Phonon-induced electron relaxation in quantum rings

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We study electron-acoustic phonon scattering and electron relaxation in quantum rings in the absence and in the presence of external magnetic fields. Electron-phonon interaction is accounted for both the deformation potential and piezoelectric coupling. At zero magnetic field and small ring radius, the deformation potential phonon scattering is orders of magnitude larger than the piezoelectric one. However, the piezoelectric coupling is found to become important when the ring radius and magnetic field increase. It can be the dominant scattering mechanism at large ring radii and/or large magnetic fields. In comparison with the quantum dot case, the acoustic phonon scattering is stronger in quantum rings.

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I. INTRODUCTION

Recent successful fabrication of self-assembled quantum rings (QRs) in nanometer dimensions has triggered a great deal of interest in theoretical and experimental research. As a matter of fact, the QR structures could lead to the development of novel devices in the fields of quantum cryptography, quantum computation, optics and optoelectronics. These nanoscale quantum objects also provide an excellent ground to investigate many fundamental physical phenomena such as quantum coherence, which is also critical for quantum information processing.

Study on electron energy relaxation in semiconductor quantum dots (QDs) has received considerable attention due to the critical importance of carrier-relaxation processes in the performance of novel semiconductor devices (e.g., lasers and infrared photo-detectors) based on QDs. One fundamental question is how the discrete density of electronic states in these nanostructures affects the relaxation time of excited electrons. It was recognized that electron-phonon scattering is severely suppressed in QDs. In a QD where the energy level separation is small as compared to the optical phonon energy, the electron-acoustic phonon interaction is dominant.

Electron relaxation in QDs due to acoustic phonon scattering has been studied extensively over the past few years. Coupling to acoustic phonons has been effectively proved to be the most important mechanism of electron relaxation from excited states in laterally confined QDs and vertically coupled quantum dots (CQDs). Both the deformation potential (DF) and the piezoelectric (PZ) acoustic phonons can scatter strongly the excited electrons. These scattering mechanisms have been recently studied for single and coupled QDs. Theoretical estimations of phonon-induced scattering rates have shown a satisfactory agreement with transport spectroscopy experiments.

In this work, we extend previous calculations of the electron-acoustic-phonon relaxation to the case of quantum rings. In particular, we consider GaAs/AlGaAs QRs. We study single electron relaxation in a QR as a necessary first step to further investigations of the few-electron and the coupled quantum ring cases. The electron is considered photo-excited or electrically injected to an excited state. The ring is laterally confined and the effects of the confinement and external magnetic fields are investigated. We took into account both DF and PZ phonon scattering mechanisms and we stress the differences between the QD and QR geometries. The scattering rate demonstrates strong oscillations as a function of the confinement and the external field.

The outline of our paper is as follows. In Sec. II we present our theoretical model and explain the numerical methods to solve the single-particle problem. In Sec. III we show the numerical calculations of the relaxation rates in the absence and in the presence of magnetic fields, separately. Based on our results, we comment about the differences in the electron relaxation processes between QRs and QDs. Finally, we conclude in Sec. IV.

II. THEORETICAL MODEL

Within the effective mass and envelope function approximation we set up the following model to study the electron relaxation time in QRs. The confinement potential of the QR in the $xy$ plane is modeled by a displaced parabolic function $V(\vec{r}) = m^* \omega_0^2 (\vec{r} \parallel - r_0)^2/2$, where $m^*$ is the electron effective mass, $\vec{r} \parallel \equiv (x, y)$, $r_0$ is the ring radius, and $\omega_0$ describes the strength of the harmonic confinement in the plane. The confinement potential in the $z$ direction $V_{QW}(z)$ is considered as a quantum well of thickness $L_z$ and barrier height $V_0$. When a magnetic field $B$ is applied in the $z$ direction, the Hamiltonian for the single electron problem is given by

$$H = -\frac{(\vec{p} + e\vec{A})^2}{2m^*} + \frac{1}{2} m^* \omega_0^2 (\vec{r} \parallel - r_0)^2 + V_0 \Theta(|z| - L_z/2),$$

