Two-nucleon Correlations in the Decay of Unbound Nuclei beyond the Drip Lines

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We discuss the two-neutron emission decay of $^{26}$O nucleus from a viewpoint of dineutron correlations between the valence neutrons. We first discuss how the dineutron correlation is realized both in the coordinate space and in the momentum space by employing a three-body model. We then discuss the decay energy spectrum as well as the angular distribution of emitted neutrons. We show that an emission of two neutrons in back-to-back directions is enhanced, which we interpret as a clear manifestation of the dineutron correlation between the valence neutrons.

**KEYWORDS:** dineutron correlations, unbound nuclei, three-body model

1. Introduction

In recent years, the dineutron and diproton correlations have attracted lots of attention in connection with physics of weakly bound nuclei [1–4]. These are spatial correlations with which two nucleons are localized in the surface region of nuclei. The mean opening angles between the valence neutrons with respect to the center of the core of Borromean nuclei have been extracted from the measured Coulomb breakup cross sections [5, 6] and found to be significantly smaller than the value for the independent neutrons, 90 degrees [5, 7, 8]. The extracted values are $\langle \theta_{12} \rangle = 65.2 \pm 12.2$ degrees for $^{11}$Li and $74.5 \pm 12.1$ degrees for $^6$He [7], clearly indicating the existence of the dineutron correlation in these nuclei.

As other probes for the dineutron and diproton correlations, a two-proton radioactivity, that is, a spontaneous emission of two protons from proton-unbound nuclei, has been considered to be a good candidate [9]. Very recently, the ground state two-neutron emissions have also been observed, e.g., in $^{16}$Be [10], $^{13}$Li [11], and $^{26}$O [12, 13]. An attractive feature of these phenomena is that the two valence nucleons are emitted directly from the ground state in contrast to the Coulomb breakup, in which a nucleus is first excited by the external electromagnetic interaction.

In this contribution, we present three-body model calculations for the two-neutron decay of the $^{26}$O nucleus [14,15]. To this end, we assume the three-body structure of $^{24}$O+$n+n$ and take into account the couplings to continuum by the Green’s function technique, which was originally invented in order to describe the continuum dipole excitations of $^{11}$Li [16].

We shall discuss a role of dineutron correlation in the decay process, and thus let us first discuss the mechanism of the dineutron correlation in the next section.
2. Dineutron correlation in the coordinate and in the momentum spaces

As has been discussed in Refs. [4, 17, 18], the dineutron correlation, that is, the spatial localization of the two-particle density, is caused by a coherent admixture of many configurations with opposite parity states. Symbolically, let us write a two-particle wave function as,

\[ \Psi(r, r') = \alpha \Psi_{ee}(r, r') + \beta \Psi_{oo}(r, r'), \]  

(1)

where \( \Psi_{ee} \) and \( \Psi_{oo} \) are two-particle wave functions with even and odd angular momentum states, respectively. Notice that \( \Psi_{ee}(r, r') = \Psi_{ee}(r, -r') \) and \( \Psi_{oo}(r, r') = -\Psi_{oo}(r, -r') \). For nuclear systems, the coefficients \( \alpha \) and \( \beta \) are such that the interference term in the two-particle density,

\[ \alpha^* \beta \Psi_{ee}^* \Psi_{oo} + \text{c.c.}, \]

is positive for \( r' = r \) while it is negative for \( r' = -r \) [18]. That is, the two-particle density is enhanced for the nearside configuration with \( r \sim r' \) as compared to the farside configuration with \( r \sim -r' \), provided that the wave functions \( \Psi_{ee}(r, r') \) and \( \Psi_{oo}(r, r') \) are mixed coherently. This is nothing but the dineutron correlation.

