Earthquake Depth-Energy Release: Thermomechanical
Implications for Dynamic Plate Theory

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Abstract. Analysis of the global centroid-moment tensor catalog reveals significant regional variations of seismic energy release to 290 km depth. The largest variations, with direction from the baseline indicated using plus and minus signs, and in decreasing order, occur at 14-25 km depths in continental transform (+), oceanic ridge/transform (+), continental rift (+), Himalayan-type (+), island arc-type (-), and Andean-type (-) margins. At 25-37 km depths, variations one-fifth the size occur in continental rift (+), island arc-type (+), Andean-type, Himalayan-type (-), oceanic ridge/transform (-), and continental transform, (-) margins. Below 37-km depth, variations one-tenth the size occur in Andean-type and Himalayan-type margins to depths of about 260 km. Energy release in island arc-type margins closely tracks the baseline to the maximum depth of earthquakes at 699 km. The maximum depth of earthquakes in Andean-type and Himalayan-type margins is 656 and 492 km, respectively, while in divergent and transform margins it is about 50 km. These variations reflect radial and lateral contrasts in thermomechanical competence, consistent with a shear-dominated non-adiabatic boundary layer some 700-km thick, capped by denser oceanic lithosphere as much as 100 km thick, or lighter continental tectosphere 170 to 260 km thick. Thus, isobaric shearing at fractally-distributed depths likely facilitates toroidal plate rotations while minimizing global energy dissipation. Shear localization in the shallow crust occurs as dislocations at finite angles with respect to the shortening direction, with a 30 degree angle being the most likely. Consequently, relatively low-angle (~30°) reverse faults, steep (~60°) normal faults, and triple junctions with orthogonal or hexagonal symmetry are likely to form in regions of crustal shortening, extension, and transverse motion, respectively. Thermomechanical theory also predicts adiabatic conditions in the mantle below about 1000-km depth, consistent with observed variations in bulk sound speed.
1.0 Introduction

A century ago, the solid mechanical properties of rocks factored highly in the development of the isostatic hypothesis, which holds that Earth’s outer layers, the crust and lithosphere, are strong and capable of supporting loads for geological time periods (Watts, 2001). Vertical motions of the lithosphere, in response to progressive sedimentary loading, volcano construction, and the advance of ice sheets, are accommodated by deformation of a subjacent weak asthenosphere (Barrell, 1914). Subsequent removal of loads, due to erosion or ablation, results in the rebound of formerly depressed areas. Thus a dynamic equilibrium between Earth’s shape, topography, and external gravity field is maintained.

In principle, the relative horizontal motions of continents suggested by Wegener (1966) are consistent with the presence of the asthenosphere. Still, the mechanism he proposed, continental ‘rafts’ plowing undeformed through oceanic crustal ‘seas’ under the influence of centrifugal forces, is at odds with the known contrast between stronger mafic oceanic crustal rocks and weaker felsic continental ones. Consequently, wide acceptance of continental drift had to await the development of the sea floor spreading hypothesis (Dietz, 1961; Hess, 1962), and recognition of Benioff (1954) zones as down going extensions of oceanic lithosphere. Thus, the continents appear to be embedded within blocks of mobile lithosphere, the horizontal motion of which is decoupled from the deeper mantle by the asthenosphere, and accommodated by its progressive creation and destruction at oceanic ridges and trenches, respectively (Isacks et al., 1968). Lithosphere is neither created nor destroyed at transform faults, where the relative motion of adjacent blocks is parallel to the fault (Wilson, 1965).

In the late 1960’s, the kinematic theory of plate tectonics (Le Pichon, 1968; McKenzie and Morgan, 1969; McKenzie and Parker, 1967; Morgan, 1968; Wilson, 1965) successfully explained global map patterns of earthquake foci and first motions (Isacks et al., 1968), and systematic variations of sea floor age across active oceanic ridges (Vine and Matthews, 1963), via the relative horizontal motion of rigid spherical caps. Attempts to reconcile this new theory with observed patterns in continental geology followed soon after (Atwater, 1970; Dewey and Bird, 1970), but the rigidity assumption has never sat well with the overall width of, and diffuse nature of deformation in, mountain belts. Other limitations of plate theory include its
inability to account for known differences in the density and strength of crustal rocks between oceans and continents, and the various thickness estimates for the crust (Rudnick and Gao, 2003) and oceanic lithosphere (Parsons and Sclater, 1977), as well as stable cratons and shields (Artemieva, 2009; Artemieva et al., 2004; Artemieva and Mooney, 2001). Moreover, it does not predict the existence of seismicity, but rather uses it to define the boundaries of, and relative motions between, its plate-like blocks. These boundaries apparently act as long-lived zones of weakness in the global plate system. The dynamic basis for plate theory and its relation to motions in the deeper mantle remain poorly understood (Bercovici et al., 2000; McKenzie, 1969; Tackley, 2000).

In order to simulate horizontal (toroidal) rotations of plate-like blocks many variations of the standard Earth model (Bercovici et al., 2000) incorporate ad hoc mechanisms for shear localization. However, because these mechanisms are based on temperature-dependent variations in viscosity, and ideal viscous materials do not manifest shear bands or dislocations, they cannot account for seismicity. This also holds for power law viscous, i.e. Reiner-Rivlin, materials. Although these materials can produce shear bands for exponents 2 ≤ n < ∞, they cannot simultaneously propagate shear waves, and hence are rheological fluids (Patton and Watkinson, 2010). Therefore, a major stumbling block to further progress on the dynamic plate problem is not a lack of data, but rather a deficiency in applied rheological theory.

Since the mid-1960’s, space-based geodesy has provided global gravity and topography datasets with ever increasing quality and resolution. Today, the tracking of satellite orbits constrains the low-frequency harmonics of Earth’s external gravity field, with half-wavelengths (λ₁/₂) greater than about 200 km (spherical harmonic degrees l < 100). Analysis of these data, combined with gravimetric survey and global topography data (Amante and Eakins, 2009), show that Earth’s gravity and topography are correlated (r = 0.6-0.7) for λ₁/₂ < 1000 (l > 20) (Wieczorek, 2007), consistent with isostasy at a regional level (Watts, 2001). It is therefore common to hear tectonic specialists talking about the effective elastic thickness of the crust and lithosphere. The gravity-topography correlation diminishes rapidly for l ≤ 20, which implies that vertical (poloidal) motions in the mantle actively support the longest wavelength gravity and geoid anomalies (McKenzie, 1967).
An outstanding question for dynamic Earth models, and the standard model in particular, is whether long-wavelength gravity anomalies can be associated unequivocally with sub-lithospheric mantle convection (Kaula, 1972; Steinberger et al., 2010) and so-called dynamic topography (Hager, 1984; Panasyuk and Hager, 2000). Here I show that key features of Earth’s gravity-topography correlation and admittance can be explained using a thermomechanical boundary layer hypothesis, which constrains the possible range of wavelengths associated with standard convection to $\lambda_{\text{c}} > 2500$ km ($l < 8$). This limit is far larger than the $\lambda_{\text{c}} > 650$ km ($l < 31$) one identified by Steinberger et al. (2010) based on empirical ‘downward continuation’ of gravity spectra and crustal thickness modeling, and wholly consistent with the scale of robust heterogeneities revealed by seismic tomography (Dziewonski et al., 1977; Dziewonski and Woodward, 1992; Gu et al., 2001).

Establishment of the Global Positioning System (GPS) in the early 1990’s makes it possible to account for ever longer-period neotectonic deformations. These new datasets have been used recently to estimate density-normalized rates of mechanical energy dissipation $\chi$ [m$^2$s$^{-1}$] in plate boundary zones, which fall in the range $4.2 \leq -\log \chi \leq 5.2$, and also in intraplate regions, which fall in the range $5.3 \leq -\log \chi \leq 7.0$ (Patton and Watkinson, 2010). As shown later in the paper, similar estimates of $\chi$ for spontaneous-failures in load hold experiments on the Mt. Scott granite (Katz and Reches, 2002, 2004) fall in the range $2.3 \leq -\log \chi \leq 5.6$. The overlap in these estimates suggests that joint analysis of neotectonic survey and rock mechanics data could constrain natural rates of energy dissipation in the broad deforming plate boundary regions, which have been so problematic for the kinematic theory.

Furthermore, by comparing these mechanical dissipation estimates to measured rates of thermal energy dissipation $\kappa$ [m$^2$s$^{-1}$] in silicates, e.g. $5.9 \leq -\log \kappa \leq 6.3$ (Clauser and Huenges, 1995; Vosteen and Schellschmidt, 2003), it is possible to define the thermomechanical competence of solid earth materials by the ratio $\kappa/\chi$. Note, for values of $\kappa/\chi < 1$, the rate of mechanical dissipation is higher than thermal dissipation, so that as a material deforms it carries its heat along. Conversely, values of $\kappa/\chi > 1$ imply that the rate of thermal dissipation is higher than mechanical dissipation, so that heat will be conducted readily...
through a material while it deforms only slightly. This, in a nutshell, is the thermomechanical rigidity hypothesis (Patton and Watkinson, 2010), which is used here to interpret observed variations in seismic energy release. This definition of rigidity retains its practical meaning as a resistance to shape changes, while shedding the highly idealized kinematic connotation of no internal deformation under loading.

Mechanical properties also factor highly in seismological models for wave propagation, and are critical for inferring first-order planetary structure, like the existence and nature of the mantle and core. As is well-known, the lack of shear wave propagation through the outer core is consistent with its existence in the liquid phase, while the propagation of shear waves in the mantle and inner core is consistent with their existence in the solid phase. Additionally, the attenuation of surface waves in the shallow asthenosphere, coincident with the low velocity zone, is consistent with the presence of a partial melt phase (Presnall and Gudfinnsson, 2008), although solid-state anelasticity also plays a role (Jackson et al., 2005; Karato, 1993).

Defining thermomechanical competence by the ratio $\kappa/\chi$ has the advantage of being testable, and free of the confounding aspects of the effective elastic thickness estimated in regional isostasy studies (Watts, 2001). For example, if the entire mantle propagates shear waves and is therefore an elastic solid, how can it make sense to further define plate-like blocks by their effective elastic thickness? Granted, the seismologic and isostatic models serve different purposes, and hence their use of the term ‘elastic’ need not be consistent. Nevertheless, a consistent definition of material competence is required for any dynamic plate theory, and I shall use thermomechanical competence throughout.

Adams and Williamson (1923), and later Birch (1952), used the relation

$$\frac{K_s}{\rho} = V_p^2 - \left(\frac{4}{3}\right)V_s^2 = V_{\phi}^2$$

(1)

where $K_s$ is adiabatic bulk incompressibility and $\rho$ is density, to study the thermodynamics of the mantle based on observed radial variations in compressional $V_p$, shear $V_s$, and bulk sound $V_{\phi}$ wave speeds. Birch inferred that the mantle below about 900-km depth was chemically homogeneous and adiabatic, while between 200- and 900-km depths it was not. Furthermore, he was among the first to suggest the
importance of mineral phase changes in the upper transitional zone. Seismological data collected since the 1950’s have done nothing but reinforce Birch’s insights (Dziewonski and Anderson, 1981). Clearly, there is something unusual about the upper mantle that must be understood thermodynamically.

Proponents of the standard Earth model often point to the apparent adiabaticity of the lower mantle as evidence for its vigorous convection. Although an adiabatic gradient can be maintained by the efficient mixing of material in viscous convection cells at high Rayleigh number (Turcotte and Oxburgh, 1967), adiabaticity also prevails in the lower reaches of a solid thermomechanical mantle, whether it is convecting or not. Furthermore, because the thermal expansivity of such mantle is inversely proportional to temperature, any convection process in the lower, hotter, mantle would likely be rather sluggish compared to that in the upper mantle. These conclusions, based on the statistical thermodynamics of non-linear elastic solids, are consistent with those based on the compressibility and rigidity of likely lower mantle mineral species (Anderson, 1989, 2007).

In summary, any dynamic plate theory should predict the plate-like nature of Earth’s outer shell, provide self-consistent mechanisms for its toroidal motion and spontaneous localization of shear, and explain the observed spectral correlation and admittance of Earth’s topography and gravity. In doing so, the nature of coupling between the plate-like blocks at the surface and poloidal motions in the mantle should be made clear. Proponents of the standard Earth model emphasize the insignificance of crustal and lithospheric strength on the length and time scales of geodynamics, and point to estimates of mantle viscosity based on glacial-isostatic rebound as evidence in favor of their view. This stance is necessary, only because the rheological theory upon which the standard model is based does not account for material strength or shear localization.

Here, I adopt the complimentary stance, and ask what are the length and time scales over which the force of gravity acts within the body of a solid thermomechanical planet. In deriving the dynamic rescaling theorem for deforming differential grade-2 (DG-2) solids (Patton and Watkinson, 2010), I have come to realize that rock strength always matters, not only for plate theory but also for the entire mantle. Thus, the
confounding semantics of rock ‘viscosity’ are moot, because a clear theoretical distinction exists between
the deformation of thermomechanical solids on the one hand, and thermoviscous fluids on the other.
Consequently, the density-normalized rates of mechanical energy dissipation measured in rock mechanics
experiments can be applied immediately to the dynamic plate problem, regardless of whether they are
termed kinematic viscosities or mechanical diffusivities. Henceforth, I shall adopt the latter term, in an
effort to discourage further semantic confusion.

