Electric Excitation of Spin Resonance in Antiferromagnetic Conductors

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Antiferromagnetism couples electron spin to its orbital motion, thus allowing excitation of electron spin transitions by an AC electric field. In a weakly-doped two-dimensional antiferromagnetic insulator on a lattice of square symmetry, whose conduction band minimum falls at the center \( \Sigma \) of the magnetic Brillouin zone (MBZ) boundary, as shown in Fig. II(a). Both the two-dimensionality and the square symmetry of this example simplify the description and make it relevant to materials such as cuprates and iron pnictides, yet neither of the two features is essential to the effect. Numerous other antiferromagnets of different crystal symmetry and effective dimensionality are discussed in Ref. [10]. Magnetic field is assumed small on the scale of the electron excitation gap \( \Delta \) and of the reorientation threshold, and thus does not perturb antiferromagnetic order.

Here, I illustrate this effect by an example, that may be relevant to a number of antiferromagnetic conductors: I study electric excitation of \textit{itinerant}-electron resonance in a weakly-doped two-dimensional antiferromagnetic insulator on a lattice of square symmetry, whose conduction band minimum falls at the center \( \Sigma \) of the magnetic Brillouin zone (MBZ) boundary, as shown in Fig. II(a). Both the two-dimensionality and the square symmetry of this example simplify the description and make it relevant to materials such as cuprates and iron pnictides, yet neither of the two features is essential to the effect. Numerous other antiferromagnets of different crystal symmetry and effective dimensionality are discussed in Ref. [10]. Magnetic field is assumed small on the scale of the electron excitation gap \( \Delta \) and of the reorientation threshold, and thus does not perturb antiferromagnetic order.

The effect is most vivid for the staggered magnetization axis \( \mathbf{n} \), pointing along the conducting plane, which is the case in several electron-doped cuprates [11, 12]. The magnetic field \( \mathbf{H} \) is nearly normal to \( \mathbf{n} \), which tends to happen due to spin-flop. It is this very geometry that I consider hereafter: orientation of the field with respect to the conducting plane may be arbitrary, as shown in Fig. II(b).

At low doping, the carriers concentrate in a small vicinity of the band minimum \( \Sigma \), and the Hamiltonian can be expanded around it. By symmetry, \( g_\perp (\mathbf{p}) \) in \( \mathcal{H}_{ZSO} \) II vanishes upon approaching the MBZ boundary, linearly in a generic case [3, 10] – and can be recast as \( g_\perp (\mathbf{p}) = g_\perp \frac{\mathbf{p} \cdot \mathbf{R}}{\mathbf{R}} \) with a constant \( \xi \), for \( \frac{\mathbf{p} \cdot \mathbf{R}}{\mathbf{R}} \ll 1 \). Here, \( \mathbf{p} \) is the component of the momentum deviation from the band minimum, locally transverse to the MBZ boundary, as shown in Fig. II(a). The length scale \( \xi \) is of the order of the antiferromagnetic coherence length \( \hbar v_F/\Delta \), and may be of the order of the lattice constant or much greater [3, 10].
where $\Omega_0 = \hbar c/2m$ is the cyclotron energy, and $H_0$ is the normal component of the field with respect to the conducting plane. This degeneracy becomes explicit upon completing the square in Eqn. (2) with respect to $[p - \frac{e}{c}A]$, or upon performing a non-uniform spin rotation

$$\Psi \rightarrow \exp \left[ i \frac{y m \xi}{\hbar} (\Omega_\perp \cdot \sigma) \right] \Psi,$$

which, in a purely transverse field ($\Omega_\parallel = 0$), eliminates $\Omega_\perp$ from the Hamiltonian altogether.

In the Landau gauge $A = (0, xH_0)$, this spin degeneracy in a transverse field acquires a simple interpretation: as shown in Fig. 2, the guiding orbit centers of the spin-up and the spin-down states split apart by the distance $\lambda \equiv 2\xi \frac{\Omega_\perp}{\hbar}$, along the $\hat{x}$ axis in real space, with the spin quantization axis chosen along $\Omega_\perp$.

