Dilatonic Dark Matter
– A New Paradigm –

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Abstract

We study the possibility that the dilaton plays the role of the dark matter of the universe. We find that the condition for the dilaton to be the dark matter of the universe strongly restricts its mass to be around 0.5 keV or 270 MeV. For the other mass ranges, the dilaton either undercloses or overcloses the universe. The 0.5 keV dilaton has the free-streaming distance of about 1.4 Mpc and becomes an excellent candidate of a warm dark matter, while the 270 MeV one has the free-streaming distance of about 7.4 pc and becomes a cold dark matter. We discuss the possible ways to detect the dilaton experimentally.

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The standard big bang cosmology has been very successful in many ways. But at a deeper level the model also raises more challenges, and may need a generalization. This is because the standard model is based on the Einstein’s theory of gravitation, which itself may need a generalization. Of course the Einstein’s theory has been a most beautiful and successful theory of gravitation. But from the logical point of view there are many indications that something is missing in the Einstein’s theory. We mention just a few:

(1) The unification of all interactions inevitably requires the existence of a fundamental spin-zero field. In fact all modern unified theories, from the Kaluza-Klein theory to the superstring, contain such a fundamental scalar field. What makes this scalar field unique is that, unlike others like the Higgs field, it couples directly to the (trace of) energy-momentum tensor of the matter field. As such it should generate a new force which will modify the Einstein’s gravitation in a fundamental way.

(2) The Newton’s constant $G$, which is supposed to be one of the fundamental constants of Nature, plays a crucial role in Einstein’s theory. But the ratio between the electromagnetic fine structure constant $\alpha_e$ and the gravitational fine structure constant $\alpha_g$ of the hydrogen atom is too small to be considered natural, $\alpha_g/\alpha_e \simeq 10^{-40}$. This implies that $G$ may not be a fundamental constant, but in fact a time-dependent parameter. If so, one must treat it as a fundamental scalar field which couples to all matter fields. Obviously this requires a drastic generalization of Einstein’s theory.

(3) In cosmology the inflation at the early stage of evolution may be unavoidable. But for a successful inflation we need a dynamical mechanism which can (not only initiate but also) stop the inflation smoothly. Unfortunately the Einstein’s gravitation alone can not provide enough attraction. Again one may need a scalar field which could generate an extra attractive force.

All these arguments, although mutually independent, suggest the existence of a fundamental scalar field which we call the dilaton which could affect the gravitation (and consequently the cosmology) in a fundamental way. In the following we discuss how the dilaton comes about in the unified field theories, and how it could provide the dark matter of the universe.

Let us first consider the $(4 + n)$-dimensional Kaluza-Klein theory whose fundamental in-
gradient is the \((4 + n)\)-dimensional metric \(g_{AB} (A, B = 0, 1, \cdots, 3 + n)\)

\[
g_{AB} = \begin{pmatrix}
    \tilde{g}_{\mu\nu} + e_0 \kappa_0 \phi_{ab} A^a_{\mu} A^b_{\nu} & e_0 \kappa_0 A^a_{\mu} \phi_{ab} \\
    e_0 \kappa_0 \phi_{ab} A^b_{\nu} & \phi_{ab}
\end{pmatrix},
\]

(1)

where \(e_0\) is a coupling constant, and \(\kappa_0\) is a scale parameter which sets the scale of the \(n\)-dimensional internal space. When the metric has an \(n\)-dimensional isometry \(G\), one can reduce the \((4 + n)\)-dimensional Einstein’s theory to a 4-dimensional unified theory[2, 3]. Indeed with \(e_0^2 \kappa_0^2 = 16 \pi G\), \(\tilde{g} = |\det \tilde{g}_{\mu\nu}|, \phi = |\det \phi_{ab}|\), and \(\rho_{ab} = \phi^{-\frac{1}{n}} \phi_{ab} (|\det \rho_{ab}| = 1)\), the \((4 + n)\)-dimensional Einstein’s theory is reduced to the following 4-dimensional Einstein-Yang-Mills theory,

\[
\mathcal{L}_0 = -\frac{1}{16 \pi G} \sqrt{\phi} \sqrt{\tilde{g}} \left[ \tilde{R} + \tilde{S} + 4 \pi G \phi^\frac{1}{2} \rho_{ab} F^a_{\mu\nu} F^b_{\mu\nu} - \frac{n - 1}{4n} \left( \frac{\partial_{\mu} \phi}{\phi} \right)^2 \right] + \frac{1}{4} \rho^{ab} \rho^{cd} (D_{\mu} \rho_{ac})(D_{\mu} \rho_{bd}) + \Lambda + \lambda (|\det \rho_{ab}| - 1) + \cdots,
\]

