Mechanism of counterflows and circulation cells in swirling flows

V N Shtern
Kutateladze Institute of Thermophysics SB RAS, 630090, Russia, Novosibirsk, Lavrentyeva str., 1
Shtern Research and Consulting, Houston, Texas, 77096, USA
E-mail: vshtern40@gmail.com

Abstract. This paper reviews counterflows, double counterflows, and circulation cells in swirling flows and argues that all these seeming paradoxical phenomena can be caused by a common swirl-decay mechanism (SDM). It is shown that SDM explains (a) the counterflow of water and oil in hydrocyclones, (b) the elongated counterflow of hot and cold air in vortex tubes, and (c) the double counterflow occurring in vortex combustion chambers. SDM also explains the development and disappearance of circulation cells, often referred to as vortex breakdown bubbles, in sealed cylindrical containers where the flow is driven by rotation of one end disk. In a few words, SDM is the following. The balance of the centrifugal force and the radial gradient of pressure in a fast-swirling flow results in that pressure at the rotation axis is smaller than pressure at the periphery. If swirl decays downstream, pressure grows along the axis. This axial gradient of pressure decelerates the near-axis flow and can reverse it thus developing a local or global counterflow. It is shown that SDM also works in two-fluid flows modeling vortex bioreactors.

1. Elongated counterflows in vortex devices
Swirling flows have some paradoxical features of practical and fundamental interest [1]. One such feature is an elongated counterflow which occupies almost the entire volume of a vortex device. Among such devices are hydrocyclones, vortex tubes, and combustion chambers discussed in more detail next.

1.1. Hydrocyclones
As the first example, figure 1 shows a schematic of a commercial hydrocyclone. Such devices are used for solid-liquid and liquid-liquid separation [2]. Oily water tangentially enters the hydrocyclone, thus developing a swirling flow, and spirals toward the opposite end. The centrifugal buoyancy shifts oil droplets to the axis where the accumulated oil paradoxically moves against the water flow – back to the entrance end wall.

This counterintuitive oil motion occurs due to the swirl decay mechanism. The fast rotation near the inlet significantly reduces pressure at the axis. The rotation decays from the inlet to the remote outlet making the near-axis pressure being larger near the outlet than that near the inlet. Thus the pressure maximum (at the periphery) and the pressure minimum (at the axis) both are located near the inlet. The
pressure drops along the sidewall from the inlet to the remote outlet and further drops along the axis from the water outlet to the oil outlet. This pressure distribution drives the counterflow depicted in figure 1.

1.2. Vortex tubes
A similar counterflow occurs in vortex tubes as figure 2 schematically shows. Highly pressurized air at the room temperature tangentially enters a vortex tube developing a swirling motion. As the air flow goes near the sidewall to the remote annular outlet, it is heated up. Peripheral air leaves the device through this outlet as an annular hot jet. The rest air makes U-turn and goes back near the axis toward the inlet end wall. During the motion, this air is cooled down and leaves the device through the central orifice in the inlet end wall as a cold jet. This energy separation effect was discovered by Ranque [3] and since vortex tubes have been widely used for spot cooling, e.g. of cutting edges and sewing needles.

![Figure 1.](image)

**Figure 1.** (color online) Schematic of counterflow in a commercial hydrocyclone.

Vortex tubes have the following seeming paradoxical features: (a) energy separation, (b) radial distribution of temperature, and (c) axial counterflow. There is yet no consensus concerning feature (a): what mechanism separates the incoming room-temperature air into the hot and cold outflows. Concerning feature (b), the centrifugal stratification typically results in a hot (lighter) fluid being located near the axis and a cold (denser) fluid being located at the periphery – oppositely to that occurs in vortex tubes. Feature (b) is due to the pressure distribution: the inlet pressure value can be three or more times the atmospheric value. This large peripheral pressure makes the density of hot air being larger than the density of cold near-axis air whose pressure is almost atmospheric.
Feature (c) is due to the swirl decay mechanism whose action is quite similar to that in hydrocyclones: The huge centrifugal force causes the pressure drop from its large peripheral value to its small near-axis value in the device-inlet vicinity. As swirl becomes significantly weaker near the hot outlet, the radial pressure gradient is smaller and the near-axis pressure is larger than those at the inlet. The resulting axial gradient of pressure drives the near-axis back flow of air.

