Score lists in \([h-k]\)-bipartite hypertournaments

S. Pirzada\(^1\), T. A. Chishti\(^2\), T. A. Naikoo\(^3\)

\(^1\)Department of Mathematics, University of Kashmir, India
\(^2\)Centre of Distance Education, University of Kashmir, India
\(^3\)Email: sdpirzada@yahoo.co.in
\(^3\)Email: tariqnaikoo@rediffmail.com

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**Abstract.** Given non-negative integers \(m, n, h\) and \(k\) with \(m \geq h > 1\) and \(n \geq k > 1\), an \([h-k]\)-bipartite hypertournament on \(m+n\) vertices is a triple \((U, V, A)\), where \(U\) and \(V\) are two sets of vertices with \(|U| = m\) and \(|V| = n\), and \(A\) is a set of \((h+k)\) - tuples of vertices, called arcs, with exactly \(h\) vertices from \(U\) and exactly \(k\) vertices from \(V\), such that any \(h+k\) subsets \(U_1 \cup V_1\) of \(U \cup V\), \(A\) contains exactly one of the \((h+k)!\) \((h+k)\) - tuples whose entries belong to \(U_1 \cup V_1\). We obtain necessary and sufficient conditions for a pair of non-decreasing sequences of non-negative integers to be the losing score lists or score lists of some \([h-k]\)-bipartite hypertournament.

1. Introduction

Hypergraphs are generalization of graphs [3]. While edges of a graph are pairs of vertices of the graph, edges of a hypergraph are subsets of the vertex set, consisting of at least two vertices. An edge consisting of \(k\) vertices is called a \(k\)-edge. A \(k\)-hypergraph is a hypergraph all of whose edges are \(k\)-edges. A \(k\)-hypertournament is a complete \(k\)-hypergraph with each \(k\)-edge endowed with an orientation, that is, a linear arrangement of the vertices contained in the hyperedge.

Instead of scores of vertices in a tournament, Zhou et al. [8] considered scores and losing scores of vertices in a \(k\)-hypertournament, and derived a result analogous to Landau’s theorem [6]. The score \(s(v_i)\) or \(s_i\) of a vertex \(v_i\) is the number of arcs containing \(v_i\) and in which \(v_i\) is not the last element, and the losing score \(r(v_i)\) or \(r_i\) of a vertex \(v_i\) is the number of arcs containing \(v_i\) and in which \(v_i\) is the last element. The score sequence (losing score sequence) is formed by listing the scores (losing scores) in non-decreasing order.

We note that for two integers \(p\) and \(q\).
The following characterizations of score sequences and losing score sequences in k-hypertournaments can be found in Zhou et al. [8].

**Theorem 1.1.** Given two non-negative integers \( n \) and \( k \) with \( n \geq k > 1 \), a non-decreasing sequence \( R = [r_1, r_2, ..., r_n] \) of non-negative integers is a losing score sequence of some k-hypertournament if and only if for each \( j \),

\[
\sum_{i=1}^{j} r_i \geq \binom{j}{k},
\]

with equality when \( j = n \).

**Theorem 1.2.** Given non-negative integers \( n \) and \( k \) with \( n \geq k > 1 \), a non-decreasing sequence \( S = [s_1, s_2, ..., s_n] \) of non-negative integers is a score sequence of some k-hypertournament if and only if for each \( j \),

\[
\sum_{i=1}^{j} s_i \geq j \left( \frac{n-1}{k-1} \right) + \left( \frac{n-j}{k} \right) - \binom{n}{k},
\]

with equality when \( j = n \).

Bang and Sharp [1] proved Landau’s theorem using Hall’s theorem on a system of distinct representatives of a collection of sets. Based on Bang and Sharp’s ideas, Koh and Ree [5] have given a different proof of Theorems 1.1 and 1.2. Some more results on scores of k-hypertournaments can be found in [4, 7].

