Spiral arms in broad-line regions of active galactic nuclei

I. Reverberation and differential interferometric signals of tightly wound cases

Jian-Min Wang¹,²,³, Pu Du¹, Yu-Yang Songsheng¹, and Yan-Rong Li¹

¹ Key Laboratory for Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, 19B Yuquan Road, Beijing 100049, China
² University of Chinese Academy of Sciences, 19A Yuquan Road, Beijing 100049, China
³ National Astronomical Observatories of China, Chinese Academy of Sciences, 20A Datun Road, Beijing 100020, China

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ABSTRACT

As a major feature of the spectra of active galactic nuclei, broad emission lines deliver information on the kinematics and spatial distributions of ionized gas located in the so-called broad-line regions (BLRs) surrounding the central supermassive black holes (SMBHs). There is growing evidence for the appearance of spiral arms in BLRs. It has been shown through reverberation mapping (RM) campaigns that the characterized radius of BLRs overlaps with that of self-gravitating regions of accretion disks. In the framework of the WKB approximation, we show robust properties of observational features of the spiral arms. The resulting spiral arms lead to the characterization of various profiles of the broad emission line. We calculated the RM and differential interferometric features of BLRs with $m = 1$ mode spiral arms. These features can be detected with high-quality RM and differential interferometric observations via such instruments as GRAVITY on board the Very Large Telescope Interferometer (VLTI). The WKB approximation will be relaxed and universalized in the future to explore more general cases of density wave signals in RM campaigns and differential spectroastrometric observations.

Key words. galaxies: active – quasars: emission lines – quasars: general – reverberation mapping

1. Introduction

Active galactic nuclei (AGNs), first discovered by Seyfert (1943), are characterized by the appearance of prominent broad emission lines in their spectra (e.g., see the composite spectra in Vanden Berk et al. 2001 and Hu et al. 2012), which typically exhibit full widths at half maximum (FWHM) of $\gtrsim 10^3\text{ km s}^{-1}$ or even a few $10^3\text{ km s}^{-1}$ (e.g., see reviews of Ho 2008; Netzer 2013). Such broad widths for the emission lines undoubtedly indicate very deep potential wells in the centers of AGNs. This was first realized by Wolger (1959), which directly motivated the establishment of the scenario that accretion onto supermassive black holes (SMBHs) powers the huge radiation energy of AGNs (Zel’Dovich 1965; Salpeter 1964; Lynden-Bell 1969). It is generally accepted that broad emission lines stem from broad-line region (BLR) gas photoionized by ionizing photons emitted from accretion disks (Osterbrock & Ferland 2006). Great progress has been made in improving our understanding of AGN and quasar activities (e.g., Netzer 2013), however, two major issues with regard to BLRs are still under debate thus far: 1) the origin of BLR gas and 2) their structure and dynamics (see a brief review in Wang et al. 2017).

A great deal of this progress is attributed to reverberation mapping (RM) campaigns since the 1980s, with the underlying principle proposed by Bahcall et al. (1972) and Blandford & McKee (1982). Photons of emission lines from structured ionized gas trace different paths to observers, leading to time lags ($\tau$) of the emission lines with respect to the ionizing photons. Such RM campaigns focusing on broad Balmer lines had detected the anticipated lags in a number of Seyfert galaxies (e.g., Peterson 1993; Peterson et al. 1998; Bentz et al. 2013; Barth et al. 2015; U et al. 2022) and quasars (e.g., Kaspi et al. 2000; Du et al. 2014, 2018a; Shen et al. 2019) over the past few decades. Along with the growing investment of observing resources and the development of analytical algorithms, the general geometry and kinematics of BLRs in some AGNs have been revealed by velocity-resolved delay analysis (e.g., Bentz et al. 2010; Denney et al. 2010; Grier et al. 2012, 2013; Du et al. 2016a; U et al. 2022), velocity-delay maps (e.g., Xiao et al. 2018; Horne et al. 2021), or dynamical modeling (e.g., Bottorff et al. 1997; Pancoast et al. 2014; Li et al. 2018; Williams et al. 2020). Many resolved BLRs have a disk-like geometry of their Hβ line region (the other have inflow or outflow, or a kind of mixture of the three configurations)¹. Moreover, the repeat RM observations of the same emission line and the RM results of the emission lines with different ionization in a few AGNs (e.g., NGC 5548, 3C 390.3, NGC 3783, NGC 7469, Mrk 817 etc.) approximately demonstrated the relation $V_{\text{FWHM}} \propto \tau^{-1/2}$, demonstrating evidence for potential of SMBHs (e.g., Peterson & Wandel 2000; Peterson et al. 2004; Lu et al. 2021), where $V_{\text{FWHM}}$ is the full-width-half-maximum (FWHM) of the emission lines. Considering the disk-like geometry of BLRs in some AGNs, this relation

¹ It turns out that high-ionization lines, such as the C iv line, favor an explanation based on the notion of origination from outflows. See Bottorff et al. (1997) for a first detailed study of C iv line in NGC 5548 observed by Hubble Space Telescope and International Ultraviolet Explorer.
probably indicates a nearly Keplerian rotation of the disk BLRs. More recently, the GRAVITY instrument mounted in Very Large Telescope Interferometer (VLTI) spatially resolved the BLRs in several AGNs (e.g., 3C 273, NGC 3783, IRAS 09149 by Gravity Collaboration et al. 2018, 2020, 2021, respectively) and also found that their BLRs are approximately characterized by Keplerian rotating disks.

