Vehicle Routing Problem with Heterogeneous Fleet, Split Delivery, Multiple Product, Multiple Trip, and Time Windows: A Case study in fuel distribution

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Abstract. The distribution of fuel in Pertamina co. Ltd Yogyakarta region is carried out using three types of vehicles, namely vehicles with a capacity of 32 kl, 24 kl, and 16 kl. In determining the distribution route, the operator chooses the gas station based on the first order and the type of vehicle determined based on the vehicle's ability to maneuver at the gas station. The determination of the route has not taken into account the time that affects the distribution, such as average speed, loading time, discharging time, and the time limit for completing the tour. So when in the high demand for fuel distribution, three gas stations are delayed from 29 gas stations in cluster I. This study analyzes the distribution of fuel in cluster 1 for Premium and Bio-Solar products. In determining the distribution route and distribution time using the VRP method by considering Heterogeneous Fleet, Split Delivery, Multiple Products, Multiple Trips, and Time Windows. The calculation is done using sequential insertion, where the selected customer is based on the smallest CT. The results of this study indicate that after processing, it was found that late customers were reduced to 1 in 3 late customers. Total mileage was reduced to 693.3 km from 701.5 km. The total tour completion time was reduced to 71,819 hours from 81,819 hours, and distribution costs increased to Rp. 20,754,119,875.00 from Rp. 18,511,210,625.00.

Keywords: Fuel Distribution, Vehicle Routing Problem, Sequential insertion, Heterogeneous Fleet-Split Delivery-Multiple Product-Multiple Trip-Time Windows.

1. Introduction
Fuel has become an essential commodity in people's lives, where fuel has become a source of energy used in society. Pertamina co. Ltd, as a producer and distributor of fuel in Indonesia, is always trying to meet consumer needs. Pertamina co. Ltd's Rewulu Depot is responsible for meeting fuel needs, especially in the Yogyakarta area. In determining the distribution route, the operator chooses the gas station based on the first order. The type of vehicle will be determined based on the vehicle's ability to maneuver at the gas station. If the vehicle load is still able to serve other gas stations, the next selected gas station is the nearest gas station based on a logical tree taking into account the vehicle's maneuverability at the gas station. The determination of the route above has not received the factors that influence the distribution time into consideration such as the average speed of the vehicle, the setup time of each location, the loading and discharging time, and the time limit for completing the tour. So when
demand on Premium and Bio-Solar products is high, there are delays of 3 gas stations from 29 gas stations. Therefore, this study is directed to propose methods of determining distribution routes to reduce or eliminate delays in the distribution of fuel.

According to [1] the determination of distribution routes can be done using Vehicle Routing Problems (VRP). Previous research [2] about the Vehicle Routing Problem Model with Characteristics of Multiple Routes, Multiple Time Windows, Multiple Product, and Heterogeneous Fleet for a Single Depot. [2] examined the delivery of fuel distribution using ships.

This study aims to solve the vehicle routing problem that considers multiple trip variants, multiple product compartments, multiple time windows, split delivery, and heterogeneous fleets using truck tank fleets. This paper is a published version of [3]. A comparison of several similar studies can be seen in Table 1.

| Author                  | Year | Topic                                           | Multiple Fleet | Split Delivery | Multiple Product | Multiple Trip | Time Windows | Order Pickup | Method                                |
|------------------------|------|-------------------------------------------------|----------------|----------------|------------------|---------------|--------------|--------------|---------------------------------------|
| Suprayogi & Priyandari | 2008 | Vehicle Routing Problem                        | No             | No             | No               | Yes           | No           | Yes          | Sequential Insertion                  |
| Arvianto et al.        | 2014 | Vehicle Routing Problem with tanker freight     | Yes            | Yes            | Yes              | Yes           | Yes          | No           | Sequential Insertion with Local Search |
| Windyarti              | 2019 | Vehicle Routing Problem                        | Yes            | No             | No               | No            | Yes          | No           | Mixed Integer Linear Programming Branch and Bound |
| Abadi et al.           | 2014 | Vehicle Routing Problem                        | No             | No             | No               | Yes           | Yes          | No           | Sequential Insertion                  |
| Park et al.            | 2016 | Inventory Routing Problem                      | Yes            | Yes            | No               | No            | Yes          | No           | Genetic Algorithm                     |
| Nugroho et al. (this study) | 2020 | Vehicle Routing Problem with a tanker truck    | Yes            | Yes            | Yes              | Yes           | Yes          | No           | Sequential Insertion                  |

2. Methods

2.1. Vehicle routing problem

According to [7], Vehicle Routing Problem or VRP is one of the most studied optimization problems related to the optimal route design for vehicles to use to serve a set of customers. Where customers can be served at time intervals. Certain vehicle travel schedules and needs to be determined. Here are four general objectives of VRP [8]:

1. Minimizing global transportation costs, related to distance and fixed costs related to vehicle use,
2. Minimizing the number of vehicles or drivers needed to serve consumer demand,
3. Balancing routes in terms of travel time and vehicle load, and
4. Minimize penalties due to unsatisfactory services to consumers, such as the inability to serve in full or late delivery.

