Reducing the Variance of Redshift Space Distortion Measurements from Mock Galaxy Catalogues with Different Lines of Sight

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ABSTRACT

Accurate mock catalogues are essential for assessing systematics in the cosmological analysis of large galaxy surveys. Anisotropic two-point clustering measurements from the same simulation show some scatter for different lines of sight (LOS), but are on average equal, due to cosmic variance. This results in scatter in the measured cosmological parameters. We use the OuterRim N-body simulation halo catalogue to investigate this, considering the 3 simulation axes as LOS. The quadrupole of the 2-point statistics is particularly sensitive to changes in the LOS, with sub-percent level differences in the velocity distributions resulting in ~ 1.5σ shifts on large scales. Averaging over multiple LOS can reduce the impact of cosmic variance. We derive an expression for the Gaussian cross-covariance between the power spectrum multipole measurements, for any two LOS, including shot noise, and the corresponding reduction in variance in the average measurement. Quadrupole measurements are anti-correlated, and for three orthogonal LOS, the variance on the average measurement is reduced by more than 1/3. We perform a Fisher analysis to predict the corresponding gain in precision on the cosmological parameter measurements, which we compare against a set of 300 extended Baryon Oscillation Spectroscopic Survey (eBOSS) emission line galaxy (ELG) EZmocks. The gain in $f\sigma_8$, which measures the growth of structure, is also better than 1/3. Averaging over multiple LOS in future mock challenges will allow the RSD models to be constrained with the same systematic error, with less than 3 times the CPU time.

Key words: cosmology; theory – large-scale structure of Universe – galaxies: statistics – methods: data analysis

1 INTRODUCTION

Measurements of the spatial distribution of galaxies in large galaxy surveys allow us to probe the expansion history of the Universe and the growth of structure, testing the ΛCDM cosmological paradigm and constraining models of dark energy. In galaxy surveys, distances are inferred from redshifts, and each galaxy redshift includes a component which is due to the peculiar velocity of the galaxy along the line of sight of the observer. This results in redshift space distortions (RSD), where structures appear flattened on large scales, due to the coherent infall of galaxies to overdense regions, while on small scales, the random virial motion of galaxies results in elongated Fingers-of-God (Kaiser 1987).

For a sample of galaxies at some effective redshift $z_{\text{eff}}$, the redshift-space clustering statistics can be used to measure the growth of structure (Guzzo et al. 2008). By fitting models of the redshift-space clustering to the data, the quantity $f\sigma_8(z_{\text{eff}})$, can be inferred, where $f = d\ln D(a)/d\ln a$ is the linear growth rate, $D(a)$ is the linear growth function, and $\sigma_8$ is the rms of the density field in spheres of radius $8\ h^{-1}\text{Mpc}$. In general relativity, the growth rate is related to the matter density through $f \approx \Omega_m^{0.55}$ (e.g. Linder 2005). Therefore, RSD measurements provide a test for general relativity.

In addition to RSD, galaxy surveys can be used to measure baryon acoustic oscillations (BAO; e.g. Cole et al. 2005; Eisenstein et al. 2005). Acoustic waves in the early Universe lead to an enhancement in the present day galaxy distribution at a characteristic BAO scale of ~ 100 $h^{-1}\text{Mpc}$. Therefore, BAO measurements from galaxy samples at different effective redshifts can be used as a standard ruler to trace out the expansion history of the Universe. If an incorrect fiducial cosmology is assumed when calculating cosmological distances from redshifts, the BAO position is scaled by the parameters $a_\parallel$ and $a_\perp$, parallel and perpendicular to the line of sight, due to the Alcock-Paczynski effect (Alcock & Paczynski 1979). Measurements of $a_\parallel$ and $a_\perp$ can be used to determine the transverse comoving distance $D_H(z_{\text{eff}})/r_{\text{drag}}$, and the Hubble distance $D_H(z_{\text{eff}})/r_{\text{drag}}$, where $r_{\text{drag}}$ is the sound horizon at the drag epoch. The Hubble distance is related to the Hubble parameter, $H(z)$, through $D_H(z) = c/H(z)$.

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To validate the models that are used in the cosmological analysis of the survey data, it is important to use accurate mock galaxy catalogues. Since the ‘true’ cosmology of the mock is known, differences between the measured and ‘true’ values of $f_{sr}g$, $D_M/r_{drag}$ and $D_H/r_{drag}$ can be used to estimate the systematic uncertainties in the measurements due to the modelling of galaxy clustering.

Typically, large N-body simulations are used, which have accurate clustering statistics down to small, non-linear scales. For the extended Baryon Oscillation Spectroscopic Survey (eBOSS; Dawson et al. 2016), mock challenges have been performed utilizing the OuterRim simulation (Heitmann et al. 2019a). The eBOSS survey, which is part of SDSS-IV (Blanton et al. 2017), targeted luminous red galaxies (LRGs, $0.6 < z < 1.0$), emission line galaxies (ELGs, $0.6 < z < 1.1$) and quasars ($0.8 < z < 2.2$) as direct tracers of the matter field. For each of these tracers, mocks have been produced using the OuterRim box, which has a comoving side length of $3 \, h^{-1}\text{Gpc}$. Despite the large volume of the simulation, variations are seen in the cosmological measurements when different lines of sight are chosen (Alam et al. 2020; Smith et al. 2020). Since the Universe is isotropic and homogeneous, all lines of sight are equally valid, and there is no choice of observer position that is ‘better’ than any other. However, the finite box size of the simulation leads to variations in the clustering statistics, due to cosmic variance. While the choice of line of sight has a very small impact on measurements of $\alpha_0$ and $\alpha_2$, offsets in the measured $f_{sr}g$ for certain lines of sight are a few percent for the ELG mocks (Alam et al. 2020), and as large as $\sim 5\%$ for the quasar mocks (Smith et al. 2020), which is at a level of more than $3\sigma$ of the modelling systematic uncertainty. Future surveys, such as the Dark Energy Spectroscopic Instrument (DESI; DESI Collaboration et al. 2016a,b), aim to measure $f_{sr}g$ to sub-percent precision. In the mock challenges for eBOSS, these differences were mitigated by averaging over different lines of sight. The effect can also be mitigated by introducing a new estimator that averages over many lines of sight, such as the spherically averaged power spectrum (Percival & White 2009). When measuring $f_{sr}g$ to the very high precision required in future surveys, it is important to understand this effect.

