Development of a thermal-visible video surveillance system based on fractional order tv-model

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Abstract. Detection and tracking of moving objects in video are an active and challenging research area of computer vision devoted to the applications of video surveillance. The several environmental conditions such as dark, foggy, snowing and rainy, and open-ended goals of the problem motivate to develop a robust surveillance system that based on the thermal-visible video spectrum fusion. However, visible-visible and thermal-thermal spectrum based surveillance model show insufficiency under such conditions. The detection of moving objects is performed by means of optical flow. This paper presents a novel fractional order total variation (TV) model in the estimation of optical flow. In particular, the presented fractional order TV-model is designed by generalizing an integer order total variation model formed by using a combination of total variation and quadratic regularization terms. However, it is challenging to solve such complex minimization problem due to the non-differentiability nature of fractional order TV-term. The fractional order derivative is discretized using the Grünwald-Letnikov derivative. The primal-dual algorithm is used as an iterative scheme to solve the resulting formulation. Finally, a number of experimental results on a fusion of visible-visible, thermal-thermal and visible-thermal video spectrum demonstrate the effectiveness of the model.

Keywords- Infrared sensors, Motion field, Total variation, Video surveillance

1. Introduction
Video surveillance is one of the dominant and active research area in the computer vision of every era. In the past decade, a significant amount of work has been done in this area while suited the scope of its real time applications such as border safety, crime preventing, unmanned vehicles, traffic management and many other issues related to public property and safety. However, most of the surveillance security systems utilize a single visible spectrum information as their input, where the video spectrum is captured in the presence of visible lights. These visible-light security system works effectively and get a certain level of accuracy in the normal weather conditions but fails under different environmental conditions like moving backgrounds, rainy, snowing, foggy, dark or low visibility and when the foreground and background objects have similar color sense. Hence, visible-light video surveillance system results in a poor motion detection as well as tracking of the objects under these
conditions. This emerges the necessity of surveillance applications to think beyond the visible spectrum and consider thermal (infrared) or thermal-visible spectrum, which is highly efficient for night vision and other environmental conditions. Thermal imager is based on infrared (IR) radiations, which have a longer wavelength compared to the visible spectrum [1].

The most important property of the thermal imagery is that the image pixel intensity remains unchanged under environment conditions because the pixel intensity value depend on the energy (radiations) emitted from the objects. However, in visible spectrum such a problem occurs very frequently due to the presence of unstable backgrounds, dust and smoke. In thermal vision system intensity or color of the image pixels is a function of the temperature of the object in a scene. Hence, the higher is the emitted energy (temperature) from the source brighter is the corresponding image. In the recent past, thermal imaging has been also frequently using in the disease detection, navigation, tourism, search and rescue, etc [2–5]. However, thermal imagery has some disadvantages also. One of them is that different objects having the different colors look similar if they are causing the equal temperature. Thus, it is very challenging to distinguish such objects but it is most unlikely that different objects may have the same temperature as their IR radiations may be different. The second disadvantage is that the infrared sensors are quite sensitive to noise when compared to the color sensors [6]. Therefore, a multispectral system that fuses the thermal and visible video is useful in providing the more information and overcome the problems of low visibility and partially occluded objects. The motion detection and tracking in thermal-visible images are carried out with the help of optical flow. A number of models with different assumptions have been presented in the computation of optical flow after the seminal work [7]. However, these models attained a satisfactory level of performance under various conditions.

In the recent-past, computer vision researchers shown a keen interest in differential variational models. Thus, optical flow estimation is more preferred by this optimization framework [8–15]. However, the accuracy of the variational methods have been continuously improved by including different environment conditions and assumptions. The algorithms presented in [8, 9] considered the gradient constancy assumption as well as the convexity term, which helps to convert into global minima and provide an increased robustness against outliers and noise. A number of the algorithms such as [8, 9, 12, 16] considered several quadratic regularization methods to demonstrate discontinuity in the motion field. However, the algorithms considered in [17–19] employed $L_1$ norm, quadratic and total variation regularization (TV) terms. Alex et al. [20] used posterior probability likelihoods to track the motion of pedestrians in thermal-visible video while Kumar et al. [21] is based on Kalman filter and fuzzy logic. Yan et al. [22] employed deep neural network to classify the active regions in the thermal and visible spectrum. However, Torabi et al. [23] have proposed a RANSAC trajectory framework for surveillance applications that employed affine transformation matrix. Apart from the thermal surveillance system, Zhang et al. [24] used TV-GAN network for face recognition in the non-light thermal videos. All these models used integer order derivatives. The performance of these integer order variational algorithms in terms of accuracy and stability can be improved by generalizing their differential order between (0, 2) [25, 26]. Nowadays, the fractional order methods have been frequently used in many real life vision applications [14, 27–29]. Nevertheless, the very rare amount of work has been contributed in the motion estimation. The most significant feature of fractional derivative is that we can find derivatives in the case of discontinuous function, while integer order methods are unable to do so. The complete theoretical explanation including properties, salient features and advantages can be found in [14,15,25,30,31].
Contribution

