Helmholtz resonator based metamaterials for sound manipulation

K Mahesh¹, R S Mini²
¹M-Tech Student, Department of Mechanical Engineering, College of Engineering Trivandrum, Thiruvananthapuram-16, Kerala, India
²Assistant Professor, Department of Mechanical Engineering, College of Engineering Trivandrum, Thiruvananthapuram-16, Kerala, India

E-mail: maheshnilambur2012@gmail.com
E-mail: miniranjithrs@gmail.com

Abstract: Metamaterials are tailored artificial structures that received wide attention now a days due to their extraordinary properties like negative density, negative bulk modulus, negative permeability, negative refractive index etc. Shape, geometry, size, orientation and arrangement of the basic units in metamaterials give them their smart properties capable of manipulating electromagnetic and acoustic waves by blocking, absorbing, enhancing, or bending waves to achieve benefits that go beyond what is possible with conventional materials. Acoustic metamaterial with Helmholtz resonators (HRs) is considered in this study. The behaviour of acoustic waves in the presence of array of Helmholtz resonators in series and parallel arrangement is studied numerically using finite element method. Series arrangement of Helmholtz resonators resulted in the extraordinary transmission of acoustic waves of certain frequencies and parallel arrangement resulted in the focusing of sound.

1. Introduction
Acoustic-wave propagation is controlled by mass density and bulk modulus of a material. In conventional materials both of these properties exhibits positive value and cannot be easily changed because the material properties are correlated with the chemical composition and bonding structures of the constituent atoms. However, if materials are made up of meta-atoms of sub wavelength scale that can enhance acoustic–matter interaction and hence it is possible for wave properties to obtain values that are not observed in nature. These are known as acoustic metamaterials and they can manipulate and control acoustic waves in ways that are not possible by conventional materials. Metamaterials with zero, or even negative refractive index offer new possibilities for acoustic imaging, lensing and controlling of acoustic waves at sub wavelength scales.

Generally metamaterials are useful for acoustic applications due to their extraordinary ability to manipulate sound and in particular they are useful for ultrasonic imaging. The conventional acoustic imaging using ultrasound exhibits lower resolution. This is due to the quickly diminishing evanescent waves which carry the sub wavelength attributes of materials. To succeed over this scenario, an acoustic super lens was proposed by Pendry et al. [1], which offered high resolution imaging of an object. This super lens is based on focusing the acoustic wave and retrieving the evanescent wave data through a flat negative-index slab. The negative bulk modulus system was experimentally demonstrated by Fang et al. [2]. They conducted an underwater ultrasonic experiment to study the transmission behavior of
Helmholtz resonators in series. Koju et al. [3] numerically demonstrated that a rigid barrier embedded with a Helmholtz Resonator can result in the extraordinary transmission of sound and it solely depends on the structure of the resonator. When acoustic waves pass through holes of sub wavelength scale, extraordinary acoustic transmission (EAT) was observed by them at resonant frequencies. They also created an acoustic lens using an array of Helmholtz resonators. Later the EAT through a solid barrier with an embedded Helmholtz resonator is experimentally demonstrated by Crow et al. [4]. From experiments they measured the amplitude and phase of transmitted wave. Maznev et al. [5] developed a metamaterial made up of soda cans that can focus sound at resonance frequency. They observed high transmission at the Helmholtz resonance frequency of the metamaterial.

Acoustic superlens was designed by Yang et al. [6] using Helmholtz-resonator-based metamaterials that can focus sound over a broad band of frequency with high resolution. They also conducted experiments which produce high resolution images of a double slits over a frequency range of 570 to 650 Hz. Wang et al. [7] developed an acoustic lens using tapered labyrinthine unit cells with sub wavelength thickness, planar profile, and broad operation bandwidth.

Later Amireddy et al. [8] experimentally demonstrated deep sub wavelength ultrasonic imaging of defects in metallic samples with a resolution of \( \frac{\lambda}{25} \) using holey-structured metamaterial lenses. In the same manner, Laurety et al. [9] developed an acoustic lens by using additive 3D printing technique. A gradient acoustic metasurface is designed by Lan et al. [10] to manipulate acoustic wave front freely. High resolution imaging was achieved by them by using metasurface made up of series of unit cells.