Moreover, there is growing evidence of the existence of substructures or the inhomogeneity on the BLR disks from RM observations. For example, a well-known phenomenon in RM is that the emission-line profiles in the mean spectra (corresponding to the entire region of line emission) and those of the root mean square spectra (RMS, corresponding to the portion of the region with response) are generally different in most AGNs (e.g., Bentz et al. 2009; Denney et al. 2009; Barth et al. 2013; Du et al. 2018b), which implies that the BLRs are inhomogeneous in terms of gas distributions. Furthermore, the sub-features in the velocity-delay maps of NGC 5548 (e.g., Xiao et al. 2018; Horne et al. 2021) explicitly indicate that there is likely to be a gas inhomogeneity in BLRs. In particular, Horne et al. (2021) recently found helical “Barber-Pole” patterns in the Lyα and C iv lines of NGC 5548, which suggests azimuthal structures in their emitting regions. Multiple-peaked and asymmetric profiles of lines also indicate complicated BLR structures. These increasing pieces of evidence potentially suggest that there exist substructures in BLRs. Questions naturally arise regarding whether the motions of the BLR substructures are chaotic or ordered and what their physical origins are.

Motivated by the observational evidence, some preliminary efforts have been made to invoke spiral arms to explain the asymmetric double-peaked broad emission lines, however, they only assume some analytical forms of the arm patterns without considering their physical origin or introducing any dynamical physics. Bergmann et al. (2003, 2017), who adopted spiral arms to explain their physical origin or introducing any dynamical physics. Motivated by the observational evidence, some preliminary efforts have been made to invoke spiral arms to explain the asymmetric double-peaked broad emission lines, however, they only assume some analytical forms of the arm patterns without considering their physical origin or introducing any dynamical physics. These efforts were made by Gilbert et al. (1999) and Storchi-Bergmann et al. (2003, 2017), who adopted spiral arms to explain the asymmetric double-peaked emission lines and their variations in some AGNs. Horne et al. (2004) calculated the transfer function of BLRs with arms for RM, but all of them assumed some analytical forms of the spirals. All these mathematical models for observational data should be derived based on the first principle in order to advance our understanding of BLR physics.

The BLR radii measured by RM span from $10^3 R_g$ to $10^4 R_g$, depending on accretion rates and SMBH masses (see Figure 6 in Du et al. 2016b; Du & Wang 2019), which overlap with outer part of accretion disks, where $R_g = G M_s/c^2 = 1.5 \times 10^{13} M_8 \text{cm}$ is the gravitational radius, $G$ is the gravitational constant, $c$ is speed of light, and $M_8 = M_8/10^8 M_\odot$ is the SMBH mass. Some pioneering works suggest that the outer regions may be self-gravitating (SG, e.g., Paczyński 1978a,b) for viscosity mechanism transferring angular momentum outward, in particular, in AGN accretion disks (see details of Shlosman & Begelman 1989). It is not difficult to give an rough estimate of the SG radius using the famous Toomre parameter $Q = k a/r_{Gor}$ (Toomre 1964), which is the criterion of the instability, where $k$ is the epicyclic frequency (equal to the angular speed $\Omega$ in a Keplerian disk), $a$ is the sound speed, and $r$ is the surface density. Using $Q = 1$, Goodman (2003) approximated $R_{SG}/R_g \approx 3.1 \times 10^9 a_{1/2}^{12} (L_{bol}/L_{Edd})^{1/2} M_8^{-2/9}$, where $a_{0.1} = a/0.1$ is the viscosity parameter and $L_{bol}/L_{Edd}$ is the Eddington ratio. Interestingly, the SG part of the disks spatially overlap BLRs generally in AGNs and this motivates the idea that BLR structures and dynamics are linked with the SG part in some way.

The ultimate fates of the SG accretion disks remain a matter of debate, however, self-regulation processes – balanced by radiation cooling and the heating internally from the dissipation of gravito-turbulence (e.g., Paczyński 1978a,b; Lin & Pringle 1987; Gammie 2001; Lodato & Rice 2004) and probably magneto-rotational instabilities as well (MRI, e.g., Balbus & Hawley 1998; Rafikov 2015) or, otherwise, externally from irradiation (driven by the inner part of accretion disk; e.g., Rice et al. 2011; Rafikov 2015) or star formation (inside the disks, e.g., Collin & Zahn 1999; Sirko & Goodman 2003; Wang et al. 2011) – are believed to maintain the disks so that they can stay in marginally stable states. In such states, non-axisymmetric perturbations (spiral structures) may inevitably grow in the SG parts, although clumps or stars may also form through condensations if the cooling time scale, $t_{cool} < \beta \Omega^{-1}$, where $\beta$ is a factor of a few (e.g., Gammie 2001; Rice et al. 2003, 2011; Kratter & Lodato 2016; Brucy & Hennebelle 2021). The aforementioned phenomenological evidence of BLR substructures and inhomogeneity suggests that they may be connected with or originated from the spiral arms in the SG part of accretion disks, at least for those AGNs with disk-like BLRs.

As the first paper in this series, we adopt the simple tight-winding approximation and use analytical formulations to discuss the observational characteristics of tightly wound cases of density waves in BLRs. The basic formulations, equilibrium states, and boundary conditions are provided in Section 2. In Section 3, observational features of the arms are discussed for RM campaigns and interferometric observations of GRAVITY/VLTI. Brief discussions and conclusions are provided in Section 4 and 5, respectively. It should be noted that the purpose of this paper is to demonstrate the general features of BLR spiral arms in observations (e.g., RM) rather than to establish a perfect model.

