\( B_s(d) - \bar{B}_s(d) \) Mixing and \( B_s \to \mu^+\mu^- \) Decay in the NMSSM with the Flavour Expansion Theorem

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Abstract

In this paper, motivated by the observation that the Standard Model predictions are now above the experimental data for the mass difference \( \Delta M_{s(d)} \), we perform a detailed study of \( B_s(d) - \bar{B}_s(d) \) mixing and \( B_s \to \mu^+\mu^- \) decay in the \( \mathbb{Z}_3 \)-invariant NMSSM with non-minimal flavour violation, using the recently developed procedure based on the Flavour Expansion Theorem, with which one can perform a purely algebraic mass-insertion expansion of an amplitude written in the mass eigenstate basis without performing diagrammatic calculations in the interaction/flavour basis. Specifically, we consider finite orders of mass insertions for neutralinos but general orders for squarks and charginos, under two sets of assumptions for the squark flavour structures (i.e., while the flavour-conserving off-diagonal element \( \delta^{LR}_{33} \) is kept in both cases, only the flavour-violating off-diagonal elements \( \delta^{LL}_{23} \) and \( \delta^{RR}_{i3} \) (\( i = 1, 2 \)) are kept in cases I and II, respectively). Our analytic results are then expressed directly in terms of the initial Lagrangian parameters in the interaction/flavour basis, making it easy to impose experimental bounds on them. It is found numerically that the NMSSM effects in both cases can accommodate the observed deviation for \( \Delta M_{s(d)} \), while complying with the experimental constraints from the branching ratios of \( B_s \to \mu^+\mu^- \) and \( B \to X_s\gamma \) decays.

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1 Introduction

It is known that Supersymmetric (SUSY) extensions of the Standard Model (SM) are well motivated by being able to provide a unification of the SM gauge couplings at high scales, a solution of the hierarchy problem, and a viable dark matter candidate \([1, 2]\). As one of the low-energy realizations of SUSY, the Minimal Supersymmetric Standard Model (MSSM) \([3]\), carrying the minimal field content consistent with observations, has received over years of continuous attentions; see e.g. Refs. \([4, 5]\) for reviews. Despite having many advantages \([4, 5]\), the MSSM needs to be extended because of the following two main motivations. The first one is due to the existence of the “µ-problem” in the MSSM \([6]\), where µ is a dimensionful parameter set by hand at the electroweak (EW) scale before spontaneous symmetry breaking. The second one is driven by the recent discovered 125 GeV SM-like Higgs boson \([7, 8]\), which has imposed strong constrains on the parameter space of MSSM \([9–11]\). Among the various non-minimal SUSY models, the Next-to-Minimal Supersymmetric Standard Model (NMSSM) \([12–15]\), the simplest extension of the MSSM with a gauge singlet superfield, not only has the capability to fix these shortcomings of the MSSM, but also alleviates the tension implied by the lack of any evidence for superpartners below the EW scale \([16]\). Specifically, this model can solve the “µ-problem” of the MSSM elegantly by generating an effective µ-term at the SUSY breaking scale \([17, 18]\). Restrictions on the Higgs sector can also be relaxed in the NMSSM, because the Higgs field can acquire a larger tree-level mass with a low SUSY scale, making accordingly quantum corrections to the observed 125 GeV Higgs boson small \([14, 15]\).

Along with the dedicated direct searches at the Large Hadron Collider (LHC) for SUSY particles \([16]\), it is also interesting and even complementary to investigate virtual effects of these hypothesized particles on low-energy processes, such as the neutral-meson mixings as well as the CP violation and rare decays of various hadrons \([19–33]\). In this paper, we shall focus on the \(Z_3\)-invariant NMSSM \([14, 15]\), a simpler scenario of NMSSM with a scale-invariant superpotential, and study its effects on \(B_s(d) - \bar{B}_s(d)\) mixing, \(B_s \rightarrow \mu^+\mu^-\) and \(B \rightarrow X_s\gamma\) decays.

With the latest lattice inputs \([34, 35]\) for the bag parameters that are used to quantify the non-perturbative hadronic matrix elements between \(|B_s(d)\rangle\) and \(|\bar{B}_s(d)\rangle\) states, the SM predictions for the mass difference between the mass eigenstates of neutral \(B_s(d)\) meson read \([34, 36]\)

\[
\Delta M_s^{SM} = 20.01 \pm 1.25 \text{ps}^{-1}, \quad \Delta M_d^{SM} = 0.630 \pm 0.069 \text{ps}^{-1},
\]  

(1.1)
both of which are now above their respective experimental values \[37, 38\]

\[
\Delta M_s^{\text{exp}} = 17.757 \pm 0.021 \text{ ps}^{-1}, \quad \Delta M_d^{\text{exp}} = 0.5064 \pm 0.0019 \text{ ps}^{-1}.
\] (1.2)

This observation has profound implications for New Physics (NP) models \[36, 39\]. As detailed in Refs. \[27, 32, 39\], this implies particularly that the constrained minimal flavour violating (CMFV) models, in which all flavour violations arise only from the Cabibbo-Kobayashi-Maskawa (CKM) matrix \[40, 41\], have difficulties in describing the current data on \(\Delta M_{s(d)}\).

Thus, in order to relax such a tension, one has to resort to scenarios with non-minimal flavour violation (NMFV), which involve extra sources of flavour- and/or CP-violation, and can provide therefore potential negative contributions to \(\Delta M_{s(d)}\) \[42–44\]. This motivates us to investigate whether the \(Z_3\)-invariant NMSSM with NMFV, in which the extra flavour violations arise from the non-diagonal parts of the squark mass matrices related to the soft SUSY breaking terms, can accommodate the observed deviation for \(\Delta M_{s(d)}\).

In SUSY models, a transition amplitude is more conveniently calculated in the interaction/flavour basis in which gauge interactions are flavour diagonal and flavour-changing interactions originate from the off-diagonal entries of the mass matrices in the initial Lagrangian before diagonalization and identification of the physics states, than in the mass eigenstate (ME) basis in which the transition amplitude is expressed in terms of the physical masses and mixing matrices. This can be achieved using two different methods. The first one is based on the well-known diagrammatic technique called the Mass Insertion Approximation (MIA) \[45–48\]. Here diagonal elements of the mass matrices are absorbed into the definition of (un-physical) massive propagators and the amplitude is, at each loop order, expanded into an infinite series of the off-diagonal elements of the mass matrices, commonly referred to as mass insertion (MI).

The second one is based on the Flavour Expansion Theorem (FET) \[49\], according to which an analytic function about zero of a Hermitian matrix can be expanded polynomially in terms of its off-diagonal elements with coefficients being the divided differences of the analytic function and arguments the diagonal elements of the Hermitian matrix. As a purely algebraic method, it offers an alternative derivation of the MIA result directly from the amplitude calculated in the ME basis, without performing tedious and error-prone diagrammatic calculations with MIs in the interaction/flavour basis \[49\]. Even in the case where there is no clear diagrammatic picture, the FET expansion can still give a consistent MIA result. This method has also been
automatized in the package MassToMI [50], facilitating the expansion of an ME amplitude to any user-defined MI order. See e.g. Refs. [43, 49, 51–53] for recent applications of this method.

In the \( \mathbb{Z}_3 \)-invariant NMSSM with NMFV, we further assume that the third-generation squarks can mix with the other two generations simultaneously, but leaving the latter two immune to each other. Such a choice is motivated by the flavour mixing effects observed in \( B_s(d) - \bar{B}_s(d) \) mixing, \( B_s \rightarrow \mu^+\mu^- \) and \( B \rightarrow X_s\gamma \) decays [22–33]. With such a specific squark flavour structures, we shall then adopt the FET procedure [49] to calculate the mass difference \( \Delta M_s(d) \) and the branching ratio of \( B_s \rightarrow \mu^+\mu^- \) decay, by considering general MI orders for squarks and charginos but finite MI orders for neutralinos. While the general MI orders have also been considered in Refs. [54, 55], only one kind of MI parameter is kept in the whole “fat propagators”. In our case, however, there exist two kinds of MI parameters in each line and the mixed arrangement of them is required. For concreteness, We call our procedure the FET expansion with different MI order and, by checking if the FET results agree with the ones calculated numerically in the ME basis, test our estimation for the optimal cutting-off MI orders. For the branching ratio of \( B \rightarrow X_s\gamma \) decay, the public code SUSY_FLAVOR [56–58] is used.

Our paper is organized as follows. In Sec. 2, after specifying the flavour structures assumed in our scenario, we introduce the FET procedure with different MI order, which is then used to calculate the \( B_s(d) - \bar{B}_s(d) \) mixing and \( B_s \rightarrow \mu^+\mu^- \) decay in Sec. 3. Detailed numerical results and discussions are then presented in Sec. 4. Our conclusions are finally made in Sec. 5. For convenience, the block terms of squarks and charginos are listed in the appendix.

2 FET with different MI order in \( \mathbb{Z}_3 \)-invariant NMSSM

2.1 Lagrangian of \( \mathbb{Z}_3 \)-invariant NMSSM

At the Lagrangian level, the \( \mathbb{Z}_3 \)-invariant NMSSM differs from the MSSM by the superpotential and the soft SUSY breaking part. The scale-invariant superpotential of NMSSM reads [59, 60]

\[
W_{\text{NMSSM}} = W_{\text{MSSM}}|_{\mu = 0} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3, \tag{2.1}
\]

where \( W_{\text{MSSM}}|_{\mu = 0} \) is the MSSM superpotential but without the \( \mu \) term [61, 62], and \( \hat{S} \) denotes the Higgs singlet superfield, while \( \hat{H}_u = (\hat{H}_u^+, \hat{H}_u^0)^T \) and \( \hat{H}_d = (\hat{H}_d^0, \hat{H}_d^-)^T \) are the two Higgs doublet superfields, with the convention \( \hat{H}_u \cdot \hat{H}_d = \hat{H}_u^+ \hat{H}_d^- - \hat{H}_u^0 \hat{H}_d^0 \). The dimensionless param-
eters $\lambda$ and $\kappa$ can be complex in general, but are real in the CP-conserving case. After the scalar component of $\hat{S}$ gets a non-zero vacuum expectation value (VEV), $\langle S \rangle = v_s/\sqrt{2}$, the second term in Eq. (2.1) generates an effective $\mu$ term, with $\mu_{\text{eff}} = \lambda v_s/\sqrt{2}$, which then solves the “$\mu$-problem” of the MSSM [14, 15].

With the scalar components of the Higgs doublet and singlet superfields being denoted by $H_u$, $H_d$, and $S$, respectively, the soft SUSY breaking Lagrangian of $Z_3$-invariant NMSSM is then given by [59, 60]

$$-L_{\text{NMSSM}}^{\text{soft}} = -L_{\text{MSSM}}^{\text{soft}}|_{\mu = 0} + m_S^2 |S|^2 + \left( \lambda A_\lambda S H_u \cdot H_d + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.} \right),$$

(2.2)

where $L_{\text{MSSM}}|_{\mu = 0}$ corresponds to the MSSM part but with the $\mu$-related term removed [61, 62]. The soft SUSY breaking mass parameter $m_S^2$ is real, while the soft SUSY breaking trilinear couplings $A_\lambda$ and $A_\kappa$ are complex in general, but are also taken to be real in the CP-conserving case, as is assumed throughout this paper.

2.2 Flavour structures of $Z_3$-invariant NMSSM

Firstly, we focus on the up- and down-squark mass squared matrices $M_\tilde{U}^2$ and $M_\tilde{D}^2$, which can be written in their most general $2 \times 2$-block form as [60]

$$M_{\tilde{q}}^2 = \begin{pmatrix} M_{\tilde{q}, LL}^2 & M_{\tilde{q}, LR}^2 \\ M_{\tilde{q}, RL}^2 & M_{\tilde{q}, RR}^2 \end{pmatrix}, \quad \tilde{q} = \tilde{U}, \tilde{D},$$

(2.3)

in the so-called super-CKM basis [48]. In the NMFV paradigm [29, 30], these two mass matrices are not yet diagonal and can introduce general squark flavour mixings that are usually described by a set of dimensionless parameters $\delta_{ij}^{AB}$, with A, B=L, R referring to the left- and right-handed superpartners of the corresponding quarks and i, j = 1, 2, 3 the generation indices [47].

