Neptune migration model with one extra planet

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ABSTRACT

We explore conventional Neptune migration model with one additional planet of mass at 0.1-2.0 $M_\oplus$. This planet inhabited in the 3:2 mean motion resonance with Neptune during planet migration epoch, and then escaped from the Kuiper belt when Jovian planets parked near the present orbits. Adding this extra planet and assuming the primordial disk truncated at about 45 AU in the conventional Neptune migration model, it is able to explain the complex structure of the observed Kuiper belt better than the usual Neptune migration model did in several respects, which are the following. (1) High-inclination Plutinos with $i \simeq 15^\circ-35^\circ$ are produced. (2) Generating the excitation of the classical Kuiper belt objects, which have moderate eccentricities and inclinations. (3) Producing the larger ratio of Neptune’s 3:2 to 2:1 resonant particles, and the lower ratio of particles in the 3:2 resonance to those in the classical belt, which may be more consistent with observations. (4) Finally, several Neptune’s 5:2 resonant particles are obtained. However, numerical experiments imply that this model is a low-probability event. In addition to the low probability, two features produced by this model may be inconsistent with the observations. They are small number of low-inclination particles in the classical belt, and the production of a remnant population with near-circular and low-inclination orbit within $a \simeq 50-52$ AU. According to our present study, including one extra planet in the conventional Neptune migration model as the scenario we explored here may be unsuitable because of the low probability, and the two drawbacks mentioned above, although this model can explain better several features which is hard to produce by the conventional Neptune migration model. The issues of low-probability event and the lack of low-inclination KBOs in the classical belt are interesting and may be studied further under a more realistic consideration.

Key Words: Kuiper Belt; Planets, migration; Resonances, orbital; Neptune
1. Introduction

Kuiper belt objects (or Edgeworth-Kuiper belt objects) are the relics of primordial planetesimal disk beyond the orbit of Neptune in our solar system. More than one thousand Kuiper belt objects (KBOs) have been observed. Dynamical models for the origin and orbital evolution of the KBOs have been proposed by many authors (For a short review of various dynamical models, see e.g. Lykawka and Mukai, 2008; Gomes, 2009). One of the models is Neptune migration model (Malhøra, 1995; Hahn and Malhotra, 2005), which invokes four migrating Jovian planets in a swarm of primordial planetesimals to explain the present structure of the Kuiper belt. Hahn and Malhotra’s (2005) model is the most updated one and their results can explain many important observed features of the KBOs. However, their model still have several weaknesses, which are (1) In accretion models (see e.g. Kenyon and Luu, 1999; Kenyon, 2002), KBOs were formed in a cluster of small bodies with initially nearly circular and coplanar orbits (eccentricity $e$ and inclination $i \lesssim 10^{-3}$). Hahn and Malhotra’s model assumes an initially dynamically hot disk (the mean value of $e$ and $i \simeq 0.1$). However, the stirring mechanism for these hot particles is unclear; (2) Few high-inclination KBOs were produced by their model. It cannot account for observations, especially for the KBOs at classical belt (the non-resonant KBOs with the semimajor axis $a$ in the range of $37 \lesssim a \lesssim 50$ AU and the perihelion $q \gtrsim 37-40$ AU) and at Neptune’s 3:2 mean motion resonance (MMR); and (3) At the end of their simulation there are plentiful KBOs at Neptune’s 2:1 MMR, which may be inconsistent with observations. The de-bias number ratio between Neptune’s 3:2 and 2:1 resonant KBOs inferred from the observation by Lykawka and Mukai’s study (Lykawka and Mukai, 2007a) is 2.8, but that ratio in Hahn and Malhotra’s simulation is about 0.4. The simulated ratio of the resonant KBOs can be influenced by several processes which were not included in their model, for example, the stochastic effect during planet migration (Zhou et al., 2002), the different migration timescales for the artificial force that drives the migration of Jovian planets (Chiang and Jordan, 2002), and the gas drag during resonance capture (Jiang and Yeh, 2004;
To improve the migration model in respect of the afore mentioned three weaknesses, we employ one additional planet with 0.1 to 2.0 \( M_\oplus \) in the Neptune migration model. As in the Neptune migration model, we start with the four Jovian planets embedded in a swarm of dynamically cold particles, which represent the KBOs. At the beginning of planet migration this extra planet is located at the 3:2 MMR with pre-migrated Neptune, and then due to the resonance capture of Neptune the extra planet migrates outward together with Neptune. Theoretical predictions and observational implications for the existence of the planets with masses of about several tenths to several Earth mass in the early outer solar system have been studied in the literature (e.g., Stern, 1991; Fernández and Ip, 1996). Moreover, the gravitational perturbation of the extra planet with mass of a few tenths to two Earth mass in the early trans-Neptunian space has already been studied in Petit et al. (1999) and Gladman and Chan (2006) for the situation without the planet migration. Petit et al. (1999) studied the perturbation of this planet-size object in the inner region of the Kuiper belt with semimajor axis \( a \) within 30-50 AU region, while Gladman and Chan (2006) explored that in the outer region of the Kuiper belt. Generally speaking, the existence of this massive object excites the primordial disk and produces high-inclination KBOs. Furthermore, in the conventional Neptune migration model, usually there are many particles trapped in Neptune’s 2:1 MMR during migration. By including one extra planet in the 3:2 MMR with Neptune during migration era, the capture rate of Neptune’s 2:1 MMR may be reduced because of the close encounter of the 2:1 resonant KBOs with this extra planet, and because of the overlap between the 2:1 MMR of Neptune and the 4:3 MMR of this extra planet. With the aid of the extra planet, we explore the possibility to improve the afore mentioned weaknesses in the conventional model. In our model we assume that this extra planet leaves the Kuiper belt in a very short timescale near the end of Neptune migration, and we artificially remove it. Otherwise it will deplete most particles and destroy the belt’s structure. Our integration continues to about 0.5 Gyr, when the distribution of all test particles is roughly stabilized.
Recently, Lykawka and Mukai (2008) also studied a putative planet in the Neptune migration model. Their extra planet with $0.3-0.7 \, M_⊕$ was located at Neptune-crossing orbit and has $a \simeq 60-80$ AU before Neptune’s migration. The extra planet was responsible for the excitation of the Kuiper belt and disk truncation at 48 AU before migration era. During or after the end of planet migration it was captured by one distant MMR with Neptune and inhabited in a stable orbit at $a \simeq 100-175$ AU with suitable $e$ and $i$. In this paper, we investigate another possible but different scenario from theirs to explain the Kuiper belt’s structure.

In the following, we use “conventional migration model” and “extra-planet model” to represent the conventional Neptune migration model and our model, respectively. We describe numerical procedures in Section 2. In Section 3 we examine the influence of resonances overlap between Neptune’s and the extra planet’s MMRs on the capture of resonant particles. In order to single out this effect, no migration is adopted. The main simulations of the extra-planet model are described in Section 4. We report the major results of the simulation in Section 5, in which comparisons between simulation results and observations are also presented. In the final part of Section 5, we explore the likelihood of the orbital evolution of the extra-planet invoked in the extra-planet model. The simulation results imply that this model is a low-probability event. We also discuss two issues in the model results, which are lack of low-inclination particles in the classical belt and the production of a remnant population with near-circular and low-inclination at near $a \simeq 50-52$ AU. Conclusions are described in Section 6.

