Slepton and Neutralino/Chargino Coannihilations in MSSM

V.A. Bednyakov

Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, 141980 Dubna, Russia; E-mail: bedny@nusun.jinr.ru

H.V. Klapdor-Kleingrothaus and E.Zaiti

Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029, Heidelberg, Germany

Within the low-energy effective Minimal Supersymmetric extension of Standard Model (effMSSM) we calculated the neutralino relic density taking into account slepton-neutralino and neutralino-chargino/neutralino coannihilation channels. We performed comparative study of these channels and obtained that both of them give sizable contributions to the reduction of the relic density. Due to these coannihilation processes some models (mostly with large neutralino masses) enter into the cosmologically interesting region for relic density, but other models leave this region. Nevertheless, in general, the predictions for direct and indirect dark matter detection rates are not strongly affected by these coannihilation channels in the effMSSM.

I. INTRODUCTION

Measurements of the cosmic microwave background radiation \( \Omega = \rho / \rho_c \approx 1 \), where \( \rho_c = 3H^2 / 8\pi G_N \) is the critical closure density of the universe, \( G_N \) is Newton’s constant and \( H = 100h \) km/sec/Mpc is the Hubble constant with \( h = 0.7 \pm 0.1 \). A variety of data ranging from galactic rotation curves to large scale structure formation and the cosmic microwave background radiation imply a significant density \( 0.1 < 0.3 \) of so-called cold dark matter (CDM). It is generally believed that most of the CDM is made of weakly-interacting massive particles (WIMPs). A commonly considered candidate for the WIMP is the lightest neutralino, provided it is the lightest supersymmetric particle (LSP) in the Minimal Supersymmetric extension of Standard Model (MSSM). Four neutralinos in the MSSM being mass eigenstates are mixtures of the bino \( \tilde{B} \), wino \( \tilde{W} \) and higgsinos \( \tilde{H}_d^0, \tilde{H}_u^0 \), and the LSP can be written as a composition \( \chi = N_{11} \tilde{B} + N_{12} \tilde{W} + N_{13} \tilde{H}_d^0 + N_{14} \tilde{H}_u^0 \), where \( N_{ij} \) are the entries of the neutralino mixing matrix. In SUSY phenomenology one usually classifies neutralinos as gaugino-like (with \( P \approx 1 \)), higgsino-like (with \( P \approx 0 \)), and mixed, where (gaugino) purity is defined as \( P = |N_{11}|^2 + |N_{12}|^2 \).

In most approaches the LSP is stable due to R-parity conservation. The neutralino, being massive, neutral and stable, often provides a sizeable contribution to the relic density. The contribution of neutralinos to the relic density is strongly model-dependent and varies by several orders of magnitude over the whole allowed parameter space of the MSSM. The neutralino relic density then impose stringent constraints on the parameters of the MSSM and the SUSY particle spectrum, and may have important consequences both for studies of SUSY at colliders and in astroparticle experiments. In light of this and taking into account the continuing improvements in determining the abundance of CDM, and other components of the Universe, which have now reached an unprecedented precision, one needs to be able to perform an accurate enough computation of the WIMP relic abundance, which would allow for a reliable comparison between theory and observation.

In the early universe neutralinos existed in thermal equilibrium with the cosmic thermal plasma. As the universe expanded and cooled, the thermal energy is no longer sufficient to produce neutralinos at an appreciable rate, they decouple and their number density scales with co-moving volume. The sparticles significantly heavier than the LSP decouple at the earlier time and decay into LSPs before the LSPs decouple themselves. Nevertheless there may exist some other next-to-lightest sparticles (NLSPs) which are not much heavier than the stable LSP. The number densities of the NLSPs have only slight Boltzmann suppressions with respect to the LSP number density when the LSP freezes out of chemical equilibrium with the thermal bath. Therefore they may still be present in the thermal plasma, and NLSP-LSP and NLSP-NLSP interactions hold LSP in thermal equilibrium resulting with significant reduction of the LSP number density and leading to
acceptable values even with a rather heavy sparticle spectrum \[8\]. These coannihilation processes can be particularly important when the LSP-LSP annihilation rate itself is suppressed.

The number density is governed by the Boltzmann equation \[8,10\]

\[
\frac{dn}{dt} + 3H n = -\langle \sigma v \rangle (n^2 - n_{eq}^2)
\]

with \(n\) either being the LSP number density if there are no other coannihilating sparticles, or the sum over the number densities of all coannihilation partners. The index “eq” denotes the corresponding equilibrium value. To solve the Boltzmann equation (1) one needs to evaluate the thermally averaged neutralino annihilation cross-section \(\langle \sigma v \rangle\). Without coannihilation processes \(\langle \sigma v \rangle\) is given as the thermal average of the LSP annihilation cross-section \(\sigma_{\chi\chi}\) times relative velocity \(v\) of the annihilating LSPs

\[
\langle \sigma v \rangle = \langle \sigma_{\chi\chi} v \rangle,
\]

otherwise, it is determined as \(\langle \sigma v \rangle = \langle \sigma_{\text{eff}} v \rangle\), where the effective thermally averaged cross-section is obtained by summation over coannihilating particles \[8,10\]

\[
\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_{eq}^i n_{eq}^j}{n_{eq} n_{eq}^i n_{eq}^j}.
\]

