On the Short Distance Part of the QCD Anomaly Contribution to the $b \to s\eta'$ Amplitude

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Abstract

In addressing the $B \to \eta'K$ puzzle, there has been a considerable hope in the literature to resolve it by the QCD anomaly contribution to the $b \to s\eta'$ amplitude. This contribution corresponds to the electroweak $b \to sg\ast g\ast$ transition followed by the off-shell gluon fusion $g\ast g\ast \to \eta'$. In the present paper we perform a critical reassessment of this issue. We show that for the hard virtual gluons in a loop there is a well defined short distance amplitude corresponding to a remnant of the QCD anomaly. However, we find that it cannot account for the measured amplitude.

In addition, we point out that the reduction of the gluon fusion vertex for the off-shell gluons is compensated by an absence of the claimed suppression in the electroweak vertex, and that some nonperturbative contributions related to the QCD anomaly may still be viable in explaining the physical $B \to \eta'K$ amplitude.

Key words: B mesons, rare decays, axial anomaly
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1 Introduction

Recent measurements [1,2,3,4,5] of two-body charmless hadronic $B$ meson decays confirm the so-called $B \to \eta'K$ puzzle. According to the PDG average [6] of CLEO, BaBar and Belle measurements, the $B \to K\eta'$ decay rate turned

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out to be unexpectedly large when compared to the rate obtained within the standard effective Hamiltonian approach using quark operators multiplied by Wilson coefficients [7]. Namely, the enhancement of

\[
Br(B^+ \rightarrow K^+ \eta') = (7.5 \pm 0.7) \cdot 10^{-5}, \\
Br(B^0 \rightarrow K^0 \eta') = (5.8^{+1.4}_{-1.3}) \cdot 10^{-5},
\]

when compared to the QCD-penguin dominated \(B \rightarrow K\pi\) rates

\[
B(B^+ \rightarrow K^0 \pi^+) = (1.73^{+0.27}_{-0.24}) \cdot 10^{-5}, \\
B(B^0 \rightarrow K^0 \pi^-) = (1.74 \pm 0.15) \cdot 10^{-5}.
\]

calls for an additional contribution to the \(K\eta'\) channel, either by performing a more complete treatment within the standard model (SM) or by invoking new physics beyond the standard model (BSM). Since the properties of the \(\eta'\)-particle are related to the QCD axial anomaly, it has been quite generally expected that the enhancement in (1) is related to this anomaly.

Among various mechanisms considered to explain the \(B \rightarrow K\eta'\) amplitude within the SM, the \(b \rightarrow s\eta'\) transition plays a distinguished role. There was an invitation in [8] to consider it as a promising short-distance (SD) mechanism. It pertains to the recently studied singlet penguin mechanism [9,10], involving two gluons fusing into \(\eta'\).

In the present paper we focus on the SD contribution to \(b \rightarrow s\eta'\) incorporating the electroweak \(b \rightarrow sg^*g^*\) transition followed by the off-shell gluon fusion \(g^*g^* \rightarrow \eta'\), where \(g^*\) denotes a virtual gluon, and \(\eta' \simeq \eta_0\) corresponds to the flavour singlet state. For low-energy gluons the \(g^*g^*\eta'\) vertex should be dominated by the QCD triangle anomaly analogously to the \(\pi^0 \rightarrow 2\gamma\) amplitude. However, for (at least) one highly virtual gluon of momentum \(q\), the \(g^*g^*\eta'\) vertex is expected to be suppressed like \(1/q^2\) similarly to the effective \(\gamma^*\gamma\pi^0\) vertex [11,12,13]. This remnant of the anomaly for hard off-shell gluons we will call the “anomaly tail” contribution. There has been a renewed interest in such contributions in view of the hadronic contributions to \((g-2)\) that they induce [14,15].

A more complete discussion on the various mechanisms presented in the literature for \(B \rightarrow \eta'K\) [16,8,17], including various versions of form factors for \(g^*g^*\eta'\) [18,19,20,9,10,21,22] will be relegated to a forthcoming paper. In these proposals, as a rule, the gluon virtuality employed in the \(g^*g^*\eta'\) transition form factor lies below \(m_b^2\). In contrast, we consider highly virtual gluons, above the \(m_b\) scale, corresponding to the mentioned anomaly tail contribution.

