SIMULATIONS OF PREGALACTIC STRUCTURE FORMATION WITH RADIATIVE FEEDBACK

MARIE E. MACHACEK
Physics Department, Northeastern University, Boston, MA 02115

GREG L. BRYAN
Physics Department, Massachusetts Institute of Technology, Cambridge, MA 02139

AND

TOM ABEL
Harvard Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

Received 2000 July 11; accepted 2000 October 19

ABSTRACT

We present results from three-dimensional hydrodynamic simulations of the high-redshift collapse of pregalactic clouds including feedback effects from a soft H$_2$ photodissociating UV radiation field. The simulations use an Eulerian adaptive mesh refinement technique to follow the nonequilibrium chemistry of nine chemical species with cosmological initial conditions drawn from a popular dark matter model. The results confirm that the soft UV background can delay the cooling and collapse of small halos (\(\sim 10^5 M_\odot\)). For reasonable values of the photodissociating flux, the H$_2$ fraction is in equilibrium throughout most of the objects we simulate. We determine the mass threshold for collapse for a range of soft-UV fluxes and also derive a simple analytic expression. Continuing the simulations beyond the point of initial collapse demonstrates that the fraction of gas which can cool depends mostly on the virial mass of the halo and the amount of soft-UV flux, with remarkably little scatter. We parameterize this relation, for use in semianalytic models.

Subject headings: cosmology: theory — galaxies: formation — hydrodynamics — methods: numerical

1. INTRODUCTION

A critical epoch in the evolution of the universe is the the postrecombination period during which the first stars form, affect their environment, and help determine how the intergalactic medium (IGM) ultimately becomes reionized. Recent observations are beginning to constrain models of this epoch. The observation of quasars at redshifts as high as \(z = 5.8\) (Fan et al. 2000) with no accompanying Gunn-Peterson absorption trough in their spectra indicates that the onset of reionization occurs quite early (\(z \gtrsim 7\)), and so the first generation of stars forms even earlier. Observations of quasar spectra (Ellison et al. 1999, 2000) suggest that metallicities possibly as high as log (C IV/H I) = -2.6 may be present even in the low column density log \(N_{\text{HI}} < 14.5\) Ly$\alpha$ forest. This too may be indicative of an active period of preenrichment of the IGM by an early generation of stars (Ostriker & Gnedin 1996). Future multiwavelength observations may be sensitive probes of this star formation epoch and the subsequent onset of reionization. A partial list of proposed observations include direct imaging of quasars and star clusters at \(z > 10\) by the Next Generation Space Telescope (Haiman & Loeb 1999; Barkana & Loeb 2000), absorption measurements in the 21 cm line using the Square Kilometer Array to identify the first epoch of massive star formation (Tozzi et al. 2000), and searches for secondary anisotropies in the cosmic microwave background by the next generation of millimeter telescopes such as ALMA to constrain properties of the reionization epoch (Benson et al. 2001). Thus it is important to understand what our best cosmological models predict for the evolution of these early star-forming structures and what governs the amount of gas that can ultimately condense into stars.

It has long been understood that a high-redshift population of stars could form naturally in hierarchical models of structure formation. In these models small masses collapse first and later merge into larger structures. The ability of these objects to form stars depends on the ability of gas within these objects to cool radiatively, reduce pressure support, and condense. However, the first structures to become self-gravitating in these models would have temperatures \(T < 10^4\) K, too cool for H I line cooling to be effective. The only other coolant available to gas with a primordial composition is molecular hydrogen. Therefore it is the complex chemistry of H$_2$ in these objects that controls whether the gas can condense to sufficiently high densities for star formation to take place.

Early analytical models of the collapse of the first small-scale structures studied the effect of H$_2$ cooling within highly idealized cloud geometries, typically a homogeneous gas cloud radiatively cooling while undergoing a uniform free-fall collapse (Saslow & Zipoy 1967; Peebles & Dicke 1968; Hirasewa 1969; Matsuda et al. 1969; Hutchins 1976; Silk 1983; Palla et al. 1983; Lepp & Shull 1984; Lahav 1986). One-dimensional simulations sought to improve upon these analytical estimates, but still had to impose the shape of the initial density profiles and mass of the virialized object as initial conditions within an assumed cloud geometry (Haiman, Toul, & Loeb 1996; Tegmark et al. 1997; Omukai & Nishi 1998; Nishi et al. 1998; Nakamura & Umemura 1999). Haiman et al. (1996) followed the dynamics of gas and cold dark matter and the H$_2$ chemistry in a single, spherical density peak and showed the importance of shell crossing between the gas and dark matter in determining the fraction of mass within an object that could collapse and the minimum mass for collapse. In the same spirit Tegmark et al. (1997) used top-hat density profiles and a simplified collapse criterion, i.e., that the cooling time

---

1 Hubble Fellow.
be less than the Hubble time for the object right after virialization, to obtain rough estimates of the critical $H_2$ mass fraction needed for cooling and the minimum mass (as a function of virialization redshift) that could produce it. However, the minimum mass to collapse determined in this way is strongly dependent on the $H_2$ cooling function used. Abel et al. (1998) and Fuller & Couchman (2000) recalculated this minimum mass for top-hat density profiles using the Tegmark et al. (1997) collapse criterion with updated $H_2$ cooling functions (Lepp & Shull 1983 and Galli & Palla 1998) and found that the minimum mass able to collapse (at virialization redshifts $\sim 30$) was reduced by as much as an order of magnitude.

Molecular hydrogen is fragile and easily photodissociated by photons whose energies lie below 13.6 eV (Stecher & Williams 1967). Thus even before reionization, radiation from the first stars could affect $H_2$ abundances at large distances (Dekel & Rees 1987). Haiman, Rees, & Loeb (1997) quantified this negative radiative feedback in more detail. They studied the collapse of homogeneous, spherical gas clouds in the presence of a soft-UV photodissociating flux assuming both ionization and chemical equilibrium within the cloud to determine which masses could still collapse. Haiman, Abel, & Rees (2000) improved on that calculation by solving spherically symmetric radiative transfer coupled with the time-dependent rate equations for the $H_2$ chemistry across truncated isothermal spheres. They also modified the collapse criterion such that a structure was said to be able to collapse and form stars if at any time during the object's history, the $H_2$ cooling time in the core became less than the time elapsed since the core's formation. This allowed the fraction of $H_2$ abundance required for cooling and collapse of the core to be lower in certain cases than previous estimates of the "critical" $H_2$ fraction based on the Tegmark et al. (1997) collapse criterion. Analytical models that include a first population of OB-like stars have been used to probe the suppression of star formation elsewhere within the host cloud due to the destruction of $H_2$ coolant by the direct radiation from the first stars (Omukai & Nishi 1999; Glover & Brand 2000).

The major drawback affecting all of the above studies whether with or without radiative feedback is their one-dimensional nature. Hierarchical structure formation that predicts the growth of structure in and along filaments is inherently three-dimensional. However, fully three-dimensional simulations are computationally challenging because of the need for large dynamic range to model simultaneously the internal structure of the first star-forming structures and the effects of gravitational tidal forces and merging on their growth. Such three-dimensional simulations have only recently become computationally feasible. Gnedin & Ostriker (1997) simulated the thermal history of the intergalactic medium and the population of first objects to collapse but had insufficient resolution to probe inside these first structures. Abel et al. (1998) used a static nested grid approach and constrained realizations of 3–4 σ primordial fluctuations in a cluster normalized CDM model to study when these structures were able to cool, but were also limited by spatial resolution (~1 kpc) from probing deeply within the peaks. Subsequent higher resolution three-dimensional simulations focused on the properties of a single collapsing cloud. Bromm, Coppi, & Larson (2000) studied the fragmentation of a $2 \times 10^9 M_\odot$ rotating disk under highly idealized initial conditions using a TREESPH code. Fuller & Couchman (2000) used an adaptive smoothing kernel smoothed particle hydrodynamics (SPH) coupled to a $\Phi^2 M$ gravity solver in a simulation box of comoving length $25 h^{-1}$ kpc to investigate the properties of the first peak to collapse. However, because of the small box size they needed to impose an overdense cold dark matter cosmology to model the formation of the cloud within its host filament. The highest spatial and mass resolution self-consistent cosmological three-dimensional simulations to date of the collapse and fragmentation of primordial clouds (Abel, Bryan, & Norman 1999, 2000; Norman, Abel, & Bryan 2000) use Eulerian adaptive mesh refinement (AMR) techniques to achieve extremely large dynamic range (currently $\geq 3 \times 10^7$). They start from cosmological initial conditions in a cluster normalized CDM model to study the detailed internal evolution of a single, collapsing cloud in interaction with its environment in a box with comoving length $L = 128$ kpc.

