Abstract

Just comparing with the scenario that the (3 + 1)-dimensional “real world” of the Calabi-Yau compactification has a tremendous landscape, we conjecture that a (4+1)-dimensional holographic theory may also hold a landscape of its vacua. Unlike the traditional studies of the AdS/CFT phenomenology where the vacua are always constructive, we discuss the proper holographic vacua and their flux compactification, starting from some general compact Einstein manifolds. The proper vacua should be restricted by (i) a consistent worldsheet theory that possesses the superconformal symmetry, (ii) some definite symmetries to keep/break the corresponding symmetries of the dual field theory, (iii) certain brane/flux configurations to cancel anomalies, and (iv) stabilities. We consider diverse fundamental parameters of the dual field theory, fixed by some special vacuum moduli.

In an opposite way, if some field theory such as QCD holds an AdS dual, it may also possess various fundamental parameters by an “landscape” of its vacuum. Different vacua may be adjacent with each other, and divided by domain walls. If the size of a single vacuum region is smaller than the visible universe, it may be testable. We discuss the consequences of this conjecture in the astrophysical environments, include but not limit to: (i) consistency with the critical energy density of the universe, (ii) the behaviors of cosmic rays, (iii) the stability and abundance of deuterons and other nuclei in the big-bang nucleosynthesis and the star burning scenarios, and (iv) the existence of strange/charm stars.

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1 Introduction

The anti-de Sitter/conformal field theory (AdS/CFT) correspondence, one of the most ambitious scenarios in string phenomenology, conjectures that a type IIB superstring theory on $\text{AdS}_5 \times S^5$ is equivalent with a $\mathcal{N} = 4 U(N_c)$ super Yang-Mills (SYM) theory in four-dimensions [1], or more generally a gravity theory on $\text{AdS}_{p+2} \times \mathcal{M}_q$ is dual to a $(p + 1)$-dimensional boundary CFT [2, 3]. The idea of “holography” [4] also pushes the applications of AdS/CFT to more realistic environments, such as QCD [5, 6], or condensed matter systems [7].

Nevertheless, the compact dimensions (thus the “vacua”) and their stabilization in AdS/CFT models, has seldom been studied systematically. On the one hand, theoretical researches always study some specific vacuum by constructive methods. For example, they break boundary supersymmetry by quotient spaces $S^5/\Gamma$ [8, 9], or the conifold construction [10, 11, 12]. On the other hand, phenomenological models which aim to approach the “real world” physics, always neglect the discussions of the compact dimensions directly. However, although difficult, the study of AdS/CFT vacua has no alternative but within the framework of flux compactification [13, 14]. Some founding works in this direction can be found in [15, 16].

The original studies of flux compactification, always aim to the Calabi-Yau threefolds. The main reason is that $\text{CY}_3$, which possesses a special holonomy of $SU(3) \subset SO(6)$, can reduce the ten-dimension critical superstring theory to some four-dimensional effective field theory which possesses $\mathcal{N} = 1$ supersymmetry. One of the properties of this scenario beyond one’s expectation, is the tremendously abundant vacua, which mainly rise from the not-very-small Betti numbers $b_2$ and $b_3$ of $\text{CY}_3$, and the various possible fluxes wrapped on it; this set of vacua is always called a “string landscape” [17]. Different vacua in the landscape hold different fundamental parameters. It was argued that the number of consistent quasirealistic flux vacua may be greater than $10^{500}$ [18], and models has been constructed to solve the cosmological constant problem using this property [19].

In this paper, we conjecture that as an analog, a holographic theory may also hold a landscape of vacua. We verify this hypothesis in two different ways, from top-down and from bottom-up. Along the first root, we discuss properties of the compact manifolds, and the restrictions of them from physical purposes. For the uncompactified dimensions to be AdS, the compact manifold should be Einstein; thus, most of our discussions are within the framework of Einstein manifolds [20]. After then, we consider the possibilities and phenomenologies of various AdS vacua, especially the properties of domain walls separate them. We also discuss the possibilities for a non-conformal boundary field theory to hold a landscape. Along the opposite root, we studied the consequences of our conjecture, if QCD (as a non-conformal boundary field theory) holds a landscape of vacua. In this case, different vacua of QCD should possess different fundamental parameters, such as the quark masses $m_q$, the running coupling constant $\alpha_S$, or the CP violating phase $\theta_{\text{QCD}}$. Another vacuum with parameters different from ours may be testable; and we estimate this possibility within several astrophysical environments.

The organization of this paper is as follows. In Sec. 2 we discuss several mathematical
and physical issues that relate to the Einstein manifolds. We consider the symmetric conditions of the string worldsheet and the dual field theories, and the properties of wrapped branes and fluxes. We also consider the stability conditions of vacua topologies. In Sec. 3 we discuss the theoretical issues to approach a QCD landscape. We consider the possibilities to break CFT, the fundamental parameters a vacuum should determine, and the deduced parameters that may relate to applications/observations. In Sec. 4 we consider how a QCD landscape affects astrophysical observations. The applications are abundant, but the studies in our paper are only tentative. We summarize our results in Sec. 5. Some mathematical supplements relatively independent to the main text are gathered in Appendix A and the validity of the orders of magnitudes estimations used in this paper, are reconsidered more carefully in Appendix B. We gather these materials together, rather than write two separate papers from either the theoretical or the astrophysical aspects, because we think neither one alone is enough to make our conjecture reasonable; however, the two roots can in fact be read separately. We always denote the indices of the extended dimensions by $\mu, \nu$, of the compact dimensions by $m, n$, and of the entire target space by $M, N$. We set $\hbar = k = c = 1$ for simplification throughout this paper.

2 Einstein manifolds and beyond

Unlike the string compactification $\text{Mink}_4 \times \text{CY}_3$, where both the four-dimensional ($- + + +$) spacetime and the compactified manifold are Ricci flat, a holographic theory may enjoy an AdS vacuum (with a dual CFT) or its generalizations (with other possible dual field theories like QCD). For the former case, the metric may be described by a product space $\text{AdS}_5 \times \mathcal{M}_5$ or $\text{AdS}_5 \times \mathcal{M}_6$, where $\mathcal{M}_q$ is some compact solution of the field equation, with its dimension depending on whether the theory is compactified from a ten-dimensional superstring theory or an eleven-dimensional M-theory. Although different ways are possible for choosing $\mathcal{M}_q$, mostly we assume it is an Einstein manifold follows $R_{mn} \propto g_{mn}$ with positive cosmological constant [21]. As a theorem by Myers, positive curvature Einstein spaces are always compact (see e.g., §6.51 of [20]).

Generally, it can be thought that the tremendous landscape of the Calabi-Yau compactification rises from the abundant type of Calabi-Yau threefold and the not-very-small Betti numbers $b_m$ of a typical one. Fluxes are quantized in $m$-cycles (which their number decided by the $b_m$). As moduli (hence the geometry) of the compact dimensions are stabilized by the fluxes, different choices of the quantized condition induce different vacua.

2.1 Einstein manifolds

Hence, to ask whether a holographic theory possesses a landscape of vacua, our questions are as follows: Whether there are abundant positive curvature Einstein 5- or 6-manifolds with different topologies? What properties (such as holonomy groups or isometry groups) do they possess? Can some of them hold not-very-small Betti numbers $b_i$? And after holding this set of manifolds in hand, the follow-up things is to filter them by some additional
physical conditions, such as special topological requirement, or stabilities of the geometry against small fluctuations.

However, we may incapable to give an all-around or up-to-date discussion for the mathematical aspect of these questions. Fortunately, some simple mathematical considerations have already given us some clues and restrictions to these questions.

Most of the time, researches of Einstein manifolds are limited to the homogeneous spaces. They are diffeomorphic to coset spaces $\mathcal{M}_q = G/H$, in which the group $G$ acts transitively on $\mathcal{M}_q$ (hence, it is the subgroup of the full isometry group), and $H$ is the isotropy subgroup of $G$ at a point in $\mathcal{M}_q$. If one is restricted to coset spaces, the complete list of Einstein manifolds is possibly explored. Some of them are discussed by physicists in the Kaluza-Klein supergravity background [21]. For example, the list of positive curvature Einstein 5-manifolds are $S^5$, $SU(3)/SO(3)$, $T^{01} = S^3 \times S^2$, $T^{11}$, and other $T^{pq} = T^{pq}/Z_r$ [22]; and of Einstein 7-manifolds are $T^7$, $S^7$, $J^7$ (squashed $S^7$), $M^{pq}$, $N^{pq}$, $Q^{pq}, S^4 \times S^3, (SU(3)/SO(3)) \times S^2$, $SO(5)/SO(3)_{\text{max}}$, and $V_{5,2}$ [23]. The $\mathcal{M}_6$ cases seem more complicated, and the already done researches are closely related to the compactified mechanisms. For some definite $\mathcal{M}_6$ and their properties, see [24, 25] and references therein.

There are also manifolds which are not coset spaces but we have systematic ways to study; e.g., the product spaces $K^3 \times T^{q-4}$ with holonomy group $SU(2)$, or the Calabi-Yau threefolds. However, these two examples are both Ricci flat. In addition, Calabi-Yau threefolds are preferred in the Mink$_4 \times CY_3$ scenario, because the holonomy group need to be $SU(3)$. We do not possess any analogous restrictions at the very beginning to discuss our cases.

The classifications of the holonomy group $\text{Hol}$ or its restricted analog $\text{Hol}^0$ (for which the loop is contractible) with homomorphism $\pi_1 \to \text{Hol}/\text{Hol}^0$, may be important for the follow-up studies. If $\text{Hol}^0$ is reducible, we have (at least locally) $T(M = M' \times M'') = T'M \oplus T''M$, and $\text{Hol}^0(M) = \text{Hol}^0(M') \cdot \text{Hol}^0(M'')$ as a de Rham decomposition [26, §3.2]. Product spaces like $K^3 \times S^1$ or $K^3 \times T^2$ are in that case. While for the irreducible cases, if the Einstein manifold is symmetric as $G/H$ in the adjoint representation, its holonomy group $\text{Hol}^0$ is just $H$ [26, §3.3]. And if it is non-symmetric, the Berger classification said that $\text{Hol}^0 = SU(3)$, $U(3)$, and $SO(6)$ for $\mathcal{M}_6$, but only $\text{Hol}^0 = SO(5)$ for $\mathcal{M}_5$ [26, §3.4]. $\text{Hol}^0 = U(3)$ gives Kähler threefolds, while $\text{Hol}^0 = SU(3)$ gives Calabi-Yau threefolds in the Mink$_4 \times CY_3$ compactifications. However, if for some reasons, we need the holonomy group of $\mathcal{M}_5$ to be smaller than $SO(5)$, we can directly rule out all the spaces which are not homogeneous.

For the $\mathcal{M}_6$ cases we may, for some reasons, prefer the six-dimensional manifold to be Kähler-Einstein. Then, the first Chern class $c_1$ of it should have a sign. The condition that $c_1$ is larger (smaller) than zero, gives positive (negative) curvature $\mathcal{M}_6$, and $c_1 = 0$ gives Calabi-Yau threefolds. In addition, we have a relation

$$V \cdot s^3 = \frac{(12\pi)^3}{3!} c_1^2,$$

where $s$ is the scalar curvature, and $V$ is the total volume of the compact manifold $\mathcal{M}_6$ [20, §11.5]. However, although a compact complex manifold with $c_1 \leq 0$ always admits a
Kähler-Einstein metric, for \( c_1 > 0 \) that statement is false \([20, \S 11.17]\). Other interesting theorems include that compact, complex manifolds with \( c_1 > 0 \) (they include the positive curvature \( \mathcal{M}_6 \) cases) have no non-trivial holomorphic \( p \)-form \([20, \S 11.24]\), and are simply connected thus that \( \text{Hol} = \text{Hol}^0 \) \([20, \S 11.26]\).

Whether there are really abundant type of positive curvature \( \mathcal{M}_q \)? If including the cases that are not homogeneous, this question is really hard to answer. However, we may have reasons to believe that they are much rarer than the negative curvature ones. While it is easy to find negative curvature Kähler-Einstein manifolds, it is hard to find a positive curvature one; in addition, the known positive curvature ones are always associated with some isometry groups \([20, \S 0.1]\). After normalizing the total volume, the scalar curvature of \( \mathcal{M}_q \) have an upper bound, which is achieved by \( S^q \) \([20, \S 12.61]\). However, this restriction may be looser for \( \mathcal{M}_5 \) than for \( \mathcal{M}_6 \). The reason is in the \( \mathcal{M}_5 \) cases, \( s \) can be arbitrarily close to zero, while in the \( \mathcal{M}_6 \) cases, they cannot.

