Chiral Perturbation Theory for $|\Delta I| = 3/2$ Hyperon Decays

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Abstract

We study the $|\Delta I| = 3/2$ amplitudes of hyperon non-leptonic decays of the form $B \to B' \pi$ in the context of chiral perturbation theory. The lowest-order predictions are determined in terms of only one unknown parameter and are consistent within errors with current data. We investigate the theoretical uncertainty of these predictions by calculating the leading non-analytic corrections. We also present an estimate for the size of the S-wave $\Lambda$ and $\Xi$ decays which vanish at leading order. We find that the corrections to the lowest-order predictions are within the expectations of naive power counting and, therefore, that this picture can be tested more accurately with improved measurements.
1 Introduction

Several papers have been devoted to the study of hyperon non-leptonic decays within the framework of chiral perturbation theory ($\chi$PT) [1, 2, 3, 4, 5]. These papers have dealt exclusively with the dominant $|\Delta I| = 1/2$ transitions and have had mixed results. At leading order in $\chi$PT, these amplitudes can be parameterized in terms of two constants, and one-loop calculations have been carried out for the leading non-analytic terms. Whereas the S-waves appear to be under control, the P-waves are not.

It has recently been pointed out that to leading order in $\chi$PT the $|\Delta I| = 3/2$ amplitudes of hyperon non-leptonic decays can be described in terms of only one parameter [6]. In view of the situation in the $|\Delta I| = 1/2$ sector, it is instructive to carry out a one-loop calculation in the $|\Delta I| = 3/2$ sector. This calculation of the corrections of $\mathcal{O}(m_s \ln m_s)$ to the leading amplitudes allows us to assess the reliability of the leading-order predictions. Two of the S-wave amplitudes (those for $\Lambda$ and $\Xi$ decays) vanish at leading order and are still zero after the non-analytic terms of $\mathcal{O}(m_s \ln m_s)$ are included. We estimate the size of these two amplitudes by looking at some non-vanishing contributions to them.

This paper is organized as follows. In Section 2 we review the basic chiral Lagrangian for heavy baryons that we use for our calculation. In Section 3 we present detailed results of our calculation of the leading non-analytic corrections. In Section 4 we discuss two types of terms that can contribute to the S-wave amplitudes for $\Lambda$ and $\Xi$ decays. In Section 5 we extract the experimental values of the $|\Delta I| = 3/2$ amplitudes for the hyperon non-leptonic decays using the latest available information on decay rates and asymmetry parameters from the Particle Data Book [7]. We find slightly different numbers from the ones published in the Particle Data Book in 1982 [8], but with similar and very large errors. We believe that there is an opportunity to improve at least some of these measurements with the large numbers of hyperon non-leptonic decays identified in two current experiments. Fermilab experiment E871, which searches for CP violation in hyperon decays, could improve the measurements of the decays of $\Lambda$ and $\Xi$ with one charged pion, and the KTeV experiment, which studies CP violation in $K \to \pi\pi$ as well as rare kaon decays at Fermilab, could improve the measurements of the decays of $\Lambda$ and $\Xi$ with a charged or neutral pion. Finally, in Section 6 we discuss our results.

2 Chiral Lagrangian

The chiral Lagrangian that describes the interactions of mesons and baryons is written down with the usual building blocks [9, 10]: the pseudo-Goldstone boson fields in the form of the matrix
\[ \Sigma = e^{i\phi/f} \], where \( f \) is the pion-decay constant in the chiral-symmetry limit and

\[
\phi = \sqrt{2} \left( \begin{array}{ccc}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\
\pi^- & \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\
K^- & \bar{K}^0 & \frac{2}{\sqrt{6}} \eta_8
\end{array} \right); \tag{1}
\]

the octet baryons in the matrix

\[
B = \left( \begin{array}{ccc}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\
\Sigma^- & \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\
\Xi^- & \Xi^0 & \frac{2}{\sqrt{6}} \Lambda
\end{array} \right); \tag{2}
\]

and the spin-\( \frac{3}{2} \) decuplet baryons. Here we follow Jenkins and Manohar \[3\] and include the baryon-decuplet fields explicitly in the Lagrangian. As they argue, the mass splitting between the octet and decuplet baryons is small compared to the scale of chiral-symmetry breaking, and this enhances the effects of the decuplet on the low-energy theory. The decuplet baryons are described by a Rarita-Schwinger field \( T^{\mu}_{abc} \), which satisfies the constraint \( \gamma_\mu T^{\mu}_{abc} = 0 \) and is completely symmetric in its \( SU(3) \) indices, \( a,b,c \). Its components are (with the Lorentz index suppressed)

\[
T_{111} = \Delta^{++}, \quad T_{112} = \frac{1}{\sqrt{3}} \Delta^+, \quad T_{122} = \frac{1}{\sqrt{3}} \Delta^0, \quad T_{222} = \Delta^-, \quad T_{113} = \frac{1}{\sqrt{3}} \Sigma^{++}, \quad T_{123} = \frac{1}{\sqrt{6}} \Sigma^{*0}, \quad T_{223} = \frac{1}{\sqrt{3}} \Sigma^{*-}, \quad T_{133} = \frac{1}{\sqrt{3}} \Xi^{*0}, \quad T_{233} = \frac{1}{\sqrt{3}} \Xi^{*-}, \quad T_{333} = \Omega^- . \tag{3}
\]

Under chiral \( SU(3)_L \times SU(3)_R \), these fields transform as

\[
\Sigma \to L \Sigma R^\dagger, \quad B \to U B U^\dagger, \quad T^{\mu}_{abc} \to U_{ad} U_{be} U_{cf} T^{\mu}_{def}, \tag{4}
\]

where \( L, R \in SU(3)_{L,R} \) and the matrix \( U \) is implicitly defined by the transformation

\[
\xi \equiv e^{i\phi/(2f)} \to L \xi U^\dagger = U \xi R^\dagger. \tag{5}
\]

We use the heavy-baryon formalism of Jenkins and Manohar \[11\] where the effective Lagrangian is written in terms of velocity-dependent baryon fields, related to the ordinary baryon fields by the transformation \[12\]

\[
B_v(x) = e^{im_B \cdot v \cdot x} B(x), \quad T^\mu_v(x) = e^{im_B \cdot v \cdot x} T^\mu(x), \tag{6}
\]

where \( m_B \) is the baryon-octet mass in the chiral-symmetry limit.
The leading-order (in the derivative expansion), chiral- and parity-invariant Lagrangian that describes the strong interactions of the pseudoscalar-meson and baryon octets as well as the baryon decuplet is given by \cite{11, 3}

\[
\mathcal{L}^s = \frac{1}{4} f^2 \left( \partial^\mu \Sigma \right)^\dagger \partial_\mu \Sigma + \text{Tr} \left( \hat{B}_v i v \cdot \mathcal{D} B_v \right) \\
+ 2D \text{Tr} \left( \hat{B}_v S_v^\mu \left\{ A_\mu, B_v \right\} \right) + 2F \text{Tr} \left( \hat{B}_v S_v^\mu \left[ A_\mu, B_v \right] \right) \\
- \hat{T}_v^\mu i v \cdot \mathcal{D} T_{\mu v} + \Delta m \hat{T}_v^\mu T_{\mu v} + C \left( \hat{T}_v^\mu A_\mu B_v + \hat{B}_v A_\mu T_{\mu v} \right) + 2 \mathcal{H} \hat{T}_v^\mu S_v \cdot A T_{\mu v} ,
\]

where \( \Delta m \) denotes the mass difference between the decuplet and octet baryons in the chiral-symmetry limit, \( S_v^\mu \) is the velocity dependent spin operator of Ref. \cite{11},

\[
\mathcal{V}_\mu = \frac{1}{2} \left( \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right), \quad A_\mu = \frac{i}{2} \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right),
\]

and

\[
\mathcal{D}^\mu B_v = \partial^\mu B_v + [\mathcal{V}^\mu, B_v] ,
\]

\[
\mathcal{D}^\mu(T_v^\nu)_{abc} = \partial^\mu(T_v^\nu)_{abc} + i \mathcal{V}^\mu_{ad}(T_v^\nu)_{dbc} + i \mathcal{V}^\mu_{bd}(T_v^\nu)_{adc} + i \mathcal{V}^\nu_{cd}(T_v^\nu)_{abd} ,
\]

\[
\hat{T}_v^\mu A_\mu B_v + \hat{B}_v A_\mu T_{\mu v} = \epsilon_{abc} (\hat{T}_v^\mu)_{cde} (A_\mu)_{eb} B_{da} + \epsilon_{abc} \hat{B}_{ad} (A_\mu)_{be} (T_v^\mu)_{cde} .
\]

