Abstract. I discuss the exciting prospects for Higgs and technicolor Goldstone boson physics at a muon collider.

INTRODUCTION

The prospects for Higgs and Goldstone boson physics at a muon collider depend crucially upon the instantaneous luminosity, $\mathcal{L}$, possible for $\mu^+\mu^-$ collisions as a function of $E_{\text{beam}}$ and on the percentage Gaussian spread in the beam energy, denoted by $R$. The small level of bremsstrahlung and absence of beamstrahlung at muon collider implies that very small $R$ can be achieved. The (conservative) luminosity assumptions for the recent Fermilab-97 workshop were:

- $\mathcal{L} \sim (0.5, 1, 6) \cdot 10^{31}\text{cm}^{-2}\text{s}^{-1}$ for $R = (0.003, 0.01, 0.1)\%$ at $\sqrt{s} \sim 100$ GeV;
- $\mathcal{L} \sim (1, 3, 7) \cdot 10^{32}\text{cm}^{-2}\text{s}^{-1}$, at $\sqrt{s} \sim (200, 350, 400)$ GeV, $R \sim 0.1\%$.

With modest success in the collider design, at least a factor of 2 better can be anticipated. Note that for $R \sim 0.003\%$ the Gaussian spread in $\sqrt{s}$, given

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2) For yearly integrated luminosities, we use the standard convention of $\mathcal{L} = 10^{32}\text{cm}^{-2}\text{s}^{-1} \Rightarrow L = 1 \text{fb}^{-1}/\text{yr}.$
by \( \sigma_{\sqrt{s}} \sim 2 \text{MeV} \left( \frac{R}{0.003\%} \right) \left( \frac{\sqrt{s}}{100 \text{GeV}} \right) \), can be comparable to the few MeV widths of very narrow resonances such as a light SM-like Higgs boson or a (pseudo-Nambu-Goldstone) technicolor boson. This is critical since the effective resonance cross section \( \sigma \) is obtained by convoluting a Gaussian \( \sqrt{s} \) distribution of width \( \sigma_{\sqrt{s}} \) with the standard s-channel Breit Wigner resonance cross section, \( \sigma(\sqrt{s}) = 4\pi \Gamma(\mu\mu) \Gamma(X)/(\hat{s} - M^2)^2 + [M\Gamma^{\text{tot}}]^2 \). For \( \sqrt{s} = M \), the result, \(^3\)

\[
\sigma \simeq \frac{\pi \sqrt{2\pi} \Gamma(\mu\mu) B(X)}{M^2 \sigma_{\sqrt{s}}} \times \left( 1 + \frac{\pi}{8} \left[ \frac{\Gamma^{\text{tot}}}{\sigma_{\sqrt{s}}} \right]^2 \right)^{-1/2}
\]

will be maximal if \( \Gamma^{\text{tot}} \) is small and \( \sigma_{\sqrt{s}} \sim \Gamma^{\text{tot}} \). \(^4\) Also critical to scanning a narrow resonance is the ability \(^2\) to tune the beam energy to one part in 10^6.

**Higgs Physics**

The potential of the muon collider for Higgs physics is truly outstanding. First, it should be emphasized that away from the s-channel Higgs pole, \( \mu^+\mu^- \) and \( e^+e^- \) colliders have similar capabilities for the same \( \sqrt{s} \) and \( \mathcal{L} \) (barring unexpected detector backgrounds at the muon collider). At \( \sqrt{s} = 500 \text{GeV} \), the design goal for a \( e^-e^- \) linear collider (\( eC \)) is \( \mathcal{L} = 50 \text{fb}^{-1} \) per year. The conservative \( \mathcal{L} \) estimates given earlier suggest that at \( \sqrt{s} = 500 \text{GeV} \) the \( \mu C \) will accumulate at least \( \mathcal{L} = 10 \text{fb}^{-1} \) per year. If this can be improved somewhat, the \( \mu C \) would be fully competitive with the \( eC \) in high energy (\( \sqrt{s} \sim 500 \text{GeV} \) running). (We will use the notation of \( \ell C \) for either a \( eC \) or \( \mu C \) operating at moderate to high \( \sqrt{s} \).)

