Integral Backstepping Based Nonlinear Control for Maximum Power Point Tracking and Unity Power Factor of a Grid Connected Hybrid Wind-Photovoltaic System

M. El Malah, A. Ba-razzouk, E. Abdelmounim, M. Madark
Department of Applied Physics, MISI Laboratory, FST Settat, University Hassan1st Settat, Morocco.

ABSTRACT
This paper proposes a novel integral backstepping-based nonlinear control strategy for a grid-connected wind-photovoltaic hybrid system. Firstly, detailed three-phase models of the hybrid system elements are presented, and then an overall state-space model is derived. Secondly, nonlinear control laws for the hybrid system’s converters are developed with the aim of ensuring maximum extraction of the available renewable energy, stabilizing the DC bus voltage and guaranteeing the operation of the hybrid system at unity power factor. The overall stability of the closed-loop system is demonstrated on the basis of Lyapunov’s stability theory. Comprehensive simulations, using the MATLAB/Simulink software environment, are carried out to assess the effectiveness of the proposed control methodology. The simulation results obtained confirm that the proposed control strategy offers high efficiency in various operating modes of the hybrid generation system.

Keyword(s):
Integral backstepping control
Maximum power point tracking
Unity power factor
Wind-PV hybrid system
Grid connection

1. INTRODUCTION
The ever-increasing demand for energy, the rapidly depleting reserves of fossil fuels and growing concerns about climate change are driving new advances in power generation from renewable resources. Due to their ecological nature and cost-effectiveness, solar photovoltaic and wind energy have become widespread energy resources. However, these resources are intermittent in nature, and thus it is impossible to provide a stable and permanent power supply directly from either one alone. This problem can be solved by efficiently integrating local energy storage elements, but the high costs and limited life span of these elements increase the production cost.

The interesting complementary behavior of wind speed and solar insolation, in terms of availability periods, has encouraged the use of PV-wind hybrid systems. Furthermore, the integration of hybrid systems into a smart grid, equipped with an intelligent energy management system to match production with use, is an appropriate approach to solve the problem of intermittency and thus reduce or completely eliminate the storage devices. The hybridization of renewable energy sources also helps to reduce the number of power converters, which have previously dedicated to each resource, and to efficiently utilize the installed converters.

Much of the academic literature on renewable energy production systems focuses mainly on their dimensioning, reliability, cost analysis and energy management [1–6]. In [2], a well-formulated method is proposed for the commercial sizing of a grid-connected PV-wind hybrid energy system. Other contributions are made on their modeling and control techniques [7–14], but the majority of the proposed control schemes are based on the classical method, such as perturb and observe or incremental conductance algorithms and proportional-integral (PI) controller [7–11]. In reality, renewable energy generation systems are non-linear whereas the PI controller is designed for linear systems, and several comparative studies have already clearly demonstrated the moderate performance of this controller compared to non-linear controllers [15–17].
A controller to track the maximum power point is required to extract the maximum amount of photovoltaic energy as well for extracting the maximum wind energy. Extensive work has been carried out on MPPT control of both solar photovoltaic and wind systems [18–21], and various nonlinear controllers have been designed to improve MPPT control in both systems [22–24].

Grid codes that take into account the integration of renewable energy include stringent requirements regarding reactive power injection. As an example, FERC’s standard interconnection agreements for energy power and other alternative technologies (Order No. 661-A) require maintaining a power factor greater than 0.95 at the interconnection point. Most of the control strategies proposed in the literature for grid-connected PV systems have covered power factor control [25, 26]. Concerning Doubly Fed Induction Generators (DFIG)-based wind energy systems, except some research works that have considered power factor control of the DFIG rotor circuit [12, 27, 28], the vast majority has focused only on stator power control [29–31].

Against this backdrop, this paper proposes an efficient power control of a grid-connected hybrid renewable energy system. The proposed hybrid system consists of a wind generator equipped with DFIG and a photovoltaic generator (PVG), as shown in Fig. 1. The energy produced by the PVG is injected into the grid through the grid side converter (GSC), and also routed to the rotor circuit of the DFIG through the rotor side converter (RSC) in the sub-synchronous operating mode. The DC-DC boost converter is used to raise the voltage of the PVG to match the DC bus voltage.

![Block diagram of the proposed PV-wind hybrid system](image-url)

The main objectives of the proposed non-linear control strategy are as follows:

- To track the maximum power point of each renewable source.
- To operate the hybrid system near unity power factor.
- To stabilize the DC link voltage.

In order to achieve these objects, the RSC controller is designed to track the MPP of the DFIG wind turbine and to inject the stator power with a power factor close to unity. The GSC controller is designed to maintain a constant DC bus voltage and to provide near-zero reactive power exchange, and the booster converter controller is designed to track the PPM of the PV generator.

The rest of this paper is organized as follows. In the second section, the mathematical models of the main system elements are presented. The third section develops the nonlinear control laws for the power converters of the hybrid system. Simulation results focusing on the validation of the proposed control strategy are given in the fourth section. The conclusions are given in the last section.
2. SYSTEM MODELING

The mathematical models of the main elements of the PV-DFIG hybrid system (turbine, DFIG, GPV…) presented in this section will be used to correctly select the system outputs, to develop accurate control laws and to verify the validity and performance of the results obtained.

