Improved quark mass density-dependent model with quark and non-linear scalar field coupling

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Abstract

The improved quark mass density-dependent model which includes the coupling between the quarks and a non-linear scalar field is presented. Numerical analysis of solutions of the model is performed over a wide range of parameters. The wave functions of ground state and the lowest one-particle excited states with even and odd parity are given. The root-mean squared radius, the magnetic moment and the ratio between the axial-vector and the vector $\beta$-decay coupling constants of the nucleon are calculated. We found that the present model is successful to describe the properties of nucleon.

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I. INTRODUCTION

Since the conjecture of Witten [1] that strange quark matter (SQM), would be more stable than the normal nuclear matter, much theoretical effort has been made on the investigation of its properties and applications [2-13]. Because of the well-known difficulty of QCD in the non-perturbative domain, many effective models reflecting the characteristics of the strong interaction are used to study the SQM. They include the MIT bag model [2-4], the quark-meson coupling (QMC) model [5], the Friedberg-Lee (FL) soliton bag model [6], the chiral SU(3) quark model [7], the quark mass density-dependent (QMDD) model [8-11] and the quark mass density- and temperature-dependent model [12, 13], etc. In this paper, we will focus our attention on the QMDD model.

The QMDD model was first suggested by G. N. Fowler, S. Raha, and R. M. Weiner. According to this model, the masses of $u, d$ quarks and strange quarks (and the corresponding anti-quarks) are given by

$$m_q = \frac{B}{3n_B}(i = u, d, \bar{u}, \bar{d}),$$  \hspace{1cm} (1)

$$m_{s, \bar{s}} = m_{s0} + \frac{B}{3n_B},$$ \hspace{1cm} (2)

where $m_{s0}$ is the current mass of the strange quark, $B$ is the vacuum energy density inside the bag, and $n_B = \frac{1}{3}(n_u + n_d + n_s)$ is the baryon number density, with $n_u, n_d, n_s$ representing the density of $u$ quark, $d$ quark, and $s$ quark. The basic hypothesis Eqs. (1) and (2) in QMDD model can easily be understand from the quark confinement mechanism. A confinement potential $k r (r^2)$ must be added to a quark system in the phenomenological effective models because the the perturbative QCD can not give us the confinement solutions of quarks. The confinement potential prevents the quark from going to infinite or to the very large regions. The large volume means that the density is small. This mechanism of confinement can be mimicked through the requirement that the mass of an isolated quark becomes infinitely large so that the vacuum is unable to support it [11, 12]. This is just the physical picture given by Eqs (1) and (2). In fact, this confinement mechanism is very similar to that of MIT bag model. But the advantage of QMDD model is that it does not need to introduce a quark confined boundary condition as that of the MIT bag model.

Although the QMDD model is successful for describing the properties of SQM [8-11], but it is still an ideal quark gas model. Compared to the usual ideal quark gas model, the
basic improvement of the QMDD model is that the quark masses depend on density and
the quark confinement mechanism is mimicked. Obviously, if we hope to investigate the
physical properties of nucleons and hyperons by means of the QMDD model, the quark-
quark interactions must be considered. Following the line of QMC model, we will introduce
quark and scalar $\sigma$ field nonlinear coupling self-consistently to improve the QMDD model
in this paper. As a first step, we ignore the $s$-quark and consider the coupling of $u$ and $d$
quarks to a non-linear scalar field only.

We hope to emphasize there are two basic differences between our improved quark mass
density-dependent (IQMDD) model and the usual QMC model suggested first by Guichon
[14] and developed by Thomas, Saito [15] and Jennings. Jin [16]. Firstly, instead of a MIT
bag for nucleon in QMC model, we do not need a MIT bag for nucleon in IQMDD model.
The constraint of MIT bag boundary condition disappears in our formulae because the quark
confinement mechanism has been established in Eqs. (1) and (2). This is very important
because it should provide a reasonable starting point for many-body calculations. Secondly,
the interaction between quark and scalar meson is limited in the bag regions for QMC model,
but for IQMDD model, this interaction is extended to the whole free space. In fact, our
model is similar to that of the F-L model, but instead of massless quarks in the F-L model,
the masses of $u$ and $d$ quarks in our model are given by Eqs. (1) and (2).

