i, j = 12 \\
0.07 & , i, j = 41 \\
0 & \text{otherwise}
\end{cases}$

$\gamma_2(v_{ij}) = \begin{cases} 
0.01 & , i, j = 34 \\
0.02 & , i, j = 23 \\
0 & \text{otherwise}
\end{cases}$

$\gamma_3(v_{ij}) = \begin{cases} 
0.08 & , i, j = 15 \\
0.04 & , i, j = 23 \\
0 & \text{otherwise}
\end{cases}$

$\gamma_4(v_{ij}) = \begin{cases} 
0.02 & , i, j = 35 \\
0 & \text{otherwise}
\end{cases}$

$\gamma_5(v_{ij}) = \begin{cases} 
0.01 & , i, j = 45 \\
0 & \text{otherwise}
\end{cases}$
Table for set of Edges:

| Edges | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\gamma_5$ | Max |
|-------|-----------|-----------|-----------|-----------|-----------|-----|
| 12    | 0.04      | 0         | 0         | 0         | 0         | 0.04|
| 15    | 0         | 0         | 0.08      | 0         | 0         | 0.08|
| 23    | 0         | 0.02      | 0         | 0         | 0         | 0.02|
| 25    | 0         | 0         | 0.04      | 0         | 0         | 0.04|
| 34    | 0         | 0.01      | 0         | 0         | 0         | 0.01|
| 35    | 0         | 0         | 0         | 0.02      | 0         | 0.02|
| 41    | 0.07      | 0         | 0         | 0         | 0         | 0.07|
| 45    | 0         | 0         | 0         | 0         | 0         | 0.01|

Hence the family $\Gamma = \{ \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 \}$ satisfies our definition 1.1.1 total colouring of fuzzy wheel graph. From the table, we got the values of $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$. Hence in this case the total chromatic number is 3.

By this Round table conference problem we find three ways to grouping the members to the subcommittees for organizing the programme. Also by this example we conclude that, if the subcommittees are odd then the number of ways of grouping the members is 3 and if the subcommittees are even then the number of ways of grouping the members is 4.

2. Conclusion

From the above example we conclude that the Fuzzy wheel graph with odd vertices has the chromatic number 3 and the graph with even vertices have the chromatic number 4. Further we can extend this result to double wheel graph.

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