A SELECTION THEOREM FOR SET-VALUED MAPS INTO NORMALLY SUPERCOMPACT SPACES

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Abstract. The following selection theorem is established:
Let $X$ be a compactum possessing a binary normal subbase $S$ for its closed subsets. Then every set-valued $S$-continuous map $\Phi: Z \to X$ with closed $S$-convex values, where $Z$ is an arbitrary space, has a continuous single-valued selection. More generally, if $A \subset Z$ is closed and any map from $A$ to $X$ is continuously extendable to a map from $Z$ to $X$, then every selection for $\Phi|A$ can be extended to a selection for $\Phi$.

This theorem implies that if $X$ is a $\kappa$-metrizable (resp., $\kappa$-metrizable and connected) compactum with a normal binary closed subbase $S$, then every open $S$-convex surjection $f: X \to Y$ is a zero-soft (resp., soft) map. Our results provide some generalizations and specifications of Ivanov’s results (see [5], [6], [7]) concerning superextensions of $\kappa$-metrizable compacta.

1. Introduction

In this paper we assume that all topological spaces are Tychonoff and all single-valued maps are continuous.

Recall that supercompact spaces and superextensions were introduced by de Groot [1]. A space is supercompact if it possesses a binary subbase for its closed subsets. Here, a collection $S$ of closed subsets of $X$ is binary provided any linked subfamily of $S$ has a non-empty intersection (we say that a system of subsets of $X$ is linked provided any two elements of this system intersect). The supercompact spaces with binary normal subbase will be of special interest for us. A subbase $S$ which is closed both under finite intersections and finite unions is called normal if for every $S_0, S_1 \in S$ with $S_0 \cap S_1 = \emptyset$ there exists $T_0, T_1 \in S$ such that $S_0 \cap T_1 = \emptyset = T_0 \cap S_1$ and $T_0 \cup T_1 = X$. A space $X$...
possessing a binary normal subbase $S$ is called *normally supercompact* \[9\] and will be denoted by $(X, S)$.

The *superextension* $\lambda X$ of $X$ consists of all maximal linked systems of closed sets in $X$. The family

$$U^+ = \{ \eta \in \lambda X : F \subset U \text{ for some } F \in \eta \},$$

$U \subset X$ is open, is a subbase for the topology of $\lambda X$. It is well known that $\lambda X$ is normally supercompact. Let $\eta_x, x \in X$, be the maximal linked system of all closed sets in $X$ containing $x$. The map $x \to \eta_x$ embeds $X$ into $\lambda X$. The book of van Mill \[9\] contains more information about normally supercompact space and superextensions, see also Fedorchuk-Filippov’s book \[3\].

If $S$ is a closed subbase for $X$ and $B \subset X$, let $I_S(B) = \bigcap\{ S \in S : B \subset S \}$. A subset $B \subset X$ is called *$S$-convex* if for all $x, y \in B$ we have $I_S(\{ x, y \}) \subset B$. An *$S$-convex map* $f : X \to Y$ is a map whose fibers are $S$-convex sets. A set-valued map $\Phi : Z \to X$ is said to be *$S$-continuous* provided for any $S \in S$ both sets $\{ z \in Z : \Phi(z) \cap (X \setminus S) \neq \emptyset \}$ and $\{ z \in Z : \Phi(z) \subset X \setminus S \}$ are open in $Z$.

**Theorem 1.1.** Let $(X, S)$ be a normally supercompact space and $Z$ an arbitrary space. Then every $S$-continuous set-valued map $\Phi : Z \to X$ has a single-valued selection provided all $\Phi(z), z \in Z$, are $S$-convex closed subsets of $X$. More generally, if $A \subset Z$ is closed and every map from $A$ to $X$ can be extended to a map from $Z$ to $X$, then every selection for $\Phi|A$ is extendable to a selection for $\Phi$.