This paper is organized as follows. The main formulae of IQMDD will be given in the
next section. The numerical results for the ground state and a number of physical quantities,
namely, the root-mean-squared (rms) charge radius $r_p$, the magnetic moment $\mu_p$, the ratio
between the axial-vector and the vector $\beta$-decay coupling constants of the nucleon $g_A/g_V$
will be presented in the third section. In the fourth section we will study the lowest one
particle excited states with even or odd parity. The last section is a summary and discussion.

II. THE IMPROVED QUARK MASS DENSITY-DEPENDENT MODEL

We now briefly outline the model and its pertinent features below. The Hamiltonian
density of the IQMDD model reads:

$$H = \psi^+[\frac{1}{i}\vec{\alpha} \cdot \vec{\nabla} + \beta(m_q + f\sigma)]\psi + \frac{1}{2}m^2 + \frac{1}{2}(\nabla\sigma)^2 + U(\sigma),$$

(3)
where \( m_q = B/3n_B \) is the mass of u(d) quark, \( \vec{\alpha} \) and \( \beta \) are the standard Dirac matrices, \( \psi \) represents the quark quantum field (color and flavor indices suppressed) satisfying the canonical anticommutation relations:

\[
\{ \bar{\psi}(\vec{r}, t), \psi(\vec{r}', t) \} = \delta^3(\vec{r} - \vec{r}'),
\]

and \( f \) is the coupling constant between the quark field \( \psi \) and the meson field \( \sigma \). The \( \sigma \) field is considered independent of time and consequently the commutator:

\[
[\pi(\vec{r}), \sigma(\vec{r}')] = 0,
\]

where \( \pi \) is conjugate field of the scalar meson field. Hence, \( \sigma \) is treated as a classical field. In Eq. (3) \( U(\sigma) \) is the self interaction potential for \( \sigma \) field, which has a phenomenological form:

\[
U(\sigma) = \frac{c_2}{2} \sigma^2 + \frac{c_3}{6} \sigma^3 + \frac{c_4}{24} \sigma^4 + B,
\]

and

\[
c_3^2 > 3c_2c_4,
\]

to ensure that the absolute minimum of \( U(\sigma) \) is at \( \sigma = \sigma_{\text{vac}} \neq 0 \). The bag constant \( B \) is introduced in order that

\[
U(\sigma_{\text{vac}}) = 0 \quad \text{and} \quad U(0) = B.
\]

Without any lose of generality, we may choose \( c_3 < 0 \), and therefore \( \sigma_{\text{vac}} > 0 \):

\[
\sigma_{\text{vac}} = \frac{3}{2c_4} \left[ -c_3 + \left( c_3^2 - \frac{8}{3} c_2 c_4 \right)^{1/2} \right].
\]

One can construct a Fock space of quark states and expand the operator \( \psi \) in terms of annihilation and creation operators on this space with c-number spinor functions \( \varphi_n^{(\pm)} \), which satisfies the Dirac equation:

\[
\left[ \frac{1}{i} \vec{\alpha} \cdot \vec{\nabla} + \beta(m_q + f\sigma) \right] \varphi_n^{(\pm)} = \pm \epsilon_i \varphi_n^{(\pm)}
\]

with superscripts \( \pm \) denoting the positive and negative energy solutions, respectively. The spinor functions \( \varphi_n \) are normalized according to

\[
\int \varphi_n^+ \varphi_n d^3r = 1.
\]
The total energy of the quark-scalar field system is given by

$$E(\sigma) = \sum_n \epsilon_n + \int \left( \frac{1}{2} (\nabla \sigma)^2 + U(\sigma) \right) d^3 r. \quad (12)$$

Minimum of $E(\sigma)$ occurs when $\sigma$ is the solution of

$$- \nabla^2 \sigma + \frac{dU(\sigma)}{d\sigma} = - f \sum_n \bar{\varphi}_n \varphi_n. \quad (13)$$

The equations of the quark field for ground state and excited states will be given in Sec. 3 and Sec. 4 respectively.

