Affect of brane thickness on microscopic tidal-charged black holes

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We study the phenomenological implications stemming from the dependence of the tidal charge on the brane thickness for the evaporation and decay of microscopic black holes. In general, the longer $L$ the weaker are the black hole life-times and the greater their maximum mass for those cases in which the black hole can grow. In particular, we again find that tidal-charged black holes might live long enough to escape the detectors and even the gravitational field of the Earth, thus resulting in large amounts of missing energy. However, under no circumstances could TeV-scale black holes grow enough to enter the regime of Bondi accretion.

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I. INTRODUCTION

As is well known by now, the conjectured existence of extra spatial dimensions [1, 2] and a sufficiently small fundamental scale of gravity has opened up the possibility that microscopic black holes can be produced and detected [3–5] at the Large Hadron Collider (LHC). In a series of papers [6–8], we have analyzed the phenomenology of a particular candidate [9] of brane-world black holes [10] in the Randall-Sundrum (RS) model [2].

Our world is thus a three-brane (with coordinates $x^{n}$, $n = 0, \ldots, 3$) embedded in a five-dimensional bulk with the metric

$$ds^2 = e^{-|y|/\ell} g_{\mu \nu} dx^\mu dx^\nu + dy^2,$$  \hspace{1cm} (1)

where $y$ parameterizes the fifth dimension and $\ell$ is a length determined by the brane tension. This parameter relates the four-dimensional Planck mass $M_p$ to the five-dimensional gravitational mass $M(5)$ and one can have $M(5) \simeq 1$ TeV $c^2$ (for bounds on $\ell$, see, e.g., Ref. [11]) and black holes with mass in the TeV range. The brane must also have a thickness, which we denote by $L$, below which deviations from the four-dimensional Newton law occur. Current precision experiments require that $L \lesssim 44 \mu m$ [12], whereas theoretical reasons imply that $L \gtrsim \ell_p M_p/M(5) \simeq 2 \cdot 10^{-19}$ m ($\ell_p$ is the four-dimensional Planck length and $\ell(5)$ is the five-dimensional Planck length). In the analysis below, the parameters $M(5)$ and $L$ are assumed to be independent of one another, but within the stated ranges.

The tidal-charged metric of Ref. [9] solves the Einstein equations projected onto the brane [13] and is given by

$$ds^2 = - A dt^2 + A^{-1} dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right),$$  \hspace{1cm} (2)

with

$$A = 1 - \frac{2 \ell_p M}{M_p r} - q \frac{\ell_p^2}{r^2},$$  \hspace{1cm} (3)

where $q > 0$ is the tidal charge. This parameter could be determined only by solving the Einstein equations in the bulk, which is a difficult problem whose analytic solution has yet to be obtained (for a perturbative study, see Ref. [8]). On general grounds, $q$ should depend upon the Arnowitt-Deser-Misner (ADM) mass $M$ in such a way that $q$ vanishes when $M \to 0$ [3, 7].

For specific forms of $q = q(M)$, tidal-charged black holes may have very long lifetimes [5], which led to the conjecture that they might be able to grow to catastrophic size within the Earth [14], contrary to the picture [15] that arises in the ADD scenario [1]. This possibility was however refuted in Ref. [16]. We also solved the system of equations which describes the time-evolution of mass and momentum for various initial conditions and parameters in the acceptable ranges and found no evidence of catastrophic growth [6, 7].

In our previous analysis, we assumed $q = q(M)$, neglecting the possible dependence of $q$ on the brane thickness $L$. In the present work, we therefore take a complementary view and assume that $q = q(M, L)$ with $q = 0$ for $L = 0$ as well as for $M = 0$. This dependence will be constrained by the experimental bounds mentioned in the beginning, along with the findings from Ref. [8]. This allows us to restrict the space of parameters to a manageable range, within which we will study the time-evolution numerically. We shall then find that tidal-charged black holes produced at the LHC would very likely evaporate instantaneously and, even for those values of the parameters which lead to an initial growth, no catastrophic scenario will arise. However, life-times are longer for larger $L$ and could be long enough to allow for black holes to escape from the detectors and result in significant amounts of missing energy.