In this paper we present the results of fully three-dimensional Eulerian AMR simulations of the formation and collapse of primordial pregalactic structure in a realistic ΩCDM cosmology. This study expands upon previous work in several important ways. First, we use a larger simulation volume (1 Mpc$^3$ comoving) to model more accurately the gravitational tidal forces and effects of merging on the evolving structures. Second, we introduce a simple model for radiative feedback. We analyze the collapse properties of individual peaks with and without the presence of this soft-UV background and compare our results with previous one-dimensional studies. Finally, we use the population of pregalactic objects evolving together in the most active structure-forming region of our simulation to develop statistics on the fraction of gas that is able to cool because of the presence of $H_2$ and the fraction of gas that is both cold and dense enough to be available for star formation as functions of cloud mass and the average soft-UV radiation field it experiences. These statistics represent the key new result of this paper. The amount of gas available for star formation in pregalactic objects is a critical input parameter into recent semianalytical attempts to model stellar feedback from the first stars on the intergalactic medium, subsequent structure formation, and the nature of reionization (Gnedin & Ostriker 1997; Ciardi et al. 2000; Ferrara et al. 2000; Ciardi, Ferrara, & Abel 2000).

While we focus on the negative effect of radiation in this paper, it should be noted that positive feedback is possible if the first objects also emit high-energy X-ray photons (Haiman et al. 2000; Oh 2000). We will address this point in future work.

This paper is organized in the following way. In §2 we describe the setup of our simulations including simulation parameters and technique, cosmological model, $H_2$ chemistry, and radiative feedback. In §3 we present the characteristics of the simulated data set of pregalactic objects identified in our simulations and used for further analysis. In §4 we use the data set as a whole to develop fitting formulae for the fraction of gas that can cool, become dense, and thus be available for star formation with and without the presence of a background $H_2$ photodissociating flux. In §5 we use radial profiles to show the internal collapse characteristics of these structures. We consider both the time evolution of a single peak for a fixed background $H_2$ photodissociating flux and also how the internal properties of a given cloud change at fixed redshift when the level of $H_2$...
photodissociating flux is varied. Armed with these examples of the internal evolution and structure of the collapsing clouds, we discuss in § 6 the neglect of self-shielding and \( \mathrm{H}^- \) photodetachment in our simulations and then use a simple analytical argument to illuminate the essential physics of the collapse. We summarize our results in § 7.

2. THE SIMULATIONS

A self-consistent three-dimensional cosmological simulation of the collapse of low-mass (10\(^7\)–10\(^8\) \( M_\odot \)) protogalaxies at high redshift (30 > \( z > 19 \)) requires both large enough simulation volume to model the gravitational tidal forces at work during the formation epoch and high spatial (and mass) resolution to probe within the small collapsing clouds. One technique that has recently been able to achieve the large dynamic range necessary to shed light on the collapse and fragmentation of these first star–producing structures is adaptive mesh refinement. The three-dimensional AMR algorithm used in our simulations is described more fully elsewhere (Bryan 1999; Bryan & Norman 1997, 1999; Norman & Bryan 1998). Briefly, the code uses an adaptive hierarchy of rectangular grid patches at various levels of resolution very similar to the algorithm described by Berger & Collela (1989). Each grid patch covers some region within its parent grid needing additional refinement and may itself become a parent grid to an even higher resolution child grid. We take the ratio of parent-to-child grid mesh spacing to be 2. The dark matter is followed using methods similar to those presented by Couchman (1991), and the gas hydrodynamics uses the artificial viscosity method of Stone & Norman (1992).

We use a flat, low matter density ΛCDM model for our simulations with \( \Omega_0 = 0.4, \Omega_b = 0.05, \Omega_\Lambda = 0.6, h = 0.65, \sigma_8 = 0.8, \) and \( n = 1 \), where \( \Omega_0 \), \( \Omega_b \), and \( \Omega_\Lambda \) are the fraction of the critical energy density carried in nonrelativistic matter, baryons, and vacuum energy, respectively, \( h \) is the dimensionless Hubble parameter in units of 100 km s\(^{-1}\) Mpc\(^{-1}\), \( \sigma_8 \) is the density fluctuation normalization in a sphere of radius 8 h\(^{-1}\) Mpc, and \( n \) is the slope of the primordial density perturbation power spectrum. The parameters are chosen to provide good consistency with observation. The fluctuation normalization is consistent with the cosmic microwave background (CMB) quadrupole as measured by COBE (Bunn & White 1997) and also with observations of the number density of galaxy clusters (White, Efstathiou, & Frenk 1993; Bond & Myers 1996). \( \Omega_b h^2 \) is consistent with big bang nucleosynthesis and the measured abundance of primordial deuterium (Copi, Schramm, & Turner 1995; Burles & Tytler 1998), and \( \Omega_\Lambda \) is consistent with the upper limit (\( \Omega_\Lambda < 0.7 \)) of Maoz & Rix (1993) and the best-fit parameters of Ostriker & Steinhardt (1995).

The simulations are initialized at \( z = 99 \) with density perturbations generated for the above ΛCDM model using the Eisenstein & Hu (1998) transfer functions in a comoving simulation volume of 1 Mpc\(^3\). This simulation volume is large enough to ensure that the fundamental mode in the box remains linear at the lowest redshifts considered here. Using a low-resolution run, the Lagrangian volume surrounding the most massive peak to form in the box by redshift \( z = 15 \) is identified as a region of active structure formation. Then in order to achieve high mass resolution in this region of interest, the simulation is reinitialized with multiple static refinement levels surrounding the chosen region such that each successive static parent grid contains the lagrangian volume of its child grid. This results in a mass resolution in the initial conditions for the region of interest of 4.78 (38.25) \( M_\odot \) for the gas (dark matter), respectively. In addition, that region is allowed to dynamically refine so that the local Jeans length is resolved at all times by at least 4 grid zones and no grid cell contains more than 4 times the initial gas mass element (4.78 \( M_\odot \)) or 10 times the initial dark matter element (382.5 \( M_\odot \)). We limit the total refinement to 14 levels within a 64\(^2\) top grid, which results in a maximum dynamic range of 1.048,576. The cov- }

oring spatial resolution of 0.95 pc at maximum refinement translates into a physical spatial resolution of 0.03 pc (0.05 pc) at \( z = 30 \) (\( z = 19 \)), respectively. We call a peak “collapsed” when it reaches maximal refinement. Once this occurs, we still wish to follow the subsequent evolution of structure in the region, but are uninterested in the details of the collapsed peak. In order to prevent the peak from collapsing further (which could cause numerical instabilities), we introduce a form of artificial pressure support. This is done by defining, for each cell, an effective pressure which is the greater of the thermal pressure and \( K g g' / 2 \Delta x^2 / \mu \), where \( \rho_0 \) is the baryon density in the cell, \( \Delta x \) is the cell width on the finest level, and \( \mu = 1.22 m_\text{H} \) is the usual mean mass per particle. The dimensionless constant \( K \) is set to 100, a value which spreads the mass over a spherical region with a radius of several cells. The form of this expression is chosen by matching the thermal energy and gravitational self-energy of the gas; notice also that it results in a polytropic equation of state. We also smooth the gravitational potential by 4\( \Delta x \), which helps to reduce the amplitude of the artificial pressure support required. Although introducing artificial pressure support causes us to lose information about the detailed morphology of the inner 1–2 pc (\( \sim 1\% \) of the virial radius) of the peak after collapse, it should not affect the determination of cooled gas fractions in § 4, since the cooling region lies outside of the collapsed H\(_2\) core.

