Recurrent neural network based optimal integral sliding mode tracking control for four-wheel independently driven robots

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Abstract
This paper investigates robust path tracking issue of the four-wheel independent driven robot (FWIDR) under time-varying system uncertainties and unavoidable external disturbances. A robust optimal integral sliding mode tracking control (OISMTC) scheme based on double feedback recurrent neural network (DFRNN) is proposed for the FWIDR system. Firstly, the presented OISMTC scheme modifies nominal optimal control part by exploiting an additional integral term to improve the tracking accuracy. Then, the designed DFRNN utilizes a double feedback loops structure to enhance the robustness against large system uncertainties by learning to approximate nonlinear systems. The adaptive law of the DFRNN is presented based on the Lyapunov theory to obtain favourable approximation performance in the presence of the time-varying operating conditions. Moreover, the asymptotic stability of the resultant FWIDR system is guaranteed by mathematical analysis. Finally, practical experiments are conducted to demonstrate the advantages of the proposed DFRNN-OISMTC method.

1 | INTRODUCTION

Mobile robots have been gradually utilized in a growing number of applications in recent decades thanks to the noteworthy dexterity, manoeuvrability and active security [1–3]. Among existing mobile robots, four-wheel independent driven robot (FWIDR), which is equipped with four independent steering and driving motors, is capable of obtaining more effectively flexible actuation and fast torque responses. Nevertheless, the dynamics of the FWIDR is regarded as a complex non-linear system with strong coupling and unknown model parameters [4]. Such variations in system parameters and external disturbances can significantly weaken control performance when the FWIDR follows a reference trajectory. Therefore, high-accuracy and robust path tracking control for the FWIDR in presence of unknown model parameters, dynamic complexities, and external disturbances is still a challenging research issue.

Many contributions have been made to obtain a satisfactory path tracking performance for mobile robots. In [5], an adaptive multiple low-level PID controller based double Q-learning algorithm is proposed for mobile robots. Erkan Kayacan et al. propose a non-linear model predictive control based on an estimated horizon technique for an articulated unmanned ground vehicle [6]. In [7], the authors present a robust output feedback control utilizing a mixed genetic algorithm to deal with path tracking issues for autonomous ground vehicles. As studied in [8], an adaptive hierarchical sliding mode control (SMC) based linear matrix inequality is proposed to address the lateral motion of the FWIDR. An extended state observer-based control method is proposed to ensure the tracking performance for a four mecanum wheeled mobile platform [9]. As one of the widely used control strategies, SMC has many significant advantages such as simple structure, fast responses and inherent robustness [10]. However, the control performance of SMC is susceptible to system uncertainties in the reaching phase, so more control effort may be required to drive the investigated system onto the sliding manifold, even leading to saturation of actuators [11].

To solve this issue, integral sliding mode tracking control (ISMTC) is presented to eliminate the reaching phase so that a prominent control performance is achieved [12]. In [13], by incorporating a disturbance observer, a novel ISMTC is proposed for the tracking control of the servo track writing. Furthermore, the optimal control technique is introduced to
combine with ISMTC to reduce control energy consumption in handling the system uncertainties. In [14], an optimal integral sliding mode control (OISMT) based on the pseudo-spectral method is designed to guarantee the anti-disturbance ability of the robotic manipulators. In [15], the authors propose a novel robust controller by integrating integral sliding mode control and optimal control to enhance the robustness for the non-linear uncertain systems. A robust OISMT scheme with an input delay is investigated for a two-wheeled inverted pendulum [16]. As for path tracking control of mobile robots, OISMT application is likely to lend strong support to deal with related control issues. However, the nominal control part of the OISMT would be less effective for the closed-loop system as the feedback error signal decreases during the sliding phase [17]. In addition, the unmeasured forces (including dynamic tyre forces and rolling resistance forces) and cumulative observation errors may cause system response deviation, thereby greatly reducing the tracking accuracy. Such issues form the motivation of our efforts due to their theoretical and practical significance.

Although the OISMT method is robust against external disturbances to realize high-performance path tracking, it is based on the exact system model, which is extremely difficult to achieve high-precision modelling in practice. One source of model uncertainties results from the change of robot payload. The mass and inertia of the mobile robot will change with the robot payload, which significantly affects lateral and longitudinal dynamics of the robot [18]. The mass uncertainties could be coped with by robust high-order SMC method. Nevertheless, this method is impractical to integrate into the FWIDR resulting from the complicated implementation process [19]. Another kind of model uncertainty is induced by the tire-road conditions. The yaw moment control manipulated by the tire lateral force that is susceptible to the tire cornering stiffness. However, many factors such as the vertical load and slip angle may affect the tire cornering stiffness such that it inevitably results in system uncertainties [20]. These obviously increases difficulties of the FWIDR system to achieve a high-performance robust dynamic control. Fortunately, the neural network is capable of approximating complex non-linear functions in the design of the controller, thereby improving the robustness of the investigated system [21]. In [22], an adaptive sliding mode control based on a radial basis function network (RBFNN) is proposed to perform vehicle dynamic stability control. In [23], by utilizing the RBFNN to approximate non-strict-feedback non-linear system, the authors propose an adaptive neural backstepping control scheme to deal with the tracking issue of time-delay systems. However, the RBFNN has poor dynamic performance since it is a static mapping. Without the blessing of tapped delays technique, the RBFNN can hardly represent dynamic mappings [24]. In [25], the authors propose an RNN based tracking controller by using a metaheuristic optimization algorithm for mobile manipulators to follow the time-varying reference trajectory. Yangming Li et al. present a novel recurrent neural network (RNN) control method using a short term memory model to improve control precision by learning from exploration [26]. A prescribed performance RNN control method is presented for certain non-linear systems to achieve better transient and steady-state performances [27]. RNNs have a good ability to process sequential or time-varying data due to its network structure of internal information feedback loops that is able to capture dynamic responses for later use. Hence, RNNs show superior robustness and efficiency on approximating dynamic functions. From an intuitive point of view, RNNs based control method provides a potential way to achieve robust tracking performance for the FWIDR system benefiting from its excellent approximation properties. However, to the best of the authors’ knowledge, there exist few pieces of research on extending suitable RNN topology to controller design for the FWIDR system, which motivates us to devote an effort in this paper.

The above-mentioned discussions motivates us to develop a robust double feedback recurrent neural network (DFRNN) based OISMT scheme for the constructed FWIDR system with time-varying system uncertainties and unavoidable external disturbances. First, a novel OISMT is investigated by incorporating the integral output error to suppresses external disturbances and achieve the optimal tracking performance for the FWIDR system. Then, by utilizing the power of the DFRNN in approximating unknown non-linear functions, a DFRNN-OISMT scheme is proposed to enhance the robustness of the FWIDR system in the presence of system uncertainties or even unknown model parameters. In addition, to facilitate the controller implementation, the FWIDR system is constructed considering dynamic model and path following model. Based on the Lyapunov theory, the presented DFRNN-OISMT scheme robustly stabilizes the FWIDR system. Implemented on a developed FWIDR, comprehensive experiments further demonstrate the superiority and robustness of the presented DFRNN-OISMT method. In summary, compared with the existing methods, the main novelties of this paper are highlighted as follows:

- A novel OISMT is designed for the FWIDR system by introducing an additional optimal integral term in the nominal control part to achieve low chattering and high accuracy path tracking performance.
- A novel adaptive law of the DFRNN based on Lyapunov theory is proposed to obtain favourable approximation performance in the presence of the time-varying operating conditions so that the designed DFRNN-OISMT can enhance the robustness against large system uncertainties.
- As far as we know, there are no methods in the literature to develop hybrid control method combining OISMT and DFRNN for the tracking control of mobile robots. The goal of this work is to fill in the gap in this interesting field.

