Implication of Dark Matter in Dwarf Spheroidal Galaxies

Hiroyuki Hirashita,1∗ Hideyuki Kamaya,2∗ and Tsutomu T. Takeuchi1∗

1 Department of Astronomy, Faculty of Science, Kyoto University, Sakyo-ku, Kyoto 606-8502

E-mail(HH): hirasita@kusastro.kyoto-u.ac.jp

2 Department of Physics, Faculty of Science, Kyoto University, Sakyo-ku, Kyoto 606-8502

(Received 1999 February 12; accepted 1999 April 9)

Abstract

We examine the correlation between physical quantities to explore the existence of dark matter (DM) in the Local Group dwarf spheroidal galaxies (dSphs). In order to clarify whether DM exists in the dSphs we compare two extreme models of their internal kinematics: [1] a tidal model (the mass of a dSph is estimated with the luminous mass, since the observed large velocity dispersions of the dSphs are attributed to the tidal force by the Galaxy), and [2] a DM model (the mass of a dSph is estimated with the virial mass, since the velocity dispersion is considered to reflect the DM potential). In both models, we find that the relation between the surface brightness of the dSphs and the tidal force by the Galaxy is consistently interpreted. This makes a critical remark about the previous studies that concluded that the tidal force is effective for the dSphs based only on model [1]. We also check the correlation between the Galactocentric distance and the tidal force. Consequently, both models are also supported in this test. Thus, we are unable to judge which of the two is more promising for the dSphs in correlation investigations. The physical process of tidal disruption is also considered. Since the timescale for the tidally perturbed stars to escape is much shorter than the orbital periods of the dSphs, it is difficult for the tidally perturbed dSphs to exist as an assembly. Thus, we suggest that the gravitational field of DM is necessary to bind the dSphs with a large velocity dispersion.

Key words: Dark matter — Galaxies: elliptical — Galaxies: evolution
1. Introduction

Recent observations have been revealing the properties of the Local Group dwarf spheroidal galaxies (dSphs). The dSphs have luminosities of order $10^5$–$10^7 L_\odot$, and are characterized by their low surface brightnesses (Gallagher, Wyse 1994 for review). The observations of such faint objects are important for several reasons. For example, in a galaxy-formation theory based on the cold dark-matter (CDM) model, low-mass galaxies are considered to be the first bound luminous objects (e.g., Blumenthal et al. 1984). Thus, the dSphs are expected to have a large amount of the CDM (see also Flin 1999 for an observational approach).

To date, the stellar velocity dispersions of dSphs have been extensively measured (e.g., Mateo et al. 1993 and references therein), which, in general, indicate a too large mass to be accounted for by the visible stars in the dSphs. In other words, dSphs generally have high mass-to-light ratios ($M_{\text{vir}}/L_V$, where $M_{\text{vir}}$ is the virial mass calculated from observed size and stellar velocity dispersion and $L_V$ is the total luminosity at the $V$ band; the 6th column in table 1). This fact can imply the presence of dark matter (DM) in these systems (e.g., Mateo et al. 1993). The existence of DM is also supported by the large spatial distribution of stars in their outer regions (Faber, Lin 1983; Irwin, Hatzidimitriou 1995).

However, the above arguments may be challenged if we take into account the tidal force exerted by the Galaxy. If a dwarf galaxy orbiting the Galaxy is significantly perturbed by the Galactic tidal force, the observed velocity dispersion of the dwarf galaxy can be larger than the gravitational equilibrium dispersion (Kuhn, Miller 1989; Kroupa 1997). Moreover, the results in Kuhn (1993) imply that the lifetimes of internally unbound satellites might be longer than one orbital period (but see Johnston 1998). The timescale for such an unbound object to survive is estimated later in this paper (subsubsection 3.2.1). This tidal picture of the dSphs also suggests that the large velocity dispersions do not necessarily show the existence of DM (but see Piatek, Pryor 1995; Oh et al. 1995). We will consider whether the observed large mass-to-light ratios of dSphs are really due to the tidal effect.

The plan of this paper is as follows. First of all, in the next section, we present correlations among the observed quantities. We summarize our results in section 3. In the same section, some discussions are also made, and we comment about the environmental effects on the dSphs.

