Density fluctuations and chiral phase transition

K Redlich¹,², B Friman², C Sasaki³

¹ Institute of Physics Theoretical Physics, University of Wrocław, 50204 Wrocław, PL
E-mail: redlich@ift.uni.wroc.pl

² Gesellschaft für Schwerionenforschung, GSI, D-64291 Darmstadt, D
E-mail: b.friman@gsi.de

³ Technische Universität München, D-85748 Garching, D
E-mail: csasaki@ph.tum.de

Abstract. Based on an effective QCD Lagrangian we discuss the properties of charge density fluctuations in the vicinity of chiral phase transition. We explore thermodynamics in the presence of spinodal phase separation. We show that appearance of spinodal decomposition in a non-equilibrium first order phase transition results in divergence of the charge density fluctuations related with the electric charge and baryon number conservation. Consequently, divergent fluctuations at the chiral phase transition are not only attributed to the critical end point but are also there along the first order phase transition if the spinodal phase separation take place. Based on the mean field dynamics, the critical exponents for these singular behavior of charge susceptibilities are also discussed.
1. Introduction

One of the objectives of experiments with ultra-relativistic heavy ion collisions is to map the QCD phase diagram and study the properties of high density strongly interacting medium. The recent results obtained within Lattice Gauge Theory (LGT) description of QCD thermodynamics at vanishing baryon density show that the phase transition from hadronic phase to quark-gluon plasma is a cross-over [1]. From effective chiral models calculations [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12], as well as from the first LGT studies at finite baryon density [13], one concludes that at finite chemical potential the cross-over transition is most likely to be converted to the first order. Consequently, there might exist a critical end point (CEP) on the QCD phase diagram as a matching point of first order and cross-over transition. Based on the universal properties of the QCD chiral phase transition, one also expects, that the CEP belongs to the 3-dimensional Ising model universality class [11, 14]. This implies, that the CEP is a particular point on the QCD phase diagram where the fluctuations of the net quark and electric charge densities are diverging [8, 11, 12].

In heavy ion phenomenology the fluctuations of conserved charges are directly accessible experimentally. Thus, a non-monotonic behavior of such fluctuations in the c.m.s collision energy in heavy ion experiments would be an ideal and transparent signal of the existence of the CEP [10]. However, such conclusion is based on the assumption that the first order phase transition appears in equilibrium. In heavy ion collisions, we are dealing with quickly expanding dynamical system, such that, any deviation from equilibrium are not excluded.

A first-order phase transition is intimately linked with the existence of a convex anomaly in the thermodynamic pressure [15, 16, 17, 18, 19]. There is an interval of energy density or baryon number density where the derivative of the pressure is positive. This anomalous behavior characterizes a region of instability in the temperature, $T$, and baryon density, $n_q$, plane. This region is bounded by the spinodal lines, where the pressure derivative with respect to volume vanishes. The above anomalous properties of the first order transition can be uncovered in non-equilibrium system.

In this work we discuss a possible influence of the spinodal phase separation on the properties of baryon number density fluctuations. We show that spinodal instabilities result in divergence of electric charge $\chi_Q$ and baryon density $\chi_{\mu\mu}$ fluctuations. Consequently, a critical behavior of charge fluctuations is not only attributed to the CEP but is also there along the first order transition if spinodal phase separation appears in a medium.

Our discussion will be based on the effective chiral model calculations, however our main conclusion on the properties of charge fluctuations in the presence of spinodal instability is quite general and is independent on the particular choice of the effective chiral Lagrangian.
2. Effective Chiral Model and Spinodal instabilities

To study the properties of the chiral phase transition in the presence of spinodal instabilities we use as an effective chiral Lagrangian the Nambu–Jona-Lasinio (NJL) model \[20\]. This model describes the interactions of quarks preserving the chiral symmetry of the massless QCD Lagrangian. To consider thermodynamics we constrain the quark interactions only to the scalar and isoscalar sectors that are controlled by the phenomenological coupling \(G_S\). For two quark flavors, three colors and for finite quark chemical potentials the Lagrangian has a well known structure \[2, 5, 21\]:

\[
\mathcal{L} = \bar{\psi} (i \slashed{\partial} - m) \psi + \bar{\psi} \mu_q \gamma_0 \psi + G_S \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \gamma_5 \psi)^2 \right],
\]

where \(m = \text{diag}(m_u, m_d)\) is the current quark mass, \(\mu_q = \text{diag}(\mu_u, \mu_d)\) is the quark chemical potential and \(\gamma\) are Pauli matrices. The momentum cut-off and the strength of the quark interaction are fixed such that to reproduce the vacuum pion parameters.