2.2. Several classes or variations of the main problems in VRP
According to [8], there are several classes or variations of the main problems in VRP, namely:
   1. Capacitated VRP (CVRP), is the simplest and most studied VRP class where the constraints are only in the form of limited vehicle capacity,
   2. Distance Constrained VRP (DCVRP), is a VRP with constraints on the allowable distance travelled,
   3. VRP with Time Windows (VRPTW), which is a VRP case where every customer has a service time limit,
   4. VRP with Pickup and Delivery (VRPPD), is a VRP with mixed services, namely shipping and taking goods in one route,
   5. VRP with Backhauls (VRPB), where the new collection can be done after all shipments are complete,
   6. Split Delivery VRP (SDVRP), where consumers are served using different vehicles,
   7. VRP with Multi Depot (MDVRP), where vendors use more than one depot to send consumers, and
   8. Periodic VRP, where delivery is done within a certain period.

2.3. Heterogeneous Fleet Vehicle Routing Problem Split Delivery Multiple Product Multiple Trip and Time Windows (HFVRPSDMPMTTW)
According to [2], in determining the delivery route to achieve the goal of minimizing shipping costs, time, and distance, there are obstacles that must be resolved, namely:
   1. Heterogeneous Fleet, the vehicle has specifications, compartments, and different types so that the capacity is also different, and each type of vehicle has a limited amount.
   2. Multiple Trip is a vehicle capable of serving one or more customers in one delivery tour.
   3. Time Windows, not only limited to depot and customer operating hours but also includes the time that affects the planning horizon such as setup time, loading time, and discharging time.
   4. Multiple Products are types of products that are sent more than one.
   5. Split Delivery is the delivery of products to customers that can be done directly with one shipment or several times of delivery.

Based on the characteristics above, the mathematical model of the Heterogeneous Fleet Vehicle Routing Problem Split Delivery Multiple Product Multiple Trip with Time Windows will be used as follows:

- **Notation:**
  - \( i \) : location index; \( i = 0 \) is depot, \( i = 1,2,...,N \) is customers
  - \( t \) : tour index; \( t = 1,2,3,...,NT \)
  - \( r \) : route index; \( r = 1,2,3,...,NR[t] \)
  - \( p \) : product index; \( p = 0,1,...,NP \)
  - \( k \) : position index; \( k = 1,2,...,NL[t,r] \)
  - \( z \) : type of vehicles index; \( z = 1,2,...,Z \)

- **Parameters:**
  - \( N \) : Number of \( i \) set customers
  - \( NP \) : Number of product type
  - \( q_{L[t,x,r,k,p]} \) : Number of demands for \( p \) product, \( k \) location, \( r \) route, \( t \) tour, and \( z \) vehicle
  - \( W_S \) : Set up time
  - \( L_T \) : Loading time
  - \( D_T \) : Discharging time
  - \( v_z \) : Vehicle speed
\[
\tau_{[t,x,r,k],[t,x,r,k+1]} \quad : \text{Travel time between } k \text{ location to } k+1, r \text{ route, } t \text{ tour, } z \text{ vehicle}
\]
\[
Qz[p] \quad : \text{Compartment capacity for } p \text{ product in vehicle } z
\]
\[
a^h_i \quad : \text{Lower bound for hole time windows } h, i \text{ locations}
\]
\[
b^h_i \quad : \text{Upper bound for hole time windows } h, i \text{ locations}
\]
\[
H \quad : \text{Horizon planning}
\]

