Research on fast switching interception of the real target

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Abstract. In order to distinguish the decoy missile from the real in the middle of the ballistic missile flight, we proposed using image recognition and sliding-mode guidance in the kinetic energy interceptor terminal guidance phase. The paper set background on the kinetic energy interceptor in the middle course of ballistic missile flight. Due to the influence of the dummy target, the preset interception point is out of alignment and interceptors require terminal guidance adjustment. In this paper, two different global fast Terminal sliding modes are used to establish the intercept model equation. By simulink simulation comparison, it is found that the sliding mode guidance proposed in this paper can quickly and effectively switch interception orbit to hit the target.

1. Introduction

Kinetic interceptor (KKV) is an interceptor missile that does not contain a warhead but directly hits the target missile with great kinetic energy. It consists of three parts: detection, guidance and power control.

At present, the interception of ballistic missiles is mainly divided into four stages: boost phase, final boost phase, mid-course, end-piece. Among them, mid-course interception is the most difficult, mainly because there are a large number of false warheads and bait released by the mother module in the mid-course, and there is no atmospheric resistance, so detection and identification is very difficult.

About accurate interception of true targets in the middle section, a lot of scholars at home and abroad related research.

In literature [1], a robust guidance law design scheme based on dynamic terminal sliding mode control theory is proposed for the interception of new high-speed maneuvering targets. The guidance law has obvious effect on eliminating chattering, but has no obvious advantage on fast guidance of interception and switching. Literature [2].

Design an improved form of nonlinear Terminal sliding mode surface, focusing on reducing miss distance. However, with the further study of the fast sliding surface, more and more new fast sliding surface are designed, and the fast convergence of the sliding surface is difficult to adapt to the new trend. In literature [3], an omni-directional true proportional guidance law is derived and designed that can intercept both the orbit and the inverse simultaneously. Although the proportional guidance can effectively intercept the mobile high speed target, the sliding-mode guidance is more rapid and accurate when the target is switched. Literature [4] refers to the concept of target interest value. By calculating the different values of interest of the true and false warhead, the target is distinguished and proportional guidance is used to hit the target. However, the interest value method is not mature and can not be used as a reference for interceptor switching terminal guidance. In literature [5], the relative motion models
of the orbits of ballistic missiles and interceptors are established, and the second-order sliding mode nonlinear guidance law based on the second-order sliding mode control theory is designed. Although this guidance law is proved to be feasible, it lacks comparison with some other fast guidance laws. Literature [6] established a mathematical model based on the open parameters of the "standard-3" interceptor, and simulated the intercepting process of the maneuvering warhead outside the atmosphere. The results show that the interceptor can accurately collide with the maneuvering target when the speed of the interceptor is lower than that of the missile. In literature [7], the optimal control theory is applied to take energy management into consideration. This guidance law is suitable for long distance interception with sufficient interception time, but it can't switch interception points quickly in the terminal guidance phase of interception. Literature [8] has realized the terminal guidance control of interceptor, effectively solved the contradiction between chattering and rapidity, and improved the dynamic and static characteristics of the system. But there was no big breakthrough in quick switching interception.

At present, radar, infrared imaging and other detection methods are widely used to identify targets. When the interceptor is close to a certain distance from the ballistic missile, the probe head can directly identify the target accurately through image processing. However, due to the short relative distance, the guidance time is severely restricted, so there are high requirements for guidance speed and accuracy. This paper set background in along the rail to intercept to research.

Mid-course interception is an effective method by putting the interceptor into the missile's predicted orbit and flying with the missile at a speed lower than that of the missile. It can provide more guidance time than the anti-orbit interception.

In the selection of guidance law, a new sliding mode guidance law is proposed. In ordinary sliding mode control, the error cannot be convergent to zero in effective time. In order to make up this shortcoming, this paper proposes the Terminal sliding mode control strategy. The global fast Terminal sliding mode control can not only satisfy the finite time convergence, but also can eliminate chattering. It has good robustness and is a very practical control strategy. Because there are many sliding-mode guidance strategies, it is difficult to explain by only using one control strategy. In this paper, a relatively fast sliding mode control strategy proposed in literature [9] was additionally referred to and compared with a classic fast Terminal sliding mode. The interception model was established and compared through simulink simulation.

