Late-time acceleration in the coupled Cubic Galileon models

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Abstract

We investigate the linearly and quadratically coupled cubic Galileon models that include linear potentials. These models may explain the late-time acceleration. In these cases, we need two equations of state parameter named the native and effective equations of state to test whether the universe is accelerating or not because there is coupling between the cold dark matter and Galileon. It turns out that there is no transition from accelerating phase to phantom phase in the future.

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I. INTRODUCTION

Observational data indicate that our universe undergoes an accelerating phase since the recent past \cite{1}. The cosmological constant could be considered as a candidate for the dark energy to explain the observational result in the ΛCDM model. However, this model has two problems of the fine tuning and the coincidence and thus, an alternative candidate was needed. One promising candidate is a dynamical dark energy model based on scalar field theory which is dubbed the quintessence \cite{2-4}. This model has a canonical kinetic term and thus, a scalar is minimally coupled to gravity. We wish to point out that the cosmological constant has a constant equation of state $\omega = p/\rho = -1$, while the scalar field model has a time-dependent equation of state with $-1 \leq \omega \leq 1$.

Recently, Planck observation \cite{5} has shown four combined data on equation of state: i) $\omega = -1.13^{+0.13}_{-0.25}$ (95%; Planck+WMAP+BAO) which is in good agreement with a cosmological constant, ii) $\omega = -1.09 \pm 0.17$ (95%; Planck+WMAP+Union2.1) that is more consistent with a cosmological constant, iii) $\omega = -1.13^{+0.13}_{-0.14}$ (95%; Planck+WMAP+SNLS) which favors the phantom phase ($\omega < -1$) at the 2σ level, and iv) $\omega = -1.24^{+0.18}_{-0.19}$ (95%; Planck+WMAP+$H_0$) which is in tension with a cosmological constant at more than the 2σ level. The last two might draw the universe into a phantom phase about at the 2σ level. However, if one uses the BAO data in addition to the CMB, there is no strong evidence for the phantom phase that is incompatible with a cosmological constant.

On the other hand, one modified gravity named the Galileon gravity was considered another model of the dynamical dark energy \cite{6}. This model is also described by a scalar field theory which contains terms of nonlinear-derivative self couplings. Turning off gravity, the Galileon action is invariant under the Galilean transformation of $\pi \rightarrow \pi + c + b_\mu x^\mu$. The field equations contain at most second derivatives of $\pi$, implying that it is surely free from the Ostrogradsky ghosts. Turning on gravity, however, breaks the symmetry. Therefore, a covariant Galilean action has been constructed \cite{7}, where the Galilean symmetry is softly broken but it preserves the shift symmetry of $\pi \rightarrow \pi + c$. Its cosmological implications have been extensively investigated to explain the late-time acceleration in \cite{8-13}.

In this work, we wish to investigate the late-time acceleration by using the linearly coupled cubic Galileon model \cite{14} together with a linear potential $V = c_1 \pi$. This model is similar to the DGP model \cite{15}. We note that adding the potential breaks the shift symmetry,
which might make the nonlinear-derivative self coupling term trivial. In this case, one needs two equations of state parameter named the native and effective equations of state to say whether the universe is accelerating or not. This is because there is coupling between the cold dark matter and Galileon. In the uncoupled case of $c_0 = 0$, the model under consideration corresponds to the cubic Galileon gravity. This has been studied in [16], which shows that the data of SN+BAO+$H_0$ prefers the Galileon gravity over the quintessence. The authors in [17] have shown an appearance of the phantom phase, but it arose from choosing negative $c_2$ and $c_3$. More recently, it turned out that the equation of state obtained from the cubic Galileon model is indistinguishable from $\omega = -1$ of the cosmological constant [18]. For $c_4 \neq 0$ and $c_5 \neq 0$, a phantom phase appeared for both $c_0 = 0$ case [19] and $c_0 \neq 0$ [14]. For a quadratic coupling with $c_4 \neq 0$ and $c_5 \neq 0$, its late-time evolution was investigated in [20] which indicates a crossing of the phantom divide line. However, we mention that these phantom phases arose from when one chooses negative coefficients $c_i (i \geq 2)$.

