Detecting the relic gravitational wave from the electroweak phase transition at SKA

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Abstract

We discuss possibilities to observe stochastic gravitational wave backgrounds produced by the electroweak phase transition in the early universe. Once the first-order phase transition occurs, which is still predicted in a lot of theories beyond the standard model, collisions of nucleated vacuum bubbles and induced turbulent motions can become significant sources of the gravitational waves. Detections of such gravitational wave backgrounds are expected to reveal the Higgs sector physics. In particular, through pulsar timing experiments planned in Square Kilometre Array (SKA) under construction, we will be able to detect the gravitational wave in near future and distinguish particle physics models by comparing the theoretical predictions to the observations.

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1 Introduction

Scientific research on gravitational wave is one of the most important subjects in physics. Detecting gravitational wave directly is essential to verify general relativity in strong gravitational fields and explore high-energy particle physics phenomena in the early universe. In other words, physics of gravitational wave is attractive for both astrophysics and particle physics. Due to a weakness of its interaction, the relic gravitational wave generated in the early universe brings us information on the early universe for what it was. We observe it as stochastic gravitational wave backgrounds. Quite recently it was reported that the relic gravitational wave originated in primordial inflation was discovered indirectly through the B-mode polarization experiment of the Cosmic Microwave Background (CMB) [1]. Therefore direct detections of the relic gravitational waves will take on increasing importance in the future.

In this paper, we discuss possible direct detections of the relic gravitational wave background produced by the first-order electroweak phase transition occurred in the early universe at around $\mathcal{O}(10^2)$ GeV. As is well known, within the Standard Model the effective potential of the Higgs field can not induce the first-order phase transition unless the Higgs mass is much lighter than the observed one [2]. In that case no gravitational wave is emitted because no latent heat is released during the transition. On the other hand however, strong first-order phase transitions are also predicted in a variety of theories beyond the Standard Model, such as supersymmetric extended models (e.g., see [3, 4]) and theories which induce a dimensional transmutation by introducing a new scalar field [5] in order to explain the electroweak symmetry breaking\(^3\). After the Higgs boson was discovered [26], we should approach various problems related the Higgs sector in detail. Therefore, particle physicists in the world tend to get momentum to tackle the physics at the electroweak phase transition head-on.

Investigations of the Higgs sector by using gravitational wave experiments are indeed exciting since we can explore particle physics through observations at cosmological scales. This kind of the verification for the Higgs sector is complementary to experiments that directly explore the theories beyond the Standard Model like the Large Hadron Collider (LHC) experiments and can be even much more powerful in some ways.

Since various experiments are planned to try to observe the gravitational waves, they cover a wide range of frequencies $10^{-9}$ Hz $\lesssim f \lesssim 10^3$ Hz. In principle future experiments such as eLISA [27] and DECIGO/BBO [28, 29, 30] have been known to detect the relic gravitational waves produced by the electroweak phase transition in future for the frequencies $10^{-7}$ Hz $\lesssim f \lesssim 10$ Hz. In this paper, we further discuss possibilities to observe

\(^3\)Originally, models with such a strong first-order phase transition have been studied in terms of baryogenesis, e.g., see also [6, 7, 8, 9] with respect to its tension with experiments and [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] for its possible modifications.
the relic gravitational waves through the pulsar timing experiments at Square Kilometre Array (SKA) under construction for the frequencies $10^{-9} \text{ Hz} \lesssim f \lesssim 10^{-4} \text{ Hz}$ [31]. The phase 1 and the phase 2 of SKA will start from 2018 and 2023, respectively [32].

In addition, so far effects by a large vacuum energy at a false vacuum on the phase transition has not been well examined. In this paper, we study the effect of the finite vacuum energy at the false vacuum in terms of cosmology.

This paper is organized as follows. In Section 2 we show model independent analyses of gravitational wave produced by the first-order electroweak phase transition. Section 3 is devoted to study the effect of the vacuum energy at the false vacuum. In Section 4, we show the experimental detectabilities of the relic gravitational wave background. Finally, in Section 5 we summarize our works.

