ABSTRACT: For the embedded Gaussian orthogonal ensemble (EGOE) of random matrices, the strength sums generated by a transition operator acting on an eigenstate vary with the excitation energy as the ratio of two Gaussians. This general result is compared to exact shell model calculations, with realistic interactions, of spherical orbit occupancies and Gamow-Teller strength sums in some $(ds)$ and $(fp)$ shell examples. In order to confirm that EGOE operates in the chaotic domain of the shell model spectrum, calculations are carried out using two different interpolating hamiltonians generating order-chaos transitions. Good agreement is obtained in the chaotic domain of the spectrum, and strong deviations are observed as nuclear motion approaches a regular regime (transition strength sums appear to follow the Dyson’s $\Delta_3$ statistic). More importantly, they shed new light on the newly emerging understanding that in the chaotic domain of isolated finite interacting many particle systems smoothed densities (they include strength functions) define the statistical description of these systems and these densities follow from embedded random matrix ensembles; some EGOE calculations to this end are presented.

1. Introduction

In the last fifteen years there has been an explosive growth in the use of random matrix theories for quantum systems particularly in the context of quantum chaos [1, 2]. Recently, working with the aim of developing a statistical theory for finite interacting many particle systems, such as atoms, molecules, nuclei, atomic clusters, metallic quantum dots etc., by incorporating the ideas of random matrices and chaos, several research groups recognized the importance of investigating the embedded random matrix ensembles in detail, i.e. the embedded Gaussian orthogonal ensemble of random matrices of $k$-body interactions (EGOE($k$)) and their various deformations [2-7]. The EGOE is introduced in the context of nuclear shell model studies [2]; The EGOE($k$) is defined in $m$-particle spaces (i.e. in the $\binom{N}{m}$ dimensional space generated by distributing $m$ fermions over $N$ single particle states) with a GOE representation in $k$-particle space for $k$-body operators (usually $k \ll m$). This ensemble and its deformed versions are studied using Monte-Carlo methods.

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with EGOE constructed in occupation number representation and alternatively assuming that realistic nuclear (similarly atomic) shell model hamiltonians are typical members of EGOE(2) and its deformations, generic features of these ensembles are inferred from exact shell model calculations. For finite interacting particle systems in the chaotic domain there are now several studies of the statistical properties (both smoothed forms and fluctuations) of energy levels, wavefunction amplitudes or equivalently transition strengths generated by action of a transition operator on an eigenstate and strength functions [1-11]. However, only in the last 2-3 years similar studies (i.e. in the context of quantum chaos) on expectation values of operators, which measure transition strength sums, as function of excitation energy have began [3,8,12-14]. Given an operator \( K = \mathcal{O}^\dagger \mathcal{O} \), the expectation values \( \langle K \rangle_E \) are the diagonal elements of \( K \) in the hamiltonian \( (H) \) diagonal basis (a more precise definition is given ahead in Eq. (3)); they give strength sums generated by the transition operator \( \mathcal{O} \) acting on the eigenstate with energy \( E \). Two major examples are single particle transfer strength sums which are expectation values of number operators that give occupancies of single particle states (they determine thermodynamic behaviour) and Gamow-Teller (GT) strength sums as function of excitation energy which are relevant in astrophysics (presupernova evolution and stellar collapse). It is expected that the smoothed \( \langle K \rangle_E \) vs \( E \) will give information about order-chaos transitions just as energies, wavefunction amplitudes and transition strengths.

Two important results given by EGOE are that in strongly interacting shell model spaces (essentially in \( \hbar \omega \) spaces) (i) the state densities take Gaussian form [9] and (ii) the bivariate strength densities take bivariate Gaussian form [10]. These results have their basis in the EGOE representation of the hamiltonian \( H \) (which is in general one plus two-body in nuclear case) and transition operators \( \mathcal{O} \). The eigenvalue density \( I(E) \) or its normalized version \( \rho(E) \) is defined by [3]

\[
I(E) = \langle \langle \delta(H - E) \rangle \rangle = d \langle \delta(H - E) \rangle = d \rho(E) ;
\]

\[
\rho(E) \xrightarrow{\text{EGOE}} \rho_g(E) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2} \left( \frac{E - \epsilon}{\sigma} \right)^2} ;
\]  

(1)

