1 Introduction

Let me start by reminding you what’s going on at all the major High Energy Physics laboratories around the world. At CERN, the LEP program is winding down, and the LHC (Large Hadron Collider) is the Lab’s future. 7 TeV protons on 7 TeV protons, a center of mass energy of 14 TeV. Four large detectors are planned. Two, ATLAS and CMS, will study high \( p_T \) physics, searching for Higgs, SUSY, etc. One, ALICE, will collide high \( Z \) nuclei (when protons aren’t being collided), and study the quark-gluon plasma. And one, LHC-B, will study bottom quark physics. It is a sobering thought that a 14 TeV accelerator will be used to study a 5 GeV object, 3 orders of magnitude down the energy scale. (But one should not forget that the Tevatron is used to study kaon physics, again 3 orders of magnitude down the energy scale.)

At DESY, the main facility is HERA, an electron-proton collider, with 800 GeV protons on 30 GeV electrons. The two principal detectors, H1 and ZEUS, study these collisions, investigating deep inelastic scattering over a kinematic range far broader than heretofore. But the proton beam will also be used, on a fixed target (wires in the fringe of the beam) for bottom quark physics, in the HERA-B experiment.

At KEK, in Japan, TRISTAN, an \( e^+ e^- \) collider operating at a center-of-mass energy of 60 GeV has been shut down, and replaced by an asymmetric \( e^+ e^- \) collider, 8 GeV on 3.5 GeV, a center of mass energy of 10 GeV, to do bottom quark physics, with the Belle experiment.

At SLAC, the SLC (SLAC Linear Collider), \( e^+ e^- \) collisions at center-of-mass energies around 90 GeV, has been shut down, and replaced by PEP-II, an asymmetric \( e^+ e^- \) collider, 9 GeV on 3 GeV, a center of mass energy of 10 GeV, to do bottom quark physics with the BaBar experiment. Thus, the study

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of the $Z^0$, a 90 GeV object, is giving way to the study of the $b$ quark, a 5 GeV object.

At Fermilab, the main facility is the Tevatron, which collides 1 TeV protons against 1 TeV antiprotons, for a center-of-mass energy of 2 TeV. There are two general purpose detectors, operated by two large collaborations, CDF and DØ. The primary goal of the running recently completed was the discovery of the top quark. Goals for the next running period (Run II) include precise measurements of top quark and $W$ boson masses, and searches for “new physics” – Higgs, SUSY, etc. But CDF has had an active program in bottom quark physics, and foresees an expanded program in Run II. A displaced vertex trigger is being implemented, in part to strengthen the $b$ physics program. DØ has done little $b$ physics so far, lacking a magnetic field in the central tracking volume. They are remedying this for Run II, and anticipate an active $b$ physics program. And serious consideration is being given to a third detector, B-TeV, which would be a dedicated bottom quark experiment.

Finally, Cornell’s Laboratory for Nuclear Studies, with a symmetric $e^+e^-$ collider (CESR), has been doing bottom quark physics for two decades.

So, bottom quark physics must be interesting, because all the major labs have it as part of their program. Why is bottom quark physics so interesting? (The cynic might argue that the labs are into bottom quark physics because it’s affordable. There is perhaps some truth in this. But it doesn’t explain LHC-B. It doesn’t explain the interest in bottom quark physics within CDF, nor SLAC’s preference for studying a 5 GeV object over a 90 GeV object.) Why is bottom quark physics interesting? A primary goal of my talk will be to answer that question for you.

Bottom quark physics can be conveniently divided into three eras;

- The Early Days – 1977-88, further divided into Discovery – 1977-80, and Roughing out the Qualitative Features – 1980-88
- Beginnings of Precision Measurements and Rare Decay Studies – 1989-98
- The ‘Factory’ Era – 1999-??

In Section 2, I’ll discuss the early days.

In Section 3, I’ll point out a change in objective that took place around 1990, and give a brief review of the flavor sector of the Standard Model.

Then, in Sections 4, 5, and 6, I’ll discuss three of the “hot topics” in $b$ physics today: determination of $|V_{ub}/V_{cb}|$, rare hadronic $B$ decays, and the radiative penguin decay $b \to s\gamma$. 
Figure 1: Dimuon mass distribution from collisions of 400 GeV protons with a nuclear target, showing the Υ states (Lederman and collaborators).

2 The Early Days

2.1 Discovery – 1977-80

The $b$ quark was discovered in its hidden form ("hidden beauty", "covered bottom") at Fermilab, in 1977, by Leon Lederman and collaborators. They measured the mass distribution of dimuon pairs from collisions of 400 GeV protons on a nuclear fixed target, and observed a structure consisting of two or more peaks in the 9.4-10.0 GeV region (see Fig. 1). The immediate (and correct) interpretation was a bound system of a quark-antiquark pair, charge $-1/3$ quarks. The bound system was named the Upsilon (Υ).

The DORIS $e^+e^-$ storage ring at DESY, at the time of the Υ discovery, had insufficient energy to produce Υ’s. The machine energy was increased, and in 1978, straining their RF, physicists at DORIS observed two narrow resonances, Υ(1S) and Υ(2S). They could go no higher.

The CESR $e^+e^-$ storage ring at LNS, Cornell, gave first luminosity to the CLEO and CUSB detectors in October, 1979. The Υ(1S) and Υ(2S) resonances were quickly located, and in December, in time to be “added in proof” to the Lab’s Christmas card, the Υ(3S) was discovered (see Fig. 2).

The three resonances, Υ(1S), Υ(2S), Υ(3S) were all narrow, with widths less than the instrumental resolution (beam energy spread). The production
rates, leptonic decay branching fractions, level spacings, all matched very well with the bound $b\bar{b}$, charge $-1/3$ quark interpretation.

While there was no doubt, by then, about the existence of the bottom quark, the studies needed to determine further properties were of its weak decays. These could not be obtained from ‘hidden beauty,’ because a bound $b\bar{b}$ system decays via the strong interaction, with $b$ and $\bar{b}$ quarks annihilating each other, forming gluons or a virtual photon. For studies of the weak decay of the bottom quark, “bare bottom,” or “naked beauty” was needed.

Bare bottom was discovered at CESR by the spring of 1980. A scan, measuring cross section for production of hadronic events vs center-of-mass energy, above the $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, revealed another resonance. This one was measurably broad (see Fig. 2), indicative of a rapid decay into $b$-flavored mesons, $\Upsilon(4S) \to B\bar{B}$. The compelling evidence for bare bottom came from the yield of muons and electrons, which also peaked at the $\Upsilon(4S)$ resonance (see Fig. 3), indicating the decay sequence $\Upsilon(4S) \to B\bar{B}$ (via the strong interaction), followed by $B \to X\ell V$ (via the weak interaction). Leptons, a tell-tale signature of a weak decay, established bare bottom.

2.2 Roughing out the Qualitative Features – 1980-88

A series of measurements, from 1980 to 1988, determined the qualitative features of the $b$ quark.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{mass_cross_section.png}
\caption{Hadronic cross section vs. center-of-mass energy, showing three narrow resonances $\Upsilon(1S,2S,3S)$, and one broad resonance $\Upsilon(4S)$ (CLEO).}
\end{figure}
2.2.1 Semileptonic Decay Branching Fraction

If the $b$ decays by a charged current interaction, $b \rightarrow W_{\text{virtual}}c$ or $b \rightarrow W_{\text{virtual}}u$, then by simple counting of the $W_{\text{virtual}}$ final states ($\bar{u}d$, $\bar{c}s$, $\tau\nu$, $\mu\nu$, $e\nu$), allowing for a factor of 3 for color for $\bar{u}d$ and $\bar{c}s$, one predicts a semileptonic decay branching fraction of $1/9$. Phase space suppresses $\bar{c}s$ and $\tau\nu$, and hadronic final state interactions enhance $\bar{u}d$ and $\bar{c}s$, leading to a theoretical prediction for the semileptonic decay branching fraction of $\approx 12\%$. Early measurements were in qualitative agreement. (Aside – now, in the precision era, the measurements appear to be 1-2\% below the theory, and that difference is not understood.)

2.2.2 Ruling out Topless Models

Giving that the bottom quark exists, is there a top quark? That was a very real question in the early 1980’s, because searches at PEP and PETRA had come up empty, and it was (then) hard to imagine that top was more than 2-3 times heavier than bottom. Producing “topless models” became an industry among theorists. Shooting them down became an experimental responsibility.