Explicit breaking of chiral symmetry, to leading order in the mass of the strange quark and in the limit \( m_u = m_d = 0 \), is introduced via the Lagrangian \cite{13}

\[
\mathcal{L}_{m_s} = a \text{Tr} \left( M \Sigma^\dagger + \Sigma M^\dagger \right) \\
+ b_D \text{Tr} \left( \hat{B}_v \left\{ \xi^\dagger M \xi^\dagger + \xi M^\dagger \xi, B_v \right\} \right) + b_F \text{Tr} \left( \hat{B}_v \left[ \xi^\dagger M \xi^\dagger + \xi M^\dagger \xi, B_v \right] \right) \\
+ \sigma \text{Tr} \left( M \Sigma^\dagger + \Sigma M^\dagger \right) \text{Tr} (\hat{B}_v B_v) \\
+ c \hat{T}_v^\mu \left( \xi^\dagger M \xi^\dagger + \xi M^\dagger \xi \right) T_{\mu v} - \tilde{\sigma} \text{Tr} \left( M \Sigma^\dagger + \Sigma M^\dagger \right) \hat{T}_v^\mu T_{\mu v} ,
\]

where \( M = \text{diag}(0, 0, m_s) \). In this limit, the pion is massless and the \( \eta_8 \) mass is related to the kaon mass by \( m_8^2 = \frac{3}{4} m_K^2 \). Furthermore, mass splittings within the baryon octet and decuplet occur to linear order in \( m_s \).

Within the standard model, the \( |\Delta S| = 1, |\Delta I| = 3/2 \) transitions are induced by an effective Hamiltonian that transforms as \( (27_L, 1_R) \) under chiral rotations:

\[
\mathcal{H}_{\text{eff}}^{(27_L, 1_R)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \left( \frac{c_1 + c_2}{3} \right) \mathcal{O}_{|\Delta I|=3/2}^{(27_L, 1_R)} + \text{h.c} .
\]

The four-quark operator \( \mathcal{O}_{|\Delta I|=3/2}^{(27_L, 1_R)} = 4 T_{jk, lm} \tilde{\psi}_L^j \gamma_\mu \psi_L^k \tilde{\psi}_L^l \gamma_\mu \psi_L^m \) has a unique chiral realization in the baryon-octet sector at leading order in \( \chi \text{PT} \) \cite{3}. Similarly, at leading-order in \( \chi \text{PT} \), there
is only one operator with the required transformation properties involving two decuplet baryon fields, and there are no operators that involve one decuplet-baryon and one octet-baryon fields. The leading-order weak chiral Lagrangian is, thus,

\[
\mathcal{L}^w = \beta_{27} T_{ij,kl} \left( \xi B_v \xi^\dagger \right)_{ki} \left( \xi B_v \xi^\dagger \right)_{lj} + \delta_{27} T_{ij,kl} \xi_{kl} \xi_{bi} \xi_{cl} \left( T^\mu_v \right)_{abc} (T_{\nu}^\mu)_{ade} + \text{h.c.} \quad (14)
\]

The non-zero elements of \( T_{ij,kl} \) that project out the \( |\Delta S| = 1, |\Delta I| = 3/2 \) Lagrangian are \( T_{12,13} = T_{21,13} = T_{12,31} = T_{21,31} = 1/2 \) and \( T_{22,23} = T_{22,32} = -1/2 \). In order to simplify notation and to parallel the discussions for the \( |\Delta I| = 1/2 \) sector of Refs. [1, 2], unlike Ref. [3], we absorb the Fermi constant, the CKM angles and the Wilson coefficients in Eq. (13) into the constants \( \beta_{27} \) and \( \delta_{27} \) in Eq. (14). The \( |\Delta I| = 1/2 \) weak Lagrangian of Ref. [2], which transforms as \( (8_L, 1_R) \), is given by

\[
\mathcal{L}^{w,8} = h_D \text{ Tr} \left( \bar{B}_v \left\{ \xi^\dagger h \xi, B_v \right\} \right) + h_F \text{ Tr} \left( \bar{B}_v \left( \xi^\dagger h \xi, B_v \right) \right) + h_C \bar{T}_v \xi^\dagger h \xi T_{\nu \mu} + \text{h.c.} , \quad (15)
\]

where \( h_{23} = 1 \) and all other elements of \( h \) vanish. Although this notation has become standard, for power counting arguments we will find it convenient to assume an explicit factor of \( G_F f_\pi^3 V_{ud} V_{us} \) multiplying \( \beta_{27} \), \( h_D \) and \( h_F \).

For purely-mesonic \( |\Delta S| = 1, |\Delta I| = 3/2 \) processes, the lowest-order weak Lagrangian can be written as

\[
\mathcal{L}^{w} = \frac{G_F}{\sqrt{2}} f_\pi^4 V_{ud} V_{us} g_{27} T_{ij,kl} \left( \partial^\mu \Sigma \right)_{ki} \left( \partial_\mu \Sigma \right)_{lj} + \text{h.c.} \quad (16)
\]

As defined in this expression, the constant \( g_{27} \) is expected to be of order one. Experimentally, it turns out that \( g_{27} \approx 0.16 \) as extracted from an analysis of \( K \to \pi \pi \) decays [14, 15, 16]. For comparison, the analogous constant for \( |\Delta I| = 1/2 \) transitions, \( g_8 \), is measured to be \( g_8 \approx 5.1 \) and is expected to be of order one when defined by the Lagrangian

\[
\mathcal{L}^{w,8} = \frac{G_F}{\sqrt{2}} f_\pi^4 V_{ud} V_{us} g_8 \text{ Tr} \left( h \partial^\mu \Sigma \right) \left( \partial_\mu \Sigma \right) + \text{h.c.} , \quad (17)
\]

which transforms as \( (8_L, 1_R) \) under chiral symmetry.

