Angular Distribution of Leptons in General $t\bar{t}$ Production and Decay

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ABSTRACT

Angular distribution of the secondary lepton in top-quark production followed by subsequent semi-leptonic decay is studied assuming general top-quark couplings. It is shown that the distribution does not depend on any possible anomalous $tbW$ couplings and is determined only by the standard $V-A$ decay vertex for any production mechanism if certain well-justified conditions are satisfied.

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Ever since the top quark was discovered [1], a lot of data have been accumulated. However, it still remains an open question if the top-quark couplings obey the Standard Model (SM) scheme of the electroweak forces or there exists a contribution from physics beyond the SM. Although it is its heaviness that prevented us from discovering this quark earlier, but once it is produced, the size of the mass is a great advantage. Namely, the huge mass \( m_t \simeq 174 \) GeV leads to a decay width \( \Gamma_t \) much larger than \( \Lambda_{\text{QCD}} \). Therefore a top quark decays immediately after being produced and the decay process is not influenced by fragmentation effects [2]. This is why the decay products could provide information on top-quark properties.

Next-generation \( e^+e^- \) linear colliders are expected to be a top-quark factory, and therefore a lot of attention has been paid to top-quark interactions in the process \( e^+e^- \rightarrow t\bar{t} \) (for a review, see [3] and the reference list there). Although usually only anomalous \( t\bar{t}\gamma/Z \) couplings have been considered, however there is a priori no good reason to assume that the decay part is properly described by the SM couplings. Therefore in a series of papers (see e.g. [4, 5, 6]) we have performed analyses of top-quark decay products assuming the most general couplings both for the production and the decay.

In Ref.[5] we have noticed an amazing fact: The angular distribution of the final leptons in \( e^+e^- \rightarrow t\bar{t} \rightarrow \ell^\pm \cdots \) is not sensitive to modification of the SM \( V-A \) decay vertex. The same conclusion was also reached by Rindani [7] through an independent calculation using the method of helicity amplitudes.\(^\dagger\) We usually suffer from too many parameters to be determined while testing top-quark couplings in a general model-independent way. Therefore, a distribution insensitive to a certain class of non-standard form factors is obviously a big advantage as it increases expected precision for the determination of other remaining relevant couplings [6].

In this short note we investigate if this interesting phenomenon appears only in the process \( e^+e^- \rightarrow t\bar{t} \rightarrow \ell^\pm \cdots \) or it could emerge within a wider class of processes. The result is remarkable: It holds in quite a general context under some natural and well-justified assumptions. In fact, we have observed that the angular distribution of leptons from decays of polarized top quark in its rest frame was free from the

\(^\dagger\)In Ref.[5] we have adopted the Kawasaki-Shirafuji-Tsai formalism [8], that will be also used here.
non-SM $tbW$ couplings \[^3\]. Since that was independent of a top-quark production mechanism, it was already a strong indication that the above decoupling would occur for any production process. Our goal here is to provide the general proof for this hypothesis through explicit calculations.

Let us consider a reaction like $1 + 2 \rightarrow t\bar{t} \rightarrow \ell^+ + X$ where the narrow-width approximation is applicable to the top quark.\[^2\] We denote the momenta of the initial particles $1, 2$ and the final lepton as $k_1, k_2$ and $p_\ell$, respectively. For such processes, one can apply the Kawasaki-Shirafuji-Tsai formula \[^8\] in order to determine the distribution of the final lepton:

$$
\frac{d\sigma}{dp_\ell} \equiv \frac{d\sigma}{dp_\ell}(1 + 2 \rightarrow t\bar{t} \rightarrow \ell^+ + X) = 2 \int d\Omega_t \frac{d\sigma}{d\Omega_t}(s_t = n) \frac{1}{\Gamma_t} \frac{d\Gamma}{dp_\ell}.
$$

