Phenomenological and theoretical developments in jet physics at the LHC *

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We review the history of jets in high energy physics, and describe in more detail the developments of the past ten years, discussing new algorithms for jet finding and their main characteristics, and summarising the status of perturbative calculations for jet cross sections in hadroproduction. We also describe the emergence of jet grooming and tagging techniques and their application to boosted jets analyses.

Keywords: QCD, jets, LHC.

1. Introduction

In the context of high energy collisions of elementary particles, ‘jets’ are bunches of particles that are detected in a contiguous region of a detector, and that stand out from the rest of the event. The reason why we are interested in this kind of object is that the simplest mechanism for producing such bunches is the production of a large momentum parton, i.e. a quark or a gluon, in the elementary collision, followed by its quasi-collinear hadronisation and decay into the observable hadrons and leptons. In this picture a jet is therefore a proxy for the original parton, and studying it allows one to probe the original elementary collision.

Experimental observation of jets is proof of the asymptotic freedom property of Quantum Chromodynamics (QCD), the theory of strong interactions: it is the fact that such interactions become weak at large energy scales that allows a quark or a gluon produced at large energy to have its own individuality for a while, and therefore seed its own jet, before the strong character of the force takes over and forces it to hadronise. Jets are therefore a ‘relic’ of the first instants after a collision, at a time \( t \ll 10^{-24} \) s, and their study allows one to image the collision’s products.

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*A precise definition of what exactly ‘bunches’ and ‘standing out’ mean is a necessary condition for performing quantitative physics studies. Formulating such a definition is a delicate and potentially complex task, and one of the reasons why a review like this one is needed to properly address this topic.
at that early time, as if taking a picture. In the following we will give a short historical introduction, and we will then review the modern approach to jets that is being used in predictions and analyses for the Large Hadron Collider (LHC), as well as the most recent advances.

2. A short history of jets

Jets were first used to study the character of the strong interaction force in the mid-seventies, when analysing events produced by the $e^+e^-$ collider PETRA at DESY. At the time QCD had just been put forward as a candidate theory for strong interactions, and experiments had the task to establish conclusively the existence of the gluon and the property of asymptotic freedom. In the context of $e^+e^-$ collisions producing, via electromagnetic interaction, quark-antiquark pairs, this meant observing events with a structure that could be assimilated to the production of three jets with large energy, and not only two back-to-back ones, the idea being that a third jet would be seeded by a gluon, emitted by either the quark or the antiquark. Various algorithms and measures were proposed, and they are reviewed for instance in Ref. 2. Suffice here to say that the ‘three-jet’ character of a (small) fraction of events could be experimentally established, providing one of the first confirmations of QCD and of asymptotic freedom.

The smallness of the strong coupling in the asymptotic, large-energy regime opens the way to performing perturbative calculations in QCD to predict the observables that are measured by experiments. Jet rates can be one such observable, and can therefore provide not only qualitative but also quantitative insight into QCD, for instance by allowing one to measure the strong coupling. However, one quickly realizes that in order to calculate jet-related quantities in perturbative QCD some care must be taken in defining the jets. Like most quantum field theories involving massless particles, QCD develops infrared and collinear singularities when either an infinitely soft particle is emitted, or when a particle is emitted collinear to another one. These singularities can be cancelled by corresponding ones in virtual contributions: this has long known to be the case for fully inclusive quantities, as shown by the Kinoshita-Lee-Nauenberg theorem. However, if an observable, e.g. a jet, is not fully inclusive and is not properly defined, the cancellation can fail to take place and the singularities will show up as divergences in the perturbative result, making it worthless: we will then say that these observables are infrared and/or collinear (IRC) unsafe. It is easy to come up with a very intuitive, and yet IRC-unsafe definition of a jet. Imagine selecting the three most energetic particles in an event, and drawing a cone around each of them. Such three jets can be measured, but their rate cannot be predicted in perturbative QCD: at some order in the perturbative series one of these three energetic particles can undergo a collinear

To the author’s best knowledge, the first person to liken the study of jets to taking a picture of a high energy physics event was Gavin Salam in his ‘Jetography’ review.
splitting into two less energetic particles, such that none of the two now qualifies as one of the three most energetic ones in the event. This particular event now displays three different jets at this perturbative order and, from the point of view of the perturbative calculation, the cancellation of the singularity against the virtual contributions will not be ensured.