where $\vec{p} = -i\hbar \nabla$, $\vec{A} = (-By/2, Bz/2, 0)$, $\Theta(z)$ is the Heavyside function and $V_0$ the conduction band offset between the GaAs well and the AlGaAs barrier.
Since the vertical confinement in the z direction is usually much stronger than the lateral one, the in-plane motion and the vertical one can be treated as decoupled. The electron wavefunction can thereby be written as a product:

\[ \psi(\vec{r}) = \psi_z(\vec{r}_\parallel) \chi(z), \]  
\[ (2) \]

where \( \vec{r} \equiv (x, y, z) \). The functions \( \psi_z(\vec{r}_\parallel) \) and \( \chi(z) \) describe the electron motion in the xy plane and in the z direction, respectively.

It is known that the present eigenvalue problem of the QR is not analytically solvable. Therefore, we evaluate the matrix elements of the in-plane ring potential on the basis of the two-dimensional parabolic QD, that is, on the Fock-Darwin basis, and diagonalize the corresponding matrix. Thus, the eigenvalues give the energy levels of the in-plane motion and the eigenvector components \( \kappa^z_n \) yield the matrix of the change of basis. The total electron eigenfunction is then a superposition of the Fock-Darwin orbitals

\[ \psi_{j,m,g}(\vec{r}) = \left( \sum_n \kappa^z_n \phi_{n,m}(x, y) \right) \chi_g(z), \]  
\[ (3) \]

where \( \phi_{n,m} \) is the n-th Fock-Darwin orbital with azimuthal angular momentum \( m \) and \( \chi_g \) is the g-th finite quantum well solution. The eigenstates can be labeled by the set of three quantum numbers \((j, m, g)\), with \( j = 0, 1, 2, \ldots \), \( m = 0, \pm 1, \pm 2, \ldots \) and \( g = 0, 1, 2, \ldots \), respectively. Throughout this paper we assume that the electron is “frozen” in the lowest energy state of the quantum well along the z direction, which means that we always consider \( g = 0 \). This assumption is justified by the fact that in normal growth condition the vertical confinement is much stronger than the in-plane one.

The electron-acoustic-phonon interaction Hamiltonian, disregarding umklapp processes, is given by

\[ H_{ep} = \sum_{\vec{q}, \lambda} M_{\lambda}(\vec{q}) (a^\dagger_{\vec{q}, \lambda} + a_{\vec{q}, \lambda}) \exp(i \vec{q} \cdot \vec{r}), \]  
\[ (4) \]

where \( M_{\lambda}(\vec{q}) \) is the scattering matrix element, \( \vec{q} \) the phonon wavevector, \( a_{\vec{q}, \lambda} \) and \( a^\dagger_{\vec{q}, \lambda} \) the phonon annihilation and creation operators, respectively, and \( \lambda \) the polarization index. At low temperature phonon absorption is negligible. The electron scattering time is given by

\[ \tau_{ij}^{-1} = \frac{2 \pi}{\hbar} \sum_{\vec{q}, \lambda} |M_{\lambda}(\vec{q})|^2 \langle |\psi_f|^2 e^{-i \vec{q} \cdot \vec{r}} |\psi_i|^2 \rangle \delta(E_f - E_i - \hbar \omega_q), \]  
\[ (5) \]

where \( E_f \) and \( E_i \) stand for the final and initial electron levels, respectively, and \( \hbar \omega_q \) is the phonon energy. We study phonon modes with a linear dispersion.

In the considered GaAs semiconductor QR, an electron interacts with the longitudinal acoustic (LA) phonon modes through a deformation potential and with both the longitudinal and the transverse acoustic (TA) phonon modes through piezoelectric fields. The total scattering matrix element is:

\[ |M(\vec{q})|^2 = |M_{LA}^{DF}(\vec{q})|^2 + \sum_{\lambda=LA,TA} |M_{\lambda}^{PZ}(\vec{q})|^2, \]  
\[ (6) \]

with the first contribution arising from the deformation potential coupling and the second one from the piezoelectric coupling. Notice that the Hamiltonian of the DF and PZ interactions are real and imaginary, respectively, which allow us to investigate these interactions separately and obtain the total contribution by simply adding up the two rates.

