

1 INTRODUCTION

Feedback control systems wherein the loops are closed through real-time networks are called Networked Control Systems (NCSs) [16, 18, 26, 29]. Advantages of using NCSs in the control area include simplicity, cost-effectiveness, ease of system diagnosis and maintenance, increased system agility and testability. However, the integration of communication real-time networks into feedback control loops inevitable leads to challenging problems such as network-induced delays and data packet losses, which can induce instability or poor performance of closed-loop control systems. Therefore, packet loss is one of the most important and special issues of NCSs.

There are two major approaches to accommodate the issue of packet loss in an NCS design. One way is that one first designs the control system without regard to the networks, and then determines a performance level that the networks should satisfy (for example, maximum allowable transfer interval) so that the closed-loop system maintains its performance (for example, stability) when some control and sensor signals are transmitted via the networks [13, 29]. The other approach is to treat the network protocol and traffic as given conditions and design the control strategies that explicitly take the network-induced issues into account [1, 21, 27, 28]. Under the assumption that the network is modeled as a switch governed by a Bernoulli process, Zhang et al [29] proposed a criterion to check whether the NCS is stable at a certain rate of packet losses, and searched for the maximum packet-loss rate under which the overall system remains stable. The method they used derives from the stability analysis for asynchronous dynamic systems. With packet-loss rate known and constant, Seiler and Sengupta [21] formulated the NCS as a Markovian jump system with two operation modes, and then applied the techniques developed for Markovian jump systems. A dynamic output feedback controller design method was proposed such that the NCS is mean square stable and has $H_\infty$ gain below certain value in terms of linear matrix inequalities (LMIs). Moreover, Yu et al [28] modeled the packet-loss process as an arbitrary but finite switching signal. This enables them to apply the theory from switched systems to stabilize the NCS. However, in the framework considered in the references mentioned above, the controller is directly connected to the actuator. That means no packets are dropped in control signals. A general framework was considered in [1], where both sampling signals and control commands are transmitted through the network and may be dropped during the transmissions. The linear quadratic Gaussian control problem was studied based on dynamic programming approach. Xiong and Lam [27] generalized the procedure in [28] to double-sided packet loss, as one of the contributions, and established stability conditions via a packet-loss dependent Lyapunov approach, as another contribution.

In the last two decades, model predictive control (MPC) has been widely adopted in industry as an effective means to deal with multivariable constrained control problems [8, 17]. The idea of MPC is stemmed from employing an explicit model of the plant to be controlled which is used to predict the future state/output behaviour over the finite time horizon. There are some research results that have been presented in MPC for NCSs. Srinivasaguta et al [22] proposed a time-stamped model predictive control algorithm for NCS when random delay is less than one sample time. Liu et al [10] proposed using a networked control predictor to take the
based on [27]. The main goal of this paper is to provide a robust stability analysis and synthesis robust predictive controller for this scheme with guaranteed cost and PDQS.

The organization of the paper is as follows. Section 2 gives the problem formulation. In Section 3, the robust output feedback predictive controller design method with input constraints using bilinear matrix inequality is presented. The approach of robust constrained networked model predictive control design with guaranteed cost and PDQS is introduced in section 4. In section 5, one benchmark example is solved by using Yalnıp BMI solvers to show the effectiveness of the proposed method. Finally, some conclusions are given.

Hereafter, the following notational conventions will be adopted: given a symmetric matrix $P = P^T$, the inequality $P > 0$ ($P \geq 0$) denotes matrix positive definiteness (semi-definiteness). "*$" denotes a block that is transposed and complex conjugate to the respective symmetrically placed one. Matrices, if not explicitly stated, are assumed to have compatible dimensions. $I$ denotes the identity matrix of corresponding dimensions. The notation $x(t+k|t)$ will be used to define, at time $t$, $k$ steps ahead, prediction of a system variable $x$ from time $t$ onwards under a specified initial state and input scenario. Note that $x(t|t) = x(t)$.