To study the thermodynamics of the NJL model we use the mean field approximation. For an isospin symmetric system the thermodynamic potential is obtained as \[21\]:

\[
\Omega(T, \mu; M)/V = \frac{(M - m)^2}{4G_S} - 12 \int \frac{d^3p}{(2\pi)^3} \left[ E(\vec{p}) - T \ln(1 - n^{(+)}(\vec{p}, T, \mu)) - T \ln(1 - n^{(-)}(\vec{p}, T, \mu)) \right],
\]

with \(M = m - 2G_S \langle \bar{\psi} \psi \rangle\) being a dynamical quark mass, \(E(\vec{p}) = \sqrt{\vec{p}^2 + M^2}\) its energy and \(n^{(\pm)}(\vec{p}, T, \mu) = \left(1 + \exp[(E(\vec{p}) \mp \mu)/T]\right)^{-1}\) are the particle/antiparticle distribution functions. The quark chemical potential \(\mu\) is expressed as an average \(\mu = (\mu_u + \mu_d)/2\).
Figure 2. The phase diagram in the temperature $T$ and quark number density $n_q$ plane in the NJL model. The filled point indicates the CEP. The full lines starting at the CEP represent boundary of the mixed phase in equilibrium. The broken-curves are the isothermal whereas the dotted ones are the isentropic spinodal lines.

The dynamical quark mass $M$ in Eq. (2), that places a role of the order parameter of chiral phase transition, is obtained self-consistently from the stationarity condition $\partial \Omega / \partial M = 0$ that leads to:

$$M = m + 24G_s \int \frac{d^3p}{(2\pi)^3} \frac{M}{E} \left[ 1 - n^+ - n^- \right].$$

(3)

In Fig. left the constituent quark mass is shown at fixed temperature $T = 30$ MeV for different values of quark chemical potentials. The behavior of $M$, seen in Fig. is typical for systems that exhibit a first order phase transition: There is no unique solution of the gap equation. Instead, there are metastable solutions that correspond to the local minima of thermodynamic potential. For finite current quark masses in the NJL Lagrangian the chiral symmetry is explicitly broken. Consequently, $M$ is not anymore an order parameter and is never zero as seen in Fig. I.

To identify the equilibrium transition from chirally broken to approximately symmetric phase one usually performs the Maxwell construction. In this case the chiral phase transition parameters are fixed such that the three extrema of the thermodynamic potential are degenerate. The location of an equilibrium transition from massive quasiparticles to almost massless quarks, calculate at fixed $T = 30$ MeV, is shown as dashed-line in Fig. I.

The non-monotonic behavior of the dynamical quark mass $M$ seen in Fig. affects any thermodynamic observable since $M(T, \mu_q)$ determines the properties of medium constituents. In particular, the thermodynamic pressure is directly obtained from the potential, $P = -\Omega$, whereas the net quark number density

$$n_q = 12 \int \frac{d^3p}{(2\pi)^3} \left[ n^+ - n^- \right].$$

(4)
Thus, in the mean field approximation the density $n_q$ has the same structure as in a non-interacting gas of massive fermions, however with the $(T,\mu)$-dependent effective quark mass $M$.

Fig. 1-right shows the inverse density dependence of thermodynamic pressure at fixed temperature. The pressure exhibits a non-monotonic structure as a consequence of the properties of the dynamical quark mass seen in Fig. 1-left. The unstable solution of the gap equation leads to mechanical instabilities in the thermodynamic pressure where its volume derivative is positive. This region appears between spinodal points characterized by the minimum and the maximum of the pressure. Outside of this region the system is mechanically stable. The volume dependence of the pressure can be studied at fixed temperature or at fixed entropy. In the first case the spinodal points are isothermal whereas in the second they are isentropic. Changing the temperature or entropy results as the isothermal or isentropic spinodal lines in $(T, n_q)$-plane. If the volume derivative of $P$ exists then the spinodal lines can be defined through the following conditions:

$$\left(\frac{\partial P}{\partial V}\right)_T = 0 \quad \text{or} \quad \left(\frac{\partial P}{\partial V}\right)_S = 0 .$$

(5)

Considering behavior of the dynamical quark mass and thermodynamic pressure one can find the phase diagram related with the chiral phase transition. Fig. 2 shows the resulting diagram in the NJL model in the $(T, n_q)$-plane that accounts for spinodal instabilities. The NJL model yields a generic, QCD like, phase diagram. It exhibits a CEP that separates cross over from the first order chiral phase transition.

