A chaotic iterative fuzzy modeling of circulating a simple sentence

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Abstract: In this paper, we propose a new model to describe variations in interpretation and perception of a simple sentence by different people. To show the understandability of a simple sentence in the prediction of future situations, the meaning of a sentence is modeled as a fuzzy if-then rule, and the fuzzy model is investigated in an iterative process. The main goal of the paper is modeling a linguistic rule. This is done using an if-then rule and its variation through one person to another. The model predicts that the interpretation reaching the final person in the following years can be chaotic and thus unpredictable.

Keywords: Sentence processor, Model, Chaos, Fuzzy rule, IF-THEN.

1. Introduction and background

People communicate with others using human languages. This ability helps people express their ideas and needs, and transform their skills. Development in science is based on transforming information and experiments \cite{1-3}. A sentence processor has a complex architecture. Every person views each sentence and attempts to interpret and comprehend it, and then he/she can make a new sentence with his or her individual concept. Rumor is a tale of explanations of events which is circulated from person to person and pertains to an object, event, or issue of public concern \cite{4,5}.

An interesting tool in the modeling of expert information is the fuzzy rule which is based on fuzzy logic \cite{6}. Fuzzy rules define grades of membership to describe linguistic states \cite{7,8}. These rules are depicted in the form of if-then rules. Fuzzy logic provides a method for representing fuzzy predictive modifiers, e.g. “median”, “small”, and “large” \cite{9,10}. A typical fuzzy logic system consists of a knowledge base, an inference engine, and a defuzzifier \cite{11,12}. Each fuzzy rule is in the form of if-then statements such as

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\[ \text{IF } x \text{ is } A \]
\[ \text{THEN } y \text{ is } B \]

where \( x \) and \( y \) are linguistic variables, and \( A \) and \( B \) are fuzzy membership functions. Fig. 1 shows the mechanism of if-then rules in a simple example form.

Fuzzy logic has many applications in various fields, such as nonlinear dynamics and chaos [11, 13]. Chaos is a complex behavior with sensitive dependence on initial conditions, which means that two initial conditions, no matter how close together, give trajectories that widely separate during the course of time [14]. Combinations of fuzzy systems and chaos have attracted much attention [15]. The generation of chaotic dynamics using fuzzy rules has been studied in [16]. Modeling of chaotic dynamics using fuzzy systems and linguistic description has been discussed in [17, 18]. In [19] modeling of chaotic dynamics using a small number of fuzzy rules and assigned characteristics such as Lyapunov exponents has been studied. This method has raised the robustness to parameter changes. Fuzzy logic is a helpful method to predict a chaotic time series [20, 21]. In [22] an adaptive fuzzy system was used to regenerate the dynamics of continuous oscillator and chaotic systems. The rules have been dedicated in regions based on stretches and branches. Many studies have investigated chaotic behavior in biological systems [23-26]. A complex Lorenz system was studied in [27]. The chaotic dynamic of visual perception was investigated in [28]. The combination of fuzzy logic and chaos seems very interesting since it can have applications in the study of biological systems.

In previous works [16-18, 29], some membership functions were used to produce a special map. However, in this paper, a linguistic rule is investigated, and iterations have been added to this rule to generate interpretation of a unique sentence in the following years or its variations from one person to another. In other words, the aim of this paper is not generating a chaotic map using fuzzy rules. Rather the main goal is modeling a linguistic rule. This is done using an if-then rule and its variation through one person to another. Using this modeling, we have found that the interpretation results reaching the final person in the following years can be chaotic and hence unpredictable. In the next section, the variation of interpretation and conception of a sentence by different people is described. A fuzzy rule models this complex system in section 3, and its different behaviors in the prediction of future situations are investigated. Finally, section 4 gives the conclusion.

2. Problem definition

A sentence is characterized by a linear sequence of words in the language. Each natural language is a primary mode of human communications [30, 31]. Functions of language include exchanging information, establishing or reinforcing social relations, and controlling the behavior of other people [32, 33]. A sentence is interpreted by each person in processing steps. The first step in the process of comprehending language is recognizing it from the perceptual input
Consider the population of a species in biology. Suppose that an expert says this sentence about the population: “If the population of a species in this year is normal, then its population in a later year will be high”. This sentence is a simple concept that can have different meanings for different people. Qualities like “normal” and “high” can be modeled using fuzzy concepts which can imply different quantities to different people. In the next section, an individual fuzzy model of language comprehension is proposed, and different predictions using this model are discussed.

3. Fuzzy model for interpreting a language rule

Consider the sentence “If the population of a species in this year is normal, then its population in a later year will be high”. Using this sentence, each person has a forecast for the population of the next year. Different people have different interpretations of the sentence, and a fuzzy rule is an efficient method to describe the information process by these people. If we define the normalized population in one year by a variable \( x \) and the normalized population in the later year by a variable \( y \), then the prediction of the population by different people during the next year can be modeled as a fuzzy if-then rule. The membership function of prediction is personal and depends on a person’s social and cultural situation.

