Response of a Kinetic Ising System to Oscillating External Fields: Amplitude and Frequency Dependence

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Abstract

The $S=1/2$, nearest-neighbor, kinetic Ising model has been used to model magnetization switching in nanoscale ferromagnets. For this model, earlier work based on the droplet theory of the decay of metastable phases and Monte Carlo simulations has shown the existence of a size dependent spinodal field which separates deterministic and stochastic decay regimes. We extend the above work to study the effects of an oscillating field on the magnetization response of the kinetic Ising model. We compute the power spectral density of the time-dependent magnetization for different values of the amplitude and frequency of the external field, using Monte Carlo simulation data. We also investigate the amplitude and frequency dependence of the probability distributions for the hysteresis loop area and the period-averaged magnetization. The time-dependent response of the system is classified by analyzing the behavior of these quantities within the framework of the distinct deterministic and stochastic decay modes mentioned above.

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I. INTRODUCTION

Hysteresis is a nonequilibrium phenomenon characteristic of metastable systems. The kinetic Ising model, for temperatures below its critical temperature $T_c$ and at non-zero external fields $H$, exhibits a metastable phase. If $H(t)$ varies periodically in time, the response of the system, the magnetization $m(t)$, lags behind the forcing field, and hysteresis occurs. Previous studies of hysteresis have been performed, both for mean-field models and for the Ising model \[1,2\]. In both cases the hysteresis loop area, $A = -\oint m(dH)$, was found to have a power-law dependence on the frequency and amplitude of $H$ for low frequencies. Also, the mean-field models exhibit a dynamic phase transition in which the period-averaged magnetization, $Q = (\omega/2\pi) \oint m dt$, changes from $Q \neq 0$ to $Q = 0$ \[3,4\]. Magnetization reversal in small ferromagnetic grains has been modeled with a kinetic Ising model in static fields \[5\]. Also, recent experiments on ultrathin ferromagnetic Fe/Au(001) films \[6\] have considered the frequency dependence of hysteresis loop areas, obtaining exponents that are consistent with those found for the two dimensional Ising model \[1\].

The model used in our study is a kinetic, nearest-neighbor Ising ferromagnet on a square lattice with periodic boundary conditions. The Hamiltonian is given by $H = -J \sum_{\langle ij \rangle} s_i s_j - H \sum_i s_i$, where $\sum_{\langle ij \rangle}$ runs over all nearest-neighbor pairs, and $\sum_i$ runs over all $N=L^2$ lattice sites. The dynamic used in this work is the Glauber \[7\] single spin-flip Monte Carlo algorithm. The system is put in contact with a heat bath at temperature $T$, and each spin can flip from $s_i$ to $-s_i$ with a probability given by \[8\]

$$W(s_i \rightarrow -s_i) = \frac{\exp(-\beta \Delta E_i)}{1 + \exp(-\beta \Delta E_i)},$$

where $\Delta E_i$ gives the change in the energy of the system if the spin flip is accepted, and $\beta = 1/k_B T$.

In contrast to earlier studies of hysteresis in Ising models, we have performed simulations in two distinct field regimes. It has recently been observed that the metastable phase in Ising models exposed to a static field $H$ decay by different mechanisms, depending on $H$ and the system size $L$ \[9\]. For fields weaker than a $T$ and $L$ dependent dynamic spinodal field $H_{DSP}$, the mean lifetime $\tau$ of the metastable phase is comparable to the standard deviation in the lifetime. This field region is termed the stochastic or single-droplet (SD) region because decay of the metastable phase proceeds by random nucleation of a single critical droplet of the stable phase. For fields stronger than $H_{DSP}$, the mean lifetime of the metastable phase is much greater than the standard deviation in the lifetime. This field region is called the deterministic or multi-droplet (MD) region because decay of the metastable phase proceeds by the nucleation, growth, and coalescence of many droplets of the stable phase. The study presented here is, to our knowledge, the first in which the effects of these two different decay mechanisms on hysteresis are considered.

In our simulations, a system of size $L$ at temperature $T$ is prepared with $m(0)=0$ and the up and down spins in random arrangement. Then, a sinusoidal field $H(t) = H_o \sin(\omega t)$ is applied, and the magnetization $m(t)$ is recorded. The field $H(t)$ is changed every attempted spin flip, allowing for a smooth variation. Using the $m(t)$ vs. $t$ data, we calculate power spectral densities (PSD) and probability distributions for the loop area $A$ and the period-averaged magnetization $Q$ for different amplitudes and frequencies of $H(t)$.
II. RESULTS

Our results are presented so as to contrast the behaviors of the hysteretic magnetization response in the SD and MD regions. All of the results shown are for $L=60$ and $T=0.8T_c$. The simulations were performed at two field amplitudes $H_o$ and several frequencies $\omega$. One value of $H_o$ was chosen such that for $\omega = 0$, the system was clearly in the SD region. Similarly, the other value of $H_o$ was chosen such that the system was definitely in the MD region. While several frequencies were used, here we show results for a value of $\omega$ for which the magnetization switches many times over the course of the entire simulated time series (although not necessarily with the same periodicity as $H(t)$). Figure 1 shows a portion of the time series in the MD regime, where the system switches over several periods from a state that oscillates with $Q < 0$, to a state that oscillates with $Q > 0$. In Fig. 2, a portion of the time series in the SD regime shows oscillations with $Q$ near plus or minus the zero-field spontaneous magnetization, punctuated by random switching events that are completed within less than one period of the field oscillation. Here, once a critical droplet forms (in a Poisson process), the stable phase quickly takes over the system.

