Coherently enhanced Raman scattering in atomic vapor

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(Dated: July 7, 2010)

We present a scheme to obtain the coherently enhanced Raman scattering in atomic vapor which is induced by a spin wave initially written by a weak write laser. The enhancement of Raman scattering is dependent on the number and the spatial distribution of the flipped atoms generated by the weak write laser. Such an enhanced Raman scattering may have practical applications in quantum information, nonlinear optics and laser spectroscopy because of its simplicity.

PACS numbers: 42.65.Dr, 42.50.Gy, 42.25.Bs

I. INTRODUCTION

The Raman scattering technique \cite{1} has enjoyed widespread applications in molecular spectroscopy and light amplification. The stimulated Raman scattering (SRS) process is well understood with a full quantum theory \cite{2}. For Raman scattering in an atomic ensemble, due to the limited number of atoms and limited interaction length, the conversion efficiency of the pump field to Stokes laser is low and the SRS regime is seldom reached. In order to obtain a high efficiency frequency conversion, a seed Stokes field is usually injected into the atomic ensemble and is used to generate the SRS in a short time.

Another method to improve conversion efficiency is coherent anti-Stokes Raman spectroscopy (CARS) \cite{3}, which utilizes the atomic coherence built in stimulated Raman process to greatly enhance the anti-Stokes component. This technique was developed into two regimes: one is femtosecond adaptive spectroscopic techniques applied to CARS (FAST CARS), which makes it easy to detect minute biological and chemical agents \cite{4,5}, another is enhanced four-wave mixing (FWM) using electromagnetically induced transparency (EIT) \cite{8} for generation of non-classical correlations, non-classical photon-pairs \cite{9-20} and single photons \cite{21,22} where the weak generated signals can avoid resonant absorption due to EIT. These works can be understood as a combined “write-read” process. In the write process, a spin wave (or an atomic coherence) is written in the atomic ensemble. In the read process, the stored spin wave is retrieved by coherent conversion from the atomic states into the anti-Stokes field.

There are many ways to prepare atomic coherence, such as EIT and stimulated Raman adiabatic passage (STI-RAP) \cite{23}. Jain et al. \cite{24} achieved a high-frequency conversion efficiency from 425 to 293 nm with the help of an atomic coherence prepared via EIT on a Raman transition. Sautenkov et al. \cite{25} demonstrated the enhancement of coherent anti-Stokes laser in Rb atomic vapor by a maximal atomic coherence prepared by fractional STI-RAP. Recently, our group observed an enhanced Raman scattering (ERS) effect by the prebuilt atomic spin wave \cite{26}. This effect can be understood as a “write-write” process shown in Fig. 1. In the first step, the Stokes field $\hat{E}_{S_1}$ is produced and a spin wave is written into the atomic ensemble by the first write field $\Omega_{W_1}$, which is the same as the first step of the “write-read” process. In the second step after some delay, differing from the CARS process, a second write field $\Omega_{W_2}$ is applied for another Raman scattering, thus another Stokes field $\hat{E}_{S_2}$ is generated.

In this article, we present a theoretical treatment for the observed ERS effect. Here, we will show that the intensity of the second Stokes field $\hat{E}_{S_2}$ can be enhanced compared to the case with no spin wave prepared initially and depends on the intensity and the spatial distribution of initially prepared atomic spin wave.

Our article is organized as follows. In Sec. II we describe our ERS based on the spin wave initially prepared by a weak write laser, and give the intensities of the Stokes laser ($\hat{E}_{S_2}$) in the counterpropagation case and in the copropagation case. In Sec. III we numerically calculate the intensities of the usual Raman scattering (URS) and ERS, which shows that the intensities of ERS are larger than that of URS and the intensity of counterpropagation case is larger that of copropagation case. In Sec. IV we discuss the ERS and URS.
conclude with a summary of our results.