One of the first attempts to define IRC-safe jet cross sections in perturbative QCD was made in 1977, when Sterman and Weinberg spelled out the necessary conditions and provided the relevant calculation for the next-to-leading order (NLO) correction to a two-jet rate in $e^+e^-$ collisions. It is worth writing out their result in detail because of its pedagogical value: using slightly modified notation, the “fraction of all events which have all but a fraction $\epsilon \ll 1$ of their energy in some pair of opposite[ly directed] cones of half-angle $\delta \ll 1$” is given by

$$f_2 = 1 - \frac{C_F \alpha_s}{\pi} \left( 4 \ln 2 \epsilon \ln \delta + 3 \ln \delta + \frac{\pi^2}{3} - \frac{7}{4} + O(\epsilon, \epsilon \ln \delta, \delta^2 \ln \epsilon) \right) \quad (1)$$

where $C_F = 4/3$ and $\alpha_s$ is the strong coupling. In this expression for the two-jet fraction one clearly recognizes the ‘no-emission’ limit, $\alpha_s \to 0$, which implies that the two-jet rate receives no higher order correction, i.e. there is never a third jet if the probability of radiating a gluon goes to zero. More generally, the two-jet rate is dominant as long as $\alpha_s$ is small. Moreover, one sees how the definition needs two resolution parameters, $\epsilon$ and $\delta$, to make it sufficiently inclusive for the real-virtual cancellation to take place: some radiation is allowed to escape, either because not sufficiently collinear or because too soft, but only a limited amount. If $\epsilon$ and $\delta$ are chosen too small one probes either very soft or very collinear radiation, re-exposing the soft and collinear singularities in the form of very large logarithms in the final perturbative result.

The Sterman-Weinberg jet definition above, while conceptually and historically important in its opening the way to the definition of IRC-safe observables in QCD, is not the most useful or practical jet observable that one can think of. For instance, it works with a predefined number of jets, while often, and even more frequently in high energy hadronic collisions, the structure of the event can vary greatly, and the task of finding out how many jets have been produced is better left to an algorithm. Since the beginning of the Eighties many different such algorithms were proposed. By ‘jet algorithm’ we mean a set of rules, possibly accompanied by parameters, for grouping the particles produced in the final state of a high energy collision into jets. Ideally, no prior assumptions on the structure of the event are made: the algorithm is simply run over all the particles in it, and it is expected to return the jets (if any). For this reason jet algorithms are also often called ‘jet finders’.

Two main classes of jet algorithms soon emerged, the cone-type and the clustering-type ones. Cone-type algorithms are an evolution of the original Sterman-Weinberg definition: jets are still expected to be cone-like agglomerations of particles, but the task of finding how many of them are present in an event and where they are is left to the algorithm. This was usually accomplished through an iter-
ative procedure, whereby the flow of energy (i.e. the sum of the four-momenta of the particles) within some cones was iterated over all placements of the cones, until a stable situation was reached – ‘stable’ usually meaning that the momentum sum of the particles inside a cone coincided with the axis of the cone itself. Cone-type algorithms are therefore a kind of top-down algorithm: they begin with pre-formed structures (albeit not necessarily a fixed number of them), and see where these can fit in the event. The other main class of jet algorithm is instead of bottom-up kind: these algorithms usually start with the elementary constituents, i.e. the list of the four-momenta of the particles in the event, and assemble them into larger structures via an agglomerative clustering procedure by iteratively combining two four-momenta, until only a certain number is left under a set of stopping rules and parameters. The clusters that have been formed in this way are then called the jets.

Algorithms from both classes have been used extensively in high energy physics in the past thirty years, cone-type ones mainly in hadronic collisions and, in the pre-LHC era, clustering-type ones mainly in $e^+e^-$ collisions. The origin of this split is to be found in the different characteristics of the two kinds of environments. In hadronic collisions many particles (from several tens up to, at the LHC, several thousands) are produced. One must therefore find an algorithm that remains reasonably fast at large multiplicities. Moreover, the noisy environment of a hadronic collision requires multiple corrections of experimental kind. These reasons pushed experimentalists to privilege, e.g. at the Fermilab Tevatron $p\bar{p}$ collider, cone algorithms, which could be implemented efficiently and returned jets with a smooth, often circular profile. In $e^+e^-$ collisions, instead, multiplicities are much lower, so that implementation speed is not a concern, and the clean environment allows for precision measurements and studies. It turned out that some clustering-type algorithms could be formulated in a way to allow for all-order resummation in perturbative QCD of some classes of terms, hence leading to more accurate predictions. Because of this, clustering-type algorithms were preferably used at the CERN LEP $e^+e^-$ collider.

It would be impossible to review here, even only superficially, all the specific algorithms that have been formulated and used in the past. I refer the interested reader e.g. to Ref. 7 for a review that mainly focuses on hadronic collisions, to Ref. 2 for one mainly concerned with $e^+e^-$ collisions, and to Ref. 1 for a more recent and modern overview.

