REVIEW OF FREE-FERMIONIC 4D STRING MODELS

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ABSTRACT

I review the main properties of four-dimensional strings constructed with free-fermions on the world-sheet. In particular I discuss possible model independent low energy predictions related to the existence of states with fractional electric charges, the computation of the string unification scale, the string model building, and the perturbative approach to supersymmetry breaking which makes the spectacular prediction of a new large dimension at the TeV scale.

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Despite the fact that progress in string theory was slow during the last couple of years compared to the situation in the early days, it remains actually the only known theory which contains quantum gravity, while simultaneously it provides a framework of unification of all fundamental interactions (gauge, scalar and Yukawa’s). At very short distances, it modifies the structure of space-time, while at energies lower than the Planck scale, all massive string excitations decouple giving rise to an effective field theory which describes the dynamics of massless modes. For the purpose of relevance to particle physics, there are two main questions:

(a) Is there a string vacuum which contains our low energy world as described by the Standard model of strong and electroweak interactions?
(b) Are there universal, model independent, string predictions at low energies?

To answer these questions, one has first to compactify the ten-dimensional heterotic superstring down to four dimensions. The string compactification procedure consists of replacing the 6 left-moving and the 22 right-moving internal coordinates by a (super) conformal field theory with central charge \( c = (9, 22) \). In the simplest constructions, the latter is a free two-dimensional theory of bosons or fermions. Here, we restrict ourselves to 4d free-fermionic strings which employ 18 left-moving and 44 right-moving 2d real fermions. The various models then correspond to the freedom of choosing the boundary conditions when the 2d fermions are parallel transported around the string. These are constrained by multiloop modular invariance which gives rise to well defined rules of construction. The main advantage of these models is their simplicity since they are obtained by tensoring Ising models which are solvable conformal field theories, allowing a straightforward derivation of the spectrum and a computation of the various couplings, correlation functions, etc. Their disadvantage is that they describe particular points of the parameter space (or their vicinity) which correspond to enhanced symmetric points of \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifolds, and it is difficult to study generic properties of continuous class of models.

The continuous parameters of four-dimensional strings consist of one fundamental mass scale given by the Regge slope \( \alpha' \) or equivalently the Planck mass, \((\alpha')^{-1/2} \sim 10^{18} \text{GeV}\), one dimensionless coupling constant \( g_4 \) determined by the dilaton vacuum expectation value (VEV), as well as several VEV’s of scalar fields called moduli which have zero potential. The relation between \( \alpha' \), \( g_4 \) and the Newton’s constant \( G_N \) is:

\[
G_N = \frac{g_4^2}{16\pi} \alpha'.
\] 

From the ten-dimensional point of view, the moduli VEV’s correspond to compactification parameters describing for instance the size and shape of the internal manifold. An important property is that they are subject to some exact discrete string symmetries called dualities which leave invariant the effective low energy Lagrangian. The determination of all scalar VEV’s is a complicate dynamical problem, closely connected to the mechanism of supersymmetry breaking, as well as to non perturbative phenomena. In addition to the continuous parameters there is also a certain number of discrete ones. Some of them will be discussed below.
Every consistent 4d string model contains in the massless spectrum the gravi
ton which appears together with a universal scalar field, the dilaton, as well as a
two-index antisymmetric tensor which is equivalent to a pseudoscalar axion. Si-
multaneously, one obtains a gauge group which can be as large as a group of rank
22.

Below, I discuss the general model independent properties and possible pre-
dictions of 4d strings, which fall into 4 categories:
1. “Exotic” representations and/or fractional electric charges.
2. Unification of couplings at a calculable unification scale.
3. No unbroken continuous global symmetries implying, in particular, that baryon
and lepton number are in general violated and proton is unstable. In fact, one of
the serious potential phenomenological problems in string model building is fast
proton decay.
4. A new large dimension at the TeV region is a spectacular prediction of pertur-
bative strings which relate its size with the scale of supersymmetry breaking.