II. THEORETICAL MODEL

In order to explain why the Raman scattering effect shown in Fig. 1 can be enhanced, we consider a three-level Raman system composed of states |1⟩, |2⟩, and |4⟩, relating to the second write field W₂. Due to the first write field W₁ is absent in the four-level system, the process is the familiar URS. When two write fields W₁ and W₂ with respective Rabi frequencies Ω₁ and Ω₂ are driven on the atomic system according to the time sequence shown in Fig. 1(b), the intensity of Stokes field ΣS₂ can be enhanced compared to the URS case due to the coherence Σ₁2 between states |1⟩ and |2⟩ is built by the first write field W₁ [24, 25]. The enhancement of the intensity of Stokes field ΣS₂ is also dependent on the spatial distribution of the flipped atoms generated by the first write field W₁.

In the undepleted pump approximation, the interaction Hamiltonian in a rotating wave frame is given by [2, 28]

\[ H = \frac{N}{V} \int d\mathbf{r} \{ \delta \Sigma_{14}(\mathbf{r}, t) - [\Omega W₂ e^{i k w₂ \cdot \mathbf{r}} \Sigma_{11} + g₂ \Sigma_{S₂}(\mathbf{r}, t) e^{i k S₂ \cdot \mathbf{r}} \Sigma_{42} + \text{H.c.}] \}, \]

(1)

where \( \delta = (\omega₁ - \omega₁) - \omega w₂ \), \( g₂ \) is the atom-field coupling constant, the Stokes field operator \( \Sigma S₂(\mathbf{r}, t) = \sqrt{\hbar \omega S₂/2}\mathbf{v}[\Sigma S₂(\mathbf{r}, t)e^{i (k S₂ \cdot \mathbf{r} - \omega S₂ t) + \text{H.c.}}] \), \( N \) is the number of atoms, and \( V \) is the quantization volume.

The collective atomic operators are \( \hat{\sigma}_{\mu \nu}(\mathbf{r}, t) = 1/NR \sum_{j=1}^{N_R} \hat{\sigma}_{\mu \nu}(t)e^{-i\omega_{\mu \nu} t} \), where \( \hat{\sigma}_{\mu \nu}(t) = \hat{\sigma}_{\nu \mu}(t) \) is the transition operator of the \( j \)th atom between states \( |\mu⟩ \) and \( |\nu⟩ \), and there is a small and macroscopic volume containing \( N_R \) (\( N_R \gg 1 \)) atoms around position \( \mathbf{r} \). The Stokes field operator \( \hat{\Sigma}_{S₂}(\mathbf{r}, t) \) obeys the wave equation

\[ \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} - i\frac{c}{2 k S₂} \overrightarrow{\nabla}^{2} \right) \hat{\Sigma}_{S₂}(\mathbf{r}, t) = ig₂N \hat{\sigma}_{24} e^{-i k S₂ \cdot \mathbf{r}}. \]

(2)

The equations of motion for the atomic operators in the Heisenberg picture are

\[ \dot{\sigma}_{24} = -i\gamma₂ \hat{\sigma}_{24} + \Omega W₂ e^{i k w₂ \cdot \mathbf{r}} \hat{\sigma}_{31} + ig₂ \hat{\Sigma}_{S₂}(\hat{\sigma}_{22} - \hat{\sigma}_{44}) e^{i k S₂ \cdot \mathbf{r}} + \hat{F}_{24}, \]

(3)

\[ \dot{\sigma}_{21} = -i\gamma₁ \hat{\sigma}_{21} + \Omega W₂ e^{-i k w₂ \cdot \mathbf{r}} \hat{\sigma}_{24}, \]

\[ \dot{\sigma}_{41} = -i\gamma₁ \hat{\sigma}_{41} - i\Omega W₂ e^{-i k w₂ \cdot \mathbf{r}} (\hat{\sigma}_{11} - \hat{\sigma}_{44}) - ig₂ \hat{\Sigma}_{S₂} \cdot e^{-i k S₂ \cdot \mathbf{r}} \hat{\sigma}_{21} + \hat{F}_{41}, \]

(4)

where \( \gamma₁ \) is the coherence \( \Sigma₁₂ \) decay rate and \( \gamma₂ \) is the decay rate of the excited state |4⟩. \( \hat{F}_{\mu \nu} \) are the Langevin noise operators for the atomic operator.

