Adaptive Finite-time Disturbance Rejection for Nonlinear Systems using an Experience-Replay based Disturbance Observer

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Abstract—Control systems are inevitably affected by external disturbances, and a major objective of the control design is to attenuate or eliminate their adverse effects on the system performance. This paper presents a disturbance rejection approach with two main improvements over existing results: 1) it relaxes the requirement of calculating or measuring the state derivatives, which are not available for measurement, and their calculation is corrupted by noise, and 2) it achieves finite-time disturbance rejection and control. To this end, the disturbance is first modeled by an unknown dynamics, and an adaptive disturbance observer is proposed to estimate it. A filtered regressor form is leveraged to model the nonlinear system and the unknown disturbance. It is shown that using this filtered regressor form, the disturbance is estimated using only measured state of the regressor. That is, contrary to the existing results on disturbance rejection, the presented approach does not require the state derivative measurements. To improve the convergence speed of the disturbance estimation, an adaptive law, equipped with experience replay, is presented. The disturbance observer is then augmented with an adaptive integral terminal sliding mode control to assure the finite-time convergence of tracking error to zero. A verifiable rank condition on the history of the past experience used by the experience-replay technique provides a sufficient condition for convergence. Compared to the existing results, neither the knowledge of the disturbance dynamics nor the state derivatives are required, and finite-time stability is guaranteed. A simulation example illustrates the effectiveness of the proposed approach.

Index Terms—Nonlinear Systems, Filtered Regressor, Adaptive Observer, Unknown Disturbance, Sliding Mode Control

I. INTRODUCTION

Disturbances can be inevitably found in almost every control system and, if not rejected, they can drastically jeopardize the system’s performance. Therefore, it has been a long stand challenge to reject disturbances in control society. Existence of persistent disturbances is one of the sources of difficulties in achieving a good system performance in applications such as marine vessels [1], active vibration suppression [2], tracking of a reference position [1], [3], and rotating mechanisms control [4]. Disturbances are not measurable in most real-world applications, but have some structures, possibly unknown, which must be leveraged by the control design to achieve a better performance. For instance, the disturbance in surprisingly large number of applications can be reasonably modeled as the output of a dynamical system, called exosystem, with unknown dynamics. For example, in systems with rotating, the disturbance source often consists of many of periodic components with unknown frequencies (e.g. engine noise in automobile and aircraft). Modeling the disturbance with an exosystem dynamics is a standard practice and has been considered by many researchers [5], [6], [7], [8].

The most common approach for disturbance cancellation is the internal model principle [9] for which the disturbance dynamics is incorporated into the controller design. A related problem is the output regulation [10] for which the system is supposed to track a reference trajectory and/or reject a disturbance with known exosystems. If the dynamics of the exosystem generating the disturbance is known, and the disturbance can be measured, these approaches can be directly used to completely reject the disturbance. However, in reality, neither the exosystem dynamics is known, nor can we measure the disturbance. In [8], an adaptive output feedback scheme with adaptive backstepping is presented to reject the disturbances by assuming that the state derivatives are measurable. In [11], [12], disturbance observers have been designed for the case where the disturbance cannot be measured, but the exosystem dynamics is assumed to be known. To relax the requirement of knowing the exosystem dynamics, adaptive state-derivative feedback techniques have been presented for both matched disturbances [5] and mismatched disturbances [6], [13]. However, to estimate the disturbance, the state derivatives are assumed to be available which usually cannot be directly sensed and must be calculated from the consecutive state measurement, which is corrupted by noise. Moreover, the history of the interaction between the disturbance and the system is not taken into account in the existing results to achieve better convergence and consequently improve the system’s performance.

Finite-time stability has attracted a surge of interest in both model-based and model-free control due to its desired properties. Specifically, variants of sliding mode control (SMC) [14], [15], such as terminal sliding mode control (TSMC) [16], [17] have been presented to guarantee the finite-time stability. Moreover, integral TSMC (ITSMC) [18] has been successfully used to achieve the finite-time stability and solve the singularity problem in TSMC. Successful applications of variants of ITSMC for robot manipulators [19] and autonomous underwater vehicles [20] have also been reported. Disturbance rejection control has also been studied using SMC and adaptive TSMC in [21], [22], [23]. However, to achieve finite-time stability, the worst-case bound of the disturbance is considered in the design, which results in unnecessary
large control efforts and excessively conservative controllers. To obviate this issue, the structure of the disturbance can be leveraged to estimate it and provide the controller with quantified and decaying disturbance bounds. This significantly improves the performance of the controller.

In this paper, we present a novel adaptive finite-time disturbance rejection controller that does not require the knowledge of the disturbance dynamics and the state derivatives. Towards this goal, we first introduce a new adaptive disturbance observer by formulating its dynamics into a filtered regressor form to overcome the shortcoming of requiring the state derivative measurements which are not usually available and their calculation is corrupted by noise. Then, we design an observer to estimate the unknown disturbance and its dynamics. Next, we present a novel experience replay-based adaptive disturbance observer, in which the history of the data collected along the system trajectories is incorporated into the update law to guarantee the exponential convergence of the disturbance estimation error under satisfying a rank condition on the history stack. This is inspired by how declarative memory (explicit memories that can be inspected and recalled consciously) in human brain stores data to reduce the number of interactions with the environment to learn it. We show that reusing the experiences increases the efficiency of data-based disturbance estimation. Finally, the disturbance observer is augmented with an adaptive ITSMC assuring that the tracking error goes to zero in finite time. The adaptive controller’s gain follows the variation tendency of the disturbance to avoid overestimating the disturbance. This is less control-energy demanding than the existing adaptive ITSMC results for disturbance rejection as they have been designed based on the maximum disturbance bound. A simulation is finally provided to verify the effectiveness of the proposed approach.

