Higgs amplitude mode in massless Dirac fermion systems

Ming Lu,1,2 Haiwen Liu,1,2 Pei Wang,1,3 and X. C. Xie1,2

1International Center for Quantum Materials and School of Physics, Peking University, Beijing 100871, China
2Collaborative Innovation Center of Quantum Matter Beijing 100871, China
3Department of Physics, Zhejiang Normal University, Jinhua 321004, China

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The Higgs amplitude mode in superconductors is the condensed matter analogy of Higgs bosons in particle physics. We investigate the time evolution of Higgs amplitude mode in massless Dirac systems, induced by a weak quench of an attractive interaction. We find that the Higgs amplitude mode in the half-filling honeycomb lattice has a logarithmic decaying behaviour, qualitatively different from the $1/\sqrt{t}$ decay in the normal superconductors. Our study is also extended to the doped cases in honeycomb lattice. As for the 3D Dirac semimetal at half filling, we obtain an undamped oscillation of the amplitude mode. Our finding is not only an important supplement to the previous theoretical studies on normal fermion systems, but also provide an experimental signature to characterize the superconductivity in 2D or 3D Dirac systems.

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I. INTRODUCTION

A conventional superconductor can be described by a charged complex order parameter $\Delta(r, t) = |\Delta(r, t)|e^{i\phi(r, t)}$. Its collective fluctuations around equilibrium including the oscillations of the phase and amplitude.\(^{1}\) The phase mode, being coupled to the electromagnetic field, moves to plasma frequency of the metal as a manifestation of Anderson-Higgs mechanism.\(^{2–4}\) The amplitude mode oscillates with the angular frequency $2\Delta_0$, analogous to the “vibration” of the longitudinal component of Higgs field in particle physics.\(^{5}\) In this sense, the amplitude mode in superconductor is sometimes also called Higgs mode or Higgs amplitude mode in the literature.\(^{1,5,7–9,13}\)

Higgs amplitude mode in superconductors, although theoretically predicted many years ago,\(^{6}\) has only been directly observed recently by the time-resolved Teraherz (THz) pump-probe technique in a clean superconducting film,\(^{7,8}\) and by measuring the excess sub-gap optical conductance in disordered films near the superconductor-insulator phase transition.\(^{9}\) The time evolution of the Higgs mode in the collisionless, dissipationless regime was studied intensely. It was revealed that the Higgs mode oscillates at a frequency of $2\Delta_\infty$ with a $1/\sqrt{t}$ decaying property in the weak coupling limit, where $\Delta_\infty$ is the asymptotic value of superconducting gap.\(^{10–13}\) However, previous works all assume that the density of states (DOS) near the Fermi level is almost a constant within the Debye cut-off energy $\omega_D$. This assumption obviously fails for honeycomb lattice or Dirac semimetals at half filling. Their DOS is either linear (2D) or quadratic (3D) at low energy, respectively, and vanishes at the Dirac point.\(^{15,20}\) Since superconductivity is strongly affected by the DOS near the Fermi level, it would be theoretically interesting to study the time evolution of Higgs mode in those systems. On the experimental side, the availability of the honeycomb optical lattice and the tunable attractive interaction by Feshbash resonance\(^{19}\) give a possible test ground for this study. Besides, the expected unique feature of the Higgs mode in superconducting Dirac semimetal can be used as an important experimental characterization to distinguish it from the normal superconductors.\(^{21,22}\)

In this paper we study the quenched dynamics in the weak coupling limit by using the Anderson pseudo-spin formalism.\(^{23}\) We find that the Higgs mode has a log-\(-\)decay behaviour in the half-filling honeycomb lattice. To understand this behaviour, we further study the pseudo-spins’ phase dynamics, and analytically solve the linearized equations of motion.\(^{11–13}\) The doped cases is also studied numerically. In the low doping limit, a double-frequency feature is found. The larger frequency increases noticeably and its peak broadens with the doping level. In the high doping limit, we are back to the $1/\sqrt{t}$ decaying property, as in a normal superconductor. When considering the 3D Dirac semimetal at neutral point, we find that the Higgs mode exhibits an undamped oscillation, with all the pseudo-spins precess synchronizely.

