X(1835) as Proton-Antiproton Bound State in Skyrme Model

Mu-Lin Yan
Interdisciplinary Center for Theoretical Study, Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

We present a review to the recent works related to interpreting the exotic particle X(1835) reported by BES as a N\bar{N}-baryonium in the Skyrme model. There are two evidences that support this interpretation: 1) There exist a classical N\bar{N}-Skyrme solution with about ~10MeV binding energies in the model; 2) The decay of this Skyrme-baryonium is caused by annihilation of p - \bar{p} inside X(1835) through the quantum tunneling effect, and hence the most favorable decay channels are X → η4π or X → η′2π. These lead to reasonable interpreting the data of BES, and especially to useful prediction on the decay mode of X(1835) for the experiment.

Key words: Exotic particle; Proton-antiproton bound state; Skyrme model; p – \bar{p}-annihilation; X(1835)-decay.

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I. INTRODUCTION

By using a sample of 5.8 × 10^7 J/ψ events collected with the BES II detector, BES collaboration studied J/ψ → γp\bar{p} at 2003, and found out an anomalous enhancement near the mass threshold in the p\bar{p} invariant-mass spectrum from this decay process. This enhancement was fitted with a subthreshold S-wave Breit-Wigner (BW) resonance function with a mass M = 1859^{+13+5}_{-10-25} MeV/c^2, a width Γ < 30MeV/c^2 (at the 90% C.L.), and a product branching fraction (BF) B(J/ψ → γX) : B(X → p\bar{p}) = [7.0 ± 0.4(stat)_{-1.4}^{+1.9}(syst)] × 10^{-5}. These observations could be naively interpreted as signals for baryonium p\bar{p} bound state, and will be denoted as X(1835) hereafter. The BES-datum fit in Ref. represents the simplest interpretation of the experimental results as indication of a baryonium resonance. As an unstable particle, the decays of X(1835) must be caused by the proton-antiproton annihilation inside the bound state X(1835). This should be regarded as a significant feature for distinguishing X(1835)’s baryonium interpretation from other ones, say glueball, hybrid, or η’’s excitation etc. In Ref., the p\bar{p}-system has been studied by means of the Skyrmion model. And an attractive potential at middle distance scale range between p and \bar{p}, and a repulsive force at short distance of p\bar{p} has been revealed. This means X(1835) can indeed be understood as a baryonium in the Skyrmion framework. Thus, to discuss the p – \bar{p} annihilations inside X(1835) in Skyrme model is legitimate. In Ref., this issue has been investigated in detail by using coherent state method in the model following the Amado-Cannata-Dedonder-Locher-Li’s studies to the annihilations of p – \bar{p} scattering. In this way, we found that B(X → η4π) >> B(X → η2π), and then we argue B(X → η′2π) (with η′ → η2π) must be much bigger than B(X → η2π). This unusual prediction provides a criteria to identify whether X(1835) is a baryonium or not. Furthermore, to search the resonance of η4π or η′2π itself at the final state invariant mass 1800 ~ 1900MeV may reveal a full resonance peak for X-particle. Obviously, it is very significant to show the resonance via X-particle’s mesonic decays both because the enhancement near the mass threshold in the p\bar{p} invariant-mass spectrum from J/ψ → γp\bar{p} decays has only shown a tail effect of the resonance and because a narrow resonance of X → η2π has never been seen. Actually, there is no clear signal of narrow resonance of η2π at 1800 ~ 1900MeV corresponding to X-particle being seen in BES II was a serious obstacle to understand the existence of X-particle at one time, because the quantum number assessment in Refs means X → ηππ should be the simplest decay mode for X-particle with the largest phase space. In this case, then, according to the analysis of Ref., to search the mesonic resonance of η4π or η′2π with (η′ → η2π) was urged. Consequently, by searching these decay processes, this obstacle was finely gotten over by a very beautiful experiment of BES II. The full resonance peak of X(1835) has been revealed in the J/ψ → γπ^+π^- channel with a statistical significance of 7.7σ. The η’ meson was detected in both ηππ and γρ channels. There are roughly 264 ± 54 events. Its mass is M_x = (1833.7 ± 6.2 ± 2.7)MeV and its width is Γ(X(1835)) = (67.7 ± 20.3 ± 7.7)MeV. The mass and width of the X(1835) are not compatible with any known meson resonance. In addition, the S-wave BW fit to the p\bar{p} invariant-mass spectrum of Ref. was improved in Ref. by further considering the final state interaction effects in X → p\bar{p} decay. The redoing corrected the original results (M ∼ 1859MeV/c^2 with Γ < 30MeV/c^2) to be M = 1831 ± 7MeV/c^2 with Γ < 153MeV/c^2 (at the 90% C.L.), which is consistent with ones observed in Ref. from X → η′ππ. This

