Effects of postselected von Neumann measurement on nonclassicality of single-photon-added coherent state

Yusuf Turek

School of Physics and Electronic Engineering, Xinjiang Normal University, Urumqi, Xinjiang 830054, China

Received: 9 September 2020 / Accepted: 30 January 2021

The effects of postselected von Neumann measurement on nonclassicality of single-photon-added coherent state (SPACS) are studied. Explicit expressions and analytical results of photon statistics including photon number distribution and Mandel $Q_m$ factor, and the squeezing parameter of the field quadrature of SPACS after postselected von Neumann measurement are investigated. The results showed that the nonclassicality of SPACS after measurement changed dramatically than the initial state. The measurement let SPACS possess more strong sub-Poissonian photon statistics in some definite coupling strength regimes and large weak values, which accompanied by low postselection probabilities.

1 Introduction

The preparation and optimization of nonclassical quantum states have great importance in quantum information processing including single-photon generation and detection [1], gravitational wave detection [2,3], quantum teleportation [4–7], quantum computation [8], generation and manipulation of atom-light entanglement [9–11], and precision measurement [12]. It is well known that the implementation of those processes depends on the generation and optimization of the related input radiation fields such as coherent states [13–15], squeezed states [16–18], even and odd coherent states [19–22], displaced and squeezed number states [23], and binomial states [24,25]. Thus, it is worthy to study the inherent properties of radiation fields to find the suitable quantum states for effectively implementing the above-mentioned quantum information processing. There are some radiation fields that initially have classical properties, but after added some photons possess nonclassical properties. The photon-added coherent states (PACSs) are typical example.

PACSs was introduced by Agarwal and Tara in 1991[26], and this state exhibits an intermediate property between a classical coherent state and a purely quantum Fock state. If we only consider the one-photon excitation of a classical coherent field, the generated state is called single-photon-added coherent state (SPACS). SPACS not only have interesting properties, but also be useful for possible future applications including the engineering of quantum states [27], quantum information protocols [28], and entangled state generation [29]. After Zavatta et al. first studied the experimental generation of the SPACS and visualized the evolution
of the quantum-to-classical transition [30], various schemes to generation [31,32] enhance the nonclassicality of SPACS widely investigated [33–35]. In general, the enhancement of nonclassicality of SPACS depends on the optimization of this state, and the optimization of quantum states is related to quantum measurement.

Measurement is a basic concept in physical science, and any information of the system can be obtained from the measurement processes. In general, a measurement is composed of three parts, i.e., measuring device, measured system, and environment. According to the requirements of quantum measurement, in a quantum measurement process, there should have been an interaction between measuring device (pointer) and measured system, and this interaction should be short enough to guarantee the measurement precision. Thus, in quantum measurement theory interaction Hamiltonian can be expressed by von Neumann Hamiltonian as $H = g \hat{A} \otimes \hat{P}$ [36]. Here, $\hat{A}$ is the operator of measured system we want to measure, $\hat{P}$ is the canonical momentum of measuring device, and $g$ represent the measuring strength. The measurement strength can decide the amount of information of the measured system after the measurement. That is to say, if the interaction strength between the measured system and apparatus (measuring device) is strong, i.e., $g \gg 1$, then we can get the information of the system we want to obtain by single trial with very small error. Whereas if the interaction strength between measured system and apparatus is very weak, i.e., $g \ll 1$, the interference between different eigenvalues of the system observable we want to measure still exists and cannot distinguish them. In weak coupling case ($g \ll 1$), we only can get very trivial information of the system by single trial [37]. However, the mission of getting enough information of the system observable in weak coupling regime introduced a new kind of quantum measurement theory, which called weak measurement.

The weak measurement, which characterized by postselection and weak value, was proposed by Aharonov, Albert, and Vaidman in 1988 [37] and considered as a generalized von Neumann quantum measurement theory. In weak measurement theory, the coupling between the pointer and the measured systems is sufficiently weak, and the obtained information by single trial is trivial. Even though the postselected weak measurement on single system provides trivial information, by repeating it on an arbitrarily large ensembles of identical system we can determine the average result with arbitrary precision [38]. One of the distinguished properties of weak measurement compared with strong measurement is that its induced weak value of the observable on the measured system can be beyond the usual range of the eigenvalues of that observable [39]. The feature of weak value is usually referred to as an amplification effect for weak signals rather than a conventional quantum measurement and used to amplify many weak but useful information in physical systems. So far, the weak measurement technique has been applied in different fields to investigate very tiny effects, such as beam deflection [40–44], frequency shifts [45], angular shifts [46], velocity shifts [47], and even temperature shift [48]. For details about the weak measurement and its applications in signal amplification processes, we refer the reader to the recent overview of the field [49,50]. In weak measurement, we only consider the evolution of unitary operator up to its first order since the interaction strength between the system and measuring device is very weak. However, if we want to connect the weak and strong measurement, check to clear the measurement feedback of postselected weak measurement and analyze experimental results obtained in nonideal measurements, the full-order evolution of unitary operator is needed [51–53], and we call this kind of measurement is postselected von Neumann measurement.

