Abstract: Due to thermal pulses, asymptotic giant branch (AGB) stars experience periods of convective mixing that provide ideal conditions for slow neutron-capture nucleosynthesis. These processes are affected by large uncertainties and are still not fully understood. By the lucky coincidence that about a quarter of all post-AGB stars turn hydrogen-deficient in a final flash of the helium-burning shell, they display nuclear processed material at the surface providing an unique insight to nucleosynthesis and mixing. We present results of non-local thermodynamic equilibrium spectral analyses of the extremely hot, hydrogen-deficient, PG 1159-type central stars of the Skull Nebula NGC 246 and the “Galactic Soccerballs” Abell 43 and NGC 7094.

Keywords: stars: abundances; stars: AGB and post-AGB; stars: atmospheres; stars: individual: WD 0044−121; stars: individual: WD 2134+25; stars: individual: WD 1751+106

1. Introduction

There are different evolutionary channels a star may go through after leaving the asymptotic giant branch (AGB). About a quarter become hydrogen (H) deficient as a result of a late flash of the helium (He)-burning shell (late thermal pulse, LTP, c.f., [1]). For stars still located on the AGB at this event (AGB final thermal pulse, AFTP, Figure 1), the H-rich envelope (with a mass of $10^{-2} M_\odot$) is mixed with the helium (He) rich intershell material ($10^{-2} M_\odot$). If the final flash happens after the departure from the AGB (envelope mass $\leq 10^{-4} M_\odot$), H is either diluted by mixing in the LTP, if the nuclear fusion is still “on”, or it is mixed down and totally consumed by the He-shell, if the star is already on the white dwarf cooling track, fusion is “off” and, thus, no entropy border exists any more. Predicted H mass fractions are 0.20 for an AFTP, 0.02 for an LTP and 0.00 for a VLTP ([1, 2], and references therein).

The three objects presented in this work belong to the spectral type of PG 1159 stars (effective temperatures of $75,000 \, K \leq T_{\text{eff}} \leq 250,000 \, K$ and surface gravities of $5.5 \leq \log(g \, / \, \text{cm/s}^2) \leq 8.0$, [1]) resulting from the H-deficient evolutionary channel. The central stars of the planetary nebulae (CSPNe) NGC 7094 (PN G066.7−28.2 [3]; CS: WD 2134−125 [4]) and Abell 43 (PN G036.0+17.6 [3]; WD 1751−106 [4]), known as spectroscopic twins, belong to the sub-type of hybrid PG 1159-stars [5] exhibiting H lines in their spectra and, thus, resulting from an AFTP. In previous analyses, $T_{\text{eff}} = 100 \pm 15 \, \text{kK}, \log(g) = 5.5 \pm 0.2$, and a deficiency in Fe and Ni of $<1$ dex were determined [6]. These stars are known to be fast rotators with rotational velocities of $v_{\text{rot}} = 46 \pm 16 \, \text{km/s}$ and $42 \pm 13 \, \text{km/s}$, respectively [7].
Figure 1. (Left): Internal structure of an AGB star (from [8]); (Right): Occurrence of AFTP, LTP, and VLTP in the Hertzsprung-Russell diagram.

WD 0044−121 [4] is the PG 1159-type central star of the Skull Nebula (NGC 246, PN G118.8−74.7 [3]) and with a mass of 0.7 $M_\odot$, it is among the most massive of this type. Previous analyses revealed $T_{\text{eff}} = 150$ kK, $\log g = 5.7$, and a Fe and Ni deficiency of about 0.25 dex. It rotates with $77^{+23}_{-17}$ km/s [7]. These stars are unique probes for AGB nucleosynthesis since they show the intershell material at the surface and exhibit a strong enough wind to prevent processes like gravitational settling and radiative levitation to tamper the original composition. The aim of this analysis is to consider atomic data of trans-iron elements ($Z \geq 30$) that became available recently ([9], and references therein) for a NLTE stellar-atmosphere calculation to determine abundances for a large set of elements to draw conclusions on the evolutionary history of these stars and on AGB nucleosynthesis.