The totally unique feature of the \( \mu C \) is the dramatic peak in the cross section \( \sigma_h \) for production of a narrow-width Higgs boson in the s-channel that occurs when \( \sqrt{s} = m_h \) and \( R \) is small enough that \( \sigma_{\sqrt{s}} \) is smaller than or comparable to \( \Gamma^{\text{tot}}_h \) \(^1\). The peaking is illustrated below in Fig. 1 for a SM Higgs (\( h_{SM} \)) with \( m_{h_{SM}} = 110 \text{GeV} \) (\( \Gamma^{\text{tot}}_{h_{SM}} \sim 3 \text{MeV} \)).

**A Standard Model-Like Higgs Boson**

For SM-like \( h \to WW, ZZ \) couplings, \( \Gamma^{\text{tot}}_h \) becomes big if \( m_h \gtrsim 2m_W \), and \( \sigma_h \propto B(h \to \mu^+\mu^-) \) [Eq. 1] will be small; s-channel production will not be useful.

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3) In actual numerical calculations, bremsstrahlung smearing is also included (see Ref. [1]).

4) Although smaller \( \sigma_{\sqrt{s}} \) (i.e. smaller \( R \)) implies smaller \( \mathcal{L} \), the \( \mathcal{L} \)'s given earlier are such that when \( \Gamma^{\text{tot}} \) is in the MeV range it is best to use the smallest \( R \) that can be achieved.
But, as shown in Fig. 1, $\sigma_h$ is enormous for small $R$ when the $h$ is light, as is very relevant in supersymmetric models where the light SM-like $h^0$ has $m_{h^0} \lesssim 150$ GeV. In order to make use of this large cross section, we must first center on $\sqrt{s} \sim m_h$ and then proceed to the precision measurement of the Higgs boson’s properties.

**FIGURE 1.** The effective cross section, $\bar{\sigma}_{h_{SM}}$, for $R = 0.01\%$, $R = 0.06\%$, and $R = 0.1\%$ vs. $\sqrt{s}$ for $m_{h_{SM}} = 110$ GeV.

For a SM-like Higgs with $m_h \lesssim 2m_W$ one expects [3] to determine $m_h$ to within $\Delta m_h \sim 100$ MeV from LHC data ($L = 300$ fb$^{-1}$) (the uncertainty $\Delta m_h$ will be even smaller if $\ell C$ data is available). Thus, a final ring that is fully optimized for $\sqrt{s} \sim m_h$ can be built. Once it is operating, we scan over the appropriate $\Delta m_h$ interval so as to center on $\sqrt{s} \simeq m_h$ within a fraction of $\sigma_{\sqrt{s}}$. Consider first the “typical” case of $m_h \sim 110$ GeV. For $m_h$ of order 100 GeV, $R = 0.003\%$ implies $\sigma_{\sqrt{s}} \sim 2$ MeV. $\Delta m_h \sim 100$ MeV implies that $\Delta m_h/\sigma_{\sqrt{s}} \sim 50$ points are needed to center within $\lesssim \sigma_{\sqrt{s}}$. At this mass, each point requires $L \sim 0.0015$ fb$^{-1}$ in order to observe or eliminate the $h$ at the 3$\sigma$ level, implying a total of $L_{tot} \leq 0.075$ fb$^{-1}$ is needed for centering. (Plots as a function of $m_{h_{SM}}$ of the luminosity required for a 5$\sigma$ observation of the SM Higgs boson when $\sqrt{s} = m_{h_{SM}}$ can be found in Ref. [1].) Thus, for the anticipated $L \sim 0.05 - 0.1$ fb$^{-1}$/yr, centering would take no more than a year. However, for $m_h \simeq m_Z$ a factor of 50 more $L_{tot}$ is required just for centering because of the large $Z \to b\bar{b}$ background. Thus, for the anticipated $L$ the $\mu C$ is not useful if the Higgs boson mass is too close to $m_Z$.

Once centered, we will wish to measure with precision: (i) the very tiny Higgs width — $\Gamma_h^{\text{tot}} = 1 - 10$ MeV for a SM-like Higgs with $m_h \lesssim 140$ GeV; (ii) $\sigma(\mu^+\mu^- \to$
$h \to X$) for $X = \tau^+\tau^-, b\bar{b}, c\bar{c}, WW^*, ZZ^*$. The accuracy achievable was studied in Ref. [1]. The three-point scan of the Higgs resonance described there is the optimal procedure for performing both measurements simultaneously. We summarize the resulting statistical errors in the case of a SM-like $h$ with $m_h = 110$ GeV, assuming $R = 0.003\%$ and an integrated (4 to 5 year) $L_{\text{tot}} = 0.4$ fb$^{-1}$. One finds 1σ errors for $\sigma B(X)$ of 8.3, 22, 15, 190% for the $X = \tau^+\tau^-, b\bar{b}, c\bar{c}, WW^*, ZZ^*$ channels, respectively, and a $\Gamma_h$ error of 16%. The individual channel $X$ results assume the $\tau, b, c$ tagging efficiencies described in Ref. [4]. We now consider how useful measurements at these accuracy levels will be.

