Origin of Neutrino Masses and Mixings

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A brief overview of the important theoretical issues in neutrino physics is presented and some of the current ideas for addressing them in the framework of the seesaw mechanism of neutrino masses are discussed. Among the topics discussed are ways to understand maximal mixing pattern among the three neutrino species, both unimaximal and bimaximal kinds. Constraints from baryogenesis on mixing patterns are noted. In the last section, the question of understanding the small masses for sterile neutrinos, which seem to be needed for various reasons is addressed.

1. INTRODUCTION

There is now strong evidence for neutrino masses and mixings from the solar and atmospheric neutrino observations. The simplest way to understand the deficits in neutrino fluxes observed in the above experiments is to assume that the incident neutrinos oscillate into another species which cannot be detected. For neutrino oscillations to take place, they must have mass and mix among themselves, with appropriate mass differences and mixing angles. As far as the accelerator searches for such oscillation effects go, with the exception of the Los Alamos experiment (LSND), all others have yielded negative results. These are of course not in contradiction with the solar and atmospheric data since they probe different ranges of masses and mixings. In fact, the negative results from the two experiments, CHOOZ and PALO VERDE provide upper limits on one of the mixing angles that has interesting implications for theories of neutrino masses. In this brief overview, I wish to draw attention to the theoretical progress in understanding mass and mixing patterns in extensions of the standard model, specifically the ones based on the seesaw mechanism that seems to provide the simplest way to understand small neutrino masses. Due to space limitations, this exposition will be brief, specially with respect to references and we refer the reader to a recent review by the author on the subject.

1.1. Four major theoretical issues in neutrino physics:

The major issues of interest in neutrino theory are driven by the following experimental results and conclusions derived from them. We will use the notation, where the flavor or weak eigenstates $\nu_{\alpha}$ (with $\alpha = e, \mu, \tau$) are expressed in terms of the mass eigenstates $\nu_i$ ($i = 1, 2, 3$) as $\nu_\alpha = \sum_i U_{\alpha i}^{MNS} \nu_i$. The $U_{\alpha i}^{MNS}$, the elements of the Maki-Nakagawa-Sakata matrix represent the observable mixing angles if the basis is such that the charged lepton masses are diagonal. In any other basis, one has $U_{\alpha i}^{MNS} = U_{\alpha i}^{\ell} U_{\nu i}$, where the matrices on the right hand side are the ones that diagonalize the charged lepton and neutrino mass matrices. For simplicity, we will drop the superscript MNS in what follows.

1.1.1. Solar neutrinos:

There is no clear winner among the various possible oscillation solutions to the solar neutrino puzzle. In our discussion below, we will consider the small angle MSW solution, specially in the case when we want to understand the LSND results in the framework of a four neutrino picture. Of particular interest now is the large angle MSW solution, since it provides one way to understand the reported flat energy distribution in the data. Vacuum oscillation requires very small mass differences and we will not dis-
cuss it here, even though it also can a viable possibility. Typical mass difference required is: $$\Delta m^2_{\nu_e-\nu_x} \sim 10^{-5} \text{ eV}^2$$, with mixing angles either maximal or small.

1.1.2. Atmospheric neutrinos:
Here evidence appears very convincing that the likely explanation involves oscillation of $$\nu_\mu$$ to $$\nu_\tau$$, with $$\Delta m^2_{\nu_\mu-\nu_\tau} \simeq 3 \times 10^{-3} \text{ eV}^2$$ and maximal mixing.

1.1.3. LSND:
The evidence for $$\nu_\mu$$ to $$\nu_e$$ oscillation from LSND needs to be confirmed by another experiment. The KARMEN experiment has eliminated part of the original LSND allowed domain. The MiniBOONE at Fermilab will either confirm or refute these observations more definitively. But taking what we have at present seriously, requires $$\nu_\mu$$ to $$\nu_e$$ mass difference $$\Delta m^2_{\nu_\mu-\nu_e} \simeq 0.2 - 2 \text{ eV}^2$$ with points around $$\Delta m^2 \simeq 6 \text{ eV}^2$$ also allowed with a small mixing (few percent level).

