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Three-dimensional couette flow of dusty fluid with heat transfer in the presence of magnetic field

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Abstract. This paper is focused on the mathematical modelling of three-dimensional couette flow and heat transfer of a dusty fluid between two infinite horizontal parallel porous flat plates in the presence of an induced magnetic field. The problem is formulated using a continuum two-phase model and the resulting equations are solved analytically. The lower plate is stationary while the upper plate is undergoing uniform motion in its plane. These plates are, respectively subjected to transverse exponential injection and its corresponding removal by constant suction. Due to this type of injection velocity, the flow becomes three-dimensional. The closed-form expressions for velocity and temperature fields of both the fluid and dust phase are obtained by solving the governing partial differentiation equations using the perturbation method. A selective set of graphical results is presented and discussed to show interesting features of the problem. It is found that the velocity profiles of both fluid and dust particles decrease due to the increase of (magnetic parameter) Hartmann number.

1. Introduction

Any fluid that contains small inert particles which are usually ignored if we consider these inert particles also the fluid along with this inert particle is known as dusty fluid. The flow of dusty fluid between the porous walls has various applications like fluid dust separators, combustion chamber walls, operation and design filters and exhaust nozzle. In order to protect certain structural failures due to frictional heat or hot gases in physical situations like re-entry of space vehicle into the earths ionosphere, protected by MHD generators, MHD couplers and bearing becomes necessary to cool them. By injecting a coolant through a porous and simultaneously removing it through the other porous wall of the channel the cooling can be achieved [1]. [2] applied the equations of motion derived by Ahmadi and Manvi [3] to study the unsteady MHD flow of conducting fluid through porous medium. Chamkha [4] discussed the hydromagnetic two-phase flow in a channel. Singh [5] [6] studied the influence of a moving magnetic field on three-dimensional Couette flow and also discussed three-dimensional flow of a hydromagnetic free convective flow past a porous plate. [7] [8] studied the oscillatory two-dimensional flow through porous medium bounded by a horizontal porous plate subjected to a variable suction velocity and also discussed the Unsteady two-fluid flow and heat transfer in a horizontal channel.

Govindharajan et al.[9] discussed the 3-D Couette flow of dusty fluid with transpiration cooling. Singh and Sharma. Singh and Verma [10] discussed the 3-D oscillatory flow through a porous medium with periodic permeability. Recently, Singh [11] studied three-dimensional Couette dusty fluid flow between two horizontal parallel porous flat plates with transverse...
sinusoidal injection at the stationary plate and its corresponding removal by constant suction. Further, they studied the velocity profiles and the skin-friction coefficients. Singh and Sharma [12] analysed the three dimensional Couette flow through a porous medium with heat transfer. Attia and Kotb [13] investigated two dimensional MHD flow between two porous, parallel, infinite, insulated, horizontal plates and the heat transfer through it when the lower plate is kept stationary and the upper plate is given a uniform velocity. Effects of such a suction velocity on various flow heat transfer problems along flat and vertical porous plates were studied extensively by Singh [14]. Gersten and Gross [15] discussed the flow and heat transfer along a plane wall with periodic suction velocity. The main objective of the present analysis is to study the three-dimensional couette flow of dusty fluid with heat transfer with the effect of an exponential injection/suction parameter in the transverse magnetic field. The perturbation technique is employed to obtain the analytical expressions for the velocity and temperature fields. The effects of different flow parameters like mass concentration of the dust particles, injection parameter and the Prandtl number on the components of the main flow velocity, cross flow velocity of both the fluid and dust particles are studied with the help of graphs. In addition, important observations on the effect of the Hartmann number on temperature fields are listed which are not studied by some of above mentioned authors.

2. Formulation of the problem

Considering the Couette flow of a viscous incompressible dusty fluid between two infinite parallel flat porous plates (see Fig. 1). A coordinate system is introduced with its origin on the lower stationary plate lying horizontally on the $x^* - z^*$ plane and the upper plate is placed at a distance $d$ which is subjected to a uniform motion $U$. The y-axis is taken perpendicular to the planes of the plates. The lower and the upper plates are assumed to be at constant temperatures $T_0$ and $T_1$, respectively, with $T_1 > T_0$. The upper plate is subjected to a constant suction $V_0$ and the lower to a transverse exponential injection velocity distribution of the form $V^*(Z^*) = V_0[1 + \varepsilon e^{\alpha z^*}]$.