This study interprets earthquake depth-energy release patterns for tectonically regionalized data from the
global centroid-moment tensor (CMT) catalog (Ekstrom and Nettles, 2011), using insights from the
thermomechanical theory of non-linear elastic DG-2 materials (Patton, 1997; Patton et al., 2000; Patton and
Watkinson, 2005; Patton and Watkinson, 2010, in review). These materials exhibit both distributed
harmonic and localized shear band modes of deformation as a function of $\kappa/\chi$. Harmonic modes alone are
possible for $0 < \kappa/\chi < \frac{1}{2}$, while both harmonic and shear band modes are possible for $\kappa/\chi > \frac{1}{2}$. This
transition occurs on a domain of thermomechanical competence lower than that associated with rigidity.
Consequently, theory predicts that even rigid materials can suffer irreversible shear band deformation.
Because shear localization liberates strain energy and increases entropy, it is reasonably identified with
earthquake faulting and damage. The spontaneous nature of this transition therefore offers a unique
opportunity to incorporate global seismicity into dynamic plate theory in a self-consistent manner.

Section 2.0 of the paper examines the map, magnitude, and depth distribution of seismicity as sampled by
the CMT catalog, and explains the method and rationale for computing earthquake depth-energy release
curves. It compares and contrasts depth variations in this signal using a six-fold classification of tectonic
margins. This section provides some background on the CMT inversion process, as compared to typical
earthquake location methods, so that the relevance of CMTs to dynamic plate theory is clear.

Section 3.0 outlines a statistical thermodynamic theory for strained inhomogeneous elastic and self-
gravitating matter configurations (Lavenda, 1995), which place severe constraints on the slope and shape of
the energy density as a function of entropy and length. This theory makes specific predictions about
temperature and pressure variations and the temperature dependence of thermal expansivity in these materials. Alone, these findings are consistent with expected variations of pressure and temperature in terrestrial planets, but offer no insight into observed variations of seismic energy release with depth in the Earth. Consequently, they provide essential foil for the thermomechanics of shear localization in DG-2 materials.

Section 4.0 outlines a statistical thermomechanical theory for DG-2 materials. It builds upon the consistency of the slope and shape of the distributed energy threshold ($\psi_D$) for DG-2 materials with the constraints derived in Section 3, and its interpretation as an elastic strain-energy function (Patton and Watkinson, 2005), to place earlier published work on these ideal materials in a very general and geologically useful context. Domains of $\kappa/\chi$ for which harmonic and shear band modes of deformation are possible are deduced using incipient modes analysis.

Section 5.0 applies thermomechanical theory to the interpretation and correlation of published data from rock mechanics experiments on the Mt. Scott Granite (2002, 2004). Not only does the localization curve ($\psi_L$) on the lower-competence harmonic domain neatly divide the sample population into macroscopically failed and un-failed groups, but it also correlates with post-loading observations of microscopic crack and macroscopic shear angles on the mixed harmonic-shear band domain at higher competence. The dynamic rescaling of lengths during sample failure is likely (Patton and Watkinson, 2010), given that the pre- and post-failure observations span the non-convex portion of the localization curve (Hobbs et al., 2011). Dynamic shear failure in these rock samples therefore minimizes energy dissipation in the combined sample-load frame system. Data from three samples suffering spontaneous shear localization are used to estimate mechanical diffusivity $\chi$.

Section 6.0 expands upon the isobaric shearing hypothesis (Patton and Watkinson, 2010) and its utility for the interpretation of Earth structure. Predicted depths to these theoretical shears, empirically calibrated via least-squares minimization of ThERM to PREM (Patton, 2001; Patton and Watkinson, 2009, 2010), are consistent with observed variations of earthquake depth-energy release. Furthermore, their fractal depth
The distribution suggests that Earth’s dynamic plate system globally minimizes energy dissipation. The structure of the lithosphere and asthenosphere, together, comprise a thermomechanical boundary layer adjacent of the surface of the planet.

Section 7.0 discusses the implications of these findings for the interpretation of rock mechanics data, pressure-temperature-time data from structural and metamorphic studies in orogens, likely source depths for common intrusive and extrusive rocks, the lateral variability of seismic wave speeds in the upper mantle and crust, and the spectral correlation and admittance of Earth’s gravity and topography.

Section 8.0 concludes the paper with the notion that coupled toroidal-poloidal motions of Earth’s plate-like blocks represent a top-down mode of convection peculiar to solid thermomechanical planets. Given the generality of the theory upon which these conclusions rest, and its remarkable correlation with datasets from a wide range of fields, it is likely that the absence of plate-like convection on other terrestrial planets can be attributed to the small size of the sample population, as well as the lack of liquid water on the known examples. Water and other volatile species tend to weaken, i.e. increase the mechanical diffusivity of, common rocks and minerals.

### 2.0 Earthquake depth-energy release

#### 2.1 Locating earthquakes

Since about 1970, earthquake hypocenters have been routinely and rapidly located using telemetric monitoring networks established by the United States Geological Survey, and many other agencies, for the purpose of mitigating earthquake hazards. These networks typically use body wave first motions to triangulate event locations within radially-symmetric elastic Earth models (Kennett and Engdahl, 1991), although for the past 15 years or so depth phases also have been used (Engdahl et al., 1998). Body waves propagate freely through solids with lateral extents much greater than the wavelength of the waves themselves. They are highly sensitive to variations in density and elastic moduli at the scale of about one-half their wavelength. Consequently, the average error for hypocenter depth in standard catalogs, like the
Preliminary Determination of Epicenters, is about 14 km (Kennett and Engdahl, 1991). Hypocenter depths for 20646 earthquakes are shown in Figure 1a.

By the time of the 1960 May 22 Mw9.5 Chile earthquake, which started the planet ringing like a bell, instrumentation had been developed that allowed the first precise studies of Earth’s free oscillations. These so called normal modes have periods as long as 54 minutes, and their observation spawned a whole new area of seismological research. Subsequent theoretical developments showed that the precise location of earthquakes, and their detailed dislocation characteristics, could be determined within the body of an Earth model by summing these normal modes (Dziewonski et al., 1981; Dziewonski and Woodhouse, 1983; Ekstrom et al., 2012). This CMT method accounts for long-period energy that is not possible via first motions of body waves alone, and does not require a radially-symmetric reference model. Furthermore, it is insensitive to lateral heterogeneity at scales less than half the wavelength of a given normal mode (Luh, 1975; Madariaga, 1972).

Since the early 1980’s, recordings of well-observed earthquakes have been routinely inverted for their CMT characteristics via normal modes summation (Dziewonski et al., 1981; Dziewonski and Woodhouse, 1983; Ekstrom et al., 2012). In this process the triangulated hypocenter is taken as the initial estimated location of a shear dislocation, which is then refined by accounting for the energy contained in the gravest and progressively higher frequency modes. Centroid depths for 20646 of these are shown in Figure 1b. The global CMT catalog, begun by Adam Dziewonski and co-workers at Harvard, is now maintained under the CMT project (Ekstrom and Nettles, 2011) at Lamont-Doherty Earth Observatory.

CMT inversion initially took into account long-period body waves (Dziewonski et al., 1981), with peak spectral energy between about 16.7 to 20 mHz, and Earth’s free oscillations with frequencies lower than 7.4 mHz (Dziewonski and Woodhouse, 1983), the so-called ‘mantle waves’. However, since 2004 normal modes with frequencies in the intermediate 7.4 to 16.7 mHz range, which are strongly affected by lateral Earth structure (Luh, 1975; Madariaga, 1972), also have been used in routine inversions (Ekstrom et al.,
As a result the practical lower limit on event magnitude, initially \( M_w > 5.5 \) and associated with a frequency cut-off above about 22 mHz, has decreased to about \( M_w > 5.0 \).

Reference models used by the CMT project include the radially-symmetric PREM (Dziewonski and Anderson, 1981) and shear attenuation model QL6 (Durek and Ekstrom, 1996), and a mantle heterogeneity model called SH8U4L8 (Dziewonski and Woodward, 1992). The latter model accounts for lateral variations in shear wave speeds at half-wavelengths as small as about 2500 km \( (l \leq 8) \), in four layers of the upper mantle, and eight layers of the lower mantle. Lateral variations of this scale represent the most robust deviations from spherical symmetry, as imaged by global tomographic models (Dziewonski et al., 1977; Gu et al., 2001). Furthermore, the majority of shear attenuation occurs in the upper mantle, above about 670-km depth (Durek and Ekstrom, 1996; Romanowicz, 1994; Romanowicz, 1995; Widmer et al., 1991).

2.2 Effects of CMT relocation

The CMT catalog, downloaded from www.globalcmt.org, includes data for 30872 earthquakes recorded during the period January 1976 through December 2010. Of these only 20646 were relocated by the CMT procedure (Ekstrom and Nettles, 2011), and therefore have quantitative error estimates. The remaining events either had their focal depths fixed by an analyst, or constrained by an inversion of short-period data. The consistent algorithmic treatment of the 20646 relocated events, and the fact that CMTs estimate the total work done by a seismic dislocation, make them ideal for the tectonic analysis presented here.

Mean standard errors for CMT latitude, longitude, and depth are 0.042°, 0.047°, and 2.82 km, respectively. The depth error estimate defines the minimum thickness of a filter used to smooth the earthquake depth-energy release curves presented here. A filter smaller than this tends to display more noise than a larger one, while larger filters discard potentially interpretable depth signal in the dataset, at least at typical crustal depths. Depth-energy release curves shown here were smoothed using either 3-km or 10-km filters.
Initial hypocenters and relocated centroids are distributed differently with both magnitude (Table 1) and depth (Table 2). Centroids generally have higher moment magnitudes than their associated hypocenters, consistent with the fact that longer-period energy is accounted for in their estimation. All 20646 relocated events appearing in the catalog are accounted for in the moment release studies presented here. Most of the seismic signal is present in the $M_w=5-6$ band, with significant signal also in the $M_w=4-5$ and $M_w=6-7$ bands. The depth distributions of hypocenters and centroids are significantly different in the crust, above about 37-km depth. Small differences in map locations exist as well, but these do not significantly alter the global map pattern of seismicity, and are not discussed further.

Figure 1 shows differences in the depth distribution of earthquake hypocenters and centroids, using a color-key linearly distributed, piecewise, between 0- and 800-km depths (Table 2). Subtle differences in color values therefore reflect real depth differences between events. The depth intervals were chosen consistent with ThERM modes $H4$, $L1$, $L2$, $L4$, $M1$, $M2$, and $M4$ (Patton and Watkinson, 2010) (Table 3).

In the upper crust, 0-14.4 km depth, there are about three times as many hypocenters as there are centroids, and the color palette ranges from red to orange (Table 2). The map distribution of events is similar for both hypocenters and centroids. Events commonly occur at oceanic ridges, oceanic trenches, and the Alpine-Himalayan belt. The centroids are distinctly deeper than their corresponding hypocenters.

In the middle crust, 14.4-25 km depth, the number of hypocenters is only about one-fifth that of centroids and the color palette ranges from orange to yellow. The map distributions of hypocenters and centroids also show significant differences. While hypocenters are common at all oceanic trenches and along the Alpine-Himalayan belt, they only rarely appear at oceanic ridges. Centroids, in addition to occurring at oceanic trenches and the Alpine-Himalayan belt, are common at oceanic ridges and also major transform boundaries, like the San Andreas-Queen Charlotte-Fairweather fault system.

In the lower crust, 25-37 km depth, hypocenters outnumber centroids by about 1.5 times, and the color palette ranges from yellow to green. Oceanic trenches are well-populated by both hypocenters and
centroids, as is the Alpine-Himalayan belt. Rare intraplate events of both types also appear. Interestingly, centroids also sparsely populate oceanic ridges, but primarily at the shallower (yellow) end of the range.

In the lithosphere, 37-99.5 km depth, the number of hypocenters and centroids is comparable, and the color palette ranges from green to blue. Hypocenters and centroids are absent at all oceanic ridges and active transform boundaries, but commonly populate all oceanic trenches as well as the Alpine-Himalayan belt. Distinct 'knots' of events appear beneath the Carpathians and the Hindu Kush, with rare events in East Africa-Madagascar. The map distributions of hypocenters and centroids are similar in most respects.

The number of hypocenters and centroids in subducting slabs transiting the upper tectosphere, 99.5-172 km depth, is comparable, and the color palette ranges from blue to purple. As in the preceding depth interval, the map distributions of events are very similar. Both hypocenters and centroids are well-represented at all oceanic trenches, but also sparsely populate the Alpine-Himalayan belt. The Carpathian and Hindu Kush 'knots' persist to these depths as well.

The number of hypocenters and centroids in subducting slabs transiting the lower tectosphere, 172-255 km depth, is comparable, and the color palette ranges from purple to pink. The maps again are very similar, with hypocenters and centroids common at all oceanic trenches. The Hindu Kush 'knot' persists to these depths as well, but the Carpathian one is absent. Some events also appear beneath the Aegean and southern Italy.

The number of hypocenters and centroids in subducting slabs transiting the asthenosphere, 255-690 km depth, is comparable, and the color palette ranges from pink to white. Again their map distributions are very similar. Hypocenters and centroids are associated almost exclusively with active subduction margins, e.g. Andes, Tonga-Lau, Malaysia, Indonesia, Japan, Aleutians, South Georgia and Sandwich Islands, except for a few events in the western Alpine Belt, particularly beneath southern Spain and Italy. The Hindu Kush 'knot' appears pink (shallow) in this depth range, consistent with the base of the tectosphere.
There are only a handful of events in the mesosphere, where the color palette is uniformly black. These events lie only a few kilometers below 690-km depth. Most appear at the Tonga trench, but a few centroids also lie between Japan and Kamchatka.

In summary, the depth distribution of earthquakes at various plate margins appears to be more consistent for centroids than it does for hypocenters. Thus, while the latter distribution could be interpreted as indicating significant differences in thickness and mechanical properties between oceanic and continental lithosphere, the former distribution suggests the opposite. In both cases, however, the distribution of earthquakes at convergent margins is distinct from that at divergent and transform margins. It is likely that some of the apparent difference between continents and ocean basins is due to the fact that travel-time models are optimized for continents (Kennett and Engdahl, 1991), where the majority of seismic receivers are located. Nevertheless, there are good rock-mechanical reasons to believe that significant differences in the mechanical properties of continents and the ocean basins exist. Therefore, a detailed study of earthquake depth-energy release from tectonically regionalized data might help quantify the nature of these differences.

