"WE FEEL THAT YOU JUST DON'T APPRECIATE THE IMPORTANCE OF WHAT WE DO HERE"
3. Nuclear Behaviour
Properties of the N-N Force

- The force is *charge symmetric*
- The force is (nearly) *charge independent*
- The force is *spin dependent*
- The force has a *non-central component*
- The force depends on the *relative velocity or momentum* of the nucleons
- The force has a *repulsive core*

- 'Exchange model': force mediated by pion exchange

- See *Phys490_latex.pdf* for more details
Isospin

Isospin was introduced by German theoretical physicist Werner Karl Heisenberg in 1932 as a way to distinguish between protons and neutrons.

- The quantum mathematics of spinors (spin up – spin down entities) can also be applied to nucleons

- In an abstract “isospace” neutrons are spin up and protons spin down!
Isospin and Nuclides

- Given a nucleus with $A$ nucleons (combined of $Z$ and $N$ nucleons):

- The third component of isospin $T_z$ describes a particular nuclide with specific $Z$ and $N$ values

- Simply: $T_z = \frac{1}{2} (N - Z)$

- Isospin $I$ is conserved in nuclear interactions
Isospin Substates Of Isobars

- By analogy with spin, an isospin $T$ state has $(2T+1)$ substates
- The $T_z$ substates correspond to states in different nuclei
Isobaric Analogue States

- **Isodoublet** states occur in odd-$A$ nuclei
- **Isotriplet** states occur in even-$A$ (even-even and odd-odd) nuclei

| Isodoublets | Isotriplets |
|-------------|-------------|
| 2.87        | 3.55        |
| 2.79        | 4.65        |
| 1.75        | 1.98        |
| 0.35        | 3.06        |
| 0           | 1.89        |
| $^1\!^9\text{Ne}_{11}$ | $^18\text{O}_{10}$ |
| $^1\!^1\text{Na}_{10}$ | $^{18}\text{Ne}_{8}$ |
| $T_z=+1/2$  | $T_z=0$     |
| $T_z=-1/2$  | $T_z=+1$    |

- $T=0$ states are isosinglets.
A=51 Mirror Nuclei

\( ^{51}_{26} \text{Fe}_{25} \)

\[ \tau = 2.87 \text{ ns} \]

\( ^{51}_{25} \text{Mn}_{26} \)

\[ \tau = 2.07 \text{ ns} \]
Mirror Nuclei: $f_{7/2}$ Shell

|   |   | 28 |   |   |
|---|---|----|---|---|
| 27 | $f_{7/2}$-shell |
| 26 |
| 25 |
| 24 | 45Cr | 46Cr | 47Cr | 48Cr | 49Cr | 50Cr | 51Cr |
| 23 | 44V | 45V | 46V | 47V | 48V | 49V |
| 22 | 42Ti | 43Ti | 44Ti | 45Ti | 46Ti | 47Ti |
| 21 | 41Sc | 42Sc | 43Sc | 44Sc | 45Sc |
| 20 | 40Ca | 41Ca | 42Ca | 43Ca |

$N=Z$ Line

Complete isobaric multiplets in $f_{7/2}$-shell…

$T=1/2$ mirror-pairs

$T=1$ isobaric triplets

$T_z = -3/2 (N=Z-3)$ members of the $T=3/2$ quadruplet - no excited states known…
Mirror Nuclei

- The force between two nucleons has the property of charge symmetry and charge independence.
- The two nuclei $^{20}\text{Na}$ and $^{20}\text{F}$ are examples of mirror nuclei.
- The numbers of protons and neutrons are exchanged.
- Reflection about the $N = Z$ line.
Independent Particle Model

- In principle, if the form of the nucleon-nucleon potential is known for bare nucleons, then the energy of a nucleon moving inside a nucleus can be calculated.

- This is a very difficult problem to solve as the nucleon interacts simultaneously with all the other nucleons.

- Use an average potential

Energy as a function of separation

\[ V_{ij} \]

repulsive core

short range attraction

\[ r = r_i - r_j \]
Independent Particle Model

- The Hamiltonian is of the form:
  \[ H = \sum (T_i + V_{ij}) \]
- It has \( 3A \) degrees of freedom and is too complicated to solve except for the lightest nuclei \( (A < 12) \)

- Instead we use an average “mean-field” potential:
  \[ H = H_{\text{mean field}} + H_{\text{residual}} \]
  where \( H_{\text{residual}} \) contains interactions between nucleons that are not accounted for by the average potential, especially interactions among valence nucleons
Nuclear Mean Free Path

- Why is it that the Independent Particle picture of nuclear motion works?

- The Pauli Exclusion Principle (PEP) gives nucleons essentially infinite mean free path.

- However, if the range of the nuclear force was 2 to 3 times longer, then nuclei could have been crystalline.
Particles in a (Potential) Box

- The short range interaction between nucleons means that each nucleon moves in an average potential
- The average separation (~ 2.4 fm) is larger than the range of the nuclear force (1.4 ~ fm)
- Nuclei cannot easily change state unless close to the Fermi surface (PEP)

Energy levels up to the 'Fermi level'
Degenerate Fermi Gas Model

- This is a simple model in which nucleons are placed in a volume $V = \frac{4\pi R^3}{3}$ and the interactions between them are ignored.

- A Fermi sea is formed, filled up to the energy corresponding to the Fermi momentum:
  \[ E_F = \frac{p_F^2}{2m} = \frac{\hbar^2 k_F^2}{2m} \]

- The binding energy per nucleon is:
  \[ B = -\frac{E}{A} = -\frac{3}{5} T_F + \frac{1}{2} V_0 \]
  where $T_F$ is the kinetic energy at the Fermi surface.
Nucleon Effective Mass

- The nuclear force has the property of **saturation** so that \( B(A,Z) \) is independent of \( A \) caused by the Pauli Exclusion Principle (PEP), its spin and isospin dependence, and (less importantly) the repulsive core.

- The nuclear separation energy \( S \) is the difference between the energy of a nucleon outside the nucleus and the energy of the Fermi level \( E_F \):

\[
S = B = -\frac{1}{5} T_F
\]

- Wrong! \( (S > 0) \) - the nucleon has an effective mass \( (m^* > m_n) \) when moving in a nucleus.
Some Nuclear Quantities

- Number density \((A/V)\) measured:
  \[ \rho \sim 0.17 \text{ fm}^{-3} (\sim 1.5 \times 10^{18} \text{ kg/m}^3) \]

- Fermi momentum:
  \[ k_F = \frac{p_F}{\hbar} \sim 1.4 \text{ fm}^{-1} \]

- Fermi energy:
  \[ E_F \sim 10 \text{ MeV} \]

- Kinetic energy of a nucleon in the nucleus:
  \[ \frac{3}{5}E_F \sim 6 \text{ MeV} \]
  corresponding to a velocity \(v/c \sim 0.14\)
Nuclear Potentials

There are two approaches:

1. An empirical form of the potential is assumed, e.g. square well, harmonic oscillator, Woods-Saxon

2. The mean field is generated self-consistently from the nucleon-nucleon interaction
Summary

- Isospin (mirror nuclei)
- Nuclear Potentials (particle in a box)