We use units with $1 = \hbar = M_p = \ell_p = \ell(5) M(5)$, where $M_p \simeq 2.2 \cdot 10^{-8}$ kg and $\ell_p \simeq 1.6 \cdot 10^{-35}$ m are related to the four-dimensional Newton constant $G_N = \ell_p/M_p$. 

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Further, $M(5) \simeq M_{\text{ew}} \simeq 1 \text{ TeV} \ (\simeq 1.8 \cdot 10^{-24} \text{ kg})$, the electro-weak scale, corresponding to $\ell(5) \simeq 2.0 \cdot 10^{-19} \text{ m}$.

II. TIDAL CHARGE AND BRANE THICKNESS

In some idealized models of the R-S model the brane thickness is taken to be zero. However this idealization is not realistic. In string theory there is a minimum length scale, which implies that there is a minimum brane thickness. Furthermore quantum corrections to any classical interactions require that the brane have a finite thickness. One simple, although not unique, way of incorporating the condition of finite brane thickness is to assume that the tidal charge is related to $M$ and $L$ according to

$$q \simeq \left( \frac{L}{\ell(5)} \right)^2 \left( \frac{M}{M(5)} \right)^{\beta} \beta^2 \ell^2 \ell(5)^2 \ ,$$

(4)

where both $\gamma$ and $\beta$ are real and positive. We further assume that they do not depend on the mass scale, and can therefore be constrained by using $M(5) \simeq 1 \text{ TeV}/c^2$ and the known bounds on $L$. For this purpose, we note that the tidal term in the metric ($A = 1 - A_t - A_N$),

$$A_t \simeq \left( \frac{\ell(5)}{\ell(5)} \right)^\gamma \left( \frac{M}{M(5)} \right)^{\beta} \beta^2 \ell^2 \ell(5)^2 \ ,$$

(5)

dominates over the usual Schwarzschild term,

$$A_N \simeq 2 \frac{M \ell_p}{M_p r^2} \ ,$$

(6)

for $r \lesssim r_c$, with

$$r_c \simeq \ell_p \frac{M_p}{M(5)} \left( \frac{L}{\ell(5)} \right)^\gamma \left( \frac{M}{M(5)} \right)^{\beta - 1} \ .$$

(7)

Although the thickness of the brane could be much less than the experimental bound on corrections to Newton’s law of gravitation, we assume the ‘worst case scenario’ in which the thickness of the brane sets the upper limit on the size of the corrections. In this way we can push the possibility of catastrophic black hole production at the LHC to the limit. Within this scenario consistency requires that $r_c$ be shorter than the length scale above which corrections to the Newton potential have not yet been detected, that is

$$r_c \ll L \ ,$$

(8)

if the black hole is “small”, in the sense that

$$R_H \ll r_c \ll L \ ,$$

(9)

where

$$R_H = \ell_p \left( \frac{M}{M_p} + \sqrt{\frac{M^2}{M_p^2} + \frac{q}{M_p}} \right) \ ,$$

(10)
is the horizon radius. For $R_H \ll r_c$, the horizon radius can then be estimated from the tidal contribution [5].

$$R_H \simeq \ell_p \left( \frac{L}{\ell(5)} \right)^{\gamma/2} \left( \frac{M}{M(5)} \right)^{\beta/2} \ ,$$

(11)

otherwise $R_H$ approaches the usual four-dimensional expression

$$R_H \simeq 2 \ell_p \frac{M}{M_p} \ .$$

(12)

The effective four-dimensional Euclidean action for such small black holes is given by [5, 17]

$$S_{(4)}^E = \frac{M_p (4 \pi R_H^4)}{16 \pi \ell_p} \simeq \ell_p M_p \left( \frac{M}{M(5)} \right)^\beta \ ,$$

(13)

with

$$M_{\text{eff}} \simeq M(5) \left( \frac{\ell(5)}{L} \right)^\gamma \ .$$

(14)

According to the area law, a black hole is classical if its mass is much larger than $M_{\text{eff}}$ [19, 20], which implies that $M_{\text{eff}} \lesssim M(5)$ for the existence of TeV-scale black holes. Since $\gamma$ and $\beta$ are positive, the above relation holds for the entire parameter range and stronger constraints could be imposed only if $L \lesssim \ell(5)$.