1. Exotic representations and/or fractional electric charges

Every gauge group factor is characterized by a positive integer $k$, the level
of the underlying 2d Kac-Moody algebra. This integer determines the correspond-
ing tree-level gauge coupling constant, $g = g_4/\sqrt{k}$, while simultaneously it restricts
the possible massless matter representations. For $k > 1$, large “exotic” representa-
tions are in general present in the spectrum, like color octets, $SU(2)$ triplets, etc.
On the contrary, $k = 1$ constructions, which include generic free-fermionic models,
guarantee that the only possible massless representations are fundamental and an-
tisymmetric of unitary groups or vectors and spinors of orthogonal groups, which
is consistent with the present experimental observations. However, in this case, the
observed electric charge quantization cannot be imposed together with a value for
the weak angle $\sin^2 \theta_W = 3/8$ at the unification scale.$^{4,5}$ Consequently, color sin-
glet states with fractional electric charges are unavoidable, unless the weak angle
is unacceptably small ($\sin^2 \theta_W \leq 3/20$). In free-fermionic models a weaker charge
quantization condition can always be imposed, namely that the electric charges of
color singlet states are half-integers.

The lightest fractionally charged particle is stable. Generically their mass
is in the TeV region, which seems to be experimentally excluded, since an esti-
mation of their relic abundancies contradicts the upper experimental bounds by
several orders of magnitude.$^6$ There are two possible ways out. The first is to make
them superheavy and use inflation to suppress their abundancies as in the case of
monopoles. The second and most natural is to confine them by a gauge group of the
hidden sector, in the same way that QCD confines the fractionally charged quarks
into integrally charged bound states.$^7$ This possibility may lead to interesting phe-
nomenological consequences related to the existence of long-lived “crypto”-baryons.$^8$
It is also conceivable that the same gauge group is also responsible for dynamical
supersymmetry breaking via gaugino and scalar condensation, in which case the confining scale must be in the TeV region.9

2. Unification of couplings

Unlike field theories, 4d strings with \( k = 1 \) imply an automatic unification of all gauge couplings without the presence of a grand unified group (GUT). Furthermore, the unification scale \( M_{st} \) is calculable by taking into account the string threshold corrections. In fact, at the one-loop level one obtains:10,11

\[
\frac{16\pi^2}{g_i^2} = \frac{16\pi^2}{g_4^2} + b_i \ln \frac{M_{st}^2}{\mu^2} + \Delta_i ,
\]

where \( \mu \) is an infrared cutoff, \( b_i \) are the one-loop \( \beta \)-function coefficients, and \( \Delta_i \) define the finite threshold corrections given by an integral over the complex modular parameter \( \tau \) of the world-sheet torus. It is important to realize that in string theory, as well as in quantum field theory, the logarithmic infrared divergence, associated with the running of low energy couplings, is due to the integration over the massless particles. When the infrared divergence, \( \text{Im} \tau \to \infty \), is regularized and it is compared to the field-theoretical \( \overline{\text{DR}} \) scheme, one finds:10

\[
M_{st} \sim 5 \times g_4 \times 10^{17} \text{GeV} .
\]

Since the unification scale is calculable, string theories, in contrast to grand unified field theories, lead in general to two low energy predictions. Both the weak angle and the strong coupling \( \alpha_s \) are in principle calculable in terms of the electromagnetic coupling constant and the Planck mass. Unfortunately, assuming the particle content of the minimal supersymmetric standard model below \( M_{st} \), one obtains \( \sin^2 \theta_W = .221 \) and \( \alpha_s = .203 \) at \( M_Z \), which are close but clearly in contradiction with the experimental values. In fact \( M_{st} \) in Eq. (3) is about a factor of 20 bigger than the desired value of the unification scale which is consistent with the low energy couplings. An obvious way to bridge this gap is by introducing an additional scale which reduces the predictive power of the theory. One possibility is to have some “unnaturally” large modulus VEV, one to two orders of magnitude larger than the Planck length, which gives rise to large threshold corrections \( \Delta_i \).12 Alternatively, one may introduce additional gauge non-singlet representations at some intermediate scale, which could either be extra matter or enhance the gauge symmetry to a grand unified type group.13 At this stage, all proposed solutions are not satisfactory, since our present understanding on the possible origin of such intermediate scales is very limited.