After presenting in Sect. 2 the electroweak \(b \rightarrow sg^*g^*\) vertex, in Sect. 3 we demonstrate how the virtual gluons are glued to the “anomaly tail” part of the \(g^*g^*\eta'\) vertex mentioned above. The resulting contribution is dubbed the
“short distance anomaly” (SDA) in what follows. In the concluding section we discuss the meaning of our results and the relation of our contributions to those existing in the literature.

2 The Flavour Changing $b \to sg^*g^*$ Transition

The flavour changing transitions into two virtual gluons were considered by two of the authors in the context of the double-penguin contributions to the $K^0 - \bar{K}^0$ [23] and $B^0 - \bar{B}^0$ mixing [24]. They have been subsequently studied by Simma and Wyler [25] in the case of rare $B$-decays. Let us now reconsider these transitions by taking the symmetric gluon momenta, so that we will be able to present the analytical expressions suitable for evaluation of hard-gluon loop integrals. The resulting $b \to sg^*g^*$ amplitude reads

$$M_{\mu\nu}^{a'd'}(b \to s g^*(p) g^*(-p)) = \frac{i\alpha_s G_F}{\pi} \frac{\sqrt{s}}{2} t^{a'd'} t^a \sum_{i=u,c,t} \lambda_i H_{i\mu\nu} b + \text{ (crossed)},$$

where $t^a$ denote colour matrices, and $\lambda_i$ are the Cabibbo-Kobayashi-Maskawa (CKM) factors. $H_{i\mu\nu}$ subsums the contribution from the box on Fig. 1(a), the contribution from the triangle on Fig. 1(b), and the contribution from the off-diagonal $b \to s$ self-energy. The $b \to s g^*(p)$ loop in Fig. 1(b) is proportional to the gluonic monopole, $(p^2 \gamma^\mu - p \cdot \gamma p^\mu) L$. This $p^2$ dependence is canceled by the $1/p^2$ dependence of the $s$-quark propagator in this reducible diagram. After combining the triangle and the self-energy contribution, the UV divergences mutually cancel, and only the monopole, triangle part contributes significantly.

By anticipating the antisymmetric structure of the $\eta'g^*g^*$ vertex into which the virtual gluons on Fig. 1 proceed, we select the relevant antisymmetric
contribution:
\[ H_i^{\mu\nu} = (-ie^{\mu\rho\sigma} p_{\rho} \gamma_{\tau} L) A_i + (\mu-\nu \text{ symmetric part}), \] (4)
where we obtain
\[
A_i = -\frac{8 M_W^2}{m_i^2 - M_W^2} \left( 1 + \frac{m_i^2}{2 M_W^2} \right) \int_0^1 dx (1-x) \ln \frac{D}{C} \left( \frac{x^2}{1-x} \right) \left[ x^2(1-x)p^2 + (x+1)m_i^2 \right] \left[ \frac{1}{1 - x} \right]
\] (5)
The abbreviations are
\[ Y_1 = \frac{1 - D}{D - C} \ln \frac{D}{C}, \quad Y_2 = \frac{1}{D - C} \ln \frac{D}{C} - \frac{1}{C}, \]
\[ D = x m_i^2 + (1-x) M_W^2 - x(1-x)p^2, \quad C = m_i^2 - x(1-x)p^2. \]
This amplitude agrees with that of \[25\] in the region of their mutual validity. Let us stress that \( A_i \) has the asymptotic behaviour \( 1/p^2 \) for high gluon momenta \( (-p^2 \rightarrow \infty) \), which is essential in order to obtain an overall finite gluon loop contribution to \( b \rightarrow s \eta' \) on Fig. 2. Another interesting limit is the leading logarithmic approximation
\[ \left( \sum_{i=u,c,t} \lambda_i A_i \right)_{L, \text{Log}} = \frac{4}{3} \lambda_c \ln \left( \frac{M_W^2}{-p^2} \right), \] (6)
where we have neglected the \( u \)-quark contribution which is CKM suppressed. It should be noted that this leading contribution comes from the triangle graph in Fig. 1(b), corresponding to the first term in (5). The box graph has no leading logarithm and is numerically small. This clearly differs from the dominance of the box part displayed on Fig. 1 of \[10\], and an explanation on the relevance of the \( b \rightarrow sg^*g^* \) amplitude (3) is in order.

