Nielsen–Olesen vortex in varying-\(\alpha\) theories

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We consider soliton solutions to Bekenstein’s theory, for which the fine structure constant \(\alpha = e^2/(4\pi hc)\) is allowed to vary due to the presence of a dielectric field pervading the vacuum. More specifically we investigate the effects of a varying \(\alpha\) upon a complex scalar field with a \(U(1)\) electromagnetic gauge symmetry subject to spontaneous symmetry breaking. We find vortex solutions to this theory, similar to the Nielsen–Olesen vortex. Near the vortex core the electric charge is typically much larger than far away from the string, lending these strings a superconducting flavour. In general the dielectric field coats the usual local string with a global string envelope. We discuss the cosmological implications of networks of such strings, with particular emphasis on their ability to generate inhomogeneous recombination scenarios. We also consider the possibility of the dielectric being a charged free field. Even though the vacuum of such a field is trivial, we find that the dielectric arrangement itself in the shape of a local string, with a quantized magnetic flux at the core — presumably borrowing these topological features from the underlying Nielsen–Olesen vortex.

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I. INTRODUCTION

The possibility that the constants of Nature may not be constant at all has long been entertained by physicists (see for instance the groundbreaking work of Dirac [3]). Historically the gravitational constant \(G\) has been the main victim of attempts to build varying constant theories. An example of a varying-\(G\) theory is Brans–Dicke theory [4], but modern string theories all display this property (eg. [5]). Another often dethroned constant is the electron charge \(e\), whose variability is envisaged by varying-\(\alpha\) theories (where \(\alpha = e^2/(4\pi hc)\) is the fine structure constant) such as the one proposed by Bekenstein [1]. A recent claim for observational evidence for a time-varying \(\alpha\) [5,6] has sparked great interest in this type of theories [7–9]. More radically a varying \(\alpha\) may result from varying speed of light (VSL) theories, typically (but not always [10]) entailing breaking of Lorentz invariance [10,11]. It appears that VSL theories may solve the cosmological implications of networks of such strings, with particular emphasis on their ability to generate inhomogeneous recombination scenarios. Should the observations of Webb et al be confirmed, no doubt most of the physical constructions employed by physicists will have to be reexamined. Bekenstein’s theory is perhaps the most conservative theory with which to interpret the new results, in the sense that it does not require giving up any truly fundamental principles, such as covariance and Lorentz invariance. Within the framework of this theory the vacuum is pervaded by a dielectric medium, screening the electric charge. The properties of this dielectric medium are determined by the electromagnetic field itself, within the context of a dynamical Lagrangian theory. Hence in the surroundings of any object with an electromagnetic energy component, there will be spatial variations in the electric charge.

The application of this theory to cosmology is clouded by the issue of determining how much of the matter in the Universe will act as a source for this dielectric medium [4] (see however [15]). Clearly one needs to understand the microphysics underlying the cosmological fluid, in particular the nature of the dark matter, in order to set up a consistent cosmological model. In this paper we therefore turn to a more concrete and simpler situation. We consider a complex scalar field with a \(U(1)\) gauge electromagnetic symmetry spontaneously broken, which couples to a dielectric field in accordance with Bekenstein’s prescription. We then consider topologically non-trivial solutions to this theory, the counterpart of the Nielsen–Olesen vortex [13]. In the standard theory such vortices have well localized concentrations of energy, along a stable string-like core. Furthermore this core constitutes a magnetic field flux tube. Hence the vortex acts as a source for the dielectric vacuum proposed in [4], leading to a varying \(\alpha\) in the vicinity of the string.

The value of \(e\) in the string core is therefore much larger (smaller) than the asymptotic value \(e_0\) (larger or smaller depending on parameter signs). If \(e\) becomes much larger we obtain a situation vaguely similar to the superconducting strings of Witten [14]. Indeed in some sense, infinite charge could amount to superconductivity. In Sec. [1] we set up the formalism, and study the asymptotics of our solution, and in Sec. [1] present the full numerical solution. A qualitative discussion of the implications of cosmological networks of such strings is presented in Sec. [17]. Such strings would combine local and global string elements in their evolution and energy loss mechanisms, as well as in their gravitational interaction with the surrounding matter. More distinctly they would generate inhomogeneities in the value of \(e\), leading, among other things, to inhomogeneous recombination scenarios.

