Balancing Weighted Substreams in MIMO Interference Channels

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Abstract—Substreams refer to the streams of each user in a system. Substream weighting, where the weights determine the prioritization order, can be important in multiple-input multiple-output interference channels. In this letter, a distributed algorithm is proposed for the problem of power minimization subject to weighted SINR constraint. The algorithm is based on two basic features, the well known distributed power control algorithm by Yates in 1995 and a simple linear search to find feasible SINR targets. The power control law used in the proposed algorithm is proven to linearly converge to a unique fixed-point.

Index Terms—MIMO, interference channel, weighted SINR, substream balancing.

I. INTRODUCTION

Prioritization, in other words desired level of fairness, is important to ensure quality-of-service (QoS) in the system. Balancing weighted substreams, data streams of the same user, can be achieved by two complementary approaches, maximization of minimum weighted SINR subject to power constraint or minimization of power subject to weighted SINR constraint, and at three different levels, prioritization between streams, users, or substreams. Explicitly, prioritization between all streams, all users, or all streams of each user can be aimed. From more to less restrictive, stream, user, and substream prioritization comes in order. Consequently, substream prioritization causes the least degradation in sum-rate, followed by user and stream prioritization for a given channel. In this work, substream prioritization is studied.

Unequal and equal weighting of SINRs are both important in practice. The former aims for the received SINRs of more important substreams to be higher than those of less important substreams. On the other hand, the latter aims for a more error resilient system. If a substream cannot be decoded, e.g., the information is lost, other substreams can be used to achieve a successful transmission with a lower quality. To this end, in this letter, a distributed algorithm is proposed to achieve desired level of SINR fairness at the substream level by using the later approach, minimizing the power subject to weighted SINR constraint. The proposed algorithm is ad-hoc in the sense that the transmit and receive beamforming vectors are initially obtained via a beamforming scheme such as SINR maximization (max-SINR) [1]. Then, the proposed algorithm is plugged and run in an ad-hoc manner. The proposed algorithm has two basic features. The power control law, the first feature, used in the algorithm is a straightforward extension of standard interference functions introduced in [2] as was also applied in [3,4]. The linear search, the second feature, used in the algorithm finds feasible SINR targets for the substreams, thus convergence of the algorithm is guaranteed. The contributions of the letter can be summarized as follows. 1) The ad-hoc nature of the proposed algorithm allows linear search for setting the SINR targets dynamically, as opposed to setting the SINR targets statically. To the best of our knowledge, setting SINR targets opportunistically in multiple-input multiple-output (MIMO) interference channels (ICs) is only studied in this letter. 2) A system with unequal substream priority can transmit a media with higher quality than a system with equal substream priority although total bit error rate (BER) of the first approach can be higher than the second. On the opposite, total BER is influential for the received media quality under equal substream priority, in other words under substream fairness condition [5]. For the later, this letter takes initiative steps in MIMO ICs in showing the effects of substream fairness on uncoded BER, SINR and sum-rate metrics. 3) Power control law in the proposed algorithm based on standard interference functions is proven to converge to a unique fixed-point. In fact, the convergence is independent from beamforming techniques and desired level of fairness.

Notation: $^\top$ and $^\dagger$ denote the transpose and complex conjugate operators. Matrices and vectors are denoted by boldface uppercase and lowercase letters, respectively. $I_M$, $I_N$, and $\text{diag}[x_1, \ldots, x_L]$ denotes all ones vector, identity matrix, zero vector or matrix, and diagonal matrix with elements $x_i$ on its diagonal, respectively. $||\cdot||_1$, $||\cdot||_\infty$ and $\min$ denote determinant, $l_1$-norm, and minimum operators, respectively, and for some given vector $x > 0$, $||\cdot||_\infty$ denotes weighted maximum norm.