Notations: In this paper, \( \mathbb{R}^n \) and \( \mathbb{R}^{n \times m} \) represent a real \( n \)–dimensional vector and a real \( n \times m \) matrix, respectively. For a matrix \( A \), \( A^T \) stands for its transpose, \( A^+ \) stands for its generalized inverse, and if matrix \( A \) has full row rank (or column rank) \( A^+ = A^T (A A^T)^{-1} \) (or \( A^+ = (A^T A)^{-1} A^T \)) stands for its pseudoinverse. \( A_{wc} \) stacks the columns of the matrix \( A \) in a vector. \( \lambda_{\min}(A) \) and \( \lambda_{\max}(A) \) represent the minimum and maximum eigenvalues of \( A \). Moreover \( (A \otimes B) \) represent the Kronecker product of \( A \) and \( B \). The function \( f(t) \) belongs to \( L_2 \) and \( L_\infty \) spaces, i.e., \( f(t) \in L_2 \) and \( f(t) \in L_\infty \), if it satisfies \( \int_0^T f(t)^T f(t) dt < \infty \) and \( \sup_{t \in \mathbb{R}} |f(t)| < \infty \), respectively.

II. A FILTERED REGRESSOR FORM FOR MODELING THE SYSTEM AND THE DISTURBANCE DYNAMICS

In this section, a nonlinear dynamical system with unknown disturbance is introduced. Then, a filtered regressor form is employed to model the nonlinear system dynamics and the disturbance exosystem dynamics.

Consider the following nonlinear dynamical systems

\[
\dot{x} = f(x) + g(x) u(x) + D \varepsilon_T \tag{1}
\]

where \( x \in \mathbb{R}^n \) is a measurable system state vector, \( f(x) \in \mathbb{R}^n \) is the drift dynamics of the system, \( g(x) \in \mathbb{R}^{n \times m} \) is the input dynamics of the system and assumed to be full column rank, and \( u(x) \in \mathbb{R}^m \) is the control input. Moreover, \( D \in \mathbb{R}^{n \times d} \) is the disturbance dynamics, and \( \varepsilon_T \in \mathbb{R}^d \) is the disturbance. It is assumed that the unknown disturbance \( \varepsilon_T \) is generated by the following dynamics

\[
\varepsilon_T = S \varepsilon_T \tag{2}
\]

where \( S \in \mathbb{R}^{d \times d} \) is an unknown matrix of appropriate dimension.

**Assumption 1**: The system (1) is stabilizable. Moreover, \( f(0) = 0 \), and \( f(x) \) and \( g(x) \) are locally Lipschitz.

**Assumption 2**: The matrix \( S \) is unknown with eigenvalues on the imaginary axis.

**Remark 1**: Note that under Assumption 2, the disturbance dynamics (2) can generate external sinusoidal disturbances and many other periodic disturbances that are common in many practical applications. Moreover, if the eigenvalues of \( S \) are located in the left-half side of the imaginary axis, it results in a temporary disturbance that its affects will go away and can be ignored. On the other hand, the eigenvalues of \( S \) cannot be in the right-hand side since it indicates an unstable exosystem; thus, the disturbance will be unbounded with infinite energy, which is not realistic.

We now present a filtered regressor form of the system dynamics (1) and the disturbance dynamics (2).

Let the functions \( f(x) \) and \( g(x) \) be parameterized as

\[
f(x) = \theta^* \xi(x) \quad g(x) = \psi^* \zeta(x) \tag{3}
\]

where \( \theta^* \in \mathbb{R}^{n \times p_\theta} \) and \( \psi^* \in \mathbb{R}^{n \times p_\psi} \) are the known weights matrices, \( \xi(x) \in \mathbb{R}^{p_\theta} \) and \( \zeta(x) \in \mathbb{R}^{p_\psi \times n} \) are the known basis functions, \( p_\theta \) and \( p_\psi \) are the dimensions of the system dynamics \( f(x) \) and \( g(x) \). Note that since \( f(x) \) and \( g(x) \) are known, \( \theta^*, \psi^*, \xi(x) \) and \( \zeta(x) \) can always be founded and also assured known. Then, from (3), the system (1) can be written as

\[
\dot{x} = \theta^* \xi(x) + \psi^* \zeta(x) u(x) + D \varepsilon_T \tag{4}
\]

or equivalently

\[
\dot{x} = \phi^* z(x,u) + D \varepsilon_T \tag{5}
\]

where \( \phi^* \in \mathbb{R}^{n \times d} \) is the known weights matrix, and \( z(x,u) = [\xi^T(x) \, \zeta^T(x)]^T \in \mathbb{R}^d \) is the regressor vector.

Inspired by (2), the filtered regressor forms of the system (1) and the disturbance dynamics (2) are given by Lemma 1 and Lemma 2, respectively.

**Lemma 1**: The system (1), (5) can be expressed as

\[
\begin{align*}
x &= \phi^* h(x) + a x + \dot{\varepsilon} + \rho(t), \\
h(x) &= -a h(x) + z(x,u), h(0) = 0, \\
l(x) &= -a x + l(0), l(0) = 0, \\
\rho(t) &= -a \rho(t), \rho(0) = x(0), \\
\varepsilon &= -a \varepsilon + \varepsilon_T, \varepsilon(0) = 0 \\
\dot{\varepsilon} &= D \varepsilon_T.
\end{align*}
\]

where \( a > 0 \), \( h(x) \in \mathbb{R}^d \) is the filtered regressor version of \( z(x,u) \), \( l(x) \in \mathbb{R}^n \) is the filtered regressor version of the state \( x \), \( \varepsilon \) is the filtered disturbance state, \( \dot{\varepsilon} \) is the filtered output disturbance.

