Nearly Tri-Bimaximal Neutrino Mixing and CP Violation

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Abstract

We point out two simple but instructive possibilities to modify the tri-bimaximal neutrino mixing ansatz, such that leptonic CP violation can naturally be incorporated into the resultant scenarios of nearly tri-bimaximal flavor mixing. The consequences of two new ansätze on solar, atmospheric and reactor neutrino oscillations are analyzed. We also discuss an interesting approach to construct lepton mass matrices under permutation symmetry, from which one may derive another nearly tri-bimaximal neutrino mixing scenario with no intrinsic CP violation in neutrino oscillations.

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1 Introduction

The atmospheric and solar neutrino oscillations observed in the Super-Kamiokande experiment have provided robust evidence that neutrinos are massive and lepton flavors are mixed. All analyses of the atmospheric neutrino deficit favor $\nu_\mu \to \nu_\tau$ as the dominant oscillation mode with the mass-squared difference $\Delta m^2_{\text{atm}} \sim 10^{-3} \text{eV}^2$ and the mixing factor $\sin^2 2\theta_{\text{atm}} > 0.85$ at the 99% confidence level. In addition, the present Super-Kamiokande and SNO data indicate that the solar neutrino anomaly is most likely attributed to the matter-enhanced $\nu_e \to \nu_\mu$ oscillation via the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism with $\Delta m^2_{\text{sun}} \sim 10^{-5} \text{eV}^2$ and $\sin^2 2\theta_{\text{sun}} \sim 0.6 - 0.98$ at the 3$\sigma$ confidence level (large-angle MSW solution). The strong hierarchy between $\Delta m^2_{\text{atm}}$ and $\Delta m^2_{\text{sun}}$, together with the small $\nu_\tau$-component in $\nu_e$ configuration restricted by the CHOOZ reactor neutrino oscillation experiment, implies that atmospheric and solar neutrino oscillations decouple approximately from each other. Each of them is dominated by a single mass scale, which can be set as $\Delta m^2_{\text{sun}} \equiv |m^2_2 - m^2_1|$ or $\Delta m^2_{\text{atm}} \equiv |m^2_3 - m^2_2|$. The mixing factors of solar, atmospheric and CHOOZ neutrino oscillations are simply given by

$$\begin{align*}
\sin^2 2\theta_{\text{sun}} &= 4|V_{e1}|^2|V_{e2}|^2, \\
\sin^2 2\theta_{\text{atm}} &= 4|V_{\mu3}|^2 \left(1 - |V_{\mu3}|^2\right), \\
\sin^2 2\theta_{\text{chz}} &= 4|V_{\mu3}|^2 \left(1 - |V_{\mu3}|^2\right),
\end{align*}$$

where $V$ is the $3 \times 3$ lepton flavor mixing matrix linking the neutrino mass eigenstates ($\nu_1, \nu_2, \nu_3$) to the neutrino flavor eigenstates ($\nu_e, \nu_\mu, \nu_\tau$). As current experimental data favor $\sin^2 2\theta_{\text{chz}} \ll \sin^2 2\theta_{\text{sun}} \sim \sin^2 2\theta_{\text{atm}} \sim \mathcal{O}(1)$, two large flavor mixing angles can be drawn from Eq. (1) in a specific parametrization of $V$: one between the 2nd and 3rd lepton families and the other between the 1st and 2nd lepton families.

So far a number of phenomenological ansätze of lepton flavor mixing with two large rotation angles, including the “democratic” ansatz and the “bimaximal” ansatz, have been proposed and discussed. In this paper we pay our particular attention to a new ansatz of the form (up to a trivial sign or phase rearrangement)

$$V_0 = \begin{pmatrix}
\frac{\sqrt{2}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix},$$

proposed recently by Harrison, Perkins and Scott. This so-called “tri-bimaximal” flavor mixing pattern predicts $\sin^2 2\theta_{\text{atm}} = 1$ and $\sin^2 2\theta_{\text{sun}} = 8/9$, consistent very well with the atmospheric neutrino oscillation data and the large-angle MSW solution to the solar neutrino problem. However, it leads also to $\sin^2 2\theta_{\text{chz}} = 0$, implying the absence of both high-energy matter resonances and intrinsic CP violation in neutrino oscillations.

