A mechanism of light transmission through metallic films is proposed, assisted by tunnelling between resonating buried dielectric inclusions. This is illustrated by arrays of Si spheres embedded in Ag. Strong transmission peaks are observed near the Mie resonances of the spheres. The interaction among various planes of spheres and interference effects between these resonances and the surface plasmons of Ag lead to mixing and splitting of the resonances. Transmission is proved to be limited only by absorption. For small spheres, the effective dielectric constant can be tuned to values close to unity and a method is proposed to turn the resulting materials invisible.

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Substantial efforts have been placed on studying light transmission through perforated metallic films since the discovery by Ebbesen et al. \cite{1} of extraordinary transmission through sub-wavelength periodic hole arrays, leading to such remarkable effects as giant funnelling of microwave radiation through metallic micrometer slits \cite{2}. Two distinct mechanisms have been identified that contribute to enhance light transmission: (i) dynamical scattering assisted by surface plasmons (SPs) and (ii) resonant transmission by coupling to propagating modes. (i) Isolated sub-wavelength holes can only support evanescent modes, but dynamical diffraction assisted by SPs in periodic hole arrays lead to transmission maxima that can beat the severe $1/\lambda^4$ dependence on wavelength $\lambda$ of the transmission derived by Bethe for single apertures \cite{3}, although there is still some controversy regarding the relative relevance of SPs and dynamical diffraction \cite{4,5,6,7}. (ii) Slits differ from holes in that the former, however small, support at least one transmission mode that can couple resonantly to external light to transmit several orders of magnitude more light than impinging directly on the slit aperture \cite{2}.

In this Letter we investigate yet another mechanism of enhanced transmission based upon hopping between dielectric inclusions that can sustain localized resonances inside metallic films. We illustrate this concept by studying light transmission through metallic Ag films that contain spherical Si inclusions arranged in square-lattice layers, which belongs to the family of periodic arrays of nanoresonators coupled via spatially homogeneous waveguides \cite{8}. The transmission is shown to take large values near the Mie resonances of the spheres, and it is only limited by metal absorption.

We first consider a single layer of inclusions [Fig. 1(a)]. Maxwell’s equations are solved rigourously for this geometry using a layer-by-layer version of the KKR multiple-scattering method \cite{9}, with lattice sums performed in real space within the metal. The complex, frequency-dependent dielectric functions of Ag and Si are taken from optical data \cite{10}.

The resonant modes of the Si spheres can be visualized through their Mie scattering coefficients inside bulk metal \cite{11}, showing prominent features in the visible and near-IR domains for a sphere radius $R = 75$ nm [Fig. 1(b)]. The corresponding transmittance spectrum exhibits peaks near those resonances [Fig. 1(c), at normal incidence] that can exceed a transmission $T = 50\%$ at wavelengths near $\lambda = 975$ nm, well above the $T = 13\%$ value expected for a homogeneous Ag film of thickness $2b = 15$ nm (i.e., the combined thickness of the thin burying layers at either sides of the spheres). Smaller separations between the sphere and the film boundaries lead to even higher transmission (e.g., $T = 70\%$ for $b = 2.5$ nm).

The transmission peaks are red-shifted with respect to the resonances in bulk Ag as a result of interaction between sphere modes and vacuum modes. However, the relevant multipoles of the inclusions and the polarization symmetry associated to those peaks can be well ascribed by comparing Figs. 1(b) and (c).

The interaction of spherical inclusions along the plane can be important at small separations between the spheres. This is illustrated in Fig. 2 which shows a contour plot of the transmittance $T$ as a function of lattice constant $a$ and wavelength. $T$ has been multiplied by $a^2/\pi R^2$ to obtain transmission cross sections per sphere. The $E1$ and $E2$ Mie resonances of Fig. 1(b) show up as two lines of transmission maxima relatively independent of $a$. However, these two modes interact and repel each other when the spheres are near touching each other (bottom part of the figure).
The SPs of the planar Ag interfaces cannot couple directly to external propagating modes. However, the presence of Si inclusions provides a source of momentum that makes this coupling possible. The coupling increases when the interaction between lattice sites (spheres) via SPs is constructive, which at normal incidence (all lattice sites are in phase) leads to the kinematical condition that SP momenta that match reciprocal lattice vectors of the square lattice of spheres are favored, that is, $\lambda n^2 + m^2 = a \sqrt{\varepsilon_{\text{Ag}}/(\varepsilon_{\text{Ag}} + 1)}$ [1], where the surface plasmon dispersion relation has been invoked. The solid curves in Fig. 2 are obtained from this equation for different combinations of integers $n$ and $m$ and they would lie near regions of transmission enhancement in hole arrays [1], but that transmission mechanism is marginal for our buried structures. Instead, a rich transmission structure shows up when the noted kinematical condition (solid curves) is at the same wavelength as a Mie resonance [see right inset of Fig. 2]: the discrete, localized Mie modes couple to the continuum of SPs to yield a transmission spectrum exhibiting a point of zero transmission, as it is well known from Fano’s theory [12] (see left inset of Fig. 2).

