Is the quantum state (an) observable?\textsuperscript{1}

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Abstract

We explore the sense in which the state of a physical system may or may not be regarded (an) observable in quantum mechanics. Simple and general arguments from various lines of approach are reviewed which demonstrate the following no-go claims: (1) the structure of quantum mechanics precludes the determination of the state of a single system by means of measurements performed on that system only; (2) there is no way of using entangled two-particle states to transmit superluminal signals. Employing the representation of observables as general positive operator valued measures, our analysis allows one to indicate whether optimal separation of different states is achieved by means of sharp or unsharp observables.

1. Introduction.

Quantum mechanics is often claimed to be a theory about ensembles only rather than about single systems. Yet there is an increasing variety of experiments exhibiting individual quantum processes which were conceived, devised and explained on the basis of this very theory. Therefore, in order to reach a proper appreciation of the scope of quantum mechanics, it is necessary to spell out the senses in which the theory does or does not apply to individual systems. In this contribution we begin with highlighting the individual aspects of quantum mechanics (Section 2) and proceed then to show why, in particular, the determination of the state of an individual system ought to be impossible for various reasons of consistency (Section 3). Rigorous arguments demonstrating this impossibility are then reviewed, using the general representation of observables as positive operator valued measures (Section 4). Finally we address briefly the question as to how well two non-orthogonal states can be discriminated (Section 5).

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2. Individual aspects of quantum mechanics.
Quantum mechanics is commonly formulated in terms of the basic duality of states and observables. Given a pair of a state operator \( \rho \) and a self-adjoint operator \( A \) representing a physical quantity, the number \( \text{Trace}[\rho A] \) is interpreted as the expectation value of the quantity if measured on an ensemble of systems all prepared in the state described by \( \rho \). Using the spectral decomposition of \( A \), this minimal statistical interpretation extends to the probability distribution \( p^A_\rho \) determined by \( \rho \) and \( A \), of which the expectation value is the first moment.

In this way the empirical meaning of certain quantum mechanical terms is fixed by making reference to the probability structure carried by the theory. But this very structure is based on the lattice of subspaces, or orthogonal projections, of the underlying Hilbert space. This observation opens up the possibility to introduce relations between states and observables which are non-probabilistic in the first instance (though equivalent, probabilistic formulations do exist): a given state \( \rho \) may be an eigenstate of a projection \( P \), meaning that one of the following equations holds:

\[
P\rho = \rho, \quad \text{or} \quad P\rho = O. \tag{1}
\]

In such a case a system prepared in the state \( \rho \) can be said to possess the \emph{real property} represented either by \( P \) or its complement \( I - P \). So \( P \) is \emph{real} in \( \rho \) if \( P\rho = \rho \), \emph{absent} in \( \rho \) if \( P\rho = O \), and \emph{indeterminate} in \( \rho \) if it is neither real nor absent. The quantum state is thus found to have the undoubtedly individual aspect of defining a valuation on the set of potential properties represented by the projections. We spell this out with reference to a pure state, represented by a unit vector \( \psi \) of the Hilbert space, and discrete observables.

Every observable \( A \) of which \( \psi \) is an eigenvector has a definite value, and the outcome of a measurement of such an observable \( A \) can be predicted with certainty to yield the corresponding eigenvalue as an outcome. It is a distinctive feature of quantum mechanics that for any state there are observables of which that state is no eigenvector. This corresponds to the situation where \( \psi \) is a (proper) superposition of eigenstates of an observable \( B \), in which case one can still (or only) assert that that observable is \emph{indeterminate} in the state \( \psi \), that is, that \( B \) does not have a definite value. Considering an ideal, repeatable
measurement of \( B = \sum b_k |\psi_k \rangle \langle \psi_k| \), then each single system of a collection of equally prepared systems can be predicted to jump into one of the eigenstates \( \psi_k \), with the probability \( p^B_{\psi}(b_k) = |\langle \psi_k | \psi \rangle|^2 \). The value of the observable \( B \) thus becomes determined dynamically for each system in the course of such a measurement and can thus be ascertained to be a real property of the system in its state after the measurement.

