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*Journal of Advanced Concrete Technology,* volume 14 (2016), pp. 444-463

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Scale-Dependent ASR Expansion of Concrete and Its Prediction coupled with Silica Gel Generation and Migration

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Received 2 June 2016, accepted 9 August 2016 doi:10.3151/jact.14.444

Abstract

This study set out to investigate the effect of gel migration through micro-pores and cracks on concrete expansion caused by alkali–silica reaction (ASR). First, sensitivity analyses with permeation of produced silica gel were conducted by using a computational scheme of multi-scale poro-mechanics. Then, silica gel movements were found to substantially affect the ASR expansion of concrete with the possibility of its scale effect being rooted in the drugging force when the silica gel is in motion. With accelerated ASR tests of mortar, the predicted scale effects were experimentally verified and the permeation of gel was inversely identified in terms of multi-scale mechanics. Here, the effect of alkali ion leaching on the scale-dependency was also discussed in view of chemo-physics as well as mechanics of produced gel migration. The measured characteristic scale-dependency in the experiment can be also simulated by nonlinear analysis of smeared cracks coupled with gel generation, migration and alkali ion leaching.

1. Introduction

The alkali silica reaction (ASR) is one of the major mechanisms leading to the deterioration of concrete composites and many researches have been conducted to predict and suppress the expansion (e.g. Kawabata et al. 2013; Kawabata and Yamada 2015; Sannoh and Torii 2014; Costa et al. 2014). Its modeling has been also studied by leading researchers (Saouma 2014; Saouma et al. 2015; Mutlon et al. 2009; Bažant and Steffens 2000; Bangert et al. 2004; Liaudat et al. 2015; Charpin and Ehrlacher 2014). Mutlon et al. (2006, 2009) conducted an experiment to detect anisotropic expansion under three-dimensional non-uniform confinement and are also developing an analytical model to consider such an anisotropic expansion of complexity with ASR.

Saouma (2014) also developed FEM models for ASR, taking the micro-chemical reactions into account, while attempting to apply models to structural levels. Furthermore, some practical methods for simulating the performance of ASR-deteriorated structural concrete are being investigated as a means of overcoming the complexity of the behavior of ASR products in the micro pores and cracks of solid concrete. Particularly in the case of multi-directionally reinforced concrete, a kinematics-based approach is assumed to simulate the internal pressure and resultant expansion due to ASR.

Meanwhile, the authors are developing a multi-scale chemo-hygral computational system (DuCOM-COM3, Maekawa et al. 2008; Maekawa and Ishida 2002), which is summarized in Fig. 1 and conducts a three-dimensional multi-scale analysis of structural concrete and also can consider recently the multi-ionic equilibrium (Elakneswaran and Ishida 2013, 2014). Among them, potassium and sodium ions in pore solution can be linked with the reaction modeling of silica gels (see Appendix).

Given the above-mentioned background, the authors set out to develop a model of ASR and the mechanistic actions that accompany multi-directional cracking (Maekawa and Fujiyama 2013; Takahashi et al. 2014), based on the poro-mechanical scheme of the solid-liquid two-phase interaction model (Biot 1963) and non-orthogonal crack-to-crack interaction modeling (Maekawa and Fukuura 2014; Maekawa et al. 2003).

In the present study, the authors focus on the ASR expansion of concrete solid with different sizes. There is no doubt that ASR is a serious issue of damages for massive concrete with light reinforcement such as gravity dams, footings and abutments of bridges. Since these structures’ scale is so large, we may assume that the produced gel’s migration of macro-scale is negligible and the provoked internal gel pressure will be solely dependent on the volume expansion of concrete composites and the gel volumes created.

However, ASR cracking and damages of reinforced concrete bridge decks, whose sizes are much smaller than those of dams and abutments and whose confinement by reinforcing bars is comparatively high, have been reported in recent years (Nakazawa and Tanaka 2013; Goto et al. 2014; Maeshima et al. 2014). Then, there is a possibility that the produced silica gels may...

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not stay at the original position where it was created and the resultant gel pressure may be reduced compared to the massive concrete cases. On this line, the authors direct their attention to the possible scale-dependent ASR expansion of concrete block without external confinement in consideration of the thin-wall or slab members.

Regarding the scale-dependent ASR expansion, the effect of alkali ion leaching has been comprehensively investigated both experimentally and analytically by Multon et al. (2016) with systematically arranged experiments (Lindgård et al. 2013). When concrete is exposed to massive pure water, sodium and potassium alkali ions which cause silica gel formation flow out through the surface of concrete blocks. As a result, less formation of silica gels develops inside concrete. Then, on the scale-dependency of ASR expansion, alkali leaching is a factor associated with the “cause of reaction” and thought to have less to do with stress field.

On the contrary, the migration of ASR gels is a factor associated with the “result of reaction” and its expansion and migration are supposed to have much to do with stress fields. As a matter of fact, the stress induced strong anisotropy of ASR expansion has been reported in the past. It is certain that both factors are related to the scale dependency of ASR expansion.

In this paper, the authors mainly focus on the latter factor of the scale effect. Then, the experiment was conducted so that the leaching effect of alkali ions becomes minimal. However, it is impossible to perfectly avoid leaching of alkali ions from the surface of concrete under wet conditions. Then in this paper, a series of sensitivity analysis coupled with ion leaching and gel migration is reported to end with discussion that both factors are of importance to cover the wide range of engineering application.

2. Consideration of ASR-gel migration

(1) Models of gel migration

The authors are developing an ASR-induced expansion model based on multi-scale poro-mechanics (Takahashi et al. 2015a, 2015b), which is summarized in Appendix (Takahashi et al. 2014, 2015b; Maekawa et al. 2015). The created total volume of ASR products is computed by numerically integrating the rate of alkali silica reaction, which is the function of sodium and potassium ions’ concentration in pore solution, water content, relative humidity and the factor related to specific surface area and the intrinsic mineral reactivity of aggregates. The created volume of silica gels is divided into two parts. One is the liquefied phase of amorphous which can exude the capillary pores and migrate through cracks like a viscous matrix agent without shear stiffness, and another one is the solid phase of the amorphous and the recrystallized solidified gel which is modeled to firmly adhere to the concrete solid.

If the former phase of silica gels would fill up the void and crack gaps, the hydrostatic pressure is assumed to develop based upon the constitutive model of solid concrete. Here, the volumetric expansion of this phase is built inside the constitutive model of the gel. This pressure rise has to be equilibrated with stresses in concrete solid. Then, the volume expansion and cracking can be introduced in concrete composite system. In general, these stress fields are not uniform and the gradient of
gel pressure is simultaneously produced according to the boundary conditions of concrete structures and their shapes and dimensions by solving the structural mechanics frame.

This pressure gradient works as the driving force of macro-scale migration of the amorphous liquefied silica gels every place to place, and this driving force becomes a source of anisotropic expansion of concrete solid, especially when some external confinement and/or reinforcement would be arranged in 3D extent. Thus, mechanically speaking, the absolute pressure of the silica gel is the source of concrete expansion and the gradient of the pressure concerned is the cause of anisotropy of expanding concrete. The latter solidified phase of the amorphous and the crystallization also creates the effective stresses with the same manner as that explained for the liquefied phase, but the migration is not allowed in this phase. The latter phase of silica gels corresponds to the recrystallized substances with calcium ion in pore solution of concrete (Katayama 2012).

