Galilean Islands in Eternally Inflating Background

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We show that the observational universe may emerge classically from a de Sitter background with low energy scale. We find, after calculating the curvature perturbation, that the resulting scenario is actually a style of the eternal inflation scenario, in which some regions will go through the slowly expanding Galilean genesis phase with the rapidly increasing energy density and become island universes, while other regions are still eternally inflating, which will make the room for more island universes to emerge.

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I. INTRODUCTION AND SUMMARY OF RESULTS

During the eternal inflation [1, 2, 3, 14], an infinite number of universes will be spawned. It is generally thought that inside an observational universe, a phase of the slow-roll inflation and reheating is required, which will set the initial condition of the “big bang” evolution, i.e. a homogeneous hot universe with the scale-invariant primordial perturbation.

In principle, the slow-roll inflation should occur in a high energy scale, which is required to insure that the amplitude of primordial density perturbation is consistent with the observations and as well as after the inflation, the reheating temperature could be suitable for a hot “big bang” evolution. In this sense, it seems that the energy scale of the eternal inflation should be enough high, or the spawning of observational universe will be island-like, which is exponentially unfavored, since it requires a large upward tunneling, e.g. [15, 16, 17].

However, the observational universe might classically emerge from a background with low energy scale, e.g. the emergent universe scenario [9]. In the emergent universe scenario, the universe originates from a static state in the infinite past, when the universe emerges, or begins to deviate from this static state, it is slowly expanding. In Ref. [10], it was for the first time observed that the scale-invariant curvature perturbation might be adiabatically generated during the slow expansion of primordial universe, also [11, 12, 13]. Thus the initial conditions of the “big bang” evolution may be set after this slowly expanding phase ends. During the slow expansion [10, 12], the null energy condition is violated, which might imply that the corresponding evolution suffers from the ghost instability.

Recently, the application of Galileon [14], or its non-trivial generalization, e.g. [15, 16, 17], to the early universe has acquired increasing attentions, e.g. see [18] for bouncing universe, and [19, 20] for Galilean genesis, in which the violation of null energy condition can be implemented stably, there is not the ghost instability, and see also earlier work about the ghost condensate [21]. In Refs. [20], it was showed by applying the generalized Galileon that the scale invariant curvature perturbation may be adiabatically generated in slowly expanding Galilean genesis phase.

Here, inspired by [9], we would like to ask a significant question, whether and how the observational universe may classically emerge from a de Sitter background with low energy scale, and what about its scenario?

We will show that the observational universe may emerge classically from a de Sitter background with low energy scale. We find, after calculating the curvature perturbation, that the resulting scenario is actually a style of the eternal inflation scenario, in which some regions will go through the slowly expanding Galilean genesis phase and become island universes, while other regions are still eternally inflating.

The outline of the paper is as follows. We firstly will introduce a model, in which the observational universe may classically emerge from a de Sitter background with low energy scale. The background evolution of the model will be presented in Sec.II, and the power spectrum of primordial perturbation will be calculated in Sec.III. Here, though the energy scale of initial background is highly low, since the universe slowly expands with rapidly increasing energy density, the initial conditions of the hot “big bang” evolution may be set. In Sec.IV, we will illustrate the resulting scenario, i.e. in an eternally inflating background, some local regions may emerge classically and become island universes. We argue that this scenario may be a viable design of the early universe, which might help to improve the current understanding to some issues relevant with the eternal inflation.

II. THE BACKGROUND

We will introduce a model, in which the observational universe may classically emerge from a de Sitter background with low energy scale. The method of building models is universal. Here, we will begin with such a Galileon Lagrangian as

\[ \mathcal{L} = -e^{4\phi/M^6} X + \frac{1}{M^6} X^3 - \frac{1}{M^7} X^2 \partial^\mu \phi - \Lambda, \]

where \( X = \partial_\mu \phi \partial^\mu \phi / 2 \) and \( M \) is a constant with mass dimension, and \( \Lambda \) sets up a de Sitter background from which the observational universe emerges.