Bipartite hypergraphs are generalization of bipartite graphs. If \( U = \{u_1, u_2, ..., u_m\} \) and \( V = \{v_1, v_2, ..., v_n\} \) are vertex sets, then the edge of a bipartite hypergraph is a subset of the vertex sets, containing at least one vertex from \( U \) and at least one vertex from \( V \). If an edge has exactly \( h \) vertices from \( U \) and exactly \( k \) vertices from \( V \), it is called an \([h,k]\)-edge. An \([h,k]\)-bipartite hypergraph is a bipartite hypergraph all of whose edges are \([h,k]\)-edges. An \([h,k]\)-bipartite hypertournament is a complete \([h,k]\)-bipartite hypergraph with each \([h,k]\)-edge endowed with an orientation, that is, a linear arrangement of the vertices contained in the hyperedge.

Equivalently, given non-negative integers \( m, n, h \) and \( k \) with \( m \geq h > 1 \) and \( n \geq k > 1 \), an \([h,k]\)-bipartite hypertournament of order \( m \times n \) consists of two vertex sets \( U \) and \( V \) with \(|U| = m\) and \(|V| = n\), together with an arc set \( E \), a set of \((h+k)\) tuples of vertices, with exactly \( h \) vertices from \( U \) and exactly \( k \) vertices from \( V \), called arcs, such that for any \( h \)-subset \( U_1 \) of \( U \) and \( k \)-subset \( V_1 \) of \( V \), \( E \) contains exactly one of the \((h+k)!\) \((h+k)\)-tuples whose \( h \) entries belong to \( U_1 \) and \( k \) entries belong to \( V_1 \). Let \( e = (u_1, u_2, ..., u_h, v_1, v_2, ..., v_k) \) be an arc in \( H \) and \( i < j \), we denote \( e(u_i, u_j) = (u_1, ..., u_i, u_j, v_1, v_2, ..., v_k) \), that
is, the new arc obtained from e by interchanging $u_i$ and $u_j$ in e. Similarly, we can have new arcs of the form $e(v_i, v_j)$ and $e(u_i, v_j)$.

For a given vertex $u_i \in U$, the score $d_H^+(u_i)$ (or simply $d^+(u_i)$) is the number of [h-k]-arcs containing $u_i$ and in which $u_i$ is not the last element. The losing score $d_H^-(u_i)$ (or simply $d^-(u_i)$) is the number of [h-k]-arcs containing $u_i$ and in which $u_i$ is the last element. Similarly, we define by $d_H^+(v_j)$ and $d_H^-(v_j)$ respectively as the score and losing score of a vertex $v_j \in V$. The losing score lists of an [h-k]-bipartite hypertournament is a pair of non-decreasing sequences $A = [a_1, a_2, ..., a_m]$ and $B = [b_1, b_2, ..., b_n]$, where $a_i$ is a losing score of some vertex $u_i \in U$ and $b_j$ is a losing score of some vertex $v_j \in V$. Similarly, the score lists are formed by listing the scores in non-decreasing order, and we denote these by $C = [c_1, c_2, ..., c_m]$ and $D = [d_1, d_2, ..., d_n]$.

2. Main results

The following two Theorems are the main results and provide a characterization of losing score lists and score lists in [h-k]-bipartite hypertournaments.

**Theorem 2.1.** Given non-negative integers $m$, $n$, $h$ and $k$ with $m \geq h > 1$ and $n \geq k > 1$, the non-decreasing sequences $A = [a_1, a_2, ..., a_m]$ and $B = [b_1, b_2, ..., b_n]$ of non-negative integers are the losing score lists of an [h-k]-bipartite hypertournament if and only if for each $p$ and $q$,

$$
\sum_{i=1}^{p} a_i + \sum_{j=1}^{q} b_j \geq \binom{p}{h} \binom{q}{k},
$$

with equality when $p = m$ and $q = n$.

**Theorem 2.2.** Given non-negative integers $m$, $n$, $h$ and $k$ with $m \geq h > 1$ and $n \geq k > 1$, the non-decreasing sequences $C = [c_1, c_2, ..., c_m]$ and $D = [d_1, d_2, ..., d_n]$ of non-negative integers are the score lists of an [h-k]-bipartite hypertournament if and only if for each $p$ and $q$,

$$
\sum_{i=1}^{p} c_i + \sum_{j=1}^{q} d_j \geq \binom{m-1}{h-1} \binom{n}{k} + q \binom{m}{h} \binom{n-1}{k-1} + \binom{m-p}{h} \binom{n-q}{k} - \binom{m}{h} \binom{n}{k},
$$

with equality when $p = m$ and $q = n$.