1.1.4. Neutrinoless double beta decay:
The most stringent limits here are from the enriched $^{76}$Ge experiment by the Heidelberg-Moscow collaboration and leads to a constraint on masses and mixing angles as: $$\sum_i U_{hi}^2 m_i \leq 0.3 \text{ eV}$$, with an uncertainty of a factor of 2 due to nuclear matrix elements.

1.1.5. $$U_{e3}$$:
The reactor experiments CHOOZ and PALO VERDE experiments imply that $$U_{e3} \lesssim 0.16$$.

The four major issues in neutrino physics can now be stated as follows:

- How does one understand the extreme smallness of the neutrino masses?
- How does one understand the maximal mixing angles among neutrinos given that there is so much similarity between quarks and leptons and that the quark mixings are small?
- Could the neutrino masses (all or some) be degenerate?
- If LSND is confirmed, the only way to reconcile all data seems to be to postulate the existence of a sterile neutrino$^3$. The question then is: how does one understand the ultralightness of the sterile neutrino? Note that since it is an electroweak singlet, its mass could take any value.

2. SMALL NEUTRINO Masses

It is well known that in the standard model the neutrino is massless due to a combination of two reasons: (i) one, its righthanded partner ($$\nu_R$$) is absent and (ii) the model has exact global $$B - L$$ symmetry. Clearly, to understand a nonzero neutrino mass, one must give up one of the above assumptions. If one blindly included a $$\nu_R$$ to the standard model as a singlet, the status of neutrino would be parallel to all other fermions in the model and one would be hard put to understand why its mass is so much smaller than that of other fermions. On the other hand, instead of adding the $$\nu_R$$, one might exploit a prevailing lore that gravity breaks all global symmetries and include in the Lagrangian Planck mass suppressed higher dimensional operators$^1$ of the form $$LHLH/M_{Pl}^2$$ (where $$L$$ is a lepton doublet and $$H$$ is the Higgs doublet). These in general lead to masses for neutrinos of order $$10^{-5} \text{ eV}$$ or less and are therefore not adequate for understanding observations. Thus a nontrivial extension of the standard model is called for.

2.1. Does $$m_\nu$$ Necessarily Imply a Right Handed Neutrino?

An interesting class of models were proposed in the 80’s where one extends the standard model without adding right handed neutrinos but extra Higgs bosons to understand small $$m_\nu$$. In one class of models, one adds a Higgs triplet with $$B - L = 2$$ with a large mass $$M \gg M_W$$. The triplet field acquires small vev of order $$v_T \sim \frac{M^2}{M}$$ via the type II seesaw mechanism$^2$. Thus, modulo the unknown origin of $$M \gg M_W$$, this provides an understanding of the small $$m_\nu$$. There is however one problem with this mechanism: to generate the baryon asymmetry of the universe, one has to employ the triplet decay and out of equilibrium decay condition of Sakharov then implies that $$M \geq 10^{13} \text{ GeV}$$. This scale is much
higher than the conventional reheating temperature in inflation models with supersymmetry\[8]. Thus while this mechanism may prove adequate for neutrino physics, it falls short of expectations within a broader picture where one attempts a unified understanding of different aspects of particle physics and cosmology.

A second approach is to attempt an understanding of small $m_\nu$, using only electroweak scale physics and radiative corrections. There are generic one loop [9] and two loop one\[10\] type models. The idea is to extend the standard model by adding $SU(2)_L$ singlet, electrically charged fields: $\eta(1,2)$ in the first case and $\eta^+$ and $h^{++}$ in the second case. The two loop mechanism has the advantage that it requires less of a fine tuning to get the desired smallness for $m_\nu$. However, both these approaches suffer from the difficulty that it is not easy to understand the origin of baryons in the universe. The reason for that is that they necessarily involve $B - L$ violating interactions at the weak scale which will necessarily erase any baryons produced.

