Design of non-conventional biaxial tensile-shear tests for the structural characterization and ductile damage assessment of ductile engineering materials

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Abstract. The paper focuses on the development and optimization of a non-conventional specimen geometry for mechanical testing of ductile materials under shear-tension loading conditions. This is of major importance, being well known from literature works, that material final failure is strongly dependent on the stress state and that the information about the material ultimate resistance is crucial in virtually any kind of machine design related safety assessment. More in detail, in this study an effective way to load the samples with a combination of shear and tension is devised, selecting and optimizing a proper specimen geometry and designing the related gripping system. The goal is to obtain stress states in test runs different as much as possible one with the other, with specimens easy to be machined, and using just a standard universal testing machine. This would make the biaxial tests available to a broad group of end users, even industrial. The state of the art of specimens for biaxial tests available in literature was used as starting point to design a novel simplified geometry and the dedicated gripping system. Numerical simulations of the prospective biaxial experiments were performed, ranging from pure shear to pure tension, to optimize the new geometry, improve its effectiveness and evaluate the material stress states and strain to fracture at the critical points that can be obtained from the tests. This local information proved to be sufficient to calibrate different damage models using the single proposed test, with proper combinations of shear to tension loads.

1. Introduction
Nowadays, new ductile metal alloys are constantly being developed, enhancing their performance to meet the industrial needs for strength reliability and manufacturing easiness. The accurate understanding of their structural performance, including the identification of ultimate strength and ductility, is crucial for their safe use. To this purpose, several ductile damage models have been devised over the years and are today available in the literature; for most formulations, material failure is assumed to be a function of the induced stress state [1,2]. All models have in common the necessity of a proper experimental investigation to achieve a satisfactory calibration on a specific material. Indeed, multiaxial stress states
are usually required, often involving multiaxial loading conditions and special testing equipment. The scenario is even worse in case of a more complex constitutive material behavior and for non-linear damage accumulation and related models, where even more sophisticated experiments are required [3–6]. Thus, there emerged the necessity of more advanced tests than the standard tensile one, that demonstrated to provide limited useful information, under uniaxial loading conditions only, and for a limited range of plastic deformation. However, inducing different stress states, in a controlled and repeatable way, is not a trivial task without employing overly complicated setups. Dedicated multiaxial testing machines or special geometries, hard to fabricate, might be required. Such complexity makes the calibration, and hence the use, of ductile damage models not suitable for the small–medium industrial companies, not devoted to research nor equipped for those kind of testing. In this regard, the chief idea of this study was to develop an effective geometry, easy to be machined and tested, ideally by using only a universal uniaxial testing machine, but nevertheless capable of inducing various stress states different one another. Thus, starting from the specimen types adopted for tension-shear testing in the literature, based on a finite element numerical verification the most promising geometry was selected, some design modification proposed and their effectiveness assessed.

2. Theoretical background

For a broad class of numerical models, ductile damage is accepted to accumulate with the increase of plastic strain, weighted on a function of the stress state, as stated in equation (1).

\[ D = \int_0^{\varepsilon_f} f(T, X) d\varepsilon_p \]  

(1)

The stress state is usually expressed using one or two scalar parameters: the stress triaxiality \( T \) and, in some formulations, the Lode parameter \( X \). These two quantities in turn depend on the invariants of the stress and deviatoric tensors as defined by equations (2) and (3) (for a more comprehensive theoretical background please refer to [2]):

\[ T = \frac{1}{3} I_1 \sqrt{3} J_2 \]  

(2)

\[ X = \frac{27}{2} \frac{J_3}{(\sqrt{3} J_2)^3} \]  

(3)

where \( I_1 \) represents the first stress invariant, \( J_2 \) and \( J_3 \) the second and third deviatoric stress invariants, respectively.