This paper is a companion paper to the eBOSS mock challenges that were performed to assess the modelling systematics in the analysis of each of the different tracers. The LRG mock challenge is presented in Rossi et al. (2020), the ELG mock challenge is found in Alam et al. (2020), and the mock challenge for the quasar clustering sample is described in Smith et al. (2020). The final cosmological interpretation of the results from all the eBOSS analyses is presented in eBOSS Collaboration et al. (2020).

The aim of this paper is to understand why $f_{sr}g$ measurements are so sensitive to the choice of line of sight, and to calculate the improvement in the measurements when averaging together multiple lines of sight. This paper is outlined as follows: We give an overview of two-point clustering in Section 2, and the simulations we use in Section 3. Section 4 investigates how the clustering statistics are affected by the velocities in the simulation. In Section 5, we derive an expression for the cross-covariance of power spectrum multipole measurements for two lines of sight, and the corresponding reduction in the variance when averaging together multiple measurements. In Section 6, we perform a Fisher analysis to quantify the reduction in the uncertainties of the measured cosmological parameters. Finally, the conclusions are summarized in Section 7.

2 TWO-POINT CLUSTERING

2.1 Correlation function

The two-point correlation function of galaxies, $\xi(r)$, provides a measurement of the probability of finding a pair of galaxies with separation $r$. It is defined as

$$\xi(r) = \langle \delta(x)\delta(x + r) \rangle,$$

where $\delta(x) = \rho(x)/\bar{\rho} - 1$ is the density contrast at position $x$, $\bar{\rho}$ is the mean density, and the angled brackets indicate an ensemble average.

In redshift space, the correlation function is anisotropic, and we can measure the 2D correlation function $\xi(s, \mu)$, where $s$ is the separation in redshift space, and $\mu$ is the cosine of the angle between the line of sight and pair separation vector. The information in the 2D correlation function can be expressed by decomposing $\xi(s, \mu)$ into Legendre multipoles,

$$\xi(s, \mu) = \sum_{\ell} \xi_{\ell}(s) L_{\ell}(\mu),$$

where $L_{\ell}(\mu)$ is the $\ell$th order Legendre polynomial. The multipoles $\xi_{\ell}(s)$ can be calculated using

$$\xi_{\ell}(s) = \frac{2\ell + 1}{2} \int_{-1}^{1} \xi(s, \mu) L_{\ell}(\mu) \, d\mu.$$  

Only the first three even multipoles are non-zero in linear theory, and we call the monopole, $\xi_0(s)$, quadrupole, $\xi_2(s)$, and hexadecapole, $\xi_4(s)$.

To compute the correlation function, the estimator of Landy & Szalay (1993) is commonly used,

$$\xi(s, \mu) = \frac{DD(s, \mu) - 2DR(s, \mu) + RR(s, \mu)}{RR(s, \mu)},$$

where $DD(s, \mu)$, $DR(s, \mu)$ and $RR(s, \mu)$ are the normalized data-data, data-random and random-random pair counts. For a periodic box, the $RR(s, \mu)$ counts can be computed analytically, and only the $DD(s, \mu)$ pair counts are needed.

2.2 Power spectrum

Alternatively, the two-point clustering statistics can be expressed in Fourier space using the power spectrum, $P(k)$, defined as

$$\langle \delta(k)\delta(k') \rangle = (2\pi)^3 \delta^{(3)}(k + k') P(k),$$

and is the Fourier transform of the correlation function. As with the correlation function, the power spectrum can be decomposed into Legendre polynomials. For a periodic box, this is given by

$$P_{\ell}(k) = (2\ell + 1) \int \frac{d^3k}{4\pi V} \delta_{g}(\mathbf{k}) \delta_{g}(-\mathbf{k}) L_{\ell}(\mathbf{k} \cdot \hat{n}) - P_{\ell}^{\text{noise}}(k),$$

where $\delta_{g}(k)$ is the overdensity of galaxies in Fourier space, $\hat{n}$ is the unit line-of-sight vector, $V$ the volume of the box and $d\Omega_{k}$ is the solid angle. The shot noise term, which is non-zero only for the monopole, is

$$P_{0}^{\text{noise}} = \frac{1}{\bar{n}_{g}},$$

where $\bar{n}_{g}$ is the mean number density of galaxies.

References

1 For the mocks in non-blind cosmologies, which do not include observational effects, the modelling systematic in $f_{sr}g$ is at a level of $\sim 1.5\%$ (Smith et al. 2020).
3 SIMULATIONS

3.1 OuterRim simulation

In this work, we use the halo catalogue from the OuterRim simulation (Habib et al. 2016; Heitmann et al. 2019a,b). The OuterRim simulation is a dark-matter-only N-body simulation, which uses a flat \( \Lambda \)CDM cosmology that is consistent with the WMAP7 measurements (Komatsu et al. 2011), with \( \Omega_m h^2 = 0.1109, \Omega_b h^2 = 0.02258, h = 0.71, \sigma_8 = 0.8 \) and \( n_s = 0.963 \). The simulation contains 10,240 particles of mass \( m_p = 1.82 \times 10^9 h^{-1} M_\odot \) within a box of comoving side length 3000 \( h^{-1} \)Mpc. Dark matter haloes are identified with a friends-of-friends (FOF) algorithm (Davis et al. 1985), with a linking length \( b = 0.168 \). We use the simulation snapshot at \( z = 1.433 \) with a mass cut \( M > 3 \times 10^{12} h^{-1} M_\odot \) applied, giving a total number density of haloes of \( 4.9 \times 10^{-4} (h^{-1} \text{Mpc})^{-3} \).

This halo catalogue is comparable in redshift and linear bias to the eBOSS quasar sample. We use the halo catalogue directly, and not the quasar mocks produced in Smith et al. (2020), since the number density of quasars is low due to the ~1% duty cycle.

3.2 EZmocks

We also utilize a set of 300 Effective Zel'dovich approximation Mocks (EZmocks; Chuang et al. 2015) which were constructed for the eBOSS ELG sample (Zhao et al. 2020). We use these mocks to quantify the uncertainties on cosmological parameter measurements for a realistic galaxy survey, and the large number of mocks improves our statistics. In each box, the density field is generated using the Zel’’dovich approximation, and galaxies are added using a parametrisation of the galaxy bias. The mocks are calibrated to the clustering measurements of the eBOSS ELGs.