Our data set consists of results from four simulations starting from identical cosmological initial conditions on the density fields. Three of the simulations follow the non-equilibrium, time-dependent evolution of nine chemical species (\( \mathrm{H}, \mathrm{H}^+, \mathrm{He}, \mathrm{He}^+, \mathrm{He}^{++}, e^-, \mathrm{H}_2, \mathrm{H}_2^+, \mathrm{H}^+ \)) using the algorithm of Anninos et al. (1997) initialized with postre-}

combination abundances (Anninos & Norman 1996). The reaction network for even this simple chemistry is complex. Numerous authors have worked to identify the dominant reactions and calculate their rate coefficients including currently Haiman et al. (1996), Abel et al. (1997), and Galli & Palla (1998). We use primarily the rate coefficients of Abel et al (1997) with the exception of rate coefficients for the processes \( \mathrm{H} + e^- \rightarrow \mathrm{H}^- + \gamma \), \( \mathrm{H}_2 + e^- \rightarrow 2 \mathrm{H} + e^- \), and \( \mathrm{H}^- + \mathrm{H}^+ \rightarrow \mathrm{H}_2^+ + e^- \), which are taken from Shapiro & Kang (1987). We have also included the three-body process \( 2 \mathrm{H} + \mathrm{H} \rightarrow \mathrm{H}_2 + \mathrm{H} \) (Orel 1987; Palla et al. 1983), although its contribution is negligible at the densities considered in this work. The small postrecombination abundance of \( \mathrm{H}_2 \) in the intergalactic medium (\( \sim 2 \times 10^{-6} \)) is too low for cooling to be effective. However, at the densities and temperatures in the interiors of collapsing protogalactic clouds, \( \mathrm{H}_2 \) formation is catalyzed by \( \mathrm{H}^- \) ions through the process

\[
\mathrm{H} + e^- \rightarrow \mathrm{H}^- + \gamma ,
\]

\[
\mathrm{H} + \mathrm{H}^- \rightarrow \mathrm{H}_2 + e^- ,
\]
and can reach abundances high enough to act as an efficient coolant allowing the first stars to form.

Once the first stars form, radiation above 13.6 eV produced by these stars is quickly absorbed by the dense neutral hydrogen surrounding them. However, the intergalactic medium is optically thin to radiation below the H I ionization limit. This radiation escapes to form a soft-ultraviolet background of radiation throughout neighboring regions with sufficient energy to photodissociate H$_2$ by means of the Solomon process,

$$H_2 + \gamma \rightarrow H^2 \rightarrow H + H,$$

thus inhibiting continued star formation (Haiman et al. 2000). H$_2^*$ is any of 76 Lyman-Werner resonances in the energy range 11.18–13.6 eV. We use a very simple model to investigate the negative feedback effects of this soft-UV radiation. The radiation field is assumed to be constant throughout the simulation with mean photon energy of 12.87 eV. In terms of the constant mean flux $F_{\text{LW}}$, the H$_2$ photodissociation rate coefficient for the Solomon process denoted in equation (3) is (Abel et al. 1997)

$$k_{\text{diss}} = 1.1 \times 10^6 F_{\text{LW}} \text{ s}^{-1}.$$  

We distinguish three of our simulations by different levels of this background flux, i.e., $F_{\text{LW}} = 0$ (first structure formation), $10^{-22}$, and $10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$, respectively. These flux levels correspond to $J_{21}$ normalizations of the spectral intensity (Haiman et al. 2000) of $J_{21} = 0$, $10^{-2.1}$, and $10^{-1.1}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$ sr$^{-1}$, respectively. We also include results from a fourth simulation that starts from the same cosmological initial conditions, but evolves varying levels of the radiative background flux. These peaks span a mass range from $3 \times 10^4$ to $7 \times 10^6 M_\odot$ and are modeled in our simulations by $\sim 1000$–$125,000$ dark matter particles, respectively. The absence of low-mass peaks in the bottom portion of Figure 1 is an artifact of our selection criterion. However, the increase in the maximum mass of the cloud with decreasing redshift is real and characteristic of the “bottom up” growth of structure in these models. Small clouds form first at high redshift, merging to form more massive structures later. There is clear evidence for mergers among the clouds we consider. See, for example, the merger of the two most massive peaks in Figure 1 into a single peak between redshifts $z = 21$ and $z = 20.5$. However, since we do not follow all of the low-mass clouds in the simulation, we do not construct complete merger trees for the sample. Thus we caution the reader that all the peaks are not statistically independent of each other. This is particularly noticeable for the high-mass peaks, which are rare in our simulation volume and may result in a decrease in the scatter of measured properties for the higher mass clouds.

We define the virial radius, $r_{\text{vir}}$, as the radius of a sphere within which the average density in the cloud exceeds 200 times the mean matter density of the universe. The virial mass, $M_{\text{vir}}$, is the total mass contained within that radius. Before cooling becomes important the mean gas-mass-weighted temperature of the cloud obeys a virial relationship between temperature, mass, and redshift similar to that observed in galaxy clusters (Bryan & Norman 1998). For a

![Figure 1](image-url)
neutral mix of primordial gas

$$T_{\text{vir}}(K) = 155 f (h^2 \Delta_e E^2)^{1/3} \left( \frac{M_{\text{vir}}}{10^7 M_\odot} \right)^{2/3},$$  \hspace{1cm} (5)

where $f = 0.8$ is a normalization factor, $h = 0.65$ is the dimensionless Hubble parameter at $z = 0$, $\Delta_e = 200$ is the collapse overdensity, and

$$E^2 = \Omega_0 (1 + z)^3 + \Omega_\Lambda \approx \Omega_0 (1 + z)^3,$$  \hspace{1cm} (6)

for $\Omega_0 = 0.4$ and $\Omega_\Lambda = 0.6$ at redshifts $30 > z > 19$. In Figure 2 we compare the mean gas-mass-weighted temperature $T_{\text{mean}}$ for the clouds in our sample to the virial temperature $T_{\text{vir}}$ defined by equation (5) for the case when no molecular hydrogen is present and for the case $F_{L,W} = 0$ when molecular hydrogen cooling is maximal. All of the clouds in our sample have virial temperatures $T_{\text{vir}} < 8000$–10,000 K, the temperature necessary for H I line cooling to become important. Peaks analyzed from the simulation without H$_2$ cannot cool and their mean temperatures are well characterized by the virial relation given in equation (5). Once cooling becomes important, the mean gas-mass-weighted temperature of the cloud will drop significantly below its virial value as demonstrated in Figure 2 by peaks from the simulation that includes the H$_2$ chemistry.

In Table 1 we compare the properties of the first protogalaxies to “collapse,” i.e., reach maximal refinement, for the simulations with H$_2$ cooling and soft-UV background flux levels $F_{L,W} = 0$, $10^{-22}$, and $10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$. Protogalactic clouds in the simulation without H$_2$ have not collapsed by redshift $z = 19$. The first structures ($F_{L,W} = 0$) in this $\Lambda$CDM model collapse early ($z \approx 30$) and are small ($r_{\text{vir}} \approx 50$ pc), low-mass (M$_{\text{vir}} \approx 1.4 \times 10^5 M_\odot$) clouds. Simulations of the minimum collapse mass by Fuller & Couchman (2000) find the minimum mass to collapse at $z = 30$ to be $\sim 3 \times 10^5 M_\odot$ (expected to be robust within about a factor of 2) thus consistent with our results. Abel et al. (2000) find that a $7 \times 10^5 M_\odot$ peak collapses first (at $z = 19$) in their high-resolution CDM simulation. This too is consistent with the minimum mass collapse curves of Fuller & Couchman (2000). Thus, despite differences in numerical method and cosmology, there appears to be general agreement that the minimum mass of the first star-forming structures is $\sim 10^5 M_\odot$. This is about an order of magnitude below the earlier estimates of Tegmark et al. (1997), primarily because of the use of improved H$_2$ cooling functions.