![Schematic of counterflow in a vortex tube.](image)

**Figure 2.** (color online) Schematic of counterflow in a vortex tube.
1.3. Vortex burners

A double counterflow can develop in a vortex burner as the photo in figure 3 illustrates [4, 5]. The tangential inlet is located close to the burner open end in the lower right corner of figure 3. Pressurized air enters through this inlet and generates a swirling motion in the burner. A paradoxical feature is that the incoming air does not immediately leave the burner through its open end. The air goes in the opposite direction, spiraling inward near the burner sidewall, reaches the dead end, turns toward the rotation axis, meets a fuel (here propane), and they combust. The fuel is supplied through a nozzle inserted in the end wall. Hot combustion products exhaust the burner through an annular region. The ambient air goes inward the burner near the axis to the end wall vicinity, where makes U-turn, and joins the annular outflow.

Three steel rods, inserted in the burner right side, visualize the temperature and velocity distributions. The rods are bright in the annular region where the hot flue gases go out. In contrast, the rods are dark near the axis and near the sidewall, where the cold air flows go inward to the dead end. Thus, there is a double counterflow: one inflow near the axis and the other inflow near the sidewall, separated by the annular outflow. This paradoxical and rather complicated flow pattern is due to the swirl decay mechanism: the converging swirling flow develops a region near the dead-end center, where pressure drops well below its atmospheric value. This region sucks inward both the ambient air (along the axis) and the injected air (along the sidewall).

![Figure 3. (color online) Photo of double counterflow in a vortex burner.](image-url)
1.4. Theory of swirl decay

To model the swirl decay effect in elongated cylindrical devices, we look a solution to the Navier-Stokes equation in the form [6-8]:

\[ v_\theta = F(r) \exp(-\lambda z) + O(\lambda^2), \quad \Psi = Q(r) \exp(-\lambda z) + O(\lambda^2), \quad v_z = r^{-1} \partial \Psi / \partial r \quad \text{and} \quad v_r = -r^{-1} \partial \Psi / \partial z. \] (1)

Here, \( v_r, v_\theta, \) and \( v_z \) are the velocity components in cylindrical coordinates \((r, \phi, z)\), \( \Psi \) is the Stokes stream function, \( \lambda \) is the decay rate in the axial direction that must be found. All the quantities are dimensionless. The length scale is the container’s inner radius \( R \) and the velocity scale is the maximal swirl velocity. The supposed weak \( z \)-dependence means that \( \lambda \ll 1 \). Product \( \lambda Re \) is not expected to be small; \( Re \) is the Reynolds number.

Substituting (1) and simple calculations reduce the Navier-Stokes equations to a system of ordinary differential equations. The no-slip at the container’s wall and the regularity at the axis are the uniform boundary conditions. This nonlinear problem has the zero solution and a value of \( \lambda Re \) must be found at which the solution is nonzero. For more details, see Section 5.2 of Ref. [4].

The bold curve in figure 4(a) shows the resulting radial distribution of the axial velocity, normalized by its maximal magnitude. The thin curve is the polynomial approximation, \( v_z/[v_{z0}] = 4r^2 - 3r^4 - 1 \). Figure 4(b) depicts the pressure distribution, where \( p_1 \) is the inlet pressure, \( p_2 \) (\( p_3 \)) is the peripheral (near-axis) pressure away from the inlet, and \( p_4 \) is the near-axis pressure at the inlet. The order, \( p_1 > p_2 > p_3 > p_4 \), explains the swirl decay mechanism.

![Figure 4](image_url)

**Figure 4.** (a) Radial profile of axial velocity and (b) pressure distribution.
Figure 5 is a photo of experimental setup for cold counterflow visualization in a model of vortex burner [4]. The pressurized air, supplied through the tangential pipe (visible in the upper right corner), does not immediately go out. First, the air goes near the sidewall to the dead end. This air motion is visualized by spiral tracks of water droplets supplied through a pipette located at the lower part of exhaust opening in figure 5.

The above-discussed flows are high-speed, Re >> 1. Now we review topological transformations governed by the swirl-decay mechanism as Re eventually increases starting with a small value. To this goal, we address a flow in a sealed cylindrical container driven by rotation of its one end-disk. This flow serves for fundamental studies of the vortex breakdown nature during more than a half-century [9].

