Modeling and analysis of the Earth pole motion with nonstationary perturbations

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Abstract. Using a numerical-analytical approach, the Earth pole motion is considered under nonstationary perturbations. A method of refined modeling of the Earth pole motion is proposed, which allows to improve the accuracy of the short-term forecast during the occurrence of irregular effects due to changes in the amplitudes of the fundamental harmonics of the oscillatory process.

1. Nonstationary perturbations in the oscillatory process of the earth's pole

High-precision forecast of spacecraft orbits is one of the important tasks of navigation. Earth orientation parameters - coordinates of the Earth pole, the difference between universal and atomic time UT1 – TAI - are included into the transformation matrix of the geocentric equatorial coordinate system into the Greenwich Terrestrial one. The coordinates of the Earth pole must be determined both in long (1–2 years) and in short (from a day to several months) time intervals. For forecasting over short time intervals, it is reasonable to approach the model of the Earth pole motion to the basic two-frequency model with constant coefficients by averaging the variable parameters in the selected interval and correcting the Chandler frequency [1 –3].

The model of the Earth pole motion considered in [4] allows one to take into account additional terms in which the fundamental frequency corresponds to the Chandler or annual frequency depending on the ratio of the amplitudes of the annual and Chandler harmonics. When the amplitude ratio changes, the main frequency in the additional terms will change quite quickly. From the results of processing data from observations and measurements of International Earth Rotation and Reference Systems Service (IERS) [1], it follows that this phenomenon is irregular. It leads to the dynamic effects described in [4]. The model of perturbed motion of the Earth relative to the center of mass, considered in [4], on the basis of which an approach to the study of basic harmonics is proposed, allows identification of the parameters of additional components of the model and applies them in the calculated procedure for predicting the motion of the Earth pole.

The construction of an autonomous model of pole motion, which makes it possible to make a forecast for rather long time intervals without correction of parameters has considerable interest in the practical aspect. In the course of research [5], it was possible to determine additional terms to the main model of the pole motion, for which the correction of coefficients is not required, i.e., corresponding to the autonomous mode of the model until the oscillating mode of the Earth pole changes.

In this case, the model of the motion of the pole can be considered as autonomous within one oscillatory mode. It’s quite problematic to make prediction of a the onset of the change in average
frequency $\psi$ and amplitude ratio changes $a_{ch}/a_h$ (the ratio of the amplitudes of the Chandler wobble $a_{ch}$ and the annual oscillation $a_h$), and its accuracy characteristics are difficult to assess. However, it is possible to determine the average frequency in the mode of adaptive functioning of the model in real time or with a small delay, which does not lead to a deterioration in the accuracy of the forecast. If we exclude the trend from data $x_p, y_p$, we’ll have the angle $\psi$ in values $\psi$ in the interval from the initial to the current point in time. For example, in the case of close amplitude values that meet the conditions $a_{ch} > a_h$, $\dot{a}_{ch} < 0$, $\dot{a}_h > 0$, the six-year modulation interval can be divided into two sub-intervals: a short (1.2 years average) with a average frequency value $\psi < N$ (N – Chandler frequency) and long (on average 5.2 years), at which the average frequency is close to the arithmetic mean of the annual and Chandler frequencies, i.e. $\psi \approx \frac{N + \nu_h}{2}$ ($\nu_h$ – frequency of one-year harmonic with a period of 1 year). This situation is observed after the spike of the 2006 phase: the mean frequency $\psi$ is close to the value $(N + \nu_h)/2$ within the limits of the calculation error.

![Fig. 1](image)

In fig. 1 the variations $\psi_{var} = \psi - \bar{N}t$ and frequency variations $\psi_{var}$, $MJD$ – Modified Julian date

In fig. 1 the variations $\psi_{var} = \psi - \bar{N}t$ and $\psi_{var}$ in relation to the six-year time interval after 2006 are shown. With increasing frequency up to 1 cycle per year, the oscillatory mode has already changed. It can be said that the oscillations switched to another mode somewhat earlier – at a frequency greater than $(N + \nu_h)/2$. The first area (before the blue line) corresponds with the average frequency $(N + \nu_h)/2$, middle area (between the colored lines) – with the linear increasing of the frequency $\psi_{var}$, quadratic dependence $\psi_{var}$ corresponds to which, and the third area – with the frequency of the annual oscillation after the change of the oscillatory mode.
Such changes can be seen in the dissipative systems [6] not just with the variations of the amplitudes of the main harmonics, but before the onset of the steady oscillatory mode.

2. Short-term forecast of the motion of the Earth pole

With the change of the oscillatory modes the motion of the Earth pole considerably differs from the motion with the “average parameters”. This necessitates the modification of the forecast model at appropriate time intervals.

![Fig. 2](image)

**Fig. 2.** The standard deviations of the forecasts for 45 days in relation to the developed model (red line) and forecasts of IERS (blue dots)

The filtering procedure and the adjustment of the algorithm can be performed using the method of "weighted" least squares. The average frequency of the Earth's pole rotation around the midpoint in one cycle before the transition process is close to the frequency of 0.9225 cycle/year. If the oscillatory mode changes to annual, then with the minimum amplitude the value of the frequency will be more than 1 cycle/year, and if the oscillatory mode doesn't change, the value will be less than 0.843 cycle/year. Thus, the simplified forecast of the motion of the Earth’s pole with the change of the oscillatory modes consists in a relative increase in the weighting coefficients of the least squares method with a pole rotation interval around the middle position immediately adjacent to the forecast interval, and also in their relative increase by the end of this interval.

In fig. 2 there’re SD of the forecast of the trajectory of the pole for 45 days compared to SD of the forecasts of IERS, published in Bulletin A [7]. The standard deviations (fig. 3) were attributed to the date of the forecast, calculated in steps equal to day. Average values of SD of the forecasts of IERS and developed model are 12 и 10 angular millisecond respectively.

Thus, the developed numerical-analytical model of oscillations of the Earth pole makes it possible to improve the accuracy of the prediction of its trajectory with irregular effects caused by the variability of the amplitudes of the fundamental harmonics of the oscillatory process.

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