As the amount of the H$_2$ photodissociating flux increases, the mass threshold for collapse increases, pushing the redshift of collapse to lower values. However, even with flux levels as high as $F_{L,W} = 10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$, H$_2$ can survive in the cores of the clouds such that objects (M$_{\text{vir}} \sim 1.26 \times 10^6 M_\odot$, $T_{\text{vir}} \sim 2262$ K) below the mass threshold for H I line cooling are still able to collapse. However, the collapse in this case is delayed until $z = 21.5$. This is consistent with the one-dimensional simulations of Haiman et al. (2000). See, in particular, their Figure 6 for $J_{21} = 10^{-1.1}$. Furthermore, the ability of a cloud to collapse does not seem to be strongly dependent on the angular momentum L of its dark matter halo, since the threshold objects we found to collapse spanned the full range of expected spin parameters $0.02 < \lambda_{\text{dm}} < 0.11$, where $\lambda = (L/E^{1/2})(GM^5/2)$, $E$ is the total energy, $M$ is the mass, and $G$ is the gravitational constant.

In order to check the numerical robustness of our results, we ran several lower mass resolution simulations ($\Delta M = 306[38] M_\odot$ for dark matter [gas]) using the same random seed for the initial conditions and region for dynamical refinement as the high mass resolution runs presented here. We also ran several different realizations of the initial conditions at this lower resolution for the $F_{L,W} = 10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$ case to ensure that there was nothing excep-
tional about the particular realization we present. We found that, although the properties of the first peak to collapse were very similar for the two mass resolutions, the higher mass resolution run delayed the redshift of first collapse to slightly lower values ($\Delta z \sim 1$). This difference in the redshift at which maximal refinement is reached, however, is already less than the cosmic variance ($\Delta z \sim 2$) in this quantity observed between simulations run with different realizations of the initial conditions. The mass threshold for collapse was also reduced by $\approx 30\%$ in the higher mass resolution run compared to the lower resolution simulation. This is slightly larger than the $\pm 20\%$ variation in mass found from simulations with fixed mass resolutions but initial conditions generated using different random seeds.

We can also address at this point what effect our initial selection of an overdense region within a given simulation has on these results. There are two parts to the answer of this question. The first is that we specifically selected an active region of structure formation because we are interested in early star formation. Our assumption was that cooling, and hence star formation, was more likely to occur first in the large halos that appear initially in overdense regions. This is confirmed by the results discussed below, showing that the cooled fractions depend mostly on halo mass. The second part of the answer is that because the cooled and cold dense fractions have such a weak dependence on redshift or local environment (or indeed, anything besides total mass), we expect these relations to show little dependence on what kind of region was selected.

4. COLD GAS FRACTION

A critical input parameter into semianalytical models that seek to include stellar as well as radiative feedback is the amount of gas in these first pregalactic objects that is available for star formation. Abel et al. (1998) estimated the fraction of gas able to cool by adding up all the mass within the radius of a $4\sigma$ peak (in three different simulations) for which the temperature in radially averaged profiles began to decrease. However, cooling alone is not sufficient to determine whether the gas is available to form stars. It must also become dense. We improve upon these previous estimates by measuring both the fraction of gas that is able to cool via molecular hydrogen cooling, $f_c$, and the fraction of gas that becomes both cold and dense, $f_{\text{cd}}$, in the clouds comprising the data sample described in §3. It is this latter fraction $f_{\text{cd}}$ of cold, dense gas that is available to form stars in these objects.

In Figure 3 we present both these statistics as functions of the cloud mass $M_{\text{vir}}$ for various levels of the background radiative flux $F_{\text{LW}}$. We define the following:

1. $f_c$ is the fraction of total gas within the virial radius with temperature $T < 0.5 T_{\text{vir}}$ and gas density $\rho > 1000 \rho_{\text{mean}}$, where $\rho_{\text{mean}}$ is the mean gas density of the universe. This is the amount of gas within the cloud that has been able to cool because of molecular hydrogen cooling.

2. $f_{\text{cd}}$ is the fraction of total gas within the virial radius with temperature $T < 0.5 T_{\text{vir}}$ and gas density $\rho > 10^{19.9} M_\odot \text{Mpc}^{-3} \sim 330 \text{ cm}^{-3}$. This is the fraction of gas within the cloud that is available for star formation.

The above criteria to determine the amount of gas in each category (cooled or both cold and dense) are applied on a cell-by-cell basis within the virial radius of each peak in the data sample described in §3. This includes both peaks that have reached maximal refinement in the simulation and those that have not. The temperature threshold has been chosen to ensure that the gas is substantially cooler than the virial temperature of the peak as given in equation (5). The density threshold for cold, dense gas corresponds to the gas density at which the baryons become important to the gravitational potential and thus to the subsequent evolution of the core. However, setting a density threshold for gas that has been able to cool through the $H_2$ chemistry but may not be dense is more subtle. We must be careful to exclude gas that is cool only because it belongs to lower mass clouds that are infalling onto the more massive structure. With the high mass resolution in our simulations, this substructure, due to merging, is resolvable. Since there is no cooling in the peaks extracted from the simulation without $H_2$, any cold gas present must be a result of infall and merging in the outer regions of the cloud. We confirm that this is indeed true in the data set without $H_2$ cooling by looking at radial profiles for the cold gas fraction in several peaks. We then used these data to determine the density threshold for cooled gas given above that first minimizes the cold infalling component. As shown in the bottom panel of Figure 3, the resultant background due to merging is negligible for all but the most massive peaks in the sample. The bump in $f_c$ in the high-mass end of the bottom panel is a result of the merger of the two most massive peaks in the box at $z \sim 20.5$ seen in Figure 1. Even in this extreme case the above density threshold keeps the infalling cold component of $f_c$ limited to a few percent. Note also that $f_{\text{cd}}$, the fraction of gas that can become dense and form stars, is zero for all peaks in the simulation without $H_2$, as expected.

![Figure 3](image-url)
Figure 3 represents one of the key results of this paper. Both the fraction of gas $f_c$ that can cool and the fraction $f_{cd}$ of cold, dense gas available for star formation are highly correlated with the pregalactic cloud mass over the mass range dominated by $H_2$ cooling. Correlation coefficients derived from a mean regression analysis of $f_c$ ($f_{cd}$) with the logarithm of the cloud mass are 0.93, 0.91, and 0.87 (0.75, 0.79, and 0.80) for radiative background fluxes $F_{LW} = 0$, $10^{-22}$, and $10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$, respectively. This logarithmic dependence of $f_c$ and $f_{cd}$ on cloud mass can be summarized in a simple fitting formula:

$$f_i = B_i \ln (M/M_{TH}) \ ,$$

for $i = c$ (cooled gas) or $i = cd$ (cold, dense gas) and $M > M_{TH}$. The best-fit slopes $B_i$ for the fraction of gas $f_c$ that cools are 0.138, 0.139, and 0.085 for radiative background fluxes of $F_{LW} = 0$, $10^{-22}$, and $10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$, respectively. However, the fraction of cold, dense gas $f_{cd}$ increases much less rapidly with mass. Best-fit slopes $B_{cd}$ for this cold, dense component are 0.058, 0.066, and 0.030 for $F_{LW} = 0$, $10^{-22}$, and $10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$, respectively. The near equality of the slopes for flux levels $F_{LW} = 0$ and $10^{-22}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$ is striking and probably reflects the fact that the cooling chemistry is the same in each case independent of radiative flux level. The apparent softening of the slopes for $F_{LW} = 10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$ is most likely an artifact of the small number of independent high-mass clouds in our simulation volume and the increased scatter for peaks near the cooling mass threshold due to secondary environmental effects. The mass thresholds $M_{TH}$ for nonzero $f_c$ and $f_{cd}$ are about equal at fixed $F_{LW}$ and increase as the level of soft UV flux is increased. This is a characterization of the negative feedback expected because of the photodissociation of $H_2$ in the lower mass clouds. Our results for the mass threshold can be parameterized as

$$M_{TH}(M_\odot) = 1.25 \times 10^5 + 8.7 \times 10^5 \left( \frac{F_{LW}}{10^{-21}} \right)^{0.47} .$$

The presence of a soft $H_2$ photodissociating flux at the levels considered here delays, but does not prevent, gas in these pregalactic objects from becoming dense because of $H_2$ cooling in their cores. The fraction of gas available for star formation due to $H_2$ cooling can be represented as

$$f_{cd} \approx 0.06 \ln (M/M_{TH}) \ ,$$

with $M_{TH}$ given by equation (8).

