Dileptons and Medium Effects in Heavy-Ion Collisions

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Abstract

We discuss the status of calculating in-medium modifications of vector-meson spectral functions in hot and dense matter, their application to dilepton spectra in ultrarelativistic heavy-ion collisions, and possible relations to chiral symmetry restoration. We emphasize the importance of constraining in-medium spectral functions by empirical information from scattering data, QCD sum rules, and lattice QCD. This is a mandatory prerequisite to arrive at reliable predictions for low-mass dileptons in heavy-ion collisions.

1 Introduction

The description of strongly interacting matter, including hadronic matter and the Quark-Gluon Plasma (QGP), is a challenging theoretical task and requires guidance from experiment. A key issue is the determination of the spectral properties of the excitations of the medium. On the one hand, this allows to identify and characterize the relevant degrees of freedom at given temperature ($T$) and baryon density ($\rho_B$). On the other hand, certain quantum-number channels bear rather direct connections to order parameters of phase transitions, so that their modifications can serve as indicators of phase changes. In the laboratory, the only way of creating high energy-density matter is to collide heavy nuclei at high energies. A large body of hadronic observables has provided ample evidence that the collision energy is largely converted into producing matter, justifying the notion of approximate (local) equilibrium. However, spectral properties of the medium as encoded in (hadronic) correlation functions require the measurement of invariant-mass spectra of decay products of strongly decaying resonances (long-lived hadrons, e.g., $J/\psi$ or $\pi^0$, decay long after the "freezeout" of the interacting system). Strong decay channels (e.g., $\rho \to \pi\pi$ or $\Delta \to N\pi$), while abundant, are subject to final-state interactions which distort the invariant-mass information of the parent particle. Nevertheless, $\pi^+\pi^-$ or $\pi^\pm p$ spectra may carry valuable information on the dilute-matter stages of a heavy-ion collision [1, 2]. Dileptons (e.g., from direct decays of the vector mesons $\rho$, $\omega$ and $\phi$) undergo negligible final-state interactions, and therefore emanate throughout the evolution of the hot and dense medium, rendering them, in principle, a direct probe of medium effects in the earlier phases [3, 4, 5]. Since the formation of sufficiently hot/dense matter in thermal equilibrium implies the presence of deconfinement and chiral symmetry restoration, the question is not so much if but how these are realized and how to deduce them from experiment.

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2 Chiral Order Parameters and Vector Mesons

In the QCD vacuum, the spontaneous breaking of chiral symmetry (SBCS) is characterized by nonzero order parameters, e.g., the scalar quark condensate, $\langle \bar{q}q \rangle \simeq (-250 \text{ MeV})^3$, the pion-decay constant, $f_\pi = 93 \text{ MeV}$, and the constituent quark mass, $m_{u,d} \simeq 350 \text{ MeV}$. Relations to hadron masses are less direct, but SBCS implies a massive splitting of chiral partners in the low-energy hadronic spectrum ($\Delta M \simeq 0.5 \text{ GeV}$). One of the cleanest examples are the isovector-vector ($V$) and -axialvector ($A$) spectral functions as measured in hadronic $\tau$ decays, cf. left panel of Fig. 1 [6]. Their integrated difference directly relates to order parameters via chiral (or Weinberg) sum rules [8],

$$f_n = - \int_0^\infty ds \frac{ds}{\pi} s^n \left[ \text{Im} \Pi_V(s) - \text{Im} \Pi_A(s) \right],$$  

$$f_{-2} = f_\pi^2 \frac{r_\pi^2}{3} - F_A, \quad f_{-1} = f_\pi^2, \quad f_0 = 0, \quad f_1 = -2\pi \alpha_s \langle \mathcal{O} \rangle$$

($r_\pi$: pion charge radius, $F_A$: axialvector formfactor in the radiative pion decay, $\pi \to l\bar{\nu}l\gamma$, $\langle \mathcal{O} \rangle \simeq (16/9)\langle \bar{q}q \rangle^2$: 4-quark condensate in factorization approximation). As discussed below (cf. Eqs. (5) and (6)), the rate of thermal dilepton emission at low mass is dominated by the $\rho$ meson, providing a direct window on its in-medium spectral function. When combined with an evaluation of the $a_1$ spectral function within a chiral approach, the Weinberg sum rules (which remain valid in the medium [9]) are a valuable tool to infer mechanisms of chiral restoration, especially when augmented with finite-$T$ lattice QCD (lQCD) results for the $f_n$ coefficients.

Useful insights may also be obtained from recent lQCD calculations of scalar and isovector quark-number susceptibilities [10]; we will return to this issue at the end of Sec. 4.

