Quantitative Characterization of Permeability Fluctuations in Sandstone

Hernán A. Makse,1 Glenn W. Davies, 2 Shlomo Havlin1,3, Plamen Ch. Ivanov,1 Peter R. King,2 and H. Eugene Stanley1

1Center for Polymer Studies and Dept. of Physics, Boston University, Boston, MA 02215 USA
2BP Exploration Operating Company Ltd., Sunbury-on-Thames, Middx., TW16 7LN, UK
3Department of Physics, Bar-Ilan University, Ramat Gan, ISRAEL

(December 31, 2021)

Abstract

Sedimentary rocks have complicated permeability fluctuations arising from the geological processes that formed them. These permeability fluctuations significantly affect the flow of fluids through the rocks. We analyze data on two sandstone samples from different geological environments, and find that the permeability fluctuations display long-range power-law correlations characterized by an exponent $H$. For both samples, we find $H \approx 0.82 – 0.90$.

I. INTRODUCTION

Permeability in sandstone can change by many orders of magnitude over very short distances [1]. Not only are there large fluctuations in permeability but the permeability can exhibit strong anisotropy. Deriving a mathematical representation to describe these spatial fluctuations in permeable rocks is a major challenge. This represents an important technological problem as hydrocarbon is frequently found in such rocks and its efficient recovery depends on the ability to describe and predict flow through such rocks, as does contaminant dispersal and control in groundwater.
Previous studies have concentrated on spatial fluctuations in porosity \[2\]. For clean, well sorted sands porosity is independent of grain size while the permeability is known to be proportional to the square of the particle radius. For poorly sorted, non-clean sands permeability is usually not a unique function of porosity \[3,4\]. Also permeability directly affects fluid flow so it is more relevant when studying hydrocarbon recovery and groundwater flow.

Permeability fluctuations have been traditionally modeled assuming correlations of finite range \[5\]. Long-range correlations in permeability significantly affect the large-scale flow in a porous medium, so it is of practical importance to distinguish between long-range and short-range correlations. In this work, we analyze detailed permeability maps, and find that the data display long-range correlations. Specifically, we investigate two different sandstone samples. One is a Triassic, fluvial trough cross bedded sandstone from Hollington near Stafford in the East Midlands of England \[4\]. The second sample is a Triassic, planar-tabular cross bedded aeolian sandstone from Locharbriggs near Dumfries, Scotland. Although the geological process is different for both samples, in both cases size segregation of particles occurs by avalanches—modified by fluid flow in one case. We find the exponent characterizing the long-range correlations in the permeability values are the same, within errors bars, for both types of sandstone: aeolian and fluvial.

II. THE SAMPLES

The Hollington sample (Sample Ho) is an example of trough cross bedding (Fig. 1). Trough cross bedding results from the migration of sand bars (accumulations of sand) along a river bed \[1\]. The current progressively moves sand from the upstream side of the sand accumulation to the downstream. On reaching the downstream side the sand will roll down this face once a critical angle is exceeded. There is a degree of turbulence or eddying at the downstream face. This results from higher flow rate at the top of the face (foresets) and lower at the base (bottomsets). Hence, grains towards the top of the downstream slope are
better sorted and have lower packing. Those at the base are more poorly sorted, resulting from a mixture of larger grains rolling down the lee face and finer material dropping out of suspension. The degree of permeability contrast between the top and base of the bed depends on a combination of stream velocity, turbulence and available grain sizes. If velocity is very high and turbulence low foresets will be formed continuously from material avalanching down the downstream face, with little contrast between lamina thickness, grain size or sorting. If turbulence and velocity fluctuate more the thickness, grain size and sorting of individual laminae will vary.

In Fig. 1 can be seen a sharp boundary between three regions in the sample. These represent where one sand bar has been overridden by another partially eroding the top of the lower unit. This has occurred twice. Two of the zones are similar in character and have higher permeability whereas the third has consistently lower permeability. We, therefore, treat the sample as having one zone of “low” and two of “high” permeability.

