Robust optimization for optimal day-ahead economic dispatch of CCHP considering wind-load uncertainty

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Abstract. Distributed wind power generators (WPGs) have been increasingly deployed into power system through combined cooling, heat, and power (CCHP) system. However, uncertain load and intermittent wind power generation adversely affect the economy of the system operation. In order to improve the economy of the CCHP operation, this paper presents a two-stage robust optimization model for the CCHP day-ahead (DA) economic dispatch considering the uncertainties of wind power generation and electric load demand. The model aims at minimizing the total operation cost. The day-ahead first stage is to decide the state and base generation of controllable generators that can withstand the worst-case scenario of the uncertainty while the real-time (RT) second stage is to adjust the generation of the generators according to the actual scenario of the uncertainty. To solve the model, duality theory, Big-M method, and column-and-constraint generation (C&CG) decomposition approach are employed to convert the model into tractable master problem and sub problem, so that a column-and-constraint generation iteration algorithm can be applied to figure out the optimal dispatch solution. Finally, experimental results reveal the effectiveness of the presented model and the employed method.

1. Introduction

Wind power has been widely integrated into power system due to its environmentally friendly feature [1]. In terms of integrating wind power, CCHP system has been regarded as an effective way. CCHP can integrate different power including cooling, heat, and electricity to meet various loads [2,3].

Research [2] proposed a CCHP model consisting micro turbine (MT), heat storage system (HSS), absorption chiller (AC), and electrical boiler (EB). The authors demonstrated that CCHP can effectively utilize various energies. Research [3] deployed an extra heat recovery system (HRS) into the CCHP, where experiments revealed the improved energy efficiency. Even though CCHP is highly effectively, uncertain load and WPG tremendously affect the economy of CCHP operation. Therefore, it is necessary to consider the uncertainty in day-ahead economic dispatch (DED) because it decides base generation (BG) and unit commitment (UC) for units.

In terms of handling uncertainty, deterministic optimization (DO), stochastic optimization (SO), and robust optimization (RO), perform best. DO optimizes based on predicted scenario. Research [4] employed DO in enabling optimal dispatch. Their experiments illustrated the tractable feature of DO in handling practical dispatch. However, research [5] indicates that the DO-based DED (D-DED) will be unfeasible if the predicted scenario encounters error. Therefore, SO that employs large-size scenarios to describe uncertainty has been proved to be more reliable than D-DED [5]. However, two
flaws still exist in SO-based DED (S-DED): massive scenarios result in computationally intensive issue; it is difficult to obtain precise probability distribution function (PDF) so that high-quality scenario is unavailable.

To overcome the issues, RO employs uncertainty set instead of predicted scenario or massive scenarios to represent uncertainty. In addition, RO only focuses on the worst-case scenario in the uncertainty set. Therefore, RO-based DED (R-DED) can avoid the unfeasibility while achieving satisfied computational efficiency. Researches [1] successfully employed RO in enabling optimal dispatch considering WPGs. Consequently, RO has been regarded as a reliable method in terms of optimizing DED.

Motivated by the above researches, this paper presents RO in enabling optimal DED of CCHP considering load-wind uncertainty. Firstly, a two-stage RO DED model is constructed. The first-stage DED is to provide optimal BG and UC that can withstand the worst-case load-wind scenario. The second-stage real-time economic dispatch (RED) is to adjust the generation after observing actual load-wind scenario. The model aims at minimizing total operational cost. Furthermore, duality theory, Big-M method, C&C decomposition, and C&C iteration are applied to figure out the model to provide the optimal DED strategy for CCHP.

The rest of the paper is organized as following: section 2 details the mathematical modelling of the DED; the solution method of the presented model is provided in section 3; section 4 discusses the case studies; section 5 summarizes the paper.

2. Two-stage robust DED for CCHP

2.1. CCHP system structure

Figure 1 illustrates operation mode of the presented CCHP. The system contains WPG, fuel cell (FC), electrical storage system (ESS), heat storage system (HSS), MT, EB, HRS, and AC. MT can generate heat power during generating electricity; EB is able to consume electricity to generate heat power. HRS will collect all the heat power and further re-allocate them. AC can convert the obtained heat power into cool power so that cool load can be satisfied. Electricity market (EM) connects distribution network and CCHP, where CCHP can buy or sell power in EM to achieve higher profit.

