Crossing the phantom divide with Ricci-like holographic dark energy

S. Lepe

Instituto de Física, Facultad de Ciencias, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile

F. Peña

Departamento de Ciencias Físicas, Facultad de Ingeniería, Ciencias y Administración, Universidad de La Frontera, Casilla 54-D, Temuco, Chile

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In this work we study the dark energy problem by adopting an holographic model proposed recently in the literature. In this model it is has been postulated an energy density \( \rho \sim R \), where \( R \) is the Ricci scalar curvature. Under this considerations, we have obtained a cosmological scenario which arises from considering two non-interacting fluids along a reasonable Ansatz for the cosmic coincidence parameter. We have adjusted the involved parameters in the model according to the observational data and showing that the equation of state for the dark energy exhibits a cross through the -1 barrier. Additionally, we have found a disagreement of these parameters in comparison with a scalar field theory approach.

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I. INTRODUCTION

The current observational data have suggest that the universe is experiencing an accelerated expansion [1–3]. In order to explain this experimental evidence, many theoretical models have been proposed in the literature. One of these models is known as dark energy problem, which is based on an unknown fluid with a negative pressure which drives the accelerated expansion. In this context, many approaches have been used to describe the dark energy such as modifications to the Einstein equations [4], scalar field models and quintessence [5], tachyonic fields [6], quintom [7] and phantom fields [8]. In a recently approach [9], it has been proposed that the current accelerated expansion could be explained through the vacuum density of a colored field, responsible for a phase transition at which the gauge SU(3)c symmetry is broken. Taking into account these considerations, it has been studied the second order electroweak transition, which could explain the accelerated evolution of the universe, under certain approximations [10]. Following this line of reasoning, holographic approaches to explain dark energy have also been interesting ideas to investigate [11]. This treatment is based on the holographic principle, which establish that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume [12]. Many works have been developed in the literature, using a holographic cut-off for the dark energy of the form \( \rho \sim H^2 \) [13] where \( H \) is the Hubble parameter. However, a recently new approach has emerged as an extension to this ideas, in which it is proposed an holographic cut-off in the density like \( \rho \sim R \), where \( R = 6 \left( 2H^2 + \dot{H} + k/a^2 \right) \) is the Ricci scalar curvature. As a consequence, the proposed cut-off is written as \( \rho = 3 \left( \lambda_1 H^2 + \lambda_2 \dot{H} \right) \), where \( \lambda_1 \) and \( \lambda_2 \) are both constants [14]. In the literature [15] and references therein), it has been considered several options for the choice of the infrared cut-off in the holographic dark energy, such as the Hubble parameter, the particle horizon, the future horizon as well of any combinations of them. All these models do not consider, at least explicitly, time derivatives terms in the Hubble parameter. By including term of the form \( \dot{H} \) it is possible to avoid the problem of causality that is present when the future horizon is used as a cut-off.

In our article, we will consider two non-interacting fluids: one of them represents dark energy in accord with the proposed cut-off and the other will play the role of usual dark matter without pressure. The present paper is organized as follows: in Sec. II, we discuss a Ricci-like holographic approach to the dark energy and where we have obtained expressions for the parameters in the model in terms of observable like the coincidence and the deceleration parameters. In Sec. III, we shall introduce an Ansatz for the cosmic coincidence parameter and where we show an explicit solutions for the Hubble parameter, the cosmic scale factor and the equation of state for the dark energy.

*Electronic address: slepe@ucv.cl

†Electronic address: fcampos@ufro.cl
In Sec. IV, we have obtained explicit values for parameters involved showing that the equation of state for the dark energy undergoes a cross through the phantom barrier. Finally, we summarize the results in Sec. V.

II. HOLOGRAPHIC RICCI DARK ENERGY

Let start our analysis by considering the non-interacting flat model (in units $8\pi G = 1$) with $\rho_1$ as the dark energy component and $\rho_2$ the dark matter component defined by:

$$3H^2 = \rho_1 + \rho_2;$$

$$\dot{\rho}_1 = 3 \left( \frac{1 + \omega_1(z)}{1 + z} \right) \rho_1,$$

$$\dot{\rho}_2 = 3 \left( \frac{1 + \omega_2(z)}{1 + z} \right) \rho_2,$$

where primes denote derivative with respect to the redshift parameter defined by $1 + z = (a/a_0)^{-1}$ where $a$ is the cosmic scale factor. Besides, $\omega_1$ and $\omega_2$ are parameters of the equation of state associated to $\rho_1$ and $\rho_2$, respectively.

