Brief Introduction to Flavor Physics

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\textbf{abstract}

We consider the standard model (SM) quark flavor sector. We study its structure in a spurionic, symmetry oriented approach. The SM picture of flavor and CP violation is now experimentally verified, hence strong bounds on beyond the SM flavor structure follow. We show how to parametrically derive such bounds, in a model independent manner, via minimal flavor violation power counting. This min-review summarizes lectures given at the ISSCSMB '08 international school. It aims to give basic tools to understand how flavor and CP violation occur in the SM and its extensions. It should be particularly useful for non-expert students who have mastered other aspects of the SM dynamics.

\section{I. INTRODUCTION}

Flavors are just replication of states with identical quantum numbers. The standard model (SM) consists of three such replications of the five fermionic representations of the SM gauge group. Flavor physics describes the non-trivial spectrum and interactions of the flavor sector. What makes flavor physics particularly interesting is that the SM flavor sector is rather unique, its special characteristic makes it testable and predictive. \footnote{Due to time limitation, this set of lectures discusses the quark sector only. Most of the concepts that are explained here can be directly applied to the lepton sector.} Let us list few of the SM unique flavor predictions:

- Contains a single CP violating parameter.\footnote{The SM contains an additional flavor diagonal CP violating parameter, the strong CP phase, however, experimental data constrains it to be smaller than $O\left(10^{-10}\right)$, hence negligibly small.}
- Flavor conversion is driven by three mixing angles.
- To leading order, flavor conversion proceeds through weak charged current interactions.
- To leading order, flavor conversion involves left handed (LH) currents.
- CP violating processes must involve all the three generations.
- The dominant flavor breaking is due to the top Yukawa coupling, hence the SM posses a large approximate global flavor symmetry [as shown below, technically it is given by $U(2)_Q \times U(2)_U \times U(1)_L \times U(3)_D$].

In the last four decades, or so, a huge effort was invested towards testing the SM predictions related to its flavor sector. Recently, due to the success of the B factories, the field of flavor physics has made a dramatic progress, culminated in Kobayashi and Maskawa winning the Nobel prize. It is now established that the SM contributions drive the observed flavor and CP violation (CPV) in nature, via the Cabibbo-Kobayashi-Maskawa \footnote{Electronic address: gilad.perez@weizmann.ac.il} description. To verify that this is indeed the case, one can allow new physics (NP) to contribute to various clean observables which can be calculated precisely within the SM. Analyses of the data before and after the B factories data have matured \cite{3,4,5} demonstrate that the NP contributions to these theoretically clean processes cannot be bigger than $O\left(30\%ight)$ of the SM contributions \cite{6,7}.
Very recently, the SM passed another non-trivial test. The neutral $D$-meson system is the only one among the four neutral meson systems ($K, D, B, B_s$) that is made of up-type quarks (for formalism see e.g. [8] and Refs. therein). But this is not the only unique aspect of this system: (i) It is the only system where long distance contributions to the mixing are orders of magnitude above the SM short distance ones [9]. (ii) It is the only system where the SM contribution to the CP violation in the mixing amplitude is expected to be below the permil level [10]. The first point means that it is extremely difficult to theoretically predict the width and mass-splitting. The second point implies that, in spite of this inherent uncertainty, $D^0 - \bar{D}^0$ mixing can unambiguously signal new physics if CPV is observed. Present data [11, 12, 13, 14, 15] implies that generic CPV contributions can be only of $\mathcal{O}(20\%)$ of the total (un-calculable) contributions to the mixing amplitudes, again consistent with the SM null prediction.

We have just given rather solid arguments for the validity of the SM flavor description. What else is there to say then? Could this be the end of the story? We have several important reasons to think that flavor physics will continue to play a significant role in our understanding of microscopical physics at and beyond the reach of current colliders.

The SM quarks furnish three representations of the SM gauge group, $SU(3) \times SU(2) \times U(1)$: $Q(3, 2)_{\frac{1}{3}} \times U(3, 1)_{\frac{2}{3}} \times U(3, 1)_{-\frac{1}{3}}$, where $Q, U, D$ stand for $SU(2)$ weak doublet, up type and down type singlet quarks respectively. Flavor physics is related to the fact that the SM consists of three replications/generations/flavors of these three representations. The flavor sector of the SM is described via the following part of the SM Lagrangian

$$\mathcal{L}^F = \bar{q}^i \slashed{D} q^j \delta_{ij} + (Y_U)_{ij} \bar{q}^i U^j H_u + (Y_D)_{ij} \bar{q}^i D^j H_d,$$

\text{II. THE STANDARD MODEL FLAVOR SECTOR}

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where \( \partial \equiv D_{\mu} \gamma^\mu \) with \( D_{\mu} \) being a covariant derivative, \( q = Q, U, D, \) within the SM with a single Higgs \( H_u = i\sigma_2 H_u^* \) (however, the reader should keep in mind that at present, the nature and content of the SM Higgs sector is unknown) and \( i, j = 1, 2, 3 \) are flavor indices.

If we switch off the Yukawa interactions the SM posses a large global, flavor symmetry, \( G_{\mathrm{SM}}, \)
\[
G_{\mathrm{SM}} = U(3)_Q \times U(3)_U \times U(3)_D .
\]

Inspecting Eq. (1) shows that the only non trivial flavor dependence in the Lagrangian is in the form of Yukawa transformation, matrices are unphysical. A simple way to see that (see e.g. [20] and Refs. therein) is to use the fact that a flavor basis transformation,
\[
Q \rightarrow V_Q Q , \quad U \rightarrow V_U U , \quad D \rightarrow V_D D ,
\]
leaves the SM Lagrangian invariant apart from redefinition of the Yukawas,
\[
Y_U \rightarrow V_Q V_U^\dagger , \quad Y_D \rightarrow V_Q V_D^\dagger ,
\]
where \( V_i \) is a 3x3 unitary rotation matrix. Each of the three rotation matrices \( V_{Q,U,D} \) contain three real parameters and six imaginary ones (the former ones correspond to the three generators of the SO(3) group and the latter correspond to the rest, six generators, of the \( U(3) \) group). We know, however, that physical observables do not depend on our choice of basis. Hence, we can use these flavor rotations to eliminate unphysical flavor parameters from \( Y_{U,D} \). Out of the 18 real parameters we can remove 9 \( (3 \times 3) \) ones. Out of the 18 imaginary parameters we can remove 17 \( (3 \times 6 - 1) \) ones. We cannot remove all the imaginary parameters due to the fact that the SM Lagrangian conserves a \( U(1)_B \) symmetry.\(^5\) Thus, there is a linear combination of the diagonal generators of \( G_{\mathrm{SM}} \) which is unbroken even in the presence of the Yukawa matrices and hence cannot be used in order to remove the extra imaginary parameter.

