Matrix Supermultiplet of $N = 2, D = 4$
Supersymmetry and Supersymmetric 3-brane

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Abstract
It is shown that the Lagrangian density of the supersymmetric 3-brane can be regarded as a component of an infinite-dimensional supermultiplet of $N = 2, D = 4$ supersymmetry spontaneously broken down to $N = 1$. The latter is described by $N = 1$ Hermitian bosonic matrix superfield $V_{mn} = V_{nm}^\dagger$, $[V_{mn}] = m + n$, $m, n = 0, 1, \ldots$, in which the component $V_{01}$ is identified with a chiral Goldstone $N = 1$ multiplet associated with central charge of the $N = 2, D = 4$ superalgebra, and $V_{11}$ obeys a specific nonlinear recursive equation providing the possibility to express $V_{11}$ (as well as the other components $V_{mn}$) covariantly in terms of $V_{01}$. We demonstrate that the solution of $V_{11}$ gives the right $PBGS$ action for the super-3-brane.

1 Introduction
Supersymmetric 3-brane gives a remarkable pattern of a theory where the partial breaking of global supersymmetry ($PBGS$) in four spacetime dimensions is occurred [1, 2]. The bosonic part of the corresponding action amounts to the “static gauge” form of the Nambu–Goto action which can be derived, for example, from the nonlinear realization of $D = 6$ relativistic symmetry in the coset space $ISO(1,5)/SO(1,3) \times SO(2)$ [3, 4]. The supersymmetric generalization of this approach is elaborated in ref. [5]. There the coset space $ISO(1,5)/SO(1,3) \times SO(2)$ is supposed to be embedded into the supersymmetric one $G/H$ involving the superbrane worldvolume coordinates $z = \{x, \theta, \bar{\theta}\}$ along with the set of the Goldstone superfields $\{\phi(z), \bar{\phi}(z), \psi_\alpha(z), \bar{\psi}_\dot{\alpha}(z), \Lambda_\alpha(z), \Xi(z)\}$ associated with the generators of spontaneously broken symmetries $\{Z = P_4 - i P_5, S_\alpha, \bar{S}_\dot{\alpha}, K_a, T\}$. The main advantage of this approach is that it allows one to find the action in terms of the Goldstone superfields on the base of the standard method of the nonlinear realization of supersymmetry in the coset superspace $G/H$ [6, 7, 8, 9]. In the case of the $N = 2 \rightarrow N = 1, D = 4$
PBGS pattern it gives the superfield action of the supersymmetric 3-brane propagating in the six-dimensional Minkowski spacetime. The component form of this action exactly coincides with the Green–Schwarz covariant action for the supermembrane obtained in ref. [2]. Note, that the final expression of the superfield action depends only on the scalar Goldstone superfields $\phi, \bar{\phi}$ and their covariant derivatives. The superfields $\psi_\alpha(z), \bar{\psi}_\dot{\alpha}(z), \Lambda_a(z), \Xi(z)$ are not involved due to the covariant constraints imposed on the corresponding Cartan forms. Nevertheless, this action is not very convenient for applications since the superfields $\phi, \bar{\phi}$ themselves have to be constrained by the nonlinear conditions $\nabla_\alpha \phi = \nabla_\dot{\alpha} \bar{\phi} = 0$, where $\nabla_\alpha, \nabla_\dot{\alpha}$ are the spinor covariant derivatives of the nonlinear realization. This makes the corresponding variation procedure rather complicated since these objects themselves depend nonlinearly on $\phi, \bar{\phi}$. The situation gets worse even more because so far we do not know exactly a full non-perturbative PBGS action of the super-3-brane derived from the nonlinear realization. Today it is specified only up to the fourth order in the superfields $\phi, \bar{\phi}$ [5].

