Abstract

Many fields of science and engineering, ranging from predicting protein structures to building machine translation systems, require large amounts of labeled data. These labeling tasks have traditionally been performed by experts; the limited pool of experts would limit the size of the datasets, and make the process slow and expensive. In recent years, there is a rapidly increasing interest in using crowds of semi-skilled workers recruited through the Internet. While this ‘crowdsourcing’ can cheaply produce large amounts of labeled data in short times, it is typically plagued by the problem of low quality. To address this fundamental challenge in crowdsourcing, we design a novel reward mechanism for acquiring high-quality data, which incentivizes workers to censor their own low-quality data. Our main results are the mathematical proofs showing that surprisingly, under a natural and desirable ‘no-free-lunch’ requirement, this is the one and only mechanism that is incentive-compatible. The simplicity of the mechanism is an additional attractive property. In preliminary experiments involving over 900 worker-tasks, we observe up to a three-fold drop in the error rates under this unique incentive mechanism.

1 Introduction

A tedious necessity in science and engineering is the labeling of large datasets. For example, in astronomy, a large number of images of galaxies need to be categorized into one of a number of morphological classes [29], or in computer science, in order to build and test a speech recognizer, a large number of speech utterances need to be labeled with the words that were spoken [18]. In the 20th century, these labeling tasks would typically be done by experts, e.g., graduate students. The limited pool of graduate students would limit the size of the datasets.

Recently, these large labeling tasks have been performed by coordinating crowds of semi-skilled workers through the Internet. This is known as crowdsourcing. Generating large labeled data sets through crowdsourcing is now inexpensive and fast. Given the current platforms for crowdsourcing [1, 8, 11, 31], the initial overhead of setting up a crowdsourcing task is minimal. Crowdsourcing has gained significant popularity in several fields including bioinformatics [5, 15, 17], medicine [25, 43], ontology [32], radiology [33], astronomy [29], environmental science [14], management [41] and computer science [3, 35]. In the engineering of intelligent systems, crowdsourcing is indispensable, because machine-learning techniques to create intelligent systems require large amounts of labeled data [10, 18, 34]. The crowdsourcing of labels is also often used to supplement automated algorithms, to perform the tasks that are too difficult to accomplish by machines alone [2, 13, 23, 27, 37].

Most workers in crowdsourcing platforms are not experts. As a consequence, labels obtained from crowdsourcing typically have a significant error [22, 38, 39]. Recent efforts have focused on developing statistical techniques to post-process the noisy labels in order to improve its quality (e.g., [7, 9, 19–21, 30, 34, 36, 40, 44]). However, when the inputs to these algorithms are erroneous, it is difficult to guarantee that the processed labels
Figure 1: Different interfaces for a task that requires the worker to answer the question “Is this the Golden Gate Bridge?”: (a) the conventional interface; (b) with an option to skip; (c) with multiple confidence levels.
the use of gold standard questions, the payment to a worker is determined solely by her own work, and unlike peer-prediction [42] or competitive markets [6], does not depend on other workers. We will call a payment mechanism “incentive compatible” if truthful actions maximize the expected payment of the worker. Finally, we note that the setup of crowdsourcing mandates the payments to be non-negative.

For some fixed threshold $T (0 < T < 1)$, we wish to incentivize the worker to skip the questions for which her confidence is smaller than $T$. For the questions for which her confidence is greater than $T$, we wish to incentivize her to select the answer that she believes is most likely to be correct. Now, the space of possible incentive-compatible mechanisms may indeed be rather wide [16, 26]. Thus, in order to narrow down our search, we impose the following natural requirement that is motivated by the practical considerations of budget constraints and discouraging spammers and miscreants [4, 22, 38, 39].

**No-Free-Lunch Axiom.** If a worker provides wrong answers to all the questions that she attempts in the gold standard, then no payment is made to the worker.

We now present an incentive-compatible mechanism that satisfies the no-free-lunch axiom. Let $\mu > 0$ be the budget, i.e., the maximum amount that can be paid to any individual worker for this task. Let $G$ be the total number of gold standard questions in the task. According to our mechanism, among the $G$ gold standard questions, if a worker has $C$ correct answers and $W$ incorrect answers (with the remaining questions skipped), then

$$\text{Payment} = \begin{cases} 
0 & \text{if } W > 0 \\
\kappa \frac{1}{1-C} & \text{otherwise,} 
\end{cases}$$

(1)

where $\kappa = \mu T^G$.

Observe that the no-free-lunch axiom is an extremely mild requirement. In fact, it is significantly weaker than imposing a zero payment on workers who answer randomly. For instance, if the questions are of binary-choice then randomly choosing among the two options for each question would result in half of the answers being correct in expectation, while the no-free-lunch axiom is applicable only when none of them turns out to be correct. On the other hand, the mechanism of Equation (1) is quite strict, imposing a zero payment even if one of the gold standard questions is answered incorrectly. We have proved that surprisingly, no other mechanism can satisfy the simple and natural requirement of no-free-lunch.

**Theorem.** The mechanism of Equation (1) is the only incentive-compatible payment mechanism that satisfies the no-free-lunch axiom.

This skip-based setting is discussed in more detail in Section 3.

One may sometimes be required to offer a certain minimum payment to each worker. In this case, denoting this minimum payment by “$M$”, the no-free-lunch axiom may be modified to making a payment of $M$ if all attempted answers in the gold standard are wrong. We have shown that in this case, the only incentive-compatible mechanism that satisfies this modified no-free-lunch axiom is the one that pays an amount $M$ in addition to that paid in Equation (1).

The following example illustrates this mechanism. On the Amazon Mechanical Turk platform, we put up a task that displayed 21 images of bridges and asked workers to identify those depicting the Golden Gate Bridge (Figure 1b). The gold standard consisted of three of the images. We set $T = \frac{2}{3}$, thus attempting to incentivize the worker to skip if her confidence was less than 67%. We also set $\mu = 5.9$ and $M = 3$ cents, thus curtailing the maximum payment to a worker to $5.9 \times \frac{33}{2} + 3 \approx 23$ cents. The error among attempted questions under the skip-based mechanism was 7% as compared to 23% under the baseline mechanism of offering a fixed payment of $M = 3$ cents and an additional 5 cents for every correct answer in the gold standard. The average payment made under the two mechanisms was almost equal, and so was the time taken by the workers.

As an aside, when $M = 0$ and $T = \frac{1}{2}$, our (unique) mechanism doubles the worker’s pay for every correct answer in the gold standard, while making the pay zero for any wrong answer. This is similar to the popular
It is worth noting that the mechanism of Equation (1) pays $\kappa$ cents when $W = 0$ and $C = 0$. A more conservative approach would be to design an incentive-compatible mechanism where a worker receives zero payment if she does not give any correct answer, a condition that we refer to as the strong-no-free-lunch condition. To this end we have shown that, unfortunately, it is impossible for any incentive-compatible mechanism to satisfy this strong-no-free-lunch condition (Section 5).

Given that we are interested in understanding the confidence of the workers for each answer, a natural extension of the above mechanism is to ask the workers to explicitly indicate their confidence for each question on a finer scale, e.g., a three-level scale responding to low, moderate and high “confidence-levels” (see Figure 1c). Consequently, we need a mechanism to incentivize them to truthfully reveal their confidence. To this end, again we took an axiomatic approach to the design of the payment mechanism and generalized the previous no-free-lunch axiom.

**Generalized-No-Free-Lunch Axiom. If the worker indicates the highest confidence-level for all the questions that she attempts to answer in the gold standard, and all of these answers turn out to be wrong, then no payment is made to the worker.**

Again, we prove that there exists one and only one incentive-compatible mechanism satisfying the generalized-no-free-lunch axiom in this setting. This mechanism is presented below. For every question, we ask the worker to indicate her confidence on the scale $0, 1, \ldots, L$. On this scale, the highest confidence-level is denoted by $L$, while a skip is denoted by 0 and represents the lowest confidence-level. For any question $i$ in the gold standard, let $x_i$ denote the evaluation of the answer provided by the worker to this question: $x_i = 0$ if the worker skipped the question, and for $j \in \{1, \ldots, L\}$, $x_i = +j$ if the question was answered correctly with confidence-level $j$, and $x_i = -j$ if the question was answered incorrectly with confidence-level $j$. Recall our notation of $G$ representing the number of gold standard questions in the task and $\mu$ representing the budget. The mechanism is:

$$\text{Payment} = \kappa \prod_{i=1}^{G} \alpha_{x_i},$$

(2)

where $\alpha_L > \alpha_{L-1} > \ldots > \alpha_0 = 0$ and $\kappa = \mu \left( \frac{1}{\alpha_L} \right)^G$ are pre-defined constants (Section 4). Observe that the mechanism is of a multiplicative form.

**Theorem.** The mechanism of Equation (2) is the only incentive-compatible payment mechanism that satisfies the generalized-no-free-lunch axiom.

If one needs to offer a minimum payment “$M$” to each worker, then each of these results continue to hold when an amount $M$ is added to the aforementioned amounts.

We conducted preliminary experiments on Amazon Mechanical Turk where we incentivized workers to indicate their confidence-levels (e.g., as in Figure 1c) using the aforementioned mechanism (Section 7). We observed that the amount of errors among the attempted questions was much lower than the baseline mechanism, but (as expected) was higher than the previously seen skip-based mechanism. We also observed that (as expected) the answers selected with higher confidence-levels were more likely to be correct. The total payment and the time taken by the workers were comparable under each of these mechanisms.

The subsequent sections in the paper present our problem setup and results formally, and in more detail. The proofs of all the theoretical results claimed in the paper can be found in the appendix.
2 Setting and Notation

In the crowdsourcing setting that we consider, one or more workers perform a task, where a task consists of multiple questions. The questions are objective, i.e., any answer can be evaluated as being correct or incorrect. Examples of objective questions include multiple-choice classification questions such as Figure 1, transcription questions such as transcribing text from audio or images, etc.

For any worker and for any question, we define her confidence about an answer as the probability, according to her, of this answer being correct. In other words, one can assume that the worker has (in her mind) a distribution over all possible answers to a question, and the confidence for an answer is the probability of that answer being correct. As a shorthand, we also define the confidence about a question as the confidence for the answer that the worker is most confident about. We assume that the worker’s confidences for different questions are independent. Our goal is that for every question, the worker should be incentivized to:

1. Skip if the confidence is below a certain pre-defined threshold, otherwise:
2. Select the answer that she thinks is most confident about, and
3. If asked, indicate a correct (quantized) value of her confidence for the answer.

We will call any algorithm that incentivizes the worker to do so as being incentive-compatible.

We consider two settings:

- **Skip-based**: For each question, the worker can either choose to ‘skip’ the question or provide an answer.
- **Confidence-based**: For each question, the worker can either ‘skip’ the question or provide an answer, and in the latter case, indicate her confidence for this answer as a number in \( \{1, \ldots, L\} \). We term this indicated confidence as the ‘confidence-level’. Here, \( L \) represents the highest confidence-level, and ‘skip’ is considered to be a confidence-level of 0.\(^1\)

One can see from the aforementioned definition that the confidence-based setting is a generalization of the skip-based setting (where the skip-based mechanism corresponds to \( L = 1 \)). The goal is to design mechanisms such that for a given set of intervals that partition \([0, 1]\), for every question we wish to incentivize the worker to indicate ‘skip’ or choose the appropriate confidence-level when her confidence for that question falls in the corresponding interval.

Let \( N \) denote the total number of questions in the task. Among these, we assume the existence of some “gold standard” questions, i.e., a set of questions whose answers are known (or can be evaluated). Let \( G (\leq N) \) denote the number of gold standard questions. The \( G \) gold standard questions are assumed to be distributed uniformly at random in the pool of \( N \) questions (of course, the worker does not know which \( G \) of the \( N \) questions form the gold standard). The payment to a worker for a task is computed after receiving her responses to all the questions in the task. The payment is based on the worker’s performance on the gold standard questions. Since the payment is based on known answers, the payments to different workers do not depend on each other, allowing us to consider the presence of only one worker without any loss in generality. We will let \( \mu \ (> 0) \) denote the budget, i.e., the maximum amount that can be paid to any individual worker for this task.

We will employ the following standard notation. For any positive integer \( K \), the set \( \{1, \ldots, K\} \) will be denoted by \( [K] \). The indicator function will be denoted by \( 1 \), i.e., \( 1\{x\} = 1 \) if \( x \) is true, and 0 otherwise. The notation \( \mathbb{R}_+ \) will denote the set of all non-negative real numbers.

\(^1\)When the task is presented to the workers, the word ‘skip’ or the numbers \( \{1, \ldots, L\} \) are replaced by more comprehensible phrases such as “I don’t know”, “moderately sure”, “absolutely sure”, etc.
Let \( x_1, \ldots, x_G \) denote the evaluations of the answers that the worker gives to the \( G \) gold standard questions, and let \( f \) denote a function that determines the payment to the worker based on these evaluations \( x_1, \ldots, x_G \). It is this function \( f \) that determines the mechanism. Note that the setup of crowdsourcing mandates the payments to be non-negative.

In the skip-based setting, \( x_i \in \{-1, 0, +1\} \) for all \( i \in [G] \). Here, “0” denotes that the worker skipped the question, “−1” denotes that the worker attempted to answer the question and that answer is incorrect, and “+1” denotes that the worker attempted to answer the question and that answer is correct. The payment function is \( f : \{-1, 0, +1\}^G \to \mathbb{R}_+ \).

In the confidence-based setting, \( x_i \in \{-L, \ldots, +L\} \) for all \( i \in [G] \). Here, we set \( x_i = 0 \) if the worker skipped the question, and for \( l \in \{1, \ldots, L\} \), we set \( x_i = l \) if the question was answered correctly with confidence \( l \) and \( x_i = -l \) if the question was answered incorrectly with confidence \( l \). The function \( f : \{-L, \ldots, +L\}^G \to \mathbb{R}_+ \) specifies the payment to be made to the worker.

Our incentive mechanisms operate under the assumption that the worker attempts to maximize her overall expected payment. \(^2\) In the sequel, the expression ‘the worker’s expected payment’ will refer to the expected payment from the worker’s point of view, i.e., the expectation will be with respect to the worker’s confidences about her answers and the uniformly random choice of the \( G \) gold standard questions among the \( N \) questions in the task. For each question \( i \in [N] \), suppose the worker indicates the confidence-level \( y_i \in \{0, \ldots, L\} \). Further, for every question \( i \in [N] \) such that \( y_i \neq 0 \), let \( p_i \) be the confidence of the worker for the answer she has selected for question \( i \), and for every question \( i \in [N] \) such that \( y_i = 0 \), let \( p_i \in (0, 1) \) be any arbitrary value. Let \( E = (\epsilon_1, \ldots, \epsilon_G) \in \{-1, 1\}^G \). Then from the worker’s perspective, the expected payment for the selected answers and confidence-levels is

\[
\frac{1}{\binom{N}{G}} \sum_{\{j_1, \ldots, j_G\} \subseteq \{1, \ldots, N\}} \sum_{E \in \{-1, 1\}^G} \left( f(\epsilon_1 y_{j_1}, \ldots, \epsilon_G y_{j_G}) \prod_{i=1}^G (p_{j_i})^{1+\epsilon_i} (1 - p_{j_i})^{1-\epsilon_i} \right).
\]

The outermost summation (average) captures the randomness arising from the unknown choice of the gold standard questions. The inner summation aggregates all possibilities regarding the correctness of the selected answers.