Another interesting connection, spelled out in Sec. [17], is the similarity between our solutions and fast-tracks, a construction found in VSL theories [12]. Such solitonic solutions to VSL allow for fast travel without a time-dilation effect. We discuss how the situation is distinctly different in the case of these strings — they still allow...
for fast travel in some sense, however they would induce a time-dilation effect of their own which has nothing to do with the special relativity effect.

The solution derived in Sec. [II] is but the simplest of many similar constructions involving solitonic solutions coupled to varying charge theories. In all of these a gauge field undergoing spontaneous symmetry breaking supplies a solitonic solution which acts as a source for a dielectric field. As a result a dielectric coating is superposed on the soliton, forcing the gauge coupling (or charge) to vary in the soliton core or near its vicinity. In Sec. [IV] we discuss the possibility of gauging the dielectric field itself. In a concluding discussion, in Sec. [VII] we also consider the possibility of non-Abelian gauge groups, and similar constructions with the morphology of monopoles and textures.

II. THE MODEL

We first describe Bekenstein’s theory, in the context of a charged complex scalar field undergoing spontaneous symmetry breaking. Let \( \phi \) be a complex scalar field with a gauged \( U(1) \) symmetry, and \( A_\mu \) be the gauge field. Let the electric charge \( e \) be a variable, with \( \epsilon = e/e_0 \) where \( e_0 \) is some fixed electric charge. Under a gauge transformation \( \delta \phi = i A_\phi \), where \( \Lambda \) is a scalar function, one should impose \( \delta A_\mu = -(\partial_\mu \Lambda)/\epsilon \), so that the derivative \( D_\mu = \partial_\mu + i e A_\mu \) transforms covariantly. The gauge invariant electromagnetic field tensor is then

\[
F_{\mu\nu} = \frac{1}{\epsilon}(\partial_\mu (\epsilon A_\nu) - \partial_\nu (\epsilon A_\mu)) \tag{1}
\]

and the Lagrangian is:

\[
\mathcal{L} = -(D_\mu \phi)^* D_\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\omega}{2\epsilon^2} \partial_\mu \epsilon \partial_\nu \epsilon \tag{2}
\]

(we are using a metric with signature \( -++ \)). The first three terms constitute the matter Lagrangian, while the last term governs the dynamics of \( \epsilon \). We adopt the proverbial Mexican hat potential:

\[
V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4, \tag{3}
\]

with \( \lambda \) and \( m^2 \) < 0 fixed parameters. The vacuum is then the circle \( |\phi| = \phi_0 = \sqrt{-m^2/(2\lambda)} \).

We first introduce a transformation which simplifies Bekenstein’s theory enormously. We note that by defining an auxiliary gauge potential \( a_\mu = \epsilon A_\mu \) and field tensor:

\[
f_{\mu\nu} = \epsilon F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \tag{4}
\]

the Lagrangian becomes

\[
\mathcal{L} = -(D_\mu \phi)^* D_\mu \phi - V(\phi) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{\omega}{2\epsilon^2} \partial_\mu \epsilon \partial_\nu \epsilon, \tag{5}
\]

in which \( D_\mu = \partial_\mu + i e a_\mu \). Hence we have eliminated the dependence on \( \epsilon \) in the matter Lagrangian apart from dividing the \( f^2 \) term by \( \epsilon^2 \). This greatly simplifies the variational problem regardless of which variables we decide to label as physical (which is essentially a matter of convention). Indeed zero variation with respect to \( \{A_\mu, \epsilon\} \) is equivalent to zero variation with respect to \( \{\phi, a_\mu, \epsilon\} \). We have also exposed an interesting similarity between this theory and Brans–Dicke changing-\( G \) theory. In the latter one multiplies the Ricci scalar (essentially a \( f^2 \) term) by a scalar field, which also does not appear explicitly elsewhere (other than in its own kinetic terms or potential).

Variation of (5) with respect to \( \phi \) produces the equation:

\[
D_\mu D^\mu \phi = \frac{\partial V}{\partial \phi^*}, \tag{6}
\]

in which we may use \( D_\mu = \partial_\mu + i e_0 a_\mu \) or \( D_\mu = \partial_\mu + i e A_\mu \). Variation with respect to \( a_\mu \) now produces straightforwardly:

\[
\partial_\mu \frac{f_{\mu\nu}}{\epsilon} \partial_\nu \epsilon = \partial_\mu \frac{F_{\mu\nu}}{\epsilon} = \partial_\mu \left[ i e_0 [\phi^* D^\nu \phi - \phi (D^\nu \phi)^*] \right] \tag{7}
\]

and finally variation with respect to \( \epsilon \) leads to:

\[
\Box \ln \epsilon = -\frac{1}{2} \frac{f^2}{\omega^2 \epsilon^2} = -\frac{1}{2\omega} F^2. \tag{8}
\]

These equations, in the \( \{A_\mu, F_{\mu\nu}\} \) representation, are nothing but Bekenstein’s equations. However the transformation we have used has simplified the derivation greatly, and it will also simplify the search for solutions in what follows.