Some other issues of \( \mathcal{M}_q \) is related to their Einstein structure moduli spaces. It is really interesting that the Einstein structure is rigid (that is, an isolated point of the moduli space) for negative curvature manifolds, but not rigid for the Ricci flat ones \([20, \S 12.73]\). The positive curvature cases, which we are interested in, are much harder to deal with. However, it should be really important to handle the moduli space of \( \mathcal{M}_q \), if we want to discuss their landscape. The special case for the Kähler-Einstein structure, and also the number of moduli, is discussed in \( \S 12.98 \) of \([20]\).

There are some powerful techniques, such as “toric variety” \([27]\), to help us study the topology of (part or all of) the Calabi-Yau threefolds. It is common for a CY3 to possess some not-very-small Betti numbers \( b_2 \) and \( b_3 \). For some special cases, the positive curvature \( \mathcal{M}_5 \) or \( \mathcal{M}_6 \) may be studies by the conifold construction, in which the conifold \( C(\mathcal{M}_q) \) can be studies properly; the Sasaki-Einstein \( \mathcal{M}_5 \)'s are already very abundant. As discussed in Sec. \( 2.2.2 \) we may not limit ourselves to the conifolds, thus we simply assume that some of them (especially the non-symmetric ones) also own these properties. Sometimes to avoid distinguishing the holonomy groups \( \text{Hol} \) and \( \text{Hol}^0 \), we assume \( \mathcal{M}_q \) to be simply connected, hence \( b_1 = 0 \); but it is in fact not needed. The hypothesis does not directly contradict with the mathematical arguments given in this subsection; and it is trusted, as we also loose some additional constraints such as complex structure or Kähler structure. Of course, complex or Kähler restraints rise from some definite physical properties of the traditional compactification, and our questions also have their own physical conditions; however, it seems not very easy to give definite (general) constraints from a physical viewpoint, or at least too early to give up possibilities for scenarios which do not possess such constraints.

### 2.2 Supersymmetric conditions

For our scenarios of the holographic landscape to work consistently, there are several different types of supersymmetric conditions. Some of them need to be held, or need to be held for definite models, but some others need to be broken for required properties. In this section, we discuss them separately.
2.2.1 Restrictions from worldsheet superconformal field theories

For a consistent ten-dimensional superstring theory, the worldsheet field theory should possess the superconformal symmetry. Consider the worldsheet theory as a two-dimensional non-linear \( \sigma \)-model. If the target manifold is Hermitian and Kähler, the worldsheet theory holds a \( \mathcal{N} = 2 \) supersymmetry; if it is hyperkähler, the worldsheet theory holds a \( \mathcal{N} = 4 \) supersymmetry, and vice versa if it is supersymmetric \[28, 29\]. While if it is Ricci flat and Kähler, the one-loop \( \beta \) function of the worldsheet theory is zero, regardless of the worldsheet supersymmetry \[30\]. We neglect the multi-loop correlation of the conformal symmetry in this paper. Absolutely, \( \text{Mink}_4 \times \text{CY}_3 \) satisfies all these requirements. However, the conditions given above are sufficient, but not necessary.

First, a \( \mathcal{N} = 1 \) worldsheet supersymmetry is enough for a consistent superstring theory. In this case, the Kähler condition is superseded by the existence of a tensor field \( J^{M N} \in C^{\infty}(T \mathcal{M}_{10} \otimes T^* \mathcal{M}_{10}) \) which satisfies \( g_{P Q} J^P_M J^Q_N = g_{M N} \) and is covariantly constant \[29\]. Notice that if in addition \( J^P_M J^M_N = -\delta^P_N \), the target manifold is Kähler, but we do not possess such conditions. To keep the worldsheet conformal symmetry, we need only \( g^{MN} \Gamma^P_{MN} = 0 \) beside Ricci flatness \[30\], a weaker condition compared with Kähler.

For our case \( \mathcal{M}_{10} = \text{AdS}_5 \times \mathcal{M}_5 \), \( J^M_N|_p \) of some point \( p \in \mathcal{M}_{10} \) generally represent as a subgroup of \( O(10) \), which is invariant under the action of \( SO(5) \times \text{Hol}^0(M_5) \). If \( \mathcal{M}_5 \) is irreducible and non-symmetric, \( \text{Hol}^0(M_5) = SO(5) \). The most simple case of \( J^M_N \) is \( \delta^M_N \). More details are given in Appendix A.1.

Although the “classical” \( \text{AdS}_5 \times S^5 \) configuration holds Ricci flatness, it does not satisfy the condition \( g^{MN} \Gamma^P_{MN} = 0 \). See Appendix A.2 for the detail calculations.

However, we may not need to take this argument too seriously. The first reason is about the applicability of worldsheet field theory, which is only an effective description in the \( g_s \rightarrow 0 \) limit. In addition, when we study the theory perturbatively (that is where the arguments of conformal symmetry come from), we give up all the heavy degree of freedom in string theory. Even if for the majority of \( \text{AdS}_5 \times \mathcal{M}_5 \), perturbative constrain is not a seriously problem, curvature singularities may exist in the moduli space. If the manifold approaches these singularities by some dynamical reasons, massive states may become massless and ruin the perturbativity \[31\]. The second reason is that the \( \text{AdS}/\text{CFT} \) correspondence in the large-\( N \) limit always possess small curvatures. Ten-dimensional supergravity is suitable in that reason, and configurations such as \( \text{AdS}_5 \times S^5 \) are indeed solutions supergravity equations. As we know seldom about how to do a string theory in some general manifold, a supergravity argument may already be enough.

2.2.2 Conditions of the dual field theories

The isometry group \( SU(4) \simeq SO(6) \) of the prototype \( \text{AdS}_5 \times S^5 \) correspondence \[1\], gives the global R-symmetry of the four-dimensional \( \mathcal{N} = 4 \) \( U(N_c) \) SYM theory. Similarly, the isometry group of some typical \( \mathcal{M}_q \) in \( \text{AdS}_5 \times \mathcal{M}_q \), gives the remained supersymmetry of the dual CFT. \( \mathcal{N} = 2 \times n_{\mathcal{M}_q} \) for type II superstring theory, and \( \mathcal{N} = n_{\mathcal{M}_q} \) for M-theory, where \( n_{\mathcal{M}_q} \) is the number of Killing spinors in \( \mathcal{M}_q \). However, while even dimensional manifolds
possess equal solutions for both orientations, when \( q \) is odd, solutions can exist only for one orientation unless for round \( q \)-spheres \([11]\). Particularly, AdS\(_5 \times M\) break half of their supersymmetry and possess only \( \mathcal{N} = nM \) (that is also true for the AdS\(_5 \times S^5 \) case, because the geometry is only constructed asymptotically).

There are already several constructive models to break \( \mathcal{N} = 4 \) supersymmetry of the boundary CFT; however, most of them still possess at least the \( \mathcal{N} = 1 \) supersymmetry. The most direct construction is for \( S^5/\Gamma \) and their blow-up manifolds \([8]\). \( \mathcal{N} = 2 \) if \( \Gamma \subset SU(2) \), \( \mathcal{N} = 1 \) if \( \Gamma \subset SU(3) \), and the CFT is still chiral if \( \Gamma \) is a complex subgroup of \( SU(4) \) \([9]\).

Another possibility is the conifold construction \([10]\). While M2/M5/D3-brane solutions can be described as the interpolation between Minkowski spaces and AdS\(_p \times S^q \), a general AdS\(_{p+2} \times M\) can be structured by locating large amounts of branes at some singularity of a conifold \( C(\mathcal{M}_q) \), and described as the interpolation between Mink\(_{p+1} \times C(\mathcal{M}_q) \) and AdS\(_{p+2} \times M\) \([11, 12]\). If the singularity is Gorenstein canonical type, \( \mathcal{M}_q \) is Einstein restricts the cone \( C(\mathcal{M}_q) \) to be Ricci flat, and the Killing spinors on \( \mathcal{M}_q \) is in one-to-one correspondence with the covariantly constant spinors on \( C(\mathcal{M}_q) \) \([32]\). If \( \mathcal{N} \geq 1 \) is needed for the dual CFT, \( C(\mathcal{M}_q) \) has to hold some special holonomy. As the Ricci flatness rule out most homogeneous manifolds, the Berger’s classification mentioned in Sec. 2.1 restricts \( C(\mathcal{M}_5) \) to \( \mathbb{R}^6 \), \( K3 \times \mathbb{R}^2 \), and CY\(_3\), and \( C(\mathcal{M}_6) \) to \( \mathbb{R}^7 \), \( K3 \times \mathbb{R}^3 \), CY\(_3 \times \mathbb{R}^1 \), and the Spin(7)-manifold. Especially, the horizon geometry of the CY\(_3\) conifolds are Einstein-Sasaki 5-manifolds, which can be described as some U(1) bundle over the Kähler-Einstein twofold with the Chern class \( c_1 > 0 \) \([33]\).

However, the more general field theories we interest in this paper, such as QCD, do not need to possess any supersymmetry. Hence, they do not need to (and, they are difficult to) be studied constructively. The \( \mathcal{N} = 0 \) supersymmetric condition generally rules out the possibilities to structure the vacua algebraic geometrically while conifold construction; in addition, maybe even the Gorenstein canonical singularities (hence, the Ricci flatness of the cone \( C(\mathcal{M}_q) \)) are not needed to locate the branes. Hence, for some general holographic field theories, the restriction for the isometry group thereafter the metric is relatively loose.

### 2.3 Fluxes, and the stabilization of moduli and topologies

The abundant Calabi-Yau vacua rise from different patterns of fluxes wrapped nontrivially on cycles in CY\(_3\). In Sec. 2.1, we argued the possibilities for Einstein 5 or 6-manifolds with positive curvature to possess not-very-small Betti numbers and various cycles. Here, we study the question, that if that is true, whether a landscape of the holographic vacua is possible or not.

#### 2.3.1 Some comparisons about flux compactification

There are several differences/comparisons between the Mink\(_4 \times CY_3\) and the AdS\(_{p+2} \times S^q\) compactifications. The most direct one is that, while the AdS\(_{p+2} \times S^q\) ones are the Freund-Rubin type \([34]\), the Mink\(_4 \times CY_3\) ones are not. We may describe the Freund-Rubin mechanism in a more modern way. By the de Rham’s theorem, given any set of integers
There exists a closed $i$-form $\omega$ which satisfies $\int_{c_n} \omega = \nu_n$, where $c_n$ are the corresponding $m$-cycles. A general orientable compact manifold $S^q$ or $M_q$ has $b_q = 1$. For the Freund-Rubin compactification of $\text{AdS}_5 \times S^5$, $\omega$ is just the dual Ramond-Ramond (RR)-field strength of the D3-branes, and we have $\int_{S^5} *F_5 = N_c$, with the number of colors $N_c$ of the gauge group of the dual $U(N_c)$ SYM theory. The radial stability of the $\text{AdS}_5 \times S^5$ configuration is discussed in e.g. [33], or in more detail in Sec. 2.3.3 with the same solution as the one calculated from the black $p$-brane supergravity [36]. The relation between the RR-charge $N_c$ and the radius of $\text{AdS}_5$ is $R \propto N_c^{1/4}$. In the Calabi-Yau cases, fluxes are compactified in the 2 and 3-cycles of $CY_3$, which their sources D4/6 (in the IIA case), D5 (in the IIB case), or NS5-branes looking as (2+1)-dimensional domain walls in $\text{Mink}_4$ [37].

The second difference is as follows. While the flux quantization in $\text{AdS}_p \times S^q$ is directly related to the gauge group of the low-dimensional theories, the branes to construct the Standard Model in $\text{Mink}_4 \times CY_3$ seem irrelevant to the ones induce compactification. That makes the quanta chosen to fix the vacua really optional for the latter case [19]. The branes to realize the gauge symmetry in $\text{Mink}_4 \times CY_3$ has to be space filling. As their fluxes have nowhere to go, their numbers (the differences between branes and anti-branes) are highly constrained by anomalies. However, since the “real world” realized in the $\text{AdS}_{p+2} \times M_q$ configurations is some holographic one, flux can goes along the radial direction of the $\text{AdS}$ space.