![Figure 1. SCHEMATIC DIAGRAM OF THE FLOW CONFIGURATION](image-url)
The boundary conditions for the problem in the dimensionless form are:

For fluid phase:

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

(1)

\[
v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{1}{\lambda} \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{f}{\tau} (u_p - u) - M^2 u
\]

(2)

\[
v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \frac{1}{\lambda} \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \frac{f}{\tau} (v_p - v)
\]

(3)

\[
v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = \frac{1}{\lambda} \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \frac{f}{\tau} (w_p - w)
\]

(4)

\[
v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{1}{\lambda F_r} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + \frac{2f}{3\tau P_r} (\theta_p - \theta)
\]

(5)

Where \( \tau \) is the relaxation time parameter of dust particles, \( f \) is the mass concentration of the dust particles and \( \lambda \) is the injection/suction parameter defined as \( \lambda = \frac{V_0 d}{u} \).

For particle phase:

\[
\frac{\partial v_p}{\partial y} + \frac{\partial w_p}{\partial z} = 0
\]

(6)

\[
v_p \frac{\partial u_p}{\partial y} + w_p \frac{\partial u_p}{\partial z} = \frac{1}{\tau} (u - u_p)
\]

(7)

\[
v_p \frac{\partial v_p}{\partial y} + w_p \frac{\partial v_p}{\partial z} = \frac{1}{\tau} (v - v_p)
\]

(8)

\[
v_p \frac{\partial w_p}{\partial y} + w_p \frac{\partial w_p}{\partial z} = \frac{1}{\tau} (w - w_p)
\]

(9)

\[
v_p \frac{\partial \theta_p}{\partial y} + w_p \frac{\partial \theta_p}{\partial z} = \frac{2(\theta_p - \theta)}{3\tau P_r \gamma}
\]

(10)

where

\[
y = \frac{\bar{y}}{d}, z = \frac{\bar{z}}{d}, u = \frac{\bar{u}}{U}, v = \frac{\bar{v}}{V_0}, w = \frac{\bar{w}}{V_0}, u_p = \frac{\bar{u}_p}{U}, v_p = \frac{\bar{v}_p}{V_0}, w_p = \frac{\bar{w}_p}{V_0}, p = \frac{\bar{p}}{\rho V_0^2},
\]

\[
\theta = \frac{T - T_0}{T_1 - T_0}, \theta_p = \frac{T_p - T_0}{T_1 - T_0}, f = \frac{N_0 m}{\rho}, \tau = \frac{\tau P_0 V_0}{d}, \tau_p = \frac{m}{k}, \gamma = \frac{C_p}{C_s}, M^2 = \frac{\sigma B_0^2}{\gamma}.
\]

Here the bar stands for dimensional quantities, \( p \) is the non-dimensional pressure, \( \bar{p} \) is the pressure, \( \rho \) is the density of the fluid, \( N_0 \) is the number density of the dust particles which is assumed to be constant, \( m \) is the mass of the dust particles, \( k \) is the Stokes drag constant which is \( 6\pi \mu r \) for spherical particles of radius \( r \), \( \tau_p \) is the relaxation time of the particle phase which is the time taken for the dust particles to adjust to the velocity of the fluid, \( \gamma \) is the ratio of specific heat of dust to that of the fluid, and \( C_p \) and \( C_s \) are the specific heat of the fluid and dust particles at constant pressure, respectively.

The boundary conditions for the problem in the dimensionless form are:

\[
y = 0 : \ u = 0, v(z) = 1 + \alpha e^{\alpha z}, w = 0, \theta = 0, u_p = 0, v_p(z) = 1 + \alpha e^{\alpha z}, w_p = 0, \theta_p = 0
\]

\[
y = 1 : \ u = 1, v(z) = 1, w = 0, \theta = 1, u_p = 1, v_p(z) = 1, w_p = 0, \theta_p = 1
\]

(11)
3. SOLUTION OF THE PROBLEM
When the amplitude in the suction velocity is very small we can assume \( u, u_p, v, v_p, w, w_p, \theta_p \) in the following form to solve the equations (1)-(10)

\[
\begin{align*}
 f(y, z) &= f_0(y) + \varepsilon f_1(y, z) + \varepsilon^2 f_2(y, z) + \ldots \\
 u(y, z) &= u_0 + \varepsilon u_1(y, z) + \varepsilon^2 u_2(y, z) + \ldots \\
 v(y, z) &= v_0 + \varepsilon v_1(y, z) + \varepsilon^2 v_2(y, z) + \ldots \\
 w(y, z) &= w_0 + \varepsilon w_1(y, z) + \varepsilon^2 w_2(y, z) + \ldots \\
 \theta(y, z) &= \theta_0 + \varepsilon \theta_1(y, z) + \varepsilon^2 \theta_2(y, z) + \ldots \\
 u_p(y, z) &= u_{p0} + \varepsilon u_{p1}(y, z) + \varepsilon^2 u_{p2}(y, z) + \ldots \\
 v_p(y, z) &= v_{p0} + \varepsilon v_{p1}(y, z) + \varepsilon^2 v_{p2}(y, z) + \ldots \\
 w_p(y, z) &= w_{p0} + \varepsilon w_{p1}(y, z) + \varepsilon^2 w_{p2}(y, z) + \ldots \\
 p(y, z) &= p_0 + \varepsilon p_1(y, z) + \varepsilon^2 p_2(y, z) + \ldots \\
 \theta_p(y, z) &= \theta_{p0} + \varepsilon \theta_{p1}(y, z) + \varepsilon^2 \theta_{p2}(y, z) + \ldots \\
\end{align*}
\]

