Scaling properties of background- and chiral-magnetically-driven charge separation in heavy ion collisions at $\sqrt{s_{NN}} = 200$ GeV

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The Anomalous Viscous Fluid Dynamics model, AVFD, is used in concert with the charge-sensitive correlator $R_{\Psi_2}(\Delta S)$ to investigate the scaling properties of background- and chiral-magnetically-driven (CME) charge separation ($\Delta S$), characterized by the inverse variance $\sigma_{R_{\Psi_2}}^{-2}$ of the $R_{\Psi_2}(\Delta S)$ distributions obtained in collisions at $\sqrt{s_{NN}} = 200$ GeV. The $\sigma_{R_{\Psi_2}}^{-2}$ values for the background are observed to be event-shape-independent. However, they scale with the reciprocal charged-particle multiplicity ($1/\langle N_{ch}\rangle$), indicating an essential constraint for discerning background from the signal and a robust estimate of the difference between the backgrounds in Ru+Ru and Zr+Zr collisions. By contrast, the $\sigma_{R_{\Psi_2}}^{-2}$ values for signal + background show characteristic $1/\langle N_{ch}\rangle$ scaling violations that characterize the CME-driven contributions. Corrections to recent $R_{\Psi_2}(\Delta S)$ measurements \cite{2} that account for the background difference in Ru+Ru and Zr+Zr collisions indicate a charge separation difference compatible with the CME. The results further suggest that $\sigma_{R_{\Psi_2}}^{-2}$ measurements for peripheral and central collisions in concert with $1/\langle N_{ch}\rangle$ scaling, provides a robust constraint to quantify the background and aid characterization of the CME.

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Ion-ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) lead to the production of a magnetized chiral relativistic quark-gluon plasma (QGP) \cite{2,6}, akin to the primordial plasma produced in the early Universe \cite{7} and several degenerate forms of matter found in compact stars \cite{3}. Pseudo-relativistic analogs include Dirac and Weyl semimetals \cite{10,11}, but not on the evolution of magnetic fields in the early Universe \cite{17,18}.

A major anomalous process predicted to occur in the magnetized QGP is the chiral magnetic effect (CME) \cite{19}. It is characterized by the vector current:

$$\vec{J}_V = \frac{N_c e \vec{B}}{2\pi^2} \mu_A, \text{for } \mu_A \neq 0,$$

where $N_c$ is the color factor, $\vec{B}$ is the magnetic field and $\mu_A$ is the axial chemical potential that quantifies the axial charge asymmetry or imbalance between right- and left-handed quarks in the plasma \cite{19,22}. Experimentally, the CME manifests as the separation of electrical charges along the $\vec{B}$-field \cite{2,11}. This stems from the fact that the CME preferentially drives charged particles, originating from the same “P-odd domain”, along or opposite to the $\vec{B}$-field depending on their charge.

The charge separation can be quantified via measurements of the first P-odd sine term $a_1$, in the Fourier decomposition of the charged-particle azimuthal distribution \cite{23}:

$$\frac{dN_{ch}}{d\phi} \propto 1 + 2 \sum_n (v_n \cos(n\Delta \phi) + a_n \sin(n\Delta \phi) + ...),$$

where $\Delta \phi = \phi - \Psi_{RP}$ gives the particle azimuthal angle with respect to the reaction plane (RP) angle, and $v_n$ and $a_n$ denote the coefficients of the P-even and P-odd Fourier terms, respectively. A direct measurement of the P-odd coefficients $a_1$, is not possible due to the strict global P and CP symmetry of QCD. However, their fluctuation and/or variance $\overline{a_1}^2$ can be measured with charge-sensitive correlators such as the $\gamma$-correlator \cite{23} and the $R_{\Psi_2}(\Delta S)$ correlator \cite{24,25}.