Throughout this paper, we assume that the third-generation squarks can mix with the other two generations simultaneously, but leaving the latter two immune to each other, so as to comply with the severe constraints from flavour and precision data [30, 63, 64]. This promotes us to consider the following two sets of squark flavour structures: while the flavour-conserving off-diagonal element $\delta_{33}^{LR}$ is kept in both cases, only the flavour-violating off-diagonal elements $\delta_{23}^{LL}$ and $\delta_{i3}^{RR}$ ($i = 1, 2$) are kept in cases I and II, respectively. In addition, to avoid the occurrence
of dangerous charge and colour breaking minima and unbounded from below directions in the effective potential \cite{65, 66}, we set all the flavour-violating off-diagonal elements in the LR and RL sectors to be zero. Then, the two mass squared matrices $M^2_U$ and $M^2_D$ in case I are given, respectively, by

$$
M^2_U = \begin{pmatrix}
M_{S1} & 0 & 0 & 0 & 0 & 0 \\
0 & M_{S1} & \delta_{23}\sqrt{M_{S1}M_{S2}} & 0 & 0 & 0 \\
0 & \delta_{23}\sqrt{M_{S1}M_{S2}} & M_{S2} & 0 & 0 & \delta_{36}M_{S2} \\
0 & 0 & 0 & M_{S1} & 0 & 0 \\
0 & 0 & 0 & 0 & M_{S1} & 0 \\
0 & 0 & \delta_{36}M_{S2} & 0 & 0 & M_{S2}
\end{pmatrix},
$$  
(2.4)

$$
M^2_D = \begin{pmatrix}
M_{S1} & 0 & -\lambda_{\text{CKM}}\delta_{23}\sqrt{M_{S1}M_{S2}} & 0 & 0 & 0 \\
0 & M_{S1} & \delta_{23}\sqrt{M_{S1}M_{S2}} & 0 & 0 & 0 \\
-\lambda_{\text{CKM}}\delta_{23}\sqrt{M_{S1}M_{S2}} & \delta_{23}\sqrt{M_{S1}M_{S2}} & M_{S2} & 0 & 0 & 0 \\
0 & 0 & 0 & M_{S1} & 0 & 0 \\
0 & 0 & 0 & 0 & M_{S1} & 0 \\
0 & 0 & 0 & 0 & 0 & M_{S2}
\end{pmatrix},
$$  
(2.5)

where $\delta_{23} \equiv \delta^{LL}_{23}$ and $\delta_{36} \equiv \delta^{LR}_{33}$. In the LL sectors, which satisfy the relation $(M^2_U)^{LL} = K^\dagger (M^2_D)^{LL} K$ (with $K$ being the CKM matrix) due to the $SU(2)_L$ gauge invariance \cite{48}, we have neglected safely the $O(\lambda_{\text{CKM}}^2)$ terms in Eq. (2.5) (and also in Eq. (2.7)), where $\lambda_{\text{CKM}}$ is one of the CKM parameters. Here we have also assumed that the first two generations of squarks are nearly degenerate in mass \cite{4}.

In case II, on the other hand, the two mass squared matrices are given, respectively, by

$$
M^2_U = \begin{pmatrix}
M_{S1} & 0 & 0 & 0 & 0 & 0 \\
0 & M_{S1} & 0 & 0 & 0 & 0 \\
0 & 0 & M_{S2} & 0 & 0 & \delta_{36}M_{S2} \\
0 & 0 & 0 & M_{S1} & 0 & 0 \\
0 & 0 & 0 & 0 & M_{S1} & 0 \\
0 & 0 & \delta_{36}M_{S2} & 0 & 0 & M_{S2}
\end{pmatrix},
$$  
(2.6)
where $\delta_{46} \equiv \delta_{13}^{\text{RR}}$ and $\delta_{56} \equiv \delta_{23}^{\text{RR}}$. In both of these two cases, a non-zero $\delta_{33}^{\text{LR}}$ is kept to reproduce the 125 GeV SM-like Higgs boson [30, 63, 64]. Here, for simplicity, we assume that all the $\delta_{ij}^{AB}$ parameters are real and hence $\delta_{ij}^{AB} = \delta_{ji}^{BA}$, due to hermiticity of the squark mass matrices.

The mass matrix for charginos in the interaction basis reads [62]

$$M_\chi = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu_{\text{eff}} \end{pmatrix}, \quad (2.8)$$

where $M_2$ is the wino mass, and $\beta = \tan^{-1}(v_u/v_d)$ is the mixing angle of the two Higgs doublets, defined in terms of their VEVs $v_u = \sqrt{2} \langle H_u \rangle$ and $v_d = \sqrt{2} \langle H_d \rangle$. The squared masses $M_{Ci}$, $M_{Pi}$ and the MI parameters $\delta_{ij}^C$, $\delta_{ij}^P$ are defined, respectively, by

$$M_{Ci} = (M_\chi^\dagger M_\chi)_{ii}, \quad M_{Pi} = (M_\chi^\dagger M_\chi)_{ii}, \quad (2.9)$$

$$\delta_{ij}^C = \frac{(M_\chi^\dagger M_\chi)_{ij}}{\sqrt{M_{Ci} M_{Cj}}}, \quad \delta_{ij}^P = \frac{(M_\chi^\dagger M_\chi)_{ij}}{\sqrt{M_{Pi} M_{Pj}}}, \quad (2.10)$$

where $i \neq j$ and the summation is not applied for the same index here.

The neutralino mass matrix is given in the basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})^T$ by [14, 43]

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -\frac{ev_d}{2 \cos \theta_W} & -\frac{ev_u}{2 \cos \theta_W} & 0 \\ 0 & M_2 & \frac{ev_d}{2 \sin \theta_W} & -\frac{ev_u}{2 \sin \theta_W} & 0 \\ -\frac{ev_d}{2 \cos \theta_W} & \frac{ev_d}{2 \sin \theta_W} & 0 & -\mu_{\text{eff}} & -\frac{\lambda v_u}{\sqrt{2}} \\ -\frac{ev_u}{2 \cos \theta_W} & -\frac{ev_u}{2 \sin \theta_W} & -\mu_{\text{eff}} & 0 & -\frac{\lambda v_d}{\sqrt{2}} \\ 0 & 0 & -\frac{\lambda v_d}{\sqrt{2}} & -\frac{\lambda v_d}{\sqrt{2}} & \sqrt{2} \kappa v_s \end{pmatrix}, \quad (2.11)$$

where $M_1$ is the bino mass, and $\theta_W$ is the weak mixing angle. Such a mass matrix indicates that the singlino $\tilde{S}$ couples only to the Higgsinos $\tilde{H}_d^0$ and $\tilde{H}_u^0$, but not to the gauginos $\tilde{B}$ and...
The squared masses $M_{Ni}$ and the MI parameters $\delta_{ij}^N$ are defined, respectively, by

$$M_{Ni} = (M^\dagger \chi_0 M_{\chi_0})_{ii}, \quad \delta_{ij}^N = \frac{(M^\dagger \chi_0 M_{\chi_0})_{ij}}{\sqrt{M_{Ni} M_{Nj}}},$$

(2.12)

where $i \neq j$ and the summation is also not applied for the same index. Diagonalization of Eq. (2.11) is rather involved and has to be in practice performed numerically [14].

### 2.3 FET expansion with different MI order

Before performing the FET expansion, one has to write down the transition amplitude in the ME basis [49]. All the relevant Feynman rules are taken from Refs. [14, 15, 62, 67, 68]. Then, the procedure of FET expansion with general/finite MI order includes the following three steps.

**Step 1.** One transforms the amplitude written in the ME basis into the intermediate result expressed in terms of the blocks $L_X(i, j)$, which are defined, respectively, as [50]

$$\sum_A (Z_U)_i A l_p (m^2_A) (Z_U)_{jA}^* A = l_p (M^2_U)_{ij} \equiv L_U(i, j),$$

(2.13)

$$\sum_A (Z_D)_i A l_p (m^2_A) (Z_D)_{jA}^* A = l_p (M^2_D)_{ij} \equiv L_D(i, j),$$

(2.14)

for the up and down squarks behaving as scalar fields,

$$\sum_A (Z^+_X)_i A l_p (m^2_A) (Z^+_X)_{jA}^* A = \sum_k (M_X)_{ik} l_p (M^\dagger_X M_X)_{kj} \equiv \sum_k (M_X)_{ik} L_C(k, j),$$

(2.15)

$$\sum_A (Z^-_X)_i A l_p (m^2_A) (Z^-_X)_{jA}^* A = \sum_k (M^\dagger_X)_{ik} l_p (M_X M^\dagger_X)_{kj} \equiv \sum_k (M^\dagger_X)_{ik} L_P(k, j),$$

(2.16)

for the charginos that behave as Dirac fermions, and

$$\sum_A (Z_{X^0})_i A l_p (m^2_A) (Z_{X^0})_{jA}^* A = \sum_k (M_{X^0})_{ik} l_p (M^\dagger_{X^0} M_{X^0})_{kj} \equiv \sum_k (M_{X^0})_{ik} L_N(k, j),$$

(2.17)

$$\sum_A (Z^+_{X^0})_i A l_p (m^2_A) (Z^+_{X^0})_{jA}^* A = \sum_k (M^\dagger_{X^0})_{ik} l_p (M_{X^0} M^\dagger_{X^0})_{kj} \equiv \sum_k (M^\dagger_{X^0})_{ik} L_N(k, j),$$

(2.18)

for the charginos that behave as Dirac fermions, and

$$\sum_A (Z_{X^0})_i A l_p (m^2_A) (Z_{X^0})_{jA} = \sum_k (M_{X^0})_{ik} l_p (M^\dagger_{X^0} M_{X^0})_{kj} \equiv \sum_k (M_{X^0})_{ik} L_N(k, j),$$

(2.19)

$$\sum_A (Z^+_{X^0})_i A l_p (m^2_A) (Z^+_{X^0})_{jA} = \sum_k (M^\dagger_{X^0})_{ik} l_p (M_{X^0} M^\dagger_{X^0})_{kj} \equiv \sum_k (M^\dagger_{X^0})_{ik} L_N(k, j),$$

(2.20)
\[ \sum_A (Z_{\chi_0})^*_{iA} m_A l_p(m_A^2)(Z_{\chi_0})_{jA} = \sum_k (M_{\chi_0})_{ik} l_p(M_{\chi_0}^T M_{\chi_0})_{kj} \equiv \sum_k (M_{\chi_0})_{ik} L_N(k, j), \quad (2.21) \]

for the neutralinos behaving as Majorana fermions. Here \( l_p(m_A^2) \) represents symbolically part of the transition amplitude that depends on the mass \( m_A \) of an internal physical particle in a Feynman diagram, at tree or loop level; for example, \( l_p(m_A^2) = 1/(q^2 - m_A^2) \) can be a propagator, with \( q^\mu \) being the momentum of particle \( A \). The unitary transformation matrices \( Z_U, Z_D, Z_\chi^+, Z_\chi^- \), and \( Z_{\chi_0} \) are introduced to diagonalize the Hermitian mass squared matrices \( M^2_U, M^2_D, M^2_\chi, M^T_\chi, \) and \( M^T_{\chi_0}, M_{\chi_0}, \) respectively. We use \( A, B, \cdots \) and \( i, j, k, \cdots \) to represent the flavour indices of field multiplets in the ME and the interaction/flavour basis, respectively.

After applying the transformation rules specified by Eqs. (2.13)–(2.21), one can see that the blocks \( L_X(i, j) \) depend only on the matrix elements of some functions with arguments being the Hermitian mass squared matrices, and can be given by the expansion [49]

\[ L_X(i, j) = \sum_{n=0}^{\infty} L_X(n; i, j), \quad (2.22) \]

where \( L_X(n; i, j) \) represents the \( n \)-th term in the MI order of the blocks \( L_X(i, j) \).

**Step 2.** During our calculation, we also encounter the case in which a Feynman diagram contains two lines involving the same particle. In such a case, the product of two blocks with MI orders specified respectively by \( m \) and \( n \), \( L_X(m; i, j)L_X(n; i, j) \), should be firstly combined into a single term with fixed MI order, such as \( L_{UU}(2n; 3; i; 3, j) = \sum_{n_1=1}^n L_U(2n_1-1; 3, i)L_U(2n-2n_1+1; 3, j) \), where \( L_{XX}(n; i; j; i', j') \) denotes the \( n \)-th term in the MI order of the block \( L_{XX}(i; j; i', j') \), with

\[ L_{XX}(i; j; i', j') = \sum_{n=0}^{\infty} L_{XX}(n; i; j; i', j'). \quad (2.23) \]

All the non-zero block terms \( L_X(n; i, j) \) and \( L_{XX}(n; i; j; i', j') \) for squarks and charginos, with the flavour structures specified in Sec. 2.2, can be easily derived and are listed in the appendix.

As the neutralino mass matrix \( M_{\chi_0} \), given by Eq. (2.11), has many non-zero elements, one can use the following recursive formulas to represent the corresponding blocks

\[ L_N(n; i_0, i_n) = \sum_{i_1, i_2, \cdots, i_{n-1}} l_N^r(i_0, i_1, \cdots, i_n) \left( \delta^N_{i_0 i_1} \sqrt{M_{N_{i_0} M_{N_{i_1}}} \right) \times \left( \delta^N_{i_1 i_2} \sqrt{M_{N_{i_1} M_{N_{i_2}}} \right) \cdots \left( \delta^N_{i_{n-1} i_n} \sqrt{M_{N_{i_{n-1}} M_{N_{i_n}}} \right), \quad (2.24) \]
where \( l_N^r(i_0, i_1, \cdots, i_n) = \frac{1}{(q^2 - M_{S_{i_0}})(q^2 - M_{S_{i_1}}) \cdots (q^2 - M_{S_{i_n}})} \), and can be re-expressed in terms of \( l_N^r(i_x) \) \((x = 0, 1, \cdots, n)\) using the “divided difference” method [50].

**Step 3.** One now needs to perform the loop-momentum integration over the products of blocks \( L_X(n; i, j) \), \( L_{XX}(n; i, j; i', j') \), and \( l_N^r(i_x) \) introduced in the last step. As only one-loop amplitudes are involved throughout this paper, this can be done using iteratively the operation \( \partial_{M_{SQ}} \text{Loop}_X \), where \( M_{SQ} \) denotes symbolically the squared mass, equaling to \( M^2 \) for scalars and to \( M\dagger M \) or \( MM^\dagger \) for fermions, and \( \text{Loop}_X \) is the \( n \)-point one-loop integrals in the Passarino-Veltman (PV) basis [69], such as \( D_{2n}, C_{2n} \) and \( B_0 \) introduced in Ref. [22].

With the aid of these three steps, one can then successfully transform a transition amplitude written initially in the ME basis into an expansion in terms of the MI parameters, up to any user-defined MI order [49, 50].

### 2.4 MI-order estimation

Although the FET procedure can provide with us the result expanded to any MI order, an optimal cutting-off should be applied due to the time costing of the programme running. Here we illustrate an efficient MI-order estimation method to get the appropriate order which was usually set by hand (order 2 or 4) in most of recent works [43, 51–53].