when \( \varepsilon = 0 \) the problem reduces to 2-D flow and are obtained as follows

\[
\begin{align*}
 u_{p0}(y) &= \frac{C_1 e^{m_1 y} + C_2 e^{m_2 y} + C_3 e^{m_3 y}}{\tau m_1 + 1} \\
 v_{p0}(y) &= \frac{C_5 e^{m_1 y} + C_6 e^{m_2 y} + C_7 e^{m_3 y}}{m_1 + b_2} \\
 w_{p0}(y) &= \frac{C_8 e^{m_1 y} + C_9 e^{m_2 y} + C_{10} e^{m_3 y}}{m_3 + b_2} \\
 w_0 &= 0, w_{p0} = 0, v_0 = 1, v_{p0} = 1 \\
\end{align*}
\]

When \( \varepsilon \neq 0 \), substituting \( \ref{12} \) in equations \( \ref{11}-\ref{10} \) comparing the coefficients of \( \varepsilon \) and neglecting the higher order terms, with the use of \( \ref{17} \) the following equations are obtained.

Fluid phase

\[
\begin{align*}
 \frac{\partial v_1}{\partial y} + \frac{\partial v_1}{\partial z} &= 0 \\
 \frac{\partial u_1}{\partial y} + \frac{\partial u_1}{\partial z} &= \frac{1}{\lambda} \left( \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) + \frac{f}{\tau} \left( u_{p1} - u_1 \right) - M^2 u_1 \\
 \frac{\partial v_1}{\partial y} &= \frac{\partial p_1}{\partial y} + \frac{1}{\lambda} \left( \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \right) + \frac{f}{\tau} \left( v_{p1} - v_1 \right) \\
 \frac{\partial w_1}{\partial y} &= \frac{\partial p_1}{\partial z} + \frac{1}{\lambda} \left( \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right) + \frac{f}{\tau} \left( w_{p1} - w_1 \right) \\
 \frac{\partial \theta_0}{\partial y} + \frac{\partial \theta_1}{\partial y} &= \frac{1}{\lambda \tau} \left( \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} \right) + \frac{2 f^2}{3 \tau \lambda} \left( \theta_{p1} - \theta_1 \right) \\
\end{align*}
\]
Particle phase:

\[
\begin{align*}
\frac{\partial v_{p1}}{\partial y} + \frac{\partial w_{p1}}{\partial z} &= 0 \quad (23) \\
v_{p1} \frac{\partial u_{p0}}{\partial y} + \frac{\partial u_{p1}}{\partial y} &= \frac{1}{\tau} (u_1 - u_{p1}) \quad (24) \\
\frac{\partial u_{p1}}{\partial y} &= \frac{1}{\tau} (v_1 - v_{p1}) \quad (25) \\
\frac{\partial w_{p1}}{\partial y} &= \frac{1}{\tau} (w_1 - w_{p1}) \quad (26) \\
v_{p1} \frac{\partial \theta_{p0}}{\partial y} + \frac{\partial \theta_{p1}}{\partial y} &= -b_1 (\theta_{p1} - \theta_1) \quad (27)
\end{align*}
\]

The boundary conditions reduces to

\[
y = 0: \ u_1 = v_1 = w_1 = 0, \theta_1 = 0, u_{p1} = 0, v_{p1} = 0, \theta_{p1} = 0.
\]

\[
y = 1: \ u_1 = v_1 = w_1 = 0, \theta_1 = 0, u_{p1} = 0, v_{p1} = 0, \theta_{p1} = 0.
\]

The above equations are linear partial differential equations representing the 3-D flow. To solve the above equations let us first solve the equations (18), (20), (21), (23), (25) and (26) for cross flow, being independent of the main flow components \(u_1\) and \(v_{p1}\), and the temperature fields \(\theta_1\) and \(\theta_{p1}\).