There were suggestions in the literature (e.g. \[17\]) that because of large cancellations observed by \[26,27,25\], the contribution of the \( b \rightarrow sg^*g^* \) mode followed by a \( g^*g^* \rightarrow \eta' \) transition would be extremely small. It should be strongly emphasized that this statement, valid for soft gluons, is wrong for hard virtual gluons, as explicated in the expressions above. Therefore, in the next section we take under scrutiny the amplitude stemming from the antisymmetric part of the electroweak amplitude on Fig. 1.
Fig. 2. The hard gluon loop contribution to the $b \to s\eta'$ transition, determined by vertices (3) and (7)

3 Short Distance Anomaly Contribution

In this section, we will consider the anomaly tail part of the effective $g^* g^* \to \eta'$ vertex to be connected to the $b \to s g^* g^*$ amplitude from the preceding section. By restricting to the hard gluons, having a virtuality above the $m_b$ scale, we aim at singling out the short distance amplitude for the $b \to s\eta'$ transition. This amplitude is of the same order in $\alpha_s$ as recently studied amplitudes with (colour) singlet penguin topology [28,29,9,10].

We need an expression for the gluonic triangle contribution for $g^* g^* \to \eta'$ on Fig. 3. Using the kinematical choice $q_1 = K/2 + p$, $q_2 = K/2 - p$ for the two gluon momenta in this figure, we get a general expression for the $g^* g^* \to \eta'$ vertex

$$N^{\alpha \beta} \left( g^* (q_1) g^* (q_2) \to \eta' (K) \right) = -i \delta^{\alpha \beta} \epsilon_{\mu \nu \alpha \beta} p_\mu K_\nu G(p^2).$$

(7)

In our case, $G(p^2)$ will turn out to be a remnant of the gluonic anomaly. A priori, the quantity $G$ depends also on the momentum $K$, but for our purposes we only need to keep $K$ to first order there, which means that $G$ is taken as independent of the $\eta'$ momentum.

Our effective vertices (3) and (7) allow us a perturbative evaluation of the amplitude displayed on Fig. 2:

$$A_{SDA} (b \to s\eta') = \int \frac{d^4 p}{(2\pi)^4} M_{\mu \nu}^{\alpha \beta} \left( b \to s g^* g^* \right) \frac{-ig^{\mu \alpha}}{p^2} N_{\alpha \beta}^{\alpha \beta'} \left( g^* g^* \to \eta' \right) \frac{-ig^{\nu \beta'}}{p^2}.$$  

(8)

Having in mind that both vertices $M$ and $N$ imply hard gluons in the loop, we arrive at the short distance representation of the singlet penguin diagram discussed in [28,29]. After substitution of (3) and (7), our result for the short
The SDA distance di-gluon mechanism reads

\[ A_{SDA}(b \rightarrow s\eta') = 2i \frac{G_F}{\sqrt{2}} (\bar{s}K \gamma \gamma L b) \sum_{i=u,c,t} \lambda_i \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \frac{\alpha_s}{\pi} A_i(p^2) G(p^2). \]  

Since \( A_i(p^2) \) and \( G(p^2) \) are already one-loop quantities, \( A_{SDA}(b \rightarrow s\eta') \) is a 3-loop amplitude.

Let us stress that eqs. (8) and (9) apply quite generally for all \( g^* g^* \eta' \) form factors \( G(p^2) \). In this paper we will use these formulae for the SDA contribution only.

The evaluation of the \( g^* g^* \rightarrow \eta' \) transition amplitude for off-shell gluons is theoretically analogous to the evaluation of the “off-shell anomaly” for the flavour triplet axial current related to \( \pi^0 \rightarrow \gamma^* \gamma^* \). A naive treatment of this photonic case leads to problems with unitarity for the \( e^+ e^- \rightarrow \gamma^* \rightarrow \pi^0 \gamma \) amplitude presented by Jacob and Wu [30], and a number of papers [11,12,13] has been devoted to its cure.