In order to prove Theorem 2.1 and Theorem 2.2, we require the following Lemmas. We note that in an [h-k]-bipartite hypertournament $H$ there are exactly $\binom{m}{h} \binom{n}{k}$ arcs, and in each arc, only one vertex is at the last entry. Therefore,
\[\sum_{i=1}^{m} d_H^-(u_i) + \sum_{j=1}^{n} d_H^-(v_j) = \binom{m}{h} \binom{n}{k}.\]

**Lemma 2.1.** If \(H\) is an \([h-k]\)-bipartite hypertournament of order \(m \times n\) with score lists \(A = [c_i]^m\) and \(B = [d_j]^n\), then

\[\sum_{i=1}^{m} c_i + \sum_{j=1}^{n} d_j = (h + k - 1) \binom{m}{h} \binom{n}{k}.\]

**Proof.** Obviously, \(m \geq h\) and \(n \geq k\). If \(a_i\) is the losing score of \(u_i \in U\) and \(b_j\) is the losing score of \(v_j \in V\), then

\[\sum_{i=1}^{m} a_i + \sum_{j=1}^{n} b_j \geq \binom{m}{h} \binom{n}{k}.\]

Now, there are \(\binom{m - 1}{h - 1} \binom{n}{k}\) arcs containing a vertex \(u_i \in U\), and \(\binom{m}{h} \binom{n - 1}{k - 1}\) arcs containing a vertex \(v_j \in V\). Therefore,

\[
\sum_{i=1}^{m} c_i + \sum_{j=1}^{n} d_j = \sum_{i=1}^{m} \left( \binom{m - 1}{h - 1} \binom{n}{k} \right) + \sum_{j=1}^{n} \left( \binom{m}{h} \binom{n - 1}{k - 1} - \binom{m}{h} \binom{n}{k} \right)
\]

\[
= m \left( \binom{m - 1}{h - 1} \binom{n}{k} \right) + n \left( \binom{m}{h} \binom{n - 1}{k - 1} - \binom{m}{h} \binom{n}{k} \right)
\]

\[
= (h + k - 1) \binom{m}{h} \binom{n}{k}.
\]

**Lemma 2.2.** If \(A = [a_1, a_2, ..., a_m]\) and \(B = [b_1, b_2, ..., b_n]\) are losing score lists of an \([h-k]\)-bipartite hypertournament \(H\), and if \(a_i < a_j\), then \(A' = [a_1, a_2, ..., a_{i-1}, a_{i+1}, ..., a_m]\) and \(B\) are losing score lists of some \([h-k]\)-bipartite hypertournament.

**Proof.** Let \(A\) and \(B\) be the losing score lists of an \([h-k]\)-bipartite hypertournament \(H\) with vertex sets \(U = \{u_1, u_2, ..., u_m\}\) and \(V = \{v_1, v_2, ..., v_n\}\) so that \(d^-(u_i) = a_i\) and \(d^-(v_j) = b_j\) \((1 \leq i \leq m, 1 \leq j \leq n)\).

If there is an \([h-k]\)-arc \(e\) containing both \(u_i\) and \(u_j\) with \(u_j\) as the last element in \(e\), let \(e' = (u_i, u_j)\) and \(H' = (H-e) \cup e'\). Clearly \(A'\) and \(B\) are the losing score lists of \(H'\).

Now, assume that for every arc \(e\) containing both \(u_i\) and \(u_j\), \(u_j\) is not the last element in \(e\). Since \(a_i < a_j\), there exist two \([h-k]\)-arcs \(e_1 = (w_1, w_2, ..., w_{i-1}, u_i, w_{i+1}, ..., w_h, z_1, z_2, ..., z_k)\) and \(e_2 = (w_1', w_2', ..., w_{h-1}', z_1', z_2', ..., z_k', u_j)\) where \(w's \in U, z's \in V, u_i \notin \{w_1, w_2, ..., w_h\}, u_j \notin \{w_1, w_2, ..., w_h\}\) and \((w_1, w_2', w_{h-1}', z_1', z_2', ..., z_k')\) is a permutation of \((w_1, w_2, ..., w_h-1, z_1, z_2, ..., z_k)\).
Now, let $e'_1 = e_1(u_i, x)$ and $e'_2 = e_2(u_j, y)$ where $x$ is any one from $\{w_1, w_2, ..., w_{h-1}, z_1, z_2, ..., z_k\}$ and $y$ is any one from $\{w'_1, w'_2, ..., w'_{h-1}, z'_1, z'_2, ..., z'_k\}$. Take $H' = (H - (e_1 \cup e_2)) \cup (e'_1 \cup e'_2)$. Then, $A'$ and $B$ are the score lists of $H'$.