2. Flow topology in a sealed cylindrical container
A flow in a sealed cylindrical container is free of unpredictable ambient disturbances, and has simple well defined boundary conditions and control parameters. These features allow for meaningful comparison of experimental [10] and numerical [11] results that helps to better understand the mechanism of vortex breakdown and other numerous topological transformations occurring in this flow [4].

2.1. Development of global counterflow
It is instructive first to consider an elongated cylinder whose length-to-radius ratio $H = 10$. Let the left end disk, located at $z = 0$, rotates with angular velocity $\Omega$ and the other walls are stationary. The flow strength is characterized by the Reynolds number, $\text{Re} = \Omega R^2/\nu$; $\nu$ is the kinematic viscosity of a fluid filling the container. Figure 6 shows numerically simulated streamlines of the meridional motion at a few characteristic values of $\text{Re}$ [12, 4].
Figure 6. Development of global meridional circulation as rotation speeds up; Re = 1 (a), 2000 (b), 4000 (c) and 5000 (d).

The flow is multicellular at Re = 1 as figure 6(a) shows. This feature agrees with the theory of creeping eddies developed by Moffatt for planar motions in a corner [13] and extended for cylindrical flows by Shankar [14] and Hills [15]. The left cell, adjacent to the rotating disk, expands while other cells shrink and disappear as Re increases. The physical reason for these topological transformations is the enhancing centrifugal force. It pushes the fast-rotating fluid to the periphery and holds it there. Thus the centrifugal force postpones the flow convergence to the axis for larger z values. As the left cell reaches the right disk, the bulk circulation becomes one-cellular, as figure 6(d) illustrates. To better observe further flow transformations near the stationary disk, now we consider a shorter container with $H = 4$.

2.2. Focusing the flow convergence near the stationary disk

Figure 7 shows that the topological metamorphosis of the meridional motion at $H = 4$ is similar to that at $H = 10$. An expected minor difference is that the number of cells in figure 7(a) is reasonably smaller than in figure 6(a). The most important difference is that figure 7 (in contrast to figure 6) also depicts the pressure distribution (grey scale, color online). Figure 7(a) shows that pressure is maximal (minimal) near the periphery (center) of the rotating disk at Re = 50. As Re increases, the location of minimal pressure shifts along the axis toward the stationary disk. This occurs due to the rotation propagation away from the rotating disk and the cyclostrophic balance, $\partial p/\partial r = \rho v_\phi^2/r$. 
Figure 7. (color online) Development of global meridional circulation as rotation speeds up at $H = 4$; $Re = 50$ (a), 500 (b), and 1500 (d).

Figure 8 depicts a nontrivial transformation of the swirl distribution as $Re$ further increases. The thin curves are contours of constant swirl velocity and the bold curves are the vortex-core boundaries. The core boundary is determined as the location of the swirl velocity maximum at a fixed $z$. 
Figure 8. (color online) Contours of constant swirl velocity and the vortex core boundaries (bold curves) at Re = 2000 (a) and 3000 (b).

Figure 9. (color online) Streamlines (a) and constant-pressure contours (b) at Re = 3000.

Figure 8(a) shows that the core radius smoothly decreases as $z$ increases at Re = 2000. In contrast, figure 8(b) reveals that the core radius sharply drops near $z = 3.2$ and a local maximum of $v_\phi$ exists near $z = 3.5$ and $r = 0.3$ at the center of closed contour in the right-lower corner of figure 8(b). This metamorphosis is produced by the flow convergence to the axis which starts focusing near the stationary
disk. The converging flow transports the angular momentum closer to the axis and thus produces the local maximum of $v_{哲学}$. 

Figure 9(a), showing the streamline pattern at $Re = 3000$, illustrates two new flow features. The first feature is the formation of the Kármán boundary layer [16] near the rotating disk where streamlines are packed. The second feature is that streamlines also start to pack near the stationary disk forming the Bödewadt boundary layer [17]. The cyclostrophic balance does not work within the Bödewadt layer, because the centrifugal force has the second-order zero at the stationary disk where $v_φ = 0$. The unbalanced radial gradient of pressure pushes the fluid toward the axis. This converging stream transports the angular momentum close to axis forming the swirl-velocity local maximum observed in figure 8(b). The cyclostrophic balance works outside of the boundary layer and generates the pressure minimum observed as a light (dark) spot in the right-lower corner of figure 9(a) (figure 9b).