2. Numerical procedures

We used the hybrid symplectic algorithm in the Mercury 6, which is an N-body integrator (Chambers, 1999), to perform our simulations. In the numerical integrations, only gravitational force is involved. Other physical processes such as accretion or fragmentation are neglected. In Section 3 and 4, our solar system consists of the Sun, the four Jovian planets, one extra planet and thousands of test particles with negligible masses. The four Jovian planets and the extra planet
interact with one another but are not perturbed by the test particles. The motion of individual particle governed by the gravitational force of the four Jovian planets and the one extra planet but not by other test particles. The test particles are discarded when colliding with any massive objects or going beyond heliocentric distance of 1500 AU. In our main simulations described in Section 4, the four Jovian planets migrate smoothly under the influence of the artificial force which is the same as that used in Hahn and Malhotra’s study (2005). This artificial force represents the gravity of all test particles. This simplification is supported by a self-consistent simulation of Hahn and Malhotra (1999). Initially the four Jovian planets are posited at a more compact orbits and then forced by artificial velocity kick $\Delta v$ in time step $\Delta t$ with the form

$$\Delta v = \frac{1}{2} \frac{\Delta a}{a} \Delta t e^{-\frac{t}{\tau}} v,$$

which causes the planet’s semimajor axis to vary as

$$a(t) \simeq a_f - \Delta ae^{-\frac{t}{\tau}},$$

where $a$ is the semimajor axis of the planet, $a_f$ is the final semimajor axis of the planet, $\Delta a$ is the planet’s radial displacement, $\tau$ is migration timescale. All simulations in Section 3 and 4, a time step of 0.5 year is used\(^{1}\), whose adequacy is discussed by Hahn and Malhotra (2005).

3. Numerical examination for resonances overlap

In the extra-planet model the orbital configuration of Neptune, the extra planet and the Twotinos (the particles in the 2:1 MMR with Neptune) forms a three-body mean motion resonance, which satisfies the relation of $n_N - 3n_E + 2n_T \simeq 0$, where $n_N$, $n_E$ and $n_T$ are the mean motion of Neptune, the extra planet, and the Twotinos, respectively and $n_N : n_E : n_T = 6 : 4 : 3$. The orbits of the three-body mean motion resonance are usually chaotic (Murray et al., 1998; Nesvorný and Morbidelli, 1998a; Nesvorný and Morbidelli, 1998b). Therefore, one additional planet located

\(^{1}\)In Section 5.3.2, we use 1-year time step to reduce computing time for the simulations containing massive particles.
at Neptune’s 3:2 MMR during planet migration may reduce Neptune’s 2:1 resonant particles due to the resonance overlap between the 2:1 MMR of Neptune and the 4:3 MMR of the extra planet. The main purpose of this section is to examine this resonance overlapping effect numerically. To simplify the situation no planet migration is employed here. All runs in this section include four Jovian planets, 1000 test particles in the 3:2 MMR and 1000 test particles in the 2:1 MMR with Neptune. The present orbits are used for the initial orbital elements of the four Jovian planets (obtained from the JPL HORIZONS system). The initial particles are uniformly distributed with semimajor axis \( a \in (38.9, 40.1) \) AU and \( (47.2, 48.4) \) AU for Neptune’s 3:2 and 2:1 MMR respectively, and have eccentricity \( e \) and inclination \( i \) following the Rayleigh distribution with mean value \( \langle e \rangle = 0.1 \) and \( \langle \sin i \rangle = \langle e \rangle / 2 \). The argument of pericentre \( \omega \), the longitude of the ascending node \( \Omega \) and the mean anomaly \( \lambda \) are randomly chosen between \( 0^\circ \) and \( 360^\circ \). Three runs among them include one extra planet with mass 0.1, 0.5 or 1.0 \( M_\oplus \) at the beginning of the integration, and one run without one extra planet is for comparison. One body in Neptune’s \( j + k : j \) MMR is defined as the body having librating resonant angle \( \phi \) around some fixed value and with amplitude \( \Delta \phi \); \( \phi \equiv (j + k) \lambda - j \lambda_N - k \varpi \), where \( j , k \) are integer, \( \lambda \) and \( \lambda_N \) are the mean longitude of the body and Neptune, respectively, and \( \varpi \) is the body’s longitude of perihelion. Initially the extra planet is set at Neptune’s 3:2 MMR \( (a = 39.5 \) AU) with zero \( e \) and \( i \); \( \omega , \Omega \) and \( \lambda \) are chosen such that this planet has resonant amplitude \( \Delta \phi \leq 110^\circ \), which provides a more stable state at Neptune’s 3:2 MMR (Levison and Stern, 1995; Nesvorny and Roig, 2000). The destruction of the structure of Neptune’s 3:2 MMR due to the perturbation of the extra planet can be demonstrated by 3:2 resonant particles. It gives a constraint on the duration of the extra planet’s stay in the 3:2 MMR with Neptune. For each run the total integration time is 8.25 Myr \( (= 5 \times 10^4 T_N \), where \( T_N \) is the orbital period of the present Neptune and is 165 yr). This integration time is much longer than the typical libration periods of Neptune 3:2 and 2:1 MMRs, which are about 100-1000 \( T_N \).

Fig. 1 shows time-averaged semimajor axis \( \langle a \rangle \) and eccentricity \( \langle e \rangle \) over a time
interval of 0.825 Myr for the test particles and the extra planet at the end of the integration. Figs. 1a-1d plot the one run without extra planet and the three runs including one extra planet with the mass of 0.1, 0.5 and 1.0 $M_{\oplus}$ respectively. We define the resonant particles to be those having resonant amplitude $\Delta\phi \leq 170^\circ$. This criterion avoids to identify most of non-resonant particles as the resonant ones due to insufficient time-sampling. At the end of the simulation, there are 15, 35, 205 and 214 Neptune’s 2:1 resonant particles for the runs containing one extra planet of 1.0 $M_{\oplus}$, 0.5 $M_{\oplus}$ and 0.1 $M_{\oplus}$, and without extra planet, respectively. These simulation results indicate that the 2:1 resonant particle number decreases as the mass of the extra planet increasing. This tendency is because the particles near Neptune’s 2:1 MMR had close-encounters with the extra planet, and because of the overlap between Neptune’s 2:1 and the extra planet’s 4:3 MMR. Both of the effects are intensified as the mass of the extra planet increases. Below the equal-perihelion line of 40.4 AU in the $\langle a \rangle$-$\langle e \rangle$ diagram (the curved dashed line in Fig. 1) Neptune’s 2:1 resonant particles suffered almost no close-encounter with the extra planet, but the 2:1 resonant particle number was still depressed as the mass of the extra planet increases. This feature displays the resonance overlapping effect more. As for Neptune’s 3:2 resonant particles in the runs containing one extra planet, all of them had close approach with this extra planet during the integration, thus the decrease of Neptune’s 3:2 resonant particles is expected. When the integration time extended to several tens of Myr for the 1.0-$M_{\oplus}$ extra-planet run, there was almost no test particle at or near Neptune’s 3:2 MMR due to the extra planet’s strong perturbation. To clarify the MMRs overlap further we perform another run for the 1-$M_{\oplus}$ extra-planet case where the initial semimajor axis of the extra planet is shifted by 1 AU farther beyond the exact 3:2 MMR of Neptune. This selection avoids the first-order MMRs of the extra planet overlapping with Neptune’s 2:1 MMR. The $\langle a \rangle$ and $\langle e \rangle$ at the end of this simulation are demonstrated in Fig. 1e. At the end of the simulation, there are 53 particles trapped in Neptune’s 2:1 MMR. It is more than those in Fig. 1d, where Neptune’s 2:1 MMR captures merely 15 particles, although the extra planet is closer to Neptune’s 2:1 MMR in Fig. 1e. Moreover, below the
equal-perihelion line of 40.4 AU the 2:1 resonant particles are more abundant in Fig. 1e than those in Fig. 1d. Therefore we conclude that the overlap of the 2:1 MMR of Neptune and the 4:3 MMR of the extra planet can reduce the resonant particles of Neptune due to the long-term gravitational perturbation of this extra planet. Reducing the resonant KBOs at Neptune’s 2:1 MMR can be achieved by setting one extra planet at Neptune’s 3:2 MMR during Neptune’s migration. It may therefore improve the conventional migration model. In the next section we explore this picture with planet migration.