The EZmock boxes are in a Planck cosmology (Planck Collaboration et al. 2014) with \( \Omega_m = 0.307115, \Omega_b = 0.048206, h = 0.6777, \sigma_8 = 0.8225 \) and \( n_s = 0.9611 \), and have a box size of 5000 \( h^{-1} \text{Mpc} \). We use boxes at \( z = 0.876 \), with a number density of \( 6.4 \times 10^{-4} (h^{-1} \text{Mpc})^{-3} \).

4 CLUSTERING STATISTICS FOR DIFFERENT LINES OF SIGHT

4.1 Clustering measurements

To transform the coordinates of a tracer from its real-space to redshift-space position requires a choice to be made for the observer position. For a periodic box, it is common to use the plane parallel approximation, where the observer is positioned at infinity along e.g. the \( x \)-axis of the simulation. The halo positions are then displaced using the \( x \)-component of the velocity. In this case, the comoving position in redshift space is given by

\[
 s = r + \frac{v_x}{H(z)} \hat{x},
\]

where \( \hat{x} \) is a unit vector parallel to the \( x \)-axis, and \( v_x \) is the \( x \)-component of proper velocity. In the case of a periodic box, periodic boundary conditions are then applied.

In real space, the clustering measurements are not impacted by the choice of observer position, since the distance between a pair of galaxies is unaffected by the line of sight. In redshift space, this is not true. The velocity used to transform to redshift space depends on the observer, so we expect to see some variation in the clustering measurements for different lines of sight.

This is illustrated in Fig. 1, where we show the power spectrum and correlation function multipoles of the OuterRim halo catalogue when transforming to redshift space with an observer at infinity in the \( x, y \) and \( z \)-direction. The power spectrum is measured using the algorithm from \( \text{twopcf} \) (Hand et al. 2018), while the correlation function is computed using the publicly available parallelized code \( \text{two_pcf} \).\(^2\) For the power spectrum, the scatter between the monopole measurements is very small, at a level of \( \sim 0.2\% \), with a larger scatter in the quadrupole, at a level of \( \sim 2\% \). For the correlation function, the differences between the measurements for different observers show much more scale dependency, with larger scatter on large scales. At a scale of \( 100 h^{-1} \text{Mpc} \), the scatter in the monopole is at a level of \( \sim 2\% \), and the scatter in the quadrupole is as large as \( \sim 10\% \), with smaller differences on small scales.

While the variations in the quadrupole measurements are surprisingly large, this scatter is expected due to cosmic variance (see Section 5). Since measurements of \( f_{\text{sr}} \) are proportional to the amplitude of the quadrupole, a large offset in the quadrupole will also lead to large offsets in measurements of \( f_{\text{sr}} \). It is therefore important to understand these large variations in the quadrupole measurements for mock challenges.

The halo catalogue from OuterRim that we are using is comparable to the eBOSS quasar sample. The variations in the clustering measurements for different observer positions is large for the quasars, but note that the results in the following subsections are not specific to any single tracer.

4.2 Pair counts

To understand how the correlation function is affected by the observer position, it is more convenient to first consider the \( DD \) pair counts, rather than the correlation function directly. The variation in the \( DD \) counts from the OuterRim halo catalogue, measured by an observer at infinity along each of the three simulation axes, is shown in Fig. 2. The left panel is in real space, where the colour indicates the standard deviation of the three measurements of \( DD(r, \mu) \), measured in bins of \( r \) and \( \mu \), divided by the mean. In real space, the variation between the three measurements is very small. The separation between each pair does not change, and the variation that is seen is due to pairs being placed in different \( \mu \) bins when observed from different directions. When integrating over \( \mu \), the total number of pairs in each bin of \( r \) is constant.

The right hand panel of Fig. 2 shows the variation in the \( DD \) counts in redshift space. For pairs with large separations, the difference in the \( DD(s, \mu) \) counts is close to zero for \( \mu \lesssim 0.75 \), but for \( \mu \gtrsim 0.75 \), the scatter is of the order of \( \sim 10\% \). The redshift-space separation, \( s \), between a pair of galaxies will be different for different observers, due to the different velocity components along each line of sight. Pairs separated along the line of sight (with \( \mu \sim 1 \)) are more strongly affected by velocities, since the separation and velocity vectors are parallel. This variation in \( DD(s, \mu) \) leads to the differences in the clustering seen in Fig. 1. Pairs with \( \mu \sim 1 \) are weighted more strongly when calculating the quadrupole (in Eq. 3, the Legendre polynomial \( L_2(\mu) = 1 \) when \( \mu = 1 \)), explaining why the differences in the quadrupole are much larger than for the monopole.

It is important to note that the differences in the \( DD \) counts are very small (less than \( 10^{-3} \)), yet this is enough to shift the amplitude of the quadrupole on large scales by as much as \( \sim 10\% \). This is illustrated by the red dotted curves in Fig. 1, where the average

\(^2\) https://github.com/lstothert/two_pcf
The pairwise velocity, \( v_{\text{pair}, x} \), in the \( x \)-direction is defined as the difference between the \( x \)-components of the individual velocities, \( v_{\text{pair}, x} = v_{1, x} - v_{2, x} \), where the \( x \)-position \( x_1 > x_2 \). This distribution looks Gaussian, but is slightly skewed towards negative velocities (i.e. infalling pairs), as expected, (e.g. Juszkiewicz et al. 1998; Tinker 2007; Bianchi et al. 2015) and the distribution looks almost identical for the three directions. The skewness of the distribution becomes larger on smaller scales, where both members of the pair reside in the same dense environment. However, the bottom panel of Fig. 3 shows the ratio to the average distribution, revealing that there are small differences, which are mostly seen in the variance of the distribution.

To summarise, for an observer at infinity, differences are seen in the measurements of the quadrupole and hexadecapole for different observer positions. This effect is due to the finite box size of the simulation, and is explained by small differences in the pairwise velocity distributions projected along the three simulation axes. As the volume is increased towards infinity, the velocity distributions will converge, and the effect will diminish. But it is important to note that even for a simulation as large as OuterRim, with a 3 \( h^{-1}\)Gpc box, the choice of observer position has a significant impact on measurements of the quadrupole, which will propagate to the measurements of \( f_{\sigma 8} \).
4.3 Velocities

In this subsection, we study in more detail the velocity distributions, which are responsible for the variations seen in the clustering measurements. Understanding the velocities is important, as it is one of the components needed in the streaming model (e.g. Fisher 1995; Scoccimarro 2004), which is often used to predict the redshift-space clustering of galaxies from a model of the real-space clustering (e.g. Reid & White 2011; Wang et al. 2014).

