Increasing fretting resistance of flexible element pack for rotary machine flexible coupling
Part 3. The influence of dynamic loads on flexible coupling flexible element stress-strain state

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Abstract. It has been carried out the analysis of stress-strain state of flexible element pack for flexible coupling as a result of the combined action of the forces from the torque transmitted by the coupling and the forces caused by misalignment of the coupled shafts. Plots of bending moments from external load and reactive forces arising in the flexible elements of the pack have been constructed. There have been obtained the analytical dependences of the deflections and rotations of the sections on the complex load. The effects of various force factors on the stress-strain state of a flexible element have been compared. Directions of increasing the fretting resistance of the flexible elements are proposed.

1. Introduction

Part 1 of the paper analyzed the causes of failure of the rotor machine flexible couplings (FC), as a result of which analyses it had been found out that the term of the flexible coupling trouble-free operation was determined by the durability of the flexible element pack (FE). The main reason for the disability of the flexible elements is the wear and tear resulted from fretting corrosion, the features of which have also been investigated. Part 2 of this work is devoted to a detailed analysis of the stress-strain state of the FE jumper between two bolts and the influence of the shaft misalignment value thereon, which fact leads to the FE pack strain. It has been found out that the FE pack should be considered unbundled. This causes a minimal mutual influence of the FEs on each other in the pack. There were obtained the following analytical dependences of internal force factors arising in the FEs, the FE linear and angular strains, specific pressure of the FE in the pack, and friction forces and power losses to overcome those forces on the values of the strain caused by the coupled shafts misalignment. It has been concluded that the main factors affecting the strength and rigidity of the plate are:

1. The combined action of bending and torsion;
2. Longitudinally bending of the compressed plates.

As disclosed in the third part of the work, it is necessary to study the peculiarities of the FE strain caused by the circumferential force from the torque transmitted by the coupling, to investigate the bending process of the FE jumper in the plane perpendicular to the FE, to analyze longitudinal-
transverse bending of the jumpers as a result of the combined action of the torque forces and the forces caused by the misalignment of the shafts coupled.

2. Basic research material

The plate strain caused by the circumferential force resulted from the torque transmitted by the coupling.

We here perform a comparative assessment of the maximum stresses in the plate from the main force factors caused by the action of force $P_2$ (the design scheme (Figure 2) is given in part 2 of this work).

1. Maximum stresses due to torque:

$$\tau_{\max} = \frac{M_{xy}}{W_K} = \frac{P_2 A_4}{\alpha \delta h^2} \sin \frac{180^\circ}{n},$$

(1)

Where $W_K = \alpha \delta h^2$ is a sectional modulus of torsion for the rectangular cross section rod; $\alpha$ is a side ratio $h/\delta$ dependent coefficient

2. Compression along axis $x$:

$$\sigma = \frac{P_{2x}}{F} = \frac{P_2 \cos \frac{180^\circ}{n}}{\delta h},$$

(2)

3. Bending in the plane $xz$:

$$\sigma'_{\max} = \frac{M}{W_y} = \frac{P_{2z} A_4}{\delta h^2} \cos \frac{180^\circ}{n},$$

(3)

4. Bending in the plane $xy$:

$$\sigma''_{\max} = \frac{M}{W_z} = \frac{P_{2y} A_4}{h \delta^2} \sin \frac{180^\circ}{n},$$

(4)

Here, the force per one FE is meant as force $P_2$. We here find the real ratios of these stresses for the conditions of this example

$$\sigma = \frac{\alpha h}{\tan 30^\circ} = \frac{0.333 \cdot 22}{0.577} = 12.7\,,$$

(5)

Where $\alpha=0.333$ – corresponds to the ratios of $h/\delta=22/0.2=110$.

$$\sigma'_{\max} = \frac{6 \alpha h}{\delta \tan 30^\circ} = \frac{6 \cdot 0.333 \cdot 22}{0.2 \cdot 0.577} = 381;$$

(6)

$$\sigma''_{\max} = \frac{6 \alpha L}{A_4} = \frac{6 \cdot 0.333 \cdot 85}{0.1} = 1698\,.$$  

(7)

Thus, if we take $\tau_{\max}$ as a unit of stress comparison, the ratio of the compared values in the accepted sequence will be: $1 : 12.7 : 381 : 1698$.

