Modified PTS-based PAPR Reduction for FBMC-OQAM Systems

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Abstract. The filter bank multicarrier with offset quadrature amplitude modulation (FBMC-OQAM) has been raised great concern in the 5G communication research. However FBMC-OQAM has also the inherent drawback of high peak-to-average power ratio (PAPR) that should be addressed. Due to the overlapping structure of FBMC-OQAM signals, it is proven that directly employing conventional partial transmit sequence (PTS) scheme proposed for OFDM to FBMC-OQAM is ineffective. In this paper, we propose a modified PTS-based scheme by employing phase rotation factors to optimize only the phase of the sparse peak signals, called as sparse PTS (S-PTS) scheme. Theoretical analysis and simulation results show that the proposed S-PTS scheme provides a significant PAPR reduction performance with lower computational complexity.

1. Introduction

5G is a next generation mobile communication beyond 2020, which is characterized by super high mobility, high energy efficiency, high spectrum efficiency, and far ahead of 4G in terms of low latency and high transmission rate. OFDM technique used in 4G is insufficient to meet the requirements of 5G scenarios due to its sensitivity to frequency offset estimation error, time misalignment and high out-of-band emission. To overcome these shortcomings, a new multicarrier modulation scheme called FBMC (Filter Bank Multicarrier) has increasingly favored by researchers [1~3]. And the combination of FBMC with OQAM modulation can bring low side lobes, achieve maximum bit rate and spectral efficiency, and be equipped with excellent time frequency localization property, which makes FBMC modulation become one of promising candidates for future 5G technique[4~6].

However, an intrinsic drawback of FBMC-OQAM systems is high peak-to-average power ratio (PAPR). Once the high-PAPR FBMC-OQAM signals drive the high-power amplifier (HPA) to operate in the nonlinear region of its characteristics curve, it may cause undesired in-band distortion and out-of-band radiation. Therefore, high PAPR will greatly deteriorate the performance of FBMC-OQAM systems.

Since the adjacent FBMC-OQAM signals are overlapped with each other, researchers have proposed some extended or hybrid methods that are based on conventional methods to tackle the PAPR problem of OFDM systems to deal with the same problem of FBMC-OQAM systems [7~11]. These conventional methods include the selective mapping (SLM) [12], partial transmit sequence (PTS) [13] and tone reservation (TR) [14]. Though these extended or joint scheme combines the advantages of the fundamental PAPR reduction schemes and can achieve better performance, it integrates the weakness of the conventional schemes.
In this paper, in order to ensure that the new scheme causes no distortion and creates no out-of-band radiation, we have introduced a novel sparse PTS scheme, termed as S-PTS for simplicity, to reduce the PAPR in FBMC-OQAM systems. For the S-PTS scheme, the first step is to divide the interdependent transmission data into several subblocks, then needs to detect and record the location of random maximum peak of the time domain FBMC-OQAM signals derived from the superposition of the subblocks. And the second step feeds back the recorded location to the subblocks and utilizes phase factors to optimize the phase of each data on the corresponding position of the subblocks. That is to decrease the superposition of the data on that location point but does not modify any data on other positions, so as to reduce the peak power of the FBMC-OQAM signals.

The rest of this paper is organized as follows. Section II presents the FBMC-OQAM system model and the conventional PTS scheme. In section III, we describe the proposed sparse PTS scheme for FBMC-OQAM systems and analyze the computation complexity. In section IV, we study the performance of the S-PTS scheme and mainly compare with C-PTS scheme through simulation. Finally, the conclusion is given in section V.

2. FBMC-OQAM System Model

2.1 FBMC-OQAM Signals

As PHYDYAS mentioned a typical FBMC-OQAM system, which needs the OQAM modulation and polyphase network to reduce the interference of adjacent sub-channels and computational complexity [15]. The transmission mode of FBMC-OQAM signals is shown in Figure 1, where the number of the subcarriers is M and the complex input symbols numbers is M, which is defined as:

\[ X_m^n = a_m^n + j b_m^n, \quad 0 \leq n \leq N - 1, \quad 0 \leq m \leq M - 1 \]  

(1)

Where \( a_m^n \) and \( b_m^n \) denote the real and imaginary parts of the mth complex symbol \( X_m^n \) on the nth subcarrier, respectively. For OQAM, the real and imaginary parts of the complex symbols are staggered in time domain by \( T/2 \), where \( T \) is the symbol period (equivalent to \( N \)). Then, the symbols are passed through a bank of transmission filters and modulated with subcarrier modulators where the spacing between every two subcarriers is \( 1/T \). In that way, the mth transmission FBMC-OQAM symbol can be obtained by adding all subcarrier signals, which can be expressed as:

\[ S_m^m(t) = \sum_{n=0}^{N-1} \left( a_m^n h(t - mT) + j b_m^n h(t - mT - \frac{T}{2}) e^{j\pi(2n+1)/2} \right) e^{j\pi(2n+1)/2} \]  