2 Model-independent analysis

When the first-order phase transition occurs, the universe makes a transition from a false vacuum state to a true vacuum state. There exists an effective potential barrier between the true and the false vacua. Then, the transition occurs due to thermal fluctuations and a quantum tunneling effect. In other words, the true vacuum bubbles are produced inside the false vacuum state. However, the bubble nucleation itself does not generate any gravitational waves because of its spherical symmetric nature. The spherical symmetry is broken when they collide through their expansion, generating stochastic gravitational waves [33]. Fine details of the colliding regions are not so important to calculate the gravitational wave production. However, the gravitational wave is rather dominated by the gross features of the evolving bubble, which depends on kinetic energies of uncollided bubble walls [34, 35]. These facts mean that so-called “the envelope approximation” should be a good approximation for evaluating the amount of the produced gravitational wave signals [36].

In addition, the bubble expansion causes a macroscopic motion of cosmic plasma. When the bubbles collide, turbulence occurs in the fluid, which can become a significant source of the gravitational wave background [38, 39].

In this section, we introduce analytical methods to study the gravitational waves produced by the first-order phase transition. We take two most important parameters, $\alpha$ and $\tilde{\beta}$, characterizing the gravitational waves from the first-order phase transition. Then we show that general model parameters sufficiently reduce to only those two parameters when we discuss signals of the relic gravitational wave background.

\footnote{On the other hand, see also a recent criticism reported by [37].}
2.1 Basics

We adopt definitions of parameters used in this section mainly by following the ones in Ref. [38]. We discuss phenomena on the basis of the Friedman-Robertson-Walker universe, in which \( a(t) \) represents scale factor of the universe. We assume that the phase transition occurs at a cosmic temperature \( T_* \) which is the order of \( \mathcal{O}(10^2) \) GeV. The gravitational wave of the frequency \( f_* \) has arrived to us to be the present frequency \( f \). Hereafter the subscript "\( \ast \)" denotes a physical quantity at the phase transition. Then, the frequency we currently observe is represented by

\[
f = f_* \frac{a_*}{a_0} = f_* \left( \frac{g_{s0}}{g_{s*}} \right)^{1/3} \frac{T_0}{T_*},
\]

where the subscript "0" means a value at the present. Here we used the adiabatic expansion of the universe (i.e., the entropy \( S \propto a^3 g_s(T) T^3 = \text{const} \)). \( g_s \) means the effective degrees of freedom,

\[
g_s(T) = \sum_{\text{boson}} g_i \left( \frac{T_i}{T} \right)^3 + \frac{7}{8} \sum_{\text{fermion}} g_i \left( \frac{T_i}{T} \right)^3
\]

where \( g_i \) counts the internal degrees of freedom of \( i \)-th particle. In the current universe, we have \( g_s(T_0 = 2.725K) \approx 3.91 \). In terms of Hubble parameter, the frequency is given by

\[
f \approx 6 \times 10^{-3} \left( \frac{g_*}{100} \right)^{1/6} \frac{T_*}{100 \text{GeV}} \frac{f_*}{H_*} \text{ mHz},
\]

where \( g_* = g_s \) for \( T \gg 1 \text{ MeV} \). Therefore we expect the typical frequency for the gravitational wave produced at the electroweak phase transition to be at around \( \sim 10^{-3} \text{ mHz} - 10^{-2} \text{ mHz} \).

The energy density of the stochastic gravitational wave background \(^5\) is calculated to be

\[
\Omega_{\text{GW}} h^2 \equiv \frac{\rho_{\text{GW}}}{\rho_c} h^2 = \Omega_{\text{GW}*} h^2 \left( \frac{a_*}{a_0} \right)^4 \left( \frac{H_*}{H_0} \right)^2 \approx 1.67 \times 10^{-5} h^{-2} \left( \frac{100}{g_*} \right)^{1/3} \Omega_{\text{GW}*}.
\]

where we used \( \rho_{\text{GW}} a_0^4 = \rho_{\text{GW}*} a_*^4, \quad \rho_c H_0^{-2} = \rho_{\text{GW}*} H_*^{-2}, \quad H_0 = 2.1332 \times h \times 10^{-42} \text{GeV} \), with \( H \) Hubble parameter and \( h \) its reduced value. The subscript "0" denotes the value at the current epoch. In the next section we will show how we can calculate \( \Omega_{\text{GW}} h^2 = \Omega_{\text{coll}} h^2 + \Omega_{\text{turb}} h^2 \) in terms of two fundamental parameters (\( \alpha \) and \( \beta \)), which is the summation of the two contributions from the bubble collision (\( \Omega_{\text{coll}} h^2 \)) and the turbulence (\( \Omega_{\text{turb}} h^2 \)).