![Figure 1. Schematic diagram of the target of the track interception.](image)

2. Establishment of the relative motion equation of missile and target
Figure 1 is a schematic diagram of the true warhead hit by missile interception switching. At the beginning, the kinetic energy interceptor intercepts the lures by mistake, and begins to intercept along the orbit before the lures reach point C. At point A, the trajectory of the real warhead is detected, the trajectory changes, and at point B, the trajectory of the real ballistic missile is intercepted.
As shown in figure 2, the relative motion equation of the missile-target is established. The center of mass of the kinetic energy interceptor is taken as the origin of coordinates, the line between the center of mass of the interceptor and the center of mass of the ballistic missile is taken as the 0x axis, the line perpendicular to 0x in the vertical plane is taken as the y axis, the upward is positive, the 0z axis is perpendicular to the x0y plane, and use the right hand theorem to determine the positive.

In reality, there is a roll Angle in the kinetic energy interceptor, but the roll Angle is small because of the attitude control system. In order to simplify this situation, the roll Angle is assumed to be 0. In this way, the movement of interceptor ballistic missile can be limited to longitudinal plane and lateral plane.

Since the motion of the kinetic energy interceptor in the longitudinal plane and the motion in the lateral plane can be expressed in a similar way, this paper focuses on the study of the motion equation in the longitudinal plane.

Use \( \dot{q}_e \) epsilon says the line of sight high and low Angle of angular velocity, the \( \sin \dot{q}_e = \frac{\dot{y}_s(t)}{R(t)} \). (t) for kinetic energy interceptor over time and the relative distance between the target; \( \dot{y}_s(t) \) as the delta as \( \Delta \) y direction delta \( \Delta t \) relative displacement for a set period of time. As if only in a short period of time \( \Delta t \), because the value of \( R(t) \gg \dot{y}_s(t) \), \( \dot{q}_e(t) \) is small, so \( \dot{q}_e(t) = \frac{\dot{y}_s(t)}{R(t)} \).

Take the differential transformation of the above formula and get

\[
\frac{\ddot{q}_e(t)}{R(t)} = -\frac{2R(t)}{R(t)} \dot{q}_e(t) - \frac{R(t)}{R(t)} \ddot{q}_e(t) + \frac{\dot{y}_s(t)}{R(t)} \quad (1)
\]

In the formula, \( \ddot{y}_s(t) = -a_{mys}(t) + a_{tys}(t) \), \( a_{mys}(t) \) represents the component of the acceleration of the kinetic energy interceptor on the 0y-axis, \( a_{tys}(t) \) represents the component of the maneuvering acceleration of a ballistic missile on the 0y-axis.

In order to design guidance law, state variables \( x_1 = \dot{q}_e(t), \ x_2 = \ddot{q}_e(t) \), by substituting \( x_1 \) and \( x_2 \) into equation (1), the relative motion state space equation in the longitudinal plane can be deduced as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-\frac{\dot{R}(t)}{R(t)} & -\frac{2\dot{R}(t)}{R(t)}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
-\frac{1}{R(t)}
\end{bmatrix} u + \begin{bmatrix}
0 \\
\frac{1}{R(t)}
\end{bmatrix} f \quad (2)
\]

In the formula, \( u = a_{mys}(t) \) is regarded as the control quantity; \( f = a_{tys}(t) \) is regarded as the disturbance quantity.

3. Relevant theories of sliding mode guidance law

The global fast sliding mode control can ensure that the system can reach the sliding mode surface in a limited time and stabilize the system in a horizontal state in a limited time. Moreover, the control lawyer
of the global fast sliding mode control is continuous, without switching items, which can eliminate chattering and has good robustness to system uncertainty and interference.

The traditional Terminal sliding mode (TSM) is

\[ s = \dot{x} + \beta x^\frac{p}{r} \]

3.1 Sliding mode surface 1 (FTSM)

In order to overcome the disadvantage of slow convergence of traditional TSM near the equilibrium position, a widely used global fast Terminal sliding (FTSM) mode is

\[ s = \dot{x} + ax + \beta x^\frac{q}{p} \]

The time taken to reach the sliding surface from the initial state is

\[ t_s = \frac{p}{\alpha (p-q)} \ln \left( \frac{\alpha x_0^\frac{p-q}{p} + \beta}{\beta} \right) \]

3.2 Sliding mode surface 2 (New FTSM)

During the interception point switching process, the kinetic energy interceptor needs to adjust to the appropriate position in a very short time. In literature (9), a new Terminal sliding mode and corresponding control method for second-order nonlinear systems with external disturbances are proposed. It can reach the sliding surface more quickly than formula (1).

\[ s = \begin{cases} x_2 + (x_1 + 1) \left[ \alpha \ln(x_1 + 1) + \beta \ln\frac{q}{p}(x_1 + 1) \right], & x_1 \geq 0 \\ x_2 + (1 - x_1) \left[ \alpha \ln(1 - x_1) + \beta \ln\frac{q}{p}(1 - x_1) \right], & x_1 \leq 0 \end{cases} \]

Can be simplified as

\[ s = x_2 + (|x_1| + 1) \left[ \alpha \ln(|x_1| + 1) + \beta \ln\frac{q}{p}(|x_1| + 1) \right] \text{sgn}(x_1) \]

Where, \( \alpha > 0, \beta > 0 \) is the design constant; p, q is positive odd and \( 0 < \frac{q}{p} < 1 \).