The signal amplification properties of weak measurement can be used in state optimization problems. Recently, the state optimization problem by using postselected von Neumann measurement has been presented widely, such as taking the Gaussian states [49,54], Hermite–Gaussian or Laguerre–Gaussian states, and nonclassical states [55,56]. The advantages of
nonclassical pointer states in increasing postselected measurement precision have been examined in recent studies [56–58]. In Ref. [59], the authors studied the effects of postselected measurement characterized by modular value [60] to show the properties of semi-classical and nonclassical pointer states considering the coherent, coherent squeezed, and Schrodinger cat state as a pointer. Most recently, the author of this paper investigated the effects of postselected von Neumann measurement on the properties of single-mode radiation fields [61] and found that postselected von Neumann measurement really changed the photon statistics and squeezing parameters of radiation fields for different weak values and coupling strengths. However, to our knowledge, the effects of postselected von Neumann measurement on nonclassicality of SPACS have not been previously investigated, neither exactly nor analytically, in any literature.

In this paper, motivated by the previous studies [58, 59, 61], we investigate the effects of postselected von Neumann measurement on nonclassicality of SPACS. In order to achieve our goal, we take the spatial degree of freedom of SPACS as measuring device and its polarization degree of freedom as measured system. First of all, we obtained the normalized final state of SPACS after postselected von Neumann measurement by taking all interaction strengths between system and measuring device into account. Then calculate the exact expressions and give numerical results of physical quantities of SPACS such as photon number distribution, \(Q_m\) factor and squeezing parameter. The numerical results showed that postselected von Neumann measurement dramatically changed the nonclassicality of SPACS. We notice that in our scheme the measurement strengths and weak values induced by postselection played an important role in changing the nonclassicality of SPACS. Here, we have to mention that SPACS is different from coherent, squeezed and Schrodinger cat states, which we studied in previous works [59, 61] and have wide applications in many quantum information processes including entangled state generation [29], quantum key distribution [62, 63], quantum digital signature [64]. Thus, the current research is worthy to study the more effective methods to the implementations of related processes.

The rest of this paper is organized as follows: In Sect. 2, we introduce the basic model setup for our scheme. In Sects. 3 and 4, we give the details about the effects of postselected von Neumann measurement on the nonclassicality of SPACS. We calculate the photon number distribution, Mandel \(Q_m\) factor and squeezing parameter of SPACS and found that postselected von Neumann measurement can change the nonclassicality of SPACS by changing the coupling strengths and weak values. We give a conclusion to our paper in Sect. 5. Throughout this paper, we use the unit \(\hbar = 1\).

## 2 Model setup

In this section, we introduce the related theories to our current research. According to the standard quantum measurement theory, the coupling interaction between system and measuring device of our scheme is taken to the standard von Neumann-type Hamiltonian [36]

\[
H = g \delta(t - t_0) \hat{\sigma}_x \otimes \hat{P}.
\]

Here, \(g\) is a coupling constant and \(\hat{P}\) is the conjugate momentum operator to the position operator \(\hat{X}\) of the measuring device, i.e., \([\hat{X}, \hat{P}] = i \hbar\). \(\hat{\sigma}_x = |H\rangle\langle V| + |V\rangle\langle H|\) is observable of the measured system we want to measure, and \(|H\rangle\) and \(|V\rangle\) are horizontal and vertical polarization of the beam with eigenvalues 1 and \(-1\), respectively. In general, to guarantee the measurement precision, the interaction time between measured system and measuring...
device is too short so that \( \int_0^T g \delta(t - t_0)dt = g \). Thus, the time evolution operator \( e^{-i \int_0^T H dt} \) of our system can be written as \( e^{-i g \hat{\sigma}_x \otimes \hat{P}} \).

The position operator \( \hat{X} \) and momentum operator \( \hat{P} \) of measuring device can be written in terms of annihilation (creation) operator, \( \hat{a} (\hat{a}^\dagger) \) in Fock space representation as

\[
\hat{X} = \sigma (\hat{a}^\dagger + \hat{a}),
\]

\[
\hat{P} = \frac{i}{2\sigma} (\hat{a}^\dagger - \hat{a}),
\]

where \( \sigma \) is the width of the beam. We know that the annihilation (creation) operator \( \hat{a} (\hat{a}^\dagger) \) obeys the commutation relation, \([\hat{a}, \hat{a}^\dagger] = 1\). By substituting Eq.(3) into the unitary evolution operator \( e^{-i g \hat{\sigma}_x \otimes \hat{P}} \), we get

