Crossing the phantom divide in brane cosmology with curvature corrections and 
brane-bulk energy transfer

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We consider the Randall-Sundrum brane-world model with bulk-brane energy transfer where
the Einstein-Hilbert action is modified by curvature correction terms: a four-dimensional scalar
curvature from induced gravity on the brane, and a five-dimensional Gauss-Bonnet curvature term.
It is remarkable that these curvature terms will not change the dynamics of the brane universe at
low energy. Parameterizing the energy transfer and taking the dark radiation term into account,
we find that the phantom divide of the equation of state of effective dark energy could be crossed,
without the need of any new dark energy components. Fitting the two most reliable and robust
SNIa datasets, the 182 Gold dataset and the Supernova Legacy Survey (SNLS), our model indeed
has a small tendency of phantom divide crossing for the Gold dataset, but not for the SNLS dataset.
Furthermore, combining the recent detection of the SDSS baryon acoustic oscillations peak (BAO)
with lower matter density parameter prior, we find that the SNLS dataset also mildly favors phantom
divide crossing.

I. INTRODUCTION

Numerous cosmological observations have confirmed that the universe is undergoing accelerated expansion. This
phenomenon was not predicted by conventional cosmology governed by general relativity with the known matter
constituents. To explain the cosmic acceleration, mysterious dark energy was proposed. There are many dark energy
models, which can be distinguished in the value and variation of the equation of state (EoS) \( w \) during the evolution
of the universe. The cosmological constant is the simplest candidate of dark energy, whose EoS \( w = -1 \) is located at
a central position among dark energy models. For quintessence [1], Chaplygin gas [2] and holographic dark energy
models [3], \( w \) always stays bigger than -1. The simplest model with \( w < -1 \) is the phantom models [4] but which will
violate not only the Weak Energy Condition but also Null Energy Condition. In general, dark energy can evolve with
the change of the EoS \( w > -1 \) to \( w < -1 \) (or vice versa). This transition is called “the phantom divide crossing”.

By current estimates, it seems likely that the phantom divide crossing occurs at the recent epoch [5], though many
of these estimations are model dependent. In particular, with an advantageous parametrization (the called CPL
parametrization) of dynamical dark energy [6], it has been found [7, 8] that most of the observational probes indeed
mildly favor dynamical dark energy crossing the phantom divide at \( z \sim 0.25 \). Moreover, in the era of structure
formation, a highly negative \( w \) makes negligible the undesirable dark energy to the total energy density. Hence even if
the observation still admits \( w = -1 \), it is still useful to construct a more general framework that can permit \( w < -1 \)[9]. However, it has been proved that the phantom divide crossing of dark energy described by the minimum coupling
single scalar field with general Lagrangian is either unstable with respect to the cosmological perturbations or realized
on the trajectories of the measure zero [10]. So most dark energy models with the phantom divide crossing either
consist of multiple scalar fields with at least one non-canonical phantom component [11] or must recourse to extending
gravity theory [12, 13]. The former is usually plagued by catastrophic UV instabilities, however it can be regarded as
an effective field description following from an underlying theory with positive energies [14]. An interesting example
where the UV pathologies are absent in phantom models is given by [15]. The latter, for example the so called 1/R
gravity [10], is severely constrained by solar system test and by cosmological observation even though it is interesting
theoretically. A novel model about \( f(R) \) gravity which pass through all tests is proposed by [17]. Other theories with
the phantom divide crossing are interacting holographic dark energy models [18], models with interactions between
dark matter and dark energy [19], model with a scale field coupled to the Gauss-Bonnet form [20], model with

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non-linear term of scalar curvature \[21\], and model considering quantum effects \[22\].

In another area of theoretical cosmology, the brane-world scenario has received a lot of interest. The well known Randall-Sundrum (RS) brane-world model \[23, 24\], inspired by D-brane ideology in string theory, envisages that our four-dimensional universe is a 3 dimensional membrane embedded in the five-dimensional bulk. The standard model particles are confined in the brane while the gravity is free to propagate into the bulk. This brane approach provides a new way of understanding the hierarchy between the four-dimensional Planck scale and the electro-weak scale. Two important generalizations of the RS model have been considered recently. The first is inspired by superstring theory which suggests the Gauss-Bonnet (GB) curvature corrections as the first and dominant quantum corrections to the Einstein-Hilbert action for a ghost-free theory \[25\]. The combined action in five dimensions gives the most general action with second-order field equation, as shown by Lovelock \[26\]. The DGP model suggests the second curvature correction term to RS model, the four-dimensional scalar curvature term. This induce gravity correction term can be interpreted as arising from a quantum effect due to the interaction between the bulk gravitons and the matter on the brane \[27\]. It is natural to study the influence of the GB correction to the DGP model. Indeed, in certain realization of string theory, the ghost-free GB term in the bulk action may naturally lead to DGP induced gravity term on the brane boundary \[28\]. It has been suggested that the combination of the two curvature corrections may shed some light on the singularity problem in the early universe \[29, 30\].