3. Jets in the LHC era

In the run up to LHC in the early 2000’s, it became clear that none of the algorithms that had been and were still in use, at the Tevatron or in preparation studies for the LHC, was entirely satisfactory. All cone-type algorithms that were available at the time were IRC-unsafe, even if some had been patched in order to shift the unsafety

\[^c\text{Or more, see e.g. Ref. 6 for an example of an algorithm performing 3 \to 2 recombinations rather than 2 \to 1 ones.}\]
to a higher order in perturbative QCD, as was the case for the MidPoint algorithm (see Ref. 8 and references therein). The (more or less severe) IRC unsafety of these algorithms was usually the product of compromises made in order to keep their running time within usable limits. As a consequence, however, the high-accuracy NLO and next-to-next-to-leading (NNLO) calculations that the theoretical community was painstakingly producing for use at the LHC risked facing measurements for observables for which no finite prediction was possible. Clustering-type algorithms from the LEP era, on the other hand, were IRC-safe but were implemented in a way that didn’t allow for graceful scaling up to the higher multiplicities of the LHC, and most of them needed very large experimental corrections. Hence they were not seriously considered by the experimental community as viable tools for LHC physics.

This situation began to change in 2005, when a fast implementation of a clustering-type algorithm, the so-called longitudinally invariant $k_t$ algorithm was proposed in Ref. 11 by G. Salam and this author. This proposal allowed one to lower the algorithmic complexity of this algorithm from $O(N^3)$ (with $N$ being the number of particles to cluster) to $O(N^2)$ and, making further use of computational geometry techniques, even to $O(N \ln N)$. For an event with $N \simeq 1000$ particles, quite typical at the LHC, this meant reducing the clustering time on a modern O(1 GHz) processor from one second to one millisecond or less per event, opening the way to practical use of this IRC-safe algorithm at the LHC.

A few years later, in 2007, G. Salam and G. Soyez put forward the SISCon algorithm, a cone-type algorithm that is IRC-safe while still being implemented in a way that is sufficiently fast for practical use. Soon thereafter, however, the pendulum of jet algorithms at the LHC would start to swing decisively in favour of clustering-type ones.

### 3.1. Jet algorithms and their implementations

In 2008 the following generalisation of the longitudinally invariant $k_t$ algorithm was presented in Ref. 13. One defines the particle-particle and the particle-beam distances respectively as

$$d_{ij} = \min(p_{t_{ij}}^2, p_{t_{ji}}^2) \left( (y_i - y_j)^2 + (\phi_i - \phi_j)^2 \right) R^2$$

$$d_{iB} = p_{t_{B_i}}^2$$

In these equations $p_t$, $y$ and $\phi$ denote the transverse momentum with respect to the beam axis, the rapidity and the azimuthal angle respectively of the particles $i$ and $j$. The ‘jet radius’ $R$ and the exponent $p$ are parameters of the algorithm. The algorithm works by calculating the $d_{ij}$’s and the $d_{iB}$’s for all particles in the event, and finding the smallest one. If this smallest distance is an inter-particle one, the two four-momenta are recombined according to a given recombination scheme. If it is a beam-distance the four-momentum concerned is simply called a jet and excluded
from further processing. The algorithm proceeds in an iterative way, recalculating distances after each step, until no particles are left in the event. One then retains all jets above a given $p_t$ cut.

When the parameter $p$ is set to one, one recovers the longitudinally invariant $k_t$ algorithm of Refs. 9, 10. The choice $p = 0$ gives an algorithm, dubbed Cambridge/Aachen,14,15 where clustering is based exclusively on the angular distance between the particles. The distance measures of these two algorithms are modelled on the physical behaviour of the QCD emission probability. For the $k_t$ algorithm this distance, the relative transverse momentum of the two particles (hence the $k_t$ name for the algorithm), is proportional to the inverse of this emission probability. For the Cambridge/Aachen algorithm, a purely angular distance is meant to mimic the (inverse of) the QCD emission probability in the presence of angular ordering.16,17 In both these cases, an algorithm formulated in this way is expected to roughly ‘walk back’ through the parton-branching process induced by QCD emissions, and therefore reconstruct approximately the original hard quark or gluon that fragmented into the final state particles.

Setting $p = -1$ means largely abandoning pretenses of a physically meaningful connection between the clustering algorithm and the QCD branching process.4 Perhaps somewhat surprisingly, this turns out to be a jet algorithm that enjoys many favourable characteristics. It has been christened ‘anti-$k_t$’ in Ref. 13 and it has been adopted as the default jet algorithm by all the experimental collaborations at the LHC. Among the useful properties of anti-$k_t$, we mention in particular IRC-safety, and the fact that it produces jets with very regular borders (often circular, if sufficiently isolated) and that do not usually extend beyond a distance $\simeq R$ from the hard particle(s) that seeds them. These latter two properties reduce sensitivity to background noise (underlying event and pile-up) and facilitate experimental corrections for these and detector-related effects.