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†email:mlyan@ustc.edu.cn
is a strong evident to support that the $p\eta$-enhancement reported in Ref.\[1\] and the resonance of $\eta'/\pi\pi$ reported by Ref.\[12\] are due to same baryonium particle with $f^G(J^{PC}) = 0^+(0^+)$. Moreover, it has been revealed also that the relative $p\eta$ decay strength is quite strong: $B(X \rightarrow p\eta)/B(X \rightarrow \pi^+\pi^-\eta') \sim 1/3$. All of these support the interpretation of that $X$-particle is a baryonium. In this talk I try to review the works in Ref.\[1\],\[2\], in which $X(1835)$ has been studied as proton-antiproton bound state in Skyrme model.

II. NUCLEON-ANTINUCLEON BOUND STATE IN SKYRME MODEL

Skyrme’s old idea \[16\] that baryons are chiral solitons has been successful in describing the static nucleon properties \[17\] since Witten’s illustration that the soliton picture of baryons is consistent with QCD in the large $N_c$-QCD theory. The Skyrme model has been widely used to discuss baryons and baryonic-system properties. Deuteron is a typical nucleon-nucleon system with baryon number 2, i.e., winding number 2, which has been extensively studied in this model. Generally, there are two Skyrmion ansatzes being used to explore this neutron-proton bound state in the Skyrme model: the product ansatz proposed by A. Jackson, A.D. Jackson and V. Pasquier \[13\], and the instanton ansatz proposed by M.F. Atiyah and N.S. Manton \[20\] for this non-trivial Skyrmon configurations. However, for the deuteron-like system $p\bar{p}$, the winding number (i.e., the baryon number in the Skyrme model) is zero, therefore the topology of the corresponding Skyrmion configurations is trivial, and only the product ansatz works. In the follows, we employ the ungroomed product ansatz to investigate the $p\bar{p}$-system.

The Lagrangian for the SU(2) Skyrme model is

$$\mathcal{L} = \frac{1}{16} F_\pi^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr}([\partial_\mu U^\dagger, (\partial_\mu U) U^\dagger]^2) + \frac{1}{8} m_c^2 F_\pi^2 \text{Tr}(U - 1),$$

where $i, j = 1, 2, 3$. The right-currents $R_\mu$ are defined via

$$R_\mu = (\partial_\mu U) U^\dagger,$$

and we express the energy in the units defined above. In this picture, the binding energies for $S\bar{S}$ (which corresponds to the classical binding energies of $p\bar{p}$) are

$$\Delta E_B = 2m_c - M(\rho),$$

where $m_c = 867$ MeV is the mass of a classical nucleon (or classical Skyrmon). A stable or quasi-stable $p\bar{p}$-binding state corresponds to the Skyrmon configuration $U_s(r, \rho_B) = U_H(r + \frac{\rho_B}{2} \hat{z}) U_H^\dagger (r - \frac{\rho_B}{2} \hat{z})$ with $\Delta E_B(\rho_B) < 0$ and $\frac{d}{d\rho}(\Delta E_B(\rho))|_{\rho=\rho_B} = 0$.

The numerical result of the static energy as a function of $\rho$ is showed in Fig. 1. From it, we find that there is a quasi-stable $p\bar{p}$-binding state:

$$\rho_B \approx 2.5 \text{ fm},$$

$$\Delta E_B(\rho_B) \approx \text{ 10 MeV}.$$

Obviously, this is a deuteron-like molecule state with rather small binding energy, and hence its mass is rather close to the threshold in proton-anti-proton mass spectrum. In this case, as long as one could further prove that this quasi-stable molecule state could have indeed a decay width, say \~{}10 MeV, then one could naturally conclude that this molecule resonance did respond for the near-threshold enhancement in $(p\bar{p})$-mass spectrum from $J/\psi \rightarrow \gamma p\bar{p}$ observed by BES, and this deuteron-like
state should be the the particle reported in experiment of ref. 11. We shall do so in the next section.

Three remarks are in order:
1) To the $p\bar{p}$-bound state, the absolute baryon number for separate $(N - \overline{N})$ system is useful for understanding the inside structure of $X$-particle in the Skyrmion framework. By the Skyrme model, for $(N - \overline{N})$-system, $U(r, \rho) \equiv U_H(r + \frac{5}{2} \hat{z})U_H^\dagger(r - \frac{5}{2} \hat{z})$. The baryon density reads

$$\rho_B(r, \rho) = \frac{\epsilon_{ijk}}{24\pi^2} Tr[(U^\dagger(r, \rho)\partial_i U(r, \rho))(U^\dagger(r, \rho)\partial_j U(r, \rho)) \times (U^\dagger(r, \rho)\partial_k U(r, \rho))],$$