If \(n_0\) denotes the nowadays number density of the relics, the relic density is given by

\[
\Omega = \frac{m_\chi n_0}{\rho_c}.
\]

Many increasingly sophisticated calculations of the relic density of neutralinos in supersymmetric models, with various approximations both in the evaluation of \(\langle \sigma_{\text{eff}} v \rangle\) and in solving the Boltzmann equation were performed \[12–20,9,21–25,5,26,27,10,11,28,8,7,29–38\]. Following \[38\] we briefly remind below of the major developments in the field.

Perhaps for the first time strong constraints for pure photino relic abundance were obtained in \[12\]. The first analysis of the general neutralino case was performed in \[13,14\]. Several other early papers subsequently appeared with more detailed and elaborate analyses. The two-neutralino annihilation into the ordinary fermion-antifermion (\(f\bar{f}\)) final states through the \(Z\)-exchange was computed in detail \[15\]. The first complete analysis of the neutralino annihilation into \(W^+W^-, Z^0Z^0\) and Higgs-pair final states was performed in \[16\]. The Higgs contribution into neutralino annihilation was first computed in \[16,17\]. For the pure gaugino-like and higgsino-like neutralinos (where several important resonances and final states are absent) all the annihilation channels were considered in \[18\]. A first complete set of expressions for the product of the cross section times velocity using the helicity amplitude technique was computed in \[19\]. The relic density calculation was made by expanding the \(\langle \sigma v \rangle\) as a power series in neutralino velocity. The angular and energy integrals in such a case can be evaluated analytically and the remaining integration over temperature was performed numerically. When expanded in the nonrelativistic limit, these give expressions for the first two coefficients of the partial wave expansion. In the early papers the partial wave expansion of the \(\langle \sigma v \rangle\) was used in most cases. The method is normally expected to give an accurate enough approximation in many regions of the model parameter space because the relic neutralino velocity is expected to be highly non-relativistic. However, it fails near \(s\)-channel resonances (quite high-energetic) and thresholds for new final states, as was first pointed out in \[20\] and further emphasized in \[21,22\]. In particular, it was shown \[21\] that due to the very narrow width of the lightest supersymmetric Higgs \(h\), in the vicinity of its \(s\)-channel exchange the error can be as large as a few orders of magnitude. Therefore a relativistic treatment of thermal averaging is necessary. A recent detailed analysis \[20\] showed that in the case of the often wide \(s\)-channel resonance exchange of the pseudoscalar Higgs boson \(A\), the expansion produces a significant error \[13\]. Furthermore subdominant channels and often neglected interference terms can also sometimes play a sizeable role.

The proper formalism for relativistic thermal averaging was developed in \[4\], and used in \[24\]. A more accurate treatment of the heat bath for both annihilating particles involving two separate thermal distributions was considered in \[23\]. A very useful compact expression for \(\langle \sigma v \rangle\) as a single integral over the cross section was for the first time derived in \[8\]. The DarkSusy code
where the relic density of neutralinos is numerically computed without the partial wave expansion approximation was developed in [11].

An additional very strong reduction of the relic abundance of WIMPs through coannihilation was first discovered in [20]. There are regions in the MSSM parameter space where higgsino-like LSP, light chargino and next-to-lightest neutralino masses become nearly degenerate and all three species can exist in thermal equilibrium. Their mutual coannihilation is often important, and even dominant [20,39]. In the coannihilation with the LSP can be involved any SUSY particle, provided its mass is almost degenerate with the mass of the LSP [20]. In the low-energy effective MSSM (effMSSM), where one ignores restriction from unification assumptions and investigates the MSSM parameter space at the weak scale [10,26,40] there is, in principle, no preference for the MSSM (effMSSM) [20,39,10]. In the coannihilation with the LSP can be involved any SUSY particle. Nevertheless due to quite reasonable and commonly used sets of free parameters, when all gaugino mass eigenvalues are calculated in terms of entries of gaugino mass matrices (µ, M), mass matrices (free parameters, when all gaugino mass eigenvalues are calculated in terms of entries of gaugino mass matrices (µ, M, tan β), the coannihilations between gauginos are expected to occur most often, in the effMSSM [20,39,10].

The relativistic thermal averaging formalism [3] was extended to include coannihilation processes in [10], and was implemented in the DarkSusy code [11] for coannihilation of charginos and heavier neutralinos. In was found [10] that for higgsino-like LSP such a coannihilation significantly decreases the relic density and rules out these LSPs from the region of cosmological interest. For the highly bino-like LSPs, the reduction of the relic density due to the coannihilation is not strong enough to avoid an overclosing of the universe.

The importance of the neutralino coannihilation with sferminos was emphasized and investigated both for sleptons [8,29] or stops [33,34,31] in the so-called constrained MSSM (cMSSM) [27,21,41] or in supergravity (mSUGRA) models [42].

