Measurement of the Bjorken Sum at very low $Q^2$

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We present new data on the polarized Bjorken sum $\Gamma_1^{p-n}(Q^2)$ at very low four-momentum transfer, $0.021 \leq Q^2 \leq 0.496$ GeV$^2$. The data were obtained in two experiments performed at Jefferson Lab: EG4 on polarized protons and deuterons, and E97110 on polarized $^3$He from which neutron data are extracted. The data cover the domain where chiral effective field theory ($\chi$EFT), the leading effective theory of the Strong Force at large distance, is expected to be applicable. The data are reasonably well described by a number of phenomenological models. The data also generally agree with $\chi$EFT calculations fulfilling the expectation that observables in which the effect of the $\Delta(1232)$ $3/2^+$ resonance is suppressed should be robustly predicted by $\chi$EFT. However, the $\Delta$ suppression also makes accurate $\Gamma_1^{p-n}(Q^2)$ measurements more challenging because of the consequent small value of $\Gamma_1^{p-n}(Q^2)$ and its enhanced sensitivity to low-$x$ contribution. Assessing that contribution with a state-of-the-art model and Regge behavior appears to significantly overestimate the contribution, underlining our lack of understanding of the nucleon spin structure at low-$x$ and low-$Q^2$.

The archetype of spin sum rules, the Bjorken sum rule [1], has played a central role in the investigation of nucleon spin structure [2]. It stands at infinite $Q^2$ the four-momentum transferred between the probing beam and the probed nucleon, and relates the nucleon flavor-singlet axial charge $g_A$ to the isovector part of the integrated spin-dependent structure function $g_1(x)$:

$$\Gamma_1^{p-n} \equiv \Gamma_1^{n} - \Gamma_1^{p} = \int_0^1 [g_1^n(x) - g_1^p(x)] dx = \frac{g_A}{6}. \quad (1)$$

Here, $x \equiv Q^2/(2M\nu)$ is Bjorken scaling variable, $M$ the nucleon mass and $\nu$ the energy transfer between the beam and the nucleon. $g_1^{p(n)}(x)$ denotes the proton (neutron) quantity. The bars over $\Gamma_1$ and the $1^-$ integral limit indicate that the elastic contribution is excluded. The value of the axial charge is well measured; $g_A = 1.2762(5)$ [3]. Measurements of $\Gamma_1$ performed at SLAC [4], CERN [5], DESY [6] and Jefferson Lab (JLab) [7, 8], by scattering polarized lepton off of polarized targets, entail finite lepton beam energy, viz finite $Q^2$. In that case, $g_1(x)$ and $\Gamma_1$ acquire a $Q^2$-dependence, which is calculable at $Q^2 \gtrsim 1$ GeV$^2$ with perturbative quantum chromodynamics (pQCD) [9], and at $Q^2 \ll 1$ GeV$^2$ with chiral effective field theory ($\chi$EFT) [10–12], an effective theory of QCD [13]. At $Q^2 \to 0$, $\Gamma_1$ relates to the Gerasimov–Drell-Hearn (GDH) sum rule [14], which has been tested and appears to be valid [4–8, 15–18]. The GDH sum rule predicts:

$$\Gamma_1^{p-n}(Q^2)_{Q^2=0} = \frac{Q^2}{8} \left( \frac{\kappa_n^2}{M_n^2} - \frac{\kappa_p^2}{M_p^2} \right), \quad (2)$$

where $\kappa_n$ and $\kappa_p$ are the anomalous magnetic moments of the neutron and proton, respectively. Since $\kappa_n^2/M_n^2 > \kappa_p^2/M_p^2$, $\Gamma_1^{p-n}(Q^2)$ is expected to depart from zero with a positive slope. Eq. (2) is assumed in the $\Gamma_1^{p-n}(Q^2)$ calculations from $\chi$EFT which predicts $Q^4$ and higher order dependences of $\Gamma_1^{p-n}(Q^2)$ that arise once $Q^2 > 0$.