2. Model and formulations

In the BLR, the ionized gas is rotating with nearly Keplerian velocity around the central SMBH. This assumption is supported by the evidence that $V_{FWHM} \text{ and } \sigma_t$ are roughly proportional to $\sim \tau_{H\beta}^{1/2}$ in some AGNs from the multiple RM campaigns (Peterson et al. 2004; Lu et al. 2021), where $\sigma_t$ is the velocity dispersion of the Hβ profiles in the RMS spectra, respectively, and $\tau_{H\beta}$ are the Hβ time lags with respect to the 5100Å continuum variations. However, the current level of accuracy with regard to the RM data does not allow us to quantitatively determine deviations from exactly Keplerian rotation, it should be reasonable to assume the disk-BLR has nearly Keplerian. As one of possible mechanisms, MRI (e.g., Balbus & Hawley 1998) drives radial motion with velocity approximated by $u/\nu_k \approx 0.1 \alpha_{0.1}$ which is much smaller than the rotation, where $\nu_k$ is the Keplerian rotation. Moreover, for the disks dominated by the central point masses, a weak self-gravity in the disks can still maintain spiral arms (e.g., Lee & Goodman 1999; Tremaine 2001).

In this paper, for simplicity, we apply the classic theory of density waves (e.g., Lin & Shu 1964, 1966; Lin & Lau 1979) to the calculation of the broad-emission-line profiles and the transfer function for RM, as well as the differential phase curve signal for GRAVITY/VLTI.

2.1. Self-gravitating disk and BLR

Following Paczyński (1978a), without specifying regulation mechanisms (dusty gas, star formation, photoionization, and ac-
cretion, etc.), we used the polytropic relation as a prescription of $Q_{\text{disk}} \sim 1$ region for general cases, which is given by $p = K_0 \rho^{n+1}/\rho$, where $p$ is the pressure, $\rho$ is the density, $K_0$ is a constant, and $n$ is the polytropic index. Fortunately, $K_0$ can be generally constrained by observations of the BLR geometry. The sound speed is given by $a_0 = (p/\rho)^{1/2} = K_0^{1/2} \rho^{1/2n}$. The vertical equilibrium admits $H = a_0 \Omega^{-1}$, and the Toomre parameter is given by $Q_{\text{disk}} = K_0 \pi G \sigma_0$, where $\sigma_0 = 2 \rho H$ is the surface density of the SG region. Here, we would like to point out that self-gravity is neglected with regard to the vertical equilibrium for simplicity. The epicyclic frequency is given by $\kappa = 2\Omega \left(1 + \frac{1}{2} \ln \Omega / \ln R \right)^{1/2} \approx \Omega$, and we adopt a Keplerian velocity here, such that $\Omega = \sqrt{GM_*/R^2} \approx 2 \times 10^{-9} M_8^{1/2} r_3^{-3/2} \text{s}^{-1}$. We thus have:

$$a_0 = \frac{K_0^{1/2}}{(2\pi G M_8^{1/2})^{1/2n}} \Omega^{1/n}, \quad (1)$$

$$\sigma_0 = \frac{2K_0^{1/2}}{(2\pi G M_8^{1/2})^{1/2n}} \Omega^{(1+n)/n}, \quad (2)$$

and

$$H = \frac{K_0^{1/2}}{(2\pi G M_8^{1/2})^{1/2n}} \Omega^{(1-n)/n}. \quad (3)$$

Given $n$, $K_0$, and $Q_{\text{disk}}$, we can derive the radial structures of BLR.

The polytropic index, $n$, is a free parameter in this paper, of which the value can be $0 \sim +\infty$. In practice, for instance, Paczyński (1978a) discussed the SG disks, with $n = 1.5$ and 3; Lubow & Ogilvie (1998) chose $n = 3$ in their work, and Korycansky & Pringle (1995) employed $n = 5$ as a typical case in the discussion of axisymmetric waves in accretion disks. In the present paper, the polytropic index $n$ controls the radius-dependent thickness of the gaseous disk, i.e., $H/R \propto R^{(n-3)/2n}$.

If $n > 3$, the disk becomes thicker at outer radius and the surface of the disk tend to be “bowl-shaped” (similar to Goad et al. 2012). The bowl shape enables the gas on the disk surface to be illuminated by the ionizing photons from the inner part of the disk, otherwise, the geometrically thin disks are not able to be sufficiently ionized.

Observational constrains on BLR gas indicate that the BLR mass of H$\beta$-emitting gas could be in a wide range from $10^3 \sim 10^5 M_\odot$, and even more massive (Baldwin et al. 2003). As a simple estimation, we integrate the surface density of the SG part of accretion disks and obtain a mass of $M_{\text{disk}} \approx 4.1 \times 10^6 \sigma_9^{4.1/3} M_8^{11/5}, \zeta_{7/10}^{7/4} M_8$ in light of the standard model of accretion disks (Shakura & Sunyaev 1973), where $M_{\text{disk}}/M_* = \mathcal{M} / M_{\text{edd}}$ is the dimensionless accretion rates, $M_*$ is the mass accretion rate, $M_{\text{edd}} = L_{\text{edd}}/c^2$ is the Eddington rates, and $L_{\text{edd}} = 1.3 \times 10^{46} M_8 \text{erg s}^{-1}$ is the Eddington luminosity. This indicates that the BLRs are the disk surface taking only tiny fraction of this SG portion. In the present paper, we assume that the ionized BLR gas is proportional to the density of the disk $\rho_{\text{ion}} \propto \rho$ (see the following Section 2.3). It is found that $M_{\text{disk}}/M_* \ll 1$, namely, the accretion disks are much lighter than the central SMBH unless for cases with extremely super-Eddington accretion rates ($\mathcal{M} \gg 1$). Although $\mathcal{M} \sim 900$ AGNs have been found from RM campaigns (Du et al. 2014, 2016a,b, 2018a), we limit the present scope for sub-Eddington accretion disks ($\mathcal{M} \leq 3$) with a nearly Keplerian rotation of the point potential of the SMBH mass.

We would point out here a possibility that enough high-density outflows emitting high ionization lines could partially shield the outer part of BLRs, making the reverberation of H$\beta$ line complicated (Dehghanian et al. 2019). If such a case occurs, the H$\beta$ line would undergo a holiday driven by the inner outflows (of C IV line) as in NGC 5548 (see Figure 7 in Pei et al. 2017), namely, a lack of reverberation would be evident. However, the H$\beta$ line has only undergone the holiday once over the last 20 years (Pei et al. 2017), indicating that such a holiday is quite rare and the obscurations are not common.