**Corollary 1.2.** Let $\Phi : Z \to X$ be an $S$-continuous set-valued map such that each $\Phi(z) \subset X$ is closed, where $X$ is a space with a binary normal closed subbase $S$ and $Z$ arbitrary. Then the map $\Psi : Z \to X$, $\Psi(z) = I_S(\Phi(z))$, has a continuous selection.

A map $f : X \to Y$ is invertible if for any space $Z$ and a map $g : Z \to Y$ there exists a map $h : Z \to X$ with $f \circ h = g$. If $X$ has a closed subbase $S$, we say $f : X \to Y$ is *$S$-open* provided $f(X \setminus S) \subset Y$ is open for every $S \in S$. Theorem 1.1 yields next corollary.

**Corollary 1.3.** Let $X$ be a space possessing a binary normal closed subbase $S$. Then every $S$-convex $S$-open surjection $f : X \to Y$ is invertible.

Another corollary of Theorem 1.1 is a specification of Ivanov’s results \[7\] (see also \[5\] and \[6\]). Here, a map $f : X \to Y$ is $A$-soft, where $A$ is a class of spaces, if for any $Z \in A$, its closed subset $A$ and any two maps $k : Z \to Y, h : A \to X$ with $f \circ h = k|A$ there exists a map $g : Z \to X$ extending $h$ such that $f \circ g = k$. When $A$ is the family of
Corollary 1.4. Let $\mathcal{A}$ be a given class of spaces and $X$ be an absolute extensor for all $Z \in \mathcal{A}$. If $X$ has a binary normal closed subbase $S$, then any $S$-convex $S$-open surjection $f : X \to Y$ is $\mathcal{A}$-soft.

Theorem 1.1 is also applied to establish the following proposition:

Proposition 1.5. Let $X$ be a $\kappa$-metrizable (resp., $\kappa$-metrizable and connected) compactum with a normal binary closed subbase $S$. Then every open $S$-convex surjection $f : X \to Y$ is a zero-soft (resp., soft) map.

Corollary 1.6. [5, 6] Let $X$ be a $\kappa$-metrizable (resp., $\kappa$-metrizable and connected) compactum. Then $\lambda f : \lambda X \to \lambda Y$ is a zero-soft (resp., soft) map for any open surjection $f : X \to Y$.

2. Proof of Theorem 1.1 and Corollaries 1.2 - 1.4

Recall that a set-valued map $\Phi : Z \to X$ is lower semi-continuous (br., lsc) if the set $\{z \in Z : \Phi(z) \cap U \neq \emptyset\}$ is open in $Z$ for any open $U \subset X$. $\Phi$ is upper semi-continuous (br., usc) provided that the set $\{z \in Z : \Phi(z) \subset U\}$ is open in $Z$ whenever $U \subset X$ is open. Upper semi-continuous and compact-valued maps are called usco maps. If $\Phi$ is both lsc and usc, it is said to be continuous. Obviously, every continuous set-valued map $\Phi : Z \to X$ is $S$-continuous, where $S$ is a binary closed normal subbase for $X$. Let $C(X, Y)$ denote the set of all (continuous single-valued) maps from $X$ to $Y$.

Proof of Theorem 1.1. Suppose $X$ has a binary normal closed subbase $S$ and $\Phi : Z \to X$ is a $S$-continuous map with closed $S$-convex values. Let $A \subset Z$ be a closed set such that every $f \in C(A, X)$ can be extended to a map $\tilde{f} \in C(Z, X)$. Fix a selection $g \in C(A, X)$ for $\Phi|A$ and its extension $\tilde{g} \in C(Z, X)$. By [9 Theorem 1.5.18], there exists a (continuous) map $\xi : X \times \exp X \to X$, defined by