### 2.2. Right handed neutrino, seesaw mechanism and neutrino mass

It seems rather compelling from the above reasoning that the small mass of neutrinos provides a strong reason to believe in the existence of an additional fermion state in nature i.e. the right handed neutrino, one per generation. The inclusion of the $\nu_R$ expands the gauge symmetry of the electroweak interactions to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry with fermion doublets $(u,d)_{L,R}$ and $(\nu, e)_{L,R}$ assigned to the left-right gauge group in a parity symmetric manner; furthermore the model has quark-lepton symmetry. More importantly, the electric charge formula for the model takes a very interesting form\[11\]: $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$. It can be concluded from this that below the scale $v_R$ where the $SU(2)_R \times U(1)_{B-L}$ symmetry breaks down to the standard model and above the scale of $M_{\nu}$, one has the relation $\Delta I_{3R} = - \frac{B-L}{2}$. This relation has the profound consequence that neutrino must be a Majorana particle and that there must be lepton number violating interactions in nature. In particular, the $\nu_L - \nu_R$ mass matrix for three generations takes the form:

$$M = \begin{pmatrix} M_{LL} & M_{LR} \\ M_{LR}^T & M_{RR} \end{pmatrix}$$

(1)

where $M_{RR} = f v_R$ is the Majorana mass matrix of the right handed neutrinos, ($f$ is the new Yukawa coupling that determines the right handed neutrino masses); $M_{LL} \approx f \frac{v_W}{v_R}$ is the induced Majorana mass matrix for the left handed neutrinos; $M_{LR} \equiv M = Y v_{\nu k}$ is the Dirac mass matrix connecting the left and the right handed neutrinos. The diagonalization of this mass matrix leads to following form for the light neutrino masses:

$$M_\nu \approx \frac{\lambda v_{\nu k}^2}{v_R} - \frac{1}{v_R} M_D^T f^{-1} M_D;$$

(2)

$f$, the Yukawa coupling matrix that is responsible for the mass matrix of the heavy right handed neutrinos characterizes the high scale physics, whereas all other parameters denote physics at the weak scale. We have called this generalized formula for neutrino masses, the type II seesaw formula to distinguish it from the one that is commonly used in literature where the first term of Eq. (2) is absent. Important feature of this formula is that both terms vanish as $v_R \to \infty$ and since $v_R \gg v_{\nu k}$, the smallness of the neutrino mass is much smaller than the charged fermion masses. As was particularly emphasized in the third paper of ref.\[1\], the dominance of V-A interaction in the low energy weak processes is now connected to smallness of neutrino masses.

If in the above seesaw formula, the second term dominates, this leads to the canonical type I seesaw formula and leads to the often discussed hierarchical neutrino masses, which in the approximation of small mixings lead to $m_\nu \simeq m_f^2/v_R$, where $f_i$ is either a charged lepton or a quark depending on the kind of model for neutrinos.

On the other hand, in models where the first term dominates, the neutrino masses are almost generation independent. Therefore, if there is indication for neutrinos being degenerate in mass\[12\] from observations, one will have to resort to type II seesaw mechanism for its understanding.

A further advantage of the right handed neutrino and seesaw mechanism is that it fits in very
nicely into grand unified frameworks based on SO(10) models. The coupling constant unification then provides a theoretical justification for the high seesaw scale and hence the small neutrino masses. Furthermore, the 16-dim. spinor representation of SO(10) has just the right quantum numbers to fit the $\nu_R$ in addition to the standard model particles of each generation.