II. SYSTEM MODEL

We consider a $K$-user IC, where there are $K$ transmitters and receivers with $M_k$ and $N_k$ antennas at node $k$, respectively. A transmitter has $d_k$ streams to be sent to its corresponding receiver. This system can be modeled as...
\[ y_k = \sum_{j=1}^{K} H_{kj} x_j + z_k, \ \forall k \in K = \{1, 2, ..., K\}, \]
where \(y_k\) and \(z_k\) are the \(N_k \times 1\) received signal vector and the zero mean unit variance circularly symmetric additive white Gaussian noise vector (AWGN) at the \(k^{th}\) receiver, respectively. \(x_j\) is the \(M_j \times 1\) signal vector transmitted from the \(j^{th}\) transmitter and \(H_{kj}\) is the \(N_k \times M_j\) channel matrix between the \(j^{th}\) transmitter and the \(k^{th}\) receiver. \(E[|x_j|^2] = p_j\) is the power of the \(j^{th}\) transmitter. The transmitted signal from the \(j^{th}\) user is \(x_j = U_j \sqrt{p_j} d_j\), where \(U_j = [u_{j1}, \ldots, u_{jd_j}]\) is the \(M_j \times d_j\) precoding matrix, \(d_j\) is \(d_j \times 1\) vector denoting the \(d_j\) independently encoded streams, and \(P_j = \text{diag}(p_{j1}, \ldots, p_{jd_j})\) is a \(d_j \times d_j\) diagonal matrix consisting of substream powers with \(\sum_{j=1}^{d_j} p_{j,l} \leq p_j\). The \(N_k \times d_k\) receiver matrix is denoted by \(V_k\).

III. PRIORITIZED SUBSTREAMS

Substream prioritization is useful for concurrent transmissions of different services, e.g., voice and media transmissions, as well as transmission of single service, e.g., different parts of the video or image can be assigned to substreams with varying importance. On the other hand, all substreams can have equal importance to have error resiliency. For both, the proposed ad-hoc algorithm in this letter can dynamically set the SINR targets. Thus, the algorithm has small rate and SINR losses.

A. Preliminaries

Max-SINR \([1]\) designs the transmit beamformer of each stream of a user separately, substreams are considered as interference on one another, thus SINR is given as \(\text{SINR}_{k,l} = \frac{\sqrt{\text{tr}(B_{k,l}^* V_k)}^2}{\text{tr}(B_{k,l}^* V_k) + \text{tr}(\text{tr}(\text{diag}(R_{k,l} - \text{diag}(H_{kj}^* P_j U_j^\dagger H_{kj})) )^2)}}, \)
where \(B_{k,l} = Q_{k,l} + I_{N_k}\), \(Q_{k,l} = \sum_{j=1}^{K} H_{kj} U_j P_j U_j^\dagger H_{kj}^\dagger - R_{k,l}\), and \(R_{k,l} = p_{k,l} H_{kj} u_{k,l} u_{k,l}^\dagger H_{kj}\) are the covariance matrices of the interference plus noise, interference, and \(l^{th}\) stream of user \(k\), respectively.

A simple technique for substream prioritization is weighting the substream-SINRs, thus substream prioritization can be achieved via joint power control and beamforming design by maximizing the minimum weighted SINRs \([6]\). Compared with stream prioritization problems, the substream problems are decoupled into \(K\) sub-problems given SINR targets are feasible, then the problem can be solved asynchronously among users. However, feasibility check is coupled among users and can be shown to be NP-hard \([4]\). Therefore, we focus on designing efficient algorithms for achieving locally optimal points. It is well known that the optimal solution to the minimization of power subject to weighted SINR constraint is achieved when the weighted SINRs are equalized, i.e., \(\text{SINR}_{k,1} = \cdots = \text{SINR}_{k,d_k} = \Gamma_k^\beta\), \(\forall k \in K\), where \(\beta_{k,l}\) are the weighting factors that reflect the priorities and \(\Gamma_k^\beta\) is the common weighted SINR target of substreams. Clearly, when \(\beta_{k,l} = 1, \forall k \in K\) and \(\forall l \in \mathcal{L} \triangleq \{1, 2, ..., d_k\}\), the problem is reduced to conventional worst SINR maximization problem. We propose a practical scheme, named ad-hoc algorithm, to balance weighted substream-SINRs. Basically, we unite the simple linear search for finding maximum possible SINR targets with the optimization problem