The coupling of liquid-solid two phases of ASR products is simply modeled with the parameter $\beta$, which is the ratio of solid phase volume to the total ASR-product as explained in Fig. 24a of Appendix and mathematically formulated by Eq. (j-k). As a matter of fact, ASR produced substances exhibit mechanical characteristics of both liquid (expressed by no shear stiffness but viscosity) and solid (no viscosity but frictional shear stiffness). The value of $\beta$ for any sol-gel substance ranges between 0 (=perfect liquid) and 1 (=perfect solid). This liquid-solid “split ratio” of $\beta$ is generally time-dependent in nature, because the phase shift of sol-gel takes time in general. Then, at the early ages, the parameter is thought to be nearby zero and to rise gradually according to aging.

It must be also noted that the recrystallization of ASR products proceeds with calcium ion in pore solutions (e.g. Katayama 2012). This chemical process may lead to the increase in the value of $\beta$, which also means the increased volume for the aging material. The recrystallized process of ASR gel injected in micro-pores and voids inside aggregates, whose phenomena are formulated in this model (see Eq. (h) in Fig. 24a of Appendix), may also have to do with the time-dependent increase in the parameter $\beta$. When the parameter $\beta$ changes from time to time, the change of gel’s volume, which was injected into crack spaces (computed by poro-mechanics formulae of Fig. 24b in Appendix), automatically lead to varying gel pressure as well, and the stress fields of solid concrete with crack damages are overlaid. Thus, the volume changes of sol-gel substances existing in crack gaps, capillary pores and inside aggregates are mechanically taken into account in the scheme of solid concrete-cracks-micro-pores and produced/varied ASR products. This parameter also implicitly represents the volume change of CSH gels.

Although the volumetric liquid-solid split ratio of $\beta$ is much associated with some microscopic chemical events and processes, Takahashi et al. (2015a) indicates the method to inversely identify this parameter mechanically from the experiments of ASR expanded concrete under non-uniformity of deviatoric stresses (see Fig. 27 of Appendix), and it was reported to be approximately 0.2 at 100–200 days after the initiation of ASR expansion. In other words, this parameter is the agent to represent the micro-events as stated previously. In this paper, the value of $\beta$ is simply set forth to be constant (=0.2), because the period of experiments to discuss the meso-scale migration of ASR products is rather short just after mixing and thought to be nearly constant from the observation of behavioral anisotropy. As a trial test, the parameter $\beta$ was changed linearly from 0.0 to 0.2 over the period of experiment in the computation, and the computed free expansion of ASR concrete was examined not to be affected much. Then, the constant value was decided to be used for simple and clear condition of simulation. The authors understand that this time-dependent “split ratio $\beta$” of the ASR products shall be further discussed as a future topic to cope with the long time range of ASR, and expect that the recrystallization process will be also linked with the time-dependent mechanical model of ASR gels over the long life (see Section (3) of Chapter 4).

The above mentioned mechanism is simultaneously solved with the mass balance requirement, the compatibility condition of deforming fields and the momentum conservation (dynamic equilibrium) as explained in Appendix. In the following paragraphs, the authors add more explanation of modeling highly related to ASR-gel migration which has much to do with scale-effect of analysis domains of concrete composites.

The liquefied amorphous phase of produced silica gels as stated above is further classified into local micro-migrant phase and the meso-scale one. The former sub-phase represents the local micro-scale gel migration from aggregates to adjacent capillary pores staying around the interface-transition zones as the former researchers fairly formulated (Charpin and Ehrlacher 2014; Muranaka and Tanaka 2013). This model may express the injection of the ASR product into voids inside aggregate as well, and mechanically reproduce the retarded volumetric expansion of concrete caused by the silica gel formation. The other is the meso and macro-scale migration through voids and cracks.

This type of gel migration is considered from the viewpoint of gel pressure gradients in the scheme of porous media (Biot 1968). The space-averaged permeability coefficient of ASR gel though cracked concrete is formulated by referring to the water permeation through cracking space described in previous studies (Appendix; Maekawa et al. 2012) as,

$$\kappa_i = \kappa \left[ 1 + \left( \frac{\varepsilon_{\beta} + \varepsilon_{\kappa}}{\alpha} \right)^2 \right]$$  \hspace{1cm} (1)

where, $\kappa_i$ is the average permeability in i-direction of an
element incorporating cracks (m/s), $\kappa'$ is the referential permeability of un-cracked concrete (m/s), $\epsilon_{ij} + \epsilon_{ik}$ means the average strain in the plane orthogonal to the $i$-direction, and $a$ is a material coefficient, $(i,j,k)$ express the axis of each orthogonal Cartesian coordinate.

The value of $(\epsilon_{ij} + \epsilon_{ik})$ represents the crack space in terms of space averaging, while the value of $a$ is identified as $1.0 \times 10^{-4}$ from previous experimental study by Wang et al. (1997). This means that ASR product’s migration is accelerated as a result of crack opening. A value of $1.0 \times 10^{-20}$ (m/s) was applied for the $\kappa'$ parameter in the previous study (Takahashi et al. 2014), which is sufficiently low to indicate that ASR gel hardly moves through cracks comparatively than condensed water. As a matter of fact, there is no testing technique to directly identify the permeability value $\kappa'$ under the constant pressure gradient of the ASR gel. Then, the numerical identification is the sole possible way at this moment to approach the microscopic mechanism of ASR gels based upon the theorem and the associated inverse analysis.

Here, it must be noted with the permeability $\kappa'$ that the micro-scale migration of silica gels related to the size of aggregates and the micro-fracture around aggregates (Hobbs et al. 1979; Zhang et al. 1999; Gao et al. 2013) is not explicitly formulated in the mechanics modeling, but represented by space-averaged parameters of meso-scale such as the smeared crack modeling. Then in this study, the authors try to explain the scale-effect of small and large concrete blocks, and try to identify the meso-scale mobility of silica gels in Eq. (1) on the basis of multi-scale poro-mechanics to allow the interaction of movable pore media and the solid matrix, which is the cracked concrete with micro-pores.

Behavioral simulation results by this modeling generally coincide with observed experimental evidences of macro and meso-scales. The possible value of $\kappa'$ parameter is most likely to range between $1.0 \times 10^{-20}$ and $1.0 \times 10^{-17}$ (m/s). In this range, primary gel migration is greatly provoked after cracking, which largely accelerates the migration though crack planes as formulated in Eq. (1). It was actually reported from micro-mechanical view point that the recrystallization of the gels is indispensable to initiate ASR-gel pressure for expansion and the gel migration could occur after micro-damage of aggregates (Rajabipour et al. 2015; Ichikawa and Miura 2007; Boukali et al. 2015). In the scheme of modeling, this micro-chemo-mechanical event is represented by the averaged volume expansion ratio and the liquid-solid parameter of $\beta$ as explained in the previous paragraphs together with the phase change of sol-gels. Although this is time-dependent in nature, the authors apply the averaged value over the time domain of the experimental program, and it was inversely obtained by conducting the anisotropic confinement tests (Takahashi et al. 2015a; Appendix).