[Figure 1]

4. Planet migration with one additional planet

4.1. Short-term (82.5 Myr) simulations

In this section we describe simulations with the planet migration. The semi-major axes of the four Jovian planets vary as Eq. (2). The initial semimajor axis and the radial displacement \(\{a_0, \Delta a\}\) for Jupiter, Saturn, Uranus and Neptune are \(\{5.4 \text{ AU}, -0.2 \text{ AU}\}\), \(\{8.7 \text{ AU}, 0.8 \text{ AU}\}\), \(\{16.2 \text{ AU}, 3.0 \text{ AU}\}\) and \(\{23.2 \text{ AU}, 7.0 \text{ AU}\}\), respectively. These values have been inferred and adopted in several papers (Malhotra, 1995; Hahn and Malhotra, 1999; Chiang and Jordan, 2002; Lykawka and Mukai, 2008). All other orbital elements for the Jovian planets are the same with the present values (adopted from the JPL HORIZONS system). The migration timescale \(\tau\) we adopted is 10 Myr (Hahn and Malhotra, 1999). In addition to the four Jovian planets, one extra planet is set at Neptune’s 3:2 MMR at the beginning of the planet migration. The eccentricity and inclination of this extra planet are arbitrarily chosen but are similar to those of the four Jovian planets. Other orbital elements \(\omega, \Omega\) and \(\lambda\) are chosen such that the extra planet has smaller resonant amplitude \(\Delta \phi\), which is usually less than 110° during our integrations. We also include 4000 test particles initially distributed from \(a = 24.0 \text{ AU}\) to 50.0 AU with the

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2To know the influence of the time-sampling on resonant particle identification, we constructed a toy model using a sinusoidal curve to model particle’s resonance libration. In this paper we define the particle with resonant amplitude \(\Delta \phi \leq 170°\) as the resonant particle. The toy model indicated that this selection recognizes \(\lesssim 1\%\) non-resonant particles as the resonant ones.
surface number density varied as $a^{-2}$. The $e$ and $i$ of particles follow the Rayleigh distribution with mean value $\langle e \rangle = 0.001$ and $\langle \sin i \rangle = \langle e \rangle / 2$. The $\omega$, $\Omega$ and $\lambda$ of the particles are randomly chosen within 0 to 360°. To clarify that our final simulation results and conclusions do not depend on the particular choice of initial particle set and extra planet, we select five particle-set a, b, c, d and e which combine with five different extra-planet A, B, C, D and E, respectively. The initial semimajor axis, eccentricity and inclination in the form of $(a, e, i)$ for the extra-planet A, B, C ,D and E are $(31.0\ AU, 0.019, 3.25^\circ)$, $(30.6\ AU, 0.071, 1.50^\circ)$, $(30.6\ AU, 0.029, 1.96^\circ)$, $(30.9\ AU, 0.016, 1.94^\circ)$ and $(30.9\ AU, 0.029, 2.42^\circ)$, respectively. For each combination of one particle set and one extra planet, we perform three runs for the extra planet with mass of $0.1\ M_\oplus$, $0.5\ M_\oplus$, $1.0\ M_\oplus$. One run without extra planet is also executed. Thus, we totally execute twenty runs and the integration time for each run is $82.5\ Myr = 5 \times 10^5\ T_N$, which is $\simeq 8\tau^3$. For all the simulations described in Section 4, the $\langle a \rangle$, $\langle e \rangle$ and $\Delta \phi$ are calculated over a time interval of $8.25\ Myr = 5 \times 10^4\ T_N$.

Fig. 2 plots an example for the semimajor axes of the four Jovian planets and one extra planet varying with time in smooth planet migration, where the extra-planet A has the mass of $1.0\ M_\oplus$. The four giant planets were driven by the artificial force while the extra planet was trapped by the 3:2 MMR with Neptune and moved outward. The resonant particle numbers varying with time for the 3:2 and 2:1 MMRs with Neptune in the runs containing the particle-set a are shown in Fig. 3. In this figure we plots the three runs with the extra planets of different masses and the one run without extra planet. Roughly speaking, during the significant migration era of about 0-20 Myr resonant particle numbers increased due to resonance capture mechanism. As Neptune parked near 30 AU, many unstable resonant particles left the MMRs gradually after about 20 Myr. Furthermore, there are several noteworthy features in this figure, (1) in the no extra-planet run, the time for the resonant particle numbers reaching the maximum value is late for the 2:1 MMR compared to that of the 3:2 MMR. This phenomenon has been mentioned in the study of Zhou et al. (2002); and (2) in Fig. 3, the humps of the 2:1 and 3:2 resonant particles are
depressed as one extra planet contained in the simulations. For the runs involving one extra planet with the masses of 0.5 $M_\oplus$ and 1.0 $M_\oplus$, the particle numbers trapped in the 2:1 MMR are always less than ten. Comparison with conventional migration model result (the run without one extra planet), the maximum resonant particle numbers of the 3:2 MMR reduce to 5.0%, 5.6% and 42% for the runs containing one extra planet of 1.0, 0.5 and 0.1 $M_\oplus$, respectively. Similar quantities for the 2:1 MMR are 0.77%, 0.66% and 38%. With the existence of the extra planet fewer particles were captured in the 2:1 MMR with Neptune because the MMRs overlapped between the 2:1 MMR with Neptune and the 4:3 MMR with the extra planet, and Neptune’s 2:1 resonant particles had close-encounters with the extra planet. The decrease for the capture rate of Neptune’s 2:1 MMR is more than that of Neptune’s 3:2 MMR as the mass of the extra planet increases. Other sixteen runs with different initial extra planets and particle sets gave similar features. Hence, We conclude that in the conventional migration model the ratio of Neptune’s 2:1 to 3:2 resonant particles can be decreased due to the presence of one additional planet, which may improve the conventional migration model. This extra planet must escape during or near the end of the planet migration. Otherwise the structure of Neptune’s 3:2 MMR would be destroyed as a result of extra planet’s perturbation. We assume that this extra planet leaves the Kuiper belt in a very short timescale. We remove it by hand and then continue the integration to about 0.5 Gyr in the next subsection.

[Figure 2, Figure 3]

4.2. Long-term (495 Myr) simulations

From the previous twenty short-term runs with about eight times migration timescale, we have understood the time variation of the resonant particles. Among the twenty runs we reproduce fifteen runs containing one extra planets, and then remove the extra planet by hand in each run at certain times during or near the end of the Neptune migration. After the removal of the extra planet, the integrations

3All the extra planets in these 82.5 Myr short-term runs were captured in the migrating Neptune’s 3:2 MMR, except the run contains the extra-planet B with the mass of 0.1 $M_\oplus$. In this run the extra planet escaped from the 3:2 MMR of Neptune at about 23 Myr.
are continued to 495 Myr (= $3 \times 10^6 T_N$). Based on Fig. 3, the particular times that we adopt to discard one extra planet are 8.25, 24.75, 33.0 and 49.5 Myr (which are $5 \times 10^4$, $1.5 \times 10^5$, $2 \times 10^5$, $3 \times 10^5 T_N$, respectively). In addition, we also extend the remaining five runs without extra planet to 495 Myr for comparison. We will discuss the results for the conventional migration and the extra-planet model in Section 5.

5. Results and discussion

5.1. Results of the 495 Myr integration and truncated disk

Taking the run containing the particle-set a as an example, we removed the $1-M_{\oplus}$ extra planet at 33 Myr, which is roughly near the end of the migration era, and expect that the ratio of Neptune's 2:1 to 3:2 resonant particle numbers will be reduced in the end of the 495 Myr simulation comparing to the conventional model results. The right panel of Fig. 4 shows the numbers of the resonant particles varying with time for this long-term integration. The left panel of Fig. 4 is just an enlarged of part of Fig. 3 for the run containing the $1.0-M_{\oplus}$ extra planet trapped in the 3:2 MMR with Neptune until 82.5 Myr integration. After removal of this additional planet, Neptune's 2:1 resonant particle numbers increased unexpectedly in several tens Myr and were in excess of the 3:2 resonant particle numbers eventually at the end of 495 Myr (the right panel of Fig. 4). The abrupt increase of the particle number in the 2:1 MMR with Neptune is because that at the time we removed the extra planet the test particles around Neptune's 2:1 MMR were abundant. As a consequence, those particles were captured by the 2:1 MMR immediately. The top panels of Fig. 5 demonstrates the orbital distribution of the test particles at the end of 495 Myr for this simulation, and the bottom panels shows those for the run without extra planet. The observed KBOs with multi-opposition are also plotted in Fig. 5. In the extra-planet model results, the excitation of initially cold test particles due to the perturbation of $1-M_{\oplus}$ extra planet is apparent in $a-e$ and $a-i$ diagrams. But there are still plentiful resonant particles in Neptune's 2:1 MMR.

