The Application of FAHP in Decisions of Pavement Maintenance

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Abstract. In this paper, a method of building the fuzzy complementary judgement matrix and checking consistency is introduced based on the knowledge of the basic theory of FAHP and the procedure to establish the mathematical model corresponded. The scope and the advantages in the problems of multi-objective decisions have also been discussed. The availability of its use in the management system in pavement maintenance is demonstrated by analyzing the optimization for maintenance. Meanwhile, the faulty is also pointed out.

1. Introduction
With the rapid growth of the high-grade highway mileages and the traffic volume as well as the increasing severity of the overload problem of transport vehicles, damages to the different extent have occurred on the pavement in succession. At the moment, the highway management department is supposed not only to keep the partially damaged roads, but also to take economic and reasonable maintenance strategies against the trend of reducing the pavement performance. In the process of rapid development of highway transportations, elements that influence the increasing amount of collection volume and calculation volume of the basic data for the pavement maintenance management decisions are gradually increasing and full of fuzziness and uncertainty. To solve this type of problem of multi-objective decisions that man determines the pros and cons of the decision schemes according to various objectives and criteria, analytic hierarchy process is generally applied, which is short of considerations of the fuzziness of the subjective judgment. People’s thinking, in which decisions are made, is different from the consistency of the judgment matrix, so it needs to be comprehensively evaluated. Therefore, some academics put forward a method called FAHP, which combines the fuzzy mathematical theories with the analytic hierarchy process and expands the applications to the fuzzy environments which possess many evaluation indexes[1-3].

2. The calculation model of FAHP

2.1. The establishment of fuzzy complementary matrix
When every two elements compare with each other in the fuzzy hierarchy analysis, their respective importance in the system can be quantitatively presented as the fuzzy judgment matrix \( A = \left( a_{ij} \right)_{n \times n} \). If the judgment matrix possesses the following two characteristics, it can be called the fuzzy complementary judgment matrix. 1) \( a_{ij} = 0.5, i=1,2,\ldots,n \); 2) \( a_{ij} + a_{ji} = 1, j=1,2,\ldots,n \).

To describe any two schemes’ relative importance to a criterion, it’s very usual to use the two scale methods in Table 1 or Table 2 to mark it quantitatively[4].
Table 1. Method of nine marks and its significance

| Scale Value $a_i$ | Definition | Explanation |
|-------------------|------------|-------------|
| 1                 | The same important | Two elements are the same important when comparing with each other. |
| 3                 | A bit more important | One element is a bit more important than the other one when comparing with each other. |
| 5                 | Obviously more important | One element is obviously more important than the other one when comparing with each other. |
| 7                 | Much more important | One element is much more important than the other one when comparing with each other. |
| 9                 | Extremely important | One element is extremely more important than the other when comparing with each other. |
| 2, 4, 6, 8        | Median     | The median of the neighboring comparison above. |
|                   | Reciprocal | If the result of the comparison between element $a_{ij}$ and element $a_{ji}$ is judgment $r_{ij}$, the result of the comparison between element $a_i$ and element $a_j$ is judgment $r_{ji}=1/r_{ij}$. |

Table 2. 0.1–0.9 Scale method and its significance

| Scale Value $a_i$ | Definition | Explanation |
|-------------------|------------|-------------|
| 0.5               | The same important | Two elements are the same important when comparing with each other. |
| 0.6               | A bit more important | One element is a bit more important than the other one when comparing with each other. |
| 0.7               | Obviously more important | One element is obviously more important than the other one when comparing with each other. |
| 0.8               | Much more important | One element is much more important than the other one when comparing with each other. |
| 0.9               | Extremely important | One element is extremely more important than the other when comparing with each other. |
| 0.1, 0.2, 0.3, 0.4 | The converse comparison | If the result of the comparison between element $a_i$ and element $a_j$ is judgment $r_{ij}$, the result of the comparison between element $eta_j$ and element $a_i$ is judgment $r_{ji}=1/r_{ij}$. |

The scale value $a_i=0.5$ suggests that two elements share the same importance; while the value $a_i \in [0.1,0.5)$ suggests that element $X_j$ is more important than element $X_i$; And the value $a_i \in [0.5,0.9)$ means that element $X_i$ is more important than element $X_j$.