**Proof**: Adding and subtracting the term \( ax \) with \( a > 0 \) to (5), one has

\[
\dot{x} = -ax + \phi^* z(x,u) + ax + D \varepsilon_T \tag{7}
\]
or equivalently
\[ x_i = -ax_i + \phi_i^T z(x, u) + ax_i + (D\epsilon)_i, \quad i = 1, \ldots, n \tag{8} \]
where \( \phi_i \) and \((D\epsilon)_i\) are the \(i\)-th rows of the weights matrix \( \phi^* \) and disturbance \( D\epsilon \), respectively.

Thus, the solution of (8) can be expressed as
\[
x_i(t) = e^{-at}x_i(0) + \int_0^t e^{-a(t-\tau)} \phi_i^T z(x, u) \, d\tau + a \int_0^t e^{-a(t-\tau)} x_i(\tau) \, d\tau + \int_0^t e^{-a(t-\tau)} (D\epsilon)_i \, d\tau \tag{9}\]
Define
\[
h(x) = \int_0^t e^{-a(t-\tau)} z(x, u) \, d\tau \tag{10}\]
\[
\hat{e}_i(x) = \int_0^t e^{-a(t-\tau)} (D\epsilon)_i \, d\tau, \quad i = 1, \ldots, n \tag{11}\]
\[
l_i(x) = \int_0^t e^{-a(t-\tau)} x_i(\tau) \, d\tau, \quad i = 1, \ldots, n \tag{12}\]
\[
\rho_i(t) = e^{-at}x_i(0), \quad i = 1, \ldots, n \tag{13}\]
\[
\epsilon_i = \int_0^t e^{-a(t-\tau)} \epsilon_i(\tau) \, d\tau, \quad i = 1, \ldots, n \tag{14}\]

Then, using (10), (12), (13) becomes
\[
x_i = \phi_i^* h(x) + al_i(x) + \hat{e}_i + \rho_i(t) \tag{15}\]
Let \( l(x) = [l_1(x), l_2(x), \ldots, l_n(x)]^T \), \( \epsilon = [\epsilon_1, \epsilon_2, \ldots, \epsilon_n]^T \), \( \tilde{\epsilon} = [\tilde{\epsilon}_1, \tilde{\epsilon}_2, \ldots, \tilde{\epsilon}_n]^T \) and \( \rho(t) = [\rho_1(t), \rho_2(t), \ldots, \rho_n(t)]^T \). The matrix form of (15) can be written as
\[
x = \phi^* h(x) + al(x) + \tilde{\epsilon} + \rho(t) \tag{16}\]

On the other hand, using (11) and (14), one has
\[
\hat{\epsilon} = D\tilde{\epsilon} \tag{17}\]

Taking derivative of \( h(x), l(x), \) and \( \epsilon \) results in
\[
h(x) = -a \int_0^t e^{-a(t-\tau)} z(x, u) \, d\tau + (z(x, u) - 0) \tag{18}\]
\[
l_i(x) = -a \int_0^t e^{-a(t-\tau)} x_i(\tau) \, d\tau + (x - 0) \tag{19}\]
\[
\hat{\epsilon} = -a \epsilon + \epsilon_T \tag{20}\]

Note that \( \rho(t) = e^{-at} \rho(0) \) gives \( \hat{\rho}(t) = -a \hat{\rho}(t) \) with \( \hat{\rho}(0) = x(0) \). This completes the proof.

Similarly to Lemma 1, a filtered regressor form for the unknown disturbance dynamics (2) are shown as the following Lemma

**Lemma 2:** The disturbance dynamics (2) can be expressed as
\[
\epsilon_T = (S + al_d) \epsilon + \rho_\Delta(t), \quad \hat{\epsilon} = -ae + \epsilon_T, \quad \epsilon(0) = 0, \quad \hat{\rho}_\Delta(t) = -a \hat{\rho}_\Delta(t), \quad \rho_\Delta(0) = \epsilon_T(0) \tag{21}\]
where \( a > 0 \) is a constant, and \( \epsilon = \int_0^t e^{-a(t-\tau)} \epsilon_T(\tau) \, d\tau \).

**Proof:** Adding and subtracting the term \( ae\epsilon \) with \( a > 0 \) to the right-hand side of the system (2), one has
\[
\hat{\epsilon}_T = -ae\epsilon_T + S\epsilon_T + ae\epsilon_T \tag{22}\]
The solution of (22) can be written as
\[
\epsilon_T = e^{-at}\epsilon_T(0) + S \int_0^t e^{-a(t-\tau)} \epsilon_T(\tau) \, d\tau + e^{-at}\epsilon_T(0) \tag{23}\]
Defining \( \epsilon = \int_0^t e^{-a(t-\tau)} \epsilon_T(\tau) \, d\tau \) and \( \rho_\Delta(t) = e^{-at}\epsilon_T(0) \), (22) becomes the first equation in (21). On the other hand, the derivative of filtered disturbance state \( \epsilon \) becomes the second equation in (21), and the derivative of \( \rho_\Delta(t) \) becomes the third equation in (21). This completes the proof.

**Remark 2:** Note that the filtered disturbance state \( \epsilon \) in the filtered regressor form of the disturbance dynamics (2) is the same as \( \epsilon \) in the filtered regressor form of the system dynamics (6). On the other hand, based on (6), \( \hat{\epsilon} = D\epsilon \) can be calculated using only the state measurements. Therefore, to estimate the disturbance in (21), we only need to estimate the unknown dynamic matrix \( S \). An observer is designed next to estimate \( S \). This is in contrast to the existing disturbance estimation results that require measurements of the state derivatives as well (3, 6).

