Regge Phenomenology of Photoproduction of $\pi^-\Delta^{++}$ and Scaling with Saturation of Trajectory

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Abstract—In this work we investigate the reaction $\gamma p \rightarrow \pi^-\Delta^{++}$ in the Reggeized model for $\pi(138) + \rho(775) + a_2(1320)$ exchanges in the $t$-channel. For a convergence of the reaction cross section at high energies the minimal forms of proton and $\Delta^{++}$ exchanges are introduced in the direct and crossed channels for gauge invariance of $\pi$ Regge-pole exchange. The role of spin-2 tensor meson $a_2$ is found to be crucial to agree with existing data at high energies. Electromagnetic multipoles of $\Delta^{++}$ baryon are analyzed in the $\Delta$ resonance region. Based on the constituents counting rule the scaled differential cross section at $E_\gamma = 4$ GeV is reproduced with the Regge trajectory saturated at large momentum transfer $-t$.

Keywords: Regge model, convergence, minimal gauge, tensor meson $a_2$, scaling

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1. INTRODUCTION

It is known that the Regge model describes hadron reactions over the resonance region without either cutoff functions or fit-parameters [1]. Meanwhile, photoproduction of pion with $\Delta$ baryon in the final state is interesting because it provides information about the internal excitation of nucleon by electromagnetic (EM) probe. In this work we investigate the reaction $\gamma p \rightarrow \pi^-\Delta^{++}$ to understand the production mechanism of the reaction at high energies and the scaling of differential cross section at wide angles by analyzing experimental data. There are plenty of data on the reaction cross sections measured up to 16 GeV photon energy [2–4]. To date these are analyzed based on the $\pi + \rho$ exchanges. However, theoretical analysis of the reaction at such high energy has not been completed yet, because the $\pi^-\Delta^{++}$ photoproduction is known to diverge due to the divergence of proton and $\Delta^{++}$ poles at high energies. For this issue we followed a special gauge prescription in previous study [5] for the convergence of proton and $\Delta$ propagations for $\pi$ exchange. Nevertheless, the data at high energy such as $E_\gamma = 16$ GeV cannot be reproduced without further contribution of $t$-channel meson exchanges. To improve such a lack in theoretical predictions existing we consider the tensor meson $a_2$ exchange and examine its role in the reaction process. At low energy, the static properties of $\Delta$ baryon are important to study the internal structure of hadrons based on the quark dynamics. We calculate contributions of electromagnetic (EM) multipole moments of $\Delta^{++}$ baryon to the total cross section with the EM form factors of $\Delta^{++}$ fully considered. On the other hand, pQCD predicts quark evidences in the photoproduction of hadron, manifesting themselves through the scaling of differential cross section at wide angles. Within the present framework we investigate scaling of the differential cross section for $\pi^-\Delta^{++}$ process by considering the saturation of the Regge trajectory at large momentum transfer $-t$.

2. THE MODEL AND OBSERVABLES

The reaction $\gamma p \rightarrow \pi^-\Delta^{++}$ at high energies is dominated by one pion exchange through the Drell process, because the reaction cross section measured in experiments shows a steep decease as the reaction energy increases. To render the $\pi$ exchange in the process gauge invariant the charge couplings of the $s$-channel
proton and \( t \)-channel \( \Delta^{++} \) poles are introduced by following the charge conservation,

\[
i M_{\pi\Delta} = \frac{f_{\pi\Delta}}{m_\pi} \bar{u}_0(p)|q^v 2pe + \frac{k \ell}{s - M_\Delta^2} e_n + \\
+ e_\Delta \frac{q_0}{u - M_\Delta^2} \left( 2p' e g_{\mu\nu} + \sum_i G^\nu_{i\mu}(p', k, e) \right) \left( s - M_N^2 \right) e_n \\
+ e_\pi \frac{2q_0}{t - m_\pi^2} \left( q - k \right) + e_\pi e_{\gamma} \mu(p),
\]

(1)

which is then used to reggeize the \( \pi \) exchange as

\[
M_\pi = M_{\pi\Delta}(t - m_\pi^2)
\times \frac{\pi \alpha_\pi(t) \left( 1 - e^{-i \pi \alpha_\pi(t)}(t) \right)}{2 \Gamma(\alpha_\pi(t) + 1) \sin \pi \alpha_\pi(t) \left( \frac{s}{m_\pi^2} \right)},
\]