Due to large detuning \( \delta \gg |\Omega w₂| \), \( \gamma₂ \), we can adiabatically eliminate the optical coherence \( \Sigma₂₄ \) and \( \Sigma₄₁ \). In the following, we consider the number of atoms scattered to |2⟩ at all times much smaller than \( N \), and obtain

\[ \dot{\sigma}_{24} = \left( 1 + i\frac{\gamma₂}{\delta} \right) \frac{\Omega W₂}{\delta} e^{i k w₂ \cdot \mathbf{r}} \hat{\sigma}_{21} + \frac{g₂}{\delta} e^{i k w₂ \cdot \mathbf{r}} \hat{\Sigma}_{S₂} \hat{\sigma}_{22}, \]

(6)

\[ \dot{\sigma}_{41} = \frac{\Omega W₂}{\delta} e^{-i k w₂ \cdot \mathbf{r}} \hat{\sigma}_{41} + i\hat{F}_{41}. \]

(7)

Using Eq. (6), we have the Stokes propagating equation

\[ \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} - i\frac{c}{2 k S₂} \overrightarrow{\nabla}^{2} \right) \hat{\Sigma}_{S₂}(\mathbf{r}, t) = i\gamma₂ \hat{\Sigma}_{S₂}(\hat{\sigma}_{21} - \hat{\sigma}_{22}) + i\hat{F}_{S₂}(t), \]

(8)

where \( \hat{F}_{S₂} = \sqrt{N \Sigma₁2} e^{i k \mathbf{r} \cdot \mathbf{r}} \) is the creation operator of the atomic spin wave where the wave vector \( \Delta k = k w₂ - k S₂ \), and \( \chi₂(\mathbf{r}, t) = g₂\sqrt{N} \Omega W₂(\mathbf{r}, t)/\delta \) is the coupling rate between the collective spin excitation \( \Sigma_{S₂} \) and the Stokes field \( \hat{\Sigma}_{S₂} \). Using Eqs. (6) and (7) we have

\[ \hat{\Sigma}_{S₂}(t) = -(\hat{\Sigma}_{S₂} - i\delta L₂) \hat{\Sigma}_{S₂} - i\chi₂ \hat{\Sigma}_{S₂}(\hat{\sigma}_{11} - \hat{\sigma}_{22}) + i\hat{F}_{S₂}(t), \]

(9)

where \( \hat{W}(t) = \hat{W}(0) \exp(-\gamma₂ t) + \exp(-\gamma₂ t) - 1 \), where \( \hat{W}(0) = \hat{\sigma}_{11}(0) - \hat{\sigma}_{22}(0) \). Therefore, \( \hat{\sigma}_{11} - \hat{\sigma}_{22} \) in Eq. (3) can be replaced by \( \hat{W}(t) \) which describes the small changes in population difference. The equations of motion (3, 4, 6) are the same as that of URS case except the initial conditions are different. In the case of URS [2, 28], the atomic medium is initially optically pumped to the ground state \( |1⟩ \), then the parameter \( \hat{W}(0) = 1 \) and the other atomic operators are zero, but in the case of ERS, the initial state is a superposition state of states \( |1⟩ \) and \( |2⟩ \), so \( \langle \hat{\sigma}_{11} \rangle \neq 0, \langle \hat{\sigma}_{22} \rangle \neq 0, \) and \( \langle \hat{\sigma}_{12} \rangle \neq 0 \).