The electron-LA phonon scattering due to the deformation potential is given by

\[ |M_{LA}^{DF}(\vec{q})|^2 = \frac{\hbar D^2}{2 \rho c \Gamma} |\vec{q}|, \]  
\[ (7) \]

for the electron-LA-phonon scattering \((\lambda = \text{LA})\), and by

\[ |M_{LA}^{PZ}(\vec{q})|^2 = \frac{32 \pi^2 \hbar e^2 h_{14}^2}{\epsilon_0^2 \rho c \Gamma} (3 q_x q_y q_z)^2 |\vec{q}|^7, \]  
\[ (8) \]

for the electron-LA-phonon scattering \((\lambda = \text{LA})\), and by

\[ |M_{TA}^{PZ}(\vec{q})|^2 = \frac{32 \pi^2 \hbar e^2 h_{14}^2}{\epsilon_0^2 \rho c \Gamma} \times \]  
\[ \left| \frac{q_x^2 q_y^2 + q_y^2 q_z^2 + q_z^2 q_x^2 - (3 q_x q_y q_z)^2}{|\vec{q}|^5} \right|, \]  
\[ (9) \]

for the electron-TA-phonon scattering \((\lambda = \text{TA})\), where \( c' \) is the transversal sound velocity. In the above Eqs. \((7), (8)\) and \((9)\) the phonon dispersions are assumed as \( \omega_{q, \lambda} = cq \) and \( \omega_{q, \lambda} = c'q \).

In order to obtain a more tractable form of the piezoelectric coupling, we performed an angular average for the longitudinal and transverse modes separately and then added the terms. This approximation leads to a minor quantitative modification on the total scattering rate in comparison with retaining the full \( \vec{q} \)-dependence. It will neither influence our main results nor the conclusions in this work. We get the following expression for the electron-piezoelectric-phonon scattering matrix elements:
TABLE I: The GaAs constants used in the paper. Unless otherwise indicated the values are taken from Ref. 29.

| parameter                      | symbol | value           |
|--------------------------------|--------|-----------------|
| effective electron mass        | $m^*$  | 0.067 $m_0$     |
| crystal density                | $\rho$ | 5.3 $\times$ 10$^3$ kg m$^{-3}$ |
| band off-set                   | $V_0$  | 3.9 $\times$ 10$^{-20}$ J   |
| longitudinal sound velocity$^{20}$ | $c$    | 3.7 $\times$ 10$^3$ m s$^{-1}$ |
| transverse sound velocity$^{20}$ | $c'$   | 3.2 $\times$ 10$^3$ m s$^{-1}$ |
| static dielectric constant    | $\varepsilon_0$ | 12.8           |
| deformation potential         | $D$    | 2.2 $\times$ 10$^{-18}$ J   |
| piezoelectric constant        | $h_{14}$ | 1.38 $\times$ 10$^9$ V m$^{-1}$ |
| piezoelectric coupling        | $P$    | 5.4 $\times$ 10$^{-20}$ J$^2$ m$^{-2}$ |

\[
|M^p_2(|\mathbf{q}\rangle)|^2 = \frac{\hbar c^2 h_0^2}{2 \rho c \Gamma} \left( \frac{12}{35} + \frac{16}{35} \right) \frac{1}{|\mathbf{q}|} = \frac{\hbar}{2 \rho c \Gamma} |\mathbf{q}|^2 P ,
\]

where we have introduced the constants $\alpha$ and $P$, which represent the ratio between the longitudinal and transverse sound velocities and the piezoelectric coupling constant, respectively.