![Figure 1: Structure of CCHP system.](image-url)
2.2. Objective and constraints

2.2.1. Uncertainty description based on box-type uncertainty set. Electricity load \( \tilde{P}_{\text{load}}(t) \) and wind power generation \( \tilde{P}_{\text{wind}}(t) \) are regarded as stochastic variables in the model. The box-type uncertainty set [6] is applied to describe \( \tilde{P}_{\text{load}}(t) \) and \( \tilde{P}_{\text{wind}}(t) \) as shown by (1)-(3).

\[
\tilde{P}_{\text{load}}(t) \in [P_{\text{load}}^{\min}(t), P_{\text{load}}^{\max}(t)], \tilde{P}_{\text{wind}}(t) \in [P_{\text{wind}}^{\min}(t), P_{\text{wind}}^{\max}(t)], t = 1, 2, \cdots, 24.
\]

(1)

\[
\sum_{t=1}^{24} \left[ \tilde{P}_{\text{wind}}(t) - P_{\text{wind}}^{\text{prev}}(t) \right] \cdot \rho_{\text{wind}}(t) + \frac{P_{\text{wind}}^{\text{prev}}(t) - \tilde{P}_{\text{wind}}(t)}{P_{\text{wind}}^{\text{prev}}(t)} \cdot \rho_{\text{wind}}(t) \leq \Gamma_{\text{wind}}
\]

(2)

\[
\sum_{t=1}^{24} \left[ \tilde{P}_{\text{load}}(t) - P_{\text{load}}^{\text{prev}}(t) \right] \cdot \rho_{\text{load}}(t) + \frac{P_{\text{load}}^{\text{prev}}(t) - \tilde{P}_{\text{load}}(t)}{P_{\text{load}}^{\text{prev}}(t)} \cdot \rho_{\text{load}}(t) \leq \Gamma_{\text{load}}
\]

(3)

Where, \( P_{\text{load}}^{\min}(t) \) and \( P_{\text{load}}^{\max}(t) \) represent minimal and maximal values of \( \tilde{P}_{\text{load}}(t) \); \( P_{\text{wind}}^{\text{prev}}(t) \) indicates predicted value of \( \tilde{P}_{\text{wind}}(t) \); \( P_{\text{load}}^{\text{prev}}(t) \) and \( P_{\text{load}}^{\text{prev}}(t) \) mean positive and negative ranges from \( P_{\text{wind}}^{\text{prev}}(t) \); \( \rho^{+}(t) \) and \( \rho^{-}(t) \) are 0-1 variables; \( \Gamma \) is adjustable budget parameter that can constrain the number of the worst-case scenarios in 24 hours.

2.2.2. Objective. The presented two-stage model for DED of CCHP is compactly shown as (4).

\[
\left\{ \begin{array}{l}
\min \left[ C_{\text{Da}}(x) + \max \left[ C_{\text{RT}}(u, y) \right] \right] \\
\text{s.t. } H_{\text{Da}}(x) = 0, G_{\text{Da}}(x) \geq 0, \\
H_{\text{RT}}(x, u, y, \Gamma) = 0, G_{\text{RT}}(x, u, y, \Gamma) \geq 0.
\end{array} \right.
\]

(4)

Where, \( x \), \( y \), and \( u \) indicate DED decision, RED decision, and stochastic variable; \( C_{\text{Da}}(x) \) denotes DED cost; \( C_{\text{RT}}(u, y) \) represents RED cost; \( H = 0 \) and \( G \geq 0 \) are the compact equality and inequality constraints. Outer function min aims at minimizing total cost. The max function is to search for worst-case scenario of \( u \) and the inner min function is to minimize RED cost under worst-case \( u \).

