Twist-induced magnon Landau levels in honeycomb magnets

Tianyu Liu\textsuperscript{1} and Zheng Shi\textsuperscript{2}

\textsuperscript{1}Max-Planck-Institut f"ur Physik komplexer Systeme, 01187 Dresden, Germany
\textsuperscript{2}Daehlem Center for Complex Quantum Systems and Physics Department, Freie Universit"at Berlin, 14195 Berlin, Germany

(Dated: April 6, 2020)

Lattice deformation resulting from elastic strain is known to spatially modulate the wave function overlap of the atoms on the lattice and can drastically alter the properties of the quasiparticles. Here we elaborate that a twist lattice deformation in two-dimensional honeycomb quantum magnet nanoribbons is equivalent to an elastic gauge field giving rise to magnon Landau quantization. When the ground state is ferromagnetic, dispersive Dirac-Landau levels are induced in the center of magnon bands, while for antiferromagnetic nanoribbons, the twist results in dispersive equidistant Landau levels at the top of magnon bands. The dispersions for both types of Landau levels are derived in the framework of the band theory.

Introduction. Strain engineering is a powerful tool in tuning properties of quantum matter, such as spin transport \cite{1, 2}, thermal conductivity \cite{3, 4}, and quantum anomaly \cite{5}. In particular, twisting one layer of bilayer graphene with respect to the other by certain “magic” angles \cite{6} results in spatial modulation of electron tunneling between the layers and produces flat “Moiré bands” responsible for the correlated insulating phase \cite{7} and the unconventional superconductivity \cite{8}. Properly tuned strain can close or open band gaps in topological quantum matter and induce phase transitions between distinct topological phases \cite{9, 10}.

Perhaps the most investigated and best understood strain effects are those associated with Dirac materials, where strain is famously equivalent to an elastic gauge field \cite{11, 12}. A circular bend in 3D Dirac/Weyl semimetals and superconductors induces a uniform pseudo-magnetic field giving rise to Dirac-Landau levels \cite{13, 14}. A uniform elastic gauge field can also be generated by twisting 3D Weyl systems around the axis on which Weyl points are located \cite{15, 16, 17, 18}. Though first theoretically proposed \cite{19} and experimentally discovered \cite{20} in graphene, the strain-induced gauge field in graphene and other 2D Dirac materials is not uniform for simple lattice deformations such as bend \cite{21, 22} or twist \cite{23}, causing some difficulty in obtaining insights of the strain-induced Landau levels (LLs).

In this Letter, we propose a simple strategy in the framework of band theory to obtain the dispersion of twist-induced LLs at Brillouin zone (BZ) corners for both ferromagnetic (FM) and antiferromagnetic (AF) honeycomb nanoribbons, whose magnon bands in the absence of twist exhibit Dirac cones and quadratic peaks, respectively. We demonstrate that the effect of twist is to relocalize the Dirac cones (quadratic peaks) from those at the BZ corners to Brillouin zone (BZ) corners. Based on this observation, we show that a correspondence between the crystal momentum of the nanoribbon and the twist-induced elastic gauge field can be drawn to explicitly give the momentum dependence of the twist-induced LLs for both ferromagnets and antiferromagnets.

Twisted Heisenberg model. We consider a Heisenberg model defined on a honeycomb lattice of spins with only nearest-neighbor interactions

\[ H = \sum_{\mathbf{r}} \sum_{i=1}^{3} J_i S_A(\mathbf{r}) \cdot S_B(\mathbf{r} + \mathbf{a}_i), \]

where \( \mathbf{r} \) denotes the position of a generic lattice site belonging to the A sublattice and vectors \((\alpha_1, \alpha_2, \alpha_3) = (\hat{x}, \hat{y}, \hat{z})\), with \( a \) being the nearest-neighbor distance, connect this site to its three nearest-neighbor sites on the B sublattice. \( J_i = J(\alpha_i) \) is the interaction strength between the spin-S located at \( \mathbf{r} \) and its \( i \)-th nearest neighbor at \( \mathbf{r} + \mathbf{a}_i \). In this Letter, we will assume isotropic interaction \( J(\alpha_i) = J \) for transparency. At sufficiently low temperature, the honeycomb magnet exhibits FM (AF) order when \( J < 0 \) (\( J > 0 \)).