\[
e^{-i g \hat{\sigma}_x \otimes \hat{P}} = \frac{1}{2} (\hat{I} + \hat{\sigma}_x) \otimes D\left(\frac{s}{2}\right) + \frac{1}{2} (\hat{I} - \hat{\sigma}_x) \otimes D\left(-\frac{s}{2}\right),
\]

where parameter \( s \) is defined by \( s := g/\sigma \) and it can characterize the measurement strengths, and \( D(\alpha) \) is a displacement operator with complex \( \alpha \) defined by \( D(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}} \). During the derivation of Eq. (4), we use the property \( \hat{\sigma}_x^2 = I \) of system observable \( \hat{\sigma}_x \). We can say that the coupling between system and pointer is weak (strong) if \( s \ll 1 (s \gg 1) \), and in this study we will consider all interaction strengths between system and measuring device.

In this paper, we consider the spatial transversal freedom of SPACS as measuring device (MD) and its polarization of freedom as measured system and study the effects of the measurement which performed on polarization part on the inherent properties of spatial part of SPACS. The SPACS is defined as

\[
|\Psi\rangle = \gamma \hat{a}^\dagger |\alpha\rangle
\]

where \( \gamma = (1 + |\alpha|^2)^{-1} \) is normalization coefficient, and \( |\alpha\rangle = D(\alpha)|\alpha\rangle \) is coherent state with coherent parameter \( \alpha = re^{i\theta} \). As proposed in original paper of Agarwal [26], this state can be produced in nonlinear processes in cavities, and its first implementation is given with nonlinear optical scheme [30]. It is well known that if we subtract on photon from coherent radiation field, the generated state still is a classical field, i.e., \( \hat{a}|\alpha\rangle = \alpha |\alpha\rangle \). However, a single-photon excitation of a coherent state changes it into something quite different. In other words, the application of the creation operator \( \hat{a}^\dagger \) changes completely the classical coherent state into a new quantum state which possesses nonclassicality.

As indicated in Fig. 1, if we assume that initially the system prepared to \( |\psi_i\rangle \) and the initial state of the measuring device is \( |\Psi\rangle \), then the evolution of the total system can be written as \( e^{-i g \hat{\sigma}_x \otimes \hat{P}} |\psi_i\rangle |\Psi\rangle \). Since our mission is to study the measurement effects on the properties
of state $|\Psi\rangle$, the normalized final state of the measuring device can be obtained by taking the postselection with postselected state $|\psi_f\rangle$ onto $e^{-i\hat{\sigma}_x \otimes \hat{P}} |\psi_i\rangle |\Psi\rangle$. After some calculation, it reads

$$|\Phi\rangle = \beta \left[ (1 + \langle \hat{\sigma}_x \rangle_w) D\left(\frac{\hat{s}}{2}\right) + (1 - \langle \hat{\sigma}_x \rangle_w) D\left(-\frac{\hat{s}}{2}\right) \right] |\Psi\rangle. \quad (6)$$

Here, the normalization coefficient $\beta$ defined by

$$\beta = \frac{1}{\sqrt{2}} \left[ 1 + |\langle \hat{\sigma}_x \rangle_w|^2 + y^2 e^{-\frac{\gamma^2}{2}} Re[(1 + \langle \hat{\sigma}_x \rangle_w^* (1 - \langle \hat{\sigma}_x \rangle_w) + e^{2\pi i m} |\alpha| (1 + (\alpha^* + s)(\alpha - s))]^{-\frac{1}{2}}, \quad (7)$$

and

$$\langle \hat{\sigma}_x \rangle_w = \frac{\langle \psi_f |\hat{\sigma}_x |\psi_i\rangle}{\langle \psi_f |\psi_i\rangle} \quad (8)$$

is the weak value of system observable $\sigma_x$. In general, the weak value $\langle \hat{\sigma}_x \rangle_w$ is complex and can be beyond the average values of eigenvalues of observable $\sigma_x$. That is to say, if the preselection state $|\psi_i\rangle$ and postselection state $|\psi_f\rangle$ are almost orthogonal, the $\langle \hat{\sigma}_x \rangle_w$ can take large values, and this feature of weak value is used as amplification of weak signals of related physical systems. In this research, we assume that initially the system is prepared to the polarization state $|\psi_i\rangle = \cos \frac{\varphi}{2} |H\rangle + e^{i\delta} \sin \frac{\varphi}{2} |V\rangle$ with $\delta \in (0, 2\pi]$ and $\varphi \in [0, \pi]$. After the interaction between the system and measuring device, the system state is postselected to $|\psi_f\rangle = |H\rangle$, and then, the weak value of the system observable $\sigma_x$ reads as

$$\langle \sigma_x \rangle_w = e^{i\delta} \tan \frac{\varphi}{2}. \quad (9)$$

From this expression, we can see that this weak value is a complex number and its value can be beyond the eigenvalues of $\hat{\sigma}_x$. However, this large weak value is accompanied by low successful postselection probability, $P_s = |\langle \psi_f |\psi_i\rangle|^2 = \cos^2 \frac{\varphi}{2}$. The interaction Hamiltonian, Eq. (1), pre- and postselected states which we introduced in this paper can be implemented in optical experiments by using polarizing beam splitter and half/quarter wave plates [65].

