Photon-Assisted Quasiparticle Transport and Andreev Transport through an Interacting Quantum Dot

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Resonant tunneling through a quantum dot coupled to superconducting reservoirs in the presence of time-dependent external voltage has been studied. A general formula of the current is derived based on the nonequilibrium Green’s function technique. Using this formula photon-assisted quasiparticle transport has been investigated for the quantum dot connected to superconductors. In addition, resonant Andreev transport through a strongly correlated quantum dot connected to a normal metallic lead and a superconducting lead is studied.

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I. INTRODUCTION

Recent advance of nano-technology has stimulated much interest in the study of quantum transport in mesoscopic structures. Among the various mesoscopic structures, devices based on quantum dot (QD) have drawn particular attention. For the quantum dot structures, electron transport is shown to be affected by the confined electrons in the dot. Very recently, novel tunable Kondo effect has been experimentally found in a single electron transistor, where the level position of the quantum dot and the tunneling rate were controlled by the external gate voltages, via Coulomb blockade.

The influence of time-varying fields on the transport through a quantum dot structure is one of potentially interesting areas. Particular applications related to this physics are photo-electron devices, such as single electron pumps, turnstiles, and photon detectors (see Ref. [1] for a review). A time-dependent potential with frequency Ω superimposed to dc bias potential can induce additional tunneling processes when electrons exchange energy by absorbing or emitting photons of energy Ω. This kind of tunneling is known as the photon-assisted tunneling (PAT). Experimental study on the PAT in the quantum dot devices, based on a configuration of a quantum dot coupled to normal metal leads (N), has been reported.[2-5]

In this paper we wish to study tunneling properties in an interacting quantum dot coupled to superconductors in the presence of an external ac voltage. For this purpose we first derive a general current formula for this system. Using this formula we investigate two examples of time-dependent transport, PAT of quasiparticles and that of the Kondo-Andreev resonance. When the quantum dot is coupled to a superconductor (S), the resonant tunneling current is strongly affected by the singular BCS density of states.[6] We find that the singularity of BCS density of states plays an important role in the photon assisted quasiparticle transport. If the coupling between the leads and the QD is sufficiently weak, the subgap transport is suppressed in the case of small dc bias, but the photon assisted tunneling combined with the asymmetric BCS density of states could allow finite electron transport. We also investigate the many-body resonance by an ac applied voltage in the N-QD-S system as an application of the derived current formula. We find new side peaks related to the photon absorption and/or emission of the strongly correlated electrons, in addition to the zero-bias peak due to the Kondo resonance with the Andreev reflection.[10]

The paper is organized as follows. In Sec. II we derive a time-dependent current formula for the quantum dot connected to superconducting leads. This is an extension of the current formula for the normal metallic leads, derived by Jauho, Wingreen, and Meir.[11] On this base, the photon-assisted quasiparticle pumping current is studied in Sec. III. The Andreev current for photon-assisted tunneling is studied in Sec. IV. Main results of this paper are summarized in Sec. V.

II. CURRENT FORMULA

We start with a general model Hamiltonian

\[ H = \sum_{\alpha} H_{\alpha} \{ c_{k\sigma}^\dagger; c_{k\sigma} \} + \sum_{\alpha} H_{T}^{\alpha} + H_{D} \{ d_{\sigma}^\dagger; d_{\sigma} \}, \]  

(1)
where \( \alpha ( \in \{ L, R \} \) denoting the left hand and right hand sides\) is the index of the leads. \( H_\alpha, H_D, \) and \( H_T^\alpha = \sum_{k \sigma} \varepsilon_k c_{k \sigma}^\dagger c_{k \sigma} + h.c. \) represent the Hamiltonians of superconducting (or normal) lead, an interaction region, and tunneling between the lead and the interaction region, respectively. The superconducting lead \( \alpha \) is characterized by the chemical potential \( \mu_\alpha \) and the gap energy \( \Delta_\alpha \). \( d_\alpha^\dagger (d_\alpha) \) is the electron creation (annihilation) operator with spin \( \sigma \) in the quantum dot. When there is a time-dependent voltage difference \( V^\alpha (t) = \mu_\alpha (t) - \mu_D \) between the lead \( \alpha \) and the quantum dot, it is convenient to perform gauge transformation. The electron creation and annihilation operator for the lead takes the form \( \hat{c}_{k \sigma}^\dagger = e^{i \phi_\alpha / 2} \hat{c}_{k \sigma}^\dagger \) and \( \hat{c}_{k \sigma} = e^{-i \phi_\alpha / 2} \hat{c}_{k \sigma} \), where \( d\phi_\alpha / dt = 2eV^\alpha / h \).