3. Model building

3.1 General properties

As explained above, string theory does not lead to a unique way to determine the vacuum, at least in perturbation theory. However, it provides a framework for
model building with rules which are much more restrictive than in ordinary field theory. The simplest models correspond to $k = 1$ constructions and they are subject to several phenomenological constraints. On the one hand they have to provide solutions to basic outstanding problems, as the generation of the observed fermion mass hierarchies, and on the other hand they should not suffer from obvious diseases, as fast proton decay, or flavor changing neutral currents, etc. These may be imposed using discrete symmetries at the level of the Standard model, or proceeding through some grand unified group. In the second case ordinary GUT’s should be modified, since $k = 1$ constructions have no higgses in the adjoint or in higher self-conjugate representations usually needed to break the GUT group.

In analogy with Calabi-Yau compactifications which lead to $E_6 \times E_8$ gauge symmetry, there is a class of free-fermionic 4d strings which lead to an intermediate step of $SO(10) \times U(1)^n \times SO(16)$. The gauge group is broken to a product of an “observable” with a “hidden” sector (modulo the $U(1)$ factors), $G_{\text{obs}} \times U(1)^n \times G_{\text{hid}}$, by 2d fermion number projections which correspond to the Wilson line breaking of $E_6$ in Calabi-Yau models. These projections also reduce the number of families to three. A general property of these constructions is that every fermion generation forms a spinor of $SO(10)$ instead of a 27 representation of $E_6$ and, thus, it contains just one more state, the right-handed neutrino, besides the known Standard model content. There are three distinct possibilities for $G_{\text{obs}}$, associated to the three different subgroups of $SO(10)$ which can be broken to the Standard model using higgses in low dimensional representations: the flipped $SU(5) \times U(1)$, the Pati-Salam left-right symmetric model $SU(4) \times SU(2)_L \times SU(2)_R$, or directly the Standard-like model $SU(3) \times SU(2)_L \times U(1)_Y \times U(1)'$.

The general strategy of this procedure, before introducing the breaking of space-time supersymmetry, is focusing two main issues: The gauge symmetry breaking and the fermion mass problem. The gauge symmetry breaking contains four steps:

(i) The breaking of the additional $U(1)$ factors. This is achieved because one particular linear combination $U(1)_A$ appears often to be anomalous at the tree-level. The anomaly is canceled by a generalization of the Green-Schwarz mechanism in four dimensions, implying that the axion is absorbed by the $U(1)_A$ gauge field which becomes massive at the one-loop level. Furthermore, the corresponding $U(1)_A$ D-term is modified by an additional “constant” proportional to the anomaly $\text{Tr}Q_A$:

$$D_A = \sum_i Q^i_A |\Phi_i|^2 + \frac{\text{Tr}Q_A}{96\pi^2\alpha'},$$  \hspace{1cm} (4)

where $\Phi_i$ are the various scalars. As a result, some of the previously flat directions are destroyed, while the supersymmetric minimization conditions of $F$ and $D$-flatness are satisfied by non vanishing VEV’s of scalar fields which in general break all extra $U(1)$’s at a calculable scale $M_A$. For typical values of $\text{Tr}Q_A \sim 1 - 100$, one finds a scale which is two to one orders of magnitude less than the string scale, $M_A \sim 10^{16} - 10^{17}$ GeV.
(ii) The breaking of the GUT-type group $G_{\text{obs}}$ to the Standard model by a Higgs mechanism, at a scale $M_{\text{GUT}} \sim 10^{16}$ GeV which could have the same or a different origin than $M_A$.