The main purpose of this paper is to discuss two simple but instructive possibilities to modify the tri-bimaximal neutrino mixing pattern in Eq. (2), such that CP violation can naturally be incorporated into the resultant scenarios of nearly tri-bimaximal flavor mixing. Two specific textures of the charged lepton mass matrix are taken into account, in order to obtain small but non-vanishing $|V_{e3}|$ or $\sin^2 2\theta_{\text{chz}}$. We find that two new ansätze have practically indistinguishable consequences on the atmospheric neutrino oscillation, but their predictions for $\sin^2 2\theta_{\text{sun}}$, $\sin^2 2\theta_{\text{chz}}$ and leptonic CP violation are rather different. We
also discuss an interesting approach to construct lepton mass matrices under permutation symmetry, from which one may derive another nearly tri-bimaximal neutrino mixing scenario with $|V_{e3}| \neq 0$ but with no intrinsic CP violation in neutrino oscillations.

2 Nearly tri-bimaximal neutrino mixing

The fact that masses of three active neutrinos are extremely small is presumably attributed to the Majorana nature of neutrino fields [9]. In this picture, the light (left-handed) neutrino mass matrix $M_\nu$ must be symmetric and can be diagonalized by a single unitary transformation:

$$U_\nu^\dagger M_\nu U_\nu^* = \text{Diag} \{ m_1, m_2, m_3 \} .$$ (3)

The charged lepton mass matrix $M_l$ is in general non-Hermitian, hence the diagonalization of $M_l$ needs a bi-unitary transformation:

$$U_\ell^\dagger M_l \tilde{U}_\ell = \text{Diag} \{ m_e, m_\mu, m_\tau \} .$$ (4)

The lepton flavor mixing matrix $V$, defined to link the neutrino mass eigenstates ($\nu_1, \nu_2, \nu_3$) to the neutrino flavor eigenstates ($\nu_e, \nu_\mu, \nu_\tau$), measures the mismatch between the diagonalization of $M_l$ and that of $M_\nu$: $V = U_\nu^\dagger U_\ell$. Note that $(m_1, m_2, m_3)$ in Eq. (3) and $(m_e, m_\mu, m_\tau)$ in Eq. (4) are physical (real and positive) masses of light neutrinos and charged leptons, respectively.

In the flavor basis where $M_l$ is diagonal (i.e., $U_\ell = 1$ being a unity matrix), the flavor mixing matrix is simplified to $V = U_\nu$. The tri-bimaximal neutrino mixing pattern $U_\nu = V_0$ can then be constructed from the product of two Euler rotation matrices:

$$R_{12}(\theta_x) = \begin{pmatrix} c_x & s_x & 0 \\ -s_x & c_x & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$R_{23}(\theta_y) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_y & s_y \\ 0 & -s_y & c_y \end{pmatrix},$$ (5)

where $s_x \equiv \sin \theta_x$, $c_y \equiv \cos \theta_y$, and so on. Taking $\theta_x = \arctan(1/\sqrt{2}) \approx 35.3^\circ$ and $\theta_y = 45^\circ$, we obtain

$$V_0 = R_{23}(\theta_y) \otimes R_{12}(\theta_x)$$

$$= \begin{pmatrix} \sqrt{2} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & \sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} .$$ (6)