The pertinent triangle amplitude was calculated first by Rosenberg [31] and reconsidered by Adler [32], who observed difficulties with divergences when inserting the triangle loop into the next loop. On the other hand, for off-shell photons, it was shown by [12,13] that the perturbative mass-independent part in the triangle diagram is canceled by the pion pole anomalous contribution. This observation was essential to resolve the Jacob-Wu paradox [30]. In our gluonic case it means that, although the quantity \( G \) in (7) obtained in a perturbative calculation of the quark-triangle loop is a priori proportional to

\[ \sum_{j=u,d,s} \int_0^1 dx \int_0^{1-x} dy \left\{ 1 + \frac{m_j^2}{Q_j} \right\}, \]  

the unity term gets canceled [12,13]. In (10) we introduced

\[ Q_j = y(1-y)q_1^2 + x(1-x)q_2^2 + 2xy q_1 \cdot q_2 - m_j^2 \]  

representing the denominator emerging from the triangle-loop integral.

For the \( g^* g^* \rightarrow \eta' \) transition the flavours \( j = u, d, s \) contribute (eventually only the s-quark contribution turns out to be important), and we obtain

\[ G = \frac{1}{f_{\eta'}} \frac{\alpha_s}{\pi} \sqrt{\frac{2}{3}} \sum_{j=u,d,s} F_j(p, K), \]  

where \( f_{\eta'} \simeq f_{\pi} \simeq 92 \text{ MeV} \) and

\[ F_j(p, K) = \int_0^1 dx \int_0^{1-x} dy \frac{m_j^2}{Q_j} = -\left( \frac{m_j^2}{-p^2} \right) \ln \left( \frac{-p^2}{m_j^2} \right). \]  

6
This represents the short distance amplitude for hard gluon virtualities far above the $\eta'$-mass. This effective vertex enters into the final gluonic loop on Fig. 2 for loop momenta $p^2 \gg K^2 = m_{\eta'}^2$. In the evaluation of the loop integral producing the short distance 3-loop contribution for the $b \to s \eta'$ amplitude, the overall renormalization scale $\mu$ will be of the order $m_b$. Taking into account that $\mu$ acts as the effective IR cut-off in our loop integral, we find that the expression (13) is slightly modified ($Q_j \to Q_j + \mu^2$ in (13) above). In the leading logarithmic approximation we obtain for the $s$-quark term

$$ F_s = - \left( \frac{m_s^2}{-p^2} \right) \ln \left( \frac{-p^2}{\mu^2} \right), $$

while the $u$- and $d$-quark contributions are completely negligible due to their small masses. On account of the CKM unitarity, the loop integral parts in (9)

$$ I_i = i \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \frac{\alpha_s}{\pi} A_i(p^2) G(p^2), $$

form the relevant GIM combination displayed on Fig. 4. This plot exhibits a reasonably mild dependence on the infra-red cut-off ($\mu$) in the gluon loop, justifying a sensible short-distance amplitude. Note that the UV convergence of the loop integral is guaranteed by the high energy behaviour of (5) and (14). Since the used form-factor (14) corresponds to the anomaly tail term, the net contribution can be termed the “short distance anomaly”, as suggested in the Introduction. Also, Fig. 4 shows the dominance of the triangle with respect to the electroweak box contribution. In the leading logarithmic approximation, we obtain from eqs. (6), (14) and (15)

$$ \left( \sum_i \lambda_i I_i \right)_{L,\text{Log}} = \frac{i}{16\pi^2 f_{\eta'}} \left[ \frac{2}{3} \left( \frac{\alpha_s}{\pi} \right)^2 \frac{4}{3} \lambda_s m_s^2 \frac{1}{6} \left\{ \ln \left( \frac{M_W^2}{\mu^2} \right) \right\}^3 \right], $$

which may be compared with the result from the (octet) penguin operator to
Fig. 4. The infra-red stability check of the anomaly tail contribution (solid line) displaying also the dominance of the triangle part (dashed line) over the box part (dotted line) from Fig. 1.

