A delayed choice quantum eraser explained by the transactional interpretation of quantum mechanics

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Abstract

This paper explains the delayed choice quantum eraser of Kim et al. [1] in terms of the transactional interpretation of quantum mechanics by John Cramer [2]. It is kept deliberately mathematically simple to help explain the transactional technique. The emphasis is on a clear understanding of how the instantaneous “collapse” of the wave function due to a measurement at a specific time and place may be reinterpreted as a gradual collapse over the entire path of the photon and over the entire transit time from slit to detector. This is made possible by the use of a retarded offer wave, which is thought to travel from the slits (or rather the small region within the parametric crystal where down-conversion takes place) to the detector and an advanced counter wave traveling backward in time from the detector to the slits. The point here is to make clear how simple the Cramer transactional picture is and how much more intuitive the collapse of the wave function becomes if viewed in this way. Also any confusion about possible retro-causal signaling is put to rest. A delayed choice quantum eraser does not require any sort of backward in time communication. This paper makes the point that it is preferable to use the Transactional Interpretation (TI) over the usual Copenhagen Interpretation (CI) for a more intuitive understanding of the quantum eraser delayed choice experiment. Both methods give exactly the same end results and can be used interchangeably.

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Complementarity, which path information and quantum erasers

Feynman 1965, in his famous lectures on physics [3] stated that the Young’s double slit experiment contains the only mystery of quantum mechanics. We may see interference, or we may know through which slit the photon passes, but we can never know both at the same time. This is what is commonly referred to as the principle of complementarity. We say two observables are complementary if precise knowledge of one implies that all possible outcomes of measuring the other are equally likely. In this case position and momentum. The fundamental enforcement of complementarity arises from correlations between the detector and the interfering particle in a way that shows up in the wave function for the system. It is not, as some undergraduate text books would have you believe, a consequence of the uncertainty principle. The Heisenberg uncertainty relation is a consequence of complementarity not the other way around. There have been many gedanken (German for thought) experiments over the years to show complementarity. The most famous being the Einstein recoiling slit, Feynman’s light scattering scheme both discussed in Feynman’s lectures on physics [3] and Wheeler’s delayed choice experiment [4].
Figure 1: The figure shows the Scully Druhl quantum eraser 2 slit arrangement. Two 3-level atoms are in place of the two slits. A laser excites either atom to the upper level $a$ which may then decay to level $b$ or $c$. If the atom decays to level $c$, the ground state, then there will be interference since there is no way to distinguish between the two atoms and so no which path information. See figure (a). The green dots represent the single slit diffraction pattern. The solid line is the intensity detected. If the atom decays to level $b$, then there is which path information and there will be no interference pattern as in figure (b). The drawings are simplified. The intensities drawn are normalized and approximate only and in fig (b) there would be a similar single slit diffraction pattern behind slit $A$.

Of particular interest here is the delayed choice quantum eraser *gedanken* experiment by Scully and Druhl 1982 [5]. This work described a basic quantum eraser experiment and a delayed choice quantum eraser arrangement. The basic quantum eraser experiment is described using two 3-level $\Lambda$–type atoms [7], in the place of two slits. See Fig. 1. The atoms start off in the ground state and then a laser pulse comes in and excites either atom $A$ or $B$. The excited atom then decays and emits a signal photon. Interference fringes are sought between these signal photons on a screen some distance away. Let the identical 3-level atoms have one upper level $a$ and two lower levels $b$ and $c$. The laser excites one of the atoms up to the level $a$ but the atom can de-excite to either state $b$ or $c$. If both atoms start off in the ground state $c$, there are two possibilities.

For a delayed choice quantum eraser [5], the 3-level atoms change to 4-level atoms with levels $a,b,c,d$, with $d$ the ground state. See Fig. 2. Instead of one exciting laser pulse there are two closely spaced pulses, which will both go to the same atom. The first laser pulse excites either atom $A$ or $B$ from the ground state $d$ to the upper level $a$. The excited atom then spontaneously decays to $c$ emitting the signal photon. The second laser pulse then excites the atom from level $c$ to level $b$, which then decays with the emission of a lower energy idler photon to the ground state. Now the atoms are inside a cleverly constructed cavity with a trap door separating them. The cavity is transparent to the signal photons and laser light but strongly reflects the idler photons. There is a detector capable of detecting the idler photons only near atom $A$. The trap door will prevent the idler photon from $B$ being detected. Now we have a choice whether to open the trap door or leave it closed. The signal photon detection is now correlated with the idler photon detection. The experiment has become a delayed choice quantum eraser, whether
Figure 2: The figure shows the Scully Druhl delayed choice quantum eraser. Two 4–level atoms, labeled A and B are in place of the two slits. The atoms are inside a double elliptic cavity with a shared focus. Both atoms and the idler detector are located at foci. The first laser pulse excites either atom to the upper level $a$, which then decays to level $c$, emitting a signal photon (green), which leaves the cavity. The second laser pulse immediately excites the same atom from level $c$ to level $b$, which then decays to the ground state $d$, emitting an idler photon (red). The idler photons cannot leave the cavity. When closed, the trapdoor prevents idler photons from atom B being detected. When the trapdoor is open, the cavity detector may detect idler photons from either atom.

