WHAT IS SUPERSYMMETRY AND HOW DO WE FIND IT?  

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Abstract
In these Lectures, we present a pedagogical introduction to weak scale supersymmetry phenomenology. A basic understanding of the Standard Model and of the ideas behind Grand Unification, but no prior knowledge of supersymmetry, is assumed. Topics covered include:

• What is supersymmetry and why do we bother with it?
• Working with a supersymmetric theory: A toy example
• Construction of supersymmetric Lagrangians
• The Minimal Supersymmetric Model
• The mSUGRA Model: A paradigm for SUSY phenomenology
• Decays of supersymmetric particles
• Production of supersymmetric particles at colliders
• Observational constraints on supersymmetry
• Supersymmetry searches at future colliders
• Constraining supersymmetry models at future colliders
• R-parity violation
• Gauge-mediated supersymmetry breaking

1 Why Is the TeV Scale Special?
The 1970’s witnessed the emergence of what has now become the Standard Model (SM) of particle physics. This is a non-Abelian gauge theory based on the gauge group \( SU(3)_C \times SU(2)_L \times U(1)_Y \). The left- and right-handed components of the matter fermions are assigned to different representations of the gauge group, thereby allowing a chiral structure for the weak interactions. It is further assumed that the gauge symmetry is spontaneously broken to the observed \( SU(3)_C \times U(1)_{em} \) symmetry by a single \( SU(2)_L \) doublet of spin zero fields that acquires a vacuum expectation value (VEV). The \( SU(2)_L \times U(1)_Y \) structure of...
electroweak interactions was strikingly confirmed with the discovery of the \( W^\pm \) and \( Z^0 \) bosons at CERN. During the last few years, the beautiful measurements of the properties of the \( Z^0 \) boson have allowed us to test electroweak theory at the \( 10^{-3} \) level. The QCD part of the SM has not been tested at the same level. Currently, QCD tests are mostly confined to the domain where the theory can be treated perturbatively. Unfortunately, this precludes the use of most of the experimental data on strong interactions; viz. the observed properties of hadrons, for QCD tests. In the future, lattice computations may change this state of affairs. It would indeed be extremely interesting if the lattice community could come up with an incisive test that could (in principle) unambiguously falsify QCD. Despite this, we should acknowledge that the SM has been spectacularly successful in accounting for a variety of experimental data spanning a vast range of energy.

Why then do we entertain the possibility of any physics beyond the SM? First, we do not really know that electroweak symmetry is broken by the VEV of a spin zero elementary field, let alone, that this electroweak symmetry breaking (EWSB) sector consists of just one \( SU(2)_L \) doublet as is assumed in the SM. Understanding the mechanism of EWSB is one of the most pressing questions of particle physics today. There are also aesthetic reasons to believe that the SM is not the complete story. It does not provide any explanation of particle masses or mixing patterns. The SM, therefore, contains a large number of arbitrary parameters. Moreover, one needs to make an \textit{ad hoc} choice of gauge group and particle multiplets. Also, the SM offers no explanation for the replication of generations. Finally, we should always keep in mind that the SM does not incorporate gravity.

Perhaps more to the point is a technical problem that arises in quantum field theories with elementary spin zero fields. Ignoring gravitational interactions (so that the vacuum energy is not relevant), the largest quantum corrections are to scalar masses: the radiative correction (\( \delta m_H \)) to the scalar boson mass diverges quadratically as the internal momentum in the loop becomes very large. This divergence is unphysical since our SM computation breaks down for loop momenta \( p^2 \sim \Lambda^2 \), where \( \Lambda \) denotes the energy scale at which the SM ceases to be an adequate description of nature. This breakdown could occur because

\( ^b \)Sometimes physicists have entertained the possibility that observed deviations between experiment and theoretical predictions point toward new physics. Frequently, the effects discussed are at the \( 2 - 3\sigma \) level. It should, however, be remembered that the chance of a large number of independent measurements deviating by \( \geq 2\sigma \) is significant; \textit{e.g.} the chance that ten independent measurements all yield agreement within \( 2\sigma \) is just 60%. Moreover, the assessment of theoretical and experimental errors is not straightforward. Here we conservatively assume that there is no significant deviation between experimental observations and SM predictions.
of form factor effects which become important at the scale \( \Lambda \), or because there are new degrees of freedom at this scale that are not part of the SM. For instance, \( \Lambda \sim M_{\text{GUT}} \) if the SM is embedded in a Grand Unified Theory (GUT) since the effects of GUT boson exchanges become important for \( p^2 \sim \Lambda^2 \). The scale \( \Lambda \) thus serves as a cut-off on loop integrals in the sense that effects not included in the SM become important above this scale, and serve to dynamically regulate the integral. In lowest order in perturbation theory, we would then write the physical scalar boson mass as,

\[
m^2_H = m^2_0 + \delta m^2_H \sim m^2_0 - g^2 \Lambda^2, 
\]

where \( m_0 \) is the bare Higgs boson mass parameter and \( g \) a dimensionless coupling constant. We will assume that all dimensionless constants and ratios are of \( O(1) \). From perturbative unitarity arguments, we believe that \( m_H \) is not larger than a few hundred GeV, so that if \( \Lambda \) is indeed as large as \( M_{\text{GUT}} \), the two terms on the right hand side of the equation, each of which is \( \sim 10^{30} \text{ GeV}^2 \), have to combine to yield an answer \( \leq 10^6 \text{ GeV}^2 \). While this possibility cannot be logically excluded, the incredible sensitivity of the theory to the input parameters is generally regarded as a shortcoming of field theories with elementary scalars.

If we turn this reasoning around, and require as a matter of principle that the theory should not require this incredible fine tuning of parameters, we would be led to conclude that

\[
\Lambda \lesssim 1000 \text{ GeV.} \tag{2}
\]

If this perturbative estimate is valid, we must conclude that new physics effects not included in the SM must manifest themselves in collisions of elementary particles at about the TeV energy scale. What form this New Physics will take is unknown. We do not even know whether it will be in the form of direct production of new particles or indication of structure (via form factors) for particles that we currently regard to be elementary. Possibilities that have been considered in the literature include technicolour, compositeness of leptons and quarks and supersymmetry. It is the last of these alternatives that forms the subject of these lectures.

It is important to note that even though we do not know what the New Physics might be, its scale has been fixed to be \( \sim 1 \text{ TeV} \). Along with the search for the Higgs boson, the only missing ingredient of the SM, the search for novel phenomena which are expected to occur at TeV energy is the primary reason for the construction of supercolliders such as the Large Hadron Collider (LHC) or a 0.5-2 TeV electron-positron collider. It is worth remarking that strong interactions in the EWSB sector could invalidate the perturbative argument that led to the bound
The search for effects of these new strong interactions at colliders poses a formidable experimental challenge.

Before closing this Section, we remark that the instability of the mass to radiative corrections is endemic to spin zero fields. Chiral symmetry and gauge symmetry, respectively protect fermions and gauge bosons from large radiative corrections to their masses. For instance, in quantum electrodynamics, the corrections to the electron mass is only logarithmically divergent, and hence, by dimensional analysis must have the form,

\[ \delta m \propto m \ln \Lambda, \]

since \( m \) is the only mass scale in the problem. Massless fermions, therefore, are protected from acquiring masses, a property that can be traced to the chiral symmetry of QED. Likewise, the photon remains massless due to the gauge symmetry. In a generic quantum field theory, however, there is no known symmetry that keeps scalars from acquiring large masses by radiative corrections without resorting to fine-tuning. There is, however, a special class of theories in which this is not necessary. The price paid is that for every known particle, one has to introduce a new partner with spin differing by \( \frac{1}{2} \). The properties of the known particles and their new partners are related by a symmetry. This symmetry is unlike any known symmetry in that it inter-relates properties of bosons and fermions. Such a symmetry is known as a supersymmetry (SUSY).

These supersymmetric partners constitute the new physics that we alluded to above. It should now be clear that if SUSY is to ameliorate the fine tuning problem, supersymmetric partners should be lighter than \( \sim 1 \) TeV, so that they can be searched for at supercolliders.

2 An Introduction to Supersymmetry

In order to describe what supersymmetric particles would look like in experiments at high energy colliders, we have to understand how they might be produced, and, if they are unstable, into what these particles decay. In other words, we have to understand their interactions. We should mention at the outset that as yet no compelling model has emerged (primarily because of our ignorance of physics at high energy scales). Nonetheless, there is a useful (albeit cumbersome) parametrization of the effective theory that can be used for phenomenological analyses. Before delving into the complications of constructing realistic SUSY field theories, we will illustrate the essential ideas of supersymmetry using a simple example first written down by Wess and Zumino.
Consider a field theory with the Lagrangian given by,
\[
\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}},
\]
where,
\[
\mathcal{L}_{\text{kin}} = \frac{1}{2} (\partial_{\mu} A)^2 + \frac{1}{2} (\partial_{\mu} B)^2 + \frac{i}{2} \bar{\psi} \partial^\mu \psi + \frac{1}{2} (F^2 + G^2),
\]
and
\[
\mathcal{L}_{\text{mass}} = -m \left[ \frac{1}{2} \bar{\psi} \psi - GA - FB \right].
\]
Here, \(A, B, F\) and \(G\) are real scalar fields, and \(\psi\) is a self-conjugate or Majorana spinor field satisfying,
\[
\psi = C \tilde{\psi}^T.
\]
where the charge conjugation matrix \(C\) satisfies
\[
C \gamma_\mu C^{-1} = -\gamma_\mu, \quad (5a)
\]
\[
C^T = C^{-1} = -C, \quad (5b)
\]
and
\[
[C, \gamma_5] = 0. \quad (5c)
\]
Notice that (4) is a constraint equation that tells us that only two of the four components of \(\psi\) are independent. This can be easily seen by projecting out the right-handed component in (4) to get
\[
\psi_R = C \gamma_0 \psi_L.
\]
Bilinears of Majorana spinors also have very special properties. For instance,
\[
\bar{\psi} \chi = \psi^T C \chi = \psi_\alpha C_{\alpha \beta} \chi_\beta = -\chi_\beta (-C_{\beta \alpha}) \psi_\alpha = \chi^T C \psi = \bar{\chi} \psi, \quad (7a)
\]
where the first minus sign in the fourth step is due to the anticommutativity of the spinor fields and the second one due to the antisymmetry (5b) of the matrix \(C\). Similarly, one can show that
\[
\bar{\psi} \gamma_5 \chi = \bar{\chi} \gamma_5 \psi, \quad (7b)
\]
\[
\bar{\psi} \gamma_\mu \chi = -\bar{\chi} \gamma_\mu \psi, \quad (7c)
\]
\[
\bar{\psi} \gamma_\mu \gamma_5 \chi = \bar{\chi} \gamma_\mu \gamma_5 \psi, \quad (7d)
\]
\[
\bar{\psi} \sigma_{\mu \nu} \chi = -\bar{\chi} \sigma_{\mu \nu} \psi. \quad (7e)
\]
Wess and Zumino observed that under the transformations,

\[ \begin{align*}
\delta A &= i\bar{\alpha}\gamma_5\psi, \\
\delta B &= -\bar{\alpha}\psi, \\
\delta \psi &= -F\alpha + iG\gamma_5\alpha + \partial\gamma_5 A\alpha + i\partial B\alpha, \\
\delta F &= i\bar{\alpha}\partial\psi, \\
\delta G &= \bar{\alpha}\gamma_5\partial\psi
\end{align*} \]  

the Lagrangian density (3) changes by a total derivative. The action then changes by just a surface term, and the equations of motion remain unchanged. Before verifying this, we note that the transformations (8) mix boson and fermion fields; i.e. the invariance of the equations of motion is the result of a supersymmetry. The parameter of the transformation \(\alpha\) is thus spinorial. Furthermore, to preserve the reality of the bosonic fields \(A, B, F\) and \(G\), as well as the Majorana nature of \(\psi, \alpha\) it itself must satisfy the Majorana property (4). To verify that (3) indeed changes by a total derivative under the transformations (8), we note that

\[ \begin{align*}
\frac{1}{2} \delta [ (\partial_\mu A)^2 ] &= (\partial^\mu A)\partial_\mu \delta A = i\partial^\mu A\bar{\alpha}\gamma_5 \partial_\mu \psi, \\
\frac{1}{2} \delta [ (\partial_\mu B)^2 ] &= -\partial^\mu B\bar{\alpha}\partial_\mu \psi, \\
\frac{i}{2} \delta [ \bar{\psi}\partial\psi ] &= \frac{i}{2} [ \delta \bar{\psi}\partial\psi + \bar{\psi}\partial\delta \psi ] \\
&= \frac{i}{2} \partial_\mu [ \delta \bar{\psi}\gamma_\mu \psi ] - \frac{i}{2} (\partial_\mu \delta \bar{\psi})\gamma_\mu \psi + \frac{i}{2} \bar{\psi}\partial\delta \psi \\
&= \frac{i}{2} \partial_\mu [ \delta \bar{\psi}\gamma_\mu \psi ] + i\bar{\psi}\partial\delta \psi,
\end{align*} \]

where in the last step we have used (7c) for the Majorana spinors \(\psi\) and \(\delta \psi\). Continuing, we have

\[ \begin{align*}
\frac{1}{2} \delta (F^2) &= iF\bar{\alpha}\partial\psi, \\
\frac{1}{2} \delta (G^2) &= G\bar{\alpha}\gamma_5\partial\psi.
\end{align*} \]

We thus find that apart from a total derivative,

\[ \delta \mathcal{L}_{\text{kin}} = -i\Box A\bar{\alpha}\gamma_5\psi + \Box B\bar{\alpha}\psi \\
+ i\bar{\psi}[-\partial F\alpha + i\partial G\gamma_5\alpha + \partial A\gamma_5\alpha + i\partial B\alpha] \\
+ iF\bar{\alpha}\partial\psi + G\bar{\alpha}\gamma_5\partial\psi. \]
Using (7a) and (7b), we see that the terms involving $\Box A$ and $\Box B$ exactly cancel, leaving the remainder which, using (7c) and (7d), can be written as a total derivative. It will be left as an exercise for the reader to verify that $\delta L_{mass}$ is also a total derivative.

In order to further understand the supersymmetric transformations, we consider the effect of two successive SUSY transformations with parameters, $\alpha_1$ and $\alpha_2$. Starting from (8a) followed by (8b) and judiciously using (7), it is simple to show that,

\[
(\delta_2 \delta_1 - \delta_1 \delta_2)A = 2i\bar{\alpha}_2 \gamma^\mu \alpha_1 \partial_\mu A,
\]

In order to find the algebra satisfied by the Majorana spinor supersymmetry generators $Q$, we write $\delta \equiv i\bar{\alpha}Q$, and find from (10) that

\[-(\bar{\alpha}_2 Q \bar{\alpha}_1 Q - \bar{\alpha}_1 Q \bar{\alpha}_2 Q)A = -\bar{\alpha}_{2b} \alpha_{1a} (Q_b \bar{Q}_a + \bar{Q}_a Q_b) A = 2i\bar{\alpha}_{2b} \alpha_{1a} (\theta A)_{ba},\]

where $a$ and $b$ are spinor indices. In the first step we have used (7a) together with the fact that the parameters $\alpha_{ia}$ anticommute amongst themselves and also with the components $Q_a$ of the SUSY generators. We will leave it to the reader to verify that the same relation holds for successive action of SUSY transformations on the fields $B$, $\psi$, $F$ and $G$. We can thus write,

\[
\{Q_a, \bar{Q}_b\} = -2(\gamma_\mu P^\mu)_{ab}
\]

where $P^\mu$ is the translation generator of the Poincaré group, and the curly brackets denote the anti-commutator. The presence of the translation generator in (11) shows that supersymmetry is a spacetime symmetry. Conservation of supersymmetry implies

\[
[Q_a, P^b] = 0,
\]

or, from Lorentz covariance,

\[
[Q_a, P^\mu] = 0.
\]

The commutators of $Q$ with the Lorentz group generators $J_{\mu\nu}$ are fixed because we have already declared $Q$ to be a spin $\frac{1}{2}$ Majorana spinor.

The supersymmetry algebra described above is not a Lie Algebra since it includes anti-commutators. Such algebras are referred to as Graded Lie Algebras. Haag, Lopuzanski and Sohnius\textsuperscript{4} have shown that (except for the possibility of neutral elements and of more than one spinorial charge $Q$\textsuperscript{20}) the algebra that we have obtained above is the most general graded Lie Algebra consistent with rather reasonable

\textsuperscript{4}For the bosonic fields, the required steps are identical to the ones above; the verification with $\psi$ involves judicious use of Fierz rearrangement and the relations (7).
physical assumptions. Models with more than one SUSY charge in the low energy theory do not lead to chiral fermions and so are excluded for phenomenological reasons. We will henceforth assume that there is just a single super-charge.

We immediately note that (12a) implies all states but the zero energy ground state (the vacuum) come in degenerate pairs, with one member of the pair being a boson and the other a fermion. Thus, in any supersymmetric theory, every particle has a partner with the same mass but with a spin differing by $\frac{1}{2}$ (since $Q$ carries $\frac{1}{2}$ unit of spin). A study of how the partners of the known SM particles would manifest themselves in experiments at colliders forms the main subject of these Lectures. But continuing our study of the basics, we note that SUSY acts independently of any internal symmetry. In other words, the generators of supersymmetry commute with all internal symmetry generators. We immediately conclude that any particle and its superpartner have identical internal quantum numbers such as electric charge, isospin, colour, etc.

In order to see how supersymmetry is realized in the model defined by (3), we note that the fields $F$ and $G$ are not dynamically independent as the Lagrangian has no kinetic terms for these fields (which, therefore, do not propagate). The Euler-Lagrange equations of these fields are,

$$F = -mB, \quad G = -mA.$$  \hfill (13)

If we substitute these back into the Lagrangian (3), we obtain,

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}A)^2 + \frac{1}{2}(\partial_{\mu}B)^2 + \frac{i}{2}\bar{\psi}\gamma^\mu\psi - \frac{1}{2}m^2(A^2 + B^2) - \frac{1}{2}\bar{\psi}\psi.$$  \hfill (14)

This Lagrangian describes a non-interacting theory and, as such, is not terribly interesting. Notice, however, that there are two real scalar fields $A$ and $B$ and one spin $\frac{1}{2}$ Majorana fermion field, all with mass $m$. We thus see that the number of bosonic degrees of freedom (two) matches the fermionic degrees of freedom (recall that (6) shows that just two of the four components of $\psi$ are dynamically independent) at each space-time point.

In order to make the model more interesting, we include an interaction term given by

$$\mathcal{L}_{\text{int}} = -\frac{g}{\sqrt{2}}A\bar{\psi}\psi + \frac{ig}{\sqrt{2}}B\bar{\psi}\gamma_5\psi + \frac{g}{\sqrt{2}}(A^2 - B^2)G + g\sqrt{2}ABF,$$  \hfill (15)

to the Lagrangian (3). The courageous reader can verify that $\mathcal{L}_{\text{int}}$ is invariant up to a total derivative under the transformations (8). Once again we can eliminate the auxiliary fields $F$ and $G$ via their Euler-Lagrange equations which get modified to,

$$F = -mB - g\sqrt{2}AB$$
$$G = -mA - \frac{g}{\sqrt{2}}(A^2 - B^2),$$

8
and obtain the total Lagrangian in terms of the dynamical fields as,

$$\mathcal{L} = \frac{1}{2} (\partial_\mu A)^2 + \frac{1}{2} (\partial_\mu B)^2 + \frac{i}{2} \bar{\psi} \gamma^\mu \psi - \frac{1}{2} m^2 (A^2 + B^2) - \frac{1}{2} m \bar{\psi} \psi$$

$$- \frac{g}{\sqrt{2}} A \bar{\psi} \psi + \frac{ig}{\sqrt{2}} B \bar{\psi} \gamma_\mu \psi - gm \sqrt{2} AB^2 - \frac{gm}{\sqrt{2}} A (A^2 - B^2)$$

$$- g^2 A^2 B^2 - \frac{1}{4} g^2 (A^2 - B^2)^2.$$  \hspace{1cm} (16)

Several features of the Lagrangian in (16) are worth stressing.

1. It describes the interaction of two real spin zero fields and a Majorana field with spin half. As before, the number of bosonic and fermionic degrees of freedom match.

2. There is a single mass parameter $m$ common to all the fields.

3. Although the interaction structure of the model is very rich and includes scalar and pseudoscalar interactions of the fermion as well as a variety of trilinear and quartic scalar interactions, there is just one single coupling constant $g$. We thus see that supersymmetry is like other familiar symmetries in that it relates the various interactions as well as masses. The mass and coupling constant relationships inherent in (16) are completely analogous to the familiar (approximate) equality of neutron and proton masses or the relationships between their interactions with the various pions implied by (approximate) isospin invariance.

2.2 How Supersymmetry Removes Quadratic Divergences

We have already mentioned that the existence of supersymmetric partners serves to remove the quadratic divergences that destabilize the scalar sector of a generic field theory. We will illustrate this cancellation
Consider the corrections to the “one point function” of the field \( A \) to first order in the coupling constant \( g \) in (16). These corrections, which are represented by tadpole diagrams shown in Fig. 1, come from trilinear couplings in the second line of the Lagrangian (16). A simple computation gives,

\[
\langle 0 | \mathcal{L}_{\text{int}} | A \rangle \sim \frac{g}{\sqrt{2}} \left\{ Tr \int \frac{d^4p}{p^2} \frac{d^4\psi}{p^2 - m_\psi} - m \int \frac{d^4p}{p^2 - m_B} - 3m \int \frac{d^4\psi}{p^2 - m_A} \right\} = \frac{g}{\sqrt{2}} \left\{ \int \frac{d^4p}{p^2} \frac{d^4\psi}{p^2 - m_\psi} 4m_\psi - m \int \frac{d^4p}{p^2 - m_B} - 3m \int \frac{d^4\psi}{p^2 - m_A} \right\},
\]

(17)

The factor 3 in the last term arises since any one of the three fields in the \( A^3 \) interactions could annihilate the external particle. Here, we have deliberately denoted the masses that enter via the propagators by \( m_A \), \( m_B \) and \( m_\psi \) although these are exactly the same as the mass parameter \( m \) that enters via the trilinear scalar couplings in Eq. (16). We first see that because all these masses are exactly equal in a supersymmetric theory, the three contributions in (17) add to zero. Thus although each diagram is separately quadratically divergent, the divergence from the fermion loop exactly cancels the sum of divergences from the boson loops. Two remarks are in order.

1. In order for this cancellation to occur, it is crucial that the \( A^3 \), \( AB^2 \) and \( A\bar{\psi}\psi \) couplings be exactly as given in (16).

2. The \textit{quadratic} divergence in the expression (17) is independent of the scalar masses, \( m_A \) and \( m_B \). It is, however, crucial that the fermion mass \( m_\psi \) is exactly equal to the mass \( m \) that enters via the trilinear scalar interactions in order for the cancellation of the quadratic divergence to be maintained. If the boson masses differ from the fermion mass \( m_\psi \), the expression in (17) is at most logarithmically divergent. As we have discussed, logarithmic divergences do not severely destabilize scalar masses.

It is also instructive to inspect the lowest order quadratic divergences in the two-point function of \( A \). The one loop contributions to the quadratic
divergences are shown in Fig. 2. It is left as an exercise for the reader to check that while each one of the diagrams in Fig. 2 is individually quadratically divergent, this divergence cancels when the three diagrams are summed. Once again, the contributions from the fermion loop cancel those from the boson loops. Moreover, this cancellation occurs for all values of particle masses. This is because trilinear scalar interactions do not contribute to the quadratic divergence that we have just computed. It is, however, crucial that the fermion Yukawa coupling \( \frac{\sqrt{2}}{2} \) is related to the quartic scalar couplings on the last line of (16).

2.3 Soft Supersymmetry Breaking

The fact that the quadratic divergences continue to cancel even if the scalar boson masses are not exactly equal to fermion masses (as implied by SUSY) is absolutely critical for the construction of phenomenologically viable models. We know from observation that SUSY cannot be an exact symmetry of nature. Otherwise, there would have to exist a spin zero or spin one particle with exactly the mass and charge of an electron. Such a particle could not have evaded experimental detection. The only way out of this conundrum (if we are to continue with these Lectures) is to admit that supersymmetric partners cannot be degenerate with the usual particles. Thus, supersymmetry must be a broken symmetry.

Does this mess up our solution to the fine-tuning problem that got us interested in SUSY in the first instance? Fortunately, it does not. We have just seen (by the two examples above) that if SUSY is explicitly broken because scalar masses differ from their fermion counterparts, no new quadratic divergences occur. We will state without proof that this is true for all processes, and to all orders in perturbation theory. It is, therefore, possible to introduce new terms such as independent additional masses for the scalars which break SUSY without the reappearance of quadratic divergences. Such terms are said to break SUSY softly. Not all SUSY breaking terms are soft. We have already seen that if \( m_\psi \neq m \), the expression in (17) is quadratically divergent. Thus additional contributions to the fermion mass in the Wess-Zumino model results in a hard breaking of supersymmetry. Similarly, any additional contribution to just the quartic scalar interactions will result in the reappearance of a quadratic divergence in the correction to \( m_A^2 \).