If only $s$-channel Higgs factory $\mu C$ data are available (i.e. no $Zh$ data from an $eC$ or $\mu C$), then the $\sigma B$ ratios (equivalently squared-coupling ratios) that will be most effective for discriminating between the SM Higgs boson and a SM-like Higgs boson such as the $h^0$ of supersymmetry are $(WW^*h)^2, (c\bar{c}h)^2, (b\bar{b}h)^2$, and $(\tau^+\tau^-h)^2$. The 1σ errors (assuming $L_{\text{tot}} = 0.4$ fb$^{-1}$ in the three-point scan centered on $m_h = 110$ GeV, or $L_{\text{tot}} = 0.2$ fb$^{-1}$ with $\sqrt{s} = m_h = 110$ GeV) for these four ratios are 15%, 20%, 18% and 22%, respectively. Systematic errors for $(c\bar{c}h)^2$ and $(b\bar{b}h)^2$ of order 5% – 10% from uncertainty in the $c$ and $b$ quark mass will also enter. In order to interpret these errors one must compute the amount by which the above ratios differ in the minimal supersymmetric model (MSSM) vs. the SM for $m_{h^0} = m_{h_{\text{SM}}}$. The percentage difference turns out to be essentially identical for all the above ratios and is a function almost only of the MSSM Higgs sector parameter $m_{A^0}$, with very little dependence on $\tan \beta$ or top-squark mixing. At $m_{A^0} = 250$ GeV (420 GeV) one finds MSSM/SM $\sim 0.5 (\sim 0.8)$. Combining the four independent ratio measurements and including the systematic errors, one concludes that a $> 2\sigma$ deviation from the SM predictions would be found if the observed 110 GeV Higgs is the MSSM $h^0$ and $m_{A^0} < 400$ GeV. Note that the magnitude of the deviation would provide a determination of $m_{A^0}$.

If, in addition to the $s$-channel measurements we also have $\ell C \sqrt{s} = 500$ GeV, $L_{\text{tot}} = 200$ fb$^{-1}$ data, it will be possible to discriminate at an even more accurate level between the $h^0$ and the $h_{\text{SM}}$. The most powerful technique for doing so employs the four determinations of $\Gamma(h \to \mu^+\mu^-)$ below:

\[
\frac{\Gamma(h \to \mu^+\mu^-)B(h \to b\bar{b})_{\mu C}}{B(h \to b\bar{b})_{\mu C}}, \quad \frac{\Gamma(h \to \mu^+\mu^-)B(h \to WW^*)_{\mu C}}{B(h \to WW^*)_{\mu C}}.
\]

5) For $\sigma B$ measurements, $L_{\text{tot}}$ devoted to the optimized three-point scan is equivalent to $\sim L_{\text{tot}}/2$ at the $\sqrt{s} = m_h$ peak.
FIGURE 2. We give \((m_{A^0}, \tan \beta)\) parameter space contours for \(\frac{\Gamma(h^0 \rightarrow \mu^+ \mu^-)}{\Gamma(h_{SM} \rightarrow \mu^+ \mu^-)}\); no-squark-mixing, \(m_{h^0}, m_{h_{SM}} = 110\) GeV.

\[
\frac{[\Gamma(h \rightarrow \mu^+ \mu^-) B(h \rightarrow ZZ^*)]_{\mu C}[\Gamma_{tot}^{h^0}]_{\mu C + \ell C}}{\Gamma(h \rightarrow ZZ^*)_{\ell C}}; \quad \frac{[\Gamma(h \rightarrow \mu^+ \mu^-) B(h \rightarrow WW^*)]_{h_{SM}^{tot}}^{\mu C}}{\Gamma(h \rightarrow WW^*)_{\ell C}}.
\]