However, many brane-world models including the RS and GB types, produce only ultra-violet modifications to general relativity, with extra-dimensional gravity dominating at high energy. DGP model may manifest nontrivial low energy extra-dimensional gravity, which results in a new branch DGP(+) with late-time acceleration \[31\]. Its generalization with the brane and bulk cosmological constants \[15\] allows \( w < -1 \) and the branch DGP(-) which has the right RS limit with \( w > -1 \), but none of these has the phantom divide crossing behavior.

Another robust way that extra-dimension can affect the low energy evolution of the universe is through the coupling between the brane and the bulk. This is analogous to the coupled dark energy scenarios \[32\], where the late-time accelerating cosmological phase is characterized by a frozen ratio of dark matter/dark energy as a result of the interaction of the dark matter with other components, such as scalar fields. It has been proved that the transfer of energy between the bulk and the brane may result in the RS model with late-time acceleration \[33\] and even the phantom divide crossing, provided that there is bulk matter \[34\] or an additional dark energy on the brane \[35\]. However, the former can not make cosmological model which is independent of bulk dynamics, and the latter can not have the phantom divide crossing by using only geometric effect. Thebrane-bulk energy transfer has also been considered in DGP model for the present universe as a global attractor using fix-points theory \[36\] and in many other different setups \[37, 38\]. The combined curvature effect and bulk contents effect on the RS model have recently been studied in \[39\].

The aim of the present work is to study the low-energy cosmological behavior based on an extended RS(II) scenario \[24\] by considering brane-bulk energy transfer and two additional curvature corrections. We obtain a closed system of three equations which determines the parameters of the desired Friedmann equation. In the low energy region, it is remarkable that the curvature corrections terms do not change the dynamics of the system. By parameterizing the energy transfer with the scale factor, we are able to exactly solve the Friedmann equation. When including the effect of the dark radiation, which is neglected in \[34, 35\], we show that the phantom divide crossing may be achieved in the absence of additional dark energy components on the brane. Furthermore, we fit the model to the two most reliable and robust SN1a datasets, the new 182 Gold dataset \[40\] and the first year Supernova Legacy Survey (SNLS) dataset \[41\], respectively complemented by the recent SDSS baryon acoustic oscillations peak (BAO) dataset.

This letter is arranged as follows: In Sec. II, we establish the most general brane world model with curvature correction terms and bulk-brane energy transfer to describe the accelerated expansion. We investigate the equation of state of effective dark energy and discuss the possibility of phantom divide crossing. In Sec. III, we fit the model to the data from dark energy observations. In the last section, we conclude with a brief summary.

II. BRANE WORLD WITH CURVATURE CORRECTIONS AND BRANE-BULK ENERGY TRANSFER

Let us consider a braneworld model. For convenience and without loss of generality we choose the extra-dimension coordinates \( y \) such that the brane is located at \( y = 0 \) and the bulk has \( Z_2 \) symmetry under the transformation \( y \rightarrow -y \). The most general action which incorporates the induced gravity and Gauss-Bonnet corrections is \[30\]

\[
S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} (\mathcal{L}_{EH} + \alpha \mathcal{L}_{GB}) + \frac{1}{2\kappa_4^2} \int_{brane} d^4x \sqrt{-\tilde{g}} \mathcal{L}_{IG},
\]

where \( g \) \( (\tilde{g}) \) and \( \kappa_5 \) \( (\kappa_4) \) are the bulk (brane) metric and bulk (brane) gravitational constant, respectively. \( L_{EH} = R - 2\Lambda \) is the five-dimensional Einstein-Hilbert Lagrangian with negative cosmological constant \( \Lambda < 0 \). The Gauss-
Bonnet curvature correction term $L_{GB}$ is written as

$$L_{GB} = R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}. $$

We can define the Gauss-Bonnet coupling $\alpha$ through string energy scale $g_s$ as $\alpha = \frac{1}{2g_s^2}$. The induced-gravity Lagrangian $L_{IG} = \tilde{R} - 2\kappa_5^2 \lambda$ consists of four-dimensional scale curvature $\tilde{R}$ and brane tension $\lambda > 0$. We can define the induced-gravity crossover length scale by $r = \frac{\kappa_5^2}{\lambda}$. For convenience, we will choose the unit that $\kappa_5 = 1$ throughout this letter. Note that by setting $r = \alpha = 0$, we can recover the RS model. The RS model with Gauss-Bonnet correction and the DGP model correspond to the case with $r = 0$ and $\alpha = 0$, respectively.