(iii) The condensation of the hidden group $G_{\text{hid}}$ at some intermediate scale $\Lambda_{\text{cond}}$, which should also confine the fractionally charged states and guarantee the observed electric charge quantization. The hidden group may also be related with the dynamical breaking of space-time supersymmetry via gaugino condensation.\textsuperscript{18,9}

(iv) The usual $SU(2) \times U(1)$ breaking which needs one pair of massless Higgs doublets. This is in general a problem in any construction, since vector-like representations tend to become massive.

Concerning the fermion masses, the strategy is the following:\textsuperscript{7} Only one generation has trilinear Yukawa couplings and acquires a mass at the lowest order in the $\alpha'$-expansion. Because of the underlying $SO(10)$ symmetry one obtains $m_b = m_\tau$ at the unification scale, which leads to a successful prediction for the bottom to tau mass ratio at low energy, after taking into account the renormalization group evolution. Furthermore, the top Yukawa coupling $\lambda_t$ is proportional to the gauge coupling at $M_{\text{st}}$ implying that the top is in general heavy. In fact an upper bound can be obtained, corresponding to the limiting case $\lambda_t = \lambda_b = \lambda_\tau = \sqrt{2}g_4$ at $M_{\text{st}}$ which is valid in the absence of any mixing.\textsuperscript{7}

$$m_t \lesssim 180\,\text{GeV}. \quad (5)$$

All other masses and mixings involving the other two generations appear in $\alpha'$-corrections which correspond to non renormalizable terms in the effective superpotential. Consequently, the corresponding Yukawa couplings are naturally suppressed by powers of $M_A(\alpha')^{1/2}$, or $M_{\text{GUT}}(\alpha')^{1/2}$, generated by the various scalar VEV's.

### 3.2 SU(5) unification

As explained above, in the context of fermionic constructions described here, ordinary $SU(5)$ unification is not possible. Its simplest variation is flipped $SU(5)$, where the up and down antiquark triplets are interchanged between the $10$ and $\bar{5}$ representations, while simultaneously the right handed electron and neutrino are interchanged between the $10$ and a singlet. Thus, every generation forms an $SO(10)$ spinor. Note that the electric charge is not anymore an $SU(5)$ generator but it mixes with an additional $U(1)$ factor which must be present.\textsuperscript{19} The gauge group $SU(5) \times U(1)$ is broken to the Standard model by the VEV of a $10 + \bar{10}$ representation along the neutral $\nu^c$, $\bar{\nu}^c$ direction. Furthermore, one needs a Higgs in the $5 + \bar{5}$ representation, embedded in a vector of $SO(10)$, to break the electroweak symmetry.

One serious problem of $SU(5)$ unification, which is a manifestation of gauge hierarchy, is that the Higgs triplets lying inside $5 + \bar{5}$ must be superheavy because they mediate fast proton decay, while the remaining Higgs doublets must be light since they play the role of Standard model higgses. In the minimal $SU(5)$ this triplet-doublet splitting requires a severe fine tuning, while flipped $SU(5)$ offers a natural solution to the problem.\textsuperscript{14} In fact, due to an allowed superpotential coupling between the two set of higgses, $10 - 10 - 5$, the anti-triplet of $10$ is paired with
the triplet of 5 and they become superheavy when SU(5) is broken, while the Higgs doublets remain massless. A further consequence of this mechanism is the absence of dimension 5 operators for proton decay, implying a life-time of the order of $10^{36}$ years with dominant decay modes $p \to \bar{\nu} \pi^+$, or $e^+\pi^0$.