[Figure 4, Figure 5]

Inspecting the results more carefully, we find other discrepancies from the results
in the extra-planet model and those in the traditional migration model. On the $a$-$e$ distribution of Fig. 5 we label the test particles according to their initial semimajor axes (Fig. 6). The colors in the $a$-$e$ diagram of the conventional migration model (the right panel of Fig. 6) show that the 2:1 MMR with Neptune contains the particles with initial semimajor axes among 35-50 AU. However, in the $a$-$e$ diagram of the extra-planet model (the left panel of Fig. 6) most particles in the 2:1 MMR have initial semimajor axes $a$ within 45-50 AU. As predicted in resonance capture theory, the eccentricities of test particles captured in 2:1 MMR get larger as the semimajor axes increase during Neptune’s migration. Therefore, in the right panel the 2:1 resonant particles with high eccentricities originated from interior region and those with low eccentricities originated from exterior region. In the left panel of Fig. 6, this feature is not clear as one extra planet with $1-M_⊕$ mass is involved in the model. Most of the 2:1 resonant particles originated from the primordial disk of 45-50 AU. In the 2:1 MMR of the extra-planet model, the absence of particles with the initial semimajor axis $a$ smaller than 45 AU is due to following causes. First, the resonances overlapped at the 2:1 MMR with Neptune and the 4:3 MMR with the extra planet during the migration era, which reduced the capture rate of the MMR as we demonstrated in Section 3. Second, some parts of the Neptune’s 2:1 resonant particles were destroyed by the close-encounters between the extra planet and the particles. Fig. 6 indicates a possible way to reduce the particles in Neptune’s 2:1 MMR under the scenario of the extra-planet model. The observed KBOs indicates an edge with semimajor axis $a$ between 45-50 AU (Trujillo and Brown, 2001a). In the extra-planet model if we truncate the initial particle disk at 45 AU, the 2:1 resonant particle number becomes small naturally (Fig. 7). During the migration epoch, the 2:1 MMR with Neptune shifted through the region of $a \approx 36$ to 47 AU. In the conventional migration model, many particles within the above region were trapped by the 2:1 MMR and finally there are plenty of 2:1 resonant populations at the simulation end. However, in the extra-planet model with the primordial disk truncated at 45 AU, the presence of the extra planet prevented most capture events in the 2:1 MMR when the 2:1 MMR passed through $a \approx 36$-47 AU in the first several
tens Myr of Neptune’s migration. With the assumption of the primordial disk’s edge, we can compare the results of the extra-planet model with the observations. It provides some constraints on this model.

[Figure 6, Figure 7]

5.2. Comparison with observed KBOs

5.2.1. $a$-e and $a$-i distributions

Fig. 7 shows that the simulation results at the end of 495 Myr for the runs containing the particle-set a in the extra-planet model and the conventional migration model with the initially truncated disk at 45 AU. The extra planet was removed at 33 Myr in the extra-planet model run. At about 0.5 Gyr the region beyond Neptune is roughly stabilized. Hence, we compare simulating particles distribution with those of the present KBOs. Observed KBOs with multi-opposition are plotted in Fig. 7. At first glance compared with the results of the traditional migration model (the bottom panels of Fig. 7), the particle orbital distributions of the extra-planet model (the top panels of Fig. 7) are more consistent with observations in several respects, (1) the classical belt has notable excitation in $a$-$e$ and $a$-$i$ spaces due to the perturbation of the extra planet during the migration era; (2) the final 2:1 resonant KBOs number is few due to the MMRs overlap, the closes-encounters during the migration epoch and the assumption of the primordial disk edge; and (3) the inclinations of the Plutinos (the KBOs at the 3:2 MMR with Neptune) distribute in a very wide range with maximum value $\simeq 30^\circ$. These three features can be seen in the other runs with extra planet having the same mass and the same escaping time. One main discrepancy between the extra-planet model results and the observations is that there are only few classical KBOs with inclination $i \lesssim 5^\circ$ (the top-right panel of Fig. 7). Most telescopes observed the KBOs near ecliptic. The probability of the detection of the KBOs at the ecliptic is roughly proportional to $1/\sin i$. Therefore, low-$i$ KBOs spend more time near the ecliptic and are more easily observed by telescopes than high-$i$ KBOs. The comparison of KBO inclination distribution between simulation’s and the observation’s should consider this telescopic selection effect.
Fig. 8 shows the ecliptic inclination distribution for the simulated particles and the observed KBOs. The ecliptic inclination distribution is the inclination distribution of particles with latitude $\beta$ near the ecliptic, which can mitigate above selection effect (Brown, 2001; Hahn and Malhotra, 2005). We choose particles with $\beta \leq 3.0^\circ$ and perihelion $q \leq 45.0$ AU. As previous understanding of the conventional migration model, it produces deficient high-inclination KBOs with inclination $i \gtrsim 5^\circ$ (the right panels of Fig. 8). In addition, the simulated particles with inclination $i \lesssim 5^\circ$ are too many. This is mainly due to the initially dynamically cold particles. As for the ecliptic inclination distribution of the extra-planet model, the simulated particles can cover most of observed high-inclination KBOs with $i \gtrsim 5^\circ$. Nevertheless, the simulated particles with $i$ within $10^\circ$-$20^\circ$ are a little more than the observed KBOs, and the low-inclination KBOs with $i \lesssim 5^\circ$ are few in the extra-planet model’s results.

[Figure 8]

Under the picture of the extra-planet model with one escaping planet from Neptune’s 3:2 MMR and an initial disk truncated at $a \approx 45$ AU, we can roughly constrain the mass and the removal time of this extra planet from the observational KBOs’ orbital distribution. Among all our runs, the model employing 1-$M_\oplus$ extra planet with the removal time near the end of the Neptune migration (33.0 Myr) gives the most consistent final particle distribution, because of following several reasons. (1) For the simulations with one extra planet of the same mass (1 $M_\oplus$), the extra planet inhabiting in the 3:2 MMR too long (0-49.0 Myr) consumed most of the 3:2 resonant particles and produced particles with too high inclination for the ecliptic inclination distribution, while it inhabiting in the 3:2 MMR too short (0-8.25 Myr) had insufficient perturbation of the Kuiper belt. (2) As for the simulation with the same removal time for the extra planet but with different masses, the excitation of the Kuiper belt was not adequate for the model with lower mass extra planet (0.1 and

\footnote{In the simulation end few particles orbited near the ecliptic in the extra-planet model results. In order to enhance the statistic meaning in the ecliptic inclination distribution, we adopted the particles with $\beta \leq 3.0^\circ$ to plot the ecliptic inclination distribution. The choice of the perihelion 45 AU roughly equals to observation limit.}
0.5 $M_\oplus$). We perform another run with 2.0-$M_\oplus$ extra planet to 82.5 Myr. At 33.0 Myr there were almost no classical KBOs with $i \leq 6^\circ$, which cannot account for the observations. (3) Increasing the extra planet’s mass and postponing its escaping time simultaneously overly excited the belt, while decreasing the extra planet’s mass and removing it earlier together had inadequate perturbation on the belt. (4) The model including the extra planet having larger mass (2.0 $M_\oplus$) with shorter inhabited time in the 3:2 MMR avoided depleting all low-inclination classical KBOs, however its 2:1 MMR with Neptune captured too many particles during the migration era after the discard of the extra planet. (5) The model with a lower mass extra planet (0.1 and 0.5 $M_\oplus$) cannot account for high-inclination Neptune’s 3:2 KBOs, even if the integration time was extended to 82.5 Myr ($\simeq 8\tau$). At this time, Neptune’s 3:2 resonant particles became too few. Hence, it is inconsistent with observations.