The current flowing into the quantum dot can be defined as the rate of change in the number of electrons in a lead \( \alpha \). The commutator of the number operator \( N_\alpha = \sum_{k \sigma \in \alpha} \hat{c}_{k \sigma}^\dagger \hat{c}_{k \sigma} \) with the Hamiltonian gives rise to the current,

\[
J_\alpha (t) = -\frac{i e}{\hbar} \text{Re} \left\{ \sum_{k \sigma \in \alpha} V^*_\alpha(t) \langle d_\alpha^\dagger \hat{c}_{k \sigma} \rangle \right\}.
\] (2)

In the case of superconducting lead, several kinds of tunneling processes, i.e., quasiparticles, Cooper-pairs, etc., can be included in the general current expression. In order to treat the system in a convenient way, we have adopted the particle-hole Nambu representations in the lead and the quantum dot such as \( \Psi_\alpha^\dagger (t) = (\hat{c}_{k \sigma}^\dagger (t) \hat{c}_{-k - \sigma} (t)) \) and \( \Psi_\alpha^L (t) = (d_\alpha^\dagger (t) d_{-\sigma} (t)) \), respectively. In the particle-hole space, new Green’s function can be defined in terms of Keldysh Green’s function,

\[
G^<_{k \sigma, \alpha, d}(t, t') = i (\Psi_{k \sigma \alpha}^\dagger (t') \otimes \Psi_{k \sigma \alpha} (t))
\] (3a)

\[
= \left\langle \left\langle \left( d_{\sigma}^\dagger (t') \hat{c}_{k \sigma} (t) \right) \left( d_{-\sigma} (t') \hat{c}_{k \sigma} (t) \right) \right\rangle \right\rangle.
\] (3b)

The time-dependent current flowing out of the lead \( \alpha \) into the quantum dot can be written as

\[
J_\alpha (t) = -\frac{2e}{\hbar} \text{Re} \left\{ \sum_{k \sigma \in \alpha} V^\alpha_{k \sigma} G^<_{k \sigma, d, \alpha}(t, t) \right\}_{11},
\] (4)

where \( \{ \cdots \}_{11} \) stands for the (1,1) component of the current matrix and the hopping matrix is \( V^\alpha_{k \sigma} = \\
\left( \begin{array}{cc}
V^\alpha_{k \sigma} & 0 \\
0 & -V^\alpha_{-k - \sigma} \\
\end{array} \right) \).

By means of Keldysh techniques for the noninteraction Hamiltonian of the leads, we have obtained Dyson’s equation

\[
G^<_{k \sigma, \alpha, d}(t, t') = i (T_C [S \Psi_{k \sigma \alpha}^\dagger (t') \otimes \Psi_{k \sigma \alpha} (t)] )
\] (5a)

\[
= \int dt_1 \left( \Psi_{k \sigma \alpha}^\dagger (t_1, t) V^\alpha_{k \sigma} G^<_{k \sigma, \alpha, d}(t_1, t') \right) + g^<_{k \sigma, \alpha, d}(t, t_1) V^\alpha_{k \sigma} G^>_{k \sigma, \alpha, d}(t_1, t'),
\] (5b)

where \( S = T_C \{ \exp[-i \int_C d \tau H_T (\tau)] \} \) is the contour-ordered \( S \)-matrix and \( T_C \) is the contour-ordering operator. \( G^>_{k \sigma, \alpha, d}(t, t') = i \langle \Psi_{k \sigma \alpha}^\dagger (t') \otimes \Psi_{k \sigma \alpha} (t) \rangle \) and \( G^\gamma_{k \sigma, \alpha, d}(t, t') = i \theta (t' - t) \{ \langle \Psi_{k \sigma} (t), \Psi_{k \sigma} (t') \rangle \} \) are the lesser and advanced Green’s matrices in the interaction region. The full Green’s functions \( G_{k \sigma, \alpha} \) in the interaction region need to be solved. The Green’s function \( g_{k \sigma, \alpha} \) represents the unperturbed Green’s function in the lead, which includes the time-dependent phase due to the chemical potential difference and the electron level variation.