The third difference is about the existence of maximum flux quanta, or the finiteness of the absolute number of flux vacua. While the vacua in $\text{Mink}_4 \times CY_3$ compactification are argued to be generally finite [18, 38], the RR-charge $N_c$ of $\text{AdS}_5 \times S^5$ can be any integers; hence, the number of the $\text{AdS}_5 \times S^5$ vacua is infinite. The reason can be understood as below. The finiteness of vacua in $\text{Mink}_4 \times CY_3$, restricts from the tadpole cancelation of the gravitational Cher-Simons corrections, rises from some global properties of the Calabi-Yau manifold. Details for type-IIB constraints are constructed, within the language of F-theory, as

$$N_{D3} + \frac{1}{(2\pi)^4\alpha'^2} \int H_3 \wedge F_3 = \frac{\chi(X_4)}{24},$$

where $N_{D3} \geq 0$ is the number of space filling D3-branes, and $\chi(X_4)$ is the Euler characteristic of the corresponding Calabi-Yau fourfold [39]. However, for the $\text{AdS}_{p+2} \times M_q$ cases which can be described by the horizon geometry of some conifolds, the restrictions should be completely local; they do not exist [12].

Fourth, the no-go theorem [40], thus the requirement of orientifold planes in $\text{Mink}_4 \times CY_3$ compactification, need to be reconsidered in the $\text{AdS}_{p+2} \times M_q$ cases. Within some definite assumptions, this theorem ensures that orientifold planes are needed in $CY_3$ to get flat or de-Sitter (dS) vacua, if nontrivial background pattern of fluxes exist. However, since we are mainly interested in $\text{AdS}$ vacuum, orientifold constructions are not as important as in the string compactification cases.
2.3.2 Various AdS vacua?

We argue some landscape of the holographic vacua by the following reason. Superstring theory has no free parameters. If some more “realistic” field theory indeed has its (exact) gravity dual, all its parameters should be fixed by the moduli of its vacua. AdS$_5 \times S^5$, or nearly all constructive vacua discussed in Sec. 2.2.2, seem too simple to accommodate so many parameters.

It is absolutely true that $q$-form flux cycled on AdS$_5 \times \mathcal{M}_q$, say, the pure Freund-Rubin compactification, is not enough. As $b_q = 1$ for oriented $\mathcal{M}_q$ and 0 for the non-oriented ones, there is only one free quantum to adjust. Within no a priori restrictions about the Betti numbers of $\mathcal{M}_q$, we may have $F_2$, $F_4$, and $H_3$ fluxes cycle on $\mathcal{M}_5$ in type-IIA theory, $F_1$, $F_3$, $F_5$, and $H_3$ fluxes cycle on $\mathcal{M}_5$ in type-IIB theory, and $F_4$ flux cycles on $\mathcal{M}_6$ in M-theory. As $b_n = b_{q-n}$ for any $\mathcal{M}_q$, the branes taken corresponding RR or Neveu-Schwarz-Neveu-Schwarz (NSNS)-charges, should always fill in the Poincaré dual cycles on $\mathcal{M}_q$, and some (3 + 1)-dimensional domain walls on AdS$_5$. Branes as (2 + 1)-dimensional domain walls on Mink$_4$ locating in definite radius of AdS$_5$ are also possible; however, the flux configurations are more complicated. To avoid the flux violating the Lorentz invariance of the dual field theory on Mink$_4$, the domain walls should not possess less dimensions. Similar brane configurations for superstring theory or M-theory, are described in [37, 41]. D5-branes wrapped on two-spheres of AdS vacua, has ever been discussed for the purpose of broken conformal symmetry (see Sec. 3.1 or the review article [42] and references therein). The domain walls themselves indeed violate the Lorentz symmetry; however, they can be sat at infinity if needed. The additional requirements may also include the supersymmetric condition of $\mathcal{M}_q$ cycles; D-brane instanton or the D-brane spatial components wrapped on them should possess a supersymmetric worldvolume. The corresponding Calabi-Yau cycles are considered in [43], in which “twist” is needed. However, we cease for more in-depth discussions for our case in this work, and leave the relevant issues to the follow-up studies.

While it is interesting to study flux compactification of some holographic theory, it is quite difficult to go along a constructive way, because central charge the dual CFT depends on the fluxes rather complicated. The first exploration is a type-IIA compactification on $T^6$ orientifold for AdS$_4$ vacua [15]. The follow-up works such as [16] are also relevant.

In [15], D4-branes carries RR-charges wrap some 2-cycles on the compact $T^6$ orientifold, and fill their (2 + 1) other dimensions at some fixed radial position of AdS$_4$. However, other configurations of D4-branes are also possible; for example, the radial AdS$_3$ inside of AdS$_4$. We will ignore the slant configurations in our discussions; they seem strange when considering the UV-IR correspondence [4], as their positions change while adjusting the energy scale of the dual CFT.

In fact, the latter configuration (in which the branes look as domain walls in the flat boundary theory) may be more natural. In our AdS$_5 \times \mathcal{M}_q$ cases, they are (2 + 1)-dimensional domain walls within Mink$_4$, or (3 + 1)-dimensional domain walls filling in the radial AdS$_3$ inside AdS$_5$. They can be shown from conifold construction; the analogous M-theory case is described in [41]. Let us set D7-branes on F-theory background Mink$_4 \times C(\mathcal{M}_7)$; D7-branes look as (2 + 1)-dimensional domain walls on Mink$_4$, and wrap 5-cycles
on 8-conifold $C(\mathcal{M}_7)$. The dual fluxes of D7-branes wrap 3-cycles on $C(\mathcal{M}_7)$. While locating a very large number of D3-branes on $C(\mathcal{M}_7)$ singularities, the spacetime deforms to $\text{AdS}_5 \times \mathcal{M}_7$, as argued in Sec. 2.2.2. After then, we can compact two-dimensions of $\mathcal{M}_7$, to get a type-IIB superstring theory in ten-dimensions. As the Mink$_4$ and the brane within it remain unchanged in this compactification, branes always look as domain walls in the dual CFT.

Hence, we conjecture that for some more “realistic” boundary theory, the dual string theory on $\text{AdS}_5 \times \mathcal{M}_q$ possesses other wrapped fluxes beside the volume form cycled on $\mathcal{M}_q$; the sources of these fluxes fix as domain walls in the holographic boundary. While the volume form cycled on the $q$-cycle directly decides the gauge group, different choices of these other quanta induce gauge theory with different fundamental parameters.

Are these choices finite or not? In Sec. 2.3.1 we discussed the finiteness of the permutation of quanta (hence the number of vacua) on Mink$_4 \times \text{CY}_3$, and the infiniteness of possible RR-charges on $\text{AdS}_5 \times S^5$. In addition, for some loose supersymmetric conditions given in Sec. 2.2.2, unless the CY$_3$ cases in which at least the toric description of topology is finite, the number of $\mathcal{M}_q$ with different topology may even diverge. Here, we assume the topology is fixed by physical reasons, and focus on the flux configuration. We guess that the choices for quanta deciding the fundamental parameters of a holographic theory, is finite if the corresponding branes display as domain walls filling radial AdS$_4$, but infinite if they are restricted in Mink$_4$. We leave the strict proof (if exists) to the follow-up studies, and give an argument as below; the proof is absolutely complicated, as the geometry is difficult to handle.

The tadpole cancelation, hence, the finiteness described in Sec. 2.3.1 can be understand by a finite energy condition [41]. The energy density, which gets contribution from both flux and space filling branes (which fill all uncompact dimensions), should be equal far away on the two sides of the domain wall. That is true for flat uncompact dimensions, in the case we show in Eq. 2; flux configuration remain unchanged while going far away from the source. That is also true for radial AdS$_4$ within AdS$_5$, but not true for the D3-branes laying on Mink$_4$ deep inside the throat of AdS$_5$; in the latter case, the flux dilutes on the boundary. Things are the same for the $(2 + 1)$-filling branes within Mink$_4$; flux dilutes on some directions. As these radial AdS$_4$ and the $(2 + 1)$-dimensional domain wall brane configurations should come together from conifold construction, we argue the holographic vacua should be infinite.

2.3.3 Stabilities

Generally, a AdS$_{p+2}$ vacua is stable, if it satisfies the Breitenlohner-Freedman bound [41]

\[ m^2 L^2 \geq -\frac{(p + 1)^2}{4}, \]

where $m$ is the scalar mass of the tachyon mode, and $L$ is the radius of the AdS space with the Ricci tensor $R_{\mu\nu} = -(p + 1)g_{\mu\nu}/L^2$. It may be difficult to discuss the stability of the vacua of some general compact manifold $\mathcal{M}_q$; the mostly discussed situations are
aimed. For the overall Ricci flat $\text{AdS}_{p+2} \times \mathcal{M}_q$, $\mathcal{M}_q = S^q$ is always stable, and the Einstein $\mathcal{M}_q = \mathcal{M}_n \times \mathcal{M}_{q-n}$ for $q < 9$ is always unstable by metric perturbations \cite{45, 46, 47}. For the free orbifold action $\text{AdS}_5 \times S^5/\mathbb{Z}_k$ with odd $k$ discussed in \cite{8}, while $k = 3$ case possesses the $\mathcal{N} = 1$ supersymmetry, $k \geq 5$ break all supersymmetry. The latter cases are unstable \cite{48}. In addition, Ref. \cite{25} gives the stable condition for 4-form flux compactified on $\text{AdS}_5$ cross some $\mathcal{N} = 0$ positive Kähler-Einstein $\mathcal{M}_6$; the discussion focuses on homogeneous spaces.

On the whole, supersymmetric conditions can help stabilize the vacua, as they give some additional restrictions \cite{45}; however, they are not absolutely needed. Maybe the discussions of the $\mathcal{N} = 0$ holographic solutions in Sec. 2.2.2 are dangerous. Or maybe a better way to construct a more “realistic” holographic theory, is to start with AdS vacua with no less than $\mathcal{N} = 1$ supersymmetry, and then break supersymmetry by some other reasons; as discussed in Sec. 2.2.2, algebraic geometrical tools (such as the properties of Einstein-Sasaki 5-manifolds) can be used in that case. It is similar to the idea of Calabi-Yau phenomenology, where some $\mathcal{N} = 1$ vacua induce the broken “real world”. Nonetheless, the relevant issues are absolutely difficult, as even the stabilization Calabi-Yau vacua is not easy to handle \cite{49}.

### 3 Holographic phenomenologies

After the more aimed issues related to Einstein manifolds considered in Sec. 2, we discussed the more phenomenological aspect of holographic vacua and their landscape, here in this section.

#### 3.1 Beyond AdS or beyond AdS$_5$

String theory in $\text{AdS}_5 \times \mathcal{M}_5$ should be the dual theory of some CFTs; the isometry group of $\text{AdS}_5$, $SO(4, 2)$, is just isomorphic to the conformal algebra of the boundary theory. Hence, to achieve the gravity dual of some more realistic field theory which is not conformal, we need the non-compact dimensions deformed from $\text{AdS}_5$.

There are several different ways already discussed, to break conformal invariance, include: (i) adding a mass deformation to a CFT, (ii) using wrapped branes – located on non-vanishing cycles of $\mathcal{M}_q$, or (iii) fractional branes – wrapped on collapsed cycles, and (iv) considering theories at finite temperature. Generally, additional branes change the blackbrane supergravity solutions, and finite temperature theories give AdS blackholes rather than AdS spaces. The first three approaches are reviewed in \cite{42}.