Fracture occurs when \( D \) reaches a critical value, conventionally set equal to 1 in most formulations. Under quasi-proportional loading conditions, as applied in this study, the average values of \( T \) and \( X \) over the plastic strain (\( \varepsilon_p \)) history can be used, as defined by equations (4) and (5).

\[ T_{av} = \frac{1}{\varepsilon_f} \int_0^{\varepsilon_f} T(\varepsilon_p) d\varepsilon_p \]  

(4)

\[ X_{av} = \frac{1}{\varepsilon_f} \int_0^{\varepsilon_f} X(\varepsilon_p) d\varepsilon_p \]  

(5)

Under these assumptions, setting \( D = 1 \), the expression (1) can be inverted, to represent a surface in the three-dimensional space \([T, X, \varepsilon_f]\), see equation (6), often called fracture locus or fracture surface, for which a unique strain at fracture can be associated to each stress state.

\[ \varepsilon_f = f^{-1}(T, X) \]  

(6)
Different forms have been proposed in the literature for expression (1). As an example, in the following equation (7) the analytical expression of \( f(T,X) \) of the damage model devised by Cortese et al. [3] is reported.

\[
f(T,X) = C_1 e^{C_2 \left( \frac{G(X,\beta,\gamma)}{G(X = 1,\beta,\gamma)} \right)^{\frac{1}{n}}} \tag{7}
\]

Where \( C_1, C_2, \beta, \gamma \) and \( n \) are material parameters and \( G(X,\beta,\gamma) \) reads:

\[
G(X) = \left[ \cos \left( \beta \frac{n}{6} - \frac{1}{3} \arccos(\gamma X) \right) \right]^{-1} \tag{8}
\]

In the following Figure 1 the tuned fracture surface of the model reported in equation (7) for a ductile Grade X65 steel alloys is shown, highlighting the locations of the different stress states and strain to fracture of the tests typically used for calibration or validation. To accurately tune this damage model, or generally speaking a damage model belonging to the class (1), four different tests have to be performed and proved to be effective, as exposed in [3]: tensile tests on round bars, on round notched bars, tensile tests in plane strain and torsion tests. As a consequence, four different geometries and two testing machines are required.

![Figure 1](image.png)

**Figure 1.** Example of a fracture surface on a Grade X65 steel. The locations of the different tests used for calibration and validation are highlighted.

### 3. Starting point, literature review and design requirements

As a starting point, the requirements for the design of an optimized multipurpose specimen geometry were identified. Firstly, the specimens should be easy to be manufactured, even with standard not CNC equipment; secondly, they should be tested using a conventional uniaxial testing machine, by inducing different multiaxial stress states in the gauge section. Moreover, the enhanced geometry must be capable to concentrate the plastic strain \( \varepsilon_p \) in the centre of the gauge section for the whole loading history and up to fracture, as well as maintaining the stress state as proportional as possible (\( T \) and \( X \) constant) during plastic deformation. Concerning the overall dimensions, in this study the specimens will be optimized to be machined from metal bars with a diameter of 20 mm.
3.1. Original “Mohr” geometry

From the several specimen geometries devised for shear or tensile – shear tests available in the literature, the ones compliant with the above design requirements were identified. A numerical verification was performed on them to quantify the constancy of the stress state and the concentration of plastic strain in the center of the gauge section. All the numerical simulations were performed using the commercial software ANSYS Workbench 2019. The models were meshed with brick elements where possible, otherwise tetrahedral elements with quadratic formulation were employed. The large displacement capability of the solver was activated. As a material model a Hollomon constitutive law of a common high strength ductile steel alloy was used. More in detail, numerical simulations on six different types of specimens were run: they were the one proposed by Miyauchi et al. [7], the ASTM B831 [8], the ASTM D7078 [9], the twin bridge proposed by Yin et al. [10], the one devised by Driemeier et al. [11] and the geometry introduced by Dunand and Mohr [12]. All of them are reported in Figure 2 and not discussed further here for the sake of conciseness. Based on the preliminary investigation, the most promising geometry was finally selected, namely the one devised by Dunand and Mohr [12], and reported in Figure 3(a). This “butterfly” shaped specimen is intended to be tested by blocking one end and applying a proper combination of horizontal and vertical forces or displacements on the other, to induce the desired shear-tensile state of stress in the critical point of the gauge section. In Figure 3(b) it is shown a scheme of the biaxial testing machine needed to load and test this specimen: the necessity of such an advanced custom equipment is a major limiting disadvantage.

