Differential elastic nucleus-nucleus scattering in complete Glauber theory

Yu.M. Shabelski and A.G. Shuvaev

Petersburg Nuclear Physics Institute, Kurchatov National Research Center
Gatchina, St. Petersburg 188300, Russia

E-mail: shabelsk@thd.pnpi.spb.ru
E-mail: shuvaev@thd.pnpi.spb.ru

Abstract

The differential elastic cross sections of $^{12}\text{C} - ^{12}\text{C}$ and $^{11}\text{Li} - ^{12}\text{C}$ nuclei are calculated in the complete Glauber theory. The role of the possible correlations connected to the shell effects in $^{11}\text{Li}$ nucleus is considered.

1 Introduction

High energy nuclear interactions are usually treated in the Glauber theory\cite{1-3}. It is highly efficient in describing the hadron-nucleus scattering where all the relevant diagrams can be summed up in the closed form. The nucleus-nucleus scattering becomes rapidly more complicated for $A > 4$ and additional simplified approximations are usually made to get the result analytically. An example of such calculations for the differential elastic and reaction cross sections for $^{11}\text{Li} - ^{12}\text{C}$ scattering can be found in Refs.\cite{4,5}

In this paper the analytical calculations of the differential elastic cross sections for $^{12}\text{C} - ^{12}\text{C}$ and $^{11}\text{Li} - ^{12}\text{C}$ scattering have been carried out in the complete Glauber theory in the generating function approach Ref.\cite{6}. The results have been obtained for several commonly used forms of the nuclear density, which allows one to get an insight to the possible nuclear correlations, in particular, to those related to the shell structure of the halo nucleus $^{11}\text{Li}$.

It is worth pointing out that it is the differential cross section rather than the integrated reaction one that is more sensitive to the nuclear density distribution, especially to its far distance parts.
2 Differential elastic cross sections

The amplitude of the elastic scattering of the incident nucleus $A$ on the fixed target nucleus $B$ is given in the Glauber theory by the expression\cite{7,8}

$$F_{AB}^{el}(q) = \frac{ik}{2\pi} \int d^2b e^{iqb} [1 - S_{AB}(b)], \quad (1)$$

where $q$ is the transferred momentum and $k$ is the mean nucleon momentum in nucleus $A$. The two-dimensional impact momentum $b$ lies in the transverse plain to the vector $k$. The function $S_{AB}(b)$ is evaluated through the sum of the independent phase shifts $\chi_{NN}(b)$ for each nucleon-nucleon scattering,

$$S_{AB}(b) = \langle A, B | \prod_{i,j} [1 - \Gamma_{NN}(b + x_i - y_j)] \rangle | A, B \rangle, \quad (2)$$

with

$$\Gamma_{NN}(b) = 1 - e^{i\chi_{NN}(b)} = \frac{1}{2\pi i k} \int d^2q e^{-iqb} f_{NN}^{el}(q). \quad (3)$$

The symbol $\langle A, B | \cdots | A, B \rangle$ denotes an average over the nucleons’ positions $x_i$ and $y_j$ in the transverse plain. Each pair $\{i, j\}$ enters the product only once, meaning that each nucleon from the projectile nucleus can scatter on each nucleon no more than once.

The elastic nucleon-nucleon amplitude, $f_{NN}$, is mainly imaginary at the beam energy about 1 GeV per nucleon, $Re f_{NN}/Im f_{NN} \lesssim 10^{-1}$. The standard parametrization is

$$f_{NN}^{el}(q) = ik \frac{\sigma_{NN}^{tot}}{4\pi} e^{-\frac{1}{2}q^2 \beta}, \quad (4)$$

where $\sigma_{NN}^{tot}$ is the total nucleon-nucleon cross section. The slope $\beta$ is related to an effective interaction radius $a = \sqrt{2\pi \beta}$. An efficient way to deal with the function \cite{2} is provided by the generating function $Z(u, v)$,

$$S_{AB}(b) = \left. \frac{1}{Z(0, 0)} \frac{\partial^A}{\partial u^A} \frac{\partial^B}{\partial v^B} Z(u, v) \right|_{u=v=0}, \quad (5)$$