The rest of this paper is organized as follows: Section 2 offers a system description of the FWIDR and formulates the control problem. In Section 3, the overall control architecture of the proposed method is given. Sections 4 and 5 provide the proposed DFRNN-OISMT scheme, including the OISMT scheme, DFRNN approach, and theoretical analysis. Section 6 gives simulations and real-life experiments. Finally, conclusions are drawn in Section 7.
2 | SYSTEM DESCRIPTION AND PROBLEM STATEMENT

2.1 | Robot dynamic model and path tracking model

Different from the structure of common mobile robots such as two-wheeled differential robots, an external yaw moment is easily generated on the FWIDR system to manipulate the robot yaw and lateral motions. Since the longitudinal dynamics of the robot can be controlled independently, we assume that the longitudinal velocity is constant. Ignoring the roll, pitch, and vertical motion, as shown in Figure 1, then the dynamic model of the FWIDR in the yaw and lateral motion is formulated as

\[
\begin{aligned}
\dot{m}v_x \dot{\beta} &= (F_{yf} + F_{yr}) - mv_x \dot{r}, \\
I_r \dot{\gamma} &= (l_f F_{yf} - l_r F_{yr}) + \Delta M_r,
\end{aligned}
\]

where \(m\) denotes the mass of the FWIDR, \(I_r\) is the yaw inertia, \(r\) represents yaw rate, \(\dot{\beta}\) is the slip angle, \(l_f\) and \(l_r\) are the distances from the front and rear axles to the centre of gravity, respectively. \(v_x\) and \(v_r\) are longitudinal velocity and lateral velocity, respectively. \(F_{yf}\) and \(F_{yr}\) are the longitudinal and lateral tire force of the \(i\)th tire with \(i = 1, 2, 3, 4 = fl, fr, rl, rr\), respectively. \(F_{yf}\) and \(F_{yr}\) are denoted as the generalized lateral tire forces of the front and rear tires, respectively, that is \(F_{yf} = F_{yf1} + F_{yf2}, F_{yr} = F_{yr3} + F_{yr4}\). \(\Delta M_r\) is the active yaw moment expressed as

\[
\Delta M_r = \sum_{i=1}^{2} F_{yi}\left(\left(-1\right)l_w \cos \delta_f + l_f \sin \delta_f\right) + \sum_{i=3}^{4} F_{yi}\left(\left(-1\right)l_w \cos \delta_r + l_r \sin \delta_r\right)
\]

where \(\delta_f\) and \(\delta_r\) denote the steering angle of the front wheel and rear wheel, respectively. \(l_w\) is half the distance between the front wheels. For simplicity, we employ \(\delta_f = -\delta_r\) for the FWIDR system.

The generalized tire lateral forces \(F_{yf}, F_{yr}\) can be described linearly with the front and rear slip angles \(\alpha_f, \alpha_r\), that is

\[
F_{yf} = z_f \alpha_f, F_{yr} = z_r \alpha_r
\]

where \(z_f, z_r\) are the cornering stiffness of the front and rear tires, respectively. \(\alpha_f, \alpha_r\) can be given as

\[
\begin{aligned}
\alpha_f &= \delta_f - \frac{l_r r}{v_x} \beta, \\
\alpha_r &= \delta_r - \beta + \frac{l_f r}{v_x}.
\end{aligned}
\]

Combining (3), (4), (5) with (1), the dynamic model of the FWIDR can be expressed as

\[
\begin{cases}
\dot{\beta} = \theta_1 \beta + \theta_2 \gamma + \theta_3 \delta_f \\
\dot{\gamma} = \theta_4 \beta + \theta_5 \gamma + \theta_6 \delta_f + \theta_7 \Delta M_r,
\end{cases}
\]

where the model parameters are defined as

\[
\begin{aligned}
\theta_1 &= -\frac{z_f + z_r}{m v_x}, \\
\theta_2 &= -\frac{z_f/l_f + z_r/l_r - m v_x^2}{m v_x^2}, \\
\theta_3 &= \frac{z_f - z_r}{m}, \\
\theta_4 &= -\frac{z_f/l_f + z_r/l_r}{l_f v_x}, \\
\theta_5 &= -\frac{z_f^2 + z_r^2}{l_r v_x}, \\
\theta_6 &= \frac{z_f/l_f + z_r/l_r}{l_f}, \\
\theta_7 &= \frac{1}{l_f}.
\end{aligned}
\]

The path tracking model for the FWIDR system is depicted in Figure 2. The lateral offset \(q_r\) represents the distance from the robot centre of gravity (CG) to the reference path. The heading error \(\phi_r\) means the error between the heading of the robot and the tangential direction of the reference path at a previewed distance \(D_p\). Then the path tracking error model of the FWIDR can be given by [28]

\[
\begin{cases}
\dot{q}_r = v_x \phi_r - v_r \beta - D_p r, \\
\dot{\phi}_r = v_x \rho_r(t) - r,
\end{cases}
\]
where \( \rho_r(t) \) is the curvature of the reference path.

Taking the system uncertainties and external disturbances into consideration, the FWIDR dynamic model incorporating the path tracking error model (7) can be formulated in a state-space form as follows:

\[
\begin{align*}
\dot{x} &= (A + \Delta A)x + Bu + d_l \\
y &= Cx,
\end{align*}
\]  

(8)

where \( x = [q, \dot{q}, \beta, r]^T \) is the state vector, \( u = [\delta_f, \Delta M_r]^T \) is the control vector. The lumped disturbance \( d_l \) defined as \( d_l = \Delta Bu + d_i \), in which \( \Delta A \) and \( \Delta B \) are the system uncertainties, \( d_i \) represents the external disturbance, \( y = [q, \dot{q}, \beta, r]^T \) is the measurement output. The system matrices can be expressed as

\[
A = \begin{bmatrix}
0 & v_x & -v_y & -D_p \\
0 & 0 & 0 & -1 \\
0 & 0 & \theta_1 & \theta_2 \\
0 & 0 & \theta_4 & \theta_5 \\
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\theta_3 & 0 \\
\theta_6 & \theta_7 \\
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}^T.
\]

**Remark 1.** The tyre of the mobile robot is viewed as an important component in the control system, the dynamic nature of the tyre exhibits inherent uncertainties when the robot accelerates and decelerates frequently on the various ground surfaces [18]. Furthermore, the model parameters of the FWIDR usually keep jumping among two or several values because of the variation of the inertia and mass along with the shifting load. These inevitably obtain system uncertainties that need to be dealt with.

**Remark 2.** The system model parameters of the FWIDR system are difficult to measure accurately and easily affected by the external environments. By performing a series of simulations and experiments, we found that \( \Delta A \) has a much greater influence on the trajectory tracking of the FWIDR compared to \( \Delta B \). Thus, \( \Delta A \) is served as the major system uncertainty during the design of controller. In subsequent sections of this paper, we employ the DFRNN to approximate the unknown part \((A + \Delta A)x\) in control design to improve the system accuracy and robustness.

2.2 Problem formulation

As demonstrated in (8), we construct the FWIDR dynamic model by incorporating the path tracking error model in the presence of the system uncertainties and external disturbances. Consequently, the control objective here is to design an OISMTC scheme based on DFRNN that guarantees the tracking errors asymptotically converge to the origin.

Before ending this section, to facilitate our controller design for the FWIDR system, we introduce the following assumption and lemma for later development.

**Assumption 1.** The lumped disturbance \( d_i \) is bounded such that \( \|d_i\| \leq d_0 \) with the constant \( d_0 > 0 \), where \( \| \cdot \| \) stands for the Euclidean norm.

**Remark 3.** For the FWIDR system in practical situation, this paper assumes 35% uncertainties in terms of the cornering stiffness. According to experiments, external disturbance \( d \) is generally less than 10. The steering angle \( \delta_f \) and yaw moment \( \Delta M_r \) have actuator saturation limits, and the yaw rate is bounded \( \mu g / r_s \), where \( \mu \) is the friction coefficient of the tire-road, \( g \) is the acceleration of gravity. Thus, referring to [29], the upper bound of the lumped disturbance \( d_i \) can be obtained.

