Abstract

In this paper, we reexamine the new physics contributions to seven well measured $B \rightarrow PP$ decays in the standard model (SM) and the general two-Higgs-doublet model (model III) and compare the theoretical predictions with the new data. Within the considered parameter space we found that: (a) the measurements of the branching ratios for $B \rightarrow \pi^+\pi^-$, $K^-\pi^+$ and $K^0\pi^+$ lead to a strong constraint on the form factor $F_{B\pi^0}(0)$: $F_{B\pi^0}(0) = 0.24 \pm 0.03$; (b) the new physics enhancements to the penguin-dominated $B \rightarrow K\pi$ and $K\eta'$ decays are significant in size, $\sim (40 - 65)\%$ w.r.t the SM predictions, and play an important role in restoring the consistency between the data and the theory.

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As is well known, one of the main objectives of B experiments is to probe for the possible effects of new physics beyond the standard model (SM). Precision measurements of the B meson system can provide an insight into very high energy scales via the indirect loop effects of the new physics \[1,2\].

Up to now, CLEO, BaBar and Belle Collaboration [3–5] have observed eighteen two-body charmless hadronic $B_{u,d}$ meson decay modes. Seven well measured $B \to PP$ (here $P$ refers to the light pseudo-scalar mesons) decays are

$$B \to \pi^\pm \pi^\mp, K\pi, K\eta'.$$  

These decay modes are closely related through the isospin symmetry and the SU(3) flavor symmetry, phenomenologically very interesting due to their key role in extracting the unitary angles $\alpha$ and $\gamma$, and the appearance of the so-called $\eta'K$ puzzle: the observed $B \to K\eta'$ decay rates [3–5] are much larger than what expected in the standard model based on the effective Hamiltonian with generalized factorization (GF) approach [6–9]. To accommodate the data, one may need an additional contribution unique to the $\eta'$ meson in the framework of the SM [10–13], or enhancements from new physics models beyond the SM [14,15].

In a previous paper [14], we considered the second possibility and calculated the new physics effects on the two-body charmless hadronic B meson decays in the general two-Higgs-doublet models (2HDM’s) [16]. In this paper, we focus on seven well measured $B \to PP$ decay modes and compare the theoretical predictions with the newest data. We still use the low-energy effective Hamiltonian with GF approach [8,9] to calculate the new physics contributions. For recent studies of $B \to PP$ decays in the SM with the QCD factorization (BBNS) and perturbative QCD (PQCD) approaches [17,18], one can see the papers [19–22] and references therein.

For the inclusive three-body decays $b \to s\bar{q}q$ with $q \in \{u,d,s\}$ the effective Hamiltonian can be written as [8],

$$H_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left\{ \sum_{j=1}^{2} C_j \left( V_{ub}V_{us}^* Q_j^s + V_{cb}V_{cs}^* Q_j^c \right) - V_{tb}V_{ts}^* \left[ \sum_{j=3}^{10} C_j Q_j + C_g Q_g \right] \right\}.$$  

The explicit expressions for all operators can be found easily for example in Ref. [8]. For $b \to d\bar{q}q$ decays, one simply makes the replacement $s \to d$. Following Ref. [8], we also neglect the effects of the electromagnetic penguin operator $Q_7\gamma$, the weak annihilation and exchange diagrams. Within the SM and at scale $M_W$, the Wilson coefficients $C_1(M_W), \cdots, C_{10}(M_W)$ at next-to-leading logarithmic order NLO) and $C_g(M_W)$ at leading logarithmic order (LO) have been given for example in Ref. [6].

In a recent paper [23], Chao et al. studied the decay $b \to s\gamma$ in model III (the third type of 2HDM’s) by assuming that only the couplings $\lambda_{tt} = |\lambda_{tt}|e^{i\theta_t}$ and $\lambda_{bb} = |\lambda_{bb}|e^{i\theta_b}$ are non-zero. They found that the constraint on $M_{H^+}$ imposed by the CLEO data of $b \to s\gamma$ can be greatly relaxed by considering the phase effects of $\lambda_{tt}$ and $\lambda_{bb}$. From the studies of Refs. [2,23], we know that for model III the parameter space

$$\lambda_{ij} = 0, \text{ for } ij \neq tt, \text{ or } bb,$$

$$|\lambda_{tt}| = 0.3, \quad |\lambda_{bb}| = 35, \quad \theta = (0^0 - 30^0), \quad M_{H^+} = (200 \pm 100)\text{GeV},$$
are allowed by the available data, where \( \theta = \theta_{bb} - \theta_{tt} \). In this paper, we calculate the new physics contributions to seven B meson decay modes in the Chao-Cheung-Keung (CCK) scenario of model III [23].

