Topology Optimization with Nozzle Size Restrictions for Material Extrusion-Type Additive Manufacturing

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Abstract Topology optimization that is tailored to additive manufacturing constraints and possibilities is an important area of research with direct implications on solution manufacturability. This paper focuses on implementing the nozzle size constraint that is associated with most material extrusion-type additive manufacturing processes, such as Fused Filament Fabrication and concrete 3D printing. The constraint is especially important for manufacturability in situations where the size of used nozzle is large in comparison to the size of the design domain. This paper suggests a new projection-based algorithm that embeds material extrusion-type primitives into the projection methodology used for material distribution approaches to topology optimization. Projection-based algorithms for continuum topology optimization have received considerable attention in recent years due to their ability to improve manufacturability in a flexible and computationally efficient manner. A formulation for single-directional primitives is presented and extended to allow the use of two-directional primitives that can simulate a more realistic nozzle movement. The proposed algorithms are demonstrated on 2D benchmark problems and are shown to satisfy the imposed nozzle size restrictions.

Keywords Topology optimization · Manufacturability · Projection · Additive Manufacturing · Material Extrusion · 3D Printing

1 Introduction

Recent decades have seen a rapid development in all additive manufacturing technologies, including material extrusion-type 3D printing. This has raised the need for new design methods that can leverage the new, increasingly complex manufacturing possibilities (Thompson et al., 2016). Topology optimization is often suggested as a design-for-additive manufacturing method since it has potential to
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generate new high-performing design solutions. It is a free-form design approach that does not require a pre-conceived notion of the final layout, and the resulting solutions are often complex. Although the technological developments of manufacturing processes have created a new fabrication paradigm, there are still associated limitations that must be considered during the design process (Liu et al., 2018). Therefore, this work seeks to implicitly embed the nozzle size constraint associated with material extrusion-type additive manufacturing processes such as Fused Filament Fabrication (FFF) (or Fused Deposition Modeling (FDM)) and concrete 3D printing.

Recently, there has been a large interest in developing topology optimization frameworks that are tailored to the possibilities and constraints afforded by additive manufacturing. A full review is beyond the scope of this paper, the reader is instead referred to Liu et al. (2018). Research has mostly focused on eliminating the overhang constraint (e.g. Gaynor and Guest (2016); Guo et al. (2017); Langelaar (2017); Allaire et al. (2017); Mass and Amir (2017)), allowing design with multiple base materials (e.g. Bendsoe and Sigmund (1999); Gaynor et al. (2014); Watts and Tortorelli (2016)), designing porous infill (Wu et al., 2017), enabling design of a structure with infills (Clausen et al., 2016), and integrating the part build direction (Liu, 2016; Chiu et al., 2018). However, most of the existing works on topology optimization and other design-for-additive manufacturing methods does not specify the targeted additive manufacturing process and must be adapted slightly to be suitable for material extrusion processes (Huang et al., 2020).

The material extrusion-type additive manufacturing processes considered in this work, most often consists of a nozzle that moves across a build plate and deposits material on a 2D slice of the design (Gibson et al., 2014). Few works have focused on implementing the specific manufacturing constraints associated with extrusion-type additive manufacturing in topology optimization frameworks. Optimization of the print path or raster direction was explored by Hoglund and Smith (2016). Here, the raster directions were implemented in density-based topology optimization as discrete orientation variables in each element. Tool path continuity was not considered. Jiang et al. (2019) later extended the algorithm to 3D. Using a level-set framework, Liu and Yu (2017) performed concurrent print-path and topology optimization by building continuous contour-offset tool paths, thus addressing the path continuity constraint. Dapogny et al. (2019) used the contour offset tool paths with a full sensitivity analysis rather than the simplification suggested in Liu and Yu (2017) and Liu and To (2017). Recently, Yu et al. (2020) suggested a level set framework for topology-optimization and raster direction within a set of patches. However, in all these works the distance between deposition paths, and thus the used nozzle size, is not controlled.

The nozzle constraint is in principle relevant for all material extrusion-type additive manufacturing processes. It is less important to consider for extrusion of higher volume fraction designs with a small nozzle compared to the size of the design domain. Here most printing processes lets the nozzle trace the edge of the design and subsequently creates infill. However, the nozzle size constraint is consequential for designs where infill cannot be created; either because the volume fraction of the design is low, or because the nozzle size is large compared to the size of the design domain. This is often the case in large-scale printing technologies such as e.g. concrete 3D printing. Therefore, the current work suggests a density-
based topology optimization framework with a nozzle size constraint embedded through the projection.

In most material extrusion-type processes, the size of the nozzle cannot be altered during the print process and therefore every feature of the realized design will consist of a discrete number of nozzle passes (Gibson et al., 2014). This process is illustrated in Fig. 1a where a 3D view of the print process is shown. The design is here realized by the nozzle placing strokes of deposited material, initially on the print bed, and subsequently on previously deposited layers. Since the nozzle size is here fixed, the nozzle strokes are deposited with a given distance such that they do not overlap.

The nozzle strokes along a 2D slice do not need to be parallel or linear. Strokes made by a nozzle pass can be placed close enough to build up a larger member size, and since the print path can deviate and change direction at any point, a built up member can split. This is illustrated in Fig. 1b that gives a top view of nozzle passes placed on a 2D slice of a design. As seen in the figure, the solid parts of a realized design must consist of an integer number of nozzle passes. For example, if a nozzle that deposits material has size $d_{\text{min,c}} = 2$ units and is printing along the length of a member, then this member can have thickness $t_{\text{member}} = 2$ units, $t_{\text{member}} = 4$ units, $t_{\text{member}} = 6$ units and so on. It is not possible for the member to have a thickness of e.g. 3 units, since the nozzle cannot pass the member and deposit 1 unit thickness.

![Fig. 1 Illustrations of the process commonly used in material extrusion-type additive manufacturing: (a) gives a 3D view of the material deposition from a nozzle of discrete size, and (b) shows how adjacent passes can build up members.](image)

Few works have considered the nozzle constraint in topology optimization algorithms. In Huang et al. (2018a,b) the nozzle size constraint was embedded in a ground-structure truss topology optimization algorithm to design for robotic spacial extrusion. In the recent work by Vantyghem et al. (2020), a 2D topology-optimized design is interpreted to create a 3D design of post-tensioned concrete structure that was subsequently realized by concrete 3D printing. In the post-processing the nozzle size constraint was enforced.