**III. THE GROUND STATE SOLUTION**

We discuss the ground state solution of the system now. Define $\varphi$ as the lowest positive energy wave function. It can be expressed as:

$$\varphi = \left( \begin{array}{c} u \\ i(\overrightarrow{\sigma} \cdot \overrightarrow{r})v \end{array} \right) \chi_m, \quad (14)$$

where $\overrightarrow{\sigma}$ are the Pauli matrices, $\chi_m = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The radial functions $u(r)$ and $v(r)$ satisfy:

$$\frac{du(r)}{dr} = -[\epsilon + m_q + f\sigma(r)]v(r), \quad (15)$$

$$\frac{dv(r)}{dr} = -2 \frac{v(r)}{r} + [\epsilon - m_q - f\sigma(r)]u(r), \quad (16)$$

the normalized condition reads:

$$4\pi \int_0^{\infty} [u^2(r) + v^2(r)] r^2 dr = 1, \quad (17)$$

and the equation of motion of $\sigma$ field Eq. (13) becomes:

$$\frac{d^2 \sigma(r)}{dr^2} + \frac{2 \sigma(r)}{r} \frac{d\sigma}{dr} = U'(\sigma(r)) + 3f[u^2(r) - v^2(r)], \quad (18)$$

Equations (15), (16) and (18) can be solved with the boundary conditions:

$$v(r = 0) = 0, \quad u(r = \infty) = 0, \quad v(r = \infty) = 0, \quad \sigma(r = \infty) = \sigma_V, \quad \sigma'(r = 0) = 0 \quad (19)$$
Before numerical calculation, we address the parameters of the model first. There are four free parameters, namely, $c_2$, $c_3$, $c_4$, and $f$ in IQMDD model. The parameters $c_2$, $c_3$ and $c_4$ fix the interaction potential $U$, while $f$ measures the coupling between the quark and the scalar field. The set of equations (15), (16) and (18) should be solved alternately until consistency was obtained using the iterative method [17]. Once the solutions of the above equations are obtained, one can calculate a number of physical quantities pertaining to the three-quark system which have been measured from experiments. Let $r_p$, $\mu_p$, and $g_A/g_V$ be, respectively, the rms charge radius, the magnetic moment, and the ratio between the axial-vector and the vector $\beta$-decay coupling constants of the nucleon. They satisfy [18]:

\[
<r_p^2> = \int \varphi^+ \varphi r^2 d^3r / \int \varphi^+ \varphi d^3r,
\]

(20)

\[
\mu_p = \frac{1}{2} \left( \int \vec{r} \times \varphi^+ \vec{\alpha} \varphi d^3r \right)_z / \int \varphi^+ \varphi d^3r,
\]

(21)

and

\[
g_A/g_V = \frac{5}{3} \int \varphi^+ \sigma z \varphi d^3r / \int \varphi^+ \varphi d^3r,
\]

(22)

By using Eq. (14), we find:

\[
<r_p^2> = 4\pi \int_0^{\infty} (u^2 + v^2) r^4 dr,
\]

(23)

\[
\mu_p = \frac{8\pi}{3} \int_0^{\infty} r^3 uvdr,
\]

(24)

and

\[
g_A/g_V = \frac{20\pi}{3} \int_0^{\infty} r^2 (u^2 - \frac{1}{3} v^2) dr.
\]

(25)

Since in the present form the model is flavor independent, the charge radius of the neutron in the model is

\[
<r_n^2> \equiv 0,
\]

(26)

and the neutron magnetic moment is

\[
\mu_n = -\frac{2}{3} \mu_p,
\]

(27)
as given by the $SU(6)$ algebra. Corrections to these relations arise only when QCD effects are included.