Fig. 4 shows the distribution of the individual velocities of each object in the box. The average of each of the three velocity components, $v_i$, is shown in bins of position along each axis, $i$. When $i = j$, (i.e. the velocity component is parallel with the axis in which it is binned) variations can be seen, which is due to the coherent motion of haloes within the large-scale structure of the finite box. When $i \neq j$, it is interesting to note that the average velocity is zero. This is because there are no transverse modes in the simulation. Another way to think about this is to consider a single spherically symmetric overdense region, which is surrounded by infalling haloes. The average $v_x$ of the haloes, in bins of $x$, will be positive for small $x$, and negative at large $x$, since the average motion of the haloes is towards the centre of the overdense region. However, the average $v_y$ and $v_z$, in bins of $x$, is zero, due to the spherical symmetry.

However, it is the pairwise velocities, and not the individual velocities, that are responsible for the variation seen in the clustering measurements. The distribution of pairwise velocities for pairs in a single bin of separation (at 100 h$^{-1}$Mpc) is shown in Fig. 3. The mean and standard deviation of the same distribution is shown in Fig. 5, but as a function of separation. The top panel shows the $x$, $y$, and $z$-components of the mean pairwise velocity, $v_{\text{pair}}$, and the difference to the average $v_{\text{pair}}$. Negative velocities indicate that the pairs are infalling, which is expected, due to the coherent infall of haloes towards dense regions. The magnitude of the infall velocity increases with decreasing pair separation, and this is consistent with similar measurements from other simulations, and the predictions from perturbation theory models (e.g. Cuesta-Lazaro et al. 2020).

Reducing variance of RSD measurements

Figure 3. Top panel: Distribution of the $x$, $y$, and $z$ components of pairwise halo velocities (coloured curves), for pairs of haloes with separation $99 < r < 101 h^{-1}$Mpc. The dotted black curve is the average distribution. Shaded regions indicate the error estimated from 512 jackknife regions. Bottom panel: Ratio to the average distribution.

Figure 4. Average of the velocity components $v_x$ (blue), $v_y$ (yellow) and $v_z$ (green), in bins of $x$ (upper panel), $y$ (middle panel) and $z$ (lower panel).
This reduction in the uncertainties is shown more clearly in Fig. 7. The upper three sets of panels show the jackknife uncertainties in the power spectrum (left) and correlation function (right), for the three individual measurements (black dotted curves), the mean of the three uncertainties (black solid curve), and the uncertainty in the combined measurement (coloured curves). The ratio is shown in the lower panels.

On large scales (i.e. small \( k \)), there is very little change in the uncertainties in the power spectrum monopole after combining the three measurements, and the ratio is very close to 1. This is not surprising, since the three measurements of the monopole are highly correlated, so very little information is gained when combining them. On small scales (large \( k \)), there is a small reduction in the uncertainties, while in the correlation function, the ratio is consistent with 1 over all scales. There is a larger reduction in the uncertainties on the hexadecapole, by a factor of \( \sim 0.75 \), in both the power spectrum and correlation function on large scales.

The most striking result is for the quadrupole, where the uncertainties are reduced by a factor ~ 0.2 on large scales. In the case that the measurements of a quantity are completely uncorrelated, combining the three lines of sight would be equivalent to increasing the volume by a factor of 3, and therefore the uncertainties in the combined measurement would be reduced by a factor of \( 1/\sqrt{3} \approx 0.58 \) (indicated by the horizontal dotted line in the lower panels of Fig. 7). It is remarkable that the gain in the quadrupole measurement is much greater than this.

In the next section, we aim to understand this result. We calculate the cross-covariance between power spectrum measurements for two different lines of sight, and show how this relates to the gain in the uncertainties when combining multiple measurements together.

## 5 CROSS-COVARIANCE BETWEEN TWO DIFFERENT LINES OF SIGHT

The measurements of galaxy clustering from a simulation are impacted by the choice made for the line of sight. In this section we estimate the cross-covariance matrix for the power spectrum multipoles, measured with two lines of sight, \( u \) and \( v \). In Section 5.1, we derive an expression for the cross-covariance in the case where \( u \) and \( v \) are orthogonal, neglecting shot noise. In Section 5.2, we generalise this for any two lines of sight, and include shot noise. We show how the cross-covariance is related to the gain in the uncertainties in the power spectrum measurements in Section 5.3.

### 5.1 Orthogonal lines of sight

The cross-covariance matrix of the power spectrum multipoles, with different lines of sight, can be written as

\[
C_{\ell \ell'ij}^{uv} = \left\{ \left[ \hat{P}_\ell^i(k_i) - \langle \hat{P}_\ell^i(k_i) \rangle \right] \left[ \hat{P}_{\ell'}^j(k_j) - \langle \hat{P}_{\ell'}^j(k_j) \rangle \right] \right\}
\]

\[= C_{\ell \ell'ij} \rho_{\ell \ell'ij}^{uv}, \tag{9}
\]

where \( C_{\ell \ell'ij} \) is the covariance matrix of power spectrum measurements with the same line of sight, and \( \rho_{\ell \ell'ij}^{uv} \) is the cross-covariance, which we aim to calculate. By rearranging Eq. 10, we can write...
Reducing Variance of RSD Measurements

The first of these terms is $\langle \hat{P}_g^{u}(k_i)\hat{P}_g^{v}(k_j) \rangle$, which can be dropped, since it is being subtracted by the second term in the numerator of Eq. 12.

