Poynting-Robertson Effect on the Lyapunov Stability of Equilibrium Points in the Generalized Photogravitational Chermnykh’s Problem

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Abstract The Poynting-Robertson (P-R) effect on Lyapunov stability of equilibrium points is being discussed in the the generalized photogravitational Chermnykh’s problem when bigger primary is a source of radiation and smaller primary is an oblate spheroid. We derived the equations of motion, obtained the equilibrium points and examined the linear stability of the equilibrium points for various values of parameter which have been used in the present problem. We have examined the effect of gravitational potential from the belt. The positions of the equilibrium points are different from the position in classical case. We have seen that due to the P-R effect all the equilibrium points are unstable in Lyapunov sense.

Key words: equilibrium points: generalized photogravitational: Chermnykh’s problem: radiation pressure: Poynting-Robertson effect

1 INTRODUCTION

The Chermnykh’s problem is a new kind of restricted three body problem which was first time studied by Chermnykh (1987). Papadakis and Kanavos (2007) given numerical exploration of Chermnykh’s problem, in which the equilibrium points and zero velocity curves studied numerically also the non-linear stability for the triangular Lagrangian points are computed numerically for the Earth-Moon and Sun-Jupiter mass distribution when the angular velocity varies. The solar radiation pressure force $F_p$ is exactly apposite to the gravitational attraction force $F_g$ and change with the distance by the same law it is possible to consider that the result of action of this force will lead to reducing the effective mass of the Sun or particle. It is acceptable to speak about a reduced mass of the particle as the effect of reducing its mass depends on the properties of the particle itself. The first order in \( \frac{V}{c} \) the radiation pressure force is given by [see Poynting (1903), Robertson (1937)]:

$$F = F_p\left\{ \frac{R}{R} - \frac{V.R}{cR^2} - \frac{V}{c} \right\}$$

Where $F_p = \frac{3L_m}{16\pi R^2c^2}$ denotes the measure of the radiation pressure force, $R$ the position vector of $P$ with respect to radiation source Sun $S$, $V$ the corresponding velocity vector and $c$ the velocity of light. In the
expression of $F_p$, $L$ is luminosity of the radiating body, while $m$, $\rho$ and $s$ are the mass, density and cross section of the particle respectively.

The first term in equation (1) expresses the radiation pressure. The second term represents the Doppler shift of the incident radiation and the third term is due to the absorption and subsequent re-emission of the incident radiation. These last two terms taken together are the Poynting-Robertson effect. The Poynting-Robertson effect will operate to sweep small particles of the solar system into the Sun at cosmically rapid rate. Chernikov (1970) discussed the position as well as the stability of the Lagrangian equilibrium points when radiation pressure, P-R drag force are included. Murray (1994) systematically discussed the dynamical effect of general drag in the planar circular restricted three body problem. Ishwar and Kushvah (2006) examined the linear stability of triangular equilibrium points in the generalized photogravitational restricted three body problem with Poynting-Robertson drag, $L_4$ and $L_5$ points became unstable due to P-R drag which is very remarkable and important, where as they are linearly stable in classical problem when $0 < \mu < \mu_{\text{Routh}} = 0.0385201$. Further the normalizations of Hamiltonian and nonlinear stability of $L_{4(5)}$ in the present of P-R drag has been studied by Kushvah, Sharma, and Ishwar (2007a,b,c).

In this paper we generalized our previous paper Kushvah (2008a) in which linear stability has been examined in the generalized photogravitational Chernykh’s problem, we have seen that the collinear points are linearly unstable and triangular points are stable in the sense of Lyapunov stability provided $\mu < \mu_{\text{Routh}} = 0.0385201$. In present paper we have obtained the equations of motion and equilibrium points, zero velocity curves and the linear stability in the generalized photogravitational Chernykh’s problem. We have found that the collinear points deviate from the axis joining the two primaries, while the triangular points are not symmetrical due to Poynting-Robertson effect. We have examined the effect of gravitational potential from the belt, oblateness effect and radiation effect on the Lyapunov stability.