### 3 Skip-based Setting

In this section, we consider the setting where for every question, the worker can choose to either answer the question or skip it; no additional information is asked from the worker. Let \( T \in (0, 1) \) be a predefined value. The goal is to design payment mechanisms that incentivize the worker to skip the questions for which her confidence is lower than \( T \), and answer those for which her confidence is higher than \( T \). \(^3\) Moreover, for the questions that she attempts to answer, she must be incentivized to select the answer that she believes is most likely to be correct.

We impose the following simple and natural requirement:

**Axiom 1 (No-free-lunch Axiom).** If all the answers attempted by the worker in the gold standard are wrong, then the payment is zero. More formally,

\[
0 < \sum_{i=1}^G 1\{x_i \neq 0\} = \sum_{i=1}^G 1\{x_i = -1\} \Rightarrow f(x_1, \ldots, x_G) = 0.
\]

\(^2\)We will consider general ‘utility functions’ later in the paper (Section 6).

\(^3\)In the event that the confidence about a question is exactly equal to \( T \), the worker may be equally incentivized to answer or skip.
One would generally expect a payment mechanism to impose the restriction of zero payment to spammers who answer randomly. For instance, in a task with binary-choice questions, a spammer is expected to have half of the attempted answers incorrect. The no-free-lunch axiom which we impose is however a significantly weaker condition, mandating zero payment only if all attempted answers are incorrect.

3.1 Payment Mechanism

Consider the payment mechanism described in Algorithm 1.

**Algorithm 1** Incentive mechanism for skip-based setting

- Inputs:
  - Threshold $T$
  - Budget $\mu$
  - Evaluations $(x_1, \ldots, x_G) \in \{-1, 0, +1\}^G$ of the worker’s answers to the $G$ gold standard questions
- Set $\alpha_{-1} = 0$, $\alpha_0 = 1$, $\alpha_{+1} = \frac{1}{T}$
- The payment is
  $$f(x_1, \ldots, x_G) = \kappa \prod_{i=1}^{G} \alpha_{x_i}$$
  where $\kappa = \mu T^G$

Note that the mechanism presented in Algorithm 1 is identical to that given by Equation (1), but under a different notation.

The following theorem shows that this mechanism indeed incentivizes a worker to skip the questions for which her confidence is below $T$, while answering those for which her confidence is greater than $T$. In the latter case, the worker is incentivized to select the answer which she thinks is most likely to be correct.

**Theorem 1.** The mechanism of Algorithm 1 is incentive-compatible and satisfies the no-free-lunch condition.

3.2 Uniqueness of this Mechanism

We prove that the mechanism of Algorithm 1 is unique.

**Theorem 2.** The mechanism of Algorithm 1 is the only incentive-compatible mechanism that satisfies the no-free-lunch condition.

3.3 The Case of a Necessary Minimum Payment

One may encounter a scenario where a certain minimum payment must necessarily be made to every worker. Let $M$ be this minimum requisite payment. We modify the no-free-lunch axiom to accommodate this necessity of minimum payment:

**No-free-lunch axiom under minimum payment $M$:** If all answers attempted by the worker in the gold standard are incorrect, then the worker is paid an amount $M$. 

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We modify the mechanism of Algorithm 1 to pay an additional amount \( M \) to each worker: the payment to a worker whose answers to the \( G \) questions are \((x_1, \ldots, x_G) \in \{-1, 0, +1\}^G\) is

\[
f(x_1, \ldots, x_G) = M + \kappa \prod_{i=1}^G \alpha_{x_i},
\]

where the constants \( \kappa \) and \( \alpha_{-1}, \alpha_0, \alpha_{+1} \) are as defined in Algorithm 1.

A simple corollary of Theorem 1 and Theorem 2 proves the incentive compatibility and uniqueness of this mechanism in this setting.

**Corollary 3.** If a minimum payment of \( M \) is required to be made to each worker, then the mechanism described by Equation (3) is the only incentive-compatible mechanism that satisfies the no-free-lunch axiom under minimum payment.

### 4 Confidence-based Setting

In this section, we will discuss incentive mechanisms when the worker is asked to select from more than one confidence-levels for every question. In particular, for some \( L \geq 1 \), the worker is asked to indicate a confidence-level in the range \( \{0, \ldots, L\} \) for every answer. Level 0 is the ‘skip’ level, and level \( L \) denotes the highest confidence. Note that we do not solicit an answer if the worker indicates a confidence-level of 0 (skip), but the worker must provide an answer if she indicates a confidence-level of 1 or higher. This makes the case of having only a ‘skip’ as considered in Section 3 a special case of this setting, and corresponds to \( L = 1 \).

We generalize the requirement of no-free-lunch to the confidence-based setting as follows.

**Axiom 2 (Generalized-no-free-lunch axiom).** If all the answers attempted by the worker in the gold standard are selected as the highest confidence-level (level \( L \)), and all of them turn out to be wrong, then the payment is zero. More formally,

\[
0 < \sum_{i=1}^G 1\{x_i \neq 0\} = \sum_{i=1}^G 1\{x_i = -L\} \Rightarrow f(x_1, \ldots, x_G) = 0.
\]

In the confidence-based setting, we require the specification of a set \( \{S_l, T_l\}_{l=1}^L \) of thresholds that determine the confidence-levels that the workers should indicate. These thresholds are generalizations the skip-based setting.

- The threshold \( T \) in the skip-based setting specified when the worker should skip a question and when she should attempt to answer. This is generalized to the confidence-based setting where for every level \( l \in [L] \), a fixed threshold \( S_l \) specifies the ‘strength’ of confidence-level \( l \): If restricted to only the two options of skipping or selecting confidence-level \( l \) for any question, the worker should be incentivized to select confidence-level \( l \) if her confidence is higher than \( S_l \) and skip if her confidence is lower than \( S_l \).

- If a worker decides to not skip a question, she must choose one of multiple confidence-levels. A set \( \{T_l\}_{l=1}^L \) of thresholds specify the boundaries between different confidence-levels. In particular, when the confidence of the worker for a question lies in \((T_{l-1}, T_{l+1})\), then the worker must be incentivized to indicate confidence-level \( (l - 1) \) if her confidence is lower than \( T_l \) and to indicate confidence-level \( l \) if her confidence is higher than \( T_l \).

\[
\text{If the worker’s confidence is exactly equal to } T_l \text{ for any } l \in [L], \text{ then she may be equally incentivized to choose level } l \text{ or level } (l - 1).
\]
We will call a payment mechanism as incentive-compatible if it satisfies the two requirements listed above, and also incentivizes the worker to select the answer that she believes is most likely to be correct for every question for which her confidence is higher than $T_1$.

The problem setting inherently necessitates certain restrictions in the choice of the thresholds. Since we require the worker to choose a higher level when her confidence is higher, the thresholds must necessarily be monotonic:

$$0 < S_1 < S_2 < \cdots < S_L < 1$$

and

$$0 < T_1 < T_2 < \cdots < T_L < 1.$$ 

Also observe that the definitions of $S_1$ and $T_1$ coincide, and hence $S_1 = T_1$. Additionally, we can show (Corollary 19 in the Appendix) that for incentive-compatible mechanisms to exist, it must be that $T_l > S_l \forall l \in \{2, \ldots, L\}$. As a result, the thresholds must satisfy

$$T_1 = S_1, \ T_2 > S_2, \ldots, T_L > S_L.$$ 

These thresholds may be chosen based on various factors of the problem at hand, for example, on the post-processing algorithms, any statistics on the distribution of worker abilities, budget constraints, etc. Designing these values is outside the scope of this paper.

### 4.1 Payment Mechanism

The proposed payment mechanism is described in Algorithm 2.

**Algorithm 2** Incentive mechanism for the confidence-based setting

- **Inputs:**
  - Thresholds $S_1, \ldots, S_L$ and $T_1, \ldots, T_L$
  - Budget $\mu$
  - Evaluations $(x_1, \ldots, x_G) \in \{-L, \ldots, +L\}^G$ of the worker’s answers to the $G$ gold standard questions

- **Set $\alpha_{-L}, \ldots, \alpha_L$ as**
  - $\alpha_L = \frac{1}{S_L}, \ \alpha_{-L} = 0$
  - For $l \in \{L-1, \ldots, 1\}$,
    $$\alpha_l = \frac{(1 - S_l)T_{l+1}\alpha_{l+1} + (1 - S_l)(1 - T_{l+1})\alpha_{-(l+1)} - (1 - T_{l+1})}{T_{l+1} - S_l}$$
    and
    $$\alpha_{-l} = \frac{1 - S_l\alpha_l}{1 - S_l}$$
  - $\alpha_0 = 1$

- **The payment is**
  $$f(x_1, \ldots, x_G) = \kappa \prod_{i=1}^{G} \alpha_{x_i}$$

  where $\kappa = \mu \left( \frac{1}{\alpha_L} \right)^G$. 

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The following theorem shows that this mechanism indeed incentivizes a worker to select answers and confidence-levels as desired.

**Theorem 4.** The mechanism of Algorithm 2 is incentive-compatible and satisfies the generalized-no-free-lunch condition.

**Corollary 5.** The mechanism of Algorithm 2 also ensures a condition stronger than the ‘boundary-based’ definition of the thresholds \( \{T_l\}_{l \in [L]} \) given earlier. Under this mechanism, for every \( l \in [L - 1] \) the worker is incentivized to select confidence-level \( l \) (over all else) whenever her confidence lies in the interval \( (T_l, T_{l+1}) \), select confidence-level 0 (over all else) whenever her confidence is lower than \( T_1 \) and select confidence-level \( L \) (over all else) whenever her confidence is higher than \( T_L \).

**4.2 Uniqueness of this Mechanism**

We prove that the mechanism of Algorithm 2 is unique.

**Theorem 6.** The payment mechanism of Algorithm 2 is the only incentive-compatible mechanism that satisfies the generalized-no-free-lunch condition.

**4.3 The Case of a Necessary Minimum Payment**

We now consider the scenario where a certain minimum payment must necessarily be made to every worker. Let \( M \) be this minimum requisite payment. We modify the generalized-no-free-lunch axiom to accommodate this necessity of minimum payment:

*Generalized-no-free-lunch axiom under minimum payment of \( M \):* If all answers attempted by the worker in the gold standard are selected as the highest confidence-level (level \( L \)), and all of them are incorrect, then the worker is paid an amount \( M \).

We modify the mechanism of Algorithm 2 to pay an additional amount \( M \) to each worker: the payment to a worker whose answers to the \( G \) questions are \( (x_1, \ldots, x_G) \in \{-L, \ldots, +L\}^G \) is

\[
f(x_1, \ldots, x_G) = M + \kappa \prod_{i=1}^{G} \alpha_{x_i},
\]

where the constants \( \kappa \) and \( \alpha_{-L}, \ldots, \alpha_{+L} \) are as defined in Algorithm 2.

A simple corollary of Theorem 4 and Theorem 6 shows the incentive compatibility and uniqueness of this mechanism in this setting.

**Corollary 7.** If a minimum payment of \( M \) is required to be made to each worker, then the mechanism described by Equation (4) is the only incentive-compatible mechanism that satisfies the generalized-no-free-lunch axiom under minimum payment.

**5 A Stronger No-free-lunch Condition: Impossibility Results**

Recall that the no-free-lunch axiom under the skip-based mechanism of Section 3 imposes a zero payment if all attempted answers in the gold standard were incorrect. However, a worker who skips all the questions may still receive a payment. The generalization under the confidence-based mechanism of Section 4 imposed a zero payment if all attempted answers in the gold standard were selected with the highest confidence-level and were
incorrect. However, a worker who marked all questions with a lower confidence level may be paid even if her answers to all the questions in the gold standard turn out to be incorrect. One may thus wish to impose a stronger requirement instead, where no payment is made to workers who make no useful contribution. This is the primary focus of this section.

Consider a skip-based setting. Define the following axiom which is slightly stronger than the no-free-lunch axiom defined previously.

**Strong-no-free-lunch:** If none of the answers in the gold standard are correct, then no payment is made. More formally, \( \sum_{i=1}^{G} 1\{x_i > 0\} = 0 \Rightarrow f(x_1, \ldots, x_G) = 0 \).

We argue that the strong-no-free-lunch axiom is only slightly stronger than the no-free-lunch axiom proposed in Section 3 for the skip-based setting. The strong-no-free-lunch condition can equivalently be written as \( \sum_{i=1}^{G} 1\{x_i \neq 0\} = \sum_{i=1}^{G} 1\{x_i = -1\} \Rightarrow f(x_1, \ldots, x_G) = 0 \). From this interpretation, one can see that to the set of events necessitating a zero payment under the no-free-lunch axiom, the strong-no-free-lunch axiom adds only one extra event, the event that the worker skips all questions. Unfortunately, it turns out that this minimal strengthening of the requirements is associated to impossibility results.

In this section we show that no mechanism satisfying the strong-no-free-lunch axiom can be incentive compatible in general. The only exception is the case when (a) all questions are in the gold standard \((G = N)\), and (b) it is guaranteed that the worker has a confidence greater than \(T\) for at least one of the \(N\) questions. These conditions are, however, impractical for the crowdsourcing setup under consideration in this paper. We will first prove the impossibility results under the strong-no-free-lunch axiom. For the sake of completeness (and also to satisfy mathematical curiosity), we will then provide a (unique) mechanism that is incentive-compatible and satisfies the strong-no-free-lunch axiom for the skip-based setting under the two conditions listed above.

Let us continue to discuss the skip-based setting. In this section, we will call any worker whose confidences for all of the \(N\) questions is lower than \(T\) as an unknowledgeable worker, and call the worker a knowledgeable worker otherwise.

**Proposition 8.** No payment mechanism satisfying the strong-no-free-lunch condition can incentivize an unknowledgeable worker to skip all questions. As a result, no mechanism satisfying the strong-no-free-lunch axiom can be incentive-compatible.

This negative result relies on trying to incentivize an unknowledgeable worker to act as desired. Since no mechanism can be incentive compatible for unknowledgeable workers, we will now consider only workers who are knowledgeable. The following proposition shows that the strong-no-free-lunch condition is too strong even for this relaxed setting.

