Calculation of Deformation and Ductility of Steel Fiber Reinforced High-Strength Concrete Columns

Hui Liu

Department of Engineering Management, China Railway Construction Corporation Limited, NO.40 Fu Xing Road, Beijing 10085, China
Email: 1009102@qq.com

Abstract. Based on the simple stress-strain relationship of steel fiber concrete, the influence of strain coordination factor on the cross section is analyzed, and the calculation equation of curvature ductility coefficient is deduced. According to the law of curvature distribution of flexural members, the curvature distribution is simplified into two-segment and three-segment linear distribution, and the corresponding calculation equations of length, load and deflection of different sections are deduced, and the calculation method based on the theoretical curve of load-deflection is proposed. Compared with the test results, the ductility coefficient, the theoretical curve of load-deflection and the test results all show a high degree of coincidence.

Keywords: Steel fiber, reinforced high strength concrete column, deformation.

1. Introduction

In recent years, high strength concrete is widely used in the field of high-rise building construction, now have confirmed that the reinforced concrete building design should not only meet the requirements of project for strength, also want to meet the requirement of ductility, and have strong ability of deformation, in order to reduce or avoid artifacts appear brittle failure in use phase, this structure will also hold strong seismic performance, energy consumption volume also decrease accordingly. In essence, concrete belongs to a kind of brittle material, and its brittleness increases with the increase of strength. Many areas in China have high incidence of earthquakes. High strength concrete structures are used in building construction, so it is of great practical significance to rationally design the ductility of structures. Adding steel fiber into concrete can not only improve the tensile, flexural and shear strength of concrete, but also optimize the toughness and deformation capacity of concrete components. In this paper, the deformation and ductility of steel fiber high strength concrete columns are analyzed theoretically, and the relevant calculation equations are derived.

2. Project Overview

The total construction area of the project is about 263500 square meters, including 11 high-rise residential buildings, 9 commercial buildings, a heat exchange station, a main entrance gate in an underground garage. The underground level of high-rise residential buildings is 1 floor, and the underground level is 2 floors, ranging from 22 to 33 floors above ground. All the businesses are 2 floors above ground, and the underground garage is 1 floor underground. HPB300 and HRB400E are mainly used in the construction stage of this project. HRB400E is used for longitudinal reinforcement of beam, column and wall, HPB300 for local wall column hooks, HRB400E for plate reinforcement, and HRB400E for hoop reinforcement of beam, column and wall edge members. The key points in the construction stage of reinforcement engineering are three links:
adjusting and controlling the purchase of reinforcement, processing and manufacturing of reinforcement and its connecting parts, and site installation.

All grade III steel bars selected in this project are HRB400E steel bars with seismic fortification requirements. The specific requirements are as follows:

1. The ratio of the actual tested value of the tensile strength of the corresponding longitudinal reinforced bars to the measured value of the yield strength is ≥1.25.

2. The ratio of the measured yield strength value to the standard yield strength value is ≤1.3.

3. The measured total elongation of HRB400E reinforcement at the maximum tensile strength is ≥9.0%.

3. Curvature Ductility Coefficient and Section Bending Moment

3.1. The Basic Assumptions

In order to facilitate the calculation process, the following basic assumptions are made for the characteristics of steel fiber concrete:

1. Before yielding, the strain distribution in the cross section conforms to the assumption of plane section. After yielding, the distribution characteristics of the average strain along the cross-section height of the steel bar are presented as a folded shape, which essentially means that the strain in the compression zone is distributed in a straight line, and the strain in the tensile zone is distributed in another straight line, and the strain coordination factor ($\varphi_u$) is used to dispatch the strain correlation.

2. The ideal elastic-plastic stress-strain curve is used to analyze the tensile and compressive reinforcement and the strengthening period.

3. The stress-strain correlation of SFRC can be divided into the following two cases:

   Compression condition:
   
   When $0 \leq \varepsilon_{fc} \leq \varepsilon_{f0}$, $\sigma_{fc} = E_{fc} \cdot \varepsilon_{fc}$ \hspace{1cm} (1)
   
   When $\varepsilon_{fc} \geq \varepsilon_{f0}$, $\sigma_{fc} = f_{fcm}$ \hspace{1cm} (2)
   
   Where, $\sigma_{fc}$ and $\varepsilon_{fc}$ correspond to the compressive stress and compressive strain of steel fiber concrete in sequence;

   $\varepsilon_{f0}$ and $\varepsilon_{fm}$ are respectively the yield pressure strain and the pressure strain peak.