The simplest of the topless models had $b$ a weak isospin singlet, decaying by flavor-mixing with $s$ and $d$. In this case, the GIM mechanism would be inoperative, and there would be flavor-changing neutral decays of $b$, in particular $b \rightarrow s\ell^+\ell^-$. Kane and Peskin derived a lower limit on the ratio
\[ \Gamma(b \rightarrow s \ell^+ \ell^-)/\Gamma(b \rightarrow c \ell \nu), \] for this topless model. CLEO (1984) and Mark J (1983) showed that the ratio was below the Kane-Peskin limit, ruling out that model.

A more complicated topless model had \( b \) a weak isospin singlet, but decaying \textit{not} by flavor mixing but by some new mechanism – exotic decays, which gave rise to enhanced yields of charged leptons and/or neutrinos and/or baryons. CLEO (1983) knocked that model off, by measuring yields of \( \mu, e, p \), and missing energy.

The last stand of topless models was a particularly ugly one due to Henry Tye. It had \( b \) in a right-handed doublet with \( c \). Its decays mimicked \( b \rightarrow c W^- \) reasonably well. However, its predicted production asymmetry, in \( e^+ e^- \rightarrow (\gamma \text{ or } Z^0) \rightarrow b \bar{b} \), was very different, in the \( \gamma - Z^0 \) interference region, from the predictions for \( b \) a left-handed doublet with \( t \). Experiments at PETRA (1985) established the left-handed doublet nature of \( b \), killing the final topless model. Although it wouldn’t be discovered for another 10 years, by 1985 it was clear that top \textit{had} to exist.

2.2.3 \( |V_{ub}/V_{cb}| \)

Does \( b \) decay predominantly to \( c(b \rightarrow c W^- \text{virtual}) \) or to \( u(b \rightarrow u W^- \text{virtual}) \)? While there was a bias favoring \( b \rightarrow c \), as of 1980 there was no strong theoretical argument favoring \( b \rightarrow c \), nor any experimental evidence.

First evidence came from the kaon yield in \( B \) decay (CLEO, CUSB, 1982), which was large, as would be expected for a \( b \rightarrow c \rightarrow s \) sequence. The yield implied \( |V_{ub}/V_{cb}|^2 < 0.15 \).

Next evidence came from the lepton momentum spectrum. Since \( u \) is lighter than \( c \), \( b \rightarrow u \ell \nu \) will have a stiffer lepton spectrum than \( b \rightarrow c \ell \nu \) (see Fig. 4). By measuring the lepton spectrum and fitting to a mix of \( b \rightarrow u \ell \nu \) and \( b \rightarrow c \ell \nu \), CLEO (1984) established that \( |V_{ub}/V_{cb}|^2 < 0.04 \). By concentrating on the endpoint region of the spectrum, with more data, CLEO (1987) established that \( |V_{ub}/V_{cb}|^2 < 0.02 \). Finally, with still more data, CLEO (1990) saw leptons beyond the \( b \rightarrow c \ell \nu \) endpoint, establishing that \( |V_{ub}/V_{cb}|^2 > 0 \).

2.2.4 \( B \) Reconstruction

Although there was no doubt about the existence of the \( b \) quark in its bare form, and thus no doubt about the existence of \( b \)-flavored hadrons, it was nonetheless important to ‘reconstruct’ them, to assemble the decay products and show that they came from the decay, e.g., of a \( B \) meson. Aside from the aesthetics of “it’s got to be there, so you must \textit{show} that it is there”, \( B \) reconstruction was needed to determine the \( B \) meson mass. CLEO did this in 1983.
2.2.5 $b$ Lifetime

The $\Upsilon(4S)$ has a mass just slightly above $B\overline{B}$ threshold. As a result, the only decay of $\Upsilon(4S)$ is $\Upsilon(4S) \rightarrow B\overline{B}$. There are no extra particles to confuse the situation. However (at a symmetric $e^+e^-$ collider) the $B$ and $\overline{B}$ are moving very slowly, with momenta $\sim 300$ MeV/$c$, $\beta \sim 0.06$. This is not a suitable environment for determining $b$ lifetime.

For $e^+e^-$ collisions at higher energies, the $b$-flavored hadrons will be moving faster, but the signal-to-noise will be less favorable (1 in 11, rather than 1 in 4), and the events will be more complicated (many particles in addition to the two $b$-flavored hadrons). For measuring the $b$ lifetime, the higher speed of the $b$-flavored hadrons overcomes the disadvantages just mentioned, and makes lifetime measurements possible. In 1983, MAC and Mark II, at PEP, made such measurements. They found the lifetime to be long, $\approx 1$ ps, implying $|V_{cb}| \approx 0.05$. Had $|V_{cb}|$ been more like $|V_{ts}| = 0.22$, the lifetime would have been a factor of 20 shorter, and probably would have then been too short to measure. This result, the long $b$ lifetime, was the first surprise in $b$ physics.

2.2.6 $B^0 - \overline{B}^0$ Mixing

It was recognized early on that the $B^0$ could, in principle, transform itself
into a $\overline{B}^0$, just as the $K^0$ transforms into a $\overline{K}^0$. The diagram is a box diagram, with $W^+, W^-$ on two parallel sides of the box, and $u, c, t, \bar{u}, \bar{c}, \bar{t}$ on the other two sides. In the limit of equal $u, c, t$ quark masses, the summed diagrams vanish, via the GIM mechanism. For “reasonable” values of the top quark mass – say 20 GeV, the rate for $B^0 - \overline{B}^0$ mixing would be immeasurably small.

In 1987, the UA1 experiment at CERN, in $p\bar{p}$ collisions, saw like-sign dilepton pairs, which they interpreted as $B^0 - \overline{B}^0$ mixing. As the extremely complicated environment of $p\bar{p}$ collisions made interpretation difficult, few people took this result seriously.

Later that same year, the ARGUS experiment at DESY, in the clean environment of $e^+e^-$ collisions at the $\Upsilon(4S)$, also saw like-sign lepton pairs. This could not be ignored. The most natural interpretation (and the correct one) was that top was a lot heavier than people had thought. This interpretation was slow in being accepted.

### 2.3 The Early Days – Summary

By 1990, there was a clear answer to “What are the basic features of the bottom quark?”.

- It is a member of a left-handed weak isospin doublet, with a (very heavy) top quark.

- It decays dominantly to the charm quark, via a charged current interaction $b \rightarrow cW_{\text{virtual}}^-$. $|V_{cb}| \approx 0.04$, so the coupling of the third generation to the second is smaller than the coupling of the second generation to the first ($|V_{us}| = 0.22$).

- Its decay to the up quark, $b \rightarrow uW_{\text{virtual}}^-$, is small but not zero, so the coupling of third generation to first is the smallest of the three couplings.

- Because the top quark is so massive, the GIM mechanism breaks down for loop and box diagrams involving $b$. One consequence is the observed large rate for $B^0 - \overline{B}^0$ mixing. Another should be measurable rates for penguin decays. Let’s look!

### 3 A Change in Objective

In “the early days,” the objective was to determine the basic features of the bottom quark. By 1990, that had been done, and the emphasis shifted.
Now, in recent times, and in the future, the objective is to use the bottom quark to probe the Standard Model, and search for physics ‘Beyond the Standard Model’.

There are two approaches to this probing and searching. These are:

- “Overdetermining the CKM Matrix”
- Measuring rates for Electroweak Penguins

I’ll examine each of these a bit later, but first a brief review of the flavor sector of the Standard Model.

3.1 The Flavor Sector of the Standard Model

Quarks come in left-handed weak isospin doublets, and decay via emission of (real or virtual) $W^\pm$ bosons.

\[
\begin{pmatrix}
  t \\
  b
\end{pmatrix}
\rightarrow
bW^+ ,
\begin{pmatrix}
  c \\
  s
\end{pmatrix}
\rightarrow
sW^+
\]

(3.1-1)

Thus $t$ decays to $b$ and a real $W^+$, while $c$ decays to $s$ and a virtual $W^+$.  

**Question:** How do the lower, lighter members of the doublets decay? The $b$ quark can’t decay $b \rightarrow tW^-$; that violates energy conservation.

**Answer:** The mass eigenstates $(b, s, d)$ and the weak interaction eigenstates $(b', s', d')$ are slightly different. The $b$ quark “flavor mixes” with $s$ and $d$ quarks, according to the CKM matrix (Cabibbo, Kobayashi, Maskawa):

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
=
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

(3.1-2)

\[
\uparrow
data.
\uparrow
mass.
\]

So, the $b$ quark (mass eigenstate) is a mixture of $b'$ (dominant component, amplitude $\propto V_{tb}$), $s'$ (amplitude $\propto V_{cb}$), and $d'$ (amplitude $\propto V_{ub}$), and decays $b \rightarrow cW_V^-$ with an amplitude proportional to $V_{cb}$, due to its $s'$ component, and decays $b \rightarrow uW_V^-$ with an amplitude proportional to $V_{ub}$, due to its $d'$ component.

