Acoustic field simulation of phased array in borehole in layered formation

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Abstract. Horizontal stratified strata is a common geological model, and it needs to be simulated numerically in order to get the propagation law of sound waves in it, usually using the finite-difference time-domain method. However, for hundreds of meters underground well section, how to improve the calculation efficiency is a question that we need consider. In this paper, a dynamic method is proposed to obtain the computational domain, which can minimize the computational domain when the simulation results are reliable. When the transmitter is located below the receiver and the nearest delamination boundary is more than 2.0 m below the instrument, all delamination media below the instrument can be ignored in the calculation. A similar strategy is used for the layered boundary located above the array receiver. In this paper, the method is used to simulate the response of phased array acoustic logging in multi-layer media.

1. Introduction
Horizontal stratigraphic strata are the most common geological structure. In order to improve the processing method of acoustic logging data for field exploration, it is necessary to study the borehole acoustic field in the horizontal multi-layer medium model. For this kind of non-uniform formation, the elastic wave equation has no analytic solution, and the Borehole Acoustic field cannot be simulated by means of real axis integration. The finite-difference time-domain method is a practical method for numerical simulation of wave fields. Stephen [1] et al. first proposed a two-dimensional finite-difference method for calculating the borehole acoustic field of a unipolar (or symmetric) source. Randall et al. [2] further developed the numerical simulation method for multipolar acoustic logging, and simulated the acoustic response in more complex cases, including non-uniform borehole diameter, horizontal stratification of formation, non-uniform axis and so on. Wang and Tang [3] proposed a non-splitting perfect matching layer for the finite difference method of elastic wave equation, which improves the effect of absorbing boundary and the computational efficiency of finite difference method. They also simulated the acoustic field of the quadruple acoustic logging while drilling in transversely ISOTROPIC formations. These two dimensional finite-difference time-domain methods, which are based on the axisymmetric assumption of the model, can greatly improve the efficiency of acoustic field response simulation.

However, it is difficult to simulate the Acoustic logging responses at different depths by using the two-dimensional axisymmetric method because of the large computation area. In the computation, only a certain part can be truncated as the computation area. It is necessary for us to consider how to minimize the computation area[4], improve the computation speed and save the memory without affecting the results of numerical simulation. In this paper, the two-dimensional symmetric acoustic field simulation of phased array is presented, and its validity and correctness are verified by an example.
2. Calculation model and calculation method
In order to investigate the effect of the distance between the interface and the sound source/receiver on the log response in a multi-layer formation, we adopt the layered medium model as shown in Fig. 1 and assume that an infinite length of liquid-filled borehole runs vertically through the horizontal layered medium. We established the cylindrical coordinates \((r, \theta, z)\), and take the well axis as the symmetrical axis. In the three-layer media model in figure 1, from top to bottom are layer 1, layer 2, and layer 3, respectively. We assume that the instrument is located in the Middle layer 2, where the lower sound source is \(L_{23}\) from the boundary of layer 3, the farthest receiver at the top is \(L_{12}\) from layer 1. In this paper, the influence of these boundary distances on the log full-wave response is analyzed by an example. The well hole is not shown in the diagram. The Blue Circle represents the sound source and the Green Circle represents the receiver farthest from the sound source.

\[ \begin{align*}
\frac{\partial V}{\partial t} &= \nabla \cdot \tau \\
\frac{\partial V}{\partial t} &= C(\nabla V + V \nabla)
\end{align*} \]

In formula (1)-(2), \(t\) represents time, and \(C\) represents the matrix of elastic coefficients. For transversely ISOTROPIC media with symmetric and \(Z\) axes in agreement, it can be expressed as

\[
C = \begin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{pmatrix}
\]

Where \(C_{12} = C_{11} - 2C_{66}\), \(C_{13} = C_{11}\), \(C_{33} = C_{11}\), \(C_{44} = C_{66}\) and \(C_{66}\) are independent elastic coefficients: when \(C_{11} = C_{33} = \lambda + 2\mu\), \(C_{13} = \lambda\), \(C_{44} = C_{66} = \mu\), the transversely isotropy degenerates to isotropy. In order to understand the above partial differential equation, a two-dimensional finite-difference time-domain method algorithm is used for numerical simulation in this paper.

2.2. Numerical simulation and example analysis
In the numerical simulation example, we will investigate the effect of the formation interface and the
distance between the transducer and the receiver response. If the reflected wave does not overlap with the direct wave or is outside the receiving window, the influence of the boundary on the simulation result of the log response can be neglected. In the simulation, the homogeneous medium can be used at the end of the interface to reduce the calculation area and improve the simulation efficiency [5]. The medium parameters in the numerical simulation are presented in Table 1, assuming a well radius of 0.1 m and a central frequency of 8 KHz.

| Medium  | Density (kg/m$^3$) | Longitudinal velocity (m/s) | Shear velocity (m/s) |
|---------|------------------|---------------------------|---------------------|
| fluid   | 1000             | 1500                      | —                   |
| layer2  | 2000             | 2200                      | 1400                |
| layer3  | 2400             | 4400                      | 2700                |