we see interference or not will depend on whether we leave the trap door open or closed. If the trap door is closed and we detect an idler photon, we know that atom A was excited. If we do not detect a photon then atom B was excited, either way we have which path information that will destroy the fringes. If the trap door is open, then we no longer have which path information since either atom could have emitted the idler photon. In principle the decision, to leave the trap door open or closed, can be made after the signal photons have been detected. The paradox is, how does the signal photon know which pattern to make, a single slit diffraction pattern or a two-slit interference pattern, if we have not yet decided to leave the trap door open or closed?

Englert, Scully and Walther [6] in 1991 constructed a very nice atom interference gedanken experiment that shows the physics in a very straightforward manner, although the experiment would be extremely difficult to perform in practice. Soon afterward in 1993, a polarization experiment by Wineland’s group [8], was the first to demonstrate an actual realization of the Scully–Druhl quantum eraser gedanken experiment. They used mercury ions in trap as the two “atoms” and observed linear $\pi$ and circularly $\sigma$ polarized light. Choosing to detect linear polarized light, corresponded to the case that the ions in a trap were in the same initial and final state. This implies that there was no which path information and so there was interference. Choosing to observe circular polarized light, corresponded to the case that the ions were in distinguishable end states after scattering a photon, so which path information was available and hence there was no interference. You could choose to observe interference or not depending on whether you chose to observe linear or circularly polarized light.

There have been many quantum optics experiments involving two photon entangled states and quantum eraser arrangements to prove the complementarity arguments above. Three of the better ones are [9] [10] [11]. One experiment in particular by Zeilinger’s group [12] is worthy of a
special note. The arm lengths in their apparatus were very long, between 55 m up to 144 Km. They point out that there is no possible communication between one photon and the other in the entangled pair because of the space-like separation between them and they assume no faster-than-light communication is possible.

The most famous real experiment of the delayed choice type is that by Kim et al. [1], using parametric down conversion entangled photons. It has drawn considerably more press than any other experiment of this type and even has a couple of online animations [13]. We choose to present our case for the transactional interpretation of quantum mechanics using the Kim experiment as our example, but any of the delayed choice quantum erasers would work just as well.

Introduction to the Transactional Interpretation of Quantum Mechanics

The transactional interpretation of quantum mechanics was written by John Cramer [2] in a review article in 1986. Cramer also wrote a short overview [14]. It is a way to view quantum mechanics that is very intuitive and easily accounts for all the well known quantum paradoxes, EPR [15], which-way detection and quantum eraser experiments, [16, 17]. Unfortunately, it has garnered little support over the years and has fallen off the radar. It deserves a much broader dissemination and part of the motivation to publish this paper was to bring Cramer’s ideas to the attention of the younger generation of physicists, who may not have heard of it before.

Absorber theory and Advanced Waves

The idea of advanced waves in classical electrodynamics started with Dirac [18] in 1938 and his derivation of the radiation reaction of a charged accelerated particle. Dirac assumed an advanced wave, since it was allowed by the relativistic wave equation, but gave no physical explanation as to where it came from. Later Wheeler and Feynman [19] wrote a couple of papers in 1945 and 1949 on absorber theory, which was their attempt to give a physical description of the origins of the advanced waves introduced by Dirac. An added motivation was to try and remove the self energy from the electron, but that was not entirely successful. The radiation reaction could be accounted for without self interaction, but vacuum polarization still presented a problem (charge renormalization). This can only be solved by eliminating the “point” particle picture. Feynman’s PhD thesis included the path integral approach to non-relativistic quantum mechanics, which was used to describe how to quantize the direct particle interaction of absorber theory [20]. Paul Davies later generalized these classical results for the relativistic case [21, 22]. Hoyle and Narlikar also did some of this relativistic work [23]. There are now three different models for absorbers which have slightly differing advanced wave behavior. Wheeler-Feynman [19], Csonka [24] and Cramer [2]. These models differ with regard to what exactly happens when there is a less than perfect absorber present. They are discussed in the very readable paperback by Nick Herbert [25]. So far, we have a working theory for classical electrodynamics and now for QED. Then Cramer spells out the general quantum version of the theory applicable to all systems not just electrons [27, 28].
Figure 3: Cramer’s wiggle diagram. The figure shows a plane-wave transaction between an emitter and an absorber particle. The black vertical lines are the world-lines for each particle. Waves from the emitter are solid lines, waves from the absorber are dotted. The retarded waves are red for both emitter and absorber and the advanced waves are blue. Red retarded waves move up toward the right. Blue advanced waves move downward to the left. Note that along the path between the emitter and absorber the waves add constructively but before the emitter and after the absorber the waves destructively interfere.