* Research Fellow of the Japan Society for the Promotion of Science.
2. Correlation among Observed Quantities

2.1. Surface Brightness and Galactocentric Distance

In this section, we consider what determines the physical condition of the dSphs. We re-examine the correlation which Bellazzini et al. (1996; hereafter B96) have presented.

First, a dimensionless form of the Galactic tidal force is defined. The Galactic potential is modeled by a spherical dark halo with a flat rotation curve of amplitude $V_{\text{rot}}$ (Honma, Sofue 1997 for the latest results). We assume that the dark halo extends up to $R_{\text{GC}} = 100$ kpc. With the above flat-rotation model of the Galactic potential, the mass of the Galaxy within the Galactocentric distance $R_{\text{GC}}$ is expressed as

$$M_G(R_{\text{GC}}) = \begin{cases} V_{\text{rot}}^2 R_{\text{GC}} & \text{for } R_{\text{GC}} \leq 100 \text{ kpc}, \\ 1.1 \times 10^{12} M_\odot & \text{for } R_{\text{GC}} \geq 100 \text{ kpc}, \end{cases}$$

where we assumed $V_{\text{rot}} = 220 \text{ km s}^{-1}$ (e.g., Burkert 1997).

We consider a spherical satellite galaxy orbiting the Galaxy. The tidal force which a star (whose mass is $m_*$) at the core radius ($r_c$; the radius where the surface brightness falls to half its central value; Binney, Tremaine 1987, p. 25) of the satellite galaxy experiences is approximately

$$F_T = \frac{G m_* M_G}{R_{\text{GC}}} r_c.$$  \hspace{1cm} (2)

The star also experiences a gravitational binding force exerted by the satellite galaxy, which is described as

$$F_B = \frac{G m_* M_s}{r_c^2},$$  \hspace{1cm} (3)

where $M_s$ is the total mass of the satellite galaxy. We then define a dimensionless tidal force identical to the definition by B96,

$$F_{T,B} = \frac{F_T}{F_B} = \left( \frac{M_G}{M_s} \right) \left( \frac{r_c}{R_{\text{GC}}} \right)^3.$$  \hspace{1cm} (4)

We estimate $M_s$ in the following two different ways:

[1] Tidal model: The large velocity dispersion of dSphs is attributed not to deep DM potentials, but to significant perturbation by the Galactic tides.

[2] DM model: A significant amount of DM exists in a dSph, and virial equilibrium in the DM potential is realized.

For the tidal model, the mass of every dSph is estimated in the following way:

$$M_s = M_L,$$  \hspace{1cm} (5)
where $M_L$ is the luminous mass calculated directly from the total luminosity at the $V$ band by assuming $M_L/L_V = 1$ (in solar units). The difference in the assumed value of $M_L/L_V$ does not affect the analyses in this paper as long as it is constant. For the DM model,

$$M_s = M_{\text{vir}},$$

where $M_{\text{vir}}$ is the virial mass derived from the observed central velocity dispersion by assuming virial equilibria (Pryor, Kormendy 1990; 4th column of table 1). We assume in the DM model (i.e., when estimating the virial mass) that mass traces light (e.g., Pryor 1992). Though we cannot judge by the data available at present whether this assumption is true for dSphs, we know that mass does not follow light in more luminous galaxies (e.g., Binney, Tremaine 1987, chap. 2). However, the assumption has been made by many previous investigations, so we make it in order to make comparisons with them.

Figure 1 shows the correlation between $\mu_V(0)$ (central surface brightness in $V$ band) and $\log F_{T,B}$ for the tidal model. We calculate the linear correlation coefficient, $r$, by

$$r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}},$$

where $(\bar{x}, \bar{y})$ is the average of all data points $(x_i, y_i)$ and $\sum_i$ means the summation of all the data. The correlation coefficient is $r = 0.91$ for the data points of the tidal model, which indicates a strong correlation. (If $r > 0.71$ for 8 data, the two quantities are correlated with a confidence level of 95%.). Thus, B96 insisted that the Galactic tidal effect had settled the present ranking in the dSph central brightness. We have indeed confirmed this result.