2.3 Regional variations

Given differences in the depth distribution of earthquakes, noted above, and their apparent correlation with tectonics, it is natural to consider regional subsets of the CMT catalog. These subsets (Figure 2) over sample the CMT catalog by about 0.8% (Table 4). This is due to the expedient method used to select event subsets, particularly at convergent margins where data density is high. It is unlikely, however, that this small discrepancy has any real impact on the conclusions of this report. A more in-depth analysis of these data, including detailed consideration of CMT dislocation characteristics, is in progress.

Earthquake depth-energy release curves $\Sigma M_w(z;t)$ present the sum of moment magnitudes $M_w$ for earthquakes, filtered for depth $z$ using a boxcar of selected thickness $t$, at every kilometer from the surface to about 700-km depth. They are an elaboration upon similar curves presented by Frohlich (1989) in his review of deep-focus earthquakes. In early work with the CMT catalog, filter thicknesses of 1, 3, 5, 7, 9,
and 11 km, were used, but did not significantly change the depth patterns shown here. The primary effect
of filter thickness, apart from curve smoothing, is to change the magnitude of these sums. Consequently,
most curves presented here are computed using a 3-km filter, which matches the mean standard depth error
of CMTs. This seems to provide adequate smoothing without discarding potential signal. Because moment
magnitude is related to scalar seismic moment $M_0$ [dyne-cm] by the formula $M_W = \frac{2}{3}\log_{10}(M_0) - 10.7$ (Aki
and Richards, 2002), these curves serve as simple dimensionless proxies for seismic energy release with
depth in the planet. How this prevalent, stochastic, and highly localized mode of energy dissipation might
be related to energy dissipation in the larger geotectonic system is of primary interest in studying these
plots.

As discussed earlier, well-observed earthquakes with $M_W > 5.0$ are relocated routinely as part of the CMT
inversion procedure, using the associated hypocenters as initial estimates. Observed differences between
hypocenter and centroid depth-energy release curves can, in part, be understood in this manner. However,
the depth-energy release pattern for hypocenters (Figure 3a) is the same for all six tectonic subsets of the
earthquakes studied here, as well as for the entire dataset. This is somewhat artificial, and probably reflects
optimizations in the quick location algorithms used for event hazard monitoring. On the other hand, the
depth-energy release curves for centroids have a more natural appearance (Figure 3b), and reveal
interpretable depth structure which apparently depends on the nature of tectonic margins.

The amplitude of these depth-energy release curves is proportional to the number of events in the given
data subset. Consequently, a normalization scheme is needed to enhance possible variations in the depth
signal they contain. Given the strong correlation of summed moment magnitude with the number of
earthquakes in each subset (Table 4), an expedient normalization scheme is to simply divide $\Sigma M_W(z;t)$ by
the number of events $N$ in the subset. Event-normalized earthquake depth-energy release curves
$\Sigma M_W(z;t)/N$ reveal significant differences between divergent and transform margins on the one hand and
convergent margins on the other. In both panels of Figure 4, the depth-release curve for the entire set of
relocated CMTs (black dashed, Table 4) serves as a baseline for these regional comparisons.
2.3.1 Divergent and transform margins

The three colored curves shown in Figure 4a correspond to event populations occurring at divergent and transform margins, as plotted in Figure 2a, c, e. There is relatively little energy release associated with the brittle upper crust. In the middle crust, from 14-25 km depths, the most seismic energy is dissipated at continental transforms and oceanic ridges, while the least is dissipated at continental rifts. At all three boundary types, the energy dissipation per event is greater than the global CMT baseline. It is possible that the greater dissipation of energy at continental transforms, compared to the oceanic crust, is due to the lesser Coulomb strength of typical felsic lithologies found there, when compared to mafic ones.

In the lower crust, from 25-37 km depths, the most seismic energy is dissipated at continental rifts, at a per-event rate higher than the global baseline. In contrast, seismic energy dissipation at oceanic ridges and continental transforms are both significantly below the global baseline, with the least dissipation occurring at continental transforms. This pattern is consistent with the observation that continental crust generally is thicker than ocean crust, and suggests aseismic creep in the lower crust of continental transforms. The similar amounts of energy dissipation in oceanic and continental crust at extensional margins, combined with their differences in thickness, again suggest that oceanic crustal rocks are stronger than continental rocks.

Below 37-km depth, all three boundaries dissipate less energy than the global per-event baseline, with the most dissipation occurring in continental rifts, and the least in oceanic ridges. Below about 50-km depth, the depth-energy release curves disappear altogether (Table 4). Coincidentally, these depths correspond to a range of pressures thought to be important for the extraction of mid-ocean ridge basalts (MORB) (Presnall and Gudfinnsson, 2008). Perhaps the fact that basalt volcanism is common at oceanic ridges and continental rifts (e.g. East Africa), and not at continental transforms, reflects decompression melting in the mantle at divergent margins.

2.3.2 Convergent margins
The three colored curves shown in Figure 4b correspond to event populations occurring at convergent margins, as shown in Figure 2b, d, f. Again, there is relatively little energy release associated with the brittle upper crust. In the middle crust, from 14-25 km depths, the most seismic energy is dissipated at Himalayan-type margins, and at a per-event rate higher than the global CMT baseline. In contrast, the dissipation occurring at island arc-type and Andean-type margins is below the per-event baseline, with the least dissipation occurring at Andean-type margins. This might reflect the juxtaposition of generally thicker and weaker continental crust on both sides of Himalayan-type margins, when compared to the other two margins types.

In the lower crust, from 25-37 km depths, the most seismic energy is dissipated at island arc-type margins, but at rates only slightly greater than the global per-event baseline. Seismic energy dissipation at Andean-type and Himalayan-type margins is below the global baseline, and while the Andean-type release curve closely parallels the baseline, the Himalayan-type curve deviates substantially.

Below 37-km depth, seismic energy dissipation in Himalayan-type margins is noticeably less than the global baseline to about 75-km depth. Earthquake depth-energy release curves for convergent margins persist to depths of 492 km at Himalayan-type margins, 656 km at Andean-type margins, and 699 km at island arc-type margins (Table 4). The apparent contrast between convergent and divergent/transform margins, earlier discerned from the seismicity maps, is clearly reflected in these depth variations.

### 2.3.3 Variations throughout the asthenosphere

Given the great depth to which seismic activity occurs at convergent margins, it makes sense to examine deviations from the global baseline throughout the asthenosphere. Again, the depth-release curve for the entire set of relocated CMTs (black dashed, Table 4) serves as a baseline for these regional comparisons. Figure 5 shows event-normalized release curves for the three convergent margin types, filtered using a 10-km boxcar. Relatively large deviations from the global baseline are apparent, particularly at depths less than about 260 km.
In the lithosphere, from 37-100 km depths, seismic energy dissipation at Himalayan-type margins is less than the baseline to about 75-km depth, as noted earlier, while that at Andean-type and island arc-type margins closely parallel the baseline. This might reflect the fact that subduction of oceanic lithosphere is ongoing at the two latter margins, but has ceased at the former.

In the upper tectosphere, from 100-175 km depths, the greatest energy dissipation occurs at Andean-type margins, and at rates higher than the global baseline. Through this range, energy dissipation at Himalayan-type and island arc-type margins closely tracks the global baseline, except at about 160-180 km depth, where dissipation in the Himalayan-type margins falls off. Coincidentally, these depths correspond to the range of pressures for which peridotite xenoliths in kimberlites show distinct planar tectonite fabrics (Boyd, 1973; James et al., 2004). Perhaps this reflects localized weakening of the mantle at these depths, consistent with isobaric shearing at mode M1 of ThERM (Patton and Watkinson, 2010) (Table 3).

In the lower tectosphere, from 175-260 km depths, seismic energy dissipation at Himalayan-type and Andean-type margins noticeably differ from the global baseline, while that at island arc-type margins closely tracks the baseline. This is consistent with the presence of continental tectosphere (Jordan, 1975) at the former two margin types, and its absence at island arc margins.

In the asthenosphere, from about 500-650 km depths, the rate of seismic energy dissipation increases. This is a depth range where mineral phase transformations are thought to be likely (Birch, 1952; Ringwood, 1991), and it is possible that these earthquakes represent a phase-transformation ‘anti-crack’ population (Green, 2005), although there are other hypotheses (Frohlich, 1989). Seismic energy dissipation tails off substantially at about 660-km depth, before disappearing altogether at about 700-km depth. This cut-off of seismicity roughly corresponds to modes M3 and M4 of ThERM (Patton and Watkinson, 2010).

In summary, many, but certainly not all, of the variations of earthquake depth-energy release from the CMT catalog correspond with the boundary layer structure of ThERM (Patton and Watkinson, 2009, 2010) (Table 3). The coincidence of inflections, triplications, and minima in these release curves with the
predicted depths of isobaric shearing modes suggests a coherent layering of thermomechanical competence with depth in the planet. Furthermore, the marked differences in event-normalized energy release between tectonic regions suggest significant lateral variations. Consequently, it makes sense to explore further the implications of this model for dynamic plate theory.

3.0 Thermodynamics of solid self-gravitating matter configurations

3.1 Preliminaries

The macroscopic notion of heat is defined as the difference between the internal energy and work performed on a system, consistent with the Joule heating experiments (Chandrasekhar, 1967). The First Law of thermodynamics is therefore

\[ dQ = dU - dW \]  

(2)

where \( dQ \), \( dU \) and \( dW \) are increments of heat, internal energy and work, respectively. Note that heat is a derived quantity, having no meaning independent of the First Law.

Guided by triaxial rock mechanics experiments, I shall account for the entropy density of strained solid materials using a simple one-dimensional elastic model. This model exhibits an unorthodox behavior consistent with Lavenda’s (1995) notion of thermodynamic symmetry breaking. I deduce the expected slope, shape, and temperature dependence of the energy density for this model, which then serves as foil for the thermomechanics of shear localization exhibited by non-linear elastic DG-2 materials. Note that while entropy appears in all statements of the Second Law, it was first formulated axiomatically by Carathéodory (ca. 1909), based on an analysis of Pfaffian differential equations, to read

\[ dQ = TdS \]  

(3)

where \( dS \) is an increment in the entropy density, and \( T \) is absolute temperature.

Consider a cylindrical test specimen of rock, with length \( l \) and diameter \( d \), placed in a loading frame for the purpose of strength characterization. Upon applying a force \( \varphi \) directed along a line parallel to the specimen’s length, it is observed that the specimen shortens by a length increment \( dl \). Consequently, the increment of work needed to shorten the cylinder from \( l+dl \) to \( l \) is given by
Because the specimen can be held under relatively small loads for long periods of time, it is reasonable to assume that it manifests a force equal and opposite to the applied load. Presumably, this reaction force arises from electromagnetic interactions in the sample’s microstructure. Furthermore, experience shows that if the cylinder were unloaded, it would likely return to its original length. This is Hooke’s law (Ut tensio sic vis, ca. 1642).

However, it is equally valid to consider this problem from a material point of view. Responding to a directed environmental load of magnitude φ, the cylinder strains by an increment \( dl \) of its overall length \( l \), and as a result distributes an increment of energy \( dU \) throughout its microstructure and mineral fabric. Therefore, I am also free to assume a macroscopic relation of the form

\[
dU = \varphi dl .
\]  

From a thermodynamic point of view, the unloaded state to which the cylinder tends to return upon unloading is somehow more likely than the loaded one, and should therefore coincide with a maximum in entropy. Consequently, any deformation of the cylinder from this ideal state must necessarily decrease the entropy of the cylinder itself, as a function of length. Consequently, I seek a relation of the form

\[
dS = -f(l)dl .
\]  

Combining the First and Second Laws, equations (2) and (3), I obtain

\[
TdS = dU - dW .
\]  

Upon substituting for the work and internal energy increments using (4) and (5) respectively, I find

\[
TdS = \varphi dl - \varphi dl \Rightarrow dS = 0
\]  

Hence, for a positive absolute temperature thermodynamics predicts no increase in entropy, consistent with the apparent lack of energy dissipation, a state of mechanical equilibrium for cylinders under small loads, and everyday experience.
3.2 Strained inhomogeneous elastic solids

Curiously, there is no need to account for heat in these experiments, despite its fundamental importance in thermodynamics. It would appear that there is no meaningful distinction to be made between heat and work for this model (Lavenda, 1995). Instead the thermodynamic potentials for work, internal energy, and entropy are all functions of a single variable. This has immediate consequences for the usual combination of the First and Second Laws, equation (7). Because heat and work are indistinguishable, and heat is already accounted for in the product of the temperature and the entropy increment through the Second Law, equation (7) can be rewritten as

\[
\frac{f(l)}{\phi} = \frac{1}{T} = -\frac{d\Delta S}{d\Delta U}. \quad (9)
\]

Here the entropy and internal energy are prefixed with deltas to distinguish these primitive functions from the classical thermodynamic potentials assumed above, which are first-order homogeneous functions. These primitive functions are inhomogeneous, which is to say that they can manifest scale-dependence, contrary to the scalability expected of classical homogeneous potentials. In the following section I explore the scale-dependence of this system, by employing power laws for initial statistical distributions in length.

The importance of statistical variability in the behavior of elastic materials, and rocks in particular, is demonstrated by modeling the entropy and energy potentials for this system as power laws in length \( l \).

Following Lavenda (1995), I define the internal energy increase as

\[
\Delta U(l) = -\frac{\sigma l^n}{m} \quad (10)
\]

and the entropy reduction as

\[
\Delta S(l) = -\frac{k\sigma l^n}{n} \quad (11)
\]

where \( k \) is Boltzmann’s constant, \( \sigma \) and \( \eta \) are positive constants independent of temperature, and \( n \) and \( m \) are positive numbers.