For comparison let us remind ourselves that the experimental values of proton mass $E = 4.69$ fm$^{-1}$, the proton magnetic moment $\mu = 2.79$ nuclear magnetons, $g_A/g_V = 1.25$, and the charge radius of proton $r_p = 0.83$ fm. To do numerical calculation, we follow Ref. [17, 18]. We fix $r_p = 0.83$ fm first. Once the value is chosen, there are only three free parameters. We have studied mostly two families of parameters for ground system state. These are the $c_2 = 0$ and the $B^{1/4} = 145$ MeV cases respectively. Each choice fixes one other parameter by a specific requirement for the shape of the potential. Therefore it is sufficient to label our results by the coupling constant $f$ and constant $c_4$.

Our results for the two families ($c_2 = 0, B^{1/4} = 145$ MeV) are summarized in three tables. Through these tables, we use the units $\hbar = c = 1$ and use fm as the fundamental unit for length.

We consider the case $B^{1/4} = 145$ MeV first. The bag constant $B$ corresponds to the difference between two minima of the $U(\sigma)$. In Table. 1 we list the bag properties as a function of the parameter $c_4$ for two values of the coupling constant $f$. The variation of bag properties with the coupling constant $f$ for several values of $c_4$ is given in Table. 2. Several features emerge from these calculations. Firstly, we note that an increase of the coupling constant produces a continuous change from a volume quark distribution for small $f$ to a surface quark distribution for large $f$. This change is illustrated in Fig. 1 where we plot the quark density $u^2 - v^2$ for three values of $f$. It should be noted that shape of the soliton field does not change significantly with $f$ for given $c_4$. The variations of the quark charge density as a function of the radius with increasing $c_4$ are plotted in Fig. 2 and Fig. 3. When $c_4$ decreases, with a fixed value of $f$, the quark charge distribution $u^2 - v^2$ change slowly from surface to volume. This is illustrated in Fig. 2 and Fig. 3 for $f = 75$ and $f = 200$, respectively. The change from volume to surface quark charge density is also evident in the variations of the values of the magnetic moment $\mu_p$ and $g_A/g_V$. For instance from Table. 2, when $c_4 = 8 \times 10^5$, the magnetic moment varies with increasing $f$ from $1.87\mu_B$ to $2.61\mu_B$, where $\mu_B$ is the Bohr magneton of the proton. Similarly, $g_A/g_V$ varies from 1.21 for small $f$ to 0.618 for large $f$. It is also found from the tables that for some $c_4$ and $f$ cases we can not found a solution for $r_p = 0.83$ fm.

The variation of the scalar field $\sigma$ as a function of the radius for $f = 50, c_4 = 2 \times 10^5$
and \( B^{1/4} = 145 \) MeV is presented in Fig. 4. The value of \( \sigma \) inside the hadron is very different from that of outside: inside \( \sigma \) is less than zero, but outside \( \sigma \) approaches \( \sigma_{\text{vac}} \). The abrupt transition of scalar field through hadron surface will contribute to the total energy remarkably, as shown in Eqs. (11) and (12).

**TABLE 1.** Variation of the properties with increasing parameters for the \( B^{1/4} = 145 \) MeV bag for the two values of \( f \).

| \( f \) | \( c_4 \) | \( \epsilon \) | \( E \) | \( \mu_p \) | \( g_A/g_V \) |
|---|---|---|---|---|---|
| 30 | \( 8 \times 10^4 \) | 1.54 | 6.54 | 2.39 | 0.904 |
|    | \( 1 \times 10^5 \) | 1.56 | 6.56 | 2.38 | 0.929 |
|    | \( 4 \times 10^5 \) | 1.74 | 6.94 | 2.25 | 1.06 |
|    | \( 8 \times 10^5 \) | 1.85 | 7.26 | 2.15 | 1.11 |
| 200 | \( 8 \times 10^4 \) | No solution |
|     | \( 1 \times 10^5 \) | No solution |
|     | \( 4 \times 10^5 \) | 1.24 | 5.23 | 2.64 | 0.603 |
|     | \( 8 \times 10^5 \) | 1.22 | 5.21 | 2.63 | 0.618 |
TABLE 2. Variation of the properties as a function of $f$ for several values of $c_4$ with $B^{1/4} = 145$ MeV.