In this way, the total electron-phonon scattering matrix element in Eq. (9) becomes,

\[
|M(\mathbf{q})|^2 = \frac{\hbar}{2 \rho c \Gamma} |\mathbf{q}| (D^2|\mathbf{q}|^2 + P).
\]

Consequently, the total scattering rate from $(j,m,g)$ to $(j',m',g')$ state at $T = 0$ can be written explicitly as,

\[
\tau_{-1}^{-1}(j,m,g) \rightarrow (j',m',g') = \frac{D^2 q_0^3 + P q_0}{2 \pi \rho c^2} \sum_{m''=0}^N \sum_{n=0}^N k_{n'}^2 k_n^2 \times 
\int_0^{\pi/2} d\theta \sin \theta \langle \phi_{n',m'} | e^{-i q_0 r \sin \theta} | \phi_{n,m} \rangle^2 \times 
|\langle \lambda_g | e^{-i q_0 z \cos \theta} | \lambda_g \rangle|^2 ,
\]

where $q_0 = (E_{j',m',g'} - E_{j,m,g})/\hbar c$ and $N$ is the number of the Fock-Darwin orbitals considered.

III. NUMERICAL RESULTS AND DISCUSSION

A quantum ring is distinctly different from a dot in that it has a not-simply-connected geometry. This property makes QRs particularly interesting, especially for studies in the presence of a vertical magnetic field. First of all, we study the energy spectrum for the in-plane motion. In Fig. 1(a) we show the energy levels as a function of (a) the ring radius at zero magnetic field and (b) the magnetic field at fixed ring radii. The ring radius is measured in units of $\lambda_0 = \sqrt{\hbar/m^* \omega_0}$ and the energies in units of $\hbar \omega_0$. For a lateral confinement $\hbar \omega_0 = 5$ meV, one gets $\lambda_0 \sim 15$ nm, which is a typical value for the ring radius reported in recent experiments. In Fig. 1(a) we reproduced the results already obtained in Ref. 21 within an identical theoretical framework. We got results in perfect agreement with the aforementioned ones, as one can see immediately comparing Fig. 1(a) to Fig. 1 in Ref. 21 in the case of zero magnetic field. Moreover, in the presence of magnetic field, we calculated the spectra for values of the ring radius other than the ones shown in Fig. 4 of Ref. 21.

It is interesting to notice the differences between the cases $B=0$ and $B \neq 0$. In the absence of magnetic field the lowest states have definite quantum numbers (although the degeneracy $\pm m$ is present). When a vertical magnetic field is present, level crossings occur due to the multiple connected geometry of QRs. The ground state changes from the state with $m=0$ to the ones with $m=-1, -2, -3,...$ with increasing magnetic field (see Fig. 1(b)). The ring eigenfunctions are piecewise defined...
as a function of $B$. This feature is not present in a QD where no level crossings occur between the ground state and the low-lying levels. They converge smoothly to the first Landau levels at large magnetic fields. Actually, identifying the crossing points in the energy spectra in a QR gives straightforward information about the specific ring topology. Notice that the crossing point density in magnetic field increases as the ring radius increases.

In what follows, we report the calculations of the electron relaxation time due to acoustic phonon scattering in the cases $B=0$ and $B\neq 0$. We consider the relaxation rates from the first and second excited states to the ground state because they are often the most relevant transitions. These transitions can be monitored experimentally (for instance, by means of pump-and-probe techniques). For an electron initially in the first excited state, the relaxation rate is given by the direct scattering rate from this state to the ground state. For electrons in the second (or higher) excited state, different relaxation channels exist and the relaxation rate can be obtained using Matthiessen’s rule.

### A. In the absence of magnetic field

In this section we study the acoustic phonon scattering and electron relaxation rate in QRs at zero magnetic field. Electron-acoustic phonon interactions in nanostructures are in general determined by the interplay of lateral confinement frequency, quantum well thickness and phonon wavelength.

