Coherent $\Theta^+$ and $\Lambda(1520)$ photoproduction off the deuteron

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Abstract

We analyze an effect of the coherent $\Theta^+\Lambda(1520)$ photoproduction in $\gamma D$ interaction near the threshold. We demonstrate that the coherence effect becomes manifest in a comparison of the $nK^+$ invariant mass distribution when the $pK^-$ invariant equals the $\Lambda(1520)$ mass. Our model calculations indicate a sizeable contribution of resonant and non-resonant background processes in the $\gamma D \rightarrow npK^+K^-$ reaction which generally exceed the contribution of the coherent resonant channel. However, we find that the coherent $\Theta^+\Lambda(1520)$ photoproduction is enhanced relative to the background processes in the forward hemisphere of the $pK^-$ pair photoproduction. Moreover, the coherence effect does not depend on the $\Theta^+$ photoproduction amplitude and is defined by the probabilities of the $\Lambda(1520)$ photoproduction and the $\Theta^+ \rightarrow NK$ transition. Therefore, this coherence effect may be used as an independent method for studying the mechanism of $\Theta^+$ production and $\Theta^+$ properties.

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I. INTRODUCTION

The first evidence for the pentaquark hadron \( \Theta^+ \) discovered by the LEPS collaboration at SPring-8 [1] was subsequently confirmed in other experiments [2]. However, some other experiments failed to find the \( \Theta^+ \) signal (for a review see [3, 4]). Since then the situation concerning the existence of the pentaquarks remained controversial. Independent studies of the manifestation of a \( \Theta^+ \) state in different processes are, therefore, urgently desired.

\( \Theta^+ \) photoproduction in the reaction \( \gamma D \rightarrow npK^+K^- \) seems to be very interesting and important [5, 6]. First, it allows to study simultaneously the \( \gamma p \rightarrow \Lambda(1520)K^+ \) and \( \gamma n \rightarrow \Theta^+K^- \) subreactions characterized by the similarity in the production mechanisms, i.e. both processes are described by the same set of the tree level Feynman diagrams [7–9]. Therefore, one hopes to define the ratio of \( \Theta^+ \) to \( \Lambda(1520) \) photoproduction with minimal uncertainty of the production mechanisms, which is important for understanding the nature of \( \Theta^+ \). Second, in case of the \( \gamma D \) interaction one can study qualitatively a new basic process - the coherent \( \Theta^+\Lambda(1520) \) photoproduction. This reaction has its own physics interest and unambiguously will shed new light to pentaquark properties and the mechanism of the \( \Theta^+ \) photoproduction.

It is commonly supposed now that the total width of the \( \Theta^+ \) is as small as \( \Gamma_{\Theta} \sim 1 \text{ MeV} \) [10], being much smaller than the total \( \Lambda^* \) decay width, \( \Gamma_{\Lambda^*} \simeq 15.6 \text{ MeV} \) [11]. (Throughout this paper, for simplicity, we use notation \( \Lambda^* \equiv \Lambda(1520) \).) This means that the most promising way for studying the coherent \( \Lambda^*\Theta^+ \) production is to analyze the invariant \( nK^+ \) mass, \( M_{nK^+} \), distribution at fixed invariant mass of the \( pK^- \) pair, \( M_{pK^-} \). The enhancement of the \( \Theta^+ \) photoproduction, when \( M_{pK^-} \) is in the vicinity of the \( \Lambda^* \) mass, would indicate the manifestation of the coherent \( \Lambda^*\Theta^+ \) photoproduction. This particular channel will appear in strong competition with the resonant and non-resonant background processes. By the notation ”resonant process” we mean, for example, the \( \Lambda^* \) photoproduction from the proton inside the deuteron, when the neutron is a spectator, and similarly the \( \Theta^+ \) photoproduction from a neutron, when the deuteron’s proton is a spectator. The notation ”non-resonant” process denotes \( K^+K^- \) photoproduction from a nucleon without excitation of \( \Lambda^* \) or \( \Theta^+ \). It is clear that the coherent photoproduction and the background processes must be analyzed together using the same theoretical approaches. This allows to define the kinematical conditions where the coherent channel manifests itself clearly above strong background processes.
The aim of the present paper is to discuss these important aspects. Our model includes the elementary subprocesses of $\gamma N \rightarrow \Lambda^* K$ and $\gamma N \rightarrow \Theta^+ \bar{K}$ reactions. For the latter ones we use a model based on the effective Lagrangian approach of Ref. [8] which is, generally speaking, similar to the models developed by other authors in Refs. [12–22]. All these approaches predict the approximate equality of the cross sections of the $\gamma n \rightarrow \Theta^+ K^-$ and $\gamma p \rightarrow \Theta^+ \bar{K}^0$ reactions. This equality may be changed into a suppression of the $\gamma p \rightarrow \Theta^+ \bar{K}^0$ transition [7, 23]. However, we are going to demonstrate that the amplitude of the coherent $\Lambda^* \Theta^+$ photoproduction, when $\Lambda^*$ is produced in the forward hemisphere in the $\gamma D$ center of mass system, is defined by the product of the $\Lambda^*$ photoproduction amplitude in $\gamma N$ interaction and the amplitude of the $\Theta^+ \rightarrow NK$ transition. In other words, the coherence effect of the $\Lambda^* \Theta^+$ photoproduction in the forward hemisphere does not depend on the $\Theta^+$ photoproduction amplitude and remains finite even if the cross section of the $\gamma p \rightarrow \Theta^+ \bar{K}^0$ reaction is vanishing. The coherence effect in the backward hemisphere is sensitive to the $\Theta^+$ photoproduction amplitude, and it is suppressed in parallel with the suppression of the $\gamma p \rightarrow \Theta^+ \bar{K}^0$ reaction.

Our paper is organized as follows. In Sec. II, we discuss the resonant $\Theta^+$ and $\Lambda^*$ photoproduction from a nucleon. In Sec. III, we consider the coherent $\gamma D \rightarrow \Lambda^* \Theta^+$ reaction. Our model is similar to the approach of Ref. [24], developed for coherent $\Theta^+ \Lambda(\Sigma^0)$ photoproduction from a deuteron. In Sec. IV, we discuss the background processes. We start thereby from an analysis of the non-resonant background in "elementary" $\gamma N \rightarrow \Theta^+ \bar{K}$ and $\gamma N \rightarrow \Lambda^* K$ reactions. Then we apply these subprocesses to an analysis of the background spectator channels. Finally, we estimate the contribution of the coherent semi-resonant processes, which differ from the coherent photoproduction by the replacement of one hyperon by $NK$ or $N\bar{K}$ pairs. The results of our numerical calculations are presented in Sec. V. The summary is given in Sec. VI. In Appendix A, we show an explicit form of the transition operators for the resonance amplitude.
II. PHOTOPRODUCTION FROM A NUCLEON

A. $\Theta^+$ photoproduction

The main diagrams for the amplitude of the resonance $\Theta^+$ photoproduction in the reaction $\gamma N \rightarrow NK\bar{K}$ are shown in Fig. 1. We neglect here the contribution resulting from the photon interacting with the final decay vertex [12]. In view of the chosen kinematics, where the invariant mass of the final $KN$ pair is near the resonance position, this is a good approximation since in the neglected graphs the $\Theta^+$ is far off-shell and the graphs of Figs. 1a-d dominate the resonance contribution. From a formal point of view gauge invariance is lost without contributions arising from the electromagnetic interaction in the decay vertex. However, following Ref. [25] for the initial photoproduction process, we construct an overall conserved current by an appropriate choice of the contact term of Fig. 1d.

In this section $k$, $p$, $q$, $\bar{q}$, and $p'$ denote the four-momenta of the incoming photon, the initial nucleon, the outgoing $K$ and $\bar{K}$ mesons, and the recoil nucleon, respectively. The standard Mandelstam variables for the virtual $\Theta^+$ photoproduction are defined by $t = (\bar{q} - k)^2$, $s \equiv W^2 = (p + k)^2$. The $\bar{K}$ meson production angle $\theta$ in the center-of-mass system (c.m.s.) is given by $\cos \theta = k \cdot \bar{q}/(|k||\bar{q}|)$, and the corresponding solid angle is $\Omega$. We consider the integrated $\Theta^+$ decay distribution. The differential cross section $\gamma N \rightarrow \Theta^+\bar{K} \rightarrow NK\bar{K}$ as a function the $\bar{K}$ meson production angle and $NK$ invariant mass, $M_{nK^+}$, at the resonance position with $M_{nK^+} = M_\Theta = 1.54$ GeV is related to the cross
section of the $\Theta^+$ photoproduction in the $\gamma N \rightarrow \Theta^+ \bar{K}$ reaction as

$$\frac{d\sigma_{f_i}}{d\Omega \, dM_{nK^+}} \bigg|_{M_{nK^+}=M_\Theta} = \frac{1}{\pi \Gamma_\Theta} \frac{d\sigma_{f_i}^{\Theta^+}}{d\Omega}$$

(1)

with $\Gamma_\Theta$ as $\Theta^+$ decay width and

$$\frac{d\sigma_{f_i}^{\Theta^+}}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_{\text{out}}}{p_{\text{in}}} \frac{1}{4} \sum_{m_i, m_f, \lambda_\gamma} |A_{m_f, m_i, \lambda_\gamma}^{\Theta^+}|^2$$

(2)

Here, $A_{f_i}^{\Theta^+}$ is the $\Theta^+$ photoproduction amplitude in the $\gamma N \rightarrow \Theta^+ \bar{K}$ reaction, $m_i$ and $m_f$ are the nucleon and $\Theta^+$ spin projections, respectively, and $\lambda_\gamma$ denotes the incoming photon helicity; $p_{\text{in}}$ and $p_{\text{out}}$ are the relative momenta in the initial and the final states in c.m.s., respectively. Further on we will concentrate on the calculation of $A_{f_i}^{\Theta^+}$. For simplicity, in this analysis we limit our consideration to the isoscalar, spin-1/2 $\Theta^+$. Generalization for higher spin [9] may be done in a straightforward manner.