To study the spectrum in an arbitrary field, it is convenient to use a different Landau gauge: $A = (-yH_0, 0)$. The spin rotation (4) removes the transverse field term, and turns the uniform longitudinal field $\Omega_\parallel$ into a spiral texture $\Omega'_\parallel$ with a constant pitch $q \equiv \frac{2m\xi}{\hbar}$ along the $y$-axis in the conducting plane:

$$\Omega'_\parallel = \Omega_\parallel \cos [qy] + \mathbf{n}_\perp \times \Omega_\parallel \sin [qy],$$

where $\mathbf{n}_\perp$ is the unit vector along $\Omega_\perp$. It is helpful to recast the cyclotron motion in terms of ladder operators as per $\frac{\delta^+ \delta^-}{\sqrt{2}} = \frac{\lambda}{l_H} - \frac{\alpha}{l_H^2}$ and $\frac{\delta^+ \delta^-}{\sqrt{2}} = \frac{\beta}{l_H}$, where $l_H = \frac{\hbar}{\sqrt{2m}}$ is the magnetic length. Now, the Hamiltonian (2) reads

$$\mathcal{H} = \Omega_0 \left[ a^+ a + \frac{1}{2} \right] - (\Omega'_\parallel \cdot \sigma),$$

where $\Omega_0 \equiv \hbar c/2m$ is the cyclotron energy, and $H_0$ is the normal component of the field with respect to the conducting plane. This degeneracy becomes explicit upon completing the square in Eqn. (2) with respect to $[p - \frac{e}{c}A]$, or upon performing a non-uniform spin rotation

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$$\mathcal{H} = \Omega_0 \left[ a^+ a + \frac{1}{2} \right] - (\Omega'_\parallel \cdot \sigma),$$
with $y$ in $\Omega_\parallel'$ of Eqn. (3) expressed via the ladder operators.

According to Eqn. (3), in the limit of a weak longitudinal field ($\Omega_\parallel \ll \Omega_0$), the spin precesses at a characteristic frequency $\Omega_\parallel$, which is small compared with the cyclotron frequency $\Omega_0$ of the orbital motion. In this limit, the splitting $\delta \mathcal{E}_n$ of the $n$-th Landau level is given simply by averaging $(\Omega_\parallel' \cdot \sigma)$ over the orbital eigenstate $|n\rangle$ of the first term in Eqn. (4), leading to

$$\delta \mathcal{E}_n = 2 \Omega_\parallel f_n \left( \frac{\xi \Omega_\parallel}{l_H \Omega_0} \right),$$

where $f_n(\zeta) = L_n(2\zeta^2) \exp[-\zeta^2]$, and $L_n(\zeta)$ is the Laguerre polynomial [14]. The degeneracy is lifted in a peculiar way: for $\Omega_\parallel \ll \Omega_0$, the splitting $\delta \mathcal{E}_n$ of the $n$-th Landau level decays and oscillates as a function of $\zeta = \frac{\xi \Omega_\parallel}{l_H \Omega_0}$, as shown in Fig. 3. For a small fixed $\Omega_\parallel \ll \Omega_\perp$, this amounts to decaying oscillations with reducing the angle between the field and the conducting plane, as shown in Fig. 1(b).

The factor $f_n(\zeta)$ in Eqn. (7) is of a simple origin. The longitudinal component $\Omega_\parallel$ hybridizes the two states in Fig. 2 and lifts their degeneracy. Indeed, the splitting vanishes as the distance $\lambda = 2 \xi \Omega_\parallel \Omega_\perp$ between the guiding orbit centers exceeds the wave function size $l_H \sqrt{n+1}$. The oscillations on the background of this decay are due to spatial oscillation of the two wavefunctions for $n > 0$.

III. ELECTRIC EXCITATION OF SPIN RESONANCE

The momentum dependence of $g_L(p)$ has a spectacular spectroscopic manifestation: excitation of spin resonance transitions by an AC electric field – the very same transitions that are normally excited by an AC magnetic field in an ESR experiment.