By varying the action in Eq. (1) with respect to the bulk metric, we obtain the field equation

$$G_{AB} + 2\alpha H_{AB} = T_{AB}|_{\text{total}}, \quad (2)$$

where $H_{AB}$ is the second order Lovelock tensor

$$H_{AB} = R R_{AB} - 2R^C_A R_{BC} - 2R^{CD} R_{ABCD} + R^{CDEF} R_{CDE} - \frac{1}{4} g_{AB} L_{GB}. $$

The total energy-momentum tensor $T_{AB}|_{\text{total}}$ is decomposed into bulk and brane components

$$T_{AB}|_{\text{total}} = T_{AB}|_{\text{bulk}} + T_{AB}|_{\text{brane}} \delta(y).$$

Here, we use the normalized Dirac delta function, $\delta(y) = \sqrt{\gamma} / g \delta(y)$. The bulk component is

$$T_{AB}|_{\text{bulk}} = -\Lambda g_{AB} + T_{AB},$$

where $T_{AB}$ denotes any possible energy-momentum in the bulk. The brane component is written as

$$T_{AB}|_{\text{brane}} = -\lambda g_{AB} - r \tilde{G}_{AB} + \tilde{T}_{AB},$$

where $\tilde{G}_{AB}$ arises from the scalar curvature in Eq. (1). The energy momentum tensor $\tilde{T}_{AB}$ represents matter on the brane with energy density $\rho$ and constant equation of state parameter $w_m$.

The five-dimensional line element in the bulk is given by

$$ds^2 = -n^2(t,y) dt^2 + a^2(t,y) \gamma_{ij} dx^i dx^j + b^2(t,y) dy^2 \quad (3)$$

where $n_{ij}$ is a 3-dimensional maximally symmetric metric whose spatial curvature is characterized by $k = 0, \pm 1$. In this letter, we are interest in spatially flat brane $k = 0$. We choose the coefficients $n(t,0) = 1$ so that $t$ is the proper time along the brane. For simplicity, we assume that the fifth dimension is static $\bar{b} = 0$ and we set $b = 1$.

To determine the Friedmann equation on the brane, we impose the junction condition for a braneworld in Gauss-Bonnet gravity. For later use, we define

$$\Phi = \frac{1}{2} \left( \frac{\dot{a}}{a} \right)^2 - \frac{a'}{a^2},$$

where prime and dot denote the derivative with respect to $y$ and $t$, respectively. The jump of the (00) and $(ij)$ components of the field equation Eq. (2) across the brane gives

$$2\left[ 1 + \frac{8}{3} \alpha (H^2 + \frac{\Phi_0}{2}) \right] \frac{a_+'}{a_0} = rH^2 - u, \quad (4)$$

$$2\frac{n'_+}{n_0} + 4\alpha \frac{n'}{a_0} + 8\alpha \Phi_0 + 16a_0 \frac{a'_+}{a_0} (H^2 + \dot{H}) = 3r (H^2 + \frac{2}{3} \dot{H}) + v, \quad (5)$$

where $2a'_+ = -2a'_-$ and $2n'_+ = -2n'_-$ are the discontinuities of the first derivatives. $H = \frac{\dot{a}}{a_0}$ is the Hubble constant on the brane and $\Phi_0 = \Phi(t,0)$. We define

$$u = \frac{1}{3} (\rho + \lambda), \quad v = (w_m \rho - \lambda).$$
One can show that Eq. (4) can be written in terms of the square of Hubble parameter $H^2$

$$4[1 + \frac{8}{3} \alpha (H^2 + \frac{\Phi_0}{2})]^2 (H^2 - \Phi_0) = [rH^2 - u]^2. \tag{6}$$

Once we know the behaviors of $\Phi_0$ and $\rho$, Eq. (6) can be solved and its solution will give the desired Friedmann equation.

