Cosmological constraints on an exponential interaction in the dark sector

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Cosmological models where dark matter (DM) and dark energy (DE) interact with each other are the general scenarios in compared to the non-interacting models. The interaction is usually motivated from the phenomenological ground and thus there is no such rule to prefer a particular interaction between DM and DE. Being motivated, in this work, allowing an exponential interaction between DM and DE in a spatially flat homogeneous and isotropic universe, we explore the dynamics of the universe through the constraints of the free parameters where the strength of the interaction is characterized by the dimensionless coupling parameter \( \xi \) and the equation of state (EoS) for DE, \( w_x \), is supposed to be a constant. The interaction scenario is fitted using the latest available observational data. Our analyses report that the observational data permit a non-zero value of \( \xi \) but it is very small and consistent with \( \xi = 0 \). From the constraints on \( w_x \), we find that both phantom (\( w_x < -1 \)) and quintessence (\( w_x > -1 \)) regimes are equally allowed but \( w_x \) is very close to ‘−1’. The overall results indicate that at the background level, the interaction model cannot be distinguished from the base \( \Lambda \)-cold dark matter model while from the perturbative analyses, the interaction model mildly deviates from the base model. We highlight that, even if we allow DM and DE to interact in an exponential manner, but according to the observational data, the evidence for a non-zero coupling is very small.

1. INTRODUCTION

According to a large number of independent astronomical surveys [1, 2, 3, 4, 5], our universe is currently expanding with an acceleration. This accelerating phase does not fit into the standard cosmological model requiring the presence of some negative pressure component fluid in the universe sector dubbed as dark energy. And from the current astronomical estimation, this so-called dark energy fluid occupies almost 68% of the total energy density of the universe. The rest 32% of this energy density is filled up by a pressureless dark matter fluid (also called as cold dark matter) and baryons, radiation. The common behaviour in both dark matter and dark energy is that, both are unknown to us by its origin, character, dynamics for instance. The above picture can be framed in terms of the \( \Lambda \)CDM cosmology where the dark energy fluid is represented by some cosmological constant, \( \Lambda > 0 \) and CDM is the cold dark matter. But, as well known, the problem with the cosmological constant [6] leads to several alternative models [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] trying to explain this accelerating phase so that the observational data can match with the theoretical models at hand.

Among various cosmological models, a particular class of models where the underlying fluids may interact with each other, widely known as interacting cosmologies, gained a significant attention to modern cosmological research. In interacting cosmologies, usually the gravitational theory is assumed to be described by the General theory of Relativity and the main two fluids of the universe describing its dark sector, namely, the dark matter and dark energy, are allowed to interact with each other\(^1\). In particular, the total fluid of the dark sector is conserved. For a detailed understanding of the interacting cosmologies, we refer to some recent reviews [18, 19].

One may note that the origin of interaction was not to explain the current accelerating universe rather its primary motivation was to find a possible explanation towards the cosmological constant problem [20] which as well known to the cosmological community, is existing since long back ago and remained silent until the dark energy era began. When the alternative \( \Lambda \)CDM models appeared in the literature, it was found that they

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raised a problem which asks “why the energy densities of dark matter and dark energy are of same order at current time?” also known as coincidence problem [21]. Consequently, it was found that the old concept of interaction between fields [20] can explain the cosmic coincidence problem [22]. Following this, a large amount of investigations [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45] have been performed. Recently a series of investigations toward the same direction comment that the astronomical data available today do not completely rule out the possibility of a non-zero interaction in the dark sector [46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59]. Additionally, some most recent articles in this context argue that the interaction in the dark sector could be a very fantastic theory that may release the tension on the local Hubble constant [51, 55], a most tallative issue in modern cosmology at present. Moreover, it has been found that the presence of interaction in the dark sector pushes the dark energy fluid into the phantom region [28, 55, 57, 58]. On the other hand, interaction cosmologies can describe, in a phenomenological way, the unified dark energy models, for instance see [60, 61]. Thus, the interacting models having the above features clearly demand for more investigations in recent years.