**III. AN ADAPTIVE DISTURBANCE OBSERVER USING MEASURED SYSTEM’S STATES**

Since the disturbance \( \epsilon_T \) cannot be measured and only the system’s state is assumed to be measurable, we design a disturbance observer using the filtered regressor form (21) as follows
\[
\hat{\epsilon}_T = (\hat{S} + al_d) \epsilon + \hat{\rho}_\Delta(t) \tag{24}\]
\[
\hat{\rho}_\Delta(t) = -a \hat{\rho}_\Delta(t), \quad \rho_\Delta(0) = \hat{\epsilon}_T(0) \]
where \( \hat{S} \) is the estimation of the disturbance weights matrix \( S \). Note that as stated in Remark 2, \( \epsilon \) is measured using only the measured states.

To design an adaptive disturbance observer, the following auxiliary dynamics are used to develop an adaptive law for \( \hat{S} \).
\[
\hat{\epsilon} = \phi^* h(x) + al(x) + \hat{\epsilon} + \rho(t) \tag{25}\]
\[
\hat{\epsilon}_T = D\hat{\epsilon} \tag{26}\]
where \( \hat{\epsilon} \) is an auxiliary variable used for estimation of the disturbance, \( \hat{\epsilon} \) is the estimated filtered disturbance, and \( \hat{\epsilon}_T \) is the estimated filtered output disturbance.

**Remark 3:** In this paper, the state \( x \) is assumed to be available for measurement. Note that in (24), \( \hat{\epsilon} \) is not actually the state estimation and is only used to measure the disturbance. Defining \( \hat{\epsilon}_T = \epsilon_T - \hat{\epsilon}_T \), \( \hat{S} = S - \hat{S} \), and \( \hat{\rho}_\Delta = \rho_\Delta - \hat{\rho}_\Delta \), and using (21), (24), one has
\[
\epsilon_T = (S + al_d) \epsilon + \rho_\Delta - (\hat{S} + al_d) \epsilon - \hat{\rho}_\Delta \tag{26}\]
Defining $\hat{e} = x - \hat{x}$, the adaptive law for $\hat{S}$ is designed as
\begin{equation}
\dot{\hat{S}}_{\text{vec}} = \Gamma (\hat{e}^T F^T \otimes D) \hat{e}^T
\end{equation}
where $\hat{S}_{\text{vec}}$ is the estimated vector obtained by stacking rows of the unknown matrix $\hat{S}$, and $F = D^T$.

The following lemmas are used in the proof of Theorem 1.

**Lemma 3**: If $f, \dot{f} \in L_\infty$ and $f \in L_p$ for some $p \in [1, \infty]$, then $f(t) \to 0$ as $t \to \infty$.

**Lemma 4**: If $\lim_{t \to \infty} f(t)$ exists and is finite, and $f(t)$ is a uniformly continuous function, then $\lim_{t \to \infty} f(t) = 0$.

**Theorem 1**: Under Assumption 2, consider the nonlinear system 
(1) with unknown disturbance dynamics (2). Then, the adaptive law (27) along with the disturbance observer (24), (25) guarantees the convergence of the disturbance estimation error $\hat{e}_T$ to zero.

**Proof**: Let $\bar{e} = \hat{e} - \hat{\hat{e}}$. From (6) and (25) it yields
\begin{equation}
\bar{e} = -a \hat{e} + D \hat{e}_T
\end{equation}

Based on (20) and (25), one has
\begin{equation}
\hat{e} = -a \hat{e} + D \hat{e}_T
\end{equation}

According to (26), (28), and (29), one has
\begin{equation}
\dot{\hat{e}} = -a \hat{e} + D \hat{e}_T
\end{equation}

Thus, the system (30) can be rewritten as
\begin{equation}
\dot{\hat{e}} = -a \hat{e} + (\hat{e}^T F^T \otimes D) \bar{S}_{\text{vec}} + D \hat{\Delta}
\end{equation}
where $\bar{S}_{\text{vec}}$ is a vector obtained by stacking rows of the matrix $\bar{S}$.

Now, consider the following Lyapunov function candidate.
\begin{equation}
V = \bar{e}^T \hat{e} + \bar{S}_{\text{vec}}^T \Gamma^{-1} \bar{S}_{\text{vec}}
\end{equation}

Using (31) and the adaptive law (27), the derivative of (32) yields
\begin{equation}
\begin{aligned}
\dot{V} &= -2a \bar{e} \hat{e} + 2a \bar{e} \hat{\Delta} \bar{S}_{\text{vec}} + 2 \bar{e} (\hat{e}^T F^T \otimes D) \bar{S}_{\text{vec}} \\
&+ \bar{S}_{\text{vec}}^T (\hat{e}^T F^T \otimes D) \hat{e} - \bar{S}_{\text{vec}}^T (\hat{e}^T F^T \otimes D) \bar{S}_{\text{vec}} \\
&- 2a \bar{e} \hat{e} + 2a \bar{e} \hat{\Delta} \\
&\leq 2a \bar{e} \hat{e} + 2a \bar{e} \hat{\Delta} + 2 \|D\| \|\hat{\Delta}\| \|\bar{S}_{\text{vec}}\| \\
&\leq -a \bar{e} \hat{e} + 4a \|D\| \|\hat{\Delta}\|^2
\end{aligned}
\end{equation}

Note that based on (21) and (24), one has $\hat{\Delta} = (\epsilon_T(t) - \hat{\epsilon}_T(t)) e^{-at}$. Since $\hat{\Delta}$ goes to zero exponentially fast, according to (26), for any $V(t) > 0$, there exists a $t_1 \geq t_0$ such that $\forall t \geq t_1$, $\dot{V} \leq 0$, this implies that $V(t)$ is bounded. Then, one knows that $e \in L_\infty$ and $\bar{S}_{\text{vec}} \in L_\infty$.

