Spin-orbit correlations in the nucleon

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We investigate the correlations between the quark spin and orbital angular momentum inside the nucleon. Similarly to the Ji relation, we show that these correlations can be expressed in terms of specific moments of measurable parton distributions. This provides a whole new piece of information about the partonic structure of the nucleon.

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I. INTRODUCTION

One of the key questions in hadronic physics is to unravel the spin structure of the nucleon, a very interesting playground for understanding many non-perturbative aspects of quantum chromodynamics (QCD). So far, most of the efforts have focused on the proper decomposition of the nucleon spin into quark/gluon and spin/orbital angular momentum (OAM) contributions, see Ref. [1] for a detailed recent review, and their experimental extraction. The spin structure is however richer.

In this Letter we provide the relation between the quark spin-orbit correlation and measurable parton distributions. In section IV, we provide an estimate of the various contributions, and conclude the paper with section V.

II. QUARK SPIN-ORBIT CORRELATION

We define the local gauge-invariant light-front operator for the quark spin-orbit correlation as

\[ \hat{\mathcal{C}}^a_\gamma = \int d^3x \frac{1}{2} \bar{\psi} \gamma^\gamma \gamma_5 (x \times \hat{D})_z \psi, \]

where \( \hat{D} = \hat{\partial} - \hat{\partial} = 2i g A \) is the symmetric covariant derivative, \( a^\pm = \sqrt{2} (a^0 \pm a^3) \) for a generic four-vector \( a \) and \( d^3x = dx^- d^2x_1 \). It can be written in the following form

\[ \hat{\mathcal{C}}^a_\gamma = \int d^3x (x^1 T^{a^1}_{q5} - x^2 T^{a^2}_{q5}), \]

where \( T^{\mu\nu}_{q5} \) is the parity partner of the QCD quark energy-momentum tensor

\[ T^{\mu\nu}_{q5} = \frac{i}{2} \bar{\psi} \gamma^\mu \gamma_5 i \hat{D}^\nu \psi. \]

Just like in the case of the generic asymmetric energy-momentum tensor \([2, 3]\), we find that the non-forward matrix elements of \( T^{\mu\nu}_{q5} \) can be parametrized in terms of five form factors (FFs)

\[ \langle p', \Lambda' | T^{\mu\nu}_{q5} | p, \Lambda \rangle = \overline{\pi}(p', \Lambda') \Gamma^{\mu\nu}_{q5} u(p, \Lambda) \]

with

\[ \Gamma^{\mu\nu}_{q5} = \frac{p^{(\mu} q^{\nu)}}{2} \tilde{A}_q(t) + \frac{p^{(\mu} q^{\nu)} \Delta}{4M} \tilde{B}_q(t) + \frac{p^{(\mu} q^{\nu)} \gamma_5}{2} \tilde{C}_q(t) + \frac{p^{(\mu} q^{\nu)} \gamma_5 \gamma_5}{4M} \tilde{D}_q(t) + M \sigma^{\mu\nu} \gamma_5 \tilde{F}_q(t), \]

where \( M \) is the nucleon mass, \( p = \frac{p' + p}{2} \) is the average four-momentum and \( t = \Delta^2 \) is the square of the four-momentum transfer \( \Delta = p' - p \). For convenience, we used the notations \( a^{(\mu} b^{\nu)} = a^\mu b^\nu + a^\nu b^\mu \) and \( a^{(\mu} b^{\nu)} = a^\mu b^\nu - a^\nu b^\mu \). Using the light-front spinors and \( \epsilon \)-identities,

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we arrive at the following expansion up to linear order in $\Delta$

\[
\langle p', \Lambda' | T^{\mu \nu}_{q_5} | p, \Lambda \rangle = \left[ P^{(\mu} \mathcal{S}^{\nu)} - P^{(\mu \nu)} + \frac{\Delta \mu \nu}{2} \right] \tilde{A}_q
\]

\[
+ \frac{P^{(\mu} \mathcal{S}^{\nu)} - P^{\mu \nu}}{2} \tilde{B}_q + \mathcal{O}(\Delta^2)
\]  

with $\epsilon_{0123} = +1$ and $S^\mu$ the covariant spin vector satisfying $P^\mu S = 0$ and $S^2 = -M^2 = -P^2$ up to terms of order $\Delta^2$ or higher. For convenience, we removed the argument of the FFs when evaluated at $t = 0$, i.e. $\tilde{X}_q = \tilde{X}_q(0)$. Substituting this expansion into the matrix element of Eq. (2) and working in the symmetric light-front frame, i.e. with $P_\perp = 0$, we find

\[
C_q^\mu \equiv \frac{\langle p, \Lambda | \mathcal{P} \mathcal{A}^\mu | p, \Lambda \rangle}{\langle p, \Lambda | \mathcal{P} | p, \Lambda \rangle} = \frac{1}{2} (\tilde{A}_q + \tilde{C}_q). 
\]