The CKM matrix is a unitary matrix, which places $n^2$ constraints on an $n \times n$ matrix.
Because the phase of each quark state is arbitrary, $2n - 1$ phases in the CKM matrix can be transformed away.

Thus, if there were only two families of quarks, ($u,d$), and ($c,s$), the CKM matrix would be a $2 \times 2$ matrix, with 4 complex elements, 8 parameters. The unitarity of the CKM matrix reduces 8 to 4, and the arbitrariness of the quark state phases reduces 4 to 1, a single parameter, the Cabibbo angle. The CKM matrix for two families is described by a single parameter, and can be made real.

But there are (at least) three families of quarks, ($u,d$), ($c,s$), and ($t,b$). The CKM matrix is a $3 \times 3$ matrix, and the arithmetic goes 9 complex elements $\rightarrow 18$ parameters $-9$, for unitarity, $-5$, for arbitrary phases $= 4$. The CKM matrix for three families is described by four parameters: 3 angles and 1 phase. This phase cannot be transformed away. If the phase is nonzero, weak decays will not be CP invariant.

Thus

3 families $\Rightarrow$ CP violation.

This was Kobayashi and Maskawa's insight in 1973, before the charm quark had been discovered, let alone any members of the third family.

3.1.1 CP Violation

CP violation was observed in neutral kaon decay in 1964, by Christenson, Cronin, Fitch and Turlay. Given Kobayashi and Maskawa's insight, that implies 3 families.

The $b$ quark, hence a third family, was observed in 1977, by Lederman and collaborators. That implies CP violation.

So, why is everyone making such a fuss about CP violation? It’s expected, observed, explained, isn’t it? There are two reasons why CP violation is now considered a “big deal”.

A) The CP violation given by the phase in the CKM matrix appears to be too small to account for the baryon-antibaryon asymmetry of the Universe at early times. Cosmology requires “New Physics,” and it must be CP-violating New Physics.

B) Measurements using CP-violating $b$ decays can help determine (overdetermine) the CKM matrix, hence probe the correctness of the Standard Model (or see New Physics).

Reason B) has CP violation in $b$ decay a useful tool for probing, searching, while reason A) has it as the primary object of study.

My own view is that too much attention is give to A) (hey, it’s got a lot of PR value), and not enough to B). But, in any case, whether you prefer to
focus on A) or B), what studies you’ll perform will be much the same. CP
violation in $b$ decay should be, and will be, studied.

3.1.2 Penguins, $B^0 - \overline{B}^0$ Mixing and the GIM Mechanism

The GIM mechanism causes flavor-changing neutral currents to vanish
at tree level. It also suppresses FCNC beyond tree level, for loop and box
diagrams. Let’s work this through for an important example, $b \to s \gamma$. (The
same argument applies to gluonic penguins $b \to s g$.)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{feynman_diagrams.png}
\caption{Standard Model Feynman diagrams for $b \to s \gamma$ decay, with CKM factors indicated.}
\end{figure}

The Feynman diagrams for $b \to s \gamma$ are shown in Fig. 5. The overall amplitude is the sum of the three diagrams, with $u$, $c$, and $t$ quark inside the loop. The CKM factors are as shown on the figure, and thus the amplitude is

$$A = A(m_u^2)V_{ub}V_{us}^* + A(m_c^2)V_{cb}V_{cs}^* + A(m_t^2)V_{tb}V_{ts}^*$$  \hspace{1cm} (3.1.2-1)

The amplitudes $A(m_u^2)$, $A(m_c^2)$, $A(m_t^2)$ depend only on masses, their flavor dependence having been removed by factoring out the CKM pieces. But, from unitarity

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0 \ (Unitarity)$$  \hspace{1cm} (3.1.2-2)

Thus, if $m_u = m_c = m_t$, then $A = 0$. That’s GIM, the cancelation of the different terms in the sum. There is suppression, and the closer the three masses are to each other, the more the suppression.

But, $m_t \gg m_c, m_u$. So, the cancelation is far from complete. The amplitude $A$ is proportional to $m_t^2$, and, since $m_t$ is a lot larger than originally expected, penguins in $b$ decay are also a lot larger than originally expected.

A similar argument applies to $B^0 - \overline{B}^0$ mixing. The Feynman diagrams are shown in Fig. 6. There is a double sum over $u$, $c$, $t$; $\pi$, $\overline{\pi}$, $\overline{t}$. If $m_u = m_c = m_t$, the sum is zero, due to the unitarity of the CKM matrix. The heavy top badly breaks GIM, with an amplitude for mixing proportional to $m_t^2$. 

11
3.2 Why is Bottom Quark Physics so Interesting?

We’re now in a position to answer the question “Why is bottom quark physics so interesting, such a good probe of New Physics?”

The answer is, “Because the top quark is so massive!”

The massive top quark gives rise to substantial $B^0 - \bar{B}^0$ mixing, and substantial rates from loop diagrams (Penguins). Both of these are powerful tools for testing the Standard Model, for searching for New Physics.

3.2.1 Using $B^0 - \bar{B}^0$ Mixing to Learn Weak Phases

Consider a decay of a neutral $B$, with $B^0$ and $\bar{B}^0$ reaching the same final state, $B^0 \rightarrow X$ and $\bar{B}^0 \rightarrow X$. Examples are $B^0 \rightarrow \psi K^0 \rightarrow \psi K^0_s$ and $\bar{B}^0 \rightarrow \psi K^0 \rightarrow \psi K^0_s$; and $B^0 \rightarrow \pi^+ \pi^- \rightarrow \pi^+ \pi^-$. A particle born as a $B^0$ has two routes to this final state: i) The direct one $B^0 \rightarrow X$, and ii) the indirect one, through $B^0 - \bar{B}^0$ mixing, $B^0 \rightarrow \bar{B}^0 \rightarrow X$. The amplitudes for these two routes will add coherently, and interfere. Similarly, the amplitudes for $B^0 \rightarrow X$ and $\bar{B}^0 \rightarrow \bar{B}^0 \rightarrow X$ will add coherently and interfere. Immediately after birth, a particle born as a $B^0$ will be a $B^0$, but, over time, it will mix into $\bar{B}^0$, and so time development is the key. By tagging particle flavor at birth, comparing $|A(B^0 \rightarrow X) + A(\bar{B}^0 \rightarrow \bar{B}^0 \rightarrow X)|^2$ with $|A(B^0 \rightarrow X) + A(\bar{B}^0 \rightarrow \bar{B}^0 \rightarrow X)|^2$, studying the time development of both, one can determine

$$\sin \left( \phi_{Mixing} + 2\phi_{B^0 \rightarrow X} \right)$$

The expected value of $\phi_{B^0 \rightarrow \psi K_s}$ is zero, while the Standard Model value of $\phi_{Mixing}$ is $2\beta$, so, studying $B \rightarrow \psi K^0_s$ is the much talked about “measurement of $\sin 2\beta$".
Note that one must study \textit{time development}. This class of measurements, time development of tagged $B^0$, $\bar{B}$ to a common final state, is the rationale behind \textit{asymmetric} $e^+e^-$ colliders at the $\Upsilon(4S)$. The asymmetric initial state energies has the center-of-mass moving in the lab, so the decay time can be measured.

\textbf{3.2.2 Electroweak Penguins as Probes of New Physics at High Mass Scales}

The Standard Model diagram for $b \rightarrow s\gamma$ is shown in Fig. 7. The photon may be emitted from any of the charged lines. The top quark internal line is shown, because it is the excess of $m_t^2$ above $m_c^2$, $m_u^2$, that breaks the GIM mechanism. The mass scale of the diagram is set by the masses of the particles in the loop – $m_t$ and $M_W$, $\sim 100$ GeV.

Consequently, contributions from New Physics (e.g., charged Higgs, SUSY particles; see Fig. 7) will show up for New Physics masses in that same range. So, penguins probe for New Physics up to masses $\sim 500$ GeV.

This argument, given for electroweak penguins, applies also to gluonic penguins. However, electroweak penguins, in particular $b \rightarrow s\gamma$, has the advantage that its rate can be calculated, within the Standard Model, and Beyond the Standard Model, to a precision of 10%.
3.2.3 Learning Weak Phases from Penguin-Tree Interference

Many rare $B$ decays involve both penguin and tree amplitudes, while some related decays are pure penguin, or pure tree. By studying relative rates and CP asymmetries, one can sort the phases out, and determine weak phases.