Quantum mechanics does also allow for the preparation of an individual system in a state \( \psi \): one may simply perform a repeatable measurement of an observable of which \( \psi \) is an eigenstate (associated with a nondegenerate eigenvalue). As long as there is a nonzero probability for the corresponding eigenvalue, there will be a chance to obtain the desired state. Hence a repeatable measurement can be used as a filter for preparing individual systems in known states.

So far we have elucidated the possibilities of obtaining information about properties or state descriptions pertaining to a single system after a measurement. The complementary question is whether, or to what extent, quantum mechanics allows one to infer, solely on the basis of information obtained from a measurement or any sequence of manipulations carried out on a single system, in which state that system was prior to the measurement. This question of individual state determination has been raised, in particular, in the context of discussions about proposals of employing entangled quantum states of spatially separated systems to effect superluminal signaling [1]. For a long time arguments against such a violation of Einstein causality were formulated on the level of more or less realistic concrete models of signaling schemes, including considerations of amplification or photon cloning processes [2-9]. More recently the issue was touched upon again in quantum information theory when it was realized that quantum correlations can be used for the safe teleportation of cryptographic keys [10]; yet another context where the question of individual state determination arises naturally is the quantum thermodynamics of black holes [11]. Meanwhile there do exist abstract and thus completely general proofs of the impossibility of individual state determination and some systematic investigations on the problem of optimizing the state inference [11,12-15]. However, these investigations seem to have remained largely unnoticed, and the production of new schemes for individual state determinations, while apriori doomed to fail, does not seem to come to an end [16]. Therefore it may be
justified to collect those simple and general arguments and show how quantum mechanics
manages to preclude individual state determination and to protect itself against the other-
wise desastrous implications. The presentation will be self-contained and occasionally uses
methods that are different from those applied in the literature. We proceed with outlining
first why individual state determination should be regarded undesirable.

3. Undesirable consequences of individual state determination.

Imagine that by some ingenious ISD-procedure it were possible to determine the state of
a quantum system, represented by a density operator $T$. It follows, first of all, that one
were able to decide whether a given quantum system is in a pure or a mixed state. (Of
course, it would also be possible to distinguish between arbitrary pairs of pure states; we
return to this case below.) This means that a single measurement on the given system
alone will suffice to find out whether or not it is entangled with some environment systems.
Consequently, one would have to realize that the usual quantum mechanics description of
states as density operators is incomplete: the same density operator $T$ would represent
two physically distinguishable situations: (1) an ensemble of systems in pure states, the
distribution of which is described by $T$; or (2) an ensemble of systems, each entangled with
another system, so that in each single case $T$ provides the exhaustive description of the
reduced state.

Due to this feature, the ISD-procedure can be used to effect superluminal signal trans-
mission. In fact, consider an Einstein-Podolsky-Rosen correlated system consisting of two
spatially separated spin-$\frac{1}{2}$ particles $A, B$ in a singlet state,

$$\Psi = \frac{1}{\sqrt{2}} (|+,z\rangle - |-,z\rangle + |-,z\rangle - |+,z\rangle). \quad (2)$$

(Here $|\pm, z\rangle$ denotes the eigenvectors of the spin-$\frac{1}{2}$ operator $s_z$.) A signal would consist
of one observer’s, Bob’s, measuring or not measuring some spin component $s_n$ on particle
$B$, while the second observer, Abner, would apply the ISD-procedure on particle $A$ to find
out whether Bob has measured or not: in the first case Abner would find $A$ in one of the
pure states $|\pm, n\rangle$, while in the latter case he would find $A$ in the mixed reduced state
$\frac{1}{2}(|+,z\rangle\langle+\rangle + |-,z\rangle\langle-\rangle) = \frac{1}{2}I$. Hence the ISD-procedure would allow an instantaneous
detection of which bit of the message was sent from the spacelike distant particle.
We note that the alphabet can be easily extended: instead of measuring or not, Bob may choose to measure one of a collection of $N$ spin components $s_{nk}$, $k = 1, \ldots, N$. In that case Abner’s ISD-procedure would have to be able only to distinguish between a finite number (at least 4) of pure states. We show next that individual state determination is indeed impossible in general and also that the unambiguous distinction even of only two non-orthogonal states must fail.

