Scalar Meson Decay Constants and the Nature of the $a_0(980)$

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The $a_0(980)$, $a_0(1450)$ and $K_0^*(1430)$ decay constants are determined using a form of QCD sum rules known to produce a very accurate determination of the $\rho$ decay constant. The ratio of $a_0(980)$ to $K_0^*(1430)$ decay constants is shown to be $\sim 0.6$, ruling out both the “loosely-bound-$K\bar{K}$-molecule” and Gribov minion scenarios for the $a_0(980)$. Solutions for the isovector scalar spectral function obtained in the literature from sum rule analyses employing a more restrictive single-resonance-plus-continuum form of the input spectral ansatz, are also investigated. These solutions, which suggest, in contrast to the present results, negligible coupling of the $a_0(980)$ to the isovector scalar density are shown to produce a very poor match between the OPE and hadronic sides of the sum rules employed here, and hence to be ruled out.

1. Introduction

Scenarios proposed in the literature for the nature of the $f_0(980)$ and $a_0(980)$ (the loosely-bound $K\bar{K}$ molecule, crypto-exotic (four-quark), unitarized quark model (UQM), and Gribov minion pictures) differ significantly in their spatial extent. Processes previously proposed to distinguish between these different scenarios (the $\gamma\gamma$ decay widths, and $\phi \rightarrow \gamma a_0, f_0$) suffer from difficult-to-quantify theoretical uncertainties associated with the necessity of modelling the non-trivial dynamics of the processes in question. In this paper we show how to determine the scalar decay constants of the scalar mesons and use this information to make progress in distinguishing between the different scenarios.

Because the various scenarios differ significantly in their spatial extent, pointlike probes such as decay constants are ideal for distinguishing amongst them. Relations between decay constants often provide non-trivial information on $SU(3)_F$ classification and/or mixing. For example, the scenario in which the $\pi$ and $K$, though having $m_K/m_\pi \approx 4.5$, are assigned to the same pseudo-Goldstone boson octet requires the approximate equality of the $\pi$ and $K$ decay constants, as observed experimentally. Similarly, $f_{K^*} = 1.1 f_\rho \approx f_\rho$ confirms the assignment of the $\rho$ and $K^*$ to the same $SU(3)_F$ multiplet, while $f_\omega^{EM} \approx f_\rho^{EM}/3$, (rather than $f_\omega^{EM} \approx f_\rho^{EM}/\sqrt{3}$, as expected for a pure octet $\omega$) confirms the near-ideal mixing of the vector meson sector. In this paper, the basic idea is to determine the various scalar meson decay constants and, using the “normal quark model state” $K_0^*(1430)$ for reference, investigate which (if either) of the known $a_0$ resonances might, given the values of their decay constants, belong to the same $SU(3)_F$ multiplet.
2. Determining The Scalar Meson Decay Constants

We first determine the decay constant of the “reference” quark model state, the $K_0^*(1430)$. With $J_{us} = (m_s - m_u)\bar{s}u$, we define $f_{K_0^*}$ via $\langle 0 | (m_s - m_u)\bar{s}u | K^+ \rangle = f_{K_0^*} m_{K_0^*}^2$. $f_{K_0^*}$ can be read off from the $K_0^*(1430)$ peak value of the spectral function, $\rho_{us}$, of the corresponding correlator, $\Pi_{us}(q^2) \equiv i \int dxe^{iqs} \langle 0 | T (J_{us}(x)J_{us}^+(0)) | 0 \rangle$. Since s-wave $K\pi$ scattering is elastic up through the $K_0^*(1430)$, the spectral function is saturated by $K\pi$ intermediate states out to $s \equiv q^2 \sim 2$ GeV$^2$. Unitarity then allows the spectral function to be expressed in terms of the timelike scalar $K\pi$ form factor, $f_{K\pi}(s)$. $f_{K\pi}$, in turn, satisfies an Omnes relation whose overall normalization is set by $K_{\ell\pi}$ data, and whose phase (appearing in the integral which defines the Omnes function) is, up to the onset of inelasticity, simply the $I = 1/2$ $K\pi$ scattering phase shift. At the largest $s$ for which it has been measured ($\simeq 2.9$ GeV$^2$), the $K\pi$ phase has essentially reached its known asymptotic value, $\pi$. By assuming (1) the absence of a possible polynomial prefactor in the Omnes relation and (2) that the phase is $\pi$ from 2.9 GeV$^2$ to $\infty$, one can thus construct the $K\pi$ part of $\rho_{us}$, and hence determine $f_{K_0^*}$. The result of this exercise is