2.3. Emergence of vortex breakdown bubbles

This pressure minimum sucks the downstream fluid and reverses the near-axis flow. The reversal first occurs in local cells (vortex breakdown bubbles, VBBs) where the axial velocity magnitude is small as figure 10 illustrates.

Figure 10(a) shows VBB1, which already emerges near $z = 1.4$, and VBB2 just emerging near $z = 3.2$ at this $Re$ value. The white spot near $z = 3.6$ and $r = 0$ indicates that the minimal pressure is located upstream of VBB2. The bold curve in figure 10(b) depicts the vortex core boundary and reveals that as the fluid goes back near the axis, the vortex core abruptly expands near $z = 3.5$ that is an indicative feature of vortex breakdown.

![Figure 10. (color online) Streamlines (a) and constant-$v_φ$ contours at $Re = 3448$.](image-url)
3. Control of vortex breakdown
The swirl decay mechanism not only explains the flow reversal, but also indicates means of its control. One evident way to suppress a velocity reversal is to reduce the swirl decay. We discuss here two ways to do this: (a) nonintrusive—by co-rotating the sidewall and (b) intrusive—by co-rotating a thin rod inserted near the axis [4]. The way (a) is tested by numerical simulation alone [18] while way (b) is tested both by experiments [19] and numerical simulations [20].

3.1. Suppression of vortex breakdown by sidewall co-rotation
Comparison of figures 11(a) and 10(a) shows that as Re increase, the number of VBBs grows and they start to merge forming an elongated double counterflow. The sidewall co-rotation reverses this trend, as figure 11 illustrates where $\Omega_s$ is the sidewall-to-disk angular velocity ratio. As $\Omega_s$ increases at fixed Re, the VBBs, presented in figure 11(a), shrink, as figure 11(b) shows, and disappears as figure 11(c) illustrates. Note that even the relatively slow co-rotation with $\Omega_s = 0.075$ totally suppresses all vortex breakdown bubbles.

![Figure 11](image)

**Figure 11.** (color online) Suppression of vortex breakdown by co-rotating sidewall whose angular velocity $\Omega_s = 0$ (a), 0.05 (b) and 0.075 (c); Re = 4000.
3.2. *Suppression of vortex breakdown by rod co-rotation*

Figure 12(a) is a schematic of the experimental setup. A glass cylinder of inner radius 7.62 cm and height 91.5 cm is filled with glycerin (77% by volume) and water (23%) solution. A central rod of radius 0.317 cm can rotate independently of the rotating bottom disk. Figure 12(b) is a photo of VBBs at Re = 2720 and $H = 3.25$ where the rod is removed. Figure 13 shows that the rod co-rotation suppresses these VBBs as Re increases at the fixed Re$_r$; Re$_r$ is the Reynolds number based on the radius and angular velocity of the rod.

*Figure 12.* (color online) (a) Schematic of experimental setup and (b) visualized VBBs.

*Figure 13.* Suppression of VBBs by the rod co-rotation. Re$_r = 0$ (a), 12 (b), 21 (c), and 29 (d). The upper row shows flow photos and the lower row shows the corresponding flow schematics. Re = 2720.
4. Vortex breakdown in a two-fluid flow

The swirl decay mechanism also works in a two-fluid flow [21]. Figure 14 shows a schematic of the experimental setup (which serves as a bioreactor model). Cylindrical glass container 1 of radius 45 mm is filled with glycerin (78%) and water (22%) solution of height \( h_g \) at rest and sunflower oil of height \( h_o \) at rest. The fluids are covered by a lid rotated by stepping motor 2 with angular velocity \( \omega \). The cylinder is embedded in glass rectangular box 3 filled with water. The flow is illuminated by pulsed laser 4 and recorded by camera 5. The Reynolds number, \( \text{Re} = \frac{\omega R^2}{\nu_o} \), characterizes the rotation strength; \( \nu_o \) is the kinematic viscosity of the oil.

Figure 14. (color online) Schematic of the experimental setup.

Figure 15. Appearance and disappearance of vortex breakdown bubble in the upper-fluid flow.

Photos in figure 15 show the flow patterns in the vertical cross-section including the rotation axis. The arrowed loops in figure 15 at \( \text{Re} = 600 \) schematically indicate the meridional circulation of the upper fluid driven by the rotating lid. At this \( \text{Re} \), the interface only slightly rises near the axis. The circulation intensifies as \( \text{Re} \) increases and the swirl decay starts to work. The oil convergence near the interface transports the angular momentum to the axis vicinity where the oil rotation intensifies. The cyclostrophic balance develops a region of significantly reduced pressure above the interface-axis intersection. The low
pressure sucks the downstream near-axis flow, decelerates it, then reverses it, and thus generates the vortex breakdown bubble (VBB) observed in figure 15 at Re = 700 near the center of the oil domain.

