THE TEXTUAL TRADITION OF NICOMACHUS’ INTRODUCTIO ARITHMETICA AND OF THE COMMENTARIES THEREON: A THEMATIC CROSS-SECTION

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Abstract
The entire textual tradition of Nicomachus’ *Introductio arithmetica* and of the commentaries and scholia thereon is investigated by using a single, unusually cryptic sentence in Nicomachus’ text as case-study. The relations between the “Recensions” of Ammonius’ lecture notes are also clearly established. Finally, the way an entire exegetical tradition has dealt with such a cryptic sentence is discussed.

Keywords: Nicomachus, Greek arithmetics, manuscript tradition, commentaries

Resumen
El presente artículo ofrece un estudio completo de la tradición textual de la *Introductio arithmetica* de Nicómaco, incluidos los comentarios y escolios a la obra, a partir del estudio de caso de una sola frase, inusualmente criptica, del texto de Nicómaco. Asimismo, se delimitan con claridad las relaciones entre las diferentes “recensiones” de las notas de lectura de Amonio. Para finalizar, se analiza la forma en que la tradición exegética ha lidiado con esta criptica sentencia.

Metadata: Nicómaco, aritmética griega, tradición manuscrita, comentarios

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THE TEXTUAL TRADITION OF NICOMACHUS’ INTRODUCTIO ARITHMETICA AND OF THE COMMENTARIES THEREON: A THEMATIC CROSS-SECTION

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1. Introduction

Nicomachus’ Introductio arithmetica (henceforth Ar.) and its commentators are a mare magnum no one has seriously studied in a complete and thorough way. As we shall briefly see in Section 4, major philological problems –without doubt the most intricate philological problems that affect the entire Greek technical corpus– make such an undertaking extremely difficult. I shall be no exception; I shall take instead an unprecedented route: I will present a thematic cross-section of the entire manuscript and exegetical record by using a single sentence in Nicomachus’ text as case-study. Of course, this sentence cannot be a chance one. I have selected an unusually cryptic statement if gauged to Nicomachus’ writing standards –so unusually cryptic and unsyntactical as to quite certainly be a gloss that has crept into the text at a deep pre-traditional stage. This happened in fact before the renowned Neoplatonic philosopher Ammonius (early 6th century) held a course on Ar. whose lecture notes make, in the form of a number of “Recensions”, the bulk of the exegetical record on Nicomachus’ treatise. An interesting fact is that a paraphrasis of the test-sentence crept in its turn, as an adjunct (henceforth “Adjunct”) to the very last sentence of Book I, into the text transmitted by a fair majority of the available manuscripts.

Thus, the present study has two philological aims and a related historical aim:

• to investigate in depth but locally the entire manuscript tradition of Ar.;
• to state clearly what are the relations between the “Recensions” of Ammonius’ lecture notes (henceforth “Rec.”);

* Reproductions of, and information on, most of the manuscripts I shall mention can be found by searching the standard database https://pinakes.irht.cnrs.fr by means of the Diktyon number I have indicated either in the list of Nicomachean manuscripts set out in the Appendix or, for those manuscripts that are not included in the list, in the main text and in the footnotes. I thank Sofia di Mambro, Paolo Fait, Luca Farina, Diether Roderich Reinsch, Stefano Serventi, Claudia Sojer, and Francesco Valerio for the logistic support, Ciro Giacomelli, Anna Gioffreda, Inmaculada Pérez Martín, and David Speranzi for the paleographic expertises, Bernard Vitrac for a critical reading of a draft of this paper.
to present a specimen of the contortions an exegetical tradition undergoes as a reaction to a cryptic text.

I shall set out the philological and exegetical record in Section 5. Before doing this, Section 2 presents the (non-trivial) mathematical background, Section 3 my test-sentence, Section 4 the philological context. On the basis of the documents set out in Section 5, Section 6 will offer a series of philological remarks; in particular, I shall outline a broad picture of the textual tradition of Ar. in form of clusters of related manuscripts. It is also to be noted that, if the test-sentence I have selected really is an interpolation, and since it is witnessed to by all manuscripts, the tradition of Ar. is rooted in an archetype. I shall also show that the so-called Rec. II of Ammonius’ lecture notes is a Byzantine recension of Rec. I, and that Rec. IV is an independently conceived Byzantine commentary. 1 Section 7 will summarize the main points of the interpretive proposals of our test-sentence set forth in the scholarly tradition of commentaries and scholia on Ar. Finally, Section 8 will show to what extent the three aims listed above have been attained. The Appendix presents my database, including the list of the manuscripts I could not check. 2

The documentary record is huge: I have collated about 100 manuscripts. Since this is not a critical edition, all variant readings must be made available to the reader, even those readings contained in manuscripts that, already at this stage of the research, are patently doomed to be discarded as apographs of extant models. For this reason, I have decided to present the variant readings in a variety of ways, none of which resorts to a canonical critical apparatus; in particular, I make systematic use of paratextual elements such as italics or underlining, and, most importantly, of colour markings. This entails that the present paper makes better sense if published online. Readers may well be disconcerted by my layout choices; I urge them to consider what a traditional critical apparatus would have looked like.

I decided to keep myself “blind” to any feature of the manuscripts I have seen that figures in portions of Ar. other than the very end of Book I, where the test-sentence lies. 3 Since my aim was not to prepare a critical edition of Ar., I had to set a range of relevant information I allowed myself availing of – I set the strictest one.

1 For the meaning of “Byzantine recension”, and for a survey of them, see F. Acerbi, “Byzantine Recensions of Greek Mathematical and Astronomical Texts: A Survey”, Estudios bizantinos 4 (2016), 133-213.

2 The overall numbers are: 83 witnesses of Ar. that include the test-sentence; 20 that do not or that I could not check (9 items in the latter category); 19 witnesses of Rec. IV, 2 of which also contain Ar.

3 I will thus disregard the results in F. Acerbi, “Eliminazioni diagrammatiche”, Scripta 13 (2020), 9-37, in which a wide-ranging campaign of eliminations is carried out by means of special methods. It goes without saying that the two approaches corroborate each other.
2. Mathematical Background

A fragment from Archytas is the first document that attests for investigations into numerical means:4

There are three means in music: one is the arithmetic, the second geometric and the third subcontrary, which they call “harmonic”. The mean is arithmetic, whenever three terms are in proportion by exceeding one another in the following way: by that which the first exceeds the second, by this the second exceeds the third. And in this proportion, it turns out that the interval of the greater terms is smaller and that of the smaller greater. The mean is geometric, whenever they [scil. the terms] are such that as the first is to the second so the second is to the third. Of these "terms" the greater and the lesser make an equal interval. The mean is subcontrary, which we call harmonic, whenever they are such that, by which part of itself the first term exceeds the second, by this part of the third the middle exceeds the third. It turns out that, in this proportion, the interval of the greater terms is greater and that of the lesser is less.

Thanks to a clever definition not coinciding with Archytas’s,5 mathematicians posterior to him carried out a complete classification of the means, listing eleven of them. The process of classification took a long while; the details of its development vary from source to source: these are Nicomachus, Ar. II.21-29; Iamblichus, in Ar. III.36-71 Vinel;6 Theon of Smyrna, Expositio, 106-111 and 113-119 Hiller;7 Pappus, Coll. III.44-57, who mentions “Nicomachus the Pythagorean”. To all these sources as well as to Archytas, a mean is a triad of numbers or “terms” (c,b,a), with a > b > c and such that b is univocally determined by a and c; by metonymy, the middle term b usually receives the same denomination of “mean”.8

4 Porphyry, in Harm. I.5, 93.5-17 Düring = fr. 47 B 2 DK; English translation by C. A. Huffman, Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King, Cambridge 2005. The “interval” between the terms is their ratio; exactly this terminological point is discussed by Porphyry in the excursus in which he quotes Archytas and other authorities. This denomination remained standard in harmonic theory.

5 Since Archytas’ harmonic mean only refers to the notion of “part” (= a divisor), it does not coincide with the item of the classification that bears the same name (this fact escaped Pappus, who identifies the two means in Coll. III.30): they do coincide only if the available ratios are epimoric.

6 Jamblique, In Nichomachi Arithmetica, ed. N. Vinel, Pisa – Roma 2014.

7 Theonis Smyrnæti Expositio rerum ad legendum Platonem utilium, ed. E. Hiller, Lipsiae 1878.

8 Greek technical jargon calls the triad μεσοτής and the middle term μέσος. The ultimate source of all modern accounts of the classification is P. Tannery, “L’arithmétique des grecs dans Pappus”, Mémoires de la Société des sciences physiques et naturelles de Bordeaux 3 (1880), 351-371, reprinted in Id., Mémoires Scientifiques, I, Toulouse – Paris 1912, 80-105: 90-98.
The means are classified as follows. Take the three differences of the terms, \( a - b, b - c, a - c \), which authors qualified by Pappus as “recent” (Coll. III.46) called “first”, “second”, and “third excess”. A mean is determined by identifying the ratio of any two of the three excesses and the ratio of two of the three terms, possibly taking the same term twice. Let us see how only 11 combinations result. Since a ratio can always be taken to be greater-to-lesser without loss of generality, there are only three possible ratios between the excesses, namely, \((a - b) : (b - c)\), \((a - c) : (a - b)\), and \((a - c) : (b - c)\). Now, the ratio \((a - b) : (b - c)\) can be greater than, equal to, or lesser than 1. If the first alternative applies, the ratio can only be the same as \(a : b\), \(b : c\), or \(a : c\), but it is easy to see that the first two cases are equivalent. If the second alternative applies, the ratio \((a - c) : (b - c)\) can only be identical to \(a : a = b : b = c : c\). If the third alternative applies, the ratio \((a - c) : (b - c)\) can indeed be the same as any of \(c : b, b : a,\) or \(c : a\). This gives \(2 + 1 + 3 = 6\) means associated to the first ratio of the excesses. Next, the ratio \((a - c) : (a - b)\) can only be greater than 1: therefore, it can only be the same as \(a : b, b : c,\) or \(a : c\), and all three cases may obtain. This gives three additional means. Finally, the ratio \((a - c) : (b - c)\) can only be greater than 1, too: therefore, again, it can only be the same as \(a : b, b : c,\) or \(a : c\), but in this case \((a - c) : (b - c) :: a : b\) is idle since it entails \(a = b\). This gives two additional means. Thus, there are \(6 + 3 + 2 = 11\) means.

The means are eleven but Nicomachus and Pappus only list ten of them. Fortunately, the two lists do not match: each of the two authors describes a mean that is missing in the other. The reason of the omission must lie in the fact that Nicomachus and Pappus relied on different Neo-Pythagorean sources, each of which closed the list as soon as a “holy” decad was completed. Nicomachus can be taken to imply this when he writes “after which the moderns discover four other ‹means› as well, making up the number ten, which, according to the Pythagorean view, is the most perfect possible” (Ar. II.22.1). Following Pappus’ order and including the mean he does not list, the eleven means are defined as follows:

| Arithmetic | \((a - b) : (b - c) :: a : a\) |
| Geometric  | \((a - b) : (b - c) :: a : b\) |
| Harmonic   | \((a - b) : (b - c) :: a : c\) |
| Fourth or subcontrary to the harmonic | \((a - b) : (b - c) :: c : a\) |
| Fifth or subcontrary to the geometric  | \((a - b) : (b - c) :: c : b\) |
| Sixth      | \((a - b) : (b - c) :: b : a\) |
| Seventh    | \((a - c) : (a - b) :: b : c\) |
| Eighth     | \((a - c) : (a - b) :: a : b\) |
| Ninth      | \((a - c) : (a - b) :: a : c\) |
| Tenth      | \((a - c) : (b - c) :: b : c\) |
| Eleventh   | \((a - c) : (b - c) :: a : c\) |
Pappus, *Coll. III.47-57*, also describes a method for generating all means by starting from the sole “minimal” geometric mean (1,1,1). The method is also described by Theon of Smyrna, *Expositio*, 106-111 Hiller, who draws it from Adrastus, who in his turn used Eratosthenes as his source, by Nicomachus, *Ar. I.23.6-II.2.2*, by Iamblichus, *in Nic. III.39-45, 112.33-114.29 Vinel*, and by Proclus, *in Ti. II, 18-20 Diehl*. Yet, in these authors the method is used to show how to generate successive geometric means by starting again from the minimal geometric mean (1,1,1). As a matter of fact, in these authors the method is not used to generate the means, but to generate species-wise the ratios of the five basic species (but the two notions are strictly related, as we shall see presently).

The method described by Pappus works as follows. Given a geometric mean \((c,b,a)\) with \(c < b < a\), one can always take “linear combinations” of the terms \(c, b,\) and \(a\) to get three new terms \(C, B,\) and \(A\). It happens that appropriate linear combinations do give rise to all means. For instance, a new geometric mean is found if we set \(C = c, B = c + b, A = c + 2b + a\). Pappus’ exposition has some gaps, but they can easily be filled; the following table provides a synopsis of the transformations required to generate the indicated mean by taking the associated linear combination of the terms of an assigned geometric mean.

| Type      | Transformation                       |
|-----------|--------------------------------------|
| Arithmetic| \(C = c + b, B = c + 2b + a, A = c + 3b + 2a\) |
| Geometric | \(C = c, B = c + b, A = c + 2b + a\)    |
| Harmonic  | \(C = c + b, B = c + 2b, A = c + 3b + 2a\) |
| Fourth    | \(C = c + b, B = c + 2b + 2a, A = c + 3b + 2a\) |
| Fifth     | \(C = c + b, B = c + 2b + a, A = c + 3b + a\) |
| Sixth     | \(C = b + a - c, B = c + 2b + a, A = 2c + 3b + a\) |
| Seventh   | \(C = c, B = b + a, A = c + b + a\)     |
| Eighth    | \(C = c + 2b, B = c + 2b + a, A = c + 3b + 2a\) |
| Ninth     | \(C = c + b, B = c + b + a, A = c + 2b + a\) |
| Tenth     | \(C = c, B = c + b, A = c + b + a\)     |
| Eleventh  | \(C = b + a, B = 2b + a, A = c + 2b + a\) |

The reader is urged to keep in mind the second transformation in the list, which is, as said, the only one used by all other sources mentioned above.

To understand how this specific transformation was used to generate all ratios, as Nicomachus does, recall that the standard classification of greater-to-lesser ratios lists five of them, as we learn from instance in Nicomachus, *Ar. I.17.8-23.3*. These are, if reduced to lowest terms and where \(n, m,\) and \(k\) are integer numbers:

- multiple ratios, of the form \(\frac{n}{1} = n\);
- superparticular ratios, of the form \(\frac{(n+1)}{n} = 1 + \frac{1}{n}\);
• multiple-superparticular ratios, of the form $k + \frac{1}{n}$, with $k > 1$;
• superpartient ratios, of the form $\frac{n + m}{n} = 1 + \frac{m}{n}$, with $1 < m < n$;
• multiple-superpartient ratios, of the form $k + \frac{m}{n}$, with $k > 1$ and $1 < m < n$.

It is clear that this partition of greater-to-lesser ratios is exclusive and exhaustive, for any such ratio can be written as $k + \frac{m}{n}$, with $k \geq 1$ and $0 \leq m < n$.

Now, any geometric mean in lowest terms can be written $(b^2, ab, a^2)$ for suitable integers $a$ and $b$, with $a > b$, prime to each other.\(^9\) This amounts to saying that there is a ratio in lowest terms associated to any geometric mean in lowest terms: if the mean is $(b^2, ab, a^2)$, the ratio is $\frac{a}{b}$\(^10\) which can be read off within the mean as the two ratios $ab/b^2$ and $a^2/ab$; neither of these ratios, however, can be in lowest terms unless $b = 1$.

With this background material, let us now see what Nicomachus does at Ar. I.23.6-II.2.2. As said, he presents a two-arm recursive prescription (πρόσταγμα) for finding the geometric means associated to all kinds of ratios, by starting from the minimal geometric mean of “equality” $(1,1,1)$, which corresponds to the ratio $(1,1)$.\(^11\) The prescription maps a mean $(z,y,x)$, whose terms are called in increasing order “first”, “second”, and “third”, into the mean $(z,z + y,z + 2y + x)$ or, by taking the original terms in reverse order (ἀναστραφέντων) $(x,y,z)$, into the mean $(x,x + y,x + 2y + z)$. Here is Nicomachus’ text (IA I.23.8): πρῶτον πρώτῳ ἴσον ποιῆσαι, δεύτερον δὲ πρώτῳ ἅμα καὶ δευτέρῳ, τρίτον δὲ πρώτῳ καὶ δυσὶ δευτέροις ἅμα καὶ τρίτῳ “make the first equal to the second, the second to the first together with the second, the third to the first and two seconds together with the third”.\(^12\)

Since, as we have just seen, any geometric mean can be written $(b^2, ab, a^2)$, the prescription can be finally formulated as follows (I skip some parentheses from now on), where $b^2 = “first
term”, ab = “second
term”, a^2 = “third
term”:

\[
\begin{align*}
\text{b^2,ab,a^2} & \rightarrow b^2, b^2 + ab, b^2 + 2ab + a^2 = b^2, (b + a) b, (b + a)^2 \\
\text{b^2,ab,a^2} & \rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a + b) a, (a + b)^2
\end{align*}
\]

\(^9\) This is proved in Elem. VIII.11.

\(^10\) This ratio will also be written $(a,b)$.

\(^11\) The philosophical gist of the procedure lies in the fact that all ratios –that is, the entire unordered world of inequality– is thereby reduced to the unifying principle of equality. In the initial sections of Book II, Nicomachus applies the inverse prescription, thereby getting from any ratio to the ratio of equality. Note that Nicomachus always uses προστάγματα in the plural, apparently taking each of the clauses in the “prescription” as a separate item. I shall stick to the singular to simplify matters.

\(^12\) The absence of the article in the Greek text is a way to formulate the prescription in full generality.
The inverse transformation \( \star \rightarrow \) amounts to compounding inversion of extremes and first-arm prescription, as follows:

\[
b^2, ab, a^2 \rightarrow a^2, ab, b^2 \rightarrow a^2, a^2 + ab, (a + b)a, (a + b)^2
\]

Therefore, on account of the above one-to-one map between means in lowest terms and ratios in lowest terms, the same two-arm prescription applied to ratios reads:

\[
(a, b) \rightarrow (b + a, b), \quad \text{that is}, \quad \frac{a}{b} \rightarrow \frac{b + a}{b} = 1 + \frac{a}{b}
\]

\[
(a, b) \star \rightarrow (a + b, a), \quad \text{that is}, \quad \frac{a}{b} \rightarrow \frac{a + b}{a} = 1 + \frac{b}{a}
\]

For example, take the mean \((1,1,1)\) and apply the prescription once; both arms give the same result: \((1,1,1) \rightarrow (1,1 + 1,1 + 2 + 1) = (1,2,4)\), which is the double ratio. Let us iterate the prescription by applying it to the double ratio itself:

\[
(1,2,4) \rightarrow (1,1 + 2,1 + 4 + 4) = (1,3,9)
\]

\[
(1,2,4) \star \rightarrow (4,2,1) \rightarrow (4,4 + 2,4 + 4 + 1) = (4,6,9)
\]

The former is the triple ratio, the latter is the sesquialter, or hemiolic, ratio.

Thus, the “direct” (ὀρθῶς) arm sends ratios of kind X into ratios of kind multiple-X because at every step it adds one unit to the original ratio, eventually keeping the resulting ratios within such a multiple-X kind; the “inverse” arm of the prescription sends ratios of kind X into ratios of a different kind because at every step it adds one unit to the inverse of the original ratio, eventually transforming it into a superpartient or multiple-superpartient ratio. All and only the five kinds of ratios are generated through this procedure, as can be seen from a rough flow chart that summarizes the entire process.
Let us note, finally, that, in its domain of application, the “direct” arm sends a ratio into a greater ratio (this is obvious, since it simply adds 1 to the original ratio), whereas the “inverse” arm may send a ratio either into a greater or into a smaller ratio (just check in the flow chart above).

3. A Concise Statement by Nicomachus: in Fact, a Gloss

Let us have a close look at the prescription:

\[ b^2, ab, a^2 \rightarrow b^2, b^2 + ab, b^2 + 2ab + a^2 = b^2, (b + a)b, (b + a)^2 \]

\[ b^2, ab, a^2 \rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a + b)a, (a + b)^2 \]

It is evident that the extremes of all the involved means are square numbers, that the largest terms of the two means in the right-hand side are the same and a square, namely, \((b + a)^2 = (a + b)^2\), and that the largest term of the original mean becomes the smallest term of the mean that results from the inverse transformation. Moreover, if we write the inverse transformation in terms of the direct transformation as done above, namely

\[ b^2, ab, a^2 \rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a + b)a, (a + b)^2 \]

and we skip the inversion step \(\rightarrow\), thus:

\[ a^2, ab, b^2 \rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a + b)a, (a + b)^2 \]

we see that in both arms of the prescription the smallest term in the mean remains the same under the transformation: in the direct arm, it is \(b^2\), in the inverse arm, it is \(a^2\).

Nicomachus also seems to have noticed some of these properties, for at Ar. I.23.15 –in the middle of a series of examples– he drops in the following remark:

\[ \varepsilonπι πασῶν δὲ τῶν διαζευχθεισῶν καὶ ἀφ’ ἧς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι “In the case of all these ‹relations› that are ‹thus› differentiated, and of the one from which both ‹of the differentiated ones are derived›, the last square remains the same; the first ‹term› is transformed into the smallest, and invariably the extremes are squares”.

The problem with this statement is that it is too concise, fairly ambiguous and under-determined by the context (there is no mention of ἔσχατος term in what precedes, nor of διαζευχθεῖσαι relations; the qualifiers “first” and “smallest” do not belong to homogeneous descriptive levels; the sense in which “the first ‹term› is transformed into the smallest” is hardly clear, and can only apply to one of the two arms of the prescription, not to both; the statement “the last square remains the same” can instead be taken to apply to both arms,

13 This remark makes the entire section Ar. I.23.15 in Hoche’s edition.
14 The adjective will only occur at Ar. II.2.2 in the present context.
as seen above, or –more straightforwardly– to the direct arm only), and so badly formulated (the same set of objects is first qualified by πασῶν and then by ἀμφότεραι) as to be unsyntactical (the presence of ἀφ’ ἦς is jarring). An exegetical problem resulted, which was “solved” in manyfold ways by the commentators of Ar. and that even interfered with the treatise’s text in most manuscripts. I have written “solved” between quotation marks because some of the “solutions” set forth by the commentators are weird or irremediably wrong. Even D’Ooge has a partly incorrect translation, since he writes “the last term is always the same and a square”, which is definitely not what the Greek text says.

As a partial excuse for the performances of the commentators, let me state clearly that I think that this sentence –despite being witnessed to by the entire manuscript tradition– is a gloss. First and foremost, this is a question of style: there is not one single passage in the entire Ar. that is so clumsily written and so cryptic, and which disrupts the line of argument in so offensive a way. I also surmise that the origin of the gloss lies in a remark by Iamblichus, where he is just commenting on what Nicomachus does: ἐπὶ δὲ πασῶν τῶν πλασσομένων σχέσεων καὶ ἀφ’ ἦς αἱ πλάσεις, οἱ ἄκροι τετράγωνοι γίνονται, says Iamblichus.15 And what he says is crystal-clear, and perfectly syntactical Greek. Someone after him thought it better to buttress Nicomachus’ text –possibly only in the margin of his own manuscript exemplar of Ar.– with an enriched version of Iamblichus’ statement. The final outcome of this brilliant idea we read in our manuscripts.

4. Philological Context

The task of editing the Introductio arithmetica is made particularly difficult by the fact that it was, after its composition in the 2nd century, the most widely used textbook of number theory for more than one millennium. For this reason, it was frequently copied: we know about 100 manuscript witnesses of it, of which nearly 90 contain the complete work and the rest fragments or excerpta.16 Not only this: there is not the slightest doubt – as we shall see ad abundantiam – that the text has undergone modifications, and that wild contamination has occurred since the earliest witnesses we have access to; we have all reasons to think that this also occurred at a pre-traditional stage. Anyway, the only edi-

15 At in Nic. III.68, 120.9-10 Vinel. I add that, had Iamblichus read section I.23.15 in his Nicomachus, he would hardly have omitted elaborating on all claims contained in the sentence.
16 A fairly complete list of manuscripts is in W. Haase, Untersuchungen zu Nikomachos von Gerasa, Dissertation, Eberhard-Karls-Universität Tübingen 1982, 319-398. The database pinakes lists 107 items, but with some misattributions; see also M. L. D’Ooge – F. E. Robbins – L. Ch. Karpinski (transl. comm.), Nicomachus of Gerasa, Introduction to Arithmetic, Ann Arbor 1926, 146-166, for a list of manuscripts and some, very partial and preliminary, investigations into a part of the manuscript tradition. My list is found in the Appendix.
tion of _Ar._ is Richard Hoche’s, who used a random and quite limited sample of manuscripts, namely, those readily available to him because they were kept in nearby German libraries. Hoche took the one carried by Gott. philol. 66 as his text of reference, relying on a wrong dating of the manuscript to 10th century.  