3. BARYOGENESIS CONSTRAINTS

In the models with a heavy right handed neutrino, there is a definite mechanism for baryogenesis via leptogenesis, which has been well discussed in the literature. It is the decay of the right handed neutrinos (denoted by $N_R$) $N_R \rightarrow \nu \ell$ that in conjunction with the CP violation in leptonic couplings generates lepton asymmetry. The lepton asymmetry is subsequently converted to baryon asymmetry via the sphaleron interactions, associated with the electroweak physics. An essential prerequisite for leptogenesis is that the interactions involving the $N_R$ must be out of equilibrium i.e. $\Gamma_{N_R} \ll H$ (H is Hubble parameter at the relevant epoch). This equation roughly translates into $\frac{H^2}{\Gamma} \leq \frac{10M_{N_R}}{h}$, $h$ being the Yukawa couplings of the relevant $N_R$, expected to be roughly of the same order of magnitude as the quark or lepton Yukawa couplings. It is not very hard to see that only the lightest of the three righthanded neutrinos (which in a hierarchical picture is predominantly of electron type i.e. $N_eR$) will have the smallest coupling $h$ and will satisfy the out of equilibrium condition. However since all the $N_R$’s are mixed among each other, the same condition also implies that $N_eR$ mixing with $N_{\mu,\tau R}$ determined by the $f$ matrix better be very small. In quantitative terms, this means that $f_{e\mu} \leq \frac{m_\mu}{m_e}$ and $f_{e\tau} \leq \frac{m_\tau}{m_e}$ or so. To see the kind of constraints they imply on the neutrino masses and mixings, let us assume type I seesaw and that in analogy with the quark sector, the mixings in the Dirac neutrino mass matrix are negligible. One then gets:

$$v R^{-1} M D M D^T \sum_i U_{e i} U_{\mu i} m_i^{-1} \leq 5 \times 10^{-3}.$$  \hspace{1cm} (3)

$$v R^{-1} M D M D^T \sum_i U_{e i} U_{\tau i} m_i^{-1} \leq 2.5 \times 10^{-4}.$$  \hspace{1cm} (4)

These in turn can put severe constraints on models for neutrino masses. In fact, there are interesting examples neutrino mass models in the literature which have difficulty satisfying Eq. (3).

4. MAXIMAL MIXING IN GAUGE MODELS

Let us consider only the three neutrino case, which is sufficient to understand data if LSND results are not included. There are in general two types mixing matrices that can fit data:

**Case (i): Unimaximal mixing**

$$U = \begin{pmatrix} 1 & \epsilon & 0 \\ -\epsilon/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \epsilon/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}.$$ \hspace{1cm} (5)

**Case (ii): Bimaximal mixing**

$$U = \begin{pmatrix} 1/\sqrt{2} & 1/2 & 0 \\ -1/\sqrt{2} & 1/2 & 0 \\ 1/2 & 1/2 & -1/\sqrt{2} \end{pmatrix}.$$ \hspace{1cm} (6)

Clearly, the neutrino mixing patterns described in the above equations bear no resemblance to that in the quark sector, in spite of the fact that at the fundamental level there is complete quark lepton symmetry. The difference must therefore be accounted for by whatever mechanism makes the neutrinos Majorana particle e.g. seesaw mechanism. Below we explore several ideas that are being explored in the literature to gain an insight into the maximal mixing. Looking at the formula for $U_{MNS} = U^T U_{\nu}$, one can think of three ways: (i) $U_{\ell} \simeq 1$ but $U_{\nu}$ is intrinsically different; (ii) at the seesaw scale, both $U_{\ell,\nu} \simeq 1$ but dynamics of renormalization group extrapolation generates the maximal mixing; (iii) $U_{\nu} \sim 1$ but $U_{\ell}$ has the maximal mixing pattern. Let us briefly describe each one and discuss their relative merits.

4.1. Maximal mixing due to $\nu_R$ texture

From the type I seesaw formula, $M_{\nu} \simeq \frac{M_D^2}{v R}$, we see that this case is very generic to grand unified theories where, one may expect to have $M_{\ell,\nu} \sim M_q$ so that the entire
mixing angle pattern arises from texture in the right handed neutrino mass matrix. There is something natural about it since the origin of the right handed neutrino mass matrix owes its origin to a different source than the quark and lepton masses. There exist examples in literature where this case is realized using symmetries or specific ansatz for the right handed neutrino mass matrix \[18,19\].