2 EQUATIONS OF MOTION AND POSITION OF EQUILIBRIUM POINTS

Let us consider the model proposed by Miyamoto and Nagai (1975), according to this model the potential of belt is given by:

$$V(r, z) = \frac{b^2 M_b}{N^3} \frac{[a r^2 + (a + 3N)] (a + N)^2}{[r^2 + (a + N)^2]^{5/2}}$$

where $M_b$ is the total mass of the belt and $r^2 = x^2 + y^2$, $a$, $b$ are parameters which determine the density profile of the belt, if $a = b = 0$ then the potential equals to the one by a point mass. The parameter $a$ “flatness parameter” and $b$ “core parameter”. where $N = \sqrt{z^2 + b^2}$, $T = a + b$, $z = 0$. Then we obtained

$$V(r, 0) = -\frac{M_b}{\sqrt{r^2 + T^2}}$$

and $V_z = \frac{M_{mx}}{(r^2 + T^2)^{3/2}}$, $V_y = \frac{M_{my}}{(r^2 + T^2)^{3/2}}$. As in Kushvah (2008a,b), we consider the barycentric rotating co-ordinate system $Oxyz$ relative to inertial system with angular velocity $\omega$ and common $z$–axis. We have taken line joining the primaries as $x$–axis. Let $m_1, m_2$ be the masses of bigger primary(Sun) and smaller primary(Earth) respectively. Let $Ox, Oy$ in the equatorial plane of smaller primary and $Oz$ coinciding with the polar axis of $m_2$. Let $r_e, r_p$ be the equatorial and polar radii of $m_2$ respectively, $r$ be the distance
of the masses and distance between primaries as unity, the unit of time i.e. time period of \( m_1 \) about \( m_2 \) consists of \( 2\pi \) units such that the Gaussian constant of gravitational \( k^2 = 1 \). Then perturbed mean motion \( n \) of the primaries is given by \( n^2 = 1 + \frac{3\Delta x}{2} + \frac{2M_1 \varepsilon_{r_1}}{(r_2 + T^2)^{3/2}} \), where \( r_2 = (1 - \mu)q_1^{2/3} + \mu^2 \). For simplicity, we set \( r = r_c = 0.9999, T = 0.01 \) for further numerical results, where \( A_2 = \frac{r_2 - r_1}{2r_2} \) is oblateness coefficient of \( m_2 \), where \( \mu = \frac{m_2}{m_1 + m_2} \) is mass parameter, \( 1 - \mu = \frac{m_1}{m_1 + m_2} \) with \( m_1 > m_2 \). Then coordinates of \( m_1 \) and \( m_2 \) are \((x_1, 0) = (-\mu, 0)\) and \((x_2, 0) = (1 - \mu, 0)\) respectively. Further, in our consideration, the velocity of light needs to be dimensionless, too, so consider the dimensionless velocity of light as \( c_a \) which depends on the physical masses of the two primaries and the distance between them. The mass of Sun \( m_1 \approx 1.989 \times 10^{30} \text{kg} \approx 332, 946m_2\) (The mass of Earth), hence mass parameter for this system is \( \mu = 3.00348 \times 10^{-6} \). In the above mentioned reference system the we determined the equations of motion of the infinitesimal mass particle in \( xy \)-plane. Now using Miyamoto and Nagai (1975) profile and Kushvah (2008a,b), then the equations of motion are given by:

\[
\dot{x} - 2n\dot{y} = U_x - V_x = \Omega_x, \tag{4}
\]

\[
\dot{y} + 2n\dot{x} = U_y - V_y = \Omega_y \tag{5}
\]

where

\[
\Omega_x = n^2 x - \frac{(1 - \mu)q_1(x + \mu)}{r_1^3} - \frac{\mu(x + \mu - 1)}{r_2^3}
- \frac{3 \mu A_2 (x + \mu - 1)}{2} - \frac{M_b x}{r_2^3 (r_2 + T^2)^{3/2}}
- \frac{W_1}{r_1^2} \left\{ \frac{(x + \mu)}{r_2^3} [(x + \mu)\dot{x} + \dot{y}] + \ddot{x} - n\dot{y} \right\},
\]