In the current work we investigate the cosmological constraints allowing an exponential interaction between dark matter and dark energy. The choice of an exponential interaction is indeed phenomenological, however it cannot be excluded on the basis of other interaction models that have been widely studied in the last couple of years. We consider such an interaction in order to investigate their ability with the observational data. For metric which describes the geometry of the universe we consider the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) line element. Moreover, the dark components are assumed to have barotropic nature. The scenario has been fitted using the latest astronomical measurements from various data sets and the markov chain monte carlo package cosmomc has been used to extract the observational constraints of the model. It is quite interesting to note that even if we allow an exponential interaction in the dark sector, the resulting scenario does not deviate much from the ΛCDM cosmology. This might be considered to be an interesting result in the field of interacting cosmologies because this reflects that although any arbitrary choice for an interaction model can be made, but the observational data may not allow a strong interaction in the dark sector.

The presentation of the manuscript is as follows. In section 2 we describe the gravitational equations of the interacting universe at the background and perturbative levels. Section 3 describes the observational data employed in this work, fitting technique, and the results of the analysis. Finally, section 4 closes the entire work with a short summary.

2. GRAVITATIONAL EQUATIONS IN AN INTERACTING UNIVERSE: BACKGROUND AND PERTURBATIONS

In this section we describe the background and perturbation equations for the interacting dark fluids. Specifically, we consider a model of our universe where the total energy density of the universe is contributed by relativistic (radiation) and non-relativistic species (baryons, pressureless dark matter and dark energy). The fluids are barotropic where dark matter and dark energy interact with each other while the radiation and baryons do not take part in the interaction. We denote \((p_i, \rho_i)\) as the pressure and energy density of the \(i\)-th component of the fluid where \(i = r, b, c, x\) respectively represent the radiation, baryons, pressureless dark matter and dark energy.

Now, considering a spatially flat FLRW line element for the universe with expansion scale factor \(a(t)\), the conservation equations for the interacting fluids follow

\[
\dot{\rho}_c + \frac{3}{a} \rho_c = -Q, \quad (1)
\]
\[
\dot{\rho}_x + \frac{3}{a} (1 + w_x) \rho_x = Q, \quad (2)
\]

where \(w_x = p_x/\rho_x\) is the equation of state parameter for the dark energy fluid which we assume to be constant. And \(Q\) is the interaction rate between the dark fluids which determines the direction of energy flow between them. For \(Q < 0\), the energy flow takes place from DE to CDM whereas the energy flow from CDM to DE is conferred by \(Q > 0\). The conservation equations for radiation and baryons are the usual ones and they respectively take the forms \(\rho_r = \rho_{r0}a^{-4}\), \(\rho_b = \rho_{b0}a^{-3}\). Here, \(\rho_{i0}\) \((i = r, b)\) is the value of \(\rho_i\) at current time for the \(i\)-th fluid.

The Hubble equation takes the form

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left( \rho_r + \rho_b + \rho_c + \rho_x \right),
\]

which together with the conservation equations for all fluids \((1)\) and \((2)\) for pressureless dark matter, dark energy respectively and two for radiation and baryons), can determine the dynamics of the universe, provided the interaction rate \(Q\), is supplied from outside. Technically, there is no such specific rule to select the forms for \(Q\) and thus some phenomenological choices are initially made and then they are tested with the astronomical data. The well known models for the interaction rates are, \(Q \propto \rho_x \) \[67\], \(Q \propto \rho_c \) \[30\], \(Q \propto (\rho_c + \rho_x) \) \[31\], \(Q \propto \dot{\rho}_x \) \[58\], \(Q \propto \rho_x^2/\rho_c \) \[57\] etc.