Transactional Interpretation

For an interaction to take place between two particles, emitter and the absorber, Cramer says the emitter must send out an offer wave. This offer wave would be half an advanced and half a retarded wave going out in all directions looking for an absorber, something to interact with. When the retarded offer wave reaches the absorber, that particle sends out a counter wave, also half retarded and half advanced. See Cramer’s wiggle diagram, Fig. 3. The advanced counter wave would travel backward in time, along the exact incident path of the original retarded offer wave (it is the complex conjugate of the retarded offer wave), thus constructive interference takes place along the path between the particles. In the one spatial dimension drawn in Fig. 3, the advanced counter wave reaches the emitter particle at the exact time when the retarded offer wave was emitted. This enables the advanced wave from the absorber to exactly cancel with the advanced wave from the emitter at the location of the emitter. Likewise, the retarded wave from the emitter will cancel the retarded wave from the absorber at the location of the absorber. Only the retarded wave from the emitter and the advanced wave from the absorber along the adjoining path are enhanced by the superposition, they do not cancel out. These waves represent the interaction between the particles.

In three spatial dimensions things are a little more complicated. Advanced and retarded waves travel in all directions not just in the direction of one absorber. Retarded waves carry on into the future and maybe absorbed at some later point in time. An advanced wave travels backward in time to the big bang. At this point it is reflected and will move forward in time as an advanced wave identical to, and π out of phase with, the incident advanced wave. This will produce a cancellation at every point along the world-line back to the point of emission of the wave. All advanced waves therefore cancel out, [26]. Note that the waves are assumed to travel at speed $c$ the speed-of-light in a vacuum, although the advanced wave is traveling backward in time, or with $-t$ [27]. Basically, in quantum terms, the regular wave function is the offer wave, the
complex conjugate wave function is the counter wave and together they give a handshake \[28\], which allows an interaction to take place.

Recently a book published by Ruth Kastner \[29\] has expounded the virtues of the transactional method with an additional twist allowing for free will. There are many examples of the use of the transactional method in the book and it is well worth a read. In this paper we make no distinction between the original Cramer Transactional Interpretation (TI) and the Kastner version of Possibilist Transaction Interpretation (PTI). Kastner’s approach \[30\], “is to consider a growing emergent universe in which the future is not set in stone but is actualized from an underlying substratum of quantum possibilities.”

Cramer’s approach means (from the authors view point) that the future is set, the past, present and future may all coexist and we simply have the illusion of flowing through time. To avoid confusion, we quote Cramer on his own interpretation \[31\];

“Let me give an example. When you use your cash card at the grocery store to pay for your purchases, the electronic handshake that occurs between the bank and the cash register insures that money is “conserved” and is neither created nor destroyed, but it does not determine what you elected to purchase. The same is true with quantum transactions, which guarantee the conservation laws but do not determine the future. The real difference between Kastner’s PTI and my TI is that for her, offer and confirmation waves exist as objects only in some multidimensional Hilbert space. In the TI the waves exist in real 3+1 dimensional space. Hilbert space was invented by theorists prone to abstraction because it was the only way they could imagine that quantum waves could be entangled. The TI explains how they can be entangled, because the multi-particle transactions allow only those subset of the waves that satisfy the conservation laws to become real transactions.”

Other’s have considered a Many-Worlds Interpretation, with every possible event happening along parallel realities in order to maintain free will. Neither Kastner nor Cramer agree with the many-worlds view \[15\]. Here, the reader is asked to make up their own mind. This paper is concerned only with; Does the transactional interpretation fit the data or not? It is found that all the usual quantum results hold and the TI is simply an alternative point of view from the Copenhagen interpretation, and the instantaneous collapsing wave function, way of thinking.