![Figure 2. Geometries preliminary tested: (a) Miyauchi [7]; (b) ASTM B831 [8]; (c) ASTM D7080 [9]; (d) Twin Bridge [10]; (e) Driemeier [11] and (f) Mohr [12].](image)
This geometry exhibits an excellent performance in concentrating the plastic strain in the center of the sample and in maintaining the stress state constant up to large strain, under very different loading conditions. As an example, please refer to the next Figures 4 and 5. In Figure 4a the finite element model of the specimen is reported, along with the scheme of the constraints and horizontal ($U_X$) and vertical ($U_Y$) displacements applied to simulate a shear-tension test, while in Figure 4b the contour map of the equivalent plastic strain is shown, for a generic combination of applied tension and shear, confirming that the maximum deformation is located in the center of the gauge section.

For the loading conditions of Figure 4, in Figure 5 the histories of triaxiality $T$ and Lode parameter $X$ over the plastic strain accumulations are reported, confirming the capability of this geometry in keeping them constant up to fracture.
3.2. Geometry optimization

Beside the necessity of a dedicated biaxial testing machine, the other main drawback of this geometry is the complexity in specimen fabrication. Indeed, the gauge section, thinner than the rest of the specimen, is defined by an Euler spiral, or clothoid, marked in dashed blue in the next Figure 6, which might be difficult to be machined accurately. Thus, the first modification was the simplification of that region. The aim was to design a new gauge section defined by a piecewise line, made of three straight segments. A parametric numerical campaign was run, to identify the optimal values of the dimensions “L” and “H” reported in Figure 6. At the same time the lateral protrusions of the gripped parts were shortened in order to avoid stress intensifications at the specimen boundaries (Figure 7(a)). The additional requirement initially set was the possibility to machine the specimens from a bar with a diameter of 20 mm: the original specimen dimensions were therefore scaled down, to fit the width into the bar diameter. The resulting modified geometry is shown in Figure 7(a), with the final values of $L = 2$ and $H = 2.5$. As an example, in Figure 7(b) it is reported the contour map of equivalent plastic strain on the new geometry under a pure shear loading condition, which does not differ appreciably from the one corresponding to the original Mohr’s geometry, confirming the effectiveness of the proposed geometrical simplification.

Figure 5. Shear-tension test: typical (a) Triaxiality ($T$) evolution and (b) Lode parameter ($X$) evolution vs equivalent plastic strain.

Figure 6. Tensile – shear specimen devised by Dunand and Mohr et al. [12] with the Euler spiral delimiting the gauge section marked in dashed blue and the proposed new piecewise curve in solid red.
Eventually, the gripped parts were thickened to host three holes on each side of the specimens since, as detailed in the following, they will serve the purpose of properly transferring both tensile and shear loads on the specimen. The resulting final geometry is reported in Figure 8.

For validation, an additional numerical campaign was run, applying the constraints directly to the holes in the gripping regions. In the following Table 1 the values of displacements that were imposed have been listed, to reproduce the loading conditions proposed by Dunand and Mohr. The corresponding histories of $T$ and $X$ are reported in Figure 9, showing again a good constancy over time.
Table 1. Applied loading conditions and corresponding values of parameters $T$ and $X$

| Condition                | $UX$ | $UY$ | $T$  | $X$  |
|--------------------------|------|------|------|------|
| Pure shear               | 4.5  | Free | 0.00 | 0.05 |
| Shear - tension 70% 30%  | 3.15 | 0.14 | 0.08 | 0.36 |
| Shear - tension 50% 50%  | 4.5  | 0.47 | 0.19 | 0.73 |
| Shear - tension 30% 70%  | 1.35 | 0.33 | 0.26 | 0.91 |
| Pure tension             | 0    | 0.47 | 0.61 | 0.99 |

Figure 9. Tension-shear tests: (a) Triaxiality $T$ evolution and (b) Lode parameter $X$ evolution vs equivalent plastic strain.