\[
\Omega_y = n^2 y - \frac{(1 - \mu)q_1 y}{r_1^3} - \frac{\mu y}{r_2^3}
- \frac{3 \mu A_2 y}{2} - \frac{M_b y}{r_2^3 (r_2 + T^2)^{3/2}}
- \frac{W_1}{r_1^2} \left\{ \frac{y}{r_2^3} [(x + \mu)\dot{x} + \dot{y}] + \ddot{y} + n(x + \mu) \right\},
\]

\[
\Omega = \frac{\mu^2 (x^2 + y^2)}{2} + \frac{(1 - \mu)q_1}{r_1} + \frac{\mu}{r_2}
+ \frac{M_b}{2r_2^2} + \frac{M_b}{(r_2 + T^2)^{1/2}}
+ W_1 \frac{[(x + \mu)\dot{x} + \dot{y}]}{2r_1^2} - n \arctan \left( \frac{y}{x + \mu} \right)
\]

\[
W_1 = \frac{(1 - \mu)(1 - \mu)}{c_a}, \quad q_1 = 1 - \frac{E}{F}, \quad \chi (\text{C.G.S. system}) = 1 - \frac{5.6 \times 10^{-5}}{a_p} \chi.
\]

The energy integral of the problem is given by \( C = 2\Omega - \dot{x}^2 - \dot{y}^2 \), where the quantity \( C \) is the Jacobi’s constant. The zero velocity curves \( C = 2\Omega(x, y) \) are presented in various frames of figure (1) for the entire range of parameters \( q_1, A_2, M_b \) details of the frames are given in the table (1). We have seen that in frame \( B(q_1 = .5) \) and \( C(q_1 = 1) \) there are closed curves around the \( L_4(5) \) so they are stable but the stability
From above, we obtained:

\[ x = -\mu \pm \left[ \frac{q_{1}}{n^{2}} \right]^{2/3} \left[ 1 + \frac{nW_{1}}{2(1 - \mu)y} + \frac{3A_{2}}{2} \right]^{2/3} - y^{2} \quad \text{and} \quad \left( 1 - 2r_{c} \right)M_{b} \left( 1 - \frac{3\mu A_{2}}{2(1 - \mu)} \right) \right]^{-2/3} \]

\[ y = 1 - \mu \pm \left[ 1 - \frac{nW_{1}}{\mu y} - \frac{5}{2}A_{2} - \frac{\mu(1 - 2r_{0})M_{b}}{(r_{c}^{2} + T^{2})^{3/2}} \right]^{-2/3} - y^{2} \]

From equations (7, 8) the value of \( y \) is always positive, hence the the equilibrium points are no-longer

| Table 1 Zero velocity curves when \( T = 0.01, \mu = 0.025 \) |
|---|
| Frame when \( M_{b} = 0.2 \) | I(\( A_{2} = 0.00 \)) | II(\( A_{2} = 0.02 \)) | III(\( A_{2} = 0.04 \)) | About the stability of \( L_{4(5)} \) |
| A (\( q_{1} = 0 \)) | No | Yes | Yes | Unstable |
| B (\( q_{1} = 0.5 \)) | very small | very small | very small | Stability reduced |
| C (\( q_{1} = 1 \)) | yes | yes | yes | Stable |
| Frame | q_{1} = 1A_{2} = 0.02 | I(\( M_{b} = 0.25 \)) | II(\( M_{b} = 0.5 \)) | III(\( M_{b} = 0.75 \)) | Stability affected by mass of the belt |
| D | low effect of belt | medium effect of belt | very high effect of belt | yes |

around \( L_{4(5)} \) disappeared so they are unstable due to P-R effect. When the P-R effect is absent as in frames \( D(q_{1} = 1, A_{2} = 0.02 I - M_{b} = 0.25, II - M_{b} = 0.5, III - M_{b} = 0.75) \) the effect of oblateness and mass of the belt is presented, we have seen that the position of ovals around the \( L_{4(5)} \) are different but still \( L_{4(5)} \) are stable.