If the membership function of the input is e.g. a bell curve (Eq. 2) and the membership function of the output is e.g. a sigmoid function (Eq. 3), then the proposed model can depict different interpretations by a person in different situations using a singleton fuzzifier and singleton defuzzifier. A singleton fuzzifier transforms crisp values of input into a fuzzy set with the membership function which is one for the input value and zero for other values, and a singleton defuzzifier transforms linguistic terms to crisp values [36]. Fig. 2 shows an example of input and output membership functions for one person. By using such membership functions, the fuzzy model can show the process of prediction for each person one year later. Then the person can predict the population of the following years by applying the fuzzy if-then rule (general sentence) iteratively. Fig. 3 illustrates the iterative process in the prediction of the population of each year from the previous ones using the fuzzy model. On the other hand, the model can show different predictions of future years by different people, since they have a different sentence processor and a different interpretation (as shown by e.g. different membership functions for each person in the model).

\[
f(x; a_x, b_x, c_x) = \frac{1}{1 + \left(\frac{x - c_x}{a_x}\right)^{2b_x}}
\]  

(2)
A bifurcation diagram of the model for changing the parameter $b_x$ is shown in Fig. 4. The number of iterations to build bifurcation diagram is 1150, and we have removed 1000 iterations of transient time in plotting the diagram. The bifurcation parameter $b_x$ is a parameter of the linguistic rule representing the interpretation of various people from a fuzzy modifier based on a person's social and cultural situation. In this paper in the assumed rule "If the population of a species in this year is normal, then its population in a later year will be high", the normal modifier is represented by a bell function. The parameter $b_x$ in this function can change the decreasing or increasing slope. To display the modifiers "normal" and "high", different membership functions could be chosen. A sample of those membership functions has been selected to describe the prediction of the population of future years by various people. Different people have a different understanding of a sentence (different if-then rules) that causes different interpretations. As an example, consider the discussed rule. Different people ascribe different meanings to this sentence depending on their personality. Changes in the personality parameter affect the prediction of the population in the later year. One person with personality parameter $b_x = 1.5$ has a complex, chaotic forecast for the following years, while the prediction of another person with personality parameter $b_x = 2$ has a less complex behavior. This shows that different people can have different forecasts of the population of future years using the same rule. A positive Lyapunov exponent in part (b) of Fig. 4 confirms the chaotic predictions. We have used Wolf's Jacobian method to calculate the Lyapunov exponent in Fig. 4b with 10000 iterations.

Here is the result of the fuzzy-related procedure in simpler algebraic words:

At first, by assigning the input to a bell function, we compute fuzzification of this value (the "if" part). Then we feed this value to the sigmoid function, and the output is computed (the "then" part). Then this process continues iteratively (Eq. 4). In other words, this sentence can be considered as a one-dimensional map (Eq. 5).

\[
f(y;a_y,c_y) = \frac{1}{1 + e^{-a_y(y-c_y)}}
\]  

\[
x_{k+1} = f_y(f_x(x_k; a_x,b_x,c_x); a_y,c_y)
\]  

\[
x_{k+1} = \frac{1}{1 + e^{-a_x \left( \frac{1}{1 + e^{a_x y - c_x}} \right)}}
\]  

\[
1 + e^{-a_x \left( \frac{1}{1 + e^{a_x y - c_x}} \right)}
\]
Map plots and cobweb plots are two useful tools to investigate the dynamical properties of a system. To study the dynamics of System (5), a cobweb plot is used as shown in Fig. 5. The map plot of a discrete system such as \( x_{k+1} = f(x_k) \) displays the function \( f \) in its domain. The cobweb plot shows the transition of the time series (black color) within the map plot (blue) [37, 38]. The red plot shows the identity line. Its intersection point with the map plot determines the equilibrium points. The map plot shows that system (5) has a single locally quadratic maximum. Part (a) of the figure shows the cobweb plot for \( b_x = 0.5 \) which has a period-4 cycle. Parts (b), (c), and (d) show the cobweb plot in the chaotic, period-6, and chaotic regions, respectively. The equilibrium point of the system is unstable in these cases since the absolute value of its derivative is larger than one. The equilibrium point of the system is only stable in \([0.0, 0.147]\) and \([4.277, 5.0]\) of the studied interval, which can be seen in the bifurcation diagram of Fig. 4. The system’s dynamic always remains in the domain \([0,1]\) in the steady state. The system has a period-doubling route to chaos at different intervals of the parameter. Also, there are some periodic windows in the bifurcation diagram.

This model can help linguists to follow variations of interpretation of a sentence in transition between people. To have a more precise model, its membership functions should change for different people (iteration).