SISCone12 is instead an algorithm of cone type, meaning that it is based on the concept of stable cones, as defined above. Finding all stable cones can be a very onerous operation: if performed combinatorially over $N$ particles this operation has complexity $O(N^2 N)$, making it totally impractical beyond a (small) handful of particles. Old cone-type algorithms typically skirted this problem by resorting to approximate geometrical methods, based for instance on using a finite number of ‘seeds’ (e.g. some of the particles in the event) as the starting point of an iterative procedure that searched for stable cones. The problem of these approximate approaches is that they cannot guarantee that all stable cones will be found: missing some stable cones means that the algorithm becomes sensitive to the addition (or removal) of soft or collinear particles, and therefore IRC-unsafe. The breakthrough of SISCone consists in a geometrical procedure that guarantees that all stable cones are found, while remaining practical in terms of time taken: $N$ particles can be

\footnote{This is the case for momenta, but not for angles: small-angle splittings are still privileged, like in the $k_t$ and the Cambridge/Aachen algorithms.}
clumped by SISCone in $O(N^2 \ln N)$ time. While slower than the clustering-type algorithms mentioned above, this is fast enough for concrete use at the LHC.

The $k_t$, Cambridge/Aachen, anti-$k_t$ and SISCone jet algorithms are all implemented in the FastJet package\cite{11,FR}. today’s de-facto standard for jet clustering for LHC phenomenology and experimental analyses. It can be downloaded from \url{http://fastjet.fr}

### 3.2. Jet areas and background subtraction

The availability in FastJet of fast implementations of IRC-safe algorithms meant that one could explore properties of jets that were beforehand either impossible or too laborious to study. One first such property is the area of a jet. Naively, the area of a jet is the surface, in the rapidity-azimuth plane, covered by the jet itself. However, since the jet is simply a collection of particles, one cannot immediately determine unambiguously such an area. Jet areas have phenomenological significance because they can be interpreted as a jet algorithm’s susceptibility to contamination from a background of diffuse, roughly uniform soft radiation in an event. Physical realisations of such a background are pile-up noise from multiple simultaneous collisions and the underlying event.$^\text{f}$

Jet areas were first studied extensively in a modern context in Ref.\cite{19} In this paper various definitions were given, and the characteristics of the resulting jet areas were studied analytically and/or numerically. The main result is the observation that jet areas are not only a characteristic of a specific jet algorithm, but also depend on the constituents of each jet: jets composed exclusively of many soft particles will often have typical areas that will differ from those of jets anchored by a few hard particles. Moreover, jet areas will ‘evolve’ as a function of the energy scale of the jet, a result of the emission of new particles via QCD radiation. These results can be

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\text{Table 1. A summary of main active area results for the four main IRC-safe algorithms $k_t$, Cambridge/Aachen, anti-$k_t$ and SISCone, taken from Refs.\cite{19} and\cite{3}. The values for the active (A) areas for 1-particle jets (1PJ) and for the magnitude of the active area fluctuations ($\Sigma$) are shown, followed by the coefficients of the respective anomalous dimensions ($D$ and $S$) in the presence of perturbative QCD radiation. All values are normalised to $\pi R^2$, where $R$ is the jet radius parameter.}

| Algorithm   | $A(1PJ)/\pi R^2$ | $\Sigma(1PJ)/\pi R^2$ | $D/\pi R^2$ | $S/\pi R^2$ |
|-------------|-------------------|------------------------|-------------|-------------|
| $k_t$       | 0.81              | 0.28                   | 0.52        | 0.41        |
| Cam/Aachen  | 0.81              | 0.26                   | 0.08        | 0.19        |
| Anti-$k_t$  | 1                 | 0                      | 0           | 0           |
| SISCone     | 1/4               | 0                      | 0.12        | 0.07        |

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\text{\footnotemark[5]This is certainly the case for clustering-type jets. In the case of cone-type jets an a priori geometrical interpretation may already be available through the definition of the cones themselves.}

\text{\footnotemark[6]To a lesser extent, since it is usually less uniform and can retain some dependence from the hard collision.}
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summarized by the values given in Table 1 for the four main IRC-safe algorithms, \( k_t \), Cambridge/Aachen, anti-\( k_t \) and SISCone. They express the average active areas (as defined in Ref. 13) for idealised jets constituted of a single hard particle and filled by a roughly uniform distribution of much softer radiation. These values can be interpreted in a physical context as follows: \( k_t \) and Cambridge/Aachen jet areas have similar sizes, on average slightly smaller than the ‘circular cone’ area \( \pi R^2 \), but fluctuate a lot from one jet to another. Furthermore, jets from these algorithms have both areas and their fluctuations that increase at large energy scales, as shown by the positive coefficients \( D \) and \( S \) that control their evolution through the equations

\[
\langle \Delta A \rangle \simeq D C_1 \frac{\alpha_S(Q_0)}{\pi b_0} \ln \frac{\alpha_S(R_{\text{pt}})}{\alpha_S(Q_0)} \quad \text{and} \quad \langle \Delta \Sigma^2 \rangle \simeq S^2 C_1 \frac{\alpha_S(Q_0)}{\pi b_0} \ln \frac{\alpha_S(R_{\text{pt}})}{\alpha_S(Q_0)} .
\]

The fluctuating and increasing jet areas implied by these equations are the characteristics that make jets from the \( k_t \) or Cambridge/Aachen algorithms difficult to correct for at the experimental level. On the other hand, one can see from Table 1 that anti-\( k_t \) jets containing a single hard particle have instead a fixed area equal to \( \pi R^2 \), with zero fluctuations and zero evolution. This result holds well at the level of physically more realistic jets, explaining why anti-\( k_t \) is an ideal algorithm to work with at the experimental level and therefore why it has been chosen as the default algorithm by the LHC collaborations.

