Analytical prediction of the blade trailing edge broadband noise of a plug fan

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Abstract. This study presents a low-order broadband noise prediction tool for centrifugal plug fans, based on analytical models using inputs from CFD simulations. Firstly, the fan used as a reference in this investigation was tested in a double reverberant room where the aerodynamic performance and the acoustic signature were measured simultaneously. Secondly, CFD RANS simulations of the fan in the test rig were performed. The outcome of the simulations is then used to feed Amiet’s analytical broadband noise model, along with wall-pressure spectral and correlation length models. Amiet’s model has been transposed to a rotating blade, adapted to the geometry of the fan. Finally, the model predictions are compared with the measurements.

1. Introduction
A plug fan is a centrifugal machine without volute, the use of which is widespread in building ventilation. As the demand for quieter systems and equipment grows, so does the need for low-order noise prediction tools for fan manufacturers. The main sources of broadband noise in a fan are mainly leading-edge noise (or turbulence-interaction noise), local flow separation, vortex shedding (only when the trailing-edge is blunt), interaction of turbulent outlet flow with the volute’s tongue (only in centrifugal fans with volute), tip vortex noise and trailing-edge noise (or self-noise) [1]. The latter is due to the scattering of the turbulence of the boundary layers at the trailing-edge. Its relative contribution to the global noise of a rotating machine can be significant when inlet turbulence is low and constitutes the focus of the present paper.

Among the different prediction tools, Amiet’s analytical model for the trailing-edge noise generated by an airfoil inside a wind-tunnel is particularly useful [2]. This model was later completed with a leading-edge back-scattering correction [3]. The model for airfoils has also been adapted for rotating blades, such as helicopter rotors [4] or low-speed fans [5, 6]. In [7], Amiet’s model was adapted to a centrifugal fan with volute, showing that for that particular case, trailing-edge noise was not the dominating source. However, to the authors’ knowledge, the adaptation of the model to a plug fan is missing.
2. Experimental and numerical setup

The study has been done on a plug fan, i.e. a backward-curved centrifugal fan without volute. It has seven blades of a constant thickness of 2 mm and an outer diameter of 360 mm (see Table 1 for more details).

| Designation          | Value |
|----------------------|-------|
| Number of blades \(B\) | 7     |
| Blade thickness \(e\) [mm] | 2     |
| Inlet diameter \(D_i\) [mm] | 224   |
| Outlet diameter \(D_o\) [mm] | 360   |
| Outlet span \(L\) [mm] | 98    |
| Chord \(c\) [mm] | 135   |
| Rotational speed \(n\) [rpm] | 1440  |

To assess the contribution of trailing-edge noise to the overall spectrum, the baseline impeller was modified with the inclusion of trailing-edge serrations. Their design parameters were an amplitude of \(2h = 24\) mm and a wavelength \(\lambda = 8\) mm (the impeller is designated accordingly as L8H12). The serrations are iron-shaped (as described in [8]), which is slightly more complex than the widespread sawtooth or sinusoidal geometry.

The fan has been tested in a double reverberant room according to standardized test category A: non-ducted at inlet and outlet (see the layout in Figure 1). The fan is mounted on a support, on the partition between both rooms, the bigger room being on the inlet side (see Figure 2). An auxiliary fan, which noise is duly filtered to avoid affecting the measurements, allows adjusting the operating point of the test fan. The volume flow rate is measured with a multi-nozzle chamber while the fan pressure rise is obtained according to ISO 5801 [9] with pressure rings in both reverberant rooms. The fan sound power levels are determined in both rooms following ISO 13347-2 [10], using a rotating microphone in the big room and 3 fixed microphones in the small one to make a spatial average of the sound pressure field in each room.

![Figure 1. Top view layout of the test facility.](image1)

![Figure 2. View of the fan assembly in the outlet pressure chamber.](image2)
CFD simulations of the test rig were also performed. The simulation model was 3D steady RANS, using the $k - \epsilon$ turbulence model and the Multiple Reference Frame (MRF) approach. The rotating domain encompasses the impeller and its interface is 23 mm away from the trailing-edge. Overall, 9.39M unstructured polyhedral volumes were used, with 5.22M of them in the rotating domain. On the blade surface, there is a boundary layer made of 5 cell layers, with a meshing size of 0.5 mm.

3. Experimental and numerical results

The best efficiency point of the fan, which corresponds to a volume flow rate of 2196 m$^3$/h, has been used as a reference for this study. The narrowband sound pressure spectra measured for this operating point, for both the baseline and the serrated impellers, is presented in Figure 3. The original spectra (not displayed here) showed some peaks due to resonances and vibroacoustic phenomena. As we are only interested in modelling the broadband noise, most of these peaks have been removed with a one-dimensional median filter, of order 14. Then, the biggest peaks which have not been treated by the filter have been trimmed manually.