Observe that equation (9) can be rearranged to represent the temperature as
\[ T = -\frac{d\Delta U}{d\Delta S}. \] (12)

Upon differentiating the primitive functions (10) and (11) with respect to length, and substituting into equation (12) I obtain

\[ T = \eta \frac{k_m}{\kappa \sigma} l^{m-n}. \] (13)

Consequently, the temperature of this model system can either increase or decrease with length, depending on whether the exponent \( m - n \) is positive or negative. The temperature is independent of length for \( m = n \).

The modulus of elasticity \( E \) for the model is defined by the derivative of the force \( \varphi \), which in turn is the derivative of the internal energy, equation (5). Upon eliminating length in this expression via the temperature relation, equation (13), I obtain

\[ E = (m-1)\eta \left( \frac{k \sigma T}{\eta} \right)^{\frac{m-2}{m-n}}. \] (14)

For \( n = 2 \), the modulus of elasticity is \( E = (m-1)k \sigma T \), and the force reduces to a generalized Hooke’s law \( \varphi = E(T)l \). This relation further reduces to a linear force-displacement law, but only when the internal energy too is quadratic in length, \( m = 2 \).

Upon inverting equation (13) to express length as a function of temperature, differentiating the result with respect to temperature, and eliminating the constants via (13) I obtain

\[ \frac{1}{l} \frac{dl}{dT} = \frac{1}{(m-n)T}. \] (15)

This expression characterizes the thermal elongation of the model, analogous to the thermal expansivity in three-dimensions. The latter material property dictates the scale over which body forces can act in the model. Consequently the model elongates upon heating for \( m > n \), shortens upon heating for \( m < n \), and is undefined for \( m = n \).
By separating the entropy and energy increments in equation (12), dividing through by a temperature increment $dT$, and employing the chain rule to express the common length dependencies for entropy, energy, and temperature, I obtain

$$\frac{d\Delta U}{dl} \frac{dl}{dT} = -T \frac{d\Delta S}{dl} \frac{dl}{dT}. \quad (16)$$

Upon substituting the derivatives of equations (10), (11), and (13) with respect to length into (16) I find

$$\frac{d\Delta U}{dT} = -T \frac{d\Delta S}{dT} = \left( \frac{\eta}{m-n} \right) \frac{l^m}{T} \quad (17)$$

For positive absolute temperature, this shows that the heat capacity of this inhomogeneous elastic system cannot be defined simultaneously as

$$C \equiv \frac{dQ}{dT} = T \frac{d\Delta S}{dT} \quad (18)$$

and

$$C \equiv \frac{dQ}{dT} = \frac{d\Delta U}{dT} \quad (19)$$

because one of these definitions always will be negative when the other is positive, and vice versa. Apart from the pathological case for $m = n$, there are two other distinct types of inhomogeneous elastic systems depending on whether $m < n$ or $m > n$. 

Variability as a function of length is inversely proportional to the exponent appearing in the primitive power laws, above. Hence, a smaller exponent means greater variability. Equation (10), defining the internal energy increase, is associated with mechanical variability. However, equation (11), defining the entropy reduction, is associated with statistical variability rather than thermal variability, because heat is not evident in this problem. Consequently, systems dominated by either mechanical variability ($m < n$) or statistical variability ($m > n$) can be identified on the basis of their heat capacity $C$, as follows:

$$C \equiv \frac{dQ}{dT} = \begin{cases} T \frac{dS}{dT}, & m < n \\ \frac{dU}{dT}, & m > n \end{cases} \quad (20)$$
These conclusions are easily substantiated by returning to thermodynamic fundamentals. Eliminating length between the primitive equations (10) and (11), I find for the case of mechanical variability that

$$\Delta S \sim -\left(\Delta U\right)^{\frac{n}{m}}. \quad (21)$$

In words, the entropy is a concave function of the internal energy. Also, for the case of statistical variability I find that

$$\Delta U \sim \left(\Delta S\right)^{\frac{m}{n}}. \quad (22)$$

In words, the internal energy is a convex function of the entropy. Furthermore, because $dS/dU < 0$, these representations are mutually exclusive. The usual symmetry of the entropy and energy representations, expected from equilibrium thermodynamics and arising from first-order homogeneity of thermodynamic potentials, is broken.

For an inhomogeneous elastic system dominated by mechanical variability ($m < n$), $dQ$ is the amount of heat evolved by the system, which leads to a decrease in entropy by an amount $dS = \frac{dQ}{T}$ (Figure 6a). Therefore fewer microscopic states are available at lower temperatures. The slope of the concave entropy density function is $-1/T$; higher temperatures are associated with flatter slopes, and lower temperatures with steeper slopes. This model elongates upon cooling. Because temperature and heat are both decreasing functions of length, the entropy density is proportional to length. Mechanical variability offers no insight for the thermodynamics of DG-2 materials.

On the other hand, in an inhomogeneous elastic system dominated by statistical variability ($m > n$), $dQ$ is the amount of heat absorbed by the system, which leads to an increase in internal energy by an amount $dU = dQ > 0$ (Fig. 6b). This corresponds to a decrease in the entropy by an amount $dS = -\frac{dQ}{T}$. Consequently there are fewer microscopic states available at higher temperatures. The slope of the convex energy density function is $-T$; higher temperatures are associated with steeper slopes, and lower temperatures with flatter ones. This model elongates upon heating. Furthermore, because temperature and heat are both increasing functions of length, energy density is inversely proportional to length. Statistical variability offers crucial insights for the thermodynamics of DG-2 materials.
Nowhere in this simple one-dimensional model has the phenomenon of shear failure been addressed. For example, if we repeatedly load our test specimen, or apply progressively higher loads, experience tells us that the specimen will, at some point, spontaneously fail, sometimes after suffering significant microphysical damage (Katz and Reches, 2004). In other words, the simple act of loading the test cylinder changes its prior microstructure and mineral fabric. Although the detailed distribution of these changes cannot be known to an outside observer, they can be treated statistically, as was appreciated by Weibull (ca. 1939). These issues are addressed in Section 4.0.

3.3 Self-gravitating matter configurations

Lavenda (1995) shows that the thermodynamics of a self-gravitating body, like a planet or star, is also subject to symmetry breaking of the type outlined above. In this case, like that of inhomogeneous elastic systems dominated by statistical variability, the energy density is given by a monotonically decreasing function of the entropy density (Figure 6b). Significantly, the energy density is inversely proportional to the body’s radius. In other words, both pressure and temperature increase with depth.

The heat capacity of a self-gravitating body is defined by

\[ C \equiv \frac{dQ}{dT} = \frac{dU}{dT} = \frac{mc^2}{T_0} \]  

where \( m \) is the mass of an elementary particle, \( c \) is the speed of light, and \( T_0 \) is a reference temperature. The energy density, and by association the stress, of a self-gravitating body plays a fundamental role in shaping the body, and in controlling its spacetime evolution, in accordance with the far reaching implications of Einstein’s (1916) theory of gravitation.

4.0 Thermomechanics of DG-2 materials

4.1 Diharmonic equation

The pure-shearing plane-strain deformation of non-linear elastic DG-2 materials is governed by the diharmonic equation (Patton, 1997)
EARTHQUAKE DEPTH-ENERGY RELEASE

\[ 0 = \alpha^2 \frac{\partial^4 \psi}{\partial x^4} + \left(1 + \alpha^2 \right) \frac{\partial^4 \psi}{\partial x^2 \partial z^2} + \frac{\partial^4 \psi}{\partial z^4} \]  

(24a)

\[ \alpha^2 = \frac{\left(1 - 2(\kappa/\chi)\right)}{(1 + 2(\kappa/\chi))} \]  

(24b)

where the ratio of thermal \( \kappa \) to mechanical \( \chi \) diffusivities is called thermomechanical competence. Note that as \( \kappa/\chi \to 0 \), equation (24) reduces to the biharmonic equation, which appears in theories of linear elastic and linear viscous materials.

4.2 Incipient modes analysis

Here incipient modes analysis (Patton and Watkinson, 2010) is used to document the deformation modes of DG-2 materials and thereby facilitate comparison with the thermodynamic properties of simple inhomogeneous elastic materials, documented in section 3. I substitute wave-like harmonic and dislocation-like shear band solutions into the differential equation (24) to identify domains were deformation modes of these types are possible. In all cases, domains of thermomechanical competence that allow real roots, or at least roots with real parts, will admit solutions of the assumed type. In both trial solutions \( \psi \) is the stress-energy function, identically satisfying incompressibility. This linear analysis in no way constrains the finite growth of the resulting structures.

For these analyses, consider a two-dimensional spatial domain in which a specific but arbitrary set of Cartesian axes are drawn through an arbitrarily chosen point. Consider also a plane strain deformation field where the velocity components \( (u, w) \) expressed in this coordinate system are assumed proportional to the distance from the origin in the following way, \( (u, w) \propto (-x, z) \).

The usual Cartesian harmonic normal modes

\[ \psi = \exp^{i(xu + wz)} \]  

(25)

suffice for the wave-like case. Here a relatively competent layer of thickness \( H^r \) has its mean position parallel to the shortening direction, and it is initially planar. In other words, the boundaries between the layer and the weaker matrix initially lie at \( z = \pm H^r/2 \). As the layer shortens in the base field, it tends to
thicken, and harmonic perturbations with amplitude $\delta^*$ might begin to develop. The growth of such undulations into observable folds or waves depends on the relative rates of their amplification versus uniform layer thickening. The wavenumber $\omega$ predicts the normalized wavelength of the incipient undulation through the relation $L^*/H^* = 2\pi/\omega$. The small scalar parameter in this case is $\varepsilon = \delta^*/H^*$, so that these deductions are rigorous only for the case of infinitesimal fold amplitude.

Substituting the harmonic trial solution (25) into (24a) I obtain the four distinct roots

$$r = \pm \omega \quad \text{or} \quad r = \pm \alpha \omega.$$  \hspace{1cm} \text{(26)}

These are all real for $0 < \kappa/\chi < \frac{1}{2}$, thus wave-like material deformations are expected to form on this domain of relatively low thermomechanical competence, in response to far-field forcing (Figure 7a, solid blue curve). Note also that for $\frac{1}{2} < \kappa/\chi < \infty$ these roots are mixed, with two real and two pure imaginary, so that harmonic disturbances are also possible on this domain of higher thermomechanical competence (Figure 7a, dotted blue curve). Together, these harmonic perturbations represent shear waves (reversible rotational distortions) propagating throughout the material (Truesdell, 1964). Furthermore, given the planetary scale implications of DG-2 thermodynamics, material coupling on the domain of lower competence could explain some aspects of long-period Love wave attenuation in the upper mantle (Table 5). The normalized wavelength of these disturbances scales as $L^*/H^* = (\kappa/\chi)^{-1}$ (Patton and Watkinson, 2005).

In the dislocation-like case, I use shear band solutions

$$\psi = \exp^{g_x x + g_z z}$$  \hspace{1cm} \text{(27)}

following the work of Hill & Hutchinson (1975) and Needleman (1979). The vector $g$, with components $g_x$ and $g_z$ in the chosen coordinate system, is normal to the incipient shear band. Consequently, the arctangent of the (real) ratio $g_x/g_z$ determines the angle between the band itself and the shortening direction. The ratio of the thickness of the incipient band $\delta^*$, to its length $L^*$, provides a suitable small scalar parameter $\varepsilon$, so that these deductions are rigorous only for the case of a vanishingly thin band.

Substituting the shear band trial solution (27) into (24a), I obtain the four distinct roots
While the first pair of roots is purely imaginary, the second pair is real for $\frac{1}{2} < \kappa/\chi < \infty$, where the rescaling modulus (24b) itself is imaginary. Consequently, dislocation-like disturbances are expected to form at angles, symmetric about the loading axis, ranging from 0 to 45 degrees for these relatively high values of thermomechanical competence (Fig. 7a, red curves). Significantly, an angle of 30 degrees corresponds with the value $\kappa/\chi = 1$, which defines the lower limit of thermomechanical rigidity. This suggests that dislocation-like disturbances should form, preferentially, at about 30 degrees with respect to the shortening direction in thermomechanically competent materials (Patton and Watkinson, 2011; Patton and Watkinson, in review). This prediction is consistent with Anderson’s 1905 theory of crustal faulting (Jordan et al., 2003).

4.3 Stress-energy density map

The thermomechanics of DG-2 materials can be represented graphically (Patton, 2005; Patton and Watkinson, 2005) (Figure 7b) using three energy thresholds, one for distributed harmonic deformations (green curve, $\psi^D = (\kappa/\chi)^{-1}$), one for intrinsic strain-energy storage (gray curve, $\psi^I = (|\alpha|)^{-1}$), and another for localized shearing deformations (orange curve, $\psi^L = (|\alpha|\kappa/\chi)^{-1}$). Taken together, these curves define a statistically stable stress-energy density map for these materials. Observe that the three threshold curves are monotonically decreasing on certain domains of thermomechanical competence, and that the stress-energy density approaches infinity (“blows up”) at the lower end of these respective domains. The distributed threshold curve has a vertical asymptote at $\kappa/\chi = 0$, while the intrinsic curve has one at $\kappa/\chi = \frac{1}{2}$. The localization threshold curve, defined as the product of the other two, consequently exhibits two asymptotes, with a distinct non-zero energy minimum between them. The presence of two distinct energy spikes in this diagram, and their diffusive connection via the dynamic rescaling theorem (Patton and Watkinson, 2010), give rise to all of the geologically interesting behavior of the DG-2 material.