The effective Lagrangians which define the Born terms for the diagrams shown in Fig. 1a - d are discussed in many papers (for references see Ref. [8]). Note that different phase conventions are often employed. Therefore, for the sake of definiteness, we list here the effective Lagrangians used in the present work:

$$\mathcal{L}_{\gamma KK} = i e (K^- \partial^\mu K^+ - K^+ \partial^\mu K^-) A_\mu$$

(3a)

$$\mathcal{L}_{\gamma \Theta \Theta} = -e \theta \left( \gamma_\mu - \frac{\kappa_\Theta}{2M_\Theta} \sigma_{\mu\nu} \partial^\nu \right) A^\mu \Theta$$

(3b)

$$\mathcal{L}_{\gamma NN} = -e \bar{N} \left( e_N \gamma_\mu - \frac{\kappa_N}{2M_N} \sigma_{\mu\nu} \partial^\nu \right) A^\mu N$$

(3c)

$$\mathcal{L}_{\Theta NN}^{[\text{pv}]} = \mp \sqrt{\frac{g_{\Theta NN}}{M_\Theta \pm M_N}} \bar{\Theta} \Gamma^\pm_\mu (\partial^\mu K) N + \text{h.c.}$$

(3d)

$$\mathcal{L}_{\Theta NN}^{[\text{ps}]} = -i \sqrt{\frac{g_{\Theta NN}}{M_\Theta \pm M_N}} \bar{\Theta} \Gamma^\pm_\mu A^\mu K N + \text{h.c.}$$

(3e)

$$\mathcal{L}_{\Theta NN}^{[\text{ps}]} = -i g_{\Theta NN} \bar{\Theta} \Gamma^\pm_\mu K N + \text{h.c.}$$

(3f)

$$\mathcal{L}_{\Theta NN}^{[\text{ps}]} = \frac{eg_{\Theta NN}}{M_\Theta \pm M_N} \bar{\Theta} \Gamma^\pm_\mu A^\mu K N + \text{h.c.}$$

(3g)

$$\mathcal{L}_{\Theta NN}^{[\text{ps}]} = \frac{eg_{\Theta NN}}{M_\Theta \pm M_N} \bar{\Theta} \Gamma^\pm_\mu A^\mu K N + \text{h.c.}$$

(3h)

where $A^\mu$, $\Theta$, $K$, and $N$ are the photon, $\Theta^+$, kaon, and the nucleon fields, respectively, $K^*$ stands for the vector kaon field; $\Gamma^\pm_\mu \equiv \Gamma^\pm_\mu (\text{with } \Gamma^+ = \gamma_5 \text{ and } \Gamma^- = 1$ for positive

1 Throughout this paper, isospin operators will be suppressed in all Lagrangians and matrix elements for simplicity. They can be easily accounted for in the corresponding coupling constants.
and negative parity, respectively), $e_p = 1$, $e_n = 0$, and $\kappa_N$ denotes the nucleon anomalous magnetic moment ($\kappa_p = 1.79$ and $\kappa_n = -1.91$), $\kappa_\Theta$ stands for the anomalous magnetic moment of $\Theta^+$ and $\kappa^*$ denotes the tensor coupling of nucleon and strange vector mesons. The superscripts "PS" and "PV" correspond to the pseudo-scalar and pseudo-vector $\Theta^+ NK$ coupling schemes. Equation (3e) describes the contact (Kroll-Ruderman) interaction in the pseudo-vector coupling scheme (see Fig. 1d), which does not appear in case of the pseudo-scalar coupling (cf. Eq. (3f)).

In calculating the invariant amplitudes we dress the vertices by form factors. In the present tree-level approach and within our chosen kinematics, only the lines connecting the electromagnetic vertex with the initial $\Theta^+ KN$ vertex correspond to off-shell hadrons. We describe the product of both the electromagnetic and the hadronic form-factor contributions along these off-shell lines by the covariant phenomenological function

$$F(M, p^2) = \frac{\Lambda^4}{\Lambda^4 + (p^2 - M^2)^2},$$

(4)

where $p$ is the corresponding off-shell four-momentum of the virtual particle, $M$ denotes its mass, and $\Lambda$ stands for the cut-off parameter. The electromagnetic current of the complete amplitude is conserved by making the initial photoproduction process gauge invariant. To this end, we apply the gauge invariance prescription by Haberzettl [25] with the modification by Davidson and Workman [26] to construct a contact term for the initial process $\gamma N \rightarrow \Theta^+ K$ free of kinematical singularities. We emphasize that contributions of the latter type are necessary even for pure pseudo-scalar coupling.

Since the coupling scheme and the $\Theta^+$ parity are unknown one has to define the corresponding parameters in such a way to get the corresponding cross sections independently of $\Theta^+$ parity and coupling scheme. We follow Ref. [8], where parameters of the model are fixed by a comparison of the resonant $\Theta^+$ photoproduction cross section and non-resonant background with experiment and it is shown that one can find such a parameter set which parallels the prediction for PS and PV couplings and for positive and negative $\Theta^+$ parity states as well, at least for the unpolarized, single and double polarization spin observables. Therefore, we can limit the present analysis to the PS coupling and a positive $\Theta^+$ parity.

The resonance amplitudes obtained for the $\gamma n$ and $\gamma p$ reactions read

$$A_{fi}^{\Theta^+} (\gamma n) = \bar{u}_\Theta(p_\Theta) \left[ M_s^\mu + M^t_\mu + M^u_\mu + M^c_\mu + M^t_\mu(K^*) \right] u_n(p) \varepsilon^\mu,$$

(5a)

$$A_{fi}^{\Theta^+} (\gamma p) = \bar{u}_\Theta(p_\Theta) \left[ M_s^\mu + M^u_\mu + M^c_\mu + M^t_\mu(K^*) \right] u_p(p) \varepsilon^\mu.$$

(5b)
The explicit forms of the transition operators $M^i_\mu$ for the $\gamma n \to \Theta^+K^-$ and $\gamma p \to \Theta^+\bar{K}^0$ reactions are exhibited in Appendix A.

For a positive $\Theta^+$ parity the coupling constant $g_{\Theta NK}$ is found from the $\Theta^+$ decay width as

$$\Gamma_\Theta = \frac{|g_{\Theta NK}|^2 p_F}{2\pi M_\Theta} (\sqrt{M_N^2 + p_F^2} - M_N).$$  \hspace{1cm} (6)

We choose a small width, $\Gamma_\Theta = 1$ MeV [10], assuming that the observed width in the invariant mass distribution is determined by the experimental resolution. The magnitude of the coupling constant $g_{\gamma KK^*}$ is extracted from the width of the $K^* \to \gamma K$ decay [11]. Its sign is fixed by SU(3) symmetry. This delivers $eg_{\gamma K^0K^*0} = -0.35$ and $eg_{\gamma K^\pm K^*\pm} = 0.23$.

The contribution of the $s$-channel (Fig. 1b) is small causing to a rather weak dependence of the total amplitude on the tensor coupling $\kappa_\Theta$ in the $\gamma \Theta \Theta$ vertex within a “reasonable” range of $0 \lesssim |\kappa_\Theta| \lesssim 0.5$ [27]. Therefore, we can choose $\kappa_\Theta = 0$. The coupling constant $g_{\Theta NK^*}$ is written as $g_{\Theta NK^*} = \alpha_\Theta g_{\Theta NK}$, where the parameter $\alpha_\Theta$ depends on the choice of the tensor coupling $\kappa^*$ in Eq. (3h) and cut-off parameters $\Lambda_K^*$ in the form factors of the $K^*$ exchange amplitude. Increasing value of $\Lambda_K^*$ leads to a decreasing $\alpha_\Theta$. Following Ref. [8] we use $\Lambda_K^* = 1.5$ GeV and $\alpha_\Theta = 1.875$ at $\kappa^* = 0$. This value of $\alpha_\Theta$ is close to the quark model estimates $\alpha_\Theta = \sqrt{3}$ [28].

Another cut-off parameter, $\Lambda_B$, defines the Born terms of the $s$-, $u$-, and $t$-channels and the current-conserving contact terms. Note that the inclusion of the $\Sigma$ and $\Lambda$ photoproduction processes [29] results in a larger ambiguity in the choice of $\Lambda_B$ which varies from 0.5 to 2 GeV depending on the coupling scheme and the method of conserving the electromagnetic current etc. The analysis of the vector meson photoproduction [30] and $\gamma n \to \Theta^+K^-$ favor a small value of the cut-off, $\Lambda_B \simeq 0.5$ GeV. For the $\gamma p \to \Theta^+\bar{K}^0$ reaction the $K^*$ exchange channel remains to be dominant at $\Lambda_B \leq 1.5$ GeV and, therefore, in this paper we use a "universal" value, $\Lambda_B \simeq 0.5$ GeV, for all Born terms.

In Fig. 2 we exhibit the differential cross sections of the reactions $\gamma n \to \Theta^+K^-$ (a) and $\gamma p \to \Theta^+\bar{K}^0$ (b) in the c.m.s. at $E_\gamma = 2$ GeV. One can see that the $t$-channel $K^*$ exchange depicted in Fig. 1e gives the dominant contribution compared to the Born terms shown in Fig. 1a - d in both reactions.
FIG. 2: The differential cross section of the reaction $\gamma n \rightarrow \Theta^+ K^-$ (a) and $\gamma p \rightarrow \Theta^+ K^0$ (b) at $E_\gamma = 2$ GeV. The notation "Born" correspond to the coherent sum of the $t$, $s$, $u$-exchange diagrams and the contact term, shown in Fig. 1a - d, respectively. Solid curves indicate the total of all contributions. The contribution of $K^*$ exchange is indicated and shown as dash-dotted curves which almost overlap with solid curves.

B. $\Lambda(1520)$ photoproduction

The main diagrams for the amplitudes of the excitation of the $\Lambda$ hyperon in the $\gamma N \rightarrow NK\bar{K}$ reaction at low energies are shown in Fig. 3. Similarly to the $\Theta^+$ photoproduction we neglect the photon interaction within the decay vertex and restore the gauge invariance by a proper choice of the contact terms. The Mandelstam variables for the virtual $\Lambda^*$ photoproduction are defined by $t = (q - k)^2$, $s \equiv W^2 = (p + k)^2$. The $K$ meson production angle $\theta$ (in $\gamma p$ c.m.s.) is given by $\cos \theta = k \cdot q / (||k||||q||)$.