Taking the block term \( L_{UU}(2n; 3, 2, 3, 2) \) in case I,

\[
L_{UU} (2n; 3, 2; 3, 2) = \frac{nM_{S_1}M_{S_2}\delta_{23}^2}{(q^2 - M_{S_1})^2(q^2 - M_{S_2})^{n+1}} \Delta_1^{n-1},
\]

where \( \Delta_1 \equiv \frac{M_{S_1}M_{S_2}\delta_{23}^2}{q^2 - M_{S_1}} + \frac{M_{S_3}^2\delta_{36}^2}{q^2 - M_{S_2}} \), as an example, and using the inequality [49]

\[
|\text{PV}_0^{(n+1)}(m_1^2, m_2^2, \cdots, m_n^2, m_{n+1}^2)| \leq \frac{1}{m_{n+1}^2} |\text{PV}_0^{(n)}(m_1^2, m_2^2, \cdots, m_n^2)|,
\]

satisfied by the PV-integrals with vanishing external momenta that are defined by

\[
\text{PV}_0^{(n)}(m_1^2, m_2^2, \cdots, m_n^2) = -i(4\pi)^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{\prod_{j=1}^{n} (q^2 - m_j^2)},
\]

with the assumption that \( n \geq 3 \) to avoid divergent integrals, we can obtain

\[
\left| \int d^4q \, L_{UU}(2n; 3, 2; 3, 2) \, L_{\text{oth}} \right| \leq n (\delta_{23}^2 + \delta_{36}^2)^{n-1} \left| \int d^4q \, L_{UU}(2; 3, 2; 3, 2) \, L_{\text{oth}} \right|,
\]

where \( L_{\text{oth}} \) is the one-loop amplitude.
where \( L_{\text{others}} \) represent the blocks related to the other types of particles, such as \( L_{\text{CC}} \). Then, for a given small constant \( 0 < c_0 < 1 \), only when

\[
(n + 1) \left( \delta_{23}^2 + \delta_{36}^2 \right)^n < c_0,
\]

(2.29)
can the terms starting from the \((2n + 1)\)-th MI order in the series expansion of the block \( L_{UU}(3, 2; 3, 2) \) be safely neglected. Thus, the cutting-off MI order for \( L_{UU}(3, 2; 3, 2) \) should be 2\( n \) at least. The same method can be applied for other blocks, and the final cutting-off MI orders for squarks and charginos can be determined accordingly.

For the neutralino blocks, Eq. (2.26) still works for estimating the required MI order. In this case, we obtain

\[
\left| \int d^4q L_N(n; i_0, j_0) L_{\text{others}} \right| \leq \sum_{i_1, i_2, \cdots , i_{n-1}} \left| \int d^4q L_N^\prime(i_0) L_{\text{others}} \delta_{i_0 i_1}^N \delta_{i_1 i_2}^N \cdots \delta_{i_{n-1} j_0}^N \sqrt{\frac{M_{N i_0}}{M_{N j_0}}} \right| \sqrt{\frac{M_{N i_0}}{M_{N j_0}}}, \tag{2.30}
\]

and

\[
\left| \int d^4q L_N(n; i_0, i_n) (M_{\chi_0})_{i_0 j_0} L_{\text{others}} \right| \leq \sum_{i_1, i_2, \cdots , i_n} \left| \int d^4q L_N^\prime(i_0) L_{\text{others}} \delta_{i_0 i_1}^N \delta_{i_1 i_2}^N \cdots \delta_{i_{n-1} i_n}^N (M_{\chi_0})_{i_0 j_0} \right| \sqrt{\frac{M_{N i_0}}{M_{N i_n}}} \sqrt{\frac{M_{N i_0}}{M_{N j_0}}} \sqrt{\frac{M_{N i_n}}{M_{N i_0}}} < c_0
\]

(2.31)

So, when \( \left| \delta_{i_0 i_1}^N \delta_{i_1 i_2}^N \cdots \delta_{i_{n-1} j_0}^N \right| < c_0 \) and \( \left| \delta_{i_0 i_1}^N \delta_{i_1 i_2}^N \cdots \delta_{i_{n-1} i_n}^N (M_{\chi_0})_{i_0 j_0} \right| < c_0 \) for fixed indices \( i_0 \) and \( j_0 \), the summation over the MI index can be terminated to the \( n \)-th order.

3 \( B_s(d) - \bar{B}_s(d) \) mixing and \( B_s \to \mu^+ \mu^- \) decay

In this section, we shall apply the procedure of FET expansion with general/finite MI order to the \( B_s(d) - \bar{B}_s(d) \) mixing and \( B_s \to \mu^+ \mu^- \) decay, within the \( \mathbb{Z}_3 \)-invariant NMSSM with NMFV.

3.1 \( B_s(d) - \bar{B}_s(d) \) mixing

The strength of \( B_s(d) - \bar{B}_s(d) \) mixing is described by the mass difference \( \Delta M_{s(d)} \), defined by [70]

\[
\Delta M_q = 2|M_{12}^q| = 2|\langle B_q| \mathcal{H}_{\text{eff}}^{\Delta B=2} |\bar{B}_q \rangle|, \quad q = s, d,
\]

(3.1)
where $M_{12}^q$ denotes the off-diagonal element in the neutral $B_q$-meson mass matrix, and the effective weak Hamiltonian can be written in a general form as [21]

$$H_{\text{eff}}^{\Delta B=2} = \sum_i C_i Q_i + \text{h.c..}$$

Within the $Z_3$-invariant NMSSM with NMFV, the following eight operators, as defined in Ref. [22], are all found to be relevant:

\begin{align*}
Q_{VLL}^1 &= (\bar{b}^\alpha \gamma_\mu P_L q^\alpha)(\bar{b}^\beta \gamma_\mu P_L q^\beta), & Q_{LR}^1 &= (\bar{b}^\alpha \gamma_\mu P_L q^\alpha)(\bar{b}^\beta P_R q^\beta), \\
Q_{VRR}^1 &= (\bar{b}^\alpha P_L q^\alpha)(\bar{b}^\beta P_R q^\beta), & Q_{LR}^2 &= (\bar{b}^\alpha \sigma_{\mu\nu} P_L q^\alpha)(\bar{b}^\beta \sigma^{\mu\nu} P_R q^\beta), \\
Q_{SLL}^1 &= (\bar{b}^\alpha P_L q^\alpha)(\bar{b}^\beta P_L q^\beta), & Q_{SLL}^2 &= (\bar{b}^\alpha \sigma_{\mu\nu} P_L q^\alpha)(\bar{b}^\beta \sigma^{\mu\nu} P_L q^\beta), \\
Q_{SRR}^1 &= (\bar{b}^\alpha P_R q^\alpha)(\bar{b}^\beta P_R q^\beta), & Q_{SRR}^2 &= (\bar{b}^\alpha \sigma_{\mu\nu} P_R q^\alpha)(\bar{b}^\beta \sigma^{\mu\nu} P_R q^\beta),
\end{align*}

(3.3)

where $\alpha$ and $\beta$ are the colour indices, $\sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]$, and $P_{L,R} = (1 \mp \gamma_5)/2$.

To the lowest order in the EW theory, the corresponding Wilson coefficients $C_i$, at the matching scale, of the operators $Q_i$ are obtained by evaluating the various one-loop box diagrams mediated by heavy particles appearing in the SM and beyond. Within the SM, only $C_{VLL}^1$ gets a non-negligible contribution from the one-loop box diagrams with up-type quarks and $W$ bosons circulating in the loops [70], and the perturbative two-loop QCD corrections to $C_{VLL}^1$ are also known [73]. In the context of $Z_3$-invariant NMSSM with NMFV, on the other hand, all the eight Wilson coefficients $C_i$ can get non-zero contributions from the additional one-loop box diagrams mediated by: 1) charged Higgs, up-quarks; 2) chargino, up-squarks; 3) gluinos, down-squarks; 4) neutralinos, down-squarks; 5) mixed gluino, neutralino, down-squarks [22, 27, 74].

With the aid of FeynArts [75] and FeynCalc [76] packages, all these Feynman diagrams can be calculated and the resulting Wilson coefficients are expressed in terms of the rotation matrices $Z_U, Z_D, Z_\chi^+, Z_\chi^-$, and $Z_{\chi_0}$, as well as the blocks $L_X(i,j)$ and $L_{XX}(i,j; i', j')$. Our results for the Wilson coefficients agree with the ones given in Refs. [27, 29, 43]. Then, following the procedure detailed in Sec. 2.3, we can transform these Wilson coefficients given in the ME basis into the FET results. Here we have made full use of the hierarchies among the CKM parameters to

\footnote{Here we do not consider the double-penguin diagrams, which involve the exchange of CP-even and CP-odd scalars, and can give significant contributions only for large values of $\tan \beta$ [22, 71, 72]. This is justified by our choice of the two sets of SUSY parameters collected in Table 2, with $\tan \beta$ being fixed at 3 and 10, respectively.}
simplify the final results. For example, when calculating \( \Delta M_s \) in case I, we encounter a term

\[
\sum_{i,j} K^*_i L_{UU}(i,j) K_j = \sum_i K^*_i L_{UU}(i,i) K_i + \sum_{i \neq j} K^*_i L_{UU}(i,j) K_j. \tag{3.4}
\]

As \( L_{UU}(i,j) \), with \( i \neq j \), does not vanish only when \( (i, j) = (2, 3) \) or \( (3, 2) \), and because of \( |K^*_i K^*_{i2}| \ll |K^*_i K^*_{22}| \), we can safely neglect the term with \( (i, j) = (2, 3) \) to get

\[
\sum_{i,j} K^*_i L_{UU}(i,j) K_j \approx \sum_i K^*_i L_{UU}(i,i) K_i + K^*_3 L_{UU}(3,2) K_{22}. \tag{3.5}
\]

Once the initial conditions for the Wilson coefficients are obtained, we need to include the renormalization group (RG) running effects from the matching scale down to the low-energy scale, at which the hadronic matrix elements are evaluated by lattice methods [34, 35]. All the relevant ingredients for this RG running can be found in Ref. [21]. In this way, we can obtain the final result of the off-diagonal element \( M_{12}^q \) and thus the mass difference \( \Delta M_{s(d)} \). For later convenience, the different contributions to \( M_{12}^q \) are split into the following form:

\[
M_{12}^q \equiv M_{12}^{(q) \text{SM}} + M_{12}^{(q) \text{CH}} + M_{12}^{(q) \text{C}} + M_{12}^{(q) \text{NG}}, \tag{3.6}
\]

where \( M_{12}^{(q) \text{SM}} \), \( M_{12}^{(q) \text{CH}} \), \( M_{12}^{(q) \text{C}} \), and \( M_{12}^{(q) \text{NG}} \) represent contributions from the SM, the charged Higgs, the charginos, as well as the neutralinos and gluinos, respectively.

### 3.2 \( B_s \rightarrow \mu^+ \mu^- \) decay

The rare decay \( B_s \rightarrow \mu^+ \mu^- \) proceeds dominantly via Z-penguin and W-box diagrams within the SM and its branching ratio is highly suppressed [70]. In the context of \( \mathbb{Z}_3 \)-invariant NMSSM with NMFV, there are in general three types of one-loop diagrams that contribute to this decay, including the various box, the Z-penguin, and the neutral-Higgs-penguin diagrams [22, 77–83]. The relevant effective weak Hamiltonian reads [82, 83]\(^2\)

\[
\mathcal{H}_{\text{eff}} = \frac{1}{16\pi^2} \sum_{X,Y=L,R} (C_{VXY} \mathcal{O}_{VXY} + C_{SXY} \mathcal{O}_{SXY}) + \text{h.c.}, \tag{3.7}
\]

\(^2\)Here we need not consider the tensor operators \( \mathcal{O}_{TX} = \langle \bar{b} \sigma^{\mu\nu} P_X s | \bar{\nu} \sigma_{\mu\nu} \mu \rangle \). While also receiving contributions from these three types of one-loop diagrams in the \( \mathbb{Z}_3 \)-invariant NMSSM with NMFV, they do not contribute to this process due to the vanishing matrix elements, \( \langle 0| b \sigma^{\mu\nu} P_X s | B_s \rangle = 0 \).
where the vector ($O_{VXY}$) and scalar ($O_{SXY}$) operators are defined, respectively, by

$$O_{VXY} = (\bar{b}\gamma_{\mu}P_X s)(\bar{\mu}\gamma^{\mu}P_Y \mu), \quad O_{SXY} = (\bar{b}P_X s)(\bar{\mu}P_Y \mu).$$

(3.8)

The branching ratio of $B_s \rightarrow \mu^+\mu^-$ decay is then calculated to be [82, 83]

$$B(B_s \rightarrow \mu^+\mu^-) = \frac{\tau_{B_s}}{16\pi} \frac{|M|^2}{M_{B_s}} \sqrt{1 - \left(\frac{2m_\mu}{M_{B_s}}\right)^2}, \quad (3.9)$$

where $\tau_{B_s}$ is the $B_s$-meson lifetime, and the squared matrix element is given by [81–84]

$$(4\pi)^4|M|^2 = 2|F_S|^2 (M_{B_s}^2 - 4m_\mu^2) + 2|F_P|^2 M_{B_s}^2 + 8|F_A|^2 M_{B_s}^2 m_\mu^2 + 8\text{Re}(F_P F_A^*) M_{B_s}^2 m_\mu,$$

(3.10)

with the scalar, pseudo-scalar, and axial-vector form factors defined, respectively, by [82]

$$F_S = i \frac{M_{B_s}^2 f_{B_s}}{4m_b + m_s} (C_{SLL} + C_{SLR} - C_{SRR} - C_{SRL}), \quad (3.11)$$

$$F_P = i \frac{M_{B_s}^2 f_{B_s}}{4m_b + m_s} (-C_{SLL} + C_{SLR} - C_{SRR} + C_{SRL}), \quad (3.12)$$

$$F_A = -i \frac{f_{B_s}}{4} (-C_{VLL} + C_{VLR} - C_{VRR} + C_{VRL}), \quad (3.13)$$

where $f_{B_s}$ is the $B_s$-meson decay constant, and $m_{b(s)}$ denotes the $b(s)$-quark running mass.