In this paper, a robust thermal-visible video surveillance system is developed by introducing a novel variational fractional order TV-model. The fusion of thermal-visible video spectrum provides a robust surveillance system against environmental conditions and overcome the problems of low visibility and partially occluded objects. The motion tracking in the image sequence is done by using the assumption of optical flow. The novelty of this work also includes: (i) The presented model provides an effective robustness against environmental conditions, which is imparted by the formulation of total variation regularization and quadratic terms. (ii) The variational functional leads to the discontinuity preserving dense optical flow due to the fusion of global and classical models of [7, 12]. (iii) A fractional order generalization of the variational integer order TV-models in (0, 2). The fractional order derivative discretization of the model into a numerical scheme is carried out with the help of Grünwald-Letnikov derivative. Finally, the discretized variational functional is numerically computed with the help of primal-dual technique. The validation of the proposed model is performed by considering the fusion of visible-visible, thermal-thermal and visible-thermal video spectrum.

2. Proposed variational fractional order TV-model in optical flow

Estimation of the optical flow \( u = (u, v) \) in the image sequences using a seminal work of Horn et al. [7] is based on the minimization of the variational model defined as

\[
F(u) = \int_{\Phi} \left( |\nabla u|^2 + |\nabla v|^2 + \lambda \left( r(u, v) \right)^2 \right) dX
\]

where, \( I_{2w} = I_2,\ r(u, v) = (u - u_0) \nabla I_{2w} + I_{2w} - I_1 \) and \( \lambda > 0 \) is known as a regularization parameter. Also, \( X = (x, y) \) and \( I_1, I_2 \subset \mathbb{R}^2 \) and \( u_0 \) represent the image frames and initial approximation of the motion in the image sequence, respectively.

The above variational functional (1) can be modified into the following form according to [18] in order to determine a more accurate, robust, discontinuity preserving and dense optical flow,

\[
F(u) = \int_{\Phi} \left( |\nabla u| + |\nabla v| + \beta|u + v| + \lambda \left( r(u, v) \right)^2 \right) dX
\]

here, \( \nabla := \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \) and \( \beta \) is an arbitrary constant. The motive behind the formulation of the variational model (2) using the total variation-L1 and quadratic terms is to impart the robustness against motion blur, noise and illumination, and improve the minimization scheme. Furthermore, the presented variational functional fused the approaches of models [7] and [12]. The model of [7] furnishes smooth and dense optical flow for each region, while the model of [12] renders more sharp boundaries and edges in the flow fields. Hence, the resulting variational functional (2) keeps more accuracy as well as gives discontinuity preserving dense optical flow.

The proposed variational fractional order TV-model in the estimation of optical flow of variational functional (2) can be written as

\[
F(u) = \int_{\Phi} \left( |D^\alpha u| + |D^\alpha v| + \beta|u + v| + \lambda \left( r(u, v) \right)^2 \right) dX
\]

where, \( D^\alpha := \left( D^\alpha_x, D^\alpha_y \right)^T \) is called the Riemann-Liouville fractional derivative [31] and \( |D^\alpha u| = \sqrt{(D^\alpha_x u)^2 + (D^\alpha_y u)^2} \). It is obvious from (3) that when \( \alpha \in \mathbb{Z}^+ \), the proposed fractional order TV-model generalizes a class of the integer order variational models.
The proposed TV-model (3) can be divided into the following three models according to [32] in order to determine the optical flow \( \mathbf{u} = (u, v) \)

\[
F_{TV-1} = \int_{\Phi} \left[ \frac{1}{2\theta} (u - \hat{u})^2 + \frac{1}{2\theta} (v - \hat{v})^2 + \lambda (r(\hat{u}, \hat{v}))^2 \right] d\mathbf{X} \tag{4}
\]