In the previous discussion we only posited the extra planet in Neptune’s 3:2 MMR at the beginning of the Neptune migration. We also attempt to put this extra planet in the 2:1 MMR with Neptune, which is another strong MMR of Neptune in the Kuiper belt. We do three more runs, where this extra planet is at Neptune’s 2:1 MMR when the Neptune’s migration starts. The same as previous runs we remove the extra planet at or near the end of the migration and then integrate to 495 Myr. The main drawback of this configuration is that at the simulation end the dearth of the Plutinos of inclination $\geq 27^\circ$ is obvious in all the three runs. It is inconsistent with the observations. Therefore, we think that setting one extra planet at Neptune’s 2:1 MMR is unlikely.

Base on the above discussion, according to the $a$-$e$, $a$-$i$ diagrams, ecliptic inclination distribution and Neptune’s 3:2 to 2:1 resonant particles’ ratio, the mass and the removal time of the extra planet are about 1.0 $M_\oplus$ and near the end of the migration ($\sim 33$ Myr). In our simulations, we merely performed several sparse points in a large parameter space for the mass and the removal time of the extra planet. Therefore, these two quantities can be only crudely determined. In the following subsections we will compare more simulation results of the runs adopting the extra planet with 1.0-$M_\oplus$ mass and 33 Myr removal time with the observation KBOs.
5.2.2. Plutinos and other resonant KBOs

In Lykawka and Mukai’s (2007a) identification of observed KBOs, there are 100 Plutinos among 622 KBOs. The number of the Plutinos provides a large fraction among total KBOs’ population. Therefore, creating Plutinos with similar orbital distributions and physical properties to observations is an important task to the theoretical models. Fig. 9 demonstrates simulated time-averaged eccentricities $\langle e \rangle$, time-averaged inclinations $\langle i \rangle$ and resonant angles $\Delta \phi$ for the Plutinos. Those quantities for observed Plutinos are also plotted in this figure (from Lykawka and Mukai, 2007a). The main discrepancy between the simulated results of the conventional migration model and those of the extra-planet model is that the extra-planet model produced many high-inclination Plutinos with inclination between $15^\circ$-$35^\circ$, which usually cannot perform by the conventional migration model (the top two panels of Fig. 9). These high-inclination Plutinos were generated due to the perturbation of extra planet and gives a more consistent $\langle e \rangle$-$\langle i \rangle$ distributions with the observations. In the bottom two panels of Fig. 9, some parts of the simulated Plutinos of the extra-planet model possess $\Delta \phi \geq 130^\circ$ that ought to be insufficient integration time in our simulation (Nesvorny and Roig, 2000). Generally, the orbital properties of the Plutinos generated from the extra-planet model are agreeable to the observations.

As for the ratio of Neptune’s 3:2 to 2:1 resonant particles, the average value of this ratio provided by the five runs of the extra-planet model is 2.0. The same quantity provided by the five runs of the conventional migration model is 0.44. Therefore, the ratio calculating from the extra-planet model is about five times larger than that of the conventional migration model, and is more consistent with the observations ($\sim 2.8$). Although in our simulations we assumed that migration process is artificially smooth, more realistic migration may not change this conclusion.

Another important ratio is the Plutino number to the number of the classical KBO. In the simulation end, we consider the particles having perihelion $q \geq 37$ AU and $a \leq 50$ AU as the classical KBOs, except those belong to the 3:2, 5:3, 7:4 and 2:1 MMRs. The average ratio of the Plutino to the classical KBO numbers is 1.5 for the five runs of the conventional migration model, while the same ratio for the
The extra-planet model is 0.098, which is closer to a de-bias observational ratio that is about 0.04 (Trujillo and Jewitt, 2001b).

Beyond $a = 50$ AU there are still abundant observed KBOs inhabiting in higher order MMRs with Neptune, e.g. the 7:3, 5:2, 3:1 and 4:1 MMRs. From Lykawka and Mukai’s (2007a) study, among 622 KBOs there are 12 KBOs located in the 5:2 MMR with Neptune, which is the major resonant population outside $a = 50$ AU. These distant resonant populations can be captured more easily during the Neptune migration if they were dynamically hot before resonance capture (Chiang et al., 2003; Hahn and Malhotra, 2005; Lykawka and Mukai, 2007b). In the extra-planet model, the perturbation of the extra planet supplies the excitation for particles in eccentricities and inclinations prior the resonance capture. Fig. 10 demonstrates that in the five runs of the extra-planet model, four runs have several Neptune’s 5:2 resonant particles (left two panels), while only one among the five runs of the conventional migration model produces 5:2 resonant particles (right two panels). Therefore, more particles were trapped at Neptune’s 5:2 MMR in the extra-planet model than those in the conventional migration model. Generally, the behaviors of simulated 5:2 resonant particles of the extra-planet model are similar with the observations, except the inclination distribution of the run containing the particle-set a (the crosses in the top-left panel of Fig. 10). For this run the 5:2 resonant particles’ inclinations are within $20^\circ$-$35^\circ$ which is higher than the observations. One possible explanation is that the observations prefer low inclination objects as we mentioned previously. For circular orbit objects, the detection probability near the ecliptic is about three times higher for the objects with $i = 10^\circ$ than for the objects with $i = 30^\circ$. One run of the conventional migration model also provided 5:2 resonant particles but their resonant amplitudes are all larger than $120^\circ$. None of the runs of the conventional migration model produced 5:2 resonant particle with small resonant amplitude, which has more stable orbit.

[Figure 9, Figure 10]
5.2.3. Extended scattering KBOs

Usually extended scattering KBOs have $a \gtrsim 50.0$ AU and $q \gtrsim 40.0$ AU (definition varies slightly in literature). These objects do not suffer close-encounters of the giant planets and form one class of KBOs. From the Minor Planet Center database (July, 10, 2007), among 717 observed KBOs ($a \geq 30$ AU) with multi-opposition there are 7 extended scattering KBOs, e.g. 2004 $XR_{190}$ ($a = 57$ AU, $q = 51$ AU), 2000 $CR_{105}$ ($a = 218$ AU, $q = 44$ AU) and Sedna ($a = 487$ AU, $q = 76$ AU). A biased abundance of this class KBOs is $7/717 \simeq 1\%$. The intrinsic ratio of the extended scattering KBOs should be larger than this value because of the larger distant of these objects. From the study of Gladman and Chan (2006), emplacing an Earth mass plant in the Kuiper belt promotes the production of the extended scattering KBOs with $50$ AU $\lesssim a \lesssim 500$ AU within several hundred Myr due to the secular perturbation of this extra planet. In the extra-planet model results, merely several particles have orbits like 2004 $XR_{190}$ with $50$ AU $\lesssim a \lesssim 80$ AU (Fig. 13). The absence of objects with orbits like 2000 $CR_{105}$ or Sedna is mainly due to that we assumed our extra planet escaping quickly near the end of the Neptune migration. Hence, the extra planet had insufficient time to influence the Kuiper belt. That the extra planet stay in the Kuiper belt more than several tens Myr would destroy the stability of Neptune’s main mean motion resonance, for example, the 3:2, 5:3 and 2:1 MMRs. Therefore, we may not invoke the picture in Gladman and Chan (2006) to produce 2000 $CR_{105}$-like or Sedna-like extended scattering KBOs in our model. Other possible mechanisms may be responsible for the construction of the extended scattering KBOs, including stellar passage (Ida et al., 2000; Kenyon and Bromley, 2004) or a distant undiscovered planet (Lykawka and Mukai, 2008).

5.2.4. Outer edge of Kuiper belt

In the extra-planet model we assume the primordial planetesimal disk with an outer edge near 45 AU in order to decrease Neptune’s 2:1 resonant particles in our scenario. Another motivation is to have a more consistent $a$-$e$ distribution of the KBOs between model results and observations. The observed $a$-$e$ distribu-
tion implies an edge at $\simeq 45$ AU. In the migration era of our model, the extra planet’s semimajor axis and eccentricity increased simultaneously through migration. Although this extra planet always inhabited at the 3:2 MMR, some initially cold particles within $40 \text{ AU} \lesssim a \lesssim 45$ AU were still disturbed by high-eccentric migrating extra planet. This causes that some particles moved outward beyond $a \simeq 45$ AU during extra planet’s migration. The extra planet had the final aphelion $\simeq 48$ AU at the migration end and cleaned most of particles with near circular orbits within $45 \text{ AU} \lesssim a \lesssim 50$ AU. Therefore, there is a group of nearly circular particles which still assemble within $a \simeq 50-52$ AU (see the right panels of Fig. 11). We will discuss whether this remnant creating by the extra-planet model could be observed in Section 5.3.4.