The resulting \(1\sigma\) error for \(\Gamma(h \rightarrow \mu^+ \mu^-)\) is \(\lesssim 5\%\). Fig. 2, which plots the ratio of the \(h^0\) to \(h_{SM}\) partial width in \((m_{A^0}, \tan \beta)\) parameter space for \(m_{h^0} = m_{h_{SM}} = 110\) GeV, shows that this level of error allows one to distinguish between the \(h^0\) and \(h_{SM}\) at the \(3\sigma\) level out to \(m_{A^0} \gtrsim 600\) GeV. Additional advantages of a \(\Gamma(h \rightarrow \mu^+ \mu^-)\) measurement are: (i) there are no systematic uncertainties arising from uncertainty in the muon mass; (ii) the error on \(\Gamma(h \rightarrow \mu^+ \mu^-)\) increases only very slowly as the \(s\)-channel \(L_{tot}\) decreases, \(^6\) in contrast to the errors for the previously discussed ratios of branching ratios from the \(\mu C\) \(s\)-channel data which scale as \(1/\sqrt{L_{tot}}\). Finally, we note that \(\Gamma_{tot}^{h^0}\) alone cannot be used to distinguish between the MSSM \(h^0\) and SM \(h_{SM}\) in a model-independent way. Not only is the error substantial (\(\sim 12\%\) if we combine \(\mu C, L = 0.4\) fb\(^{-1}\) \(s\)-channel data with \(\ell C, L = 200\) fb\(^{-1}\) data) but also \(\Gamma_{tot}^{h^0}\) depends on many things, including (in the MSSM) the squark-mixing model. Still, deviations from SM predictions are

\(^6\) This is because the \(\Gamma(h \rightarrow \mu^+ \mu^-)\) error is dominated by the \(\sqrt{s} = 500\) GeV measurement errors.
generally substantial if \( m_{A^0} \lesssim 500 \text{ GeV} \) implying that the measurement of \( \Gamma_{h}^{\text{tot}} \) could be very revealing.

We note that the above errors and results hold approximately for all \( m_h \lesssim 150 \text{ GeV} \) so long as \( m_h \) is not too close to \( m_Z \).

Precise measurements of the couplings of the SM-like Higgs boson could reveal many other types of new physics. For example, if a significant fraction of a fermion’s mass is generated radiatively (as opposed to arising at tree-level), then the \( h f \bar{f} \) coupling and associated partial width will deviate from SM expectations [5]. Deviations of order 5% to 10% (or more) in \( \Gamma(h \to \mu^+ \mu^-) \) are quite possible and, as discussed above, potentially detectable.

The MSSM \( H^0, A^0 \) and \( H^\pm \)

We begin by recalling [3] that the possibilities for \( H^0, A^0 \) discovery are limited at other machines. (i) Discovery of \( H^0, A^0 \) is not possible at the LHC for all \((m_{A^0}, \tan \beta)\): e.g. if \( m_{\tilde{t}} = 1 \text{ TeV} \), consistency with the observed value of \( B(b \to s\gamma) \) requires \( m_{A^0} > 350 \text{ GeV} \), in which case the LHC might not be able to detect the \( H^0, A^0 \) at all, and certainly not for all \( \tan \beta \) values. If \( \tan \beta \lesssim 3 \), detection might be possible in the \( H^0, A^0 \to t\bar{t} \) final state, but would require \( \lesssim 10\% \) systematic uncertainty in understanding the absolute normalization of the \( t\bar{t} \) background. Otherwise, and certainly for \( \tan \beta \gtrsim 3 \), one must employ \( b\bar{b}A^0, b\bar{b}H^0 \) associated production, first analyzed in Refs. [6,7] and recently explored further in [8,9]. There is currently considerable debate as to what portion of \((m_{A^0}, \tan \beta)\) parameter space can be covered using the associated production modes. In the update of [7], it is claimed that \( \tan \beta \gtrsim 5 (\gtrsim 15) \) is required for \( m_{A^0} \sim 200 \text{ GeV} (\sim 500 \text{ GeV}) \). Ref. [8] claims that still higher \( \tan \beta \) values are required, \( \tan \beta \gtrsim 20 (\tan \beta \gtrsim 30) \), whereas Ref. [9] claims \( \tan \beta \gtrsim 2 (\gtrsim 4) \) will be adequate. (ii) At \( \sqrt{s} = 500 \text{ GeV}, e^+e^- \to H^0A^0 \) pair production probes only to \( m_{A^0} \sim m_{H^0} \lesssim 230 - 240 \text{ GeV} \). (iii) A \( \gamma\gamma \) collider could potentially probe up to \( m_{A^0} \sim m_{H^0} \sim 0.8\sqrt{s} \sim 400 \text{ GeV} \), but only for \( L_{\text{tot}} \gtrsim 150 - 200 \text{ fb}^{-1} \) [10].