In (1) \( \langle \langle \cdots \rangle \rangle \) denotes trace (similary \( \langle \cdots \rangle \) denotes average), the \( \epsilon, \sigma \) and \( d \) are centroid, width (\( \sigma^2 \) is variance) and dimensionality respectively. Note that \( \epsilon = \langle H \rangle, \sigma^2 = \langle (H - \epsilon)^2 \rangle \), ‘\( G \)’ stands for Gaussian and the bar over \( \rho(E) \) indicates ensemble average (smoothed) with respect to EGOE. The strength \( R(E, E') \) generated by a transition operator \( \mathcal{O} \) in the \( H \)-diagonal basis is \( R(E, E') = | \langle E' | \mathcal{O} | E \rangle |^2 \). Correspondingly the bivariate strength density \( I_{\text{biv};\mathcal{O}}(E, E') \) or \( \rho_{\text{biv};\mathcal{O}}(E, E') \) which is positive definite and normalized to unity is [10]

\[
I_{\text{biv};\mathcal{O}}(E, E') = \langle \langle \mathcal{O}^\dagger \delta(H - E') \mathcal{O} \delta(H - E) \rangle \rangle = I'(E') | \langle E' | \mathcal{O} | E \rangle |^2 I(E) \]

\[
= \langle \langle \mathcal{O}^\dagger \mathcal{O} \rangle \rangle \rho_{\text{biv};\mathcal{O}}(E, E') ;
\]

\[
\rho_{\text{biv};\mathcal{O}}(E, E') \xrightarrow{\text{EGOE}} \rho_{\text{biv};\mathcal{O}}(E, E') = \rho_{\text{biv}-g;\mathcal{O}}(E, E')
\]  

(2)
The bivariate Gaussian $\rho_{\text{biv}}(E, E')$ in (2) is defined by the centroids and variances of its marginal densities and a bivariate correlation coefficient. Though the EGOE forms in (1,2) are derived by evaluating the averages over fixed-$m$ spaces, in large number of numerical shell model examples it is verified that [2,10,15,16] they apply equally well in fixed- $m$, $mT$ and $mJT$ spaces. In practice the so-called Edgeworth corrections are added to the Gaussian forms in (1,2). Before proceeding further two remarks are in order:

1. Level and strength fluctuations for EGOE follow [2] the GOE fluctuations, i.e. nearest spacing distribution is Wigner distribution, Dyson-Mehta $\Delta_3(L)$ statistic follow the GOE $\ln(L)$ behaviour for large $L$ and strength fluctuations are of Porter-Thomas (P-T) type. The $\Delta_3$ form is tested ahead in Fig. 3 and recent tests of P-T form for EGOE are given in Fig. 1.

2. EGOE smoothed forms (1,2) (and (3) ahead) gave birth to the so called statistical nuclear spectroscopy (see [9,10,15-17] and references therein) and there are recent studies of this in atoms [11], molecules and solids [13] and mesoscopic systems [3-5].

The purpose of this paper is to first point out that EGOE, via (1,2) gives rise to a statistical theory for the smoothed forms for transition strength sums and the theory operates in the chaotic domain of the spectrum. Shell model tests in $(ds)^6$ space for occupancies and GT strength sums in $(fp)^6$ space are carried out. The EGOE theory and the shell model studies are described in Sect. 2. In order to confirm that the agreement between shell model and EGOE theory is a consequence of chaoticity of the shell model spectrum, GT strength sums and occupancies are calculated using two different interpolating hamiltonian that generate order-chaos transitions. Results of these calculations form Sect. 3. Results in Sects. 2, 3 give nuclear physics examples for the newly emerging understanding that in the chaotic domain of isolated finite interacting many particle systems smoothed densities, defined by EGOE, give rise to the statistical description of these systems. Further remarks on this important generic result for quantum chaos in isolated finite interacting particle systems are given (together with the results from a EGOE calculation for occupancies) in the concluding Section 4.

2. EGOE theory for transition strength sums and shell model tests

One important by product of (2) is that the transition strength sum density $\langle \langle O^\dagger O \delta(H - E) \rangle \rangle$, which is a marginal density of the bivariate strength density, takes a Gaussian form as the marginal of a bivariate Gaussian is a Gaussian. Therefore, using (1), it is immediately seen that transition strength sums vary with excitation energy as ratio of Gaussians. Given $K = O^\dagger O$, the transition strength sum density is the expectation value density denoted by $\rho_K(E)$ and then [16,13],

$$\langle K \rangle^E = [d(m)\rho(E)]^{-1} \left[ \sum_{\alpha \in E} \langle E\alpha \mid K \mid E\alpha \rangle \right] I_K(E)/I(E) = \rho_K(E)/\rho(E)$$

$$\text{EGOE} \frac{\rho_K(E)/\rho(E)}{\rho_K;G(E)/\rho_G(E)} = \rho_K;G(E)/\rho_G(E) \ ,$$