(2)

where \(\tilde{R}\) and \(\tilde{S}\) are the scalar curvature of \(\tilde{g}_{\mu\nu}\) and \(\phi_{ab}\), \(\Lambda\) is the \((4 + n)\)-dimensional cosmological constant, \(\lambda\) is a Lagrange multiplier. But notice that the above Lagrangian has a crucial defect. First the metric \(\tilde{g}_{\mu\nu}\) does not represent the Einstein’s gravitation because \(\tilde{g}\) does not describe the proper 4-dimensional volume element. Furthermore the \(\phi\)-field appears with a negative kinetic energy, and thus can not be treated as a physical field. To cure this defect one must perform the following conformal transformation, and introduce the Einstein metric \(g_{\mu\nu}\) and the dilaton field \(\sigma\) by [3]

\[
g_{\mu\nu} = \sqrt{\phi} \tilde{g}_{\mu\nu}, \quad \sigma = \frac{1}{2} \left( \frac{n + 2}{n} \right) \ln \phi.
\]

(3)

With this the Lagrangian (2) is written as

\[
\mathcal{L}_0 = -\frac{\sqrt{\phi}}{16 \pi G} \left[ R + \frac{1}{2} (\partial_{\mu} \sigma)^2 + 4 \pi G e^{\alpha \sigma} \rho_{ab} F^a_{\mu\nu} F^b_{\mu\nu} \right] + \frac{1}{4} (D_{\mu} \rho^{ab})(D_{\mu} \rho_{ab}) + \Lambda (|\det \rho_{ab}| - 1) + \cdots,
\]

(4)

where \(R\) and \(S\) are the scalar curvature of \(g_{\mu\nu}\) and \(\rho_{ab}\), \(\alpha\) and \(\beta\) are the coupling constants given by \(\alpha = \sqrt{(n + 2)/n}\) and \(\beta = \sqrt{n/(n + 2)}\). This shows that in the Kaluza-Klein theory the dilaton appears as the volume element of the internal metric which, as a component of the metric \(g_{AB}\), must couple to all matter fields. In the superstring theory the dilaton appears as
the massless scalar field that the mass spectrum of the closed string must contain. After the full string loop expansion, the 4-dimensional effective Lagrangian of the massless modes has the following form in the string frame

\[ L_S = -\frac{\sqrt{\tilde{g}}}{\alpha'} \left[ \tilde{C}_g(\varphi)\tilde{R} + \tilde{C}_\varphi(\varphi)(\partial_\mu \varphi)^2 - \frac{\alpha'}{4} \tilde{C}_1(\varphi)(F_{\mu\nu}^a)^2 + \cdots \right], \tag{5} \]

where \( \alpha' \) is the string slope parameter, \( \tilde{g}_{\mu\nu} \) is the string frame metric, \( \varphi \) is the string dilaton, and \( \tilde{C}_i(\varphi) \) \((i = g, \varphi, 1, 2, 3, \cdots)\) are the dilaton coupling functions to various fields. At present their exact forms are not known beyond the fact that in the limit \( \varphi \) goes to \(-\infty\) they should admit the following loop expansion

\[ \tilde{C}_i(\varphi) = e^{-2\varphi} + a_i + b_i e^{2\varphi} + c_i e^{4\varphi} + \cdots. \tag{6} \]

Now introducing the Einstein metric \( g_{\mu\nu} \) with a conformal transformation and replacing the original dilaton field \( \varphi \) with a new one \( \sigma \), one may put the Lagrangian (5) into the following standard form

\[ L_S = -\frac{\sqrt{g}}{\alpha'} \left[ R + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{\alpha'}{4} C_1(\sigma) F_{\mu\nu}^a F_{\mu\nu}^a + \cdots \right]. \tag{7} \]

Notice that in the standard form the dilaton coupling function to gravity \( \tilde{C}_g(\varphi) \) and the self coupling function \( \tilde{C}_\varphi(\varphi) \) disappear completely with the redefinition of the fields. Only the coupling functions to the other matter fields remain.