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The evolution of background is determined by the field equation
\[
\left( -e^{4\phi/M^2} + \frac{15}{M^8} X^2 + \frac{24}{M^4} H \dot{\phi} X \right) \ddot{\phi} \\
+ 3 \left( -e^{4\phi/M^2} + \frac{3}{M^8} X^2 \right) H \dot{\phi} \\
+ \left( -\frac{4}{M} e^{4\phi/M^2} + \frac{6 H \dot{\phi}^2}{M^7} + \frac{18 H^2 \phi^2}{M^7} \right) X = 0
\] (2)
and the Friedmann equation
\[
3H^2 M_p^2 = -e^{4\phi/M^2} X + \frac{5}{M^8} X^3 + \frac{6}{M^4} X \dot{\phi}^2 H + \Lambda,
\] (3)
where $M_p^2 = \frac{1}{8\pi G}$. When $H > \frac{\ddot{\phi}}{\dot{\phi}^2}$, the universe is in eternally inflating regime, and the background is highly inhomogeneous, as will be confirmed in Sec.IV. Thus initially we require
\[
H \ll \frac{\dot{\phi}}{\dot{\phi}}.
\] (4)
The field equation (2) approximately becomes
\[
\left( -e^{4\phi/M^2} + \frac{15}{M^8} X^2 \right) \ddot{\phi} - \frac{4}{M} e^{4\phi/M^2} X \simeq 0.
\] (5)
The solution is
\[
e^{4\phi/M^2} = \left( \frac{5}{4} \right)^{1/4} \frac{1}{M(t^*_s - t)}.
\] (6)
\[
\ddot{\phi} = \frac{\dot{\phi}}{t^*_s - t}.
\] (7)
This implies $e^{4\phi/M^2} X = \frac{5}{M^4} X^3$. Thus Eq.(3) is simplified as,
\[
H^2 M_p^2 \simeq \frac{H}{M^2 (t^*_s - t)^5} + \Lambda/3 = \frac{HM_p^2}{x^4 (t^*_s - t)} + \Lambda/3,
\] (8)
where we define
\[
x = M^{1/2} M_p^{1/2} (t^*_s - t)
\] (9)
for convenience. Initially $|t| \gg |t^*_s|$, the first term in right-hand side of Eq.(8) is negligible, which indicates that initially the universe is in a de Sitter state. However, with the lapse of time, we will have $\frac{HM_p^2}{x^4 (t^*_s - t)} \simeq \Lambda$. The corresponding time is
\[
t_C \simeq \frac{-1}{H_0^{1/5} M_p^{2/5} M_p^{2/5}} = \left( \frac{H_0^{1/5}}{M_p^{2/5} M_p^{2/5}} \right) t_0.
\] (10)
where $H_0 = \sqrt{\frac{\Lambda}{3}/M_p} = -1/t_0$. Thus after $t_C$, the universe will deviate from the de Sitter state and begin to the evolution of genesis. This is just the model which we require.

A. The slowly expanding evolution

We will detailed illuminate how the background evolves in this model. During $t < t_C$,
\[
H = \left[ 1 + \frac{1}{H_0 x^4 (t^*_s - t)} \right]^{1/2} H_0 \simeq H_0
\] (11)
is almost constant. Thus
\[
a \sim e^{H_0 (t_C - t_0)},
\] (12)
which seems indicate that the universe is exponentially expanding. However, in term of Eq.(11) and $H_0 |t_0| = 1$, we have
\[
H_0 (t_C - t_0) = \left( 1 - \frac{H_0^{4/5}}{M_p^{2/5} M^{2/5}} \right) H_0 |t_0| < 1,
\] (13)
which implies that the time that this phase lasts is shorter than one efold. Thus during this period the universe is actually slowly expanding.