(2) Non-uniform cracks in uniaxial restraint test

To determine the effect of gel migration, first, the authors drew on the results of a previous study by Takahashi et al. (2015a). In that study, heavily uniaxial restraint experiments were performed to examine the anisotropy of ASR expansion. ASR expansions were tested under several magnitudes of confinement and highly anisotropic behaviors were observed. The experiment is summarized in Appendix where the deformational results are shown in Fig. 27. Subsequently, it was shown that the changes in the length of the specimens could be successfully predicted by using the above-mentioned poromechanical approach which allows the ASR product in motion though cracks and micro-pores.

Here, the critical point of discussion in this paper is the transverse directional strain and cracking. When the smaller longitudinal restraint is given to the ASR concrete, the transverse strain is increased. But, when much higher compressive restraint is set forth, the transverse strain is tremendously reduced. This complexity cannot be explained only by the single-phase constitutive model of concrete continuum. Although it was not covered by the paper (Takahashi et al. 2015a), the crack patterns arising during the expansion tests were also recorded. In the present study, the uniaxial restraint case is first examined and discussed. An image of the specimen is shown in Fig. 2. The test conditions were as described in the previous study (Takahashi et al. 2015a).

The crack patterns on four faces of 112 days after casting are shown in Fig. 3. We can see that the cracks exhibit a locality. The main large cracks are found in the longitudinal direction, while the other cracks are localized around the center of each surface. Here, it must be noted that the transverse direction is under compressive stress state due to the confinement by steel plates at-
tached at the end edges of the specimen, but that the cracking is introduced in the transverse direction. As stated previously, if this tensile fracture under total compressive stress field is allowed by the constitutive law of a “single continuum”, it is the violation of thermodynamics (Chen and Saleeb 1982). This anisotropy shall be treated as a micro structural problem and the scale effect must be also explained within the scope of this scheme.

(3) Trial simulations with/without gel migration
This anisotropic behavior (see Appendix) was simulated with the multi-scale mechanics model. The conditions of the simulations were the same as those of the previous study (Takahashi et al. 2015a), but considered are two cases with different gel permeability coefficients, $\kappa^*$, to study the sensitivity of the gel migration model; one value is $1.0 \times 10^{-21}$ (m/s), which means that there is substantially no gel migration, while the other is $1.0 \times 10^{-14}$ (m/s), in which case the gel can migrate to some extent when we consider the time scale of several weeks.

Figure 4 shows the principal strain distributions obtained as a result of the above two simulations. From Fig. 4, a clear difference can be seen with and without gel migration. The larger principal strains are observed in the center of a surface in the gel migration case (Fig. 4b), which indicates that the crack locality is in the middle of specimen, while this strain distribution cannot be observed in the case of no gel migration (Fig. 4a), which means that the cracks are rather uniform.

Realistic expansion behavior can be obtained by considering the gel migration. From these simulations, the process as shown in Fig. 5 can be found and suggested. With the uniaxial restraint of expansion in the longitudinal direction, an anisotropic stress field is created. The anisotropy of the stress can cause not only anisotropic expansion of the whole specimen, but also local gel movement in the transverse direction around the center part of the specimen. The local gel movement can lead to the locality of the cracks, after which the principal strain develops as shown in Fig. 4b. This locality cannot be expressed without considering the silica gel migration. Herein, the gel migration seems to play a substantial role in ASR expansion of small or middle scale structural concrete of complexity.

Anisotropy in expansion is treated as a material property for large concrete volume but it appears that the anisotropy should be treated as a structural behavior, even in a laboratory-scale smaller specimen in which the local gel pressure is not uniform. There might be an analogy to heat hydration of cement and the resultant thermal expansion of concrete block (discussed in Chapter 3 again). If concrete is infinitely small, the heat energy is easily conducted and released to the external environment. Then, the thermal expansion becomes nil due to considerably small temperature rise. But, if it is of greater size, heat energy liberation hardly occurs and the temperature rises adiabatically and results in higher thermal expansion. Both heat energy and silica gel can be in motion under the gradient of temperature as well as the gel pore pressure. In the next session, the effect of gel migration on ASR expansion is further studied quantitatively through analysis and experiment.

3. Sensitivity analyses with gel permeability
To investigate the effect of gel migration on ASR expansion of concrete, sensitivity analyses were conducted. Referring to the results of an experiment carried out as part of a previous study (Muranaka and Tanaka 2013), free expansion of the prism specimen was simulated. The specimen size was 100 × 100 × 400 mm and the mix proportion was as listed in Table 1. After sealing for one day, the specimen was placed in an environment in which the temperature is 40°C and the relative humidity (RH) is 100%. As a result, the expansion as shown in Fig. 6 was observed. In the simulations, the same dimension, mix proportion and environmental condition were set with the analytical mesh as shown in Fig. 7.

Figure 8 shows the results of the sensitivity analyses while varying the permeability coefficient parameter denoted by $\kappa^*$ in Eq. (1). Here, in order to identify the reactivity of aggregate used, the coefficient of the
chemical reaction rate with sodium and potassium ions (denoted by $k$ in Eq. (a) of Appendix) is set to $1 \times 10^{-8}$ based on a pre-analysis of inverse calculation. The result shows that the expansion progress changes with the permeability coefficient even though the same ASR chemical reaction and the same resultant volume of the produced gels are assumed. The initial ASR expansion is almost the same, because this stage corresponds to the liquefied movable gel injection into the capillary pores and this micro-migrant phase is hardly related to the permeability which governs the meso-scale migration of the liquefied amorphous silica gel phase as discussed in Chapter 2.

Furthermore, there is a large gap between the expansion arising with a permeability coefficient of $1.0 \times 10^{-19}$ (m/s) and that with a coefficient of $1.0 \times 10^{-18}$ (m/s). The results of simulations show that the gel movement through the referential volume of cracked concrete has a major effect on the ASR expansion progress as explained in Chapter 2. Here, it must be noted again that the computed expansion of the specimen is space-averaged and the local expansion of each element is not uniform, because the specimen behavior is rather structural one.

The smaller expansion may be caused by the exuding of the ASR gel from the surface of the specimen as explained in Chapter 2. The gel pressure rises inside the specimen while the surface remains at atmospheric pressure. ASR gel is thus easily exuded from the surface because the distance between the surface and the core center is less and this leads to the gel pressure effectively dropping in the specimen. In this specific case, a smaller permeability coefficient, such as $1.0 \times 10^{-20}$ (m/s) might be an appropriate value for expressing the experimental trends. In the case of $1.0 \times 10^{-18}$ (m/s) in Fig. 8, the slight recovery of the expansion can be seen. This is attributed to the balance of the chemical reaction and the mechanics of gel leaching that the rate of producing the gel becomes less than the rate of the silica gel migration. Then, when the permeability would be infinitely large (no migration), the former factor is always larger than the latter one. Then, no recovery of expansion takes place logically.

