Abstract

The amplitude analysis has been performed on the $\pi^0\pi^0$ final state obtained in the $\pi^-p$ charge exchange process. The $\pi^0\pi^0$ scattering amplitudes have been obtained for the S and D waves by the Chew-Low extrapolation and the partial wave analysis. Breit Wigner parameters have been obtained for $f_0(1370)$ and $f_2(1270)$. I=0 S wave $\pi^0\pi^0$ scattering phase shift has been obtained below $KK$ threshold. They agree well with the $\pi^+\pi^-$ standard phase shift data below 650GeV and deviate by about 10 degrees from the standard data above 650MeV. They show a different behavior from those of Cason and others. The $\pi^0\pi^0$ phase shift data have been analyzed by the IA method. Resonance parameters have been obtained to be $m_\sigma = 588 \pm 12\text{MeV}$ and $\Gamma_\sigma = 281 \pm 25\text{MeV}$ with $r_c = 2.76 \pm 0.15\text{GeV}^{-1}$. They are excellently in agreement with those obtained in the reanalysis on the $\pi^+\pi^-$phase shift data.

Introduction: The scalar meson $\sigma$, a chiral partner of pion as the Nambu-Goldstone Boson has long been expected[1] to be found. A mass of $\sigma$ is predicted to be twice of that of the constituent quark mass, $m_q$ in the NJL model, the low energy effective theory of QCD, i.e. $2m_q = 500 - 700\text{MeV}$. It was so pity that the existence of it had long been rejected in the analysis[2] of the $\pi\pi$ scattering phase shift data. The situation has been improved for its existence in recent studies[3,4] of re-analyses of the $\pi\pi$ phase shift data.[5] $\sigma$ has revived on the PDG table[6] after 20 years of vanishing. S.Ishida et al.[4] have, especially, shown the clear evidence of $\sigma$ in their reanalysis of the $\pi\pi$ phase shift data by the interference amplitude (IA) method with introduction of a negative background phase in the analysis. The introduction of the negative background is not a simply symptomatic treatment for the analysis, but has reasoning on the experimental fact that the S wave I=2 $\pi\pi$ scattering phase
shift[5] is negative and has also reasoning in the theoretical consideration[4,7] that the negative background phase has an origin in the compensating $\lambda\phi^4$ contact interaction term requested by the current algebra and PCAC.

It might be vital important to see $\sigma$ in the production process, as well. $\sigma$ has been shown[8] in the analysis of the $\pi^0\pi^0$ system produced in the $pp$ central collision process at 450 GeV/c performed by the NA12/2 experiment at CERN. The variant mass and width method[9] were used for the analysis. The similar analysis[10] has been performed on the $\pi\pi$ final state produced in the $J/\Psi$ decay into $\omega\pi^+\pi^-$ obtained by the DM2 collaboration[11] at DCI. The $\pi\pi$ mass distribution as well as angular distributions have been well reproduced in the analysis.

A sizable accumulation of $\pi\pi$ events produced in the central collision process has been treated[12], so far, to be of a background process according to the conclusion from the traditional analyses[2] of $\pi\pi$ phase shift data. Though the comments[13] were stated on the preliminary results[14] of the analysis of the $\pi^0\pi^0$ scalar state below 1GeV in the concluding remarks at Hadron’95, they lose now their physics ground. The comment[15] should also be retracted, which was given in the summary talk at Hadron’97 to the results of the reanalysis[4] on the $\pi^+\pi^-$ phase shift data, without notice of the essential points of the analysis. Discussions more in detail can be found in the reports at Hadron’97 and at this workshop[16].

It can be seen apparent deviations of the $\pi^0\pi^0$ phase shift data[17] from the $\pi\pi$ phase shift data[5](the standard $\pi\pi$ phase shift data, so called). Meanwhile, we performed an amplitude analysis on the $\pi^0\pi^0$ final state produced in the $\pi^-p$ charge exchange reaction, $\pi^-p \rightarrow \pi^0\pi^0n$ at KEK. $\pi^0\pi^0$ scattering phase shifts have been obtained and analyzed. We present here the results of our amplitude analysis on the $\pi^0\pi^0$ final state and of analysis of the scattering phase shifts with the IA method.