Previous works on the Galilean models have shown phantom phases, depending on the choice of coefficients. Hence, it is very curious to check if a phantom phase happens really in the Galilean models because phantom behavior is quite interesting and it is possibly bad for the future evolution of the universe. Hence, we hope to find a phantom phase in the linearly and quadratically coupled cubic Galileon model that include linear potential. However, we have found no phantom phase when we did make a complete computation. We wish to understand why there is no phantom by comparing it with the Brans-Dicke theory with a linear potential. If one suppresses a nonlinear-derivative self coupling term of $(\nabla \pi)^2 \Box \pi$ capturing a decisive feature of the Galileon gravity, the linearly coupled cubic Galileon model with the potential is similar to the Brans-Dicke cosmology with the power-law potential $\Phi^\alpha$ where one could observe a future crossing of the phantom divide line only for $\alpha > 1$ [21]. Thus, it seems that choosing the linear potential $V = c_1 \pi$ does not provide the phantom phase because this potential breaks the shift symmetry. Explicitly, the addition of the linear potential did not make the nonlinear-derivative self coupling term of $(\nabla \pi)^2 \Box \pi$ matter and thus, it did not play a role in the late-time evolution. Very recently, a model of Slotheon gravity with $V = V_0 e^{-\frac{c_1 \pi}{\rho}}$ and $c_0 = c_2 = c_3 = c_4$ has provided the cosmic acceleration but not the phantom phase in the late-time evolution [13].

Inspired by the above motivation, we will focus on observing whether the phantom phase appears in the future when we use the linearly and quadratically coupled cubic Galileon...
models that include linear potentials.

II. EVOLUTION EQUATIONS

The covariant Galileon action can be written as

\[ S_{cG} = \int d^4x \sqrt{-g} \left[ \frac{M^2_{pl} R}{2} + \frac{1}{2} \sum_{i=1}^{5} c_i \mathcal{L}_i + \mathcal{L}_m - \frac{c_G}{M^3_{pl}} T^{\mu \nu} \partial_\mu \pi \partial_\nu \pi - \frac{c_0}{M_{pl}^3} T \right], \] (1)

where \( c_{1-5} \) are arbitrary dimensionless constants, and \( M_{pl} \) is the reduced Planck mass, and \( M^3 = M_{pl} H_0^2 \) to make the \( c \)’s dimensionless. \( \mathcal{L}_m \) denotes the Lagrangian for the cold dark matter and \( T^{\mu \nu} \) (its trace \( T \)) represent the energy-momentum tensor. The Galileon action is usually classified into three classes: the uncoupled Galileon with \( c_0 = c_G = 0 \); the linearly coupled Galileon with \( c_0 \neq 0 \) and \( c_G = 0 \); the derivative coupled Galileon with \( c_0 = 0 \) and \( c_G \neq 0 \).

We are working in the Jordan frame where the explicit coupling between \( \pi \) and \( T \) is removed when we use a metric redefinition \[14\]. Among three classes of the Galileon model, we focus on the linearly coupled cubic Galileon (lcG) model as

\[ S_{lcG} = \int d^4x \sqrt{-g} \left[ \left( 1 - 2 c_0 \frac{\pi}{M_{pl}} \right) \frac{M^2_{pl} R}{2} - \frac{c_2}{2} (\nabla \pi)^2 - \frac{c_3}{M^3} (\nabla \pi)^2 \Box \pi - V(\pi) + \mathcal{L}_m \right], \] (2)

where the linear potential of \( V(\pi) = c_1 \pi \) is introduced for our late-time evolution. Here we demand that \( c_i \ (i \geq 2) \) are positive to avoid the phantom scalar field. The case of \( c_0 = 0 \) corresponds to the cubic Galileon gravity \[18\]. This model with \( V(\pi) = 0 \) was extensively studied in \[14\] and the uncoupled case of \( c_0 = 0, c_2 = 1 \) and \( c_3/M^3 = \alpha/2M_{pl}^3 \) was investigated in \[16\]. In addition to the choice of \( c_2 = 1 \), we use \( \alpha \) instead of \( c_3 \) which shows a decisive feature of the Galileon gravity when we compare it with the quintessence and the Brans-Dicke cosmology.