An explicit calculation shows that the 9 real parameters correspond to 6 masses and 3 CKM mixing angles, while the imaginary parameter corresponds to the CKM celebrated phase. To see that, we can define a mass basis where \( Y_{U,D} \) are both diagonal. This can be achieved by applying a bi-unitary transformation on each of the Yukawas:
\[
Q^{u,d} \rightarrow V_Q^{u,d} Q^{u,d} , \quad U \rightarrow V_U U , \quad D \rightarrow V_D D ,
\]
which leaves the SM Lagrangian invariant apart from redefinition of the Yukawas,
\[
Y_U \rightarrow V_Q Y_U V_U^\dagger , \quad Y_D \rightarrow V_Q Y_D V_D^\dagger ,
\]
the difference between the transformations used in Eqs. (3,4) and the ones above (5,6) is in the fact that each component of the \( SU(2) \) weak doublets (denoted as \( Q^u \equiv U_L \) and \( Q^d \equiv D_L \)) transforms independently. This manifestly breaks the \( SU(2) \) gauge invariance, hence, such a transformation makes sense only for a theory in which the electroweak symmetry breaking via the Higgs mechanism. Applying the above transformation amounts to "moving" to the mass basis. The SM flavor Lagrangian, in the mass basis, is given by (in a unitary gauge),
\[
L_m^\mathrm{NC} = \left( \bar{q}^i \partial \gamma^j \delta_{ij} \right)_{\mathrm{NC}} \equiv \left( \bar{q}_L \gamma^\mu \gamma^5 t_L \right) \left( \begin{array}{ccc}
\gamma_a & 0 & 0 \\
0 & \gamma_c & 0 \\
0 & 0 & \gamma_b \\
\end{array} \right) \left( \begin{array}{c}
\bar{u}_R \\
\bar{c}_R \\
\bar{t}_R \\
\end{array} \right) (v + h) + (u, c, t) \leftrightarrow (d, s, b) + \frac{g_2}{\sqrt{2} m_L} y_{ij}^{\mathrm{CKM}} V^{\mathrm{CKM}}_{ij} d_L w^+ + \text{h.c.},
\]
where the subscript NC stands for neutral current interaction for the gluons, the photon and the \( Z \) gauge bosons, \( W^\pm \) stands for the charged electroweak gauge bosons, \( h \) is the physical Higgs field, \( v \sim 176 \text{ GeV} \), \( m_i = y_i v \) and \( V^{\mathrm{CKM}} \) is the CKM matrix
\[
V^{\mathrm{CKM}} = V_Q^{u} V_Q^{d} .
\]

\[^4\] At the quantum level a linear combination of the diagonal \( U(1)'s \) inside the \( U(3)'s \) which corresponds to the axial current is anomalous.

\[^5\] More precisely only the combination \( U(1)_{B-L} \) is non-anomalous.
In general the CKM is a $3 \times 3$ unitary matrix, with 6 imaginary parameters. However, as evident from Eq. 7, the charged current interactions are the only terms which are not invariant under individual quark vectorial $U(1)^6$ field redifinitions,

$$u_i, d_j \rightarrow e^{i\theta_{u_i, d_j}}.$$ (9)

The diagonal part of this transformation corresponds to the classically conserved baryon current while the non-diagonal, $U(1)^5$, part of the transformation can be used to remove 5 out of the 6 phases, leaving the CKM matrix with a single physical phase. Notice also that a possible permutation ambiguity for ordering the CKM entries is removed since we have ordered the fields in Eq. (7) according to their masses, light fields first. This exercise of explicitly identifying the mass basis rotation is quite instructive, and we have already learned several important issues regarding how flavor is broken within the SM (we shall derive the same conclusions using a spurion analysis in a symmetry oriented manner below in section III).

- Flavor conversions only proceed via the three CKM mixing angles.
- Flavor conversion is mediated via the charged current electroweak interactions.
- The charge current interactions only involve LH fields.

Even after removing all the unphysical parameters there are various possible forms for the CKM matrix. For example, a parameterization used by the particle data group (PDG) [21], is given by

$$V_{\text{CKM}} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{KM}}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{KM}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{KM}}} & s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{KM}}} \\
s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{KM}}} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{KM}}} & c_{12}s_{13}
\end{pmatrix},$$ (10)

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$. The three $\sin \theta_{ij}$ are the three real mixing parameters while $\delta_{\text{KM}}$ is the Kobayashi-Maskawa phase.

### B. CP violation

The SM predictive power picks up once CPV is considered. We have already proven that the SM flavor sector consists of a single CP violating parameter. Once presented with a SM Lagrangian where the Yukawa matrices are given in a generic basis, it is not-trivial to determine whether CP is violated or not. This is even more challenging when discussing beyond the SM dynamics where new CP violating sources might be present. A brute force way, to establish that CP is violated would be to show that no field redefinitions would render a real Lagrangian. For example consider a Lagrangian with a single Yukawa matrix,

$$\mathcal{L}^Y = Y_{ij} \bar{\psi}_L \phi \psi_R^j + Y_{ij}^* \bar{\psi}_R \phi^\dagger \psi_L^i,$$ (11)

where $\phi$ is a scalar and $\psi_X^i$ is a fermion field. A CP transformation exchanges the operators

$$\bar{\psi}_L \phi \psi_R^j \rightarrow \bar{\psi}_R \phi^\dagger \psi_L^i,$$ (12)

but leaves their coefficients, $Y_{ij}$ and $Y_{ij}^*$, unchanged, since CP is a linear unitary non-anomalous transformation. This means that CP is conserved if

$$Y_{ij} = Y_{ij}^*.$$ (13)