(2)

with the trajectory \( \alpha_\pi(t) = 0.7(t - m_\pi^2) \) and the collection of transverse terms, i.e., \( k_i G^\nu_{i\mu}(p', k, k) = 0 \) in the \( \Delta \) pole, which contains the divergent terms at high energy. The minimal gauge requires removal of the transverse components \( k \cdot \ell \) and \( G^\nu_{i\mu}(p', k, k) \) in Eq. (1) by redundancy for gauge invariance of \( \pi \) Regge-pole [5].

We now ask what is the next contribution to pion exchange in the \( t \)-channel. For this question we investigate roles of vector meson \( \rho \) and tensor meson \( a_2 \) Regge pole exchanges independently, although these are exchange degenerate in the \( \rho \) trajectory. For the \( \rho \) exchange we consider the well known form of the \( \rho N\Delta \) coupling here. But for the case of \( a_2 N\Delta \) there is no information about either the coupling constants or the interaction Lagrangian at present. In this work by considering spin-parity conservation of \( a_2 N\Delta \) coupling we construct a new Lagrangian of the form as

\[
\mathcal{L}_{a_2 N\Delta} = i \frac{f_{a_2 N\Delta}}{m_{a_2}} \Delta^\lambda \left( g_{\lambda\mu} \partial_\nu + g_{\lambda\nu} \partial_\mu \right) \gamma_5 N a_2^{\mu\nu} + h.c.
\]

(3)

Here we use the identity to determine the \( f_{a_2 N\Delta} \) coupling constant [6],

\[
f_{a_2 N\Delta} = -\frac{3 f_{\rho N\Delta}}{m_\rho} \tag{4}
\]

based on the fact that the coupling constant \( g_{\gamma\pi\rho} = 0.224 \) is the same order of magnitude as \( g_{\gamma\pi\rho} = -0.276 \) [5].

Figures 1 and 2 show the results in total and differential cross sections and beam polarization up to \( E_\gamma = 16 \) GeV. We use the radiative decay coupling constants from the Particle data Group and the meson-baryon coupling constants \( f_{\rho N\Delta} = 2.0 \), and \( f_{\rho N\Delta} = 8.57 \) with the trajectories and phases as given in Ref. [5]. Recall that the \( a_2 N\Delta \) coupling constant is not a free parameter, but determined once the \( \rho N\Delta \) coupling is chosen. The total cross section with the minimal gauge shows a good behavior of convergence at high energies. The role of \( a_2 \) exchange is clear rather than the \( \rho \), and even indispensable to agree with the differential cross section and beam polarization at 16 GeV in Fig. 2, as compared to the dotted curves which are the cases without \( a_2 \) exchange.

Figure 3 shows the validity of the present model at low energy with only the charge couplings as discussed above. But the model could also be applied to study electromagnetic (EM) multipole moments of \( \Delta^{++} \) as we consider the \( \gamma\Delta\Delta \) coupling vertex to have the four EM moments

\[
e_\mu \Gamma_{\gamma\Delta\Delta}^{\lambda\mu\nu}(p', k, p) = -\epsilon_\Delta(g_{\lambda\sigma} \epsilon - \epsilon_{\lambda\sigma} \gamma^\sigma - \gamma_{\lambda} \epsilon^\sigma + \gamma_{\lambda} \epsilon^\sigma) \\
+ \frac{e}{4M_\Delta} \left[ \kappa_{\lambda\sigma} \gamma_{\lambda\sigma} + \gamma_{\lambda} k_{\lambda} k_{\sigma} \right] \left[ \sigma, k \right] \\
- \frac{e\ell_{\Delta}}{4M_\Delta} \left[ k_{\lambda} k_{\sigma} \epsilon - \frac{1}{2} \ell (\epsilon_{\lambda} k_{\sigma} + \epsilon_{\sigma} k_{\lambda}) \right],
\]

(5)
with \( e^{\Delta^+} = 2 \), \( \kappa^{\Delta^+} = 4.34 \), \( \chi^{\Delta^+} = 12.34 \), and \( \lambda^{\Delta^+} = 6.8 \) taken from Ref. [7]. For a better agreement with data we use \( e \) and \( \kappa \) in this case and present the result in Fig. 4 where the propagations of proton with \( e \) and \( \kappa \) with spin-3/2 projection operator are fully considered. Of course, the cross section without the minimal gauge should diverge, as can be seen over \( E_{\gamma} = 1.6 \) GeV. Nevertheless, we notice that our model works good enough to test these EM multipoles below the region convergent.