II. MODEL AND FORMALISM

We start by considering the negative-U Hubbard model on honeycomb lattice:

$$\hat{H} = - \sum_{\langle ij, \sigma \rangle} \hat{a}_{i\sigma}^\dagger \hat{b}_{j\sigma} + h.c. - U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \mu \sum_{i \sigma} \hat{n}_{i\sigma}$$

(1)

where $\hat{a}_i$ ($\hat{b}_i$) is the on-site annihilation operator on sub-lattice $A$ ($B$); $\hat{n}_{i\sigma}$ is the number operator on lattice site $i$ with spin index $\sigma$; $\mu$ is chemical potential and $U$ is the the on-site attractive interaction. We choose the nearest-neighbor hopping as the energy unit throughout this paper.

To study the dynamics, we write out the corresponding mean-field Hamiltonian in $\mathbf{k}$-space after a unitary transformation: $\hat{a}_{\mathbf{k}\sigma} = \frac{1}{\sqrt{2}}(e^{i\mathbf{k}\mathbf{r}} \hat{c}_{\mathbf{k}\sigma} + \hat{d}_{\mathbf{k}\sigma})$, $\hat{d}_{\mathbf{k}\sigma} = \frac{1}{\sqrt{2}}(-\hat{c}_{\mathbf{k}\sigma} + \hat{d}_{\mathbf{k}\sigma}^{\dagger})$.
\[ e^{-i\theta_k \hat{a}_{\mathbf{k}\sigma}}: \\
H_{MF} = -\sum_{\mathbf{k}} (\mu - |\gamma_k|) \hat{c}_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} (\mu + |\gamma_k|) \hat{d}_{\mathbf{k}\sigma}^\dagger d_{\mathbf{k}\sigma} \\
- \Delta^*(t) \sum_{\mathbf{k}} \left( \hat{c}_{\mathbf{k}+}^\dagger c_{\mathbf{k}-} + \hat{d}_{\mathbf{k}+}^\dagger d_{\mathbf{k}-} + h.c. \right) \] (2)

where \( \hat{a}_{\mathbf{k}\sigma} \) is the Fourier component of \( \hat{a}_i \) \( (\hat{b}_i) \); \( e^{i\theta_k} = \gamma_k / |\gamma_k| \) with \( \gamma_k = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \) and \( \delta \) being the three real space nearest-neighbour vectors; the time-dependent order parameter \( \Delta(t) = \frac{U}{N_c} \sum_{\mathbf{k}} \langle a_{\mathbf{k}+}^\dagger a_{\mathbf{k}-} \rangle = \frac{U}{N_c} \sum_{\mathbf{k}} \langle b_{\mathbf{k}+}^\dagger b_{\mathbf{k}-} \rangle \), in which \( N_c \) is the number of unit cells and \( \langle \cdots \rangle \) denotes the time dependent quantum-mechanical expectation value.

We define two sets of Anderson pseudo-spins: \( \hat{S}_k^{(\pm)} = \frac{1}{2} \left( \hat{c}_{\mathbf{k}+}^{(\pm)} \sigma_{\mathbf{k}+}^{(\pm)} \right) \), \( \hat{S}_k^{(-)} = \frac{1}{2} \left( \hat{d}_{\mathbf{k}+}^{(\pm)} \sigma_{\mathbf{k}+}^{(\pm)} \right) \), with their corresponding local fields \( \mathbf{b}_k(\pm)(t) = (\Delta^R(t), \Delta^I(t), \mu \mp |\gamma_k|) \). It is straightforward to check that the pseudo-spin operators satisfies the commutation relationship of the angular momentum (with \( \hbar = 1 \)). Using the above definition, the Hamiltonian can be written as the sum of the “Zeeman energy” of pseudo-spins in their corresponding local fields:

\[ H_{MF} = -2 \sum_{\mathbf{k},i=\pm} \mathbf{b}_k^{(i)} \cdot \hat{S}_k^{(i)} \] (3)