$$\rho_K(E) = \langle K\delta(H - E) \rangle = d^{-1} I_K(E) = d^{-1} \langle \langle K\delta(H - E) \rangle \rangle ; \ K = O^\dagger O \ .$$
Fig. 1. (a) Distribution of renormalized shell model transition strengths compared with the Porter-Thomas (P-T) form. The locally averaged strengths \( R(E_i, E_f) \) are calculated using the EGOE bivariate Gaussian form (2). Shell model (SM) calculations are in 223 dimensional \((ds)^{m=5, J=5/2, T=1/2}\) space with hamiltonian defined by Wildenthal interaction [8] and the transition operator is the two-body part of the hamiltonian obtained after substracting the configuration isospin centroid part. The SM+EGOE result is in good agreement with P-T. (b) Number of principal components (NPC) and information entropy \( (S^{info}) \) versus energy \( (E) \) for a strength distribution in 307 dimensional \((ds)^{m=6, J=2, T=0}\) space. The hamiltonian is Kuo interaction with \(^{17}\)O single particle energies [24] and the transition operator is same as in (a) but for the Kuo interaction. The exact shell model results are compared with the GOE and EGOE predictions; the EGOE formulas (they use P-T) are given in [7]. (c) Same as (b) but for wavefunctions.
In deriving (3) it is assumed that the smoothed form of $\rho_K(E)/\rho(E)$ reduces to ratio of smoothed forms of $\rho_K(E)$ and $\rho(E)$. This result ignores the fluctuations in both $\rho_K(E)$ and $\rho(E)$ and the r.m.s error due to neglect of the fluctuations is given in terms of the number of principal components (NPC) or the inverse participation ratio for the transition operator $C$. Note that (3) takes into account $(K, H)$ and $(K, H^2)$ correlations which define the centroid $\epsilon_K$ and width $\sigma_K$ of $\rho_K(E)$; $\epsilon_K = \langle KH \rangle / \langle K \rangle$ and $\sigma_K^2 = \langle KH^2 \rangle / \langle K \rangle - \epsilon_K^2$. First discussions of the EGOE result in (3) are in [16, 17, 19]. Statistical models that are inappropriate, are applied recently in the study of GT strength sums as function of excitation energy in $(ds)$-shell [12] although there are several studies of GT strengths and strength sums in statistical nuclear spectroscopy [14, 21, 22].