**Lemma 2.3.** Let $A = [a_1, a_2, ..., a_m]$ and $B = [b_1, b_2, ..., b_n]$ be non-decreasing sequences of non-negative integers satisfying (1). If $a_m < \binom{m-1}{h-1} \binom{n}{k}$, then there exists $r (1 \leq r \leq m-1)$ such that $A'/r = [a_1, a_2, ..., a_{r-1}, a_{m+1}]$ is non-decreasing and $A'$ and $B$ satisfy (1).

**Proof.** Let $r$ be the maximum integer such that $a_{r-1} < a_r = a_{r+1} = ... = a_{m-1}$ with $a_0 = 0$ if $r = 1$.

To show that $A'$ and $B$ satisfy (1), we need to prove that for each $p (r \leq p \leq m-1)$,

$$\sum_{i=1}^{p} a_i + \sum_{j=1}^{q} b_j > \binom{p}{h} \binom{q}{k},$$

(3)

As $a_m < \binom{m-1}{h-1} \binom{n}{k}$, we have

$$\sum_{i=1}^{m-1} a_i + \sum_{j=1}^{n} b_j = \binom{m}{h} \binom{n}{k} - a_m$$

$$> \binom{m}{h} \binom{n}{k} - \binom{m-1}{h-1} \binom{n}{k}$$

$$= \left[ \left( \binom{m}{h} - \binom{m-1}{h-1} \right) \binom{n}{k} \right]$$

$$= \left( \frac{m}{h} \right) \binom{n}{k}.$$  

This shows that for $r = m-1$, (3) is true.

Now, assume that $r \leq m-2$. Then (3) holds for $p = m-1$.

If there exists $p_0 (r \leq p_0 \leq m-2)$ such that

$$\sum_{i=1}^{p_0} a_i + \sum_{j=1}^{q} b_j = \binom{p_0}{h} \binom{q}{k},$$

choose $p_0$ as large as possible.

Since

$$\sum_{i=1}^{p_0+1} a_i + \sum_{j=1}^{q} b_j > \binom{p_0+1}{h} \binom{q}{k},$$

therefore

$$a_{p_0} = a_{p_0+1} = \left( \sum_{i=1}^{p_0+1} a_i + \sum_{j=1}^{q} b_j \right) - \left( \sum_{i=1}^{p_0} a_i + \sum_{j=1}^{q} b_j \right)$$

$$> \left( \frac{p_0+1}{h} \right) \binom{q}{k} - \left( \frac{p_0}{h} \right) \binom{q}{k} = \left( \frac{p_0}{h-1} \right) \binom{q}{k}.$$  

Thus, it follows that

$$\sum_{i=1}^{p_0} a_i + \sum_{j=1}^{q} b_j = \sum_{i=1}^{p_0} a_i + \sum_{j=1}^{q} b_j - a_{p_0}$$
\[
\left( \frac{p_0}{h} \right) \left( \begin{array}{c} q \\ k \end{array} \right) - \left( \frac{p_0}{h-1} \right) \left( \begin{array}{c} q \\ k \end{array} \right) \\
\left( \frac{p_0-1}{h} \right) + \left( \frac{p_0-1}{h-1} \right) \left( \begin{array}{c} q \\ k \end{array} \right) - \left( \frac{p_0}{h-1} \right) \left( \begin{array}{c} q \\ k \end{array} \right) \\
\left( \frac{p_0-1}{h-1} \right) - \left( \frac{p_0-1}{h-2} \right) \left( \begin{array}{c} q \\ k \end{array} \right) \\
\left( \frac{p_0-1}{h} \right) \left( \begin{array}{c} q \\ k \end{array} \right),
\]

a contradiction with the hypothesis on A and B. Hence (3) holds.