for which the closed expression have been obtained in Ref.\cite{6}

$$Z(u, v) = e^{W_y(u,v)}, \quad z_y = 1 - \frac{1}{2} \frac{\sigma_{NN}^{tot}}{a^2}, \quad (6)$$

$$W_y(u,v) = \frac{1}{a^2} \int d^2x \ln \left( \sum_{M \leq A, N \leq B} \frac{z_y^M}{M!N!} [a^2 u \rho_A^1(x - b)]^M [a^2 v \rho_B^1(x)]^N \right). \quad (7)$$
The transverse densities entering this formula are expressed through the three-dimensional nucleon distributions in the colliding nuclei,

\[ \rho_{A,B}^{\perp}(x_\perp) = \int dz \rho_{A,B}(z,x_\perp), \quad \int d^2 x_\perp \rho_{A,B}^{\perp}(x_\perp) = 1. \]

The function \( W_y(u,v) \) goes as the series built of the overlaps,

\[ t_{m,n}(b) = \frac{1}{\alpha^2} \int d^2 x \left[ a^2 \rho_A^{\perp}(x - b) \right]^m \left[ a^2 \rho_B^{\perp}(x) \right]^n, \] (8)

with \( m \leq A \) and \( n \leq B \). For relatively light nuclei, say, \( A, B \lesssim 20 \), the generating function can be explicitly differentiating with respect to \( u, v \) variables yielding the final \( S_{AB}(b) \) function. For more heavy nuclei the differentiating amounts with a rather good accuracy to sending these variables to \( A \) and \( B \) values (see Ref.[6] for details).

The density distribution in a relatively light nuclei with the atomic weight \( A \lesssim 20 \) is well parametrized by the harmonic oscillator function

\[ \rho_A(r) = \rho_0 \left[ 1 + \frac{1}{6} (A - 4) \frac{r^2}{\lambda^2} \right] e^{-r^2/\lambda^2}, \] (9)

with \( \rho_0 \) being the normalization, and the factor \( \lambda \) being adjusted to match the nuclear mean square radius, \( R_{\text{rms}} = \sqrt{r^2_A}, r^2_A = \int d^3 r r^2 \rho_A(r) \). The simple form of this parametrization enables one to get analytically all the overlaps relevant to \(^{12}\text{C} - ^{12}\text{C} \) scattering. For more complicated densities the overlaps can be calculated numerically.

There exist several parametrizations for \(^{11}\text{Li} \) nucleon density, which are more complex due to the halo structure. The density is supposed to be the sum of two separate parts for the central core and for the surrounding it neutron halo.\(^{10,12} \) Below we use the particular parametrization of the form\(^{13} \)

\[ \rho(r) = N_c \rho_c(r) + N_v \rho_v(r), \] (10)

\( N_c \) is the number of the nucleons in the core, and \( N_v \) is the number of the valence neutrons in the halo. The core density is taken in Gaussian form,

\[ \rho_c(r) = \frac{1}{\pi^{3/2} a_c^3} e^{-r^2/a_c^2}, \quad a_c = \sqrt{2/3} R_c, \]

where \( R_c \) is the core mean square radius. The halo is also parametrized in the Gaussian forms, which are different for the various shell structures supposed for it,

\[ \rho_v^{G}(r) = \frac{1}{\pi^{3/2} a_G^3} e^{-r^2/a_G^2}, \quad \rho_v^{O}(r) = \frac{2}{3 \pi^{3/2} a_O^3} r^2 e^{-r^2/a_O^2}, \quad \rho_v^{2S}(r) = \frac{2}{3 \pi^{3/2} a_{2S}^3} \left( \frac{r^2}{a_{2S}^2} - \frac{3}{2} \right)^2 e^{-r^2/a_{2S}^2}, \]

\[ a_G = \sqrt{2/3} R_v, \quad a_O = \sqrt{2/5} R_v, \quad a_{2S} = \sqrt{2/7} R_v, \] (11)
where \( R_v \) is the halo mean square radius. Whilst more complicated, these forms also admit the straightforward analytical evaluation of \( t_{m,n}(b) \) functions.