The delayed choice quantum eraser by Kim et al.

First we briefly explain the experiment and the observed results. The experimental arrangement can be seen in Fig. 4. An argon laser (\(\lambda_p = 351.1\ nm\)) is passed through a double slit and illuminates a type II phase matching nonlinear crystal of \(\beta\)-Barium Borate BBO (\(\beta - BaB_2O_4\)) The slit \(A\) allows region \(A\) of the crystal to be illuminated and slit \(B\) allows only region \(B\) of the crystal to be illuminated. This small region is about 0.3 mm long which we take to be the slit width \(a\). The separation \(d\) of the two regions is about 0.7 mm as specified in the paper \[1\]. So we may discuss regions \(A\) and \(B\) of the crystal just as well as the original 2 slits. Parametric down conversion will occur at both sites and from the one pump photon will emerge two photons, a signal and an idler. Note that all possible frequencies are created \(\nu_p = \nu_s + \nu_i\). We are selecting two of the same frequency, or equivalently, twice the pump wavelength \(\lambda_s = 702.2\ nm\).
signal and idler photons represent the e-ray and o-ray of the nonlinear crystal.

These photons are momentum entangled and are created essentially at the same time. The probability for a downconversion event is slight, so we may assume that there is only one entangled pair of photons in the system at any given time. Different wavelengths of signal and idler photons exit the crystal at different angles. The required wavelengths are selected by restricting the exit angle. Usually a small range of wavelengths would be selected. For convenience we track only one wavelength, but we should bear in mind that there will be a small bandwidth of wavelengths which will affect the interference pattern of the signal photons and change the visibility of the fringes accordingly. The bandwidth can also be changed using filters in front of the detectors. The detectors will have a less than perfect efficiency which will also affect the fringe visibility. The efficiency of the detectors was not mentioned in the experiment however, and neither was the effective bandwidth.

The signal photons are sent through a lens, of focal length \( f \) (not specified in the paper [1]) and then focussed onto a screen where they can be detected by detector \( D_0 \). The detector scans, via stepper motor, along the \( x \)-axis to build up a pattern. The lens is used to create the far field condition at the detector so we expect a Fraunhofer type pattern which is built up over time. The idler photons, from region \( A \) and \( B \) of the crystal, are sent in the direction of a Glen-Thompson prism (a wedge mirror in figure 3. is used for convenience) which separates them into different paths. The idler photons from region \( A \) hit BSA and are either reflected or transmitted. The reflected photons will be detected by \( D_3 \). The transmitted photons will be reflected by mirror MA and then either transmitted through the beamsplitter BS to detector \( D_2 \) or reflected by BS into detector \( D_1 \). The idler photons from \( B \) hit BSB and are either reflected or transmitted. The reflected photons will be detected by \( D_4 \). The transmitted photons will be reflected by mirror MB and then either transmitted through the beamsplitter BS to detector \( D_1 \) or reflected by BS into detector \( D_2 \).

The time of flight from the crystal to the detector \( D_0 \) for the signal photons is 8 ns shorter than for the idler photons which go in the direction of the beamsplitters and were eventually detected by detectors \( D_3, D_4 \) or by \( D_1 \) or \( D_2 \). The equivalent path length is approximately 2.5 m. We assume that all the detector path lengths, \( D_1 - D_4 \), are the same and equal to 2.5 m. This path length will introduce a constant phase shift into each joint detection. It is also assumed that all mirror reflection angles are the same in both paths so that no additional phase shift differences need to be considered. Since all the phase shifts are considered equal they will cancel out and will not effect the overall interference pattern.

All the detectors are linked to a coincidence counter and the interference patterns are recorded. The intensity pattern recorded at \( D_0 \) shows no interference when there is a coincidence between \( D_0 \) and \( D_3 \) or \( D_4 \). In these cases, we have which path information, since \( D_3 \) only records idler photons from slit \( A \) and \( D_4 \) only records idler photons from slit \( B \). Since the signal and idler photons come from the same region of the crystal, we would then know through which path the signal photons came and we expect no interference.