The $\gamma$-correlator measures charge separation as:

$$\gamma_{\alpha\beta} = \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_2) \rangle, \quad \Delta \gamma = \gamma_{OS} - \gamma_{SS},$$

where $\Psi_2$ is the azimuthal angle of the 2nd-order event plane which fluctuates about the RP, $\phi$ denote the particle azimuthal emission angles, $\alpha, \beta$ denote the electric charge (+) or (−) and SS and OS represent same-sign (+, −) and opposite-sign (+, −) charges.

The $R_{\Psi_2}(\Delta S)$ correlator \cite{24,25} measures charge separation relative to $\Psi_2$ via the ratio:

$$R_{\Psi_2}(\Delta S) = C_{\Psi_2}(\Delta S)/C_{\Psi_2}^0(\Delta S),$$

where $C_{\Psi_2}(\Delta S)$ and $C_{\Psi_2}^0(\Delta S)$ are correlation functions that quantify charge separation $\Delta S$, approximately parallel and perpendicular (respectively) to the $\vec{B}$-field. The charge shuffling procedure employed in constructing these correlation functions ensures identical properties for their numerator and denominator, except for the
charge-dependent correlations, which are of interest \[24, 25\]: \( C_{q_{2}}(\Delta S) \) measures both CME- and background-driven charge separation while \( C_{q_{2}}(\Delta S) \) measures only background-driven charge separation. The inverse variance \( \sigma_{R_{q_{2}}}^{2} \) of the \( R_{q_{2}}(\Delta S) \) distributions serves to quantify the charge separation \[24, 25, 26\].

A vexing ongoing debate is whether the charge-sensitive \( R_{q_{2}}(\Delta S) \) correlator shows the requisite response and sensitivity necessary to (i) discern and characterize CME- and background-driven charge separation and (ii) pin down the influence of the background difference in collisions of Ru+Ru and Zr+Zr isotopes. The latter is crucial for resolving the ambiguity reported for recent STAR measurements \[1\] which sought to determine a possible CME-driven charge separation difference for these isotopes. Here, we employ the AVFD model \[24, 30\] to chart the \( R_{q_{2}}(\Delta S) \) correlators’ response to varying degrees of signal and background, primarily in Au+Au collisions, to evaluate its efficacy for detecting and characterizing CME-driven charge separation in the presence of realistic backgrounds. We find characteristic scaling patterns for the background and scaling violations for signal + background that (i) discern between CME- and background-driven charge separation and (ii) allow a robust estimate of the background difference for the Ru+Ru and Zr+Zr isotopes. Corrections to recent STAR \( R_{q_{2}}(\Delta S) \) measurements \[1\] which accounts for this background difference, give results that suggest a CME-driven charge separation that is larger in Ru+Ru than in Zr+Zr collisions.

The AVFD model, which includes realistic estimates for charge-dependent backgrounds such as resonance decays and local charge conservation (LCC) is known to give good representations of the experimentally measured particle yields, spectra, \( v_{2} \), etc \[31\]. Thus, it provides an essential benchmark for evaluating the interplay between possible CME- and background-driven charge separation in actual data. The model simulates charge separation resulting from the combined effects of the CME and the background. An in-depth account of its implementation can be found in Refs. \[24\] and \[31\]. In brief, the second-generation Event-by-Event version of the model, called E-by-E AVFD, uses Monte Carlo Glauber initial conditions to simulate the evolution of fermion currents in the QGP, in concert with the bulk fluid evolution implemented in the VISHNU hydrodynamic code \[32\], followed by a URQMD hadron cascade stage. Background-driven charge-dependent correlations result from LCC on the freeze-out hypersurface and resonance decays. A time-dependent magnetic field \( B(\tau) = \frac{B_{0}}{1+(\tau/\tau_{B})^{2}} \), acting in concert with a nonzero initial axial charge density \( n_{5}/s \), is used to generate a CME current (embedded in the fluid dynamical equations), leading to a charge separation along the magnetic field. The peak values \( B_{0} \), obtained from event-by-event simulations \[33\], are used with a relatively conservative lifetime \( \tau_{B} = 0.6 \text{ fm/c} \). The initial axial charge density, which results from gluonic topological charge fluctuations, is estimated based on the strong chromo-electromagnetic fields in the early-stage plasma. The present work uses the input scaling parameters \( n_{5}/s \) and LCC to regulate the magnitude of the CME- and background-driven charge separation.