This is, however, not a basis independent statement. Since physical observables do no depend on a specific basis choice it is enough to find a basis in which the above relation holds.6

Sometimes the brute force way is tedious and might be quite complicated. A more systematic approach would be to identify a phase reparamaterization invariant or basis independentd quantity, that vanishes in the CP conserving limit. As discovered in [22], for the SM case one can define the following quantity

$$C^{\text{SM}} = \det [Y_D Y_D^\dagger, Y_U Y_U^\dagger],$$ (14)

6 It is easy to show that in this example, in fact, CP is not violated for any number of generations.
and the SM is CP violating if and only if
\[ \Im(C^{\text{SM}}) \neq 0. \]  
(15)

It is trivial to prove that only if the number of generations is three or more then CP is violated. Hence, within the SM, where CP is broken explicitly in the flavor sector, any CP violating process has to involve all the three generations. This is a strong requirement which leads to several sharp predictions. Furthermore, all the CPV observables are correlated since they are all proportional to a single CP violating parameter, \( \delta^{\text{KM}} \). Finally, it is worth mentioning that CP violating observables are related to interference between different processes and hence are measurements of amplitude ratios. Thus, in various known cases they turn out to be cleaner and easier to interpret theoretically.

C. The flavor puzzle

Now that we have precisely identified the SM physical flavor parameters it is interesting to ask what is their experimental value (using \( \overline{\text{MS}} \))\cite{21}:

\[
\begin{align*}
    m_u &= 1.5\ldots3.3\text{MeV}, \quad m_d = 3.5\ldots6.0\text{MeV}, \quad m_s = 150^{+30}_{-40}\text{MeV}, \\
    m_c &= 1.3\text{GeV}, \quad m_b = 4.2\text{GeV}, \quad m_t = 170\text{GeV}, \\
    V_{ud}^{\text{CKM}} &= 0.97, \quad V_{us}^{\text{CKM}} = 0.23, \quad V_{ub}^{\text{CKM}} = 3.9 \times 10^{-3}, \\
    V_{cd}^{\text{CKM}} &= 0.23, \quad V_{cb}^{\text{CKM}} = 1.0, \quad V_{tb}^{\text{CKM}} = 41 \times 10^{-3}, \\
    V_{td}^{\text{CKM}} &= 8.1 \times 10^{-3}, \quad V_{ts}^{\text{CKM}} = 39 \times 10^{-3}, \quad V_{tb}^{\text{CKM}} = 1, \quad \delta^{\text{KM}} = 77^\circ,
\end{align*}
\]

(16)

where \( V_{ij}^{\text{CKM}} \) corresponds to the magnitude of the \( ij \) entry in the CKM matrix, \( \delta^{\text{KM}} \) is the CKM phase, only uncertainties bigger than 10% are shown, numbers are shown to a 2-digit precision and the \( V_{ti}^{\text{CKM}} \) entries involve indirect information [a detailed description and Refs. can be found in the PDG\cite{21}].

Inspecting the actual numerical values for the flavor parameters, given in Eq. (16) shows a peculiar structure. Most of the parameters, apart from the top mass and the CKM phase, are small and hierarchical. The amount of hierarchy in the flavor sector can be characterized by looking at two different classes of observables:

- Hierarchies between the masses, which are not related to flavor converting processes - as a measure of these hierarchies we can just estimate what is the size of the product of the Yukawa coupling square differences

\[
\left( m_l^2 - m_u^2 \right) \left( m_u^2 - m_d^2 \right) \left( m_d^2 - m_s^2 \right) \left( m_s^2 - m_b^2 \right) \left( m_b^2 - m_t^2 \right) / v^{12} = O \left( 10^{-19} \right).
\]

(17)

- Hierarchies in the mixing which mediate flavor conversion, this is related to the tiny misalignment between the up and down Yukawas - one can quantify this effect in a basis independent fashion as follows. A CP violating quantity, associated with \( V_{CKM} \), that is independent of parametrization\cite{22}, \( J_{\text{KM}} \), is defined through

\[
\Im \left[ V_{ij}^{\text{CKM}} V_{kl}^{\text{CKM}} \left( V_{il}^{\text{CKM}} \right)^* \left( V_{kj}^{\text{CKM}} \right)^* \right] = J_{\text{KM}} \sum_{m,n=1}^3 \epsilon_{km} \epsilon_{jn} = c_{12} c_{23}^2 c_{13}^2 s_{12} s_{23} s_{13} \sin \delta^{\text{KM}} \approx \lambda^6 A^2 \eta = O \left( 10^{-5} \right).
\]

(18)

where \( i,j,k,l = 1,2,3 \). We see that even though \( \delta^{\text{KM}} \) is of order unity the resulting CP violating parameter is small since it is "screened" by small mixing angles. If any of the mixing angles is a multiple of \( \pi/2 \) then the SM Lagrangian becomes real. Another, explicit way to see that \( Y_t \) and \( Y_D \) are quasi aligned is via the Wolfenstein parametrization of the CKM matrix, where the four mixing parameters are \( (\lambda, A, \rho, \eta) \) with \( \lambda = |V_{us}| = 0.23 \) playing the role of an expansion parameter\cite{23}:

\[
V_{\text{CKM}} = \begin{pmatrix}
    1 & \lambda & A \lambda^3 (\rho - i \eta) \\
    -\frac{\lambda^2}{\lambda} & 1 - \frac{\lambda^2}{\lambda} & A \lambda^2 \\
    A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix} + O(\lambda^4).
\]

(19)

Basically, to zeroth order, the CKM matrix is just a unit matrix!

As we shall discuss further below, both kind of hierarchies described in the bullets lead to suppression of CPV. Thus, a nice way to quantify the amount of hierarchies both in masses and mixing angles is to compute the value of the reparameterization invariant measure of CPV introduced in Eq. (15)

\[
C^{\text{SM}} = J_{\text{KM}} \times \left( m_u^2 - m_d^2 \right) \left( m_u^2 - m_s^2 \right) \left( m_d^2 - m_b^2 \right) \left( m_s^2 - m_t^2 \right) / v^{12} = O \left( 10^{-23} \right).
\]

(20)
This tiny value of \( C_{\text{SM}} \) that characterizes the flavor hierarchy in nature would be of order 10% in theories where \( Y_{U,D} \) are generic order one complex matrices. The smallness of \( C_{\text{SM}} \) is something that many flavor models beyond the SM try to address. Furthermore, SM extensions that have new sources of CPV tend not to have the SM built in CP screening mechanism. Thus, they give too large contributions to the various observables that are sensitive to CP breaking. Therefore, these models are usually excluded by the data, which is consistent with the SM predictions.