Figure 1: Left panel: vector and axialvector spectral functions from hadronic $\tau$ decays [6] with model fits using vacuum $\rho$ and $a_1$ spectral functions plus perturbative continuia [7]; right panel: schematic scenarios for chiral symmetry restoration in hot and dense matter.
3 Vector Mesons in Medium

The most common approach \[5\] to evaluate medium effects on vector-meson properties consists of evaluating effective interactions with surrounding hadrons from the heat bath, leading to in-medium self-energy insertions which are resummed in the propagator as (here for the $\rho$)

$$D_\rho(M, q; \mu_B, T) = \left[ M^2 - (m^{(0)}_\rho)^2 - \Sigma_{\rho \pi \pi} - \Sigma_{\rho B} - \Sigma_{\rho M} \right]^{-1}.$$ (3)

The different contributions may be classified as being due to (a) modifications of the pions in the $\pi \pi$ decay, (b) direct $\rho$-baryon couplings (e.g., $\rho$-$N(1520)$ excitation) and (c) direct $\rho$-meson couplings (e.g., $\rho + \pi \rightarrow a_1$ excitations), cf. Fig. 2. The key to a reliable description of the in-medium spectral function lies in constraining the effective vertices by independent empirical information. Besides hadronic and radiative branching ratios (e.g., $N(1520) \rightarrow \rho N, \gamma N$), more comprehensive information is encoded in scattering data, e.g., $\pi N \rightarrow \rho N$ (lower left panel in Fig. 3), $\gamma N$ absorption (both representing leading order in nuclear density, $\varrho_N$), or $\gamma A$ absorption (upper left panel in Fig. 3). The latter is directly proportional to the $\rho$ spectral function at finite density, $\sigma_{\gamma A}^{abs}(q_0) \propto \text{Im} D_\rho(M = 0, q; \varrho_N)$. In addition, QCD sum rules provide further checks by relating in-medium quark- and gluon condensates to dispersion integrals over the calculated spectral functions, cf. right panel of Fig. 3.

A selection of $\rho$ spectral functions in cold nuclear matter is compiled in Fig. 4. All calculations qualitatively agree in that the main effect is a substantial broadening, accompanied by a small upward mass shift. The largest effects are found in the calculation of Ref. \[11\] (upper left panel), which includes an in-medium pion cloud and direct $\rho$-$N$ interactions and has been constrained by both nuclear photoabsorption and $\pi N \rightarrow \rho N$ scattering. The results agree within 30% with a calculation where the $\rho$-$N$ interactions are based on comprehensive $\pi$-$N$ phase shift analyses \[16\] (upper right panel). This agreement is not surprising to the extent that the leading-density term is constrained by the same empirical data, and that higher orders are suppressed. The calculations of Ref. \[17\] (lower left panel) include the dressing of the pion cloud but only the $N(1520)$ resonance for the direct $\rho$-$N$ coupling; consequently, the medium effects are somewhat less pronounced. This also applies to Ref. \[13\] (lower right panel), where the focus was on $S$-wave $\rho$-$N$ scattering (i.e., no $P$-wave resonances included). It is noteworthy that the predicted width and mass estimates of Refs. \[11, 12\], $(m_\rho, \Gamma_\rho) \simeq (0.8, 0.45)$ GeV at nuclear saturation density, $\varrho_N = 0.16 \text{ fm}^{-3}$ (upper left panel in Fig. 4), are in good agreement with the "allowed" range inferred from QCD sum rules (cf. right middle panel in Fig. 3).
Figure 3: Constraints on in-medium $\rho$ mesons from photoabsorption on nuclei (upper left panel) [14], $\pi N \rightarrow \rho N$ scattering (lower left panel) [13], and QCD sum rules (right panels) [15].

In Ref. [18] the in-medium electromagnetic (e.m.) correlation function, $\Pi_{\text{em}}$ (which governs the thermal dilepton rate), has been evaluated using a virial expansion in pion- and nucleon-densities coupled with chiral reduction formulae. In the soft-pion limit in a pion gas, the isovector channel exhibits the so-called chiral mixing effect [19],

$$\Pi_{V,A} = (1 - \varepsilon) \Pi_{V,A} - \varepsilon \Pi_{A,V}, \quad \varepsilon = T^2/6f_\pi^2,$$

indicating that the $\rho$-resonance peak is not broadened albeit quenched. More elaborate calculations lead to substantial strength below the $\rho$-mass, with sizable contributions from baryons [18].

Different approaches [12, 18, 20] thus agree that medium modifications due to nucleons (baryon density, $\rho_B$) are more important than due to pions (temperature, $T$), at comparable density. Pions are, after all, Goldstone bosons implying reduced interaction strength.
Figure 4: Comparison of $\rho$-meson spectral functions in cold nuclear matter within the many-body approaches of Refs. [11, 12] (upper left panel), [16] (upper right panel), [17] (lower left panel) and [13] (lower right panel).

4 Dilepton Spectra in Heavy-Ion Collisions

The thermal emission rate of dileptons per unit 4-volume and 4-momentum is given by [21]

\[
\frac{dN_{ll}}{d^4xd^4q} = -\frac{\alpha_{em}^2}{\pi^3M^2} f^B(q_0; T) \text{Im}\Pi_{em}(M, q_0; \mu_B, T) .
\] (5)

At low mass, $M \leq 1$ GeV, where radiation from hadronic matter prevails, the (free) e.m. spectral function is saturated by the light vector mesons $\rho$, $\omega$ and $\phi$ (vector dominance model, VDM),

\[
\text{Im}\Pi_{em} = C_\rho \text{Im}D_\rho + C_\omega \text{Im}D_\omega + C_\phi \text{Im}D_\phi .
\] (6)

In phenomenological fits, the prefactors $C_V = m_V^4/g_V^2$ approximately reproduce the e.m. decay branchings in the naive quark model, i.e., $C_\rho : C_\omega : C_\phi = 9 : 1 : 2$, illustrating the prevalent role of the $\rho$-meson. In most approaches, (a form of) VDM is assumed to hold in the medium, although this has been challenged recently [22].

