A Phase Space Path Integral
for (2+1)-Dimensional Gravity

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Abstract
I investigate the relationship between the phase space path integral in (2+1)-dimensional gravity and the canonical quantization of the corresponding reduced phase space in the York time slicing. I demonstrate the equivalence of these two approaches, and discuss some subtleties in the definition of the path integral necessary to prove this equivalence.

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Over the past several years, (2+1)-dimensional general relativity has become a popular model in which to explore the conceptual foundations of quantum gravity \[1\]. But although a few papers have been written about path integral methods \[2, 3, 4, 5, 6, 7, 8, 9\], the primary focus of research has been on various forms of canonical quantization. The purpose of this paper is to briefly describe a phase space path integral, both to display its equivalence to canonical quantization on a reduced phase space and to underline the assumptions needed to demonstrate this equivalence.

In one sense, the results of this paper are obvious: there exist fairly general formal proofs of the equivalence of phase space path integration and reduced phase quantization \[10,11,12\]. Indeed, the path integral for general relativity may be defined by the requirement that it give the correct reduction to the physical phase space \[13,14,15\]. But these proofs involve subtle assumptions about measures, gauge-fixing procedures, and ranges of integration \[16\], and they are particularly tricky when one deals with general relativity, a theory in which the Hamiltonian constraint plays a rather peculiar role \[17\]. It is therefore useful to explore a simple model in which all of the details can be made explicit.

In this paper I concentrate on (2+1)-dimensional gravity on a manifold with the topology \(\mathbb{R} \times \Sigma\), where \(\Sigma\) is a closed orientable surface of genus \(g > 0\). The Hamiltonian reduction of this model in the York time slicing has been completely analyzed by Moncrief \[18,19\]. Thanks to the finite number of physical degrees of freedom of (2+1)-dimensional general relativity, a simple and complete description of the reduced phase space is possible. It should be apparent, however, that much of the analysis presented here also applies, at least formally, to 3+1 dimensions as well.

The canonical action for (2+1)-dimensional gravity is

\[
I_{\text{grav}} = \int d^3x \sqrt{-(3)g} (3)R = \int dt \int_\Sigma d^2x (\pi^{ij} \dot{g}_{ij} - N^i \mathcal{H}_i - N\mathcal{H}),
\]

where the momentum and Hamiltonian constraints take the form

\[
\mathcal{H}_i = -2\nabla_j \pi^j_i \\
\mathcal{H} = \frac{1}{\sqrt{g}} \pi_{ij} (\pi^{ik} \pi^{jl} - \pi^{ij} \pi^{kl}) - \sqrt{g} R.
\]

The phase space path integral is

\[
Z = \int [d\pi^{ij}] [dg_{ij}] [dN^i] [dN] \exp \{iI_{\text{grav}}[\pi, g]\},
\]

but the first class constraints \(\mathcal{H}_\mu = (\mathcal{H}, \mathcal{H}_i)\) generate a set of transformations that must be gauge fixed in order for this expression to be well-defined. For gauge conditions \(\chi^\mu = 0\), the path integral becomes

\[
Z = \int [d\pi^{ij}] [dg_{ij}] [dN^i] [dN] [\delta][\mathcal{H}_\mu, \chi^\nu] \det |\{\mathcal{H}_\mu, \chi^\nu\}| \exp \{iI_{\text{grav}}[\pi, g]\},
\]

*Here, \(g_{ij}\) and \(R\) refer to the induced metric and scalar curvature of a time slice, while the spacetime metric and curvature are denoted \((3)g_{\mu\nu}\) and \((3)R\). Roman indices \(i, j, \ldots\) are spatial indices, raised and lowered with the spatial metric \(g_{ij}\); Greek indices \(\mu, \nu, \ldots\) are spacetime indices.*
where \{ , \} denotes the Poisson bracket. Our goal is to evaluate this integral, reducing it to a quantum mechanical path integral over the finitely many physical degrees of freedom of \((2+1)\)-dimensional gravity.