In the presence of lattice deformation, the most important effect can be incorporated to the Heisenberg model (Eq. 1) by amending the nearest-neighbor interaction to

\[ J_i = J \exp(-\beta \frac{\delta_i}{\alpha_i}), \]

where \( \delta_i \) is the bond length associated with the \( i \)-th nearest neighbor after the deformation and \( \beta \) is the Grüneisen parameter of order unity \cite{31, 32}. Without loss of generality, we take \( \beta = 1 \) for the following analytical derivations and numerical simulations.

In particular, for the twist deformation illustrated in Fig. 1(a), a nanoribbon is twisted around \( x \) axis in such a way that a lattice site originally located at position \( \mathbf{r} = (x, y, 0) \) is relocated to \( \mathbf{r} + \mathbf{u}(\mathbf{r}) = (x, y \cos \lambda x, y \sin \lambda x) \), where \( \lambda = \Omega/L \) measures the rotational angle of a \( y \)-direction chain per unit length along the \( x \) direction. Consequently, the bond length after the twist becomes

\[ \delta_i = \left[ \alpha_i^2 + \lambda^2 \alpha_x^2 \left( y^2 + y a_{i,y} \right) \right]^{1/2}. \]

The resulting nearest-neighbor interactions are then the exponentially decaying

\[ J_1 = J_2 = J \exp\{1 - [1 + \frac{1}{2} \lambda^2 (y^2 + \frac{9}{2} y)]^{1/2}\} \]

and

\[ J_3 = J. \]

Specifically, for a narrow nanoribbon with sufficiently small twist, the bond length can be estimated as

\( a \) for the fol-
\( \delta_1 = \alpha_i [1 + \lambda^2 \alpha^2_x (y^2 + y \alpha_{i,y}) / 2\alpha^2_y], \) giving rise to nearest-neighbor interactions

\[
J_3 = J_2 = J - \frac{3}{8} \lambda^2 (y^2 + \frac{3}{2} y) J, \quad J_3 = J.
\]

**Honeycomb ferromagnets.**—When the honeycomb magnet nanoribbon exhibits FM order \((J < 0)\), the Heisenberg Hamiltonian (Eq. 1) can be second-quantized by the Holstein-Primakoff transformation \([39]\)

\[
| \chi \rangle = \sum_{\alpha_i} a^\dagger_{\alpha_i} b_{\alpha_i} + a_{\alpha_i} b^\dagger_{\alpha_i} S^\dagger_{\alpha_i} (r) = S - a^\dagger_{\alpha_i} a_{\alpha_i} S^\dagger_{\beta_i} (r) = (2S - b^\dagger_{\beta_i} b_{\beta_i}) / 2b_{\beta_i}
\]

and \( S_{\alpha_i} \), where \( a_{\alpha_i} \) is the magnet annihilation operator associated with the \( A (B) \) sublattice. The resultant magnon tight-binding Hamiltonian to the bilinear order reads

\[
H^F = \sum_{\alpha_i} J_i S(\alpha_{i} b_{r+\alpha_i} + a_{\alpha_i} b^\dagger_{\alpha_i} - a^\dagger_{\alpha_i} a_{\alpha_i} - b_{r+\alpha_i} b^\dagger_{\alpha_i}),
\]

where the FM ground state energy \( E^G_F = \sum_{\alpha_i} J_i S^2 \) has been deduced from the Heisenberg Hamiltonian. By applying Fourier transform of the basis \( (a_{\alpha}, b_{\alpha})^T = N_{ac}^{-1 / 2} \sum_k e^{i k \cdot r} (a_k, b_k)^T \), where \( N_{ac} \) is the number of unit cells, we obtain the Bloch Hamiltonian matrix

\[
H_k = \sum_{\alpha_i} J_i S [\cos(k \cdot \alpha_i) \sigma_x - \sin(k \cdot \alpha_i) \sigma_y - \sigma^0],
\]

where \( \sigma_x, \sigma_y \), and \( \sigma^0 \) are the Pauli matrices and the identity matrix defined in the basis \( (a_k, b_k)^T \). Without lattice deformation, the Bloch Hamiltonian dispersion \( \epsilon_k / |J| = 3 \pm \sqrt{3} \cos(k z_a) + 4 \cos(\frac{k z_a}{\sqrt{3}}) \cos(\frac{2k z_a}{\sqrt{3}}) \}