The vanishing of the (1,3) element in $V_0$ assures an exact decoupling between solar ($\nu_e \to \nu_\mu$) and atmospheric ($\nu_\mu \to \nu_\tau$) neutrino oscillations. The corresponding neutrino mass matrix $M_\nu$ takes the form

$$M_\nu = V_0 \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V_0^T.$$
\[
\begin{pmatrix}
A_\nu - B_\nu - C_\nu & C_\nu & -C_\nu \\
C_\nu & A_\nu & B_\nu \\
-C_\nu & B_\nu & A_\nu
\end{pmatrix},
\]

(7)

where

\[
A_\nu = \frac{m_3^2 + m_1 + 2m_2}{6},
\]

\[
B_\nu = \frac{m_3^2 - m_1 + 2m_2}{6},
\]

\[
C_\nu = \frac{m_2 - m_1}{3}.
\]

(8)

If \(m_1 \approx m_2\) holds, one may arrive at a simpler texture of \(M_\nu\) with

\[
A_\nu \approx \left(\frac{m_3 + m_1}{2}\right),
\]

\[
B_\nu \approx \left(\frac{m_3 - m_1}{2}\right)\quad \text{and} \quad C_\nu \approx 0.
\]

Let us comment briefly on the mathematical structure of \(M_\nu\) obtained in Eq. (7). Indeed \(M_\nu\) can be decomposed as

\[
M_\nu = A_\nu I_A + B_\nu I_B + C_\nu I_C,
\]

(9)

where

\[
I_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

\[
I_B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},
\]

\[
I_C = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}.
\]

(10)

Such a structure of the neutrino mass matrix is very similar to that giving rise to the bimaximal flavor mixing \([10]\). Note that the diagonalization of \(M_\nu\) requires an orthogonal matrix which is able to diagonalize \(I_B\) and \(I_C\) simultaneously. This orthogonal matrix is just \(V_0\) given in Eq. (6). Although the decomposition of \(M_\nu\) shown above is by no means unique, it might have a meaningful interpretation in an underlying theory of neutrino masses with specific flavor symmetries.

The tri-bimaximal neutrino mixing pattern will be modified, if \(U_l\) deviates somehow from the unity matrix. This can certainly happen, provided that the charged lepton mass matrix \(M_l\) is not diagonal in the flavor basis where the neutrino mass matrix \(M_\nu\) takes the form given in Eq. (7). As \(U_\nu = V_0\) describes a product of two special Euler rotations in the real (2,3) and (1,2) planes, the simplest form of \(U_l\) which allows \(V = U_\nu^\dagger U_l\) to cover the whole 3 × 3 space should be \(U_l = R_{12}(\theta_x)\) or \(U_l = R_{31}(\theta_z)\) (see Ref. \([11]\) for a detailed discussion).

To make CP violation incorporated into \(V\), we adopt the complex rotation matrices:

\[
R_{12}(\theta_x, \phi_x) = \begin{pmatrix} c_x & s_x e^{i\phi_x} & 0 \\ -s_x e^{-i\phi_x} & c_x & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

\[
R_{31}(\theta_z, \phi_z) = \begin{pmatrix} c_z & 0 & s_z e^{i\phi_z} \\ 0 & 1 & 0 \\ -s_z e^{-i\phi_z} & 0 & c_z \end{pmatrix}.
\]

(11)
In this case, we arrive at lepton flavor mixing of the pattern

\[
V_{(x)} = R_{12}^{\dagger}(\theta_x, \phi_x) \otimes V_0
\]

\[
= \left( \begin{array}{ccc}
\frac{1}{\sqrt{6}} (2c_x + s_x e^{i\phi_x}) & \frac{1}{\sqrt{3}} (c_x - s_x e^{i\phi_x}) & -\frac{1}{\sqrt{2}} s_x e^{i\phi_x} \\
\frac{1}{\sqrt{6}} (c_x - 2s_x e^{-i\phi_x}) & \frac{1}{\sqrt{3}} (c_x + s_x e^{-i\phi_x}) & -\frac{1}{\sqrt{6}} c_x \\
\frac{1}{\sqrt{6}} (s_x + 2c_x e^{-i\phi_x}) & -\frac{1}{\sqrt{3}} (c_x - s_x e^{-i\phi_x}) & \frac{1}{\sqrt{2}} c_x \\
\end{array} \right),
\]  