In order to obtain the time-dependent Green’s functions in the superconducting leads, we take out the time-dependent phase associated with a voltage difference in the electron operators as we mentioned above. Assuming \( \Delta_\alpha \) to be independent on the external fields in the superconducting leads, we can separately consider the time variations of the energy levels in the leads and in the interaction region (for example, \( \varepsilon_k (t) = \varepsilon_k + V^\alpha_{k \sigma} \cos \Omega t \)). The total phase \( \phi_\alpha (t) \) includes the phase due to the time variation of the energy levels. The Green’s functions of a superconducting lead have a form such as \( g_{k \sigma, \alpha}(t, t') = R_z (\phi_\alpha (t)) g_{k \sigma, \alpha}(t - t') R_z (\phi_\alpha (t')) \), where \( R_z (\phi_\alpha) = \exp[-i (\phi_\alpha / 2) \sigma_\alpha] \) is a rotation operator in the two-dimensional complex space. The electron operators \( c \) transform to the quasi-particle operators \( \gamma \) by means of the Bogoliubov-Valatin transformation,

\[
\left( \begin{array}{c}
c_{k \sigma} \\
c^\dagger_{-k - \sigma}
\end{array} \right) = \left( \begin{array}{cc}
u_k & u_k \\
-v_k & u_k
\end{array} \right) \left( \begin{array}{c}
\gamma_{k \sigma} \\
\gamma_{-k - \sigma}^\dagger
\end{array} \right)
\] (6)
where $f_{\alpha}(E_k)$ is the Fermi distribution function in the lead $\alpha$. One can define the matrix coherence factors $U_k^\alpha$ and $V_k^\alpha$ given by

$$ U_k^\alpha = \begin{pmatrix} u_k^2 & -u_kv_k & v_k^2 \\ -u_kv_k & u_k^2 & v_k \\ v_k^2 & u_k & u_k^2 \end{pmatrix} \quad \text{and} \quad V_k^\alpha = \begin{pmatrix} v_k^2 & u_kv_k & u_k^2 \\ u_kv_k & u_k & u_k \\ u_k^2 & v_k & u_k^2 \end{pmatrix}. \tag{8} $$

The elements of the matrix coherence factors are given by $u_k^2 = 1/2(1 + \varepsilon_k/E_k)$ and $v_k^2 = 1/2(1 - \varepsilon_k/E_k)$. Here, $E_k = \sqrt{\varepsilon_k^2 + |\Delta_0|^2}$. For normal metallic leads ($\Delta_0 = 0$), $\varepsilon_k$ becomes the electron energy, and the matrix coherence factors are reduced to $U_k^\alpha = \theta(\varepsilon_k) P_+ + \theta(-\varepsilon_k) P_-$ and $V_k^\alpha = \theta(-\varepsilon_k) P_+ + \theta(\varepsilon_k) P_-$, where $P_{\pm} = 1/2(\pm \sigma_z)$ are the polarization matrices and $\sigma_z$ represent the Pauli (unit) matrices. The time-dependent coupling matrix can be written in a compact form as $\Gamma_{S}^\alpha(E_k, t, t') = 2\pi N_0^S(E_k) V_{k_\sigma}^\alpha(t_k) U_{k_\sigma}^\alpha(t') V_{k_\sigma}^\alpha(t')$, where $W_{k_\sigma}^\alpha(E_k) = U_{k_\sigma}^\alpha(\varepsilon_k, E_k) + V_{k_\sigma}^\alpha(\varepsilon_k, -E_k)$ and $N_0^S(E_k)$ is the density of states of the superconducting lead $\alpha$. As a consequence, the time-dependent current for the systems coupled to two superconducting leads can be written as