From the point of view of the issue under our investigation, the third case, namely, bending a FE in the plane $xz$, where the ratio $\sigma'_{\max}/\tau_{\max} = 381$, is worthy of attention. The fourth case is of no interest since it is not accompanied by the deflection of the plate in the $z$-axis direction. The second case (axial compression of the plate) is associated with the loss of stability. It will be considered separately. The influence of the plate rotation can be neglected. Therefore, only two cases need to be considered:

1. Bending a flexible element (FE) in the plane $xz$;

2. Longitudinally bending.
2.1. Bending a flexible element (FE) in the plane xz
Calculation scheme is represented in Fig. 1. Static equilibrium equation:

\[ \sum X = H - P_{2x} = 0 \Rightarrow H = \bar{P}_{2x}; \]  
\[ \sum Y = V_A - V_B = 0 \Rightarrow V_A = V_B; \]  
\[ \sum m_B = \bar{P}_{2x} \Delta_A + V_A \ell - m_A - m_B = 0. \]

**Figure 1.** To the calculation of the jumper plate affected by the force of \( P_2 \) between two bolts.

The calculation scheme remains unchanged, but instead of the vertical force \( P_1 \), the horizontal force \( \bar{P}_{2x} \) acts in this case, which fact should affect the values of the reactive forces since the equation \( \sum m_B = 0 \) here includes the moment \( \bar{P}_{2x} \Delta_A \). This moment is an external load, which was absent in the previous two examples in terms of force. In this case, the external load also includes the force \( P_1 \), which entered the vertical reactions of the supports. It is this force that causes the displacements \( \Delta_A \) and \( \theta_A \) to appear. But the force \( P_2 \) is a real external force from the torque transmitted by the coupling.
This is an active external load. Therefore, in addition to the coefficients at unknowns \( X_1 = V_A \) and \( X_2 = m_A \), the free terms will also appear in the equations of strain compatibility:

\[
\Delta_{1\rho} = \frac{\ell^2}{2EI} \bar{P}_{2z} \Delta_A, \quad \Delta_{2\rho} = -\frac{\ell^2}{2EI} \bar{P}_{2z} \Delta_A.
\]  

These expressions are obtained by multiplying the \( M_P \) plot from the external load by individual plots \( \bar{M}_1 \) i \( \bar{M}_2 \) (Figure 1). The remaining coefficients: \( \delta_1 = \frac{\ell \theta_A}{3EI} \); \( \delta_2 = \frac{\ell}{EI} \); \( \delta_3 = \delta_4 = -\frac{\ell^2}{EI} \), and also \( X_1 = V_A \); \( X_2 = m_A \); \( \Delta_1 = \Delta_A \); \( \Delta_2 = -\theta_A \) - the minus sign here means that the directions of \( \Delta_2 \) and \( \theta_A \) are opposite. Then the canonical strain equations take the forms:

\[
\frac{1}{3} P_1 \ell^3 - \frac{1}{2} m_A \ell^2 + \frac{1}{2} \bar{P}_{2z} \Delta_A \ell^2 = EI \Delta_A;
\]

\[
-\frac{1}{2} P_2 \ell^2 + m_A \ell = -\bar{P}_{2z} \Delta_A \ell = EI \theta_A.
\]

The solution of these equations together with the equation of statics \( \Sigma M_A = 0 \) gives:

\[
V_A = V_B = \frac{6EI}{\ell^3} (2\Delta_A - \theta_A \ell); \quad m_A = \frac{2EI}{\ell^2} (3\Delta_A - 2\theta_A \ell) + \bar{P}_{2z} \Delta_A; \quad m_B = \frac{2EI}{\ell^2} (3\Delta_A - \theta_A \ell).
\]

As in the previous example, these results were obtained, from the strain compatibility equations:

\[
EI \theta_{x=l} = EI \theta_i = m_B \ell - V_B \ell^2; \quad EI \omega_{x=l} = EI \Delta_i = \frac{1}{2} m_B \ell^2 - \frac{1}{6} V_B \ell^3.
\]

Moving on to the calculations, we here find the numerical values for all reactions. Almost all the values are the same as in the previous example. The reason is the same boundary conditions: \( \Delta_A = 0.5 \text{ mm} \); \( \theta_A = 0.0025 \text{ rad} \).