When the coincidence counts are between \( D_0 \) and \( D_1 \) there is an interference pattern. The beamsplitter BS mixes the idler photons from both regions and we have now erased the which path information. There is also an interference pattern when there is a coincidence between \( D_0 \) and \( D_2 \) but this pattern differs from the previous one by a phase shift of \( \pi \). In other words if
Figure 4: The figure shows the set up for the Kim et al. delayed choice experiment. All three beamsplitters, BSA, BSB and BS, are 50:50 lossless beamsplitters. A pump laser is incident on two slits A and B which also corresponds to two different small regions within a BBO ($\beta - BaB_2O_4$) crystal for parametric down conversion. We assume that the signal and idler photons are the same frequency and are both half the pump frequency. Photons from region A are colored red and photons from region B are colored blue for tracking convenience only. The signal photons (marked s) from both regions go to detector $D_0$ where an interference pattern may or may not be observed. The idler photons (marked i) from both regions are separated by a wedge mirror (a prism was used in the actual experiment) and then go to beamsplitters BSA and BSB respectively. Idler photons from A alone are recorded by detector $D_3$ by reflection from BSA and idler photons from B alone are detected by $D_4$ by reflection from BSB. If the idler photons are transmitted through BSA or BSB then they are mixed by the third beamsplitter BS and can be detected by either detector $D_1$ or $D_2$. The idler photons at these detectors no longer carry any which path information. All detectors then go to a coincidence counter. The diagram is meant to illustrate the same arm lengths for the red and blue idler photons, the reflections from the mirrors and beamsplitters are not accurately drawn with correct angles and refraction is not included.
one pattern shows a co-sinusoidal interference the other will be sinusoidal. The experiment is considered a delayed choice quantum eraser since the signal photons path length is shorter than the idler photons. It would seem that the signal photons are detected first, then we make a selection of which coincidence detections to look at, and depending on that choice we see or do not see interference of the signal photons. The paradox being, how can you influence the signal photon, basically tell it to interfere or not, by making a choice of detector \( D_1 - D_4 \), 8 ns after the signal photon has already been detected by \( D_0 \). This however is the wrong way to think about this problem. If looked at in the correct way there is no paradox.

These observations can easily be explained in terms of the transactional interpretation of quantum mechanics as follows. A brief account of this experiment is given in the book by Kastner [29], we give a bit more detail here.

**Transactional interpretation derivation**

Let us start with a few preliminaries. The three beamsplitters in the experiment are all 50:50 lossless beamsplitters. When a photon wavepacket goes through one of these beamsplitters there is no loss so one would expect the probability amplitude of the wave function to remain unaltered.

\[
|\psi|^2 = |r\psi + t\psi|^2 = [|r|^2 + |t|^2 + (r^*t + rt^*)]|\psi|^2
\]  

This means that the amplitude reflection and transmission coefficients obey,

\[
|r|^2 + |t|^2 = 1 \\
|r|^2 = |t|^2 = \frac{1}{2} \\
r^*t + rt^* = 0
\]

hence \( r = \frac{i}{\sqrt{2}} \) and \( t = \frac{1}{\sqrt{2}} \).  

We take all the beamsplitters to be identical for convenience. It will be assumed that each optical path length for the idler photons, is the same and any phase changes due to mirror reflections have been compensated for. An offer wave will go out from the slits and get absorbed by a detector. The detector will then send back an advanced wave (backwards in time) along the same path as the incident wave to the slits to *handshake* and confirm the interaction. Only then does the photon actually leave the slit region. The offer wave is a momentum entangled two-photon state (or bi-photon). The possible transactions will depend on the detector configuration which generates the counter wave. We will go through the process step by step.

The original offer wave from the slits comes from the pump laser beam, we will take this to be,

\[
\psi = \frac{\alpha}{\sqrt{2}} (|A_p\rangle + |B_p\rangle)
\]

where the subscript \( p \) stands for pump. The \( \alpha \) is the single slit diffraction pattern, a sinc function of the usual kind. \( A \) and \( B \) stand for the photon wave functions from the two slits, of plane wave type. Parametric downconversion inside the \( \beta \)-barium borate (BBO) crystal duplicates each pump photon into a signal and an idler photon. The offer wave then becomes,

\[
\psi = \frac{\alpha}{\sqrt{2}} (|A_s\rangle|A_i\rangle + |B_s\rangle|B_i\rangle)
\]
We select both the signal and idler photons of half the pump frequency, by restricting the exit angle from the crystal. Even so there will be a small spread in frequency, and thus wavelength, which will cause the fringe visibility to be less than perfect. However, we will continue thinking of the photons wave functions as simple monochromatic plane waves for simplicity. It is easy to generalize the end result for more than one wavelength.

We skip the details of the parametric downconversion process but they can be found here [32, 33, 34]. The first reference refers to 5 basic quantum experiments and has simple theory accessible to undergraduates [32]. The second reference has more theory but still some experiment, and is geared more for graduates and researchers [33] and the last reference is a theory paper again for researchers [34].