$$ J_\alpha(t) = \frac{2e}{h} \int_{-\infty}^{t'} dt' \int \frac{dE_k}{2\pi} \operatorname{Im} \left\{ \sum_{\sigma} e^{iE_k(t-t')} \Gamma_{S}^\alpha(E_k, t, t_1) \left[ -\mathcal{G}_\sigma^\alpha(t_1, t, t_1) + f_{\alpha}(E_k) \mathcal{G}_\sigma^\alpha(t_1, t_1) \right] \right\}_{11}, \tag{9} $$

with the time-dependent coupling matrix

$$ \Gamma_{S}^\alpha(E, t, t') = \frac{\Gamma_{S}^\alpha}{\sqrt{E^2 - \Delta_0^2}} \begin{pmatrix} e^{-i\phi(t)/2} & 0 & e^{i\phi(t)/2} \\ 0 & 1 & 0 \\ e^{i\phi(t)/2} & 0 & e^{-i\phi(t)/2} \end{pmatrix} \begin{pmatrix} |E| & \text{sign}(E) \Delta_0 \\ \text{sign}(E) \Delta_0 & |E| \end{pmatrix} \begin{pmatrix} e^{i\phi(t')/2} & 0 \\ 0 & e^{-i\phi(t')/2} \end{pmatrix}. \tag{10} $$

The coupling constant, which is assumed to be independent of energy and spin, is defined by $\Gamma_{S}^\alpha = 2\pi |V_{k_\sigma}^\alpha|^2 N^\alpha(0)$ with $N^\alpha(0)$ being the normal density of states in the Fermi level.

In the limit $\Delta_0 = 0$, the current expression given in Eq. (9) reduces to the formula obtained by Jauho, Wingreen and Meir for normal metallic leads. In principle, the current formula of Eq. (9) can describe various transport processes, associated with the superconducting leads. The time dependent transport physics for a quantum dot connected to the superconducting leads with the applied time-dependent voltage can be studied based on Eq. (9) once the interaction in the quantum dot is determined.

### III. PHOTON-ASSISTED QUASIPARTICLE CURRENT

On the basis of the formula derived in the previous section, we now study the resonant quasiparticle tunneling process in the presence of an oscillating external ac voltage. We assume that the quantum dot is weakly coupled to the superconducting leads. The Coulomb charging energy ($U \sim e^2/C$), where $C$ is the total capacitance of the quantum dot, and the level spacing of the discrete electronic states are much greater than the energy gap. Therefore, in the weak coupling limit $\Gamma \ll \Delta_0 \ll E_k$, the subgap transport due to Andreev reflection is negligible due to large charging energy in the quantum dot. Recently, Oosterkamp, Kouwenhoven, Koolen, Vaart, and Harmand observed photon induced pumping of the dc current via the 0D state in the normal single electron transistor. They also observed photon sideband resonances. Compared to the case of normal metallic leads, here in the case of the superconducting leads much enhanced currents are shown in the photon-induced pumping and the direction of the current changes, due to the singular BCS density of states.

We have calculated the time-averaged current, in the presence of sinusoidal external voltage difference between the lead $\alpha$ and the quantum dot with the frequency $\Omega$ and the oscillation amplitude $V_{ac}^{\alpha}$. By neglecting the off-diagonal terms of the general current expression which are related to the Andreev transport, and averaging with respect to time, the dc current can be written as

$$ J_{\alpha}^{Q} = \frac{ie}{h} \sum_{\sigma} \int \frac{d\varepsilon}{2\pi} \left\{ \Gamma_{S}^\alpha(\varepsilon) \mathcal{G}_{\sigma}^\alpha(\varepsilon) + K_{S}^\alpha(\varepsilon) \left( \mathcal{G}_{\sigma}^\alpha(\varepsilon) - \mathcal{G}_{\sigma}^\alpha(\varepsilon) \right) \right\}, \tag{11} $$