Several layers of spheres can interact in a single film producing splitting and shifting of the Mie resonances, as shown in Fig. 3. The main 975-nm feature of Fig. 1(c) is subdivided here into a series of $N$ peaks in a film containing $N$ layers. The coupling between Mie modes at consecutive layers is the same as in previously introduced photonic molecules that arise from the interaction of Mie modes in neighboring dielectric particles [13]. The tight-binding approach used in that context becomes a natural tool to understand the hopping of light between contiguous spheres in our case. In this spirit, an analogy with electron transmission between localized states in quantum systems can be established by considering a simple tight-binding model [13] in which a resonant mode trapped in each layer is associated to a state $|i\rangle$ (with $i = 1, 2, ..., N$). The dynamics is governed by the Hamiltonian

$$H = (\omega_b + i\Sigma_b)(|1\rangle\langle 1| + |N\rangle\langle N|) + \sum_{i=2}^{i=N-1} (\omega_b + i\Sigma_b)|i\rangle\langle i| + \sum_{i=1}^{i=N-1} (V|i\rangle\langle i + 1| + h.c.),$$

where $\omega_b$ and $\Sigma_b$ are the energy and width of the resonances, and $V$ is the hopping amplitude, which expresses the magnitude of the interaction between spheres. For internal layers the width $\Sigma_b$ accounts for dissipation into bulk metal. The states $|1\rangle$ and $|N\rangle$ interact with the continuum of light modes on the left and right hand sides of the film, respectively, described by frequency dependent self-energy $\Sigma_{\omega}(\omega)$. The transmittance in the model is defined by the squared absolute value of the transition
FIG. 3: Transmittance through Ag films containing different numbers \( N \) of buried planar arrays of Si spheres of radius \( R = 75 \) nm under normal incidence conditions. The planar unit cell is a square of side \( a = 250 \) nm, which prevents interaction between spheres within each plane. Solid curves are exact solutions of Maxwell’s equations. Broken curves are obtained from the tight-binding model explained in the text [Eq. (1)].

amplitude and can be written as [14]

\[
T = 4|\Sigma_s(\omega)g_{1,N}(\omega + i0^+)|^2, \tag{1}
\]

where \( g_{1,N}(z) = \langle 1| (z - H)^{-1}|N\rangle \). The parameters of the Hamiltonian have been chosen to produce the best fit to the transmittance obtained from numerical solution of Maxwell’s equations: \( \Sigma_b = -0.002\omega_b, V = 0.03\omega_b \), and a linear function coupling to vacuum given by \( \Sigma_s(\omega) = -0.005\omega_b - 0.03(\omega - \omega_b) \). Note that radiative coupling via non-vanishing \( \text{Im}\{V\} \) can produce changes in the fine shapes [15] that lie beyond the scope of this work.

The results derived from Eq. (1) are shown in Fig. 3 as dashed curves. Characteristic features of the exact transmittance (solid curves) are well reproduced, including peak splitting, peak asymmetry coming from the energy dependence of \( \Sigma_s \), and narrowing of the resonances with increasing number of layers. This model corroborates that the main mechanism producing transmission is tunnelling between electromagnetic modes trapped in each layer.

The dielectric function of Ag near the main resonance of Fig. 3 is essentially negative (\( \epsilon_{\text{Ag}} = -49 + 0.6i \)), so that coupling among the spheres and between the spheres and the film planar surfaces is mediated by evanescent modes. However, the small imaginary part of \( \epsilon_{\text{Ag}} \) plays a leading role in reducing the absolute transmittance \( \text{Im}\{\epsilon_{\text{Si}}\} \ll 1 \).

This is made clear in Fig. 4 which has been calculated under the same conditions as in Fig. 3 but setting the imaginary part of the dielectric constant of Ag to zero.

A Mie cavity mode inside a lossless bulk metal must have infinite lifetime because light can neither escape through nor be absorbed by the metal. Therefore, the width of the resonance in the lossless film for \( N = 1 \) (6.72 nm FWHM, as obtained from Fig. 1) has to remain the same (indeed, the sum of the FWHM in the two peaks for \( N = 2 \) is 6.72 nm as well). This leakage contribution to peak broadening adds up to the intrinsic width of the Mie resonance in the lossy bulk metal [4.53 nm FWHM as obtained from the E1 peak in Fig. 1(b)], yielding a total width of 11.25 nm, which compares well with the value of 11.83 nm observed for the actual lossy metallic film \( (N = 1 \) solid curve in Fig. 3).
(i.e., $\varepsilon_{\text{eff}} = 1$) if metal absorption is compensated by doping the dielectric with active atoms under inversion population conditions. We have proposed a specific design for such invisible material at wavelengths in the visible, but structures insensitive to microwaves should be also realizable due to smaller absorption in this range and using reradiating active elements in the millimeter scale.

In conclusion, we have shown that light transmission through metallic films can be enhanced by coupling to resonances localized in dielectric inclusions, paving the way towards driving light deep inside metals. Light circuits based upon this mechanism can be envisioned. When the inclusions are small compared to the wavelength, the resulting metamaterial can be made invisible (i.e., $\varepsilon_{\text{eff}} = 1$) if metal absorption is compensated by doping the dielectric with active atoms under inversion population conditions. We have proposed a specific design for such invisible material at wavelengths in the visible, but structures insensitive to microwaves should be also realizable due to smaller absorption in this range and using reradiating active elements in the millimeter scale.

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