If the discrete nozzle size is not considered in the design algorithm, post-processing is required to realize the design. Figure 2 gives the example of a 2D cantilever beam. The design is obtained with a large minimum length-scale, ensuring a layout with low complexity and mostly uniform member sizes as seen in Fig. 2a. Figure 2b, gives the same structure, overlaid with a large constant nozzle size.
constraint in a post-processing step. If this design is to be realized, e.g. by concrete 3D printing, it is seen that a significant portion of the structural material falls outside the realized beam. In Fig. 2 the computationally evaluated performance (here the compliance $C$) is given before and after post-processing. As expected, the performance is clearly altered for this example. It is generally well known that post-processing can be problematic for topology-optimized designs (see e.g. Jewett and Carstensen (2019) for examples in concrete).

![Fig. 2 Cantilever beam designed without a nozzle size constraint. In (a) the topology obtained using a large minimum length scale ($r_{\text{min,c}} = 2.0$) is shown, and (b) shows a post-processing step that allows printing by a nozzle of radius $r_{\text{nozzle}} = 2.0$.](image)

This paper proposes to include a nozzle size constraint via the projection or filtering operation in a density-based topology optimization framework. The method uses an extension of the Heaviside projection method for Discrete Objects (Ha and Guest, 2014; Guest, 2015) to include print directional primitives in a 2D design. The suggested primitives have two distinct regions that describe (i) the center part of a nozzle pass that cannot overlap with adjacent passes, and (ii) a thin bonding region where overlapping is allowed. The size of both center and bonding regions are chosen by the user and thought to be dependent on the specifics of the used material extrusion process.

The paper is structured as follows; for completeness a short description of the Heaviside projection method for Discrete Object projection is given. This is followed by an extension that allows designing with single-directional nozzle movement. Since most material extrusion-type additive manufacturing processes are characterized by (at least) two-directional nozzle movement along a 2D slice of the design, an extension is subsequently suggested to allow designing with two-directional primitives. Here, nonlinear weighting functions are used to let the magnitudes of the design variables determine in which direction the nozzle is moving.

2 The Heaviside Projection Method for Discrete Objects

Discrete Object Heaviside projection (Ha and Guest, 2014; Guest, 2015) was suggested to design layouts of discrete objects within a matrix, e.g. to include stiff reinforcing features in a compliant material or discrete compliant features within
Discrete Object projection is based on the Heaviside projection method (Guest et al., 2004), in which the design problem is split into two spaces: a design variable space where the optimization is done, and a physical representation (or finite element space) where the equilibrium conditions and structural performance are evaluated. These two spaces are connected through the projection of the design variables onto the finite elements. In the most simple form, the projection is constructed such that an active design variable creates a circular feature in the physical space.

In Discrete Object Heaviside projection (Ha and Guest, 2014), an active design variable creates two features in the physical space as shown in Fig. 3. In Fig. 3a, the spilt of the design problem into two spaces is illustrated. Here the design variable space is indicated by spheres and the physical representation by the grey blocks. The placement of the design variables can be chosen freely (see e.g. Guest and Smith Genut (2010)), and will in this work coincide with the nodal locations of the finite elements. The design variable \( \phi_i \) in Fig. 3 (indicated with a black sphere) is actively projecting and creating two features in the physical representation; a circular feature with radius \( r_{\text{min},c} \) and a concentric enclosure with thickness \( t_E \). A top view is illustrated in Fig. 3b to clearly show the projected shapes. In standard Discrete Object projection (Ha and Guest, 2014) the algorithm is constructed such that \( r_{\text{min},c} \) dictates the size of the discrete objects that are projected and the minimum object spacing is prescribed by the enclosure thickness \( t_E \).

In Ha and Guest (2014), Discrete Object projection is implemented by performing two separate projections of the design variables onto the finite elements. For element \( e \), a projection is performed by defining a neighborhood set for the considered feature. In this work, the neighborhood set for the circular feature is denoted \( N_c^e \) and contains all design variables \( i \) that are located within the object...
radius $r_{\text{min},c}$ of the element centroid $\bar{x}^e$. $N^e_E$ defines the neighborhood containing all design variables within in the concentric enclosure:

$$i \in N^e_c \text{ if } ||x_i - \bar{x}^e|| \leq r_{\text{min},c},$$

$$i \in N^e_E \text{ if } r_{\text{min},c} < ||x_i - \bar{x}^e|| \leq r_{\text{min},c} + t_E,$$

where $x_i$ is the location of design variable $i$.

The mapping onto the finite elements is done separately for each feature type of each element by computing the weighted average or linear filtering (Bruns and Tortorelli, 2001; Bourdin, 2001) of the design variables in the neighborhood set. The weighted averages $\mu^e_c$ and $\mu^e_E$ are expressed as

$$\mu^e_c = \frac{\sum_{i \in N^e_c} w_c(x_i - \bar{x}^e) \cdot \phi_i}{\sum_{i \in N^e_c} w_c(x_i - \bar{x}^e)},$$

$$\mu^e_E = \frac{\sum_{i \in N^e_E} w_E(x_i - \bar{x}^e) \cdot \phi_i}{\sum_{i \in N^e_E} w_E(x_i - \bar{x}^e)}.$$

In Eqs (3-4), $w_c(x_i - \bar{x}^e)$ and $w_E(x_i - \bar{x}^e)$ are herein uniform weighting functions defined as

$$w_c(x_i - \bar{x}^e) = \begin{cases} 1 & \text{if } x_i \in N^e_c \\ 0 & \text{otherwise} \end{cases},$$

$$w_E(x_i - \bar{x}^e) = \begin{cases} 1 & \text{if } x_i \in N^e_E \\ 0 & \text{otherwise} \end{cases}.$$

To obtain binary solutions, the averaged design variables ($\mu^e_c$ and $\mu^e_E$) are passed through a Heaviside function to obtain the feature element densities $\rho^e_c$ and $\rho^e_E$:

$$\rho^e_c = 1 - e^{-\beta \mu^e_c} + \mu^e_c e^{-\beta},$$

$$\rho^e_E = 1 - e^{-\beta \mu^e_E} + \mu^e_E e^{-\beta}.$$

Here $\beta \geq 0$ dictates the curvature of the regularization that approaches the Heaviside function as $\beta$ approaches infinity. The feature densities are therefore bounded by 0 and 1.