\[
F_{TV-u} = \int_{\Phi} \left[ |D^\alpha u| + \beta |u| + \frac{1}{2\theta} (u - \hat{u})^2 \right] d\mathbf{X} \tag{5}
\]

\[
F_{TV-v} = \int_{\Phi} \left[ |D^\alpha v| + \beta |v| + \frac{1}{2\theta} (v - \hat{v})^2 \right] d\mathbf{X} \tag{6}
\]

here, \( \theta \) is known as the tightness parameter, which tells us the closeness of \( \hat{u} \) and \( \hat{v} \) from \( u \) and \( v \), respectively. In TV-1 variational model, \( u \) and \( v \) are considered as fixed whereas \( \hat{u} \) and \( \hat{v} \) unknowns. The variational functionals given in TV-\( u \) and TV-\( v \) are simplified using the theory proposed in [33].

According to the Euler-Lagrange method of integral calculus, minimization of the unknowns \( (\hat{u}, \hat{v}) \) of (4) results into the following system of equations

\[
v - 2\lambda \theta \tau_0 I_2^y = 2\lambda \theta I_2^x I_2^y \hat{u} + (1 + 2\lambda \theta (I_2^y)^2) \hat{v}
\]

\[
u - 2\lambda \theta \tau_0 I_2^x = (1 + 2\lambda \theta (I_2^x)^2) \hat{u} + 2\lambda \theta I_2^x I_2^y \hat{v}
\]

where, \( I_t = I_{2w} - I_1 \) and \( \tau_o = I_t - u_o I_2^x - v_o I_2^y \). Hence, the simplified iterative expressions for the variables \( \hat{u} \) and \( \hat{v} \) can be defined as

\[
\hat{u} = \frac{(1 + 2\lambda \theta (I_2^y)^2)u - 2\lambda \theta (I_2^x I_2^y)\nu}{D}
\]

\[
\hat{v} = \frac{2\lambda \theta (I_2^x I_2^y)u - (1 + 2\lambda \theta (I_2^x)^2)v + 2\lambda \theta \tau_0 I_2^y}{-D}
\]

where, \( D \) represents the determinant of (7).

In order to minimize the variational functional TV-\( u \) using the theory of model [33], it is necessary to discretized the fractional derivative of order \( \alpha \). The derivative discretization is carried out with the help of Grunewald-Letnikov definition [30] as

\[
D_x^\alpha u_{i,j} = \sum_{p=0}^{W-1} w_p^{(\alpha)} u_{i+p,j} \quad \text{and} \quad D_y^\alpha u_{i,j} = \sum_{p=0}^{W-1} w_p^{(\alpha)} u_{i,j+p}
\]

(10)

where, \( W \) is called the window mask size and

\[
w_p^{(\alpha)} = (-1)^p S_p^\alpha \quad \text{and} \quad S_p^\alpha = \frac{\Gamma (\alpha + 1)}{\Gamma (p + 1) \Gamma (\alpha - p + 1)}
\]

Here, \( \Gamma (\alpha) \) denotes the Gamma function.

In order to solve the discretized equations of TV-\( u \), using the primal-dual algorithm as proposed in [34], we assign the pairwise component \( (i,j) \) of \( u \) and \( \hat{u} \) in \( P \) and \( Q \) such that

\[
P_{(j-1)n+i} = u_{i,j} \quad \text{and} \quad Q_{(j-1)n+i} = \hat{u}_{i,j}
\]

(11)
where, \( N \in \mathbb{N}^2 \) and \( P, Q \in \mathbb{R}^N \). Here, \( n \) is the amount of pixels in the image. Hence, the derivative discretization of fractional order \( \alpha \) of \( u \) defined as

\[
A^{(\alpha)}_q X = \begin{cases} 
(\sum_{p=0}^{N-1} w_p^{(\alpha)} P_{q+p}, \sum_{p=0}^{N-1} w_p^{(\alpha)} P_{q+np})^T \\
\text{if}(q \text{ mod } n) \neq 0 \quad \text{and} \quad q \leq N - n \\
(0, \sum_{p=0}^{N-1} w_p^{(\alpha)} P_{q+np})^T \\
\text{if}(q \text{ mod } n) = 0 \quad \text{and} \quad q < N - n \\
(\sum_{p=0}^{N-1} w_p^{(\alpha)} P_{q+p}, 0)^T \\
\text{if}(q \text{ mod } n) = 0 \quad \text{and} \quad q > N - n \\
(0, 0)^T \\
\text{if}(q \text{ mod } n) \neq 0 \quad \text{and} \quad q > N - n
\end{cases} 
\]  