The standard view of the exegetical record on _Ar._ is that one ancient commentary proved so successful that we have four different versions of it, referred to as Recensions I-IV. Detail is as follows. The Neo-Platonic philosopher Ammonius held a course on _Ar._ whose lecture notes were published by his pupils. The edited notes have the form of a running commentary organized in “lemmas”, which clarify single statements in the text. As a matter of fact, the comments on the technical portions of _Ar._ frequently feature lists of numerical examples (ratios, figured numbers, means, etc.), exactly as _Ar._ does. This happens in general in mathematical and astronomical commentaries of Late Antiquity, which mainly comprise explanations of the _technologia_, examples, scholarly and antiquarian digressions.  

Every Recension is divided in two Books, as _Ar._ is. Rec. I, which both the manuscripts and modern scholarship assign to John Philoponus, was edited in the 1860s by Hoche in a series of three _Schulprogramme._

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17 Nicomachi Geraseni pythagorei Introductionis arithmeticae libri II, Lipsiae 1866.

18 It was in fact penned only in the late 13th century, all the while using an imitative script: G. De Gregorio – G. Prato, “Scrittura arcaizzante in codici profani e sacri della prima età paleologa”, Römische Historische Mitteilungen 45 (2003), 59-101; and, most recently, F. Acerbi – A. Gioffreda, “Manoscritti scientifici della prima età paleologa in scrittura arcaizzante”, _Scripta_ 12 (2019), 9-52: 26 and 36 for the dating to late 13th century.

19 P. Tannery, “Rapport sur une mission en Italie du 24 Janvier au 24 Février 1886”, _Archives et Missions scientifiques et littéraires, 3e série_ 13 (1888), 409-455, reprinted in Id., _Mémoires Scientifiques_, II, Toulouse – Paris 1912, 269-331: 302-310. See also L. G. Westerink, “Deux commentaires sur Nicomaque: Asclépius et Jean Philopon”, _REG_ 82 (1968), 526-555; and L. Tarán, _Asclepius of Tralles, Commentary to Nicomachus’ Introduction to Arithmetic_, Transactions of the American Philosophical Society 59.4, Philadelphia 1969, 5-20, in particular as to the relations between Rec. I and Rec. III.

20 For a first orientation on Ammonius’ school and on the main characters mentioned below, see the entries in R. Goulet (ed.), _Dictionnaire des philosophes antiques_, 7 vol., Paris 1994-2018.

21 On the structure of the Neoplatonic commentaries, see the synthesis by Ph. Hoffmann, “What was Commentary in Late Antiquity? The Example of the Neoplatonic Commentators”, in M. L. Gill – P. Pellegrin (eds.), _A Companion to Ancient Philosophy_, Malden (Mass.) 2006, 597-622. On the way these lecture notes were published, see the seminal M. Richard, “ΑΠΟ ΦΩΝΗΣ”, _Byzantion_ 20 (1950), 191-222.

22 For the exegetical practices of Eutocius, one of Ammonius’ pupils, see most recently F. Acerbi, “Commentari, scolii e annotazioni marginali ai trattati matematici greci”, _Se-T_ 10 (2012), 135-216. with bibliography.

23 For a _specimen novae editionis criticae_, unfortunately not achieved, see Haase, _Untersuchungen_ (cit. n. 16), 401-447.
Hoche himself, when editing Rec. I, listed the differences between Rec. I and Rec. II for the first book of Ar. in the preface of his 1865 Schulprogramm;\textsuperscript{24} in doing so, he relied on exactly one manuscript, namely, Zeitz, Stiftsbibliothek 67. Hoche never published a similar list for the scholia to the second book. Almost a century later, A. Delatte edited the Rec. II scholia for Book II, using Vat. gr. 1411 and Athens, EBE 1238, as witnesses.\textsuperscript{25} All witnesses of Rec. II anchor the scholia on the text by means of numerals; this graphic convention, despite being found in Hoche’s edition, is only adopted by a small number of witnesses of Rec. I. In the manuscripts, Rec. II is duly ascribed to Philoponus. More on Rec. II in Section 6 below.

Rec. III is available in a critical edition.\textsuperscript{26} Manuscripts and scholars concur in ascribing it to Asclepius of Tralles, a pupil of Ammonius and possibly a schoolmate of Philoponus.

As we shall see in Section 6,\textsuperscript{27} Rec. IV is not a recension of Ammonius’ lecture notes\textsuperscript{28} but an independent commentary, to be assigned with certainty to the Byzantine period because of its prolixity and of its paraphrastic structure, to such an extent that large portions of it offer a self-contained argument.

The pattern of association of Ar. with its several commentaries is quite complex: there are about 40 manuscripts in which one of the Recensions is copied as a self-contained text (either following or preceding the Nicomachean treatise, or even in absence of it), about 20 manuscripts in which the Recension lies in the margins. About 40 manuscripts carry Ar. with or without fully-fledged commentaries in form of scholia; many of these have scholia unrelated to any of the Recensions.

\textsuperscript{24} Ἰωάννου γραμματικοῦ Ἀλεξανδρέως (τοῦ Φιλοπόνου) εἰς τὸ πρῶτον τῆς Νικομάχου ἀριθμητικῆς εἰσαγωγῆς, ed. R. Hoche, 2 fasc., Gymnasium zu Wesel 1864-65, ii-xv. This edition is totally unreliable, as we shall see.

\textsuperscript{25} A. Delatte, Anecdota Atheniensia et alia. Tome II. Textes grecs relatifs à l’histoire des sciences, Liège – Paris 1939, 129-187.

\textsuperscript{26} Tarán, Asclepius (cit. n. 19), who based himself on Ambros. B 77 sup., ff. 102r-149v, and Monac. gr. 431, ff. 98r-114v (ca. 1330; Diktyon 44879), a copy of which is Par. gr. 2376, ff. 1r-56r. The “Notes to the Text” in Tarán’s edition, 73-81, give a fairly clear idea of aims, methods and philosophical stand of Rec. I and III.

\textsuperscript{27} See the discussion of the entire textual tradition in F. Acerbi, “La tradition manuscrite de la « Recension IV » du commentaire à l’Introductio arithmetica de Nicomaque”, submitted. This tradition is split into two branches, whose ancestors are extant manuscripts.

\textsuperscript{28} The same assessment is in Tarán, Asclepius (cit. n. 19), 20. As a matter of fact, it is enough to read Rec. IV to grasp the difference from the other Recensions. Ammonius is never referred to in Rec. IV.
5. Texts

The first text I present below is of course Nicomachus, *Ar.* I.23.15, in all manuscripts I was able to check and that do contain the end of Book I (82 items). I shall partition the textual record into five groups, according to distinctive variant readings.

The exegetical record comprises the following items:

1) Ammonius’ lecture notes, Rec. I (Philoponus) in a limited set of manuscripts.
2) Ammonius’ lecture notes, Rec. II (anonymous), in the text of most of the extant manuscripts.
3) Ammonius’ lecture notes, Rec. III (Asclepius), in Tarán’s edition.
4) The anonymous Byzantine commentary called “Recension IV”, in my own critical text. 29
5) The Adjunct to the text of the *Introductio arithmetica* in the same manuscripts as those used for *Ar.* I.23.15 (this is subsection 5.1), and a scholium thereon (subsection 5.2). I shall partition the textual record of the Adjunct into six groups, according to distinctive variant readings.
6) The scholia in Matrit. 4678 and other manuscripts (with glosses).
7) Glosses and scholia in Monac. gr. 482 and Laur. Plut. 28.35.
8) Four scholia in Ambros. G 62 sup. and other manuscripts; the scholia are ascribed to Michael of Ephesus, to an anonymous from the town of Δυρράχιον (present-day Durrës), to Eustratius of Nicaea, and to a judge Nicholas Dishypatos, respectively.
9) Four scholia in Vat. gr. 256 and other manuscripts.
10) Glosses and scholia by Isaak Argyros in Norimb. Cent. V app. 36. 30
11) A scholium eventually included in some Rec. I manuscripts and ascribed in most of them to some Theodoros/Demetrios Protocensor. 31
12) Glosses, and scholia to the Adjunct, in Hamburg, S-UB, philol. 88, Vat. gr. 2387, and related manuscripts.
13) Dishypatos’ and the anonymous from Dirrachios’ (*sic*) scholia in Vat. gr. 198 and in a copy of it.
14) Glosses and scholia in Marc. gr. Z. 320 and related manuscripts.
15) Glosses and a scholium to the Adjunct by a later hand in Par. gr. 2373.
16) Glosses and scholia in Kharkow, UL, 269-p, 369-c and related manuscripts.
17) Glosses and scholia in Marc. gr. Z. 316 and 319.
18) Glosses and a scholium in Laur. Conv. Soppr. 30.

29 Acerbi, “La tradition” (cit. n. 27).
30 First edited, with some misreadings, in C. F. A. Nobbe, *Codicum Quelferbytani et Norimbergensis scholia graeca ad librum I. Isagoges Nicomacheae nunc primum edita*, Lipsiae 1862, 18.
31 It is partly edited by Hoche, Ἰωάννου (cit. n. 24), xiv-xv, who calls the author Theodoros.
In my editions, I regularize punctuation and accents, and neglect variants of spelling such as movable ny, –σσ– vs. –ττ–, geminate or simple ny, etc. Numerals are transcribed without apex, ordinals carry the termination at the exponent. I frequently omit the anchoring lemma of the scholia; otherwise, it is within angular brackets <**>. Each textual unit comprises the list of the manuscripts used, the text itself, and the critical apparatus, which is always negative. The variant readings recorded within the text are keyed to the witnesses by means of the number that characterizes the latter’s shelfmark. The infralinear glosses are inserted in the text, within braces. Other conventions are explained in due course. A complete list of the manuscripts employed, with additional information on each of them, is set out in the Appendix.

Nicomachus, Ar. I.23.15, in the manuscripts

The textual record is partitioned into five groups, according to distinctive variant readings; these are marked in red; the first text has no such marking. Singular readings are recorded in the apparatus; manuscripts marked by the same colour share interesting singular readings.

A

Ambros. G 62 sup., f. 43r; Ambros. P 121 sup., f. 130v; Bucur., BAR gr. 620, pp. 220-221; Gott. philol. 66, ff. 138v-139r; Guelf. 36 Gud. gr., f. 17r-v; Hamburg, S-UB, philol. 89, p. 75; Laur. Conv. Soppr. 30, f. 42v; Laur. Plut. 28.35, f. 31r; Laur. Plut. 58.29, f. 152v; Leiden, BRU, Voss. gr. Qº 23, ff. 29v-30r; Marc. gr. Z. 309, f. 193r; Marc. gr. Z. 316, f. 60v; Marc. gr. Z. 318, f. 20v; Marc. gr. Z. 320, f. 52r-v; Marc. gr. Z. 333, f. 61r; Marc. gr. Z. 514, f. 24v; Marc. gr. Z. 592, f. 36v; Matrit. 4678, f. 25v; Monac. gr. 482, f. 132v; Mutin. α.T.8.14, f. 27r-v; Mutin. α.U.9.7, f. 38r; Neapol. III.C.1, ff. 19v-20r; Norimb. Cent. V app. 36, f. 18r; Oxon. Bodl. Holkham 71, f. 278r-v; Oxon. Bodl. Laud. gr. 44, p. 89; Par. Coislin 174, f. 58r; Par. gr. 2107, f. 84v; Par. gr. 2374, f. 21r-v; Par. gr. 2376, f. 78r; Par. gr. 2450, f. 111r; Par. gr. 2479, f. 91v; Par. gr. 2480, pp. 339-340; Par. gr. 2531, f. 80r-v; Par. suppl. gr. 450, ff. 34v-35r; Roma, Arch. S. Paolo 24C, f. 53v; Scorial. Σ.ΙΙ.15, f. 171r-v; Scorial. Σ.ΙΙ.12, f. 20v; Scorial. XI.9, ff. 46v-47r; Vat. Barb. gr. 273, f. 122r; Vat. gr. 186, f. 243r; Vat. gr. 195, f. 42v; Vat. gr. 198, f. 19r; Vat. gr. 1026, f. 63v; Vindob. phil. gr. 62, f. 24r; Vindob. phil. gr. 220, f. 61r-v

ἐπὶ πασῶν δὲ τῶν διαζευγθεισῶν καὶ ἀφ’ ἧς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Ambros. P 121 sup.: ἀκρως scr. sed –οι s.l. m.1 | Bucur., BAR gr. 620: ἀμφότερα τετράγωνος ὁ αὐτὸς μένει | Gott. philol. 66: ἀμφότερα | Laur. Plut. 58.29: πάντως | s.l. γρ. πάντες | Leiden, BRU, Voss. gr. Qº 23: ἀμφότερων | Marc. gr. Z. 309: ἀμφότεροι | Marc. gr. Z. 514: τετράγωνος ἐσχάτως μένει (marg. γρ. ὰσαύτως) | Neapol. III.C.1: –γον– (pr.) | Par. gr. 2374: διαζευχθεισῶν σχέσεων
ἐπὶ πασῶν δὲ τῶν διαξευθεισῶν καὶ ἀφ’ ἧς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος ὡσαύτως μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Ambros. I 8 sup.: ὡσαύτως ∩ ὁ αὐτὸς s.l. | Athens, EBE 1115 & Vat. gr. 256: ὡσαύτως ∩ γρ. καὶ ὁ αὐτός s.l. | Marc. gr. Z. 319: πάντες | Marc. gr. Z. 317: διαλεχθεισῶν καὶ διαξευθεισῶν | Marc. gr. Z. 595 & Par. gr. 2762: διαλεχθεισῶν sed –ζευ– s.l. | ὁ (pr.) ∩ ἡ | Par. gr. 2372: ἁμφότερα

ἐπὶ πασῶν δὲ τῶν διαξευθεισῶν καὶ ἀφ’ ἧς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος [om. ὁ αὐτός] μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Mutin. a.W.3.1: μεταβαίνειν

Ambros. B 77 sup., f. 57v; Berol. Phillipps 1549, f. 84r; Bucur., BAR gr. 520, p. 118; Cizensis 67, f. 51v; Erlangen, UL A.8, f. 38v; Leiden, BRU, BPG 74G, f. 24v; Monac. gr. 76, f. 247v; Oxon. Lincoln gr. 33, f. 109v; Par. gr. 2063, f. 33r; Par. gr. 2377, f. 84r; Vat. gr. 197, f. 15v; Vat. gr. 1411, f. 76r

ἐπὶ πασῶν δὲ τῶν διαξευθεισῶν καὶ ἀφ’ ἧς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος | –ος | –ων s.l. | πάντες | –ων s.l. | Berol. Phillipps 1549 & Par. gr. 2377: marg. ἐν ἕλλοις οὗτως, ὁ μὲν πρῶτος τετράγωνος ὁ αὐτός μένει, ὁ δὲ ἔσχατος εἰς τὸν ἐλάττονα μεταβαίνει | Erlangen, UL A.8: om. τὸν

Kharkow, UL 269-p, 369-c, f. 87v; Oxon. New Coll. 299, f. 55v; Par. gr. 2481, f. 18v; Par. gr. 2483, p. 152; Scorial. PII.3, f. 431v; Vat. gr. 1051, f. 20v

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ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν καὶ ἀφ’ ἥς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος ὢν αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἑλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Kharkow, UL, 269-p, 369-c & Par. gr. 2483: marg. ἐν ἄλλοις οὕτως· ὁ μὲν πρῶτος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ ἔσχατος εἰς τὸν ἑλάττονα μεταβαίνει | Par. gr. 2483: s.l. (ἀμφο)τέρων m. rec. | Vat. gr. 1051: ἔσχατος s.l. | Scorial. P.II.3: ἀμφοτέρων

Glosses to Ar. I.23.15

Vat. gr. 195, f. 42v
ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν καὶ ἀφ’ ἥς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος {λόγος δηλονότι δ ἢ ὁ θ} εἰς τὸν ἑλάττονα {** ἐπίτριτος ** ὁ ι** ἐπίτριτος} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

**COMMENTSARIES**

C.1. Recension I

In Hoche’s edition, this is sect. I.ρπϛ, but the numbering has almost no manuscript authoritatity. Singular readings are recorded within the text; recurrent clusters of witnesses are identified by the same colour. A detailed comparison of Texts C.1-4 will be carried out in Section 6.2. The first three manuscripts contain only Rec. I.

Ambros. I 83 inf., f. 89r-v (m. rec.); Angel. gr. 1, f. 33r-v; BNCF, Fondo Naz. (olim Magliab.) II.III.37, f. 46r-v; Bucur., BAR gr. 620, pp. 220-222; Gott. philol. 66, ff. 138v-139v; Hamburg, S-UB, philol. 89, pp. 368-370; Laur. Plut. 28.35, f. 29r; Marc. gr. Z. 333, f. 61v; Monac. gr. 76, ff. 158v-159r; Monac. gr. 482, ff. 218v-219r; Mutin. a.T.8.14, f. 27v; Mutin. a.U.9.7, f. 38r-v; Par. Coislin 174, f. 102r; Par. gr. 2480, pp. 339-341; Par. gr. 2483, pp. 145-146; Par. gr. 2531, f. 79v; Roma, Arch. S. Paolo 24C, ff. 53v-54r; Scorial. P.II.3, ff. 431v-432r, 432v-433r; Vat. gr. 198, f. 19r

ρπϛ. διαζευχθεῖσα εἰσὶ σχέσεις αἱ τε {τε om. 28.35 482} κατ’ ὰρθὸν ἐπιζευγνύμεναι {ἔπε— 620} καὶ αἱ κατὰ ἀναστροφὴν συμπλεκόμεναι· διεξευκται γὰρ τοῦ ὀρθοῦ τὸ ἀναστραμμένον {ἀνα— 76}· ἑκάστης μὲν σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι {τ. ε. 28.35 482 : τεταγμένοι sed corr. m.1 1 : ἐπιτεταγμένοι 2531}, δι’ ἣν αἰτίαν ἐροῦμεν {ἐ. α. 89 2483 P.II.3 198}. ἀλλ’ {ὀκὶ 83} ὁ μὲν ἔσχατος ὁ αὐτὸς μένει {ὡς αὐτὸς P.II.3}, φησίν· οὐ πρόεισι γὰρ {γὰρ om. P.II.3} ἐπ’ {εἰς 620} ἐπεπίκα τοῦ ἀναστραμμένου μονάδος {ἐπεὶ 89 2483 198 · ἐπεὶ adj. add. 2531} ἐπὶ {ὁικ 83 37} ἐσχατος ὡν· ὁ δὲ πρῶτος ἐκ τῆς ἀναστροφῆς πρὸς τὸν ἑλάττονα πρόεισι μέχρι μονάδος, ἐπεὶ {ἐπι 83} καὶ {ἀεὶ add. 2531} ἀπὸ {ὁικ 37} μονάδος {ἐπεὶ — μονάδος om. 76} ἠρξατο. τούτῳ δὲ δῆλον ἐκ τε τῶν {τῶν om. P.II.3} προειληφυίων σχέσεων κάκι {καὶ ἐκ 83 37 2483 198 : καὶ 333 : καὶ 76} τῶν {καὶ αὐτῶν P.II.3} λέγεσθαι μελλουσών. πάντως {πάντων 89 2483 198} δὲ, φησίν {φησίν om. 2483 quod lemma fecit}, οἱ ἄκροι τετράγωνοι· {πάντως — τετράγωνοι om. P.II.3

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sed perperam incl. 6a} ἐπὶ μὲν {ακτῶν add. P.II.3} ἡμιολίων, τοῦ δ καὶ ς {δις 2531} καὶ θ {τοῦ δ ς ιβ 83 37}, ὁ δ καὶ ς {ο om. 83 37 333 2483 P.II.3} θ· ἐπὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ς καὶ ις, ὁ θ καὶ ς {τοῦ θ ιβ 83 333 28.35 482}. διὰ τοῦτο οἱ άκροι τούτων τετράγωνοι εἰσίν. ἐπεὶ οὖν ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ς καὶ ις καὶ ιϛ {τοῦ θ ιβ ιϛ 83 37}, ὁ θ καὶ ιϛ {τοῦ θ ιβ ιϛ, ὁ θ καὶ ιϛ 333 : καὶ ιϛ s.l. m. rec. α.T.8.14}. καὶ τῶν ἀλλήλων τὸν αὐτὸν λόγον ἔχοντες, οἱ άκροι αὐτῶν τετράγωνοι εἰσίν. ἐδείχθη γὰρ παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἄριθμοι ἐλάττον εἰσὶν πρὸς ἀλλήλους τὸν αὐτὸν λόγον, οἱ άκροι αὐτῶν τετράγωνοι εἰσίν. ἀλλ' ίσως εἴποι τις ἄν· ὅτι γε add. 333 : s.l. m. rec. a.T.8.14} οὐκ εἰσίν ἐλάττονες τοῦ δ καὶ ιϛ καὶ ιϛ {τοῦ θ ιβ ιϛ, ὁ θ καὶ ιϛ 333 : καὶ ιϛ s.l. m. rec. α.T.8.14}. οὐδὲ γάρ εἰσίν πυθμένες τῶν ἡμιολίων· ἢδον γάρ, ὁ γ {τρίτος 2483 P.II.3} τοῦ β ἡμιολίος ἐστίν. πρὸς οὐ λέγομεν ὅτι δύο μὲν ἐφεξῆς ἐσχάτοις θεομίλους εὔφησκονται, ὡς τὰ β καὶ τὰ {τὰ om. 333} γ. τρεῖς δὲ ἐφεξῆς {ἐφεξῆς om. a.U.9.7 28.35 P.R.II.3} ἐλάττονες τοῦ δ καὶ τοῦ τοῦ om. 1 620 89 198 {θού τοῦ δ καὶ θ 333} οὐδέποτε. ὡσαύτως δὲ add. 28.35 482 καὶ δύο μὲν ἐπιτρίτους, τὸν γ καὶ τὸν δ, εὑρεῖν ἐστί, τρεῖς δὲ ἐφεξῆς ὑποκάτω τοῦ θ καὶ τοῦ {τοῦ om. 37} ιβ καὶ τοῦ ιϛ ἀδύνατον. καὶ ἐπὶ τῶν ἀλλῶν ὡσαύτως· εἰκότως οὖν οὗτοι οὐ {οὗτοι καὶ 89 198 : οὐ om. 76 174 333 : οὐ del. m. rec. a.T.8.14} τοὺς άκρους τετραγώνους ἔχουσιν.

Marc. gr. Z. 333: ἐκ τῶν ἐπιμερῶν κατ' εὐθεῖαν ληφθέντων γεννῶνται οἱ πολλαπλασιεπιμερεῖς, ἀντεστραμμένως δὲ οἱ ἑτερογενεῖς ἐπιμερεῖς, ἤγουν ἑτεροειδεῖς· τὸ γὰρ ἐπιδιμερὲς καὶ τὸ ἐπιτριμερὲς καὶ τὸ ἐπιτετραμερὲς καὶ τὸ ἐπιπενταέβδομον ὁμογενῆ μέν εἰσιν ὡς πάντα ἐπιμερῆ, ἑτεροειδῆ δέ· ἕτερον γάρ ἐστι τὸ ἐπιδιμερὲς τοῦ ἐπιτριμεροῦς, καὶ τοῦτο τοῦ ἐπιπενταέβδομου extr. e Rec. IV add. in fine

C.2. Recension II

The text displays substantial variants Hoche did not record; they are shared by all witnesses unless otherwise stated. Singular readings are recorded within the text; recurrent clusters of witnesses are identified by the same colour.