Fortunately, there exists another approach which significantly simplifies these computation problems. In contrast to the previous approach to the system under consideration this one is based on the linear realization of supersymmetry [10, 12]. It strongly resembles the PBGS theory of the supersymmetric D3-brane (see, e.g. [11]), where the hidden supersymmetry is described by the $N = 2, D = 4$ vector supermultiplet. It turns out that in the case of the supersymmetric 3-brane the corresponding Goldstone supermultiplet can be described by the $N = 2$ tensor supermultiplet constrained by the suitable nilpotence conditions [12]. Solving them we get the south for action in terms of the linear $N = 1$ Goldstone superfield. It was shown in ref. [12] that this action can be equivalently transformed into the chiral Goldstone superfield action [10] with the help of a chiral–linear superfield duality transformation. This fact, indeed, radically simplifies the computation problems since in this case we should vary only the unconstrained superfield of the linear realization $\phi(x + 2i\theta\bar{\theta}, \theta)$. At last, the obvious advantage of this approach as compared with the aforementioned one is that it provides the possibility to get a full non-perturbative form action of the super-3-brane.

Nevertheless, there is a problem which remains unsolved even in the framework of this approach. Actually, from the papers [10, 12] we know how to construct the Lagrangian density of the super-3-brane in terms of the linear $N = 1$ Goldstone superfield and how to transform it with the help of the duality transformations into the final form action depending on the chiral Goldstone superfield $\phi$. However, implementing such a procedure we inevitably lose a straightforward contact between the Lagrangian and its transformation law with respect to the $N = 2, D = 4$ supersymmetry. Precisely, we do not know today how explicitly it transforms with respect to the variation of $\phi$?

In this article we are going to show that there exists a simple answer to this question. In fact, the required transformation law can be derived from the new infinite-dimensional matrix $N = 2$ supermultiplet which incorporates both the chiral Goldstone multiplet $\phi$ and the Lagrangian density $\mathcal{L}$ as its independent scalar $N = 1$ components.
2 Extended vector $N = 2$, $D = 4$ supermultiplet

Let us begin with the central charge extension of the $N = 2$, $D = 4$ Poincaré superalgebra

\begin{align}
\{Q_\alpha, Q_{\dot{\alpha}}\} &= -2P_{a\dot{a}}, \quad \{S_\alpha, S_{\dot{\alpha}}\} = -2P_{a\dot{a}}, \\
\{Q_\alpha, S_\beta\} &= 2\varepsilon_{\alpha\beta}Z, \quad \{Q_{\dot{\alpha}}, S_{\dot{\beta}}\} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{Z},
\end{align}

where the other (anti)commutators are supposed to vanish. In what follows the $N$ superspace notations for the generators will be used

Now let us consider the $N$ extended vector $\phi$ which obeys Bianchi identities

\begin{align}
W = \phi - \omega^\alpha W_\alpha - \frac{1}{4}\omega^2 \bar{\phi}^2 - i\omega^\alpha \bar{\omega}^{\dot{\alpha}} \partial_{a\dot{a}} \phi + \frac{i}{2} \omega^2 \bar{\omega} \partial^{a\dot{a}} \partial_\alpha W_\alpha
\end{align}

which obeys Bianchi identities

\begin{align}
D^{(\omega)} W - \bar{D}^{2} \bar{W} = 0, \quad D^{(\omega)} A \partial W + \bar{D}^{(\omega)} \bar{D} \bar{W} = 0,
\end{align}

and possesses the following transformation laws

\begin{align}
\delta W = c - (\eta^\alpha S_\alpha + \bar{\eta}_{\dot{\alpha}} \bar{S}_{\dot{\alpha}}) W, \quad ZW = 1
\end{align}

with respect to the central charge and the second supersymmetry transformations. Here the standard conventions for the $N = 1$ spinor covariant derivatives are assumed

\begin{align}
D_\alpha = \partial_\alpha^{(\phi)} - i\bar{\theta}\alpha \partial_{a\dot{a}}, \quad \bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}}^{(\bar{\phi})} + i\theta\dot{\alpha} \partial_{a\dot{a}}, \quad D^{(\omega)} = D_\alpha |_{\theta = \omega}.
\end{align}