We have already induced wrapped branes in Sec. 2.3.2, for the properties of various flux compactification patterns. We may generally prefer these wrapped branes and fluxes induced by them to stabilize the moduli of $\mathcal{M}_q$, but not alter its geometry (such as its holonomy group). Maybe a redefinition of the covariant derivative $D_\mu = \partial_\mu + \omega_\mu \rightarrow \partial_\mu + \omega_\mu + A_\mu$ is needed, where $\omega_\mu$ is the spin connection, and $A_\mu$ is the external gauge fields given by the wrapped branes; the operation is described in \cite{42}. This scenario seems more reasonable in the large-$N_c$ limit, where the D3-branes Freund-Rubin compactification dominate the
geometry. However, things become specious for some more “realistic” theories, such as QCD which possesses $N_c = 3$. It is difficult to believe the three D3-branes lead the near-AdS product geometry, while other branes perturb the exact values of moduli. This may be a general problem when construction QCD from Large-$N_c$ QCD, which is not easy to resolve. Similar problem rises from adding flavor branes, which can only be down in a false assumption – the probe limit (exact quenched approximation) $N_f \ll N_c$. For the reason above, we leave this problem to the follow-up studies.

How should $\mathcal{M}_5$ changes while deforming AdS$_5$? If requirements, such as the Ricci flatness discussed in Sec. 2.2.1 are needed, they may be related to each other. Generally, deformations of AdS$_5$ may ruin the product structure, thus make the definition of a “vacuum” ambiguous; however, in the minimum models, AdS$_5$ and $\mathcal{M}_5$ may simply be independent with each other. For example, the temperature of a dual field theory is always described by the black hole temperature. Take the near horizon geometry of a black 3-brane solution [36] (in this case, the Einstein frame and the string frame are same with each other)

$$\begin{align*}
    ds^2 &= H^{-1/2}(-f dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + H^{1/2}[f^{-1} dr^2 + r^2 d\Omega_5^2(\theta_1, \ldots \theta_5)], \\
    &= H^{-1/2}(-f dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + H^{1/2}[f^{-1} dr^2 + r^2 d\Omega_5^2(\theta_1, \ldots \theta_5)],
\end{align*}
$$

(4)

where $H = 1 + (R/r)^4$ and $f = 1 - (r_0/r)^4$, we have the uncompactified dimensions an AdS blackhole solution

$$\begin{align*}
    r^2 R^2 (-f dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + R^2 f^{-1} dr^2,
\end{align*}
$$

(5)

with its Ricci scalar $20/R^2$, just like the extreme AdS$_5$ case

$$\begin{align*}
    r^2 R^2 (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + R^2 f^{-1} dr^2.
\end{align*}
$$

(6)

Notice that the phase transition between Eq.(5) and (6), is just the hard-wall description of the confinement-deconfinement phase transition [50, 51], we argue that a (Hawking-Page type) phase transition, though maybe changes the topology of the background spacetime, is possible to be irrelevant with the compactified dimensions.

### 3.2 Fundamental parameters

The fundamental parameters of QCD may include: the quark masses $m_q (q = u, d, c, s, t, d)$, the coupling constant $\alpha_s$, and the phase $\theta_{QCD}$. If the QCD vacuum corresponds to a dual gravity theory, all these parameters should be decided by the moduli of the compact dimensions. To discuss quark masses, one may prefer to include the Higgs mechanism to the holographic dual; we neglect the details for these considerations, and simply admit most Standard Model results if needed. As QCD are always related to other Standard Model sectors, by the Cabibbo-Kobayashi-Maskawa matrix, or other scenarios relate the strong, weak, and electromagnetic interactions, one may prefer for example the $SU(5)$ grand unification theory rather than the QCD itself, corresponds to some dual gravity
theory. In this case, QCD may arise from some spontaneous symmetry breaking processes dual to tachyon condensation \[52\]. One may also prefer to discuss the landscape of some other realistic systems, such as superfluidity and superconductivity \[7\]. We neglect all these possibilities, and discuss only the landscape of QCD itself here in this paper.

As the coupling constant $\alpha_s$ is running, it is a little difficult to consider its relationship with the moduli. In the perturbative region, one always prefers to describe $\alpha_s(E)$ by the formula

$$\alpha_s(E) = -\frac{2\pi}{(-11 + 2n_f/3) \ln(E/\Lambda_{QCD})},$$

where $n_f = 6$ is the number of flavors, and treat $\Lambda_{QCD}$ as a “fundamental” parameter. It seems strange to generalize $\Lambda_{QCD}$ to the non-perturbative regions. Running coupling constants may be understood by the UV-IR correspondence \[4\] in the dual gravity theory; however, quantitative considerations are still difficult.

D-brane physics relates $g_{YM}$ and the phase $\theta$ to the dilaton-axion $\tau$ by

$$\tau = \frac{4\pi i}{g_{YM}^2} + \frac{\theta}{2\pi} = \frac{i}{g_s} + \frac{\chi}{2\pi}.$$

More specifically, the conformal coupling constant of $\text{AdS}_5 \times S^5$ has its relationship with the geometry $g_{YM}^2 = 4\pi g_s = R^4/\alpha'^2 N_c$, where $R$ is the radius of $\text{AdS}_5$, and $N_c$ is the number of D3-branes. We may conjecture in the QCD case that while the dilaton-axion $\tau$ decides $\alpha_s$ and $\theta_{QCD}$, other moduli decide parameters such as quark masses. It seems that the dilaton-axion should not be constant in the radial direction for a running $\alpha_s$. We also conjecture that the moduli is decided by the minimum of some potential $V$ (maybe resemble the one in terms of the D-terms and the superpotential, used widely in the Calabi-Yau compactification), and $V_{\text{min}}$ is to some extent related to the cosmological constant term hence the radius $R$ of the AdS space. We may further assume that the relation $4\pi \alpha_s = R^4/\alpha'^2 N_c$ can be generalized (at least in some definite energy scale/AdS radius) in the $\text{AdS}_5 \times \mathcal{M}_q$, and also the non-AdS cases discussed in Sec. 3.1.

What is the behavior of the domain walls considered in Sec. 2.3.2? The fundamental parameters possess different values in different sides of the domain wall, as the moduli do. The potential $V_{\text{min}}$ should also be different in each side. Naïvely, one may thought the domain walls are infinitely thin, as they are D-branes with codimension one. However, it is not possible because we deal with quantum geometry rather than the classical one \[27\], in which distances and topologies become meaningless in small scales. One may turn to think that the domain walls have thickness of the string scale $l_s$, or the five-dimensional Planck length $l_p^5 = l_p^{q+3}/\text{volume}(\mathcal{M}_q)$, where $l_p$ is the Planck length in ten or eleven dimensions. As $l_p = g_s^{1/4} \alpha'^{1/2} = g_s^{1/4} l_s$, we have $l_p^3 = (g_{YM}^4/16\pi^2) \cdot l_s^3/\text{volume}(\mathcal{M}_q)$, and $\text{volume}(\mathcal{M}_q) \sim R^4$. In addition, $l_p^3 = 15 R^3/128\pi^4 N_c^2$ especially in the $\text{AdS}_5 \times S^5$ case. An alternative consideration of the domain wall thickness is given in Appendix B.1.

The metastable vacuum of larger potential should transform/tunnel to the more stable one. Unlike the bubble nucleation cases, there is no barrier for this phase transition, and the velocity of the interface should finally tend to the speed of light $c$ \[53\]. However, if the
potential disparity is really tiny, but the domain wall is cumbersome, this limit may be difficult to achieve. To roughly estimate the motion of the domain wall, we assume that it is at rest in the beginning, and ask its velocity \( v \) after time \( t \).

All energy rises from the difference of potential \( \Delta V_{\min} \)'s, should transform to the kinetic energy of the domain wall. As the increase of energy is proportional to the sweeping distance \( s \), the domain wall possesses a constant acceleration and \( s = vt/2 \). By assuming that the domain wall mass density \( \mu_{\text{brane}} \) is the same order of the tension of the branes, in the Newtonian limit, energy conservation gives

\[
\Delta V_{\min} \cdot s = \frac{v^2}{2} \mathcal{T}_3,
\]

for some \( D_p \)-brane moving in \((p+1)\)-dimensional spacetime, where the brane tension \( T_p \) in string frame and \( T_p = g_s^{(3-p)/4} T_p \) in Einstein frame. For branes wrapped on \( m \)-cycles of compact dimensions, we estimate the effective tension \( T_{p-m} \sim T_p \cdot \text{volume}(m\text{-cycle}) \sim T_p R^m \). The validity of the equivalence between \( \mu_{\text{brane}} \) and \( T_3 \), is discussed in Appendix 2.3.2.

For branes filling the AdS_4 within AdS_5 discussed in Sec. 2.3.2, we have

\[
\Delta V_{\min} \cdot \frac{vt}{2} = \frac{v^2}{2} \mathcal{T}_3 \sim \frac{v^2}{2} \frac{(4\pi)^{m/4+1} N_c^{m/4}}{g_{\text{YM}}^2 l_s^4} \Delta V \cdot t.
\]

Hence

\[
v \sim \frac{(2\pi)^{m+3}}{(4\pi)^{m/4+1}} \frac{g_{\text{YM}}^2 l_s^4}{N_c^{m/4}} \Delta V \cdot t.
\]

How can we estimate \( V_{\min} \) and \( \Delta V_{\min} \)? Absolutely, \( V_{\min} \) has a dominate contribution, rises from the negative curvature property of AdS_5. If \( \Delta V_{\min} \) is also in this order of magnitude, we have \( \Delta V_{\min} \sim 1/R_5 = 1/N_c^{5/4} g_{\text{YM}}^{5/2} l_s^5 \) and \( v \sim (2^{m/2+1} \pi^{m/4+2} / N_c^{m+5/4} g_{\text{YM}}^{1/2} t/l_s) \). Chosen \( t \sim 10^{10} \) yr as the age of the Universe, and \( l_s = \alpha'^{1/2} \sim 1 \) fm as the typical size of a hadron, we have \( t/l_s \sim 10^{14} \) and \( v \gg 1 \). Here, \( l_s \) is chosen instinctively from the Nambu string, or more accurately by the Regge slope; \( \alpha' \sim (1 \text{ GeV})^{-2} \) and \( l_s \sim 0.3 \) fm. The reasonability of the estimations \( l_s \) and \( t/l_s \), are discussed in more detail in Appendix B.1. Hence, the Newtonian approximation break down, and domain walls should move at the speed of light. However, it is possible that this contribution of \( V_{\min} \) cancels for different vacua, and \( \Delta V_{\min} \) rises from other corrections.

One contribution is the intrinsic energy density of the vacuum, \( \rho_{\text{vac}} \). For the scenario of zero-point energy fluctuation cutting off at Planck scale, we have \( \rho_{\text{vac}} = \eta/l_p^4 \) in four dimensions, and \( \eta/I_p^4 \) in five dimensions, where \( I_p \) is the effective lower dimensional Planck length separately. To solve the cosmological constant problem, one need \( \eta = 10^{-120} \) in four dimensions. Assuming \( \Delta V_{\min} \) has the same order of magnitude of \( \rho_{\text{vac}} \), we have

\[
v \sim \frac{2^{m/2+23/3} \pi^{3m/4+16/3}}{N_c^{m/4-25/12} g_{\text{YM}}^{1/2} \eta / I_p},
\]

for some \( D_p \)-brane moving in \((p+1)\)-dimensional spacetime.
in the AdS$_5 \times M_5$ case. For example, for branes wrapped on 3-cycles, $m = 3$ and $v \sim (3.38 \times 10^6 N_c^{1/3} / g_{YM}^{1/2}) (\eta t/l_s)$. As $t/l_s \sim 10^{41}$, if $\eta \ll 10^{-47}$, one have $v \ll 1$ and the domain wall is non-relativistic. It seems possible when comparing with the cosmological constant case, $\eta \sim 10^{-120}$. However, for the statistical explanations of tiny $\eta$ [19, 18], most other vacua possess $\eta \sim 1$, and a typical $\Delta V_{\text{min}}$ is not such small. Deeper reasoning is needed for the more detailed estimation of $\eta$ in the holographic cases.

Therefore, the domain wall filling the radial AdS$_4$ within AdS$_5$ can be either relativistic or non-relativistic, depends on the magnitude of $\Delta V$; we hold definite reasons to rule out neither cases. We skip to consider the other kind of domain walls discussed in Sec. 2.3.2 such as the $(2 + 1)$-filling branes within Mink$_4$; the observational effects and the dynamical properties seem impalpable in the boundary description.

There is something else to declare, for the non-relativistic branes discussed above. What should happen if a relativistic particle (for example, a proton or a neutron) goes across the domain wall? As discussed above, the fundamental parameters are different in the two sides; hence, the properties (such as mass or the charge radius) of the particles should also change. We hypothesize that the energy of the particles stays the same while passing through the domain wall.

