Chirality-dependent planar Hall effect in inhomogeneous Weyl semimetals

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The planar Hall effect (PHE), the appearance of an in-plane transverse voltage in the presence of co-planar electric (E) and magnetic (B) fields, occurs in regular Weyl semimetals (WSMs) as one of the fundamental manifestations of chiral anomaly represented by a E \cdot B term. We demonstrate here that PHE can survive in an inhomogeneous Weyl semimetal (IWSM) even in the absence of the aforesaid anomaly. Using a semiclassical Boltzmann transport theory, we show that PHE could also appear in an IWSM due to the strain-induced chiral gauge potential, which couples to the Weyl fermions of opposite chirality with opposite sign. Our study shows a resultant phase shift in the current associated with opposite chirality Weyl nodes, which, remarkably, leads to a finite chirality-dependent planar Hall effect (CPHE) in the IWSMs. Interestingly, we show that a small tilt in the Weyl node can generate a pure CPHE even in the absence of an applied magnetic field. The CPHE finds an important implication in `chiralitytronics`. We also discuss the experimental feasibility of these novel effects of strain in type-I IWSMs.

Introduction:- Theoretical predictions and experimental discoveries of Weyl semimetals (WSMs) have led to an explosion of activities in the area of three-dimensional topological systems in recent years. Weyl semimetals appear as topologically nontrivial conductors where the spin-nondegenerate valence and conduction bands touch at isolated points in momentum space, the so-called "Weyl nodes" \cite{1}. In normal time reversal symmetry (TRS) broken Weyl semimetal there is a finite separation (b) between the Weyl nodes in momentum space, whereas there can also be a finite separation (b\textsubscript{0}) along the energy axis if it additionally breaks the inversion symmetry (IS) \cite{3,12,9,11}. Recently, it has been shown that these separations (b, b\textsubscript{0}) can be varied both spatially as well as temporally by means of lattice deformations originating from a mechanical strain or inhomogeneous magnetization \cite{12,10}. In an inhomogeneous Weyl semimetal (IWSM) or strain-induced WSM, the space- and time-dependent b and/or time-dependent b\textsubscript{0} generate chiral pseudomagnetic field (B\textsubscript{5} = \nabla \times b) and pseudoelectric field (E\textsubscript{5} = -\nabla b\textsubscript{0} - \partial t b) which, unlike the real electromagnetic fields, couple with opposite signs to opposite chiralities \cite{12,15,17}.

In the presence of external fields, one of the most celebrated examples of anomaly in a regular Weyl semimetal is the chiral anomaly \cite{3,18,19,17,22,23}. Interestingly, in the case of an inhomogeneous Weyl semimetal, where both ordinary and pseudo-electromagnetic fields are present, a new type of anomaly appears. The conservation laws, in an IWSM, become modified in the presence of pseudo-fields and take the new form \cite{12,13,15}:

\[ \partial_t \rho_5 + \nabla \cdot j_5 = \frac{e^2}{2 \pi^2 \hbar^2} (E \cdot B + E_5 \cdot B_5), \]  
\[ \partial_t \rho + \nabla \cdot j = \frac{e^2}{2 \pi^2 \hbar^2} (E \cdot B_5 + E_5 \cdot B), \]

where \( \rho \) and \( \rho_5 \) are the total electron and chiral density, respectively, and \( j \) and \( j_5 \) are current densities associated to \( \rho \) and \( \rho_5 \) respectively, \( \rho_5 = \rho_R - \rho_L \), represents the difference between the charge densities associated with the right- and left-handed Weyl fermions. Here, Eq. 1 expresses the famous chiral anomaly, which implies non-conservation of separate electron numbers of opposite chirality for Weyl fermions in the presence of parallel real electromagnetic (E \cdot B) or pseudo-electromagnetic fields (E\textsubscript{5} \cdot B\textsubscript{5}). On the other hand, it is clear from the Eq. 2 that the total charge conservation is violated if either E\textsubscript{5} \neq 0 or B\textsubscript{5} \neq 0 in the presence of real external fields. This apparent charge-density non-conservation which results in pumping of charge between the bulk and the edge of the system gives a novel kind of anomaly in an IWSM \cite{15,25}. For a TRS-broken but inversion symmetric IWSM having no temporal variation of b, there will be no pseudo-electric field present and the violation of total charge conservation is apparently due to the term \( \propto E \cdot B_5 \).