Furthermore, by integrating (33) from both sides, one has
\begin{equation}
\begin{aligned}
a \int_0^t \bar{e} \hat{e} d\tau &\leq -\int_0^t \bar{V} d\tau + \int_0^t 4a \|D\| \|\hat{\Delta}\|^2 \|\bar{S}_{\text{vec}}\|^2 d\tau \\
&\leq V(0) - V(t) + \int_0^t 4a \|D\| \|\hat{\Delta}\|^2 \|\bar{S}_{\text{vec}}\|^2 d\tau
\end{aligned}
\end{equation}

The last integral is bounded since $\hat{\Delta}$ goes to zero exponentially fast, which implies $a \int_0^t \bar{e} \hat{e} d\tau < \infty$, thus, $\bar{e} \in L_2$. Based on Assumption 2, $\epsilon_T \in L_\infty$. Then, using (17) and (20), one has $\bar{e} \in L_\infty$. From $\bar{e} \in L_\infty$ and $\bar{e} \in L_\infty$, (30) concludes that $\hat{e} \in L_\infty$, which together with $\hat{e} \in L_2$ and Lemma 3 implies $\bar{e} \to 0$ as $t \to \infty$. Note that $\bar{e} \to \epsilon$ as $t \to \infty$, because $\hat{e} = \epsilon - \hat{\epsilon} \to 0$ as $t \to \infty$. Note also that $\epsilon = \int_0^t e^{-a(t-\tau)} \epsilon_T(\tau) d\tau$ and $\hat{\epsilon} = \int_0^t e^{-a(t-\tau)} \hat{\epsilon}_T(\tau) d\tau$. Then, $\lim_{t \to \infty} (\epsilon - \hat{\epsilon}) = \lim_{t \to \infty} \int_0^t e^{-a(t-\tau)} (\epsilon_T - \hat{\epsilon}_T) d\tau = 0$. From Lemma 2, $\hat{\epsilon}_T = \epsilon_T - \hat{\epsilon}_T \to 0$ as $t \to \infty$.

Therefore, the disturbance estimation error $\hat{e}_T$ converges to zero. This completes the proof.

**Remark 4**: Note that although Theorem 1 shows that $\hat{e}_T \to 0$, it cannot guarantee that $\hat{S} \to S$. An experience-replay based adaptive disturbance observer is designed next to estimate $S$ accurately and make the convergence speed much faster.

IV. EXPERIENCE-REPLAY BASED ADAPTIVE DISTURBANCE OBSERVER

Inspired by [24, 27, 28] which used the experience replay for system identification, the experience-replay technique is used to improve the convergence speed of the disturbance observer. Note that the term $\rho_{\Delta}(t)$ goes to zero exponentially fast; therefore, one can choose a large enough $a$ such that after a short time $t_0$, the impact of $\rho_{\Delta}(t)$ is ignored. The experience replay stores past data in a history stack and reuse them in the disturbance estimation law as
\begin{equation}
\begin{aligned}
\dot{\hat{S}}_{\text{vec}} &= \Gamma (\hat{e}^T F^T \otimes D) \hat{e} \\
&+ \kappa \Gamma \sum_{i=1}^n Y^T_i (x(t_i) - x(t_i - \Delta t) - \mathcal{L}_i - \hat{Y}_i - \hat{\hat{Y}}_{\text{vec}})
\end{aligned}
\end{equation}

where $\hat{S}_{\text{vec}}$ and $\bar{S}_{\text{vec}}$ are obtained by stacking rows of the unknown matrix $\hat{S}$ and $\bar{S}$, respectively, $\Delta t$ is a positive constant denoting the size of the window of integration, $\kappa \in \mathbb{R}^n$ is a constant, and $\Gamma$ is a positive definite gain matrix. $t_i \in [0, t]$ are the time points which are between the $t_0$ and the current time, $Y_i = Y(t_i), \hat{Y}_i = \hat{Y}(t_i)$, and $\mathcal{L}_i = \mathcal{L}(t_i)$.

\begin{equation}
Y(t) = \begin{cases}
0, & t \in [t_0, t_0 + \Delta t] \\
\int_{t_0}^t (\epsilon_T(\tau))^T F^T \otimes D) d\tau, & t > t_0 + \Delta t
\end{cases}
\end{equation}

\begin{equation}
\hat{Y}(t) = \begin{cases}
0, & t \in [t_0, t_0 + \Delta t] \\
\int_{t_0}^t \epsilon_T(\tau) d\tau, & t > t_0 + \Delta t
\end{cases}
\end{equation}

\begin{equation}
\mathcal{L}(t) = \begin{cases}
0, & t \in [t_0, t_0 + \Delta t] \\
\int_{t_0}^t \epsilon_T(\tau) d\tau, & t > t_0 + \Delta t
\end{cases}
\end{equation}

For any $t > t_0 + \Delta t$, integrating (5) yields
\begin{equation}
\int_{t_0}^t \dot{e}(\tau) d\tau = \int_{t_0}^t \epsilon_T(\tau) d\tau + D \int_{t_0}^t \epsilon_T(\tau) d\tau
\end{equation}

Using (36)-(39), one has
\begin{equation}
X(t) - X(t - \Delta t) = Y(t) S_{\text{vec}} + \mathcal{L}(t) + \hat{Y}(t)
\end{equation}

where $S_{\text{vec}}$ is the stacking rows of the unknown matrix $S$.