$$\xi(x, F) = \bigcap \{I_S(\{x, a\}) : a \in F\} \cap I_S(F),$$

where $\exp X$ is the space of all closed subsets of $X$ with the Vietoris topology. This map has the following properties for any $F \in \exp X$: (i) $\xi(x, F) = x$ if $x \in I_S(F)$; (ii) $\xi(x, F) \in I_S(F)$, $x \in X$. Because each $\Phi(z)$, $z \in Z$, is a closed $S$-convex set, $I_S(\Phi(z)) = \Phi(z)$, see [9 Theorem 1.5.7]. So, for all $z \in Z$ we have $h(z) = \xi(\tilde{g}(z), \Phi(z)) \in \Phi(z)$. Therefore, we obtain a map $h : Z \to X$ which is a selection for $\Phi$ and $h(z) = g(z)$ for all $z \in A$. It remains to show that $h$ is continuous. We can show that the subbase could be supposed to be invariant with
respect to finite intersections. Because $\xi$ is continuous, this would imply conti-
nuity of $h$. But instead of that, we follow the arguments from the proof of [9, Theorem 1.5.18].

Let $z_0 \in Z$ and $x_0 = h(z_0) \in W$ with $W$ being open in $X$. We 
may assume that $W = X \setminus S$ for some $S \in \mathcal{S}$. Because $x_0$ is the 
intersection of a subfamily of the binary family $S$, there exists $S^* \in \mathcal{S}$ 
containing $x_0$ and disjoint from $S$. Since $S$ is normal, there exist 
$S_0, S_1 \in \mathcal{S}$ such that $S \subset S_1 \setminus S_0$, $x_0 \in S^* \subset S_0 \setminus S_1$ and $S_0 \cup S_1 = X$. 
Hence, $x_0 \in (X \setminus S_1) \cap \Phi(z_0)$. Because $\Phi$ is $\mathcal{S}$-continuous, 
there exists a neighborhood $O_1(z_0)$ of $z_0$ such that $\Phi(z) \cap (X \setminus S_1) \neq \emptyset$ for every $z \in O_1(z_0)$. Observe that $\bar{g}(z_0) \in X \setminus S_1$ provided $\Phi(z_0) \cap S_1 \neq \emptyset$, otherwise $x_0 \in I\Phi\{\bar{g}(z_0), a\} \subset S_1$, where $a \in \Phi(z_0) \cap S_1$. Consequently, we 
have two possibilities: either $\Phi(z_0) \subset X \setminus S_1$ or $\Phi(z_0)$ intersects both $S_1$ 
and $X \setminus S_1$. In the first case there exists a neighborhood $O_2(z_0)$ with 
$\Phi(z) \subset X \setminus S_1$ for all $z \in O_2(z_0)$, and in the second one take $O_2(z_0)$ 
such that $\bar{g}(O_2(z_0)) \subset X \setminus S_1$ (recall that in this case $\bar{g}(z_0) \in X \setminus S_1$). 
In both cases let $O(z_0) = O_1(z_0) \cap O_2(z_0)$. Then, in the first case we 
have $h(z) \in \Phi(z) \subset X \setminus S_1 \subset X \setminus S$ for every $z \in O(z_0)$. In 
the second case let $a(z) \in \Phi(z) \cap (X \setminus S_1)$, $z \in O(z_0)$. Consequently, $h(z) \in 
I\Phi\{g(z), a(z)\} \subset X \setminus S_1 \subset S_0 \subset X \setminus S$ for any $z \in O(z_0)$. Hence, $h$ is 
continuous.

When the set $A$ is a point $a$ we define $g(a)$ to be an arbitrary point 
in $\Phi(a)$ and $\bar{g}(x) = g(a)$ for all $x \in X$. Then the above arguments 
provide a selection for $\Phi$. \hfill $\square$