Berol. Phillipps 1549, ff. 84v-88r; Cizensis 67, ff. 35v-36r; Oxon. Lincoln gr. 33, f. 110r; Par. gr. 2377, f. 87v-88r; Scorial. Y.I.12, f. 45r-v; Vat. gr. 1411, f. 60r

<ρ νς. διαζευχθεῖσαι σχέσεις εἰσίν αἰ τε κατ’ ὀρθότητα τῶν προτέρων ἐκκειμένων ἀπογεννώμεναι καὶ αἰ {οι om. 67} κατὰ ἀναστροφὴν· διεξεύχεται γάρ τοῦ ὀρθοῦ τὸ ἀνεστραμμένον· ἐφ’ ἑκάστης οὖν φησὶ σχέσεως οἱ άκροι πάντως {ἐς 1549 2377} εἰσί τετράγωνοι, δι’ ἥν αἰτίαν ἔστεντον ὁ υπόλογος ἐν πάσαις ταῖς κατ’ ὀρθότητα ἐκκειμέναις ταῖς κατ’ ἀντιστροφήν, τετράγωνος ὢν ὁ αὐτὸς μένει, τούτεστιν ὁ υπόλογος ἐν πάσαις ταῖς ἐκθέσεις ταῖς κατ’ ὀρθότητα ἐκκειμέναις ταῖς κατ’ ἀντιστροφήν, τετράγωνος ὡν ὁ αὐτὸς μένει, τούτεστιν υπόλογος, οἷον ἐν ἡμιολίῳ σχέσει ορθῶς ἐκκειμένη {ἐνοι
67} ἦσαν ὁ δ ς θ, καὶ πάντως ὑπόλογος ἦν ἐν τούτοις ὁ δ (τοῦτον γάρ φησιν ἐσχάτον)· ἀντιστραφέντες [–στρέφοντες 33] δ’ ἦσαν οὕτως θ δ, καὶ πάλιν ὁ δ, εἰ καὶ οὕτως τε- τακται, ἀλλὰ πάλιν ὑπόλογος ἦστι πρὸς τὸν ϛ· ὁ δὲ πρῶτος, ἦγουν {ήτοι 33} οἱ μέγιστοι πρόλογοι, εἰ ἀπὸ τοῦτον ἄρχοιτο δηλαδή {δηλονότι 33} ἡ τῆς ἐξῆς σχέσεως πρόβα- σις, ὑπόλογος γίνεται ἐν ἕκεινη. οἷον ἐστώ ἡ ῥηθεῖσα ἡμιόλιος σχέσις ἀντεστραμμένη, ἦγουν {οἷον 33} θ ϛ δ· ἐκ ταύτης γίνεται ἄλλη σχέσις τῆς ἑξῆς πρόβασις, ἐπεί οὗν ὃ ἦν ἐν μὲν ἐκείνοις πρόλογος ἦν ὁ θ καὶ μέγιστος {–ον 1549 2377 Υ.Ι.12 1411}, ἐν δὲ τούτοις καὶ ὑπόλογος καὶ ἐλάχιστος, πάντες δὲ, φησίν, οἱ ἄκροι τετράγωνοι· ἐνδέκειθαι γάρ τῶν ἡμιο- λίων, τοῦ δ καὶ θ {δ ϛ θ 33}, δ καὶ θ τετράγωνοι {–ος 67}· ἐπεὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιϛ καὶ ιϛ, δ καὶ θ· ἐνδέκειθαι γάρ τῶν ἡμιο- λίων, τοῦ δ καὶ θ {δ ϛ θ 33}, ἐδείχθη γάρ τῶν ἡμιο- λίων, τοῦ δ καὶ θ {δ ϛ θ 33}, δ καὶ θ τετράγωνοι {–ος 67}· τοῦτο δὲ, φησίν, οἱ ἄκροι τετράγωνοι· ἐνδέκειθαι γάρ τῶν ἡμιο- λίων, τοῦ δ καὶ θ {δ ϛ θ 33}, δ καὶ θ τετράγωνοι {–ος 67}· ἐπεὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιϛ καὶ ιϛ, δ καὶ θ· ἐνδέκειθαι γάρ τῶν ἡμιο- λίων, τοῦ δ καὶ θ {δ ϛ θ 33}, δ καὶ θ τετράγωνοι {–ος 67}· ἐπεὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιϛ καὶ ιϛ, δ καὶ θ· ἐνδέκειθαι γάρ τῶν ἡμιο- λίων, τοῦ δ καὶ θ {δ ϛ θ 33}, δ καὶ θ τετράγωνοι {–ος 67}·
C.4. Recension IV

My own critical text. B and E are the sigla I have assigned to the two branches of the manuscript tradition, and coincide with extant manuscripts. It turns out that E is a recension of B. Segments of text drawn from Ar. are in red; variant readings are in blue.

C.5.1 The Adjunct to the text of the Introductio arithmetica in some manuscripts

Same conventions as for Ar. I.23.15 above.

A

Ambros. B 77 sup., f. 58r; Ambros. G 62 sup., f. 43r-v; Bucur., BAR gr. 620, pp. 224-225; Gott. philol. 66, f. 141r in Rec. I, f. 141r-v (m. rec. κείμενον); Laur. Conv. Soppr. 30, f. 43r; Laur. Plut. 32 These statements are proved in Acerbi, “La tradition” (cit. n. 27).
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28.35, f. 31v; Laur. Plut. 58.29, f. 153r; Marc. gr. Z. 317, f. 23v; Marc. gr. Z. 316, f. 61r-v; Marc. gr. Z. 318, f. 21r; Marc. gr. Z. 319, f. 39r; Marc. gr. Z. 320, f. 52v; Marc. gr. Z. 330, f. 302r; Marc. gr. Z. 333, f. 61v; Marc. gr. Z. 514, f. 24v; Marc. gr. Z. 595, f. 32v; Monac. gr. 482, f. 133r; Mutin. a.U.9.7, f. 38v (marg.); Neapol. III.C.6, f. 32r; Oxon. Bodl. Holkham 71, f. 278v; Oxon. Bodl. Laud. gr. 44, p. 90; Par. Coislin 174, f. 102v (marg. Rec. I); Par. gr. 2373, f. 17r; Par. gr. 2376, f. 78v; Par. gr. 2762, f. 41r-v; Par. gr. 2480, pp. 343-345, 346 (in Rec. I); Par. gr. 2531, f. 80v (marg.); Par. suppl. gr. 450, f. 35r; Roma, Arch. S. Paolo 24C, f. 55r marg. Rec. I; Scorial. Σ.II.15, f. 171v; Scorial. Χ.I.9, f. 47rv; Vat. gr. 1026, ff. 63v–65r (m. rec.); Vat. gr. 1040, f. 20v; Vat. gr. 1051, f. 20v (m. rec.); Vat. Ottob. gr. 310, f. 142r; Vindob. phil. gr. 62, f. 24r

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἄλληλαις γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσης εἰς τὸν ἐλάττονα τῆς γενομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν.

Ambros. B 77 sup. γενομένης | γεννηθ– | Ambros. G 62 sup.: marg. μέχρι τούτου εὑρίσκεται τὸ πρῶτον βιβλίον τὸ δ’ ἄλλο περιττόν | ἐπαλλήλως | om. ὁ (ult.) | Laur. Conv. Suppr. 30: om. ὁ αὐτός | Laur. Plut. 28.35: μὲν τοῖς | ἐπαλλήλως | om. ὁ (ult.) | Marc. gr. Z. 316 & Scorial. X.I.9: γεννωμένης μεταβαίνει | Marc. gr. Z. 317: om. ἐκκειμένων | Marc. gr. Z. 318: om. ὁ αὐτός | Marc. gr. Z. 319: λεγομένων | Marc. gr. Z. 320: ἀλλήλας | καὶ ὁ s.l. m. rec. | μειζῶν | ex ὁ αὐτός fecit ὁ (ult.) | Mutin. a.U.9.7: ἄλληλα(πρωτετέρως) | ἐπαλλήλως | om. ὁ (ult.) | Neapol. III.C.6: om. ἐστιν | Oxon. Bodl. Holkham 71: μὲν τὴν | –γεν– | ubique | om. ὁ (ult.) | Mutin. a.U.9.7: ἄλληλα(πρωτετέρως) | ἐπαλλήλως | om. ὁ (ult.) | Par. gr. 2376: om. ὁ αὐτός | Par. gr. 2762: αὐτόν | Par. gr. 2531: ἀλλήλας | ἀμφοτέραις | Par. suppl. gr. 450: ἐπαλλήλως | om. ὁ (ult.) | Vat. gr. 1026: om. ὁ (ult.) | Vindob. phil. gr. 62: ἀλλήλας

B

Ambros. I 8 sup., f. 34r; [Athens, EBE 1115, p. 59]; Bonon BU 2263, f. 16v; Hamburg, S-UB, philol. 88, f. 24r; Kharkow, UL, 269-p, 369-c, f. 87v; Leiden, BRU, Periz. Qº 39, f. 23v; Mutin. a.W.3.1, f. 18r; Oxon. Bodl. Selden Supra 20, f. 17r; Par. gr. 2372, ff. 25v-26r; Par. gr. 2479, f. 92v (m. rec. κείμενον); Par. gr. 2483, p. 153; Scorial. Σ.II.15: μὲν τὴν | λεγομένων | –γεν– | ubique | γενομένης | om. ὁ (ult.) | Vat. gr. 1026: om. ὁ (ult.) | Vindob. phil. gr. 62: ἀλλήλας

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντες οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἄλληλαις γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσης εἰς τὸν ἐλάττονα τῆς γενομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν.
Athens, EBE 1115: deficit usque ad metaβαίνει | ἀμφοτέρες (male legit comp. Vat. gr. 256) | Leiden, BRU, Periz. Q 39: om. ἐν | Par. gr. 2372: ἀπογεννώσεις | Par. gr. 2479: om. τῆς (pr.) | Scorial. P.II.3: ἀπὸ γεννέσης | γενομένης

C

Vat. gr. 196, f. 15r

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων {om. ἐκθέσεων} πάντως οἱ άκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἀλλήλαις γενομένων, καὶ ο ἐν πρώτος τοῦ ἀπὸ γε

νώσεως εἰς τὸν ἑλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτὸς ἐστίν.

ex ἐκκειμένων fecit ἐκθέσεων m. rec.

D

Hamburg, S-UB, philol. 89, p. 368 (marg.); Par. gr. 2374, f. 21v; Scorial. Y.III.12, f. 21r; Vat. gr. 198, f. 19r (marg. keyed to the first word of Ar. I.23.15)

ἐπὶ πασῶν {om. μέντοι} τῶν ἐκκειμένων σχέσεων πάντως μὲν οἱ άκροι τετράγωνοι εἰ-

σιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἀλλήλαις μηκυνθεῖσων, καὶ ο ἐν πρώτος τῆς ἀπογε

νώσης εἰς τὸν ἑλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ ο μείζων τετράγωνος ὁ αὐτὸς ἐστίν.

Par.gr.2374: ἐκθέσεων | πάντες | Scorial. Y.III.12: [[ἐλήφθησαν]] ante ἐπ’ ἀλλήλαις | des. μηκυνομένων

E

Berol. Phillipps 1549, f. 84v; Bucur., BAR gr. 520, p. 118; Erlangen, UL A.8, ff. 38v-39r; Cizensis 67, f. 51v; Monac. gr. 76, f. 248r; Oxon. Lincoln gr. 33, f. 110r; Par. gr. 2377, f. 84v; Vat. gr. 1411, f. 76v

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων θέσεων πάντως μὲν οἱ άκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἀλλήλαις γινομένων, καὶ ο μὲν πρώτος τῆς ἀπογε

νώσης εἰς τὸν ἑλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτὸς ἐστίν.

Bucur., BAR gr. 520: γεννηθήσας | Cizensis 67: ἐλάχιστον | Erlangen, UL A.8: γενομένης | Par. gr. 2377: γενομένης

F

Ambros. P 121 sup., f. 130v; Guelf. 36 Gud. gr., f. 17v; Leiden, BRU, BPG 74G, f. 25r; Marc. gr. Z. 309, f. 193v; Marc. gr. Z. 592, f. 37r; Neapol. III.C.1, f. 20v; Norimb. Cent. V app. 36, f. 18v; Par. Coislin 174, f. 58v; Par. gr. 2107, f. 85r; Par. gr. 2450, f. 111v; Vat. Barb. gr. 273, f. 122v; Vindob. phil. gr. 220, f. 62r

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ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἀλλήλας γενομένων, καὶ ο ἡ μὲν πρῶτος τῆς ἀπογεννώσης εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθεῖσαι ὁ ἐσχατος καὶ μείζων τετράγωνοι ἴσα εἰσίν.

Ambros. P 121 sup.: καὶ ὁ μείζων | τετράγωνος ὡσαύτως {ὁ αὐτὸς s.l. m. rec.} ἐστίν | Leiden, BRU, BPG 74G: τετράγωνος ὡσαύτως ἐστίν | Marc. gr. Z. 592: τῶν παρ’ αὐτῶν | Neapol. III.C.1: –γεν– ubique | καὶ [[(οὐ]] μείζων | τετράγωνος ὡσαύτως ἐστίν | Norimb. Cent. V app. 36: om. πάντως | καὶ ὁ μείζων | Par. Coislin 174: om. καὶ (sec.) | τετράγωνος ὡσαύτως ἐστίν | Par. gr. 2107: πλ(ευ) ρ(ῶν) | totum locum fenestrat | Par. gr. 2450: τετράγωνος ὡσαύτως ἐστί | Vat. Barb. gr. 273: τετράγωνος ὡσαύτως ὡσαύτως ἐστίν | Vindob. phil. gr. 220: καὶ ού μείζων | τετράγωνος ὡσαύτως ἐστίν

Glosses to the Adjunct

Guelf. 36 Gud. gr. {marg. μέχρι τούτου εὑρίσκεται τὸ πρῶτον βιβλίον τὸ δ’ ἄλλο περιττόν} ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλ(ευ)ρ(ῶν) τετράγωνοι αὐτῶν ἐπ’ ἀλλήλας {s.l. –ων} γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσης εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθεῖσαι ὁ ἐσχατος καὶ μείζων τετράγωνοι {s.l. –ς} ὡσαύτως εἰσίν {marg. αὐτὸς ἐστὶν}.

C.5.2 The scholium to the Adjunct in Recension II manuscripts

Berol. Phillipps 1549, f. 88r-v; Cizensis 67, f. 36r; Monac. gr. 76, f. 160r-v; Oxon. Lincoln gr. 33, f. 110v; Par. gr. 2377, f. 88r-v; Vat. gr. 1411, f. 60r

<ρ>νθ {om. 76}. <πάντως οἱ ἄκροι> τοὺς μὲν ἄκρους πασῶν τῶν τοιούτων σχέσεων τετραγώνους {–ος sed –ους s.l. 1549} ἔφησεν εἶναι, καὶ ἡμεῖς εἶπομεν δείκνυσθαι τοῦτο ἀναγκαίως καὶ παρὰ τῷ γεωμέτρῃ. τοὺς δὲ μέσους φησιν ἐκ τῶν πλευρῶν ἑνὸς πλευρᾶς ἐπὶ τὴν τοῦ ἑτέρου. δείκνυσι γὰρ καὶ περὶ τούτου ὁ γεωμέτρης ὅτι δύο μεταξὺ τετραγώνων αριθμῶν {ἀ. τ. 33} εἷς ἀνάλογον {–ος 67} ἐμπίπτει, καὶ οὗτος ὑπὸ τῶν πλευρῶν τῶν τετραγώνων πολλαπλασιαζομένων ἐπ’ ἀλλήλας γίνεται.

C.6. The scholia in Matrit. 4678, f. 25v; cf. Kharkow, UL, 269-p, 369-c, f. 87v [a], Laur. Conv. Soppr. 30, ff. 42v-43r [a], Marc. gr. Z. 309, f. 194r [ab], Oxon. Bodl. Holkham 71, f. 278r [a], Par. gr. 2483, p. 149 [a], Scorial. P.II.3, f. 432r-v [a], Vat. gr. 1026, ff. 63v-64r [ab]

a. ἐφ’ ὃν ἔξθετο σχέσεων ἐν τοῖς ὑποδείγμασιν, ἐὰν τάξωμεν πρῶτον τὴν ἡμιολίαν, εἶτα τὴν ἐπίτριτον καὶ ἐξῆς τὸν ἐπιτέταρτον, εὐρήσομεν {–σαμ– 30} ὅτι ὁ μὲν τῆς ἡμιολίας
πρῶτος τετράγωνος ἐφ’ έαυτοῦ μένει, ὁ δὲ ἐσχατός τετράγωνος μεταβαίνων εἰς τὸν ἐπιτρίτον ἐλάττονα ὡντα τοῦ ἡμιολίου πρῶτος αὐτοῦ γίνεται. καὶ πάλιν αὐτὸς μὲν ὁ πρῶτος τοῦ ἐπιτρίτου οὐ μετέρχεται εἰς τὸν ἐπιτέταρτον, {usque huc legi nequit 71} ὁ δὲ ἐσχατός τοῦ ἐπιτρίτου μετερχόμενος πρῶτος γίνεται εἰς ὑπετέταρτον ἐλάττονα {–ον 4678} ὡντος τοῦ ἐπιτρίτου.

Kharkow, UL, 269-p: lemma ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν | Par. gr. 2483: lemma ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν | marg. σημείωσον ὅτι ξένου | marg. οὐκ ἐστι τοῦ Φιλοπόνου m. rec. | Kharkow, UL, 269-p & Laur. Conv. Soppr. 30 & Oxon. Bodl. Holkham 71 & Par. gr. 2483 & Scorial. P.II.3: ἡμιόλιος masc. ubique | om. τὴν (sec.) | ἐφ’ ἐαυτὸν (ἐαυτοῦ 30) | ἐλάττω | om. μετερχόμενος | Marc. gr. Z. 309: ἐλάττω | om. ὁ πρῶτος | Vat. gr. 1026: om. ὁ (ter.)

b. <πάντως δὲ οἱ ἄκροι τετράγωνοι.> οὐ πάντες τῶν ἔχοντων ὁμοίως τὰς σχέσεις, ἀλλὰ τῶν ἐκκειμένων ἐνταῦθα. τὸ μέντοι “τοὺς πρῶτους μὴ μεταβάινειν τοὺς δὲ ἐσχάτους μεταβαίνειν εἰς τοὺς ἐλάττους” καὶ ἐπὶ τῶν μὴ ἐχόντων τοὺς <ἄκρους> τετραγώνους συμβαίνει, ἐξεθέμεθα σαφήνειας χάριν ὑποδείγματα ἐπὶ διαγραμμάτων καὶ τῶν ἐχόντων τοὺς ἄκρους τετραγώνους καὶ τῶν μὴ ἐχόντων. οἷον τῶν μὲν ἐχόντων ταῦτα: *** τῶν δὲ μὴ ἐχόντων πάλιν ταῦτα: η, ιβ, ιϛ, *ιϛ* κ κε | λβ μ ν, ν ξ οβ | Vat. gr. 1026: πάντων δὲ τῶν | ἐξεθέμεθα σαφήνειας | τάδε δὲ τῶν | τάδε dein des.

Glosses to the Adjunct

Marc. gr. Z. 309, f. 193r
ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. ἤγουν τῆς μονάδος καὶ δυάδος} καὶ ἄφ’ ἢς ἀμφότεραι, ὁ μὲν ἐσχατὸς τετράγωνος {s.l. ὁ δ} ὁ αὐτὸς μένει, ὁ δὲ πρῶτος {s.l. τετράγωνος πρῶτος η μονάς} εἰς τὸν ἐλάττωνα {s.l. τὸν θ· οὗτος γὰρ ἡμιόλιος} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Vat. gr. 1026, f. 63v
ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. τὴν μονάδα καὶ δυάδα καὶ τριάδα καὶ τετράδα τὰς} ἐκ τῶν γ’ μονάδων γεννηθεὶσας ... τῆς ἀντιστροφῆς αὐτῶν [...] καὶ ἄφ’ ἢς ἀμφότεραι, ὁ μὲν ἐσχατὸς τετράγωνος {s.l. ὁ δ} ὁ αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττωνα {s.l. τὸν θ· οὗτος γὰρ ἡμιόλιος, ὁ δ’ ἡμιόλιος λόγος ἐλάττων} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

C.7. Glosses and scholia in Monac. gr. 482, f. 132v-133r, and Laur. Plut. 28.35, f. 31r-v [only the glosses]

Signs anchor the scholia on the underlined syntagms. The quotation of Ar. I.23.15 is in red. Cf. Texts C.8a and C.9a for the readings in blue.
Ar. I.23.15

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. ἀναλογιῶν} καὶ ἀφ’ ἢς {s.l. ἀναλογίας} ἀμφότεραι, οὐ μὲν ἐσχάτος {s.l. οἱ ἐλάττων ἦγουν διὸ ἄκροι} τετράγωνον οὗ αὐτὸς μένει, οὐ δὲ πρῶτος {s.l. ήγουν μείζον} εἰς τὸν ἐλάττονα μεταβαίνει, πάντως {s.l. ήγουν ἡλίαν ἀνάγκης} δὲ οἱ ἄκροι τετράγωνοι {s.l. σignum quadratum}.

a. διαζευχθείσας λέγει τὴν μονάδα καὶ δυάδα καὶ τετράδα τὰς ἐκ τῶν τριῶν μονάδων γεννηθείσας κατὰ τὸ πρόσταγμα τὸ λέγον “πρῶτον πρώτῳ ἴσον ποίησον”, καὶ πάλιν τὰς γεννηθείσας ἀπ’ αὐτῶν τῆς τε μονάδος καὶ τριάδος {lege δυάδος} καὶ τετράδος, αἰτινές εἰσι μία, τρεῖς, θ, καὶ τὰς ἐκ τῆς ἀντιστροφῆς αὐτῶν τῶν δ, β, α, αἰτινές εἰσι δ, ς, θ. ἐπὶ πασῶν οὖν τούτων τῶν διαζεύξεων καὶ ἀφ’ ἢς πρώτης διαζεύξεως, τούτεστι τῶν τριῶν μονάδων, ὁ μὲν ἔσχατος τετράγωνος (οἷον ὁ δ) ὁ αὐτὸς μένει, ὁ δὲ πρῶτος (ητὶ ἡ μονάς-καὶ γὰρ αὐτὴ πάντα δυνάμει) εἰς τὸν ἐλάττονα μεταβαίνει, ἦτοι τὸν θ. ὁ γὰρ θ ἡμιόλιος, ὁ δὲ ἡμιόλιος λόγος ἐλάττων τοῦ διπλασίου, ὥσπερ καὶ ὁ ὀπίτριτος τοῦ ἐλάττονος.

b. ἐδείχθη παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὀσὶ πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσί.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν {s.l. τῶν τετραγώνων} ἐπ’ ἀλλήλως γενομένων, καὶ ὁ μὲν πρῶτος {s.l. μείζων ὅρος} τῆς ἀπογεννώσης {s.l. ἐκθέσεως} εἰς τὸν ἐλάττονα τῆς γεννηθείσης, ἤτοι τὸν ἐλάττων οὗ ἐσχάτος τετράγωνος αὐτὸς ἐστιν.

C.8. Four scholia in Ambros. G 62 sup., ff. 65v-66v [abcd], Marc. gr. Z. 309, ff. 193v-194r [ab], Marc. gr. Z. 316, f. 61v-v [c], Marc. gr. Z. 318, ff. 20v-21r [abc], Marc. gr. Z. 319, ff. 39v-40r [bc], Marc. gr. Z. 333, f. 61v [a{part.}c], Mutin. a.T.8.14, f. 27v [m. rec.; a{part.}c], Scorial. X.I.9, ff. 46v-47r [c]

In items a. and d., the quotation of Ar. I.23.15 is in red. Cf. Texts C.7a and C.9a for the first reading in blue in item a. In item d., integrations in lacuna, some of which exempli gratia, are enclosed in brackets. Because of its length, I have introduced paragraphs in item d.

a. tit. {τοῦ} φιλοσόφου Ἐφεσίου σχόλιον εἰς τὸ ῥητὸν τὸ *** 62 : τοῦ φιλοσόφου Ἐφεσίου σχόλιον 318 διαζευχθείσας λέγει τὴν μονάδα καὶ δυάδα καὶ τετράδα τὰς ἐκ τῶν τριῶν μονάδων γεννηθείσας κατὰ τὸ πρόσταγμα τὸ λέγον “πρῶτον πρώτῳ ἴσον ποίησον”, καὶ πάλιν τὰς γεννηθείσας ἀπ’ αὐτῆς τῆς μονάδος καὶ δυάδος καὶ τετράδος, αἰτινές εἰσιν α γ θ, καὶ τὰς εξ ἀναστροφῆς αὐτῶν τῆς δ β α, αἰτινές εἰσι δ ς θ {καὶ τὰς — δ ς θ om. 62 318}. ἐπὶ πασῶν
οὖν τούτων τῶν διαζεύξεων καὶ ἄρ’ ἢς πρώτης διαζεύξεως, τούτεστι τῶν τριῶν μονά-
δων, ο μὲν ἔσχατος τετράγωνος (οἷον ο δ) ο αὐτός μένει, ο δὲ πρῶτος (PerPixel η μονάς· καὶ
gάρ αὕτη πάντα δυνάμει) εἰς τὸν ἐλάττωνα μεταβαίνει, ήτοι τὸν θ· ὁ γὰρ θ ἡμιόλιος, ο δ’
ἡμιόλιος λόγος ἐλάττων του διπλασίου, ὥσπερ καὶ ο (PerPixel om. 318) ἐπίτριτος του ἡμιολίου.