4. Impossibility of individual state determination.

4.1. State inference from measurement outcomes. The determination of a system’s state requires that some measurement is performed. Every measurement can be represented by some observable whose values are exhibited as the measurement outcomes. In order to decide from the outcomes whether or not the system was in a given state $\varphi$, there needs to be at least one outcome which occurs with certainty if the state was $\varphi$, and which will certainly not occur if the state was some state $\psi$ different from $\varphi$. Hence the probability for that outcome must be $p_\varphi = 1$ for the state $\varphi$ and $p_\psi = 0$ for $\psi$. Since the quantum mechanical probabilities are expectations of some positive operators $E$ representing the event in question, it follows that $\langle \varphi | E \varphi \rangle = 1$ and $\langle \psi | E \psi \rangle = 0$. These equations are equivalent to

$$E \varphi = \varphi \quad \text{and} \quad E \psi = O \quad (3)$$

(cf. remark [17]). Therefore,

$$\langle \varphi | \psi \rangle = \langle E \varphi | \psi \rangle = \langle \varphi | E \psi \rangle = 0, \quad (4)$$

which is to say that $\varphi$ and $\psi$ are mutually orthogonal. It follows that no measurement exists which would allow one to distinguish unequivocally between any (even a single) pair of non-orthogonal states.

It may be noted that in this argument the usual assumption is not made that the operator $E$ is a projection. For the numbers $\langle \xi | E \xi \rangle$ to represent probabilities, $\xi$ being any unit vector, it is formally necessary and sufficient that $E$ is an operator bounded between the unit ($I$) and null ($O$) operators. These operators are known as effects (as compared to properties). An effect for which there are states such that (3) is satisfied
belongs to the class of *approximate properties* [18]; for $E$ it can be asserted that in the state $\varphi$, $E$ is real while for $\psi$ the complement $I - E$ is real.

**4.2. Measurement theoretical formulation.** There is a measurement theoretical version of the argument which exploits the unitarity of the quantum dynamics. This formulation has the advantage of allowing a direct confrontation with the concrete model proposals put forward in favor of individual state determination [1,16]. In the quantum mechanical description of a measurement process, the object system, originally in state $\varphi$, is coupled with an apparatus (or probe), originally in state $\phi$, by means of a unitary operator $U : | \varphi \phi \rangle \mapsto U | \varphi \phi \rangle$. A measurement will be completed once a pointer observable of the probe has been registered. Hence there should be a complete collection $Q_k$ of projection operators in the probe’s Hilbert space (such that $\sum_k Q_k = I$) whose expectation values in the state $U | \varphi \phi \rangle$ give the probabilities for the occurrence of the outcomes $k$. This *measurement scheme*, constituted by a probe, a unitary coupling, and a pointer, qualifies as a measurement of some observable of the object in the sense that the pointer frequency distributions can be interpreted as probabilities in the object’s Hilbert space. Indeed, to any $Q_k$ there exists an operator $E_k$ associated with the object such that the following *probability reproducibility condition* is satisfied:

$$\langle U\varphi\phi\mid Q_k U\varphi\phi \rangle = \langle \varphi \mid E_k \varphi \rangle. \quad (5)$$

for all states $\varphi$. The completeness condition $\sum_k Q_k = I$ is inherited by the $E_k$ so that $\sum_k E_k = I$. Further, all $E_k$ are positive operators since the expectation values in (5) are non-negative. The map $k \mapsto E_k$ thus constitutes a positive operator valued measure, which represents the measured observable. If the $E_k$ happen to be projections, they define an observable in the ordinary sense, which may be called a *sharp observable*; otherwise one is dealing with an *unsharp observable*. [An account of the general representation of observables as operator valued measures can be found in Ref. 18.]