Here, the DM model is tested by the same method as B96, except for mass estimations. We derive the mass by using the virial theorem, $M_s = M_{\text{vir}}$. Figure 2 shows the relation between $\mu_V(0)$ and $F_{T,B}$ for the DM model. In this model, we find that the two quantities are less correlated ($r = 0.48$) than in the tidal model. A lower correlation is naturally expected for the DM model, since it is the DM in a dSph, not the tidal force of the Galaxy, that determines the dynamical condition of the dSphs.

We note that B96 only tested the tidal model. Since the DM model as well as the tidal model can be consistently interpreted by the same method as B96, we cannot conclude that the tidal force is effective in determining the physical conditions for the dSphs. Indeed, B96 commented in their subsection 3.3 that their analyses do not necessarily show that that tidal force is important for the dSphs.

Thus, there is nothing to choose between the two models in the $\mu_V(0)-F_{T,B}$ relations. We, therefore, need further
considerations to determine which of the two models is applicable for dSphs. The further examination is discussed in the next subsection.

2.2. Tidal Force and Galactocentric Distance

In this subsection we present a correlation between the dimensionless tidal force (\( \log F_{T,B} \)) and the Galactocentric distance (\( \log R_{GC} \)) for each model. This correlation is shown in figures 3 and 4 for the tidal model and the DM model, respectively.

2.2.1. The \( \log F_{T,B} - \log R_{GC} \) relation: results

First, the \( \log F_{T,B} - \log R_{GC} \) relation is plotted in figure 3 using the tidal model (\( M_s = M_L \)). In this model, the correlation coefficient is \( r = -0.94 \) and the linear regression is

\[
\log F_{T,B} = -4.12 \log R_{GC} + 6.55.
\]  

We obtain a strong correlation between the two quantities. One interpretation of the regression is that the tidal force is more effective for dSphs nearer to the Galaxy. However, this interpretation is not unique, as shown in subsubsection 2.2.2.

Next, the same relation is presented in figure 4 by using the DM model (\( M_s = M_{vir} \)). The two quantities are marginally correlated (\( r = -0.68 \)). The linear regression for this model becomes

\[
\log F_{T,B} = -1.50 \log R_{GC} - 0.105.
\]

2.2.2. Interpretation of the regressions

In the tidal model, the correlation presented in the previous subsubsection is stronger than that in the DM model. The regression and the stronger correlation in the tidal model between \( F_{T,B} \) and \( R_{GC} \) (figure 3) may mean that the tidal forces are more effective for dSphs located nearer to the Galaxy (subsubsection 2.2.1), but there is a known correlation between the luminosity and the Galactocentric distance, both of which are directly observable. Since we determine the mass of a dSph by luminosity in the tidal model, the strong correlation can reflect a correlation between luminosity and the Galactocentric distance. Indeed, B96 (their figure 1) showed that \( \mu_V(0) \) and \( R_{GC} \) are correlated. Because \( F_{T,B} \), comprising a few observed quantities, is not directly observable, the good correlation
between \( F_{T,B} \) and \( R_{GC} \) has only a secondary meaning, showing one of the interpretations on the relation among observed quantities.

According to Caldwell et al. (1998), the relations among the surface brightness, luminosity, and metallicity of dSphs belonging to the M81 Group show no difference from the relations of the Local Group dSphs. This may imply that the physical properties of dSphs are determined by their intrinsic conditions, not by their environments. However, there is still a possibility that the environments influence the dSphs so that the relations among surface brightness, luminosity, and metallicity are conserved.

In the DM model, we find that in equation (9)

\[ F_{T,B} \propto R_{GC}^{-1.50}. \]

The error (\( \sigma_c \)) of the slope of regression (9) is estimated from the propagation of error (e.g., Hoel 1971),

\[ \sigma_c = \sqrt{\frac{\sigma^2 \Delta}{n}}. \]

where \( n \) and \( \sigma^2 \) mean the number of data and the dispersion of data \( \log F_{T,B} \), respectively. Here, \( \Delta \) is defined by

\[ \Delta \equiv n \sum_{i=1}^{n} R_i^2 - \left( \sum_{i=1}^{n} R_i \right)^2. \]

The data set \( \{ R_i \}_{i=1}^{n} \) is composed of \( n \) (here, \( n = 8 \)) data of \( R_{GC} \); \( \{ R_i \}_{i=1}^{n} \).