In the most popular mSUGRA model [42] SUSY breaking occurs in a hidden sector and is communicated to observable sectors via gravitational interactions. The model has a minimal set of parameters: m0, m1/2, A0, tan β and sign(µ). Here m0 is the universal scalar mass, m1/2 is the universal gaugino mass and A0 is the universal trilinear mass, all evaluated at M_GUT, tan β is the ratio of Higgs field vacuum expectation values and µ is a Higgs parameter of the superpotential. In particular, within the framework of the mSUGRA, it was found [24,28] that at large tan β, indeed large new regions of model parameter space gave rise to reasonable values for the CDM relic density, due to off-resonance neutralino annihilation through the broad A and H Higgs resonances. There are strong correlations of sfermion, Higgs boson and gaugino masses in mSUGRA originating from unification assumptions. In regions of mSUGRA parameter space where χ and τ (or other sleptons) were nearly degenerate (at low m0), coannihilations could give rise to reasonable values of the relic density even at very large values of m1/2, at both low and high tan β [8,31]. In addition, for large values of the parameter A0 or for non-universal scalar masses, top or bottom squark masses could become nearly degenerate with the χ, so that squark coannihilation processes can become important as well [33,34]. Therefore due to slepton and squark coannihilation effects, the relic density can reach the cosmologically interesting range of 0.1 < Ωh^2 < 0.3 [3]. In the mSUGRA LSP is naturally almost pure bino-like as was first noticed in [43] from the point of view of low-energy SUSY and CDM.

Having in mind investigation of future prospects for direct and indirect detection of LSPs we follow the most phenomenological (general) view, not bounded by theoretical restrictions from sfermion/gaugino/Higgs mass unifications, etc. To this end we need maximally general and accurate calculations of the relic density within the low-energy effective MSSM scheme (effMSSM) [20,39,10]. The only available high-level tools for these calculations was the DarkSusy code (the best code to our knowledge at the moment, this paper was started). Unfortunately the code calculates only neutralino with neutralino/chargino coannihilations (NCC), which is not sufficient in the case of bino-like LSPs, when neutralino-slepton coannihilation (SLC) and neutralino-squark coannihilation are claimed to be dominant [20,39,41,43]. This paper is aimed at a comparative study of NCC and SLC channels, exploration of relevant changes in the relic density and investigation of their consequences for detection of cold dark matter particles in the effMSSM.

II. THE effMSSM APPROACH

As free parameters in the effMSSM we use [41] the gaugino mass parameters M1, M2; the entries to the squark and slepton mixing matrices m^2_{Q}, m^2_{U}, m^2_{D}, m^2_{R}, m^2_{L} for the 1st and 2nd generations...
and $m_{\tilde{Q}_i}^2, m_{\tilde{T}_i}^2, m_{\tilde{D}^c_i}^2, m_{\tilde{u}^c_i}^2, m_{\tilde{d}^c_i}^2$ for the 3rd generation, respectively; the 3rd generation trilinear soft couplings $A_t, A_b, A_{t\tau}$; the mass $m_A$ of the pseudoscalar Higgs boson, the Higgs superpotential parameter $\mu$, and $\tan \beta$. To reasonably reduce the parameter space we assumed $m_{\tilde{U}}^2 = m_{\tilde{D}}^2 = m_{\tilde{Q}}^2; m_{\tilde{T}}^2 = m_{\tilde{D}^c}^2; m_{\tilde{u}^c}^2 = m_{\tilde{D}^c}^2; m_{\tilde{d}^c}^2 = m_{\tilde{L}}^2$ and have fixed $A_t = A_{t\tau} = 0$. The remaining parameters defined our effMSSM parameter space and were scanned randomly within the following intervals:

-1 TeV < $M_t$ < 1 TeV, -2 TeV < $M_2, \mu, A_t$ < 2 TeV, $1.5 < \tan \beta < 50,$
50 GeV < $M_A < 1000$ GeV, 10 GeV$^2$ < $m_{\tilde{Q}}^2, m_{\tilde{L}}^2, m_{\tilde{Q}_i}^2, m_{\tilde{L}_i}^2$ < 10$^6$ GeV$^2$.

We have included the current experimental upper limits on sparticle masses as given by the Particle Data Group [14]. As in [14], we have used the limit $m_{\tilde{\chi}_1^\pm} \geq 85$ GeV for the lighter chargino mass. The current limits on the rare $b \to s\gamma$ decay [14] and the $g-2$ limits [14] have also been imposed. In agreement with a flat accelerating universe [3], we assume $0.1 < \Omega h^2 < 0.3$ for the cosmologically interesting region. The calculations of the neutralino-nucleon cross sections, and direct and indirect detection rates follow the description given in [3,14].