Following the same procedure as in the SM, it is straightforward to calculate the new \( \gamma \)-, \( Z^0 \)- and gluonic penguin diagrams induced by the exchanges of charged-Higgs bosons appeared in model III (for details of the calculations, see Ref. [14]). In the NDR scheme, by using the input parameters as given in Eqs.(3) and setting \( \mu = 2.5 \) GeV, we find that:

\[
C_1 = 1.1245, \quad C_2 = -0.2662, \quad C_3 = 0.0186, \quad C_4 = -0.0458, \\
C_5 = 0.0113, \quad C_6 = -0.0587, \quad C_7 = 0.0006, \quad C_8 = 0.0007, \\
C_9 = -0.0096, \quad C_{10} = 0.0026, \quad C_{g}^{\text{eff}} = 0.3364
\]  

(4)

where \( C_{g}^{\text{eff}} = C_{8G} + C_{5} \).

For the \( B \rightarrow PP \) decay modes considered here, the decay amplitudes as given in Ref. [8] will be used without further discussion about details. We focus on estimating the new physics effects on those seven well measured decay modes. In the NDR scheme and for \( SU(3)_C \), the effective Wilson coefficients can be written as [3]

\[
C_i^{\text{eff}} = \left[ 1 + \frac{\alpha_s}{4\pi} \left( i T^V + \gamma_V^T \log \frac{m_b}{\mu} \right) \right] C_j + \frac{\alpha_s}{24\pi} A_i (C_t + C_p + C_g) + \frac{\alpha_{ew}}{8\pi} B_i C_e,
\]  

(5)

where \( A_i = (0, 0, -1, 3, -1, 3, 0, 0, 0, 0) \), \( B_i = (0, 0, 0, 0, 0, 0, 1, 0, 1, 0) \), \( \hat{r}_V \) and \( \gamma_V \) contain the process-independent contributions from the vertex diagrams [1][4]. The function \( C_t, C_p, \) and \( C_g \) describe the contributions arising from the penguin diagrams of the current-current \( Q_1,2 \), the QCD operators \( Q_3-Q_6 \), and the tree-level diagram of the magnetic dipole operator \( Q_{8G} \), respectively. The explicit expressions of the functions \( C_t, C_p, \) and \( C_g \) can be found for example in Ref. [14].

In the generalized factorization approach, the effective Wilson coefficients \( C_i^{\text{eff}} \) will appear in the decay amplitudes in the combinations,

\[
a_{2i-1} \equiv C_{2i-1}^{\text{eff}} + \frac{C_{2i}^{\text{eff}}}{N_c^{\text{eff}}}, \quad a_{2i} \equiv C_{2i}^{\text{eff}} + \frac{C_{2i-1}^{\text{eff}}}{N_c^{\text{eff}}}, \quad (i = 1, \ldots, 5)
\]  

(6)

where the effective number of colors \( N_c^{\text{eff}} \) is treated as a free parameter varying in the range of \( 2 \leq N_c^{\text{eff}} \leq \infty \), in order to model the non-factorizable contribution to the hadronic matrix elements.

In the B rest frame, the branching ratios \( \mathcal{B}(B \rightarrow PP) \) can be written as

\[
\mathcal{B}(B \rightarrow XY) = \tau_B \frac{|p|}{8\pi M_B^2} |M(B \rightarrow XY)|^2,
\]  

(7)

where \( \tau_B = 1.653 \) ps and 1.548 ps for \( B = B_u^- \) and \( B_d^0 \) [24], respectively. \( p_B \) is the four-momentum of the B meson, \( M_B = 5.279 \) GeV is the mass of \( B_u \) or \( B_d \) meson. In the numerical calculations we use the same input parameters (the masses, gauge couplings, decay constants, form factors, etc.) as in Ref. [14]. Particularly, the elements of CKM matrix in the Wolfenstein parametrization are: \( A = 0.81, \lambda = 0.2205, \rho = 0.12, \eta = 0.34, \)
which corresponds to $\gamma = 71^\circ$ and $\sin 2\beta = 0.79$ favored by the global fit and the new measurements \cite{25}.