We will adopt for the dark energy component $\rho_1$ the holographic Ricci approach given by:

$$\rho_1 = 3 \left( \lambda_1 - \lambda_2 (1 + z) \frac{\dot{H}}{H} \right) H^2,$$

where $\lambda_1$ and $\lambda_2$ are adjustable constants. The additional term $\dot{H}$ included in equation (4) gives a non-constant coincidence parameter, which represents a difference with the typical $\rho_1 \sim H^2$ holographic cut-off proposed in the literature [13] (besides, as we stated in the introduction, this derivative term of $H$ avoids the causality problem). We will use this fact for doing a fit of $\lambda_1$ and $\lambda_2$ in accord to the current observational data.

In terms of the deceleration parameter defined by:

$$q + 1 = (1 + z) \frac{\dot{H}}{H},$$

Equation (5) can be written in the form:

$$\rho_1 = 3 \left[ \lambda_1 - \lambda_2 (q + 1) \right] H^2.$$

By combining of equations (6) and (11), the energy density $\rho_2$ becomes:

$$\rho_2 = 3 \left[ (1 + z) + \lambda_2 (q + 1) \right] H^2,$$

From equations (6) and (7) it is possible to establish the following constraint for the deceleration parameter $q(z)$:

$$\frac{\lambda_1}{\lambda_2} - \frac{1}{\lambda_2} < q(z) + 1 < \frac{\lambda_1}{\lambda_2},$$

As a consequence that both $\rho_1$ and $\rho_2$ satisfy the weak energy condition. Besides, the constraint indicates that $q(z)$ have a strong dependence on the allowed values of $\lambda_1$ and $\lambda_2$, considered either by theoretical models or by observational data.

- The coincidence parameter.
  
  The coincidence parameter is defined by the ratio:

$$r = \frac{\rho_2}{\rho_1} = \frac{(1 - \lambda_1)/\lambda_2 + (q + 1)}{\lambda_1/\lambda_2 - (q + 1)}.$$
By using the previous definition, it is possible to express the deceleration parameter $q(z)$ in terms of coincidence parameter $r$ by:

$$q(z) + 1 = \frac{1}{\lambda_2} \left[ \lambda_1 - (1 + r(z))^{-1} \right],$$  \hspace{1cm} (10)

The current observational data have shown that $q(0) < 0$. This condition imposes a constrain over the possible values of the adjustable constants of the model, given by:

$$\lambda_1 - \lambda_2 < [1 + r(0)]^{-1},$$  \hspace{1cm} (11)

By combining equations (10) and (11), this constrain takes the following form:

$$(\lambda_1 - \lambda_2) - [1 + r(0)]^{-1} = \lambda_2 q(0) < 0 \implies \lambda_2 > 0,$$  \hspace{1cm} (12)

and from this relationship, together with equation (11), we have also obtained $\lambda_1 > 0$. Then, by combining equations (10) and (12) it is possible to obtain the following expression for the adjustable parameters of the present model:

$$\lambda_1 = \frac{1}{1 + r(\infty)} + \frac{1 + q(\infty)}{q(\infty) - q(0)} \left[ \frac{1}{1 + r(0)} - \frac{1}{1 + r(\infty)} \right],$$  \hspace{1cm} (13)

and

$$\lambda_2 = \frac{1}{q(\infty) - q(0)} \left[ \frac{1}{1 + r(0)} - \frac{1}{1 + r(\infty)} \right],$$  \hspace{1cm} (14)

With the previous expressions and by using the current observational data, it is possible to fix the values $\lambda_1$ and $\lambda_2$ for our model. If we consider, at early times, the cosmic evolution driven by dark matter like dust, the value $q(\infty)$ is equal to $1/2$.