We have performed several other tests on the $F_{LW} = 0$ sample to check whether the above results are biased as a result of our treatment of the data. First, in order to establish that the above relationship is not an artifact of the evolution of the most massive peaks across several epochs, we removed the two most massive peaks (identified at $z = 20$) and their progenitors from each redshift and refitted the data. The correlation of $f_c$ ($f_{cd}$) with the logarithm of halo mass is essentially unchanged, 0.92 (0.75) compared to 0.93 (0.75) for the full data set, respectively. While the best-fit slopes $B_c$ and $B_{cd}$ steepen slightly over those for the full data set, they too agree within the standard errors of the fit. Second, we may test whether halo mass, independent of redshift, is the key factor in determining the cooled and cold, dense fractions. First a straightforward plot of $f_c$ or $f_{cd}$ as a function of redshift has significantly more scatter than Figure 3 and in fact mirrors Figure 1, suggesting that the epoch dependence observed is due solely to the redshift evolution of the halo masses themselves. To test this further, we divide the $F_{LW} = 0$ sample into two redshift bins, $z > 24$ and $z < 24$ and for each plot $f_c$ ($f_{cd}$) as a function of the halo mass. Qualitatively the distributions merge seamlessly one into the other with no evidence of an epoch-dependent break. A regression analysis of each bin does reveal that the high-redshift bin has more scatter and a lower correlation coefficient than the low-redshift bin. However, for these high redshifts there are fewer halos and the halo masses lie close to the cooling mass threshold, where secondary effects are more likely to be important. If we restrict each redshift bin to the same mass range near threshold, the slopes (and correlation coefficients) agree within the standard errors of the fits. Thus we are confident that halo mass is the most significant factor in determining the fractions of gas that can cool and the cold, dense gas available for star formation and that this dependence is well described by the fitting formulae given in equations (7) and (9) above.

5. UNDERSTANDING PEAK FORMATION AND COLLAPSE

In this section we use spherically averaged radial profiles of the pregalactic cloud properties to illustrate the internal dynamics of its collapse. We first consider the evolution with redshift of a collapsing pregalactic object when exposed to a fixed level of photodissociative flux. We then consider how the internal properties of a given cloud change at fixed redshift when the level of the photodissociating flux changes.

5.1. Evolution with Redshift

In three of the panels of Figure 4 we show the evolution of the gas density, temperature, and $H_2$ mass fraction from $z = 30$ to maximal refinement at $z = 21.5$ for the first object to collapse at a radiative flux level $F_{LW} = 10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$ (see Table 1). In the fourth panel (lower right) we show the evolution of the cooling and dynamical timescales for $25 \leq z \leq 21.5$ after cooling begins. Starting at large radius, the upper set of curves represent the $H_2$ cooling time $t_{cool}$ given by

$$t_{cool} = \frac{1.5n_g k_b T}{\Lambda n_{H_2} m_{H_2}} ,$$

where $n_g$, $n_{H_2}$, and $n_{H_2}$ are number densities for the gas, neutral hydrogen, and $H_2$, respectively, $k_b$ is the Boltzmann constant, $T$ is the temperature, and $\Lambda$ is the $H_2$ cooling function. The lower set of curves at large radii show the dynamical timescale $t_{dyn}$, where

$$t_{dyn} = \left( \frac{3\pi}{16G\rho} \right)^{1/2}$$

for the same redshifts. The solid (dot-dashed) horizontal line is the Hubble time,

$$t_{Hubble} = \frac{2}{3H_0} [\Omega_m(1 + z)^3]^{-1/2} ,$$

for $z = 21.5$ (25), respectively, and the heavy dashed line is the difference between the two, i.e., the elapsed time between the onset of cooling and collapse. The vertical line locates the virial radius $r_{vir} = 141$ pc at $z = 21.5$ when the structure has grown to a mass of $1.26 \times 10^6 M_\odot$. The increases in density, temperature, and $H_2$ mass fraction seen outside the
virial radius at $\sim 1$ kpc are a result of averaging over other massive structures forming nearby.

During the initial stages of evolution (from $30 \leq z \leq 25$) $\text{H}_2$ abundances are too low to be an effective coolant and the cloud grows from a mass of $\sim 6.5 \times 10^4 \ M_\odot$ to $5.3 \times 10^5 \ M_\odot$ through accretion and merging from within its host filament. The radial profiles within the core are well described by constant densities and temperatures that increase as expected as the mass of the object increases, while the outer edges of the profiles steepen with decreasing $z$. The $\text{H}_2$ mass fractions are also roughly constant in the core (mirroring the density profiles) and grow with decreasing $z$. By $z = 25$ the core temperature has risen to about 2900 K, the core gas density has reached $\sim 3 \times 10^{17} \ M_\odot \ Mpc^{-3}$ $\approx 10$ cm$^{-3}$, the $\text{H}_2$ mass fraction is $\sim 2.3 \times 10^{-5}$, and the core has broadened from about 10–20 pc. For $F_{\text{LW}} = 10^{-21}$ erg$^{-1}$ cm$^{-2}$ Hz$^{-1}$ the photodissociation timescale,

$$t_{\text{diss}}(s) = k_{\text{diss}}^{-1} = 9 \times 10^{-9} / F_{\text{LW}},$$

is $9 \times 10^{12}$ s, much shorter than other timescales in the problem. Thus we can understand the $\text{H}_2$ mass fraction in the core by a simple equilibrium argument. If we assume that $\text{H}_2$ formation is dominated by the $^1\text{H}^- \text{process}$ given in equations (1) and (2), the $\text{H}_2$ equilibrium number density is given by

$$n_{\text{H}_2}^{\text{equil}} \approx \frac{k_{\gamma} n_\text{H} n_e}{k_{\text{diss}}},$$

where $k_{\gamma} \approx 1.8 \times 10^{-18} T^{0.88}$ cm$^3$ s$^{-1}$ over this temperature range, and $n_\text{H} (n_e)$ is the number density of neutral hydrogen (electrons), respectively. At $z = 25$ equation (14) predicts an equilibrium $\text{H}_2$ mass fraction $n_{\text{H}_2}^{\text{equil}} \sim 2.6 \times 10^{-5}$ in the core that is nearly identical to the value above found in our simulation.

By $z = 24$ the $\text{H}_2$ mass fraction has reached $\sim 4 \times 10^{-5}$ and gas in the central region has begun to cool. As cooling continues the core density steepens, evolving into a $\sim r^{-2}$ profile at $z = 21.5$ in the inner 10 pc. The gas density reaches number densities $n \geq 4 \times 10^3$, exceeding the dark matter density for $r < 1$ pc. Notice, however, that most of the cooling occurs in the region $2 < r < 20$ pc, outside this very dense central core. The temperature profile at collapse is very similar to that found by Abel, Bryan, & Norman (2000) showing with decreasing radius cosmic infall, shocking, and a cooling flow. We also find for this object that the cooling time becomes comparable to the dynamical time over most of the cooling flow region just prior to collapse, and that the outer radius of the cooling region corresponds closely to the radius at which the cooling time has dropped below the elapsed time (Fig. 4, heavy dashed line) in agreement with the collapse criterion of Haiman et al. (2000).

Although spherically averaged radial profiles are easy to picture, it is important to remind ourselves that the evolution of these objects is fully three-dimensional and they do not collapse in isolation from their surroundings. In Figure 5 we show images of the log of the projected dark matter density, the baryonic density, and the temperature at...
FIG. 5.—Two-dimensional projections of the dark matter density (top), gas density (middle), and temperature (bottom) for a region 680 pc (proper) on a side at $z = 21.5$ in the simulation with $F_{\text{LW}} = 10^{-22}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$. The gray scale is logarithmic. The region contains the two most massive clouds, $2.4 \times 10^6 M_\odot$ (upper structure) and $2.2 \times 10^6 M_\odot$ (lower structure) formed in the simulation.

