2D Inversion of Frequency-domain Electromagnetic Data Generated by Line Current Source

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Abstract. The inversion of electromagnetic data is an inherently non-unique and unstable problem due to the data noises and incomplete data coverage, which means different geoelectrical models could fit the observed data at the same level. A stable solution of an ill-posed inverse problem can be obtained by the regularization methods in which some desired structure are imposed to stabilize the inverse problem. We have developed an algorithm to invert frequency-domain electromagnetic data generated by a line current source for a 2D model of the earth. To stabilize the inversion, we adopt a smoothness-constraint approach for the model parameters and adjust the regularization parameter objectively using an adaptive regularization parameter technique. For calculating the sensitivities using any of the accurate methods will be very time-consuming, we applied the adjoint-equation approximate method. By means of the numerical examples using synthetic data, we have demonstrated that the inversion method can be effectively to reconstruct the subsurface resistivity structure.

1. Introduction
The frequency-domain controlled-source electromagnetic method has long been used to obtain information about the subsurface resistivity distribution. In recent years, there have been many efforts and successes in developing efficient modeling and inversion algorithms. Although 2.5D and 3D modeling and inversion algorithms are now available in [1-2] (Mitsuhata and Uchida, 2002; Commer and Newman, 2008), they are still expensive and time consuming. Consequently, many electromagnetic surveys are still being done along a single profile and hence 2D inversion analysis still plays an important role as a primary interpretation tool in real applications.

The finite element method (FEM) and finite difference method (FDM) are the two most common techniques applied to forward modeling of electromagnetic data [3-6], In the case of a simple homogenous medium, the FDM is equivalent to the FEM with linear basis functions. However, the two methods are not equivalent for inhomogeneous models. The FEM is advantageous for frequency-domain electromagnetic problems in geophysics, because it can be easier to implement and compute than the FDM method. Thus, we discuss the FEM for solving the frequency-domain controlled-source electromagnetic forward problem.

Recently, progress in the electromagnetic inverse problem has been made. We present a smoothness-constraint technique, realized by model parameter transformation functions, to address this issue. Thereby, the electrical conductivity updates during the inversion process are restricted such that non-realistic results are suppressed.
The main difference between our inversion method and those previous works is our smoothness-constrained inversion approach applied to solve the 2D frequency-domain electromagnetic data generated by line current source. Firstly, we explain our modeling code which uses the FEM and is designed to deal with the line current source. In order to check the validity of the code, we compare the numerical results with the analytical solutions. Secondly, we develop a smoothness-constrained least-squares inversion method and describe the adjoint-equation method for calculating the sensitivities. Finally, our inversion method in this study is tested on synthetic data to verify its performance.

2. Forward Modeling
The earth model considered in this paper is shown in figure 1. The y-direction is set to be the strike direction, and the resistivity is invariant along the y-axis, varying only in the x-z plane. Assuming a time harmonic dependence of $e^{-i\omega t}$, the electric field about a line current source at the surface of the earth is a TE field [7] (Ward and Hohmann, 1988), described by

$$
\frac{\partial}{\partial x} \left( \frac{1}{io\mu} \frac{\partial E_y}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{io\mu} \frac{\partial E_y}{\partial z} \right) + \sigma E_y = I \delta(x-x_0) \delta(z-z_0)
$$

(1)

where $E_y$ is the y-direction electric field, $I$ is the line current source at the surface, $\delta$ denotes the unit Dirac function, $\omega$ is the angular frequency, $\sigma$ is the spatially variable electrical conductivity, and $\mu (=4\pi \times 10^{-7} \text{H/m})$ is the magnetic permeability of free space.

For the outer boundary conditions, we have adopted infinite boundary conditions, also recognize the electric field has been attenuated to zero in the outer boundary, and this will not significantly affect the results. So equation (1) with the following problem is equivalent

$$
F(E_y) = \iint_{\Omega} \left\{ \frac{1}{2} \frac{1}{io\mu} \left( \nabla E_y \right)^2 - \frac{1}{2} \sigma E_y^2 - 2I \delta(x-x_0) \delta(z-z_0) \right\} dx dz
$$

(2)

where $\Omega$ is the computing domain.

To test the forward modeling algorithm, we applied the code to a homogeneous half-space. For a nonmagnetic homogeneous half-space, the analytical response can expressed as

$$
E_y = \frac{io\mu_0 I}{\pi (k_1^2 - k_0^2) x^2} \left[ ik_0 x K_i \left( ik_0 x \right) - ik_1 x K_i \left( ik_1 x \right) \right]
$$

(3)

where $K_i$ is the modified Bessel function of the second kind of order 1.