Simulated AVFD events were generated for varying degrees of signal and background for a broad set of centrality selections in Au+Au and isobar collisions for analysis with the \( R_{q_{2}}(\Delta S) \) correlator. Here, it is noteworthy that the Monte Carlo Glauber parameters employed in the AVFD calculations for the isobars are similar to those used in the centrality calibrations reported in Ref. \[3\]: cross-checks ensured good agreement between the experimental and simulated \( N_{ch}\)-distributions for both isobars.

The event selection and cuts mimic those used in the analysis of experimental data \[1\]. Charged particles with transverse momentum \( 0.2 < p_{T} < 2.0 \text{ GeV/c} \) are used to construct \( \Psi_{2} \). Each event is subdivided into two sub-events with pseudorapidity \( 0.1 < \eta < 1.0 \) (E) and \( -1.0 < \eta < -0.1 \) (W) to obtain \( \Psi_{2}^{E} \) and \( \Psi_{2}^{W} \) and their associated centrality-dependent event-plane resolution factors. The \( R_{q_{2}}(\Delta S) \) distributions are determined for charged particles with \( 0.35 < p_{T} < 2.0 \text{ GeV/c} \), taking care to use \( \Psi_{2}^{W} \) for particles within the range \( 0.1 < \eta < 1.0 \) and \( \Psi_{2}^{E} \) for particles within the range \(-1.0 < \eta < -0.1 \) to avoid possible self-correlations, as well as to reduce the influence of the charge-dependent non-flow correlations. The resulting distributions are corrected \( [R_{q_{2}}(\Delta S''\prime)] \) to account for the effects of particle-number fluctuations and the event-plane resolution \[24\]. The sensitivity of \( R_{q_{2}}(\Delta S) \) to variations in the elliptic flow \( (v_{2}) \) magnitude at a selected centrality, is also studied using event-shape selection via fractional cuts on the distribution of the magnitude of the \( q_{2} \) flow vector \[24\]: for a given centrality, the magnitude of \( v_{2} \) is increased(decreased) by selecting events with larger(smaller) \( q_{2} \) magnitudes. This analysis aspect is performed with three sub-events \((A[\eta < -0.3], B[\eta < 0.3], \text{and } C[\eta > 0.3])\) using the procedures outlined earlier and \( q_{2} \) selection in sub-event B.

Figure 1 shows a representative comparison of the distributions obtained for signal \( (S_{q_{2}}) \) + background \( (B_{q_{2}}) \) \( [n_{5}/s = 0.1 \text{ and LCC=33\%}] \) and background without signal \( \text{[LCC=33\% and } n_{5}/s = 0.0] \) in 30-40\% (a) and 60-70\% (b) central Au+Au collisions. They show the expected concave-shaped distributions for background and signal + background respectively. For the 60-70\% centrality cut, similar distributions are indicated for background and signal + background, suggesting a loss of sensitivity to the signal in these peripheral collisions. Such a loss will result if the \( B \)-field is approximately randomly oriented to \( \Psi_{2} \) in these collisions. For the 30-40\% centrality cut, Fig. 1(a) shows a narrower distribution for signal + background than for background. This narrowing indicates that the CME signal increases the magnitude of
FIG. 1. Comparison of the $R_{\Psi_2}(\Delta S)$ distributions for signal + background (solid circles) and background without signal (solid squares) for 30-40% (a) and 60-70% (b) Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

FIG. 2. $\sigma_{R_{\Psi_2}}^2$ vs. $1/\langle N_{ch} \rangle$ [(a) and (b)] and $f_{\text{CME}}$ vs. $1/\langle N_{ch} \rangle$ [(c) and (d)] for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, for two different parameter sets for signal and background as indicated. The dotted lines are drawn to guide the eye. The $f_{\text{CME}}$ values in (c) and (d) characterize the fraction of the charge separation which is CME-driven following Eq. (4) evaluated with the inverse variance ($\sigma_{R_{\Psi_2}}^{-2}$) of the respective distributions. For the 30-40% central collisions shown in Fig. 1 (a), $f_{\text{CME}} \approx 60\%$. This value is a good benchmark of the sensitivity of the $R_{\Psi_2}(\Delta S)$ correlator to CME-driven charge separation of this level of signal ($n_{S}/s = 0.1$) in the presence of charge-dependent background (LCC = 33%) in Au+Au collisions.