We remark that the establishment of those interactions in the current literature followed from their agreement with the observational data and their stabilities at large-scale, and thus a new interaction appearing in the literature should be equally welcomed. In this work we propose the following interaction
\[
Q = 3H\xi\rho_x \exp \left( \frac{\rho_x}{\rho_c} - 1 \right),
\]
where \(\xi\) is the coupling strength of the interaction. One can see that in terms of the coincidence parameter \(r = \rho_c/\rho_x\), the interaction \(Q\) can be recast as \(Q = 3H\xi\rho_x \exp \left( \frac{r}{2} - 1 \right)\), and thus, for \(r \to \infty\), \(Q \approx 3H\xi\rho_x\) while for \(r \to 1\), \(Q \approx 3H\xi\rho_x\). Those limits have been studied extensively in the bibliography, see \([65]\) and references therein. In general, the majority of the interaction models are linear functions on the energy densities. There are a few nonlinear models \([18]\) which however do not provide the linear interactions in the limit. On the other hand, for the exponential interaction \([3]\), its linear and nonlinear behaviour are still retained. As one can see, for \(r \to 1\), and \(r \to \infty\) it mimics the linear interaction scenario \(Q \propto \rho_x\), while on the other hand, it may also provide quadratic terms in the interaction rate as the first corrections in the linear case. One can check that the Taylor series expansion of \([3]\) around \(\rho_x = 0\), one gets
\[
Q \propto \rho_x + \frac{\rho_x^2}{\rho_c} + ...
\]
In Fig. 1, we describe the qualitative evolution of the exponential interaction model \([3]\), denoted by \(Q_e\) for different values of the coupling parameter. We also made a comparison between the interaction models. From the comparison, we see that the model \(Q_2\) always presents a very different behaviour in compared to the exponential model as well as with other interaction models. A common behaviour we notice from the analysis is that, the exponential model \([3]\) behaves similarly to other two interaction models \((Q_1, Q_3)\); however, the exponential model leaves a notable deviation around a very small neighbourhood of \(z = 0\). We also observe from Fig. 1 that, for large redshifts the exponential interaction is only differentiated from other two interaction models \((Q_1, Q_3)\) only for large coupling parameter.

Now, for any cosmological model, one must ensure its stability in the large scale of the universe, and thus, we need to study the perturbation equations. In order to do that, we consider the perturbed FLRW metric with scalar mode \(k\) given by \([62,63,64]\)

\[
d\!s^2 = a^2(\tau) \left[-(1 + 2\phi) d\tau^2 + 2\partial_iBd\tau dx^i \right. \\
\left. + \left((1 - 2\psi)\delta_{ij} + 2\partial_i\partial_j E\right) dx^i dx^j \right],
\]
where \(\tau\) is the conformal time and the quantities \(\phi, B, \psi, E\) represent the gauge-dependent scalar perturbations.

The perturbation equations for the metric \([5]\) follow \([65,66,67]\)

\[
\nabla_\nu T^\mu_\nu = Q^\mu_A, \quad \sum_A Q^\mu_A = 0,
\]
where we have used \(A\) just to represent the fluid (either dark matter or dark energy); \(Q^\mu_A = (Q_A + \delta Q_A)u^\mu + F^\mu_A\) in which \(Q_A\) is the energy transfer rate and \(F^\mu_A = a^{-1}(\partial^\mu f_A)\) is the momentum density transfer relative to the four-velocity \(u^\mu\). Let us note that following the
earlier works [66, 67] the momentum transfer potential is specialized to be the simplest physical choice, which becomes zero in the rest frame of the dark matter, that means, we have the following equation $k^2 f_A = Q_A(\theta - \theta_c)$. 

Now, introducing $\delta_A = \delta \rho_A / \rho_A$, as the density perturbation for the fluid $A$, and assuming no anisotropic stress (i.e. $\pi_A = 0$), in the synchronous gauge, that means with the conditions $\phi = B = 0$, $\psi = \eta$, and $k^2 E = -h/2 - 3\eta$), the explicit perturbation equations (density and velocity perturbations) can be written as [65] [66] [67].

\[ \delta_A' + 3\mathcal{H} (c_{sA}^2 - w_A) \delta_A + 9\mathcal{H}^2 (1 + w_A) \left( c_{sA}^2 - c_{\alpha A}^2 \right) \frac{\theta_A}{k^2} + (1 + w_A) \theta_A - 3 (1 + w_A) \psi' + (1 + w_A) k^2 (B - E') = a \rho_A (\delta Q_A - Q_A \delta_A) + \frac{aQ_A}{\rho_A} \left[ \phi + 3\mathcal{H} (c_{sA}^2 - c_{\alpha A}^2) \frac{\theta_A}{k^2} \right], \]

