Directed XOR/XNOR logic circuit implemented by microring resonators: simulation and demonstration

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Abstract. A directed logic circuit consisting of two silicon microring resonators which can perform XOR and XNOR operations has been proposed. These two operations are evaluated as the optical signal is directed along the circuit, and the results appear at some specific output ports of the circuit. A potential advantage of this new logical paradigm is that it has markedly less state delay than traditional logic circuits. Devices based on the above proposal are fabricated on an 8-inch silicon-on-insulator wafer. The microring resonators in the circuit are modulated through thermo-optic effect. Two electrical modulating signals applied to the microring resonators represent the two operands of the logical operation respectively. Bitwise XOR and XNOR operations at 20 kbit/s are demonstrated. The scattering matrix method is adopted to analyse the transmission spectra of the circuit. The symmetrical characteristics of the spectra are well described by this model. The potential applications of this device are given.

1. Introduction

Directed logic is a newly proposed logical paradigm in which the Boolean functions are calculated through the directed propagation of light [1–3]. Contrast to traditional implementation of optical logic, the directed logic is specially adapted to the features and promises of optics since it depends on the propagation of light other than the nonlinear interactions between light and materials. Directed logic circuit is a network consisting of several simple elements, each of which performs switching function on its input optical signal. Which operation to be performed is determined by a separate Boolean input applied to the corresponding element. The cumulative effect of the switching operations of all the elements yields the value of the function in question. In other words, the Boolean function is evaluated as the optical signal is directed along the switching network and the result appears at some specific output port. Compared to traditional digital logic circuit, directed logic has markedly less state delay because the inputs that determine the states of the switching elements do not pass through the preceding elements in the circuits. Therefore, all elements can perform their switching functions simultaneously and the results are given instantaneously [1, 2].

As pointed out in [1], the XOR operation is indispensable in all encoding and decoding schemes such as label processing, parity checking, data encryption/decryption, and pseudo-random binary sequence (PRBS) generation. Here we report a directed logic architecture consisting of two cascaded...
microring resonators (MRRs) which can perform XOR and XNOR operations. The switching functions of the proposed directed logic circuit are performed by MRRs, which are widely used to function as filters [4], modulators [5], switches [6], and optical logic gates [7–9]. The XOR logic gate introduced here can be regarded as a newly-added item to the function library of MRRs and it can benefit from the potential high operation speed and low power consumption features of the MRR modulator [10–11].

2. Principle of the proposed logic circuit
The proposed architecture is schematically shown in figure 1. A continuous optical wave with the wavelength of \( \lambda \) is modulated by an electrical pulse train \( X \) using MRR 1 (we regard the electrical pulse train \( X \) as a sequence of Boolean 0s and 1s, taking no account of their actual amplitudes here for simplicity), the optical pulse trains that appear at the through and drop ports of MRR 1 are \( \bar{X} \) and \( X \) respectively ( \( \bar{X} \) is the bitwise inverse of \( X \) ). It should be noted that the input optical wave is assumed to be directed to the drop port of MRR 1 when the electrical modulating signal is 1. In other words, the MRR 1 resonates at the wavelength of \( \lambda \) when the applied electrical signal is at high level. So does the MRR 2.

![Figure 1](image_url)

**Figure 1.** The proposed directed logic circuit as an XOR and XNOR calculator (CW: Continuous Wave; EPT: Electrical Pulse Train; OPT: Optical Pulse Train).

If an optical pulse train \( X \) is fed into MRR 2 from its input port solely and is modulated by an electrical pulse train \( Y \) bit-by-bit (here we assume that no signal is fed into MRR 2 from its add port), the optical pulse trains that appear at the through and drop ports of MRR 2 will be \( X \cdot \bar{Y} \) and \( X \cdot Y \), respectively, also taking the aforementioned assumption (the input optical wave is directed to the drop port of MRR 2 when the electrical modulating signal is 1). The symbol \( \cdot \) represents the logical operation and here.