![Figure 3. Narrowband sound pressure spectra at fan outlet](image)

The serrations on the trailing-edge reduce broadband noise, yielding a reduction of around 2 dB. Furthermore, a previous investigation with leading-edge serrations applied to the same fan did not yield a broadband reduction over all the measured frequencies (see [11] for more details). This suggests that, in our case, trailing-edge noise is more significant than leading-edge noise, and justifies that the present study is focused on the modelling of the former mechanism.

The comparison between the experimental and numerical fan curves is shown in Figure 4. Simulations systematically underpredict the fan pressure, with a relative error of 9.7% at the best efficiency point. In this type of fan, the pressure side is on the convex side of the blade and the suction side is on the opposite side. The vectors on the mid-span plane (Figure 5) show that the blades guide the flow quite well, with a small detachment close to the leading-edge that reattaches quite fast.

The visualisation of the relative velocity magnitude on a normal plane (Figure 6) shows that the flow detaches close to the front plate. We are interested in the suction side of the blade, where the boundary layer is thicker and more turbulent, and thus should contribute dominantly to the blade trailing edge noise. The velocity profile was extracted along a normal line situated midspan of the blade at a distance equal to 10% of the chord, upstream of the trailing edge (Figure 7). This simplified extraction neglects the thickening of the boundary layer close to the front plate, as well as the flow detachment which is present in the area.
The streamwise component of the velocity is much larger than the other two (respectively 6 times bigger than the spanwise and 21 times bigger than the normal), which can be neglected. From the velocity profile we can obtain the free stream velocity $U_e$ and compute $U_{99}$, the boundary layer thickness $\delta$, the displacement thickness $\delta^*$ and the momentum thickness $\theta$. The wall shear stress $\tau_w$ and the wall pressure gradient in the streamwise direction $dP/dx$, have also been obtained at the point 10% of the chord from the trailing-edge. It is to be noted that $\tau_w$ also corresponds to the maximum along the normal line, which is to be preferred in the presence of an adverse pressure gradient ([12]). The boundary layer parameters are listed in Table 2.

| Designation | $U_e$ [m/s] | $U_{99}$ [m/s] | $\delta$ [mm] | $\delta^*$ [mm] | $\theta$ [mm] | $\tau_w$ [Pa] | $dP/dx$ [Pa/m] |
|-------------|-------------|----------------|---------------|-----------------|---------------|---------------|----------------|
| Value       | 21.1        | 20.9           | 5.8           | 1.3             | 0.7           | 0.7           | 2720           |
4. Analytical model

Assuming a large aspect ratio $L/b$, the simplified expression for the far-field acoustic PSD of Amiet’s trailing-edge model becomes:

$$S_{pp} = \left(\frac{\omega x_3 b}{2\pi c_0 S_0}\right)^2 \cdot 2\pi L \cdot I \left(\frac{\omega}{U_c}, \frac{k x_2}{S_0}\right) \Phi_{pp}(\omega)$$

(1)

where $I$ is the tridimensional radiation integral, $L$ is the blade span length, $b$ is the half of chord length ($b = c/2$), $c_0$ is the speed of sound, $U_c$ is the convection velocity (estimated as $U_c = 0.7U_e$ but not directly accessible), $k = kc/2$ (where $k$ is the acoustic wavenumber), $l_y$ is the correlation length and $S_0$ is the corrected distance for convection effects (defined in [2]). The same formulation and notations as in [3] have been used throughout this paper. The coordinates $x_1, x_2, x_3$ follow, respectively, the streamwise, spanwise and normal directions. Amiet’s frame of reference is centred on the trailing-edge of the blade, which is simplified as a zero thickness flat plate tangent to the blade at the trailing edge.

The correlation length $l_y$ has been estimated using the empirical model proposed in Figure 7 of [13], which averages different measurements on airfoils and fan blades (see the redimensionalized curve in Figure 8). We also need to estimate the wall pressure spectrum $\Phi_{pp}$, for which we have used two different models (Figure 9):

- Rozenberg’s model, which can be found on Equation 12 from [14]
- The empirical non-dimensional spectrum, shown in Figure 6 of [13], which averages many experiments on both airfoils and fan blades and is redimensionalized for our case

![Figure 8. Correlation length, dimensionalized from [13].](image1)

![Figure 9. Wall-pressure models, dimensionalized for the current case.](image2)

We can consider that the circular motion of a blade is locally tangent to a translating motion. The noise is calculated for an angle of the blade trailing-edge $\phi = \Omega t$, where $\Omega$ is the angular velocity and $t$ the time. The reference frames are displayed in Figure 10.