Our main task is then to calculate the Wilson coefficients $C_{VXY}$ and $C_{SXY}$. Within the SM, only $C_{VLL}$ gets a non-negligible contribution\(^3\), and we shall use the fitting formula in Eq. (4) of Ref. [87] to get the numerical result for it. The additional contributions to $C_{VXY}$ and $C_{SXY}$ from the $Z_3$-invariant NMSSM with NMFV are calculated by ourselves with the aid of FeynArts and FeynCalc packages. Then, the FET procedure with general/finite MI order is applied for the NMSSM contributions, in exactly the same way as for $B_s(d) - \bar{B}_s(d)$ mixing.

It should be noted that the branching ratio given by Eq. (3.9) is the so-called “theoretical” branching ratio, which corresponds to the decay time $t = 0$, while the time-integrated branching ratio measured at experiments is given by [88–90]

$$\overline{B}(B_s \rightarrow \mu^+\mu^-) = 1 + \frac{A_{\Delta\Gamma_s}y_s}{1 - y_s^2} B(B_s \rightarrow \mu^+\mu^-), \quad (3.14)$$

\(^3\)When the small external momenta are taken into account, the SM $W$-box and $Z$-penguin diagrams also generate contributions to the Wilson coefficients $C_{SXY}$, besides from the Higgs-penguin diagrams [85, 86].
where $A_{\Delta \Gamma_s}$ is a time-dependent observable [88–90], and $y_s$ is related to the decay width difference $\Delta \Gamma_s$ between the two $B_s$-meson mass eigenstates, defined by

$$y_s \equiv \frac{\Gamma_L^s - \Gamma_H^s}{\Gamma_L^s + \Gamma_H^s} = \frac{\Delta \Gamma_s}{2 \Gamma_s},$$

(3.15)

with $\Gamma_L^s$ ($\Gamma_H^s$) denoting the lighter (heavier) eigenstate decay width and $\Gamma_s = \tau_{B_s}^{-1}$ the average decay width of $B_s$ meson. In the absence of beyond-SM sources of CP violation, which is assumed throughout this paper, both $A_{\Delta \Gamma_s}$ and $y_s$ will take their respective SM values [88–91], and the two branching ratios are then related to each other via a simple relation

$$\overline{B}(B_s \to \mu^+\mu^-) = \frac{1}{\tau_{B_s} \Gamma_H^s} B(B_s \to \mu^+\mu^-),$$

(3.16)

which holds to a very good approximation [87].

4 Numerical results and discussions

After getting the analytic FET results for $B_{s(d)} - \bar{B}_{s(d)}$ mixing and $B_s \to \mu^+\mu^-$ decay, we now proceed to analyze numerically the parameter space of $Z_3$-invariant NMSSM with NMFV that is allowed under these experimental constraints.

4.1 Choice of input parameters

Firstly, we collect in Table 1 part of the input parameters used throughout this paper. For the $B_{s(d)}$-mixing bag parameters and the decay constants $f_{B_{s(d)}}$, we take the values provided by the FNAL/MILC collaboration [34] and the averages by the Particle Data Group [37], respectively.

The relevant model parameters of $Z_3$-invariant NMSSM with NMFV include

$$M_1, M_2, M_3, M_{S1}, M_{S2}, \mu_{\text{eff}}, \tan \beta, A_\lambda, A_\kappa, \lambda, \kappa, \delta_{23}^{\text{LL}}, \delta_{33}^{\text{LR}}, \delta_{13}^{\text{RR}}, \delta_{23}^{\text{RR}},$$

(4.1)

where $M_3$ is the gluino mass. In this paper, we shall consider two sets of fixed parameters that are collected in Table 2 and are both characterized by a large $\lambda$ and a small $\tan \beta$, to avoid suppressing the NMSSM-specific contributions to the SM-like Higgs mass [14, 15], but allow the remaining parameters $M_{S2}, \delta_{23}^{\text{LL}}, \delta_{33}^{\text{LR}}, \delta_{13}^{\text{RR}}$, and $\delta_{23}^{\text{RR}}$ to vary freely.
Table 1: Summary of part of the input parameters used throughout this paper.

| QCD and EW parameters [37] | $G_F [10^{-5} \text{ GeV}^{-2}]$ | $\alpha_s(M_Z)$ | $M_W [\text{GeV}]$ | $\sin^2 \theta_W$ |
|-----------------------------|-------------------------------|----------------|----------------|----------------|
| $1.1663787$ | $0.1181(11)$ | $80.379$ | $0.2312$ |

| Quark masses [GeV] [37] | $m_b(m_b)$ | $m_c(m_c)$ | $m_t$ |
|------------------------|------------|------------|------|
| $4.15^{+0.04}_{-0.03}$ | $1.275^{+0.025}_{-0.035}$ | $173.0(4)$ |

| $B$-meson parameters [38] | $M_{B_d}[\text{GeV}]$ | $M_{B_s}[\text{GeV}]$ | $1/\Gamma_{Hr}[\text{ps}]$ |
|--------------------------|------------------------|------------------------|--------------------------|
| $5.280$ | $5.367$ | $1.609(10)$ |

| CKM parameters [92] | $\lambda_{\text{CKM}}$ | $A$ | $\bar{\rho}$ | $\bar{\eta}$ |
|---------------------|------------------------|-----|--------------|--------------|
| $0.2251(4)$ | $0.831^{+0.021}_{-0.031}$ | $0.155(8)$ | $0.340(10)$ |

Table 2: Two set of fixed parameters, all being defined at the scale 1 TeV, for the $Z_3$-invariant NMSSM with NMFV. They are given in units of “GeV” except for $\tan \beta$, $\lambda$, and $\kappa$.

| $M_1$ | $M_2$ | $M_{\tilde{g}}$ | $\sqrt{M_{S1}}$ | $\mu_{\text{eff}}$ | $\tan \beta$ | $A_\lambda$ | $A_\kappa$ | $\lambda$ | $\kappa$ |
|-------|-------|----------------|-----------------|------------------|---------------|-------------|-------------|---------|---------|
| Scenario A | 500 | 1000 | 2100 | 1600 | 200 | 3 | 650 | $-10$ | 0.67 | 0.1 |
| Scenario B | 500 | 1000 | 2100 | 1600 | 200 | 10 | 2000 | $-100$ | 0.3 | 0.2 |

The set of fixed parameters in scenario A is similar to that of the scenario TP3 in Ref. [93], with $\lambda$ being close to the perturbative limit but still avoiding running into a Landau pole well below the GUT scale [15]. The one in scenario B is, however, featured by a large $A_\lambda$, which is closely related to the charged-Higgs mass [15]. In both of these two scenarios, the obtained masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$, as well as the mass splitting $\Delta m(\tilde{\chi}_2^0, \tilde{\chi}_1^0)$ are all compatible with the mass bounds set by the LHC through combined searches for charginos and neutralinos [94, 95]. In addition, the choice $\mu_{\text{eff}} = 200 \text{ GeV}$ not only complies with the lower bounds on chargino and neutralino masses, but also ensures that no tree-level fine tuning is necessary to achieve the EW symmetry breaking [15].

The LHC direct searches have also led to stringent limits on the masses of stops, sbottoms, and gluinos [96–103]. The recent result [103] has shown that, for pair produced stops decaying into top quarks, stop masses up to 940 GeV are already excluded in the phenomenological MSSM with a wino-like next-to-lightest supersymmetric particle, while the excluding limit can
Figure 1: Allowed regions (blue dots) for the parameters $M_{S2}$ and $\delta_{33}^{LR}$ required to match the SM-like Higgs mass in scenarios A (left) and B (right), respectively. The gray dots indicate that the SM-like Higgs mass is not in the range $122 \text{ GeV} \leq m_{\text{SM-like Higgs}} \leq 128 \text{ GeV}$.

be up to 860 GeV in scenarios with a Higgsino-like lightest supersymmetric particle. Obviously, being based on simplified models, these bounds are obtained without considering the most general squark flavour structures, and can be relaxed when taking into account these mixings [104]. But for a conservative analysis, we shall assume that the mass of lightest squark is above 940 GeV, while the gluino mass is fixed at 2100 GeV [105].

Scenario A will give a light singlet scalar with mass around 90 GeV and a 125 GeV SM-like Higgs, while the charged-Higgs mass is around 650 GeV, which may be beneficial for describing the branching ratio of $B \to X_s \gamma$ decay [106, 107]. The SM-like Higgs predicted in scenario B is, on the other hand, the lightest among the neutral scalars, and the charged Higgs with mass around 2 TeV can make its effect on the $B \to X_s \gamma$ decay marginal. Taking together with the values of $\tan \beta$, we can say that both of these two scenarios make the Higgs-penguin effects negligible for both $B_{s(d)} - \bar{B}_{s(d)}$ mixing [43] and $B_s \to \mu^+\mu^-$ decay [82].

The parameters $\delta_{33}^{LR}$ and $M_{S2}$ are chosen to get the SM-like Higgs mass in the range $122 \text{ GeV} \leq m_{\text{SM-like Higgs}} \leq 128 \text{ GeV}$ [108]. The allowed regions for $\delta_{33}^{LR}$ and $M_{S2}$, obtained with the aid of the package NMSSMCALC [59, 109–114], are shown in Figure 1. It can be seen that $800^2 \text{ GeV}^2 \leq M_{S2} \leq 2000^2 \text{ GeV}^2$ and $-0.6 \leq \delta_{33}^{LR} \leq 0.5$ in scenario A, while $1500^2 \text{ GeV}^2 \leq M_{S2} \leq 2000^2 \text{ GeV}^2$ and $\delta_{33}^{LR}$ is only allowed to be around 0.2 in scenario B.
These bounds will be taken into account in the following numerical analyses.

4.2 FET result with optimal MI order

4.2.1 Cutting-off MI-order estimation

We first estimate the optimal cutting-off MI order for neutralinos. Here the MI parameters $\delta_{ij}^N$ depend on the six $Z_3$-invariant NMSSM parameters $M_1, M_2, \mu_{\text{eff}}, \tan \beta, \lambda, \text{ and } \kappa$. Keeping for the moment $\lambda$ and $\kappa$ as free variables, but with $\tan \beta = 3$ and 10 corresponding respectively to the two scenarios defined in Table 2, we find that $\delta_{15}^N = \delta_{25}^N = 0$ and $\delta_{12}^N = -0.007$. The dependence of the other MI parameters $\delta_{ij}^N$ on $\lambda$ and $\kappa$ are displayed in Figure 2. From the magnitudes of these $\delta_{ij}^N$ shown and based on the criterion specified in Sec. 2.4, one can see that the effects of $\delta_{12}^N, \delta_{13}^N$, and $\delta_{34}^N$ are negligible with the MI order higher than 1, and that of $\delta_{14}^N$ and $\delta_{23}^N$ can be neglected starting from the third MI order (even from the second MI order for $\delta_{23}^N$ in scenario B); while the MI orders higher than 3 should be kept for $\delta_{24}^N, \delta_{35}^N$, and $\delta_{45}^N$. When the parameters $\lambda$ and $\kappa$ are fixed at the values in scenarios A and B, it is further found that the values of $(M_{\chi_0})_{in,3}/\sqrt{M_{N_{in}}}$, for $i_n = 1, 2, 3$, are much smaller than for $i_n = 4, 5$, and all...
Figure 3: Variation of $\Delta M_s$ with respect to $\delta_{23}^{LL}$ in case IA (upper panel) and $O_{\Delta M_s}$ with respect to $\delta_{23}^{RR}$ and $\delta_{13}^{RR}$ in case IIA (lower panel). Here $\sqrt{M_{S2}}$ is fixed at 1100 (left), 1300 (middle), and 1500 GeV (right) in both cases. In case IA, $\delta_{35}^{LR}$ is set to be $-0.15$ by considering the cutting-off MI orders of 2 (dotted blue), 4 (dot-dashed blue), 6 (dashed blue), and 8 (solid blue), respectively.

the terms involving $L_N(n; 3, i_n)(M_{\chi_0})_{i_n3}$ can be, therefore, discarded safely for $i_n = 1, 2, 3$.

As the MI parameters for charginos are all less than 0.6 in scenarios A and B, the optimal cutting-off MI order for chargino and squark is determined to be 8 using Eq. (2.29), when the squark MI parameters are less than 0.6 and the given small parameter $c_0$ is set to be 0.1.

4.2.2 MI-order comparison for $B_s(d) - \bar{B}_s(d)$ mixing

After obtaining the optimal cutting-off MI orders, we now check the convergence of the FET results with different MI orders. Let us first discuss the $B_s(d) - \bar{B}_s(d)$ mixing. The upper panel in Figure 3 shows the variation of $\Delta M_s$ with respect to $\delta_{23}^{LL}$ by considering the cutting-off MI order for squarks from 2 to 8 in case IA, in which the set of fixed parameters in scenario A is used under the case I assumption for the squark flavour structures (and similar definitions apply to the cases IB, IIA, and IIB, to be mentioned below). Here the results with MI order of 2 are similar to what are usually considered in the MIA method. One can see that $\Delta M_s$ varies with respect to $\delta_{23}^{LL}$ only slowly for MI order of 2 but decreases obviously for higher MI orders.
Figure 4: Variation of $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ with respect to $\delta_{23}^{LL}$ for $\delta_{33}^{LR} = -0.15$ in case IA. Here $\sqrt{M_{S2}}$ is fixed at 1100 (left), 1300 (middle), and 1500 GeV (right), respectively. The dotted, dot-dashed, and solid blue curves represent the results with squark cutting-off MI order of 2, 4, and 6, respectively.