Substituting (40) into (35) yields
\[
\dot{\hat{S}}_{\text{vec}} = \Gamma (\dot{e}^T F^T \otimes D)^T \dot{e} + \kappa \Gamma \sum_{i=1}^{n} Y_i^T \dot{Y}_i \hat{S}_{\text{vec}} \tag{41}
\]

From the adaptive law (45) and (41), the time is divided into two phases. In the initial phase, the collected data is insufficient to satisfy a richness condition on the history stack. After a finite period of time, the observer switches to the second phase, where the history stack is sufficiently rich. To assure that the observer switches to the second phase in finite time, sufficiently rich data are required to be collected after a finite period of time as discussed in the following assumption.

Assumption 3: The system (1) is sufficiently excited over a finite duration of time. Specifically, there exist a positive constant \( \omega \) and time \( T > t_0 + \Delta t \) for any \( t > T \), such that \( \lambda_{\text{min}}(\sum_{i=1}^{n} Y_i^T Y_i) > \omega \).

Remark 5: Compared to the adaptive law (27), (41) has an extra term which depends on the history of data collected over time.

Theorem 2: Under Assumptions 1-3, consider the nonlinear system (1) with the unknown disturbance (2). Then, the adaptive control law (41) along with the disturbance observer (24) guarantee that the unknown dynamic matrix estimation error \( \hat{S} \) and estimation error \( \tilde{e}_T \) converge to zero exponentially fast.

Proof: Consider the following Lyapunov function candidate

\[
V = \dot{e}^T \dot{e} + \hat{S}_{\text{vec}}^T \Gamma^{-1} \hat{S}_{\text{vec}} \tag{42}
\]

Under Assumption [3] the system (1) only requires to be exciting up to time \( T \), after which the exciting data recorded during \( t \in [t_0, T] \) is used for all \( t > T \).

Then, using (31) and the adaptive law (41), during \( t \in [T, \infty) \), the derivative of (42) yields

\[
\dot{V} = -2a \dot{e}^T \dot{e} + \dot{e}^T (\dot{e}^T F^T \otimes D) \hat{S}_{\text{vec}}
+ \hat{S}_{\text{vec}}^T \Gamma^{-1} \Gamma \hat{S}_{\text{vec}} - 2 \kappa \dot{S}_{\text{vec}}^T \sum_{i=1}^{n} Y_i^T \dot{Y}_i \hat{S}_{\text{vec}}
- 2 \dot{a} \dot{e}^T \dot{e} - 2 \kappa \dot{S}_{\text{vec}}^T \sum_{i=1}^{n} Y_i^T \dot{Y}_i \hat{S}_{\text{vec}} \tag{43}
\]

According to Assumption 3, \( \lambda_{\text{min}}(\sum_{i=1}^{n} Y_i^T Y_i) > 0 \) for any \( t \in [T, \infty) \). This implies that \( \sum_{i=1}^{n} Y_i^T Y_i \) is positive.

Let \( \eta(t) = [\dot{e}^T, \hat{S}_{\text{vec}}^T]^T \). From (43) one has

\[
\eta(t) \leq \sqrt{\sigma_1 \sigma_2} \eta(T) \exp(-\lambda_1(t-T)) \tag{44}
\]

where \( \sigma_1 = \max\{1, \lambda_{\text{max}}(\Gamma^{-1})\} \) and \( \sigma_2 = \min\{1, \lambda_{\text{min}}(\Gamma^{-1})\} \). Thus, the error \( \dot{e} \) and the estimation error \( \hat{S} \) converge to zero exponentially fast. Note that \( \dot{e} \) goes to \( e \) exponentially fast because \( \dot{e} = e - \tilde{e} \) goes to zero exponentially fast. Thus, we can obtain \( \tilde{e}_T = e_T - \hat{e}_T \to 0 \) exponentially fast. This completes the proof.

Remark 6: Condition (44) shows that the convergence rate depends on \( \omega \) and \( a \). Using an appropriate data selection algorithm for adding new samples to the history stack and removing the old ones to increase the minimum eigenvalue of \( \sum_{i=1}^{n} Y_i^T Y_i \) can significantly improve the convergence speed.

V. Adaptive Finite-Time Control Law Design and Stability Analysis

In this section, a finite-time disturbance rejection controller is presented by incorporating the integral terminal sliding mode control (ITSMC) with the proposed disturbance observer. Using Theorem 2, the variation of tendency of disturbance is known and will be leveraged in the control design; therefore, to guarantee the stabilization of the system, the controller’s gain does not need to set to a high value in contrast to [22], [29].

Let define \( x_d \) as the reference trajectory of the system (1) and assume that \( x_d \) is available for the control purpose. Thus, the tracking error is defined as

\[
e_t = x - x_d \tag{45}
\]

To develop the ITSMC, the sliding surface \( \sigma \) is defined as

\[
\sigma = e_a + e_t \tag{46}
\]

where \( e_t = \int_0^t \text{sign}(e_x)(\tau) \, d\tau \).

To reject the disturbance, the following controller is designed as

\[
u = g^+(x)(-f(x) + \dot{x}_d - D \dot{e}_T - \text{sign}(e_x) - k(t) \text{sign}(\sigma)) \tag{47}
\]

where the adaptive controller’s gain \( k(t) \) is defined as

\[
k(t) = k_0 + k_1 ||\tilde{e}|| e^{-\lambda_1 t} \tag{48}
\]

where \( k_0 \) is a small positive constant, \( k_1 \geq ||F|| ||D|| \), and \( \lambda_1 \) is a positive value defined in Theorem 2.

The following lemma is used in the proof of Theorem 3.