The isovector structure of $\Gamma_1^{p-n}$ simplifies its theoretical calculation compared to the individual nucleon case. In particular, the suppression of the contribution of the $\Delta(1232)$ $3/2^+$ excitation—which degrades the convergence of the $\chi$EFT perturbative series—should make the $\chi$EFT prediction of $\Gamma_1^{p-n}(Q^2)$ robust [19]. While this expectation is consistent with earlier data [7, 8], measurements of another observable in which the $\Delta$ is suppressed, the Longitudinal-Transverse interference polarizability $\delta_{LT}(Q^2)$ [20] showed that there the argument failed. This perplexing outcome triggered both $\chi$EFT calculations more challenging because of the consequent small value of $\Gamma_1^{p-n}(Q^2)$ and its enhanced sensitivity to low-$x$ contribution.
| $Q^2$ [GeV$^2$] | $\Gamma_1^{p-n}$ |
|----------------|-----------------|
| 0.021          | 0.00522± (0.00029 stat) ± (0.00124 syst.) |
| 0.024          | 0.00087± (0.00036 stat) ± (0.00129 syst.) |
| 0.029          | 0.01255± (0.00310 stat) ± (0.00151 syst.) |
| 0.035          | 0.00396± (0.00295 stat) ± (0.00163 syst.) |
| 0.042          | 0.00802± (0.00331 stat) ± (0.00160 syst.) |
| 0.050          | 0.00937± (0.00350 stat) ± (0.00179 syst.) |
| 0.059          | 0.01033± (0.00371 stat) ± (0.00207 syst.) |
| 0.071          | 0.01311± (0.00406 stat) ± (0.00233 syst.) |
| 0.084          | 0.01068± (0.00439 stat) ± (0.00234 syst.) |
| 0.101          | 0.01217± (0.00417 stat) ± (0.00256 syst.) |
| 0.120          | 0.02764± (0.00469 stat) ± (0.00297 syst.) |
| 0.144          | 0.01935± (0.00519 stat) ± (0.00268 syst.) |
| 0.173          | 0.02664± (0.00512 stat) ± (0.00274 syst.) |
| 0.205          | 0.02437± (0.00608 stat) ± (0.00289 syst.) |
| 0.244          | 0.04644± (0.00630 stat) ± (0.00294 syst.) |
| 0.292          | 0.05003± (0.00620 stat) ± (0.00260 syst.) |
| 0.348          | 0.05328± (0.00682 stat) ± (0.00266 syst.) |
| 0.416          | 0.04774± (0.00732 stat) ± (0.00290 syst.) |
| 0.496          | 0.07697± (0.00883 stat) ± (0.00698 syst.) |

| $Q^2$ [GeV$^2$] | $\Gamma_1^{n-p}$ |
|----------------|-----------------|
| 0.035          | 0.00882± (0.00056 stat) ± (0.00175 syst.) |
| 0.057          | 0.01185± (0.00105 stat) ± (0.00231 syst.) |
| 0.079          | 0.01312± (0.00138 stat) ± (0.00271 syst.) |
| 0.100          | 0.01387± (0.00139 stat) ± (0.00315 syst.) |
| 0.150          | 0.02251± (0.00129 stat) ± (0.00511 syst.) |
| 0.200          | 0.03088± (0.00209 stat) ± (0.00580 syst.) |
| 0.240          | 0.03895± (0.00225 stat) ± (0.00532 syst.) |

TABLE I: $\Gamma_1^{p-n}$ from EG4 (top) and EG4/E97110 (bottom).