The results with MI orders of 6 and 8 are nearly identical, which justifies the validity of our MI-order estimation, and hence the cutting-off MI order 8 or 6 is optimal.

In order to show the convergence of the FET results in the case II assumption for squark flavour structures, we consider the ratio

$$O_{\Delta M_s} = \frac{\Delta M_s|_{\text{MI-order}=6}}{\Delta M_s|_{\text{MI-order}=8}},$$

which gives the difference between $\Delta M_s$ by considering the squark MI orders of 6 and 8, respectively. As shown in the lower panel of Figure 3, $O_{\Delta M_s}$ is nearly 1 in the whole area in case IIA, implying that the convergence has been verified. Similar observations are also made for $\Delta M_d$ in case IIB and for $\Delta M_d$ in all the four cases.

4.2.3 MI-order comparison for $B_s \rightarrow \mu^+\mu^-$ decay

For our prediction of the time-integrated branching ratio $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ in the $Z_3$-invariant NMSSM with NMFV, the slepton mass squared matrices are set to be diagonal, with all diagonal elements being given by $M_{S1}$. As an example of convergence checking, we show in Figure 4 the variation of $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ with respect to $\delta_{23}^{LL}$ for $\delta_{33}^{LR} = -0.15$ in case IA. Being affected by $\delta_{13}^{RR}$ and $\delta_{23}^{RR}$ quite weakly [43, 115], the convergence of $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ in case II is not shown here. It can be seen from Figure 4 that the FET result obtained with squark MI order of 2 has no significant deviation from that with higher MI orders, which is obviously different from what is observed in the case of $B_{s(d)} - \bar{B}_{s(d)}$ mixing.
When the off-diagonal element $\delta_{LL}^{23}$ is zero, there exists no squark MI contribution and the NMSSM contributions come only from the charged Higgs and the diagonal part of $M^2_{\tilde{D}}$. Even in this case, $\overline{B}(B_s \rightarrow \mu^+\mu^-)$ can be obviously enhanced, putting it to be larger than the LHCb measurement, $(3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$ [116]. As a reference, our prediction within the SM is $(3.6 \pm 0.3) \times 10^{-9}$, obtained by using the fitting formula in Eq. (4) of Ref. [87] with the updated input parameters listed in Table 1 and the decay constant $f_{B_s}$ from Ref. [37].

4.2.4 FET vs. mass diagonalization

From the previous analyses, we can see that, in some regions of the NMSSM parameter space, the usually adopted MIA is not adequate by considering only the first one or two MI orders, and higher orders in the MI expansion must be considered. It is also shown that the FET results with optimal cutting-off MI orders do demonstrate good convergence. In this subsection, we show that these results also agree well with the ones calculated in the ME basis with exact diagonalization of the mass matrices that is achievable only numerically [49, 53].

As an example, we show in Figure 5 the NMSSM contributions to Re($2M^{(s)}_{12}$) from different gauginos (see Eq. (3.6) for their definitions) obtained with these two methods. Here only the NMSSM contribution to Re($2M^{(s)}_{12}$) is shown in case IIA, because in this case only the parameters $\delta_{13}^{RR}$ and $\delta_{23}^{RR}$ from $M^2_{\tilde{D}}$ are involved and they do not contribute to Re($2M^{(s)}_{12}$). It can be clearly seen that the FET results with squark MI order of 8 agree with the ones calculated numerically in the ME basis very well.

4.3 Constraints on the $Z_3$-invariant NMSSM parameters

As mentioned in the Introduction, the SM predictions for the mass differences $\Delta M_s$ and $\Delta M_d$ are now larger than their respective experimental values. For the time-integrated branching ratio $\overline{B}(B_s \rightarrow \mu^+\mu^-)$, on the other hand, the 2017 LHCb measurement\(^4\), $(3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$ [116], is in reasonable agreement with the SM prediction, $(3.6 \pm 0.3) \times 10^{-9}$. This will impose much more stringent constraints on NP [91].

As an example, search for this decay has also been performed by the ATLAS collaboration [122]. The 2017 LHCb measurement includes the LHC Run 2 data and represents the first single-experiment observation of this decay, with a significance of 7.8 standard deviations [116].
The NMSSM contributions to $\text{Re}(2M^{(s)}_{12})$ in case IA from different gauginos (upper panel) and to $\text{Re}(2M^{(s)\text{NG}}_{12})$ in case IIA (lower panel). Here $\delta_{33}^\text{LR}$ is fixed at $-0.15$ in case IA, and $\sqrt{M_S}$ is set to be 1100 (left), 1300 (middle), and 1500 GeV (right), respectively. The dashed curves represent the FET results with squark MI order of 8 in both cases, while the dotted in the upper panel and the solid in the lower panel correspond to the ME calculations.

scenarios with extended Higgs sectors or SUSY theories [123–126]. Measurements of its CP- and isospin-averaged branching ratio by the BaBar [127–129] and Belle [130, 131] collaborations lead to the following combined result [107]

$$
(B_{s\gamma}^{\text{exp}})_{E_\gamma>1.6\text{GeV}} = (3.27 \pm 0.14) \times 10^{-4},
$$

(4.3)

which is in excellent agreement with the state-of-the-art SM prediction [132]

$$
(B_{s\gamma}^{\text{SM}})_{E_\gamma>1.6\text{GeV}} = (3.36 \pm 0.23) \times 10^{-4}.
$$

(4.4)

Here the photon energy cutoff $E_\gamma > 1.6\text{GeV}$ is defined in the decaying meson rest frame. The charged-Higgs contribution in the NMSSM (or MSSM) belongs to the case of Model-II considered in Ref. [107], and always interferes with the SM one in a constructive manner, when the charged-Higgs mass is below 1 TeV. Then, the extra one-loop SUSY contributions should
Figure 6: Allowed regions for $\delta^{LL}_{23}$ and $\delta^{LR}_{33}$ in cases IA (upper panel) and IB (lower panel), respectively. Here $\sqrt{M_{S2}}$ is set to be 1100 (1700) (left), 1300 (1800) (middle), and 1500 (1900) GeV (right) in case IA (IB). The red dot area represents the allowed region from $B_{s\gamma}$, while the green and the blue paliform area shows the allowed region from $\Delta M_d$ and $\Delta M_s$, respectively. The gray areas are already excluded by direct squark searches performed at the LHC [96–103].

involve a cancellation with that from the charged Higgs, so as to comply with the $B \to X_s\gamma$ constraint [22, 133]. As the SM and charged-Higgs contributions to $B_{d\gamma}$, the branching ratio of $B \to X_d\gamma$ decay, are both suppressed by the CKM factor $|K_{31}/K_{32}|^2$ with respect to $B_{s\gamma}$, while the contributions from the squark MI parameters are not affected by this factor, it is expected that $\delta^{LL}_{13}$ in $M_G^2$ will be strongly constrained by $B_{d\gamma}$. As a result, we have set $\delta^{LL}_{13}$ to be zero in Sec. 2.2 from the very beginning.

In the following, we shall exploit the 95% C.L. bounds from $\Delta M_s$, $\Delta M_d$, $\bar{B}(B_s \to \mu^+\mu^-)$, and $B_{s\gamma}$, to set the allowed regions for the NMSSM parameters $M_{S2}$, $\delta^{LL}_{23}$, $\delta^{LR}_{33}$, $\delta^{RR}_{13}$, and $\delta^{RR}_{23}$.

4.3.1 Results in case I

Firstly, we show in Figure 6 the allowed regions for squark MI parameters $\delta^{LL}_{23}$ and $\delta^{LR}_{33}$ in case I. Here three choices of $M_{S2}$ from 1100$^2$ to 1500$^2$ GeV$^2$ in case IA and from 1700$^2$ to 1900$^2$ GeV$^2$ in case IB are made to match the SM-like Higgs mass shown in Figure 1. The
bound from $\overline{B}(B_s \to \mu^+\mu^-)$ is satisfied in the whole parameter regions displayed in Figure 6 and is, therefore, not shown. One can see that the excluded area from direct squark searches at the LHC [96–103] becomes reduced with increasing $M_{S2}$, and the allowed region from $B_{s\gamma}$ moves slightly to the larger positive side of $\delta^{LL}_{23}$ when $M_{S2}$ increases in both of these two cases.

The allowed region for $\delta^{LL}_{23}$ from $\Delta M_s$ also becomes reduced when $M_{S2}$ increases, and only the one with large magnitude of $\delta^{LL}_{23}$ survives in case IA. It is particularly observed that, for $M_{S2} = 1100^2$ GeV$^2$, there exists no allowed region for $\delta^{LL}_{23}$ and $\delta^{LR}_{33}$ in this case. In case IB, however, the allowed region for $\delta^{LL}_{23}$ from $\Delta M_s$ becomes much larger and is almost independent of $M_{S2}$. The bound from $B_{s\gamma}$ also shows that a larger $\delta^{LL}_{23}$ is required in case IA, while in case IB a smaller $\delta^{LL}_{23}$ is favored. These features can be explained by the following observations: in scenario A, there exist considerably large and positive contributions to $\Delta M_s$ and $B_{s\gamma}$ from the charged Higgs, and a large $\delta^{LL}_{23}$ is needed to provide large but negative contributions, so as to cancel the charged-Higgs effects [22, 133]. In scenario B, on the other hand, the positive contributions to $\Delta M_s$ and $B_{s\gamma}$ from the charged Higgs become much smaller, and only a small $\delta^{LL}_{23}$ is needed. However, the allowed regions for $\delta^{LL}_{23}$ and $\delta^{LR}_{33}$ from $\Delta M_d$ are large in both cases, because the main contribution to $\Delta M_d$ from the charged Higgs is small and large extra NMSSM contributions are always needed to reconcile the deviation between experiment and theory.

After taking into account the 95\% C.L. bounds from $\Delta M_s$, $\Delta M_d$, $B_{s\gamma}$, $\overline{B}(B_s \to \mu^+\mu^-)$, as well as the SM-like Higgs mass, we find numerically that the squark MI parameters $\delta^{LL}_{13} > 0.45$ and $|\delta^{LR}_{33}| \sim 0.15$, and the allowed region is severely small in case IA. In case IB, on the other hand, while the parameter $\delta^{LL}_{23}$ can be smaller, the allowed region is relatively larger.

4.3.2 Results in case II

In Figure 7, we show the allowed regions for $\delta^{RR}_{13}$ and $\delta^{RR}_{23}$ in case II. Here, to match the SM-like Higgs mass, the squark MI parameter $\delta^{LR}_{33}$ is set to be $-0.4$ in case IIA and $0.2$ in case IIB. As the branching ratio $\overline{B}(B_s \to \mu^+\mu^-)$ is nearly not affected by $\delta^{RR}_{13}$ or $\delta^{RR}_{23}$ [43, 115], the bound from this observable is not considered in this case.

As already observed in case I, the excluded area from direct squark searches also becomes reduced with increasing $M_{S2}$, and the bounds from $\Delta M_d$ and $\Delta M_s$ become stronger for a larger $M_{S2}$ in case IIA. Especially, the bound from $\Delta M_s$ is so strong that it is no longer compatible with that from the squark mass set by the LHC [96–103]. In addition, the whole area of $\delta^{RR}_{13}$ and $\delta^{RR}_{23}$ is allowed by $B_{s\gamma}$. In case IIB, however, while the allowed region from $\Delta M_d$ stays nearly
the same, the one from $\Delta M_s$ shrinks slowly when $M_{S2}$ increases. The allowed region, with a positive $\delta^{RR}_{23}$, under the bound of $B_{s\gamma}$ begins to emerge only when $M_{S2}$ is about 1900$^2$ GeV$^2$.

After considering the 95% C.L. bounds from $\Delta M_s$, $\Delta M_d$, $B_{s\gamma}$, as well as the SM-like Higgs mass, there exists no allowed region for $\delta^{RR}_{13}$ and $\delta^{RR}_{23}$ in case IIA. In case IIB, however, the allowed area of $\delta^{RR}_{13}$ and $\delta^{RR}_{23}$ exists only when $M_{S2}$ is larger than about 1900$^2$ GeV$^2$.

5 Conclusion

In this paper, motivated by the observation that the SM predictions are now above the experimental data for the mass difference $\Delta M_{s(d)}$, and the usual CMFV models have difficulties in reconciling this discrepancy, we have investigated whether the $Z_3$-invariant NMSSM with NMFV, in which the extra flavour violations arise from the non-diagonal parts of the squark mass matrices, can accommodate such a deviation, while complying with the experimental constraints from the branching ratios of $B_s \rightarrow \mu^+\mu^-$ and $B \rightarrow X_{s\gamma}$ decays.
Instead of using the usually adopted MIA method, we have calculated the NMSSM contributions to \( \Delta M_{s(d)} \) and \( \mathcal{B}(B_s \to \mu^+\mu^-) \), using the recently developed FET procedure, which allows to perform a purely algebraic MI expansion of a transition amplitude written in the ME basis without performing tedious and error-prone diagrammatic calculations in the interaction/flavour basis. Specifically, we have considered finite MI orders for neutralinos but general MI orders for squarks and charginos, under the following two sets of assumptions for the squark flavour structures: while the flavour-conserving off-diagonal element \( \delta_{33}^{LR} \) is kept in both cases, only the flavour-violating off-diagonal elements \( \delta_{23}^{LL} \) and \( \delta_{33}^{RR} \) \((i = 1, 2)\) are kept in cases I and II, respectively. In this way, our analytic results are then expressed directly in terms of the initial Lagrangian parameters in the interaction/flavour basis, making it easy to impose experimental bounds on them. We have also presented an efficient method to estimate the optimal cutting-off MI orders for neutralinos, charginos, and squarks.