In a one-dimensional discrete dynamical system with a single locally quadratic maximum, the Feigenbaum constant has a universal value of approximately 4.66. In such a case, the Schwarzian derivative is negative [39]. We have calculated the Schwarzian derivative in the studied interval of parameter \( b_x \). The results showed that it is negative. Fig. 6 shows this derivative in \( b_x = 2.6 \) and \( b_x = 4.5 \), respectively.

Also, in a period-doubling bifurcation into chaos, the Feigenbaum number is constant [40]. A zoomed view of the bifurcation diagram of System (5) in \( b_x \in [0.88, 1] \) is shown in Fig. 7. The figure reveals that the bifurcation of period-4 to period-8 happens at \( b_x = 0.900274 \), the bifurcation of period-8 to period-16 at \( b_x = 0.960422 \), the bifurcation of period-16 to period-32 at \( b_x = 0.971971 \), the bifurcation of period-32 to period-64 at \( b_x = 0.974364 \), and the bifurcation of period-64 to period-128 at \( b_x = 0.974870 \).

The Feigenbaum constant can be calculated from

\[
\delta = \lim_{n \to \infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}}
\]
where $a_n$ is the parameter for which $n$th period-doubling occurs. This constant in the studied interval is

$$\delta_1 = \frac{a_2 - a_1}{a_3 - a_2} = 5.208069962767323$$

$$\delta_2 = \frac{a_3 - a_2}{a_4 - a_3} = 4.826159632260817$$

$$\delta_3 = \frac{a_4 - a_3}{a_5 - a_4} = 4.729249011857605$$

The Feigenbaum constant approaches its limit of $4.669202$.

The model can be discussed from two aspects as shown in Fig. 8. In the first aspect, assume a person has used the constant rule every year with constant parameters. Also, the case can be considered in which a society has people with the same interpretability of a sentence. In such a case, based on the bifurcation diagram of Fig. 4a, there are many parameter values of the assumed memberships ($b_x$) that give chaotic dynamics. In the second point of view, by transforming the sentence from one person to another, the parameter of the membership function which determines the interpretation of one person from the sentence is varied. When a person hears a sentence, he tries to understand and interpret it. Then he transmits his understanding of the sentence to the next person who has a different membership parameter $b_x$. Fig. 4a has shown that there are many cases in which the understanding of a person from one sentence is unpredictable and chaotic.

4. Conclusion

This paper proposes a fuzzy model to describe variations in the prediction of future situations using a simple sentence. Application of the sentence “If the population of a species in this year is normal, then its population in a later year will be high” in the prediction of the species population of the following years is investigated using a fuzzy model. Bifurcation analysis of the model shows that forecasts using the sentence can have different dynamics, such as periodic and chaotic behavior. Note that the application of this rule does not say anything about the actual population; only the person’s perception of what it might be in later years. The real population dynamic might be complex or simple. The paper has modeled the sentence interpretation and shown that the model can give chaotic dynamics. In other words, the main goal of this paper was modeling a linguistic rule. This was done using an if-then rule and its variation through one person to another. Using this modeling, we found that the interpretation that reaches the final person in the following years can be chaotic and hence unpredictable. This model can help linguists to follow
variations of interpretation of a sentence in transition between various people. To have a more precise model, its membership functions should be changed for different people (iteration).

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Fig. 1. Mechanism of if-then rules.

Fig. 2. Input and output membership functions for one person with parameters $a_x = 0.36, b_x = 3.2, c_x = 0.5, a_y = 9, c_y = 0.5$.

Fig. 3. Prediction of the population in each year from the population of the previous year.

Fig. 4. a) Bifurcation diagram and b) Lyapunov exponent of the proposed model with respect to changing personality parameter $b_x$ with the other parameters constant $a_x = 0.36, c_x = 0.5, a_y = 9, c_y = 0.5$.

Fig. 5. Cobweb plot of System (5) for a) $b_x = 0.5$ and initial value $x_0 = 0.36$. b) $b_x = 1.2$ and initial value $x_0 = 0.93$. c) $b_x = 2.25$ and initial value $x_0 = 0.97$. d) $b_x = 4.25$ and initial value $x_0 = 0.05$.

Fig. 6. The Schwarzian derivative of System (5) in the range of the $x$ variable with a) $b_x = 2.6$. b) $b_x = 4.5$.

Fig. 7. A zoomed view of the bifurcation diagram of System (5) for $b_x \in [0.88, 1]$.

Fig. 8. The two aspects of bifurcations in Model (5). a) the first aspect. b) the second aspect.
Fig. 2.

\[ x_k \rightarrow \text{Bell function} \rightarrow \mu_{k+1} \rightarrow \text{Sigmoid function} \rightarrow x_{k+1} \]
Fig. 4.
Fig. 5.

![Graph a](image1)

![Graph b](image2)

Fig. 6.
Fig. 7.
Biography

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