Assume the write field \( \Omega w₂ \) corresponding to a focused beam and the Fresnel number \( \tilde{F} = A/\lambda S₂ L \) (\( A \) is the cross-sectional area, and \( L \) is the cell length) is of the order of unity; then only a single transverse spatial mode contributes strongly to emission along the direction of the write field \( \Omega w₂ \). Therefore the above model can be simplified as a one-dimensional model, the propagating quantized Stokes field \( \hat{\Sigma}_{S₂} \) obeys the equation of motion [2, 28]

\[ \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) \hat{\Sigma}_{S₂}(z, t) = i\gamma₂ \hat{\Sigma}_{S₂}(\hat{\sigma}_{21} - \hat{\sigma}_{22}), \]

(11)

\[ \partial_z \hat{\Sigma}_{S₂} = -G \hat{\Sigma}_{S₂} - i \hat{W}(t) \chi₂(t) \hat{\Sigma}_{S₂} + i \hat{F}_{S₂}, \]

(12)

where \( G \) is the parameter \( \delta \) and \( G \). Using the moving frame \( t' = t - z/c \), \( z' = z \), the solution is [2, 28].
\[ \hat{S}_{a_2}^\dagger (z', t') = e^{-\Gamma_2(t')} \hat{S}_{a_2}^\dagger (z', 0) - i \int_0^{t'} W(t'') \chi_2 (t'') e^{-[\Gamma_2(t'') - \Gamma_2(t'')] H_2 (z', 0, t', t'') \hat{\Delta}_S (0, t'') dt'' + e^{-\Gamma_2(t')} \int_0^{t'} G_{S_2} (z', z'', t', 0) \hat{S}_{a_2}^\dagger (z'', 0) dz'' + \int_0^{t'} e^{-[\Gamma_2(t'') - \Gamma_2(t'')] \hat{F}_{S_2}^\dagger (z', t'') dt'' + \int_0^{t'} e^{-[\Gamma_2(t'') - \Gamma_2(t'')] G_{S_2} (z', z'', t', t'') \hat{F}_{S_2}^\dagger (z', t'') dz'' dt'', \]  
\[ \hat{E}_S (z', t') = \hat{E}_S (0, t') + \chi_2 (t') e^{-\Gamma_2(t')} \int_0^{t'} W(t'') \chi_2 (t'') e^{-[\Gamma_2(t'') - \Gamma_2(t'')] \hat{E}_S (0, t'') G_{S_2} (z', 0, t', t'') dt'' + i \chi_2 (t') e^{-\Gamma_2(t')} \int_0^{t'} H_2 (z', z'', t', 0) \hat{S}_{a_2}^\dagger (z'', 0) dz'' + i \chi_2 (t') e^{-\Gamma_2(t')} \int_0^{t'} e^{-[\Gamma_2(t'') - \Gamma_2(t'')] \int_0^{z'} H_2 (z', z'', t', t'') \hat{F}_{S_2}^\dagger (z'', t'') dz'' dt'', \]  
(13)

where

\[ H_2 (z', z'', t', t'') = I_0 (2 \sqrt{[p_2 (t') - p_2 (t'')] z' - z''}, \]
\[ G_{S_2} (z', z'', t', t'') = \frac{c (z' - z'')}{p_2 (t') - p_2 (t'')} G_{S_2} (z', z'', t', t''), \]
\[ G_{S_2} (z', z'', t', t'') = \sqrt{\frac{p_2 (t') - p_2 (t'')}{c (z' - z'')}} \times I_1 (2 \sqrt{[p_2 (t') - p_2 (t'')] z' - z''}), \]  
(15)

Here, \( \Gamma_2 (t') = \int_0^{t'} \Gamma S_2 (t'') dt'' \) and \( p_2 (t') = \int_0^{t'} W(t'') \chi_2 (t'')^2 dt'' \), and \( I_n (x) \) is the modified Bessel function of the first kind of order \( n \). This solution is also for pencil-shaped atomic ensemble for URS case when \( W (0) = 1 \) [2, 28].