We assume \(v_1, u_{p1}, w_{p1}, p_1\) of the following form

\[
v_1(y, z) = v_{11}(y)e^{\alpha z} \quad (28)
\]

\[
w_1(y, z) = \left(\frac{1}{\alpha}\right) v'_{11}(y)e^{\alpha z} \quad (29)
\]

\[
v_{p1}(y, z) = vp_{11}(y)e^{\alpha z} \quad (30)
\]

\[
w_{p1}(y, z) = \left(\frac{1}{\alpha}\right) vp'_{11}(y)e^{\alpha z} \quad (31)
\]

\[
p_1(y, z) = p_{11}(y)e^{\alpha z} \quad (32)
\]

here the prime denotes differentiation with respect to \(y\). Equations (28)-(32) are chosen so as to satisfy the continuity equations (18) and (23). Substituting the above equations into (20), (21), (25), and (26) also applying the transformed boundary conditions the solution of \(v_1, v_{p1}, w_{p1}, w_{p1}\) are obtained as,

\[
p_1(y, z) = \left( c_1 \cos \alpha y + c_2 \sin \alpha y \right) e^{\alpha z} \quad (33)
\]

\[
v_1(y, z) = \left( c_3 e^{r_{1y}} + c_4 e^{r_{2y}} + c_5 e^{r_{3y}} + c_6 e^{r_{4y}} + k_0 d_1 \cos \alpha y - k_0 d_2 \sin \alpha y \right) e^{\alpha z} \quad (34)
\]

\[
w_1(y, z) = \frac{1}{\alpha} \left( c_{3r1} e^{r_{1y}} + c_{4r2} e^{r_{2y}} + c_{5r3} e^{r_{3y}} - k_0 d_1 \sin \alpha + ak_0 d_2 \cos \alpha \right) e^{\alpha z} \quad (35)
\]

\[
v_{p1}(y, z) = \left( \frac{c_{3r1} e^{r_{1y}}}{r_1} + \frac{c_{4r2} e^{r_{2y}}}{r_2} + \frac{c_{5r3} e^{r_{3y}}}{r_3} + \frac{ak_0 d_1}{1 + \alpha^2 \tau^2} \right) \left( \cos \alpha y + \alpha \tau \sin \alpha y \right) \left( \sin \alpha y - \alpha \tau \cos \alpha y \right) e^{\alpha z} \quad (36)
\]

\[
w_{p1}(y, z) = \frac{1}{\alpha} \left( \frac{c_{3r1} e^{r_{1y}}}{r_1} + \frac{c_{4r2} e^{r_{2y}}}{r_2} + \frac{c_{5r3} e^{r_{3y}}}{r_3} + \frac{ak_0 d_1}{1 + \alpha^2 \tau^2} \right) \left( \alpha \tau \cos \alpha y - \sin \alpha y \right) + \frac{k_0 d_2}{1 + \alpha^2 \tau^2} \left( \sin \alpha y - \alpha \tau \cos \alpha y \right) e^{\alpha z} \quad (37)
\]
Similarly assuming,

\[ u_1(y, z) = u_{11}(y)e^{az} \]  
\[ u_{p1}(y, z) = u_{p11}(y)e^{az} \]  
\[ \theta_1(y, z) = \theta_{11}(y)e^{az} \]  
\[ \theta_{p1}(y, z) = \theta_{p11}(y)e^{az} \]  

Corresponding boundary conditions

\[ y = 0 : u_{11} = 0, \theta_{11} = 0, u_{p11} = 0, \theta_{p11} = 0 \]
\[ y = 1 : u_{11} = 0, \theta_{11} = 0, u_{p11} = 0, \theta_{p11} = 0 \]  

using the boundary conditions equations (42) and also using equations (38),(39),(40) and (41) following set of equations are obtained,

\[ u_1(y, z) = \left\{ \frac{L_1e^{ry} + M_1e^{zy} + N_1e^{zy} + L_2e^{(r_1+m_1)y} + L_3e^{(r_2+m_2)y} + L_4e^{(r_2-m_2)y} + L_5e^{(r_3+m_1)y} + L_6e^{(r_2-m_2)y} + \theta_1(y, z) = \left\{ \frac{R_1e^{s_1y} + R_2e^{s_2y} + R_3e^{s_3y} + M_1e^{(r_1+m_3)y} + M_2e^{(r_1+m_4)y} + M_3e^{(r_2+m_3)y} + M_4e^{(r_2+m_4)y} + M_5e^{(r_3+m_3)y} + M_6e^{(r_3+m_4)y} + M_7e^{m_3y}(\sin y + N_3\cos y) + M_9e^{m_4y}(\cos y + N_3\sin y) + M_{10}e^{m_5y}(\cos y - N_1\sin y) + M_{11}e^{(r_1-b_1)y} + M_{12}e^{(r_2-b_1)y} + M_{13}e^{(r_1-b_1)y} + M_{14}e^{(r_2-b_1)y} + M_{15}e^{(r_2-b_1)y} + M_{16}e^{(r_1+b_1)y} + M_{17}e^{(r_2+b_1)y} + M_{18}e^{(r_2+b_1)y}e^{az} + \right. \]  
\]  