2. Methodology

Sound transmission characteristics of array of HR embedded acoustic metamaterial is studied using finite element software, COMSOL multiphysics. Enhanced transmission of sound was observed by Koju et al. when a Helmholtz resonator was embedded within a rigid barrier. Extraordinary transmission occurs at the resonance frequency and the transmitted waves goes through a phase change between 0 and \( \pi \). In this study, influence of arrangement of double neck HR (necks on both sides of the cavity) arrays, in series and parallel modes within a rigid barrier is considered for the analysis. The HR system consists of two necks of length, \( l \), and cross-sectional area \( A \). The two necks are protruding from a closed cavity of volume, \( V \), as shown in figure 1(a). It is similar to a simple spring- mass system because the air in the neck acts as a mass and the air in the cavity act as a spring. The resonant frequency of the double necked Helmholtz resonator system is given by,

\[
fr = \frac{v}{2\pi} \sqrt{\frac{2A}{L'V}}
\]  

(1)

where \( v \) is the speed of sound in air and \( L' \) is the corrected neck length. The corrected neck length, \( L' \), is longer than the actual neck length, \( l \), and it varies with the radius of neck opening, \( r \). \( L' \) is given by,

\[
L' = l + 1.5r
\]  

(2)

For validation, two configurations of HRs used by Koju et al [4] was simulated using COMSOL multiphysics and the results are compared. Each of them consists of a double necked HR embedded in a reflecting barrier. The resonant frequency, determined for each case using equation (1) is same. Each configuration of HR differed by the arrangement of necks. In the first case, the two necks are in line where as in the second case the necks are at offset position (figure 1(b)). The amplitude and phase of the transmitted waves is extracted using COMSOL multiphysics. For simulations a 2.6 m long waveguide was modeled with the HR positioned at the center of the waveguide. The geometric parameters used for creating the model is given in Table 1. The wave guide along with the resonator is shown in figure 1(c).
Table 1. Geometrical parameters of model

| Model Parameters       | Value    |
|------------------------|----------|
| Cavity length          | 0.02 m   |
| Cavity diameter        | 0.1 m    |
| Neck length            | 0.05 m   |
| Neck radius            | 0.0079 m |
| Waveguide length       | 2.6 m    |
| Waveguide radius       | 0.05 m   |

Figure 1- Schematic view of a) Helmholtz resonator with straight arrangement of necks. b) Helmholtz resonator with offset arrangement of necks c) waveguide with Helmholtz resonator embedded

In the simulations, the walls of HR and waveguide are selected as sound hard boundary walls. The simulation is carried out in a discrete frequency range of 200 Hz to 500 Hz in steps of 10 Hz. For all the simulations the highest frequency of wave chosen is well below the cut-off frequency, \( f_c \)

\[
f_c = \frac{0.92 \nu}{\pi d}
\]

where \( d \) is the diameter of waveguide.

A plane acoustic wave of 0.2 Pa amplitude is given as the input at the left end of the waveguide, and the right end of the waveguide is set as a non-reflecting boundary (matched boundary condition), from which output is taken. The amplitude and phase of the output signal is determined from the complex transmission function by dividing the complex output signal obtained from the waveguide in the presence and absence of Helmholtz resonator. For the finite element simulation free tetrahedral element is used as the mesh element with an element size of 0.015m.