Marc. gr. Z. 309: om. tit. | ποιῆσαι | ἀπ’ αὐτῶν τῆς τε μονάδος | Marc. gr. Z. 333: om. tit. | καὶ
tὴν δυ. καὶ τὴν τε. | des. ποιῆσαι (sic) dein inc. c | Mutin. a.T.8.14: tit. εἰς τὸ ἐπὶ πασῶν δὲ τῶν
dιαζευχθεῖσων ἐξήγησις Εὐστρατίου Νικαίας | καὶ τὴν δυ. καὶ τὴν te. | des. ποίησον dein inc. c
b. tit. εἰς τὸ αὐτὸ κρείττων ἐξήγησις 309: εἰς τὸ αὐτὸ του Δυρραχίου 318 : σχόλιον του
Δυρραχίου 319

ἐφησαν ἀνωτέρω ὅτι ληφθῆσονται {–σεται 309 319} ἡμῖν “ἀπαντα τάς τής ἀνισότητος
eἰδή” ἐκ τινων προσταγμάτων. ἐπάγει τοίνυν παρακολούθημα τι {–ματα 309} των δια-
ζευγνυμένων καὶ ἀπογεννωμένων ύπό {ἀπὸ 309 319} τινος σχέσεως ὀρθῶς {ὡς ἡμιολίους 319}
κειμένης ἢ ἀντιστρόφως, καὶ φησιν ὅτι “ἐπί πασῶν τῶν διαζευχθεῖσων καὶ ἄρ’ ἢς”
sχέσεως ἀπετελέσθησαν ο μὲν ἔσχατος ἀριθμὸς τῶν ἀπογεννωμένων

εἰς τῆς πρώτης {ἐκείνης add. 309 319} σχέσεως τετράγωνων καὶ {τε καὶ ο 319} αὐτός

εὑρεθήσεται, ὁ δὲ πρῶτος ἀριθμὸς ἄρ’ ἢς {σχέσεως add. 319} αἱ διαζευχθεῖσαι ἔσχατος ἀριθμὸς τῶν ἀπογεννωμένων
τῶν ἀπογεννηθεισῶν {–εἰς 309} ἀπογεννηθεισῶν μὲν ἐλάττων εὑρίσκεται ἐν τῇ εξ αὐτῆς
{–οὐ 309} διαζευχθείσης, καὶ προκείσθω ἡμῖν σχέσις ἀριθμεῖσθαι {ἡ ἡμιόλιος m. rec. 318 : θ ἡμιόλιος 319} θ δ’
ποίησον οὖν πρῶτον πρώτῳ ἴσον, δεύτερον πρώτῳ ἴσον καὶ δευτέρῳ ἅμα {πρώτῳ ἅμα καὶ δευτέρῳ 309 : πρώτῳ ἅμα καὶ δευτέρῳ 319}, καὶ τρίτον πρῶτῳ ἅμα καὶ δυτίκα δευτέρῳ {ἄμα 319} εἰς τὸ αὐτὸ κρείττων ἐξήγησις 318 : Εὐστρατίου τοῦ Νικαίας

καὶ τὴν δυ. καὶ τὴν τε. | des. ποίησον dein inc.
πρώτης γεγονυῖα τῆς {τῆς om. 316 319 X.I.9} ἰσότητος, ὁρθῶς μὲν κειμένη τὴν α {τῶν πρῶτον 319} γ θρηματισμὸν {–σίαν 316 319 X.I.9} σχέσιν ἀπεκύησεν ἀναστραφείσα δὲ τὴν δ θ ἰσότητος. ἐκείνη μὲν οὖν ἀφ’ θ ἀμφότεραι, αὐτὰ δὲ {δὲ om. 319} αἱ διαζευχθέ− 

πρώτης γεγονυῖα τῆς {τῆς om. 316 319 Χ.I.9} ἰσότητος, ὀρθῶς μὲν κειμένη τὴν α {τῶν πρῶτον 319} γ θ τριπλασίας {–σίαν 316 319 Χ.I.9} σχέσιν ἀπεκύησεν ἀναστραφεῖσα δὲ τὴν δ ϛ ἠμιολίαν. ἐκείνη μὲν οὖν ἀφ’ ἧς ἀμφότεραι, αὐτίκα δὲ {δέ om. 319} γεννῶσα ληφθῇ, τὸν lac. 62 μὲν ἔσχατον ὁ ἡμιόλιον τὸ τριπλάσιον· οὗτοι γὰρ ὑστερογενεστέροι τῆς ἀνισότητος τῆς σχέσεως, τὸ δὲ αὐτὸ τοῦτο καὶ {καὶ om. 316 319 X.I.9} ταῖς διεζευγμέναις συμβαίνει, ὡστε γὰρ τῷ παρόντι {ἀρκεῖ — παρόντι om. 333 α.T.8.14} ὁ πρῶτος τῶν ὅρων ὁ δ’ ἐλάττων ἐσχάτου τῆς σχέσεως, τὸ δὲ αὐτὸ τοῦτο καὶ {καὶ om. 316 319 Χ.I.9} ταῖς διεζευγμέναις συμβαίνει, ὡστε γὰρ τῷ παρόντι {ἀρκεῖ — παρόντι om. 333 α.T.8.14} γίνεται, εἴθ’ ὁ μείζων εἴθ’ {εἴτε bis 333 α.T.8.14} ἐσχάτου τῆς σχέσεως. τὸ δὲ αὐτὸ τοῦτο καὶ {καὶ om. 316 319 Χ.I.9} ταῖς διεζευγμέναις συμβαίνει, ὡστε γὰρ τῷ παρόντι {ἀρκεῖ — παρόντι om. 333 α.T.8.14} γίνεται, εἴθ’ ὁ μείζων εἴθ’ {εἴτε bis 333 α.T.8.14} ἐσχάτου τῆς σχέσεως. 

Marc. gr. Z. 333 & Mutin. a.T.8.14: a dein tit. ἄλλως ἡ ἀρχή | τὸν [ἱμίλιον a.T.8.14] τριπλάσιον τῶν ἠμιλίων· οὗτοι γὰρ (ὁ ἠμιλιοῦς λέγω καὶ ὁ τριπλάσιος) ύστερογένεστεροι

d. εἰς τὸ αὐτὸ Νικόλαον κριτοῦ τοῦ Δισυπάτου
eiπόν ο ὁ ὁνομαστικά “ἀπό της ἰσότητος πρώτης οἷον μητρὸς τινος καὶ ῥίζης γεννᾶσθαι τα ποικίλα εἴδη τῆς ἀνισότητος” καὶ παραδόσεως κανόνα τοῦτον ποιητικὸν τὸν λέγοντα “πρῶτον πρῶτον ἰσούς ποίησον, δεύτερον πρῶτῳ ἅμα καὶ δευτέρῳ, τρίτον ἰσούς πρῶτῳ καὶ τρίτῳ καὶ δυσενθέρεος”, εξέθετο παραδειγματικῶς τρεῖς μονάδας, καὶ κατὰ τὸν πρώτης γεγονυῖα τῆς {τῆς om. 316 319 X.I.9} ἰσότητος, ὁρθῶς μὲν κειμένη τὴν α {τῶν πρῶτον 319} γ θρηματισμὸν {–σίαν 316 319 X.I.9} σχέσιν ἀπεκύησεν ἀναστραφείσα δὲ τὴν δ θ ἰσότητος. ἐκείνη μὲν οὖν ἀφ’ θ ἀμφότεραι, αὐτίκα δὲ {δὲ om. 319} αἱ διαζευχθέ−
παραδοθέντα κανόνα ἑδειξεν ἕξ αὐτῶν γεννᾶσθαι πρῶτον τὸν ἀ β δ διπλασίαν λόγον· τούτῳ (ήγουν δ β α) ἀποτελείσθαι τὸν δ θ ἡμιόλιον, ὡς λόγους καὶ ἀς σχέσεις διεξευγμένους καλεί, κατὰ τὸ ἐκεῖ δικαίων διὰ τὸ ἐτερογενές· ἢ μὲν γὰρ τριπλάσιος ὑπὸ τὴν πολυπλασίαν ἀνάγεται ἢ δὲ ἡμιόλιος ἐπὶ τὴν ἐπιμόριον, τὰ δὲ ἐτερογενὲς διεξεύκται πάντα καὶ ἀπηλλοτρίωνται ἀλλήλων ἐκατέραν (σχέσιν). τούτων δὴ τῶν διεξευγμένων ὀρθομενών τε καὶ ἀναστραφμένων ἐδίδαξε γίνεσθαι γεννητικὴν ἐτέρων διεξευγμένων σχέσεων, ὡστε τὴν μὲν α θ δ κε διπλασιεφήμισυν ἐν δὲ τῇ ἀνεστραμμένῃ τῇ δ α ἡμιολίαν ἐν μὲν τῇ κατ’ ὄρι(θον) θεσθαι ἀπογεννᾶν τῇ δ θ τῇ ἑπτῆσθαι ἀπὸ μὲν ἑτερων τῆς δ διπλασίας, ἐν δὲ τῇ ἐμοὶ δοκοῦν ἑτερογενές. ἡ μὲν γὰρ τριπλάσιος ὑπὸ τὴν πολυπλασίαν ἀνάγεται ἡ δὲ ἡμιόλιος ἐπὶ τὴν ἐπιμόριον, τὰ δὲ ἑτερογενὲς διέζευκται πάντα καὶ ἀπηλλοτρίωνται ἀλλήλων {σχέσιν}· τούτων δὴ τῶν διεξευγμένων ὀρθομενών τε καὶ ἀναστραφμένων ἐδίδαξε γίνεσθαι γεννητικὴν ἑτέρων διεξευγμένων σχέσεων. ταύτῃ οὖν τῇ μεθόδῳ χρησάμενος ὁ Νικόμαχος εἰς παράστασιν τῆς προχωρήσεως τῆς ἀνισότητος καὶ δείξας παραδειγματικῶς – ἐπὶ τῆς ἐν ἡμιολίῳ λόγῳ τῷ δ ϛ θ σχέσεως τῷ γεννωμένῳ ἀπὸ τοῦ διπλασίου τοῦ δ β α – συνίστασθαι ἐκ ταύτης ἀπὸ μὲν τοῦ ἐλάττονος ὅρου τοῦ δ – ὅτε καὶ ὀρθῶς {κεῖται} – τὸν {δ ι κ}ε διπλασιεφήμισυν, ἐκ δὲ τοῦ μείζονος ὅρου τοῦ θ – ὅτε καὶ ἀνέστραπται – γεννᾶσθαι τὸν θ ιε κε ἐπιδιμερῆ ἢ ἐπιδίτριτον σχέσιν. ταύτῃ οὖν τῇ μεθόδῳ χρησάμενος ὁ Νικόμαχος εἰς παράστασιν τῆς προχωρήσεως τῆς ἀνισότητος καὶ δείξας παραδειγματικῶς – ἐπὶ τῆς ἐν ἡμιολίῳ λόγῳ τῷ δ ϛ θ σχέσεως τῷ γεννωμένῳ ἀπὸ τοῦ διπλασίου τοῦ δ β α – συνίστασθαι ἐκ ταύτης ἀπὸ μὲν τοῦ ἐλάττονος ὅρου τοῦ δ – ὅτε καὶ ὀρθῶς {κεῖται} – τὸν {δ ι κ}ε διπλασιεφήμισυν, ἐκ δὲ τοῦ μείζονος ὅρου τοῦ θ – ὅτε καὶ ἀνέστραπται – γεννᾶσθαι τὸν θ ιε κε ἐπιδιμερῆ ἢ ἐπιδίτριτον σχέσιν.
τὸν αὐτὸν δὴ τρόπον καὶ ἐφ᾽ ἐκατέραν τῶν διαζευγμένων, ὅταν καὶ αὐτὰ δηλονότι ληφθοῦσιν έτέρων γεννητικαὶ ἐν τῇ κατ᾽ ὀρθόν ή κατ᾽ ἀναστροφήν θέσει, τὸ αὑτὸ συνε- νεχθῆσεται, οἷον ἐπὶ τῆς ἐν τριπλασίῳ λόγῳ σχέσεως τῆς α γ θ, ἐν τῇ ἐπὶ τῆς ἐπιτριτείου σχέσεως τῆς α β δ ἢ α γ θ ἢ δ ϛ θ. ἀλλὰ τούτου τὸν λόγον ως κεκομψευμένον μὲν ἐπαινῶ, εἰ δὲ σημαίνει τὴν διάνοιαν τοῦ κειμένου, συνιδεῖν οὐκ ἔχω· ᾖ γὰρ ἄν, εἰ κατὰ τοῦτον τὸν νοῦν ἐξελήφθη, τὸ ῥήτωρ τοῦ κατὰ τὴν δισεπιτριτείου σχέσεως [110 θ'] ἔχειν".
Glosses in Marc. gr. Z. 318

Ar. I.23.15

ἐπὶ πασῶν [s.l. σχέσεων δηλονότι ἐπιμερῶν καὶ πολλαπλασιεπιμορίων] δὲ τῶν διαζευχθείσων [s.l. τῶν προλεχθείσων σχέσεων] καὶ ἀφ’ ἓς [s.l. σχέσεως δηλονότι τῆς ἐπιμορίου] ἀμφότεραι, οἱ μὲν ἐσχατοὶ τετράγωνοι ὁ αὐτὸς μένει, ὁ δὲ πρῶτος [s.l. λόγος] εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοί εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἀλλήλαις γενομένων, καὶ ὁ μὲν πρῶτος [s.l. τουτέστιν ὁ γεννῶν αὐτὸν ἀπαιστοῦ γεννωμένου ἐλάττων τῆς ἀπογεννήσης ἐκ τὸν ἐλάττονα μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἐσχατος καὶ μείζων τετράγωνος ἐστιν.

C.9. Four scholia in Vat. gr. 256, f. 316r-v [abcd], Ambros. I 8 sup., ff. 33r-34r [abcd], and Athens, EBE 1115, pp. 58-59 [abcd], Oxon. Bodl. Selden Supra 20, ff. 16v-17r [bd], Par. gr. 2479, f. 92r (m. rec.) [bd], Vat. gr. 186, f. 243r [a]

In Ambros. I 8 sup. and Vat. gr. 256, the scholia are anchored on the indicated words; in Vat. gr. 186, the reference sign is located at the beginning of Ar. I.23.15; anchoring and reference signs in Athens, EBE 1115 are the same as those of Vat. gr. 256. The quotation of Ar. I.23.15 is in red. Cf. Texts C.7a and C.8a for the reading in blue in Text a.

a. Ἐφεσίου φιλοσόφου σχόλιον

<πασῶν.> διαζευχθείσας λέγει τὴν μονάδα καὶ δυάδα καὶ τετράδα τὰς ἐκ τῶν τριῶν μονάδων γεννηθείσας κατὰ τὸ πρόσταγμα τὸ λέγον “πρῶτον πρώτῳ ἴσον ποίησον”, καὶ πάλιν τὰς γεννηθείσας αὕτων τῆς τριάδος καὶ δυαδος καὶ τετράδος, αὕτης εἰς μία, τρεῖς, θ, καὶ τὰς ἐκ τῆς ἀντιστροφῆς τετράδος, αὕτης εἰς δ ἄν, αὐτίνες εἰς δς θς. ἐπὶ πασῶν οὖν τῶν τετράγων τῶν διαζευχθείσων καὶ ἀφ’ ἓς τῆς ἀπογενέσθαις, τουτέστι τῶν τριῶν μονάδων, ὁ μὲν ἐσχατος τετράγωνος ὁ διπλασιεπιμοριων (οἷον ὁ δ) ὁ αὐτὸς μένει, ὁ δὲ πρῶτος (ἤτοι ἡ μονάς· καὶ γὰρ αὕτη πάντα δυνάμει) εἰς τὸν ἐλάττωνα μεταβαίνει, ἤτοι τὸν θ· ὁ γὰρ ὁ θμίλιος, ὁ δὲ ἡμιόλιος λόγος ἐλάττων τοῦ διπλασίου, ὡσπερ καὶ ὁ ἐπίτριτος τοῦ ἡμιολίου.

Ambros. I 8 sup.: om tit. | –περ καὶ — ἡμιολίου legi nequit | Athens, EBE 1115: δυάδα καὶ τριάδα | Vat. gr. 186: om tit. | ἐννέα | δ β β | τουτέστιν ὁ δ | εἰς τὸν ἐλάττωντα μεταβαίνει

b. <διαζευχθείσων.> διαζευχθείσας λέγει τὰς ἀφ’ ἓς ἡμιολίου τοῦ τοῦ ἡμιολίου τοῦ τοῦ ἡμιολίου (ὁ γὰρ ὁ θμίλιος, ὁ δὲ ἡμιόλιος λόγος ἐλάττων τοῦ διπλασίου, ὡσπερ καὶ ὁ ἐπίτριτος τοῦ ἡμιολίου)

<i>Texts C.7a and C.8a for the reading in blue in Text a.</i>
The textual tradition of Nicomachus’ *Introductio arithmetica*

Épimorión, ósôn eî túchô toû ëmiolióny genoméewn. tô dè “ôph’ ëh àmívôterai” perî aûthês tôs ( tôs om. 20 2479) épimoriónô pherî aî diàzexevêisai genôvassai. fêñsi oûn ëti èn (ën om. 2479) àmívôterai aîs diàzexevêisai eîs èstî kai ë oûthôs (ô add. 20 2479) èsçhatoç (ô te kai add. 20 2479) tetràégwosô, ëôstepr ën th ëdiçèpirôfî th th iê kai th diplasieifîmîsë th ëth i ke (ië 20 2479) eîs èstîn ëôpôs èsçhatoç (ë. ë. 20 2479) th dè thès àpogènînësës thàstas thàs ñchêsës ëmiolión (ëmioliónu 20 2479) prôôtôs tetràégwos (hûgon oûn meîzûn aûthês th th-tóttûn ãâr ànûsërw prôôtôn èsthêkën ë oûkômaçhôs), ëû tôs fêñsi (seq. meîzûn ãôn hic hab. 20 2479) òn tô ëmioló thô dè ë th meîzûn ãôn èthôs th èn èstîn ën tô ñchêsës ën th ëdiçèpirôfî th th iê kai ën thàstî ãâr th ñchêsë kai (kai om. 20 2479) th òh ëthôs ãînestî èn (dè add. 20 2479) th ëmioló ëû meîzûn ãôn (ãînestî 20 2479), kai èptî thàs lôîpôn thà ñchêsës ëûrêsi (th ènàlôgôn).

Oxon. Bodl. Selden Supra 20 & Par. gr. 2479: èôfôtôûn | ëgnènìðesìs ñchêsës | àntesèstramëmënu th tô d kai th 2479 | add. in fine e Rec. ñìaçèvêisë (hûgon ëû ñchêsë aû thèt thèrèn èpèvñû- ñmënu kai aî kàta ànàstrôfô th ñmënu kàta èn ànàstrôfô th àntesèstramëmënu

<tnótnoi.> èdëèîhë parà tô ãwëmètth ëûì ëûn têres àrîthmîin ëlâîñsîtôi ëswî prôs àlllîlous thàs aûthûs lôîgôn èxòntes, oû àkroû aûthûs ñchêprûwûmëîs ènìs.

Ambros I 8 sup.: à tînës vóratûs

To the Adjunct

d. <mnêsoi.> oû ñmësoi phêñi tôs èkèkëmëwn èkèðêsëwë èk tôs ëplewûw aûthûs tôs àkrwôn gî- ñontai ep’ àlllîlais genômëwn. oûñ èpit tôs ëmiolòw tôs dêz th òh mësos ãôn èk tôs ëplewûw tôs àkrwôn gîñontai, toutèstî tôs d kai tôs th- ëplewrà ãâr tôs d ëh duâs, kai ëplewrà tôs th ë triâs, aîtînès ap’ àlllîlais ñînômënu poioûsî tôs mësos tôs ñç dîs ãâr ç, ç. kai èptî tôs àlllûn ëmôtûs ëûrêsi (th àntesèstramëmënu ñchêsë tûta. ëplewrà tôs ke tâ ç, tôs pa tà th- èk tôs ëplewûw tôs dûw tôs pa kai tôs ke (lèwo ìè d’è ñom. 20 1115) th- tûta ãâr aî ëplewûai gîñêntai o ñmëso o me tôs ëplewûw èp’ àlllîlais genômëwn- pëntákis gîr th, me. pàliin ëplewrà tôs ke tâ ç, tôs dè rôc tâ id- tèssariskaidêkaton ãâr tôs rôc tâ id- tèssariskaidêkaton ãâr id, rôc. hî ëplewûa ìn tôs ke (tâ ç èlwû d’è) kai è ëplewrà tôs rôc (lèwo d’è tô id) ñûnëdlônta prôs àlllîla àpòtèlûsû tôs o- pëntákis ãâr tô id gîñontai o.

Oxon. Bodl. Selden Supra 20 & Par. gr. 2479: tô d, tôs ç kai tôs th, kai (kai om. 20) òh ç | ãînestî | tôns ç om. 20 | èp’ àlllîlais | ñes. ëûrêsi (tûta 20)

C.10. Isaak Argyros’ glosses and scholium in Norimb. Cent. V. App. 36, f. 18r

A sign anchors the scholium on the underlined word; the infralinear glosses are also transcribed.

Ar. I.23.15
èptî pàsòw dè tôw ñìaçèvêisìs (s.l. hûgon tôw àpò miàs tînòs ñchêsës àpogènînìðëi- sëwë) kai àph’ ëh àmívôterai (s.l. hûgon dûw), o mun èsçhatoç (s.l. hûgon th èpûlògûs thès
ἀπογεννῆσας τετράγωνος ὁ αὐτὸς [s.l. ἦγουν ὑπόλογος] μένει [s.l. ἦγουν ἐν τῇ μιᾷ δηλονότι], ὁ δὲ πρῶτος [s.l. ἦγουν ὁ πρόλογος] εἰς τὸν ἐλάττονα [s.l. ἦγουν τὸν ὑπόλογον τῆς ἑτέρας] μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

diaξενυθείσας φησὶ τὰς ἀπὸ μιᾶς τινος σχέσεως ἀπογεννηθείσας ἑτέρας β σχέσεις τὴν τε ὑπόλογον κατ’ ὀρθότητα καὶ τὴν κατ’ ἀντιστροφήν, ἢ τοῦτο ἐγενετοὶ ὑπόλογος μένει ἐν τῇ μιᾷ δηλονότι, ὁ δὲ πρῶτος ἐν τῇ ἐλάττωσιν ὑπόλογον τῆς ἑτέρας μεταβάλλει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

diαζευχθείσας φησὶ τὰς ἀπὸ μιᾶς τινος σχέσεως ἀπογεννηθείσας ἑτέρας β σχέσεις τὴν τε δηλονότι κατ’ ὀρθότητα καὶ τὴν κατ’ ἀντιστροφήν, ἢ τοῦτο ἐγενετοὶ ὑπόλογος μένει ἐν τῇ μιᾷ δηλονότι, ὁ δὲ πρῶτος ἐν τῇ ἐλάττωσιν ὑπόλογον τῆς κατ’ ἀντιστροφῆς μεταβάλλει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

σχόλιον {σχόλιος 620 66 482 24C} Θεοδώρου πρωτοκένσωρος ἐξαγωγὸν ἐνταῦθα τετράγωνον τῶν αὐτῶν μένοντα τὸν ἑκατέρον ἐκείνου ἐκ 

C.11. Theodoros/Demetrios Protocensor’s scholium

The distinctive differences between the two versions of the scholium are in red; note, however, the variant reading shared by the family of Roma, Arch. S. Paolo 24C and Version b. In Version b, the manuscripts marked in blue only contain Rec. I.
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C.12. Glosses, and scholia to the Adjunct, in Hamburg, S-UB, philol. 88, ff. 23v-24r, and Vat. gr. 2387, f. 48v; cf. Marc. gr. Z. 317, f. 23r-v, Marc. gr. Z. 595, f. 32r-v, Par. gr. 2762, f. 40v

A sign in the Hamburg manuscript anchors each scholium on the underlined word; the infralinear glosses in Hamburg, S-UB, philol. 88 and in Marc. gr. Z. 317 are also transcribed.

Ar. I.23.15
ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν [διαλεχθεισῶν καὶ διαζευχθεισῶν 317 : τοῦ τριπλασίου καὶ τοῦ ἐπιμορίου καὶ τῆς σχέσεως αὐτῆς τῆς διπλασίου marg. 317] καὶ ἀφ’ Ἦς ἀμφότεραι, ὁ μὲν ἐσχάτος τετράγωνος μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα [s.l. κατὰ τὴν σχέσιν τῷ γάρ πρῶτον ὑπόκειται ἡμιολίον, τὸ δεύτερον ἐπίτηρτον 88] μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Adjunct
ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντες οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἀλλήλας γενομένων, καὶ ὁ μὲν πρῶτος [marg. ἤγουν ὁ μεῖζων ὅρος 317] τῆς ἀπογεννώσης εἰς τὸν ἐλάττονα [s.l. κατὰ τὴν σχέσιν 88] τῆς γινομένης [marg. ἤγουν τῆς εξ ἀναστροφῆς γινομένης 317] μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἐσχάτος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν.