In the next section, we study the effects of postselected von Neumann measurement on nonclassicality of SPACS by taking into account the signal amplification effect induced by postselection and weak value.

3 Effects of photon statistics

In this section, in order to investigate the effects of postselected von Neumann measurement on photon statistics of SPACS, we check the photon number distribution and Mandel $Q_m$ factor of SPACS after measurement.

3.1 Photon number distribution

In this subsection, we check the effect of postselected von Neumann measurement on photon number distribution of SPACS. The probability of finding $n$ photons under the final state $|\Phi\rangle$ of SPACS after the measurement is given by

$$P(n) = |\langle n |\Phi\rangle|^2. \quad (10)$$
The explicit expression of $P(n)$ can be obtained by substituting $|\Phi\rangle$, which is given in Eq. (6), to the above expression, and its corresponding numerical results are shown in Fig. 2. From previous studies, it can be deduced that the initial SPACS have sub-Poissonian distribution [26], and the solid black curve in Fig. 2a represents the photon number distribution of initial SPACS. In Fig. 2a we check the effects of interaction strengths on photon number distribution, the result showed that with increasing interaction strength, the photon number distribution of SPACS become broader and occurs oscillation in definite photon regions. We know that weak value has amplification property on weak signals when pre- and postselection of system state is almost orthogonal. In Fig. 2b, we display the photon number distribution $P(n)$ of SPACS as a function of photon numbers $n$ for different weak values. As indicated in Fig. 2b, in weak measurement regime, the probability of finding $n$ photons decreased as it increases the weak value, and photon distribution curves become narrower than the initial state. This implies that in the weak measurement regime, after postselected von Neumann measurement, the sub-Poissonian property of SPACS is increased with large weak values.

3.2 The Mandel $Q_m$ factor of SPACS

The photon variance of initial SPACS is less than its mean photon number so that it has non-classicality, which is characterized by sub-Poissonian photon statistics [26]. We know that if the underling $P$- function does not possess or negative Wigner function is negative then the corresponding radiation field has nonclassicality, and SPACS defined in Eq. (5) possess both of them [26]. However, in this study, in order to check the effects of postselected von Neumann measurement on nonclassicality of SPACS, we choose the experimentally accessible method—Mandel $Q_m$ factor. This parameter is very useful to characterize the nonclassicality of any radiation field and it was introduced by Mandel in 1979 [66]. According to his study, if any distribution which is narrower than Poissonian distribution must correspond to a non-classical radiation field, and this nonclassicality of that radiation field can be characterized by Mandel $Q_m$ factor. The definition of Mandel $Q_m$ factor is

$$Q_m = \frac{\langle (\hat{a}^\dagger \hat{a})^2 \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 - \langle \hat{a}^\dagger \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle}. \quad (11)$$

It is clear from the above expression that $Q_m = 0$ for coherent state and $Q_m = -1$ for Fock state. The negativity of $Q_m$ ($-1 \ll Q_m < 0$) is a sufficient condition for the field to be nonclassical [67]. However, if $Q_m > 0$ we can’t make a conclusion about the nonclassicality of the field.
The Mandel $Q_m$ parameter of SPACS after postselected von Neumann measurement as a function of coherent state parameter $r$ for different coupling strengths $s$ with fixed weak value ($\varphi = \frac{\pi}{9}$) in (a), and for different weak values with fixed coupling strength ($s = 0.1$) in (b). Here, $\delta = 0$, $\theta = \frac{\pi}{4}$.

The $Q_m$ parameter for initial pointer state represented by $|\Psi_1\rangle$ in Eq. (5) is given by

$$Q_{m,\Psi} = -\gamma^2 \frac{1 + 2|\alpha|^2 + 2|\alpha|^4}{1 + 3|\alpha|^2 + |\alpha|^4}.$$  \hspace{1cm} (12)

Furthermore, the $Q_{m,\Phi}$ can be obtained by calculating the expectation values in Eq. (11) under the final normalized state of the pointer $|\Phi_1\rangle$ after measurement. Since the explicit expression of $Q_{m,\Phi}$ is too cumbersome to show here, we only give its numerical results in Fig. 3. In Fig. 3a, we plot $Q_{m,\Phi}$ as a function of coherent state parameter $r$, and the solid black curve ($s = 0$) represents the initial state case which is described by $Q_{m,\Psi}$. It is showed that the nonclassicality of photon-added coherent state is attenuated as coupling strength $s$ increases for definite weak value. In Fig. 3b, we check the effects of different weak values on Mandel $Q_{m,\Phi}$ parameter for fixed coupling strength. As indicated in Fig. 3b, the nonclassicality of photon-added coherent state is increased as increasing the weak value in weak coupling regime ($s < 1$), and these results keep in accordance with numerical results of photon number distribution which is presented in Fig. 2. From Fig. 3a and b, we can induce that in weak coupling regime the photon statistics of SPACS gradually possess more sub-Poissonian as increasing the weak value which accompanied by low postselection probability.