1 PRELIMINARIES AND PROBLEM FORMULATION

The framework of NCS considered in the paper is depicted in Fig. 1. Let the polytopic model of the plant to be controlled be described by the following linear discrete time difference equation

$$x(t + 1) = A(\xi)x(t) + B(\xi)u(t),$$

$$y(t) = Cx(t)$$

(1)

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^l$ denote the state, control input, and output respectively. The matrices $A(\xi), B(\xi)$ belong to convex and bounded set $S$, which is a polytop with $N$ vertices $S_1, S_2, \ldots, S_N$ that can be formally defined as follows

$$S := \left\{ A(\xi) \in \mathbb{R}^{n \times n}, B(\xi) \in \mathbb{R}^{m \times m} : A(\xi) = \sum_{i=1}^{N} \xi_i A_i, \quad B(\xi) = \sum_{i=1}^{N} \xi_i B_i, \quad \sum_{i=1}^{N} \xi_i = 1, \xi_i \geq 0 \right\}.$$  (2)

Matrices $A_i, B_i$ and $C$ are known matrices with constant elements and appropriate dimensions.

Consider the following output feedback predictive control algorithm as the controller of NCS

$$u(t+k) = u(t+k|t) = \sum_{j=k}^{N_u} F_{kj}[y(t+j|t) - w(t+j|t)]$$  (3)
where \( k \in \{0, 1, \ldots, N_u - 1\} \); \( F_{kj} \in \mathbb{R}^{m 	imes l} \) denotes output feedback gain matrices; \( u(t + k|t) \), \( y(t + k|t) \), and \( w(t + k|t) \in \mathbb{R}^l \) for \( k \geq 1 \) denote, respectively, the input, output, and desired reference predictions at time instant \( t + k \). Based on [27], the packet-loss process in this paper is influenced by the transmission of data. If data are lost at one sampling time, at next sampling time there is negligible network-induced delay (time delay is within sampling time of NCS) to the actuator, and as a result, the packet-loss occurs. The sensor and the controller only send data at time \( t + k \), if it takes values in \( \mathbb{Z} = \{1, 2, \ldots, s, s + 1, \ldots, t + s + l, t + s + 1, \ldots, t + s + l + l_{p_{\max}}\} \) is submitted and implemented at a buffer device with length \( l_{p_{\max}} \) in the actuator. The buffer device is used to store the newest control sequence \( U_s(t_s) \) transmitted successfully from MPC to actuator at sampling time \( t_s \in \mathbb{Z} \). At time instant \( t \in (t_s, t_s + l_{p_{\max}}) \), the packet loss occurs, and control action \( u(t) \) corresponding to the current sampling time from control sequence \( U_s(t_s) \) in the buffer device will be applied to the actuator. The main goal of this paper is to design a predictive controller (3) with input constraints (4) so that, control action \( u(t) \) from control sequence \( U_s(t_s) \) robustly stabilizes NCS and ensures input constraints and guaranteed cost of the following cost function (over the infinite optimization horizon) \( J = \sum_{t=0}^{\infty} J(t) \), and where

\[
J(t) = \sum_{k=0}^{N_u} x^T(t + k|t)Q_k x(t + k|t) + \sum_{k=0}^{N_u-1} u^T(t + k)R_k u(t + k),
\]

and \( Q_k \in \mathbb{R}^{n \times n}, R_k \in \mathbb{R}^{m \times m} \) are positive semidefinite (definite) and definite matrices, respectively for all \( k \) (\( Q_k = q_k I, R_k = r_k I, q_k \geq 0, r_k > 0 \)) and \( I \) is the unitary matrix.