$z = 21.5$ for a portion of the dynamically refined region in the $F_{\text{LW}} = 10^{-22}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$ simulation. This region contains the two most massive structures in the simulation. The image is 680 pc (proper) on a side and is projected along the $y$-axis. The masses (virial radii) for these protogalaxies are $2.4 \times 10^6 M_\odot$ (174 pc) for the upper cloud and $2.2 \times 10^6 M_\odot$ (170 pc) for the lower cloud, while their proper center-to-center separation is only $\sim 300$ pc. Thus their outer regions are already beginning to merge.

The temperature map shows an intricate pattern of asymmetrical shocks with maximum temperatures around 5400 K surrounding elongated knots of cooled gas at the clouds’ centers. These cooling knots have temperatures of a few hundred kelvins and are the regions of highest $H_2$ concentration and thus gas density. The dark matter halos, containing $\sim 42,000 (37,000)$ particles for the upper (lower) structures, respectively, clearly show substructure near their centers due to the prior infall and merger of lower mass clouds that have not yet completely relaxed. This merger activity is responsible in part for the aspherical geometry of the cooling knots and corresponding gas density.

5.2. Changing the Photodissociating Flux

In Figure 6 we compare the spherically averaged radial profiles of dark matter density, gas density, temperature, and $H_2$ mass fraction at fixed redshift ($z = 21.5$) but with varying levels of $H_2$ photodissociating flux for the $2.2 \times 10^6 M_\odot$ (lower) structure of Figure 5. The cloud has only been able to maximally refine in the simulation with $F_{\text{LW}} = 0$, although it is close to collapse in the $F_{\text{LW}} = 10^{-22}$ case also. This is in spite of the fact that the mass of this object is above the masses of the first objects to collapse in both the $F_{\text{LW}} = 10^{-22}$ and $10^{-21}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$ simulations (see Table 1). Probably this is because of the strong effect of tidal interactions and high level of merging activity in and surrounding this object. The features located at 300 pc and 1 kpc in the profiles reflect the presence of the upper, $2.4 \times 10^6 M_\odot$ protogalaxy of Figure 5 and the $1.26 \times 10^6 M_\odot$ structure we analyzed earlier in Figure 4, respectively. The dark matter density profiles are essentially the same for all of the simulations, independent of fraction or radiative flux. This is to be expected as long as the gravitational potential is dominated by the dark matter. The dark matter density only steepens away from this common profile in the inner $\sim 2$ pc of the $F_{\text{LW}} = 0$ case when the gas density equals and then exceeds the dark matter density. For the $F_{\text{LW}} = 0$ case the gas density steepens in the inner several parsecs with gas number densities reaching $\sim 10^4$, characteristic of the formation of a dense collapsed core. The cooling region extends inward from about 40 pc until the temperature drops to the minimum value possible under $H_2$ cooling, $\sim 200$ K, at around 3 pc. The $H_2$ mass fraction has saturated at $\sim 10^{-3}$. The sharp drops in temperature that correspond to enhancements in the $H_2$ mass fraction at 9 and 25 pc are the result of averaging over infalling clouds that were able to form $H_2$ in their cores and cool before merging with this more massive object. As the radiative flux level is increased, the negative feedback becomes apparent. The maximum gas density drops in the central region and forms a homogeneous core whose core radius also increases as the flux is increased. The radius of the cooling region moves inward and cooling is suppressed because of the decreased amount of $H_2$ coolant. The cloud is clearly in photodissociation equilibrium for the high-flux ($F_{\text{LW}} = 10^{-22}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$) simulation.
10^{-21}) case with the H_{2} fraction given by equation (14). For
the lower flux case, the H_{2} fraction is somewhat smaller than
the photodissociation equilibrium value in the inner 10 pc be-
cause of the influence of recombination on the formation
of H^{-} ions, and thus the H_{2} formation rate. We also
see, as expected, that there is no cooling at all for the case in
which H_{2} is absent.

We can gain a better understanding of the above radial
profiles and their scaling properties by comparing the rele-
vant physical timescales in the problem. These times are
shown for both the case (\textit{top panel}) and the
\textit{lower panel}) in Figure 7. In addition to
profiles for the Hubble, dynamical, photodissociation,
and cooling times defined earlier, we include profiles for the
sound crossing time \( t_{\text{cross}} \), the recombination time \( t_{\text{rec}} \) given by

\[ t_{\text{rec}} = 7.7 \times 10^{3} T^{0.64}/n_{e}, \tag{15} \]

and the H_{2} formation time \( t_{H_{2}} \) given by

\[ t_{H_{2}} = \frac{n_{H_{2}}}{k_{\text{H}_{2}} n_{e} n_{e}}. \tag{16} \]

All times are in seconds. We also show that the two-body
relaxation time \( t_{\text{relax}} \) (eq. [8-71] in Binney & Tremaine
1987) for the dark matter particles is long compared to all
other timescales, confirming that two-body numerical
effects are unimportant.

For both cases the photodissociation and H_{2} formation
times are much shorter than the Hubble time everywhere
within the virial radius. In the high-flux case they are also
much shorter than the recombination time, so that pho-
dissociation equilibrium \( t_{\text{diss}} = t_{H_{2}} \) is established
throughout the cloud (see eq. [14]). Even so, enough H_{2}
forms for the cooling time to drop below the Hubble time in the inner
40 pc—core collapse is not far off. The low-flux case is
similar in the outer parts of the cloud: the recombina-
tion time is long and the H_{2} fraction is governed by its equi-
librium value. However, within 10 pc recombination
becomes faster than photodissociation. This reduces the
amount of H^{-} allowed to form and suppresses H_{2} formation
below the photodissociation equilibrium value.

However, enough H_{2} has formed that cooling and collapse
proceed faster than photodissociation \( t_{\text{H}_{2}} < t_{\text{diss}} \) and
\( t_{\text{cool}} < t_{\text{diss}} \) at these small radii. We also find here—as in
Abel et al. (2000)—that \( t_{\text{cross}} < t_{\text{dyn}} \) in the inner regions
of the cloud, so that the collapse proceeds hydrostatically.

6. DISCUSSION

Now that we have detailed examples of the collapse
properties of clouds in our simulations, we are better able to
assess the importance of neglected processes in our simu-
lations. In this section we discuss in more detail why the
neglect of self-shielding and H\textsuperscript{−} photodetachment should not significantly change our results. We then use a simple analytical model, similar to that used by Tegmark et al. (1997), to illustrate the essential physics of the collapse.

6.1. Neglected Processes

We first return to the issue of H\textsubscript{2} self-shielding. Although this effect could, in principle, be important for our objects, we argue here that its actual impact is slight. The maximum column densities of H\textsubscript{2} we encounter in our simulations are around \(10^{17}\) \(\text{cm}^{-2}\), while a column density of \(5 \times 10^{14}\) \(\text{cm}^{-2}\) is sufficient to start blocking the soft-UV flux. So, at first glance, it appears that self-shielding should be important; however, this value has been derived for a static gas distribution. The Lyman-Werner bands consist of a large number of individual lines whose width is dominated (in our case) by Doppler broadening.

There are substantial velocity gradients within all of our simulated objects, even those which appear to be relaxed. These take the form not just of radial inflow but also of large-scale disordered velocity fields (i.e., turbulence). Typical rms line-of-sight velocities are around 3–4 km s\(^{-1}\). These values are larger than the \(\sim 2\) km s\(^{-1}\) line widths as a result of thermal Doppler broadening. This means that even if a given parcel of gas is behind a large column of molecular hydrogen, it may still be photodissociated if it has a velocity shift relative to the blanketing material. There is very little continuum opacity.

It is a difficult radiative transfer calculation to determine the precise effect of these velocity shifts; however we may consider two limiting cases. The first is the previously mentioned static case in which velocities are negligible compared to the thermal Doppler widths (\(v/b < 1\)) and an H\textsubscript{2} column density of \(5 \times 10^{14}\) \(\text{cm}^{-2}\) is sufficient to start affecting the photodissociation rate. In the other extreme, in which \(v/b > 1\), the rate is not significantly reduced until a substantial fraction of the 11–13 eV range is blocked by the Lyman-Werner lines. Since this does not occur until the damping wings of the lines become important, it requires a much larger molecular hydrogen column density, of order \(10^{23}\) \(\text{cm}^{-2}\) (Draine & Bertoldi 1996; Glover & Brand 2000).