Assuming equilibrium transition there is a coexistence phase that ends at the CEP. However, accounting for expected instabilities due to a convex anomaly one can distinguish the metastable from mechanically unstable regions that are separated by the
spinodal lines. From the thermodynamic relation
\[
\left( \frac{\partial P}{\partial V} \right)_T = \left( \frac{\partial P}{\partial V} \right)_S + \frac{T}{C_V} \left[ \left( \frac{\partial P}{\partial T} \right)_V \right]^2,
\]
(6)
it is clear that the isentropic spinodal lines are lying inside the isothermal spinodals and that both lines coincide at \( T = 0 \). In the mean field approximation the isothermal spinodals are matched with the cross over transition at the CEP whereas isentropic spinodals appear below CEP. The last property is modified when including quantum fluctuations due to changes in critical exponents of the specific heat with constant volume \( C_V \) [19]. In the mean field approximation the \( C_V \) is finite, whereas it diverges in the quantum system. Consequently, from Eq. (6) the isothermal and isentropic conditions are equivalent at the CEP.

3. Charge fluctuations in the presence of spinodal phase separation

We have already mentioned in the introduction that fluctuations of conserved charges are excellent observables to study the critical properties in QCD medium related with chiral phase transition. In statistical physics fluctuations of conserved charges are quantified by corresponding susceptibilities, \( \chi_{\mu\mu} = \partial n_q / \partial \mu \). Thus, fluctuations of net quark number density \( \chi_{\mu\mu} \) measures response of the density to the change in quark chemical potential.

Fig. 3-left shows the evolution of the net quark number fluctuations along a path of fixed \( T = 50 \) MeV in the \( (T, n_q) \)-plane. When entering the coexistence region, there is a singularity in \( \chi_{\mu\mu} \) that appears when crossing the isothermal spinodal lines, where the fluctuations diverge and the susceptibility changes sign. Between the spinodal lines, the susceptibility is negative. This implies an instability of the baryon number fluctuations when crossing the transition between the chirally symmetric and broken phases.
The behavior of $\chi_{\mu\mu}$ seen in Fig. 3 is a direct consequence of the thermodynamic relation

$$\left( \frac{\partial P}{\partial V} \right)_T = -\frac{n_q^2}{V} \frac{1}{\chi_{\mu\mu}},$$

(7)

which connects the pressure derivative with the susceptibilities. Along the isothermal spinodal lines the pressure derivative in (7) vanishes. Thus, for non-vanishing density $n_q$, $\chi_{\mu\mu}$ must diverge to satisfy (7). Furthermore, since the pressure derivative $\partial P/\partial V|_T$ changes sign when crossing the spinodal line, there must be a corresponding sign change in $\chi_{\mu\mu}$, as seen in Fig. 3 left. Indeed, the negative specific heat in low energy nuclear collisions has been reported as the first experimental evidence for the liquid-gas phase transition [23]. Due to the linear relation between $\chi_{\mu\mu}$, the isovector susceptibility $\chi_I$ and the charge susceptibility $\chi_Q$ [8, 10], the charge fluctuations are also divergent at the isothermal spinodal line. Thus, in heavy-ion collisions, fluctuations of the baryon number and electric charge could show enhanced fluctuations, as a signal of the spinodal decomposition [19, 22]. The spinodal phase separation can also lead to fluctuations in strangeness [17] and isospin densities.

In the case of an equilibrium first order phase transition, the density fluctuations do not diverge. The fluctuations increase as one approaches the CEP along the first order transition and decrease again in the cross over region. This led to the prediction of a non-monotonous behavior of the fluctuations with increasing beam energy as a signal for the existence of a CEP [8, 10]. We stress that strictly speaking this is relevant only for the idealized situation where the first order phase transition takes place in equilibrium. In the more realistic non-equilibrium system one expects fluctuations in a larger region of the phase diagram, i.e., over a broader range of beam energies, due to the spinodal instabilities [19].

The singular behavior of the net quark number susceptibility at the CEP and isothermal spinodals can be quantified by the corresponding critical exponents. Fig. 4 shows the $\chi_{\mu\mu}$ calculated in the NJL model in the vicinity of the CEP and spinodals as a function of the reduced quark chemical potential $t = (\mu - \mu_c)/\mu_c$ at fixed temperature $T = 30\text{MeV}$. There is a clear scaling of $\chi_{\mu\mu} \sim t^{-\gamma}$, with the universal critical exponents $\gamma$. At the isothermal spinodal line this exponent is found to be $\gamma = 1/2$, while $\gamma = 2/3$ at the CEP in agreement with the mean-field results [11, 12]. Thus, the singularities at spinodals yield a somewhat stronger divergence as they join at the CEP [22].