\[ \theta_A' + \mathcal{H} (1 - 3c_{sA}^2) \theta_A - \frac{c_{sA}^2}{1 + w_A} k^2 \delta_A - k^2 \phi = \frac{a}{(1 + w_A)\rho_A} \left[ (Q_A\theta - k^2 f_A) - (1 + c_{sA}^2) Q_A\theta_A \right], \]

where the prime is the differentiation with respect to the conformal time $\tau$; $\mathcal{H}$ is the conformal Hubble parameter; $c_{sA}^2$, $c_{\alpha A}^2$, are respectively the adiabatic and physical sound velocity for the fluid $A$ related as $c_{sA}^2 = \rho_A' / (w_A \rho_A) = w_x + w_x' / (\rho_A' / \rho_A)$; $\theta = \theta_{\mu}^\mu$ is the volume expansion scalar. To avoid any kind of instabilities, $c_{sA}^2 \geq 0$ has been assumed. For cold dark matter, since $w_c = 0$, thus, one has $c_{sA}^2 = 0$. On the other hand, for dark energy fluid we assume $c_{sA}^2 = 1$ [65] [66] [67]. Now, one can write down the density and the velocity perturbations for the dark energy and cold dark matter as

\[ \delta_x' = -(1 + w_x) \left( \theta_x + \frac{h'}{2} \right) - 3\mathcal{H} (c_{s,x}^2 - w_x) \left[ \delta_x + 3\mathcal{H} (1 + w_x) \frac{\theta_x}{k^2} \right] - 3\mathcal{H} w_x' \frac{\theta_x}{k^2} \]

\[ + \frac{aQ}{\rho_x} \left[ -\delta_x + \frac{\delta Q}{Q} + 3\mathcal{H} (c_{s,x}^2 - w_x) \frac{\theta_x}{k^2} \right], \]

\[ \theta_x' = -\mathcal{H} (1 - 3c_{s,x}^2) \theta_x + \frac{c_{s,x}^2}{1 + w_x} k^2 \delta_x + \frac{aQ}{\rho_x} \left[ \theta_x - (1 + c_{s,x}^2) \theta_x \right], \]

\[ \delta_c' = - \left( \theta_c + \frac{h'}{2} \right) + \frac{aQ}{\rho_c} \left( \delta_c - \frac{\delta Q}{Q} \right), \]

\[ \theta_c' = -\mathcal{H} \theta_c, \]

where $\delta Q/Q$ includes the perturbation term for the Hubble expansion rate $\delta H$. One may note that in the evolution equation for $\theta'_c$, no interaction term is present. This is because, since for the cold dark matter species, $c_{sA}^2 = 0$ has been assumed, and $k^2 f_A = Q_c (\theta - \theta_c)$, thus, the term inside the third brace of the right hand side of eqn. (??) actually vanishes. Now, for the interaction model (3), the explicit evolution for density and velocity perturbations are
$\delta'_x = -(1 + w_x) \left( \theta_x + \frac{k'}{2} \right) - 3 \mathcal{H} (c_{sx}^2 - w_x) \left[ \delta_x + 3 \mathcal{H} (1 + w_x) \frac{\theta_x}{k^2} \right] - 3 \mathcal{H} w_x' \frac{\theta_x}{k^2}$

$+ 3 \mathcal{H} \xi \exp \left( \frac{2}{\rho_c} - 1 \right) \left[ \frac{\rho_x}{\rho_c} (\delta_x - \delta_c) + \frac{\theta + h'/3}{3 \mathcal{H}} + 3 \mathcal{H} (c_{sx}^2 - w_x) \frac{\theta_x}{k^2} \right],$

$\theta'_x = -\mathcal{H} (1 - 3c_{sx}^2) \theta_x + \frac{c_x^2}{1 + w_x} k^2 \delta_x + 3 \mathcal{H} \xi \exp \left( \frac{2}{\rho_c} - 1 \right) \left[ \frac{\rho_x}{\rho_c} (\delta_x - \delta_c) - \frac{\theta + h'/3}{3 \mathcal{H}} \right],$