The signal photons are sent to the detector $D_0$. The idler photons are sent to the beamsplitter setup. The path lengths in the experiment are arranged so that the signal photons reach detector $D_0$ before the idler photons reach their final destination. So if the signal photon is detected at position $x$ on the screen, then our offer wave becomes

$$\psi = \frac{\alpha}{\sqrt{2}} \left( \langle x | A_s \rangle | A_i \rangle + \langle x | B_s \rangle | B_i \rangle \right).$$  \hfill (5)

A simple fourier transform of a slit with a constant electric field will give the single slit diffraction amplitude $\alpha$ in the form

$$\alpha = \text{sinc}(k_x a/2)$$ \hfill (6)

where $a$ is the slit width and $k_x = k \sin \theta$ and the angle $\theta$ is the angular displacement from the center of the slits to the position $x$ on the screen. For the paraxial ray approximation this would be

$$k_x = k \sin \theta = \frac{kx}{f} = \frac{\pi x}{\lambda f}$$ \hfill (7)

where $f$ is the focal length of the lens which is taken to be roughly the slit screen distance and $\lambda$ is the wavelength of the signal photons and we have used $k = 2\pi/\lambda$. Hence

$$\alpha = \text{sinc} \left( \frac{kxa}{2f} \right) = \text{sinc} \left( \frac{\pi xa}{\lambda f} \right)$$ \hfill (8)

We will now assume that

$$\langle x | A_s \rangle = e^{ik_x d_A}$$
$$\langle x | B_s \rangle = e^{ik_x d_B}$$ \hfill (9)

where $d_A$ and $d_B$ are the distances from the crystal regions $A$ and $B$ to the screen at position $x$. Also we assume that the slit separation can be given by $d = d_A - d_B$. The offer wave can now be written as,

$$\psi_{ow} = \frac{\alpha}{\sqrt{2}} \left( e^{ik_x d_A} | A_i \rangle + e^{ik_x d_B} | B_i \rangle \right).$$ \hfill (10)

Note that we have now dealt with the signal photons and only have to concern ourselves with the idler photon detection. Three cases follow:
Case 1:

Assume the idler photon will be detected at detector $D_1$. The offer wave produced by passing photons through the beamsplitters will be

$$\psi_{ow} = \frac{\alpha}{\sqrt{2}} (e^{i k_x d_A t} A_i + e^{i k_x d_B t^2} B_i)$$

(11)

the $A_i$ idler photon is transmitted through BSA and reflected from BS to reach $D_1$. The $B_i$ idler photon is transmitted through BSB and transmitted through BS to reach $D_1$. See Fig. 4 for details of the paths. We have assumed that the extra path length in traveling through the beamsplitters is the same for both photons $A_i$ and $B_i$, otherwise we would need additional phase factors to account for the path length difference. The counter wave produced by detector $D_1$ will be the complex conjugate wave traveling backward in time towards the slits,

$$\psi_{cw}^* = \frac{\alpha^*}{\sqrt{2}} (e^{-i k_x d_A t} A_i + e^{-i k_x d_B t^2} B_i)$$

(12)

The probability that this transaction will occur then becomes,

$$\psi_{cw}^* \psi_{ow} = \frac{1}{2} |\alpha|^2 \left[ |r|^2 |t|^2 \langle A_i | A_i \rangle + |t|^4 \langle B_i | B_i \rangle + |t|^2 \left( r^* t \langle A_i | B_i \rangle e^{-i k_x d} + r t \langle B_i | A_i \rangle e^{i k_x d} \right) \right]$$

(13)

Let the amplitudes $\langle A_i | A_i \rangle = \langle B_i | B_i \rangle = 1$, $\langle A_i | B_i \rangle = \eta_1^{1/2} \exp(-i \phi)$ and complex conjugate $\langle B_i | A_i \rangle = \eta_1^{1/2} \exp(i \phi)$, where $\eta_1$ represents the detector efficiency of $D_1$ which is most likely less than unity. The detector efficiency has been incorporated into the probability amplitude for convenience only. Then we may write,

$$\psi_{cw}^* \psi_{ow} = \frac{1}{2} |\alpha|^2 \left[ |r|^2 |t|^2 + |t|^4 \right] + \eta_1 \left( r^* t e^{-i (k_x d + \phi)} + r t e^{i (k_x d + \phi)} \right)$$

(14)