For the seven well measured $B \rightarrow PP$ decay modes, currently available measurements as reported by CLEO, BaBar and Belle Collaborations \cite{3,4} and their averages are listed in Table \ref{tab:1}. The data have been changed greatly when compared with those in year 2000:

- For ratio $B(B \rightarrow \pi^+\pi^-)$, the new BaBar result is $(4.1 \pm 1.0 \pm 0.7) \times 10^{-6}$ instead of the old $(9.3^{+2.8}_{-2.1}^{+1.2}_{-1.4}) \times 10^{-6}$. The average therefore decreased to $(4.4 \pm 0.9) \times 10^{-6}$.

- The BaBar measurement of $B(B \rightarrow K^0\pi^0)$ is only $\sim 8 \times 10^{-6}$, the average therefore becomes much smaller than two years ago.

- For the ratio $B(B \rightarrow K^0\eta')$, both the BaBar and Belle result are much smaller than CLEO’s measurement, the average is only $(56 \pm 10) \times 10^{-6}$ and clearly smaller than the branching ratio of $B \rightarrow K^+\eta'$ decay.

In the SM and GF approach, the decay $B \rightarrow \pi^+\pi^-$ is very simple and receives contributions from the dominated tree diagram, the QCD and electroweak penguin diagrams, and depend on one form factor $F_0^{B\pi}(0)$ only, as can be seen from the decay amplitude \cite{3}:

$$
\mathcal{M}(\bar{B}^0 \rightarrow \pi^+\pi^-) = -i \frac{G_F}{\sqrt{2}} f_{\pi} F_0^{B\pi}(m_{\pi}^2) \left(m_B^2 - m_{\pi}^2\right) \\
\times \left\{ V_{ub} V_{td}^* a_1 - V_{tb} V_{id}^* \left[ a_1 + a_{10} + 2 (a_6 + a_8) \frac{m_{\pi}^2}{(m_b - m_a)(m_a + m_d)} \right] \right\} 
$$

The new physics contribution to this decay in the model III is only 2.5\% and thus can be neglected. In the SM, there is no other contribution to this decay mode and therefore it seems to be a clean decay mode to determine the form factor $F_0^{B\pi}(0)$. For $\gamma \approx 71^\circ$ as indicated by the global fit, the interference between the tree and penguin-diagrams is constructive, and the measured branching ratio of this decay leads to a small $F_0^{B\pi}(0)$

$$
F_0^{B\pi}(0) = 0.21 \pm 0.03. \tag{9}
$$

Which is clearly smaller than the values from Lattice-QCD or Light-cone QCD sum rules: $F_0^{B\pi}(0) = 0.30 \pm 0.04$, or $0.28 \pm 0.05$ as given in Ref. \cite{26} and Ref. \cite{27}, respectively. For $F_0^{B\pi}(0) = 0.3$ and $\gamma = 71^\circ$, however, the SM prediction for the branching ratio is $(7.5 - 10.7) \times 10^{-6}$ for $N_{\ell}^\ell = 2 - \infty$ in the GF approach and about $9 \times 10^{-6}$ in the BBNS approach \cite{28}, which is clearly too large to be consistent with the data. We thus believe that the form factor $F_0^{B\pi}(0)$ should be apparently smaller than 0.3.

On the other hand, the QCD-penguin-dominated $B \rightarrow K^-\pi^+$ and $K^0\pi^+$ decays also depend on the form factor $F_0^{B\pi}(0)$, as can be seen from the decay amplitudes as given in Ref. \cite{3}. But one should be very careful to extract the form factor $F_0^{B\pi}(0)$ from these two decay modes because of (a) the neglected rescattering and other non-factorized contributions to these two decays may be large; and (b) the new physics contributions in the model III are also large, $\sim 50\%$ respect to the SM predictions. In the GF approach, the measured branching ratios of these two decays prefer a larger $F_0^{B\pi}(0)$: $F_0^{B\pi}(0) = 0.30 \pm 0.03$ if the new physics contributions to these two decay modes are not included. But
\[ F_0^{B\pi}(0) = 0.24 \pm 0.03. \] (10)

if the measured branching ratios of \( B \to \pi^+\pi^-, K^-\pi^+, K^0\pi^+ \) decays and the new physics enhancements are all taken into account. Although the central value of the form factor in Eq.(10) is still smaller than \( F_0^{B\pi}(0) = 0.28 \pm 0.05 \) as given in Refs. 26,27, they are compatible within errors. In the numerical calculations, we will use \( F_0^{B\pi}(0) = 0.24 \pm 0.03. \)