FIG. 3: Tree level diagrams for the reaction $\gamma N \rightarrow \Lambda^* K \rightarrow NK\bar{K}$. 
For the description of the $\Lambda^*$ excitation with $J^P = \frac{3}{2}^-$ we use the following effective Lagrangians [30, 31]

$$\mathcal{L}_{\Lambda^* NK} = \frac{g_{\Lambda^* NK}}{M_{\Lambda^*}} \bar{\Lambda}_\mu^* \theta^{\mu\nu}(Z) (\partial_\nu \bar{K}) \gamma_5 N + \text{h.c.} ,$$  

(7a)

$$\mathcal{L}_{\gamma \Lambda^* NK} = -i \frac{g_{\Lambda^* NK}}{M_{\Lambda^*}} \bar{\Lambda}_\mu^* \gamma_5 \Lambda^\mu \bar{K} N + \text{h.c.} ,$$  

(7b)

$$\mathcal{L}_{\Lambda^* NK^*}^\pm = i \frac{g_{\Lambda^* NK^*}}{M_{\Lambda^*}} \bar{\Lambda}_\mu^* \theta^{\mu\nu}(Y) \gamma^\lambda F_{K\lambda\nu} N + \text{h.c.} ,$$  

(7c)

where $\Lambda^*$ is the $\Lambda(1520)$ field, $M_{\Lambda^*}$ denotes the $\Lambda^*$ mass, $F_{K}^{\mu\nu}$ is related to the vector $K^*$ meson field as $F_{K}^{\mu\nu} = \partial^\nu K^*\mu - \partial^\mu K^*\nu$. The operator $\theta^{\mu\nu}(X)$ is a function of the "off-shell" parameter $X$: $\theta^{\mu\nu}(X) = g_{\mu\nu} - \left(\frac{1}{2} + X\right)\gamma_\mu \gamma_\nu$. In this paper we consider such a kinematics where the invariant mass of the outgoing $NK$ pair is close to $M_{\Lambda^*}$, $\Lambda^*$ is almost on-shell, and therefore, the contribution from terms proportional to $\gamma_\mu \gamma_\nu$ in $\theta^{\mu\nu}(X)$ disappears. This means that $\theta^{\mu\nu}(X)$ may be replaced by $g_{\mu\nu}$. We assume a vanishing value of the anomalous magnetic moment of $\Lambda^*$ and, therefore, neglect the $\Lambda^*\gamma$ interaction, and, correspondingly, the contribution of the $u$-channel shown in Fig. 3c. All vertices are dressed by the form factors similarly to the case of the $\Theta^+$ photoproduction with the same cut-off parameters.

The amplitudes for the $\gamma p \rightarrow \Lambda^* K^+$ and $\gamma n \rightarrow \Lambda^* K^0$ reactions read

$$A_{\Lambda^* f_i}(\gamma p) = \bar{u}_{\Lambda^*}(p^*_\Lambda) \left[ M^s_{\sigma\mu} + M^t_{\sigma\mu} + M^c_{\sigma\mu} + M^t_{\sigma\mu}(K^*) \right] u_p(p) \varepsilon^\mu, \quad (8a)$$

$$A_{\Lambda^* f_i}(\gamma n) = \bar{u}_{\Lambda^*}(p^*_\Lambda) \left[ M^s_{\sigma\mu} + M^t_{\sigma\mu}(K^*) \right] u_n(p) \varepsilon^\mu. \quad (8b)$$

The explicit transition operators $M^t_{\sigma\mu}$ for these reactions are listed in Appendix A.

The coupling constant $g_{\Lambda^* NK}$ is found from the $\Lambda^*$ decay width,

$$\Gamma_{\Lambda^* \rightarrow NK} = \frac{|g_{\Lambda^* NK}|^2 p_F^3}{6\pi M_{\Lambda^*}^2} \left( \sqrt{M_N^2 + p_F^2} - M_N \right),$$  

(9)

where $p_F$ is $\Lambda^* \rightarrow N\bar{K}$-decay momentum. Taking $\Gamma_{\Lambda^* \rightarrow NK} \simeq 0.45 \times 15.6$ MeV [11], one finds $|g_{\Lambda^* NK}| = 32.6$.

Analog to the above considered $\Theta^+$ photoproduction we denote $g_{\Lambda^* NK^*} = \alpha_{\Lambda^*} g_{\Lambda^* NK}$. The parameter $\alpha_{\Lambda^*}$ must be defined by a comparison of calculated cross sections with experimental data at $E_\gamma \sim 2$ GeV. However, the available experimental data for the $\gamma p \rightarrow \Lambda^* K^+$ reaction cover the energy range $E_\gamma = 2.8 - 4.8$ (GeV) [32], beyond the applicability of the effective Lagrangian formalism. Thus, in this region the total cross section decreases with energy as $E_\gamma^{-2.1}$, whereas the amplitudes of Eq. (8) predict a strong increase. The energy
dependence at high energy is reasonably well described by the Regge phenomenology. Since the Λ* decay angular distribution supports the dominance of the t-channel natural parity exchange processes, one can assume that the dominant contribution to the Λ* photoproduction at high energy comes from the leading K* trajectory [33]. The corresponding amplitude is obtained from the t-channel K* meson exchange in Eq. (8) by the Reggezation of the K* meson exchange propagator, i.e.

\[ \frac{1}{t - M_{K*}^2} \rightarrow \gamma(t) \left( \frac{s}{s_0} \right)^{\alpha(t)}, \]  

(10)

where \( \alpha(t) = \alpha(0) + \alpha' t \) is the Regge trajectory and \( \gamma(t) \) denotes the normalization function

\[ \gamma(t) = C_R (\text{Tr}[RR^\dagger])^{-1}, \]

\[ R = \bar{u}_\Lambda^\sigma(p_\Lambda^*) \left[ \varepsilon^{\mu\nu\alpha\beta} k^\nu q^\alpha (q'_{\sigma} \gamma^\beta - q'_{\sigma} g^\beta) \right] u_n(p) \varepsilon^\mu, \]  

(11)

with \( q' = p_{\Lambda^*} - p \). In the following we assume that at energies near the threshold, the production amplitude is defined by the effective Lagrangian model of Eq. (8), \( A_{\text{eff.}L.}^{\Lambda^*} \), whereas at high energies it is described by the Regge phenomenology, \( A_R^{\Lambda^*} \), as

\[ A^{\Lambda^*} = A_{\text{eff.}L.}^{\Lambda^*} \theta(E_0 - E_\gamma) + A_R^{\Lambda^*} \theta(E_\gamma - E_0). \]  

(12)

We take \( E_0 = 2.3 \text{ GeV} \) as matching point between the two regimes. The choice of parameters in Eq. (11) as \( s_0 = 1 \text{ GeV} \), \( \alpha(t) = -0.1+0.9t \) and \( C_R = 29.6 \) gives a satisfactory description of the high energy data, as exhibited in Fig. 4 for the differential cross section at \( E_\gamma = 3.7 \text{ GeV} \).

In Fig. 5 we show the energy dependence of the total cross section. The dot-dashed curve is the fit of the data \( \sigma \simeq 6.55 \left( \frac{E_\gamma}{\text{GeV}} \right)^{-2.1} (\mu\text{b}) \) from [32]. For illustration we also show the cross section calculated with a constant amplitude where the energy dependence is defined by the phase space volume alone. The strength parameter \( \alpha_{\Lambda^*} \) is adjusted by fitting the calculated cross section to the experimental extrapolation (dot-dashed curve) at the normalization point. Two solutions \( \alpha_{\Lambda^*} = +0.372 \) and \(-0.657 \) result in two different energy dependencies of the cross section at low energy. Both solutions exceed the experimental data above the normalization point. The solution with positive \( \alpha_{\Lambda^*} \) at low energies is close to the pure phase space dependence shown by the long-dashed curve.

In Fig. 6 we show the differential cross sections of the Λ* photoproduction at \( E_\gamma = 2 \text{ GeV} \). The differential cross sections of the \( \gamma p \rightarrow \Lambda^* K^+ \) reaction for positive \( \alpha_{\Lambda^*} \) together
FIG. 4: Differential cross section of the reaction $\gamma p \rightarrow \Lambda^+ K^+$ at $E_\gamma = 3.7$ GeV. Experimental data from Ref. [32].

FIG. 5: The total cross section of the reaction $\gamma p \rightarrow \Lambda^* K^+$ as a function of the photon energy. The experimental data are taken from Ref. [32]. The dot-dashed curve is the fit of this data $\sigma \approx 6.55 E_\gamma^{-2.1} (\mu b)$. The long dashed curve represents the cross section when the amplitude is taken to be constant. The solid curves corresponds to the amplitude of Eq. (12). The signs "±" corresponds to the sign of $\alpha_{\Lambda^*}$. The dashed curve describes the extrapolation of the effective Lagrangian model to the high energy region.

with the separate contributions of the Born and $K^*$ exchange channels are shown in Fig. 6a. In case of the $\gamma n \rightarrow \Lambda^* K^0$ reaction, shown in Fig. 6b by the solid curve, the Born term (s-channel exchange) is negligible. In the $\gamma p$ reaction, the interplay of the Born terms and the $K^*$ exchange amplitude is important at forward angles that leads to a dependence of the total cross section on the sign of $\alpha_{\Lambda^*}$ (see Fig. 6b). However, as we will see later, in the coherent $\gamma D \rightarrow \Lambda^* \Theta^+$ reaction the region of backward angles of the $K^+$ photoproduction...
FIG. 6: (a) The differential cross section of the $\gamma p \rightarrow \Lambda^* K^+$ reaction at $E_\gamma = 2$ GeV. The notation "Born" corresponds to the coherent sum of the $t, s$-exchange diagram and the contact term, shown in Fig. 3a, b, and d, respectively. (b) The differential cross section of the $\gamma n \rightarrow \Lambda^* K^0$ reaction (solid curve) and $\gamma p \rightarrow \Lambda^* K^+$ reaction (dashed and dot-dashed curves). The symbol "±" indicates the sign of $\alpha_{\Lambda^*}$.

gives the main contribution and, therefore, the final result is not sensitive to the choice of the solution. Nevertheless, for further consideration we chose the solution with positive $\alpha_{\Lambda^*}$ because it describes better the total $K^+ K^-$ production in $\gamma p$ interaction at low energies.