Let us consider $\rho$ first. Since the left-hand side of Eq. (2) is divergence-free, the total energy momentum tensor is conserved in the bulk

$$\nabla_A T^A_{\text{total}} = 0.$$ Its zero component is

$$\dot{T}_{00} + 3\frac{\dot{a}}{a}(T_{00} - T_{11}) - (n' + 3\frac{a'}{a})T_{50} + T_{55} - [\dot{\rho} + 3(1 + w_m)H\rho] \delta(y) = 0. \tag{7}$$

Integrating around $y = 0$ and using the $Z_2$ symmetry ($T_{50}(t, +0) = -T_{50}(t, -0)$), we can determine the evolution of $\rho$

$$\dot{\rho} + 3(1 + w_m)H\rho = 2T_{05}. \tag{7}$$

This implies that the energy conservation law on the brane is broken. Another method to obtain the semi-energy conservation law Eq. (7) is to solve the junction conditions Eq. (4) and Eq. (5). Their exact solutions are complicated but we can make further simplification by assuming $\alpha$ to be small. This is a reasonable assumption since we are interested in the effect of GB correction on the late-time evolution of the universe. The solutions of the Eq. (4) and Eq. (5) up to the first order in $\alpha$ are

$$\frac{a'}{a} + \frac{a}{a} = 1 + 2rH^2 - u + \frac{1}{6} v (rH^2 - u) \left[ r^2 H^4 + u^2 - 2H^2 (6 + ru) \right] \alpha, \tag{8}$$

$$\frac{n'}{n_0} = \frac{1}{2} \left( 2u + v + rH^2 + 2rH \right) + \frac{1}{6} \left( r^3 H^6 - 3rH^4 (-4 + ru + rv + r^2 H) + u(8 + 5ru + 2rv) \right) - 3H^2 \left[ 4v + u(8 + 5ru + 2rv) + 4r(4 + ru)H \right] \alpha. \tag{9}$$

Note that Eq. (4) is a cubic equation for the discontinuity $\frac{a'}{a_0}$ which has one real solution, the other two are complex. Since we require our cosmological equations to have the right $\alpha \to 0$ limit, we take only the real solution. Substituting Eq. (8) and Eq. (9) into the (05) component of the field equation Eq. (2)

$$3(n' \frac{a'}{a} - \frac{a'}{a})(1 + 4\alpha \Phi) = T_{05}, \tag{10}$$

we recover (up to the first order of $\alpha$) the same semi-conservation law Eq. (7).

Let us consider the function $\Phi$. For brane cosmology with Gauss-Bonnet correction, $\Phi$ is just the first integral satisfying the constraint equation

$$\Phi + 2\alpha \Phi^2 = \frac{\Lambda}{6} + \frac{C}{a^2},$$

where $\frac{C}{a^2}$ is the dark radiation term. We point out an important observation that the above first integral can be recovered from a general differential equation with nontrivial $T_{05}$ and $T_{55}$

$$\psi + 4\frac{\dot{a}}{a} + 4T_{05} \frac{a'}{a} + 4\frac{\dot{a}}{a} (T_{55} - \Lambda) = 0, \tag{11}$$

where

$$\psi = 6(\Phi + 2\alpha \Phi^2).$$

Notice that we only consider the solution having the right $\alpha \to 0$ limit

$$\Phi = \frac{-3 + \sqrt{3\sqrt{3 + 4\alpha \psi}}}{12\alpha}. \tag{12}$$
The differential equation Eq. (1) is obtained by substituting $\frac{a'}{a}$ from Eq. (10) into the (55) component of the field equation Eq. (2)

$$3 \left( \frac{a'}{a} \right)^2 - \frac{1}{a^2} (1 + 4a) - 3 \Phi = T_{55} - \Lambda.$$ 

Note that for vanishing $T_{05}$ and $T_{55}$, the constraint equation Eq. (11) gives a bulk equation. But for generic values of $T_{05}$, the constraint equation depends on the discontinuity of the first derivative on the brane. Moreover, we point out that the physical meaning of $\psi$ can be understood as the effective bulk cosmological constant. To see this, we rewrite Eq. (11) as

$$\dot{\psi}_1 + 4 \frac{\dot{a}}{a} \psi_1 + 4T_{05} \frac{a'}{a} + 4 \frac{a'}{a} T_{55} = 0 \quad (13)$$

where $\psi_1 = \psi - \Lambda$ takes the role as the correction to the bulk cosmological constant. We can easily see that, for $T_{05} = T_{55} = 0$, the only correction to bulk cosmological constant $\Lambda$ is the dark radiation term $\sim a^{-4}$. For a nontrivial bulk, the term $4T_{05} \frac{a'}{a} + 4HT_{55}$ in Eq. (11) will give further corrections to $\Lambda$. We will come back to discuss this point later.