[Figure 11]

5.3. Discussion

5.3.1. Assumption of truncation disk

In the extra-planet model, we assume the primordial disk having an edge at about 45 AU. Several processes may produce this edge prior to planet migration (see Gomes et al., 2004 and references therein), for example, a passing star or nearby stars photoevaporated.

5.3.2. Removal of extra planet

In our simulation the extra planet was discarded artificially. We postulate that it leaves the Kuiper belt within a short timescale less than several Myr and does not influence the KBOs anymore. We execute 500 runs to investigate the possibility of this event. In each run we merely consider the Sun, four Jovian planets, and one 1-$M_\oplus$ extra planet at pre-migrated Neptune’s 3:2 MMR with zero $e$, $i$ and different $a$ within 0.6 AU of exact resonance position. Other three orbital angles are randomly chosen between $0^\circ$ and $360^\circ$. Test particles are not included. Other orbital configurations for the Jovian planets and integration time are the same with the runs in Section 4.1. After 82.5 Myr integration, there are 486 survived extra planets, which are all have perihelion $q \leq 50$ AU and will still influence KBOs.
Within 82.5 Myr, only 13 extra planets escaped from the solar system due to the close approach of one or multi Jovian planets, and the remaining one collided with Jupiter. The resonant amplitudes $\Delta \phi$ of these 14 objects are all larger than $90^\circ$ in the early stage of migration. Therefore, in the extra-planet model if we invoke an extra planet leaving the Kuiper belt quickly near the end of the Neptune migration, the probability is less than $14/500 \approx 0.03$.

The assumption of particles with negligible mass in the our simulation loses gravitational effect on the extra planet. A $1-M_\oplus$ mass extra planet embedded in a primordial planetesimal disk with mass of several tens $M_\oplus$ would suffer the dynamical friction, which circularizes the orbits of the extra planet and may reduce the chance to scatter the extra planet by Jovian planets. Furthermore, considering massive particles renders the stochastic migration, which decreases the capture probability for the extra planet during the migration (Zhou et al., 2002; Chiang et al., 2007). To explore these phenomena, we perform several runs using massive particles. We do six runs with 4000 equal-mass particles, which distribute with $10 \text{ AU} \leq a \leq 45 \text{ AU}$, and small $e$ and $i$ as in Section 4.1. The surface mass density $\sigma = 0.14(l/40 \text{ AU})^{-2} \text{ g cm}^{-2}$, which provides mass of $\approx 80 M_\oplus$ for initial disk. These particles interact with planets but not themselves. The initial orbits for the four Jovian planets are the same as those in Section 4.1, but no artificial force is adopted here. One extra planet with $1-M_\oplus$ mass having zero $e$, $i$ and different $a$ in each run is set at the 3:2 MMR with pre-migrated Neptune. In addition, we perform another same six runs but with 16000 equal-mass particles, which gives higher resolution. The total integration time for each run is 82.5 Myr. In all twelve runs, none of the extra planets was trapped by Neptune’s 3:2 MMR during the main migration epoch of $t \lesssim 20 \text{ Myr}$. Most of the extra planets were captured when they migrated near the end of disk. The left and middle panels of Fig. 12 show two typical examples for the runs containing 4000 and 16000 massive particles, respectively. We think that the extra planet was hardly captured in the 3:2 MMR in the early epoch of the migration is because of large difference between the migration speed of the extra planet and that of the 3:2 MMR. Hence, the extra planet merely migrated across the
3:2 MMR. When the extra planet was near disk edge, both the migration speed of the 3:2 MMR and the extra planet are slow, and then the extra planet was captured more easily. Furthermore, we find that the capture occurring after 20 Myr becomes easier when each particle possesses smaller mass. We believe that this is due to the enhancement of capture ability as the Neptune migration is more smooth (Chiang et al., 2007). In our higher resolution runs with the 16000 particles, each particle has mass of $5 \times 10^{-3} M_\oplus$, which is still larger than reality by several orders (mass $\approx 10^{-6} M_\oplus$ for 100 km size object). Instead of increasing particle number, which requires huge computing time, we reduce the total mass of primordial disk to investigate a less noisy migration for planets. We perform same six runs as previous with 16000 particles for each run, but initial disk mass has only 15 $M_\oplus$. In all the six runs, at the beginning the extra planet was captured by Neptune’s 3:2 MMR until about 10 Myr; after that, the extra planet left MMR and migrated outward to about 40 AU. The right panels of Fig. 12 show one run with low-mass planetesimal disk. We think whether the extra planet can be captured or not depends on the smoothness of the Neptune migration, the dynamical friction on the extra planet, and the migration speed of the 3:2 MMR and that of the extra planet. Our simulations here merely imply that the capture of the extra planet by the 3:2 MMR during migration may be possible in a more realistic simulation. Concerning for the escape of the extra planet in the extra-planet model, the extra planet in the massive-disk simulation indeed has larger eccentricity during capture state, but the value of $e$ still $\lesssim 0.1$, which is hard for its scattering by the Jovian planets. It is unclear that in a more realistic case how large the eccentricity can reach during capture event.

In conclusion, more study is needed to better understand the interesting orbital evolution of the extra planet invoked in the extra-planet model in the future. In the present work, we just consider it as a low-probability event.

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5The extra planet was removed from the integration as its heliocentric distance is larger than 1500 AU.

6The main purpose of the massive particle runs is to explore the extra-planet’s dynamics in a massive disk. In Fig. 12, Neptune’s final position does not locate near 30 AU. Parking Neptune at suitable position depends on the initial orbital configuration of the four Jovian planets and the behavior of primordial disk. We think our main conclusions in the difference between the convention migration and the extra-planet model results would be retained under reasonable initial planets’
5.3.3. Issue of low-inclination classical KBOs

Although the extra-planet model can generate several observational features which are hard produced in the conventional migration model, there is still one main discrepancy between the extra-planet model results and the observations. The extra-planet model produces too few low-inclination KBOs with \( i \lesssim 5^\circ \) in the classical belt, even if the ecliptic inclination distributions are used for comparison. In this model, we used massless particles to represent the KBOs in our simulations. At the time of several tens Myr that the extra planet was removed, \( \sim 60\% \) of initial particles remains in the trans-Neptunian region. If the initial primordial disk has mass of \( 30 \, M_\oplus \) (within 24-45 AU), these remaining particles have mass of \( \sim 18 \, M_\oplus \). From accretion theory, only \( \sim 2-5\% \) of these particles/planetesimals have size \( \sim 100 \) km, while the major disk mass was occupied by the particles/planetesimals having size with \( \sim 0.1-10 \) km, which will then collide to dust grains and are removed from the Kuiper belt in Gyr timescale (Kenyon et al., 2008). In this period these 100 km size objects suffer dynamical friction from small bodies. The cooling timescale of the dynamical friction for the big bodies is

\[
 t_{dy} \sim i^4 \frac{M_\oplus^{1.5}}{M \sigma (Ga)^{0.5} f} \approx 5 \times 10^4 \left( \frac{0.6}{f} \right) \left( \frac{i}{5^\circ} \right)^4 \left( \frac{10^{-6} M_\oplus}{M} \right) \left( \frac{a}{40 \, AU} \right)^{1.5} \text{Gyr}, \tag{3}
\]

where \( i \) and \( a \) are inclination and semimajor axis of big body, respectively; \( \sigma = 0.14(a/40 \, AU)^{-2} \) g cm\(^{-2} \) is the surface mass density of initial disk; \( M \) is the mass of big body, which is \( \sim 10^{-6} M_\oplus \) for the body of 100-km size; \( f \) is the ratio of remaining disk as we remove the extra planet; \( G \) is the gravitational constant. This cooling timescale of \( 5 \times 10^4 \) Gyr is much larger than 4.5 Gyr, the age of our solar system. Therefore, the influence of the dynamical friction may not compensate for the shortage of the low-inclination particles. Whether physical collision between the bodies of 100-km size and numerous bodies of smaller size with total mass of about tens Earth mass alleviates the deficiency of the low-inclination part may be addressed in the future works.
5.3.4. Remnant population near $a \simeq 50 - 52$ AU