A minimal $SU(5) \times U(1)$ field-theoretical toy model should contain, in addition to the three families of $SO(10)$ spinors decomposed as $F_i = (10, 1/2)$, $\bar{F}_i = (\bar{5}, -3/2)$, and $l_i^c = (1, 5/2)$, $i = 1, 2, 3$, a pair of GUT higgses $H = (10, 1/2)$ and $\bar{H} = (\bar{10}, -1/2)$, a pair of light higgses $h = (5, -1)$ and $\bar{h} = (\bar{5}, 1)$ forming a vector of $SO(10)$, as well as three $SO(10)$ singlets $\phi_i = (1, 0)$. The most general trilinear superpotential, which is invariant under the discrete $Z_2$ symmetry $H \to -H$, is:

$$W = \lambda_d F \bar{F} h + \lambda_u F \bar{f} \bar{h} + \lambda_e \bar{f} F h + \lambda_\nu F \bar{H} \phi + \lambda_\phi \phi^3 + \lambda_\mu h \bar{h} \phi + \lambda_1 H \bar{H} h + \lambda_2 \bar{H} H \bar{h}, \quad (6)$$

where the generation indices were omitted for simplicity. The first five couplings in Eq. (3) give rise to the fermion masses, while the last two are responsible for the triplet-doublet splitting. In particular, $\lambda_d, u, e$ generate masses for down quarks, up quarks and charged leptons, respectively, while $\lambda_u$ gives also Dirac masses to neutrinos. In contrast to the ordinary $SU(5)$, down quarks and leptons belong to different multiplets and get masses from different Yukawa couplings. As a result, one looses the relation $m_d = m_e$ at $M_{GUT}$, which is successful for the third generation but problematic for the other two. Instead, one now obtains $m_\nu = m_u$. However, the couplings $\lambda_\nu$ and $\lambda_\phi$ generate a generalized see-saw mechanism for the right-handed neutrinos which form superheavy Dirac eigenstates with the singlets $\phi$. Diagonalizing the resulting mass matrix of $\nu_i, \nu_i^c$ and $\phi_i$, one also finds tiny Majorana masses for the left-handed neutrinos of the order of $m_\nu^2 < \phi > /M_{GUT}^2$.

Although simple and economic, the field theoretical $SU(5) \times U(1)$ model is not a real grand unified theory. It introduces two independent gauge coupling constants, and one looses the mass relation $m_b = m_\tau$. On the contrary, in the context of 4d strings gauge coupling unification is automatic, while the successful relation $m_b = m_\tau$ is retained as a consequence of the underlying $SO(10)$ structure in the trilinear couplings of these constructions. This symmetry is broken in the massive string sector so that the corresponding physically unrealistic relations for the lighter generations, which acquire masses through non-renormalizable interactions, are expected to be violated.

An explicit example of a three-generation flipped $SU(5)$ model with the above properties was derived using the fermionic constructions of 4d strings. The total gauge group is $G_{\text{obs}} \times U(1)^4 \times G_{\text{hid}}$, with $G_{\text{obs}} = SU(5) \times U(1)$ and $G_{\text{hid}} = SO(10) \times SU(4)$. The massless spectrum consists of 4 complete families + 1 antifamily, 4 pairs of $5 + \bar{5}$ higgses, 10 singlets charged under $U(1)^4$, 5 neutral singlets, as well as “hidden” matter which contains 5 $SU(5) \times U(1)$ singlets transforming as $\{(10, 1) + (\bar{1}, 6)\}$ under $G_{\text{hid}}$, and 6 $SU(5)$ singlets transforming as $\{(1, 4) + (\bar{1}, 4)\}$ and having fractional electric charges $\pm 1/2$. The pair of GUT higgses $H + \bar{H}$ is provided by one of the families and the antifamily, while one linear combination of the four $U(1)$’s appears to be anomalous. Also, the trilinear superpotential is easily derived
and it exhibits the desired properties. As a result, the analysis of gauge symmetry breaking follows the general description presented above.