Furthermore, the form factor \( F_0^{BK}(0) \) cannot deviate too much from \( F_0^{B\pi}(0) \), otherwise the \( SU(3) \) flavor symmetry will be broken badly. As indicated by the data and theoretical considerations, it is a good approximation to take \( F_0^{BK}(0)/F_0^{B\pi}(0) = f_k/f_\pi \) as a measure of \( SU(3) \) symmetry breaking. We then find that

\[ F_0^{BK}(0) = 0.29 \pm 0.04. \] (11)

In the GF approach, the QCD-penguin-dominated \( B \to K\eta' \) decays depend on the form factors \( F_0^{BK}(0) \) and \( F_0^{B\eta'}(0) \),

\[ F_0^{B\eta'}(0) = F_0^{B\pi}(0) \left[ \frac{\sin \theta_8}{\sqrt{6}} + \frac{\cos \theta_0}{\sqrt{3}} \right] = 0.10 \pm 0.02, \] (12)

for \( F_0^{B\pi}(0) = 0.24 \pm 0.03, \theta_0 = -9.1^\circ \) and \( \theta_8 = -22.2^\circ \) in the two-angle mixing scheme 29.

In Table I, we show the theoretical predictions for the branching ratios of seven studied decay modes. We use the form factors as given in Eqs.(10,11,12), and keep all other input parameters the same as those used in Ref. 14. The branching ratios collected in Table I are the averages of the corresponding \( B \) and anti-\( B \) meson decay rates. The ratio \( \delta B \) describes the new physics correction on the decay rates and is defined as

\[ \delta B(B \to XY) = \frac{B(B \to XY)^{III} - B(B \to XY)^{SM}}{B(B \to XY)^{SM}}. \] (13)

From Table I and Table II, we find that

- By using \( F_0^{B\pi}(0) = 0.24 \pm 0.03 \), our predictions for \( B(B \to \pi^+\pi^-) \) in both the SM and model III are in agreement with the data.

- For \( B \to K\pi \) decays, the SM predictions seem smaller than the measurements. The results under the BBNS approach are similar 22. For the \( B \to K^0\pi^0 \) decay, specifically, the SM prediction is about half of the measured decay rate, and the new physics enhancement is essential for the theoretical prediction to become consistent with the data.

- By taking into account the uncertainties of those input parameters as given explicitly in Ref. 14, we find numerically that

\[ B(B \to K^+\eta') \approx B(B \to K^0\eta') = \begin{cases} (10 - 40) \times 10^{-6} \text{ in SM}, \\ (17 - 57) \times 10^{-6} \text{ in Model III}. \end{cases} \] (14)

where the uncertainties of those input parameters have been considered. It is easy to see that the SM predictions in the GF approach is about half of the measured value.
In BBNS approach, the theoretical predictions for the corresponding branching ratios are also much smaller than the experimental data \[22\]. The new physics enhancement can boost the theoretical predictions close to the lower part of the measured values, but still leaves a moderate space for additional contributions.

Since 1997, the unexpected large $\eta'$ production has been widely discussed in the literature \[10–15,20–22,28\]. For the sake of completeness, we here make a brief comment on some typical interpretations in the framework of the SM.

- Atwood and Soni \[10\] gave arguments for the need of enhanced $b\to sg^*$ decays followed by $g^*\to \eta'g$ via the QCD gluon anomaly. Taking a constant $gg\eta'$ vertex form factor $H(0,0,m_{\eta'}^2)$, the observed large branching ratio $B\to \eta'X_s$ can be achieved. But as pointed by Hou and Tseng \[10\], Kagan and Petrov \[15\], if one considers the running of $\alpha_s$, and the $m_{\eta'}^2/(q^2-m_{\eta'}^2)$ dependence of the $gg\eta'$ coupling, the result presented in \[10\] will be reduced greatly.