**Lemma 1.** (Barbalat Lemma [30]) Given a uniformly continuous function \( f(t) : \mathbb{R} \to \mathbb{R} \), \( t \geq 0 \), if the limit \( \lim_{t \to \infty} \int_0^t f(t) \, dt \) exists and is finite, then it is derived that \( \lim_{t \to \infty} f(t) = 0 \).

3 Overall control architecture

Based on the constructed FWIDR system, our control scheme is proposed to achieve robust path tracking. It can be found that the primary control challenges of the FWIDR system are: (1) the unmeasured induced forces and cumulative observation errors; (2) system uncertainties or unknown system parameters. To solve the above-mentioned challenges, a DFRNN-OISMTC scheme is presented, and the corresponding control framework is shown in Figure 3. It consists of (1) an OISMTC scheme, incorporating the integral output error to improve the tracking accuracy of the FWIDR system; (2) a DFRNN-OISMTC scheme, utilizing the advantages of the DFRNN in approximating unknown non-linear functions to enhance the robustness of the resultant system in the presence of unknown model parameters.

4 Optimal integral sliding mode control

OISMT is combining the optimal controller with the integral sliding mode tracking controller, which ensures tracking accuracy and robustness with less control effort. As for tracking control of mobile robots, OISMT has a relatively good control performance against external disturbances. In this section, a novel OISMT method will be designed to achieve optimal tracking performance. The nominal optimal control law using additional error integral feedback is first designed and the integral sliding mode control law is then performed. The control
input \( u(t) \) for the FWIDR system (8) will be designed as the combination of an optimal nominal control law \( u_0(t) \) and a switching control law \( u_s(t) \). Hence, the overall control law \( u(t) \) can be expressed as

\[
u(t) = u_0(t) + u_s(t).
\]  

(9)

### 4.1 Optimal control law design using additional error integral feedback

To design the nominal controller, we consider the FWIDR system by neglecting the uncertain part; that is, the nominal system can be expressed as follows:

\[
\dot{x}(t) = Ax + B u_0(t).
\]  

(10)

The nominal control \( u_0(t) \) is regarded as an admissible control law if the equilibrium point of the closed-loop system (10) is asymptotic stability [31]. According to [32], proportional-like nominal control law, which is directly proportional to the error signal, is the most commonly used method for the OISMTC design. Nevertheless, it suffers from poor ability to track the desired trajectory due to the decrement of the error signal, especially in mobile robots applications. To address this problem, a novel optimal nominal control law using additional error integral feedback is presented as follows:

\[
u_0(t) = G_p x(t) + G_i \int_0^t y(t) dt,
\]  

(11)

where \( G_p \in \mathbb{R}^{2 \times 4} \) and \( G_i \in \mathbb{R}^{2 \times 2} \) are the feedback gain matrix. The integral term in the controller (11) is based on the measurement output since the slip angle \( \beta \) and the yaw rate \( r \) are difficult to measure accurately, and measurement data are difficult to obtain in real time. By adding the measurement output integral action, the designed nominal control law \( u_0(t) \) has the capacity of improving the tracking accuracy for the investigated system.

To obtain the optimal gain \( G_p \) and \( G_i \) in (11), we employ the augmented system method proposed in [33]. Thus, the nominal FWIDR dynamics (10) is combined with the integral output error can be expressed as

\[
\begin{align*}
\dot{\hat{x}}(t) &= \hat{A} \hat{x}(t) + \hat{B} u_0(t) \\
\hat{y}(t) &= \hat{C} \hat{x}(t),
\end{align*}
\]  

(12)

where the augmented term \( \hat{x}(t) = [x^T \ b^T]^T \), \( \hat{y}(t) = [y^T \ b^T]^T \), in which \( b(t) = \int_0^t y(s) ds \). The system matrix \( \hat{A} = [A \ 0] \), \( \hat{B} = [B \ 0] \), \( \hat{C} = [0 \ 1] \) with the identity matrix \( I \).

The integrated quadratic performance index of the OISMTC is defined as

\[
J = \frac{1}{2} \int_0^\infty \left( \hat{y}(t)^T Q \hat{y}(t) + u_0(t)^T R u_0(t) \right) dt,
\]  

(13)

where \( Q \in \mathbb{R}^{4 \times 4} \) and \( R \in \mathbb{R}^{2 \times 2} \) are the positive semi-definite matrix and symmetric matrix.

The nominal control law \( u_0(t) \) which minimizes (13) is given as

\[
u_0(t) = -R^{-1}B^T P \hat{x} = -G \hat{x} = -[G_p \ G_i] \hat{x},
\]  

(14)
where the composite matrix $G = [G_p G_v] \in \mathbb{R}^{2\times6}$. $P$ is unique, symmetric and positive definite, which fulfills the following algebraic Riccati align
\[
-PA - A^T P - PBR^{-1}B^T P - Q = 0. \tag{15}
\]

Thus, the closed-loop system dynamics is obtained as
\[
\dot{x}(t) = (A - BG)x(t). \tag{16}
\]

Remark 4. The traditional OISMTC in [32] is improved where an additional error integral feedback action is integrated into the nominal control part. The use of a quadratic performance index in predicting the optimal controller gain to enhance the transient state and control performance without increasing the computational complexity.

The nominal control law $u_n$ in (14) asymptotically tracks the reference trajectory for the nominal FWIDR dynamics (12). However, the above nominal control law $u_n$ cannot guarantee the excellent tracking performance and robustness for the closed-loop system in the presence of the system uncertainties, the external disturbances, and even the unknown system matrix. How to tackle this problem will be discussed in the subsequent sections.

### 4.2 Integral sliding mode control

Note that the designed nominal control law $u_n$ does not take into consideration of the unknown system disturbances and external disturbances, which cannot guarantee the robustness for the FWIDR system response performance. For this purpose, we propose a switching control law $u_i$ to synthesize with the nominal control law $u_n$ such that the reachability of the designed integral sliding manifold is ensured and the overall control system is robust.

Define $A' = A + \Delta A$, then the system model (8) can be rewritten as follows:
\[
\dot{x} = A'x + Bu + d_i. \tag{17}
\]

To obtain a robust control scheme for the FWIDR system in (17), an integral sliding surface $\sigma$ is designed as follows:
\[
\sigma(x, t) = \Pi \left\{ x(t) - x(t_0) - \int_{t_0}^{t} (Ax + Bu_i)dt \right\}, \tag{18}
\]

where $\Pi \in \mathbb{R}^{2\times4}$ denotes a projection matrix that must fulfill the $\Pi B$ is uniformly invertible. $x(t_0)$ stands for the initial error states. Note that the integral sliding is able to eliminate the reaching phase such that the invariance is achieved in the entire system response, the term $\Pi x(t_0)$ guarantees that $\sigma = 0$ when $t = t_0$, $u_n$ is the nominal control law, which is used to achieve the control performance for the nominal system in the absence of lumped disturbance.

Here, the switching control law $u_i$ is designed as
\[
u_i = -\varepsilon \Pi B^{-1}\varepsilon \text{sign}(\sigma(t)) - M(A' - A)x, \tag{19}
\]

where $\varepsilon > 0$, sign$(\cdot)$ is signum function, and $M \in \mathbb{R}^{2\times4}$ satisfies $BM = I$. From (9), (14) and (19), one can obtain the control law $u(t)$ as follows:
\[
u(t) = -G\dot{x} - \Pi B^{-1}\varepsilon \text{sign}(\sigma(t)) - M(A' - A)x, \tag{20}
\]

where $G$ is optimal control gain to stabilize and optimize the nominal system, $\varepsilon$ and $k$ are the switching control gain to eliminate the matched uncertainties for the resultant system.

