Internal Excitations and Dissipative Damping of Quantum Hall Skyrmions

H.A. Fertig\textsuperscript{1}, L. Brey\textsuperscript{2}, R. Côté\textsuperscript{3}, and A.H. MacDonald\textsuperscript{4}

1. Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506-0055.
2. Instituto de Ciencia de Materiales (CSIC), Universidad Autónoma C-12 28049 Madrid, Spain.
3. Université de Sherbrooke, Département de Physique, Sherbrooke, Québec, Canada J1K 2R1
4. Department of Physics, Indiana University, Bloomington, IN 47405

We propose an intrinsic maximum speed for dissipationless E cross B drift of Skyrmion quasiparticles in quantum Hall ferromagnets. When this speed is exceeded, Skyrmions can radiate spin-waves by making internal excitations which allow total spin to be conserved. Our proposal is illustrated by a time-dependent Hartree-Fock approximation calculation of the excitation spectrum for a Skyrmion bound to an impurity.

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Studies of low-energy excitations in quantum Hall systems continue to be a rich source of new physical phenomena \cite{1}. Recently, it has been found that spin-polarized incompressible quantum Hall states may support charged quasiparticles that carry non-trivial spin textures \cite{8} (“spin-texture quasiparticles”, or STQ’s). These textured quasiparticles are a natural generalization of skyrmion states that arise in the context of the non-linear sigma model \cite{1}, and have a spin density that may be characterized by a winding number of precisely unity. They carry a charge in quantum Hall ferromagnets because \cite{2,3,10} of the commensurability relations between magnetic flux and charge density required for incompressibility. In the ground state, deviation from the incompressible filling factor \( \nu \) is accomplished by introducing STQ’s which are responsible for a rapid degradation of the system’s spin polarization \cite{8}. This effect has been observed near \( \nu = 1 \) in several recent experiments \cite{11–13} that directly probe the spin density of the two-dimensional electron gas. In this letter we address the dynamics of STQ’s. We propose an intrinsic limit for the speed of dissipationless E cross B drift of STQ’s. When this speed is exceeded, STQ’s can radiate spin-waves by making internal excitations which allow total spin to be conserved.

Since STQ’s are finite size solitons, classical descriptions \cite{8} suggest they will have unusual localized excitations. Indeed, it follows from microscopic considerations \cite{8} that STQ’s of both positive and negative charge have an internal quantum number \( K = 0, 1, \ldots \) which specifies the number of reversed spins in their interiors. (The \( K = 0 \) STQ’s are the undressed minority spin electron and majority spin hole states.) It follows that the dependence of the energy of STQ states on Zeeman energy has the form \( \epsilon_K = U_K + K\tilde{g} \) so that their energetic ordering is dependent on the Zeeman coupling strength, \( \tilde{g} = g^* \mu_B B \). Existing experiments \cite{11} are consistent with \( K = 3 \) for the lowest energy STQ at \( \nu = 1 \) under typical experimental circumstances.

In two-dimensions, a non-interacting charged particle in a strong perpendicular magnetic field responds to an external potential which is smooth on the scale of its cyclotron orbit radius by drifting the orbit guiding center along an equipotential \cite{11}. This dissipationless E cross B drift plays an important role in the theory of quantum Hall effect transport phenomena and is, in particular, the basis for the network model \cite{11} of localization behavior in a strong magnetic field. For the case of a constant electric field \( E \) in the \( \hat{x} \) direction, the guiding center drifts in the \( \hat{y} \) direction with velocity \( v_{dr} = cE/B \). In a quantum treatment, electronic states within a Landau level are distinguished by wavevector \( Q_y \) related (up to a gauge dependent constant) to the \( x \)-coordinate of the guiding center by \( X = \ell^2 Q_y \), where \( \ell = \sqrt{\hbar c/eB} \) is the magnetic length.