As an example, consider $B^- \rightarrow K^- \pi^0$, $B^+ \rightarrow K^+ \pi^0$, and $B^\pm \rightarrow K^0 \pi^\pm$. The first two involve both penguin and tree amplitudes, while the last is pure penguin. The amplitudes for the three processes are

\[
A(B^- \rightarrow K^- \pi^0) = A_P + A_T e^{i\phi_W} e^{i\phi_S}
\]
\[
A(B^+ \rightarrow K^+ \pi^0) = A_P + A_T e^{-i\phi_W} e^{i\phi_S}
\]
\[
A(B^\pm \rightarrow K^0 \pi^\pm) = A_P
\]

where $\phi_W$ is the difference in weak phase between penguin and tree, and $\phi_S$ is the difference in strong phase between penguin and tree. Squaring amplitudes, one sees, for the first two modes

\[
|A|^2 = A_P^2 + 2A_P A_T [\cos \phi_W \cos \phi_S \mp \sin \phi_W \sin \phi_S] + A_T^2
\]

while the square for the third mode is just $|A_P|^2$.

Thus, the rate difference, i.e., CP asymmetry, gives $\sin \phi_W \sin \phi_S$, while the rate sum, compared with the third mode, gives $\cos \phi_W \cos \phi_S$. Of course, there are complications to the naive picture just presented, due to electroweak penguins, color-suppressed trees, and long distance rescatterings. But, the decays do depend on strong and weak phases, roughly as indicated, and by studying several rare decays one can learn weak phases.

3.3 How Does One Determine Elements of the CKM Matrix?

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

(3.3-1)

Rates for nuclear beta decay, compared to the rate for muon decay, gives a very precise determination of the magnitude of $V_{ud}$. Kaon and hyperon decay rates give good determinations of the magnitude of $V_{us}$. Assorted studies of charm decay give rough measurements of $|V_{cd}|$ and $|V_{cs}|$. However, since the third generation is only weakly coupled to the first two, these studies determine only a single parameter, $\lambda = \sin \theta_{Cabbibo}$.

Studies of $b$ decay determine two more parameters. In particular, the rate for $b \rightarrow c\ell\nu$ determines $|V_{cb}|$, and the rate for $b \rightarrow u\ell\nu$ determines $|V_{ub}|$.  

14
Can $|V_{ts}|, |V_{td}|$ be determined from studies of top decay? *Not soon!* The rate for $t \rightarrow W^+ s$ is proportional to $|V_{ts}|^2$, and the rate for $t \rightarrow W^+ d$ is proportional to $|V_{td}|^2$. Measuring those rates would give $|V_{ts}|$ and $|V_{td}|$. But the expected value for $|V_{ts}|^2$ is $\sim 2 \times 10^{-3}$, while that for $|V_{td}|^2$ is $\sim 10^{-4}$, while $|V_{tb}|^2 \approx 1$, giving a dominant decay $t \rightarrow Wb$. It will be a while (quite a while) before top decay branching fractions at the $10^{-3} - 10^{-4}$ level are measured.

So, for the foreseeable future, the situation is this. We can determine three magnitudes in the CKM matrix – $\lambda, |V_{cb}|, |V_{ub}|$ – from tree-level processes, theoretically secure, relatively free from possible New Physics contributions, reliably giving what they claim to determine. All else will come from loops, boxes, places where New Physics is likely to enter. Thus, if an “overdetermination of the CKM matrix” finds an inconsistency, that does not mean a problem with the CKM matrix, but rather that the relation to the CKM matrix of some measurable has been changed by New Physics. For example, if sin $2\beta$ as determined from the time development of tagged $B^0 \rightarrow \psi K_s^0$ disagrees with expectations, that would mean that the phase of the $B^0 - \overline{B^0}$ mixing amplitude is not sin $2\beta$, but has been altered by New Physics contributions to mixing.

Let’s rewrite the CKM matrix in a $b$-centric fashion. Taking $|V_{cb}| = 0(\lambda^2), |V_{ub}| = 0(\lambda^3)$, and enforcing unitarity, we have, correct to $0(\lambda^3)$

$$
\begin{pmatrix}
1 - \lambda^2/2 & \lambda & |V_{ub}|e^{-i\gamma} \\
-\lambda & 1 - \lambda^2/2 & |V_{cb}|
\end{pmatrix}
$$

(3.3-2)

Since $\lambda$ is already determined with high precision, this form makes apparent the urgency of good determinations of $|V_{cb}|$ and $|V_{ub}|$.

$|V_{cb}|$ is obtained from measurements of the $B$ meson lifetime, and either the rate for $B \rightarrow D^* \ell \nu$ extrapolated to the point where $D^*$ is at rest, or the rate for $B \rightarrow X \ell \nu$ inclusive, plus information on the $b$ quark mass and HQET Operator Product Expansion parameter $\lambda_1$. The $b$ lifetime and semileptonic decay branching fraction are well measured. CLEO has in hand data on $B \rightarrow D^* \ell \nu$, and on moments of hadronic mass and lepton energy in $B \rightarrow X \ell \nu$ sufficient for $\pm 4\%$ determinations of $|V_{cb}|$, separately by each method. For now, $|V_{cb}| = (39.5 \pm 3.6) \times 10^{-3}$.

$|V_{ub}|$ is less well determined, and $\gamma$ even less well determined.
4 What is $|V_{ub}/V_{cb}|$?

4.1 Limitations of Previous Approaches

In Section 2.2.3, I described progress during the early days in placing upper-limits on, and finally establishing a nonzero value for, $|V_{ub}/V_{cb}|$. All the approaches tried then had serious limitations. The kaon yield approach was really a measurement of $|V_{cb}|$, limiting $|V_{ub}|$ by $|V_{cb}|'$s deviation from 1.0. Since the total number of kaons produced per $b \to cW\nu$ decay is uncertain at greater than the ten percent level, this approach was quickly discarded, once it was realized that $|V_{ub}/V_{cb}|$ was in the sub-ten-percent range.

Fitting the measured lepton spectrum in $B$ semileptonic decay to the predicted spectra for $b \to c\ell\nu$ and $b \to u\ell\nu$ hits its limit because, with the $b \to u\ell\nu$ rate less than 5% of the $b \to c\ell\nu$ rate, minor errors in modeling of the $b \to c\ell\nu$ spectrum cause major errors in the $b \to u\ell\nu$ yield. This approach has also been discarded.

The endpoint approach avoids sensitivity to the $b \to c\ell\nu$ modeling because it limits the focus to the lepton momentum range where $b \to c\ell\nu$ is small or zero. But here there is sensitivity to the modeling of $b \to u\ell\nu$. The fraction of the $b \to u\ell\nu$ spectrum in the endpoint windows cannot be reliably calculated, and its uncertainty limits accuracy of $|V_{ub}|$ by this method to $\sim 20\%$. While results from this approach are currently one of the two ways now giving useful results, future improvements to the 10% range and below seem unlikely. (Note added in proof: Leibovich, Low, and Rothstein, hep-ph/9909404 v2, show how to determine the fraction, using a measurement of the photon spectrum from $b \to s\gamma$.)

4.2 Neutrino Detection

The difficulty in studying $b \to u\ell\nu$ is the neutrino. If that particle were ‘detected’, its momentum measured, then the decay would cause no problems. Consequently, several of us in CLEO are attempting a new approach, ‘detecting’ the neutrino in a semileptonic decay via the missing 4-momentum in the event. Given a ‘detected’ neutrino, one can then carry out full reconstruction of exclusive semileptonic decays, or look at the mass distribution in inclusive semileptonic decays.

4.2.1 Exclusive Decays $B \to \pi\ell\nu$, $B \to \rho\ell\nu$

With the neutrino ‘detected’, the decays $B \to \pi\ell\nu$, $B \to \rho\ell\nu$, $B \to \omega\ell\nu$ have no undetected particles, and so the standard $B$ reconstruction technique is applicable. The summed energy of the decay products of the can-
didate $B$ are compared with the beam energy, giving a difference $\Delta E$ which should peak at zero. The summed vector momenta of the decay products of the candidate $B$, $P_{\text{cand}}$, are used to calculate the “beam constrained mass” $M_{m\ell\nu} = \sqrt{E_{\text{beam}}^2 - P_{\text{cand}}^2}$, which should peak at the $B$ mass.

We completed and published an analysis for $B \to \pi\ell\nu$, and $B \to \rho/\omega\ell\nu$ some time ago (PRL 77, 5000 (16 Dec. 1996)), based on a $4fb^{-1}$ data sample. The plots of mass and energy difference are shown in Fig. 8. The branching fraction accuracy (statistical plus systematic) gave a 12% uncertainty in $V_{ub}$, and that uncertainty should fall as $1/\sqrt{L}$.