**Theorem 1.** Consider the FWIDR system (8) under the requirement that the pair $(A, B)$ is controllable. If the control law (20) is adopted and $\varepsilon > \Pi \|d_i\|$, then the states of the FWIDR system asymptotically track the reference trajectory.

**Proof 1.** We select the Lyapunov function candidate as follows:
\[
V_1 = \frac{1}{2} \sigma^T \sigma. \tag{21}
\]

Taking the derivation of (21) with respect to time yields
\[
\dot{V}_1 = \sigma^T \dot{\sigma} = \sigma^T \Pi (A'x + Bu + d_i - (Ax + Bu_i)). \tag{22}
\]

Substituting (20) into (22), one can obtain
\[
\dot{V}_1 = \sigma^T \Pi \left\{ (A' - A)x + Bu_i + d_i \right\} = \sigma^T \left\{ -\varepsilon \text{sign}(\sigma(t)) + \Pi d_i \right\} = -\varepsilon \|\sigma(t)\| + \|\sigma(t)\| \Pi d_i \leq \|\sigma(t)\|(-\varepsilon + \Pi \|d_i\|) < 0.
\]

Then, it can be concluded that the Lyapunov function $V_1$ will decrease gradually and the sliding surface $\sigma$ will asymptotically converge to zero, which means that the tracking error will tend to zero. This completes the proof.

Remark 5. The discontinuous term sign$(\cdot)$ is applied to the sliding mode implementations due to the imprecision of system modelling and disturbances. However, it leads to an undesired chattering phenomenon (high-frequency switching between two control inputs), which may impose a vibration or perturbation on the system responses [34]. An alternative approach for chattering reduction is to use a boundary layer technique via replacing the sign$(\cdot)$ by its continuous approximation. Then, a practical control law is can be described as follows:
\[
u(t) = -G\dot{x} - \Pi B^{-1}\varepsilon \|\sigma(t)\| + \eta - M(A' - A)x, \tag{24}
\]

where $\eta$ denotes a small positive scalar.
Remark 6. In conventional OISMT works, the nominal control part is directly proportional to the state error signals such that it is easy to expand to various applications. However, for the FWIDR system, it leads to poor tracking property with a decrement of the error signal due to the unmeasured induced forces and cumulative observation errors. This paper introduces an additional optimal integral term in the nominal control part, so the novel designed OISMTC is capable of maintaining low chattering and high trajectory tracking performance for the FWIDR system.

The designed control law $u(t)$ asymptotically tracks the reference trajectory for the FWIDR. However, if the dynamic system matrix $A'$ in (20) has some uncertainties or might be unknown, it is extremely difficult to obtain the proposed ideal control law (20). To cope with this problem, one can use DFRNN to estimate the unknown part of the FWIDR system model.

5 | DFRNN BASED OPTIMAL INTEGRAL SLIDING MODE CONTROL

As the complex FWIDR system runs in a time-varying external environment, the system parameters of the dynamics model are difficult to measure accurately. Benefiting from the power of neural networks in approximating unknown non-linear functions, in this section, a DFRNN is introduced to approximate the unknown system part $A'x$. Then, a DFRNN-OISMTC scheme is designed to make the path tracking of the FWIDR more robust.

5.1 | Structure of DFRNN

The RNN has been widely applied in controller design due to its capacity to approximate dynamic mappings with good accuracy and remarkable learning speed for its structure of internal feedback. DFRNN is an enhancement of RNN, which consists of internal feedback and external feedback to capture both the internal state information and the output signal through time delays to achieve better approximation property. The structure of the DFRNN is shown in Figure 4, which comprises an input layer, a hidden layer, and an output layer. As shown in Figure 4, one recurrent feedback (purple lines) is embedded in the hidden layer, and other feedback (olive lines) is connected the neuron between input layer and output layer. Thus, the DFRNN is capable of capturing the dynamic response with two feedback loops to achieve better approximation performance. The symbol $Z^{-1}$ in Figure 4 denotes a backward shift operator, which means the output signal is sent to the input layer after one cycle. The detailed description of each layer is explained as follows:

Layer 1 (Input Layer): The input layer of the DFRNN is composed of artificial input neurons, which are related to the input signals $X = [x_1, x_2, ..., x_m]^T$ and the output of the last cycle $Y_l$. It brings the initial data into the system for further processing by subsequent layers of neurons. The output $\xi = [\xi_1, \xi_2, ..., \xi_m]^T$ of this layer is expressed as

$$\xi = XW_f Y_l,$$

(25)

where $W_f = [w_{f1}, w_{f2}, ..., w_{fm}]$ denotes the feedback weight vector connecting the output and input layers.
Layer 2 (Hidden Layer): This layer has a looping capability such that its input consists of the output of the input layer as well as the output from a previous calculation performed by this layer. A Gaussian function is selected as the activation function for each neuron in this layer. Then, the output \( Z = [z_1, z_2, \ldots, z_n] \) of this layer is calculated as

\[
Z = e^{\gamma}, g = [g_1, g_2, \ldots, g_n]^T, g_j = \sum_{i=1}^{m} \frac{(z_i + w_{yj} - y_{ij})^2}{\lambda^2},
\]

where \( W_o = [w_{y1}, w_{y2}, \ldots, w_{yn}] \) is the internal feedback weight vector. \( Z = [z_1, z_2, \ldots, z_n]^T \) denotes the state vector. \( z_i \) denotes the state of the \( i \)th neuron in the last cycle. \( y_{ij} \) and \( \lambda_{ij} \) represent the centre and width of the Gaussian function, separately.

Layer 3 (Output Layer): The output layer is the last layer of the neurons that produces the results for given inputs. This layer neurons usually do not have an activation function. The output \( Y = [y_1, y_2, \ldots, y_n]^T \) of this layer is given as

\[
Y = W_oZ,
\]

where \( W_o \in \mathbb{R}^{n \times n} \) is the matrix form of output layer weights.

To derive the DFRNN, we can rewrite the output of the neural network as follows:

\[
Y(X, W_f, W_b, W_o, \Gamma, \Lambda) = W_o \cdot Z(X, W_f, W_b, \Gamma, \Lambda),
\]

where

\[
\begin{bmatrix}
\gamma_{11} & \ldots & \gamma_{1n} \\
\vdots & \ddots & \vdots \\
\gamma_{n1} & \ldots & \gamma_{nn}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\lambda_{11} & \ldots & \lambda_{1n} \\
\vdots & \ddots & \vdots \\
\lambda_{n1} & \ldots & \lambda_{nn}
\end{bmatrix}
\]

are Gaussian function parameter matrices.

Remark 7. Compared with the regular RBFNNs, which have poor dynamic performance without the blessing of tapped delays technique, the DFRNN has a good capacity to process sequential or time-varying data due to its network structure of internal information feedback loops that is able to capture dynamic responses for later use. In addition, the neurons in the hidden layers of the DFRNN provide a set of centre and width with arbitrary initial values for each activation function to make the network have more strong fitting ability. Moreover, the DFRNN can capture the dynamic response with two feedback loops to achieve better approximation performance. Therefore, DFRNN shows superior robustness and efficiency on approximating dynamic functions.

5.2 | Design of DFRNN-OISMTC

Considering the excellent approximate performance of DFRNN, the unknown part \( A'x = \varsigma \) of the FWIDR dynamic model can be approximated by the DFRNN, that is

\[
\dot{x} = \varsigma + Bu + d_i.
\]

Based on the universal approximation theory [35], any continuous function is able to be approximated by a single hidden layer forward neural network with an arbitrary precision. We assume that there exist optimal weights \( W^*_f, W^*_b, W^*_o \) and optimal parameters \( \Gamma^*, \Lambda^* \) in order to facilitate the mathematical expression and derivation of the subsequent content. The unknown part is approximated as

\[
\varsigma = W^*_oZ^* + \epsilon,
\]

where \( \epsilon \) denotes the mapping error.