From the Kaiser formula, the density contrast of galaxies in redshift space is related to the real-space matter field through

$$\delta_g^{uv}(k) = b \left( 1 + \beta \cos^2 \theta_u \right) \delta_m(k),$$

with $b$ the linear bias, $\theta_u$ the cosine of the angle between the wave vector $k$ and line of sight $u$, and $\beta = f/b$ where $f$ is the linear growth rate. An identical equation can also be written that relates $\delta_g^{u}(k')$ to the matter field in terms of angle $\theta_v$. Also, because of homogeneity we use Eq. 5, and therefore

$$\left\langle \delta_g^{u}(k)\delta_g^{v}(k') \right\rangle = (2\pi)^3 b^2 \left( 1 + \beta \cos^2 \theta_u \right) \left( 1 + \beta \cos^2 \theta_v \right) \rho_m(k).$$

For the case in which the two lines of sight are perpendicular (which we emphasise by setting $u = x$ and $v = y$), we can put these ingredient together, using $\cos \theta_v = \sin \theta_x \sin \phi_x$ and $d\Omega_x = d\phi_x d\theta_x \sin \theta_x$. Finally, $\hat{\rho}_{ij}^{XY}$ can be written as

$$\hat{\rho}_{ij}^{XY}(\beta) = \delta_j^{X} K_{ij}^{XY}(\beta).$$

Figure 7. Left: Errors in the power spectrum multipole measurements from OuterRim, estimated using 512 jackknife subsamples. The first 3 panels show the errors in the monopole, quadrupole and hexadecapole, respectively. Black dotted curves show the errors for the 3 different lines of sight, and the black solid curve is the average of these 3. The coloured curves show the errors when the 3 lines of sight are combined. The ratios are shown in the lower panel, indicating the reduction in errors when combining the lines of sight. The horizontal lines indicate ratios of 1 and 1/\(\sqrt{3}\). Right: same as the left panels, but for the correlation function measurements.

\[ \frac{\rho_{ij}^{XY}}{\rho_{ij}^{XX}} = \frac{\left[ \hat{P}_{ij}^{u}(k_i) - \langle \hat{P}_g^{u}(k_i) \rangle \right] \left[ \hat{P}_{ij}^{v}(k_j) - \langle \hat{P}_g^{v}(k_j) \rangle \right]}{\frac{\langle \hat{P}_g^{u}(k_i) \hat{P}_g^{v}(k_j) \rangle}{\rho_g^{XY}}} \]  

(11)

\[ \rho_{ij}^{XY}(\beta) = \delta_j^{X} K_{ij}^{XY}(\beta) \]  

(17)
\[ \kappa_{\ell \ell'}(\beta) = \int d\phi_x d\theta_x \sin \theta_x \left( 1 + \beta \cos^2 \theta_x \right)^2 \left( 1 + \beta \sin^2 \theta_x \cos^2 \theta_x \right)^2 \frac{\ell_{\ell'}(\cos \theta_x) \ell_{\ell'}(\sin \theta_x \cos \phi_x)}{4} \] 

(18)

\[ \kappa_{\ell \ell'}(\beta) = 2\pi \int d\theta_x \sin \theta_x \left( 1 + \beta \cos^2 \theta_x \right)^4 \frac{\ell_{\ell'}(\cos \theta_x) \ell_{\ell'}(\cos \theta_x)}{16} \] 

(19)

Here, where we have neglected shot noise, the cross-covariance only depends on \( \beta \), and is independent of \( k \). \( \kappa_{\ell \ell'}^{xy} \) is shown in Fig. 8. For the monopole (\( \rho_{00}^{xy} \)), the cross-correlation reaches 1 if there is no RSD (\( \beta = 0 \)), which is as expected, and decreases as \( \beta \) is increased.

For the quadrupole (\( \rho_{22}^{xy} \)), the cross-correlation is negative, which indicates that there is an anti-correlation between the quadrupole measurements for two lines of sight.

The limits as \( \beta \rightarrow 0 \) and \( \beta \rightarrow \infty \) for the different multipoles are

\[ \lim_{\beta \rightarrow 0} \rho_{00}^{xy}(\beta) = 1 \quad \lim_{\beta \rightarrow \infty} \rho_{00}^{xy}(\beta) = \frac{3}{35} \]

\[ \lim_{\beta \rightarrow 0} \rho_{22}^{xy}(\beta) = -\frac{1}{2} \quad \lim_{\beta \rightarrow \infty} \rho_{22}^{xy}(\beta) = -\frac{33}{5810} \] 

(20)

\[ \lim_{\beta \rightarrow 0} \rho_{44}^{xy}(\beta) = \frac{3}{8} \quad \lim_{\beta \rightarrow \infty} \rho_{44}^{xy}(\beta) = \frac{5239}{199080} \]

These cross-correlations are related to the scatter between the power spectrum (and correlation function) measurements on large scales (as shown in Fig. 1 for the OuterRim haloes). Very little power spectrum (and correlation function) measurements on large scales when being observed from different directions. When RSD is included, the variations in the quadrupole are due to the velocity distributions. When the velocity is increased, the overall amplitude of the quadrupole increases, but the relative difference between the measurements decreases, due to a reduction in the anti-correlation. Despite this, it is still the direction where the rms of the velocity distribution is largest that will have the strongest quadrupole.

### 5.2 General line of sight and shot noise

The cross-covariance derived in the previous section can be generalised for any two lines of sight, which are not necessarily orthogonal. Taking the line of sight \( u \) as being directed along the reference \( x \)-axis (\( u = (1, 0, 0) \)), we can write the unit wavevector as \( \hat{k} = (\cos \theta_u, \sin \theta_u \cos \phi_u, \sin \theta_u \sin \phi_u) \). As before, \( \theta_u \) is the angle between \( \hat{k} \) and \( \hat{u} \), and \( \phi_u \) is the azimuthal angle around \( \hat{u} \). In the same coordinate system we can also write the second line of sight, \( v \), as \( \hat{v} = (\cos \theta_v, \sin \theta_v \cos \phi_v, \sin \theta_v \sin \phi_v) \), where \( \theta_v \) is the angle between \( \hat{v} \) and \( \hat{u} \), and \( \phi_v \) is the azimuthal angle of \( v \) around \( u \). The cosines of the angles, \( \cos \theta_u = \hat{k} \cdot \hat{u} \) and \( \cos \theta_v = \hat{k} \cdot \hat{v} \), are therefore related through

\[ \cos \theta_v = \cos \theta_u \cos \theta_{uv} + \sin \theta_u \cos \phi_u \sin \theta_{uv} \cos \phi_{uv} \]

\[ + \sin \theta_u \sin \phi_u \sin \theta_{uv} \sin \phi_{uv} \]

\[ = \cos \theta_u \cos \theta_{uv} + \sin \theta_u \sin \theta_{uv} \cos \phi_u \sin \phi_{uv} \]

(21)

where, in the last equality, we have used that \( \phi_v = \phi_{uv} - \phi_u \).

