Covapixels

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Abstract

We propose and discuss the summarization of superpixel-type image tiles/patches using mean and covariance information. We refer to the resulting objects as covapixels.

Keywords: Covapixels, image processing, image segmentation, neural networks, superpixels.

1 Introduction

Various approaches have been examined for reducing the input complexity of data to be processed by multilayer learning architectures. This is particularly of interest for the processing of large, high-resolution images by neural-inspired networks. Specifically, there is need to reduce the effective pixel-complexity of large high-definition and ultra-high definition images to permit practical training on large image datasets.

The simplest approach for reducing pixel complexity is to perform simple tile-based decimation, e.g., perform bicubic scaling to a lower resolution. A more sophisticated approach involves performing a non-regular decimation into non-rectangular tiles, referred to as superpixels [6], that are constrained to conform to salient structures of the image. Figures 1 and 2 provide representative examples.

In this paper we briefly introduce an alternative approach, based on the notion of covapixels for the compressed representation of images and other forms of large structured pieces of information. In the following section we define an example of a covapixel representation in which the analog of a superpixel may take the form of a pair \((\vec{\mu}, C)\), where \(\vec{\mu}\) is a \(k \times 1\) vector and \(C\) is \(k \times k\) symmetric (hermitian) nonnegative-definite matrix.

2 Covapixels

The motivation underpinning use of superpixels is the desire to obtain the data-reduction advantages of tile-based resolution reduction while minimizing the potential loss of important detail information. Unfortunately, the irregular boundaries of superpixels can have the effect of introducing spurious detail information.

More specifically, the imposition of an artificial size or area constraint on superpixels will tend to introduce relatively complex and artificial segmentation boundaries within large homogeneous areas of a given image. In other words, spurious feature entropy is introduced into the covapixel decomposition of the image.

Upon further reflection, it becomes clear that whatever generalization of a pixel is defined must somehow produce a relatively homogeneous spatial tessellation in which each generalized pixel encodes a region that is as locally uniform as possible in terms of image detail. The problem that arises with superpixels...

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1Superpixel tessellations depicted in figures were computed using the online segmentation tool of [9].

2This matrix may generally be represented in a variety of simplified or compressed forms, e.g., as the triangular Cholesky square root.
Figure 1: Example of an image of an exterior location (top) and its decomposition into superpixels (bottom). Note the arbitrary superpixel boundaries in relatively uniform regions of the sky and pavement.
Figure 2: Example of an image of an interior location (**left**) and its decomposition into superpixels (**right**). Note the arbitrary superpixel boundaries in relatively uniform regions of the shower and rug.
The representational complexity of their boundaries, which then increases the input complexity of whatever system/network is expected to manipulate and process them.

We propose to address many of the limitations of previous methods for image complexity reduction by decomposing images in a way that encodes local image information in a simpler form that admits use of standard operators used in tracking and control applications. Specifically, we represent a covapixel as a vector and matrix pair that summarizes a given tile/region of an image in which, for example, the vector defines the location (and possibly other attributes) of the region while the matrix encodes some measure of the spatial extent of the region and/or the distribution associated with some measure of the content (feature attributes) of the region.

The reason for adopting a mean and covariance representation is to permit the processing system/network to use data fusion operators such as the Kalman filter (KF) update (and its inverse/information form) [3]; Covariance Intersection (CI); Covariance Union (CU); Covariance Addition (CA); and their variants 3 ([7, 1]) for most or all internal processing of covapixel information. More intuitively, the complex boundary of a superpixel is replaced with a mean and covariance statistical representation that can be interpreted (though necessarily so) as a Gaussian probability distribution, e.g., as informally depicted in Figure 3.

The key feature of the mean-covariance covapixel representation is that its information is in a form that can be directly used by standard data fusion operators as depicted in Figure 4.

3 Discussion

We have briefly introduced and discussed a generalization of pixel information that can be expressed in the mean and covariance form widely used in filtering and control applications. This generalization is referred to as a covapixel because it can encode arbitrary state information in terms of a state vector and an associated error covariance matrix 4.

Although we have focused on applications to image processing, the replacing of scalars with mean and covariance pairs could even be applied to the elements of matrices and tensors, e.g., to maintain covariance

3 See the appendices of [7] for a unified discussion of CI, CU, and CA.

4 It should be noted that the term covariance matrix does not necessarily denote the second central moment of a probability distribution. Specifically, it may represent an upper bound on that moment [7] or could be interpreted to define an elliptical/ellipsoidal bounded region/volume [2].
estimates of numerical error that accrues from the application of linear algebra operators. This of course suggests the potential use of units of this kind in neural network-type architectures which would intrinsically process mean and covariance pairs rather than scalars.

One of the key potential benefits of covapixels is their simpler parametric representation relative to the complex boundaries of superpixels. It can be anticipated that this simpler representation will reduce the amount of spurious implicit feature detail that can result from complex boundaries. The critical question that remains to be answered is whether the loss of precise boundary detail results in a loss of salient information that undermines the practical utility the approach\(^5\).

**References**

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\(^5\)If this is the case, then an alternative in the opposite direction would be to represent superpixel information in the form an adjacency matrix representing a graph that may include non-rigid joints, which would incur a significant increase in complexity but would admit various tools from linear algebra [5] to be applied for the dynamic maintenance of tessellation structures, e.g., in video applications.