Generally, Eq. (6) has three solutions of $H^2$, but only two of them are left for small $\alpha$:

$$H^2 = \frac{6 + 3ru \mp \phi}{3r^2} + \frac{\alpha}{9r^4} \{32[\mp 9r^2u + 6ru\phi \mp 2\phi(\mp 6 + \phi)] \pm 8\sqrt{(\pm 18 + 3ru + \phi) \psi \pm r \psi^2}\}, \quad (14)$$

where we have used Eq. (12) and $\phi = \sqrt{6 + 6ru - r^2 \psi}$. These two branches recover the two branches of DGP model when $\alpha \to 0$. One can find three equations (7), (11), and (14) consisting of a closed system for variables $\rho$, $\Phi$ and $H$, which determines the evolution of the universe (provided the bulk energy-momentum tensor is known). However, these equations are entangled in a complicated way. In the following, we will seek the simplest low-energy effective theory which can be analytically solved.

We would like to make some assumptions here. First, we consider constraints on $\rho$ and $H^2$. In low energy region, we can assume

$$\rho \ll \lambda, \rho \ll \frac{\Lambda}{\alpha^3}, H^2 \ll \frac{\lambda}{r}.$$ 

(15)

This low energy region is self-consistent and can be easily realized. The simplest example is the case where the brane tension and the bulk cosmological constant are very large. Moreover, it should be noticed that in the $\alpha \to 0$ and $r \to 0$ limits Eq. (15) is consistent with RS low energy region

$$\rho \ll \lambda, H \ll r, \lambda \rho \sim H^2, \Lambda \sim \lambda^2$$ 

(16)

where the last constraint comes from the well known RS fine-tuning mechanics. Noticing that the present Hubble rate $H_0$ is much smaller than the string energy scale $g_s$ and $H^2/H_0^2$ is close to 1 at late time, we have $\alpha H^2 \ll 1$. Hence the region $\rho \ll \frac{\lambda}{\alpha M}$ does not suggest new energy constraint beyond Eq. (16). Similar consideration can be applied to the $rH^2 \ll \lambda$ constraint in Eq. (15) if we set $rH \sim 1$ in DGP$(\pm)$ brane [23]. Second, to derive a cosmological system that is largely independent of the bulk dynamics, usually one can assume that on the brane the contribution of $T_{55}$ relative to the bulk vacuum energy is much less important than the brane matter relative to the brane vacuum energy, or schematically $\frac{T_{55}}{H\rho} \ll \frac{\lambda}{\alpha}$. Thus, $T_{55}$ can be omitted in the low energy region. Third, the semi-conservation law Eq. (7) implies $T_{05} \lesssim H\rho$ in general. Fourth, at late time, the dark radiation may be assumed to be much smaller than bulk cosmological constant. By taking into to account the above assumptions, we find that Eq. (13) can be simplified as

$$\dot{\psi}_1 + 4H \psi_1 + 4T_{05} A = 0,$$ 

(17)

where $A$ is a constant defined by

$$A = -\frac{\lambda}{6} (1 - \frac{\lambda^2}{27\alpha}).$$

It should be noticed that $T_{05} A \ll HA$. Returning to the discussion about the correction to the bulk cosmological constant. We can now conclude that the correction to the bulk cosmological constant $\psi_1$ is much smaller than the
bulk cosmological constant, \( \psi_1 \ll \Lambda \). This is reminiscent of \( \rho \ll \lambda \) on the brane, and can be understood as bulk-brane duality. Thus, Eq. \ref{eq:14} can be expanded in terms of \( \rho \) and \( \psi_1 \) (keeping only the first order terms):

\[
H^2 = B\rho + C\psi_1 + D,
\]

where we absorb \( \lambda \) and \( \Lambda \) into the constant coefficients

\[
B = \frac{E \mp 6}{3rE^2} + \frac{2\alpha}{9r^3E^3} \{32(E \mp 6)E^2 - 9\rho[\pm 8\rho \Lambda(8 + 3r\lambda) \pm 32\lambda(4 + r\lambda) \pm 3r^2\Lambda^2]\},
\]

\[
C = \pm \frac{1}{E} + \frac{\alpha}{E^3r^2} \{-96(E\lambda \mp 6) - 32r\lambda(E \mp 9) + 8r^2\Lambda(2E \mp 27) \mp 3r^4\Lambda^2\},
\]

\[
D = \mp \frac{E + r\lambda}{3r^2} + \frac{\alpha}{9Er^4} \{64\rho(6 \mp 6E/(6 \mp E)) \mp 32r^2\lambda^2 \pm 8r^2[\pm \Lambda(3 \mp 4E + r\lambda) \pm 4\Lambda^2]\},
\]

with \( E = \sqrt{6(6 + 2r\lambda - r^2\Lambda)} \).