The Locharbriggs sample (Sample Lo) is shown in Fig. 2. The sedimentary rock was formed by windblown sand [6]. The wind transports the sand to the crest of the dune. When the sand exceeds a critical angle it avalanches. In these avalanches the fine and coarse grains systematically segregate. The resulting layers are clearly seen in Fig. 2. In this case the eddying fluid (air) on the leeward face has insufficient energy to influence the avalanching process resulting in the foresets being planar rather than convex. Note that in this system grain size range and permeability is relatively constant parallel to the laminations. This mechanism of grain size segregation is explained in more detail in Ref. [7]. Again merging of dunes gives rise to sharp transitions in permeability, hence there are two zones. We notice also a strong anisotropy in the high permeability zone of this sample.

### III. CORRELATIONS

The permeability of the rock of both samples was measured by standard mini-permeametry [8]. The mini-permeameter essentially measures the pressure required to force
a given flow rate of gas into the rock. The permeability maps are shown in Figs. 1b and 2b. It is clear that permeability varies significantly within a very short scale. The permeability may not be an independent random process. To measure the size of the fluctuations we plot the permeability histograms for the Sample Ho in Fig. 3. The distributions for the high and low permeability regions are separated. The high permeability zone has typical permeability 3300mD, the low permeability zone has typical permeability 30mD. It should be noted that for uncompacted, well sorted, clean, quartzite sandstone, local permeability is proportional to the square of the grain radius. The high permeability zone consists of interbedded fine and coarse grain material and hence has a much higher variability. The low permeability zone is more homogeneous, consisting of more exclusively fine grained material.

Next the spatial correlations in permeability were measured. We study the correlations of the permeability field $k(i, j)$ $(i, j = 1, \ldots, n_x, n_y)$ along the $x$ and $y$ directions (see Fig. 1b). To this end, we first integrate the permeability variables along both directions separately, by calculating the “net displacements” $x_j(\ell)$ and $y_i(\ell)$

$$x_j(\ell) \equiv \sum_{i=1}^{\ell} (k(i, j) - \overline{k(j)}) \quad [j = 1, \ldots, n_y], \quad (1a)$$

$$y_i(\ell) \equiv \sum_{j=1}^{\ell} (k(i, j) - \overline{k(i)}) \quad [i = 1, \ldots, n_x], \quad (1b)$$

where $\overline{k(j)} = (1/n_x) \sum_{i=1}^{n_x} k(i, j)$ and $\overline{k(i)} = (1/n_y) \sum_{j=1}^{n_y} k(i, j)$. Then we calculate the variance $V_x(\ell) \equiv \langle (x(\ell))^2 - \overline{x(\ell)}^2 \rangle^{1/2}$ and $V_y(\ell) \equiv \langle (y(\ell))^2 - \overline{y(\ell)}^2 \rangle^{1/2}$ as a function of the lag $\ell$. The spatial average over a window of size $\ell$ is denoted by the overbar, and the disorder average over different displacements $(x_j$ and $y_i$) is denoted by the angular brackets. The scaling behavior of the variance $V(\ell) \sim \ell^H$ can distinguish between short and long range correlations. For uncorrelated permeability variables, $H = 1/2$, while $1/2 < H < 1$ indicates persistent long-range correlations among the variables. The correlation exponent $H$ describes the “roughness of the permeability landscape”. The method described so far is the standard rms fluctuations analysis which however, is known to fail in the following situations: (i) when the signal is nonstationary, and (ii) when the signal is highly correlated $H \simeq 1$. 
In case (i), the method detects spurious correlations due to the patchiness of the signal [12], while in case (ii) the method gives smaller effective exponents (in particular when small samples are used) because the variance has an upper bound $V(\ell) < \ell$ and therefore the method cannot detect fluctuations with exponent $H \geq 1$ [12]. Apart from possible nonstationarities, in our case we find that the permeability values are strongly correlated.