**Proof of Theorem 2.1. Necessity.** Let A and B be the losing score lists of an \([h,k]\)-bipartite hypertournament \(H(U, V)\). For any \(p\) and \(q\) with \(h \leq p \leq m\) and \(k \leq q \leq n\), let \(U_1 = \{u_1, u_2, \ldots, u_p\}\) and \(V_1 = \{v_1, v_2, \ldots, v_q\}\) be the set of vertices such that \(d^- (u_i) = a_i\) for each \(1 \leq i \leq p\), and \(d^- (v_j) = b_j\) for each \(1 \leq j \leq q\). Let \(H_1\) be the \([h,k]\)-bipartite subhypertournament formed by \(U_1\) and \(V_1\). Then

\[
\sum_{i=1}^{m} a_i + \sum_{j=1}^{n} b_j \geq \sum_{i=1}^{m-1} \overline{d}_{H_1}(u_i) + \sum_{j=1}^{n} \overline{d}_{H_1}(v_j) = \left( \frac{p}{h} \right) \left( \begin{array}{c} q \\ k \end{array} \right).
\]

**Sufficiency.** We induct on \(m\) and keep \(n\) fixed. For \(m = h\), the result is obviously true. Therefore, let \(m > h\), and similarly \(n > k\).

Now, \(a_m = \sum_{i=1}^{m} a_i + \sum_{j=1}^{n} b_j - \left( \sum_{i=1}^{m-1} a_i + \sum_{j=1}^{n} b_j \right)\)

\[
\leq \left( \frac{m}{h} \right) \left( \begin{array}{c} n \\ k \end{array} \right) - \left( \frac{m-1}{h} \right) \left( \begin{array}{c} n \\ k \end{array} \right)
\]

\[
= \left[ \left( \frac{m}{h} \right) - \left( \frac{m-1}{h} \right) \right] \left( \begin{array}{c} n \\ k \end{array} \right)
\]

\[
= \left( \frac{m-1}{h-1} \right) \left( \begin{array}{c} n \\ k \end{array} \right).
\]

We consider the following two cases.

**Case 1.** \(a_m = \left( \frac{m-1}{h-1} \right) \left( \begin{array}{c} n \\ k \end{array} \right)\).

So, \(\sum_{i=1}^{m-1} a_i + \sum_{j=1}^{n} b_j = \sum_{i=1}^{m-1} a_i + \sum_{j=1}^{n} b_j - a_m\)

\[
= \left( \frac{m}{h} \right) \left( \begin{array}{c} n \\ k \end{array} \right) - \left( \frac{m-1}{h-1} \right) \left( \begin{array}{c} n \\ k \end{array} \right)
\]

\[
= \left[ \left( \frac{m}{h} \right) - \left( \frac{m-1}{h-1} \right) \right] \left( \begin{array}{c} n \\ k \end{array} \right) = \left( \frac{m-1}{h-1} \right) \left( \begin{array}{c} n \\ k \end{array} \right).
\]

By induction hypothesis \([a_1, a_2, \ldots, a_{m-1}]\) and B are losing score lists of an \([h,k]\)-bipartite hypertournament \(H' (U', V)\) of order \(m-1 \times n\). Construct an \([h,k]\)-bipartite hypertournament \(H\) of order \(m \times n\) as follows. In \(H'\), let \(U' = \{u_1, u_2, \ldots, u_{m-1}\}\) and \(V = \{v_1, v_2, \ldots, v_n\}\). Adding a new vertex \(u_m\), for each \((h+k)\)-tuple containing \(u_m\), arrange \(u_m\) on the last entry. Denote \(E_1\) to be the set of
and B are losing score lists of H. Let \( E(H) = E(H') \cup E_1 \). Clearly, A and B are losing score lists of H.

**Case 2.** \( a_m < \binom{m-1}{h-1} \binom{n}{k} \).

Applying Lemma 2.3 repeatedly on A and keeping B fixed until we get a new non-decreasing list \( A' = [a_1', a_2', \ldots, a_m'] \) in which now \( a'_m = \binom{m-1}{h-1} \binom{n}{k} \).

By Case 1, A' and B are the losing score lists of an \( [h-k] \)-bipartite hypertournament. Now, apply Lemma 2.2 on A' and B repeatedly until we obtain the initial pair of non-decreasing lists A and B. Then by Lemma 2.2, A and B are the losing score lists of an \( [h-k] \)-bipartite hypertournament.