For the numerical analyses, we have considered two sets of NMSSM parameters that are denoted, respectively, by scenarios A and B in Table 2. They are both characterized by a large \( \lambda \) and a small \( \tan \beta \), to avoid suppressing the NMSSM-specific contributions to the SM-like Higgs mass, and also make the Higgs-penguin effects negligible for \( B_{s(d)} - \bar{B}_{s(d)} \) mixing and \( B_s \to \mu^+\mu^- \) decay. Together with the two assumptions for squark flavour structures, there are totally four different cases, IA, IB, IIA, and IIB, to be discussed. Firstly, after getting the optimal cutting-off MI orders for neutralinos, charginos, and squarks using our estimation rules, we have verified the convergence of the FET results obtained with the corresponding MI orders. Then, taking \( \text{Re}(2M_{12}^2) \) in cases IA and IIA as an example, we have demonstrated that the FET results with optimal cutting-off MI orders agree well with the ones calculated in the ME basis with exact diagonalization of the mass matrices that is achievable only numerically.

Finally, after considering the 95% C.L. bounds from the observables \( \Delta M_s, \Delta M_d, \mathcal{B}(B_s \to \mu^+\mu^-), B_{s7} \), as well as the SM-like Higgs mass, we have discussed the allowed regions for the parameters \( M_{S2}, \delta_{23}^{LL}, \delta_{33}^{LR}, \delta_{13}^{RR}, \) and \( \delta_{23}^{RR} \). It is found that only large values of \( \delta_{23}^{LL} \), with \( |\delta_{33}^{LR}| \sim 0.15 \), are allowed in case IA, and the allowed region for \( \delta_{23}^{LL} \) becomes reduced when \( M_{S2} \) increases from 1100^2 to 1500^2 GeV^2. In case IB, on the other hand, with \( \delta_{33}^{LR} \) being fixed at about 0.2, relatively smaller magnitude of \( \delta_{23}^{LL} \) is found to be allowed, and the allowed region for \( \delta_{23}^{LL} \) becomes almost independent of \( M_{S2} \). In case IIA, with \( \delta_{33}^{LR} \) being fixed at \(-0.4\), there exists no allowed region for \( \delta_{13}^{RR} \) and \( \delta_{23}^{RR} \), because of the strong bound from \( \Delta M_s \). In case IIB, on the contrary, the allowed region for \( \delta_{13}^{RR} \) and \( \delta_{23}^{RR} \), with \( \delta_{33}^{LR} \) being set to be about 0.2, exists
only when $M_{S2}$ is larger than about $1900^2 \text{ GeV}^2$.

As a final remark, we should mention that with the experimental progress in direct searches for SUSY particles as well as the more and more precise theoretical predictions for these observables, the NMSSM effects on these low-energy flavour processes can be further exploited.

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Appendix: Block terms for squarks and charginos

In this appendix, we list all the non-zero block terms for squarks and charginos. For convenience, we introduce the notations $\Delta_1 \equiv \frac{M_{S1}M_{S2}\delta_{23}^2}{q^2-M_{S1}} + \frac{M_{S2}^2\delta_{36}^2}{q^2-M_{S2}}$ and $\Delta'_1 \equiv \frac{M_{S1}M_{S2}(1+\lambda_{CKM})\delta_{23}^2}{q^2-M_{S1}}$ in case I, and $\Delta_2 \equiv \frac{M_{S1}M_{S2}(\delta_{16}^2+\delta_{56}^2)}{q^2-M_{S1}}$ in case II. In the following, $n = 1, 2, 3, \cdots$, denotes the MI-order index.

For up-type squarks, the non-zero block terms are given as

\begin{align}
L_U(0; i,i) &= \begin{cases} 
\frac{1}{q^2-M_{S1}}, & i = 1, 2, 4, 5 \\
\frac{1}{q^2-M_{S2}}, & i = 3, 6 
\end{cases} 
, 
\quad (1.1)

L_U(2n-2; 3,3) &= \frac{1}{(q^2 - M_{S2})^n} \Delta_1^{n-1}, 
\quad (2.2)

L_U(2n-1; 3,i) &= \begin{cases} 
\frac{\sqrt{M_{S1}M_{S2}\delta_{23}^2}}{(q^2-M_{S1})(q^2-M_{S2})^n} \Delta_1^{n-1}, & i = 2 \\
\frac{M_{S2}\delta_{36}^2}{(q^2-M_{S2})^{n+1}} \Delta_2^{n-1}, & i = 6 
\end{cases} 
, 
\quad (3.3)

L_U(2n; i,j) &= \begin{cases} 
\frac{M_{S1}M_{S2}\delta_{23}^2}{(q^2-M_{S1})(q^2-M_{S2})^n} \Delta_1^{n-1}, & (i,j) = (2,2) \\
\frac{M_{S2}\sqrt{M_{S1}M_{S2}\delta_{23}^2\delta_{56}}}{(q^2-M_{S1})(q^2-M_{S2})^{n+1}} \Delta_1^{n-1}, & (i,j) = (2,6), (6,2) \\
\frac{M_{S2}^2\delta_{36}^2}{(q^2-M_{S2})^{n+2}} \Delta_1^{n-1}, & (i,j) = (6,6) 
\end{cases} 
, 
\quad (4.4)
\end{align}
\[ L_{UU}(2n; 3, i; 3, j) = \sum_{n_1=1}^{n} L_U(2n_1 - 1; 3, i)L_U(2(n + 1 - n_1) - 1; 3, j), \quad i, j = 2 \text{ or } 6, \quad (5) \]

\[ L_{UU}(2n - 1; i, j; 3, j') = \sum_{n_1=0}^{n-1} L_U(2n_1; i, j)L_U(2(n - n_1) - 1; 3, j'), \quad i, j = 1-6 \text{ and } j' = 2 \text{ or } 6, \quad (6) \]

\[ L_{UU}(2n - 2; i, j; i', j') = \sum_{n_1=0}^{n-1} L_U(2n_1; i, j)L_U(2(n - 1 - n_1); i', j'), \quad i, j, i', j' = 1-6, \quad (7) \]

in case I.

For down-type squarks, the non-zero block terms are given as,

\[ L_D(2n - 1; 3, i) = \begin{cases} \frac{-\sqrt{M_{S1}M_{S2}}\lambda\delta_{23}}{(q^2 - M_{S1})(q^2 - M_{S2})} \Delta_{1}^{n-1}, & i = 1 \\ \frac{\sqrt{M_{S1}M_{S2}}\delta_{23}}{(q^2 - M_{S1})(q^2 - M_{S2})} \Delta_{1}^{n-1}, & i = 2 \end{cases}, \quad (8) \]

\[ L_{DD}(2n; 3, i; 3, i') = \sum_{n_1=1}^{n} L_D(2n_1 - 1; 3, i)L_D(2(n + 1 - n_1) - 1; 3, i'), \quad i = 1 \text{ or } 2, \quad (9) \]

in case I, and

\[ L_D(2n - 1; 6, i) = \frac{\sqrt{M_{S1}M_{S2}}\delta_{6}}{(q^2 - M_{S1})(q^2 - M_{S2})^{n}} \Delta_{2}^{n-1}, \quad i = 4 \text{ or } 5, \quad (10) \]

\[ L_{DD}(2n; 6, i; 6, i') = \sum_{n_1=1}^{n} L_D(2n_1 - 1; 6, i)L_D(2(n + 1 - n_1) - 1; 6, i'), \quad i = 4 \text{ or } 5, \quad (11) \]

in case II.

The non-zero block terms for charginos include

\[ L_X(2n - 2; i, i) = \frac{1}{(q^2 - M_{X_i})^{n}(q^2 - M_{X_{i'}})^{n-1}}(\delta_{12}^{X}\sqrt{M_{X_1}M_{X_2}})^{2n-2}, \quad (12) \]

where \((i, i') = (1, 2)\) or \((2, 1)\),

\[ L_X(2n - 1; 1, 2) = \frac{1}{(q^2 - M_{X_1})^{n}(q^2 - M_{X_2})^{n}}(\delta_{12}^{X}\sqrt{M_{X_1}M_{X_2}})^{2n-1}, \quad (13) \]

\[ L_{XY}(2n - 2; i, i; j, j) = \sum_{n_1=0}^{n-1} L_X(2n_1; i, i)L_Y(2(n - 1 - n_1); j, j), \quad (14) \]
where \((i, j) = (1, 1), (1, 2), (2, 1),\) or \((2, 2),\)

\[
L_{XY}(2n - 1; i, i; 1, 2) = \sum_{n_1 = 0}^{n-1} L_X(2n_1; i, i)L_Y(2(n - n_1) - 1; 1, 2),
\]

(15)

where \(i = 1\) or \(2,\) and

\[
L_{XY}(2n; 1, 2; 1, 2) = \sum_{n_1 = 1}^{n} L_X(2n_1 - 1; 1, 2)L_Y(2(n + 1 - n_1) - 1; 1, 2).
\]

(16)

Here the subscripts \(X\) and \(Y\) can be \(C\) or \(P.\)

References

[1] H. P. Nilles, Supersymmetry, Supergravity and Particle Physics, Phys. Rept. 110 (1984) 1–162.

[2] H. E. Haber and G. L. Kane, The Search for Supersymmetry: Probing Physics Beyond the Standard Model, Phys. Rept. 117 (1985) 75–263.

[3] S. Dimopoulos and H. Georgi, Softly broken supersymmetry and SU(5), Nucl. Phys. B193 (1981) 150–162.

[4] S. P. Martin, A Supersymmetry primer, hep-ph/9709356. [Adv. Ser. Direct. High Energy Phys.18,1(1998)].

[5] D. J. H. Chung, L. L. Everett, G. L. Kane, S. F. King, J. D. Lykken, and L.-T. Wang, The Soft supersymmetry breaking Lagrangian: Theory and applications, Phys. Rept. 407 (2005) 1–203, [hep-ph/0312378].

[6] J. E. Kim and H. P. Nilles, The \(\mu\)-Problem and the Strong CP Problem, Phys. Lett. 138B (1984) 150–154.

[7] ATLAS Collaboration, G. Aad et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B716 (2012) 1–29, [arXiv:1207.7214].
[8] CMS Collaboration, S. Chatrchyan et al., *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, Phys. Lett. B716 (2012) 30–61, [arXiv:1207.7235].

[9] A. Djouadi and J. Quevillon, *The MSSM Higgs sector at a high $M_{SUSY}$: reopening the low tan$\beta$ regime and heavy Higgs searches*, JHEP 10 (2013) 028, [arXiv:1304.1787].

[10] A. Djouadi, L. Maiani, G. Moreau, A. Polosa, J. Quevillon, and V. Riquer, *The post-Higgs MSSM scenario: Habemus MSSM?*, Eur. Phys. J. C73 (2013) 2650, [arXiv:1307.5205].

[11] A. Djouadi, L. Maiani, A. Polosa, J. Quevillon, and V. Riquer, *Fully covering the MSSM Higgs sector at the LHC*, JHEP 06 (2015) 168, [arXiv:1502.05653].

[12] P. Fayet, *Supergauge invariant extension of the Higgs mechanism and a model for the electron and its neutrino*, Nucl. Phys. B90 (1975) 104–124.

[13] J. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski, and F. Zwirner, *Higgs bosons in a nonminimal supersymmetric model*, Phys. Rev. D39 (1989) 844.

[14] M. Maniatis, *The Next-to-Minimal Supersymmetric extension of the Standard Model reviewed*, Int. J. Mod. Phys. A25 (2010) 3505–3602, [arXiv:0906.0777].

[15] U. Ellwanger, C. Hugonie, and A. M. Teixeira, *The Next-to-Minimal Supersymmetric Standard Model*, Phys. Rept. 496 (2010) 1–77, [arXiv:0910.1785].

[16] S. Strandberg, *SUSY: a review of the results from the LHC experiments, 39th International Conference on High Energy Physics (ICHEP 2018) Seoul, Korea, July 4-11, 2018*, .

[17] R. Barbieri, L. J. Hall, Y. Nomura, and V. S. Rychkov, *Supersymmetry without a Light Higgs Boson*, Phys. Rev. D75 (2007) 035007, [hep-ph/0607332].

[18] L. J. Hall, D. Pinner, and J. T. Ruderman, *A Natural SUSY Higgs Near 126 GeV*, JHEP 04 (2012) 131, [arXiv:1112.2703].

[19] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager, and L. Silvestrini, *Universal unitarity triangle and physics beyond the standard model*, Phys. Lett. B500 (2001) 161–167, [hep-ph/0007085].
[20] A. J. Buras, P. H. Chankowski, J. Rosiek, and L. Slawianowska, $\Delta M(s)/\Delta M(d)$, $\sin 2\beta$ and the angle $\gamma$ in the presence of new $\Delta F = 2$ operators, Nucl. Phys. B619 (2001) 434–466, [hep-ph/0107048].

[21] A. J. Buras, S. Jager, and J. Urban, Master formulae for $\Delta F = 2$ NLO QCD factors in the standard model and beyond, Nucl. Phys. B605 (2001) 600–624, [hep-ph/0102316].

[22] A. J. Buras, P. H. Chankowski, J. Rosiek, and L. Slawianowska, $\Delta M_d,s,B_0^{d,s} \rightarrow \mu^+\mu^-$ and $B \rightarrow X_s\gamma$ in supersymmetry at large $\tan \beta$, Nucl. Phys. B659 (2003) 3, [hep-ph/0210145].

