The sign of the day-night asymmetry for solar neutrinos

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A qualitative understanding of the day-night asymmetry for solar neutrinos is provided. The greater night flux in $\nu_e$ is seen to be a consequence of the fact that the matter effect in the sun and that in the earth have the same sign. It is shown in the adiabatic approximation for the sun that for all values of the mixing angle $\theta_V$ between 0 and $\pi/2$, the night flux of neutrinos is greater than the day flux. Only for small values of $\theta_V$ where the adiabatic approximation badly fails does the sign of the day-night asymmetry reverse.

It was pointed out a long time ago [5] that as a result of the matter effect in the earth it is possible that the flux of neutrinos at night is different from that in the day. Calculations made for a variety of situations [2–4] almost always gave a greater flux at night than during the day. This note is designed to explain the sign of the day-night asymmetry.

Most calculations until recently concerned values of the vacuum mixing angle $\theta_V < 45^\circ$ such that the $\nu_e$ flux at earth was less than half of the expected flux so that most of the arriving neutrinos were $\nu_x$ (that is, $\nu_\mu$ or $\nu_\tau$). It was then often said that the earth effect was to change $\nu_x$ to $\nu_e$ and $\nu_e$ to $\nu_x$ so that there were more $\nu_e$ at night because there were more $\nu_x$ to start with [5]. This explanation is fundamentally wrong.

That this is wrong is obvious from noting in recent calculations [2] a positive asymmetry persists when $\sin^2 2\theta_V = 1$, corresponding to maximal mixing. This point has been discussed in detail recently [5]. Furthermore, even if $\theta_V > 45^\circ$ there is still a positive asymmetry as can be seen, for example, from Figs. 1 and 2 in Ref. [5].

We start by assuming the adiabatic approximation for the neutrinos traversing the sun and that $\Delta m^2/2E$ is much less than the matter effect near the center of the sun where the neutrinos originate. In this case the neutrinos emerge from the sun in the upper vacuum mass eigenstate

$$\nu_2 = \sin \theta_V \nu_e + \cos \theta_V \nu_x.$$  \hspace{1cm} (1)

There are no oscillations between the sun and the earth so that the $\nu_e$ flux arriving at the earth is $\sin^2 \theta_V F_0$, where $F_0$ is the expected flux without oscillations in the sun. When the neutrinos go through the earth, the state $\nu_2$ is mixed with

$$\nu_1 = \cos \theta_V \nu_e - \sin \theta_V \nu_x.$$  \hspace{1cm} (2)

Thus, the neutrinos that emerge at night are in a coherent mixture ($a \nu_2 + b \nu_1$). The night flux then depends on the relative sign and phase of $a$ and $b$. For neutrinos going through the mantle of the earth a good approximation is a constant density $N_e$, where $N_e$ is the electron density. In the $\nu_1$-$\nu_2$ representation the propagation in the earth is given by

$$i \frac{\partial}{\partial t} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \mathcal{H} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix},$$

where

$$\mathcal{H} = \frac{1}{2} \begin{pmatrix} B & A \\ A & -B \end{pmatrix},$$  \hspace{1cm} (3)

and

$$A = \sqrt{2} G_F N_e \sin 2\theta_V,$$

$$B = \sqrt{2} G_F N_e \cos 2\theta_V - \Delta_0,$$

$$\Delta_0 = \frac{\Delta m^2}{2p},$$  \hspace{1cm} (4)

with $\Delta m^2 \equiv m_2^2 - m_1^2$ and $p$ being the momentum of the neutrino that is approximately equal to its energy $E$. The state that emerges then is

$$\nu_2(t) = -i \sin 2\theta_M \sin \lambda t |\nu_1\rangle + (\cos \lambda t - i \cos 2\theta_M \sin \lambda t) |\nu_2\rangle,$$  \hspace{1cm} (5)

where

$$\tan 2\theta_M = -\frac{A}{B},$$

$$\lambda = \frac{1}{2} \sqrt{A^2 + B^2}.$$  \hspace{1cm} (6)