Thus, to determine the quark spin-orbit correlation, one has to measure the $\tilde{A}_q(t)$ and $\tilde{C}_q(t)$ FFs, which are analogous to the axial-vector FF $G_A^q(t)$. The $\tilde{B}_q(t)$ and $\tilde{D}_q(t)$ FFs, which are induced by the pseudoscalar FF $G_0^q(t)$, are not needed since they contribute only to higher-$x$-momentums of $T^{\mu \nu}_{q_5}$, as one can see from the expansion $\Pi(p', \Lambda') \gamma_5 u(p, \Lambda) = \frac{\Delta \mu \nu}{2M} + \mathcal{O}(\Delta^2)$.

### III. LINK WITH PARTON DISTRIBUTIONS

Like in the case of the energy-momentum tensor, there is no fundamental probe that couples to $T^{\mu \nu}_{q_5}$ in particle physics. However, it is possible to relate the various FFs to specific moments of measurable parton distributions. From the component $T^{\mu \nu}_{q_5}$, we find

\[
\int dx \tilde{H}_q(x, \xi, t) = \tilde{A}_q(t),
\]

\[
\int dx \tilde{E}_q(x, \xi, t) = \tilde{B}_q(t),
\]

where $\tilde{H}_q(x, \xi, t)$ and $\tilde{E}_q(x, \xi, t)$ are the GPDs parametrizing the non-local twist-2 axial-vector light-front correlator $\Gamma_{qA}^+$

\[
\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} \langle p', \Lambda' | \mathcal{P} (z^-) \gamma^+ \gamma_5 \mathcal{W} (z^-) | p, \Lambda \rangle = \frac{1}{2\pi} \Pi(p', \Lambda') \Gamma_{qA}^+ u(p, \Lambda)
\]

with $\mathcal{W} = \mathcal{P} \exp[i g \int_{z^-/2}^{z^-} dy^- A^+(y^-)]$ a straight light-front Wilson line and

\[
\Gamma_{qA}^+ = \gamma^+ \gamma_5 \tilde{H}_q(x, \xi, t) + \frac{\Delta \mu \nu}{2M} \tilde{E}_q(x, \xi, t),
\]

the skewness variable being given by $\xi = -\Delta^+ / 2P^+$. The relations for the other FFs can be obtained using the following QCD identity

\[
\tilde{\mathcal{A}}^{\mu \nu} \gamma_5 i \mathcal{D}^{\nu} \psi = 2m \tilde{\mathcal{A}}^{\mu \nu} \gamma_5 \psi - \epsilon^{\mu \nu \alpha \beta} \partial_\alpha (\tilde{\mathcal{A}}_\beta \psi),
\]

where $m$ is the quark mass. Taking the matrix elements of both sides and using some Gordon and $\epsilon$-identities, we find

\[
\tilde{C}_q(t) = \frac{m}{2M} H_1^q(t) - F_1^q(t),
\]

\[
\tilde{D}_q(t) = \frac{m}{2M} H_2^q(t) - F_2^q(t),
\]

\[
\tilde{F}_q(t) = \frac{m}{2M} H_3^q(t) - \frac{1}{2} G_E^q(t).
\]

where the electric Sachs FF is given by $G_E^q(t) = F_1^q(t) + \frac{1}{4M^2} F_2^q(t)$. The FFs on the right-hand side parameterize the vector and tensor local correlators as follows

\[
\langle p', \Lambda' | \mathcal{P} \gamma^+ \gamma_5 \psi | p, \Lambda \rangle = \Pi(p', \Lambda') \Gamma_{qV}^+ u(p, \Lambda),
\]

\[
\langle p', \Lambda' | \mathcal{P} \sigma^{\mu \nu} \gamma_\gamma \gamma_5 \psi | p, \Lambda \rangle = \Pi(p', \Lambda') \Gamma_{qT}^+ u(p, \Lambda)
\]

with

\[
\Gamma_{qV}^+ = \gamma^+ F_1^q(t) + \frac{im\Delta^+}{2M} F_2^q(t),
\]

\[
\Gamma_{qT}^+ = \frac{\Delta \mu \nu}{4M^2} H_2^q(t) + \frac{im\Delta^+}{4M^2} H_3^q(t) + \frac{iM^2}{M^4} \gamma_5 H_3^q(t).
\]