$$f_{K_0^*} m_{K_0^*}^2 = 0.0842 \pm 0.0045 \text{ GeV}^3. \tag{1}$$

The input theoretical assumptions are supported by the following observations: (1) the determination of $m_s$ associated with a finite energy sum rule (FESR) analysis of $\Pi_{us}$, using $\rho_{us}$ as generated above, is extremely stable, and produces an extremely good match between hadronic and OPE sides; (2) the resulting $m_s$ value is reproduced by a recent analysis based on flavor breaking in hadronic $\tau$ decays (which involves NO such additional theoretical assumptions).

To determine the $a_0(980)$ and $a_0(1450)$ decay constants, we employ a form of FESR tested in the isovector vector channel and shown to produce a determination of the $\rho$ decay constant, using only OPE information, with experimental $\alpha_s$ values as the dominant input, accurate to within experimental errors. The general FESR relation for a typical correlator $\Pi$ is $f_{s_{th}} \int_{s_{th}}^{s_0} ds w(s) \rho(s) = \frac{\langle 0 | T (J_{us}(x)J_{us}^+(0)) | 0 \rangle}{\delta_{s_{th}}} \int_{s_{th}}^{s_0} ds w(s) \Pi(s)$, with $w(s)$ any function analytic in the region of the contour, $s_{th}$ the physical threshold, and $\rho(s)$ the corresponding spectral function. $s_0$ is to be chosen large enough that the OPE can be reliably employed on the RHS. Weight functions satisfying $w(s_0) = 0$, which cut out the region of the integral over the circle near the timelike real axis, have been shown to produce sum rules very well satisfied, even down to (surprisingly) low scales $s_0 \sim 2$ GeV$^2$.

Since, in the isovector scalar channel, the $a_0(980)$ and $a_0(1450)$ are well separated, it is sufficient to take for the hadronic spectral ansatz an incoherent sum of two Breit-Wigner resonance forms using PDG values for the masses and widths. The decay constants are to be fit in the FESR analysis, which works by using analyticity, together with qualitative non-perturbative input (the known resonance positions and widths), to essentially “measure” the decay constants in terms of $\alpha_s$ (the $D = 0$ OPE terms dominate at scales $s > 2$ GeV$^2$). We work with the correlator of the scalar density $J_{ud} \equiv (m_s - m_u)\bar{d}u$, the mass factor being chosen so as to cancel in the ratio of $a_0$ to $K_0^*(1430)$ decay constants. The ratio then reduces to that of the matrix elements of the $\bar{d}u$ and $\bar{s}u$ densities which, since these densities are members of the same $SU(3)_F$ octet, must reduce to 1 in the $SU(3)_F$ limit for an $a_0$ lying in the same multiplet as the $K_0^*(1430)$.
On the OPE side of our FESR’s, the dominant \( D = 0 \) part of the OPE is known to four-loop order\([2,8]\), and the small higher \( D \) terms are also known out to \( D = 6 \)[2]. Instanton contributions are determined using the instanton liquid model\([9]\). A more detailed description of the OPE input and the method of calculation can be found in Ref. \([7]\).

Fitting the \( a_0 \) decay constants using the OPE as described above, one finds

\[
\begin{align*}
    f_{a_0} m_{a_0}^2 & = 0.0447 \pm 0.0085 \text{ GeV}^3, \\
    f_{a_0'} m_{a_0'}^2 & = 0.0647 \pm 0.0123 \text{ GeV}^3.
\end{align*}
\]

