Critical current density of domain wall oscillation due to spin-transfer torque

Tomohiro Taniguchi\(^1,2\)† and Hiroshi Imamura\(^1\)∗

\(^1\) Nanosystem Research Institute, National Institute of Advanced Industrial Science and Technology, 1-1-1 Umezono, Tsukuba, Ibaraki 305-8568, Japan
\(^2\) Institute of Applied Physics, University of Tsukuba, 1-1-1 Tennou-dai, Tsukuba, Ibaraki 305-8573, Japan

E-mail: †tomohiro-taniguchi@aist.go.jp, ∗h-imamura@aist.go.jp

Abstract. The domain wall oscillation due to spin-transfer torque was studied by numerically solving the Landau-Lifshitz-Gilbert (LLG) equation. For a domain wall whose rotation angle \(\theta_{\text{max}}\) is less than 180°, we found the existence of the critical current density above which the magnetization dynamics are induced. We studied the dependence of the critical current density on the rotation angle \(\theta_{\text{max}}\) and found that the critical current density is proportional to \(180° - \theta_{\text{max}}\).

The domain wall oscillation due to spin-transfer torque has drawn great interest because of its potential application to spin-electronics devices such as spin-transfer torque driven nano-oscillator [1, 2]. Franchin \textit{et al.} studied the dependence of the oscillation frequency \(\nu\) of the 180° domain wall on the electric current density \(j_e\) by solving the Landau-Lifshitz-Gilbert (LLG) equation both numerically and analytically, and found that \(\nu \propto j_e\) and \(\nu \propto j_e^2\) for the small and large current regions, respectively [1]. They also found the static compression of the domain wall where the spatial variation of the magnetization \(M, |\partial M/\partial x|\), is constant in time.

To manipulate the spin-transfer torque driven nano-oscillator, the domain oscillation should be converted to the oscillation of the electric resistance. It is well known that the magnetoresistance effect due to a domain wall is proportional to \(1/d^2\), where \(d \propto 1/|\partial M/\partial x|\) is the thickness of the wall [3, 4]. Since the compression of the wall is constant in time for 180° wall, as shown by Franchin \textit{et al.}, the oscillation of the 180° wall cannot be detected electrically. The magnetoresistance effect due to the domain wall in the current-confined-path (CCP) current-perpendicular-to-plane (CPP) spin valve has been studied extensively [5–10]. In CCP-CPP spin valve, the rotation angle of the domain wall is easily controlled by applying the magnetic field [11]. Recently, by solving the LLG equation numerically, Matsushita \textit{et al.} studied the oscillation of the thickness of the domain wall confined in the CCP-CPP spin valve when the rotation angle of the domain wall is less than 180° [2]. Their results open the possibility of the realization of the spin-transfer torque driven nano-oscillator.

In this paper, we study the domain wall oscillation due to spin-transfer torque where the rotation angle of the wall \(\theta_{\text{max}}\) is less than 180°. By numerically solving the LLG equation, we found the existence of the critical current density above which the magnetization dynamics in the wall are induced. We study the dependence of the critical current density on the angle \(\theta_{\text{max}}\), and found that the critical current density is proportional to \(180° - \theta_{\text{max}}\). The dependence of
the critical current density on the Gilbert damping constant and the width of the system are also studied.

We consider the spin-transfer torque driven magnetization dynamics in the one-dimensional domain wall where the rotation angle of the wall is given by $\theta_{\text{max}}$, as shown in Fig. 1. The domain wall lies over $0 \leq x \leq L$, where $L = Na$ is the width of the system, and $N + 1$ and $a$ are the number of the magnetizations in the wall and the lattice constant, respectively. The magnetizations in the wall are coupled via the exchange interaction, and without the electric current, the direction of the magnetizations in the wall are coupled via the exchange interaction, and without the electric current density is $j_e$. As shown in Fig. 2 (a), the steady precession of the $x$ and $y$-components of the magnetizations is induced where the precession frequency is $11.9$ GHz. The magnetizations in the wall are compressed along the $x$-direction, as