$\delta'_c = - \left( \theta_c + \frac{h'/2}{2} \right) + 3 \mathcal{H} \xi \frac{\rho_x}{\rho_c} \exp \left( \frac{2}{\rho_c} - 1 \right) \left[ \delta_c - \delta_x - \frac{\rho_x}{\rho_c} (\delta_x - \delta_c) - \frac{\theta + h'/3}{3 \mathcal{H}} \right],$

$\theta'_c = - \mathcal{H} \theta_c,$

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Parameter & Prior (IDE) \\
\hline
$\Omega_b h^2$ & [0.005, 0.1] \\
$\tau$ & [0.01, 0.8] \\
$n_s$ & [0.5, 1.5] \\
$\log(10^{10} A_s)$ & [2.4, 4] \\
$100 \theta_{MC}$ & [0.5, 10] \\
$w_x$ & – \\
$\xi$ & [0.2, 1] \\
\hline
\end{tabular}
\caption{Summary of the flat priors on the parameters for the interacting model \cite{3}.}
\end{table}

3. OBSERVATIONAL DATA, FITTING TECHNIQUE AND THE RESULTS

The observational data, methodology and the results of the exponential interaction model are described in that Section.

We consider several observational data to constrain the current interaction model as follows:

- Cosmic microwave background radiation from Planck \cite{68, 69}. The data is recognized as Planck TTTEEE+low TEB.

- Baryon acoustic oscillation (BAO) distance measurements from the 6dF Galaxy Survey (6dFGS) (redshift measurement at $z_{\text{eff}} = 0.106$) \cite{20}. Main Galaxy Sample of Data Release 7 of Sloan Digital Sky Survey (SDSS-MGS) ($z_{\text{eff}} = 0.15$) \cite{21}, CMASS and LOWZ samples from the latest Data Release 12 (DR12) of the Baryon Oscillation Spectroscopic Survey (BOSS) ($z_{\text{eff}} = 0.57$) \cite{22} and (\$z_{\text{eff}} = 0.32$) \cite{72}.

- Redshift space distortion (RSD) data from CMASS sample ($z_{\text{eff}} = 0.57$) \cite{73} and the LOWZ sample ($z_{\text{eff}} = 0.32$) \cite{73}.

- The weak gravitational lensing (WL) data from the Canada–France–Hawaii Telescope Lensing Survey (CFHTLenS) \cite{74, 75}.

- Joint light curve analysis (JLA) sample \cite{76} from in the redshift interval $0 < z < 2$ \cite{77}.

- Latest cosmic chronometers (CC) measurements spanned in the redshift interval $0 < z < 2$ \cite{77}.

- The current estimated value of the Hubble parameter from the Hubble space telescope (HST) yielding $H_0 = 73.02 \pm 1.79$ km/s/Mpc with 2.4% precision \cite{78}. We identify this data as HST.

We use the markov chain monte carlo package cosmomc \cite{79, 80} to constrain the model. This is an efficient simulation where the convergence of the model parameters is based on the Gelman-Rubin statistics \cite{81} that may result in a sufficient convergence of all model parameters. The parameters space for the IDE scenario is

$$P_2 = \left\{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, w_x, \xi, n_s, \log(10^{10} A_S) \right\},$$

which is eight dimensional. Here, $\Omega_b h^2$, $\Omega_c h^2$, are the baryons and cold dark matter density respectively; $100 \theta_{MC}$, is the ratio of sound horizon to the angular diameter distance, $\tau$, is the optical depth; $w_x$ is the equation of state parameter for dark energy; $\xi$ is the coupling strength; $n_s$, $A_S$, are respectively the scalar spectral index, and the amplitude of the initial power spectrum.

Now let us come to the observational constraints on the model. To constrain the entire interacting scenario we have used four different observational data, namely,

- Planck TTTEEE + low TEB (CMB),
- CMB + BAO + RSD,
- CMB + BAO + HST,
- CMB + BAO + RSD + HST + WL + JLA + CC.
Using the priors for the model parameters summarized in Table I and then performing a likelihood analysis using cosmomc, in Table II, we summarize the results. In Fig. 2: One dimensional posterior distributions of some selected parameters of the interacting model have been shown for different combined analysis employed in this work.