I name this phenomenon Zeeman Electric-Dipole Resonance (ZEDR) to note its similarity with Electric-Dipole Spin Resonance (EDSR) in semiconductors and semiconductor heterostructures with spin-orbit coupling [15].

A. Resonance in a quantizing field

To study the effect for discrete Landau levels, notice that a uniform AC electric field $E'_y$ along the $\hat{y}$-axis couples to the $y$-component $ey = el_H a^+ a$ of the electron dipole moment. With $E'_y$, the Hamiltonian (2) reads

$$\mathcal{H} = \Omega' a^+ - (\Omega_\parallel' \cdot \sigma) - \frac{a^+ a}{\sqrt{2}} l_H E'_y. \tag{8}$$

In the absence of a longitudinal component $\Omega_\parallel$, the last term in (5) induces only the cyclotron resonance: spin-conserving electric dipole transitions between the adjacent Landau levels, with the matrix element $M_{CR}$

$$M_{CR} = \langle n+1, \sigma | e y E'_y | n, \sigma \rangle = e l_H E'_y \sqrt{n+1/2}, \tag{9}$$

whose scale is set by the Larmore radius $l_H \sqrt{n+1}$.

A small longitudinal component $\Omega_\parallel \ll \Omega_0$ changes this picture, as $(\Omega_\parallel' \cdot \sigma)$ couples the electron spin to its orbital motion. As a result, the $n$-th Landau level eigenstate $|n\rangle$ with spin projection $\alpha$ on the direction of $\Omega_\parallel$ acquires a small admixture of other states $|m\beta\rangle$, and the AC electric field begins to induce a number of previously forbidden transitions.

Here, I restrict myself to spin-flip transitions within the same Landau level [16], excited by an AC electric field as shown in Fig. 3. Treating the admixture of other Landau levels to the first order in $(\Omega_\parallel' \cdot \sigma)$, one finds [17] the ZEDR matrix element

$$M_{ZEDR} = \langle n\uparrow \uparrow | e y E'_y | n\downarrow \downarrow \rangle,$$

$$M_{ZEDR} = -2 e \xi E'_y \frac{\Omega_\parallel}{\Omega_0} \frac{\Omega_\perp}{\Omega_0} L_n(2\zeta^2) \exp[-\zeta^2], \tag{10}$$

where $\zeta = \frac{\xi \Omega_\parallel}{l_H \Omega_0}$. Apart from the dependence on the orientation of the field with respect to the conducting plane and to the staggered magnetization, ZEDR matrix elements are defined simply by the length scale $\xi$. Being at least of the order of the lattice spacing, in a weakly-coupled spin density wave antiferromagnet $\xi \sim \hbar v_F/\Delta$ (see Refs. 9, 10) may reach a 10 nm scale [13]. At the same time, the ESR matrix elements are defined by the
Compton length $\lambda_C = \frac{h}{m_0 \alpha} \approx 0.4$ pm. The characteristic ratio of the ZEDR matrix elements to those of ESR can thus be estimated as $\frac{\delta}{\lambda_C} = \frac{\frac{1}{2} \frac{\xi}{a_B}}{\Delta} \approx 53$ pm is the Bohr radius, and $\Delta = \frac{\hbar e}{|n|} \approx 137$ is the inverse fine structure constant. Thus, the ZEDR absorption exceeds that of ESR by about $\left( 137 \cdot \frac{\alpha}{a_B} \right)^2$, which amounts to at least four orders of magnitude.

B. Resonance in a continuous spectrum

Now, consider a situation, where the DC magnetic field $\mathbf{H}$ couples only to the electron spin, but not to its orbital motion, which is the case for a field along the conducting plane. According to Eqn. (2), the field splits the conduction band into two subbands $\mathcal{E}_\pm(p)$

$$\mathcal{E}_\pm(p) = \frac{p_y^2}{2m} \pm \sqrt{\Omega^2 + \left( \frac{p_y \xi}{\hbar} \right)^2} \Omega_\perp,$$

and the AC field induces transitions between them.