2. Observations and Model Atmospheres

Our analysis is based on high-resolution observations from the far ultraviolet (FUV) to the optical wavelength range. The log for the observations retrieved from the Barbara A. Mikulski Archive for Space Telescopes (MAST) and the European Southern Observatory (ESO) Science Archive is shown in Table 1. For the calculation of synthetic spectra, we employ the Tübingen NLTE Model Atmosphere Package (TMAP\(^1\), [10–12]) working under the assumption of a plane-parallel geometry and hydrostatic and radiative equilibrium. For WD 2134+125 and WD 1751+106, we consider opacities of 31 elements from H to barium Ba. The models for WD 0044−121 include the elements H, He, C, N, O, F, Ne, Mg, Ar, Ca, Fe, and Ni. To analyse transiron-element abundances, we added them individually in line-formation calculations in which the temperature structure is kept fixed and the occupation numbers for the levels are calculated. We performed test calculations to confirm that these elements do not affect the atmospheric structure (see also ([9], and references therein)). Atomic data for H, He, and the light metals (atomic weight $Z < 20$) is retrieved from the Tübingen Model Atom Database (TMAD\(^2\), [10]), for the iron group (Ca-Ni) elements, we used Kurucz’s line lists\(^3\) [13,14], and the transiron-element data is available via the Tübingen Oscillator Strength Service (TOSS\(^4\)). For all elements with $Z \geq 20$, a statistical approach is used based on super lines and super levels [10].

\(^1\) http://astro.uni-tuebingen.de/~TMAP
\(^2\) http://astro.uni-tuebingen.de/~TMAD
\(^3\) http://kurucz.harvard.edu/atoms.html
\(^4\) http://dc.g-vo.org/TOSS
Table 1. Observation log for WD 2134+125, WD 1751+106, and WD 0044−121.

| Object          | Instrument | Dataset/ Prog. ID | Start Time (UT) | Exp. Time (s) |
|-----------------|------------|-------------------|-----------------|---------------|
| WD 2134+125     | FUSE       | P1043701000       | 2000-11-13 08:53:28 | 22,754        |
|                 | STIS       | O8MU020100        | 2004-06-24 20:43:30 | 650           |
|                 | STIS       | O8MU020200        | 2004-06-24 22:19:29 | 656           |
|                 | STIS       | O8MU020300        | 2004-06-24 23:55:29 | 655           |
|                 | UVES       | 167.D−0407(A)     | 2001-08-21 02:00:03 | 300           |
|                 | STIS       | 167.D−0407(A)     | 2001-09-20 01:40:00 | 300           |
| WD 1751+106     | FUSE       | B0520201000       | 2001-07-29 20:41:47 | 11,438        |
|                 | FUSE       | B0520202000       | 2001-08-03 22:18:20 | 9528          |
|                 | GHRS       | Z3GW0304T         | 1996-09-08 07:00:34 | 4243          |
|                 | UVES       | 167.D−0407(A)     | 2001-06-18 05:03:38 | 300           |
|                 | UVES       | 167.D−0407(A)     | 2001-07-26 01:27:48 | 300           |
| WD 0044−121     | FUSE       | E1180201000       | 2004-07-12 17:01:47 | 6505          |
|                 | STIS       | O8O7010100        | 2004-05-28 10:11:51 | 1967          |
|                 | STIS       | O8O7010200        | 2004-05-28 11:31:56 | 2736          |
|                 | UVES       | 167.D−0407(A)     | 2002-09-06 09:32:22 | 300           |
|                 | UVES       | 165.H−0588(A)     | 2000-12-07 02:26:29 | 300           |

a: Far Ultraviolet Spectroscopic Explorer; b: Space Telescope Imaging Spectrograph; c: Goddard High Resolution Spectrograph; d: UV-Visual Echelle Spectrograph.

3. Preliminary Results

For accurate abundance determinations, the precise knowledge of the stellar parameters is an inevitable requirement. We redetermined the surface gravity using our best fit for the observed line wings and depth increments of He II λλ 4100.1, 4338.7, 4859.3, 5411.5 Å and H I λλ 4101.7, 4340.5, 4861.3 Å. The temperature determination is based on the ionization equilibrium of O V/O VI using O V λ 1371.3 Å and O VI λλ 1080.6, 1081.2, 1122.3, 1122.6, 1124.7, 1124.9, 1126.3, 1290.1, 1290.2, 1291.8, 1291.9. The redetermination of \( v_{\text{rot}} \) results in lower values for both hybrid PG 1159 stars. The new values are summarized in Table 2.

Table 2. Parameters of WD 2134−125, WD 1751+106, and WD 0044−121.

|          | WD 2134−125 | WD 1751+106 | WD 0044−121 |
|----------|-------------|-------------|-------------|
| \( T_{\text{eff}} / \text{kK} \) | 115 ± 5     | 115 ± 5     | 150 ± 10    |
| \( \log (g / \text{cm/s}^2) \) | 5.4 ± 0.1   | 5.5 ± 0.1   | 5.7 ± 0.1   |
| \( v_{\text{rot}} / \text{km/s} \) | 28 ± 5      | 18 ± 5      | 75 ± 15     |
| \( d / \text{kpc} \) | 2.38 ±0.59  | 2.94 ±0.82  | 1.08 ±0.22  |
| \( M / M_\odot \) | 0.64 ±0.06  | 0.60 ±0.09  | 0.74 ±0.19  |
| \( M_{\text{ini}} / M_\odot \) | 3.30 ±0.65  | 2.87 ±0.71  | 3.91 ±0.48  |
| \( \log (L / L_\odot) \) | 4.04 ±0.32  | 3.91 ±0.32  | 4.27 ±0.59  |