In practice, the output of the DFRNN, which is regarded as the estimated value of \( \varsigma \), can be expressed as

\[
\hat{\varsigma} = \tilde{W}_o \cdot \tilde{Z}(X, \tilde{W}_f, \tilde{W}_b, \tilde{\Gamma}, \tilde{\Lambda}).
\]

Define \( Z^* = \tilde{Z} + \bar{Z} \), \( W^*_o = \tilde{W}_o + \bar{W}_o \). Then, the approximation error between the optimal and the estimated value is obtained as follows:

\[
\varsigma - \hat{\varsigma} = W^*_oZ^* + \epsilon - \tilde{W}_o \tilde{Z} = W^*_o(\tilde{Z} + \bar{Z}) - \tilde{W}_o \tilde{Z} + \epsilon = \tilde{W}_o \tilde{Z} + \bar{W}_o \bar{Z} + \epsilon = \bar{W}_o \bar{Z} + \epsilon' + \bar{Z},
\]

where \( \epsilon' = \bar{W}_o \bar{Z} + \epsilon \) is the lumped approximation error.

To derive the parameters \( \tilde{W}_f, \tilde{W}_b, \tilde{W}_o, \Gamma, \Lambda \) of the DFRNN online by Lyapunov-based adaptive laws, we use the Taylor expansion technique to transform the non-linear function \( \bar{Z} \) into a partially linear form, that is

\[
\bar{Z} = \frac{\partial Z}{\partial W_f} |_{\tilde{W}_f = \bar{W}_f}(W^*_f - \bar{W}_f)
\]

and

\[
\frac{\partial Z}{\partial W_b} |_{\tilde{W}_b = \bar{W}_b}(W^*_b - \bar{W}_b)
\]

\[
\begin{bmatrix}
\frac{\partial Z}{\partial \Gamma} |_{\Gamma = \tilde{\Gamma} - \hat{\Gamma}} + \frac{\partial Z}{\partial \Lambda} |_{\Lambda = \Lambda - \hat{\Lambda}} \\
\end{bmatrix}
\]

(32)

\[
= \Delta Z_{zf} + \Delta Z_{zb} + \tilde{Z}f + \tilde{Z}b + \tilde{Z}t \cdot \tilde{\Gamma} + \tilde{Z} \cdot \tilde{\Lambda} + O,
\]

where \( O \) denotes the high-order term. \( \Delta Z_{zf}, \Delta Z_{zb}, \Delta Z_{t} \) and \( \Delta Z_{\Lambda} \) are given as follows:

\[
\begin{bmatrix}
\Delta Z_{zf} = \begin{bmatrix}
\frac{\partial Z_1}{\partial W_f} & \frac{\partial Z_1}{\partial W_f} & \cdots & \frac{\partial Z_n}{\partial W_f}
\end{bmatrix} \mid_{\tilde{W}_f = \bar{W}_f}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta Z_{zb} = \begin{bmatrix}
\frac{\partial Z_1}{\partial W_b} & \frac{\partial Z_1}{\partial W_b} & \cdots & \frac{\partial Z_n}{\partial W_b}
\end{bmatrix} \mid_{\tilde{W}_b = \bar{W}_b}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta Z_{t} = \begin{bmatrix}
\frac{\partial Z_1}{\partial \Gamma} & \frac{\partial Z_1}{\partial \Gamma} & \cdots & \frac{\partial Z_n}{\partial \Gamma}
\end{bmatrix} \mid_{\Gamma = \tilde{\Gamma}}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\Delta Z_{\Lambda} = \begin{bmatrix}
\frac{\partial Z_1}{\partial \Lambda} & \frac{\partial Z_1}{\partial \Lambda} & \cdots & \frac{\partial Z_n}{\partial \Lambda}
\end{bmatrix} \mid_{\Lambda = \tilde{\Lambda}}
\end{bmatrix}
\]
In a combination of (31) and (32), we have
\[
\begin{align*}
\zeta - \xi &= \hat{V}_{\partial} \dot{Z} + \hat{V}_{\partial} Z + \ell' \\
&= \hat{V}_{\partial} \dot{Z} + \hat{V}_{\partial} (\Delta Z_{af}) \cdot \hat{V}_{f} + \Delta Z_{ab} \cdot \hat{V}_{b} \\
&\quad + \Delta Z_{\hat{\Gamma}} + \hat{\Gamma} + \Delta Z_{\hat{\Lambda}} + \hat{\Lambda} + \ell'',
\end{align*}
\]
where \( \ell' = \ell + \hat{V}_{\partial} \Delta \) is the lumped approximation error bounded as \( |\ell'| \leq \ell'' \), where \( \ell'' \) is a positive constant.

As the system parameters of the FWIDR are difficult to measure accurately and easily affected by the external environment, the proposed OISMC method (20) cannot obtain ideal tracking performance. Here, with the DFRNN approximator, a novel DFRNN-OISMTC scheme is designed as below
\[
u(t) = -G \hat{x} - (PB)^{-1} \varepsilon \hat{\sigma}(\hat{\sigma}(t)) - M\zeta - \hat{\Lambda}x,
\]
where the control parameters \( \hat{V}_{\partial}, \hat{V}_{f}, \hat{V}_{b}, \hat{\Gamma}, \hat{\Lambda} \) are tuned online as follows:
\[
\begin{cases}
\hat{V}_{\partial} = -\alpha_1 \hat{\sigma}^T \Pi Z \\
\hat{\Gamma} = -\alpha_2 \hat{\sigma}^T \Pi \hat{V}_{\partial} \Delta \hat{Z}_{\hat{\Gamma}} \\
\hat{\Lambda} = -\alpha_3 \hat{\sigma}^T \Pi \hat{V}_{\partial} \Delta \hat{Z}_{\hat{\Lambda}} \\
\hat{V}_{f} = -\alpha_4 \hat{\sigma}^T \Pi \hat{V}_{\partial} \Delta \hat{Z}_{af} \\
\hat{V}_{b} = -\alpha_5 \hat{\sigma}^T \Pi \hat{V}_{\partial} \Delta \hat{Z}_{ab},
\end{cases}
\]
where \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \) and \( \alpha_5 \) are constant positive parameters.

Remark 8. The designed DFRNN structure is employed in an optimal integral sliding mode controller to approximate the unknown system dynamics, and the weights with arbitrary initial values are updated online by Lyapunov-based adaptive laws (36) to get favourable approximation performance. For instance, the output layer weight \( \hat{V}_{\partial} \) is updated as \( \hat{V}_{\partial}(t) = \hat{V}_{\partial}(t-1) + \hat{V}_{\partial}(t) = \hat{V}_{\partial}(t-1) + \hat{V}_{\partial}(t) \), and the adaptive law \( \hat{V}_{\partial}(t) = -\alpha_1 \hat{\sigma}^T \Pi Z \), where \( t \) is the current time.

Remark 9. In the existing studies, the gradient descent algorithm is employed during backpropagation to update the weight parameters for the RNN. However, it suffers from a vanishing gradient or exploding gradient due to all of the neurons far back in time [306]. Differing from the above method, this paper has the parameters of DLRNN updated online based on the Lyapunov stability theorem. This not only avoids the vanishing gradient or exploding gradient problem but also ensures system stability.

Remark 10. It is noted that time-delays as in [377] may destabilize the stability of RNNs when the maximum of time-delays is exceed. The instability of the network depends on the degree of time-delays [386]. By solving transcendental equations, the upper bounds of time-delays can be estimated. Thus, if the time-delays is smaller than the upper bounds, then the global stability of the investigated RNNs can be guaranteed [39]. This method is likely to lend strong support to solve the time-delay issues.