In order to explain the ERS, we compare the intensity of the ERS with that of the URS. Using Eq. (14), the intensity at the end of the atomic cell is given by

\[ I_{S_2} (t') = \frac{\hbar \omega_{S_2}}{L} \langle \hat{S}_{S_2}^\dagger (L, t') \hat{E}_{S_2} (L, t') \rangle \]
\[ = \frac{\hbar \omega_{S_2}}{c L} \chi_2^2 (t') e^{-2 \text{Re} \Gamma_2 (t')} \int_0^L H_2 (L, z', t', 0) \int_0^L H_2 (L, z'', t', 0) \langle \hat{S}_{a_2} (z', 0) \hat{S}_{a_2}^\dagger (z'', 0) \rangle dz' dz'', \]  
\[ + 2 \frac{\hbar \omega_{S_2}}{c} \chi_2^2 (t') \gamma_{S_2} \int_0^{t'} e^{-2 \text{Re} \Gamma_2 (t') - \Gamma_2 (t'')} \left[ \frac{\langle p_2 (t') - p_2 (t'') \rangle L}{c} \right] - I_2^2 \left( \frac{\langle p_2 (t') - p_2 (t'') \rangle L}{c} \right) dt'', \]  
(16)

where the first and second terms are from the third and fourth terms in Eq. (14). According to the commutation relation of spin wave, the term \( \langle \hat{S}_{a_2} (z', 0) \hat{S}_{a_2}^\dagger (z', 0) \rangle \) in Eq. (15) is

\[ \langle \hat{S}_{a_2} (z', 0) \hat{S}_{a_2}^\dagger (z'', 0) \rangle = L \delta (z' - z'') \]
\[ + \langle \hat{S}_{a_2}^\dagger (z'', 0) \hat{S}_{a_2} (z', 0) \rangle. \]  
(17)

For URS, \( \langle \hat{S}_{a_2}^\dagger (z', 0) \hat{S}_{a_2} (z'', 0) \rangle \) equals zero because no initial spin wave is prepared, but \( \langle \hat{S}_{a_2}^\dagger (z', 0) \hat{S}_{a_2} (z'', 0) \rangle \) ≠ 0 for ERS. The term \( \langle \hat{S}_{a_2} (z', 0) \hat{S}_{a_2} (z'', 0) \rangle \) due to the initially written spin wave, is the key for the appearance of the ERS for the write field \( \Omega_{W_2} \). For convenience, we assume the write fields intensity being constant \( \langle \Omega_{W_2} (t') = \Omega_{W_2} (0) \rangle \), after being switched on at \( t' = 0 \), so \( \Gamma_2 (t') = \Gamma \gamma_{a_2} t' \), and \( p_2 (t') = \eta (t') \chi_2^2 t' \) with \( \eta (t') = W (0) (1 - 1/2 \gamma_{a_2}^2 t') - 1/2 \gamma_{a_2}^2 \). In the case of URS (considering a Raman system composed of states |1, 2, and |4), no initial Stokes field is externally incident on the ensemble and no initial spin wave is written into the ensemble, so the initial conditions for the Stokes field and the spin wave are

\[ \langle \hat{E}_S (0, t') \hat{E}_{S_2} (0, t') \rangle = 0, \quad \langle \hat{S}_{a_2}^\dagger (z', 0) \hat{S}_{a_2} (z', 0) \rangle = 0. \]  
(18)

Then from Eqs. (16)-(18) the intensity of the Stokes field in URS case is
\[ I_{S_2}(t') = \frac{\hbar \omega S_2 \chi_2^2 L}{c} \int_0^{t'} dt'' e^{-2\gamma_{S_2}(t'-t'')} \times [I_0(2 \sqrt{K_p})^2 - I_1(2 \sqrt{K_p})^2 + e^{-2\gamma_{S_2}t'}] \times [I_0(2 \sqrt{\eta(t')^2 \chi_2^2 L}) - I_1(2 \sqrt{\eta(t')^2 \chi_2^2 L})^2]], \]

where \( K_p = \frac{L}{c} \chi_2^2 |\eta(t')t'' - \eta(t'')t'| \), and \( W(0) = 1 \) in the case of the URS.