We now seek solutions similar to the Nielsen–Olesen vortex in this theory. We therefore introduce the ansatz \( \phi = \chi(r)e^{i n \theta} \) and \( a_\theta = a(r) \) with all other components for \( a_\mu \) set to zero. We define a magnetic field out of the \( f_{\mu\nu} \) tensor, so that \( b = \nabla \times a \). Hence the magnetic field is aligned with the \( z \) direction and has value:

\[
b = b_z = \frac{1}{r} \frac{d}{dr} [ra]. \tag{9}
\]

The dynamical equations are:

\[
\frac{1}{r} \frac{d}{dr} \left( r \frac{d\chi}{dr} \right) - \left[ \left( \frac{n}{r} - e_0 a \right)^2 + m^2 + 2\lambda \chi^2 \right] \chi = 0, \tag{10}
\]

which is unmodified, and

\[
\frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} (ra) \right) + 2e_0 \left( \frac{n}{r} - e_0 a \right) \chi^2 = 0 \tag{11}
\]

and

\[
\frac{1}{r} \frac{d}{dr} \left( r \frac{d\ln \epsilon}{dr} \right) = -\frac{1}{2\omega} b_z^2. \tag{12}
\]
To investigate the asymptotic behaviour far from the core, first recall that the scalar field takes on the constant value \( \chi = \sqrt{-m^2/\lambda} \) for \( r \to \infty \). From Eq. (10) we then see that
\[
a = \frac{n}{r e_0},
\]
(13)
which also agrees with Eq. (11). From this we deduce that the flux of \( b \) is quantized:
\[
\int b \cdot dS = \oint a \cdot dl = \frac{2\pi n e_0}{e_0} \quad (14)
\]
but not the flux of \( B = b/\epsilon \) in which \( \epsilon \) takes the role of a magnetic permeability. We also find the asymptotic solution for \( \epsilon \):
\[
\epsilon = \left( \frac{r}{r_0} \right)^{\frac{2}{\omega}}
\]
(15)
where \( I \) is the integral of \( B^2 \) over a string cross section. Hence \( \epsilon \) can only either go to zero (if \( \omega > 0 \)) or infinity (if \( \omega < 0 \)) far away from the string — corresponding to a logarithmic divergence in \( \psi \); not surprising since (12) has sources at spatial infinity (at \( z = \pm \infty \)). Furthermore the energy in the \( \epsilon \) field diverges away from the string. These features are well known properties of global strings. Indeed we have a local or gauged string (in the field \( \phi \)) superposed on a global string (in the field \( \epsilon \)).

In a cosmological setting these seemingly pathological divergences are naturally removed by the scale of curvature of the strings. Then the difference between the asymptotic and core values of the electric charge is roughly of the order of \( (r_c/r_0)^{-I/\omega} \), where \( r_c \) and \( r_0 \) are the curvature and core radii of the string respectively.

If we require that \( \psi \) has a positive definite energy, then \( \omega > 0 \) in which case the charge at the core should be much higher than its asymptotic value. It is in this case that we can claim a similarity between our construction and superconducting strings [14]. In some loose sense a diverging electric charge should be equivalent to superconductivity. Indeed applying an electric field upon a conductor in the interior of this string subjects the charge carriers to a force proportional to \( \epsilon \). Hence the electric force applied to them is much larger than normal. If the resistance to which they are subject does not change, we can then ignore it — and it is in this sense that these strings could maintain persistent currents and therefore be labelled superconducting. Note that this is just a loose analogy: effects such as the expulsion of magnetic fields from the interior of the string are not present in this case.