Weyl semimetals exhibit several fascinating anomaly-related transport properties \cite{18,24,29,35}. One such phenomenon is the occurrence of positive longitudinal magneto-conductivity (LMC) and the associated planar Hall effect (PHE) due to the proper chiral anomaly E \cdot B \cite{27,31}. But a TRS-breaking IWSM, where E\textsubscript{5} = 0, presents a more fascinating ground for the search of novel transport signatures due to the presence of the anomaly \( \sim E_5 \cdot B_5 \). The LMC in the presence of pseudo-fields has recently been studied where it is found to get enhanced by chiral pseudomagnetic effect \cite{14}. On the other hand, the planar Hall effect, to the best of our knowledge, has not yet been discussed in the context of an IWSM. Therefore, it is a natural to ask how PHE behaves in the presence of pseudo-fields as well as in the presence of this novel anomaly. Recent experimental realizations \cite{36,38} of IWSMs add to the quest for this search.

Using the semiclassical Boltzmann transport theory, we studied the planar Hall effect for a TRS broken but inversion symmetric type-I inhomogeneous Weyl semimetal...
with spatially varying $\mathbf{b}$. Our results show that PHE can exist even without chiral anomaly and solely due to an applied strain, which manifests itself by an anomaly proportional to $\mathbf{E} \cdot \mathbf{B}_5$. We also find that a chirality-dependent conductivity (CDC), defined as the difference of contributions from the two nodes of different chiralities, is finite when $\mathbf{B}$ and $\mathbf{B}_5$ are simultaneously present in a nontilted WSM. Remarkably, in case of tilted type-I WSM, we observe the occurrence of a finite CDC even in the absence of real magnetic field.

**Boltzmann formalism in an inhomogeneous Weyl Semimetal:** The momentum space Hamiltonian for a linearized tilted Weyl node can be expressed as

$$H_{\chi} = \hbar v_F (\chi \mathbf{k} \cdot \sigma + \gamma_\chi \mathbf{k} \sigma_0) - \mu,$$

where $v_F$ is the Fermi velocity, $\chi$ is the chirality associated with the Weyl node, $\sigma$ represents the vector of Pauli matrices, $\sigma_0$ is the identity matrix, and $\gamma_\chi$ is the tilt parameter along arbitrary $\mathbf{k}$-direction. We choose that tilting is along the $k_x$-direction for the rest of our work without any loss of generality [39]. In this work, we consider the time reversal broken WSM with two Weyl cones tilted exactly by an opposite amount i.e., $\gamma_\chi = \gamma_\chi \mathbf{k}_b$. Here we restrict our discussion for type-I WSM ($\gamma_\chi < 1$) where we always have the point-like Fermi surface at the Weyl node. However, we’ll use the natural unit for rest of this work, that is, $\hbar = c = 1$.

One of the key concepts in developing the formalism for planar Hall effect in an IWSM is that, in presence of external magnetic field as well as static strain applied to these systems, chiral charges feel different effective fields given by $\mathbf{B}_\chi = \mathbf{B} + \chi \mathbf{B}_5$ according to their chiralities [14][25][40]. Now, we briefly present the semi-classical formula for the planar Hall conductivity (PHC) and longitudinal magneto-conductivity (LMC). We have performed our calculations taking $B \sim 0 \sim 3T$, $B_5 \sim 0 \sim 3T$ and considering temperature $T \ll (\sqrt{B}, \sqrt{B_5}) \ll \mu$. The Landau quantization can be neglected in these low magnetic field ($\mathbf{B}, \mathbf{B}_5$) regime. The transport properties of electrons can be understood using the phenomenological Boltzmann kinetic equation [41]:

$$\frac{\partial}{\partial t} \mathbf{r} + \mathbf{v}_\chi \cdot \nabla \mathbf{r} + \mathbf{k}_\chi \cdot \nabla \mathbf{k}_\chi = I_{\text{coll}}[f^{\chi}_{r,k,t}],$$

where on the right hand side $I_{\text{coll}}[f^{\chi}_{r,k,t}]$ is the collision integral, which, within relaxation time approximation, takes the analytical form as: $I_{\text{coll}}[f^{\chi}_{r,k,t}] = (f_{eq} - f^{\chi}_{eq})/\tau (k)$ with $\tau (k)$ is the relaxation time. $f^{\chi}_{eq}$ is the electron distribution function, $\mathbf{k}_\chi$ is the quasimomentum and $f_{eq}$ is the equilibrium Fermi-Dirac distribution function that describes electron distribution in the absence of any external field. Here, we ignore the momentum dependence of the relaxation time for simplicity.