On other hand, the acceleration of the expansion is given by:

$$\ddot{a} / a = -q(z) H^2(z),$$  \hspace{1cm} (15)

By using the expression for the deceleration parameter given in equation (10), it can be written in the form:

$$\ddot{a} / a = \frac{1}{\lambda_2} \left[ \frac{1}{1 + r(z)} - (\lambda_1 - \lambda_2) \right] H^2(z),$$  \hspace{1cm} (16)

and expression (16) reveals that only a positive acceleration is consistent with the constrain given in equation (12), i. e., there is no transition from a decelerated regime to an accelerated one.

The equation of state.

In what follow, we will analyze the equation of states in our model. Let consider the definitions for the dark energy component $\rho_1$ and the dark matter $\rho_2$ given in equations (2) and (3) together with the definition of the coincidence parameter $r$ given by equation (9). By combining these equations, we obtain the following expression:

$$\omega_1 - \omega_2 = -\frac{1}{3} \left( 1 + z \right) \frac{\dot{r}}{r},$$  \hspace{1cm} (17)

By using equation (11), (9) and (5), it is straightforward to show that:

$$1 + \omega_1(z) = \frac{2}{3} \left( q(z) + 1 \right) - \frac{1}{3} \left( 1 + z \right) \frac{\dot{r}(z)}{1 + r(z)},$$  \hspace{1cm} (18)

$$1 + \omega_2(z) = \frac{2}{3} \left( q(z) + 1 \right) + \frac{1}{3} \left( 1 + z \right) \frac{\dot{r}(z)}{r(z) [1 + r(z)]}.$$  \hspace{1cm} (19)
Considering \( \omega_2 (z) = 0 \) for the dark matter, we obtain:

\[
\dot{r} (0) = [1 - 2q (0)] r (0) [1 + r (0)], \tag{20}
\]

so that \( \dot{r} (0) \) it is determined from the observational data for \( q (0) \) and \( r (0) \).

From equations (18) and (20) it is possible to obtain an expression for \( \omega_1 (0) \) in the form:

\[
\omega_1 (0) = \frac{1}{3} \frac{\dot{r} (0)}{r (0)} = \frac{1}{3} [1 - 2q (0)] [1 + r (0)]. \tag{21}
\]

The set of equation of states given by (18) and (19), together with the definition of the deceleration parameter given in equation (5), show explicitly the inhomogeneous character of our model for the dark energy problem [16].

In the next section, after choosing a reasonable Ansatz for the coincidence parameter, we will give an explicit solutions for \( H (z) \) and the cosmic scale factor and we will discuss the cosmology in this model.

### III. AN ANSÄTZ FOR THE COINCIDENCE PARAMETER

Let consider the following Ansatz for the coincidence parameter

\[
r (z) = r_0 + \epsilon_0 z (1 + z)^{-1}, \tag{22}
\]

where \( r (0) = r_0 \), \( \dot{r} (0) = \epsilon_0 \) and \( r (\infty) = r_0 + \epsilon_0 \)

This type of parametrization has been considered in previous works concerning to an interacting scheme between dust and a holographic dark energy density described by \( \rho_1 \sim H^2 \) [17].

Starting from the definitions given in equations (11) and (14) and by using the proposed Ansatz, it is possible to obtain the following solution for the Hubble parameter

\[
H (z) = H (0) (1 + z)^{\frac{1}{\lambda_2}} \left[ \left( \frac{1 + r_0}{\epsilon_0} \right) \frac{1 + z_s}{z - z_s} \right]^{1/\lambda_2(1+r_0+\epsilon_0)},
\]

where it is straightforward to verify the following limit:

\[
H (z \to \infty) \to (1 + z)^{\frac{1}{\lambda_2} \left[ \frac{\lambda_1 - 1/(1+r_0+\epsilon_0)}{\lambda_2} \right]}, \tag{24}
\]

so that,

\[
H (z \to \infty) \to \infty \iff \lambda_1 > (1 + r_0 + \epsilon_0)^{-1}, \tag{25}
\]

\[
\Rightarrow (1 + r_0 + \epsilon_0)^{-1} < \lambda_1 < \lambda_2 + (1 + r_0)^{-1}, \tag{26}
\]

where the inequality given in equation (11) has been used to obtain the expression (26).