### III. SPURION ANALYSIS OF THE SM FLAVOR SECTOR & MINIMAL FLAVOR VIOLATION

In this part we shall try to be more systematic in understanding the way flavor is broken within the SM. We shall develop a spurion, symmetry oriented description for the SM flavor structure and also generalize it to NP models with similar flavor structure that goes under the name minimal flavor violation (MFV).

#### A. Spurion understanding of the SM flavor breaking

It is clear that if we set the Yukawa couplings of the SM to zero we restore the full global flavor group, \( G_{\text{SM}} = U(3)_Q \times U(3)_U \times U(3)_D \). In order to be able to better understand the nature of flavor and CPV within the SM, in the presence of the Yukawa terms, we can use a spurion analysis as follows. Let us formally promote the Yukawa matrices to spurion fields, which transform under \( G_{\text{SM}} \) in a manner that makes the SM invariant under the full flavor group (see e.g. [24] and Refs. therein). From the flavor transformation given in Eqs. (3,4) we can read the representation of the various fields under \( G_{\text{SM}} \) (see illustration in Fig. 1).

**Fields:** \( Q(3,1,1), U(1,3,1), D(1,1,3) \);

**Spurions:** \( Y_{U}(3,3,1), Y_{D}(3,1,3) \).

The flavor group is broken by the "background" value of the spurions \( Y_{U,D} \) which are bi-fundamentals of \( G_{\text{SM}} \). It is instructive to consider the breaking of the different flavor groups separately (since \( Y_{U,D} \) are bi-fundamentals the breaking of quark doublet and singlet flavor groups are linked together, so this analysis only give partial information to be completed below). Consider the quark singlet flavor group, \( U(3)_U \times U(3)_D \), first. We can construct a polynomial of the Yukawas with simple transformation properties under the flavor group. For instance, consider the objects

\[
A_{U,D} \equiv Y_{U,D}^\dagger Y_{U,D} - \frac{1}{3} \text{tr} \left( Y_{U,D}^\dagger Y_{U,D} \right) I_3.
\]

Under the flavor group \( A_{U,D} \) transform as

\[
A_{U,D} \rightarrow V_{U,D} A_{U,D} V_{U,D}^\dagger.
\]

Thus, \( A_{U,D} \) are adjoints of \( U(3)_{U,D} \) and singlets of the rest of the flavor group [while \( \text{tr}(Y_{U,D}^\dagger Y_{U,D}) \) are flavor singlets]. Via similarity transformation we can bring \( A_{U,D} \) to a diagonal form, simultaneously. Thus, we learn that the background value of each of the Yukawa matrices transform under \( G_{\text{SM}} \) down to a residual \( U(1)_{U,D}^3 \), first. We can construct a polynomial of the Yukawas with simple transformation properties under the flavor group. For instance, consider the objects

\[
A_{Q^u,Q^d} \equiv Y_{U,D}^\dagger Y_{U,D} - \frac{1}{3} \text{tr} \left( Y_{U,D}^\dagger Y_{U,D} \right) I_3.
\]

However, in this case the breaking is more involved since \( A_{Q^u,d} \) are adjoint of the same flavor group. This is a direct consequence of the \( SU(2) \) weak gauge interaction which relates the two components of the \( SU(2) \) doublets. This actually motivates one to extend the global flavor group as follows. If we switch off the electroweak interactions the SM global flavor group is actually enlarged to

\[
G_{\text{weakless}}^{\text{SM}} = U(6)_Q \times U(3)_U \times U(3)_D,
\]

since now each \( SU(2) \) doublet, \( Q_i \) can be split into two independent flavors, \( Q_i^{u,d} \) with identical \( SU(3) \times U(1) \) gauge quantum numbers [25]. This limit, however, is not very illuminating since it does not allow for flavor violation at all. To make a progress it is instructive to distinguish the \( W^\pm \) neutral current interactions from the \( W^\pm \) charged current interactions...
Flavor conversion occurs because of the fact that in general we cannot diagonalize simultaneously $A$ which characterize the breaking of the individual LH flavor symmetries, LH charged current interactions. These transitions are mediated by the spurion backgrounds, $g_2^\pm$, since the Yukawa matrices are bi-fundamentals – The LH and RH flavor breaking are tied together. The full breaking of the quark singlet groups is rather trivial. It is, however, linked to the more involved LH flavor breaking to obtain the complete picture of how flavor is broken within the SM. As we saw the flavor breaking within the SM occurs only when $g_2^\pm$ has a background value. Unlike $Y_{U,D}$, $g_2^\pm$ is a special spurion in the sense that its eigen values are degenerate as required by the weak gauge symmetry. Hence, it breaks the $U(3)^u \times U(3)^d$ down to a diagonal group which is nothing but $U(3)$. We can identify two bases where $g_2^\pm$ has an interesting background value: The weak interaction basis where the background value of $g_2^\pm$ is simply a unit matrix\(^8\)

\[ (g_2^\pm)_{\text{int}} \propto 1_3, \]

and the mass basis where (after removing all unphysical parameters) the background value of $g_2^\pm$ is the CKM matrix\(^9\)

\[ (g_2^\pm)_{\text{mass}} \propto V^{\text{CKM}}. \]

Now we are at position to understand the way flavor conversion is obtained in the SM. Three spurions must participate in the breaking, $Y_{U,D}$ and $g_2^\pm$. Since $g_2^\pm$ is involved it is clear that generation transition has to involve LH charged current interactions. These transitions are mediated by the spurion backgrounds, $A_{Q^u,Q^d}$ [see Eq. (24)] which characterize the breaking of the individual LH flavor symmetries,

\[ U(3)^u \times U(3)^d \rightarrow U(1)^3 \times U(1)^3. \]