\[
\min_{\beta_{k,l}} \sum_{k,l} p_{k,l} \text{subject to}
\]

\[ \text{SINR}_{k,l} \geq \Gamma_k^\beta, p_{k,l} > 0, \sum_{l=1}^{d_k} p_{k,l} \leq p_k, \forall k \in K \text{ and } \forall l \in \mathcal{L} \]

that can be solved via conventional distributed power control algorithm \([2] [3] [4]\) with the maximum power constraint. The well known distributed power control algorithm with maximum power per user \(p_k\) constraint \([2]\) is given as \(p_k = \frac{\Gamma_k^\beta}{p_k^{n-1}} p_k\), where superscript \(n\) is the iteration number, \(p_k^n\) is the power, and \(\text{SINR}_{k,1}^\beta\) is the SINR of user \(k\). Basically, a user increases its power if its SINR is below its SINR target and vice versa. Clearly the SINR target can be unmet due to the maximum power constraint. The goal of the proposed algorithm is to achieve substream prioritization while causing the least sum-rate degradation. Therefore, power saving is not the primary concern of our proposed algorithm. By directly following the steps in \([2]\), the standard interference function for our problem is given as \([3]\)

\[
I_{k,l}(p) = \Gamma_{k,l}\delta_{k,l},
\]

where \(p = [p_{1,1}, \ldots, p_{1,d_k}, \ldots, p_{K,1}, \ldots, p_{K,d_k}]^T\) is the transmitted power vector of the system, \(\Gamma_{k,l} = \beta_{k,l}^\Gamma\), and \(\delta_{k,l} = \frac{p_{k,l}}{\text{SINR}_{k,l}^\Gamma}\).

Finally, joint optimization of beamforming vectors and power allocation is a challenging problem for schemes where SINR targets are dynamically determined. The degrees of freedom (DoF) essentially drop to zero for schemes where SINR targets are preset \([46]\). Whereas DoF is not zero for our scheme since opportunistic maximum SINR search is performed, together with achieving fairness between data streams. This is shown in sum-rate simulation results in Section \([15]\) where the DoF loss is not significant compared to the conventional max-SINR where fairness is not achieved.

B. Proposed Algorithm

The proposed ad-hoc algorithm in Table \([1]\) opportunistically searches for feasible SINR targets for substreams. The algorithm can run synchronously among users. In Table \([1]\), \(P_k^n = [p_{k,1}^n, \ldots, p_{k,d_k}^n]\) is the power vector, \(p_{k,l}^n\) is the power at iteration \(n\), \(\beta_{k,l}^n = \beta_{k,l} / \sum_{l=1}^{d_k} \beta_{k,l}\) is the normalized weighting factor for the \(l^{th}\) substream of the \(k^{th}\) user, and \(\epsilon\) is set to \(10^{-3}\). \(P_j^{n-1}\) and \(B_k^{n-1}\) are the previously defined terms with the iteration numbers. \(R_{k,l}^n = H_{kk}\ k u_{k,l} u_{k,l}^\dagger H_{kk}\) is akin to a covariance matrix, \(1 = [1, \ldots, 1]^T\) is all ones vector, \(\delta_{k} \triangleq [\delta_{k,1}, \ldots, \delta_{k,d_k}]^T\) and similarly \(\text{SINR}_k\) is the vector of substream-SINRs for user \(k\). \(p_{k,\text{total}}\) is a variable used in the simulation that can have maximum value \(p_k\), and the limit variable is an upper bound for the iteration number of power control law.

Linear search: Since the proposed algorithm is ad-hoc, the maximum SINR achieved after beamforming is the upper bound to the maximum SINR achieved after the proposed
ad-hoc algorithm. For simplicity, consider the substream fairness constraint case, where \( \beta_{k,l} = 1, \forall l \in \{1, 2\} \). Thus \( \Gamma^C_k = \Gamma_k \), where \( \Gamma_k \) is average SINR of user \( k \). Clearly setting average SINR as the target is a good starting point for searching as will be shown in Example 2.