5.0 Application to rock mechanics
The foregoing analysis can be applied to the interpretation of data from rock mechanics experiments on cylindrical specimens. Three facts make this possible. First, the earlier analysis of statistical variability (Figure 6b) shows that the energy density for strained elastic solids must be a monotonically decreasing function of the entropy, which in turn must be an inverse function of length. Thus, the diameter/length ratio ($d/l$) of a cylindrical specimen serves as the abscissa for plotting these data (Table 6). Second, because the intrinsic stress-energy density threshold ($\psi'$, Figure 7b) arises from the differential normal-stress term of the DG-2 constitutive equation (Patton, 1997; Patton and Watkinson, 2005; Patton and Watkinson, 2010) it makes sense to plot normalized differential stress (NDS) from the experiments as the ordinate. Using Katz & Reches’ (2002, 2004) definition of $NDS = (\sigma_1 - \sigma_3)/586$ and their identification of $NDS \approx 0.96$ as a critical threshold for spontaneous sample failure, it is a simple matter to see that the number $\zeta = 4\sqrt{3}$, common in the theory of DG-2 materials (Patton and Watkinson, 2010), reconciles their conclusion with this thermomechanical theory. The final and seemingly obvious fact is that the deformation of any rock sample in any load frame is possible only because of the thermomechanical rigidity of the loading frame itself. Thus, the deformed samples and the loading frame comprise complementary parts of a larger, and more interesting, thermomechanical system. The samples plot at $(\kappa/\chi, \psi) = (d/l, \zeta (\sigma_1 - \sigma_3)/586)$, while the loading frame plots at $(\kappa/\chi, \psi) = (1, 1)$.

Fourteen samples of the medium-grained Mt. Scott granite were loaded to predetermined values of axial stress $\sigma_1$, at confining pressure $\sigma_3 = 41$ MPa, and held for predetermined periods, before the loads were slowly released. Twelve samples that did not suffer macroscopic failure are plotted as green circles (Figure 7b). Three of these samples (105, 124, & 125, Figure 7b, inset) were reloaded to failure at higher loads, as indicated by gold diamonds and thin dashed lines. Three other samples loaded to failure also are plotted as gold diamonds. Three samples (#104, #106 and #110), plotted as red diamonds, spontaneously failed during their hold periods. Given the standard error in measured Coulomb strength for the Mt. Scott granite ($586 \pm 16$ MPa), indicated by the thin dashed lines plotted above and below the distributed and localized threshold curves (inset), all the failed samples plot above the localization threshold.
Katz & Reches (2002, 2004) observed the microscopic and macroscopic damage to several samples after loading. They report two populations of microscopic cracks. Population A are intragranular tension cracks (Figure 7a, red dashed curve) with angles, in relation to the loading direction, in the range 0º – 10º, while Population B are intergranular shear fractures (Figure 7a, red solid curve) with angles in the range 11º – 40º. They also report angles of macroscopic shear failures, which tend to increase with confining pressure. The macroscopic shear angles correspond to thermomechanical competences in the range 0.6 < $\kappa/\chi$ < 0.9. These values are consistent with ‘internal friction’ values for Anderson-type brittle faulting. Additionally, the angle of the macroscopic shear developed in specimen #110, which suffered spontaneous failure during its hold period, can be connected to its pre-failure loading state by a tie line LB-LY’, tangent to the localization curve. Thus, the macroscopic failures observed in these triaxial loading experiments appear to be consistent with the predictions of dynamic rescaling (Patton and Watkinson, 2010). In contrast, the microscopic Population B shear fractures correspond to the range 0.54 < $\kappa/\chi$ < 2.9. This spread might reflect the inhomogeneous distribution of lengths in the samples prior to loading, the effect of confining pressure, or both.

Data from the three spontaneously failed samples can be used to estimate the mechanical diffusivity as $\chi = \frac{l^2}{\tau}$. The resulting values fall in the range $2.3 \leq -\log \chi \leq 5.6$. These estimates are in or below the range $-\log \chi \leq 5.3$-5.7, predicted by theory (Patton and Watkinson, 2010), and overlap with ranges estimated from GPS strain rates, $4.3 \leq -\log \chi \leq 7$, GPS differential velocities, $4.4 \leq -\log \chi \leq 5.2$, and structural data, $4.2 \leq -\log \chi \leq 4.7$. This correlation suggests that combined analysis of data from experimental rock mechanics and GPS surveys will factor highly in further development of dynamic plate theory.

### 6.0 Isobaric shearing hypothesis

In a self-gravitating fluid body, pressure $p$ increases with depth according to the hydrostatic relation $p = \rho gz$, where $\rho$ is mass density, $g$ is gravitational acceleration, and $z$ is depth. Similarly, pressure increases with depth in a solid body according to a lithostatic relation, given by the tensor trace of its stress-energy density. The depth to which differential stresses can persist in any planet, then, depends on the definition of its stress-energy density. However, because even terrestrial planets are spheroidally-shaped, which
reflects the combined effects of their self-gravity and rotational momentum, the magnitude of these differential stresses must be small. Furthermore, given that temperature increases with depth (Figure 6b), and that microphysical mechanisms for solid-state creep are thermally activated, even these small differential stresses must diminish rapidly with depth. In the static case, they decay entirely, and the pressure becomes for all intents and purposes hydrostatic. Consequently, the maintenance of differential stresses at depth in a planet requires some dynamic process (McKenzie, 1967). For more than 40 years, this process has been assumed to conform to the assumptions of the standard Earth model (Bercovici et al., 2000; Bunge et al., 1997; Tackley, 2000). However, based on the foregoing thermomechanical analysis, it is likely that geodynamics is much more interesting than heretofore recognized.

With pressure and temperature as boundary conditions on a self-gravitating planet, and confining pressures much larger than potential differential stresses, it is hard to argue that material strength matters, except for the fact that the *dynamic rescaling theorem* (Patton and Watkinson, 2010) focuses deformation on the smallest crystalline structures of a solid system. This is necessary, so that the global dissipation of energy is minimized. An immediate consequence of this prediction is that shear waves can be propagated throughout a thermomechanical mantle, whereas in a viscous one, no such propagation is possible. Absent this theorem, however, one must accept the geodynamicist’s approximation, that over long time and length scales the mantle is effectively viscous. As plausible as this might sound, it is impossible to falsify. Moreover, it is inconsistent with the fact that rocks loaded in the laboratory exist in the solid-state. Thus, it is clear that the failure of the standard Earth model arises solely from a theoretical deficiency. On the other hand, with the dynamic rescaling theorem, the only substantive differences between deformation of a rock sample in the laboratory, and tectonic deformation of the Earth, are the relative magnitude of the confining pressure and effect of global conservation laws. In other words, shear localization at the planetary-scale must account for the incompatibility of rectilinear motions with the spheroidally curved geometry of the planet itself, while in the laboratory this is of no concern.

The statistically stable thermomechanics of non-linear elastic DG-2 materials are not altered by pressure. Consequently, even under extreme pressures the locus of its three energy thresholds and documented
scaling relationships can be expected to hold. Therefore, a thermomechanical Earth model can be formed simply by scaling up to Earth’s radial structure and applying pressure and temperature boundary conditions at its surface (Patton and Watkinson, 2009, 2010). The result (Figure 8) immediately predicts a variation of pressure and temperature expected for Earth’s mantle. Furthermore, it predicts that the outer colder parts of the mantle should be thermomechanically rigid, thermodynamically isothermal, and subject to brittle shear localization, while the deeper hotter parts should be thermomechanically ductile and thermodynamically adiabatic. Adiabaticity prevails as $\kappa/\chi \rightarrow 0$, consistent with depths in the lower mantle, and coincidentally where Birch (1952) showed it to pertain on the basis of seismic wave speed variations (Equation 1). The asthenosphere of a thermomechanical Earth is not adiabatic, because differential stresses (normal stress differences) there are large enough to cause shear localization. Moreover, the vanishing of seismicity at the asthenosphere-mesosphere boundary, ~700 km deep, reflects this fundamental change in thermodynamic conditions. For comparison, the variation in earthquake depth-energy release (Figure 5) is plotted in the last panel of Figure 8, where the most significant deviations from the per-event global baseline clearly correlate with the low velocity zone, and depths consistent with Earth’s lithosphere and tectosphere.

This model, being self-similar or fractal, can also be used to correlate crustal and upper mantle observations. Figure 9 depicts ‘crustal overtones’ of ThERM. For comparison, the variation in earthquake depth-energy release for all six margin types considered in this paper (Figure 4) are plotted in the last panel of Figure 9, where the most significant deviations from the global per-event baseline correlate to the seismic lid and crust. What does this remarkable correlation mean for dynamic plate theory?

For a self-gravitating solid elastic body, like a terrestrial planet, we can anticipate some degree of interplay between the inhomogeneous statistical distribution of length scales in the body, and the distribution of thermal and compositional lengths over which the body force of gravity might act. This interplay is expressed particularly in the structure of the thermomechanical boundary layer that forms adjacent to the cold surface of the planet, but also by the fact that elastic shear waves are propagated throughout the mantle. Thus, in order for any portion of a thermomechanical planet to suffer deformation, there must be measurable contrasts in material competence, as well as concentrated body forces. Furthermore, the
mechanical diffusivity $\chi$ must be greater than the thermal diffusivity $\kappa$ in deforming portions of this complex system. Consequently, the thermomechanical boundary layer that forms will always be $\zeta$ times thicker than the purely thermal one. Depth variations in seismic energy release on such a planet are therefore to be expected. Finally, given that pressure and temperature variations are explicitly predicted by theory, it is reasonable to suppose that variations in material competence will exhibit strong dependencies on bulk composition and volatile content.

In summary, the energy density function for a self-gravitating, inhomogeneous elastic body, dominated by statistical variability ($m > n$), must be a monotonically decreasing convex function of the radial coordinate and the entropy reduction. Furthermore, the entropy reduction itself is an inverse function of length. Heat absorbed by such a system will tend to increase the internal energy, but correspondingly decrease the entropy. Such a body will not readily evolve heat, except when regional conditions favor a return to more classical thermodynamics. In these subsystems, the energy density function would necessarily exhibit a positive slope. The monotonically increasing branches of the intrinsic and localization thresholds (dashed curves, Figure 7b) exhibit these characteristics, and also correlate with depths in the asthenosphere where magma generation generally is thought to take place (Figures 8 and 9). Magmatic differentiation provides a mechanism for generating density contrasts between continental and oceanic crust, the tectosphere, and average mantle.

These considerations are quite general and place, once and for all, the stress-energy density thresholds of DG-2 materials (Patton and Watkinson, 2005) in a coherent thermodynamic context. Consequently, the behavior of these ideal materials can be correlated with the pressure, temperature, age, and geometry of geological structures observed in outcrops, orogens, and terrestrial planets. For example, the outer parts of such planets are predicted to be relatively cold, competent, and subject to dynamic shear localization (“brittle”), while the inner parts are predicted to be relatively hot, incompetent (“ductile”), and structurally simple. For Earth, this is reflected in the remarkable correlation of spherically symmetric elastic models, like PREM (Dziewonski and Anderson, 1981), with the predicted depth distribution of isobaric shears in a body with a ~100-km thick lithosphere (Patton, 2001; Patton and Watkinson, 2009, 2010). As
demonstrated here, this correlation also holds for variations in earthquake depth-energy release (Figures 3-5). The following section discusses the implications of these findings for the interpretation of various geophysical data sets, and dynamic plate theory in general.

7.0 Discussion

7.1 Laboratory measured versus theoretical viscosity

Spontaneous shear dislocations are characteristic modes of deformation for solids under loading, which are related non-linearly to the strength of the loaded material. In a loading frame in the laboratory, rock strength can be reduced to a steady-state rate of energy dissipation in shear, i.e. ‘viscosity’. This is possible only because the loading frame itself is effectively rigid (‘stiff’) by comparison. But workers conducting these experiments, and theoreticians interpreting their results for geodynamic models, surely understand that the materials involved are solids. So why is the standard Earth model based on viscous fluid dynamics?

Another important macroscopic property of rocks is density, which is related to chemical composition and mineralogy via specific gravity. Perhaps, then, a better measure of rock ‘strength’ is the ratio of a rock’s ‘viscosity’ to its density. This ratio has dimension \( L^2 T^{-1} \), which has been called ‘diffusivity’. This latter term is descriptive, in that energy is being dissipated by the imposed deformation, and unburdened by the fluid mechanical connotations of the term ‘viscosity’.

For example, the rate of energy dissipation in the shearing of a rheological fluid depends on the ratio of its molecular viscosity (\( \mu \)) and its density (\( \rho \)). Although we might call this quantity the ‘shear stress diffusivity’, one usually hears the term kinematic viscosity (\( \nu = \mu/\rho \)).

Similarly, the rate of heat dissipation in a substance can be expressed as the ratio of its thermal conductivity (\( k \)) to the product of its heat capacity (\( C_p \)) and density (\( \rho \)), also having dimension \( L^2 T^{-1} \). This quantity is usually called thermal diffusivity (\( \kappa = k/\rho C_p \)), but perhaps some would prefer the term ‘thermal viscosity’?
Finally, the rate of energy dissipation in a deforming rheological solid can be expressed as the ratio of its ‘viscosity’ to its density, which might also be called ‘kinematic viscosity’. However, because this brings to mind many ideas that have naught to do with deforming solids, it does little to dispel the lexical confusion I am attempting to address. Consequently, based on my study of DG-2 materials, I suggest the term ‘normal-stress diffusivity’. However, in a pinch one might simply try mechanical diffusivity ($\chi = d^2/\tau$), as it provides a descriptive counterpoint to the thermal diffusivity, above. Semantics aside, the crucial thing here is that geologists acknowledge the fundamental theoretical difference between the mode of energy dissipation in rheological fluids, and the possible modes of energy dissipation in rheological solids. The theory discussed here is parameterized by the ratio $\kappa/\chi$, which might be called thermomechanical competence. Regardless of its name, it provides new insight into the nature of Earth’s plate-like blocks, and their relative horizontal and vertical motions.