So far, we have neglected the shot noise. Calling the real space RSD displacements along the \( u \) and \( v \) lines of sight \( \delta_{uv} \) and \( \delta_{uv} \), the shot noise contribution to Eq. 16 is simply the Poisson shot noise term, \( 1/\bar{n} \) (where \( \bar{n} \) is the density), modulated by a phase shift term \( e^{i\bar{\psi}_{uv} \hat{k} \cdot \hat{u}} \), where \( \bar{\psi}_{uv} = k \cdot \hat{u} = k \cos \theta_u \) (and a similar expression for \( \bar{\psi}_{uv} \)). This is the characteristic function of the Gaussian random variable, \( \bar{\psi}_{uv} \), which has a mean of zero, and variance \( \sigma_{\bar{\psi}_{uv}}^2 \). Hence the shot noise contribution reads \( e^{-\sigma_{\bar{\psi}_{uv}}^2/2}/\bar{n} \), where \( \sigma_{\bar{\psi}_{uv}} \) is given by

\[ \sigma_{\bar{\psi}_{uv}}^2 = \left( k^2 \bar{\psi}_{uv} - k^2 \bar{\bar{\psi}}_{uv} \right)^2 \]

(22)

\[ = k^2 \left( \bar{\psi}_{uv}^2 \right) + k^2 \left( \bar{\bar{\psi}}_{uv}^2 \right) - 2k^2k_i \bar{\psi}_{uv} \bar{\psi}_{uv} \]

(23)

From linear perturbation theory, the displacement \( \bar{\psi}_{uv} \) due to redshift space distortions is given in Fourier space by

\[ \bar{\psi}_{uv}(k) = i f k \bar{k} \bar{\delta}(k) \]

(24)

We therefore have, taking the Fourier transform,

\[ \langle \bar{\psi}_{uv} \rangle = \frac{f^2}{(2\pi)^3} \int d^3k k^2 \bar{\psi}_{uv} \bar{\bar{\psi}}_{uv}(k) \]

\[ = \frac{f^2}{(2\pi)^3} \int dk \bar{\psi}_{uv}(k) \int_0^{2\pi} d\phi_v \int_0^\pi \sin \theta_u d\theta_u \cos \theta_u \]

\[ \langle \cos \theta_u \cos \theta_{uv} \rangle + \sin \theta_u \sin \theta_{uv} \cos \phi_u \sin \phi_{uv} \]

\[ = f^2 \sigma_{\bar{\psi}_{uv}}^2 \]

(25)

where the one-dimensional rms of the displacement field is

\[ \sigma_{\bar{\bar{\psi}}_{uv}}^2 = \frac{1}{6\pi^2} \int dk \bar{\bar{\psi}}_{uv}(k) \]

(26)

Since \( \cos \theta_{uv} = \cos \theta_{uv} = 1 \), we also have \( \langle \bar{\psi}_{uv}^2 \rangle = \langle \bar{\bar{\psi}}_{uv}^2 \rangle = f^2 \sigma_{\bar{\psi}_{uv}}^2 \), and therefore, \( \sigma_{\bar{\bar{\psi}}_{uv}}^2 = \left( k^2 + k^2 - 2k^2 \cos \theta_{uv} \right) f^2 \sigma_{\bar{\psi}_{uv}}^2 \). In the case that the two lines of sight are orthogonal (\( \cos \theta_{uv} = 0 \)), \( \bar{\psi}_{uv} \) and \( \bar{\bar{\psi}}_{uv} \) do not correlate. The shot noise contribution is largest (\( 1/\bar{n} \)) when \( f = 0 \), as expected, and vanishes as \( f \rightarrow \infty \).

Finally, putting everything together, and using \( d\Omega_u = d\phi_ud\theta_u \sin \theta_u \), the cross-covariance \( \rho_{\ell \ell'}^{uv}(\beta, b, h) \) can be written as

\[ \rho_{\ell \ell'}^{uv}(\beta, b, h) = \delta^K \kappa_{\ell \ell'}^{uv}(k_i, \beta, b, h) \]

(27)

with, at leading order in \( (\bar{n} \bar{\bar{\psi}}_{uv}(k))^{-1} \) (e.g. Meiksin & White 1999; MNRAS 000, 1–13 (2020))
Reducing Variance of RSD Measurements

Its covariance matrix, $C$, (dropping indices for conciseness) is

$$\langle \hat{P}^{\ell \ell, \text{los}} \rangle = \left\langle \left( \frac{\beta^x + \beta^y}{2} \right)^2 \right\rangle$$

and the reduction in the uncertainty is $\sqrt{\rho^{\ell \ell, \text{los}}}$.

5.3.3 All lines of sight

The above can be extended to three orthogonal lines of sight by writing the average measurement as

$$\rho^{\ell \ell, \text{los}} = \frac{\beta^x + \beta^y + \beta^z}{3}.$$ (36)

Following the same logic, it can be shown that the gain in the variance is

$$\rho^{\ell \ell, \text{los}} = \frac{1 + 2 \rho^{xy}}{3}. \tag{37}$$

For the case of no shot noise, the limits are

$$\lim_{\beta \rightarrow 0} \rho^{\ell \ell, \text{los}}(\beta) = 1$$

$$\lim_{\beta \rightarrow +\infty} \rho^{\ell \ell, \text{los}}(\beta) = \frac{41}{105}$$

$$\lim_{\beta \rightarrow 0} \rho^{\ell \ell, \text{los}}(\beta) = 0$$

$$\lim_{\beta \rightarrow +\infty} \rho^{\ell \ell, \text{los}}(\beta) = \frac{2872}{8715}$$

$$\lim_{\beta \rightarrow 0} \rho^{\ell \ell, \text{los}}(\beta) = \frac{7}{12}$$

$$\lim_{\beta \rightarrow +\infty} \rho^{\ell \ell, \text{los}}(\beta) = \frac{104779}{298620}.$$ (38)

In the limit as $\beta \rightarrow 0$, the relative variance of the monopole is 1, as expected, since the three monopole measurements in real space are identical. For the quadrupole, the relative variance is reduced to 0.

The predicted gain in the uncertainties ($\sqrt{\rho^{\ell \ell, \text{los}}(\beta)}$) for the multipole measurements is shown by the solid curves in Fig. 9, for the case of 3 orthogonal lines of sight, with no shot noise. We also measure the gain in uncertainties from a set of 100 Gaussian random fields, shown by the points, where redshift-space distortions are added using the Kaiser formula with different values of $\beta$. The measured gain in the uncertainties from the Gaussian random fields is in good agreement with the prediction. The red vertical dashed line indicates the value of $\beta$ for the OuterRim halo catalogue, and the gain is consistent with the gain seen on large scales in Fig. 7.