### 2.2. Equations and boundary conditions

#### 2.2.1. Perturbation equations

We adopt the formalisms and notations used in Lin & Lau (1979). For an $m$-fold axisymmetric perturbation, $(u, v, \sigma) = (u_0, v_0, \sigma_0) + (u_1, v_1, \sigma_1) e^{i(\omega t - m\phi)}$, where $u$ and $v$ are the radial and azimuthal velocities, and $\sigma$ is the surface density. The parameters with the subscript “0” correspond to the equilibrium state (we take $u_0 = 0$ in this paper), which are functions of the radius.

We have the perturbation of the equations given in Appendix A:

$$\frac{1}{R} \frac{d}{dR} \left(R \sigma_0 u_1 \right) - \frac{i}{R} \sigma_0 v_1 + i(\omega - m\Omega) \sigma_1 = 0, \quad (4)$$

$$i(\omega - m\Omega) u_1 - 2 \sigma_0 v_1 = - \frac{d}{dR} \left( \psi_1 + h_1 \right), \quad (5)$$

and

$$\frac{k^2}{2\Omega} u_1 + i(\omega - m\Omega) v_1 = \frac{i}{R} \psi_1 + h_1, \quad (6)$$

yielding the following differential equation:

$$\frac{d^2}{dR^2} \left( h_1 + \psi_1 \right) + A \frac{d}{dR} \left( h_1 + \psi_1 \right) + B(h_1 + \psi_1) = -Ch_1, \quad (7)$$

where $h_1 = \sigma_0^2 \sigma_1 / \sigma_0$. Here, the coefficients $A, B, C$ are given in Appendix B.

The Poisson equation of the SG portion of the accretion disks reads $\nabla^2 \psi = 4\pi G \rho \delta(z)$, through vertical integration, yielding

$$\psi_1 = \frac{1}{2R} \int \Sigma dR, \quad (8)$$

where $\delta(z)$ is the Dirac-$\delta$ function, $\Sigma = 2\pi G \sigma_0 / a_0^2$, and $a_0 \equiv 1$ is the sign function of wave vector ($k$) for trailing and leading waves, respectively. The Poisson equation holds approximately in the order of $(H/R)^2$. Combining the perturbation equations, we have the equation of the reduced enthalpy ($U$):

$$\frac{d^2 U}{dR^2} + k_1^2 U = 0, \quad (9)$$

where

$$U = H_{11} \left[ \frac{k_1^2 (1 - \nu^2)}{\sigma_0 R} \right]^{-1/2} \exp \left[ \frac{i}{2} \int \Sigma dR \right], \quad (10)$$

and

$$k_1^2 = \left( \frac{a_0}{\kappa} \right)^2 \left( Q_{\text{disk}}^2 - 1 + \nu^2 \right), \quad \nu = \frac{\omega - m\Omega}{\kappa}. \quad (11)$$

Bertin (2014) presents more detailed derivations of the above equations (7 and 9). Equation (9) works approximately in the order of $H/R$, which is in agreement with that of the Poisson equation. As a first application of density waves in BLR, we retain this order of approximation for simplicity.
2.2.2. Boundary conditions

In the context of spiral galaxies, the outer boundary conditions are imposed by radiation conditions (Lin & Lau 1979). Considering that the dynamics of dusty and dust-free gas will be very different due to radiation pressures (either from the local or central part) of accretion disks. The boundary could be distinguished by the dust sublimation radius. For the present SG disk, the outer boundary is fixed at the inner edge (inward is dust-free) of dusty torus, where the waves are evanescent. Although the dusty torus is generally not spatially resolved (except for NGC 1068, which shows a near-infrared cavity with a sharp edge by Gravity Collaboration et al. 2020), fortunately, the RM campaigns of near-infrared continuum emissions show that the inner edge of torus is about \( R_{\text{torus}} \approx 0.1 L_{\odot}^{0.5} \) pc (Suganuma et al. 2006; Koshida et al. 2014; Minezaki et al. 2019; Lyu et al. 2019). In the latter, \( L_{\odot} \) is the \( V \)-band luminosity in units of \( 10^{43} \) erg s\(^{-1}\) (see the latest version in Minezaki et al. 2019). It was long believed that the outer edge of a BLR is simply the inner edge of the torus (e.g., Netzer & Laor 1993; Suganuma et al. 2006; Czerny & Hryniewicz 2011). Therefore, we can obtain the outer boundary of the BLR:

\[
R_{\text{out}} = 205.9 \eta_{0.1}^{1/2} \delta_{L_{\odot}}^{1/2} \text{M}_\odot, \tag{12}
\]

from \( R_{\text{out}} = R_{\text{torus}} \), the bolometric luminosity is \( L_{\text{bol}} = \epsilon L_V = \eta_{\odot} L_{\odot} \delta_{L_{\odot}} \), \( \epsilon = 10 \eta_{0.1} \) is the bolometric luminosity correction factor, and \( \eta = 0.1 \eta_{0.1} \) is the radiative efficiency. This is in agreement with the observation of NGC 1068 (Gravity Collaboration et al. 2020) by GRAVITY onboard VLTI. We adopt a vanishing perturbation of density at the outer boundary. For the simplest case of the inner boundary, we assume

\[
dU/dR = 0, \tag{13}
\]

just as in Lau & Bertin (1978). We set the inner radius to be 10% of the outer radius and we checked that the detailed value of inner radius does not change the general features of spiral arms. With the boundary conditions, it becomes a eigenvalue problem for solving Eq. (9). Then we can obtain the perturbation of the surface density (more details in, e.g., Lau & Bertin 1978; Lin & Lau 1979).