Finally we note that a similar approach for the $\Lambda^*$ photoproduction based on the effective Lagrangian formalism was developed in the recent paper [7]. Differences consist in a different choice of the form factors and parameters, which results in slightly different predictions for the differential and total cross sections. This difference may be resolved experimentally.

III. REACTION $\gamma D \rightarrow \Lambda^* \Theta^+$

The tree level diagrams for the coherent $\gamma D \rightarrow \Lambda^* \Theta^+$ photoproduction are shown in Fig. 7. First of all note that the amplitudes from the charge and neutral meson exchange shown in Figs. 7a and c and/or b and d give a constructive interference in the total cross section. That is because in the elementary amplitudes of $\gamma N \rightarrow \Lambda^* K$ and $\gamma N \rightarrow \Theta^+ \bar{K}$ reactions the dominant contribution comes from the $K^*$ exchange. The different signs in $\gamma K^0 \bar{K}^0$ and $\gamma K^{*+} K^-$ vertices are compensated by the different signs in $n \Theta^+ K^-$ and $p \Theta^+ \bar{K}^0$ interactions. The latter is a consequence of the assumed isospin $I = 0$ of the pentaquark.

The amplitudes of the coherent $\Lambda^* \Theta^+$ photoproduction are expressed through the transition operators of the "elementary" processes $\gamma N \rightarrow \Lambda^* K$ and $\gamma N \rightarrow \Theta^+ \bar{K}$ shown in Fig. 7a,c
FIG. 7: Tree level diagrams for the reaction $\gamma D \to \Lambda^* \Theta^+$. The exchange of charged and neutral mesons are shown in (a,b) and (c,d), respectively.

and b,d, respectively, as

$$A_{(a,c)} = g_{\Theta NK} \int \frac{d^4p}{(2\pi)^4} \bar{u}_\Theta \gamma_5 \frac{1}{q^2 - M_K^2} \bar{u}_\Lambda^\sigma \mathcal{M}_{\sigma\mu} \frac{\not{\tau} + M}{p^2 - M^2} \Gamma_D \frac{\not{\tau} + M}{p'^2 - M^2} U_D\epsilon^\mu, \quad (13a)$$

$$A_{(b,d)} = -\frac{g_{\Lambda^* NK}}{M_{\Lambda^*}} \int \frac{d^4p}{(2\pi)^4} \bar{u}_\Theta \mathcal{M}_{\mu} \frac{1}{q^2 - M_K^2} \bar{u}_\Lambda^\sigma q_\sigma \gamma_5 \frac{\not{\tau} + M}{p^2 - M^2} \Gamma_D \frac{\not{\tau} + M}{p'^2 - M^2} U_D\epsilon^\mu, \quad (13b)$$

where the transition operators $\mathcal{M}$ are described in the previous section, $\Gamma_D$ and $U_D$ stand for the deuteron $np$ coupling vertex and the deuteron spinor, respectively, $p' = p_D - p$ and $q$ is the momentum of the exchanged kaon.

Following Ref. [24] we assume that the dominant contribution to the loop integrals comes from their imaginary parts which may be evaluated by summing all possible cuttings of the loops, as shown in Fig. 8. Calculating the imaginary parts we use the following substitutions

FIG. 8: Diagrammatic representation of cutting (indicated in by crosses) the loop diagrams.

for the propagators of the on-shell particles (shown by crosses)

$$\frac{1}{q^2 - M_K^2} \to 2\pi \delta(q^2 - M_K^2),$$

$$\frac{\not{\tau} + M}{p^2 - M^2} \to 2\pi (\not{\tau} + M) \delta(p^2 - M^2) \quad (14)$$
and the identity

\[ \int d^4p \delta(p^2 - M^2) = \int \frac{d^3p}{2E} \]  

(15)

with \( E^2 = p^2 + M^2 \). We also use the standard representation of the product of the deuteron vertex function and the attached nucleon propagator through the non-relativistic deuteron function

\[ \Gamma_D \frac{\bar{u}_1(p)\bar{u}_2(p_D - p)}{U_D} = \sqrt{2M_D} \psi_{m_D,m_1m_2} , \]  

(16)

where \( \psi_{m_D,m_1m_2} \) is the deuteron wave function with the spin projection \( m_D \) and the nucleons spin projections \( m_1 \) and \( m_2 \). By using Eqs. (14) - (16), one can express the principal parts of the invariant amplitudes in Eq. (13) as

\[ A^P_{(a,c)} = g_{\Theta NK} \sum_{m_1m_2} \left[ \bar{u}_{\Theta}(p_{\Theta})\gamma_5 u_{m_1}(r) \right] \cdot \left[ \bar{u}_\Lambda^*(p_\Lambda)M^*_{a\mu} \epsilon^\mu u_{m_2}(r) \right] S^{\Lambda^*}_{m_1m_2} , \]  

(17a)

\[ A^P_{(b,d)} = -\frac{g_{\Lambda^*NK}}{M_\Lambda} \sum_{m_1m_2} \left[ \bar{u}_{\Theta}\Theta^\mu u_{m_1}(r)\epsilon^\mu \right] \cdot \left[ \bar{u}_\Lambda^* q_{\sigma} \gamma_5 u_{m_2}(r) \right] S^{\Theta^+}_{m_1m_2} , \]  

(17b)

where \( r = p_D/2 \), and

\[ S^{\Lambda^*}_{m_1m_2} = I^i_{m_1m_2}(p_{\Theta}) + I^j_{m_1m_2}(k - p_\Lambda) , \quad S^{\Theta^+}_{m_1m_2} = I^i_{m_1m_2}(p_\Lambda) + I^j_{m_1m_2}(k - p_{\Theta}) , \quad \]

\[ I^i_{m_1m_2}(p_X) = i \frac{\sqrt{2M_D}}{16\pi} \int \frac{dp}{E_{p_X}} \theta(1 - |a_{i,j}(p,p_X)|) \phi_{m_D,m_1m_2}(p,a(p,p_X)) , \]

\[ a_i(p,p_X) = \frac{2EE_X + M^2_X - M^2}{2pp_X} , \quad a_j(p,p_X) = \frac{2EE_X - M^2_X + M^2}{2pp_X} , \]

\[ \phi_{m_D,m_1m_2}(p,a) = \sqrt{4\pi} \left( \frac{1}{2} m_1 \frac{1}{2} m_2 |m_D| \right) \left( u_0(p) + \frac{1}{\sqrt{8}} (3a^2 - 1)(1 - 3 \delta_{m_D0}) u_2(p) \right) , \]  

(18)

where \( M^2_X = E^2_X - p^2_X \) and \( u_l \) with \( l = 0,2 \) is the radial deuteron wave function in the momentum space, normalized as

\[ \int \frac{d^3p}{(2\pi)^3} \Phi(p) = 1 , \]

where

\[ \Phi(p) = 4\pi \left( u_0^2(p) + u_2^2(p) \right) . \]  

(19)

In deriving Eqs. (17) we neglect the weak dependence of the "elementary" amplitudes of \( \gamma N \to \Lambda^* K \) and \( \gamma N \to \Theta^+ \bar{K} \) on \( p \) (see Figs. 2 and 4), compared to the sharp \( p \) dependence
FIG. 9: The differential cross section of the $\gamma D \rightarrow \Lambda^* \Theta^+$ reaction. The notations $\gamma D \rightarrow \Lambda^*(\Theta^+)$ and $\gamma D \rightarrow \Theta^+(\Lambda^*)$ correspond to the diagrams in Fig. 7a, c and b, d, respectively.

of $\Phi(p)$. In our calculation we use the deuteron wave function for the "realistic" Paris potential [34]. We checked that the final result does not depend on the fine structure of the deuteron wave function and practically does not depend on the choice of the potential.

The differential cross section of the coherent $\Lambda^* \Theta^+$ photoproduction reads

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 S P_{\text{in}}} \left( \frac{P_{\text{out}}}{S P_{\text{in}}} \right) |A_{a,c} + A_{b,d}|^2,$$

where $S, P_{\text{in}}$ and $P_{\text{out}}$ are the square of the total energy, momenta in initial and the final states in $\gamma D$ c.m.s., respectively; averaging and summing over the spin projections in the initial and the final states are assumed. Note that the interference between amplitudes $A_{a,c}$ and $A_{b,d}$ is negligible and they can be summed incoherently.

In Fig. 9 we show the differential cross section of the reaction $\gamma D \rightarrow \Lambda^* \Theta^+$ at $E_\gamma = 2$ GeV as a function of the angle between the beam direction and direction of flight of $\Lambda^*$ in the $\gamma D$ c.m.s. The non-monotonous behaviour of the cross section is completely defined by the spectral functions $S^{\Lambda^*}$ and $S^{\Theta^+}$ in Eqs. (17a) and (17b), respectively. The spectral functions $S^{\Lambda^*}$ and $S^{\Theta^+}$ have sharp peaks in forward ($\theta_{\gamma \Lambda^*} \simeq 27.5^\circ$) and backward ($\theta_{\gamma \Lambda^*} \simeq 152.5^\circ$) hemispheres, respectively.
IV. BACKGROUND CONTRIBUTION

Since the $\Lambda^*$ and $\Theta^+$ are unstable baryons, the typical experiment for studying the coherent $\gamma D \rightarrow \Lambda^*\Theta^+$ process must include a simultaneous measurement of the $pK^-$ and $nK^+$ invariant masses. Therefore, the question is whether the predicted cross section of the coherent $\Lambda^*\Theta^+$ photoproduction is large enough to be seen above the background of competing resonance and non-resonance processes in the $\gamma D \rightarrow npK^+K^-$ reaction.