In the case of ERS, like URS, no Stokes field is externally incident on the ensemble, but the atomic ensemble contains a spin wave \( S_{\dagger 1} \) written by the first write field \( \Omega_{W_1} \), which is the form is \( \tilde{S}_{\dagger 1}(z', t_1) = \sqrt{\gamma_{S_2}(z', t_1)} e^{i\Delta k z} \), where \( t_1 \) is the duration of the first write pulse. When the second write field \( \Omega_{W_2} \) is driven on the ensemble, the initial conditions are given by

\[ \langle \hat{S}_{\dagger 1}(0, t') | \hat{S}_{\dagger 2}(0, t') \rangle = 0, \quad \langle \hat{S}_{\dagger 2}(z', 0) | \hat{S}_{\dagger 2}(z', 0) \rangle \neq 0. \]

Then from Eqs. \( 16 \), \( 17 \), and \( 20 \), the intensity of the Stokes field in the ERS case is

\[ I_{S_2}(t') = I_{S_{20}}(t') + I_{\text{add}}(t'), \]

\[ I_{\text{add}}(t') = \frac{\hbar \omega S_2 \chi_2^2 L}{c} e^{-2\gamma_{S_2}t'} \int_0^L H_2(L, z', t') \int_0^L dz' dz'' H_2(L, z'', t', 0) \langle \tilde{S}_{\dagger 2}(z', 0) S_{\dagger 2}(z'', 0) \rangle \]

where \( I_{S_{20}}(t') \) is the usual Raman intensity as Eq. \( 19 \) except here \( W(0) \neq 1 \) in ERS case, and \( I_{\text{add}}(t') \) is the additional intensity generated by the initially prepared spin wave.

Next we investigate the additional intensity \( I_{\text{add}}(t') \). We first analyze the term \( \langle \tilde{S}_{\dagger 2}(z', 0) S_{\dagger 2}(z'', 0) \rangle \). We consider two different propagation geometries: copropagating or counterpropagating of this two write fields \( \Omega_{W_1} \) and \( \Omega_{W_2} \). For the copropagating case, we have \( S_{\dagger 2}(z', 0) = S_{\dagger 2}(z', t_1) \), so

\[ \langle \tilde{S}_{\dagger 2}(z', 0) S_{\dagger 2}(z'', 0) \rangle = n(z', t_1) \delta(z' - z''), \]

and for two write fields counter-propagating case, we have \( S_{\dagger 2}(z', 0) = S_{\dagger 2}(L - z', t_1) \), so

\[ \langle \tilde{S}_{\dagger 2}(z', 0) S_{\dagger 2}(z'', 0) \rangle = n(L - z', t_1) \delta(z' - z''), \]

with

\[ n(z', t_1) = \int_0^{t_1} e^{-2\gamma c \text{Re}[\Gamma_{S_1}]} / \chi_2^2 L I_0(2 \sqrt{\chi_2^2 L t' / c})^2 dz', \]

where \( \gamma_1 = \chi_2^2 L t' / c \), \( \zeta = \chi_2^2 L t' / c \), \( 0 \leq t \leq t_1 \) are the dimensionless strengths, \( \text{Re}[Z] \) denotes the real part of a complex number \( Z \), and the coefficient \( \chi_2^2 L / \text{Re}[\Gamma_{S_1}] \) is the order of optical depth. Figure \( 2 \) shows the spatial distribution of the flipped-atom number prepared by the first write field \( \Omega_{W_1} \) from Eq. \( 25 \), where the flipped density becomes larger toward the end part of the atomic ensemble, and the flipped density increases with increment of the dimensionless strength \( \gamma_1 \).