The $\nu_e$ emerging probability is

$$P_{2e} = |\langle \nu_e | \nu_2(t) \rangle|^2 = \sin^2 \theta_V + \sin 2\theta_M \sin (2\theta_M + 2\theta_V) \sin^2 \lambda t.$$  \hspace{1cm} (7)

From Eq. (5) it is seen that when $\Delta_0 \gg \sqrt{2} G_F N_e$, $\tan 2\theta_M$ has a small positive value given by $A/\Delta_0$; as $\Delta_0$ decreases till it is much smaller than $\sqrt{2} G_F N_e$, the value of $2\theta_M$ approaches $\pi - 2\theta_V$, corresponding to $\tan 2\theta_M = -\tan 2\theta_V$. For this whole range of $\theta_M$ it follows from Eq. (6) that the emerging $\nu_e$ probability is always greater than $\sin^2 \theta_V$ for all values of $\theta_V$ between zero and $\pi/2$. For the maximum of the oscillation in

*One of us (L.W.) admits to having said this once.
Eq. (7), i.e., \( \sin^2 \lambda t = 1 \), there exists a value of \( \theta_M \) such that the emerging night flux equals \( F_0 \), the no-oscillation flux; this corresponds to
\[
\tan 2\theta_M = \cot \theta_V, \tag{8}
\]
which occurs for all \( \theta_V \) if
\[
\Delta_0 = \sqrt{2} G_F N_e \equiv \Delta_0^{\text{max}}. \tag{9}
\]

We will neglect the first of these since if \( \Delta m^2 \) is so large the earth effect will be very small. If the adiabatic approximation fails then the state that arrives at the earth will be a mixture of \( \nu_1 \) and \( \nu_2 \). Except for values of \( \Delta m^2 \) well below \( 10^{-8} \) eV\(^2 \) this mixture will be incoherent \( \dagger \) with a probability \( 1 - P_e \) for \( \nu_2 \) and \( P_e \) for \( \nu_1 \). Here \( P_e \) is the “jumping probability” given approximately by \( \dagger \)
\[
P_e = \frac{e^{-\gamma \sin^2 \theta_V} - e^{-\gamma}}{1 - e^{-\gamma}}, \tag{11}
\]
where
\[
\gamma = 2 \pi r_0 \Delta_0, \quad r_0 = R_{\text{sun}}/10.54 = 6.60 \times 10^4 \text{ km}. \tag{12}
\]
The electron neutrino flux at earth is then
\[
D = (1 - P_e) \sin^2 \theta_V + P_e \cos^2 \theta_V, \tag{13}
\]
and the night flux is given by
\[
N = (1 - P_e) P_{2e} + P_e (1 - P_{2e}) \tag{14}
\]
So one has
\[
N - D = (1 - 2P_e) \left( P_{2e} - \sin^2 \theta_V \right). \tag{15}
\]
Clearly \( N \) is greater than \( D \) if \( P_e < 1/2 \), since \( P_{2e} > \sin^2 \theta_V \) from Eq. (7). Thus the \( N < D \) situation occurs only for \( P_e > 1/2 \).

\[\dagger\]These are analogous to Pauli spin vectors for this 2-component system.

FIG. 1. Schematic view of the evolution of \( \nu_2 \) in matter. The vector 2 representing the initial state of \( \nu_2 \) is precessing around the heavy mass eigenstate \( M \) in matter.

The results may be understood from Fig. 1. The state \( \nu_2 \) is represented by the vector 2 while the heavy eigenvector in matter is \( M \). In matter the vector 2 precesses about the vector \( M \) arriving at \( 2' \) at the midpoint of the precession. Eq. (8) corresponds to
\[
2\theta_M = \pi/2 - \theta_V \tag{10}
\]
and one can see directly that \( 2' \) then coincides with the vector \( \nu_e \).

The sign of the day-night effect now clearly is seen to depend on the fact that the vector \( M \) is displaced from 2 in the direction of \( \nu_e \), which follows from the fact that \( A/\Delta_0 \) is positive. The reason for this is that the matter effect in the sun which makes \( \Delta_0 \) positive has the same sign as that in the earth. It may be noted that this means that if \( \bar{\nu}_e \) were originating instead of \( \nu_e \) the asymmetry would have the same sign.