![Figure 8: B mass and energy difference plots for $B \to \pi\ell\nu$ and $B \to \rho/\omega\ell\nu$ (CLEO).](image)

The big issue is model dependence – how the branching fraction for the exclusive modes are related to $V_{ub}$. Of course, they are proportional to $|V_{ub}|^2$, with the constants of proportionality related to form factors. It is through the uncertainty in the form factors that model dependence enters. For the 1996 analysis, we estimated this at $\pm20\%$ in $|V_{ub}|$. This will improve with more data, which will allow measurement of the $q^2$ dependence of the decays, providing constraints on models for form factors. It will also improve with better form factor calculations, from lattice gauge QCD and other techniques. Finally, studies of the decays $D \to \pi\ell\nu$, $D \to \rho\ell\nu$, where $|V_{cd}|$ is quite well known, can also help. One can expect an accuracy in $|V_{ub}|$ from CLEO’s existing, $14fb^{-1}$ data sample, in the $\pm15\%$ range, or better, depending on how much progress can be made on the model dependence.
4.2.2 Inclusive Decays, $B \to X_u \ell \nu$

Given a ‘detected’ neutrino, and a (really) detected charged lepton, one can calculate the mass of the hadronic system $X$ in the decay $B \to X \ell \nu$:

$$M_X^2 = M_B^2 + M_{\ell \nu}^2 - 2E_B E_{\ell \nu} + 2P_B P_{\ell \nu} \cos \theta_{\ell \nu, B}$$

All quantities in this equation are known except $\theta_{\ell \nu, B}$, the angle between the $B$ meson and the $\ell \nu$ system (everything evaluated in the lab frame). The total lack of knowledge of $\theta_{\ell \nu, B}$ results in a smearing in the determination of $M_X^2$, which is reasonably small since $P_{\ell \nu}$ is small ($\sim$300 MeV/c).

The game plan, then, is to measure the $M_X^2$ distribution, given neutrino and charged lepton, and then fit that distribution with a sum of $b \to u \ell \nu$ and $b \to c \ell \nu$. The contributions from $b \to c \ell \nu$ will include $D$, $D^*$, and heavier stuff. The contributions from $b \to u \ell \nu$ will dominantly be below the $D$ meson mass, consisting of $X_u$ objects like $\pi$, $\rho$, $A_1$, $A_2$, etc. A calculation of the expected $X_u$ mass distribution is possible, for example from a naive spectator model, or more properly from HQET and OPE. If one could measure $M_X^2$ to high precision, separating $b \to u \ell \nu$ from $b \to c \ell \nu$ would be easy, and an inclusive measurement of $|V_{ub}|$, with relatively little model dependence, would be possible.

Unfortunately, the measurement of $M_X^2$ so far achieved has rather poor resolution, due to the inaccuracy in determining the neutrino vector momentum. This inaccuracy is not so much from the inaccuracy in measurement of individual particles, but rather from ‘messups’ (inefficiencies in detecting charged particles and photons, false tracks and photons), and also from undetectable particles ($K$-long, neutrons, second neutrinos in the event). The consequence of the poor resolution in $M_X^2$ is that there is a low-mass-side tail to $b \to c \ell \nu$, which swamps the $b \to u \ell \nu$ contribution.

An analysis has been completed (Scott Roberts’ Ph.D. thesis, University of Rochester, 1997), but not submitted for journal publication. To suppress the $b \to c \ell \nu$ component, we required $P_\ell > 2.0$ GeV/c, a momentum bite a factor of 2 bigger than the $P_\ell > 2.3$ GeV/c typical of an endpoint analysis. The choice of 2.0 GeV/c was a compromise between reducing model dependence (wanting a lower momentum cut) and suppressing $b \to c \ell \nu$ (wanting a higher momentum cut).

The measured $M_X^2$ distribution is shown in Fig. 9. The fitted components from $b \to c \ell \nu$ and $b \to u \ell \nu$ are also shown. In the region where $b \to u \ell \nu$ is substantial, the $b \to c \ell \nu$ background is about twice the $b \to u \ell \nu$ signal. Taking faith in our modeling of the $b \to c \ell \nu$ background (though allowing a systematic error for its uncertainty), we obtained a fit, from a 5$fb^{-1}$ data sample, which
Figure 9: $M^2_X$ distribution from decay $B \rightarrow X\ell\nu$. The curves are $b \rightarrow u\ell\nu$ (shaded) and $b \rightarrow c\ell\nu$ (dashed curve) contributions, and their sum (Scott Roberts’ thesis).

gives $|V_{ub}|$ to ±16% – statistical plus systematic. We did not carry out a careful study of model dependence, but since the lepton momentum bite is twice as large as that for the endpoint analysis, one would expect a model dependence that is twice as small – ±10% instead of ±20%.

The value of $|V_{ub}|$ obtained from the fit is quite reasonable, and the combined nominal error, (±16% with ±10%) are competitive. But the plot is certainly not very convincing. The $b \rightarrow c\ell\nu$ component is just too large in the $b \rightarrow u\ell\nu$ signal region. And we would like to push the lepton momentum cut down, say to 1.8 or 1.6 GeV/c, which would make the $b \rightarrow c\ell\nu$ background several times larger. So, our plan is not to publish this analysis, but to work on it some more – a lot more.

- We will use the full CLEO II data sample of 14 fb$^{-1}$, a factor of 3 increase from that in Scott Roberts’ analysis. (This is the easy one.)
- We will improve the accuracy with which neutrinos are ‘detected’ and their momenta determined, by upgrading our algorithm for distinguishing between showers in the electromagnetic calorimeter caused by photons and by hadrons; by improving various aspects of charged particle
tracking; and by pushing to lower momentum our electron identification capabilities (we veto events with more than one charged lepton, hence more than one neutrino).

- Finally, we will study the correctness of our simulation of the \( b \to c\ell\nu \) component, to be sure we are correctly modeling the low-mass tail. (For example, we will fake \( K \)-long events by finding events with \( K \)-shorts, then pitching the \( K \)-short, and see if the \( M_X^2 \) spectra so obtained for data and Monte Carlo agree.)

The original motivation for neutrino ‘detection’ was for studying inclusive decays, with its use for exclusive decays an afterthought. We still view the inclusive approach as the best hope for a measurement of \( |V_{ub}| \) to \( \pm 10\% \) accuracy.

5 Rare Hadronic \( B \) Decays

5.1 Introduction

I should start this section by saying what I mean, and indeed what is typically meant, by “rare”, as it refers to \( B \) decays. A “rare” \( B \) decay is one which involves penguin or box diagrams. With this definition, it is easy to see why the field of rare \( B \) decays is ahead of the field of rare kaon decays, why \( b \to s \) processes have been studied, while \( s \to d \) processes much less so. The CKM factor for \( b \to s \) penguins is \( V_{tb}V_{ts}^* \), while that for the dominant, \( b \to c \) is \( V_{cb} \).

\[
|V_{tb}V_{ts}^*|/|V_{cb}| \sim 1
\]

For kaon decays, the penguin with top quark in the loop has a CKM factor \( V_{ts}V_{td}^* \), while that for the dominant, \( s \to u \) tree is \( V_{us} \).

\[
|V_{ts}V_{td}^*|/|V_{us}| \sim 3 \times 10^{-6}
\]

The branching fractions for rare \( B \) decays are typically 5-6 orders of magnitude larger than those for rare kaon decays – \( 10^{-5} \) vs \( 10^{-11} \) – 10^{-12}.

As we saw in Section 3.2.3, rare decays involving penguins often also involve \( b \to u \) trees (see Fig. 10). The example given there was \( B \to K\pi \), a “Cabibbo-allowed penguin”, i.e., \( b \to s \) penguin. The tree diagram there is the “Cabibbo-suppressed \( b \to u \) tree”, \( b \to uW^-_V, W^-_V \to \bar{u}s \). The Cabibbo-suppression is in the decay of the virtual \( W \), \( W^- \to \bar{u}s \), rather than the Cabibbo-allowed decay \( W^- \to \bar{u}d \). This same mix of Cabibbo-allowed penguin plus (sometimes) Cabibbo-suppressed \( b \to u \) tree occurs for \( B \to K^*\pi \), \( B \to K\rho, B \to K\omega, B \to K^*\rho \). The Cabibbo-allowed penguin diagram contributes to all of these, while the Cabibbo-suppressed \( b \to u \) tree is absent for the charge modes involving neutral \( K \) or \( K^* \).

There is also a class of decays involving Cabibbo-allowed \( b \to u \) trees, i.e., \( b \to uW^-_V, W^-_V \to \bar{u}d \), and Cabibbo-suppressed penguins, i.e. \( b \to d \) penguins.
Figure 10: The Penguin and $b \to u$ Tree diagrams contributing to rare hadronic $B$ decays. (a) Cabibbo-allowed penguin; (b) Cabibbo-suppressed $b \to u$ tree; (c) Cabibbo-allowed $b \to u$ tree; (d) Cabibbo-suppressed penguin.