\(^5\)It is related with the strain \( \sqrt{S_{\text{GW}}} \) to be \( \Omega_{\text{GW}} h^2 = 3.132 \times 10^{35} (f/\text{Hz})^3 (\sqrt{S_{\text{GW}}}/\text{Hz}^{-1/2})^2 \).
2.2 Fundamental parameters, $\alpha$ and $\tilde{\beta}$

We introduce two important parameters $\alpha$ and $\tilde{\beta}$ to discuss model-independent analyses. At a finite temperature, the bubble nucleation rate of the phase transition is represented by \[ \Gamma(T) = \Gamma_0(T) e^{-S(T)} \simeq \Gamma_0(T) e^{-\frac{S_3}{T}}, \quad (2.5) \]

where $\Gamma_0(T)$ has units of energy to the fourth power and is typically represented by $\Gamma_0(T) \sim T^4$. $S_3$ stands for the euclidean action of the system $[40, 41]$, \[
S_3(T) = \int 4\pi r^2 \left[ \frac{1}{2} \left( \frac{d\phi_b}{dr} \right)^2 + V_{\text{eff}}(\phi_b, T) \right]. \quad (2.6)
\]

Notice that $S_3$ becomes time-independent at a high temperature $[41]$. $V_{\text{eff}}(\phi, T)$ means the effective potential of the field $\phi$ at a finite temperature $T$. $\phi_b$ represents a bubble profile of the field $\phi$. $r$ denotes a radius in the polar coordinates. Then the bubble profile is obtained by solving the bounce equation, \[
\frac{d^2\phi_b}{dr^2} + \frac{2}{r} \frac{d\phi_b}{dr} - \frac{\partial V}{\partial \phi_b} = 0, \quad (2.7)
\]

with \[
\left. \frac{d\phi_b}{dr} \right|_{r=0} = 0, \quad (2.8)
\]

and \[
\phi_b|_{r=\infty} = 0. \quad (2.9)
\]

Since the bubble nucleation rate has an exponential dependence, a key is a behavior of $S_3/T$. By taking the time derivative of the action, we define \[
\beta \equiv -\left. \frac{dS}{dt} \right|_{t_*}. \quad (2.10)
\]

In a neighborhood of $t_*$, we naturally expect a series expansion to be $S(t) = S(t_*) - \beta(t - t_*) + \ldots$. Here we introduce a dimension-less parameter to express the time derivative of the action, \[
\tilde{\beta} \equiv \frac{\beta}{H_*} = T_* \left. \frac{dS}{dT} \right|_{T_*} = T_* \frac{d}{dT} \left( \frac{S_3}{T} \right) \bigg|_{T_*}, \quad (2.11)
\]

where we used a property of the adiabatic expansion of the universe, $dT/dt = -TH$. This $\tilde{\beta}$ is one of the most important parameters to characterize the shape of the gravitational
wave spectrum. It is sufficient to look at the relationship used to determine the typical value of this, being able to percolate properly even for the exponentially-expanding universe\textsuperscript{6}.

\[
\frac{\Gamma}{H^4} \sim O(1).
\]  
(2.12)

Using this condition, it is possible to estimate the value of \( \tilde{\beta} \) for each model.