| $c_4$  | $f$ | $\epsilon$ | $E$  | $\mu_p$ | $g_A/g_V$ |
|-------|-----|------------|------|---------|-----------|
| $8 \times 10^5$ | 15  | 2.18       | 8.16 | 1.87    | 1.21      |
|       | 30  | 1.85       | 7.26 | 2.14    | 1.11      |
|       | 50  | 1.56       | 6.47 | 2.39    | 0.936     |
|       | 75  | 1.41       | 5.97 | 2.49    | 0.794     |
|       | 100 | 1.24       | 5.70 | 2.55    | 0.720     |
|       | 200 | 1.22       | 5.21 | 2.61    | 0.618     |
| $4 \times 10^5$ | 15  | 2.11       | 8.01 | 1.96    | 1.22      |
|       | 30  | 1.74       | 6.94 | 2.25    | 1.06      |
|       | 50  | 1.49       | 6.28 | 2.43    | 0.868     |
|       | 75  | 1.36       | 6.07 | 2.55    | 0.743     |
|       | 100 | 1.30       | 5.81 | 2.57    | 0.680     |
|       | 200 | 1.24       | 5.23 | 2.63    | 0.603     |
| $1 \times 10^5$ | 15  | 1.96       | 7.62 | 2.08    | 1.17      |
|       | 30  | 1.56       | 6.56 | 2.39    | 0.929     |
|       | 50  | 1.41       | 6.09 | 2.48    | 0.765     |
|       | 75  | 1.33       | 5.77 | 2.50    | 0.677     |
|       | 100 |            |      |         | No solution |
|       | 200 |            |      |         | No solution |
| $8 \times 10^4$ | 15  | 1.94       | 7.61 | 2.09    | 1.16      |
|       | 30  | 1.54       | 6.54 | 2.39    | 0.904     |
|       | 50  |            |      |         | No solution |
|       | 75  |            |      |         | No solution |
|       | 100 |            |      |         | No solution |
|       | 200 |            |      |         | No solution |

Another family of parameters considered is characterized by $c_2 = 0$. In this case $U(\sigma)$ has an inflection point at $\sigma = 0$, and only one minimum. We vary $c_3$, $c_4$ and $f$ subject to the bag size constraint $r_p = 0.83$ fm. Our numerical results for this family are summarized
in Table. 3. We find that when $f$ increases, the character of the bag changes from volume confinement to surface confinement again. Similarly, we also find that for a given $f$, when $c_4$ increases, the magnetic moment decreases. In summary, we find in both cases:

1) The quark energy $\epsilon$ and the ratio $g_A/g_V$ decreases with increasing parameters $f$ and $c_4$.

2) On the other hand, the magnetic moment $\mu$, has just the opposite behavior: it is a decreasing function of the parameter $c_4$ and an increasing function of the parameter $f$.

3) The total energy of the ground state decreases as a function of $f$ and increases as a function of $c_4$. The rise of the total energy $E$ is caused by a contribution from $3\epsilon$ mainly.

4) An increase of the coupling constant produces a continuous change from a volume quark distribution to a surface quark distribution, and an decrease of $c_4$ can reproduce same results.
TABLE 3. Variation of the properties as a function of $f$ and $c_4$ with $c_2=0$.

