Vehicle oscillation taking into account the rheological properties of the suspension

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Abstract. The forced oscillations of a four-axle vehicle with a double spring suspension are considered. The motion of a system with six degrees of freedom can be represented with sufficient accuracy by a system with two degrees of freedom. Therefore, the body of the vehicle has two degrees of freedom: sideways movement and wagging (jumping and galloping carts will be neglected). It is assumed that the rheological properties of the spring (suspension) are different and obey the hereditary theory of Boltzmann-Volterra viscoelasticity. As the core of heredity, the Koltonov-Rzhantsyn core is used, which has weakly singular features of the Abel type. Effective computational algorithms for solving problems based on the use of quadrature formulas have been developed. For numerical calculation, a computer program has been compiled, the results of which are presented in the form of graphs. The influence of the rheological properties of the suspension on the forms of vertical and angular movement of the body is studied. It was found that due to the suspension viscosity, the amplitude of vertical and angular oscillations decreases and the frequency increases.

1. Introduction
Dynamic vibration dampers, as some additional devices introduced into the initial design schemes of vibration protection systems, can be considered as one of the means of controlling the state of the object of protection. It is shown that mathematical models of oscillatory systems in the form of structural schemes of dynamically equivalent automatic control systems have certain advantages in comparison with conventional approaches based on the use of differential equations. Dynamic quenching in structural models is interpreted as the introduction of additional negative feedback circuits. Such chains are formed on the basis of structural transformations of the original model according to the rules of parallel and serial connection of the spring [1-3].

A problem of estimation of dynamic properties of technical objects subject to action of vibration loads can be solved by using numerical schemes in the form of a mechanical oscillatory systems with several degrees of freedom, which gives some opportunity to assess the forms of dynamic interactions and definition of requirements to appropriate architectural solutions, which depend on properties of constituent elements. A number of issues related to the construction of design schemes for technical objects, the features of their element base, and the possibilities for forming and evaluating dynamic States of technical objects are discussed in [4-7].

Studies of forced oscillations on a straight section of the track are related to the fact that the vehicle is moving along a certain guide, which has inevitable irregularities [8-10]. Fluctuations in the
vehicle caused by irregularities may be undesirable depending on the frequency of these irregularities along the path and the speed of the crew.

2. Problem Statement
Consider the forced oscillations of a four-axle vehicle that has a double spring suspension (Fig. 1.). The motion of a system with six degrees of freedom can be represented with sufficient accuracy by a system with two degrees of freedom. Therefore, we assume that the vehicle body has two degrees of freedom: jumping and galloping [11].

To study the vibrations of the sprung parts of the vehicle, the following designations are used:

- $m_k$ — body mass; $I_k$ — body inertia moment when galloping; $c_{11}, c_{12}$ — vertical stiffness of the Central suspension of the cart; $z_k$ and $\phi_k$ — vertical and angular movements of the body, respectively; $L_1 + L_2$ — body base.

After calculating the kinetic and potential energy and using the Lagrange equation, the equilibrium equations of the problem under consideration in the elastic formulation are derived [12-16].

The problem is complicated when studying transients in inherently deformable systems. The main difficulty here is to find solutions to a system of weakly singular integro-differential equations under given initial conditions and arbitrary external loads. An important practical interest is the study of the behavior of systems under pulsed loads and free damping oscillations caused by specified initial conditions. Using the Boltzmann-Volterra principle when taking into account the rheological properties of the suspension, we obtain a system of integro-differential equations describing the problem under consideration in the viscoelastic formulation [17,18]:

![Figure 1. Schematic diagram of a four-axle vehicle with a double spring suspension.](image)

\[
\begin{align*}
(m_k \ddot{z}_k + c_{11}(1 - R_{11}^*)(z_k + L_1\phi_k) + c_{12}(1 - R_{12}^*)(z_k - L_2\phi_k) &= Q(t) ; \\
I_k \ddot{\phi}_k + c_{11}(1 - R_{11}^*)(L_1^2\phi_k + L_1z_k) + c_{12}(1 - R_{12}^*)(L_2^2\phi_k - L_2z_k) &= 0
\end{align*}
\]

where $R_{1p}^* w = \int_{0}^{t} R_{1p} (t - \tau) w(\tau) d\tau$ — the integral operator with the kernel relaxation $R_{1p}(t) = \epsilon_p t^\alpha e^{-\beta_p t}; p = 1,2.$

3. Solution Method
By entering dimensionless values $\bar{t} = \frac{t}{t_0}$; $\bar{z}_k = \frac{z_k}{z_0}$; $\bar{\phi}_k = \frac{\phi_k}{\phi_0}$ keeping the previous notation, we get
\begin{equation}
\begin{cases}
\ddot{z}_k + (1 - R_{11}^r)(A_1 z_k + A_3 \varphi_k) + (1 - R_{12}^r)(A_2 z_k - A_4 \varphi_k) = q(t); \\
\dot{\varphi}_k + (1 - R_{11}^s)(B_1 \varphi_k + B_3 z_k) + (1 - R_{12}^s)(B_2 \varphi_k - B_4 z_k) = 0
\end{cases}
\end{equation}

where \( A_1 = \frac{\dot{c}_1 z_k}{m_k}; \quad A_2 = \frac{\dot{c}_2 z_k}{m_k}; \quad A_3 = \frac{\dot{c}_1 z_k}{m_k \bar{c}_0}; \quad A_4 = \frac{\dot{c}_2 z_k}{m_k \bar{c}_0}; \)

\[ q(t) = \frac{\dot{c}_0}{m_k} Q(t); \]

\[ B_1 = \frac{\dot{c}_1 z_k^2}{l_k}; \quad B_2 = \frac{\dot{c}_2 z_k^2}{l_k}; \quad B_3 = \frac{\dot{c}_1 z_k}{l_k \bar{c}_0}; \quad B_4 = \frac{\dot{c}_2 z_k}{l_k \bar{c}_0} \]

Suppose that

\[ z_k(t) = z_k(t) = 0 \quad \text{and} \quad \varphi_k(t) = \dot{\varphi}_k(t) = 0 \quad \text{pri} \quad t = 0. \quad (2) \]

System (1) is solved by a method based on the use of a quadrature formula [19]. Integrating the system (1) twice in time in the interval \([0; t]\) and taking into account the initial

\begin{equation}
\begin{cases}
z_k = \int_{0}^{t} (t-s)q(s) ds - \int_{0}^{t} G_{11}(t-s) [A_1 z_k(s) + A_3 \varphi_k(s)] ds - \\
\quad - \int_{0}^{t} G_{12}(t-s) [A_2 z_k(s) - A_4 \varphi_k(s)] ds;
\end{cases}
\end{equation}

\[ \varphi_k = - \int_{0}^{t} G_{11}(t-s) [B_1 \varphi_k(s) + B_3 z_k(s)] ds - \int_{0}^{t} G_{12}(t-s) [B_2 \varphi_k(s) - B_4 z_k(s)] ds. \]

Assuming \( t_n = n \cdot \Delta t, \ n = 1, 2, 3, \ldots (\Delta t \text{-time step}) \) in (3), and replacing the integrals with quadrature trapezoid formulas, we have:

\begin{equation}
\begin{cases}
z_{kn} = \sum_{i=0}^{n-1} D_j (t_n - t_i) q_i - A_1 \sum_{i=0}^{n-1} D_j G_{11}(t_n - t_i) z_{ki} - A_2 \sum_{i=0}^{n-1} D_j G_{12}(t_n - t_i) z_{ki} - \\
\quad - A_3 \sum_{i=0}^{n-1} D_j G_{11}(t_n - t_i) \varphi_{ki} + A_4 \sum_{i=0}^{n-1} D_j G_{12}(t_n - t_i) \varphi_{ki};
\end{cases}
\end{equation}