Finally, the element densities obtained from the two feature projections are combined to give the element density in the physical representation of the design:

$$\rho^e = \frac{\rho^e_c (2 - \rho^e_E)}{2}.$$

Equation (9) is constructed such that if an element only receives active projection from the circular feature ($\rho^e_c = 1$ and $\rho^e_E = 0$) then the element density will be $\rho^e = 1$ and hence the element will be stiff. Opposite, if an element only receives active projection from the concentric enclosure ($\rho^e_c = 0$ and $\rho^e_E = 1$) then the resulting element density is $\rho^e = 0$ and the element will be of the compliant phase. If an element is not receiving active projecting from either space ($\rho^e_c = 0$ and $\rho^e_E = 0$) the element density will also be $\rho^e = 0$. If an element receives feature projection from both the circular and enclosure feature ($\rho^e_c = 1$ and $\rho^e_E = 1$) it will get an element density of $\rho^e = 0.5$ and become inefficient when a standard stiffness penalization of intermediate densities is used (see section 4.1).
3 Object Projection for Single-Directional Printing

This work proposes to achieve an embedded discrete nozzle size constraint by making the following assumptions; (i) the center part of the extruded material has a discrete member size that cannot be altered or overlap with other nozzle passes, and (ii) a bonding region is present on each side of the non-overlapping center. The bonding region is allowed to overlap with bonding regions created from other passes. The center and bonding regions within the nozzle passes are illustrated in Fig. 4 where the red material is the center non-overlapping part of a nozzle pass whereas the green illustrates the bonding regions.

This work suggests to use an altered form of the enclosure domain from standard Discrete Object projection (Ha and Guest, 2014) to create a bonding zone. If the printing process is primarily associated with the nozzle moving along a single direction, the bonding layer is placed on each side of the non-overlapping feature along this direction.

Figure 5 shows top views of the new projection features for printing processes where the nozzle moves primarily (a) horizontally and (b) vertically. Here, the radius of the circular features \( r_{\text{min,c}} \) describes the non-overlapping size printed by the nozzle and \( t_h = t_b \) or \( t_h = t_v \) gives the bonding thickness allowed on each side. It should be noted that the herein suggested features does not correctly represent a nozzle with the intended diameter at the start and end of a nozzle pass.

Since no changes have been made to the circular feature, the projection that leads to \( \rho_e^c \) is obtained by using Eq.s (1), (3), (5) and (7) as above. The new bonding regions are implemented by changing the enclosure domain neighborhood from Eq. (2):

\[
i \in N_h^c \quad \text{if} \quad \left\| x_i - \left\{ \bar{x}_e^c \pm \bar{y}_e^c \left( r_{\text{min,c}} + \frac{t_b}{2} \right) \right\} \right\| \leq \frac{t_b}{2}, \quad (10)
\]

\[
i \in N_v^c \quad \text{if} \quad \left\| x_i - \left\{ \bar{x}_e^c \pm \bar{y}_e^c \left( r_{\text{min,c}} + \frac{t_v}{2} \right) \right\} \right\| \leq \frac{t_v}{2}, \quad (11)
\]

For single-direction printing, the user picks the one of these neighborhoods that corresponds to the primary printing direction of the considered printer. The chosen neighborhood (Eq. (10) or Eq. (11)) is used to compute the averaged design.
variables in Eq. (4) and passed through the Heaviside function to give the feature densities for the bonding layers \( \rho_e^b = \rho_e^h \) or \( \rho_e^b = \rho_e^v \) by Eq. (8).

In this work, the properties of the bonding layer is assumed to be equal to the properties of the printed material. The non-overlapping requirement of the circular features is therefore embedded by defining the element densities in the physical representation as:

\[
\rho_e^c = \rho_e^c + \rho_e^b - 1 - \rho_e^c \rho_e^b.
\]  (12)

Equation (12) considers an element in the physical space to have full stiffness if either \( \rho_e^c = 1 \) or \( \rho_e^b = 1 \) while the other is zero. This is different from the classic Discrete Object formulation in Eq. (9) where the combination of \( \rho_e^c = 1 \) with \( \rho_e^b = 0 \) results in a soft matrix element stiffness. Additionally, it is worth noting that in this work a void element will be created if both \( \rho_e^c = 0 \) and \( \rho_e^b = 0 \). If both features are actively projecting onto an element, the stiffness will be reduced to 0.5 when using both Eq.s (9) and (12). Since a stiffness penalization method is used herein (see section 4.1), a 0.5 stiffness is uneconomical material use and the optimizer will be encouraged to make changes in the following iterations. A comparison between element densities obtained in the physical representation by using Eq.s (9) and (12) is given in Tab. 1.

4 Solution Algorithm

The proposed algorithm is used to solve the 2D benchmark problems of the cantilever and MBB beams, and the compliant inverter mechanism. All example prob-
Table 1 Combinations of $\rho^c$ and $\rho^E$ or $\rho^b$ and the resulting $\rho^e$ from classic Discrete Object projection (Eq. (9)) and the herein proposed single-directional printing (Eq. (12)).

| Combination | $\rho^c$ | $\rho^E$ or $\rho^b$ | Eq. (9) | Eq. (12) |
|-------------|---------|---------------------|--------|---------|
| a           | 1       | 0                   | 1      | 1       |
| b           | 0       | 1                   | 0      | 1       |
| c           | 1       | 1                   | 0.5    | 0.5     |
| d           | 0       | 0                   | 0      | 0       |

lems for single-directional printing considers the following problem formulation:

\[
\begin{align*}
\text{minimize} \quad & f = L^T d \\
\text{subject to} \quad & K(\phi)d = F \\
& \sum_{e \in \Omega} (\rho^c_e + \rho^b_e) v^e \leq V_{\text{max}} \\
& \phi_{\text{min}} \leq \phi_i \leq \phi_{\text{max}} \quad \forall \ i \in \Omega
\end{align*}
\]  

(13)

Here $\phi_i$ are the independent design variables, $F$ are the nodal forces, $d$ are the nodal displacements and $K$ is the global stiffness matrix. For minimum compliance problems $L = F$, whereas for a minimum displacement objective $L$ is a vector with value one at the displacement(s) of concern and zeros at all other locations. The allowable volume of material within the design domain is denoted $V_{\text{max}}$, and $v^e$ is the volume of element $e$. It is worth noting that the volume constraint differs slightly from the formulation that is typically used. Here, a simple sum of the projected feature densities is used to evaluate the total amount of material use in the design domain.