2.1. Aerodynamic Power Conversion

Wind turbine dynamics is modeled in this paper on the basis of the following expressions [12]:

\[
P_{\text{aer}} = \frac{1}{2} C_p(\lambda, \beta) \rho \pi R^2 V^3
\]

\[
C_p(\lambda, \beta) = c_1 \left( \frac{c_2}{\lambda_i} - c_3 \beta - c_4 \right) e^{\frac{-c_5}{\lambda_i}} + c_6 \lambda
\]

\[
\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08 \beta} - \frac{0.035}{\beta^3 + 1}
\]

(1)

where:

- \( P_{\text{aer}} \): Aerodynamic power extracted by the turbine.
- \( C_p \): Wind turbine power coefficient.
- \( R \): Blade radius.
- \( \Omega_t \): angular speed of the turbine
- \( \lambda \): Tip speed ratio.
- \( \beta \): Blade pitch angle.
- \( \rho \): Air density.
- \( V \): Wind speed.
- \( c_i \): Constants (\( c_1 = 0.5176 \); \( c_2 = 116 \); \( c_3 = 0.4 \); \( c_4 = 5 \); \( c_5 = 21 \); \( c_6 = 0.0068 \))

Fig. 2 illustrates the characteristics of the power coefficient of the wind turbine at different values of the blade pitch angle.

![Figure 2. \( C_p(\lambda) \) characteristics at different values of \( \beta \).](image)

The pitch control is provided to protect the wind turbines against turbulence and excessive overload, under normal conditions \( \beta = 0 \).

2.2. Photovoltaic Generator Model

The equivalent electrical circuits of a PV cell shown in the previous figure can be modeled using the expressions below [23]:

\[
I_{\text{cell}} = I_{\text{ph}} - I_{\text{sat}} \left( \exp \left( \frac{q V_{\text{cell}} + I_{\text{cell}} R_{\text{se}}}{V^2} \right) - 1 \right) - \frac{V_{\text{cell}} + I_{\text{cell}} R_{\text{se}}}{R_{\text{sh}}}
\]

\[
I_{\text{ph}} = \frac{E}{E_{\text{ref}}} \left( I_{\text{sc}} + K_i \left( T - T_{\text{ref}} \right) \right)
\]

\[
I_{\text{sat}} = I_{\text{rs}} \left( \frac{1}{T_{\text{ref}}} \right)^3 \exp \left( \frac{-q E_{\text{go}}}{V^2} \right) \left( \frac{1}{T} - \frac{1}{T_{\text{ref}}} \right)
\]

(2)

where:

- \( E \): Solar irradiation in W/m².
- \( E_{\text{go}} \): Band-gap energy of the Si solar cell.
- \( E_{\text{ref}} \): Reference irradiation (1kW/m²).
- \( I_{\text{cell}} \): Current across the cell.
If $N_p$ denotes the number of parallel strings in a PV generator, and each string contains $N_s$ cells in series, the expression of generator current $(I_{pv})$ versus generator voltage $(V_{pv})$ can be derived as follows:

$$V_{pv} = N_s V_{cell}$$

$$I_{pv} = N_p I_{ph} - N_p I_{sat} \left[ e^{\left(\frac{N_p V_{pv} + N_s I_{pv} R_{se}}{q N_s k T} - 1\right)} - \frac{N_p V_{pv} + N_s I_{pv} R_{se}}{N_s R_{se}} \right]$$ (3)

In this paper, a PV generator made up of ten SM55 panels connected in series is considered. Electrical specifications for one panel are given in the following table.

**Table 1. Electrical Specifications for SM55 Solar Panel**

| Parameter                  | Value |
|----------------------------|-------|
| Maximum power              | 55W   |
| Current at the maximum power point | 3.15A |
| Voltage at the maximum power point | 17.4V |
| Maximum current (short circuit output) | 3.45A |
| Maximum voltage (open circuit) | 21.7V |
| Current temperature coefficient | 1.2 mA/°C |
| Number of series cells $N_s$ | 36    |
| Number of parallel modules $N_p$ | 1     |

The power-voltage characteristics of the PV generator for different levels of solar irradiation are shown in Fig. 3. The coordinates of the maximum power points are shown in the zoomed portions of this figure and will be used to verify the accuracy of the simulation results.

### 2.3. Boost Converter and Inverters Models

The circuit diagram of the boost converter, interfacing the PV generator with the DC bus, is presented in the following figure.

![Circuit diagram of the boost converter](image)

Its dynamics can be described by the following relationships:

$$\begin{align*}
\frac{dV_{pv}}{dt} &= \frac{I_{pv} - I_L}{C_p} \\
\frac{dI_L}{dt} &= \frac{V_{pv} - (1 - u_s) V_b}{L_p}
\end{align*}$$ (4)

where: $C_p$, $L_b$ and $I_L$ represent the capacitance, the inductance and inductance current, the switching signal $(u_s)$ can take on only two possible states, $u_s=0$ (switch open) and $u_s=1$ (switch closed). The electrical losses in the booster converter are generally negligible, the photovoltaic power is therefore conserved and we can write:
where: \( P_{pv} = V_{pv}I_{pv} \) is the photovoltaic power and \( I_{b} \) is the current at the output of the booster converter.

The relationship between the three-phase voltages and the DC-bus voltage of one of the two inverters (GSC for example) is expressed as follows [16]:

\[
\begin{bmatrix}
V_{ga} \\
V_{gb} \\
V_{gc}
\end{bmatrix}
= \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
S_{ga} \\
S_{gb} \\
S_{gc}
\end{bmatrix}
+ \begin{bmatrix}
V_c \\
V_{gb} \\
V_{gc}
\end{bmatrix}
\]

