Holographic Type II
Goldstone Bosons

Karl Landsteiner
Instituto de Física Teórica, UAM-CSIC

I. Amado, D. Arean, A. Jimenez-Alba, K.L., L. Melgar, I. Salazar-Landea
[arXiv:1302.5641, arXiv:1307.8100]

Crete CTP, 07-10-2013
Outline

• Goldstone theorems
• Field theoretical model
• Holographic model(s)
• Landau criterion
• Summary
Goldstone Theorems

• Spontaneously broken continuous symmetry

\[ \lim_{k \to 0} \omega(k) = 0 \]

• At least one mode

• No constraint on power: \( \omega(k) \propto k^n \)

• Lorentz symmetry:
  - \( \omega(k) = ck \)

• One mode for every broken generator
Goldstone Theorems

• No Lorentz symmetry

• State: temperature $T$, density $\mu$

• Principally: non-relativistic, Lifshitz, ...

• Classification:

  [Nielsen-Chadha '74]

  • Type I: $\omega \alpha k^{2n+1}$
  • Type II: $\omega \alpha k^{2n}$
Goldstone Theorems

- Chadha-Nielsen
  \[ N_I + 2N_{II} \geq N_{BG} \]

- Brauner-Watanabe-Murayama
  \[ \langle [Q_a, Q_b] \rangle = B_{ab} \]
  \[ N_I + N_{II} = N_{BG} - \frac{1}{2} \text{rank}(B_{ab}) \]

- Brauner-Murayama-Watanabe, Nicolis-Piazza, Kapustin
  ("massive" Goldstone)
  \[ \omega = q\mu \]
Field Theory Model

T. Schafer, D. T. Son, M. A. Stephanov, D. Toublan and J. J. M. Verbaarschot, [hep-ph/0108210]
V. A. Miransky and I. A. Shovkovy, [hep-ph/0108178]

\[ \mathcal{L} = (\partial_0 - i\mu) \Phi^\dagger (\partial_0 + i\mu) \Phi - \overline{\partial} \Phi^\dagger \overline{\partial} \Phi - M^2 \Phi^\dagger \Phi - \lambda^4 (\Phi^\dagger \Phi)^2 \]

Doublet of U(2) \[ \phi = (\phi_1, \phi_2)^T \]

\( \mu \) inside overall U(1)

\[ \begin{align*}
\omega_1^2 &= \frac{\mu^2 - M^2}{3\mu^2 - M^2} p^2 + O(p^4), \\
\omega_2^2 &= 6\mu^2 - 2M^2 + O(p^2), \\
\omega_3^2 &= p^2 - 2\mu \omega_3, \quad \omega_3 \approx \frac{p^2}{2\mu}, \\
\omega_4^2 &= p^2 + 2\mu \omega_4. \quad \omega_4 = 2\mu
\end{align*} \]

Type I Goldstone
Type II Goldstone
“Massive Goldstone”
Holography

- Global on boundary = local in Bulk
- $U(2)$ gauge fields + scalar in doublet
- gauged model,
- Chemical potential only in $U(1)$
- Holographic Goldstone modes = Quasinormal Modes
- Simplest: $U(2)$ generalization of HHH
  [Hartnoll-Herzog-Horowitz]
- Decoupling limit
Holography

3D AdS
Schwarzschild BH

Maxwell +
Charged Scalar

Quasinormal Modes:
Holography

3D AdS
Schwarzschild BH

Maxwell + Charged Scalar

Quasinormal Modes:
Holography

3D AdS
Schwarzschild BH

Maxwell +
Charged Scalar

Quasinormal Modes:

\[ \omega(k) = \pm \Omega(k) - i \Gamma(k) \]

Horizon

“infalling”
Holography

U(2) decomposition:

\[
\sigma_- = \frac{1}{2}(\sigma_0 - \sigma_3) = \begin{pmatrix}
0 & 0 \\
0 & 1
\end{pmatrix}
\]

\[
\sigma_1 = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]

\[
\sigma_2 = \begin{pmatrix}
0 & i \\
-i & 0
\end{pmatrix}
\]

\[
\sigma_+ = \frac{1}{2}(\sigma_0 + \sigma_3) = \begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}
\]

broken:

unbroken:
Holography

Normal phase (high T)

\[ \omega = -iDk^2 \quad D = \frac{3}{4\pi T} \]