Jet areas can be evaluated for realistic jets using the FastJet package. The possibility to know, for each jet, its susceptibility to contamination from a roughly uniform background opens naturally the way to exploit this property to correct for this. In practice, this can be used to subtract contamination from underlying event and pile-up radiation effects by subtracting from each jet’s transverse momentum a quantity \( \rho A \), where \( A \) is a jet’s active area and \( \rho \) is a transverse momentum density of soft radiation, per unit rapidity and azimuth, as measured in each event:

\[
p_{\text{corrected}} = p_{\text{raw}} - \rho A .
\]

This procedure successfully corrects a jet’s transverse momentum for contamination, jet-to-jet area fluctuations and event-to-event background level fluctuations, and only leaves residual uncertainties related to point-to-point fluctuations of the background in a single event (as one cannot measure it properly inside a hard jet) and back-reaction effects and fluctuations.

In these equations \( \alpha_S \) denotes the strong coupling, \( b_0 \) the first coefficient of the QCD beta function, \( C_1 \) a color factor, \( Q_0 \) an initial low scale where ideally no extra radiation is present in the jet, \( p_{\text{pt}} \) the transverse momentum of the original emitting particle, and therefore of the jet.

SISCone areas also have no fluctuations, however they do increase with increasing energy scales. More importantly, this result does not hold as well as for anti-\( k_t \) at the realistic jets level, where the ‘split-merge’ step of SISCone, needed to deal with overlapping stable cones, produces jets of different areas and therefore induces fluctuations that are absent in the ideal 1-particle jet.

Back-reaction is related to the fact that the constituents of a jet can be modified by whether the clustering is performed in the presence of neighbouring particles or not. A further merit of anti-\( k_t \) jets is that they are minimally sensitive to this effect.
4. Perturbative predictions for jet cross sections

Higher order perturbative corrections to jet production are obviously important for precision phenomenology and measurements. Not only do they give more accurate predictions for the production cross sections, but they also give a first peek into the substructure of the jets, that can now be constituted of more than a single quark or gluon.\footnote{Realistic jets are of course composed of many constituents as a result of the hadronisation of quarks or gluons, but this substructure is not under the control of perturbative QCD.}

Among the main processes involving jets and of interest at the LHC we find inclusive jet production and the production of a number of jets in association with an electroweak vector boson or a Higgs boson. We list below the main higher order perturbative calculations that have been performed in QCD for these processes. For brevity, we often mention only the reference that appeared first in the literature\footnote{This is especially true for the NLO calculations, many of which are quite old and have therefore been repeated a number of times, using different techniques.}, but it is worth bearing in mind that for calculations of this level of complexity a second independent result is usually equally valuable, even if it comes much later.

For inclusive jet hadroproduction, NLO corrections to one-,\textsuperscript{21,22} two-,\textsuperscript{23} three-,\textsuperscript{24} four-\textsuperscript{25}and five-jet\textsuperscript{26} rates have been known since some time. Four-\textsuperscript{25}and five-jet\textsuperscript{26} rates have been calculated to NLO more recently. No full NNLO calculation for inclusive jet hadroproduction has yet been completed, but first partial results for the one- and two-jet rates have appeared in Ref.\textsuperscript{27}.

NLO corrections for hadroproduction of an electroweak boson $V$ plus $n$ jets have been calculated a long time ago for $n = 1$\textsuperscript{28} and $n = 2$\textsuperscript{29} NLO corrections for $V + n$ jets with $n \geq 3$ have become available only more recently, thanks to the advent of new techniques that have greatly simplified the calculation of one-loop amplitudes. They are now known for $n = 3$\textsuperscript{30,31} $n = 4$\textsuperscript{32} and $n = 5$\textsuperscript{33} NNLO corrections for $V + 1$ jet have been calculated very recently in Refs.\textsuperscript{34,35}.

Finally, Higgs plus one jet is known to NLO\textsuperscript{36} and to NNLO\textsuperscript{37,38} while Higgs plus two jets was calculated to NLO in Ref.\textsuperscript{39} for the gluon-gluon fusion process and in Ref.\textsuperscript{40} for the vector boson fusion (VBF) process. $H + 3j$ is also known to NLO for the gluon-gluon fusion process\textsuperscript{41} and for the VBF process\textsuperscript{42} NNLO corrections to $H + 2j$ production for the VBF process have been presented recently in Ref.\textsuperscript{43}.