Our solution also has vague similarities to the dilatonic string of [19], for which the string mass per unit length is a function of a scalar field.
III. NUMERICAL SOLUTIONS OF THE MODEL

Equations (11)–(12), together with the asymptotic values at \( r = 0 \) and \( r = r_c \), constitute a boundary value problem. For the sake of numerically solving this problem, it is convenient to make the following change of variables so as to avoid singularities:

\[
a \to v = ar, \\
\epsilon \to \psi = \ln \epsilon.
\]

We also reduce the problem to first order by introducing the new variables

\[
\sigma = \frac{d\chi}{dr}, \quad \eta = \frac{d\psi}{dr}.
\]

The new set of equations suitable for numerical implementation is

\[
\frac{d\chi}{dr} = \sigma, \\
\frac{d\sigma}{dr} = -\sigma + \left( \frac{n - e_0 v}{r} \right)^2 + m^2 + 2\lambda^2 \chi, \\
\frac{dv}{dr} = br, \\
\frac{db}{dr} = 2\eta b - \left( 2e_0 \frac{n - e_0 v}{r} \chi \right) e^{2\psi}, \\
\frac{d\psi}{dr} = \eta, \\
\frac{d\eta}{dr} = -\frac{\eta}{r} - \frac{1}{2\omega} b^2 e^{-2\psi}.
\]

We first check our code on the Nielsen–Olesen vortex with constant \( \epsilon \). The results for the scalar field and the magnetic field compare well with the original work [13], and are shown as the dashed lines in Figure 1. We then allow the \( \epsilon \) to vary, and incorporate Equations (13)–(15), and are shown as the solid lines in Figure 1.

IV. QUALITATIVE DISCUSSION OF A COSMOLOGICAL NETWORK

Varying-\( \alpha \) strings, if formed at phase transitions, would have a complex evolution. It is conceivable that the string core would still be governed by the Nambu–Goto action. Also, presumably these strings, when crossing, would still intercommute (although this fact should be checked by numerical simulations). However, in addition to intercommuting, the dielectric field would act as a long-range force between the strings, creating a double mechanism for string interaction. Hence we should have something like a local string network, acting as a source for a global string network, the two networks being driven by their usual interaction mechanisms plus a complex interaction between the two.

Energy loss processes would again combine local and global string elements. The core string should develop small scale structure, via intercommutation, thereby emitting gravitational radiation. On the other hand the dielectric field would now supply a channel for the string to lose energy via the emission of scalar radiation. A combination of processes peculiar to local and global strings should therefore push these strings towards a scaling solution, but clearly we may expect such scaling solutions to be distinctly different from the usual ones.

The interactions of these strings and the other matter in the universe would also be rather peculiar. Gravitationally we would find a combination of the effects of local strings (and their conical flat space) and the more complex global string gravitational fields. However, predicting the density fluctuations in this scenario as a simple superposition of global and local fluctuations (that is the total spectrum as a weighted average of the separate spectra) is clearly too gross an approximation. The local and global string networks will be highly correlated, and have a strong feedback effect upon each other. Whatever the gravitational effects of these strings upon the surrounding matter will be, they have to be determined by simulations along the lines of [16,17] specifically applied to varying-\( \alpha \) string networks. Note that unlike conventional super-conducting cosmic strings we would not expect the equation of state of these strings to differ from standard ones.

However what would truly distinguish these strings, and their possible cosmological effects, is the fact that the electric charge varies in their vicinity. For a straight string the electron charge variation away from the core would be given by:

\[
\frac{\Delta e}{e} = \frac{\beta}{\omega \pi} G \mu \ln \frac{r}{r_0},
\]

in which for the \( I \) defined after equation (13) we used \( I = \beta G \mu \). Here \( \mu \) is the string mass per unit length, and \( \beta \) is the fraction of this mass in the magnetic field flux tube. These fluctuations are therefore of the order of the gravitational potential induced by the strings.

Hence, in addition to acting as gravitational seeds for structure formation, these strings would affect any electromagnetic processes in their neighbourhood. A topical example is recombination. In the vicinity of these strings the hydrogen binding energy would suffer spatial variations, leading to inhomogeneous recombination. The implications of a homogeneous changing-\( \alpha \) upon recombination and CMB anisotropy were studied in [18,19]. A network of changing-\( \alpha \) strings would provide additional effects.
V. A COMPARISON OF VARYING-$\alpha$ STRINGS AND FAST TRACKS

Although there is a parallel between the solutions found in this paper and fast-tracks, there is a crucial difference. Fast-tracks are string-like solutions to some covariant VSL theories such that the speed of light is much higher near the string core. Hence observers may move along the string core much faster than the asymptotic value of $c = c_{\infty}$. Moreover such “super-$c_{\infty}$” speeds need not be relativistic, that is, they may still be much smaller than the local value of $c$, so that such observers would not be subject to time-dilation effects. Fast-tracks are what space travel is begging for: fast, “superluminal” travel, free from time dilation. It can be shown that a change of units transforms fast-tracks into wormholes.