The position equilibrium points are given by putting \( \Omega_{x} = \Omega_{y} = 0 \) i.e.,

\[
\begin{align*}
\frac{n^{2}x}{r_{1}^{2}} - \frac{(1 - \mu)q_{1}(x + \mu)}{r_{1}^{2}} - \frac{\mu(x + \mu - 1)}{r_{2}^{2}} - \frac{3\mu A_{2}(x + \mu - 1)}{r_{2}^{2}} - \frac{M_{b}x}{(r^{2} + T^{2})^{3/2}} - \frac{W_{1}ny}{r_{1}^{2}} = 0, \\
\frac{n^{2}y}{r_{1}^{2}} - \frac{(1 - \mu)q_{1}y}{r_{1}^{2}} - \frac{\mu y}{r_{2}^{2}} - \frac{3\mu A_{2}y}{r_{2}^{2}} - \frac{M_{b}y}{(r^{2} + T^{2})^{3/2}} - \frac{W_{1}n(x + \mu)}{r_{1}^{2}} = 0
\end{align*}
\]

when \( (W_{1} \neq 0) \), from equations (7, 8) we obtained:

\[
\begin{align*}
 r_{1} &= q_{1}^{1/3} \left[ 1 - \frac{nW_{1}}{6(1 - \mu)y} - \frac{A_{2}}{2} \right. \\
 & \left. \quad + \frac{(1 - 2r_{c})M_{b} \left( 1 - \frac{3\mu A_{2}}{2(1 - \mu)} \right)}{3(r_{c}^{2} + T^{2})^{3/2}} \right] \\
 r_{2} &= 1 + \frac{\mu(1 - 2r_{c})M_{b}}{3(r_{c}^{2} + T^{2})^{3/2}} + \frac{nW_{1}}{3\mu y}
\end{align*}
\]

From above, we obtained:

\[
\begin{align*}
x &= -\mu \pm \left[ \frac{q_{1}^{1/3}}{n^{2}} \left[ 1 + \frac{nW_{1}}{2(1 - \mu)y} + \frac{3A_{2}}{2} \right]^{1/2} - y^{2} \right]^{-1/2} \\
 x &= 1 - \mu \pm \left[ \frac{1 - nW_{1}^{2}}{\mu y} - \frac{5}{2}A_{2} - \frac{\mu(1 - 2r_{0})M_{b}}{(r_{c}^{2} + T^{2})^{3/2}} \right]^{-2/3} - y^{2} \right]^{1/2}
\end{align*}
\]
then from equations (4) and (5) we obtained the triangular equilibrium points as:

\[
x = -\mu + \frac{q_{1}^{2/3}}{2}(1 - A_2) - \frac{nW_1 \left[ \mu q_{1}^{2/3} - 2(1 - \mu) \right]}{6\mu(1 - \mu)y_0} \\
\quad + \frac{(1 - 2r_c)M_b \left\{ 1 - \frac{3\mu A_2}{(1 - \mu)} \right\} q_{1}^{2/3} - 1}{3 \left( r_c^2 + T^2 \right)^{3/2}}
\]

(13)

\[
y = \pm \frac{q_{1}^{2/3}}{2} \left[ 4 - q_{1}^{2/3} + 2 \left( q_{1}^{2/3} - 2 \right) A_2 \right] \\
\quad - \frac{2nW_1 \left( q_{1}^{2/3} - 2 \right)}{3\mu(1 - \mu)y_0} \\
\quad - \frac{4(2r_c - 1)M_b \left\{ \left( q_{1}^{2/3} - 3 \right) - \frac{3\mu A_2(q_{1}^{2/3} - 3)}{2(1 - \mu)} \right\}}{3 \left( r_c^2 + T^2 \right)^{3/2}}
\]

(14)

All these results are similar with Szebehely [1967], Ragos and Zafiropoulos [1995], Kushvah [2008a,b] and others.