DED cost (5) involves MT generation \( C_{\text{MT}} \), FC generation \( C_{\text{FC}} \), and transaction in day-ahead EM \( C_{\text{DA}}^{\text{Grid}} \). RED cost (6) contains wind power curtailment \( C_{\text{wind}} \), transaction in real-time EM \( C_{\text{EM}}^{\text{RT}} \), and regulation of controllable generators (CGs). CGs indicate MT, FC, and transaction.

\[
\begin{align*}
C_{\text{Da}}(x) &= C_{\text{MT}} + C_{\text{FC}} + C_{\text{DA}}^{\text{Grid}}, C_{\text{MT}} = \sum_{t=1}^{24} [a_{\text{MT}} P_{\text{MT}}(t) + b_{\text{MT}}] \\
C_{\text{FC}} &= \sum_{t=1}^{24} [a_{\text{PC}} P_{\text{PC}}(t) + b_{\text{PC}}], C_{\text{DA}}^{\text{Grid}} = \sum_{t=1}^{24} [\lambda_{\text{DAC}} P_{\text{DA}}(t) - \lambda_{\text{DAC}}^{\text{DA}} P_{\text{DA}}(t)] \\
C_{\text{RT}}(u, y) &= C_{\text{RT}}^{\text{up}} + C_{\text{RT}}^{\text{down}} + C_{\text{RT}}^{\text{up}} + C_{\text{RT}}^{\text{down}} + C_{\text{RT}}^{\text{wind}} + C_{\text{RT}}^{\text{EM}} \\
C_{\text{RT}}^{\text{up}} + C_{\text{RT}}^{\text{down}} &= \sum_{t=1}^{24} [\lambda_{\text{RT}}^{\text{up}} P_{\text{RT}}^{\text{up}}(t) + \lambda_{\text{RT}}^{\text{down}} P_{\text{RT}}^{\text{down}}(t) + \lambda_{\text{RT}}^{\text{up}} P_{\text{RT}}^{\text{up}}(t) + \lambda_{\text{RT}}^{\text{down}} P_{\text{RT}}^{\text{down}}(t)] \\
C_{\text{wind}} &= \sum_{t=1}^{24} \lambda_{\text{wind}} [P_{\text{wind}}(t) - P_{\text{wind}}(t)] C_{\text{EM}}^{\text{RT}} = \sum_{t=1}^{24} \lambda_{\text{wind}} [P_{\text{wind}}(t) - P_{\text{wind}}(t)]
\end{align*}
\]

(5)

(6)

Where, \( a \), \( b \), and \( \lambda \) are cost coefficients; \( P_{\text{CG}}(t) \) is BG of CGs; \( P_{\text{CG}}^{\text{up}}(t) \) and \( P_{\text{CG}}^{\text{down}}(t) \) are up-regulation and down-regulation of CGs; \( P_{\text{Buy}}(t) \), \( P_{\text{Sell}}(t) \), \( P_{\text{Buy}}^{\text{RT}}(t) \), and \( P_{\text{Sell}}^{\text{RT}}(t) \) are bought sold power in day-ahead EM and real-time EM; \( C_{\text{CG}}^{\text{up}} \) and \( C_{\text{CG}}^{\text{down}} \) are up-regulation and down-regulation costs of CGs; \( P_{\text{wind}}^{\text{RT}}(t) \) is rejected wind power.
2.2.3. Controllable generator operation constraint. Constraints of CGs involve limitation of generation, ramp limitation, and regulation limitation, which are shown as (7).

\[
\begin{align*}
0 & \leq P_{CG} (t) \leq S_{CG} (t) P_{CG}^{\max}, \quad -R_{CG}^{Dn} \leq P_{CG} (t) - P_{CG} (t-1) \leq R_{CG}^{Up}, \quad 0 \leq P_{CG} (t) \leq R_{CG}^{Up} \\
0 & \leq P_{CG} (t) + P_{CG}^{\max} - P_{CG} (t) - S_{CG} (t) P_{CG}^{\max}, \quad 0 \leq P_{CG} (t) \leq R_{CG}^{Dn} \\
R_{CG}^{Dn} & \leq P_{CG} (t) + P_{CG}^{\max} (t) - P_{CG} (t) - [P_{CG} (t-1) + P_{CG}^{\max} (t-1) - P_{CG} (t-1)] \leq R_{CG}^{Up}
\end{align*}
\]

Where, \( P_{CG}^{\min} \) and \( P_{CG}^{\max} \) denote the minimal and maximal generation of CGs; \( R_{CG}^{Up} \) and \( R_{CG}^{Dn} \) represent limitations of up-ramp and down-ramp; \( S_{CG} (t) \) indicate state of CGs, where “1” means “on” and “0” represents “off”.