Lemma 5: Consider the following system

\[
\dot{x} = f(x), \quad f(0) = 0, \quad x \in \mathbb{R}^n \tag{49}
\]

Let \( V(x) \) be defined as a positive definite continuous function which satisfies

\[
\dot{V}(x) + a_1 V^{a_2}(x) \leq 0 \tag{50}
\]

where \( a_1 > 0 \) and \( 0 < a_2 \leq 1 \). Thus, \( x \) converges to the equilibrium point in finite time.

The following theorem presents a finite-time control law for disturbance rejection control using the proposed disturbance observer.

Theorem 3: Under Assumptions 1-3, consider the nonlinear system (1) with the unknown disturbance (2). The control law (47) along with the adaptive disturbance observer (24) and adaptive law (41) ensures that the tracking error \( e_t \) converges to zero in finite time.

Proof: After collecting rich data (i.e., after \( t > T \)), we use the experience replay to assure the convergence of disturbance
estimation error to zero. From (1) and (47), the derivative of the sliding mode surface $\sigma$ can be given as

$$\dot{\sigma} = -k(t) \text{sign}(\sigma) + (\tilde{e}^T F^T \otimes D) \dot{S}_{\text{vec}} \quad (51)$$

Consider the following Lyapunov function candidate as

$$V = \sigma^T \sigma \quad (52)$$

Then, the derivative of (52) is

$$\dot{V} = -2(k(t) ||\sigma||^2) + 2\sigma(\tilde{e}^T F^T \otimes D) \dot{S}_{\text{vec}} \leq -2(k(t) ||d_A|| ||\sigma||) \leq -2(k(t) ||d_A||) V^\frac{1}{2} \quad (53)$$

where $||d_A||$ is the bound of $(\tilde{e}^T F^T \otimes D) \dot{S}_{\text{vec}}$, and $k(t)$ is designed as (48). From Theorem 2, the disturbance estimation error converges to zero exponentially fast under Assumption 2. Based on Theorem 2, $k \omega \geq \lambda_1$. Therefore, one has

$$k(t) = k_0 + k_1 ||\tilde{e}|| e^{-\lambda_1 t} > k_1 ||\tilde{e}|| e^{-\lambda_1 t} \geq ||d_A|| \quad (54)$$

Substituting (54) into (53), the Lyapunov function candidate (52) satisfies the finite-time stability condition (50) in Lemma 5. Therefore, for any initial condition $\sigma(0) \neq 0$, the system (1) reaches the sliding manifold $\sigma(t) = 0$ in finite time. Then, using (46), one has

$$e_s = -\int_0^t \text{sign}(e_s(\tau)) d\tau \quad (55)$$

which implies that the system (1) converges to zero along $\sigma(0) \neq 0$ in finite time after the system reaches the sliding manifold $\sigma(t) = 0$ in finite time (31), [22]. Therefore, the tracking error $e_s$ converges to zero in finite time. This completes the proof.

**Remark 8:** If the rich data is not collected at the time of the control design, i.e., the condition of Assumption 3 is not satisfied, the proposed controller (47) can be modified as follows by adding another phase to it to make sure that before Assumption 3 is satisfied, the system remains stable.

$$u = \begin{cases} 
    g^+(x)(-f(x) + \dot{x}_d - D\dot{\varepsilon}_T + h e_s) & t \leq T \\
    g^+(x)(-f(x) + \dot{x}_d - D\dot{\varepsilon}_T - \text{sign}(e_s) - k(t) \text{sign}(\sigma)) & t > T 
\end{cases} \quad (56)$$

where $h$ is a positive constant.

Before rich data is collected, one has

$$\dot{e}_s = \dot{x} - \dot{x}_d = -h e_s + D\dot{\varepsilon}_T$$

$$= -h e_s + (\tilde{e}^T F^T \otimes D) \dot{S}_{\text{vec}} + D\tilde{p}_\Delta \quad (57)$$

Consequently, it is clear that the system (1) is stable during data collection according to the convergence of disturbance estimation error to zero, and $\tilde{p}_\Delta$ goes to zero exponentially fast.

The schematic of the finite-time disturbance rejection using the experience-replay approach is shown in Fig.1.

![Fig. 1. Framework of finite-time disturbance rejection using experience-replay approach.](image)

**VI. SIMULATION**

In this section, we present an example to illustrate the effectiveness of the proposed control scheme.

**Example 1:** Consider the following nonlinear system as

$$\begin{align*}
\dot{x}_1 &= x_1 + x_2 - x_1(x_1^2 + x_2^2) + u_1 + \varepsilon_{t_1} \\
\dot{x}_2 &= -x_1 + x_2 - x_2(x_1^2 + x_2^2) + u_2 + \varepsilon_{t_2}
\end{align*} \quad (58)$$

where $x_1$ and $x_2$ are the states of the system, and $\varepsilon_{t_1}$ and $\varepsilon_{t_2}$ are the unknown disturbances. Let $z(x, u) = [x_1 x_2 x_1(x_1^2 + x_2^2) x_2(x_1^2 + x_2^2)]^T$, $u = [u_1 u_2]$, and $\varepsilon_T = [\varepsilon_{t_1} \varepsilon_{t_2}]^T$. Then, (58) can be written as

$$\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = 
\begin{bmatrix}
1 & 1 & -1 & 0 & 0 & 1 & 0 \\
-1 & 1 & 0 & -1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
z(x, u) \\
\varepsilon_T
\end{bmatrix} \quad (59)$$

The disturbance dynamics can be expressed as

$$\dot{\varepsilon}_T = 
\begin{bmatrix}
0 & \beta \\
-\beta & 0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_T \\
\varepsilon_T
\end{bmatrix} = 
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_T \\
\varepsilon_T
\end{bmatrix} \quad (60)$$

with $\beta$ as an unknown parameter.