where $\chi$EFT can be well tested. The data are from the experiments EG4 (polarized proton and deuteron targets, henceforth called “EG4”, or “proton” and “deuteron”) and E97110 (polarized 3He target, henceforth called “E97110” or “3He”). The experimental and analysis descriptions, including the extraction of the individual integrals $\Gamma_1^{p,n}$, are reported in Refs [17, 18]. To reach the $x = 0$ limit of integral (1) requires infinite energy. The measurements reached down to $x \approx 10^{-3}$, with the lower $x$ contributions to $\Gamma_1^{p,n}$, $\Gamma_1^d$ and $\Gamma_1^{3\text{He}}$ estimated using a parameterization of previous data [22]. Since $\Gamma_1 \propto Q^2$ in the $Q^2 \to 0$ limit, $\Gamma_1 \approx 0$ at very low $Q^2$ and the importance of the low-$x$ contribution cannot be assessed using the ratio of the measured integral over the estimated full integral. Therefore, in contrast to the earlier publications on $\Gamma_1^{p,n}$ and $\Gamma_1^{n-p}$, we do not provide this ratio. Instead, we will assess in the later part of this article the role of the low-$x$ region by fitting the measured and full integrals, and comparing the resulting fit parameters.

The proton and deuteron data, analyzed at common $Q^2$ values, are combined as $\Gamma_1^{p-n} = 2\Gamma_1^p - \Gamma_1^d / (1 - 1.5\omega_d)$ with the deuteron D-state probability $\omega_d = 0.05 \pm 0.01$ [23] and $\Gamma_1^d$ understood as “per nucleus”. We call the values obtained this way “the EG4 data”. The proton and neutron (3He) data were analyzed at different $Q^2$ values. Since the proton data have finer $Q^2$-bins, they were first combined into the same number of bins as for the neutron (3He) data, and then linearly interpolated to the neutron (3He) data $Q^2$ values. We call the values obtained this way “the EG4/E97110 data”. The two resulting (semi-independent) data sets for $\Gamma_1^{p-n}$ are reported in Table I and shown in Fig. 1, along with data from previous experiments at larger $Q^2$ [4–8]. With the new data, the world data set for $\Gamma_1^{p-n}$ now spans nearly 3 orders of magnitude in $Q^2$. Also shown in Fig. 1 are the latest $\chi$EFT calculations [11, 12] and several models. The Burkert-Ioffe model (green line) is an extrapolation of deep inelastic scattering (DIS) data based on vector meson dominance combined with a parameterization of the resonance contribution [24]. The Pasechnik et al. model [25] (dot-dashed line) applies analytical perturbation theory (APT) to an earlier model [26] that used the smooth $Q^2$-dependence of $g_1 + g_2$ to extrapolate DIS data to low $Q^2$. APT extends the applicability of pQCD by suppressing the Landau pole of $\alpha_s^{pQCD}(Q^2)$ at $Q^2 = \Lambda_{QCD}^2$ [27] ($\alpha_s^{pQCD}$ is $\alpha_s$ computed perturbatively). Finally, the light-front holographic QCD (LFHQCD) method [28] has been used to compute $\alpha_s(Q^2)$ [29], the effective charge that folds into $\alpha_s$ the higher-twists and hard gluon radiation effects on $\Gamma_1^{p-n}$ [30, 31]. Then, $\Gamma_1^{p-n}$ is obtained using $\Gamma_1^{p-n} = \frac{\alpha_s}{6}(1 - \frac{2g_1}{\pi})$. This is depicted in Fig. 1 by the red line.

The $\Gamma_1^{p-n}$ formed using the deuteron (EG4) or the neutron from 3He (E97110) agree, indicating that for this
TABLE II: Best fit of the world data (EG4, EG4/E97110, EG1b, E94010/EG1a) on $\Gamma_{1}^{p-n}(Q^2)$ using a fit function $bQ^2 cQ^4$. The fit is performed up to $Q^2 = 0.244$ GeV$^2$, the maximum range of the E97110 data. The "uncor" uncertainty designates the point-to-point uncorrelated uncertainty. It is the quadratic sum of the statistical uncertainty and a factor for the systematic uncertainty determined so that $\chi^2/d.o.f. = 1$ for the best fit, see Appendix. The "cor" uncertainty is the correlated uncertainty estimated from the remaining factor for the systematic uncertainty. Also listed are results of fits to the predictions from $\chi$EFT and models.