where $\mathcal{G}$ denotes the $(1,1)$ component of the Green’s function matrix $\mathcal{G}$. $\Gamma_{S}^\alpha(\varepsilon) = \int d\varepsilon' \Gamma_{\alpha}^\alpha(\varepsilon') \mathcal{G}_{\sigma}^\alpha(\varepsilon - \varepsilon')$ and $K_{S}^\alpha(\varepsilon) = \int d\varepsilon' \Gamma_{\alpha}^\alpha(\varepsilon') f_{\alpha}(\varepsilon') Q_{\sigma}^\alpha(\varepsilon - \varepsilon')$. The dimensionless BCS factor is $\mathcal{G}_{\sigma}^\alpha(\varepsilon) = |\varepsilon|/\sqrt{\varepsilon^2 - |\Delta_0|^2}$. The effects of
photons and emission processes are included in the probability function $Q^\pm_\alpha(\varepsilon) = \sum_n J^\pm_n(\lambda_n) \delta(\varepsilon \pm \mu_\alpha \pm n\Omega) = eV_{\alpha}/\hbar\Omega$.

To describe the interacting quantum dot we consider the Anderson impurity model $H_D = \sum_{\sigma} \varepsilon_{\sigma} d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow$ for $\varepsilon_{\sigma} = \varepsilon_{\sigma} - \varepsilon_{\sigma}$. In general, the Green’s functions of the quantum dot can be found from Dyson’s equation $G^{r,a} = G^{0,a}_0[1 + \Sigma^{r,a}]$ and $G^< = [1 + \Sigma^G]^< + G^G$. By using the equation of motion technique and the mean field approximation\footnote{For the photon-assisted tunneling, we obtain the approximate Green’s function $G^\alpha_\sigma(\varepsilon) \approx e^{i\Omega / 2} (\sum_{\alpha} \Gamma^\alpha_\sigma(\varepsilon) K^\alpha_\sigma(\varepsilon) - G^\alpha_\sigma(\varepsilon))$, where $\Gamma^\alpha_\sigma(\varepsilon) = \sum_{\alpha} \Gamma^\alpha_\sigma(\varepsilon)$ and $K^\alpha_\sigma(\varepsilon) = \sum_{\alpha} K^\alpha_\sigma(\varepsilon)$. This approximation guarantees automatically the current conservation.} for the photon-assisted tunneling, we obtain the approximate current formula derived in Ref. 17 for the quasiparticle resonant tunneling of a quantum dot connected to superconductors.

The time averaged current through the system becomes

$$J^Q = \frac{e}{\hbar} \sum_{\sigma} \int d\varepsilon \tilde{G}(\varepsilon) A^Q_\sigma(\varepsilon), \quad (12)$$

where $\tilde{G}(\varepsilon) = (K^0_\sigma(\varepsilon) \Gamma^R_\sigma(\varepsilon) - K^R_\sigma(\varepsilon) \Gamma^L_\sigma(\varepsilon)) / \Gamma(\varepsilon)$ and $A^Q_\sigma(\varepsilon) = -\text{Im}[G^\sigma_\sigma(\varepsilon)]/\pi$. In the absence of the ac voltage, Eq. (12) reduces to the current formula derived in Ref. 17 for the quasiparticle resonant tunneling of a quantum dot connected to superconductors.

After neglecting the higher-order correlation functions the approximate Green’s function $G^\alpha_\sigma(\varepsilon)$ can be written as

$$G^\alpha_\sigma(\varepsilon) = \frac{1 - \langle n_{\sigma} \rangle}{\varepsilon - \varepsilon_{\sigma} - \Sigma^\alpha_\sigma(\varepsilon)} + \frac{\langle n_{\sigma} \rangle}{\varepsilon - \varepsilon_{\sigma} - U - \Sigma^\alpha_\sigma(\varepsilon)}, \quad (13)$$

where the self-energy is $\Sigma^\alpha_\sigma(\varepsilon) = -i\Omega S(\varepsilon) / 2$. Here we do not take into account the Kondo-like correlations because it is not important in the case of both leads being superconductors. The occupation number in the nonequilibrium state can be obtained from the relation $\langle n_{\sigma} \rangle = (1/2\pi i) \int d\varepsilon G^\sigma_\sigma(\varepsilon)$. Then, the current and the occupation number of the quantum dot are solved self-consistently.