[23] A. J. Buras, P. H. Chankowski, J. Rosiek, and L. Slawianowska, Correlation between $\Delta M_s$ and $B^{0}_{d,s} \rightarrow \mu^+\mu^-$ in supersymmetry at large $\tan \beta$, Phys. Lett. B546 (2002) 96–107, [hep-ph/0207241].

[24] A. J. Buras, Relations between $\Delta M(s,d)$ and $B(s,d) \rightarrow \mu\bar{\mu}$ in models with minimal flavor violation, Phys. Lett. B566 (2003) 115–119, [hep-ph/0303060].

[25] G. Hiller, B physics signals of the lightest CP odd Higgs in the NMSSM at large $\tan \beta$, Phys. Rev. D70 (2004) 034018, [hep-ph/0404220].

[26] F. Domingo and U. Ellwanger, Updated Constraints from B Physics on the MSSM and the NMSSM, JHEP 12 (2007) 090, [arXiv:0710.3714].

[27] W. Altmannshofer, A. J. Buras, and D. Guadagnoli, The MFV limit of the MSSM for low $\tan \beta$: Meson mixings revisited, JHEP 11 (2007) 065, [hep-ph/0703200].

[28] R. N. Hodgkinson and A. Pilaftsis, Supersymmetric Higgs Singlet Effects on B-Meson FCNC Observables at Large $\tan \beta$, Phys. Rev. D78 (2008) 075004, [arXiv:0807.4167].

[29] M. Arana-Catania, S. Heinemeyer, M. J. Herrero, and S. Penaranda, Higgs Boson masses and B-Physics Constraints in Non-Minimal Flavor Violating SUSY scenarios, JHEP 05 (2012) 015, [arXiv:1109.6232].

[30] M. Arana-Catania, S. Heinemeyer, and M. J. Herrero, Updated Constraints on General Squark Flavor Mixing, Phys. Rev. D90 (2014), no. 7 075003, [arXiv:1405.6960].
[31] F. Domingo, *Update of the flavour-physics constraints in the NMSSM*, Eur. Phys. J. C76 (2016), no. 8 452, [arXiv:1512.02091].

[32] M. Blanke and A. J. Buras, *Universal Unitarity Triangle 2016 and the tension between $\Delta M_{s,d}$ and $\varepsilon_K$ in CMFV models*, Eur. Phys. J. C76 (2016), no. 4 197, [arXiv:1602.04020].

[33] F. Domingo, H. K. Dreiner, J. S. Kim, M. E. Krauss, M. Lozano, and Z. S. Wang, *Updating Bounds on R-Parity Violating Supersymmetry from Meson Oscillation Data*, JHEP 02 (2019) 066, [arXiv:1810.08228].

[34] Fermilab Lattice, MILC Collaboration, A. Bazavov et al., *$B^0_s$-mixing matrix elements from lattice QCD for the Standard Model and beyond*, Phys. Rev. D93 (2016), no. 11 113016, [arXiv:1602.03560].

[35] S. Aoki et al., *Review of lattice results concerning low-energy particle physics*, Eur. Phys. J. C77 (2017), no. 2 112, [arXiv:1607.00299].

[36] L. Di Luzio, M. Kirk, and A. Lenz, *Updated $B_s$-mixing constraints on new physics models for $b \to s\ell^+\ell^-$ anomalies*, Phys. Rev. D97 (2018), no. 9 095035, [arXiv:1712.06572].

[37] Particle Data Group Collaboration, M. Tanabashi et al., *Review of Particle Physics*, Phys. Rev. D98 (2018), no. 3 030001.

[38] HFLAV Collaboration, Y. Amhis et al., *Averages of $b$-hadron, $c$-hadron, and $\tau$-lepton properties as of summer 2016*, Eur. Phys. J. C77 (2017), no. 12 895, [arXiv:1612.07233].

[39] M. Blanke and A. J. Buras, *Emerging $\Delta M_d$-Anomaly from Tree-Level Determinations of $|V_{cb}|$ and the Angle $\gamma$*, Eur. Phys. J. C79 (2019), no. 2 159, [arXiv:1812.06963].

[40] N. Cabibbo, *Unitary Symmetry and Leptonic Decays*, Phys. Rev. Lett. 10 (1963) 531–533.

[41] M. Kobayashi and T. Maskawa, *CP Violation in the Renormalizable Theory of Weak Interaction*, Prog. Theor. Phys. 49 (1973) 652–657.
[42] J. Foster, K.-i. Okumura, and L. Roszkowski, *Probing the flavor structure of supersymmetry breaking with rare B-processes: A Beyond leading order analysis*, JHEP 08 (2005) 094, [hep-ph/0506146].

[43] J. Kumar and M. Paraskevas, *Distinguishing between MSSM and NMSSM through $\Delta F = 2$ processes*, JHEP 10 (2016) 134, [arXiv:1608.08794].

[44] D. Ghosh, P. Paradisi, G. Perez, and G. Spada, *CP Violation Tests of Alignment Models at LHCII*, JHEP 02 (2016) 178, [arXiv:1512.03962].

[45] L. J. Hall, V. A. Kostelecky, and S. Raby, *New Flavor Violations in Supergravity Models*, Nucl. Phys. B267 (1986) 415–432.

[46] J. S. Hagelin, S. Kelley, and T. Tanaka, *Supersymmetric flavor changing neutral currents: Exact amplitudes and phenomenological analysis*, Nucl. Phys. B415 (1994) 293–331.

[47] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, *A Complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model*, Nucl. Phys. B477 (1996) 321–352, [hep-ph/9604387].

[48] M. Misiak, S. Pokorski, and J. Rosiek, *Supersymmetry and FCNC effects*, Adv. Ser. Direct. High Energy Phys. 15 (1998) 795–828, [hep-ph/9703442].

[49] A. Dedes, M. Paraskevas, J. Rosiek, K. Suxho, and K. Tamvakis, *Mass Insertions vs. Mass Eigenstates calculations in Flavour Physics*, JHEP 06 (2015) 151, [arXiv:1504.00960].

[50] J. Rosiek, *MassToMI - A Mathematica package for an automatic Mass Insertion expansion*, Comput. Phys. Commun. 201 (2016) 144–158, [arXiv:1509.05030].

[51] A. Dedes, M. Paraskevas, J. Rosiek, K. Suxho, and K. Tamvakis, *Rare Top-quark Decays to Higgs boson in MSSM*, JHEP 11 (2014) 137, [arXiv:1409.6546].

[52] H. Eberl, E. Ginina, A. Bartl, K. Hidaka, and W. Majerotto, *The decays $h^0 \rightarrow b\bar{b}$ and $h^0 \rightarrow c\bar{c}$ in the light of the MSSM with quark flavour violation*, JHEP 06 (2016) 143, [arXiv:1604.02366].
[53] A. Crivellin, Z. Fabisiewicz, W. Materkowska, U. Nierste, S. Pokorski, and J. Rosiek, *Lepton flavour violation in the MSSM: exact diagonalization vs mass expansion*, JHEP **06** (2018) 003, [arXiv:1802.06803].

[54] E. Arganda, M. Herrero, X. Marcano, R. Morales, and A. Szynkman, *Effective lepton flavor violating $H \ell_i \ell_j$ vertex from right-handed neutrinos within the mass insertion approximation*, Phys. Rev. **D95** (2017), no. 9 095029, [arXiv:1612.09290].

[55] M. J. Herrero, X. Marcano, R. Morales, and A. Szynkman, *One-loop effective LFV $Z \ell_k \ell_m$ vertex from heavy neutrinos within the mass insertion approximation*, Eur. Phys. J. **C78** (2018), no. 10 815, [arXiv:1807.01698].

[56] J. Rosiek, P. Chankowski, A. Dedes, S. Jager, and P. Tanedo, *SUSY_FLAVOR: A Computational Tool for FCNC and CP-violating Processes in the MSSM*, Comput. Phys. Commun. **181** (2010) 2180–2205, [arXiv:1003.4260].

[57] A. Crivellin, J. Rosiek, P. H. Chankowski, A. Dedes, S. Jaeger, and P. Tanedo, *SUSY_FLAVOR v2: A Computational tool for FCNC and CP-violating processes in the MSSM*, Comput. Phys. Commun. **184** (2013) 1004–1032, [arXiv:1203.5023].

[58] J. Rosiek, *SUSY FLAVOR v2.5: a computational tool for FCNC and CP-violating processes in the MSSM*, Comput. Phys. Commun. **188** (2015) 208–210, [arXiv:1410.0606].

[59] J. Baglio, R. Gröber, M. Mühlleitner, D. T. Nhung, H. Rzehak, M. Spira, J. Streicher, and K. Walz, *NMSSMCALC: A Program Package for the Calculation of Loop-Corrected Higgs Boson Masses and Decay Widths in the (Complex) NMSSM*, Comput. Phys. Commun. **185** (2014), no. 12 3372–3391, [arXiv:1312.4788].

[60] B. C. Allanach et al., *SUSY Les Houches Accord 2*, Comput. Phys. Commun. **180** (2009) 8–25, [arXiv:0801.0045].

[61] J. Rosiek, *Complete Set of Feynman Rules for the Minimal Supersymmetric Extension of the Standard Model*, Phys. Rev. **D41** (1990) 3464.

[62] J. Rosiek, *Complete set of Feynman rules for the MSSM: Erratum*, hep-ph/9511250.
[63] K. Kowalska, *Phenomenology of SUSY with General Flavour Violation*, JHEP **09** (2014) 139, [arXiv:1406.0710].

[64] K. De Causmaecker, B. Fuks, B. Herrmann, F. Mahmoudi, B. O’Leary, W. Porod, S. Sekmen, and N. Strobbe, *General squark flavour mixing: constraints, phenomenology and benchmarks*, JHEP **11** (2015) 125, [arXiv:1509.05414].

[65] U. Ellwanger and C. Hugonie, *Constraints from charge and color breaking minima in the (M+1)SSM*, Phys. Lett. **B457** (1999) 299–306, [hep-ph/9902401].

[66] J. A. Casas, *Charge and color breaking*, hep-ph/9707475. [Adv. Ser. Direct. High Energy Phys.21,469(2010)].

[67] F. Franke and H. Fraas, *Neutralinos and Higgs bosons in the next-to-minimal supersymmetric standard model*, Int. J. Mod. Phys. **A12** (1997) 479–534, [hep-ph/9512366].

[68] U. Ellwanger, J. F. Gunion, and C. Hugonie, *NMHDECAY: A Fortran code for the Higgs masses, couplings and decay widths in the NMSSM*, JHEP **02** (2005) 066, [hep-ph/0406215].

[69] G. Passarino and M. J. G. Veltman, *One Loop Corrections for $e^+e^- \rightarrow \mu^+\mu^-$ Annihilation Into $\mu^+\mu^-$ in the Weinberg Model*, Nucl. Phys. **B160** (1979) 151–207.

[70] A. J. Buras, *Weak Hamiltonian, CP violation and rare decays*, hep-ph/9806471.

[71] C. Hamzaoui, M. Pospelov, and M. Toharia, *Higgs mediated FCNC in supersymmetric models with large $\tan \beta$*, Phys. Rev. **D59** (1999) 095005, [hep-ph/9807350].

[72] A. Dedes, *The Higgs penguin and its applications: An Overview*, Mod. Phys. Lett. **A18** (2003) 2627–2644, [hep-ph/0309233].

[73] A. J. Buras, M. Jamin, and P. H. Weisz, *Leading and Next-to-leading QCD Corrections to $\epsilon$ Parameter and $B^0 - \bar{B}^0$ Mixing in the Presence of a Heavy Top Quark*, Nucl. Phys. **B347** (1990) 491–536.

[74] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, *Effects of supergravity induced electroweak breaking on rare $B$ decays and mixings*, Nucl. Phys. **B353** (1991) 591–649.
[75] T. Hahn, *Generating Feynman diagrams and amplitudes with FeynArts 3*, *Comput. Phys. Commun.* **140** (2001) 418–431, [hep-ph/0012260].

[76] V. Shtabovenko, R. Mertig, and F. Orellana, *New Developments in FeynCalc 9.0*, *Comput. Phys. Commun.* **207** (2016) 432–444, [arXiv:1601.01167].

[77] C. Bobeth, A. J. Buras, F. Kruger, and J. Urban, *QCD corrections to $\bar{B} \to X_d \nu \bar{\nu}$, $\bar{B}_{d,s} \to \ell^+\ell^-$, $K \to \pi \nu \bar{\nu}$ and $K_L \to \mu^+\mu^-$ in the MSSM*, *Nucl. Phys.* **B630** (2002) 87–131, [hep-ph/0112305].

[78] C. Bobeth, T. Ewerth, F. Kruger, and J. Urban, *Analysis of neutral Higgs boson contributions to the decays $\bar{B}_s \to \ell^+\ell^-$ and $\bar{B} \to K \ell^+\ell^-$*, *Phys. Rev.* **D64** (2001) 074014, [hep-ph/0104284].

[79] P. H. Chankowski and L. Slawianowska, $B_{d,s}^0 \to \mu^-\mu^+$ decay in the MSSM, *Phys. Rev.* **D63** (2001) 054012, [hep-ph/0008046].

[80] C.-S. Huang, W. Liao, Q.-S. Yan, and S.-H. Zhu, $B_s \to \ell^+\ell^-$ in a general 2HDM and MSSM, *Phys. Rev.* **D63** (2001) 114021, [hep-ph/0006250]. [Erratum: Phys. Rev.D64,059902(2001)].

[81] G. Isidori and A. Retico, $B_{s,d} \to \ell^+\ell^-$ and $K_L \to \ell^+\ell^-$ in SUSY models with nonminimal sources of flavor mixing, *JHEP* **09** (2002) 063, [hep-ph/0208159].