lowest order in the leading logarithmic approximation

\[ M_{\text{Peng}} = \lambda_{c} \frac{G_{F}}{\sqrt{2}} \left( -\frac{2}{3} \frac{\alpha_{s}}{\pi} \ln \left( \frac{M_{W}^{2}}{\mu^{2}} \right) \left( \bar{s}t^{a} \gamma^{\mu} L b \right) \left( \bar{q} t^{a} \gamma_{\mu} q \right) \right). \]  

(17)

4 Discussion and Conclusions

In the present letter we attempt to clarify a possible role of the \( b \to s \eta' \) transition in explaining the \( B \to \eta' K \) amplitude, and in particular the role of the QCD anomaly in obtaining the \( b \to s \eta' \) amplitude. The result above shows that we are able to successfully distinguish the short distance \( b \to s \eta' \) amplitude (SDA) related to the QCD axial anomaly. Thereby, the involved form of the flavour-changing vertex for hard gluons is in clear contrast to the result obtained by [10] for soft gluons, where the box (Fig. 1(a)) instead of the triangle (Fig. 1(b)) dominates. This enhancement of the electroweak vertex for the hard off-shell gluons is compensated by a suppression in the two gluon fusion to \( \eta' \) vertex. The net \( b \to s \eta' \) amplitude coming from this anomaly tail turns out to be dominated by the strange quark contribution. In the present approach it corresponds to the quark triangle contribution to the \( \eta' g^* g^* \) coupling for the highly off-shell gluons. We note that the SDA result vanishes in the chiral limit \( m_{s} \to 0 \).
Thus, demonstrating that we have obtained a contribution which is different from those already existing in the literature, we can attempt to compare it to some related amplitudes. A simple comparison of our singlet amplitude (9) at the leading logarithm level (16) to the ordinary penguin in (17):

$$A_{\text{Peng}} \sim f_\pi G_F \left( \frac{\alpha_s}{\pi} \right) \ln \left( \frac{M_W^2}{\mu^2} \right)$$

(18)

gives the ratio

$$\frac{A_{\text{SDA}}}{A_{\text{Peng}}} \sim \left( \frac{2}{3} \right)^{3/2} \frac{\alpha_s}{\pi} \left( \frac{m_s}{4\pi f_{\eta'}} \right)^2 \ln \left( \frac{M_W^2}{\mu^2} \right)^2,$$

(19)

where we didn’t explicate some additional factors in $A_{\text{SDA}}$ and $A_{\text{Peng}}$ which cancel in the ratio. This ratio is at the level of a few percent, but depends strongly on what one takes for the involved parameters. How this estimate will be modified within a more proper treatment of renormalization group equations is also an issue to be further investigated. We have done a simple estimate which shows that short distance QCD corrections do not change this result substantially.

To conclude, the above demonstrates that purely short-distance anomalous (SDA) aspects of the $b \to s\eta'$ vertex are marginal in explaining the $B \to \eta'K$ puzzle. There was a similar fate for the electroweak $s\bar{d} \to \gamma^*\gamma^* \to \mu\bar{\mu}$ contribution to the short-distance $K \to \mu\bar{\mu}$ amplitude [33]. There is an additional experience in the analogous off-shell photon case [14], where the corresponding QED quark triangle overshoots the measured value of the related form factor.

In our case, the smallness of the anomaly contribution to the $b \to s\eta'$ can be ascribed to the depletion of the QCD off-shell triangle vertex. Accordingly, the expectation of [8], as we understand it, turns out not to be fulfilled, and we seem to be in compliance with the conclusions by [10] that the singlet penguins do not do the job and that a combination of several effects within the Standard model would be necessary. In addition, our observation of the absence of the suppression in the electroweak vertex, when we depart from truly soft gluons, opens a window for some other contributions which resemble the QCD anomaly. For example, ref. [34] advocated that the $B \to \eta'K$ puzzle could be explained by the additional complicated non-perturbative quark-gluon interactions related to the anomaly. However, at present these interactions are merely parametrized by a phenomenological coupling. Such an extra piece to the usual effective Hamiltonian, appearing due to non-perturbative aspects of the QCD anomaly, has to be justified yet. We intend to come back to a more complete treatment of the $B \to \eta'K$ amplitude in a forthcoming paper.
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