The Einstein equation is derived from \[21\] as

\[ M^2_{pl} \left( 1 - \frac{2c_0}{M_{pl}^2} \right) G_{\mu \nu} = \left[ -2 M_{pl} c_0 (\nabla_\mu \nabla_\nu - g_{\mu \nu} \nabla^2) \pi + T_{\mu \nu} + T^\alpha_{\mu \nu} + T^V_{\mu \nu} + T^m_{\mu \nu} \right], \] (3)

where \( G_{\mu \nu} = R_{\mu \nu} - R g_{\mu \nu}/2 \) is the Einstein tensor and the energy-momentum tensors are
defined to be \[ T_{\mu\nu} = \left[ \nabla_\mu \pi \nabla_\nu \pi - \frac{1}{2} g_{\mu\nu} (\nabla \pi)^2 \right], \] (4)

\[ T^\alpha_{\mu\nu} = \frac{\alpha}{M_{3 \text{pl}}^3} \left[ \nabla_\mu \pi \nabla_\nu \pi \Box \pi + g_{\mu\nu} \nabla^\alpha (\nabla \pi)^2 - \nabla_{(\mu} \pi \nabla_{\nu)} (\nabla \pi)^2 \right], \] (5)

\[ T^V_{\mu\nu} = \frac{c_1}{2} \pi g_{\mu\nu}. \] (6)

Here the energy-momentum tensor \( T^m_{\mu\nu} \) for a pressureless matter of \( \mathcal{L}_m \) is given by

\[ T^m_{\mu\nu} = \rho_m u_\mu u_\nu \] (7)

with \( u_\mu \) the four velocity.

The Galileon equation takes the form

\[ \Box \pi + \frac{\alpha}{M_{3 \text{pl}}^3} \left[ (\Box \pi)^2 - (\nabla_\mu \nabla_\nu \pi)^2 - R^\mu\nu \nabla_\mu \pi \nabla_\nu \pi \right] - V_\pi - c_0 M_{3 \text{pl}} R = 0, \] (8)

where \( V_\pi \) denotes the derivative with respect to \( \pi \). To study its cosmological implications, it would be better to convert Eq. (3) into a standard form of the Einstein equation

\[ G_{\mu\nu} = M_{3 \text{pl}}^{-2} \left( T^\pi_{\mu\nu} + \frac{T^m_{\mu\nu}}{1 - 2 \frac{c_0}{M_{3 \text{pl}}}} \right), \] (9)

where \( T^\pi_{\mu\nu} \) denotes the energy-momentum tensor from all \( \pi \)'s contributions \times \((1 - 2c_0 \pi / M_{3 \text{pl}})^{-1}\).

Importantly, the Bianchi identity obtained by acting \( \nabla^\mu \) on Eq. (9) leads to the total conservation-law as \[ \nabla^\mu T^\pi_{\mu\nu} + T^m_{\mu\nu} \nabla^\mu \left( \frac{1}{1 - 2 \frac{c_0}{M_{3 \text{pl}}}} \right) = 0 \] (10)

when one uses the conservation-law for the cold dark matter

\[ \nabla^\mu T^m_{\mu\nu} = 0. \] (11)