exhibits two Dirac cones \([\text{Fig. 1(b)}]\) at the BZ corners for a fictitious lattice deformation that alters the nearest-neighbor interactions according to \( J_1 = J_2 = J + \delta J \) and \( J_3 = J \), where the variation \( \delta J \) is a constant, an effective Dirac theory can be obtained by linearizing the Bloch Hamiltonian (Eq. 4) in the vicinity of BZ corners as

\[
h \nu = h \nu_0 \left( q_z + \eta \frac{2 \delta J}{3a} \right)^{\sigma_x + \nu_0^2 q_y} \sigma_y - (3JS + 2\delta JS) \sigma^0,
\]

where the magnon velocity is \( (\nu_x, \nu_y) = \frac{3JSa}{2h} (-1, 1) \). We note that \( \delta J \) has two effects. On the one hand, it shifts the two Dirac cones uniformly in the energy dimension by an amount of \(-2\delta JS\). This effect is rather trivial and can be greatly suppressed by a Zeeman field \( B_z = -2\delta JS / g_B \mu_B \), where \( \mu_B \) is the Bohr magneton. Therefore, we will neglect this effect in the following. On the other hand, \( \delta J \) displaces the two Dirac cones oppositely in the momentum dimension by \( \nu_0 \frac{2 \delta J}{3a} \). Although an electric field can also alter the positions of magnon Dirac cones according to the Aharonov-Casher effect \([30]\) [41].
We apply numerical simulations on the tight-binding Hamiltonian of a zigzag nanoribbon with exponentially decaying interactions (Eq. 2) and find that Eq. 6 well captures the dispersion of the LLs as illustrated in Fig. 3(a). For a fixed momentum slightly away from the Dirac point \( K \), we also test the \( \sqrt{\lambda a} \) dependence of the first few Dirac-Landau levels. We find that dispersive Dirac-Landau levels are induced on right (left) of valley \( K (K') \) (Fig. 2b), reflecting the valley sensitivity of the emergent vector potential. We note that the dispersive Dirac-Landau levels only reside in the vicinity of Dirac cones. This observation is best demonstrated by the fact that the first three LLs \( (n=0, \pm 1, \pm 2) \) associated with each valley are not connected through the bulk. Instead, the edge spectral function [Fig. 2d], which considers the 20% lattice sites on the edges, reveals that LLs originating from different valleys are connected by edge states.

To obtain more insights on the twist-induced LLs, we now present the results of our numerical simulations. We apply numerical simulations on the tight-binding Hamiltonian of a zigzag FM nanoribbon with quadratically decaying interactions (Eq. 2) and find that Eq. 6 well captures the dispersion of the LLs as illustrated in Fig. 3(a). For a fixed momentum slightly away from the Dirac point \( K \), we also test the \( \sqrt{\lambda a} \) dependence of the first few Dirac-Landau levels. We find that dispersive Dirac-Landau levels are induced on right (left) of valley \( K (K') \) (Fig. 2b), reflecting the valley sensitivity of the emergent vector potential. We note that the dispersive Dirac-Landau levels only reside in the vicinity of Dirac cones. This observation is best demonstrated by the fact that the first three LLs \( (n=0, \pm 1, \pm 2) \) associated with each valley are not connected through the bulk. Instead, the edge spectral function [Fig. 2d], which considers the 20% lattice sites on the edges, reveals that LLs originating from different valleys are connected by edge states.

To obtain more insights on the twist-induced LLs, we now derive the dispersion of these LLs using quadratically decaying interactions in Eq. 2 which is a good approximation when the twist is sufficiently small. A more generic derivation regarding large twist is given in the supplemental material (SM). Since the twist-induced vector potential \( \hat{A}(y) = \eta \frac{2\hbar}{3a} \delta J(y) \hat{z} \) is to relocate the Dirac cones, for a specific momentum that is \( q_x \) on the right of \( K \) point, the Dirac point is shifted to this momentum by \( \delta \hat{A} = -\frac{\hbar}{2a} \lambda^2 y^2 \), whose curl \( \delta \hat{B} = -\partial_y \delta \hat{A} = \frac{\hbar}{2a} \lambda^2 y = \frac{\hbar}{2a} \lambda (4aq_x)^{1/2} \) results in the dispersive Dirac-Landau levels

\[
\epsilon_{\text{LL}_n}(q_x) = -3JS + \text{sgn}(n) \sqrt{n} \frac{\hbar}{2a} \lambda (4aq_x)^{1/2} \sin(n) \sqrt{n},
\]