![FIG. 4: (Color online) The spin splitting of the conduction band, sketched after Eqn. (12) in a small vicinity of the band minimum at point $\Sigma$.](image)

According to [2], a purely transverse field ($\Omega_\parallel = 0$) lifts the Kramers degeneracy by splitting the two degenerate subbands by the ‘distance’ $\delta p_y = \frac{\hbar}{2m} \xi \Omega_\perp$ along the $p_y$-axis:

$$\mathcal{H} = \frac{p_y^2}{2m} + \frac{1}{2m} \left[ p_y - \frac{m \xi}{\hbar} (\Omega_\perp \cdot \sigma) \right]^2. \quad (12)$$

Illustrated in Fig. 4 this is, indeed, a momentum-space counterpart of the real-space splitting in Fig. 2.

In the continuous spectrum, ZEDR may be treated simply as being induced by the term $\delta\mathcal{H}_{\text{ZEDR}} = \frac{e}{2} A_\parallel^\star (\Omega_\perp \cdot \sigma)$. Its matrix element between the states with the spin along and against the direction of the effective magnetic field $(\Omega_\parallel + \frac{\delta H}{\hbar} \xi \Omega_\perp)$ is equal to

$$|\langle \uparrow | \delta\mathcal{H}_{\text{ZEDR}} | \downarrow \rangle|^2 = \left[ \frac{e \xi E_\omega^\star}{\hbar \omega} \right]^2 \frac{\Omega_\perp^2 \Omega_\parallel^2}{\Omega_\parallel^2 + \left( \frac{\delta p_x}{\hbar} \Omega_\perp \right)^2}, \quad (13)$$

where I used the relation $\langle \downarrow | (\hat{n} \cdot \sigma) | \uparrow \rangle = n_+ \equiv n_x + i n_y$ for an arbitrary unit vector $\hat{n}$.

The ZEDR absorption $P_{\text{ZEDR}}^\omega$ is given, according to the Fermi golden rule, by the product of the modulo squared [13] of the matrix element of $\delta\mathcal{H}_{\text{ZEDR}}$ by the AC field frequency $\omega$, and by $\pi n$ with the subsequent summation over the Fermi surface, yielding

$$P_{\text{ZEDR}}^\omega = \frac{m}{\pi} \frac{|e \xi E_\omega^\star|^2}{16} \frac{\sin^2 \theta \cos^2 \theta \left( \frac{\omega_H}{\omega} \right)^4}{\sqrt{\left( \frac{\omega_H}{\omega} \right)^2 \left[ \cos^2 \theta + 2 \mu m \xi^2 \sin^2 \theta \right] - 1} \left( \frac{\omega_H}{\omega} \right)^2 \cos^2 \theta}, \quad (14)$$

where $\mu$ is the electron chemical potential counted from the bottom of the band, and $\omega_H \equiv 2 \Omega$. The result [13] is presented in a form, corresponding to sweeping the magnitude of the DC field at a fixed angle $\theta$ to the staggered magnetisation $\mathbf{n}$ and at a fixed frequency $\omega$. In agreement with Eqn. (13), the ZEDR matrix elements are again defined simply by the lengthscale $\xi$.

The lineshape described by Eqn. (14) is intrinsically broadened: according to Eqn. (11), in a magnetic field
电激射达到磁共振的条件是满足 

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\frac{\Delta}{\epsilon_F} \ll 1,
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将上述方程代入得到

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\frac{\Delta}{\epsilon_F} \ll 1,
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通过方程 (9) 确定 Zeeman 电偶极子共振的存在条件，其中 

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方程 (10) 说明 Zeeman 电偶极子共振的存在条件是满足 

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haps employed to manipulate and monitor carrier spin with electric field.