**Proposition 9.** When \(G < N\), there exists no mechanism that is incentive-compatible for knowledgeable workers and satisfies the strong-no-free-lunch condition.

Given this impossibility result for \(G < N\), we are left with \(G = N\) which means that the true answers to all the questions are known apriori. This condition is clearly not applicable to a crowdsourcing setup; nevertheless, it is mathematically interesting and may be applicable to other scenarios such as testing and elicitation of beliefs about future events.

Proposition 10 below presents a mechanism for this case and proves its uniqueness. We previously saw that an unknowledgeable worker cannot be incentivized to skip all the questions (even when \(G = N\)). Thus, in our payment mechanism, we do the next best thing: Incentivize the unknowledgeable worker to answer only one question, that which she is most confident about, while incentivizing the knowledgeable worker to answer questions for which her confidence is greater than \(T\) and skip those for which her confidence is smaller than \(T\).

**Proposition 10.** Let \(C\) be the number of correct answers and \(W\) be the number of wrong answers (in the gold standard). Let the payment be zero if \(W > 0\) or \(C = 0\), and be \(\frac{\mu T^G d}{T T^G}\) otherwise.
Under this mechanism, when \( G = N \), an unknowledgeable worker is incentivized to answer only one question, that for which her confidence is the maximum, and a knowledgeable worker is incentivized to answer the questions for which her confidence is greater than \( T \) and skip those for which her confidence is smaller than \( T \). Furthermore, when \( G = N \), this is the only mechanism that obeys the strong-no-free-lunch condition and is incentive-compatible for knowledgeable workers.

The following proposition shows that the strong-no-free-lunch condition is not applicable to the confidence-based setting (\( L > 1 \)). The strong-no-free-lunch condition is still defined as in the beginning of Section 5, i.e., the payment is zero if none of the answers are correct.

**Proposition 11.** When \( L > 1 \), for any values of \( N \) and \( G (\leq N) \), it is impossible for any mechanism to satisfy the strong-no-free-lunch condition and be incentive-compatible even when the worker is knowledgeable.

### 6 General Utility Functions

In this section, we consider a setting where the worker, instead of maximizing her expected payment, aims to maximize the expected value of some utility function of her payment.

Consider any function \( U : \mathbb{R}_+ \to \mathcal{I} \), where \( \mathcal{I} \) is any interval on the real number line. We will require the function \( U \) to be strictly increasing and to have an inverse. Examples of such functions include \( U(x) = \log(1 + x) \) with \( \mathcal{I} = \mathbb{R}_+ \), \( U(x) = \sqrt{x} \) with \( \mathcal{I} = \mathbb{R}_+ \), and \( U(x) = 1 - e^{-x} \) with \( \mathcal{I} = [0, 1] \).

Consider the problem setting of Section 4 (of which, the setting of Section 3 is a special case). Suppose that instead of aiming to maximize the expected payment, the worker has some utility \( U \) for any payment made to her, and that she aims to maximize this utility. In other words, for any payment \( f \) made to the worker (based on the evaluation of her answers to the gold standard questions), her utility for this payment is \( U(f) \). The worker aims to maximize the expected value of \( U(f) \), where the expectation is with respect to her beliefs regarding correctness of her answers and the uniformly random distribution of the \( G \) gold standard questions among the set of \( N \) questions. The function \( U \) is assumed to be known to the worker as well as the system designer.

Recall the notation \( \{x_i\}_{i=1}^G \), \( \{\alpha_j\}_{j=-L}^L \) and \( \kappa \) from Algorithm 2. Also recall the (generalized-)no-free-lunch axiom which mandates a zero payment if, in the gold standard, (all attempted questions are marked as the highest confidence \( L \) and) the answers to all the attempted questions are incorrect.

**Theorem 12.** For a worker who aims to maximize function \( U \) of the payment, the one and only mechanism that is incentive-compatible and satisfies the (generalized-)no-free-lunch axiom is

\[
\text{Payment}(x_1, \ldots, x_G) = U^{-1}\left(\kappa \prod_{i=1}^G \alpha_{x_i} + U(0)\right),
\]

where the constants \( \{\alpha_j\}_{j=-L}^L \) and \( \kappa \) are as defined in Algorithm 2.

The value of \( \kappa \) should be small enough that \( \kappa (\alpha_L)^G \) is no larger than the length of the interval \( \mathcal{I} \).

**Corollary 13.** If a minimum payment of \( M \) is required, then the one and only mechanism that is incentive-compatible and satisfies the (generalized-)no-free-lunch axiom with minimum payment \( M \) is

\[
\text{Payment}(x_1, \ldots, x_G) = U^{-1}\left(\kappa \prod_{i=1}^G \alpha_{x_i} + U(M)\right),
\]

where the constants \( \{\alpha_j\}_{j=-L}^L \) and \( \kappa \) are as defined in Algorithm 2.
The rest of this paper assumes that the worker aims to maximize her expected payment. This assumption is a special case of the setting of the present section, and the utility function for this special case is \( U(x) = x \).

### 7 Experiments

In this section, we describe practical experiments that we performed using the Amazon Mechanical Turk crowdsourcing platform [1]. Before delving into details, we would like to emphasize the following caveats. When a worker encounters a mechanism for only a small amount of time (at most a few tens or a few hundred tasks in typical research experiments) and for a small amount of money (at most a few dollars in typical crowdsourcing tasks), we cannot expect the worker to completely understand the mechanism and act precisely as required. For instance, we wouldn’t expect our experimental results to change significantly even upon moderate modifications in the promised amounts, and furthermore, we certainly expect the outcomes to be noisy. Incentive compatibility kicks in when the worker encounters a mechanism across a longer term, for example, when a proposed mechanism is adopted as a standard for a platform, or when higher amounts are involved, such as in a high-value auction. This is when we would expect workers or other entities (e.g., bloggers or researchers) to spend time and effort in designing strategies to game the mechanism. The theoretical guarantee of incentive compatibility then prevents such gaming. Our intention for conducting the experiments was to perform a sanity check on the designed mechanisms. In the experiments, we observe a substantial reduction in the error-rates while receiving no negative comments from the workers, suggesting that these mechanisms can work; the fundamental theory underlying the mechanisms ensures that the system cannot be gamed in the long run.

We will first briefly describe the Amazon Mechanical Turk crowdsourcing platform. In this platform, individuals or businesses (called ‘requesters’) can post tasks, and any individual (called a ‘worker’) may complete the task over the Internet in exchange for some pre-specified payment. The payment may comprise of two parts: a fixed component which is identical for all workers performing that task, and a ‘bonus’ which may be different for different workers and is paid at the discretion of the requester.

We designed nine experiments (tasks) ranging from image annotation to text and speech recognition, which are described in detail below and illustrated in Figure 2. For each experiment, we compared a baseline mechanism of paying a fixed amount per correct answer, our skip-based mechanism and our confidence-based mechanism. For each mechanism in each experiment, we specified the requirement of 35 workers independently performing the task. This amounts to a total of 945 worker-tasks (315 worker-tasks for each mechanism). We also set the following constraints for a worker to attempt our tasks: the worker must have completed at least 100 tasks previously, and must have a history of having at least 95% of her prior work approved by the respective requesters. In each experiment, we offered a certain small fixed payment (in order to attract the workers in the first place) and executed the variable (incentive) part of our mechanisms via a bonus payment. Figure 3 depicts an example of the instructions given to the worker.

Figure 4 plots, for the baseline, skip-based and confidence-based mechanisms for all nine experiments, the (i) fraction of questions that were answered incorrectly, (ii) fraction of questions that were attempted, (iii) the average payment to a worker (in cents), and (iv) break up of the answers in terms of the fraction of answers in each confidence level. The payment for various tasks plotted in Figure 4 is computed as the average of the payments across 100 (random) selections of the gold standard questions, in order to prevent any distortion of the results due to the randomness in the choice of the gold standard questions. The time taken by the workers to submit any completed task, from the time we put the task up, was nearly the same across the three payment mechanisms. This attests to an absence of self-selection of workers across the three mechanisms. We also received many positive comments (and no negative comments) from the workers. Examples of comments that we received: “I was wondering if it would possible to increase the maximum number of HITs I may complete for you. As I said before, they were fun to complete. I think I did a good job completing them, and it would be great to complete some more for you.”; “I am eagerly waiting
for your bonus.”; “Enjoyable. Thanks.”

The complete data, including the interface presented to the workers in each of the tasks, the results obtained from the workers, and the ground truth solutions for these tasks, are available on the website of the first author.

A Recognizing the Golden Gate Bridge

A set of 21 photographs of bridges were shown to the workers, and for each photograph, they had to identify if it depicted the Golden Gate Bridge or not. An example of this task is depicted in Figure 2a, and the instructions provided to the worker under the three mechanisms are depicted in Figure 3. The fixed amount offered to workers was $M = 3$ cents for the task, and the bonus was based on 3 gold standard questions. We compared (a) the baseline mechanism with 5 cents for each correct answer in the gold standard, (b) the skip-based mechanism with $\kappa = 5.9$ and $\frac{1}{7} = 1.5$, and (c) the confidence-based mechanism with $\kappa = 5.9$ cents, $L = 2$, $\alpha_2 = 1.5$, $\alpha_1 = 1.4$, $\alpha_0 = 1$, $\alpha_{-1} = 0.5$, $\alpha_{-2} = 0$. The results of this experiment are presented in Figure 4a.

B Transcribing Vehicles’ License Plate Numbers from Photographs

This task presented the workers with 18 photographs of cars and asked them to transcribe the license plate numbers from each of them (source of photographs: http://www.coolpl8z.com). An example of this task is depicted in Figure 2b. The fixed amount offered to workers was $M = 4$ cents for the task, and the bonus was based on 4 gold standard questions. We compared (a) the baseline mechanism with 10 cents for each correct answer in the gold standard, (b) the skip-based mechanism with $\kappa = 0.62$ and $\frac{1}{3} = 3$, and (c) the confidence-based mechanism with $\kappa = 3.1$ cents, $L = 2$, $\alpha_2 = 2$, $\alpha_1 = 1.95$, $\alpha_0 = 1$, $\alpha_{-1} = 0.5$ $\alpha_{-2} = 0$. The results of this experiment are presented in Figure 4b. When evaluating, in the worker’s answers as well as in the true solutions, we converted all text to upper case, and removed all spaces and punctuations. We then declared a worker’s answer to be in error if it did not have an exact match with the true solution.

C Classifying Breeds of Dogs

This task required workers to identify the breeds of dogs shown in 85 images (source of images: [10, 24]). For each image, the worker was given ten breeds to choose from. An example of this task is depicted in Figure 2c. The fixed amount offered to workers was $M = 5$ cents for the task, and the bonus was based on 7 gold standard questions. We compared (a) the baseline mechanism with 8 cents for each correct answer in the gold standard, (b) the skip-based mechanism with $\kappa = 0.78$ and $\frac{1}{2} = 2$, and (c) the confidence-based mechanism with $\kappa = 0.78$ cents, $L = 2$, $\alpha_2 = 2$, $\alpha_1 = 1.66$, $\alpha_0 = 1$, $\alpha_{-1} = 0.67$, $\alpha_{-2} = 0$. The results of this experiment are presented in Figure 4c.

D Identifying Heads of Countries

Names of 20 personalities were provided and had to be classified as to whether they were ever the (a) President of the USA, (b) President of India, (c) Prime Minister of Canada, or (d) neither of these. An example of this task is depicted in Figure 2d. The fixed amount offered to workers was $M = 2$ cents for the task, and the bonus was based on 4 gold standard questions. While the ground truth in most other multiple-choice experiments had approximately an equal representation from all classes, this experiment was heavily biased with one of the classes never being correct and another being correct for just 3 of the 20 questions. We compared (a) the baseline mechanism with 2.5 cents for each correct answer in the gold standard, (b) the skip-based mechanism with $\kappa = 0.25$ and $\frac{1}{3} = 3$, and (c) the confidence-based mechanism with $\kappa = 1.3$ cents, $L = 2$, $\alpha_2 = 2$, $\alpha_1 = 1.95$, $\alpha_0 = 1$, $\alpha_{-1} = 0.5$, $\alpha_{-2} = 0$. The results of this experiment are presented in Figure 4d.
E  Identifying Flags

This was a relatively long task, with 126 questions. Each question required the workers to identify if a displayed flag belonged to a place in (a) Africa, (b) Asia/Oceania, (c) Europe, or (d) neither of these. An example of this task is depicted in Figure 2e. The fixed amount offered to workers was $M = 4 \text{ cents}$ for the task, and the bonus was based on 8 gold standard questions. We compared (a) the baseline mechanism with $\kappa = 0.2$ and $\frac{1}{T} = 2$, and (c) the confidence-based mechanism with $\kappa = 0.2 \text{ cents}$, $L = 2$, $\alpha_2 = 2$, $\alpha_1 = 1.66$, $\alpha_0 = 1$, $\alpha_{-1} = 0.67$, $\alpha_{-2} = 0$. The results of this experiment are presented in Figure 4e.

F  Distinguishing Textures

This task required the workers to identify the textures shown in 24 grayscale images (source of images: [28, Dataset 1: Textured surfaces]). For each image, the worker had to choose from 8 different options. Such a task has applications in computer vision, where it aids in recognition of objects or their surroundings. An example of this task is depicted in Figure 2f. The fixed amount offered to workers was $M = 3 \text{ cents}$ for the task, and the bonus was based on 4 gold standard questions. We compared (a) the baseline mechanism with 10 cents for each correct answer in the gold standard, (b) the skip-based mechanism with $\kappa = 3.1$ and $\frac{1}{T} = 2$, and (c) the confidence-based mechanism with $\kappa = 3.1 \text{ cents}$, $L = 2$, $\alpha_2 = 2$, $\alpha_1 = 1.66$, $\alpha_0 = 1$, $\alpha_{-1} = 0.67$, $\alpha_{-2} = 0$. The results of this experiment are presented in Figure 4f.

G  Transcribing Text from an Image: Film Certificate

The task showed an image containing 11 (short) lines of blurry text which the workers had to decipher. We used text from a certain certificate which movies releasing in India are provided. We slightly modified its text in order to prevent workers from searching a part of it online and obtaining the entire text by searching the first few transcribed lines on the internet. An example of this task is depicted in Figure 2g. The fixed amount offered to workers was $M = 5 \text{ cents}$ for the task, and the bonus was based on 2 gold standard questions. We compared (a) the baseline mechanism with 20 cents for each correct answer in the gold standard, (b) the skip-based mechanism with $\kappa = 5.5$ and $\frac{1}{T} = 3$, and (c) the confidence-based mechanism with $\kappa = 12.5 \text{ cents}$, $L = 2$, $\alpha_2 = 2$, $\alpha_1 = 1.95$, $\alpha_0 = 1$, $\alpha_{-1} = 0.5$, $\alpha_{-2} = 0$. The results of this experiment are presented in Figure 4g. When evaluating, in the worker’s answers as well as in the true solutions, we converted all text to upper case, and removed all spaces and punctuations. We then declared a worker’s answer to be in error if it did not have an exact match with the true solution.