3 LYAPUNOV STABILITY

In this section we will examine the P-R effect on the linear stability conditions. In mathematics, the notion of Lyapunov’s stability occurs in the study of dynamical systems. In simple terms, if all solutions of the dynamical system that start out near an equilibrium point of Lyapunov’s stability occur in the study of dynamical systems. In simple terms, if all solutions of the dynamical system that start out near an equilibrium point stay near it forever, then it is Lyapunov’s table. Let the position of any equilibrium point is \((x*, y*)\) the taking \(x = x* + \alpha, y = y* + \beta\), where \(\alpha = \xi e^{\lambda t}, \beta = \eta e^{\lambda t}\) are the small displacements \(\xi, \eta, \lambda\) are parameters, then the equations of perturbed motion corresponding to the system of equations (4), (5) are as follows:

\[
\dot{\alpha} - 2n\ddot{\beta} = \alpha \Omega_{x*}^* + \beta \Omega_{y*}^* + \dot{\alpha} \Omega_{x*}^* + \dot{\beta} \Omega_{y*}^* 
\]

(15)

\[
\ddot{\beta} + 2n\dot{\alpha} = \alpha \Omega_{y*}^* + \beta \Omega_{y*}^* + \dot{\alpha} \Omega_{y*}^* + \dot{\beta} \Omega_{y*}^* 
\]

(16)

where suffix * is corresponding to the equilibrium points.

\[
(\lambda^2 - \lambda \Omega_{x*}^*) \xi + \left[ -(2n + \Omega_{x*}^*) \lambda - \Omega_{x*}^* \right] \eta = 0
\]

(17)

\[
[(2n - \Omega_{y*}^*) \lambda - \Omega_{y*}^*] \xi + \left( \lambda^2 - \lambda \Omega_{y*}^* - \Omega_{y*}^* \right) \eta = 0
\]

(18)

this system has singular solution if,

\[
\left| \begin{array}{ll}
\lambda^2 - \lambda \Omega_{x*}^* - \Omega_{x*}^* & -(2n + \Omega_{x*}^*) \lambda - \Omega_{x*}^* \\
(2n - \Omega_{y*}^*) \lambda - \Omega_{y*}^* & \lambda^2 - \lambda \Omega_{y*}^* - \Omega_{y*}^*
\end{array} \right| = 0
\]
at the equilibrium points:

\[ a = 3 \frac{W_1}{r_{1*}^2}, \]
\[ b = 2n^2 - f_* - \frac{3\mu A_2}{r_{2*}^2} + \frac{3M_b T^2}{(r_2^2 + T^2)^{5/2}} + \frac{2W_1^2}{r_{1*}^4}, \]
\[ c = -a(1+\epsilon), \]
\[ e = \frac{\mu}{r_{2*}^2} A_2 + \frac{\mu}{r_{1*}^2 r_{2*}^2} \left( 1 + \frac{5 A_2}{2r_{2*}^2} \right) y_*^2 \]
\[ + \frac{3M_b \left( \frac{r_*^2 y_*^2}{r_{1*}^2} - T^2 \right)}{(r_2^2 + T^2)^{5/2}} \]
\[ d = (n^2 - f_*) \left[ n^2 + 2f_* - \frac{3\mu A_2}{r_{2*}^5} + \frac{3M_b T^2}{(r_2^2 + T^2)^{5/2}} \right] \]
\[ + 9\mu(1-\mu) y_*^2 \left[ \frac{q_1}{r_{1*}^2 r_{2*}^2} + \frac{3M_b}{(r_2^2 + T^2)^{5/2}} \left\{ \frac{\mu q_1}{r_{1*}^2} + \frac{(1-\mu) \left( 1 + \frac{5 A_2}{2r_{2*}^2} \right)}{r_{2*}^2} \right\} \right] \]
\[ - \frac{6\mu n W_1 y_*}{r_{1*}^4} \left\{ \left( x_* + \mu \right) \left( x_* + \mu - 1 \right) + y_*^2 \right\} \]
\[ + \frac{3M_b \left[ x_* (x_* + \mu) + y_*^2 \right]}{(r_2^2 + T^2)^{5/2}} \]