### 3 \( |\Delta I| = 3/2 \) Amplitudes to \( \mathcal{O}(m_s \ln m_s) \)

There are two terms in the amplitude for the decay \( B \to B' \pi \), corresponding to S- and P-wave contributions. In our calculation we refer exclusively to the \( |\Delta I| = 3/2 \) component of these amplitudes. We use the heavy-baryon approach and follow Ref. [2] to write the amplitude in the form

\[
i\mathcal{M}_{B \to B' \pi} = G_F m_\pi^2 \bar{u}_{B'} \left( A_{BB' \pi}^{(S)} + 2k \cdot S_v A_{BB' \pi}^{(P)} \right) u_B , \quad (18)
\]
where \(k\) is the outgoing four-momentum of the pion. The \(|\Delta I| = 3/2\) amplitudes satisfy the isospin relations

\[
\mathcal{M}_{\Sigma^+ \to n\pi^+} - \sqrt{2} \mathcal{M}_{\Sigma^+ \to p\pi^0} + 2 \mathcal{M}_{\Sigma^- \to n\pi^-} = 0,
\]

\[
\mathcal{M}_{\Lambda \to n\pi^0} - \sqrt{2} \mathcal{M}_{\Lambda \to p\pi^-} = 0,
\]

\[
\mathcal{M}_{\Xi^0 \to \Lambda\pi^0} - \sqrt{2} \mathcal{M}_{\Xi^- \to \Lambda\pi^-} = 0,
\]

and, therefore, only four of them are independent. We have chosen to present \(\Sigma^+ \to n\pi^+\), \(\Sigma^- \to n\pi^-\), \(\Lambda \to p\pi^-\) and \(\Xi^- \to \Lambda\pi^-\) because these are the same ones used in the discussion of \(|\Delta I| = 1/2\) transitions \([1, 2]\).

We follow the notation of Jenkins \([2]\) to write the S- and P-wave decay amplitudes at the one-loop level in the form

\[
\mathcal{A}^{(S)}_{BB'\pi} = \frac{1}{\sqrt{2}} f_\pi \left[ \alpha^{(S)}_{BB'} + \left( \bar{\beta}^{(S)}_{BB'} - \bar{\lambda}^{(S)}_{BB'\pi}\alpha^{(S)}_{BB'} \right) \frac{m_K^2}{16\pi^2 f_\pi^2} \frac{m_N^2}{\mu^2} \right],
\]

\[
\mathcal{A}^{(P)}_{BB'\pi} = \frac{1}{\sqrt{2}} f_\pi \left[ \alpha^{(P)}_{BB'} + \left( \bar{\beta}^{(P)}_{BB'} - \bar{\lambda}^{(P)}_{BB'\pi}\alpha^{(P)}_{BB'} \right) \frac{m_K^2}{16\pi^2 f_\pi^2} \frac{m_N^2}{\mu^2} + \gamma_{BB'}\alpha^{(P)}_{BB'} \right],
\]

where \(f_\pi \approx 92.4\ \text{MeV}\) is the physical pion-decay constant; \(\alpha_{BB'}\) and \(\bar{\beta}_{BB'} = \beta_{BB'} + \beta'_{BB'}\) represent contributions from tree-level and one-loop diagrams, respectively, shown in Figures \([1, 2, 3, 4]\); \(\lambda_{BB'\pi}\) arises from baryon and pion wave-function renormalization as well as the renormalization of the pion-decay constant; and \(\gamma_{BB'}\) results from the one-loop corrections to the propagator in the P-wave diagrams.\([4]\) One-loop decay graphs involving only octet baryons contribute to \(\beta_{BB'}\), whereas those with internal decuplet-baryon lines yield \(\beta'_{BB'}\).

At leading order in \(\chi PT, O(1)\), there are contributions to the amplitudes from the tree-level Lagrangian in Eq. \((14)\). They arise from the diagrams displayed in Figure \([4]\) and are given by

\[
\alpha^{(S)}_{\Sigma^+n} = -\frac{3}{2}\beta_{27}, \quad \alpha^{(S)}_{\Sigma^-n} = \beta_{27}, \quad \alpha^{(S)}_{\Lambda p} = 0, \quad \alpha^{(S)}_{\Xi^-\Lambda} = 0,
\]

\[
\alpha^{(P)}_{\Sigma^+n} = \left( -\frac{1}{2} D - \frac{3}{2} F \right) \frac{\beta_{27}}{m_\Sigma - m_N}, \quad \alpha^{(P)}_{\Sigma^-n} = F \frac{\beta_{27}}{m_\Sigma - m_N},
\]

\[
\alpha^{(P)}_{\Lambda p} = \frac{1}{\sqrt{6}} D \frac{\beta_{27}}{m_\Sigma - m_N}, \quad \alpha^{(P)}_{\Xi^-\Lambda} = -\frac{1}{\sqrt{6}} D \frac{\beta_{27}}{m_\Xi - m_\Sigma},
\]

\(1\) A similar contribution to P-wave amplitudes in the \(|\Delta I| = 1/2\) sector was not included in the calculation of Jenkins \([2]\) and was pointed out by Springer \([4]\).
Figure 1: Tree-level diagrams for (a) S-wave and (b) P-wave hyperon non-leptonic decays. Solid (dashed) lines denote baryon-octet (meson-octet) fields. A solid dot (hollow square) represents a strong (weak) vertex. In all figures, the strong vertices are generated by $\mathcal{L}^s$ in Eq. (7). Here the weak vertices come from $\mathcal{L}^w$ in Eq. (14).

Figure 2: One-loop diagrams contributing to S-wave hyperon non-leptonic decay amplitudes, with weak vertices from $\mathcal{L}^w$ in Eq. (14). Double (single) solid-lines represent baryon-decuplet (baryon-octet) fields. where the strong vertices in the P-wave graphs are from $\mathcal{L}^s$ in Eq. (7). These results correspond to those obtained in Ref. [6] without the heavy-baryon formalism.

At next order in $\chi$PT, we will have amplitudes of $\mathcal{O}(m_s)$ arising both from one-loop diagrams with lowest-order vertices and from counterterms. At present, there is not enough experimental input to determine the value of the counterterms. For this reason, we follow the approach that has been used for the $|\Delta I| = 1/2$ amplitudes [1, 2] and calculate only those terms of $\mathcal{O}(m_s \ln m_s)$. These terms are uniquely determined from the one-loop amplitudes because they cannot arise from local counterterm Lagrangians. $\chi$PT purists may argue that this is an incomplete calculation. However, our calculation will suffice to estimate the robustness of the leading-order predictions. With a complete calculation at next-to-leading order, it would be possible to fit all the amplitudes but we feel that there is nothing to be learned from that exercise given the large number of free parameters available.

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2The coupling $\beta_{27}$ is proportional to the constant $b_{27}$ in Ref. [3].
3We have not constructed a complete list of the operators that occur at next-to-leading order. However, we have verified that there are at least six new terms so that, at $\mathcal{O}(m_s)$, there are more unknown constants than there are measurements. This is analogous to what happens for $|\Delta I| = 1/2$ amplitudes [17].
We present our results for the $\mathcal{O}(m_s \ln m_s)$ terms separating the contributions from different types of diagrams. From one-loop diagrams involving only octet baryons, shown in Figures 2 and 3, we obtain

$$\begin{align*}
\beta_{\Sigma^+ n}^{(S)} &= \left( \frac{23}{8} - \frac{5}{4} D^2 - 3DF + \frac{9}{4} F^2 \right) \beta_{27}^{(S)} , \\
\beta_{\Sigma^- n}^{(S)} &= \left( -\frac{23}{12} + \frac{5}{6} D^2 + 2DF - \frac{3}{2} F^2 \right) \beta_{27}^{(S)} ,
\end{align*}$$