H  Transcribing Text from an Image: Script of a Play

The task showed an image containing 12 (short) lines of blurry text which the workers had to decipher. We borrowed a paragraph from Shakespeare’s play ‘As You Like It.’ We slightly modified the text of the play in order to prevent workers from searching a part of it online and obtaining the entire text by searching the first few transcribed lines on the internet. An example of this task is depicted in Figure 2h. The fixed amount offered to workers was 5 cents for the task, and the bonus was based on 2 gold standard questions. We compared (a) the baseline mechanism with $M = 20 \text{ cents}$ for each correct answer in the gold standard, (b) the skip-based mechanism with $\kappa = 5.5$ and $\frac{1}{T} = 3$, and (c) the confidence-based mechanism with $\kappa = 12.5 \text{ cents}$, $L = 2$, $\alpha_2 = 2$, $\alpha_1 = 1.95$, $\alpha_0 = 1$, $\alpha_{-1} = 0.5$, $\alpha_{-2} = 0$. The results of this experiment are presented in Figure 4h. When evaluating, in the worker’s answers as well as in the true solutions, we converted all text to upper case, and removed all spaces and punctuations. We then declared a worker’s answer to be in error if it did not have an exact match with the true solution.
I Transcribing Text from Audio Clips

The workers were given 10 audio clips which they had to transcribe to text. Each audio clip was 3 to 6 seconds long, and comprised of a short sentence, e.g., “my favourite topics of conversation are sports, politics, and movies.” Each of the clips were recorded in different accents using a text-to-speech converter. An example of this task is depicted in Figure 2i. The fixed amount offered to workers was $M = 5$ cents for the task, and the bonus was based on 2 gold standard questions. We compared (a) the baseline mechanism with 20 cents for each correct answer in the gold standard, (b) the skip-based mechanism with $\kappa = 5.5$ and $1/T = 3$, and (c) the confidence-based mechanism with $\kappa = 12.5$ cents, $L = 2$, $\alpha_2 = 2$, $\alpha_1 = 1.95$, $\alpha_0 = 1$, $\alpha_{-1} = 0.5$, $\alpha_{-2} = 0$.

The results of this experiment are presented in Figure 4i.

8 Conclusion

We design a reward mechanism for crowdsourcing to ensure collection of high-quality data. Under a very natural no-free-lunch axiom, we mathematically prove that surprisingly, our mechanism is the only feasible reward mechanism. In preliminary experiments, we observe a significant drop in the error rates under this unique mechanism as compared to basic baseline mechanisms, suggesting that our mechanism has the potential to work well in practice.

Our mechanisms offer some additional benefits. The pattern of skips or confidence levels of the workers provide a reasonable estimate of the difficulty of each question. In practice, the questions that are estimated to be more difficult may now be delegated to an expert or to more non-expert workers. Secondly, the theoretical guarantees of the mechanism may allow for better post-processing of the data, incorporating the confidence information and improving the overall accuracy. The simplicity of the rules of our mechanisms may facilitate an easier adoption among the workers.

In conclusion, given the uniqueness in theory, simplicity, and good performance in practice, we envisage our ‘multiplicative’ mechanisms to be of interest to practitioners as well as researchers in the various disciplines of science and engineering that employ crowdsourcing.

Acknowledgements

We thank John C. Platt, Christopher J. C. Burges and Christopher Meek for many inspiring discussions. We also thank John C. Platt and Martin J. Wainwright for helping in proof-reading and polishing parts of the manuscript. This work was done when the first author was an intern at Microsoft Research.

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Figure 2: Various tasks on which the payment mechanisms were tested. The interfaces shown are that of the baseline mechanism, i.e., without the skipping or confidence choices.
a  **Baseline Mechanism**

Identify the Golden Gate Bridge of San Francisco, California in the 21 photos

*** Instructions for BONUS (Read Carefully) ***
• There are three questions whose answers are known to us, based on which the bonus is calculated
• \( \text{BONUS (cents)} = 5 \times \text{number of questions out of these that you correctly answer} \)

b  **Skip-based multiplicative mechanism**

Identify the Golden Gate Bridge of San Francisco, California in the 21 photos

If you are not sure about any answer, then mark "I’m not sure"
You need to mark at least something for every question, otherwise your work will be rejected

*** Instructions for BONUS (Read Carefully) ***
• You start with 5.9 cents of bonus for this HIT
• There are three questions whose answers are known to us, based on which the bonus is calculated
• For each of these questions you answer CORRECTLY, your bonus will INCREASE BY 50% (every 1 cent will become 1.5 cents)
• If you answer any of these questions WRONG, your bonus will become ZERO
• So for questions you are not sure of, mark the "I'm not sure" option: this does not affect the bonus

c  **Confidence-based multiplicative mechanism**

Identify the Golden Gate Bridge of San Francisco, California in the 21 photos

For each answer, you also need to indicate how sure you are about that answer
If you are not sure about any answer, then mark "I don't know"
You need to mark at least something for every question, otherwise your work will be rejected

*** Instructions for BONUS (Read Carefully) ***
• If any answer marked "absolutely sure" is wrong, your bonus will become ZERO for this entire HIT (you do not get any bonus for this HIT)
• For every answer marked "absolutely sure" that is correct, your bonus will INCREASE BY 50% (every 1 cent will become 1.5 cents)
• For every answer marked "moderately sure" that is wrong, your bonus will be HALVED (every 1 cent will become half a cent)
• For every answer marked "moderately sure" that is correct, your bonus will be INCREASE BY 40% (every 1 cent will become 1.4 cents)
• Marking "I don't know" for any answer does not change your bonus

Figure 3: The instructions displayed to the worker under the three mechanisms for the task of identifying the Golden Gate Bridge.
Figure 4: The error-rates and payments in the nine experiments. Each individual bar in the plots corresponds to one mechanism in one experiment and is generated from 35 distinct workers (this totals to 945 worker-tasks).
APPENDIX: Proofs

In this section, we provide proofs of the theoretical results presented in the paper. Although the skip-based setting is a special case of the confidence-based setting, we prove the results separately for it since the proofs of the skip-based setting are much simpler and may offer valuable insights.

Recall from Section 2 that for any question, we assume that the worker has a certain “confidence” about any answer that she has in mind. The confidence for an answer is formally defined as the probability, according to the belief of the worker, of that answer being correct. The confidence of a worker for a question is defined as the confidence for the answer that she believes is most likely to be correct.

A Skip-based Setting

A.1 Proof of Working of Algorithm 1

Proof of Theorem 1. The mechanism satisfies the no-free-lunch condition since the payment is zero when there are one or more wrong answers. It remains to show that the mechanism is incentive-compatible.

We will first assume that, for every question that the worker does not skip, she selects the answer which she believes is most likely to be correct. Under this assumption we will show that the worker is incentivized to skip the questions for which her confidence is smaller than \( T \) and answer if it is greater than \( T \). Finally, we will show that the mechanism indeed incentivizes the worker to select the answer which she believes is most likely to be correct for the questions that she doesn’t skip.

Let us first assume that \( G = N \), i.e., all questions are included in the gold standard. Let \( p_1, \ldots, p_N \) be the confidences of the worker for to questions \( 1, \ldots, N \) respectively. Further, let \( p(1) \geq \cdots \geq p(m) > T > p(m+1) \geq \cdots \geq p(N) \) be the ordered permutation of these confidences (for some number \( m \)). Let \( \{1, \ldots, N\} \) denote the corresponding permutation of the \( N \) questions. If the mechanism is incentive-compatible, then the expected payment received by this worker should be maximized when the worker answers questions \( (1), \ldots, (m) \) and skips the rest. Under the mechanism proposed in Algorithm 1, this action fetches the worker an expected payment of

\[
\kappa \frac{p(1)}{T} \cdots \frac{p(m)}{T}.
\]

Alternatively, if the worker answers the questions \( \{i_1, \ldots, i_z\} \), with \( p_{i_1} < \cdots < p_{i_y} < T < p_{i_{y+1}} < \cdots p_{i_z} \), then the expected payment is

\[
p_{i_1} \cdots p_{i_z} \frac{\kappa}{T^z} = \kappa \frac{p_{i_1}}{T} \cdots \frac{p_{i_z}}{T} \leq \kappa \frac{p_{i_1}}{T} \cdots \frac{p_{i_y}}{T} \leq \kappa \frac{p(1)}{T} \cdots \frac{p(m)}{T}.
\]

where (6) is because \( \frac{p_{i_j}}{T} \leq 1 \) \( \forall j > y \) and holds with equality only when \( z = y \). Inequality (7) is a result of \( \frac{p_j}{T} \geq 1 \) \( \forall j \leq m \) and holds with equality only when \( y = m \). It follows that the expected payment is (strictly) maximized when \( i_1 = (1), \ldots, i_z = (m) \) as required.

The case of \( G < N \) is a direct consequence of the result for \( G = N \). When \( G < N \), from a worker’s point of view, the set of \( G \) questions is distributed uniformly at random in the superset of \( N \) questions. However, for every set of \( G \) questions, the relations (5), (6), (7) and their associated equality/strict-inequality conditions hold. The expected payment is thus (strictly) maximized when the worker answers the questions for which her confidence is greater than \( T \) and skips those for which her confidence is smaller than \( T \).
Finally, one can see that for every question that the worker chooses to answer, the expected payment increases with an increase in her confidence. Thus, the worker is incentivized to select the answer that she thinks is most probable.

A.2 Proof of Uniqueness of Algorithm 1

The proof of uniqueness (Theorem 2) is based on a certain condition necessitated by incentive-compatibility, that we state and prove in the form of Lemma 14 below. The proof of Theorem 2 is presented subsequently. We begin by introducing an additional piece of notation.

Let \( g : \{-1, 0, 1\}^N \rightarrow \mathbb{R}_+ \) represent the expected payment given an evaluation of all the \( N \) answers, when the identities of the gold standard questions are unknown. Here, the expectation is with respect to the (uniformly random) choice of the \( G \) gold standard questions. If \( (x_1, \ldots, x_N) \in \{-1, 0, 1\}^N \) are the evaluations of the worker’s answers to the \( N \) questions then the expected payment is

\[
g(x_1, \ldots, x_N) = \frac{1}{N!} \sum_{(i_1, \ldots, i_G) \subseteq \{1, \ldots, N\}} f(x_{i_1}, \ldots, x_{i_G}) .
\]

Notice that when \( G = N \), the functions \( f \) and \( g \) are identical.

The following lemma presents a condition that must necessarily be satisfied by any incentive-compatible mechanism. This lemma does not require the no-free-lunch condition.

**Lemma 14.** Any incentive-compatible mechanism must satisfy, for every question \( i \in \{1, \ldots, G\} \) and every \((y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G) \in \{-1, 0, 1\}^{G-1}\),

\[
T f(y_1, \ldots, y_{i-1}, 1, y_{i+1}, \ldots, y_G) + (1 - T) f(y_1, \ldots, y_{i-1}, -1, y_{i+1}, \ldots, y_G) = f(y_1, \ldots, y_{i-1}, 0, y_{i+1}, \ldots, y_G) .
\]

**Proof of Lemma 14.** First we consider the simpler case of \( G = N \). In the set \( \{y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G\} \), for some \( (\eta, \gamma) \in \{0, \ldots, G - 1\}^2 \), suppose there are \( \eta \) elements with a value 1, \( \gamma \) elements with a value -1, and \( (G - 1 - \eta - \gamma) \) elements with a value 0. Let us assume for now that \( i = \eta + \gamma + 1 \), \( y_1 = 1 \), \ldots, \( y_{\eta+1} = -1 \), \ldots, \( y_{\eta+\gamma} = -1 \), \( y_{\eta+\gamma+2} = 0 \), \ldots, \( y_G = 0 \).

Suppose the worker has confidences \( (p_1, \ldots, p_{\eta+\gamma}) \in (T, 1]^{\eta+\gamma} \) for the first \( (\eta + \gamma) \) questions, a confidence of \( q \in (0, 1] \) for the next question, and confidences smaller than \( T \) for the remaining \( (G - \eta - \gamma - 1) \) questions. The payment mechanism must incentivize the worker to answer the first \( (\eta + \gamma) \) questions and skip the last \( (G - \eta - \gamma - 1) \) questions; for question \( \eta + \gamma + 1 \), it must incentivize the worker to answer if \( q > T \) and skip if \( q < T \). Supposing the worker indeed attempts the first \( (\eta + \gamma) \) questions and skips the last \( (G - \eta - \gamma - 1) \) questions, let \( x = \{x_1, \ldots, x_{\eta+\gamma}\} \in \{-1, 1\}^{\eta+\gamma} \) denote the evaluation of the worker’s answers to the first \( (\eta + \gamma) \) questions. Define quantities \( \{r_j\}_{j \in [\eta+\gamma]} \) as \( r_j = 1 - p_j \) for \( j \in \{1, \ldots, \eta\} \), and \( r_j = p_j \) for
\[
j \in \{\eta + 1, \eta + \gamma\}. \text{ Incentive-compatibility necessitates}\]
\[
q \sum_{x \in \{-1,1\}^{\eta + \gamma}} \left( f(x_1, \ldots, x_\eta, -x_{\eta+1}, \ldots, -x_{\eta+\gamma}, 1, 0, \ldots, 0) \prod_{j \in [\eta + \gamma]} r_j^{1-x_j} (1 - r_j)^{1+x_j} \right) \\
+ (1 - q) \sum_{x \in \{-1,1\}^{\eta + \gamma}} \left( f(x_1, \ldots, x_\eta, -x_{\eta+1}, \ldots, -x_{\eta+\gamma}, -1, 0, \ldots, 0) \prod_{j \in [\eta + \gamma]} r_j^{1-x_j} (1 - r_j)^{1+x_j} \right) \\
q < T \sum_{x \in \{-1,1\}^{\eta + \gamma}} \left( f(x_1, \ldots, x_\eta, -x_{\eta+1}, \ldots, -x_{\eta+\gamma}, 0, 0, \ldots, 0) \prod_{j \in [\eta + \gamma]} r_j^{1-x_j} (1 - r_j)^{1+x_j} \right).
\]
\[
(10)
\]

The left hand side of this expression is the expected payment if the worker chooses to answer question \((\eta + \gamma + 1)\), while the right hand side is the expected payment if she chooses to skip it. For any real-valued variable \(q\), and for any real-valued constants \(a, b \text{ and } c\),
\[
a q \leq b \Rightarrow ac = b.
\]