### 7.2 Implications

There are good reasons to think that the observed earthquake depth-energy release signal is real. First, while it is known that hypocenter locations are strongly influenced by reference model, centroid locations are not. This is because hypocenter locations are triangulated using body wave travel times, while centroids are located using a summation of long-period body wave and free-oscillation modes. In fact, the relocation algorithm for centroids is remarkably insensitive to small scale a priori structure (Aki and Richards, 2002). Furthermore, earthquakes are but one, relatively minor, mode of energy dissipation for the planet, and hence must at some level be related to more general deformation processes in a self-gravitating body (Chao and Gross, 1995; Chao et al., 1995). If the observed coincidence between the earthquake depth-energy release curves and ThERM is real, then it provides crucial support for the hypothesis that Earth’s tectonic plates are part of a thermomechanical boundary layer, as much as 700-km thick, that has developed over the course of Earth's history (Patton, 2001; Patton and Watkinson, 2009).

This coincidence has interesting implications for post-orogenic extensional collapse of orogens, particularly when compared with recent reviews of pressure-temperature-time paths for metamorphic rocks. Maximum pressure estimates, based on equilibrium mineral phase assemblages, for mid-crustal schists and gneisses...
also commonly coincide with ThERM shear modes (Brown, 2007). A striking example of this is the depth-
pressure classification of detachment faults exposed in metamorphic core complexes in the northwestern
United States and southwestern Canada, where maximum pressures coincide with H4, L1, and L2 (Patton
and Watkinson, in review).

If observed variations in earthquake depth-energy release are interpreted as proxies for strength, or more
specifically for depth and lateral contrasts in thermomechanical competence ($\kappa/\chi$), then the weakest parts of
the crust globally are located between 15- and 25-km depths (Figure 3b). Regionally the weakest crust is
found in continental transform, oceanic ridge/transform, continental rift, and Himalayan-type convergent
margins (Figures 4, 5). The appearance of ocean crust in this list probably reflects its relative thinness,
rather than its intrinsic weakness. The upper and lower boundaries of this weak layer occur at depths
consistent with predicted shearing modes H4 and L1 of ThERM (Patton and Watkinson, 2010). Finally,
there are additional triplications and slope changes at about 37-km depth, coincident with mode L2 of
ThERM, also suggestive of a vertical change in thermomechanical competence. As mentioned above,
metamorphic tectonites with pressures of this magnitude are common in collapsed orogens (Brown, 2008).
It is likely that the integration of crustal pressure-temperature-time data with centroid moment release
studies at a regional scale will be a potentially fruitful avenue for future research.

Significant differences in the number of hypocenters and centroids occur to about 40-km depth (Table 2).
This simply reflects the depth range most affected by the procedural relocation of events in the CMT
catalog. However, tectonically notable differences exist between the map and depth distributions of
hypocenters and centroids above about 25-km depth, roughly coincident with mode L1 of ThERM (Patton
and Watkinson, 2010). Centroids populate both convergent and divergent margins throughout this typical
'crustal' depth range. In contrast, hypocenters largely occur above 15-km depth, with rare events as deep as
25 km at divergent margins. Taken together, these distributions suggest a global maximum seismogenic
crustal thickness of about 25 km (Figure 4a, b).
Using a thermal diffusivity of $10^{-6}$ m$^2$s$^{-1}$, typical for silicates (Vosteen and Schellschmidt, 2003), the equivalent thermal age for the 25-km-thick seismogenic crust is about 20 Ma. Presumably, this is the time needed for decompression melting processes at oceanic ridges to settle down to a steady-state (Crosby et al., 2006). Recent estimates of pressure and temperature ranges for MORB extraction, based on natural compositions, are about 1.2-1.5 GPa and 1250-1280 °C (Presnall and Gudfinnsson, 2008). These pressures correspond to depths of about 40-50 km, consistent with the depth extent of seismic energy release at oceanic ridges (Figure 4a, orange curve). Furthermore, these workers propose that fracturing of newly formed lithosphere induces the explosive formation and escape of CO$_2$ vapor, which drives MORB volcanism, while the source region for material forming the oceanic lithosphere extends no deeper than about 140 km. In this and other models, the MORB source region therefore lies comfortably below $L_2$.

Hypocenters and centroids are common in the Alpine-Himalayan belt to depths of about 100 km, roughly coincident with mode $L_4$ of ThERM, with rare events as deep as about 172 km, coincident with mode $M_1$. Events beneath the Hindu-Kush occur to about 260-km depth, consistent with mode $M_2$. All events deeper than about 260 km appear to be associated with down going slabs from past or present subduction of oceanic lithosphere. Note also that anelastic tomography (Romanowicz, 1994) reveals significant lateral variations in the attenuation of shear waves in the uppermost 250 km of the mantle, which are correlated with oceanic ridges and continental shields. The shear attenuation pattern below this depth apparently shifts, correlating with the global distribution of volcanic hotspots. Consequently, regions of Earth's asthenosphere underlying oceanic ridges and hotspots attenuate more low-frequency seismic energy than do cratons and shields, which can remain seismically active to similar depths. This is the same depth range as that potentially impacted by material coupling of surface waves in the thermomechanical boundary layer of ThERM (Table 5).

Using a typical thermal diffusivity of $10^{-6}$ m$^2$s$^{-1}$, the equivalent thermal ages for 100-km thick lithosphere, 172-km thick upper tectosphere, and 255-km thick lower tectosphere are about 310 Ma, 940 Ma and 2Ga, respectively. In contrast, the equivalent thermal age for a layer 690-km thick is about 15 Ga, more than three times the age of the Earth and comparable to the age of the universe. Clearly, it is implausible that
the structure of the asthensphere is solely due to thermal diffusion processes. Consequently, some dynamic convective (advective?) process is needed to maintain differential stresses to these depths (McKenzie, 1967).

Geologic history has been divided into ocean basin time (0-200 Ma), plate tectonic time (200-950 Ma), and “pre-tectonic” time (950-2300 Ma), on the basis of marine and continental geology (Moores and Twiss, 1995). Also, based on Brown's (2008) recent review, the equivalent thermal ages, above, correspond to the amalgamation times for supercontinents Pangea, Rodinia, and Nuna. Reconciling the global smoothness of thermal (and viscous) diffusion with the spatial complexity and local non-smoothness of tectonic deformation processes requires a model that incorporates a self-consistent mechanism for strain localization with depth. Isobaric shearing in the asthenosphere offers a more comprehensive explanation for these first-order observations, than does the standard Earth model. It is time for the strength of solid earth materials to be included in dynamic plate theory.

As noted in Section 1.0, the spectral characteristics of Earth’s gravity-topography correlation and admittance, combined with the adiabaticity of the lower mantle, are consistent with vigorous convection in the sublithospheric mantle of the standard Earth model, at lateral scales $\lambda_{\beta} > 1000 \text{ km}$. This conclusion complements those of regional isostasy, where near surface loads are supported by the flexural rigidity of the crust and lithosphere, at lateral scales $\lambda_{\beta} < 1000 \text{ km}$. This latter range of wavelengths, down to about 50 km, has been called the ‘diagnostic waveband of flexure’ ((Watts, 2001), pg. 178). Furthermore, the inviscid (i.e., mechanically indeterminate) nature of the asthenosphere in flexural isostasy does not conflict with the fluid nature of the sublithospheric mantle in the standard model. Consequently, it is reasonable to conclude that mantle convection actively supports Earth’s longest-wavelength gravity and geoid anomalies (Hager, 1984; McKenzie, 1967; Panasyuk and Hager, 2000; Richards and Hager, 1984; Steinberger et al., 2010). Based on recent crustal thickness and flexural modeling, Steinberger et al (2010) suggest that Earth’s gravity anomalies with $\lambda_{\beta} \geq 650 \text{ km} \ (l \leq 30)$ are probably due to sources in the sublithospheric mantle, while those at shorter wavelengths have sources predominantly in the lithosphere. This downward
revision of the upper limit for the diagnostic waveband therefore provides more room for the operation of
the standard model, including mantle plumes.

Given the presence of a thermomechanical boundary layer at Earth’s surface, some 700-km thick, as
suggested by this study, the interpretation of gravity-topography spectra should be revisited. Based on
earlier incipient modes analysis, a competent layer near the surface of a dynamic thermomechanical planet
can be expected to develop low-amplitude material waves with wavelengths ranging from 2 to \( \zeta \) times the
layer thickness. Assuming the depths of the isobaric shears (Table 3) define a series of layer thicknesses,
the equivalent ranges of angular degree can be worked out (Table 5). Coincidentally, the long-wavelength
limit for folding of a layer M2 thick is \( l = 22 \), roughly coincident with the roll off in the gravity-topography
correlation (Wieczorek, 2007). Furthermore, the empirical limit suggested by Steinberger et al (2010) is
bracketed by the short-wavelength M4 and the long-wavelength M1 limits, and coincident with the short-
wavelength M3 limit. In other words, the correlation they propose is supported by these findings, provided
that the lithosphere is taken to include the continental tectosphere. However, the implications of
thermomechanical theory for convection in the deep mantle are dramatically different from those of the
standard model. For example, the long-wavelength M3 and M4 limits are \( l = 8 \) (Table 5). Consequently,
gravity and geoid anomalies only with \( \lambda_{\chi} > 2500 \text{ km} (l < 8) \) can be unequivocally associated with lower
mantle convection. Coincidentally, this is the scale of robust lateral variations imaged by seismic
tomography (Dziewonski et al., 1977; Dziewonski and Woodward, 1992; Gu et al., 2001). Furthermore,
given the inverse temperature dependence of thermal expansivity for inhomogeneous elastic solids, any
convection process in the lower mantle is likely to be rather sluggish. Detailed study of these intriguing
spectral observations is in progress. Finally, material coupling in these layers is likely to have a measurable
impact on the attenuation of intermediate and long-period surface waves, consistent with the predicted non-
adiabaticity of the upper mantle (Durek and Ekstrom, 1996; Romanowicz, 1995).

8.0 Conclusion

Global plate motions are the result of a complex planetary-scale rock mechanics experiment, where the
motive force of gravity drives thermomechanically competent oceanic lithosphere into the relatively
weaker rock units of the continents. Under extreme pressures, isobaric shearing at discrete depths facilitates toroidal plate motions, i.e., Euler rotations of spheroidal caps, while minimizing global energy dissipation. The fractal depth distribution of these disclinations is dictated by the statistically-stable thermomechanics of shear localization in inhomogeneous non-linear elastic self-gravitating solids, and scales with the thickness of mature oceanic lithosphere, where the greatest density contrasts and gravitational body forces reside. At low pressures, shear localization in crustal rocks occurs as dislocations at finite angles with respect to the shortening direction, with a 30 degree angle being the most likely. Consequently, relatively low-angle (~30°) reverse faults, steep (~60°) normal faults, and triple junctions with orthogonal or hexagonal symmetries are likely to form in regions of crustal shortening, extension, and transverse motion, respectively. In convergent plate boundary regions, this results in the overall thickening and seismogenic deformation of weaker rocks in the zone. Once convergence ceases, these regional high-potential energy welts relax over 30 Ma periods, often exposing equilibrium assemblages of middle- and lower-crustal rocks at the surface which are bounded by crustal scale shear zones. Deep crustal migmatites are produced by dehydration anatexis during shortening, and heat release following gravitational collapse drives further melting and differentiation of the crust. Equilibrium pressures in these crystalline cores are consistent with exhumation from 25- and 37-km depths, coincident with the L1 and L2 isobaric shears of ThERM. At extensional plate margins, brittle failure of crustal rocks depressurizes the subjacent mantle, resulting in its partial fusion and the subsequent eruption of basaltic lavas. These lavas cool and solidify, preserving a record of the ambient magnetic field. At transform margins, seismic energy dissipation is limited to about 25-km depth, but can be distributed over a wide region, depending on the competence of the rocks on either side of the fault. Wider shear zones are likely in continental regions, because of the lower competence of felsic crust. The dominance of toroidal motions at Earth’s surface can be attributed to the combination of liquid water, which has a profound weakening effect on common rocks and minerals, and the localization of shear at discrete depths within the thermomechanical Earth. The absence of plate tectonics on other terrestrial planets is simply due to the small number of known examples. Furthermore, the correlation and admittance of Earth’s gravity-topography spectra can be reconciled with this novel thermomechanical theory. In consequence, mantle convection, in the sense of the standard model, is possible only in the deep mantle at a scale $\lambda_5 > 2500$ km ($l < 8$), consistent with the scale of robust lateral
variations imaged by seismic tomography. Finally, the temperature dependence of thermal expansivity for this thermomechanical Earth makes vigorous convection in the lower mantle unlikely. The dominant mode of thermal convection for the planet, therefore, looks and acts like a dynamic version of plate tectonics.

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References

Adams, L.H., Williamson, E.D., 1923. The composition of the Earth's interior. Smithsonian Report, 241-260.

Aki, K., Richards, P.G., 2002. Quantitative Seismology, 2nd ed. University Science Books, Sausalito, CA.

Amante, C., Eakins, W., 2009. ETOP01 1 arc-minute global relief model: procedures, data sources and analysis. NOAA, Boulder, p. 25.

Anderson, D.L., 1989. Theory of the Earth. Blackwell Scientific Publications, Boston.

Anderson, D.L., 2007. New Theory of the Earth. Cambridge University Press, Cambridge.

Artemieva, I.M., 2009. The continental lithosphere: Reconciling thermal, seismic, and petrologic data. Lithos 109, 23-46.

Artemieva, I.M., Billien, M., Leveque, J.-J., Mooney, W.D., 2004. Shear wave velocity, seismic attenuation, and thermal structure of the continental upper mantle. Geophysical Journal International 157, 607-628.

Artemieva, I.M., Mooney, W.D., 2001. Thermal thickness and evolution of Precambrian lithosphere: A global study. Journal of Geophysical Research 106, 16387-16414.