The non-renormalizable terms in the effective superpotential were computed and analyzed up to 8th order in the $\alpha'$-expansion by two different groups.\textsuperscript{20,21} It turns out that there is an appropriate choice of scalar VEV’s which leads to reasonable phenomenology. In particular, all triplets become superheavy, while there remain one or two pairs of massless doublets. Furthermore, one obtains fermion masses having the correct orders of magnitude. Despite many interesting features, this model has two main defects. The generic problem of the unification scale which cannot be lowered easily using the field content present in the spectrum, and the problem of fixing the arbitrariness in the choice of scalar VEV’s used to explain the fermion mass hierarchies.

3.3 Pati-Salam unification

A different maximal subgroup of $SO(10)$, which can be broken to the Standard model without adjoint higgses, is $SU(4) \times SU(2)_L \times SU(2)_R$. The fermion generations form again spinors of $SO(10)$, which are now decomposed as $F_i = (\bar{4}, 2, 1)$ and $\bar{F}_i = (\bar{4}, 2, 1)$, containing the left-handed and right-handed components, respectively. The gauge symmetry is broken to the Standard model by a pair of higgses $H = (4, 1, 2)$ and $\bar{H} = (\bar{4}, 1, 2)$. Furthermore, the two electroweak higgses form the representation $h = (1, 2, 2)$, which emerges from the vector of $SO(10)$ together with a sextet $D_6 = (6, 1, 1)$.

The properties of this model are very similar to those of flipped $SU(5)$. On the one hand, the couplings $\lambda_{1,2}$ in the superpotential of Eq. (6), realizing the triplet-doublet splitting, are replaced by $\lambda_1 HHD_6 + \lambda_2 \bar{H}\bar{H}D_6$ which pairs the Higgs triplets with those of sextets into superheavy states. On the other hand, the first three couplings $\lambda_{d,u,e}$ are replaced by a single term $\lambda F\bar{F}h$ which gives masses to all fermions. Thus, one recovers the $SO(10)$ relation $m_d = m_e$ together with $m_u = m_\nu$. Finally, the see-saw mechanism for the right-handed neutrinos works in the same way as before through the couplings $\lambda_{\nu,\phi}$. A first instance of a string derived model with these properties was presented in Ref.15.

4. Perturbative supersymmetry breaking

One of the most important problems in string theory, which establishes the connection with our low energy world, is the breaking of space-time supersymmetry. In the context of the effective field theory, this breaking can be parameterized by introducing additional low energy parameters which determine for instance the mass-splitting inside supermultiplets. On the contrary, in string theory, there is no independent parameter related to the scale of supersymmetry breaking and the situation is very restrictive. It turns out that in perturbation theory this scale is necessarily linked with the size of an internal compactified dimension.\textsuperscript{22} Thus, since supersymmetry breaking must occur at energies close to the weak scale to protect the gauge hierarchy, 4d strings predict a new dimension in the TeV region!
Such a large dimension appears to have a lot of theoretical problems. The most serious is that all couplings blow up very rapidly above the decompactification scale, just by naive dimensional analysis since the effective field theory becomes higher dimensional, and perturbation theory breaks down. From the four-dimensional point of view the problem appears with the production of an infinite tower of Kaluza-Klein (KK) excitations, corresponding to the components of the momentum along the compactified directions, which increase indefinitely the $\beta$-functions.

This is the main reason why this possibility had not received attention and supersymmetry breaking is usually assumed to occur non-perturbatively. However, it was recently suggested that there is a way out of the large coupling problem in a particular class of 4d string models, which include orbifold compactifications. This opens the exciting possibility of lowering part of the massive string spectrum at energies accessible to future accelerators. The main observation is that in these models the KK-excitations are organized in multiplets of $N = 4$ (spontaneously broken) supersymmetry, leading to cancellations of large corrections among particles of different spin. In particular $\beta$-functions remain small and gauge couplings evolve logarithmically up to the Planck scale, despite the presence of an infinite tower of modes which fill up the “desert”. Furthermore, the chiral character of the theory implies the existence of a new class of states, called twisted, which have no field theory analog as they are not accompanied by KK-excitations.