and the total baryon number of the system with any separation $\rho$ is zero, i.e.,

$$B(\rho) \equiv \int d^3 r \rho_B(r, \rho) = 0. \quad (13)$$

But obviously the absolute baryon number for separate $(N - \overline{N})$ is nonzero,

$$|B(\rho)| \equiv \int d^3 r |\rho_B(r, \rho)| \neq 0. \quad (14)$$

From the bottom panel of Fig.2, one can see when $N$ and $\overline{N}$ are separated away largely, saying $\rho > 1.5 fm$, $|B(\rho)| \approx 2$, and it is almost $\rho$-independent. When $\rho < 1.5 fm$, $|B(\rho)|$ decreases sharply, and when $\rho < 0.5 fm$ the "baryonic matters" in the system are almost annihilated away, i.e., $|B(\rho)| \to 0$. The curve of function $|B(\rho)|$ indicates that the configurations of $N\overline{N}$-Skyrmion with $\rho_{N\overline{N}} > 1.5 fm$ are molecular states of $N - \overline{N}$, and ones with $\rho_{N\overline{N}} < 0.5 fm$, they are mesonic solutions. The upper panel of Fig.2 shows the curve of the static energy of Skyrmion-baryoniums (the same as Fig.1). Obviously, since the distance between $p$ and $\overline{p}$ inside $X$ $\rho_B \approx 2.5 fm$ (see Eq. 10) is much larger than ~ 1.5 fm, and $|B(\rho_B)| \approx 2$, we conclude this baryonium configuration corresponding to the enhancement belongs to a molecular bound state consisted of proton and antiproton.

2) In above, the discussion on the binding energies is somehow qualitative. Only the classical soliton energies of $p\overline{p}$ have been taken into account. Even though they should be the leading order of large $N_c$ expansion, the semi-classical quantum correction to the $p\overline{p}$ energies may decrease the small binding energies so as to vanish. Similar problem occurred in the discussions to deuteron by using $\gamma_5$-groomed product skyrmion ansatz [12]. There were two methods to get over this difficulty for deuteron case: changing the ansatz [21], or improving the calculations of the classical soliton energies to include the contributions from the higher order of $N_c$-expansion (or higher order of derivative expansion) [21]. The Skyrmie model with 6-order derivative term has been explored in order to calculate the the quark spin contents in proton [22,23], and hence the model has been fixed. It should be interesting to pursue the the semi-classical quantum corrections to the $p\overline{p}$ in this generalized Skyme model. Namely, one could learn how to quantize the classical baryonium solutions without topology charge (see Eq. 10) semi-classically.

3) Eq. (10) is the SU(2)-Skyrmion Lagrangian. It can be extended into SU(3)-one when $U(x)$ becomes into $3 \times 3$ matrix function, the Wess-Zumino term is added, and the the SU(2) chiral symmetry breaking term in 10 changes to SU(3)-one [24,25]. The real world has three light flavors, and hence there are three flavor contents at least in the "sea" of SU(2)-nucleons [25]. The hyper-baryons with strange quantum number, of course, only emerge in the SU(3)-Skyrmion theory. The fact, however, that SU(2)-Skyrmion can rather rightly describe the properties of real nucleons indicates the three flavor "sea" effects of nucleons have been partly (at least) covered by SU(2)-Skyrmion. The ($p - \overline{p}$)-system discussed in this paper is flavor singlet and the baryon number free, and hence the
III. A PHENOMENOLOGICAL MODEL WITH A SKYRMION-TYPE POTENTIAL

In order to further catch the features of the Skyrmion prediction to $p\bar{p}$ and to derive the decay width of this quasi-stable particle, we employ the potential model induced from the skyrmion picture of $p\bar{p}$-interactions for the nucleons. For over fifty years there has been a general understanding of the nucleon-nucleon interaction as one in which there is, in potential model terms, a strong repulsive short distance core together with a longer range weaker attraction. The attractive potential at the middle range binds the neutron and the proton to form a deuteron. In comparison with the skyrmion result on the deuteron [24–29], we notice several remarkable features of the static energy $M(\rho)$ and the corresponding $(p\bar{p})$-potential $V(p\bar{p})$. Firstly, the potential is attractive at $\rho > 2.0$ fm, similar to the deuteron case. This is due to the reason that the interaction via pseudoscalar $\pi$-meson exchange is attractive for both quark-quark $qq$ and quark-antiquark $q\bar{q}$ pairs. Physically, the attractive force between $p$ and $\bar{p}$ should be stronger than that of $pn$. Therefore the fact that our result of the $p\bar{p}$-binding energy (see eq. (11)) $\Delta E_B(p\bar{p}) \approx 10$ MeV is larger than that of deuteron ($2.225$ MeV) is quite reasonable physically. Secondly, there is a static Skyrmion energy peak at $\rho \approx 1$ fm in Fig. 1. This means that the corresponding potential between $p$ and $\bar{p}$ is repulsive at that range. This is an unusual and also an essential feature. The possible explanation for it is that the skyrmions are extended objects, and there would emerge a repulsive force to counteract the deformations of their configuration shapes when they close to each other. Similar repulsive potential has also been found in previous numerical calculation [33]. Thirdly, a well potential at middle $\rho$-range is formed due to the competition between the repulsive and attractive potentials mentioned above, similar to the deuteron case. But the depth should be deeper than that in the deuteron case, as argued in a QCD based discussion [24]. The $p$ and $\bar{p}$ will be bound to form a baryonium in this well potential. Finally, the potential turns to decrease quickly from $\sim 2000$ MeV to zero when $\rho \to 0$. This means that there is a strong attractive force at $\rho \approx 0$. Physically, the skyrmions are destroyed at this $\rho$ range, and $p\bar{p}$ are annihilated.