Now let us take the Fourier transform of \( \Psi(r, r') \), that is,

\[ \tilde{\Psi}(k, k') = \int \! dr dr' e^{i k \cdot r} e^{i k' \cdot r'} \Psi(r, r'). \]

(2)

Notice that there is a factor \( i^l \) in the multipole decomposition of \( e^{i k \cdot r} \), and that \( (i^l)^2 \) is +1 for even values of \( l \) and −1 for odd values of \( l \). This leads to [14]

\[ \tilde{\Psi}(k, k') = \alpha \tilde{\Psi}_{ee}(k, k') - \beta \tilde{\Psi}_{oo}(k, k'). \]

(3)

If one constructs a two-particle density in the momentum space with this wave function, the interference term therefore acts in the opposite way to that in the coordinate space. That is, the two-particle density in the momentum space is hindered for \( k \sim k' \), while it is enhanced for \( k \sim -k' \). This could be intuitively understood also from a point of view of uncertainty relation between coordinate and momentum.

Figure 1 shows the two-particle density for the \( ^6\text{He} \) nucleus in the coordinate space (the left panel) and in the momentum space (the right panel), obtained with the three-body model.

**Fig. 1.** The two-particle density of the \( ^6\text{He} \) nucleus in the coordinate space (the left panel) and in the momentum space (the right panel). The density in the coordinate space is plotted as a function of \( |r| = |r'| = r \) and the angle between \( r \) and \( r' \), while density in the momentum space is plotted as a function of \( |k| = |k'| = k \) and the angle between \( k \) and \( k' \). Weight factors of \( 8\pi^2 r^4 \sin \theta \) and \( 8\pi^2 k^4 \sin \theta \) are multiplied to the densities, and thus the units in the plots are fm\(^{-2}\) and fm\(^2\) for the left and the right panels, respectively.
of Ref. [3]. To plot the density, we set $|r| = |r'|$ and $|k| = |k'|$. One can see that the density in the coordinate space is enhanced for small values of the angle between $r$ and $r'$. In contrast, the density in the momentum space is enhanced for large values of the angle between $k$ and $k'$.

This leads to an interesting idea on two-nucleon emission. That is, two nucleons are confined inside a nucleus as a spatially compact dinucleon-like cluster, and they are thus first emitted together in a similar direction. Outside a nucleus, since the two nucleons do not bound, they move according to the momentum distribution which they originally have before the emission, that is, the two nucleons fly apart in the opposite direction. This behavior has actually been seen in a recent time-dependent three-body model calculation for two-proton decay of $^{6}$Be [19].

3. Two-neutron decay of $^{26}$O

Let us now apply the three-body model to the two-neutron decay of $^{26}$O and discuss how the dineutron correlation affects its dynamics. To this end, we use a Woods-Saxon potential between a valence neutron and the core nucleus, $^{24}$O, in which the parameters are chosen in order to reproduce the energy of the $d_{3/2}$ resonance state of $^{25}$O at 770 keV [20]. For the two-body pairing interaction between the valence neutrons, we employ a density-dependent contact interaction [3], whose parameters are adjusted to reproduce the ground state energy of $^{27}$F using a similar three-body model of $^{25}$F+$n+n$.

3.1 Decay energy spectrum

We first discuss the decay energy spectrum. With the three-body model for $^{26}$O, we compute it for a given angular momentum $I$ as,

$$\frac{dP_I}{dE} = \sum_k |\langle \Psi_k^{(I)} | \Phi_{\text{ref}}^{(I)} \rangle|^2 \delta(E - E_k),$$

(4)

where $\Psi_k^{(I)}$ is a solution of the three-body model Hamiltonian with the angular momentum $I$ and the energy $E_k$, and $\Phi_{\text{ref}}^{(I)}$ is the wave function for a reference state with the same angular momentum. For the reference state, we use the uncorrelated two-neutron state in $^{27}$F with the $|1d_{3/2} \otimes 1d_{3/2}|(IM)\rangle$ configuration, which is dominant in the ground state of $^{27}$F, since $^{26}$O was produced in the experiment of Ref. [12] with the proton knockout reaction of $^{27}$F. The actual calculations for the decay energy spectrum are done using the Green’s function technique [16], by fully taking into account the continuum effects. See Refs. [14,15] for details.