Thus, it is noteworthy that \( \mu^+\mu^- \to H^0, A^0 \) in the \( s\)-channel potentially allows production and study of the \( H^0, A^0 \) up to \( m_{A^0} \sim m_{H^0} \lesssim \sqrt{s} \). To assess the potential, let us (optimistically) assume that a total of \( L_{\text{tot}} = 50 \text{ fb}^{-1} \) (5 yrs running at \( \mathcal{L} = 1 \times 10^{33} \)) can be accumulated for \( \sqrt{s} \) in the 250 – 500 GeV range. (We note that \( \Gamma_{A^0}^{\text{tot}} \) and \( \Gamma_{H^0}^{\text{tot}} \), although not big, are of a size such that resolution of \( R \gtrsim 0.1\% \) will be adequate to maximize the \( s\)-channel cross section, thus allowing
for substantial $\mathcal{L}$.)

There are then several possible scenarios. (a) If we have some preknowledge or restrictions on $m_{A^0}$ from LHC discovery or from $s$-channel measurements of $h^0$ properties, then $\mu^+\mu^- \rightarrow H^0$ and $\mu^+\mu^- \rightarrow A^0$ can be studied with precision for all $\tan\beta \gtrsim 1 - 2$. (b) If we have no knowledge of $m_{A^0}$ other than $m_{A^0} \gtrsim 250 - 300$ GeV from LHC, then we might wish to search for the $A^0, H^0$ in $\mu^+\mu^- \rightarrow H^0, A^0$ by scanning over $\sqrt{s} = 250 - 500$ GeV. If their masses lie in this mass range, then their discovery by scanning will be possible for most of $(m_{A^0}, \tan\beta)$ parameter space such that they cannot be discovered at the LHC (in particular, if $m_{A^0} \gtrsim 250$ GeV and $\tan\beta \gtrsim 4 - 5$). (c) Alternatively, if the $\mu C$ is simply run at $\sqrt{s} = 500$ GeV and $L_{\text{tot}} \sim 50$ fb$^{-1}$ is accumulated, then $H^0, A^0$ in the $250 - 500$ GeV mass range can be discovered in the $\sqrt{s}$ bremsstrahlung tail if the $b\bar{b}$ mass resolution (either by direct reconstruction or hard photon recoil) is of order $\pm 5$ GeV and if $\tan\beta \gtrsim 6 - 7$ (depending on $m_{A^0}$). Typical peaks are illustrated in Fig. 3.\footnote{SUSY decays are assumed to be absent in this and the following figure.}

Finally, once the closely degenerate $A^0, H^0$ are discovered, it will be extremely interesting to be able to separate the resonance peaks. This will probably only be possible at a muon collider with small $R \lesssim 0.01\%$ if $\tan\beta$ is large, as illustrated in Fig. 4.

We note that the above results assume that SUSY decays of the $H^0$ and $A^0$ do

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3}
\caption{$N(\ell\ell)$ in the $m_{b\bar{b}} \pm 5$ GeV interval vs. $m_{b\bar{b}}$ for $\sqrt{s} = 500$ GeV, $L_{\text{tot}} = 50$ fb$^{-1}$, and $R = 0.1\%$; peaks are shown for $m_{A^0} = 120, 300$ or $480$ GeV, with $\tan\beta = 5$ and $20$ in each case.}
\end{figure}
not have a large net branching ratio for $m_{A^0}, m_{H^0} \lesssim 500$ GeV. If SUSY decays are significant, the possibilities and strategies for $H^0, A^0$ discovery at all machines would have to be re-evaluated.