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[1] Semiconductor Spintronics and Quantum Computation, edited by D. D. Awschalom, D. Loss, N. Samarth (Springer, Berlin 2002).
[2] Concepts in Spin Electronics, edited by S. Maekawa (Oxford University Press, New York 2006).
[3] I. Žutić, J. Fabian, S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004).
[4] S. Methfessel and D. C. Mattis, in Magnetic Semiconductors, Handbuch der Physik Vol. 18, edited by S. Flügge (Springer-Verlag, Berlin, 1968), p. 1.
[5] T. Jungwirth, J. Sinova, J. Mašek, J. Kučera, A. H. MacDonald, Rev. Mod. Phys. 78, 809 (2006).
[6] Spin Dependent Transport in Magnetic Nanostructures, edited by S. Maekawa and T. Shinjo (Advances in Condensed Matter Science, Taylor & Francis, 2002).
[7] E. I. Rashba, Physica E 20, 189 (2004).
[8] S. A. Brazovskii and I. A. Luk’yanchuk, Zh. Eksp. Teor. Fiz. 96, 2088 (1989) [Sov. Phys. JETP 69, 1180 (1989)].
[9] R. Ramazashvili, Phys. Rev. Lett. 101, 137202 (2008).
[10] R. Ramazashvili, Phys. Rev. B 79, 184432 (2009).
[11] A. N. Lavrov, H. J. Kang, Y. Kurita, T. Suzuki, Seiki Komiya, J. W. Lynn, S.-H. Lee, Pengcheng Dai, and Yoichi Ando, Phys. Rev. Lett. 92, 227003 (2004).
[12] M. Matsuda, Y. Endoh, K. Yamada, H. Kojima, I. Tanaka, R. J. Birgeneau, M. A. Kastner and G. Shirane, Phys. Rev. B 45, 12548 (1992).
[13] At first sight, the longitudinal and the transverse term in Eqn. (2) transform differently under inversion $I$. However, $\xi$ also has odd parity (see Refs. [9, 10]), thus removing the issue.
[14] I. S. Gradshtein, I. M. Ryzhik, Table of Integrals, Series, and Products (Academic Press, 2000).
[15] E. I. Rashba, V. I. Sheka, in Landau Level Spectroscopy, edited by G. Landwehr and E. I. Rashba (Elsevier Science Publishers B.V., 1991).
[16] In a weak field, transitions to other Landau levels are suppressed by even powers of $\frac{\xi_p}{\hbar} < 1$.
[17] ZEDR has been predicted in Ref. [19]. However, the result for absorption in a quantizing field, given in this article, overlooks the fact, that both the Landau level splitting and the absorption vanish in a purely transverse field.
[18] For comparison, cyclotron resonance matrix elements in a magnetic field of one Tesla are defined by $l_H \approx 26$ nm.
[19] R. Ramazashvili, Zh. Eksp. Teor. Fiz. 100, 915 (1991) [Sov. Phys. JETP 73, 505 (1991)].
[20] A. Shengelaya, H. Keller, K. A. Müller, B. I. Kochelaev, K. Conder, Phys. Rev. B 63, 144513 (2001).
[21] At the same field $\Omega_0 \sim \frac{\Delta^2}{\epsilon_F}$, the relevant values of $\frac{\xi_p}{\hbar}$ become of the order of unity, and the expansion of $g_{\perp}(p)$ breaks down together with the analytic results above.
[22] E. Fawcett, Rev. Mod. Phys. 60, 209 (1988).
[23] Ravi K. Kummeruru & Yeong-Ah Soh, Nature 452, 859 (2008), and references therein.
[24] F. Ronning et al., Phys. Rev. B 67, 165101 (2003).
[25] N. P. Armitage et al., Phys. Rev. Lett. 88, 257001 (2002).
[26] F. Keffer and C. Kittel, Phys. Rev. 85, 329 (1952).
[27] H. Zabel, J. Phys.: Condens. Matter 11, 9303 (1999).
[28] Yugo Oshima, PhD thesis, Kobe University (2003).
[29] M. R. Trunin, Usp. Fiz. Nauk 175, 1017 (2005) [Physics-Uspekhi 48, 479 (2005)].
[30] A. Shekhter, M. Khodas, A. M. Finkelstein, Phys. Rev. B 71, 165329 (2005).
[31] M. Duckheim and D. Loss, Nature Physics 2, 195 (2006).