   Among them [1]:
   
   $\varepsilon_{f0} = (1.3 + 0.01f_{fcm} + 0.96V_f \frac{l}{d}) \times 10^{-3}$ \hspace{1cm} (3)
   
   $\varepsilon_{fm} = \varepsilon_{cu} + 0.0025(V_f \frac{l}{d})^{0.85}$ \hspace{1cm} (4)

   $f_{fcm}$, $\varepsilon_{cu}$, $E_{fc}$ and $f_{fcm}$ correspond to the cubic compressive strength (unit MPa), ultimate compressive strain (take 0.03), elastic modulus (MPa) and bending compressive strength (MPa) of SFRC in turn.

   $f_{fcm} = f_{cm}[1 + 0.126(V_f \frac{l}{d})^{0.74}]$ \hspace{1cm} (5)

   $f_{cm}$ represents the flexural compressive strength held by matrix concrete, and $f_{cm}$ of conventional concrete is 1.1 times that of $f_{fc}$.

   Tensile condition:
When \( 0 \leq \varepsilon_f \leq \varepsilon_y^f, \sigma_f = E_{fc} \cdot \varepsilon_f \) \hspace{1cm} (6)

When \( \varepsilon_f \geq \varepsilon_y^f, \sigma_f = f_{\text{fm}} \) \hspace{1cm} (7)

Type;
\[ \sigma_f, \varepsilon_f, \varepsilon_y^f \text{ and } \varepsilon_u^f \text{ correspond successively to the tensile stress, tensile strain, yield tensile strain and ultimate tensile strain of steel fiber concrete.} \]

Among them:
\[ \varepsilon_y^f = 0.0002 + 0.096V_f \frac{l}{d} \times 10^{-4} \] \hspace{1cm} (8)

3.2. The Bending Moment and Curvature When the Crack is about to Form

When \( \sum M = 0 \), there are:
\[ k_f = \frac{1 + 2n \cdot \frac{A_s}{bh} + 2 \frac{N}{bhf_f}}{2 + 4n \cdot \frac{A_s}{bh} + 2 \frac{N}{bhf_f}} \] \hspace{1cm} (9)

Therefore, the curvature can be derived \( (\phi_f) \) :
\[ \phi_f = \frac{\varepsilon_f}{(1-k_f)h} = \frac{f_{\beta}}{(1-k_f)hE_{fc}} \] \hspace{1cm} (10)

And
\[ M_f = M_s + M_N + M_t \] \hspace{1cm} (11)

\( M_s, M_N \) and \( M_t \) are the bending moments corresponding to the tensile and compression reinforcement, the bending moments held by axial force \( N \), and the bending moments held by concrete in the tensile zone respectively.

3.3. Bending Moment \( (M_y) \) and Curvature \( (\phi_f) \) When Tensile Longitudinal Reinforcement Begins to Yield

Under conventional working conditions, when the tensile reinforcement yields, the stress distribution in the tensile zone is no longer linear and may be divided into two parts with corresponding heights of \( \beta_1 \) and \( \beta_2 \) in turn. When the tensile reinforcement begins to yield, the strain \( (\varepsilon_f) \) of the compression boundary will show two conditions, that is, \( \varepsilon_f \leq \varepsilon_{f0} \) and \( \varepsilon_f > \varepsilon_{f0} \), and the stress distribution in the corresponding pressure area shows two kinds of triangular and trapezoidal patterns. Factors such as the ratio of longitudinal reinforcement and the content of steel fiber affect the actual stress type of the cross section.