**Remark.** If \( h = 1 \), \( k = 1 \), we get the definition of scores in bipartite tournaments and Theorem 2.1 gives

\[
\sum_{i=1}^{p} a_i + \sum_{j=1}^{q} b_j \geq \binom{p}{1} \binom{q}{1} = pq,
\]

which is the characterization of score lists due to Beineke and Moon [2].

**Proof of Theorem 2.2.** Let \([c_1, c_2, \ldots, c_m]\) and \([d_1, d_2, \ldots, d_n]\) be score lists of an \([h-k]\)-bipartite hypertournament \( H(U, V) \), where \( U = \{u_1, u_2, \ldots, u_m\} \) and \( V = \{v_1, v_2, \ldots, v_n\} \) with \( d_H(u_i) = c_i \) for \( i = 1, 2, \ldots, m \), and \( d_H(v_j) = d_j \) for \( j = 1, 2, \ldots, n \). Clearly, \( d^+(u_i) + d^-(u_i) = \binom{m-1}{h-1} \binom{n}{k} \) and \( d^+(v_j) + d^-(v_j) = \binom{m}{h} \binom{n-1}{k} \).

Let \( a_{m+1-i} = d^-(u_i) \) and \( b_{n+1-j} = d^-(v_j) \).

Then \([a_1, a_2, \ldots, a_m]\) and \([b_1, b_2, \ldots, b_n]\) are the losing score lists of \( H \). Conversely, if \([a_1, a_2, \ldots, a_m]\) and \([b_1, b_2, \ldots, b_n]\) are the losing score lists of \( H \), then \([c_1, c_2, \ldots, c_m]\) and \([d_1, d_2, \ldots, d_n]\) are the score lists of \( H \). Hence it is sufficient to show that conditions (1) and (2) are equivalent provided

\[
c_i + a_{m+1-i} = \binom{m-1}{h-1} \binom{n}{k}
\]

and

\[
d_j + b_{n+1-j} = \binom{m}{h} \binom{n-1}{k}.
\]

First, assume (2) holds. Then

\[
\sum_{i=1}^{p} a_i + \sum_{j=1}^{q} b_j = \sum_{i=1}^{p} \left\{ \binom{m-1}{h-1} \binom{n}{k} - c_{m+1-i} \right\} + \sum_{j=1}^{q} \left\{ \binom{m}{h} \binom{n-1}{k-1} - d_{n+1-j} \right\}
\]

\[
= p \left( \binom{m-1}{h-1} \binom{n}{k} + q \right) \binom{m}{h} \binom{n-1}{k-1} - \left[ \sum_{i=1}^{m} c_i + \sum_{j=1}^{n} d_j - \sum_{i=1}^{m-p} c_i - \sum_{j=1}^{n-q} d_j \right]
\]

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\[ \geq p \left( \binom{m-1}{h-1} \binom{n}{k} \right) + q \left( \binom{m}{h} \binom{n-1}{k-1} \right) - \left( h + k - 1 \right) \left( \binom{m}{h} \binom{n}{k} \right) \]
\[+ (m-p) \left( \binom{m-1}{h-1} \binom{n}{k} \right) + (n-q) \left( \binom{m}{h} \binom{n-1}{k-1} \right) \]
\[= \left( \binom{m-(m-p)}{h} \binom{n-(n-q)}{k} \right) - \left( \binom{m}{h} \binom{n}{k} \right) \]
with equality when \( p = m \) and \( q = n \). Thus, (1) holds.
Now, when (1) holds, using a similar argument as above, we can prove that (2) holds.
This completes the proof of the Theorem.

**Corollary 2.1.** Given non-negative integers \( m, n, h \) and \( k \) with \( m \geq h > 1 \) and \( n \geq k > 1 \), the non-decreasing sequences \( A = [a_i]_m^1 \) and \( B = [b_j]_n^1 \) of non-negative integers are the losing score lists of an \([h-k]\)-bipartite hypertournament if and only if for each \( p \) and \( q \),
\[ \sum_{i=1}^{p} a_i + \sum_{j=1}^{q} b_j \leq \left( \binom{m}{h} \binom{n}{k} \right) - \left( \binom{m-p}{h} \binom{n-q}{k} \right), \]

**Proof.** This follows from Theorem 2.1.

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