(24)

$$\beta_{\Lambda p}^{(S)} = 0 , \\
\beta_{\Xi^- \Lambda}^{(S)} = 0 ,$$

Figure 3: One-loop diagrams contributing to P-wave hyperon non-leptonic decay amplitudes, with weak vertices from $\mathcal{L}_w$ in Eq. (14).
\[
\begin{align*}
\beta_{\Sigma^+ n}^{(P)} &= \left( \frac{29}{12} D + \frac{29}{8} F - \frac{19}{36} D^3 - \frac{29}{12} D^2 F - \frac{31}{12} D F^2 + \frac{15}{4} F^3 \right) \frac{\beta_{27}}{m_{\Sigma} - m_N}, \\
\beta_{\Sigma^- n}^{(P)} &= \left( -\frac{29}{12} F + \frac{2}{9} D^2 F + 2 DF^2 - 2F^3 \right) \frac{\beta_{27}}{m_{\Sigma} - m_N}, \\
\beta_{\Lambda p}^{(P)} &= \frac{1}{\sqrt{6}} \left( -\frac{29}{12} D + \frac{16}{9} D^3 + 2D^2 F - 2DF^2 \right) \frac{\beta_{27}}{m_{\Sigma} - m_N}, \\
\beta_{\Xi^- \Lambda}^{(P)} &= \frac{1}{\sqrt{6}} \left( \frac{29}{12} D - \frac{16}{9} D^3 + 2D^2 F + 2DF^2 \right) \frac{\beta_{27}}{m_{\Xi} - m_{\Sigma}}.
\end{align*}
\]
\[ \lambda_N = \frac{17}{6} D^2 - 5DF + \frac{15}{2} F^2, \quad \lambda'_{N} = \frac{1}{2} C^2, \]
\[ \lambda_{\Lambda} = \frac{7}{3} D^2 + 9F^2, \quad \lambda'_{\Lambda} = C^2, \]
\[ \lambda_{\Sigma} = \frac{13}{3} D^2 + 3F^2, \quad \lambda'_{\Sigma} = \frac{7}{3} C^2, \]
\[ \lambda_{\Xi} = \frac{17}{6} D^2 + 5DF + \frac{15}{2} F^2, \quad \lambda'_{\Xi} = \frac{13}{6} C^2, \]
\[ \lambda_{\pi} = -\frac{1}{3}, \quad \lambda_{f} = -\frac{1}{5}. \] (32)

Finally, for the P-waves, we must also include one-loop corrections to the propagator that appears in tree-level pole diagrams. These corrections have been partially addressed for the \( |\Delta I| = 1/2 \) amplitudes by Springer [4]. We find that it is sufficient to take into account the one-loop renormalization of the baryon masses to correctly include these non-analytic corrections to the hyperon decay amplitudes. This is true for both the usual terms of \( \mathcal{O}(m_s \ln m_s) \), as well as terms of \( \mathcal{O}(\pi m_s^{1/2}) \). The latter correspond to the baryon-mass corrections that are proportional to \( \pi m_K^3 \) in the calculation of Ref. [13]. We find

\[ \gamma_{\Sigma^+ N} = \gamma_{\Sigma^- N} = \gamma_{\Lambda p} = \frac{\mu_{\Sigma N}}{m_{\Sigma} - m_N}, \quad \gamma_{\Xi^- \Lambda} = \frac{\mu_{\Xi \Sigma}}{m_{\Xi} - m_{\Sigma}}, \] (33)

where

\[ \mu_{XY} = -\left(\bar{\beta}_X - \bar{\beta}_Y\right) \frac{m_K^3}{16\pi f^2} \]
\[ + \left[ (\bar{\gamma}_X - \bar{\gamma}_Y - \bar{\lambda}_X \alpha_X + \bar{\lambda}_Y \alpha_Y) m_s + (\bar{\lambda}'_X - \bar{\lambda}'_Y) \Delta m \right] \frac{m_K^2}{16\pi^2 f^2} \ln \frac{m_K^2}{\mu^2}, \] (34)

and

\[ \alpha_N = -2(b_D - b_F) - 2\sigma, \quad \alpha_{\Sigma} = -2\sigma, \quad \alpha_{\Xi} = -2(b_D + b_F) - 2\sigma, \] (35)

\[ \bar{\beta}_N = \frac{5}{3} D^2 - 2DF + 3F^2 + \frac{4}{9\sqrt{3}}(D^2 - 6DF + 9F^2) + \frac{1}{3} C^2, \]
\[ \bar{\beta}_{\Sigma} = 2D^2 + 2F^2 + \frac{16}{9\sqrt{3}} D^2 + \left(\frac{10}{9} + \frac{8}{9\sqrt{3}}\right) C^2, \] (36)
\[ \bar{\beta}_{\Xi} = \frac{5}{3} D^2 + 2DF + 3F^2 + \frac{1}{9\sqrt{3}} (D^2 + 6DF + 9F^2) + \left(1 + \frac{8}{9\sqrt{3}}\right) C^2, \]
\[ \bar{\gamma}_N = \frac{23}{9} b_D - \frac{25}{9} b_F - b_D \left( \frac{4}{3} D^2 + 12 F^2 \right) + b_F \left( \frac{2}{3} D^2 - 4DF + 6F^2 \right) + \frac{52}{9} \sigma - 2\sigma \lambda_N + \frac{1}{3} c C^2 - 2\tilde{\sigma} \lambda_N^f , \]
\[ \bar{\gamma}_\Sigma = 2 b_D - b_D \left( 6D^2 + 6F^2 \right) - b_F (12DF) + \frac{52}{9} \sigma - 2\sigma \lambda_\Sigma + \frac{8}{9} c C^2 - 2\tilde{\sigma} \lambda_\Sigma^f , \]
\[ \bar{\gamma}_\Xi = \frac{43}{9} b_D + \frac{25}{9} b_F - b_D \left( \frac{4}{3} D^2 + 12 F^2 \right) - b_F \left( \frac{2}{3} D^2 + 4DF + 6F^2 \right) + \frac{52}{9} \sigma - 2\sigma \lambda_\Xi + \frac{20}{9} c C^2 - 2\tilde{\sigma} \lambda_\Xi^f . \]

The parameters that appear in the strong Lagrangian, Eq. (7), can be measured in semi-leptonic hyperon decays. For our numerical estimates we will employ the values

\[ D = 0.61 \pm 0.04 , \quad F = 0.40 \pm 0.03 , \quad C = 1.6 , \quad \mathcal{H} = -1.9 \pm 0.7 , \]

which were obtained from a three-parameter fit to semi-leptonic hyperon decays including non-analytic corrections from octet- and decuplet-baryon loops [13].

The values of the parameters that appear in Eq. (12) may be obtained by fitting the tree-level expressions for the baryon masses [including terms of up to \( O(m_s) \)] to the physical masses. At this order, the baryon-octet masses are not independent, and instead they satisfy the Gell-Mann-Okubo relation. Similarly, the baryon-decuplet masses satisfy Gell-Mann’s equal-spacing rule. All this implies that it is possible to extract these parameters in more than one way from the physical masses. The different parameter sets obtained are equivalent to \( O(m_s) \). We choose the following,

\[ b_D m_s = \frac{3}{8} (m_\Sigma - m_\Lambda) \approx 0.0290 \text{ GeV} , \]
\[ b_F m_s = \frac{1}{4} (m_N - m_\Xi) \approx -0.0948 \text{ GeV} , \]
\[ c m_s = \frac{1}{2} (m_\Omega - m_\Delta) \approx 0.220 \text{ GeV} , \]
\[ \Delta m - 2 (\bar{\sigma} - \sigma) m_s = m_\Delta - m_\Sigma \approx 0.0389 \text{ GeV} , \]

where the fourth parameter is the only combination of \( \Delta m , \sigma m_s \) and \( \bar{\sigma} m_s \) which occurs in Eq. (34).

4 Beyond the Leading Non-Analytic Corrections

In the previous section we have calculated the terms of \( O(m_s \ln m_s) \) that occur in the \( |\Delta I| = 3/2 \) hyperon non-leptonic decay amplitudes. To this order we have found that the S-wave amplitudes for \( \Lambda \) and \( \Xi \) decays vanish, whereas the experimentally measured amplitudes do not. In this section we discuss two examples of terms that contribute to these amplitudes and use them as an
estimate of their size in $\chi$PT. A prediction for these amplitudes is not possible at present due to the unknown constants that occur at the order at which we first find a non-zero result, $O(m_s)$.