Now, the skin-friction components \( T_x, T_z \) can be calculated in the main flow and transverse directions as follows,
the influence of accelerating the motion of the fluid. So, we can say that the presence of dust particles in the fluid has

concentration. The velocity profile maintain an increase trend near lower plate and attain its

profile decrease with an increase in the injection parameter for any value of dust particle mass

and dust decrease with an increase in the mass concentration of the dust particles. Also, the

4. Results and discussion

shown easily that

\[ T_z = \frac{dT^* z}{\mu U} = \left( \frac{du_1}{dy} \right)_{y=0} + \varepsilon \left( \frac{du_{11}}{dy} \right)_{y=0} e^{\alpha z} \]

\[ T_x = (A_{m1} + B_{m2}) + \varepsilon \{ L_1 r_1 + M_{r2} + N_{r3} + L_1 (r_1 + m_1) + L_2 (r_1 + m_2) + L_3 (r_2 + m_1) \]

\[ + L_4 (r_2 + m_2) + L_5 (r_3 + m_1) + L_6 (r_3 + m_2) + L_7 (\alpha^2 + N_0 m_1) + L_8 (\alpha^2 + N_1 m_2) \]

\[ + L_9 (m_1 \alpha - N_0 \alpha) + L_{10} (m_2 \alpha - N_1 \alpha) + L_{11} (r_1 - 1/\tau) + L_{12} (r_2 - 1/\tau) + L_{13} (r_3 - 1/\tau) \]

\[ + L_{14} (\alpha^2 - N_2 / \tau) - L_{15} (N_2 \alpha + \alpha / \tau) + L_{16} (m_1 - 1/\tau) - L_{17} (m_2 - 1/\tau) \]

\[ + L_{18} (2/\tau) e^{\alpha z} \]  

(47)

\[ T_y = \frac{dT^* z}{\mu V_0} = \varepsilon \left( \frac{du_{11}}{dy} \right)_{y=0} e^{\alpha z} \]

\[ T_z = \frac{dT^* z}{\mu V_0} = \varepsilon \{ c_3 t_2^2 + c_4 t_2^2 + c_5 t_3^2 - k_1 d_1 a^2 \} e^{\alpha z} \]  

(48)

From the temperature field the heat transfer coefficient in terms of Nusselt number along y

direction is obtained as

\[ N_u = (A_{m3} + B_{m4}) + \varepsilon \{ R_1 s_1 + R_2 s_2 + R_3 s_3 + M_1 (r_1 + m_3) + M_2 (r_1 + m_4) \]

\[ + M_3 (r_2 + m_3) + M_4 (r_2 + m_4) + M_5 (r_3 + m_3) + M_6 (r_3 + m_4) \]

\[ + M_7 (\alpha^2 + N_3 m_3) + M_8 (\alpha^2 + N_4 m_4) + M_9 (m_3 - N_3 \alpha) + M_{10} (m_4 - N_4 \alpha) \]

\[ + M_{11} e^{(r_1 - b_1) y} + M_{12} e^{(r_2 - b_1) y} + M_{13} e^{(r_2 - b_1) y} + M_{14} e^{-b_1 y} (a \sin y + N_3 \cos y) \]

\[ + M_{15} e^{-b_1 y} (a \cos y - N_3 \sin y) + M_{16} e^{-(b_1 + 1/\tau) y} \]

\[ + M_{17} e^{(m_2 - 1/\tau) y} + M_{18} e^{(m_4 - 1/\tau) y} \} e^{\alpha z} \]  

(49)

The Nusselt number distribution along z direction,

\[ \theta_1 (y, z) = \{ R_1 e^{r_1 y} + R_2 e^{r_2 y} + R_3 e^{s_2 y} + M_1 e^{(r_1 + m_2) y} + M_2 e^{(r_1 + m_4) y} + M_3 e^{(r_2 + m_2) y} \]

\[ + M_4 e^{(r_2 + m_4) y} + M_5 e^{(r_2 + m_2) y} + M_6 e^{(r_2 + m_4) y} + M_7 e^{m_2 y} (a \sin y + N_3 \cos y) \]

\[ + M_8 e^{m_2 y} (a \cos y - N_3 \sin y) + M_9 e^{m_2 y} (a \cos y - N_3 \sin y) \]

\[ + M_{10} e^{m_2 y} (a \cos y - N_3 \sin y) + M_{11} e^{(r_1 - b_1) y} + M_{12} e^{(r_2 - b_1) y} + M_{13} e^{(r_2 - b_1) y} \]

\[ + M_{14} e^{-b_1 y} (a \sin y + N_3 \cos y) + M_{15} e^{-b_1 y} (a \cos y - N_3 \sin y) \]

\[ + M_{16} e^{-(b_1 + 1/\tau) y} + M_{17} e^{(m_2 - 1/\tau) y} + M_{18} e^{(m_4 - 1/\tau) y} \} \]  