Flavor conversion occurs because of the fact that in general we cannot diagonalize simultaneously $A_{Q^u,Q^d}$ and $g_2^\pm$, where the misalignment between $A_{Q^u}$ and $A_{Q^d}$ is precisely characterized by the CKM matrix. This is illustrated in Fig. 3 where it is shown that the flavor breaking within the SM goes through collective breaking\(^9\) a term often used in the context of little Higgs models (see e.g.\(^27\) and Refs. therein). We can now combine the LH and RH quark flavor symmetry breaking to obtain the complete picture of how flavor is broken within the SM. As we saw the breaking of the quark singlet groups is rather trivial. It is, however, linked to the more involved LH flavor breaking since the Yukawa matrices are bi-fundamentals – The LH and RH flavor breaking are tied together. The full breaking is illustrated in Fig. 1.

\[ G^{\text{SM}} = U(3)^Q \times U(3)^U \times U(3)^D \]

\[ Y_U(3,\bar{3},1), \quad Y_D(3,1,\bar{3}) \]

**FIG. 1:** The SM flavor symmetry breaking by the Yukawa matrices.

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\(^7\) To get to this limit, formally, one can think of a model where the Higgs field is an adjoint of $SU(2)$ and a singlet of color and hypercharge. In this case the Higgs VEV preserves a $U(1)$ gauge symmetry and the $W^\pm$ would therefore still remain massless. However, the $W^\pm$ will acquire masses of the order of the Higgs VEV and, therefore, charged current interactions would be suppressed.

\(^8\) Note that the interaction basis is not unique since $g_2^\pm$ is invariant under flavor transformation where $Q^u$ and $Q^d$ are rotated by the same amount, see more in the following.
FIG. 2: Breaking of the $U(3)_{U,D}$ groups by the Yukawa matrices which form an appropriate LH (RH) flavor group singlet (adjoint+singlet).

\[
U(3)_U \Rightarrow U(1)^3_U \quad U(3)_D \Rightarrow U(1)^3_D
\]

\[
U(3)_Q \text{ (diag.)}
\]

\[
Y_U Y_U^\dagger \quad Y_D Y_D^\dagger
\]

\[
U(3)_{Q_u} \Rightarrow U(1)^3_{Q_u} \Rightarrow U(1)_B \iff U(1)^3_{Q_d} \iff U(3)_{Q_d}
\]

\[V_{CKM}
\]

\[g_2^\pm
\]

FIG. 3: $U(3)_{Q_u,d}$ breaking by $A_{Q_u,d}$ and $g_2^\pm$.

B. A comment on description of flavor conversion in physical processes

The above spurion structure allows us to describe SM flavor converting processes. However, the reader might be confused since we have argued above that flavor converting processes must involve the three spurions, $A_{Q_u,d}$ and $g_2^\pm$. It is well known, that the rates for charge current processes, which are described via conversion of down quark to an up one (and vis a versa), say like beta decay or $b \to u$ transitions are only suppressed by the corresponding CKM entry, or $g_2^\pm$. What happened to the dependence on $A_{Q_u,d}$? The key point here is that in a typical flavor precision measurement the experimentalists produce mass eigenstate (for example a neutron or a $B$ meson), and thus the fields involved are chosen to be in the mass basis. For instance a $b \to c$ process is characterized by producing a $B$ meson which decays into a charmed one. Hence, both $A_{Q_u,d}$, $A_{Q_u}$ participate, being forced to be diagonal, but in a nonlinear way (i.e strictly speaking this transition cannot be described in a basis independent fashion by some simple insertion of powers of $A_{Q_u}$ and $A_{Q_d}$). Physically we can characterize it by writing an operator

\[
O_{b \to c} = \bar{c}_{\text{mass}} (g_2^\pm)^{\text{ch}_{\text{mass}}} b_{\text{mass}}
\]

where both the $b_{\text{mass}}$ and $c_{\text{mass}}$ quarks are mass eigenstate. Note that this is consistent with the transformation rules for the extended gauge group, $G_{\text{exten}}^{SM}$ given in Eqs. (26,27), where the fields involved belong to different representations of the extended flavor group.

The situation is different when flavor changing neutral current processes are considered. In such a case a typical measurement involves mass eigenstate quarks belonging to the same representation of $G_{\text{exten}}^{SM}$. For example, processes that mediate $B^0_d \to B^0_d$ oscillation due to the tiny mass difference $\Delta m_d$ between the two mass eigenstates (which was measured for the first time by the ARGUS experiment [28]) are described via the following operator, omitting the spurion structure for simplicity,

\[
O_{\Delta m_d} = (b_{\text{mass}} d_{\text{mass}})^2.
\]

Obviously, this operator cannot be generated by SM processes since it violates the $G_{\text{exten}}^{SM}$ symmetry. Since it involves flavor conversion (it violates $b$ number by two units, hence denoted as $\Delta b = 2$ and belongs to $\Delta F = 2$ class of FCNC processes) it has to have some power of $g_2^\pm$. A single power of $g_2^\pm$ connects LH down quark to a LH up one, so the
leading contribution should go like $\bar{D}_L (g_2^{±})^{ik} D_L (g_2^{±}^{*})^{kj} (i, k, j = 1..3)$ which implies, as expected that this process is mediated at least via one loop. This would not work as well since we can always rotate the down quark fields into the mass basis and simultaneously rotate also the up type quarks (away from their mass basis) so that $g_2^{±} \propto 1_3$. These manipulations define the interaction basis which is not unique [see Eq. (28)]. Therefore, the leading flavor invariant spurion that mediates FCNC transition would have to involve the up type Yukawa spurion as well. A naive guess would be

$$O_{\Delta m_d} \propto \left[ \bar{b}_{\text{mass}} (g_2^{±})_{mass} (A_{Q^u})_{kl} (g_2^{±}^{*})_{mass} d_{\text{mass}} \right]^2$$

$$\sim \left[ \bar{b}_{\text{mass}} \left( m_t^2 (V_{\text{CKM}})_{tb} (V_{\text{CKM}})^{*}_{td} + m_c^2 (V_{\text{CKM}})_{cb} (V_{\text{CKM}})^{*}_{cd} \right) d_{\text{mass}} \right]^2,$$  

where it is understood that $(A_{Q^u})_{kl}$ is evaluated in the down quark mass basis (obviously tiny corrections of order $m_t^2$ are neglected in the above). This expression captures the right flavor structure and is correct for large class of SM extensions. However, it is actually incorrect in the SM case. The reason is that within the SM the flavor symmetries are badly broken by the large top quark mass [26]. The SM corresponding amplitude consist of a rather non-trivial, non-linear function of $A_{Q^u}$ instead of the above naive expression (see e.g. [29] and Refs. therein), which assumes only the simplest polynomial dependence of the spurions. The SM amplitude for $\Delta m_d$ is described via a box diagram and two out of the four power of masses are cancelled, since they appear in the propagators.