The simplest realization of the minimal supersymmetric standard model in the context of this mechanism was worked out recently and it was shown to be very restrictive, even in the absence of an explicit string construction. Quarks and leptons should be identified with twisted states having no KK-excitations. Moreover, at the tree-level, the only source of supersymmetry breaking is a common gaugino mass, $m_{1/2} = 1/2R$, determined by the compactification radius $R$, while all scalar masses and trilinear scalar couplings vanish, $m_0 = A = 0$. On the other hand, there is a simple choice for the Higgs sector, which offers a natural solution to the $\mu$-problem. The second Higgs doublet, characteristic of the supersymmetric standard model, can be identified with the first massive KK-mode of the first Higgs doublet carrying opposite hypercharge. In this way, one obtains a Higgs mixing superpotential term with a coupling $\mu$ equal to the gaugino mass, together with a non-universal soft breaking Higgs potential:

$$m_{1/2} = \mu = \frac{1}{2R}, \quad m_0 = A = B = 0 \quad V_{\text{soft}} = -\mu^2 h_1^2 + 3\mu^2 h_2^2.$$

(7)

Note that although supersymmetry is broken, its breaking scale determined by $R$ is arbitrary. In fact $R$ is given by the VEV of a modulus field which corresponds to a flat direction of the tree-level scalar potential, even in the presence of supersymmetry breaking. This situation is reminiscent of the no-scale supergravity models.

This mechanism of supersymmetry breaking has an additional interesting feature which allows the dynamical determination of scales by minimizing the full one-loop effective potential with respect to the modulus and Higgs fields. It is the
absence of quadratic divergences in the one-loop cosmological constant ($StrM^2 = 0$) due to the underlying $N = 4$ supersymmetry of the massive modes. It follows that the effective potential of the Higgs and modulus fields is independent of the Planck mass at the renormalizable level. As a result, a new scale $Q_0$ is dynamically generated through the running of the renormalization group equations, defined as the energy where the mass-squared of the Higgs becomes negative and triggers electroweak symmetry breaking. Both the Higgs VEV and $R^{-1}$ are then proportional to $Q_0$ which can be hierarchically smaller than the string scale, depending on the value of the top Yukawa coupling.

Starting with the initial conditions of Eq. (7) and replacing the minimization with respect to the modulus by the phenomenological condition on the value of the $Z$-mass, one is left with one free parameter, the top Yukawa coupling. An analysis of both theoretical and experimental constraints shows that there is an allowed region,

$$140 \text{ GeV} \lesssim m_t \lesssim 155 \text{ GeV},$$

(8)

where the whole particle spectrum is given as a function of the top mass.\(^{24}\) The lower bound comes from the present experimental limits on supersymmetric Higgs detection, while at the upper bound the value of the supersymmetry breaking scale goes to infinity. In the lower bound the masses of scalar quarks and gluinos are lying in the range of 200-300 GeV, while the masses of scalar leptons, neutralinos, charginos and higgses in the range of 50-150 GeV. Also, the compactification scale is around 200 GeV. Finally, the values of $\tan \beta$ are in the range between 1.7 and 4, while the lightest supersymmetric particle is a neutralino in the whole region of Eq. (8).

A characteristic signature of this mechanism is the existence of Kaluza-Klein excitations for gauge bosons and higgses at low energy. In particular the lightest one is an excited photon with mass $1/R$, and it could be accessible to future accelerators with a very clear signal in the $l^+l^-$ channel. The present experimental limits on the size of extra dimensions follow from the analysis of effective four-fermion interactions induced by the exchange of KK-modes, and imply that $R^{-1}$ can be as low as 200 GeV.\(^{24}\)

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