From the computed internal gel pressures, the gel migration behaviors can be inversely studied. By Darcy’s law applied to the analysis, the possible length of the ASR gel migration is simply calculated by Eq. (2) as,

$$L(t) = \int_0^t \! v \, dt = \int_0^t \! \left( \frac{\kappa \cdot (p / \rho)}{\tau} \right) \, dt$$

where, $L(t)$ is the possible total length of gel migration over time $t$, $v$ is the gel velocity through cracks, $p$ is the

Table 1 Mix proportions for sensitivity analyses (Muranaka and Tanaka 2013).

| W/C (%) | W (kg/m³) | C (kg/m³) | S (non-reactive) (kg/m³) | G (reactive andesite) (kg/m³) | Na₂Oeq (kg/m³) |
|---------|-----------|-----------|--------------------------|------------------------------|---------------|
| 45      | 169       | 376       | 775                      | 925                          | 8.0           |

Fig. 6 Measured expansion (Muranaka and Tanaka 2013).

Fig. 7 Finite element mesh for sensitivity analyses.

Fig. 8 Free expansion with different gel permeability.

Fig. 9 Calculated possible distance of ASR migration through cracks.
gel pressure at the center of specimen, $l$ is the mean distance from the center to the surface (≈ 50 mm in this case), $r_c$ is the volume ratio occupied by cracks in calculated meshes. Here, it is assumed that the pressure gradient is almost uniform, starting from the surface, to simplify the calculations, although there will most likely be a pressure distribution through the specimen.

Figure 9 shows the calculated estimated lengths of gel migration for different permeability coefficients. As the permeability increases, the gel migration length does too. It can be shown that the gel can move by several tens of millimeters over several hundred days. When we focus on the permeability of $1.0 \times 10^{-19}$ (m/s) and $1.0 \times 10^{-18}$ (m/s), we can see a large gap between the distances resulting from these two values, which is similar to the expansion as shown in Fig. 8. For the case of $1.0 \times 10^{-18}$ (m/s), the amount of movement reaches several centimeters prior to 100 days, meaning that the ASR gel is easily exuded from the surface of the specimen measuring $100 \times 100 \times 400$ mm. The relationship between the size of the specimen and the amount of movement of the gel seems to be an important factor in these expansion simulations, too.

The discussion based on the simulated results reveals the possibility of size dependency in ASR averaged expansion of concrete block. If the ASR expansion is affected by the gel being exuded from the surface, then the size of the specimen is the dominant factor affecting the expansion, which is similar to the heat release and thermal expansion as stated in the previous chapter. A schematic image of the size dependency is shown in Fig. 10. In the smaller specimens, the gel easily reaches the specimen surface under the gel pressure gradient, while the driving force of the expansion is also reduced. As an analogy, heat is also easily released from a small specimen in the case of thermal expansion.

These discussions mean that different scale’s specimens exhibit different degrees of space-averaged expansion, which can be said to be a result of the scale effect of the ASR expansion. In addition, if any size dependency is detected in the experiments, we may be able to estimate the permeability. If no size dependency is observed, then the permeability has no effect on the ASR expansion and the way of modeling adopted in this study, that is, the poro-mechanical approach, would not be suitable for the prediction of ASR. We need some conclusion of the model strategy especially for reinforced concrete of small/middle scales. This is the motivation background of this study in view of the durability mechanics.

4. Free expansion experiment with specimens of different sizes

(1) Experimental program

In the previous chapter, the scale effect of ASR expansion of concrete solids was pre-discussed. Based on this concept, an experimental study was conducted. Mortar cube specimens of different sizes were fabricated and their averaged expansions were measured under the AS accelerated reaction. In order to avoid comparatively small-scale gel motion around aggregates as mentioned in Chapter 1 and to produce uniform field of deformation as much as possible, coarse aggregates were not used in considering the purpose of this study.

From the results of sensitivity analyses described in the previous chapter, it would appear that, as a material age, the silica gel can migrate by up to several centimeters over several tens of days. Thus, the authors arranged four sizes of mortar, namely, 1 cm, 2 cm, 4 cm, and 10 cm, as the test series. Table 2 lists the mix proportions of the concrete specimens. The fine aggregate is the reactive andesite, which was prepared by crushing the coarse aggregate used in previous studies (Muranaka and Tanaka 2013), whose experiment is shown in Fig. 6. Three specimens of each size were targeted. After six

| W/C (%) | W (kg/m³) | C (kg/m³) | S (reactive andesite) (kg/m³) | Na₂Oeq (kg/m³) |
|---------|-----------|-----------|-------------------------------|---------------|
| 50      | 296       | 591       | 1331                          | 11.61         |

Table 2 Mix proportions of mortar for experiment.

Fig. 10 Scale dependency in ASR expansion and thermal expansion.
days of wet sealed curing, the specimens with thin wet gauzes to protect the surfaces were entirely and tightly wrapped by the closed plastic films and placed in the humid chamber of 95-99% in order to minimize drying as much as possible and alkaline elution from their surfaces so that the same rate of alkaline silica reaction is reproduced regardless of the size of specimens. About 0.035-0.040 g/cm$^2$ of water was initially contained in the wet gauze for surface protection and when the gauze was dried during the measurement of about 10min including the preparatory processes, additional water drop about 0.010-0.020 g/cm$^2$ was supplied to keep it wet. The gauze was replaced with the new one of every 7 weeks and 4-6 times of the experimental program.

The wrapped specimens were placed at an elevated temperature of 40°C and RH 95-99% to accelerate the reaction. During the acceleration, no moisture and alkali movement into and out (leaching) from the wrapped specimens are allowed. Figure 11 shows the specimens of different sizes, while Fig. 12 shows the specimen curing during the ASR-acceleration period. The weight change over the experimental program was monitored to indicate how fairly the specimens were sealed on vapor and ions, and was controlled within 0.1% of the accuracy limit of the measurement device. The alkali ion leaching to the 1mm thin gauze is expected to be negligibly small, because the leached ion was tried to remain inside the thin gauze where the alkali ion concentration must rise. It should be mentioned that neither dew condensation nor dripping loss of dropped water took place at the measurement of deformation because of the entire sealing of the whole specimens against vapor. As ion leaching through the wrapping sheet replaced at the maintenance was not directly measured in the experiment, the authors will conduct the trial analysis to show an extreme case where concrete would be exposed to perfectly pure water in assessing the analytical results and showing that the ion leaching has little influence on the point of discussion in the later Chapter 5.

Given that some of the specimens used in this experiment were smaller than the standardized test specimens and the environmental conditions were quite humid, no commonly used existing methods were applicable to the measurement of the averaged expansion of these concrete blocks. In this study, an image-scanning method was adopted to measure the block expansion in progress. The equipment used for the measurements was a 3D shape-measuring machine (Fig. 13). Two feature points on certain surfaces were predesignated and the distance between those two points was measured throughout the test period (Fig. 14). The minimum error of the measuring instrument was around 2 μm. It takes just 1-2 minutes for each measurement.