According to the numerical scales mark above, elements $a_1$, $a_2$, ..., $a_n$ compare with each other respectively. Then the result is just like the following fuzzy complementary judgment matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad (1)$$

2.2. Determination of the weight of each hierarchy evaluation index

The main task of the mutual sorting of hierarchies is to define the extent, to which the two elements of hierarchies up and down influence each other. It’s usually represented as an index value in the mathematical model. Below are the calculating steps of the index weight\(^5\).

1) The product $M_i$ of each row is calculated in the judgment matrix: $M_i = \prod_{j=1}^{n} a_{ij}$

2) The nth root $M_i$ of is calculated: $W_i = \sqrt[n]{M_i}$

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3) The vector is regularized, \( W_i = \frac{w_i'}{\sum_{i=1}^{n} w_i} \) which can make it satisfy the requirement \( \sum_{i=1}^{n} W_i' = 1 \), then \( W = [W_1, W_2, W_3, W_4, W_5] \).

4) The greatest characteristic root \( \lambda_{\text{max}} \) of the judgment matrix is calculated: \( \lambda_{\text{max}} = \sum_{i=1}^{n} \frac{(AW)_i}{nW_i} \).

2.3. The testing method of consistency of fuzzy complementary judgment matrix
Whether the weight value from the calculation above is reasonable and whether it can be used to evaluate the rationality of the judgment matrix established and handled by the decision makers should be further compared and judged to examine the consistency. If the judgment matrix obtained is a consistency matrix, it suggests that the qualitative judgments made by the decision makers logically meet the requirement of transitivity. Although in these decisions there are only comparisons between any two of them, the qualitative judgments are consistent and acceptable. To the contrary, they should be corrected.

Therefore, the evaluation indexes \( C.I \) and \( C.R \) also need to be calculated. It is regulated, when \( C.R < 0.10 \), that the fuzzy complementary judgment matrix \( A \) is defined to meet the requirement of consistency and acceptance. The smaller the \( C.R \) is, the greater the consistency is. When the value is zero, it means the complete consistency of the judgment matrix\(^4\). \( C.I = \frac{\lambda_{\text{max}}-n}{(n-1)} \), \( C.R = C.I/R.I \), \( R.I \) is the correction value shown in Table 3.

| Order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|-------|---|---|---|---|---|---|---|---|---|----|----|
| \( R.I \) | 0 | 0 | 0.54 | 0.9 | 1.12 | 1.26 | 1.36 | 1.41 | 1.46 | 1.49 | 1.52 |

2.4. The establishment of fuzzy comprehensive evaluation index matrix
Members in an experts group figure out the proportion of the risk elements in each hierarchy with the analytic hierarchy process and analyze it connecting with the practical situations. Besides, they also evaluate the condition of each risk element in the element level, so they get a judgment matrix \( U \). Then the general formula in the reference\(^6\), which is used to obtain the weights of fuzzy complementary judgment matrix, is shown below:

\[
U_i = \frac{\sum_{j=1}^{n} a_{ij} \cdot n}{n(n-1)} (i = 1, 2, ..., n)
\]  

(2)

This formula fully includes the advantages and the judging information of the fuzzy consistency judgment matrix. Its amount of calculation is small, which is convenient for the computer programming. Besides, it’s very easy to be applied to the practical project.

2.5. The fuzzy comprehensive evaluation results
Firstly the evaluation index weight value \( u_i \) of each hierarchy is worked out, and then the evaluation index matrix \( U \) of each hierarchy is further worked out, which is the fuzzy comprehensive evaluation set. At last, the fuzzy comprehensive evaluation results through the formula \( R = \nabla \ast U \ast \nabla^T \) are figured out.