There are also 4 Diffusion modes

I. Amado, M. Kaminski and K. L., [arXiv:0903.2209 [hep-th]]
Holography

\[ \Psi'' + \left( \frac{f'}{f} + \frac{2}{\rho} \right) \Psi' + \frac{\chi^2}{f^2} \Psi - \frac{m^2}{f} \Psi = 0, \]
\[ \chi'' + 2 \frac{\rho}{\rho} \chi' - 2 \frac{\Psi^2}{f} \chi = 0, \]
\[ \xi'' + 2 \frac{\rho}{\rho} \xi' = 0, \]
\[ \chi = \bar{\mu}_\chi - \frac{\bar{n}_\chi}{\rho} + O \left( \frac{1}{\rho^2} \right) \]
\[ \xi = \bar{\mu}_\xi - \frac{\bar{n}_\xi}{\rho} + O \left( \frac{1}{\rho^2} \right) \]
\[ \Psi = \frac{\psi_1}{\rho} + \frac{\psi_2}{\rho^2} + O \left( \frac{1}{\rho^3} \right) \]
\[ \text{b.c.s: } \bar{\mu}_\chi = \bar{\mu}_\xi = \bar{\mu}. \]
\[ \psi_1 = 0. \]
\[ \psi_2 \propto \langle O \rangle \]
\[ \Psi'' + \left( \frac{f'}{f} + \frac{2}{\rho} \right) \Psi' + \frac{\chi^2}{f^2} \Psi - \frac{m^2}{f} \Psi = 0 , \]

\[ \chi'' + \frac{2}{\rho} \chi' - \frac{2\Psi^2}{f} \chi = 0 , \]

\[ \xi'' + \frac{2}{\rho} \xi' = 0 , \]

**Scalar**

**\( \sigma^- \) Sector**

**\( \sigma^+ \) Sector**

**Charge in \( \sigma_3 \) Sector:**

(Charge in \( \sigma_0 \) sector always >0)

triggers 2nd phase transition to p-wave

[I. Amado, D. Arean, A. Jimenez-Alba, L. Melgar, I. Salazar-Landea, arXiv: 1309.5986]
Holography

“σ-” Sector:

Broken phase, Type I (4th sound)  \( \omega = v_s k + (b - i\Gamma_s)k^2 \)
Holography

“σ_{1,2}” Sector:

Broken phase, Type II \[ \omega = (B - iC)k^2 \]
Broken phase, Type II \( \omega = (B - iC)k^2 \)

The temperature dependence of \( B \) and \( C \) is plotted in Figure x8. The value at \( T = T_c \) is given by the same value as in the ungauged model and in fact can also be cross checked by calculating the scalar mode dispersion relation in the unbroken phase at \( T = T_c \) since the QNMs must be continuous through the phase transition. We find a rather surprising dependence of \( B \) with the temperature. It starts at a finite value at the transition and then it rises rather sharply and falls off slower. It reaches a minimum at \( T \approx \frac{w}{4}T_c \) at which we found the change of sign in the residue of current correlators. We also find another peak around \( T \approx \frac{w}{4}T_c \). We expect that it is again related with the instability found in the gauge sector around that temperature. It would also be interesting to calculate \( B \) using an alternative method, such as the sound velocity can be calculated from thermodynamic considerations alone. In order to do this one would need to formulate the hydrodynamics of type II Goldstone modes. We are however not aware of such a hydrodynamic formulation and leave this for future research.

Additional unstable mode: p-wave condensate
Holography

“σ-” Sector:
Conductivities related to type I

We see that the resulting system of equations is now completely decoupled. We only have two diagonal conductivities \( \sigma^{++} \) and \( \sigma^{-+} \) corresponding to the unbroken \( U \) diffusive sector and a mode which is associated to the broken \( U \) coupling to the condensate. The former is the same as in the unbroken phase and of no further interest for us. The latter is again the well-studied \( U \) sqwave superconductors. Its conductivity has been already calculated in \([w]\) To check our numerics we have recalculated it and in Figure uu we show its behavior. It coincides completely with \([w]\). The real part shows the characteristic of superconductivity. Numerically this can be seen through the \( \omega \) behavior in the imaginary part. The Kramers-Kronig relation (see in appendix) implies then infinite DC conductivity. The real part of the AC conductivity also exhibits a temperature-dependent gap.

Figure uu: Real part (left) and imaginary part (right) of the conductivity as a function of frequency. The plots correspond to temperatures in the range \( T/T_c \approx 0.9 \). As expected, the plots reproduce the ones of \([w]\).

4.5 Conductivities in the \( l \) sector
The relevant equations for the \( l \) sector read

\[
\begin{align*}
\tau &= f a (1) x o f a (1), \\
\omega^2 f - \Psi^2 o \Theta^2 f &= a (1) x, \text{l97m} \\
\tau &= f a (2) x o f a (2), \text{l98m}
\end{align*}
\]

These equations obey the symmetry \( a (1) x \rightarrow a (2) x, a (2) x \rightarrow -a (1) x \). In general, this behavior is also typical of translation invariant charged media, in which accelerated charges cannot relax. However, working in the probe limit we effectively break translation invariance and therefore the infinite DC conductivity is a genuine sign of superconductivity.