At this stage, we introduce the flat Friedmann-Robertson-Walker (FRW) metric as

\[ ds^2 = -dt^2 + a^2(t) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right), \] (12)

where \( a(t) \) is the scale factor. Then, we write the Einstein equation (3) and the Galileon equation (8) as

\[ 3 \left( 1 - 2 \frac{c_0}{M_{3 \text{pl}}} \right) M_{3 \text{pl}}^2 H^2 = \rho_m + 6c_0 M_{3 \text{pl}} H \tilde{\pi} + \frac{\tilde{\pi}^2}{2} \left( 1 - 6 \frac{\alpha}{M_{3 \text{pl}}^3} H \tilde{\pi} \right) + V(\pi), \] (13)

\[ \left( 1 - 2 \frac{c_0}{M_{3 \text{pl}}} \right) M_{3 \text{pl}}^2 \left( 2 \dot{H} + 3H^2 \right) = 2c_0 M_{3 \text{pl}} \left( \tilde{\pi} + 2H \tilde{\pi} \right) - \frac{\tilde{\pi}^2}{2} \left( 1 + 2 \frac{\alpha}{M_{3 \text{pl}}^3} \tilde{\pi} \right) + V(\pi), \] (14)

\[ \ddot{\pi} + 3H \dot{\pi} - \frac{3\alpha}{M_{3 \text{pl}}^3} \tilde{\pi} \left( 3H^2 \tilde{\pi} + \dot{H} \tilde{\pi} + 2H \dot{\pi} \right) + V_\pi + 6M_{3 \text{pl}} c_0 \left( 2H^2 + \dot{H} \right) = 0. \] (15)
In the case of \( c_0 = 0 \), the above all equations reduce to Eqs. (2)-(4) of Ref. \[16\]. We can rewrite Eqs. (13) and (14) as the standard forms of the Friedman equations

\[
3 M_{\text{pl}}^2 H^2 = \frac{\rho_{\text{m}}}{\left(1 - \frac{2 c_0}{M_{\text{pl}}^2} \right)} + \left\{6 c_0 M_{\text{pl}} H \dot{\pi} + \frac{\dot{\pi}^2}{2} \left(1 - \frac{6 \alpha}{M_{\text{pl}}^3} H \ddot{\pi}ight) + V(\pi)\right\},
\]

\[
M_{\text{pl}}^2 \left(2 \dot{H} + 3 H^2\right) = -\left\{-2 c_0 M_{\text{pl}} (\ddot{\pi} + 2 H \dot{\pi}) + \frac{\dot{\pi}^2}{2} \left(1 + \frac{2 \alpha}{M_{\text{pl}}^3} \ddot{\pi}\right) - V(\pi)\right\},
\]

which can also be found from \[9\] directly. Since Eqs. (16) and (17) are the first and second Friedman equations, respectively, we can read off the energy density and pressure for the Galileon as

\[
\rho_{\pi} = \frac{1}{\left(1 - \frac{2 c_0}{M_{\text{pl}}^2} \right)} \left\{6 c_0 M_{\text{pl}} H \dot{\pi} + \frac{\dot{\pi}^2}{2} \left(1 - \frac{6 \alpha}{M_{\text{pl}}^3} H \ddot{\pi}\right) + V(\pi)\right\},
\]

\[
p_{\pi} = \frac{1}{\left(1 - \frac{2 c_0}{M_{\text{pl}}^2} \right)} \left\{-2 c_0 M_{\text{pl}} (\ddot{\pi} + 2 H \dot{\pi}) + \frac{\dot{\pi}^2}{2} \left(1 + \frac{2 \alpha}{M_{\text{pl}}^3} \ddot{\pi}\right) - V(\pi)\right\}.
\]

Now we express the total conservation-law (10) in terms of density and pressure for the Galileon and matter density as

\[
\dot{\rho}_{\pi} + 3 H (\rho_{\pi} + p_{\pi}) = -\frac{2 c_0}{M_{\text{pl}}^3} \rho_{\text{m}} \left(1 - \frac{2 c_0}{M_{\text{pl}}^2} \right)^2,
\]

which is very similar to the Brans-Dicke cosmology \[21\]. We observe that the left-hand side of Eq. (20) implies an energy (matter) transfer between Galileon and cold dark matter.