In order to calculate planar Hall effect in the presence of real as well as pseudomagnetic fields, we apply the electric field along the $x$-axis and the real magnetic field in the $xy$ plane i.e., $\mathbf{E} = E\hat{x}$ and $\mathbf{B} = B \cos \theta_b \hat{x} + B \sin \theta_b \hat{y}$. We apply the strain in such a way that the pseudomagnetic field is also lying on the $xy$-plane i.e., $\mathbf{B}_5 = B_5 \cos \theta_b \hat{x} + B_5 \sin \theta_b \hat{y}$. Here, $\theta_b$ is the angle between electric field and real (pseudomagnetic) field. Now in the presence of the effective magnetic field $\mathbf{B}^\chi = (\mathbf{B} + \chi \mathbf{B}_5)$ and electric field $\mathbf{E}$, the semiclassical equations of motion are given below [42][44]:

$$\dot{\mathbf{r}}_\chi = D^\chi_k [\mathbf{v}_k + e(\mathbf{E} \times \Omega^\chi_k) + e(\mathbf{v}_k \cdot \Omega^\chi_k) \mathbf{B}_\chi],$$

$$\dot{\mathbf{k}}_\chi = D^\chi_k [e \mathbf{E} + e(\mathbf{v}_k \times \mathbf{B}_\chi) + e^2 (\mathbf{E} \cdot \mathbf{B}_\chi) \Omega^\chi_k].$$

where $\mathbf{v}_k = \partial \mathbf{r}_{eq}/\partial \mathbf{k}$ is the group velocity and $D^\chi_k = [1+e(\mathbf{B}_\chi \cdot \Omega^\chi_k)]^{-1}$ depicts the modification of the phase space volume in the simultaneous presence of magnetic field and Berry curvature $\Omega^\chi_k$ [43]. The higher order corrections due to the inhomogeneous Fermi velocity is neglected in this work [44][45][47]. Considering spatially uniform field in steady state condition, retaining only the contributions due to linear response, the general expression of the longitudinal conductivity and planar Hall conductivity associated with each Weyl node with chirality $\chi$ is calculated, respectively, as

$$\sigma_{xx}^\chi = \frac{-e^2}{2\pi^2} \int d^3k D^\chi_k [v^2_0 + 2e v_x (\mathbf{B}_\chi \cdot \hat{x}) (\mathbf{v}_k \cdot \Omega^\chi_k)] + e^2 (\mathbf{B}_\chi \cdot \hat{x})^2 (\mathbf{v}_k \cdot \Omega^\chi_k)^2 (\partial f_{eq} / \partial k_x)$$

$$\sigma_{yx}^\chi = \frac{-e^2}{2\pi^2} \int d^3k D^\chi_k [v_x v_y + e v_y (\mathbf{B}_\chi \cdot \hat{y}) (\mathbf{v}_k \cdot \Omega^\chi_k)] + e^2 (\mathbf{B}_\chi \cdot \hat{y}) (\mathbf{v}_k \cdot \Omega^\chi_k)^2 (\partial f_{eq} / \partial k_y).$$

From Eqs. (9, 10), we now define the total conductivity $\sigma_{ij}^T$ and chirality dependent conductivity $\sigma_{ij}^\chi$: as follows:

$$\sigma_{ij}^T = \sigma_{ij}^{+1} + (-1)\sigma_{ij}^{-1}$$

**Conductivities in an inhomogeneous Weyl Semimetal:** Following the formalism described in the previous section, we have analytically calculated the total LMC and PHC in case of non-tilted as well as tilted type-I WSMs. The expressions for total LMC ($\sigma_{xx}^T$) and PHC ($\sigma_{yx}^T$) for nontilted case are given by

$$\sigma_{xx}^T = \frac{\sigma^\mu_{xx} \tau + \sigma^\nu_{xx} T}{6\pi e^2} + \sigma^{\nu_5}_{xx} B^2 + \sigma^{\nu_5}_{xx} B_5^2 + \frac{7\sigma^{\nu_5}_{xx} \tau}{6\pi e^2 T} (B^2 \cos^2 \theta_b + B_5^2 \cos^2 \theta_s),$$

$$\sigma_{yx}^T = \frac{7\sigma^{\nu_5}_{xx} \tau}{6\pi e^2 T} (B^2 \sin 2\theta_b + B_5^2 \sin 2\theta_s).$$

