Coherent “metallic” resistance and medium localisation in a disordered 1D insulator

Martin Moško,1,2 Pavel Vagner,2,1 Michal Bajdich,1 and Thomas Schäpers2

1Institute of Electrical Engineering, Slovak Academy of Sciences, Dúbravská cesta 9, 841 04 Bratislava, Slovakia
2Institut für Schichten und Grenzflächen, Forschungszentrum Jülich GmbH, 52425 Jülich, Germany

(Dated: March 22, 2022)

It is believed, that a disordered one-dimensional (1D) wire with coherent electronic conduction is an insulator with the mean resistance \( \langle p \rangle \approx e^{2L/\xi} \) and resistance dispersion \( \Delta_p \approx e^{L/\xi} \), where \( L \) is the wire length and \( \xi \) is the electron localisation length. Here we show that this 1D insulator undergoes at full coherence the crossover to a 1D “metal”, caused by thermal smearing and resonant tunnelling. As a result, \( \Delta_p \) is smaller than unity and tends to be \( L/\xi \)-independent, while \( \langle p \rangle \) grows with \( L/\xi \) first nearly linearly and then polynomially, manifesting the so-called medium localisation.

PACS numbers: 73.23.-b, 73.61.Ey

Any coherent electron wave in a one-dimensional (1D) wire with uncorrelated disorder is exponentially localized \( \xi \). The resistance \( p \) of such a disordered 1D wire should therefore increase exponentially with the wire length, \( L \).

In an ensemble of macroscopically identical wires the resistance fluctuates from wire to wire owing to disorder randomness, so it is natural to study the mean resistance \( \langle p \rangle \), where the angular brackets denote the average over the ensemble. Landauer found at zero temperature \( \langle p \rangle \approx 0.5 \{ \exp(2L/\xi) - 1 \} \),

\[
\langle p \rangle = 0.5 \{ \exp(2L/\xi) - 1 \},
\]

(1)

where \( \xi \) is the electron localisation length (throughout this paper, \( p \) is a dimensionless resistance in units \( h/2e^2 \)). He also found that \( \langle 1/p \rangle \) diverges and noted that \( \langle p \rangle \) is not representative of the ensemble. Anderson et al. \( \xi \) therefore defined the typical resistance \( \rho_t \equiv \exp \langle f \rangle - 1 \), where \( f = \ln(1 + \langle p \rangle) \) and \( \langle f \rangle \) is the ensemble average of \( f \). They showed for zero temperature that \( \langle f \rangle = L/\xi \), i.e.,

\[
\rho_t = \exp(L/\xi) - 1.
\]

(2)

The variable \( f \) exhibits giant fluctuations, the dispersion \( \Delta_f \equiv \langle (\rho^2) - \langle \rho^2 \rangle \rangle^{1/2} / \langle p \rangle \) can be shown to grow as \( \xi \)

\[
\Delta_p \approx \exp(L/\xi).
\]

(3)

However, \( \Delta_f \propto 1/\sqrt{L} \), which means that the variable \( f \) self-averages and \( \rho_t \) is representative of the ensemble \( \xi \).

The above formulae hold also for a quasi-1D wire with many 1D channels \( \xi \). Thus, for \( T = 0 \) K and \( L > \xi \) any 1D wire is insulating, i.e., both the resistance and fluctuations grow exponentially with \( L/\xi \).

Thouless \( \xi \) argued that at nonzero temperatures the exponential rise with \( L \) would not be apparent since the inelastic collisions would cause electrons to hop from one localised state to another one. This would cause \( \rho \propto L \) and the 1D localisation would be manifested solely by a characteristic temperature dependence of hopping. All this would happen if the electron transport time through the wire exceeds the inelastic scattering time.

At even higher temperatures, the electron coherence length \( L_\phi \) would become smaller than \( \xi \). This would give rise to the “metallic” resistance \( \langle p \rangle \approx L/\xi \) and weak localisation (see e.g. Ref. \( \xi \)). The crossover between the “metallic” resistance and hopping, first predicted in Ref. \( \xi \), has been observed in the quasi-1D wires \( \xi \).

Therefore, the disordered 1D insulator described by eqs. \( \xi - \xi \) is believed to exist at appropriately low temperatures, i.e., without hopping and at full coherence \( \xi \), \( \xi \), and the crossover to the 1D “metal” seems to occur only if inelastic collisions are present.