Are there other soft SUSY breaking terms in the toy theory that we have been considering? Recall that the combinatorial factor 3 in the last term in (17). This tells us that the contribution of the \( A \) loop from the trilinear \( A^3 \) interaction is exactly three times bigger than the

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\*There are additional quadratic divergences in the two point function from the tadpoles of Fig. 1, which, as we have just seen, separately cancel.
contribution from the $B$ loop from the $AB^2$ interaction (the coupling constants for these interactions are exactly equal). Thus, there will be no net quadratic divergence in the expression (17) even if we add a term of the form,

$$L_{\text{soft}} = k(A^3 - 3AB^2)$$

to our model, where $k$ is a dimensional coupling constant. Obviously, this interaction does not give a quadratically divergent correction to the one loop contribution to $m_A^2$. It is an example of a soft supersymmetry breaking interaction term. We remark that this term can be written in terms of $S = \frac{A+iB}{\sqrt{2}}$ as

$$L_{\text{soft}} = \sqrt{2}k(S^3 + \text{h.c.})$$

(18a)

while an arbitrary splitting in the masses of $A$ and $B$ can be incorporated by including a term,

$$L_{\text{soft}} = m^2(S^2 + \text{h.c.})$$

(18b)

into the Lagrangian. It will turn out that super-renormalizable terms that are analytic in $S$ are soft while terms that involve products of $S$ and $S^*$ (except supersymmetric terms such as $S^*S$ already present in (16)) result in a hard breaking of SUSY. The reader can, for instance, easily check that an interaction proportional to $(S^2S^* + \text{h.c.}) = 2(A^2 + B^2)A$ leads to a quadratically divergent contribution to the expression in (17).

Although we have illustrated the cancellation of quadratic divergences with just a couple of examples, it is important to stress that this is a general feature of supersymmetric theories. The reader is also urged to verify that the quadratic divergence cancels in the one loop tadpole and mass corrections to the $B$ field. Furthermore, we have already noted that this cancellation of quadratic divergences is true to all orders in perturbation theory. The SUSY resolution of the fine-tuning issue rests upon this important property of supersymmetric models.

3 Construction of Supersymmetric Lagrangians

3.1 Non-Gauge Theory

The fields in the model we have been considering can be re-written in terms of

$$S = \frac{1}{\sqrt{2}}(A + iB)$$

$$\psi$$

$$F = \frac{1}{\sqrt{2}}(F + iG)$$

12
where $S$, $\psi$ and $F$ transform into one another under the SUSY transformations (8) which can be re-written as:

\[
\begin{align*}
\delta S &= -\sqrt{2}\bar{\alpha}\psi_L, \quad (20a) \\
\delta \psi_L &= -\sqrt{2}F\alpha_L + \sqrt{2}\bar{\theta}S\alpha_R, \quad (20b) \\
\delta F &= \sqrt{2}\bar{\alpha}\partial_\psi L \quad (20c)
\end{align*}
\]

Thus $(S, \psi_L$ and $F$) together constitute an irreducible supermultiplet in exactly the same way that the proton and neutron form a doublet of isospin. Further, analogous to the isospin formalism that treats the nucleon doublet as a single entity, there is a formalism known as the superfield formalism\textsuperscript{21} that combines all three components of the supermultiplet into a superfield $\hat{S}$. Since only one chiral component of the Majorana spinor $\psi$ enters the transformations, such superfields are referred to as (left) chiral superfields. Further, because the lowest spin component of the multiplet has spin zero, this superfield is known as a left chiral scalar superfield. It is easy to check that the Hermitean conjugate of a left chiral superfield is a right chiral superfield. There are, of course, other irreducible multiplets of supersymmetry just as there are other representations of isospin symmetry.

The superfield formalism\textsuperscript{21} is the most convenient way of discussing how to write supersymmetric Lagrangians. As we do not have time to discuss it during these Lectures, we will content ourselves by stating clearly (but without proof) those features that will be useful to us.

1. There is a multiplication rule $\hat{S} = \hat{S}_1\hat{S}_2$ which allows us to compute the components of the “product superfield” in terms of the components of $\hat{S}_1$ and $\hat{S}_2$, and

2. The product of two (and hence, several) left (right) chiral superfields is itself a left (right) chiral superfield, but the product of a left chiral superfield and a right chiral superfield is neither a left nor a right chiral superfield.

The strategy for the construction of supersymmetric Lagrangians is straightforward once we observe from (8d,e), or equivalently from (20c) that the $F$ component of a left-chiral superfield changes by a total derivative under a SUSY transformation. By (2) above, since any product of left-chiral superfields is itself a left-chiral superfield, any analytic function $f(\hat{S}_1, \hat{S}_2, ..., \hat{S}_N)$ of left chiral superfields is a composite left chiral superfield. The $F$ component of this composite superfield is thus a function of the component fields in $\hat{S}_1, \hat{S}_2, ..., \hat{S}_N$ which changes by a total derivative under supersymmetry transformations. This function, therefore, has exactly the properties that we want from a supersymmetric

\textsuperscript{e}Since $\psi_R$ is not independent of $\psi_L$, we only have to specify how $\psi_L$ transforms.
Lagrangian density. It is a function of various fields $S_i, \psi_i$ and $\mathcal{F}_i$ that remains invariant up to a total derivative under SUSY, and is thus a candidate for our Lagrangian. The function $f$ is referred to as the superpotential. The reason that it has to be analytic is that if it involves both $\hat{S}_i$ and $\hat{S}_i^*$, it will no longer be a chiral superfield and its $\mathcal{F}$-component will no longer be a SUSY invariant.

The Lagrangian density, i.e. the $\mathcal{F}$-component of the superpotential, can be readily computed using the rules for superfield multiplication. The computation is somewhat tedious. It is a function of the component fields $S_i, \psi_i$ and $\mathcal{F}_i$ of the superfields $\hat{S}_i$ that appear in the superpotential. The auxiliary fields $\mathcal{F}_i$ can be eliminated using the algebraic constraints (analogous to the equation below (15)) from their Euler-Lagrange equations. The resulting Lagrangian takes the form,

$$L = -\sum_i \left| \frac{\partial f}{\partial \hat{S}_i} \right|^2 \hat{S}_i^2 = \sum_{i,j} \left\{ \bar{\psi}_i \left[ 1 - \gamma_5 \right] \frac{1}{2} \left( \frac{\partial^2 f}{\partial \hat{S}_i \partial \hat{S}_j} \right)_{\hat{S}_i = S_i} \psi_j + h.c. \right\}$$

The terms involving derivatives of the superpotential are functions of just the scalar fields $S_i$ since in the expression we set $\hat{S}_i = S_i$ after differentiation. The first term in (21) is the scalar potential while the second term describes the interaction of the scalars with the fermions. Notice that the bilinear terms in the superpotential become mass terms for both the scalars and the fermions.

It is apparent from the first term in (21) that a term of degree $n$ in the superpotential leads to a Lagrangian density with mass dimension $d = 2(n - 1)$. For the theory to be power-counting renormalizable, we must have $2(n - 1) \leq 4$, and the superpotential at most cubic in the superfields.

Before proceeding further, let us illustrate the use of (21) by a simple example where the superpotential is a function of just one superfield. Choose

$$f = \frac{1}{2}m\hat{S}^2 + \frac{1}{3}g\hat{S}^3.$$  

Then, using (21) it is easy to see that

$$L = -\left| m\hat{S} + g\hat{S}^2 \right|^2 - \frac{1}{2} \left\{ \bar{\psi} \left[ 1 - \gamma_5 \right] \left( m + 2gS \right) \psi + h.c. \right\},$$

which, using (19) reduces to the Lagrangian (16) except that the kinetic energy terms are missing.
These have their origin in a different source. For our purposes it is sufficient to recall that we saw that they were separately supersymmetric, so that the Lagrangian in (21) only needs to be supplemented \(^\text{\textsuperscript{\textdagger}}\) by,

\[
L_{\text{kin}} = \sum_i (\partial_{\mu} S_i)^\dagger (\partial^\mu S_i) + \frac{i}{2} \sum_i \bar{\psi}_i \gamma^\mu \psi_i, \tag{23}
\]

which are the canonically normalized kinetic energies for complex scalar and Majorana fermion fields. The Lagrangian given by the sum of (21) and (23) is the most general globally supersymmetric Lagrangian for non-gauge theories. We now turn to the corresponding formula for the Lagrangian in gauge theory.

3.2 The Lagrangian for Supersymmetric Gauge Theories

In order to write down a locally gauge invariant supersymmetric Lagrangian, we have to introduce a gauge covariant derivative. As in the usual Yang-Mills construction of gauge theories, this is done by introducing a set of massless vector fields which, under gauge transformations, transform as the adjoint representation of the gauge group. In a supersymmetric theory, this cannot be done without also including some additional fermions to match the gauge bosons that we had to introduce. We have to introduce a complete supermultiplet of gauge potentials. However, this supermultiplet differs from the multiplet (19) of the Wess Zumino model, in that the gauge field (unlike the scalar field \(S\) in the chiral supermultiplet) is real. As a result, the gauge supermultiplet is neither a left nor a right chiral superfield. Although we have not shown this in these Lectures, it can be demonstrated that all but three components of the gauge supermultiplet can be chosen to be zero. \(^\text{\textsuperscript{\textdaggerdbl}}\) This choice of the supermultiplet is known as the Wess–Zumino gauge in the literature. In this gauge, the gauge supermultiplet consists of \((V_\mu, \lambda, D)\), where \(V_\mu\) is the usual Yang–Mills gauge potential, \(\lambda\) is a Majorana spinor field, and \(D\), like the field \(F\) in (19) is an auxiliary non–propagating field that can be algebraically eliminated via its Euler–Lagrange equations.

\(^\text{\textsuperscript{\textdagger}}\)We are oversimplifying at this point. The Lagrangian for the kinetic terms as well as the one in (21) involves the auxiliary fields \(F\) (see Eq. (3), as an example of this). It is only after the auxiliary fields are eliminated that we end up with a sum of (21) and (23).

\(^\text{\textsuperscript{\textdaggerdbl}}\)In general, a real superfield has more than three non-vanishing components. In a SUSY gauge theory, however, the gauge parameter itself can be chosen as the scalar component of a chiral superfield. By a judicious choice of this gauge-parameter superfield, all but three of the components of the original real superfield can be gauged away. This choice is not supersymmetric, and vanishing components are resurrected by a SUSY transformation. A combination of a SUSY and a gauge transformation leaves the form of the gauge field unaltered.
Notice that once again the number of dynamical bosonic degrees of freedom (two for the gauge field) matches the number for the dynamical fermionic degrees of freedom (two for the Majorana fermion $\lambda$), in agreement with our general considerations. This new fermion, called the gaugino, is the supersymmetric partner of the gauge boson and so, under gauge transformations, transforms as a member of the adjoint representation of the gauge group.

As before, we will content ourselves by presenting a general formula for the couplings of “matter” particles and their superpartners to gauge bosons and gauginos in a globally supersymmetric gauge theory. Matter particles belong to chiral supermultiplets such as (19), while the gauge bosons and their gaugino partners reside in the gauge multiplet that we have just introduced. After elimination of the auxiliary fields $F_i$ and $D_A$, the globally supersymmetric Yang-Mills Lagrangian takes the form,

$$L = \sum_i (D_\mu S_i) d^\mu (D_\mu S_i) + \frac{i}{2} \sum_i \bar{\psi}_i \gamma_\mu D_\mu \psi_i$$

$$-\frac{1}{4} \sum_A F_{\mu\nu A} F^{\mu\nu A} + \frac{i}{2} \sum_A \bar{\lambda}_A \gamma_\mu D_\mu \lambda_A$$

$$-\sqrt{2} \sum_{i,A} \left[ S_i^\dagger (g_{iA} t_A) \bar{\psi}_i \frac{1}{2} \gamma_\mu \lambda_A + h.c. \right]$$

$$-\frac{1}{2} \sum_A \left[ \sum_i S_i^\dagger g_{iA} t_A S_i + \xi_A \right]^2$$

$$-\sum_i \left[ \frac{\partial f}{\partial S_i} \right]^2$$

$$-\frac{1}{2} \sum_{i,j} \left\{ \bar{\psi}_i \left[ 1 - \gamma_5 \right] \left( \frac{\partial^2 f}{\partial S_i \partial S_j} \right)_{S_i = S_j} \psi_j + h.c. \right\}$$

Here, $S_i$ ($\psi_i$) denotes the scalar (Majorana fermion) component of the $i$th chiral superfield, $F_{\mu\nu A}$ is the Yang-Mills gauge field, $\lambda_A$ is the Majorana gaugino superpartner of the corresponding gauge boson and $\xi_A$ are constants which can be non-zero only for $U(1)$ factors of the gauge group. In anticipation of simple grand unification, we will set these to zero. The last two lines of (24) come from the superpotential interactions and are identical to the Lagrangian in Eq. (21).

We note the following:

1. The first two lines are the gauge invariant kinetic energies for the components of the chiral and gauge superfields. The derivatives that appear are gauge covariant derivatives appropriate to the particular representation in which the field belongs. For example, if
we are talking about SUSY QCD, for quark fields in the first line of Eq. \( \text{Eq. (24)} \) the covariant derivative contains triplet SU\( (3)_C \) matrices; \( \text{i.e. } D_\mu = \partial_\mu + ig_\alpha \frac{\lambda_\alpha}{4} V_\mu^\alpha \), whereas the covariant derivative acting on the gauginos in the following line will contain octet matrices. These terms completely determine how all particles interact with gauge bosons.

2. The third line describes the interactions of gauginos with matter and Higgs multiplets (we will soon see that quarks, leptons as well as Higgs bosons (and their superpartners) belong to chiral supermultiplets). Notice that these interactions are also determined by the gauge couplings. Here \( t_{\alpha A} \) is the appropriate dimensional matrix representation of the group generators for the \( \alpha \)th factor of the gauge group, while \( g_\alpha \) are the corresponding gauge coupling constants (one for each factor of the gauge group). Matrix multiplication is implied. To see that these terms are gauge invariant, recall that \( \psi_{iR} \) which is fixed by the Majorana condition, transforms according to the conjugate representation to \( \psi_{iL} \).

3. Line four describes quartic couplings of scalar matter. Notice that these are determined by the gauge interactions. The interactions on this line are referred to as \( D \)-terms.

4. Finally, the last two lines in Eq. \( \text{Eq. (24)} \) describe the non-gauge, superpotential interactions of matter and Higgs fields, such as the Yukawa interactions responsible for matter fermion masses in the SM. Since these interactions do not involve any spacetime derivatives, choosing the superpotential to be a globally gauge invariant function of superfields is sufficient to guarantee the gauge invariance of the Lagrangian. For a renormalizable theory, the superpotential must be a polynomial of degree \( \leq 3 \).

Since the procedure that we have described is crucial for the construction of SUSY models, we summarize by presenting a recipe for constructing an arbitrary supersymmetric gauge theory.

\( (a) \) Choose a gauge group and the representations for the various supermultiplets, taking care to ensure that the theory is free of chiral anomalies. Matter fermions and Higgs bosons form parts of chiral scalar supermultiplets, while gauge bosons reside in the real gauge supermultiplet.

\( (b) \) Choose a superpotential function which is a globally gauge invariant polynomial (of degree \( \leq 3 \) for renormalizable interactions) of the various left chiral superfields.

\( (c) \) The interactions of all particles with gauge bosons are given by the usual “minimal coupling” prescription.
(d) Couple the gauginos to matter via the gauge interactions given in (24).

(e) Write down the additional self interactions of the scalar matter fields as given by (24).

(f) Write down the non–gauge interactions of matter fields coming from the superpotential. The form of these is as given by (21), or equally well, by the last two lines of (24).

The final step is to write down soft supersymmetry breaking terms which are crucial for the construction of realistic models.

Before closing our discussion of exact supersymmetry we briefly comment (without proof) on how supersymmetry protects scalar masses from large radiative corrections. Using perturbation theory, it can be shown\(^{23}\) that radiative corrections can alter masses and couplings in the superpotential only through wave function renormalization, provided supersymmetry is unbroken: in other words, if any parameter in the superpotential is zero to begin with, it will not be generated at any order in perturbation theory unless quantum corrections in the propagator induce mixing in the kinetic energy terms (D-terms) of the superfields. \(^{21,23}\) This statement is most easily proven using supergraphs, \(^{21}\) which is a diagramatic technique that keeps the underlying supersymmetry manifest in the course of the calculation. We have seen, however, that the mass parameter for the scalar component of a chiral superfield (the Higgs field is just such a scalar) arises from the superpotential and hence, gets radiatively corrected only due to the (at most logarithmically divergent) wave function renormalization. In terms of a calculation involving usual Feynman graphs with components of the superfield, this is equivalent to a cancellation between graphs involving internal boson loops and those involving loops of the fermionic partners of the bosons. Individually, these contributions are all very large, but supersymmetry leads to a precise cancellation of the bosonic and fermionic contributions, order by order in perturbation theory. If supersymmetry is broken at a scale \(\Lambda_{SUSY}\), the cancellations are not complete and we are left with \(\delta m^2 \sim O(\Lambda^2_{SUSY})\), which fixes \(\Lambda_{SUSY}\) to be smaller than \(O(1)\) TeV, as we have already seen.

\(^{23}\)What is really shown is that radiative corrections can only induce \(D\)-terms, and not \(F\)-terms. If off-diagonal bilinear \(D\)-terms are induced by radiative corrections, “new” superpotential interactions may be generated. We should also caution that some \(D\)-terms can be rewritten as \(F\)-terms. Such terms may also be radiatively generated. Generally, these involve non-renormalizable interactions. However, there are interesting cases\(^3\) where renormalizable superpotential interactions may be generated in the effective low energy theory obtained by integrating out super-heavy fields.
3.3 Supersymmetry Breaking

a. Spontaneous Supersymmetry Breaking

Supersymmetry cannot be an exact symmetry of nature. Aesthetically, it would be most pleasing if SUSY is spontaneously broken. As with gauge symmetry breaking, this would then preserve the coupling constant relationships needed for the cancellations of quadratic divergences but break the unwanted degeneracy between the masses of particles and their superpartners.

The action of any symmetry transformation on a field operator \( \phi \) can be schematically written as,

\[
\delta \phi = [\bar{\alpha}Q, \phi]. \tag{25a}
\]

In order for the symmetry not to be spontaneously broken, the symmetry generator \( Q \) should annihilate the vacuum, so that

\[
\langle 0 | \delta \phi | 0 \rangle = 0. \tag{25b}
\]

If this is not the case, the symmetry will be spontaneously broken.

If \( Q \) above is a generator of supersymmetry it is clear that \( \delta \phi \) must be a spin zero field (otherwise rotational invariance automatically ensures \( \langle 0 | \delta \phi | 0 \rangle = 0 \), and we obtain no new information), or from Eq. (20), \( \phi = \psi \). We thus see that in order to break supersymmetry spontaneously without breaking Lorentz invariance we must have

\[
\langle 0 | F | 0 \rangle \neq 0,
\]

where \( F \) is the auxiliary field of a chiral supermultiplet.

We now recall that the auxiliary fields \( F_i \) are algebraically eliminated via their Euler-Lagrange equations. This then suggests a way of breaking SUSY spontaneously: choose the superpotential so that the system of equations,

\[
\langle 0 | F_i | 0 \rangle = 0,
\]

is inconsistent. This is equivalent to the statement that the set of equations,

\[
\langle 0 | \left[ \frac{\partial f}{\partial S_i} \right]_{\tilde{s}_i = \bar{s}_i} | 0 \rangle = 0, \tag{26}
\]

has no consistent solution. This mechanism, due to O’Raifeartaigh,\(^{25}\) is also known in the literature as \( F \)-type breaking. As an example, we leave it to the reader to check that the superpotential,

\[
f(\hat{X}, \hat{Y}, \hat{Z}) = \lambda(\hat{X}^2 - \mu^2)\hat{Y} + m\hat{X}\hat{Z},
\]

leads to the spontaneous breakdown of SUSY.
Although we have not discussed the transformation of real superfields in detail, we should mention that supersymmetry is also spontaneously broken if the corresponding auxiliary field (denoted by \(D\)) develops a vacuum expectation value. This gives us the other known way of breaking SUSY spontaneously: \(D\)-term or Fayet-Illiopoulos breaking.

b. Practical Supersymmetry Breaking

Much as we would like to have it, a compelling model where SUSY is broken spontaneously has not yet been constructed. From many phenomenological analyses, it is fortunate that one does not need to know the details of the physics of SUSY breaking since they are not currently understood. The best that we can do at present is to provide a useful parametrization of SUSY-breaking effects. Our guiding principle is that the SUSY breaking terms should not destabilize scalar masses by reintroducing the quadratic divergences that SUSY was introduced to eliminate in the first place. In other words, SUSY breaking effects can be incorporated by including all soft SUSY breaking masses and interactions consistent with the known symmetries (Poincaré invariance, SM gauge invariance and any other global symmetries that we might impose).

Girardello and Grisaru have classified all renormalizable soft SUSY breaking operators. For our purposes, it is sufficient to know that these consist of:

- explicit masses for the scalar members of chiral multiplets; \(i.e.\) squarks, sleptons and Higgs bosons;
- an independent gaugino mass for each factor of a direct product gauge group: for instance, we would have masses \(M_1\), \(M_2\) and \(M_3\) for the \(U(1)_Y\), \(SU(2)_L\) and \(SU(3)_C\) gauginos, respectively;
- new super-renormalizable scalar interactions: for each trilinear (bi-linear) term in the superpotential of the form \(C_{ijk}\hat{S}_i\hat{S}_j\hat{S}_k\) \((C_{ij}\hat{S}_i\hat{S}_j)\), we can introduce a soft supersymmetry breaking scalar interaction \(A_{ijk}C_{ijk}\hat{S}_i\hat{S}_j\hat{S}_k\) \((B_{ij}C_{ij}\hat{S}_i\hat{S}_j)\) where the \(A\)'s and \(B\)'s are constants.

These terms are often referred to as \(A\)- and \(B\)-terms, respectively.

We have already seen examples of the cubic and quadratic soft breaking interactions at the end of Sec. 2 (see Eq. (18)). The scalar and gaugino masses obviously serve to break the undesired degeneracy between the masses of sparticles and particles. We will see later that the explicit trilinear scalar interactions mainly affect the phenomenology of the third family.

4 The Minimal Supersymmetric Model

We now have the necessary background to begin discussing particle physics in a supersymmetric world, where SUSY is somehow (softly) broken at the weak scale. We start with a discussion of what we will term
as the Minimal Supersymmetric Model (MSSM). As the name suggests, it is the simplest phenomenologically viable supersymmetric theory in that it contains the fewest number of new particles and new interactions.

4.1 Field Content

We have already seen that in order to construct supersymmetric theories, we have to introduce a partner for every particle of the SM, with a spin differing by $\frac{1}{2}$ but with the same internal quantum numbers. Furthermore, we have seen that matter fermions and Higgs bosons are members of chiral scalar supermultiplets. Thus, the SUSY partners of matter fermions must have spin zero bosons as their partners. The dynamical matter fields of our model are thus given by,

$$
\left( \begin{array}{c}
\nu \\
e
\end{array} \right)_L, \left( \begin{array}{c}
e_R \\
u_L \\
d
\end{array} \right)_L, \left( \begin{array}{c}
u_R \\
d_R
\end{array} \right),
$$

(27a)

and

$$
\left( \begin{array}{c}
\tilde{\nu}_L \\
\tilde{\nu}_L
\end{array} \right), \left( \begin{array}{c}
\tilde{\nu}_R \\
\tilde{\nu}_R
\end{array} \right), \left( \begin{array}{c}
\tilde{\nu}_L \\
\tilde{\nu}_L
\end{array} \right), \left( \begin{array}{c}
\tilde{\nu}_R \\
\tilde{\nu}_R
\end{array} \right),
$$

(27b)

for the first family. The other families are copies of this exactly as in the SM: SUSY sheds no light on the reason for the replication of generations. Note that for the lepton (quark) doublet there is a scalar lepton or slepton (scalar quark or squark) doublet, and likewise for the singlets. Thus, corresponding to each massive Dirac fermion $f$, there are two complex SUSY fields $\tilde{f}_L$ and $\tilde{f}_R$, the partners of the left and right chiral projections of the fermion. This is in keeping with our counting of the number of degrees of freedom — a massive Dirac fermion has four degrees of freedom corresponding to two spin states of the particle and antiparticle. Note also that $\tilde{f}_L$ and $\tilde{f}_R$ have exactly the same gauge quantum numbers as their SM fermion partners.

In the gauge field sector, we have a gauge supermultiplet for each factor of the gauge group; i.e., the dynamical fields are,

$$
A_0, \ A_\mu, \ g_\mu
$$

(28a)

and

$$
\tilde{\lambda}_0, \ \tilde{\lambda}, \ g.
$$

(28b)

\(^1\)We warn the reader that the term MSSM does not have a standard usage in the literature. Therefore, one should always pay attention to explicit and implicit assumptions that are made by each group of authors analysing the MSSM.
The vector fields above are the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge potentials whereas the fermion fields are the spin $\frac{1}{2}$ Majorana supersymmetric partners of these fields. Like the gauge fields, these fermion fields transform as the adjoint representation of the appropriate group factor. Once $SU(2)_L \times U(1)_Y$ is broken, fields with the same spin and charge can mix to form the observed particles (e.g. $\gamma$ and $Z^0$) as discussed in the next subsection.