We consider three types of background processes. One is the photoproduction of a $K^+K^-$ pair in a $\gamma p$ interaction when the neutron is a spectator. This process includes the resonant $\gamma p \rightarrow \Lambda^*K^+ \rightarrow pK^-K^+$ photoproduction and the non-resonant $\gamma p \rightarrow pK^+K^-$ reaction shown in Fig. 10a and b, respectively.

![Tree level diagrams for background processes.](image)

FIG. 10: Tree level diagrams for background processes. a - d: non-coherent spectator channels, e and f: coherent semi-resonant background processes.

Similarly, a $K^+K^-$ pair can be produced in a $\gamma n$ interaction, when the proton is a spectator. The corresponding processes are depicted in Fig. 10c and d.

The third process is the coherent "background" when the $K^+K^-$ pair is produced in a $\gamma N$ interaction and one of the kaons together with the second nucleon forms the outgoing $\Theta^+$ or $\Lambda^*$, as shown in Fig. 10 e and f, respectively. We denote it as a coherent semi-resonant background.
A. Spectator channels

First, let us consider the $K^+K^-$ photoproduction in a $\gamma D$ interaction where the neutron or proton are merely spectators. As an input, we have to describe the elementary processes $\gamma p \to pK^+K^-$ and $\gamma n \to nK^+K^-$ which consist of the resonant and non-resonant parts.

1. $\gamma p \to pK^+K^-$

The dominant contribution to the non-resonant part in $\gamma p$ reactions comes from the virtual vector meson decay and $\Lambda(1405)$ excitation [8, 22] as depicted in Fig. 11a and b. The contribution from excitations of other hyperons is strongly suppressed since they are far off-shell. The vector meson channel $\gamma p \to Vp \to pK^+K^-$, where $V = \phi, \rho, \omega$ has been analyzed in detail in Ref. [8]. In the present study we use this model where the vector mesons are produced through the Pomeron and meson ($\pi, \eta, \sigma$) exchanges with the same parameters. The only difference with Ref. [8] is that now we do not use a cut on the invariant mass of the $K^+K^-$ pair around the $\phi$ meson mass.

We parameterize the amplitude of the virtual $\Lambda(1405)$ excitation through the $K^*$ exchange process. This assumption is supported by the $K^*$ exchange dominance in $\Lambda^*$ and $\Theta^+$ photoproduction and allows to reduce the number of unknown parameters. The amplitude of this channel reads

$$ A_{fi}^\Lambda' \equiv \bar{u}(p') \mathcal{M}_{\mu i}^\Lambda' u(p) \varepsilon^\mu, $$

$$ \mathcal{M}_{\mu i}^\Lambda' = -i \frac{e g_{\gamma KK'} g'_{\Lambda' \Lambda}}{M_{K^*}(t - M_{K^*}^2)} \varepsilon^{\mu \alpha \beta} k_\nu q_\alpha \frac{\langle \mathcal{P}_{\Lambda' \Lambda} \rangle_{\gamma 5} \gamma^\beta}{p_{\Lambda'}^2 - M_{\Lambda'}^2 + i\Gamma_{\Lambda'} M_{\Lambda'}} F_{K^*}(t), \quad (21) $$

where $\Lambda' \equiv \Lambda(1405)$, $\Gamma_{\Lambda'} = 50$ MeV is the total decay width of $\Lambda'$ [11], $F_{K^*}(t)$ is the $K^*$ exchange form factor, the constant $g'$ is a product of two coupling constants $g_{\Lambda' NK}$ and

![Fig. 11: Background processes for the $\gamma p \to pK^+K^-$ reaction. (a): vector meson contribution. (b): virtual $\Lambda(1405)$ excitation.](image-url)
The choice $g' \simeq 7.8$ gives the correct value of the total yield of $K^+K^-$ mesons at $E_\gamma \sim 2$ GeV. Note that the interference between the resonance and non-resonance channels in the total cross section is rather weak and, therefore, they can be added incoherently. Thus, the total cross section of the $\gamma p \rightarrow pK^+K^-$ reaction reads

$$
\frac{d\sigma}{d\Omega dM_{pK^-}} = \left( \frac{d\sigma}{d\Omega} \right)^{\gamma p \rightarrow \Lambda^* K^+} F^{\Lambda^*}(M_{pK^-}) + \frac{1}{64\pi^2} \frac{p_{out}}{s} \frac{q_F}{16\pi^3} \int \left( |A_{fi}(\gamma p)|^2 + |A_{fi}'|^2 \right) d\Omega_F, \quad (22)
$$

where $\Omega$ is the solid angle of the $K^-$ meson photoproduction in the $\gamma p$ c.m.s., $q_F$ is the momentum of the $K^-$ meson in the c.m.s. of the $pK^-$-pair, $\Omega_F$ is the $K^-$ meson solid angle in this system. Summing and averaging over the spin projection in the initial and the final states is to be included. $F^{\Lambda^*}(M_{pK^-})$ stands for the $\Lambda^*$ decay distribution which is obtained straightforwardly from the general expression of the $\gamma p \rightarrow pK^+K^-$ amplitude with the virtual excitation of a $\Lambda^*$ hyperon,

$$
F^{\Lambda^*}(M_x) = \frac{\Gamma_{\Lambda^* \rightarrow pK^-}}{\Gamma_{\text{tot}}} \frac{2M_x M_{\Lambda^*} \Gamma_{\text{tot}}}{\pi(M_x^2 - M_{\Lambda^*}^2)^2 + (\Gamma_{\text{tot}} M_{\Lambda^*})^2}, \quad (23)
$$

where $\Gamma_{\text{tot}} = 15.6$ MeV and $\Gamma_{\Lambda^* \rightarrow pK^-} = (0.45/2) \times \Gamma_{\text{tot}}$ [11].

![Figure 12](attachment:image.png)

**FIG. 12**: The $pK^-$ invariant mass distribution in the $\gamma p \rightarrow pK^+K^-$ reaction at $E_\gamma = 2$ GeV. The resonant channel, vector meson and $\Lambda(1405)$ contributions are shown by thin solid, long dashed and dashed curves, respectively.
The $pK^-$ invariant mass distribution at $E_\gamma = 2$ GeV integrated over $\Omega$ is shown in Fig. 12. One can see that the $\Lambda(1405)$ excitation contributes at $M_{pK^-}$ below the $\Lambda^*$ resonance position, and the vector meson channels contribute mainly at large $M_{pK^-}$, above $M_{\Lambda^*}$. The partial contributions to the total $\gamma p \rightarrow pK^+K^-$ cross section are the following: $\sigma(\Lambda^*) \simeq 0.19 \mu b$, $\sigma(V) \simeq 0.17 \mu b$ and $\sigma(\Lambda(1405)) \simeq 0.07 \mu b$. The total cross section $\sigma_{\text{tot}} \simeq 0.43 \mu b$ is in agreement with the experimental data of Ref. [35]: $\sigma_{\text{tot}}^{\exp} = (0.47 \pm 0.12) \mu b$ at $E_\gamma = 2 - 2.5$ (GeV).

2. $\gamma n \rightarrow nK^+K^-$

In this case the non-resonance part is dominated by the vector meson excitation and, therefore, the $nK^+$ invariant mass distribution may be written in obvious notation as

$$\frac{d\sigma}{d\Omega dM_{nK^+}} = \left( \frac{d\sigma}{d\Omega} \right)_{\gamma n \rightarrow \Theta^+K^-} F^{\Theta^+}(M_{nK^+}) + \frac{1}{64\pi^2} \frac{1}{s} \frac{q_F}{p_{\text{in}}^2} \int |A_{\gamma n}^{\Theta^+}(\gamma n)|^2 d\Omega_F$$

with

$$F^{\Theta}(M_x) = \frac{1}{2\pi} \frac{2M_x M_\Theta \Gamma_\Theta}{(M_x^2 - M_\Theta^2)^2 + (\Gamma_{\text{tot}} M_\Theta)^2}. \quad (25)$$

We will also use the Gaussian distribution taking into account the small $\Theta^+$ decay width and the finite experimental resolution

$$F^G_{\Theta^+}(M_x) = \frac{1}{2\sigma\sqrt{2\pi}} e^{-\frac{(M_x - M_\Theta)^2}{2\sigma^2}}. \quad (26)$$

The $nK^+$ invariant mass distribution at $E_\gamma = 2$ GeV integrated over $\Omega$ is shown in Fig. 13. One can see the sharp peak of $\Theta^+$ excitation. In case of a Gaussian $\Theta^+$ decay distribution the peak is modified. The height of the peak is reduced by the factor $\sigma/\Gamma_\Theta$ and the width becomes proportional to $\sigma$.

3. Spectator reactions $\gamma D \rightarrow pK^+K^-(n)$ and $\gamma D \rightarrow nK^+K^-(p)$

The differential cross section of the $\gamma D \rightarrow pK^+K^-(n)$ reaction, where the neutron is a spectator, reads

$$\frac{d\sigma_{\gamma p,(n)}}{d\Omega dM_{pK^-} dM_{nK^+}} = \left( \frac{d\sigma}{d\Omega dM_{pK^-}} \right)_{\gamma p \rightarrow pK^+K^-} W_{nK}(M_{nK^+}) ,$$

$$W_{nK}(M_{nK^+}) = 2M_{nK^+} \int \frac{dP_n}{(2\pi)^3 \sqrt{1 + P_n^2/M_n^2}} \delta(M_{nK^+}^2 - (p_n + q)^2) \Phi(P_n) , \quad (27)$$
where we neglect the smooth dependence of \( d\sigma^{\gamma p \rightarrow pK^+K^-} \) on \( p_n \) in comparison to the sharp \( p_n \) dependence of the momentum distribution in the deuteron, \( \Phi(p_n) \), defined in Eq. (19).