Dissipative electronic motion by a distance \( \ell^2 Q_y \) along the electric field can take place only if the electronic energy \( (cE\ell^2 Q_y) \) and momentum \( hQ_y \) change are transferred to long wavelength phonon excitations of the host semiconductor. These transitions are possible only if \( v_{dr} \) exceeds semiconductor sound speed, a condition which appears to be connected \cite{11} with the breakdown of the quantum Hall effect at large current densities. The recent discovery of STQ’s implies that the quasiparticles relevant to quantum Hall transport phenomena near \( \nu = 1 \) (in weak disorder systems) are not bare electrons or holes but \( K \neq 0 \) STQ’s. In the presence of a smooth disorder potential the Landau levels for STQ’s will also exhibit dissipationless E cross B drift \cite{11}. In the STQ case, however, dissipation is possible at sufficiently large drift velocities even without phonon emission. A STQ can move along the electric field and conserve energy and momentum by emitting a long wavelength spin wave and reducing its internal quantum number \( (K \rightarrow K - 1) \) to conserve total spin. This dissipative process is not available to bare \( K = 0 \) quasiparticles in the absence of spin-orbit coupling and is related to spin-charge coupling in quantum Hall ferromagnets \cite{11,3,10}. The energy conservation condition for transitions with momentum transfer \( hQ_y \) is:
The right-hand-side of Eq. (1) is the energy of a long-wavelength spin-wave \( \epsilon Q_y \) with wavevector \( Q_y \). This equation has solutions if \( v_d \) exceeds

\[
v_{\text{max}} = \sqrt{\left( 16\pi \rho_s \epsilon^2 (\tilde{\gamma} + \epsilon \kappa - \epsilon K) \right)^{1/2}}. \tag{2}
\]

\( v_{\text{max}} \) is the speed limit for dissipationless E cross B drift of \( K \neq 0 \) STQ's [23]. This process provides a mechanism for the descriptions [25] of STQ dynamics.

To illustrate the importance of this new dissipative process for STQ's we have evaluated the longitudinal process for STQ's we have evaluated the longitudinal energy of a long-wavelength spin-wave [18] with wavevector \( Q_y \), and the number of these states in a given energy interval increases with increasing system size as the continuous spectrum of the thermodynamic limit is approached. The excitation energy of the single very low energy mode below the the Zeeman gap is independent of system size and we identify this as a transition between different internal states of the STQ, and zero-energy excitations which correspond to transitions between different angular momenta states and hence to translations of the STQ with a given internal quantum number. To identify transitions between different internal energy states we have picked out the portion of the spectrum which is independent of the finite radius of the electron disk used for our Hartree-Fock calculations as illustrated for a particular value of \( \tilde{\gamma} \) in Fig. 2(a). Spin-wave states have a minimum energy [18] equal to \( \tilde{\gamma} \) and the number of these states in a given energy interval increases with increasing system size as the continuous spectrum of the thermodynamic limit is approached. The excitation energy of the single very low energy mode below the the Zeeman gap is independent of system size and we identify this as a transition between different internal states of the STQ. An analysis of the residue of this pole shows that the excitation is one in which the transverse (perpendicular to the Zeeman field) component of the spin-polarization becomes time-dependent (as expected from its conjugate \( \chi \) relationship to \( K \)), although there is also a small “breathing” motion in the charge density. We remark that the existence of these low-energy excitations of STQ's is not apparent in the ‘band’ structure of the broken symmetry HF mean-field ground state. They are captured only when the time and space dependence of the Hartree and exchange local fields is included in the response function calculation. In a classical calculation these modes would be gapless but a finite gap is expected [18] in quantum calculations because of the microscopic size of the STQ's. Similar low-energy modes have been identified in lattice simulations of easy-plane ferromagnets [24].
FIG. 1. Lowest energy \( m = 0 \) excitations for a system with finite thickness \( W = 0.5 \ell_0 \), as a function of (a) inverse system size, and (b) \( \tilde{g} \). The line in (a) at the left indicates the Zeeman gap. A very low energy mode appears below this gap. The solid line in (b) indicates how this mode is expected to behave when quantization of \( K \) is properly accounted for (see text).