In certain sense, this phase is similar to the slow expansion studied in Refs.11,13, also the slow contraction [22]. Here, $H \sim H_0 + \frac{1}{(t^*_s - t)}$, while in Refs.11,13, $H \sim H_0 + \frac{1}{t^*_s - t}$.

When $t \simeq t_C$, the universe will deviate from the de Sitter background and begin to the evolution of genesis. During $t > t_C$,
\[
H \simeq \frac{1}{x^4 (t^*_s - t)}
\] (14)
is rapidly increasing. Thus
\[
a \sim e^{\int H dt} \sim exp \left( \frac{1}{x} \right).
\] (15)
During this period, since $x \gg 1$, the universe is still slowly expanding. However, different from that during $t < t_C$, the energy density of universe during this period will rapidly increase until the end of the slowly expanding phase.

When $x \simeq 1$, the slow expansion ends. The definition [22] of $x$ gives
\[
t_c = \mathcal{O}(t_s) \simeq -\frac{1}{\sqrt{MM_p}}.
\] (16)
We assume that at $t_c$ the reheating will happen and the available energy of field will be rapidly released into the radiation. Hereafter, the local universe will begin the evolution of hot “big bang” model. Eq.(12) gives
\[
H_c \simeq \frac{1}{x_c^4 (t^*_s - t_c)} \simeq \sqrt{MM_p}.
\] (17)
Thus at this time, the energy density of Galileon field is $\rho_G \simeq M_p^3 M \gg \Lambda$. In Sec.III.B, we will see the observation requires $M/M_p \sim 1/10^{10}$. Thus this energy will be enough for the reheating of universe.
We plot the evolutions of $a$ and $H$ with respect to the time in the inset panel of Fig.1, in which the parameters $\mathcal{M} = M_P$ and $\Lambda = M_P^2/10^8$ are taken, We have $t_C/|t_e| \simeq -6$ from Eqs. (10) and (16), which is consistent with Fig.1. Here, the values of the parameters used are only to conveniently plotting the background evolution. In principle, we could have a broader choice of the range of parameters, e.g. $\Lambda$ is equal to or smaller than the value of the current cosmological constant.

B. The violation of null energy condition

The statement of the null energy condition is equivalent to $\epsilon > 0$, where $\epsilon = -\frac{\dot{H}}{H^2}$. Here, in term of Eq. (3), we find $\dot{H} > 0$ through the entire evolution, which implies that during the slow expansion, the null energy condition is violated all along.

Here, $H$ is determined by Eq. (3), which is slightly complicated. However, for different phases, we approximately have

$$|\epsilon| = \left| \frac{\dot{H}}{H^2} \right| \simeq \frac{\mathcal{M} M_P}{H_0^2} / x^6, \quad \text{during} \quad t < t_C, \quad (18)$$

$$x^4, \quad \text{during} \quad t_C < t < t_e, \quad (19)$$

which implies that during $t < t_C$, $|\epsilon|$ is increasing with the time, while it is decreasing during $t_C < t < t_e$. When $t = t_C$, $|\epsilon|$ arrives at its maximal value

$$|\epsilon| \simeq \frac{M_2^{2/5} \mathcal{M}^{2/5}}{H_0^{4/5}} \gg 1. \quad (20)$$

Physically, initially the universe is in a slow expanding phase with $H \simeq H_0$ and $H$ being gradually increased, thus initially $|\epsilon| \ll 1$ and will become larger and larger, while after $t > t_C$ the universe is in the slowly expanding Galilean genesis phase with rapidly increasing $H$, thus $|\epsilon|$ will be smaller and smaller. When $t_e \sim O(t_e)$, we have $|\epsilon| \simeq 1$, the genesis phase ends. We plot the evolution of $\epsilon$ with respect to the time in Fig.1.

Though during the slow expansion, the null energy condition is violated all along, we will see that there is not the ghost instability.