- Halperin and Zhitnitsky \[13\] argued that the dominant contribution to $\eta'$ production is due to the Cabibbo favored $b\to (\bar{c}c)_1s$ process followed by the transition $(\bar{c}c)_1\to \eta'$, i.e. the ”intrinsic charm” component of $\eta'$. But according to the explicit calculations in Refs. \[8,9\], this mechanism can not give a good explanation for the measured $B\to K\eta'$ decay rates.

- Yuan and Chao \[11\] argued that the inclusive $\eta'$ production in B decays may dominantly come from the Cabibbo favored $b\to (\bar{c}c)_8s$ process where $\bar{c}c$ pair is in a color-octet configuration, and followed by the nonperturbative transition $(\bar{c}c)_8\to \eta'X$.

- The authors of Ref. \[12\] proposed di-gluon fusion mechanism. It seems that this mechanism could enhance $\eta'$ production. But because of our ignorance about the form factor of $g^*g^*\eta'$ vertex, there are large uncertainties in calculation.

- Recently, M.Z.Yang and Y.D.Yang reconsidered the di-gluon mechanism and gave a calculation for $B\to \eta'(P$ in the BBNS approach. They computed the vertex $g^*g^*\eta'$ in perturbative QCD, and found that the branching ratios of $B\to K\eta'$ are really enhanced and in agreement with the data. But, as pointed in Ref. \[22\], the consistency of Yang’s perturbative calculation is questionable due to the end point behavior.

From above discussions, one can understand that it is still an open question for us to interpret the large branching ratios of $B\to K\eta'$ decays. Further investigations for various possible mechanisms are welcome.

Furthermore, because of the isospin symmetry between the $u$ and $d$ quarks, the decays $B\to K^+\eta'$ and $K^0\eta'$ should theoretically have similar branching ratios. The known new mechanisms in the SM or the new physics models also contribute to these two decay modes in very similar way. The measurements of the BaBar and Belle collaboration, however, show a clear difference between these two decay rates. We do not know how to interpret this difference. One may need something new to resolve this problem if it is confirmed by the forthcoming data.

In short, we reexamined the branching fractions of seven well measured $B\to PP$ decay modes in the SM and model III, and compared the theoretical predictions with the new
data. Within the considered parameter space we found that: (a) the measurements of the branching ratios for $B \rightarrow \pi^+\pi^-$, $K^-\pi^+$ and $K^0\pi^+$ lead to a strong constraint on the form factor $F_{B\pi}^B(0)$: $F_{B\pi}^B(0) = 0.24 \pm 0.03$; (b) the new physics enhancements to the penguin-dominated $B \rightarrow K\pi$ and $K\eta'$ decays are significant in size, $\sim (40 - 65)\%$ $w.r.t$ the SM predictions, and play an important role in restoring the consistency between the data and the theory.

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TABLE I. Measurements of the branching ratio $\mathcal{B}(B \to PP)$ (in units of $10^{-6}$) as reported by CLEO, BaBar and Belle Collaborations. The last column lists the world average.

| Decay Mode | CLEO          | BaBar         | Belle         | Average       |
|------------|---------------|---------------|---------------|---------------|
| $B^0 \to \pi^+\pi^-$ | $4.3^{+1.8}_{-1.5}$ $\pm$ 0.5 | $4.1 \pm 1.0$ $\pm$ 0.7 | $5.6^{+2.9}_{-2.0}$ $\pm$ 0.4 | $4.4 \pm 0.9$ |
| $B^0 \to K^+\pi^-$ | $17.2^{+2.5}_{-2.4}$ $\pm$ 1.2 | $16.7 \pm 1.6$ $\pm$ 1.3 | $19.3^{+3.4}_{-3.2}$ $\pm$ 1.5 | $17.3 \pm 1.5$ |
| $B^+ \to K^+\pi^0$ | $11.6^{+3.0}_{-2.7}$ $\pm$ 1.3 | $10.8^{+2.1}_{-1.9}$ $\pm$ 1.0 | $16.3^{+3.3}_{-3.1}$ $\pm$ 1.6 | $12.1 \pm 1.7$ |
| $B^+ \to K^0\pi^+$ | $18.2^{+4.0}_{-4.0}$ $\pm$ 1.6 | $18.2^{+3.3}_{-3.0}$ $\pm$ 2.0 | $13.7^{+5.7}_{-4.8}$ $\pm$ 1.9 | $17.4 \pm 2.6$ |
| $B^0 \to K^0\pi^0$ | $14.6^{+5.9}_{-5.1}$ $\pm$ 2.4 | $8.2^{+3.1}_{-2.7}$ $\pm$ 1.2 | $16.0^{+7.2}_{-5.9}$ $\pm$ 2.7 | $10.7 \pm 2.7$ |
| $B^+ \to K^+\eta'$ | $80^{+10}_{-9}$ $\pm$ 7 | $70 \pm 8$ $\pm$ 5 | $79^{+12}_{-11}$ $\pm$ 9 | $75 \pm 7$ |
| $B^0 \to K^0\eta'$ | $89^{+18}_{-16}$ $\pm$ 9 | $42^{+13}_{-11}$ $\pm$ 4 | $55^{+19}_{-16}$ $\pm$ 8 | $56 \pm 10$ |