As a result,
\[
T \sum_{x \in \{-1,1\}^{\eta + \gamma}} \left( f(x_1, \ldots, x_\eta, -x_{\eta+1}, \ldots, -x_{\eta+\gamma}, 1, 0, \ldots, 0) \prod_{j \in [\eta + \gamma]} r_j^{1-x_j} (1 - r_j)^{1+x_j} \right) \\
+ (1 - T) \sum_{x \in \{-1,1\}^{\eta + \gamma}} \left( f(x_1, \ldots, x_\eta, -x_{\eta+1}, \ldots, -x_{\eta+\gamma}, -1, 0, \ldots, 0) \prod_{j \in [\eta + \gamma]} r_j^{1-x_j} (1 - r_j)^{1+x_j} \right) \\
- \sum_{x \in \{-1,1\}^{\eta + \gamma}} \left( f(x_1, \ldots, x_\eta, -x_{\eta+1}, \ldots, -x_{\eta+\gamma}, 0, 0, \ldots, 0) \prod_{j \in [\eta + \gamma]} r_j^{1-x_j} (1 - r_j)^{1+x_j} \right) = 0.
\]
\[
(11)
\]

The left hand side of (11) represents a polynomial in \((\eta + \gamma)\) variables \(\{r_j\}_{j=1}^{\eta + \gamma}\) which evaluates to zero for all values of the variables within a \((\eta + \gamma)\)-dimensional solid ball. Thus, the coefficients of the monomials in this polynomial must be zero. In particular, the constant term must be zero. The constant term appears when \(x_j = 1 \forall j\) in the summations in (11). Setting the constant term to zero gives
\[
T f(x_1 = 1, \ldots, x_\eta = 1, -x_{\eta+1} = -1, \ldots, -x_{\eta+\gamma} = -1, 1, 0, \ldots, 0) \\
+ (1 - T) f(x_1 = 1, \ldots, x_\eta = 1, -x_{\eta+1} = -1, \ldots, -x_{\eta+\gamma} = -1, -1, 0, \ldots, 0) \\
- f(x_1 = 1, \ldots, x_\eta = 1, -x_{\eta+1} = -1, \ldots, -x_{\eta+\gamma} = -1, 0, 0, \ldots, 0) = 0
\]
\[
(12)
\]
as desired. Since the arguments above hold for any permutation of the \(G\) questions, this completes the proof for the case of \(G = N\).

Now consider the case \(G < N\). In the set \(\{y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G\}\), for some \((\eta, \gamma) \in \{0, \ldots, G - 1\}^2\) with \(\eta + \gamma < G\), suppose there are \(\eta\) elements with a value 1, \(\gamma\) elements with a value \(-1\), and \((G - 1 - \eta - \gamma)\) elements with a value 0. Let us assume for now that \(i = \eta + \gamma + 1, y_1 = 1, \ldots, y_\eta = 1, y_{\eta+1} = -1, \ldots, y_{\eta+\gamma} = -1, y_{\eta+\gamma+2} = 0, \ldots, y_G = 0\).
Suppose the worker has confidences \( \{p_1, \ldots, p_{\eta+\gamma}\} \in (T, 1]^{\eta+\gamma} \) for the first \((\eta + \gamma)\) of the \(N\) questions, a confidence of \(q \in (0, 1] \) for the next question, and confidences smaller than \(T\) for the remaining \((N - \eta - \gamma - 1)\) questions. The payment mechanism must incentivize the worker to answer the first \((\eta + \gamma)\) questions and skip the last \((N - \eta - \gamma - 1)\) questions; for the \((\eta + \gamma + 1)\)th question, the mechanism must incentivize the worker to answer if \(q > T\) and skip if \(q < T\). Supposing the worker indeed attempts the first \((\eta + \gamma)\) questions and skips the last \((N - \eta - \gamma - 1)\) questions, let \(x = \{x_1, \ldots, x_{\eta+\gamma}\} \in \{-1, 1\}^{\eta+\gamma}\) denote the the evaluation of the worker’s answers to the first \((\eta + \gamma)\) questions. Define quantities \(\{r_j\}_{j \in [\eta+\gamma]}\) as \(r_j = 1 - p_j\) for \(j \in \{1, \ldots, \eta\}\), and \(r_j = p_j\) for \(j \in \{\eta + 1, \eta + \gamma\}\). Incentive-compatibility necessitates

\[
q \sum_{x \in \{-1, 1\}^{\eta+\gamma}} \left( g(x_1, \ldots, x_{\eta}, -x_{\eta+1}, \ldots, -x_{\eta+\gamma}, 1, 0, \ldots, 0) \prod_{j \in [\eta+\gamma]} \frac{1-x_j}{r_j} \left(1 - r_j\right)^{1+x_j} \right) \\
+ (1 - q) \sum_{x \in \{-1, 1\}^{\eta+\gamma}} \left( g(x_1, \ldots, x_{\eta}, -x_{\eta+1}, \ldots, -x_{\eta+\gamma}, -1, 0, \ldots, 0) \prod_{j \in [\eta+\gamma]} \frac{1-x_j}{r_j} \left(1 - r_j\right)^{1+x_j} \right) \\
\begin{aligned}
q < T \\
q \leq c \Rightarrow \forall \eta \leq b \Rightarrow ac = b, \text{ we get that} \\
Tg(x_1 = 1, \ldots, x_{\eta} = 1, -x_{\eta+1} = -1, \ldots, -x_{\eta+\gamma} = -1, 1, 0, \ldots, 0) \\
+ (1 - T)g(x_1 = 1, \ldots, x_{\eta} = 1, -x_{\eta+1} = -1, \ldots, -x_{\eta+\gamma} = -1, -1, 0, \ldots, 0) \\
- g(x_1 = 1, \ldots, x_{\eta} = 1, -x_{\eta+1} = -1, \ldots, -x_{\eta+\gamma} = -1, 0, 0, \ldots, 0) = 0.
\end{aligned}
\]

(13)

The proof now proceeds via induction on the quantity \((G - \eta - \gamma - 1)\), i.e., on the number of skipped questions in \(\{y_1, \ldots, y_i-1, y_{i+1}, \ldots, y_G\}\). We begin with the case of \((G - \eta - \gamma - 1) = G - 1\) which implies \(\eta = \gamma = 0\). In this case (14) simplifies to

\[
Tg(1, 0, \ldots, 0) + (1 - T)g(-1, 0, \ldots, 0) = g(0, 0, \ldots, 0).
\]

(15)

Applying the expansion of function \(g\) in terms of function \(f\) from (8) gives

\[
T\left(c_1 f(1, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)\right) + (1 - T)\left(c_1 f(-1, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)\right) = \left(c_1 f(0, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)\right)
\]

(16)

for constants \(c_1 > 0\) and \(c_2 > 0\) that respectively denote the probabilities that the first question is picked and not picked in the set of \(G\) gold standard questions. Cancelling out the common terms on both sides of the equation, we get the desired result

\[
Tf(1, 0, \ldots, 0) + (1 - T)f(-1, 0, \ldots, 0) = f(0, 0, \ldots, 0).
\]

(17)

Next, we consider the case when \((G - \eta - \gamma - 1)\) questions are skipped in the gold standard, and assume that the result is true when more than \((G - \eta - \gamma - 1)\) questions are skipped in the gold standard. In (14), the functions \(g\) decompose into a sum of the constituent \(f\) functions. These constituent functions \(f\) are of two types: the first where all of the first \((\eta + \gamma + 1)\) questions are included in the gold standard, and the second where one or
more of the first \((\eta + \gamma + 1)\) questions are not included in the gold standard. The second case corresponds to situations where there are more than \((G - \eta - \gamma - 1)\) questions skipped in the gold standard and hence satisfies our induction hypothesis. The terms corresponding to these functions thus cancel out in the expansion of (14). The remainder comprises only evaluations of function \(f\) for arguments in which the first \((\eta + \gamma + 1)\) questions are included in the gold standard: since the last \((N - \eta - \gamma - 1)\) questions are skipped by the worker, the remainder evaluates to

\[
Tc_3f(y_1, \ldots, y_{\eta+\gamma}, 1, 0, \ldots, 0) + (1 - T)c_3f(y_1, \ldots, y_{\eta+\gamma}, -1, 0, \ldots, 0)
= c_3f(y_1, \ldots, y_{\eta+\gamma}, 0, 0, \ldots, 0)
\]

(18)

for some constant \(c_3 > 0\). Dividing throughout by \(c_3\) gives the desired result.

Finally, the arguments above hold for any permutation of the first \(G\) questions, thus completing the proof.

\[\square\]

**Proof of Theorem 2.** We will first prove that any incentive compatible mechanism satisfying the no-free-lunch condition must make a zero payment if one or more answers in the gold standard are incorrect. The proof proceeds by induction on the number \(S\) of skipped questions in the gold standard. Let us assume for now that in the \(G\) questions in the gold standard, the first question is answered incorrectly, the next \((G - 1 - S)\) questions are answered by the worker and have arbitrary evaluations, and the remaining \(S\) questions are skipped. The proof proceeds by an induction on \(S\). Suppose \(S = G - 1\). In this case, the only attempted question is the first question and the answer provided by the worker to this question is incorrect. The no-free-lunch condition necessitates a zero payment in this case, thus satisfying the base case of our induction hypothesis. Now we prove the hypothesis for some \(S\) under the assumption of it being true when the number of questions skipped in the gold standard is \((S + 1)\) or more. From Lemma 14 (with \(i = G - S - 1\)) we have

\[
Tf(-1, y_2, \ldots, y_{G-S-2}, 1, 0, \ldots, 0) + (1 - T)f(-1, y_2, \ldots, y_{G-S-2}, -1, 0, \ldots, 0)
= f(-1, y_2, \ldots, y_{G-S-2}, 0, 0, \ldots, 0)
\]

(19)

\[
= 0 .
\]

(20)

where (20) is a consequence of our induction hypothesis since \(f(-1, y_2, \ldots, y_{G-S-2}, 0, 0, \ldots, 0)\) corresponds to the case when the last \((S + 1)\) questions are skipped and the first question is answered incorrectly. Now, since the payment \(f\) must be non-negative and since \(T \in (0, 1)\), it must be that

\[
f(-1, y_2, \ldots, y_{G-S-2}, 1, 0, \ldots, 0) = 0
\]

(21)

and

\[
f(-1, y_2, \ldots, y_{G-S-2}, -1, 0, \ldots, 0) = 0 .
\]

(22)

This completes the proof of our induction hypothesis. Furthermore, each of the arguments above hold for any permutation of the \(G\) questions, thus proving the necessity of zero payment when any one or more answers are incorrect.

We will now prove that when no answers in the gold standard are incorrect, the payment must be of the form described in Algorithm 1. Let \(\kappa\) be the payment when all \(G\) questions in the gold standard are skipped. Let \(C\) be the number questions answered correctly in the gold standard. Since there are no incorrect answers, it follows that the remaining \((G - C)\) questions are skipped. Let us assume for now that the first \(C\) questions are answered correctly and the remaining \((G - C)\) questions are skipped. We repeatedly apply Lemma 14, and the
fact that the payment must be zero when one or more answers are wrong, to get

\[
f(1, \ldots, 1, 1, 0, \ldots, 0) = \frac{1}{T} f(1, \ldots, 1, 1, 0, \ldots, 0) - \frac{1 - T}{T} f(1, \ldots, 1, -1, 0, \ldots, 0) \quad (23)
\]

\[
f(1, \ldots, 1, 0, 0, \ldots, 0) = \frac{1}{T} f(1, \ldots, 1, 0, 0, \ldots, 0) \quad (24)
\]

\[
\vdots
\]

\[
f(0, \ldots, 0) = \frac{1}{T^C} f(0, \ldots, 0) \quad (26)
\]

\[
f(0, \ldots, 0) = \frac{1}{T^C} \kappa. \quad (27)
\]

In order to abide by the budget, we must have the maximum payment as \( \mu = \kappa \frac{1}{T^C} \). It follows that \( \kappa = \mu T^G \). Finally, the arguments above hold for any permutation of the \( G \) questions, thus proving the uniqueness of the mechanism of Algorithm 1.

A.3 Proof for The Case of a Necessary Minimum Payment

Proof of Corollary 3. Adding \( M \) to all instances of the payment function \( f \) and average payment \( g \) mentioned in the proofs of Theorem 1 and Theorem 2 gives the desired results.

B Confidence-based Setting

In this section, for the sake of brevity, the notation \( q \in (T_m, T_{m+1}) \) will mean

\[
q \in \begin{cases} 
[0, T_1] & \text{if } m = 0 \\
(T_L, 1] & \text{if } m = L \\
(T_m, T_{m+1}) & \text{otherwise}
\end{cases}
\]

B.1 Proof of Working of Algorithm 2

We first state and prove three properties that the constants \( \{\alpha_l\}_{l=-L}^L \) defined in Algorithm 2 must satisfy. We will use these properties subsequently in the proof of Theorem 4.

Lemma 15. For every \( l \in \{0, \ldots, L - 1\} \)

\[
T_{l+1} \alpha_{l+1} + (1 - T_{l+1}) \alpha_{-(l+1)} = T_{l+1} \alpha_l + (1 - T_{l+1}) \alpha_{-l}, \quad (28)
\]

and

\[
S_{l+1} \alpha_{l+1} + (1 - S_{l+1}) \alpha_{-(l+1)} = \alpha_0 = 1. \quad (29)
\]

Lemma 16. \( \alpha_L > \alpha_{L-1} > \cdots > \alpha_{-L} = 0. \)

Lemma 17. For any \( m \in \{1, \ldots, L\} \), any \( p > T_m \) and any \( z < m \),

\[
p \alpha_m + (1 - p) \alpha_{-m} > p \alpha_z + (1 - p) \alpha_{-z}, \quad (30)
\]
and for any \( m \in \{0, \ldots, L - 1\} \), any \( p < T_{m+1} \) and any \( z > m \),
\[
p\alpha_m + (1-p)\alpha_{-m} > p\alpha_z + (1-p)\alpha_{-z}.
\]

**Proof of Lemma 15.** Algorithm 2 states that \( \alpha_{-l} = \frac{1 - \alpha_l S_l}{1 - S_l} \) for all \( l \in [L] \). A simple rearrangement of the terms in this expression gives (29).