In order to study the domain of validity of (3), for the occupancies $\langle n_\alpha \rangle^E_{\hbar}$ shell model calculations in 307 dimensional $(2s1d)^m=6,J=2,T=0$ space are carried out using the Rochester - Oak Ridge shell model code and for the GT strength sums in 814 dimensional $(1f2p)^m=6,J=0,T=0$ space using the NATHAN code [23] of the Strasbourg-Madrid group. In the $(ds)$-shell studies the hamiltonian employed is $H = h(1) + V(2)$ defined by Kuo’s [24] two-body matrix elements ($V(2)$) and $^{17}$O single particle energies ($h(1) \iff \epsilon_{d_{5/2}} = -4.15$ MeV, $\epsilon_{d_{3/2}} = 0.93$ MeV, $\epsilon_{s_{1/2}} = -3.28$ MeV). Similarly in the $(fp)$ study the so called KB3 interaction [23] is employed with $h(1) \iff \epsilon_{f_{7/2}} = 0.0$ MeV, $\epsilon_{f_{5/2}} = 6.5$ MeV, $\epsilon_{p_{3/2}} = 2$ MeV, $\epsilon_{p_{1/2}} = 4$ MeV. The expectation value density $\rho_{n_\alpha,G}$ for the number operators $n_\alpha$ in $(ds)$ shell and $\rho_{K(GT),G}$ for the $K(GT)$ operator that generates GT strength sums in $(fp)$ shell are constructed in terms of their centroids and widths and similarly the state density Gaussian. Then using (3) the smoothed form of GT strength sum as function of excitation energy is constructed and compared with exact shell model results. From Fig. 2 it is seen that the EGOE result (3) describes very well the shell model results except at the edges of the spectra. The reason for the deviation at the edges is well known - here the states are not chaotic (sufficiently complex). In the $(ds)$ shell example the $K$-density centroid $\epsilon_K$, width $\sigma_K$, skewness $\gamma_{1,K}$ and excess $\gamma_{2,K}$ (the $\gamma_1$ and $\gamma_2$ are shape parameters that measure deviations from the Gaussian form) are $\epsilon_K = -35.08$ MeV, $\sigma_K = 9.63$ MeV, $\gamma_{1,K} = 0.08$ and $\gamma_{2,K} = -0.09$ for $d_{5/2}$ density and $-29.50$ MeV, $10.24$ MeV, $-0.08, -0.17$ respectively for the $d_{3/2}$ density. Similarly for the state density the parameters are $\epsilon = -32.78$ MeV, $\sigma = 10.24$ MeV, $\gamma_1 = 0.05$ and $\gamma_2 = -0.18$. In the $(fp)$ shell example, $\epsilon_K = 11.12$ MeV, $\sigma_K = 8.65$ MeV, $\gamma_{1,K} = 0.09$, $\gamma_{2,K} = -0.18$, $\epsilon = 9.51$ MeV, $\sigma = 8.62$ MeV, $\gamma_1 = 0.10$ and $\gamma_2 = -0.19$. Firstly the $|\gamma_1|$ and $|\gamma_2|$ values (being much less than 0.3) clearly show that all the densities are close to Gaussian. Moreover $\hat{\sigma} = \sigma_K/\sigma \sim 1$. The scaled centroid shifts $\Delta_K = (\epsilon_K - \epsilon)/\sigma$ are $-0.225$ and 0.32 for the $d_{5/2}$ and $d_{3/2}$ densities while it is 0.187 for the $(fp)$ shell GT example. With $\langle K \rangle^E$ given as ratio of Gaussians and that $\hat{\sigma} \sim 1$ imply that in the middle of the spectrum $\langle K \rangle^E \sim \langle K \rangle \left\{1 + \hat{\Delta}_K \hat{E}\right\}$; $\hat{E} = (E - \epsilon)/\sigma$, i.e $\langle K \rangle^E$ is linear in energy. This linear form and its polynomial extensions are used in the past [21, 22, 25]. The value of $\hat{E}$ for the ground state are $-2.6$ and $-3.1$ for $(ds)$ and $(fp)$ shell examples respectively. The range of validity (from the center of the spectrum) of the linear form for $\langle K \rangle^E$ is much smaller for $(ds)$-shell than in $(fp)$-shell as $\hat{\Delta}_K$ for the former is larger than for the later; see [23]. Fig. 2 showing good agreement between shell model results for $\langle n_\alpha \rangle^E$ and $\langle K(GT) \rangle^E$ with realistic interactions and the EGOE prediction (3), gives a justification to our assertion that EGOE starts operating from the region of the onset of chaos for
the strength sums as well. Next we study this question via \( \langle n_\alpha \rangle^E \) and \( \langle K(\text{GT}) \rangle^E \) when the hamiltonian changes thorough a parameter from a symmetry preserving, i.e. regular hamiltonian to a chaotic one.

Fig. 2. (a) Occupation numbers for \( d_{5/2} \) and \( d_{3/2} \) orbits vs excitation energy \( (E) \) in the same example as in Fig. 1b. Shown are also the shell model state and occupation number densities (histograms) compared to the EGOE Gaussian forms (continuous curves) given by (1,3). (b) GT strength sum versus excitation energy \( (E) \) for the 814 dimensional six particle \( (fp) \)-shell space with \( J = 0, T = 0 \). The exact shell-model results for the realistic KB3 interaction \(^23\) are compared with the EGOE predictions given by (3).

3. Results with MF and SU(4) interpolating hamiltonians
For further confirming the conclusions from Fig. 2, shell model calculations are performed in the 325 dimensional \((ds)^{m=8, J=T=0}\) space for occupancies and GT strength sums with two different interpolating hamiltonians. First set of calculations use the spherical shell model mean-field (MF) hamiltonian \(h(1)\) as the unperturbed hamiltonian \(H_0\) and in this case the occupation number operators commute with \(H_0\),

\[
H_\lambda(MF) = h(1) + \lambda V(2) = H_0 + \lambda(H_{SM} - H_0)
\]

Note that \(H_{SM} = h(1) + V(2)\); the \(h(1)\) is defined by \(^{17}\)O single particle energies and \(V(2)\) by Kuo’s two-body matrix elements as in the Sect. 2 \((ds)\)-shell example. In the figures in Figs. 3a-d the calculations with (4) are denoted by \(\text{MF}\). It is easily seen that spherical configurations (generated by distributing the nucleons in the three \((ds)\) shell orbits) are eigenstates for \(H_0\). Therefore for \(\lambda = 0\) in (4), the spectrum will have degeneracies. In the second set of calculations the \(SU(4) - ST\) scalar part \(H_{SU(4) - ST}\) of \(H_{SM}\) is used as the unperturbed hamiltonian,

\[
H_\lambda(SU(4)) = H_{SU(4) - ST; scalar} + \lambda(H_{SM} - H_{SU(4) - ST; scalar}) = H_0 + \lambda(H_{SM} - H_0)
\]