TABLE II. Branching ratios (in units of $10^{-6}$) of seven $B \to PP$ decay modes in the SM and Model III by using $F_0^{B^0}(0) = 0.21$ (the first entries), 0.24 (the second entries), 0.27 (the third entries), and assuming $N_{c eff}^f = 2$, $\theta = 0^\circ$ and $M_{H^+} = 200$ GeV. The last column lists the world average data.

| Decay Mode | SM | Model III | $\delta B$ [%] | Data |
|------------|----|-----------|----------------|------|
|             | 2  | 3         | $\infty$       | 2   | 3         | $\infty$ | $N_{c eff}^f = 3$ |
| $B^0 \to \pi^+\pi^-$ | 3.66 | 4.16 | 5.24 | 3.75 | 4.26 | 5.37 | 2.5 | 4.4 $\pm$ 0.9 |
|             | 4.78 | 5.43 | 6.85 | 4.90 | 5.56 | 7.01 | 2.5 |
|             | 6.05 | 6.87 | 8.66 | 6.20 | 7.04 | 8.87 | 2.5 |
| $B^+ \to K^+\pi^0$ | 4.93 | 5.53 | 6.85 | 7.12 | 8.01 | 9.98 | 44.9 | 12.1 $\pm$ 1.7 |
|             | 6.44 | 7.22 | 8.95 | 9.29 | 10.5 | 13.0 | 44.9 |
|             | 8.15 | 9.14 | 11.3 | 11.8 | 13.2 | 16.5 | 44.9 |
| $B^0 \to K^+\pi^-$ | 7.18 | 7.96 | 9.66 | 10.8 | 12.0 | 14.7 | 51.3 | 17.3 $\pm$ 1.3 |
|             | 9.37 | 10.4 | 12.6 | 14.1 | 15.7 | 19.2 | 51.3 |
|             | 11.9 | 13.2 | 16.0 | 17.9 | 19.9 | 24.3 | 51.3 |
| $B^+ \to K^0\pi^+$ | 8.12 | 9.46 | 12.5 | 12.1 | 14.1 | 18.4 | 48.9 | 17.4 $\pm$ 2.6 |
|             | 10.6 | 12.4 | 16.3 | 15.9 | 18.4 | 24.1 | 48.9 |
|             | 13.4 | 15.6 | 20.6 | 20.1 | 23.3 | 30.5 | 48.9 |
| $B^0 \to K^0\pi^0$ | 2.90 | 3.32 | 4.25 | 4.56 | 5.21 | 6.66 | 56.9 | 10.7 $\pm$ 2.7 |
|             | 3.80 | 4.34 | 5.56 | 5.96 | 6.80 | 8.69 | 56.9 |
|             | 4.81 | 5.49 | 7.03 | 7.54 | 8.61 | 11.0 | 56.9 |
| $B^+ \to K^+\eta'$ | 9.69 | 12.2 | 18.2 | 16.3 | 20.1 | 29.0 | 64.7 | 75 $\pm$ 7 |
|             | 12.7 | 16.0 | 23.8 | 21.3 | 26.3 | 37.9 | 64.7 |
|             | 16.0 | 20.2 | 30.5 | 27.0 | 33.3 | 47.9 | 64.7 |
| $B^0 \to K^0\eta'$ | 9.33 | 12.0 | 18.3 | 15.6 | 19.5 | 30.0 | 62.8 | 56 $\pm$ 10 |
|             | 12.2 | 15.6 | 23.9 | 20.4 | 25.5 | 37.4 | 62.8 |
|             | 15.4 | 19.8 | 30.2 | 25.8 | 32.2 | 47.3 | 62.8 |