It is also necessary to account for the Coulomb scattering. The Coulomb amplitude reads

\[
f_C(q) = -\frac{1}{2\pi} M_{AB} Z_A Z_B e^2 \int d^3 z e^{iqz} V_c(z), \quad V_c(z) = \int d^3 x d^3 y \frac{\rho_A(x)\rho_B(y-z)}{|x-y|}
\]

where \( q = p - p' \) is the momentum transfer, \( M_{AB} \) is reduced mass of the A and B nuclei, \( Z_A, Z_B \) are their charge. Passing to the form factors one gets

\[
f_C(q) = -2M_{AB} Z_A Z_B e^2 \frac{\rho_A(q)\rho_B(q)}{q^2}, \quad \rho_{A,B}(q) = \int d^3 x e^{iqx} \rho_{A,B}(x),
\]

(12)

the core form factor having been taken for the halo nucleus. The elastic differential cross section is evaluated through the scattering amplitude \( f \) as

\[
d\sigma = |f|^2 \frac{2\pi}{k^2} \sin \theta d\theta = -\frac{\pi}{k^2} dt, \quad dt = 2k^2 \sin \theta d\theta,
\]

where \( \theta \) and \( k \) are the scattering angle and the momentum conjugated to the relative distance in the center of mass system, \( k = (M_{APB} - M_{BPA})/(M_A + M_B), \quad t = (p - p')^2 = 2k^2(1 - \cos \theta) \). The amplitude \( f \) is a sum of the Coulomb and the short range strong ones, thus

\[
d\sigma \frac{dt}{dt} = \frac{\pi}{k^2} |f_C(q) + F_{AB}^e(q)|^2 = \frac{1}{4\pi} \left| 2\pi \frac{k}{k} f_C(q) + i\phi(q) \right|^2,
\]

\[
\phi(q) = \int d^2 b e^{iqb}[1 - S_{AB}(b)].
\]

Given the fact that in the one photon exchange approximation the Coulomb amplitude is real we have

\[
\frac{d\sigma}{dt} = \frac{1}{4\pi} \left[ 4\pi^2 \frac{k^2}{k^2} f_C^2(q) + \phi^2(q) \right].
\]

(13)

3 Numerical calculations

The parameters of NN amplitude have been chosen to be \( \sigma_{NN}^{tot} = 43 \text{ mb}, \quad \beta = 0.2 \text{ fm}^2 \), the NN total cross section being taken as an average over \( pp \) and \( pn \) values.

We start from \(^{12}\text{C} - ^{12}\text{C} \) scattering. The distribution has been used with the value of the mean square radius \( R_{\text{rms}} = 2.49 \text{ fm} \) fitted from Monte-Carlo simulation of
a $^{12}\text{C} - ^{12}\text{C}$ collision in Ref.[13]. The results of the calculations are presented in Fig. 1. The figure exhibits 2 maxima and 2 minima between them at the transferred momentum interval $-t = 0.001 - 0.15\text{GeV}^2$ (the point $-t = 0$ is excluded for the singular Coulomb contribution). As is seen from the figure the values the cross section takes at the minima are due to the Coulomb interaction only. This is a consequence of the fact that the real part of the strong NN amplitude is neglected.

The Fig. 2 presents the results for the scattering of the exotic halo nucleus $^{11}\text{Li}$ on $^{12}\text{C}$ target obtained with the three parametrizations (11) with $R_c = 2.50 \text{ fm}$, $R_v = 5.04 \text{ fm}$, $N_c = 9$, $N_v = 2$. Since the core part is common for all of them the Coulomb contributions are identical and the difference should be attributed to various shell structures supposed for the neutrons in the halo. To expose this difference more explicitly the enlarged fragment of Fig. 2 is presented in Fig. 3.

4 Conclusion

We have carried out the calculations of the differential cross sections of $^{12}\text{C} - ^{12}\text{C}$ and $^{11}\text{Li} - ^{12}\text{C}$ scattering in the complete Glauber theory with account for the Coulomb contribution. The Coulomb scattering plays an important role in the nucleus-nucleus collisions especially at the small transverse momenta. In the case of $^{11}\text{Li} - ^{12}\text{C}$ scattering we have considered 3 parametrization for the halo taken from Ref.[13]. The results show a notable difference between the obtained cross sections.

References

[1] R.J. Glauber, _Cross-sections in deuterium at high-energies_ Phys. Rev. **100**, 242 (1955).