Marc. gr. Z. 317 & Marc. gr. Z. 595 & Par. gr. 2762: α β δ | ὑπογεννᾷ | ὡς ὁ ἐσχάτος (cum 317)

b. οἷον ὁ δ καὶ ο ὁ θ εἰσί τετράγωνοι. πλευρὰ γοῦν τοῦ {τῶν 2387} δ εἰσὶ τὰ β, τοῦ {τῶν 2387} δ ἐπὶ πλευρὰ τὰ γ, ἄτινα (ἡγουν τὰ β καὶ γ) μιχθέντα ἐγένοντος μ. δ. γ. γρ. τρις, γ., ὡς καὶ οὔτος τετράγωνος ἐστιν ἀλλ’ ἀνισόπλευρος διὰ τὸ τὴν μίαν πλευρὰν διαφέρειν τῆς ἑτέρας μονάδι. ὡσάυτως ἄτινα (ἡγουν τὰ τρία καὶ τὰ τέσσαρα) μιγνέτα ἐποίησαν τὸν μέσον αὐτῶν. πλευρὰ γοῦν τοῦ θ εἰσὶ τὰ γ, τοῦ δὲ ὡς καὶ τὰ δ. οὔτος τετράγωνος μέσον τοῦ θ. ἤγουν τοῦ ἤβ. τρις γ. ὡς καὶ ὁ θ. ἄτινα (ἡγουν τὰ τρία καὶ τὰ τέσσαρα) μιγνέτα ἐποίησαν τὸν μέσον αὐτῶν. πλευρὰ γοῦν τοῦ θ ὡς καὶ τὰ δ. ἄτινα (ἡγουν τὰ τρία καὶ τὰ τέσσαρα) μιγνέτα ἐποίησαν τὸν μέσον αὐτῶν. πλευρὰ γοῦν τοῦ θ ὡς καὶ τὰ δ. ἄτινα (ἡγουν τὰ τρία καὶ τὰ τέσσαρα) μιγνέτα ἐποίησαν τὸν μέσον αὐτῶν.
C.13. Dishypatos’ and the one from Dirrachios’ (sic) scholia in Vat. gr. 198, f. 19r, and Hamburg, S-UB, philol. 89, pp. 368-369

a. εἰς τοῦτο ὁ μὲν Δισύπατος Νικήτας λέγει ὅτι διαζευχθείσας λέγει ὁ Νικόμαχος τὴν τε ἐξ ὀρθῆς καὶ τὴν ἐξ ἀντεστραμμένης γεννηθείσας ἀπό τῆς μίας καὶ πρώτης σχέσεως. λέγει γοῦν ὃ τε ἐσχάτως οἴονε υπόλογος ὁ καὶ ἀρχικώτερος ὁ αὐτὸς μένει ἐν τῇ ἐξ ὀρθῆς ἀπογεννηθείσῃ σχέσει, δηλαδὴ ἐν τῇ ἐξ ἀντεστραμμένης ἀποκυηθείσῃ σχέσει· γίνεται γάρ ὁ πρόλογος υπόλογος. οἶον ἔστιν α β δ διπλασία σχέσις. ἐκ ταύτης ὀρθῆς μὲν γεννᾶται ἀ γ θ τριπλασία σχέσις, ἀντεστραμμένης δὲ ἂς ἆς ἡμιολία σχέσις. ὥστε ὃ ἐσχάτως καὶ ἐπὶ τῇ τριπλασίᾳ καὶ ἐπὶ τῇ ἡμιολίᾳ ὁ αὐτὸς μένει (ήγουν ὁ α) καὶ ἐν τῷ τριπλασίῳ ας (ήγουν ὁ α). ἐκείνης γὰρ ὁ πρόλογος ἦν ὁ πρόλογος καὶ μείζων τῶν ἄλλων ἀριθμῶν ἐν τῇ διπλασίᾳ καὶ ἐλάσσον ἀριθμός· ἐλάσσος γάρ ὁ τοῦ δ ἂς τοῦ θ.

b. ὁ δὲ Διρραχιοῦ φησίν ὡς σκόπησον τὰς διεζευγμένας σχέσεις ἀπὸ τῆς ας, οἴον ἀπὸ διπλασίου α β δ διπλασίαν α γ θ τριπλασίαν ας (ήγουν ὁ α) καὶ ἐν τῷ τριπλασίῳ ας (ήγουν ὁ α) καὶ ἐν τῇ διπλασίᾳ καὶ ἐπὶ τῇ ἡμιολίᾳ ὁ πρῶτος μένει (ήγουν ὁ δ), ὁ δὲ ἐν τῷ τριπλασίῳ καὶ ἐπὶ τῇ ἡμιολίᾳ ὁ πρῶτος μένει (ήγουν ὁ δ). οἶον ἔστιν τῶν ἄλλων ἀριθμῶν ἐν τῇ διπλασίᾳ καὶ ἐλάσσον ἀριθμός· ἐλάσσος γὰρ ὁ δ τοῦ ϛ ἢ τοῦ θ.

C.14. Glosses and a scholium in Marc. gr. Z. 320, f. 52r-v (Gregoras’ hand), Par. gr. 2450, f. 111r, Vindob. phil. gr. 220, ff. 61r-62r, Neapol. III.C.1, f. 20v, Vat. Barb. gr. 273, f. 122r-v [a]

A sign anchors the scholium on the underlined word in Par. gr. 2450 and Vat. Barb. gr. 273; the infralinear glosses of these two manuscripts and of Vindob. phil. gr. 220 are also transcribed; Neapol. III.C.1 does not have infralinear glosses; all glosses and scholia stem from Marc. gr. Z. 320, which only anchors the first scholium; the sign is also found in the accompanying table (f. 51v), followed by ἐπὶ πασῶν δὲ τῶν διαζευχθεῖσῶν. Quotations of Ar. I.23.15 are in red.

Ar. I.23.15
ἐπὶ πασῶν δὲ τῶν διαζευχθεῖσων {s.l. τῶν προλεχθεῖσων σχέσεων 220 : ἐπιμορίων καὶ ἐπιμερῶν 2450 273 : τῶν προλεχθησῶν σχέσεων 220} καὶ ἄφ’ ἕς {s.l. τῆς πολυπλασίου
2450 273 : non habent 320 220) ἀμφότεραι, ὁ μὲν ἐσχατος τετράγωνος {s.l. ὁ ἄκρος ἐκάτερος 320 2450 273 : ὁ ἄκρος 220} ὁ αὐτὸς μένει {s.l. ἀμετάβατος μένει 320 2450 273 : ἀμετάβατος 220}, ὁ δὲ πρῶτος {s.l. ἤγουν ὁ μείζων λόγος 320 2450 273 : non habent 220} εἰς τὸν ἑλάττονα {s.l. ἤγουν τὸν ἑπιμόριον καὶ τὸν ἑπιμερῆ· ἤττων γὰρ οὗτος τοῦ πολλαπλασίου 320 2450 220 : — ἐπιμερῆ· ἤγουν *τρίτον* 273} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

a. πάσαι αἱ διαζευχθεῖσαι σχέσεις εἰσίν αἱ ἐπιμόριοι καὶ ἐπιμερεῖς· ἢ δὲ ἢ ἀφ’ ἦς {s.l. ἀφ’ ἦς III.C.1} ἀμφότεραι ἐστίν ἡ πολλαπλάσιος, ἢς πάσας ὡς ὁρᾶς ἐξεθέμεθα {ἐν τῷ ἀριστῷ μέρει 320 III.C.1 220}, {inc. alia m. 320} ὡν πάντες οἱ ἄκροι τετράγωνοι, ὁ μὲν οὖν {οὖν om. III.C.1 220} ἐσχατος τετράγωνος (ἡτοι {ἡτοι III.C.1 220} ἐκάτερος τῶν ἄκρων) ὁ αὐτὸς μένει {[s.l. 320]} καὶ οὐ μεταβαίνει ἐκ {ἐκ s.l. 2450} τοῦ εἶναι τετράγωνος. ὁ δὲ πρῶτος ἢτοι ὁ μείζων {[s.l. 320]} εἰς τὸν ἑλάττονα μεταβαίνει {[s.l. 320]}, ἢτοι ἀπὸ τοῦ πολλαπλασίου εἶδους εἰς τὸ ἑπιμόριον, καὶ ἀπὸ τοῦ ἑπιμορίου αὐθίς εἰς τὸ ἑπιμερὲς· ἤττων γὰρ οὗτος τοῦ πολλαπλασίου εἰς τὸ ἑπιμέριον καὶ ἐπιμερῆς. ἤττως δὲ ἡ ἐκάστη σχέσις ὀρθῶς μὲν κειμένη γεννᾷ {γεννᾶται III.C.1 220} ἐπιμόριον, ὥστε ἑκάστη τῆς γεννηθείσης ἑκάστης σχέσεως, ἀναστρεφομένη δὲ {δὲ om. 273} ποιεῖ ἑκάστης σχέσεως ἑλάττων καὶ αὐτῆς οὔσης, ὥστε ἐκ μιᾶς σχέσεως διαζεύγνυνται δύο σχέσεις.

b. (marg. inf. 320 : m. rec. marg. sup. 2450 : post a III.C.1 220) γρ. οἱ μὲν ἔσχατοι τετράγωνοι οἱ αὐτοὶ μένουσι, τουτέστι κατὰ τὴν ποιότητα {ποσότητα III.C.1} τοῦ τετραγωνισμοῦ, καὶ οὐ κατὰ τὴν ποσότητα τῶν ἐν αὐτοῖς μονάδων.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν {s.l. τῶν ἄκρων 320 2450 273 : ἤγουν τῶν ἄκρων 220} ἐπὶ ἀλλήλας γενομένων {s.l. τῶν ἄκρων 320 2450 273 : ἤγουν τῶν ἄκρων 220}, καὶ ὁ μὲν πρῶτος {s.l. ὁ μείζων 320 2450 273 220} τῆς ἐπιμορίας τῆς ἐπιμεροῦς τῆς ἐπίστομης {s.l. ἤγουν τῆς ἐπιμορίου καὶ ἑξῆς τῆς ἐπιμορίους 320 2450 273 220} μεταβαίνει, ἐν ἀμφοτέραις δὲ τοῖς ἐπεξεργασίας ωσαύτως ὡσαύτως {ex ὁ αὐτός fecit m. rec. 320} ἐστίν.

C.15. Glosses and a scholium to the Adjunct by a later hand in Par. gr. 2373, f. 17r

A sign anchors the scholium on the underlined word; the infralinear glosses are also transcribed.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι {s.l. εἰσίν} ἐκ τῶν πλευρῶν αὐτῶν ἐπὶ ἀλλήλας γενομένων {s.l. πολλαπλασιασθεισῶν},
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C.16. Glosses and scholia in Kharkow, UL, 269-p, 369-c, ff. 87v-88r, and in Par. gr. 2483, pp. 149, 152-153; cf. Laur. Conv. Soppr. 30, f. 43r [a], Oxon. Bodl. Holkham 71, f. 278r [a], Scorial. P.II.3, ff. 433v-434r, 436v-437r [ab]

A sign in 2483 anchors the scholium on the underlined word; the infralinear glosses are also transcribed. Quotations of *Ar*. I.23.15 are in red.

*Ar*. I.23.15

> ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. ἤγουν τῶν ἀντεστραμμένων} καὶ ἀφ’ ἧς ἀμφότεραι {s.l. αἱ ἀντεστραμμέναι καὶ αἱ ὀρθῶς}, οἷον ἐξαχθευτεὶς καὶ τῶν ἄκρων τετράγωνοι.

C.1 [2483 P.II.3 only]

C.6a + a. <πάντως δὲ οἱ ἄκροι τετράγωνοι.> οἷον ὡς ἐπὶ παραδείγματος {–ατι 30} ἡμιολίου {–λιοι 30 : ἡμιϊ 71} ἐκκείσθωσαν ὅροι δ ϛ θ, καὶ ἑξῆς ἐπίτριτοι θ ἔσχατον, ὅπως δὲ καὶ ἐάν τὸν πρῶτον τετράγωνον ὄψει πρῶτον ἐν τῇ ἐπιτετάρτῳ σχέσει. ὁμοίως δὲ καὶ ἐὰν τὸν ἐλάττονα τῆς {τὸν 71} ἐπιτετάρτῳ σχέσεως βουληθῇς ἐν τῇ ἐπιτετάρτῳ σχέσει, ὃς ἐλάττων ἐστι {καὶ 2483 Ρ .II.3} τοῦ ἐπιτετάρτου.

To the Adjunct

b. ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων, τοιτετίστι καὶ τῶν ὀρθῶς καὶ ἀντεστραμμένως κειμένων, ἐκθέτου τῷ ἀναλογικῷ καὶ ὀρθῷ καὶ ἀντεστραμμένῳ δ ϛ θ καὶ ἀντεστραμμένῳ {οὗτος} ἡμιολίου {–λιοι 30 : ἡμιϊ 71} ἐκκείσθωσαν ὅροι δ ϛ θ, καὶ ἑξῆς ἐπίτριτοι θ ἔσχατον, ὅπως δὲ καὶ ἐάν τὸν πρῶτον τετράγωνον τῆς {τὸν 71} ἐπιτετάρτῳ σχέσεως βουληθῆσαι ἐν τῇ ἐπιτετάρτῳ σχέσει, ὃς ἐλάττων ἐστι {καὶ 2483 Ρ .II.3} τοῦ ἐπιτετάρτου.
ἐν δύο καὶ τριῶν ἐγένετο ὁ ϛ μέσος, πλευρῶν ὄντων τοῦ δ καὶ θ τῶν δύο ἄκρων τετραγώνων. πάλιν ὡς ἐξ ἀλλής ἀρχής ἢ κειμένη ἀνάλογια ἀντεστραμμένος ή θ ϛ γεννᾶ τὴν θ ιε κε. ὁ πρῶτος οὖν φησὶ τῆς ἀπογεννώσης {–έσεως Ρ.ΙΙ.3} σχέσεως μεταβαίνει εἰς τὸν ἐλάττονα, ἦγουν ο θ γίνεται ἐλάττων μείζων ὧν ἐπὶ τῆς πρώτης. ἐν ἀμφοτέραις δὲ {ἐλάττων — ἐν bis P.ΙΙ.3} ταῖς γεννηθείσαις ἀναλογίαις, δηλονότι ἕκ τε τῆς ὀρθῶς κειμένης καὶ μή, τούτου τής δ ι κε, θ ιε κε, ὑπάρχει ὁ αὐτός καὶ εἷς καὶ ἔσχατος καὶ μείζων, ἦγουν ὁ κε· καὶ ἔσχατος γὰρ κατὰ τὴν {τὴν om. 269-p} τάξιν ἐν ἀμφοτέραις καὶ μείζων ὑπάρχει τῇ ποσότητι {ὁ αὐτός καὶ εἷς δηλαδή add. Ρ.ΙΙ.3 : m. rec. 2483}. Kharkow, UL, 269-p, 369-c & Par. gr. 2483: sicut novum scholium rursus ἀναλογίαις — ποσότητι ὑπάρχει (τῇ δ ι κε | ὑπάρχει ὁ ἔσχατος καὶ μείζων | κατὰ τὴν τάξιν καὶ ἐν | om. τῇ ποσότητι) sed erasit m. rec. 2483 et post ποσότητι add. ὁ αὐτός καὶ εἷς δηλαδή
c. τὰ μὲν λέγεται καὶ ἔστιν, ὡς ὁ ἄνθρωπος· τὰ δὲ οὔτε λέγεται οὔτε ἐστίν, οἷον τὰ μηδαμῇ μηδαμῶς· τὰ δὲ ἔστι μὲν οὐ λέγεται δέ, ὡς τὰ ἐν τῇ Λιβύῃ θηρίᾳ· τὰ δὲ λέγεται μὲν οὔκ ἔστι δέ, ὡς εἰ τὰ εἰποί τὸ τετράγωνον {τρέγωνον 269-p} ἢ τὸ τρίγωνον ἵσον τῷ κύκλῳ κατὰ τὴν χώρησιν, καὶ ἄλλα τοιαῦτα.

C.17. Glosses and scholia in Marc. gr. Z. 316, ff. 60v-61v, Scorial. X.I.9, ff. 46v-47r, 47v-48r, Marc. gr. Z. 319, f. 40r [b]

A sign anchors the scholium on the underlined word; the infralinear glosses are also transcribed.

Ar. I.23.15 ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν καὶ ἄρ’ ἡς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος {s.l. ἦγουν πρόλογος 316} ὁ αὐτός μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι. Marc. gr. Z. 316, f. 60v, Scorial. X.I.9, f. 47r, margin in both manuscripts a. καὶ γὰρ ἀποδέδεικται γραμμικῶς ὅτι έὰν τρεῖς ἀριθμοὶ ἐλάχιστοι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἐχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσι κατὰ τὸ πόρισμα τοῦ δευτέρου θεωρημάτου τοῦ ἐκ τῆς Εὐκλείδου.

margin in Marc. gr. Z. 316, f. 61r, main text, lesser size script in Scorial. X.I.9, ff. 46v-47r C.8c

Adjunct ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν {s.l. τῶν ἄκρων} ἐπ’ ἀλλήλαις γενομένον {s.l. ήτοι πρὸς ἀλλήλας πολυπλασιασθεισῶν}, καὶ ὁ μὲν πρῶτος {s.l. κατὰ τὴν θέσιν} τῆς ἀπογεννώσης {s.l. τῆς διπλασίας φέρε εἰπεῖν ἐντεστραμμένης ἢ καὶ ὀρθῶς κειμένης : –ῳ –ῳ Χ.I.9} εἰς [ 119 ]
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τὸν ἐλάττονα τῆς γεννωμένης [s.l. σχέσεως οίον τῆς ήμιολίου] μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἐσχάτος [s.l. τῇ θέσῃ] καὶ μείζων τετράγωνος [s.l. τῷ λόγῳ, πρόλογος ὡν] ὁ αὐτὸς [s.l. ἤγουν πρόλογος] ἐστίν.

after the end of Book I, lesser size script in Marc. gr. Z. 316, f. 61v; main text, lesser size script in Scorial. Χ.Ι.9, ff. 47v-48r; main text, after Texts C.8bc in Marc. gr. Z. 319, f. 40r

b. παραδειγματιστέον ἐπὶ τῶν διπλασίων πῶς οἱ μέσοι γεννῶνται ἐκ τῶν πλευρῶν τῶν ἀκρῶν πρὸς ἀλλήλας πολυπλασιασθεῖσιν. ἔστωσαν γὰρ α β δ, ὁ γοῦν μέσος (ηγουν ὁ β) γεννᾶται ἐκ τῶν πλευρῶν τῶν ἀκρῶν (ητοι τοῦ α καὶ τοῦ β δ 319) πρὸς ἀλλήλας πολυπλασιασθεῖσιν. πλευρὰ γὰρ τῆς μὲν μονάδος ἡ μονάς (δτετράς 319), τετράγωνον γοῦν τούτων πολυπλασιασθεῖσάς γοῦν ταύτας 319 πρὸς ἀλλήλας γεννᾶται ὁ μέσος ἀπαξ γὰρ τὰ δύο, δύο. καὶ ἐπὶ τῶν λοιπῶν ὡςαύτως. ὁρὰ δὲ καὶ ἐπὶ ἐτέρων ἐκθέσεων [-eos 319] τὸ λεγόμενον.

C.18. Glosses and a scholium in Laur. Conv. Soppr. 30, ff. 42v-43r, and Oxon. Bodl. Holkham 71, f. 278r-v [only the scholia]

Ar. I.23.15 ἐπὶ πασῶν δὲ τῶν διαζευχθεῖσιν [s.l. ἐπιμορίων καὶ ἐπιμερῶν] καὶ ἄρ’ ἢς ἀμφότεραι [s.l. τῆς πολυπλασίου], ὁ μὲν ἐσχάτος [s.l. ἄκρος] τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος [s.l. μείζων λόγος] εἰς τὸν ἐλάττονα [s.l. λόγον] μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

C.6a + C.16a ὁ ρκϛ ἔχει τὸν πα καὶ ἑαυτοῦ ἔννατον, ἤτοι με. ὁ δὲ ροϛ ἔχει τὸν ρκϛ. πεντάκις δὲ ιδ, ο. ἀπὸ δὲ τοῦ ἐλάττονα κε ρ ροϛ διπλασιεπιτετράπεμτοι. ὁ γὰρ ὁ τὸν κε ἔχει δις καὶ τέσσαρα αὐτοῦ πέμπτα, καὶ ὁ ροϛ τὸν δ αὐτοῦ πέμπτα· παντάκις γὰρ ὁ δ δις καὶ τεσσάρακις [τετράκις 71] ιδ, υς δις τά καὶ νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροϛ. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ἔσον ὡς δὶς τὰ ο νς, ροストレス.
6. Philological Remarks

6.1. Nicomachus’ Introductio arithmetica

In the previous Section I have collected the textual data related to two sentences of Ar.: section I.23.15 and an adjunct that is attached to the end of Book I in most manuscripts and that Hoche prints in his apparatus. The Adjunct is nothing but a paraphrase, in reverse order, of Ar. I.23.15. The following table compares Version A of I.23.15 and Version C.5.1.A of the Adjunct, the latter suitably reassembled (the numbers restore the order).

| Ar. I.23.15                                      | Adjunct                                      |
|-------------------------------------------------|----------------------------------------------|
| ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν                  | ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων 1   |
| καὶ ἀφ’ ἕς ἀμφότεραι,                           | ἐν ἀμφότεραι δὲ ταῖς γεννηθείσαις 4         |
| ὁ μὲν ἐσχατος τετράγωνος ο ἀυτός μένει,       | ὁ ἐσχατος καὶ μείζων τετράγωνος ο ἀυτός ἐστιν 5 |
| ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει,      | καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσης εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, |
| πάντως δὲ οἱ ἄκροι τετράγωνοι                   | πάντως οἱ ἄκροι τετράγωνοι εἰσίν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ’ ἀλλήλαις γενομένων, 2 |

As said, Ar. I.23.15 is unanimously witnessed to, whereas fifteen manuscripts out of eighty-four do not have the Adjunct. Among them, we find the two earliest witnesses of Ar.: Matrit. 4678 and Par. gr. 2479. In the latter, the Adjunct is added by a hand of end 13th – beginning 14th century (see Section 6.3 below). These facts appear to exclude the possibility that the Adjunct is part of the original Ar., even if a wild plot featuring a wholesale and very early omission of the Adjunct in the exemplars read by Iamblichus and Ammonius (who do not read it at the end of their Ar.) and its surfacing again in late 13th-century copies cannot be ruled out as utterly unreal. In this scenario, Ar. I.23.15 is a clumsy attempt at restoring, in the wrong place, the original, and much clearer, final sentence. The Adjunct is in fact the clearest yet concise interpretation of I.23.15 in the entire exegetical record; its real nature of adjunct is made plain by the second clause in sentence (2) above, a well-known property that, however, has no bearing on the context of I.23.15.

33 Maybe I should be less categorical: after all, our Text C.3 is the last scholium of Rec. III, and the scholium before it comments on Ar. I.23.11. Thus, nothing can be said about the position of the clause explained in Text C.3. Note, moreover, that C.3 only comments on one of the three claims in I.23.15. On these grounds, the reader is urged to conceive a scenario even wilder than the one I have just sketched.

34 See Section 7 below. There remains just one designation quirk in the Adjunct.
From the philological point of view, *Ar.* I.23.15 and the Adjunct raise important methodological problems. First and foremost, if I.23.15 is an interpolation, then the manuscript tradition of *Ar.* admits of an archetype, defined (as it should) by the set of non-original readings shared by the entire tradition: this set may well be a singleton.\(^{35}\) Of course, the existence of an archetype presupposes adherence to the genealogical paradigm; if contamination takes place, this cannot be taken for granted. Well, the Adjunct shows what “contamination” might have meant in our case. For in Vat. gr. 2297 + Roma, Arch. S. Paolo 24C, an important witness of the late 13th century,\(^{36}\) the Adjunct lies in the margin of Rec. I, which in its turn lies in the margins of *Ar.*; in its apograph Gott. philol. 66, the Adjunct is relocated at the end of Book I of Rec. I, but a later hand copies it again, preceded by κείμενον, in a blank space in the same page and in the subsequent, blank, page. Well, in Par. gr. 2480, a slavish copy of the *Gottingensis*, we read the Adjunct both at the end of Rec. I and at the end of *Ar.*, whereas in another conformal copy of it, namely, Bucur., BAR gr. 620, the Adjunct can only be read as a last sentence of *Ar.*.\(^{37}\) On the other hand, in manuscripts such as Par. gr. 2107, the Adjunct is marked for deletion by a later hand, and might well have been deleted in later copies.