An example of a non-vanishing contribution at $O(m_s)$ is that from a counterterm already discussed in Ref. [6]. In the heavy-baryon formalism, it corresponds to

$$L_1^w = \frac{\tilde{\beta}_{27}}{m_N} T_{ij,kl} \left[ (\xi B_v)_{kn} (B_v \xi^\dagger)_{ni} - (\xi B_v)_{kn} (B_v \xi^\dagger)_{ni} \right]_i + \text{h.c.} ,$$

and gives rise to the contributions

$$\alpha_{\Sigma^+}^{(S)} = 0 , \quad \alpha_\Lambda^{(S)} = -\sqrt{\frac{3}{2}} \tilde{\beta}_{27} \frac{m_\Lambda - m_N}{m_N} ,$$

$$\alpha_{\Sigma^0}^{(S)} = -\tilde{\beta}_{27} \frac{m_\Sigma - m_N}{m_N} , \quad \alpha_{\Xi^0}^{(S)} = \sqrt{\frac{3}{2}} \tilde{\beta}_{27} \frac{m_\Xi - m_\Lambda}{m_N} .$$

Eq. (40) has been normalized so that $\tilde{\beta}_{27}$ is naturally of the same order as $\beta_{27}$. As remarked in Ref. [6], this operator reproduces the current-current form of vacuum saturation for the S-waves if one takes $\tilde{\beta}_{27} \approx -0.32 \sqrt{f_\pi} G_F m_\pi^2$. We will find in the following section that a lowest-order fit to S-wave $\Sigma$ decays gives $\beta_{27} \approx -0.07 \sqrt{f_\pi} G_F m_\pi^2$, and, therefore, the vacuum-saturation value for $\tilde{\beta}_{27}$ is five times larger than expected. There is no reason, however, to trust the vacuum-saturation approximation, and we prefer to use $\tilde{\beta}_{27} \approx \beta_{27}$ to estimate these terms. In the literature one finds that vacuum-saturation calculations often include a meson-pole diagram for the P-waves. In particular, Ref. [19] claims that these pole diagrams are important to fit experiment. Within the framework of $\chi$PT, these contributions are suppressed with respect to the ones we calculate by factors of $m_\pi^2/m_K^2$ or $m_d/m_s$. We see that the $\chi$PT predictions are completely at odds with those of vacuum saturation for hyperon decays.

Other non-vanishing contributions to the S-wave $\Lambda$ and $\Xi$ decays come from the one-loop diagrams in which the weak transition involves only mesons, as in Figure 4. These diagrams give calculable contributions that are formally of $O(m_s^2 \ln m_s)$ and, therefore, should be smaller than the contributions of $O(m_s \ln m_s)$. The corresponding diagrams for $|\Delta I| = 1/2$ transitions were included in the calculation of Jenkins [2], who argued that the large value of $g_8$ in Eq. (17) (corresponding to her $h_\pi$) could compensate their being of higher order. At present we do not have a detailed understanding of the $|\Delta I| = 1/2$ enhancement in kaon decays so we cannot really rule out Jenkins’ argument. However, it is also possible that whatever is responsible for the enhancement of $|\Delta I| = 1/2$ kaon decays will also enhance $|\Delta I| = 1/2$ baryon decays in a similar way, invalidating Jenkins’ argument. In other words, the coefficients $h_D$ and $h_F$ of Eq. (13)

\[\text{We have used } m_N \text{ as a normalization scale, but this is not meant to imply that this is a heavy-baryon mass correction. It could just as well be the chiral-symmetry breaking scale, } \Lambda_{\chi_{SB}}.\]
are also enhanced with respect to the expectation of naive dimensional analysis. In either case, the numerical results of Jenkins confirm that the non-analytic contributions from the diagrams analogous to those in Figure 4 are important for the $|\Delta I| = 1/2$ amplitudes. Notice that these terms do not involve unknown parameters so they can be quantified.

Returning to the $|\Delta I| = 3/2$ amplitudes, we want to calculate from the diagrams in Figure 4 the terms proportional to the constant $g_{27}$ of Eq. (16). In this case, the dimensionless constant $g_{27} \approx 0.16$, unlike $g_8$, is suppressed with respect to the expectation from naive dimensional analysis. These terms, of $\mathcal{O}(m_s^2 \ln m_s)$, would be further suppressed by the small value of $g_{27}$ and completely negligible if Jenkins’ argument to include the analogous terms is correct. However, if the $|\Delta I| = 1/2$ enhancement is universal, the large value of $g_8$ is not responsible for the importance of these terms in the calculation of Jenkins. In this case, we expect that these terms could be equally important for the $|\Delta I| = 3/2$ amplitudes. A posteriori, we find that these terms are indeed as important as those of $\mathcal{O}(m_s \ln m_s)$ as it happened in the calculation of the $|\Delta I| = 1/2$ amplitudes. We will return to this discussion in Section 6. Here we present the analytical expression for these terms.

\[\text{We disagree with the expressions presented in Ref. [2] for these terms (those proportional to } h_\pi \text{ in Ref. [2]). This disagreement, however, does not affect our present discussion. We will present our results for } |\Delta I| = 1/2 \text{ transitions elsewhere.}\]
The contributions to the amplitudes of Eqs. (20) and (21) from diagrams involving only octet baryons are

$$\beta_{\Sigma^+n}^{(S)} = -\left[(3D^2 + 9DF)(m_\Lambda - m_N) + (7D^2 - 9DF)(m_\Sigma - m_N)\right]\gamma_{27},$$

$$\beta_{\Sigma^-n}^{(S)} = \left[(-3 - D^2 + 12DF - 9F^2)(m_\Sigma - m_N) + (2D^2 + 6DF)(m_\Lambda - m_N)\right]\gamma_{27},$$

$$\beta_{\Lambda p}^{(S)} = -\frac{1}{\sqrt{6}}(9 + 13D^2 + 18DF + 27F^2)(m_\Lambda - m_N)\gamma_{27},$$

$$\beta_{\Xi^-\Lambda}^{(S)} = \frac{1}{\sqrt{6}}(9 + 13D^2 - 18DF + 27F^2)(m_\Xi - m_\Lambda)\gamma_{27},$$

$$\beta_{\Sigma^+n}^{(P)} = \left(\frac{1}{3}D^3 + \frac{7}{3}D^2F + DF^2 - F^3\right)\gamma_{27},$$

$$\beta_{\Sigma^-n}^{(P)} = \left(\frac{1}{3}D - \frac{1}{3}F - D^3 - \frac{5}{3}D^2F - 3DF^2 + 3F^3\right)\gamma_{27},$$

$$\beta_{\Lambda p}^{(P)} = \frac{1}{\sqrt{6}}\left(-\frac{1}{3}D - F + \frac{5}{3}D^3 + 9D^2F + 5DF^2 + 3F^3\right)\gamma_{27},$$

$$\beta_{\Xi^-\Lambda}^{(P)} = \frac{1}{\sqrt{6}}\left(-\frac{1}{3}D + F + \frac{5}{3}D^3 - 9D^2F + 5DF^2 - 3F^3\right)\gamma_{27}.$$

From diagrams involving decuplet baryons, we find

$$\beta_{\Sigma^+n}^{(S)} = C^2\left[\frac{3}{2}(m_\Sigma - m_N) - 2(m_\Delta - m_N) + m_{\Sigma^*} - m_N\right]\gamma_{27},$$

$$\beta_{\Sigma^-n}^{(S)} = C^2\left[-\frac{23}{9}(m_\Sigma - m_N) + \frac{4}{3}(m_\Delta - m_N) - \frac{2}{3}(m_{\Sigma^*} - m_N)\right]\gamma_{27},$$

$$\beta_{\Lambda p}^{(S)} = \frac{5}{2\sqrt{6}}C^2(m_\Lambda - m_N)\gamma_{27},$$

$$\beta_{\Xi^-\Lambda}^{(S)} = \frac{7}{2\sqrt{6}}C^2(m_\Xi - m_\Lambda)\gamma_{27},$$

$$\beta_{\Sigma^+n}^{(P)} = -(\frac{10}{27}\mathcal{H} + \frac{2}{3}D + \frac{10}{9}F)C^2\gamma_{27},$$

$$\beta_{\Lambda p}^{(P)} = -\frac{1}{\sqrt{6}}\left(\frac{10}{27}\mathcal{H} + \frac{8}{9}D + 4F\right)C^2\gamma_{27},$$

$$\beta_{\Xi^-\Lambda}^{(P)} = \frac{1}{\sqrt{6}}\left(\frac{5}{9}D - \frac{4}{3}F\right)C^2\gamma_{27}.$$ 

(45)

We have used the notation \(\gamma_{27} \approx 2.5 \times 10^{-9}\).