3. 2D Inversion
Inverse problem of frequency-domain electromagnetic data is ill-posed and non-linear, which can be solved with some regularization methods. The following parametric functional is minimized in the regularization

$$
P(m) = \phi(m) + \beta \cdot S(m),
$$

(4)

where $\phi(m)$ is a misfit functional and $S(m)$ is a stabilizing functional, and $\beta$ is a regularization parameter which controls the trade-off between these two contributions in a minimization process. Determination of the regularization parameter (Lagrangian multiplier), which balances the minimization of the data misfit and model roughness, may be a critical procedure to achieve both resolution and stability. In our implementation, we adopted the adaptive regularization parameter algorithm, which is defined as
where $k$ is $k$ th iteration. Just as for the the adaptive approach,

4. Numerical Experiment

4.1. First Model

The sizes of the surrounding homogeneous space and the two rectangular blocks are 6000m $\times$ 3000m and 500m $\times$ 500m, respectively. The resistivity of the surrounding half space is 500 $\Omega \cdot m$, whereas the resistivity of the left rectangular body is 1000 $\Omega \cdot m$ and the resistivity of the right rectangular body is 250 $\Omega \cdot m$, as shown in figure 1(a). Synthetic data are generated by finite element method. The mesh used for the inversion consists of 52 columns and 26 rows, giving rise to 1352 blocks to invert for. We choose a homogeneous model of resistivity of 500 $\Omega \cdot m$, locate 25 receivers at the surface at an interval 250m, and use 9 frequencies (0.125, 0.25, 0.5, 2, 4, 8, 16, 32, 64, 128Hz). Figure 1(b) shows the inversion result corresponding to lower and upper resistivity bounds of 320 and 660 $\Omega \cdot m$, respectively. From the inversion result, we can clearly identify a low resistivity anomaly and a high anomaly in the inverted section. It is clear that the smoothness-constrained inversion algorithm for frequency-domain electromagnetic data can be effectively recover simple geo-electrical model.

![Figure 1](image-url)
In the experiment, we evaluate the whole process by inverting the data after adding different levels of noise. The results of the experiment are summarized in Table 1 and we can conclude that this inversion algorithm has worked well in estimating the true noise level.

4.2. Second Model

Figure 2(a) shows a model which is the same as the one tested in [8] (Sasaki, 1989). In this model, a conductive layer of 5 is embedded in a host medium of 50, but this layer is disconnected or faulted over the range approximately 0–2000m in the horizontal coordinate. There also exist one low-resistivity (10) and one higher resistivity (100) body just below the surface. The stations are located every 500m along a profile from -8500m to 8500m in the horizontal coordinate, resulting in 35 sites in total. There are a total number of nine frequencies, which are 0.125, 0.25, 0.5, 2, 4, 8, 16, 32, 64 and 128Hz. Synthetic data are generated also by finite element method. The total number of inversion blocks is generated by 40×28(=1120), Figure 2(b) shows inversion result. The conductive layer and a fault over 7.5–12km are imaged very well for the inversion. From the above model study, it is confirmed that the 2D smoothness-constrained inversion of frequency-domain electromagnetic data can efficiently recover complex subsurfaces.

**Figure 2.** A performance test of 2D smoothness-constrained inversion for the model as same as one tested by Sasaki (1989). (a) The 2D true model. (b) Inversion result for frequency-domain electromagnetic data.
5. Conclusions

We have developed a smoothness-constrained least-squares method applied to solve the 2D frequency-domain electromagnetic data generated by line current source. We believe that the method may be applied equally for large-scale 2D, 2.5D, or 3D frequency-domain controlled-source electromagnetic data. In the numerical examples presented, we have demonstrated that the inversion method can be effectively to reconstruct the subsurface resistivity structure. Further work will be concentrated on extension of this inversion method to the 3D frequency-domain controlled-source electromagnetic data. The implementation of the method to the 3D case, if successful, may be a useful inversion tool in understanding a wide variety of electromagnetic earth phenomena.

Acknowledgments

This project has been supported by the Open Research Fund of Hunan Provincial Key Laboratory of Hydropower Development Key Technology(Grant NO. PKLHD201702) and supported by the Research Foundation of Education Bureau of Hunan Province, China(Grant NO. 19C0562). The authors would like to thank the anonymous reviewers and editors for their helpful comments that led to significant improvement in this paper.

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