If the invariant mass of the \( nK^+ \) pair is not fixed then the integration over \( M_{nK^+} \) leads to the obvious result

\[
\int dM_{nK^+} \frac{d\sigma^{\gamma p \rightarrow pK^+K^-}}{d\Omega dM_{pK^-} dM_{nK^+}} \simeq \left( \frac{d\sigma}{d\Omega dM_{pK^-}} \right)^{\gamma p \rightarrow pK^+K^-}.
\]  

(28)

When the invariant mass is fixed then the function \( W_{nK}(M_{nK^+}) \) becomes important and, moreover, it mainly defines the dependence of the cross section on \( M_{nK^+} \). Indeed, let us assume that the momentum distribution in a deuteron behaves like a delta function, i.e. \( \Phi(p) \simeq (2\pi)^3 \delta(p) \). Then one gets

\[
W_{nK}(M_{nK^+}) \simeq 2M_{nK^+} \delta(M_{nK^+}^2 - (M_N^2 + M_K^2 + 2E_K + M_N)).
\]

(29)

That is, the distribution \( W_{nK}(M_{nK^+}) \) has a peak around the point \( M_{nK^+0} \simeq \sqrt{M_N^2 + M_K^2 + 2E_K + M_N} \) which is determined by the energy of the \( K^+ \) meson in the laboratory system. On the other hand, this energy depends on the invariant mass of the \( pK^- \) pair and the angle of the \( K^+ \) production in the \( \gamma p \) c.m.s. In reality, the distribution function

FIG. 13: The \( nK^+ \) invariant mass distribution in the \( \gamma n \rightarrow nK^+K^- \) reaction at \( E_\gamma = 2 \text{ GeV} \). The symbol "BW" means the Breit-Wigner \( \Theta^+ \) decay distribution of Eq. (25). The dashed curve corresponds to a Gaussian distribution of the \( \Theta^+ \) decay width \( \sigma = 5 \text{ MeV} \).
reads

\[ W_{nK}(M_{nK^+}) = 2M_{nK^+} \int \frac{p^2}{8\pi^2q_L \sqrt{1 + p^2/M_N^2}} \Phi(p) \theta(1 - |a|), \]

\[ a = \frac{2 \sqrt{(q_L^2 + M_K^2)(p^2 + M_K^2)} + M_N^2 + M_K^2 - M_{nK^+}^2}{2pq_L}, \quad (30) \]

where \( q_L \) is the momentum of \( K^+ \) meson in laboratory system. The distribution function \( W_{nK} \) is shown in Fig. 14a as a function of \( M_{nK^+} \) at fixed angle of \( pK^- \) pair photoproduction, \( \theta_{\gamma(pK^-)} \) (in \( \gamma D \) c.m.s.) for three different invariant masses of the \( pK^- \) pair: \( M_{pK^-} = 1.52, 1.57 \) and 1.47 GeV. The choice of \( \theta_{\gamma(pK^-)} = 27.5^\circ \) corresponds to the position of the maximum of the coherent \( \gamma D \to \Lambda^*\Theta^+ \) photoproduction cross section at forward angles (see Fig. 9). This angle corresponds to the backward \( K^+ \) photoproduction in \( \gamma p \to \Lambda^*K^+ \): \( \theta_{\gamma K^+} \simeq 119^\circ \) in the \( \gamma p \) c.m.s.

![Fig. 14](image)

**FIG. 14:** (a) The invariant mass distribution function \( W_{nK} \) as a function of \( M_{nK^+} \) at \( \theta_{\gamma(pK^-)} = 27.5^\circ \) and fixed values of \( M_{pK^-} \). (b) The invariant mass distribution function \( W_{pK} \) as a function of \( M_{nK^+} \) at \( \theta_{\gamma(pK^-)} = 152.5^\circ \) and fixed values of \( M_{pK^-} \).

The differential cross section of the \( \gamma D \to nK^+K^-(p) \) reaction, where the proton is spectator, may be obtained from Eq. (27), using the substitution \( n \to p, K^+ \to K^- \) and \( M_{nK^+} \to M_{pK^-} \),

\[ \frac{d\sigma^{sp.(p)}}{d\Omega dM_{pK^-}dM_{nK^+}} = \left( \frac{d\sigma}{d\Omega dM_{nK^+}} \right)^{\gamma n \to nK^+K^-} W_{pK}(M_{pK^-}). \quad (31) \]

The essential difference is that now we analyze the dependence of the distribution function \( W_{pK} \) not on \( M_{pK^-} \) but on the invariant mass \( M_{nK^+} \). This dependence is included in \( W_{pK} \).
implicitly through the dependence of the momentum of $K^-$ on $M_{nK^+}$ and therefore, in
general, we have no narrow peak structure of $W_{pK}$ as a function of $M_{nK^+}$. As an example, in
Fig. 14b we show the distribution $W_{pK}$ as a function of $M_{nK^+}$ at fixed values of $M_{pK^-} = 1.52,$
1.57 and 1.47 GeV and $\theta_{\gamma(pK^-)} = 152.5^\circ$. One can see a broad maximum at $M_{pK^-} = 1.52$
GeV and an almost monotonic behaviour at 1.47 and 1.57 GeV.

B. Coherent semi-resonant background

The amplitude of the process shown in Fig. 10e is calculated similarly to the amplitude
of the coherent $\Lambda^*\Theta^+$ photoproduction described by Eq. (17a). The corresponding cross
section reads

$$
\frac{d\sigma^e}{d\Omega dM_{pK^-} dM_{nK^+}} = \frac{1}{64\pi^2 s} \frac{1}{p_{\text{in}} p_{\text{out}}} \cdot \frac{\bar{F}}{2} \int d\Omega_F |A_e|^2 F^{\Theta}(M_{nK^+}) ,
$$

$$
A_e = g_{\Theta NK} \sum_{m_1m_2} [\bar{u}_\Theta (p_\Theta) \gamma_5 u_{m_1}(r)] \cdot [\bar{u}_{\Lambda^*} (p_{\Lambda}^*) M^{\gamma p_{\Lambda}^* pK^-} \varepsilon^\mu u_{m_2}(r)] S_{m_1m_2}^{\Lambda^*} ,
$$

where $p_{\text{in}}, p_{\text{out}}$ are the momenta of the proton and $pK^-$ pair in $\gamma p$ c.m.s., $\Omega$ and $\Omega'$ are the
solid angles of the $pK^-$ pair in $\gamma D$ and $\gamma p$ reactions, respectively, $\bar{q}_F$ is the momentum of
$K^-$ meson in the rest frame of the $pK^-$ pair, $\Omega_F$ is the solid angle of $K^-$ in this frame. The
additional factor $1/2$ assumes renormalization of the flux in the $\gamma D$ system compared to the $\gamma p$
interaction. The function $F^{\Theta}(M_{nK^+})$ is defined in Eq. (25). Averaging and summing
over the spin projections in initial and the final states, respectively, have to be performed.
Actually, here we have a sum of two cross sections. One is the contribution of the virtual vector meson and another one is the contribution of the virtual $\Lambda(1405)$ excitation.

Similarly, one can write the cross section of the process shown in Fig. 10f as

$$
\frac{d\sigma^f}{d\Omega dM_{pK^-} dM_{nK^+}} = \frac{1}{64\pi^2 s} \frac{1}{p_{\text{in}} p_{\text{out}}} \cdot \frac{\bar{F}}{2} \int d\Omega_F |A_f|^2 F^{\Lambda^*}(M_{pK^-}) ,
$$

$$
A_f = -\frac{g_{\Lambda^* NK}}{M_{\Lambda^*}^{\gamma}} \sum_{m_1m_2} [\bar{u}_\Theta M^{\gamma p_{\Lambda^*} pK^-} \varepsilon^\mu u_{m_1}(r)] \cdot [\bar{u}_{\sigma} q_\sigma \gamma_5 u_{m_2}(r)] S_{m_1m_2}^{\Lambda^*} ,
$$

where the function $F^{\Lambda^*}(M_{pK^-})$ is defined in Eq. (23) and other notations are similar to the
previous case.

Let us now compare the contribution of the coherent $\Lambda^*\Theta^+$ photoproduction and the coherent semi-resonant background described by Eqs. (32) and (33) in the vicinity of the
FIG. 15: Comparison of the coherent $\Lambda^*\Theta^+$ photoproduction (solid curve) and coherent semi-resonant background (dashed curve) depicted in Fig. 10e,f.

$\Theta^+$ and $\Lambda^*$ resonance position

$$\frac{d\sigma^{ch.}}{d\Omega} = \int_{M_{\Lambda^*}+\Delta}^{M_{\Lambda^*}+\Delta} \int_{M_{\Theta^*}+\Delta}^{M_{\Theta^*}+\Delta} dM_{pK^-} dM_{nK^+} \frac{d\sigma^{\gamma D \to \Lambda^*\Theta^+}}{d\Omega} F_{\Lambda^*}(M_{pK^-}) F_{\Theta^+}(M_{nK^+}),$$

$$\frac{d\sigma^{ch.bg.}}{d\Omega} = \int_{M_{\Lambda^*}-\Delta}^{M_{\Lambda^*}+\Delta} \int_{M_{\Theta^*}-\Delta}^{M_{\Theta^*}+\Delta} dM_{pK^-} dM_{nK^+} \left( \frac{d\sigma^e}{d\Omega dM_{pK^-} dM_{nK^+}} + \frac{d\sigma^f}{d\Omega dM_{pK^-} dM_{nK^+}} \right),$$

where $\Delta = 20$ MeV. In Fig. 15 we show result of such a comparison. One can see that the coherent background contribution has local maxima caused by the spectral functions $S$, but the values of these contributions at the peak positions are much smaller compared to the coherent process. Therefore, the dominant background contribution comes from the spectator processes.

V. RESULTS AND DISCUSSION

As pointed out above, the coherent $\Lambda^*\Theta^+$ photoproduction seems to be accessible most effectively by a search for a sharp $\Theta^+$ peak in the invariant $nK^+$ mass distribution at fixed invariant masses of the $pK^-$ pair

$$\frac{d\sigma^{\gamma D \to npK^+K^-}(M_0)}{d\Omega dM_{nK^+}} = \int_{M_0-\Delta}^{M_0+\Delta} dM_{pK^-} \frac{d\sigma^{\gamma D \to npK^+K^-}}{d\Omega dM_{nK^+} dM_{pK^-}}.$$ (35)
In our further analysis we choose $M_0 = 1.52, 1.57,$ and $1.47$ GeV and $\Delta = 20$ MeV. One can expect that the coherent photoproduction appears at $M_0 = M_{\Lambda^*} = 1.52$ GeV and it is suppressed relative to the strong background when we go above or below this point. Since the cross section of the coherent photoproduction at $E_\gamma = 2$ GeV has bumps at $\theta_{\gamma\Lambda^*} \approx 27.5^\circ$ and $152.5^\circ$ in $\gamma D$ c.m.s. (see Fig. 9), then it is natural to expect that the regions around these angles are more favored for a manifestation of the coherence effect.