2.2.4. Storage system constraint. This paper strictly follows research [6] to construct constraints of electricity storage system and heat storage system. Please refer to the constraints in research [6].

2.2.5. Electricity market constraint. EM constraint is represented as (8).

\[
\begin{align*}
S_{Buy}^{DA} (t) + S_{Sell}^{DA} (t) & \leq 1, 0 \leq P_{Buy}^{DA} (t) \leq S_{Buy}^{DA} (t) P_{EM}^{\max}, \quad 0 \leq P_{Sell}^{DA} (t) \leq S_{Sell}^{DA} (t) P_{EM}^{\max} \\
0 & \leq P_{Buy}^{RT} (t) \leq S_{Buy} (t) P_{EM}^{\max}, \quad 0 \leq P_{Sell}^{RT} (t) \leq S_{Sell} (t) P_{EM}^{\max} \\
0 & \leq P_{Buy}^{DA} (t) + P_{Buy}^{RT} (t) \leq S_{Buy} (t) P_{EM}^{\max}, \quad 0 \leq P_{Sell}^{DA} (t) + P_{Sell}^{RT} (t) \leq S_{Sell} (t) \cdot P_{EM}^{\max}
\end{align*}
\]

Where, \( S_{Buy}^{DA} (t) \) and \( S_{Sell}^{DA} (t) \) denote 0-1 buying power state and 0-1 selling power state; \( P_{EM}^{\max} \) indicates limitation of exchanged power.

2.2.6. Power balance constraint. DA power balance is constrained by (9) while that of RT is restricted by (10).

\[
\begin{align*}
P_{TR} (t) + P_{RC} (t) + P_{Dis} (t) + P_{wind}^{\max} (t) + P_{Buy} (t) = P_{load}^{\max} (t) + P_{Sell} (t) + P_{EB} (t) \\
Q_{EM}^{H} (t) + Q_{EB}^{H} (t) + Q_{wind}^{H} (t) = Q_{load} (t) + Q_{Chu} (t), \quad O_{load} (t) = O_{TR} (t) + O_{EB} (t) + O_{HSS} (t) \\
P_{RT} (t) + P_{RT}^{EM} (t) - P_{MT}^{\max} (t) + P_{MT}^{\max} (t) - P_{MT}^{\max} (t) + P_{Buy} (t) + P_{Sell} (t) + P_{Div} (t) \\
+P_{Chu} (t) + P_{RT}^{EB} (t) - P_{Chu}^{EM} (t), \quad Q_{MT}^{RTH} (t) + Q_{EB}^{RTH} (t) + Q_{Div}^{RTH} (t) = Q_{load} (t) \\
+Q_{Chu} (t) + Q_{Chu}^{EM} (t) - Q_{Chu}^{EM} (t), \quad O_{load} (t) = O_{CG}^{H} (t) + O_{RT}^{H} (t) + O_{HSS} (t)
\end{align*}
\]

Where, \( P_{load}^{\max} (t) \), \( Q_{load} (t) \), and \( O_{load} (t) \) denote electricity, heat, and cooling loads; \( Q_{CG}^{H} (t) \) indicates specific heat flow from certain CG; \( O_{CG} (t) \) represents certain cool flow from specific CG; \( P_{load} \) means constant residential load; \( Q_{RT}^{RTH} (t) \) is stochastic load; \( Q_{RT}^{H} (t) \) and \( O_{RT}^{H} (t) \) are certain heat or cool flow from certain CG in RT stage.

3. Solution method

C&CG decomposition and duality theory [6] are employed to convert model (1)-(10) into a master problem (MP) and a sub problem (SP), which are shown as (11). Furthermore, bi-linear term \( u^T \xi \) in SP is derived by Big-M method [6] as (12) so that MP and SP with mixed integer linear programming formulation can be achieved. Finally, C&CG iteration [6] is applied to solve MP and SP as figure 2.
Step 1 (Initialization): Set initial scenario \( u_i \) and convergence gap \( \delta \). Initialize upper bound \( U_0 = +\infty \), lower bound \( \underline{L}_0 = -\infty \), and iteration number \( k = 1 \).