This problem is statically indeterminate. The degree of static uncertainty is 2, so here; we also use the expressions for the known \( \Delta_A \) and \( \theta_A \) as the strain compatibility equations:

When \( Q_{(x)} = -V_B \); \( M_{(x)} = m_B - V_B x \), then \( EI \theta_{(x)} = m_B x - \frac{V_B x^2}{2} \), \( EI \omega_{(x)} = m_B x^2 - \frac{V_B x^3}{6} \).

For intersection \( A \):

\[
EI \theta_A = m_B \ell - V_B \ell^2; \quad EI \Delta_A = \frac{m_B \ell^2}{2} - \frac{V_B \ell^3}{6}.
\]

The solution of these equations together with the equations of statics results in the followings:

\[
V_A = V_B = \frac{6EI}{\ell^3} (2\Delta_A - \theta_A \ell);
\]

\[
V_B = V_A = \frac{6EI}{\ell^3} (2\Delta_A - \theta_A \ell).
\]
\[ m_b = \frac{2EI}{\ell^2} (3\Delta_A - \Theta_1 \ell); \]  
(22)

\[ m_A = \bar{P}_{2A} \Delta_A + \frac{2EI}{\ell^2} (3\Delta_A - 2\Theta_2 \ell). \]  
(23)

The force \( P_{2x} \) per one plate is equal to: \( \bar{P}_{2x} = \frac{P_2 \cos \frac{180}{2n} \cdot \ell}{n} = \frac{4790 \cdot 0.866}{2 \cdot 6.50} = 6.9 \, \text{N} \), where \( P_2 = \frac{400 \cdot 10^3}{83.5} = 4790 \, \text{N} \) is a circumferential force. Here, it is understood that the force of \( P_2 \) compresses each pack ingoing in the course of rotation and stretches each pack that goes away. More specifically, the packs experience, respectively, centrifugal compressing and eccentric stretching. The material of the plates is plastic steel, which equally works in tension and compression. Therefore, the force \( P_2 \) is equally divided between the compressed and the stretched packs.

On the conditions of this example we have:

\[ V_A = V_B = \frac{6 \cdot 2 \cdot 10^3 \cdot 0.0147}{83.5^3} (2 \cdot 0.5 - 0.0025 \cdot 83.5) = 0.024 \, \text{N}; \]  
(24)

\[ m_A = 6.9 \cdot 0.5 + \frac{2 \cdot 2 \cdot 10^3 \cdot 0.0147}{83.5^3} (3 \cdot 0.5 - 2 \cdot 0.0025 \cdot 83.5) = 4.365 \, \text{N} \cdot \text{mm}; \]  
(25)

\[ m_A = \frac{2 \cdot 2 \cdot 10^3 \cdot 0.0147}{83.5^3} (3 \cdot 0.5 - 0.0025 \cdot 83.5) = 1.089 \, \text{N} \cdot \text{mm}; \]  
(26)

\[ H = 6.9 \, \text{N}. \]  
(27)

In the intersection A, it is necessary to distinguish between the unknown reference moment \( X_2 = m_A = 4.365 \, \text{N} \cdot \text{mm} \) acting clockwise and the external moment acting counterclockwise. Thus, the total moment in the intersection A or the bending moment \( M_{\text{tot}} = -4.365 + 3.45 = -0.915 \, \text{N} \cdot \text{mm} \). This value can be obtained from the equation of bending moments. To control the correctness of constructing the plot of the bending moments, we here calculate the moments in the utmost intersections by adding (composing) the ordinates of the plot of the bending moments from the external load with the product of the extra unknowns \( X_1 = V_A \) and \( X_2 = m_A \) by the corresponding ordinates of the individual plots.