A change of coordinates is needed to express the position in Amiet’s frame of reference $(x_1, x_2, x_3)$ with respect to the fixed frame of reference $(X, Y, Z)$ and the angles $\phi$ and $\zeta$ (blade trailing edge angle). For an axial machine, it can be quite complex (e.g. [6, 15]), as there are one translation and up to three rotations involved. However, given the simpler geometry of our case, the problem can be solved in two dimensions (the coordinates $x_2$ and $Z$ are equivalent).
Only one rotation and one translation are required. When these are performed, the following equations for the change of coordinates are derived:

\[
x_1 = X \cos(\phi + \zeta) + Y \sin(\phi + \zeta) - r \cos(\zeta) \tag{2}
\]

\[
x_2 = Z \tag{3}
\]

\[
x_3 = Y \cos(\phi + \zeta) - X \sin(\phi + \zeta) + r \sin(\zeta) \tag{4}
\]

To obtain the total PSD of the far-field acoustic pressure, we need to average the PSD of Equation 1 over all azimuthal positions \( \phi \):

\[
S_{pp_{b}} = \frac{B}{2\pi} \int_{0}^{2\pi} \left( \frac{\omega_{e}(\phi)}{\omega} \right)^{2} S_{pp}(\phi, \omega_{e})d\phi \tag{5}
\]

Accounting for a Doppler-effect correction there is the Doppler factor \( \frac{\omega_{e}(\phi)}{\omega} \), \( B \) is the number of blades, and \( S_{pp} \) is the acoustic PSD obtained from Equation 1. As the Mach number is \( M = 0.06 \) and we are interested in modelling the broadband noise, the Doppler factor can be assumed close to 1. The integral is calculated numerically using the trapezoidal rule. A convergence study has shown that the interval \([0, 2\pi]\) can be accurately discretized with 10 angular steps for all frequencies. Increasing this number has hardly any effect on the result of the integration.

5. Model validation

The sound power level needs to be calculated to compare it with the experimental results. For this, the sound pressure level is integrated over a hemisphere which points towards the positive Z coordinate and therefore encompasses the outlet fan side (see Figure 2). The inlet has not been considered because the sound transmission is much more complex due to the propagation of the sound waves through the inlet nozzle. A convergence study showed that the results converge with 27 points over the hemisphere (so the points are evenly distributed with a point every 30 degrees in both polar angle and azimuth). Another numerical assessment showed that \( L_{w} \) converges with a radius of the hemisphere \( R \) of 3 m, which ensures the conditions of acoustic and geometrical far-field. The following equation has been used to calculate the sound power level:

\[
L_{w} = 10 \log_{10} \left( \frac{1}{N} \sum_{i=1}^{N} 10^{0.1L_{p,i}} \right) + 10 \log_{10} (4\pi R^{2}) + 3 \tag{6}
\]
where 3 dB are added to take into account the reflection of the sound on the wall between both reverberant rooms. This is true when the source is close to the wall in terms of acoustic wavelength.

Finally, the sound power spectra have been computed with both Rozenberg’s model and the average empirical wall pressure spectrum. The narrowband spectra have been converted to one-third octave bands so they can be compared with the experimental results. The comparison is shown in Figure 11. Both analytical models reproduce the trend of the experimental results, but strongly underpredict the sound power, Rozenberg’s model being more inaccurate. This underprediction is believed a result of the fact that only the trailing-edge noise is considered in the analytical formulation, even though other sources may also contribute to the far-field noise. The substantial inaccuracy of the sound prediction could be due to the uncertainty of the wall-pressure models. Besides, the inaccuracy of the CFD results, induced by a rather coarse meshing in the blade boundary layer, could also contribute to the underprediction. The origin of the discrepancies is currently under investigation.

![Comparison of analytical and experimental power spectra](image)

**Figure 11.** One-third octave sound power level at outlet

6. Conclusions and future works

An analytical model to predict the trailing-edge noise of a plug fan has been developed. The outcome of CFD RANS simulations of the fan has been used as an input to estimate the spanwise correlation length and the spectrum of the wall-pressure fluctuations. A transposition of Amiet’s model to the centrifugal impeller has been implemented. The predicted sound pressure level is averaged for all angular positions of a blade and multiplied by the number of blades to get the far-field sound power level of the impeller, which is then compared with experimental results.

Two different analytical models have been evaluated, which differ on how the wall pressure spectrum is estimated (either with Rozenberg’s model or with an empirical spectrum average). Both models strongly underpredict the sound power level generated by the impeller, the second model being better than the first one. The underprediction could be due to the inaccuracy of the wall pressure spectra estimation at the trailing edge. In the future, this will be tackled by performing LBM simulations of the fan, which will also allow assessing the accuracy of the low-order RANS simulation.
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