We apply numerical simulations on the tight-binding Hamiltonian of a zigzag FM nanoribbon with quadratically decaying interactions (Eq. 2) and find that Eq. 6 well captures the dispersion of the LLs as illustrated in Fig. 3(a). For a fixed momentum slightly away from the Dirac point \( K \), we also test the \( \sqrt{\lambda a} \) dependence of the first few Dirac-Landau levels. We find that dispersive Dirac-Landau levels are induced on right (left) of valley \( K (K') \) (Fig. 2b), reflecting the valley sensitivity of the emergent vector potential. We note that the dispersive Dirac-Landau levels only reside in the vicinity of Dirac cones. This observation is best demonstrated by the fact that the first three LLs \( (n=0, \pm 1, \pm 2) \) associated with each valley are not connected through the bulk. Instead, the edge spectral function [Fig. 2d], which considers the 20% lattice sites on the edges, reveals that LLs originating from different valleys are connected by edge states.

To obtain more insights on the twist-induced LLs, we now derive the dispersion of these LLs using quadratically decaying interactions in Eq. 2 which is a good approximation when the twist is sufficiently small. A more generic derivation regarding large twist is given in the supplemental material (SM).
The Hamiltonian of a zigzag AF nanoribbon can be obtained by Bogoliubov transformation [41] as
\[ \mathcal{H}_k = \sum_i J_i S[\cos(k \cdot \alpha_i) \tau^x - \sin(k \cdot \alpha_i) \tau^y + \tau^0], \]
where \( \tau^{x,y} \) and \( \tau^0 \) are Pauli matrices and identity matrix defined in basis \((a_k, b_k^\dagger)^T\) and a constant term \(-\sum_i J_i S\) altering the Néel state energy is temporarily ignored [11]. Without lattice deformation, the dispersion of \( \mathcal{H}_k \) can be obtained by Bogoliubov transformation [11] as \( \xi_k/J_S = 3\sqrt{1 - |\xi_k|^2} \), which is doubly degenerate with \( \xi_k = \frac{3}{2} \sum_i e^{ik \cdot \alpha_i} \). \( \xi_k \) exhibits two quadratic peaks at BZ corners \( K' \) as illustrated by the blue curve in Fig. 1(b). For a zigzag nanoribbon [Fig. 1(c)], the band \( \xi_k \) of the infinite system becomes a set of bands [Fig. 4(a)].

For a fictitious lattice deformation that alters the nearest-neighbor interactions according to \( J_1 = J_2 = J + \delta J \) and \( J_3 = J \) with a constant \( \delta J \), the magnon dispersion in the vicinity of BZ corners is [11]
\[ \varepsilon_q = (3JS + 2\delta JS) - \frac{3JS}{8} a^2 \left[ \left( q_x + \eta \frac{2\delta J}{3a_J} \right)^2 + q_y^2 \right]. \]

Similar to the FM case, a constant \( \delta J \) can shift the quadratic peaks identically in the energy dimension and oppositely in the momentum dimension. The former can be compensated by an external magnetic field, while the latter can still be interpreted as an emergent vector potential \( \mathcal{A} = \eta \frac{2\delta J}{3a_J} \) same as that in ferromagnets. In the presence of twist deformation, \( \delta J = \delta J(y) \) varies along the \( y \) direction. But as long as \( \delta J(y) \) varies slowly on the lattice scale, the effect of \( \delta J(y) \) can still be treated as a vector potential \( \mathcal{A} \) shifting the quadratic peaks and producing LLs.

To support our argument, we numerically calculate the band structure of a zigzag AF nanoribbon with exponentially decaying interactions employed. We find dispersive equidistant LLs on the right (left) of quadratic peak \( K \) (\( K' \)) [Fig. 4(b)]. These LLs are 4-fold degenerate except for the zeroth LL, which is doubly degenerate. We have read off the LL energies \( \varepsilon_{LL_n} \) of the first few LLs marked by the crosses in Fig. 4(c) and find the sequence \( \varepsilon_{LL_n} - \varepsilon_{LL_0} \) indeed shows the expected \( n \) dependence on LL index \( n \). This is consistent with the quadratic dispersion (Eq. 9) at BZ corners. The bulk spectral function [Fig. 4(c)] confirms the bulk origin of the LLs with best resolution for the first three LLs \((n = 0, \pm 1, \pm 2)\) of each quadratic peak, which are connected by edge states rather than through the bulk as illustrated by the edge spectral function [Fig. 4(d)].