4.1 Penalization of Intermediate Densities

The Solid Isotropic Material with Penalization (SIMP) method (Bendsøe, 1989) is used to guide the design to a 0–1 solution. Therefore the following expression relates the element stiffness matrices to the topology:

\[
K^e(\phi) = (\rho^0_e + \rho_{\text{min}}) K^e_0.
\]

Here $\eta \geq 1$ is the exponent penalty term, $K^e_0$ is the element stiffness matrix of a pure solid element and $\rho_{\text{min}}$ is a small positive number required to maintain positive definiteness of the global stiffness matrix. In this work $\rho_{\text{min}} = 10^{-3}$ is used.

4.2 Sensitivities

The sensitivities of the objective function are calculated as follows:

\[
\frac{\partial f}{\partial \phi_i} = \sum_{e \in \Omega} \frac{\partial f}{\partial \phi^e_i} \frac{\partial \phi^e_i}{\partial \phi_i}.
\]

(15)
The partial derivative of the objective function $f$ with respect to the element density $\rho^e$ is problem dependent and calculated using the adjoint method. The partial derivative of the element density with respect to the design variables follows the chain rule. The sensitivity of Eq. (12) is

$$\frac{\partial \rho^e}{\partial \phi_i} = (1 - 1.5 \rho^0) \frac{\partial \rho^c}{\partial \phi_i} + (1 - 1.5 \rho^e) \frac{\partial \rho^b}{\partial \phi_i}, \quad (16)$$

where

$$\frac{\partial \rho^c}{\partial \phi_i} = (\beta e^{-\beta \mu^c} + e^{-\beta}) \frac{\partial \mu^c}{\partial \phi_i}, \quad (17)$$

$$\frac{\partial \rho^b}{\partial \phi_i} = (\beta e^{-\beta \mu^b} + e^{-\beta}) \frac{\partial \mu^b}{\partial \phi_i}. \quad (18)$$

The partial derivatives of $\mu^c$ and $\mu^b$ are found by differentiating Eq.s (3-4), respectively.

The sensitivity of the volume constraint is

$$\frac{\partial c}{\partial \phi_i} = \left( \frac{\partial \rho^c}{\partial \phi_i} + \frac{\partial \rho^b}{\partial \phi_i} \right) v^e. \quad (19)$$

4.3 Optimizer

All problems are solved using the Method of Moving Asymptotes (MMA) as the optimization algorithm (Svanberg, 1987). A continuation method is applied to the SIMP exponent penalty to transform the problem from a relaxed, unpenalized state ($\eta = 1$) to the penalized, near discrete formulation. This is common practice in topology optimization as it is known to help avoid convergence to undesirable local minima. Herein, an increment of $\Delta \eta = 0.5$ is used every 50 iterations till $\eta_{\text{max}} = 5.0$. For the cantilever and the MBB beam problems no continuation is applied to the Heaviside parameter (Guest et al., 2011) and a constant value of $\beta = 50$ is taken. The compliant mechanism design problems proved more sensitive and thus a continuation method was also applied to $\beta$, with $\Delta \beta = 1.1^k$ where $k$ is the iteration number and $\beta_{\text{max}} = 50$ as e.g. done in Guest et al. (2004). All problems are solved using four node quadrilateral elements, a uniform initial distribution of material, and design variable bounds of $\phi_{\text{min}} = 0$ and $\phi_{\text{max}} = 1$.

5 Numerical Examples

The design domains for the three example problems considered herein are illustrated in Fig. 6. Symmetry is employed on the MBB, and inverter design problems as shown by the dashed lines to model (and design) only half of the domain. All problems assume plane stress conditions. The material is assumed to be isotropic with Young’s modulus and Poisson’s ratio are chosen as $E = 1.0$ and $\nu = 0.3$, respectively.

The cantilever problem (Fig. 6a) has $L = 40$, $H = 25$ and $P = 1$, whereas the MBB problem (Fig. 6b) has $L = 60$, $H = 20$ and $P = 1$. For both problems, a volume constraint of $V_{\text{max}} = 50\%$ is used.
Fig. 6 Design domains for (a) the cantilever beam-, (b) the MBB beam-, and (c) the compliant mechanism design problem.

The compliant mechanism inverter problem (Fig. 6d) uses the following parameter magnitudes: \( L = 120, P = 1, V_{\text{max}} = 30\%, k_{\text{in}} = 1 \) and \( k_{\text{out}} = 10^{-3} \).

5.1 Cantilever Beams

Figure 7 gives the result obtained for the cantilever beam problem when designing with a predominantly horizontal printing direction and a nozzle that is characterized by \( r_{\text{min,c}} = 1.0 \) and \( t_{\text{h}} = 1.0 \). The design is conducted on a 240 x 150 mesh. In Fig. 7a the density distribution is given as computed by Eq. (12). The elements within the center and bonding regions are seen to achieve full stiffness. However, a small interface of slightly lower density elements can be detected around the dif-
Fig. 7 Cantilever beam designed for a horizontal printing direction with $r_{\text{min,c}} = 1.0$ and $t_h = 1.0$; (a) gives the element density $\rho^e$ obtained from Eq. (12) and used in the finite element evaluation of the design, and in (b) and (c), the feature densities for the center and bonding regions, $\rho^e_c$ and $\rho^e_h$, are given. The design variable $\phi_i$ distribution is shown in (d), and (e) illustrates the required minimum distance between actively projecting design variables to avoid overlapping.
different feature densities. This is an artifact of choosing circular features to describe the bonding regions and is found to be more predominant on finer meshes.