Another important parameter is a quantity that represents how much latent heat is released at the phase transition. In the symmetry phase, we denote the false vacuum energy density and the thermal energy density to be \( \epsilon(T) \), and \( \rho_{\text{rad}}(T) \), respectively. Then, the parameter \( \alpha = \alpha(T) \) is defined by

\[
\alpha \equiv \frac{\epsilon(T)}{\rho_{\text{rad}}(T)}.
\]  
(2.13)

Here, the energy density of the false vacuum is represented by

\[
\epsilon \equiv \Delta V_{\text{eff}} - T \Delta s = \Delta V_{\text{eff}} - T \frac{\partial \Delta V_{\text{eff}}}{\partial T},
\]  
(2.14)

where

\[
\Delta V_{\text{eff}} = \Delta V_{\text{eff}}(T) \equiv V_{\text{eff}}(\phi_{\text{false}}, T) - V_{\text{eff}}(\phi_{\text{true}}, T),
\]  
(2.15)

with \( \phi_{\text{true}} \) and \( \phi_{\text{false}} \) being the field values at the true and false vacua, respectively. Also, the energy density of radiation \( \rho_{\text{rad}} \) is given by

\[
\rho_{\text{rad}}(T) = \frac{\pi^2}{30} g_* T^4.
\]  
(2.16)

Using those two parameters (\( \alpha \) and \( \tilde{\beta} \)), a peak spectrum of the gravitational wave \( \tilde{\Omega} h^2 \) at a peak frequency \( \tilde{f} \) is represented by [42]

\[
\tilde{f}_{\text{coll}} \simeq 5.2 \times 10^{-3} \frac{T_*}{H_* 100 \text{GeV}} \left( \frac{g_*}{100} \right)^{1/6} \text{mHz},
\]  
(2.17)

\[
\tilde{\Omega}_{\text{coll}} h^2 \simeq 1.1 \times 10^{-6} \kappa^2 \left( \frac{H_*}{\beta} \right)^2 \left( \frac{\alpha}{1 + \alpha} \right)^2 \frac{v_b^3}{0.24 + v_b^2} \left( \frac{100}{g_*} \right)^{1/3},
\]  
(2.18)

\[
\tilde{f}_{\text{turb}} \simeq 3.4 \times 10^{-3} \frac{u_s}{v_b} \frac{T_*}{H_* 100 \text{GeV}} \left( \frac{g_*}{100} \right)^{1/6} \text{mHz},
\]  
(2.19)

\[
\tilde{\Omega}_{\text{turb}} h^2 \simeq 1.4 \times 10^{-4} u_s^5 v_b^2 \left( \frac{H_*}{\beta} \right)^2 \left( \frac{100}{g_*} \right)^{1/3}.
\]  
(2.20)

Here the subscript “coll” and “turb” denote the values in cases of the bubble collision and the turbulence, respectively. In these expressions, the bubble velocity \( v_b \), the fluid velocity \( u_s \) and the efficiency factor \( \kappa \) are expressed as a function of \( \alpha \) to be [42]

\[
v_b(\alpha) = \frac{1}{\sqrt{3}} + \sqrt{\alpha^2 + 2\alpha \frac{2}{3}},
\]  
(2.21)

\textsuperscript{6}There are also another evaluations such as \( \int dt \left( \frac{\Gamma}{H^4} \right) \sim O(1) \) appeared in other works. However, we have checked that this difference does not change our conclusion.
\[
\begin{align*}
  u_s(\alpha) &= \sqrt{\frac{\kappa\alpha}{\frac{4}{3} + \kappa\alpha}}, \quad (2.22) \\
  \kappa(\alpha) &= \frac{1}{1 + 0.715\alpha} \left(0.715\alpha + \frac{4}{27\sqrt{3\alpha}} \right). \quad (2.23)
\end{align*}
\]

In case of the bubble collision, the entire spectrum has been also calculated analytically. Using an envelope approximation, the full spectrum of the gravitational wave from the bubble collision is given by [43]

\[
\Omega_{\text{coll}}(f)h^2 = \tilde{\Omega}_{\text{coll}}h^2 \frac{(a + b)f^a}{bf^{a+b} + af^{a+b}}, \quad (2.24)
\]

where the value of \(a\) and \(b\) lie in the range \(a \in [2.66, 2.82]\), and \(b \in [0.90, 1.19]\). In case of the strong first-order phase transition, by a numerical simulation using a large number of colliding bubbles, the authors of [43] obtained \(a \approx 2.8\), and \(b \approx 1\). In Eq. (2.24) it is easily found that \(\Omega_{\text{coll}}h^2(f) = \tilde{\Omega}_{\text{coll}}h^2\) at \(f = \tilde{f}\). There is a remark that the formulae given here are available only when \(\tilde{\beta}\) is sufficiently large [43].