At each frequency, the output from an empty wave guide is taken as a reference signal \( S_{\text{ref}}(f) \). Then at same set of frequencies the output signal from the waveguide with HR is taken \( S_{HR}(f) \). \( T(f) \) is the complex transmission function obtained from the ratio,

\[
T(f) = \frac{S_{HR}(f)}{S_{\text{ref}}(f)}
\]

The magnitude of transmission function represents the transmission amplitude at each frequency whereas the phase of transmission function gives the relative phase delay. For both configurations of Helmholtz resonator (offset and straight arrangement of necks) the graph between transmission and frequency is plotted in figure 2. The results obtained for Koju et al [4] is also plotted in the figure. As expected peak frequency is obtained at 350 Hz and it is very much similar to the theoretical value (349 Hz). From this it is clear that the EAT of metamaterial is solely based on structural parameters (cavity length, cavity radius, neck radius, neck length) of HR embedded barrier and resonance frequency. It does not depend up on the orientation of necks and line of sight of acoustic waves. By this validation we confirmed that our numerical model is correct. So we expanded our study to different arrangement of Helmholtz resonators.
2.1. Horizontal array of Helmholtz resonators with normal arrangement and offset arrangement of necks

In order to study the effect Helmholtz resonator array on the sound transmission characteristics, seven HRs are arranged in series for both normal (figure 3 (a)) and offset placement of necks (figure 3 (b)). Similar to the earlier study a plane acoustic wave of 0.2 Pa amplitude is given as the input at the left end, where the at the right end is set as a matched boundary, from which output is taken. The transmittance through wave guide is measured using the equation (4). The simulation is carried out in a discrete frequency range of 200 Hz to 500 Hz in steps of 10 Hz.

![Figure 2](image_url)

Figure 2- Comparison between simulation and literature results of normal HR arrangement and offset HR arrangement.

2.2. Vertical arrangement of Helmholtz resonators

The influence of vertical arrangement of Helmholtz resonators is studied by placing seven Helmholtz resonators in parallel mode in a rectangular waveguide. The cross sectional dimensions of the waveguide is 0.088 m x 0.62 m. All seven HR has equal cavity volume of $1.57 \times 10^{-4} m^3$. One neck is protruding from each side of the resonator barrier with a cross sectional area of $2 \times 10^{-4} m^2$. The neck is cylindrical in shape with a radius 0.0079 m. The neck length of HRs reduces from the center HR to the outer HRs with lengths 5.96 cm, 5.64 cm, 5.28 cm and 5.00 cm respectively. The acoustic waves passes through the central HR (having the highest neck length), is phase delayed compared to the acoustic waves passes through the outer HRs results in focusing. The schematic diagram of HRs in the rectangular waveguide, in vertical arrangement, is shown in figure 4.

![Figure 3](image_url)

Figure 3- Helmholtz resonators for a) straight arrangement of necks b) offset arrangement of necks
3. Result and Discussion

In the case of horizontal array of Helmholtz resonators extraordinary transmission is observed for both configuration at four different frequencies. For normal arrangement of necks EAT occurs at 268 Hz, 344 Hz, 406 Hz and 448 Hz. For each of these frequencies corresponding transmittance values are 0.75, 0.88, 0.92, and 0.91 respectively. The increase and decrease behaviour of transmittance is mainly due to the phase shift occurring before and after the Helmholtz resonance frequency. Diffractive interference is the reason behind the continuous phase shift of acoustic waves. For offset arrangement of necks, EAT occurs at 254 Hz, 333 Hz, 398 Hz and 438 Hz. For each these frequencies corresponding transmission values are 0.59, 0.94, 0.55, and 0.78 respectively. Similar to the normal arrangement it also exhibits continuous phase shift and it leads to the sudden decline in transmittance within the extraordinary transmission frequencies.

An acoustic lens is created and simulated its frequency response for a frequency range of 200-500 Hz. At 338 Hz the lens showed extraordinary transmitting and focusing capabilities. The pressure intensity plot obtained from the simulation is shown in figure 6. Plane wave is given as input at the left end of the waveguide and the focusing happens at a distance of 1.33 m in front of the lens.
4. Conclusion
Straight as well as offset arrangement of necks resulted in the transmission of acoustic waves over a broad range of frequencies. By increasing number of Helmholtz resonators in the horizontal arrangement we can increase the number of transmitted frequencies with extraordinary transmission. The vertical arrangement showed more practical significance. Since this kind of configuration enabled the focusing of sound to a particular point. But this focusing characteristics is observed over a narrow band of frequency. An important practical application of this type of arrangement is in the field of defect detection, which require the persistence of focusing over a broader range.

5. References
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