It is evident from Eqs. (9, 10) that $\Delta \sigma_{xx}^T = \sigma_{xx}^T (\mathbf{B}) - \sigma_{xx}^T (\mathbf{B} = 0)$ and $\sigma_{yx}^T$ for a normal WSM (where $B_5 = 0$) varies respectively as $B^2 \cos^2 \theta_b$ and $B^2 \sin 2\theta_b$ with respect to applied magnetic field. The strain manifests itself by generating $B_5$ and enhances the LMC [14] as well as the PHC, which are illustrated.
The chirality-dependent longitudinal magnetoconductivity and planar Hall conductivity (CPHC) in case of non-tilted IWSMs are calculated as

\[
\sigma_{yx}^C = B B_5 \frac{e^4 v_F^3 \tau}{15 \pi^2 \mu^2} (4 \cos \theta_c \cos \theta_s + \frac{1}{2} \sin \theta_c \sin \theta_s) (11)
\]

\[
\sigma_{yx}^C = \frac{\tau e^4 v_F^3 \tau}{60 \pi^2 \mu^2} B B_5 \sin (\theta_c + \theta_s). (12)
\]

From Eqs. (11) (12) it is clear that both \(\sigma_{yx}^C\) and \(\sigma_{yx}^T\) vanish in the absence of \(\mathbf{B}\) or \(\mathbf{B}_5\). The underlying physics can be understood as follows. The chirality dependent chemical potential \(\mu_x\) in presence of \(\mathbf{E} \cdot \mathbf{B}_5\) term is given as [14],

\[
\mu_x = \left[ \mu^3 - \frac{3}{2} v_F^3 e^2 \tau (\mathbf{E} \cdot \mathbf{B}_5) \right]^{\frac{1}{2}} (13)
\]

A TRS broken Weyl semimetal in equilibrium i.e., in absence of external field, have \(\mu_+ = \mu_- = \mu\). The change in absolute value of the chemical potential has different values in the two nodes i.e., \(|\Delta \mu_+| \neq |\Delta \mu_-|\) only if both \(\mathbf{B}\) and \(\mathbf{B}_5\) are present. A finite chirality imbalance in the absolute value of chemical potential change between the nodes of opposite chirality causes \(\sigma_{yx}^C \neq \sigma_{yx}^T\). This leads to a finite CDC which is one of the salient features in our present study. We have noticed that CDC has linear dependence on \(\mathbf{B}\) and \(\mathbf{B}_5\), contrary to the quadratic dependence of total conductivity. We present our numerical results of angular dependence \(\theta_s\) of \(\sigma_{yx}\) in Fig. 1. for \(\theta_s = \pi/2\) in the left panel i.e., \(\mathbf{B}_5 \perp \mathbf{E}\) and \(\theta_s = 0\) in the right panel i.e., \(\mathbf{B}_5 \parallel \mathbf{E}\). The solid red line in each panel displays the results for CPHC and blue dotted line displays the total PHC respectively. The important difference between CPHC and PHC is that the former has \(2\pi\) periodicity whereas later has \(\pi\) periodicity with \(\theta_s\). This holds true also for tilted IWSMs, discussed later. The black and green line correspond to \(\sigma_{yx}^C\) and \(\sigma_{yx}^T\) respectively and they are related by \(\sigma_{yx}^C(\theta_s) = \sigma_{yx}^T(\theta_s \pm \pi)\). Therefore, a finite \(\mathbf{B}_5\) field effectively modify the phase difference between the two opposite chiral nodes, leading to a finite CPHC. Moreover, \(\sigma_{yx}^C(\theta_s)\) is an even function (i.e., \(\sigma_{yx}^C(\theta_s) = \sigma_{yx}^C(-\theta_s)\)) and odd function (i.e., \(\sigma_{yx}^C(\theta_s) = -\sigma_{yx}^C(-\theta_s)\)) for \(\theta_s = \pi/2\) and \(\theta_s = 0\) respectively. Consequently, this leads to a finite CPHC at \(\theta_s = n\pi\) (in left panel of Fig. 1) and \(\theta_s = (2n + 1)\pi/2\) (in right panel of Fig. 1), where the total PHC remains zero. This is one of the striking results here: the situation is akin to the pure valley current in valleytronics [15] and we call this effect pure CPHE. The important point we have noticed is that a pure CPHE occurs only for \(\theta_s = \pi/2\) or \(\theta_s = 0\) (see Eq. 10) and Eq. 12).