The bottom line of all of this is that the Adjunct may have (or actually has) entered or left the text of *Ar.* by contamination with the exegetical apparatus and by collation initiatives of copyists or scholars. Of course, the same might well have happened in the case of *Ar.* I.23.15: imagine two, or even more, “real” but lost witnesses that served as roots of the tradition, and such that only some of them have I.23.15 in the text, whereas the others either have it in the margin or do not have it at all, but in all of their copies I.23.15 has been included in *Ar.*, possibly by contamination with the exegetic apparatus or with the exemplars that do have I.23.15. Yet, if this *might* have occurred, then it *might* also *have not*. And if we want to keep within the stemmatic paradigm, these

\(^{35}\) On the meaning of “archetype” and “contamination”, see, most recently, the excellent discussion in P. Trovato, *Everything You Always Wanted to Know about Lachmann’s Method. A Non-Standard Handbook of Genealogical Textual Criticism in the Age of Post-Structuralism, Cladistics, and Copy-Text*, Storie e linguaggi 7, Padova 2017\(^2\), sect. 3, passim, with bibliography; and also Ph. Roelli (ed.), *Handbook of Stemmatology. History, Methodology, Digital Approaches*, Berlin 2020, secs. 2 and 4.1.

\(^{36}\) See C. Sojer, “Un codice dimenticato di Nicomaco di Gerasa, riscritto su testi innografici e sul *Commento a Giovanni* di Cirillo di Alessandria”, *Nέα Ρώμη* 8 (2011), 199-217; and C. Hofstetter, “Un nouveau témoin complet de l’*Introduction arithmétique* de Nicomaque de Gerasa”, *Nέα Ρώμη* 15 (2018), 177-192, on this manuscript.

\(^{37}\) For these manuscripts, check I.23.15, Version A, and Texts C.1, C.5.1A and C.11a, and cf. Section 6.3 below.
possible-world scenarios are excluded by definition: if there is an innovation shared by
the entire tradition, then the stemma has one single root, called “archetype”. All of this
shows, a fact that is frequently forgotten, that stemmatics is a formal method grounded
on a strict principle of parsimony.\textsuperscript{38} Since \textit{any} kind of innovation can be supposed to be
transmitted by contamination in a suitably wild scenario, the alternative to stemmatics
is in fact no method at all –even in the case of \textit{Ar.}, whose \textit{varia lectio} displays a massive
contamination even at pre-traditional stages. The fact that \textit{some} wild scenarios actually
occurred does not mean that \textit{any} wild scenario has: counterexamples are a warning,
not a proof.

Both \textit{Ar.} I.23.15 and the Adjunct display several distinctive variant readings, which
realize a partition of the textual tradition of \textit{Ar.}; some of these subsets of manuscript
witnesses are very small (see Versions \textit{CE} of I.23.15 and \textbf{C.5.1D} of the Adjunct), some
obviously characterize the revised text of \textit{Ar.} attached to Rec. II (Version \textit{D}, with some
manuscripts entering the group by polygenesis, and \textbf{C.5.1E}). Some of the variants are
or include sheer mistakes (Versions \textit{BCE} and \textbf{C.5.1CDF}); some offer a very bewildering
text (\textbf{C.5.1D}); some are inherited and some have an obvious polygenetic nature. But
many of the variant readings perfectly fit the syntactical and mathematical context: this
shows how difficult is to apply the genealogical method in the topmost portion of the
stemma of \textit{Ar.} The most important such variant (\textit{D} and \textbf{C.5.1B}) has πάντες instead of
πάντως in both I.23.15 and the Adjunct, a reading that is also witnessed to in one of the
running commentaries on \textit{Ar.} To them we now turn our attention.

6.2. The “Recensions” of Ammonius’ lecture notes

I first set out our extracts of the four “Recensions” in three comparative tables; these
are Texts \textbf{C.1} to \textbf{C.4} above. The first two tables account for the evolution Rec. III → Rec.
I → Rec. II; the third table compares Rec. I and Rec. IV (in this case, similar yet differ-
tently formulated argumental items are in red, an important variant reading is in blue).
Colour markings highlight the changes from Rec. III to Rec. I (modified items: orange;
items absent in the other Recension: blue) and from Rec. I to Rec. II (different items:
green; an important variant reading is in blue). Textual units corresponding to each
other are located in facing cells. Take in due account the fact that the text of Rec. I and
II is not a critical text.

\textsuperscript{38} Thus, the archetype is a formal object, which need not coincide with a manuscript. But
if it did, such a manuscript must be non-extant.
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Rec. III, sect. I.ρνζ

| diαξευδθείσαι εἰςι σχέσεις αἳ τε κατ’ ὅρθον ἐπίζευγνόμεναι καὶ αἱ κατὰ ἀναστροφὴν
| διεξεπεῖται γάρ τοῦ ὅρθοῦ τὸ ἀνεστραμμένον· ἐκάστης μὲν οὖν σχέσεως οἱ ἄκροι
| πάντως εἰσὶ τετράγωνοι, δ’ ἦν αὐτίαν ἐρούμεν·

| ἀλλ’ οἱ μὲν ἔσχατος οὗτος μένει, φησίν· οὐ πρόεισι
| γάρ ἐὰν ἤτερον ἥνεμοινονος· αὐξάμενος, ἐσχάτος ὡς
| ὁ δὲ πρῶτος ἐκ τῆς ἀναστροφῆς πρὸς τὸν ἐλάττονα
| πρὸς τὸν ἑκάστου μέχρι μονάδος, ἐπειτὶ καὶ ἀπὸ μονάδος ἡμερότατο.
| τοῦτο δὲ ἐδήλων ἐκεῖ ἐκ τῶν προειληφυῖαν σχέσεων
| καὶ τῶν λέγουσθαι μελλούσων.

| οἱ γὰρ ἄκροι πάντως τετράγωνοι εὑρεθήσονται, ὡς
| ἐπὶ μὲν τῶν ἡμιολίων τοῦ δ καὶ τοῦ ζ καὶ τοῦ θ, ὁ δ καὶ ὁ θ· ἐπὶ δὲ τῶν ἐπιτρίτων τοῦ δ καὶ τοῦ ιβ καὶ τοῦ
| ιϛ, ὁ δ καὶ ὁ ιϛ· καὶ ἐπὶ πάντων τῶν εἰδῶν τετράγωνοι
| εἰσίν οἱ ἄκροι, καὶ τοῦτο εὐλόγως,

| ἐπειδή δεδεικτεὶ γραμμικῶς ὅτι ἐὰν τρεῖς ἀριθμοὶ
| ἐλάχιστοι ἦσαν πρὸς ἀλλήλους τὸν αὐτὸν λόγον
| ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσίν. ἐπεὶ οὖν
| καὶ ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ θ καὶ ὁ ιϛ
| εἰσίν, διὰ τοῦτο οἱ ἄκροι τετράγωνοι εἰσίν.

| ἐδείχθη γάρ παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ
| ἐλάχιστοι ἦσαν πρὸς ἀλλήλους τὸν αὐτὸν λόγον
| ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσίν. ἐπεὶ οὖν
| ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ θ καὶ ὁ ιϛ εἰσίν, διὰ τοῦτο
| οἱ ἄκροι τούτων τετράγωνοι εἰσίν.

| ἀλλ’ ἴσως εἴποι τις ὅτι οὐκ εἰσίν ἐλάχιστοι, οὐδὲ
| γὰρ πυθμένες εἰσίν τῶν ἡμιολίων· ἰδοὺ γάρ, ὁ γ τοῦ
| β ἡμιόλιος.

| φαμέν ὅτι δύο μὲν ἀριθμοὶ ἡμιόλιοι εὑρίσκονται, ὡς ὁ β καὶ ὁ γ, τρεῖς δὲ πυθμένες
| εἰσίν τῶν ἡμιολίων· ἵνα γάρ, ὁ γ τοῦ
| β ἡμιόλιος εὑρίσκει.

| ἀλλ’ ἴσως εἴποι παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ
| ἐλάχιστοι ἦσαν πρὸς ἀλλήλους τὸν αὐτὸν λόγον
| ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσίν. ἐπεὶ οὖν
| ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ θ καὶ ὁ ιϛ εἰσίν, διὰ τοῦτο
| οἱ ἄκροι τούτων τετράγωνοι εἰσίν.

| καὶ ἐπὶ πάντων τῶν ἄλλων ὡσάκτως, ὠστε εἰκότως
| τοὺς ἄκρους τετράγωνον ἐχουσίν.

| καὶ ἐπὶ τῶν ἄλλων ὡσάκτως· εἰκότως οὖν οὕτως οὐ
| τοὺς ἄκρους τετράγωνον ἐχουσίν.
διαζευγθεῖσαι εἰς σχέσεις αἱ τε κατ’ ὀρθόν ἐπιζευγνωμέναι καὶ αἱ κατὰ ἀναστροφήν συμπλεκόμεναι· διέζευκται γὰρ τοῦ ὀρθοῦ τὸ ἀνεστραμμένον· ἑκάστης μὲν οὖν σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι, δι’ ἥν αὐτὴν ἐρούμενον·

ἀλλ’ ὁ μὲν ἔσχατος ὁ αὐτὸς μένει, φησίν· οὐ πρόεισι γὰρ ἐπὶ τοῖς ἑπταβιούς ἢ ἀπὸ ἀποβιούς ἐσχάτος· ἑκάστης μὲν οὖν σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι, δι’ ἥν αὐτὴν ἐρούμενον·

πάντως δέ, φησίν, οἱ ἄκροι τετράγωνων· ἐπὶ μὲν ἡμιολίων, τοῦ δ καὶ ϛ καὶ θ, ὁ δ καὶ θ· ἐπὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιβ καὶ ιϛ, ὁ θ καὶ θ· καὶ ἐπὶ πάντων ὀμιλίων· καὶ τούτῳ εὐλόγως·

ἐδείχθη γὰρ παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάττονες τοῦ δ καὶ θ ἐπὶ τῶν ἡμιολίων· οἱ ἄκροι αὐτῶν ἐλαττούμενοι· οὐδὲ γάρ εἰσί πυθμένες τῶν ἡμιολίων· ὁ δ καὶ θ καὶ θ· πρὸς ὃ λέγομεν ὅτι δύο μὲν ἡμιολίοι εὑρίσκονται, ὡς τὰ β καὶ τὰ γ, τρεῖς δὲ ἐπὶ τῶν ἡμιολίων· εὐφράστως καὶ δύο μὲν ἐπιτρίτους, τὸν γ καὶ τὸν δ. ἐυθαῦτα, τρεῖς δὲ ἐφεξῆς· ἐπὶ τῶν θ καὶ τοῦ ιβ καὶ τοῦ ιϛ ἐν τούτῳ·

καὶ ἐπὶ τῶν ἄλλων· εἰκότως οὐκ ὁ πάντως εἰς τετραγώνων ἐξούσια·
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Rec. I, sect. Iρης

διαζευχθέναι εἰσὶ σχέσεις αἳ τε κατ' ὀρθὸν ἐπὶ πάσης τῆς ὅρθης καὶ ἀντιστραμμένης λήψεως – διέζευγται γὰρ ἢ κατ' εὐθείαν καὶ κατὰ ἀντιστροφὴν λήψις – ἅρ' ἡς τῆς ὅρθης καὶ ἀντιστραμμένης λήψεως ἁμφότερα γεννάνται αἱ σχέσεις, εἰ μὲν κατ' εὐθείαν ληθή, γεννάτα ἡ πολλαπλασιεύσεως σχέσεις, εἰ δὲ ἀντιστραμμένης, ἡ ἐπιμερής.

Rec. IV

οἷς μὲν ἐσχάτων ὦν τετράγωνων, οἱ δὲ ἐσχάτων δὲ ἐστιν δίτη, εἰ ἀντιστρεφόμενα καὶ κατακλυσμένα· διέζευκται γὰρ τοῦ ὀρθοῦ τὸ ἀνεστραμμένον· ἑκάστης μὲν οὖν σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι, δι’ ἣν αὐτίκα ἐρούμενον εἰπεῖ·

μὲν ἐσχάτης πάντως ὄρθρος, ὦν εἰς τὸ ἱμιολίον, οὐκ εἰς ἱμιολίον ἐπὶ πάσης τῆς ὅρθης καὶ ἀντιστραμμένης λήψεως – διέζευγται γὰρ ἢ κατ' εὐθείαν καὶ κατὰ ἀντιστροφὴν λήψις – ἅρ' ἡς τῆς ὅρθης καὶ ἀντιστραμμένης λήψεως ἁμφότερα γεννάνται αἱ σχέσεις, εἰ μὲν κατ' εὐθείαν ληθή, γεννάτα ἡ πολλαπλασιεύσεως σχέσεις, εἰ δὲ ἀντιστραμμένης, ἡ ἐπιμερής.

πάντως δὲ, φησὶν, οἱ ἄκροι τετράγωνων· ἐπὶ μὲν ἡμιολίων, τοῦ δ καὶ ϛ καὶ θ, ὁ δ καὶ ὁ θ· ἐπὶ δὲ τῶν ἡμιολίων ἡμίολιον τοῦ δ καὶ ϛ καὶ θ, τρεῖς δὲ ἐπὶ τῶν ἡμιολίων ἡμίολιον τοῦ δ καὶ ϛ καὶ θ πρῶτος ἐλάττων ἐστὶ τοῦ ἐπιτρίτου καθ' ὃν τρόπον καὶ ὁ ἐπίτριτος τοῦ ἡμιολίου. οὗτος δὲ ὁ λόγος καὶ ἡ μετάβασις τοῦ πρῶτου κατ' ἀντιστροφὴν εἰς τὸν ἐλάττονα ἐπὶ μόνων τῶν ἐπιμορίων σῴζει.

πάντες δὲ οἱ ἄκροι τετράγωνων· ἐπὶ μὲν ἡμιολίων, τοῦ δ καὶ ϛ καὶ θ, ὁ δ καὶ ὁ θ· ἐπὶ δὲ τῶν ἡμιολίων ἡμίολιον τοῦ δ καὶ ϛ καὶ θ, τρεῖς δὲ ἐπὶ τῶν ἡμιολίων ἡμίολιον τοῦ δ καὶ ϛ καὶ θ πρῶτος ἐλάττων ἐστὶ τοῦ ἐπιτετάρτου καθ' ὃν τρόπον καὶ ὁ ἐπιτετάρτος τοῦ ἡμιολίου. οὗτος δὲ ὁ λόγος καὶ ἡ μετάβασις τοῦ πρῶτου κατ' ἀντιστροφὴν εἰς τὸν ἐλάττονα ἐπὶ μόνων τῶν ἐπιμορίων σ وأوضح.
The first two tables display a typical pattern for the evolution Rec. III → Rec. I → Rec. II: local rewritings and growth by inflation. That this was the case in the transition from Rec. III to Rec. I has already been shown by Westerink and Tarán, who also surmised –rightly in my opinion– that Rec. III is more faithful to Ammonius’ lecture notes than Rec. I, and that the latter might even have used the former as its sole source. In our case, Philoponus adds a (wrong) clarification of the part of Ar. I.23.15 left uncommented by Rec. III (second cell in the table), smooths out the syntax and regularizes the lexicon, and unduly (but not unreasonably given the context) adds a crucial negation in the final sentence.

The second table shows, a fact that is confirmed by a complete collation of the text as I have undertaken, that Rec. II is a recension of Rec. I according to the same principles as are used by Philoponus to modify Asclepius’ text: local rewritings and growth by inflation. In particular, Rec. II radically rewrites Rec. I’s wrong clarification (second cell), adds some specific explanations, makes some expressions more explicit, deletes the final negation, aligns the quote from Ar. with its own text of Ar., changes the pseudo-citation of the Elements (where no theorem with such an enunciation exists) to a wording more faithful to a formulation consistently adopted in the Euclidean treatise. There can be no doubt, in my opinion, that Rec. II is a Byzantine recension of Rec. I. Tannery surmised that the reviser was Isaak Argyros, to whom some scholia are expressly ascribed in the margins of Vat. gr. 1411, which, as far as my documentary record goes, is in fact the only independent witness of Rec. II. Tannery’s hypothesis must be false exactly for the reason he adduces: were Argyros the reviser, he would not have kept his own name by

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39 Westerink, “Deux commentaires” (cit. n. 19), and Tarán, Asclepius (cit. n. 19), 8-17.
40 Several late witnesses perceived the problem and deleted this negation: check Text C.1.
41 Qualifier πάντες instead of πάντως, at the beginning of the third cell: see I.23.15, Version D. The same reading is adopted by Rec. IV.
42 The designation “among the numbers that have the same ratio as them” instead of “that have the same ratio to each other”.
43 Tarán, Asclepius (cit. n. 19), 18; and M. Roueché, “Why the Monad is Not a Number: John Philoponus and In De Anima 3”, JÖB 52 (2002), 95-133: 116-118, claim that Rec. II was composed in Late Antiquity, but their arguments rest on weak bases.
44 On this 14th-century Byzantine scholar see now A. Gioffreda, Tra i libri di Isacco Argiro, Transmissions 4, Berlin 2020.
45 Tannery, “Rapport” (cit. n. 19), 302; contra, Tarán, Asclepius (cit. n. 19), 18. The scholia ascribed to Argyros are at Vat. gr. 1411, ff. 49v (nr. 73), 59v (nr. [1]55), 95r (nr. 17), 102v (nr. 53). The former two are edited in F. Acerbi, “I problemi aritmetici attribuiti a Demetrio Cidone e Isacco Argiro”, Estudios bizantinos 5 (2017), 131-206, Testo 21; the latter two in Delatte, Anecdota (cit. n. 25), 139.21-26 and 173.19-174.3.
46 Check I.23.15, Version D, Texts C.2, C.5.1E, C.5.2, and see Acerbi, “I problemi” (cit. n. 45).
the side of his own additions. We have also read Argyros’ autograph scholium in Text C.10 above, and it does not coincide with the corresponding exegetical unit of Rec. II.

The last table shows that Rec. IV is a not a redaction of Ammonius’ lecture notes (or a Byzantine recension of such a redaction) as the other “Recensions” are, but an independently conceived writing. In our case, Rec. IV employs different wordings of crucial statements (red colour above), offers a very long, and different exegesis of the crux in Ar. I.23.15 (second cell), and does not even allude to the aporia of the *pythmenes* on which the others spend a rhetorical question.47

As our specimen shows, Rec. IV is a commentary-paraphrase that is not organised in “lemmas” as the other Recensions are; we read in it –dismembered among uncountably many repetitions and examples– a fair amount of Ar., so that the argument of Rec. IV can sometimes be followed without having Ar. at hand. This is impossible in the other Recensions. Thus, as a literary product, Rec. IV stands mid-way between the other Recensions and Iamblichus’ *in Nicomachi Arithmetican*, which is in fact a recension of Ar. It is fairly obvious that the author of Rec. IV has largely drawn from the exegetical tradition, the other Recensions included.48 There is not the slightest doubt, in my opinion, that Rec. IV is a Byzantine writing, and a fairly typical one because of its prolixity and of its paraphrastic habit; a similar literary product is Nicholas Cabasilas’ commentary on Book III of Ptolemy’s *Almagest*.49

The author of Rec. IV is anonymous. If he has to be a Byzantine scholar, as I think he has to, we might make a case for excluding Michael Psellus –for his name would hardly have disappeared– and the 12th-century scholars who gathered around Anna Comnena, for we read scholia to Ar. I.23.15 ascribed to Michael of Ephesus and Eustratius and they are fairly different from the text of Rec. IV (see the next Section).50 It is possible that Rec. IV is the product of the renewed interest in philosophy and the sciences that characterized the Nicaean and early Paleologan periods; a *terminus ante quem* is set by the copy of the earliest manuscript witness of the text –Ambros. H 58 sup. (gr. 438; *Diktyon* 47 Ammonius summarizes here Iamblichus, *in Nic.* III.52-55, 116.12-29 Vinel.

48 Cf. Tarán, *Asclepius* (cit. n. 19), 20.

49 On the mathematical skills of this important theologian, see F. Acerbi – I. Pérez Martín, “Les études géométriques et astronomiques à Thessalonique d’après le témoignage des manuscrits: de Jean Pédiásimos à Démétrios Kydônès”, *Byzantion* 89 (2019), 1-35: 13-17. Cabasilas’ commentary is unedited.

50 For a first orientation on 12th-century scholarship, see A. Kaldellis, “Classical Scholarship in Twelfth-Century Byzantium”, in Ch. Barber – D. Jenkins (eds.), *Medieval Greek Commentaries on the Nicomachean Ethics*, Leiden – Boston 2009, 1-43. The seminal study on Anna Comnenen’s circle is R. Browning, “An Unpublished Funeral Oration on Anna Comnena”, *PCPhS*, n.s. 8 (1962), 1-12.
42871)– dated to between 1295 and 1315.\textsuperscript{51} Only two witnesses of Rec. IV have its text laid out in the form of a framing commentary on \textit{Ar.}, namely, Laur. Plut. 58.29 and Ambros. B 77 sup.; it is obvious that the latter is a conformal copy of the former (check also the apparatus of I.23.15, Versions A and D).

6.3. The scholia

The exegetical record in form of isolated scholia is exceptionally rich (recall that \textit{Ar.} I.23.15 makes three claims); for this reason, I was induced to create so many C-subdivisions. The presence of specific scholia may suggest filiations between manuscripts, but it must be stated clearly that shared scholia may originate either in slavish copy or in collation of another exemplar. Thus, the scholia shared by several witnesses can only provide the basis for a supporting criterion of filiation between manuscripts.

I set out in the following table such scholia as have an author name attached or as are transversal to my C-subdivisions; brackets mean either “anonymous” (round), or “penned by a later hand” [square], or “partial copy” \{braces\}:

| ms          | Mich. | Dyyr. | Eustr. | Dishyp. | Prot. | Arg. | C.6a | C.14a | C.16a | C.17b | C.18 |
|-------------|-------|-------|--------|---------|-------|------|------|------|------|------|------|
| Ambros. I 8 sup. | (C.9a) |       |        |         |       |      |      |      |      |      |      |
| Ambros. G 62 sup. | C.8a  | (C.8b) | C.8c   | C.8d   |       |      |      |      |      |      |      |
| Athens, EBE 1115 | C.9a  |       |        |         |       |      |      |      |      |      |      |
| Bucur., BAR gr. 620 |       |       |        |         |       |      |      |      |      |      |      |
| Gott. philol. 66 |       |       |        |         |       |      |      |      |      |      |      |
| Hamburg, S-UB, philol. 89 | (C.13) | (C.13) |       |         |       |      |      |      |      |      |      |
| Kharkow, UL 269-p |       |       |        |         |       |      |      |      |      |      |      |
| Laur. Conv. Soppr. 30 |       |       |        |         |       |      |      |      |      |      |      |
| Laur. Plut. 28.35 | (C.7a) |       |        |         |       |      | C.11a|      |      |      |      |
| Marc. gr. Z. 309 | (C.8a) | (C.8b) |       |         |       |      |      |      |      |      |      |
| Marc. gr. Z. 316 |       |       | C.8c   |         |       |      |      |      |      |      |      |
| Marc. gr. Z. 318 | C.8a  | C.8b  | C.8c   |         |       |      |      |      |      |      |      |
| Marc. gr. Z. 319 | C.8b  | C.8c   |         |         |       |      |      |      |      |      |      |
| Marc. gr. Z. 320 |       |       |         |         |       |      |      |      |      |      |      |
| Marc. gr. Z. 333 | (C.8a) |       | C.8c   |         | (C.11b)|      |      |      |      |      |      |
| Matritr. 4678 |       |       |         |         |       |      |      |      |      |      |      |
| Monac. gr. 76 |       |       |         |         |       | (C.11b)|      |      |      |      |      |
| Monac. gr. 482 | (C.7a) |       |       |         |       |      |      |      |      |      |      |

\textsuperscript{51} A. Turyn, \textit{Dated Greek Manuscripts of the thirteenth and fourteenth centuries in the Libraries of Italy}, 2 vol., Urbana – Chicago – London 1972, vol. 1, 88-90.
The textual tradition of Nicomachus’ *Introductio arithmetica*

| ms                  | Mich. | Dyrr. | Eustr. | Dishyp. | Prot. | Arg. | C.6a | C.14a | C.16a | C.17b | C.18 |
|---------------------|-------|-------|--------|---------|-------|------|------|-------|-------|-------|------|
| Mutin. a.T.8.14     |       |       |        |         | (C.11b)|      |      |       |       |       |      |
| Mutin. a.U.9.7      |       |       | C.11b  |         |       |      |      |       |       |       |      |
| Neap. III.C.1       |       |       |        |         |       |      |      |       |       |       |      |
| Nor. Cent. V. App. 36|       |       |        |         |       |      |      |       |       | C.10  |      |
| Oxon. Bodl. Holkh. 71|       |       |        |         |       |      | *   | *     | *     |       |      |
| Par. Coislin 174    |       |       |        |         |       |      | C.11b|       |       |       |      |
| Par. gr. 2450       |       |       |        |         |       |      |      | *     |       |       |      |
| Par. gr. 2480       |       |       | C.11a  |         |       |      |      |       |       |       |      |
| Par. gr. 2483       |       |       |        |         |       |      | *   | *     |       |       |      |
| Par. gr. 2531       |       |       | C.11b  |         |       |      |      |       |       |       |      |
| Roma Abb. S Paolo 24C|       |       |        |         |       |      | C.11a|       |       |       |      |
| Scorial. PII.3      |       |       |        |         |       |      | *   | *     |       |       |      |
| Scorial. X.I.9      |       |       | C.8c   |         |       |      |      |       |       |       |      |
| Vat. Barb. gr. 273  |       |       |        |         |       |      |      |       |       |       |      |
| Vat. gr. 186        |       |       |        | (C.9a)  |       |      |      |       |       |       |      |
| Vat. gr. 198        |       |       | [C.13] |         |       |      |      |       |       |       |      |
| Vat. gr. 256        |       |       | C.9a   |         |       |      |      |       |       |       |      |
| Vat. gr. 1026       |       |       |        |         |       |      |      |       |       |       |      |
| Vindob. phil. gr. 220|       |       |        |         |       |      |      |       |       |       |      |

As the table shows, particularly successful have been five scholia of assigned author, three of which can be found in a fair number of manuscripts, either anonymous or with ascribed authorship. Two of these five scholia come from the 12th century scholars mentioned above, namely, Michael of Ephesus and Eustratius of Nicaea; two others were composed—apparently in the context of a scholarly debate—by an anonymous reader from the town of Δυρράχιον (present-day Durrës), and by a judge Nicholas Dishypatos, of whom no more information appears to be available (Texts C.7-9 and C.13). 52 Michael’s and Eustratius’ exegeses were added by a later hand, partly and misattributed, in Mutin. a.T.8.14, and in its copy Marc. gr. Z. 333 (check also Text C.11b); Eustratius’ alone also figures in Marc. gr. Z.