If an optical pulse train \( \bar{X} \) is fed into MRR 2 from its add port solely and is modulated by an electrical pulse train \( Y \) bit-by-bit (here we assume that no signal is fed into MRR 2 from its input port), the optical pulse trains that appear at the through and drop ports of MRR 2 will be \( \bar{X} \cdot Y \) and \( \bar{X} \cdot \bar{Y} \), respectively. If two optical pulse trains \( X \) and \( \bar{X} \) are fed into MRR 2 from its input and add ports simultaneously and both of them are modulated by an electrical pulse train \( Y \) synchronously, we can obtain two optical pulse trains \( X \cdot \bar{Y} + \bar{X} \cdot Y \) and \( X \cdot Y + \bar{X} \cdot \bar{Y} \) at the through and drop ports of MRR 2, respectively, as shown in figure 1. The symbol \( + \) represents the logical operation or, which is actually implemented by the multiplexing function of MRR 2.

It is well known in Boolean algebra that the XOR and XNOR operations of two logical variables \( X \) and \( Y \) can be calculated by the formulas \( X \oplus Y = X \cdot \bar{Y} + \bar{X} \cdot Y \) and \( X \odot Y = X \cdot Y + \bar{X} \cdot \bar{Y} \), where the symbols \( \oplus \) and \( \odot \) represent the XOR and XNOR operators respectively. Therefore, the proposed circuit shown in figure 1 can be used as an XOR/XNOR calculator.
3. Fabrication and characterization of the device
The working principle shown in the last section indicates that we can execute the XOR and XNOR operations simultaneously using the circuit in figure 1. Here in this part, we introduce the fabrication and characterization of the device. We adopt the silicon platform to fabricate the device due to its compact size, low power consumption and potential high speed [10–11]. The characterization procedure is composed of the static and the dynamic test. The former is for determining the working points, including the working wavelength and the tuning voltage representing logical 1 for each MRR. And the latter is for verifying the dynamic operation of the circuit as an XOR/XNOR calculator.

3.1. Fabrication of the device
The device schematically shown in figure 1 is fabricated on an 8-inch silicon-on-insulator wafer with 220-nm-thick top silicon (Si) layer and 2-μm-thick buried dioxide (BOX) layer. The cross-section of Si waveguides is 400×220 nm². The radii of the ring waveguides are both 10 μm. Gaps between the ring and the straight waveguides are 400 nm. Two 200-nm thick titanium micro-heaters are employed to tune the MRRs thermally, as shown in the inset of figure 2. Aluminium wires with the width of 50 μm and pads with the size of 150×150 μm² are fabricated after the heaters are done. The micrograph of the device is shown in figure 2, which has an effective area of about 1.5×0.6 mm². Details on the fabrication process are given in [12].

3.2. Characterization of the device
The characterization of the device consists of static test and dynamic test. The former is for determining the working points and the latter is for verifying the dynamic operation of the circuit. A lensed single-mode fiber is used to couple light from an amplified spontaneous emission (ASE) source or a tunable laser source into the chip. Another lensed fiber is used to couple light from the chip to an optical spectrum analyser (OSA) or a photodiode for detection.

3.2.1. Static test for determining the working points. The broadband light is coupled into the device through the input lensed fiber. The output light from the through port of the circuit is collected by the output lensed fiber and fed into the OSA. The same procedure is used to obtain the spectrum of the drop port. Results of the foregoing tests are shown in figure 3. After obtaining the static spectra of the two output ports, two tunable voltage sources are adopted to provide voltages to the two heaters, respectively. When a MRR is heated up, the effective refractive index of the ring waveguide increases, and it results in a redshift of the resonant wavelength of the corresponding MRR.

As we get the highest extinction ratio in the region near 1536 nm when no electrical signals are applied (see figure 3), the working wavelength will be chosen in this region, which is enlarged to
shown in figure 4(a) and figure 6(a). According to the principle aforementioned, a minimum (representing a logical 0 output) should be given at the through port when two applied electrical signals are both at low level (representing two logical 0 inputs) or high level (representing two logical 1 inputs). 1538.072 nm is chosen as the working wavelength because a minimum appears there when the electrical inputs are both 0 V (see figure 4(a)). With an appropriate voltage applied to corresponding MRR alone (4.70 V for MRR 1 and 5.04 V for MRR 2), it resonates at 1538.072 nm and a maximum appears there, as shown in figure 4(b) and 4(c). With two voltages of 4.70 V and 5.04 V applied to the corresponding MRRs simultaneously, a minimum appears at 1538.072 nm again, as shown in figure 4(d). In summary, voltages of 4.70 V and 5.04 V are applied to MRR 1 and MRR 2 respectively when the operands of the XOR operation are 1s, and no voltages are applied to them when the operands are 0s.