From the Hamiltonian, we can get the equations of motion of pseudo-spins: \( \frac{\partial}{\partial t} \hat{S}_k^{(i)}(t) = -2\mathbf{b}_k^{(i)} \times \hat{S}_k^{(i)}(t) \), where \( i = \pm \) and \( \hat{S}_k^{(i)}(t) \equiv \langle \hat{S}_k^{(i)}(t) \rangle \) are the expectation value of Anderson pseudo-spin operators. The time dependent gap can be written using pseudo-spins as: \( \Delta(t) = \frac{U}{2N_c} \sum_{\mathbf{k},i=\pm} \left( \hat{S}_k^{(i)x} + i\hat{S}_k^{(i)y} \right) \).

For simplicity, we can also label the pseudo-spins by energy state \( \epsilon_j \) rather than \( \mathbf{k} \), so that we can combine the two sets of pseudo-spins as a single set. Explicitly, the equations of motion and time dependent gap can be rewritten as:

\[ \frac{\partial}{\partial t} \mathbf{S}_j(t) = -2\mathbf{b}_j(t) \times \mathbf{S}_j(t) \] (4)
\[ \Delta(t) = \frac{U}{2N_c} \sum_j \left( \mathbf{S}_j^x(t) + i\mathbf{S}_j^y(t) \right) \] (5)

with:

\[ \mathbf{b}_j(t) = (\Delta^R(t), \Delta^I(t), \epsilon_j) \] (6)

where \( \epsilon_j \in (-\omega_D, \omega_D) \), and \( \mathbf{S}_j \) can be view as the classical spin with length \( \frac{1}{2} \). Writing like this, the additional DOS information is needed. It satisfies \( D(\epsilon) \propto |\epsilon - \mu| \), for we have a 2D linear dispersion near the Dirac point before superconducting, see [FIG.1(c)].

The quenched dynamics is as follows: at \( t \leq 0 \), the system is in equilibrium with the initial interacting strength \( U_i \). From the spin Hamiltonian, the initial spins are parallel to their local fields [FIG. 1(a)]. At \( t = 0^+ \), we change the interaction strength abruptly to make the system out of equilibrium. The pseudo-spins start to precess around their local fields, while the local fields also change due to their dependence on pseudo-spins. (c) The half filling case: \( \mu = 0 \), where \( \epsilon_k \equiv \pm |\gamma_k| \). (d) The high doping limit: \( \mu \gg \Delta_{0f} \). (e) The low doping limit: \( \mu \sim \Delta_{0f} \).

The quenched dynamics is as follows: at \( t \leq 0 \), the system is in equilibrium with the initial interacting strength \( U_i \). From the spin Hamiltonian, the initial spins are parallel to their local fields [FIG. 1(a)]. At \( t = 0^+ \), we change the interaction strength to \( U_f \), then the local fields change immediately for the sudden change of \( \Delta(t) \). Therefore, the current spin configuration is no longer stable. According to equation (4), they will precess around their local fields[FIG.1(b)], which in turn will change the gap and the local fields simultaneously by equation (5) and (6) . We denote \( \Delta_{0i} \) and \( \Delta_{0f} \) as the corresponding equilibrium gap when the interaction strength are \( U_i \) and \( U_f \), respectively. In the following, they are used to describe the quenched dynamics for convenience.

III. THREE DOPING CASES FOR HONEYCOMB LATTICE

We consider the dynamics of three doping cases for honeycomb lattice as shown in Fig. 1(c, d, e): half filling, high doping limit and low doping limit.