52 Dishypatos mentions the anonymous scholar from Durrës and dubs him “most Aristotelian”. As far as these two characters are concerned, proximity searches in the TLG give no results that may provide serious grounds for an identification. Tannery, “Rapport” (cit. n. 19), 329, first put these four scholia to scholarly attention; however, he could not read the epithet that identifies the author of the first scholium, which can be found in the Martini-Bassi catalogue. The pieces of information that come from secondary literature have recently been compiled in I. Nesseris, *Η Παιδεία στην Κωνσταντινούπολη κατά τον 12ο αιώνα*, PhD Thesis, University of Ioannina 2014, vol. 1, 286-289. On Michael of Ephesus’ mathematical activities see F. Acerbi – B. Vitrac, “Les mathématiques de Michel d’Éphèse”, *REB* 80 (2022).
316 and in its conformal apograph Scorial. X.I.9 (see also Text C.17b); the latter two scholia were transcribed in severely abridged form by Malachias in his Vat. gr. 198 (he also calls the judge “Niketas”), and in Hamburg, S-UB, philol. 89 therefrom (check also Texts C.1 and C.5.1D). These four scholia are compiled together only in Ambros. G 62 sup., a very important witness of Ar., which alone hands Dishypatos’ text down to us in its longer version; the four Bessarion manuscripts Marc. gr. Z. 309, 316, 318 (a “patrimonial” manuscript), 319 offer a different text of those of the above scholia they happen to preserve.

As for the fifth scholium in this group, it was in fact the most successful scholium on one single claim of Ar. I.23.15, and was eventually included in some Rec. I manuscripts; in most of them, it is ascribed to some Theodoros/Demetrios Protocensor (Text C.11). Theodoros’ and Michael of Ephesus’ scholia, the latter here anonymous, were transmitted from Monac. gr. 482 to its copy Laur. Plut. 28.25 (check also Texts C.1, C.5.1A, and C.7). In the former manuscript, Theodoros’ scholium is attached to Rec. I, which was penned by Neophytos Prodromenos as a whole after Ar.; Michael’s annotation was instead located in due place at the end of Book I of Ar.; the copyist of Laur. Plut. 28.35 relocated Rec. I in the margins of Ar., so that he eliminated Michael’s scholium while keeping ‘Theodoros’. The other manuscripts that assign the scholium to Theodoros all stem from Roma, Arch. S. Paolo 24C (cf. Section 6.1 above). The ascription to Demetrios was spread from Mutin. α.U.9.7 to its (quite incorrect) conformal copy Par. gr. 2531 (have a look at the layout at the end of Book I) and, most notably, to some Rec. I manuscripts, among which I have checked Angel. gr. 1 and BNCF, Fondo Naz. (olim Magliab.) II.III.37 and Vat. gr. 2262. The scholium lies anonymous in Mutin. α.T.8.14 and in its copy Marc. gr. Z. 333 (see also Text C.8ac); the text is of the Demetrios-type.

Nikephoros Gregoras apposed glosses, a scholium (Text C.14a) and diagrammatic scholia to Ar. I.23.15 in his own recension manuscript Marc. gr. Z. 320. We find these items in the direct copies Ambros. P 121 sup. (only the diagrams), Par. gr. 2450 (a conformal apograph of which is Vat. Barb. gr. 273), Vindob. phil. gr. 220 (a copy of which is Neap. III.C.1); see also the transmitted correction in Texts C.5.1AF. Isaak Argyros’ scholium (Text C.10) is instead unique to his own fair copy Norimb. Cent. V. App. 36. A distinctive reading (Adjunct, Version F) and diagrams show that Argyros used Par. gr. 2107 as his model; a conformal apograph of the Paris manuscript is also Guelf. 36 Gud. gr. (cf. C.5.1 gloss).

As for anonymous scholia, Text C.6a is shared by Matr. 4678 and Vat. gr. 1026; together with Text C.16a, it is also shared by Kharkow, UL 269-p, 369-c and by its copies Par. gr. 2483 and Scorial. P.II.3. These two texts together with Text C.18 are further shared by the magnificent Oxon. Bodl. Holkham 71 and by its likely copy Laur. Conv. Soppr. 30. The Kharkow manuscript shares with Oxon. New Coll. 299 and with Vat. gr. 1051 and its conformal copy Par. gr. 2481 a distinctive mistake in Ar. I.23.15 (Version E).
The following table sets out the scholia to the Adjunct:

| ms                    | Text |
|-----------------------|------|
| Ambros. I 8 sup.      | C.9bd|
| Athens, EBE 1115      | C.9bd|
| Berol. Phillipps 1549 | C.5.2|
| Cizensis 67           | C.5.2|
| Hamburg, S-UB, philol.88 | C.12|
| Marc. gr. Z. 317      | C.12 |
| Marc. gr. Z. 595      | C.12 |
| Monac. gr. 76         | C.5.2|
| Oxon. Bodl. Selden Supra 20 | C.9bd|
| Oxon. Lincoln Coll. gr. 33 | C.5.2|
| Par. gr. 2373         | [C.15] |
| Par. gr. 2377         | C.5.2|
| Par. gr. 2479         | [C.9bd]|
| Par. gr. 2762         | C.12 |
| Vat. gr. 256          | C.9bd|
| Vat. gr. 1411         | C.5.2|
| Vat. gr. 2387         | C.12 |

The second column singles out all representatives of Rec. II I was able to check. As said, they all stem from Vat. gr. 1411. As for the third column, the indicated scholia and related material (inclusive of Michael’s scholium C.9a) were transmitted by conformal copy from Vat. gr. 256 to Athens, EBE 1115 (check also Text 5.C.1B), and by collation, along with the Adjunct itself C.5.1B, from Oxon. Bodl. Selden Supra 20 to Par. gr. 2479 (here a later hand). The fourth column displays a group of tightly related manuscripts, further linked by the mistake in I.23.15, Version C.

The information gathered in this Section and an analytic look at the varia lectio recorded in Section 5 (this is summarized in the tables of the Appendix) allow outlining the following schematization of a part of the textual tradition of Ar. The two large continents, connected by several bridges, represent the presence (absence on the left) of the Adjunct; the sign “>” connects an apograph to its (likely or obvious) model. The manuscripts are identified by the number that characterizes their signature; those marked in red have Theodoros/Demetrios Protocensor’s scholium. Enclaves delimited by broken lines also depend on extratextual criteria.53

53 These are: presence of the “Soterichos” scholium located between the two Books of Ar. (blue); Rec. II ascribed to Proclus (orange); diagrammatic apparatus (green: see Acerbi,
7. Exegetical stances

It is important to recall that Ar. I.23.15 makes three claims, whose meaning has been explained in Section 3 above. How these claims, along with the context-setting clause, were interpreted by the scholiasts is best discussed by taking the Adjunct as a reference; in the following table, the claims of the Adjunct are singled out, and I.23.15 is set out facing and disassembled (cf. Section 6.1 above, where the complementary operation is performed):

“Eliminazioni” [cit. n. 3]). See the Appendix for details. On the “Soterichos” scholium see C. Hofstetter, “Le mystérieux Soterichos: contribution à l'établissement de l'histoire du texte d'un commentaire byzantin à la psychogonie de Platon”, RHT 13 (2018), 59-91.
Thus, as seen in Section 3 above, two “disjoined” numerical means are “generated” from an assigned mean (this is the Context: rows nr. 1-2 above): the extreme terms of these means are “always square numbers” (Claim 1: row nr. 5 above), the “first” (and largest) term of the “generating” mean “is transformed into the smaller ‹term› of the resulting ‹mean›” (Claim 2: row nr. 4 above), the “last and largest” term is the same in the resulting means (Claim 3: row nr. 3 above). In the Adjunct, we also get for free an enrichment of Claim 1: the middle term of a geometric mean is the geometric mean of the extremes. What is at issue here, then, are the numerical means and their terms. No mention of ratios, which, however, are the backbone of the general context of Ar. I.23.6-17 in virtue of the identification of ratios and means,54 as seen in Section 2 above. This imperfect match could not be swallowed by several readers, who insisted in forcing Ar. I.23.15 in the unsuitable mould of ratios. If an unsuitable mould might well prove harmless in point of exegesis, the real issue in interpreting I.23.15 is simple: which of the two extremes is “the first”, and which is “the last”. The alternative is not settled by the wording of I.23.15, as we have repeatedly seen. Moreover, it is clear that all of the following constraints cannot be filled at the same time: (1) the “first” and the “last” term cannot have the same position in a mean; (2) all Claims must refer to both “resulting” means; (3) if something within a specific kind is “transformed into ‹anything› smaller” in the same kind, it cannot be the smallest such item around. Add to all of this the diabolic touch of interchanging the extremes of the mean in the shift from the “direct” to the “inverse” arm of the prescription. The result is the exegetical diffraction I shall summarize in what follows.

Let us start with the Context. Most interpreters agree in the above interpretation, usually by conceiving of what is disjoined as means-as-ratios (σχέσεις) or means-as-proportions (ἀναλογίαι): see Texts C.1, C.2, C.4 (Rec. I, II, IV), C.5.1D (a variant reading in the Adjunct), C.7 gloss, C.8c and gloss, C.8bd = C.13ab, C.9b, C.10 and gloss, C.12

54 When defining the means at Ar. I.23.7, Nicomachus calls them ἐκθέσεις “setting-outs” (wiz., of numerical terms), and this denomination will be picked up in the Adjunct.
gloss, C.14a and gloss, C.16b, C.18 gloss. The exceptions are Texts C.3 (Rec. III), which does not tackle this point, Texts C.6 glosses, C.7a = C.8a = C.9a, which quite incorrectly think that what is disjoined are terms such as the unit, the dyad, etc., and C.16 gloss, which identifies “disjoined” with “inverse”. Finally, note that the Adjunct disconnects the two phrases that define the Context, and distributes them between the Claims.

As for the Claims, let us start on safe ground, namely, with Claim 1. Rec. I-IV (Texts C.1-4) shift from a cautious “proved by means of lines (= rigorously)” to a questionable “proved by the geometer/in geometry”, cite a pseudo-Euclidean enunciation (modified by Rec. II, as seen in Section 6.2; the formulation of Rec. I/III is also adopted in Texts C.7b and C.9c, C.17a, C.18), and offer one or more basic examples. Rec. I-III, along with Texts C.6b (very sketchily), C.17a, C.18, and, in a different formulation, Argyros’ scholium C.10, also present and solve the aporia of the pythmenes: the property holds for means in lowest terms, but each mean contains two instances of the ratio it corresponds to, and neither instance can be in lowest terms (cf. Section 2 above), that is, those terms that are the pythmenes of the ratio.55 The scholium to the Adjunct C.5.2 explains, by quoting the enunciation of Elem. VIII.11, that the middle term referred to by the Adjunct is the product of the “sides” (= square roots) of the extremes, whereas Texts C.9d, C.12b, C.14 gloss to the Adjunct, C.15, C.16ab, C.17c present a series of examples of this very last statement. Texts C.8, C.11, C.13 (Malachias’ rewriting of C.8db), C.14, C.15 do not discuss this Claim.

Interpreting Claim 2 was a thorny issue. Rec. III (Text C.3) is silent on it. Philoponus (Text C.1), who could not plagiarize anyone, concocts an explanation that is just a smoke screen: “the first drifts towards the unit, for it also originated from the unit”. The reviser of Rec. II (Text C.2) cannot be happy with this and offers a correct explanation; he adopts, however, a terminology that others also applied in this context: the largest term of a mean is read as the “antecedent” of the ratio associated to it, the smallest term of the mean being associated with the “consequent” of the “same” ratio. As explained in Section 2, the two terms are “antecedent” and “consequent” of different albeit equivalent ratios, neither of which is in lowest terms. This fact slightly complicates matters. The smaller-term reading can be found in Texts C.8d = C.13a, C.9b, C.10, C.12a, C.15 gloss, C.16b, C.17 gloss; the terminology of “antecedent” and “consequent”, which obviously also fits Claim 3, is further adopted in Texts C.8c, C.8d = C.13a, C.10, C.11, C.17 glosses.

Rec. IV (Text C.4) introduces a major misconception to us: the first term of the generating mean is not transformed into a smaller term but into a smaller ratio. As said at the

55 In general, a pythmen is the smallest number, or set of numbers, or ratio, to which a certain property applies. The concept was already well-developed in the times of Plato: R. 546c1-3.
end of Section 2, this can only be true, and not even in every instance, for the “inverse” arm of the prescription, contrary to the explicit statement of *Ar*. I.23.15 that the property must hold for both arms. We also find the ratio-reading in Texts C.6a and glosses, C.7a and glosses = C.8a = C.9a, C.8c, which however combines it with the smaller-term reading, C.8 gloss, C.12 glosses, C.14a and glosses, C.16a, C.18 gloss.

Text C.8b appears to endorse the smaller-term interpretation but gets entangled in an unwelcome confusion because it writes the generating mean of its own basic example in reverse order: 9,6,4 instead of 4,6,9. Of course, this move has the effect of inverting the referent of the pronouns “first” and “last” in *Ar*. I.23.15. This problem disappears in Malachias’ radical rewriting in Text C.13b, where the numbers of the example are also different.

Let us finally come to Claim 3. The situation with Rec. III (Text C.3) and Rec. I (Text C.1) is the same as above: the former is silent, the latter deploys a smoke screen. Rec. II (Text C.2) adopts two basic interpretations: (1) Claim 3 applies to both arms of the prescription and not to the direct arm only (see the discussion in Section 2 above); (2) as a consequence, the “last” term is the smallest one in a mean, or, by using the wording of Rec. II, is the “consequent” of the lesser of the two ratios that figure in the mean. In its not-so-clear-cut formulation, Rec. IV seems to put forward both (1) and (2). The same position as Rec. II we find in Texts C.7 glosses, possibly (the line of argument is very confused; one must induce from the interpretive stance for Claim 3, but note that the final remark of Text C.8d seems to imply the greatest-term reading) in C.8b, and in C.8cd, C.10, C.11, C.12a, C.16a.

The Adjunct C.5.1 adopts (1) above but also the reading that (3) the “last” term is the greatest one in a mean: as a consequence, the two greatest terms in the resulting means are identical (this is also the reading I prefer). The same position we find in Texts C.6b glosses, C.7a = C.8a = C.9a, in Malachias’ radical rewriting of C.8b in Text C.13b, and again in Texts C.9b, C.16b, C.17 glosses. Texts C.8d, and C.14 and glosses appear to set forth the idea that the last square remains the same as a square (“as for quality”), not as of its numerical value (“as for quantity”).

Texts C.6a, C.13a (Malachias’ rewriting of C.8d), C.15 do not really tackle the issue.

All in all, we may be charitable with our commentators and scholiasts: *Ar*. I.23.15 *really is* an ambiguous sentence. The scholia that definitely beat around the bush are a handful, among which Philoponus’ Rec. I. Noteworthy points are: Malachias’ rewritings of Dishypatos’ and the man from Dyrrachios’ scholia; the clear hints of a scholarly debate lingering over *Ar*. I.23.15; the metonymic misinterpretation that makes, for instance, Michel of Ephesus think that the referents’ set of the pronouns “last”, “first”, and
“smallest” may contain both terms and ratios; the systematic identification of means and ratios, refined in some scholiasts by the use of the technical terms “antecedent” and “consequent”; the moderate use of numerical examples; the citation of a phantom-theorem in the Elements.

8. Conclusion

Let us finally sketch, by referring to the three aims listed at the beginning of this paper, the main results of this tour de force.

Philological aim 1. Some order has been made in the textual tradition of Ar. It is shown that the tradition is rooted in an archetype to be located between Iamblichus and Ammonius because of the presence, in all manuscripts, of the interpolation Ar. I.23.15. It is also shown that a text I have called “the Adjunct” and that is witnessed to by most manuscripts is a clarification of Ar. I.23.15 that has crept into the text, certainly after Ammonius. Some of these manuscripts allow following the process of absorption of the Adjunct by the text. The varia lectio associated to I.23.15 and to the Adjunct, along with the pattern of presence and the varia lectio of the scholia thereon, allow outlining a first map of the witnesses of Ar. Yet, much of the varia lectio perfectly fits the syntactical and mathematical context; this shows how difficult is to apply the genealogical method in the topmost portion of the stemma of Ar.

Philological aim 2. The relationships between the so-called Recensions of Ammonius’ lecture notes are settled once and for all: only Rec. III and Rec. I deserve this appellation; Rec. II is a Byzantine recension of Rec. I; Rec. IV is an independently conceived writing of Byzantine origin. As for Rec. III and Rec. I, my impression is that Philoponus simply reworked Asclepius’ set of scholia, with a close look at his own Nicomachus. It is of some interest, and even slightly bewildering, that Ammonius/Asclepius did not comment on the most problematic claims of Ar. I.23.15.

Historical aim. Byzantine scholars hotly debated about the meaning of Ar. I.23.15 and of the Adjunct; the exegetical documentary record is very rich, some scholia being transmitted in fairly different redactions. The scholarly debate maybe found its akmē in the 12th century, maybe within the circle of scholars gathered around Anna Comnena (after all, they had to study some number theory for commenting on Aristotle’s Nicomachean Ethics), and could be taken to be responsible for some of the pre-traditional modifications that affect Nicomachus’ text and for part of the rich apparatus of scholia in form of syllogistic diagrams that characterizes some manuscript witnesses.56

56 See Acerbi, “Eliminazioni” (cit. n. 3), on this.
**Appendix. The documentary record**

**Manuscripts of Nicomachus, *Introductio arithmetica*, that contain the end of Book I (83)**

The four central columns set out the Version of *Ar*. I.23.15, the number of the Text number of the associated scholium, if any, the Version of the Adjunct, the presence or absence of glosses to I.23.15, respectively. The last two columns give the Hoche’s and D’Ooge’s *sigla*. Later hand texts are within brackets; within braces are texts that can only be read partly.

| ms          | copyist | date     | Book I  | Book II | I.23.15 | Txt | Adj | gl | remarks                                                                 |
|-------------|---------|----------|---------|---------|---------|-----|-----|----|-------------------------------------------------------------------------|
| Ambros. I 8 sup. (gr. 451) *(Diktyon 42887)* | 12th ex. – 13th in. | 1v-34r | 34r-73r | B       | 9      | B   | Y   | inc 4.7 αὕτη marg sch sch [Michael of Ephesus] related to Vat. gr. 256 |
| Ambros. B 77 sup. (gr. 105) *(Diktyon 42327)* | 14th ex. – 15th in. | 2r-58r | 58v-101r| D       | 3      | 4   | A   | 1r-58r, 58v-101v Rec IV 102r-132r, 132r-149v des mut Rec III copy of Laur. Plut. 58.29 as of *Ar* + Rec IV |
| Ambros. G 62 sup. (gr. 404) *(Diktyon 42822)* | 13th ex. – 14th in. | 28r-43v | 48r-65v | A       | 8      | A   | N   | sch Michael of Ephesus, Δυρραχίου, Eustratius, Dishypatos Soterichos splits *Ar*. |
| Ambros. P 121 sup. (gr. 640) *(Diktyon 43117)* | 15th | 111r-130v | 130v-152v | A       | N      | F   | N   | copy of Marc. gr. Z. 320 |
| Athens, EBE 1115 *(Diktyon 3411)* | 14th ex. | 1-59 | 60-134 | B       | 9      | [B] | N   | marg mix comm sch Michael of Ephesus copy of Vat. gr. 256 partial lacuna on p. 59 affects Adj |
| Bonon. BU 2263 *(Diktyon 74179)* | 16th | 1r-16v | 17r-34r | C       | N      | B   | N   | many diagrams m rec related to Hamburg, S-UB, philol. 88 and Vat. gr. 2387 |

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| ms                        | copyist         | date   | Book I | Book II | I.23.15 | Tnt   | Adj | gl | remarks                                                                 |
|--------------------------|-----------------|--------|--------|---------|---------|-------|-----|----|--------------------------------------------------------------------------|
| Berol. Phillipps 1549    | Valeriano Albini| 16th²  | 1r-84v | 89r-159r| D       | 2     | 5.2 | E  | N  | 1r-88v, 89v-159v Rec II incl Ar sch εν ἄλλοις stems from Vat. gr. 1411 after rec |
| (Diktyon 9450)           |                 |        |        |         |         |       |     |    |                                                                          |
| Bucuresti, BAR gr. 520   |                 | 18th   | 87-118 | 119-154 | D       | N     | E   | N  | nearly all diagrams missing                                               |
| (Litzica 210) (Diktyon 10593) |               |        |        |         |         |       |     |    |                                                                          |
| Bucuresti, BAR gr. 620   |                 | 18th   | 11-234 | 235-430 | A       | 1     | 11a | A  | N  | sch Theodoros protocensor conformal copy of Gott. philol. 66               |
| (Litzica 183) (Diktyon 10693) |             |        |        |         |         |       |     |    |                                                                          |
| Erlangen, UL A.8         |                 | 17th   | 1r-39r | 39r-54v | D       | N     | E   | N  | s.l. Rec II anchoring numbers Rec II (missing) ascr. Proclus stems from Vat. gr. 1411 |
| (Diktyon 14298)          |                 |        |        |         |         |       |     |    |                                                                          |
| Gott. philol. 66          |                 | 13th ex.| 1r-140r| 142r-265r| A       | 1     | 11a | [A]| N  | 1r-141r, 142r-266r Rec I sch Theodoros protocensor Adj in Rec I, Adj m rec copy of Roma, Arch. S. Paolo 24C |
| (Diktyon 17413)          |                 |        |        |         |         |       |     |    |                                                                          |
| Guelf. 36 Gud. gr.        |                 | 16th in.| 1r-17v | 17v-32v | A       | N     | F   | N  | Guelf. 43 + 37 + 36 + 40 Gud. gr. annotated by Matteo Macigni systematic copy of Par. gr. 2107 |
| (Diktyon 72080)          |                 |        |        |         |         |       |     |    |                                                                          |
| Hamburg, S-UB, philol. 88 | Alexios         | ca 1294| 1r-24r | 24v-52v | C       | 12    | B   | Y  | sch; cf. Vat. gr. 2387 no compl of Vat. Barb. gr. 164                   |
| (Diktyon 32406)          |                 |        |        |         |         |       |     |    |                                                                          |
| Hamburg, S-UB, philol. 89 | Francesco Gozzadino | 1633 | 1-76  | 77-152  | A       | 1     | 13  | [D]| N  | 161-373, 399-548 Rec I sch Dishyptatos; sch Διρραχίου Adj marg Soterichos splits Rec I conformal copy of Vat. gr. 198 |
| (Diktyon 32407)          |                 |        |        |         |         |       |     |    |                                                                          |