Our analyses show that the observational data allow a very small interaction in the dark sector which is consistent with the non-interaction limit, $\xi = 0$. One stringent point we notice is that, for the observational data CMB + BAO + HST, $\xi = 0$ is not allowed at least within 68% confidence-level (CL), but in the 95% CL, the non-interacting scenario is recovered. The lowest coupling strength as observed from Table II is attained for the final combined analysis (CMB + BAO + RSD + HST + WL + JLA + CC) where $\xi = 0.0058 \pm 0.0058$ at 68% CL. In fact, for this particular combined analysis, $\xi < 0.0143$

| Parameters | CMB | CMB+BAO+RSD | CMB+BAO+HST | CMB+BAO+RSD+HST +WL+JLA+CC |
|------------|-----|-------------|-------------|----------------------------|
| $\Omega m$ | 0.2214 0.019 | 0.00014+0.0034 | 0.02226 | 0.02226+0.00060 |
| $\Omega m$ | 0.1154 0.027 | 0.00019 | 0.1118 | 0.1143+0.00057 |
| $\Omega m$ | 1.04066 0.0038 | 0.00054 | 1.04095 | 1.04079+0.00070 |
| $n_s$ | 0.9712 0.0055 | 0.0044+0.0088 | 0.9750+0.0083 | 0.9769+0.0044 |
| $\tau$ | 0.072 0.018 | 0.018+0.035 | 0.075 | 0.081+0.019 |
| $\ln(10^{10}A_s)$ | 3.09 0.036 | 0.035+0.068 | 3.092 | 3.104+0.036 |
| $w_x$ | -0.9961 0.0624 | -0.037+0.072 | -0.9756 | -1.0860+0.0530 |
| $\xi$ | 0.0081 0.0029 | 0.0028+0.0128 | 0.0062 | 0.0062+0.0029 |
| $\Omega_m0$ | 0.309 0.022 | 0.017+0.029 | 0.300 | 0.280+0.016 |
| $\sigma_8$ | 0.919 0.050 | -0.112+0.174 | 0.966 | 0.969+0.053 |
| $H_0$ | 67.01 1.55 | 1.18+2.36 | 67.04 | 70.02+1.22 |

FIG. 2: One dimensional posterior distributions of some selected parameters of the interacting model have been shown for different combined analysis employed in this work.

TABLE II: Observational constraints at 68% (1σ), 95% confidence-levels (2σ) on the model parameters for the interacting scenario with constant dark energy equation of state have been displayed using the observational analyses shown in the table. We recall that here $\Omega_m0$ is the current value of $\Omega_m(= \Omega_b + \Omega_c)$.
at 95% CL, and $\xi < 0.0172$ at 99% CL, which definitely suggest for a weak interaction scenario. The suggestion of weak interaction is also followed by other observational combinations in this work. Additionally, concerning the observational constraints on the dark energy equation of state, we have some different observations. As from Table 11 one can see that for the first two analyses, namely, CMB alone and CMB + BAO + RSD, the dark energy state parameter is found to be of quintessence type while for the remaining two analyses, its phantom character is suggested. Moreover, we note that for the analysis, CMB + BAO + HST, $w_x < -1$, is preferred in 68% CL. The addition of other external data sets, namely WL, JLA and CC into this data set (i.e. CMB + BAO + HST) shrinks the parameters space for $w_x$ constraining, $w_x = -1.0168^{+0.0407}_{-0.0331}$ (at 68% CL) which shows that the quintessence regime is also not excluded but of course the dark energy state parameter is close to the cosmological constant limit, $w_x = -1$. We further note that for all the observational data sets, $w_x$ is actually very close to the cosmological constant boundary $w_x = -1$. Since the coupling strength is very small and $w_x$ is close to $-1$ boundary, thus, one can find that the current interaction model is quite close to that of the non-interacting ΛCDM cosmological model.