From equations (22) and (23), it is possible to obtain the following expression for the Hubble parameter:

\[
\frac{\ddot{H}}{H} (z) = \frac{1}{\lambda_2} \left[ \frac{\lambda_1}{1 + z} - \frac{1}{\epsilon_0} \left( \frac{1 + z_s}{z - z_s} \right) \right], \tag{27}
\]

By using the equations (23) and (27), the acceleration can be written in the form:

\[
\frac{\ddot{a}}{a} (z) = H^2 (z) \left[ 1 - \frac{1}{\lambda_2} \left( \lambda_1 - \frac{1}{\epsilon_0} \frac{(1 + z_s)(1 + z)}{z - z_s} \right) \right], \tag{28}
\]
Finally, by using equations (18), (23) and (27) it is possible to write the equation of state corresponding to \( \omega_1 \) in the form:

\[
1 + \omega_1 (z) = \frac{2}{3\lambda_2} \left[ \lambda_1 - 1 \frac{(1 + z_s)(1 + z)}{\epsilon_0 (1 + z_s)} \right] - \frac{1}{3} \frac{\epsilon_0 (1 + z_s)}{(1 + r_0) (1 + z_s) + \epsilon_0 z},
\]

and

\[
1 + \omega_1 (z \to \infty) \to \frac{2}{3\lambda_2} \left[ \lambda_1 - (1 + r_0 + \epsilon_0)^{-1} \right] - \frac{1}{3} \frac{\epsilon_0}{1 + r_0 + \epsilon_0} \frac{1}{z}.
\]

If we analyze the expression obtained for the Hubble parameter, it is possible to notice that exist a singularity at the value \( z = z_s = \epsilon_0 (1 + r_0 + \epsilon_0)^{-1} - 1 \). This is a type III singularity: at a finite value of the scale factor both energy density and pressure diverge (18). However, this singularity does not occur in the future evolution given that, before this point is achieved, the coincidence parameter given in equation (22) has vanished ( \( r (\tau) = 0 \) for \( \tau = \epsilon_0 (r_0 + \epsilon_0)^{-1} - 1 > z_s \)). Therefore, the evolution cross the phantom divide but no experience a future singularity.

A final calculations, just for completeness, gives the solutions for the scale factor when \( z \to \infty \) and \( z \to z_s \). The expressions are, respectively

\[
a(t) = a(0) \left[ \alpha \beta (\lambda_1 / \lambda_2 - \beta) H(0)(t - t_0) + 1 \right]^{1/(\lambda_1 / \lambda_2 - \beta)} \implies \dot{a}(t) > 0
\]

where we have used equation (11) and the definitions given by:

\[
\alpha = \left( \frac{1 + r_0}{\epsilon_0} \right) (1 + z_s), \quad \beta = \frac{1}{\lambda_2 \epsilon_0} (1 + z_s).
\]

and

\[
a(t) = a(t_s) - a(0) \left[ (\beta + 1) C \right]^{1/(\beta + 1)} (t_0 + t_s - t)^{1/(\beta + 1)}
\]

where \( C \) is a constant and \( t_s \sim [a(t_s)/a(0) - 1]^{\beta + 1} \).

**IV. THE OBSERVATIONAL DATA**

In order to fit the adjustable parameter of our model, \( \lambda_1 \) and \( \lambda_2 \), we have used the following observational data: \( r_0 \approx 0.37 \) from Refs. [1, 2] and \( q_0 \approx -0.7 \) from Ref. [3]. By replacing these values in equation (20) we get the value \( \epsilon_0 \approx 1.22 \). Under these consideration, the adjustable parameters take the values \( \lambda_1 \approx 0.8 \) and \( \lambda_2 \approx 0.3 \). Additionally, we have obtained the following limits for the equation of state \( \omega_1 (z \to \infty) \approx -0.08 \to \omega_1 (0) \approx -1.096 \). In this sense, we can pointed out as a prediction of our model, that the equation of state of the dark energy cross the phantom divide.

This type of behavior was obtained before, by using other approaches such as quintom cosmology [4], nevertheless, as a difference and an advantage in comparison with the quintom approach, our model does not required a phantom field as an input.