### 3 ROBUST MODEL PREDICTIVE CONTROL WITH INPUT CONSTRAINTS

In this section, we recall some results of the robust MPC design with input constraints from papers [15, 25]. At sampling time \( t := t_s \in \mathbb{Z} \), the predicted states of the system (1) for the instant \( t + k - t_s \in \{0, 1, \ldots, N_u - 1\} \) are given by

\[
x(t + k - t_s|t_s) = A_t \xi x(t + k|t_s) + B_t \xi u(t + k|t_s).
\]

Let us define stacked vectors with future states and desired references in corresponding forms as follows

\[
x_f(t) = [x^T(t) \ldots x^T(t + N_y - 1|t)],
\]

\[
\nu(t) = [u^T(t) \ldots u^T(t + N_u - 1|t)].
\]

Considering \( \nu(t) = [u^T(t) u^T(t + 1) \ldots u^T(t + N_u - 1)] \), state model prediction is obtained as follows

\[
A_f(\xi)x_f(t + 1) = A_v(\xi)x(t) + B_f(\xi)\nu(t)
\]

where

\[
A_f(\xi) = \begin{bmatrix}
I & 0 & 0 & \ldots & 0 & 0 \\
-A(\xi)1 & 0 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & -A(\xi) I
\end{bmatrix}
\]
where the guaranteed cost control law for the closed-loop system (13) if and only if there exists a Lyapunov function $V_0(t) = x_f(t)^T P_0(t) x_f(t)$ such that the following condition holds

$$\Delta V_0(t) + J(t) \leq 0.$$  

Moreover, summarizing (16) from initial time $t_0$ to $t \to \infty$, the following inequality is obtained

$$-V_0(t_0) + J \leq 0.$$  

Definition 3 and inequality (17) imply that $J_0 = V_0(t_0)$.

**THEOREM.** The closed loop system (13) is robustly stable with guaranteed cost $J_0$ and parameter dependent quadratic stability if and only if there exist matrices $H_0 \in R^{nN_u \times n(N_u + 1)}$, $P_0(\xi) = P_0^T(\xi) > 0$ and gain matrix $F_0$ such that the following bilinear matrix inequality holds [15].

$$W(\xi) \leq 0$$

where

$$W(\xi) = D(\xi) + \sum_{k=0}^{n-1} A_k(\xi) - \sum_{k=0}^{n-1} B_k(\xi) F_0 C.$$

Consider system (9) where the control $\nu(t)$ is constrained to evolve in the following set

$$\Gamma = \{ \nu(t) \in R^{mnN_u}; \|\nu_u\| \leq \bar{u}_i; i_d = i + (j-1)m; \}

i = 1, \ldots, m; j = 1, \ldots, N_u \}$$

To derive sufficient stability conditions for input constraints for (13), we consider that the positive invariant region [19], with respect to closed-loop system motion can be defined by the ellipsoidal Lyapunov function set given by $V_0(t)$ as follows

$$\Omega(P_0(\xi)) = \{ x_f(t) \in \{ x_f(t) \in R^{mnN_u}; x_f(t)^T P_0(t) x_f(t) \leq \theta \}$$

where $\theta$ is a positive real parameter which determines the size of $\Omega(P_0(\xi))$.

Consider $D_{iq} F_0$ denotes the $i_d-th$ row of matrix $F_0$ where $D_{iq} = [0 \ldots 0 \ldots 1 \ldots 0]$ and define

$$\Omega(P_0(\xi)) = \{ x_f(t) \in R^{mnN_u}; \| D_{iq} F_0 C x_f(t) \| \leq \bar{u}_i; i_d = i + (j-1)m; i = 1, \ldots, m; j = 1, \ldots, N_u \}$$

The condition of input constraints reduces to LMI given by the following theorem [25].
number packet loss $l_p(t) \in \ell$ is a discrete state, $x_f(t)$ is the state of continuous part. Activity function is defined by closed-loop output feedback model predictive controls $MPC_i$, in (29).

At sampling time $t := t_s(l_p = 0)$, model predictive control $MPC_i$, defined in Section 2 is used to compute control sequence $U_i(t)$. If no packet loss at $t + 1$, $MPC_0$ is applied. Otherwise, jump to $MPC_1$, it means that one packet lost.