For $\beta \neq 1$, one then has that $r_c = L$ corresponds to a critical mass

$$M_c = M(5) \left( \frac{L}{\ell(5)} \right)^{\frac{1}{\gamma - 1}} \ .$$

(15)

Further, for $\beta \neq 2$, the condition that $r_c = R_H$ leads to

$$M \simeq M_H \equiv M(5) \left[ \left( \frac{M(5)}{M_p} \right)^2 \left( \frac{\ell(5)}{L} \right)^{\gamma} \right]^{\frac{1}{\gamma - 1}} \ ,$$

(16)

whereas for $\beta = 2$ one finds no constraint on $M$, but $R_H \ll r_c$ implies that

$$\left( \frac{M_p}{M(5)} \right)^2 \left( \frac{L}{\ell(5)} \right)^{\gamma} \gg 1 \ ,$$

(17)

which is true for all $\gamma > 0$.

For $\beta \neq 1$ and $\beta \neq 2$, $M_c$ and $M_H$ are properly defined as above, and the condition [5] implies

$$\left( \frac{M}{M(5)} \right)^{\beta - 1} \ll \left( \frac{L}{\ell(5)} \right)^{1 - \gamma} \ .$$

(18)

We then have two cases: (i) for $0 < \beta < 1$,

$$M \gtrsim M_c \ ,$$

(19)

while, (ii) for $\beta > 1$,

$$M \lesssim M_c \ .$$

(20)
Similarly, one can analyze the lower bound in Eq. (9), and again find two separate cases: (a) for $\beta < 2$ we get

$$M \lesssim M_H ,$$

and, (b) for $\beta > 2$,

$$M \gtrsim M_H .$$

A. $\beta > 1$

In this range, the tidal term grows with $M$ faster than the Newtonian term, and Eq. (8) becomes the upper bound (20) on the maximum black hole mass, namely

$$M \lesssim M_c ,$$

and the condition $M_c \gg M_{(5)}$ holds if

$$0 \lesssim \gamma < 1 .$$

We must then analyse the three cases (a), (b) and $\beta = 2$ separately.

1. $1 < \beta < 2$

In this case, a “small” black hole must have a mass in the range

$$M_{(5)} \ll M \ll \min\{M_H , M_c\} .$$

Requiring that $M_H \gg M_{(5)}$ implies that

$$\left( \frac{M_p}{M_{(5)}} \right)^{\frac{2}{\beta}} \left( \frac{L}{\ell_{(5)}} \right)^{\frac{\gamma}{\beta}} \gg 1 ,$$

which is true for all positive values of $\gamma$, whereas $M_c \gg M_{(5)}$ leads to the condition in Eq. (24).

We must however notice that, for $\beta \to 1^+$ and $\gamma$ in the above range, the critical mass $M_c \to \infty$ and an infinitely massive black hole would be of the tidal kind, thus ruling out the Schwarzschild geometry. We therefore require that $M_c \lesssim M_\odot \simeq 10^{54}$ TeV (the mass of the sun).

2. $\beta = 2$

The black hole mass must be smaller than the critical mass

$$M \ll M_c \simeq M_{(5)} \left( \frac{L}{\ell_{(5)}} \right)^{1-\gamma} ,$$

and $M_c \gg M_{(5)}$ again leads to Eq. (24).