Atwater, T., 1970. Implications of plate tectonics for the Cenozoic tectonic evolution of western North America. BGS A 81, 3513-3536.

Barrell, J., 1914. The strength of the Earth's crust. VI. Relations of isostatic movements to a sphere of weakness - the asthenosphere. Journal of Geology 22, 655-683.

Benioff, H., 1954. Orogenesis and deep crustal structure: additional evidence from seismology. Bulletin Geological Society America 65, 385-400.

Bercovici, D., Ricard, Y., Richards, M.A., 2000. The relation between mantle dynamics and plate tectonics: a primer, in: Richards, M.A., Gordon, R.G., van der Hilst, R.D. (Eds.), The history and dynamics of global plate motions. American Geophysical Union, Washington D.C., pp. 5-46.

Birch, F., 1952. Elasticity and constitution of Earth's interior. Journal of Geophysical Research 57, 227-286.

Boyd, F.R., 1973. A pyroxene geotherm. Geochimica et Cosmochimica Acta 37, 2533-2546.

Brown, M., 2007. Metamorphic conditions in orogenic belts: A record of secular change. International Geology Review 49, 193-234.

Brown, M., 2008. Characteristic thermal regimes of plate tectonics and their metamorphic imprint throughout Earth history: when did Earth first adopt a plate tectonics mode of behavior? GSA Special Publication 440, 97-128.

Bunge, H.-P., Richards, M.A., Baumgardner, J.R., 1997. A sensitivity study of three-dimensional spherical mantle convection at 10^8 Rayleigh number: effects of depth-dependent viscosity, heating mode, and an endothermic phase change. Journal of Geophysical Research 102, 11991-12007.

Chandrasekhar, S., 1967. An Introduction to the Study of Stellar Structure. Dover Publications, Mineola, NY.

Chao, B.F., Gross, R.S., 1995. Changes in the Earth's rotational energy induced by earthquakes. Geophysical Journal International 122, 776-783.

Chao, B.F., Gross, R.S., Dong, D.-N., 1995. Changes in global gravitational energy induced by earthquakes. Geophysical Journal International 122, 784-789.
Clauser, C., Huenges, E., 1995. Thermal conductivity of rocks and minerals, AGU Reference Shelf 3: A Handbook of Physical Constants - Rock Physics and Phase Relations AGU, Washington D.C., pp. 105-125. Crosby, A.G., McKenzie, D.P., Sclater, J.G., 2006. The relationship between depth, age and gravity in the oceans. Geophysical Journal International 166, 553-573. Dewey, J.F., Bird, J.M., 1970. Mountain belts and the new global tectonics. Journal of Geophysical Research 75, 2625-2647. Dietz, R.S., 1961. Continent and ocean basin evolution by spreading of the sea floor. Nature 190, 854-857. Durek, J.J., Ekstrom, G., 1996. A radial model of anelasticity consistent with long-period surface-wave attenuation. Bulletin of the Seismological Society of America 86, 144-158. Dziewonski, A.M., Anderson, D.L., 1981. Preliminary reference Earth model. Physics of the Earth and Planetary Interiors 25, 297-356. Dziewonski, A.M., Chou, T.-A., Woodhouse, J.H., 1981. Determination of earthquake source parameters from waveform data for studies of global and regional seismicity. Journal of Geophysical Research 86, 2825-2852. Dziewonski, A.M., Hager, B.H., O'Connell, R.J., 1977. Large-scale heterogeneities in the lower mantle. Journal of Geophysical Research 82, 239-255. Dziewonski, A.M., Woodhouse, J.H., 1983. An experiment in systematic study of global seismicity: centroid-moment tensor solutions for 201 moderate and large earthquakes of 1981. Journal of Geophysical Research 88, 3247-3271. Dziewonski, A.M., Woodward, R.L., 1992. Acoustic imaging at the planetary scale, in: Emert, H., Harjes, H.-P. (Eds.), Acoustical Imaging. Plenum, New York, pp. 785-797. Einstein, A., 1916. Die Grundlage der allgemeinen Relativitatstheorie, in: Hawking, S. (Ed.), A Stubbornly Persistent Illusion: The Essential Scientific Writings of Albert Einstein. Running Press, Philadelphia, pp. 46-98. Ekstrom, G., Nettles, M., 2011. The Global Centroid-Moment-Tensor (CMT) Project. Lamont Doherty Earth Observatory-Columbia University. Ekstrom, G., Nettles, M., Dziewonski, A.M., 2012. The global CMT project 2004-2010: Centroid-moment tensors for 13017 earthquakes. Physics of the Earth and Planetary Interiors 200, 1-9. Engdahl, E.R., Van der Hilst, R.D., Buland, R., 1998. Global teleseismic earthquake relocation with improved travel times and procedures for depth determination. Bulletin of the Seismological Society of America 88, 722-743. Frohlich, C., 1989. The nature of deep-focus earthquakes. Annual Reviews of Earth and Planetary Science 17, 227-254. Green, H.W., 2005. New Light on Deep Earthquakes. Scientific American, 1-9. Gu, Y.J., Dziewonski, A.M., Su, W., Ekstrom, G., 2001. Models of mantle shear velocity and discontinuities in the pattern of lateral heterogeneities. Journal of Geophysical Research 106, 11169-11199. Hager, B.H., 1984. Subducted slabs and the geoid: constraints on mantle rheology and flow. Journal of Geophysical Research 89, 6003-6015. Hess, H.H., 1962. History of the ocean basins, in: Engel, A.E.J., James, H.L., Leonard, B.F. (Eds.), Petrological Studies: A Volume in Honor of A.F. Buddington. Geological Society of America, Boulder, CO, pp. 599-620.
Hill, R.E.T., Hutchinson, J.W., 1975. Bifurcation Phenomena in the Plane Tension Test. Journal of the Mechanics and Physics of Solids 23, 239-264.

Hobbs, B.E., Ord, A., Regenauer-Lieb, K., 2011. The thermodynamics of deformed metamorphic rocks: A review. Journal of Structural Geology 33, 758-818.

Isacks, B., Oliver, J., Sykes, L.R., 1968. Seismology and the new global tectonics. Journal of Geophysical Research 73, 5855-5899.

Jackson, I., Webb, S., Weston, L., Boness, D., 2005. Frequency dependence of elastic wave speeds at high temperature: a direct experimental demonstration. Physics of the Earth and Planetary Interiors 148, 85-96.

James, D.E., Boyd, F.R., Schutt, D.L., Bell, D.R., Carlson, R.W., 2004. Xenolith constraints on seismic velocities in the upper mantle beneath southern Africa. Geochemistry Geophysics Geosystems 5, 1-32.

Jordan, T.H., 1975. The continental tectosphere. Reviews of Geophysics and Space Physics 13, 1-12.

Jordan, T.H.C., Beroza, G., Cornell, C.A., Crouse, C.B., Dieterich, J., Frankel, A., Jackson, D.D., Johnston, A., Kanamori, H., Langer, J.S., McNutt, M.K., Rice, J., Romanowicz, B.A., Sieh, K., Somerville, P.G., 2003. Living on an Active Earth: Perspectives on Earthquake Science. The National Academies Press, Washington, D.C.

Karato, S., 1993. Importance of Anelasticity in the Interpretation of Seismic Tomography. Geophysical Research Letters 20, 1623-1626.

Katz, O., Reches, Z., 2002. Pre-failure damage, time-dependent creep and strength variations of a brittle granite, 5th International Conference on Analysis of Discontinuous Deformation. Balkema, Rotterdam, Ben-Gurion University, Israel, pp. 1-5.

Katz, O., Reches, Z., 2004. Microfracturing, damage and failure of brittle granites. Journal of Geophysical Research 109, 10.1029/2002JB001961.

Kaula, W.M., 1972. Global gravity and tectonics, in: Robertson, E.C., Hays, J.F., Knopoff, L. (Eds.), The Nature of the Solid Earth. McGraw-Hill, New York, pp. 385-405.

Kennett, B.L.N., Engdahl, E.R., 1991. Travel times for global earthquake location and phase identification. Geophysical Journal International 105, 429-465.

Lavenda, B.H., 1995. Thermodynamics of Extremes. Albion Publishing, Chichester.

Le Pichon, X., 1968. Sea-floor spreading and continental drift. Journal of Geophysical Research 73, 3661-3697.

Luh, P.C., 1975. Free oscillations of the laterally inhomogeneous Earth: Quasidegenerate multiplet coupling. Geophysical Journal of the Royal Astronomical Society 32, 203-218.

Madariaga, R., 1972. Torsional free oscillations of the laterally heterogeneous Earth. Geophysical Journal of the Royal Astronomical Society 27, 81-100.

McKenzie, D.P., 1967. Some remarks on heat flow and gravity anomalies. Journal of Geophysical Research 72, 6261-6273.

McKenzie, D.P., 1969. Speculations on the consequences and causes of plate motions. Geophysical Journal of the Royal Astronomical Society 18, 1-32.

McKenzie, D.P., Morgan, W.J., 1969. Evolution of triple junctions. Nature 224, 125-133.

McKenzie, D.P., Parker, R.D., 1967. The north Pacific: An example of tectonics on a sphere. Nature 216, 1276-1280.

Moores, E.M., Twiss, R.J., 1995. Tectonics. W. H. Freeman and Company, New York.
Morgan, W.J., 1968. Rises, trenches, great faults, and crustal blocks. Journal of Geophysical Research 73, 1958-1982.

Needleman, A., 1979. Non-Normality and Bifurcation in Plane Strain Tension and Compression. Journal of the Mechanics and Physics of Solids 27, 231-254.

Panasyuk, S.V., Hager, B.H., 2000. Models of isostatic and dynamic topography, geoid anomalies, and their uncertainties. Journal of Geophysical Research 105, 28199-28209.

Parsons, B., Sclater, J.G., 1977. An analysis of the variation of ocean floor bathymetry and heat flow with age. Journal of Geophysical Research 82, 803-827.

Patton, R.L., 1997. On the general applicability of relaxation modes in continuum models of crustal deformation, Department of Geology. Washington State University, Pullman, p. 129.

Patton, R.L., 2001. Earth's potentiosphere. EOS Transactions AGU Fall Meeting Supplement 82, T41B-0860.

Patton, R.L., 2005. Non-linear viscoelastic models of rock folding and faulting, in: Hancock, H., Fisher, L., Baker, T., Bell, T., Blenkinsop, T.G., Chapman, L., Cleverley, J.S., Collins, B., Duckworth, R., Evins, P., Ford, P., Oliver, A., Oliver, N., Rubenach, M., Williams, P. (Eds.), STOMP: Structure, Tectonics and Ore Mineralization Processes.

James Cook, Townsville, QLD, Australia, p. 103.

Patton, R.L., Manoranjan, V.S., Watkinson, A.J., 2000. Plate formation at the surface of a convecting fluid, XIIIth International Congress on Rheology. The British Society of Rheology, Cambridge, U.K., pp. 167-169.

Patton, R.L., Watkinson, A.J., 2005. A viscoelastic strain energy principle expressed in fold-thrust belts and other compressional regimes. Journal of Structural Geology 27, 1143-1154.

Presnall, D.C., Gudfinnsson, G.H., 2008. Origin of the oceanic lithosphere. Journal of Petrology 49, 615-632.

Richards, M.A., Hager, B.H., 1984. Geoid anomalies in a dynamic Earth. Journal of Geophysical Research 89, 5987-6002.

Ringwood, A.E., 1991. Phase transformations and their bearing on the constitution and dynamics of the mantle. Geochimica et Cosmochimica Acta 55, 2083-2110.
Romanowicz, B., 1994. Anelastic tomography: a new perspective on upper mantle thermal structure. Earth and Planetary Science Letters 128, 113-121.

Romanowicz, B., 1995. A global tomographic model of shear attenuation in the upper mantle. Journal of Geophysical Research 100, 12375-12394.

Rudnick, R.L., Gao, S.S., 2003. Composition of the Continental Crust, Treatise on Geochemistry. Elsevier, pp. 1-64.

Steinberger, B., Werner, S.C., Torsvik, T.H., 2010. Deep versus shallow origin of gravity anomalies, topography and volcanism on Earth, Venus and Mars. Icarus 207, 564-577.

Tackley, P.J., 2000. The quest for self-consistent generation of tectonic plates in mantle convection models, in: Richards, M.A., Gordon, R.G., van der Hilst, R.D. (Eds.), The history and dynamics of global plate motions. American Geophysical Union, Washington, D.C., pp. 47-72.

Truesdell, C., 1964. The natural time of a viscoelastic fluid: Its significance and measurement. Physics of Fluids 7, 1134-1142.

Turcotte, D.L., Oxburgh, E.R., 1967. Finite amplitude convective cells and continental drift. Journal of Fluid Mechanics 28, 29-42.

Vine, F.J., Matthews, D.H., 1963. Magnetic anomalies over oceanic ridges. Nature 199, 947-949.

Vosteen, H.-D., Schellschmidt, R., 2003. Influence of temperature on thermal conductivity, thermal capacity and thermal diffusivity for different types of rock. Physics and Chemistry of the Earth 28, 499-509.

Watts, A.B., 2001. Isostasy and Flexure of the Lithosphere. Cambridge University Press, Cambridge.

Wegener, A., 1966. The origin of continents and oceans, 4th ed. Dover, New York.

Wessel, P., Smith, W.H.F., 1998. New improved version of the Generic Mapping Tools released. EOS Transactions American Geophysical Union 76, 579.

Widmer, R., Masters, G., Gilbert, F., 1991. Spherically symmetric attenuation within the Earth from normal mode data. Geophysical Journal International 104, 541-553.

Wieczorek, M.A., 2007. Gravity and topography of the terrestrial planets, in: Schubert, G. (Ed.), Treatise on Geophysics. Elsevier.