(12)

or of the pattern

\[
V_{(z)} = R_{31}^{\dagger}(\theta_z, \phi_z) \otimes V_0
\]

\[
= \left( \begin{array}{ccc}
\frac{1}{\sqrt{6}} (2c_z - s_z e^{i\phi_z}) & \frac{1}{\sqrt{3}} (c_z + s_z e^{i\phi_z}) & -\frac{1}{\sqrt{2}} s_z e^{i\phi_z} \\
\frac{1}{\sqrt{6}} (c_z - 2s_z e^{-i\phi_z}) & \frac{1}{\sqrt{3}} (c_z + s_z e^{-i\phi_z}) & -\frac{1}{\sqrt{6}} c_z \\
\frac{1}{\sqrt{6}} &(s_z + 2c_z e^{-i\phi_z}) & \frac{1}{\sqrt{2}} c_z \\
\end{array} \right).
\]  

(13)

It is obvious that \(V_{(x)}\) and \(V_{(z)}\) represent two nearly tri-bimaximal flavor mixing scenarios, if the rotation angles \(\theta_x\) and \(\theta_z\) are small in magnitude. The complex phase \(\phi_x\) in \(V_{(x)}\) or \(\phi_z\) in \(V_{(z)}\) is the source of leptonic CP violation in neutrino oscillations.

### 3 Constraints on mixing factors and CP violation

As the mixing angle \(\theta_x\) or \(\theta_z\) arises from the diagonalization of \(M_l\), it is expected to be a simple function of the ratios of charged lepton masses. Then the strong mass hierarchy of charged leptons naturally assures the smallness of \(\theta_x\) or \(\theta_z\), as one can see later on.

Indeed a proper texture of \(M_l\) which may lead to the flavor mixing pattern \(V_{(x)}\) is

\[
M_{(x)}^l = \begin{pmatrix}
0 & C_l & 0 \\
C_l^* & B_l & 0 \\
0 & 0 & A_l
\end{pmatrix},
\]

(14)

where \(A_l = m_\tau, B_l = m_\mu - m_e,\) and \(C_l = \sqrt{m_e m_\mu} e^{i\phi_x}\). The mixing angle \(\theta_x\) in \(V_{(x)}\) is then given by

\[
\tan(2\theta_x) = 2 \frac{\sqrt{m_e m_\mu}}{m_\mu - m_e}.
\]

(15)

On the other hand, a proper texture of \(M_l\) which may give rise to the mixing pattern \(V_{(z)}\) reads as follows:

\[
M_{(z)}^l = \begin{pmatrix}
0 & C_l & 0 \\
0 & B_l & 0 \\
C_l^* & 0 & A_l
\end{pmatrix},
\]

(16)

where \(A_l = m_\tau - m_e, B_l = m_\mu,\) and \(C_l = \sqrt{m_e m_\tau} e^{i\phi_z}\). The mixing angle \(\theta_z\) in \(V_{(z)}\) turns out to be

\[
\tan(2\theta_z) = 2 \frac{\sqrt{m_e m_\tau}}{m_\tau - m_e}.
\]

(17)

