Electromagnetically Induced Transparency in strongly interacting Rydberg Gases

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We develop an efficient Monte-Carlo approach to describe the optical response of cold three-level atoms in the presence of EIT and strong atomic interactions. In particular, we consider a "Rydberg-EIT medium" where one involved level is subject to large shifts due to strong van der Waals interactions with surrounding Rydberg atoms. We find excellent agreement with much more involved quantum calculations and demonstrate its applicability over a wide range of densities and interaction strengths. The calculations show that the nonlinear absorption due to Rydberg-Rydberg atom interactions exhibits universal behavior.

The effect of electromagnetically induced transparency (EIT) [1] in light-driven multi-level systems continues to play a pivotal role in quantum and nonlinear optics. Enabling slow light propagation and thus long photon interaction times at low loss levels [2, 3], EIT media provide a promising route to applications in optical communication and quantum information science. Optical nonlinearities, however, typically arise from higher order light-atom interactions, such that realizations of such applications at very low light intensities [4] remain challenging. In this respect, recent experimental studies of EIT in cold Rydberg gases [2, 3] are opening up new perspectives for nonlinear optics on a few photon level. Exploiting the exaggerated properties of Rydberg atoms and, in particular, the strong interactions among the atoms, nonlinear phenomena can be greatly enhanced in ultracold Rydberg gases.

Recently, several different methods have been used to study laser-driven interacting gases [10-14]. A simultaneous treatment of EIT and long-range interactions, however, poses additional challenges. Common mean-field approaches, as successfully applied to radiation trapping effects on EIT [10], are found to fail [11] due to the large strength and enormous range of the van der Waals interaction, which lead to non-negligible correlations between the atoms. On the other hand, exact descriptions, based on Hilbert space truncation by interaction-blocked many-body states [12, 13], are also inapplicable since there always remains an exponentially large number of many-body states involved in the interaction-free probe transition.

Here we present a theoretical approach that allows to obtain the fully correlated steady state populations via classical Monte Carlo sampling. This is shown to yield the nonlinear optical response to classical light fields in the presence of arbitrarily strong atomic interactions. A comparison to reduced density matrix calculations shows very good agreement for small and moderate densities. Upon proper scaling the simulation results reveal a universal behavior of the nonlinear absorption, which illustrates the role of Rydberg atom interactions in the emergence of dissipative photon-photon interactions.

The considered Rydberg-EIT level scheme is shown in Fig. 1. The signal laser couples the ground state |1⟩ to a low lying state |2⟩ with Rabi frequency Ω1. State |2⟩ is coupled by a strong control laser (Ω2 > Ω1) to a highly excited Rydberg state |3⟩. The resulting dynamics of a gas composed of N such independent atoms is, thus, governed by the Hamiltonian

\[
H_0 = \sum_{i=1}^{N} \left( \Delta_1 |2⟩⟨2| + (\Delta_1 + \Delta_2) |3⟩⟨3| - \Omega_1 |2⟩⟨1| - \Omega_2 |3⟩⟨2| + h.c. \right).
\]

In addition, the intermediate state |2⟩ radiatively decays with a rate γ, whereas spontaneous decay of the long-lived Rydberg level can safely be neglected. For resonant driving, each atom settles into a dark state, |d⟩ ≈ Ω2 |1⟩ - Ω1 |3⟩, which is immune to the laser coupling and radiative decay [3]. Consequently, the complex susceptibility χ_{12} of the lower transition vanishes and the medium becomes transparent.

![Fig. 1. Illustration of the considered three-level ladder scheme. Two laser fields successively couple the states |1⟩, |2⟩ and |3⟩. For isolated atoms, EIT is realized on two-photon resonance Δ1 = -Δ2. The van der Waals interaction shift C_6/r_6^6 between atoms in the Rydberg state |3⟩ modifies this ideal transparency and leads to a nonlinear optical response of the medium.](image-url)
In the presence of van der Waals interactions

$$U = \sum_{i<j} \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6} \langle 3,3_j|\langle 3,3_j |$$

(2)

between atoms in state $|3\rangle$ the situation becomes more complex. Note that the van der Waals coefficient $C_6 \sim n^{11}$ strongly increases with the atom’s principal quantum number $n$. For typical values $n \sim 60$ this yields an enhancement of about $10^{11}$ compared to interactions in low-lying states, implying a drastically different excitation dynamics. On the one hand, the resulting level shifts lead to a strong suppression of Rydberg excitation\cite{13}, due to an excitation blockade of close atoms\cite{10}. On the other hand, the interactions perturb the atomic dark states\cite{17,18}, and admix the dissipative intermediate state\cite{9,11}. To account for these effects, we start from the von Neumann equation

$$i\dot{\rho}^{(N)} = [H_0 + U] - i\mathcal{L}[\rho^{(N)}],$$

(3)

for the $N$-body density matrix $\dot{\rho}^{(N)}$ of the gas. The Lindblad operator $\mathcal{L}$ describes spontaneous decay of the intermediate state, but can also include finite laser band widths, denoted by $\gamma_{12}$ and $\gamma_{23}$, respectively.