The time from the initial state to the sliding surface is

\[ T = \frac{p_0}{\varphi(p_0 - q_0)} \ln \left( \frac{\alpha x_0^\frac{p_0-q_0}{p_0} + \beta}{\gamma} \right) \]

This paper will compare the application of FTSM and the new FTSM in interception point switching through simulation experiments.

4. Design of sliding mode guidance law

In this paper, the new sliding mode control in literature [9] is adopted as the guidance strategy, and the sliding mode guidance law is derived in detail by using the mathematical model in formula (2).

Select the Terminal sliding mode dynamics in document (9)

\[ s = x_2 + (|x_1| + 1) \left[ \alpha \ln(|x_1| + 1) + \beta \ln\frac{q}{p}(|x_1| + 1) \right] \text{sgn}(x_1) \] (4)

In the formula, \( \alpha > 0, \beta > 0 \) is the design constant; p, q is positive odd and \( 0 < \frac{q}{p} < 1 \).
At the same time, the longitudinal sliding mode approach law is selected
\[
\dot{s} = -k_1 s - k_2 sgn (s) \tag{5}
\]
In the formula, \(k_1 = \text{const} > 0, \ k_2 = \text{const} > 0\).

By combining the parameters of formula (2), formula (4) and formula (5), the final guidance expression of the kinetic energy interceptor in the longitudinal plane can be obtained as
\[
amy_s = R\{-\frac{R}{R^2}x_1 - 2R^2 x_2 + k_1 s + k_2 sgn(s) + |\dot{x}_1|\left[\alpha \ln(|x_1| + 1) + \beta ln^{q/p}(|x_1| + 1)\right] + \frac{\beta q \ln^{(q-p)/p}(|x_1| + 1)}{p} + \frac{f}{R}\} \tag{6}
\]

Based on the stability theory of Lyapunov in modern control theory, the Lyapunov function is obtained
\[
V = \frac{s^2}{2} \tag{7}
\]
Equation (7) is differentiated with respect to time, and equation (3) is combined to obtain
\[
\dot{V} = s\{-\frac{\dot{R}}{R}x_1 - 2\frac{\dot{R}}{R} x_2 - \frac{\alpha my_s}{R} + |\dot{x}_1|\left[\alpha \ln(|x_1| + 1) + \beta ln^{q/p}(|x_1| + 1)\right] \nonumber \\
+ [\alpha + \frac{\beta q \ln^{(q-p)/p}(|x_1| + 1)}{p} + \frac{f}{R}]\} \tag{8}
\]
Substitute formula (6) into formula (8) to obtain
\[
amy_s = R\left(k_1 s + k_2 sat(s)\right) + \{-2\dot{R}(0) + R\alpha + R|x_1|\left[\alpha \ln(|x_1| + 1) + \beta ln^{q/p}(|x_1| + 1)\right] \nonumber \\
+ \frac{R q \ln^{(q-p)/p}(|x_1| + 1)}{p}\}q_\beta \tag{9}
\]
\(\dot{R}(0)\) refers to the initial relative speed at which the interception point is switched.

The motion of the kinetic energy interceptor in the lateral plane is similar to that in the longitudinal plane
\[
amz_s = -R_1\left(k_1 s' + k_2 sat(s')\right) + \{2\dot{R}_1(0) - R_1 \alpha \nonumber \\
- R_1|x_1|\left[\alpha' \ln(|x_1'| + 1) + \beta' ln^{q'/p'}(|x_1'| + 1)\right] \nonumber \\
- \frac{\beta' q' \left(q'-p'\right)/p'\left(|x_1'| + 1\right)}{p'}\}q_\beta \nonumber
\]
In order to carry out the comparison test, sliding mode surface 1 (formula 3) is selected for comparison in this paper. In the same way, the nonlinear terminal sliding mode guidance law of the longitudinal plane kinetic energy interceptor is obtained when FTSM is used for sliding mode control
\[
am_{ys} = R_2\left(k_3 s + k_4 sat(S)\right) + \{-2\dot{R}_2(0) + R_2 \alpha + R_2 \frac{\beta q_1 \left(q'/p_1-1\right)}{p_1}X_1\}q_{\beta} \tag{10}
\]
The nonlinear terminal sliding mode guidance law of the kinetic energy interceptor in the lateral plane under FTSM is
\[
am_{zs} = -R_3\left(k_3 s' + k_4 sat(S')\right) + \{2\dot{R}_3(0) - R_3 \alpha' - \frac{\beta' q' \left(q'-p'\right)/p'_1\left(|x_1'| + 1\right)}{p'_1}\}q_\beta \tag{11}
\]