The left panel of Fig. 2 shows the decay energy spectrum of $^{26}$O for the $I = 0$ (the dashed line) and $I=2$ (the solid line). For a presentation purpose, we have introduced a width of 0.21 MeV to the spectrum. For comparison, we also show the spectrum for the uncorrelated case by the dotted line, which gives the same spectrum both for $I = 0$ and $I = 2$. For the uncorrelated case, the spectrum has a peak at $E = 1.54$ MeV, that is twice the single-particle resonance energy, 0.77 MeV. With the pairing interaction between the valence neutrons, the peak energy is shifted towards lower energies. The energy shift is larger in $I = 0$ than in $I = 2$. That is, the peak in the spectrum appears at $E = 0.148$ MeV ($\Delta E = -1.392$ MeV) for $I = 0$ and at $E = 1.354$ MeV ($\Delta E = -0.186$ MeV) for $I = 2$.

The fact that the $2^+$ state appears at an energy slightly smaller than the unperturbed energy is a natural consequence of the three-body model. In standard textbooks of nuclear physics, it is shown that the energy shift due to a pairing interaction, $v(r_1, r_2) = -g \delta(r_1-r_2)$,
Fig. 2. (the left panel) The decay energy spectrum for the two-neutron emission decay of $^{26}$O. The dashed and the solid lines are for the $0^+$ and $2^+$ states, respectively. The dotted line shows the uncorrelated spectrum obtained by ignoring the interaction between the valence neutrons. (the right panel) The decay energy spectrum obtained by superposing the $I = 0$ and $I = 2$ components as is indicated in the figure. The dashed line is the same as the one in the left panel, that is, the decay energy spectrum for the pure $I = 0$ configuration. The experimental data, normalized to the unit area, are taken from Ref. [12].

is evaluated for a single-$j$ orbit as,

$$\Delta E_I = \langle [jj]|(IM)\rangle - g \delta(r_1 - r_2)|[jj]|(IM)\rangle = -gF_r \frac{(2j+1)^2}{8\pi} \left( \begin{array}{ccc} j & j & I \\ 1/2 & -1/2 & 0 \end{array} \right)^2,$$

where $F_r$ is the radial integral of the four single-particle wave functions. If one applies this formula to the $^{26}$O nucleus and sets $j = d_{3/2}$, one obtains

$$\Delta E_{I=0} = -\frac{16}{8\pi} gF_r \cdot \frac{1}{4^2}, \quad \Delta E_{I=2} = -\frac{16}{8\pi} gF_r \cdot \frac{1}{20}.$$

This equation predicts $\Delta E_{I=2} = -0.278$ MeV for $\Delta E_{I=0} = -1.392$ MeV, that is, $\Delta E_{I=0}/\Delta E_{I=2} = 5$. This value is compared to the calculated value of $\Delta E_{I=0}/\Delta E_{I=2} = 7.48$ obtained with the present three-body model. Even though the ratio $\Delta E_{I=0}/\Delta E_{I=2}$ in the three-body model somewhat deviates from the simple estimates of Eq. (6) due to the many-body continuum effects, the small energy shift for the $2^+$ state can be well understood by these formulas derived for the single-$j$ model with the residual pairing interaction. Notice that the $2^+$ energy never exceeds the unperturbed energy, and thus it must be smaller than 1.54 MeV for the $^{26}$O nucleus.

The right panel of Fig. 2 shows the decay energy spectrum obtained by including the contribution of the $I = 2$ configuration. To this end, we take a linear combination of the $I = 0$ and $I = 2$ contributions, that is,

$$\frac{dP}{dE} = (1 - \gamma) \frac{dP_{I=0}}{dE} + \gamma \frac{dP_{I=2}}{dE}.$$

The actual value of $\gamma$ would depend on the details of the wave function of $^{27}$F as well as the reaction dynamics of the proton knock-out reaction of $^{27}$F with which the initial state of $^{26}$O was prepared in the experiment of Ref. [12]. Here we arbitrarily take $\gamma = 0.1$. For comparison, the figure also shows the pure $I = 0$ component, which is the same as that shown in the
Fig. 3. The probability distribution with respect to the opening angle of the momentum vectors of the emitted two neutrons from $^{26}\text{O}$. The solid and the dashed lines show the correlated and uncorrelated results, respectively.

left panel. One can see that the experimental data are reproduced slightly better by mixing the $I=2$ component, although the error bars are large and one may not draw a definite conclusion.