C. The SM approximate symmetry structure

In the above we have considered the most general breaking pattern. However, as we have discussed the essence of the flavor puzzle is the large hierarchies in the quark masses, the eigen values of $Y_{U,D}$ and their approximate
alignment. Going back to $A_{Q^u,d}$ [defined in Eqs. [23,24]], the spurions that mediate the SM flavor conversions, we can write them as

$$A_{U,D} = \text{diag} \left(0,0,y_{t,b}^2\right) - \frac{y_{t,b}^2}{3} \mathbf{1}_3 + \mathcal{O}\left(\frac{m_{c,s}^2}{m_{t,b}^2}\right), \quad A_{Q^u,d} = \text{diag} \left(0,0,y_{t,b}^2\right) - \frac{y_{t,b}^2}{3} \mathbf{1}_3 + \mathcal{O}\left(\frac{m_{c,s}^2}{m_{t,b}^2}\right) + \mathcal{O}(\lambda^2),$$

where in the above we took advantage of the fact that $\frac{m_{c,s}^2}{m_{t,b}^2} < \lambda^2 = \mathcal{O}(10^{-5} - 4.2)$. The large hierarchies in the quark masses is translated to an approximate residual RH $U(2)_U \times U(2)_D$ flavor group which implies that RH currents which involve light quarks are very small.

We have so far only briefly discussed the role of flavor changing neutral currents (FCNCs). In the above we have argued, both based on an explicit calculation and in terms of a spurion analysis, that at tree level there are no flavor violating neutral currents, since they must be mediated through the $W^\pm$ couplings or $g_2^\pm$. In fact, this situation, which is nothing but the celebrated GIM mechanism [16], goes beyond the SM to all models where all LH quarks are $SU(2)$ doublets and all RH ones are singlets. The Z-boson might have flavor changing couplings in models where this is not the case.

Can we guess what is the leading spurion structure that induces FCNC within the SM, say which mediates the $b \rightarrow d\nu\bar{\nu}$ decay process, via an operator $O_{b\rightarrow d\nu\bar{\nu}}$? The process changes $b$ quark number by one unit (belongs to $\Delta F = 1$ class of FCNC transitions). It clearly has to contain down type, LH, quark fields (let us ignore the lepton current which is flavor-trivial, for effects related to neutrino masses and lepton number breaking in this class of models see e.g. [30]). Therefore, using the argument presented when discussing $\Delta m_{ud}$ [see Eq. (33)], the leading flavor invariant spurion that mediate FCNC would have to involve the up type Yukawa spurion as well

$$O_{b\rightarrow d\nu\bar{\nu}} \propto \bar{d}^\dagger L_3 d_3 (A_{Q^u})_{ik} g_2^\pm \bar{u} L_1 \times \nu \bar{\nu}.$$  

The above considerations demonstrate how the GIM mechanism removes the SM divergencies from various one loop FCNC processes. These, naively, are expected to be log divergent. The reason is that the insertion of

$$\sum \theta \mathcal{O}(\nabla U) \sim \frac{g_4^2}{16\pi^2 M_W^2} \langle \nu \bar{\nu} \rangle \left(\langle V^{CKM}\rangle_{tb} (V^{CKM})_{td}^* \bar{d} L \times \nu \bar{\nu}\right).$$

where we have added a one loop suppression factor and expected weak scale suppression. This rough estimation actually reproduces the SM result up to a factor of about 1.5 (see e.g. [29,31]). We, thus, find that down quark FCNC amplitudes are expected to be highly suppressed due to the smallness of the top off-diagonal entries of the CKM entries. Parameterically we find the following suppression factor for transition between the $i$th and $j$th generations:

$$b \rightarrow s \propto \left|\langle V^{CKM}\rangle_{tb} (V^{CKM})_{ts}\right| \sim \lambda^2,$$

$$b \rightarrow d \propto \left|\langle V^{CKM}\rangle_{tb} (V^{CKM})_{td}\right| \sim \lambda^3,$$

$$s \rightarrow d \propto \left|\langle V^{CKM}\rangle_{td} (V^{CKM})_{ts}\right| \sim \lambda^5,$$

where for the $\Delta F = 2$ case one needs to simply square the parameteric suppression factors. This simple exercise illustrates how powerful is the SM FCNC suppression mechanism. The gist of it is that the rate of SM FCNC processes is small since they occur at one loop, and more importantly due to the fact that they are suppressed by the top CKM off-diagonal entries, which are very small. Furthermore, since

$$|V_{ts,td}| \gg \frac{m_{c,u}^2}{m_t^2},$$

in most cases the dominant flavor conversion effects are expected to be mediated via the top Yukawa coupling.  

We can now understand how the SM uniqueness related to suppression of flavor converting processes arises:

\[\text{For simplicity we only consider cases with hard GIM where the dependence on mass differences is polynomial. There is a large class of amplitudes, for example processes that are mediated via penguin diagrams with gluon or photon lines, where the quark mass dependence is more complicated and may involve logarithms. The suppression of the corresponding amplitudes goes under the name soft GIM [29].}\]

\[\text{This definitely is correct for CP violating processes or any ones which involve the third generation quarks. It also, generically, holds for new physics MFV models. Within the SM, for CP conserving processes which involve only the first two generations one can find exceptions, for instance when considering the kaon and D meson mass differences, $\Delta m_{D,K}$.}\]
• RH currents for light quarks are suppressed due to their small Yukawa couplings (them being light).
• Flavor transition occurs to leading order only via LH charged current interactions.
• To leading order, flavor conversion is only due to the large top Yukawa coupling.