Observing the three equations Eq. \ref{eq:7}, Eq. \ref{eq:17}, and Eq. \ref{eq:18}, we find that the curvature corrections only affect the constants \( A, B, C, D, E \), and they will not vanish in the \( \alpha \to 0 \) and \( r \to 0 \) limits. This remarkable result immediately tells us that the curvature corrections will not change the form of the desired Friedmann equation, and the corresponding evolution of the universe.

To obtain the explicit solution we need to know the form of energy transfer \( T_{05} \). Unfortunately, it is not yet available and obviously depends on the mechanism which produces the energy transfer. We consider the ansatz \( T_{05} = THa^v \) (\( T \) is a constant) which was used in \cite{34, 35} for RS brane world. A justification for the ansatz has been analyzed in \cite{35} through a simple model where the bulk content is a relativistic fluid, slowly moving along the fifth dimension. Recently, the ansatz has also been used in \cite{46}. Then we find

\[
\rho = Fa^{-3} + \frac{2T}{3+v}a^v
\]

where we take \( w_m = 0 \) for dark matter, and

\[
\psi_1 = Ga^{-4} + \frac{2T\lambda(27 + \alpha\lambda^2)}{81(4 + v)}a^v
\]

where \( G \) is an integration constant. Notice that the last term in Eq. \ref{eq:20} denotes the energy flow into \( (\frac{2T}{3+v} > 0) \) or out of \( (\frac{2T}{3+v} < 0) \) the brane. Substituting Eq. \ref{eq:20} and Eq. \ref{eq:21} into Eq. \ref{eq:18}, we have

\[
H^2 = \Omega_{0m}a^{-3} + \Omega_{0v}a^v + \Omega_{0d}a^{-4} + D,
\]

where

\[
\Omega_{0m} = BF, \quad \Omega_{0v} = B \frac{2T}{3 + v} + C \frac{2T\lambda(27 + \alpha\lambda^2)}{81(4 + v)}, \quad \Omega_{0d} = CG.
\]

To impose the vanishing effective cosmological constant on the brane, we choose \( D = 0 \). It just recovers RS fine-tuning when \( \alpha \to 0 \). Finally, we can write

\[
H^2 = \Omega_{0m}a^{-3} + \Omega_{0d}a^{-4} + \Omega_{0v}a^v.
\]

This is similar to the well known result in RS model where the Friedmann equation can be generalized by adding the term depending on the brane-bulk energy flow. The curvature corrections only affect the coefficients \( \Omega_{0m}, \Omega_{0d} \) and \( \Omega_{0v} \). However, it should be pointed out that, they may play an important role, for example, when the constant \( G \) in dark radiation term and the parameter \( T \) which characterizes the brane-bulk energy exchange are not big enough to provide desired nontrivial cosmological behavior. In particular, the dark radiation term is usually neglected if one considers the large scale factor at late time. In RS model, the reason that we can keep it is that indeed we do not know the constant \( G \) which reflects the bulk geometry (Notice that it takes the role of the bulk black hole mass in a Schwarzschild-AdS\(_3 \) geometry \cite{44}). In our model, furthermore, the curvature corrections embodied in constant \( C \) may strengthen the need for keeping the dark radiation term. For an explicit example, we assume \( G \ll 6\lambda(a) \) so that the dark radiation term can be omitted \( \Omega_{0d}a^{-4} \ll \Omega_{0m}a^{-3} \) in RS case where \( C = \frac{1}{3}, \ B = \lambda \). For the dark radiation term to be important \( \Omega_{0d}a^{-4} \sim \Omega_{0m}a^{-3} \), we need \( G \sim \frac{BF}{a} \ll 6\lambda(a) \) i.e. \( \frac{BF}{a} \ll 6\lambda \). Considering the induced gravity correction and using lower branch for simplicity, we find that the condition is satisfied when \( r \gg \frac{38\lambda}{54\lambda^2 + \lambda} \), which does
FIG. 1: The EoS $w_{\text{eff}}$ (red line) with prior $z_T = 0.2$, $v = 1$ and deceleration factor $q$ (green line) with prior $z_T = 0.2$, $v = 1$, $\Omega_{0m} = 0.25$ versus redshift $z$. One can find that $w_{\text{eff}}$ crosses -1, and the $q$ crosses 0 at $z \sim 0.5$ and the order of magnitude $q|_{z=0} \sim -1$.