[82] A. Dedes, J. Rosiek, and P. Tanedo, *Complete One-Loop MSSM Predictions for $B^0 \to \ell^+\ell^-$ at the Tevatron and LHC*, *Phys. Rev.* **D79** (2009) 055006, [arXiv:0812.4320].

[83] H. Dreiner, K. Nickel, W. Porod, and F. Staub, *Full 1-loop calculation of BR($B_{s,d}^0 \to \ell\bar{\ell}$) in models beyond the MSSM with SARAH and SPheno*, *Comput. Phys. Commun.* **184** (2013) 2604–2617, [arXiv:1212.5074].

[84] C. Bobeth, T. Ewerth, F. Kruger, and J. Urban, *Enhancement of $B(\bar{B}_{d} \to \mu^+\mu^-)/B(\bar{B}_{s} \to \mu^+\mu^-)$ in the MSSM with minimal flavor violation and large $\tan \beta$*, *Phys. Rev.* **D66** (2002) 074021, [hep-ph/0204225].

[85] X.-Q. Li, J. Lu, and A. Pich, *$B_{s,d}^0 \to \ell^+\ell^-$ Decays in the Aligned Two-Higgs-Doublet Model*, *JHEP* **06** (2014) 022, [arXiv:1404.5865].
[86] P. Arnan, D. Bečirević, F. Mescia, and O. Sumensari, *Two Higgs doublet models and $b \rightarrow s$ exclusive decays*, *Eur. Phys. J.* **C77** (2017), no. 11 796, [arXiv:1703.03426].

[87] C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, E. Stamou, and M. Steinhauser, *$B_{s,d} \rightarrow \ell^+\ell^-$ in the Standard Model with Reduced Theoretical Uncertainty*, *Phys. Rev. Lett.* **112** (2014) 101801, [arXiv:1311.0903].

[88] K. De Bruyn, R. Fleischer, R. Knegjens, P. Koppenburg, M. Merk, A. Pellegrino, and N. Tuning, *Probing New Physics via the $B^0_s \rightarrow \mu^+\mu^-$ Effective Lifetime*, *Phys. Rev. Lett.* **109** (2012) 041801, [arXiv:1204.1737].

[89] A. J. Buras, R. Fleischer, J. Girrbach, and R. Knegjens, *Probing New Physics with the $B_s \rightarrow \mu^+\mu^-$ Time-Dependent Rate*, *JHEP* **07** (2013) 77, [arXiv:1303.3820].

[90] R. Fleischer, R. Jaarsma, and G. Tetlalmatzi-Xolocotzi, *In Pursuit of New Physics with $B^0_{s,d} \rightarrow \ell^+\ell^-$*, *JHEP* **05** (2017) 156, [arXiv:1703.10160].

[91] W. Altmannshofer, C. Niehoff, and D. M. Straub, *$B_s \rightarrow \mu^+\mu^-$ as current and future probe of new physics*, *JHEP* **05** (2017) 076, [arXiv:1702.05498].

[92] S. Descotes-Genon and P. Koppenburg, *The CKM Parameters*, *Ann. Rev. Nucl. Part. Sci.* **67** (2017) 97–127, [arXiv:1702.08834].

[93] F. Staub, P. Athron, U. Ellwanger, R. Gröber, M. Mühlleitner, P. Slavich, and A. Voigt, *Higgs mass predictions of public NMSSM spectrum generators*, *Comput. Phys. Commun.* **202** (2016) 113–130, [arXiv:1507.05093].

[94] CMS Collaboration, A. M. Sirunyan et al., *Search for new physics in events with two soft oppositely charged leptons and missing transverse momentum in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *Phys. Lett.* **B782** (2018) 440–467, [arXiv:1801.01846].

[95] CMS Collaboration, A. M. Sirunyan et al., *Combined search for electroweak production of charginos and neutralinos in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *JHEP* **03** (2018) 160, [arXiv:1801.03957].

[96] CMS Collaboration, A. M. Sirunyan et al., *Search for top squarks and dark matter particles in opposite-charge dilepton final states at $\sqrt{s} = 13$ TeV*, *Phys. Rev.* **D97** (2018), no. 3 032009, [arXiv:1711.00752].

37
[97] **ATLAS** Collaboration, M. Aaboud et al., *Search for dark matter and other new phenomena in events with an energetic jet and large missing transverse momentum using the ATLAS detector*, *JHEP* **01** (2018) 126, [arXiv:1711.03301].

[98] **ATLAS** Collaboration, G. Aad et al., *Search for Scalar Charm Quark Pair Production in pp Collisions at $\sqrt{s} = 8$TeV with the ATLAS Detector*, *Phys. Rev. Lett.* **114** (2015), no. 16 161801, [arXiv:1501.01325].

[99] **CMS** Collaboration, A. M. Sirunyan et al., *Search for top squark pair production in pp collisions at $\sqrt{s} = 13$ TeV using single lepton events*, *JHEP* **10** (2017) 019, [arXiv:1706.04402].

[100] **CMS** Collaboration, A. M. Sirunyan et al., *Search for supersymmetry in proton-proton collisions at 13 TeV using identified top quarks*, *Phys. Rev.* **D97** (2018), no. 1 012007, [arXiv:1710.11188].

[101] **CMS** Collaboration, A. M. Sirunyan et al., *Search for the pair production of third-generation squarks with two-body decays to a bottom or charm quark and a neutralino in proton–proton collisions at $\sqrt{s} = 13$ TeV*, *Phys. Lett.* **B778** (2018) 263–291, [arXiv:1707.07274].

[102] **ATLAS** Collaboration, M. Aaboud et al., *Search for a scalar partner of the top quark in the jets plus missing transverse momentum final state at $\sqrt{s}=13$ TeV with the ATLAS detector*, *JHEP* **12** (2017) 085, [arXiv:1709.04183].

[103] **ATLAS** Collaboration, M. Aaboud et al., *Search for top-squark pair production in final states with one lepton, jets, and missing transverse momentum using 36 fb$^{-1}$ of $\sqrt{s} = 13$ TeV pp collision data with the ATLAS detector*, *JHEP* **06** (2018) 108, [arXiv:1711.11520].

[104] G. Brooijmans et al., *Les Houches 2017: Physics at TeV Colliders New Physics Working Group Report*, in 10th Les Houches Workshop on Physics at TeV Colliders (PhysicsTeV 2017) Les Houches, France, June 5-23, 2017, 2018. arXiv:1803.10379.

[105] U. Ellwanger and C. Hugonie, *The semi-constrained NMSSM satisfying bounds from the LHC, LUX and Planck*, *JHEP* **08** (2014) 046, [arXiv:1405.6647].
[106] Q.-Y. Hu, X.-Q. Li, and Y.-D. Yang, $B^0 \to K^{*0}\mu^+\mu^-$ decay in the Aligned Two-Higgs-Doublet Model, *Eur. Phys. J.* C77 (2017), no. 3 190, [arXiv:1612.08867].

[107] M. Misiak and M. Steinhauser, Weak radiative decays of the $B$ meson and bounds on $M_{H^\pm}$ in the Two-Higgs-Doublet Model, *Eur. Phys. J.* C77 (2017), no. 3 201, [arXiv:1702.04571].

[108] J. Cao, Y. He, L. Shang, W. Su, and Y. Zhang, Natural NMSSM after LHC Run I and the Higgsino dominated dark matter scenario, *JHEP* 08 (2016) 037, [arXiv:1606.04416].

[109] K. Ender, T. Graf, M. Mühlleitner, and H. Rzehak, Analysis of the NMSSM Higgs Boson Masses at One-Loop Level, *Phys. Rev.* D85 (2012) 075024, [arXiv:1111.4952].

[110] T. Graf, R. Grober, M. Mühlleitner, H. Rzehak, and K. Walz, Higgs Boson Masses in the Complex NMSSM at One-Loop Level, *JHEP* 10 (2012) 122, [arXiv:1206.6806].

[111] M. Mühlleitner, D. T. Nhung, H. Rzehak, and K. Walz, Two-loop contributions of the order $\mathcal{O} (\alpha_t \alpha_s)$ to the masses of the Higgs bosons in the CP-violating NMSSM, *JHEP* 05 (2015) 128, [arXiv:1412.0918].

[112] S. F. King, M. Mühlleitner, R. Nevzorov, and K. Walz, Exploring the CP-violating NMSSM: EDM Constraints and Phenomenology, *Nucl. Phys.* B901 (2015) 526–555, [arXiv:1508.03255].

[113] A. Djouadi, J. Kalinowski, and M. Spira, *HDECAY: A Program for Higgs boson decays in the standard model and its supersymmetric extension*, *Comput. Phys. Commun.* 108 (1998) 56–74, [hep-ph/9704448].

[114] J. M. Butterworth et al., *THE TOOLS AND MONTE CARLO WORKING GROUP Summary Report from the Les Houches 2009 Workshop on TeV Colliders*, arXiv:1003.1643.

[115] L. Silvestrini, Searching for new physics in $b \to s$ hadronic penguin decays, *Ann. Rev. Nucl. Part. Sci.* 57 (2007) 405–440, [arXiv:0705.1624].
[116] **LHCb** Collaboration, R. Aaij et al., *Measurement of the $B^0_s \to \mu^+\mu^-$ branching fraction and effective lifetime and search for $B^0 \to \mu^+\mu^-$ decays*, Phys. Rev. Lett. **118** (2017), no. 19 191801, [arXiv:1703.05747].

[117] **CDF** Collaboration, T. Aaltonen et al., *Search for $B^0_s \to \mu^+\mu^-$ and $B^0 \to \mu^+\mu^-$ decays with the full CDF Run II data set*, Phys. Rev. **D87** (2013), no. 7 072003, [arXiv:1301.7048]. [Erratum: Phys. Rev.D97,no.9,099901(2018)].

[118] **D0** Collaboration, V. M. Abazov et al., *Search for the rare decay $B_s \to \mu\mu$, Phys. Rev. D**87** (2013), no. 7 072006, [arXiv:1301.4507].

[119] **LHCb** Collaboration, R. Aaij et al., *Measurement of the $B^0_s \to \mu^+\mu^-$ branching fraction and search for $B^0 \to \mu^+\mu^-$ decays at the LHCb experiment*, Phys. Rev. Lett. **111** (2013) 101805, [arXiv:1307.5024].

[120] **CMS** Collaboration, S. Chatrchyan et al., *Measurement of the $B^0 \to \mu^+\mu^-$ Branching Fraction and Search for $B^0 \to \mu^+\mu^-$ with the CMS Experiment*, Phys. Rev. Lett. **111** (2013) 101804, [arXiv:1307.5025].

[121] **CMS, LHCb** Collaboration, V. Khachatryan et al., *Observation of the rare $B^0_s \to \mu^+\mu^-$ decay from the combined analysis of CMS and LHCb data*, Nature **522** (2015) 68–72, [arXiv:1411.4413].

[122] **ATLAS** Collaboration, M. Aaboud et al., *Study of the rare decays of $B^0_s$ and $B^0$ into muon pairs from data collected during the LHC Run 1 with the ATLAS detector*, Eur. Phys. J. **C76** (2016), no. 9 513, [arXiv:1604.04263].

[123] A. J. Buras, M. Misiak, M. Munz, and S. Pokorski, *Theoretical uncertainties and phenomenological aspects of $B \to X_s\gamma$ decay*, Nucl. Phys. **B424** (1994) 374–398, [hep-ph/9311345].

[124] T. Hurth, *Present status of inclusive rare B decays*, Rev. Mod. Phys. **75** (2003) 1159–1199, [hep-ph/0212304].

[125] T. Hurth and M. Nakao, *Radiative and Electroweak Penguin Decays of B Mesons*, Ann. Rev. Nucl. Part. Sci. **60** (2010) 645–677, [arXiv:1005.1224].
[126] A. Paul and D. M. Straub, *Constraints on new physics from radiative B decays*, JHEP 04 (2017) 027, [arXiv:1608.02556].

[127] **BaBar** Collaboration, B. Aubert et al., *Measurement of the $B \to X_s \gamma$ branching fraction and photon energy spectrum using the recoil method*, Phys. Rev. D77 (2008) 051103, [arXiv:0711.4889].

[128] **BaBar** Collaboration, J. P. Lees et al., *Exclusive Measurements of $b \to s \gamma$ Transition Rate and Photon Energy Spectrum*, Phys. Rev. D86 (2012) 052012, [arXiv:1207.2520].

[129] **BaBar** Collaboration, J. P. Lees et al., *Precision Measurement of the $B \to X_s \gamma$ Photon Energy Spectrum, Branching Fraction, and Direct CP Asymmetry $A_{CP}(B \to X_s\gamma)$*, Phys. Rev. Lett. 109 (2012) 191801, [arXiv:1207.2690].

[130] **Belle** Collaboration, T. Saito et al., *Measurement of the $\bar{B} \to X_s \gamma$ Branching Fraction with a Sum of Exclusive Decays*, Phys. Rev. D91 (2015), no. 5 052004, [arXiv:1411.7198].

[131] **Belle** Collaboration, A. Abdesselam et al., *Measurement of the inclusive $B \to X_{s+d} \gamma$ branching fraction, photon energy spectrum and HQE parameters*, in Proceedings, 38th International Conference on High Energy Physics (ICHEP 2016): Chicago, IL, USA, August 3-10, 2016, 2016. arXiv:1608.02344.

[132] M. Misiak et al., *Updated NNLO QCD predictions for the weak radiative B-meson decays*, Phys. Rev. Lett. 114 (2015), no. 22 221801, [arXiv:1503.01789].

[133] S. Jager, *Supersymmetry beyond minimal flavour violation*, Eur. Phys. J. C59 (2009) 497–520, [arXiv:0808.2044].