\textbf{Proof of Corollary 1.2.} Since each $\Psi(z)$ is $\mathcal{S}$-convex, by Theorem 1.1 
it suffices to show that $\Psi$ is $\mathcal{S}$-continuous. To this end, suppose that 
$F_0 \in \mathcal{S}$ and $\Psi(z_0) \cap (X \setminus F_0) \neq \emptyset$ for some $z_0 \in Z$. Then $\Phi(z_0) \cap 
(X \setminus F_0) \neq \emptyset$, for otherwise $\Phi(z_0) \subset F_0$ and $\Psi(z_0)$, being intersection 
of all $F \in \mathcal{S}$ containing $\Phi(z_0)$, would be contained in $F_0$. Since $\Phi$ is 
$\mathcal{S}$-continuous, there exists a neighborhood $O(z_0) \subset Z$ of $z_0$ such that 
$\Phi(z) \cap (X \setminus F_0) \neq \emptyset$ for all $z \in O(z_0)$. Consequently, $\Psi(z) \cap (X \setminus F_0) \neq \emptyset$, 
$z \in O(z_0)$.

Suppose now that $\Psi(z_0) \subset X \setminus F_0$. Then $\Psi(z_0) \cap F_0 = \emptyset$, so there 
exists $S_0 \in \mathcal{S}$ with $\Phi(z_0) \subset S_0$ and $S_0 \cap F_0 = \emptyset$ (recall that $\mathcal{S}$ is binary). 
Since $\mathcal{S}$ is normal, we can find $S_1, F_1 \in \mathcal{S}$ such that $S_0 \subset S_1 \setminus F_1$, 
$F_0 \subset F_1 \setminus S_1$ and $F_1 \cup S_1 = X$. Using again that $\Phi$ is $\mathcal{S}$-continuous to 
choose a neighborhood $U(z_0) \subset Z$ of $z_0$ with $\Phi(z) \subset X \setminus F_1 \subset S_1$ for all 
z \in U(z_0). Hence, $\Psi(z) \subset S_1 \subset X \setminus F_0$, $z \in U(z_0)$, which completes the 
proof. \hfill $\square$

\textbf{Proof of Corollary 1.3.} Let $X$ possess a binary normal closed subbase 
$\mathcal{S}$, $f : X \to Y$ be an $\mathcal{S}$-open $\mathcal{S}$-convex surjection, and $g : Z \to Y$ be
a map. Since $f$ is both $S$-open and closed (recall that $X$ is compact as a space with a binary closed subbase), the map $\phi: Y \to X$, $\phi(y) = f^{-1}(y)$, is $S$-continuous and $S$-convex valued. So is the map $\Phi = \phi \circ g : Z \to X$. Then, by Theorem 1.1, $\Phi$ admits a continuous selection $h : Z \to X$. Obviously, $g = f \circ h$. Hence, $f$ is invertible. 

**Proof of Corollary 1.4.** Suppose $X$ is a compactum with a normal binary closed subbase $S$ such that $X$ is an absolute extensor for all $Z \in A$. Let us show that every $S$-open $S$-convex surjection $f : X \to Y$ is $A$-soft. Take a space $Z \in A$, its closed subset $A$ and two maps $k : Z \to Y$, $h : A \to X$ such that $k|A = f \circ h$. Then $h$ can be continuously extended to a map $\tilde{h} : Z \to X$. Moreover, the set-valued map $\Phi : Z \to X$, $\Phi(z) = f^{-1}(k(z))$, is $S$-continuous and has $S$-convex values. Hence, by Theorem 1.1, there is a selection $g : Z \to X$ for $\Phi$ extending $h$. Then $f \circ g = k$. So, $f$ is $A$-soft. 

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3. PROOF OF PROPOSITION 1.5 AND COROLLARY 1.6

**Proof of Proposition 1.5.** According to Corollary 1.4, it suffices to show that $X$ is a Dugundji space (resp., an absolute retract) provided $X$ is a $\kappa$-metrizable (resp., $\kappa$-metrizable and connected) compactum with a normal binary closed subbase $S$ (recall that the class of Dugundji spaces coincides with the class of compact absolute extensors for 0-dimensional spaces, see [8]). To this end, we follow the arguments from the proof of [12] Proposition 3.2]. Suppose first that $X$ is a $\kappa$-metrizable compactum with a normal binary closed subbase $S$. Consider $X$ as a subset of a Tychonoff cube $I^\kappa$. Then, by [10] (see also [12] for another proof), there exists a function $e : T_X \to T_{I^\kappa}$ between the topologies of $X$ and $I^\kappa$ such that:

(e1) $e(\emptyset) = \emptyset$ and $e(U) \cap X = U$ for any open $U \subset X$;
(e2) $e(U) \cap e(V) = \emptyset$ for any two disjoint open sets $U, V \subset X$.