The qualitative features of the proton-neutron potential for the deuteron can be well described by a simple phenomenological model of a square well potential [30, 31, 32] with a depth which is sufficient to bind the $pn^3S_1$ state with a binding energy of $-2.225$ MeV. Numerically, the potential width $a_{pn}$ is about 2.0 fm, and the depth is about $V_{pn} = 36.5$ MeV. Similarly, from the above illustrations on the features of the potential between $p$ and $\bar{p}$ based on the Skyrmion picture, we now construct a phenomenological potential model for the $p\bar{p}$ system, as shown in Fig. 3, and it will be called as skyrmion-type potential hereafter.

We take the width of the square well potential, denoted as $a_{p\bar{p}}$, as close to that of the deuteron, i.e., $a_{p\bar{p}} \sim a_{pn} \sim 2.0$ fm. According to QCD inspired considerations [23, 24, 33], the well potential between $q$ and $\bar{q}$ should be double (or more) attractive than the $qq$-case, i.e., the depth of the $p\bar{p}$ square well potential is $V_{p\bar{p}} \simeq 2V_{pn} = 73$ MeV. The width for the repulsive force revealed by the Skyrme model can be fitted by the decay width of the baryonium, and we take it to be $\lambda = 1/(2m_p) \sim 0.1$ fm, the Compton wave length of the bound state of $p\bar{p}$. The square barrier potential begins from $\rho \sim \lambda$, and the height of the potential barrier, which should be constrained by both the decay width and the binding energy of the baryonium, is taken as $2m_p + h$, where $h \sim m_p/4$. At $\rho \sim 0$, $V(p\bar{p})(\rho) \sim -c \delta(\rho)$ with a constant $c > 0$.

Analytically, the potential $V(\rho)$ is expressed as follows

$$V(\rho) = 2m_p - c\delta(\rho) + V_c(\rho), \quad (15)$$

where

$$V_c(\rho) = \begin{cases} h = m_p/4, & 0 < \rho < \lambda, \\ -V_{p\bar{p}} = -73 \text{ MeV}, & \lambda < \rho < a_{p\bar{p}}, \\ 0, & \rho > a_{p\bar{p}}. \end{cases} \quad (16)$$

With this potential, the Schrödinger equation for S-wave bound states is

$$-\frac{1}{2(m_p/2)} \frac{\partial^2}{\partial \rho^2} u(\rho) + [V(\rho) - E] u(\rho) = 0, \quad (17)$$

where $u(\rho) = \rho \psi(\rho)$ is the radial wave function, $m_p/2$ is the reduced mass. This equation can be solved analytically, and there are two bound states $u_1(\rho)$ and $u_2(\rho)$ (see Fig. 4 and Fig. 5): $u_1(\rho)$ with binding energy $E_1 < -V_{p\bar{p}} = -73$ MeV is due to $-c\delta(\rho)$-function potential mainly, and $u_2(\rho)$ with binding energy $E_2 > -73$ MeV is due to the attractive square well potential at middle range mainly. $u_1(\rho)$ is the vacuum state. And, clearly, $u_2(\rho)$ should correspond to a deuteron-like molecule state and it may be interpreted as the new $p\bar{p}$ state.
resonance reported by BES\textsuperscript{[1]}: $X(1835)$. It is also expected that corresponding binding energies $-E_2$ in the potential model provided in above $\Delta E_B(p_B)$ (see eq.\textsuperscript{[11]}) are all in agreement of the data within errors of BES \textsuperscript{[1]}. By fitting experimental data, we have

\begin{align}
E_1 &= -(2m_p - m_{n_0}) \simeq -976 \text{ MeV}, \\
E_2 &= -17.2 \text{ MeV}.
\end{align}