4 Effects of squeezing parameter

In this section, we study the effects of postselected von Neumann measurement on squeezing parameter of SPACS. In general, the squeezing parameter of radiation field is defined as [67]

$$S_\phi = (\Delta X_\phi)^2 - \frac{1}{2}$$  \hspace{1cm} (13)

where

$$X_\phi = \frac{1}{\sqrt{2}}(\hat{a}e^{-i\phi} + \hat{a}^\dagger e^{i\phi}), \quad \phi \in [0, 2\pi]$$  \hspace{1cm} (14)

is the quadrature operator of the field with

$$[\hat{X}_\phi, \hat{X}_{\phi + \frac{\pi}{2}}] = i$$  \hspace{1cm} (15)
Fig. 4  The squeezing parameter $S_{\phi}$ of SPACS after postselected von Neumann measurement. Here, $\delta = 0$, and (a) $\psi = \frac{\pi}{9}$, $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$; (b) $s = 1$, $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$; (c) $r = 2$, $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$; (d) $\psi = \frac{\pi}{9}$, $\theta = 0$, $\phi = \frac{\pi}{2}$.

and

$$\langle \Delta X_{\phi} \rangle^2 = \langle \Phi | X_{\phi}^2 | \Phi \rangle - \langle \Phi | X_{\phi} | \Phi \rangle^2.$$  \hspace{1cm} (16)

From the definition of $S_{\phi}$, it can be seen that $S_{\phi} \geq -\frac{1}{2}$. If $-\frac{1}{2} \leq S_{\phi} \leq 0$, then there has a squeezing effect of corresponding quadrature of the radiation field. For initial pointer state $|\Psi\rangle$ of SPACS, the squeezing parameter $S_{\phi,\Psi}$ is given by

$$S_{\phi,\Psi} = \gamma^4 \left[ 1 - |\alpha|^2 \cos 2(\phi - \theta) \right].$$  \hspace{1cm} (17)

From this expression, it can be deduced that when $|\alpha|^2 \cos 2(\phi - \theta) > 1$, there will occur a squeezing of the corresponding quadrature of SPACS. It is well known that the coherent state is a minimum uncertainty state and its squeezing parameter is equal to zero. However, after added on photon, its squeezing feature changed dramatically since the generated new radiation field possesses nonclassicality.

The explicit expression of squeezing parameter $S_{\phi}$ of SPACS after postselected von Neumann measurement can be calculated by using the final state $|\Phi\rangle$ and Eq. (13). Since its exact expression is too cumbersome to show here, in this paper we just analyzed the corresponding numerical results. As detailed in Figs. 4 and 5, the postselected von Neumann measurement changed the squeezing effect of photon-added coherent state significantly. In Fig. 4a, we plot the $S_{\phi,\Psi}$ vs coherent state parameter $r$ for various coupling strengths. The black solid curves in Fig. 4a represents the squeezing parameter of initial SPACS, $|\Psi\rangle$, and it indicated that the maximum squeezing of initial SPACS is just reached 1.3 dB (21.6% squeezing) near $r = 2$. The other curves in Fig. 4a denote the squeezing parameter of the field quadrature of $X_{\phi=\frac{\pi}{2}}$ after postselected measurement with various coupling strengths. It can be seen that the squeezing effect of SPACS is increased after postselected measurement for some regions. In Fig. 4b and c, we check the effects of different weak values on squeezing effect of SPACS for fixed coupling strength and fixed coherent state parameter $r$. The results
showed that weak values induced by postselected von Neumann measurement can increase the squeezing effect of SPACS for definite coupling strengths compared to initial SPACS (see the black solid curve in Fig. 4a). As mentioned above, the squeezing effects of initial SPACS are phase dependent and if only when the condition $\theta = \phi$ is satisfied, then there will occur the squeezing of corresponding quadrature of the SPACS. However, as indicated in Fig. 4d, the condition $\theta = \phi$ is no more needed to realize the quadrature squeezing of SPACS after postselected von Neumann measurement.