The errors are dominated by the estimate of the uncertainty associated with truncating the dominant \( D = 0 \) part of the OPE at 4-loop order. The quality of agreement between the OPE and hadronic side which results is shown in Figure 1 for the weight choice \( w(s) = (1 - s/s_0) (2 - s/s_0) \) (chosen to reduce the sensitivity to the less-well-known instanton contributions). The dotted line is the OPE side of the sum rule and the dashed-dotted line the hadronic side obtained using the results of Eqs. (2). If the \( a_0(980) \) is very diffuse compared to \( K_0^*(1430) \) (the loosely-bound \( K \bar{K} \) molecule scenario), one should find a much smaller decay constant; if very compact (the minion scenario), a much larger decay constant. The results of Eqs. (3) rule out both of these scenarios. Two additional possibilities need to be considered in more detail to make this conclusion definitive in the molecule case. The \( a_0(980) \) spectral strength is proportional to the square of the decay constant. To see if this small-decay-constant scenario is plausible, we set the coefficient of the \( a_0(980) \) Breit-Wigner to zero by hand and re-optimize the \( a_0(1450) \) decay constant. The best fit obtained from this exercise is shown by the solid line in Figure 1; the match to the OPE side is clearly terrible. This failure cannot be cured by using a broad background, rather than narrow resonance contribution, in the region below the \( a_0(1450) \). Indeed, the \( \pi \eta \) matrix element of the scalar density can be computed unambiguously to leading order in the chiral expansion using Chiral Perturbation Theory (ChPT). Given this matrix element, the corresponding background contribution to the spectral function can be obtained using unitarity. Setting the \( a_0(980) \) resonance contribution to zero, and optimizing the \( a_0(1450) \) decay constant in the presence of the resulting background, one obtains a “best” fit almost identical to that given by the solid line. Multiplying the ChPT-generated background contribution by a factor of 5 to allow (generously) for higher (chiral) order contributions, one obtains the “best” fit shown by the dashed line. Clearly no version of the loosely-bound molecule scenario corresponds to a good match to the OPE side, thus ruling out this scenario. The relation between the \( a_0(980) \) and \( K_0^*(1430) \) decay constants given by the results above is, in contrast, exactly what one would expect in the UQM scenario if the \( a_0(980) \) were a roughly equal admixture of a normal quark model meson core and a loosely bound two-meson component. Because of the additional hidden strange pair present in the \( a_0(980) \) in the cryptoexotic scenario, a natural expectation would be to find an \( a_0(980) \) decay constant significantly smaller than that of the \( K_0^*(1430) \). A calculation of the decay constant in this scenario would, however, be welcome and, since the results above represent an (albeit indirect) “measurement” of the scalar meson decay constants (basically in terms of \( \alpha_s \), which is very well known at the scales in question), would serve to provide a further, highly non-trivial test of the cryptoexotic scenario.
We conclude by discussing a recent claim that the $a_0(980)$ coupling to $J'_{ud} = (m_u + m_d)\bar{d}u$ is small, and that the spectral distribution is dominated by a contribution with $m \simeq 1.5$ GeV[10]. This claim (which clearly conflicts with the results above) is based on a Laplace sum rule analysis of the $J'_{ud}$ correlator (which differs from the correlator considered above by the overall multiplicative factor $[(m_u + m_d)/(m_s - m_u)]^2$) assuming a form for the spectral function consisting of a single resonance plus an OPE-generated “continuum” beyond some “continuum threshold”, $s_0$. The resonance mass and $s_0$ are fit in the sum rule analysis, whose validity, apart from the question of the suitability of the form of the spectral ansatz, relies only on analyticity and the applicability of the OPE, as in the sum rules above. IF these assumptions are valid, and IF the spectral function resulting from the fitting procedure is physical, then FESR’s analogous to those above must also be valid. Testing the spectral solution of Ref. [10] by means of the resulting FESR, one finds the results shown in Figure 2. The dotted line again represents the OPE side, and the dashed-dotted line the hadronic side, of the FESR. One immediately sees that, although the solution of Ref. [10] may represent the “best” fit within the restricted form of the spectral ansatz employed, the quality of the OPE/hadronic match is, in fact, very poor, and, moreover, far inferior to that of the two-resonance form discussed above. The results of Ref. [10], and the apparent contradiction with the results obtained here would, therefore, appear to be the artifact of the use of an overly-restrictive form for the the spectral ansatz.

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