In order to distinguish between two non-orthogonal states $\varphi, \psi$ of the object, there needs to be a projection operator $Q$ in the probe’s Hilbert space such that the corresponding probability is 1 for $\varphi$ and 0 for $\psi$: thus,

$$QU \mid \varphi \phi \rangle = U \mid \varphi \phi \rangle \quad \text{and} \quad QU \mid \psi \phi \rangle = 0. \quad (6)$$
It follows that

\[ \langle \varphi \psi \mid \varphi \varphi \rangle = \langle U \varphi \varphi \mid U \psi \psi \rangle = \langle QU \varphi \varphi \mid U \psi \psi \rangle = \langle U \varphi \varphi \mid QU \psi \psi \rangle = 0. \quad (7) \]

The first equality is due to the unitarity of \( U \), and we conclude again that the state discrimination works only for orthogonal state pairs.

This argument is equivalent to the preceding one: if \( E \) is the effect associated with \( Q \) via (5), then (6) yields (3) and vice versa.

4.3. **Why one cannot perform measurements with no state changes.** If one could, one would be able to repeat the same measurement and obtain the statistics of the measured observable by manipulating a single system. Assume a measurement were to leave unchanged all states of the object system. This is to say that the expectation values of all observables \( B \) of the object would remain unchanged, irrespective of what the measurement outcome was. Hence the conditional expectation values \( \langle B \rangle_k \) would have to coincide with the original ones, i.e.,

\[ \langle B \rangle_k \equiv \frac{\langle U \varphi \varphi \mid B Q_k U \varphi \varphi \rangle}{\langle U \varphi \varphi \mid Q_k U \varphi \varphi \rangle} = \langle \varphi \mid B \varphi \rangle \quad (8) \]

for all states \( \varphi \), all \( B \), all \( k \). Since the right-hand-side and the numerator of the left-hand-side are sesquilinear functionals of \( \varphi \), it follows that the expression in the denominator must be independent of \( \varphi \),

\[ \langle U \varphi \varphi \mid Q_k U \varphi \varphi \rangle = \lambda_k. \quad (9) \]

According to Eq. (5) the measured observable is represented by the operators \( E_k \), which, as a consequence of (9), are \( E_k = \lambda_k I \) if (8) is to hold. But for such a trivial observable the probabilities are the same for all states, so that the measurement scheme in question does provide no information at all about the system.

This reasoning can be refined so as to rule out the following proposal for ISD [16]. Suppose a measurement with finitely many outcomes can be performed such that the changes of states associated with the outcomes \( k \) are described by invertible maps. Then it is possible to conceive of further measurements with the same property which, if applied to the system after the first, principal measurement, could lead to a reversal of the state change...
that had occurred in the first instance. Thus, with some nonzero probability one would have restored the object system’s initial state; and by reading the reset measurement’s outcome, one would know this and could repeat the principal measurement on the same system, in the same initial state. It would appear that one eventually could collect the statistics of sufficiently many measurements and infer the state of the individual system. (Un)fortunately, the scheme just sketched must fall under the general category of measurement schemes referred to in subsection 4.2 and is therefore doomed to fail to serve its purpose. To see why it must fail, we observe that if a measurement procedure leads to no state change for one of its outcomes \( k \), say, then in view of Eqs. (8), (9) the corresponding effect \( E_k \) is constant; therefore the probability for that outcome does not depend on the object’s initial state. In other words, if in a sequence of measurements there is a successful reset event, all previous information about the object state is lost.