It is useful to start with a decomposition of the fields \(g_{ij}\) and \(\pi^{ij}\). Let us assume for now that \(\Sigma\) has genus \(g > 1\); the case of the torus \((g = 1)\) will be discussed briefly below. A standard result of Riemann surface theory, the uniformization theorem, implies that any metric on \(\Sigma\) can then be written in the form \(\Sigma\) [20, 21, 22]

\[
g_{ij} = f^* e^{2\lambda} \bar{g}_{ij}(\tau),
\]

(5)

where \(f\) is a diffeomorphism (typically generated by a vector field \(\xi^i\)), \(\lambda\) is a conformal factor, and the \(\bar{g}_{ij}(\tau)\) are a finite-dimensional family of fixed reference metrics of constant curvature \(k = -1\), parametrized by \(6g - 6\) moduli \(\tau_r\). A related parametrization can be given for the momenta \(\pi^{ij}\):

\[
\pi^{ij} = \frac{1}{2} g^{ij} \pi + \sqrt{g}(PY)^{ij} + \sqrt{\bar{g}} p_r \Psi^{(r)ij},
\]

(6)

where \(P\) is an operator taking vectors \(\xi^i\) to traceless tensors \(h_{ij}\),

\[
(P \xi)^{ij} = \nabla_i \xi_j + \nabla_j \xi_i - g_{ij} \nabla k \xi^k,
\]

\[
(P^t h)_i = -2 \nabla^j h_{ij},
\]

(7)

and the \(\Psi^{(r)}\) are a basis of \(\ker P^t\), that is, transverse traceless tensors, or in the language of Riemann surfaces, quadratic differentials. Note that the dimension of \(\ker P^t\) is \(6g - 6\), the same as the dimension of the moduli space; indeed, the \(\Psi^{(r)}\) are a basis for the tangent space of the moduli space of \(\Sigma\) [18, 22].

In terms of these new variables, the constraints (2) become

\[
\mathcal{H}_i = \sqrt{g}(P^t PY)_i - \nabla_i \pi
\]

\[
\mathcal{H} = -\frac{1}{2} \pi^2 e^{-2\lambda} + \sqrt{g} e^{-2\lambda} p_r p_s \bar{g}^{ij} \bar{g}^{kl} \Psi^{(r)ij} \Psi^{(s)kl}
\]

\[
+ \sqrt{g} e^{-2\lambda} \bar{g}^{ij} \bar{g}^{kl} (PY)_{ik} (PY)_{jl} + \sqrt{g} (2 \bar{\Delta} \lambda - k),
\]

(8)

where \(\bar{\Delta}\) is the Laplacian with respect to \(\bar{g}_{ij}\). We would like to change variables in the path integral from \((g_{ij}, \pi^{ij})\) to \((\tau_r, \lambda, f, p_s, \pi^i, Y^i)\). To find the Jacobian for this transformation, we follow the approach introduced in string theory by Alvarez [22, 23]. (For an application to the covariant path integral in quantum gravity, see [24].) We start by defining inner products

\[
\langle \delta g, \delta g \rangle = \int_{\Sigma} d^2 x \sqrt{g} g^{ij} \delta g_{ik} \delta g_{jl}
\]

\[
\langle \delta \xi, \delta \xi \rangle = \int_{\Sigma} d^2 x \sqrt{g} \bar{g}^{ij} \delta \xi_i \delta \xi_j
\]

\[
\langle \delta \lambda, \delta \lambda \rangle = \int_{\Sigma} d^2 x \sqrt{g} (\delta \lambda)^2
\]

(9)
on the tangent space to the space of metrics on \( \Sigma \). Corresponding to the parametrization (5), an arbitrary infinitesimal deformation of the metric \( g_{ij} \) admits an orthogonal decomposition\[^2\]

\[
\delta g_{ij} = 2(\delta \tilde{\lambda})g_{ij} + (P(\delta \tilde{\xi}))_{ij} + \delta \tau_r T^{rs} \Psi^{(s)}_{ij},
\]

where the last term is the orthogonal projection of the modular deformation of \( g_{ij} \) onto \( \ker P^\dagger \), that is,

\[
T^{rs} = \langle \chi^{(r)}(u), \Psi^{(u)} \rangle \langle \Psi^{(u)}, \Psi^{(s)} \rangle^{-1}, \quad \text{with} \quad \chi^{(r)}_{ij} = \frac{\partial g_{ij}}{\partial \tau_r}.
\]

Now consider the simple Gaussian path integral

\[
1 = \int [dg_{ij}] e^{i\langle \delta g, \delta g \rangle} = \int d^n(\delta \tau) [d(\delta \tilde{\lambda})][d(\delta \tilde{\xi})] J e^{i(\delta \tilde{\xi}, P^\dagger P \delta \tilde{\xi})} e^{8i(\delta \tilde{\lambda}, \delta \tilde{\xi})} e^{i\delta \tau_r \delta \tau_s T^{rs} T^{uv} \langle \Psi^{(u)}, \Psi^{(v)} \rangle}.
\]