![Figure 3](image3.png)

**Figure 3.** The transmission spectra obtained from the through and drop ports of the circuit.

We can find from figure 4(b) and 4(c) that the two MRRs without applied voltages have different resonant wavelengths (1536.188 nm for MRR 1 and 1535.898 nm for MRR 2). Such a slight difference mainly originates from the fabrication errors, which have no effect on the operation of the device except for the different tuning voltages required for two MRRs.

![Figure 4](image4.png)

**Figure 4.** Response spectra at the through port of the circuit. Voltages applied to MRR 1 and MRR 2 are (a) both 0 V; (b) 4.70 V and 0 V; (c) 0 V and 5.04 V; (d) 4.70 V and 5.04 V, respectively.

3.2.2. Dynamic XOR operation at the through port. The dynamic performance of the device is shown in figure 5(a-c). Two pseudo-random binary sequence $2^4$-1 non-return-to-zero signals with a data rate of 20 kbit/s are converted to two analog voltage signals bit-by-bit according to the rule presented above and then applied to the corresponding MRRs. Clearly, a logical 0 output is obtained when the two applied signals are both at high level or both at low level. And a logical 1 output is obtained otherwise. It indicates that the device performs the XOR operation correctly at the through port.
3.2.3. Dynamic XNOR operation also at the through port. According to the principle aforementioned, the XNOR operation can be achieved at the drop port at the same time when XOR operation is carried out at the through port. But due to the structure of waveguides we choose, the fabricated device has a low extinction ratio (about 3 dB as shown in figure 3) at the drop port, which makes the logical operations at the drop port inefficient. Nevertheless, we can achieve XNOR operation at the through port as well, just by choosing different working wavelength and tuning voltages corresponding to logical 1 inputs for two MRRs.

The working wavelength is still chosen from the region around 1536 nm. According to the principle of the XNOR operation, a maximum should be given when two operands are both 0s. 1536.335 nm is chosen as the working wavelength since a maximum appears there when the electrical inputs are both 0 V (see figure 6(a)). With a voltage of 3.58 V applied to MRR 1 alone, it is tuned to resonate far away from 1536.335 nm and a minimum appears there (see figure 6(b)). With a voltage of 2.58 V applied to MRR 2 alone, it resonates a little to the right of 1536.335 nm. The signal downloaded to the drop port of MRR 1 is further downloaded to the drop port of MRR 2 and therefore a minimum is achieved at the through port (see figure 6(c)). With two voltages of 3.58 V and 2.58 V applied to two corresponding MRRs simultaneously, a maximum appears at 1536.335 nm as shown in figure 6(d). The dynamic performance of the XNOR operation is shown in figure 5(d-f). It shows that the XNOR operation is carried out correctly at the through port as well.

![Figure 5](image_url)

**Figure 5.** Signals applied to two MRRs and detected at the through port of MRR 2 when executing the XOR operation (a-c) and the XNOR operation (d-f). Signals applied to (a) MRR 1 and (b) MRR 2 and (c) the result of XOR operation at the through port. Signals applied to (d) MRR 1 and (e) MRR 2 and (f) the result of XNOR operation at the through port as well.

![Figure 6](image_url)

**Figure 6.** Response spectra at the through port of the circuit. Voltages applied to MRR 1 and MRR 2 are (a) both 0 V; (b) 3.58 V and 0 V; (c) 0 V and 2.58 V; (d) 3.58 V and 2.58 V respectively.
4. Scattering matrix model and spectrum evolution