**Example 1.** Assume \( \text{SINR}_{1,1}^0 = 5 \) and \( \text{SINR}_{1,2}^0 = 10 \) are achieved for the \( k^{th} \) user after beamforming. Thus for substream fairness, SINR target is set to \( \Gamma^1_k = \Gamma^2_k = 7.5, \forall l \in \{1, 2\} \), where superscript denotes the iteration number \( m = 1 \), and achieved SINR after the first iteration is denoted by \( \text{SINR}^1_{k,l} \). Please note that iteration number \( m = 0 \) is not needed in Algorithm 1 thus it is omitted. Ideally \( \text{SINR}^2_{k,1} = \text{SINR}^2_{k,2} = 7.5 \) must be achieved after the ad-hoc algorithm. However, due to the distributed nature of the algorithm, and especially when the substreams are highly unbalanced, the optimal SINR target may not be achieved.

Step 10 of the algorithm is the most critical part where the substream powers are updated in order from the substream with the lowest to the highest \( \delta_{k,l} \). In this way, the substream with the lowest \( \delta_{k,l} \) can definitely reach the SINR target, while the substream with the highest \( \delta_{k,l} \) reaches to a maximum possible SINR value by using the remaining power budget of the user \( k \). In the next iteration, the target SINR is the average of these achieved SINRs, thus the algorithm keeps iterating until the convergence of substream-SINRs. The convergence plot of the proposed algorithm is given in [3].

**Example 2.** As explained previously, the substream with the highest SINR is guaranteed to achieve the SINR target \( \Gamma^m_k \) in the \( m^{th} \) iteration. Consider Example 1 After the first iteration, the second substream is guaranteed to achieve the target \( \text{SINR}^2_{k,1} = \Gamma^1_k = 7.5 \). Meanwhile, assume that only \( \text{SINR}^2_{k,1} = 5.5 \) can be achieved for the first substream. Then the SINR target for the next iteration is \( \Gamma^2_k = 6.5 \). After the second iteration, \( \text{SINR}^2_{k,2} = 6.5 \) is again guaranteed to be achieved and assume only \( \text{SINR}^2_{k,1} = 5.75 \) can be achieved by expending the remaining power. Then for the third iteration, the SINR target is \( \Gamma^3_k = 6.125 \). The SINR target keeps dropping until the substream-SINRs are equal, thus convergence is guaranteed.

Finally, in the high SNR regime, e.g., at 30 dB, high number of iterations for power control law, i.e., while loop between the lines 6 and 14 in Algorithm 1 can be required. An upper bound can be set and the parameter \( \epsilon \) can be tuned for avoiding high number of iterations. For further details, the reader is referred to the first author’s website [5].

**IV. Numerical Results**

Numerical results for MIMO ICs with \( K = 3, M_k = 4, N_k = 4 \), and \( d_k = 2 \) are presented in this section. \( 10^3, \ 10^4, \ 10^5 \), and \( 10^6 \) random ICs are tested for SNR values 0, 5, 10, and 15 dB, respectively. Channel coefficients are generated by i.i.d. zero-mean unit-variance complex Gaussian variables, QPSK modulation is used, and iteration number is set to 16. The sum-rate is defined as \( R_{\text{sum}} = \sum_{k=1}^{K} \sum_{l=1}^{r} p_k \sum_{y=1}^{B} (1 + \text{SINR}_{k,l}) \). Finally, the same type of filter structures are used at the transmitters and receivers, i.e., max-SINR filter is used for both transmit and receive filters.

**A. Balancing weighted substreams**

In Table II for \( \beta_{k,1} = 1 \) and \( \beta_{k,2} = 6, \forall k \in \mathcal{K} \), SINR values and ratios before (SINR\(_{k,l}^0\)) and after (SINR\(_{k,l}^m\)) Algorithm I are presented at 10 dB. Please note that given predetermined priorities \( \beta_{k,l} \), optimal balancing of weighted substreams can be achieved if the substream-SINR proportions are already smaller than the priority proportions, i.e., \( \frac{\text{SINR}_{k,2}^m}{\text{SINR}_{k,1}^m} < \frac{\beta_{k,2}}{\beta_{k,1}} \).