3. $\beta > 2$

The black hole is now “small” if

$$\max\{M_H , M_{(5)}\} \ll M \ll M_c .$$

The condition that $M_H \ll M_c$ then implies the new constraint

$$\left( \frac{M_{(5)}}{M_p} \right)^{\frac{2}{\beta}} \ll \left( \frac{L}{\ell_{(5)}} \right)^{\frac{\gamma+\beta-2}{\beta}} ,$$

which holds for all values of $\gamma$ in the range given in Eq. (24).

B. $0 < \beta < 1$

In this case, the tidal term grows with $M$ more slowly than the Newton potential, and we obtained

$$M \gtrsim M_c .$$

This also implies that the critical mass $M_c$ must be smaller than $M_{(5)}$, which results in the bound (24). The black hole is then small for

$$M_{(5)} \ll M \ll M_H ,$$

where $M_H$ is again given in Eq. (16). A necessary condition is that $M_H \gg M_{(5)}$ which holds for $\gamma$ in the range given in Eq. (24).

C. $\beta = 1$

Both $A_t$ and $A_N$ grow linearly with $M$, and (7) reads

$$r_c \simeq \ell_p \frac{M_p}{M_{(5)}} \left( \frac{L}{\ell_{(5)}} \right)^{\gamma} ,$$

which does not depend on $M$. Eq. (8) leads to

$$\left( \frac{L}{\ell_{(5)}} \right)^{\gamma-1} \ll 1 .$$

which again constrains $\gamma$ to the range given in Eq. (24).

III. TIME-EVOLUTION

The time evolution of the black hole mass is obtained by summing the evaporation and accretion rates [8, 13],

$$\frac{dM}{dt} = \left. \frac{dM}{dt} \right|_{evap} + \left. \frac{dM}{dt} \right|_{acc} .$$

Evaporation occurs via the Hawking effect [18] and is described in the microcanonical picture [6, 19]. There are
two mechanisms by which microscopic black holes accrete mass: one due to the collisions with the atomic and sub-atomic particles encountered as they sweep through matter, and one due to the gravitational force the black hole exerts on surrounding matter once it comes to rest. The latter is known as Bondi accretion and is appreciable only when the horizon radius is greater than atomic size \[ \text{[21]} \]. Eq. (34) and the equation for the time-evolution of the momentum,

\[
\frac{dp}{dt} = \frac{p}{M} \frac{dM}{dt} \bigg|_{\text{evap}}.
\]

form a system which can be solved numerically to obtain \( M(t) \) and \( p(t) \).

In the following we shall evolve a black hole produced with a typical initial mass \( M(0) \approx 10^{15} \text{ TeV/cm}^2 \approx 10^{-23} \text{ kg} \) and momentum \( p(0) \) varied in the range from 1 MeV/cm to 5 TeV/cm \[ \text{[22]} \]. We will analyze values for the parameter \( \beta > 0 \) in each of the different ranges considered in Section \( \text{[II]} \) and

\[
0 \lesssim \gamma < 1,
\]

The brane thickness \( L \) will be varied in the range \( 10^{-13} \mu\text{m} \lesssim L \lesssim 44 \mu\text{m} \), with the corresponding critical mass \( M_\text{c} \) given in Eq. (14). Our results are given in Tables \( \text{[III]} \) in which \( M_\text{univ} \) is the black hole mass after a time of the same order of magnitude as the present age of our Universe (\( 10^{18} \text{ sec} \)), \( R_\text{H} \) and \( R_\text{EM} \) the corresponding values of the horizon and capture radius, \( M_\text{E} \) the mass reached after traveling the Earth’s diameter, \( R_\text{E} \) the corresponding capture radius, \( v_\text{E} \) the time to travel the Earth’s diameter and \( v_\text{E} \) the velocity at that point.