Towards the goal of proving (28), we will first prove an intermediate result:
\[
\alpha_l > 1 > \alpha_{-l} \forall l \in \{L, \ldots, 1\}.
\]

The proof proceeds via an induction on \( l \in \{L, \ldots, 2\} \). The case of \( l = 1 \) will be proved separately. The induction hypothesis involves two claims: \( \alpha_l > 1 > \alpha_{-l} \) and \( T_l \alpha_l + (1 - T_l) \alpha_{-l} > 1 \). The base case is \( l = L \) for which we know that \( \alpha_L = \frac{1}{S_L} > 1 > 0 = \alpha_{-L} \) and \( T_l \alpha_l + (1 - T_l) \alpha_{-l} = \frac{T_l}{S_l} > 1 \). Now suppose that the induction hypothesis is true for \((l+1)\). Rearranging the terms in the expression defining \( \alpha_l \) in Algorithm 2 and noting that \( 1 > T_{l+1} > S_l \), we get
\[
\alpha_l = \frac{(1 - S_l) (T_{l+1} \alpha_{l+1} + (1 - T_{l+1}) \alpha_{-(l+1)}) - (1 - T_{l+1})}{(1 - S_l) - (1 - T_{l+1})}
\]
\[
> \frac{(1 - S_l) - (1 - T_{l+1})}{(1 - S_l) - (1 - T_{l+1})}
\]
\[
= 1. \tag{35}
\]

From (29) we see that the value 1 is a convex combination of \( \alpha_l \) and \( \alpha_{-l} \). Since \( \alpha_l > 1 \) and \( S_l \in (0, 1) \), it must be that \( \alpha_{-l} < 1 \). Furthermore, since \( T_l > S_l \) we get
\[
T_l \alpha_l + (1 - T_l) \alpha_{-l} > S_l \alpha_l + (1 - S_l) \alpha_{-l}
\]
\[
= 1. \tag{37}
\]

This proves the induction hypothesis. Let us now consider \( l = 1 \). If \( L = 1 \) then we have \( \alpha_L = \frac{1}{S_L} > 1 > 0 = \alpha_{-L} \) and we are done. If \( L > 1 \) then we have already proved that \( \alpha_2 > 1 > \alpha_{-2} \) and \( T_2 \alpha_2 + (1 - T_2) \alpha_{-2} > 1 \).

An argument identical to (33) onwards proves that \( \alpha_1 > 1 > \alpha_{-1} \).

Now that we have proved \( \alpha_l > \alpha_{-l} \forall l \in [L] \), we can rewrite the expression defining \( \alpha_{-l} \) in Algorithm 2 as
\[
S_l = \frac{1 - \alpha_{-l}}{\alpha_l - \alpha_{-l}}.
\]

Substituting this expression for \( S_l \) in the definition of \( \alpha_l \) in Algorithm 2 and making some simple rearrangements gives the desired result (28).

**Proof of Lemma 16.** We have already shown (32) in the proof of Lemma 15 above that \( \alpha_l > 1 > \alpha_{-l} \forall l \in [L] \).

Next we will show that \( \alpha_{l+1} > \alpha_l \) and \( \alpha_{-(l+1)} < \alpha_{-l} \forall l \geq 0 \). First consider \( l = 0 \), for which Algorithm 2 sets \( \alpha_0 = 1 \), and we have already proved that \( \alpha_1 > 1 > \alpha_{-1} \).

Now consider some \( l > 0 \). Observe that since \( S_l \alpha_l + (1 - S_l) \alpha_{-l} = 1 \) (Lemma 15), \( S_{l+1} > S_l \) and \( \alpha_l > \alpha_{-l} \), it must be that
\[
S_{l+1} \alpha_l + (1 - S_{l+1}) \alpha_{-l} > 1. \tag{39}
\]

From Lemma 15, we also have
\[
S_{l+1} \alpha_{l+1} + (1 - S_{l+1}) \alpha_{-(l+1)} = 1. \tag{40}
\]
Subtracting (39) from (40) we get
\[ S_{l+1}(\alpha_{l+1} - \alpha_l) + (1 - S_{l+1})(\alpha_{-(l+1)} - \alpha_{-l}) < 0. \] (41)
From Lemma 15 we also have
\[ T_{l+1}\alpha_{l+1} + (1 - T_{l+1})\alpha_{-(l+1)} = T_{l+1}\alpha_l + (1 - T_{l+1})\alpha_{-l} \] (42)
\[ \Rightarrow T_{l+1}(\alpha_{l+1} - \alpha_l) + (1 - T_{l+1})(\alpha_{-(l+1)} - \alpha_{-l}) = 0. \] (43)
Subtracting (41) from (43) gives
\[ (T_{l+1} - S_{l+1})[(\alpha_{l+1} - \alpha_l) + (\alpha_{-l} - \alpha_{-(l+1)})] > 0. \] (44)
Since \( T_{l+1} > S_{l+1} \) by definition, it must be that
\[ \alpha_{l+1} - \alpha_l > \alpha_{-(l+1)} - \alpha_{-l}. \] (45)
Now, rearranging the terms in (42) gives
\[ (\alpha_{l+1} - \alpha_l)T_{l+1} = -(\alpha_{-(l+1)} - \alpha_{-l})(1 - T_{l+1}). \] (46)
Since \( T_{l+1} \in (0, 1) \), it follows that the terms \((\alpha_{l+1} - \alpha_l)\) and \((\alpha_{-(l+1)} - \alpha_{-l})\) have opposite signs. Using (45) we conclude that \( \alpha_{l+1} - \alpha_l > 0 \) and \( \alpha_{-(l+1)} - \alpha_{-l} < 0 \). \(\blacksquare\)

**Proof of Lemma 17.** Let us first prove (30). First consider the case \( z = m - 1 \). From Lemma 15 we know that
\[ T_m\alpha_{m-1} + (1 - T_m)\alpha_{-(m-1)} = T_m\alpha_m + (1 - T_m)\alpha_m \]
\[ \Rightarrow 0 = T_m(\alpha_m - \alpha_{m-1}) + T_m(\alpha_{-(m-1)} - \alpha_{-m}) - (\alpha_{-(m-1)} - \alpha_{-m}) \]
\[ < p(\alpha_m - \alpha_{m-1}) + p(\alpha_{-(m-1)} - \alpha_{-m}) - (\alpha_{-(m-1)} - \alpha_{-m}), \] (47)
where (47) is a consequence of \( p > T_m \) and Lemma 16. A simple rearrangement of the terms in (47) gives (30).
Now, for any \( z < m \), recursively apply this result to get
\[ p\alpha_m + (1 - p)\alpha_{-m} > \]
\[ p\alpha_{m-1} + (1 - p)\alpha_{-(m-1)} \]
\[ > p\alpha_{m-2} + (1 - p)\alpha_{-(m-2)} \]
\[ \vdots \]
\[ > p\alpha_z + (1 - p)\alpha_{-z}. \]

Let us now prove (31). We first consider the case \( z = m + 1 \). From Lemma 15 we know that
\[ T_{m+1}\alpha_m + (1 - T_{m+1})\alpha_{-m} = T_{m+1}\alpha_{m+1} + (1 - T_{m+1})\alpha_{-(m+1)} \]
\[ \Rightarrow 0 = T_{m+1}(\alpha_{m+1} - \alpha_m) + T_{m+1}(\alpha_{-(m+1)} - \alpha_{-m}) - (\alpha_{-m} - \alpha_{-(m+1)}) \]
\[ > p(\alpha_{m+1} - \alpha_m) + p(\alpha_{-m} - \alpha_{-(m+1)}) - (\alpha_{-m} - \alpha_{-(m+1)}), \] (48)
where (48) is a consequence of \( p < T_{m+1} \) and Lemma 16. A simple rearrangement of the terms in (48)
Proof of Theorem 4. The choice of $\alpha_{-L} = 0$ made in Algorithm 2 ensures that the payment is zero whenever any answer in the gold standard evaluates to $-L$. This ensures that the no-free-lunch condition is satisfied. We now prove incentive-compatibility.

Define $E = (e_1, \ldots, e_G) \in \{-1, 1\}^G$ and $E_{\setminus 1} = (e_2, \ldots, e_G)$. Suppose the worker has confidences $p_1, \ldots, p_N$ for her $N$ answers. For some $(s(1), \ldots, s(N)) \in \{0, \ldots, L\}^N$ suppose $p_i \in (T_{s(i)}, T_{s(i)+1}) \forall i \in \{1, \ldots, N\}$, i.e., $s(1), \ldots, s(N)$ are the correct confidence-levels for her answers. Consider any other set of confidence-levels $s'(1), \ldots, s'(N)$. When the mechanism of Algorithm 2 is employed, the expected payment (from the point of view of the worker) on selecting confidence-levels $s(1), \ldots, s(N)$ is

$$E[\text{Pay}] = \frac{1}{N} \sum_{(j_1, \ldots, j_G) \subseteq \{1, \ldots, N\}} \sum_{G_{E_{\setminus 1}}} \prod_{i=1}^G \alpha_{e_i s(j_i)} (p_{j_i})^{1+\epsilon_i} (1 - p_{j_i})^{1-\epsilon_i}$$

$$= \frac{1}{N} \sum_{(j_1, \ldots, j_G) \subseteq \{1, \ldots, N\}} \sum_{G_{E_{\setminus 1}}} (p_{j_1} \alpha_{s(j_1)} + (1 - p_{j_1}) \alpha_{-s(j_1)}) \prod_{i=2}^G \alpha_{e_i s(j_i)} (p_{j_i})^{1+\epsilon_i} (1 - p_{j_i})^{1-\epsilon_i}$$

$$= \frac{1}{N} \sum_{(j_1, \ldots, j_G) \subseteq \{1, \ldots, N\}} \prod_{i=1}^G (p_{j_i} \alpha_{s(j_i)} + (1 - p_{j_i}) \alpha_{-s(j_i)})$$

$$> \frac{1}{N} \sum_{(j_1, \ldots, j_G) \subseteq \{1, \ldots, N\}} \prod_{i=1}^G (p_{j_i} \alpha_{s'(j_i)} + (1 - p_{j_i}) \alpha_{-s'(j_i)})$$

which is the expected payment under any other set of confidence-levels $s'(1), \ldots, s'(N)$. The last inequality is a consequence of Lemma 17.

An argument similar to the above also proves that for any $m \in \{1, \ldots, L\}$, if allowed to choose between only skipping and confidence-level $m$, the worker is incentivized to choose confidence-level $m$ over skip if her confidence is greater $S_m$, and choose skip over level $m$ if her confidence is smaller than $S_m$. Finally, from Lemma 16 we have $\alpha_L > \cdots > \alpha_{-L} = 0$. It follows that the expected payment (51) is strictly increasing in each of the values $p_1, \ldots, p_N$. Thus the worker is incentivized to report the answer that she thinks is most likely to be correct.

Proof of Corollary 5. The proof of Theorem 4 also proves this claim.
B.2 Proof of Uniqueness of Algorithm 2

We will first define one additional piece of notation. Let \( g : \{-L, \ldots, L\}^N \rightarrow \mathbb{R}_+ \) denote the expected payment given an evaluation of the \( N \) answers, where the expectation is with respect to the (uniformly random) choice of the \( G \) gold standard questions. If \((x_1, \ldots, x_N) \in \{-L, \ldots, L\}^N\) are the evaluations of the worker’s answers to the \( N \) questions then the expected payment is

\[
g(x_1, \ldots, x_N) = \frac{1}{(G)^{N}} \sum_{(i_1, \ldots, i_G) \subseteq \{1, \ldots, N\}} f(x_{i_1}, \ldots, x_{i_G}).
\]  

(53)

Notice that when \( G = N \), the functions \( f \) and \( g \) are identical.

The proof of uniqueness (Theorem 6) is based on a certain condition necessitated by incentive-compatibility, that we state and prove in the form of Lemma 18 below. The proof of Theorem 6 is presented subsequently. Note that this lemma does not require the generalized-no-free-lunch condition.

**Lemma 18.** Any incentive-compatible mechanism must satisfy, for every question \( i \in \{1, \ldots, G\} \), every \((y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G) \in \{-L, \ldots, L\}^{G-1} \), and every \( m \in \{1, \ldots, L\} \),

\[
T_m f(y_1, \ldots, y_{i-1}, m, y_{i+1}, \ldots, y_G) + (1 - T_m) f(y_1, \ldots, y_{i-1}, -m, y_{i+1}, \ldots, y_G)
\]

\[
= T_m f(y_1, \ldots, y_{i-1}, m - 1, y_{i+1}, \ldots, y_G) + (1 - T_m) f(y_1, \ldots, y_{i-1}, -(m - 1), y_{i+1}, \ldots, y_G)
\]

(54)

and

\[
S_m f(y_1, \ldots, y_{i-1}, m, y_{i+1}, \ldots, y_G) + (1 - S_m) f(y_1, \ldots, y_{i-1}, -m, y_{i+1}, \ldots, y_G)
\]

\[
= f(y_1, \ldots, y_{i-1}, 0, y_{i+1}, \ldots, y_G).
\]

(55)

Note that (54) and (55) coincide when \( m = 1 \), since \( T_1 = S_1 \) by definition.