Considering its decay width will be derived soon (see eq.\textsuperscript{23}), we conclude that the near-threshold narrow enhancement in the $p\bar{p}$ invariant mass spectrum from $J/\psi \to \gamma p\bar{p}$ might be interpreted as a state of protonium in this potential model.

\[\begin{aligned}
\text{FIG. 4: The Wave Function } u_1(\rho).
\end{aligned}\]

In the skyrmion-type potential of $p\bar{p}$, there are two attractive potential wells: one is at $\rho \sim 0$ and the other is at middle scale, together with a potential barrier between them. At $\rho \sim 0$, the baryon- and anti-baryon annihilates. Naturally, we postulate that the bound states decay dominantly by annihilation and, therefore, we can derive the width of protonium state $u_2(\rho)$ by calculating the quantum tunnelling effect for $u_2(\rho)$ passing through the potential barrier. By WKB-approximation, the tunnelling coefficient (i.e., barrier penetrability) reads\textsuperscript{[22]}

\begin{align}
T_0 &= \exp \left[-2 \int_0^\lambda \sqrt{m_p(h - E_2)} \right] \\
&= \exp \left[-2\lambda \sqrt{m_p(h - E_2)} \right].
\end{align}

In the square well potential from $\lambda$ to $a_{p\bar{p}}$, the time-period $\theta$ of round trip for the particle is

\[\theta = \frac{2 [a_{p\bar{p}} - \lambda]}{v} = [a_{p\bar{p}} - \lambda] \sqrt{\frac{m_p}{V_{p\bar{p}} + E_2}}.\]

Thus, the state $u_2(r)$'s life-span is $\tau = \theta T_0^{-1}$, and hence the width of that state reads

\[\Gamma \equiv \frac{1}{\tau} = \frac{1}{a_{p\bar{p}} - \lambda} \sqrt{\frac{V_{p\bar{p}} + E_2}{m_p}} \exp \left[-2\lambda \sqrt{m_p(h - E_2)} \right].\]

Numerically, substituting $E_2 = -17.2 \text{ MeV}$, $a_{p\bar{p}} = 2.0 \text{ fm}$ into\textsuperscript{[22]}, we obtain the prediction of $\Gamma$:

\[\Gamma \simeq 15.5 \text{ MeV},\]

which is a reasonable number when comparing it with the experimental data\textsuperscript{[1]}. This result indicates also that the $(p\bar{p})$-collision times per second in $X$ are about

\[\nu = \frac{\Gamma}{h} \simeq 2.35 \times 10^{22} \text{ times/second}.\]

This reflects the possibility of $(p\bar{p})$-collisions inside $X$-particle in the sense of quantum theory. When that possibility $P = 1 = \nu \tau$, one reobtains the lifetime of $X$ as $\tau = 1/\nu = 1/\Gamma$. Noting since the binding energy $E_2$ is rather small (comparing with $2m_p$), the annihilations which cause $X(1835)$ to be unstable are almost at rest.

Two remarks are in order:

1) Because there are adjustable parameters ($c$, $h$, $\lambda$, $V_{p\bar{p}}$, $a_{p\bar{p}}$) in our potential model, it is no doubt to fit the renewed experimented data in Ref.\textsuperscript{[12]}.

2) The quantum numbers of $X$-particle corresponding to $u_2(\rho)$ are $I^G(J^{PC}) = 0^+(0^{-+})$, and hence the particle corresponding to $u_1(\rho)$ must be of $I^G(J^{PC}) = 0^+(0^{-+})$ also. Considering the range for nonzero $u_1(\rho)$ is of $\rho \sim 0$ (see Fig.4), and then $|B(\rho \sim 0)| \sim 0$ (see Fig. 2), the $u_1(\rho)$-particle must be a meson (a point-like particle) rather than a baryonim. Thus, the possible candidates are $\eta$ or $\eta'$. Moreover, the gluon contents for $u_1(\rho)$-particle, $u_2(\rho)$-particle $X(1835)$ and $(p\bar{p})$ should be same. Thus, only $\eta'$ is available, and then we conjectured in above that the particle corresponding to $u_1(\rho)$ is $\eta'$.\textsuperscript{[13]}

\[\begin{aligned}
\text{FIG. 5: The Wave Function } u_2(\rho).
\end{aligned}\]
IV. PROTON-ANTIPROTON ANNIHILATION INSIDE X(1835) AND ITS MESONIC DECAYS