3. The analysis of the application example
The four alternative schemes mentioned in the pavement maintenance management decisions are regarded as the research objects, which will be optimized and sorted. At the same time, the fuzzy analytic hierarchy process is used to evaluate them comprehensively. The main elements influencing the schemes decision are classified into four categories, which are the pavement performance evaluation, the traffic volume, the road age and the status and function of the road sections in the road network. They are considered as the primary element level.

In general, the top level of the decision-making hierarchy structure is the optimized objective, which is the objective level; all the alternative schemes are in the bottom level, which is the decision level; in the middle are all the elements influencing the decisions, which is the criterion level (it can be multilevel). In this paper, the research objects’ corresponding hierarchy structure of program analysis is shown in Table 4.
Table 4. Hierarchical chart of program analysis

| Objective Level | Schemes Optimization and Sorting U of Highway Maintenance Management |
|-----------------|-------------------------------------------------|
| Criterion level | Evaluation of the pavement performance U₁     |
| Decision level  | Maintenance scheme 1                           |
|                 | Traffic volume U₂                               |
|                 | Road age U₃                                     |
|                 | Road function U₄                                |
|                 | Maintenance scheme 2                            |
|                 | Maintenance scheme 3                            |
|                 | Maintenance scheme 4                            |

1) The judgment matrix A is determined through scoring after investigating 10 experts with questionnaire survey. According to the qualitative and quantitative empirical analysis of experts, the relative value of the importance of two elements in the same element level is figured out with the method of nine marks, and the weights matrix is obtained, which is shown in Table 5.

Table 5. Fuzzy complementary judgment matrix of index weight U-U₁

| U₁  | U₁₁ | U₁₂ | U₁₃ | U₁₄ | Wᵢ  | λₘₓ  | C,R  |
|-----|-----|-----|-----|-----|------|------|------|
| U₁  | 1   | 3   | 1   | 1/3 | 0.201|      |      |
| U₂  | 1/3 | 1   | 1/3 | 1/5 | 0.077| 4.044| 0.0161<0.1 |
| U₃  | 1   | 3   | 1   | 1/3 | 0.201|      |      |
| U₄  | 3   | 5   | 3   | 1   | 0.521|      |      |

2) The fuzzy judgment matrix Uᵢ of the four alternative maintenance schemes is established respectively and successively according to the order - evaluation of pavement performance, traffic volume, road age, the status and function of the road sections in the road network, which is shown from Table 6 to Table 9. And it’s scored with the 0.1~0.9 scale method[4].

Table 6. Fuzzy Complementary Judgment Matrix of Pavement Performance index U₁

| U₁  | U₁₁ | U₁₂ | U₁₃ | U₁₄ | Wᵢ  | λₘₓ  | C,R  |
|-----|-----|-----|-----|-----|------|------|------|
| U₁₁ | 0.5 | 0.4 | 0.4 | 0.6 | 0.242|      |      |
| U₁₂ | 0.6 | 0.5 | 0.5 | 0.7 | 0.275|      |      |
| U₁₃ | 0.6 | 0.5 | 0.5 | 0.7 | 0.275|      |      |
| U₁₄ | 0.4 | 0.3 | 0.3 | 0.5 | 0.208|      |      |

Table 7. Fuzzy Complementary Judgment Matrix of Traffic Volume Index U₂

| U₂  | U₂₁ | U₂₂ | U₂₃ | U₂₄ | Wᵢ  | λₘₓ  | C,R  |
|-----|-----|-----|-----|-----|------|------|------|
| U₂₁ | 0.5 | 0.3 | 0.7 | 0.6 | 0.258|      |      |
| U₂₂ | 0.7 | 0.5 | 0.9 | 0.8 | 0.325|      |      |
| U₂₃ | 0.3 | 0.1 | 0.5 | 0.4 | 0.192|      |      |
| U₂₄ | 0.4 | 0.2 | 0.6 | 0.5 | 0.225|      |      |