According to the co-propagating case and the counterpropagating case, the additional intensities \( I_{\text{add}}(t') \) can be expressed as \( I_{\text{add-cop}}(t') \) and \( I_{\text{add-counter}}(t') \). Thus the intensities of Stokes field \( \hat{S}_{\dagger 2} \) are written as

\[ I_{S_{2-cop}}(t') = I_{S_{20}}(t') + I_{\text{add-cop}}(t'), \]

\[ I_{S_{2-counter}}(t') = I_{S_{20}}(t') + I_{\text{add-counter}}(t'). \]

The spatial distribution will result in different ERS intensities \( I_{S_{2-cop}}(t') \) and \( I_{S_{2-counter}}(t') \) for the copropagating case and the counterpropagating case, respectively. The ratio \( I_{\text{add-counter}}(t') / I_{\text{add-cop}}(t') \) of the additional intensities \( I_{\text{add-counter}}(t') \) and \( I_{\text{add-cop}}(t') \) is shown in Fig. \( 3 \) from which we know that the additional intensity of the counter-propagating case is much larger that of the copropagating case. The ratio increases with increment of the dimensionless strength \( \eta(t')^2 \chi_2^2 L t' / c \), and also increases with increment of the dimensionless strength \( \zeta_1 \).
III. NUMERICAL ANALYSIS

In this section, we will numerically calculate the intensities of URS \( I_{SG} (t') \), of ERS in the co-propagating case \( I_{SG-cw} (t') \), and of ERS in the counter-propagating case \( I_{SG-counter} (t') \) using Gaussian-shaped write fields. Assume the write fields \( \Omega_{W_1} \) and \( \Omega_{W_2} \) have the following Gaussian shapes,

\[
\Omega_{W_1} (t') = \Omega_{W_{10}} [e^{-30(t'/T_1-0.5)^2} - e^{-7.5}],
\]

\[
\Omega_{W_2} (t') = \Omega_{W_{20}} [e^{-30(t'/T_2-0.5)^2} - e^{-7.5}],
\]

where \( T_1 \) and \( T_2 \) are the pulse durations of write fields \( W_1 \) and \( W_2 \), respectively, and \( \Omega_{W_{10}} \) and \( \Omega_{W_{20}} \) are the coefficients. From Eq. (16), we can obtain the corresponding intensities for differently initial conditions. The parameters \( \Gamma_1 \) and \( \Gamma_2 \) from Eq. (16) for the two-write-fields copropagating case are calculated according to

\[\langle S_{a_1} (z'', T_1) S_{a_1}^* (z'', T_1) \rangle = e^{-2Re[\Gamma_1(T_1)]} \int_0^{z''} G_{S_1} (z'', z''' , T_1, 0) dz''',\]

\[+ L^2 e^{-2Re[\Gamma_1(T_1)]} \int_0^{z''} G_{S_1}^2 (z'', z''' , T_1, 0) dz''' + 4 \gamma_1 L \int_0^{T_1} e^{-2Re[\Gamma_1(T_1)-\Gamma_1(t)']} \int_0^{z''} G_{S_1} (z'', z''' , T_1, 0) dz''' dt',\]

\[+ 2 \gamma_L L^2 \int_0^{T_1} e^{-2Re[\Gamma_1(T_1)-\Gamma_1(t)']} dt' \int_0^{T_1} e^{-2Re[\Gamma_1(T_1)-\Gamma_1(t)']} \int_0^{z''} G_{S_1}^2 (z'', z''' , T_1, t') dz''' dt',\]

where \( \gamma_L = \gamma_1 |\Omega_{W_{10}}|^2/\Delta^2 - i|\Omega_{W_{10}}|^2/\Delta \), and \( \bar{p}_1 = (g_1 \sqrt{\Omega_{W_{10}}}/\Delta)^2 \). For two write fields counterpropagating case, \( \langle S_{a_2} (z', 0) S_{a_2}^* (z''', 0) \rangle = \langle S_{a_1} (L - z'', T_1) S_{a_1}^* (L - z''', T_1) \rangle \).