In the figures in Figs. 3a-d calculations with (5) are denoted by \(SU(4)\). Note that the GT operator commutes with the \(SU(4)\) hamiltonian \(H_{SU(4) - ST}\). For \(H = H_{SU(4) - ST}\), the eigenvalues and eigenvectors are given easily by \(SU(4) - ST\) algebra. The eigenstates are labelled, for a given number \(m\) of valence nucleons, by \(L, S, J, T\) and the \(U(4)\) irreducible representations (irreps) \(\{f\} = \{f_1 f_2 f_3 f_4\}\) or the \(SU(4)\) irreps \(\{F\} = \{F_1 F_2 F_3\}\) where \(f_1 + f_2 + f_3 + f_4 = m\), \(f_1 \geq f_2 \geq f_3 \geq f_4 \geq 0\), \(F_1 = (f_1 + f_2 - f_3 - f_4)/2\), \(F_2 = (f_1 - f_2 + f_3 - f_4)/2\), \(F_3 = (f_1 - f_2 - f_3 + f_4)/2\). Construction of \(H_{SU(4) - ST}\) part of a given \(H\) are given in \([26]\).

\[
H_{SU(4) - ST} = \frac{1}{2}(n-1)(n-2) E(0, \{0\} 00) - n(n-2) E(1, \{1\} \frac{1}{2} \frac{1}{2})
\]

\[
+ \frac{1}{8} \left[ n(2n-9) + G_2 + 2(S^2 + T^2) \right] E(2, \{2\} 11)
\]

\[
- \frac{1}{8} \left[ n - G_2 + 2(S^2 + T^2) \right] E(2, \{2\} 00)
\]

\[
+ \frac{1}{8} \left[ n(n+3) - G_2 + 2(S^2 - T^2) \right] E(2, \{11\} 10)
\]

\[
+ \frac{1}{8} \left[ n(n+3) - G_2 - 2(S^2 - T^2) \right] E(2, \{11\} 01)
\]

In (6) \(n\) is number operator, \(G_2\) is \(U(4)\) quadratic Casimir operator with eigenvalues \(\langle G_2 \rangle^{\{f\}} = f_1(f_1 + 3) + f_2(f_2 + 1) + f_3(f_3 - 1) + f_4(f_4 - 3)\) and \(S^2\) and \(T^2\) are operators with eigenvalues \(S(S+1)\) and \(T(T+1)\). Construction of \(H_{SU(4) - ST}\) requires the values for the centriod energies (determined by the given \(H\)) \(E(m, \{f\} ST) = \langle H \rangle^{m\{f\} ST}\). For the \((ds)\)-shell \(H_{SM}\) hamiltonian (defined after (4)), they are \(E(0, \{0\} 00) = 0\), \(E(1, \{1\} \frac{1}{2} \frac{1}{2}) = -2.302\text{MeV}\), \(E(2, \{2\} 11) = -4.176\text{MeV}\), \(E(2, \{2\} 00) = -2.975\text{MeV}\), \(E(2, \{11\} 10) = -8.360\text{MeV}\) and \(E(2, \{11\} 01) = -7.048\text{MeV}\). Using these, \(H_\lambda\) is constructed and then occupancies and
\[ \langle K(GT) \rangle^E \] are calculated for various \( \lambda \) values. The SU(4) − ST algebra gives for \( \lambda = 0 \),
\[ \langle K(GT) \rangle^E = \frac{2}{3} \left( \langle C_2(SU(4)) \rangle^F - S(S + 1) \right); \]
\[ \langle C_2(SU(4)) \rangle^F = F_1(F_1 + 4) + F_2(F_2 + 2) + F_3^2. \]
In addition, the allowed \( \{ f \} S \) values for the \((ds)\)-shell example are:
- \( \{2222\}0; \{3221\}1; \{3311\}2; \{332\}1; \{4221\}1; \{422\}2; \{431\}1, 2, 3; \{44\}0, 2, 4; \{511\}0; \{521\}1, 2; \{53\}1, 3; \{611\}1; \{62\}2. \]
With these results, the GT curve for \( \lambda = 0 \) case is constructed as a check of the shell model calculations. Each of the \( \{ f \} S \) states will have several \( L = S \) states and therefore the states here have degeneracies.