Using directly observable quantities, \( F_{T,B} \) is expressed as

\[ F_{T,B} \propto \frac{\theta^2}{v^2}, \]

where we used the relations \( M_G \propto R_{GC}, M_s \propto r_c v^2 \) (virial equilibrium), and \( \theta = r_c/R_{GC} \) (\( v \) and \( \theta \) mean the internal velocity dispersion and the angular size of a dSph, respectively). Assuming the error of \( \theta \) and \( v \) to be 10\% and 20\%, respectively (Mateo et al. 1993), \( \sigma_F \) is estimated as \( \sigma_F \simeq 0.24 \). This leads to \( \sigma_c \simeq 0.40 \). Thus, we obtain the regression

\[ F_{T,B} \propto R_{GC}^{-1.50 \pm 0.40}. \]

If the flat rotation curve of the Galaxy is taken into account, \( M_G \propto R_{GC} \). Using equation (10), we obtain

\[ F_{T,B} \propto M_s^{-1} R_{GC}^{-2}, \]

where we define the mass density of satellite dwarf galaxies as

\[ \rho_s = \frac{M_s}{r_c^3}. \]
If the density of a dSph is determined by DM contained in itself, $\rho_s$ is independent of $R_{GC}$. Using relations (14) and (15), we obtain

$$\rho_s \propto R_{GC}^{-0.50\pm0.40}.$$  

(17)

This shows that it cannot be determined whether $\rho_s$ depends on $R_{GC}$ strongly or weakly. We need a larger sample of galaxies to judge whether the density of the dSphs is internally or environmentally determined.

Burkert (1997) also examined the relation between the mass density and the Galactocentric distance with a smaller sample than ours. This argument supports both the tidally perturbed picture and the DM-dominated picture of the dSphs.

In summary, we have no evidence which model gives a better kinematical picture of the dSphs. This implies that a statistical study of the physical quantities of the dSphs may be unable to prove the existence of DM in the dSphs. Thus, a straightforward examination of the physical process of tidal disruption is necessary. The subject of tidal heating has been investigated in many researches (e.g., Weinberg 1994; Kundić, Ostriker 1995; Gnedin, Ostriker 1997).

3. Summary and Discussions

3.1. Summary

We re-examined the correlations between the physical quantities of the Local Group dSphs. We estimated the mass of the dSphs by the following two ways:

[1] tidal model (the mass of a dSph is estimated with the luminous mass), and

[2] DM model (the mass of a dSph is estimated with the virial mass).

In the tidal model, there is strong correlation between the surface brightnesses of the dSphs and the tidal force by the Galaxy, as shown by B96. In the DM model, as expected, a correlation is barely found between the two quantities. This is consistent for the DM model, since the internal kinematic conditions of dSphs are determined only by their internal DM. We also examined the correlation between the Galactocentric distance and the dimensionless tidal force defined in equation (4). Here, we obtained a strong correlation in the tidal model, while we found a weak correlation in the DM model. For the two models, the strengths of the derived correlations support both models. In other words, we cannot determine which of the two is superior by examining the correlations mentioned
above. We note that B96 does not necessarily assert that the tidal force is the dominant factor for determining the kinematical conditions in the dSphs (their subsection 3.3). However, many papers cite B96 to show that the tidal forces determine the structure of the dSphs based only on their subsections 2.3–2.4 and figure 3. We stress again that we can never judge the importance of the tidal effect from figure 3 of B96.

Our approach is an alternative to the discussion by Burkert (1997), who compared the DM-dominated picture and tidally perturbed picture of the Local Group dSphs using a smaller sample than ours. We have demonstrated that the superiority of the model can never be judged from statistical analyses of the presently available data. Thus, we should resort to methods other than statistical methods.

3.2. Discussions

3.2.1. Tidal origin of the large velocity dispersions?

If circular orbits are assumed for the dSphs, the tidal forces from the Galaxy are constant in time. Both figures 3 and 4 show that $F_{T,B} < 0.1$ for all dSphs, even when the tidal model is adopted. Hence, the effect of tidal heating proves to be too small to account for the large velocity dispersions in the dSphs if the orbits are circular. The same view is found in figure 1 of Pryor (1996).