As will be shown later, the effects due to the tails parts of the spectrum given in Eq. (2.24) on experimental detectabilities are quite small. Hence, even if we do not adopt full expressions for the spectrum in the turbulent case, which has not been known analytically, our results should not change significantly only in the current purposes. Therefore we may take \(\Omega_{\text{turb}}h^2(f) = \tilde{\Omega}_{\text{turb}}h^2(f)\) for any \(f\)’s approximately as a full spectrum for the turbulent case.

### 3 Effects of the vacuum energy at the false vacuum

In the previous section, we adopted the parametrizations, in which we took a limit that the vacuum energy is completely negligible. However, here we carefully check possible effects on the productions of the relic gravitational wave background.

First, we investigate how the percolation is influenced by the vacuum energy. Here we consider typical cases in the Minimal Supersymmetric Standard Model (MSSM) as a specific example (see Appendix A for the details). If we ignore the vacuum energy, we have obtained \(T_* \sim 103.1\text{GeV}\) in order to complete the percolation as is shown in Fig. 1.

Next we incorporate the effect of the vacuum energy in this setup. For a concrete calculation, we calculate \(\Gamma/H^4\) by parameterizing the vacuum energy \(\Lambda_{\text{vac}} \equiv \Delta V_{\text{eff}}(T = 0)\). As seen in the previous section, the bubble nucleation is determined by the value of the euclidean action. However, there is no effect from the vacuum energy on the nucleation because the constant term is renormalized in the definition of the nucleation rate. Only the Hubble parameter, \(H^2 = \rho/(3m_{\text{pl}}^2)\) with \(m_{\text{pl}}\) Planck mass, should be changed by adding the vacuum energy to the total energy density \(\rho = V_{\text{eff}}(\phi, T) + \rho_{\text{rad}}\). The result is plotted in Fig. 2. As seen in this figure, the effect is only a mild change on \(T_*\) with a
small difference by the order of $O(0.1) \text{GeV}$. That is because the bubble nucleation has the exponential dependence on $T$, which is the dominant contribution to possibly change $\Gamma/H^4$.  

\[ \int dt \Gamma H^4 \sim 1. \]  

In this section, we discuss detectabilities of the relic gravitational wave background produced at the electroweak phase transition by using two fundamental parameters $\alpha$ and $\tilde{\beta}$. In case of the phase transition at the electroweak scale, the useful experiments should be eLISA [27], Ultimate DECIGO [28, 29, 30] and SKA [31]. The sensitivities of the experiments are summarized in Refs. [30, 44]

\[ \int dt \Gamma H^4 \sim 1. \]  

\[ \int dt \frac{\Gamma}{H^4} \sim 1. \]  

As was mentioned in the previous footnote, there is another evaluation $\int dt \frac{\Gamma}{H^4} \sim 1$. We have also calculated transition temperature in those two cases, i.e., no vacuum energy $\Lambda_{\text{vac}} = 0$, and $\Lambda_{\text{vac}} = (500)4 \text{GeV}$. In those cases the corresponding transition temperatures are $101.91 \text{GeV}$ and $101.73 \text{GeV}$, respectively. Therefore, although there seems to exist a small difference between $\Gamma/H^4$ and $\int dt \frac{\Gamma}{H^4}$, it does not change our conclusions so much.