Using our earlier results Eq(2) for the amplitudes $r$ and $t$ of the lossless beamsplitters and

$$e^{\pm i \pi/2} = \cos \pi/2 \pm i \sin \pi/2 = \pm i$$

(15)

we get,

$$\psi_{cw}^* \psi_{ow} = \frac{1}{4} |\alpha|^2 \left[ 1 + \eta_1 \cos(k_x d + \phi + \pi/2) \right]$$

$$= \frac{1}{4} |\alpha|^2 \left[ 1 + \eta_1 \cos \left( \frac{\pi d}{\lambda f} + \phi + \pi/2 \right) \right]$$

(16)

It is more general to leave the result in this form. However the Kim paper [1] goes on to simplify further, uses $\eta_1 = 1$ for perfect detection and writes,

$$\psi_{cw}^* \psi_{ow} = \frac{1}{2} |\alpha|^2 \cos^2 \left[ \frac{k_x d}{2} + \frac{\phi}{2} + \frac{\pi}{4} \right]$$

(17)

where $\alpha$ is given by Eq.(8) and $k_x$ is given by Eq.(7). In the last step we used the double angle formula for $\cos 2\beta = 2 \cos^2 \beta - 1$. This is the coincidence result between detector $D_1$ together with detector $D_0$ and shows interference.

Using our result Eq.(16) it is easy to generalize to a small spread of wavelengths (bandwidth=$\Delta \lambda$) by using a computer code to plot the equation and summing the interference patterns for $\lambda$.
\( \lambda \pm \Delta \lambda, \lambda \pm \Delta \lambda/2 \) and \( \lambda \pm \Delta \lambda/4 \). This will give a quite accurate interference pattern which will match the experimental data very well. If you also include the detector efficiency \( \eta_1 \) then you can match the experimental fringe visibility almost exactly. This is easy to do with a symbolic manipulation code like Mathematica, which also plots the results for you.

**Case 2:**

When the idler photons are detected at \( D_2 \) the offer wave becomes,

\[
\psi_{ow} = \frac{\alpha}{\sqrt{2}} \left( e^{ik_x d_A t^2} |A_i\rangle + e^{ik_x d_B t^r} |B_i\rangle \right)
\]  

(18)

Note that the \( A_i \) photon is transmitted by both BSA and BS, and the \( B_i \) photon is transmitted by BSB but reflected by BS to reach \( D_2 \). See Fig. 4 for details. The detector produces a counter wave which is the complex conjugate of the offer wave above,

\[
\psi_{cw}^* = \frac{\alpha^*}{\sqrt{2}} \left( e^{-ik_x d_A t^*} \langle A_i | + e^{-ik_x d_B t^* r^*} \langle B_i | \right)
\]  

(19)

Using the same manipulations as before, leaving the detector efficiency as unity, the joint probability detection of coincidence counts between \( D_0 \) and \( D_2 \) becomes,

\[
\psi_{cw}^* \psi_{ow} = \frac{|\alpha|^2}{2} \left| t \right|^2 \left[ \left| t \right|^2 \langle A_i | A_i \rangle + |r|^2 \langle B_i | B_i \rangle + \left( t^* r \langle A_i | B_i \rangle e^{-ik_x d} + r^* t \langle B_i | A_i \rangle e^{ik_x d} \right) \right]
\]

\[
= \frac{|\alpha|^2}{4} \left[ 1 + \frac{i}{2} e^{-i(k_x d + \phi)} - \frac{i}{2} e^{i(k_x d + \phi)} \right]
\]

\[
= \frac{|\alpha|^2}{4} \left[ 1 + \cos \left( k_x d + \phi - \frac{\pi}{2} \right) \right]
\]

\[
= \frac{|\alpha|^2}{2} \cos^2 \left( \frac{k_x d}{2} + \frac{\phi - \pi}{4} \right)
\]  

(20)

which also shows interference. The factor \( \alpha \) is given by Eq.(8). Note that this interference is \( \pi \) out of phase with the interference pattern obtained from the coincidence count between \( D_0 \) and \( D_1 \). This is easier to see in the cosine result rather than the \( \cos^2 \) result. That means if the interference with \( D_1 \) is co-sinusoidal then this interference would be sinusoidal. This is exactly what was observed in the experiment [1].