The approximation that the state emerging from the sun is \( \nu_2 \) may fail for two reasons:

1. \( \Delta m^2 \) is large enough that the matter effect does not dominate even near the center of the sun where the neutrinos originate.
2. The adiabatic approximation fails.

FIG. 2. Probabilities of observing a \( \nu_e \) for a \( \nu_e \) originating from the center of the sun during the day (dashed curve) and night (solid).

In Fig. 2 we show night and day fluxes \( N \) and \( D \), respectively, for the maximum asymmetry case corresponding to Eq. (7). The observed night flux in any experiment depends upon the location, the time of year, and the time of night. Detailed results for different experiments are given in Refs. 2–4. Here to get the qualitative behaviour, we consider the mantle with \( N_e = 2.5 N_A/\text{cm}^3 \) and average over the traveling distance \( ct \) of the neutrinos through the earth between 0 and 1.5\( R_E \), where \( R_E \)
is the radius of the earth. From the figure, one can see that the night flux is greater than the day flux except for very small values of $\sin^2 \theta_V$.

As long as the matter oscillation wave length $\ell_m$ is less than $R_E$, the $\sin^2 \lambda t$ term averages to about $1/2$; since the maximum of the oscillation yields the flux $F_0$, the difference $N - D = \frac{1}{2} (1 - \sin^2 \theta_V)$, which holds for large values of $\theta_V$. When $\ell_m \sim R_E$ there is sensitivity to the oscillations depending upon the particular way the night is defined. This is illustrated for our particular assumption by the oscillation for values between $\sin^2 \theta_V = 0.05$ and 0.5. When $\sin^2 \theta_V$ is small, $\frac{1}{2} (1 - \sin^2 \theta_V) \approx \frac{1}{2}$ and so the oscillations are roughly about a value of $N = \frac{1}{2}$. For neutrinos going through the core there is a more complicated oscillation possibility [3]. For smaller values of $\theta_V$ the value of $\ell_m$ gets much larger than $R_E$ so that $N - D$ decreases rapidly between $\sin^2 \theta_V = 0.05$ and 0.01. Finally, the adiabatic approximation fails for $\sin^2 \theta_V < 0.01$ where the jumping probability $P_e$ significantly rises, and $D$ becomes greater than $N$ for $\sin^2 \theta_V \lesssim 0.001$. The day-night asymmetry defined as

$$A_{DN} = \frac{N - D}{N + D}$$

reaches a minimum value of $-0.007$ for $\sin^2 \theta_V = 0.001$.

In Fig. 3 we show the day-night asymmetry $A_{DN}$ for three values of $\Delta_0$, corresponding to a range of 9 in energy for fixed $\Delta m^2$, or a range of 9 in $\Delta m^2$ for a fixed energy.

For values of $\Delta m^2/2E$ larger than $\Delta_0^{max}$ (dashed curve) the vector $2'$ is above $\nu_e$ as in Fig. 1; as a result, at the peak of the oscillation its $\nu_e$ component is less than maximal. Furthermore, $\theta_M$ is approximately proportional to $\theta_V$ so that as $\theta_V$ gets smaller and the day flux decreases, so does $N - D$. For $\Delta m^2/2E$ smaller than $\Delta_0^{max}$ (dash-dotted curve), the vector $2'$ is below $\nu_e$. The main difference between the dashed and the dash-dotted curves is that the failure of the adiabatic approximation occurs for larger values of $\theta_V$ for the case of smaller $\Delta m^2$.

**Conclusion:** In this paper, we have tried to provide a qualitative understanding of the day-night asymmetry for solar neutrinos, in particular, its sign. We have explored the general behaviour as a function of $\theta_V$ and $\Delta m^2/E$ without consideration of fitting present solar neutrino data. The greater flux at night is seen to be a consequence of the fact that the matter effect in the sun has the same sign as that in the earth. The sign of the asymmetry is reversed only for very small values of $\theta_V$ where the adiabatic approximation fails badly (jumping probability greater than 0.5); the magnitude of the asymmetry with the opposite sign is extremely small.

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