We show that the crossover to another 1D “metal” occurs at full coherence due to the thermal smearing and resonant tunneling. It results in the wire resistance rising with \( L/\xi \) first nearly linearly and then polynomially owing to the medium localisation. In addition, \( \Delta_p \) becomes smaller than unity and tends to be \( L/\xi \)-independent.

The insulator - metal transition at \( T = 0 \) K has recently been found in a 1D solid with specially correlated disorder \( \xi \). In our work the disorder is uncorrelated and the reported insulator - “metal” crossover, albeit also coherent, is driven by low nonzero temperatures.

We consider a 1D wire with disorder described by potential \( V(x) = \sum_{j=1}^N \gamma \delta(x - x_j) \), where \( \gamma \delta(x - x_j) \) is the \( \delta \)-shaped impurity potential of strength \( \gamma \), \( x_j \) is the \( j \)-th impurity position randomly chosen along the wire, and \( N \) is the number of impurities. If the 1D electrons tunnel through disorder coherently, the electron wave function \( \Psi_k(x) \) is the solution of the tunneling problem

\[
\left[ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \Psi_k(x) = \varepsilon \Psi_k(x),
\]

(4)

\[
\Psi_k(x \to 0) = e^{ikx} + r_k e^{-ikx}, \quad \Psi_k(x \to L) = t_k e^{ikx},
\]

(5)

where \( \varepsilon = \hbar^2 k^2 / 2m \) is the electron energy, \( k \) is the wave vector, \( m \) is the effective mass, \( r_k \) is the electron reflection amplitude, and \( t_k \) is the electron transmission amplitude. In the boundary conditions \( \xi \) the electron impinging disorder from the left is considered. The beginning and end of the wire are set at \( x = 0 \) and \( x = L \), respectively.

We find the amplitude \( t_k \) by means of the transfer matrix (TM) method. The TM of disorder \( V(x) \) reads \( \xi \).
\[
T = (1/t_k^* \ - r_k/t_k) = T(x_N) \ldots T(x_2)T(x_1), \quad (6)
\]
where
\[
T(x_j) = \left(1 - i\Omega/k \ - i\Omega e^{-2ikx_j}/1 + i\Omega/k \right) \quad (7)
\]
is the TM of the δ-barrier at position \(x_j\) and \(\Omega = m\gamma/\hbar^2\). We evaluate the right hand side of eq. (6) numerically.

We therefore obtain the transmission probability \(T = |t_k|^2\) and evaluate numerically the two-terminal conductance
\[
G = \int_0^\infty d\varepsilon \left[ -\frac{d}{d\varepsilon} \right] T(\varepsilon), \quad (8)
\]
where \(f(\varepsilon)\) is the Fermi distribution. To calculate the wire resistance, instead of \(\rho = 1/G\) we use the formula
\[
\rho = 1/G - 1/f(0), \quad (9)
\]
where \(1/f(0)\) is the contact resistance (\(\simeq 1\) at low \(T\)). At zero temperature equation (8) gives \(\rho = R(\varepsilon_F)/T(\varepsilon_F)\), where \(R = 1 - T\) and \(\varepsilon_F\) is the Fermi energy. Equations (1) and (2) were derived \(\text{[2, 3, 4, 6]}\) by averaging the formula \(\rho = R(\varepsilon_F)/T(\varepsilon_F)\). To extend that work to \(T > 0\) K means to average eq. (9) (see comment \(\text{[10]}\) for details).

We parametrize disorder by parameters \(R(k_F)\) and \(N_I\), where \(R(k) = \Omega^2/(k^2 + \Omega^2)\) is the reflection probability for a single δ-barrier and \(N_I\) is the 1D impurity density. (Note that the final results do not depend on the choice of \(R(k)\) as we assume very small \(k_BT/\varepsilon_F\). For \(R(k) \ll 1\) we can also ignore fluctuations of \(R(k)\) from impurity to impurity.) As the positions \(x_j\) are mutually independent, the distances \(a = x_j+1 - x_j\) between the neighboring impurities in a given wire are selected at random from the distribution \(P(a) = N_I \exp[-N_Ia]\).

In Fig. \(\text{[1]}\) we show the averaged 1D resistance versus the wire length and temperature. The results were obtained for disorder with parameters \(R(k_F) = 0.01\) and \(N_I = 10^7 \text{ m}^{-1}\) and for the electron gas parameters typical of the GaAs wire: \(m = 0.067 m_0\) and \(\varepsilon_F = 35 \text{ meV}\).