Evaluating the integrals over \( \delta \tau, \delta \tilde{\lambda}, \) and \( \delta \tilde{\xi} \), we easily find that the Jacobian \( J \) is

\[
J_g = \det|P^\dagger P|^{1/2} \det|T| \det|\langle \Psi^{(u)}, \Psi^{(v)} \rangle|^{1/2}.
\]

This Jacobian was derived by considering integrals on the tangent space to the space of metrics, but a simple argument shows that it is equal to the Jacobian for the integral over the \( g_{ij} \)\[^3\]. The \( \pi^{ij} \) integral gives a similar Jacobian,

\[
J_\pi = \det|P^\dagger P|^{1/2} \det|\langle \Psi^{(u)}, \Psi^{(v)} \rangle|^{1/2},
\]

which combines with \( J_g \) to give a total Jacobian

\[
J = \det|P^\dagger P| \det|T| \det|\langle \Psi^{(u)}, \Psi^{(v)} \rangle| = \det|P^\dagger P| \det|\chi^{(u)}, \Psi^{(v)}|.
\]

If we now change variables and integrate over \( N^\mu \), the path integral (14) becomes

\[
Z = \int d^n p d^n \tau \det|\langle \chi^{(u)}, \Psi^{(v)} \rangle| \times \int [d(\pi/\sqrt{g})][d\lambda][dY][d\xi] \det|P^\dagger P| \delta[\lambda^\mu] \delta[\mathcal{H}_\nu/\sqrt{g}] \det|\{\mathcal{H}_\mu, \chi^\nu\}| e^{iI_{\text{grav}}[p, \tau, \lambda, \pi]}.\]

The factors of \( 1/\sqrt{g} \) in the delta functionals are somewhat conventional, but can be viewed as coming from the inner products (14) and the rule

\[
\int [da] e^{i(a,b)} = \delta[b].
\]

\[^1\] \( \delta \tilde{\lambda} \) and \( \delta \tilde{\xi} \) are infinitesimal Weyl transformations and diffeomorphisms, up to linear shifts.
The integral over $Y_i$ is now straightforward; from (8),

$$\int [dY_i] \det |P^i P| \delta[\mathcal{H}_i/\sqrt{g}] = 1,$$

where from now on we set $Y_i = (P^i P)^{-1} \nabla_i \pi / \sqrt{g}$.

To proceed further, we must make a partial choice of gauge fixing. The momentum constraints $\mathcal{H}_i$ generate ordinary spatial diffeomorphisms, and their treatment is straightforward. The Hamiltonian constraint $\mathcal{H}_i$ on the other hand, does not generate time reparametrizations, as one might naively expect, although the corresponding transformations are related on shell [17]. Nevertheless, $\mathcal{H}$ generates an invariance that must be gauge fixed [25]. Following Moncrief, let us do so by choosing the York time slicing [26]

$$\chi = \pi / \sqrt{g} - T = 0,$$  \hspace{1cm} (15)

where $T$ is a time coordinate. Observe that with this gauge choice, $\nabla_i \pi = 0$, and hence $Y_i = 0$. Note also that $\chi$ is a spatial scalar, so $\{\mathcal{H}_i, \chi\} = 0$ when $\chi = 0$. The determinant $\det |\{\mathcal{H}_i, \chi^\nu\}|$ therefore splits into a product

$$\det |\{\mathcal{H}, \chi\}| \det |\{\mathcal{H}_i, \chi^j\}|.$$

The first term is easily evaluated, using the canonical commutators

$$\{g_{ij}(x), \pi^{kl}(x')\} = \delta_{ij} \delta_{kl} (x - x'),$$  \hspace{1cm} (16)

and yields

$$\det |\{\mathcal{H}, \chi\}| = \det \left| -\Delta + \frac{1}{g} \pi_{ij} \pi^{ij} \right| = \det \left| e^{-2\lambda} \left( -\bar{\Delta} + \frac{T^2}{2} e^{2\lambda} + e^{-2\lambda} p_r p_s g^{ij} g^{kl} \Psi^{(r)}_i \Psi^{(s)}_j \right) \right|$$

at $Y_i = 0$ and $\pi / \sqrt{g} = T$. We could next choose a gauge condition $\chi^i = 0$ and evaluate the remaining determinant, but we need not do so. Almost everything in the path integral is invariant under spatial coordinate transformations; the only remaining $\xi$-dependent terms are

$$\int [d\xi] \delta[\chi^i] \det |\{\mathcal{H}_i, \chi^j\}| = 1.$$

The path integral (14) thus simplifies to

$$Z = \int d^3 p \, d^3 \tau \, \det |\langle \chi^{(u)}, \Psi^{(v)} \rangle| \int [d\lambda] \delta[\mathcal{H}/\sqrt{g}] \det |\{\mathcal{H}, \chi\}| e^{i \mathcal{H}_{\text{grav}}[p, \tau, \lambda]}$$  \hspace{1cm} (18)

with $\det |\{\mathcal{H}, \chi\}|$ given by (17).