5.3.4 All lines of sight

One can go further and average over all possible lines of sight, $\omega$, of the solid angle $\Omega$.

$$\rho^{\ell \ell, \text{all-los}} = \frac{1}{4\pi} \int d\Omega \rho^{\ell \ell, \omega}.$$ (38)

As before, we have

$$\left( \langle \hat{P}^{\ell \ell, \text{all-los}} \rangle \right)^2 = \frac{1}{(4\pi)^2} \int d\Omega \int d\Omega' \left( \hat{P}^{\ell \ell, \omega} \right) \left( \hat{P}^{\ell \ell, \omega'} \right). \tag{39}$$
Following from statistical isotropy, a reference line of sight \( u \) can be chosen, such that

\[
\left\langle \left( \rho^{\text{all-los}} \right)^2 \right\rangle = \frac{1}{4\pi} \int d\Omega \left( \rho^{\mu} \rho^{\nu} \right),
\]

and therefore, when averaging over all lines of sight, the gain in the variance is

\[
\rho^{\text{all-los}} = \frac{1}{2} \sum_{\mu=0}^{\nu=1} \int d\Omega \sin \theta_{\mu\nu} \rho^{\mu\nu}.
\]

Already, with 3 orthogonal lines of sight, the relative variance of the quadrupole is pinned down to 0 in the limit \( \beta \to 0 \). When averaging over all lines of sight, the limits are

\[
\begin{align*}
\lim_{\beta \to 0} \rho^{\text{all-los}}_{00ii}(\beta) &= 1 \\
\lim_{\beta \to \infty} \rho^{\text{all-los}}_{22ii}(\beta) &= \frac{9}{25} \\
\lim_{\beta \to 0} \rho^{\text{all-los}}_{44ii}(\beta) &= 0 \\
\lim_{\beta \to \infty} \rho^{\text{all-los}}_{44ii}(\beta) &= \frac{155584}{7838775}.
\end{align*}
\]

This also pins down the relative variance of the hexadecapole to 0 as \( \beta \to 0 \), while there is also a reduction in the limits as \( \beta \to \infty \) for all multipoles.

The gain in the uncertainties when averaging all lines of sight is shown by the dotted curves in Fig. 9. Compared to the case of 3 orthogonal lines, there is very little change in the monopole, with some improvement in the quadrupole. For the hexadecapole, the improvement is much more striking, where the relative gain in the uncertainties is pulled very close to zero even at large values of \( \beta \). While averaging over all lines of sight greatly improves the hexadecapole measurement, this comes at the cost of inflating the cross-terms in the covariance matrix. The monopole and quadrupole of the power spectrum averaged over all lines of sight (and also the monopole and hexadecapole) are fully correlated, such that the covariance matrix is singular.

6 FISHER ANALYSIS

In this section, we propagate the cross-covariance between power spectrum measurements down to the measurements of the cosmological parameters.

Cosmological parameters are estimated through a \( \chi^2 \) minimisation,

\[
\chi^2(p) = [P - M(p)]^T C^{-1} [P - M(p)],
\]

where \( P \) is the measured power spectrum, and \( M(p) \) is the model, which depends on parameters \( p \), and \( C \) is the covariance matrix. Taking the derivative of \( \chi^2(p) \) with respect to \( p \) yields

\[
0 = \left[ \frac{\partial M(p)}{\partial p} \right]^T C^{-1} [P - M(\hat{p})].
\]

At first order around the true parameter value \( p_0 \) we can write \( M(\hat{p}) = M(p_0) + \frac{\partial M(p)}{\partial p} \cdot (\hat{p} - p_0) \), with all derivatives with respect to \( p \) taken at \( p_0 \). Without loss of generality, we set \( p_0 = 0 \) and \( M(p_0) = 0 \). Eq. 44 can therefore be rearranged to give

\[
\left[ \frac{\partial M(p)}{\partial p} \right]^T C^{-1} \left[ \frac{\partial M(p)}{\partial p} \right] \hat{p} = \left[ \frac{\partial M(p)}{\partial p} \right]^T C^{-1} P.
\]

Writing the Fisher information as

\[
F = \left[ \frac{\partial M(p)}{\partial p} \right]^T C^{-1} \left[ \frac{\partial M(p)}{\partial p} \right],
\]

we obtain

\[
\hat{p} = F^{-1} \left[ \frac{\partial M(p)}{\partial p} \right]^T C^{-1} P.
\]

The covariance of the measured parameters, when the power spectra
measurements are calculated with respect to the same line of sight, is simply
\[
\langle \bar{p}_u \bar{p}_v^T \rangle = F^{-1} \left( \frac{\partial M(p)}{\partial p} \right)^T C^{-1} \frac{\partial M(p)}{\partial p} F^{-1} = F^{-1}.
\]
For two different lines of sight \(u\) and \(v\), the covariance is
\[
\langle \bar{p}_u \bar{p}_v^T \rangle = F^{-1} \left( \frac{\partial M(p)}{\partial p} \right)^T C^{-1} \left( \frac{\partial M(p)}{\partial p} \right) F^{-1} = F^{-1} C_{uv} F^{-1},
\]
where \(C_{uv}\) is the cross-covariance between the two different lines of sight \(u\) and \(v\) for measurements of the power spectrum, which is estimated in Section 5. The same calculations derived in Section 5.3 can be used to calculate the gain in variance on the cosmological parameters, when averaging over multiple lines of sight.

### 6.1 Measurements from EZmocks

We test the Fisher forecasts by comparing the gain in uncertainties of the cosmological parameters against measurements from the eBOSS ELG EZmock boxes described in Section 3.2.

The power spectrum in the periodic box is measured for the 300 EZmocks, with RSD applied along the x, y, and z-directions. We show the gain in the uncertainties in the power spectrum measurements, when averaging together the three lines of sight, in Fig. 10, where the uncertainty is measured from the error on the mean. To estimate the error on the error, the 300 mocks are split into 6 sets of 50, and the standard deviation is calculated between the errors evaluated for each set. The dotted lines show the theoretical prediction (from Eqs. 27-29), where \(\beta\) and \(\delta\) are the mean values obtained by fitting the measured power spectrum monopoles, quadrupoles and hexadecapoles with a linear Kaiser model damped by a Lorentzian and hexadecapole, testing the prediction on large, linear scales. For \(\beta\) on small scales, there is some small disagreement in the quadrupole, measured in the mocks. Since the mocks are affected by shot noise, when averaging together the three lines of sight, in Fig. 10, where the uncertainty is measured from the error on the mean. To estimate the error on the error, the 300 mocks are split into 6 sets of 50, and the standard deviation is calculated between the errors evaluated for each set. The dotted lines show the theoretical prediction (from Eqs. 27-29), where \(\beta\) and \(\delta\) are the mean values obtained by fitting the measured power spectrum monopoles, quadrupoles and hexadecapoles with a linear Kaiser model damped by a Lorentzian.