From the Eqs. (3), (5) it follows that the chiral $N = 1$ superfields $\phi$, $W_\alpha$ transform as

\begin{align}
\delta \phi &= f + \eta^\alpha W_\alpha, \quad f = c - 2i\eta^\alpha \theta_\alpha, \\
\delta W_\alpha &= \frac{1}{2} \bar{D}^2 \bar{\phi} \eta_\alpha - 2i \partial_{a\dot{a}} \bar{\phi} \bar{\eta}_{\dot{\alpha}},
\end{align}

where $W_\alpha$ is supposed to be constrained by the reality condition

\begin{align}
D_\alpha W_\alpha + \bar{D}_{\dot{\alpha}} W^{\dot{\alpha}} = 0.
\end{align}

From Eqs. (6) we see that owing to the presence of the inhomogeneous term in the r.h.s. (5) the lowest component of the vector supermultiplet $\phi$ acquires the pure shift term $f$.

\footnote{The conjugation conditions are as follows}

\begin{align}
(\partial_\alpha)^\dagger = -\bar{\partial}_{\dot{\alpha}}, \quad (\partial_{a\dot{a}})^\dagger = \partial_{a\dot{a}}, \quad (D_\alpha)^\dagger = \bar{D}_{\dot{\alpha}}, \quad (D_{a\dot{a}})^\dagger = -\bar{D}_{\dot{\alpha}} D_\alpha, \quad (D^\alpha D_\alpha)^\dagger = \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}}.
\end{align}
in its transformation law. This implies that the superfield $D_\alpha \phi$ is shifted on the term proportional to $\eta_\alpha$ under the action of the $S$-supersymmetry transformations. Hence, this superfield can be treated as the Goldstone fermion of spontaneously broken $N = 2$ supersymmetry keeping the $N = 1$ supersymmetry unbroken. This is the typical pattern of PBGS theory in which the corresponding “longwavelength” effective action is appeared. In the most cases these actions can be derived straightforwardly from covariant constraints imposed on the supermultiplets incorporating a proper Goldstone multiplet (see, e.g. [13] and refs. therein). In our case, however, this approach becomes inapplicable because the corresponding linear representation of the $N = 2$, $D = 4$ supersymmetry is unknown. Let us try to improve this situation.

To do this we, first, should solve the constraint (7) in terms of $N = 1$ gauge real prepotential $L$

$$W_\alpha = -\frac{i}{4} \bar{D}^2 D_\alpha L, \quad \bar{W}_\dot{\alpha} = (W_\alpha)^\dagger = \frac{i}{4} D^2 \bar{D}_{\dot{\alpha}} L, \quad (8)$$

and then introduce the “extended” vector $N = 2$ supermultiplet

$$\delta L = \bar{f} \phi + \bar{f} \phi + \eta^\alpha Z_\alpha + \bar{\eta}_{\dot{\alpha}} \bar{Z}^{\dot{\alpha}},$$  \quad (9)
$$\delta Z_\alpha = \bar{f} W_\alpha - \frac{1}{2} \eta_{\dot{\alpha}} \bar{D}^2 \bar{F} - \bar{\eta}^\dot{\alpha} \bar{D}_\dot{\alpha} D_\alpha L,$$ \quad (10)
$$\delta F = f \phi + \eta^\alpha \Psi_\alpha,$$ \quad (11)
$$\delta \Psi_\alpha = f W_\alpha - \frac{1}{2} \eta_{\dot{\alpha}} \bar{D}^2 \mathcal{L} - 2i \bar{\eta}^{\dot{\alpha}} \partial_{\alpha \dot{\alpha}} F,$$ \quad (12)

where the auxiliary chiral $N = 1$ superfields $F, Z_\alpha, \Psi_\alpha$ are involved to provide the closure of the transformations (9)–(12) off-shell. The key point of our investigation is that the transformations (9)–(12) allow the complex Bianchi identity

$$D^\alpha \Psi_\alpha + \bar{D}_{\dot{\alpha}} \bar{Z}^{\dot{\alpha}} = 0. \quad (13)$$