To overcome the limitations of the rms method, we will analyze the spatial correlations of the permeability by using the detrended fluctuation analysis (DFA) of Ref. [11] and the wavelet analysis [13].

The results for the permeability correlations for the Sample Ho are shown in Fig. 4. In Fig. 4a we show the correlations for both high and low permeability zones combined measured in the $x$ and $y$ directions. In this case, before calculating the variance, the permeability is normalized by dividing by the standard deviation calculated independently for each direction. The data are consistent with power-law correlations; using the DFA method, we find $H_x = 0.89 \pm 0.06$ and $H_y = 0.90 \pm 0.06$. Results for the correlations along the $x$ direction are shown separately for the high and low permeability zones in Fig. 4b. The correlations are satisfactorily modeled by a power law where $H_x \simeq 0.89$, independent of the magnitude of the overall permeability. These values are confirmed, within the error bars, using the wavelet analysis. We find that $H_x = 0.82 \pm 0.06$ and $H_y = 0.84 \pm 0.06$.

As seen in Fig. 2 the high permeability zone of sample Lo presents strong anisotropy with anisotropic axes $(x', y')$ not coincident with the coordinate frame $(x, y)$ (see Fig. 2b). We calculate the variance along the $y'$ direction (parallel to the direction of the crests) and find using the DFA method $H = 0.85 \pm 0.06$ (Fig. 3); a value that is consistent with the findings for the Sample Ho. Using wavelet analysis, we find $H = 0.84 \pm 0.06$. Along the $x'$ direction a periodic morphology is observed with a wave length of about 60 mm. This introduces a characteristic length scale so that no scale invariance power law correlations are expected along this direction. The existence of this laminar periodic structure is consistent with a depositional model of sand dune dynamics [7].
IV. DISCUSSIONS

Spatial fluctuations in rock permeability exist, and require quantitative methods to describe them. We have shown that permeability correlations in two different rock samples can be well described using a power law. Further, the exponent is quite similar for the two samples. The essential physical feature in the formation of these two samples is avalanching of the sand grains, which gives rise to segregation and alternation in the fine and coarse material. It is possible that this mechanism could explain the apparent universality in the power law correlation for these rock types.

These spatial fluctuations have significant consequences for prediction of, e.g., hydrocarbon recovery or contaminant transport in groundwater [3, 14]. The fact that there exist long range correlations implies that the spread in contaminant transport could occur be much faster than would be predicted from a short range correlation model.

The authors would like to thank BP Exploration Operating Company for financial support and permission to publish this paper. H. A. M. wish to thank S. Tomassone for valuable discussions. S. H. wish to thank the Israel Science Foundation for financial support.
REFERENCES

[1] J. R. L. Allen, *Principles of Physical Sedimentology* (George Allen and Unwin, London 1985).

[2] T. A. Hewett, SPE 15386, 61st Ann. Tech. Conf. of SPE, New Orleans, Louisiana (1986); T. A. Hewett and R. A. Behrens, SPE 18326, 63st Ann. Tech. Conf. of SPE, Houston, Texas (1988).

[3] M. Sahimi, Rev. Mod. Phys. **65**, 1393 (1993).

[4] A. C. Brayshaw, G. W. Davies, and P. W. M. Corbett, in *Advances in Fluvial Dynamics and Stratigraphy*, edited by M. Dawson (1995).

[5] L. Smith and F. W. Schwartz, Water Resour. Res. **16**, 303 (1979).

[6] R. A. Bagnold, *The physics of blown sand and desert dunes* (Chapman and Hall, London 1941).

[7] H. A. Makse, S. Havlin, P. R. King, and H. E. Stanley (submitted).

[8] C. Halvorsen and A. Hurst, in *Advances in Core Evaluation Accuracy and Prediction*, edited by P. F. Worthington (Gordon and Breach Science Publishers, 1990).