(2)

Where \( h(t) \) is impulse response of the prototype filter with KT length and K is the overlapping factor[15]. The FBMC-OQAM symbol duration is extended by KT and the transmission symbol rate is still \( 1/T \), which means that K consecutive output symbols overlap with each other in the time domain. The overlapping structure of FBMC-OQAM signals is described in Figure 2. Then the length of FBMC-OQAM signal \( S(t) \) that are derived from summation of the M FBMC-OQAM symbols is \( (M+K-1/2)/T \), i.e.,

\[ S(t) = \sum_{m=0}^{M-1} S_m^m(t), \quad 0 \leq t \leq (M + K - \frac{1}{2})T \]  

(3)
2.2 Analysis of PAPR in FBMC-OQAM System

We need to consider multiple overlapped data blocks when reducing the PAPR of FBMC-OQAM systems, rather than each block independently like OFDM system. Thus, the general definition of PAPR for OFDM systems is not applicable to FBMC-OQAM systems. Then, we divide the main value of $S(t)$ into $M$ intervals equally with the time duration $T$. Thus, the PAPR of each interval is defined by

$$PAPR_i = 10 \log_{10}\left(\frac{\max_{0 \leq s < T} |S(t)|^2}{E[|S(t)|^2]}\right), \quad i = 0, 1, \ldots, M - 1$$

(4)

Where $E[|S(t)|^2]$ denotes the expectation, i.e., the average power of FBMC-OQAM signals. Similar to OFDM system, we still utilize complementary cumulative distribution function (CCDF) to measure the PAPR performance of FBMC-OQAM system, which is defined as the probability that PAPR exceeds a given threshold $PAPR_0$. 

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**Figure 1** The transmission mode of the FBMC-OQAM signals

**Figure 2** Structure of the FBMC-OQAM signals
2.3 Conventional PTS Scheme for FBMC-OQAM

For the conventional PTS (C-PTS) scheme, the subcarriers are divided into V subblocks and each subblock has \( \frac{N}{V} \) subcarriers. Then, each disjoint subblock \( S_m v(t) \) is multiplied with corresponding phase rotation factor \( b_v, v=1,2,...,V \), and the elements in the vector \( b_v \) are usually selected from the set \( \{-1,1\} \).

\[
S^m(t) = \sum_{v=1}^{V} b_v S^m_v(t) \tag{5}
\]

The phase vector is chosen for the signals with least PAPR, which is shown as

\[
\arg \min_{b_v} \max_{v} \left| \sum_{v=1}^{V} b_v S^m_v(t) \right|^2 \tag{6}
\]

In general, an exhaustive searching method is used in Equation (6) to find the optimal solution, and the phase optimization is applied to each data of FBMC-OQAM signals, which is bound to cause a large amount of computation. Moreover, affected by the overlapping structure of consecutive FBMC-OQAM signals, directly applying the C-PTS scheme to reduce the PAPR of FBMC-OQAM systems is not very effective.

In addition, in order to improve the effect of the C-PTS scheme to reduce PAPR, we divide the overlapped filtered signals into several segments and then pick out an optimal phase rotation factors with exhaustive searching method in each segment. And the simulation results will be presented in section IV.

3. PAPR Reduction Based on Sparse PTS Scheme

In this section, we propose a sparse PTS scheme to reduce the PAPR of the FBMC-OQAM signals with better PAPR reduction performance and lower complexity than the conventional PTS scheme.

3.1 Proposed S-PTS scheme

Similar to the conventional PTS scheme, the preliminary procedure for the proposed S-PTS scheme is to transmit the interdependent input signals in packet mode. Thus, the \( m \)th original FBMC-OQAM signals can also be expressed as follows:

\[
S^m(t) = \sum_{v=0}^{V-1} \sum_{n=0}^{N/V-1} [a^m_n + v(N/V)] h(t - mT) + j b^m_{n + v(N/V)} h(t - mT - \frac{T}{2}) e^{j(n + v(N/V))\frac{2\pi f}{T} + \frac{\pi}{2}} \tag{7}
\]

![Figure 3 Structure diagram of FBMC-OQAM system combined with S-PTS scheme](image-url)
Figure 3 illustrate a block diagram. The novel procedure is to detect and record the location of maximum peak of the time domain signals $S(t)$ and employ phase factors to optimize the phase of the data on the recorded position of the subblocks. Then, we set an iteration value $I$ to detect the position of peak signals repeatedly in order to reduce the peak power of FBMC-OQAM signals effectively.