(50)

from equations (47)-(50) for the main flow skin friction and heat transfer coefficients can be shown easily that \( N_u = T_z \) for \( P_\tau = 1 \).

4. Results and discussion

4.1. Main flow and velocity profile

From Fig.2 and Fig.3, it is evident that irrespective of value \( \lambda \) the velocity profile of fluid and dust decrease with an increase in the mass concentration of the dust particles. Also, the profile decrease with an increase in the injection parameter for any value of dust particle mass concentration. The velocity profile maintain an increase trend near lower plate and attain its maximum value closer to the lower plate and thereafter it decreases steadily and reaches steady state.

Further, it is clear that the fluid and dust particles behave in the same manner. But the profiles of the dust are at a lower height as compared with the fluid. The velocity profiles of a clean fluid increases steadily where as in dusty fluid, it increase steadily near the lower plate and a decreasing is seen as it approaches the other plate. This phenomenon can be attributed to the presence of the dust particles. So, we can say that the presence of dust particles in the fluid has the influence of accelerating the motion of the fluid.
Figure 2. Variation of main flow fluid and dust velocity with y

Figure 3. Variation of main flow fluid and dust velocity with z

Figure 4. Variation of main flow velocity for various values of M for (a) fluid and (b) dust
4.2. Cross flow velocity profiles

![Graph of cross flow velocity profiles for y](image1)

![Graph of cross flow velocity profiles for z](image2)

**Figure 5.** Variation of cross flow for velocity y for various values of λ values (a) fluid (b) dust

![Graph of cross flow velocity profiles for y](image3)

![Graph of cross flow velocity profiles for z](image4)

**Figure 6.** Variation of cross flow for velocity z for various values of f with different λ values (a) fluid (b) dust
The cross flow velocity component \( w \) is due to the transverse exponential injection velocity distribution applied through the porous plate at rest. The cross flow velocity increases with an increase in the concentration of the dust particles at a point which is located little away from the mid-way between the two plates and thereafter reverses its trend. The velocity profiles increase steadily near the lower plate and reach the maximum value at a point a little away from the lower plate.

4.3. Skin friction components \( T_x \) and \( T_z \)  
The variations of the skin friction components \( T_x \) and \( T_z \) in the main flow and transverse directions respectively are shown in Tables 1 and 2. We conclude that for a given value of the mass concentration and relaxation time of the dust particles, the main flow skin friction component \( T_x \) increases with an increase in the injection parameter. For a given value of the injection parameter the main flow skin friction component \( T_x \) increases with an increase in the mass concentration of the dust particles, whereas a reverse trend is seen when the relaxation time of the dust particle increases. We find an appreciable increase in the \( T_x \) value even for a
Table 1. Variations of $T_x$ and $T_z$ for $Pr=0.7$ and $Pr=7.0$ with various values of $\lambda$

| $\lambda$ | $T_x$         | $T_z$         |
|----------|--------------|--------------|
| 0.4      | 3.5677       | 0.00000002090|
| 0.6      | 3.6939       | 0.00000002300|

Table 2. Variations of $T_x$ and $T_z$ for $Pr=0.7$ and $Pr=7.0$ with various values of $\lambda$

| $\tau$ | $T_x$         | $T_z$         |
|--------|--------------|--------------|
| 0.7    | 0.8846       | 0.0000014100 |
| 0.8    | 0.8590       | 0.000004885 |
| 0.9    | 0.8270       | 0.000002002 |

4.4. Nusselt number

Nusselt number Nu for both air and water decreases with an increase in the injection parameter for a given value of the mass concentration of the dust particles. The Nusselt number for both air and water decreases with an increase in the relaxation time of the dust particles. It is to be noted that for a clean fluid the heat transfer coefficient Nu decreases with increase of the injection parameter $\lambda$ for both air and water. It is observed that heat transfer coefficient is much lower in the case of water ($Pr = 7.0$) than in the case of air ($Pr = 0.7$) for both dusty fluid and clean fluid cases.
Figure 9. Variation of Nu with $\lambda$ for various values of $f$ with (a) $Pr=0.7$ and (b) $Pr=7.0$