In above, we have pointed out that $N\bar{N}$-annihilation causes the $p\bar{p}$ baryonium $X(1835)$ to be unstable. In order to find out what decay processes are of the most favorable channels, we now pursue the $N\bar{N}$ annihilations inside $X(1835)$ in the Skyrme model. As a nucleon model inspired by QCD, the Skyrme model has a very useful advantage in describing $N\bar{N}$ annihilation: this effective theory provides a self-contained dynamics that encompasses nonlinear processes such as meson production, baryon excitation, and annihilation. The Skyrme model requires no additional dynamical assumption, such as the ad hoc dynamical behavior of the color confinement wall that must be assumed in bag models. The studies of annihilation in the Skyrme model have suggested that annihilation proceeds very rapidly when the baryon and antibaryon collide and that the product of this rapid annihilation is a pion pulse or coherent pion wave. The annihilation of $N\bar{N}$ scattering process at rest has been investigated by Amado, Cannata, Dedonder, Locher, Shao and Lu (ACDLSL) 3 10 11 in the Skyrme model by using the coherent state method. When the proton and antiproton collided, they will be annihilated into mesons rapidly. $(p\bar{p})$-annihilation at rest but without considering the $P$- and $G$-parity has been investigated by using coherent state method in 3 10 11. In the follows, we introduce ACDLSL’s coherent state of Refs. 3 10 11 briefly, and then, the studies in Ref. 2 which lead to reveal the most favorable channels for $X$-decays.

A. coherent state for annihilation of $N\bar{N}$ scattering process at rest.

1) Coherent state is the eigenstate of annihilation operator, $a|\lambda\rangle = \lambda |\lambda\rangle$:

$$|\lambda\rangle = e^{-|\lambda|^2/2} e^{\alpha^* x} |0\rangle,$$

which is useful for the quantum optics, e.g., to describe the pulse from rapid radiations of photons in laser 34. To free quantum scalar field $\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}} (a_k e^{-ik\cdot x} + a_k^* e^{ik\cdot x})$, the normalized quantum state $|f\rangle$ defined by

$$|f\rangle = \exp \left[ -\frac{1}{2} \int d^3k |f(k)|^2 + \int d^3k f(k) a_k^* \right] |0\rangle,$$

which is the coherent state of $\phi(x)$, i.e.,

$$\phi^{(+)}(x)|f\rangle = \varphi(x)|f\rangle, \quad \text{with} \quad \varphi(x) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}} e^{-ik\cdot x} f(k).$$

2) The coherent state with fixed 4-momentum and isospin 3 10 11: The Fig.2 shows when the distance between Skyrmion and anti-Skyrmion $\rho$ less than $\sim 1\,fm$, $|B(\rho)|$ will decrease sharply. This means the pions radiated from the annihilation of $N\bar{N}$ form a purse. The processes are very rapid, and are similar to the photon radiated from the laser. Considering this feature of $N\bar{N}$-annihilation, a coherent state description to $N\bar{N}$ scattering annihilation has been suggested in Ref. 2, and a coherent state with fixed 4-momentum and isospin 3 10 11 has been formed by Amado, Cannata, Dedonder, Locher, Shao and Lu (ACDLSL) as follows

$$|K, I, I_z\rangle_{ACDLSL} = \int \frac{d^4x}{(2\pi)^{3/2}} \frac{d\Omega_\alpha}{4\pi} e^{i K \cdot x} |f, x, \hat{n}, 2\rangle Y_{I_z}^*(\hat{n})$$

where

$$|f, x, \hat{n}, 2\rangle = |e^{F(x, \hat{n})} - F(x, \hat{n}) - 1|0\rangle,$$

$$F(x, \hat{n}) = \int d^3 k e^{-i k \cdot x} f(k) a_k^* \hat{n},$$

$$|f(k)|^2 = \frac{C k^2}{(k^2 + \alpha^2)^2(\omega_k^2 + \gamma^2)^2\omega_k^2}$$

with $\alpha = \gamma = 2m_x$. Then the $\pi^0$-radiations from $|K, I, I_z\rangle_{ACDLSL}$ can be discussed by calculating the mean number of charged $\pi^\pm$ and $\pi^0$ ($\mu = \pm, 0)$:

$$\hat{N}_\mu = \text{ACDLSL}_{(K, I, I_z)} \int d^3 k a_{\mu, k}^* a_{\mu, k} |K, I, I_z\rangle_{ACDLSL}.$$

The function $|f(k)|^2$ in eq. 30 comes from the following considerations provided in Ref. 11. The $\pi$-field equation is:

$$\left( \nabla^2 - \frac{\partial^2}{\partial t^2} - \mu^2 \right) \Phi(r, t) = S(r, t).$$