**TABLE II**

| User | \( \text{SINR}^0_{k,1} \) | \( \text{SINR}^0_{k,2} \) | Ratio\(_0\) | \( \text{SINR}^m_{k,1} \) | \( \text{SINR}^m_{k,2} \) | Ratio\(_m\) |
|------|-----------------|-----------------|---------|-----------------|-----------------|---------|
| 1    | 8.89            | 22.53           | 2.53    | 4.49            | 26.93           | 6       |
| 2    | 8.09            | 21.54           | 2.66    | 4.23            | 25.40           | 6       |
| 3    | 8.13            | 19.49           | 2.40    | 3.95            | 23.67           | 6       |

**B. Achieving substream fairness**

For max-SINR with and without Algorithm I uncoded BER results in Fig. 1 and stream-SINRs and sum-rates concurrently in Fig. 2 are plotted where weighting factors are set to \( \beta_{k,l} = 1 \), \( \forall k \in \mathcal{K} \) and \( \forall l \in \mathcal{L} \). For max-SINR without Algorithm I power is simply split equally among the substreams of each user. SINR and sum-rate results, and SNR values are in linear scale, bits per channel use, and in dB scale, respectively. SINR legends in Fig. 2 follow the same order in Fig. 1(a). The second substream-SINRs of max-SINR without Algorithm I achieve values around 45 at 15 dB, but not plotted for brevity. Substream fairness is achieved at the cost of a reasonable sum-rate degradation as seen in Fig. 2. The proposed algorithm whose objective is substream fairness can achieve stream fairness in ergodic sense as seen in Fig. 2 with lower algorithmic complexity and less information exchange than the algorithms whose objectives are stream fairness.
In this section, the power control law used in Algorithm I is proven to converge to a unique fixed-point at a linear rate under appropriate assumptions by using contractive interference functions introduced in [8]. For further details regarding this section, the reader is referred to [8]. Next, necessary steps are taken to prove interference functions (1) are contractive.

The interference function (1) can be rewritten as
\[
I_{k,l}(p) = \sum_{s=1}^{d_k} \sum_{j=1}^{K} T_{k,l}^{j,s} p_{j,s} + N_{k,l} = \frac{1}{G_{k,l}}, \quad G_{k,l} = \|v_{k,l}^\top H_{k,j} u_{j,s}\|^2, \quad \text{and}
\]

\[
T_{k,l}^{j,s} = \begin{cases} 0 & \text{if } (k, l) = (j, s), \\ \frac{v_{k,l}^\top G_{k,l}^{j,s}}{G_{k,l}} & \text{else}. \end{cases}
\]

Define \( \mathbf{T} \) as a \( Kd \times Kd \) matrix with entries in (2), where \( a \) and \( b \) in \( \mathbf{T}(a, b) \) denote the row and column indices, respectively [3].

**Theorem 1.** If \( \|\mathbf{T}\|_\infty < 1 \) for some \( \mathbf{v} > 0 \), then interference functions (1) are \( c \)-contractive interference functions with \( c = \|\mathbf{T}\|_\infty \).

**Proof:** The interference functions satisfy the contractivity condition with \( c = \|\mathbf{T}\|_\infty \)
\[
I_{k,l}(p + c\mathbf{v}) = I_{k,l}(p) + \epsilon \sum_{s=1}^{d_k} \sum_{j=1}^{K} T_{k,l}^{j,s} v_j \\
\leq I_{k,l}(p) + \epsilon \|\mathbf{T}\|_\infty v_k.
\]

An easily verifiable but a more conservative choice for \( \mathbf{v} \) can be the \( \mathbf{v} = 1 \) option. In this case, the row sums or the spectral radius of the matrix \( \mathbf{T} \) should be less than 1 [8]. We refer the interested reader to the first author’s website [5] for a more comprehensive treatment on the convergence of power control law.

VI. CONCLUSION

A distributed ad-hoc algorithm that balances weighted substream-SINRs has been developed. The algorithm guarantees feasible SINR targets opportunistically via its ad-hoc and linear search features. Via contractive interference functions, the power control law in the proposed algorithm is proven to linearly converge to a unique fixed-point under appropriate assumptions.

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