We have evaluated the relic density of the LSP under ignoring of any coannihilation (IGC), taking into account only NCC or SLC separately, as well as including both coannihilation channels (BCC). To this end in our former code [14] DarkSusy procedures of $\langle \sigma v \rangle$ evaluation and solution of Boltzmann equation were implemented. All coannihilations with two-body final states that occur between neutralinos, charginos and sleptons, as long as their masses are $m_i < 2m_{\tilde{\chi}_1}$ were included. The Feynman amplitudes for NCC and SLC were taken from DarkSusy [11] and [14,8], respectively. We calculated $\langle \sigma v \rangle$ and $\Omega h^2$ following the relevant DarkSusy routines [14], which we have merged with code [14] in a way that guarantees the correct inclusion of SLC.

If all sleptons, neutralinos and charginos in question are substantially heavier than the LSP ($m_i \geq 2m_{\tilde{\chi}_1}$) and no way for coannihilations, the resulting relic density $\Omega h^2 = \Omega h^2_{\text{NCC}} = \Omega h^2_{\text{BCC}} = \Omega h^2_{\text{SLC}}$ is equal to $\Omega h^2$ of former results obtained without any coannihilations (with $\langle \sigma v \rangle = \langle \sigma v_{\text{NCC}} \rangle$). When at least one of coannihilation channels (NCC or SLC) is indeed relevant, the $\Omega h^2_{\text{IGC}}$ (ignorance of any coannihilation) is calculated with

$$\langle \sigma v \rangle_{\text{IGC}} = \langle \sigma v_{\tilde{\chi}_1} \rangle \left( \frac{n_{\text{eq}}}{n_{\text{eq}}^\text{IGC}} \right)^2,$$

where $n_{\text{eq}}$ includes all open coannihilation channels. This formula allows a comparative study of results, relevant to one or both coannihilation channels, always delivering a decreasing ratio $\Omega h^2_{\text{NCC}} / \Omega h^2_{\text{IGC}} < 1$ in accordance with [3,14] and sometimes contrary to [14,8]. We introduced $\Omega h^2_{\text{COA}}$ as a common notation for $\Omega h^2_{\text{BCC}}, \Omega h^2_{\text{NCC}}$ or $\Omega h^2_{\text{SLC}}$.

### III. RESULTS AND DISCUSSIONS

We performed our calculations in the effMSSM approach given above and results of our considerations (scatter plots) are presented in Figs. [10] and Figs. [11] for neutralino relic density and CDM observables, respectively.

#### A. Relic density

The general view of the reduction effect on the relic density (RD) due to NCC, SLC and BCC are shown in Fig. [11] as ratios $\Omega h^2_{\text{COA}} / \Omega h^2_{\text{IGC}}$ together with comparison of NCC against SLC in the form of the ratio $\Omega h^2_{\text{SLC}} / \Omega h^2_{\text{NCC}}$. On the basis of our sampling (50000 models tested) the maximum factor of RD decrease due to NCC is about $2 \cdot 10^{-3}$ for $m_{\chi} \approx 200$ GeV, while SLC reduces the RD maximally by a factor of $8 \cdot 10^{-4}$ for $m_{\chi} \approx 300$ GeV. Both reduction factors are roughly of the same order of about $10^{-3}$. These results depend on the applied experimental limits on the second-lightest neutralino, chargino and stau masses. If there were no limits implemented on their masses, the factor of relative RD reduction due to NCC could reach a maximum value of $2 \cdot 10^{-5}$ for models with $m_{\chi} \approx 40$ GeV. But in our case, the current experimental limits are $m_{\tilde{\chi}_1^\pm} > 85$ GeV and $m_{\tilde{\tau}} > 81$ GeV, and therefore the critical LSP mass that enables non-negligible NCC and SLC contributions is also of the same order ($m_{\chi} \geq 80$ GeV).
FIG. 1. Effects of neutralino-chargino/neutralino (NCC) and slepton-neutralino (SLC) coannihilations. Panels a)–d) display ratios $\Omega_{\text{BCC}}/\Omega_{\text{IGC}}$, $\Omega_{\text{NCC}}/\Omega_{\text{IGC}}$, $\Omega_{\text{SLC}}/\Omega_{\text{IGC}}$, and $\Omega_{\text{NCC}}/\Omega_{\text{SLC}}$, respectively. The maximal reduction factors for both NCC and SLC are of the order of $10^{-3}$.

FIG. 2. The same as in Fig. 1 a), b) and c), but plotted together. Here $\Omega_{\text{BCC}}/\Omega_{\text{IGC}}$, $\Omega_{\text{NCC}}/\Omega_{\text{IGC}}$, $\Omega_{\text{SLC}}/\Omega_{\text{IGC}}$, and $\Omega_{\text{NCC}}/\Omega_{\text{SLC}}$ are marked with squares, triangles, and stars, respectively. Therefore, a square filled with a star (triangle) depicts a model that is only affected by SLC (NCC), while the other coannihilation channel in the majority of models gives negligible contribution.
From Fig. 2 one can see that the reduction of RD by coannihilations is mainly due to either NCC or SLC. The other channel of coannihilation plays no role or leads only to a much smaller further reduction. Although other coannihilation processes besides NLSP-LSP can in principal occur (including the next-to-NLSP (NNLSP) and next-to-NNLSP, etc), Fig. 3 demonstrates that a stau \( \tilde{\tau} \) as a NLSP indeed entails a dominant SLC effect, while a next neutralino \( \tilde{\chi}_2 \) or chargino \( \tilde{\chi}^\pm \) as a NLSP indeed entails a dominant NCC effect.