$\pi^0\pi^0$ scattering amplitudes in the $\pi^-p$ charge exchange process: An amplitude analysis has been performed on the $\pi^0\pi^0$ final state produced in the $\pi^-p$ charge exchange process, $\pi^-p \rightarrow \pi^0\pi^0n$ at 8.95 GeV/c. The data were collected by the Benkei spectrometer of the E135 experiment at 12 GeV proton synchrotron at KEK (Tsukuba). The Benkei spectrometer had a high resolution performance with wide geometrical acceptance for charged particles and gammas. The neutral events of four gammas with no charged particle have been used for the present analysis. The gammas produced in a liquid hydrogen target hit by the negative pion beam were detected by a total absorption hodoscope spectrometer consisted with active converters (AC) and main radiators (MR) of lead glass. Positions of gammas were determined by tracks recorded in wire chamber planes placed between AC and MR. Recoil neutrons were not detected. Reader can refer details of the system elsewhere[18]. Reconstructed $\pi^0$ signals were selected in the region, $|M_{2\gamma} - M_{\pi^0\pi^0}| \leq 40$MeV. The mass resolution for $\pi^0$ detection was 13MeV. The missing mass squared against $\pi^0\pi^0$ was selected in the region between 0.3 and 1.21 GeV$^2$. The effec-
Fig. 1. (a) Effective mass distribution of the $\pi^0\pi^0$ system in $\pi^-p \rightarrow \pi^0\pi^0n$. Data are corrected by the acceptance. (b) $t$ distribution of $\pi^-p \rightarrow \pi^0\pi^0n$.

tive mass distribution of the $\pi^0\pi^0$ system is shown in Fig.1a). A broad peak is seen below 1GeV with a clear peak around 1.3GeV. The $t$ distribution in Fig.1b) shows that the one pion exchange dominates in the process. We may write the cross section for the one pion exchange process, as follows,

$$\frac{d\sigma}{dm_{\pi\pi}d\cos \theta dt} \sim \frac{m_{\pi\pi}^2 4m_N^2 - t}{|p_1| (m_\pi^2 - t)^2} T_{\pi\pi}^2(m_{\pi\pi}^2, \cos \theta, t)$$  \hspace{1cm} (1)

where $|p_1| = \sqrt{m_{\pi\pi}^2/4 - m_\pi^2}$ is the pion momentum in the $\pi\pi$ center of mass system. $\theta$ is the scattering angle of the $\pi\pi$ system in the $\pi^+\pi^- \rightarrow \pi^0\pi^0$ scattering or the azimuth angle in the G-J frame. $T_{\pi\pi}$ is the off mass shell scattering amplitude of the pion.

The $\pi^0\pi^0$ system is in a state with even angular momentum and even isospin. The on mass shell scattering amplitude is written with S and D waves, as follows,

$$T_{\pi\pi}(m_{\pi\pi}^2, \cos \theta, m_\pi^2) = A_S + A_D \times \sqrt{5} \times \frac{3\cos^2 \theta - 1}{2}$$  \hspace{1cm} (2)

where $A_S$ and $A_D$ are the on mass shell amplitudes for S and D waves, respectively. A linear form is used for the extrapolation[19] of the off mass shell amplitude to the on mass shell one.

$$T_{\pi\pi}^2(m_{\pi\pi}^2, \cos \theta, t) = (1 + \alpha(m_\pi^2 - t))T_{\pi\pi}^2(m_{\pi\pi}^2, \cos \theta, m_\pi^2)$$  \hspace{1cm} (3)

where $\alpha$ is the extrapolation coefficient. Then, the cross section becomes

$$\sigma \sim \frac{m_{\pi\pi}^2 4m_N^2 - t}{|p_1| (m_\pi^2 - t)^2} \left(1 + \alpha(m_\pi^2 - t)\right)$$  \hspace{1cm} (4)
Fig. 2. Results of the partial wave analysis. (a) Intensity distribution of the S wave amplitude square. (b) Intensity distribution of the D wave amplitude square. (c) Extrapolation coefficient $\alpha$. (d) Relative phase $\delta$ between S and D wave amplitudes, $\delta = \phi_D - \phi_S$ below 1.0GeV and $\delta = \phi_S - \phi_D$ above 1GeV.

$$\times \left( A_S^2 + 5A_D^2 \left( \frac{3\cos^2 \theta - 1}{2} \right)^2 + 2\sqrt{5} |A_S| |A_D| \frac{3\cos^2 \theta - 1}{2} \cos \delta \right)$$

where $\delta$ is the relative phase between $A_S$ and $A_D$. The results of PWA are shown in Fig.2a)-d). A broad peak is seen below 1GeV in the S wave intensity distribution (Fig.2a)). A rapid fall around 0.9GeV may be due to the interference of the S wave with $f_0(980)$ which appears as a sharp variation of the phase motion in Fig.2d). A clear peak around 1.3GeV in Fig.2a) may correspond to $f_0(1370)$. No structure is seen around 1.5GeV. A clear peak of $f_2(1270)$ is seen in the D wave intensity distribution (Fig.2b)). A week but significant intensities are seen below 1GeV. It might be interesting to note that the similar events are recognized in the intensity distributions of the $\pi^+\pi^-$ and $\pi^0\pi^0$ final states produced in the $pp$ central collision production[20]. The Breit-Wigner amplitude parameters are obtained for the peaks around 1.3GeV in the S and D waves with the relativistic form. Parameters obtained for masses and widths of $f_0(1370)$ and $f_2(1270)$ are as follows,