[2] R. J. Glauber and G. Matthiae, _High-energy scattering of protons by nuclei_, Nucl. Phys. B **21**, 135 (1970).

[3] W. Czyz and L. C. Maximon, _High-energy, small angle elastic scattering of strongly interacting composite particles_, Annals Phys. **52**, 59 (1969).

[4] G. D. Alkhazov and V. V. Sarantsev, _Sensitivity of Cross Sections for Elastic Nucleus-Nucleus Scattering to Halo Nucleus Density Distributions_, Phys. At. Nucl. **75**, 1544 (2012) arXiv:1107.0533.
[5] G. D. Alkhazov and V. V. Sarantsev, *Sensitivity of reaction cross sections to halo nucleus density distributions*, Phys. At. Nucl. 77, 912-916 (2014) [arXiv:1312.0782](https://arxiv.org/abs/1312.0782).

[6] Y. M. Shabelski and A. G. Shuvaev, *Generating function for nucleus-nucleus scattering amplitudes in Glauber theory*, Phys. Rev. C 104, no.6, 064607 (2021) [arXiv:2104.04943](https://arxiv.org/abs/2104.04943).

[7] C. Pajares and A. V. Ramallo, *Effects of the multiple scattering structure in the propagation of hadronic properties in nucleus-nucleus collisions*, Phys. Rev. D 31, 2800 (1985).

[8] V. M. Braun and Y. M. Shabelski, *Multiple Scattering Theory for Inelastic Processes*, Int. J. Mod. Phys. A 3, 2417 (1988).

[9] G. D. Alkhazov, A. V. Dobrovolsky and A. A. Lobodenko, *Matter density distributions and radii of light exotic nuclei from intermediate-energy proton elastic scattering and from interaction cross sections*, Nucl. Phys. A 734, 361 (2004).

[10] S. Ilieva, F. Aksouh, G. D. Alkhazov, L. Chulkov, A. V. Dobrovolsky, P. Egelhof, H. Geissel, M. Gorska, A. Inglessi, R. Kanungo, et al. *Nuclear-matter density distribution in the neutron-rich nuclei 12,14 Be from proton elastic scattering in inverse kinematics*, Nucl. Phys. A 875, 8 (2012).

[11] A. A. Korsheninnikov, E. Y. Nikolskii, C. A. Bertulani, S. Fukuda, T. Kobayashi, E. A. Kuzmin, S. Momota, B. G. Novatskii, A. A. Ogloblin, A. Ozawa, et al. *Scattering of radioactive nuclei 6 He and 3 H by protons: Effects of neutron skin and halo in 6 He, 8 He, and 11 Li*, Nucl. Phys. A 617, 45 (1997).

[12] M. V. Zhukov, B. V. Danilin, D. V. Fedorov, J. M. Bang, I. J. Thompson and J. S. Vaagen, *Bound state properties of Borromean Halo nuclei: He-6 and Li-11*, Phys. Rept. 231, 151 (1993).

[13] M. A. M. Hassan, M. S. M. Nour El-Din, A. Ellithi, E. Ismail and H. Hosny, *The effect of halo nuclear density on reaction cross-section for light ion collision*, Int. J. Mod. Phys. E 24, 1550062 (2015).

[14] C. Merino, I. S. Novikov and Y. M. Shabelski, *Nuclear Radii Calculations in Various Theoretical Approaches for Nucleus-Nucleus Interactions*, Phys. Rev. C 80, 064616 (2009).
Figure 1: Differential elastic cross section of $^{12}\text{C}-^{12}\text{C}$ scattering at the energy about 1 GeV per nucleon. The dash-dotted line shows the pure Coulomb scattering, the solid line is for the sum of the Coulomb and strong scatterings, Eq. (13).
Figure 2: Differential elastic cross section of $^{11}$Li–$^{12}$C scattering at the energy about 1 GeV per nucleon. The different lines are for the sum of the strong and Coulomb cross sections for the various halo density parametrizations [11]. The dotted line is for $\rho_G$, the dashed line is for $\rho_O$, the solid line is for $\rho^{2S}_v$. The dash-dotted line stands for the pure Coulomb cross section.
Figure 3: The enlarged fragment of Fig.2. The notations are the same as in Fig.2.