4.4. Why the EPR-signaling scheme doesn’t work. We consider the 2-letter alphabet version of the signaling scheme where \( Bob \) measures either \( s_x \) or \( s_y \). Then \( Abner \) either receives one of the states \(| \pm, x \rangle \) or one of \(| \pm, y \rangle \). \( Abner \) may measure any observable of particle \( A \) which might allow him to infer, at least with some probability, in which of the two sets the state of \( A \) is. Hence there should be an outcome, represented by a positive operator \( E \), whose occurrence \( Abner \) would interpret as indicating that the state was one of \(| \pm, x \rangle \). Similarly, the occurrence of any other outcome, the totality of which are represented by \( I - E \), would be interpreted as indicating that the state was one of \(| \pm, y \rangle \). Assuming equal \textit{apriori} probabilities for the two sets of states, the total probability for a correct inference is

\[
\frac{1}{2} \left( \frac{1}{2} \langle +, x | E | +, x \rangle + \frac{1}{2} \langle -, x | E | -, x \rangle \right) \\
+ \frac{1}{2} \left( \frac{1}{2} \langle +, y | (I - E) | +, y \rangle + \frac{1}{2} \langle -, y | (I - E) | -, y \rangle \right) \\
= \frac{1}{2} \text{Trace} \left[ E \cdot \frac{1}{2} \left( | +, x \rangle \langle +, x | + | -, x \rangle \langle -, x | \right) \right] \\
+ \frac{1}{2} \text{Trace} \left[ (I - E) \cdot \frac{1}{2} \left( | +, y \rangle \langle +, y | + | -, y \rangle \langle -, y | \right) \right] \\
= \frac{1}{2} \text{Trace} \left[ E \cdot \frac{1}{2} I \right] + \frac{1}{2} \text{Trace} \left[ (I - E) \cdot \frac{1}{2} I \right] = \frac{1}{2}. \tag{10}
\]

Thus correct and wrong inferences are equally likely; there is no way of telling what ob-
servable was measured by the distant observer. It would not even help if Abner and Bob agreed that every bit of a message would be sent in a large number of copies: if Bob had sent the $x$ letter, Abner would receive a finite ensemble of particles which were (roughly) equally distributed over the states $|+\rangle_x$, $|-\rangle_x$; and this would not be distinguishable from the corresponding ensemble emerging from sending the $y$ letter. Thus the message is blurred as soon as one tries to employ statistical procedures.

This fact has provoked the proposal that a particle incoming at the receiver’s end should be subjected to an amplification process by which hopefully its state could be cloned and determined from the ensemble of copies. In other words, the question is whether there exists some form of amplification procedure, followed by a measurement performed on the amplified system, which would allow Abner to infer with certainty whether that individual system was in one of the states $|+\rangle_x$, $|+\rangle_y$, say. However, the process thus described constitutes a measurement procedure performed on these non-orthogonal states, so that the no-go arguments of the preceding subsections immediately apply to this situation.

5. Optimal state discriminations.

Being unable to achieve perfect state inferences, then how well can one distinguish between two non-orthogonal states [12-15]? Given one of the non-orthogonal states $\varphi, \psi$, with equal apriori probabilities, one may ask which measurement would maximize the probability of correct inferences. Thus, again, there ought to be an outcome, represented by a positive operator $E$, whose occurrence is more likely in the state $\varphi$ than in $\psi$, so that one would infer that the state was $\varphi$. Accordingly, the probability for an outcome corresponding to $I - E$ is more likely if the state was $\psi$ rather than $\varphi$. The total probability of a correct inference is

$$p = \frac{1}{2} \langle \varphi | E \varphi \rangle + \frac{1}{2} \langle \psi | (I - E) \psi \rangle$$

$$= \frac{1}{2} \left( 1 + \text{Trace} \left[ E \cdot (|\varphi\rangle\langle \varphi| - |\psi\rangle\langle \psi|) \right] \right). \quad (11)$$