5 Current Experimental Values

From the measurement of the decay rate and the decay-distribution asymmetry parameter \(\alpha\), it is possible to extract the value of the S- and P-wave amplitudes for each hyperon decay \([10]\). Using
the numbers from the 1998 Review of Particle Physics [7], we find the results presented in Table 1. Here \( s \) and \( p \) are related to \( \mathcal{A}_{BB'\pi}^{(S,P)} \) by

\[
s = \mathcal{A}^{(S)}, \quad p = -|k|\mathcal{A}^{(P)},
\]

where \( k \) is the pion three-momentum in the rest frame of the decaying baryon. These numbers are very similar to those quoted by Jenkins [2] (whose conventions we follow), because there are no new experimental measurements. Our minor differences are due to the fact that we use masses in the isospin-symmetry limit instead of physical masses. From the amplitudes in Table 1, we can extract the \( |\Delta I| = 3/2 \) components using the relations

\[
S_3^{(\Lambda)} = \frac{1}{\sqrt{3}}(\sqrt{2}s_{\Lambda^{+}\rightarrow n\pi^0} + s_{\Lambda^{-}\rightarrow p\pi^-}) ,
\]

\[
S_3^{(\Xi)} = \frac{2}{3}(\sqrt{2}s_{\Xi^0\rightarrow \Lambda\pi^0} + s_{\Xi^-\rightarrow \Lambda\pi^-}) ,
\]

\[
S_3^{(\Sigma)} = -\sqrt{\frac{2}{15}}(s_{\Sigma^+\rightarrow n\pi^+} - \sqrt{2}s_{\Sigma^0\rightarrow p\pi^0} - s_{\Sigma^-\rightarrow n\pi^-}) .
\]

and corresponding ones for the P-wave amplitudes. Similarly, one obtains the \( |\Delta I| = 1/2 \) components of the amplitudes (for \( \Lambda \) and \( \Xi \) decays) using the expressions

\[
S_1^{(\Lambda)} = \frac{1}{\sqrt{3}}(s_{\Lambda^{+}\rightarrow n\pi^0} - \sqrt{2}s_{\Lambda^{-}\rightarrow p\pi^-}) ,
\]

\[
S_1^{(\Xi)} = \sqrt{\frac{2}{3}}(s_{\Xi^0\rightarrow \Lambda\pi^0} - \sqrt{2}s_{\Xi^-\rightarrow \Lambda\pi^-}) ,
\]

and analogous ones for the P-waves. The \( |\Delta I| = 1/2 \) rule for hyperon decays can be seen in the ratios shown in Table 2. The experimental values for \( S_3 \) and \( P_3 \) are listed in the column labeled “Experiment” in Table 3.

Table 1: Experimental values for S- and P-wave amplitudes. In extracting these numbers, final-state interactions have been ignored and isospin-symmetric masses used.

| Decay mode       | \( s \)       | \( p \)       |
|------------------|---------------|---------------|
| \( \Sigma^+ \rightarrow n\pi^+ \) | 0.06 ± 0.01   | 1.85 ± 0.01   |
| \( \Sigma^+ \rightarrow p\pi^0 \)  | -1.48 ± 0.05  | 1.21 ± 0.06   |
| \( \Sigma^- \rightarrow n\pi^- \)  | 1.95 ± 0.01   | -0.07 ± 0.01  |
| \( \Lambda \rightarrow p\pi^- \)   | 1.46 ± 0.01   | 0.53 ± 0.01   |
| \( \Lambda \rightarrow n\pi^0 \)   | -1.09 ± 0.02  | -0.40 ± 0.03  |
| \( \Xi^- \rightarrow \Lambda\pi^- \)| -2.06 ± 0.01  | 0.50 ± 0.02   |
| \( \Xi^0 \rightarrow \Lambda\pi^0 \)| 1.55 ± 0.02   | -0.33 ± 0.02  |
Table 2: Ratios of $|\Delta I| = 3/2$ to $|\Delta I| = 1/2$ amplitudes, derived from Table 1. Here $S_\Sigma^{-} = s_{\Sigma^{-}\to n\pi^{-}}$ and $P_\Sigma^{(\Sigma)} = p_{\Sigma^{+}\to n\pi^{+}}$.

|              | Value                  |
|--------------|------------------------|
| $S_3^{(A)}/S_1^{(A)}$ | 0.026 ± 0.009          |
| $P_3^{(A)}/P_1^{(A)}$   | 0.031 ± 0.037          |
| $S_3^{(\Xi)}/S_1^{(\Xi)}$ | 0.042 ± 0.009         |
| $P_3^{(\Xi)}/P_1^{(\Xi)}$ | -0.045 ± 0.047       |
| $S_3^{(\Sigma)}/S_1^{(\Sigma)}$ | -0.055 ± 0.020       |
| $P_3^{(\Sigma)}/P_1^{(\Sigma)}$ | -0.059 ± 0.024       |

6 Discussion

In this section we present a numerical comparison of our results with experiment. We start with the tree-level, lowest-order in $\chi$PT, calculation. At this order, there are four non-zero predictions that depend only on the parameter $\beta_27$ (and on parameters from the strong sector that have been determined elsewhere). We can extract the value of $\beta_27$ from each of these four amplitudes, and compare these values to test the consistency of the framework. We find, in units of $\sqrt{2} f_{\pi} G_F m_{\pi}^2$,

$$\beta_27 = \begin{cases} 
-0.068 \pm 0.024 & \text{from } S_3^{(\Sigma)} \\
0.12 \pm 0.15 & \text{from } P_3^{(A)} \\
0.040 \pm 0.042 & \text{from } P_3^{(\Xi)} \\
0.23 \pm 0.10 & \text{from } P_3^{(\Sigma)} 
\end{cases} \quad (49)$$

We have used the experimental values listed in Table 3, and the errors we quote for $\beta_27$ do not include any estimate of theoretical errors. The results in Eq. (49) are not inconsistent, but given the large experimental errors, it would be premature to say that the fit is good.