### 3.3 Nuclear properties affected by various $\alpha_s$ and $m_q$

A variation of the QCD coupling constant $\alpha_s$ or quark masses $m_q$, causes several consequences. Some of them are list as below.

Masses of hadrons and nuclei alter while varying the fundamental parameters. In the chiral limit where quark and pion masses are simply neglected, only the change of $\alpha_s$ (or $\Lambda_{\text{QCD}}$) plays a role; however, chiral assumption is not a good assumption for our purpose. The relationship between mass of pion meson and fundamental parameters, may be estimated by the Gell-Mann-Oakes-Renner relation [57]. Roughly we have

$$m_{\pi}^2 = \frac{m_u + m_d}{f_\pi^2} \langle 0|q\bar{q}|0 \rangle \sim (m_u + m_d)\Lambda_{\text{QCD}},$$

(14)

as the geometric mean between weak and strong scales; the coupling of the axial current to pion $f_\pi \sim \Lambda_{\text{QCD}}$, and $\langle 0|q\bar{q}|0 \rangle \sim \Lambda_{\text{QCD}}^3$ [58, 59]. Masses of protons and neutrons can be alters while strange quark mass $m_s$ changes, as the strange sea may contribute 1/5 of the nucleon mass; however, the dependence of $u$ and $d$ quark masses are weaker [58, 60]. The strange quark on-shell mass $m_s = 95 \pm 25 \text{ MeV}$ is absolutely cannot be neglected.

As the nucleon masses alter, the proton-neutron mass difference and neutron lifetime also changes. $m_n - m_p$ can be approximately estimated by

$$m_n - m_p = m_d - m_u - \xi \alpha \Lambda_{\text{QCD}},$$

(15)

where $\alpha$ is the fine structure constant, and $\xi$ is a free parameter which satisfies $\xi \alpha \Lambda_{\text{QCD}} = 0.76 \text{ MeV}$ at present [61, 59]. The neutron lifetime $\tau_n$ depend on this difference by

$$\frac{1}{\tau_n} = \frac{1}{60} \left( 1 + 3 g_2^2 G_F^2 m_e^5 \left[ \sqrt{q^2 - 1} (2q^4 - 9q^2 - 8) + 15 \ln \left( q + \sqrt{q^2 - 1} \right) \right] \right),$$

(16)
where \( q = (m_n - m_p)/m_e \), and \( G_F \) is the Fermi constant \([60]\). It may happen in some cases, where neutron is in fact stable.

A variation of the neutron lifetime \( \tau_n \), is related to a variation of the \( n \rightarrow p + e^- + \bar{\nu}_e \) reaction rate. Similarly, the reaction rates of \( p + n \rightarrow ^2D + \gamma \) and \( ^2D + ^2D \rightarrow ^3T + p \) are also changed \([59]\).

In addition, the stabilities of light nuclei alter. Intuitively deuterons tend to unbind while \( \alpha_S \) (thus also \( \Lambda_{QCD} \)) decreases; dineutrons and diprotons tend to be stable while \( \alpha_S \) increases \([62]\). However, for a quantitative estimation, the critical parameter to control the nuclear binding energies dominated by pion exchange is \( c \equiv \sqrt{(m_u + m_d)/\Lambda_{QCD}} \) \([63]\). If for some definite \( c \), the binding energy \( E_B < 0 \), the correspondent nucleus is unstable. Deuterons becomes unstable if \( c \) decreases by a factor of 0.77. Deneutrons become stable if \( c \) increases by 2.6, while deprotons becomes stable if it increases by 3.2. Nevertheless, a first principle estimation of \( E_B \) is still lacking. By assuming a constant \( \Lambda_{QCD} \), the variation \( \delta E_B/E_B \) may depend on \( \sigma, \omega \)-mesons and nucleon mass changes separately by contributions proportional to \( \delta m_h/m_h (h = \sigma, \omega, N) \) \([64]\).

The stabilities of high-Z nuclei are also relevant to \( \alpha_S \) \([65]\). By precondition the liquid drop model, the stabilized condition is

\[
\frac{Z^2}{A} < \frac{4\pi r_0^3}{3e^2 T},
\]

where \( A \) is the atomic number, \( Ze \) is the charge, \( r_0 \sim 10^{-13} \text{cm} \) is the nuclear radius, and \( T \) is the surface tension of the nucleus; in a first approximation we may assume \( T \propto g_{YM}^2 = 4\pi\alpha_S \). Hence unstable nuclei become stable while \( \alpha_S \) increases, and stable nuclei become unstable while \( \alpha_S \) decreases.

Others also argue that the variation of \( \alpha_S \) is related to the single-particle resonance shift \( \Delta E_0 \) by \( \Delta E_r/V_0 \simeq \Delta \alpha_S/\alpha_S \), where \( V_0 \sim 50 \text{MeV} \) denotes the depth of the nuclear potential well \([66]\). Or the proton gyromagnetic ratios \( g_p \) depends on fundamental parameters by

\[
g_p = g_p(m_q = 0) \left( 1 + \sum_q \zeta_q \frac{m_q}{\Lambda_{QCD}} \right),
\]

where \( \zeta_q \) is free parameters to denote that this equation is only a linear approximation \([58]\).

4 Astrophysical applications

The AdS/CFT phenomenology in the astrophysical context is only at its infancy. The influences of AdS/CFT to the cosmological QCD phase transition are discussed in \([67]\); and a relation between the strange quark stars and the Kovtun-Son-Starinets bound, a direct result from the finite temperature AdS/CFT, is discussed in \([68]\).

In Sec. 2 and 3, we are engaged in a top-down scenario to discuss whether a holographic theory can possess a landscape of vacua. That scenario, though exciting, is at most a conjecture with a huge number of logical and technical uncertainties. In this section,
we try to give a bottom-up argument of how can a holographic field theory (especially QCD) with divergent landscape affects our real world. Does it have some observable applications? Can it solve definite experimental/observational problems? Different sorts of constraints of fundamental parameters are reviewed in [69]. Astrophysical environments have their own advantages for these questions, as they possess large spatial scales, which may include domain walls; most terrestrial experiments can only constrain the variation of the fundamental constants within some definite timescale. Notwithstanding, maybe a better background to discuss this problem, is within the areas of nuclear/RHIC physics [6], or condensed matter systems [7], in which the AdS/CFT phenomenologies are studies more deeply. Because of the professional background of the authors, we limit our discussions in the context of astrophysics, and leave the relevant issues list above to the follow-up studies. Unlike the “multiverse” discussions caused by Calabi-Yau-kind landscape, which mainly focus on various gravity-related parameters such as the cosmological constant [70, 71], the applications of the AdS/CFT (or simply QCD) landscape seem more abundant. However, we flung off here only some superficial arguments considered within few possible environments. Hopefully, more all-around and deep-inside discussions will come soon.

4.1 What can we predict?

Most former constraints of fundamental constants, limit on their variations within some definite timescale. Moreover, mostly, authors assume that they vary smoothly. For our purpose, the minima of vacua expectation value are fixed, for some definite topology of the extra dimensions, and quantum numbers of fluxes. Thus, the expected values of the fundamental parameters are also explicit. By admitting these preconditions, several phenomenological consequences are possible:

Firstly, smoothly varied constants while time elapses are still possible. Although the minima are definite, vacua may only tend to it, and that may be a long-term process. This idea applies widely in string inflationary models, and should also be practicable in late universe. The special condition that all parameters are dependent on a single dilaton field, and the correspondent constraint from big-bang nucleosynthesis (BBN), are discussed in [72, 73]. However, as mostly for this case, the discussions are similar to what given in [69] and references therein, we neglect to discuss their consequences here.

Secondly, even if all vacua are in their minima, fundamental constants in local universe can also change within some timescale. They change discontinuously. That happens, if a domain wall with its dynamical properties discussed in Sec. 3.2 sweeps us in some definite ancient epoch. The domain wall can be either relativistic, or non-relativistic. To clarify, we roughly distinguish two kind of restraints: (i) The ones focus on local changes of parameters, such as BBN predictions, stabilities of nuclei, or some other terrestrial experiments. (ii) The ones focus on far away objects such as pulsars or quasars, and the conformability of their observational properties with models. In the latter case, time elapsing is reconstructed by the long conveyance of photons. Formerly, both kinds of constraints preconceive a spatial-independent but time-dependent variation. In our situation, non-relativistic domain walls are suitable for both restraints; however, relativistic ones cannot be restrained
by the second kind of scenarios. In that case, the other side of the domain wall is always out of our observable universe. In addition, non-relativistic domain walls are in some sense difficult to understand. According to the discussions of Sec. 3.2, the controlled parameter $\eta$ need to be fine-tuned, to avoid the velocity to be too small; in this case, the relevant domain walls seem too nearby.

Thirdly, the fundamental parameters may take different values in different part of the universe, and the (non-relativistic) domain walls separate them. Few former constraints focus on this possibility; local restraints are mostly invalid, as the domain walls are nearly stationary. The consistent conditions needed, and also the consequences of this possibility, is the major point we discuss in this section.

4.2 A consistent condition

Generally, domain walls are precluded in the universe, because their total masses easily dominate over the matter and radiation densities. Notice that the energy density is proportional to $a(t)^0, a(t)^{-1}, a(t)^{-2}, a(t)^{-3}, \text{ and } a(t)^{-4}$ for the cosmological constant, domain walls, cosmic strings, matter (non-relativistic point particles), and radiation, where $a(t)$ is the scale factor, domain walls easily dominate when $a(t)$ becomes large.

For our purpose, we need the energy density of the domain walls to be subordinate to the critical density $\rho_c = 3H^2/8\pi G \simeq 1.03 \times 10^{-26} \text{ kg} \cdot \text{m}^{-3}$ (by choosing the Hubble constant $H = 74.2 \text{ kg} \cdot \text{s}^{-1} \text{Mpc}^{-1}$). However, the brane tension

$$T_3 \sim \frac{(4\pi)^{m/4+1} N_c^{m/4} \ 1}{(2\pi)^{m+3} \ g_{YM}^2 \ l_s^4}$$

(19)

given in Sec. 3.2 is for $(3 + 1)$-dimensional branes in $\text{AdS}_5$. We do not know how to calculate the $(2 + 1)$-dimensional “holographic” tension. To give the right dimensions, two possible choices are $T_{dw} = T_3^{3/4}$ and $T_3 l_p$, where $l_p$ is the five-dimensional Planck length; the validity of the estimation of $T_{dw}$, is discussed in Appendix B.3.

For domain walls separated by a typical distance $d_{dw}$, their contribution to the energy density is around $\rho_{dw} = T_{dw}/d_{dw}$. However, for the choices of $T_{dw}$ listed above, $d_{dw}$ always seems too large. For example, by assuming $l_s \sim 1 \text{ fm}$ as the typical nuclear size, for the case $T_{dw} = T_3^{3/4} \simeq \kappa/l_s^3$, $d_{dw} < \rho_c$ demand $d_{dw} > 6.9\kappa \times 10^6 \text{ Mpc}$. For example, for branes wrapped on 3-cycles, $m = 3$ and $d_{dw} > 4.9 \times 10^4 N_c^{9/16} / g_{YM}^{3/2} \text{ Mpc}$, which is already larger than the scale of the visible universe.

Therefore, if our estimation of domain wall tension is reasonable, to avoid dominating the energy density, the typical distance $d_{dw}$ should be really large; thus different QCD vacua should not be testable in visible universe. However, it is entirely possible that the “holographic” tension in boundary field theory, should be calculated in other ways. In that case, the critical density may not be a strict restraint.
4.3 Cosmic rays

Cosmic rays travel long distances to earth. If the regions they travel hold different fundamental parameters comparing with the local universe, the “landscape” of QCD vacua should leave clues at observatories. As we argued in Sec. 3.2, energy possessed by the cosmic ray particles remains the same while crossing the domain wall, although some other parameters change. Generally, to give meaningful restraints to confirm/rule out the landscape, the size of the regions (or the typical distances between the domain walls) should not be too large, otherwise all possible observations come from the same region; or too small, that the divergence is fully averaged.