It is clearly seen from the results in Figs. 3a,b that the EGOE smooth form is not a good approximation to the exact results in the case of regular motion. For \( \lambda \sim 0 \) there are several (approximately) good quantum numbers with nearby levels carrying different sets of quantum numbers and therefore expectation values show large fluctuations as a function of excitation energy. The order-chaos transition as \( \lambda \) increases is clearly illustrated by the spectral rigidity \( \Delta_3 \) in Figs. 3c,d (also by the distribution of nearest neighbour level spacings as shown in [14]). It is seen that occupancies and GT strength sums behave rather like the \( \Delta_3 \) statistic, approaching more slowly the EGOE and GOE limits respectively.

This similarity is probably due to the fact that both \( \Delta_3 \) and strength sums are related to long-range correlations between the energy levels or wavefunctions. Thus, in the quantum chaotic domain transition strength sums, independent of the hamiltonian, follow EGOE forms and we have statistical spectroscopy in the chaotic domain.

There is another important observation that follows from Figs. 3a-d, i.e. as the interacting particle system becomes chaotic, expectation values take smoothed forms (within GOE fluctuations [20]) and hence described by the smoothed densities. For \( \lambda \gg \lambda_c \) (\( \lambda_c \) corresponds to order-chaos border and we determine \( \lambda_c \) by using \( \chi^2_\lambda \) which is the mean square deviation of the exact shell model strength sum from the EGOE smoothed form [14]; see Fig. 4) they take the EGOE form given by (3). This generic result is of central interest in quantum chaos studies of finite interacting particle systems as discussed in the next section.

4. Further results and concluding remarks

Besides the nuclear shell model results for GT strength sums and occupation numbers presented in Figs. 2-4 (GT results for the SU(4) case are reported in [14] recently), the behaviour of occupancies as a many particle hamiltonian makes order-chaos transitions is studied recently by several groups: (i) using a 20 member EGOE(1+2) in 330 dimensional \( N = 11, m = 4 \) space with the MF hamiltonian (4); \( h(1) \) is defined by the single particle energies \( \epsilon_i = i + (1/i) ; \ i = 1, 2, \ldots, 11 \) and \( V(2) \) is EGOE(2) [2]; (ii) using the four interacting electrons Ce atom [11]; (iii) using a symmetrized coupled two-rotor model [13]; (iv) using EGOE(1+2) as in (i) but in the 924 dimensional \( N = 12, m = 6 \) space with 25 members [27]. Most significant conclusion of all these studies is that transition strength sums show quite different behaviour in regular and chaotic domains of the spectrum. In order to make this argument clear results of (iv) are shown in Fig. 5; here occupancies for the single particle states are calculated for various values of the interpolating parameter \( \lambda \).