Step 2 (Solve MP): Input scenario set \( u_i \) into MP for solving. Record optimal solution \( (x, y) \), optimal value \( \alpha \) of objective, and \( c^\top x \). Update lower bound \( \underline{L}_k = \alpha, i = 1, 2, \ldots, k \).

Step 3 (Solve SP): Input \( x_i \) into SP for solving. Record optimal solution \( (u, y) \) and optimal value \( \beta \) of objective. Set \( u_k \) as \( k+1 \) worst scenario. Update upper bound \( U_k = \beta + c^\top x \).

Step 4 (Check Convergence): If \( U_k - L_k \leq \delta \), terminate algorithm and record optimal value \( v \) as expected cost. Otherwise, set \( k = k + 1 \), add RED variable \( y_{k+1} \) to \( u_{k+1} \) and new constraints: \( \Theta \geq d \gamma_{k+1} + e \beta_{k+1}, Cx + Dy_{k+1} = i \).

\[ \Theta \geq d \gamma_{k+1} + e \beta_{k+1}, Cx + Dy_{k+1} = i \]

Return to Step 2.

4. Case study

This section shows the experimental results to illustrate the effectiveness of the R-DDED. The data including cost coefficient, predicted load and WPG follow research [6].

4.1. Comparison of different methods

Table 1 lists the results of D-DDED, S-DDED, and R-DDED with \( \Gamma = 12 \) and 30% prediction error. In terms of computational time, S-DDED is the highest as it has to consider massive scenarios; D-DDED and R-DDED are comparable because they only focus on single scenario. Regarding economy, R-DDED achieves the best performance. This is because it balances the DA cost and RT cost so that minimal total cost is achieved.

| Method | Time (s) | DA Cost ($) | Actual RT Cost ($) | Actual Total Cost ($) |
|--------|----------|-------------|--------------------|----------------------|
| D-DDED | 8.5      | 5973.5      | 609.1              | 6582.6               |
| S-DDED | 1782.3   | 6010.8      | 562.3              | 6573.1               |
| R-DDED | 7.6      | 6203.8      | 366.5              | 6570.3               |

4.2. Impact of budget and prediction error

The results of R-DDED with different \( \Gamma \) and prediction error are respectively illustrated in left and right sides Table 2. The table shows that both higher \( \Gamma \) and higher error lead to more iteration that results in higher computational time. In addition, it is obvious that both the parameters cause in higher DA cost and total cost.
Figure 3: Comparison of Different Budgets and Errors.

In addition, Figure 3 shows that higher $\Gamma$ is able to achieve lower WPG curtailment probability and cost because higher $\Gamma$ causes better robustness so that WPG can be further utilized. On the other hand, even though higher error leads to higher cost, it cannot result in better WPG utilization as higher $\Gamma$. Instead, worse error causes worse WPG utilization. Therefore, it is important to select a suitable $\Gamma$ while achieving prediction as precise as possible.

Table 2: Comparison of Different Budgets and Errors

| $\Gamma$ | Iteration | Time (s) | DA Cost | Total Cost | Error | Iteration | Time (s) | DA Cost | Total Cost |
|---------|-----------|----------|---------|------------|-------|-----------|----------|---------|------------|
| 12      | 3         | 8        | 6204 $  | 6570 $     | 30%   | 3         | 8        | 6204 $  | 6570 $     |
| 16      | 5         | 29       | 6460 $  | 6889 $     | 40%   | 6         | 18       | 6636 $  | 7384 $     |
| 20      | 6         | 45       | 6487 $  | 6925 $     | 50%   | 8         | 57       | 7001 $  | 8011 $     |

5. Conclusion

This paper presents a two-stage R-DED for CCHP considering load-wind uncertainty. The day-ahead first stage decides BG and UC while the real-time second stage adjusts the generation to correct the strategy. Based on C&CG decomposition, Big-M method, duality theory, and C&CG iteration, the presented model is figured out. Compared to D-DED and S-DED, R-DED achieves satisfied economy and efficiency. In addition, experiments show that budget parameter and prediction error have significant impact on the strategy so that it is important to select suitable budget while achieving prediction as accurate as possible.

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