Generally, at sampling time $t := t_s + l_p$ ($1 \leq l_p \leq l_{p_{\max}}$), $MPC_p$ is applied. If no packet loss at $t + 1$, jump to $MPC_0$. Otherwise, if $l_p < l_{p_{\max}}$, jump to $MPC_{p+1}$ and it means that $l_p + 1$ packets are lost. The predicted states of $MPC_p$ for the instant $t + k$, $k = 0, \ldots, N_u - 1$, are given by

$$x(t + k + 1|t) = A(x(t + k|t)) + B(x(t))u(k),$$

$$u(k) = \begin{cases} u(t_s + l_p + k) & \text{if } 0 \leq k \leq N_u - l_p - 1, \\ u(t_s + N_u - 1) & \text{if } N_u - l_p \leq k \leq N_u - 1. \end{cases}$$

Based on equation (3), $u(t_s + l_p + k)$ for $0 \leq k \leq N_u - l_p - 1$ can be rewritten as follows

$$u(t_s + l_p + k) = \sum_{j=l_p+k}^{N_u} F_{i(l_p+k),j} C x(t_s + j|t_s).$$

Substituting $j$ by $i + l_p$ to (24), we obtain

$$u(t_s + l_p + k) = \sum_{i=k}^{N_u-l_p} F_{i+l_p+i|t_s} C x(t_s + l_p + i|t_s)$$

for $0 \leq k \leq N_u - l_p - 1$.

If the following condition

$$x(t_s + l_p + i|t_s) = x(t_s + l_p + i|t_s + l_p) = x(t + i|t)$$

holds for $0 \leq i \leq N_u - l_p$, then the state model of $MPC_p$ is obtained in the form of (9) with the following control algorithm

$$
\begin{align*}
\nu(t, l_p) &= F_{i(l_p),l_p} C \eta(t) + F_{i,l_p} C x(t) + F_{i,f} C f x(t+1) \\
&= F_{i(l_p),l_p} C \eta(t) + F_{i,l_p} C x(t) + F_{i,f} C f x(t+1)
\end{align*}
$$

where $F_{i,l_p}$ is created by rearrange elements of gain matrix $F_0$ and has the following structure

$$
F_{i,l_p} = \begin{bmatrix}
F_{i(l_p),l_p} & \cdots & F_{i(l_p,N_u-1),l_p} & F_{i,l_p} & 0 & \cdots & 0 \\
F_{i(l_p+1),l_p} & \cdots & F_{i(l_p+1,N_u-1),l_p} & F_{i(l_p+1),l_p} & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & F_{i(N_u-1),l_p} & 0 & \cdots & 0 \\
0 & \cdots & 0 & F_{i(N_u-1),l_p} & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & F_{i(N_u-1),l_p} & 0 & \cdots & 0
\end{bmatrix}
$$

Fig. 2. Schematic representation of a hybrid automaton

**Theorem 6.** The inclusion $\Omega(P_0(\xi)) \subseteq L(F_0)$ is for output feedback control equivalent to [25]

$$\begin{bmatrix}
P_{0}(\xi) \\
D_{i_0} F_0 C m
\end{bmatrix}^* \lambda_{i_d}^0 \geq 0$$

(22)

for all $i_d = i + (j - 1)m$; $i = 1, \ldots, m$; $j = 1, \ldots, N_u$ and $\lambda_{i_d}^0 \in (0, \infty)$.

Without regard to the networks (the packet-loss problem is not considered in the controller design), if conditions (18) and (22) hold, then guaranteed MPC controller (3) is robustly stabilizes uncertain system (1) and guarantees input constraints (4). To fix the packet-loss problem, the compensation mechanism introduced in Section 2 is used. The first $l_{p_{\max}}$ elements of vector $\nu(t)$ are encapsulated into one packet so-called control sequence $U_s(t) = \{u(t_s), u(t_s+1), \ldots, u(t_s+l_p), \ldots, u(t_s+l_{p_{\max}})\}$ and sent to the buffer device in the actuator. However, control sequence $U_s(t_s)$ obtained from the above MPC controller design(without regard to the networks) can induce instability or poor performance (the guaranteed cost and the input constraints) of the NCS. This problem will be solved in the next section.

