Estimation of the energy contributions of a hydroelectric power station using space parameter exploration

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Abstract. From the study of nonlinear dynamics, the study of chaotic dynamics in deterministic systems has been developed. If a system is deterministic, its behavior would be easy to predict. However, there are systems in which their behavior seems to be unpredictable due to the complexity of the dynamics of the system, which is reflected in an aperiodic behavior. This article presents the application of parameter exploration techniques for the study of aperiodic deterministic systems using time constraint estimation through the calculation of the Lyapunov exponent in order to estimate future system values. This technique is developed on a part of the energy matrix of the Colombian system, especially with hydroelectric production.

1. Introduction
The study of chaotic dynamics in deterministic systems has become very popular in recent decades, emerging from the study of nonlinear dynamics; this is due to the amazing discoveries that his study has thrown [1]. Of course, it would be natural to think that, if a system is deterministic, its behavior would be easy to predict; however, there are systems in which their behavior seems not to be predictable, not due to lack of determinism, but due to the complexity of the system dynamics which is reflected in an aperiodic behavior that results in an increase in the complexity of the analysis [2]. This phenomenon can be observed in systems where very similar initial conditions show totally different behaviors, this due to the exponential divergence between nearby trajectories [3].

The dynamics of the attractor exhibits a sensitive dependence on the initial conditions that means that two trajectories that start very close to each other, quickly diverge from each other and will have completely different futures, which indicates that a long-term prediction of a chaotic system in which small uncertainties at the entrance are amplified extremely quickly becomes practically impossible, in [4] and [5] techniques for predicting values of chaotic systems are raised through the study of parameter exploration, however, exploration of parameters is like walking through the jungle, an arduous path where it is possible to find all kinds of behaviors, intersecting limit cycles, intermittent chaos, noisy periodicity and even strange attractors [6], therefore, the need arises use some criteria for the exploration of parameters which allows to reconstruct the space of states d and a system, in which a periodicity is observed that makes possible the prediction of behaviors of said system [7].

In this article we propose a technique for estimating values using time-limited parameter exploration using Lyapunov exponents.
2. The phase spaces
The physical space, phase space or phase diagram is a mathematical construction that allows to represent the set of conjugated positions and moments of a particle system. More technically, the phase space is a differentiable variety of even dimension, such that the coordinates of each point represent both the generalized positions and their corresponding conjugate moments. That is, phase space is a way of representing a dynamic system which consists of a space that has as many dimensions as the number of variables needed to specify the state of the original system. Each coordinate axis of this space represents one of the variables that make up the system. In essence, the phase space represents, with a certain vector structure, the set of possible states in which the modeled system may be [8]. In this space, the graph of the various points obtained from the values of the variables for different times provides a description of the state of the system over time. The orbit of a particular state will be represented by a curve or trajectory in this space. This representation of the system allows a qualitative description of the temporal evolution of the model we are studying.

2.1. Limit cycle
If we have a closed trajectory in phase space for which we can guarantee the existence of at least one other trajectory that can become it when time tends to infinity (positive infinity or negative infinity), we say that said trajectory is a limit cycle; behavior that is common in some non-linear systems. By Jordan's curvature theorem, each closed path divides the plane into two regions, the interior and exterior of the curve, from where, given a limit cycle and a path in its interior that approaches the cycle when the time tends to infinity, there will be a neighborhood around the limit cycle such that all the trajectories that begin inside it will approach the limit cycle [9]. In the event that all neighboring trajectories approach the limit cycle as time approaches infinity, it is known as a stable manifold, attractor or attractor limit cycle (ω-cycle limit). On the other hand, if all the trajectories in the vicinity of approximate as time tends to negative infinity, then it is known as an unstable limit cycle. Figure 1 shows an example of a stable or attractor limit cycle, obtained from the Van der Pol oscillator.