Our tensor FFs are related to the ones used in Refs. [8, 9] in the following way

\[
A_{T10}^q(t) = -\frac{1}{2} \left[ H_1^q(t) + \frac{1}{4M^2} H_2^q(t) \right] + H_3^q(t),
\]

\[
B_{T10}^q(t) = \frac{1}{4} \left[ H_1^q(t) + H_2^q(t) \right] + H_3^q(t),
\]

\[
\tilde{A}_{T10}^q(t) = -\frac{1}{4} H_2^q(t).
\]

The quark spin-orbit correlation is therefore given by the simple expression

\[
C_q^\gamma = \frac{1}{2} \int dx x \tilde{H}_q(x, 0, 0) - \frac{1}{2} \left[ F_1^q(0) - \frac{m}{2M} H_1^q(0) \right]
\]

which is the analogue in the parity-odd sector of the Ji relation [5] for the quark OAM

\[
L_q^\gamma = \frac{1}{2} \int dx x [H_q(x, 0, 0) + E_q(x, 0, 0)] - \frac{1}{2} G_A^\gamma(0).
\]

From the components $T^{\mu \nu}_{q_5}$ with $j = 1, 2$ we also find the following relations

\[
\int dx x \tilde{G}_1^q(x, \xi, t) = -\frac{1}{2} \left[ \tilde{B}_q(t) + \tilde{D}_q(t) \right],
\]

\[
\int dx x \tilde{G}_2^q(x, \xi, t) = -\frac{1}{4} \left[ \tilde{A}_q(t) + \tilde{C}_q(t) \right] + \left( 1 - \xi^2 \right) \tilde{F}_q(t),
\]

\[
\int dx x \tilde{G}_3^q(x, \xi, t) = -\frac{1}{4} \tilde{F}_q(t),
\]

\[
\int dx x \tilde{G}_4^q(x, \xi, t) = -\frac{1}{2} \tilde{F}_q(t),
\]

using the parametrization of Ref. [10] for the twist-3 quark helicity GPDs. As a check, we inserted in Eqs. [22] the expressions for $\int dx x \tilde{G}_1^q(x, \xi, t)$ derived in Ref. [10] within the Wandzura-Wilczek approximation and consistently recovered Eqs. [13] in the
massless quark limit. The analogue of the Penttinen-Polyakov-Shuvaev-Strikan relation \[10\] \[11\]

\[ L_q^q = - \int dx x G_2(x, 0, 0) \]  

(29)

in the parity-odd sector is therefore

\[ C_q^q = - \int dx x [\tilde{G}_2^q(x, 0, 0) + 2\tilde{G}_4^q(x, 0, 0)]. \]  

(30)

For completeness, we note that the quark spin-orbit correlation can also be expressed in terms of GTMDs

\[ C_q^\perp = \int dx d^2k_\perp \frac{k_\perp^2}{M^2} G_{11}^q(x, 0, k_\perp^2, 0, 0). \]  

(31)

The only difference with the expression given in Ref. \[2\] is that here it is obtained using a direct straight Wilson line instead of staple-like one \[12\] \[14\].

**IV. DISCUSSION**

In the previous section, we have obtained three different expressions for the quark spin-orbit correlation in terms of parton distributions. From an experimental point of view, Eq. (23) is clearly the most useful one. By equating the right-hand side of Eq. (23) with the right-hand sides of Eqs. (30) and (31), we obtain two new sum rules among parton distributions.

In order to determine the quark spin-orbit correlation, we need to know three quantities. The first quantity is the Dirac FF evaluated at \( t = 0 \) which simply corresponds to the valence number, namely \( F_1^u(0) = 2 \) and \( F_1^d(0) = 1 \) in a proton, and therefore does not require any experimental input. The second quantity is the tensor FF \( H_1^q(0) \) which is not known so far. However, since it appears multiplied by the mass ratio \( m/4M \approx 0.25\% \) for \( u \) and \( d \) quarks in Eq. (23), we do not expect it to contribute significantly to \( C_q^q \). The last quantity is the second moment of the quark helicity distribution

\[ \int_{-1}^1 dx x \tilde{H}_q(x, 0, 0) = \int_{-1}^1 dx x [\Delta q(x) - \Delta \bar{q}(x)]. \]  

(32)

Contrary to the lowest moment \( \int_{-1}^1 dx [\Delta q(x) + \Delta \bar{q}(x)] \), this second moment cannot be extracted from deep inelastic scattering (DIS) polarized data without additional assumptions about the polarized sea quark densities. The separate quark and antiquark contributions can however be obtained \( e.g. \) in a combined fit to inclusive and semi-inclusive DIS. From theLeader-Sidorov-Stamenov (LSS) analysis of Ref. \[15\], we obtain

\[ \int_{-1}^1 dx x \tilde{H}_u(x, 0, 0) \sim 0.19, \]  