### III. BACKGROUND EVOLUTION

To solve three coupled equations (13)-(15), we introduce the new variables

\[
x = \frac{\dot{\pi}}{\sqrt{6 H M_{\text{pl}}}}, \quad y = \frac{\sqrt{V}}{\sqrt{3 H M_{\text{pl}}}}, \quad \epsilon = -\frac{6 H \ddot{\pi}}{M_{\text{pl}}^3},
\]

\[
\lambda = -M_{\text{pl}} \frac{V_{\pi}}{\sqrt{V}}, \quad z = -\frac{2 \pi}{M_{\text{pl}}}, \quad \eta = \ln \left[\frac{a}{a_0}\right],
\]

where \( a_0 \) is the present Ho\'ll parameter.
where \( \eta \) is introduced instead of the scale factor \( a \). Equations (13)-(15) are transformed into the first-order coupled equations

\[
x' = \frac{-1}{1 + \alpha \epsilon} \left\{ 3x + \frac{3}{2} \alpha \epsilon x - \sqrt{\frac{6}{2}} \lambda y^2 + 2\sqrt{6c_0} + \frac{H'}{H} \left( x + \frac{3}{2} \alpha \epsilon x + \sqrt{6c_0} \right) \right\}, \tag{23}
\]

\[
y' = -y \left( \frac{H'}{H} + \sqrt{\frac{6}{2}} \lambda x \right), \tag{24}
\]

\[
\epsilon' = \epsilon \left( 2 \frac{H'}{H} + \frac{x'}{x} \right), \tag{25}
\]

\[
\lambda' = \sqrt{6} \lambda^2 x (1 - \Gamma), \tag{26}
\]

\[
z' = -2\sqrt{6} x, \tag{27}
\]

\[
\frac{H'}{H} = \frac{A}{B}, \tag{28}
\]

where \( \Gamma = \frac{VV_{\pi\pi}}{V^2_\pi} \) and prime (’') denotes the differentiation with respect to \( \eta \). Here, \( A \) and \( B \) are given as

\[
A = -3(2 + 4\alpha \epsilon + \alpha^2 \epsilon^2) x^2 + \sqrt{6} \left( \lambda y^2 \alpha \epsilon - 2c_0 \alpha \epsilon - 4c_0 \right) x
+ 6(1 + \alpha \epsilon)(y^2 - 1 - c_0 z) + 12c_0 \lambda y^2 - 48c_0^2, \tag{29}
\]

\[
B = \alpha^2 \epsilon^2 x^2 + 4\sqrt{6}c_0 \alpha \epsilon x + 4(1 + c_0 z)(1 + \alpha \epsilon) + 24c_0^2. \tag{30}
\]

Importantly, the total conservation-law (20) can be rewritten as the new variables as

\[
\dot{\rho}_\pi + 3H \left( 1 + \omega_{\pi}^{\text{nat}} \right) \rho_\pi = H \left( -2\sqrt{6} \frac{c_0 x \rho_m}{(1 + c_0 z)^2} \right), \tag{31}
\]

where the native equation of state is defined to be

\[
\omega_{\pi}^{\text{nat}} = \frac{p_\pi}{\rho_\pi} = \frac{-2\sqrt{6} c_0 x \left( 2 + \frac{H'}{H} + \frac{x'}{x} \right) + x^2 \left( 1 - \frac{1}{3} \epsilon \frac{H'}{H} - \frac{1}{3} \alpha \epsilon \frac{x'}{x} \right) - y^2}{2\sqrt{6}c_0 x + x^2(1 + \alpha \epsilon) + y^2}. \tag{32}
\]