### Table I: Time-evolution of black hole mass for large initial momentum \( p(0) \), \( L = 10^{-4} \mu\text{m}, \beta = 1.1, \gamma = 0.1 \) which result in \( M_\text{c} = 4 \cdot 10^{24} \text{ TeV/cm}^2 \).

| \( p(0) \) (MeV/c) | \( M(0) \) (TeV/c) | \( M_\text{univ} \) (kg) | \( R_\text{EM} \) (m) | \( R_\text{H} \) (m) | \( t_\text{E} \) (sec) | \( v_\text{E} \) (km/sec) |
|-------------------|-----------------|-----------------|-------------|-------------|---------------|----------------|
| 5.0 \cdot 10^{-4} | 1.8 \cdot 10^{-24} | 1.3 \cdot 10^{-18} | 0.1 | 1.3 \cdot 10^{5} | | |
| 1.0 \cdot 10^{-4} | 2.0 \cdot 10^{-25} | 1.3 \cdot 10^{-18} | 5.2 | 2.6 \cdot 10^{1} | | |
| 1.0 \cdot 10^{-3} | 2.0 \cdot 10^{-26} | 1.3 \cdot 10^{-18} | 52 | 2.6 \cdot 10^{2} | | |
| 1.0 \cdot 10^{-4} | 2.0 \cdot 10^{-27} | 1.3 \cdot 10^{-18} | 4.9 \cdot 10^{2} | 26 | | |

### Table II: Time evolution of black hole mass for small initial momentum \( p(0) \), \( L = 10^{-4} \mu\text{m}, \beta = 1.1, \gamma = 0.1 \) which result in \( M_\text{c} = 4 \cdot 10^{24} \text{ TeV/cm}^2 \).

| \( p(0) \) (MeV/c) | \( M(0) \) (TeV/c) | \( M_\text{univ} \) (kg) | \( R_\text{EM} \) (m) | \( R_\text{H} \) (m) |
|-------------------|-----------------|-----------------|-------------|-------------|
| 100 | 11.0 | 8.4 \cdot 10^{-15} | 2.0 \cdot 10^{-16} | 9.2 \cdot 10^{-30} |
| 10 | 11.0 | 1.8 \cdot 10^{-15} | 1.3 \cdot 10^{-16} | 3.9 \cdot 10^{-30} |
| 1.0 | 11.0 | 3.9 \cdot 10^{-16} | 9.4 \cdot 10^{-17} | 1.7 \cdot 10^{-39} |

### Table III: Time evolution of black hole mass as function of brane thickness \( L \) for \( \beta = 1.1, \gamma = 0.1 \) and initial conditions \( M(0) = 11 \text{ TeV/cm}^2 \) and \( p(0) = 100 \text{ MeV/c} \). N/A means that the black hole mass does not grow.

| \( \gamma \) | \( M_\text{c} \) (TeV/c) | \( M_\text{univ} \) (kg) | \( R_\text{EM} \) (m) | \( R_\text{H} \) (m) |
|----------|-----------------|-----------------|-------------|-------------|
| 0.1 | 4 \cdot 10^{-24} | 8.4 \cdot 10^{-15} | 2.0 \cdot 10^{-16} | 9.2 \cdot 10^{-30} |
| 0.3 | 2 \cdot 10^{-26} | 8.4 \cdot 10^{-15} | 2.0 \cdot 10^{-16} | 9.2 \cdot 10^{-30} |
| 0.5 | 4 \cdot 10^{-24} | 8.4 \cdot 10^{-15} | 2.0 \cdot 10^{-16} | 9.2 \cdot 10^{-30} |
| 0.7 | 4 \cdot 10^{-24} | N/A | N/A | N/A |

### Table IV: Time evolution of black hole mass as a function of \( \gamma \) for \( \beta = 1.1, L = 10^{-4} \mu\text{m} \) and initial conditions \( M(0) = 11 \text{ TeV/cm}^2 \) and \( p(0) = 100 \text{ MeV/c} \). N/A means that the black hole mass does not grow.

### A. Rapidly decaying solutions

The first important result is that, for \( 0 < \beta < 1 \) and \( 1.3 \lesssim \beta \) (this lower bound slightly varies depending on the values of \( \gamma \) and \( L \)), the black hole decays almost instantly. In fact, the decay time is less than \( 10^{-10} \text{ sec} \) and the black hole would not exit the detector inside which it was produced.