![Figure 1. Van der Pol oscillator limit cycle.](image)

2.2. Dissipative system and attractor of a system
A dynamic system is said to be dissipative if the volume of any set in the phase space decreases over time. To better understand this concept, let's take a set of points $S_0$ in the phase space, this set has a volume that we denote by $V_0$. Now consider all the points belonging to $S_0$ each as initial conditions of different trajectories. Then letting the system evolve for a time $T$ we will have that the trajectories that started in $S_0$ have evolved forming a new set $S_T$, which has a volume $V_T$. If for every set $S_0$ and for all time $T$ $V_0 > V_T$ is fulfilled, that is, the volume in the phase space is contracted under the action of the system of differential equations, it is said that the system, which determines these differential equations, is dissipative. Therefore, in any dissipative system, if we start with a set where there is a wide variety of initial conditions, the trajectories eventually converge to a set of smaller and smaller volume until they become a set of null volume. An important object in the study of dynamic systems and that in particular
characterizes dissipative systems, is that of attractor. An attractor \( A \) is a set of zero volume in the phase space that satisfies the following conditions [10].

- \( A \) is invariant; that is, if \( x(t) \) is a path such that \( x(0) \in A \), then \( x(t) \in A \) for all time \( t \).
- There exists a set \( U \), which contains \( A \), such that if \( x(0) \in U \) the distance from \( x(t) \) to \( A \) tends to 0 as \( t \) tends to \( \infty \). The largest \( U \) that satisfies this property is called the base of the attractor.
- \( A \) is minimal; that is, there is no proper subset of \( A \) that satisfies the two previous conditions.

There are different types of attractors depending on the size of the system in the phase space (number of dependent variables that make up the system). For example, a bounded solution of an autonomous differential equation on the line must converge to a point, which is called an attractive equilibrium point. For autonomous differential equations in the plane the bounded solutions can converge to an equilibrium point and a new type of behavior is possible: they can converge to a closed curve called periodic orbit or limit cycle. Finally, for systems of equations with 3 or more dependent variables, the behaviors of the solutions are varied and there are sets of different forms that attract the trajectories of these systems.

2.3. Attractor reconstruction

Suppose we are interested in studying the dynamics and evolution of a phenomenon in which \( m \) variables interact, that is \( m \) time-dependent variables determine each state \( X \); that is, \( X(t) = (x_1(t), x_2(t), x_3(t), \ldots, x_m(t)) \). This phenomenon in mathematical terms will be represented by a system of equations that is initially unknown. Actually, the information you have about the system are measurements of one of the \( m \) variables, let’s assume it is the first \( x_1(t) \). Therefore there are values \( s_0 = x_1(\tau), s_1 = x_1(t+\tau), s_2 = x_1(t+2\tau), s_3 = x_1(t+3\tau), \ldots, s_k = x_1(k\tau) \) taken in \( \tau \) length intervals, this sequence of measurements is called time series. With the values \( s_i \) it is built the called vectors with delay defined by Equation (1).

\[
Y_i = (s_i, s_{i-\tau}, \ldots, s_{i-2d\tau}).
\] (1)

The value \( \tau \) is called the delay time and the value \( d \) corresponds to the size of the system attractor. For example, if the attractor is a particular state, a point in the phase space (space of \( n \) dimensions), then \( d = 0 \); if the attractor is a set of states that form a curve in the phase space, then \( d = 1 \) [11].

2.4. Prediction of behavior and exponent of Lyapunov

A characteristic of the chaotic systems is their high sensitivity to the change in the initial conditions of the system, where very similar initial conditions yield totally different behaviors, this due to the exponential divergence between nearby trajectories. The initial conditions greatly affect the attractors since it can happen that despite the fact that two trajectories that start very close, they quickly move away from each other, having totally different futures, which indicates that a long-term prediction of a chaotic system in the which small uncertainties in the input are amplified extremely quickly becomes practically impossible, therefore it is necessary to determine the moment at which the prediction loses reliability, as illustrated in Figure 2.