4.3.1 Protons

The energy scale of the Greisen-Zatsepin-Kuzmin (GZK) cutoff [74, 75] may be altered, as the masses of proton, π-meson, and the Δ+ resonance may change in different regions. For the reaction \( p + \gamma_{\text{CMB}} \rightarrow \Delta^+ \rightarrow p' + \pi^0 \) (or \( n + \pi^+ \)), four-momentum conservation gives \( E_{\gamma_{\text{CMB}}} + E_p = E_{\Delta^+} \) and \( p_{\gamma_{\text{CMB}}} + p_p = p_{\Delta^+} \) gives \( m_p^2 + 2(E_p E_{\gamma_{\text{CMB}}} - p_p \cdot p_{\gamma_{\text{CMB}}}) = m_{\Delta^+}^2 = (m_p + m_{\pi^0})^2 \), thus \( E_p = (m_{\pi^0}^2 + 2m_p m_{\pi^0})/4E_{\gamma_{\text{CMB}}} \) for head-on collisions. For the 2.7 K cosmic microwave background (CMB) photons, their typical energy \( E_{\gamma_{\text{CMB}}} \) is around 1.1 meV, thus we have the cutoff energy \( E_p \sim 6 \times 10^{19} \text{ eV} \). As we have already observed the almost isotropic CMB radiation, the variation of \( E_{\gamma_{\text{CMB}}} \) by the “landscape” is at most a second-order correction. The cutoff energy scale \( E_p \) should indeed changes if \( m_p \) or \( m_\pi \) alter.

In current, the existence of GZK cutoff has already been confirmed, but its quantitative properties still hold several uncertainties [76, 77]. By assuming that there exist a sharp cutoff accurately locates at \( E_p \), the distance of the nearest domain walls should be larger than the mean free path of particles with energy a little above \( E_p \). The required distance is roughly 100 Mpc [78]. However, the existence of a sharp cutoff may be a too strong condition, as a typical ultra-high-energy cosmic ray (UHECR) source seems within the 100 Mpc distance [76], and the colliding angles posit randomly. While losing this requirement, GZK cutoff is no longer a strict restraint, as a variation of \( m_p \) or \( m_\pi \) can at most alters \( E_p \) with one order of magnitude.

If the UHECRs in fact come from further sources, the scale of the regions can still be only around 100 Mpc if a sharp cutoff exists. Nevertheless, additional selection principles should be required, to ensure that the local “vacuum” possesses the smallest \( E_p \) comparing with its adjacent regions. The other possibility is that a variation of \( m_p \) or \( m_\pi \) also alters the cross section of \( p + \gamma_{\text{CMB}} \rightarrow \Delta^+ \); however, careful calculations are needed to give further estimations.

4.3.2 Neutrons

Although mostly being neglected, an alternative probability is that UHECRs are in fact neutrons; air shower experiments such as Pierre Auger or ARGO-YBJ cannot distinguish protons and neutrons. One skip this possibility for several reasons: Firstly, as its mean
lifetime is only 885.7 s, an GZK neutron can only travels 550 kpc (by chosen \( E_p = 6 \times 10^{19} \text{ eV} \)), which is much smaller than the distance of a mainstream proton source. Secondly, as the Fermi acceleration mechanism of cosmic rays is an electromagnetic phenomenon \[79\], a neutral particle is hard to accelerate within it. We reconsider this possibility here, because a different QCD vacuum may alter the neutron lifetime \( \tau_n \), as discussed in Sec. \[3.3\] .

As neutrons cannot be influenced by the intergalactic magnetic fields, one may think this can help explaining the isotropy of cosmic rays. However, it is unlikely to be so. First, selection principles are needed to guarantee that the local vacuum possesses the smallest \( \tau_n \), while in other parts of the universe neutrons hold longevity. Second, it is hard to understand why neutron composition surpasses proton, if we believe some alternative cosmic ray producing mechanisms, such as the decay of heavy particles. Third, one should explain why UHECRs seem coincident with the supergalactic plane \[76\].

### 4.3.3 An alternative

An alternative possibility is that we indeed live within a domain wall. This scenario seems too ambitious. However, as we still lack a reasonable estimation of domain wall thickness, and we have some ways to widen it, as discussed in Appendix \[B.1\] we cannot rule it out intuitively. If its thickness is of order the string length \( l_s \sim 1 \text{ fm} \), or the five-dimensional Planck length \( l_p \), this case may not happen.

One clue is that we all live within the supergalactic plane. In the mainstream models, large-scale structures like filaments (planes), haloes or voids, are understood as direct consequences of nonlinear gravitational effect, and they have already been resulted from N-body simulations. The coincidence of UHECRs with the supergalactic plane is easily explained; mass collapse in the structure formation produces objects such as active galactic nuclei (AGNs) and gamma-ray bursts (GRBs), and the latter ones are thought to be the sources of UHECRs. In our scenario, a proper vacuum is a one where the potential \( V \) get its minimum \( V_{\text{min}} \), and a domain wall is a region where two proper vacua conjuncts (maybe some smoothness conditions, or the “domain wall visualizing” methods, make it really wide); hence it possesses some large \( V \), and some different vacua. The angle distribution of UHECR patterns may be understood, if in the normal vacua located at \( V_{\text{min}} \), the fundamental parameters are disadvantageous for UHECRs to transport; for example, maybe the mean free path of the reaction \( p + \gamma_{\text{CMB}} \rightarrow \Delta^+ \rightarrow p^0 + \pi^0 \) or \( p'' + e^+ + e^- \) in these vacua is very small. The \( e^+e^- \) pair production case is interesting, as its cutoff energy is only \( 4 \times 10^{17} \text{ eV} \) in our vacua; however, its cross section is too low to be considered. If generally in the bulk, the \( e^+e^- \) cross section is larger, UHECRs can only come from direction within the domain wall. In this case, stellar originations of UHECRs are not insisted on. As we always lack of plausible ways to observe these regions, we never know their physics and fundamental parameters. Additional considerations should be needed, to explain why stars or galaxies never appear in the vacua close to \( V_{\text{min}} \); some tentative discussions are given in Sec. \[4.3.2\] Despite the fact that our scenario has nothing to do with the galaxy rotation curves, it may even explain the dark matter puzzle. If the particles (for examples, protons or neutrons) in the voids of \( V_{\text{min}} \) are heavier than which within the domain walls, they can
contribute the additional “dark” masses.

4.4 Abundances of low-Z nuclei

As already been discussed in Sec. 3.3, a different QCD vacuum may possess different hadronic masses, different neutron lifetime, different reaction rates, or different binding energies of light nuclei. Here we discuss how they affect astrophysical observations.

4.4.1 Big-bang nucleosynthesis

BBN gives maybe the tightest bound for variations of fundamental parameters. Starting from [62], several works focus on the question of in what regions of variations of $\Lambda_{\text{QCD}}$ or quark masses $m_q$, can BBN give a consistent result with observations. Fundamental parameters mainly affect BBN through (i) the deuteron binding energy and (ii) the neutron-proton mass difference. A comprehensive discussion of the dependence of several deduced and fundamental parameters is given in [80].

The observation of the primordial abundances of several light elements, constrain the parameters of BBN. These elements mainly include $^2\text{D}$, $^3\text{He}$, $^4\text{He}$, and $^7\text{Li}$. Most measurements aim to objects within the solar system, such as meteors, lunar soil, the atmosphere of Jupiter, or the local universe, such as the interstellar medium (ISM), the Pop I stars, and the galactic and extragalactic HII regions, which hold little use for our purpose. Nevertheless, the abundance of deuterium $^2\text{D}$ can also be measured by the quasar absorption line. This gives constraints for the fundamental parameters in the BBN era, at redshift $z \sim 3$. The up-to-date observational results can be found in [81] and references therein.

Generally, the measurement of the primordial $^4\text{He}$ mass fraction $Y_p$, or some other local abundances, seem more accurate than the quasar observations of $^2\text{D}/H$. $^2\text{D}/H$ fluctuates from about $1.5 \times 10^{-5}$ to $4 \times 10^{-5}$, for several quasars of redshift from 2 to 3.5. In addition, even after including the observational errors, these measurements are still inconsistent with each other, and several of them are out of the weighted mean value $^2\text{D}/H = (2.63 \pm 0.31) \times 10^{-5}$ [81]. One possible explanation is that only quasars with low metallicity are suitable for this measurement; however, the residual metal component can still affect the observational values. The other possibility is that $^2\text{D}/H$ indeed fluctuates here and there, which hints that fundamental parameters hold a landscape in different part of the universe.

Here we estimate the variation of other parameters, if $^2\text{D}/H$ intrinsically fluctuates. Notice that the deuterium abundance is extremely sensitive to the nucleon mass $m_N = (m_p + m_n)/2$, and $\partial \ln(^2\text{D}/H)/\partial \ln m_N = 3.5$ [80]. For $^2\text{D}/H$ varies between $(1.5 \sim 4) \times 10^{-5}$, $m_N$ changes from 799.75 MeV to 1058.42 MeV, with the local value 938.92 MeV corresponds to $^2\text{D}/H = 2.63 \times 10^{-5}$. Similarly, as $\partial \ln(^2\text{D}/H)/\partial \ln(m_d - m_u) = -2.9$, we have $(m_d - m_u)_{\text{max}}/(m_d - m_u)_{\text{min}} \sim 1.48$; and as $\partial \ln(^2\text{D}/H)/\partial \ln(m_d + m_u) = 17$, $(m_d + m_u)_{\text{max}}/(m_d + m_u)_{\text{min}} \sim 1.06$. In addition, we should emphasis that the fluctuations of $m_N$, $m_d - m_u$, and $m_d + m_u$ are only some upper limits, for some given distribution of $^2\text{D}/H$. The value of $\partial \ln(\bullet)/\partial \ln(\bullet)$ calculated in [80] assume that all parameters vary independently, but the observed $^2\text{D}/H$ is the aggregative effect for all kind of variations.
Moreover, the divergence of $^2$D/H given by the quasar observations, is incapable to give the upper limits of the variation of fundamental parameters. A selected effect should be taken into account, that all possible observations aim to regions where quasars can be produced. It was argued that mostly the brightest quasars are hosted by the largest galaxies in the early universe, and they end up today as central galaxies in rich clusters [82]. Even if we assume that the local environment (such as the value of fundamental parameters) of quasars is similar to ours, it is still possible that we all posit in regions where structure formation are easier than others do.

4.4.2 Stellar evolution

A different QCD vacuum can absolutely alter the stellar evolution scenario in several ways. However, as most classical theories in this field rely on numerical methods, which hold several free parameters, quantitative discussions of our issue may be really difficult.

The star formation properties, include the shape of the Hayashi track, should generally be unaltered by another QCD vacuum. Gravitational collapse process is mainly caused by gravitational and electromagnetic phenomena (the latter one is the origin of dissipation), which are independent of the strong interaction.

The burning of stars, in which energy releases by nuclear fusion reaction, should be affected if the fundamental QCD parameters change. The observational Hertzsprung-Russell (HR) diagram may not rule out the existence of these vacua, as its data points are all sampled from nearby stars. We may firstly assume that the processes of proton-proton chain and CNO cycle are still the most important ones. We neglect to discuss the CNO cycle, as the influences of other QCD vacua are hard to estimate from the fundamental parameters. In the proton-proton chain reaction, which mostly dominates in stars with masses lower than about $1.2 M_\odot$ in our QCD, is bottlenecked by the $^1$H+$^1$H $\rightarrow$ $^2$D+$e^+ + \nu_e$ reaction. Notice that the cross section of this reaction has a factor

$$f(x) = (x^2 - 1)^{1/2} \left( \frac{x^4}{30} - \frac{3x^2}{20} - \frac{2}{15} \right) + \frac{x^4}{4} \log \left[ x + (x^2 - 1)^{1/2} \right] ,$$

where $x = (2m_p - m_D)/m_e$ and $m_D$ is the mass of $^2$D without electrons [83], the reaction rate relies sensitively on the proton and deuterium masses. If $x$ is smaller in other QCD vacua, the lifetime of low mass main sequence stars, should be tremendously longer.

It is possible that in some QCD vacua, stars cannot even be ignited, either because the reaction rates of the proton-proton chain and CNO cycle are both too small, or because these processes are not possible for those parameters. The observational effect should be dark voids. It is not entirely impossible, as ever been roughly discussed in Sec. 4.3.3 in the background of cosmic rays; however, as an parallel idea confronts the ordinary structure formation scenario, detailed discussions are needed for its consistency and reasoning.