We end this sub-section with just a few remarks on the possibilities for production of $H^0 A^0$ and $H^+ H^-$ pairs at a high energy $\mu C$ (or $e C$). Since $m_{A^0} \gtrsim 1$ TeV cannot be ruled out simply on the basis of hierarchy and naturalness (although fine-tuning is stretched), it is possible that energies of $\sqrt{s} > 2$ TeV could be required for pair production. If available, then it has been shown [11,12] that discovery of $H^0 A^0$ in their $b\bar{b}$ or $t\bar{t}$ decay modes and $H^+ H^-$ in their $t\bar{b}$ and $b\bar{t}$ decays will be easy for expected luminosities, even if SUSY decays are present. As a by-product, the masses will be measured with reasonable accuracy.

Regardless of whether we see the $H^0, A^0$ in $s$-channel production or via pair production, one can measure branching ratios to other channels, including supersymmetric pair decay channels with good accuracy. In fact, the ratios of branching ratios and the value of $m_{A^0} \sim m_{H^0} \sim m_{H^\pm}$ will be measured with sufficient accuracy that, in combination with one gaugino mass, say the chargino mass (which will also presumably be well-measured) it will be possible [11] to discriminate with incredible statistical significance between different closely similar GUT scenarios for the GUT-scale soft-supersymmetry-breaking masses. Thus, Higgs pair produc-
tion could be very valuable in the ultimate goal of determining all the soft-SUSY-breaking parameters.

Finally, entirely unexpected decays of the heavy Higgs bosons of SUSY (or other extended Higgs sector) could be present. For example, non-negligible branching ratios for $H^0, A^0 \rightarrow \tau\tau + c\bar{c}$ FCNC decays are not inconsistent with current theoretical model-building ideas and existing constraints [13]. The muon collider $s$-channel $\mu^+\mu^- \rightarrow H^0, A^0$ event rate is sufficient to probe rather small values for such FCNC branching ratios.

**Verifying Higgs CP Properties**

Once a neutral Higgs boson is discovered, determination of its CP nature will be of great interest. For example, direct verification that the SM Higgs is CP-even would be highly desirable. Indeed, if a neutral Higgs boson is found to have a mixed CP nature (implying CP violation in the Higgs sector), then neither the SM nor the MSSM can be correct. In the case of the SM, one must have a multi-doublet (or more complicated) Higgs sector. In the case of the MSSM, at least a singlet Higgs boson (as in the NMSSM) would be required to be present in addition to the standard two doublets.

One finds that the $\gamma\gamma$ and $\mu^+\mu^-$ single Higgs production modes provide the most elegant and reliable techniques for CP determination. In $\gamma\gamma$ collisions at the $eC$ (a $\gamma\gamma$ collider is not possible at the $\mu C$), one establishes definite polarizations $\vec{e}_{1,2}$ for the two colliding photons in the photon-photon center of mass. Since $L_{\gamma\gamma h} = \vec{e}_{1} \cdot \vec{e}_{2} E + (\vec{e}_{1} \times \vec{e}_{2}) \cdot O$, where $E$ and $O$ are of similar size if the CP-even and CP-odd (respectively) components of the $h$ are comparable. There are two important types of measurement. The first [14] is the difference in rates for photons colliding with $++$ vs. $--$ helicities, which is non-zero only if CP violation is present. Experimentally, this difference can be measured by simultaneously flipping the helicities of both of the initiating back-scattered laser beams. The second [14–16] is the dependence of the $h$ production rate on the relative orientation of transverse polarizations $\vec{e}_{1}$ and $\vec{e}_{2}$ for the two colliding photons. In the case of a CP-conserving Higgs sector, the production rate is maximum for a CP-even (CP-odd) Higgs boson when $\vec{e}_{1}$ is parallel (perpendicular) to $\vec{e}_{2}$. The limited transverse polarization that can be achieved at a $\gamma\gamma$ collider implies that very high luminosity is needed for such a study.

In the end, a $\mu^+\mu^-$ collider might well prove to be the best machine for directly
probing the CP properties of a Higgs boson that can be produced and detected in the s-channel mode [17,18]. Consider transversely polarized muon beams. For 100% transverse polarization and an angle $\phi$ between the $\mu^+$ transverse polarization and the $\mu^-$ transverse polarization, one finds

$$\sigma(\phi) \propto 1 - \frac{a^2 - b^2}{a^2 + b^2} \cos \phi + \frac{2ab}{a^2 + b^2} \sin \phi,$$