| $c_4$ | $f$  | $\epsilon$ | $E$  | $\mu_p$ | $g_A/g_V$ |
|-------|------|-------------|------|---------|-----------|
| $1 \times 10^4$ | 15 | 1.64 | 7.00 | 2.31 | 1.00 |
|       | 30 | 1.38 | 6.17 | 2.50 | 0.769 |
|       | 50 | 1.29 | 5.77 | 2.57 | 0.667 |
|       | 75 | 1.25 | 5.53 | 2.60 | 0.621 |
|       | 100 | 1.24 | 5.43 | 2.61 | 0.601 |
|       | 150 | 1.22 | 5.31 | 2.63 | 0.583 |
| $2 \times 10^4$ | 15 | 1.64 | 7.00 | 2.23 | 1.05 |
|       | 30 | 1.38 | 6.17 | 2.48 | 0.810 |
|       | 50 | 1.29 | 5.77 | 2.57 | 0.692 |
|       | 75 | 1.25 | 5.53 | 2.60 | 0.635 |
|       | 100 | 1.24 | 5.43 | 2.63 | 0.609 |
|       | 150 | 1.22 | 5.31 | 2.63 | 0.587 |
| $4 \times 10^4$ | 15 | 1.83 | 7.36 | 2.16 | 1.12 |
|       | 30 | 1.48 | 6.41 | 2.46 | 0.857 |
|       | 50 | 1.34 | 5.97 | 2.55 | 0.723 |
|       | 75 | 1.28 | 5.71 | 2.60 | 0.652 |
|       | 100 | 1.25 | 5.57 | 2.63 | 0.620 |
|       | 150 | 1.24 | 5.51 | 2.61 | 0.592 |

IV. THE ONE PARTICLE EXCITED STATES

The one particle excited states for F-L model had been calculated by Saly and Sundaresan [19]. In this section, we use their method to study the one particle excited state for IQMDD model.

A. First excited state with even parity

In this configuration, we shall consider two quarks to be in the ground state, while one quark will be placed in the first excited level $\epsilon_1$. Then the system of equations to be solved is
\[
\frac{du(r)}{dr} = -[\epsilon_0 + m_q + f\sigma(r)]v(r), \tag{28.a}
\]
\[
\frac{dv(r)}{dr} = -2\frac{v(r)}{r} + [\epsilon_0 - m_q - f\sigma(r)]u(r), \tag{28.b}
\]
\[
\frac{du_1(r)}{dr} = -[\epsilon_1 + m_q + f\sigma(r)]v_1(r), \tag{28.c}
\]
\[
\frac{dv_1(r)}{dr} = -2\frac{v_1(r)}{r} + [\epsilon_1 - m_q - f\sigma(r)]u_1(r), \tag{28.d}
\]
\[
4\pi \int_0^\infty [u^2(r) + v^2(r)]r^2dr = 1, \tag{28.e}
\]
\[
4\pi \int_0^\infty [u_1^2(r) + v_1^2(r)]r^2dr = 1, \tag{28.f}
\]
\[
\frac{d^2\sigma(r)}{dr^2} + \frac{2}{r} \frac{d\sigma(r)}{dr} = U'(\sigma) + 2f[u^2(r) - v^2(r)] + f[u_1^2(r) - v_1^2(r)]. \tag{28.g}
\]

The set of equations (28.a)-(28.g) is to be solved with the boundary conditions:

\[
v(r = 0) = 0, v_1(r = 0) = 0, u(r = \infty) = 0, v(r = \infty) = 0,
\]
\[
v_1(r = \infty) = 0, u_1(r = \infty) = 0, \sigma(r = \infty) = \sigma_V, \sigma'(r = 0) = 0. \tag{29}
\]

### B. Lowest energy with odd-parity state

To obtain the lowest energy odd-parity state, we place two quarks in the ground state of even-parity and one quark in the lowest odd-parity state. In this case the system of equation is:

\[
\frac{d\tilde{u}(r)}{dr} = -[\tilde{\epsilon}_0 + m_q + f\sigma(r)]\tilde{v}(r), \tag{30.a}
\]
\[
\frac{d\tilde{v}(r)}{dr} = -2\frac{\tilde{v}(r)}{r} + [\tilde{\epsilon}_0 - m_q - f\sigma(r)]\tilde{u}(r), \tag{30.b}
\]
\[
\frac{d\tilde{u}_1(r)}{dr} = [\tilde{\epsilon}_1 - m_q - f\sigma(r)]\tilde{u}_1(r), \tag{30.c}
\]
\[
\frac{d\tilde{v}_1(r)}{dr} = -2\frac{\tilde{u}_1(r)}{r} - [\tilde{\epsilon}_1 + m_q + f\sigma(r)]\tilde{v}_1(r), \tag{30.d}
\]
\[
4\pi \int_0^\infty [\tilde{u}^2(r) + \tilde{v}^2(r)]r^2dr = 1, \tag{30.e}
\]
\[
4\pi \int_0^\infty [\tilde{u}_1^2(r) + \tilde{v}_1^2(r)]r^2dr = 1, \tag{30.f}
\]
\[
\frac{d^2 \sigma (r)}{dr^2} + 2 \frac{d \sigma (r)}{dr} = U'(\sigma) + 2f[\tilde{u}^2 (r) - \tilde{v}^2 (r)] + f[\tilde{u}_1^2 (r) - \tilde{v}_1^2 (r)],
\]

where \( \tilde{u}_1 (r), \tilde{v}_1 (r) \) are the components of wave function \( \varphi \):

\[
\varphi = \begin{pmatrix}
\tilde{x} \\
0
\end{pmatrix}
\]

The set of equations (30.a)-(30.g) is to be solved with the boundary conditions:

\[
\tilde{v} (r = 0) = 0, \quad \tilde{u}_1 (r = 0) = 0, \quad \tilde{u} (r = \infty) = 0, \quad \tilde{v} (r = \infty) = 0,
\]

\[
\tilde{u}_1 (r = \infty) = 0, \quad \tilde{v}_1 (r = \infty) = 0, \quad \sigma (r = \infty) = \sigma_v, \quad \sigma' (r = 0) = 0.
\]

For simplicity, we consider \( B^{1/4} = 145 \) MeV, \( r_p = 0.83 \) fm case only.

**TABLE 4. Dependence of the solutions on \( c_4 \) for \( f=30 \).**

| Ground state | Even-parity state | odd-parity state |
|-------------|-----------------|-----------------|
| \( c_4 \)   | \( \epsilon \) E | \( \epsilon_0 \) \( \epsilon_1 \) \( E_+ \) | \( \tilde{\epsilon}_0 \) \( \tilde{\epsilon}_1 \) \( E_- \) |
| 8 \times 10^5 | 1.85 7.26 2.02 3.92 9.11 1.87 3.10 8.70 |
| 6 \times 10^5 | 1.81 7.16 1.99 3.99 9.17 1.84 3.11 8.71 |
| 4 \times 10^5 | 1.76 7.09 1.95 4.12 9.29 1.80 3.11 8.74 |
| 2 \times 10^5 | 1.63 6.72 1.82 4.16 9.13 No solution |
| 1 \times 10^5 | 1.56 6.56 1.72 4.43 9.36 No solution |
| 8 \times 10^4 | 1.54 6.54 1.58 4.41 9.84 No solution |

**TABLE 5. Dependence of the solutions on \( f \) for \( c_4=4 \times 10^5 \).**

| Ground state | Even-parity state | odd-parity state |
|-------------|-----------------|-----------------|
| \( f \)     | \( \epsilon \) E | \( \epsilon_0 \) \( \epsilon_1 \) \( E_+ \) | \( \tilde{\epsilon}_0 \) \( \tilde{\epsilon}_1 \) \( E_- \) |
| 15 | 2.09 7.93 No solution No solution |
| 20 | 1.95 7.53 2.18 3.80 9.17 2.01 3.21 8.82 |
| 25 | 1.85 7.32 2.05 3.91 9.21 1.88 3.14 8.79 |
| 30 | 1.76 7.09 1.95 4.12 9.29 No solution |
| 35 | 1.66 6.79 1.80 4.32 9.29 No solution |
| 40 | 1.59 6.59 1.59 4.28 9.60 No solution |
| 50 | 1.49 6.30 1.48 4.19 9.63 No solution |
Our results are summarized in Tables 4, 5 and Figs. 5, 6. In Table 4, we fix $r_p = 0.83$ fm, $B^{1/4} = 145$ MeV, $f = 30$ and show the dependence of ground state energy $E$, the first even-parity excited state energy $E_+$, and the first odd-parity state energy $E_-$ on $c_4$. The dependence of $E$, $E_+$, $E_-$ on $f$ for $c_4 = 4 \times 10^5$ is shown in Table 5. Remember that the experimental values of (uud) system are $E = 4.69$ fm$^{-1}$, $E_+ = 7.35$ fm$^{-1}$ and $E_- = 7.67$ fm$^{-1}$ and compare our results with that given by Table VI, VII of Ref. [19] for F-L model with massless quarks, we find that the agreement with experiments for IQMDD model is better than that of F-L model.