**4 ROBUST NETWORKED MODEL PREDICTIVE CONTROLL DESIGN**

In this section, a necessary and sufficient robust stability conditions to robust networked MPC design with guaranteed cost control law for the case of lost packet are presented. Main result of this section on robust networked MPC design can be summarized in Theorem 8.

Due to the compensation mechanism, the control action $u(t)$ corresponding to the current sampling time from control sequence $U_s(t_s)$ in the buffer device will be applied to the actuator at every sampling time $t \in (t_s; t_s + l_p(t_s))$ ($l_p(t_s)$ packets are lost). This process is considered as a switched system with schematic representation of a hybrid automaton in Fig. 2. Hybrid state of the hybrid automat is $x_h(t) = (l_p(t), x_f(t))$ in which
According to mathematical induction (a method of mathematical proof), condition (26) holds for all \( 1 \leq l_p \leq l_p^{\text{max}} \), if we prove that, (26) holds for the cases of \( l_p = \{1, 2\} \), and with assumption that (26) holds for \( l_p - 1 \), it will hold for \( l_p \).

Due to the result of solving difference equation (1), \( x(t_n+1|t_n) = x(t_n+1|t_n+1) \) holds for \( i = 0 \). Using (7) and (23) for \( i = 1 \), we obtain \( x(t+2|t) = x(t+2|t+1) = A(\xi)x(t+1|t)+B(\xi)u(t+1) \). Sequentially repeating the previous step for \( i = 2, \ldots, N_a - 1 \), it is easily to show that \( x(t+i+1|t) = x(t+i+1|t+1) \). Condition (26) holds for \( l_p = 1 \).

Repeating all steps and using results in the above proof for \( l_p = 1 \), we can prove that, condition (26) holds for \( l_p = 2 \).

Now, (26) is supposed to hold for \( l_p - 1 \). We have to show that, (26) will hold for \( l_p \). Indeed, for \( i = 0 \), we have \( x(t+i|t) = x(t+i|t+1) = A(\xi)x(t+i+1|t)+B(\xi)u(t+i+1) \). Because (26) holds for \( l_p - 1 \), then \( x(t+i|t+1) = x(t+i+1|t+1) \). As a result, \( x(t+|t+1) = x(t+|t+1) \). By the same way for \( i = 1, \ldots, N_a - l_p - 1 \), (26) is true for \( l_p \). The proof of (26) is obtained.

Applying control algorithm (27) into model prediction (9), the closed-loop MPC of \( MPC_{l_p} \) is obtained as follows

\[
A_{cf}(\xi, l_p)x_f(t+1) = A_{cx}(\xi, l_p)x(t)
\]

with the schematic representation of hybrid automaton in Fig. 2 is robust stable with guaranteed cost if and only if the following conditions hold

\[
x_f^T(t+1)P_{l_p}x_f(t+1) - x_f^T(t)P_{l_p}x_f(t) + J(t) \leq 0,
\]

where

\[
A_{cf}(\xi, l_p) = A_f(\xi) - B_f(\xi)F_{1_p}C_f,
\]

\[
A_{cx}(\xi, l_p) = A_x(\xi) + B_f(\xi)F_{1_p}C_x.
\]

Let us define the following parameter-dependent Lyapunov functional candidate for the closed-loop feedback of \( MPC_{l_p} \).

\[
V_{l_p}(t) = x_f^T(t)\mathbf{P}_{l_p}(\xi)x_f(t),
\]

\[
\mathbf{P}_{l_p}(\xi) \in R^{N_x \times N_x} : \mathbf{P}_{l_p}(\xi) = \text{diag}\{P_{l_p}(\xi), \ldots, P_{l_p}(\xi)\},
\]

with the guaranteed cost in the closed-loop feedback for NCS is rewritten as the follows.

\[
J(t) = x^T(t)Q_0x(t) + x_f^T(t+1)Q_fx_f(t+1) + (F_{1_p}C_x(t) + F_{1_p}C_f(t+1))^T R_x (F_{1_p}C_x(t) + F_{1_p}C_f(t+1)) =
\]

\[
\eta^T(t)\left( Q + C_{m_r}^T F_{l_p}^T R F_{l_p} C_m \right) \eta(t)
\]

where \( Q_f = \text{diag}\{Q_1, Q_2, \ldots, Q_{N_u}\} \).