In the extra-planet model results of Fig. 11, the remnant populations near $a \simeq 50-52$ AU have near-circular and low-inclination orbits with $e \lesssim 0.05$ and $i \lesssim 5^\circ$, respectively. To estimate how many objects in the remnant may be observed so far, we choose another cold classical population with $a$ within 42-45 AU, $i \leq 5^\circ$ and $q \geq 40$ AU in the results of the extra-planet model. This cold classical population has similar small $e$ and $i$ with the above remnant population, but smaller heliocentric distance. In the our five runs of the extra-planet model, the average number ratio of the remnant population to this cold classical population is 0.93. With this ratio, and assuming these two populations have equal size objects and average semimajor axes 43.5 AU and 51.0 AU, we can roughly estimate the observational number of objects for the remnant population to be

$$\frac{N_r}{N_c} \times \left(\frac{51.0}{43.5}\right)^4 = 0.93,$$

where $N_r$ and $N_c$ are the observational number of objects for the remnant population and the cold classical KBOs under above criterion; $N_c = 174$ from the Minor Planet Center database July, 10, 2007 for the KBOs with multi-opposition; the second term in left hand side is the correction factor due to the observational bias of flux. The $N_r$ provided by this estimate is 86, which is inconsistent with the observations, where we do not observe any KBO near $a \simeq 50$-52 AU with near-circular and low-inclination orbit so far. In conclusion, based on the assumption of equal size objects and Eq. (4), the remnant population predicted by the extra-planet model seems to contradict the present observations.

6. Conclusions

We explore the conventional Neptune migration model with one extra planet in the 3:2 MMR with Neptune during migration. After the extra planet excites the primordial disk to a suitable state, we assume that this planet escapes from the disk in a short timescale and does not influence disk particles anymore. In all the our runs of the extra-planet model if we assume the primordial disk truncated at 45
AU, the extra planet having the mass of $\sim 1.0 \, M_\oplus$ and escaping roughly near the end of Neptune's migration produces the KBOs' orbital distribution which is most consistent with the observations. The extra-planet model generates several observational features of the Kuiper belt that cannot be explained via the conventional Neptune migration model. These features are that (1) the high-inclination Plutinos with $i \simeq 15^\circ$-$35^\circ$; (2) the excitation of the classical KBOs, which have moderate eccentricities and inclinations; (3) the larger ratio of Neptune's 3:2 to 2:1 resonant particles, and the lower ratio of the Plutino to the classical KBO numbers, which may be more consistent with observations; and (4) several 5:2 resonant particles. However, the simulation results in the discussion imply that this model is a low-probability event. In the extra-planet model, to understand the orbital evolution of the extra planet embedded in a massive disk, more realistic simulations and some analytic works are needed. In addition to the low probability issue, two features produced by the extra-planet model may be inconsistent with the observations. They are the small number of low-inclination particles in the classical belt, and the production of a remnant population with near-circular and low-inclination orbit within $a \simeq 50$-$52$ AU. Although the extra-planet model can explain several features which are unable to produce by the conventional Neptune migration model, it destroys the low-inclination population in the classical belt, which is the easiest and a natural outcome of the conventional Neptune migration model. Since our simulations consist of particles with negligible mass, one may wonder whether including the mass of particles could change this conclusion or not. Taking into account the mass of particles, we found that the dynamical friction suffered by the bodies of $\sim 100$-km size seems unlikely to generate more low-inclination particles in Gyr timescale, as indicated in Eq. (3). Whether the effect of physical collision between objects of different sizes mitigates this shortage has yet to explore. As for the remnant population, based on the assumption of equal size for all objects and Eq. (4), we may observe several tens of KBOs in this population according to the results of the extra-planet model, but to date no KBOs are observed to be with near-circular and low-inclination orbit near $a \simeq 50$-$52$ AU.
In summary, according to our present study, including one extra planet in the conventional Neptune migration model as the scenario we explored here may be unsuitable because of the low probability, and the two drawbacks mentioned above, although this model can explain better several features which are hard to produce by the conventional Neptune migration model. The issues of the low probability and lack of low-inclination classical KBOs are interesting and will be investigated further under a more realistic consideration.

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References

Brown, M.E., 2001. The inclination distribution of the Kuiper Belt. Astron. J. 121, 2804-2814.

Chambers, J.E., 1999. A hybrid symplectic integrator that permits close encounters between massive bodies. Mon. Not. R. Astron. Soc. 304, 793-799.

Chiang, E.I., Jordan, A.B., 2002. On the Plutinos and Twotinos of the Kuiper belt. Astron. J. 124, 3430-3444.

Chiang, E.I., Jordan, A.B., Millis, R.L., Buie, M.W., Wasserman, L.H., Elliot, J.L., Kern, S.D., Trilling, D.E., Meech, K.J., Wagner, R.M., 2003. Resonance occupation in the Kuiper belt: Case examples of the 5:2 and Trojan resonances. Astron. J. 126, 430-443.

Chiang, E., Lithwick, Y., Murray-Clay, R., Buie, M., Grundy, W., Holman, M., 2007. A brief history of trans-neptunian space. In: Reipurth,
B., Jewitt, D., Keil, K. (Eds.), Protostars and Planets V Compendium. Univ. of Arizona Press, Tucson, pp. 895911.

de La Fuente Marcos, R. and de La Fuente Marcos, C., 2008. Confined chaotic motion in three-body resonances: trapping of trans-Neptunian material induced by gas-drag. Mon. Not. R. Astron. Soc. 388, 293-306.

Fernández, J.A., Ip, W.-H., 1996. Orbital expansion and resonant trapping during the late accretion stages of the outer planets. Planet. Space Sci. 44, 431-439.

Gladman, B., Chan, C., 2006. Production of the extended scattered disk by rogue planets. Astrophys. J. 643, L135-L138.

Gomes, R.S., 2009. On the origin of the Kuiper belt. Celest. Mech. Dyn. Astron., DOI: 10.1007/s10569-009-9186-5.

Gomes, R.S., Morbidelli, A., Levison, H.F., 2004. Planetary migration in a planetesimal disk: why did Neptune stop at 30 AU? Icarus 170, 492-507.

Hahn, J.M., Malhotra, R., 1999. Orbital evolution of planets embedded in a planetesimal disk. Astron. J. 117, 3041-3053.

Hahn, J.M., Malhotra, R., 2005. Neptune’s migration into a stirred-up Kuiper belt: A detailed comparison of simulations to observations. Astron. J. 130, 2392-2414.

Ida, S., Larwood, J., Burkert, A., 2000. Evidence for early stellar encounters in the orbital distribution of Edgeworth-Kuiper belt objects. Astron. J. 528, 351-356.

Jiang, I.-G., Yeh, L.-C., 2004. The drag-induced resonant capture for Kuiper belt objects. Mon. Not. R. Astron. Soc. 355, L29-L32.
Kenyon, S.J., 2002. Planet formation in the outer Solar System. Publ. Astron. Soc. Pacific 114, 265-283.

Kenyon, S.J., Bromley, B.C., 2004. Stellar encounters as the origin of distant Solar System objects in highly eccentric orbits. Nature 432, 598-602.

Kenyon, S.J., Luu, J.X., 1999. Accretion in the early Kuiper belt. II. Fragmentation. Astron. J. 118, 1101-1119.

Kenyon, S.J., Bromley, B.C., O’Brien, D.P., Davis, D.R., 2008. Formation and collisional evolution of Kuiper Belt objects. In: Barucci, M.A., Boehnhardt, H., Cruikshank, D.P., Morbidelli, A. (Eds.), The Solar System Beyond Neptune. Univ. Arizona Press, Tucson, pp. 293-313.

Levison, H.F., Stern, S.A., 1995. Possible origin and early dynamical evolution of the Pluto-Charon binary. Icarus 116, 315-339.

Lykawka, P.S., Mukai, T., 2007a. Dynamical classification of trans-Neptunian objects: Probing their origin, evolution, and interrelation. Icarus 189, 213-232.