$$\Delta a = \begin{pmatrix} 0 & \sigma_{21}(1-R_{21}) + \sigma_{32} & \sigma_{31}(1-R_{31}) \\ -\sigma_{21} + \sigma_{31} - \sigma_{32}R_{23} & -\sigma_{21}(1-R_{21}) - \sigma_{31}R_{31} & \sigma_{21}R_{21} + \sigma_{32}(1-R_{32}) \\ \sigma_{21} - \sigma_{31} + \sigma_{32}R_{23} & -\sigma_{21} + \sigma_{31} + \sigma_{32}R_{23} & -\sigma_{21}R_{21} - \sigma_{31}(1-R_{31}) - \sigma_{32}(1-R_{32}) \end{pmatrix},$$

(5)

to the original coefficient matrix $a$ in eq.\cite{4}, where $\sigma_{\alpha\beta} = (|a_{\alpha\beta}| - a_{\alpha\beta})/2$ and $R_{\alpha\beta}$ are free parameters. With this definition of the $\sigma_{\alpha\beta}$ any negative rate coefficient $a_{\alpha\beta}$ is set to zero, while the additional terms compensate for the according changes of the steady states. Consequently, the parameters $R_{\alpha\beta}$ are chosen such that the transformed rate equations $\dot{\rho}^{(i)} = (a + \Delta a)\rho^{(i)}$ yield steady states identical to those of the original equation $\dot{\rho}^{(i)} = a\rho^{(i)}$. Explicitly, this condition gives

$$R_{21} = a_{33}(a_{21} + a_{22}) + a_{23}(a_{31} - a_{32})$$

$$R_{31} = a_{22}(a_{31} + a_{33}) + a_{32}(a_{21} - a_{23})$$

$$R_{32} = a_{21}(a_{23} + a_{33}) + a_{23}(a_{31} + a_{32})$$

(6a)

(6b)

(6c)

for the transformation coefficients in eq.\cite{5}.

Having established a proper rate equation description for the single particle dynamics, the approach can be extended to interacting atoms. As shown in\cite{21}, this is straightforwardly accomplished by replacing the local detunings and, hence, the individual atomic transition rates become dynamical variables that depend on the entire many-body state of the system. As a result one obtains a many-body rate equation where the dynamics of the $i$th atom is governed by its individual transition rates $a_{\alpha\beta}^{(i)}$ which depend on the actual Rydberg atom configuration through the local detuning $\Delta_2^{(i)}$. Although this rate equation still covers the exponentially large number of all $3^N$ many-body states, it can be efficiently solved via classical Monte-Carlo sampling. Starting from the initial state with all atoms in their ground state $|\alpha\rangle = |1\rangle$, the steady state is obtained by performing repeated random transitions according to the probabilities $a_{\alpha\beta}^{(i)} = \delta \cdot a_{\alpha\beta}^{(i)}$ to make a transition within a time step $\delta t$. In this way, we are able to obtain the fully correlated $N$-body state-distribution of the particles, and in particular the average level populations $\rho_{\alpha\alpha} = N^{-1} \sum_i \rho_{\alpha\alpha}^{(i)}$. Since

$$\text{Im}(\rho_{12}) = \frac{\gamma}{\Omega_1} \rho_{22},$$

(7)

the Monte-Carlo approach also allows to determine the imaginary part of the complex optical susceptibility

$$\chi_{12} = \frac{2\mu^2\Omega_0}{\epsilon_0\hbar\Omega_1} \rho_{12},$$

(8)
Monte Carlo calculations and the reduced-density matrix expansion [11] give consistent results for both the real and imaginary part of $\rho_{12}$ (and hence of $\chi_{12}$). As expected from a low-density expansion the latter tends to deviate with increasing densities, with significant deviations occurring only at rather high densities $\gtrsim 10^{10} \text{cm}^{-3}$. As we show in the following, the Monte-Carlo calculations yield the expected high density limit, and should thus applicable for arbitrary densities.

As the density increases the imaginary part of $\chi_{12}$ starts to saturate while the real part develops a maximum and goes to zero at high densities. In fact, this behavior can be understood from simple arguments. At very high densities, a single Rydberg atom strongly shifts a large number of surrounding atoms out of resonance. As a consequence the Rydberg laser appears far detuned for the majority of atoms, such that they act as an effective two-level medium. Taking the limit $\Delta_2^{(i)} \rightarrow \infty$ the asymptotic high-density steady state value of $\rho_{12}$, hence, approaches

$$\text{Im}[\rho_{12}^{(\infty)}] = \frac{\Omega_1 \gamma}{g_{12} \gamma^2 + g_{23} \Omega_1}$$

where $g_{12} = (\gamma + \gamma_{12})/\gamma$, $g_{23} = (2\gamma + 3\gamma_{23})/(\gamma + \gamma_{23})$. This simple limit is indicated by the horizontal arrow in Fig. 3 and gives very good agreement with the Monte Carlo simulations.