(33)

\[ \int_{-1}^1 dx x \tilde{H}_d(x, 0, 0) \sim -0.06, \]  

(34)

**TABLE I:** Comparison between the lowest two axial moments \( \Delta^{(1)q} = \int_{-1}^1 dx x^2 H_q(x, 0, 0) \) for \( q = u, d \) as predicted by the naive quark model (NQM), the light-front constituent quark model (LFCQM) and the light-front chiral quark-soliton model (LF\( \chi \)QSM) at the scale \( \mu^2 \approx 0.26 \) GeV\(^2\) \[14\], with the corresponding values obtained from the LSS fit to experimental data \[13\] at the scale \( \mu^2 = 1 \) GeV\(^2\). Note that these estimates are however not particularly reliable since it follows from the new HERMES and COMPASS data on multiplicities that the fragmentation functions given in Ref. \[17\] and used in the LSS analysis are presumably not correct \[18\]. Further experimental data and dedicated analyses are therefore required.

| Model        | \( \Delta^{(0)u} \) | \( \Delta^{(0)d} \) | \( \Delta^{(1)u} \) | \( \Delta^{(1)d} \) |
|--------------|---------------------|---------------------|---------------------|---------------------|
| NQM          | \( 4/3 \)           | \(-1/3\)            | \( 4/9 \)           | \(-1/9\)            |
| LFCQM        | 0.995               | \(-0.249\)          | 0.345               | \(-0.086\)          |
| LF\( \chi \)QSM | 1.148              | \(-0.287\)          | 0.392               | \(-0.098\)          |
| Exp.         | 0.82               | \(-0.45\)           | \sim 0.19           | \sim 0.06           |

The second moment of the quark helicity distribution being a valence-like quantity with suppressed low-\( x \) region, we may expect phenomenological quark model predictions to be more reliable for this second moment than for the lowest one. In Table I we provide the first two moments of the \( u \) and \( d \) quark helicity distributions obtained within the naive quark model (NQM), the light-front constituent quark model (LFCQM) and the light-front chiral quark-soliton model (LF\( \chi \)QSM) at the scale \( \mu^2 \approx 0.26 \) GeV\(^2\), see Ref. \[16\]. From these estimates, we may safely expect a negative quark spin-orbit \( C_q^q \) for both \( u \) and \( d \) quarks, meaning that the quark spin and kinetic OAM are, in average, anti-correlated. This has to be contrasted with the model results obtained in Ref. \[2\] where the quark spin and canonical OAM are, in average, correlated.

As a final comment, let us mention that we could also have defined a quark correlation using only the symmetric part of \( T^{\mu\nu}_{4q} \), just like what Ji did with the energy-momentum tensor for the total angular momentum \( \vec{3} \). In this case, the tensor

\[ M^{\mu\nu}_{q5} = x^\nu \frac{1}{2} T^{(\mu\nu}_{5_{q}} \left] x^\sigma \frac{1}{2} T^{(\mu\nu}_{5_{q}}, \]  

(35)

is conserved and the charge

\[ \frac{\vec{C}_q^q}{\Lambda} = \int dx^3 x M^{4_{q5}} \]  

(36)

is time-independent. We then simply have

\[ C_q^q \equiv \frac{(p_{5_{q5}})(p_{5_{q5}})}{(p_{5_{q5}})(p_{5_{q5}})} = \frac{1}{2} \Lambda q(0), \]  

(37)

\[ = \frac{1}{2} \int_{-1}^1 dx x \tilde{H}_q(x, 0, 0). \]  

(38)
Note that, contrary to the case of the energy-momentum tensor, the charges \( \int d^3x \, T_{q5}^{+\nu} \) and \( \int d^3x \, \frac{1}{2} T_{q5}^{+(+\nu)} \) are different. This comes from Eq. (12) which shows that the antisymmetric part contributes to the charge when \( m \neq 0 \).

V. CONCLUSIONS

We provided a local gauge-invariant definition of the quark spin-orbit correlation, which is a new independent piece of information about the nucleon spin structure. We derived several expressions for the expectation value of this correlation in terms of measurable parton distributions, leading to new sum rules. Using estimates from fits to available experimental data and phenomenological quark models, we concluded that the quark spin-orbit correlation is very likely negative, meaning that the quark spin and kinetic orbital angular momentum are, in average, anti-correlated. However, a more precise determination of this quark spin-orbit correlation requires further experimental data and dedicated analyses in order to disentangle quark and antiquark contributions to the helicity distribution. Tensor form factors are also in principle needed, but the corresponding contribution can safely be neglected for light quarks. We also believe that Lattice QCD is able to make significant contributions in this matter.

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