4.2. Radiative magnification of mixing angles

This is a very interesting where dynamics plays an essential role in understanding the maximal mixings. The basic idea is that at the seesaw scale, all mixings angles are small, a situation is quite natural if the pattern of f Yukawa coupling is similar to the usual standard model ones. Since the observed neutrino mixings are weak scale observables, one must extrapolate\[20\] to the seesaw scale mass matrices to the weak scale and recalculate the mixing angles.

The extrapolation formula is

\[ \mathcal{M}_\alpha(M_Z) = I M_\alpha(v_R) I \]  \hspace{1cm} (7)

where

\[ I_{\alpha \alpha} = \left( 1 - \frac{h^2}{16\pi^2} \right) \]  \hspace{1cm} (8)

Note that since \( h_\alpha = \sqrt{2} m_\alpha/v_wk \) (\( \alpha \) being the charged lepton index), in the extrapolation only the \( \tau \)-lepton makes a difference. In the MSSM, this increases the \( \mathcal{M}_{\tau \tau} \) entry of the neutrino mass matrix and essentially leaves the others unchanged. It was shown in a recent paper\[21\] that if the muon and the tau neutrinos are nearly degenerate in mass at the seesaw scale, and in supersymmetric theories, the \( \tan \beta \geq 5 \), the radiative corrections can become large enough so that at the weak scale the two diagonal elements of \( \mathcal{M}_\nu \) which were nearly equal but different at the seesaw scale become so degenerate that the mixing angle becomes maximal and a solution to the atmospheric neutrino deficit emerges even though at the seesaw scale, the mixing angles were small. This happens only if the experimental observable \( \Delta m^2_{23} \leq 0 \) i.e. the usual hierarchical pattern. This possibility can be tested in experiments such as the ones being contemplated in future neutrino factories. This mechanism will become more compelling if the LSND results are confirmed since in that case the \( \nu_\mu \) and \( \nu_\tau \) are nearly degenerate or evidence for a small neutrino hot dark matter component emerges.

An interesting question is whether this mechanism can be extended to the case of three generations and whether it can explain the bimaximal pattern also. This question was investigated in the ref.\[21\] and it was found that the answer to the first question is yes and to the second, it is “no”. Furthermore for the first case to work, one must explain the solar neutrino puzzle by the small angle MSW mechanism if one has to be consistent with the CHOOZ-PALO-VERDE constraint.

4.3. Lopsided unification

This basic idea of this class of models is that in the seesaw formula, \( M^D \) and \( M_{RR} \) are generic with very little generation mixing but \( M_\ell \) is special. The reason this is consistent with grand unification of quarks and leptons is that in some GUT theories such as \( SU(5) \), one has the relation \( M_d = M^T_d \). What this means is that the lepton mixings are related to mixings among the right handed down quarks, which are not observable and thus need not be small. As a result, the \( U_\ell \) can harbor large mixing angles without contradicting quark-lepton unification. If fermion masses are asymmetric as in \( SU(5) \) theories or in \( SO(10) \) theories with \( 120 \) type Higgs bosons, then one could realize this scheme\[22\]. Of course, the largeness of the mixing angles must be put in by hand.

4.4. Bimaximal mixing

In this subsection, let us briefly discuss the bimaximal mixing and their theoretical origin. It has been realized\[23\] that the neutrino mass matrix that leads to bimaximal mixing pattern is:

\[ \mathcal{M}_\nu = \begin{pmatrix} \delta & F & F \\ F & A & -A \\ F & -A & A \end{pmatrix} \]  \hspace{1cm} (9)

There are two subcases of this mass matrix with very different experimental tests; case (i) \( \delta \ll A \ll F \): In this case, the neutrino mass hierarchy is inverted i.e. \( \Delta m^2_{23} \geq 0 \). On the other hand, there is another case, i.e. case (ii) where \( \delta \ll
existence of mirror matter in the universe, that neutrino and (ii) another based on the possible
which could then be a candidate for the sterile
leptons has a standard model singlet neutrino,

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reasoning has been pursued in recent literature to
understand the ultralightness of the sterile neutrino
for the mass matrix[24]. This is however a
weak scale theory and does not possess the
advantages of seesaw type models. There have been
attempts to derive these patterns from grand unified
theories[25].