not violate the low energy region Eq. (13). Another example can be given by considering the relationship between the accelerated expansion and the brane-bulk energy flow. The accelerated expansion is characterized by the deceleration parameter $q = -\frac{\ddot{a}}{a H^2}$. Using $a = a_0 (1 + z)$ and taking $H^2|_{z=0} = 1$, we can rewrite the Friedmann equation as

$$H^2 = \Omega_{0m} (1 + z)^3 + \Omega_{0v} (1 + z)^{-v} + (1 - \Omega_{0m} - \Omega_{0v})(1 + z)^4$$

where we have absorbed $a_0$ into $\Omega_{0m}$ and $\Omega_{0v}$. The deceleration parameter can be given as

$$q = -1 + \frac{1}{2} \frac{d \log H^2}{d \log (1 + z)}.$$

Now we will omit the terms which decrease more quickly than matter density at late time, then the dark radiation term is absent and one can find that the present accelerated expansion $q|_{z=0} < 0$ needs $v > -3$ and $\Omega_{0v} > \frac{1}{3+v}$. This suggests that in RS case (noticing $\lambda > 0$ in RS(II) brane world), the bulk energy must flow into the brane (the later term in Eq. (20) is positive, i.e. $T > 0$). However, if we consider the curvature correction (still the induced gravity correction and using the lower branch), we find that accelerated expansion may be achieved when the bulk energy flows out of the brane, which only needs $|\frac{\delta}{\delta \nu}| < \frac{3+v}{4(1+z)^3+\lambda}$ that can be easily realized.

Let us consider whether or not this model is permitted to cross the phantom divide and what is favored by fitting the model to observational datasets. We find that the combination of the last two terms in Eq. (22) may achieve the phantom divide crossing, without the need of any other dark energy components. The EoS of effective dark energy can be written as

$$w_{\text{eff}} = -1 + \frac{1}{3} \frac{d \log [H^2 - \Omega_{0m} (1 + z)^3]}{d \log (1 + z)}.$$

It is easy to see that $w_{\text{eff}}$ is a constant if the dark radiation term is omitted $\Omega_{0v} = 1 - \Omega_{0m}$ or the brane-bulk energy transfer is trivial $\Omega_{0v} = 0$. But if both of them are taken into account, we may achieve the phantom divide crossing in very broad parameter space. Explicitly, let us assume $w_{\text{eff}} = -1$ at $z_T$, then

$$w_{\text{eff}}|_{z=0} = -1 + \frac{4v [1 - (1 + z_T)^{4+v}]}{3[v + 4(1+z_T)^{4+v}]}.$$

When $z_T > 0$, one can find $w_{\text{eff}}|_{z=0} < -1$ if $v > 0$. This implies that the brane-bulk energy flow increases with the expansion of the universe. For an explicit example of the phantom divide crossing and current accelerated expansion, see Fig. 1.

III. OBSERVATION OF THE PHANTOM DIVIDE CROSSING

Now we use the observational datasets to test our cosmological model. We will use the new 182 gold supernova Ia data ($0 < z < 1.76$) and the first year Supernova Legacy Survey (SNLS) dataset ($0 < z < 1$), combined with the recent
BAO measurement from SDSS to fit our model. Recently these data have been widely used in the fittings of different braneworld cosmological models [47]. There are other dark energy observational probes, including the 3-year WMAP CMB shift parameter [48], the X-ray gas mass fraction in clusters [49], and the linear growth rate of perturbations at $z = 0.15$ as obtained from the 2dF galaxy redshift survey [50]. However, since our model is effective at low energy, we will not use the 3-year WMAP CMB shift parameter focusing on the high redshift region. Besides, for simplicity, we do not adopt other probes of dark energy which have large relative errors compared with SnIa, SNLS, CMB and BAO probes [8].

For the supernova Ia data, the measured quantity is the bolometric magnitude $m$

$$m = \bar{M} + 5 \log_{10}(D_L)$$

where $\bar{M}$ is the Hubble-parameter-free absolute magnitude

$$\bar{M} = M - 5 \log_{10} \left( \frac{H_0^{-1}}{M_{pc}} \right) + 25.$$ 

$M$ is the absolute magnitude, and

$$D_L = (1 + z) \int_0^z dz' \frac{H_0}{H(\Omega_{0m}, \Omega_{0v}, v)}$$

is the Hubble free luminosity distance ($H_0 d_L/c$). The data points of the Gold dataset are given after implementing correction for galactic extinction, $K$-correction and light curve width-luminosity correction, in terms of the distance modulus

$$u_{\text{Gold}}^{\text{obs}}(z_i) = m_{\text{Gold}}^{\text{obs}}(z_i) - M.$$ 

For SNLS datasets, also presents for each point, the stretch factor $s$ used to calibrate the absolute magnitude and the rest frame color parameter $c$ which mainly measures host galaxy extinction by dust. Thus, the distance modulus in this case depends apart from the absolute magnitude $M$, on two additional parameters $\alpha$ and $\beta$

$$u_{\text{SNLS}}^{\text{obs}}(z_i) = m_{\text{SNLS}}^{\text{obs}}(z_i) - M + \alpha(s_i - 1) - \beta c_i.$$ 