(12)

for \( q = 1, 2, \ldots, N \), where, \( A^{(\alpha)}_q \in \mathbb{R}^{N \times 2} \). Thus, the problem specified discretize representation of the TV-\( u \) variational functional is given as

\[
E_{TV-u} = \sum_{q=1}^{N} \| A^{(\alpha)}_q P - \beta P \| + \frac{1}{2\theta} \| P - Q \|^2
\]  

(13)

The above expression is similar to the image denoising model as given in [33]. Hence, its solution in terms of primal-dual algorithm as described in [34], can be written as

\[
u^{p+1} = \frac{u^p - \tau_p \text{div}\alpha d^{p+1} + \tau_p \frac{1}{\beta} \hat{u}}{1 + \frac{1}{\beta} \tau_p}
\]  

(14)

where, \( d \) is an unknown variable related to the problem of dual. The similar steps would be adopted to solve the functional of TV-\( v \).

3. Experiments, results and discussions

3.1. Datasets

Experimental datasets demonstrate a significant role in assessing the performance of any algorithm. However, there is a lack of infrared or thermal image datasets, but few datasets are available. This work discusses the following databases which have considered based on their different properties,

- **Visible dataset**: Home and Mans [35]
- **Thermal dataset**: Home and Mans [35]

The reference and target image frames of the given database are illustrated in Figs. 1-2. The further characterization along with various features and conditions would be found in [35].

3.2. Performance measure:-

The validation of the performance is also done by evaluated the STD (standard deviation) and color map of the flow field [36]. This standard deviation illustrates the subpixel accuracy in the motion field.
3.3. Experimental discussions

All the algorithms have been evaluated on the MATLAB platform of version R2019a in a windows workstation of 128GB RAM. The estimation process, analysis and comparisons are briefly demonstrated by performing different experiments on a variety of image sequences where reference and target frames are considered as visible-visible, thermal-thermal and visible-thermal, respectively. The five scale warping method is used to deal large motion. The number of iterations, $N_{\text{warp}}$, $N_{\text{scale}}$ and the different parameters are chosen based on the properties of the data sets. The minimum values of few parameters are set as $\lambda = 100$, $\theta = 0.5$ and $\beta = 0.8$. All the fractional derivatives are obtained by using a window framework of order $3 \times 3$. The experimental results are illustrated both in quantitative as well as qualitative forms. Optical flow vector plots and color maps are considered to demonstrate the qualitative results. In vector representation of output results, the movement of a target is illustrated by the divergence of vectors while in the color maps different color indicates multiple motion and homogeneous color represents either static background or large displacement. The quantitative results are given in the form of statistical error.

In the first experiment, the statistical result (STD) is estimated by varying the values of $\alpha$ in the range of $(0, 2)$ for the given image sequences by considering the reference and target image frames as visible-visible, thermal-thermal and visible-thermal, respectively. The relationship between $\alpha$ and STD are shown in the third row of Figs. 1-2, whereas after $\alpha = 1.6$, the STD results are more unstable. The smaller is the STD result value presents the accuracy in the estimation. Therefore, the optimal fractional order for these image sequences is chosen by considering the lowest statistical error as shown in the Figs. 1-2 that shows the stability of the results. It is observed that in case of the Home image sequence, the optimal fractional order for the fusion of the visible-visible, thermal-thermal and visible-thermal spectrum is 0.8, 0.6 and 0.8, respectively. The optimal fractional order for the Mans image sequence in the same combination is 0.8, 1 and 0.8.

The next experiment is performed on Home outdoor image sequence which contains total 2107 frames both in visible and thermal spectrum. The frame numbers 670 and 685 of size $320 \times 240$ pixels are considered as the reference and target image, respectively. This image sequence represents the front view of a building under shadow where multiple objects are moving independently and some of them are occluded. The estimated results for the visible-visible spectrum in terms of the optical flow vector plot and color map corresponding to their optimal fractional order and the reference image are shown in the first column of Fig. 1. The second and third column of Fig. 1 illustrate the results for thermal-thermal and visible-thermal spectrum, respectively. It is clear from the Fig. 1 that the proposed fractional order TV-model provides efficient good results in all three cases. The color maps are dense and clearly indicate the movement of different objects, which can be justified from the vector plots. However, the occluded objects can be seen easily in the thermal-thermal and visible-thermal spectrum results. The effect of noise and outliers (shadow, etc) is negligible while considering the TV-regularization term. Thus, the proposed algorithm provides a better results under various conditions.