$$M_{f_0(1370)} = 1278 \pm 5 \ \text{MeV}, \quad \Gamma_{f_0(1370)} = 197 \pm 8 \ \text{MeV},$$
$$M_{f_2(1270)} = 1286 \pm 7 \ \text{MeV}, \quad \Gamma_{f_2(1270)} = 161 \pm 14 \ \text{MeV}.$$
The $\pi^0\pi^0$ scattering amplitudes below $K\bar{K}$ threshold are expressed by the S wave I=0 and I=2 phase shifts, $\delta_0^S$ and $\delta_2^S$, respectively, as follows, $A_2^S \sim \sin^2(\delta_0^S - \delta_2^S)$. The $\delta_0^S - \delta_2^S$ distribution is deduced by normalizing the maximum value of $|A_S|^2$ to be 1. The experimental $\delta_2^S$ is shown to be well reproduced by the hard core type phase shift, $\delta_2^S = -r_c^2 |p_1|$, with core radius $r_c = 0.17\text{fm} = 0.87\text{GeV}^{-1}$.[4] By using this $\delta_2^S$ we can obtain $\delta_0^S$, shown in Fig.3. $\delta_0^S$ values are in good agreement with the $\pi^+\pi^-$ standard phase shift data below 650MeV. They appear somewhat higher by about 10 degrees than the standard phase shift above 650MeV. They are different as a whole from those of Cason and others[17], which have currently been used for phase shift analyses. Recently, Kaminski and others[21] have obtained $\pi^+\pi^-$ phase shifts in their reanalysis of the polarization data. Their up-flat solution shows apparent deviation from ours. The phase difference $\delta_0^S - \delta_2^S$ at the neutral Kaon mass is obtained in our analysis with the value 42.5$\pm$3$^\circ$, which is consistent with the prediction from the CP violation parameters in the K-decay,[22] 40.6$\pm$3$^\circ$.

**$\pi^0\pi^0$ phase shift analysis:** $\pi^0\pi^0$ phase shifts data are analyzed by the IA method[4], which describes the process with a few physically clear parameters. S matrix is written with phase shift, $\delta(s)$ and scattering amplitude, $a(s)$, as follows, $S = e^{2i\delta(s)} = 1 + 2ia(s)$. $\delta(s)$ is the sum of phase shifts come from resonances concerning, $\delta_R$'s and background phase shift, $\delta_{BG}$ below $K\bar{K}$ threshold. $f_0(980)$ and $\sigma$ are taken as resonances. Then, $\delta(s) = \delta_R + \delta_{BG} = \delta_{f_0} + \delta_{\sigma} + \delta_{BG}$. The relativistic Breit-Wigner is taken for $a(s)$, $a(s) = \sqrt{s}\Gamma_R(s)/(m_R^2 - s - i\sqrt{s}\Gamma_R(s))$. Total S matrix is written as, $S = S_RS_{BG} = S_{f_0}S_{\sigma}S_{BG}$. The unitarity condition for $S$ is satisfied by each $S$ matrix, automatically. The negative phases are taken for $\delta_{BG}$ which
are expressed by the hard core, \( \delta_B = -r_c|p_1| \). The solid line presented in Fig.3c) is the best fitted curve obtained below \( K\bar{K} \) threshold by fitting with parameters, resonant mass \( m_\sigma \), resonant width \( \Gamma_\sigma \) and hard core radius \( r_c \). The data are excellently reproduced by the curve. The \( \chi^2 \) is obtained with \( \chi^2/n_{n.o.f} = 20.4/12 \). Parameters obtained are,

\[
m_\sigma = 588 \pm 12\text{MeV}, \quad \Gamma_\sigma = 281 \pm 25\text{MeV} \text{ and } r_c = 2.76 \pm 0.15\text{GeV}^{-1}. \quad (6)
\]

These values are in good agreement with those which we have obtained in the reanalysis on the standard \( \pi^+\pi^- \) phase shift data. A dotted line in Fig.3c) shows also the result of fitting with \( r_c = 0 \) (i.e. without negative background), that is the same with the case of the traditional analysis. The \( \chi^2/n_{n.o.f} \) is obtained with 85.0/13, worse than our best fit. The parameter obtained with \( r_c = 0 \) are \( m_\sigma' = 890 \pm 16\text{MeV} \) and \( \Gamma_\sigma' = 618 \pm 51\text{MeV} \).

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