One may interpret Eq. (31) as

\[
\dot{\rho}_\pi + 3H \left( 1 + \omega_{\pi}^{\text{nat}} + \frac{2\sqrt{6} c_0 x}{3 (1 + c_0 z)^2} \frac{\rho_m}{\rho_\pi} \right) \rho_\pi = 0. \tag{33}
\]

From this equation, one can obtain the effective equations of state as

\[
\omega_{\pi}^{\text{eff}} = \omega_{\pi}^{\text{nat}} + \frac{2\sqrt{6}}{3} \frac{c_0 x}{1 + c_0 z} \frac{1 - \Omega_m}{\Omega_\pi}, \tag{34}
\]

where the two density parameter \( \Omega_\pi \) are given by

\[
\Omega_\pi = \frac{\rho_\pi}{\rho_c} = \frac{2\sqrt{6}c_0 x + x^2(1 + \alpha \epsilon) + y^2}{1 + c_0 z}, \quad \Omega_m = 1 - \Omega_\pi = \frac{\rho_m}{\rho_c (1 + c_0 z)}. \tag{35}
\]
Here $\rho_c = 3M_{pl}^2H^2$ is the critical energy density and we used the relation

$$\frac{1}{1 + c_0 z} \frac{\rho_m}{\rho_\pi} = \frac{\Omega_m}{\Omega_\pi}. \quad (36)$$

In case of $c_0 = 0$, we have $\omega_\pi^{nat} = \omega_\pi^{eff}$, which implies that $\omega_\pi^{eff}$ is not necessary to describe the uncoupled Galileon model. Since $\omega_\pi^{eff}$ and $\Omega_m$ blow up at $z = -1/c_0$ for $c_0 < 0$, we require $c_0 > 0$.

We rewrite the first Friedmann equation (13) in terms of the new variables as

$$\Omega_m = 1 - \frac{2\sqrt{6}c_0 x + x^2 (1 + \alpha\epsilon) + y^2}{1 + c_0 z}, \quad (37)$$

which was used to find the initial values of the evolving variables. According to the Planck mission [5], the current dark matter content is $\Omega_0^m = 0.315$. We take this value as an initial condition for the numerical evolution. After solving the first-order coupled equations, one finds the background evolution for the uncoupled case ($c_0 = 0$). As is depicted in Fig. 1, we observe the accelerating universe of $\omega_\pi^{nat} \geq -1$ in the future. Furthermore, Fig. 2 shows a typical background evolution for the linearly coupled case ($c_0 \neq 0$). In this case, we find the accelerating universe which is shown by $\omega_\pi^{nat} \geq -1$ as well as $\omega_\pi^{eff} \geq -1$. Definitely, there is no signal to give a phantom phase with $\omega_\pi^{nat} < -1$ in the future.

IV. QUADRATIC COUPLING

Recently, there was a work for the quadratic coupling which shows a crossing of the phantom divide [20]. To see whether this happens really or not, we consider the action without $L_4$ and $L_5$ for simplicity

$$S_{qcG} = \int d^4x \sqrt{-g} \left[ \left( 1 - 4c_0 \frac{\pi^2}{M_{pl}^2} \right) \frac{M_{pl}^2 R}{2} - \frac{c_2}{2} (\nabla\pi)^2 - \frac{c_3}{M^3} (\nabla\pi)^2 \Box \pi - V(\pi) + L_m \right]. \quad (38)$$

Following the computation steps of the previous section, two equations that are different from the linearly coupled case are

$$x' = \frac{-1}{1 + \alpha\epsilon} \left\{ 3x + \frac{3}{2} \alpha\epsilon x - \sqrt{6} \lambda y^2 - 4\sqrt{6}c_0 z + \frac{H'}{H} \left( x + \frac{3}{2} \alpha\epsilon x - 2\sqrt{6}c_0 z \right) \right\}, \quad (39)$$