Keeping into account (13) one can easily find out that the transformations (9)–(12), considered together with (6), are closed off-shell

$$[\delta_1, \delta_2] \{ L, F, Z_\alpha, \Psi_\alpha \} = 2i(\eta_1^\alpha \bar{\eta}_{\dot{\alpha}}^2 - \eta_2^\alpha \bar{\eta}_{\dot{\alpha}}^1) \partial_{\alpha \dot{\alpha}} \{ L, F, Z_\alpha, \Psi_\alpha \}. \quad (14)$$

The next step is to realize that, apart from the global $N = 2$ supersymmetry, there exist the local $N = 2$ supertransformations

$$\delta F = M, \quad \delta \Psi_\alpha = \Omega_\alpha, \quad \delta Z_\alpha = \Sigma_\alpha, \quad \delta \mathcal{L} = N + \bar{N}, \quad (15)$$

which preserve this supermultiplet. To prove this we should identify the superfield parameters $\{ M, N, \Omega_\alpha, \Sigma_\alpha \}$ with components of a “complexificated” $N = 2$ vector supermultiplet

$$\delta M = \eta^\alpha \Omega_\alpha, \quad \delta \Omega_\alpha = -\frac{1}{2} \bar{D}^2 \bar{N} \eta_\alpha - 2i \partial_{\alpha \dot{\alpha}} M \bar{\eta}^{\dot{\alpha}}, \quad (16)$$
$$\delta N = \eta^\alpha \Sigma_\alpha, \quad \delta \Sigma_\alpha = -\frac{1}{2} \bar{D}^2 \bar{M} \eta_\alpha - 2i \partial_{\alpha \dot{\alpha}} N \bar{\eta}^{\dot{\alpha}}, \quad (17)$$
restricted by the condition
\[ D^\alpha \Omega_\alpha + \bar{D}_\dot{\alpha} \bar{\Sigma}^\dot{\alpha} = 0. \] (17)

Having at our disposal these parameters one can easily verify that the redefined superfields
\[ \mathcal{L}' = \mathcal{L} + N + \bar{N}, \quad F' = F + M, \quad \Psi'_\alpha = \Psi_\alpha + \Omega_\alpha, \quad Z'_\alpha = Z_\alpha + \Sigma_\alpha \] (18)

have the same transformation laws as \{\mathcal{L}, F, Z_\alpha, \Psi_\alpha\}.

Now we see, that all these ingredients gathered together indicate that the set of the \( N = 1 \) superfields \{\phi, \mathcal{L}, F, Z_\alpha, \Psi_\alpha\} actually can be considered as a new off-shell gauge supermultiplet of the \( N = 2, D = 4 \) supersymmetry. Note that it appears only in the framework of the PBGS theory. It does not exist beyond this theory due to the vanishing of the \( f \)-dependent terms in the r.h.s. of the Eqs. (6), (9)–(12). It is also very important to understand that all the subsidiary degrees of freedom \( F, \Psi_\alpha, Z_\alpha \) as well as those describing the superspin-0 representation of \( \mathcal{L} \) can be finally transformed away owing to the gauge symmetry (15), (18). Therefore, on the mass-shell we have the same content of the superfields as that of the vector \( N = 2 \) supermultiplet. Nevertheless, there is an essential difference between them. In contrast to the vector \( N = 2 \) supermultiplet the new one includes both the physical superfields \( \phi \) and \( \mathcal{L} \) among its components. As we will see in the next section this significantly simplifies the process of constructing of the corresponding effective action.

3 Recursive equation

In order to do this we follow the standard prescription of refs. [11, 12, 13] and introduce the recursive equation
\[ \mathcal{L} = \phi \bar{\phi} + \frac{1}{16} \left( \frac{(D^\alpha \phi)(D_\alpha \phi)(D_{\dot{\alpha}} \bar{\phi})(D_{\dot{\alpha}} \bar{\phi})}{1 - \frac{i}{4}D^\alpha W_\alpha + \frac{1}{4}}, \quad A = -2(\bar{\partial}^{\alpha \dot{\alpha}} \phi)(\partial_\alpha \partial_{\dot{\alpha}} \phi) - \frac{1}{4}(D^2 \phi)(\bar{D}^2 \bar{\phi}). \] (19)