When the stress in the compression zone is triangular in distribution \( (\varepsilon_f \leq \varepsilon_{f0}) \) [2]:
\[ D = \frac{1}{2} \sigma_{fc} \cdot k_yh_yb = \frac{1}{2} k_y^2bhy_yE_{fc} \cdot (1-k_y) \] \hspace{1cm} (12)
\[
\sigma_y = \begin{cases} 
E_s \varepsilon_y = \frac{k_y h_0 - a^2}{(1 - k_y)h_0} f_y, & \text{when } \varepsilon_y \leq \varepsilon_y^y \\
f_y & \text{when } \varepsilon_y > \varepsilon_y^y 
\end{cases}
\] (13)

\[
\beta_2 = \frac{\varepsilon_y^y}{\varepsilon_y^y} \cdot (1 - k_y) h_0 = \frac{f_{ftm}}{E_f s} f_s \frac{(1 - k_y) h_0}{f_y} = n \frac{f_{ftm}}{f_y} (1 - k_y) h_0
\]

\[
\beta_1 = h - k_y h_0 - \beta_2
\]

\[
z = \frac{1}{2} f_{ftm} \beta_2 \cdot b + f_{ftm} \beta_1 b
\] (16)

The equation \( \sum X = 0 \) is reduced to the following two cases:

First, when the compression reinforcement does not yield:

\[
\left( \frac{1}{2n} + n\alpha_4 \alpha_2 - \frac{1}{2} \alpha_4^2 n - \alpha_2 \right) k_y^2 + (\alpha_4^2 n + \alpha_2 + \alpha_3 + \rho + \rho') k_y + \left( n\alpha_4 \alpha_2 - \alpha_3 \alpha_2 - \frac{1}{2} \alpha_4^2 n - \alpha_3 - \rho - \lambda \rho' \right) = 0
\] (17)

So let's solve for \( k_y \).

In the second case, when the yield of the pressurized steel bar has occurred, the corresponding equation of \( k_y \) is:

\[
A' k_y^2 + B' k_y + C' = 0
\] (18)

In the above equation:

\[
A' = \frac{1}{2n} + n\alpha_4 \alpha_2 - \frac{1}{2} \alpha_4^2 n - \alpha_2
\]

\[
B' = \alpha_4^2 n + \alpha_2 + \alpha_3 + \rho - \rho' + \alpha_2 \lambda - 2n\alpha_4 \alpha_2
\]

\[
C' = n\alpha_4 \alpha_2 - \alpha_3 \alpha_2 - \frac{1}{2} \alpha_4^2 n - \alpha_3 - \rho + \rho'
\] (21)

When the stress distribution in the compression zone is trapezoidal (\( \varepsilon_f > \varepsilon_{f0} \)), the compression zone can be highly refined into two parts, namely, \( \beta_4, \beta_1 \): [3]

\[
\beta_4 = \frac{\varepsilon_{f0}}{\varepsilon_y^y} (1 - k_y) h_0 = n \frac{f_{ftm}}{f_y} (1 - k_y) h_0
\]

\[
\beta_3 = k_y h_0 - \beta_4 = k_y h_0 (1 + n \frac{f_{ftm}}{f_y}) - nh_0 \frac{f_{ftm}}{f_y}
\]

\[
D = bf_{ftm} \beta_3 + \frac{b}{2} f_{ftm} \beta_4 = bf_{ftm} [k_y h_0 (1 + n \frac{f_{ftm}}{f_y}) - \frac{1}{2} nh_0 \frac{f_{ftm}}{f_y}]
\] (24)

When the compression reinforcement does not yield, the corresponding equation of \( k_y \) is:
When the reinforcement yields, there are:

\[
\begin{align*}
\alpha_3 + & \frac{1}{2} n \alpha_1^2 + \rho - \rho' + \frac{1}{2} n \alpha_2^2 + \alpha_2 \lambda_2 + \lambda \rho = 0 \\
\end{align*}
\]

(25)

3.4. The Corresponding Bending Moment (Mu) and Curvature (\(\phi_f\)) When the Cross-Section Reaches the Limit State

At this point, assuming that the plane section structure is damaged, strain distribution along the section height is assumed to be linear, and strain coordination factor (\(\alpha_u\)) is used to schedule the strain relationship.