The algorithm of S-PTS scheme involves the following steps:

Detect the location point of maximum peak of FBMC-OQAM signals and record it as $P_i$, as shown in Figure.3. Assume the total number of iteration as $I (i=1,2,...,I)$. Look for the data $Div (v=1,2,...,V)$ at the corresponding position $P_i$ in the subblocks and each data is rotated with different phase rotation factors $b_v \in \{-1,1\}$, so as to keep these data phase from being consistent at the same time and modify their summation at this position $P_i$. Find out a best one ($b_1, b_2,..., b_V$) from $2V$ phase rotation factor combinations to achieve the least PAPR for signals. Then multiply the data $Div (v=1,2,...,V)$ by the best combination ($b_1, b_2,..., b_V$) to obtain the new data $(b_1Di_1, b_2Di_2,..., b_VDi_V)$ with the minimum summation. Use $(b_1Di_1, b_2Di_2,..., b_VDi_V)$ to update the data at the current position $P_i$ and generate the optimized subblocks.

Repeat the step a to step d until the iteration meets the value of $I$. Finally, the time domain FBMC-OQAM signals with the least PAPR that are derived from the superposition of the optimized subblocks are selected for transmission.

The difference between the C-PTS and S-PTS schemes lies in that all the data are taken into account to select the optimal phase rotation combination in the C-PTS scheme, while the proposed S-PTS scheme only executes phase optimization operation for few data with high peak, without changing any data on other positions. This means that computational complexity of the proposed S-PTS scheme would be much lower than that of the C-PTS scheme. In addition, since the occurring probability of the maximum peak of FBMC-OQAM signals is relatively low, only a few maximum peaks need to be processed with phase optimization so that its impact on the average power of FBMC-OQAM signals is imperceptible. It is explained theoretically that our proposed S-PTS scheme can make the PAPR of FBMC-OQAM signals decrease significantly.

### 3.2 Computational Complexity Analysis of S-PTS scheme

In this subsection, we analyze the computational complexity of the proposed S-PTS scheme by comparing with the C-PTS scheme. We mainly analyze the computation of phase optimization operation in both schemes, ignoring the computation of the same and small part. In order to find out the best combination of phase factors from $2V$ candidates which are derived from exhaustive search, the proposed S-PTS scheme requires $IV2^V$ complex multiplications and $2I(V-1)2^V$ complex additions, while the C-PTS scheme requires $NMV2^V$ complex multiplications and $2NM(V-1)2^V$ complex additions. Finally, when determining the transmission signals with the least PAPR, the S-PTS and C-PTS schemes require $IV$ and $NMV$ complex multiplications, $2I(V-1)$ and $2NM(V-1)$ complex additions, respectively. Then, the numbers of required real multiplications and real additions for the two schemes are listed in Table 1.

| Scheme   | Real multiplications | Real additions  |
|----------|----------------------|-----------------|
| S-PTS    | $IV(2^V+1)$          | $2I(V-1)(2^V+1)$|
| C-PTS    | $NMV(2^V+1)$         | $2NM(V-1)(2^V+1)$|

From the Table 1 we can see that the number of iteration $I$ is directly related to the computational complexity in the proposed S-PTS scheme. The probability of maximum peak of FBMC-OQAM signal is relatively low, which means the iteration times $I$ is much less than the value of $NM$ in the C-PTS scheme. Therefore, the computational complexity of the S-PTS scheme will be far less than that of the C-PTS scheme. Similarly, compared to the C-PTS scheme, the recovery of signals phase for the PTF scheme at the receiver also only needs a small amount of calculation.
3.3 Analysis of Side Information in S-PTS Scheme
As we all know, PTS schemes need to send side information (SI) about the phase rotation factors to recover the original phase of signals at the receiving end. For the proposed S-PTS scheme, the side information increases with the increasing of iteration number I, while the side information increases with the increasing of segment number S. Compared with the C-PTS scheme, the superior PAPR reduction performance of the proposed S-PTS scheme is not at the expense of adding additional side information. And the simulation results will be presented in section IV.