5. Inside the jets: jet substructure

Clustering-type algorithms of agglomerative kind, like $k_t$, Cambridge/Aachen and anti-$k_t$, allow one to inspect the clustering history of each jet, i.e. the order in which its constituents were clustered with each other, and at what values of the distance measure $d_{ij}$ this happened. When this distance measure has physical meaning, as is the case for $k_t$ or Cambridge/Aachen, this information can be exploited to acquire
knowledge about the splitting processes that originated the constituents of the jets, and therefore the nature of the initiating particle.

5.1. Early attempts

The first attempt to exploit the substructure of clustering-type jets was made by M. Seymour in 1993. In Ref. [44] he studied a heavy Higgs decaying into a pair of $W$ bosons, with one $W$ decaying leptonically and the other hadronically. He then looked at the two subjets obtained by undoing the last clustering step of a $k_t$ algorithm with $R = 1$: these two subjets should correspond to the two quarks from the hadronically decaying $W$, since the $d_{ij}$ distance between them is generally the largest one (because proportional to the $W$ mass) and therefore they cluster last in the $k_t$ clustering sequence. Performing this unclustering, and looking in appropriate invariant mass and angular distance windows of the two subjets, effectively tags the $W$ boson. Ref. [44] observed that a better mass resolution could be obtained in this way rather than directly clustering with cone algorithms with much smaller radii ($R \sim 0.25$): this latter approach could still allow one to directly identify the subjets, but the large radiation loss from small jets leads to a poorer mass resolution.

Further early work that exploited a jet’s substructure was performed in Refs. [45, 46]. In these papers the observation that the distribution of the $k_t$ distance $d_{ij}$ between the two candidate $W$ subjets is close to the $W$ mass for real $W$ boson decays, but lower for generic, high-mass QCD jets, was used to significantly reduce the background. This approach is often referred to as ‘Y-splitter’.

5.2. Renaissance: the BDRS tagger for boosted Higgs

In a landmark paper [47] published in 2008, Butterworth, Davison, Rubin and Salam (BDRS) presented a jet substructure-based analysis for the search at the LHC of a boosted (i.e. with large transverse momentum $p_t \gg m_H$) Higgs boson decaying into a bottom-antibottom pair. Such a search faces a huge background from standard QCD jets, and had previously been deemed unfeasible. BDRS approached the problem by requiring that the Higgs be highly boosted: this reduces the signal, but reduces the background even more. Moreover, a boosted Higgs tends to decay in a collimated way in the laboratory frame, with all decay products to be found within a single jet with radius $\sim 2m_H/p_t \simeq 1$ for $p_t > 200$ GeV. This allows one to devise the following strategy: look for hard ‘fat’ (i.e. large radius) jets as possible candidates, and analyse their substructure in order to tag those that exhibit a two-prong structure, telltale sign of the decay of the Higgs into the $b\bar{b}$ pair.

In practice, this was achieved in BDRS the following way. An event is first clustered with a jet algorithm with a large radius $R$. This jet algorithm can be quite generic, since the constituents of the hard jet of interest can then be reclustered

\cite{1See e.g. Section 5.3.1 of Ref. [1] for a detailed analytical discussion.
with an algorithm whose clustering history is physically meaningful and can be exploited to analyze the jet substructure. The choice made in BDRS was to use the Cambridge/Aachen (C/A) algorithm\cite{14,15} for the reclustering, and therefore exploit the angular distances of the constituents. The C/A algorithm does not have the property that the decay products of a massive particle tend to cluster last, as is the case for the $k_t$ algorithm. BDRS had therefore to devise a way to identify this ‘relevant splitting’ in the whole clustering sequence. This was done by performing successive declusterings and, for each step, studying the asymmetry of the two momenta and the ‘mass drop’ of the jet: a declustering step involving not too-asymmetric momenta and producing a big mass drop is interpreted as having split the jet formed by the two decay products of the Higgs. One has therefore effectively tagged a jet containing the Higgs\cite{29} On the other hand, if the tagging conditions are never met, the jet is interpreted as QCD background. This tagger (which is also known as the ‘mass drop tagger’) was also complemented by an additional refinement, meant to increase the resolution of the Higgs mass. Dubbed filtering, this procedure breaks the Higgs-tagged jet into smaller subjets of radius $R_{filt}$, and eventually retains only the three hardest ones. This allows one to retain the decay products of the Higgs and at most a single gluon emitted by then, and discard much of the underlying event contamination. A procedure of this kind, meant to ‘clean’ a jet of radiation not pertaining to the hard process of interest, is often referred to as grooming. The BDRS procedure described above is illustrated schematically in figure 1, taken from Ref. 47.