In fact, for \(R(k) < 1\) and \(N_I \gg 2\pi k_F\) (weak low-density disorder) our results depend only on the parameters \(L/\xi\) and \(T/\xi\), independently on the choice of \(R(k_F)\), \(N_I\), \(m\), and \(\varepsilon_F\). Here \(g(\varepsilon_F) = 1/(\pi\hbar\varepsilon_F)\) is the density of energy levels. For weak low-density disorder the localisation length is just the elastic mean free path, \(\xi = (N_I B_T(k_F))^\frac{1}{2}\). For the above parameters \(\xi = 10 \mu\text{m} \text{ and } T/\xi \simeq 1 \text{ K}\).

In the top panel of Fig. \(\text{[1]}\) we reproduce at \(T/\xi = 0\) the exponential growth \(\rho_0\) and \(\rho_1\). However, at \(T/\xi > 0\) both \(\rho_0\) and \(\rho_1\) tend to grow with \(L/\xi\) much slower than predicted by eqs \(\text{[1]}\) and \(\text{[2]}\). At \(T/\xi > 1\) they exhibit up to \(L/\xi \leq 2\) the metallic dependence \(\rho = \rho_T = L/\xi\) with a nonlinear correction which is \(\leq 0.4L/\xi\). For larger \(L/\xi\) this nonlinear growth is still far much slower than \(\exp(L/\xi)\) and in general not \(\exp(\text{const. } L)\) (see Fig.3).

The bottom panel of Fig. \(\text{[1]}\) shows the dispersions \(\Delta_\rho\) and \(\Delta_g\), where \(g = 1/\rho\). At \(T = 0\) K, \(\Delta_\rho\) follows the exponential rise \(\text{[3]}\) and \(\Delta_g\) diverges. As \(T\) increases, \(\Delta_\rho\) and \(\Delta_g\) decrease below unity and variation with \(L\) tends to disappear, unlike other transport regimes in which \(\Delta_\rho\) and \(\Delta_g\) always vary with \(L\) in some typical way \(\text{[7]}\).

To provide insight we now approximate eq. \(\text{[5]}\) by
\[
G \simeq \frac{1}{4k_BT} \int_{\varepsilon_F + 2k_BT}^{\varepsilon_F - 2k_BT} d\varepsilon \ T(\varepsilon). \quad (10)
\]
A typical \(T(\varepsilon)\) dependence is shown in Fig. \(\text{[2]}\). It consists of narrow peaks separated on average by energy \(k_BT_\xi = 1/(\varepsilon_F L)\). Some of these peaks have maximum close to unity due to the resonant tunneling across disorder, the rest are the much lower peaks with a negligible area compared to the area below the resonant peaks. The mean distance between the resonant peaks is \(k_BT_\xi\) and the mean number of such peaks in the energy win-
involves at least one resonant peak. This happens for \( T > T_{\xi} \), i.e., the crossover temperature is \( T > T_{\xi} \), where \( T_{\xi} \) is defined by \( \ln(1 + \rho_{\xi}) \equiv (\ln(1 + \rho)) \). Thus, a physically allowed exponential form of \( \rho(L) \) has to coincide with equation \( (\ln(1 + \rho)) = \text{const} \times L \), which however cannot fit the sublinear curve in the inset. The resistance thus rises with \( L \) polynomially rather than exponentially. This is neither the strong localisation (the SL is manifested by eqs. (1 - 3)) nor the weak localisation (the WL occurs at \( L/\xi \ll 1 \)), but somewhere between. We therefore speak about the medium localisation, which means a nonexponential decay of resonant transmission with \( L/\xi \).

What happens for extremely large \( L/\xi \)? Inset to the right panel shows the typical resistance for \( T \ll T_{\xi} \). Here the rise \( \propto \exp(\text{const} L) \) reappears at \( L/\xi \gtrsim 50 \). For \( T > T_{\xi} \) neither \( \rho_{\xi} \) nor \( \langle \rho \rangle \) is numerically feasible, but for \( L/\xi \gtrsim 40 \) we find \( \rho_{\xi} \approx \langle \rho \rangle \propto (L/\xi)^{3/2} \exp(L/4\xi) \) by using other approach not reported here. In the “infinite” wire also the resonant transmission is finally damped exponentially.