III. THE PERTURBATION

We will study the curvature perturbation in this model. The quadratic action of the curvature perturbation $\mathcal{R}$ is

$$S_2 \sim \int d^3 x \sqrt{\epsilon_s} \frac{a^2 Q_R}{c_s^2} \left( R^2 - c_s^2 \left( \partial \mathcal{R} \right)^2 \right), \quad (21)$$

FIG. 1: The evolution of $\mathcal{R}$ with respect to the time is plotted, in which $|t_e| = \frac{1}{\sqrt{\mathcal{M} M_P}}$ is given by Eq. (10), and $\mathcal{M} = M_P$ and $\Lambda = M_P^2/10^8$ are taken. The inset panel is the evolutions of $a$ and $1/H$, in which initially $a_{ini} \simeq 1/H_0$, and for clarity $a$ has been divided by $10^4$ and $1/H$ by $10^3$. Here, $t_C/|t_e| \simeq -6$, which is given by Eq. (10). We see that during $t < t_C$, the universe slowly expands with $H \simeq H_0$, and during this period initially $|\epsilon| \ll 1$ and then will gradually increase, up to $|\epsilon| \gg 1$, while during $t_C < t < t_e$, the universe is still slowly expanding but with rapidly increasing $H$ and decreasing $|\epsilon|$, which has been calculated in the uniform field gauge in e.g. Refs. [15], [17]. Here, we have [20]

$$Q_R = \left[ -e^{4\varphi}/\mathcal{M} + \frac{3X^2}{\mathcal{M}^2} + \frac{8X(\varphi + H \dot{\varphi}) - \frac{8X^4}{\mathcal{M}^4 M_P^4}}{\left( \frac{2X^2}{\mathcal{M}^2} - H \right)^2} \right] X, \quad (22)$$

$$c_s^2 = \frac{-e^{4\varphi}/\mathcal{M} + \frac{3X^2}{\mathcal{M}^2} + \frac{8X(\varphi + H \dot{\varphi}) - \frac{8X^4}{\mathcal{M}^4 M_P^4}}{\left( \frac{2X^2}{\mathcal{M}^2} - H \right)^2}}{-e^{4\varphi}/\mathcal{M} + \frac{3X^2}{\mathcal{M}^2} + \frac{12H X^3}{\mathcal{M}^3} + \frac{12X^4}{\mathcal{M}^4 M_P^4}}, \quad (23)$$

both of which are determined by the evolution of background.

A. The evolutions without the ghost

We will firstly investigate whether there is the ghost instability around the corresponding backgrounds. Before $t_e \sim O(t_e)$, the universe is slowly expanding. During this period the Galileon field is dominated, which contributes the curvature perturbation $\mathcal{R}$. Here, $Q_R > 0$ and $c_s^2 > 0$ are required for the avoidance of the ghost instability. The evolution of field is determined by Eqs. (9) and (11). We observe that in the numerator of (22), the terms $\sim 1/x^6$ are dominated and other terms are negligible, while in the denominator of (22), during $t < t_C$, $H \simeq H_0$ is dominated, and during $t_C < t < t_e$, $H$ is
determined by Eq. (11). Thus we approximately have
\[ Q_\gamma \simeq \frac{\dot{M} M^3_{\text{BH}}}{H_0^3}, \quad \text{during } t < t_C, \]  
(24)
\[ M_{\text{pl}}^2 x^4, \quad \text{during } t_C < t < t_\epsilon. \]  
(25)

The similar calculations give \( c_s^2 \sim 1.4 \) during \( t < t_\epsilon \). Thus the background is ghost-free during the slow expansion. The case is similar to that in ghost condensation mechanism, e.g. see Ref. [21] for the ekpyrotic universe. Here, it is significant to notice \( Q_\gamma \simeq M_{\text{pl}}^2 |\epsilon| \).