Note that at forward and backward angles of the $pK^-$ pair photoproduction, $\theta_{\gamma(pK^-)}$, some of spectator processes shown in Fig. 10 are suppressed dynamically. To illustrate this point let us consider the dependence of $\cos \theta_{\gamma K^-}$ in $\gamma p \to \Lambda^* K^+$ photoproduction and $\cos \theta_{\gamma K^+}$ in $\gamma n \to \Theta^+ K^-$ photoproduction as a function of $\cos \theta_{\gamma\Lambda^*}$ in the $\gamma D \to \Lambda^* \Theta^+$ reaction at $E_\gamma = 2$ GeV.

Consider first $\gamma D \to npK^+K^-$ photoproduction at a forward angle of the $pK^-$ pair at $\theta_{\gamma(pK^-)} \approx 27.5^\circ$ and $E_\gamma = 2$ GeV. The corresponding invariant mass distributions for $M_0 = 1.52, 1.57$ and $1.47$ are shown in Fig. 17a, b and c, respectively.

At $M_0 = M_{\Lambda^*}$, the background is dominated by the resonant $\Lambda^*$ photoproduction in the spectator mechanism shown in Fig. 10a. The next important contribution comes from the non-resonant spectator channel (Fig. 10b). The shape of the background spectrum

![FIG. 16: The dependence of $\cos \theta_{\gamma K^-}$ in $\gamma p \to \Lambda^* K^+$ photoproduction and $\cos \theta_{\gamma K^+}$ in $\gamma n \to \Theta^+ K^-$ photoproduction as a function of $\cos \theta_{\gamma\Lambda^*}$ in the $\gamma D \to \Lambda^* \Theta^+$ reaction at $E_\gamma = 2$ GeV.](image)
FIG. 17: The $nK^+$ invariant mass distribution in the $\gamma D \rightarrow npK^+K^-$ reaction at fixed values of the $pK^-$ invariant mass. The angle of the $pK^-$ pair photoproduction in $\gamma D$ c.m.s., $\theta_{\gamma(pK^-)} = 27.5^\circ$ and $E_{\gamma} = 2$ GeV. (a) $M_{pK^-} = 1.52\pm0.02$ GeV; (b) $M_{pK^-} = 1.57\pm0.02$ GeV, (c) $M_{pK^-} = 1.47\pm0.02$ GeV. Notations ”sp.$(\gamma p \rightarrow \Lambda^*K^+)$” and ”sp.$(\gamma p \rightarrow pKK)$” correspond to the processes depicted in Fig. 10a and b, respectively; ”$\gamma D \rightarrow \Theta^+pK^-$” corresponds to the coherent background shown in Fig. 10e, ”$\gamma D \rightarrow \Lambda^*\Theta^+$” corresponds to the coherent $\Lambda^*\Theta^+$ photoproduction (Fig. 7).

has a resonance like behavior with the center close to the mass of $\Theta^+$ and a width of about 15 MeV. This behaviour is defined by the spectral distribution function $W_{nK}$ (or the deuteron momentum distribution) in Eq. (27) and the kinematics (see Fig. 14a). At $M_{pK^-} = 1.52$ GeV, $W_{nK}$ has a sharp peak at $M_{nK^+} \simeq 1.54$ GeV. For $M_{pK^-} = 1.57$ and 1.47 GeV the peak position is shifted to lower or higher masses, respectively. Similarly, one can see the corresponding shift in the background contribution at $M_0 = 1.57$ and 1.47 GeV, shown in Figs. 17b and c. Here, the background is dominated by the non-resonant spectator channels. Its value is almost similar for all considered values of $M_0$ being much smaller than the total background at $M_0 = 1.52$ GeV.

At $M_0 = 1.52$ GeV, the height of the peak of the coherent $\Lambda^*\Theta^+$ channel is about one third of the total background contribution. This ratio decreases for $M_0 = M_{\Lambda^*} \pm 70$ MeV. Thus, a summary plot of the total $nK^+$ invariant mass distribution for three fixed intervals of the $pK^-$ invariant mass is shown in Fig. 18. One can conclude that, since the width of the coherent photoproduction is much smaller than the effective width of the background, this contribution can be extracted experimentally under the condition of a high resolution measurement of the $nK^+$ invariant mass.

In case of a energy resolution comparable to the width of the background peak one has to
FIG. 18: A summary plot of the total $nK^+$ invariant mass distribution in the $\gamma D \rightarrow npK^+K^-$ reaction at three fixed intervals of the $pK^-$ invariant mass with $M_0 = 1.52, 1.57$ and $1.47$ (GeV) at $\theta_{\gamma(pK^-)} = 27.5^o$ and $E_\gamma = 2$ GeV.

smear this peak. The simplest way to do it is integrating the $nK^+$ invariant mass distribution

over $\Omega$ in the forward hemisphere of the $pK^-$ pair photoproduction. The corresponding predictions for $M_0 = 1.52$ GeV and a summary plot for three values of $M_0$ are shown in Figs. 19a and b, respectively. One can see that again at $M_0 = M_\Lambda^*$ the background is
dominated by the resonance $\Lambda^*$ photoproduction where the neutron is a spectator. But the shape of the background is quite different from the previous case. Instead of the narrow peak one observes a monotonous increase of the background contribution. This behavior allows to extract the sharp $\Theta^+$ peak of the coherent $\Lambda^*\Theta^+$ photoproduction. The peak becomes negligible at $M_0 = M_{\Lambda^*} \pm 70$ MeV, as shown in Fig. 19b. Here one can also see the prediction for a Gaussian smearing of the $\Theta^+$ peak with $\sigma = 5$ MeV.

Consider now the backward hemisphere of the $pK^-$ pair photoproduction in the reaction $\gamma D \rightarrow npK^+ K^-$, say for $\theta_{\gamma(pK^-)} \simeq 152.5^\circ$. The corresponding invariant mass distributions at different $M_0$ are exhibited in Fig. 20. Now, the dominant contribution to the background comes from the spectator resonant $\Theta^+$ photoproduction, depicted in Fig. 10c. The other channels are rather weak. At $M_0 = 1.52$ GeV the background contribution is enhanced by the distribution function $W_{pK}$ which at $M_{nK^+} \simeq 1.54$ GeV is much greater for $M_0 \simeq M_{\Lambda^*}$ (see Fig. 14b). The coherent contribution of the $\Lambda^*\Theta^+$ photoproduction is a factor of four smaller than the background contribution.

The summary plot of the total invariant mass distribution of the $nK^+$ for three fixed intervals of the $pK^-$ invariant mass is displayed in Fig. 21. One can see a strong increase of the invariant mass distribution at $M_0 = 1.52$ GeV. But this increase is caused mainly by the properties of the distribution function $W_{pK}$. Here, we have no striking qualitative effect of the coherent $\Lambda^*\Theta^+$ photoproduction. Therefore, studying the coherent $\Lambda\Theta^+$ photoproduction seems to be difficult in this kinematical region.

Now we would like to make three comments. First, since in the forward hemisphere of

FIG. 20: The same as in Fig. 17 but for $\theta_{\gamma(pK^-)} = 152.5^\circ$. Notations "sp.$(\gamma n \rightarrow \Theta^+ K^+)$" and "sp.$(\gamma n \rightarrow nKK)$" corresponds to the processes depicted in Fig. 10c and d, respectively.
FIG. 21: The same as in Fig. 18 but for \( \theta_{\gamma(pK^-)} = 152.5^\circ \).

FIG. 22: The differential cross section of \( \Lambda^*\Theta^+ \) photoproduction with (solid curve) and without (dashed curve) contribution of the \( \gamma p \to \Theta^+ K^0 \) subprocess.

The \( \Lambda^* \) photoproduction the dominant contribution comes from the backward angles of the \( K^+ \) photoproduction in the elementary \( \gamma p \to \Lambda^* K^+ \) subprocess, our predictions are not sensitive to the choice of the solution for the coupling strength \( \alpha_{\Lambda^*} \) discussed in Sec. II (see Fig. 6 b).

Second, in our analysis we have assumed that the \( \Theta^+ \) photoproduction from the nucleon is dominated by the \( t \)-channel \( K^* \) exchange process. This assumption leads to a similarity of the \( \Theta^+ \) photoproduction from the neutron and proton. A violation of this similarity (or a suppression of the photoproduction from the proton, with keeping the cross section of the \( \gamma n \to \Theta^+ K^- \) on the same level) discussed recently [9, 23] would result in a suppression of the process shown in Fig. 7d. As a consequence, the coherent cross section of \( \Lambda^*\Theta^+ \)
photoproduction would be suppressed around the second peak at backward angles of the $pK^-$ pair photoproduction, shown in Figs. 9 and 15 leaving the first peak at forward angles photoproduction without change. The corresponding calculation of the differential cross section with and without contribution of the $\gamma p \to \Theta^+ \bar{K}^0$ subprocess is presented in Fig. 22. Since the coherent $\Lambda^*\Theta^+$ photoproduction is determined by the first peak, our main result shown in Fig. 19 remains unchanged.

Third, the “bump-like” structure of the differential cross section of the coherent $\gamma D \to \Lambda^*\Theta^+$ reaction is caused mainly by the spectral functions $S$ in Eqs. (17). Thus in Eq. (17a), the amplitude of the $\Theta^+ \to nK^+$ transition is a smooth function compared to the spectral function $S_{\Lambda^*}$ independently on the properties of $\Theta^+$. Therefore, our predictions remain to be valid for the $J^P = \frac{3}{2}^+$ of $\Theta^+$, considered in recent Ref. [9].