(3)

where the coupling of the $h$ to muons is given by $h\pi(a + ib\gamma_5)\mu$, $a$ and $b$ being the CP-even and CP-odd couplings, respectively. If the $h$ is a CP mixture, both $a$ and $b$ are non-zero and the asymmetry

$$A_1 \equiv \frac{\sigma(\pi/2) - \sigma(-\pi/2)}{\sigma(\pi/2) + \sigma(-\pi/2)} = \frac{2ab}{a^2 + b^2},$$

(4)

will be large. For a pure CP eigenstate the cross section difference

$$A_2 \equiv \frac{\sigma(\pi) - \sigma(0)}{\sigma(\pi) + \sigma(0)} = \frac{a^2 - b^2}{a^2 + b^2},$$

(5)

is +1 or -1 for a CP-even or CP-odd $h$, respectively. Since background processes and partial transverse polarization will dilute the statistics, further study will be needed to fully assess the statistical level of CP determination that can be achieved in various cases.

### Exotic Higgs Bosons

If there are doubly-charged Higgs bosons, $e^-e^-\rightarrow\Delta^{--}$ probes $\lambda_{ee}$ and $\mu^-\mu^-\rightarrow\Delta^{--}$ probes $\lambda_{\mu\mu}$, where the $\lambda$'s are the strengths of the Majorana-like couplings [19–21]. Current $\lambda_{ee,\mu\mu}$ limits are such that factory-like production of a $\Delta^{--}$ is possible if $\Gamma^\text{tot}_{\Delta^{--}}$ is small. Further, a $\Delta^{--}$ with $m_{\Delta^{--}} \lesssim 500$–1000 GeV will be seen previously at the LHC (for $m_{\Delta^{--}} \lesssim 200$–250 GeV at TeV33) [22]. For small $\lambda_{ee,\mu\mu,\tau\tau}$ in the range that would be appropriate, for example, for the $\Delta^{--}$ in the left-right symmetric model see-saw neutrino mass generation context, it may be that $\Gamma^\text{tot}_{\Delta^{--}} \ll \sigma_{\sqrt{s}}$, leading to $\sigma_{\ell^-\ell^-\rightarrow\Delta^{--}} \propto \lambda_{\ell\ell}^2/\sqrt{s}$. Note that the absolute rate for $\ell^-\ell^-\rightarrow\Delta^{--}$ yields a direct determination of $\lambda_{\ell\ell}^2$, which, for a $\Delta^{--}$ with very small $\Gamma^\text{tot}_{\Delta^{--}}$, will be impossible to determine by any other means. The relative branching ratios for $\Delta^{--}\rightarrow e^-e^-,\mu^-\mu^-,\tau^-\tau^-$ will then yield values for the remaining $\lambda_{\ell\ell}$'s.

Because of the very small $R = 0.003\% - 0.01\%$ achievable at a muon collider, $\mu^-\mu^-$

8) For small $\lambda_{ee,\mu\mu,\tau\tau}$, $\Gamma^\text{tot}_{\Delta^{--}}$ is very small if the $\Delta^{--}\rightarrow W^-W^-$ coupling strength is very small or zero, as required to avoid naturalness problems for $\rho = m_W^2/[\cos^2\theta_W m_Z^2]$. 
collisions will probe weaker $\lambda_{\mu\mu}$ coupling than the $\lambda_{ee}$ coupling that can be probed in $e^-e^-$ collisions. In addition, it is natural to anticipate that $\lambda_{\mu\mu}^2 \gg \lambda_{ee}^2$. A more complete review of this topic is given in Ref. [23].

**PROBES OF NARROW TECHNICOLOR RESONANCES**

In this section, I briefly summarize the ability of a low-energy muon collider to observe the pseudo-Nambu-Goldstone bosons (PNGB’s) of an extended technicolor theory. These narrow states need not have appeared at an observable level in $Z$ decays at LEP. Some of the PNGB’s have substantial $\mu^+\mu^-$ couplings. Thus, a muon collider search for them will bear a close resemblance to the light Higgs case discussed already. The main difference is that, assuming they have not been detected ahead of time, we must search over the full expected mass range.