(Fig. 10c,d). Examples include $B \to \pi \pi$, $B \to \pi \rho$, $B \to \pi \omega$, $B \to \rho \rho$. In fashion similar to the ‘allowed penguin, suppressed tree’ class, there are particular modes for which the Cabibbo-suppressed penguin is absent, e.g. $B^\pm \to \pi^\pm \pi^0$.

So, Penguin-Tree interference is the rule rather than the exception in rare hadronic $B$ decays. And the exceptions, modes which are pure allowed penguins, or pure allowed $b \to u$ trees, help to sort out the interference.

As mentioned in Section 3.2.3, the simple picture is complicated by electroweak penguins $b \to sZ^0$ (we’ve been talking about gluonic penguins $b \to sg$), color-suppressed trees, long distance rescattering. It will require careful study of many rare decays before a precise value of the CKM phase $\gamma$ can be obtained. But as we will see, some qualitative information can already be obtained.

5.2 The Data Sample

CLEO has 10 million events of the form $e^+e^- \to \Upsilon(4S) \to B + \bar{B}$, and has recently completed analysis of several of the rare decay modes. The reconstruction is conventional, with the summed energy of the decay products of the candidate $B$ compared with the beam energy, and the summed vector momenta of the decay products of the candidate $B$ used to calculate the beam-constrained mass.
There are substantial backgrounds to rare $B$ decays, not from the dominant $b \to c$ tree decays, but from the continuum background process $e^+ e^- \to q\bar{q}$, $q = u, d, c, s$. These backgrounds are 2-jet-like, and are suppressed by a maximum likelihood fit, inputting many ‘shape variables’.

Examples of $B$ mass plots and $\Delta E$ plots are shown in Fig. 11.

![Figure 11: Plots of (left) $\Delta E \equiv E_{\text{cand}} - E_{\text{beam}}$, and (right) $M_{\text{cand}} \equiv \sqrt{E_{\text{beam}}^2 - P_{\text{cand}}^2}$, for the decay $B \to \eta' K^\pm$ (CLEO). The filled histograms are from $\eta' \to \eta \pi^+ \pi^-$, $\eta \to \gamma\gamma$, while the unfilled are from $\eta' \to \rho^0\gamma$.](image)

5.3 Results

5.3.1 $B \to K\pi, \pi\pi, KK$

Results for these modes are given in Table 1. All four $B \to K\pi$ modes have been convincingly seen. Only one of the three $B \to \pi\pi$ modes has been convincingly seen, though the evidence for $B \to \pi^\pm\pi^0$ is fairly good. No $B \to KK$ mode has been seen, nor were they expected to be.
Table 1: Branching fractions for $B$ decays to $K\pi$, $\pi\pi$, and $KK$.

| Mode $B$ | $B(10^{-6})$ | Mode $B$ | $B(10^{-6})$ |
|----------|---------------|----------|---------------|
| $K^+\pi^-$ | $17 \pm 3$ | $K^0\pi^+$ | $18 \pm 5$ |
| $K^+\pi^0$ | $12 \pm 3$ | $K^0\pi^0$ | $15 \pm 6$ |
| $\pi^+\pi^-$ | $4.3 \pm 1.6$ | $\pi^+\pi^0$ | $5.6 \pm 3.0$ |
| $\pi^0\pi^0$ | $< 9.3$ |
| $K^+K^-$ | $< 1.9$ | $K^+\bar{K}^0$ | $< 5.1$ |

5.3.2 B Decays Involving $\eta$ or $\eta'$

Results for the decay $B \to (\eta$ or $\eta') (K$ or $K^*)$ are shown in Table 2. One sees that $B \to \eta'K$ is big, much larger than all the others. $B \to \eta K^*$ is seen, and is larger than $B \to \eta K$.

Table 2: Branching fractions for $B$ decays to $(\eta$ or $\eta')$ plus $(K$ or $K^*)$.

| Mode $B$ | $B(10^{-6})$ | Mode $B$ | $B(10^{-6})$ |
|----------|---------------|----------|---------------|
| $B^0 \to \eta'K^0$ | $80 \pm 12$ | $B^0 \to \eta'K^{*0}$ | $< 87$ |
| $B^0 \to \eta K^0$ | $88 \pm 19$ | $B^0 \to \eta K^{*0}$ | $< 20$ |
| $B \to \eta'K^0$ | $83 \pm 11$ | $B \to \eta K^*$ | $< 22$ |
| $B^0 \to \eta'K^0$ | $< 7.1$ | $B^0 \to \eta K^{*0}$ | $27 \pm 10$ |
| $B \to \eta K^0$ | $< 9.5$ | $B \to \eta K^0$ | $14 \pm 5$ |
| $B \to \eta'K^0$ | $< 5.2$ | $B \to \eta K^*$ | $18 \pm 5$ |

The interpretation of these results is far from clear.

- The $\eta'$ could perfectly well contain a $c\bar{c}$ component, and if it did, a Cabibbo-allowed $b \to c$ tree could contribute (as it does for $B \to \psi K$). This situation would lead to enhanced branching fractions for both $B \to \eta'K$ and $B \to \eta'K^*$.
- As pointed out by Lipkin, there will be contributions from the gluonic penguins $b \to sg$, $g \to ss\pi$ and $b \to sg$, $g \to u\bar{u}/d\bar{d}$, and these diagrams will interfere. Lipkin argues that this will enhance $B \to \eta'K$ and $B \to \eta K^*$ relative to $B \to \eta'K^*$ and $B \to \eta K$. 

23
The data show some features of both suggestions, but at present there is no quantitative understanding.

5.3.3 Decays $B \to \text{Pseudoscalar Vector}$

Only a smattering of these have been seen so far, e.g. $B^\pm \to \omega\pi^\pm$, $B \to \rho\pi$, $B \to K^*\pi$. CLEO’s analyses of the Pseudoscalar-Vector modes are finished for the full data sample of 10 million $B\bar{B}$ events for about half of the decay modes. Results are given in Table 3.

| Mode       | $\mathcal{B}(10^{-6})$ | Mode       | $\mathcal{B}(10^{-6})$ |
|------------|-------------------------|------------|-------------------------|
| $\pi^\pm\rho^0$ | $10.4 \pm 4.0$       | $K^\pm\rho^0$ | $< 17.3$             |
| $\pi^0\rho^0$     | $< 5.5$              | $K^0\rho^0$   | —                      |
| $\pi^\pm\rho^\mp$ | $27.6 \pm 8.9$     | $K^\pm\rho^\mp$ | $< 32.3$             |
| $\pi^0\rho^\mp$   | $< 42.6$             | $K^0\rho^\mp$ | —                      |
| $\pi^\pm\omega$   | $11.3 \pm 3.4$      | $K^\pm\omega$ | $(3.2 \pm 2.3)$, $< 7.9$ |
| $\pi^0\omega$     | $< 5.8$              | $K^0\omega$   | $(10 \pm 5)$, $< 21$  |
| $\pi^+K^{*0}$     | $< 15.9$             | $K^-K^{*0}$   | $< 5.3$               |
| $\pi^0K^{*0}$     | $< 3.6$              | $K^0K^{*0}$   | —                      |
| $\pi^+K^{*-}$     | $(22 \pm 9)$        | $K^+K^{*-}$   | —                      |
| $\pi^0K^{*-}$     | $< 31.0$             | $K^0K^{*-}$   | —                      |

5.4 Search for Direct CP Violation in $B$ Decay

If some $B$ decay has contributions from two (or more) amplitudes $A_1$, $A_2$, with relative weak phase $\phi_W$, and relative strong phase $\phi_S$, i.e., a total decay amplitude $A = |A_1| + |A_2|e^{i\phi_S}e^{i\phi_W}$, then there will be direct CP violation in the decay, which will show up as a charge asymmetry

$$ A = \frac{\Gamma(B \to X) - \Gamma(B \to X)}{\Gamma(B \to X) + \Gamma(B \to X)} $$ (5.4-1)

If $|A_2| \ll |A_1|$, then

$$ A \approx 2 \frac{|A_2|}{|A_1|} \sin \phi_S \sin \phi_W $$ (5.4-2)
For penguin-tree interference, one expects $|A_2/A_1| \approx 0.2$. It’s less clear what to expect for $\sin \phi_S$, but in the absence of some enhancement due to long range rescattering it will be small, probably less than 0.25. So we expect $A$ less than 0.1. CLEO results, for five decay modes that have been convincingly seen, and are self-tagging, are given in Table 4. There are no surprises. All asymmetries are consistent with zero. There is not yet sufficient sensitivity to see CP violations at the level expected. Since the errors are dominantly statistical, and are based on 10 million $B\bar{B}$ pairs, it will likely be a while before nonzero asymmetries are established.