Figures 7b-c gives the feature density distributions $\rho^c_c$ and $\rho^h_c$, respectively. For the non-overlapping nozzle passes in Fig. 7b, the $\rho^c_c$ distribution is as expected seen to be disconnected between passes. The bonding feature regions in Fig. 7c instead show that in $\rho^h_h$ merging of bonding layers is allowed.

Figure 7e gives the design variable distribution obtained for the design. As seen, this distribution gives an indication of the nozzle movement and can be used to trace the path planning. A clear disconnect between passes is seen. The distance between actively projecting design variables in two different passes must be at least $2r_{min,c} + t_h$. This minimum separation distance is required to avoid an element receiving active projection from both feature phases as illustrated in Fig. 7f.

In Fig. 8a the results from Fig. 7 are summarized and compared to a design obtained with the nozzle moving primarily in the vertical direction (Fig. 8b). Here, the center non-overlapping part of a pass is illustrated with red, whereas the bonding regions are indicated with green. The compliance obtained at $\eta_{max}$ are given for both designs. As can be seen by comparing the designs in Fig. 8, the one-way restriction of the nozzle movement significantly affects the design. For the cantilever design case, the horizontally moving nozzle allows an intuitive design of the top and bottom members but is very restrictive for the center part of the design domain. In contrast, a predominantly vertically moving nozzle results in many small nozzle strokes along the top and bottom members. As a result, the achieved compliance is higher for a vertical printing direction than when primarily restricting to a horizontal nozzle movement.

![Fig. 8 Cantilever beams designed with $r_{min,c} = 1.0$ for primarily (a) horizontal printing with $t_h = 1.0$, and (b) vertical printing with $t_v = 1.0$. Here red gives the center non-overlapping region, whereas the bonding regions are illustrated with green.](image)

5.2 MBB Beams

The results of the MBB beam design problem obtained with different nozzle size restrictions and for primarily horizontal and vertical nozzle movement are shown in Figs. 9 and 10, respectively. All results are obtained on a $480 \times 160$ mesh and are
found to fulfill the non-overlapping restrictions. The ratio of the center-to-bonding region is here held constant at $r_{\text{min},c}/t_h = 1.0$ and the feature sizes are varied from small to large. As expected, the designs with obtained with horizontal-directional primitives contain long nozzle strokes along the upper and lower members, but requires many short stroke passes to built up the center region of the MBB beam design domain. The opposite is seen for the designs obtained with vertical-directional primitives. Further, as expected, the complexity of the design is found to decrease when the nozzle size is increased causing a slight elevation of the compliance.

5.3 Compliant Inverter Mechanism

The proposed method allows the designer to specify the ratio of the center-to-bonding region of a nozzle pass. Figure 11 gives the results of the inverter mechanism designed for two different ratios. The total pass width is held constant at $r_{\text{min},c} + t_h = 3.0$ but in (a) $r_{\text{min},c} = 1.0$, whereas in (b) $r_{\text{min},c} = 2.0$. Both de-
signs are obtained for a primarily horizontal nozzle movement and on a $240 \times 120$ mesh. The output displacement $d_{out}$ evaluated at $\eta_{max}$ are given for both cases.

As can be seen by comparison of the two results in Fig. 11, the smaller bonding region used for the design in (b) results in a lower number of connecting bonding interfaces than seen in the design in (a). This is especially caused by the very few strokes needed to make the left half of the design. In the left half of the design domain, the horizontal nozzle movement is suited to capture the design features. However, on the right side of the design in both Fig. 11a and b, the horizontal nozzle movement has problems with creating the diagonal members that connect the output displacement to the rest of the structure. The design in (a) takes advantage of the increased flexibility offered by the larger bonding region and can therefore build this diagonal member up with fewer nozzle strokes than needed for (b) (7 as opposed to 9 in each of the diagonal members on the right). The resulting output displacement is therefore higher in (a) than in (b).
6 Extension to Two-Directional Primitives

As seen in the presented single-directional results, the algorithm with one-directional primitives can easily design members and features along the primary printing and some diagonal directions. However, in the orthogonal direction, the design results are often forced to be built up of many small nozzle strokes. As most material extrusion 3D printing processes can print both horizontally and vertically along the slice of a design, an extension of the formulation to include two-directional primitives is suggested.

The extension to include two-directional primitives is herein suggested by introducing a new set of design variables \( \{ \phi_D \}_i \). These design variables control the printing direction. Figure 12 schematically shows how the problem now has three spaces (instead of the standard two spaces shown in Fig. 3): a printing direction space \( \{ \phi_D \}_i \), a circular object placement space \( \{ \phi_c \}_i \), and the physical representation. The proposed algorithm is constructed such that an active design variable from the circular feature space \( \{ \phi_c \}_i = 1 \) will result in a circular feature in the physical representation. In addition, a bonding layer will be created. The direction of the bonding layer depends on the value of the print direction design variable \( \{ \phi_D \}_i \); if \( \{ \phi_D \}_i = 0 \) a horizontal boundary feature will be placed and \( \{ \phi_D \}_i = 1 \) results in a vertical bonding feature.

The choice of printing direction is implemented using nonlinear weighting functions. The same formulation is used for the nonlinear weighting functions as in Carstensen and Guest (2018) where two-phase minimum and maximum length-scale control was embedded through the projection. The hyperbolic tangent func-
Fig. 12 Schematic illustrations of how two-directional primitives are included by introducing a new design variable space \( \{ \phi_D \} \). When \( \{ \phi_c \} = 1 \), a circular object is placed in the physical representation and a bonding region primitive is placed for (a) horizontal–, and (b) vertical nozzle movement when \( \{ \phi_D \} = 0 \) and \( \{ \phi_D \} = 1 \), respectively.

The nonlinear weighting functions used herein are defined by:

\[
    w_h(\{ \phi_D \}) = \frac{1 + \alpha_h}{1 + \alpha_h \cdot e^{-4 \ln(\alpha_h)\{ \phi_D \}}},
\]

\[
    w_v(\{ \phi_D \}) = \frac{1 + \alpha_v}{1 + \alpha_v \cdot e^{-4 \ln(\alpha_v)\{1 - \phi_D\}}},
\]

where \( \alpha_h \) and \( \alpha_v \) are user defined parameters.