First of all, we calculate the scattering rate as a function of the ring radius $r_0$ for different lateral confinement frequencies. The scattering rate from the first excited state $(0,\pm1,0)$ to the ground state $(0,0,0)$ is shown in Fig. 2(a) for a fixed ring thickness $L_z = 5$ nm. The scattering rate $\tau^{-1}$ shows an oscillatory behavior as a function of the ring radius. For very small $r_0$ (corresponding to quantum dots), the total scattering rate is small and decreases as the confinement frequency $\omega_0$ increases. With increasing the ring radius, the scattering rate increases pronouncedly for small $\omega_0$. As is evident in Fig. 2(b), for the considered confinement frequencies, the DF acoustic phonon scattering is much larger than the PZ phonon scattering for small ring radius. This is a well known result for QDs where the DF phonon scattering is dominant. However, our calculation shows that the PZ phonon scattering becomes much larger than that of the DF in QRs (large $r_0$). For each $\hbar\omega_0$, there is a clear crossover point from which the PZ scattering rate starts to dominate over the DF one. Therefore, in QRs, the PZ effects are important and cannot be neglected as it is usually done in QDs. As one can see from Eq. (12), the relative contributions of DF and PZ scattering are determined by the prefactor before the integral. At small $q_0$ the PZ coupling is dominant over the DF one. Actually, by increasing the ring radius the energy levels get closer and closer (see Fig. 1(a)), and consequently $q_0$ becomes smaller and smaller. This is the reason why after a certain $r_0$ the PZ coupling becomes the dominant scattering mechanism. We will return and comment more on this point in the next section.

Although the PZ and DF scattering rates are of different features, their behavior in the QD limit (i.e., $r_0 \to 0$) is similar. They decreases rapidly as the lateral confinement frequency increases. The reason is that the orthogonality of the electron states leads the factor $\langle \psi_f | \exp(-i \mathbf{q} \cdot \mathbf{r}) | \psi_i \rangle$ to vanish rapidly when the phonon wavelength is shorter than the lateral confinement length.

![Fig. 2: (Color online). (a) The total scattering rate from $(0,\pm1,0)$ to $(0,0,0)$ state as a function of the ring radius for different lateral confinements $\hbar\omega_0$. The vertical well thickness is $L_z = 5$ nm. (b) The contributions of the DF and PZ couplings are plotted separately.](image)

In Fig. 3 the effect of the QR thickness $L_z$ on the phonon scattering rates is studied. The DF, PZ, as well as total scattering rates are given as a function of the ring radius for three different $L_z$ at fixed lateral confinement $\hbar\omega_0 = 5$ nm. The scattering curves exhibit strong oscillations for small ring radius. The oscillation in the electron-phonon scattering rate is originated by the match of the localized electron wavelength to the wavelengths of the emitted phonons in the scattering as discussed in Ref. 10 for the QD case. In the calculation, the electron wave-
function is decoupled in the $z$ direction and in the $xy$ plane. Thus, such a match depends on the projection of the phonon wavevector onto the $xy$ plane due to its 3D characteristic. For large $L_z$, the electron-phonon scattering elements depend more strongly on the projection angle of the phonon wavevector onto the $xy$ plane. Consequently, the oscillation in the scattering rate is stronger for larger $L_z$. It reaches maxima (minima) when the electron wave function is in-phase (anti-phase) with the phonon wave. The scattering rate can be suppressed by orders of magnitude due to the anti-phase relation between the electron wave function and the phonon wave in the ring. This effect has been proposed as a possible way to control the coherence time for single and coupled QDs.\textsuperscript{15,18,19} With increasing the ring radius, the phonon scattering is enhanced pronouncedly and the oscillations disappear.

From Fig. 4 one can notice that for large ring radius the two relaxation rates $\tau_{r,1}^{-1}$ and $\tau_{r,2}^{-1}$ are very similar and of the order of 10 ns$^{-1}$. For small ring radius, interesting features appear. The relaxation rate $\tau_{r,1}^{-1}$ shows a striking dip at $r_0 \sim 0.38\lambda_0$ and $\tau_{r,2}^{-1}$ has two minima at $r_0 \sim 0.38\lambda_0$ and $r_0 \sim 0.52\lambda_0$ where the relaxation rate is reduced by orders of magnitude. This effect results from the anti-phase relation between the electron and phonon wave functions as discussed above. Although it is extremely difficult to control QR geometry in the fabrication process, in the same spirit of Refs. 15,18,19, we confirm that a geometry-induced control of the relaxation time is feasible also in the case of QRs of small radius.