Here $\Gamma_t$ is the top total width, $d\Gamma/dp_\ell$ is the spin-averaged top width $d\Gamma/dp_\ell \equiv d\Gamma/dp_\ell(t \rightarrow b\ell^+\nu)$ in the CM frame of $t\bar{t}$ pair, and $d\sigma(s_t = n)/d\Omega_t$ is the top-quark angular distribution

$$
\frac{d\sigma}{d\Omega_t}(s_t = n) \equiv \frac{d\sigma}{d\Omega_t}(1 + 2 \rightarrow t\bar{t}; s_t = n)
$$

with its polarization vector $s_t$ being replaced with the so-called “effective polarization vector” $n$

$$
n_\mu = -\left[ g_{\mu\nu} - \frac{p_t p_\nu}{m^2_t} \right] \sum \int d\Phi \tilde{B} \Lambda_+ \gamma_5 e^\nu B
$$

where the spinor $B$ is defined such that the matrix element for $t(s_t) \rightarrow \ell^+ + \cdots$ is expressed as $\tilde{B}u_t(p_t, s_t)$, $\Lambda_+ \equiv p_t + m_t$, $d\Phi$ is the relevant final-state phase-space element, and $\sum$ denotes the appropriate spin summation.

Equation (1) could be re-expressed in terms of the rescaled energy and the direction of the lepton, $x$ and $\Omega_\ell$:

$$
\frac{d\sigma}{dx d\Omega_\ell} = 2 \int d\Omega_t \frac{d\sigma}{d\Omega_t}(s_t = n) \frac{1}{\Gamma_t} \frac{d\Gamma}{dx d\Omega_\ell},
$$

where $x$ is defined by the $t\bar{t}$ CM-frame lepton-energy $E_\ell$ and $\beta \equiv \sqrt{1 - 4m^2_t/s}$ as

$$
x \equiv \frac{2E_\ell}{m_t} \sqrt{(1 - \beta)/(1 + \beta)}.
$$

\[^2\] As the ratio of the top-quark width to its mass is of the order of $\Gamma_t/m_t \simeq \mathcal{O}(10^{-2})$, the approximation is well justified.
It is natural to adopt $k_1$ direction as the $z$ axis to express $d\sigma/(dx d\Omega_t)$, while for the width $d\Gamma/(dx d\Omega_t)$, since it is invariant under a three-dimensional orthogonal transformation, we can use its form calculated in the frame where the top-quark-momentum ($p_t$) direction is chosen as the $z$ axis in the integrand on the right-hand side of eq.(3).

The width in such a frame has been calculated in terms of $x$, $\omega \equiv (p_t - p_\ell)^2/m_t^2$ and the azimuthal angle $\phi$ in [4], assuming $m_\ell = m_b = 0$ and the most general decay couplings

$$\Gamma^\mu_{W tb} = -\frac{g}{\sqrt{2}} V_{tb} \bar{u}(p_\ell) \left[ \gamma^\mu (f^L_1 P_L + f^R_1 P_R) - \frac{i \sigma^{\mu\nu} k_\nu}{M_W} (f^L_2 P_L + f^R_2 P_R) \right] u(p_t), \quad (4)$$

$$\bar{\Gamma}^\mu_{W tb} = -\frac{g}{\sqrt{2}} V^*_{tb} \bar{v}(p_b) \left[ \gamma^\mu (\bar{f}^L_1 P_L + \bar{f}^R_1 P_R) - \frac{i \sigma^{\mu\nu} k_\nu}{M_W} (\bar{f}^L_2 P_L + \bar{f}^R_2 P_R) \right] v(p_\ell), \quad (5)$$

where $P_L/R = (1 \mp \gamma_5)/2$, $V_{tb}$ is the $(tb)$ element of the Kobayashi-Maskawa matrix and $k$ is the momentum of $W^\pm$ boson. \[^3\] as

$$\frac{1}{\Gamma_t} \frac{d\Gamma}{dx d\omega d\phi} = \frac{1 + \beta}{2\pi \beta} \frac{3B_t}{W} \omega \left[ 1 + 2\text{Re}(f^R_2)\sqrt{r} \left( \frac{1}{1-\omega} - \frac{3}{1+2r} \right) \right] \quad (6)$$

where $r \equiv (M_W/m_t)^2$, $B_t \equiv \Gamma/\Gamma_t$, $W \equiv (1 - r)^2(1 + 2r)$, $x$ and $\omega$ are restricted as