In Fig. 3 we also show the dependence of the matter fluctuation amplitude $\sigma_8$ with different model parameters which clearly shows that $\sigma_8$ is correlated with the coupling strength $\xi$ and also with the CDM density parameter $\Omega_{m0}$. Certainly, a higher coupling in the dark sector allows higher values of $\sigma_8$. One important feature we observe is that, the parameter $\sigma_8$ takes larger values (for all combined analyses) in presence of an interaction in the dark sector while in absence of the interaction, $\sigma_8$ takes lower values\(^2\). The allowance of interaction may increase the values of $\sigma_8$, is already explored in \(^3\). This is the first evidence which demonstrates that the exponential interaction \(^4\) which although allows a very small coupling between the dark sectors but might present a slight different behaviour compared to the non-interacting ΛCDM cosmological model. That is

\(^2\) The estimations of $\sigma_8$ for the non-interacting ΛCDM model using different observational data are enlisted in \(^5\).
FIG. 4: 68% and 95% confidence level dependence of the matter fluctuation amplitude $\sigma_8$ with various model parameters in presence of the exponential interaction in the dark sector. Here too we have shown the figures for different combined analysis as in other plots. From the above figures we find that $\sigma_8$ is uncorrelated with $w_x$, but the remaining combinations do exhibit the correlations.

something which has been derived analytically for a class of general cosmological models [83]. Moreover, in Fig. 5 we show the effects of the coupling parameter on the evolution of the Hubble rate as well as on the density parameters for DM and DE. From this figure (Fig. 5) one can see that as the coupling strength increases, the model deviates from the $\Lambda$-cosmology, as expected, see again [83].

We now move to the analysis of the model at the perturbative level. The plots have been displayed for the single analysis CMB + BAO + RSD + HST + WL + JLA + CC. At first we measure the effects of the coupling strength on the CMB TT and matter power spectra both shown in Fig. 6 which shows that higher coupling strength is equivalent to significant deviation from the $w_x$CDM cosmology. The deviation is much pronounced from the matter power spectra (right panel of Fig. 6). However, since the estimated values of the coupling strength is small (see Table II), thus, it is expected that the model is close to that of the $w_x$CDM cosmology, however, practically that is not true. In order to understand that deviation, in Fig. 7 we demonstrate the relative deviation of the model from the $\Lambda$CDM model through the CMB TT (left panel of Fig. 7) and matter power spectra (right panel of Fig. 7). In both panels, we see that the interaction model mildly deviates from the $\Lambda$CDM cosmology and such a mild deviation is only detected from the analyses of the model at the perturbative level – not from the analyses at the backgour level. That means the deviation, however small it is, is not detectable only from the estimations of the coupling parameter $\xi$ and the dark energy equation of state, $w_x$ – the analyses at the level of perturbations are necessary.

Thus, according to the observational data employed in this work, one may notice that a nonzero value of the coupling parameter $\xi$ for the present exponential interaction model (3) is allowed, however, the evidence for a non-zero coupling is very small; see the one dimensional posterior distribution for $\xi$ displayed in Fig. 2. And following this, a very mild deviation of the exponential interaction model from the non-interacting $w_x$CDM cosmology (and from the $\Lambda$CDM cosmology too) is also allowed by the data, whilst such a deviation is only realized from the analyses at the perturbative level.
ΛCDM = 0
ξ = 0
ξ = 0.1
ξ = 0.2
ξ = 0.3

Ω_c (ξ = 0)
Ω_x (ξ = 0)
Ω_c (ξ = 0.1)
Ω_x (ξ = 0.1)
Ω_c (ξ = 0.2)
Ω_x (ξ = 0.2)
Ω_c (ξ = 0.3)
Ω_x (ξ = 0.3)

FIG. 5: The evolution of the Hubble rate (left panel) and the density parameters for CDM and DE (right panel) for different coupling strengths of the exponential interaction model have been shown for the parameters fixed from the mean values of the combined analysis CMB + BAO + RSD + HST + WL + JLA + CC.

FIG. 6: The plots show how the coupling strength affects the CMB spectra (left panel) and the matter power spectra (right panel). We note that while drawing the plots we take the mean values of the remaining parameters from the combined analysis CMB + BAO + RSD + HST + WL + JLA + CC.

4. SUMMARY AND CONCLUSIONS

An interacting scenario between a pressureless dark matter and a dark energy fluid where both of them have constant barotropic state parameters, has been studied. The background geometry is described by the usual FLRW line-element with no curvature.