Now we move to our discussions on the tilted type-I IWSM. Here, our analytical calculation reveals the total LMC and PHC as

\[
\sigma_{yx}^C = \frac{e^4 v_F^3 \tau}{15 \pi^2 \mu^2} (1 + 2\gamma_{\parallel}) \left( B^2 \cos^2 \theta_s + B_5^2 \cos^2 \theta_s \right)
\]

\[
+ \frac{e^4 v_F^3 \tau}{60 \pi^2 \mu^2} (1 + 7\gamma_{\parallel})(B^2 \sin^2 \theta_s + B_5^2 \sin^2 \theta_s) + \frac{e^4 v_F^3 \tau}{15 \pi^2 \mu^2} \left( \frac{23}{15} \gamma_{\parallel}^2 \right) B \cos \theta_s, (14)
\]

\[
\sigma_{yx}^T = \frac{e^4 v_F^3 \tau}{40 \pi^2 \mu^2} \left( \frac{11}{2} \gamma_{\parallel} + \gamma_{\parallel}^2 \right) B \sin \theta_s
\]

\[
+ \frac{e^4 v_F^3 \tau}{15 \pi^2 \mu^2} \left( \frac{11}{2} \gamma_{\parallel} + \gamma_{\parallel}^2 \right) B \sin \theta_s (15)
\]
In a tilted IWSM, it is evident from Eqs.\cite{14,15} that the PHC as well as LMC are enhanced due to the chiral pseudomagnetic effect in presence of $B_5$. A finite PHC and also LMC can exist even in the absence of applied magnetic field, which is solely due to the $\mathbf{E} \cdot \mathbf{B}_5$-type anomaly in an IWSM. To find the exact nature of the magnetic field and angular dependencies of PHC and LMC, we have performed detailed numerical calculations. It shows that, in a regular WSM (where $B_5 = 0$), $\Delta \sigma_{yx}^T$ and $\sigma_{yx}^C$ varies respectively as $B \cos \theta_b$ and $B \sin \theta_b$ with respect to applied magnetic field. On the other hand, in an IWSM, PHC varies as $B_5^2 \cos \theta_s$ whereas LMC follows $B_5^2 \cos^2 \theta_s$ with respect to the pseudomagnetic field.

The analytical form of chirality-dependent longitudinal magneto-conductivity (CLMC) and planar Hall conductivity (CPHC) in case of tilted-type I WSMs are as follows:

$$
\sigma_{xx}^C = \frac{4\pi^2 \mu^2}{15\pi^2} B B_5 \cos \theta_b \cos \theta_s + \frac{e^2}{6\pi^2} \left( \frac{23}{5} \gamma_x + \gamma_x^2 \right) B_5 \cos \theta_s + \frac{4e^2v_F^2}{30\pi^2} \left( 1 + 7\gamma_x^2 \right) B_5 \sin \theta_b \sin \theta_s \tag{16}
$$

$$
\sigma_{yx}^C = \frac{7e^2v_F^2}{20\pi^2} \frac{7}{3} + \gamma_x^3 B_5 \sin(\theta_b + \theta_s) + \frac{11}{2\pi^2} \frac{e^2v_F^2}{\gamma_x} B_5 \sin \theta_s \tag{17}
$$