The first results for PNGB’s at a muon collider appear in Refs. [24] and [25]. Here I summarize the results for the lightest $P^0$ PNGB as given in Ref. [24]. Although the specific $P^0$ properties employed are those predicted by the extended BESS model [24], they will be representative of what would be found in any extended technicolor model for a strongly interacting electroweak sector. The first point is that $m_{P^0}$ is expected to be small; $m_{P^0} \lesssim 80$ GeV is preferred in the BESS model. Second, the Yukawa couplings and branching ratios of the $P^0$ are easily determined. In the BESS model, $\mathcal{L}_Y = -i \sum_f \lambda_f \bar{f} \gamma_5 f P^0$ with $\lambda_b = \sqrt{2} \frac{m_b}{v}$, $\lambda_\tau = -\sqrt{6} \frac{m_\tau}{v}$, $\lambda_\mu = -\sqrt{6} \frac{m_\mu}{v}$. 

**FIGURE 5.** $L_{\text{tot}}$ required for a $5\sigma$ $P^0$ signal at $\sqrt{s} = m_{P^0}$. 

![Graph](image-url)
Note the sizeable $\mu^+\mu^-$ coupling. The $P^0$ couplings to $\gamma\gamma$ and $gg$ from the ABJ anomaly are also important. Overall, these couplings are not unlike those of a light Higgs boson. Not surprisingly, therefore, $\Gamma^{\text{tot}}_{P^0}$ is very tiny: $\Gamma^{\text{tot}}_{P^0} = 0.2, 4, 10$ MeV for $m_{P^0} = 10, 80, 150$ GeV, respectively, for $N_{TC} = 4$ technicolor flavors. For such narrow widths, it will be best to use $R = 0.003\%$ beam energy resolution.

![Graph](image)

**FIGURE 6.** $L_{\text{tot}}$ required to scan indicated 5 GeV intervals and either discover or eliminate the $P^0$ at the 3$\sigma$ level.

For the detailed tagging efficiencies etc. described in [24], the $L_{\text{tot}}$ required to achieve $\sum_k S_k/\sqrt{\sum_k B_k} = 5$ at $\sqrt{s} = m_{P^0}$, after summing over the optimal selection of the $k = b\bar{b}$, $\tau^+\tau^-$, $c\bar{c}$, and $gg$ channels (as defined after tagging), is plotted in Fig. 5. Very modest $L_{\text{tot}}$ is needed unless $m_{P^0} \sim m_Z$. Of course, if we do not have any information regarding the $P^0$ mass, we must scan for the resonance. The (very conservative, see [24] for details) estimate for the luminosity required for scanning a given 5 GeV interval and either discovering or eliminating the $P^0$ in that interval at the 3$\sigma$ level is plotted in Fig. 6. If the $P^0$ is as light as expected in the extended BESS model, then the prospects for discovery by scanning would be excellent. For example, a $P^0$ lying in the $\sim 10$ GeV to $\sim 75$ GeV mass interval can be either discovered or eliminated at the 3$\sigma$ level with just $0.11$ fb$^{-1}$ of total luminosity, distributed in proportion to the luminosities plotted in Fig. 6. The $L$ that could be achieved at these low masses is being studied [26]. A $P^0$ with $m_{P^0} \sim m_Z$ would be much more difficult to discover unless its mass was approximately known. A 3$\sigma$ scan of the mass interval from $\sim 105$ GeV to 160 GeV would require about 1 fb$^{-1}$ of integrated luminosity, which is more than could be comfortably achieved for the
DISCUSSION AND CONCLUSIONS

There is little doubt that a variety of accelerators will be needed to explore all aspects of the physics that lies beyond the Standard Model and accumulate adequate luminosity for this purpose in a timely fashion. For any conceivable new-physics scenario, a muon collider would be a very valuable machine, both for discovery and detailed studies. Here we have reviewed the tremendous value of a muon collider for studying any narrow resonance with $\mu^+\mu^-$ (or $\mu^-\mu^-$) couplings, focusing on neutral light Higgs bosons and the Higgs-like pseudo-Nambu-Goldstone bosons that would be present in almost any technicolor model. A muon collider could well provide the highest statistics determinations of many important Higgs or PNGB fundamental couplings. In particular, it might provide the only direct measurement of the very important $\mu^+\mu^-$ coupling. Measurement of this coupling will very possibly allow discrimination between a SM Higgs boson and its light $h^0$ SUSY counterpart. Comparison of the $\mu^+\mu^-$ coupling to the $\tau^+\tau^-$ coupling (one may be able to approximately determine the latter from branching ratios) will also be of extreme interest. For Higgs physics, developing machine designs that yield the highest possible luminosity at low energies, while maintaining excellent beam energy resolution, should be a priority.

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