**FIG. 3.** Ratio \( \Omega h_{\text{NCC}}^2 / \Omega h_{\text{SLC}}^2 \) versus \( m_{\chi} \). Stars indicate that the \( \tilde{\tau} \) is the NLSP, triangles mean that the light chargino \( \tilde{\chi}^\pm \) is the NLSP, small filled squares mark the models where the second-lightest neutralino \( \tilde{\chi}_2 \) is the NLSP. One sees that if \( \tilde{\tau} \) is the NLSP, the SLC necessarily dominates, while \( \tilde{\chi}_2 \) or \( \tilde{\chi}^\pm \) being the NLSP always leads to dominant NCC.

**FIG. 4.** The same as in Fig. 3, but versus \( m_{\text{NLSP}} - m_{\chi} \).
From Fig. 4 one can notice that mass differences $m_{\tilde{\tau}} - m_\chi < 20$ GeV lead to a RD reduction factor of 0.5–0.005; 20 GeV < $m_{\tilde{\tau}} - m_\chi$ < 40 GeV lead to the factor of 0.8–0.01 and mass differences 40 GeV < $m_{\tilde{\tau}} - m_\chi$ < 100 GeV can still lead to factors smaller than 0.3 due to SLC. Mass differences $m_{\tilde{\chi}^\pm} - m_\chi < 2$ GeV lead to a RD decrease by factors of 0.1–0.005; 2 GeV < $m_{\tilde{\chi}^\pm} - m_\chi$ < 40 GeV lead to the factor of 0.9–0.02 due to NCC. For both kinds of NLSPs, the coannihilation effect may become negligible if $m_{NLSP} - m_\chi$ ≥ 30 GeV, and necessarily becomes negligible if $m_{NLSP} - m_\chi$ ≥ 100 GeV. Therefore, future increase of the lower mass limits for all possible NLSP (at Tevatron or LHC) can, in principle, strongly reduce the importance of the effect of any of the coannihilation channels.

Although we have implemented the coannihilation opening threshold of $m_i = 2m_\chi$, it was found that a SLC-reducing factor less than 0.5 (0.1) occurs only for $m_{\tilde{\tau}} < 1.12 (1.05) m_\chi$. Accordingly, a NCC-reducing factor less than 0.5 (0.1) appears for $m_{\tilde{\chi}^\pm} < 1.16 (1.10) m_\chi$ and $m_{\tilde{\chi}_2} < 1.11 (1.08) m_\chi$. Therefore for all channels of coannihilation, relevant effects occur if the mass difference between the coannihilation partner and the LSP is within 10–15%. This is in an agreement with previous considerations [20,8,30,10,32,29,34,31].

From Fig. 4 one can also see that charginos and neutralinos come to lie close in mass to the LSP more often than staus (and other sleptons). This is an expected result of correlations in the gaugino sector of the effMSSM, as mentioned in the Introduction, which explains the NCC dominance over SLC seen from Figs. 2 and 3. If one manages to construct a SUSY model where the LSP mass is almost always degenerate with one of the slepton masses (as for example, in mSUGRA models with bino-like neutralinos) the dominant coannihilation channel will be SLC.

**FIG. 5.** Illustration of the shifting of effMSSM models inside and outside the cosmologically interesting range $0.1 < \Omega h^2 < 0.3$ due to NCC and SLC. Models with $\Omega h^2_{IGC}$, $\Omega h^2_{SLC}$, $\Omega h^2_{NCC}$ and $\Omega h^2_{BCC}$ are marked with empty circles, filled circles, small dots, and empty squares, respectively. Therefore, a superposition of all those symbols corresponds to a model which is totally untouched by coannihilation. A black-framed filled circle marks a model which is untouched by SLC ($\Omega h^2_{SLC} = \Omega h^2_{IGC}$), but shifted down due to NCC. If the corresponding $\Omega h^2_{BCC}$ (which is equal to $\Omega h^2_{NCC}$) remains within this range, it still presents in the figure below the black-framed filled circle as an empty square with a black dot inside. By analogy, an empty square with a filled circle inside gives a model which was shifted into the region due to SLC only ($\Omega h^2_{BCC} = \Omega h^2_{SLC}$), and if the corresponding $\Omega h^2_{IGC}$ also is in the cosmologically viable range, it is located above the filled square as an empty circle with a dot inside. One can notice that a quite big amount of models is shifted out of $0.1 < \Omega h^2 < 0.3$ due to NCC (grey circles).