Wilson, J.T., 1965. A new class of faults and their bearing on continental drift. Nature 207, 343-347.
Figure captions

**Figure 1.** Plots of a) hypocenter and b) centroid depths for the 20646 algorithmically relocated earthquakes from the global CMT catalog (Ekstrom and Nettles, 2011), with origin times in the period January 1976 through December 2010. With increasing depth, the color breaks coincide with the isobaric shears H4, L1, L2, L4, M1, M2, and M4, of the ThERM (Patton and Watkinson, 2009, 2010). The CMT relocation procedure generally maps crustal seismicity to deeper levels, as indicated by the contrasting orange and yellow hues of ocean ridge earthquakes.

**Figure 2.** Plots of centroid depths for earthquakes in six tectonic settings (Table 4): a) continental transform; b) Himalaya-type convergence; c) oceanic ridge/transform; d) island arc-type convergence; e) continental rift; and f) Andean-type convergence. These subsets of the CMT catalog are used to compute the earthquake depth-energy release curves appearing in Figures 3-5.

**Figure 3.** Plots of seismic depth-energy release $\Sigma M_w (z; 3)$, smoothed using a 3-km thick boxcar filter and color-keyed by tectonic setting (Table 4, Figure 2): a) Hypocenter depth-release curves exhibit sharp peaks at about 10- and 30-km depths and appear somewhat artificial, most likely due to the quick-epicenter location routines used by the USGS and ISC; b) Centroid depth-release curves span a broad range of depths and exhibit peak energy release at 17- to 18-km depths. The peak amplitude of most curves is proportional to the number of earthquakes in that tectonic setting. About $\frac{3}{4}$ of the earthquakes in this catalog occur in island arc-type convergence zones.

**Figure 4.** Plots of seismic depth-energy release $\Sigma M_w (z; 3) / N$, smoothed using a 3-km thick boxcar filter, normalized by the number of events $N$, and color-keyed by tectonic setting (Table 4, Figure 2): a) depth-release curves at divergent and transform margins extend no deeper than about 50 km, and exhibit large positive and negative excursions from the global average at typical crustal levels. The deepest earthquakes coincide with the MORB source region (Presnall and Gudfinnsson, 2008); b) depth-release
curves at convergent margins extend to depths of about 700 km, and also exhibit large positive and negative excursions from the global average at typical crustal levels.

**Figure 5.** Color-keyed plots of seismic energy release with depth $\sum M_w(z;10)/N$, normalized by the number of events $N$ in convergent margins, and smoothed with a boxcar filter 10-km thick. While the seismic energy release with depth in island arc-type convergent margins (purple) closely approximates the global baseline (black dashed), the Himalaya-type (green) and Andean-type (blue) margins display marked deviations in depth ranges. Positive and negative excursions of these curves likely indicate lateral variations mantle strength, and suggest stratification consistent with modes $L_4$, $M_1$, and $M_2$ of ThERM (Patton and Watkinson, 2009).

**Figure 6.** Graphical summary of the statistical thermodynamics of strained inhomogeneous elastic and self-gravitating matter configurations: a) mechanical variability bears little insight for DG-2 materials; b) statistical variability offers crucial insight for the thermodynamics of shear localization in DG-2 materials.

**Figure 7.** Correlation of experimental data from fourteen samples of the Mt. Scott granite, subjected to load-hold analysis (Katz and Reches, 2002, 2004), with the predictions of statistical thermodynamics and deformation modes of DG-2 materials: a) Normalized differential stresses (NDS) on intact samples ($\psi = \zeta$ NDS) are plotted as functions of diameter/length ($d/l$) for load-hold (green circles, red diamonds) and load-to-failure tests (orange diamonds). Three samples (red diamonds) spontaneously failed during the designated hold period. All macroscopically failed samples (diamonds) plot above the localization threshold curve ($\psi^L$, orange) given the range of standard error in Coulomb strength for the Mt. Scott granite (586 ± 16 MPa, dashed curves, inset); b) Populations of microscopic cracks with respect to the loading direction, observed post-loading, fall in the range $\frac{1}{2} < \kappa/\chi < 2.9$, while macroscopic shears fall in the range $0.6 < \kappa/\chi < 0.9$. The thermomechanically rigid portion of this system is the stiff load frame itself, which by definition plots at $\kappa/\chi = 1$. The wide range of microscopic crack angles probably reflects the prior geometry of sample grain-size and fabric.
Figure 8. Comparison of ThERM (Patton and Watkinson, 2009), scaled to a fundamental thickness of 99.54 km representing mature oceanic lithosphere, with depth variations in compressional $V_p$ and shear $V_s$ wave speeds, density $\rho$ (Dziewonski and Anderson, 1981), and normalized seismic depth-energy release $\Sigma M_w(z)/N$ (Figure 4b). The thickness of cratons and shields (Artemieva and Mooney, 2001) correlate with the $M_1$ and $M_2$ shears of ThERM (Patton and Watkinson, 2009), while the depth cut-off of seismicity, at about 700-km depth (Frohlich, 1989), correlates with $M_{3-4}$. Given that thermomechanical competence generally decreases with depth in ThERM, subducting slabs are likely to freely enter the lower mantle, in contrast to the mesosphere hypothesis (Isacks and Molnar, 1969; Isacks et al., 1968). The re-orientation of pressure and tension axes in deep earthquake focal mechanisms are therefore likely to result from anti-crack shear ruptures in metastable spinel mineral species at very high pressures (Green and Burnley, 1989). The depth cut-off of seismicity might result from a rapid decrease in entropy density at 700-km depth (Patton and Watkinson, in review).

Figure 9. Comparison of the crustal ‘overtones’ of ThERM (Patton and Watkinson, 2009), scaled to a thickness of $F \zeta^{-1} = 14.4$ km representing the brittle crust, with depth variations in compressional $V_p$ and shear $V_s$ wave speeds, density $\rho$ (Dziewonski and Anderson, 1981), and normalized seismic depth-energy release $\Sigma M_w(z)/N$ (Figure 4). The greatest variations of earthquake depth-energy release correlate with the low-velocity zone, as well as several distinct levels in the crust, lithosphere, and tectosphere. In regions where the tectosphere is absent, the asthenosphere resides subjacent to the crust.
Energy Density ($\Delta U$), Entropy Density ($\Delta S$); Radius ($R$)

**a)** MECHANICAL ($m < n$)

- Heat capacity $T(dS/dT)$
- Fewer states at lower $T$
- Elongation upon cooling

**b)** STATISTICAL ($m > n$)

- Heat capacity $dU/dT$
- Fewer states at higher $T$
- Elongation upon heating
Stress−Energy Density ($\psi$)

- $\psi = \zeta (\sigma_1 - \sigma_3)/586$

Wavelength ($\kappa/\chi$)$^{-1}$

Thermomechanical Competence ($\kappa/\chi$)

- Macroscopic Shear (#110; $\sigma_3 = 41$ MPa)
- Macroscopic Shears ($\sigma_3 = 14$–28 MPa)
- POP. A (Intragranular Cracks), Axial Split ($\sigma_3 \sim 0$ MPa)
- POP. B (Shear Cracks)

Angle (arctan($\kappa/\chi$); degrees)

- $\kappa/\chi = d/l$

Macrosopic Shears ($\sigma_3$)

- Mt. Scott Granite

DISLOCATION MODES

HARMONIC MODES
| $M_w$ range | Hypocenters | Centroids | Difference |
|------------|-------------|-----------|------------|
| 0-1        | 199         | -         | (199)      |
| 1-2        | -           | -         | -          |
| 2-3        | -           | -         | -          |
| 3-4        | 4           | -         | (4)        |
| 4-5        | 3849        | 2102      | (1747)     |
| 5-6        | 15262       | 16053     | 791        |
| 6-7        | 1317        | 2182      | 865        |
| 7-8        | 14          | 292       | 278        |
| 8-9        | 1           | 16        | 15         |
| 9-10       | -           | 1         | 1          |
| Total      | 20646       | 20646     | 0          |
Table 2. Hypocenter and centroid statistics by depth

| Depth range | Hypocenters | Centroids | Difference | Color range |
|-------------|-------------|-----------|------------|-------------|
| 0-14.4      | 3127        | 1031      | (2096)     | red-orange  |
| 14.4-25     | 1051        | 5268      | 4217       | orange-yellow|
| 25-37       | 5667        | 3471      | (2196)     | yellow-green|
| 37-99.5     | 5511        | 5495      | (16)       | green-blue  |
| 99.5-172    | 2520        | 2605      | 85         | blue-purple |
| 172-255     | 941         | 929       | (12)       | purple-pink |
| 255-690     | 1828        | 1838      | 10         | pink-white  |
| 690-800     | 1           | 9         | 8          | black       |
| Total       | 20646       | 20646     | 0          |             |
Table 3. Depths to isobaric shears of ThERM (Patton and Watkinson, 2010)

| Shear | Depth* (km) | Pressure** (GPa) |
|-------|-------------|------------------|
| H1    | 3.6         | 0.10             |
| H2    | 5.3         | 0.14             |
| H3    | 13.8        | 0.37             |
| H4    | 14.4        | 0.39             |
| L1    | 25          | 0.67             |
| L2    | 37          | 1.04             |
| L3    | 96          | 2.96             |
| L4    | 100         | 3.00             |
| M1    | 172         | 5.40             |
| M2    | 255         | 8.27             |
| M3    | 663         | 23.4             |
| M4    | 690         | 24.5             |

*F = 99.54 km; ζ = 4√3

**Based on least-squares misfit to PREM (Dziewonski and Anderson, 1981)
Table 4. Centroid-moment magnitude statistics* by tectonic setting

| Setting                  | N         | ΣMw       | Max. Z (km) | Plot color  |
|--------------------------|-----------|-----------|-------------|-------------|
| All relocated events     | (20646)   | (112877)  | 699         | black-dashed|
| Continental transform    | 203       | 1094      | 47          | yellow      |
| Oceanic ridge/transform  | 1056      | 5610      | 49          | orange      |
| Continental rift         | 99        | 2291      | 53          | red         |
| Himalaya-type convergence| 1112      | 5957      | 492         | green       |
| Island arc-type convergence| 15821    | 86879     | 699         | purple      |
| Andean-type convergence  | 2511      | 13887     | 656         | blue        |
| Total (0.8% over sample) | 156       | 2841      |             |             |

*Correlation coefficient for (N, ΣMw) population is r = 0.999899, with intercept y = -64.4
Table 5. Predicted ranges of spherical harmonic degree and frequency for material waves propagating in the boundary layer structure of a thermomechanical Earth

| Layer | Depth (km) | Degree** (l) | Spheroidal† (mHz) | Toroidal‡ (mHz) |
|-------|------------|--------------|-------------------|-----------------|
|       | d          | λ = ζd       | λ = 2d            | Low  | High |
| L1    | 25         | 231          | 794               | 25.6 | -    | 23.0 | -    |
| L2    | 37         | 156          | 538               | 15.5 | -    | 15.3 | -    |
| L3    | 96         | 60           | 208               | 6.50 | 21.0 | 7.13 | 23   |
| L4    | 100        | 57           | 200               | 6.28 | 20.0 | 6.93 | 22.4 |
| M1    | 172        | 34           | 116               | 4.24 | 12.0 | 4.38 | 13.2 |
| M2    | 255        | 22           | 79                | 3.08 | 8.43 | 3.00 | 9.20 |
| M3    | 663        | 8            | 29                | 1.43 | 3.75 | 1.35 | 3.74 |
| M4    | 690        | 8            | 30                | 1.43 | 3.85 | 1.35 | 4.00 |

† Isobaric shears of ThERM (Patton and Watkinson, 2009, 2010) with F=99.54 km
‡ Graphical estimates from Figure 8.8 of (Aki and Richards, 2002)
§ Graphical estimates from Figure 8.7 of (Aki and Richards, 2002)

\[ l = \left( 2 \pi R_{\text{in}} / \lambda \right) - (1/2) \]

\[ R_{\text{in}} = 6371 \text{ km} \]
Table 6. Experimental results of load-hold tests on Mt. Scott granite (from Katz and Reches, 2002, 2004), with estimates of mechanical diffusivity. Sample diameter $d = 25.4 \text{ mm}$.

| Sample | $l$ (mm) | $\tau$ (min) | Max. NDS | -log $\chi^*$ | Comments |
|--------|----------|--------------|----------|--------------|----------|
| 101    | 66.9     | -            | 1.05     | -            | load to failure |
| 102    | 63.5     | 95           | 1.03     | -            | load hold |
| 103    | 93.8     | -            | 1.02     | -            | load to failure |
| 104    | 93.7     | 61           | 1.05     | 5.6          | spontaneous failure |
| 105    | 99.0     | 180          | 0.80     | -            | load hold (1) |
| 106    | 96.1     | 1.25         | 1.01     | 3.9          | spontaneous failure |
| 108    | 100.2    | 180          | 0.86     | -            | load hold |
| 109    | 98.1     | 180          | 0.93     | -            | load hold |
| 110    | 94.5     | 0.03         | 0.96     | 2.3          | spontaneous failure |
| 112    | 90.9     | -            | 0.98     | -            | load to failure |
| 113    | 96.1     | 180          | 0.96     | -            | load hold |
| 114    | 96.6     | 180          | 0.88     | -            | load hold |
| 115    | 93.4     | 360          | 0.91     | -            | load hold |
| 116    | 97.1     | 180          | 0.78     | -            | load hold |
| 117    | 89.4     | 180          | 0.54     | -            | load hold |
| 123    | 95.4     | 180          | 0.57     | -            | load hold |
| 124    | 76.0     | -            | 0.95     | -            | load hold (1) |
| 125    | 83.2     | 180          | 0.93     | -            | load hold (1) |

*Computed using $\chi = \frac{l^2}{\tau}$