The centrality dependence of $\sigma_{R_{\Psi_2}}^2$ is summarized for Au+Au collisions in Fig. 2 for two different parameter sets for signal and background as indicated. To highlight the scaling property of the background, $\sigma_{R_{\Psi_2}}^{-2}$ is plotted vs. $1/\langle N_{ch} \rangle$, where $\langle N_{ch} \rangle$ is the mean number of charged particles employed to evaluate $R_{\Psi_2}(\Delta S)$ at the centrality of interest. Figs. 2 (a) and (b) show that the background scales as $1/\langle N_{ch} \rangle$, indicating that the observation of this scaling for the experimental $\sigma_{R_{\Psi_2}}^{-2}$ measurements would be a strong indication for background-driven charge separation with very little if any, room for a CME contribution. Figs. 2 (a) and (b) also indicate comparable background and signal + background $\sigma_{R_{\Psi_2}}^{-2}$ values for large and small $\langle N_{ch} \rangle$. This similarity suggests that background-driven charge separation dominates over the CME-driven contributions in the most central and peripheral collisions. Thus, the $\sigma_{R_{\Psi_2}}^{-2}$ measurements for peripheral and central collisions can be leveraged with $1/\langle N_{ch} \rangle$ scaling to give a quantitative estimate of the background over the entire centrality span.

The $\sigma_{R_{\Psi_2}}^{-2}$ values, shown for signal + background in Figs. 2 (a), and (b), indicate characteristic positive deviations from the $1/\langle N_{ch} \rangle$ scaling observed for the background. This apparent scaling violation gives a direct signature of the CME-driven contributions to the charge separation. They are quantified with the $f_{\text{CME}}$ fractions (cf. Eq. (4) shown in Figs. 2 (c) and (d). The indicated $f_{\text{CME}}$ values peak in mid-central collisions but reduce to zero at large and small $\langle N_{ch} \rangle$, i.e., central and peripheral collisions. They further indicate that, for these collisions, the $R_{\Psi_2}(\Delta S)$ correlator is sensitive to CME-driven charge separation even for a small signal ($n_{S}/s = 0.05$) in the presence of significant charge-dependent background (LCC = 40%).
FIG. 4. $q_2$ dependence of the $R_{q_2}(\Delta S''')$ distributions for background without signal [(a), (b) and (c)] and signal + background [(d), (e) and (f)]. The respective panels show the $q_2$-selected $R_{q_2}(\Delta S''')$ distributions [(a) and (d)], the corresponding $v_2$ values [(b) and (e)], and the $\sigma_{R_{q_2}}$ values [(c) and (f)] extracted from the distributions in (a) and (d).

The background $\sigma_{R_{q_2}}^{-2}$ values for Au+Au and Ru+Ru collisions are compared in Fig. 5. The results for Ru+Ru collisions show the same 1/⟨$N_{ch}$⟩ scaling observed for Au+Au. However, they indicate that, for the same centrality, the $\sigma_{R_{q_2}}^{-2}$ values for Ru+Ru collisions are larger than those for Au+Au, suggesting a lowering of the sensitivity to the signal in collisions for the isobars.