In this study, two surfaces of each specimen were selected for continuous measurements. There were three specimens, so a total of six lengths were measured for each specimen size. The measurements were conducted three times on each surface, with the average value of the three measurements being taken as the final value. To eliminate the effect of thermal expansion and shrinkage, the specimens were held at 20°C for 2.5 hours for cooling by keeping the entire sheet wrapping before measurement of the size. During measurement including preparatory works, specimens were inevitably exposed to a room ambient condition as shown in Fig. 13. Then, the short-term drying and drift temperature change of the small ranges may not be avoided even though the measurement took about 20min. With the preliminary test, the length change during the measurement was controlled within 50-100μ. It is within the range of limit accuracy for the measuring device and within 10% of the overall volumetric deformation. This is also within the reproducibility of the experiment as shown in Fig. 15. The base lengths were measured after the material had aged for one day and the calculated expansion strain was compared for the different specimen sizes.
To ensure the accuracy of the measurements obtained by the image-scanning method, the 10-cm specimen was also subjected to measurements with a mechanical contact gage. Note that 10 cm is the minimum size which can be measured by using the contact gage method.

(2) Experimental results

Figure 15 shows the free expansions measured for the mortar specimens. The averaged values for the six surfaces of each size are shown in the figure and error range in the figure shows the standard deviation of each measurement. The solid lines represent the results obtained with the image-scanning method, while the dotted lines represent the results obtained with the contact gage method. By comparing the results of the measurements obtained for the 10-cm cube specimens with the two measuring methods, the experimental trends are almost the same. The expansions measured with the image scanning method were shown to be sufficiently reliable. The progress of the expansion in this experiment was generally found to be faster than the result as shown in Fig. 6, despite the same andesite being used in both experiments. This difference may result from the different reaction rates due to the specimens having different aggregate sizes, specific surface areas and initial alkali contents.

Here, there is a clear difference in the expansion amounts depending on the sizes of the specimens. The expansion observed with the 10-cm specimens is the largest, while that observed for the 4-cm specimen is the second largest among all four sizes. The expansions of the 1-cm and 2-cm specimens are obviously smaller than that of the 4-cm specimens. Thus, it may be said at least qualitatively and experimentally that scale effect of ASR expansion of concrete block holds even if the deviation of measurement around the mean value is taken into account.

Figure 16 shows the crack appearance on the specimens after experiment. Visible cracks larger than 0.1mm are drawn in the figure. Multi-directional visible cracks appeared on bigger specimens, while it is quite rare in smaller specimens. The difference depends on the specimen size is observed here as well. In the experiments, gel exudations on the specimen surfaces were observed regardless of appearance of visible cracks for every size of the specimens, which may affect the degree of expansion. This is numerically simulated as explained in the latter Section (3).

In Fig. 15, the expansion in the 1-cm specimen is slightly larger than that in the 2-cm specimen. The water provided by initially wet gauze sheets might be relatively greater for the smaller specimens and these differences in the expansion could be caused by the difference in internal water state. This difference is not known whether it is within the scatter of reproducibility of experiment or not. Even though it is supposed to be a limitation of this experiment, the size-dependency can be said to exist. This will be again discussed with the computational simulation of perfect sealing in the later section together with the effect of leaching of alkali ion into the thin gauze.

(3) Simulations with the multi-scale poro-mechanical model

The authors attempted to simulate the experimental trends with the calculation models. The expansions in the experiments were simulated and the sensitivity of ASR gel permeation was investigated. The prepared meshes are shown in Fig. 17 in consideration of the space-discretization for coupled mass and momentum conservations (Ohno et al. 2012). As a matter of fact, the gel phase pressure in Eq. (k) of Fig. 24a in Appendix tends to sharply change close to the surfaces where denser finite elements are arranged. The same mix proportions and the environmental conditions as those used in the experiment were set in the simulations.

Figure 18 shows the simulated space-averaged expansions for different permeability coefficients.
alkali silica reaction rate of the specimens (see \( k \) in Appendix) was determined to be \( 2 \times 10^{-8} \) as a result of the sensitivity analyses, which is twice of the value obtained with the simulations in the previous study as shown in Fig. 8. As a matter of fact, the used fine aggregate, which was made by crushing the coarse aggregates of the previous study, has reasonably higher reactivity because of the finer particles’ size with larger amount of specific surface areas.

The different relations of the expansions for different sizes are observed for different permeability coefficients. First, for an extremely low permeability, which is \( 1.0 \times 10^{-23} \) (m/s), all of the sizes exhibit the same large expansions because the internal gels cannot easily migrate through the cracks. Thus, the gels firmly remain around fine aggregates and without any gel migration, no scale effect can be observed and the silica gel volume is fully used for the concrete volumetric expansion. In contrast, all of the sizes exhibit smaller expansions for a permeability of \( 1.0 \times 10^{-17} \) (m/s) because the gels are easily exuded accompanying quick reduction of gel pressure.

Figure 19 shows the exuded gel’s volume normalized by the original concrete volume for each specimen size. The larger the specimen is, the smaller gel volume is released compared to the original volume of concrete blocks, i.e., the leached gel is about 0.8% of concrete volume for smaller size, but for 10cm case, 0.2%. This can be a simple explanation of the scale-effect, too. This computational result qualitatively matches with the experimental cracking patterns as shown in Fig. 16. For the large concrete block, the internal gel pressure grows much and comparatively larger gels remain inside the concrete. Finally, the local strain of cement paste matrix exceeds the tensile rupture limit. In the case of smaller specimens, gel leaching is comparatively large as explained above, and the expansion of cement paste phase

Fig. 18 Simulated size effects of ASR expansion with different gel permeability.
by the inflated gels becomes less. Then, the tensile cracking limit is not violated with any cracking. This is also simulated by the multi-scale modeling as well.

Among the values of permeability, the value between $1.0 \times 10^{-18}$ (m/s) and $1.0 \times 10^{-19}$ (m/s) seems to produce fair scale-effect trends that are quantitatively similar to those observed in the experiments as well. Until around 80 days of the material age, the simulation result with $1.0 \times 10^{-18}$ (m/s) has a fair agreement with the experiment. After 80 days, however, the value of $1.0 \times 10^{-19}$ (m/s), which means the harder permeation of silica gel, rather brings about the better prediction (see Fig. 18 as well).

Here, it should be noted that the kinematics of ASR gels is not stable but varies since the recrystallization of the amorphous liquefied gel with calcium ion proceeds simultaneously, that is to say, with the passage of time, the composition of the produced ASR gel can change, together with its physical characteristics (Katayama 2012). Thus, the viscosity is supposed to decrease as the gel crystallizes over time. This can result in a lower permeability. Current models tentatively regard the gel permeability and other properties as being constants but, in the future, the time dependency of the physical properties of the ASR gels should be considered to further generalize the modeling.

By simply setting an appropriately constant permeability for the poro-mechanical models, we reproduce the characteristic trends in the size dependency of the ASR expansion. Here, it is clearly shown that the gel migration is one of the dominant factors in ASR expansion and should be considered in the simulations of ASR phenomena especially for thin structural members of small or medium dimensions. In addition, from the experiment using several sizes of specimen, we may find an appropriate value for the gel permeability. Thus, the applicability of the poro-mechanical approach was verified with both experimental and analytical studies.