It is obvious that this expression is maximal if $E$ is the spectral projection associated with the positive eigenvalue of the operator $|\varphi\rangle\langle \varphi| - |\psi\rangle\langle \psi|$. The eigenvalues are easily
determined to be $\pm \sqrt{1 - |\langle \varphi | \psi \rangle|^2}$, so that the maximal available probability is

$$p_{\text{max}} = \frac{1}{2} \left[ 1 + \sqrt{1 - |\langle \varphi | \psi \rangle|^2} \right].$$

As an example, in the case of a spin-$\frac{1}{2}$ system and with $\varphi = | +, x \rangle$, $\psi = | +, y \rangle$ one finds $p_{\text{max}} = \frac{1}{4} [2 + \sqrt{2}] \approx 0.85$.

There are other ways of formulating an optimization problem. For instance, one may conceive of a measurement scheme which allows one to infer $\varphi$ and $\psi$ with certainty from some distinct outcomes, but at the price that there are some further outcomes from which no unique inferences are possible. Thus there should be (at least) three outcomes, represented by the probe projections $Q_1, Q_2, Q_3$ (where $Q_1 + Q_2 + Q_3 = I$), such that

$$Q_2 U \varphi \varphi = 0, \quad Q_1 U \psi \psi = 0. \tag{13}$$

Under these circumstances it is evident that the state must have been $\varphi$, resp. $\psi$, if the outcome is 1, resp. 2. According to Eq. (4) there are three positive operators $E_1, E_2, E_3$ associated with the object system, which allow one to rewrite Eqs. (13) as

$$E_2 \varphi = 0, \quad E_1 \psi = 0. \tag{14}$$

We wish to maximize the probability of correct inferences (assuming again equal apriori probabilities),

$$p = \frac{1}{2} \langle \varphi | E_1 \varphi \rangle + \frac{1}{2} \langle \psi | E_2 \psi \rangle. \tag{15}$$

There is a constraint: the residual operator $E_3 = I - E_1 - E_2$ must be positive,

$$\langle \xi | (I - E_1 - E_2) \xi \rangle \geq 0 \quad \text{for all states} \; \xi. \tag{16}$$

For the sake of simplicity we consider only the case where the object’s Hilbert space is two-dimensional (e.g., a spin-$\frac{1}{2}$ system). Then the conditions (14) imply that $E_1, E_2$ are of the form

$$E_1 = e_1 (1 - |\psi\rangle\langle \psi |), \quad E_2 = e_2 (1 - |\varphi\rangle\langle \varphi |). \tag{17}$$

Note that the positivity condition (16) forbids $e_1, e_2$ to assume their maximal value 1; this is to say that the operators $E_1, E_2, E_3$ cannot be projections. In other words, they constitute an unsharp observable of the object [18]. Inserting (17) into (15) yields

$$p = \frac{1}{2} (e_1 + e_2) \left( 1 - |\langle \varphi | \psi \rangle|^2 \right), \tag{18}$$
and the positivity constraint (16) is found to be

$$0 \leq 1 - (e_2 + e_2) + e_1 e_2 \left(1 - |\langle \varphi | \psi \rangle|^2\right),$$

(19)

Noting that the probability $p$ increases with increasing $e_1, e_2$, we solve (19) for $e_1$,

$$e_1 \leq \frac{1 - e_2}{1 - e_2 w}, \quad w \equiv (1 - |\langle \varphi | \psi \rangle|^2),$$

(20)

and insert this into (18):

$$p \leq \frac{1}{2} f(e_2) w, \quad f(e_2) = \frac{1 - e_2^2 w}{1 - e_2 w}.$$

(21)

The function $f(e_2)$ is maximal at $e_2 = \frac{1}{w} [1 - \sqrt{1 - w}]$, for which the maximal value of $e_1$ according to (20) is $e_1 = e_2$. Therefore the maximal value of $p$ is

$$p_{\text{max}} = 1 - |\langle \varphi | \psi \rangle| \quad \text{at} \quad e_1 = e_2 = \frac{1}{1 + |\langle \varphi | \psi \rangle|}.$$

(22)

This reproduces the result of Jaeger and Shimony [15] and underlines it with the observation that optimal state inference in the present sense and case is achieved with a measurement of an unsharp rather than sharp observable.