Finally, we hope to point, as shown in Tables 4 and 5, that the allowed range of parameters is large for the existence of the first even-parity excited state. The only restriction is that the height of the potential well $f \sigma_V$ must be greater than the quark eigenvalues $\epsilon_1$. But for odd-parity states, the allowed range of parameters restricts severely. We cannot find solution in many cases. To show this point more transparently, for odd-parity solutions, we fix $r_p = 0.83$ fm, $B^{1/4} = 145$ MeV, $f = 30$ and draw the curves of total quark density $[2(u^2 - v^2) + \tilde{u}_1^2 - \tilde{v}_1^2]$ vs. r and scalar field $\sigma$ vs. r for different $c_4 = 4 \times 10^5, 6 \times 10^5$ and $8 \times 10^5$ in Fig. 5 respectively. We find no solutions exist for smaller values of $c_4$. The same curves for fix $c_4 = 4 \times 10^5$ but different $f = 20$ and 25 are shown in Fig. 6. We also find no solutions exist for higher values of $f$.

V. SUMMARY AND DISCUSSION

After introducing the quark and non-linear scalar field coupling, we suggest an improved quark mass density-dependent model. We obtain soliton solutions of ground state and excited states for the coupled equations for quark and scalar fields satisfying the required boundary conditions by numerical method. Since the most interesting question is whether the results of these calculations resemble the physics we are trying to describe, we concentrate on the dependence of the results on the phenomenological parameters introduced in this model. We present these dependence in the form of tables and figures for the ground state and low-lying excited states. The wave functions of quark are given. By using the wave function of ground state, we have calculated the rms charge radius, the magnetic moment and the ratio between the axial-vector and the vector $\beta$-decay coupling constant of the nucleon and compared these values with experiment. We find the results given by IQMDD
model are in agreement with experiment.

Since the boundary condition of MIT bag model has been given up in IQMDD model, the many-body calculation beyond mean field approximation can easily be carried out in this model.

We note that the study of this paper is still limited at zero temperature and u, d quarks. Since the spontaneous breaking symmetry of $U(\sigma)$ will be restored at finite temperature, it is of interest to extend our discussion to finite temperature and study the effect of s quarks for hyperons. Working on this topic is in progress.

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FIG. 1

FIG. 1: quark density $u^2 - v^2$ versus radius for $f=15, 40, 200$ for $B^{1/4} = 145$ MeV.
FIG. 2: Quark density $u^2 - v^2$ versus radius for $c_4 = 2 \times 10^5$ and $8 \times 10^5$, $B^{1/4}=145$ MeV and $f=75$. 
FIG. 3: The same as Fig.2, but $f=200$. 
FIG. 4: The soliton field $\sigma$ versus radius for $f=50$ and $c_4=2 \times 10^5$, $B^{1/4}=145$ MeV
FIG. 5: Dependence of the odd-parity solutions on the parameter $c_4$. Other parameters are kept at $f=30$, $r_p=0.83$ fm for $B^{1/4} = 145$ MeV. No solutions exist for smaller values of $c_4$. 

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FIG. 6

FIG. 6: Dependence of the odd-parity solutions on the parameter $c_4$. Other parameters are kept at $f=30$, $r_p=0.83$ fm for $B^{1/4} = 145$ MeV. No solutions exist for higher values of $f$. 

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