Consider the set valued map $r : I^\kappa \to X$ defined by

(1) $r(y) = \bigcap \{I_S(\overline{U}) : y \in e(U), U \in T_X\}$ if $y \in \bigcup \{e(U) : U \in T_X\}$ and $r(y) = X$ otherwise,

where $\overline{U}$ is the closure of $U$ in $X$. According to condition (e2), the system $\gamma_y = \{U \in T_X : y \in e(U)\}$ is linked for every $y \in I^\kappa$. Consequently, $\omega_y = \{S \in S : U \subset S \text{ for some } U \in \gamma_y\}$ is also linked. This implies $r(y) = \bigcap \{S : S \in \omega_y\} \neq \emptyset$ because $S$ is binary.

Claim. $r(x) = \{x\}$ for every $x \in X$.

Suppose there is another point $z \in r(x)$. Then, by normality of $S$, there exist two elements $S_0, S_1 \in S$ such that $x \in S_0 \setminus S_1$, $z \in S_1 \setminus S_0$ and
\(S_0 \cup S_1 = X\). Choose an open neighborhood \(V\) of \(x\) with \(\overline{V} \subset S_0 \setminus S_1\). Observe that \(x \in e(V)\), so \(z \in I_S(\overline{V}) \subset S_0\), a contradiction.

Finally, we can show that \(r\) is upper semi-continuous. Indeed, let \(r(y) \subset W\) with \(y \in \mathbb{I}^r\) and \(W \in T_X\). Then there exist finitely many \(U_i \in T_X\), \(i = 1, 2, \ldots, k\), such that \(y \in \bigcap_{i=1}^{k} e(U_i)\) and \(\bigcap_{i=1}^{k} I_S(\overline{U_i}) \subset W\). Obviously, \(r(y') \subset W\) for all \(y' \in \bigcap_{i=1}^{k} e(U_i)\). So, \(r\) is an usco retraction from \(\mathbb{I}^r\) onto \(X\). According to \([1]\), \(X\) is a Dugundji space.

Suppose now, that \(X\) is connected. By \([9]\), any set of the form \(I_S(F)\) is \(S\)-convex, so is each \(r(y)\). According to \([9]\) Corollary 1.5.8, all closed \(S\)-convex subsets of \(X\) are also connected. Hence, the map \(r\), defined by (1), is connected-valued. Consequently, by \([1]\), \(X\) is an absolute extensor in dimension 1, and there exists a map \(r_1 : \mathbb{I}^r \to \exp X\) with \(r_1(x) = \{x\}\) for all \(x \in X\), see \([2]\) Theorem 3.2]. On the other hand, since \(X\) is normally supercompact, there exists a retraction \(r_2\) from \(\exp X\) onto \(X\), see \([9]\) Corollary 1.5.20]. Then the composition \(r_2 \circ r_1 : \mathbb{I}^r \to X\) is a (single-valued) retraction. So, \(X \in AR\). \(\square\)

Proof of Corollary 1.6. It is well known that \(\lambda\) is a continuous functor preserving open maps, see \([3]\). So, \(\lambda X\) is \(\kappa\)-metrizable. Moreover, \(\lambda X\) is connected if so is \(X\). On the other hand, the family \(S = \{F^+ : F\text{ is closed in } X\}\), where \(F^+ = \{\eta \in \lambda X : F \in \eta\}\), is a binary normal subbase for \(\lambda X\). Observe that \(\lambda f\) is \(S\)-convex because \((\lambda f)^{-1}(\nu) = \bigcap\{f^{-1}(H)^+ : H \in \nu\}\) for every \(\nu \in \lambda Y\). Then, Proposition 1.5 completes the proof. \(\square\)

The next proposition shows that the statements from Proposition 1.5 and Corollary 1.6 are actually equivalent. At the same time it provides more information about validity of Corollary 1.4.