The mean field values of these critical exponents extracted from the numerical studies shown in Fig. 4 can be also derived analytically from the Ginzburg-Landau potential [22] that is a good approximation of any effective chiral models in a vicinity of chiral phase transition.

The critical exponents shown in Fig. 4 are renormalized when including quantum and thermal fluctuations, but the smooth evolution of the singularity from the spinodal lines to the CEP, illustrated in Fig. 3 right, is expected to be generic being model independent.
4. Conclusions

We have shown that the net quark number fluctuations diverge at the isothermal spinodal lines of the first order chiral phase transition. As the system crosses this line, it becomes unstable with respect to spinodal decomposition. The unstable region is in principle reachable in non-equilibrium systems, created e.g. in heavy ion collisions. This means that large fluctuations of the baryon or electric charge density are expected not only at the second order CEP but also at a non-equilibrium first order transition.

Acknowledgments

K.R. acknowledges partial support of the Gesellschaft für Schwerionenforschung (GSI) and the Polish Ministry of National Education (MEN).

References

[1] M. Cheng et al., hep-lat/0710.0354.
[2] T. Kunihiro, Phys. Lett. B 271 395 (1991).
[3] J. Berges and K. Rajagopal, Nucl. Phys. B 538, 215 (1999).
[4] M. A. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov and J. J. M. Verbaarschot, Phys. Rev. D 58, 096007 (1998). T. Hatsuda and T. Kunihiro, Prog. Theor. Phys. 74, 765 (1985).
[5] O. Scavenius, A. Mocsy, I. N. Mishustin and D. H. Rischke, Phys. Rev. C 64, 045202 (2001); A. Mocsy, F. Sannino and K. Tuominen, Phys. Rev. Lett. 92, 182302 (2004).
[6] M. Stephanov, Acta Phys.Polon. B 35, 2939 (2004); Prog. Theor. Phys. Suppl. 153, 139 (2004); Int. J. Mod. Phys. A 20, 4387 (2005).
[7] H. Fujii, Phys. Rev. D 67, 094018 (2003). H. Fujii and M. Ohtani, Phys. Rev. D 70, 014016 (2004).
[8] M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. Lett. 81, 4816 (1998).
[9] Y. Hatta and M.A. Stephanov, Phys. Rev. Lett. 91, 102003 (2003)
[10] C. Sasaki, B. Friman and K. Redlich, Phys. Rev. D 75, 074013 (2007); J. Phys. G 32, S283 (2006).
[11] Y. Hatta and T. Ikeda, Phys. Rev. D 67, 014028 (2003).
[12] B. J. Schaefer and J. Wambach, Phys. Rev. D 75, 085015 (2007).
[13] C. R. Allton, et al., Phys. Rev. D 68, 014507 (2003); Phys. Rev. D 71, 054508 (2005). Z. Fodor and S. D. Katz, JHEP 0203, 014 (2002); JHEP 0404, 050 (2004).
[14] S. Ejiri, F. Karsch and K. Redlich, Phys. Lett. B 633, 275 (2006).
[15] P. Chomaz, M. Colonna and J. Randrup, Phys. Rept. 389, 263 (2004).
[16] D. Bower and S. Gavin, J. Heavy Ion Physics 15, 269 (2002). J. Randrup, J. Heavy Ion Physics 22, 69 (2005).
[17] V. Koch, A. Majumder and J. Randrup, Phys. Rev. C 72, 064903 (2005).
[18] J. Randrup, Phys. Rev. Lett. 92, 122301 (2004).
[19] C. Sasaki, B. Friman and K. Redlich, Phys. Rev. Lett. 99, 232301 (2007).
[20] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); Phys. Rev. 124, 246 (1961).
[21] For reviews and applications of the NJL model to hadron physics, see e.g., U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991); S. P. Klevansky, Rev. Mod. Phys. 64 (1992) 649; T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994); M. Buballa, Phys. Rept. 407, 205 (2005).
[22] C. Sasaki, B. Friman and K. Redlich, arXiv:0712.276 [hep-ph].
[23] M. D’Agostino et al., Phys. Lett. B 473, 219 (2000).