In Fig. 5 all calculated relic densities ($\Omega h^2_{IGC}$, $\Omega h^2_{SLC}$, $\Omega h^2_{NCC}$ and $\Omega h^2_{BCC}$) are depicted in the cosmologically interesting region $0.1 < \Omega h^2_{COA} < 0.3$. There is a quite big amount of models (mostly with lower LSP masses) which are completely unaffected by coannihilation. When at least
one of coannihilation channels is relevant, the RD decreases and some cosmologically unviable models with $\Omega h^2_{IGC} > 0.3$ enter the cosmologically interesting range $0.1 < \Omega h^2_{COA} < 0.3$, due to NCC (squares with a dot in the figure), SLC (filled squares), or both NCC and SLC (empty squares). There are also models which enter the less interesting region for LSP to be CDM ($\Omega h^2_{COA} < 0.1$). The largest amount of models was shifted out due to NCC (filled circles), and a relatively small amount of models was shifted out due to SLC (circles with dots), or both NCC and SLC (open circles). Contrary to mSUGRA, in the effMSSM with SLC and NCC we can not find a possibility to derive any cosmological upper limit for $m_\chi$. There are cosmologically interesting LSPs within the full mass range $12 \text{ GeV} < m_\chi < 720 \text{ GeV}$ (Fig. 5) accessible in our scan irrelevantly to neglecting or inclusion of any coannihilation channels in question.

Cosmologically interesting LSPs occur with arbitrary compositions when coannihilations are ignored (Fig. 6), the inclusion of NCC rules out all the models with higgsino-like LSPs, and SLC further tends to rule out LSPs with mixed composition, so that only LSPs with $P > 0.6$ remain as dominant CDM candidates. While NCC is important both for higgsino-like and gaugino-like LSPs the SLC mainly affects only gaugino-like LSPs. In general our estimations (Fig. 6) are in accordance with [8] when bino-like LSPs and SLC are concerned and in accordance with [10] when NCC effect is concerned.

B. Detection rates

Now we briefly consider the influence of NCC and SLC on prospects for indirect and direct CDM neutralino detection. Figure 7 displays the expected indirect detection rates for upgoing muons produced in the Earth by neutrinos from decay products of $\chi\chi$ annihilation which takes place in the core of the Earth or of the Sun. We compare the rate predictions for cosmologically interesting LSPs when the RD is evaluated with or without coannihilations taken into account. We have seen before that the RD in most models with $m_\chi \leq 250 \text{ GeV}$ is untouched both by SLC and NCC, because the difference $m_{NLSP} - m_\chi$ is too large to yield significant effects, therefore the corresponding detection rates are not influenced (depicted in the figures as filled squares with dots inside). For $\chi\chi$ annihilation in the Earth upgoing muon detection rates merely lie within the range $10^{-19} \text{ m}^{-2} \cdot \text{yr}^{-1} < \Gamma_\mu < 5 \cdot 10^{-9} \text{ m}^{-2} \cdot \text{yr}^{-1}$ as long as $m_\chi \leq 250 \text{ GeV}$. When $m_\chi \geq 250 \text{ GeV}$, some of the models with $0.1 < \Omega h^2_{IGC} < 0.3$ are ruled out from the cosmological interesting range ($\Omega h^2_{COA} < 0.1$; Fig. 6) mainly due to NCC (Fig. 6). Others models with $\Omega h^2_{IGC} > 0.3$ are shifted inside this region (Fig. 6 and Fig. 7) mainly due to SLC. In total, for $m_\chi \geq 250 \text{ GeV}$ one finds
$10^{-19} \text{ m}^{-2} \cdot \text{yr}^{-1} < \Gamma_{\text{BCC}} < 5 \cdot 10^{-7} \text{ m}^{-2} \cdot \text{yr}^{-1}$, when the RD is evaluated with coannihilations are taken into account and, $10^{-19} \text{ m}^{-2} \cdot \text{yr}^{-1} < \Gamma_{\text{IGC}} < 4 \cdot 10^{-6} \text{ m}^{-2} \cdot \text{yr}^{-1}$ when coannihilations are neglected. The large values of the detection rates of $\chi\chi$ annihilation in the Earth are slightly decreased (from $10^{-6} \text{ m}^{-2} \cdot \text{yr}^{-1}$ to $10^{-7} \text{ m}^{-2} \cdot \text{yr}^{-1}$) only for heavy LSPs $m_\chi > 500 \text{ GeV}$ in accordance with the fact that the corresponding models are ruled out from the cosmologically interesting range. The few models with maximal detection rates at a level of $10^{-4} \text{ m}^{-2} \cdot \text{yr}^{-1}$ ($m_\chi < 500 \text{ GeV}$) are found to be untouched.

**FIG. 7.** Indirect detection rate for upgoing muons from $\chi\chi$ annihilation in the Earth (a) and the Sun (b). As in Fig. 5, empty circles, grey circles, black dots, and squares correspond to $0.1 < \Omega h^2_{\text{IGC}}, \Omega h^2_{\text{SLC}}, \Omega h^2_{\text{NCC}}, \Omega h^2_{\text{BCC}} < 0.3$, respectively. NCC slightly decreases the detection rates for models with $m_\chi \geq 500 \text{ GeV}$.