After \( t_\epsilon \sim O(t_\ast) \), the universe will be full of the radiation, and begin the evolution of hot “big bang” model. During this period, Eq. (4) become
\[ 3H^2 M_{\text{pl}}^2 \simeq \rho_{\text{rad}}, \]  
(26)
where \( \rho_{\text{rad}} \sim 1/a^4 \) is dominated. However, the Galileon field still exists, though its energy density is negligible.

The perturbation of the Galileon field has been calculated in Ref. [23]. The field \( \varphi \) is ghost-free requires \( Q_{\delta \varphi} > 0 \), in which
\[ Q_{\delta \varphi} = -\dot{e}^{4\varphi/M} + \frac{3X^2}{M^8} + \frac{8X}{M^4} (\ddot{\varphi} + H \dot{\varphi}) - \frac{8X^4}{M^4 M_{\text{pl}}^2}. \]  
(27)

During the reheating, the available energy of Galileon field is rapidly released into the radiation, it is reasonably imagined that after the reheating we have \( \dot{\varphi} \ll \varphi^2/M \) and \( X \ll M^4 \), i.e. \( \dot{\varphi} \ll M^2 \). Thus the absence of the ghost requires that
\[ e^{4\varphi/M} < \frac{X^2}{M^8} \]  
(28)
has to be satisfied, which equals to \( e^{4\varphi/M} < \varphi^2/M^2 \). \( \dot{\varphi}/M^2 \ll 1 \) implies that the condition (28) is equivalent to \( \varphi/M \ll -1 \). The corresponding value of \( \varphi \) at the time \( t_C \) can be obtained from Eqs. (10) and (11), which is
\[ \frac{\varphi_C}{M} \sim - \ln \left( \frac{M^3_{\text{pl}} H_0}{M^3_{\text{BH}}} \right)^{1/5} \ll -1. \]  
(29)
Thus if after the reheating the Galileon field can be reset in a region from which it evolves initially, there will be not the ghost instability during the hot “big bang” evolution.

B. The power spectrum consistent with the observations

After affirming that the background is ghost-free, we will calculate the power spectrum of the curvature perturbation \( \mathcal{R} \) generated during the slow expansion. The equation of \( \mathcal{R} \) is
\[ u''_k + \left( c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0, \]  
(30)
after we define \( u_k \equiv zR_k \), which can be derived from [21], where \( ' \) is the derivative for the conformal time \( \eta = \int dt/a \simeq t/a \) and \( z^2 \equiv 2a^2 Q_\gamma /c_s^4 \).

When \( k^2 \gg z''/z \), the perturbation mode is leaving the horizon, and hereafter it freezes out. Here, \( a \) is almost unchanged, which implies \( z \sim \sqrt{Q} \). Thus with Eqs. (24) and (25), we can write \( z''/z \) during different periods as
\[ \frac{z''}{z} = \frac{(\sqrt{Q})''}{\sqrt{Q}} \simeq \left( \nu^2 - \frac{1}{4} \right) / (\eta_s - \eta)^2, \]  
(31)
where \( \nu^2 = 49/4 \) for \( k < k_C \) and \( \nu^2 = 9/4 \) for \( k > k_C \), and \( k_C \) is the comoving wave number of the perturbation mode leaving the horizon at \( t = t_C \). Thus Eq. (30) is approximately a Bessel equation with the \( \mathcal{R} \) horizon
\[ 1/\mathcal{R}_H = a \sqrt{\frac{z}{z'}} \sim t_\ast - t, \]  
(32)
which is right during \( t < t_C \) and \( t > t_C \).