The nonlinear weighting functions are illustrated in Fig. 13, where it is seen that a low value of \( \{ \phi_D \} \) will result in \( w_h(\{ \phi_D \}) \to 1 \) and \( w_v(\{ \phi_D \}) \to 0 \). The
algorithm is constructed such that this combination gives a horizontal bonding feature in the physical space. Similarly, if \( \{\phi_D\}_i \) is close to 1 then \( w_h(\{\phi_D\}_i) \to 0 \) and \( w_v(\{\phi_D\}_i) \to 1 \) and the resulting feature in the physical space will be vertical.

![Fig. 13 Nonlinear weighting functions as defined by Eq.s (20-21) with \( \alpha_h = \alpha_v = 0.15 \).](image)

The extension is numerically implemented by having three separate projections on to the finite elements; one for the circular feature, and for the horizontal and vertical bonding layers, respectively. The averaged design variables for the circular feature projection only depends on the circular design variables \( \{\phi_c\}_i \) and is calculated as follows:

\[
\mu_e^c = \frac{\sum_{i \in N_e^c} w(x_i - \bar{x}^e) \cdot \{\phi_c\}_i}{\sum_{i \in N_e^c} w(x_i - \bar{x}^e)}
\]  

Since a bonding layer can only be created around a circular feature, the averaged design variables for the horizontal and vertical bonding layers depends on both design variable spaces. They are defined using nonlinear weighting functions from Eq.s (20-21) as

\[
\mu_h^e = \frac{\sum_{i \in N_h} w(x_i - \bar{x}^e) \cdot \{\phi_c\}_i \cdot w_h(\{\phi_D\}_i)}{\sum_{i \in N_h} w(x_i - \bar{x}^e)}
\]  
\[
\mu_v^e = \frac{\sum_{i \in N_v} w(x_i - \bar{x}^e) \cdot \{\phi_c\}_i \cdot w_v(\{\phi_D\}_i)}{\sum_{i \in N_v} w(x_i - \bar{x}^e)}
\]

The averaged design variables are passed through the Heaviside function in a standard manner (Eq. (7)) to give the feature densities \( \rho_e^c \), \( \rho_h^e \) and \( \rho_v^e \), respectively. The feature densities are combined to give the element density using the following expression:

\[
\rho^e = \rho_e^c + \rho_h^e + \rho_v^e - 1.5 \rho_e^c (\rho_h^e + \rho_v^e).
\]  

The element density in Eq. (25) is constructed such that an element \( e \) will have full stiffness if only one of the feature densities is active (\( \rho_e^c = 1 \) or \( \rho_h^e = 1 \)).
or $\rho^c = 1$) while the others are 0. If both the circular feature density and one of the bonding layer densities are 1, then the element will have reduced stiffness that is inefficient when used in combination with SIMP. Table 2 gives the different combinations of the feature densities and the resulting element density in the physical representation obtained by Eq. (25).

Table 2 Combinations of $\rho^c$ and the resulting $\rho^e$ from Eq. (25).

| Combination | $\rho^c$ | $\rho^c_h$ | $\rho^c_v$ | $\rho^e$ |
|-------------|---------|-----------|-----------|---------|
| a           | 1       | 0         | 0         | 1       |
| b           | 0       | 1         | 0         | 1       |
| c           | 0       | 0         | 1         | 1       |
| d           | 1       | 1         | 0         | 0.5     |
| e           | 1       | 0         | 0.5       | 0.5     |
| f           | 0       | 0         | 0         | 0       |

6.1 Sensitivities

The sensitivities of the element densities in Eq. (25) with respect to the two design variable spaces $\{\phi_D\}_i$ and $\{\phi_c\}_i$ are calculated separately and follows the chain rule.

$$
\frac{\partial \rho^c}{\partial \{\phi_c\}_i} = (1 - 1.5 \rho^c_h + \rho^c_v) \frac{\partial \rho^c_h}{\partial \{\phi_c\}_i} + (1 - 1.5 \rho^c_v) \left( \frac{\partial \rho^c_h}{\partial \{\phi_D\}_i} + \frac{\partial \rho^c_v}{\partial \{\phi_c\}_i} \right)
$$

$$
\frac{\partial \rho^e}{\partial \{\phi_D\}_i} = (1 - 1.5 \rho^c) \left( \frac{\partial \rho^c_h}{\partial \{\phi_D\}_i} + \frac{\partial \rho^c_v}{\partial \{\phi_D\}_i} \right)
$$

For the circular features the partial derivative of the feature density with respect to $\{\phi_c\}_i$ is calculated as above. For the bonding features, the following is used:

$$
\frac{\partial \rho^c_h}{\partial \{\phi_c\}_i} = \frac{\partial \rho^c_h}{\partial \mu^c_h} \frac{\partial \mu^c_h}{\partial \{\phi_c\}_i},
$$

$$
\frac{\partial \rho^c_v}{\partial \{\phi_c\}_i} = \frac{\partial \rho^c_v}{\partial \mu^c_v} \frac{\partial \mu^c_v}{\partial \{\phi_c\}_i},
$$

where the partial derivatives $\frac{\partial \rho^c_h}{\partial \mu^c_h}$ and $\frac{\partial \rho^c_v}{\partial \mu^c_v}$ are as in Eq. (18). The partial derivatives of the averaged design variables with respect to $\{\phi_c\}_i$ are found by differentiation of Eqs. (23-24):

$$
\frac{\partial \mu^c_h}{\partial \{\phi_c\}_i} = \frac{w(x_i - \bar{x}_c)}{\sum_{i \in N_h^c} w(x_i - \bar{x}_c)} \cdot w_h(\{\phi_D\}_i),
$$

$$
\frac{\partial \mu^c_v}{\partial \{\phi_c\}_i} = \frac{w(x_i - \bar{x}_c)}{\sum_{i \in N_v^c} w(x_i - \bar{x}_c)} \cdot w_v(\{\phi_D\}_i).
$$
Similarly, the partial derivatives of the bonding feature densities with respect to \( \{ \phi_D \} \), are calculated by

\[
\frac{\partial \rho_h}{\partial \{ \phi_D \}_i} = \frac{\partial \rho_h}{\partial \mu_h} \cdot \frac{\partial \mu_h}{\partial \{ \phi_D \}_i},
\]