**FIG. 8.** Neutralino-proton scattering cross sections for scalar (spin-independent) interaction (a) and axial (spin-dependent) interaction (b). The notations as in Fig. 7.
In the case of indirect detection of upgoing muons from $\chi\chi$ annihilation in the Sun one has in general a similar behavior for models with $m_\chi \geq 250 - 300$ GeV. The only noticeable difference is in the absolute predictions for detection rates for models with $m_\chi > 600$ GeV where instead of $\Gamma^{\mu}_{IGC} < 10^{-4}$ m$^{-2}$·yr$^{-1}$ one expects the rates to be $\Gamma^{\mu}_{BCC} < 3 \times 10^{-5}$ m$^{-2}$·yr$^{-1}$. The highest predicted detection rates of $10^{-1}$ m$^{-2}$·yr$^{-1}$ are again correlated to a few models which are untouched by coannihilation.

Figure 8 shows neutralino-proton scattering cross sections for the scalar (spin-independent) and the axial (spin-dependent) interactions. As in the previous figures the models with $m_\chi \leq 250$ GeV are hardly affected by coannihilation, and for the majority of those models both neutralino-proton and neutralino-neutron scattering cross sections reach values $\sigma \leq 10^{-17}$ GeV$^{-2}$ with the maximal cross section of order $10^{-15}$ GeV$^{-2}$. Cosmologically interesting models with $m_\chi \geq 250$ GeV were influenced by coannihilations as discussed above, and the maximal value of the neutralino-nucleon cross-section decreases from $10^{-15}$ GeV$^{-2}$ to $5 \times 10^{-16}$ GeV$^{-2}$ for the models with $m_\chi > 500$ GeV. In total, independently of neglection or inclusion of NCC and SLC the maximal scalar scattering neutralino-nucleon cross section reaches $10^{-16}$–$10^{-15}$ GeV$^{-2}$.

The spin-dependent neutralino-nucleon cross sections are typically higher than the spin-independent ones, and we have found the maximal values at $10^{-10}$ GeV$^{-2}$ for the axial neutralino-proton and $10^{-11}$ GeV$^{-2}$ for the axial neutralino-neutron scattering for the models which are untouched by the coannihilations. The majority of cosmologically interesting models yields axial neutralino-proton scattering cross sections in the range $5 \cdot 10^{-16}$ GeV$^{-2} < \sigma < 2 \cdot 10^{-12}$ GeV$^{-2}$ and axial neutralino-neutron scattering cross sections in the range $2 \cdot 10^{-16}$ GeV$^{-2} < \sigma < 8 \cdot 10^{-13}$ GeV$^{-2}$.

![Fig. 9. Event rate for direct neutralino detection in a $^{73}$Ge detector. As in Fig. 5, empty circles, grey circles, black dots, and squares correspond to $0.1 < \Omega h^2_{IGC}, \Omega h^2_{SLC}, \Omega h^2_{NCC}, \Omega h^2_{BCC} < 0.3$, respectively. NCC slightly decreases the maximal event rates for models with $m_\chi \geq 500$ GeV, but the models with smaller LSP mass are untouched by the coannihilations.](image)

Figure 9 shows the expected direct detection event rates calculated for a $^{73}$Ge detector when NSS, SLC, and BCC are taken into account. For models with $m_\chi \leq 250$ GeV coannihilations of any kind play no role. The optimistic estimations of the event rate for models with $m_\chi \geq 400$ GeV are slightly decreased due to NCC.
Within the low-energy effective MSSM we calculated the neutralino relic density (RD) taking into account both slepton-neutralino (SLC) and neutralino-chargino/neutralino (NCC) coannihilation channels. To this end we have implemented in our code \cite{40} the relic density part (with neutralino-chargino coannihilations) of the DarkSusy code \cite{11} supplied with the adopted code of \cite{8} (calculating slepton-neutralino coannihilations).

We have shown that in effMSSM the maximum factors of RD decrease due to NCC as well as due to SLC can reach $10^{-3}$, as long as the lower limits for $m_{\tilde{\tau}}$ and $m_{\tilde{\chi}^\pm}$ are similar (both being of order of 80 GeV). We conclude that both coannihilation effects are comparable in the effMSSM. For the majority of models affected by coannihilations and successfully passed all relevant accelerator, cosmological and rare-decay constraints it was observed that either NCC or SLC alone produces significant reduction of RD while the other coannihilation channel gives considerably smaller or zero reduction effect. Both coannihilations (NCC and/or SLC) are found to produce non-negligible effect only if the relevant NLSP mass is smaller than $1.15m_\chi$. The type of the NLSP determines the dominant coannihilation channel. Due to the fact that the effMSSM more often favors neutralino and chargino, but not sleptons to be the NLSP (the NLSP-LSP mass differences in general are systematically larger for sleptons than for gauginos) the NCC is the more often dominant coannihilation channel in agreement with \cite{10}. Only LSFs with purity $P > 0.6$ remain CDM candidates of cosmological interest. Some models with $\Omega h^2 > 0.3$ under neglect of coannihilation enter into the cosmologically interesting region merely due to SLC, and some other models shift out of the region below $\Omega h^2 < 0.1$, merely due to NCC. In the effMSSM, contrary to mSUGRA \cite{8}, both coannihilations do not imply new cosmological limits on the mass of the LSP. We noticed that the most optimistic predictions for neutralino-nucleon cross sections, indirect and direct detection rates for cosmologically interesting models are untouched by these coannihilations. Only for large $m_\chi \geq 500$ GeV, the respectively high values are slightly reduced, because of the NCC rules out corresponding models from the cosmological interesting region $0.1 < \Omega h^2_{IGC} < 0.3$.