(6)

where \( V_{ga}, V_{gb} \) and \( V_{gc} \) are the secondary voltages of GSC grid connection transformer. \( S_{ga}, S_{gb} \) and \( S_{gc} \) are the switching signals of the GSC (that can take on two possible states: 0 or 1). Thus, the model of the GSC with L-filter is given by the following system of equations:

\[
\begin{align*}
\frac{di_{ga}}{dt} & = -\frac{R_f}{L_f} i_{ga} + \frac{R_f}{L_f} i_{gb} + \frac{V_c}{L_f} \left( 2S_{gb} - S_{ga} - S_{gc} \right) \\
\frac{di_{gb}}{dt} & = -\frac{R_f}{L_f} i_{gb} + \frac{R_f}{L_f} i_{gc} + \frac{V_c}{L_f} \left( 2S_{gc} - S_{ga} - S_{gb} \right) \\
\frac{di_{gc}}{dt} & = -\frac{R_f}{L_f} i_{gc} + \frac{R_f}{L_f} i_{ga} + \frac{V_c}{L_f} \left( 2S_{ga} - S_{gb} - S_{gc} \right)
\end{align*}
\]

(7)

where \( i_{ga}, i_{gb}, i_{gc} \) are the three phase GSC currents, \( R_f \) and \( L_f \) are the resistance and the inductance of the filter. The three-phase control model is expressed as follows:

\[
\begin{align*}
\frac{di_{ga}}{dt} & = -\frac{R_f}{L_f} i_{ga} + \frac{V_c}{L_f} \left( 2S_{gb} - S_{ga} - S_{gc} \right) \\
\frac{di_{gb}}{dt} & = -\frac{R_f}{L_f} i_{gb} + \frac{V_c}{L_f} \left( 2S_{gc} - S_{ga} - S_{gb} \right) \\
\frac{di_{gc}}{dt} & = -\frac{R_f}{L_f} i_{gc} + \frac{V_c}{L_f} \left( 2S_{ga} - S_{gb} - S_{gc} \right)
\end{align*}
\]

(8)

where \( k_{ga}; k_{gb} \) and \( k_{gc} \) are the three phase input signals of PWM utilized to generate \( S_{gi} \), as the schematic diagram of Fig.7 shows.

Vector control is a powerful tool for designing simple control of a three-phase system. This tool is used in this work, for this objective, the three-phase electrical quantities are transformed into an arbitrary dq-reference frame, the transformation matrix is:

\[
T(\theta) = \frac{2}{3} \begin{bmatrix}
\cos(\theta_p) & \cos(\theta_p - \frac{2\pi}{3}) & \cos(\theta_p + \frac{2\pi}{3}) \\
\sin(\theta_p) & -\sin(\theta_p - \frac{2\pi}{3}) & -\sin(\theta_p + \frac{2\pi}{3})
\end{bmatrix}
\]

(9)

where \( \theta_p \) is an arbitrary angular position of the d-q frame. The model of the GSC with L-filter in the dq frame is given by:

\[
\begin{align*}
\frac{dl_{dg}}{dt} & = \omega_s l_{dq} - \frac{R_f}{L_f} l_{dg} + \frac{V_{ag}}{L_f} - \frac{V_c}{L_f} K_{dg} \\
\frac{dl_{dq}}{dt} & = -\omega_s l_{dg} - \frac{R_f}{L_f} l_{dq} + \frac{V_{ag}}{L_f} - \frac{V_c}{L_f} K_{dg}
\end{align*}
\]

(10)

where: \( \begin{bmatrix} V_{ag} \\ V_{bg} \end{bmatrix} = T(\theta) \begin{bmatrix} V_{ga} \\ V_{gb} \end{bmatrix} ; \begin{bmatrix} l_{dg} \\ l_{dq} \end{bmatrix} = T(\theta) \begin{bmatrix} i_{ga} \\ i_{gb} \end{bmatrix} \) and \( K_{dg} \) is the control signal of the d-q frame with respect to the \( \alpha \)-axis of stationary reference frame attached to the stator.
2.4. DFIG Models

The DFIG stator winding is directly connected to the grid, while the rotor winding is connected via a PWM bidirectional back-to-back inverters (Fig. 1). The dynamic relationships between voltage, current and flux in an arbitrary reference frame $dq$ are expressed as follows [12, 27]:

$$
\begin{align*}
V_{ds} &= R_s I_{ds} + \frac{d\phi_{ds}}{dt} - \omega_s \varphi_{qs} \\
V_{qs} &= R_s I_{qs} + \frac{d\phi_{qs}}{dt} + \omega_s \varphi_{ds} \\
V_{dr} &= R_r I_{dr} + \frac{d\varphi_{dr}}{dt} - \omega_r \varphi_{qr} \\
V_{qr} &= R_r I_{qr} + \frac{d\varphi_{qr}}{dt} + \omega_r \varphi_{dr}
\end{align*}
$$

where:

$$
\begin{align*}
\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} &= T(\theta_s) \begin{bmatrix} V_{sa} \\ V_{sb} \end{bmatrix} \quad \begin{bmatrix} I_{ds} \\ I_{qs} \end{bmatrix} &= T(\theta_r) \begin{bmatrix} I_{ra} \\ I_{rb} \end{bmatrix} \quad \begin{bmatrix} \phi_{ds} \\ \phi_{qs} \end{bmatrix} = T(\theta_s) \begin{bmatrix} \varphi_{sa} \\ \varphi_{sb} \end{bmatrix} \\
\begin{bmatrix} V_{dr} \\ V_{qr} \end{bmatrix} &= T(\theta_r) \begin{bmatrix} V_{ra} \\ V_{rb} \end{bmatrix} \quad \begin{bmatrix} I_{dr} \\ I_{qr} \end{bmatrix} &= T(\theta_r) \begin{bmatrix} I_{ra} \\ I_{rb} \end{bmatrix} \quad \begin{bmatrix} \varphi_{dr} \\ \varphi_{qr} \end{bmatrix} = T(\theta_r) \begin{bmatrix} \varphi_{ra} \\ \varphi_{rb} \end{bmatrix}
\end{align*}
$$

$\omega_r = \frac{d\theta_r}{dt}$ and $\theta_r$ is the angular position of the $d$-$q$ frame with respect to the $\alpha$-$\beta$ reference frame attached to the rotor.