The permeability value inversely derived from the experiment of mortar is around $1.0 \times 10^{-18}$ (m/s) or $1.0 \times 10^{-19}$ (m/s) as stated previously, which is larger than the optimal value shown in the previous chapter for concrete, that is around $1.0 \times 10^{-20}$ (m/s). This difference can be also explained by the time-dependent viscosity of the produced gels. As mentioned previously, the reaction rate of the fine aggregates used in this study is higher than that in the previous one with coarse aggregates (Fig. 6), and the reactivity with sodium and potassium ions in pore solution is about double. Then, the time of convergence of expansion is about 140 days. However, the case of concrete with larger coarse aggregates as shown in Fig. 6 and Fig. 8 takes time of about 300 days. The longer duration of the reaction brings about higher order of gel solidification as a whole.

(4) Broken symmetry in ASR-induced expansion – Check of computational consistency –

Figure 20 shows the internal silica gel pressure distributions in the calculations with the permeability value of $1.0 \times 10^{-18}$ (m/s) which is within the appropriate for the experiment. The figure shows the gel pressure distributions projected in the cross sections at the material age of 70 days after casting. It can be seen that there are clear differences in the remaining pressure in the center part of the specimen in different sizes. As mentioned before in this paper, the larger specimens can hold the expansion pressure even after 70 days of material age, while the gel pressure for expansion almost disappears in the smaller ones. This can be said as the direct reason of the scale effect on the expansion of concrete with ASR.

Slightly non-uniform distribution of the silica gel pressures can be seen in the case of greater specimens (i.e. 10 cm and 4 cm) and furthermore, the symmetry of pressure distribution is a little broken despite of the six plane-symmetry of the finite element meshes. As a matter of fact, the tangential stiffness matrix of the coupled system of solids and pore media as formulated in Appendix exhibits non-symmetry. Then, the anisotropic solution of pressure fields of the reality can be included in nature (Kojic and Bathe 2005) even though the geometry and boundary conditions of the targeted solid are of perfect symmetry.

In order to investigate this broken symmetry, the transitions of gel pressure distributions in 10 cm cube and their principal strain distribution on surfaces are shown in Fig. 21. Here, obviously non-uniformity of the gel pressures is shown.
pressure profile starts to develop when surface cracks happen at around 30 days of the material age. At the very beginning of expansion, broken symmetry of gel pressure appears and it affects to the following analysis and the asymmetric gel pressure distribution remains throughout the calculation.

Once localized crack happens in the process of ASR expansion, gel migration as well as other mass transports can be accelerated. The locality would govern the succeeding reaction and structural mechanics and it may result in the local crack patterns as shown in Fig. 16. This is some sort of bifurcation of the equilibrium solution from the perfect symmetry of pressure field, and nicely coincides with the nature. The broken symmetry of expansion can be also seen in the profile of resultant strain fields as shown in Fig. 21 as well. The magnitude of broken symmetry is not so great. It is attributed to higher viscosity of ASR gels.

From a theoretical viewpoint, the magnitude of this asymmetry must depend on the gel permeability. Figure 22 shows the mean value and the coefficient of variation for the computed expansive strains on the 6 surface planes of the 10cm cube at 70 days with different gel permeability denoted by $\kappa^*$. The coefficient of variation stands for the standard deviation divided by the mean value. As the mean expansion decreases with the greater gel permeability, the larger variation holds computation-ally accompanying the broken symmetry accelerated.

![Fig. 21 Gel pressure distribution transition with time.](image)

![Fig. 22 Asymmetry in surface strain with different gel permeability.](image)
This simulation fits the theoretical consistency. Here, it should be noted that searching of the numerical solution to satisfy the governing equation was conducted under multi-core parallel processing of computation. The multi-core process definitely introduces a very small perturbation of digit to the target solution. Then, it is not necessary to artificially introduce any perturbation in geometry and/or material properties in space for the trigger of bifurcation (broken symmetry).

The computationally obtained coefficient of variation ranges 2% or less as shown in Fig. 22. This is the theoretically possible variation under the perfect uniformity of material properties over the volume. Then, the real variation must be greater than 2% because purely perfect uniformity of material property in 3D extent never exists in the first place, and as a matter of fact, the natural perturbation is the trigger of broken symmetry in reality. As shown in Fig. 15, the variation of expansive strain measured is about 10-20%, which is reasonably larger than the theoretical minimum and may reflect the non-uniformity of material property. The computation-oriented variation of solution can be ignored in discussing the applicability of the modeling.

It is concluded in view of theoretical soundness that the proposed scheme of ASR simulation certainly satisfies the requirement of mechanics to allow the broken symmetry of deformation even though the solution target has the perfect symmetry of geometry. This theoretical consistency is of great importance for the ASR damaged structural analysis for massive concrete, large-scale blocks and so on. In other words, when the ASR expansion would be quite simply modeled just similar to the thermal expansion of materials, the solution of broken symmetry is never detected. If so, it would be a violation of nature.

5. Effect of leaching of alkali ions

Since the silica gel migration is the main focal point of this paper, the authors tried to avoid the impact of alkali ion leaching in experiments as stated in Chapter 1. Then, the exuded ion was forced to remain inside the wet gauze wrap so that the gradient of ion concentration...
is kept mild or hopefully zero. Recently, Yamada et al. (2016) developed the advanced alkali wrapping method to prevent alkali ion leaching by initially including the sodium ion in gauze sheets. It is reported that the mere gauze wrapping cannot perfectly stop leaching, and some difference of ASR expansion was seen with and without alkali wrapping. Although the experiment of this paper is thought to mainly correspond to the alkali silica gel migration apart from the ion leaching (see Chapter 4), it will be also productive to consider the ion leaching with ASR product migration as well on the scale effect under the experimental conditions in this study.

Then, in order to discuss how greatly the leaching of ion and exudation of silica gel influence on the scale effect, the authors conducted the sensitivity analysis of the extreme case where concrete is exposed to perfectly pure condensed water and the maximum alkali ion leaching is expected to develop as shown in Fig. 23a. This is quite opposite case of the experiment in Chapter 4. It is possible to perfectly keep the boundary ion concentration to be zero with no convection of condensed water in the analysis. But in experiments, this idealized boundary is hardly reproduced.

Figures 23b and 23c show the computed ASR expansion of concrete with and without the idealized leaching impact by solving the already verified multi-ion modeling (see Chapter 2, Nakarai et al. 2006 and Takahashi and Ishida 2014). As discussed in Chapter 3, the simulation of larger gel permeability ($\kappa^* = 10^{-18}$) brings about better correlation at the early age when amorphous phase of silica gel is thought be dominant like viscous liquid. On the contrary, crystallization proceeds with calcium in pore solution at the older ages, and the silica gel’s motion is thought to be hard enough to correspond to the smaller permeability in the analysis as shown in Fig. 23c with $\kappa^* = 10^{-19}$.