$$0 \leq \omega \leq 1 - r, \quad 1 - x(1 + \beta)/(1 - \beta) \leq \omega \leq 1 - x, \quad (7)$$

$$r(1 - \beta)/(1 + \beta) \leq x \leq 1. \quad (8)$$

To find eq.(6) we have assumed the standard $V - A$ coupling for $W \rightarrow \ell \nu_\ell$ and kept only SM contribution and the interference terms between the SM and non-SM parts. Since we have neglected $b$-quark mass, only $f^R_2$ interferes with the SM.

For eq.(4), the effective vector $n$ defined in eq.(2) takes the following form \[^5\] as

$$n^\mu = \left( g^{\mu\nu} - \frac{p_\ell^\mu p_\ell^\nu}{m_\ell^2} \right) \frac{m_t}{p_\ell p_\ell^+} (p_{\ell^+})_\nu. \quad (9)$$

\[^3\] It is worth to mention that the form factors for top and anti-top quark satisfy

$$f_1^{L,R} = \pm \bar{f}_1^{L,R}, \quad f_2^{L,R} = \pm \bar{f}_2^{R,L},$$

where upper (lower) signs are those for $CP$-conserving (-violating) contributions \[^6\], assuming $CP$-conserving Kobayashi-Maskawa matrix. Therefore all the form factors contain both $CP$-conserving and $CP$-violating components. Since $W$ is on-shell, two extra form factors that are needed to describe the decay vertices do not contribute.
It is worth to emphasize that this form is exactly the same as the one given in the SM\, [10, 11]. Namely $n^\mu$ did not receive any non-standard corrections even though our calculation assumed the most general top-quark decay vertex parameterization.

Changing one of the independent variables from $\omega$ to $\theta$ (the angle between $p_t$ and $p_\ell$) in the differential top-quark width (3) through

$$\omega = 1 - x \frac{1 - \beta \cos \theta}{1 - \beta},$$

we have

$$\frac{d\Gamma}{dx d\Omega_\ell} = \frac{\beta x}{1 - \beta} \frac{d\Gamma}{dx d\omega d\phi}.$$  

Substituting this expression into eq.(8), we are led to

$$\frac{d\sigma}{dx d\Omega_\ell} = \frac{2\beta x}{1 - \beta} \int d\Omega_\ell \frac{d\sigma}{d\Omega_\ell}(s_t = n) \frac{1}{\Gamma_t} \frac{d\Gamma_\ell}{dx d\omega d\phi}$$

$$= \frac{3B_t}{\pi W} \frac{1 + \beta}{1 - \beta} x$$

$$\times \int d\Omega_\ell \frac{d\sigma}{d\Omega_\ell}(s_t = n) \omega \left[ 1 + 2 \text{Re}(f_2^R) \sqrt{r} \left( \frac{1}{1 - \omega} - \frac{3}{1 + 2r} \right) \right].$$

Once we have this formula, we may choose the lepton direction as the $z$ axis for $d\Omega_\ell$ integration. In this frame, the top-quark polar angle $\theta_t$ is equivalent to $\theta$. So, in the following, we will use eq.(10) with $\theta$ replaced by $\theta_t$.

Let us derive constraints on $x$. Equation (7) implies:

- $0 \leq \omega \leq 1 - r \implies r(1 - \beta)/(1 - \beta \cos \theta_t) \leq x \leq (1 - \beta)/(1 - \beta \cos \theta_t)$
- $1 - x(1 + \beta)/(1 - \beta) \leq \omega \leq 1 - x$

$$\implies x \leq x(1 - \beta \cos \theta_t)/(1 - \beta) \leq x(1 + \beta)/(1 - \beta)$$

The latter constraint is trivially satisfied. So, when we perform $x$ integration first for a fixed $\theta_t$ in order to derive the lepton angular distribution, its upper and lower bounds are

$$x_+ = (1 - \beta)/(1 - \beta \cos \theta_t), \quad x_- = r(1 - \beta)/(1 - \beta \cos \theta_t).$$