\[ \varphi_{kn} = - B_1 \sum_{i=0}^{n-1} D_j G_{11}(t_n - t_i) \varphi_{ki} - B_2 \sum_{i=0}^{n-1} D_j G_{12}(t_n - t_i) \varphi_{ki} - \\
\quad - B_3 \sum_{i=0}^{n-1} D_j G_{11}(t_n - t_i) z_{ki} + B_4 \sum_{i=0}^{n-1} D_j G_{12}(t_n - t_i) z_{ki}; \]

where \( q_i = q(t_i); \quad z_{ki} = z_k(t_i); \quad \varphi_{ki} = \varphi_k(t_i); \quad D_0 = \frac{\Delta t}{2}; \quad D_j = \Delta t, \ j = 1, 2, 3, \ldots, n - 1 \)

\[ G_{1p}(t_n - t_i) = t_n - t_i - \int_{0}^{t_n-t_i} (t_n - t_i - \tau) R_{1p}(\tau) d\tau, \quad p = 1, 2. \]
4. Results
To conduct a computational experiment, a computer program has been compiled, the numerical results of which are presented in the form of graphs. Numerical calculations were performed. The following initial data were used:

\[ \Delta t = 0.05; \quad A_1 = 1.2; \quad A_2 = 1.6; \quad A_3 = 3.5; \quad A_4 = 3.5; \quad B_1 = 3.8; \quad B_2 = 4.2; \quad B_3 = 1.5; \quad B_4 = 1.5; \quad \alpha_1 = \alpha_2 = 0.25; \quad \beta_1 = \beta_2 = 0.05. \]

For pictures 2 and 3, respectively, show the shape of the vertical and angular movements of the body when the external load is constant, \( q(t) = 0.6 \). Here and in the future, a solid line indicates the graph for the elastic \( (\varepsilon_i = 0) \), a dotted line \( (\varepsilon_i = 0.05) \), and a dotted line \( (\varepsilon_i = 0.1) \) for the viscoelastic suspension. The graph shows that under constant load, the oscillatory process occurs near the creep curve. Taking into account the rheological properties of the suspension leads to a decrease in the amplitude of vertical and angular movements of the body from the static equilibrium position. Increasing the frequency of body vibrations leads to a phase shift. Over time, the viscoelastic properties of the suspension significantly affect the amplitude and frequency.

![Figure 2](image2.png)

**Figure 2.** Vertical vibrations of the body under constant load.

![Figure 3](image3.png)

**Figure 3.** Angular oscillations of a body under the action of constant load.

Figures 5 and 6 show, respectively, the forms of vertical and angular movements of the body caused by a periodic piecewise constant driving force (Fig.4).
Figure 4. Form of the driving force.

The graph shows that taking into account the rheological properties of the suspension leads to a decrease in the amplitude of vertical and angular movements of the body. Increasing the frequency of body vibrations leads to a phase shift.

Figure 5. Vertical vibrations of the body under the action of piecewise constant load.

Figure 6. Angular oscillations of the body under the action of piecewise constant load.
Figure 7. Vertical fluctuations of the body under the influence of periodic load.

Figure 8. Angular fluctuations of the body under the influence of periodic load.

5. Conclusion
Figures 7, 8 shows the forms of vertical and angular movements of the body, respectively, when it is subjected to a periodic load: $q(t) = 0.6\sin2\pi t + 0.8\cos2\pi t$. The graph shows that taking into account the rheological properties of the suspension leads to a decrease in vertical and angular movements. At the same time, changes in the frequency of vertical vibrations of the body are insignificant, and angular fluctuations are significant.

The use of schemes that allow obtaining the solution of the problem in closed form or using well-studied algorithms of type (4) is of great interest. The results obtained allow us to conclude that it is appropriate to take into account the hereditary deformable properties of the suspension to reduce the amplitude of vehicle vibrations during transients.

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