Theorem 2. Consider the FWIDR system (8) with the designed sliding surface (18) under the assumption 1-3. The control parameters \( \hat{V}_{\partial}, \hat{V}_{f}, \hat{V}_{b}, \hat{\Gamma}, \hat{\Lambda} \) are updated by (36). When the presented control law in (35) is employed, the stability of the FWIDR can be guaranteed.

Proof 2. To show the stability of the FWIDR system, the following Lyapunov function candidate is constructed
\[
V_2 = \frac{1}{2} \sigma^T \sigma + \frac{1}{2 \alpha_1} \hat{V}_{\partial}^T \hat{V}_{\partial} + \frac{1}{2 \alpha_2} \hat{\Gamma}^T \hat{\Gamma} + \frac{1}{2 \alpha_3} \hat{\Lambda}^T \hat{\Lambda} + \frac{1}{2 \alpha_4} \hat{V}_{f}^T \hat{V}_{f} + \frac{1}{2 \alpha_5} \hat{V}_{b}^T \hat{V}_{b}.
\]

The time derivative of \( V_2 \) yields
\[
\dot{V}_2 = \sigma^T \dot{\sigma} + \frac{1}{\alpha_1} \hat{V}_{\partial}^T \hat{V}_{\partial} + \frac{1}{\alpha_2} \dot{\Gamma}^T \hat{\Gamma} + \frac{1}{\alpha_3} \dot{\Lambda}^T \hat{\Lambda} + \frac{1}{\alpha_4} \dot{V}_{f}^T \hat{V}_{f} + \frac{1}{\alpha_5} \dot{V}_{b}^T \hat{V}_{b}.
\]

The combination of (9), (35) and (36) results in
\[
\dot{V}_2 = \sigma^T \dot{\sigma} + \frac{1}{\alpha_1} \hat{V}_{\partial}^T \hat{V}_{\partial} + \frac{1}{\alpha_2} \dot{\Gamma}^T \hat{\Gamma} + \frac{1}{\alpha_3} \dot{\Lambda}^T \hat{\Lambda} + \frac{1}{\alpha_4} \dot{V}_{f}^T \hat{V}_{f} + \frac{1}{\alpha_5} \dot{V}_{b}^T \hat{V}_{b}.
\]
6 | EXPERIMENT VERIFICATION

6.1 | Experimental setup

As illustrated in Figure 5, the developed FWIDR is equipped with a rigid chassis and four active steer-drive wheels. Each wheel has two degrees of freedom to roll and turn actively so that it exhibits superior control flexibility. The specifications of the FWIDR are shown in Table 1. In the experiments, the saturation limit for the steering angle and yaw moment are specified as 40 Nm and 20°, respectively. The hardware architecture of the FWIDR is shown in Figure 6, which consists of the following three main modules: 1) Perception module, by using various sensors such as laser, inertial measurement unit, ultrasonic, it endows the robot with the ability to perceive, comprehend, and reason about the surrounding environment; 2) Decision module, it has capacity of generating a series of desired motions which satisfy dynamic constraints and possibly optimize the robotic movement; 3) Movement control module, it is aimed at tracking a time-varying reference trajectory within the robot configuration space, the key components of which include motion controller, sever drives for actuating a robot. The perception-decision-control cycle is a feedback loop manipulating the mobile robot to the goal. To implement the proposed control method, we generated a real-time control code in Matlab on the industrial computer of the FWIDR system. The Gauss-Newton non-linear method is adopted to solve the optimization problem, which is carried out in Matlab function invfreqz. The cycle of the whole system, including perception, decision, and movement control modules, is about 100 ms. Among them, the time consumed by the control algorithm is about 10 ms.

As for the implementation of the proposed DFRNN-OISMTC scheme, we conduct a lot of simulations and experiments to find the most suitable control parameters through trial and error. The corresponding parameters are set as \( Q = \text{diag}[30 \ 10], R = \text{diag}[5 \ 5, \ 2, 2, 2, 2 \ 4], \), \( \Pi = \begin{bmatrix} 0.2 & 0.1 & 1 & 1 & 0.2 & 0.3 \end{bmatrix} \). The longitudinal velocity \( v_x \) of the FWIDR is 1.5 m/s. For the DFRNN, particle swarm optimization algorithm is adopted to search the optimal fractional orders benefiting from its high efficiency, low computational burden and excellent optimization.

### Table 1 The FWIDR parameters

| Description | Symbol | Value |
|-------------|--------|-------|
| Robot mass  | \( m \) | 700 kg |
| Robot yaw inertia | \( I_y \) | 130 KgM² |
| Distance from front axle to CG | \( l_f \) | 0.48 m |
| Distance from rear axle to CG | \( l_r \) | 0.56 m |
| Front cornering stiffness | \( \zeta_f \) | 3000 N/rad |
| Rear cornering stiffness | \( \zeta_r \) | 3000 N/rad |

\[
\begin{align*}
\dot{V}_2 &= \sigma^T \Pi^{'} \left( \zeta - \xi \right) + \Pi \text{sign}(\sigma) + d_t \\
&+ \frac{1}{\alpha_1} \dot{W}_a^T \dot{W}_a + \frac{1}{\alpha_2} \Gamma^T \dot{\Gamma} + \frac{1}{\alpha_3} \dot{\Lambda}^T \dot{\Lambda} \\
&+ \frac{1}{\alpha_4} \dot{W}_b^T \dot{W}_b + \frac{1}{\alpha_5} \dot{W}_b^T \dot{W}_f. \\
\end{align*}
\]

Substituting (34) into (39), we can obtain

\[
\begin{align*}
\dot{V}_2 &= \sigma^T \Pi^{'} \left( \zeta - \xi \right) + \Pi \text{sign}(\sigma) + d_t \\
&+ \frac{1}{\alpha_1} \dot{W}_a^T \dot{W}_a + \frac{1}{\alpha_2} \Gamma^T \dot{\Gamma} + \frac{1}{\alpha_3} \dot{\Lambda}^T \dot{\Lambda} \\
&+ \frac{1}{\alpha_4} \dot{W}_b^T \dot{W}_b + \frac{1}{\alpha_5} \dot{W}_b^T \dot{W}_f. \\
\end{align*}
\]

Based on the adaptive law presented in (36), we have

\[
\begin{align*}
\dot{V}_2 &= \sigma^T \Pi^{'} \left( \zeta - \xi \right) + \Pi \text{sign}(\sigma) + d_t \\
&\leq -\| \sigma \| (\varepsilon - \Pi^{'} \xi + d_t). \\
\end{align*}
\]
The number of the neurons in the hidden layer is selected as 6. The initial values of the weights $W_f^r$, $W_h^r$, $W_o$ are set to zeros. The initial values of the $\Gamma$, $\Lambda$ are set as $\Gamma = \text{ones}(1,24)$, $\Lambda = \text{ones}(1,24)$. The parameters in the adaptive law (36) are specified as $\alpha_1 = 0.04$, $\alpha_2 = 0.01$, $\alpha_3 = 11000$, $\alpha_4 = 0.0025$ and $\alpha_5 = 0.3$.

6.2 Results and discussions

Scenario (1): To verify the superiorities of the proposed optimal nominal controller with additional error integral feedback, we conducted the simulation experiments compared with the traditional optimal controller [32]. In this scenario, we simulate the FWIDR system by neglecting the uncertain part, that is the nominal system is expressed as (10).