Let us define the theoretical distance modulus

$$u_{\text{th}}(z_i) = m_{\text{th}}(z_i) - M = 5 \log_{10}(D_L) - 5 \log_{10} \left( \frac{H_0^{-1}}{M_{pc}} \right) + 25.$$ 

We shall assume that the supernova Ia measurements come with uncorrelated Gaussian errors $\sigma_i^2$ (including flux uncertainties, intrinsic dispersion of SnIa absolute magnitude and peculiar velocity dispersion) in which case the likelihood function is given by the $\chi^2$ distribution

$$\chi^2(\Omega_{0m}, \Omega_{0v}, v) = \sum_i \frac{[u_{\text{obs}}(z_i) - u_{\text{th}}(z_i)]^2}{\sigma_i^2}$$

where $N = 182$ for Gold datasets, and $N = 115$ for SNLS.

For BAO measurement, we shall use the model independent measurement of the parameter [51]

$$A = \Omega_{0m}^{\frac{1}{2}} \left( \frac{H_0}{H} \right)^{\frac{1}{2}} \left[ \frac{1}{0.35} \int_0^{0.35} \frac{H_0}{H} dz \right] = 0.469 \pm 0.017$$

to construct an additional term in the $\chi^2$ equation Eq. (23)

$$\chi^2_{BAO} = \frac{[A(\Omega_{0m}, \Omega_{0v}, v) - 0.469]^2}{0.017^2}.$$ 

The theoretical model parameters are determined by minimizing the $\chi^2$ and $\chi^2 + \chi^2_{BAO}$.

In order to investigate the dependence of the resulting best fits on the prior of $\Omega_{0m}$, we will consider two cases $\Omega_{0m} = 0.2$ and $\Omega_{0m} = 0.3$ instead of marginalizing over $\Omega_{0m}$. The range between the two cases includes the current best fit value of $\Omega_{0m}$ based on WMAP and SDSS which is $\Omega_{0m} = 0.24 \pm 0.02$ [52]. Considering the errors using the covariance matrix method [52], we show the best fit form of $w_{\text{eff}}$ for each dataset category in Fig. (2). The corresponding $\chi^2$ contours in the remaining two parameter space is shown in Fig. (3).
FIG. 2: The best fit form of $w_{eff}(z)$ for different dataset category for both $\Omega_{dm} = 0.2$ and $\Omega_{dm} = 0.3$. The categories are: Gold182 dataset and it with BAO (row 1), SNLS and it with BAO (row 2). The dashed line in each panels represents the phantom divide.

FIG. 3: The 68% and 95% confidence contours in the $\Omega_{\nu} - v$ parameter space for each dataset category for both $\Omega_{dm} = 0.2$ and $\Omega_{dm} = 0.3$. The cross point of dashed lines in each panels represents $\Lambda CDM$.

IV. SUMMARY

We have studied the cosmological dynamics of the general RS brane world scenario with brane-bulk energy transfer and two curvature correction terms. For small Gauss-Bonnet coupling, we obtain a closed system of three equations having two branches that correspond to the two branches in the DGP model. They describe the evolution of the Hubble rate, the energy density and the time-dependent effective bulk cosmological "constant". We find that in the low energy region Eq. (14), these two branches have equivalent dynamics as that of the RS model with the brane-bulk energy exchange. Furthermore, we have shown that the phantom divide crossing can be achieved when the dark radiation term is presented and the brane-bulk energy flow increases with the expansion of the universe, without the need of any additional dark energy components on the brane [34], or bulk matter [35], or even exotic phantom material [4]. From the fitting, it has been revealed that the model indeed has a small tendency of phantom divide crossing for the Gold dataset, but not for the SNLS dataset. This is consistent with the analysis in [7,8] for CPL parametrization of dynamical dark energy. We have further shown that the BAO constraint with the lower matter density prior mildly changes the tendency of SNLS dataset and favors the $w$ crossing $-1$. 
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