Tidal heating may be effective for the elliptical orbit with a sufficiently small perigalactic distance. However, Piatek and Pryor (1995) showed by numerical simulations that the velocity dispersions of dSphs are little affected by tidal forces, even when the tidal forces effectively disrupt the outer regions of the dSphs. Moreover, the escape timescale, $t_{\text{esc}}$, defined by the time needed for the tidally heated star to escape to $l = 1$ kpc from a dSph, is estimated as

$$t_{\text{esc}} = 10^8 \left( \frac{l}{1 \text{kpc}} \right) \left( \frac{v_{\text{esc}}}{10 \text{ km s}^{-1}} \right)^{-1} \text{ yr},$$

(18)

where $v_{\text{esc}}$ is the escape velocity of the dSph. This estimate is consistent with that of Johnston (1998). The estimated escape velocity is an order of magnitude smaller than the orbital timescale of dSphs ($\sim 1$ Gyr). Thus, the significantly tidal-perturbed stars in dSphs go their separate ways and cannot be observed as an assembly. Therefore, we suggest that tidal disruption seems to be unlikely to explain the large velocity dispersions of the dSphs.

There have been some simulations which indicated that the dSph systems are tidally disrupted in the age of the Universe by the collective effect of successive passages of the pericenter (Kuhn, Miller 1989; Oh et al. 1995; Kroupa
We note that in the early epoch of the Universe, when the scale factor of the Universe is smaller than that of today and the expansion of the Universe is not negligible, the distance between a dSph and the Galaxy is smaller, which leads to a stronger tidal force. Thus, further simulation is needed to take into account the expansion effect (see Mishra 1985 for treatment of the cosmic expansion). If such an effect is included, the dSphs may be quickly disrupted in the galaxy-formation epoch without the presence of the DM to bind themselves.

For further discussions and reviews concerning the tidal heating of dSphs, see Pryor (1996).

### 3.2.2. Other environmental effects?

The ram pressure of the gaseous Galactic halo may have an influence on the dSphs. Here, we briefly estimate the ram pressure force of the halo gas acting on the orbiting dSphs.

Songaila (1981) estimated the mass density of the gaseous halo to be $\rho_{\text{halo}} \sim 10^{-27} \text{ g cm}^{-3}$. Using this value, the ram pressure force by the gaseous halo is estimated as

$$\rho_{\text{halo}} V_{\text{rot}}^2 (\pi r_c^2) \sim 10^{30} \left( \frac{\rho_{\text{halo}}}{10^{-27} \text{ g cm}^{-3}} \right) \left( \frac{V_{\text{rot}}}{220 \text{ km s}^{-1}} \right)^2 \left( \frac{r_c}{300 \text{ pc}} \right)^2 \text{ dyn}. \quad (19)$$

Since the tidal force by the Galaxy at the perigalactic distance $R_p$ is estimated as (equation 2)

$$\frac{G M_G M_s r_c}{R_p^3} = V_{\text{rot}}^2 M_s r_c \sim 10^{30} \left( \frac{M_s}{10^7 M_\odot} \right) \left( \frac{r_c}{300 \text{ pc}} \right) \left( \frac{R_p}{30 \text{ kpc}} \right)^{-2} \text{ dyn}, \quad (20)$$

the ram pressure is comparable to the tidal force. This means that even in the situation where the tidal force is effective, the ram pressure is also effective. Therefore, we should not conclude that only the tidal force has established the present surface brightnesses of the dSphs. It may be possible that the density gradient of the gaseous Galactic halo may produce the correlation which B96 presented. Einasto et al. (1974) pointed out the correlation between the morphological types of companion galaxies and their distances from the parent galaxies. They also suggested that the density of coronal gas in the halo $\rho_c$ is proportional to $R_{\text{GC}}^{-2}$.

Actually, we need a more precise treatment of the ram pressure, which takes into account various factors for dSphs; the fraction of the gas mass to the total mass, the internal distribution of the gas, the form of the gas (dense or diffuse state), etc. The process of ram-pressure stripping is extensively investigated by numerical simulations by Portnoy et al. (1993). The application of this process to the Galactic halo should be addressed by future work.