The Lagrangian (7) looks very much like the Lagrangian (4). In both cases the dilaton appears as a fundamental scalar field. Of course there are some differences. One is the form of the dilatonic coupling functions to various matter fields. In the Kaluza-Klein theory they have simple exponential forms, whereas in the string theory their explicit forms are not known. Another is the mass of the dilaton. In the Kaluza-Klein theory the dilaton can easily acquire a mass, but in the superstring theory it remains massless to all orders of perturbation. But these differences may not be so serious as it appears. To understand this, notice that the Lagrangian (4) is valid only at the tree level. So with the quantum correction in the Kaluza-Klein theory, the difference in the dilaton coupling functions between the two theories becomes insignificant. As for the mass of the dilaton, there is no fundamental principle which can keep
it massless, even enough the perturbative expansion leaves it massless in the string theory. So it could acquire a mass through some unknown non-pertubative or topological mechanism. From this one may conclude that as far as the dilaton is concerned the string theory and the Kaluza-Klein theory give us practically the same effective Lagrangian, at least in the low energy approximation. In both cases the dilaton comes in as the spin-zero partner of the Einstein’s spin-two graviton. So in the unified theories one must take the dilatonic modification of the Einstein’s theory seriously, whether one likes it or not.

As this point it should also be mentioned that the Brans-Dicke theory is very similar to the above unified theories, as far as the dilaton is concerned[6]. In fact the Brans-Dicke theory can be viewed as a 5-dimensional Kaluza-Klein theory, so that the unified theories could be regarded as the “generalized” Brans-Dicke theories. There are a few characteristic features common in all these theories:

1) It is the Einstein metric $g_{\mu\nu}$, not the string frame metric $\tilde{g}_{\mu\nu}$, which describes the massless spin-two graviton and thus the Einstein’s gravitation[3]. In fact the $\tilde{g}_{\mu\nu}$ is a strange mixture of the spin-two graviton and spin-zero dilaton which does not even describe a mass eigenstate. This tells that, when one wants to compare the theory with the Einstein’s gravitation, one must use the Einstein frame.

2) The unified theories could easily accommodate a successful inflation, since the dilaton could play the role of the inflaton[7, 8]. More significantly the dilaton could be the dark matter of the universe, because in the unified cosmology the dilatonic matter could easily become the dominant matter of the universe[9].

3) The dilaton describes a “fifth force” which modifies the Einstein’s gravitation[10]. This implies that an apparent violation of the equivalence principle must take place in the unified theories. So an important issue in these theories is how to minimize the modification of the Einstein’s theory and the violation of the equivalence principle to an acceptable level[5, 11].

Now we discuss the dilatonic dark matter. To see how the dilaton reaches the thermal equilibrium notice that the dominant interaction modes of the dilaton with other matter fields are the creation(annihilation) process quark + gluon to quark + dilaton, and the scattering process quark + dilaton to quark + dilaton. Normally the dilatonic coupling strength would
be $\alpha m_q/m_p$, where $\alpha$ is the coupling constant, $m_p$ is the Planck mass, and $m_q$ is the mass of the quarks. But notice that at high temperature (at $T \gg m_q$), the coupling strength becomes $\alpha T/m_p$. With this one can easily estimate the dilaton creation (and annihilation) cross section $\sigma \simeq g^2 \alpha^2 (T/m_p)^2 \times 1/T^2$, so that the creation rate $\Gamma$ is given by

$$\Gamma \simeq n_q \sigma v \simeq g^2 \alpha^2 \left(\frac{T}{m_p}\right)^2 \times T.$$  

(8)

Similarly the scattering cross section $\sigma$ is given by $\sigma \simeq \alpha^4 (T/m_p)^4 \times 1/T^2$, with the following interaction rate $\Gamma$

$$\Gamma \simeq n_q \sigma v \simeq \alpha^4 \left(\frac{T}{m_p}\right)^4 \times T.$$  

(9)

On the other hand the Hubble expansion rate $H$ in the early universe is given by $H \simeq T^2/m_p$.

From this we conclude that the dilaton is thermally produced from the beginning, and decouples with the other sources at around the Planck scale with the decoupling temperature $T_d$ given by

$$T_d \simeq \frac{m_p}{\alpha^{3/2}}.$$  

(10)

Notice that the dilaton decouples with the other sources at around the same time as the graviton does. This is indeed what one would have expected, since the dilaton is nothing but the scalar counterpart of the Einstein’s graviton.