Fig. 1(b) illustrates how the internal excitation energy depends on \( \tilde{g} \) for a fixed system size and provides additional support for our interpretation of this pole. It appears in our response functions both for large Zeeman coupling and for small Zeeman coupling where the ground state has \( K \neq 0 \). The collective mode frequency vanishes precisely at \( \tilde{g} = g_c \), where \( g_c \) is the value for which \( K \neq 0 \) STQ’s first become stable in the Hartree-Fock approximation. As \( \tilde{g} \) decreases below \( g_c \), we expect a series of level crossings in which STQ’s with larger internal quantum number become the ground state \( \tilde{g} \). Although the Hartree-Fock ground state calculations do not respect the quantization of \( K \), comparison with exact diagonalization calculations suggests that accurate estimates of \( U_K \) are obtained by imposing the condition \( K = \sum_m |u_m|^2 = \text{integer} \) as a constraint on the trial wavefunction in Eq. (3). The energy difference between the ground and lowest excited STQ states obtained by this procedure is illustrated as a solid line in Fig. 1(b). Although the scale of the internal excitation energies found in our response function calculations agrees with the scale associated with this sequence of level crossings, we do not appear to see vanishing gaps at level crossing positions. We remark in passing that the existence of these low energy internal excitations should be important in the behavior of the magnetization of a quantum Hall system doped away from \( \nu = 1 \) as a function of \( \tilde{g} \) at temperatures below the Zeeman gap. In particular cusps associated with level crossings between isolated STQ states with different numbers of reversed spins may survive weak STQ-STQ interactions and finite temperatures.

We now turn to an analysis of our results for response functions in the presence of an impurity potential. Specifically we consider the effect of an ionized donor set back from the electron layer. Effects of such impurities on electromagnetic absorption have been observed experimentally \[22\]. Here we analyze this situation taking into account that such impurities may bind textured quasiparticles, using our TDHFA method.

FIG. 2. Electromagnetic absorption spectrum computed using the TDHFA for various values of \( \tilde{g} \). Each symbol represents a delta-function peak with relative weights for power absorption given by the abscissa. For \( \tilde{g} = 0.025 \) the spin-texture has collapsed, and there is only one delta-function peak in the absorption spectrum. Lower values of \( \tilde{g} \) all have non-trivial spin-textures. Data shown is for \( M_{\text{max}} = 120 \) and a finite well width of \( 0.5 \ell_0 \) is included in the calculation. An negatively charged impurity is located \( 2.0 \ell_0 \) from the electron plane.

In the absence of a spin-texture, the response of a \( K = 0 \) quasiparticle bound to an impurity to a time-dependent electric field is easy to understand. The quasiparticle can absorb one quantum of orbital angular momentum, and a single sharp line is found in the absorption spectrum. The frequency of this line is independent of the strength of the Zeeman coupling, \( \tilde{g} \). Physically, this absorption line corresponds to exciting the quasiparticle into an orbiting state around the impurity.

The presence of a spin texture makes this situation far
more rich and interesting. Fig. 2 illustrates the absorption spectrum for various values of $\tilde{g}$. As can be seen, both the magnitude and the peak frequency of the absorption is dependent on $\tilde{g}$, a direct consequence of spin-charge coupling intrinsic to spin textures in the quantum Hall system. (A similar sensitivity of the absorption line-shape to $\tilde{g}$ also occurs for excitations to higher Landau levels [23].) When $\tilde{g}$ is moderately large (but not so large as to completely collapse the spin texture) and the impurity potential is not too strong, there is a sharp line in the absorption spectrum that may be interpreted as exciting the STQ into an orbit around the impurity center. Generally, the energy of this excited state is sensitive to the Zeeman coupling.

For weaker Zeeman coupling and stronger impurity potentials, such that the energy of the orbiting state rises above the Zeeman gap, the speed limit for the skyrmion is exceeded. The resulting dissipation broadens the absorption peak, as illustrated in Fig. 2. We have confirmed that the absorption illustrated here is indeed a broadened peak in the thermodynamic limit, and not a series of sharp peaks, by computing the absorption for larger system sizes (up to $M_{\text{max}} = 960$). We find that the peak height actually decreases with increasing system size, and that the number of states participating in the absorption increases.

In summary, we have demonstrated, using a time-dependent Hartree-Fock approach, that spin-charge coupling leads to a new dissipation channel for spin-polarized quantum Hall systems. The effect is a result of the spin texture of the quasiparticles, which allows for low-energy internal modes. The effect is shown explicitly in the electromagnetic absorption by quasiparticles bound to impurities.

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