**Proposition 3.1.** For any class \(\mathcal{A}\) the following statements are equivalent:

(i) If \(X\) is a compactum possessing a normal binary closed subbase \(S\), then any open \(S\)-convex surjection \(f : X \to Y\) is \(\mathcal{A}\)-soft.

(ii) The map \(\lambda f : \lambda X \to \lambda Y\) is \(\mathcal{A}\)-soft for any compactum \(X\) and any open surjection \(f : X \to Y\).

**Proof.** (i) \(\Rightarrow\) (ii) Let \(X\) be a compactum and \(f : X \to Y\) be an open surjection. It is easily seen that \(\lambda f\) is an open surjection too. We already noted that \(S = \{F^+ : F \subset X\text{ is closed}\}\) is a normal binary closed subbase for \(\lambda X\) and \(\lambda f\) is a \(S\)-convex and open map. Hence, by (i), \(\lambda f\) is \(\mathcal{A}\)-soft.

(ii) \(\Rightarrow\) (i). Suppose \(X\) is a compactum possessing a normal binary closed subbase \(S\), and \(f : X \to Y\) is an \(S\)-convex open surjection. To show that \(f\) is \(\mathcal{A}\)-soft, take a space \(Z \in \mathcal{A}\), its closed subset \(A\) and
two maps $h : A \to X$, $g : Z \to Y$ with $f \circ h = g|A$. So, we have the following diagram, where $i_X$ and $i_Y$ are embeddings defined by $x \to \eta_x$ and $y \to \eta_y$, respectively.

\[
\begin{array}{ccc}
A & \overset{h}{\longrightarrow} & X \\
\downarrow{id} & & \downarrow{f} \\
Z & \overset{g}{\longrightarrow} & Y
\end{array}
\]

Since, by (ii), $\lambda f$ is $A$-soft, there exists a map $g_1 : Z \to \lambda X$ such that $h = g_1|A$ and $\lambda f \circ g_1 = g$. The last equality implies that $g_1(Z) \subset (\lambda f)^{-1}(Y)$. According to [9, Corollary 2.3.7], there exists a retraction $r : \lambda X \to X$, defined by

\[
r(\eta) = \bigcap\{F \in S : F \in \eta\}.
\]

Consider now the map $\bar{g} = r \circ g_1 : Z \to X$. Obviously, $\bar{g}$ extends $h$. Let us show that $f \circ \bar{g} = g$. Indeed, for any $z \in Z$ we have

\[
g_1(z) \in (\lambda f)^{-1}(g(z)) = (f^{-1}(g(z)))^+.
\]

Since $f$ is $S$-convex, $I_S(f^{-1}(g(z))) = f^{-1}(g(z))$, see [9, Theorem 1.5.7]. Hence, $f^{-1}(g(z))$ is the intersection of the family $\{F \in S : f^{-1}(g(z)) \subset F\}$ whose elements belong to any $\eta \in (\lambda f)^{-1}(g(z))$. It follows from (2) that $r(\eta) \in f^{-1}(g(z))$, $\eta \in (\lambda f)^{-1}(g(z))$. In particular, $\bar{g}(z) \in f^{-1}(g(z))$. Therefore, $f \circ \bar{g} = g$. \hfill \square

The following corollary follows from Corollary 1.4 and Proposition 3.1.

**Corollary 3.2.** If $X$ is a compactum with a binary normal closed sub-base $S$ such that $\lambda X$ is an absolute extensor for a given class $A$, then any open $S$-convex surjection $f : X \to Y$ is $A$-soft.

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