In view of the hierarchy of three charged lepton masses (i.e., \(m_e \ll m_\mu \ll m_\tau\)), we obtain \(s_x \approx \sqrt{m_e/m_\mu}\) and \(s_z \approx \sqrt{m_e/m_\tau}\) to a good degree of accuracy. Numerically, we find \(\theta_x \approx 3.978^\circ\) and \(\theta_z \approx 0.972^\circ\) by using the inputs \(m_e = 0.511\) MeV, \(m_\mu = 105.658\) MeV, and \(m_\tau = 1.777\) GeV [12].
Now let us calculate the mixing factors of solar, atmospheric and reactor neutrino oscillations. With the help of Eqs. (1) and (12) or (13), we arrive straightforwardly at

\[
\sin^2 2\theta_{\text{sun}}^{(x)} = \frac{8}{9} \left( 1 - \frac{3}{4} s_x^2 - s_x c_x \cos \phi_x + \frac{3}{2} s_x^3 c_x \cos \phi_x - 2 s_x^2 c_x^2 \cos^2 \phi_x \right),
\]
\[
\sin^2 2\theta_{\text{atm}}^{(x)} = 1 - s_x^4,
\]
\[
\sin^2 2\theta_{\text{chz}}^{(x)} = 1 - c_x^4;
\]

and

\[
\sin^2 2\theta_{\text{sun}}^{(z)} = \frac{8}{9} \left( 1 - \frac{3}{4} s_z^2 + s_z c_z \cos \phi_z - \frac{3}{2} s_z^3 c_z \cos \phi_z - 2 s_z^2 c_z^2 \cos^2 \phi_z \right),
\]
\[
\sin^2 2\theta_{\text{atm}}^{(z)} = 1,
\]
\[
\sin^2 2\theta_{\text{chz}}^{(z)} = 1 - c_z^4.
\]

Allowing \( \phi_x \) and \( \phi_z \) to take arbitrary values, we find that the minimal and maximal magnitudes of \( \sin^2 2\theta_{\text{sun}}^{(x)} \) and \( \sin^2 2\theta_{\text{sun}}^{(z)} \) are

\[
\left[ \sin^2 2\theta_{\text{sun}}^{(x)} \right]_{\min} = \frac{8}{9} \left( 1 - \frac{3}{4} s_x^2 - s_x c_x + \frac{3}{2} s_x^3 c_x - 2 s_x^2 c_x^2 \right),
\]
\[
\left[ \sin^2 2\theta_{\text{sun}}^{(x)} \right]_{\max} = \frac{8}{9} \left( 1 - \frac{3}{4} s_x^2 + s_x c_x - \frac{3}{2} s_x^3 c_x - 2 s_x^2 c_x^2 \right);
\]

and

\[
\left[ \sin^2 2\theta_{\text{sun}}^{(z)} \right]_{\min} = \frac{8}{9} \left( 1 - \frac{3}{4} s_z^2 - s_z c_z + \frac{3}{2} s_z^3 c_z - 2 s_z^2 c_z^2 \right),
\]
\[
\left[ \sin^2 2\theta_{\text{sun}}^{(z)} \right]_{\max} = \frac{8}{9} \left( 1 - \frac{3}{4} s_z^2 + s_z c_z - \frac{3}{2} s_z^3 c_z - 2 s_z^2 c_z^2 \right);
\]

respectively. A numerical illustration of \( \sin^2 2\theta_{\text{sun}}^{(x)} \) and \( \sin^2 2\theta_{\text{sun}}^{(z)} \) as functions of \( \phi_x \) and \( \phi_z \) is shown in Fig. 1, from which \( 0.816 \leq \sin^2 2\theta_{\text{sun}}^{(x)} \leq 0.938 \) and \( 0.873 \leq \sin^2 2\theta_{\text{sun}}^{(z)} \leq 0.903 \) can be obtained. Note that \( \sin^2 2\theta_{\text{atm}} = 1.000 \) holds in both scenarios. In addition, we get \( \sin^2 2\theta_{\text{chz}}^{(x)} \approx 0.01 \) and \( \sin^2 2\theta_{\text{chz}}^{(z)} \approx 0.0006 \). Therefore two nearly tri-bimaximal neutrino mixing patterns are practically indistinguishable in the atmospheric neutrino oscillation experiment. It is possible to distinguish between them in the solar neutrino oscillation experiment. They can unambiguously be distinguished with the measurements of \( |V_{e3}| \) and CP or T violation in a variety of long-baseline neutrino oscillation experiments.