Before closing this section it is worth adding an observation about the choice made by BDRS of using the C/A algorithm. This choice forced them to introduce a procedure to identify the ‘relevant splitting’, while the $k_t$-algorithm would naturally have given this as the last step in the clustering. However, the advantage of using C/A resides in the fact of working with angular distances: once a Higgs has been successfully tagged (step 2 in figure 1), the use of C/A means that the radius $R_{bb}$ of the two cones shown in the figure will be of the same size as the angular distance between them. In turn, this means that the angular ordering property of QCD ensures that most of the radiation emitted by the Higgs decay products will be

\cite{In the BDRS paper, $b$-tags of the two jets into which the candidate Higgs is expected to have decayed are also required.}
contained in these two cones (since it must be emitted at angles smaller than the
angle $R_{bb}$ of the emitting dipole), and one is therefore assured of collecting it all in
the end.

5.3. Taggers and groomers and their experimental validation

The BDRS paper has spawned a mini-industry of jet substructure-related tools
and analyses, with dozens of tools having been proposed since 2008, and many of
them having been validated at the experimental level. A full review of all these
developments is impossible to achieve here, but we will try to convey the main
ideas and describe the most commonly used tools. To this end, it is worth bearing
in mind the main goal of a tagging/grooming procedure: identify jets produced by
specific final states (often boosted heavy particles decaying into hadrons), while
discarding radiation that cannot be traced back to the original particle of interest,
so as to improve resolution in the reconstruction of its mass. Two main classes of
algorithms have emerged that can achieve this goal, those based on the substructure
of a jet (usually probed through successive declustering of a preexisting jet, or
through specifically crafted clustering algorithms), and those based on the pattern
of radiation within a jet (often also referred to as jet shapes).

BDRS belongs to the first class, and it achieves its goal by implementing two
separate procedures, mass drop (and asymmetry) criteria during declustering (for
tagging), and filtering (for grooming). Generalisations (and variants) of BDRS-like
procedures can also lead to taggers for three rather than two prongs, to be used for
top tagging. Early examples have been given in Refs. [48–50], as well as in Ref. [51].
Two other early algorithms that belong to the jet substructure class and that have
been widely used are pruning[52,53] and trimming[54].

Pruning works from the bottom up, modifying the way particles are clustered: at
each $1 \rightarrow p$ recombination step, protojets are clustered only if
$$\min(p_{t1},p_{t2})/p_{tp} > z_{cut} \quad \text{or} \quad \Delta R_{12} < D_{cut},$$
and typical values for the parameters are $z_{cut} \simeq 0.1$ and $D_{cut} \simeq 1$. Pruning tries therefore to veto recombinations where either one of the
particles is much softer than the other, or there is a large angular distance between
the particles, properties that can characterise either soft radiation, or emissions
from unrelated processes (e.g. underlying event or pileup). Complemented with a
procedure to tag a pruned jet that can originate from a heavy particle, e.g. if after
pruning its mass is within a given window, pruning can act at the same time as a
tagger and a groomer.

Trimming works instead from the top down, taking a jet and splitting it into
a number of smaller subjets of smaller radius. Of these subjets only those with a
transverse momentum larger than a certain fraction $f_{cut}$ of the full jet transverse
momentum are retained, and are then recombined into a ‘trimmed’ jet. One can see
that trimming works in a way which is very similar to filtering[57], differing in using
a cut on the subjets’ momentum rather than on selecting a number of the hardest
ones, and like pruning it can act at the same time as a tagger and a groomer if
supplemented with a tagging criterion.

We mentioned above that a second class of taggers is based on the study of the pattern of radiation within a jet. The most widely used member of this class is probably $N$-subjettiness. This jet shape is constructed as follows. Given a jet $J$, $N$ candidate subjets are identified and the jet shape variable $\tau_N$ is then calculated as

$$\tau_N = \frac{1}{R \sum_{k \in J} p_{tk}} \sum_{k \in J} p_{tk} \min_{i \in \text{subjets}} \{\Delta R_{ik}\},$$

(6)

where the $p_{tk}$ are the transverse momenta of all constituents of the jet $J$, and the $\Delta R_{ik}$ represent the distance in the rapidity-azimuth plane between the constituent $k$ and the subjet $i$. $R$ is the original jet radius. One can see from the definition above that the values of $\tau_N$ will be distributed between zero and one. It is moreover readily apparent that $\tau_N$ will be minimized (i.e. $\tau_N \approx 0$) when all radiation is aligned with the $N$ candidate subjets directions, whereas large values of $\tau_N$ will signal that there are at least $N + 1$ subjets in the jet. These considerations suggest that the value of the ratio $\tau_N/\tau_{N-1}$ is a good variable to discriminate jets with at least $N$ subjets from background jets with $N - 1$ or fewer subjets. In practice, $\tau_2/\tau_1$ and $\tau_3/\tau_2$ ratios are usually employed to tag boosted $W$ bosons (two-prong decays) and boosted top quarks (three-prong decays) respectively.