The third experiment is performed on Mans image sequence where multiple objects are moving fast and two of them are occluded by tree. This image sequence consisting of total 3012 frames both in visible and thermal spectrum, and each of size $320 \times 240$ pixels. This image sequence has captured using a street camera placed on a circular pole. The reference and target image are considered as the frame numbers 64 and 80 in all the three cases. The estimated results for visible-visible, thermal-thermal and visible-thermal spectrum
corresponding to their optimal fractional order (0.8, 1 and 0.8) have been represented in Fig. 2. In Fig. 2, the occlusion effect can be observed in case of the visible-visible spectrum while it is removed in thermal-thermal and visible-thermal spectrum, respectively. However, the color maps clearly demonstrate the discontinuity between different objects and provide smooth and dense flow inside a region. The quantitative results illustrated in Fig. 3 in terms of STD demonstrate the superiority of the proposed TV-model compare to [23,37]. However, all the algorithms have computed at the same parameter settings.

Figure 1. Results of the estimated Optical flow in case of the Home image sequence for the visible-visible spectrum (first column), thermal-thermal spectrum (second column) and visible-thermal spectrum (third column). The reference and target image frames are shown in first two rows, third and fourth row contain their corresponding α order and vector plots, and color maps are illustrated in the fifth row.
Figure 2. Output qualitative results in case of the Mans image sequence for the visible-visible spectrum (in first column), thermal-thermal spectrum (in second column) and visible-thermal spectrum (in third column). The reference and target image frames are shown in first two rows, third and fourth row contain their corresponding $\alpha$ order and vector plots and color maps are illustrated in the fifth row.

4. Conclusions and future remark

In this work, a robust video surveillance system has developed in the fusion of thermal-visible spectrum by introducing a novel variational fractional order TV-model, which is performed using a total variation $L_1$ norm and quadratic terms. The motion tracking of targets in the multi-spectral image sequence has performed using optical flow. Experimental results from the fusion of thermal-thermal and visible-thermal spectrum are good under several environmental conditions. The optical flow color maps are dense and preserves discontinuity. It is also observed that for $\alpha \in Z^+$, the proposed model generalizes a class of variational integer order. The introduced TV-model would be a great relax to the feature selection algorithms.
Figure 3. Comparative study of the STD for algorithm 1 (proposed model) v/s Liu et al. [37] model (algorithm 2) v/s Torabi et al. [23] model (algorithm 3) for the visible-thermal image sequence.

References

[1] Berg A 2016 Detection and Tracking in Thermal Infrared Imagery PhD thesis Linköping University Electronic Press.

[2] Kim J H, Starr J W and Lattimer B Y 2015 Firefighting robot stereo infrared vision and radar sensor fusion for imaging through smoke Fire Technology 51(4) 823–845.

[3] Mambou S J, Maresova P, Krejcar O, Selamat A and Kuca K 2018 Breast cancer detection using infrared thermal imaging and a deep learning model Sensors 18(9) 2799.

[4] Yakimenko O A, Kaminer I I, Lentz W J and Ghyzel PA 2002 Unmanned aircraft navigation for shipboard landing using infrared vision IEEE Transactions on Aerospace and Electronic Systems 38(4) 1181–1200.

[5] Yu T, Mo B, Liu F, Qi H and Liu Y 2019 Robust thermal infrared object tracking with continuous correlation filters and adaptive feature fusion Infrared Physics & Technology 98 69–81.

[6] Gade R and Moeslund T B 2014 Thermal cameras and applications: a survey Machine vision and applications 25(1) 245–262.

[7] Horn B and Schunck B 1981 Determining optical flow Artificial Intelligence 17 185–203.

[8] Black M J and Anandan P 1996 The robust estimation of multiple motions: Parametric and piecewise smooth flow Computer Vision and Image Understanding 63 (1) 75–104.

[9] Brox T, Bruhn A, Papenberg N and Weickert J 2004 High accuracy optical flow estimation based on a theory for warping European Conference on Computer Vision (ECCV 2004) (4) 25–36.

[10] Kumar P, Kumar S and Raman B 2015 A vision based motion estimation in underwater images International Conference on Advances in Computing, Communications and Informatics (ICACCI) 1179–1184.