Finally, we come to the electroweak symmetry breaking sector. In the SM, the $SU(2)_L \times U(1)_Y$ symmetry is broken by a single doublet of Higgs fields which acquires a non–vanishing value in the ground state. Moreover, the same field, by virtue of its Yukawa interactions with fermions gives rise to a mass term for all the fermions of the SM. Technically, this is possible because the doublet (the complex conjugate of the doublet) can couple to the $T_3 = +\frac{1}{2}$ ($T_3 = -\frac{1}{2}$) fermions in a gauge invariant way. In a supersymmetric theory, however, Yukawa interactions come from a superpotential which, as we have seen, cannot depend on a field as well as its complex conjugate. As a result, any doublet can give mass either to a $T_3 = +\frac{1}{2}$ or a $T_3 = -\frac{1}{2}$ fermion, but not both. Thus, in order to give masses to all the fermions, we are forced to introduce two Higgs doublet chiral superfields $\hat{h}_u$ and $\hat{h}_d$ which interact with $T_3 = +\frac{1}{2}$ and $T_3 = -\frac{1}{2}$ fermions, respectively. This sector then consists of the dynamical boson fields with hypercharge $Y_{\bar{h}_u} = 1$, $Y_{\hat{h}_d} = -1$,

$$
\begin{bmatrix}
\hat{h}_u^+ \\
\hat{h}_u^0
\end{bmatrix}, \quad \begin{bmatrix}
\hat{h}_d^+ \\
\hat{h}_d^0
\end{bmatrix},
$$

and their fermionic partners (the Higgsino doublets)

$$
\begin{bmatrix}
\tilde{h}_u^+ \\
\tilde{h}_u^0
\end{bmatrix}, \quad \begin{bmatrix}
\tilde{h}_d^+ \\
\tilde{h}_d^0
\end{bmatrix}.
$$

The fermion spinor fields that appear are Majorana. The charge shown corresponds to that of its left chiral component; the right-handed part has the opposite charge. Notice also that the upper component of the doublet $\hat{h}_d$ has been written with an electric charge $Q = -1$. In other words, we have taken the $\hat{h}_u$ and $\hat{h}_d$ doublets to transform as the $\bar{2}$ and $2^*$ representations, respectively. Since these are equivalent, it should be clear that this is done only for convenience.

### 4.2 Interactions

The supersymmetric interactions for these fields can now be readily worked out using Eq. (24). The interactions of the matter and Higgs

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3Specifically, this makes it easy to embed these fields into the $\bar{5}$ and $5^*$ representations of $SU(5)$ if the theory is embedded in a GUT.
fields (and their superpartners) with gauge bosons and gauginos are as

given by the first four lines, and are thus model-independent, except

for the constant $\xi$ which we set to zero. In particular, because of su-

persymmetry, the gauge couplings also determine all the interactions of

gauginos. Given the field content, model dependence arises via the choice

of the superpotential $f$ which, for the MSSM, is taken to be,

\[
\begin{align*}
    f_{\text{MSSM}} &= \mu (\hat{h}_u^0 \hat{h}_d^0 + \hat{h}_u^+ \hat{h}_d^-) \hat{U}^c + f_u (\hat{u} \hat{h}_d^0 - \hat{d} \hat{h}_u^0) \hat{U}^c \\
    &+ f_d (\hat{d} \hat{h}_d^0 + \hat{u} \hat{h}_u^0) \hat{D}^c + f_e (\hat{e} \hat{h}_d^0 + \hat{e} \hat{h}_u^0) \hat{E}^c + \ldots.
\end{align*}
\]

Since the superpotential is a function of only the left–chiral superfields,

we work with the (left–handed) conjugates of the $SU(2)_L$ singlet fermions

and their partners, which together constitute a left chiral supermultiplet

with the quantum numbers of the representation conjugate to that of the

usual (right-handed) singlet fermions. In Eq. (30), $\hat{u}$ and $\hat{d}$ denote the

$SU(2)_L$ components of the doublet quark superfield. A similar notation

is used for leptons. The minus sign in the second term is because it is the

anti-symmetric combination of two doublets that forms an $SU(2)_L$
singlet. Since $\hat{h}_d$ is defined to transform according to the $2^*$ represen-
tation, the symmetric combination appears in other terms. Finally, $f_u$,

$f_d$ and $f_e$ are the coupling constants for the Yukawa interactions that

give rise to first generation quark and lepton masses. The ellipses denote

similar terms for other generations.

The observant reader will have noticed that we have not written the most general $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant renormalizable superpotential in Eq. (30). In particular, we could have included the terms given by,

\[
    g_1 = \sum_i \mu_i \hat{L}_i \hat{h}_u + \sum_{i,j,k} \left[ \lambda_{ijk} \hat{L}_i \hat{E}_j \hat{E}_k^- + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k^c \right],
\]

and,

\[
    g_2 = \sum_{i,j,k} \lambda''_{ijk} \hat{U}_i^c \hat{D}_j^c \hat{D}_k^c.
\]

in the superpotential $f$. In the Eq. (31a) and (31b), $i, j$ and $k$ denote generation indices, while the $\lambda$’s are coupling constants. We have, for brevity, not expanded out the gauge invariant product of doublets in Eq. (31a).

The Lagrangian interactions can now be obtained by substituting the appropriate superpotential into Eq. (24). It is easy to check that the terms obtained from $g_1$ and $g_2$ lead to the violation of lepton and baryon number conservation, respectively. This can also be seen directly from
the superpotential. For instance, with the usual assignment of lepton number of one unit to $\hat{L}$ and $\hat{E}$ (so that $f_{MSSM}$ remains invariant), $g_{1}$ clearly is not globally invariant under the corresponding $U(1)$ transformations. In other words, the MSSM framework in which the couplings in $g_{1}$ and $g_{2}$ are all set to zero, assumes that there are no renormalizable baryon or lepton number violating operators in the superpotential.

In addition to the supersymmetric interactions discussed above, we also need to include soft supersymmetry breaking interactions. These include an independent mass (or, allowing for flavour mixing, mass matrix) for each squark, slepton and Higgs boson multiplet: i.e. 9 squark + 6 slepton + 2 Higgs boson masses neglecting inter-generation mixing, a gaugino mass for each of the $SU(3)_{C}$, $SU(2)_{L}$ and $U(1)_{Y}$ gauginos, and finally the trilinear and bilinear $A$- and $B$-terms for scalars. Again ignoring inter-generation mixing, there are nine $A$-terms for the nine Yukawa interactions in the superpotential (30) but just one $B$-term.

We further assume that baryon and lepton number are conserved by all renormalizable interactions of the MSSM, and set the trilinear and bilinear soft SUSY breaking terms corresponding to operators in $g_{1}$ and $g_{2}$ to zero. The MSSM thus contains thirty soft SUSY breaking parameters together with the supersymmetric parameter $\mu$ in addition to the arbitrary parameters of the SM. One of these parameters can be eliminated in favour of $M_{W}$, so that we have thirty independent SUSY parameters left over.

With these assumptions, it is easy to check that $R$-parity, defined to be $+1$ for leptons, quarks, gauge bosons and Higgs bosons, and $-1$ for their supersymmetric partners, is automatically conserved in the interactions of gauge bosons and gauginos as given by the first four lines of (24). Whether or not it is a good symmetry depends on the choice of the superpotential and the soft SUSY breaking interactions. The reader can easily verify using (24) that the interactions from the $f_{MSSM}$ term in the superpotential also conserve $R$-parity, while those from $g_{1}$ or $g_{2}$ do not. Thus $R$-parity is conserved by the renormalizable interactions (including soft SUSY breaking terms) of the MSSM. It is assumed that $R$-parity invariance is an exact symmetry of the model. This has important implications as we will see later.

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\footnote{We will leave it as an exercise for the reader to figure out how many parameters there would be if we allowed inter-generation mixing.}

\footnote{For the MSSM fields, it is easy to check that $R = (-1)^{2S+L+B}$, where $L$ and $B$ denote the lepton- and baryon-number, respectively and $S$ is the spin. Since the MSSM conserves $B$, $L$ and angular momentum, $R$ is conserved. Spontaneous $R$ violation via a VEV of a doublet sneutrino is excluded by the measurement of the $Z$ width at LEP.}
4.3 Mass Eigenstates in the MSSM

*SUSY Scalars:* The scalar partners $\tilde{f}_L$ and $\tilde{f}_R$ have the same electric charge and colour, and so can mix if $SU(2)_L \times U(1)_Y$ is broken. It is simple to check that the gauge interactions conserve chiral flavour in that they couple only left (right) multiplets with one another, i.e. $\tilde{f}_L$ couples only to $\tilde{f}_L$ (and $\tilde{f}_L$) via gauge boson (gaugino) interactions. Unless this “extended chiral symmetry” is broken, there can be no $\tilde{f}_L - \tilde{f}_R$ mixing. This symmetry is, however, explicitly broken by the Yukawa interactions in the superpotential. We thus conclude that $\tilde{f}_L - \tilde{f}_R$ mixing is proportional to the corresponding Yukawa coupling and hence to the corresponding fermion mass. For the purposes of collider signals that will be our main concern, this mixing is generally negligible except for third generation sfermions. We will, therefore, neglect this intra-generational mixing for the first two generations, and for simplicity, also any inter-generational mixing.

*SUSY Fermions:* The gauginos and Higgsinos are the only spin-$\frac{1}{2}$ fermions. Of these, the gluinos being the only colour octet fermions, remain unmixed and have a mass $m_{\tilde{g}} = |M_3|$. Electroweak gauginos and Higgsinos of the same charge can mix, once electroweak gauge invariance is broken. The MSSM mass matrices can be readily worked out using Eq. (24) and Eq. (30). We first focus on the mass terms in the charged gaugino–Higgsino sector. Since the field operator for the charged eigenstate must be a Dirac spinor, we first combine the Majorana gauginos (Higgsinos) into a Dirac gaugino (Higgsino) field with definite charge

$$\tilde{\lambda} = \frac{1}{\sqrt{2}} (\tilde{\lambda}_1 + i \tilde{\lambda}_2) \quad (32a)$$

and

$$\tilde{\chi} = P_L \tilde{h}_d^+ - P_R \tilde{h}_u^+ \quad (32b)$$

where $P_L$ and $P_R$ are the left and right chiral projectors, respectively. We stress again that $\tilde{h}_d^+$ and $\tilde{h}_u^+$ are Majorana spinors whose left chiral components have the charge denoted by the spinor. The right chiral component is fixed by (6), and because of the complex conjugation, has the opposite charge as the left–chiral component. Hence, $P_R \tilde{h}_u^+$ is negatively charged, so that $\tilde{\chi}$ is indeed a Dirac field with charge $Q = -1$.

The interactions can then be written in terms of the spinors $\tilde{\lambda}$ and $\tilde{\chi}$ and the corresponding mass matrix read off from the bilinear terms involving

\[m\]

\[m\]**Without the assumption of $R$-parity conservation there would also be mixing between the $\hat{h}$ and $\hat{L}$ supermultiplets. Such a mixing, which is absent in the MSSM, can have significant phenomenological impact.**
these fields. In the Lagrangian, these take the form,

\[- (\tilde{\lambda}, \tilde{\chi})^T \left[ M_{\text{(charge)}} P_L + M_{\text{(charge)}}^T P_R \right] \left( \begin{array}{c} \tilde{\lambda} \\ \tilde{\chi} \end{array} \right) \]  

(33a)

with,

\[ M_{\text{(charge)}} = \left( \begin{array}{cc} M_2 & -gv_d \\ -gv_u & -\mu \end{array} \right) \]  

(33b)

In Eq. (33b), the $-\mu$ entry comes from the superpotential whereas the off-diagonal entries come the interactions involving the Higgs supermultiplet and the gauginos (the third term in Eq.(24)); this trilinear term becomes an off-diagonal mass term if the scalars acquire VEVs. In (33b), $g$ and $g'$ are the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants, respectively. Finally, $M_2$ is the soft SUSY breaking $SU(2)_L$ gaugino mass.

The mass matrices for the neutral gaugino-Higgsino sector can be similarly worked out. In this case, we find that the Lagrangian contains the terms

\[- \frac{1}{2} \begin{pmatrix} \tilde{h}_u^0 & \tilde{h}_d^0 & \tilde{\lambda}_3 \end{pmatrix} \left[ M_{\text{(neutral)}} P_L + M_{\text{(neutral)}}^T P_R \right] \begin{pmatrix} \tilde{h}_u^0 \\ \tilde{h}_d^0 \\ \tilde{\lambda}_3 \end{pmatrix} \]  

(34a)

with

\[ M_{\text{(neutral)}} = \left( \begin{array}{ccc} 0 & \mu & -\frac{gv_d}{\sqrt{2}} \\ \mu & 0 & -\frac{gv_u}{\sqrt{2}} \\ -\frac{gv_d}{\sqrt{2}} & -\frac{gv_u}{\sqrt{2}} & M_1 \end{array} \right) \]  

(34b)

The sources of the various terms are more or less as in (33b). Note that in addition to the soft supersymmetry breaking mass term $M_2$ for the $SU(2)_L$ gaugino which also appears in (33b), there is now an independent mass term $M_1$ for the $U(1)_Y$ gaugino. Note also that $R$-parity conservation precludes any mixing of the charged leptons (neutrinos) with the charged (neutral) gaugino-Higgsino sector.

The mass eigenstates can now be obtained by diagonalizing these matrices. In the MSSM, the charged Dirac Higgsino (composed of the charged components of the doublets $\tilde{h}_u$ and $\tilde{h}_d$) and the charged gaugino (the partner of the charged $W$ boson) mix to form two Dirac charginos.

\[^{\text{n}}\text{If an eigenvalue of the mass matrix for any state } \psi \text{ turns out to be negative, one can always define a new spinor } \psi' = \gamma_5 \psi \text{ which will have a positive mass. For neutralinos, } \psi' \text{ should be defined with an additional factor } i \text{ to preserve its Majorana nature after the } \gamma_5 \text{ transformation.} \]
while the two neutral Higgsinos and the neutral $SU(2)_L$ and $U(1)_Y$ gauginos mix to form four Majorana neutralinos $\tilde{Z}_1 \ldots \tilde{Z}_4$, in order of increasing mass. In general, the mixing patterns are complex and depend on several parameters: $\mu$, $M_{1,2}$ and $\tan \beta \equiv \frac{v_u}{v_d}$, the ratio of the vacuum expectation values of the two Higgs fields introduced above. If either $|\mu|$ or $|M_1|$ and $|M_2|$ are very large compared to $M_W$, the mixing becomes small. For $|\mu| \gg M_W, |M_{1,2}|$, the lighter chargino is essentially a gaugino while the heavier one is a Higgsino with mass $|\mu|$; also, the two lighter neutralinos are gaugino-like while $\tilde{Z}_{3,4}$ are dominantly Higgsinos with mass $\sim |\mu|$. If instead, the gaugino masses are very large, it is the heavier chargino and neutralinos that become gaugino-like.

Without further assumptions, the three gaugino masses are independent parameters. It is, however, traditional to assume that there is an underlying Grand Unification, and that these masses derive from a common gaugino mass parameter defined at the unification scale. The differences between the various gaugino masses then come from the fact that they have different interactions, and so undergo different renormalization when these are evolved down from the GUT scale to the weak scale. The gaugino masses are then related by,

$$\frac{3M_1}{5\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3}. \quad (35)$$

Here the $\alpha_i$ are the fine structure constants for the different factors of the gauge group. With this GUT assumption, $\tilde{W}_1$ and $\tilde{Z}_{1,2}$ will always be substantially lighter than gluinos. It is for this reason that future $e^+e^-$ colliders operating at $\sqrt{s} \simeq 500-1000$ GeV are expected to be competitive with hadron supercolliders such as the LHC which has much higher energy. We also mention that for not too small values of $|\mu|$, $M_{1,2}$ the lightest neutralino tends to be dominantly the hypercharge gaugino.

The Electroweak Symmetry Breaking Sector: Although this is not in the mainstream of what we will discuss, we should mention that because there are two doublets in the MSSM, after the Higgs mechanism there are five physical spin zero Higgs sector particles left over in the spectrum. Assuming that there are no CP violating interactions in this sector, these are two neutral CP even eigenstates ($h$ and $H$, with $m_h \leq m_H$) which behave as scalars as far as their couplings to fermions go, a neutral "pseudoscalar" CP odd particle $A$, and a pair of charged particles $H^\pm$.

The Higgs boson sector of the MSSM is greatly restricted by SUSY. At tree-level, it is completely specified by the parameters $m_{H_u}^2$, $m_{H_d}^2$, $\mu$ and $B$. The soft masses and the $B$-parameter can be eliminated in favour of the VEVs (or equivalently, $M_W^2 = \frac{1}{2}g^2(v_u^2 + v_d^2)$ and $\tan \beta$) and one of the physical Higgs boson masses, usually chosen to be $m_A$. The parameters $\tan \beta$ and $\mu$ also enter into the SUSY fermion mass matrices, so that the SUSY Higgs sector is completely characterized by just one.
additional parameter. In particular, the Higgs quartic self-couplings are all given by those on line four of Eq. (24) and so are fixed to be $O(g^2)$. This leads to the famous (tree-level) bound, $m_h < \min[M_Z, m_A] |\cos 2\beta|$. This bound receives important corrections from $t$ and $\tilde{t}$ loops because of the rather large value of the top Yukawa coupling and the bound is weakened to about 120-130 GeV depending on the value of $m_t$. Thus, in contrast to early expectations, $h$ may well escape detection at LEP2. It is worth mentioning that if we assume that all couplings remain perturbative up to the GUT scale, then the mass of the lightest Higgs boson is bounded by 145-150 GeV in any weak scale SUSY model. The physics behind this is the same as that behind the bound $m_{H_{SM}} \lesssim 200$ GeV on the mass of the SM Higgs boson, obtained under the assumption that the Higgs self-coupling not blow up below the GUT scale; the numerical difference between the bounds comes from the difference in the evolution of the running couplings in SUSY and the SM. An $e^+e^-$ collider operating at a centre of mass energy $\sim 300$ GeV would thus be certain to find a Higgs boson if these arguments are valid.

4.4 Implications of R-parity Conservation

In any realistic SUSY theory, the existence of scalar quarks and leptons admits the possibility of gauge-invariant, renormalizable baryon and lepton number violating interactions and so forces us to impose additional global symmetries. To see this, note that if all the dimensionless couplings $\lambda$, $\lambda'$ and $\lambda''$ that occur in Eq. (31a) and Eq. (31b) are of similar strength as the gauge couplings, and if supersymmetric particles are indeed lighter than $\sim 1$ TeV, we would be led to conclude that the proton would decay at the weak rate at complete variance with our very existence! Furthermore, the decays $\mu \to e\gamma$ and $\mu \to ee\bar{e}$, or processes such as $\mu N \to eN$ on which there are stringent bounds from experiment, would certainly have been observed. This situation is quite different from the SM where gauge invariance guarantees the absence of any renormalizable lepton- or baryon-number violating interactions. Within the MSSM, we introduce a discrete symmetry (R-parity invariance) to ensure that both $g_1$ and $g_2$ vanish. Other alternatives will be discussed toward the end of these Lectures.

The conservation of R-parity has important implications for phenomenology.

- SUSY (R-odd) particles have their own identity and do not mix with R-even SM particles. We will refer to these as sparticles and denote them with twiddles.
- Sparticles can only be pair produced in collisions of ordinary particles.

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- A sparticle must decay into an odd number of sparticles.
- As a result, the lightest supersymmetric particle (LSP) must be absolutely stable.

There are strong limits on the existence of stable or even very long-lived (τ ≳ age of the universe) charged or coloured particles. Such particles would have been produced in the Big Bang and would bind with ordinary particles resulting in exotic heavy atoms or nuclei. For masses up to about 1 TeV, estimates of their expected abundance are in the range $O(10^{-10} - 10^{-6})$ whereas the empirical abundance is $≈ O(10^{-12} - 10^{-29})$ depending on the element whose exotic isotope is searched for. This null result is taken to imply that a stable LSP must be a weakly interacting, neutral sparticle. Within the MSSM, the LSP can then only be either the lightest neutralino or one of the scalar neutrinos. We will see later that a sneutrino is disfavoured if we also assume that the LSP also forms the dark matter in our galactic halo. On the other hand, it has been shown that a stable neutralino is a promising candidate for cosmological cold dark matter. In a supergravity theory, the SUSY partner of the graviton could also be the LSP. Unless it is extremely light, however, it couples to other particles only with gravitational strength couplings so that it effectively decoupled for the purposes of collider phenomenology. In this case, the next lightest sparticle plays the role of the LSP and the constraints discussed above apply to it as long as the lifetime for its decay into the gravitino exceeds the age of the universe. If this is not the case, or if $R$-parity is not conserved, the “effective LSP” may even be charged or coloured. Throughout most of these Lectures we will assume that the LSP is the lightest neutralino.

We note here that regardless of what the LSP is (as long as it is neutral and lives to travel at least a few metres), the LSP’s produced in the decays of sparticles in SUSY events behave like neutrinos in the experimental apparatus: i.e. they escape without depositing any energy. Thus, in any model where $R$-parity is assumed to be conserved, apparent missing energy ($E_T$) and an imbalance of transverse momentum ($p_T$) are generally regarded as characteristic signatures of SUSY.

4.5 Is the MSSM a Practical Framework for SUSY Phenomenology?

The MSSM is the simplest framework for SUSY phenomenology. A big advantage of this framework is that except for the assumption of $R$-parity conservation (and, of course, supersymmetry!), we have assumed very little else: a minimal particle content, Poincaré invariance and gauge invariance. The price that we have to pay for such an agnostic view

\[\text{Moroi}\] has pointed out that late decay of this effective LSP can potentially spoil the successful predictions of Big Bang Nucleosynthesis.
is the large number of free parameters to parametrize SUSY breaking. We saw that even in the simplified case where we neglect generational mixing between sfermions, there were 30 new parameters. This is not necessarily a problem for SUSY phenomenology if we are studying a SUSY process where just a few sparticles are relevant: if this is so, only a handful of these thirty parameters would be relevant for the analysis of the process in question. We will, however, see that this is generally not the case. Typically, heavy sparticles decay into lighter sparticles which further decay until this cascade terminates in the stable LSP. This, of course, means that to describe completely even a single SUSY reaction may require the knowledge of properties of several particles (the parent, along with all the daughters that are part of the decay cascade) which, in turn, depend on the large number of MSSM parameters. This renders the MSSM rather unwieldy for many phenomenological analyses.

Assuming grand unification ameliorates the situation to some extent: there are then only two scalar masses per generation of sfermions in $SU(5)$ and only one gaugino mass parameter, but the parameter space is still too large. In the future, a deeper understanding of the mechanism of supersymmetry breaking may relate the many SUSY breaking parameters of the MSSM, resulting in a significant reduction of the parameter space. For the present, however, we have to rely on assumptions about the nature of physics at the high energy scale to reduce the number of parameters. It is important to keep in mind that these assumptions may prove to be incorrect. For this reason, one should always be careful to test the sensitivity of phenomenological predictions to the various assumptions, particularly when using the models to guide our thinking about the design of future experiments.

Historically, inspired by supergravity model studies, many early phenomenological studies assumed that all squarks (often, sleptons were either assumed to be degenerate with squarks, or to have masses related to $m_{\tilde{q}}$) were degenerate except for $D$-term splitting. They also incorporated the GUT assumption (35) for gaugino masses. The masses and couplings of all sparticles were determined by the SM parameters together with relatively few additional (SUSY) parameters which were frequently taken to be,

$$m_{\tilde{g}}, m_{\tilde{q}}, (m_{\tilde{\ell}}), \mu, \tan \beta, A_t, m_A.$$ (36)

The parameter $A_t$ mainly affects top squark phenomenology, and so, was frequently irrelevant. Other $A$-terms, being proportional to the light fermion masses, are irrelevant for collider phenomenology.

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$p$An example of this would be $e^+e^- \rightarrow \tilde{\mu}_R\tilde{\mu}_R$, if $\tilde{\mu}_R \rightarrow \mu\tilde{Z}_1$ all the time. In this case, the SUSY reaction can be completely described by $m_{\tilde{\mu}_R}$ and $m_{\tilde{Z}_1}$.
In view of the fact that additional assumptions are necessary, and further, that assumptions based on supergravity models are incorporated into phenomenological analyses, it seems reasonable to explore the implications of these models more seriously. Toward this end, we describe the underlying framework in the following section.

5 The mSUGRA framework: A Model Paradigm

When supersymmetry is promoted to a local symmetry, additional fields have to be introduced. The resulting theory, which includes gravitation is known as supergravity (SUGRA). It is not our purpose here to study SUGRA models in any detail. Our purpose is solely to provide motivation for an economic and elegant framework that has recently become very popular for phenomenological analysis and to carefully spell out its underlying assumptions.