We can now use the remaining delta functional to evaluate the integral over $\lambda$. From eqn. (8),

$$\mathcal{H}/\sqrt{g} = 2 e^{-2\lambda} \left( \bar{\Delta} \lambda - \frac{k}{2} - \frac{T^2}{4} e^{2\lambda} + \frac{1}{2} e^{-2\lambda} p_r p_s g^{ij} g^{kl} \Psi^{(r)}_i \Psi^{(s)}_j \right).$$  \hspace{1cm} (19)
Moncrief has shown that the equation $\mathcal{H} = 0$ has a unique solution $\lambda = \bar{\lambda}(p, \tau, T)$ \[18\]. Hence

$$\delta[\mathcal{H}/\sqrt{g}] = \det \left[ \frac{\delta}{\delta \lambda}(\mathcal{H}/\sqrt{g}) \right]^{-1} \delta[\lambda - \bar{\lambda}],$$

and the determinant is easily seen to exactly cancel \[17\] when $\mathcal{H} = 0$. We thus obtain

$$Z = \int d^n p d^n \tau \det |\langle \chi^{(u)}, \Psi^{(v)} \rangle| e^{iI_{grav}[p, \tau]},$$

where $\bar{I}$ is the action evaluated at the solution of the constraints.

Finally, let us consider the remaining determinant $\det |\langle \chi^{(u)}, \Psi^{(v)} \rangle|$. I have defined the reduced phase space coordinate $p_r$ by the decomposition \[6\]. Moncrief, on the other hand, uses a slightly different parametrization,

$$\bar{p}_r = \int_{\Sigma} d^2 x e^{2\lambda(\pi^{ij} - \frac{1}{2}g^{ij}\pi)} \frac{\partial \bar{g}_{ij}}{\partial \tau_r} = \langle p_s \Psi^{(s)}, \chi^{(r)} \rangle = \langle \chi^{(r)}, \Psi^{(s)} \rangle p_s.$$ \[21\]

By changing variables to $\bar{p}_r$, we can write the path integral \[20\] as

$$Z = \int d^n \bar{p} d^n \tau e^{iI_{grav}[\bar{p}, \tau]}.$$ \[22\]

An explicit expression for the action $\bar{I}$ may be obtained by inserting eqns. \[5\] and \[6\] into the term $\pi^{ij}\dot{g}_{ij}$ in eqn. \[1\]; a simple computation gives

$$I_{grav}[\bar{p}, \tau] = \int dT \left( \bar{p}_r \frac{d\tau_r}{dT} - H(\bar{p}, \tau, T) \right), \quad H = \int_{\Sigma} d^2 x \sqrt{\bar{g}} e^{2\lambda(\bar{p}, \tau, T)}.$$ \[23\]

This is precisely Moncrief’s reduced phase space action \[18\], and eqn. \[22\] is exactly the right quantum mechanical path integral for the corresponding reduced phase space quantum theory. To the extent that an ordinary quantum mechanical path integral for a system with finitely many degrees of freedom is equivalent to canonical quantization, we have therefore reproduced the quantum theory described in reference \[1\].

A similar analysis is possible when $\Sigma$ is a torus, although a few complications occur. Decompositions of the form \[5\] and \[6\] again exist; the $\bar{g}_{ij}$ are now a two-parameter family of metrics of curvature $k = 0$, normalized to unit volume. The operator $P^\dagger P$ now has zero modes, however, corresponding to conformal Killing vectors; its determinant is thus identically zero. A similar phenomenon occurs in string theory, where a careful analysis shows that it causes no problems (see \[23\]), essentially because the zero modes should be omitted from the decompositions \[6\] and \[10\]. A further complication arises from the existence of classical configurations with $p_r = 0$, for which the constraint $\mathcal{H} = 0$ has solutions only when $T$ vanishes. For these configurations, the gauge choice \[15\] is no longer admissible. Such exceptional solutions also complicate the analysis in the Chern-Simons formulation \[28\], but they form a set of measure zero in the space of solutions, and can probably be safely omitted from
the path integral. If we do so, we again recover expressions of the form (22) and (23), and the path integral again reproduces canonical quantization.