We found that the Fe abundance for both hybrid PG 1159 stars needed to be corrected. The lines of Fe VII and Fe VIII appear weaker in the synthetic spectra due to the higher \( T_{\text{eff}} \) and rotational broadening that was not taken into account in previous works with the result that we find Fe to be less deficient \([\text{Fe}] = −0.79 \) for WD 2134−125 and \([\text{Fe}] = −0.43 \) for WD 1751+106, \([X] = \log (\text{mass fraction/solar mass fraction}) \). Due to the challenge of its fast rotation, no unambiguous identification of Fe lines was possible for WD 0044−121. Based on Fe VII λ 1141.4 Å and Fe VIII λλ 1006.1, 1148.2 Å an upper limit of the super-solar value \([\text{Fe}] = 0.33 \) is reasonable. No clear identification of trans-iron element lines was possible for any of the three stars. For the two hybrid PG 1159 stars, the strongest computed lines were used to determine upper abundance limits for Zn, Ga, Ge, Kr, Zr, Te, I, and Xe. This was not possible for WD 0044−121, since the atomic data for the most prominent ionization stages of these elements in this high temperature range is still lacking. The abundance determinations...
are summarized in Figure 2. Mass and luminosity were determined using He-burning post-AGB tracks [15] (Figure 3). Using the flux calibration of [16], we find spectroscopic distances for the stars. The values are summarized in Table 2.

Figure 2. Photospheric abundances (estimated errors $+/−0.1$ dex) of WD 2134−125, WD 1751+106, and WD 0044−121. Arrows indicate upper limits. In the lower panel, the abundances in $[X/O] = \log \left( \frac{X}{O} \right)_{\text{surf}} - \log \left( \frac{X}{O} \right)_{\text{solar}}$ with the number fractions of element X and O are compared to the yields of the models from [17,18] for different initial masses.

Figure 3. Positions of the CSPNe of NGC 7094, A66 43, and NGC 246 (within their error ellipses) and related objects in the $\log T_{\text{eff}} − \log g$ plane compared with evolutionary tracks (labeled with the respective masses in $M_\odot$) of VLTP stars ([15], full lines) and of hydrogen-burning post-AGB stars ([19], calculated with initial solar metallicity; dashed lines).
4. Discussion

In our detailed spectroscopic analysis, we could confirm the high $T_{\text{eff}}$ and low $\log g$ of WD 0044−121 which places this star close to the Eddington limit for PG 1159 stars (Figure 3) and approves its classification as one of the heaviest stars of this type. In return, this causes difficulties due to instabilities in calculating model atmospheres for this star. As improvement, models including wind effects will be employed in a further analysis of all these stars. The Fe deficiency is not explained by nucleosynthesis models [17,18] that predict solar Fe abundances. A speculative reason for low Fe abundances is the conversion into Ni and heavier elements due to neutron capture [20]. An enhanced Ni/Fe ratio or a clear enhancement of trans-iron elements would indicate this. Unfortunately, no lines of these elements were identified. Given the high distances above and below the galactic plane, the progenitors of these stars could also belong to a low metallicity halo population. We included the low metallicity models for the Small Magellanic Cloud [18] in Figure 2 for comparison but they do not reproduce the negative [Fe/O] values for these stars and therefore cannot be consulted to explain the Fe deficiency. Parallaxes of $(0.431 \pm 0.061)''$ and $(0.615 \pm 0.059)''$ for WD 1751+106 and WD 2134−125, respectively, were published in the second data release of the Gaia mission from which we can derive distances of $(2.32 \pm 0.33)$ kpc and $(1.63 \pm 0.16)$ kpc. The values lie below our spectroscopically measured distances but for WD 1751+106, the values still agree within the error limits. This might lead to the speculation of potentially too high surface gravity values.

5. Conclusions

In our NLTE spectral analysis of the CSPNe WD 2134−125, WD 1751+106, and WD 0044−121, we improved the determination of the stellar parameters and found, for the first two objects, upper abundance limits for s-process elements that have never been analysed in any of these stars before. WD 0044−121 shows no clue for s-process element enhancement which leads to the speculation that the s-process becomes less effective for higher mass and metallicity. Unfortunately, the abundance limits cannot be used to constrain the nucleosynthesis models. In a forthcoming analysis, we plan to search for the radioactive element technetium and for s-process signatures in the ejected nebula material.

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