A correlation may be produced in the formation epoch of the Local Group. For example, a Galactic wind in its initial starburst phase may affect the star-formation histories of dSphs (van den Bergh 1994; Hirashita et al.
Since different star-formation histories mean different ‘supernova histories,’ and supernovae may have played a dominant role in determining the structure parameters (for example, luminosity profiles: Larson 1974; Saito 1979; see also Hirashita 1999), the difference in star-formation histories may lead to the correlation which B96 pointed out.

We note that in the DM context of the dSphs, the suggestion by Hirashita et al. (1997) has another meaning. According to Mac Low and Ferrara (1999), dwarf galaxies cannot necessarily lose all of their gas by supernova heating in the presence of a DM potential. Hence, the ram pressure by the Galactic wind may be necessary for gas depletion in the dSphs.

3.2.3. DM in dSphs

[1] Although we concluded in subsubsection 3.2.1 that the large mass-to-light ratio is not explained by only the tidal force, it becomes possible if the DM content is considered. Considering that tidally disrupted dSphs survive only $\sim 10^8$ yr (subsubsection 3.2.1), the DM content in dSphs is needed to bind a dSph as a system. As implied in subsubsection 3.2.2, ram-pressure stripping by the Galactic gaseous halo or supernovae in the dSphs may be effective for determining the physical parameters of the dSphs. Even in this context, the DM content is necessary to maintain the dSphs as a bound system.

[2] In section 2, the linear regression (9) and correlation coefficient in figure 4 indicate that $\rho_s$ may correlate a little with the Galactocentric distance (subsubsection 2.2.2). Since this implies that the density is determined by the internal structure of the dSphs, this is consistent with the DM-dominant picture of dSphs (Oh et al. 1995).

Here, we note that Hirashita et al. (1998) explained the mass-luminosity relation of the dwarf spheroidal galaxies in the context of galaxy formation in the DM potential. Finally, we should stress again that the analyses by B96 necessarily showed the dominance of the tidal force in determining the kinematical conditions of the Local Group dSphs. Considering the above [1] and [2] as well as the result in section 2, the DM-dominated dSphs are no less probable than the tidally perturbed dSphs.

We first thank Prof. C. Pryor, the referee, for his careful reading and making valuable comments, which improved this paper very much. We are grateful to Profs. S. van den Bergh, T. E. Armandroff, M. Bellazzini for useful comments about environmental effects on satellite galaxies. Their comments at the IAU meeting were helpful to
improve the discussions in the paper. We also thank Profs. M. Saitö, Drs. S. Mineshige, and T. T. Ishii for helpful comments. All of us acknowledge the Research Fellowship of the Japan Society for the Promotion of Science for Young Scientists. We made extensive use of the NASA’s Astrophysics Data System Abstract Service (ADS).

References

Bellazzini M., Fusi Pecci F., Ferraro F.R. 1996, MNRAS 278, 947 (B96)

Binney J., Tremaine S. 1987, Galactic Dynamics (Princeton University Press, Princeton)

Blumenthal G.R., Faber S.M., Primack J.R., Rees M.J. 1984, Nature 311, 517

Burkert A. 1997, ApJ 474, L99

Caldwell N., Armandroff T.E., Da Costa G.S., Seitzer P. 1998, AJ 115, 535

Caldwell N., Armandroff T.E., Seitzer P., Da Costa G.S. 1992, AJ 103, 840

Einasto J., Saar E., Kaasik A. 1974, Nature 252, 111

Faber S.M., Lin D.N.C. 1983, ApJ 266, L17

Flin P. 1999, in The Stellar Content of Local Group Galaxies, ed P. Whitelock, R. Cannon (Kluwer, Dordrecht) in press

Gallagher J.S. III, Wyse R. F. G. 1994, PASP 106, 1225

Gnedin O.Y., Ostriker J.P. 1997, ApJ 474, 223

Hirashita H. 1999, ApJ in press

Hirashita H., Kamaya H., Mineshige S. 1997, MNRAS 290, L33

Hirashita H., Takeuchi T.T., Tamura N. 1998, ApJ 504, L83

Hoel P.G. 1971, Introduction to Mathematical Statistics, 4th edition (Wiley, New York) ch7