[11] Kumar P and Kumar S 2016 A modified variational functional for estimating dense and discontinuity preserving optical flow in various spectrum International Journal of Electronics and Communications 70(3) 289–300.

[12] Nagel H H and Enkelmann W 1986 An investigation of smoothness constraints for the estimation of displacement vector fields from image sequences IEEE Transactions on Pattern Analysis and Machine Intelligence 8 565–593.

[13] Niese R, Hamadi A A, Farag A, Neumann H and Michaelis B 2012 Facial expression recognition based
on geometric and optical flow features in colour image sequences IET Computer Vision 6(2) 79–89.

[14] Chen D, Sheng H, Chen YQ and Xue D 2013 Fractional-order variational optical flow model for motion estimation Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences 371(1990) 20120148.

[15] Bardeji S G, Figueiredo I N and Sousa E 2017 Optical flow with fractional order regularization: variational model and solution method Applied Numerical Mathematics 114 188–200.

[16] Weickert J 1998 On discontinuity-preserving optic flow Computer Vision and Mobile Robotics Workshop.

[17] Werlberger M, Trobin W, Pock T, Wodel A, Cremers D and Bischof H 2009 Anisotropic huber-L1 optical flow British Machine Vision Conference (BMVC) vol 1.

[18] Zach C, Pock T and Bischof H 2007 A duality based approach for realtime TV-L1 optical flow Pattern Recognition p 214–223.

[19] Drulea M and Nedevschi S 2011 Total variation regularization of local-global optical flow 14th International Conference on Intelligent Transportation p 318–323.

[20] Leykin A and Hammoud R 2006 Robust multi-pedestrian tracking in thermal-visible surveillance videos Conference on Computer Vision and Pattern Recognition Workshop (CVPRW’06) p 136–136.

[21] Kumar P, Mittal A and Kumar P 2006 Fusion of thermal infrared and visible spectrum video for robust surveillance International Conference on Computer Vision, Graphics and Image Processing p 528–539.

[22] Yan Y, Zhao H, Kao F J, Vargas VM, Zhao S and R J 2018 Deep background subtraction of thermal and visible imagery for pedestrian detection in videos International Conference on Brain Inspired Cognitive Systems p 75–84.

[23] Torabi A, Massé G and Bilodeau G A 2012 An iterative integrated framework for thermal–visible image registration sensor fusion, and people tracking for video surveillance applications Computer Vision and Image Understanding 116(2) 210–221.

[24] Zhang T, Willem A, Yang S and Lovell B 2018 Tv-gan: Generative adversarial network based thermal to visible face recognition International Conference on biometrics p 174–181.

[25] Miller K S and Ross B 1993 An introduction to the fractional calculus and fractional differential equations (New York:Wiley).

[26] Oldham K B 1974 The fractional calculus Elsevier.

[27] Kumar P, Kumar S and Raman B 2016 A fractional order variational model for the robust estimation of optical flow from image sequences Optik 127(20) 8710–8727.

[28] Pu Y F, Zhou J L and Yuan X 2010 Fractional differential mask: a fractional differential-based approach for multiscale texture enhancement IEEE Transactions on Image Processing 19(2) 491–511.

[29] Tian D, Xue D and Wang D 2015 A fractional-order adaptive regularization primal–dual algorithm for image denoising Information Sciences 296 147–159.

[30] Miller K S 1995 Derivatives of noninteger order Mathematics magazine 183–192.

[31] Riemann B 1876 Versuch einer allgemeinen auffassung der integration und differentiation Gesammelte Werke 62.

[32] Chambolle A 2004 An algorithm for total variation minimization and applications Journal of Mathematical Imaging and Vision 20 (1–2) 89–97.

[33] Rudin L I, Osher S and Fatemi E 1992 Nonlinear total variation based noise removal algorithms Physica D: Nonlinear Phenomena 60(1) 259–268.

[34] Chen D, Chen YQ and Xue D 2013 Fractional-order total variation image restoration based on primal-dual algorithm Abstract and Applied Analysis 2013.

[35] Davis J W and Sharma V 2007 Background-subtraction using contour-based fusion of thermal and visible imagery Computer Vision and Image Understanding 106(2) 162–182.

[36] Barron J L, Fleet D J and Beauchemin S 1994 Performance of optical flow techniques International Journal of Computer Vision 12 43–77.

[37] Liu Q, He Z, Li X and Zheng Y 2019 Ptb-tir: A thermal infrared pedestrian tracking benchmark IEEE Transactions on Multimedia 22(3) 666–675.