The strength of CP or T violation in neutrino oscillations, no matter whether neutrinos are Dirac or Majorana particles, is measured by a universal and rephasing-invariant parameter \( J \) [13], defined through the following equation:

\[
\text{Im} \left( V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^* \right) = J \sum_{\gamma,k} (\varepsilon_{\alpha\beta\gamma} \varepsilon_{ijk}) ,
\]

in which the Greek subscripts run over \((e, \mu, \tau)\), and the Latin subscripts run over \((1, 2, 3)\). Considering two lepton mixing scenarios proposed above, we obtain

\[
J^{(x)} = \frac{1}{6} s_x c_x \sin \phi_x ,
\]
\[
J^{(z)} = \frac{1}{6} s_z c_z \sin \phi_z .
\]
For illustration, we typically take $\phi_x = \phi_z = 90^\circ$. Then we arrive at $J(x) \approx 0.0115$ and $J(z) \approx 0.0028$, respectively. The former could be determined from the CP-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions or from the T-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions in a long-baseline neutrino oscillation experiment, if the terrestrial matter effects are insignificant or under control.

4 Further discussions and remarks

We have discussed two simple possibilities to construct the charged lepton and neutrino mass matrices, from which two nearly tri-bimaximal neutrino mixing patterns can naturally emerge. Both scenarios are compatible with the large-angle MSW solution to the solar neutrino problem, although their numerical predictions for the mixing factor $\sin^2 2\theta_{\text{sun}}$ may be different from each other. Two lepton mixing patterns are practically indistinguishable in the atmospheric neutrino oscillation experiment, but their consequences on $|V_{e3}|$ and leptonic CP violation are different and distinguishable. Only one of them is likely to yield an observable CP- or T-violating asymmetry in long-baseline neutrino oscillation experiments.

There are certainly other possibilities to modify the tri-bimaximal neutrino mixing ansatz, such that non-vanishing $|V_{e3}|$ (and CP violation) can naturally be incorporated into the resultant scenarios of nearly tri-bimaximal mixing. For illustration, we follow an interesting approach proposed in Ref. [14] to consider charged lepton and neutrino mass matrices of the form

$$M_l M_l^\dagger = \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix},$$

$$M_\nu M_\nu^\dagger = \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y^* & 0 & x \end{pmatrix}. \quad (24)$$

Clearly $M_l M_l^\dagger$ is invariant under cyclic permutation of three generation indices [14], and $M_\nu M_\nu^\dagger$ has four texture zeros [15]. Note that $y$ was assumed to be real in Ref. [8]. One will see later on that $|V_{e3}| \neq 0$ may non-trivially result from the phase of $y$. The Hermitian matrices $M_l M_l^\dagger$ and $M_\nu M_\nu^\dagger$ can be diagonalized as follows:

$$U_l^\dagger M_l M_l^\dagger U_l = \text{Diag}\left\{ m_{e}^2, m_{\mu}^2, m_{\tau}^2 \right\},$$

$$U_\nu^\dagger M_\nu M_\nu^\dagger U_\nu = \text{Diag}\left\{ m_1^2, m_2^2, m_3^2 \right\}. \quad (25)$$

It is straightforward to obtain

$$U_l = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix}, \quad (26)$$

where $\omega = \exp(+i2\pi/3)$ and $\bar{\omega} = \exp(-i2\pi/3)$; and

$$U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} e^{i\phi} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} e^{-i\phi} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (27)$$
where $\varphi = \arg(y)$. Then the lepton flavor mixing matrix $V = U_l^\dagger U_{\nu}$, which describes the mismatch between the diagonalization of $M_l M_l^\dagger$ and that of $M_{\nu} M_{\nu}^\dagger$, takes the form