- **Variables**
  - \(NV\) : Number of vehicles
  - \(NTz\) : Number of tours by \(z\) vehicle
  - \(NR[t,z]\) : Number of routes in the \(t\) tour by \(z\) vehicle
  - \(NL[t,z,r]\) : Number of locations on the \(r\) route in the \(t\) tour by \(z\) vehicle
  - \(L_{t,z,r,k}\) : Location at \(k\) position, \(r\) route, in \(t\) tour by \(z\) vehicle
  - \(b_{L,t,z,r,k,p}\) : Load delivered on \(r\) route, \(t\) tour by \(z\) vehicle for \(p\) product (unit in volume)
  - \(Y_{L,t,z,r,k,p}\) : The proportion of \(p\) product shipments on \(r\) route, \(t\) tour, \(z\) vehicle, and \(k\) location
  - \(Wp\) : Travel time
  - \(Wt_{L,t,z,r,k}\) : Waiting time for vehicle in location at \(k\) position, \(r\) route, in \(t\) tour by \(z\) vehicle
  - \(J_{tL[t,z,r,k]}\) : Arrival at \(k\) location, \(t\) tour, \(z\) vehicle, and \(r\) route
  - \(WLT\) : Loading time
  - \(WDT\) : Discharging time
  - \(J_{S[I,t,z,NL[t,z,r]]}\) : Finished visit time at \(NL[t,z]\) position on the \(t\) tour, \(z\) vehicle, and \(r\) route
  - \(J_{m[I,t,z,NL[t,z,r]]}\) : Departure at \(NL[t,z]\) position on the \(t\) tour, \(z\) vehicle, and \(r\) route
  - \(CT[t,z]\) : Completion time for \(t\) tour by \(z\) vehicle
  - \(TCT\) : Total completion time
  - \(RCT\) : Range of completion time
  - \(TCD\) : Total distribution cost
  - \(C_{SK}\) : Vehicle fixed cost
  - \(C_{BB}\) : Fuel cost
  - \(C_L\) : Loading/unloading cost per unit
  - \(C_G\) : Driver’s cost for each visit
  - \(C_M\) : Accommodation costs
  - \(C_R\) : Retribution cost per vehicle per tour

- **Problem HFVRPSDMPMTTW** can be formulated as follows:
  \[
  \text{Min } NV = \sum_{NT} NTz \quad (1)
  \]
  \[
  \text{Min } TCT = \sum_{t=1}^{NT} CT[t,z] \quad (2)
  \]
  \[
  \text{Min } RCT = \max\{CT[t,z]\} - \min\{CT[t,z]\} \quad (3)
  \]
  \[
  \text{Min } TCD = \sum_{r=1}^{RR} \sum_{i=0}^{N} \sum_{j=1}^{N+1} C_{SKz} + C_g N_R
  
  + \sum_{r=1}^{RR} \sum_{z=0}^{2} C_{BB} V_z W_p + 3 C_M H N_v
  + 2 \sum_{r=1}^{RR} \sum_{i=0}^{N} q_i C_{I} N_r + C_R N_v \quad (4)
  \]

subject to
The steps of forming the distribution route used to refer to the model used by [2] by modifying the departure step. The steps for forming a route are as follows:

\[ L_{t,x,r,1} = L_{t,x,r,NL[t,x]} = 0 \]  \hspace{1cm} (5)  
\[ L_{t,x,r,k} = i \]  \hspace{1cm} (6)  
\[ NTz = NV \]  \hspace{1cm} (7)  
\[ L_{t,x,r,k} = L_{t,x,r-1,NL[t,x,r-1]} = 0 \]  \hspace{1cm} (8)  
\[ L_{t,x,r,k} \geq 1 \]  \hspace{1cm} (9)  
\[ b_{Lt,x,r,p} \leq Q[p] \]  \hspace{1cm} (10)  
\[ b_{Lt,x,r,p} \leq b_{Lt,x,r,p} + Q[p] \]  \hspace{1cm} (11)  
\[ b_{Lt,x,r,p} \leq \sum_{k=2}^{NL[t,x]-1} q_{Lt,x,k,p} \cdot y_{Lt,x,k,p} \]  \hspace{1cm} (12)  
\[ \sum_{t=1}^{NT} y_{Lt,x,k,p} = 1 \]  \hspace{1cm} (13)  
\[ Q[p] = Q[p] - q[i \cdot p] \]  \hspace{1cm} (14)  
\[ \sum_{h=1}^{n} (a_i^h \leq T_i \leq b_i^h) \]  \hspace{1cm} (15)  
\[ Wp = \tau[L(t,x,r,k)][L[t,x,r,k+1]] \]  \hspace{1cm} (16)  
\[ WLT = b_{Lt,z,r,k,p} \]  \hspace{1cm} (17)  
\[ WDT = b_{Lt,z,r,k,p} \]  \hspace{1cm} (18)  
\[ CT[t] = W_0 \sum_{r=1}^{NL[t,x]} \sum_{t=1}^{NL[t,x]} L_{t,x,r,k} + (WDT) \]  \hspace{1cm} (19)  
\[ + WLT) \sum_{r=1}^{NL[t,x]} \sum_{t=1}^{NL[t,x]} b_{Lt,x,p} \]  \hspace{1cm} (20)  
\[ + \sum_{r=1}^{NL[t,x]} \sum_{t=1}^{NL[t,x]-1} \tau[L[t,x,r,k]][L[t,x,r,k+1]] \]  \hspace{1cm} (21)  
\[ + \sum_{r=1}^{NL[t,x]} \sum_{t=2}^{NL[t,x]-1} W_{Lt,z} \]  \hspace{1cm} (22)  
\[ CT[t] = JS_L[t,x,r,NL[t,x]] \]  \hspace{1cm} (23)  
\[ Jm_L[t,x,r,NL[t,x]] = JS_L[t,x,r,NL[t,x]] \]  \hspace{1cm} (24)  
\[ Jm_L[t,x,r,NL[t,x]] = JS_L[t,x,r,NL[t,x]] + Wp \]  \hspace{1cm} (25)  
\[ Wt_{Lt,z,r,k} = \begin{cases} b_i^h - JtL[t,z,r,k], & b_i^h \geq JtL[t,z,r,k] \\ 0, & b_i^h \leq JtL[t,z,r,k] \end{cases} \]  \hspace{1cm} (26)  
\[ CT[t,x] \leq H \]  \hspace{1cm} (27)  
\[ TCT = \sum_{t=1}^{NT} CT[t,x] \]  \hspace{1cm} (28)
Step 0: 
  a. Initiate the number of customers that have not been served and the number of tours, the number of vehicles, and the time of completion of the tour is 0 (not yet formed).

Step 1: 
  a. Establishment of the first tour and first route for the vehicle (ntz = ntz + 1) with the initial vehicle load (blt, z, r, p = 0).

Step 2: 
  a. Insert the customer that has not been served into the route that has been formed.
  b. Set departure time (jml [t, z, r, nl [t, z, r]]) = 0. (first vehicle)
  c. Set departure time (jml [t, z, r, nl [t, z, r]]) = setup time (ws) + loading time (wlt) of the previous vehicle. (2nd vehicle and so on)
  d. Calculate loading time (wlt) using equation (17), discharging time (wdt) using equation (18) and calculate tour completion time (ct) based on equation (19) for each vehicle.
  e. If ct> h, continue to step 8. If ct<h, select the customer who gives the shortest tour completion time and proceed to step 3.

Step 3: 
  a. Calculate the vehicle load (blt, z, r, P) based on Equation (12) and vehicle compartment (Q [p]) based on Equation (14).
  b. If customer request q [i * p]> Q [p], then the request will be fulfilled by using another vehicle (split delivery).
  c. Update the number of customers and proceed to step 4.

Step 4: 
  a. If all customers have been served, continue to step 9. Instead, continue to step 5.

Step 5: 
  a. Set the departure time (jml [t, z, r, NL [t, z, r]]) based on Equation (21)
  b. Calculate loading time (WLT) using Equation (17), discharging time (WDT) using Equation (18) and calculate tour completion time (CT) based on Equation (19) for each vehicle.
  c. If CT <H and q [i * p] <Q [p], proceed to step 6. If CT <H and q [i * p]> Q [p], continue to step 7. If CT> H, proceed to step 8.

Step 6: 
  a. Calculate the vehicle load (blt, z, r, P) based on Equation (12) and vehicle compartment (Q [p]) based on Equation (14).
  b. If customer request q [i * p]> Q [p], then the request will be fulfilled using another vehicle (split delivery).
  c. Update the number of customers and proceed to step 4.

Step 7: 
  a. Establish the next route (r = r + 1) for the vehicle (ntz = ntz + 1). Return to step 1.

Step 8: 
  a. Establish the next tour (t = t + 1) for the first vehicle route (ntz = ntz + 1). Return to step 1.

Step 9: 
  a. Set the number of vehicles based on Equation (7). Calculate the total CT based on Equation (2), calculate the time span based on Equation (3).