6. Conclusion.

*Is the quantum state (an) observable?* In physics one wants to find out about the state of a system; and all one can do is to perform measurements of some observables. There is no way to obtain a unique determination of an unknown state from a single measurement outcome. The reliability of the state discrimination decreases with increasing similarity between the two states under consideration, measured in terms of their inner product.

It may be worth recalling that there are measurement procedures of a single observable which give complete statistical information about the state of a system [18]. Such an informationally complete observable is necessarily unsharp. In fact for any sharp observable $A$, the vectors $\psi$ and $e^{if(A)} \psi$ represent in general different states [if $\psi$ is not an eigenvector of $f(A)$]; but these states have the same $A$ distributions.

We conclude that the quantum state is not an observable but not unobservable.
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References and Notes.
1. N. Herbert, *Found. Phys.* **12**, 1171 (1982).
2. D. Dieks, *Phys. Lett.* **92A**, 271 (1982).
3. P. Eberhard, *Nuovo Cim.* **46B**, 392 (1978).
4. G.C. Ghirardi, A. Rimini, T. Weber, *Lett. Nuovo Cim.* **27**, 293 (1980).
5. R.J. Glauber, in *New Techniques and Ideas in Quantum Measurement Theory*, ed. D.M. Greenberger, The New York Academy of Sciences, New York, 1986, pp. 336-372, esp. p. 362ff.
6. L. Mandel, *Nature* **304**, 188 (1983).
7. W.K. Wootters, W.H. Zurek, *Nature* **299**, 802 (1982).
8. The question of the peaceful coexistence of quantum mechanics and relativity has been reviewed by A. Shimony, in S. Kamefuchi et. al. (eds.), *Foundations of Quantum Mechanics in the Light of New Technology*, Tokyo: The Physical Society of Japan, 1984.
9. A formulation of the no-signaling proof that comprises general amplification and cloning procedures is given in H. Scherer, P. Busch, *Phys. Rev.* **47**, 1647 (1993).
10. A. Peres, *Quantum Theory: Concepts and Methods*, Kluwer Academic Publishers, Dordrecht, 1993, Chapter 9.
11. M.G. Alford, S. Coleman, J. March-Russell, *Nuclear Physics* **B351**, 735 (1991).
12. I.D. Ivanovic, *Physics Letters A* **123**, 257 (1987).
13. D. Dieks, *Physics Letters A* **126**, 303 (1988).
14. A. Peres, *Physics Letters A* **128**, 19 (1988).
15. G. Jaeger, A. Shimony, *Phys. Lett. A* **197**, 83 (1995).
16. A. Royer, *Phys. Rev. Lett.* **73**, 913 (1994); Erratum: ibid., **74**, 1040 (1995).
17. We prove the following mathematical fact which will be used in several instances throughout this paper: for an operator $E$ with $0 \leq \langle \xi | E \xi \rangle \leq 1$ for all states $\xi$, the relation $\langle \varphi | E \varphi \rangle = 1$, resp. $\langle \psi | E \psi \rangle = 0$ (for states $\varphi, \psi$) is equivalent to $E \varphi = \varphi$, resp. $E \psi = 0$. The latter equations are obviously sufficient for the former. Their necessity follows from the observation that $\langle \xi | F \xi \rangle = \langle \sqrt{F} \xi | \sqrt{F} \xi \rangle = \| \sqrt{F} \xi \|^2$, where $F$ is either $E$ or $I - E$.
18. P. Busch, M. Grabowski, P. Lahti, *Operational Quantum Physics*, Springer-Verlag, Berlin, 1995.