Figure 3 numerically shows the intensities of three cases \( I_{SG}(t'), I_{SG-cw}(t') \), and \( I_{SG-counter}(t') \) using Eqs. (16) and (32). In Fig. 4, we choose the initial population difference \( W(0) = 0.99 \), and the other experimental parameters are the same as that were used in Ref. 26. When \( N g_1^2 |\Omega_{W_{10}}|^2 T_1 L/(c \Delta^2) = 8.5 \), the extent of enhancements agrees with our recent experimental results [26], where \( N g_1^2 |\Omega_{W_{10}}|^2 T_1 L/(c \Delta^2) \) is the order of the optical depth. When only the write light \( \Omega_{W_2} \) is turned on, a usual Raman scattering occurs, as shown in the line marked with circles in Fig. 4. When the write fields \( \Omega_{W_1} \) and \( \Omega_{W_2} \) are turned on according to the timing diagram in Fig. 4(b) and are in the copropagation configuration, an enhancement in \( \hat{E}_{S_2} \) occurs, where the experimental result corresponds to the line marked with square and the theoretical numerical calculation corresponds to the solid line in Fig. 4(a). When the write fields \( \Omega_{W_1} \) and \( \Omega_{W_2} \) are in the counterpropagation configuration, a much bigger enhancement effect in \( \hat{E}_{S_2} \) is shown in Fig. 4(b), where the line marked with pentagram (the solid line) is from the experimental (theoretical) data. From Fig. 4 it is easily seen that the intensity of two write fields counterpropagating case is larger than that of two write fields copropagating case. The reason why the counter-propagating case has a larger enhancement effect than the copropagating case is as follows. In the copropagating case, when the second write field \( \Omega_{W_2} \) enters the atomic medium, it first encounters a very small number of the flipped atoms, which is equivalent to a small seed for amplification. On the other hand, in the case of counterpropagating, once the second write field \( \Omega_{W_2} \) enters the medium, it immediately encounters a maximum number of the flipped atoms and starts to amplify it all the way through the atomic medium, which is equivalent to a large seed for amplification.
FIG. 4: (Color online) The intensities of Stokes fields versus the dimensionless time for (a) two write fields copropagating case and for (b) two write fields counterpropagating case. The solid lines are from theoretical results. The lines marked with “o”, “square,” and “pentagram” are from experimental data which describe the usual Raman ($\Omega_1 = 0$), the copropagation case of ERS, and the counterpropagation case of ERS, respectively. The parameters are as follows: $W(0) = 0.99$, $N_0^2 |\Omega_1|^2 T_1 L/(c \Delta^2) = 8.5$, $|\Omega_2|/|\Omega_1| = 1.56$, $\Delta = 1.2$ GHz, $\delta = 1$ GHz, $\gamma_1 = 10$ KHz, $\gamma_1 = 2\pi \times 5.746$ MHz, and $\gamma_2 = 2\pi \times 6.605$ MHz.

IV. DISCUSSION

In this section, let us compare the enhanced Raman process with the usual Raman process. Assume the classical write field does not undergo depletion, and then the usual Raman process is generated from spontaneous Raman scattering at very small times to transient SRS at moderately small times, and finally to steady-state SRS. That is to say, the spontaneous Stokes scattering which is from the vacuum acts as the source term to generate SRS, so the usual Raman process need a period of time to SRS and its phase is random. But the enhanced Raman process is generated based on the prepared spin wave and it directly and quickly to the SRS from the initiation. The phase of the enhanced Stokes laser is from the spin wave and is nonrandom. The atomic spin wave will act like an input seed to the Raman amplification process in the same way as the input Stokes field would, so the intensity of Stokes field $\hat{E}_S$ is enhanced.

Our scheme can also be explained by the language of nonlinear optics. After the first write laser is driven on the atomic ensemble, the atomic ensemble is turned into a new medium within a certain coherence time, where the nonlinear coefficient is larger compared to no initial coherence case. Then when another write laser is driven on the atomic ensemble, an enhanced Stokes field will occur.

V. CONCLUSION

In conclusion, we theoretically demonstrated an enhanced Raman effect, which is another mechanism to realize SRS instantaneous and to increase the conversion efficiency. The conversion efficiency of the second write laser is high due to the initially prepared spin wave by the first write field, and the enhancement of the Stokes field intensity in two-write-fields counterpropagating case is much larger than that in the two-write-fields copropagating case. The ERS is useful in quantum information, and in nonlinear optics and to detect minute biological and chemical agents.

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[30] data from http://steck.us/alkalidata.