Figure 1. The schematic view of the 1-dimensional domain wall in which the rotation angle of the wall is $\theta_{\text{max}} \leq 180^\circ$. The electric current $j_e$ is applied along to the $x$-axis.
Figure 2. (a) The time evolution of the domain wall dynamics above the critical current density. (b) The magnetization alignment at $t = 0$ (dotted line) and $t = 10$ ns (solid line) above the critical current density. (c) The time evolution of the domain wall dynamics below the critical current density. (d) The magnetization alignment at $t = 0$ (dotted line) and $t = 10$ ns (solid line) below the critical current density.

shown in Fig. 2 (b). These dynamics are similar to those found in Ref. [1]. On the other hand, by decreasing the electric current density, the magnetization dynamics relax and stop in a few nano-second, as shown in Fig. 2 (c), where $j_e = 11.4 \times 10^{10}$ A/m$^2$. In this case, although the magnetization in the wall first rotate around the $z$-axis, as shown in Fig. 2 (d), the wall is not compressed. From Figs. 2, it was found that the electric current density should be larger than a certain value (critical current density) to induce the steady precession of the magnetizations. It should be noted that the critical current density of the 180$^\circ$ wall is zero [1].

The physics behind the above results are as follows. Before applying the electric current, the magnetizations inside the wall vary uniformly, i.e., the relative angle between the neighboring magnetizations is constant, $\theta_{\text{max}}/N$, and the exchange energy $\propto -\sum_{i=0}^{N-1} \mathbf{m}_i \cdot \mathbf{m}_{i+1}$ is minimized. By applying the electric current, due to the spin-transfer torque, the $x$ and $y$-components of the magnetizations try to rotate around the $z$-axis, as shown in Figs. 2 (d). Then, for $\theta_{\text{max}} < 180^\circ$, the relative angles between the magnetizations at the boundaries ($x = 0, L$) and that inside the wall ($0 < x < L$) increase compared to the equilibrium alignment, which increases the exchange energy of the system. Thus, to induce the magnetization dynamics, the electric current density should be larger than the critical value which is determined by the increase of the exchange energy. This is the origin of the critical current density. On the other hand, when $\theta_{\text{max}} = 180^\circ$, the relative angle, and therefore the exchange energy in the wall is constant during the rotation of the magnetization around the $z$-axis, and thus, the critical current density is zero.

We studied the dependence of the critical current density on the rotation angle $\theta_{\text{max}}$, the Gilbert damping constant $\alpha$, and the width of the system $L$ numerically, as shown in Figs. 3 (a), (b), and (c), respectively. From Fig. 3 (a), where $\alpha = 0.02$ and $L = 30$ nm, it is clear that
The critical current density is proportional to $180^\circ - \theta_{\text{max}}$, and is zero for $\theta_{\text{max}} = 180^\circ$. The value of the current density for $\theta_{\text{max}} = 170^\circ$ is more than $50 \times 10^{10}$ A/m², which is comparable to the current density available in experiments. From Fig. 3 (b), where $\theta_{\text{max}} = 178^\circ$ and $L = 30$ nm, we find that the critical current density is approximately proportional to the Gilbert damping constant $\alpha$, and remains finite in the limit of $\alpha \to 0$. As shown in Fig. 3 (c), where $\theta_{\text{max}} = 178^\circ$ and $\alpha = 0.02$, the critical current density increases rapidly with decreasing the width of the system $L$, and reach more than $10^{12}$ A/m² when $L$ is on the order of a few nano-meter. Since this value is much higher than the electric current density available in experiments, we cannot expect to observe the domain wall oscillation for a sufficiently thin wall.

In conclusion, we studied the domain wall oscillation due to spin-transfer torque by numerically solving the LLG equation. For a domain wall whose rotation angle $\theta_{\text{max}}$ is less than $180^\circ$, we found the existence of the critical current density above which the magnetization dynamics are induced. We also found that the critical current density is (i) proportional to $180^\circ - \theta_{\text{max}}$, (ii) approximately proportional to $\alpha$, and (iii) increased rapidly with decreasing the width of the system.

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