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**4 Detectability of relic gravitational wave**

In this section, we discuss detectabilities of the relic gravitational wave background produced at the electroweak phase transition by using two fundamental parameters $\alpha$ and $\tilde{\beta}$. In case of the phase transition at the electroweak scale, the useful experiments should be eLISA [27], Ultimate DECIGO [28, 29, 30] and SKA [31]. The sensitivities of the experiments are summarized in Refs. [30, 44]

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There is also a limit from non-detections of extra radiation through the CMB observation such as the Planck satellite experiments as an additional constraint. The extra radiation like the stochastic gravitational background can be measured as a deviation of the effective number of the neutrino species \( N_{\nu,\text{eff}} \) from three to be \( \Delta N_{\nu} = N_{\nu,\text{eff}} - 3 \). Then the energy fraction of the extra radiation at present can be expressed by \( \Omega_{\text{extra}} h^2 = 5.108 \times 10^{-6} \Delta N_{\nu} \). So far the Planck collaborations have reported that observationally they had an upper bound on \( \Delta N_{\nu} \), to be \( \Delta N_{\nu} \lesssim 1. \) \(^{45}\). Then we obtain an upper bound on the energy fraction of the relic gravitational wave background, \( \Omega_{\text{GW}} h^2 < 5.108 \times 10^{-6} \left( \frac{\Delta N_{\nu}}{1} \right) \). (4.1)

This is effective over a broad range of frequencies for \( 10^{-17} \text{Hz} \lesssim f \), \(^8\) which is wider than the one obtained by big-bang nucleosynthesis (BBN).

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8 Or the range is represented in terms of the comoving wave number to be \( 10^{-2} \text{ Mpc}^{-1} \lesssim k \) through \( f = 1.535 \times 10^{-3}\text{Hz}(k/10^{10}\text{Mpc}^{-1}) \).
Figure 4: Signals of the relic gravitational wave background in case of the bubble collision. The band regions mean the peak signals $\tilde{\Omega}h^2$ for $T = 70\text{GeV}$, and $T = 100\text{GeV}$ from the left to the right, respectively. The broken power means the corresponding full spectrum whose peak is located at $\Omega h^2 = \tilde{\Omega} h^2$. The model parameters are changed to be $\{\alpha, \tilde{\beta}\} = \{0.1, 0.1\}, \{0.1, 10^4\}, \{10, 0.1\}, \{10, 10^4\}$. We assumed $g_* = 106.75$.

In Fig. 5, we plot the signals of the relic gravitational wave background sourced by the turbulence. Contrary to the case of the bubble collision, it is notable that the peak frequency depends on both $\alpha$ and $\tilde{\beta}$. Of course, the peak signal is also determined by both $\alpha$ and $\tilde{\beta}$. The turbulence makes an important contribution to the signal and has larger detectable parameter regions$^9$.

Figure 5: Same as Fig. 4, but for the case of the turbulence.

We scanned parameter regions in terms of detectabilities in the $(\alpha, \tilde{\beta})$ plane. In Fig. 6 the case of the bubble collision is plotted. Here, we consider only the case of $T_\ast = 100\text{GeV}$.

$^9$By recent hydrodynamic simulations, e.g., [37, 47, 48], it was pointed out that its contribution might be smaller.
It displays the three regions that can be detected in three experiments (Ultimate DECIGO, eLISA and SKA). The excluded regions by the Plank [Eq. (4.1)] is also plotted at the bottom. Top regions are covered by the WD-WD noise. We have checked that the allowed region does not change much even if we consider the corresponding tail of the full spectrum shown in Eq. (2.24). In Fig. 7, we also plot the case of the turbulence.

![Figure 6: Detectabilities in the ($\alpha, \tilde{\beta}$) plane for the signals sourced by the bubble collision. From the top to the bottom, the cases for Ultimate DECIGO, eLISA, SKA are plotted, respectively. The excluded regions by the Plank constraint (Eq. 4.1) are also plotted. The WD-WD noise means the region where signals are covered by the foreground noise by the WD-WD binaries.](image)

5 Conclusions

After the discovery of the Higgs boson, particle physicists in the world tend to get momentum to tackle the physics at the electroweak phase transition head-on and approach various serious problems related the Higgs sector in detail. Therefore, it is attractive to revisit a variety of possible scenarios for the electroweak phase transition. We have examined detectabilities of the stochastic gravitational wave background produced by the first-order electroweak phase transition in a general setup with carefully considering effects of the vacuum energy on the expansion of the universe.