A. Half filling

Without loss of generality, we choose the initial gap \( \Delta_{0i} \) to be real. The particle-hole symmetry guarantees the gap to be real throughout the evolution\(^{10} \). The problem is to solve a system of coupled differential equations (4) with the initial condition: \( \mathbf{S}_j(0) = \)

![Figure 1](image_url)
review the literature for the normal superconductors, claiming that similar behaviour has been pointed out in the previous decays as a logarithmic decaying property in the present case, while it matches with the \(2\Delta\) envelope. The result is shown in FIG. (2): the data is well fitted by a log-decay function:

\[
\frac{\Delta(t)}{\Delta_0} = a + \frac{2b\delta\Delta_0 \cos(c\Delta_0 f t + d)}{\Delta_0 f \ln(e\Delta_0 f t)}
\]

where\(\Delta_0\) is the gap and local fields are related with the pseudo-spins as:

\[
\Delta(t) = \frac{\epsilon_j}{2\sqrt{\Delta_0^2 + \epsilon_j^2}} \mathcal{S}_j(t)
\]

and\(\mathcal{S}_j(t) = (\Delta(t), 0, \epsilon_j)\). The DOS in the half filling case is proportional to \(|\epsilon|\).

We numerically simulate the equation (4) with \(N = 50000\) energy levels and the Debye cut-off energy \(\omega_D = 0.5\). The method we use is the Runge-Kutta of the 8-th order with an adjustable time step to meet a sufficient high precision. Other numbers of energy levels are also tried to verify that the results are unaffected by the finite size effect. We also adopt the weak coupling limit\((\Delta_0 f \ll \omega_D)\) and the weak quench limit\((\delta\Delta_0 = \Delta_0 - \Delta_0 f \ll \Delta_0)\). To satisfy this, we quench from \(\Delta_0 i = 0.013\) to \(\Delta_0 f = 0.012\). The result is shown in FIG. (2): the data is well fitted by a log-decay function:

\[
\frac{\Delta(t)}{\Delta_0} = a + \frac{2b\delta\Delta_0 \cos(c\Delta_0 f t + d)}{\Delta_0 f \ln(e\Delta_0 f t)}
\]

The fitted parameter are: \(a = 0.9975, b = 1.091, c = 1.994, d = 0.2554, e = 22.36\). We find that \(c = 2a\) is almost exactly satisfied, which means that \(\Delta(t)\) oscillates with the \(2\Delta_\infty\) angular frequency, indicating it is the Higgs amplitude mode. However, the mode has a logarithmic decaying property in the present case, while it decays as \(1/\sqrt{t}\) in the normal superconductors. This slow decaying behaviour suggests the Higgs mode in the half-filling superconducting honeycomb lattice has a much longer lifetime than that in the usual superconductors. We also note that \(a\) is slightly smaller than 1, meaning \(\Delta_\infty < \Delta_0 f\). Explicitly, we find \(1 - a \approx \delta\Delta_0^2/3\Delta_0^2\). The similar behaviour has been pointed out in the previous literature for the normal superconductors, claiming that the difference is of order \(\delta\Delta_0^2/6\Delta_0^2\).

The slower decaying property compared with normal superconductors can be qualitatively understood by studying the phase dynamics of the single pseudo-spin on different energy levels. \(\Delta_0 i = 0.013\) and \(\Delta_0 f = 0.012\) at half filling. The solid lines are for \(D(\epsilon) = 1\), while the dashed lines are for \(D(\epsilon) \propto |\epsilon|\). (a) The precession phases \(\phi_j\) for \(\epsilon_j = 2\Delta_0 f, 4\Delta_0 f\). They are almost linear for the large time dynamics and \(\phi_j\) for constant DOS has larger “phase slope”. (b) The time averaged precession frequency \(\omega_j\). For constant DOS, \(\omega_j\) coincides with quasiparticle energy spectrum. For linear DOS case, the flatter dispersion of \(\omega_j\) gives rise to in a weaker dephasing, therefore a slower decay of the amplitude.