(32)

\[
\frac{\partial \rho_v}{\partial \{ \phi_D \}_i} = \frac{\partial \rho_v}{\partial \mu_v} \cdot \frac{\partial \mu_v}{\partial \{ \phi_D \}_i}.
\]

(33)

Here the partial derivatives of the averaged design variables with respect to \( \{ \phi_D \} \) are

\[
\frac{\partial \mu_h}{\partial \{ \phi_D \}_i} = \frac{w(x_i - \bar{x})}{\sum_{i \in N_i} w(x_i - \bar{x})} \cdot \{ \phi_c \}_i \cdot \frac{\partial w_h(\{ \phi_D \}_i)}{\partial \{ \phi_D \}_i},
\]

(34)

\[
\frac{\partial \mu_v}{\partial \{ \phi_D \}_i} = \frac{w(x_i - \bar{x})}{\sum_{i \in N_i} w(x_i - \bar{x})} \cdot \{ \phi_v \}_i \cdot \frac{\partial w_v(\{ \phi_D \}_i)}{\partial \{ \phi_D \}_i}.
\]

(35)

The sensitivity of the nonlinear weighting functions are found by differentiation of Eq.s (20-21) as

\[
\frac{\partial w_h(\{ \phi_D \}_i)}{\partial \{ \phi_D \}_i} = \frac{1 + \alpha_h}{(1 + \alpha_h \cdot e^{-4 \ln(\alpha_h)\{ \phi_D \}_i})^2} \cdot 4 \alpha_h \ln(\alpha_h) e^{-4 \ln(\alpha_h)\{ \phi_D \}_i},
\]

(36)

\[
\frac{\partial w_v(\{ \phi_D \}_i)}{\partial \{ \phi_D \}_i} = \frac{1 + \alpha_v}{(1 + \alpha_v \cdot e^{-4 \ln(\alpha_v)\{ \phi_D \}_i})^2} \cdot (-4) \alpha_v \ln(\alpha_v) e^{-4 \ln(\alpha_v)\{ \phi_D \}_i}.
\]

(37)

6.2 Numerical Examples

All example problems with two-directional primitives considers the following problem formulation:

\[
\begin{align*}
\text{minimize } & \quad f = L^T d \\
\text{subject to } & \quad K(\{ \phi_c \}, \{ \phi_v \}) d = F \nonumber \\
& \quad \sum_{\mu \in \Omega} (\rho_c + \rho_c + h_v) v^{\mu} \leq V_{\text{max}} \nonumber \\
& \quad \phi_{\text{min}} \leq \{ \phi_c \}_i \leq \phi_{\text{max}} \quad \forall i \in \Omega \nonumber \\
& \quad \phi_{\text{min}} \leq \{ \phi_D \}_i \leq \phi_{\text{max}} \quad \forall i \in \Omega.
\end{align*}
\]

Equation (38) is solved using the same continuation and Heaviside parameters as implemented for the single-directional designs. However, it is found that crips results are more easily obtained when the maximum SIMP exponent penalty is driven to \( \eta_{\text{max}} \approx 1.0 \).

The user-defined parameters \( \alpha_h \) and \( \alpha_v \) in the nonlinear weighting functions (Eq.s (20-21)) are for all examples chosen as \( \alpha_h = \alpha_v = 0.15 \). This value is found to work well on all tested problems. However, the algorithm is found to be sensitive to the choice of \( \alpha_h \) and \( \alpha_v \). Generally, if the chosen values are very small the algorithm will not place the boundary layer around features that does not connect members. For low \( \alpha \)-values, the boundary region is e.g. seen to be omitted around all diagonal members in cantilever designs. Opposite, if the magnitudes of \( \alpha_h \) and
Fig. 14: Cantilever beam designed with the new two-direction nozzle primitives. (a) gives the combined result where red is the non-overlapping center of a nozzle pass and green is the bonding region. The design variable distributions are given in (b) as \( \{\phi_c\} \), and (c) as \( \{\phi_D\} \). The element density distribution computed by Eq. (25) is shown in (d). The feature density distribution for the non-overlapping center \( (\rho_e^c) \) is shown in (e) and in (f) and (g) the feature density distributions are given for the horizontal \( (\rho_e^h) \) and vertical \( (\rho_e^v) \) bonding features, respectively.
If $\alpha$ are too large, the algorithm will actively place both horizontal and vertical features from an active design variable.

Figure 14 shows the result obtained for the cantilever beam with $r_{\text{min},c} = 1.0$ and $t_b = 1.0$. In (a) the structure is illustrated with red showing the non-overlapping region and green illustrating the bonding regions. The result is seen to fulfill the nozzle size restrictions while allowing both horizontal and vertical nozzle movement. The boundary region is present around the center nozzle pass at most locations throughout the design domain. However, it is seen to be mission at curves and sharp corners. The current construction of the algorithm does not allow for a totally free nozzle movement in all directions since it is restricted to use only two directional primitives. Including additional primitives in future works can possibly alleviate this problem.

In Fig. 14b-c the design variable distributions for the placement ($\{\phi_c\}_i$) and direction ($\{\phi_D\}_i$) spaces are shown. The placement space design variable distribution in (b) is seen to be a 0–1 distribution that depicts the printing path. Opposite, the design variable distribution that controls the bonding feature direction in (c) is not 0–1 but contains large regions where the design variables take a mid-range value. This is in fact mostly the case in regions of the design space where $\{\phi_c\}_i$ is inactive. The construction of the algorithm ensures that the two design variable spaces result in 0–1 feature density distributions as shown in Fig. 14e-g. These combine to a 0–1 element density distribution by using Eq. (25) which is shown in Fig. 14d.

To compare the performance of the proposed algorithms with single- and two-directional primitives, Fig. 15 gives the convergence histories for all cantilever beam designs with $r_{\text{min},c} + t_b = 2.00$ presented in the current paper. In addition, Tab. 3 lists the performances and measures of non-discreteness and computational time. As seen in Fig. 15 stable convergences is achieved for all designs. Since $\eta$-continuation to a higher $\eta_{\text{max}}$ (7.0 as opposed to 5.0) is found to work better in the two-directional primitive framework, the used number of iterations is as anticipated seen to be higher for this case.