As Re further increases, the suction attracts VBB downward while the interface continues to rise near the axis. The interface and VBB nearly meet, as figure 15 shows at Re = 900, and thus block the oil convergence to the axis. The minimal pressure rises killing the suction. In turn, the absence of suction kills the VBB. There is no VBB in figure 15 at Re = 1100.

Figure 16 shows the corresponding transformations of velocity distribution $v_z(z)$ at $r = 0$. The empty (filled) circles present velocity values in the upper (lower) fluid. The bold horizontal line indicates the interface location. The dashed horizontal line indicates the boundary between the lower-fluid cells. We focus here on the upper-fluid (oil) flow near the axis shown by the empty circles in figure 16.
The oil velocity profile has the single maximum at Re = 400. At Re = 500, the profile has two local maxima separated by the local minimum. The minimum value decreases, but yet remains positive at Re = 600. It becomes negative at Re = 700. This means that the axial velocity reverses inside the VBB observed in figure 15 at Re = 700. Next, the minimum location shifts closer to the interface as figure 16 illustrates at Re = 900. Finally, the minimum value becomes positive, i.e., the VBB disappears as figure 16 illustrates at Re = 1100.

Figures 15 and 16 together show that the swirl decay mechanism works in a two-fluid flow similarly as it works in a single-fluid flow.

Conclusion
This review demonstrates that global counterflows, occurring in vortex devices, and local counterflows in vortex breakdown bubbles all can be caused by a common physical reason here referred to as the swirl decay mechanism (SDM). SDM involves (a) the cyclostrophic balance, (b) the weakening of fluid rotation downstream, and (c) formation of a local minimum of pressure. SDM explains all features of vortex breakdown, discussed in these review, including single and two-fluid flows as well as means of vortex breakdown control.

Acknowledgments
The work was partially supported by Russian Scientific Foundation (Project № 19-19-00083).

References
[1] Shtern V 2012 Counterflows: Paradoxical Fluid Mechanics Phenomena (New York: Cambridge University Press)
[2] Schultz S, Gorbach G and Piesche M 2009 Chem. Eng. Sci. 64 3935–52
[3] Ranque G 1933 J. de Physique et Radium 4 112S
[4] Shtern V 2018 Cellular Flows: Topological Metamorphoses in Fluid Mechanics (New York: Cambridge University Press)
[5] Borissov A A, Shtern V N, Gonzalez H and Yurausquin A 2010 Proc. 8th Int. Symp. High Temp. Air Combustion and Gasification (Poland, Poznan) 297–304
[6] Shtern V and Borissov A 2010 Phys. Fluids 22 063601
[7] Shtern V and Borissov A 2010 Phys. Fluids 22 083601
[8] Borissov A and Shtern V 2010 Int. J. of Energy for Clean Environment 11 203–25
[9] Vogel H U 1968 Max-Planck-Inst. für Strömungsforschung (Gottingen: Bericht) 6
[10] Esquidier MP 1984 Exp. Fluids 2 189–96
[11] Lopez JM 1990 J. Fluid Mech. 221 533–52
[12] Herrada M A, Shtern V N and Torregrosa M M 2015 J. Fluid Mech. 766 590–610
[13] Moffatt H K 1964 J. Fluid Mech. 18 1–18
[14] Shankar PN 1998 Phys. Fluids 10 540–9
[15] Hills CP 2001 Phys. Fluids 13 2279–86
[16] Kármán T 1921 Z. angew. Math. und Mech. 1 233–52
[17] Bödewadt UT 1940 Z. angew. Math. und Mech. 20 241–53
[18] Shtern V N, Torregrosa V V and Herrada M A 2012 Phys. Fluids 24 043601
[19] Husain H, Shtern V and Hussain F 2003 Phys. Fluids 15 271–9
[20] Herrada M A and Shtern V N 2003 Phys. Rev. E 68 041202
[21] Naumov I V, Glavny V G, Sharifullin B R and Shtern V N 2019 Phys. Rev. Fluids 4 054702