Applying theory of switched system for discrete system [11, p.129] and using Lemma 4, the switched system

Note the switched control algorithm for the case of lost packet is given in Fig. 2.

Necessary and sufficient condition for robust stability with guaranteed cost for the NCS with packet loss is given by the following Lemma.

Lemma 7. Control sequence \( U_s(t_s) \) is the guaranteed cost control law for the NCS with packet loss if and only if the following condition holds

\[
B_{l_p}^q(\xi) = A_{l_p}^q(\xi, l_p)(\bar{P}_{l_p}(\xi) + Q_f)A_{l_p}(\xi, l_p) - P_{l_p}(\xi) + Q_0 + (F_{1_p}C_x + F_{1_p}C_f A_{l_p}(\xi, l_p))^T R_x \times (F_{1_p}C_x + F_{1_p}C_f A_{l_p}(\xi, l_p)) \leq 0
\]

where

\[
i_p = \{0, l_p + 1\}, \quad \bar{P}_{l_p}(\xi) \in R^{N_x \times N_x}
\]

and

\[
\bar{P}_{l_p}^q(\xi) = \text{diag}\{P_{l_p} - P_{l_p}, \ldots, P_{l_p} - P_{l_p}, P_{l_p}(\xi)\}.
\]

Robust stability condition (35) is not directly applicable due to its numerical complexity. In the following theorem the novel formulation of robust stability condition is developed, which provide LMI for MPC robust stability analysis and BMI for MPC robust design.

Theorem 8. Control sequence \( U_s(t_s) \) robustly stabilizes the NCS with loss packet process \( t_s \) and ensures the guaranteed cost \( J_0 \), input constraints (4) if and only if there exist matrices \( H_{l_p} \in R^{N_x \times N_x} \), \( P_{l_p}(\xi) = P_{l_p}^q(\xi) > 0 \), and gain matrices \( F_{l_p} \) such that the following bilinear matrix inequality (BMI)

\[
W_{l_p}^q(\xi) = D_{l_p}^q(\xi) + A_{m}(\xi, l_p)H_{l_p} + H_{l_p}^T A_{m}(\xi, l_p) + Q + C_{m_r}^T F_{l_p}^T R F_{l_p} C_m \leq 0;
\]

and the following linear matrix inequality (LMI)

\[
\begin{bmatrix}
P_{l_p}(\xi) & * \\
D_{l_p}F_{l_p}C_m & \lambda_{l_p}^T
\end{bmatrix} \geq 0; \quad \lambda_{l_p}^T \in \langle \frac{Q}{2} \rangle
\]

where

\[
i_d = i + (j - 1)m, \quad j = 1, \ldots, N_u - l_p
\]
hold for all $0 \leq l_p \leq l_{p_{\text{max}}}$. 

Where $D_{lp}^p \in R^{(N_v+1)\times N_v}$ and 

\[ D_{lp}^p(\xi) = \text{diag}\{-P_{lp}, P_l - P_{lp}, \ldots, P_l - P_{lp}, P_{lp}\}(\xi), \]

\[ A_{nl}(\xi,l_p) = [A_{cx}(\xi,l_p) - A_{cf}(\xi,l_p)]. \] \hspace{1cm} (39)