Following the technique we have developed for ferromagnets, we immediately find the momentum dependence of the twist-induced vector potential and the resulting gauge field as \( \mathcal{A}_x = -\frac{\hbar}{2} q_x \) and \( \mathcal{B}_z = \frac{\hbar}{2a_J} \lambda (4a q_x)^{1/2} \), respectively. The latter results in the dispersive equidistant LLs
\[ \varepsilon_{LL_n}(q_x) = \sqrt{(3JS)^2 - 2n \frac{\hbar}{\lambda} \mathcal{B}_z q_x^2} \]
\[ = 3JS - 3JS \lambda a \sqrt{4a q_x^2} |n|. \]

We apply numerical simulations of the tight-binding Hamiltonian of a zigzag AF nanoribbon with quadratically decaying interactions (Eq. 2). As illustrated in Fig. 5(a), we find Eq. 10 well captures the LL dispersion between the quadratic peak at \( K \) and the maximally displaced quadratic peak (dashed blue curve), which is \( q_x^m = \frac{\lambda a W^2}{\hbar^2} \) on the right of quadratic peak \( K \). We also examine the linear dependence on \( \lambda a \) of the first few LLs at a fixed momentum slightly away from quadratic peak \( K \). These results are summarized in Fig. 5(b).

Conclusions. We have studied the Landau level dispersion of twisted nanoribbons of ferromagnetic and antiferromagnetic honeycomb magnets. We elucidate that these Landau levels are pulled out by the twist-displaced magnon Dirac cones (quadratic peaks) from those located at Brillouin zone corners such that a correspondence can be constructed between the crystal momentum of the nanoribbon and the twist-induced gauge field, from which the dispersion of Landau levels can be explicitly derived for the ferromagnetic (antiferromagnetic) nanoribbons.
Our proposal may be carried out with honeycomb ferromagnets CrX₃ (X=F, Cl, Br, I) \[12, 43\] and antiferromagnet MnPS₃ \[44\]. The required Zeeman field canceling the twist-induced onsite energy may be provided by a fine-tuned array of magnetic force microscope tips \[45\], and the dispersive Landau levels can be imaged by neutron scattering \[46\].

The authors are indebted to R. Moessner, M. Franz, and H. Kondo for insightful discussions. ZS is supported in part by project A02 of the CRC-TR 183.