\( \sigma^{-} \) Sector: (just HHH model)
Holography

“σ_{1,2}” Sector:
Conductivities related to type II, diagonal

Off-diagonal σ’s: no δ-function pole!
Holography

Fate of the diffusion modes in broken phase "σ-" Sector: gapped (pseudo) diffusion

\[ \omega = -i\gamma - iDk^2 \]
Holography

Fate of the diffusion modes in broken phase

“$\sigma$-” Sector: gapped (pseudo) diffusion

$$\omega = -i\gamma - iDk^2$$
**Holography**

Fate of the diffusion modes in broken phase

\[ \sigma_{1,2} \text{ - Sector: 2 gapped modes} \quad \omega = \pm \Omega - i\Gamma \]

\[ \begin{array}{c}
\text{Re}(\omega) \\
-6 & -4 & -2 & 2 & 4 & 6 \\
\end{array} \]

\[ \begin{array}{c}
\text{Im}(\omega) \\
-0.5 & -0.4 & -0.3 & -0.2 & -0.1 \\
\end{array} \]

\[ \begin{array}{c}
T=0.8T_c \\
T=0.5T_c \\
\end{array} \]
Holography

“Massive” Goldstone $Re(\omega) \approx 1.1q\mu$

![Graph showing real and imaginary parts of $\omega$ vs. $\mu/\mu_c$]
Landau criterion

- Superflow = spatial gauge field on boundary
- Critical superfluid velocity
  - $T=0$ via boosts $\omega(p) + \vec{p}.\vec{S} \leq 0 \Rightarrow S_c = \min \frac{\omega(p)}{p}$
  - $T>0$ more complicated
- Basic idea: negative energy, instability
- QNMs:
  $$\Re(\omega(p, S, T)) \leq 0 \quad \Im(\omega(p, S, T)) \geq 0$$
Landau criterion

Type I Goldstone
Landau criterion

Weak Coupling

(courtesy: A. Schmitt)

[Alford, Mallavarpu, Schmitt, Stetina] to appear
Landau criterion

Type I Goldstone

Stripes?
Landau criterion

Type II Goldstone

The study was motivated by the appearance of Type II NG bosons in the model as detailed in [p], which according to the aforementioned criterion should be responsible for driving the system out of the superfluid phase for arbitrarily small supervelocity. Taking advantage of the fact that the usual U(1) holographic wave superconductor is contained in [e] we have revisited the Landau criterion for holographic Type II NG bosons in Section 6, pointing out that when addressing the problem of the stability of the superfluid at finite supervelocity the analysis of the free energy does not give the correct answer. The QNM spectrum presents a tachyon at finite momentum for temperatures \( T^* < T < \tilde{T} \) being \( \tilde{T} \) the temperature at which the system is supposed to pass through a phase transition to an homogeneous and stable superfluid with nonvanishing supervelocity according to the FlEl calculation. Hence, the homogeneous superfluid is stable only for \( T < T^* \) and \( \gamma \approx \pi \) the velocity of sound vanishes. This condition can be easily seen to be equivalent to the Landau criterion and signals the existence of a critical velocity above which the superfluid is not stable anymore. Indeed, in the simple language of Section 6, the dispersion relation from the moving frame can be written as:

\[
\omega = \frac{\epsilon}{2|\mathbf{k}|^2} \left[ \frac{1}{\epsilon - \epsilon_0} \right]^2
\]

Does not support finite superflow!
Superconductor but not Superfluid
Landau criterion

Phase diagram based on Landau criterion

C.P. Herzog, P.K. Kovtun, and D.T. Son. Phys.Rev., D79:066002
Pallab Basu, Anindya Mukherjee, and Hsien-Hang Sieh, Phys.Rev., D79:045010

Stripes ?

Landau criterion via QNMs
Ungauged Model

What if SU(2) only global in AdS bulk?

- U(1) gauge field + scalar doublet
- Global symmetry sufficient to chose vacuum
- SU(2) = “Outer Automorphism”
- Decomposition: HHH-superfluid + scalar
- Still type II Goldstone?
- Massive?
Massless mode in 2nd scalar!

Dispersion relation: \( \omega = (b + ic)k^2 \)
Ungauged Model

Massive mode:

Does not obey theory!! $\omega \neq q\mu$

Effective U(2) symmetric action only for lower Energies!
Summary

- Type II Goldstone QNMs!
- Compare to weak coupling
- Universality of Pseudo-diffusion?
- “Un-gauged” model: no SU(2) gauge fields, violates some Theorems
- Backreacted models
  1st, 2nd, 4th sound modes etc.
- Superflow: striped phases?
Summary

• Type II Goldstone QNMs!
• Compare to weak coupling
• Universality of Pseudo-diffusion?
• "Un-gauged" model:
  no SU(2) gauge fields, violates some Theorems
• Backreacted models
  1st, 2nd, 4th sound modes etc.
• Superflow: striped phases?

THANK YOU!
Drude Peak

Due to gapped “Pseudo” diffusion mode!
Holography

Conductivities related to type II, off-diagonal
no Superconductor!