D. Minimal flavor violation

So far we have focused only on the SM flavor structure and developed a spurion description of the SM flavor breaking. We can, however, extend our above analysis to include also an important class of SM extensions, denoted as minimal flavor violation (MFV) [32] which includes, among others, various extended Higgs models [32, 33], supersymmetric models [34, 35] and under some assumptions warped extra dimension models [26, 36] (for reviews see e.g [31] and Refs. therein). As we shall see, the models which belong to the MFV class enjoy much of the protection against large flavor violation that we have found to exist in the SM case and therefore tend to be consistent with current flavor precision measurements.

The basic idea can be described in the language of effective field theory (EFT) without the need of referring to a specific framework. MFV models can have a very different microscopical dynamics, however, by definition they all have a common origin of flavor breaking, the SM Yukawa matrices. After integrating out the NP degrees of freedom we expect to obtain a low energy EFT which involves only the SM fields and bunch of higher dimension, Lorentz and gauge invariant, operators suppressed by the NP scale ΛMFV. Since flavor is broken only via the SM Yukawas then we can study the most general flavor breaking of the MFV framework by the simple following prescription: We should construct the most general set of higher dimensional operators, which in addition of being Lorentz and gauge invariant, are required also to be flavor invariant, using the spurion analysis that we have introduced in the previous part.

If we are interested in SM processes where the typical energy scale is much smaller than ΛMFV and the NP is not strongly coupled then we expect that the dominant non-SM flavor violation would arise from the lowest order higher dimension operators. For processes involving quark fields, the leading operators are of dimension six. For instance, we expect that the leading flavor violating operators that mediate ΔF = 2 processes would involve only LH fields,

\[ \mathcal{L}_{\Delta F=2}^{MFV} = \frac{1}{(\Lambda_{MFV})^2} [Q_i (a_u A_{Q_u} + a_d A_{Q_d}) Q_j]^2 + \ldots \]  

(39)

where in the above we have written, for simplicity, the leading polynomial of the Yukawas instead of just a quadratic term as presented here. The experimental information is obtained by looking at the dynamics (masses, mass differences, decay, time evolution etc...) of down type mesons hence we can just look at the form that \( \mathcal{L}_{\Delta F=2}^{MFV} \) takes in the down quark mass basis. By definition \( A_{Q_u} \) is diagonal and does not mediate flavor violation, however, in the down type basis, \( A_{Q_d} \) is not diagonal and is given by

\[ (A_{Q_d})_{\text{down}} = V^{\text{CKM}} \text{diag} \left( 0, 0, y_t^2 \right) (V^{\text{CKM}}) \dagger - \frac{y_t^2}{3} I_3 + \mathcal{O} \left( \frac{m_3^2}{m_t^2} \right) \sim y_t^2 \left( V^{\text{CKM}} \right)_{ti} (V^{\text{CKM}})_{tj}^\ast , \]

(40)

where in the above we took advantage of the approximate \( U(2) \) symmetry limit discussed in the previous subsection. As expected, we find that within the MFV framework FCNC processes are suppressed by roughly the same amount as the SM processes, and therefore are typically, at least to leading order, consistent with present data. This need not to be the case when new flavor diagonal CP violating sources are allowed [26, 37].

11 Note that in the presence of NP, we do not generally expect that \( y_t^\pm \) would be the only object that mediates the breaking of \( U(3)_{Q_u} \times U(3)_{Q_d} \), hence there is no advantage in using \( g^{\text{SM}}_{\text{exten}} \) representations in this case, nor \( y_t^\pm \) as a spurion.

12 Since the top Yukawa is of order unity and possibly also the bottom one this might not be a justified assumption. Generically, one would expect to get a generic polynomial of the Yukawas instead of just a quadratic term as presented here. The general case can be dealt with by resumming the contributions from the large eigenvalue via non-linear sigma model techniques, which allow one to separate the large and small terms in the Yukawa matrices [26]. This leads to somewhat richer structure than discussed here. For simplicity we shall only focus on the linear MFV case where only the leading terms in the polynomial expansion are considered.
FIG. 5: On the left (right) the allowed range for the new physics contributions to $\Delta F = 2$ processes in the $B^0_d$ system. The constraints are shown in the $h_d - \sigma_d$ plan prior to (after) 2004 \cite{4}. The color scale represents the confidence level (1 means only unknown theoretical uncertainties included). Measurements of SM, tree-level, CPV observables, which are unlikely to be affected by new physics, led to an order of magnitude improvement in the constraints of non-SM contributions. It is established that the SM contributions are the dominant ones.

E. Beyond MFV & the 2004 ”revolution”

It is interesting to note that only fairly recently has the data begun to disfavor models with only LH currents, but with new sources of flavor and CPV \cite{3,4}, characterized by a CKM-like suppression $\cite{38,39}$. In fact this is precisely the way that one can test the success of Kobayashi-Maskawa mechanism for flavor and CP violation $\cite{4,5,6,7,40}$. Below we focus on NP in $\Delta F = 2$ processes which are clean to interpret theoretically. In addition, for simplicity, we only focus on the $B_d$ system where the improvement in constraining new data was particularly dramatic. The NP contributions to $B^0_d$ mixing can be expressed in terms of two parameters, $h_d$ and $\sigma_d$ defined by

$$M^d_{12} = (1 + h_d e^{2i \sigma_d}) M^{d,SM}_{12},$$

(41)

where $M^{d,SM}_{12}$ is the dispersive part of the $B_d^0 - \bar{B}_d^0$ mixing amplitude in the SM.

FIG. 6: The allowed region, shown in grey, in the $x_{12}^{NP}/x_{12} - \sin \phi_{12}^{NP}$ plane. The pink and yellow regions correspond to the ranges predicted by, respectively, the linear MFV and general MFV classes of models $\cite{12}$.