This result is not surprising, but it is worthwhile to review the assumptions needed to reach this conclusion. Four in particular are important:

1. **Gauge fixing:** The gauge choice (15) is the phase space version of the York time slicing condition, in which the mean (extrinsic) curvature \( \text{Tr}K = T \) is used as a time coordinate. Moncrief has shown that for solutions of the field equations in 2+1 dimensions, this is a good global coordinate choice, at least in the domain of dependence of an initial spacelike surface \( \Sigma \) of genus \( g > 1 \). But the path integral involves a sum over all spacetimes, including geometries for which \( \text{Tr}K = T \) is certainly not a good time slicing. This is not necessarily a contradiction, since the relationship between \( \pi \) and \( \text{Tr}K \) also breaks down off shell, but there is clearly more to be understood here. A useful starting point might be a careful analysis of the exceptional \( p_r = 0 \) solutions for genus 1, which are classical solutions for which the gauge choice (15) fails.

2. **Conformal anomalies:** My derivation of the reduced phase space path integral (22) involved no determinants of the form \( \det |e^{2\lambda}| \). Depending on the precise choice of the measure, however, such terms could appear. For example, it is not obvious a priori whether the integral over the lapse function should give \( \delta [\mathcal{H}/\sqrt{g}] \) or, say, \( \delta [\mathcal{H}] \); these two possibilities differ by such a determinant.

Similar determinants appear as conformal anomalies in noncritical string theory, where it has been argued that they lead to a Liouville action for \( \lambda \) [27],

\[
I = \int_{\Sigma} d^2x \sqrt{\bar{g}} \left( \frac{1}{2} \lambda \Delta \lambda + k \lambda + \mu_0^2 e^{2\lambda} \right).
\]

Since the value of \( \lambda \) is fixed by a delta functional in our path integral, such a term would not lead to new degrees of freedom, but it would change the action (23). For a torus universe, \( k = 0 \) and the Hamiltonian constraint requires that \( \lambda \) be constant, so the only effect of a Liouville term would be to multiplicatively renormalize the reduced phase space Hamiltonian \( \mathcal{H} \). For genus \( g > 1 \), however, the effect would be more significant.

3. **Lapse integration:** A crucial element of this derivation was the appearance of a delta functional \( \delta [\mathcal{H}/\sqrt{g}] \) coming from the integration over the lapse \( N \). Such a term requires an integration range from \( -\infty \) to \( \infty \) and a Lorentzian signature \( iN\mathcal{H} \) rather than \( N\mathcal{H} \) in the exponent. As Teitelboim has noted [29], this is not a unique choice: for instance, one could instead define a “causal” amplitude by integrating over only positive values of \( N \). It should be clear that such an amplitude is not equivalent to that coming from reduced phase space quantization.
4. **Choices of time slicing:** This paper has considered only one special choice of time slicing. In canonical quantization, it is not at all clear that different choices lead to equivalent quantum theories [30]. In the path integral formalism, on the other hand, a general theorem due to Fradkin and Vilkovisky states that the path integral is independent of the gauge-fixing function $\chi$ [11, 15]. It would be interesting to try to apply this theorem to a concrete example in 2+1 dimensions, and to work out implications for canonical quantization. In particular, one might compare this slicing with Teitelboim’s “proper time gauge” $\partial_t N = 0$ [31]. For the torus universe, this gauge is classically equivalent to the York slicing, but the two need not be equivalent off shell, so a comparison could be illuminating.

Finally, let me briefly discuss the extension of this analysis to 3+1 dimensions. As in 2+1 dimensions, the spatial metric admits a decomposition of the form (5), where the metrics $\bar{g}_{ij}$ can be determined by the Yamabe condition $R(\bar{g}) = \text{const.}$ along with some suitable spatial gauge condition [21, 32]. In contrast to the (2+1)-dimensional model, the $\bar{g}_{ij}$ now form an infinite-dimensional space, whose properties are not fully understood. Nevertheless, one can at least formally extend many of the results of this paper. The decompositions analogous to (4) and (10) are discussed in reference [33], and the corresponding Jacobians have been derived, albeit in a rather different context, in references [3] and [24]. It is not hard to show that the Jacobian analogous to $\det|P|P|$ is again cancelled by terms coming from the integration over $Y$, and that in the York time slicing [15], the determinant $\det \{H, \chi\}$ is cancelled, up to possible conformal determinants, when one integrates the delta functional $\delta[H]$. What is missing, however, is an explicit description of the resulting (infinite dimensional) reduced phase space and the measure corresponding to that of (22), i.e., a three-dimensional version of moduli space.

**Acknowledgements**

This work was supported in part by National Science Foundation grant PHY-93-57203 and Department of Energy grant DE-FG03-91ER40674.

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