**Proof.** Sufficiency. Considering $H^p_{lp} = [H^p_{lp,x}, H^p_{lp,f}]$, where $H^p_{lp,x} \in R^{N_v \times N_v}$ and $H^p_{lp,f} \in R^{N_v \times N_s}$, the inequality (37) can be rewritten as 

\[ W_{lp}^p(\xi) = \begin{bmatrix} W_{11}(\xi) & W_{12}(\xi) \\ W_{12}(\xi) & W_{22}(\xi) \end{bmatrix} \leq 0 \] \hspace{1cm} (40)

where 

\[ W_{11}(\xi) = -P_{lp}(\xi) + H_{lp}^p + A_{cx}(\xi,l_p) + A_{cf}(\xi,l_p)H_{lp,x} + Q_0 + CT F_{lp,x} R F_{lp,x} C, \]

\[ W_{12}(\xi) = -H_{lp}^p + A_{cf}(\xi,l_p) + A_{cx}(\xi,l_p)H_{lp,f} + CT F_{lp,f} R F_{lp,f} C_f, \]

and $W_{22}(\xi) = P_{lp}^T(\xi) - H_{lp}^p + A_{cf}(\xi,l_p) + A_{cx}(\xi,l_p)H_{lp}^p + Q_f + CT F_{lp,f} R F_{lp,f} C_f$. 

Since the matrix $L = [I A_{lp}^T(\xi,l_p)]$ has full row rank, multiplying the left of (40) by $L$ and the right by $L^T$, (35) is obtained. It means that, the sufficiency is proved.

**Necessity.** Suppose that there exist symmetric positive definite matrices $P_{lp}(\xi)$ and $P_{lp}(\xi)$ such that robust stability condition (35) holds; necessarily, there exists a scalar $\beta_{lp}^+ > 0$ such that 

\[ A_{lp}^T(\xi,l_p)(\tilde{P}_{lp}^p(\xi) + \beta_{lp}^+ I) A_{lp}(\xi,l_p) - P_{lp} \leq 0. \] \hspace{1cm} (41)

Applying Schur complement formula to (41), we obtain 

\[ \begin{bmatrix} -P_{lp}(\xi) & A_{lp}^T(\xi,l_p)(\tilde{P}_{lp}^p(\xi) + \beta_{lp}^+ I) \\ \ast & -P_{lp}^T(\xi) + \beta_{lp}^+ I \end{bmatrix} \leq 0 \] \hspace{1cm} (42)

Taking $H_{lp,x} = -\frac{1}{2} \beta_{lp}^+ (A_{lp}^{-1}(\xi,l_p))^T A_{lp}(\xi,l_p)$ and $H_{lp,f} = (A_{lp}^{-1}(\xi,l_p))^T (\tilde{P}_{lp}^p(\xi) + \frac{1}{2} \beta_{lp}^+ I)$, after some manipulations the following inequality is obtained.

\[ \begin{bmatrix} W_{11,lp}(\xi) & W_{12,lp}(\xi) \\ W_{12,lp}(\xi) & W_{22,lp}(\xi) \end{bmatrix} \leq 0 \] \hspace{1cm} (43)

where

\[ W_{11,lp}(\xi) = W_{12}(\xi), \quad W_{22,lp}(\xi) = W_{22}(\xi) \] and

\[ W_{12,lp}(\xi) = W_{11}(\xi) + \beta_{lp}^+ A_{lp}^T(\xi,l_p) A_{lp}(\xi,l_p). \]

Because $\beta_{lp}^+ A_{lp}^T(\xi,l_p) A_{lp}(\xi,l_p) \geq 0$, then the inequality (40) resp. the inequality (37) is obtained which proves the necessity. For guaranteed cost the proof goes the analogical way as given above.

To prove condition (38) for input constraints, see [25]. Theorem 8 is proved.

Note that (37) is affine to $\xi$. If $W_{lp,j}^p \leq 0, j = 1, \ldots, N$, is feasible with respect to unknown $P_{lp,j} = P_{lp,j}^T > 0, P_{lp,j} = P_{lp,j}^T > 0, H^p_{lp,j}$, and $F_{lp}$ for all $0 \leq l_p \leq l_{p_{\text{max}}}$, then the control sequence $U_0(t_k)$ guarantees robust stability and guaranteed cost for NCS with predictive control (3) within the convex set defined by (2). Therefore, BMI robust stability condition “if and only if” in (37) reduces to sufficient condition.