Once the dilaton acquires a mass, it becomes unstable and decays to the ordinary matter. A typical decay process is the two photon process and the fermion pair production process described by the following interaction Lagrangian

$$L_{int} \simeq -\frac{\alpha}{4} \sqrt{16\pi G} \phi F_{\mu\nu}F^{\mu\nu} - \beta \sqrt{16\pi G} m \phi \bar{\psi} \psi.$$  

(11)

For the two photon process we obtain the following life-time at the tree level\(^\text{[12]}\)

$$\tau_1 \simeq \frac{16}{\alpha^2} \left(\frac{m_p}{\mu}\right)^2 \frac{1}{\mu}.$$  

(12)

where $\mu$ is the mass of the dilaton. Similarly for the pair production we obtain

$$\tau_2 \simeq \frac{1}{2\beta^2} \left(1 - 4 \frac{m^2}{\mu^2}\right)^{-3/2} \left(\frac{m_p}{m}\right)^2 \frac{1}{\mu} \geq \frac{5.38}{\beta^2} \left(\frac{m_p}{\mu}\right)^2 \frac{1}{\mu}.$$  

(13)

A more detailed calculation which includes all possible decay channels allowed in the standard electroweak model gives us Fig.1 for the dilaton life-time with respect to its mass. Notice that
here we have assumed $\alpha \simeq \beta \simeq 1$ for simplicity, but it should be kept in mind that in reality the coupling constants could turn out to be much smaller\cite{12}.

To estimate how much the dilaton contributes to the matter density of the present universe one must estimate the number density of the dilaton at present time. From the entropy conservation of the universe one can easily estimate the present temperature $T_\phi$ of the dilaton. Based on the standard electroweak theory one finds

$$T_\phi \leq \left( \frac{3.91}{106.75} \right)^{1/3} T_0 \simeq 0.91^\circ K, \quad (14)$$

where $T_0$ is the present temperature of the background radiation. Notice that again this is the temperature of the graviton at present time. From this one can estimate the number density $n_0$ of the dilaton at present. Assuming that the dilaton is stable one has

$$n_0 = \frac{\zeta(3)}{\pi^2} T_\phi^3 \simeq 7.5 / cm^3. \quad (15)$$

But obviously, the massive dilaton can not be stable, and the number density of the dilaton $n(\mu)$ must crucially depend on its mass. So for the dilaton to provide the critical mass of the universe one must have

$$\rho(\mu) = n(\mu) \times \mu = n_0 e^{-t_0/\tau(\mu)} \times \mu \simeq 10.5 \ h^2 \ keV/cm^3. \quad (16)$$

where $t_0$ is the age of the universe, $\tau(\mu)$ is the life-time of the dilaton, and $h$ is the Hubble constant (in the unit of 100Km/sec Mpc). A numerical calculation with $t_0 \simeq 1.5 \times 10^{10}$ years shows that there are two mass ranges, $\mu \simeq 0.5$ keV or $\mu \simeq 270$ MeV, which can make the dilaton a candidate of the dark matter in the universe. In Table I the interesting physical quantites are shown for different values of $h$.

Notice that with $h \simeq 0.6$ the mass becomes 0.5 keV or 270 MeV. Also notice that the $\rho(\mu)$ starts from zero when $\mu = 0$ and reaches the maximum value at $0.5 \text{ keV} < \mu < 270 \text{ MeV}$ and again decreases to zero when $\mu = \infty$. This means that when $\mu < 0.5 \text{ keV}$ or $\mu > 270 \text{ MeV}$ the dilaton undercloses the universe, but when $0.5 \text{ keV} < \mu < 270 \text{ MeV}$ it overcloses the universe. From this one may conclude that the dilaton with $0.5 \text{ keV} < \mu < 270 \text{ MeV}$ is not acceptable because this is incompatible with the cosmology. In view of the fact that the dilaton must exist
in all the unified field theories, the above constraint on the mass of the dilaton should provide us an important piece of information in search of the dilaton.

Now we discuss the possibility of the dilatonic dark matter in more detail:

a) $\mu \simeq 0.5$ keV. In this case the available decay channel is the $\gamma\gamma$ process. So the life-time is given by $\tau \simeq 4.0 \times 10^{26}$ years, which tells that it is almost stable. To determine whether this dilaton could serve as a hot or cold dark matter, one must estimate the free-streaming distance $\lambda$ of the dilaton. The dilaton becomes non-relativistic around $T \simeq \mu/3 \simeq 0.17$ keV, long before the matter-radiation equilibrium era. In terms of time this corresponds to

$$t_{NR} \simeq 1.2 \times 10^7 \times \left(\frac{keV}{\mu}\right)^2 \left(\frac{T_\phi}{T_0}\right)^2 \text{sec} \simeq 1.88 \times 10^6 \text{sec.}$$

From this one obtains

$$\lambda \simeq 0.16 \left(\frac{keV}{\mu}\right) \left(\frac{T_\phi}{T_0}\right) \left[\ln \left(\frac{t_{EQ}}{t_{NR}}\right) + 2\right] \text{Mpc}$$

$$\simeq 1.4 \text{ Mpc.}$$ (18)