[9] H.-O. Peitgen and D. Saupe, eds., *The Science of Fractal Images* (Springer-Verlag, New York 1988); T. Vicsek, *Fractal Growth Phenomena*, 2nd ed. (World Scientific, Singapore 1991); A. Bunde and S. Havlin, eds., *Fractals in Science* (Springer-Verlag, Berlin 1994); A.-L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge 1995).

[10] The exponent $H$ is called the Hurst exponent for the associated fractional Brownian motion (fBm) defined by Eq. (1). However, it should be noted that the permeability is a fractional Gaussian noise (fGn).

[11] C.-K. Peng, *et al.*, Phys. Rev. E **49**, 1685 (1994).
[12] H. Leschhorn and L.-H. Tang, Phys. Rev. Lett. 70, 2973 (1993); H. A. Makse and L. A. N. Amaral, Europhys. Lett. 31, 379 (1995).

[13] J. F. Muzy, E. Bacry, and A. Arneodo, Phys. Rev. Lett. 67, 3515 (1991). A. Arneodo, E. Bacry, P. V. Graves, and J. F. Muzy, *ibid* 74, 3292 (1995).

[14] S. Prakash *et al.* Phys. Rev A 46, R1724 (1992).
FIGURES

FIG. 1. (a) Photograph of one of the slabs for the Sample Ho. The sample consists of two slabs of 474 mm by 276 mm and 10 mm thick. Three faces at height $z = 0$, $z = 10$, and $z = 20$ mm were used to study the permeability pattern, from which the $z = 0$ face is shown in this figure. Unfortunately the measurements of one face were corrupted by instrumentation error and so only three faces could be used. (b) Permeability map of 1a. The permeability was measured every 10mm in the $x$ direction and every 4mm in the $y$ direction. Therefore, a grid of $n_x = 48$ by $n_y = 69$ permeability values was obtained.

FIG. 2. (a) Photograph of the face $z = 0$ of the Sample Lo. The sample consists of two slabs of 448mm by 246mm and 10 mm thick, and again only three faces were used in this study. (b) Permeability map of 2a. The permeability was measured every 12mm and 4mm in the $x$ and $y$ directions, respectively, so that a grid of $n_x = 38$ by $n_y = 61$ was obtained. Notice the strong anisotropy of this sample manifested by the crests elongated along the $y'$ direction.

FIG. 3. Normalized permeability distributions for the Sample Ho corresponding to (a) low permeability zone, and (b) high permeability zone. In both figures we plot the distributions corresponding to three different faces of the sample. The distributions are fitted by Gaussian forms. We notice the large difference in the mean value of the permeability between the low and high permeability zones.
FIG. 4. Log-log plot of the variances of the permeability calculated using the DFA method. (a) Variances $V_x(\ell)$ and $V_y(\ell)$ along the $x$ and $y$ directions respectively averaged over the three different faces of the sample, and over the high and low permeability zones together for $V_x(\ell)$ and over the high permeability zone for $V_y(\ell)$. The power law relationship between the variance and the separation distance $\ell$ is characterized by exponents $H_x = 0.89 \pm 0.06$ and $H_y = 0.90 \pm 0.06$. The exponents are the same within error bars indicating the isotropy of the correlations in the $xy$ plane. (b) Variance $V_x(\ell)$ calculated along the $x$ direction for the high and low permeability zones, separately. Data are averaged over the three different faces of the sample. Both set of data are consistent with a power law $H_x \simeq 0.89$, showing that the spatial correlations are the same in both zones.

FIG. 5. Log-log plot of the variance calculated along the $y'$ for the high permeability zone of Sample Lo, averaged over the three different faces of the sample. Along the $y'$ direction, a correlation exponent of $H = 0.85 \pm 0.06$ is found. However, along the $x'$ direction, a periodic pattern is observed. Thus the anisotropy in this sample is manifested in a change of behavior from long-range correlation scaling along $y'$ to periodic morphology along $x'$. 