$$V = \begin{pmatrix}
\frac{1}{\sqrt{6}}(1 + e^{-i\varphi}) & \frac{1}{\sqrt{3}}(1 - e^{i\varphi}) & \frac{1}{\sqrt{6}}(\bar{\omega} + \bar{\omega}e^{-i\varphi}) \\
\frac{1}{\sqrt{6}}(\omega + \bar{\omega}e^{-i\varphi}) & \frac{1}{\sqrt{3}}(1 - e^{i\varphi}) & \frac{1}{\sqrt{6}}(\omega - \bar{\omega}e^{i\varphi}) \\
\frac{1}{\sqrt{6}}(\bar{\omega} + \omega e^{-i\varphi}) & \frac{1}{\sqrt{3}}(\omega - \bar{\omega}e^{i\varphi}) & \frac{1}{\sqrt{6}}(\bar{\omega} - \omega e^{i\varphi})
\end{pmatrix}.$$  

(28)

The tri-bimaximal neutrino mixing (up to a trivial sign or phase rearrangement [8]) can then be reproduced from $V$ in the limit $\varphi = 0$. The nearly tri-bimaximal mixing scenario obtained in Eq. (28) leads to

$$\sin^2 2\theta_{\text{sun}} = \frac{8}{9} \cos^2 \frac{\varphi}{2},$$
$$\sin^2 2\theta_{\text{atm}} = \frac{8}{3} \sin^2 \left(\frac{\varphi}{2} + \frac{2\pi}{3}\right) \left[1 - \frac{2}{3} \sin^2 \left(\frac{\varphi}{2} + \frac{2\pi}{3}\right)\right],$$
$$\sin^2 2\theta_{\text{chz}} = \frac{8}{3} \sin^2 \frac{\varphi}{2} \left(1 - \frac{2}{3} \sin^2 \frac{\varphi}{2}\right).$$  

(29)

We plot the changes of $\sin^2 2\theta_{\text{sun}}$, $\sin^2 2\theta_{\text{atm}}$ and $\sin^2 2\theta_{\text{chz}}$ as functions of $\varphi$ in Fig. 2, where the experimental upper bound $\sin^2 2\theta_{\text{chz}} < 0.1$ has been taken into account. One can see that the allowed ranges of $\varphi$ are $0 \leq \varphi \leq 0.125\pi$ (or $22.5^\circ$) and $2\pi \geq \varphi \geq 1.875\pi$ (or $337.5^\circ$). Accordingly, we obtain $0.939 \leq \sin^2 2\theta_{\text{atm}} \leq 1$ and $0.962 \leq \sin^2 2\theta_{\text{atm}} \leq 1$. We also obtain $0.855 \leq \sin^2 2\theta_{\text{sun}} \leq 0.889$ for both ranges of $\varphi$.

Note that $J = 0$ holds exactly, although $V$ is complex. Therefore no intrinsic CP violation could be observed in neutrino oscillation experiments, if the tri-bimaximal neutrino mixing pattern or its revised version in Eq. (28) were correct. The result $J = 0$ makes such a nearly tri-bimaximal ansatz less interesting. Of course, the complex phases in $V$ may have significant effects on the neutrinoless double beta decay, if neutrinos are Majorana particles.

Finally let us remark that both the tri-bimaximal mixing pattern and its possible extensions require some peculiar flavor symmetries to be imposed on the charged lepton and neutrino mass matrices. It is likely that one of the three nearly tri-bimaximal neutrino mixing patterns under discussion serves as the leading-order approximation of a more complicated flavor mixing matrix. For the time being, however, such simple ansätze are very instructive and useful to explore the main features of lepton flavor mixing and CP violation through neutrino oscillations. We expect that more delicate neutrino oscillation experiments in the near future can help pin down the explicit pattern of neutrino mixing, from which one may get some insight into the underlying flavor symmetry and its breaking mechanism responsible for the origin of both lepton masses and leptonic CP violation.