It is clearly seen, for example from Fig. 5, that below the region of onset of chaos.
Fig. 3. (a) Occupation numbers for $d_{5/2}$ and $d_{3/2}$ orbits vs excitation energy ($E$) for the MF and SU(4) interpolating hamiltonians given by (4) and (5). Shell model results are compared with the EGOE predictions (continuous curves) given by (3). (b) same as (a) but for GT strength sums. (c) Averaged spectral rigidity $\Delta_3(L)$ for the eigenvalues of the MF hamiltonian (4). Error bars give the standard deviation of the $\Delta_3$ average over overlapping intervals of length $L$. The dashed curves are for Poisson and the continuous curves are for GOE. (d) same as (c) but for the SU(4) hamiltonian (5). For the $\lambda = 1$ case the parameters $(\epsilon, \sigma, \gamma_1, \gamma_2)$ for the state, $d_{5/2}$, $d_{3/2}$ and GT densities are ($-52.59$ MeV, $13.15$ MeV, $0.10$, $0.03$), ($-55.30$ MeV, $12.31$ MeV, $0.01$, $-0.06$), ($-48.70$ MeV, $13.42$ MeV, $0.07$, $0.05$) and ($-48.37$ MeV, $12.64$ MeV, $0.06$, $-0.1$) respectively.
Fig. 4. \( R(\lambda) = \chi^2(\lambda)/\chi^2(1) \) vs \( \lambda \). Results for MF and SU(4) calculations are shown for \( d_{5/2} \) occupancies and GT strength sums. From the observations in the studies in [14], a plausible definition that the \( H_\lambda \) system is chaotic is given by the condition \( R(\lambda) \leq 1.5 \). Then, \( \lambda_c \) is defined by \( R(\lambda_c) = 1.5 \). The \( \lambda_c \) values for the four cases are shown in the figure. The difference in the two \( \lambda_c \) values in the case of occupancies is easily understood in terms of the norms [13, 14] of \( H_0 \) and \( H_1 = H_{SM} - H_0 \); however this is not the case with GT strength sums. Thus it appears that a complete theory for \( \lambda_c \), in case of strength sums, may come from the study of \( \Delta_3 \), strength fluctuations, NPC and information entropy for the hamiltonian and the transition operator involved.
Fig. 5. Occupation numbers for a 25 member EE(1+2) ensemble, defined by the Hamiltonian $h(1) + \lambda \{V(2)\}$, in the 924 dimensional $N = 12, m = 6$ space (see text); in the figure $N$ is denoted as $N$. Details of matrix construction etc. are given in [27]. Results are shown for the lowest 5 single particle states and for six values of $\lambda$. In the calculations occupation numbers are averaged over a bin size of 0.1 in $(E - \epsilon)/\sigma$; $\epsilon$ is centroid and $\sigma$ is width. The spectra of all the ensemble members are first zero centered and scaled to unit width and then the ensemble average is carried out. The estimate of [4] gives $\lambda_c \sim 0.05$ for order-chaos border in the present EE(1+2) example. It is clearly seen that once chaos sets in, the occupation numbers take stable smoothed forms. See [27] for further details.
occupation numbers show strong fluctuations (in the regular ground state domain perturbation theory applies). In this region there is no equilibrium distribution for $I_K(E)$, $I(E)$ and other densities. However in the chaotic domain (see \cite{3} and also Fig. 4 for methods of determining $\lambda_c$) the densities can be replaced by their smoothed forms as in (1)-(3). Therefore there is a statistical mechanics, defined by various smoothed densities, operating in the quantum chaotic domain of finite interacting particle systems (this is also the essence of statistical nuclear spectroscopy \cite{9,10,15-17}; see \cite{28} for arguments for statistical spectroscopy in atoms). In fact in favourable situations, it is possible to introduce thermodynamic concepts (effective temperatures and chemical potentials etc.) in the chaotic domain. In order to firmly establish the universality of these results, it is essential to carry out numerical studies for a wide variety of interacting particle systems and investigate various deformed EGOE’s in detail.

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References

[1] T. Ghur, A. Müller-Groeling and H.A. Weidenmüller, Phys. Rep. 299, 189 (1998).
[2] T. A. Brody, J. Flores, J. B. French, P. A. Mello, A. Pandey and S. S. M. Wong, Rev. Mod. Phys. 53, 385 (1981).
[3] V.V. Flambaum and F.M. Izrailev, Phys. Rev. E 56, 5144 (1997).
[4] Ph. Jacques and D.L. Shepelyansky, Phys. Rev. Lett. 79, 1837 (1997); B. Georgeot and D.L. Shepelyansky, Phys. Rev. Lett. 81, 5129 (1998); P.G. Silvestrov, Phys. Rev. E 58, 5629 (1998).
[5] B.L. Altshuler, Y. Gefen, A. Kamenev and L.S. Levitov, Phys. Rev. Lett. 78, 2803 (1997); C. Mejía-Monasterio, J. Richert, T. Rupp and H.A. Weidenmüller, Phys. Rev. Lett. 81, 5189 (1998); X. Leyronars, J. Tworzydlo and C.W.J. Beenakker, Phys. Rev. Lett. 82, 4894 (1999).
[6] C.W. Johnson, G.F. Bertsch and D.J. Dean, Phys. Rev. Lett. 30, 2749 (1998).
[7] V.K.B. Kota and R. Sahu, Phys. Lett. B 429, 1 (1998).
[8] V. Zelevinsky, B.A. Brown, N. Frazier and M. Horoi, Phys. Rep. 276, 85 (1996).
[9] K.K. Mon and J. B. French, Ann. Phys. (N.Y.) 95, 90 (1975); J.B. French and V.K.B. Kota, Phys. Rev. Lett. 51, 2183 (1983).
[10] J.B. French, V.K.B. Kota, A. Pandey and S. Tomsovic, Phy. Rev. Lett. 58, 2400 (1987); Ann. Phys. (N.Y.) 181, 235 (1988).
[11] V.V. Flambaum, A.A. Gribakina, G.F. Gribakin and M.G. Kozlov, Phys. Rev. A 50, 267 (1994); V.V. Flambaum, A.A. Gribakina, G.F. Gribakin and I.V. Ponomarev, Phys. Rev. E 57, 4933 (1998); V.V. Flambaum, A.A. Gribakina and G.F. Gribakin, Phys. Rev. A 58, 230 (1998).
[12] N. Frazier, B.A. Brown, D.J. Millener and V. Zelevinsky, Phys. Lett. B 414, 7 (1997).
[13] F. Borgonovi, I. Guarnieri, F.M. Izrailev and G. Casati, Phys. Lett. A 247, 140 (1998).
[14] V.K.B. Kota, R. Sahu, K.Kar, J.M.G. Gómez and J. Retamosa, Phys. Rev. C 60, 051306 (1999).
The mean square deviation $\hat{\Sigma}^2(E)$ at energy $E$ in the smoothed expectation value $\overline{\mathcal{M}(E)} = \langle \mathcal{O} | \mathcal{O} \rangle_E$ follows by using the Porter-Thomas assumption for strength fluctuations. In terms of the bivariate strength density $\rho_{\text{biv}}(E,E')$ and the state density $\rho'(E')$, it is easy to show that