5. Sterile neutrinos

If the existence of the sterile neutrino[26] becomes confirmed say, by a confirmation of the
LSND observation of $\nu_\mu - \nu_e$ oscillation or directly by SNO neutral current data, a key theoretical
challenge will be to construct an underlying theory that embeds the sterile neutrino along with
the others with appropriate mixing pattern, while naturally explaining its ultralightness.

If a sterile neutrino was introduced into the standard model, the gauge symmetry would not
forbid its bare mass, implying that there would be no reason for it mass to be small. It is a common
experience in physics that if a particle has mass lighter than normally expected on the basis of
known symmetries, then it is an indication for the existence of new symmetries. This line of
reasoning has been pursued in recent literature to understand the ultralightness of the sterile neutrino
by using new symmetries beyond the standard model.

There are several suggestions for this new symmetry that might help us to accommodat an ultralight $\nu_s$ and in the process lead us to new classes of extensions of physics beyond the standard model.

Two examples are: (i) one based on the $E_6$ grand unification model, where the $27$-dim. representation which is used to represent the quarks and leptons has a standard model singlet neutrino, which could then be a candidate for the sterile neutrino and (ii) another based on the possible existence of mirror matter in the universe, that

we discuss later.

A completely different way to understand the ultralightness of the sterile neutrino is to introduce large extra dimensions in a Kaluza-Klein framework and not rely on any symmetries. In these models, the sterile neutrino is supposed to live in the bulk; therefore if the bulk size is of order milli-eV$^{-1}$ as is generally contemplated, the sterile neutrino can be identified as one of the KK excitations of a massless bulk neutrino. For a discussion of this approach, see [27].

Let us now briefly touch on the mirror universe model for the ultralight neutrino[28,29] since the idea has implications far beyond the domain of neutrino physics. The basic idea is that there is a complete duplication of matter and forces in the universe (i.e. two sectors in the universe with matter and gauge forces identical prior to symmetry breaking) as suggested , say by the string theories. The mirror sector of the model will then have three light neutrinos which will not couple to the Z-boson and would not therefore have been seen at LEP. Denoting the fields in the mirror sector by a prime over the standard model fields, the $\nu'_\mu$ can qualify as the sterile neutrinos of which we now have three. The lightness of $\nu'_\mu$ is dictated by the mirror $B' - L'$ symmetry in a manner parallel to what happens in the standard model. Thus the ultralightness of the sterile neutrinos is understood in a most “painless” way.

The two “universes” are assumed to communicate only via gravity or other forces that are equally weak. This leads to a mixing between the neutrinos of the two universes and can cause oscillations between $\nu_e$ of our universe to $\nu'_e$ of the parallel one in order to explain for example the solar neutrino deficit. The atmospheric neutrino deficit in this case can be explained by using the usual $\nu_\mu - \nu_\tau$ oscillation.

There are two versions of this model: one where the masses in both sectors are identical for corresponding particles[29]. This is called the symmetric miror model. On the other hand, one can envision a scenario where after symmetry breaking all masses scale by a common factor[30]. The second one has certain cosmological advantages, that mirror matter can become the dark matter of the universe.
6. CONCLUSION

In conclusion, the seesaw mechanism appears by far to be the simplest way to understand the small neutrino masses. A grand unified framework such as the one based on the SO(10) can then help us to understand the high seesaw scale as a consequence of gauge coupling unification. This also provides a way to understand the origin of baryons in the universe. Our understanding of mixings on the other hand, is at a preliminary level. A particular challenge to theorists is to understand the so called bimaximal mixing pattern, which will emerge as the favorite, if the solar neutrino deficit is to be solved via the large angle MSW or the vacuum oscillation. Finally we comment on the status of the sterile neutrino and understanding of its tiny mass in a "natural" framework.

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