According to \(\sum X = 0\):

\[
\begin{align*}
N &= D + f_y A - Z - f_y A_s \\
D &= b f_{cm} k_u h_0 (1 - \frac{\epsilon_{y0}}{\epsilon_{fm}}) + b f_{cm} \frac{\epsilon_{y0}}{\epsilon_{fm}} k_a h_0 \\
Z &= b f_{t} \frac{\epsilon_{y0}}{\phi_u \epsilon_{fm}} k_u h_0 + b f_{cm} k_u h_0 (\frac{\epsilon_y^{u} - \epsilon_{y0}}{\phi_a \epsilon_{fm}}) \\
\end{align*}
\]

(27)

(28)

(29)

Symmetric reinforcement:

\[
\begin{align*}
k_u &= \frac{N / b h_0}{(1 - \frac{1}{2} \frac{\epsilon_{y0}}{\epsilon_{fm}}) f_{cm} - \frac{1}{2} \frac{\epsilon_{y0}}{\phi_u \epsilon_{fm}} - \frac{f_{cm}}{\phi_a \epsilon_{fm}} (\frac{\epsilon_y^{u} - \epsilon_{y0}}{\epsilon_{fm}})} \\
\end{align*}
\]

(30)

The limit curvature after adjustment using the strain coordination factor (\(\phi_u\)) is:

\[
\phi_u = \frac{\epsilon_{u} \epsilon_{fm}}{k_u h_0} \\
\]

(31)

The value interval of \(\phi_u\) is usually 1.1~1.3. Based on the comparison and analysis of the experimental results, \(\phi_u = 1.2\).

In the calculation of ultimate bending moment (Mu), the calculation diagram is unified with the calculation of normal section strength, and the stress in compression and tensile zones is simplified into equivalent rectangular distribution. The equivalent pressure in the compression section is the bending compressive strength (\(f_{cm}\)) of steel fiber concrete. The actual tensile zone elevation is h-x_0, and h-x is taken after the equivalence.
In the case of symmetric reinforcement, the following equation can be obtained from \( \sum X = 0 \):

\[
M_u = N\left(\frac{h}{2} - \frac{x}{2}\right) + f_{y1}A_s(h_0 - x') + \frac{1}{2} f_{fm}(h - x)bh
\]

(32)

3.5. Curvature Ductility Coefficient

\[
\beta_\phi = \frac{\phi_x}{\phi_y}
\]

(33)

4. Load-Deflection Theory Curve

4.1. Simplified Curvature Profile

The stress zone constructed by steel fiber concrete was refined into three section regulation (cracking section \(a-a'\), yield section \(b-b'\), and danger section \(c-c'\)). Therefore, the curvature of \(a-a'\), \(b-b'\) and \(c-c'\) section regulates the constructed curvature distribution features. In order to enhance the simplicity of the deflection calculation equation, the following assumptions are made for the curvature distribution diagram: at the initial yield of \(c-c'\) interface, the curvature presents a symmetric distribution along the length direction and the middle line, and both ends are composed of two broken lines with different slopes. When the \(c-c'\) section reaches the limit curvature, the curvature is symmetrical to the midline distribution, and the two ends are respectively composed of three slopes or other.

4.2. Calculation Equation of Characteristic Points of Load-Deflection Theory Curve

(1) When the bending moment (\(M_c\)) of the mid-span section is equal to the yielding moment (\(M_y\)), there are: [6]

\[
M_f = N \cdot y_f + \frac{1}{2} P_y \cdot a
\]

(34)

\[
M_y = N \cdot \Delta_y + \frac{1}{2} P_y \cdot l
\]

(35)

After interpreting the correlation between load and deflection, there are:

\[
y_f = \int_0^a \phi(x) dx + a \int_0^b \phi(x) dx = \phi_x \left( \frac{al}{2} - \frac{a^2}{2} \right) + \phi_y \left( \frac{al}{2} - \frac{a^2}{6} \right)
\]

(36)

\[
\Delta_y = \int_0^a \phi(x) dx = \phi_x \left( \frac{l^2}{3} - \frac{al}{6} - \frac{a^2}{6} \right) + \phi_y \left( \frac{al}{6} + \frac{l^2}{6} \right)
\]

(37)

Based on the above equation, it can be concluded that:

\[
\phi_x a^3 - 2l(\phi_x + \phi_y)a^2 + [l^2 (\phi_x + 2\phi_y) + 6M / N]a - \frac{6M_f l}{N} = 0
\]

(38)

So we can figure out the length \(a\) held by the elastic region, and we can figure out \(P_y\) and \(\Delta_y\).