Eq. (19) completely fixes the gauge freedom of the extended vector supermultiplet, however, the global \( N = 2 \) supersymmetry remains unbroken. To prove this we have to come back to the equations (6), (9)–(12). Analyzing them we observe that the transformations (11), (12) admit quite simple nonlinear solutions
\[ F = \frac{1}{2} \phi^2, \] (20)
\[ \Psi_\alpha = \phi W_\alpha. \] (21)

Varying these expressions in accordance with (6) we recover the transformation laws (11), (12) and the further constraints
\[ \mathcal{L} = \phi \bar{\phi} - \frac{i}{4}(W^\alpha D_\alpha \mathcal{L} - Z^\alpha D_\alpha \phi), \] (22)
\[ Z_\alpha = \bar{\phi} W_\alpha - \frac{1}{2}(\bar{D}^\alpha \mathcal{L}) \partial_{\alpha \dot{\alpha}} \phi - \frac{i}{4}(\bar{D}^\alpha \bar{\phi}) D_\alpha \bar{D}_{\dot{\alpha}} \mathcal{L}. \] (23)
Note, that when deriving these “second-class” constraints (22), (23) the Bianchi identity (13) must be taken into account. At the first sight these constraints manifestly contradict the condition of chirality of the superfield $Z_\alpha$, which was postulated in the previous section. In addition, the equation (22) does not seem to be real as it should be. However, these discrepancies can be removed simultaneously by the making use of the “nilpotence” conditions following from the recursive equation (19)

$$
(L - \phi \bar{\phi})D_\alpha \phi = (L - \phi \bar{\phi})\bar{D}_\bar{\alpha} \bar{\phi} = 0.
$$

Let us emphasize that these conditions serve as the crucial step towards the recursive equation (19). Really, as it follows from (24) the explicit solution of the Eq. (21), (23) can be written as

$$
Z_\alpha = -\frac{i}{8} \bar{D}^2 D_\alpha V, \quad \Psi_\alpha = -\frac{i}{8} \bar{D}^2 \bar{D}_\bar{\alpha} \bar{V},
$$

where

$$
V = 2\bar{\phi}L - \phi \bar{\phi}^2.
$$

It is easy to see now that for the given $Z_\alpha$ the recursive equation (22) is reduced to its “canonical” form (19). So, we have proved the covariance of the conditions (19), (22), (23) with respect to $N = 2, D = 4$ supersymmetry. The explicit solution of the Eq. (19) will be obtained in the section 5. Here, however, it is instructive to pay our attention to the following remarkable fact resulting from our considerations.

4 Infinite-dimensional matrix $N = 2$ supermultiplet

Looking at the Eqs. (26), one cannot pass over a very unexpected interpretation of this solution. No matter whether there are composite solutions for the components of the extended vector supermultiplet or not, in fact, we always have the possibility to avoid the presence of the fermion superfields in the supermultiplet we have just constructed. To do this one should, firstly, incorporate the general $N = 1$ superfield $V$ into the content of the supermultiplet. Then, demanding the closure of the corresponding bracket $N = 2$ transformations off-shell we arrive at the transformation law

$$
\delta V = 2fL + 2f\bar{F} - i\eta^\alpha \bar{D}^2 D_\alpha G + i\bar{\eta}_\alpha D^2 \bar{D}_\bar{\alpha} H,
$$

where

$$
G = 2\phi^2 L - \frac{4}{3} \phi^3 \bar{\phi}, \quad H = 4\phi \bar{\phi} L - 3\phi^2 \bar{\phi}^2.
$$