In order to further study the effects of weak values on phase catch condition of squeezing parameter of SPACS after postselected von Neumann measurement, in Fig. 5 we plot the squeezing parameter $S_{\phi}$ by changing weak values and quadrature phase $\phi$, respectively. It is visible by comparing Fig. 5a and b that the condition $\theta = \phi$ is not necessary for realizing the squeezing effect of field quadratures of SPACS after postselected von Neumann measurement. We notice that as shown in Fig. 4a, near $r = 2$ the $X_{\phi=\pi/2}$ quadrature of initial state reached its maximum squeezing (see the black solid curve in Fig. 4a). Thus, in Fig. 5c and d, we fixed coherent state parameter ($r = 2$) and checked the quadrature squeezing of $S_{\phi,\phi}$ vs weak values and quadrature phase $\phi$ for various moderate coupling strengths, respectively. We found that the $X_{\phi=\pi/2}$ quadrature of the SPACS has always squeezing compared with $r = 4$ case (Fig. 5b), whereas the $X_{\phi=0}$ quadrature always has no squeezing. In a word, as indicated in Fig. 5c and d, when the system has anomalous weak values and moderate coherent state parameters such as $r$ equal to two ($r = 2$) with $\theta = \pi/2$, the $X_{\phi=\pi/2}$ quadrature of measuring device always has squeezing for moderate coupling strengths $s$.

The above results indicated that after the postselected measurement the squeezing of the quadrature of SPACS is changed dramatically with increasing weak values and coupling strengths $s$ between system and measuring device compared to no interaction case (see the black solid curve in Fig. 4d) for moderate coherent state parameters. The maximum squeezing
reached 3.98dB (66.3% squeezing) corresponding to $(\Delta X_{\phi})^2 = 0.2$ than the initial state which possesses maximum squeezing with $(\Delta X_{\phi})^2 = 0.37$ (21.6% squeezing).

5 Conclusion and remarks

In this study, we investigated the effects of postselected von Neumann measurement on non-classicality of SPACS. In order to achieve our goal, first of all we derived explicit expression of SPACS after measurement by considering all coupling strengths between system and measuring device. We checked the photon number distribution, Mandel $Q_m$ factor and squeezing parameter of SPACS after postselected von Neumann measurement for various system parameters. Our numerical results are given according to the exact expressions of corresponding physical quantities, and they showed that postselected von Neumann measurement changed the nonclassicality, which characterized by sub-Poissonian photon distribution of SPACS dramatically. We found that with increasing weak values the photon number distribution becomes narrower, the negativity of Mandel $Q_m$ factor and squeezing parameter are increased than the initial state. Our numerical results also showed that after postselected von Neumann measurement, the squeezing parameter of SPACS is no more suffered the rigid phase matching condition as the initial state.

We anticipate that the presented optimization scheme of nonclassicality of SPACS in this paper would be helpful to provide other effective methods to implement the related practical problems in quantum information processing as mentioned in this paper. Furthermore, in this paper we only consider one photon excitation of coherent radiation field, and it is a simple case of PACSs. Thus, effects of postselected von Neumann measurement on radiation field properties of more that one-photon excitation of coherent radiation fields are still worth to study. Work along this line is in progress, and results will be presented in near future.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant No. 11865017), the Introduction Program of High Level Talents of Xinjiang Ministry of Science and the Natural Science Foundation of Xinjiang Uyghur Autonomous Region (Grant No. 2020D01A72).