Here $S(r, t)$ is the source of the pion field $\Phi$. Inspired by the Skyrmin calculations 33 38, a very simple spherically symmetric form of $S(r, t)$ has been suggested in Ref. 2 11 by Amado, Cannata, Dedonder, Locher and Shao (i.e., ACDLS-ansatz) as follows

$$S(r, t) = \text{ACDLS}_{(r, t)} = \left\{ \begin{array}{ll}
0, & \text{if } t < 0, \\
S_0 \frac{e^{-\gamma t}}{\sqrt{2\pi}} e^{-r^2/2}, & \text{if } t > 0.
\end{array} \right.$$ (32)

By using $f(k) = -i \sqrt{2\pi} S(k, \omega_k)/(2\omega_k)$ and

$$S(k, \omega_k) = \int \frac{d^3r dt}{(2\pi)^2} \exp(-ik \cdot r + i\omega_k t) S(r, t),$$

we get the eq. 30.

B. Coherent states with fixed $G$- and $P$-parities

We address that the coherent state $|K, I, I_z\rangle_{ACDLSL}$ is $G$- and $P$-parities free, therefore it can not be used to discuss the $X(1835)$-decays. In Ref. 2, $X(1835)$ has been treated as meson radiation source with $f^G(J^{PC}) =$
the light quark flavor symmetry breaking to Goldstone particle with

It is easy to check the following equations

C. $\pi$- and $\eta$-radiations from $|K, I, I_z\rangle_{GP}$:

The probability of $(N_\pi, \pi, N_\eta)$-radiations from $|K, I, I_z\rangle_{GP}$ is:

where $I(K)$ is the normalization factor:

where $I(K, m, n) = \sum_{m+n \geq 2; \text{m even, n odd}} \frac{16 I(K, m, n)}{m!n!} F(m, I)$

and $F(m, I) = \int \frac{d\Omega_n d\Omega_q}{4\pi} Y_{Iz}(\mathbf{n}^I) Y_{Iz}(\mathbf{n}^I) (|\mathbf{n}^I\rangle |\mathbf{n}^I\rangle^m)

Noting the branching fraction $B(X(1835) \rightarrow m\pi + n\eta) \propto P(m, n)$, the ratios of $B(X \rightarrow \eta 4\pi)/B(X \rightarrow \eta 2\pi)$, etc can be calculated:

These are desirous results. The process of $X \rightarrow \eta 4\pi$ is a very favorable channel, and the branching fraction of the simplest decay channel $X \rightarrow \eta 2\pi$ is suppressed by about 1 orders comparing with one of $X \rightarrow \eta 2\pi$.

D. An intuitive picture:

1) Naively, the number of valence quarks in $X(1835)$ (or $\eta(1450)$) is equal to the number of valence quarks in $(\eta\pi\pi)$.

2) However, the gluon content for $(\eta\pi\pi)$ and $(\eta\pi\pi\pi)$ are different. Skyrmion tell us the gluon mass-percentage in the proton (or antiproton) is larger than that for the $\pi$ or $\eta$.

3) This indicates that there are some "redundant gluons" that are left over in the process of $X \equiv (g\bar{g}) \rightarrow \eta 2\pi$.

Consequently, the process should be expressed as

$$X \equiv (g\bar{g}) \rightarrow \eta 2\pi G.$$
where $G$ represents the "redundant gluons". Going further
\[
X \equiv (\eta \bar{p}) \rightarrow \eta 2\pi G
\]
\[
\rightarrow \pi \pi
\]
(41)

Therefore, the most favorable $X$-particle decay process should be $X \rightarrow \eta 4\pi$, instead of $X \rightarrow \eta 2\pi$. Most likely, $G$ and $\eta$ could combine to form the meson $\eta'$, in which case the process (41) becomes
\[
X \rightarrow 2\pi (\eta G) = 2\pi \eta' \rightarrow \eta \pi \pi
\]
(42)

Eq. (42) is just $(X \rightarrow \eta 4\pi)$, where the factor of $\eta' \rightarrow \eta \pi \pi$ is the dominate channel to be considered (i.e., $B(\eta' \rightarrow \eta \pi \pi) \simeq 65\%$). In this view, the process of $(X \rightarrow \eta 2\pi)$ will be almost forbidden and the $(X \rightarrow \eta 4\pi)$ or $(X \rightarrow \eta' 2\pi)$ would be most favorable, i.e.,
\[
B(X \rightarrow \eta' 2\pi) \gg B(X \rightarrow \eta 2\pi).
\]
(43)

Since $m_{\eta'} \gg m_{\eta}$, this is a very unusual result. This result comes from that $X(1835)$ is a baryon-antibaryon bound state, and its decay is caused by $N - \bar{N}$ annihilations.