$$\frac{\hat{\Sigma}^2(E)}{(\overline{\mathcal{M}(E)})^2} = \frac{2}{d'} \times \left\{ \int dE' \frac{\rho_{\text{biv}}(E,E')}{\rho'(E')} \right\}^{-1} \int dE' \left( \frac{\rho_{\text{biv}}(E,E')}{\rho'(E')} \right)^2$$

See V.K.B. Kota, Proc. Symp. Nucl. Phys. (Published by Library and Information Services, B.A.R.C., Bombay, India) 39A, 208 (1997), for a numerical example. Note that $\hat{\Sigma}^2(E)/(\overline{\mathcal{M}(E)})^2 = 2/3(NPC)_E$ and the EGOE formula for $(NPC)_E$ is given by Eq. (6) of Ref. [7].

[15] J.B. French and V.K.B. Kota, Ann. Rev. Nucl. Part. Sci. 32, 35 (1982).
[16] V.K.B. Kota and K. Kar, Pramana - J. Phys. 32, 647 (1989).
[17] V.K.B. Kota and D. Majumdar, Z. Phys. A 351, 365 (1995); Z. Phys. A 351, 377 (1995); Nucl. Phys. A 604, 129 (1996).
[18] J. Karwowski, Int. J. Quantum Chem. 51, 425 (1994); D.B. Waz and J. Karwowski, Phys. Rev. A 52, 1067 (1995); J. Planelles, F. Rajadell and J. Karwowski, J. Phys. A 30, 2181 (1997).
[19] J.B. French, V.K.B. Kota and J.F. Smith, University of Rochester Report No. UR-1122 (ER40425-245) (1989).
[20] The mean square deviation $\hat{\Sigma}^2(E)$ at energy $E$ in the smoothed expectation value $\overline{\mathcal{M}(E)} = \langle \mathcal{O} | \mathcal{O} \rangle_E$ follows by using the Porter-Thomas assumption for strength fluctuations. In terms of the bivariate strength density $\rho_{\text{biv}}(E,E')$ and the state density $\rho'(E')$, it is easy to show that

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[21] K. Kar and S.S.M. Wong, Phys. Lett. B 98, 393 (1981); K. Kar, Phys. Rev. C 28, 2446 (1983); K. Kar, J. Phys. G 9, 735 (1983); S. Sarkar and K. Kar, J. Phys. G 14, L123 (1988); K. Kar, A. Ray and S. Sarkar, Astrophys. J. 434, 662 (1994); S. Sarkar and K. Kar, Phys. Lett. B 387, 227 (1996).
[22] S. Sarkar and K. Kar, Phys. Rev. C 40, 1826 (1989).
[23] G. Martinez-Pinedo, A.P. Zuker, A. Poves and E. Caurier, Phys. Rev. C 55, 187 (1997); E. Caurier, G. Martinez-Pinedo, F. Nowacki, J. Retamosa and A. P. Zuker, Phys. Rev. C 59, 2033 (1999).
[24] T.T.S. Kuo, Nucl. Phys. A 103, 71 (1967).
[25] J.P. Draayer, J.B. French and S.S.M. Wong, Ann. Phys. (N.Y.) 106, 472 (1977); Ann. Phys. (N.Y.) 106, 503 (1977).
[26] T.R. Halemane, K. Kar and J.P. Draayer, Nucl. Phys. A 311, 301 (1978).
[27] V.K.B. Kota, R. Sahu, K.Kar, J.M.G. Gómez and J. Retamosa, in preparation (2000).
[28] V.V. Flambaum, A.A. Gribakina, G.F. Gribakin and I.V. Ponomarev, Physica D 131, 205 (1999).