4. Simulation Results
In this section, extensive simulations are conducted to assess the PAPR reduction performance of the proposed S-PTS scheme. Table II shows the simulation parameters for FBMC-OQAM system.

| Table 2 Simulation Parameters |
|-------------------------------|
| Modulation mode               | 4QOAM            |
| Number of subcarriers         | \( N = 64 \)     |
| Number of sub-blocks          | \( V = 4 \)      |
| Phase rotation factor         | \( b_i = \{1, -1\} \) |
| Number of iteration (S-PTS)   | \( I = 6, 8, 12, 16 \) |
| Number of segment (C-PTS)     | \( S = 1, 8, 16 \) |

Figure 4 shows the PAPR reduction of the S-PTS scheme with \( I = 6, 8, 12, 16 \) and the C-PTS scheme with \( S = 1, 8, 16 \), respectively. The curve “Original” indicates the FBMC-OQAM signals without any PAPR reduction schemes. It is obvious that increasing I is more effective to decrease the value of PAPR for the S-PTS scheme. If the CCDF is fixed at the value of 10^{-3}, the PAPR values of the proposed S-PTS scheme for \( I = 6, 8, 12, 16 \) are about 2.5dB, 2.7dB, 3.1dB, 3.4dB smaller than the original FBMC-OQAM signals, respectively. Therefore, the S-PTS scheme could significantly reduce the PAPR of the FBMC-OQAM signals.

**Figure 4 CCDFs of the proposed S-PTS and C-PTS schemes**

Furthermore, we compare the PAPR reduction performances of the S-PTS scheme with that of the C-PTS scheme in Figure 4. First of all, we can see that when we directly apply the C-PTS scheme without dividing segments (\( S = 1 \)), it is not very effective to reduce the PAPR of the FBMC-OQAM signals. And the value of PAPR for the C-PTS scheme decrease faster as the increasing of segments S. Nevertheless, at CCDF=10^{-3}, the PAPR of the proposed S-PTS scheme with \( I = 8 \) is about 7.6dB, smaller than 7.7dB of the C-PTS with \( S = 8 \). Besides, the PAPR of the proposed S-PTS scheme with
I=16 is about 7.0dB, smaller than 7.5dB of the C-PTS with S=16. That is to say, in the case where the amount of side information is the same, the proposed S-PTS scheme can achieve better PAPR reduction performance than the C-PTS scheme. In addition, the PAPR of the proposed S-PTS scheme with I=12 is 7.3dB, less than 7.5dB of the C-PTS with S=16, which indicates that our proposed S-PTS scheme can achieve superior PAPR reduction performance when it has less side information.

Moreover, we substitute the simulation parameters into the equations in Table I to calculate the numbers of the real multiplications and real additions, as shown in Table III.

**Table 3 The Computational Complexity of the Two Schemes**

| Scheme | Real multiplications | Real additions | PAPR performance |
|--------|----------------------|----------------|------------------|
| C-PTS  | S=1                  | 69 632         | 104 448          | 8.6   |
|        | S=8                  | 69 632         | 104 448          | 7.7   |
|        | S=16                 | 69 632         | 104 448          | 7.5   |
|        | I=6                  | 408            | 612              | 7.8   |
| S-PTS  | V=4                  |                |                  |       |
|        | I=8                  | 544            | 816              | 7.6   |
|        | I=12                 | 816            | 1 224            | 7.3   |
|        | I=16                 | 1 088          | 1 632            | 7.0   |

As can be seen in the Table III, the computational complexity of our proposed S-PTS scheme with various parameters are much smaller than that of the C-PTS scheme.

Figure 5 shows the peak canceling signals of the proposed S-PTS scheme with I=16 and original signals. It can be found that, peak powers of the original signal appear in a few points, majority of points have lower powers, which means that the calculations of our proposed S-PTS scheme will not be too complicated. And the data with the exception of peak signals in black curve coincides with the data in red curve and is covered in the simulation. Obviously, the whole amplitude of the red curve is lower than 2.5, which means our proposed S-PTS can efficiently reduce the PAPR of the FBMC-OQAM signals.

**Figure 5 peak canceling signals with PTF scheme and original signals**

Therefore, in general, our proposed S-PTS scheme has more excellent PAPR reduction performance and lower computational complexity compared with the C-PTS scheme.

5. Conclusions

In this paper, we propose a novel sparse PTS scheme with low complexity to reduce the PAPR of FBMC-OQAM systems. In order not to cause distortion and waste of frequency resources, the key of
this scheme is to use the typical phase rotation factors to optimize the phase of the peak signals derived from peak detection of the filtered signals. Simulation results show that, compared with the C-PTS scheme, our proposed not only achieves better PAPR reduction performance but also has lower computational complexity.

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7. References

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