![FIG. 3](image-url)

**FIG. 3:** (Color online). The total, DF and PZ scattering rates from $(0, \pm 1, 0)$ to $(0, 0, 0)$ state as a function of the ring radius for different vertical confinements $L_z$. The lateral confinement is $\hbar \omega_0 = 5$ meV.

We calculated the relaxation rates $\tau_{r}^{-1}$ for the transitions from the first and second excited states to the ground state as shown in Fig. 4 for the QRs with $\hbar \omega_0 = 5$ meV and $L_z = 5$ nm. For an electron in the first excited state $(0, \pm 1, 0)$, its relaxation rate $\tau_{r,1}^{-1}$ to the ground state coincides with the direct scattering rate between the two states, as already mentioned. For an electron in a higher state, its relaxation to the ground state is in general a multichannel process. In the specific case under investigation, an electron in the second excited state $(0, \pm 2, 0)$ can relax to the ground state by direct transition $(0, \pm 2, 0) \rightarrow (0, 0, 0)$ or indirect transition $(0, \pm 2, 0) \rightarrow (0, \pm 1, 0) \rightarrow (0, 0, 0)$. Using Matthiessen’s rule, the relaxation rate for the second excited state can be written as

$$\tau_{r,2}^{-1} = \tau_{r,1}^{-1} + \tau_{r,2,1}^{-1} + \tau_{r,2,2}^{-1} = \tau_{(0, \pm 2, 0) \rightarrow (0, 0, 0)} + \tau_{r,2,1}^{-1} + \tau_{r,2,2}^{-1}.$$ \textsuperscript{(13)}
and DF phonon scattering in this region is of approximately the same contribution. This again confirms that differently from the QD case, the PZ phonon scattering is critical in the QRs. The scattering rate decreases rapidly with increasing $q_0$ and the DF phonon scattering becomes dominant over the PZ one (about two orders of magnitude). Strong scattering at small energies implies that the transition probability for an electron from one state to its adjacent level in energy is much larger than to other levels. As a consequence, relaxation from a higher excited state to the ground state is basically a multi-scattering process through all the intermediate states between them.

**B. In the presence of magnetic field**

In this section we study the effects of a magnetic field $B$ on the electron-phonon scattering and the electron relaxation time in a QR. As shown in Fig. 1(b), an external magnetic field in the $z$-direction alters drastically the electron energy spectrum.

For instance, the ground state in the case $r_0 = 2\lambda_0$ is defined by the orbitals: $(0, 0, 0)$ for $0 \leq B < 1.35$, $(0, -1, 0)$ for $1.35 \leq B < 3.14$, $(0, -2, 0)$ for $3.14 \leq B < 4.77$ and so on. Hence, the ring eigenstates as a function of magnetic field are piecewise. In Ref. 34 the effect of a time varying magnetic field are shown to produce non-adiabatic changes upon the electron states of the QR, leading to strong non-linear effects in the time-depending magnetization.

In what follows, we will refer to the ground state, the first and second excited states as G.S., F.E.S. and S.E.S, respectively, rather than as a set of three quantum numbers. From now on we concentrate on a well defined ring geometry with $L_z = 5$ nm and $\hbar\omega_0 = 5$ meV. For fixed ring radii $r_0 = 2\lambda_0$ and $r_0 = 3\lambda_0$, we calculated the scattering rates among the lowest three states (i.e., the S.E.S., F.E.S. and G.S.) as a function of magnetic field. The results are shown in Fig. 6 and Fig. 7 respectively. The contributions of the DF and PZ phonons to the scattering rates are given separately.

![FIG. 5: (Color online). The scattering rate for the transitions between two low-lying adjacent states as a function of the energy difference of the two levels. In the inset the DF and PZ contributions for all the possible transitions are given separately.](image5.png)

![FIG. 6: (Color online). The scattering rates as a function of the magnetic field for a ring of radius $r_0 = 2\lambda_0$ for (a) the DF and (b) the PZ couplings.](image6.png)
the increasing number of level crossings as \( r_0 \) becomes larger. Notice that in this case the oscillations are due to the in-plane level crossings at which \( q_0 \rightarrow 0 \) rather than to the anti-phase relation between the electron and phonon wavefunctions in the QD case\(^{15}\).