5. Simulations and analysis

5.1. Parameter setting
In order to simplify the model, the kinetic energy interceptor is designed to deflect only in the longitudinal plane because the guidance law formula in the lateral plane of the kinetic energy interceptor is only different from that in the longitudinal plane. In order to better study the guidance law at different
distances, two groups of relative distances were selected for simulation comparison.

Take $\alpha = 1$, $\beta = 1$, $p = 5$, $q = 3$. When the target was initially clearly detected and guidance began, the relative distance between the interceptor and the target missile was 1000m and 300m respectively, and the relative speed between the interceptor and the missile was 200m/s. The initial velocity of the interceptor is 3km/s, the initial velocity of the missile is 3.2km/s, and the interceptor is in front of the flight path of the missile. Set $K_1=0.1$, $k_2=k_3=k_4=0.1$.

Matlab/Simulink data simulation, this paper uses M- function and integration module to establish the simulation model. Set the initial value of angular velocity to 0 and the initial deflection angle to 0.1.

5.2. Simulation result

5.2.1. The relative distance is 1000m.

![Deflection Angle comparison in 1000m.](image1)

![Deflection Angle Velocity comparison in 1000m.](image2)

![Deflection Angle Acceleration comparison in 1000m.](image3)

![Deflection Displacement comparison in 1000m.](image4)
5.2.2. The relative distance is 300m.

![Deflection Angle comparison in 300m.](image1)

![Deflection Angle Velocity comparison in 300m.](image2)

![Deflection Angle Acceleration comparison in 300m.](image3)

![Deflection Displacement comparison in 300m.](image4)

5.3. Analysis of simulation results

At a given time, the angular acceleration in the two groups of experimental data tends to 0, and the lateral deflection distance curve is gradually parallel to the X-axis. When the angular acceleration approaches 0, it means that the rudder with complex deflection is able to complete the specified work without applying rudder deflection force. When the lateral deflection distance is parallel to the X-axis, it means that the lateral deflection distance reaches the predetermined value and does not change. This proves that KKV is capable of terminally intercepting the target at a relative distance of 1000m and 300m within a given finite intercept time.

The simulation results of the guidance law derived by the two sliding mode control strategies are different. When the initial deflection Angle is given to be 0.1, the FTSM curves in figure 3 and figure 7 are under the new FTSM curve, indicating that the deflection Angle of FTSM is larger under the same parameters. This conclusion can also be reflected by the fact that the deflection displacement of FTSM in figure 6 and figure 10 is larger than that of the new FTSM. At the same time, figure 5 reflects that the maximum deflection Angle acceleration of the two guidance laws is approximately equal at 1000m, that is, the power of the steering gear is almost the same. Although FTSM lags behind the new FTSM in the process of approaching to 0, its deflectable distance is further. Therefore, it is better to choose FTSM guidance strategy when the terminal
guidance time is sufficient.

When the relative distance is 300m, FTSM shows advantages in deflection Angle, deflection Angle acceleration and deflection distance, but it can be seen from figure 9 that, at the initial guidance, the initial deflection acceleration of FTSM is nearly 4 times that of the new FTSM. This puts forward a higher requirement on the maximum deflection power of the steering gear and reduces the robustness of the interceptor. It can be seen from the solid lines in figure 10 and figure 8 that the new FTSM can still effectively adjust the deflection displacement and deflection speed at a small power, so it is better to choose the new FTSM guidance strategy when the terminal guidance time is less.

6. Conclusion
In this paper, trajectory interception is used to intercept the target in the middle course of a ballistic missile. By studying the sliding-mode terminal guidance law to intercept the warhead, the fast target switching is realized. The following conclusions are obtained through calculation and simulation:

1. The two sliding-mode guidance laws proposed in this paper can effectively solve the problem of short interception time and high precision in the process of switching between real and false warheads. This shows that the sliding-mode guidance law has a large scope in identifying and tracking the terminal guidance of the real warhead.

2. When the terminal guidance time is sufficient, the advantage of FTSM is greater. Compared with the new FTSM, the FTSM control strategy can achieve a higher lateral deflection distance at the same maximum deflection power.

3. The advantage of the new FTSM is greater when the terminal guidance time is less. Compared with the FTSM, the new FTSM can reach the deflection position quickly at a lower maximum deflection power.

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