![Fig. 15](image.png)

Fig. 15 Convergence histories for cantilever beams designed with $r_{\text{min},c} + t_b = 2.00$ prescribed by; a minimum length-scale requirement (Fig. 2a), horizontal (Fig. 8a) and vertical (Fig. 8b) print primitives, and with two-directional primitives (Fig. 14a).
As expected, it is clear from the plot in Fig. 15 that the lowest compliance is obtained for a design without nozzle restrictions. This is also seen by the compliance values listed in Tab. 3. To ease comparison, the compliance is evaluated for all final designs at $\eta = 5.0$. It is seen that implementing the nozzle restrictions causes the compliance to increase. The highest compliance is achieved for single-directional design with vertical nozzle movement. The implementation of two-directional nozzle movement lowers the compliance and achieves a better performance than the manual post-processing from Fig. 2b. However, the cost of using the proposed frameworks are increases in non-discreteness of the solutions (the percentage of intermediate or ‘grey’ elements) and in computational time. In Tab. 3, the non-discreteness is evaluated with a tolerance of $\text{tol} = 0.05$, and the computational time is normalized by the time used to achieve the conventional solution. Implementation of the single directional primitives is found to cause a computational time increase of about 30%. The computational time required for the two-directional framework is as expected seen to be much higher as it requires doubling the number of design variables.

### Table 3

Comparision of compliance, non-discreteness (‘grey’ elements) and computation time for cantilever results obtained with $r_{\text{min},c} + t_b = 2.00$. The ‘grey’ elements are evaluated with $\text{tol} = 0.05$ and the computational time is normalized to the conventional solution.

| Print Primitives | Fig. | $r_{\text{min},c}$ | $t_b$ | $C (\eta = 5)$ | ‘Grey’ Elements | Time Ratio |
|------------------|------|-------------------|-------|----------------|----------------|------------|
| N/A              | 2a   | 2.00              | –     | 47.94          | 11%            | 1.00       |
| N/A post-processed | 2b   | 2.00              | –     | 56.42          | 16%            | –          |
| Horizontal       | 8a   | 1.00              | 1.0   | 56.48          | 13%            | 1.28       |
| Vertical         | 8b   | 1.00              | 1.0   | 60.41          | 19%            | 1.30       |
| Two-directional  | 14a  | 1.00              | 1.0   | 56.00          | 17%            | 3.72       |

The results of the MBB beams designed with two-directional primitives is given in Fig. 16. Here the ratio of $r_{\text{min},c}/t_b$ is held constant at 1.0 and the size of the used primitives is varied. For all designs, the nozzle size restrictions are fulfilled. Again, the algorithm has some difficulty placing the bonding region of diagonal members with an angle close to 45°. As a consequence, these diagonal members will either be built up of a sequence of horizontal and vertical nozzle strokes (see e.g. center members of Fig. 16c) or the bonding region will be omitted.

Figure 17 gives the two-directional compliant inverter designs. The designs are obtained with $r_{\text{min},c} + t_b = 3.00$ where in (a) $r_{\text{min},c} = 1.00$, and in (b) $r_{\text{min},c} = 2.00$. Both designs fulfill the nozzle size restrictions. It is interesting to note, that the flexibility afforded by the larger bonding region in (a) results in diagonal members built up by a higher number of horizontal-vertical primitive sequences than observed in (b).
Fig. 16 MBB beams designed with two-directional primitives with \( r_{\text{min},c}/t_b = 1.00 \) where (a) \( r_{\text{min},c} = t_b = 0.25 \), (b) \( r_{\text{min},c} = t_b = 0.50 \), and (c) \( r_{\text{min},c} = t_b = 0.75 \).

7 Conclusions

A technique is proposed for restricting member sizes in topology optimization to conform to a discrete number of nozzle passes as imposed by a material extrusion-type additive manufacturing process. The deposited material is assumed to consist of a non-overlapping core and bonding regions where overlapping of adjacent nozzle strokes is permitted. The material properties of these two regions is assumed to be the same. The proposed framework allows the designer to prescribe length scales that conforms with the specifics of the extrusion-type additive manufacturing process intended for fabrication of a design. The proposed approach thus relies on additive manufacturing engineers to identify the size of the bonding regions associated with material deposition processes.

If the used manufacturing technology mainly allows nozzle movement along a single main direction, a single-primitive framework is suggested. Here, a single set of design variables are independently projected to create both center and bonding regions in the physical representation. The obtained designs fulfill the nozzle size restrictions and are found to replicate known benchmark solutions well along their main nozzle movement direction. However, the designs are also seen to require a large number of small nozzle strokes for members that tend to be found in the
opposite direction. This tendency causes an increase the non-linearity of the design space and can hence require some tuning of initial parameters.

Additionally, an algorithm with two-directional primitives is suggested for manufacturing processes that allow a more free nozzle movement. A new set of design variables is defined and the magnitude of these determine the direction of the print primitive. Since the number of design variables is larger, a higher computational cost is observed. The algorithm is also somewhat sensitive to parameter choices. However, due to the added freedom in the nozzle movement, the tests performed herein indicates that quality solutions are more easily obtained when designing with two primitives as opposed to one.

Despite this preliminary success, the proposed approach does have two primary disadvantages in that it does not consider the anisotropy nor the continuity constraint which are both highly relevant for material extrusion-type additive manufacturing. Although not addressed in the current work, it is interesting to note that designing with two primitives typically results in more continuity than observed when designing with one. Generally, a smaller number of nozzle strokes are found in the two-directional primitive solutions than in the single-primitive results. Moreover, the number of strokes needed to realize a design is herein found to decrease with the size of the bonding features. It is speculated that in future extensions of the current work, the continuity constraint can be further implicitly addressed by prescribing a lower stiffness in bonding regions as opposed to the stiffness in core of a nozzle stroke. An extension to design with two materials (as e.g. in Bendsoe and Sigmund (1999); Gaynor et al. (2012); Watts and Tortorelli (2016)) will possibly encourage the optimizer to place the center of a nozzle pass along the main load paths and hence result in more continuous features.
Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Replication of Results

The paper contains all details needed to replicate the presented results.

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