V. SUMMARY

In this talk, the main contents of Refs. [4, 5] have been revived with some commentary. The main conclusion of

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[1] J.Z. Bai et al. (BES Collaboration), Phys. Rev. Lett. 91 (2003) 022001.
[2] E. Fermi and C.N. Yang, Phys. Rev. 76, 1739 (1949).
[3] A. Datta and P.J. O’Donnel, Phys. Lett. B567, 273 (2003).
[4] Mu-Lin Yan, Si Li, Bin Wu, and Bo-Qiang Ma, Phys. Rev. D 72 (2005) 034027, hep-ph/0405087.
[5] Gui-Jun Ding and Mu-Lin Yan, Phys. Rev. C 72 (2005) 015208, hep-ph/0502127.
[6] Gui-Jun Ding, Jia-Lun Ping and Mu-Lin Yan, Baryon-Antibaryon Enhancements in Quark Models, hep-ph/0510013.
[7] Gui-Jun Ding, Rong-Gang Ping and Mu-Lin Yan, Production of $X(1835)$ as Baryonium with Sizable Gluon Content, hep-ph/0511186 to appear in Euo Phys Jour. A.
[8] Chong-Shou Gao and Shi-Lin Zhu, Commun. Theo. Phys. 42, 844 (2004).
[9] R.D. Amado, F. Cannata, J-P. Dedonder, M.P. Locher, and B. Shao, Phys. Rev. Lett. 72, 970 (1994).
[10] R.D. Amado, F. Cannata, J-P. Dedonder, M.P. Locher, and B. Shao, Phys. Rev. C50, 640-651, 1994, hep-ph/9403374.
[11] Yang Lu and R.D. Amado, Phys. Lett. B 357, 446 (1995); Phys. Rev. C 52, 2158 (1995).
[12] M. Ablikim et al. (BES Collaboration), Phys. Rev. Lett. 95 (2005) 262001, hep-ex/0508025.
[13] S. Eidelman et al. (Particle Data Group), Phys. Lett. B 952 (2004) 1.
[14] B.S. Zou and H.C. Chiang, Phys. Rev. D 69, 034004 (2004).
[15] A. Sibirtsev et al., Phys. Rev. D 71, 054010 (2005).
[16] T.H.R. Skyrme, Proc. Roy. Soc. A 260 (1961) 127.
[17] G.S. Adkins and C.R. Nappi, Nucl. Phys. B 233 (1984) 109.
[18] E. Witten, Nucl. Phys. B 223 (1983) 422 and 433.
[19] A. Jackson, A.D. Jackson and V. Pasquier, Nucl. Phys. A 432 (1985) 567.
[20] M.F. Atiyah and N.S. Manton, Phys. Lett. B222, 438 (1989); Commun. Math. Phys. 153, 422 (1993). bibitemmanohar.
[21] M. Lacombe B. Loiseau, R.Vinh Mau and W.N. Cottingham, hep-ph/0109319.
[22] Z.Y.Sun, K.Chen and M.L.Yan, Science in China A37 (1994)70.
[23] B.A.Li and M.L.Yan, Phys. Lett., B282(1992) 435.
[24] E. Guadagnini, Nucl. Phys., B236 (1984) 35; A.V. Manohar, Nucl. Phys., B248 (1984) 19; M. Chemtob,
[25] M.L. Yan, Phys. Rev. D35 (1987) 2803; G.W. Wu, M.L. Yan and K.F. Liu, Phys. Rev. D43 (1991) 185.

[26] E. Braaten and L. Carson, Phys. Rev. Lett. 56 (1985) 1897.

[27] A.J. Schramm, Phys. Rev. C37, (1988) 1799.

[28] M.F. Atiyah and N.S. Manton, Phys. Lett. B 222 (1989) 438; Commun. Math. Phys. 152 (1993) 391.

[29] R.A. Leese, N.S. Manton, and B.J. Schroers, Nucl. Phys. B 442 (1995) 228.

[30] S. Ma, Rev. Mod. Phys. 25 (1953) 853.

[31] L. Hulthen and M. Sugawara, Handbuch der Physik 39 (1959) 1.

[32] L.I. Schiff, Quantum Mechanics (McGraw-Hill Book Company, Inc., 1949).

[33] Y. Lu and R.D. Amado, Phys. Rev. C 54 (1996) 1566.

[34] K. Maltman and N. Isgur, Phys. Rev. D 29 (1984) 952.

[35] A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D 12 (1975) 147.

[36] R.J. Glauber, Phys. Rev., 130, 2529 (1963); ibid., 131, 2766 (1963). R.J. Glauber, B.S. Skagerstam, Coherent States, World Scientific Publishing Co., Singapore, 1985.

[37] H.M. Sommermann, R. Seki, S. Larson and S.E. Koonin, Phys. Rev. D45, 4303 (1992).

[38] B. Shao, N.R. Walet and R.D. Amado, Phys. Lett. B303, 1 (1993).