To quantitatively understand the fitting equation (7), we solve equations of motion (4) by linearizing it around \(\mathcal{S}_j^0 \equiv \left(\frac{\Delta_0 f}{2\sqrt{\Delta_0^2 + \epsilon_j^2}}, 0, \epsilon_j\right)\) and \(b_j^0 \equiv (\Delta_0 f, 0, \epsilon_j)\):

\[
\frac{\partial}{\partial t} \delta \mathcal{S}_j^0(t) = 2\epsilon_j \delta \mathcal{S}_j^0(t)
\]

\[
\frac{\partial}{\partial t} \delta \mathcal{S}_j^y(t) = \frac{\epsilon_j}{\sqrt{\Delta_0^2 f + \epsilon_j^2}} \delta \Delta(t) + 2\Delta_0 f \delta \mathcal{S}_j^x(t) - 2\epsilon_j \delta \mathcal{S}_j^z(t)
\]

\[
\frac{\partial}{\partial t} \delta \mathcal{S}_j^z(t) = -2\Delta_0 f \delta \mathcal{S}_j^x(t)
\]

where \(\delta \Delta(t) \equiv \Delta(t) - \Delta_0 f\) and \(\delta \mathcal{S}_j(t) \equiv \mathcal{S}_j(t) - \mathcal{S}_j^0\). The above coupled differential equation can be solved by Laplace transform: \(\mathcal{L}[\mathcal{S}_j(t)] \rightarrow \tilde{\mathcal{S}}(s)\). In the thermodynamic and the weak coupling limit, we arrive at the final
form of $\delta \Delta (s)$:

$$\delta \Delta (s) = \frac{\delta \Delta_0}{2 \Delta_{0f}} \left( \frac{1}{\left( \frac{s}{2 \Delta_{0f}} \right)^2 + 1} \right) \tan^{-1} \left( \frac{s}{2 \Delta_{0f}} \right)$$

By inverse Laplace transform, we can get the approximate form of $\Delta(t)$ (see Appendix A):

$$\Delta(t) \approx \Delta_f + 2 \delta \Delta_0 \cos 2 \delta \Delta_0 t \ln 4 \Delta_f t$$

B. Doping cases

In the high doping limit ($\mu \gg \Delta_{0f}$) as illustrated in Fig. 1(d). The system without attractive interaction is basically a normal metal, therefore we expect the Higgs mode will have the square-root decaying behaviour. To verify this, we choose $\mu = 0.12 = 10 \Delta_{0f}$ and simulate equation (4)-(6) with other parameters equal to those in the half-filling case. The result is shown in Appendix B. We can see $|\Delta(t)|$ indeed decays as $1/\sqrt{t}$, with the oscillation frequency equals to $2 \Delta_{\infty}$.

To see how the mode change from the logarithmic decay to the $1/\sqrt{t}$ decay, we investigate the low doping limit where $\mu \sim \Delta_{0f}$ [Fig. 1(e)]. By simulating equation (4)-(6) with several different values of $\mu$, we find there are two frequencies in the low doping case: one is the Higgs frequency $2 \Delta_{\infty}$, the other is slightly larger than the first one, resulting in a beat pattern as shown in FIG.4 (a). As $\mu$ increases, we find both frequencies increase. However, the Higgs frequency increases only slightly, while the larger frequency increases more remarkably and the peak broadens [Fig.4 (b)]. Physically, the decay of the Higgs mode is due to its interaction with the bottom part of the particle-hole continuum.11, 14. As we doped away from half filling, those states most responsible for the damping increase, resulting a faster decaying behaviour. When $\mu$ is large enough (about $2 \Delta_{0f}$), the second peak can hardly be discerned and the transform from the logarithmic decay to square-root decay accomplishes. We also find a very interesting empirical formula, associating the difference of the two frequencies $\delta \omega$ with the chemical potential $\mu$: $\delta \omega / \Delta_{0f} = 2 \left( \frac{\mu}{\Delta_{0f}} \right)^2$.