Since the errors in the measurements of the P-wave amplitudes are larger than those in the S-wave amplitudes, we can take the point of view that we will fit the value of $\beta_27$ to the S-wave $\Sigma$ decay and treat the P-wave amplitudes as predictions. Recall that in the analysis of the $|\Delta I| = 1/2$ amplitudes, the two parameters that occur at lowest order in $\chi$PT theory are also extracted from a fit to S-wave amplitudes, and the P-wave amplitudes are then treated as
Table 3: Summary of results for $|\Delta I| = 3/2$ components of the S- and P-wave amplitudes. We use the parameter values $\beta_{27} = \delta_{27} = \tilde{\beta}_{27} = -0.068 \sqrt{2} f_{\pi} G_F m_s^2$ as discussed in the text. In the Theory columns with the “Octet” and “Decuplet” headings, each $P_3$ entry is written as a sum of two numbers, where the second one results from the baryon-mass renormalization in the tree-level P-wave diagrams. Each of the $g_{27}$ terms is also given as a sum of two numbers, the first one coming from diagrams with octet baryons only and the second one from graphs involving the decuplet.

| Amplitude | Experiment | Theory |
|-----------|------------|--------|
|           |            | Tree  | Octet | Decuplet | $g_{27}$ term | $\tilde{\beta}_{27}$ term |
|           |            | $O(1)$| $O(m_s \ln m_s)$ | $O(m_s \ln m_s)$ | $O(m_s^2 \ln m_s)$ | $O(m_s)$ |
| $S_3^{(A)}$ | $-0.047 \pm 0.017$ | 0     | 0     | 0        | 0.063–0.018    | 0.027 |
| $S_3^{(\Xi)}$ | $0.088 \pm 0.020$ | 0     | 0     | 0        | -0.051–0.033   | -0.036 |
| $S_3^{(\Sigma)}$ | $-0.107 \pm 0.038$ | -0.107| -0.089| -0.084   | 0.003+0.079    | 0.029 |
| $P_3^{(A)}$ | $-0.021 \pm 0.025$ | 0.012 | 0.011–0.006 | -0.015–0.045 | 0.003–0.006 | 0 |
| $P_3^{(\Xi)}$ | $0.022 \pm 0.023$ | -0.037| -0.055+0.031| 0.046+0.019 | -0.001–0.000 | 0 |
| $P_3^{(\Sigma)}$ | $-0.110 \pm 0.045$ | 0.032 | 0.031–0.016 | -0.047–0.124 | -0.003+0.002 | 0 |

predictions. In that case, the predictions for the P-wave amplitudes are completely wrong, differing from the measurements by factors of up to 30 $[1, 2]$. We show our results for the $|\Delta I| = 3/2$ amplitudes in the column labeled “Tree, $O(1)$” in Table 3. These lowest-order predictions for the P-wave amplitudes are not impressive, but they do have the right order of magnitude and they differ from the central value of the measurements by at most three standard deviations in the four cases where the predictions are non-zero.

At lowest order, two of the S-wave amplitudes vanish, $S_3^{(A)}$ and $S_3^{(\Xi)}$. This, of course, only indicates that these two amplitudes are predicted to be smaller than $S_3^{(\Sigma)}$ by about a factor of three since there are non-vanishing contributions from operators that occur at next order, $O(m_s/\Lambda_{\chi\text{SB}})$ [for example, Eq. (40)]. Again, this is not in conflict with experiment.

Beyond leading order in $\chi$PT, there are too many unknown coefficients for the theory to be predictive. This makes it possible to fit all the amplitudes at next-to-leading order, but this fit is not particularly instructive. In this paper we limit ourselves to study the question of whether the lowest-order predictions are subject to large higher-order corrections. To address this issue, we look at our one-loop calculation of the $O(m_s \ln m_s)$ corrections. In Table 3 we present these corrections in two columns, using a subtraction scale $\mu = 1$ GeV. The column marked “Octet, $O(m_s \ln m_s)$” is the result of those diagrams in which only octet baryons are allowed in the loops. (Contributions from the renormalization of the wave-function and decay constant of the pion are included in this column.) These results depend only on the constant $\beta_{27}$ and we use the
value $\beta_{27} = -0.068 \sqrt{2} f_{\pi} G_F m_{\pi}^2$ from the leading-order fit to $S^{(\Sigma)}_3$ (of course they also depend on the parameters from the strong sector, but these are already determined). For the P-waves we have split the results into the sum of two terms, the second one corresponding to baryon-mass renormalization in the tree-level pole diagrams [the $\gamma_{BB'}$ of Eq. (21)] and the first one to everything else. We find it instructive to examine these two terms separately because, as we noted in Section 3, the second term was not included in the calculation of the $|\Delta I| = 1/2$ amplitudes of Jenkins [2] We find that the $\gamma_{BB'}$ terms have very similar values to other terms of the same order. However, when combined with the other terms they can change the relative size of corrections to different amplitudes substantially (see also the column in Table 3 that shows the decuplet loops).

We can see in Table 3 that some of the loop corrections are as large as the lowest-order results even though they are expected to be smaller by about a factor of $M_K^2 / (4\pi f_{\pi}^2)$. By studying Eqs. (24) and (25), it is possible to see that each amplitude receives several contributions from different diagrams that combine to give the polynomials in $D$ and $F$. These terms add up constructively in some cases and nearly cancel out in others, resulting in deviations of up to an order of magnitude from the power counting expectation. This is an inherent flaw in a perturbative calculation where the expansion parameter is not sufficiently small.

The one-loop corrections are all much smaller than their counterparts in $|\Delta I| = 1/2$ transitions, where they can be as large as 15 times the lowest-order amplitude in the case of the P-wave in $\Sigma^+ \rightarrow n\pi^+$ [2]. In that case, Jenkins noted that this was due to an anomalously small lowest-order prediction arising from the cancellation of two nearly identical terms [2].

An important argument in deciding that the $O(m_s \ln m_s)$ corrections are a good estimate for the size of the complete next-to-leading-order corrections is that these are non-analytic terms that cannot arise from counterterms. In our calculation we find that there is another type of non-analytic correction at one-loop, of $O(\pi m_s^{3/2})$. These terms have the same origin as the terms proportional to $m_K^3$ that occur in the analysis of baryon masses [4] that appear in Eq. (34). Numerically, we find that they are not important for the $|\Delta I| = 3/2$ transitions, and for this reason we do not discuss them in detail. They only affect $S^{(\Sigma)}_3$ where they decrease the size of the $O(m_s \ln m_s)$ correction by about 10%.

Next we consider the contributions of loop diagrams with decuplet baryons. It has been emphasized in Ref. [11] that the decuplet plays a special role in heavy-baryon $\chi$PT and that these terms can, therefore, be significant. Our results depend on one additional constant, $\delta_{27}$, which cannot be fit from experiment because it does not contribute to any of the observed weak decays of a decuplet baryon. To illustrate the effect of these terms, we choose $\delta_{27} = \beta_{27}$, a choice consistent with dimensional analysis and with the normalization of Eq. (14). We present these results in

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6 From our results, it appears that the inclusion of these terms could significantly affect the discussion of the $|\Delta I| = 1/2$ amplitudes. We will present our results for that case elsewhere.
the column marked “Decuplet, $\mathcal{O}(m_s \ln m_s)$” in Table 3. For the P-waves, we have again split the contributions from $\gamma_{BP}'$ from everything else. The comments regarding the results for octet-baryon loops also apply here. We see that the decuplet-loop contributions could be important and occur at the same level as the octet-loop contributions. To some extent this result follows from our choice for $\delta_{27}$, but it is illustrative of the special role of the decuplet in the formalism of Ref. [11]. Notice, for example, that the analogous parameters in the $|\Delta I| = 1/2$ sector $h_D$, $h_F$ and $h_C$ of Eq. (17) end up being of the same order after they are fitted to the experimental amplitudes [2].

At this point in the calculation, with terms of order $\mathcal{O}(1)$ and terms of order $\mathcal{O}(m_s \ln m_s)$, we still find that $S_{3}^{(A)}$ and $S_{3}^{(\Xi)}$ are zero. We now discuss the non-zero contributions to these two amplitudes derived in Section 4.