The $\sigma_{R_{q_2}}^{-2}$ values extracted for background and signal + background at a given centrality, were checked to establish their sensitivity to variations in the magnitude of the anisotropic flow coefficient $v_2$. For this, as discussed earlier, event-shape selection via fractional cuts on the distribution of the magnitude of the $q_2$ flow vector was used. Representative results for the sensitivity of $\sigma_{R_{q_2}}^{-2}$ to a change in the magnitude of $v_2$ [at a given centrality] are shown in Fig. 5 for background without signal [(a), (b) and (c)] and signal + background [(d), (e) and (f)] for Au+Au collisions. The respective panels show the $q_2$-selected $R_{q_2}(\Delta S''')$ distributions [(a) and (d)], the corresponding $v_2$ values [(b) and (e)], and the $\sigma_{R_{q_2}}$ values [(c) and (f)] extracted from the distributions shown in (a) and (d). They indicate that, while $v_2$ shows a sizable increase with $q_2$ (cf. panels (b) and (e)), the corresponding $\sigma_{R_{q_2}}^{-2}$ values (cf. panels (c) and (f)) are insensitive to $q_2$ regardless of background or signal + background.

Similar patterns of insensitivity have been observed for the $q_2$-selected $\sigma_{R_{q_2}}^{-1}$ measurements reported for Ru+Ru and Zr+Zr collisions [1]. Notably, the reported insensitivity spans a $v_2$ range (from low to high $q_2$) much larger than the measured difference between the $v_2$ flow coefficients for the two isobars at a given centrality [1], indicating that the $v_2$ difference between the isobars does not lead to an added difference in their $\sigma_{R_{q_2}}^{-2}$ values. Contributing factors to this insensitivity could stem from (i) an effective $\Delta\eta$ gap between the event-plane and the interest particles that suppresses the charge-dependent non-flow correlations and (ii) the charge shuffling employed in the denominator of the correlation functions that comprise the $R_{q_2}(\Delta S''')$ correlator [24, 25]. The latter eliminates the charge-independent flow correlations and reduces the charge-dependent non-flow correlations.

The ratio of the inverse variance for the two isobars ($\sigma_{R_{q_2}}^{-2}$/Zr+Zr) can also benchmark CME-driven charge separation, which is more prominent in collisions of Ru+Ru than Zr+Zr [1]. However, such a ratio must be corrected to account for the background difference between the two isobars. Since $\sigma_{R_{q_2}}^{-2}$ is $q_2$-independent and the background scales as 1/⟨$N_{ch}$⟩, a robust estimate for the correction factor at a given centrality is the ratio of the respective ⟨$N_{ch}$⟩ values for the two isobars. The protocol for the STAR blind-analysis precluded the application of this correction to the $R_{q_2}(\Delta S''')$ measurements reported in Ref. [1], leading to an ambiguity in the interpretation of measurements that sought to determine a possible CME-driven charge separation difference between the two isobars. Fig. 5 shows the corrected ratios obtained using the $\sigma_{R_{q_2}}^{-1}$/Zr+Zr data reported for several centrality selections in Ref. [1]. The ⟨$N_{ch}$⟩-scaled ratios greater than 1.0 are consistent with more significant CME-driven charge separation in Ru+Ru collisions than Zr+Zr collisions.

In summary, AVFD model simulations that incorporate varying degrees of CME- and background-driven
charge separation are used to study the scaling properties of charge separation in heavy ion collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). The inverse variance \( \sigma_{R_{2}}^{-2} \) of the \( R_{2} (\Delta S) \) distribution, that characterize the charge separation, indicate a linear dependence on \( 1/\langle N_{ch} \rangle \) which is an essential constraint for discerning background from the signal and a precise estimate of the difference between the backgrounds in Ru+Ru and Zr+Zr collisions. By contrast, the \( \sigma_{R_{2}+}^{-2} \) values for signal + background show characteristic deviations from the \( 1/\langle N_{ch} \rangle \) scaling, which serve to characterize the CME-driven contributions to the charge separation. Corrections to recent \( R_{2} (\Delta S) \) measurements \(^1\) that account for the background difference in Ru+Ru and Zr+Zr collisions, indicate a charge separation difference between the isobars compatible with the CME. The study further suggests that \( \sigma_{R_{2}+}^{-2} \) measurements for peripheral and central collisions can be leveraged with \( 1/\langle N_{ch} \rangle \) scaling to quantify the background and aid characterization of the CME in a wealth of available systems.

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