When the quantum dot is coupled to the superconductors, the pumping current due to quasiparticles can flow even when the ac-fields are applied to both sides of the leads. In the case of the normal lead, on the other hand, the electron pumping current can flow only when the ac-field is applied between the dot and one lead. In the normal lead case, when the ac-fields are applied to both sides of the leads, the current can not flow. Figure 1 displays the zero-bias pumping current can flow only when the ac-field is applied between the dot and one lead, whereas a single photon process dominates, giving rise to a greater current, if the dot level lies in the BCS gap $\delta = 0\Delta$ and $1\Delta$. In regions A and B the two-photon process becomes dominant, whereas a single photon process dominates, giving rise to a greater current, if the dot level lies in the BCS gap $\delta = 0\Delta$ and $1\Delta$.

IV. PHOTON-ASSISTED ANDREEV CURRENT

In the present section we will investigate the Andreev resonant tunneling through a strongly correlated quantum dot coupled to a normal metal and to a superconductor. We consider that an ac voltage is applied between the normal metallic lead and the quantum dot only. Generalizing the ansatz of Ng\cite{Ng} to the case of a superconducting lead \footnote{We consider that an ac voltage is applied between the normal metallic lead and the quantum dot only. Generalizing the ansatz of Ng\cite{Ng} to the case of a superconducting lead \footnote{In obtaining the Andreev current we have assumed that the ac voltage difference is applied only on the normal side and that the noninteracting self-energies}} (which leads to current conservation,) the time averaged current in the presence of sinusoidal external ac voltage on the normal lead can be obtained from Eq. (3) considering the Andreev reflections,

$$J^A_\alpha = \frac{ie}{\hbar} \int \frac{d\varepsilon}{2\pi} \text{Tr} \left\{ \sigma \epsilon_{\sigma}^G(\varepsilon) |\Sigma^\sigma_\sigma(\varepsilon), K^\alpha_\sigma(\varepsilon) | G^\sigma_\sigma(\varepsilon) \right\}, \quad (14)$$

where $K^\alpha_\sigma(\varepsilon) = \left( \begin{array}{cc} K^{\pm}_N(\varepsilon) & 0 \\ 0 & K^{\pm}_N(\varepsilon) \end{array} \right)$ with $K^{\pm}_N(\varepsilon) = \int d\varepsilon' f_\alpha(\varepsilon') Q^\pm_\alpha(\varepsilon - \varepsilon')$. In obtaining the Andreev current we have assumed that the ac voltage difference is applied only on the normal side and that the noninteracting self-energies
are in the limit $\Omega, k_B T, eV \ll \Delta, U$. In the absence of the ac voltage, our equation of (14) can be shown to be reduced to the current formula derived by Fazio and Raimondi[21]. The Green’s functions can be solved by the equation of motion technique, taking into account Kondo-like correlations[24]. In the infinite $U$ limit the Green’s function matrix has the form

$$(\mathcal{E} - \mathcal{E}_\sigma - \Sigma'_{\sigma}(\varepsilon)) \mathbf{G}_{\sigma}^r(\varepsilon) = \mathbf{1} - (\mathbf{n}), \quad (15)$$

where $\mathcal{E}_\sigma = \begin{pmatrix} \varepsilon_\sigma & 0 \\ 0 & -\varepsilon_-\sigma \end{pmatrix}$ and the self-energy can be written as $\Sigma(\varepsilon) = \Sigma_N^r(\varepsilon) + \Sigma_S^r(\varepsilon)$, with $\Sigma_N^r$ and $\Sigma_S^r$ denoting the self-energy contribution from the normal metal and the superconductor, respectively. The effects of the PAT and Kondo correlations are included in $\Sigma_{N,11}^r(\varepsilon)$ as $[2]$

$$\Sigma_{N,11}^r(\varepsilon) = \sum_{n=-\infty}^{\infty} J_n^r(\lambda_N)eV - n\Omega + i0^+. \quad (16)$$