References

1. G.S. Buller, R.J. Collins, Meas. Sci. Technol 21, 12002 (2010). https://doi.org/10.1088/0957-0233/21/1/012002
2. H. Grote, M. Weinert, R.X. Adhikari, C. Affeldt, H. Wittel, Opt. Express 24, 20107 (2016). https://doi.org/10.1364/OE.24.020107
3. F.Y. Khalili, H. Miao, Y. Chen, Phys. Rev. D 80, 042006 (2009). https://doi.org/10.1103/PhysRevD.80.042006
4. S.J. van Enk, O. Hirota, Phys. Rev. A 64, 022313 (2001). https://doi.org/10.1103/PhysRevA.64.022313
5. H. Jeong, M.S. Kim, J. Lee, Phys. Rev. A 64, 052308 (2001). https://doi.org/10.1103/PhysRevA.64.052308
6. G.J. Milburn, S.L. Braunstein, Phys. Rev. A 60, 937 (1999). https://doi.org/10.1103/PhysRevA.60.937
7. S.L. Braunstein, H.J. Kimble, Phys. Rev. Lett. 80, 869 (1998). https://doi.org/10.1103/PhysRevLett.80.869
8. T.C. Ralph, A. Gilchrist, G.J. Milburn, W.J. Munro, S. Glancy, Phys. Rev. A. 64, 042319 (2003). https://doi.org/10.1103/PhysRevA.64.042319
9. L. Li, Y.O. Dudin, A. Kuzmich, Nature 498, 466 (2013). https://doi.org/10.1038/nature12227
10. B. Hacker, S. Welte, S. Daiss, A. Shaukat, S. Ritter, L. Li, G. Rempe, and G. Nat. Photon. 13, 110 (2019). https://doi.org/10.1038/s41566-018-0339-5
11. C.A. Muschik, K. Hammerer, E.S. Polzik, J.I. Cirac, Phys. Rev. A 73, 062329 (2006). https://doi.org/10.1103/PhysRevA.73.062329
12. W.J. Munro, K. Nemoto, G.J. Milburn, S.L. Braunstein, Phys. Rev. A 66, 023819 (2002). https://doi.org/10.1103/PhysRevA.73.023819
13. R.J. Glauber, Phys. Rev. 131, 2766 (1963). https://doi.org/10.1103/PhysRev.131.2766
14. U.M. Titulaer, R.J. Glauber, Phys. Rev. 145, 1041 (1966). https://doi.org/10.1103/PhysRev.145.1041
15. D. Stoler, Phys. Rev. D 4, 155 (1971). https://doi.org/10.1103/PhysRevD.4.155
16. P.D. Walls, Nature 306, 141 (1983). https://doi.org/10.1038/306141a0
17. R.J. Glauber, Phys. Rev. 131, 2766 (1963). https://doi.org/10.1103/PhysRev.131.2766
18. U.L. Andersen, T. Gehring, C. Marquardt, G. Leuchs, Phys. Scripta. 91, 053001 (2016). https://doi.org/10.1088/0031-8949/91/5/053001
19. C. Monroe, D.M. Meekhof, B.E. King, D.J. Wineland, Science 272, 113 (1996). https://doi.org/10.1126/science.272.5265.1131
20. A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri, P. Grangier, Nature 448, 784 (2007). https://doi.org/10.1038/nature06054
21. H. Yuen, Phys. Rev. A 13, 2226 (1976). https://doi.org/10.1103/PhysRevA.13.2226
22. V.V. Dodonov, V.I. Man'ko, D.E. Nikonov, Phys. Rev. A 51, 3328 (1995). https://doi.org/10.1103/PhysRevA.51.3328
23. H.P. Yuen, Phys. Rev. A 13, 2226 (1976). https://doi.org/10.1103/PhysRevA.13.2226
24. D. Stoler, B.E.A. Saleh, Opt. Acta 32, 345 (1985). https://doi.org/10.1080/713821735
25. T.C. Lee, Phys. Rev. A 31, 1213 (1985). https://doi.org/10.1103/PhysRevA.13.1213
26. G.S. Agarwal, K. Tara, Phys. Rev. A 43, 492 (1991). https://doi.org/10.1103/PhysRevA.43.492
27. A.P. Lund, H. Jeong, T.C. Ralph, M.S. Kim, Phys. Rev. A 70, 020101 (2004). https://doi.org/10.1103/PhysRevA.70.020101
28. J. Wenger, R. Tualle-Brouri, P. Grangier, Phys. Rev. Lett. 92, 153601 (2004). https://doi.org/10.1103/PhysRevLett.92.153601
29. Y. Li, H. Jing, M.-S. Zhan, Phys. B-At Mol Opt. 39, 2107 (2006). https://doi.org/10.1088/0953-4075/39/9/001J
30. A. Zavatta, S. Viciani, M. Bellini, Science 306, 660 (2004). https://doi.org/10.1126/science.1103190
31. Y. Li, H. Jing, M.S. Zhan, Phys. Lett. A 372, 4177 (2008). https://doi.org/10.1016/j.physleta.2008.03.061
32. M. Barbieri, N. Spagnolo, M.G. Genoni, F. Ferreyrol, R. Blandino, M.G.A. Paris, P. Grangier, R. Tualle-Broui, Phys. Rev. A 82, 063833 (2010). https://doi.org/10.1103/PhysRevA.82.063833
33. V.V. Dodonov, M.A. Marchiolli, Y.A. Korennoy, V.I. Man’ko, Y.A. Moukhin, Phys. Rev. A 58, 4087 (1998). https://doi.org/10.1103/PhysRevA.58.4087