In the case of our strings the situation is rather different. As $\alpha$ changes near the string core so will change the time rates associated with all electromagnetic processes. In particular an atomic clock, ticking at a rate $\tau \propto 1/\alpha^2$, will tick differently. Biological processes, being electromagnetic, also tick to this rate. If the charge decreases towards the string core, then alpha is smaller, and so the time scales $\tau$ of all electromagnetic processes increase. Unfortunately this situation is realized in the case $\omega < 0$, for which the dielectric energy density is not positive definite. Nonetheless let us consider further this case.

Using coordinate time we know that “super-$c_{\infty}$” speeds cannot be achieved near the string. However, since $e \to 0$, it would then be possible to “pickle” observers moving along the string, since $\tau \to \infty$. If we were to measure speeds along the string in atomic clock units, then we could indeed measure “super-$c_{\infty}$”:

The point is that natural organisms would be able to travel very large distances within their perceived time scales.

However the use of such strings for space travel would still lead to twin paradox effects: clearly a round trip would cause a large difference in ages between sedentary and nomadic twins. Curiously enough such a time dilation effect has nothing to do with relativistic speeds. It is simply given by

$$\Delta t = \Delta t_0 \int \frac{dt}{\alpha^2}. \tag{26}$$

In this respect the varying-$\alpha$ strings considered here are distinctly different from VSL fast-tracks.

VI. GAUGED DIELECTRIC FIELD

We have noted that the dielectric coating surrounding our modified Nielsen–Olesen vortex is like a global string superposed on the usual gauged string (made up of charged scalar field and a magnetic flux tube). Further symmetry would be enforced if the dielectric itself were charged, that is if we replaced Bekenstein’s real scalar field $\psi = \ln \epsilon$, by a complex field $\psi$, such that $\epsilon = e^{\psi}$. Upon gauging the $U(1)$ symmetry associated with $\psi$ we therefore arrive at the Lagrangian:

$$\mathcal{L} = -(D^\mu \phi)^* D_\mu \phi - V(\phi) - \frac{1}{4} f_{\mu \nu} f^{\mu \nu} \epsilon \omega \left[ (\tilde{D}_\mu \psi)^* (\tilde{D}^\mu \psi) + \frac{1}{4} G_{\mu \nu} G^{\mu \nu} \right], \tag{27}$$

in which $\tilde{D}_\mu = \partial_\mu + igB_\mu$, where $g$ the charge of the dielectric field, $B^\mu$ is the photon associated with the dielectric, and $G_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ the corresponding field tensor. The equations for the scalar field $\phi$ and the gauge field $a_\mu$ remain unaffected, but the equation for $\psi$ is now:

$$\tilde{D}_\mu \tilde{D}^\mu \psi = -\frac{1}{4\omega} f^2 e^{-2\psi} \frac{\psi}{|\psi|} \tag{28}$$

and for $B_\mu$

$$\partial_\nu G^{\nu \mu} = j_\psi^\mu = ig[\psi^* \tilde{D}^\mu \psi - \psi (\tilde{D}^\mu \psi)^*]. \tag{29}$$

Under cylindrical symmetry these equations may be solved using the same ansatz for $\phi$ and $a_\mu$ as before, and the counterparts for the dielectric: $\psi = \xi(r)e^{im\theta}$, and $B_\phi = B(r)$ (with all other components of $B_\mu$ set to zero). The new equations are

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{d\xi}{dr} \right) - \left( \frac{m}{r} - gB \right) \xi + \frac{f^2 e^{-2\xi}}{4\omega} = 0, \tag{30}$$

$$\frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} (rB) \right) + 2g \left( \frac{m}{r} - gB \right) \xi^2 = 0. \tag{31}$$

Given that $f^2$ is confined to the (local) string core, the asymptotics for the new fields are similar. While we still have $\chi = \sqrt{-\frac{\alpha^2}{\alpha^2}}$ and $a = n/(r_{\phi 0})$, for $r \to \infty$, we find that $\xi$ may go to any constant (if $e_0$ is the asymptotic value of the electric charge, $\xi \to 0$), while

$$B = \frac{m}{rg} \tag{32}$$

Overall we find that the dielectric behaves like a local string superposed on the usual Nielsen–Olesen vortex. Its magnetic flux, associated with the gauge field $B_\mu$, is quantized, with a quantum number $m$. This is particularly curious as there is nothing topological in the nature of the dielectric string. Somehow it borrows these features from the topological nature of the $\phi$-string sourcing it. The $\psi$ field does not have a potential, only kinetic terms plus a source at the string. Thus the $\psi$ field can take on any covariantly constant value far away from the string, which amounts to constant $|\psi|$, and a phase equal to $m\theta$.