A model of the redshift-space power spectrum is fit to the measured power spectra of each mock, in order to obtain the best fit values of the cosmological parameters. We use the the TNS model (Taruya et al. 2010), using Regularized Perturbation Theory (RegPT) at 2-loop order (Taruya et al. 2012), as described in de Mattia et al. (2020). An analytic Gaussian covariance matrix is used, following the method of Grieb et al. (2016). Two different sets of fits are performed. In the first set, the fits are applied over the k-range 0.03 < \(k\) < 0.12 hMpc\(^{-1}\) for the monopole, quadrupole and hexadecapole, testing the prediction on large, linear scales. For this range we use a linear Kaiser model, with a Lorentzian Finger-of-God term. In the second set of fits, the k-range is extended to 0.03 < \(k\) < 0.20 hMpc\(^{-1}\) for all three multipoles, which tests the prediction on smaller scales, where the linear approximation begins to break down.

### 7 CONCLUSIONS

To validate the models used in the cosmological analysis for large galaxy surveys, and to estimate systematics, it is essential to utilize realistic mock catalogues. The cosmological parameters that
are measured can be compared against the true value, since the cosmology of the mock is known. In the eBOSS survey, mock challenges have been performed for the LRGs, ELGs and quasars, which utilized the OuterRim N-body simulation, to assess the modelling systematics.

As shown in Alam et al. (2020) and Smith et al. (2020), there is scatter in the measurements of the cosmological parameters for different choices of the line of sight. This is due to cosmic variance within the mock, which has a finite size. However, for the quasar sample, the shifts in the measurements of $f/\sigma_8$ are as large as ~ 5%. For the eBOSS mock challenges, it is important that this effect is understood, and especially important for validating models at the high precision needed for future surveys, such as DESI.

We investigate the impact the line of sight has on the power spectrum and correlation function measurements from the OuterRim simulation, with observers positioned in the $x$, $y$ and $z$-directions. The halo catalogue used is comparable in redshift and linear bias of the eBOSS quasar sample, where this effect is large, but our results from this study are not specific to any individual tracer. While the variations between the monopole measurements are small, a much larger scatter is seen in the quadrupole. On large scales, the scatter between the correlation function quadrupole measurements is as much as ~ 10%.

Since the amplitude of the quadrupole depends on the velocities, we investigate the velocity distributions in the simulation, along each of the 3 axes. Variations in the pairwise velocity distributions are small, resulting in differences of less than 0.1% in the $DD(s, \mu)$ pair counts (for bins with $\mu$ close to 1, which are affected the most by velocities). These small variations are amplified when the correlation function is calculated. It is primarily the rms of the velocity distribution (in each bin of $s$) that is responsible for the variations in the quadrupole, with a large rms resulting in a larger amplitude (ie. the quadrupole is more negative). The small variations seen in the velocity distributions are expected from cosmic variance.

The variations in the two-point clustering measurements can be mitigated by averaging together measurements taken using different lines of sight. We derive an expression for the cross-covariance for two power spectrum multipole measurements, assuming linear theory, in the simplified case of orthogonal lines of sight with no shot noise (Eqs. 17-19), and in the more general case of any lines of sight, including shot noise (Eqs. 27-29). Monopole measurements are highly correlated, with a weak correlation for the hexadecapole, while the quadrupole measurements are anti-correlated.

We show how the cross-covariance is related to the reduction in the variance when averaging together multiple lines of sight. In the case of averaging together 3 orthogonal lines of sight, the reduction is given by Eq. 37. In the OuterRim halo catalogue, there is only a very small reduction in the variance for the monopole. In the limit that there is no RSD, the variance is unchanged, since the three measurements are identical. For the quadrupole, the anti-correlation results in large gains in the variance, much larger than a factor of $1/3$ (or $1/\sqrt{3}$ in the uncertainties) that would be expected if the measurements were uncorrelated. In the limit that there is no RSD, the relative variance goes to 0. In the OuterRim halo catalogue, on large scales, the variance in the combined quadrupole measurement is ~ 25 times smaller than the individual measurements, resulting in uncertainties that are more than 5 times smaller.

A reduction in the variance of the power spectrum measurements also results in a reduction in the uncertainties on the measured cosmological parameters. We measure this for the eBOSS ELG sample, using a set of 300 EZmocks boxes. We measure the reduction in the uncertainties of $f/\sigma_8$, $D_M/r_{\text{drag}}$ and $D_H/r_{\text{drag}}$, and compare this with the gain predicted from performing a Fisher analysis. When performing the fit to scales of 0.12 Mpc$^{-1}$, we expect small gains in the uncertainties on $D_M/r_{\text{drag}}$ and $D_H/r_{\text{drag}}$ of ~ 0.90 and ~ 0.77, respectively, with much larger gain in $f/\sigma_8$ of ~ 0.39 due to the measurements being strongly anti-correlated. Extending the fit to smaller scales of 0.2 Mpc$^{-1}$, the anti-correlation for $f/\sigma_8$ is reduced, resulting in gains that are only slightly better than a factor of $1/\sqrt{3}$. The Fisher forecasts show reasonable agreement with the measurements from the mocks.

By averaging together the clustering measurements from the 3 orthogonal lines of sight, large gains can be made in the variance of the quadrupole, and hence $f/\sigma_8$ measurements, which are better than a factor of 1/3. In mock challenges, when validating models, it is therefore important to average these measurements together. This is particularly true if a single mock is used, which was the case for the OuterRim mocks used in eBOSS. The effect is stronger for tracers, such as quasars, which are more strongly biased, and have a smaller value of $\beta$. For a set of many mock catalogues, where the total volume is large, it is still worthwhile to combine the results measured from multiple lines of sight. This will allow tighter constraints to be placed on the models, without needing to generate more mocks.

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DATA AVAILABILITY

The OuterRim simulation snapshot used in this work is publicly available at https://cosmology.alcf.anl.gov/outerrim. The EZmock catalogues and their clustering measurements will be shared on reasonable request to the corresponding author with permission of Cheng Zhao.

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