In the column marked “$g_{27}$ term, $\mathcal{O}(m_s^2 \ln m_s)$” of Table 3 we present the result from the one-loop diagrams in which the weak transition occurs in a vertex that involves only mesons, Figure 4. We present separate results for the contributions from loops with only octet baryons (first term) and loops with octet and decuplet baryons. The non-analytic part of these diagrams is uniquely determined in terms of $g_{27}$ measured in $K \rightarrow \pi\pi$ decays and of couplings from the strong interaction Lagrangian. Since there can be contributions of $\mathcal{O}(m_s)$ from counterterms such as Eq. (40), we cannot expect these terms to be the dominant ones. They do indicate, however, that $\chi$PT produces a non-zero $S_{3}^{(A)}$ and $S_{3}^{(\Xi)}$ at the right level.

If we take, for example, $\tilde{\beta}_{27} = \beta_{27}$ in Eq. (40), we obtain terms of $\mathcal{O}(m_s)$ that are also of the same order as the measured $S_{3}^{(A)}$ and $S_{3}^{(\Xi)}$. We show these numbers in the column labeled “$\tilde{\beta}_{27}$ term, $\mathcal{O}(m_s)$” in Table 3. We conclude that these two amplitudes can be accommodated naturally in $\chi$PT, but they cannot be predicted at present.

It is intriguing that some of the terms proportional to $g_{27}$ are as large as they are in Table 3. Formally they are of $\mathcal{O}(m_s^2 \ln m_s)$, and we have argued in Section 4 that we expect them to be smaller than the terms of $\mathcal{O}(m_s \ln m_s)$. For the same reason, we would also expect them to be smaller than the terms of $\mathcal{O}(m_s)$ proportional to $\tilde{\beta}_{27}$. But a glance at Table 3 shows that this is not the case. A careful study of our results in Sections 3 and 4 indicates that if we compare the terms proportional to $g_{27}$ with the terms proportional to $\beta_{27}$, the former are “suppressed” by factors of order one times $m_s/f_\pi$. This explains the relative importance of these terms both in our calculation and in that of Jenkins [4]: although they are indeed proportional to an additional power of $m_s$, the scale is set by $f_\pi$ instead of $4\pi f_\pi$. From studying the diagrams of Figure 4 it is obvious that there are no additional factors of $4\pi$ associated with these terms relative to other one-loop terms. Also, since the weak transitions arise from the leading-order weak Lagrangian in the meson and baryon sectors, there are no relative factors of the scale of chiral symmetry breaking. The different relative importance of these terms in the different amplitudes is again due
to the polynomials in $D$ and $F$. For example, the contribution of $g_{27}$ terms to $S_3^{(\Sigma)}$ from baryon-octet loops looks relatively small. But if we calculate this number for the decay $\Sigma^+ \to n\pi^+$, we find that it is an order of magnitude larger. From our calculation, it is not possible to decide whether the $m_s/f_\pi$ factor is simply a numerical peculiarity of these diagrams, or whether it signals a breakdown of chiral perturbation theory for hyperon decays in the sense that there are some higher order terms where the scale is set by $f_\pi$.

For completeness, we summarize our results for arbitrary values of $\beta_{27}$, $\delta_{27}$ and $\tilde{\beta}_{27}$ in Table 4. The terms proportional to $g_{27}$ are the same as in Table 3 since this constant is fixed.

Finally, it is worth pointing out that it is possible to obtain a good fit to experiment with just the terms considered in this paper. A least-squares fit to the four amplitudes that are not zero at $O(m_s \ln m_s)$ yields $\chi^2 \approx 1.2$, and the extracted values of $\beta_{27}$ and $\delta_{27}$ lead to the theoretical numbers shown in Table 4. Similarly, a one-parameter fit to $S_3^{(\Lambda)}$ and $S_3^{(\Xi)}$ has $\chi^2 \approx 2.8$, and the result is also shown in Table 4.

In conclusion, we have presented a discussion of $|\Delta I| = 3/2$ amplitudes for hyperon non-leptonic decays in $\chi$PT. At leading order these amplitudes are described in terms of only one parameter. This parameter can be fit from the observed value of the S-wave amplitudes in $\Sigma$ decays. After fitting this number, we have made certain predictions for the other amplitudes, some quantitative (the P-waves) some qualitative (the other S-waves). We have used our one-loop calculation to discuss the robustness of the predictions. Our predictions are not contradicted by current data, but current experimental errors are too large for a meaningful conclusion. We have shown that the one-loop non-analytic corrections have the relative size expected from naive power counting. The combined efforts of E871 and KTeV could give us improved accuracy in the measurements of some of the decay modes that we have discussed and allow a more quantitative comparison of theory and experiment.

Table 4: $|\Delta I| = 3/2$ components of the S- and P-wave theoretical amplitudes. Only contributions proportional to $\beta_{27}$, $\delta_{27}$ or $\tilde{\beta}_{27}$ are tabulated here.

| Amplitude | Tree $O(1)$ | Octet $O(m_s \ln m_s)$ | Decuplet $O(m_s \ln m_s)$ | $\tilde{\beta}_{27}$ term $O(m_s)$ |
|------------|-------------|--------------------------|---------------------------|---------------------------------|
| $S_3^{(\Lambda)}$ | 0           | 0                        | 0                         | $-0.399 \tilde{\beta}_{27}$     |
| $S_3^{(\Xi)}$   | 0           | 0                        | 0                         | $0.528 \tilde{\beta}_{27}$     |
| $S_3^{(\Sigma)}$| 1.581 $\beta_{27}$ | 1.316 $\beta_{27}$   | 1.466 $\beta_{27} - 0.230 \delta_{27}$ | $-0.428 \tilde{\beta}_{27}$     |
| $P_3^{(\Lambda)}$| $-0.174 \beta_{27}$ | $-0.072 \beta_{27}$    | $0.870 \beta_{27} + 0.025 \delta_{27}$ | 0                               |
| $P_3^{(\Xi)}$   | 0.546 $\beta_{27}$ | 0.359 $\beta_{27}$    | $-0.610 \beta_{27} - 0.358 \delta_{27}$ | 0                               |
| $P_3^{(\Sigma)}$| $-0.475 \beta_{27}$ | $-0.223 \beta_{27}$  | $2.466 \beta_{27} + 0.069 \delta_{27}$ | 0                               |
Table 5: Results of least-squares fits to the $|\Delta I| = 3/2$ components of the S- and P-wave amplitudes, compared with experiment. The parameter values preferred by the fits are $\beta_{27} = -0.033$, $\delta_{27} = -0.103$, and $\tilde{\beta}_{27} = 0.283$, all in units of $\sqrt{2} f_\pi G_F m_\pi^2$.

| Amplitude | Experiment | Theory |
|-----------|------------|--------|
| $S_3^{(A)}$ | $-0.047 \pm 0.017$ | $-0.068$ |
| $S_3^{(\Xi)}$ | $0.088 \pm 0.020$ | $0.066$ |
| $S_3^{(\Sigma)}$ | $-0.107 \pm 0.038$ | $-0.120$ |
| $P_3^{(A)}$ | $-0.021 \pm 0.025$ | $-0.023$ |
| $P_3^{(\Xi)}$ | $0.022 \pm 0.023$ | $0.027$ |
| $P_3^{(\Sigma)}$ | $-0.110 \pm 0.045$ | $-0.066$ |

Acknowledgments This work was supported in part by DOE under contract number DE-FG02-92ER40730. The work of A. El-Hady was supported in part by DOE under contract number DE-FG02-87ER40371. G. V. thanks the Special Center for the Subatomic Structure of Matter at the University of Adelaide for their hospitality while part of this work was done. We thank John F. Donoghue, Xiao-Gang He, K. B. Luk and Sandip Pakvasa for helpful discussions.

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