It thus appears reasonable to employ the derived high-density limit as a natural scale for $\chi_{12}$. In addition we can use the interaction blockade of Rydberg excitation [16] to rescale the gas density, i.e. the abscissa in Fig. 3. Comparing the Rydberg atom density $\rho_{\text{Ryd}} = \rho_0 N^{-1} \sum_i \rho_{12}^{(i)}$ obtained from the Monte Carlo calculation to the corresponding value $\rho_{\text{Ryd}}^{(0)}$ for vanishing interactions one obtains

FIG. 2. Calculated probe beam absorption spectra for a Rubidium Rydberg-EIT medium at various densities. The lines show results of the Monte-Carlo simulations compared to quantum calculations based on a reduced-density matrix expansion [11] (symbols). The atoms are resonantly ($\Delta_2 = 0$) excited to 55S Rydberg states. The probe and coupling beams have Rabi frequencies of $\Omega_1 = 1 \text{MHz}$ and $\Omega_2 = 2 \text{MHz}$, respectively, with linewidths of $\gamma_{12} = \gamma_{23} = 100 \text{kHz}$.

where $\mu_{12}$ denotes the dipole moment of the probe transition and $\varepsilon_0$ is the permittivity of vacuum. In addition, the resonant real part of $\chi_{12}$ can be obtained from the nonlinear absorption spectrum using the Kramers-Kronig relations.

In the following we consider the specific case of a Rubidium Rydberg gas. The atoms are excited by the probe laser ($\Omega_1$) from the $|S_{1/2}\rangle = |1\rangle$ ground state to the $|P_{3/2}\rangle = |2\rangle$ intermediate state, while the coupling laser resonantly ($\Delta_2 = 0$) drives the transition between $|P\rangle$ and $|nS_{1/2}\rangle = |3\rangle$ Rydberg state with $\Omega_2 = 2 \text{MHz}$. We include the intermediate state decay of $\gamma = 6.1 \text{MHz}$ and further assume realistic linewidths [9] of $\gamma_{12} = \gamma_{23} = 100 \text{kHz}$ for both beams.

FIG. 3. Complex susceptibility $\chi_{12}$ as a function of the gas density $\rho_0$ under EIT conditions, $\Delta_1 = \Delta_2 = 0$. The reduced density matrix calculations (symbols) start to deviate from the Monte Carlo results (lines) for densities $\gtrsim 10^{10} \text{cm}^{-3}$. Remaining parameters are identical to those of Fig. 2.
the fraction

\[
    f_{bl} = \frac{\rho_{\text{ryd}}^{(0)}}{\rho_{\text{ryd}}} - 1
\]

(10)
of suppressed Rydberg excitations. Re-expressing the total atomic density through eq. (10) and scaling the probe beam absorption by eq. (9), yields a universal dependence of \( \tilde{\chi}_{12} = \gamma_{12}/\chi_{12}^{(\infty)} \) on the laser parameters, atomic density and interaction strength. We have verified this behavior for a wide range of parameters, some of which are exemplified in Fig. 4 for \( \gamma_{12} = \gamma_{23} = 0 \) (Fig. 4b). Indeed, all data points collapse on a single curve (Fig. 3) described by

\[
    \tilde{\chi}_{12} = \frac{f_{bl}}{1 + f_{bl}}.
\]

This simple formula nicely illustrates the effects of excitation blocking on the optical susceptibility of the Rydberg-EIT medium: For \( f_{bl} < 1 \) one finds a nonlinear absorption proportional to the blockade fraction, which, however, saturates at the two-level limit \( \chi_{12}^{(\infty)} \) for \( f_{bl} \gtrsim 1 \), i.e. at the onset of a strong excitation blockade. Since \( f_{bl} \sim \Omega_{1}^{2} \) for weak Rydberg excitation \( \ell^{2} \), this implies a finite cubic nonlinearity, as observed in \( \ell^{3} \).

In conclusion, we have presented a numerical method that permits to explore the optical response of an EIT medium in the presence of strong atomic interactions. In particular, we considered, Rydberg-Rydberg atom interactions, which yield strong photon-photon interactions, manifested in a nonlinear \( \Omega_{1} \)-dependence of the optical susceptibility. At small atomic densities our Monte Carlo results are in excellent agreement with reduced-density matrix calculations, but in addition are shown to be applicable over a much wider range of densities and interaction strengths. Beyond the present calculations, the described approach permits efficient simulations of very large atom numbers, which in future studies should enable detailed comparisons with experiments. Besides illuminating the effect of Rydberg interactions, the revealed universal behavior of the probe absorption may be of use in experiments, by permitting quick estimates of optical nonlinearities and allowing to determine the medium absorption from pure population measurements. While we have primarily focussed on nonlinear absorption phenomena the developed approach can also be used to study the optical response of far-detuned EIT media and, thus, to explore prospects for realizing strong coherent photon-photon interactions.

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