### B. Growing solutions

For \( 1 < \beta \lesssim 1.3 \), the mass of the black hole can grow and a typical example is displayed in Fig. [I].

In Table [II] we show the evolution of the masses of the black holes as a function of the initial momentum \( p_0 \) for \( \gamma = 0.1, \beta = 1.1 \) and \( L = 10^{-4} \mu\text{m} \). The critical mass in this case is constant and approximately equal to its maximum allowed value \( 4 \cdot 10^{24} \text{ TeV/cm}^2 \). The black

### Table V: Time evolution of black hole mass as function of \( \beta \) for \( \gamma = 0.1, L = 10^{-4} \mu\text{m} \) and initial conditions \( M(0) = 11 \text{ TeV/cm}^2 \) and \( p(0) = 100 \text{ MeV/c} \). N/A means that the black hole mass does not grow.

| \( \beta \) | \( M_\text{c} \) (TeV/c) | \( M_\text{univ} \) (kg) | \( R_\text{EM} \) (m) | \( R_\text{H} \) (m) |
|----------|-----------------|-----------------|-------------|-------------|
| 1.1 | 4 \cdot 10^{-24} | 8.4 \cdot 10^{-15} | 2.0 \cdot 10^{-16} | 9.2 \cdot 10^{-30} |
| 1.2 | 4 \cdot 10^{-24} | N/A | N/A | N/A |
At this point accretion stops and the black hole evaporates rapidly. The data we present simulates a scenario in which the whole Universe is filled with matter of density equal to the Earth’s. We notice that the final mass increases with the value of the initial momentum. The highest attainable mass in this time frame is of the order of $10^{-14}$ kg, for which both the electromagnetic radius and horizon radius are orders of magnitude smaller than atomic size. Bondi accretion therefore does not occur.

Table III shows the dependence of the mass $M$ on the brane thickness up to $L = 10^{-4} \mu m$, since for $L$ larger, the critical mass would be greater than the mass of the sun. The data shows that $M_c$, $M_{\text{minv}}$, $R_{\text{EM}}$, and $R_H$ all increase with $L$ and, for $L \lesssim 10^{-9} \mu m$ the black holes evaporate instantly. The maximum values of $R_{\text{EM}}$ and $R_H$ are again many orders of magnitude smaller than atomic size and the black holes remain far from Bondi accretion. We note here that the the range $10^{-4} \mu m \ll L$ is not excluded. The parameter $\gamma$, for instance, can be slightly increased in order to keep $M_c$ within limits for all values of $L$ smaller than $44 \mu m$, as one can see from Fig. 2. Nevertheless, $M_c$, $R_{\text{EM}}$, and $R_H$ will not vary more than two orders of magnitude from the values presented in Table III.

The study of the dependence on $\gamma$ does not show strong variations in the final values at $10^{18}$ sec. Table IV shows that increasing $\gamma$ to 0.7 still leads to instantaneous decay. The disappearance of a peak in $M(t)$ is more sensitive to $\beta$, as can be seen from Table V.

### IV. CONCLUSIONS

In this investigation we have continued our study of the microscopic black holes which could be produced at the LHC, based on the model presented in Refs. [5, 6] and the description of brane-world black holes given in Ref. [9]. In particular, we have extended the treatment of Ref. [7] by allowing the tidal charge to depend on the brane thickness. Conditions for the black holes to be tidal and phenomenologically acceptable were then used to determine the range of the parameters $\gamma$ and $\beta$ in Eq. (4). Subsequently, the time evolution of the black hole mass and momentum were obtained numerically.

We find that tidal black holes would evaporate (almost) instantly, except for $1 < \beta \lesssim 1.3$. Inside this range, black holes cannot grow to catastrophic size, but might live long enough to escape the detectors and result in significant amounts of missing energy.
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