Here, the impact of alkali leaching can be seen on the expansion of 1.0 and 2.0 cm small concrete specimens, but comparatively smaller impact of alkali leaching is indicated for 7 and 10cm cases. Figure 23d shows the specific alkali leaching (total exudation of potassium and sodium ion normalized by the initial alkalinity of the concrete dosage) computed. The rapid leaching develops up to 50% or more for 1cm specimen. If the ASR is computationally terminated, 80-90% of the alkalinity is simulated at 150 days for normal concrete. According to logics, infinitely large specimen shall specify zero specific leaching and in reality of 10cm, it just comes up to 10-20%. It must be noted again that these values correspond to the leaching of comparatively early ages (about 100 days) owing to the accelerated experiments in this paper, and may consistently match the past experiences and analytical studies (Lindgård et al. 2013).
Within the experimental conditions in this study, the effect of leaching to the gauze sheet on the concrete volume expansion is thought not to be large according to the sensitivity analysis with the extreme boundary conditions so far from the reality of the experiment in this paper. For the larger size, the alkali leaching impact is hardly seen. But, if the rate of alkali silica reaction is not so rapid, the leaching impact is thought to become comparatively greater. In the study by Multon and Sellier (2016), alkali ions had already leached before the primary volumetric expansions, while the simulated expansion starts just after the exposure of concrete finite elements to the pure condensed water and at the very beginning of leaching. That’s why scale effects by not alkali leaching but gel migration could be detected both in experiments and simulations of this paper. The scale effect of expansion is also the function of temperature and relative humidity as well.

In consideration of these factors, it can be concluded that ASR product migration cannot be ignored not only in simulating the anisotropy of ASR expansion under deviatoric stress fields with non-uniform confinement (Section (3) in Chapter 2; Takahashi et al. 2015a), but also in simulating the scale effect of ASR expansion, especially when the rate of reaction is comparatively rapid than the rate of ion leaching. At the same time, it is of importance to consider the ion leaching as well especially when the rate of AS reaction is comparatively slower than the rate of ion leaching, and when the long-term expansion is targeted, too.

6. Conclusions

In this study, the effect of gel migration on ASR expansion was investigated. Trials and sensitivity analyses regarding the permeability coefficient were conducted and the possibility of the scale effect of the ASR expansion due to the exuding of silica gels from the surface was identified in the results of the simulations. An experiment addressing ASR-induced expansion with mortar specimens of different sizes was conducted to confirm the expected size dependency. The expansions were measured with an image-scanning method and the expected dependency of the differences in the expansions on size was observed in the experiments.

The measured trends can be reproduced by setting an appropriate permeability for the models. The validity of the poro-mechanical approach for ASR phenomena was computationally confirmed in this study. As for the scale-dependency of expansion, the leaching of alkali ions in pore solution is also the critical factor of chemical reaction aspect of isotropy as well as the produced gel migration of mechanical aspect of anisotropy. Thus, these aspects and their coupling are to be further discussed in future development.

Acknowledgements

The authors express their deepest appreciation to Prof. K. Torii of Kanazawa University and Prof. I. Iwaki of Nihon University for their valuable advices and suggestions. This study was financially supported by JSPS KAKENHI Grant No. 15H05531 and Cross-ministerial Strategic Innovation Promotion Program (SIP).

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Appendix
ASR-Cracked Concrete interacting model and
experimental verification
The ASR-induced expansion model used in this study is
formulated based on multi-scale poro-mechanics by
Takahashi et al. (2014, 2015b), which has been also
used widely in geotechnical engineering applications
such as the consolidation and liquefaction of soil foun-
dations (Biot 1963; Okhovat et al. 2009). In the field of
structural concrete mechanics, the multi-scale mechan-
ics has been applied to fatigue life assessment of rein-
forced concrete members with stagnant water (Maekawa
et al. 2015). Here, the ASR gel is treated as a medium
filling cracks and micro-voids and migrating over the
volume through voids and cracks. The point of formul-
ation is to combine the kinematics of pore media with the
solid skeleton deformation and fracture.
Figure 24a shows the computational constitutive ma-
terial’s modeling of ASR gel generation and its con-
tribution to expansion. Based on the chemical equations

\[
\begin{align*}
\Delta G_{\text{ASR}} &= R_{\text{ASR}} - \Delta t \cdot M_{\text{gel}} \quad \text{[Gel volume]} \\
C_{\text{ASR},0} &= 2.0 \cdot \Delta G_{\text{ASR}} \cdot \rho_{\text{gel}} \cdot 1.0E+6 / M_{\text{gel}} \quad \text{(f)} \\
W_{\text{ASR},0} &= 8.4 \cdot \Delta G_{\text{ASR}} \cdot \rho_{\text{gel}} \cdot 1.0E+6 / M_{\text{gel}} \quad \text{(g)} \\
& \quad \text{[d:Time step interval], } M_{\text{gel}}: \text{Molecular mass of ASR gel} \\
& \quad \rho_{\text{gel}}: \text{Density of ASR gel[kg/m}^3]\end{align*}
\]

Fig. 24a Overall calculation scheme for ASR gel generations and stress formations (Takahashi et al. 2014, 2015b).
for ASR, the rate of ASR is formulated as a function of the alkali concentration, updated free water in the unit referential volume, and the reactive aggregate’s content, as expressed by Eq. (a). The authors intended to apply for the logical and simplest linear formulae in terms of these factors related to ASR. As a matter of fact, provided that the alkali-ion and moisture would be lost inside the unit referential volume, the rate of reaction is to be computationally terminated as they do in reality. Then, the content of alkalinity in the unit referential volume implies the path-dependency of the reaction.

The control coefficient of the alkali silica reaction rate denoted by $k$ in Eq. (a) is arranged to identify the characteristics of each aggregate with different minerals through inverse sensitivity analyses, because the reactivity can change greatly with the aggregate type as well as the aggregate size (or specific surface area). Thus, this parameter represents the reactivity of aggregate phase. The effects of the relative humidity (RH) is formulated in Eq. (b) based upon the experimental facts that the ASR almost terminates below 80% of the relative humidity. The temperature dependency is also taken into account in Eq. (c) based upon the conventional Arrhenius principle. This formula indicates that the reaction is descending according to drying specified by RH and is accelerated under elevated temperature. These formulae qualitatively match the reality as well.

The generated ASR gel volume is calculated by using the volume-compatibility Eq. (e) with reference to $X_2Si_2O_5(H_2O)_{8.4}$ as the ASR gel molecular formula for each alkali, $X$ (= Na or K) and 1.6 kg/m$^3$ as the ASR gel molecular formula for the volume-compatibility Eq. (e) with reference to $\rho_{gel}$. This linear formulae linked with Eq. (a) are obvious in consideration of the mass balance and the definition of time interval denoted by $\Delta t$. The consumed alkali and water are also calculated in terms of the mass conservation. The mass change rate expressed by Eq. (f) and Eq. (g) is included in the sink term of the discretized mass conservation and computationally solved step by step (Maekawa et al. 2008).

Central to the theorem is that the water and alkali contents of the pore solution, which are control parameters for computing the reaction rate by Eq. (d), are set as global variables for both the thermodynamic analytical system (DuCOM; Maekawa et al. 2008) and the 3D meso-scale structural analytical system (COM3; Maekawa et al. 2003). These global schemes are to search nonlinear finite element solutions to satisfy the equilibrium of mass and the momentum conservation.

In the scheme of DuCOM, the multi-ionic approach developed by Elakneswaran and Ishida (2013, 2014) is also built-in to compute the mass balances of sodium and potassium in considering the ion-mutual interaction of relative solubility. Based on this composition of mass and momentum conservations, strong coupling between the material properties and the mechanical phenomena is realized in the simulation of the life of structural concrete.