The reference trajectory is $x_d = \begin{bmatrix} 2\sin 2t \\ 4\cos 3t \end{bmatrix}$; therefore, the tracking error becomes $e_s = \begin{bmatrix} x_1 - 2\sin 2t \\ x_2 - 4\cos 3t \end{bmatrix}$. The actual value of $\beta$ is assumed to be $\beta = 2$, and the parameters $a$ and $\Gamma$ are selected as $a = 2$ and $\Gamma = 50$.

Now, the adaptive law (27) is used to estimate the disturbance. Fig. 2 shows that the estimation of the disturbance $\dot{\varepsilon}_T$ goes to the actual disturbance $\varepsilon_T$. Fig. 3 shows the convergence of the error for all elements of the matrix $S$ in (60).

Then, the experience-replay based adaptive law (41) is used to estimate the disturbance. Fig. 4 shows that the estimation of the disturbance $\dot{\varepsilon}_T$ converges to the actual $\varepsilon_T$. Fig. 5 shows the convergence of the error for all elements of the matrix $S$ in (60). Finally, we use the adaptive ITSMC (47) along with the experience-replay based adaptive law (41) for tracking the reference trajectory $x_d$. Fig. 6 shows that the tracking errors $e_{x_1} = x_1 - 2\sin 2t$ and $e_{x_2} = x_2 - 4\cos 3t$ converge to zero in finite time.
Comparing Figs. 4-5 to Figs. 2-3, one can conclude that the experience-replay based adaptive observer has much faster convergence speed than the case without using experience replay.

Example 2: Consider the following two-mass-spring system as shown in Fig. 7, which can be used to model a large number of practical systems, including deformable objects’ movement and vibration of mechanical systems [32]. This system is controlled via \( u_1, u_2 \) and disturbed by an external force \( w \), where \( m_1 \) and \( m_2 \) denote masses, and \( k_1 \) and \( k_2 \) are spring constants. Defining \( x = [y_1, \dot{y}_1, y_2, \dot{y}_2]^T \) as the system state, where \( y_1 \) and \( \dot{y}_1 \) are the displacement and velocity of mass \( m_1 \), respectively, \( y_2 \) and \( \dot{y}_2 \) are the displacement of and velocity of mass \( m_2 \), respectively. Then, the system dynamics with an unknown disturbance are described as

\[
\dot{x} = Ax + Bu + Dw
\]  

where

\[
A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1 + k_2}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ \frac{-k_2}{m_2} & 0 & \frac{-k_2}{m_2} & 0 \\ \end{bmatrix},
\]

\[
B = \begin{bmatrix} 0 & \frac{1}{m_1} & 0 & 0 \\ 0 & 0 & \frac{1}{m_1} & 0 \\ \end{bmatrix}^T,
\]

\[
D = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ \end{bmatrix}^T.
\]
The dynamics of the unknown disturbance can be expressed as

\[
\dot{w} = \begin{bmatrix} 0 & -\beta \\ \beta & 0 \end{bmatrix} w = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}
\] (65)

The system parameters are \( m_1 = 1 \text{kg}, \ m_2 = 1 \text{kg}, \ k_1 = 1 \text{N/s}, \) and \( k_2 = 1 \text{N/s}. \)

The actual value of \( \beta \) is assumed to be \( \beta = 1.5, \) and the parameters \( a \) and \( \Gamma \) are selected as \( a = 3 \) and \( \Gamma = 50. \)

The reference trajectory is \( x_d = [\sin t \ \cos t \ \cos t \ -\sin t]^T, \) and the tracking error is \( e_x = x - x_d = [e_{x_1} \ e_{x_2} \ e_{x_3} \ e_{x_4}]^T. \)

Now, the adaptive law (27) is used to estimate the disturbance. Fig. 8 shows that the estimation of the disturbance \( \hat{w} \) goes to the actual disturbance \( w. \) Fig. 9 shows the convergence of the error for all elements of the matrix \( S \) in (60).

Then, the experience-replay based adaptive law (41) is used to estimate the disturbance. Fig. 10 shows that the estimation of the disturbance \( \hat{w} \) converges to the actual \( w. \) Fig. 11 shows the convergence of the error for all elements of the matrix \( S \) in (60). Finally, the adaptive ITSMC (47) along with the experience-replay based disturbance adaptive law (41) is used for tracking the reference trajectory \( x_d. \) Fig. 12 shows that the tracking errors \( e_{x_1}, e_{x_2}, e_{x_3}, e_{x_4} \) converge to zero in finite time.

Comparing Figs. 10-11 to Figs. 8-9, one can conclude that the experience replay based adaptive observer has much faster convergence speed than the case without using experience replay.
These results confirm that the proposed approach successfully estimates the disturbance as well as its dynamics, and the proposed adaptive ITSMC successfully tracks the reference trajectory.

VII. CONCLUSION

For a class of systems with unknown disturbance, an adaptive observer was presented to estimate the disturbance. The proposed approach assures that the disturbance estimation error as well as the disturbance exosystem dynamics identification error go to zero exponentially fast. To achieve this goal, a filtered regressor form is presented to model both the system dynamics and the disturbance dynamics. This allows us to estimate the disturbance without requiring the measurement of disturbance or state derivatives. Using the experience-replay based adaptive law, convergence of unknown disturbance dynamics to the actual dynamics is guaranteed. Then, an integral terminal sliding mode controller is presented to assure that the tracking error goes to zero in finite time. The future work will consider a stochastic framework to take into account the measurement noise and will also consider output feedback control design for disturbance rejection.

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