Step 10: 
  a. Calculate the TCT based on Equation (4)

3. Result and Discussion
Based on calculation using sequal insertion method, the route of each truck tanker, total distance and completion hour was solved and it can be summarized on Table 2 below.
Table 2. Route recapitulation of sequential insertion method

| No | Route                          | Vehicle Capacity (Kl) | Total Distance (Km) | Completion Time (Hour) |
|----|--------------------------------|-----------------------|--------------------|------------------------|
| 1  | Depot-S18-S03-S20-S21          | 32                    | 33.8               | 3.998                  |
| 2  | Depot-S27-S29-44.552.18        | 32                    | 26.3               | 3.661                  |
| 3  | Depot-S24-S25-S02-S01          | 32                    | 44                 | 4.92                   |
| 4  | Depot-S19-S04                  | 32                    | 23                 | 3.796                  |
| 5  | Depot-S04-S09-S21-S07          | 32                    | 43                 | 5.629                  |
| 6  | Depot-S07-S10-S08              | 24                    | 45                 | 5.566                  |
| 7  | Depot-S08-S17-S11-S15          | 32                    | 83.3               | 7.371                  |
| 8  | Depot-S26-S01                  | 32                    | 40.3               | 5.829                  |
| 9  | Depot-44.552.06-S05-S13        | 24                    | 82.6               | 7.407                  |
| 10 | Depot-44.5561.12-S12           | 32                    | 93.2               | 7.826                  |
| 11 | Depot-S13-S23                  | 24                    | 88                 | 7.826                  |
| 12 | Depot-S14-S06                  | 24                    | 90.8               | 8.197                  |

Table 2 shows the results of the route recapitulation using the insertion sequence method. The vehicle with the smallest capacity will result in faster tour completion time. However, a smaller vehicle capacity will require a greater number of vehicles to meet demand. As well as the start time of vehicle departure does not start simultaneously, causing the start time to be longer for the next vehicle. Therefore, larger capacity vehicles are preferred to meet customer demand. The following comparison of distribution routes using the sequential insertion method and the original can be seen in Table 3.

Table 3. Comparison of sequential insertion method with the original value (Pertamina co. LTD)

| No | Variable                                          | Sequential Insertion Method | Original Value       |
|----|---------------------------------------------------|------------------------------|----------------------|
| 1  | Total Distance                                    | 693.3                        | 701.5                |
| 2  | Number of Vehicles                                | 12                           | 13                   |
| 3  | Total Completion Time                              | 71.819                       | 81.819               |
| 4  | Completion Time Range                              | 4,536                        | 6,252                |
| 5  | Total Distribution Cost                           | Rp. 20,754,119,875           | Rp. 18,511,210,625   |
| 6  | Total number of customers who are served late      | 1                            | 3                    |

Based on the above comparison using the sequential insertion method produces a total distance of 693.3 km. It shows that the vehicle mileage using the sequential insertion method is smaller than the distance of the original, which is 701.5 km. The total number of customers who are served late using the sequential insertion method is 1. The total number of customers can be 0 if the total vehicle becomes 13, and there is one vehicle that does not have a maximum load of 16 kl capacity vehicles that only carry eight kl. However, this is better than before, namely that three customers have not been served. The sequential insertion method produces total tour completion time (TCT) and tour completion period (RCT) of 71.819 hours and 4.536 hours. It is caused by a smaller mileage, and a smaller number of vehicles (NV) compared to the original. The total distribution cost (TCD) using the sequential insertion method is Rp. 20,754,119,875.00. It shows that the cost of distribution using the sequential insertion method is greater than that of the original. It is due to the number of vehicles with a capacity of 32 kl more and the number of fixed costs for a 32 kl capacity vehicle of Rp 1,800,000,000.00.

4. Conclusion

We have solved the Vehicle Routing Problem in fuel distribution by using the sequential insertion method. The solution to the problem is done by selecting gas stations based on the smallest tour completion time. In this research, tour completion time is influenced by many factors, namely vehicle start time, setup time, loading time, discharging time, travel time, and waiting time. The case discussed
in this study is the distribution of fuel at the Pertamina co. Ltd depot. Calculation results show that the total tour completion time is smaller than 71.819 hours from 81.819 hours. It is influenced by the selection of gas stations and the type of vehicle used so that the total distance and number of vehicles becomes less. Also, the number of customers served late decreased from 3 stations to 1 station. However, the sequential insertion method produces greater distribution costs due to the number of vehicles with a capacity of 32 kl has a larger fixed cost despite the larger capacity of 32 kl. Further research can be suggested as follows: (1) Need to consider other products and vehicle availability, (2) Need to consider customer goals in the block area, (3) Need to consider using a settlement method, (4) Using another method to solve the problem, like simulations and the Clark and Wright method, (5) Need to consider using metaheuristic, so it can be used for larger data, and route formation can be faster.

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