The electromagnetic torque $T_{em}$ is given by:

$$
T_{em} = -p \frac{M}{L_s} (\varphi_{ds} I_{qr} - \varphi_{qs} I_{dr})
$$

where $p$ is the number of pole pairs $p$ within the DFIG stator.

A stator flux-oriented vector control strategy is utilized in this paper. When the $d$-axis is aligned with the stator flux vector, $(\varphi_{ds} = \varphi_s; \varphi_{qs} = 0)$, the stator currents become:

$$
\begin{align*}
I_q &= -\frac{M}{L_s} I_{qr} \\
I_d &= \frac{\varphi_s}{L_s} - \frac{M}{L_s} I_{dr}
\end{align*}
$$

If the stator resistance drop is neglected compared to stator terminal voltages, the voltages can also be simplified as follows:

$$
\begin{align*}
V_{qs} &= V_s = \varphi_s \omega_s \\
V_{ds} &= 0 & V_{ag} &= \varphi_s = mV_s \\
V_{dg} &= mV_{ds} = 0
\end{align*}
$$

where $m$ is the transformation ratio of $T_r$, these voltages are therefore aligned with the quadrature axis. The active and reactive powers are decoupled and simplified as follows:

$$
\begin{align*}
P_s &= V_{ds} I_{ds} + V_{qs} I_{qs} = V_s \frac{M}{L_s} I_{qr} \\
Q_s &= V_{qs} I_{ds} - V_{ds} I_{qs} = V_s \frac{M}{L_s} I_{dr} + \frac{1}{\omega_s} \frac{V_s^2}{2} & P_g &= V_{dg} I_{dg} = V_{ag} I_{ag} = V_{ag} I_{ag} \\
Q_g &= V_{ag} I_{dg} - V_{dg} I_{ag} = V_{ag} I_{ag}
\end{align*}
$$

And the rotor currents derivatives can be deduced from (11):

$$
\begin{align*}
\frac{dI_{dr}}{dt} &= \alpha_r I_{dr} - \frac{R_r}{L_r \sigma} I_{dr} + \frac{V_r}{L_r \sigma} K_{dr} \\
\frac{dI_{qr}}{dt} &= -\alpha_r I_{qr} - \frac{R_r M}{\omega_s L_s L_r \sigma} V_s - \frac{R_r}{L_r \sigma} I_{qr} + \frac{V_c}{L_r \sigma} K_{qr}
\end{align*}
$$

where: $[K_{dr} \quad K_{qr}] = T(\theta_r) [k_{ra} \quad k_{rb} \quad k_{rc}]$, $\omega_s = \omega_s - p \Omega_r$, the leakage coefficient $\sigma = 1 - M^2/L_s L_r$.

The expression of the electromagnetic torque, (12), in the synchronous reference frame becomes:

$$
T_{em} = -p \frac{M}{L_s} \varphi_s I_{qr}
$$
2.5. Overall State Space Model

According to (5), hybrid system DC-bus voltage is governed by:

$$\frac{dV_c}{dt} = \frac{1}{C} \left( K_{dg} I_{dg} + K_{ag} I_{ag} + \frac{P_{pv}}{V_c} - I_{rc} \right)$$  \hspace{1cm} (18)

where $I_{rc}$ is the DC-current absorbed by the RSC. Then, according to (4), (10), (16) and (18) an overall state space representation of the investigated hybrid system in synchronous d-q frame is given by:

$$\begin{align*}
\frac{dI_{dr}}{dt} &= \omega_r I_{qr} - \frac{R_r}{L_r} I_{dr} + \frac{V_c}{L_r} K_{dr} \\
\frac{dI_{qr}}{dt} &= -\omega_r I_{dr} - \frac{\omega_r M}{\omega_s L_s} V_s - \frac{R_r}{L_r} I_{qr} + \frac{V_c}{L_r} K_{qr} \\
\frac{dV_{pv}}{dt} &= I_{pv} - I_L \\
\frac{dI_L}{dt} &= V_{pv} - (1 - U) V_c \\
\frac{dI_{dg}}{dt} &= \omega_s I_{dq} - \frac{R_f}{L_f} I_{dg} - \frac{V_c}{L_f} K_{dg} \\
\frac{dI_{aq}}{dt} &= -\omega_s I_{dq} - \frac{R_f}{L_f} I_{aq} + \frac{V_g}{L_f} - \frac{V_c}{L_f} K_{aq} \\
\frac{dV_c}{dt} &= \frac{1}{C} \left( K_{dg} I_{dg} + K_{ag} I_{ag} + \frac{P_{pv}}{V_c} - I_{rc} \right)
\end{align*}$$  \hspace{1cm} (19)

3. HYBRID SYSTEM CONTROLLER DESIGN

The GSC controller (Fig. 1) is designed to maintain a constant DC link voltage and to operate the GSC at unity power factor regardless of the direction of rotor power flow. The RSC controller is designed to ensure maximum extraction of wind energy and to inject the stator power with near unity power factor, while the boost converter controller is designed to operate the PV generator at its maximum power point. The voltage and frequency of electrical grid are assumed to be stable, and the stator flux is estimated as follows:

Figure 8. Control schematic diagram
\[ \dot{\theta}_s = \int (v_{sd} - R_s i_{sd}) dt \]  
\[ i \in \{a, b, c\} \]  
\[ (20) \]

The stator transformation angle, \( \theta_s \), is obtained using a 2nd order phase locked loop (PLL). The PLL proposed in this paper uses the quadrature component of the stator flux as feedback on the progress of synchronization, as illustrated by the following block diagram [12]:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{block_diagram.png}
\caption{Block diagram of the PLL.}
\end{figure}

### 3.1. Hybrid system Outputs and Their References.

The DFIG rotor dynamics is governed by Newton’s second law expressed as follows:
\[ \frac{d^2 \omega_r}{dt^2} = \frac{T_{a/r}}{J} + \frac{T_{em}}{J} - F \Omega_r \]  
\[ (21) \]
where, \( J \) is the turbine-DFIG combined inertia, \( F \) is the coefficient of the total viscosity (Turbine and DFIG), \( T_{a/r} \) is the aerodynamic torque applied to DFIG rotor, \( T_{em} \) is the algebraic value of the electromagnetic torque of DFIG and \( \Omega_r \) its rotor speed.