| Data set               | $(b \pm \text{uncor} \pm \text{cor})$ [GeV$^{-2}$] | $(c \pm \text{uncor} \pm \text{cor})$ [GeV$^{-4}$] |
|-----------------------|---------------------------------------------------|--------------------------------------------------|
| World data            | $0.186 \pm 0.017 \pm 0.030$                      | $-0.124 \pm 0.046 \pm 0.080$                    |
| GDH Sum Rule [14]     | 0.0618                                            | 0.07                                            |
| $\chi$EFT Bernard et al. [11] | 0.07                                              | 0.3                                            |
| $\chi$EFT Alarcón et al. [12] | 0.066(4)                                           | 0.25(12)                                       |
| Burkert-Ioffe [24]    | 0.09                                              | 0.3                                            |
| Paschenek et al. [25] | 0.09                                              | 0.4                                            |
| LFHQCD [26]           | 0.177                                             | -0.087                                         |

The $b$ and $c$ values are obtained by fitting the theoretical results in the same way as the data. We note that for LFHQCD $b$ and $c$ can be calculated; $b_{\text{LFH}} = \frac{4\pi\alpha}{3(2\pi)^2} = 0.177$ and $c_{\text{LFH}} = \frac{4\pi4}{8(2\pi)^2} = -0.074$, with $M_{\rho}$ the $\rho$-meson mass. (The fit to the LFHQCD prediction yields $b = -0.067$, about 10% lower than expected due to the truncation of the fit polynomial.)

Although there is an overall agreement between data and predictions, the data points are generally above the theoretical calculations, except LFHQCD (continuous red line in Fig. 1). That deviation makes both the value of $c$ to disagree with the $\chi$EFT expectations, and the value of $b$ to be larger than $b_{\text{GDH}}$. The best fit yields $b = 0.186 \pm 0.017 \pm 0.031$ GeV$^{-2}$, about 3.5 standard deviations above $b_{\text{GDH}}$. Rather than indicating a violation in the isovector sector of the GDH sum rule, a generic relation of quantum field theory, this likely comes from the unmeasured low-$x$ contribution to $\Gamma_{1}^{p-n}$. Although we have estimated that contribution, its is difficult to know its associated uncertainty because neither data nor firm theoretical guidance exist. Since many resonances that contribute to $\Gamma_{1}^{p-n}$ cancel in $\Gamma_{1}^{n}$, notably the $\Delta$ resonances, the low-$x$ contribution contributes more to $\Gamma_{1}^{p-n}$ than to $\Gamma_{1}^{n}$. In fact, fitting the measured part of $\Gamma_{1}^{p-n}$ from EG4 before adding the assessed low-$x$ contribution yields $b_{\text{no low-}} = 0.093 \pm 0.032$ (see Table III), which shows that a 100% variation on the low-$x$ contribution would make $b$ from EG4 consistent with $b_{\text{GDH}}$. This implies that the low-$x$ contribution is significantly overestimated or might even have the wrong sign. Alternatively, the finding that $b > b_{\text{GDH}}$ could come from a systematic effect in the proton data since the EG4 and EG4/E97110 data sets partly share the same proton results. However, the earlier $\Gamma_{1}^{p-n}$ data [7] (open diamonds in Fig. 1) already suggested the higher trend. Another possibility is that the contributions to $\Gamma_{1}^{p-n}$ from deuteron and $^3$He both have, coincidentally, a similar systematic shift in spite of the different types of nuclear corrections, e.g. due to coherent or n-body breakup contributions. Finally, it could be that $\Gamma_{1}^{p-n}(Q^2)$ has a quicker departure from the slope predicted by GDH sum rule than expected.

The LFHQCD prediction matches the data well, although $b_{\text{LFH}} = 0.177$ is, like the data, in tension with $b_{\text{GDH}}$. The good agreement of LFHQCD with the data adds to the successful descriptions of many observables, including hadron spectroscopy [33, 34], form factors [35], GPDs [36], polarized quark distributions [37] and $\alpha_s$ [29].