It was recognised rather early that it is very difficult to construct globally supersymmetric models where SUSY is spontaneously broken at the weak scale. This led to the development of geometric hierarchy models where SUSY is broken in a “hidden” sector at a scale \( \mu_s \gg M_W \). This sector is assumed to interact with ordinary particles and their superpartners (the “observable” sector) only via exchange of superheavy particles \( X \). This then suppresses the couplings of the Goldstone fermion (which resides in the hidden sector) to the observable sector: as a result, the effective mass gap in the observable sector is \( \mu \sim \frac{\mu_s^2}{M_X} \) which can easily be \( \lesssim 1 \) TeV even if \( \mu_s \) is much larger.

An especially attractive realization of this idea stems from the assumption that the hidden and observable sectors interact only gravitationally, so that the scale \( M_X \sim M_{\text{Planck}} \). This led to the development of supergravity GUT models of particle physics. Because supergravity is not a renormalizable theory, we should look upon the resulting Lagrangian, with heavy degrees of freedom integrated out, as an effective theory valid below some ultra-high scale \( M_{\text{GUT}} \) or \( M_{\text{Planck}} \), in the same way that chiral dynamics describes interactions of pions below the scale of chiral symmetry breaking. Remarkably, this Lagrangian turns out to be just the same as that of a globally supersymmetric \( SU(3)_C \times SU(2)_L \times U(1)_Y \) model, together with soft SUSY breaking masses and \( A \)- and \( B \)-parameters of \( O(M_{\text{Weak}}) \).

The economy of the minimal supergravity (mSUGRA) GUT framework stems from the fact that because of the assumed symmetries, \( ^9 \) here the term minimal refers to a technical assumption: the canonical choice of kinetic energy terms for matter and gauge fields. In this case, there is a global \( U(N) \) symmetry in a model with \( N \) chiral super-multiplets. However, since supergravity is a non-renormalizable theory, in principle, kinetic terms may also arise from higher dimensional operators, and
various soft SUSY breaking parameters become related independent of the details of the hidden sector and the low energy effective Lagrangian can be parametrized in terms of just a few parameters. For instance, the chiral multiplets all acquire the same soft SUSY breaking scalar mass $m_0$. Likewise, there is a universal $A$-parameter, common to all trilinear interactions. The GUT assumption, of course, implies that the soft SUSY breaking gaugino masses are related as in Eq. (35). The only role played by the “minimal” in mSUGRA is to provide a rationale for the universality of soft SUSY breaking parameters. It is worth stressing that the universality does not follow from an established principle such as general covariance. For phenomenological purposes, one can forget about the origins of the model and simply view it as a version of the MSSM with universal scalar and gaugino masses and $A$-parameters at some ultra-high scale.

The universality of the scalar masses does not imply that the physical scalar masses of all sfermions are the same. The point is that the parameters in the Lagrangian obtained by integrating out heavy fields should be regarded as renormalized at the high scale $M_X$ at which these symmetries are unbroken. If we use this Lagrangian to compute processes at the 100 GeV energy scale relevant for phenomenology, large logarithms $O(\ln \frac{M_X}{M_W})$ due to the disparity between the two scales invalidate the perturbation expansion. These logarithms can be straightforwardly summed by evolving the Lagrangian parameters down to the weak scale. This is most conveniently done using renormalization group equations (RGE).

The renormalization group evolution leads to a definite pattern of sparticle masses, evaluated at the weak scale. For example, gauge boson-gaugino loops result in increased sfermion masses as we evolve these down from $M_X$ to $M_W$ while superpotential Yukawa couplings (which are negligible for the two lightest generations) have just the opposite effect. Since squarks have strong interactions in addition to the electroweak interactions common to all sfermions, the weak scale squark masses are larger than those of sleptons. Neglecting Yukawa couplings in the RGE, we have to a good approximation,

$$m^2_{\tilde{q}} = m^2_0 + m^2_\tilde{q} + (5 - 6)m^2_1 + D - terms, \quad (37a)$$

$$m^2_{\tilde{\ell}} = m^2_0 + m^2_{\tilde{\ell}} + (0.15 - 0.5)m^2_1 + D - terms. \quad (37b)$$

In Eq. (37b), $m_1$ is the common gaugino mass at the scale $M_X$. Notice that squarks and sleptons within the same $SU(2)_L$ doublets are split only need not take the canonical form.

These running masses evaluated at the sparticle mass, or more crudely, at a scale $\sim M_Z$, are not identical to, but are frequently close to the physical masses which are given by the pole of the renormalized propagator.
by the $D$-terms, whose scale is set by $\frac{1}{2}M_X^2$. In contrast, various flavours of left- (and separately, right-) type squarks of the first two generations are essentially degenerate, consistent with flavour changing neutral current (FCNC) constraints from the observed properties of $K$, $D$ and $B$ mesons. The difference in the coefficients of the $m_1^2$ terms reflects the difference between the strong and electroweak interactions alluded to above. Although we have not shown this explicitly, $\tilde{\ell}_R$ which has only hypercharge interactions tends to be somewhat lighter than $\tilde{\ell}_L$ as well as $\tilde{\nu}_L$ unless D-term effects are significant. Since $m_3 = (2.5 - 3)m_1$, it is easy to see that squark and slepton masses are related by,

$$m_{\tilde{q}}^2 = m_{\tilde{\ell}}^2 + (0.7 - 0.8)m_{\tilde{g}}^2, \quad (38)$$

Here, $m_{\tilde{q}}^2$ and $m_{\tilde{\ell}}^2$ are the squared masses averaged over the squarks or sleptons of the first (or second) generation. In the second term, the unification of gaugino masses has been assumed.

The Yukawa couplings of the top family are certainly not negligible. These Yukawa interactions tend to reduce the scalar masses at the weak scale. The RGE effects from these can overcome the additional $m_t^2$ in Eq. (37a), so that $t_L$ and $t_R$ tend to be significantly lighter than other squarks (of course, by $SU(2)_L$ invariance, the soft-breaking mass for $\tilde{b}_L$ is the same as that for $\tilde{t}_L$). In fact, we can say more: because $\tilde{t}_R$ receives top quark Yukawa corrections from both charged and neutral Higgs loops in contrast to $\tilde{t}_L$ which gets corrections just from the neutral Higgs, its squared mass is reduced (approximately) twice as much as that of $\tilde{t}_L$. Moreover, as we have already mentioned, these same Yukawa interactions lead to $t_L - t_R$ mixing, which further depresses the mass of the lighter of the two $t$-squarks (sometimes referred to as the stop) which we will denote by $\tilde{t}_1$. In fact, care must be exercised in the choice of input parameters: otherwise $m_1^2$ is driven negative, leading to the spontaneous breakdown of electric charge and colour. For very large values of $\tan\beta \sim \frac{m_t}{m_b}$, bottom and tau Yukawa couplings are also important; these affect $b$-squark and $\tau$ slepton masses and mixings in an analogous way.

The real beauty and economy of this picture comes from the fact that these same Yukawa radiative corrections drive electroweak symmetry breaking. Since the Higgs bosons are part of chiral supermultiplets, they also have a common mass $m_0$ at the scale $M_X$ and undergo similar renormalization as doublet sleptons due to gauge interactions: i.e. these positive contributions are not very large. The squared mass $m_{h_u}^2$ of the Higgs boson doublet which couples to the top family, however, receives large negative contributions (thrice those of the $t_L$ squark since there are three different colours running in the loop) from Yukawa interactions.

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This is a non-trivial observation since alternative mechanisms to suppress FCNC based on different symmetry considerations have been proposed.

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and so can become negative, leading to the correct pattern of gauge symmetry breaking. Furthermore, because $f_t > f_b$, $\tan \beta > 1$. While this mechanism is indeed very pretty, it is not a complete explanation of the observed scale of spontaneous symmetry breakdown since it requires that $m_0$, the scalar mass at the very large scale $M_X$ be chosen to be $\leq 1$ TeV; in other words, the small dimensionless ratio $m_0/M_X$ remains unexplained. Also, for some choices of model parameters, it is possible to get ground states where $SU(3)_C$ is broken.

Let us compare the model parameters with our list (36) for the MSSM. Here, we start with GUT scale parameters, $m_0$, $m_{\tilde{g}}$, $A_0$, $B_0$ and $\mu_0$. The weak scale parameter $\mu$ (actually, $\mu^2$) is adjusted to give the experimental value of $M_Z$. It is convenient to eliminate $B_0$ in favour of $\tan \beta$ so that the model is completely specified by just a four parameter set (with a sign ambiguity for $\mu$),

$$m_0, m_{\tilde{g}}, \tan \beta, A_0, sgn(\mu),$$

(39)

(together with SM parameters) without the need of additional ad hoc assumptions as in the MSSM.

The mSUGRA model leads to a rather characteristic pattern of particle masses and mixings. We have already seen that the first two generations of squarks are approximately degenerate, while the lighter of the $t$-squarks, and also $\tilde{b}_L$ can be substantially lighter. If $\tan \beta$ is large, the lighter of the two tau states will be considerably lighter than other sleptons. Also, from Eq. (38) it follows that sleptons may be significantly lighter than the first two generations of squarks if $m_{\tilde{g}} \approx m_{\tilde{q}}$, and have comparable masses if squarks are significantly heavier than gluinos. We also see that gluinos can never be much heavier than squarks. Furthermore, because the top quark is very massive, the value of $|\mu|$ obtained from the radiative symmetry breaking constraint generally tends to be much larger than the electroweak gaugino masses, so that the lighter (heavier) charginos and neutralinos tend to be gaugino-like (Higgsino-like). As a result, except when $\tan \beta$ is very large, the additional Higgs bosons $H$, $A$ and $H^\pm$ also become very heavy, and $h$ couples like the SM Higgs boson.

We should stress that while the mSUGRA GUT framework provides a very attractive and economic picture, it hinges upon untested assumptions about the symmetries of physics at very high energies. It could be that the GUT assumption is incorrect though this would then require the unification implied by the observed values of gauge couplings at LEP to be either purely fortuitous, or due to some sort of string unification. It could be that the assumption of universal scalar masses (or $A$-parameters) is wrong. Recall that our arguments for this hinged upon the existence of an additional global $U(N)$ symmetry among the $N$ chi-
ral multiplets. This is, perhaps, reasonable as long as we are near the Planck scale where gravitation presumably dominates gauge or Yukawa interactions. Non-universal masses could result if this $U(N)$ is broken by the explicit introduction of non-canonical kinetic terms for chiral supermultiplets. We should also remember that in the absence of a theory about physics at the high scale, we do not have a really good principle for choosing the scale $M_X$ at which the scalar masses are universal. In practice, most phenomenological calculations set this to be the scale of GUT symmetry breaking where the gauge couplings unify. If, instead, this scale were closer to $M_{Planck}$, the evolution between these scales could result in non-universal scalar masses at $M_{GUT}$: this could have significant impact, particularly on the condition for electroweak symmetry breaking.

This mismatch between the GUT scale and the scale at which scalar masses are assumed to unify also yields a novel source of lepton flavour violation in SUSY GUTS. The point is that if lepton and slepton vertices are not diagonalized by the same rotation, the lepton-slepton-gaugino vertex will not conserve lepton flavour. This is not an issue if sleptons have universal masses since any rotation leaves the identity matrix invariant. Barbieri and Hall have pointed out that if the scale at which scalar masses unify is substantially larger than the GUT scale, radiative corrections due to large top quark Yukawa interactions split the third generation slepton (defined to be the slepton in the same supermultiplet as the top) mass from that of other sleptons. The resulting mismatch of the lepton and slepton mass matrices, they note, leads to leptonic flavour violation that might be detectable in the next round of experiments.

Despite these shortcomings, this framework at the very least should be expected to provide a useful guide to our thinking about supersymmetry phenomenology. In spite of the fact that it is theoretically rather constrained, it is consistent with all experimental and even cosmological

\footnote{We warn the reader that there is a folk theorem that says that effective field theories for the classical ground states of superstring theories cannot have continuous global symmetries except for Peccei-Quinn type symmetries associated with axion-like fields; \textit{i.e.} all other continuous symmetries (except, possibly, an accidental symmetry of lower dimensional operators) are gauge symmetries. If this is the case, then the introduction of a global symmetry to obtain universal SUSY breaking parameters as discussed above would be questionable. This theorem has, however, been proven using 2D superconformal theory on the string world sheet. It is presently unclear whether such a proof would survive the recent theoretical developments where non-perturbative effects play a critical role. The reader may, of course, take the view that the mSUGRA model is not derived from string theory, in which case these considerations are not relevant. It is, perhaps, also worth mentioning that while many supersting models lead to non-universal SUSY breaking parameters at the string scale, universal soft-breaking parameters are at least possible in the so-called dilaton dominated scenario. I am grateful to Fernando Quevedo for discussions about global symmetries in string theory.}
constraints and even, as we will see, contains a candidate for galactic 
and cosmological dark matter. Indeed mSUGRA provides a reasonably 
flexible yet tractable framework whose underlying assumptions, as we 
will see, can be subject to direct tests at future colliders.

6 Decays of Supersymmetric Particles

Before we can discuss signatures via which sparticle production might 
be detectable at colliders, we need to understand how sparticles decay. 
We will assume that $\tilde{R}$-parity is conserved and that $\tilde{Z}_1$ is the LSP.

6.1 Sfermion Decays

We have seen in Sec. 4 that gauginos and Higgsinos couple sfermions 
to fermions. Since we have also assumed that $\tilde{Z}_1$ is the LSP, the decay 
$\tilde{f}_{L,R} \to f \tilde{Z}_1$ ($f \neq t$) is always allowed. Depending on sparticle masses, the decays

$$\tilde{f}_{L,R} \to f \tilde{Z}_i, \quad \tilde{f}_L \to f' \tilde{W}_i$$

(40)

to other neutralinos or to charginos may also be allowed. The chargino 
decay modes of $\tilde{f}_R$ only proceed via Yukawa interactions, and so are neg-
ligible for all but t-squarks, except for large values of $\tan \beta$ for which the 
effects of the bottom and tau Yukawa interactions become important.

Unlike sleptons, squarks also have strong interactions, and so can 
also decay into gluinos via,

$$\tilde{q}_{L,R} \to q \tilde{g},$$

(41)

if $m_\tilde{q} > m_\tilde{g}$. Unless suppressed by phase space, the gluino decay mode 
of squarks dominates, so that squark signatures are then mainly de-
termined by the decay pattern of gluinos. If $m_\tilde{q} < m_\tilde{g}$, squarks, like 
sleptons, decay to charginos and neutralinos. The important thing 
to remember is that sfermions dominantly decay via a two-body mode.

The various partial decay widths can be easily computed using the 
Lagrangian we have described above. Numerical results may be found in 
the literature for both sleptons$^5$ and squarks$^8$ and will not be repeated 
here. The following features, however, are worthy of note:

- The electroweak decay rates are $\sim \alpha m_f$ corresponding to lifetimes 
of about $10^{-22} \left( \frac{m_\tilde{q}}{\text{GeV}} \right)$ seconds. Thus sfermions decay without 
leaving any tracks in the detector. The reader can check that the 
same is true for the decays of other sparticles discussed below.

$^u$The Yukawa coupling of the upper (lower) member of the weak doublet is given by $f_f = \frac{g_\nu_f}{\sqrt{2} M_W \sin \beta} \left( \frac{g_\nu_f}{\sqrt{2} M_W \cos \beta} \right)$, where $m_f$ is the mass of the fermion $f$ which may be either a 
quark or a lepton.
• Light sfermions directly decay to the LSP. For heavier sfermions, other decays also become accessible. Decays which proceed via the larger $SU(2)_L$ gauge coupling are more frequent than those which proceed via the smaller $U(1)_Y$ coupling (we assume Higgs couplings are negligible). Thus, for $\tilde{f}_L$, the decays to charginos dominate unless they are kinematically suppressed, whereas $\tilde{f}_R (f \neq t)$ mainly decays into the neutralino with the largest hypercharge gaugino component.

• Very heavy sleptons (and squarks, if the gluino mode is forbidden) preferentially decay into the lighter (heavier) chargino ($\tilde{f}_L$ only) and the lighter neutralinos $\tilde{Z}_{1,2}$ (the heavier neutralinos $\tilde{Z}_{3,4}$) if $|\mu| (m_\tilde{g})$ is very large. This is because $\tilde{W}_{1,2}$, $\tilde{Z}_{1,2}$ ($\tilde{W}_2, \tilde{Z}_{3,4}$) are the sparticles with the largest gaugino components.

Top Squark Decays: We have seen that $t$-squarks are different in that (i) the mass eigenstates are parameter-dependent mixtures of $\tilde{t}_L$ and $\tilde{t}_R$, (ii) $\tilde{t}_1$, the lighter of the two states may indeed be much lighter than all other sparticles (except, of course, for phenomenological reasons, the LSP) even when other squarks and gluinos are relatively heavy, and (iii) top squarks couple to charginos and neutralinos also via their Yukawa components. As a result the decay patterns of $\tilde{t}_1$ can differ considerably from those of other squarks. Yukawa interactions may also be important for $b$-squarks and $\tau$-sleptons if $\tan \beta$ is very large.

The decay $\tilde{t}_1 \to t\tilde{g}$ will dominate as usual if it is kinematically allowed. Otherwise, decays to charginos and neutralinos, if allowed, form the main decay modes. Since $m_t$ is rather large, it is quite possible that the decay $\tilde{t}_1 \to t\tilde{Z}_1$ is kinematically forbidden, and $\tilde{t}_1 \to b\tilde{W}_1$ is the only tree-level two body decay mode that is accessible, in which case it will obviously dominate. If the stop is lighter than $m_{\tilde{W}_1} + m_b$ and has a mass smaller than about 125 GeV (which, we will see, is in the range of interest for experiments at the Tevatron), the dominant decay mode of $\tilde{t}_1$ comes from the flavour-changing $\tilde{t}_1 \to c\tilde{c}_L$ loop level mixing induced by weak interactions, and the decay $\tilde{t}_1 \to c\tilde{Z}_1$ dominates its allowed tree level decays into (at least) four-body final states. If $m_{\tilde{t}_1} \sim 175 - 225$ GeV, the three-body decays $\tilde{t}_1 \to b\tilde{W}_1$ may be accessible, with the two body decays $\tilde{t}_1 \to b\tilde{Z}_1$ and $\tilde{t}_1 \to t\tilde{Z}_1$ still closed. The rate for this three body decay then has to be compared with the two body loop decay to assess the decay pattern of $\tilde{t}_1$. Unfortunately, this branching fraction is sensitive to the model parameters, and no general statement is possible.
6.2 Gluino Decays

Since gluinos have only strong interactions, they can only decay via

$$\tilde{g} \rightarrow \tilde{q}_{L,R} \bar{q}_{L,R},$$  \hspace{1cm} (42)

where the squark may be real or virtual depending on squark and gluino masses. If $m_{\tilde{g}} > m_{\tilde{q}}$, $\tilde{q}_L$ and $\tilde{q}_R$ are produced in equal numbers in gluino decays (except for phase space corrections from the non-degeneracy of squark masses). In this case, since $\tilde{q}_R$ only decays to neutralinos, neutralino decays of the gluino dominate. If, as is more likely, $m_{\tilde{g}} < m_{\tilde{q}}$, the squark in Eq. (42) is virtual and decays via Eq. (40), so that gluinos decay via three body modes,

$$\tilde{g} \rightarrow q\bar{q}Z_i, q\tilde{q}W_i.$$

(43)

In contrast to the $m_{\tilde{g}} > m_{\tilde{q}}$ case, gluinos now predominantly decay into charginos because of the large $SU(2)_L$ gauge coupling, and also into the neutralino with the largest $SU(2)_L$ gaugino component. For small values of $|\mu| (\ll |\mu_t|)$, these may well be the heavier chargino and the heaviest neutralino; if instead $\mu$ is relatively large, as is generally the case in the mSUGRA framework, the $\tilde{W}_1$ and $\tilde{Z}_2$ decays of gluinos frequently dominate.

We should also point out that our discussion above neglects differences between various squark masses. As we have seen in the last section, however, third generation squarks $t_1$ and $b_1 \sim b_L$ may in fact be substantially lighter than the other squarks. It could even be $\tilde{g} \rightarrow b\bar{b}_1$ and/or $\tilde{g} \rightarrow t\bar{t}_1$ are the only allowed two-body decays of the gluino in which case gluino production will lead to final states with very large $b$-multiplicity, and possibly also hard, isolated leptons from the decays of the top quark or the $t/b$ squark. Even if these decays are kinematically forbidden, branching fractions for decays to third generation fermions may nonetheless be large because of enhanced $t_1$ and $b_1$ propagators (recall that the decay rates roughly depend on $1/m_{\tilde{q}}^4$) with qualitatively the same effect.

Finally, we note that there are regions of parameter space where the radiative decay,

$$\tilde{g} \rightarrow gZ_i,$$  \hspace{1cm} (44)

can be important. This decay, which occurs via third generation squark and quark loops, is typically enhanced relative to the tree-level decays if the neutralino contains a large $h_u$ component (which has large Yukawa couplings to the top family).
6.3 Chargino and Neutralino Decays

Within the MSSM framework, charginos and neutralinos can either decay into lighter charginos and neutralinos and gauge or Higgs bosons, or into $\tilde{f}\bar{f}$ pairs if these decays are kinematically allowed. We will leave it as an exercise for the reader to make a list of all the allowed modes and refer the reader to the literature\cite{30,59} for various formulae and numerical values of the branching fractions. If these two-body decay modes are all forbidden, the charginos and neutralinos decay via three body modes,

$$\tilde{W}_i \rightarrow f\bar{f}Z_j, \tilde{W}_2 \rightarrow f\bar{f}\tilde{W}_1$$  \hfill (45)

$$\tilde{Z}_i \rightarrow f\bar{f}\tilde{Z}_j \text{ or } f\bar{f}\tilde{W}_1,$$  \hfill (46)

mediated by virtual gauge bosons or sfermions (amplitudes for Higgs boson mediated decays, being proportional to fermion masses are generally negligible except when the corresponding Yukawa couplings are enhanced). Typically, only the lighter chargino and the neutralino $\tilde{Z}_2$ decay via three body modes, since the decays $\tilde{Z}_{3,4} \rightarrow \tilde{Z}_1Z$ or $\tilde{Z}_1h$ and $\tilde{W}_2 \rightarrow W\tilde{Z}_1$ are often kinematically accessible. Of course if the $\tilde{Z}_2$ or $\tilde{W}_1$ are heavy enough they will also decay via two body decays: these decays of $\tilde{Z}_2$ are referred to as “spoiler modes” since, as we will see, they literally spoil\cite{11} the clean leptonic signal via which $\tilde{Z}_2$ may be searched for.

For sfermion masses exceeding about $M_W$, $W$-mediated decays generally dominate the three body decays of $\tilde{W}_1$, so that the leptonic branching for its decays fraction is essentially the same as that of the $W$; i.e. 11% per lepton family. An exception occurs when $\mu$ is extremely large so that the LSP is mainly a $U(1)_Y$ gaugino and $\tilde{W}_1$ dominantly an $SU(2)_L$ gaugino. In this case, the $WW_1\tilde{Z}_1$ coupling is considerably suppressed: then, the amplitudes for $\tilde{W}_1$ decays mediated by virtual sfermions may no longer be negligible, even if sfermions are relatively heavy, and the leptonic branching fractions may deviate substantially from their canonical value of 11%.

One may analogously expect that $\tilde{Z}_2$ decays are dominated by (virtual) $Z^0$ exchange if sfermion masses substantially exceed $M_Z$. This is, however, not true since the $Z^0$ couples only to the Higgsino components of the neutralinos. As a result, if either of the neutralinos in the decay $\tilde{Z}_2 \rightarrow \tilde{Z}_1ff$ has small Higgsino components the $Z^0$ contribution may be strongly suppressed, and the contributions from amplitudes involving relatively heavy sfermions may be comparable. This phenomenon is common in the mSUGRA model where $|\mu|$ is generally much larger than

\*The decay to Higgs does not yield leptons, whereas the decay to $Z$ has additional backgrounds from SM $Z$ sources.
the electroweak gaugino masses, and $\tilde{Z}_1$ and $\tilde{Z}_2$ are, respectively, mainly
the hypercharge and $SU(2)_L$ gauginos. If, in addition, $m_{\tilde{q}} \sim m_{\tilde{g}}$, we see from Eq. (38) that sleptons are much lighter than squarks, so that
the leptonic decays $\tilde{Z}_2 \to \ell\bar{\nu}_{\ell} \tilde{Z}_1$, which lead to clean signals at hadron
colliders, may be considerably enhanced. There are, however, other
regions of parameter space, where sleptons are relatively light, but the
amplitudes from virtual slepton exchanges interfere destructively with
the $Z^0$-mediated amplitudes, and lead to a strong suppression of this
declay. Of course, the branching fraction for the three-body decay is
tiny if two-body “spoiler modes” $Z_2 \to ZZ_1$ or $Z_2 \to h\tilde{Z}_1$ are kinematically
allowed. For basically the same reasons the decay $\tilde{Z}_2 \to W_1 f\bar{f}'$
which is mediated by virtual $W$ exchange, even though it is kinematically
disfavoured, can sometimes be competitive with the LSP decay mode of $\tilde{Z}_2$.