The transmission spectra of the proposed structure are obtained using the scattering matrix method [13]. The whole device can be classified into three subsidiary sets according to their functions [3], as shown in figure 7. The first set consists of four coupling areas which cause energy exchanges between the straight waveguides and the ring waveguides. These coupling areas are described by four scattering matrixes, whose elements are assumed to be independent of wavelength [14]. The second set consists of two ring waveguides and two straight waveguides, which cause phase shifts and attenuations to the optical fields passing through them. The third set is the crossing of two straight waveguides, which possibly causes scattering when light passes through it. The scattering matrix describing the energy exchange in coupling area I is in the form of

$$
\begin{pmatrix}
E_{p1} \\
E_{p2}
\end{pmatrix} = \begin{pmatrix}
t_1 & k_1 \\
-k_1^* & t_1^*
\end{pmatrix}
\begin{pmatrix}
l \\
E_{p1}
\end{pmatrix}
$$

(1)

where the complex mode amplitudes are normalized such that their squared magnitudes correspond to the modal powers. The input wave is chosen to be $l$ so that all the field amplitudes are normalized to it. We assume that the four coupling areas are identical.

Figure 7. Scattering matrix model of the proposed circuit (Z_I~Z_II: coupling areas I~IV; S_I~S_II: straight waveguides I~II; R_I~R_II: ring waveguides I~II).

The simulation result of the transmission spectra of the through and drop ports of the device based on the above model is shown in figure 8, which agrees well with the experimental result as shown in figure 3. The scattering at the crossing results in the ripples in the transmission spectra of the device.

Figure 8. The transmission spectra obtained by scattering matrix modelling.
It can be noted in both figures 3 and 8 that the spectra show mirror symmetry, with the axis of symmetry located around 1536 nm. This is not caused by the radius deviation of the two MRRs, since we can still observe such kind of symmetry even when we adopt two identical radii in the numerical model.

Further investigation shows that such a property of mirror symmetry is related to the length difference between the two straight waveguides $S_I$ and $S_{II}$.

**Figure 9.** Typical spectra obtain through numerical model when the length difference between $S_I$ and $S_{II}$ are (a-b) odd multiple of one-half the circumference of the ring waveguide; (c-d) even multiple of one-half the circumference of the ring waveguide; (e-f) other values.

When the length difference between $S_I$ and $S_{II}$ is odd multiple of one-half the circumference of the ring waveguide, the spectra of the circuit show translational symmetry (see figure 9 (a) and (b)). In figure 9 (a) and (b), the length differences between $S_I$ and $S_{II}$ are $C/2$ and $-C/2$ respectively, where
$C$ means the circumference of the ring waveguide. In such situations the periods of the spectra equal the free spectral range (FSR) of the MRR. When the length difference between $S_I$ and $S_{II}$ is even multiple of one-half the circumference of the ring waveguide, the spectra of the circuit show translational symmetry too (see figure 9 (c) and (d)), but with a period of $2^*\text{FSR}$. In figure 9 (c) and (d), the length differences between $S_I$ and $S_{II}$ are $C$ and $-C$ respectively. In all other situations where the length difference between $S_I$ and $S_{II}$ is not integral multiple of one-half the circumference of the ring waveguide, the spectra show mirror symmetry (see figure 9 (e) and (f)), as demonstrated in figure 3 and figure 8. In figure 9 (e) and (f), the length differences between $S_I$ and $S_{II}$ are $C/4$ and $-C/4$ respectively. It should be noted here that the aforementioned mirror symmetry is a localized property. When we view the spectra in a wider wavelength region, translational symmetry will turn up again, with a much larger period than FSR.

Such a structure (see figure 1 and figure 7) can be regarded as a Mach-Zehnder interferometer (MZI) as a whole, while using the microring resonators as the splitting and combining units. In traditional MZI, only the behaviour of the two arms is wavelength-dependent. But for the structure presented in this paper, the splitting and combining units are also wavelength-dependent. That is why it shows such varied spectra in figure 9.

5. Conclusion

Directed logic circuit which can carry out the XOR and XNOR operations is proposed and experimentally demonstrated. The symmetry of the spectra is well described by the scattering matrix model. In order to achieve low latency of the directed logic circuit, the cavity photon lifetime should be minimized. An alternative way is to choose other kind of switching elements instead of resonators. This device can be used as a modulator with the function of scrambling to facilitate the timing recovery circuit of the receiver by eliminating long sequences of 0 or 1 only. It can also be regarded as a modulator with the function of XOR encryption.

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