Figure 7 shows the results of the tracking errors for the nominal system. By analyzing Figure 7, it can be found that both methods can stabilize the lateral error and heading error, but for our optimal control method, the performances of the path tracking errors responses are significantly improved. The tracking errors obtain faster response with lower overshoots and lower steady-state errors using our optimal nominal control method than using the traditional one. The sideslip angle and yaw rate results are presented in Figure 8. From this figure, it is shown that both methods can maintain the sideslip angle and yaw rate in reasonable regions, but our optimal control method for the nominal system can clearly enhance their tracking performance though improving the response speed and reducing the overshoots. At 1.1 s, the sideslip angle of the nominal system under the traditional optimal control peaks 0.1225 rad, which indicates an apparent overshoot response. The nominal control inputs are shown in Figure 9. To sum up, our optimal control method by introducing an additional integral term accumulates the output error over time and this can be used to compute enough control inputs to obtain the high dynamic response performance of the nominal system.

To adequately demonstrate the advantages of the proposed DFRNN-OISMTM scheme, the following methods are taken for experimental comparison: traditional ISMTC [40], OISMTM [41], and OISMTM with commonly-used RBFNN [42]. We performed experiments under different reference trajectories and various road conditions (i.e. flat hard road ground and rough
slippery concrete ground). The reference trajectory for the FWIDR is generated by using piecewise Bézier curves with continuous curvature. Moreover, the experiments are implemented on the developed FWIDR with varying extra payloads, and the specific loads are unknown. Given this context, we provide the following scenarios for demonstration.

Scenarios (2): The FWIDR is controlled to track continuous trajectories on the flat hard road ground to verify the superiorities of the proposed control method. The response results of the proposed path tracking control method are presented in Figures 10–15. Figures 10 and 11 illustrate the robot trajectories and tracking errors of the employed comparison control methods, respectively. It is observed that four control schemes ensure effective tracking of the reference trajectory. However, our proposed DFRNN-OISMTC scheme provides the best response performances such as smaller overshoot and fewer oscillations of the tracking errors. Initial errors of the FWIDR system emerge at the start pose [0,0.1,0] on the ground, then the...
system responds quickly, and the lateral error and head error are reduced to an acceptable level. Meantime, thanks to the additional integral term in optimal integral sliding mode control strategy, the maximum values of the tracking errors are significantly decreased. The response results of the slip angle and yaw rate are shown in Figure 12. It can be found that, compared with the other three controllers, the proposed DFRNN-OISMTC greatly reduces the oscillations and enhances the response speed. As shown in Figure 13, the sliding surfaces approach zero using three sliding mode control methods, and it is derived that the oscillation of the sliding surfaces around the equilibrium point is effectively suppressed in terms of the DFRNN-OISMTC method. Figure 14 shows the control inputs including steering angle and external yaw moment. It can be seen that the control inputs amplitude of the proposed method is the smallest among the four compared controllers. To offer a compensative view of the DFRNN, the norm of the parameters, including the weight $\tilde{W}_a$, the feedback coefficient $\tilde{W}_f$, $\tilde{W}_h$, the width $\tilde{\Gamma}$, and the centre $\tilde{\Lambda}$, are shown in Figure 15. It can be found that all these network parameters are tuned online by the proposed adaptive law (36) and finally converge to the optimal values. Thanks to this adaptive mechanism, the system uncertainties are effectively suppressed to improve path tracking performance.

To analyze the results quantitatively, we provide the quantitative criterions of the tracking errors in terms of root mean square error (RMS), integrated square error (ISE) and standard deviation (STD). The experimental results are given in Figure 16. We can see that the proposed DFRNN-IOSMTC scheme achieves the highest tracking accuracy of both lateral error and heading error. The RMS guaranteed by the proposed scheme are 46.11%, 64.64% and 71.53% of the ones provided by the ISMTC, OISMTC and RBFNN-OISMTC for the lateral error $q_e$, respectively. For the heading error $\phi_e$, the percentages are 57.59%, 83.38% and 91.41%. The ISE provided by the newly proposed scheme are 22.98%, 31.70% and 45.44% of the ones guaranteed by the ISMTC, OISMTC and RBFNN-OISMTC for $q_e$, respectively. For $\phi_e$, the percentages are 37.47%, 73.73% and 64.72%. In the meantime, the STD ensured by our scheme are only 43.97%, 54.66% and 63.88% of the ones provided by the ISMTC, OISMTC and RBFNN-OISMTC for $q_e$, respectively. For $\phi_e$, the percentages are 57.30%, 72.66% and 78.92%. Thus, the obtained results
FIGURE 17 Trajectories of the FWIRA in scenario 3

FIGURE 18 Tracking errors in scenario 3

FIGURE 19 Responses of sideslip angle and yaw rate in scenario 3

FIGURE 20 Sliding surfaces in scenario 3

FIGURE 21 Control inputs in scenario 3

demonstrate that the proposed DFRNN-OISMTC scheme can ensure that the FWIDR follows the reference trajectory in real-time, and it yields higher precision and lower oscillation than the other three schemes.

Scenario (3): We test the tracking performance of the proposed control method on the rough slippery concrete ground that is partially covered by oil and water. The experimental results are presented in Figures 17–22. analyzing Figures 17 and 18, it can be found that the proposed DFRNN-OISMTC achieves preferable convergence properties and path tracking performance among four control schemes. The FWIDR starts at an initial lateral error 0.1 m and an initial heading error 0.08 rad. As depicted in Figure 18(a), the following lateral error $q_e$ drops quickly at the beginning and converges to a relatively small level in a few seconds. In the steady-state, the lateral error and heading error of our controller are bounded by $\pm 6$ mm and $\pm 0.2$ deg, respectively, which are smaller than the other compared controllers. As shown in Figure 18(b), the following heading error
ϕ, keeps oscillating near zero after a certain steering adjustment. Moreover, as shown in Figure 20, the DFRNN-OISMTC can manipulate the designed sliding mode surfaces to zero within a smaller vibration range due to the adaptive law (36), thereby the chattering phenomena can be greatly alleviated. As exemplified in Figure 19, the slip angle and yaw rate of all four controllers are limit into the acceptable ranges, while the presented controller significantly improves the tracking accuracy. Figure 21 illustrates the corresponding steering angle and external yaw moment. The norm of the adaptive network parameters $\hat{W}_o, \hat{W}_f, \hat{W}_h, \hat{\Gamma}, \hat{\Lambda}$ are shown in Figure 22. It is clear that the parameters of the DLRNN are updated quickly to approximate the non-linear part, and they are eventually stable at a certain constant. Therefore, the DFRNN-OISMTC scheme can obtain the high-performance path following of the FWIDR under complicated lumped disturbances.

To accurately evaluate the control performance of the proposed scheme, we calculated the RMS, ISE and STD in terms of the lateral error and heading error, respectively. As illustrated in Figure 23, the compared three control schemes can guarantee successful following of the reference trajectory under the rough slippery concrete ground. However, the control accuracy and robustness against uncertainties are not as good as our designed DFRNN-OISMTC control scheme. To be specific, the RMS of $q_e$ ensured by the ISMTC, OISMTC and RBFNN-OISMTC are 231.61%, 177.40% and 149.08% of our designed control scheme, respectively. For $\phi_e$, the percentages are 171.43%, 135.53% and 123.94%. Similar results can be achieved of ISE and STD. Therefore, the proposed control scheme is useful for the stabilization and error mitigation of the FWIDR system under system uncertainties, which demonstrates the advantages of the DFRNN-OISMTC method.