**Proof of Lemma 18.** First consider the case of \( G = N \). For every \( j \in \{1, \ldots, i - 1, i + 1, \ldots, G\} \), define

\[
r_j = \begin{cases} 
1 - p_j & \text{if } y_j \geq 0 \\
p_j & \text{if } y_j < 0.
\end{cases}
\]

Define \( E_{\backslash i} = \{\epsilon_1, \ldots, \epsilon_{i-1}, \epsilon_{i+1}, \ldots, \epsilon_G\} \). For any \( l \in \{-L, \ldots, L\} \) let \( \Lambda_l \in \mathbb{R}_+ \) denote the expected payment (from the worker’s point of view) when her answer to the \( i \)th question evaluates to \( l \):

\[
\Lambda_l = \sum_{E_{\backslash i} \in \{-1, 1\}^{G-1}} \left( f(y_1 \epsilon_1, \ldots, y_{i-1} \epsilon_{i-1}, l, y_{i+1} \epsilon_{i+1}, \ldots, y_G \epsilon_G) \prod_{j \in [G] \setminus \{i\}} \frac{1 - \epsilon_j}{r_j - \frac{1}{2}} (1 - r_j) \frac{1 + \epsilon_j}{r_j + \frac{1}{2}} \right). 
\]

(56)

Consider a worker who has confidences \( \{p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_G\} \in (0, 1)^{G-1} \) for questions \( \{1, \ldots, i - 1, i + 1, \ldots, G\} \) respectively, and for question \( i \) suppose she has a confidence of \( q \in (T_m, T_{m+1}) \). For question \( i \), we must incentivize the worker to select confidence-level \( m \) if \( q > T_m \), and to select \((m - 1)\) if \( q < T_m \). This necessitates

\[
q \Lambda_m + (1 - q) \Lambda_{-m} \leq_T q \Lambda_{m-1} + (1 - q) \Lambda_{-(m-1)}.
\]

(57)
Also, for question $i$, the requirement of level $m$ having a higher incentive as compared to skipping when $q > S_m$ and vice versa when $q < S_m$ necessitates

\[ q\Lambda_m + (1 - q)\Lambda_{-m} \begin{cases} \leq & q < S_m \\ \geq & q > S_m \end{cases} \Lambda_0. \tag{58} \]

Now, note that for any real-valued variable $q$, and for any real-valued constants $a$, $b$ and $c$,

\[ aq \begin{cases} \leq & q < c \\ \geq & q > c \end{cases} b \implies ac = b. \]

Applying this fact to (57) and (58) gives

\[ (T_m\Lambda_m + (1 - T_m)\Lambda_{-m}) - (T_m\Lambda_{m-1} + (1 - T_m)\Lambda_{-(m-1)}) = 0, \tag{59} \]
\[ (S_m\Lambda_m + (1 - S_m)\Lambda_{-m}) - \Lambda_0 = 0. \tag{60} \]

From the definition of $\Lambda_l$ in (56), we see that the left hand sides of (59) and (60) are both polynomials in $(G - 1)$ variables $\{r_j\}_{j \in [G] \setminus \{i\}}$ and take a value of zero for all values of the variables in a $(G - 1)$-dimensionall solid ball. Thus, each of the coefficients (of the monomials) in both polynomials must be zero, and in particular, the constant terms must also be zero. Observe that in both these polynomials, the constant term arises only when $\epsilon_j = 1 \forall j \in [G] \setminus \{i\}$ (which makes the exponent of $r_j$ to be 0 and that of $(1 - r_j)$ to be 1). Thus, setting the constant term to zero in the two polynomials results in

\[ T_m f(y_1, \ldots, y_{i-1}, m, y_{i+1}, \ldots, y_G) + (1 - T_m) f(y_1, \ldots, y_{i-1}, -m, y_{i+1}, \ldots, y_G) = T_m f(y_1, \ldots, y_{i-1}, m-1, y_{i+1}, \ldots, y_G) + (1 - T_m) f(y_1, \ldots, y_{i-1}, -(m-1), y_{i+1}, \ldots, y_G) \tag{61} \]
and

\[ S_m f(y_1, \ldots, y_{i-1}, m, y_{i+1}, \ldots, y_G) + (1 - S_m) f(y_1, \ldots, y_{i-1}, -m, y_{i+1}, \ldots, y_G) = f(y_1, \ldots, y_{i-1}, 0, y_{i+1}, \ldots, y_G) \tag{62} \]

thus proving the claim for the case of $G = N$.

Now consider the case when $G < N$. In order to simplify notation, let us assume $i = 1$ without loss of generality (since the arguments presented hold for any permutation of the questions). Suppose a worker’s answers to questions $\{2, \ldots, G\}$ evaluate to $(y_2, \ldots, y_G) \in \{-L, \ldots, L\}^{G-1}$, and further suppose that the worker skips the remaining $(N - G)$ questions. By going through arguments identical to those for $G = N$, but with $f$ replaced by $g$, we get the necessity of

\[ T_m g(m, y_2, \ldots, y_G, 0, \ldots, 0) + (1 - T_m) g(-m, y_2, \ldots, y_G, 0, \ldots, 0) = T_m g(m-1, y_2, \ldots, y_G, 0, \ldots, 0) + (1 - T_m) g(-(m-1), y_2, \ldots, y_G, 0, \ldots, 0) \tag{63} \]
and

\[ S_m g(m, y_2, \ldots, y_G, 0, \ldots, 0) + (1 - S_m) g(-m, y_2, \ldots, y_G, 0, \ldots, 0) = g(0, y_2, \ldots, y_G, 0, \ldots, 0). \tag{64} \]

We will now use this result in terms of function $g$ to get an equivalent result in terms of function $f$. For some $S \in \{0, \ldots, G - 1\}$, suppose $y_{G-S+1} = 0, \ldots, y_G = 0$. The remaining proof proceeds via an induction on $S$.
We begin with $S = G - 1$. In this case, (63) and (64) simplify to
\[ T_m g(m, 0, \ldots, 0) + (1 - T_m) g(-m, 0, 0, \ldots, 0) = T_m g(m - 1, 0, \ldots, 0) + (1 - T_m) g(-(m - 1), 0, \ldots, 0) \] (65)
and
\[ S_m g(m, 0, \ldots, 0) + (1 - S_m) g(-m, 0, \ldots, 0) = g(0, 0, \ldots, 0) \] (66)
Applying the definition of function $g$ from (53) leads to
\[ T_m (c_1 f(m, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)) + (1 - T_m) (c_1 f(-m, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)) = T_m (c_1 f(m - 1, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)) + (1 - T_m) (c_1 f(-(m - 1), 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)) \] (67)
and
\[ S_m (c_1 f(m, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)) + (1 - S_m) (c_1 f(-m, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)) = (c_1 f(0, 0, \ldots, 0) + c_2 f(0, 0, \ldots, 0)) \] (68)
for constants $c_1 > 0$ and $c_2 > 0$ that respectively denote the probabilities that the first question is picked and not picked in the set of $G$ gold standard questions. Cancelling out the common terms on both sides of the equation, we get the desired results
\[ T_m f(m, 0, \ldots, 0) + (1 - T_m) f(-m, 0, \ldots, 0) = T_m f(m - 1, 0, \ldots, 0) + (1 - T_m) f(-(m - 1), 0, \ldots, 0) \] (69)
and
\[ S_m f(m, 0, \ldots, 0) + (1 - S_m) f(-m, 0, \ldots, 0) = f(0, 0, \ldots, 0) \] (70)
Next, we consider the case of a general $S \in \{0, \ldots, G - 2\}$ and assume that the result is true when $y_{G-S} = 0, \ldots, y_G = 0$. In (63) and (64), the functions $g$ decompose into a sum of the constituent $f$ functions. These constituent functions $f$ are of two types: the first where all of the first $(G - S)$ questions are included in the gold standard, and the second where one or more of the first $(G - S)$ questions are not included in the gold standard. The second case corresponds to situations where there are more than $S$ questions skipped in the gold standard, i.e., when $y_{G-S} = 0, \ldots, y_G = 0$, and hence satisfies our induction hypothesis. The terms corresponding to these functions thus cancel out in the expansion of (63) and (64). The remainder comprises only evaluations of function $f$ for arguments in which the first $(G - S)$ questions are included in the gold standard: since the last $(N - G + S)$ questions are skipped by the worker, the remainder evaluates to
\[ T_m c_3 f(y_1, \ldots, y_{i-1}, m, y_{i+1}, \ldots, y_G) + (1 - T_m) c_3 f(y_1, \ldots, y_{i-1}, -m, y_{i+1}, \ldots, y_G) = T_m c_3 f(y_1, \ldots, y_{i-1}, m - 1, y_{i+1}, \ldots, y_G) + (1 - T_m) c_3 f(y_1, \ldots, y_{i-1}, -(m - 1), y_{i+1}, \ldots, y_G), \]
\[ S_m c_3 f(y_1, \ldots, y_{i-1}, m, y_{i+1}, \ldots, y_G) + (1 - S_m) c_3 f(y_1, \ldots, y_{i-1}, -m, y_{i+1}, \ldots, y_G) = c_3 f(y_1, \ldots, y_{i-1}, 0, y_{i+1}, \ldots, y_G) \]
for some constant $c_3 > 0$. Dividing throughout by $c_3$ gives the desired result.
Finally, the arguments above hold for any permutation of the first $G$ questions, thus completing the proof.
We now show that the restriction $T_l > S_l$ was necessary when defining the thresholds in Section 4.

**Corollary 19.** Incentive-compatibility necessitates $T_l > S_l \forall l \in \{2, \ldots, L\}$, even in the absence of the generalized-no-free-lunch axiom.

**Proof.** First observe that the proof of Lemma 18 did not employ the generalized-no-free-lunch axiom, neither did it assume $T_1 > S_1$. We will thus use the result of Lemma 18 to prove our claim.

Suppose the confidence of the worker for all but the first question is lower than $T_1$ and that the worker decides to skip all these questions. Suppose the worker attempts the first question. In order to ensure that the worker selects the answer that she believes is most likely to be true, it must be that

$$f(l, 0, \ldots, 0) > f(-l, 0, \ldots, 0) \forall l \in [L].$$

We now call upon Lemma 18 where we set $i = 1$, $m = l$, $y_2 = \ldots, y_G = 0$. Using the fact that $T_l > T_{l-1} \forall l \in \{2, \ldots, L\}$, we get

$$T_l f(l, 0, \ldots, 0) + (1 - T_l) f(-l, 0, \ldots, 0) = T_l f(l - 1, 0, \ldots, 0) + (1 - T_l) f(-l - 1, 0, \ldots, 0)$$

$$> T_{l-1} f(l - 1, 0, \ldots, 0) + (1 - T_{l-1}) f(-l - 1, 0, \ldots, 0)$$

$$= T_{l-1} f(l - 2, 0, \ldots, 0) + (1 - T_{l-1}) f(-l - 2, 0, \ldots, 0)$$

$$> T_{l-2} f(l - 2, 0, \ldots, 0) + (1 - T_{l-2}) f(-l - 2, 0, \ldots, 0)$$

$$\vdots$$

$$> T_1 f(1, 0, \ldots, 0) + (1 - T_1) f(-1, 0, \ldots, 0)$$

$$= f(0, \ldots, 0)$$

$$= S_l f(l, 0, \ldots, 0) + (1 - S_l) f(-l, 0, \ldots, 0).$$

Since $f(l, 0, \ldots, 0) > f(-l, 0, \ldots, 0)$, we have our desired result.

**Proof of Theorem 6.** We will first prove that any incentive compatible mechanism that satisfies the no-free-lunch condition must give a zero payment when one or more questions are selected with a confidence $L$ and turn out to be incorrect. Let us assume for now that in the $G$ questions in the gold standard, the first question is answered incorrectly with a confidence of $L$, the next $(G - 1 - S)$ questions are answered by the worker and have arbitrary evaluations, and the remaining $S$ questions are skipped. The proof proceeds by an induction on $S$. If $S = G - 1$, the only attempted question is the first question and this is incorrect with confidence $L$. The no-free-lunch condition necessitates a zero payment in this case, thus satisfying the base case of our induction hypothesis. Now we prove the hypothesis for some $S$ under the assumption that the hypothesis is true for every $S' > S$. From Lemma 14 with $m = 1$, we have

$$T_1 f(-L, y_2, \ldots, y_{G-S-1}, 1, 0, \ldots, 0) + (1 - T_1) f(-L, y_2, \ldots, y_{G-S-1}, -1, 0, \ldots, 0)$$

$$= T_1 f(-L, y_2, \ldots, y_{G-S-1}, 0, 0, \ldots, 0) + (1 - T_1) f(-L, y_2, \ldots, y_{G-S-1}, 0, 0, \ldots, 0)$$

$$= f(-L, y_2, \ldots, y_{G-S-1}, 0, 0, \ldots, 0)$$

$$= 0,$$

where (79) is a consequence of our induction hypothesis given the fact that $f(-L, y_2, \ldots, y_{G-S-1}, 0, 0, \ldots, 0)$ corresponds to the case when the last $(S+1)$ questions are skipped and the first question is answered incorrectly with confidence $L$. Now, since the payment $f$ must be non-negative and since $T \in (0, 1)$, it must be that

$$f(-L, y_2, \ldots, y_{G-S-1}, 1, 0, \ldots, 0) = 0$$

(80)
and

\[ f(-L, y_2, \ldots, y_{G-S-1}, -1, 0, \ldots, 0) = 0. \tag{81} \]

Repeatedly applying the same argument to \( m = 2, \ldots, L \) gives us

\[ f(-L, y_2, \ldots, y_{G-S-1}, m, 0, \ldots, 0) = 0 \quad \forall m \] and

\[ f(-L, y_2, \ldots, y_{G-S-1}, -m, 0, \ldots, 0) = 0 \quad \forall m. \]

This completes the proof of our induction hypothesis. Observe that each of the aforementioned arguments hold for any permutation of the \( G \) questions, thus proving the necessity of zero payment when any one or more answers are incorrect.

We will now prove that when no answers in the gold standard are incorrect with confidence \( L \), the payment must be of the form described in Algorithm 1. Let \( \kappa \) denote the payment when all \( G \) questions in the gold standard are skipped, i.e.,

\[ \kappa = f(0, \ldots, 0). \]

Now consider any \( S \in \{0, \ldots, G-1\} \) and any \((y_1, \ldots, y_{G-S-1}, m) \in \{-L, \ldots, L\}^{G-S} \). The payments \( \{f(y_1, \ldots, y_{G-S-1}, m, 0, \ldots, 0)\}_{m=-L}^{L} \) must satisfy the \((2L-1)\) linear constraints arising out of Lemma 18 and must also satisfy \( f(y_1, \ldots, y_{G-S-1}, -L, 0, \ldots, 0) = 0 \). This comprises a total of \( 2L \) linearly independent constraints on the \((2L+1)\) values \( \{f(y_1, \ldots, y_{G-S-1}, m, 0, \ldots, 0)\}_{m=-L}^{L} \). The only set of solutions that meet these constraints are

\[ f(y_1, \ldots, y_{G-S-1}, m, 0, \ldots, 0) = \alpha_m f(y_1, \ldots, y_{G-S-1}, 0, 0, \ldots, 0), \]

where the constants \( \{\alpha_m\}_{m=-L}^{L} \) are as specified in Algorithm 2. Applying this argument \( G \) times, starting from \( S = 0 \) to \( S = G-1 \), gives

\[ f(y_1, \ldots, y_G) = \kappa \prod_{j=1}^{G} \alpha_{y_j}. \]

Finally, the budget requirement necessitates \( \mu = \kappa (\alpha_L)^G \), which mandates the value of \( \kappa \) to be \( \mu \left( \frac{1}{\alpha_L} \right)^G \). This is precisely the mechanism described in Algorithm 2.

\[ \square \]

**B.3 Proof for The Case of a Necessary Minimum Payment**

*Proof of Corollary 7.* Adding \( M \) to all instances of the payment function \( f \) and average payment \( g \) mentioned in the proofs of Theorem 4 and Theorem 6 gives the desired results.

\[ \square \]

**C A Stronger No-free-lunch Condition: Impossibility Results**

*Proof of Proposition 8.* If the worker skips all questions, then the expected payment is zero under the strong-no-free-lunch axiom. On the other hand, in order to incentivize knowledgeable workers to select answers whenever their confidences are greater than \( T \), there must exist some situation in which the payment is strictly larger than zero. Suppose the payment is strictly positive when questions \( \{1, \ldots, z\} \) are answered correctly, questions \( \{z+1, \ldots, z'\} \) are answered incorrectly, and the remaining questions are skipped. If the confidence of the unknowledgeable worker is in the interval \((0, T)\) for every question, then an attempt to answer questions \( \{1, \ldots, z'\} \) fetches her an expected payment that is strictly positive. Thus, this unknowledgeable worker is incentivized to answer at least one question.