4. Design of the gripping system and final numerical simulations

In the previous section, all the numerical simulations were run applying loads and constrains directly to the specimen, as could be done using the biaxial equipment of Figure 2(b). In order to generate variable multiaxial stress states using a simple uniaxial testing machine a proper gripping system was devised. At first, the authors came up with the design reported in Figure 10, where the specimen can be mounted at different angles with respect to the axial displacement imposed by the uniaxial machine to the two prismatic grips, to induce exactly the horizontal to vertical displacements of Table 1. This was achieved connecting the grips to the testing machine through simple forks sporting two pins connected to properly angled sets of holes. However, the preliminary numerical simulations immediately showed a lack of stiffness of the system uniaxial machine/grips, in the direction perpendicular to the application of the load, which completely modified the ratios of actual applied loads, thus changing the resulting induced $T$ and $X$. 
Figure 10. Design of the dedicate gripping system: preliminary attempt, starting from the loading combinations proposed by Dunand and Mohr

Consequently, the authors switched towards a solution consisting in an equally spaced array of holes every 15°, leaving the assembly free to align along the loading direction, by connecting the fork to the grips using a single hole instead of two. Only for the two extremal positions, $\alpha = 0^\circ$ and $\alpha = 90^\circ$ corresponding to pure shear and pure tension respectively, two pins are employed to constrain the position during test run as rigidly as possible. The final design is shown in Figure 11.

Figure 11. Final optimized version of the gripping system for tension – shear tests. The equally spaced positions allow to apply different loading combinations, from pure shear to plane strain tension.

The choice of adopting an array of equally spaced holes has been done to regularly vary the loading conditions. In this case, a FE model involving the whole assembly was necessary to retrieve the stress states actually induced in the specimen for each configuration, which are also affected by the rigid rotation and elastic stiffness of the two grips. Conversely, should a specific stress state be desired, the position of the holes on the grips could be found via FEA through an iterative procedure. In the following Figure 12 the evolution of $T$ and $X$ over the loading history is reported, proving the effectiveness of the new simplified geometry, in conjunction with the devised gripping system. It is worth noting that the stress states can be broadly varied, and that even for the mixed tension-shear combinations, corresponding to the intermediate configurations connected through a single hole and identified with the $\alpha$ angle with respect to the pure shear position ($\alpha = 0^\circ$), fairly constant values of $T$ and $X$ were obtained, confirming that the assembly of sample and grips align with the loading direction and then remain stable, with no excessive rotations.
Finally, the location of the different stress states that can be induced are reported onto the $T - X$ domain in the next Figure 13 as blue triangular markers. They are well spaced and distributed, aligned along the plane stress curved marked in dashed red, and allow to fully sweep the domain. This is this a remarkable result, achieved using only one specimen geometry, which can be therefore be adopted to calibrate the damage models presented in section 2. Furthermore, they have been compared with the loading conditions proposed by Dunand and Mohr, marked in Figure 13 with bigger green, red and blue dots. Finally, in the same graph, there were reported for comparison the stress states of the four different tests previously employed by the authors for damage models tuning, shown in Figure 1: tensile test on smooth and notched round bars (RB and RNB respectively), tensile test in plane strain condition and torsion test. It is worth noting how out of the four different stress states two of them can be reproduced using the modified single geometry proposed here, further confirming its capability in inducing different stress states into the material and hence its effectiveness in being employed for models calibration or validation.
Figure 13. Location of the stress–state that can be induced through the modified tension-shear geometry and gripping system (blue triangles), compared with those achievable by the loading conditions proposed by Dunand and Mohr (round dots), and those related to the four different geometries currently employed by the authors.

5. Conclusions
From a literature study, the most suitable biaxial tension-shear specimen geometry, intended for ductile fracture investigation, is identified and subsequently modified to be easily realized and tested using a conventional uniaxial testing machine. To this purpose a dedicated gripping system has been additionally designed, capable of clamping the specimen at different angles, thus applying different combinations of tension and shear on it. This to simplify the calibration procedure of several classes of ductile damage models, usually particularly complex and often requiring special equipment to be fulfilled.

Numerical simulations proved the capabilities of the proposed testing methodology in reproducing very different proportional loading stress states, by simply varying the loading conditions applied to the samples. The information of the strain to fracture associated to these stress states, that could be experimentally retrieved performing this kind of tests, was demonstrated to be sufficient for an accurate tuning of generic damage models based on triaxiality and Lode angle parameters, proving the ultimate strength of ductile materials can be characterized using a unique test and a conventional testing facility.

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