Figure 3 shows the PSDs for the entire time series of Figs. 1 and 2. The first peak in the MD data is at the driving frequency $f=(\omega/2\pi)$ of the external field. The higher-frequency peaks are located at integer multiples of $f$. For the SD data the resolution at low frequencies is not fine enough to show even the first peak. However, other data in the SD regime using $H(t)$ with shorter periods do show a peak at the driving frequency. Longer time series are needed to resolve the low-frequency part of the spectrum. Both the MD and SD data show large low-frequency components, indicating the slow dynamics of the magnetization reversal in both regimes. In the high-frequency part of the spectrum, the PSDs have slopes on the log-log plot of approximately two, corresponding to exponential short-time correlations.

Figure 4 shows the probability distributions of the hysteresis loop areas $A$ for the data of Figs. 1 and 2. The distribution for the MD regime shows a single wide peak, since $m(t)$ lags behind $H(t)$ by a nearly constant phase factor. The distribution in the SD regime has a narrow peak near zero as $m(t)$ oscillates near the positive or negative spontaneous magnetization for most of the length of the run. The long tail at higher $A$ values corresponds to the occasional rapid switching events.

Figure 5 shows the distributions of the period-averaged magnetization $Q$ for the data in Figs. 1 and 2. The distributions for both the SD and the MD regions show a double-peaked structure. The two sharp peaks in the SD data are due to $m(t)$ oscillating near the spontaneous magnetization values during most of the run. The distribution for the MD regime shows two peaks as well, but each peak is much wider than in the SD case. The positions of the peaks in both distributions have been found to be frequency dependent as well as amplitude dependent. In both cases the asymmetry of the distribution is an effect of the finite length of the time series.

III. CONCLUSION

Our results show distinct differences between the multi-droplet region and the single-droplet region. This study shows that the nature of the response of an Ising system depends not only on the competition between the two time scales: the oscillation period of the
external field, and the lifetime of the metastable state. The response also depends on the
mode by which the system switches magnetization states, which could depend not only on
the field amplitude and frequency, but also on the temperature and system size. In future
studies, we plan to use longer time series to obtain better low-frequency resolution in the
PSDs and better statistics for the probability distributions of $A$ and $Q$.

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FIGURES

FIG. 1. Magnetization $m(t)$ and external field $H(t)$ vs. time $t$ in the MD region with $(2\pi/\omega) = 276$ Monte Carlo steps per spin (MCSS), $H_o = 0.3J$, and the lifetime in static field $\tau_{\omega=0} \approx 55$ MCSS. The darker line shows $m(t)$ and the lighter line denotes $H(t)$. The total length of the time series is approximately $2 \times 10^5$ MCSS. The ratio of the period of $H$ to $\tau_{\omega=0}$ is approximately 5.

FIG. 2. Magnetization $m(t)$ and external field $H(t)$ vs. time $t$ in the SD regime with $(2\pi/\omega) = 5000$ MCSS, $H_o = 0.1J$, and $\tau_{\omega=0} \approx 1000$ MCSS. The darker line shows $m(t)$ and the lighter line denotes $H(t)$. The total length of the time series is approximately $4 \times 10^5$ MCSS. The ratio of the period of $H$ to $\tau_{\omega=0}$ is approximately 5.

FIG. 3. Power spectral densities (PSDs) for the time series of Figs. 1 and 2. The magnetization is sampled every 0.1 MCSS, so the Nyquist frequency is 5 MCSS$^{-1}$. The lowest frequency that can be resolved for both PSDs is $0.2 \times 10^{-4}$ MCSS$^{-1}$.

FIG. 4. Probability distributions of the hysteresis loop areas, $A=-\oint m dH$. $A$ is calculated for the MD (●) and SD (□) regions from the data of Figs. 1 and 2. We have used bins of size 0.1 for the area values for both data sets. Note that the loop areas have been normalized by $4H_o$, the maximum possible loop area. The number of events (periods) for the SD data and MD data is different; the MD distribution uses 759 events and the SD distribution uses 83 events.

FIG. 5. Probability distributions of the period-averaged magnetization, $Q=(\omega/2\pi)\oint m dt$. $Q$ is calculated for the MD and SD regions from the data in Figs. 1 and 2. The size of the bins used and the quality of the statistics are the same as in Fig. 4.