Here, whatever the top-quark production mechanism is, $d\sigma(s_t = n)/d\Omega_\ell$ depends on $p_\ell$ only through $n$ vector and, however, the effective vector $n$ has no
dependence in our case as it is directly seen in eq. \( (\text{9}) \) when the lepton mass is neglected. Consequently \( d\sigma(s_t = n)/d\Omega_t \) has no \( x \) dependence at all and the non-SM decay part of \( d\sigma/d\Omega_t \) is proportional to

\[
\int_{x_+}^{x_-} dx x\omega\left(\frac{1}{1 - \omega} - \frac{3}{1 + 2r}\right).
\]

It is not hard to confirm that this integral vanishes. That is, the non-standard-decay contribution disappears from the lepton angular distribution for any top-quark production mechanism:

\[
\frac{d\sigma}{d\Omega_t} = \int dx \frac{x}{1 - \beta} \int d\Omega_t \frac{d\sigma}{d\Omega_t}(s_t = n) \frac{1 + \beta}{\pi} \frac{3B_t}{W} \omega,
\]

\[
= \frac{2m_t^2 B_t}{\pi s} \int d\Omega_t \frac{1}{(1 - \beta \cos \theta_t)^2} \frac{d\sigma}{d\Omega_t}(s_t = n),
\]

(13)

where \( d\sigma/d\Omega_t(s_t = n) \) contains only information on the production process. The last form of this equation is the same as the one given by Arens and Sehgal within the SM \( \text{(10)} \).

Summarizing, we have shown that:

If the following conditions

- the top-quark decay is described by the sequential processes \( t \rightarrow W^+b \rightarrow b\ell^+\nu_l \),
- narrow-width approximation is applied for \( t \) and \( W \),
- only linear terms in non-standard form factors are kept,
- \( b \) quarks and final leptons are treated as massless,

are satisfied, then

- linear corrections proportional to \( f_2^R \) in the angular distribution of leptons

\[
\frac{d\sigma}{d\cos \theta_{\ell}}(1 + 2 \rightarrow t\bar{t} \rightarrow \ell^+ + X)
\]

vanish for any \( t\bar{t} \) production process. So, only \( V-A \) structure of the top-quark decay influences the leptonic angular distribution.

There are a few comments in order here.
Non-standard effects are often parameterized in terms of $SU(2) \times U(1)$ gauge symmetric, local and dim.6 effective operators \[12, 13\]. Notice however that the above theorem holds in a more general context than just the scenario of effective operators: Since $t \rightarrow W^+ b$ is a 2-body decay, all relevant momentum products are fixed by the on-mass-shell conditions. Therefore whatever the origin of $f_R^2$ and $\bar{f}_L^2$ is,\(^4\) they are just constant numbers, and the proof goes through. Observed deviation from the angular distribution, eq.(13), could indicate that $t \rightarrow W^+ b$ is not the main decay channel of the top quark.

An analogous conclusion applies also for the $\ell^-$ angular distributions from $\bar{t}$ decays, i.e., disappearance of $\bar{f}_L^2$.

As it was shown in \[5\], the effective polarization $n$-vector for the final $b$-quark distribution receives an additional contribution from anomalous decay vertex and therefore the angular distributions of $b$ quarks are sensitive to modifications of the SM top-quark decay vertex in contrast to the case of $\ell^\pm$.

In conclusion, we have proved that the lepton angular distribution in the processes $1 + 2 \rightarrow t\bar{t} \rightarrow b\ell\nu_{\ell}X$ is independent of any anomalous $tbW$ couplings regardless what is the production mechanism. Therefore the distribution is sensitive only to non-standard effects that enter the production process, and the number of unknown top-quark couplings that parameterize the distribution is reduced. We believe that for that reason the angular distribution will be useful while testing top-quark couplings at future colliders.

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\(^4\)For instance, they could be loop-generated form factors. Notice that box-diagrams lead to 3-body decays and therefore such corrections are beyond our theorem, also emission of real photons or gluons is not included in our scheme. See \[14\] for QCD corrections to the top-quark decays.
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