It should be pointed out that outer boundary conditions could be revised for individual AGNs if the spatially resolved conditions are different from the present. The adopted conditions are based on the argument that any H\( \beta \) photons will be made extinct by dusty gas within the torus no matter the tides are outflows (K"onigl & Kartje 1994; Elitzur & Shlosman 2006), clumpy structures (Nenkova et al. 2008), or classical continue torus (Antonucci 1993). On the other hand, accretion disks could extend outward and correspond to the mid-plane of torus, while density waves also extend (but depends on the local radiation pressure). In some AGNs, ALMA (Atacama Large Millimeter/submillimeter Array) observations show spiral arms, as in a few Seyfert galaxies (Combes et al. 2019), and it would be interesting to test if they are consistent with those in BLRs. Recent interferometric observations of NGC 1068 show a much more complicated structure (Gámez Rosas et al. 2022) as well as the counter-rotating disk from 0.2 to 7 pc by ALMA observations (Imanishi et al. 2018; Impellizzeri et al. 2019). In this paper, the simplest conditions are taken for the outer boundary physics.

2.3. Line profiles and 2D transfer functions

The emissivity distributions in BLRs are still unclear from observations. From photoionization, the locally optimally emitting clouds model suggests that the line emission irradiates most efficiently from a relatively narrow range of the ionization parameter, \( U_{\text{ion}} = Q_{\text{ion}}/4\pi R_{\text{eff}}^2 \), where \( Q_{\text{ion}} \) is the number of ionization photons, \( n_{\text{ion}} = \rho_{\text{ion}}/m_H \) is the number density of hydrogen, \( \rho_{\text{ion}} \sim \rho \left[ \sigma_0(R) + \sigma_1(R, \varphi) \right] / 2H = \left[ \sigma_0(R) + \sigma_1(R, \varphi) \right] \Omega / 2\Omega_0 \) is the ionized hydrogen density and assumed to be proportional to the density of the disk, and \( m_H \) is the mass of hydrogen atom (Baldwin et al. 1995; Korista et al. 1997; Korista & Goad 2000). For simplicity, we assume that the emissivity (reprocessing coefficient) is a Gaussian function of ionization parameter as:

\[
\Xi_R \propto \frac{1}{\sqrt{2\pi \sigma_U}} e^{-\left( U_{\text{ion}} - U_c \right)^2/2\sigma_U^2}, \tag{14}
\]

where \( U_c = U_{\text{ion}}(R_c) \) is the ionization parameter corresponding to the most efficient line emission at radius \( R_c \) and \( \sigma_U = \Delta R_{\text{ion,max}} - \Delta R_{\text{ion, min}} \) is a parameter that controls the range of line emission. Actually, the form of Eq. (14) is a simplified version of the popular model (e.g., Pancost et al. 2014; Li et al. 2018). The typical BLR radii are on average smaller than the inner edges of tori by factors of \( 4 \sim 5 \) (Koshida et al. 2014; Minezaki et al. 2019), thus we adopted \( R_c = 1/4R_{\text{torus}} \), and assume \( \sigma_U = 0.20 \) (corresponding to a not very compact line-emitting region).

Given the configuration of the disk-like BLR in Section 2.1, the emission-line profile can be expressed as

\[
F_\lambda(\lambda) = \int_0^{R_{\text{in}}} \frac{dR}{R_a} \int_0^{2\pi} d\varphi \int_0^\infty dU \Xi_R \delta \left( \lambda - \lambda_0 \left( 1 + \frac{V + n_{\text{obs}}}{c} \right) \right), \tag{15}
\]

where \( \nu \) is the velocity of emitting gas, \( n_{\text{obs}} = (0, \sin i_0, \cos i_0) \) is the vector of line of sight, and \( i_0 \) is the inclination angle, Blandford & McKeel (1982) developed the linear reverberation technique to map BLRs. Denoting the ionizing continuum light curve and broad-line light curve at a velocity, \( \nu \), of the line profile as \( L_c(t) \) and \( L_L(t, \nu) \), respectively, we have:

\[
L_L(\nu, t) = \int_{-\infty}^{\infty} d\nu' L_c(\nu') \Psi_\nu(t, \nu - \nu'), \tag{16}
\]

where \( \Psi_\nu(t, \nu) \) is the 2D transfer function (velocity-delay map), \( \Psi_\nu(t, \nu) = 0 \) for \( t < 0 \), and \( \Psi_\nu(t, \nu) \geq 0 \) for \( t \geq 0 \). The RM campaigns obtained \( L_L(t, \nu) \) and \( L_c(t) \), which can be used to infer \( \Psi_\nu(t, \nu) \). With the geometric and kinematic configurations of BLRs with density waves, the 2D transfer function can be obtained via:

\[
\Psi_\nu(t, \nu) = \int dR g(R, \nu) \frac{Z_{\text{eff}}}{4\pi R^2} \delta \left( \nu - \frac{\nu + n_{\text{obs}}}{c} \right), \tag{17}
\]

where \( g(R, \nu) \) is the projected one-dimensional velocity distribution function. With Equation (17), features of density waves can be calculated and then compared with observations.

In principle, high-fidelity data from RM campaigns can be used to generate 2D transfer functions through the maximum entropy (Horne et al. 2004) or the improved Pixon-based method (Li et al. 2021). Spiral arms as prominent inhomogeneous components of BLRs can be directly tested avoiding uncertainties of explanations in light of complicated profiles alone.