In the simulations, \(v/b > 1\) so we are likely to be closer to the second regime than the first. Even if it were marginally important, there are three further reasons to think that self-shielding is unimportant. First, these large column densities occur at late times, once the core has already collapsed. For most of the evolution the objects are clearly in the optically thin regime. Second, the maximum column density quoted above is to the center of the object, but the cooling front lies outside of the H\textsubscript{2} core, and so beyond most of the covering material. Finally, once a sufficient amount of H\textsubscript{2} has formed to produce a substantial blanketing column, the cooling time has dropped below the photodissociation time, so that photodissociation is no longer important (and thus even if the soft-UV flux is attenuated, the subsequent evolution of the gas would be unchanged). So while the results described earlier are properly lower limits to the cooled fraction because of our neglect of self-shielding, a full radiative transfer calculation should produce nearly identical results.

Let us also reexamine whether H\textsuperscript{−} photodetachment (which we neglect) could significantly affect the results. In order to bracket the size of this effect we consider two spectral shapes for the radiation field, \(J \sim v^z\) with \(z = 1\) and \(-1\). The softer spectrum, \(z = 1\), is the spectral slope expected in this energy range from a typical OB star. The steeper slope, \(z = -1\), is the type of spectral shape one might expect for a background radiation field built up from a number of sources. The cross section (in \(\text{cm}^2\)) for H\textsuperscript{−} photodetachment is given by (Shapiro & Kang 1987; Abel et al. 1997)

\[
\sigma_{\text{detach}} = 7.928 \times 10^5 (v - v_{\text{th}})^{3/2} \frac{1}{v^2} \quad \text{for } v > v_{\text{th}},
\]

where \(v_{\text{th}} = 0.755\) eV/\(h\nu\), and \(h\nu\) is Planck’s constant. The H\textsuperscript{−} photodetachment rate coefficient \(k_{\text{detach}}\) can then be calculated:

\[
k_{\text{detach}} = \int_{v_{\text{th}}}^{\infty} 4\pi J(v) \frac{h\nu}{v^2} \sigma_{\text{detach}}(v) dv,
\]

with \(h\nu v_H = 13.6\) eV. For the two spectral shapes given above we find \(k_{\text{detach}} = 1.9 \times 10^{-12} F_{\text{LW}}\) s\(^{-1}\) for \(z = 1\) and \(6.4 \times 10^{-11} F_{\text{LW}}\) s\(^{-1}\) for \(z = -1\), where \(F_{\text{LW}}\) is the radiation flux in \(\text{ergs}^{-1}\ \text{cm}^{-2}\ \text{Hz}^{-1}\).

The dominant processes other than photodetachment that determine the abundance of H\textsuperscript{−} are shown in equations (1) and (2) with rate coefficients \(k_{\gamma}\) and \(k_d\), respectively. Furthermore, the processes proceed sufficiently rapidly compared to the H\textsubscript{2} chemistry that \(n_{\text{H\textsuperscript{−}}}\) is well approximated by its equilibrium value (Abel et al. 1997) such that (including photodetachment)

\[
n_{\text{H\textsuperscript{−}}} \approx \frac{k_{\gamma} n_{\text{H\textsuperscript{−}}} n_e}{k_d + k_{\text{detach}}}.
\]

We see that H\textsuperscript{−} photodetachment can be neglected as long as \(k_{\text{detach}} \ll k_d n_{\text{H\textsuperscript{−}}}\). This leads to a condition on the hydrogen number density \(n_{\text{H\textsuperscript{−}}} \gg k_{\text{detach}}/k_d\) that can be checked against the conditions found within the collapsing structures. For gas temperatures found in our simulations, \(k_d \approx 1.43 \times 10^{-9} \text{cm}^3\ \text{s}^{-1}\) (Abel et al. 1997). Thus photodetachment is unimportant provided that \(n_{\text{H\textsuperscript{−}}} \approx 0.045(F_{\text{LW}}/10^{-2})\) for the steeper \(z = -1\) spectrum or, a much weaker constraint, that \(n_{\text{H\textsuperscript{−}}} \approx 1.4 \times 10^{-3}(F_{\text{LW}}/10^{-2})\) for the softer \(z = 1\) spectrum.

Let us check these constraints against the hydrogen number densities found within the first peak to collapse at our highest flux level \((F_{\text{LW}} = 10^{-21} \text{ergs}^{-1} \text{cm}^{-2} \text{Hz}^{-1})\) shown in Figure 4. At \(z = 25\), before cooling commences, hydrogen number densities in the core region where H\textsubscript{2} will form already exceed \(n_{\text{H\textsuperscript{−}}} \approx 7\). This number density satisfies the constraint for the steepest spectrum \((z = -1)\) by more than 2 orders of magnitude, so that the neglect of H\textsuperscript{−} photodetachment does not significantly affect our results.

6.2. An Analytic Estimate of the Collapse Threshold

Analytical arguments often provide qualitative insight into the key physical principles at work in a complex process. Since we have seen that the H\textsubscript{2} fraction is close to photodissociation equilibrium throughout our objects, we use these equilibrium abundances to develop analytical estimates of the critical H\textsubscript{2} fraction and mass thresholds for collapse. We adopt the simplified collapse criterion, similar to that of Tegmark et al. (1997), that an object will collapse if the cooling time \(t_{\text{cool}}\), given by equation (10), is less than the Hubble time \(t_{\text{Hubble}}\), defined in equation (12). Thus

\[
\frac{1.5 n_g k_b T}{\Lambda n_{\text{H\textsuperscript{−}}} n_{\text{H\textsuperscript{−}}}^2} \leq \frac{2}{3} H_0^3 [\Omega_0 (1 + z)^3]^{-1/2}.
\]
The equality gives the critical number density of $H_2$ necessary for collapse. Solving for $n_{h_2}^{\text{crit}}$, we obtain

$$n_{h_2}^{\text{crit}} = 2.25 \frac{n_g k_B T}{n_h \Lambda} H_\odot \Omega_0^{1/2} (1 + z)^{3/2}. \quad (21)$$

In this equation, $\Lambda$ is, in our case, the Lepp & Shull (1983) cooling function. In the low-density limit ($n_{h_2} \ll 10^4$ cm$^{-3}$) for temperatures $300 \, K \leq T \leq 5000 \, K$ of interest here, it is well approximated by $\Lambda \approx 6.0 \times 10^{-11} T_{k_B}^{-1/2} e^{-730/T}$ for $T_{k_B}^{-1} \approx 1.8 \times 10^{-7} h$, such that

$$n_{h_2}^{\text{crit}} \approx 1.3 \times 10^{-7} h e^{-730/T} \Omega_0^{1/2} (1 + z)^{3/2}. \quad (22)$$

If we now equate this critical abundance of $H_2$ required to satisfy our collapse criterion to the photodissociation equilibrium abundance from equation (14) and solve for $F_{\text{LW}}$, we obtain an expression for the critical flux that just allows cooling. Therefore they are susceptible to the destruction of molecular hydrogen from soft-UV flux in the $11-13$ eV range (the Lyman-Werner bands). This radiation comes from the very first generation of stars, and so its amplitude is unclear; we test four cases, ranging from zero flux to complete $H_2$ destruction. The initial conditions are drawn from a $\Lambda$CDM cosmological model, and the simulations evolve the nonequilibrium rate equations for 12 species of hydrogen and helium. Our main conclusions may be summarized as follows:

1. A soft-UV flux does delay cooling and star formation in the cores of clouds in this mass range. An increasing amount of radiation delays collapse until later redshifts when larger objects have collapsed. However, even without any radiative feedback, objects with masses less than $\sim 1.4 \times 10^3 M_\odot$ (at $z \sim 30$) do not cool efficiently because of their low $H_2$ fraction.

2. The amount of cold, dense gas which is available for star formation in the cores of the collapsing objects depends primarily on two numbers, with relatively little scatter: the flux of soft-UV radiation and the mass of the cloud. We show the cold, dense fraction increases logarithmically with the mass of the halo, and we provide a fit for this relation as well (eq. [9]).