We should also mention that, if the parameter $\tan \beta$ is large, bottom
and tau Yukawa interactions can considerably modify the decay pat-
terns of charginos and neutralinos. This happens partly because $\tilde{b}_1$ and
$\tilde{\tau}_1$ masses are reduced with respect to those of other squarks and slep-
tons, but also because coupling to Higgsino components of $\tilde{W}_1$ and $\tilde{Z}_1$ is
not negligible. For $\tan \beta \gtrsim 25 - 30$, the branching fraction for the decay
$\tilde{W}_1 \to \tau \nu \tilde{Z}_1$ can substantially exceed that of $\tilde{W}_1 \to e \nu \tilde{Z}_1$ or $\tilde{W}_1 \to \mu \nu \tilde{Z}_1$
declays. Likewise, $\tilde{Z}_2 \to b\bar{b} \tilde{Z}_1$ may be the dominant decay mode of $\tilde{Z}_2$
while the decay $\tilde{Z}_2 \to \tau \nu \tilde{Z}_1$ can occur much more rapidly than its de-
cay to $e$ or $\mu$. It could even be that the stau becomes so light that the
declays $\tilde{W}_1 \to \tilde{\tau}_1 \nu$ and $\tilde{Z}_2 \to \tilde{\tau}_1 \tau$ become accessible, and being the only
two-body modes dominate the decay of charginos and neutralinos.

Finally, we note that there are regions of parameter space where the
rate for the two body radiative decay

$$\tilde{Z}_2 \to \tilde{Z}_1 \gamma$$

which is mediated by $ff$ and gauge boson-gaugino loops may be compa-
rollable to that for the three body decays. These decays are important
in two different cases: (i) if one of the neutralinos is photino-like and the
other Higgsino-like, both $Z^0$ and sfermion mediated amplitudes are small
since the photino (Higgsino) does not couple to the $Z$-boson (sfermion),
and (ii) both neutralinos are Higgsino-like and very close in mass (this
occurs for small values of $|\mu|$); the strong suppression of the three-body
phase space then favours the two-body decay. We mention here that nei-

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Footnotes:

64 There are, however, other regions of parameter space, where sleptons are relatively light, but the
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two-body modes dominate the decay of charginos and neutralinos.

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ther of these cases is particularly likely, especially within the mSUGRA framework.

6.4 Higgs Boson Decays

Unlike in the SM, there is no clear dividing line between the phenomenology of sparticles and that of Higgs bosons, since as we have just seen, Higgs bosons can also be produced via cascade decays of heavy sparticles. Higgs boson decay patterns exhibit a complex dependence on model parameters. Unfortunately, we will not have time to discuss these here, and we can only refer the reader to the literature. We will, therefore, confine ourselves to mentioning a few points that will be important for later discussion.

In SUGRA models, all but the lightest Higgs scalar tend to be (but are not always) rather heavy and so are not significantly produced either in sparticle decay cascade decays or directly at colliders. An important exception occurs for very large values of $\tan \beta$ for which $A$, and hence, also $H$ and $H^\pm$ may be within the kinematic reach of future colliders or even LEP2.

Within the more general MSSM framework, the scale of their masses is fixed by $m_A$, which is an independent parameter. If $m_A$ is large ($\gtrsim 200$ GeV), $h$ (which has a mass smaller than $\sim 130$ GeV) behaves like the SM Higgs boson, while $H$, $A$ and $H^\pm$ are approximately decoupled from vector boson pairs. The phenomenology is then relatively simple: the decay $h \rightarrow b\bar{b}$ which occurs via $b$-quark Yukawa interactions dominates, unless charginos and/or neutralinos are also light; then, decays of $h$ into neutralino or chargino pairs, which occur via the much larger gauge coupling, is dominant unless $\tan \beta$ is very large. The invisible decay $h \rightarrow Z_1Z_1$, is clearly the one most likely to be accessible, and has obvious implications for Higgs phenomenology. These supersymmetric decay modes are even more likely for the heavier Higgs bosons, particularly if their decay to $t\bar{t}$ pairs is kinematically forbidden; this is especially true for $h$ which cannot decay to vector boson pairs, but also for $H$ since its coupling to $V V$ pairs ($V = W, Z$) is suppressed when it is heavy. The decays $A \rightarrow hZ$ and $H \rightarrow hh$ can be important, while $H \rightarrow AA$ is usually inaccessible. Finally, charged Higgs bosons $H^+$ mainly decay via the $tb$ mode unless this channel is kinematically forbidden. Then, they mainly decay via $H^+ \rightarrow Wh$, or if this is also kinematically forbidden, via $H^+ \rightarrow c\bar{s}$ ( $\tan \beta \sim 1.2$) or $H^+ \rightarrow \tau\nu$ ( $\tan \beta > 1.2$).

7 Sparticle Production at Colliders

Since $R$-parity is assumed to be conserved, sparticles can only be pair produced by collisions of ordinary particles. At $e^+e^-$ colliders, sparticles
such as charged sleptons and sneutrinos, squarks and charginos) with
significant couplings to either the photon or the $Z$-boson can be produced
via $s$-channel $\gamma$ and $Z$ processes, with cross sections comparable with
$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, except for kinematic and statistical factors. Selectron
and electron sneutrino production also occurs via $t$-channel neutralino
and chargino exchange, while sneutrino exchange in the $t$-channel will
contribute to chargino pair production. Neutralino production, which
proceeds via $Z$ exchange in the $s$-channel and selectron exchange in
the $t$ and $u$ channels, may be strongly suppressed if the neutralinos are
gaugino-like and selectrons are relatively heavy. Cross section formulæ
as well as magnitudes of the various cross sections may be found e.g. in
Baer et. al.

At hadron colliders, the situation is somewhat different. Since sparticle
production is a high $Q^2$ process, the underlying elementary SUSY
process is the inelastic collision of quarks and gluons inside the proton. In
other words, it is the hard scattering partonic cross section that is
computable within the SUSY framework. This cross section is then con-
voluted with parton distribution functions to obtain the inclusive cross
section for SUSY particle production. Thus, unlike at electron-positron
colliders, only a fraction of the total centre of mass energy is used for
sparticle production. The balance of the energy is wasted in the un-
derlying low $p_T$ event which only contaminates the high $p_T$ signal of
interest.

Squarks and gluinos, the only strongly interacting sparticles, have
the largest production cross sections unless their production is kinemat-
ically suppressed. These cross sections are completely determined in
terms of their masses by QCD and do not depend on the details of the
supersymmetric model. QCD corrections to these have also been com-
cuted. Squarks and gluinos can be also be produced in association
with charginos or neutralinos via diagrams involving one strong and one
electroweak vertex. Finally, $\tilde{W}_i$ and $\tilde{Z}_j$ can be produced by $q\bar{q}$ annihi-
lation via processes with $W$ or $Z$ exchange in the $s$-channel, or squark
exchange in the $t$ (and, for neutralino pairs only, also the $u$) channel.

The cross sections for various processes at a 2 TeV $p\bar{p}$ collider (cor-
responding to the energy of the Main Injector (MI) upgrade of the
Tevatron) are illustrated in Fig. 3, while those for a 14 TeV $pp$
collider (the LHC) are shown in Fig. 4. We have illustrated our results
for (a) $m_{\tilde{q}} = m_{\tilde{g}}$, and (b) $m_{\tilde{q}} = 2m_{\tilde{g}}$ and fixed other parameters at the
representative values shown. These figures help us decide what to search
for. While squarks and gluinos are the obvious thing to focus the ini-
tial search on, we see from Fig. 3 that at even the MI (and certainly at
any higher luminosity upgrade that might be envisioned in the future),
the maximal reach is likely to be obtained via the electroweak produc-
tion of charginos and neutralinos, provided of course that their decays

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Figure 3: Total cross sections for various sparticle production processes by $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV.
lead to detectable signals. In contrast, we see from Fig. that gluino (and, possibly, squark) production processes offer the best opportunity for SUSY searches at the LHC for gluino masses up to 1 TeV even if squarks are very heavy.

8 Simulation of Supersymmetry Events

Once produced, sparticles rapidly decay into other sparticles until the decay cascade terminates in a stable LSP. It is the end products of these decays that are detectable in experiments and which, it is hoped, can provide experimental signatures for supersymmetry. The evaluation of these signatures obviously entails a computation of the branching fractions for the decays of all the sparticles, and further, keeping track of numerous cascade decay chains for every pair of parent sparticles. Many groups have constructed computer programs to calculate these decay processes. For any set of MSSM parameters (or alternatively, for a SUGRA parameter set), a public access program known as ISASUSY (ISASUGRA) which can be extracted from the Monte Carlo program ISAJET lists all sparticle and Higgs boson masses as well as their decay modes along with the corresponding partial widths and branching fractions.

Event generator programs provide the link between the theoretical framework of SUSY which provides, say, cross sections for final states with quarks and leptons, and the long-lived particles such as π, K, γ, e, µ etc. that are ultimately detected in real experiments. Many authors have combined sparticle production and decay programs to create parton level event generators which may be suitable for many purposes. More sophisticated generators include other effects such as parton showers, heavy flavour decays, hadronization of gluons and quarks, a model of the underlying event, etc. These improvements have significant impact upon detailed simulations of, for instance, the jets plus isolated multi-lepton signal from squark and gluino production at the LHC. General purpose SUSY event generators available today are: ISAJET, SPYTHIA.

\footnote{This conclusion crucially depends on the validity of the gaugino mass unification condition Eq. which implies that gluinos are much heavier than $\tilde{W}_1$ and $\tilde{Z}_2$.}

\footnote{The current version of ISAJET (v. 7.29) allows the reader to input independent soft SUSY breaking masses for each of the fermion multiplets as well as independent masses for the three gauginos. Unless the user explicitly specifies, ISAJET assumes that sfermions of the first two generations with the same gauge quantum numbers have the same soft SUSY breaking masses; it also incorporates gaugino mass unification as the default. In addition the user has to specify $\mu$, $\tan \beta$, $m_A$ along with the three third generation $A$-terms. In other words, all thirty MSSM parameters but the six $A$-terms for the first two generations, which are usually irrelevant for phenomenology can be independent inputs. This allows for simulation of a large variety of theoretical scenarios.}
Figure 4: Total cross sections for various sparticle production processes by $pp$ collisions at $\sqrt{s} = 14$ TeV.
and SUSYGEN\textsuperscript{78} ($e^+e^-$ collisions only). A detailed discussion of these programs and their virtues and shortcomings is beyond the scope of these Lectures. We will instead refer the interested reader to the literature\textsuperscript{79} and to the documentation that accompanies these codes.

9 Observational Constraints on Supersymmetry

The non-observation of any supersymmetric signal at either LEP\textsuperscript{80} or at the Tevatron\textsuperscript{81,82,83} provides the most direct lower limits on sparticle masses. Indirect limits may come from virtual effects of SUSY particles on rare processes (e.g. flavour changing neutral currents or proton decay) or from cosmological considerations such as an over-abundance of LSP’s, resulting in a universe that would be younger than the age of stars. While these indirect limits can be important, they are generally sensitive to the details of the model: non-observation of loop effects could be a result of accidental cancellation with some other new physics loops (so care must be exercised in extracting limits on sparticle masses); proton decay\textsuperscript{84} is sensitive to assumptions about GUT scale physics while the cosmological constraints\textsuperscript{85} can be simply evaded — the price is the loss of a promising dark matter candidate — by allowing a tiny violation of $R$-parity conservation which would have no impact on collider searches. We do not mean to belittle these constraints which lead to important bounds in any given framework (for instance, minimal SUGRA $SU(5)$), but should also recognise that these bounds are likely to be more model-dependent than direct constraints from collider experiments. It is, however, only for reasons of time that we will confine ourselves to direct limits from colliders.

The cleanest limits on sparticle masses come from experiments at LEP. The agreement of $\Gamma_Z$ with its expectation in the SM gives\textsuperscript{2} essentially model-independent lower limits of 30-45 GeV on the masses of charginos, squarks, sneutrinos and charged sleptons whose couplings to $Z^0$ are fixed by gauge symmetry. These limits\textsuperscript{a} do not depend on how sparticles decay. Likewise, the measurement of the invisible width of the $Z^0$ which gives the well-known bound on the number of light neutrino species, yields a lower limit on $m_{\tilde{\nu}}$ only 2-5 GeV below $M_{Z^0}$ if the sneutrino decays invisibly via $\tilde{\nu} \rightarrow \nu Z_1$, even if only one of the sneutrinos is light enough to be accessible in $Z^0$ decays.\textsuperscript{b} In contrast, the

\textsuperscript{a}The same considerations also exclude spontaneous $R$-violation via a vev of a doublet sneutrino because the associated Goldstone boson sector would then have gauge couplings to $Z^0$ and make too large a contribution to $\Gamma_Z$.

\textsuperscript{b}Experiments searching for neutrino-less double beta decay are designed to detect the recoil of the nucleus. If stable sneutrinos are the LSP and their density is large enough to form all of the galactic dark matter their flux would be high enough to be detectable via elastic scattering from nuclei in these experiments. As a result, sneutrinos with masses between 12-20 GeV
bounds on neutralino masses are very sensitive to the model parameters because for large $|\mu|$, as we have already pointed out, the neutralino may be dominantly a gaugino with strongly suppressed couplings to the $Z^0$.

LEP experimentalists also perform direct searches for sparticles whose decays frequently lead to extremely characteristic final states. For instance, slepton (squark) pair production followed by the direct decay of the sfermion to the LSP leads to a pair of hard, acollinear leptons (jets) together with $\not{p}_T$. Chargino production can lead to events with acollinear jet pairs, a lepton + jet + $\not{p}_T$ and also acollinear leptons + $\not{p}_T$. Such event topologies are very distinctive and do not occur in the SM if the centre of mass energy is below the $WW$ threshold. Thus the observation of just a handful of such events would suffice to signal new physics. During the past year the LEP energy has been increased in steps from $M_Z$ to 130-140 GeV to 161 GeV and beyond. Currently, the highest energy of LEP operation is 172 GeV. For $\sqrt{s}>2M_W$, $WW$ events contaminate the SUSY signal. Strategies for separating the signal from SM background are discussed in the next Section.

From a non-observation of any SUSY events in the data sample of $11\, pb^{-1}$ that has been accumulated at 172 GeV, lower limits $m_{\tilde{W}}^1 > 84-86$ GeV, $m_{\tilde{e}_R} > 70$ GeV (the bound on the smuon mass is a little weaker) have already been deduced. A $t$-squark below 63-75 GeV, depending on the stop mixing angle is also excluded, assuming $\tilde{t}_1 \rightarrow c\tilde{Z}_1$. Finally, assuming that the GUT unification condition for $M_1$ and $M_2$ is valid, the L3 and ALEPH collaborations have deduced a 95% lower limit, $m_{\tilde{Z}_1} > 24.6$ GeV, if $m_\phi \geq 200$ GeV. With a larger data sample (as will be expected in the next run) LEP2 experiments should be able to probe charged sparticles up to 80-90% of the kinematic limit.

The search for squarks and gluinos is best carried out at hadron colliders by searching for $E_T$ events from $\tilde{q}\tilde{q}$, $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{g}$ production. The final states from the cascade decays of gluinos and squarks leads to events consisting of several jets plus possibly leptons and $E_T$. For an integrated luminosity of about 10-20 pb$^{-1}$ on which the analyses of the Run IA of the CDF and D0 experiments are based, the classic $E_T$ channel offers the best hope for detection of supersymmetry. The non-observation of $E_T$ events above SM background expectations (after cuts to increase the signal relative to background) allows Tevatron experimentalists to infer a lower limit of 173 GeV on $m_{\tilde{g}}$. This bound improves to 229 GeV if squarks and gluinos are assumed to have the same mass. Since then, the CDF and D0 experiments have collectively accumulated and about 1 TeV are excluded. The Kamiokande experiment from a non-observation of high energy solar neutrinos produced by the annihilation of gravitationally accumulated sneutrinos in the sun exclude $3\text{ GeV} \leq m_\tilde{\nu} \leq 25\text{ GeV}$. These limits, when combined with the LEP bounds clearly disfavour the sneutrino as the stable LSP.
about \( \sim 200 \, pb^{-1} \) of integrated luminosity, and should begin to be sensitive to various multilepton signatures which we will discuss when we address prospects for SUSY searches in the future. We should mention though that already with the data sample of Run I, non-observation of any events in the dileptons + jets + \( E_T \) channel allows these collaborations to infer bounds very close to bounds from the \( E_T \) searches. The D0 Collaboration has also excluded \( \tilde{t}_1 \) with a mass between 60 and 90 GeV, assuming \( \tilde{t}_1 \rightarrow c\tilde{Z}_1 \) and that \( m_{\tilde{t}_1} - m_{\tilde{Z}_1} \) is sizeable.

Before closing this section, we briefly remark about potential constraints from "low energy" experiments, keeping in mind that these may be sensitive to model assumptions. The measurements of the inclusive \( b \rightarrow s\gamma \) decay by the CLEO experiment and its agreement with SM expectations constrain the sum of SUSY contributions to this process. Supersymmetry also allows for new sources of CP violation in gaugino masses or \( A \)-parameters. These phases, which must be smaller than \( \sim 10^{-3} \) in order that the electric dipole moment of the neutron not exceed its experimental bound, are set to zero in the MSSM.

Finally, we note that because SUSY, unlike technicolour, is a decoupling theory, the agreement of the LEP data with SM expectations is not hard to accommodate. We just have to make the sparticles heavier than 100-200 GeV. This would, of course, make it difficult to accommodate "anomalies" in the LEP data unless some sparticles are rather light. The anomalies of yesteryear, however, seem to be fading away.

10 Searching for Supersymmetry at Future Colliders and Super.colliders

10.1 \( e^+e^- \) Colliders

We saw in the last Section that the LEP2 collider has been successfully operated above the \( WW \) threshold. During the next run due to begin in July 1997, experiments should accumulate \( \sim 100 \, pb^{-1} \) of integrated luminosity at \( \sqrt{s} = 184 \) GeV, and so, should be able to probe charginos and sleptons up to about 85-90 GeV. The signals for sparticles are much the same as discussed in the last Section. The significant difference is that while SM backgrounds can be easily removed below the \( WW \) threshold, the separation of the SUSY signal from \( W \)-pair production requires more effort. This should not be very surprising since the \( W \) is a heavy particle and its decays can lead to both acollinear dilepton + \( E_T \) as well as \( jets + \ell + E_T \) and \( jets + E_T \) event topologies. Another possible complication to be kept in mind as we search for heavier sparticles is that cascade decay channels may begin to open up. This should not pose too much of a problem, however, since the energy has been increased in

\(*The related process \( b \rightarrow s\ell \bar{\ell} \) has also been discussed.*
stages. For example, one would expect to see chargino production before the production of sleptons which are heavy enough to decay to charginos sets in.

Signals for sparticle production at LEP2 have been studied in great detail assuming that sparticles decay directly to the LSP. Below the $WW$ threshold, they are readily detectable in exactly the same way as at LEP. Above that, the production of $W^+W^-$ pairs, which has a very large cross section $\sim 18$ pb (compared to 0.2 pb for smuons and $\sim 10$ pb for charginos with mass about $M_W$) is a formidable background. The situation is not as bad as it may appear on first sight. For $WW$ events to fake sleptons, both $W$'s have to decay to the particular flavour of leptons, which reduces background by two orders of magnitude. Further rejection of background may be obtained by noting that while slepton events are isotropic, the leptons from $W$ decay exhibit strong backward-forward asymmetry. Thus by selecting from the sample of acollinear $\mu^+\mu^-$ events those events where the fast muon in the hemisphere in the $e^-$ beam direction has the opposite sign to that expected from a muon from $W$ decay, it is possible to reduce the background by a factor of five, with just 50% loss of signal.

The strategy for charginos is more complicated and will not be detailed here. We will only mention that here the clean environment of electron-positron colliders plays a crucial role. The idea is to make use of the kinematic differences between the two-body decay of the $W$ into a massless neutrino, and the three body decay of the chargino into the massive LSP. Using the cuts detailed in Ref.97, it should be possible to detect charginos up to within a few GeV from the kinematic limit in the mixed lepton plus jet channel.

Neutralino signals, as we should by now anticipate, are sensitive to model parameters. A recent analysis within the framework of the SUGRA models describes strategies to optimize these signals, and also separate them from other SUSY processes. This analysis also discusses signals from cascade decays of sparticles.

Higher energy electron-positron colliders will almost certainly be linear colliders, since synchrotron radiation loss in a circular machines precludes the possibility of increasing the machine energy significantly beyond that of LEP2. Several laboratories are evaluating the prospects for construction of a 300-500 GeV collider, whose energy may later be increased to 1 TeV, or more: these include the Next Linear Collider (NLC) program in the USA, the Japanese Linear Collider (JLC) program in Japan, the TESLA and CLIC programs in Europe, and VLEPP in the former Soviet Union. The search for the lightest charged sparticle, be it the chargino or the slepton (or perhaps the $t_1$) should proceed along the same lines as at LEP2 and discovery should be possible essentially all the way to the kinematic limit. Of course, because
production cross sections rapidly decrease with energy, a luminosity of 10-30 fb$^{-1}$/yr will be necessary.

While these studies have conclusively demonstrated that the lightest of the “visible” sparticles will be easily discovered at the Linear Collider, cascade decays have to be incorporated for a study of the heavier sparticles. Recent studies[103,104] within the mSUGRA framework have examined the prospects for discovering various sparticles at a 500 GeV Linear Collider. It is shown that with an integrated luminosity of $\sim 20$ fb$^{-1}$, it should be possible to discover charged sleptons (and also sneutrinos if they decay visibly), charginos, $\tilde{t}_1$ and squarks essentially all the way up to the kinematic limit even if these do not directly decay to the LSP. A machine with a centre of mass energy of about 700-1000 GeV should be able to search for charginos up to 350-500 GeV, and so, assuming the gaugino mass unification condition, will cover the parameter space of weak scale supersymmetry.

It is also worth mentioning that one can exploit the availability of polarized beams to greatly reduce SM backgrounds: for example, the cross section for $WW$ production which is frequently the major background is tiny for a right-handed electron beam. While the availability of polarized beams and the clean environment of $e^+e^-$ collisions clearly facilitates the extraction of the signal, we will see below that these capabilities play a really crucial role for the determination of sparticle properties which, in turn, serves to discriminate between models. The availability of polarization does not, however, appear to be crucial for SUSY discovery.[105]

We should stress that $e^+e^-$ colliders are ideal facilities to search for Higgs bosons. At LEP2, one can typically search for Higgs bosons with a mass up to about $\sqrt{s} \approx 100$ GeV; an $e^+e^-$ collider operating at 300 GeV would be virtually guaranteed to find one of the Higgs bosons if the MSSM framework, with its weakly coupled Higgs sector, is correct, although it may not be possible to distinguish this from the Higgs boson of the SM. Indeed if no Higgs boson is discovered at a 500 GeV Linear Collider, many accepted ideas about unification of interactions may have to be re-evaluated. In contrast, we will see that if the LHC is operated only at its lower value of the design luminosity (10 fb$^{-1}$/yr), the discovery of a Higgs boson cannot be guaranteed even with relatively optimistic (but not unrealistic) assumptions about detector capabilities. It appears that several years of LHC operation with a luminosity of 100 fb$^{-1}$/yr are necessary to reasonably ensure the discovery of at least one of the MSSM Higgs bosons.

10.2 Future Searches at Hadron Colliders

Tevatron Upgrades
The CDF and D0 experiments have already collected an integrated luminosity of about 200 pb$^{-1}$, to be compared with 10-20 pb$^{-1}$ for the data set on which the $E_T$ analyses described in the last Section were based. It is thus reasonable to explore whether an analysis of this data can lead to other signatures for supersymmetry. Of course, the size of the data sample will increase by yet another order of magnitude after about a year of MI operation, and by significantly more if the TeV33 upgrade, with its design luminosity of $\sim 10^{-25}$ fb$^{-1}$/yr, comes to pass.

Gluinos and Squarks: While the increase in the data sample will obviously result in an increased reach via the $E_T$ channel$^{107,108}$, we have already seen that the cascade decays of gluinos and squarks lead to novel signals ($n$ jets plus $m$ leptons plus $E_T$) via which one might be able to search for SUSY. Since the gluino is a Majorana particle, it decays with equal likelihood to positive or negative charginos: the leptonic decays of the chargino can then lead to events with two isolated same-sign (SS) charged leptons$^{110,111}$, together with jets plus $E_T$.

If instead of one of the charginos we have a leptonically decaying neutralino, trilepton event topologies result. While other topologies will also be present, the SS and 3$\ell$ events are especially interesting because (after suitable cuts$^{112,113}$) the SM physics backgrounds$^d$ are estimated to be 2.7 fb and 0.7 fb, respectively, for $m_t = 175$ GeV. The corresponding signal cross sections, together with the cross sections in other channels, are illustrated in Fig. 5 which has been obtained using ISAJET 7.13. These include signals from all sparticle sources, not just gluinos and squarks.

Before closing, we remind the reader that for large values of $\tan\beta$ charginos and neutralinos preferentially decay into the third generation particles. In fact, we have seen that they may even exclusively decay to the stau family. In this case, multilepton signals may be greatly reduced, although there are potentially new signatures involving $b$-jets and $\tau$-leptons via which to search for SUSY. Prospects for these searches are currently under investigation.