2.4. Signals for GRAVITY/VLTI

As a powerful technique, spectroastrometry (SA) developed from “Differential Speckle Interferometry” (Beckers 1982; Petrov 1989; Rakshit et al. 2015) measures the center of photons and therefore greatly improve the spatial resolution. The
spectrum with GRAVITY/VLTI can reach an unprecedentedly high spatial resolution of ~ 10µas. It had been successfully applied to 3C 273 for the geometry and kinematics of its BLR (Gravity Collaboration et al. 2018). The spiral arms developed from density waves may show some signatures that can be detected by GRAVITY/VLTI. The detailed scheme for spectroastrometry technique is described in Rakshit et al. (2015) and Songsheng et al. (2019). Below, we outline a brief description for the sake of completeness. Given the surface brightness distribution, we have the photon center of the source at wavelength

$$\lambda :$$

$$\epsilon(\lambda) = \frac{\int O(\alpha, \lambda) d^2 \alpha}{\int O(\alpha, \lambda) d^2 \alpha},$$

where $$O(\alpha, \lambda) = O_{\ell} + O_{\delta}$$ is the surface brightness distribution of the source contributed by the BLR and continuum emissions, and $$\alpha$$ is the angular displacement on the celestial sphere. Given the geometry and kinematics of a BLR, its $$O_{\ell}$$ can be calculated for the broad emission line with the observed central wavelength, $$\lambda_{\text{cen}}$$, via

$$O_{\ell} = \int \frac{2 \kappa F_c}{4\pi R^2} f(R, \nu) \delta(\alpha - \alpha') \delta(\lambda - \lambda') d^3 R \, d^3 \nu,$$ (19)
where \( \lambda' = \lambda_0 n_0 / (1 + u \cdot n_0/c) \) includes the gravitational shift due to the central SMBH, \( R_S = 2 R_g \) is the Schwarzschild radius, \( \gamma_0 = \{1 - v^2/c^2\}^{-1/2} \) is the Lorentz factor, \( \alpha' = [R = (R \cdot n_0)/n_0]/D_\lambda \), \( R \) is the distance to the central SMBH, \( f(R, \nu) \) is the velocity distribution of BLR clouds at \( R \), and \( F_c \) is the ionizing flux. By introducing the fraction of the emission-line flux to the total (\( \epsilon \)), we have

\[
\epsilon(\lambda) = \lambda \epsilon(\lambda),
\]

where

\[
\epsilon(\lambda) = \int \alpha(\lambda') d\lambda', \quad \lambda = \int \alpha(\lambda') d\lambda', \quad F(\lambda) = \int \alpha(\lambda') d\lambda',
\]

and

\[
F_{\text{tot}}(\lambda) = F(\lambda) + F_c(\lambda).
\]

For an interferometer with a baseline \( B \), a non-resolved source, with a global angular size smaller than its resolution limit \( \lambda/B \), takes the interferometer phase

\[
\phi(\lambda, \lambda) = -2 \pi \alpha \cdot \epsilon(\lambda) - \epsilon(\lambda),
\]

where \( u = B/\lambda \) is the spatial frequency, and \( \lambda_0 \) is the wavelength of a reference channel. Given the geometry and kinematics of the BLR, the spectroastronomical signals can therefore be calculated.

### 3. Results: SA and RM signals

It is known that the nearly-Keplerian disks dominated by the potential of central sources favor the \( m = 1 \) mode, namely, a single spiral arm (Adams et al. 1989; Shen et al. 1990; Lee et al. 2019). We can obtain a value for \( \omega \) from the eigenvalue problem of Equation (9) as well as the perturbed component (\( \sigma_1/\sigma_0 \)) for density waves. In this paper, we only focus on the general parameter in following calculations, rather than \( K \) and \( \Lambda \). The dispersion relation can be used to give a rough estimate to the tightness of winding of the arms, which is expressed by

\[
\omega = \frac{2 \pi G M}{R^2 (1 - \nu^2)} = \frac{2 \pi G M}{R^2 (1 - \nu^2)}.
\]

Pitch angles of spiral arms are determined by \( \tan i = 1/R \) for given \( n \), \( K_0 \), and \( Q_{\text{disk}} \), and \( k_0 R \) can be representative of the global pitch angles. In order to conveniently show the spiral arms, we use the proxy of pitch angles defined by

\[
k = \frac{2 \pi G M}{R^2 (1 - \nu^2)} = \frac{2 \pi G M}{R^2 (1 - \nu^2)}.
\]

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Fig. 2. Characterized profiles of BLR-disk ($\sigma_i + \sigma_f$) with one-arm structure (in blue) and differential phase curves (in orange). The panels are for different orientation and azimuthal angles. Dashed-lines represent emission from an homogeneous disk, and the solid lines from the disk with spiral-arm. In the calculations, we take the perturbations of surface density ($\sigma_i/\sigma_0$)$_{\text{max}} = 0.2$. The differences between the solid and dashed lines are provided as residuals with the corresponding colors. The differential phases can be measured by GRAVITY, and the residual to the homogenous disk can be also detected for the BLR with the present parameters. Less massive black holes can be measured by GRAVITY+ (the next generation of GRAVITY).

rian disks in Welsh & Horne (1991) and Wang et al. (2018). A Keplerian disk shows a symmetric bell-like feature. For clarity, in Figure 3, we only plot the response of the density perturbations ($\sigma_i$). For comparison, an example of the transfer function for the unperturbed disk is shown by Figure D.1 in Appendix D. As shown in Figure 3, the major influences of the spiral arm are the variations of the bell’s waist. We find that the higher and lower density perturbations (wave crest and wave trough, see also Figure 1) generate positive and negative signals (stronger and weaker responses) with superposition to the transfer functions of Keplerian disks, respectively. In fact, there is a certain deficit of the right waist in the bell-like transfer function of NGC 5548 as shown in (Xiao et al. 2018). When $\theta = 0^\circ$, the positive signals are mainly located in near side (smaller time lags) and the negative signals tend to be at larger time lags. Along with $\theta$ increasing from $0^\circ$, the positive and negative signals rotate clockwise. Measuring the rotation of the features is one of the keys to test the presence and dynamics of the spiral arms in BLRs. For cases with $Q_{\text{disk}} = 1 + \Delta Q$, where $\Delta Q < 0.3$, we have carried out the calculations and found the general patterns of arms do not change significantly for given $k_0 R$. Thus, we omit these figures.