\[ \square \]

*Proof of Proposition 9.* Consider a (knowledgeable) worker who has a confidence of \( p \in (T, 1] \) for the first question, \( q \in (0, 1) \) for the second question, and confidences in the interval \((0, T)\) for the remaining questions. Suppose the worker attempts to answer the first question (and selects the answer the believes is most likely to be correct) and skips the last \((N-2)\) questions as desired. Now, in order to incentivize her to answer the second
Now, the strong-no-free-lunch condition implies
\[ f \] and hence \( p \). Now suppose a (knowledgeable) worker has a confidence of \( c \) for some constants \( f \) included in the gold standard. Since \( f \) in the gold standard, and the probability that the first (or, second) but not the second (or, first) questions are lower than \( g \), the payment mechanism must satisfy
\[
pg(1, 1, 0, \ldots, 0) + (1 - p)qg(-1, 1, 0, \ldots, 0) + p(1 - q)g(1, -1, 0, \ldots, 0) \\
+ (1 - p)(1 - q)g(-1, -1, 0, \ldots, 0) \quad \frac{q}{q > T} \leq p(1, 0, 0, \ldots, 0) + (1 - p)g(-1, 0, 0, \ldots, 0) .
\] (82)

For any real-valued variable \( q \), and for any real-valued constants \( a, b \) and \( c \),
\[
aq \frac{q < c}{q > c} b \Rightarrow ac = b .
\]

As a result,
\[
pTg(1, 1, 0, \ldots, 0) + (1 - p)Tg(-1, 1, 0, \ldots, 0) + p(1 - T)g(1, -1, 0, \ldots, 0) \\
+ (1 - p)(1 - T)g(-1, -1, 0, \ldots, 0) - pg(1, 0, 0, \ldots, 0) - (1 - p)g(-1, 0, 0, \ldots, 0) = 0 .
\] (83)

The left hand side of this equation is a polynomial in variable \( p \) and takes a value of zero for all values of \( p \) in a one-dimensional box \((T, 1]\). It follows that the monomials of this polynomial must be zero, and in particular the constant term must be zero:
\[
Tg(-1, 1, 0, \ldots, 0) + (1 - T)g(-1, -1, 0, \ldots, 0) - g(-1, 0, 0, \ldots, 0) = 0 .
\] (84)

Now, the strong-no-free-lunch condition implies \( f(-1, -1, 0, \ldots, 0) = f(-1, 0, \ldots, 0) = f(0, \ldots, 0) = 0 \), and hence \( g(-1, -1, 0, \ldots, 0) = g(-1, 0, 0, \ldots, 0) = 0 \). Since \( T \in (0, 1) \), we have
\[
0 = g(-1, 1, 0, \ldots, 0) \\
= c_1 f(-1, 1, 0, \ldots, 0) + c_2 f(-1, 0, \ldots, 0) + c_3 f(1, 0, \ldots, 0) ,
\] (85)

for some constants \( c_1 > 0 \) and \( c_2 > 0 \) that represent the probability that the first two questions are included in the gold standard, and the probability that the first (or, second) but not the second (or, first) questions are included in the gold standard. Since \( f \) is a non-negative function, it must be that
\[
f(1, 0, \ldots, 0) = 0 .
\] (87)

Now suppose a (knowledgeable) worker has a confidence of \( p \in (T, 1] \) for the first question and confidences lower than \( T \) for the remaining \((N - 1)\) questions. Suppose the worker chooses to skip the last \((N - 1)\) questions as desired. In order to incentivize the worker to answer the first question, the mechanism must satisfy for all \( p \in (T, 1] \),
\[
0 < pg(1, 0, \ldots, 0) + (1 - p)g(-1, 0, \ldots, 0) - g(0, 0, \ldots, 0) \\
= pc_3 f(1, 0, \ldots, 0) + pc_4 f(0, 0, \ldots, 0) + (1 - p)c_3 f(-1, 0, \ldots, 0) + (1 - p)c_4 f(0, 0, \ldots, 0) - f(0, 0, \ldots, 0) \\
= 0 ,
\] (88)

where \( c_3 > 0 \) and \( c_4 > 0 \) are some constants. The final equation is a result of the strong-no-free-lunch condition and the fact that \( f(1, 0, \ldots, 0) = 0 \) as proved above. This yields a contradiction, and hence no incentive-compatible mechanism \( f \) can satisfy the strong-no-free-lunch condition when \( G < N \) even when allowed to address only knowledgeable workers.

Finally, as a sanity check, note that if \( G = N \) then \( c_2 = 0 \) in (86). The proof above thus doesn’t hold when \( G = N \).
Proof of Proposition 10. We will first show that the mechanism works as desired.

First consider the case when the worker is unknowledgeable and her confidences are of the form $T > p(1) ≥ p(2) ≥ p(3) ≥ \cdots ≥ p(G)$. If she answers only the first question, then her expected payment is

$$\kappa \frac{p(1)}{T}.$$  

Let us now see her expected payment if she doesn’t follow this answer pattern. The strong-no-free-lunch condition implies that if the worker doesn’t answer any question then her expected payment is zero. Suppose the worker chooses to answer questions $\{i_1, \ldots, i_z\}$. In that case, her expected payment is

$$\kappa p_{i_1} \cdots p_{i_z} T^z = \kappa \frac{p_{i_1} \cdots p_{i_z}}{T^z}$$  \hspace{1cm} (89)

$$\leq \kappa \left( \frac{p(1)}{T} \right)^z$$  \hspace{1cm} (90)

$$\leq \kappa \frac{p(1)}{T},$$  \hspace{1cm} (91)

where (91) uses the fact that $p(1) < T$. The inequality in (91) becomes an equality only when $z = 1$. Now when $z = 1$, the inequality in (90) becomes an equality only when $i_1 = (1)$. Thus the unknowledgeable worker is incentivized to answer only one question – the one that she has the highest confidence in.

Now consider a knowledgeable worker and suppose her confidences are of the form $p(1) ≥ \cdots ≥ p(m) > T > p(m+1) ≥ \cdots ≥ p(G)$ for some $m ≥ 1$. If the worker answers questions $(1), \ldots, (m)$ as desired, her expected payment is

$$\kappa \frac{p(1) \cdots p(m)}{T}.$$  

Now let us see what happens if the worker does not follow this answer pattern. The strong-no-free-lunch condition implies that if the worker doesn’t answer any question then her expected payment is zero. Now, if she answers some other set of questions, say questions $\{i_1, \ldots, i_z\}$ with $p(1) ≤ p_{i_1} < \cdots < p_{i_y} ≤ p(m) < p_{i_{y+1}} < \cdots p_{i_z} ≤ p(G)$. The expected payment in that case is

$$\kappa \frac{p_{i_1} \cdots p_{i_z}}{T^z} = \kappa \frac{p_{i_1} \cdots p_{i_z}}{T}$$  \hspace{1cm} (92)

$$\leq \kappa \frac{p_{i_1} \cdots p_{i_y}}{T}$$  \hspace{1cm} (93)

$$\leq \kappa \frac{p(1) \cdots p(m)}{T},$$  \hspace{1cm} (94)

where inequality (93) is a result of $\frac{p_{i_j}}{T} ≤ 1 \forall j > y$ and holds with equality only when $y = z$. Inequality (94) is a result of $\frac{p(1)}{T} ≥ 1 \forall j ≤ m$ and holds with equality only when $y = m$. Thus the expected payment is maximized when $i_1 = (1), \ldots, i_z = (m)$ as desired. Finally, the payment strictly increases with an increase in the confidences, and hence the worker is incentivized to always consider the answer that she believes is most likely to be correct.

We will now show that this mechanism is unique.

The necessary conditions derived in Lemma 14, when restricted to $G = N$ and $(y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G) \neq \{0\}^{N-1}$, is also applicable to the present setting. This is because the strong-no-free-lunch condition assumed here is a stronger condition than the no-free-lunch axiom considered in Lemma 14, and moreover, $(y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G) \neq \{0\}^{N-1}$ avoids the use of unknowledgeable workers in the proof of Lemma 14. It follows that for every question $i \in \{1, \ldots, G\}$ and every $(y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G) \in \{-1, 0, 1\}^{G-1} \setminus \{0\}^{G-1}$,
it must be that
\[ Tf(y_1, \ldots, y_{i-1}, 1, y_{i+1}, \ldots, y_G) + (1 - T)f(y_1, \ldots, y_{i-1}, -1, y_{i+1}, \ldots, y_G) = f(y_1, \ldots, y_{i-1}, 0, y_{i+1}, \ldots, y_G). \] (95)

We claim that the payment must be zero whenever the number of incorrect answers \( W > 0 \). The proof proceeds by induction on the number of correct answers \( C \). First suppose \( C = 0 \) (and \( W > 0 \)). Then all questions are either wrong or skipped, and hence by the strong-no-free-lunch condition, the payment must be zero. Now suppose the payment is necessarily zero whenever \( W > 0 \) and the total number of correct answers is \((C - 1)\) or lower, for some \( C \in [G-1] \). Consider any evaluation \((y_1, \ldots, y_G) \in \{-1, 0, 1\}^G\) in which the number of incorrect answers is more than zero and the number of correct answers is \( C \). Suppose \( y_i = 1 \) for some \( i \in [G] \), and \( y_j = -1 \) for some \( j \in [G] \setminus \{i\} \). Then from the induction hypothesis, we have \( f(y_1, \ldots, y_{i-1}, -1, y_{i+1}, \ldots, y_G) = f(y_1, \ldots, y_{i-1}, 0, y_{i+1}, \ldots, y_G) = 0 \). Applying (95) and noting that \( T \in (0, 1) \), we get that \( f(y_1, \ldots, y_{i-1}, 1, y_{i+1}, \ldots, y_G) = 0 \) as claimed. This result also allows us to simplify (95) to: For every question \( i \in \{1, \ldots, G\} \) and every \((y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G) \in \{-1, 0, 1\}^{G-1} \setminus \{0\}^{G-1}\),
\[ f(y_1, \ldots, y_{i-1}, 1, y_{i+1}, \ldots, y_G) = \frac{1}{T} f(y_1, \ldots, y_{i-1}, 0, y_{i+1}, \ldots, y_G). \] (96)

We now show that when \( C > 0 \) and \( W = 0 \), the payment must necessarily be of the form described in the statement of Proposition 10. The proof again proceeds via an induction on the number of correct answers \( C \geq 1 \). Define a quantity \( \kappa > 0 \) as
\[ \kappa = Tf(1, 0, \ldots, 0). \] (97)

Now consider the payment \( f(1, y_2, \ldots, y_G) \) for some \((y_2, \ldots, y_G) \in \{0, 1\}^{G-1} \setminus \{0\}^{G-1} \) with \( C \) correct answers. Applying (96) repeatedly (once for every \( i \) such that \( y_i = 1 \)), we get
\[ f(1, y_2, \ldots, y_G) = \frac{\kappa}{T^C}. \] (98)

Unlike other results in this paper, at this point we cannot claim the result to hold for all permutations of the questions. This is because we have defined the quantity \( \kappa \) in an asymmetric manner (97), in terms of the payment function when the first question is correct and the rest are skipped. In what follows, we will prove that the result claimed in the statement of Proposition 10 indeed holds for all permutations of the questions.

From (96) we have
\[ f(0, 1, 0, \ldots, 0) = Tf(1, 1, 0, \ldots, 0) \]
\[ = f(1, 0, 0, \ldots, 0) \]
\[ = \kappa. \] (99)

Thus the payment must be \( \kappa \) even if the second answer in the gold standard is correct and the rest are skipped. In fact, the argument holds when any one answer in the gold standard is correct and the rest are skipped. Thus the definition of \( \kappa \) is not restricted to the first question alone as originally defined in (97), but holds for all permutations of the questions. This allows the other arguments above to be applicable to any permutation of the questions. Finally, the budget constraint of \( \mu \) fixes the value of \( \kappa \) to that claimed, thereby completing the proof. \( \square \)

**Proof of Proposition 11.** Proposition 10 proved that under the skip-based setting with the strong-no-free-lunch condition, the payment must be zero when one or more answers are incorrect. This part of the proof of Proposi-
tion 10 holds even when $L > 1$. It follows that for any question, the penalty for an incorrect answer is the same for any confidence-level in $\{1, \ldots, L\}$. Thus the worker is incentivized to always select that confidence-level for which the payment is the maximum when the answer is correct, irrespective of her own confidence about the question. This contradicts our requirements.

D General Utility Functions

Proof of Theorem 12. We will first verify that the proposed payment is always non-negative and satisfies the (generalized-)no-free-lunch axiom. Recall from Theorem 4 that for every evaluation $\{x_1, \ldots, x_G\}$ for which the (generalized-)no-free-lunch axiom mandates a zero payment, the value of $\kappa \prod_{i=1}^{G} \alpha_{x_i}$ is zero. It follows that the payment $U^{-1}\left(\kappa \prod_{i=1}^{G} \alpha_{x_i} + U(0)\right) = U^{-1}(0 + U(0)) = 0$. Further, recall that the value of $\kappa \prod_{i=1}^{G} \alpha_{x_i}$ in Algorithm 2 is never smaller than zero. Since the function $U$ is increasing, so is $U^{-1}$, and hence the payment is always non-negative.

We will now prove that the proposed payment is incentive-compatible. To this end, observe that the utility of the proposed payment is

$$U(\text{Payment}) = U\left(U^{-1}\left(\kappa \prod_{i=1}^{G} \alpha_{x_i} + U(0)\right)\right) = \kappa \prod_{i=1}^{G} \alpha_{x_i} + U(0).$$

Theorem 4 and Corollary 7 show that the expectation of this function is behaves exactly as required for incentive-compatibility.

We will now prove uniqueness of this mechanism. Replacing $f(\cdot)$ by $U(\text{Payment}(\cdot))$ in the proofs of Theorem 6 and Corollary 7, we get that the function $U(\text{Payment})$ must be of the form

$$U(\text{Payment}(x_1, \ldots, x_G)) = \kappa \prod_{i=1}^{G} \alpha_{x_i} + c,$$

for some constant $c$, where the constants $\{\alpha_{x_j}\}_{j=1}^{L}$ and $\kappa$ are as defined in Algorithm 2. In other words, the payment must be of the form

$$\text{Payment}(x_1, \ldots, x_G) = U^{-1}\left(\kappa \prod_{i=1}^{G} \alpha_{x_i} + c\right).$$

When the evaluations $x_1, \ldots, x_G$ are such that the (generalized-)no-free-lunch applies, we need Payment = 0. It follows that $c = U(0)$. $\square$

Proof of Corollary 7. The proof is identical to the proof of Theorem 12, except that the zero payment is replaced by a payment $M$. $\square$