Charginos and Neutralinos: The electroweak production of charginos

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$^d$In addition, there are always detector-dependent instrumental backgrounds from misidentification of jets or isolated pions as leptons, mismeasurement of the sign of the lepton charge, etc. that a real experimentalist has to contend with.
Figure 5: Cross sections (in fb) at the Tevatron ($\sqrt{s} = 1.8$ TeV) for various event topologies after cuts described in Ref. 112 from which this figure is taken. We take $\mu = -m_{\tilde{g}}$, $\tan \beta = 2$, $A_t = A_b = -m_{\tilde{q}}$ and $m_A = 500$ GeV. The $E_T$ cross sections are labelled with diamonds, the 1-$\ell$ cross sections with crosses, the $\ell^+\ell^-$ cross sections with x's and the SS ones with squares. The dotted curves are for the 3-$\ell$ cross sections while the dashed curves show the cross sections for 4-$\ell$ events. For clarity, error bars are shown only on the lowest lying curve; on the other curves, these error bars are significantly smaller. The $m_{\tilde{g}} = 150$ GeV case in b is excluded even by the LEP constraints on the $Z$ width, since in this case the sneutrino mass is just 26 GeV.
and neutralinos, we have seen, offers yet another channel for probing supersymmetry, the most promising of which is the hadron-free trilepton signal from the reaction \( p\bar{p} \rightarrow \tilde{W}_1 \tilde{Z}_2 X \), where both the chargino and the neutralino decays leptonically. In fact, we saw in Fig. 3 that for very large integrated luminosities, this channel potentially offers the maximal reach for supersymmetry (since the opposite sign (OS) dilepton signal from \( \tilde{W}_1 \tilde{W}_1 \) production suffers from large SM backgrounds from \( WW \) production). It was first emphasized by Arnowitt and Nath\(^{114}\) that, with an integrated luminosity of \( \sim 100 \text{ pb}^{-1} \), this signal would be observable at the Tevatron even if resonance production of \( \tilde{W}_1 \tilde{Z}_2 \) is suppressed. A subsequent analysis\(^{111}\) showed that the signal may even be further enhanced in some regions of parameter space due to enhancements in the \( \tilde{Z}_2 \) leptonic branching fractions, as discussed in Sec. 6. Detailed Monte Carlo studies\(^{115},\,107,\,65,\,66,\,108\) including effects of experimental cuts were performed to confirm that Tevatron experiments should indeed be able to probe charginos via this channel. Indeed analyses\(^{116}\) by the CDF and D0 collaborations, from a non-observation of a signal in this channel, have obtained upper limits on the trilepton cross section. The resulting limit on the chargino mass is below that obtained from LEP. This analysis may be viewed as leading to the best (direct) lower limit on \( m_{\tilde{Z}_2} \) over some ranges of model parameters. It is possible that in the future Tevatron experiments will probe parameter regions that may not be accessible at LEP2.

We stress that the trilepton signal, which depends on the neutralino branching fractions, is sensitive to the model parameters, and it is not possible to simply state the reach in terms of the mass of the chargino. For favourable values of parameters, experiments at the MI should be able to probe charginos heavier than 100 GeV, corresponding to \( m_{\tilde{g}} \gtrsim 300 \sim 350 \text{ GeV} \) (at TeV33, up to \( \sim 500-600 \text{ GeV} \) where the two-body spoiler decays of \( \tilde{Z}_2 \) become accessible, and for light sleptons, even up to 600-700 GeV); on the other hand, there are other regions of parameter space where the leptonic branching fraction of the neutralino is strongly suppressed,\(^{65,\,66}\) and there is no signal for charginos as light as 45-50 GeV even at the TeV33 upgrade of the Tevatron. The signal may also be suppressed\(^{69}\) because charginos and neutralinos decay exclusively into stau (and \( \tilde{\nu}_\tau \)). Thus, while this channel can probe significant regions of the parameter space of either the MSSM or SUGRA, the absence of any signal in this channel will not allow one to infer a lower limit on \( m_{\tilde{W}_1} \).

Top Squarks: We have seen that \( \tilde{t}_1 \), the lighter of the two top squarks, may be rather light so that it may be pair-produced at the Tevatron even if other squarks as well as the gluino are too heavy. If its decay to chargino is allowed, the leptonic decay of one or both stops leads to events...
with one or two isolated leptons together with jets plus $E_T$ — exactly the same event topologies as for the top quark search. Thus $t\bar{t}$ production is the major background to the $t$-squark search. Because $\sigma(t\bar{t}) \sim 10\sigma(t_1\bar{t}_1)$ for a top and stop of the same mass, stop signals are detectable at the Tevatron only if the stop is considerably lighter than the top. In an early analysis, it was shown that with an integrated luminosity of around 100 pb$^{-1}$, the stop signal should be detectable at the Tevatron if $m_{\tilde{t}_1} \lesssim 100$ GeV, provided $b$-jets can be adequately tagged. Since then, LEP experiments have obtained strong bounds on the chargino mass, leaving only a small range of parameters where this analysis is applicable. The D0 collaboration have searched their data sample of $\sim 75$ pb$^{-1}$ for $\tilde{t}_1\tilde{t}_1 \rightarrow b\tilde{W}_1^+b\tilde{W}_1^- \rightarrow bb\gamma e^-e^-$ events and found that an order of magnitude larger data sample is needed to attain the sensitivity that is needed for this search.

If the chargino is heavy, the stop will instead decay via $\tilde{t}_1 \rightarrow c\tilde{Z}_1$ and stop pair production will be signalled by dijet plus $E_T$ events, and hence looks like the squark signal, but without any cascaded decays. An analysis by the D0 collaboration excludes 60 GeV $< m_{\tilde{t}_1} < 90$ GeV if the LSP mass is smaller than 25-50 GeV.

Mrenna et. al. using cuts optimized to detect heavier stops, find that, with an integrated luminosity of $2$ fb$^{-1}$, it should be possible to explore stops as heavy as 160 GeV if they decay via the chargino mode. They claim a reach of 200 GeV with a data sample of $25$ fb$^{-1}$ that may be available at TeV33. If chargino is too heavy for the decay $\tilde{t}_1 \rightarrow b\tilde{W}_1$ to be accessible but the stop mass is in the 180-250 GeV range, the three body decay $\tilde{t}_1 \rightarrow bW\tilde{Z}$ may be kinematically accessible. Since its branching ratio is sensitive to model parameters, one has to examine how this decay compares to the loop decay $\tilde{t}_1 \rightarrow c\tilde{Z}_1$ in order to assess the viability of this signal.

Sleptons: The best hope for slepton detection appears to be via the clean OS dilepton plus $E_T$ channel. But even here, there is a large irreducible background from WW production as well as possible contamination of the signal from other SUSY sources such as chargino pair production. It was concluded that at the MI it would be very difficult to see sleptons from off-shell $Z$ production, i.e. if $m_{\tilde{\ell}} \lesssim 50$ GeV. Sleptons in this mass range are obviously already excluded by LEP experiments. At TeV33, experiments would probe sleptons with masses up to about 100 GeV, given an integrated luminosity of $25$ fb$^{-1}$.

\textsuperscript{4}M. Mangano (private communication) has also performed this analysis with a more detailed simulation and more realistic assumptions about $b$-tagging capabilities.

\textsuperscript{f}The sensitivity would be improved when channels involving $e$ and $\mu$ are analysed and combined. Enhanced $b$-tagging capability that should be available during the next run will also improve the sensitivity.
**SUSY Searches at the LHC**

While it is certainly possible that SUSY may be discovered at a luminosity upgrade of the Tevatron, we have seen that there are parameter ranges where a SUSY signal may evade detection even if sparticles are not very heavy. In order to cover the complete parameter-space of weak scale SUSY either a linear collider operating at a centre of mass energy \( \sim 0.7 - 1 \) TeV or the LHC is necessary.

We see from Fig. 4 that, at the LHC, squarks and gluinos dominate sparticle production up to gluino masses beyond 1 TeV. It is thus reasonable to focus most attention on these although, of course, signals should be looked for in all possible channels. For reasons of brevity, and because the ideas involved in LHC searches are qualitatively similar to those described above, we will content ourselves with just presenting an overview of the LHC reach, and refer the interested reader to the vast amount of literature that already exists for details.

As before, the cascade decays of gluinos and squarks result in \( n \)-jet plus \( m \)-leptons plus \( E_T \) events \(^{12,13,14}\) where \( m = 0 \) corresponds to the classic \( E_T \) signal. Of the multilepton channels, the SS and \( m \geq 3 \) channels suffer the least from SM backgrounds. It is worth keeping in mind that at the LHC, many different sparticle chains contribute to a particular event topology, and further, that the dominant production mechanism for any particular channel depends on the model parameters. For instance, gluino pair production (with each of the gluinos decaying to a chargino of the same sign) is generally regarded as the major source of SS dilepton events; notice, however, that the reaction \( pp \rightarrow \tilde{b}_L \tilde{b}_L X \rightarrow t\tilde{W}^-_1 t\tilde{W}^+_1 X \) may also be a copious source of such events, since now the leptons can come either from top or chargino decays (recall that we had noted that \( \tilde{b}_L \) may be relatively light). It is, therefore, necessary to simultaneously generate all possible sparticle processes in order to realistically simulate a signal in any particular event topology. This is possible using ISAJET. However, this raises another issue which is especially important at the LHC. If we see a signal in any particular channel, can we uncover its origin? We will return to this later, but for now, focus ourselves on the SUSY reach of the LHC.

The ATLAS collaboration \(^4\) at the LHC has done a detailed analysis of the signal in the \( E_T \) as well as in the SS dilepton channels. They found that gluinos as light as 300 GeV should be easily detectable in the \( E_T \) channel. \(^5\) Then requiring rather stiff cuts, \( E_T > 900 \) GeV, \( p_T(jet_1, jet_2, jet_3) > 200 \) GeV, \( p_T(jet_4) > 100 \) GeV along with a cut \( S_T > 0.2 \) on the transverse sphericity, they find that with an integrated

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\(^9\)The capability of LHC experiments to detect a signal from relatively light gluinos is important to ensure that there is no window of masses where SUSY signals may escape detection at both the Tevatron MI and the LHC.
luminosity of 10 $fb^{-1}$, it should be possible to search for gluinos with a mass up to 1.3 TeV (2 TeV) for $m_{\tilde{q}} = 2m_{\tilde{g}}$ ($m_{\tilde{q}} = m_{\tilde{g}}$). This reach is altered by about $\pm 300$ GeV if the integrated luminosity is changed by an order of magnitude. Very similar results for the reach in the $B_T$ channel have also been obtained within the context of the SUGRA framework, although the event selection criteria used are quite different.

In the same-sign dilepton channel, the ATLAS collaboration concludes that the reach of the LHC will be 900-1400 GeV (for $m_{\tilde{q}} = 2m_{\tilde{g}}$) or 1100-1800 GeV (for $m_{\tilde{q}} = m_{\tilde{g}}$), where the lower (higher) number corresponds to a luminosity of 1 $fb^{-1}$ (100 $fb^{-1}$).

Prospects for SUSY detection in the SS and other multilepton channels (with or without real Z bosons) have also been discussed. An analysis of the multilepton signals within the context of SUGRA models has also been performed. The greatest SUSY reach is obtained in the $1\ell$ channel. The reason is that a large fraction of events from a heavy gluino or squark contain at least one lepton from cascade decays. By exploiting the presence of hard jets and large $E_T$ in SUSY events, these authors have been able to devise cuts to reduce SM backgrounds from $W \rightarrow \ell\nu$ and $t\bar{t}$ production to a manageable level. Assuming an integrated luminosity of 10 $fb^{-1}$, a gluino mass reach of $\sim 2.3$ (1.6) TeV is claimed for $m_{\tilde{g}} \sim 400-500$ GeV ($m_{\tilde{g}} \sim 1-1.3$ TeV). More importantly, it has been shown that for $m_{\tilde{g}} \leq 400-500$ GeV ($m_{\tilde{g}} \leq 1-1.3$ TeV), there should be an observable signal in several (OS, SS, 3$\ell$) channels if $\tan \beta < \sim 10$. Otherwise, any signal in the $B_T$ or $1\ell$ channel cannot be attributed to gluino and squark production in the mSUGRA framework. For very large values of $\tan \beta$, sparticle decay patterns may be considerably modified, and the situation needs to be reassessed. We do not, however, expect SUSY would evade detection at the LHC.

Within the SUGRA framework, the LHC should, in the clean trilepton channel, be able to probe $\tilde{W}_1\tilde{Z}_2$ production all the way up to where spoiler modes of $\tilde{Z}_2$ become accessible if $\mu < 0$ and $\tan \beta$ is not too large. Then it is possible to find a set of cuts that cleanly separate the $\tilde{W}_1\tilde{Z}_2$ event sample from both SM backgrounds as well as other sources of SUSY events. This will prove to be important later. For positive values of $\mu$, signals are readily observable for rather small and large values of $m_{\tilde{0}}$ in the intermediate range 400 GeV $\lesssim m_{\tilde{0}} \lesssim 1000$ GeV, this signal is suppressed because of the suppression of the leptonic $\tilde{Z}_2$ branching fraction emphasized earlier.

As at Tevatron upgrades, the OS dilepton channel offers the best opportunity for slepton searches. At the LHC, it should be possible to detect sleptons up to about 250-300 GeV, although excellent jet vetoing capability will be needed to detect the signal for the highest masses.
Table 1: Estimates of the discovery reach of various options of future hadron colliders. The signals have mainly been computed for negative values of $\mu$. We expect that the reach in especially the all $\rightarrow 3\ell$ channel will be sensitive to the sign of $\mu$.

| Signal | Tevatron II | Main Injector | TeV33 | LHC |
|--------|-------------|---------------|-------|-----|
| $H_T(q \gg \tilde{g})$ | $\tilde{g}(210)/\tilde{g}(185)$ | $\tilde{g}(270)/\tilde{g}(200)$ | $\tilde{g}(340)/\tilde{g}(200)$ | $\tilde{g}(1300)$ |
| all $\rightarrow 3\ell$ ($\tilde{q} \gg \tilde{g}$) | $\tilde{g}(180)$ | $\tilde{g}(260)$ | $\tilde{g}(430)$ | |
| $H_T(q \sim \tilde{g})$ | $\tilde{g}(300)/\tilde{g}(245)$ | $\tilde{g}(350)/\tilde{g}(265)$ | $\tilde{g}(400)/\tilde{g}(265)$ | $\tilde{g}(2000)$ |
| $l^{\pm}l^\mp(q \sim \tilde{g})$ | $\tilde{g}(180 - 230)$ | $\tilde{g}(320 - 325)$ | $\tilde{g}(385 - 405)$ | $\tilde{g}(1000)$ |
| all $\rightarrow 3\ell$ ($\tilde{q} \sim \tilde{g}$) | $\tilde{g}(240 - 290)$ | $\tilde{g}(425 - 440)$ | $\tilde{g}(550)$ | $\tilde{g}(1000)$ |
| $\tilde{t}_1 \rightarrow c\tilde{Z}_1$ | $\tilde{t}_1(80 - 100)$ | $\tilde{t}_1(120)$ | $\tilde{t}_1(150)$ | |
| $\tilde{t}_1 \rightarrow b\tilde{W}_1$ | $\tilde{t}_1(80 - 100)$ | $\tilde{t}_1(120)$ | $\tilde{t}_1(180)$ | |
| $\Theta(\tilde{t}_1\tilde{t}_1^* \rightarrow \gamma\gamma)$ | $\tilde{t}_1(250)$ | |

The SUSY reach of possible future hadron colliders is summarized in Table 1.

Several comments are worth noting:

- In some places, two sets of numbers are given for the reach. These correspond to results from different analyses more fully described in the review from where this Table is taken. Basically, the more conservative number also requires the signal to be larger than 25% of the background, in addition to exceeding the 5$\sigma$ level. Also, the two analyses do not use the same cuts.

- The multilepton rates in the Table are shown for negative values of $\mu$ and $\tan(\beta) = 2$. For other parameters, especially for $\mu > 0$, the trilepton rates may be strongly suppressed due to a suppression of the $\tilde{Z}_2$ branching fraction discussed above. Notice also that at TeV33, the reach in the leptonic channels exceeds that in the $H_T$ channel. At the TeV33 upgrade, hadronically quiet trilepton events may be observable all the way up to the spoiler modes for favourable ranges of model parameters. It is, however, important to remember that supersymmetry may escape detection at these facilities even if sparticles are relatively light.

- At the LHC, gluinos and squarks are detectable to well beyond 1 TeV in the $l\ell$ and $H_T$ channels, and up to 2 TeV if their masses are roughly equal. Thus the LHC should be able to probe the complete parameter space of weak scale SUSY, at least within the assumed framework. Moreover, there should be observable signals in the multilepton channels if a signal in the $H_T$ channel is to be
attributed to supersymmetry.

- Tevatron upgrades will not probe sleptons significantly beyond the reach of LEP2, whereas the LHC reach may be comparable to that of the initial phase of linear colliders.

- Tevatron upgrades should be able to detect $\tilde{t}_1$ with a mass up to 120 GeV at the MI and up to 150-180 GeV at TeV33. It has also been pointed out, assuming that $\tilde{t}_1 \to c\tilde{Z}_1$ is its dominant decay, that it should be possible to search for $\tilde{t}_1$ at the LHC via the two photon decay of the scalar $\tilde{t}_1\tilde{t}_1$ bound state, in much the same way that Higgs bosons searches (to be discussed next) are carried out.

**Higgs Bosons:** At the LHC, MSSM Higgs bosons are dominantly produced by $gg$ fusion (via loops of quarks and squarks), and for some parameter ranges, also via $gg \rightarrow b\bar{b}H$ reactions. Vector boson fusion, which in the SM dominates these other mechanisms for large Higgs masses ($m \gtrsim 900$ GeV) is generally unimportant, since the couplings of heavy Higgs bosons to $VV$ pairs is suppressed. Unfortunately, we do not have much time to discuss various strategies that have been suggested for the detection of the Higgs sector of SUSY. Over much of the parameter space, all the Higgs bosons except the lightest neutral scalar, $h$, are too heavy to be of interest, although for some ranges of MSSM parameters, signals from $H \rightarrow \gamma\gamma, \tau\tau, \mu\mu, 4\ell$ and $A \rightarrow \gamma\gamma, \tau\tau, \mu\mu$ may be observable. The two photon decay mode is the most promising channel for $h$ detection at the LHC. The regions of parameter space where there is some signal for an MSSM Higgs boson either at the LHC or at LEP2 have been nicely summarized in the technical report of the CMS Collaboration, assuming that sparticles are too heavy to be produced via Higgs boson decays. The most striking feature of their analysis is that despite optimistic detector assumptions, if the LHC is operated only at its low design luminosity option, there are regions of parameter space where there may be no signal for any of the Higgs bosons either at the LHC or at LEP2. Part of this hole may be excluded by analyses of rare decays such as $b \rightarrow s\gamma$ mentioned earlier.

It has been suggested that Higgs boson signals may also be detectable in this hole region via $t\bar{t}h$ production where a lepton from $t$ decay may be used to tag the event so that $h$ can then be detected via $h \rightarrow \gamma\gamma$.

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\*\*This is not necessarily a good assumption. The branching fractions for the SUSY decays can be quite substantial for large regions of parameter space, and can reduce the signals via which the Higgs bosons are usually searched for and increase the parameter space hole referred to below. Sometimes, however, they lead to novel signals for Higgs boson searches which can then refill some of the hole region. Of course, for almost all cases where SUSY Higgs decays are important, it should also be possible to detect the sparticles at the LHC. It is only Higgs boson detection that may be more difficult.\*\*
its dominant $b\bar{b}$ decay. This would require efficient $b$ tagging with the high luminosity option for the LHC. Whether this is technically possible is not clear at this time. A different analysis indicates that the with an integrated luminosity of 300 $fb^{-1}$ the LHC would be "guaranteed" to find at least one of the MSSM Higgs bosons over essentially the entire range of parameters not covered by LEP2 via signals in the $h \rightarrow \gamma\gamma$ and $A, H \rightarrow \tau\tau$ channels if the data from the ATLAS and CMS experiments are combined. It is instructive to note that there are substantial regions of parameter space where it would be possible to detect more than one Higgs boson of the MSSM.

It is clear that Higgs boson searches at the LHC pose a formidable experimental challenge. In contrast, these would be relatively easy at a linear collider, the first of many examples of the complementary nature of these facilities.

10.3 The SUSY Reach of Various Facilities: A Recapitulation

Because the mSUGRA model is completely determined by just four parameters, it provides a compact framework for comparing the prospects for SUSY detection via disparate SUSY processes and at different experimental facilities. Within this framework, the scale of sparticle masses is mainly determined by $m_0$ and $m_{\tilde{\chi}}^2$ so that the $m_0 - m_{\tilde{\chi}}^2$ plane, for fixed values of $A_0$, $\tan \beta$ and $\text{sgn} \mu$ provides a convenient panorama for comparing the capabilities of various future facilities, as shown in Fig. 6. Here, we have chosen $\tan \beta = 2$, $A_0 = 0$ and $\mu > 0$. Except perhaps for very large values of $\tan \beta$, the qualitative features illustrated are only weakly sensitive to this choice.

The bricked (hatched) region is excluded by theoretical (experimental) constraints. The region below the lines labeled MI and TeV33 is where experiments at the Tevatron should be able to discover SUSY, assuming an integrated luminosity of 2 and 25 $fb^{-1}$. The discovery region is a composite of several possible discovery channels, although the $E_T$ and clean 3f channels dominate the reach. To help orient the reader, we have also shown contours for gluino and squark masses of 1 TeV.

The upper solid line of Fig. 6 shows the boundary of the corresponding region at the LHC which essentially coincides with the discovery region in the $1\ell + \text{jets} + E_T$ channel. Similarly, the solid line labeled NLC500 denotes the reach of the NLC operating at $\sqrt{s} = 0.5$ TeV, as obtained using ISAJET. It consists of three parts: the horizontal portion at $m_{\tilde{\chi}} \sim 300$ GeV essentially follows the $m_{\tilde{\chi}}^2 = 250$ GeV contour, while the rising portion below $m_0 = 200$ GeV follows the $m_{\tilde{\chi}} = 250$ GeV contour. The reach drops when $m_{\tilde{\chi}} = m_{\tilde{\chi}}$ because the daughter electron becomes too soft. An observable signal from $e^+e^- \rightarrow \tilde{Z}_1 \tilde{Z}_2$ makes
Figure 6: The SUSY reach for various facilities as given by the mSUGRA model. We take \( \tan \beta = 2, A_0 = 0 \) and \( \mu > 0 \).

up the intermediate portion of the contour. The dashed-dotted contours mark the boundaries of the region where \( \tilde{W}_1 \) and/or \( \tilde{e}_R \) are kinematically accessible at NLC1000 or 1500. Although new backgrounds from two-to-three- and four-particle production processes have not been evaluated, we believe that this region closely approximates the reach of the NLC operating at these higher energies.

The following observations are worthy of emphasis:

- We again see that though TeV33 can probe an interesting region of parameter space, there is a significant range of parameters, consistent with qualitative upper bounds on sparticle masses from fine tuning arguments, where there is no observable signal. The discovery reach of TeV33 is not overwhelmingly larger than that of the Main Injector (MI).

- The large reach of the LHC is evident from this figure. We also see that the LHC provides a significant safety margin over the upper limits expected from fine-tuning arguments.

- For the purposes of assessing the SUSY reach (and for this purpose alone), we see that an \( e^+e^- \) collider operating between 1 - 1.5 TeV has a reach similar to that of the LHC.

11 Beyond SUSY Discovery: More Ambitious Measurements

We have seen that if the minimal SUSY framework that we have adopted is a reasonable approximation to nature, experiments at supercolliders should certainly be able to detect signals for physics beyond the SM. If
we are lucky, such signals might even show up at LEP2 or at Tevatron upgrades. We will then have to figure out the origin of these signals. If the new physics is supersymmetry, it is likely (certain at the LHC) that there will simultaneously be signals in several channels. While the observation of just one or two of these signals would convince the believers, others would probably demand stronger evidence. It is not, however, reasonable to expect that we will immediately detect all (or even several of) the super-partners. Thus, it is important to think about just what information can be obtained in various experiments, information that will help us to elucidate the nature of the underlying physics. Towards this end, we would like to be able to:

- measure any new particle’s masses and spins, and
- measure its couplings to SM particles; these would serve to pin down its internal quantum numbers.

More ambitiously, we may ask:

- Assuming that the minimal framework we have been using is correct, is it possible to measure the model parameters? Is it possible to actually provide tests for, say, the mSUGRA framework, and thus also test the assumptions about the physics at the GUT or Planck scale that are part and parcel of this picture?
- At hadron colliders, especially, where several new sparticle production mechanisms may be simultaneously present, is it possible to untangle these from one another?
- As mentioned in Sec. 1, like any other (spontaneously broken) symmetry, supersymmetry, though softly broken, implies relationships between the various couplings in the theory. Is it possible to directly test supersymmetry by experimentally verifying these coupling constant relationships?