(2) When the mid-span section reaches the limit state, there are:

\[
M_f = N \cdot y_f + \frac{P_y}{2} \cdot a
\]

(39)
\[ M_y = N \cdot y_y + \frac{P_u}{2} (a' + b') \quad (40) \]

\[ M_u = N \Delta u_0 + \frac{P_u}{2} l \quad (41) \]

\[ y'_y = \frac{a'^2}{3} \phi_j + \frac{1}{2} a'b' (\phi_y + \phi_f) + a'l_p \phi_u \quad (42) \]

\[ y_y = \left( \frac{1}{3} b'^2 + \frac{1}{2} a'b' \right) \phi_j + \left( \frac{1}{3} a'^2 + \frac{1}{6} b'^2 + \frac{1}{2} a'b' \right) \phi_y + \left( a' + b' \right) l_p \phi_u \quad (43) \]

\[ \Delta u_0 = \left( \frac{1}{3} b'^2 + \frac{1}{2} a'b' \right) \phi_j + \left( \frac{1}{3} a'^2 + \frac{1}{6} b'^2 + \frac{1}{2} a'b' \right) \phi_y + \left( l - \frac{1}{2} l_p \right) l_p \phi_u \quad (44) \]

In the type, a', b', yf', yy, \Delta u_0, Pu is an unknown quantity, it can measure the solution of equations. To simplify the calculation, the equivalent equation is as follows:

\[ \frac{M_u - M_y}{N} = \frac{1}{2} \phi_j l_p^2 + \frac{P_u}{2N} l_p \quad (45) \]

\[ \frac{M_y - M_f}{N} = \left( \frac{1}{3} \phi_j + \frac{1}{6} \phi_f \right) b'^2 + \phi_j l_p b' - \frac{P_u}{2N} b' \quad (46) \]

\[ \frac{M_f}{N} = \frac{1}{3} \phi_j a'^2 + \frac{1}{2} (\phi_j + \phi_f) a'b' + \phi_f a'l_p + \frac{P_u}{2N} a' \quad (47) \]

The solution of the equivalent system of equations was obtained by using the iterative method. Py and Pu values are similar, so Py=Pu was set in the first round. Generally, high accuracy results can be obtained after about 4 rounds of iteration.

4.3. Ductility Factor of Displacement

\[ \beta_\Delta = \Delta_u / \Delta_y \quad (48) \]

By measuring and calculating the values corresponding to different characteristic points on the load-deflection curve, the theoretical load-deflection curve can be sketched, and the actual measurement curve of the same specimen can be drawn together for comparative analysis. As shown in figure 1, after observation, we determined that there was a high coincidence between the measured value and the test results, indicating that the calculation method derived in this paper was highly practical.
5. Conclusion
In this paper, the moment and curvature of the interface are calculated from three periods: cross section cracking, yielding of tensile longitudinal reinforcement and ultimate compressive strain state of edge concrete. Strain coordination factor is used to adjust the stress distribution state of the cross section. When $\varphi_u=1.2$, the accuracy of calculation results can be guaranteed to meet the design requirements. The theoretical curve of load-deflection is sketched and compared with the actual measurement situation. It is found that the coincidence of the two is very high, which indicates that the calculation method proposed in this paper and the corresponding calculation equation of relevant indexes can effectively present the real situation.

Reference
[1] Shan D M, Yang Q N, Mao M J, et al. 2019 Influence of steel fiber content on the punching performance of reinforced concrete bridge panel *Earthquake Disaster Prevention Technology* 14(03) 584-590.
[2] Zhang J W, Li C, Feng C J, et al. 2019 Study on seismic performance of HRB600 reinforced steel fiber reinforced high strength concrete columns *Journal of Architectural Structure* 40(10) 113-121.
[3] Yu H B and Li Y 2019 Experimental study on axial compression of reinforced concrete columns with industrial recycled steel fiber *Journal of Jilin Jian Zhu University* 36(03) 1-6.
[4] Yu H J 2018 Research on concrete damage and reinforcement technology *Green Building Materials* (06) 245-247.
[5] Zhao Y R, Wang Z H, Xie Y P, et al. 2018 Experimental study on bending Performance and fatigue life of steel fiber reinforced concrete beams *Concrete* (05) 42-45+50.
[6] Gao D Y, You P B, Tang J Y, et al. 2012 Calculation of shear capacity of steel fiber reinforced concrete shear wall with concrete filled steel tube frame Journal of Building Structures 39(06) 10-20.