Figure 10. Variation of Nu with $\tau$ for various values of $f$ with (a) $Pr=0.7$ and (b) $Pr=7.0$

Figure 11. Variation of Nu with $\lambda$ for various values of $f$ with (a) $Pr=0.7$ and (b) $Pr=7.0$
5. Conclusion
This problem of three-dimensional Couette flow and heat transfer of a dusty fluid between two infinite horizontal parallel porous flat plates in the presence of transverse magnetic field was solved analytically using the perturbation method. The lower stationary plate was subjected to transverse exponential injection while the upper plate was uniformly moving to corresponding constant suction. A selective set of graphical results was presented and discussed to show interesting features of the flow and heat transfer situation. It was found that.
- The effect of mass concentration parameter on the main flow fluid- and particle-phase velocities $u$ and $u_p$ are similar. But this effect is opposite to the skin friction components $T_x$, $T_z$ and the Nusselt number $Nu$.
- The effect of the injection parameter on the main flow fluid- and particle-phase velocities $u$ and $u_p$ and the Nusselt number $Nu$ is the same but this effect is opposite to that of the skin friction components $T_x$ and $T_z$.
- The effect of relaxation time parameter on the skin friction components $T_x$, $T_z$ and the Nusselt number $Nu$ is the same. - The effect of magnetic parameter $M$ on the mainflow fluid and particle phase are similar and decreasing in all.

Appendix

$$m_1=\frac{-\left(\lambda + (M^2 + \alpha)\tau\right) + \sqrt{\left(\lambda + (M^2 + \alpha)\tau\right)^2 - 4(1 - \lambda\tau)\alpha}}{2(1 - \lambda\tau)}$$

$$m_2=\frac{-\left(\lambda + (M^2 + \alpha)\tau\right) + \sqrt{\left(\lambda + (M^2 + \alpha)\tau\right)^2 - 4(1 - \lambda\tau)\alpha}}{2(1 - \lambda\tau)}$$

$$m_3=\frac{-(1 - \lambda\tau)}{\lambda} - m_1 - m_2$$

$$a_1=\frac{1}{\tau m_1 + 1}, \quad a_2=\frac{1}{\tau m_2 + 1}, \quad a_3=\frac{1}{\tau m_2 + 1}, \quad a_4=e^{-1/\tau}, \quad a_5=\frac{1}{m_1 + b_2}, \quad a_6=\frac{1}{m_2 + b_2}, \quad a_7=\frac{1}{m_3 + b_2}$$

$$r_1=\frac{(1 + f + M^2)\lambda + \tau \alpha^2 + \sqrt{(1 + f)\lambda + \tau \alpha^2)^2 + 4\alpha^2(1 - \lambda\tau) + M^2}}{2(1 - \lambda\tau)}$$

$$r_2=\frac{(1 + f + M^2)\lambda + \tau \alpha^2 - \sqrt{(1 + f)\lambda + \tau \alpha^2)^2 + 4\alpha^2(1 - \lambda\tau) + M^2}}{2(1 - \lambda\tau)}$$

$$r_3=\lambda - \frac{1}{\tau} - r_2 - r_1, \quad d_1=k(c_2 - \alpha\tau c_1) + \alpha(c_1 + \alpha\tau c_2), \quad d_2=\alpha(c_2 - \alpha\tau c_1) - k(c_1 + \alpha\tau c_2),$$

$$k=\frac{a_{12} - a_{11}\alpha^2}{a_{12} - \alpha^2}, \quad k_0=\frac{\lambda\alpha}{\tau\alpha^2(k_1)}$$

$$L_1=\frac{(r_1 + m_1)^3 + b_0(r_1 + m_2)^3 + b_2(r_1 + m_2) + b_3}{k_3}$$

$$L_3=\frac{(r_1 + m_1)^3 + b_0(r_1 + m_2)^3 + b_2(r_1 + m_2) + b_3}{k_3}$$

$$L_4=\frac{(r_1 + m_1)^3 + b_0(r_1 + m_2)^3 + b_2(r_1 + m_2) + b_3}{k_3}$$

$$L_5=\frac{(r_1 + m_1)^3 + b_0(r_1 + m_2)^3 + b_2(r_1 + m_2) + b_3}{k_3}$$

$$L_6=\frac{(r_1 + m_1)^3 + b_0(r_1 + m_2)^3 + b_2(r_1 + m_2) + b_3}{k_3}$$

$$t_0=3m_1 + b_0, \quad t_1=3m_1^2 + 2b_0m_1 + b_2, \quad t_2=m_1^3 + 2b_0m_1^2 + b_2m_1 + b_3, \quad N_0=\frac{t_2 - \alpha^2 t_0}{t_1 - \alpha^2}$$