The author would like to thank H. Fritzsch for useful discussions and comments. This work was supported in part by the National Natural Science Foundation of China.
References

[1] Y. Fukuda et al., Phys. Lett. B 436 (1998) 33; Phys. Rev. Lett. 81 (1998) 1158; 81 (1998) 1562; 82 (1999) 1810; 85 (2000) 3999.

[2] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 87 (2001) 071301.

[3] L. Wolfenstein, Phys. Rev. D 17 (1978) 2369; S.P. Mikheyev and A.Yu. Smirnov, Sov. J. Nucl. Phys. 42 (1985) 913.

[4] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B 420 (1998) 397; Palo Verde Collaboration, F. Boehm et al., Phys. Rev. Lett. 84 (2000) 3764.

[5] H. Fritzsch and Z.Z. Xing, Phys. Lett. B 372 (1996) 265; Phys. Lett. B 440 (1998) 313; Phys. Rev. D 61 (2000) 073016.

[6] F. Vissani, hep-ph/9708843 (unpublished); V. Barger, S. Pakvasa, T.J. Weiler, and K. Whisnant, Phys. Lett. B 437 (1998) 107; D.V. Ahluwalia, Mod. Phys. Lett. A 13 (1998) 2249; H. Fritzsch and Z.Z. Xing, Phys. Lett. B 440 (1998) 313; A. Baltz, A.S. Goldhaber, and M. Goldhaber, Phys. Rev. Lett. 81 (1998) 5730; T. Kitabayashi and M. Yasue, Nucl. Phys. B 609 (2001) 61; K.S. Babu and R.N. Mohapatra, hep-ph/0201170, and references therein.

[7] For a review of various neutrino mixing ansätze, see: G. Altarelli and F. Feruglio, Phys. Rep. 320 (1999) 295; H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. 45 (2000) 1; S.M. Barr and I. Dorsner, Nucl. Phys. B 585 (2000) 79.

[8] P.F. Harrison, D.H. Perkins, and W.G. Scott, hep-ph/0202074 (to be published in Phys. Lett. B). See also W.G. Scott, Nucl. Phys. B (Proc. Suppl.) 85 (2000) 177; P.F. Harrison and W.G. Scott, hep-ph/0203209.

[9] T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto, KEK, Japan, 1979; M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by D. Freedman and P. van Nieuwenhuizen (North Holland, Amsterdam, 1979), p. 315; R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[10] Z.Z. Xing, Phys. Rev. D 64 (2001) 093013.

[11] C. Jarlskog, in CP Violation, edited by C. Jarlskog (World Scientific, Singapore, 1989), p. 3; H. Fritzsch and Z.Z. Xing, Phys. Rev. D 57 (1998) 574.

[12] Particle Data Group, D.E. Groom et al., Eur. Phys. J. C 15 (2000) 1.

[13] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039.

[14] See, e.g., P.F. Harrison and W.G. Scott, Phys. Lett. B 333 (1994) 471; S.L. Adler, Phys. Rev. D 59 (1999) 015012;

[15] See, e.g., H. Fritzsch, Phys. Lett. B 73 (1978) 317; Nucl. Phys. B 155 (1979) 189; P. Ramond, R.G. Roberts, and G.G. Ross, Nucl. Phys. B 406 (1993) 19; H. Fritzsch and Z.Z. Xing, Nucl. Phys. B 556 (1999) 49.
Figure 1: The mixing factors $\sin^2 2\theta_{\text{sun}}^{(i)}$ and $\sin^2 2\theta_{\text{sun}}^{(i)}$ against arbitrary values of $\phi_x$ and $\phi_z$ in two nearly tri-bimaximal neutrino mixing patterns.

Figure 2: The mixing factors $\sin^2 2\theta_{\text{sun}}$, $\sin^2 2\theta_{\text{atm}}$ and $\sin^2 2\theta_{\text{chz}}$ against arbitrary values of $\varphi$ in a nearly tri-bimaximal neutrino mixing scenario.