Repeating this process step by step we end up with the following infinite-dimensional matrix $N = 1$ superfield

$$
V_{mn} = V^\dagger_{nm} = \frac{2^{m+n-2}}{m!n!} \left( mn\phi^{m-1}\bar{\phi}^{n-1}L + (1 - mn)\phi^m \bar{\phi}^n \right), \quad m, n \geq 0.
$$
One can check that this superfield transforms as follows

\[ \delta V_{mn} = 2fV_{m-1,n} + 2\tilde{f}V_{m,n-1} - \frac{i}{8}f^\alpha \tilde{D}^2 D_\alpha V_{m,n+1} + \frac{i}{8}\tilde{\eta}_\alpha D^2 \tilde{D}^\alpha V_{m+1,n}, \]

\[ \delta V_{0n} = 2\tilde{f}V_{0,n-1} + \frac{i}{8}\tilde{\eta}_\alpha D^2 \tilde{D}^\alpha V_{1,n}, \]

when \( \phi \) and \( \mathcal{L} \) are varied in accordance with Eqs. (6), (9), (25), (26). Moreover, examining the transformation laws (30) we reveal that they are closed off-shell (just in accordance with the superalgebra (1)) for all the components \( V_{mn} \) treating as new independent \( N = 1 \) superfields when the corresponding matrix elements are normalized as follows

\[
\begin{pmatrix}
V_{00} & V_{01} & V_{02} & \cdots \\
V_{10} & V_{11} & V_{12} & V_{13} \\
V_{20} & V_{21} & V_{22} & \cdots \\
\vdots & \quad & \quad & \ddots
\end{pmatrix}
= \begin{pmatrix}
1/4 & \tilde{\phi}/2 & \tilde{F} & \cdots \\
\phi/2 & \mathcal{L} & \tilde{V} & \tilde{G} \\
\tilde{F} & \tilde{V} & H & \cdots \\
\quad & \quad & \quad & \ddots
\end{pmatrix},
\]

with \( V_{00} \) being chiral. Thus, beginning with either the finite-dimensional \( N = 2 \) vector supermultiplet (6) or its extended analog (9)–(12) collecting both the bosonic and fermionic components we necessarily finish with the infinite-dimensional \( N = 2 \) matrix supermultiplet \( V_{mn} \) composed of the bosonic components only. It is obvious, that by the definition \( V_{mn} \) proves to be infinite-reducible and can be “truncated” by the spinor covariant derivatives up to the vector \( N = 2 \) supermultiplet (6). Its advantage, however, is that it immediately leads to the extended vector \( N = 2 \) supermultiplet which gives the algorithmic procedure of constructing the “canonical” form of the recursive equation (19). Otherwise, it may be merely guessed. Hence, regarding the super-3-brane itself, we have proved the existence of some new fundamental representation of the PBGS theory which the effective Lagrangian density along with the corresponding chiral Goldstone superfield belongs to.

Notice, that earlier a very resembling procedure was demonstrated in the framework of \( N = 4, D = 4 \) PBGS theory [14]. There the \( N = 2, D = 4 \) Born–Infeld superfield Lagrangian density was revealed among the components of an infinite-dimensional off-shell supermultiplet providing PBGS. Just as \( V_{mn} \) this supermultiplet encompasses the infinite set of the worldvolume superfields obeying an infinite set of covariant constraints. In contrast to our case these constraints are solved there only iteratively.

5 Lagrangian density

A closed form of the Lagrangian density for the supersymmetric 3-brane can be established uniquely from the recursive equation (19) rewritten preliminary in the following more convenient form

\[
L = \frac{1}{16} \frac{(D\phi)^2(D\tilde{\phi})^2}{1 - \frac{A}{16}D^a \tilde{D}^2 D_\alpha \mathcal{L} + \frac{A}{2}}, \quad L \equiv \mathcal{L} - \phi \tilde{\phi}.
\]
This equation can be solved exactly since the numerator of (32) contains the maximal number of the Grassmann spinors $D_\alpha \phi$, $D_\dot{\alpha} \bar{\phi}$. Therefore in the denominator we should leave only the terms which do not include these spinors. Thus we get the following “effective” equation

$$X \equiv D^\alpha \bar{D}^2 D_\alpha L = \frac{1}{16} \frac{(D^\beta D^2 D_\beta (D\phi)^2 (D\bar{\phi})^2)_{\text{eff}}}{1 + \frac{A}{2} - \frac{X}{16}}, \quad (33)$$