When our prediction is to be compared with experiments, one should pay attention, at least, the following two points. First, an energy spread in the beam photon may change the shape of the background, which is mainly determined by the quasi-free $\Lambda^*$ production. However, our conclusion indicated by Fig. 19 is not changed qualitatively. Second, the shape of the background is sensitive to the acceptance of the measurement. In particular, the effect of coherent $\Lambda^*\Theta^+$ production may be significantly suppressed when the detector does not have acceptance to detect $pK^-$ pair in the forward angles. In contrast, the acceptance to the forward $pK^-$ like one in the case of LEPS of SPring-8 [1] may make the effect more pronounced.

VI. SUMMARY

In summary we analyzed the coherent $\Lambda^*\Theta^+$ photoproduction in $\gamma D$ interaction with taking into account different background processes. We found that the behavior and the strength of the background processes depend strongly on the kinematics where the momentum distribution in the deuteron plays a key role. Thus, at fixed angle of the $pK^-$ photoproduction the $nK^+$ invariant mass distribution of the background processes looks like a narrow peak with maximum around the $\Theta^+$ mass. This behaviour hampers the extraction of the coherent process at finite invariant mass resolution. Most promising is an experimental analysis of the distributions integrated over the $pK^-$ production angles in the forward hemisphere of c.m.s. In this case the background processes increase monotonously
with $M_{nK^+}$ in the vicinity of $M_{\Theta^+}$, which allows to extract the coherent $\gamma D \to \Lambda^* \Theta^+$ channel even with finite invariant mass resolution. We demonstrated that the coherent $\Theta^+ \Lambda(1520)$ photoproduction does not depend on the $\Theta^+$ photoproduction amplitude, but rather it is defined by the probabilities of the $\Lambda(1520)$ photoproduction and the $\Theta^+ \to NK$ transition. Therefore, this effect may be used as an independent method for studying the mechanism of $\Theta^+$ production and $\Theta^+$ properties.

Our model estimates for the $\gamma D$ reaction may be considered as an example why the $\Theta^+$ peak is seen under certain experimental conditions and why it does not appear above the strong background in other ones.

Finally, we note that the predicted process of the coherent $\Lambda^* \Theta^+$ photoproduction may be studied experimentally at the electron and photon facilities at LEPS of SPring-8, JLab, Crystal-Barrel of ELSA, and GRAAL of ESFR.

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**APPENDIX A: TRANSITION OPERATORS FOR THE RESONANCE AMPLITUDES**

1. The $\Theta^+$ photoproduction amplitude

We show here the explicit expressions for the transition operators $M_\mu$ in Eq. (5) for a positive $\Theta^+$ parity and the PS coupling scheme.

The specific parameters for the form factor in Eq. (4) are defined by

$$F_s = F(M_N, s), \quad F_u = F(M_{\Theta}, u), \quad \text{and} \quad F_t = F(M_{K^+}, t). \quad (A1)$$

In addition, we need the form factor combinations

$$\tilde{F}_{tu} = F_t + F_u - F_tF_u \quad \text{and} \quad \tilde{F}_{su} = F_s + F_u - F_sF_u \quad (A2)$$
to construct the contact terms $\mathcal{M}_{\mu}^{c}$ given below that make the initial photoproduction amplitude gauge invariant [25, 26]. The four-momenta in the following equations are defined according to the arguments given in the reaction equation

$$\gamma(k) + N(p) \rightarrow \Theta^+(p_{\Theta}) + \bar{K}(\bar{q}) .$$

(A3)

a. $\gamma n \rightarrow \Theta^+ K^-$

$$\mathcal{M}_{\mu}^{i} = \frac{i e g_{\Theta \gamma \gamma} (k_{\mu} - 2 \bar{q}_{\mu}) \gamma_{5}}{t - M_{K^+}^{2}} F_{i} ,$$

(A4a)

$$\mathcal{M}_{\mu}^{s} = i e g_{\Theta \gamma \gamma} \frac{\bar{q} + \gamma_{5}}{s - M_{N}^{2}} \left( i \frac{k_{\mu}}{2 M_{N}} \sigma_{\mu \nu} k^{\nu} \right) \gamma_{5} F_{s} ,$$

(A4b)

$$\mathcal{M}_{\mu}^{u} = i e g_{\Theta \gamma \gamma} \left( \gamma_{\mu} + i \frac{\kappa_{\Theta}}{2 M_{\Theta}} \sigma_{\mu \nu} k^{\nu} \right) \left( \frac{p_{\Theta} - \gamma_{5}}{u - M_{\Theta}^{2}} \right) F_{u} ,$$

(A4c)

$$\mathcal{M}_{\mu}^{c} = i e g_{\Theta \gamma \gamma} \gamma_{5} \left[ \frac{(k - 2 \bar{q})_{\mu}}{t - M_{K^+}^{2}} (\bar{F}_{iu} - F_{i}) + \frac{(2 p_{\Theta} - k)_{\mu}}{u - M_{\Theta}^{2}} (\bar{F}_{iu} - F_{u}) \right] .$$

(A4d)

The transition operator of $t$-channel $K^*$ exchange amplitude is given by

$$\mathcal{M}_{\mu}^{i}(K^*) = \frac{e g_{\gamma K K^*} g_{\Theta \gamma K^*} \varepsilon_{\mu \nu \lambda \beta} k^{\alpha} \bar{q}^{\beta}}{M_{K^*} M_{\Theta} + M_{N}^{2}} \left[ \gamma^{\nu} - i \frac{\sigma^{\nu \lambda}}{M_{\Theta} + M_{N}^{2}} (p_{\Theta} - p_{\nu}) \right] F(M_{K^*}, t) .$$

(A5)

b. $\gamma p \rightarrow \Theta^+ \bar{K}^0$

$$\mathcal{M}_{\mu}^{s} = i \frac{e g_{\Theta \gamma \gamma}}{M_{\Theta} + M_{N}^{2}} \gamma_{5} \left( \frac{\bar{q} + \gamma_{5}}{s - M_{N}^{2}} \right) \left( \gamma_{\mu} + i \frac{k_{\mu}}{2 M_{N}} \sigma_{\mu \nu} k^{\nu} \right) F_{s} ,$$

(A6a)

$$\mathcal{M}_{\mu}^{u} = i \frac{e g_{\Theta \gamma \gamma}}{M_{\Theta} + M_{N}^{2}} \left( \gamma_{\mu} + i \frac{\kappa_{\Theta}}{2 M_{\Theta}} \sigma_{\mu \nu} k^{\nu} \right) \left( \frac{p_{\Theta} - \gamma_{5}}{u - M_{\Theta}^{2}} \right) F_{u} ,$$

(A6b)

$$\mathcal{M}_{\mu}^{c} = i \frac{e g_{\Theta \gamma \gamma}}{M_{\Theta} + M_{N}^{2}} \gamma_{5} \left[ \frac{(2 p + k)_{\mu}}{s - M_{N}^{2}} (\bar{F}_{su} - F_{s}) + \frac{(2 p_{\Theta} - k)_{\mu}}{u - M_{\Theta}^{2}} (\bar{F}_{su} - F_{u}) \right] .$$

(A6c)

2. $\Lambda^*$ photoproduction amplitude

We show here the explicit expressions for the transition operators $\mathcal{M}_{\sigma \mu}$ in Eq. (8) for the reactions $\gamma p \rightarrow \Lambda^* K^+$ and $\gamma n \rightarrow \Lambda^* K^0$. 

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\(a. \quad \gamma p \rightarrow \Lambda^* K^+\)

\[
\mathcal{M}_{t,\sigma} = i \frac{e g_{\Lambda^* NK} M_{\Lambda^*}^3 (2q_{\mu} - k_{\mu})(k_{\sigma} - q_{\sigma})\gamma_5}{t - M_{K^+}^2} F_t, \quad (A7a)
\]

\[
\mathcal{M}_{s,\mu}^s = -i \frac{e g_{\Lambda^* NK} M_{\Lambda^*}^3 q^{\sigma} \gamma_5 \hat{p} + \hat{k} + M_N}{s - M_N^2} \left( \gamma_{\mu} + i \frac{\kappa p}{2M_N} \sigma_{\mu\nu} k^\nu \right) F_s, \quad (A7b)
\]

\[
\mathcal{M}_{c,\mu}^c = i \frac{e g_{\Lambda^* NK} M_{\Lambda^*}^3 (2q_{\mu} - k_{\mu})(k_{\sigma} - q_{\sigma})\gamma_5}{t - M_{K^+}^2} \left[ \frac{(2q - k)_{\mu}(k - q)_{\sigma}}{t - M_{K^+}^2} (F_{ts} - F_t) - \frac{(2p + k)_{\mu}}{s - M_N^2} (F_{ts} - F_s) \right] + g_{\sigma\mu} \tilde{F}_{ts}. \quad (A7c)
\]

The corresponding form factors are defined by

\[
F_s = F(M_N, s), \quad F_t = F(M_{K^+}, t), \quad \text{and} \quad \tilde{F}_{ts} = F_t + F_s - F_tF_s. \quad (A8)
\]

The transition operator of \(t\)-channel \(K^*\) meson exchange amplitude is given by

\[
\mathcal{M}_{t,\mu}^t(K^*) = \frac{eg_{\gamma\gamma K^*} g_{\Lambda^* NK^*} \varepsilon_{\nu\mu\beta\gamma} k^\nu q^\alpha}{M_{K^*} M_{\Lambda^*}} \left[ q'_{\gamma\sigma} - \bar{q}'_{\gamma\sigma\beta} \right] F(M_{K^*}, t) \quad (A9)
\]

with \(q' = p_{\Lambda^*} - p\).

\(b. \quad \gamma n \rightarrow \Lambda^* K^0\)

\[
\mathcal{M}_{s,\mu}^s = -i \frac{e g_{\gamma NK} M_{\Lambda^*}^3 q^{\sigma} \gamma_5 \hat{p} + \hat{k} + M_N}{s - M_N^2} \left( i \frac{\kappa p}{2M_N} \sigma_{\mu\nu} k^\nu \right) F_s. \quad (A10)
\]

The \(t\)-channel \(K^*\)-exchange operator is defined by Eq. (A9) with appropriate coupling constant \(g_{\gamma\gamma K^*}\).

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