Grooming techniques like filtering, pruning and trimming, as well as taggers (both of the jet-substructure and of the radiation-pattern kind) for two- and three-prongs, have been extensively validated on data by some LHC experiments, notably ATLAS and CMS, and are now being used in measurements and searches.

5.4. Maturity: theoretical understanding and recent developments

The taggers and groomers described above, mass drop in BDRS, trimming and pruning, as well as many others, were largely developed with the help of Monte Carlo event generators like PYTHIA and HERWIG. Initial theoretical insight was tested using simulated but realistic events, the performance of the tool was assessed and improved through an iterative process, and the results could be compared to those of other approaches. This process has led to considerable understanding of the behaviour of the many taggers and groomers introduced in the past few years. However, it also has shortcomings. The performances that are observed in this way could depend critically on any one of the many details of the simulation of the Monte Carlo, possibly preventing a full understanding of what really matters and hindering further refinements. Perhaps more importantly, Monte Carlo simulations can be very time consuming, and often one cannot fully explore the parameters’ phase space: an unexpected behaviour of a tool for a given choice of parameters

$^a$The $N$ subjets could be determined by minimizing $\tau_N$ over all possible subjet directions or, in a computationally less intensive manner, by running an exclusive $k_t$ algorithm and forcing it to return exactly $N$ jets, as done in Ref. 55.
may therefore be missed. Because of this, some recent papers, e.g. Refs. [59][61] have started to analyse the behaviour of jet substructure tools from first principles in QCD, i.e. using analytical (resummed) perturbative calculations.

An explicit example of the importance of a proper theoretical understanding of taggers and groomers is given by the analysis of the mass drop tagger, trimming and pruning performed in Ref. [60]. This paper showed that the behaviour of the three tools as observed in Monte Carlo simulations could be reproduced to a very good extent using analytical calculation, hence ensuring an optimal understanding. Non-perturbative effects could be studied, and in some cases the analytical results even suggested adjustments to the design of the tools. Perhaps more importantly from the phenomenological point of view, the three tools could also be easily studied in a wider region of their parameters’ phase space. It then became readily apparent that, while the three tools perform quite similarly in terms of suppression of background QCD jets with large transverse momentum ($p_t \sim 3$ TeV) and with fairly large invariant masses, $m > 300$ GeV, important differences can arise for smaller masses, leading to a potential background-shaping effect for trimming and pruning. While previous Monte Carlo simulations had never been extended into this region, the analytical study was able to easily uncover this effect.

As a second example of the role of analytical calculations in the study and development of jet substructure tools, we wish to mention ‘soft drop declustering’. [61] This procedure has been designed and engineered from the very beginning with the help of resummed perturbative calculations. It is a tagger/groomer that aims to remove soft and large-angle radiation, and generalises and simplifies the mass drop tagger: given a jet of radius $R$ with two constituents, soft drop removes the softer one unless

$$\frac{\min(p_{t1}, p_{t2})}{p_{t1} + p_{t2}} > \frac{z_{\text{cut}}}{r} \left( \frac{\Delta R_{12}}{R_0} \right)^{\beta}.$$ \hspace{1cm} (7)

This procedure can be applied in a recursive way while successively declustering jets with more than two constituents. $z_{\text{cut}}$ and $\beta$ are parameters of the algorithm, and Ref. [61] has studied analytically their dependence in a number of different observables. An interesting characteristic of ‘soft drop’ is that, according to the value of $\beta$, it can behave either as a groomer (for $\beta > 0$, which only removes soft radiation) or a tagger (for $\beta \leq 0$, which also removes soft-collinear radiation, leading to the tagging of two-prong decays). For $\beta = 0$ soft drop behaves like the (modified) mass drop tagger introduced in Ref. [60].

6. Conclusions

The field of jet physics has seen much progress in the past ten years. Standardised algorithms, infrared and collinear safe, are now used by all experimental col-

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*In a perhaps somewhat counterintuitive way, the mass drop tagger is simplified in ‘soft drop’ by removing the ... mass drop condition.*
laborations, facilitating comparisons of measurements and allowing for meaningful comparisons with theoretical calculations. Beyond the jets as observables, their substructure and the pattern of radiation within them are now used to make the analyses of final states in high energy collisions ever more powerful: tagging of massive objects producing boosted jets, and subtraction of soft-radiation contamination from underlying event and pileup, are prime examples.

Among the recent developments, the shift away from ‘design by trial and error’ using Monte Carlo event generators, and towards ‘design by theoretical understanding’ that exploits first principle calculations in QCD, promises to advance the field even further in the coming years.

The exploitation of the LHC in the coming decade will certainly see no lack of new jet-based tools at the disposal of phenomenologists and experimentalists alike, in the quest for ever more precise measurements and, hopefully, discoveries.

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