We have shown that the relic gravitational wave background produced at the electroweak epoch will be observed by the future experiments, such as SKA, eLISA and DECIGO. In particular, the small $\tilde{\beta}$ regions, which is naturally predicted in some particle physics models such as MSSM, will be able to be searched by SKA very near future.
Figure 7: Same as Fig. 6, but for the signals sourced by the turbulence.

Acknowledgments

We would like to thank Kenta Hotokezaka, Kunihito Ioka, and Eibun Senaha for useful discussions. This work is partially supported by the Grant-in-Aid for Scientific research from the Ministry of Education, Science, Sports, and Culture, Japan, Nos. 25.2309 (Y.K), and 21111006, 22244030, 23540327, 26105520 (K.K.). The work of K.K. is also supported by the Center for the Promotion of Integrated Science (CPIS) of Sokendai (1HB5804100).

A Scalar sector in MSSM

In this section we show details of the adopted models which are motivated by MSSM [3]. Those models are used to calculate specific physical variables in Sec 3. We refer only a scalar potential required here to be $V(\phi, T) = V_{\text{eff}}(\phi, T)$ with a constant vacuum energy $\Lambda_{\text{vac}} = \Delta V_{\text{eff}}(T = 0)$. The effective potential $V_{\text{eff}}(\phi, T)$ is then represented by [3]

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + V_1(\phi, T) + V_2(\phi, T).$$

(A.1)

The tree level potential $V_0(\phi)$ depends only on the scalar field.

$$V_0(\phi) = - \frac{m_H^2}{4} \phi^2 + \frac{m_H^2}{v^2} \phi^4,$$

(A.2)

with $m_H$ being the Higgs mass and $v$ being the vev of the Higgs field.

$V_1(\phi, T)$ is calculated at the thermal one-loop level. The following is a result of the high temperature expansion,

$$V_1(\phi, T) = \frac{T^2}{2v^2} \left( \frac{m_H^2}{4} + \frac{5m_W^2}{6} + \frac{5m_Z^2}{12} + m_t^2 \right) \phi^2$$
\[-T \left( E_{SM} + 2N_c \left( \frac{m_{\text{stop}}^2 + \Pi_{\text{stop}}}{12\pi} \right)^{3/2} \right) \phi^3, \quad (A.3)\]

where the number of colors \( N_c = 3 \), with \( m_t \), \( m_{\text{stop}} \), \( m_W \) and \( m_Z \) being masses of the top quark, the scalar top quark, the \( W \) boson and the \( Z \) boson, respectively. Here we introduced

\[
E_{SM} = \frac{1}{3} \left( \frac{2m_W^2 + m_Z^2}{2\pi v^3} \right), \quad (A.4)
\]

\[
m_{\text{stop}}^2 = -m_U^2 + \left( 0.15 \frac{m_Z^2}{v^2} \cos(2\beta_{\text{MSSM}}) + \frac{m_t^2}{v^2} \right) \phi^2, \quad (A.5)
\]

\[
\Pi_{\text{stop}} = \frac{4g_s^2}{9} T^2 + \frac{h_t^2}{6} (1 + \sin^2 \beta_{\text{MSSM}}) T^2 + \left( \frac{1}{3} - \frac{1}{18} \cos(2\beta_{\text{MSSM}}) \right) g'^2 T^2, \quad (A.6)
\]

where \( g' \) and \( g \) are the \( U(1)_Y \times SU(2)_L \) gauge coupling constants, \( g_s \) is the strong gauge coupling constant, \( h_t \) is the top Yukawa coupling, \( m_U^2 \) is the model parameter of the soft-mass squared, and \( \beta_{\text{MSSM}} \) is an angle defined by \( \tan \beta_{\text{MSSM}} \) to be the ratio of two Higgs’ vevs. Finally, \( V_2(\phi, T) \) is calculated by the two loop effect by incorporating the effect of weak boson and scalar top quark (stop),

\[
V_2(\phi, T) = \frac{\phi^2 T^2}{32\pi^2} \left( \frac{51g^2}{16} - 3h_t^4 \sin^4(\beta_{\text{MSSM}}) + 8g_s^2 h_t^2 \sin^2(\beta_{\text{MSSM}}) \right) \log \left( \frac{\Lambda}{\phi} \right). \quad (A.7)
\]

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