Table 4: CP Asymmetries for five rare decay modes.

| Mode   | $A$     |
|--------|---------|
| $B \to K^\pm \pi^\mp$ | $-0.04 \pm 0.16$ |
| $B \to K^{\mp} \pi^\pm$ | $-0.29 \pm 0.23$ |
| $B \to K^{0} \pi^0$ | $+0.18 \pm 0.24$ |
| $B \to K^{0} \eta'$ | $+0.03 \pm 0.12$ |
| $B \to \omega \pi^\mp$ | $-0.34 \pm 0.25$ |

5.5 Interpretation of Rare Hadronic $B$ Decays

Now that CLEO has roughed out the rare $B$ decay terrain, what does it all mean? Recall that in Section 3.2.3, the motivation for studying rare $B$ decays was given as using them to determine weak phases. What can the existing rare $B$ decay data tell us about weak phases, in particular, about $\gamma$, the phase of $V_{ub}^*$?

CLEO’s visiting theorist George Hou and CLEO members Jim Smith and Frank Würthwein have addressed that question. They assume naive factorization, use effective-theory matrix elements, and ignore annihilation type diagrams. With these assumptions, (and some more, mentioned below) they are able to express the amplitudes for all two-body rare $B$ decays in terms of a relatively small number of parameters.

The quark-level process $b \to q_1 \overline{q}_2 q_3$ is described, in effective theory, by ten parameters $a_1 \ldots a_{10}$. These are calculable within a QCD framework, and Hou et al. take two sets of values from the literature.

The binding of $q_1 \overline{q}_2$ into mesons is described by decay constants $f$, ($f_\pi$, $f_K$, $f_{K^*}$, $f_\rho$, $f_\omega$, $f_\phi$) which are known.

The binding of $q_3$ and the spectator antiquark into a meson is described by form factors. For $B \to PP$, there is a single form factor $F_0^{BB}$ (but $P = \pi, K$),
while for $B \to PV$ there are two more, $F_{BP}^B$, $P = \pi$, $K$, and $A_{B\pi}^B$, $V = \rho$, $\omega$. Hou et al. lean on SU(3), with breaking, to relate $F_{BK}^B$ to $F_{B\pi}^B$, and $A_{B\omega}^B$ to $A_{B\rho}^B$. They also use the relation $F_{BP}^B = F_{B1}^P$, valid at $q^2 = 0$. They thus describe the decays of interest with just two form factors $F_{B\pi}^B$ and $A_{B\rho}^B$, rather than six.

Two of the penguin terms ($a_6$, $a_8$) depend on the quark mass ($m_s$ or $m_d$) and Hou et al. allow a free parameter $R_{su}$ to describe this dependence.

Hou et al. thus use five free parameters: $F_{B\pi}^B$, $A_{B\rho}^B$, $R_{su}$, $|V_{ub}/V_{cb}|$, and $\gamma$. They constrain $|V_{ub}/V_{cb}|$ by including the difference from its measured values, $0.08 \pm 0.02$, as a term in $\chi^2$.

They fit 14 branching fractions: $K^{-}\pi^+$, $K^{-}\pi^0$, $\bar{K}^0\pi^-$, $\bar{K}^0\pi^0$, $\pi^+\pi^-$, $\pi^-\pi^0$, $\rho^0\pi^-$, $\omega\pi^-$, $\rho^0\pi^\pm$, $K^*\pi^+$, $\omega K^-$, $\omega \bar{K}^0$, $\phi K^-$, and $\phi \bar{K}^0$. They leave the $\eta'$ and $\eta$ decay modes out of the fit, as something strange is happening with these modes. The $\chi^2$ of the fit, as a function of $\gamma$, is shown in Fig. 12. The fit gives $\gamma = 114 \pm 23$ degrees.

![Figure 12: $\chi^2$ of fit to 14 rare hadronic $B$ decay branching fractions, as a function of $\gamma$, the phase of $V_{ub}$ (Hou-Smith-Würthwein).](image)
The error just quoted is that from the branching fraction errors only, and does not include anything for theoretical uncertainty. Those must be estimated and included before a serious number for $\gamma$ can be quoted. However, from this exercise, so far, we can see that the data contain information sufficient for a precise determination of $\gamma$, given adequate theoretical understanding. Further, they argue for a large value of $\gamma$. I’ll take the liberty of assuming that the theoretical error won’t be more than $\sim \pm 50^\circ$, and interpret the rare $B$ results as saying $\gamma > 60^\circ$.

6 The Radiative Penguin Decay $b \to s\gamma$

In Section 3.2.2, I argued that electroweak penguin processes, in particular $b \to s\gamma$, probe for New Physics up to masses $\sim 500$ GeV. What’s been learned so far?

6.1 The Exclusive Decay $B \to K^*(890)\gamma$

The observation of $B \to K^*(890)\gamma$, in 1993, was the first clear observation of a penguin process. That analysis combined conventional $B$ reconstruction techniques with continuum suppression techniques, and used a likelihood ratio approach for further evidence. While the existence of the radiative penguin process $b \to s\gamma$ was clearly established by this analysis, it did not provide a good measurement of the inclusive rate (the theoretically interesting quantity), since the theoretical estimates of $\Gamma(B \to K^*\gamma)/\Gamma(b \to s\gamma)$ ranged from 5% to 90%. A direct measurement of $b \to s\gamma$ was called for.

6.2 Branching Fraction for $b \to s\gamma$

The inclusive decay $b \to s\gamma$ gives a monoenergetic photon in the $b$ quark rest frame. That monoenergetic line is Doppler broadened by the motion of the $b$ quark in the $B$ meson frame, and the motion of the $B$ meson in the lab frame. But it remains a relatively narrow distribution. In Fig. 13, I show the photon energy distribution expected from $b \to s\gamma$, along with that expected from other $B$ decay processes. The $b \to s\gamma$ decays extend beyond those from other $B$ decay processes and a study of the photon spectrum above 2.0 GeV should cleanly give $b \to s\gamma$.

But wait. There are other curves shown on Fig. 13. One is the photon spectrum from initial state radiation in continuum production, $e^+e^- \to q\bar{q}\gamma$. The other, the spectrum of $\gamma$’s from $\pi^0$ decay in continuum production, $e^+e^- \to q\bar{q} \to \pi^0 X \to \gamma\gamma X$. The sum of these two processes is more than
two orders of magnitude larger than $b \rightarrow s \gamma$, at the $b \rightarrow s \gamma$ peak. Continuum suppression is absolutely essential.

In our 1995 measurement of the rate for $b \rightarrow s \gamma$, we used two different methods for continuum suppression. The first used eight carefully chosen event-shape variables. While no individual variable has strong discriminating power, each possesses some. We combined the eight variables into a single variable $r$, which tends toward +1 for $b \rightarrow s \gamma$ and tends towards −1 for ISR and $q\bar{q}$. We used a neural network for the task of combining the eight variables into a single variable. This was CLEO’s first use of a neural network, and was single handedly pushed through the collaboration by Jesse Ernst, against strong opposition, much of it from his thesis advisor (me). That neural networks are now used extensively, and intelligently, within CLEO can be attributed to Jesse’s good understanding of the strengths and limitations of the technique.

The second method for continuum suppression has been dubbed “pseudoreconstruction”. In it, a high energy photon is combined with a kaon ($K^\pm$ or $K^0_s$) and 1-4 pions (of which one may be a $\pi^0$), and tested for consistency with being a reconstructed $B$. (A $\chi^2$ composed of $B$ mass and $B$ energy, $\chi^2_B$,
is used for this test.) For those events with a pseudoreconstructed $B$, $\theta_{tt}$, the angle between the thrust axis of the candidate $B$ and the thrust axis of the rest of the event, gives additional discrimination against continuum background. In pseudoreconstruction, often one does not have the totally correct combination of particle (hence the “pseudo”), but this is not important (here), because the method is used only to suppress background, and not for a mode-by-mode $B$ reconstruction analysis.

In our 1995 result, we performed two separate analyses, the event-shape analysis and the pseudoreconstruction analysis, and averaged the branching fractions obtained from each (allowing for a small amount of event overlap). That result, $B(b \to s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$, was based on a data sample of $3.0 \, fb^{-1}$.