3. The evolution of these objects is relatively straightforward. With a negligible soft-UV radiation field, the formation of $H_2$ continues until it is suppressed by the removal of electrons from the core due to recombination. However, by the time this occurs, generally enough $H_2$ has formed to allow core collapse. When radiative feedback is important ($F_{\text{LW}} \gtrsim 5 \times 10^{-24}$ ergs$^{-1}$ cm$^{-2}$ Hz$^{-1}$), the destruction and formation processes are sufficiently fast that the $H_2$ fraction is in equilibrium (eq. [14]) and recombination plays little role until after core collapse begins. This allows the derivation of an analytic estimate for the collapse threshold which comes close to matching the full simulation results.

Finally, there are a number of other results that deserve further investigation. One of these is that the collapse redshift does not seem to depend on the angular momentum of the halo (see § 3). Another is that we do not appear to witness fragmentation in these objects. The few times in which we clearly see more than one collapsed core in a halo appears to be because of the merging of a number of halos, each of which already contained a core.

We also examined a number of processes which we omitted from the simulation, including $H_2$ self-shielding and $H^-$ photodetachment, none of which appear to be
important for this calculation. These results were derived within the context of a currently popular $\Lambda$-dominated cosmological model. However, at these high redshifts, the effective $\Omega$ should be close to unity, so the most important parameter is the amplitude of the power spectrum on these scales. Relatively small changes to the amplitude (such as would arise from the current uncertainty in $\sigma_8$) are unlikely to have a major impact on the results presented here, although it should always be kept in mind that we are extrapolating the power spectrum to scales significantly smaller than constrained by observations.

REFERENCES

Abel, T., Anninos, P., Norman, M. L., & Zhang, Y. 1998, ApJ, 508, 518
Abel, T., Anninos, P., Zhang, Y., & Norman, M. L. 1997, New A, 2, 181
Abel, T., Bryan, G. L., & Norman, M. L. 1999, in Proc. MPA/ESO Conf., Evolution of LSS; from Recombination to Garching, ed. A. J. Banday, R. K. Sheth, L. N. da Costa (Garching: MPA-ESO)
———. 2000, ApJ, 534, 57
Anninos, P., & Norman, M. L. 1996, ApJ, 460, 556
Anninos, P., Zhang, Y., Abel, T., & Norman, M. L. 1997, New A, 2, 209
Barkana, R., & Loeb, A. 2000, ApJ, 534, 57
Benson, A. J., Nusser, A., Sugiyama, N., & Lacey, C. G. 2001, MNRAS, 320, 153
Berger, M. J., & Collela, P. 1989, J. Comput. Phys., 82, 64
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press)
Bond, J. R., & Myers, S. T. 1996, ApJS, 103, 63
Bromm, V., Coppi, P. S., & Larson, R. B. 2000, 522, 5L
Bryan, G. L., & Norman, M. L. 1997, in ASP Conf. Ser. 123, Computational Astrophysics, ed. D. A. Clarke & M. J. West (San Francisco: ASP), 363
———. 1998, ApJ, 495, 80
———. 1999, in Workshop on Structured Adaptive Mesh Refinement Grid Methods, IMA Volumes in Mathematics 117, ed. N. Chrisochoides (New York: Springer), 165
Bunn, E. F., & White, M. 1997, ApJ, 480, 6
Burles, S., & Tytler, D. 1998, ApJ, 499, 699
Ciardi, B., Ferrara, A., & Abel, T. 2000, ApJ, 533, 594
Ciardi, B., Ferrara, A., Governato, F., & Jenkins, A. 2000, MNRAS, 314, 611
Copi, C. J., Schramm, D. N., & Turner, M. S. 1995, Science, 267, 192
Couchman, H. 1991, ApJ, 368, L23
Dekel, A., & Rees, M. J. 1987, Nature, 326, 455
Draine, B. T., & Bertoldi, F. 1996, ApJ, 468, 269
Eisenstein, D. J., & Hu, W. 1998, ApJ, 496, 605
Eisenstein, D. J., & Hut, P. 1998, ApJ, 498, 137
Ellison, S. L., Lewis, G. F., Pettini, M., Sargent, W. L. W., Chaffee, F. H., & Irwin, J. 1999, PASP, 111, 946
Ellison, S. L., Songaila, A., Schaye, J., & Pettini, M. 2000, AJ, 120, 1175
Fan, X., et al. 2000, AJ, 120, 1167
Ferrara, A., Ciardi, B., Marri, S., & Todini, P. 2000, preprint (astro-ph/0001010)
Fuller, T. M., & Couchman, H. M. P. 2000, 544, 6
Galli, D., & Palla, F. 1998, A&A, 335, 403
Glover, S. C. O., & Brand, P. W. J. L. 2000, MNRAS, submitted (astro-ph/0005576)
Gnedin, N. Y., & Ostriker, J. P. 1997, ApJ, 486, 581
Haiman, Z., Abel, T., & Rees, M. J. 2000, ApJ, 534, 11
Haiman, Z., & Loeb, A. 1999, in AIP Conf. Proc. 470, After the Dark Ages: When Galaxies Were Young (the Universe at $2 < z < 5$), ed. S. S. Holt & E. P. Smith (New York: AIP), 34
Haiman, Z., Rees, M. J., & Loeb, A. 1997, ApJ, 476, 458
Haiman, Z., Thoul, A. A., & Loeb, A. 1996, ApJ, 464, 523
Hirashita, T. 1969, Prog. Theor. Phys., 42(3), 523
Hutchings, J. B. 1976, ApJ, 205, 103
Lahav, O. 1986, MNRAS, 220, 259
Lepp, S., & Shull, J. 1983, ApJ, 270, 578
———. 1984, ApJ, 280, 465
Maoz, D., & Rix, H. W. 1993, ApJ, 416, 425
Matsuda, T., Sato, H., & Takeda, H. 1969, Prog. Theor. Phys., 42(2), 219
Nakamura, F., & Unemura, M. 1999, ApJ, 515, 239
Nishi, R., Susa, H., Uehara, H., Yamada, M., & Omukai, K. 1998, Mem. Soc. Astron. Italiana, 69, 395
Norman, M. L., Abel, T., & Bryan, G. 2000, in The First Stars, Proc. of MPA/ESO Workshop, ed. A. Weiss, J. Abel, & J. Hill (Heidelberg: Springer), 250
Norman, M. L., & Bryan, G. L. 1998, in Numerical Astrophysics 1998, ed. S. Miyama & K. Tomisaka (Dordrecht: Kluwer), 19
Oh, S. P. 1999, preprint (astro-ph/0005262)
Omukai, K., & Nishi, R. 1998, ApJ, 508, 141
———. 1999, ApJ, 518, 64
Orel, A. E. 1987, J. Chem. Phys., 87, 314
Ostriker, J. P., & Gnedin, N. Y. 1996, ApJ, 472, L63
Ostriker, J. P., & Steinhardt, P. J. 1995, Nature, 377, 600
Palla, F., Salpeter, E. E., & Stahler, S. W. 1983, ApJ, 271, 632
Peebles, P. J. E., & Dicke, R. H. 1968, ApJ, 154, 891
Pasachoff, W., & Zepf, D. 1967, Nature, 216, 976
Shapiro, P. R., & Kang, H. 1987, ApJ, 318, 32
Silk, J. 1983, MNRAS, 205, 705
Stecher, T. P., & Williams, D. A. 1967, ApJ, 149, L29
Stone, J. M., & Norman, M. L. 1992, ApJS, 80, 753
Tegmark, M., Silk, J., Rees, M. J., Blanchard, A., Abel, T., & Palla, F. 1997, ApJ, 474, 1
Tozzi, P., Madau, P., Meiksin, A., & Rees, M. J. 2000, ApJ, 528, 597
White, S. D. M., Efstathiou, G., & Frenk, C. S. 1993, MNRAS, 262, 1023

This work is supported in part by NSF grant AST-9803137 under the auspices of the Grand Challenge Cosmology Consortium. NASA also supported this work through Hubble Fellowship grant HF-0110401-98A from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555. The computations used the SGI Origin2000 at the National Center for Supercomputing Applications. We acknowledge useful discussions with Zoltan Haiman.