Certainly this is a very interesting number, which tells that the 0.5 keV dilaton becomes an excellent candidate of a warm dark matter.

b) $\mu \simeq 270$ MeV. In this case the available decay channels are the $\gamma\gamma$, $e^+e^-$, and $\mu^+\mu^-$ processes (the $\nu\bar{\nu}$ processes are assumed to be negligible). The decay processes $\gamma\gamma, \mu^+\mu^-$ are dominant at this energy level, and have almost the same decay width (See Table I). The corresponding life-time is given by $\tau \simeq 1.1 \times 10^9$ years, so that only a fraction of the thermal dilaton survives now. For this dilaton one has

$$t_{NR} \simeq 1.82 \times 10^{-5} \text{sec,}$$

and the corresponding free-streaming distance becomes

$$\lambda \simeq 7.35 \text{ pc.}$$

Clearly this dilaton becomes a good candidate for a cold dark matter.

Now the important question is how one could detect the dilaton. It seems very difficult to detect it through the dilatonic fifth force, because the range of the fifth force would be about $10^{-8}$ cm (for $\mu = 0.5$ keV) or about $10^{-13}$ cm (for $\mu = 270$ MeV). Perhaps a more promising
way is to use the two photon decay process, which produces two mono-energetic X-rays of $E \simeq 0.25$ keV or $E \simeq 135$ MeV with the same polarization. With the local halo density of our galaxy $\rho_{\text{HALO}} \simeq 0.3$ GeV/cm$^3$ one can easily find the local dilaton number density to be $\bar{n} \simeq 5.83 \times 10^5$/cm$^3$ for $\mu = 0.5$ keV and $\bar{n} \simeq 0.11$/cm$^3$ for $\mu = 270$ MeV. In both cases the local velocity of the dilaton is about $10^{-3}$ c. So it is very important to look for the above X-ray signals from the sky (with the Doppler broadening of $\Delta E \simeq 10^{-3}E$) or to perform a Sikivie-type X-ray detection experiment with a strong electromagnetic field to enhance the dilaton conversion, although the long life-time (for $\mu = 0.5$ keV) or the low local number density (for $\mu = 270$ MeV) of the dilaton could make such experiments very difficult. For the $\mu = 270$ MeV dilaton one could also look for the $\mu^+\mu^-$ decay process.

One might try to detect the dilaton from the accelerator experiments. The dilaton has a clear decay signal, but the production rate should be very small due to the extreme weak coupling. So one need a huge luminosity to produce the dilaton from the accelerators. There are, of course, other (indirect) ways to test the existence of the dilaton. For example, it may be worth to look for the impacts of the dilaton in the stellar evolution and the supernovae explosion. We will discuss these in a separate paper[12].

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Table 1: The dilatonic dark matter and its mass, decay widths, and total life-time for different values of $h$.

| $h$ | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|-----|-----|-----|-----|-----|-----|
| $\mu$ (keV) | 0.224 | 0.350 | 0.504 | 0.686 | 0.896 |
| $\tau_{tot}$ ($10^{26}$ years) | 44.2 | 11.6 | 3.88 | 1.54 | 0.69 |
| $\mu$ (MeV) | 274.8 | 272.7 | 271.0 | 269.6 | 268.3 |
| $\Gamma_{\gamma\gamma}$ ($10^{-40}$ MeV) | 87.15 | 85.15 | 83.57 | 82.26 | 81.11 |
| $\Gamma_{e^+e^-}$ ($10^{-55}$ MeV) | 9.60 | 9.53 | 9.47 | 9.42 | 9.38 |
| $\Gamma_{\mu^+\mu^-}$ ($10^{-40}$ MeV) | 107.71 | 103.36 | 99.74 | 96.78 | 94.17 |
| $\Gamma_{tot}$ ($10^{-40}$ MeV) | 195.86 | 188.55 | 183.30 | 179.04 | 175.28 |
| $\tau_{tot}$ ($10^{10}$ years) | 0.107 | 0.111 | 0.114 | 0.116 | 0.119 |

Figure Caption

Figure 1: The dilatonic dark matter and its mass, decay widths, and total life-time for different values of $h$ based on the standard electroweak model. The result is obtained with the assumption that the dilatonic coupling constants to the ordinary matters are of the order one.
FIGURE

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Graph showing the relationship between \( \log \tau \) [years] and \( \log \mu [\text{MeV}] \).}
\end{figure}

Fig. 1