[1] B. Huang, K.-H. Jin, B. Cui, F. Zhai, J. Mei, and F. Liu, Nat. Commun. 8, 1 (2017).
[2] C. Sahin, G. Vignale, and M. E. Flatté, Phys. Rev. Materials 3, 014401 (2019).
[3] X. Meng, T. Pandey, J. Jeong, S. Fu, J. Yang, K. Chen, A. Singh, F. He, X. Xu, J. Zhou, et al., Phys. Rev. Lett. 122, 155901 (2019).
[4] J. A. Seijas-Bellido, R. Rurali, J. Íñiguez, L. Colombo, and C. Melis, Phys. Rev. Materials 3, 065401 (2019).
[5] M. Cifuentes-Quintal, O. de la Peña-Seaman, R. Heid, R. de Coss, and K-P. Bohnen, Phys. Rev. B 94, 085401 (2016).
[6] R. Bistritzer and A. H. MacDonald, Proc. Natl. Acad. Sci. U.S.A. 108, 12233 (2011).
[7] Y. Cao, V. Fatemi, A. Demir, S. Fang, S. L. Tomarken, J. Y. Luo, J. D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, E. Kaxiras, et al., Nature 556, 80 (2018).
[8] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, Nature 556, 43 (2018).
[9] Z. Zhu, M. Li, and J. Li, Phys. Rev. B 94, 155121 (2016).
[10] D. Shao, J. Ruan, J. Wu, T. Chen, Z. Guo, H. Zhang, J. Sun, L. Sheng, and D. Xing, Phys. Rev. B 96, 075112 (2017).
[11] S. Guan, Z.-M. Yu, Y. Liu, G.-B. Liu, L. Dong, Y. Lu, Y. Yao, and S. A. Yang, npj Quantum Mater. 2, 1 (2017).
[12] S. Owerre, J. Phys.: Condens. Matter 30, 245803 (2018).
[13] W. Zhang, K. Luo, Z. Chen, Z. Zhu, R. Yu, C. Fang, and H. Weng, npj Comput. Mater. 5, 1 (2019).
[14] J. Mutch, W.-C. Chen, P. Went, T. Qian, I. Z. Wilson, A. Andreev, C.-C. Chen, and J.-H. Chu, Sci. Adv. 5, eaav9771 (2019).
[15] F. Guinea, M. Katsnelson, and A. Geim, Nat. Phys. 6, 30 (2010).
[16] N. Levy, S. Burke, K. Meaker, M. Panlasigui, A. Zettl, F. Guinea, A. C. Neto, and M. Crommie, Science 329, 544 (2010).
[17] M. A. Vozmediano, M. Katsnelson, and F. Guinea, Phys. Rep. 496, 109 (2010).
[18] M. C. Rechtsman, J. M. Zeuner, A. Tünnermann, S. Wolte, M. Segev, and A. Szameit, Nat. Photonics 7, 153 (2013).
[19] A. Cortijo, Y. Ferreirós, K. Landsteiner, and M. A. Vozmediano, Phys. Rev. Lett. 115, 177202 (2015).
[20] D. Pikulin, A. Chen, and M. Franz, Phys. Rev. X 6, 041021 (2016).
[21] A. G. Grushin, J. W. Venderbos, A. Vishwanath, and R. Ilan, Phys. Rev. X 6, 041046 (2016).
[22] A. Cortijo, D. Kharzeev, K. Landsteiner, and M. A. Vozmediano, Phys. Rev. B 94, 241405 (2016).
[23] H. Sumiyoshi and S. Fujimoto, Phys. Rev. Lett. 116, 166601 (2016).
[24] V. Arjona, E. V. Castro, and M. A. Vozmediano, Phys. Rev. B 96, 081110 (2017).
[25] T. Liu, D. Pikulin, and M. Franz, Phys. Rev. B 95, 041201 (2017).
[26] T. Liu, M. Franz, and S. Fujimoto, Phys. Rev. B 96, 224518 (2017).
[27] G. Massarelli, G. Wachtel, J. Y. Wei, and A. Paramekanti, Phys. Rev. B 96, 24516 (2017).
[28] T. Matsushita, T. Liu, T. Mizushima, and S. Fujimoto, Phys. Rev. B 97, 134519 (2018).
[29] T. Kobayashi, T. Matsushita, T. Mizushima, A. Tsuruta, and S. Fujimoto, Phys. Rev. Lett. 121, 207002 (2018).
[30] E. M. Nica and M. Franz, Phys. Rev. B 97, 024520 (2018).
[31] Y. Ferreiros and M. A. Vozmediano, Phys. Rev. B 97, 054404 (2018).
[32] T. Liu and Z. Shi, Phys. Rev. B 99, 214413 (2019).
[33] T. Liu (2020), 2002.09289.
[34] F. Guinea, A. Geim, M. Katsnelson, and K. Novoselov, Phys. Rev. B 81, 035408 (2010).
[35] Y. Chang, T. Albash, and S. Haas, Phys. Rev. B 86, 125402 (2012).
[36] S. Stuj, P. Jacobse, V. Juričić, and C. M. Smith, Phys. Rev. B 92, 075424 (2015).
[37] D.-B. Zhang, G. Seifert, and K. Chang, Phys. Rev. Lett. 112, 096805 (2014).
[38] M. M. Nayga, S. Rachel, and M. Vojta, Phys. Rev. Lett. 123, 207204 (2019).
[39] T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).
[40] Y. Alaronov and A. Casher, Phys. Rev. Lett. 53, 319 (1984).
[41] See Supplemental Material for details.
[42] B. Huang, G. Clark, E. Navarro-Moratalla, D. R. Klein, R. Cheng, K. L. Seyler, D. Zhong, E. Schmidgall, M. A.
McGuire, D. H. Cobden, et al., Nature 546, 270 (2017).

[43] S. S. Pershoguba, S. Banerjee, J. Lashley, J. Park, H. Ågren, G. Aeppli, and A. V. Balatsky, Phys. Rev. X 8, 011010 (2018).

[44] Y. Shiomi, R. Takashima, and E. Saitoh, Phys. Rev. B 96, 134425 (2017).

[45] Y. Martin and H. K. Wickramasinghe, Appl. Phys. Lett. 50, 1455 (1987).

[46] B. Brockhouse, Phys. Rev. 106, 859 (1957).