4. Discussions

4.1. Tight-winding approximation

Employing the traditional WKB approximation (e.g., Lin & Lau 1979), for the first time we applied the theory of density waves to BLRs for broad emission line profiles, differential phase curves, and velocity-delay maps. The validity of this approximation can be simply estimated by comparing the global pitch angle proxy of $k_0 R$, as shown in Figure 1 with detailed calculations forthcoming in Du et al. (2022 in preparation) which relaxes the WKB approximation for the more loosely wound cases. We find that the difference of pitch angles can be less than $\lesssim 20\%$ for $k_0 R = 5$. It is generally believed the WKB approximation is good enough.
for $k_0 R \gtrsim 5$, which is consistent with the conclusion in Lin & Lau (1979).

Moreover, the non-linear effects have been extensively studied by Lee & Goodman (1999), who draw a conclusion that single-armed density waves can exist even in nearly Keplerian disks with only weak self-gravity. The pitch angle of the arms can be significantly larger with non-linear effects. Relaxing the WKB approximation schemed by Adams et al. (1989) will generate more general results for AGN BLR issues, as we show in a forthcoming paper (Du et al. 2022 in preparation). On the other hand, some simulations show that $m = 1$ is also favored when the disk mass is comparable with the central SMBHs (e.g., Lodato & Rice 2004; Kratter & Lodato 2016). In this context, the disk will be thicker than the present cases so that the Poisson equation has more complicated expression than Eq. (8). Density waves will be modified by the radial self-gravity of the disks.

4.2. Observational appearance

The present profiles of broad emission lines can be conveniently compared with observations. The suggested spiral arms for asymmetric profiles of some AGNs (e.g., Eracleous & Halpern 1994; Storchi-Bergmann et al. 2017) could originate from the results of self-gravity instability. The observational appearance should be tested by examining individual AGNs and the statistical properties of large AGN samples. It is common to see asymmetric profiles for Palomar-Green quasars and, statistically, the asymmetries are significantly correlated with the strengths of Fe II (which clearly depends on accretion rates $\dot{M}$; see Figure 5 in Boroson & Green 1992). This has a clear implication, namely, that the homogeneity of ionized gas distributions is governed by accretion rates. Marziani et al. (2003, 2009, 2010) investigated the line profiles of low- and intermediate-redshift AGNs, concluding that the line asymmetry changes systematically along the so-called quasar “eigenvector 1 sequence.” In practice, the parameters of $n$, $K_0$, and $Q_{\text{disk}}$ may depend directly on $\dot{M}$, resulting in the dependence of the arms on the accretion rate, which could probably explain these phenomena. We also note that asymmetries of Hβ profiles are changing with time from red to blue asymmetries – or the reverse, which could be naturally explained by the pattern motion of $m = 1$ mode spiral arms. The high-fidelity RM of AGNs, which employs high spectral resolution and homogeneous cadence, will finally be capable of revealing the substructures of the BLRs through detailed 2D transfer functions based on the response to the ionizing sources (Welsh & Horne 1991; Horne et al. 2004; Wang et al. 2018; Songsheng et al. 2019, 2020) as well as the spectroastrometric observations of GRAVITY/VLTI with the predicted characteristics. Detailed comparisons with the observations will be deferred to a future paper.

However, the true situation may be more complicated than the simplified model adopted here, which could make the den-
sity waves (spiral arms) more complex, especially in the cases of radiation pressure-driven warped disks (Pringle 1996), MRI-driven turbulence, other instabilities (e.g., a brief review in Ogilvie & Latter 2013), or star formation (Shlosman & Begelman 1989; Collin & Zahn 1999; Gammie 2001; Collin & Zahn 2008; Wang et al. 2011, 2012). Magnetic fields could be very important in some cases. Moreover, fast cooling could make the disk suffer from violent instability, condense into discrete clouds, and even generate filamentary spiral pattern (e.g., Gammie 2001; Rice et al. 2003, 2011; Kratter & Lodato 2016; Brucy & Hennebelle 2021). However, on the one hand, heating caused by MRI or radiation from inner region may balance the cooling effect. On the other hand, from the perspective of observation, high-resolution spectroscopy in Arav et al. (1997, 1998) provides a lower limit to the number of clouds in BLRs and excludes the possibility that BLRs would consist of "discrete" clouds. No matter how complicated the physics is in BLRs, future detections will advance understanding the mystery of the BLRs.

4.3. Relation between BLRs and the accretion disks

The origin of BLRs and their relation with accretion disks are still a puzzle. In the present paper, we assume that the BLR is the illuminated surface layer of the accretion disk (in the SG region). Such kind of assumption was also adopted by, for instance, Goad et al. (2012). It is only in n = 3 cases that the gas on the surface can be illuminated by the central ionizing photons. It should be noted that radiation pressure from the disk may push up the height of this region (e.g., Emmering et al. 1992; Murray et al. 1995; Chiang & Murray 1996; Czerny & Hryniewicz 2011; Elvis 2017; Baskin & Laor 2018) and provide a covering factor that is large enough to explain the BLR observations, which will ease the restriction of polytropic index. However, a thicker disk may potentially provide stability against non-axisymmetric perturbation and reduce the lifetimes of the spiral arms (Ghosh & Jog 2021). An observation noted in Horne et al. (2021) indicates azimuthal structures in BLRs, which adds a constraint that the lifetime of the arms cannot be too short. The influence of the thick BLR layers should be investigated both theoretically and observationally in future.