When \( k^2 \ll z''/z \), i.e. the perturbation is deep inside the \( \mathcal{R} \) horizon, \( u_k \) oscillates with a constant amplitude. The quantization of \( u_k \) is well-defined, which sets its initial value,
\[ u_k \sim \frac{1}{\sqrt{2k}} e^{-ik_0}. \]  
(33)
When \( k^2 \ll z''/z \), the solution of \( u_k \) is
\[ u_k = \frac{\sqrt{\pi}}{2} e^{i2\pi} \sqrt{\eta_s - \eta} H^{(1)}_{\eta H / 2} (k(\eta_s - \eta)) \]
\[ \simeq \frac{4 e^{i3\pi/2 \Gamma(3/2)}}{\sqrt{2\Gamma(3/2)}} (k(\eta_s - \eta))^3 \]  
(34)
for \( k < k_C \) and
\[ u_k = \frac{\sqrt{\pi}}{2} e^{i\pi/2} \sqrt{\eta_s - \eta} H^{(1)}_{\eta H / 2} (k(\eta_s - \eta)) \]
\[ \simeq \frac{e^{i\pi/2}}{\sqrt{2k}} (k(\eta_s - \eta)) \]  
(35)
for \( k > k_C \), respectively. The perturbation spectrum is
\[ \mathcal{P}_\mathcal{R} = \frac{k^3}{2\pi^2} \frac{|u_k|^2}{z'}, \]  
(36)
which is
\[ \mathcal{P}_\mathcal{R} \simeq \frac{M^2 H_0^2 a^4}{k^4} = \frac{M^2}{H_0^2} \left( \frac{k_0}{k} \right)^4, \quad \text{for } k < k_C, \]  
(37)
\[ \simeq \frac{M}{M_{\text{pl}} x^4}, \quad \text{for } k > k_C, \]  
(38)
where \( k_0 = aH_0 \) is the comoving wave number of the perturbation mode leaving the horizon at certain time \( t_0 \). We see that in the region \( k < k_C \), the spectrum is highly red tilt, while in the region \( k > k_C \), it is scale-invariant, but its amplitude will increase \( \sim 1/x^4 \). We
also may write Eq. (38) as $P_{R}(k > k_{C}) \approx |\epsilon|^{2} \frac{M_{P}}{M_{P}}$ in term of Eqs. (14) and (19).

The evolution of $R$ outside the horizon is

\[ R \sim D_{1} \] is constant mode

or \[ D_{2} \int \frac{dn}{z^{2}} \] is changed mode,

where the increase or decay of the $D_{2}$ mode is dependent on the evolution of $z$. We find that during $t < t_{C}$ the spectrum of $R$ is dominated by the constant mode, while during $t > t_{C}$ the spectrum of $R$ is dominated by the increasing mode, which is

\[ R \sim \int \frac{dn}{Q_{R}} \sim \frac{1}{x^{3}}, \]

which is consistent with Eq. (38).

Here, a significant thing is though during $t < t_{C}$ the amplitude of perturbation leaving has left the horizon is constant, it will synchronously increase with the perturbation leaving the horizon during $t > t_{C}$ after $t > t_{C}$. Noting that for the perturbation mode outside the horizon, only is its amplitudes increasing, but the tilt of the spectrum is not altered.

When $|\epsilon| \sim 1$ or $x \simeq 1$, the change of $a$ begins to become not negligible, as has been mentioned in Sec.II. Thus the increasing of the perturbation amplitude will come to a halt shortly after $t_{e} \sim O(t_{e})$. This implies that the power spectrum of $R$ should be that calculated around $t_{e}$. In term of Eq. (38), noting $x_{e} \simeq 1$, we have

\[ P_{R} \sim \frac{\mathcal{M}}{M_{P}}. \]

The observations give $P_{R}^{1/2} \sim 1/10^{5}$, which requires $\mathcal{M}/M_{P} \sim 1/10^{10}$. Thus in this model, the parameter $\mathcal{M}$ may be fixed by the observations, there is only a free parameter, i.e. $\Lambda$.

We numerically solved Eq. (30), and plotted the evolution of the amplitude of the curvature perturbation in Fig. 2 and the resuling perturbation spectra for the different values of $\Lambda$ in Fig. 3. In Fig. 2, we see that the perturbation spectrum is almost scale invariant for $k > k_{C}$, and is $\sim 1/k^{4}$ for $k < k_{C}$, the amplitude of perturbation spectrum increases with the time, but the shape of the spectrum is not altered. Both Figs. 2 and 3 are consistent with analytical results in Eqs. (37) and (38).