### 11.1 Mass Measurements

**$e^+e^-$ colliders:** The clean environment of $e^+e^-$ colliders as well as the very precise energy of the beam allows for measurements of sparticle masses. We will briefly illustrate the underlying ideas with a simple example. It is easy to show that the total cross section for smuon production has the characteristic $P$-wave threshold dependence,

$$
\sigma(\tilde{\mu}\bar{\tilde{\mu}}) \propto \left(1 - \frac{4m_{\tilde{\mu}}^2}{s}\right)^{3/2},
$$

Since the energy of any linear collider is likely to be increased in several steps to the TeV scale, one may hope that this will be less of a problem there. The lighter sparticles will be discovered first. Knowledge about their properties thus obtained should facilitate the untangling of the more complex decays of heavier sparticles.
and further, that the energy distribution of the daughter muon from the decay of the smuon, assuming only direct decays to the LSP, is flat and bounded by

$$\frac{m_{\mu}^2 - m_{\tilde{\mu}}^2}{2(E + p)} \leq E_\mu \leq \frac{m_{\mu}^2 - m_{\tilde{\mu}}^2}{2(E - p)}$$

(48)

with $E(p)$ being the energy (momentum) of the smuon. We thus see that the energy dependence of the smuon cross section gives a measure of the smuon mass, while a measurement of the end points of the muon energy spectrum yields information about $m_\mu$ as well as $m_{\tilde{\mu}}$.

Of course, theoretically these relations are valid for energy and momentum measurements made with ideal detectors without any holes and with perfect energy and momentum resolutions. In real detectors there would be smearing effects as well as statistical fluctuations. It has been shown, taking these effects into account, that with an integrated luminosity of 100 pb$^{-1}$, experiments at LEP2 should be able to determine the smuon mass within 2-3 GeV. At Linear Colliders such as the JLC, an integrated luminosity of 20 fb$^{-1}$ will be sufficient to determine the smuon and LSP masses to within 1-2 GeV. The availability of polarized beams is extremely useful to obtain pure signal samples.

If sleptons are heavy but charginos light, a study of the reaction $e^+e^- \rightarrow \tilde{W}_1\tilde{W}_1 \rightarrow jj\tilde{Z}_1 + \ell\nu\tilde{Z}_1$ should allow the determination of $m_{\tilde{W}_1}$ and $m_{\tilde{Z}_1}$ with a precision of $\lesssim$ 3 GeV, both at LEP2 (where an integrated luminosity of about 1 fb$^{-1}$ would be necessary) and at the JLC, where good jet mass resolution is crucial. Finally, it has also been shown that with the availability of beam polarizations at linear colliders it should be possible to determine squark masses with a precision of $\sim$ 5 GeV even if these decay via MSSM cascades, assuming that their decays to gluinos are not allowed: in particular, it should be possible to determine the splittings between $L$- and $R$-type squarks with good precision.

Precision mass measurements are also possible at Linear Colliders even if sparticles cascade decay. Since the end-points of the lepton energy spectrum in Eq. (13) do not depend on the stability of the daughters, the end points of the electron energy spectrum from the process $e^+e^- \rightarrow \tilde{\nu}_e\tilde{\nu}_e \rightarrow e^-\tilde{W}_1^+ e^+\tilde{W}_1^- \rightarrow e^+e^-\mu^\pm jj + E_T$ provides an opportunity for simultaneous measurement of $m_{\tilde{\nu}_e}$ and $m_{\tilde{W}_1^-}$. With a left-handed electron beam (conservatively assumed to be 80% polarized) these masses are shown to be measurable to better than 1.5%. A measurement of the $b$-jet energy in is a $e^+e^- \rightarrow \tilde{t}_1\bar{\tilde{t}}_1 \rightarrow bW_1\bar{b}W_1$ events has been shown to yield the stop mass to 6% (better if $m_{\tilde{W}_1}$ is independently determined). For a discussion of other mass measurements including from three body decay of the chargino, we refer the reader to...
Hadron Colliders: Can one say anything about sparticle masses from SUSY signals at hadron colliders? Despite the rather messy event environment, it is intuitively clear that this should be possible if one can isolate a single source of SUSY events from both SM backgrounds as well as from other SUSY sources: a study of the kinematics would then yield a measure of sparticle masses within errors determined by the detector resolution. We have already seen that it is indeed possible to isolate a relatively clean sample of \( p\bar{p} \to W_1\tilde{Z}_2 \to \ell\ell' + \not{E}_T \) events at the LHC. The end-point of the \( m_{\ell\ell} \) distribution, it has been shown, yields an accurate measure of \( m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \).

At the 1996 Snowmass Workshop, an mSUGRA model parameter set \((m_0, m_{\tilde{A}}, A_0, \tan\beta, \text{sgn}\mu) = (200, 100, 0, 2, -1)\) (all mass parameters are in GeV), which leads to a rather light sparticle spectrum, was chosen to facilitate a comparison of the capabilities of TeV33, NLC and the LHC for studying SUSY. We will refer to this as the Snowmass Point. The LHC subgroup showed that for this point at least, it is possible to isolate a very pure sample of gluino pair events. They identified an important decay chain \( \tilde{g} \to b\tilde{b}_1 \to bb\tilde{Z}_2 \to bb\ell^+\ell^- \tilde{Z}_1 \) which has a combined branching fraction of 25%. Thus gluino pair production leads to very distinctive events with multiple \( b \)-jets and several leptons. Even allowing for \( b \)-tagging efficiency and other experimental cuts, they estimated that there should be \( \sim 272K \) such events per LHC year! This enormous data sample enabled them to infer that a determination of the end point of the dilepton mass distribution will only be limited by the error in the absolute calibration of the electromagnetic calorimeter, which they estimated to be 50 MeV. These authors also went on to show how \( m_{\tilde{g}} - m_{\tilde{b}_1} \) can be determined to 10% for this value of mSUGRA parameters.

A measurement of the gluino mass would be especially important at hadron colliders since gluinos cannot be pair produced by tree-level processes at \( e^+e^- \) colliders. The best technique for this has been proposed by Paige also at the 1996 Snowmass Workshop. He showed that the distribution in the variable,

\[
M_{eff} = |p_{T1}| + |p_{T2}| + |p_{T3}| + |p_{T4}| + \not{E}_T
\]

defined as the scalar sum of the transverse energies of the four hardest jets plus \( \not{E}_T \), yields a measure of the gluino/squark mass scale defined as \( M_{SU3Y} = \min(m_{\tilde{g}}, m_{\tilde{u}R}) \) to a precision of about 10%. The choice of \( m_{\tilde{u}R} \) to represent the squark mass scale is arbitrary. Since this method appears to require only a moderately clean SUSY sample, it should be possible to determine \( M_{SU3Y} \) for gluinos and squarks as heavy as 1.5 TeV after about three years of low luminosity LHC operation.
Hinchliffe et al. studied several different mSUGRA cases to assess the prospects for other SUSY measurements at the LHC. They showed, for instance, that it might be possible to measure the mass of $h$ produced in gluino and squark cascades via its $b\bar{b}$ decay. They estimate a precision of $\pm 1$ GeV on this measurement. For details about this, and techniques for other measurements, we refer the reader to this important study.

11.2 Determination of Spin at $e^+e^-$ colliders

If sparticle production occurs via the exchange of a vector boson in the $s$-channel, it is easy to check that the sparticle angular distribution is given by, $\sin^2 \theta$ for spin zero particles, and $E^2(1 + \cos^2 \theta) + m^2 \sin^2 \theta$ for equal mass spin $\frac{1}{2}$ sparticles. Thus if sparticles are produced with sufficient boost, the angular distribution of their daughters which will be relatively strongly correlated to that of the parent, should be sufficient to distinguish between the two cases. Chen et al. have shown that, with an integrated luminosity of 500 $pb^{-1}$, it should be possible to determine the smuon spin at LEP2. A similar analysis has been performed for a 500 GeV linear collider.

11.3 Tests of the mSUGRA Framework and Determination of Model Parameters

We begin by recalling that within the mSUGRA GUT framework, the four parameters, $m_0$, $m_\frac{1}{2}$, $\tan \beta$ and $A_0$, together with the sign of $\mu$ completely determine all the sparticle masses and couplings. Since the number of observables can be much larger than the number of parameters, there must exist relations between observables which can be subjected to experimental tests. In practice, such tests are complicated by the fact that there are experimental errors, and further, it may not be possible to cleanly separate between what, in principle, should be distinct observables; e.g. cross sections for $E_T$ events from $\tilde{g}\tilde{g}$, $\tilde{g}\tilde{q}$ and $\tilde{q}\tilde{q}$ sources at the LHC.

Because of the clean experimental environment, the simplicity of the initial state and the availability of polarized beams, many tests can be most cleanly done at $e^+e^-$ colliders, where we have already seen that it is possible to determine sparticle masses with a precision of 1-2%. The determination of the selectron and smuon masses will allow us to test their equality $m_{\tilde{e}_L} = m_{\tilde{\mu}_L}$, $m_{\tilde{e}_R} = m_{\tilde{\mu}_R}$ at the percent level — the same may be done with staus, though with a somewhat smaller precision. This is a test of the assumed universality of slepton masses. A comparable precision is obtained even if the sleptons do not directly decay to the LSP.
A different test may be possible if both $\tilde{\ell}_R$ and $\tilde{W}_1$ are kinematically accessible and a right-handed electron beam is available. It is then possible to measure $m_{\tilde{Z}_1}$, $m_{\tilde{W}_1}$, $\sigma_R(\tilde{\ell}_R\tilde{\ell}_R)$ and $\sigma_R(\tilde{W}_1\tilde{W}_1)$ (note that the chargino cross section for right-handed electron beams has no contribution from sneutrino exchange!). These four observables can then be fitted to the four MSSM parameters $\tan \beta$ and the electroweak gaugino masses $M_1$ and $M_2$. In practice $\mu$ may be rather poorly determined if the chargino is dominantly a gaugino, $\frac{m_{\tilde{\ell}_R}}{M_2}$ is rather precisely obtained so that it should be possible to test the gaugino mass unification condition at the few percent level, given $50 \text{ fb}^{-1}$ of integrated luminosity. It may further be possible to determine $m_{\tilde{\nu}_e}$ by measuring chargino production with a left-handed electron beam. This is of interest because the difference between the squared masses of the sneutrino and $\tilde{\ell}_L$ is a direct test of the $SU(2)_L$ gauge symmetry for sleptons. For further details and other interesting tests, we refer the reader to the original literature.

One would naively assume that analogous tests are much more difficult at the LHC. This is because measurement of individual sparticle masses (as opposed to mass differences), which as we have just seen allows us to directly test various mSUGRA assumptions at Linear Colliders, appears to be difficult. One straightforward approach is to search for correlations amongst various signals that might be observed (along with bounds on signals that are not seen) at various colliders. To make these correlations apparent, it is convenient to display various signals as predicted by the model in the $m_0$ $- m_{\frac{1}{2}}$ plane for fixed sets of values of $\tan \beta$ and $A_0$. One would then attempt to zero in on the parameters by looking for regions of the plane for which the model predictions agree with the rates for all signals that are seen in experiments at the Tevatron, LEP2 and the LHC. One would also check that predictions for the rates for signals that are not seen are indeed below the sensitivity of these experiments. Such a plot would resemble Fig. except with many more contours.

A systematic study of how LHC data could serve to test the mSUGRA framework was begun by Baer et. al. A measurement of $m_3$ or $m_{\tilde{\tau}_2} - m_{\tilde{\tau}_1}$ as discussed above would roughly fix $m_{\tilde{\tau}_1}$, while a measurement of $m_q$ (or the gluino squark mass difference) would yield an estimate of $m_0$. A determination of the relative cross sections for different event topologies can also yield information about the underlying parameters. For instance, the ratio of the cross sections for multilepton plus jets plus $E_T$ events and for multijet plus $E_T$ events without leptons is significantly larger if the leptonic branching fractions of charginos and

\footnote{Feng and Strassler have shown that, with $1 \text{ fb}^{-1}$ of data, a test of this relation at the 20% level may also be possible at LEP2.}
neutralinos is enhanced. Indeed such an observation would suggest that sleptons are relatively light (probably even light enough to be produced via cascade decays of gluinos and squarks) which, in turn, may lead us to infer that $m_0$ is not very large. It was shown that while $m_0$ and $m_{1/2}$ may be roughly determined from LHC data, a determination of $\tan \beta$ or $A_0$ is difficult. The LHC subgroup study of the Snowmass Point showed that it should be possible to determine $m_0$, $m_{1/2}$ and $\tan \beta$ with roughly the same precision as at Linear Colliders. In fact, their determination of $m_{1/2}$ from the neutralino mass difference was more precise than the one obtained by the NLC Group. In contrast, the direct measurement of slepton masses yielded a better determination of $m_0$ at the NLC.

The reader may wonder whether the precision obtained by the LHC study is somehow special to the particular choice of parameters. To some extent, this is indeed the case. The sparticle spectrum for this point is so light that the SUSY event rate is enormous, so that it is possible to make extremely stringent cuts to isolate clean signal samples. Prospects for these measurements for other parameter choices were examined by Hinchliffe et al. It was found that $m_{1/2}$ could typically be measured to a few percent — even for the extreme case of gluinos and squarks as heavy as 1 TeV, a precision of 10% was obtained. The precision with which other parameters can be extracted depends on the scenario. It was found that, for the most part, it is possible to obtain allowed ranges of $\tan \beta$ and $m_0$. Sometimes two solutions were obtained, in which case more detailed measurements would be necessary to discriminate between them. These studies underscore the capabilities of LHC experiments and contain the first steps towards an effective measurement strategy at the LHC.

At the LHC, it should be possible to falsify the mSUGRA framework (or, for that matter, any other framework specified by a small number of parameters) by a standard $\chi^2$ analysis. If it is not possible to find a consistent set of parameters that accommodates all the data (and we are convinced that our experimental friends have not made a mistake!) the assumed framework will need to be modified or discarded. If this turns out to be the case, the direct measurements of several sparticle masses (and as we shall see, some mixing angles) at Linear Colliders will directly point to the correct theoretical picture. The broader issue of how to proceed from LHC data to determine the underlying theory appears less obvious.

\footnote{For this study, it was reasonable to assume that the lighter Higgs boson mass (which is 68 GeV) would be measured at LEP2. This constraint helps to pin down the value of $\tan \beta$ for this set of input parameters.}
11.4 Identifying Sparticle Production Mechanisms at the LHC

At $e^+e^-$ colliders where the centre of mass energy is incrementally increased, it may be reasonable to suppose that it is unlikely (except, perhaps, for the sfermion degeneracy expected in the mSUGRA model) that several particle thresholds will be crossed at the same time. It would thus be possible to focus on just one new signal at a time, understand it and then proceed to the next stage. The situation at the LHC will be quite different. Several sparticle production processes will simultaneously occur as soon as the machine turns on, so that even if it is possible to distinguish new physics from the SM, the issue of untangling the various sparticle production mechanisms will remain. For example, even if we attribute a signal in the $E/T + \text{jets}$ channel to sparticle production, how would we tell whether the underlying mechanism is the production of just gluinos or a combination of gluinos and squarks?

Some progress has already been made in this direction. We have already seen that the $\tilde{W}_1\tilde{Z}_2$ source of trileptons can clearly be isolated from other SUSY processes. The opposite sign dilepton signal from slepton production is probably distinguishable from the corresponding signal from chargino pair production since the dileptons from slepton production always have the same flavour. To tell whether squarks are being produced in addition to gluinos, at least two distinct strategies have been suggested. The first makes use of the fact that there are more up quarks in the proton than down quarks. We thus expect many more $\tilde{u}_L\tilde{u}_L$ and $\tilde{g}\tilde{u}_L$ events as compared to $\tilde{d}_L\tilde{d}_L$ and $\tilde{d}_L\tilde{g}$ events at the LHC. As a result, any substantial production of squarks in addition to gluinos will be signalled by a charge asymmetry in the same-sign dilepton sample: cascade decays of gluinos and squarks from $\tilde{q}\tilde{q}$ and $\tilde{g}\tilde{q}$ events lead to a larger cross section for positively charged same sign dileptons than for negatively charged ones. This has since been confirmed by detailed studies by the ATLAS collaboration where the SS dilepton charge asymmetry is studied as a function of $m_{\tilde{g}}/m_{\tilde{q}}$, and shown to monotonically disappear as this ratio becomes small. Another method for distinguishing gluino from squark $\text{and}$ gluino production relies on the jet multiplicity in the $E/T$ sample. The idea is to note that $\tilde{q}_R$, which are produced as abundantly as $\tilde{q}_L$, frequently decay directly to the LSP via $\tilde{q}_R \to q\tilde{Z}_1$ and so lead to only one jet (aside from QCD radiation). In contrast, gluinos decay via $\tilde{g} \to q\tilde{W}_i$ or $\tilde{g} \to q\tilde{Z}_i$, so that that gluino decays contain two, and frequently more, jets from their cascade decays. Thus the expected jet multiplicity is lower if squark production forms

\footnote{Here, we tacitly assume that squarks will not be much lighter than gluinos.}

\footnote{The extent to which this channel is contaminated by other SUSY sources has not been explicitly checked.}
a substantial fraction of the $E_T$ sample. Of course, since $\langle n_{\text{jet}} \rangle$ (from gluino production) depends on its mass, some idea of $m_{\tilde{g}}$ is necessary for this strategy to prove useful. A detailed simulation\textsuperscript{122} shows that the mean value of the $n_{\text{jet}}$ distribution increases by about $\frac{1}{2}$ unit, when the squark mass is increased from $m_{\tilde{q}} = m_{\tilde{g}}$ by about 60-80%.

Cascade decays of gluinos and squarks can result in the production of the Higgs bosons of supersymmetry. It is, therefore, interesting to ask whether these can be detected in the data sample which has already been enriched in SUSY events. Neutral Higgs bosons might be detectable\textsuperscript{121} via an enhancement of the multiplicity of central $b$-jets in the $E_T$ or same sign dilepton SUSY samples. Some care must be exercised in drawing conclusions from this because such enhancements may also result because third generation squarks happen to be lighter than the other squarks.\textsuperscript{122} It has also been shown\textsuperscript{122,139} that it may also be possible to reconstruct a mass bump in the $m_{\tilde{b}_2}$ distribution if there is a significant branching fraction for the decay $\tilde{Z}_2 \to h\tilde{Z}_1$ and $h$ is produced in events with no other $b$-jets, since otherwise we would have a large combinatorial background. The charged Higgs boson, if it is light enough, may be identifiable via the detection of $\tau$ lepton enhancements in SUSY events\textsuperscript{122} or even in $t \to bH^+$ decays;\textsuperscript{133} it should, however, be kept in mind that such light charged Higgs bosons also contribute to the $b \to s\gamma$ decays.

We also saw that in the recent case studies for the LHC\textsuperscript{137,139} it was frequently possible to isolate specific SUSY production and decay chains by judicious choice of cuts. An example of this is gluino pair production which yielded a measurement of the mass difference, $m_{\tilde{g}} - m_{\tilde{b}_1}$, as discussed above. It appears that while the complexity and variety of potential SUSY signals precludes us from writing down a general algorithm that can be used to identify the production mechanisms that may be present in the LHC data sample, by studying the features of a given data set we will be able to infer a considerable amount. These sort of studies have only just begun, and considerable work remains to be done. In this respect, at least, it appears that the analysis of data from Linear Colliders will be simpler.

We stress that the complex cascade decay chains of gluinos and squarks may be easier to disentangle if we already have some knowledge about the masses and couplings of the lighter charginos and neutralinos that are produced in these decays. While it is indeed possible that $\tilde{W}_1$ may be discovered at LEP2 and that its mass is determined there, it is likely that we may have to wait for experiments at the Linear Collider to be able to pin down the couplings, and, perhaps, even for discovery\textsuperscript{a}.

\textsuperscript{a}These may be directly produced with large cross sections or may lead to enhancement of gluino decays to third generation fermions as discussed in Sec. 68.
of $\tilde{W}_1$ and $\tilde{Z}_2$. In this case, a reanalysis of the LHC data in light of new information that may be gained from these experiments may prove to be very worthwhile: it may thus be necessary to archive this data in a form suitable for subsequent reanalysis. Once again, we see the complementary capabilities of $e^+e^-$ and hadron colliders.

11.5 Direct Tests of Supersymmetry

We have already seen that supersymmetry, like any other symmetry, implies relationships between various dimensionless couplings in the theory even if it is softly broken. For example, the fermion-sfermion-gaugino (or, since the Higgs multiplet also forms a chiral superfield, the Higgs-Higgsino-gaugino) coupling is completely determined by the corresponding gauge coupling. A verification of the relation would be a direct test of the underlying supersymmetry. We emphasize that such a test would be essentially model independent as it relies only on the underlying global supersymmetry, and not on any details such as assumptions about physics at the high scale or even the sparticle content. In practice, such tests are complicated by the fact that the gauginos (or the Higgs bosons $h_u$ and $h_d$ and the corresponding Higgsinos, or for that matter, the sfermions $\tilde{f}_L$ and $\tilde{f}_R$) are not mass eigenstates, so that the mixing pattern has to be disentangled before this test can be applied. This will require an accurate measurement of several observables which can then be used to disentangle the mixing and also simultaneously to measure the relevant coupling.

Feng et al. 141 have argued that such a test can be performed via a determination of chargino properties. As we have seen, the charged gaugino and the corresponding Higgsino can mix only if electroweak symmetry is broken. This is the reason why the off-diagonal terms in the chargino matrix of Eq. (33b) are equal to $-\sqrt{2}M_W \cos \beta$ and $-\sqrt{2}M_W \sin \beta$, respectively. Assuming that the chargino is a mixture of just one Dirac gaugino and one Dirac Higgsino, the most general mass matrix would contain four parameters: the two diagonal elements and the two off-diagonal ones. These latter can always be parametrized by $-\sqrt{2}M_W \cos \beta$ and $-\sqrt{2}M_W \sin \beta$. It is the SUSY constraint on the Higgs-Higgsino-gaugino coupling that forces $M_W^\chi = M_W$. A determination of four independent quantities that depend only on these parameters could then be used to see if the SUSY constraint $M_W^\chi = M_W$ is valid. How best to do this determination depends on what the underlying parameters are. Here we will merely say that these SUSY tests can be

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\[\text{This could especially be the case if the spoiler decay modes of the neutralino are accessible.}\]

\[\text{These relations are corrected by radiative corrections which are generally expected to be smaller than a few percent.}\]
done at about the 30% level at a 500 GeV Linear Collider, and refer the reader to the original paper for further information.

A more precise test has recently been suggested by Nojiri et. al. These authors suggest that an accurate measurement of the cross section and angular distribution of electrons produced via $e^+e^- \rightarrow \tilde{e}_R\tilde{e}_R$ could lead to a 2% measurement of the electron-selectron-hypercharge gaugino coupling if an integrated luminosity of 100 $fb^{-1}$ is accumulated at Linear colliders. In any SUSY model, this coupling, at tree level, should be equal to the $U(1)_Y$ gauge coupling $g'$ up to a Clebsch-Gordan coefficient. It is instructive to note that a 2% measurement of this coupling begins to be sensitive to radiative corrections which, in turn, would be sensitive to sparticles that may be beyond the kinematic reach of the machine.

We are not aware of any proposal for an analogous test at the LHC.

12 Beyond Minimal Models

Up to now, we have mainly confined our analysis to the MSSM framework. Even then, as we saw in Sec. 4, the unmanageably large number of free parameters required us to make additional assumptions in order to obtain tractable phenomenology. It is clearly impractical to seriously discuss the phenomenology of the many possible extensions of the MSSM framework that have been considered. Here, we will first list some of the ways in which this framework may be modified, leaving it to the reader to figure out the implications for phenomenology. Thinking about this will also help to view our previous discussion in proper perspective. We will then select two of these modifications (for reasons explained below) and discuss their phenomenological implications in more detail.

The MSSM framework may be extended or modified in several ways, roughly arranged in order of increasing “non-minimality”.

1. We may give up the exact universality of the gaugino masses at the GUT scale. Threshold corrections due to unknown GUT, and perhaps even gravitational interactions would certainly yield model-dependent corrections which preclude exact unification. It is also conceivable that there is, in fact, no grand unification at all, but the apparent unification of couplings inferred from LEP experiments is a result of string type unification; in this case, the gaugino masses unify at the string scale which is generally somewhat larger than $M_{GUT}$. We have already noted that even in SUGRA type models, we do not really know the exact scale at which scalar masses unify.

A somewhat different test of SUSY which yields a similar precision when the chargino is mainly gaugino-like has also been discussed in this study. J. Feng and N. Polonsky have observed that this latter test becomes an order of magnitude more precise if the sneutrino mass is independently known to a few GeV.
We also remark that the additional assumption of minimal kinetic energy terms is crucial for obtaining universal scalar masses. Any deviation from the assumed universality of scalar masses will, at the very least, modify the conditions of radiative symmetry breaking. The pattern of scalar masses may also be modified by $D$-terms if the gauge group contains additional factors. Naively, one would think that if these additional symmetries are broken at a sufficiently large scale, this would have no impact upon low energy physics. This is not always the case. These terms can significantly alter the pattern of sparticle masses, and hence, impact upon production and decays of sparticles. Detailed measurements of scalar masses and branching ratios in future experiments can potentially lead to the discovery of new physics, at energy scales to which we may not have direct access at colliders during our lifetimes.

2. $R$-parity may be explicitly broken by superpotential interactions $g_1$ and $g_2$ in Eq. (31a) and Eq. (31b).