4.4. Self-gravity of the accretion disks

To maintain density waves needs $Q_{\text{disk}} \sim 1 - 1.3$ (e.g., Lodato & Rice 2004) driven by several mechanisms mentioned in Section 1, however, it is expected to distinguish them from observations. Star formation in the self-gravitating disks could support the state of $Q_{\text{disk}}$ as suggested by (e.g., Shlosman & Begelman 1989; Collin & Zahn 1999; Thompson et al. 2005; Wang et al. 2011). As a self-regulation mechanism, higher star formation will decrease the surface density of the disks, while increasing $Q_{\text{disk}}$; whereas lower star formation rates increase the surface density, decreasing $Q_{\text{disk}}$. Except for releases of gravitational energy of the accreting gas, star formation, and supernovae explosion will supply additional energies to this region. As independent evidence of this, AGNs and quasars are known to be metal-rich, thus providing observational constraints on these processes. It would be interesting to test the potential dependence of broad emission line profiles on the metallicity (Z). As a hint, we see that the asymmetries of Hβ profiles are strongly correlated with Fe II strength ($R_{\text{Fe}}$) (see Figure 5 in Boroson & Green 1992), while $R_{\text{Fe}}$ is a proxy of the accretion rates (e.g., Boroson & Green 1992; Marziani et al. 2003; Hu et al. 2012; Shen & Ho 2014) correlating with Z.

Finally, self-gravity has been neglected in the vertical direction of the BLR with regard to its height (i.e., in the equation of $H = a_{\text{eff}}\Omega^2$. A sophisticated treatment of the vertical structure will include self-gravity as well as radiation pressure from viscosity dissipation and star formation (and supernovae explosion). We leave this aspect to a future paper.

5. Conclusions

There is mounting evidence of the presence of spiral arms in BLRs of AGNs. In this paper, using the WKB approximation, we start from the perturbation equations to study dynamics and structures of ionized gas in the SG regions around SMBHs, which constitute the major parts of BLRs. We calculate the major properties of density waves excited by the m = 1 mode. The features of density waves can be detected by asymmetric profiles, differential interferometric signals (by GRAVITY/GRAVITY+ onboard VLTI), and 2D transfer functions from RM campaigns. In particular, the patterns of spiral arm in the 2D transfer functions are unique due to the rotation motion of the spiral arm.

These features help us to better understand the physical connection between SG disks and BLRs in AGNs. Using the hypothesis that SG regions maintain the Toomre constant $Q \sim 1$, we show that the excited density waves arising from perturbations of SG disks are responsible for inhomogeneous distributions of BLRs. It is possible to observationally test the density waves in BLRs with the current instruments. Our preliminary results show that density waves in BLRs provide a new avenue for studying BLR structures and dynamics as well as for resolving long-standing issues related to BLRs.

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Appendix A: Basic equations

We start from the ideal fluid equations in the cylindrical coordinates \((R, \phi, z)\). For reader’s convenience, we list the classical equations which can be found from Lau & Bertin (1978), Lin & Lau (1979), and Binney & Tremaine (2008). The continuity equation is expressed as:

\[
\frac{\partial \rho}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} (R \rho u) + \frac{1}{R} \frac{\partial}{\partial \phi} (\rho v) = 0, 
\]

and the motion equations are

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial R} + v \frac{\partial u}{\partial \phi} - \frac{\nu^2}{R} = - \frac{1}{R} \frac{\partial}{\partial R} (\psi + h), 
\]

and

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial R} + v \frac{\partial v}{\partial \phi} + \nu u = \frac{1}{R} \frac{\partial}{\partial \phi} (\psi + h), 
\]

where \(\psi\) is the potential. We should mention that the viscosity is neglected in above equations. This is valid for the quasi-Keplerian rotation disk as in the standard disk model (Shakura & Sunyaev 1973).

Appendix B: Coefficients

The coefficients are given as follows:

\[
A = - \frac{1}{R} \frac{d \ln \rho}{d \ln R}, \quad \alpha = \frac{\kappa^2 (1 - \nu^2)}{\sigma_0 R}, \quad \nu = \frac{\omega - m \Omega}{\kappa}; 
\]

\[
B = - \frac{m^2}{R^2} \frac{4 m \Omega R' (R'' \kappa)}{\kappa^2 (1 - \nu^2)} + \frac{2 m \Omega}{R \kappa} \frac{d \ln \left(\frac{\kappa^2 \sigma_0 \Omega}{\kappa^2 \nu}\right)}{d \ln R}, \quad C = - \frac{\kappa^2 (1 - \nu^2)}{a_0^2}. 
\]

Appendix C: Wave numbers of spiral arms

We present the dependence of \(k_0 R\) on the parameter \(K_0\) and the polytropic index \(n\) in Figure C.1. \(k_0 R\) decreases with \(K_0\) and \(n\) increase. In addition, \(k_0 R\) increases slightly if \(\dot{M}\) and \(M_*\) increase. The current dependence on \(\dot{M}\) and \(M_*\) in the present model is mainly caused by the dependence of the inner and outer boundaries on these two parameters (see Eq. 12). In reality, \(K_0\), \(n\), and even \(Q_{\text{disk}}\) may rely on \(\dot{M}\) and \(M_*\) more directly.

Appendix D: Example of the transfer function for unperturbed disk

In Figure 3 of Section 3, we present the transfer functions of the spiral arms (\(\sigma_1\)). Here, for comparison, we provide an example of the transfer function of the unperturbed disk (\(\sigma_0\)) in Figure D.1.
**Fig. C.1.** $\overline{k_0 R}$ as a function of $K_0$ and $n$. The color represents the logarithmic value of $\overline{k_0 R}$. The red solid, dashed, and dotted lines represent $\overline{k_0 R} = 5, 10, 20$, respectively (corresponding to the cases in Figure 1).

**Fig. D.1.** Example of the transfer function for unperturbed disk ($\sigma_0$).