We mention that we obtain the $2^+$ state of $E=1.338$ MeV if we use the pairing interaction which yields the ground state energy of 5 keV rather than 148 keV. This value is similar to the previous result, $E=1.354$ MeV, and we thus conclude that the energy of the $2^+$ state is much less sensitive to the $nn$ interaction as compared to the $0^+$ state. This implies that the $2^+$ state of $^{26}\text{O}$ should definitely appear around $E=1.3$ MeV as long as the three-body picture is correct.

3.2 Angular distribution of the emitted neutrons

Let us next discuss the angular distribution of the emitted neutrons. The dotted line in Fig. 3 shows the distribution obtained without including the $nn$ interaction, which is almost symmetric around $\theta_{12}=\pi/2$. In the presence of the $nn$ interaction, the angular distribution becomes highly asymmetric, in which the emission of two neutrons in the opposite direction (that is, $\theta_{12}=\pi$) is enhanced, as is shown by the solid line. The average angle is calculated to be $\langle \theta_{12} \rangle = 115.3^\circ$. This behavior should reflect properties of the resonance wave function of $^{26}\text{O}$, as we have discussed in Sec. 2. That is, because of the continuum couplings, several configurations with opposite parity states mix coherently, which leads to an enhancement of $\theta \sim 0$ in the coordinate space and of $\theta \sim \pi$ in the momentum space (for the $^{26}\text{O}$ nucleus, it is mainly due to the interference between the $l=0$ and $l=1$ components, as higher partial waves are considerably suppressed inside the centrifugal barrier for $e_1 \sim e_2 \sim E_{g.s.}/2=0.07$ MeV). See also Ref. [21]. We can therefore conclude that, if an enhancement in the region of $\theta \sim \pi$ in the angular distribution was observed experimentally, that would make a clear evidence for the dineutron correlation in this nucleus.

4. Summary

We have discussed a role of dineutron correlation in the two-neutron emission decay of the neutron unbound nucleus $^{26}\text{O}$. To this end, we have used the three-body model with a contact neutron-neutron ($nn$) interaction. In the absence of the $nn$ interaction, the two va-
Valence neutrons occupy the $d_{3/2}$ resonance state at 0.77 MeV in $^{25}\text{O}$ and thus the unperturbed energy for $^{26}\text{O}$ is 1.54 MeV above the $2\pi$ threshold. In the presence of the $\text{nn}$ interaction, the energy is shifted towards low energies. For the ground state, the experimental data suggest that the energy is shifted almost to the threshold. On the other hand, we have shown that the $2^+$ state should appear at around $E = 1.35$ MeV and this value does not change much even if we change the $\text{nn}$ interaction to vary the ground state energy from 150 keV to 5 keV. This $2^+$ energy is close to, but slightly smaller than, the unperturbed energy and thus the energy shift from the unperturbed energy is much smaller than the energy shift for the $0^+$ state. We have argued that this is a typical spectrum well understood by the single-$j$ model with the pairing residual interaction.

We have also discussed that the density distribution of a three-body resonance state in $^{26}\text{O}$ is strongly reflected in the angular distribution of the emitted neutrons. In particular, the emission of the two neutrons in back-to-back angles is enhanced in the angular distribution, that can be interpreted as a clear evidence for the dineutron correlation. That is, the coherent admixture of many configurations with opposite parity states leads to the dineutron correlation, with which the two valence neutrons tend to have momenta in the opposite directions.

So far, the angular distribution for the two-neutron decay of $^{26}\text{O}$ has not yet been measured experimentally. It would be extremely intriguing if it will be measured at new generation RI beam facilities, such as the SAMURAI facility at RIKEN [22]. A measurement of the $2^+$ state would also be useful in order to understand the ground state properties of the unbound $^{26}\text{O}$ nucleus.

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