Scenario (4): To further verify the robustness of the proposed DFRNN-OISMTC method, the experiments are implemented on the developed FWIDR with varying extra payloads, and the specific loads are unknown. The corresponding path tracking performance and control signals are shown in Figures 24–29. Figures 24 and 25 depict trajectories and tracking errors in the lateral and heading direction, respectively. The FWIDR runs from the start pose [0,0.1,0] on the ground, where the initial lateral error and heading error are 0.1 m and 0.016 rad, respectively. It can be clearly seen that the tracking error at the beginning
drops quickly and converges to a relatively small level in a few seconds. As shown in Figure 27, the designed sliding mode surfaces based on our controller keep relatively smaller. The proposed method significantly alleviated the chattering phenomena of the FWIDR system, therefore the control behaviour much better matches the reference trajectory. Figures 26 and 28 show the responses of tracking and control inputs, respectively. The degree of fluctuation of the control inputs using the DFRNN-OISMTC is less than other controllers. The evolution of the adaptive network parameters $\hat{W}_o, \hat{W}_f, \hat{W}_h, \hat{\Gamma}, \hat{\Lambda}$ are presented in Figure 29, and these parameters are updated online to approximate the non-linear system function. It can be found that these parameters are quickly tuned to the optimal value to compensate the non-linear part in the model efficiently and accurately due to the designed the adaptive law. Thus, the proposed DFRNN-OISMTC scheme has a strong robustness to the system.
CONCLUSIONS

This paper proposes a DFRNN-OISMTC scheme for robust path tracking of the FWIDR system under unmoulded system parameters and unavoidable external disturbances. By adding an additional integral term, a novel OISMTC scheme is designed to guarantee an accurate path tracking. Then, the FWIDR system is approximated by virtue of the DFRNN to deal with large system uncertainties such that the robustness of the investigated system is enhanced. Moreover, the adaptive law is presented based on the Lyapunov theory to guarantee the asymptotic stability. Comparative experiments are conducted verify the effectiveness and superiorities of the proposed DFRNN-OISMTC scheme.

The time-delay issue is not the topic of this paper since we focus on the robust control of the FWIDR under system uncertainties and external disturbances. However, as for the robotic applications, the input-delayed might weaken the control accuracy, and even result in system instability. The time delays estimation (TDE) technique guarantees an attractive model-free structure for the time delays system. In the future, we will try to further enhance the trajectory tracking performance of the investigated system by combing the TDE technique.

REFERENCES

1. Wang, Z., et al.: Distributed regular polygon formation control and obstacle avoidance for non-holonomic wheeled mobile robots with directed communication topology. IET Contr. Theory Appl. 14(9), 1113–1122 (2019)
2. Niroui, F., et al.: Deep reinforcement learning robot for search and rescue applications: Exploration in unknown cluttered environments. IEEE Robot. Autom. Lett. 4(2), 610–617 (2019)
3. Santos J, et al.: Remote control of an omnidirectional mobile robot with time-varying delay and noise attenuation. Mechatronics 52, 7–21 (2018)
4. Rongrong, W., Hui, Z., Junmin, W.: Linear parameter-varying controller design with optimal feedback control for nonlinear uncertain systems. Trans. Inst. Meas. Control 41(5), 1331–1340 (2019)
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FIGURE 30 The performance indexes of tracking errors in scenario 4. (a) Lateral error \( q_e \), (b) heading error \( \phi_e \).
17. Surjade, P. V., Tiwari, A. P., Shimjith, S. R.: Robust optimal integral sliding mode controller for total power control of large PHWRs. IEEE Trans. Nucl. Sci. 65(7), 1331–1344 (2018)

18. Park, B. S., et al.: Adaptive output-feedback control for trajectory tracking of electrically driven non-holonomic mobile robots. IET Control Theory Appl. 5(6), 830–838 (2011)

19. Ni, J., Hu, J., Xiang, C.: Robust control in diagonal move steer mode and experiment on an X-by-wire UGV. IEEE/ASME Trans. Mechatronics 24(2), 572–584 (2019)

20. Hu, C., et al.: Robust H∞ output-feedback control for path following of autonomous ground vehicles. Mech. Syst. Signal Proc. 70, 414–427 (2016)

21. Elhakki, O., Shojaei, K.: Observer-based neural adaptive control of a platoon of autonomous tractor-trailer vehicles with uncertain dynamics. IET Contr. Theory Appl. 14(14), 1898–1911 (2020)

22. Ji, X., et al.: A vehicle stability control strategy with adaptive neural network sliding mode theory based on system uncertainty approximation. Veh. Syst. Dyn. 56(6), 923–946 (2018)

23. Yang, Y., Niu, Y.: Event-triggered adaptive neural backstepping control for nonstrict-feedback nonlinear time-delay systems. J. Franklin Inst. 357(8), 4624–4644 (2020)

24. Fei, J., Chen, Y.: Dynamic terminal sliding-mode control for single-phase active power filter using new feedback recurrent neural network. IEEE Trans. Power Electron. 35(9), 9906–9924 (2020)

25. Khan, A. H., et al.: Tracking control of redundant mobile manipulator: An RNN based metaheuristic approach. Neurocomputing 400, 272–284 (2020)

26. Li, Y., Li, S., Hannaford, B.: A model-based recurrent neural network with randomness for efficient control with applications. IEEE Trans. Ind. Inform. 15(4), 2054–63 (2018)

27. Ni, J., et al.: Prescribed performance fixed-time recurrent neural network control for uncertain nonlinear systems. Neurocomputing 363, 351–365 (2019)

28. Chen, C., et al.: Hierarchical adaptive path-tracking control for autonomous vehicles. IEEE Trans Intell. Transp. Syst. 16(5), 2900–2912 (2015)

29. Hu, C., et al.: Output constraint control on path following of four-wheel independently actuated autonomous ground vehicles. IEEE Trans. Veh. Technol. 65(6), 4033–4043 (2015)

30. Wang, J., et al.: Integral sliding mode control using a disturbance observer for vehicle platoons. IEEE Trans Ind. Electron. 67(8), 6639–6648 (2019)

31. Peng, H., et al.: A symplectic instantaneous optimal control for robot trajectory tracking with differential-algebraic equation models. IEEE Trans. Ind. Electron. 67(5), 3819–3829 (2019)

32. Zhang, H., et al.: Nearly Optimal integral sliding-mode coconsensus control for multiagent systems with disturbances. IEEE Trans. Syst. Man Cybern.-Syst. (2019), https://doi.org/10.1109/TSMC.2019.2944259

33. Liu, R., Li, S.: Optimal integral sliding mode control scheme based on pseudo-spectral method for robotic manipulators. Int. J. Control 87(6), 1131–1140 (2014)

34. Wang, Y., et al.: Fault-tolerant control for in-wheel-motor-driven electric ground vehicles in discrete time. Mech. Syst. Signal Proc. 121, 441–454 (2019)

35. He, S., et al.: Adaptive optimal control for a class of nonlinear systems: The online policy iteration approach. IEEE Trans. Neural Netw. Learn. Syst. 31(2), 549–558 (2019)

36. Turabieh, H., Mafarja, M., Li, X.: Iterated feature selection algorithms with layered recurrent neural network for software fault prediction. Expert Syst. Appl. 122, 27–42 (2019)

37. Hu, J., et al.: Robust adaptive sliding mode control for discrete singular systems with randomly occurring mixed time-delays under uncertain occurrence probabilities. Int. J. Syst. Sci. 51(6), 987–1006 (2020)

38. Liu, L., et al.: Quasi-consensus control for a class of time-varying stochastic nonlinear time-delay multiagent systems subject to deception attacks. IEEE Trans. Syst. Man Cybern. Syst. (2020)

39. Sun, B., et al.: Synchronization of discrete-time recurrent neural networks with time-varying delays via quantized sliding mode control. Appl. Math. Comput. 375, 125093 (2020)

40. Hu, C., Wang, R., Yan, F.: Integral sliding mode-based composite nonlinear feedback control for path following of four-wheel independently actuated autonomous vehicles. IEEE Trans. Transp. Electrif. 2(2), 221–230 (2016)

41. Chen, S. A., et al.: Improved optimal sliding mode control for a non-linear vehicle active suspension system. J. Sound Vibr. 395, 1–25 (2017)

42. Xia, R., et al.: Neural network based integral sliding mode optimal flight control of near space hypersonic vehicle. Neurocomputing 379, 41–52 (2020)

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