Figures 2 and 3 show a comparison of the acoustic log responses of phased array acoustic sources in layered and homogeneous media respectively, with red curves representing layered media and black curves representing homogeneous media. When the sound source is 0.5 m away from the bottom boundary $L_{23}$, the reflected and direct waves in the log response cannot be separated, so in the sampling window of the log (usually<5ms), the results of wave field simulation in stratified and homogeneous media are quite different. Therefore, the boundary of the lower layer cannot be neglected in the simulation. In the finite difference calculation, the boundary of the lower layer should be included in the calculation area to ensure the accuracy of the calculation. As the boundary of the lower layer increases, the reflected wave is gradually separated from the direct wave. When $L_{23}$ is 2.0 m, the reflected wave is completely separated from the direct wave, and the response of the layered medium and the homogeneous medium is identical in the time window of 0-5 ms. The existence of the lower layer boundary can be neglected in the numerical forward calculation, the lower truncation boundary of the calculation area can be set as the sound source position, thus the calculation area can be reduced, the calculation memory can be saved and the speed can be increased.

For the distance relationship between the upper boundary and the most remote receiver, the same strategy can be used to determine the position of the upper boundary in the computational domain according to the acoustic reciprocity principle. To sum up, in order to improve the processing speed and save memory in numerical simulation, it is necessary to optimize the range of truncation area.

![Fig. 2](image1.png)

(a) Lower Boundary Distance $L_{23}$=0.5m
(b) Lower Boundary Distance $L_{23}$=2.0m

Fig. 2 Comparison of phased array source response of layered medium and homogeneous medium
All layered media can be reduced to the figure 1 model, taking into account only the distance between the lower boundary and the emitter, and the position of the upper boundary with the farthest receiver. Based on the results of the numerical simulation and discussion above, the interception of the upper and lower boundary of the computational domain can be divided into the following cases: If $L_{12}$ or $L_{23}$ is larger than 2.0 m, the corresponding upper or lower boundary can be completely ignored, thus, the location of the farthest receiver or source can be set as the truncation boundary of the computational region; if $L_{12}$ or $L_{23}$ is less than 2.0 m, the location of the upper or lower boundary can be used as the truncation boundary of the computational region. In this way, the scope of the calculation area can be minimized, as shown in figure 3.

Finally, this paper uses this regional optimization strategy to realize the continuous simulation of logging curves at various depth points. Assuming that the model is a multi-layered medium with a thickness of only 3.0 m, the acoustic log response at each depth is numerically simulated using the boundary selection method mentioned above, the time-slowness similarity correlation method is used to extract the velocity of the simulated wave curve, and the result is consistent with the input velocity parameters of the model. Figure 4 is a continuous simulation of the horizontal multi-layer logging response p-wave and s-wave velocity extraction results, black is p-wave velocity, red is s-wave velocity, It can be seen from figure 5, the speed extracted from the simulation results is basically consistent with that of the model. There are two main reasons for the deviation, one is that the model has a thin layer; the other is that the interlayer velocity varies greatly. This proves the feasibility of the method of computing region selection.

3. Discussion and conclusion

When simulating the response of Acoustic logging in multi-layer formation [6], it is an effective way to improve the simulation speed and save the computational memory by selecting the smallest numerical calculation area with proper optimization scheme. The results show that when the distance between the source and the farthest receiver is greater than 2.0 m, the corresponding formation boundary can be neglected in numerical simulation, the axial position of the source or the farthest receiver can be used as the boundary of the calculation area; otherwise, if the formation boundary is less than 2.0 m from the source or the farthest receiver, the calculation area should contain the boundary. The results of numerical simulation show that the method can meet the needs of logging curve simulation and velocity extraction accuracy. In the future work, this method will be used to simulate and analyze the borehole acoustic field of anisotropic layered model.
References
[1] STEPHEN R A, CARDO-CACAS F, CHENG C H. Finite-difference synthetic acoustic logs [J]. Geophysics, 1985, 50(10): 1588–1609.
[2] RANDALL C J, SCHEIBNER D J, WU P T. Multiple borehole acoustic waveforms: Synthetic logs with beds and borehole washout [J]. Geophysics, 1991, 56(11):1757–1769.
[3] Tang X M, Patterson D J. 2009. Single-well S-wave imaging using multicomponent dipole acoustic-log data. Geophysics, 74(6):211-223.
[4] Tang X M, Wei Z T. 2012. Single-well acoustic reflection imaging using far-field radiation characteristics of a borehole dipole source. Chinese J. Geophys. (in Chinese),55(8):2798-2807.
[5] HE X, HU H, GUAN W. Fast and slow flexural waves in a deviated borehole in homogeneous and layered anisotropic formations [J]. Geophys. J. Int., 2010, 181(1): 417–426.
[6] Deng Ying, Chen Jie, Yu Qijiao. Sound field simulation method of multipole source in layered medium. Applied Acoustics, 2017, 36(6): (555-558)