Operating at the maximum power point of the turbine (i.e. \( \lambda = \lambda_{opt} \) and \( C_\rho = C_{\rho,max} \)) implies that:
\[ P_{max} = \frac{1}{2} C_{p,max} \rho \pi R^2 (V)^3 = \frac{1}{2} C_{p,max} \rho \pi R^2 \left( \frac{\Omega_r}{\lambda_{opt}} \right)^3 \]  
\[ (22) \]

The optimal aerodynamic torque, \( T_{a/r, opt} \), is therefore such that:
\[ T_{a/r, opt} = \frac{P_{max}}{\Omega_r} \]  
\[ = \frac{G \rho_0 R^2 (\Omega_r \lambda_{opt})^2}{2} \]  
\[ (23) \]
where \( K_{opt} = \frac{1}{2 G^3 \lambda_{opt}^2} C_{p,max} \rho \pi R^2 \) and \( G \) is the gearbox ratio.

So, the optimal electromagnetic torque, \( T_{em, opt} \), is such that:
\[ (24) \]

\[ (\text{23}) \Rightarrow T_{em, opt} = F \Omega_r - T_{a/r, opt} \]

Then, the quadrature current reference is as follows:
\[ I_{qr, ref} = \frac{I_s}{p \Theta_{q0}} \left( K_{opt} \Omega_r - F \Omega_r \right) \]  
\[ \Rightarrow \frac{dI_{qr, ref}}{dt} = \frac{-I_s}{p \Theta_{q0}} \frac{d \Omega_r}{dt} \left( 2K_{opt} \Omega_r - F \right) \]  
\[ (25) \]

In order to achieve operation at unitary power factor and according to (15):
\[ \begin{align*}
Q_{s, opt} &= -V_s M I_{dr, ref} + \frac{V_s^2}{\omega_s L_s} = 0 \\
Q_{g, opt} &= V_g I_{dg, ref} = 0
\end{align*} \]  
\[ (26) \]

This implies that:
\[ I_{dr, ref} = \frac{V_s}{M_{opt}} \quad \text{and} \quad I_{dg, ref} = 0 \]  
\[ (27) \]

The power derivative of the PV generator with respect to its voltage, \( \frac{\partial P_{pv}}{\partial V_{pv}} \), is chosen as a controlled output. When the solar generator is operating in its maximum state (Fig. 4), this output becomes zero:
\[ \left( \frac{\partial P_{pv}}{\partial V_{pv}} \right) = 0 \]  
\[ (28) \]

### 3.2. RSC Control Laws

Let us define the error \( e_1 \) between the quadrature component of the rotor current and its desired value:
\[ e_1 = I_{qr} - I_{qr, ref} \]  
\[ (29) \]

\( e_1 \) derivative with respect to time, using (19), is:
\[ \frac{de_1}{dt} = -\omega_r I_{dr} - \frac{M_{pv}}{L_s} V_{pv} - \frac{R_s}{L_s} i_{qr} + \frac{K_{opt} V_{pv}}{L_s} \frac{dI_{qr, ref}}{dt} \]  
\[ (30) \]

The first Lyapunov function candidate (LFC) is defined as:
\[ V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} \sigma_1^2 \]  
\[ (31) \]

where \( \sigma_1 = c_{11} \int_0^t e_1(\tau) d\tau \), \( c_{11} \) is the integral action design parameter. \( V_1 \) time-derivative is as follows:
\[
\frac{dv_t}{dt} = \varepsilon_1 (\frac{de_t}{dt} + c_1 \sigma_t)
\]  

(32)

If we choose a dynamic quadrature component of RSC control signal, \(K_{qr}\), as follows:

\[
K_{qr} = \frac{L_r \sigma}{V_c} (-c_1 e_t - c_1 \sigma_t + \omega_r l_{dr} + \frac{R}{L_r \sigma} I_{qR} + \frac{g_{Mv_b}}{L_r \sigma} + \frac{d l_{qr, ref}}{dt})
\]  

(33)

where \(c_1\) is a strictly positive design parameter. \(V_t\) time-derivative becomes:

\[
\frac{dv_t}{dt} = -c_1 \varepsilon_1^2
\]  

(34)

Similarly, the error between \(l_{dr}\) and its desired value is defined as follows:

\[
e_2 = l_{dr} - l_{dr, ref}
\]  

(35)

Its derivative with respect to time, according to (19) and (27), is:

\[
\frac{de_2}{dt} = \omega_r l_{qr} - \frac{R}{L_r \sigma} l_{dr} + \frac{K_{dc} V_c}{L_r \sigma}
\]  

(36)

The LFC is as follows:

\[
V_2 = \frac{1}{2} \varepsilon_2^2 + \frac{1}{2} \sigma_2^2
\]  

(37)

where \(\sigma_2 = c_2 \int_0^t e_2(t) \, dt\), \(c_2\) is the integral action design parameter. \(V_2\) time-derivative is as follows:

\[
\frac{dv_2}{dt} = \varepsilon_2 \frac{de_2}{dt} + c_2 \sigma_2
\]  

(38)