where

$$(D^\beta D^2 D_\beta (D\phi)^2 (D\bar{\phi})^2)_{\text{eff}} = (D^2 \phi)^2 (D^2 \bar{\phi})^2 + 64(D^a \phi)(\partial^{aa} \phi)(\partial^{\sigma\dot{\sigma}} \bar{\phi})(\partial_{\sigma\dot{\sigma}} \bar{\phi}) + 16(D^2 \phi)(D^2 \bar{\phi})(\partial^{a\dot{a}} \phi)(\partial_{a\dot{a}} \bar{\phi}) \quad (34)$$

Solving (33) and choosing the solution with the nonsingular behaviour in the limit $\phi = 0$

$$X = 8 \left( 1 + \frac{A}{2} - \sqrt{1 + A + B} \right), \quad (35)$$

$$B = (\partial^{a\dot{a}} \phi)(\partial_{a\dot{a}} \bar{\phi})(\partial^{\sigma\dot{\sigma}} \phi)(\partial_{\sigma\dot{\sigma}} \bar{\phi}) - (\partial^{a\hat{a}} \phi)(\partial_{a\hat{a}} \bar{\phi})(\partial^{\sigma\dot{\sigma}} \phi)(\partial_{\sigma\dot{\sigma}} \bar{\phi}),$$

we finally find the following expression for the Lagrangian density

$$\mathcal{L} = \phi \bar{\phi} + \frac{1}{8} \frac{(D^a \phi)(D_a \phi)(D_{\dot{a}} \bar{\phi})(D^{\dot{a}} \bar{\phi})}{1 + \frac{A}{2} + \sqrt{1 + A + B}}. \quad (36)$$

This is the canonical form of the Lagrangian density for the super-3-brane obtained in a framework of another approach in refs. [10, 12]. Note that the corresponding action

$$S = \int d^4x d^4\theta \mathcal{L}, \quad (37)$$

is manifestly invariant with respect to the unbroken supersymmetry transformations while under the broken one it is shifted in accordance with Wess–Zumino theory on the surface term $\int d^4x d^4\theta (\bar{\phi}^\dagger \phi + \bar{\phi}^\dagger \phi + \eta^a Z_a + \bar{\eta}_{\dot{a}} \bar{Z}^{\dot{a}})$.

### 6 Conclusions

Thus, we have shown that the most natural way of description of the supersymmetric 3-brane is achieved when both the chiral Goldstone $N = 1$ superfield and the Lagrangian density are embedded into the infinite-dimensional matrix $N = 2$ supermultiplet $V_{mn}$ restricted by the conditions (19), (29). It is proved that he latter appears in the framework of $PBGS$ theory and solves ultimately the problem of constructing of the supersymmetric 3-brane action in its closed form.

Nevertheless, many problems remain outside this work. One of them is the problem of connection of this approach with the nonlinear realization of the $N = 2$, $D = 4$ supersymmetry in the superspace [5, 15]. Having ensured such a connection we will have an
opportunity to test the general principle of the Nambu–Goldstone theories: the uniqueness of the Goldstone interaction arising in models with various mechanisms of spontaneous breaking of supersymmetry. As to the system considered here it would be desirable to understand how the chiral Goldstone multiplet of the linear realization $\phi$ is interlinked with that of the nonlinear realization $[5]$, and how a universal closed form action of this realization looks like?

Related question is whether this approach will allow us to solve a problem of spontaneous breaking of $D = 6$ Lorentz symmetry in target space. From the general reasons we know that the part of this symmetry related to the transverse directions should be realized nonlinearly, in Goldstone mode fashion $[5]$. Unfortunately, up to now we do not know how the corresponding Goldstone worldvolume superfields are involved into the residual PBGS action.

At last, there is the most intrigue question: whether the PBGS theory we have considered here can be promoted to the case of $AdS_6$ background, as it has been done, for example, for the $N = 1, D = 4$ supermembrane in ref. $[16]$?

We hope to answer these questions in future.

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