The speciality of this work is the consideration of an exponential interaction in the dark sector, and then to see how an exponential interaction affects the entire dynamics of the universe as it is expected that the exponential character of the interaction rate might affect the background and perturbative evolutions in an extensive way. We note that the exponential interaction is the simplest generalization of the linear interaction scenario \cite{29, 52}. Thus, allowing such an interaction in the dark sectors, we fit the entire interacting scenario using the markov chain monte carlo package \texttt{cosmomc} \cite{79, 80} which is equipped with a converging diagnostic \cite{81}. Interestingly enough, we find that even if we allow such an exponential nature of the interaction rate in the dark sector, the observational data, at present, do not allow the resulting scenario beyond the ΛCDM model at least at the background level.

To analyze the model we have constrained the entire interacting scenario using different observational data. We find that the coupling strength, ξ, estimated by all the analyses is low and hence a weak interaction limit (i.e. ξ ∼ 0) is suggested. We also find that, for all the analyses, ξ = 0 can be recovered within 68% CL (for the analysis with CMB + BAO + HST, ξ = 0 is recovered at 95% CL). Thus, one can clearly see that a non-interacting \( w_x \)CDM cosmology is positively recovered by the observational data. Now, concerning the dark energy state parameter, we find that its quintessential and phantom characters are both allowed but indeed, all the estimations are close to the cosmological constant boundary. In particular, for the analysis with CMB alone and CMB + BAO + RSD, the mean values of \( w_x \) are quintessential while for the rest two analyses, that means with CMB + BAO + HST and the final combination, CMB + BAO + RSD + HST + WL + JLA + CC, the dark energy state parameter exhibits its phantom behavior. The estimated value of the dark-energy-
state-parameter for the final combination has been constrained to be, \( w_x = -1.0168^{+0.0407}_{-0.0331} \) (at 68% CL). Hence, one can safely state that the overall interacting picture at the background level is close to the non-interacting \( \Lambda \text{CDM} \) model. But, it is quite important to mention that only from the background evolution, the characterization of any cosmological model is not concrete. We have a number of evidences which clearly demonstrate that the interaction model is distinguished from \( \Lambda \text{CDM} \) model. The first evidence (it might be considered to be a weak one) comes from the constraints on the matter fluctuation amplitude \( \sigma_8 \) and later from the evolutions in the CMB temperature and matter power spectra (this is a strong evidence). The values of \( \sigma_8 \) for all the analyses performed in this work are very large in compared to the Planck’s estimation \([5]\) while one may also note that the errors bars in \( \sigma_8 \) are also very large in compared to what Planck estimated \([5]\), and hence, one might argue that although the allowed mean values of \( \sigma_8 \) are very large for the present interacting model, but within 68% CL, they can be close to the Planck’s values. For all the analyses, one can find that the 68% regions of \( \sigma_8 \) are, 0.807 < \( \sigma_8 < 0.969 \) (CMB), 0.794 < \( \sigma_8 < 1.023 \) (CMB + BAO + RSD), 0.848 < \( \sigma_8 < 1.022 \) (CMB + BAO + HST) and 0.807 < \( \sigma_8 < 0.943 \) (CMB + BAO + RSD + HST + WL+ JLA + CC).

From the analysis at the perturbative level, we see that the model is indeed distinguished from the non-interacting \( \Lambda \text{CDM} \) and \( w_x \text{CDM} \) models, which is pronounced from the matter power spectra (right panel of Fig. 6) in compared to the temperature anisotropy in the CMB spectra (left panel of Fig. 6). Such deviation is also clearly reflected from the relative deviation of the interacting model with respect to the \( \Lambda \text{CDM} \) model displayed in Fig. 7. The left panel of Fig. 7 indicates the relative deviation in the CMB spectra while the right panel stands for the relative deviation in the matter power spectra. However, such deviation is not much significant.

Thus, in summary, we find that an exponential interaction, a choice beyond the usual choices for the interaction rates, is astronomically bound to assume weak coupling strength and the overall scenario stays within a close neighbourhood of \( w_x \text{CDM} \) as well as the \( \Lambda \text{CDM} \) model too.

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