## 5 Examples

In this section, we present the results of numerical calculations and simulations for a numerical example to demonstrate the effectiveness of the proposed method, namely its ability to cope with robust stability, guaranteed cost, and input constraints without complex computational load. Numerical calculations have been realized by using PEN-BMI.

The discrete model of double integrator turns to (1) where

\[ A_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = [1 \ 0] \]

and uncertainty matrices are

\[ A_{1u} = \begin{bmatrix} 0.01 & 0.01 \\ 0.02 & 0.03 \end{bmatrix}, \quad B_{1u} = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix} \]

For the case when number of uncertainty is $p = 1$ the number of vertices is $N = 2^p = 2$, the matrices (2) corresponding to two working points $1.wp$ and $2.wp$ are calculated as follows

\[ \begin{bmatrix} A_1 = A_0 + A_{1u} \\ A_2 = A_0 - A_{1u} \end{bmatrix} = \begin{bmatrix} B_0 + B_{1u} \\ B_0 - B_{1u} \end{bmatrix}. \]

Considering with prediction horizon and control horizon as $N_u = N_y = 8$. We assume that the packet-loss upper bound $l_{p_{\text{max}}} = N_u = 1 = 7$, which means that up to 87.5% of the packets, can be lost during the network transmissions.

Applying Theorem 8 with parameters $\theta = 30$ and $\varpi = 1$ for the input constraints, and $(Q = qI; R = rI)$ 

\[ q = \{q_i\}_{i=0}^\infty = \{0.001; 0.0025; 0.005; 0.0075; 0.01; 0.015; 0.075; 0.1\}, \]

\[ r = \{r_i\}_{i=0}^\infty = \{1; 10; 100; 1000; 10^4; 10^5; 10^7\} \]

for the cost function, the gain matrix $F_0$ of predictive control algorithm (3) is obtained as follows

\[ 10^{-2} \begin{bmatrix} -1.85 \cdot 10^{-4} \cdot 10^{-11} \cdot 9.83 \cdot 8.33 \cdot 6.52 \cdot 4.84 \cdot 2.37 \cdot 0.24 \\ 0 \cdot 0.72 \cdot 4.38 \cdot 6.14 \cdot 6.55 \cdot 6.2 \cdot 5.25 \cdot 3.97 \cdot 2.63 \\ 0 \cdot 0 \cdot 1.35 \cdot 2.27 \cdot 4.48 \cdot 5.55 \cdot 5.71 \cdot 5.29 \cdot 4.78 \\ 0 \cdot 0 \cdot 0 \cdot 3.96 \cdot 6.01 \cdot 6.36 \cdot 5.46 \cdot 6.41 \cdot 7.24 \\ 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 2.70 \cdot 4.43 \cdot 2.44 \cdot 2.68 \cdot 7.97 \\ 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 2.17 \cdot 7.18 \cdot 4.76 \cdot 7.55 \\ 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0.03 \cdot 0.04 \end{bmatrix} \]
stable and guarantees input constraints. In Fig. 5, there is comparison between the case of packet loss and no packet loss in the network at the first operating point. Comparison between the results obtained by design method without considering packet loss and proposed method at the first operating point is shown in Fig. 6. It shows that, the design method without considering packet also stabilizes NCS, but gives less performance than the proposed method.

6 CONCLUSION

The stabilization of networked predictive control system with packet-loss was studied in this paper. The packet-loss process is arbitrary and bounded by the control horizon of model predictive control. Networked predictive control systems with packet loss are modeled as switched linear systems. This enables us to apply the theory of switched systems to establish the stability condition of networked predictive control systems. The stabilizing controller design is based on sufficient robust stability conditions formulated as a solution of bilinear matrix inequality BMI. The effectiveness of the proposed method was illustrated by a numerical example and simulations.

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