In conclusion, we presented new data on the Bjorken sum $\frac{d}{dQ^2}(Q^2) = 0.021 \leq Q^2 \leq 0.496$ GeV$^2$ range, which should cover well the domain of applicability of $\chi$EFT. Overall, the $\chi$EFT corrections to the leading order GDH contribution are in the right direction and improve the agreement with the data significantly. The overall consistency of $\chi$EFT and the data contrasts with the difficulties encountered by $\chi$EFT to predict the individual $\Gamma_{1}^{p-n}$ [18] and $\Gamma_{1}^{n}$ [17]. This supports the expectation that $\chi$EFT offers robust prediction for quantities in which the $\Delta$ resonance contribution is suppressed [19]. However, that suppression also makes accurate measurements of $\Gamma_{1}^{p-n}$ challenging since it increases the relative importance of the low-$x$ contribution compared to the $\Gamma_{1}^{p-n}$, and since it makes the magnitude $\Gamma_{1}^{p-n}$ much smaller than that of the individual $\Gamma_{1}^{p-n}$. In fact, at $Q^2 = 0$, the isovector part of the GDH sum rule predicts $\frac{d}{dQ^2}(Q^2) = 0.0618$ GeV$^2$ compared to the isoscalar prediction $\frac{d}{dQ^2}(Q^2)|_0 = 0.97$ GeV$^2$. A future high-energy (up to $\nu = 12$ GeV) measurement of the GDH...
sum at $Q^2 = 0$ on both the proton and the deuteron \[38\] and will help constrain the low-$x$ contribution and may re-

solve that sign issue. The future $Q^2 = 0$ data associ-
ciated with the present ones will therefore improve the testing of $\chi$EFT. Finally, the $P_{1,-}^{-n}$ data agree well with the

phenomenological models \[24, 25\], and especially with the

LFHQCD \[28\]. Aside from testing non-perturbative de-
scriptions of the strong force, the data will be useful for

extracting the QCD running coupling $\alpha_s (1)$ in the

strong, yet near-conformal, regime of QCD.

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Appendix: Fit statistical studies

To compare the data sets to each other and determine how well $b$ and $c$ are determined from the data, we performed fits over different subset of the data. In addition, to assess the possible influences of higher order $Q^2$-terms and of point-to-point correlated uncertainties, we also used fit functions allowing for a constant offset or addition, to assess the possible influences of higher order terms in the fit functions. The results are given in Table III. We remark that the fit to the world data and that of the EG4+EG4/E97110 combined data sets yield similar results (Table III) revealing the dominate nature of the new data at low $Q^2$. The amount of systematic correlation between the data points being difficult to estimate, we assumed that the fraction of the systematic uncertainty needed to obtain a $\chi^2/d.o.f = 1$ is part of the point-to-point uncorrelated uncertainty. For the fit to the world data, this fraction is 64% and was added quadratically to the statistical uncertainty. This yields the first uncertainty quoted in Tables II and III (uncor). The uncor is obtained by re-performing the fit with the data points systematically shifted by the remaining 36% of the systematic uncertainty. The cor uncertainty indicates how compatible the data set is with a statistical dispersion around the fit function and only reflects a fraction of the systematic effects, most of them (64%) being included in uncor. The fit $\chi^2$ is computed with the uncor uncertainty. Irrespective of the fit function used, the $bQ^2 + cQ^4$ function is used to determine $b$ and $c$. If the quadratic sum of the statistical and the entire systematic uncertainties is too small, to reach $\chi^2/d.o.f. = 1$, then $cor = 0$ and $\chi^2/d.o.f. > 1$. The fractions of point-to-point uncorrelated systematic uncertainty are 100% for EG4 (no low-x), 100% for EG4, 83% for EG4/E97110, 80% for EG4+EG4/E97110 and 64% for the word data. The results in Table III indicate that the large effect of the unmeasured low-$x$ contribution, as already mentioned. The value for $c$ is consistent with zero for our main result, but depends strongly on the fit form. Like $b$, it is also strongly dependent on the low-$x$ contribution. While in most fits the central value of $c$ has the opposite sign to that predicted by $\chi$EFT, the signs agree if an offset $a$ is allowed or if $b$ is enforced. Adding a $Q^6$ term does not help with the $b > b$ and the c sign issues. Although the EG4 and EG4/E97110 data sets agree, as do all their fit coefficients $a$, $b$, $c$ and $d$, there is a possible small systematic offset, viz $a \neq 0$, for the EG4/E97110 data, perhaps because He nuclear corrections become more difficult to control at lower $Q^2$ \[39\], but more likely because that data set does not reach a $Q^2$ value as low as the EG4 set. The delicate low-$x$ contribution assessment is therefore less suppressed by the $P_{1,-}^{-n}(0) = 0$ constraint for the EG4/E97110 data set than the EG4 one. In other words, should we be extrapolating the data to $Q^2 = 0$ to determine $a$, the extrapolation with EG4 data would be better constrained since they reach lower $Q^2$.