Lykawka, P.S., Mukai, T., 2007b. Origin of scattered disk resonant TNOs: Evidence for an ancient excited Kuiper belt of 50 AU radius. Icarus 186, 331-341.

Lykawka, P.S., Mukai, T., 2008. An outer planet beyond Pluto and the origin of the trans-Neptunian belt architecture. Astron. J. 135, 1161-1200.

Malhotra, R., 1995. The origin of Pluto’s orbit: Implications for the solar system beyond Neptune. Astron. J. 110, 420-429.
Murray, N., Holman, M., Potter, M., 1998. On the origin of chaos in the asteroid belt. Astron. J. 116, 2583-2589.

Nesvorný, D., Morbidelli, A., 1998a. An analytic model of three-body mean motion resonances. Celest. Mech. Dynam. Astron. 71, 243-271.

Nesvorný, D., Morbidelli, A., 1998b. Three-body mean motion resonances and the chaotic structure of the asteroid belt. Astron. J. 116, 3029-3037.

Nesvorný, D., Roig, F., 2000. Mean motion resonances in the trans-Neptunian region I. The 2:3 resonance with Neptune. Icarus 148, 282-300.

Petit, J.-M., Morbidelli, A., Valsecchi, G.B., 1999. Large scattered planetesimals and the excitation of the small body belts. Icarus 141, 367-387.

Stern, S.A., 1991. On the number of planets in the outer solar system - Evidence of a substantial population of 1000-km bodies. Icarus 90, 271-281.

Trujillo, C.A., Brown, M.E., 2001a. The radial distribution of the Kuiper belt. Astrophys. J. 554, L95-L98.

Trujillo, C.A., Jewitt, D.C., 2001b. Properties of the trans-Neptunian belt: Statistics from the Canada-France-Hawaii telescope survey. Astron. J. 122, 457-473.

Zhou, L.-Y., Sun, Y.-S., Zhou, J.-L., Zheng, J.-Q., Valtonen, M., 2002. Stochastic effects in the planet migration and orbital distribution of the Kuiper belt. Mon. Not. R. Astron. Soc. 336, 520-526.
Figure 1: Time-averaged semimajor axis $\langle a \rangle$ and eccentricity $\langle e \rangle$ at the end of 8.25 Myr simulation over a time interval of 0.825 Myr. (b), (c) and (d) are the runs including one extra planet in Neptune’s 3:2 MMR with the mass of 0.1, 0.5 and 1.0 $M_{\oplus}$, respectively. (a) is the run without extra planet for comparison. Non-resonant particles are represented by green dots. Neptune’s 3:2 and 2:1 resonant particles are marked by blue crosses. Red filled circle marks the extra planet. The perihelion of 40.4 AU is plotted by dashed line. Two vertical dashed lines represent the locations of the exterior 3:2 and 2:1 MMR with Neptune. In figure (e), one extra planet with 1.0 $M_{\oplus}$ is set one AU farther beyond the exact Neptune’s 3:2 MMR at the beginning.
Figure 2: Semimajor axes variation with time during the planet migration for the four Jovian planets and the extra-planet A with the mass of 1.0 $M_{\oplus}$ as an example. The smooth migration of the four Jovian planet are driven by the artificial force. The extra planet was captured in Neptune’s 3:2 MMR and moved outward together with Neptune.
Figure 3: Neptune’s 3:2 and 2:1 resonant particle numbers changing with time for the particle-set a. The three runs with different masses extra planet and the one run without extra planet are shown. The time interval for resonant particle identification is 8.25 Myr. Dashed and solid lines indicate the 3:2 and 2:1 resonant particle numbers, respectively. The runs with the extra planet of 0.1 $M_\oplus$, 0.5 $M_\oplus$ and 1.0 $M_\oplus$ are indicated with filled circles, stars and open circles, respectively. The run without one additional planet is shown by crosses. In the runs containing 0.5-$M_\oplus$ and 1.0-$M_\oplus$ extra planets, the 2:1 resonant particle numbers are always less than ten.
Figure 4: Neptune’s 3:2 and 2:1 resonant particle numbers varying with time. The left panel is just an enlarged version of part of Fig. 3 for the run containing 1.0-$M_{\oplus}$ extra planet. We reproduce the same run but removed this additional planet at 33 Myr, and then continued integration to 495 Myr. Right panel plots this long-term integration and starts from $t = 33$ Myr. Dashed and solid lines represent Neptune’s 3:2 and 2:1 resonant particle numbers, respectively.
Figure 5: The orbital elements for the runs containing the particle-set a and the 1.0-\(M_\oplus\) extra-planet A. This figure plots the orbital distribution of test particles (red crosses) at the end of 495 Myr for the extra-planet model (top panels) and the conventional migration model (bottom panels), respectively. The extra-planet A was removed at 33.0 Myr in the extra-planet model. Neptune’s 3:2 and 2:1 MMR are indicated with vertical lines. The perihelia of 30, 35 and 40 AU are shown with curved dashed lines. The observed KBOs with multi-opposition are also plotted with green circles (from the Minor Planet Center database of July, 10, 2007).
Figure 6: This figure plotting the orbital elements which are the same as the left two panels of Fig. 5, but we label particles with their initial semimajor axes. Crosses (red), triangles (green), squares (purple) and circles (blue) indicate the particles with initial semimajor axes from 24-35 AU, 35-40 AU, 40-45 AU and 45-50 AU, respectively.
Figure 7: The same figure as Fig. 5 but with initial disk truncated at $a = 45$ AU. The 5:2 MMR with Neptune is also labeled with vertical dashed line.
Figure 8: This figure showing the ecliptic inclination distribution for the results of the extra-planet model (left panels) and those of the conventional migration model (right panels) at the simulation end of 495 Myr. Top to bottom figures are the runs containing the particle-set a, b, c, d and e, respectively. The extra planets with 1.0 $M_\oplus$ were removed at 33 Myr in the extra-planet model. We chose particles with ecliptic latitudes $\beta \leq 3.0^\circ$ and perihelion $q \leq 45.0$ AU. Vertical axes represent particle number normalized to total particle number under above constraints. Simulated results and observed KBOs with multi-opposition are plotted by blue dashed and
Figure 9: This figure showing time-averaged orbital elements $\langle e \rangle$, $\langle i \rangle$ and Neptune’s 3:2 resonant amplitude $\Delta \phi$ for simulated Plutinos (blue symbols) at the end of 495 Myr simulation, where the runs containing particle-set a, b, c, d and e are labeled by crosses, squares, open circles, triangles and diamonds, respectively. Left two panels are the results of the extra-planet models, where the 1.0-$M_\oplus$ extra planet was removed at 33 Myr. Right two panels are the results of the conventional migration model. 100 observed Plutinos are also plotted with red filled circles from the identification of Lykawka and Mukai (2007a).
Figure 10: The same with allocation of Fig. 9 but for Neptune’s 5:2 resonant particles.
Figure 11: The $a$-$e$ distribution for the extra-planet model with one 1.0-$M_\oplus$ extra planet removing at 33 Myr. From top to bottom are the runs containing particle-set a, b, c, d and e, respectively. The left and the right panels are the same figures but with different scale for semimajor axis. Simulated particles and observed KBOs are shown with red crosses and green circles, respectively. Three vertical lines indicate Neptune’s 3:2, 2:1 and 5:2 MMR. Perihelia of 30, 35 and 40 AU are represented by dashed curves.
Figure 12: Semimajor axes $a$, the eccentricities $e$ and Neptune’s 3:2 resonant angles $\phi$ varying with time for the three massive-disk runs. Left panels show one run with 4000 massive particles and disk mass having 80 $M_\oplus$ as initial conditions. Middle panels are one run consisting of 16000 massive particles and the disk mass of 80 $M_\oplus$. A low-mass-disk run with 15 $M_\oplus$ and 16000 massive particles is shown in right panels. $t-a$ diagrams plot semimajor axis of the extra planet (blue curve) and that of Neptune’s 3:2 MMR (red curve) changing with time. The capture of the extra planet by the 3:2 MMR can be identified by librating resonant angles.