Table 8. Fuzzy Complementary Judgment Matrix of Road Age Index U₃

| U₃  | U₃₁ | U₃₂ | U₃₃ | U₃₄ | Wᵢ  | λₘₓ  | C,R  |
|-----|-----|-----|-----|-----|------|------|------|
| U₃₁ | 0.5 | 0.6 | 0.5 | 0.8 | 0.283|      |      |
| U₃₂ | 0.4 | 0.5 | 0.4 | 0.7 | 0.250|      |      |
| U₃₃ | 0.5 | 0.6 | 0.5 | 0.8 | 0.283|      |      |
| U₃₄ | 0.2 | 0.3 | 0.2 | 0.5 | 0.208|      |      |

Table 9. Fuzzy Complementary Judgment Matrix of Road Function Index U₄

| U₄  | U₄₁ | U₄₂ | U₄₃ | U₄₄ | Wᵢ  | λₘₓ  | C,R  |
|-----|-----|-----|-----|-----|------|------|------|
| U₄₁ | 0.5 | 0.7 | 0.6 | 0.4 | 0.267|      |      |
| U₄₂ | 0.3 | 0.5 | 0.4 | 0.2 | 0.200|      |      |
| U₄₃ | 0.4 | 0.6 | 0.5 | 0.3 | 0.233|      |      |
| U₄₄ | 0.6 | 0.8 | 0.7 | 0.5 | 0.300|      |      |

According to the four sorting vectors of maintenance schemes which are from every single element above, in the criterion level, the evaluation index weight U of schemes optimization sorting criteria of
the highway pavement maintenance management can be:

\[
U = \begin{bmatrix}
0.242 & 0.275 & 0.275 & 0.208 \\
0.285 & 0.325 & 0.192 & 0.225 \\
0.283 & 0.250 & 0.283 & 0.208 \\
0.267 & 0.200 & 0.233 & 0.300
\end{bmatrix}
\]  \quad (3)

According the evaluation results of criterion level, the fuzzy comprehensive evaluation results of the objective level can be obtained from the formula \( R = U \times W \):

\[
R = \begin{bmatrix}
0.242 & 0.275 & 0.275 & 0.208 \\
0.285 & 0.325 & 0.192 & 0.225 \\
0.283 & 0.250 & 0.283 & 0.208 \\
0.267 & 0.200 & 0.233 & 0.300
\end{bmatrix} \begin{bmatrix}
0.201 & 0.077 & 0.201 & 0.521
\end{bmatrix} = \begin{bmatrix}
0.2335 & 0.2327 & 0.2414 & 0.2722
\end{bmatrix}
\]

\[ R = U \times W \]  \quad (4)

According to the maximum subordination principle\(^{[7-8]}\), the optimization sorting of each alternative scheme is just like this: 0.2722(Scheme 4)>0.2414(Scheme 3)>0.2335(Scheme 1)>0.2327(Scheme 2). Therefore, Scheme 4 is the optimal choice.

4. Conclusion

This paper introduces the mathematical model of FAHP, analyzes its calculation theory and applies this method to the decisions optimization of pavement maintenance management to sort the schemes optimization. And it makes full use of the information in the alternatives, combining the qualitative analysis and quantitative analysis organically to get the best scheme. So it is proved that this method can be generally used in the evaluation of some complicated systems and in the multi-objectives decisions of schemes optimization.

While FAHP has solved this problem- the existing difference between the consistency of matrix and people's thinking, it's still not perfect enough and needs to be improved. This can be specifically divided into the following three aspects:

- If the index weight of various schemes is determined only according to the consistency of the fuzzy complementary judgment matrix when people make analysis and decisions with this method, the result can be less persuasive;
- Because of the quantitative limit of experts taking part in the evaluation and the differences in the individual subjectivity, there are limitations in the data collections to a certain extent, which may also influence the final evaluation results;
- With the increasing hierarchies of the structural layer, the calculation can be more and more complicated. Then the programming software will be needed to solve the problems.

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