The latter burning phases of stars are also altered for different QCD parameters. We give up the quantitative discussions, as the feedback is hard to estimate. Nevertheless, there is one possibility to mention. Several astrophysical events are difficult to comprehend from theoretical levels; for example, supernovae never explode in computer simulations. Is it
possible that they explode because they locate in regions with a different QCD? However, this explanation is generally unlikely to be so, because one know example (SN 1054; the Crab Nebula) is really nearby.

The final stage of stars is also different for different QCD. In reality, stars end as white dwarfs, neutron stars or black holes, with the former two possess some upper mass limits, called the Chandrasekhar mass limits. For the case of the electron-degenerate matter, the mass $M_{\text{max}} \propto (\mu_em_H)^{-2}$, where $\mu_e$ is the molecular weight per electron and $m_H$ is mass of the hydrogen atom, depends but is not very sensitive to the QCD parameters.

4.5 Quark matter

In our “real world”, the energy per baryon number of $^1\text{H}$, $^{12}\text{C}$ and $^{56}\text{Fe}$ is $E_h = 938.8$, 931.5, and 930.4 MeV respectively. In the case of only $u$ and $d$ quarks, from nuclear observations we know that nuclear matter is absolutely more stable than quark matter. It was conjectured that the true zero temperature and pressure ground state of is the “strange quark matter”, the quark-gluon plasma (QGP) of $u$, $d$, and $s$ quarks [84]. The main reason is that the additional strange freedom lowers the Fermi energy. In case of the MIT bag model [85, 86], one has $\alpha_S = 0$ and $m_q = 0$ (especially $m_s = 0$). Assuming that the system is electrically neutral, thus $2n_u/3 - n_d/3 - n_s/3 - n_e = 0$ and $n_e \sim 0$; and the pressure $p_F/4\pi^2$ is equal for the $(ud)$ or $(uds)$ QGP, where $p_F = \hbar(3\pi^2n_q)^{1/3}$ is the Fermi momentum. Notice that $p_{F,u} : p_{F,d} = 1 : 2^{1/3}$ for the $(ud)$ case, and $p_{F,u} : p_{F,d} : p_{F,s} = 1 : 1 : 1$ for the $(uds)$ case. The average quark kinetic energy is generally proportional to $p_{F,q}$ of that particle, thus we have

$$\frac{E_{\text{uds}}}{E_{ud}} = \left(\frac{\frac{4}{3} + \frac{1}{3} + \frac{1}{3}}{\frac{4}{3} + \frac{2}{3} \cdot 2^{1/3}}\right)^{1/4} \approx 0.887. \quad (21)$$

Assuming that $E_{ud} \sim E_h$, we have $\Delta E = E_h - E_{uds} \sim 100$ MeV. Hence if $m_s \lesssim 100$ MeV, strange quark matter is more stable than hadronic matter. These matter ground state conjecture is consistent with the nowadays constraint is $m_s = 95 \pm 25$ MeV.

The existence of charm quarks is generally ruled out from the ground state conjecture. Charm quark seems too heavy, which a typical mass $m_c = 1.25 \pm 0.05$ GeV. As and $p_{F,u} : p_{F,d} : p_{F,s} : p_{F,c} = 1 : 2^{1/3} : 2^{1/3} : 1$ for the $(uds)$ QGP, roughly we have

$$\frac{E_{\text{udsc}}}{E_{ud}} = \left(\frac{\frac{1}{6} + \frac{2^{1/3}}{3} + \frac{2^{1/3}}{3} + \frac{1}{6}}{\frac{1}{3} + \frac{2}{3} \cdot 2^{1/3}}\right)^{1/4} \approx 0.810. \quad (22)$$

Thus, we need $m_c + m_s/2 \lesssim 200$ MeV to make the “charm quark matter” the true ground state, which is absolutely impossible.

However, within our discussions of QCD landscape, it is possible that in some other vacuum $m_c$ is not so heavy, thus the charm quark matter is in fact more stable. Or maybe in some vacuum, $E_{ud} < E_h$, therefore hadron states cannot even exist in the zero temperature and pressure case.
4.5.1 Strange stars or charm stars

The strange star is a theoretical model of compact star, which hypothesizes that compact star is composed of strange quark matter [87, 88]. It is a direct corollary of the conjecture that the strange quark matter is the true ground state of matter. It is argued that the strange star, rather than the neutron star, is the true origin of pulsar. Both the strange and the neutron star possess some Chandrasekhar mass limit of $M_\star \sim 1M_\odot$ at the radius $R_\star \sim 10\text{ km}$; however, the smaller the lighter strange stars, but the larger the neutron ones. In general, one can calculate the strange star mass-radius relationship, using the Tolman-Oppenheimer-Volkoff equation and some proper equation of state (EoS), integrating from the center of the star, and indicating the surface as the radius where $P = 0$.

For some QCD vacuum in which $E_{udsc} < E_h$, the hypothesized star should in turn be the “charm star”. Charm stars have ever been discussed in the realistic QCD [89], which do not possess stable charm matter. Hence, in their case, charm stars exist when the central energy density of the star is really high, and $P = 0$ is no longer a reasonable assumption. They conclude that charm stars are generally unstable by perturbations, thus should not exist. In our case, for vacua where $m_c$ and $E_{udsc}$ are much smaller, charm stars can exist even for really small masses and radii.

However, one may feel difficult to discriminate between strange stars and charm stars far away in the sky. Consider the EoS of the MIT bag model, hence the mass-radius relationship of the star. In the first approximation of $m_s = m_c = 0$, this EoS is independent of the flavor number, and both strange and charm quark matter, or even the $(ud)$ QGP, hold the EoS of $P = (\rho - 4B)/3$, where $B$ is the bag constant. This EoS should generally be suitable even if $m_q \neq 0$, as when $m_q$ becomes dynamically important, its abundance decrease. $B$ should absolutely depends on $m_q$ and $\alpha_S$; however, in a straightforward understanding of the bag model, $m_q = \alpha_S = 0$ is assumed. Therefore, strange stars and charm stars possess similar mass-radius relationships; the only different is that for the charm star cases, the effective bag constant $B$ is different from what estimated from baryon resonance states on earth.

The other possibility is that the surface properties are different for strange and charm stars; however, it is also not likely to be so. The quark surface should be really thin, which possesses a typical strong length scale of order 1 fm; however, electronic distribution should be more diffused. Thereby a strong electric field exists near the star surface. Both the cases of strange star surface with [88] or without [90] the hadronic “crust”, have ever been considered. Intuitively, charm stars should hold a much stronger electric field than the strange stars; as $n_u \neq n_d$ and $n_s \neq n_c$, the overall neutralized condition makes $n_e$ larger in charm quark matter. In contrast, strange quark matter have $n_u : n_d : n_s \simeq 1 : 1 : 1$. Detailed calculations show that $n_e^{(udsc)} \sim 100n_e^{(uds)}$ for the same baryonic number density (see Fig. 3 of [89], for example). Quantitatively, one always estimates the surface configuration by the Thomas-Fermi model. Our discussion limits to the bare quark star case, in which the crust does not exist. While $n = p_F^3/3\pi^2$ for fermions like electrons or quarks, equilibrium condition gives the chemical potential at infinite $\mu_\infty = p_{F,e} - V_z = p_{F,q} - V_q = 0$, where $V_e (V_q)$ is the electrostatic potential for electrons (quarks). Poisson
equation gives
\[ \frac{d^2V_e}{dz^2} = \begin{cases} 
4\alpha(V_e^3 - V_q^3)/3\pi & z \leq 0 \\
4\alpha V_q^3/3\pi & z > 0 
\end{cases}, \]  
where \( \alpha \) is the fine structure constant, and \( z \) is the height measured from the stellar surface.

It possesses a solution
\[ V_e = \frac{3V_q}{\sqrt{6\alpha/\pi V_q z + 4}} \]  
for \( z > 0 \), and an electric field \( E_e = -dV_e/dz \). Notice that \( V_q \) is only related to the quark matter density, which is insensitive to \( n_e \) or the existence of charm quark, so do \( V_e \) and the electric field \( E_e \).

4.5.2 Strangelets or charmlets

Strangelets have already been discussed widely in the issue of the cosmological QCD phase transition, in [84] and the follow-up works. They may also be produced in ultra-relativistic heavy ion collisions. Reference [91] gives a detailed study of charmlets; however, as charm quarks are too heavy, charmlets exist in reality should always be unstable. Estimated from perturbative QCD, an attractive force exists for large enough quark mass \( m_q \); the author explain this phenomenon by a MIT bag model with the Casimir energy of the bag. In some other vacua, \( m_c \) is not so heavy, and charmlets may be stable or at least easily produced. In the first case, charmlets may be the relic of the cosmological QCD phase transition.

In general, light charmlets or charm resonance states, even if exist in nearby vacua, cannot diffuse to our vacua. The reason has already been discussed in Sec. 3.2. When charmlets go through the domain wall, the fundamental parameters change, and the nuggets decay to more stable matter.

An interesting thing is that maybe in some vacua, the (ud) QGP is the true ground state. Generally, in this case, hadronic matter are not stable, and stars formed by the gravitational collapse cannot burn. There are two possibilities. On the one hand, maybe at high temperature and low pressure regions, QGP is also more stable than hadronic matter. The cosmological QCD phase transition cannot happen in this case, and the late universe is either full of quark nuggets/quark stars, or full of black holes. The latter case happens when single pieces of quark matter leave after the big-bang, have their masses surpass the Chandrasekhar limit. On the other hand, if the hadronic matter is more stable at high temperature, hadronization indeed happens in the early universe. In this case, if the potential barrier to transform from hadronic matter to quark matter is low, the transformation goes smoothly while gravitational collapse, and the final state is small quark nuggets. Larger objects are not possible, as dissipations do not exist in this system. However, if the barrier is high, transformation may only happens within some violent process, as at really high pressure we always have the (udscbt) QGP the most stable one. As these violent processes should not happen within our visible universe, this possibility is excluded. However, it may be a little early to rule out the other possibilities, that there are regions in the universe where (small or large) quark pieces floating in the sky. They behave like dark voids.
5 Discussions and conclusion

In this paper, we discussed the possibilities that whether QCD, or more generally some holographic $(3 + 1)$-dimensional gauge theories, can possess a “landscape” of its vacua. We first limited our discussions to some boundary CFTs, for which the compact manifolds are Einstein. An Einstein 5- or 6-manifold $\mathcal{M}_q$, which needs not to be homogeneous, may have some not-very-small Betti numbers $b_m$. Therefore, if fluxes wrap different patterns on cycles of it, the moduli are also different; a landscape of the holographic vacua should rise for this reason. We examined several relevant issues for this conjecture from the theoretical viewpoint. The geometry of $\text{AdS}_5 \times \mathcal{M}_q$ should also possess some symmetric conditions, such as the worldsheet superconformal symmetry, or definite supersymmetries of the dual field theories. However, it seems that even the $\text{AdS}_5 \times S^5$ violates the one-loop worldsheet conformal symmetry. The isometry group of $\mathcal{M}_q$ decides the R-symmetry of the holographic theory, which is not imposed in our case. In addition, we considered the anomaly cancelation, the no-go theorem, the brane/flux configurations, and the vacua stabilities. We focused on the $(2 + 1)$-dimensional domain walls within Mink$_4$, and also the $(3 + 1)$-dimensional domain walls filling the radial AdS$_4$ inside AdS$_5$. We considered the possibilities to break the AdS$_5$ geometry, or equivalently the conformal symmetry of the boundary theory, and argued that the confinement-deconfinement phase transition may not affect the vacua configurations. After then, we applied our conjecture of the “holographic landscape” directly to QCD, for which the fundamental parameters such as $m_q$, $\alpha_S$ or $\theta_{\text{QCD}}$ should depend on the moduli of the compact dimensions. We studied the properties of the domain walls, such as their masses, thicknesses, and their dynamical properties; they may be both relativistic and non-relativistic. We discussed how this “QCD landscape” affects nuclear physics; they may alter the hadronic masses, the reaction rates, and the stabilities of the nuclei.