$$\frac{H'}{H} = \frac{A_{qc}}{B_{qc}}, \quad (40)$$
FIG. 1: Evolutions for the uncoupled Galileon with $c_0 = 0$. $\eta = \ln[a/a_0] < 0 (\eta > 0)$ denote the past (future) and $\eta = 0$ represents the present time with $a = a_0$. These include density parameter $\Omega_m$ of cold dark matter (blue-dotted) and density parameter $\Omega_\pi$ of Galileon (green-dotted-dashed). Red-solid curve denotes the equation of state $\omega_\pi^{\text{nat}}$ for Galileon. We impose $\Omega_0^m = 0.315$ at $\eta = 0$ as an initial condition and $\alpha = 1.0$. The other initial conditions are given by $\epsilon_0 = 5.0$, $x_0 = 0.01$, $y_0 = 0.8272847152$, $\lambda_0 = 0.1$ and $z_0 = 1.0$.

where $A_{qc}$ and $B_{qc}$ are given as

$$A_{qc} = -3(2 + 4\alpha\epsilon + \alpha^2\epsilon^2 - 32c_0\alpha\epsilon - 32c_0)x^2 + \sqrt{6}\left(\lambda y^2\alpha\epsilon + 4c_0z\alpha\epsilon + 8c_0z\right) x$$
$$+ 6(1 + \alpha\epsilon)(y^2 - 1 + c_0z^2) - 24c_0z\lambda y^2 - 192c_0^2z^2, \quad (41)$$

$$B_{qc} = \alpha^2\epsilon^2x^2 - 8\sqrt{6}c_0z\alpha\epsilon x + 4\left(1 - c_0z^2\right)(1 + \alpha\epsilon) + 96c_0^2z^2. \quad (42)$$

Also, the native and effective equations of state are given as

$$\omega_\pi^{\text{nat}} = \frac{\rho_{q\pi}}{p_{q\pi}} = \frac{4\sqrt{6}}{3} \frac{c_0zx\left(2 + \frac{\pi'}{\pi} + \frac{x'}{x}\right) - 16c_0x^2 + x^2\left(1 - \frac{1}{3}\alpha\epsilon\frac{\pi'}{\pi} - \frac{1}{3}\alpha\epsilon\frac{z'}{z}\right) - y^2}{-4\sqrt{6}c_0zx + x^2(1 + \alpha\epsilon) + y^2}, \quad (43)$$

$$\omega_\pi^{\text{eff}} = \omega_\pi^{\text{nat}} - \frac{4\sqrt{6}}{3} \frac{c_0zx}{1 - c_0z^2} \frac{1 - \Omega_\pi}{\Omega_\pi}. \quad (44)$$
FIG. 2: Evolutions for the linearly coupled Galileon with $c_0 = 0.1$. These include density parameter $\Omega_m$ of cold dark matter (blue-dotted) and density parameter $\Omega_\pi$ of Galileon (green-dotted-dashed) as functions of $\eta = \ln[a/a_0]$. Black-solid (red-dotted-dashed) curves denote the equation of state $\omega_\pi^{\text{nat}}$ ($\omega_\pi^{\text{eff}}$) for Galileon. We choose $\Omega^0_m = 0.315$ at $\eta = 0$ and $\alpha = 1.0$. The initial conditions are $\epsilon_0 = 0.1, x_0 = 0.1, y_0 = 1.14456772, \lambda_0 = 0.1$ and $z_0 = 10.0$.

We note that the first Friedmann equation for the initial condition is slightly modified to be

$$\Omega_m = 1 - \frac{-4\sqrt{6}c_0zx + x^2(1 + a\epsilon) + y^2}{1 - c_0z^2}. \quad (45)$$

We observe from Fig. 2 that there is no crossing of the phantom divide in the future. There is no essential difference between the linearly coupled and quadratically coupled Galileon models.