Finally, it is possible to compare our results with those ones reported in Ref. [19]. In this work, authors described the same problem, but considering a scalar field theory based on quantum vacuum fluctuations as a physical constituent of dark energy, in order to visualize the constants \( \lambda_1 \) and \( \lambda_2 \) under a dynamical scope. They claim that the origin of \( \rho = 3 \left( \lambda_1 H^2 + \lambda_2 \dot{H} \right) \) in the framework of the holographic principle is in a sense kinematical, i.e., it lacks any dynamical support. The constants \( \lambda_1 \) and \( \lambda_2 \) are, in principle, arbitraries and in this sense, without dynamics. Besides, they have obtained the following expression for the adjustable parameter \( \lambda_1 = N/8\pi - 2 \) and \( \lambda_2 = N/16\pi - 2 \) where \( N \) is the difference between the bosonic and fermionic fundamental modes present in the theory. They have calculated that the value \( N \approx 100 \) is a quite realistic result if it is taken the limit \( \omega_1 \sim -1 \). However, according to our fitted values for \( \lambda_1 \) and \( \lambda_2 \), it is possible to obtain the value \( N \approx 25 \). Thus, our fitted set of parameter \( \{\lambda_1, \lambda_2\} \) is in clear disagreement with those ones which comes from the scalar field theory aforementioned.


V. FINAL REMARKS

In this work we have discussed a holographic model for the dark energy component inspired in \( \rho_1 \sim R \) where \( R = 6 \left( 2H^2 + \dot{H} + k/a^2 \right) \) is the Ricci scalar curvature. We have obtained from it an early quintessence behavior for the dark energy and a late phantom evolution, that is, a quintom-like scheme. A reasonable Ansatz for the coincidence parameter was used and in terms of it we found explicit values for the parameters which characterizes the model. Although the predictions of the \( \Lambda \)CDM model have a remarkable consistency with the current observational data, the dynamical model that we have used, it is mildly favored by the observations because it is possible to explain the barrier -1 crossing.

Finally, we did not found a good agreement with a scalar field theory approach about the values of these parameters.

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[1] E. Komatsu et al, arXiv: 1001.4538 [astro-ph.CO].
[2] D. Larson et al, arXiv: 1001.4635 [astro-ph.CO].
[3] A. G. Reiss et al, Astrophys. J. 607, 665 (2004).
[4] Damien A. Easson, Int. J. Mod. Phys. A 19, 5343 (2004).
[5] B. Ratra and J. Peebles, Phys. Rev. D 37, 321 (1988); De-Chang Dai, S. Dutta and D. Stojkovic, Phys. Rev. D 80, 063522 (2009).
[6] A. Sen, JHEP 07, 065 (2002); L. R. W. Abramo and F. Finelli, Phys. Lett. B 575, 165 (2003).
[7] E. Elizalde et al, Phys. Rev. D 70, 043539 (2004); Yi-Fu Cai, E. N. Saridakis, M. R. Setare and Jun-Qing Xia, arXiv: 0909.2726; Bo Feng, Xiu-Lian Wang and Xin-Min Zhang, Phys. Lett. B 607, 35 - 41 (2005).
[8] R. R. Cadwell, Phys. Lett. B 545, 23 (2002); S. Nojiri and S. Odintsov, Phys. Lett. B 562, 147 (2003).
[9] E. Greenwood, E. Halstead, R. Poltis and D. Stojkovic, Phys. Rev. D 79, 103003 (2009).
[10] D. Stojkovic, G. Starkman and R. Matsuo, Phys. Rev. D 77, 063006 (2008).
[11] Bo Hu and Yi Ling, Phys. Rev. D 73, 123510 (2006).
[12] Yun Soo Myung, Phys. Lett. B 610, 18 (2005).
[13] Miao Li, Phys. Lett. B 603, 1 (2004); Miao Li and Yi Wang, Phys. Lett. B 687, 243 (2010).
[14] L. N. Granda and A. Oliveros, arXiv: 0810.3149 [gr-qc].
[15] S. Nojiri and S.D. Odintsov, Gen. Rel. Grav. 38, 1285 (2006).
[16] S. Nojiri and S.D. Odintsov, Phys. Rev D 72, 023003 (2005).
[17] Anjan A. Sen and D. Pavón, Phys. Lett. B 664, 7 (2008).
[18] S. Nojiri, S.D. Odintsov and S. Tsujikawa, Phys. Rev D 71, 063004 (2005).
[19] B. Broda and M. Szanecki, arXiv: 0910.5145 [gr-qc].