Then, the stabilizing direct component of RSC control signal is chosen as:

\[
K_{dr} = \frac{L_r \sigma}{V_c} (-c_2 \varepsilon_2 - c_2 \sigma_2 + \frac{R}{L_r \sigma} l_{dr} - \omega_r l_{qr})
\]  

(39)

where \(c_2\) is a strictly positive design parameter. With the above choice, \(V_2\) time-derivative becomes:

\[
\frac{dv_2}{dt} = -c_2 \varepsilon_2^2
\]  

(40)

### 3.3. GSC Control Laws

The error between \(l_{dg}\) and its desired value is defined as follows:

\[
e_3 = l_{dg} - l_{dg, ref}
\]  

(41)

Its time-derivative, using (19) and (27), is:

\[
\frac{de_3}{dt} = \omega_p l_{ag} - \frac{R}{L_f} l_{dg} - \frac{V_c}{L_f} K_{dg}
\]  

(42)

The LFC is defined as follows:

\[
V_3 = \frac{1}{2} \varepsilon_3^2 + \frac{1}{2} \sigma_3^2
\]  

(43)

where \(\sigma_3 = c_3 \int_0^t e_3(t) \, dt\), \(c_3\) is the integral action design parameter. Then, \(V_3\) time-derivative is as follows:

\[
\frac{dv_3}{dt} = \varepsilon_3 \frac{de_3}{dt} + c_3 \sigma_3
\]  

(44)

Then, the stabilizing direct component of GSC control signal is chosen as:

\[
A_1 K_{dg} = B_1
\]  

(45)

where: \(A_1 = \frac{V_c}{L_f}\), \(B_1 = c_3 \varepsilon_3 + c_3 \sigma_3 + \omega_p l_{ag} - \frac{R}{L_f} l_{dg}\), with \(c_3\) is a strictly positive design parameter.

Through this choice, the LFC time-derivative becomes:

\[
\frac{dv_3}{dt} = -c_3 \varepsilon_3^2
\]  

(46)

The DC bus stabilization error is defined as follows:

\[
e_4 = V_c - V_{c, ref}
\]  

(47)

where \(V_{c, ref}\) is a constant reference voltage. \(e_4\) time-derivative, using (19), is:

\[
\frac{de_4}{dt} = \frac{1}{C} (K_{dg} I_{dg} + K_{ag} I_{ag} + \frac{p_{pv}}{V_c} - l_{rc})
\]  

(48)

A similar LFC is defined as:

\[
V_4 = \frac{1}{2} \varepsilon_4^2 + \frac{1}{2} \sigma_4^2
\]  

(49)

where \(\sigma_4 = c_4 \int_0^t e_4(t) \, dt\), \(c_4\) is the integral action design parameter. Then, \(V_4\) time-derivative is as follows:

\[
\frac{dv_4}{dt} = \varepsilon_4 \frac{de_4}{dt} + c_4 \sigma_4
\]  

(50)

Then, \(K_{dg}\) and \(K_{ag}\) are chosen as follows:

\[
A_3 K_{dg} + A_4 K_{ag} = B_2
\]  

(51)

where: \(A_3 = \frac{l_{dg}}{C}, A_4 = \frac{l_{ag}}{C}, B_2 = -c_4 e_4 - c_4 \sigma_4 + \frac{1}{C} (l_{rc} - \frac{p_{pv}}{V_c})\), and \(c_4\) is a strictly positive design parameter.

Subsequently, \(V_4\) time-derivative becomes:
where: \( A_2 = 0 \)

### 3.4. Boost Control Law

**Step 1**: The PV-MPPT error is defined:

\[
e_\varepsilon = \frac{\partial p_v}{\partial v_p} \left( \frac{\partial p_v}{\partial v_p} \right)_{ref} = \frac{\partial (v_p p_v)}{\partial v_p} = I_{pv} + V_{pv} \frac{\partial i_p}{\partial v_p}
\]

According to (19), \( e_\varepsilon \) derivative with respect to time is:

\[
d \frac{d \varepsilon}{dt} = \left( V_{pv} \frac{\partial^2 v_p}{\partial v_p^2} + 2 \frac{\partial i_p}{\partial v_p} \right) \frac{dv_p}{dt} = \left( V_{pv} h + 2 \frac{\partial i_p}{\partial v_p} \right) \left( \frac{V_{pv} l_p}{c_{pv}} \right) = \frac{f}{c_{pv}} (I_{pv} - I_L)
\]

where: \( h = \frac{\partial^2 v_p}{\partial v_p^2} \) and \( f = V_{pv} h + 2 \frac{\partial i_p}{\partial v_p} \)

The first LFC is defined as:

\[
V_5 = \frac{1}{2} \varepsilon_\varepsilon^2 + \frac{1}{2} \varepsilon_5^2
\]

where: \( \varepsilon_5 = c_{51} \int_0^t \varepsilon_\varepsilon(t) \, dt \), \( c_{51} \) is the integral action design parameter. And, \( V_5 \) time-derivative is:

\[
\frac{d V_5}{dt} = \varepsilon_5 \frac{d \varepsilon}{dt} + c_5 \varepsilon_5
\]

To make \( \dot{V}_5 \) negative, \( I_L \) is adopted as a virtual control provided that its desired value is:

\[
\alpha_1 = I_L + \frac{c_{pv}}{f} (c_5 \varepsilon_5 + c_5 \varepsilon_5^2)
\]

where \( c_5 \) is a strictly positive design parameter.