More recently, we’ve combined the two continuum suppression techniques into a single, unified analysis. For all events containing a high energy photon, we compute the neural net variable $r$. For the subset of events that pseudoreconstruct, with very loose requirements, we also calculate $\chi^2_B$ and $\cos \theta_{tt}$. For these events, we feed $\chi^2_B$, $\cos \theta_{tt}$, and $r$, into another neural network, obtaining a new net variable $r_{comb}$. We assign a weight to each event, based on $r_{comb}$ for pseudoreconstructed events, and on $r$ for those events which fail to reconstruct. In this way we’ve analyzed a $4.7 \, fb^{-1}$ data sample. The photon energy spectrum obtained is shown in Fig. 14. The branching fraction obtained is

$$B(b \to s\gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$$

where the errors, in order, are statistical, systematic, and model dependent. This number is in excellent agreement with the NLO prediction of $(3.28 \pm 0.33) \times 10^{-4}$ of Chetyrkin, Misiak and Münz.

The comparison of experimental result with Standard Model prediction can be (has been) used to place restrictions on New Physics. For example, our conservative upper limit on the branching fraction, $4.5 \times 10^{-4}$, rules out a charged Higgs with Model II coupling for Higgs masses less than 200 GeV. (In SUSY, there would be additional particles, which could contribute with opposite sign, so the limitation is more complicated. However, a hunk of SUSY parameter space is ruled out.)

CLEO now has $14 fb^{-1}$, 3 times the integrated luminosity used in the analysis just described. What’s holding us back? Well, look at the three errors on the branching fraction. Reducing the statistical error by $1/\sqrt{3}$ will do little good unless systematics and model dependence can be beaten down. That takes more time.
Figure 14: The measured photon energy spectrum. (a) On-$\Upsilon(4S)$ yield (points); scaled off-$\Upsilon(4S)$ yield (grey); contribution from $b$ decays other than $b \to s \gamma$ (black). (b) Net yield (points) compared with spectator model prediction for $b \to s \gamma$.

6.3 CP Asymmetry in $b \to s \gamma$

The CP asymmetry in $b \to s \gamma$, $A$, defined by

$$A = \frac{|A(b \to s \gamma)|^2 - |A(\bar{b} \to \bar{s} \gamma)|^2}{|A(b \to s \gamma)|^2 + |A(\bar{b} \to \bar{s} \gamma)|^2},$$

is very small, less than 1%, in the Standard Model. So, observing a nonzero value would be clear evidence for New Physics.

Suppose, in addition to the Standard Model decay amplitude for $b \to s \gamma$, $A_{SM}$, there is a New Physics amplitude, which differs in weak phase from $A_{SM}$.
by $\theta_W$, and in strong phase by $\theta_S$. Then

\[
A(b \to s\gamma) = A_{SM} + A_{New} e^{i\theta_S} e^{i\theta_W};
\]

\[
A(\bar{b} \to \bar{s}\gamma) = A_{SM} + A_{New} e^{i\theta_S} e^{-i\theta_W}.
\]

The $(b/\bar{b})$ averaged branching fraction, $B$, is

\[
B = \frac{1}{2} \left[ |A(b \to s\gamma)|^2 + |A(\bar{b} \to \bar{s}\gamma)|^2 \right] \approx A_{SM}^2 (1 + 2\rho \cos \theta_S \cos \theta_W + \rho^2),
\]

where $\rho = A_{New}/A_{SM}$. The CP asymmetry $A \approx 2\rho \sin \theta_S \sin \theta_W$.

If one is sensitive to branching fraction differences of 20%, then one can detect New Physics amplitudes that are 10% of the Standard Model amplitude, if $\theta_W$ is near zero or 180 degrees, but cannot detect New Physics amplitudes smaller than 45% of the SM amplitude, if $\theta_W$ is near 90°. For $\theta_W$ near 90°, $A \approx 2\rho \sin \theta_S$. So, if one were sensitive to CP asymmetries of 0.10, then one would have sensitivity to this New Physics for $\rho \sin \theta_S > 0.05$.

So, there is a portion of New Physics parameter space, albeit small, where New Physics will show up as a CP asymmetry, but not as a branching fraction difference. This is discussed in general by A. Kagan and M. Neubert (hep-ph/9803368), and as applied to SUSY by Aoki, Cho, and Oshimo (hep-ph/9811251). Asymmetries in the 0.05-0.20 range are mentioned.

How might CLEO measure CP asymmetries in $b \to s\gamma$? By pseudoreconstruction! But wait a minute, didn’t I just say, in Section 6.2, that “In pseudoreconstruction, often one does not have the totally correct combination of particles (hence the ‘pseudo’), but this is not important, because the method is used only to suppress background …”? Well, yes. It still isn’t necessary to get the totally correct combination of particles, but it is necessary to get the flavor – $b$ or $\bar{b}$ – right. It turns out we get the flavor right about 92% of the time. It is straightforward to correct for the 8% mistake rate, a 19% scaling up of the measured asymmetry. With the 4.7$fb^{-1}$ data sample used for the most recent branching fraction analysis, we obtain a corrected asymmetry of

\[
A = 0.16 \pm 0.14 \pm 0.05
\]

So, no evidence for CP violation, but errors that are uncomfortably large. The errors shown are statistical and systematic, in that order. With relatively little work, the systematic error can be reduced substantially, so even with 3 times the luminosity (our in-hand 14$fb^{-1}$), the measurement will be statistics limited. An error of ±0.08 should be straightforward to achieve. We’re looking for ways to push that down, giving consideration to lepton tagging as a possibility.
7 Summary

In “the Early Days”, the basic features of the bottom quark were established:

- A left-handed doublet with a very heavy top.
- Decaying dominantly to charm, $b \rightarrow cW^{−}$. Coupling to the second generation, $|V_{cb}| \approx 0.04$, smaller than the coupling between second generation and first, $|V_{us}| = 0.22$.
- Decay to up, $b \rightarrow uW^{−}$, suppressed relative to decay to charm, but not zero.

In “Recent Times”, the emphasis is on testing the Standard Model, searching for New Physics. There are two approaches: measuring rates for electroweak penguins, and “overdetermining the CKM matrix”. Let’s see where we now stand on each, and where we are going.

On electroweak penguins, the branching fraction for $b \rightarrow s \gamma$ has been measured to $±17\%$, and is in good agreement with the Standard Model. In the near future, with data already in hand, the accuracy should be improved, to $±10\%$. At that point, the error on the measurement will be about equal to the error on the theoretical prediction, and further progress will be slower in coming.

The CP asymmetry has been measured to an accuracy of $±0.14$, and should soon improve to $±0.08$. Further improvements are straightforward, as the error is purely statistical, and the large data samples to be accumulated by BaBar, Belle, and CLEO, in the next 3-4 years, should give a sensitivity to asymmetries in the 0.05 range.

The electroweak process $b \rightarrow s \ell^{+}\ell^{−}$ has not yet been seen, though seeing it may not be far off. Very large data samples will be required to study the various distributions that this 3-body final state makes available. Possibly this is the role for hadron colliders.

Concerning “overdetermining the CKM matrix”, I would argue that the CKM matrix has now been “determined”. Figure 15 shows the famous unitarity triangle, the one from the unitarity condition obtained by multiplying the first column of the CKM matrix by the complex conjugate of the third column:

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} \approx V_{ub}^{*} - \lambda |V_{cb}| + V_{td} = 0 \quad (7-1)$$

The base of the triangle, $\lambda |V_{cb}|$, is known to $±10\%$ (soon to be $±4\%$). The left-hand leg, $|V_{ub}|$, is known to $±25\%$. The one sigma error band for $|V_{ub}|$ is shown. The angle $\gamma$ probably lies between $60^\circ$ and $90^\circ$, the lower limit coming

32
from the Hou, Smith, Würthwein analysis of CLEO data, the upper limit from $B_s - \bar{B}_s$ mixing. I’ve shown more conservative limits on Fig. 15, 45° and 110°. That, I claim, “determines” the CKM matrix. With that, one can predict the angle $\beta$ to be $20^\circ \pm 5^\circ \pm 2^\circ$, where the first error comes from the uncertainty in $|V_{ub}|$, and the second from the uncertainty in $\gamma$. (Note that the uncertainty in the prediction of $\beta$ comes dominantly from $|V_{ub}|$, not $\gamma$.)

The first “overdetermination” of the CKM matrix comes from the CP violating parameter $\epsilon$ in neutral kaon decay. The band it defines nicely intersects the allowed region.

The next “overdetermination” will be BaBar and Belle’s measurements of $\beta$. With 30 fb$^{-1}$ data samples, they expect to measure $\beta$ to $\pm 5^\circ$. It will be interesting to see how their results compare with CLEO’s prediction of $(20 \pm 5)^\circ$, and also how their error on measured $\beta$ will compare with CLEO’s predictions based on improved measurement of $|V_{ub}|$. Interesting times ahead!

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