34. A. Zavatta, S. Viciani, M. Bellini, Phys. Rev. A 72, 023820 (2005). https://doi.org/10.1103/PhysRevA.72.023820
35. D. Kalamidas, C.C. Gerry, A. Benmoussa, Phys. Lett. A 372, 1937 (2008). https://doi.org/10.1016/j.physleta.2007.10.089
36. J. von Neumann, Mathematical Foundations of Quantum Mechanics (Princeton University Press, Princeton, 1955)
37. Y. Aharonov, D.Z. Albert, L. Vaidman, Phys. Rev. Lett. 60, 1351 (1988). https://doi.org/10.1103/PhysRevLett.60.1351
38. J. Tollaksen et al., New J. Phys. 12, 013023 (2010). https://doi.org/10.1088/1367-2630/12/1/013023
39. Y. Aharonov, D. Rohrlich, Quantum Paradoxes- Quantum Theory for the Perplexed (Wiley-VCH, Weinheim, 2005)
40. O. Hosten, P. Kwiat, Science 319, 787 (2008)
41. L. Zhou, Y. Turek, C.P. Sun, F. Nori, Phys. Rev. A 88, 053815 (2013). https://doi.org/10.1103/PhysRevA.88.053815
42. M. Pfeifer, P. Fischer, Opt. Express 19, 16508 (2011)
43. P.B. Dixon, D.J. Starling, A.N. Jordan, J.C. Howell, Phys. Rev. Lett. 102, 173601 (2009). https://doi.org/10.1103/PhysRevLett.102.173601
44. D.J. Starling, P.B. Dixon, A.N. Jordan, J.C. Howell, Phys. Rev. A 80, 041803 (2009). https://doi.org/10.1103/PhysRevA.82.041803
45. D.J. Starling, P.B. Dixon, A.N. Jordan, J.C. Howell, Phys. Rev. A 82, 063822 (2010). https://doi.org/10.1103/PhysRevA.82.063822
46. O.S. Magnaño Loaiza, M. Mirhosseini, B. Rodenburg, R.W. Boyd, Phys. Rev. Lett. 112, 200401 (2014). https://doi.org/10.1103/PhysRevLett.112.200401
47. G.I. Viza, J. Martínez-Rincón, G.A. Howland, H. Frostig, I. Shomroni, B. Dayan, J.C. Howell, Opt. Lett. 38, 2949 (2013)
48. P. Egan, J.A. Stone, Opt. Lett. 37, 4991 (2012)
49. A.G. Kofman, S. Ashhab, F. Nori, Phys. Rep. 520, 43 (2012). https://doi.org/10.1016/j.physrep.2012.07.001
50. J. Dressel, M. Malik, F.M. Miatto, A.N. Jordan, R.W. Boyd, Rev. Mod. Phys. 86, 307 (2014). https://doi.org/10.1103/RevModPhys.86.307
51. Y. Aharonov, A. Botero, Phys. Rev. A 72, 052111 (2005). https://doi.org/10.1103/PhysRevA.72.052111
52. A. Di Lorenzo, J.C. Egues, Phys. Rev. A 77, 042108 (2008). https://doi.org/10.1103/PhysRevA.77.042108
53. A.K. Pan, A. Matzkin, Phys. Rev. A 85, 022122 (2012). https://doi.org/10.1103/PhysRevA.85.022122
54. K. Nakamura, A. Nishizawa, M.-K. Fujimoto, Phys. Rev. A 85, 012113 (2012). https://doi.org/10.1103/PhysRevA.85.012113
55. B. de Lima Bernardo, S. Azevedo, A. Rosas, Opt. Commun. 331, 194 (2014). https://doi.org/10.1016/j.optcom.2014.06.008
56. Y. Turek, H. Kobayashi, T. Akutsu, C.-P. Sun, Y. Shikano, New J. Phys. 17, 083029 (2015). https://doi.org/10.1088/1367-2630/17/8/083029
57. S. Pang, T.A. Brun, Phys. Rev. Lett. 115, 120401 (2015). https://doi.org/10.1103/PhysRevLett.115.120401
58. Y. Turek, W. Maimaiti, Y. Shikano, C.-P. Sun, M. Al-Amri, Phys. Rev. A 92, 022109 (2015). https://doi.org/10.1103/PhysRevA.92.022109
59. Y. Turek, T. Yusufu, Phys. J. D 72, 202 (2018). https://doi.org/10.1140/epjd/e2018-90258-8
60. Y. Kedem, L. Vaidman, Phys. Rev. Lett. 105, 230401 (2010). https://doi.org/10.1103/PhysRevLett.105.230401
61. Y. Turek, Chinese Phys. B 29, 090302 (2020). https://doi.org/10.1088/1674-1056/ab9f23
62. M. Miranda, D. Mundarain, Quant. Inf. Proc 16, 298 (2017). https://doi.org/10.1007/s11128-017-1752-2
63. Y. Wang, W.S. Bao, H.Z. Bao et al., Phys. Lett. A 381, 1393 (2017). https://doi.org/10.1016/j.physleta.2017.01.058
64. J.J. Chen, C.H. Zhang, J.M. Chen, C.M. Zhang, Q. Wang, Quant. Inf. Proc (2020). https://doi.org/10.1007/s11128-020-02695-5
65. A. Hariri, D. Curic, L. Giner, J.S. Lundeen, Phys. Rev. Lett. 100, 032119 (2019). https://doi.org/10.1103/PhysRevLett.100.032119
66. L. Mandel, Opt. Lett. 4, 205 (1979). https://doi.org/10.1364/OL.4.000205
67. G.S. Agarwal, Quantum Optics (Cambridge University Press, Cambridge, 2013)