Notice that none of the points made in Section V, regarding the cosmological implications of local strings coupled to an ungauged dielectric, apply to the strings considered in this section. The variations in $e$ experienced in the surroundings of these strings are confined to
microphysical distances, and have no direct cosmological implications.

It could also happen that $\phi$ and $\psi$ are coupled to the same $U(1)$ gauge field. Then

$$L = -(D^\mu \phi)^* D_\mu \phi - V(\phi) - \frac{f_{\mu\nu}f^{\mu\nu}}{4e^2} - \omega(D_\mu \psi)^* (D^\mu \psi),$$

leading to equations:

$$D_\mu D^\mu \phi = \frac{\partial V}{\partial \phi^*},$$

$$D_\mu D^\mu \psi = -\frac{1}{4\omega} f^2 e^{-2|\psi|} \frac{\psi}{|\psi|},$$

and the gauge-field equation:

$$\partial_\nu \frac{f_{\mu\nu}}{e^2} = j_\nu^\phi + j_\nu^\psi,$$

$$j_\mu^\phi = i e a [\phi^* D^\nu \phi - \phi (D^\nu \phi)^*],$$

$$j_\nu^\psi = i e a [\psi^* D^\nu \psi - \psi (D^\nu \psi)^*].$$

Studying the asymptotics of these equations we find that in this case the quantum number $m$ associated with $\psi$ would have to be the same as $n$. Indeed we have that $\phi = \chi(r)e^{in\theta}$, with $\chi = \sqrt{-m^2/2a}$, and $a = n/(re_0)$; but now we should also have $\psi = \xi(r)e^{in\theta}$ with $\xi(r)$ going to any constant. More generally it could be that the charge of the $\psi$ field is $g = ke$, where $k$ is an integer (or more generally a rational number), in which case $m = kn$.

**VII. CONCLUDING REMARKS**

In this paper we studied the counterpart of the Nielsen–Olesen vortex in Bekenstein’s varying-$\alpha$ theory, by means of analytical asymptotic methods, and numerically. We found that such strings are covered by the dielectric medium characterizing Bekenstein’s theory. This coating, in effect, looks like a global string superposed upon the local string core. The electric charge would thus vary (typically increase) as the string core is approached.

We then discussed possible cosmological implications of such strings. Clearly their networks will be much more complex than just the superposition of a local and a global string network. We pointed out the main aspects in which their dynamics and energy loss mechanisms will be more complex. Structure formation in these theories will also have more to it than just a superposition of results known to be true for the two types of network. In addition we highlighted a peculiar feature of these networks: their ability to generate inhomogeneities in the electric charge, and consequently (among other things) to generate inhomogeneous reionization scenarios.

In a brief section we compared these strings with fast-tracks: solitonic solutions to VSL theories along which fast travel without time-dilation effects is possible. We showed that while in some sense fast travel along these strings is possible, in those cases one cannot evade a time-dilation effect. Curiously enough this time dilation effect is present even if observers do not exceed non-relativistic speeds. It is an effect merely due to the fact that the pace of atomic clocks depends upon $\alpha$, and slows down accordingly near the string core.

Finally, we initiated an exploration of other solitonic solutions in these theories. We considered the possibility that the dielectric field itself might be a gauged. We found that in such a case, even if the dielectric is not endowed with a potential, it acquires topological features, e.g. quantization of its associated magnetic field flux in the string core.

Although in this paper we restricted ourselves to gauged $U(1)$ symmetries it is possible to generalize our constructions to non-abelian symmetry groups. Indeed counterparts to Bekenstein’s theory associated with strong interactions were discussed in [18]. Following such generalizations it would be possible to construct monopoles and textures (associated with $O(3)$ and $SU(2)$ gauge symmetries), covered by similar dielectric coatings. The only type of soliton which apparently could not be associated with changing-charge theories are domain walls, for which there is no associated gauge symmetry. We defer to future work the scrutiny of these more complicated solitons.

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