$$L_7=\frac{k_7}{(t_1 - \alpha^2)(\alpha^2 + N_0^2)}, \quad L_8=\frac{k_8}{(t_4 - \alpha^2)(\alpha^2 + N_0^2)}, \quad L_9=\frac{-k_9}{(t_1 - \alpha^2)(\alpha^2 + N_0^2)}$$
\begin{align*}
L_{10} &= \frac{-k_{10}}{(t_4 - \alpha^2)(\alpha^2 + N_1^2)}, \quad L_{11} = \frac{k_{11}}{(r_1 - 1/\tau) + b_0(r_1 - 1/\tau)^2 + b_2(r_1 - \frac{1}{\tau}) + b_3} \\
L_{12} &= \frac{k_{12}}{(r_1 - 1/\tau) + b_0(r_2 - \frac{1}{\tau})^2 + b_2(r_2 - \frac{1}{\tau}) + b_3}, \quad N_2 = t_8 - \alpha^2t_6 \\
L_{14} &= \frac{-k_{14}}{(t_7 - \alpha^2)(\alpha^2 + N_2^2)}, \quad L_{15} = \frac{(t_7 - \alpha^2)(\alpha^2 + N_2^2)}{\alpha^2} \\
L_{16} &= \frac{(m_1 - 1/\tau) + b_0(m_1 - \frac{1}{\tau})^2 + b_2(m_1 \frac{1}{\tau}) + b_3}{k_{16}} \\
L_{17} &= \frac{(m_2 - 1/\tau) + b_0(m_2 - \frac{1}{\tau})^2 + b_2(m_2 \frac{1}{\tau}) + b_3}{k_{17}} \\
L_{18} &= \frac{k_{18}\tau^3}{(r_1 + m_1)\tau + 1}, \quad G_1 = \frac{g_1\tau}{g_4\tau}, \quad G_2 = \frac{g_2\tau}{g_5\tau} \\
G_3 &= \frac{(r_1 + m_2)\tau + 1}{g_7\tau}, \quad G_4 = \frac{g_4\tau}{g_9\tau}, \quad G_5 = \frac{g_5\tau}{g_8\tau} \\
G_6 &= \frac{(m_1 + 1)^2 + \alpha^2\tau^2}{r_3\tau}, \quad G_7 = \frac{(m_2 + \frac{1}{\tau})^2 + \alpha^2\tau^2}{g_9\tau}, \quad G_8 = \frac{(m_2 + \frac{1}{\tau})^2 + \alpha^2\tau^2}{g_1\tau} \\
G_9 &= \frac{(m_3 + \frac{1}{\tau})^2 + \alpha^2\tau^2}{g_1\tau}, \quad G_{10} = \frac{(m_2 + \frac{1}{\tau})^2 + \alpha^2\tau^2}{g_1\tau}, \quad G_{11} = \frac{(m_2 + \frac{1}{\tau})^2 + \alpha^2\tau^2}{g_1\tau} \\
G_{12} &= \frac{(m_3 + \frac{1}{\tau})^2 + \alpha^2\tau^2}{g_1\tau}, \quad G_{13} = \frac{g_1\tau}{g_4\tau}, \quad G_{14} = \frac{g_4\tau}{g_5\tau}, \quad G_{15} = \frac{g_5\tau}{g_6\tau} \\
H_1 &= \frac{r_1 + m_3 + b_1}{r_1 + m_4 + b_1}, \quad H_2 = \frac{-h_1}{h_2}, \quad H_3 = \frac{h_3}{h_4}, \quad H_4 = \frac{r_2 + m_4 + b_1}{r_2 + m_4 + b_1} \\
H_5 &= \frac{r_3 + m_3 + b_1}{r_3 + m_4 + b_1}, \quad H_6 = \frac{h_9}{h_{10}}, \quad H_7 = \frac{h_{10}}{h_7}, \quad H_8 = \frac{h_7}{h_8} \\
H_9 &= \frac{h_9}{h_8}, \quad H_{10} = \frac{(h_9 + h_{10})^2 + \alpha^2}{h_7}, \quad H_{11} = \frac{h_7}{r_1}, \quad H_{12} = \frac{h_7}{r_2}, \quad H_{13} = \frac{h_7}{r_3} \\
H_{14} &= \frac{h_9}{h_{10}}, \quad H_{15} = \frac{(h_9 + h_{10})^2 + \alpha^2}{r_1}, \quad H_{16} = \frac{h_{11}}{r_1}, \quad H_{17} = \frac{h_{11}}{r_2}, \quad H_{18} = \frac{h_{11}}{r_3} \\
H_{18} &= \frac{-h_{18}}{\alpha}
\end{align*}

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