\[ M_{x=0} = V_A \ell - m_1 \ell + \bar{P}_{2A} \Delta_A = 0.024 \cdot 83.5 - 4.365 + 3.45 = 1.089 \, \text{N} \cdot \text{mm}; \]  
(28)

\[ M_{x=A} = -m_1 \ell + \bar{P}_{2A} \Delta_A = -4.365 + 3.45 = -0.915 \, \text{N} \cdot \text{mm}. \]  
(29)

Abscissa of the inflection point of the bent plate:

\[ \ell_1 = \frac{m_2}{V_B} = \frac{1.089}{0.024} = 45.4 \, \text{mm}. \]  
(30)

Displacement at a special point \((x=\ell_1)\):

\[ \theta_1 = \frac{45.4}{2 \cdot 10^3 \cdot 0.0147} \left( 1.089 - \frac{1}{2} \cdot 1.195 \cdot 45.4 \right) = 0.0084 \, \text{rad}; \]  
(31)

\[ \omega_1 = \frac{45.4^2}{2 \cdot 2 \cdot 10^3 \cdot 0.0147} \left( 1.089 - \frac{1}{3} \cdot 0.024 \cdot 45.4 \right) = 0.253 \, \text{mm}. \]  
(32)

The control calculations:
\[
\theta_{\text{v,1}} = \frac{83.5}{2 \cdot 10^3 \cdot 0.0147} \left(1.089 - \frac{1}{2} \cdot 0.024 \cdot 83.5 \right) = 0.00247 \text{ rad} ;
\]

(33)

\[
w_{\text{v,1}} = \frac{83.5^2}{2 \cdot 2 \cdot 10^3 \cdot 0.0147} \left(1.089 - \frac{1}{3} \cdot 0.024 \cdot 83.5 \right) = 0.499 \text{ mm} ;
\]

(34)

\[
\sum M_A = -V_B \ell + m_A + m_A + \bar{P}_2, \Delta_A = -\frac{6EI}{\ell^2} (2\Delta_A - \theta_0, \ell) \ell +
\]

\[
+ \frac{2EI}{\ell^2} (3\Delta_A - \theta_0, \ell) - \bar{P}_2, \Delta_A + \bar{P}_2, \Delta_A = 0.
\]

(35)

Figure 1 shows the plot of bending moments and the plots of displacements from the force \( P_2 \) for the plate between the two bolts.

In this example, the force \( P_{2x} \) effect on the plates is considered, and it is meant that the force \( P_1 \) has already strained the pack, and as a result of its action the plates have got the strains of \( \Delta_A \) and \( \theta_0, A \). Therefore, the problem of the combined action of the forces \( P_1 \) and \( P_2 \) has been solved. This synthesis showed that the force \( P_2 \) effect on the process of bending the plates is negligible as compared to the effect of the force \( P_1 \) due to the small bending moment from the force \( P_2 \). The eccentricity of the force \( P_2 \) that makes up its arm in the expression of the bending moment is very small. For this reason, the specific pressure between the plates, the work of the friction forces, and the work of the strain forces, in fact, will be the same as those from the force \( P_1 \).

2.2. Longitudinal-transverse bending of the plates

The calculation scheme is presented in Figure 1. We here use an approximate calculation method that gives fairly accurate results [1]. Assume that the deflected axis of the rod at longitudinal-transverse loading takes the form of a sinusoid:

\[
w_{(\text{v})} = f_I \sin \frac{\pi x}{\ell},
\]

(36)

Were \( w_{(\text{v})} \) is the full deflection consisting of the deflection \( w_{(\text{v})} \) arising from the action of a single transverse load and the additional deflection \( w_{(\text{aa})} \) resulted from the longitudinal force \( H \);

\[
f_I = \frac{f}{1-H/P_e} \text{ is maximum full deflection in the middle of clearance } (x=0.5\ell); f \text{ is the deflection from the transverse loading at } x=0.5\ell; P_e = \frac{\pi^2EI}{(\nu f)^2} \text{ is the Euler force that is numerically equal to the critical force at a specified method of fixing the ends of the plate, in our case, rigid fixing on the both ends; } 
\]

\[
\nu=0.5 \text{ is a length reduction coefficient for the compressed rod with a rigid termination of its ends.}
\]