It is interesting to make a direct comparison with the QD case. In Fig. 8 the different scattering contributions are calculated for a QD with \( \hbar \omega_0 = 5 \text{ meV} \) and \( L_z = 5 \text{ nm} \) in vertical magnetic fields, \textit{i.e.} the same thickness and confinement frequency as the ring studied above. For magnetic fields up to 5 Tesla, the PZ phonon scattering rates in the QD are always orders of magnitude smaller than the DF ones, as one can see in Fig. 8. Once more, this confirms that the phonon scattering mechanism in QRs is significantly different the QDs. Such a difference is amplified in the presence of magnetic fields. In the QDs, the PZ phonon scattering can be neglected in the presence of \( B \). In the QRs, on the contrary, the PZ interactions are as important as the DF ones.

In Fig. 9 and Fig. 10 we show the relaxation rates from the F.E.S. to G.S. and from the S.E.S. to G.S. as a function of \( B \) for \( r_0 = 2\lambda_0 \) and \( r_0 = 3\lambda_0 \), respectively. In both cases the relaxation rate from the F.E.S. to G.S. shows dips at the level crossings between the two states, where the phonons (with \( q_0 \rightarrow 0 \)) involved in the scattering are of vanishing density of states. This mechanism is distinctly different from the suppression of the scattering rate due to the anti-phase relation between the electron and the phonon wave functions as was shown in Fig. 4.

Variation of the relaxation rate from the second excited state to the ground state in magnetic fields is quite small and no dips appear. The reason is that \( \tau_{r_1}^{-1} \) results from a multi-scattering process and it is calculated according to Matthiessen’s rule in Eq. 13. Due to the fact that the crossing points in the energy levels for the G.S., F.E.S. and S.E.S. take place at different magnetic fields, the direct scattering rates \( \tau_{r_1}^{-1} \) and \( \tau_{r_2}^{-1} \) are never zero simultaneously.

Finally, in Fig. 11 we show the total scattering rates in the presence of \( B \) between two different levels as a function of the energy difference between the corresponding levels instead of magnetic field. Both the DF and PZ phonons are included. We observe that the scattering rates in magnetic fields depends strongly on the
energy difference between the levels considered. But, at the same energy difference, the different scattering rates are in the same order of magnitude independently on the initial and final states. Furthermore, when we compare Figs. 11(a) and 11(b) of different ring radii (see also Fig. 5), we find that the curves in the different figures are very similar to each other, i.e., the values of the scattering rates and the positions of the maximum and minimum are almost the same. We can, thereby, confirm that the energy difference between the two states involved in the transitions is a crucial parameter in determining the acoustic phonon scattering rates. Changing of the ring radius and/or the magnetic field affects essentially the energy separation between different levels and, consequently, the phonon scattering rate.

IV. CONCLUSIONS

We studied the acoustic-phonon-induced relaxation of an excited electron in a single GaAs quantum ring. We investigated the effects of the ring geometry and external magnetic fields on the relaxation from the first and second excited states to the ground state. Our calculations show that electron-phonon scattering strongly depends on the ring lateral and vertical dimensions and on the external fields. We took into account both the deformation potential and piezoelectric field couplings and shown that they both give important contributions to the electron relaxation. The piezoelectric interaction, which is often negligible in quantum dots, represent the major source of scattering in quantum rings of large ring radius or when a high magnetic field is applied. For small ring radii, the deformation potential coupling prevail, being similar to the quantum dot case. Nevertheless, we have shown that the electron-phonon scattering rate between any two levels are mainly determined by the energy difference between them. In magnetic fields, a multiple scattering process leads to the relaxation time in the order of $10^{-1}$ nano-second for an electron from the second excited state to the ground state in QRs.

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1. J. M. Garcia, G. Medeiros-Ribeiro, K. Schmidt, T. Ngo, J. L. Feng, A. Lorke, J. P. Kotthaus, and P. M. Petroff, Appl. Phys. Lett. 71, 2014 (1997).
2. A. Lorke, R. J. Luyken, A. O. Govorov, J. P. Kotthaus,