V. DISCUSSION AND SUMMARY

For the 2D superconducting Dirac fermion case, the quenched process can be realized on the two-component cold Fermi gases trapped in a honeycomb optical lattice, with an attractive Hubbard $U$ tunable by the Feshbach resonance. The Higgs mode in this case can be detected with the rf-absorption techniques. As for the Higgs amplitude mode in 3D case, the observation is made possible by the recent discovery of superconduc-
In the thermodynamic limit and weak coupling limit, we have \( \sum_{j} f(\epsilon_j) \propto \int_{0}^{\infty} f(\epsilon) \epsilon d\epsilon \approx \int_{0}^{\infty} f(\epsilon) d\epsilon \). After the integration, we get equation (9) in the main text. Using the similarity theorem \( \mathcal{L}^{-1} \left[ f(s/n) \right] = a f(at) \), we need only to find the the inverse Laplace transform of \( f(s) = 1/(s^2 + 1) \tan^{-1} s \). We achieve this by evaluating the Bromwich integral:

\[
f(t) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} ds \tilde{f}(s) e^{st}
\]

where \( \gamma \) should be larger than the real part of any poles in the integrand.

FIG. 6. (color online) The contouir of the integral. The red cross represents the pole at \( s = 0 \), the red points are branch points at \( s = \pm i \), the red lines are the two branch cuts.

We choose the contour shown in Fig. 6, and use Cauchy’s integral theorem to evaluate the Bromwich integral \( C_0 \) marking in blue. The Jordan’s lemma tells us the contributions from big arcs \( \Gamma_1, \Gamma_2, \Gamma_3 \) are zero, and it is easy to verify that the integrals along the small arcs \( \gamma_1 \) and \( \gamma_2 \) have no contributions either. The only remaining parts are the pole at origin and line integrals \( C_1 \) to \( C_4 \). So we have:

\[
f(t) = \theta(t) - 4I_2(t)
\]

We use the contour in Fig. 7 to evaluate equation(A4), and the only remaining contribution is from the line in-
integral $\gamma_1$. To the leading order, we have:

$$I_2(t) = \Re \left[ e^{it} \int_0^{2a} e^{-2y^2} dy \right]$$  \hspace{1cm} (A5)

For large enough $t$, the above integral can be conducted by using a result by A. Erdlyi\textsuperscript{29}, thus we obtain equation (10) in the main text.

FIG. 7. (color online) The contour for $I_2(t)$, $a$ is a small real positive number, the integral $I_2(t)$ (blue) is replaced by the contour in red, while integration along $\gamma_2$ and $\gamma_3$ are zero.

Appendix B: High doping limit case

We choose $\mu = 0.12$ in this case. Because the exact particle-hole symmetry is absent when $\mu \neq 0$, $\Delta(t)$ will acquire a time-depended phase during the evolution, thus we plot the amplitude $|\Delta(t)|$ in the figure. We fit the data using the following equation provided in many literatures\textsuperscript{11–13}:

$$\frac{\Delta(t)}{\Delta_{0f}} = a + \frac{2b\delta \Delta_0}{\pi^2 \Delta_{0f} \sqrt{\Delta_{0f} t}} \cos \left( c\Delta_{0f} t + d \frac{\pi}{4} \right)$$  \hspace{1cm} (B1)

The fitting parameters are: $a = 1.0050$, $b = 0.5142$, $c = 2.0101$, $d = 0.9827$. We see $c = 2a$ is almost exactly satisfied, indicating this is the Higgs amplitude mode. However, $a$ is slightly greater than 1, meaning $\Delta_\infty$ is slightly greater than $\Delta_{0f}$. This is not so surprising because the relation $\Delta_\infty \approx \Delta_{0f} - \delta \Delta_0^2/6\Delta_{0f}$ is obtained under the strictly constant density of state condition. In conclusion, in the high doping limit, the system behaves as a normal metal without interaction, resulting the $1/\sqrt{t}$ decaying property of the amplitude $|\Delta(t)|$.

FIG. 8. (color online) High doping limit with $\mu = 0.12$, other parameters are same as those in FIG. 2 in the main text. The numerical data (blue) is well fitted by equation (B1).

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