The $T$- and $\varrho_B$-dependent emission rates are to be convoluted over the space-time history of a heavy-ion reaction. In the left panel of Fig. 4 predictions based on an in-medium $\rho$ spectral function, coupled with a thermal fireball model for central $In(158\text{AGeV})-In$ collisions at SPS,
are compared to new NA60 data [11]. The latter have reached a level of precision that allows to subtract known background sources to isolate the dilepton excess spectra. The calculations, supplemented with (small) contributions from in-medium \( \omega \) and \( \phi \) decays [23] as well as QGP emission, describe the data well [24]. The absolute yield carries an uncertainty due to the fireball lifetime (\( \tau_{FB} \approx 7 \text{ fm/c} \)) of about 25%, but the shape and relative contributions of the spectrum are robust. The calculations imply that the underlying \( \rho \) spectral function is essentially "melted" in the vicinity of the critical temperature, \( T_c \approx 175 \text{ MeV} \), with no significant mass shift at smaller temperatures. In fact, close to \( T_c \), the pertinent e.m. emission rate resembles the perturbative QGP rate which is suggestive for a form of quark-hadron duality [12]. This conclusion is corroborated by a hydrodynamic convolution of the rates following from the chiral virial approach (right panel of Fig. 5) [25], where the low-mass enhancement is accounted for but the lack of \( \rho \)-broadening results in a \( \sim 40\% \) overestimate of the data at the free \( \rho \)-mass. When folding the chiral virial rates over an expanding fireball model [26], both hadronic and QGP spectra agree well with the hydrodynamic results of Ref. [25]. The fireball model of Ref. [27] deviates from these findings in that (i) the low-mass NA60 data are fitted with a \( \rho \) spectral function based on pion-gas effects only [28], (ii) the intermediate-mass region (IMR, \( M > 1 \text{ GeV} \)) is dominated by QGP radiation. In the approaches underlying Fig. 5, 4-pion type annihilations (e.g., \( \pi + a_1 \rightarrow \mu^+ \mu^- \)), which follow from the e.m. correlator in free space (cf. left panel of Fig. 6), significantly feed into the IMR. Lower and upper estimates based on the free 4-\( \pi \) part of the e.m. spectral function and on the naive chiral mixing formula (Eq. (4) assuming \( \varepsilon(T_c) \equiv 1/2 \)), provide a band consistent with the NA60 data (upper solid curves in Fig. 6 right panel) [24]; the chiral virial approach is consistent with this band (Fig. 5 right panel) [25].

The NA60 data are not yet sensitive to extract information on medium modifications of \( \omega \) and \( \phi \) mesons [23] as included in the left panel of Fig. 5. It is interesting to note that a recent measurement of photon-induced \( \omega \)-production off nuclei in the \( \pi^0 \gamma \) decay channel [29] is indicative for an in-medium reduced \( \omega \)-mass, together with significant broadening. Moreover, 2-flavor finite-T lQCD calculations find that when approaching the (putative) critical point by increasing the quark-chemical potential, the isoscalar susceptibility (\( \omega \) channel) develops...
Figure 6: Left panel: decomposition of the free vector-isovector spectral function into $2\pi$ and $4\pi$ components as measured in $\tau$ decays [6]; right panel: $4\pi$ contributions to intermediate-mass dileptons in NA60 [4] with/-out (upper/lower dashed line) medium effects [24].

A rather sharp maximum while the isovector one ($\rho$ channel) behaves smoothly [10]. The latter is consistent with hadronic many-body calculations of the $\rho$ meson which exhibit strong broadening without significant mass shift under both RHIC and SPS conditions. At the same time, the effect of $\sigma$-$\omega$ mixing, together with a softening of a $\sigma$ mode at the critical point, could induce a dropping-mass component in the $\omega$ spectral function.

5 Conclusions

The study of in-medium vector mesons provides rich information on spectral properties of QCD matter including its phase structure. "Quality control" in hadronic model approaches by imposing empirical and theoretical constraints is mandatory to allow for meaningful applications to ultrarelativistic heavy-ion collisions. Recent high-precision dilepton data at SPS energies are consistent with hadronic many-body calculations which predict a "melting" of the $\rho$ resonance close to the expected phase boundary. The full potential of the data, including transverse-momentum and centrality dependencies, has yet to be exploited. Connections to chiral symmetry restoration through the axialvector spectral function should be pursued with high priority, both theoretically and experimentally ($\pi^{\pm}\gamma$ spectra?). Future measurements at low and high net baryon density at RHIC and the Compressed Baryonic Matter experiment at GSI will be essential to enlarge the scope across the QCD phase diagram.

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