**IV. GALILEAN ISLAND IN ETERNALLY INFLATING BACKGROUND**

We have showed that the observational universe may classically emerge from a de Sitter background with low energy scale. We will see what about the resulting scenario.

For $k < k_{C}$, the amplitude of the curvature perturbation is determined by Eq. (37), which is highly red tilt. This result implies that the amplitude of the perturbation will rapidly increases with scale $1/k$. When $k/a = \sqrt{M_{P} \Lambda}$, we have $P_{R} \sim 1$. We define the time when this perturbation mode leaves the horizon as $t_{Eter}$. When the corresponding perturbation mode leaves the
horizon, i.e. \( k/a \approx H_R \), from Eq. (32), we have
\[
t_* - t_{\text{Eter}} \simeq \frac{1}{\sqrt{M H_0}}.
\]
Thus noting \( t_* = 1/\sqrt{M M_P} \ll 1/\sqrt{M H_0} \), we have
\[
t_{\text{Eter}} \sim -\frac{1}{\sqrt{M H_0}} \sim \sqrt{\frac{H_0}{M}} t_0,
\]
and the corresponding field value \( \varphi_{\text{Eter}} \) can be derived from Eq. (6),
\[
\varphi_{\text{Eter}} = M \ln \frac{1}{M(t_* - t)} \sim -M \ln \left( \frac{M}{H_0} \right).
\]
Thus during \( t < t_{\text{Eter}} = \sqrt{\frac{H_0}{M}} t_0 \), or equivalent \( \varphi < \varphi_{\text{Eter}} \), the energy density \( \rho_\varphi \) of local regions will be randomly walking. This in certain sense implies that the global universe is actually in an eternal inflating state, i.e. some regions have gone or are going through the Galilean genesis phase, but other regions are still in inflationary regime, the inflation never completely ends.

The classical evolutions of local universes begin only after \( t_{\text{Eter}} \), not \( t_0 \) as used in Eq. (19). However, since \( t_0 < t_{\text{Eter}} < t_C \), the result is not affected.

In principle, the eternal inflation will occur in any region of space where the amplitude of the density perturbation \( \sim 1 \). In slow-roll inflation model with single normal field, \( P_R \sim 1 \) implies that the perturbation \( \delta \phi \) of inflaton \( \phi \) is the same order as its classical rolling \( \Delta \phi \sim \phi/H \) in unite of Hubble time, since \( R = \frac{H}{\varphi} \delta \phi \sim \delta \phi/\Delta \phi \).