In an opposite way, we studied how can a “QCD landscape” affects the astrophysical observations. As domain walls may be non-relativistic, another vacuum of QCD may be within the visible universe; if they are not, we can also consider the properties of the other multiverses of QCD, just as what is done in [70, 71]. We first considered whether the mass contribution of the domain walls exceeds the critical density of the universe. Most of the case, domain walls are really dangerous; however, it is not enough reasonable that they should be completely ruled out. We then discussed the properties of cosmic rays affecting by this landscape, include the GZK cutoff, the neutron lifetime, and an alternative explanation of the coincidence of the cosmic ray anisotropic spatial distribution with the supergalactic plane. The GZK cutoff depends on, but is not very sensitive to, a different QCD vacua; the variation of the neutron lifetime seems helpless to explain the observations. The alternative explanation may loose the constraints for the origins of the UHECRs; however, as too ambitious the scenario is, it needs to be studied more seriously. We also considered how the QCD landscape affects BBN and the star burning scenarios. As only the abundance of deuterium $^2\text{D}$ can be measured in far away part of the universe, only it can restrain the QCD landscape; the constraint is really loose. A different QCD vacuum can also affect the stellar evolution; for example, the time spent for the proton-proton
chain reaction depends on the deuterium mass \( m_D \) very sensitively. The Chandrasekhar mass limits of white dwarfs and neutron stars are also altered, but not very sensitive to the QCD landscape. In addition, whereas the “strange quark matter” may be the true matter ground state of our QCD \[84\], charm or \((ud)\) QGP may be the ground states of other QCDs; thus, as an alternative, charm stars or \((ud)\) quark stars may exist in those QCDs. However, the mass-radius relationships and the surface properties of those objects seem really similar to our strange stars.

Recently, Denef and Hartnoll discussed the landscape in condensed matter systems, which results a statistical distribution of critical superconducting temperatures \[92\]. In here, we compare briefly the original “string landscape”, their “atomic landscape”, and our “QCD landscape”. First, whereas the relativistic quantum critical theories are conformal, our QCD landscape should break conformal symmetry by some mechanics. Second, the original Calabi-Yau compactification possesses a \( \mathcal{N} = 1 \) supersymmetry, the “atomic landscape” in M-theory by the Sasaki-Einstein 7-manifolds holds a \( \mathcal{N} = 2 \) supersymmetry; however, it seems that our “QCD landscape” is not restricted by supersymmetry conditions. Third, as the “atomic landscape” discussed in \[92\] is limited to the Freund-Rubin compactification, only the background electromagnetic four-flux contributes to the vacua field equations; thus their statistics of the landscape rises from the different topologies of the compact dimensions. However, the original Calabi-Yau vacua, or the QCD vacua discussed in this paper, are more abundant because of different flux configurations.

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A Mathematical supplements

A.1 About \( J^M_N \)

The holonomy we use in the context is a very special one: the holonomy of a Riemannian manifold of the Levi-Civita connection on the tangent bundle of \( \mathcal{M}_n \). In this context, \( \text{Hol}(\mathcal{M}_n) \) is a subgroup of \( O(n) \).

In our case, \( J^M_N \in C^\infty(T\mathcal{M}_{10} \otimes T^*\mathcal{M}_{10}) \) is a covariantly constant tensor field thus \( \nabla_P J^M_N = 0 \), to possess worldsheet supersymmetry under

\[
\delta \varphi^M = J^M_N \varepsilon \psi^N \\
\delta (J^M_N \psi^N) = [-i \partial \varphi^M + \frac{1}{2} \Gamma^M_{NP} J^N_Q J^P_R (\overline{\psi^Q} \psi^R)] \varepsilon.
\]

Let \( H = \text{Hol}_p(\mathcal{M}_{10}) \) be the holonomy group of a point \( p \in \mathcal{M}_{10} \), which acts on a tensor field by parallel transport its vector bases separately. We denote the restricted analog of
$H$ to be $H^0$. The necessary and sufficient condition for the existence of a tensor field $J^M_N$, is that $J^M_N|_p$ is fixed by the action of $H$ on $T\mathcal{M}_{10} \otimes T^*\mathcal{M}_{10}$ \cite{26} §2.5.2.

Generally, we have $H^0 = SU(5)$, $U(5)$, $SO(10)$ in the Berger classification. For a manifold which is not complex, the unitary groups such as $SU(5)$ or $U(5)$ can be generally ruled out. In our case, the configuration $AdS_5 \times \mathcal{M}_5$ gives $H^0 = SO(5) \times \text{Hol}^0(\mathcal{M}_5)$; and if $\mathcal{M}_5$ is irreducible and non-symmetric, the Berger classification restricts $\text{Hol}^0(\mathcal{M}_5)$ to be $SO(5)$.

As the condition $g_{PQ}J^P_M J^Q_N = g_{MN}$ gives $JJ^T = J^T J = 1$, we have $J \in O(10)$. Hence, a suitable $J^M_N$ belongs to a subgroup of $O(10)$, which is invariant under the action of $H$. We limit our discussions to the simply connected cases $\mathcal{M}_{10}$ for simplicity; thereafter $H = H^0$. We also assume $\det(J) = 1$. The most general $H^0 = SO(10)$ constrains $J^M_N = \delta^M_N$ as the only possibility. The additional restriction $J^P_M J^M_N = -\delta^P_N$ in \cite{29} forces $H^0$ to be a subgroup of $U(5)$, hence gives Kähler manifolds. In our cases, we restrict $J$ to be invariant under the action of $SO(5) \times \text{Hol}^0(\mathcal{M}_5)$.

### A.2 About $g^{MN}\Gamma^P_{MN} = 0$

Generally, Kähler manifolds always satisfy $g^{MN}\Gamma^P_{MN} = 0$. The reason is that the only non-zero components for $g$ is $g_{\alpha\bar{\beta}}$ and $g_{\bar{\alpha}\beta}$, but the Levi-Civita connection has no mixed indices.

$AdS_5 \times S^5$ metric does not possess $g^{MN}\Gamma^P_{MN} = 0$, although it holds Ricci flatness. By choosing the coordinates

$$ds^2 = \frac{r^2}{R^2} (-dr^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2(\theta_1, \ldots, \theta_5), \quad (26)$$

we have

$$g^{MN}\Gamma^P_{MN} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-5r}{R^2} & 4 \sin \theta_1 \cos \theta_1 & 3 \sin \theta_2 R^2 \cos^2 \theta_1 \cos \theta_2 & 2 \sin \theta_3 R^2 \cos^2 \theta_1 \cos^2 \theta_2 \cos \theta_3 \\ 0 & 4 \sin \theta_1 \cos \theta_1 & \sin \theta_2 R^2 \cos^2 \theta_1 \cos \theta_2 \cos \theta_3 & \sin \theta_3 R^2 \cos^2 \theta_1 \cos^2 \theta_2 \cos^2 \theta_3 \cos \theta_4 \\ 0 & 3 \sin \theta_2 R^2 \cos^2 \theta_1 \cos \theta_2 & \sin \theta_3 R^2 \cos^2 \theta_1 \cos^2 \theta_2 \cos \theta_3 & \sin \theta_4 R^2 \cos^2 \theta_1 \cos^2 \theta_2 \cos^2 \theta_3 \cos \theta_4 \\ 0 & 2 \sin \theta_3 R^2 \cos^2 \theta_1 \cos^2 \theta_2 \cos \theta_3 & \sin \theta_4 R^2 \cos^2 \theta_1 \cos^2 \theta_2 \cos^2 \theta_3 \cos \theta_4 & 0 \end{pmatrix}. \quad (27)$$

### B Orders of magnitudes reconsidered

#### B.1 $l_s \sim 1$ fm?

$l_s = \alpha^{1/2} \sim 1$ fm comes from the studies of the Nambu string \cite{55}, or the Regge slope. It is the fundamental scale of the QCD string. However, $t/l_s \sim 10^{41}$, which is used in Sec. 3.2.
may not be reasonable. The reason is that the “time” in (4 + 1)-dimensional gravity corresponds to the boundary QCD, may not equal to the “time” of our (3 + 1)-dimensional gravity.

It was generally said in Sec. 3.2 that all free parameters of QCD, such as \( m_q \) or \( \alpha_s \), are decided by the moduli of the compact manifold. However, this judgement may be a little too naïve; the quark masses \( m_q \), or the (3 + 1)-dimensional “time”, are completely the gravitational effect. Thereafter the question turns to be: how can we induce gravity in the boundary theory?

One known method to solve this question, is to add the “Planck brane” as some (field) UV cutoff [93, 94, 95]. As a generalization of the Randall-Sundrum (RS) I model [96], this brane induces gravity, whereas QCD is located at some other branes in the more IR regions. Unlike the generalized RS I models, QCD is not fixed at the “TeV brane” of RS I, as ever discussed in [93]. In this case, the string scale is of order the (3 + 1)-dimensional Planck length \( l_{p,4} = \sqrt{G} = 1.6 \times 10^{-20} \text{ fm} \) at the radius of the “Planck brane”, but of order 1 fm at the QCD visualizing radius. However, the time \( t \sim 10^{10} \text{ yr} \) is also measured at the “Planck brane” (which is located at the gravity IR, by the UV-IR relation [17]), and is tremendously redshifted; by pulling it back to the QCD radius, one has \( t \sim (l_{p,4}/1 \text{ fm}) \times 10^{10} \text{ yr} \) and a much suppressed \( t/l_s \sim 10^{21} \).

Nevertheless, one may think there are some inconsistencies for the above considerations. By estimating in regions of larger radius, at the “Planck brane” for instance, the value of \( t/l_s \) is much larger. That may in fact hint that for the AdS\(_4\) branes inside AdS\(_5\), the part away from the throat is more easily accelerated. However, this distortion of branes is hard to understand; points located at different radial positions is not really independent, they are only “holographic images” with each other [93]. It seems a too tough issue, which we left to the follow-up studies.

In Sec. 3.2 we also argued that the domain walls may have the thickness of the string scale \( l_s \). It is true if the “domain wall visualizing” radius is at the same position as the QCD brane. However, it may generally not need. Just aswhat discussed above, if the “domain wall visualizing” radius is deeply inside the throat of AdS, the string scale there, thereafter the domain wall thickness, can be much larger. The other possibility to widen the boundary domain walls, is to consider slant branes, which we fall to discuss in this paper. Domain walls with thickness much wider than \( l_s \), is needed in the scenarios of Sec. 4.3.3.

**B.2 \( \mu_{\text{brane}} \sim T_3 \)?**

The relationship \( \mu_{\text{brane}} \sim T_3 \) is used in Sec. 3.2 to consider whether the domain walls are relativistic or non-relativistic. By the Newton’s second law, the transverse wave equation of a stretched (one-dimensional) string gives \( v = \sqrt{T/\mu} \), where \( T \) is the tension and \( \mu \) is the mass per unit length. Thus one possesses the relation \( \mu \sim T \) if the wave is relativistic. \( \mu \sim T \) is also a general assumption in the study of cosmic strings/superstrings.
B.3 \( T_{dw} \sim T_3^{3/4} \) or \( T_{dw} \sim T_3 l_p \)?

\( T_{dw} \sim T_3^{3/4} \) or \( T_{dw} \sim T_3 l_p \) can be obtained by dimensional analysis, where \( T_3 \) is the tension of the \((3 + 1)\)-dimensional branes perpendicular to the boundary of \( \text{AdS}_5 \), and \( T_{dw} \) is the holographic brane tension. It was used in the estimations of Sec. 4.2 in which the domain walls are too heavy to be visible in our universe. However, these two seem not the only answers.

On the one hand, similar to the RS I model, or the case of Appendix B.1, the right answer of \( T_{dw} \) should depend on in what \( \text{AdS} \) radius the holographic domain wall visualizing itself. The domain walls should be really light if the radius is deeply inside the throat. On the other hand, as Eq. (19) gives \( T_3 \propto l_s^{-4} \), \( T_3 \) itself is much smaller in the (field) IR regions, because the string length \( l_s \) is much larger there. Hence, \( T_{dw} \) is suppressed in that region, even if \( T_{dw} \sim T_3^{3/4} \) or \( T_{dw} \sim T_3 l_p \) is still valid. As discussed in Sec. 4.2, a smaller \( T_{dw} \) is needed, if we want another vacua to be visible, and the contribution of the domain walls to the energy density of the universe is consistent with the critical density.

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