where \( f_* = \frac{(1-\mu) q_0}{r_{1*}^2} + \frac{\mu}{r_{2*}^2} \left( 1 + \frac{3 A_2}{2r_{2*}^2} \right) + \frac{3M_b}{(r_2^2 + T^2)^{5/2}} \). The points \( L_1, L_2, L_3 \) no longer lie along the line joining the primaries, since the condition is not satisfied for them, so taking \( y \to 0, \frac{W_1}{y} \to 0 \) because \( y \gg W_1, x \gg W_1 \), from (19) we have \( r_1 \approx \left[ \frac{q_0}{W_1} \right]^{1/3} \). Since \( f_* > 1 \) and characteristic equation (19) positive root for collinear points so they are unstable. Using Ferrari’s theorem the roots of characteristic equation (19) are given by:

\[ \lambda_i = -\frac{(a + A)}{4} \pm \sqrt{\left( \frac{a + A}{4} \right)^2 - B} \]  (20)

where \( A = \pm \sqrt{8t - 4b + a^2} \), and \( B = \left( \frac{b}{2} + \alpha_1 a^2 \right) \left( 1 \pm \sqrt{1 + 8\alpha_1} \right) + \frac{1 + e}{\sqrt{1 + 8\alpha_1}}, \) \( \alpha_1 = \frac{(1+e)(1+e^2 - b + d)}{2(\beta + \mu)} > 0 \) from this the characteristic roots are given by:

\[ \lambda_{1,2} = -\frac{a \left( 1 + \sqrt{1 + 8\alpha_1} \right)}{4} \]
\[ \pm \sqrt{\frac{a^2 \left( 1 + \sqrt{1 + 8\alpha_1} \right)}{16} - B_1} \]  (21)
\[ \lambda_{3,4} = -\frac{a \left( 1 - \sqrt{1 + 8\alpha_1} \right)}{4} \]
\[ \pm \sqrt{\frac{a^2 \left( 1 - \sqrt{1 + 8\alpha_1} \right)}{16} - B_2} \]  (22)

where

\[ B_1 = \left( \frac{b}{2} + \alpha_1 a^2 \right) \left( 1 + \sqrt{1 + 8\alpha_1} \right) - \frac{1 + e}{\sqrt{1 + 8\alpha_1}}, \]
\[ B_2 = \left( \frac{b}{2} + \alpha_1 a^2 \right) \left( 1 - \sqrt{1 + 8\alpha_1} \right) + \frac{1 + e}{\sqrt{1 + 8\alpha_1}}. \]
from equations\(^{(21,22)}\), we found that at least one of the roots \(\lambda_i (i = 1, 2, 3, 4)\) have a positive real part due to P-R effect thus the triangular equilibrium points are unstable in the sense of Lyapunov stability result are similar to Chernikov (1970) and Kushvah (2008b).

4 CONCLUSION

We have seen the collinear points \(L_1, L_2, L_3\) no longer lie along the line joining the primaries and they are unstable in classical case. If \(q_1 = 0.5, q_1 = 1\) there are closed curves around the \(L_{4(5)}\) so they are stable but the stability range reduced\([see frame \(B(q_1 = 0.5)\) in figure (11)] due to P-R effect. When the P-R effect is absent as in frames \(D(q_1 = 1, A_2 = 0.02I - M_b = 0.25, II - M_b = 0.5, III - M_b = 0.75)\) the effect of oblateness and mass of the belt is presented, we have seen that the positions of ovals around the \(L_{4(5)}\) are different but still \(L_{4(5)}\) are stable. In frame \(A(q_1 = 0)\) the closed curves around \(L_{4(5)}\) disappeared so they are unstable due to P-R effect.

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Fig. 1  This figure show the zero velocity curves for $\mu = 0.025, T = 0.01$, frames (A-I to III) $q_1 = 0.00, M_b = 0.02, A_2 = 0.00 - 0.04$, (B-I to III) $q_1 = 0.50, M_b = 0.02, A_2 = 0.00 - 0.04$, (C-I to III) $q_1 = 0.75, M_b = 0.25, 0.5, 0.75$, $A_2 = 0.02$