Here, we will see that \( P_R \sim 1 \) similarly means that the perturbation of field is the same order as its classical rolling. The perturbation of the Galilean field has been calculated in Ref. [24]. When \( k^2 \ll z_\delta \varphi^2/\delta \varphi \), we have
\[
\delta \varphi_k \simeq \frac{\Gamma(\frac{5}{2})}{\sqrt{2}} \frac{e^{\pi}}{a \sqrt{2k}} Q_\delta \varphi \Gamma(\frac{3}{2}) (k(\eta_* - \eta))^2,
\]
where \( z_\delta \varphi = a \sqrt{Q_\delta \varphi} \) and
\[
Q_\delta \varphi \simeq \frac{M_\varphi^2}{M^2} x^4,
\]
which is given by Eq. (27). Thus the average square of the amplitude of field fluctuations is
\[
< \delta \varphi_k^2 > = \frac{1}{8\pi} \int_{a H_\delta \varphi/c}^{a H_\varphi} |\delta \varphi_k|^2 d^3k \sim \int_{a H_\delta \varphi/c}^{a H_\varphi} \frac{a^2 M^4}{k^3} dk \sim \frac{M^4}{H^2 R},
\]
where
\[
1/H_\delta \varphi = a \left| \frac{z_\delta \varphi}{z_\varphi} \right|^{1/2} \sim t_* - t
\]
corresponds to the horizon of \( \gamma_\varphi \), which means that for the perturbation being leaving the horizon, we have \( k/a \approx H_\delta \varphi \). We actually have \( H_R \approx H_\varphi \). The classical rolling of Galilean field is approximately
\[
\Delta \varphi = \varphi / H_0 \sim \frac{M H_R}{H_0},
\]
where \( \varphi \) is given by Eq. (7). Thus for \( k < \sqrt{M H_0} \), we have
\[
\frac{\sqrt{< \delta \varphi_k^2 >}}{\Delta \varphi} \sim \frac{M H_0}{H_R^2} > 1.
\]
Thus during \( t < t_{\text{Eter}} \), or equivalent \( \varphi < \varphi_{\text{Eter}} \), the perturbation of field is larger than its classical rolling, which implies that the field is randomly jumping. Thus in some local regions of the global universe, the field will jumped to the regime of \( \varphi > \varphi_{\text{Eter}} \), and the local universe will go through the evolution of slowly expanding Galilean genesis, while in other regions the field will again jump back and is still in random jumping. This result again indicates that the global universe is actually in an eternally inflating regime. When \( t \sim t_{\text{Eter}} \), we have
\[
\frac{\varphi}{M} = \sqrt{\frac{M}{H_0}} H_0 \gg H_0,
\]
which insures the rationality of \( \varphi \).

We summarized the resulting scenario in Fig. 4. During the eternal inflation, some regions of global universe will go through the slowly expanding Galilean genesis phase and become island universes, while other regions are still eternally inflating, which will make the room for more island universes to emerge. The island universes generally has the initial conditions \( a_{ini} \approx 1/H_0 \) and \( H_{ini} \approx H_0 \), which is consistent with Ref. [24]. During different periods after \( t_{\text{Eter}} \), the values of \( H \) and \( |\epsilon| \) are summarized in Tab.I. We see that during \( t_{\text{Eter}} < t < t_C \), the island universe is in a slowly expanding phase with almost constant \( H \approx H_0 \), while during \( t_C < t < t_e \), it is in the Galilean genesis phase with rapidly increasing \( H \) and

| Period        | \( H \)          | |\( \epsilon |\) |
|---------------|------------------|------|
| \( t = t_{\text{Eter}} \) | \( H_0 \)         | \( < 1 \) |
| \( t_{\text{Eter}} < t < t_C \) | \( H_0 \)         | \( \frac{M H^4 M^4 H_0^4}{H_0^5} \gg 1 \) |
| \( t = t_C \) | \( \gtrsim H_0 \) |      |
| \( t_C < t < t_e \) | \( \frac{M^2 R^2 (t_* - t)}{H_0^5} \gtrsim 1 \) |
| \( t = t_e \) | \( \sqrt{M M_P} \gg H_0 \) | \( \sim 1 \) |
the scale-invariant primordial perturbation may be generated during this period. At $t = t_c$, the genesis phase of island universe ends, the available energy of field will be released into the radiation, and the universe reheats, e.g. [27], [20]. Hereafter, in the corresponding local region, or islands, the evolution of hot “big bang” model begins.

Here, we call this local thermalized universe as island universe. During different periods after $t_E$, the values of $H$ is summarized in Tab.I.

Here, $\Lambda$ is regarded as constant, which sets up a de Sitter background from which the observational universe emerges. The model with $\Lambda$ being a landscape of effective potential of Galileon field $\varphi$ or other fields is certainly far unfavored. In certain sense, our scenario is a significant supplement to the phenomenology of the eternal inflation, which might help to improve the current understanding for the measure problem. The relevant issue will be discussed elsewhere. The introduction of AdS bounce in eternally inflating background has similar result [33], and also [34], [35].

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