Note however, that the limit of extremely long coherent 1D segments is experimentally irrelevant. In general, the resistance \( \gtrsim 10^{9} \Omega \) is hardly measurable owing to intrinsic physical limitations (e.g. the rf noise [8]). On the insulating side of the crossover, one can thus measure \( \rho_{\xi} \approx e^{L/\xi} \) in principle for \( L/\xi \lesssim 12 \). On the “metallic” side, both \( \langle \rho \rangle \) and \( \rho_{\xi} \) reach \( 10^{9} \Omega \) for \( L/\xi \gtrsim 30 \) (not shown in Fig. 3), so they could be measurable for \( L/\xi \lesssim 30 \).

In Ref. [14] the resistance of a single 1D channel was measured for various \( L > \xi \) in a series of GaAs quantum wires prepared by a method allowing to reproduce macroscopic parameters from wire to wire. In Fig. 4 the
The experimental data grow superlinearly. This means that in the measured 1D wires \( L/\xi \gtrsim 2 \mu \text{m} \) and the value \( \xi \approx 1.5 \mu \text{m} \) can be realized through many different combinations of parameters \( N_l \) and \( R_t(\kappa_p) \), but the results remain the same provided the disorder is weak. So we do not need to consider sample-dependent details. Open circles are the experimental data from inset (a) reduced by the contact resistance (the two-terminal resistance at \( L \to 0 \)). To prove the superlinear rise with \( L \) clearly, the experimental data are compared with the linear fit \( \rho = L/\xi \). The best fit is obtained for \( \xi = 0.6 \mu \text{m} \) (dashed line). However, it overestimates all experimental data for \( L \lesssim 2 \mu \text{m} \) and the value \( \xi = 0.6 \mu \text{m} \). The superlinear rise of the measured data is systematically above \( \rho = L/\xi \) and cannot be ascribed to the data dispersion. The main panel shows that the simulated mean resistance, standard deviation and resistance distribution fit the experiment. For a perfect comparison experiments with very large ensembles of wires are needed.

If decoherence were present in the experiment, it would cause the dependence \( \rho = L/\xi \), which is not observed. This means that in the measured 1D wires \( L/\xi > 8 \mu \text{m} \), which is order of magnitude more than in a related 2D electron gas at the same temperature. Why is \( L/\xi \) so large? At low temperatures decoherence is due to the electron-electron interaction. In the (ballistic) 2D electron gas the interaction of two electrons fulfills the conservation laws \( \varepsilon(k_1) + \varepsilon(k_2) = \varepsilon(k'_1) + \varepsilon(k'_2) \) and \( k_1 + k_2 = k'_1 + k'_2 \), which allow the energy exchange and decoherence. In the 1D system such conservation laws prohibit any energy exchange, which might be the reason for large \( L/\xi \). One might think that \( L/\xi \lesssim \xi \) since the diffusion is controlled by exponential localization. In our case \( L/\xi \) can exceed \( \xi \) many times as the resistance of segment \( L/\xi \) grows with \( L/\xi \) weakly superlinearly, not exponentially. A full calculation of \( L/\xi \) in the 1D wire is beyond the Fermi-liquid theory and has not yet appeared.

In summary, due to the thermal smearing and resonant tunnelling, the disordered 1D insulator shows at full coherence the crossover to the 1D “metal”. The resistance of the 1D “metal” grows with \( L/\xi \) first nearly linearly and then polynomially due to the medium localisation. This is in contrast to the expectation that the resistance of the coherent 1D system grows with \( L/\xi \) exponentially if \( L/\xi > 1 \). The 1D “metal” shows the resistance dispersion which is almost \( L/\xi \)-independent and smaller than unity, again in contrast with the expected (exponential) growth. Such 1D “metal” should be observable in any coherent 1D system of length \( L/\xi \lesssim 30 \), longer coherent segments are experimentally irrelevant. The crossover temperature is \( T_0 = 0.25T_\xi \), in the GaAs wires \( T_0 \approx 0.1 - 1 \text{ K} \).

M. M was supported by the APVT Agency grant APVT-20-021602. P. V. was supported by the EC fellowship HPMFCT-2000-00702 and VEGA grant 2/3118/23.

* Electronic address: martin.mosko@savba.sk

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