Therefore, the equation of the total deflections in the longitudinal-transverse bending will be:

\[
w_{(\text{s})} = \frac{w_{(\text{v})}}{1-H/P_e}.
\]

(37)

In our example:

\[
w_{(\text{s})} = \frac{x^2}{2EI} \left(m_B - \frac{1}{3} V_B x \right); \\
H=6.9 \text{ N};
\]

(38)

\[
P_e = \frac{\pi^2 \cdot 2 \cdot 10^3 \cdot 0.0147}{(0.5 \cdot 83.5)^2} = 16.65 \text{ N};
\]

(40)
\[ w_{0.5/} = \left( \frac{0.5 \cdot 83.5}{2 \cdot 2 \cdot 10^3 \cdot 0.0147} \right)^2 \left( 1.089 - \frac{1}{3} \cdot 0.024 \cdot 0.5 \cdot 83.5 \right) = 0.224 \text{ mm} ; \quad (41) \]

\[ w_{0.25/} = \left( \frac{0.25 \cdot 83.5}{2 \cdot 2 \cdot 10^3 \cdot 0.0147} \right)^2 \left( 1.089 - \frac{1}{3} \cdot 0.024 \cdot 0.25 \cdot 83.5 \right) = 0.044 \text{ mm} ; \quad (42) \]

\[ w_{(x=0.5)} = \frac{0.224}{1 - \frac{6.9}{16.65}} = 0.383 \text{ mm} ; \quad (43) \]

\[ w_{(x=0.25)} = \frac{0.044}{1 - \frac{6.9}{16.65}} = 0.075 \text{ mm} . \quad (44) \]

Full deflection at the inflection point: \((x=\ell_1=45.4 \text{ mm}):\)

\[ w_{(x=45.4 \text{ mm})} = \frac{0.253}{1 - \frac{6.9}{16.65}} = 0.432 \text{ mm} . \quad (45) \]

Thus, the presence of the compressive longitudinal force \(H\) leads to the significant, namely about 1.9 times, increase in the deflection of the plates as compared to the deflection of the same only from the transverse load. In addition, in the absence of a longitudinal force, the plates bend, for example, performing a convex upward that is, towards the eccentricity of the force \(P_2c\). Due to longitudinally bending, the direction should change to the opposite, because at the ends of the plate, the force \(H\) compressors create a pair of forces the moment of which determines the direction of the deflections. Although in this case, the options are also possible. For example, it can be argued that the bending moments from the transverse load would give impulses to the deflection of the plate in its direction before longitudinally bending begins. In any case, this is not significant. It is important that the compression of the plates along their axes enhances their strain, and this is an additional factor affecting the wear of the plates.

3. Conclusion

The study of the stress-strain state of the flexible elements of the MCK type coupling was carried out, and the problem of their longitudinal - transverse bend was solved, as well as there was given a comparative estimation of their deflections due to the longitudinally transverse bending and also the force caused by the misalignment of the coupled shafts. The influence of the circumferential force from the torque transmitted by the flexible coupling is small in comparison with the influence of the force from the coupled shafts misalignment.

The overall conclusion is as follows: the main factors that condition the wear of the flexible elements due to friction are the bending forces caused by the inaccuracy of the shafts coupled with the help of the flexible coupling and also by the longitudinal-transverse bending from the circumferential force.

On the basis of the conducted research, the following recommendations can be formulated to improve the fretting resistance of the flexible elements:

- Reducing the length of the pack by optimally increasing the number of clamping bolts;
- Increasing the length of the spacer to reduce the angles of rotation at the edges of the flexible element packs and thereby bring down the bending effect;
- Reducing the angular and linear errors of the coupled shafts through improving the manufacture technology and the accuracy of assemblage.

References

[1] Feodosiev V 1971 Resistance des materiaux (Moscow: Mir Edition) p 572