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**TABLE III:** Best fits of $T_1^{2\,\rightarrow\,0}(Q^2)$ using the functions $bQ^2 + cQ^4$, $a + bQ^2 + cQ^4$ or $bQ^2 + cQ^4 + dQ^6$ applied to various data sets: EG4 data without unmeasured low-$x$ estimate, EG4 data with low-$x$ estimate, EG4/E97110 data with low-$x$ estimate, and the world data (EG4, EG4/E97110, EG1b, E94010/EG1a). The fits are performed up to $Q^2 = 0.244$ GeV$^2$, the maximum range of the E97110 data. The line in bold indicates our main result.

| Data set                        | $(a \pm \text{uncor} \pm \text{cor})$ (GeV$^{-2}$) | $(b \pm \text{uncor} \pm \text{cor})$ (GeV$^{-4}$) | $(c \pm \text{uncor} \pm \text{cor})$ (GeV$^{-4}$) | $(d \pm \text{uncor} \pm \text{cor})$ (GeV$^{-6}$) | $\chi^2/d.o.f.$ |
|--------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|----------------|
| EG4 (no low-$x$)               | NA                                               | 0.093 $\pm$ 0.032 $\pm$ 0.000                     | $-0.137 \pm 0.191 \pm 0.000$                      | NA                                               | 1.24           |
| EG4                            | NA                                               | 0.170 $\pm$ 0.032 $\pm$ 0.000                     | $-0.046 \pm 0.191 \pm 0.000$                      | NA                                               | 1.04           |
| EG4/E97110                     | NA                                               | 0.191 $\pm$ 0.032 $\pm$ 0.021                     | $-0.155 \pm 0.137 \pm 0.056$                      | NA                                               | 1.00           |
| EG4$^*$                         | NA                                               | 0.183 $\pm$ 0.032 $\pm$ 0.019                     | $0.116 \pm 0.108 \pm 0.039$                       | NA                                               | 1.00           |
| World data (2.9 $\pm$ 2.9 $\pm$ 0.2) $\times$ 10$^{-5}$ | $0.104 \pm 0.074 \pm 0.004$                     | $0.212 \pm 0.322 \pm 0.013$                      | NA                                               | 1.05           |
| World data (7.1 $\pm$ 3.4 $\pm$ 0.1) $\times$ 10$^{-3}$ | $0.045 \pm 0.074 \pm 0.019$                     | $0.366 \pm 0.282 \pm 0.053$                      | NA                                               | 0.12           |
| World data (4.4 $\pm$ 2.1 $\pm$ 0.3) $\times$ 10$^{-3}$ | $0.088 \pm 0.049 \pm 0.015$                     | $0.233 \pm 0.197 \pm 0.038$                      | NA                                               | 0.82           |
| World data (3.0 $\pm$ 2.5 $\pm$ 0.4) $\times$ 10$^{-3}$ | $0.104 \pm 0.019 \pm 0.020$                     | $0.203 \pm 0.080 \pm 0.054$                      | NA                                               | 0.65           |

EG4$^*$ indicates data with low-$x$ estimate.
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