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| ms | copyist | date   | Book I   | Book II  | I.23.15 | Txt | Adj | gl | remarks | H | D |
|----|---------|--------|----------|----------|---------|-----|-----|----|---------|---|---|
| Kharkow, UL 269-p, 369-c (Diktyon 12442) | 14th a.m. | 67r-87v | 88r-102v | E | 6a 16 | B | Y | sch; sch ἐν ἄλλοις 102v Def. 138.9-10+ lac 48.4 ἄρ|θημα – 53.23 προχωροῦσιν, 63.1 πολλαπλασίου – 67.19 καὶ, 75.16 ψυχογονίαν – 81.11 ς, 107.7 ὠκτάκως – 119.21 τε, 123.15 καὶ – 126.10 ἰσον after ff. 83, 86, 88, 95, 96, resp correct folios order 97, 99, 98, 101, 100 | | |
| | | | | | | | | | | | |
| Laur. Conv. Soppr. 30 (Diktyon 15795) | 13th ex. | 9v-43r | 43v-68r | A | 6a 16a 18 | A | Y | sch; several hands annot Nikephoros Gregoras prol m rec des. 77.5 περὶ possibly a copy of Oxon. Bodl. Holkham 71 | | |
| Laur. Plut. 28.35 (Diktyon 16216) | 14th | 2r-31v | 31v-44v | A | 1 11a | A | Y | des. 116.9 διατηρήσουσι 2r-30r, 30r-50r Rec I sch Theodoros protocensor copy of Monac. gr. 482 | | |
| Laur. Plut. 58.29 (Diktyon 16447) | 14th² | 97r-153r | 154r-196v | A | 4 | A | N | 95v-196v Rec III Ia + Rec IV Psellos [cf. Soterichos] indep. of Ar. | | |
| Leid., BRU, Periz. Qe 39 (Diktyon 37878) | 14th | 1r-23v | 23v-40v | B | N | B | N | incomplete set of diagrams several hands | 39 |
| Leid., BRU, BPG 74G (Diktyon 37728) | 16th a.m. | 4r-25r | 25v-48r | D | N | F | N | sch; 53v-65r Rec I extr related to Par. gr. 2450 | | |
| Leiden, BRU, Voss. gr. Qe 23 (Diktyon 38130) | 14th a.m. | 1r-30r | 31r-62v | A | N | N | N | lac 140.9 καὶ – 143.11 οἶον des 146.17 διὰ τεσσάρων | | |
| Marc. gr. Z. 309 (coll. 300) (Diktyon 69780) | 14th m. | 183v-193v | 193v-204v | A | 6 8ab | F | Y | sch [Michael of Ephesus], [Δυρραχίου] possibly related to Par. gr. 2107 | | |
| ms                | copyist       | date          | Book I   | Book II  | I.23.15 | Txt | Adj gl | remarks                                                                 | H | D |
|-------------------|---------------|---------------|----------|----------|---------|-----|--------|--------------------------------------------------------------------------|---|---|
| Marc. gr. Z. 316 (coll. 670) (Diktyon 69787) |                | 14th a.m.     | 5r-61v   | 62r-126v | A       | 8c 17 | A Y    | prol des. 77.5 περὶ sch Eustatius 129r-244v, 245r-340r Rec I             |   |   |
| Marc. gr. Z. 317 (coll. 637) (Diktyon 69788) |                | 14th m.       | 1r-23v   | 23v-49v  | B       | 12   | A N    | no comm related to Marc. gr. Z. 595                                      |   |   |
| Marc. gr. Z. 318 (coll. 994) (Diktyon 69789) |                | 14th²         | 1r-21r   | 21r-44v  | A       | 8abc | A Y    | sch Michael of Ephesus, Δυρραχίου, Eustatius Soterichos indep. of Αἰρ. possibly related to Ambros. G 62 sup. and Vat. gr. 198 |   |   |
| Marc. gr. Z. 319 (coll. 1029) (Diktyon 69790) |                | 13th m.       | 1r-39r   | 40r-75v  | BD      | 8bc 17b | A N    | sch Δυρραχίου, Eustatius                                                |   |   |
| Marc. gr. Z. 320 (coll. 638) (Diktyon 69791) |                | 1320-30       | 19r-52v  | 53r-92v  | A       | 14   | A Y    | Gregor's recension manuscript                                           |   |   |
| Marc. gr. Z. 330 (coll. 1029) (Diktyon 69801) |                | 14th a.m.     | 288r-302r | 302r-314v | B       | N    | A N    | no sch                                                                  |   |   |
| Marc. gr. Z. 333 (coll. 644) (Diktyon 69804) | Bessarion      | ca 1426       | 39v-61v  | 62r-82v  | A       | 1 part 8ac 11b | A N | 39r-61v, 62r-83r mix sch sch [Demetrios] protocensor [Michael of Ephesus], [Eustatius] final sentence Rec IV copy of Mutin. α.Τ.8.14 as of sch |   |   |
| Marc. gr. Z. 514 (coll. 771) (Diktyon 69985) |                | 14th ex.      | 6r-24v   | 25r-44r  | A       | N    | A N    | no comm, no sch, no diagr                                               |   |   |
| Marc. gr. Z. 592 (coll. 529) (Diktyon 77063) | Angelos Vergetios | 16th m.       | 1r-37r   | 37v-75v  | A       | N    | F N    | no comm, no sch, no diagr related to Par. gr. 2107                      |   |   |
| Matrit. 4678 (Diktyon 40155) |                | 14th ex.      | 7r-32v   | 32v-61r  | B       | 12   | A Y    | no comm related to Marc. gr. Z. 317                                     |   |   |
| Matrit. 4678 (Diktyon 40155) |                | 1060-80       | 4r-26v   | 27r-57v  | A       | 6    | N N    | sch 4-8 13th p.m. ad 25.9 πολλά–4r sch Chortasmenos long dittography at end Book I |   |   |

[141]
| ms                      | copyist                                 | date     | Book I            | Book II           | I.23.15 | Txt | Adj | gl | remarks                                                                 |
|-------------------------|-----------------------------------------|----------|-------------------|-------------------|---------|-----|-----|----|--------------------------------------------------------------------------|
| Monac. gr. 76 (Diktyon 44520) | Bartolomeo Zanetti 1-93, Camillo Zanetti 277-453 main 5 Mon.27 94-276 | ca 1550  | 220r-248r         | 248r-276r         | D       | N   | [1] | [4] | [5.2] | [11b] | 1r-61r, 62r-93v Rec III I.a + Rec IV 94r-160v, 161r-220r Rec I+ sch [Demetrios] protocensor likely partial copy of Cizensis 67 likely partial copy of Marc. gr. Z. 397 |
| Monac. gr. 482 (Diktyon 44930) | Rec. I Neophytos Prodromenos | 13th p.m. 14th m. | 96r-133r         | 133v-169v         | A       | [1] | 7   | [11a] | A | Y | 173r-220r, 220v-252v Rec I sch [Michael of Ephesus] sch Theodoros protocensor in Rec I |
| Mutin. α.T.8.14 (Punt. 105) (Diktyon 43425) | marg Ioannes Chortasmenos | 14th 1397-1402 | 1r-28r            | 28r-54v           | A       | 1   | [8ac] | 11b | N | N | 1r-28r, 28v-54v Rec I incl Ar sch [Demetrios] protocensor m rec sch [Michael of Ephesus], (Eustratius) integr m rec Rec I almost no primary diagrams |
| Mutin. α.U.9.7 (Puntoni 56) (Diktyon 43474) | 14th 15th in. | 1v-38v | 39r-75r | A | 1 | 11b | [A] | N | marg Adj Rec I 1v-39r, 39r-75r Rec I sch Demetrios protocensor ff. 1-12 15th in. (ad I.7.5), diff. model correct folios order 11, 8-10 lac 8.8 καθὰ – 9.9 ἀριθμητική |
| Mutin. α.W.3.1 (Punt. 245) (Diktyon 43532) | Paulos | 15th | 1r-18r | 18r-37v | C | N | B | N | 38r-88v, 89r-107v Rec IV no diagrams related to Hamburg, S-UB, philol. 88 and Vat. gr. 2387 |
| Neapol. III.C.1 (Diktyon 46277) | Petrus Creticus | ca 1433 | 1r-20v | 21r-41v | A | 14 | F | N | copy of Vindob. phil. gr. 220 |
| Neapol. III.C.6 (Diktyon 46282) | 14th m. | 1r-32r | 32r-61r | B | N | A | N | some diagrams possibly thorn away |
| ms | copyist | date | Book I | Book II | I.23.15 | Txt | Adj | gl | remarks | H | D |
|---|---|---|---|---|---|---|---|---|---|---|
| Norimb. Cent. V. App. 36 (Diktyon 46676) | Isaak Argyros | ca 1370 | 1r-18v | 18v-37v | A | 10 | F | Y | sch copy of Par. gr. 2107 | N | N |
| Oxon. Bodl. Holkham 71 (Diktyon 48139) | | 13th | 265r-278v | 279r-293v | A | 6a | 16a | 18 | sch related to Laur. Conv. Soppr. 30 | |
| Oxon. Bodl. Laud. gr. 44 (Diktyon 48266) | | 16th ex. | 25-90 | 91-188 | A | N | A | A | inc mut 17.18 ἐκθέσεσι προχειρισμένων almost no sch or diagrams in Book I copy of Par. suppl. gr. 450 | |
| Oxon. Bodl. Selden Supra 20 (Diktyon 48465) | | 14th | 40r-43v, 1r-17r | 17r-39r | B | 9bd | B | N | 8.10 εὐχρηστά – 18.5 σωρηδόν, 22.19 δὲ usque ad finem note correct folios order | 42 |
| Oxon. Lincoln Coll. gr. 33 (Diktyon 48687) | | 16th | 68r-110r | 111r-143v | D | 2 | 5.2 | E | N | 68r-110v, 111r-143v Rec II incl Ar stems from Vat. gr. 1411 | L |
| Oxon. New Coll. 299 (Diktyon 48768) | | 15th | 52r-55v | 55v-59r | E | N | N | N | O |
| Par. Coislin 174 (Diktyon 49313) | | 14th, 15th | 41r-58v | 59r-79r | A | N | [1] | [11b] | F | N | 81r-103r, 103r-118v Rec I (15th) Adj Ar et marg Rec I sch Demetrios protocensor | P9 |
| Par. gr. 2063 (Diktyon 51692) | | 14th | 9v-33v | 34r-55r | D | N | N | N | des. 140.17 αὐταῖς | |
| Par. gr. 2107 (Diktyon 51736) | | 14th | 59r-85r | 85r-112v | A | N | F | N | |
| Par. gr. 2372 (Diktyon 52004) | John Chionopoulos | 15th a.m. | 1r-26r | 26r-54r | B | N | B | N | almost no diagrams prol follows Ar | P11 |
| Par. gr. 2373 (Diktyon 52005) | | 14th | 1r-17r | 17r-35r | B | [15] | A | Y | sch m rec related to Vat. gr. 1040 | P8 |
| Par. gr. 2374 (Diktyon 52006) | John Hydruntinos | 16th | 1r-21v | 22r-42v | A | N | D | N | very peculiar diagrams; no sch possibly related to Cantab. UL Kk.V.28 | P7 |
The textual tradition of Nicomachus’ Introductio arithmetica

| ms            | copyist          | date       | Book I       | Book II      | I.23.15 | Txt | Adj | gl | remarks                                                                 |
|---------------|------------------|------------|--------------|--------------|---------|-----|-----|----|--------------------------------------------------------------------------|
| Par. gr. 2376 |                  | 16th a.m.  | 57r-78v      | 78v-101v     | A       | 3   | A   | N | 1r-34r, 34r-56r Rec III, for which conformal copy of Monac. gr. 431 Ar conformal copy of Marc. gr. Z. 318 many syll diagrams |
| (Diktyon 52008) |                  |            |              |              |         |     |     |    |                                                                          |
| Par. gr. 2377 |                  | 16th       | 1r-84v       | 89r-160r     | D       | 2   | 5.2 | N | 1r-88v, 89r-161v Rec II incl Ar sch εν ἄλλος conformal copy of Berol. Phill. 1549 |
| (Diktyon 52009) |                  |            |              |              |         |     |     |    |                                                                          |
| Par. gr. 2450 | anonymus G       | ca 1335    | 92r-111v     | 111v-131v    | A       | 14  | F   | Y | 1 sch m rec Gregoras’ recension                                           |
| (Diktyon 52082) |                  |            |              |              |         |     |     |    |                                                                          |
| Par. gr. 2479 |                  | 12th p.m.  | 1r-92v       | 110r-201v    | A       | [9bd]| [B] | N | m rec marg sch & marg Adj Soterichos splits Ar collated to Oxon. Bodl. Selden Supra 20 |
| (Diktyon 52111) |                  |            |              |              |         |     |     |    |                                                                          |
| Par. gr. 2480 | Ioannes from Siatista | ca 1733 | 1-345        | 1-319        | A       | 1   | 11a | A | 1-346, 1-319 Rec I sch Theodoros protocensor Adj also in Rec I copy of Gott. philol. 66 |
| (Diktyon 52112) |                  |            |              |              |         |     |     |    |                                                                          |
| Par. gr. 2481 |                  | 15th       | 1r-19r       | 20r-24v, 41r-53v | E   | N   | N   | N | conformal copy of Vat. gr. 1051                                          |
| (Diktyon 52113) |                  |            |              |              |         |     |     |    |                                                                          |
| Par. gr. 2483 |                  | 14th       | 5-153        | 154-313      | E       | 1   | 6a  | 16 | 3-154, 154-293 Rec I sch εν ἄλλος 315-317 Def. 138.9-10+ copy of Kharkow, UL, 269-p, 369-c as for Ar |
| (Diktyon 52115) |                  |            |              |              |         |     |     |    |                                                                          |
| Par. gr. 2531 | Michael Souliardos | 15th ex.  | 42r-80v      | 81r-124r     | A       | 1   | 11b | [A] N | marg Adj marg Rec I (part) sch Demetrios protocensor copy of Mutin. α.U.9.7 |
| (Diktyon 52163) |                  |            |              |              |         |     |     |    |                                                                          |
| Par. gr. 2762 |                  | 15th a.m.  | 13r-41v      | 41v-73v      | B       | 12  | A   | N | related to Vat. gr. 196 possibly a copy of Marc. gr. Z. 595               |
| (Diktyon 52398) |                  |            |              |              |         |     |     |    |                                                                          |

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| ms                        | copyist      | date     | Book I       | Book II      | I.23.15  | Txt | Adj | gl | remarks                                                                                                                                                                                                 |
|--------------------------|--------------|----------|--------------|--------------|----------|-----|-----|----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Par. suppl. gr. 450      |              | 15th     | 6r-35r       | 35v-68v      | A        | N   | A   | N  | almost no sch or diagrams in Book I (indirect and partial) copy of Monac. gr. 482                                                                                                                      |
| Scorial. P.II.3 (gr. 23)  |              | 16th p.m.| 262r-437r    | 437r-594v    | E        | 1   | 6a  | 16 | 262r-437r, 437r-594v Rec I+ incl. Ar. copy of Par. gr. 2483                                                                                                                                              |
| Scorial. Σ.II.15 (gr. 95) |              | 13th ex. | 160r-171v    | /            | A        | N   | A   | N  | several hands                                                                                                                                                                                          |
| Scorial. Y.III.12 (gr. 282)|             | 14th     | 1r-21r       | 21r-42r      | A        | N   | D   | N  | Adj singled out as such many diagrams missing in indent                                                                                                                                               |
| Scorial. X.I.9 (gr. 351)  |              | 1544     | 4r-48r       | 49r-91v      | A        | 8c  | 17  | A  | prol des. 77.5 περὶ sch Eustratius 95r-205r, 206r-266r Rec I copy of Marc. gr. Z. 316                                                                                                               |
| Vat. Barb. gr. 273       | Lorenzo Ciatti| 16th     | 91r-122v     | 123r-157v    | A        | 14  | F   | Y  | conformal and systematic copy of Par. gr. 2450                                                                                                                                                         |
| Vat. gr. 186              |              | 13th     | 196r-243v    | 245r-286r    | A        | 9a  | N   | N  | sch [Michael of Ephesus]                                                                                                                                                                              |
| Vat. gr. 195              |              | 1342     | 1r-43r       | 43v-84v      | A        | N   | B   | Y  | sch poor diagrams                                                                                                                                                                                     |
| Vat. gr. 196              |              | 14th     | 1r-15r       | 15r-30v      | B        | N   | C   | N  | des. 147.1 εἰσαγωγῆς                                                                                                                                                                                 |
| Vat. gr. 197              |              | 16th     | 1r-32r       | 32v-68v      | D        | N   | N   | N  | inc. 3.3 ἀκίνητα correct quires order 1, 3, 4, 2, 5                                                                                                                                                     |
| Vat. gr. 198              | Malachias    | 1360-70  | 2r-19v       | 21r-33v      | A        | 1   | 13  | [D]| marg Adj; 2r-19v, 19v-33v marg Rec I+ sch Dishypatos; sch Διρραχίου Soterichos splits Ar related to Marc. gr. Z. 318                                                                                 |

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| ms                     | copyist               | date       | Book I     | Book II    | I.23.15 | Txt | Adj | gl | remarks                                      | H | D        |
|------------------------|-----------------------|------------|------------|------------|---------|-----|-----|----|----------------------------------------------|---|----------|
| Vat. gr. 256           |                       | 14th in.   | 298r-316v  | 316v-336r  | B       | 9   | B   | N  | marg mix comm sch Michael of Ephesus related to Ambros. I 8 sup. |   |          |
| (Diktyon 66887)        |                       |            |            |            |         |     |     |    |                                                              |   |          |
| Vat. gr. 1026          |                       | 14th       | 41r-63v    | 65v-84v    | A       | 6   | [A] | Y  |                                                              |   |          |
| (Diktyon 67657)        |                       |            |            |            |         |     |     |    | Adj m rec same scholia as Matrit. 4678                      |   |          |
| Vat. gr. 1040          |                       | 14th       | 1r-20v     | 20v-42v    | B       | N   | A   | N  | 1r-4r restor. ad 19.12 ἵωα                                    |   |          |
| (Diktyon 6761)         |                       | 15th       |            |            |         |     |     |    |                                                              |   |          |
| Vat. gr. 1051          | Michael of Ephesus    | 13th ex. – | 1r-20v     | 21r-40v    | E       | N   | [A] | N  |                                                              |   |          |
| (Diktyon 67682)        | (Matrit. 4678)        | 14th in.   |            |            |         |     |     |    |                                                              |   |          |
| Vat. gr. 1411          |                       | 14th ex.   | 61r-76v    | 76v-92v    | D       | 2   | 5.2 | E  |                                                              |   |          |
| (Diktyon 68042)        |                       |            |            |            |         |     |     |    | 43r-60r, 93r-106r Rec II encl Ar                              |   |          |
| Vat. gr. 2297 + Roma, Arch. S. Paolo 24C | 13th p.m. | 3r-9v + 1r-54r | 56r-111r  | A        | 1     | 11a | [A] | N  | 1r-55r, 56r-111r Rec I sch Theodoros protocensor Adj marg Rec I |   |          |
| (Diktyon 68928 + 55907) |                       |            |            |            |         |     |     |    |                                                              |   |          |
| Vat. gr. 2387          |                       | 13th p.m.  | 32r-48v    | 49r-67v    | C       | 12  | B   | N  | sch; cf. Hamburg, S-UB, philol. 88                           |   |          |
| (Diktyon 69018)        |                       |            |            |            |         |     |     |    |                                                              |   |          |
| Vat. Ottob. gr. 310    |                       | 14th       | 124r-142r  | 142r-160r  | B       | N   | A   | N  | copy of Vat. gr. 1040                                        | V|          |
| (Diktyon 65553)        |                       |            |            |            |         |     |     |    |                                                              |  |          |
| Vindob. phil. gr. 62   |                       | 15th p.m.  | 1r-24r     | 24r-47v    | A       | N   | A   | N  | no diagrams                                                  |  |          |
| (Diktyon 71176)        |                       |            |            |            |         |     |     |    |                                                              |  |          |
| Vindob. phil. gr. 220  | Nikephoros            | 15th a.m.  | 21r-62r    | 63r-104v   | A       | 14  | F   | Y  |                                                              |  |          |
| (Diktyon 71334)        |                       |            |            |            |         |     |     |    | copy of Marc. gr. Z. 320                                     |  |          |
| Zeitz, Stiftsbibliothek 67 | John Argyropoulos    | 15th       | 36v-51v    | 51v-69r    | D       | 2   | 5.2 | E  | 3r-36r, 69v-96r Rec II encl Ar related to Vat. gr. 1411       | C|          |
| (Cizensis 67, Diktyon 72776) |                   |            |            |            |         |     |     |    |                                                              |  |          |

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| ms | copyist | date | Book I | Book II | remarks |
|----|---------|------|--------|---------|---------|
| Athens, EBE 1238  
*(Diktyon 3534)* | | 18th in. | 1r-73v | 74r-136r | marg Rec II ascr. Proclus  
ms. B D’Ooge  
possibly a copy of Athon. Meg. Laur. Θ.189 |
| Athon. Meg. Laur. Θ.189  
*(Eustr. 1051)*  
*(Diktyon 28070)* | | 17th | 1r- | -68v | marg Rec II ascr. Proclus |
| Berol. Phillipps 1469  
*(Diktyon 9370)* | | 14th | 1r-4v | | des 7.9 προδίαβεβαιωθέντα |
| Cantab. UL Gg.I.2 (gr. 1397)  
*(Diktyon 12190)* | | 15th | 21r-24r | | allegedly excerpts, but no segment of text mentioned in the catalogue is contained in Ar |
| Cantab. UL Kk.V.28 (gr. 2070)  
*(Diktyon 12211)* | Angelos Vergetios | 15th m. | 1r- | -88v | no scholia except on first three folios; nº 43 D’Ooge  
same spurious incipit as Par. gr. 2374 |
| Cantab. UL Ll.V.4 (gr. 2202)  
*(Diktyon 12224)* | | 17th | 225-253 | | marg Rec II; des mut Rec II I.πε.3 ἀρτιοπερίττου  
partial copy of Oxon. Lincoln gr. 33, as stated in the ms  
(cf catalogue) |
| Gelibolu, BK 6  
*(Diktyon 17158)* | | 17th | | | no information available |
| Kozane, DB 42  
*(Diktyon 36946)* | | 1705 | 97r- | -209v | further information not available |
| Istanbul, TSM G.I.5  
*(Diktyon 33950)* | | 14th | 9r-35v | 35v-63v | 64r-184v Rec II |
| London, BL, Harley 6295  
*(Diktyon 39691)* | Ioannes | 15th p.m. | 124v-126v | | 136.2 ύπο τῶν ἄκρων usque ad finem |
| Marc. gr. 591 (coll. 1035)  
*(Diktyon 70062)* | | 16th in. | 90r-106v | 107r-119v | 1-33.16 οἱ μὲν οὖν, 75.12 δηλονότι – 102.3 οἱ τι  
conformal copy of Vindob. phil. gr. 174 |
| ms                      | copyist                          | date   | Book I | Book II     | remarks                                      |
|-------------------------|----------------------------------|--------|--------|-------------|----------------------------------------------|
| Monac. gr. 238          | Boemundos                        | 1348–66| 1r-19r |             | des 50.2 πρὸς τῷ ὅλῳ stems from model of Roma, Arch. S. Paolo 24C + Vat. gr. 2297 copied in Southern Italy |
| Oxon. Bodl. Auct. T.4.4 | Boemundos                        | 15th-16th | 329r-343r | 280r-287ν 344r-ν | 1-37.7 εἴη, 75.12 δηλονότι – 99.15 μῆκος conformal copy of Vindob. phil. gr. 174 |
| Par. gr. 2375           | Constantinos Palearcappas        | 16th   | 1-45   |             | des 45.4 τὸ ἴσον ἐστίν partial and non exclusive copy of Par. gr. 2483 |
| Scorial. T.I.11         | Constantinos Palearcappas        | 16th   | 206-210 |             | des 10.19 ὅν[τον]                                    |
| Scorial. T.II.6         | Michael Myrocephalites Antonios Eparchos | 16th m. | 1r-98r | 103r-114v | I des 58.7 ἐπιπένθεκτος, II des. 147.1 εἰσαγωγή Book I copy of Ambros. B 77 sup. Book II comes from a different ms same desinit as Vat. gr. 196 |
| Tyranabos, DB 10 + 17   | Michael Myrocephalites Antonios Eparchos | 1675   | 115r-138v + 1r-44r | 44r-109v | I.1-18.10 (17.5), I.11.3 (27.4) usque ad finem marg Rec II ascr. Proclus |
| Vat. gr. 199            |                                  | 14th   | 1r, 2r-v |             | 1-2.12 κόσμῳ, 1-6.9 ἀλῆθειαν                                    |
| Vat. gr. 1709           |                                  | 15th a.m. | 203r-210v |             | des. 28.15 τοῦ γ· καὶ conformal copy of Norimb. Cent. V. App. 36 |
| Vindob. phil. gr. 174   | copyist AG                       | 13th p.m. | 133v-140v | 141r-145v | correct folios order 133, 135-140, 134, 141-145 1-37.7 εἴη, 75.12 δηλονότι – 102.3 οἱ i |