It is evident from the above equations that even in the absence of $\mathbf{B}$, $\sigma_{ij}^C$ is finite, becomes independent of $\mu$ and depends only on the odd powers of $\gamma_x$. This is an unique effect of a TRS broken tilted type-I WSM. It vanishes if the opposite chiral nodes are tilted in the parallel direction. A finite CDC implies a difference of $\sigma_{ij}^C$ between the two nodes i.e., $\sigma_{ij}^C(\theta_s) \neq \sigma_{ij}^C(\theta_s)$. The angular $\theta_s$ dependence of $\sigma_{yx}^C$ of a tilted IWSM with $B = 0$ is shown in the left panel of Fig.\ref{fig:2}. PHC in the nodes of different chirality are related: $\sigma_{yx}^C(\theta_s) = \sigma_{yx}^C(\theta_s \pm \pi)$. As we discussed, this relation causes the novel effect of pure CPHE. We show the variation of $\sigma_{yx}^T(B = 0)$ and $\sigma_{yx}^C(B = 0)$ with $\theta_s$ for a finite $\gamma_{xx}$ in the right panel of Fig.\ref{fig:3}. It is evident that $\sigma_{yx}^C(B = 0)$ (also even function of $\theta_s$) is maximum while $\sigma_{yx}^T(B = 0)$ vanishes at $\theta_s = \pi/2$.

We now discuss the finite $\mathbf{B}$ effect on a pure CPHE of tilted IWSM. We show the density plot $\sigma_{yx}^C(\theta_s = \pi/2)$ and $\sigma_{yx}^C(\theta_s = \pi/2)$ with $B$ and $\theta_b$ respectively in the left and right panel of Fig.\ref{fig:3}. The expressions of $\sigma_{yx}^C(\theta_s = \pi/2)$ and $\sigma_{yx}^C(\theta_s = \pi/2)$ are given: $\sigma_{yx}^C(\theta_s = \pi/2) = C_1 B B_5 \cos \theta_b + C_2 B_5$ and $\sigma_{yx}^T(\theta_s = \pi/2) = C_3 B_5 \sin \theta_b + C_4 B \sin \theta_b$ (see Eq.\ref{15,17}), where $C_i$’s are constants independent of $B$ and $B_5$. Therefore, a pure CPHE occurs at $\theta_b = n \pi$, shown in Fig.\ref{fig:3}. However, the expressions of $\sigma_{yx}^C(\theta_s = 0)$ is given: $\sigma_{yx}^C = C_1 B B_5 \sin \theta_b$ and $\sigma_{yx}^T(\theta_s = 0) = \sigma_{yx}^T(\theta_s = \pi/2)$. Hence, different from non-tilted IWSM, a pure CPHE is absent for $\theta_s = 0$ and only emerge for $\theta_s = \pi/2$.

The possibility of a pure CPHE, even in the absence of real magnetic fields, looks very promising. The CPHE results in the separation and accumulation of left and right chiral fermions on opposite surfaces of WSM as shown in Fig.\ref{fig:4}. The chirality population difference at one surface is given as: $\delta n = \tau F \sigma_{yx}^C E / e$. From Ref.\cite{49}, we take an electric field $|E| = 10 V/mm$, $|B|$ and $|B_5| = 1T$ and assuming $\tau \sim 1ps$, we find chirality population difference to be $\sim 450/\mu m^2$ and $\sim 1500/\mu m^2$ for $\gamma_x = 0$ and $\gamma_x = 0.2$, respectively. This chirality-polarization leads to an unequal optical activity on the opposite surfaces. This would manifest in a difference of absorption of left and right handed circularly polarised light at the two surfaces. Consequently, the optical activity can be detected via circular dichroism experiment\cite{49}.

**Conclusion:** To summarize, using semiclassical Boltzmann formalism, we show that the total PHC gets enhanced due to chiral pseudomagnetic effect in a type-I WSM. In a IWSM, applied strain induces a new anomaly $\propto \mathbf{E} \cdot \mathbf{B}_5$ by generating pseudomagnetic field ($\mathbf{B}_5$), which is different from the proper chiral anomaly ($\propto \mathbf{E} \cdot \mathbf{B}$). This novel anomaly creates an apparent charge-density non-conservation in the system. As a consequence, there is a pumping of charges between the bulk and the edge of the system which causes a finite PHE that can be detected easily. We provide analytical and numerical calculations to suggest some definite signatures of total conductivities for experimental verification.
We further demonstrate that a pure CPHE is possible in IWSMs. In a non-tilted IWSM, CPHE occurs in the absence of both applied magnetic field and pseudomagnetic field. Moreover, in a tilted IWSM, CPHE can be found in the absence of B. We discuss its possible experimental verifications and provide explanations for its existence. Our study opens up another avenue to utilize the chiral degrees of freedom of a WSM for real applications.

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