![Figure 2. Sensitivity of a system to the initial conditions.](image_url)
The number \( \lambda \) is usually called an exponent of Lyapunov, however, this is a "loose" terminology since an \( n \)-dimensional system has \( n \) exponents of Lyapunov which are defined as follows. Consider the evolution of an infinitesimal sphere (in phase space) of disturbed initial conditions. During its evolution the sphere is distorted in an infinitesimal ellipsoid, where \( \delta_k(t), k = 1, 2, 3, n \) denote the length of the main axis of the ellipsoid, therefore. Equation (2):

\[
\delta_k(t) \approx \delta_k(0)e^{\lambda_k t},
\]

where \( \lambda_k \) are the exponents of Lyapunov; for large values of \( t \), the diameter of the ellipsoid is controlled by the most positive value of \( \lambda_k \), therefore the expression Equation (2) denotes the largest Lyapunov exponent. When a system has a positive Lyapunov exponent, a phenomenon known as a time horizon occurs, in which all predictions beyond this time will be wrong, as shown in Figure 2. The time horizon can be estimated at through Equation (3).

\[
t_{\text{horizone}} \approx \frac{1}{\lambda} \ln \left( \frac{a}{\| \delta \|} \right),
\]

where \( a \) represents the tolerance of the measure, \( \delta \) is the error associated with the measurement and \( \lambda \) the exponent of Lyapunov, the really problematic aspect of the expression Equation (3) is the linear dependence of the error, if a prediction has a tolerance in the range of \( a \) can be considered as an acceptable estimate, however, the prediction becomes intolerable when \( \| \delta \| > a \) [12].

3. Analysis and results

Before carrying out the calculations necessary for the reconstruction of the phase space through the exploration of parameters, it is necessary to know the behavior of the system, and even more important to determine if the system has chaotic behavior, be understood by chaotic system that deterministic system that presents a chaotic behavior Figure 3 shows the behavior of the energy contributions made by a hydroelectric power plant for a year.

![Figure 3. Measurement of energy contributions of a hydroelectric power plant for 365 days.](image-url)
Once the chaotic behavior of the system has been determined, it is possible to use techniques such as the Hurst exponent to reaffirm the chaos present in the system and the possible reconstruction, however, observing Figure 3 it is evident the aperiodic behavior that it presents; therefore, it is possible to proceed with the reconstruction of the phase space using parameter exploration, for this the phase space is reconstructed by varying the parameter τ known as the delay time and setting the embedding space of the attractor to a value of 15. Figures 4 show the attractor reconstruction using delay values of 5, 10, 14 and 18 respectively.

Based on Figure 4, it is possible to observe the deformation of the trajectories of the system, to the point that predicting its trajectory becomes almost impossible, this is because the Lyapunov exponent of the original system is positive in nature. Therefore, there is a point at which it is impossible to try to predict the behavior of the system as shown in Figure 2.

In order to estimate the energy contributions of the hydroelectric power station, the reconstructed phase space in Figure 1 is used as a reference, and Equation (3) is used, with a tolerance of $1 \times 10^{-8}$, with this value it was obtained that it is possible to predict the behavior of the system up to 1.7 days, from this moment on any prediction will exceed the pre-established tolerance.

![Attractor reconstruction](image)

**Figure 4.** Attractor reconstruction (a) $\tau=5$ (b) $\tau=10$ (c) $\tau=14$ (d) $\tau=18$.

4. Conclusions
The correct reconstruction of the phase space of a chaotic system is strongly linked to the estimation of the parameter $\tau$ known as the delay time, the correct estimation of this parameter provides high reliability in the results derived from it. Due to the aperiodicity of some real systems, the estimation of long-term future values is a highly complex task, since these systems are susceptible to small disturbances in their initial conditions. As a result of the analysis of the behavior of the energy inputs of a hydroelectric plant during a year in order to determine an optimal value for the time delay and to know the maximum time in which a prediction can maintain a high degree of reliability, it was obtained that with a tolerance of $1 \times 10^{-8}$, it is possible to predict the behavior of the system up to 1.7 days.
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