To constrain deviations from the SM in these processes one can use measurements which are directly proportional to $M^d_{12}$ (magnitude and phase). The relevant observables in this case are the neutral $B^0_d$ mass difference, $\Delta m_d$ and the CPV in decay with and without mixing in $B_d^0 \rightarrow \psi K, S_{\psi K}$. These processes are characterized by hard GIM
suppression and proceed, within the SM, via one loop [see Eqs. (36,37)], and in the presence of new physics they can be written as (see e.g. [20]):
\[
\begin{align*}
\Delta m_d & = \Delta m_{d}^{\text{SM}} |1 + h_d e^{2i\sigma_d}|, \\
S_{\psi K} & = \sin \left[ 2\beta + \arg \left( 1 + h_d e^{2i\sigma_d} \right) \right].
\end{align*}
\] (42)

The fact that the SM contribution Eq. (37) to these processes involve the CKM elements which are not measured directly prevents one from independently constraining the NP contributions. Indeed, prior to 2004, the experimental data yield the following rather weak constraints on $h_d$ and $\sigma_d$ [4],
\[
h_d \lesssim 6 \quad \text{and} \quad 0 \lesssim 2\sigma_d \lesssim \pi.
\] (43)

The situation was dramatically improved when BaBar and Belle experiments managed to measure CPV processes which, within the SM, are mediated via tree level amplitudes. The information extracted from these CP asymmetries in $B^\pm \to DK^\pm$ and $B \to \rho\rho$ is probably hardly affected by new physics. The most recent bounds (ignoring $2\sigma$ anomaly in $B \to \tau\nu$) are [41]
\[
h_d \lesssim 0.3 \quad \text{and} \quad \pi \lesssim 2\sigma_d \lesssim 2\pi,
\] (44)

which is very similar to the bound obtained just after 2004 when the new measurements became public. Fig. 5 shows the allowed range in the $h_d - \sigma_d$ plan before and after 2004 taken from [4]. Similar but less dramatic progress was made in the kaon and $B_s$ systems. Furthermore, a recent progress in measurements of CPV in $D^0 \to \bar{D}^0$ mixing led to an important improvement on the new physics constraints. However, in this case the SM contributions are unknown [9] and the only robust SM prediction is the absence of CPV [10]. The three relevant physical quantities related to the mixing can be defined as
\[
y_{12} \equiv |\Gamma_{12}|/\Gamma, \quad x_{12} \equiv 2|M_{12}|/\Gamma, \quad \phi_{12} \equiv \arg(M_{12}/\Gamma_{12}),
\] (45)

where $M_{12}, \Gamma_{12}$ are the total dispersive and absorptive part of the $D^0 \to \bar{D}^0$ amplitude respectively. In Fig. 6 we show (in grey) the allowed region in the $x_{12}^{\text{NP}}/x - \sin \phi_{12}^{\text{NP}}$ plane. $x_{12}^{\text{NP}}$ corresponds to the new physics contributions and $x \equiv \frac{m_2 - m_1}{\Gamma}$, where $m_i, \Gamma$ being the neutral $D$ meson mass eigenstates and average width respectively. The pink and yellow regions correspond to the ranges predicted by, respectively, the linear MFV and general MFV classes of models [12]. We see that the absence of observed CP violation removes a sizable fraction of the possible new physics parameter space, in spite of the fact that the magnitude of the SM contributions cannot be computed!

**IV. CONCLUSIONS**

In these rather laconic and far from being inclusive set of lectures, we have tried to develop a basic understanding of the SM flavor structure and some of its extensions. The idea is to provide the readers with simple symmetry oriented principles, to understand the way flavor violation is mediated within the standard model (SM). The hope is that the methods described above would allow non-experts to understand the unique SM flavor structure, and the power counting for suppression of various flavor changing processes. Furthermore, understanding the SM mechanism for suppressing flavor changing neutral currents also allows one to quickly estimate which models beyond the SM are likely to be excluded by current measurements. In addition the analysis presented might even help the reader to identify viable models and directions for future tests.

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[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
C. Csaki, A. Falkowski and A. Weiler, arXiv:0806.3757 [hep-ph].

[37] G. Colangelo, E. Nikolidakis and C. Smith, Eur. Phys. J. C 59, 75 (2009) [arXiv:0807.0801 [hep-ph]]; P. Paradisi and D. Straub, arXiv:0906.4551 [hep-ph]; M. Gorbahn, S. Jager, U. Nierste and S. Trine, arXiv:0901.2065 [hep-ph]; L. Mercolli, arXiv:0903.4633 [hep-ph]; L. Mercolli and C. Smith, Nucl. Phys. B 817, 1 (2009) [arXiv:0902.1949 [hep-ph]]; W. Altmanshofer, A. J. Buras and P. Paradisi, Phys. Lett. B 669, 239 (2008) [arXiv:0808.0707 [hep-ph]].

[38] S. Davidson, G. Isidori and S. Uhlig, Phys. Lett. B 663, 73 (2008) [arXiv:0711.3376 [hep-ph]].

[39] K. Agashe, G. Perez and A. Soni, Phys. Rev. D 71, 016002 (2005) [arXiv:hep-ph/0408134]; Phys. Rev. Lett. 93, 201804 (2004) [arXiv:hep-ph/0406101].

[40] Y. Grossman, Y. Nir and M. P. Worah, Phys. Lett. B 407, 307 (1997) [arXiv:hep-ph/9704287]; A. G. Cohen, D. B. Kaplan, F. Lepeintre and A. E. Nelson, Phys. Rev. Lett. 78, 2300 (1997) [arXiv:hep-ph/9610252]; J. P. Silva and L. Wolfenstein, Phys. Rev. D 55, 5331 (1997) [arXiv:hep-ph/9610208]; N. G. Deshpande, B. Dutta and S. Oh, Phys. Rev. Lett. 77, 4499 (1996) [arXiv:hep-ph/9608231]; J. M. Soares and L. Wolfenstein, Phys. Rev. D 47, 1021 (1993); G. Barenboim, G. Eyal and Y. Nir, Phys. Rev. Lett. 83, 4486 (1999) [arXiv:hep-ph/9905397]; G. Eyal, Y. Nir and G. Perez, JHEP 0008, 028 (2000) [arXiv:hep-ph/0008009]; S. Laplace, Z. Ligeti, Y. Nir and G. Perez, Phys. Rev. D 65, 094040 (2002) [arXiv:hep-ph/0202010].

[41] M. Bona et al., arXiv:0909.5065 [hep-ph]; V. Tisserand, arXiv:0905.1572 [hep-ph]; Update $h_d - \sigma_d$ plot can be found at URL: http://ckmfitter.in2p3.fr/plots_Moriond09.