**Step2**: The variable error between virtual control and its desired value is:

\[
e_\varepsilon = I_L - \alpha_1
\]

According to (55), (58) and (59), \( V_5 \) time-derivative (57) becomes:

\[
\frac{d V_5}{dt} = \varepsilon_5 \frac{d \varepsilon}{dt} + c_{51} \varepsilon_5
\]

\[
\varepsilon_\varepsilon \text{ time-derivative, using (19), is as follows:}
\]

\[
\frac{d \varepsilon}{dt} = \frac{c_{pv}}{f} \left[ f \left( c_5 \frac{d \varepsilon}{dt} + c_{51} \varepsilon_5 \right) - \frac{d \varepsilon_5}{dt} (c_5 \varepsilon_5 + c_{51} \varepsilon_5^2) \right] + \frac{\partial i_p}{\partial v_p} \frac{dv_p}{dt}
\]

\[
\frac{d f}{dt} = 3 \frac{d v_p}{dt} h + V_{pv} \frac{d h}{dt} \text{ and } \frac{d h}{dt} = \frac{\partial^3 i_p}{\partial v_p^3} \frac{dv_p}{dt}
\]

Let us now consider an augmented LFC such as:

\[
V_6 = V_5 + \frac{1}{2} \varepsilon_\varepsilon^2
\]

Its time-derivative is:

\[
\frac{d V_6}{dt} = \frac{d V_5}{dt} + \varepsilon_\varepsilon \frac{d \varepsilon}{dt}
\]

Then the command \( u \) is chosen as follows:

\[
u = \frac{b_0}{V_c} \left( -c_6 \varepsilon_6 + \frac{d \alpha_1}{dt} + f \frac{\varepsilon_\varepsilon}{c_{pv}} \right) - \frac{V_{pv}}{V_c} + 1
\]

where \( c_6 \) is a strictly positive design parameter. With the above choice, \( V_6 \) time-derivative becomes:

\[
\frac{d V_6}{dt} = -c_5 \varepsilon_5^2 - c_6 \varepsilon_6^2
\]

### 3.5. Overall Stability Analysis

Let us define an overall LFC such as:

\[
V_T = V_6 + \sum_{i=1}^{4} V_i = \sum_{i=1}^{6} \frac{\varepsilon_i^2}{2} + \sum_{i=1}^{5} \frac{c_i^2}{2}
\]
Its time-derivative, according to (34), (40), (46), (52) and (65), is:
\[
\frac{dV_T}{dt} = \frac{dV_6}{dt} + \sum_{i=1}^{4} \frac{dV_i}{dt} = -\sum_{i=1}^{6} c_i e_i^T
\]
(67)

Hence \( V_T \) is a positive definite function and has a negative definite derivative, consequently, the tracking errors are asymptotically stable and converge to zero in the Lyapunov approach.

4. SIMULATION RESULTS

The proposed system (Fig.1) has been implemented in the MATLAB-Simulink software on the basis of the models presented in section 2, the Simulink block diagram is given in Fig. 10. The control laws developed in section 3 are evaluated in this section, and to highlight the performances obtained, the simulation results are compared to those obtained using the P&O control (for the boost converter) and the PI control (for the RSC and GSC). The main parameters of the system, as well as those of the controllers, are summarized in Table I.

![Figure 10. Hybrid system model in Matlab/Simulink environment.](image)

**Table 2. Parameters of Controlled system**

| Hybrid system | DFIG rated power | \( P_{r}=1kW \) | Air density | \( \rho=1.22kg/m^3 \) | Filter resistance | \( R_f=0.5655\Omega \) |
|---------------|-----------------|----------------|-------------|----------------|------------------|-------------------|
| Line to line voltage | \( U_{dc}=190V \) | Blade radius | \( R_{o}=0.9m \) | Rotor resistance | \( R_{r}=0.88\Omega \) |
| DFIG pole pair number | \( p=3 \) | Viscous coefficient | \( F=64.10^3Nms/rad \) | Stator resistance | \( R_s=1.1\Omega \) |
| Maximal power coefficient | \( C_{p,max}=0.48 \) | Gearbox ratio | \( G=1.3 \) | Filter inductance | \( L_f=12.6mH \) |
| Optimal Tip speed ratio | \( \lambda_{opt}=8.1 \) | Boost converter inductor | \( L_B=20mH \) | Mutual inductance | \( M=90.1mH \) |
| DC-bus voltage reference | \( V_{dc,ref}=300V \) | PV-array capacitor | \( C_p=2000\mu F \) | Rotor inductance | \( L_s=93.1mH \) |
| DC-bus capacitor | \( C=3000\mu F \) | Switching frequency | \( f_s=10kHz \) | Stator inductance | \( L_s=93.1mH \) |
| Controllers | \( c_1=7x10^{-6}; \ c_2=7x10^{-5}; \ c_3=6x10^{-3}; \ c_4=57x10^{-3}; \ c_5=3x10^{-3}; \ c_6=65x10^{-3}; \ c_7=10^{-2}; \ c_8=10^{0}; \ c_9=10^{0}; \ c_{10}=10^{0} \) |

The hybrid system was tested under variations in solar radiation and wind speed shown in Figs. 11 and 12, respectively, and instantaneous random variations are incorporated into the wind speed profile to account for the effect of turbulence [32].

![Figure 11. Solar radiation profile](image)

![Figure 12. Wind velocity profile](image)

The first simulation result, Fig. 13, clearly shows that the PLL has reached its objective (i.e. \( \varphi_{qi}\equiv0 \)), and that the results obtained with the integral backstepping control are more accurate. Fig. 14 shows that the direct-axis component of the stator voltage, neglected in the design of the controllers (see equation (14)), is indeed almost null; its value does not exceed 0.3V with the proposed control and \( \pm 2V \) with the PI control. Fig. 15 shows the effectiveness of each of the controllers in keeping the quadrature component of the rotor current close to its reference value, the accuracy provided by the proposed non-linear controller at this level has allowed...
to obtain the optimal torque tracking accuracy shown in Fig. 16(a). Figs 17 and 18 confirm the efficiency of the nonlinear MPPT strategy, the power coefficient is kept close to its maximum value ($C_p_{\text{max}}=0.48$) and the tip speed ratio has been kept close to its optimal value ($\lambda_{\text{opt}}=8.1$) despite wind speed variations. The rotor speed illustrated in Fig. 19 reveals that the hybrid system was also evaluated during the transition from super-synchronous to sub-synchronous mode. The effort provided by the proposed RSC-controller to track the reference of the rotor direct-axis current (Fig. 20(a)), as expected, kept the reactive power injected by the DFIG stator close to zero, as shown in Fig. 21(a).