3. SUSY breaking may possibly occur at relatively low energies and not at $\sim 10^{10}$ GeV as in SUGRA type models where gravity is the messenger of SUSY breaking. Models where SUSY breaking occurs at relatively low energy ($\sim 10 - 100$ TeV) and is communicated by ordinary gauge interactions have recently received a lot of attention. In these Gauge Mediated Low Energy Supersymmetry Breaking (GMLESB) models the mass patterns, and hence the phenomenology, are considerably different from mSUGRA.

4. There could be additional chiral superfields even in the low energy theory: new generations (with heavy neutrinos), additional Higgs multiplets, or a right-handed sneutrino superfield. We certainly do not need new generations or new Higgs doublets, as they may spoil the apparent unification of couplings. Higgs fields in higher representations cause additional problems if they develop a VEV. Higgs singlets cannot be logically excluded, and are interesting because they allow for new quartic Higgs boson couplings, though one would have to understand what keeps them from acquiring GUT or Planck scale masses. A singlet right-handed sneutrino (note that this is not the superpartner of the usual neutrinos) is an interesting possibility since it occurs in $SO(10)$ GUT models, and also, because it allows for spontaneous breaking of $R$-parity conservation.

5. Finally, we could consider models with extended low energy gauge groups — either left-right symmetric models or models with additional $Z$ bosons.

For want of time, we will confine ourselves to items (2) and (3) above. This is not because these extensions are necessarily more likely to be
correct than the other. We consider $R$-parity violation because there are no sacred symmetry principles that forbid these interactions. We incorporated $R$-parity conservation only because we were motivated to do so for phenomenological reasons. The conservation of $R$-parity gave us valuable freebies (such as a candidate for cold dark matter) but this does not mean that it is necessarily right. We will soon see that it is possible to build perfectly acceptable models where $R$-parity is not conserved.

On another note, we will see that models where SUSY breaking occurs at the $\text{PeV}$ scale and is communicated to SM particles and their superpartners by gauge interactions are, like $\text{mSUGRA}$, very economic in the sense that the low energy theory can be simply parametrized. These models are very ambitious in that their goal is not only to include a mechanism for transmission of SUSY breaking, but also to obtain SUSY breaking dynamically. While this goal is yet to be realized in a compelling manner, they represent a viable alternative to the conventional picture. The resulting collider signatures, however, differ in important ways from mSUGRA expectations. From our point of view, this alone is reason enough to pay special attention to this framework.

12.1 $R$-parity Violation

In some sense, including $R$-parity violating interactions results in the minimal extension of the MSSM because it does not require the introduction of any new particles. Notice, however, that a general analysis of this requires the introduction of 48 new parameters in the superpotential of Eq. (31a) and (31b): $3\mu'$s, $9\lambda'$s, $27\lambda''$s and $9\lambda''$s. There are relations amongst these couplings in theories with larger symmetries; e.g. GUTs. Many (but not all) products of the baryon- and lepton-number violating couplings are strongly constrained by the non-observation of proton decay. In fact, it is usually assumed that only one of $B$ or $L$ violation is possible, since (assuming sparticles are heavier than the proton) the only spin 1 particles into which the proton can decay are leptons; i.e. the proton will be stable if either $B$ or $L$ is conserved. In phenomenological analyses, it is customary (and in light of the large number of new parameters, convenient) to assume that one of the couplings dominates. Even so, several of the couplings are strongly constrained.

In a very nice analysis, Barger et. al. have studied the implications from various experiments — $\beta$-decay universality, lepton universality, $\nu_e e$ scattering, $e^+ e^-$ forward-backward asymmetries and $\nu_e$ deep-inelastic scattering — for these new interactions. They find strong constraints on the lepton-number violating couplings, assuming that only one of the couplings is non-zero: for instance, they find that of the constraints from non-observation of $\mu \rightarrow e \gamma$ or $\mu \rightarrow 3e$ decays and $\mu N \rightarrow e N$ processes are
\( \lambda \)-type couplings, only \( \lambda_{131} \) and \( \lambda_{133} \) can exceed 0.2 (compare this with the electromagnetic coupling \( e = 0.3 \)) for a SUSY scale of 200 GeV, though several \( \lambda' \) and many more of the \( \lambda'' \) interactions can exceed this value. Dimopoulos and Hall\(^1\) have, from the upper limit on the mass of \( \nu_e \), obtained a strong bound (< \( 10^{-3} \)) on \( \lambda_{133} \). The same argument yields a stringent bound\(^2\) on \( \lambda'_{133} \) and a less restrictive, but significant, bound on \( \lambda'_{122} \). First generation baryon number violating interactions are strongly constrained from the non-observation of \( n - \bar{n} \) oscillations or \( NN \to KKX \).\(^3\) Constraints\(^4\) from rare \( B \) processes such as \( B^+ \to K^+K^0 \) as well as neutral \( K \) and \( D \) meson mixing limit \( \lambda'' \) couplings involving the third family. These couplings are also constrained by the precision measurements\(^5\) of \( Z^0 \) properties at LEP.

The reason to worry about all this is that if \( R \)-parity is not conserved, both sparticle production cross sections as well as decay patterns may be altered. For instance, if \( \lambda' \) interactions are dominant (with \( i = 1 \)), squarks can be singly produced as resonances \(^6\) in \( ep \) collisions at HERA\(^7\) or in the case of \( \lambda'' \) interactions, at hadron colliders.\(^8\) The production rates will, of course, be sensitive to the unknown \( R \)-parity violating couplings. Likewise, if \( R \)-parity violating couplings are large compared to gauge couplings, these \( R \)-violating interactions will completely alter sparticle decay patterns.

Even if all the \( \lambda \)'s are too small (relative to gauge couplings) to significantly affect the production and decays of sparticles (other than the LSP), these interactions radically alter the phenomenology because the LSP decays visibly, so that the classic \( E_T \) signature of SUSY is no longer viable. Even so, sparticle detection should not be a problem in the clean environment of \( e^+e^- \) colliders. In fact, LEP should be able to probe regions of parameters not explorable in the MSSM since signals from LSP pair production can now be detected.\(^9\)

The viability of SUSY detection at hadron colliders would clearly be sensitive to details of the model. Two extreme cases where the LSP decays either purely leptonically into \( e \)'s or \( \mu \)'s and neutrinos via \( \lambda \)-type couplings, or when it always decays into jets via \( \lambda'' \) couplings have been examined for their impact on Tevatron\(^{10,11,12}\) and LHC\(^{13,14,15}\) searches for supersymmetry. The signals, in the former case, are spectacular since

\(^*\)The H1 and ZEUS experiments at the HERA collider have reported an excess of events in high energy \( e^+p \to e^+ + X \) scattering at very high values of \( Q^2 \). Many theorists have suggested that this may be an indicator of some novel physics, a popular interpretation being an s-channel resonance in positron-quark scattering. This could be a spin zero particle with lepton and quark quantum numbers, the lepto-quark, or a scalar quark with \( \lambda' \) type \( R \)-violating interactions. It is not clear, however that the results of the two experiments are mutually any more compatible\(^16\) than the reported deviation from the SM. Unfortunately, it will take a couple of years for the situation to be clarified.
the decays of each LSP yields two leptons in addition to any other leptons from direct decays of $\tilde{W}_1$ or $\tilde{Z}_2$ produced in the gluino or squark cascade decays. With an integrated luminosity of 100 $pb^{-1}$ that has already been accumulated, experiments at the Tevatron should be able to probe gluinos as heavy as 500-600 GeV. In the other case where the LSP decays purely hadronically, gluino and squark detection is much more difficult than in the MSSM. The reason is that the $E_T$ signal is greatly reduced since neutrinos are now the only sources of $E_T$. In fact, if squarks are heavy, there may well be no reach in this channel even at the Main Injector. Further, the multilepton signals from cascade decays are also degraded because the jets from LSP decays frequently spoil the lepton isolation. Indeed if squarks are heavy, none of the SUSY signals would be observable in this run of the Tevatron; even the Main Injector will then not probe gluino masses beyond $\sim 200$ GeV ($350$ GeV, if $m_{\tilde{q}} = m_{\tilde{g}}$).

At the LHC, attention had mainly been focussed on the same-sign dilepton signal from gluino pair production. In the case where the LSP decays purely leptonically, the gluino mass reach exceeds 1 TeV. A natural question to ask is what happens if the LSP only decays hadronically into three jets. In particular, is it possible that SUSY can escape detection in LHC experiments even though sparticle masses are below 1 TeV? In a recent study using ISAJET, it was shown that even in this unfavourable case, experiments at the LHC would be able to detect SUSY signals in the $1\ell, \ell^+\ell^-, \ell^{\pm}\ell^{\mp}$ and $3\ell$ plus multijet channels if gluinos or squarks are lighter than 1 TeV. This study assumed that particle masses and mixing patterns are exactly as in the mSUGRA model, the only difference being that the LSP decayed into three quarks via the $\lambda''_{221}$ coupling. It thus seems unlikely that weak scale supersymmetry will escape detection at the LHC.

12.2 Gauge-Mediated Low Energy Supersymmetry Breaking

The set-up in this class of models is similar in several respects to the SUGRA type models that we discussed in Sec. 5. Supersymmetry is again assumed to be dynamically broken in a hidden sector of the theory. This sector is coupled to a “messenger sector” (which then feels the effects of SUSY breaking) by a new set of gauge interactions. Some particles in the messenger sector are assumed also to have SM gauge interactions. These then induce soft SUSY breaking masses for the sfermions, gauginos and the Higgs bosons. The effective SUSY breaking scale for the observable sector is thus suppressed by $M_{\text{mess}}$ rather than $M_{\text{Planck}}$ as for gravity-mediated SUSY breaking. We thus expect this scale to be $\sim \frac{4\pi}{\alpha} \times \mu^2/M_{\text{mess}}$, where $\mu$ is the induced SUSY breaking scale in the messenger sector, and $\alpha$ is the fine structure constant for the relevant SM gauge interaction. The effective SUSY breaking scale in the observable
sector can be 100-1000 GeV even if $\mu_s$ and $M_{mess}$ are as small as tens to hundreds of TeV.

If the effective SUSY breaking scale as well as the particle content of the low energy theory is the same in GMLESB and mSUGRA models, what difference does all this make? The main difference is that the mass of the gravitino, $m_{\tilde{G}} \sim \mu^2_s M_{\text{Planck}}$ is comparable to the weak scale in SUGRA type models, but is tiny if $M_{mess}$ is small. For instance, for $M_{mess} \sim 100$ TeV, $m_{\tilde{G}} \sim 1$ eV. But if gravitinos interact only with gravitational strength, why should we care? The point is that gravitinos, like W bosons, get their mass via the (super)-Higgs mechanism. As a result, the coupling of the longitudinal component of the gravitino of energy $E$ is enhanced by a factor $\frac{E}{m_{\tilde{G}}}$ in exactly the same way that the coupling of the longitudinal W boson is enhanced by $\frac{E}{M_W}$. In other words, the effective “dimensionless” coupling of longitudinal gravitinos is $\sim \frac{E}{M_{\text{Planck}}} \times \frac{E}{m_{\tilde{G}}}$, where the first factor corresponds to the usual gravitational coupling and the second factor to the additional enhancement. For $E \sim 100$ GeV (corresponding to the weak scale) and $m_{\tilde{G}} = 1$ eV, it is easy to check that this coupling is about $10^{-6}$. The width of a particle of mass 100 GeV that decays into its superpartner and a longitudinal gravitino via this coupling is $\sim 10^{-10}$ GeV, corresponding to a lifetime of $\sim 10^{-13}$ seconds! Thus interactions of very light longitudinal gravitinos are obviously relevant for particle physics, and often even for collider phenomenology.

We do not have the time to delve into details of this class of models. We will, instead, focus our attention on the simplest version of this model which we will use to illustrate the differences in the phenomenology. We refer the reader to Dimopoulos et. al. for details of this framework as well as variants of the minimal model. This paper also spells out the underlying assumptions.

The messenger sector of the minimal GMLESB model consists of one set of “quark” and “lepton” superfields in the $5+5^\ast$ representation of $SU(5)$ (the inclusion of a complete representation ensures that the successful prediction of $\sin^2 \theta_W$ is not disturbed) coupled to a singlet via a superpotential $f = \lambda_1 \tilde{S} \tilde{q} \tilde{q} + \lambda_2 \tilde{S} \tilde{l} \tilde{l}$. The scalar and auxiliary components of $\tilde{S}$ acquire VEVs via their interactions with the hidden sector, the latter signalling the breakdown of SUSY. SM gauge interactions induce masses for the gauginos of the observable sector via one loop quantum
corrections. If \( F \ll \langle S \rangle^2 \), these are given by,\(^{47}\)

\[
m_{\tilde{\chi}_i} = \frac{\alpha_i}{4\pi} \Lambda, \quad (49a)
\]

where \( \Lambda = \langle F \rangle \langle S \rangle \). The chiral scalars feel the effects of SUSY breaking only via these gaugino masses, so that SUSY breaking squared scalar masses, which are induced as two-loop effects, are given by,

\[
m^2_{\text{scalar}} = 2\Lambda^2 \left[ C_3 \left( \frac{\alpha_3}{4\pi} \right)^2 + C_2 \left( \frac{\alpha_2}{4\pi} \right)^2 + \frac{3}{5} \left( \frac{Y}{2} \right)^2 \left( \frac{\alpha_1}{4\pi} \right)^2 \right], \quad (49b)
\]

with \( \alpha_3 \) given in terms of the usual hypercharge coupling \( g' \) by \( \alpha_3 = \frac{g'^2}{8\pi^2} \). \( C_3 = \frac{4}{3} \) for colour triplets and zero for colour singlets while \( C_2 = \frac{3}{4} \) for weak doublets and zero for weak singlets. SUSY breaking \( A \)- and \( B \)-parameters are induced only at two-loop order and are small.

We see that the gaugino masses obey the GUT relation (35) although the underlying physics is quite different. It is also straightforward to see that squarks are heavier than gluinos in this minimal framework. The sfermion mass patterns are quite different from those in mSUGRA. The squarks are the heaviest, followed by \( \tilde{\ell}_L \) followed by \( \tilde{\ell}_R \); numerically, \( m_{\tilde{q}}^2 : m_{\tilde{\ell}_L}^2 : m_{\tilde{\ell}_R}^2 \simeq 11.6 : 2.5 : 1 \). We should regard these masses as being defined at the scale \( M_{\text{mess}} \) and evolve these to the weak scale as before. Of course, if \( M_{\text{mess}} \sim 100 \text{ TeV} \), the effect of the evolution on these masses is not as important as in mSUGRA. Radiative breaking nonetheless occurs as before because the Higgs boson mass parameters at the messenger scale are much smaller than the corresponding \( t \) and \( b \)-squark masses. Since the \( A \)-parameter is only induced at higher loops, we can take \( A \) to be zero at \( Q = M_{\text{mess}} \). Also as before, we eliminate \( B \) in favour of \( \tan \beta \) so that the model is completely specified by the parameter set,

\[(\Lambda, \tan \beta, M_{\text{mess}}, \text{sgn} \mu)\]

We see that \( \Lambda \) sets the scale for sparticle masses, and is thus the most important of these parameters. As in the mSUGRA framework, we expect that the phenomenology is not very sensitive to \( \tan \beta \) or \( \text{sgn} \mu \). The messenger mass \( M_{\text{mess}} \) only serves to determine the scale at which the mass relations are to be used as boundary conditions, so that any dependence on it is, presumably, logarithmic.

It is instructive to note that the gauge-mediation ansatz automatically guarantees that squarks (and also sleptons) with the same gauge quantum numbers will have the same mass. In particular, the first two

\[^{v}\text{This may not yield a small value of} B \text{at the messenger scale. If we incorporate the constraint} B(M_{\text{mess}}) \simeq 0, \text{then we will find that} \tan \beta \text{is large.} \]^{166}\)

\[^{166}\text{We do not include this constraint for reasons discussed elsewhere.} \]^{166}\)
generations of $\tilde{q}_L$ (also, separately, $\tilde{q}_R$) are degenerate without invoking the need for additional symmetries such as the global $U(N)$ that was needed in mSUGRA. These models are, therefore, advertised as naturally being free of flavour changing neutral current problems. That this is being somewhat oversold can be seen by noting that the messenger “leptons” and one of the MSSM Higgs doublets has the same quantum numbers. There is, therefore, no reason why the messenger “sleptons” cannot couple to the quarks in the same way as the usual Higgs scalar. Then, we will have more than one scalar with Yukawa coupling to the same quarks, which, as is well known, leads to FCNC problems. Even in these models a discrete symmetry seems necessary to prevent these couplings. A definite disadvantage of this framework vis-à-vis mSUGRA is that we lose $\tilde{Z}_1$ as a candidate for cold dark matter.

Phenomenologically, the major difference comes from the fact that $\tilde{Z}_1$, which is frequently lighter than all sparticles other than the gravitino, is now unstable, and can decay via $\tilde{Z}_1 \rightarrow \gamma \tilde{G}$, and possibly also via $\tilde{Z}_1 \rightarrow Z \tilde{G}$ or $\tilde{Z}_1 \rightarrow H_i \tilde{G}$ ($H_i = h, H, A$). The branching fraction for other sparticles to directly decay to the gravitino are small, since, as we saw, the effective dimensionless gravitino coupling was much smaller than any of the gauge couplings. Thus heavy sparticles cascade decay to lighter sparticles exactly as in the MSSM (with masses and mixing angles fixed to be as given by the GMLESB model) until the next to lightest superparticle (NLSP) is reached. The NLSP is, however, unstable and, depending on the messenger scale, may decay inside the detector. If this is $\tilde{Z}_1$, it will decay via $\tilde{Z}_1 \rightarrow \gamma \tilde{G}$; the gravitino escapes undetected, so that SUSY event topologies are characterized by multijet plus multilepton plus $E_T$ together with two photons (not both of which will be necessarily detected in the experimental apparatus) in the final state. It is worth mentioning that the lifetime of $\tilde{Z}_1$ can be rather long so that the photons need not emerge from the primary vertex in the event. In fact, the gap between the primary and secondary vertices can yield a measure of the messenger scale. At the LHC where event rate is generally not a problem, it may be experimentally easier to measure this gap via the subdominant decay $\tilde{Z}_1 \rightarrow G e^+ e^-$ of the neutralino.

The CDF experiment has seen one event which caused a considerable amount of excitement amongst the champions of these models. Specifically, they saw an event with an isolated $e^+ e^-$ pair together with a pair of hard, isolated photons and $E_T$. This event was interpreted as the selectron pair production with the subsequent decay $\tilde{e} \rightarrow e \tilde{Z}_1 \rightarrow e \gamma \tilde{G}$ of each selectron. To see if this explanation is viable, Baer et. al. recalled our discussion about continuous global symmetries in Sec. 5.

If the NLSP lives long enough so that it decays outside the detector, collider phenomenology will be essentially the same as in the MSSM.
computed the cross sections in the various event topologies that would be expected at the Tevatron as a function of $\Lambda$ which, we remind the reader, sets the scale of sparticle masses. As in the mSUGRA framework, they found that multijet plus multileptons (plus photon) events occur at a significantly larger rate than clean multilepton events without jet activity. If $\Lambda$ is adjusted so that one $e^+e^-\gamma\gamma$ event is expected in the current run of the Tevatron, they showed that several tens of $\gamma\gamma$ plus multijet plus multilepton and isolated $\gamma$ plus multijet plus multilepton events should also have been present after experimental cuts. SM backgrounds to these characteristic event topologies are small, and it is difficult to imagine how these events could have escaped detection. This conclusion, it is argued, is valid even if the messenger sector is more complicated. The GMLESB explanation of the CDF event, therefore, appears to be unlikely. Indeed a very recent analysis by the D0 Collaboration finds that the $E_T$ spectrum for the $pp \rightarrow \gamma\gamma + X$ channel at the Tevatron appears to be in complete agreement with SM expectation. In particular, there is no excess at the high $E_T$ end as would be expected in the GMLESB framework.

Before closing, we should also mention that the NLSP need not necessarily be $\tilde{Z}_1$. Since it is unstable, the cosmological constraints that we have discussed do not apply so that it may even be charged. In fact, for large values of $\tan\beta$, $\tilde{\tau}_1$ is often the NLSP and decays via $\tilde{\tau}_1 \rightarrow \tau \tilde{G}$. In this case, all SUSY events would contain multiple, isolated $\tau$ leptons in the final state instead of photons and collider signals would be correspondingly altered.

13 Concluding Remarks

We have seen that experiments at the LHC should be able to explore essentially the whole parameter space of weak scale supersymmetry if we require that sparticles provide the degrees of freedom that stabilize the electroweak symmetry breaking sector. While experiments at Tevatron upgrades or LEP2 will explore substantial regions of this parameter space, and maybe even discover sparticles, a non-observation of any signal should not be regarded as disheartening: the expected mass scale is several hundred GeV up to a TeV, and so may well not be accessible except at supercolliders. Electron-positron linear colliders, with a centre of mass energy of 500-1000 GeV should also be able to discover sparticles (almost certainly so if the frequently assumed unification condition for gaugino masses is correct). Linear colliders are the ideal facility for the discovery and subsequent detailed study of Higgs bosons.

\footnote{Although we have not discussed muon colliders in these Lectures, it is worth mentioning that because of the larger value of $m_\mu$, MSSM (and SM) Higgs boson can be produced as $s$-channel resonances at these machines. It has been shown that at a 500 GeV muon collider,
The mSUGRA model that we have described in Sec. 5 provides a consistent and calculable framework for SUSY phenomenology. It is consistent with all accelerator, astrophysical and cosmological data, with grand unification, and can incorporate (though not explain) the observed pattern of electroweak symmetry breaking. Furthermore, because SUSY is a decoupling theory in that virtual effects of sparticles become suppressed if their masses are much larger than $M_Z$, the observed agreement of the SM with LEP constraints is simply incorporated. These models also provide a natural candidate for cold dark matter.

We have seen, however, that the mSUGRA framework is based on extrapolation of the symmetries of physics at very high scales. It is important to keep in mind that one or more of its underlying assumptions may prove to be incorrect. This is especially important when considering the design of future high energy physics facilities. While it is reasonable to use the model as a guide, it is important to examine just how sensitively the various signals depend on these assumptions. The important thing, however, is that these assumptions will be testable in future experiments. For instance, even a partial determination of the pattern of sparticle masses, about which we will get information from experiments at the LHC and at Linear Colliders, will serve to guide us to the physics of SUSY breaking. We will be able to learn whether the ideas underlying mSUGRA, GMLESB, or other alternatives that we have not been able to discuss are correct. All experimental measurements — not just sparticle masses — will be useful for this purpose. We may learn about gaugino-Higgsino mixing via knowledge of their decay patterns, while a study of third generation sfermions may provide information about their intra-generational mixing (which again may serve to discriminate between models).

Experiments at supercolliders are essential both for a complete exploration of the entire parameter space, and for the elucidation of any new phenomena that might be discovered. Experiments at the LHC and TeV Linear Colliders will unambiguously discover or exclude weak scale supersymmetry. Together, these facilities will allow a comprehensive study of sparticle properties (if SUSY is discovered) which, in turn, will yield information about physics at higher energy scales. Even if SUSY turns out not to be 'The Answer, we will almost certainly learn something new, and probably unexpected, in these experiments.

Before closing, we should remind ourselves that SUSY theories, in spite of all the attention that they have received, are not a panacea. $h$ should be distinguishable from the SM Higgs boson over a wide range of parameters, and further, that it should be possible to discover $H$ and $A$ if their mass is smaller than $\sqrt{s}$. The integrated luminosity, beam resolution, as well as machine and detector features that are needed for these measurements have been delineated in this study to which we refer the interested reader.
Supersymmetry really addresses a single (but very important) issue: how is electroweak symmetry broken? It does not shed any light on the other shortcomings of the SM. For example, SUSY has nothing to say about the pattern of fermion masses and mixings, the replication of generations, the choice of the gauge group or of the particle multiplets. While there are new sources of CP violation in SUSY theories, it is fair to say that SUSY models do not really explain the origin of this. Finally, even in SUSY theories, the cosmological constant needs to be severely fine-tuned to be consistent with observation. Supersymmetric theories also cause new problems not present in the SM. We should ask:

- Why are baryon and lepton number conserved at low energy when it is possible to write dimension four $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions that allow for their non-conservation? Perhaps this tells us something about symmetries at the high scale.

- Why is the supersymmetric parameter $\mu \sim M_{Weak}$?

- What is the origin of SUSY breaking and why are SUSY breaking parameters fifteen orders of magnitude smaller than the Planck scale?

- Why are CP violation and FCNC from new SUSY sources so small?

We do not know the answers to these and probably several other questions, although many interesting suggestions exist in the literature. Perhaps clues to some of these questions lie in the unknown mechanism of SUSY breaking. We really need guidance from experiment in order to know which directions are fruitful for theory to pursue. We should, of course, always keep open the possibility that it is not supersymmetry, but some totally different mechanism that is responsible for stabilizing the electroweak scale. Only experiments can tell whether weak scale supersymmetry is realized in nature. What is clear, however, is that the exploration of the TeV scale will provide essential clues for unravelling the nature of electroweak symmetry breaking interactions. We must look to see what we find.

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