V. DISCUSSIONS

First of all, we observe that the native and effective equations of state do not cross the phantom divide of $\omega = -1$ in the linearly (quadratically) coupled cubic Galileon models. This means that there is no essential difference between the Brans-Dicke cosmology and
FIG. 3: Evolutions for the quadratically coupled Galileon with $c_0 = 0.01$. These include density parameter $\Omega_m$ of cold dark matter (blue-dotted) and density parameter $\Omega_\pi$ of Galileon (green-dotted-dashed) as functions of $\eta = \ln[a/a_0]$. Black-solid (red-dotted-dashed) curves denote the equation of state $\omega_{\pi}^{\text{nat}} (\omega_{\pi}^{\text{eff}})$ for Galileon. We choose $\Omega_m^0 = 0.315$ at $\eta = 0$ and $\alpha = 1.0$. The other initial conditions are $\epsilon_0 = 0.1, x_0 = 0.1, y_0 = 0.8209750301, \lambda_0 = 0.1$ and $z_0 = 0.0$.

The cubic Galileon models. In other words, the term of $(\nabla \pi)^2 \Box \pi$ showing a feature of the Galileon gravity did not contribute significantly to deriving the late-time acceleration. To explain it, let us compare our result with Ref. [14] where acceleration was found, even though the potential $V = c_1 \pi$ was not introduced. The reason seems to be clear because this potential breaks the shift symmetry of $\pi \rightarrow \pi + c$. The addition of the potential did not make the nonlinear-derivative self coupling term of $(\nabla \pi)^2 \Box \pi$ matter and thus, it did not play a role in the late-time evolution.

At this stage, one may ask which one is an observable quantity between $\omega_{\pi}^{\text{eff}}$ and $\omega_{\pi}^{\text{nat}}$ in the Jordan frame [21, 22]. It is well known that the Jordan frame is a physical frame because of a minimal coupling to matter. However, this frame gives rises to the non-conservation of continuity equation (20) which shows that $\rho_m$ plays the role of a source to generate a new dark fluid. Even though $\omega_{\pi}^{\text{nat}}$ indicates a genuine equation of state for a Galileon-fluid, it
cannot satisfy the continuity equation. On the other hand, although $\omega_{\pi}^{\text{eff}}$ is not a genuine equation of state for a Galileon-fluid (because it contains cold dark matter), it satisfies the continuity equation. This implies that each of them is not a perfect observable for the linearly (quadratically) coupled Galileon models. Therefore, we have to use both $\omega_{\pi}^{\text{nat}}$ and $\omega_{\pi}^{\text{eff}}$ to show the presence of a crossing of the phantom divide. If the phantom phase is observed from both, one believes that it really happens in the evolution of the linearly (quadratically) coupled cubic Galileon gravity models. Otherwise, one is hard to confirm the appearance of the phantom phase in the Jordan frame. Surely, our analysis shows the disappearance of any phantom phase in the coupled cubic Galileon models.

Finally, we mention that there is the equation \[ \dot{\omega}_{\text{tot}}^{\text{nat}} = -1 \frac{2\dot{H}}{3H^2} = -1 \frac{2H'}{3H} = \frac{p_{\pi}}{\rho_{m} + \rho_{\pi}}, \]

which defines the total equation of state

\[ \omega_{\text{tot}} = -1 - \frac{2\dot{H}}{3H^2} = -1 - \frac{2H'}{3H} = \frac{p_{\pi}}{\rho_{m} + \rho_{\pi}}. \] (47)

The evolution of $\omega_{\text{tot}}$ is given in Figs. 4 and 5 for linearly coupled and quadratically coupled cases, respectively. These also show that there is no phantom phase in the future.

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FIG. 4: Evolution for $\omega_{\text{tot}}\,^{(47)}$ as a function of $\eta = \ln[a/a_0]$ for the linearly coupled cubic Galileon model. All other conditions are the same as the Fig. 2 caption does show.

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FIG. 5: Evolution for $\omega_{\text{tot}}$ (47) as a function of $\eta = \ln[a/a_0]$ for the quadratically coupled cubic Galileon model. All other conditions are the same as the Fig. 3 caption does show.

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