Honma M., Sofue Y. 1997, PASJ 49, 453

Irwin M., Hatzidimitriou D. 1995, MNRAS 277, 1354

Johnston K.V. 1998, ApJ 495, 297

Kroupa P. 1997, New Astron. 2, 139

Kuhn J.R. 1993, ApJ 409, L13

Kuhn J.R., Miller R.H. 1989, ApJ 341, L41

Kundić T., Ostriker J. P. 1995, ApJ 438, 702

Larson R.B. 1974, MNRAS 169, 229

Lee M.G., Freedman W., Mateo M., Thompson I., Roth M., Ruiz M.-T. 1993, AJ 106, 1420

Mac Low M.-M., Ferrara A. 1999, ApJ 513, 142
Mateo M., Olzewski E.W., Pryor C., Welch D.L., Fischer P. 1993, AJ 105, 510

Mishra R. 1985, MNRAS 212, 163

Oh K.S., Lin D.N.C., Aarseth S.J. 1995, ApJ 442, 142

Piatek S., Pryor C. 1995, AJ 109, 1071

Portnoy D., Pistinner S., Shaviv G. 1993, ApJS 86, 95

Pryor C. 1992, in Morphological and Physical Classification of Galaxies, ed G. Longo, M. Capaccioli, G. Busarello (Kluwer, Dordrecht) p163

Pryor C. 1996, in Formation of the Galactic Halo, ed H. Morrison, A. Sarajedini, ASP Conf. Ser. 92, p424

Pryor C., Kormendy J. 1990, AJ 100, 127

Saito M. 1979, PASJ 31, 193

Songaila A. 1981, ApJ 248, 945

van den Bergh S. 1994, ApJ 428, 617

Vogt S.S., Mateo M., Olszewski E.W., Keane M.J. 1995, AJ 109, 151

Weinberg, M. D. 1994, AJ 108, 1414

Zaritsky D., Olszewski E.W., Schommer R.A., Peterson R.C., Aaronson M. 1989, ApJ 345, 759
Table 1. Local Group dwarf spheroidal galaxies.*

| Name       | $R_{GC}$ (kpc) | $L_V$ ($10^5 L_\odot$) | $r_c$ (pc) | $M_{vir}$ ($10^7 M_\odot$) | $M_{vir}/L_V$ | $\mu_V$ (0) (mag arcsec$^{-2}$) | Ref$^*$ |
|------------|----------------|-------------------------|------------|----------------------------|---------------|--------------------------------|--------|
| Ursa Minor | 66             | 3.0                     | 290        | 3.9                        | 130           | 25.1                           | 1      |
| Draco       | 76             | 2.5                     | 190        | 5.2                        | 210           | 25.2                           | 1      |
| Sculptor    | 78             | 16                      | 200        | 1.4                        | 8.8           | 24.1                           | 1      |
| Carina      | 89             | 2.9                     | 210        | 1.1                        | 38            | 25.2                           | 1      |
| Sextans     | 91             | 8.3                     | 380        | 2.6                        | 31            | 25.5                           | 1      |
| Fornax      | 133            | 250                     | 640        | 12                         | 4.8           | 23.2                           | 1      |
| Leo II      | 219            | 9.9                     | 220        | 1.1                        | 11            | 24.0                           | 2      |
| Leo I       | 270            | 40.9                    | 260        | 0.47                       | 1.2           | 22.3                           | 3, 4, 5|

* See text for the definitions of quantities.

† (1) Mateo et al. 1993; (2) Vogt et al. 1995; (3) Lee et al. 1993 (4) Caldwell et al. 1992; (5) Zaritsky et al. 1989.
Fig. 1. Relation between $\mu_V(0)$ (central surface brightness in the $V$ band) and $F_{T,B}$ (dimensionless tidal force) for the tidal model for each dSph in the Local Group. In this model, the mass of each dSph is estimated by the luminous mass. The correlation coefficient is 0.91.

Fig. 2. Same as figure 1, but for the dark-matter model. The mass of each dSph is estimated by the virial mass. The correlation coefficient is 0.48.

Fig. 3. Correlation between $F_{T,B}$ (dimensionless tidal force) and $R_{GC}$ (Galactocentric distance) for the tidal model. The correlation coefficient is $-0.94$. The line represents the linear regression.

Fig. 4. Same as figure 3, but for the dark-matter model. The correlation coefficient is $-0.68$. 