**Case 3:**

If the idler photon is detected at either \( D_3 \) or \( D_4 \) then the corresponding offer waves would be,

\[
\psi_{ow} = \frac{\alpha r}{\sqrt{2}} \left( e^{ik_x d_A} |A_i\rangle + e^{ik_x d_B} |B_i\rangle \right)
\]  

(21)

and the counter waves would be

\[
\psi_{cw3}^* = \frac{\alpha^* r^*}{\sqrt{2}} \langle A_i | e^{-ik_x d_A} \quad \text{for detector } D_3
\]

\[
\psi_{cw4}^* = \frac{\alpha^* r^*}{\sqrt{2}} \langle B_i | e^{-ik_x d_B} \quad \text{for detector } D_4
\]  

(22)
The probability of a coincidence count between $D_0$ and $D_3$ becomes,

$$\psi^*_{cw3}\psi_{ow} = |\alpha|^2 |r|^2 \langle A_i | A_i \rangle = \frac{|\alpha|^2}{4}$$

(23)

which shows no interference only a single slit diffraction pattern. The probability of a coincidence count between $D_0$ and $D_4$ becomes,

$$\psi^*_{cw4}\psi_{ow} = |\alpha|^2 |r|^2 \langle B_i | B_i \rangle = \frac{|\alpha|^2}{4}$$

(24)

which likewise shows no interference. Again, the single slit diffraction amplitude $\alpha$ is given by Eq (8). This also agrees with the experimental results of Kim et al. [1].

Conclusions

The main aim of this paper is to draw attention to the transactional interpretation of quantum mechanics by John Cramer and get as wide a dissemination as possible. The TI by Cramer [2], gives a simple and intuitive picture for wave function collapse distributed over the entire path of the interacting system. In the case of the Kim experiment [1], the wave function would collapse along the entire path between the slits (or the regions $A$ and $B$ of the down converting crystal) and the detectors and it would happen in a way distributed over time not in an instant. The TI picture rules out the possibility of any backward in time signals using quantum delayed choice experiments. In fact it makes clear the idea is nonsense since the advanced counter wave from the detector must travel the entire distance back to the slit in order for the photon (from the slit) to make the trip in the first place. The choice is really no longer delayed since the photon knows where it will end up because of the advanced wave coming backwards in time to confirm the interaction or handshake as Cramer puts it.

However, it only takes one experimental observation to disprove a theory. John Cramer and Nick Herbert [35] considered several experimental possibilities of nonlocal quantum signaling (retro-causal signals) involving path entangled systems and in all cases found that the complementarity between two-photon interference and one-photon interference blocks any potential nonlocal signal [36]. The whole Copenhagen way of thinking about an instantaneous wave function collapse, at a certain time at a certain place, is no longer needed. The wave function collapse is the most confusing aspect of quantum mechanics and is simply resolved using the TI method of Cramer, or PTI of Kastner.

Advanced waves are natural solutions to relativistic wave equations. In order to use this theory for the nonrelativistic case it is necessary to think of two Schrödinger equations. One Schrödinger equation for the wave function $\psi$ and one for its complex conjugate $\psi^*$ which becomes the advanced wave. In a way this makes sense if we think of the Schrödinger equation as a square root version of the relativistic Klein Gordon equation.

Furthermore, work by Hogarth [37] and Hoyle and Narlikar (HN) [38, 39, 40] has paved the way to a new version of direct particle interaction gravitational theory, which is fully Machian, incorporates advanced waves and has Einstein’s theory as a special case. The HN theory may be quantized as in their book [40] using the path integral technique pioneered by Feynman [20].

It is interesting to note that the mass field $m(x)$ in HN theory looks similar to the source field $S(x)$ introduced by Schwinger [41]. Wheeler never gave up on the absorber theory, which is a
direct particle interaction (action–at–a–distance) theory. It simply wasn’t popular at the time and dropped off the radar much like Cramer’s transactional interpretation. Gerard t’Hooft found a way to renormalize Yang Mills field theories in a way similar to QED and most physicists took that path. We believe the works of Cramer, Wheeler–Feynman, Hoyle–Narlikar, and Schwinger’s source theory, are very much related. How source theory is related to the Feynman path integrals is explained by Schweber [42]. It should be noted that Schwinger was able to derive the Casimir force using the source field method in which there are no nontrivial vacuum fields [43] [44]. The action at a distance theories are well worth study and may lead to a consistent picture of quantum gravity. Radiation reaction can be dealt with using the half retarded half advanced absorber picture. Many QED results thought to be vacuum fluctuation related can in fact be derived by considering source fields instead, including the Lamb shift and particle self energy [43]. The only outstanding problem is the vacuum polarization (renormalization of charge). HN [40] fix this problem by eliminating the “point” particle and introducing a minimum length scale equal to the gravitational radius of the particle $2Gm/c^2$. The size of the particles derived from source theory are never specified [41].

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