Figure 13. Stator flux; (a) PLL with integral backstepping control; (b) PLL with PI control

Figure 14. Stator voltage; (a) PLL with integral backstepping control; (b) PLL with PI control

Figure 15. Quadrature component of the rotor current; (a) Integral backstepping control; (b) PI control

Figure 16. Electromagnetic torque; (a) Integral backstepping control; (b) PI control

Figure 17. Power coefficient; (a) Integral backstepping control; (b) PI control

Figure 18. Tip speed ratio; (a) Integral backstepping control; (b) PI control
Fig. 22 (a) shows the perfect regulation of photovoltaic generator voltage, ensured by the proposed boost converter controller, which has allowed to reach with great precision the maximum power points zoomed in Fig. 4 of the PVG power-voltage characteristics (as illustrated in Fig. 23 (a)).

Fig. 23. Photovoltaic Power; (a) Integral backstepping control; (b) P&O

The GSC controller tried to keep the DC bus voltage constant during the simulation period. The deviation from the reference obtained with the proposed non-linear control stays within ± 0.02V (Fig. 24 (a)), but it can exceed ± 5V when using the PI control (Fig. 24 (b)). Fig. 25 (a) shows that the proposed controller also allowed the GSC to operate with a power factor close to unity. On the other hand, Fig. 25 (b) shows that the PI controller has failed to effectively control the power injected at this stage. As a result, total injected power is almost entirely active, as can be seen in Fig. 26 (a). Fig. 27 shows that the total current injected into one phase of the three-phase grid, the total current generated is injected with a power factor close to unity independently of the rotor current behavior (Fig. 28).

Overall, the proposed non-linear controllers have demonstrated their effectiveness in meeting all the stated objectives with superior performances compared to those obtained using the conventional PI controller.
5. CONCLUSION

Non-linear controllers have been proposed for a grid-connected photovoltaic-wind hybrid system in this paper. The proposed control strategy has been designed to ensure an efficient integration of wind and solar photovoltaic energy through the optimal extraction of energy from both sources and the fulfillment of grid interconnection requirements. The modeling of the system elements as well as the design of the controllers are addressed in detail. The effectiveness of the proposed control strategy has been confirmed by numerical simulations and its performances are compared to those obtained with conventional controls. Future work will focus on the practical implementation of this control strategy as well as improving the control of other PV-wind hybrid systems.
Integral Backstepping Based Nonlinear Control for MPPT & UPF of a Grid ….. (M. El malah et al)

BIOGRAPHY OF AUTHORS

Mohammed El Malah was born in Morocco on November 02, 1980. He received the Master’s degree in Automatic, Signal Processing and Industrial Computing from Science and Technical Faculty, Hassan 1st University, Settat, Morocco in 2015. His research consists in the control of the linear and nonlinear systems with use of the advanced controller. Currently, he is preparing his PhD titled “Nonlinear control of renewable energy generation systems” in the Laboratory of System Analysis and Information Processing at Hassan 1st University.

Abdellatifah Ba-razzouk received the Master’s degree (M.Sc.A.) in industrial electronics from the Université du Québec à Trois-Rivières (UQTR), Quebec, Canada, in 1993, and the Ph.D. degree in electrical and computer engineering from the École Polytechnique de Montréal, Quebec, Canada, in 1998. From 1997 to 2003, he was a Lecturer in “motors modelling and control” at the Department of Electrical and Computer Engineering, UQTR. In September 1998, he joined the Hydro-Quebec Industrial Research Chair on Power and Electrical Energy, UQTR, where he has been a Professional Research Scientist working on “high-performance intelligent control of electrical drives”. Since June 2009, he is a Professor in electrical engineering with the Department of Applied Physics and a Researcher affiliated to “Systems Analysis and Information Processing Laboratory”, both at the Faculté des Sciences et Techniques, Université Hassan 1er of Settat, Morocco. His research interests include high performance control of adjustable speed drives, parameter identification and adaptive control of electrical motors, neural networks, real-time embedded control systems, renewable energy systems, modelling and computer aided design, and real-time simulation of power electronics systems using multiprocessors platforms.

Elhassane Abdelmounim received his PhD in applied Spectral analysis from Limoges University at science and technical Faculty, France in 1994. In 1996, he joined, as Professor, the applied physics department of the science and technical faculty of Hassan 1st University, Settat, Morocco. His current research interests include digital signal processing and machine learning. He is currently coordinator of a Bachelor of Science in electrical engineering and researcher in “ASTI” System Analysis and Information Technology Laboratory at science and technical faculty, Hassan 1st University, Settat, Morocco.
Mhamed Madark was born in Settat, Morocco, in 1987. He is a Ph.D. student. He received his B.Sc. degree in mathematics engineering from the Science and Technical Faculty of Hassan 1st University, Settat, Morocco, in 2013, the M.Sc. degree in Automatic, Signal Processing and Industrial Computing from the Science and Technical Faculty of Hassan 1st University, Settat, Morocco, in 2015. His research interests include the adaptive robust nonlinear control of the induction machine and robust nonlinear controller design for three phase grid connected PV systems.