When our paper was almost finished we saw the preprints \cite{7,37,38}. In \cite{7} a new sophisticated code microMegas for calculations of the relic density in the MSSM is presented. The main characteristics of this code includes complete tree-level matrix elements for all subprocesses; all coannihilation channels with neutralinos, charginos, sleptons, squarks and gluinos; loop-corrected Higgs masses and widths. All calculations are performed with CompHEP \cite{11}. In \cite{37} the relic density of neutralinos in the mSUGRA was calculated on the basis of annihilation diagrams, involving sleptons, charginos, neutralinos and third generation squarks. The code CompHEP \cite{11} was used, too. It was found in \cite{37} that coannihilation effects are only important on the edges of the model parameter space, where some amount of fine-tuning is necessary to obtain a reasonable relic density. This paper is mostly aimed at prospects for SUSY search with various $e^+e^-$ and hadron colliders and pays no attention on the interplay between different coannihilation channels.

In addition, a full set of exact, analytic expressions for the annihilation of the lightest neutralino pairs into all two-body tree-level final states in the framework of minimal SUSY is now available \cite{38}. This set of expressions does not rely on the partial wave expansion, includes all the terms and is valid both near and further away from resonances and thresholds for new final states. Further extension of this approach to coannihilation processes together with the above-mentioned CompHEP-based codes will supply perhaps one of most powerful tools for complete relic density calculations.

V.B. thanks the Max Planck Institut fuer Kernphysik for the hospitality and RFBR (Grant 00–02–17587) for support.

\begin{thebibliography}{10}
\bibitem{1} A. T. Lee \textit{et al.} (MAXIMA Collab.), Astrophys. J. \textbf{561}, L1 (2001); C.B. Netterfield \textit{et al.} (BOOMERANG Collab.), \texttt{astro-ph/0104460}; N.W. Halverson \textit{et al.} (DASI Collab.), \texttt{astro-ph/0104489}; P. de Bernardis \textit{et al.}, \texttt{astro-ph/0105299}; A. Melchiorri, \texttt{astro-ph/0201237}.
\bibitem{2} W. L. Freedman, Phys. Rept. \textbf{333}, 13 (2000).
\bibitem{3} P. de Bernardis, \texttt{astro-ph/0004404}; A. Balbi, \texttt{astro-ph/0005124}.
\bibitem{4} E.W. Kolb and M.S. Turner, \textit{The Early Universe}, Addison-Wesley (1990); M.S. Turner, \texttt{astro-ph/0108102}.
\bibitem{5} G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. \textbf{267} (1996) 195.
\end{thebibliography}
For current status of CMSSM see J. R. Ellis, K. Olive, and Y. Santoso, hep-ph/0202110.

S. Mizuta and M. Yamaguchi, Phys. Lett. B 298, 120 (1993).

H. Baer, C. Balazs, and A. Belyaev, hep-ph/0202076.

T. Nihei, L. Roszkowski, and R. Ruiz de Austri, hep-ph/0202009.

S. Mizuta and M. Yamaguchi, Phys. Lett. B 298, 120 (1993).

V. A. Bednyakov and H. V. Klapdor-Kleingrothaus, Phys. Rev. D 63, 095005 (2001); Phys. Rev. D 62, 043524 (2000); V. A. Bednyakov, H. V. Klapdor-Kleingrothaus, and H. Tu, Phys. Rev. D 64, 075004 (2001).

For current status of CMSSM see J. R. Ellis, K. Olive, and Y. Santoso, hep-ph/0202110.

A. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. 49, 970 (1982); R. Barbieri, S. Ferrara, and C. Savoy, Phys. Lett. B 119, 343 (1982); L.J. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D 27, 2359 (1983).

L. Roszkowski, Phys. Lett. B 262, 59 (1991).

D. E. Groom et al. Eur. Phys. J. C 15, 1 (2000).

M. S. Alam et al., (CLEO Collab.), Phys. Rev. Lett. 74, 2885 (1995); K. Abe et al., (Belle Collab.), hep-ex/0107065.

H. N. Brown et al. (Muon g-2 Collab.), Phys. Rev. Lett. 86, 2227 (2001).

Toby Falk, private communication

CompHEP A. Pukhov et al., hep-ph/9908288. http://theory.sinp.msu.ru/~pukhov/calchep.html