We can confirm the time reversal symmetry that $\Sigma_{N,22}^r(\varepsilon) = -\Sigma_{N,11}^r(-\varepsilon)$. In the large $\Delta$ limit $(\Omega, \varepsilon \ll \Delta)$, $\Sigma_S^r$ can be written as $[2]$

$$\Sigma_S^r = \begin{pmatrix} G^S \ln[W/\Delta]/2\pi & -G^S/2 \\ -G^S/2 & -G^S \ln[W/\Delta]/2\pi \end{pmatrix}. \quad (17)$$

The off-diagonal terms are directly related to the Andreev transport, whereas the effect of the diagonal term normalizes the energy level of the quantum dot as $\varepsilon_{\sigma,-\sigma} = \varepsilon_{\sigma,-\sigma} + G^S \ln[W/\Delta]/2\pi$ in the large $\Delta$ limit.

Differential conductance $dJ^A/dV$ as a function of the dc bias is shown in Fig.3. The zero-bias peak in the figure originates from the Kondo effect associated with the Andreev reflection. The enhancement of the zero-bias resonance by the superconductor is already studied[10, 22]. We find that new small peaks appear near the zero-bias peak, due to the PAT of the many-body resonance. These peaks are located where the voltage is the multiple of $\Omega/2$, instead $\Omega$, differently from the case for $N-QD-N$[20, 23]. This means that the many-body resonance in the superconducting system changes from the normal Kondo resonance because of the Andreev reflection. The Kondo resonance in the superconducting system seems to have an effective charge 2 instead of e due to the proximity effect. This proximity coupling seems to allow the side peaks at $2eV = n\Omega$ with $n$ being an integer.

### V. CONCLUSION

We have studied resonant transport through an interacting quantum dot coupled to superconductors in the presence of external microwave field. We derived a general formula of the current for this system, adopting the nonequilibrium Green’s function technique. We have seen that the singularity of BCS density of states plays an important as well as interesting role in the quasiparticle PAT. Influence of microwave fields on the resonant Andreev transport through a strongly correlated quantum dot has also been studied. In addition to a zero-bias peak due to many-body resonance, side peaks are found at the bias voltage corresponding to the multiples of the half of the ac oscillation frequency.

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FIG. 1. Zero-bias quasiparticle current of the S-QD-S in units of $e\Gamma/h$ for asymmetric gap energy; $\Delta_L = 1.0$, $\Delta_R = 1.5$. The other parameters taken here are $\Gamma = \Gamma^L = \Gamma^R = 0.01$, $k_B T = 0.1$, $U = 20$, $\lambda_L = \lambda_R = 1.0$, $\Omega = 1.0$, in units of $\Delta_L$. Diagrams of the inset displays the dominant two photon processes for several cases denoted by A, B, C and D.
FIG. 2. Photon induced pumping of the quasiparticle current for the \(S\)-\(QD\)-\(S\) in units of \(e\Gamma/h\) for a small dc bias with two identical superconductors. \(\mu_L = -\mu_R = eV/2, \Gamma = \Gamma^L = \Gamma^R = 0.01, k_B T = 0.1, U = 20, \lambda_L = \lambda_R = 1.0, \Omega = 1.5\) (solid line), \(\Omega = 1.0\) (dotted line), and \(\Omega = 0.5\) (dash-dotted line) in units of \(\Delta = \Delta_L = \Delta_R\). Dominant processes with \(\Omega = 1.5\) are depicted in the diagrams of the inset for several values of the energy level in the \(QD\).
FIG. 3. The differential conductance for the $N$-$QD$-$S$ structure as a function of dc voltage in the units of $4e^2/h$ is plotted at the low temperature ($k_B T = 1 \times 10^{-4}$). In the absence of the ac field (solid line), the differential conductance has the zero-bias peak due to many-body correlation. For the ac fields of $\lambda_N = 1$ with frequency $\Omega = 0.001$ (dotted line) and $\Omega = 0.0015$ (dash-dotted line), the side peaks at $2eV = n\Omega$ are found in addition to the reduced zero-bias peaks. Here $\Gamma^N = \Gamma^S = 0.02$, $\tilde{\epsilon}_\sigma = \tilde{\epsilon}_{-\sigma} = -0.04$ on the scale of bandwidth $W$. 