On the basis of generated gel volume, the stress formation can be automatically calculated by simultaneously solving the stress equilibrium and the deformation compatibility similar to the thermal stress analysis of structures. Here, some parts of the created gels can contribute to stress formation, but the other parts do not. The silica gel is partially absorbed into the capillary pores (Muranaka and Tanaka 2013) and the amount of absorbed gel can be calculated by using Kelvin’s Eq. (h), which is a function of the gel pressure and the surface tension of the pore solution. The resultant remaining gel may substantially contribute to the stress formation. Regarding the gel-oriented internal pressure, the authors consider the solid-liquid characteristics of ASR gels.

To express the state in which a solid and liquid coexist, parameter $\beta$ is introduced, to indicate the ratio of the solidified phase volume to those of the total ASR gel. Under certain updated stress states, the solidified part of the produced ASR gel can expand around aggregate particles uniformly even under an anisotropic pressure distribution and creates the partial stresses given by Eq. (j), in which $V_{sw}/3$ indicates the term of free solid expansion strain in all directions and $V_{crack,i}$ is the smeared crack strain in $i$-th direction of the Cartesian coordinate. Then, the effective stress component is associated with the deviation of the two strain components.

The liquefied part of gels expands without shear rigidity under the isotropic pressure expressed by Eq. (k), in which $\left(\ V_{sw} - \Sigma V_{crack,i}\right)$ means the deviation of the ASR volume expansion and the increased volume of concrete solid skeleton. Then, this deviation may create the hydro-static pressure. This hydro-static stress formulation is exactly the same as that of the pore water pressure developing inside the soil foundation. According to the Biot’s concept (1963) of the effective stresses, the total pressure can be computed as the simple sum of those partial pressure components as given by Eq. (i).

The constitutive modeling equations shown in Fig. 24a are installed into the set of global equilibrium equations lined up in Fig. 24b. The equilibrium conditions can be written in terms of momentum conservation equations. Equation (m) expresses the averaged equilibrium condition of combined solid skeleton and ASR gels staying in pore spaces including capillary pores and crack spaces. Then, the dragging force does not explicitly appear, and the cracked concrete constitutive model (Maekawa et al. 2003) is substituted into the computation of stresses $\sigma_y$ from the assumed strain fields.

Equation (o) formulates the deviatoric conditions of equilibriums of the concrete solid and ASR gel phases. In other words, this is the ASR gel’s equation of motion which can be fictitiously seen from the concrete skeleton. Then, the dragging force denoted by $1/k \cdot w$ appears in Eq. (o), where $w$ and $w$ are the relative displacement and speed of ASR gel phase measured from the mother concrete. This dragging force has to be equilibrated with the pressure gradient as written in Eq. (1) and the pressure is linked with the creation of gels of
larger volume than those of aggregates. Parameter $\beta$ is tentatively assumed to be 0.2 for the first assumption with a sensitivity analysis. The volumetric stiffness of the ASR gel is tentatively supposed to be the same as that of the condensed water.

On the other hand, ASR-induced expansion tests were conducted in order to investigate the anisotropy of the expansions (Takahashi et al., 2015a). Prism specimens were made in the size of $10\times10\times40$ cm (Fig. 23a) with the aggregate which is reactive with alkali and additional sodium hydroxide was applied to accelerate ASR. The mix proportion is shown in Table 3. The reactive

**The affinity: Porous material (skeleton) with water**

- Soil skeleton: $\rho = (1-n)\rho_c + n\rho_f$
- Pore water: $\rho = (1-n)\rho_c + m\rho_f$
- Total stress:
  - Uncracked: $\sigma_{ij} = \sigma_{ij}^C + \frac{p}{\lambda} I_{ij}$
  - Cracked: $\sigma_{ij} = \sigma_{ij}^C + \frac{p}{\lambda} I_{ij} - \frac{1}{\lambda} p$

**Dynamic equilibrium equations for a saturated skeleton**

1. Equilibrium in direction $x_3$
   - $\sigma_{11,1} + \sigma_{22,2} + \sigma_{33,3} + \rho g = (1-n)\rho u_{3,tt} + n\rho(u_3 + w_3/n)_t$
   - $\sigma_{ij,tt} + \rho g = \rho u_{ij,tt} + \rho w_{ij,tt}$

2. Generalization for each coordinate axis
   - $\sigma_{ij} = \rho(u_{ij,tt} + \rho w_{ij,tt})$

**Dynamic equilibrium equations for fluid materials in the skeleton**

1. Viscosity --- resistant force exerted by contact with skeleton
   - From simple assumption; without any acceleration (static movement)
     - $\rho g = w_{3,tt}$
     - $\rho u_{ij,S} = \frac{w_{ij,tt} - u_{ij}}{\kappa_{ij}}$

2. Dynamic equilibrium
   - $\rho u_{ij,tt} - \frac{1}{\kappa} w_{ij,tt} + \rho g = (u_{ij,tt} + w_{ij,tt}/n)$

Fig. 24b Fluid-skeleton modeling (Maekawa et al., 2015).
aggregates were originally evaluated as harmless. After 28 days sealed curing, specimens were subjected to three levels of restraints. The first is free expansion case, the second is the one with restraint by a steel bar, and the last is the case with 6.2 MPa of additional stress by pre-stressing. After inducing these restraints, specimens were stored inside the ASR-activated environments, i.e., the temperature is kept constant at 40°C and the specimens were wrapped by wet clothes in order to supply enough water which may be consumed through ASR. Under these conditions, strains on the surfaces of the specimens and weight change were measured periodically. By using a contact-type strain gauge with chip targets attached on the specimens, space averaged strain were measured in both longitudinal and vertical directions (Fig. 25).

Figure 26 shows the weight changes of the specimens. The results of 3 specimens are almost the same. It means that the water absorption from wet clothes mostly coincides with each other. Then, the degree of ASR and the corresponding moisture states are thought to be similar among these cases. Figure 27 shows the development of the strains in both longitudinal and transverse directions. The isotropic free expansion case indicates approximately 800μ by volume. Significant anisotropy in expansion was observed in the case with restraint as shown in Fig. 27b. The absolute expansion is smaller than that of the free case in the longitudinal direction, while it is larger than the free case in the vertical direction. It is due to the restraint by steel bar and such a behavior is well-known as anisotropic effect caused by liquidity of ASR gel.

The response of the case with additional compressive stresses (Fig. 27c) differs from the simple restraint case. Not only in the longitudinal direction but also in the vertical one, the expansion is considerably decreased in comparison with other cases. This fact cannot be simply explained only by the change of the ASR-gel movement directions. It appears that some of the generated ASR-gel is not obviously active but firmly fixed on the aggregates. The gel absorption into the pores of concrete

| Table 3 Mix proportion. |
|-------------------------|
| W/C | W (kg/m³) | C (kg/m³) | S (kg/m³) | G (kg/m³) | Na₂O (kg/m³) |
| 55% | 170 | 309 | 853 | 983 | 6.1 |

![Fig. 25 Dimensions of specimens.](image)

![Fig. 26 Weight change of specimens.](image)

![Fig. 27 Measured expansions in each restraint condition.](image)
mixture may lead to less expansion in such a large stress conditions.