Let me now discuss the reaction at large momentum transfer. According to the dimensional analysis based on the pQCD calculation, differential cross sections at the production angle \( \theta = 90^\circ \) become angle independent and the energy dependence is given by the powers of energy squared, \( s^{2n} \). Here \( n \) is total number of particles participating in the reaction. In photoproductions of hadrons, \( n = 9 \) and the differential cross section multiplied by the factor of \( s^7 \), therefore, leads to an energy independence, i.e., scaling.

In Regge models the energy dependence of a differential cross section for the exchange of \( \alpha(t) \) trajectory is given by

\[
\frac{d\sigma}{dt} \approx s^{2\alpha(t)-2}.
\]

At wide angles, or alternatively large \(-t\), the trajectory of Regge-pole should saturate to a limiting value and it is known that \( \alpha(t) \rightarrow -1 \) for the case of meson [8]. In such kinematical conditions the differential cross section in Eq. (6) is expected to behave as \( s^{-4} \) in energy and, as a result, it should diverge with \( s^3 \) when scaled by \( s^7 \) factor. For this reason we need to introduce a form factor to suppress it.

For the saturation of a linear trajectory \( \alpha(t) \) we use a simple parameterization of the square root function [8]

\[
\alpha^*(t) = c_1 + c_2 \sqrt{t_1 - t},
\]

where the coefficients \( c_1 \) and \( c_2 \) are determined by the boundary conditions \( \alpha^*(t_0) = \alpha(t_0) \) and \( d\alpha^*(t_0)/dt = d\alpha(t_0)/dt \) at the saturation point \( t_0 \) we choose to make the trajectory saturating to \(-1 \) with \( t_0 < t_1 \). Then \( t_1 \) is the initial point of the square root function we take for calculation.

In Fig. 5 we show the saturation of \( \pi \) trajectory by choosing \( t_0 = -0.5, t_1 = -0.49 \), of \( \rho \) by \( t_0 = -1, t_1 = -0.99 \), and \( \Delta^+ \) by \( t_0 = -0.9, t_1 = -0.89 \) in unit of GeV, which lead them definitely saturating to \(-1 \) up to \(-t = 20 \) GeV. The (red) crosses calculated by Sergeenko for the \( \rho \) trajectory are given for comparison [9]. For \( \pi^{\Delta^+} \) photoproduction the result of the scaled differential cross section is presented in Fig. 6.
to compare with data at $E_\gamma = 4$ GeV from the SLAC experiment. We used the form factor of the monopole type

$$F(t) = \left(1 - \frac{t}{\Lambda^2}\right)^{-1},$$

(8)
as an overall factor of the production amplitude in Eq. (6), and obtained a good fit to experimental data at the cutoff mass 1.6 GeV. The dashed curve is from the trajectory unsaturated and the dotted one is the case of the trajectory saturated, but without the form factor. Thus, it shows a divergence over data, as expected.

3. CONCLUSIONS

In summary, we investigated the reaction $\gamma p \rightarrow \pi^+ \Delta^+$ from threshold to the Regge realm based on the Reggeization of the $t$-channel exchanges. At high energies up to $E_\gamma = 16$ GeV a good convergence of cross section is obtained with the minimal gauge for $\pi$ exchange, and the role of tensor meson $a_2$ is found to be indispensable to reproduce the data on differential cross section and beam polarization. The present approach yields an agreement with data on total cross section in the resonance region, in which case the role of $\Delta^{++}$ EM multipole moments is further analyzed with the $\gamma \Delta \Delta$ vertex fully considered. A saturation of the trajectory is applied to describe the scaled differential cross section at large $-t$. Such numerical consequences in good agreement with existing data confirm the validity of the present framework with great potential in the study of hadron reactions up to high energy and wide angle.

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