Local symmetries of non-expanding horizons

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Abstract

Local symmetries of a non-expanding horizon have been investigated in the first-order formulation of gravity. When applied to spherically symmetric horizons, only a U(1) subgroup of the Lorentz group survives as a residual local symmetry that one can make use of in constructing an effective theory on the horizon.

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In this short paper, we explore local symmetries of a non-expanding horizon (NEH), a three-dimensional null hypersurface which imitates properties of a black hole horizon. The precise definition of an NEH can be found in [1, 2]. For our present purpose, it is sufficient to characterize an NEH to be a lightlike hypersurface $\Delta$ imbedded in spacetime such that the unique (up to scaling by a function) lightlike, real vector field $l$ tangential to $\Delta$ is an expansion, shear- and twist-free. Since $l$ is also normal to $\Delta$, it is geodesic as well. These properties of $\Delta$ are independent of the scaling of $l$ \cite{1, 3}. Let us further assume that $\Delta$ is topologically equivalent to $S \times \mathbb{R}$ where $S$ is a 2-sphere. To understand local symmetries, it is imperative that the gravitational dynamics be described by the first-order formulation of gravity, namely with tetrad-connection variables so that we are also able to decipher those Lorentz transformations which survive as symmetries on this null surface. Incidentally, since the definition of an NEH is very general with a minimal number of local conditions, our symmetry analysis, as done in the following paragraphs, will survive even for Killing and event horizons. This is because, NEH and Killing horizon boundary conditions are imposed over a region of spacetime. Boundary conditions hold only on $\Delta$ for the NEH (and in an isolated horizon) while Killing horizon conditions are valid on and in the neighborhood of the horizon. In contrast, event horizon conditions are global. Thus, we expect that our symmetry analysis with the restricted set of NEH boundary conditions will continue to hold for the Killing and event horizons.

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The first-order connection formulation is equivalent to general relativity with metric variables once one solves the zero torsion equation of motion. However, the tetrad-connection variables are more natural since not only they keep track of diffeomorphisms but also of the local Lorentz symmetries. Gravity is invariant, apart from diffeomorphisms, under the local Lorentz group, $\text{SL}(2, \mathbb{C})$, which is explicit in the first-order formulation only. Here our specific interest is to investigate whether the definition of an NEH (more precisely the boundary conditions on it) leads to breaking of the bulk local Lorentz symmetries on $\Delta$. This suspicion is motivated by the very well known examples of breaking of other local symmetries such as diffeomorphisms through boundary conditions in general relativity. Consider the following case in general relativity with and without boundary: the metric variables contain both the gauge and physical degrees of freedom of which the gauge part is usually removed by diffeomorphisms. In the presence of boundary, however, we must consider only those diffeomorphisms which preserve the boundary conditions. This leads to new physical degrees of freedom associated with that boundary. In the well-known cases, the physical quantities like mass and angular momentum associated with a boundary can be thought of as global charges arising out of some broken diffeomorphism.

We systematically analyze to find out if there is indeed breaking of the local Lorentz symmetry on an NEH. Based on the residual gauge group that we find through a kinematical analysis, we propose an effective theory on the horizon. This is worked out using the pre-symplectic structure on the covariant phase space of first-order theory. It is supposed that the subsequent quantization of that theory with loop quantum gravity in the bulk would yield the quantum states of a black hole. There is a recent upsurge of interest in such effective theories, where an $\text{SU}(2)$ Chern–Simons theory has been proposed [4, 13, 14] in correlation with previous works [8–12] as the effective quantum theory on the horizon in contrast to $U(1)$ theory [5–7]. However, all of these analyses are based on isolated horizon definitions which need some more geometric structures than those of our prime interest which are the NEH conditions. In this medley, we put the relevance of our paper through the symmetry reduction mechanism on the NEH as a conclusive answer, from $\text{SL}(2, \mathbb{C}) \rightarrow \text{ISO}(2) \ltimes \mathbb{R}$. A further, rather dramatic reduction to $U(1)$ follows due to some special properties of the lie algebra $\text{iso}(2)$.

First, let us see how an NEH $\Delta$ reduces the local Lorentz symmetry. Being expansion, shear- and twist-free, certain Newman–Penrose coefficients $\kappa_{\text{NP}}, \rho, \sigma$ vanish on $\Delta$: $\kappa_{\text{NP}}$ vanishes because the null-normal $l$ is a geodesic vector field, $\rho$ vanishes because the expansion of $l$ vanishes and $\sigma$ vanishes because $l$ is shear-free also. These conditions are satisfied only on $\Delta$. However, the Newman–Penrose coefficients are sensitive to the local Lorentz transformations [15]

\begin{align*}
  l &\mapsto \xi l, n \mapsto \xi^{-1} n, m \mapsto m, \\
  l &\mapsto l, n \mapsto n, m \mapsto e^{i\theta} m, \\
  l &\mapsto l, n \mapsto n - cm - \bar{c} m + c \ell, \\
  l &\mapsto l - bm - \bar{b} m + b \ell, n \mapsto n, m \mapsto m - \bar{b} n,
\end{align*}

where $\xi, \theta, c, b$ are smooth functions on $\Delta$. Under (1), (2) and (3), $\kappa_{\text{NP}}, \rho, \sigma$ transform respectively as

\begin{align*}
  \kappa_{\text{NP}} &\mapsto \xi^2 \kappa_{\text{NP}}, \quad \rho \mapsto \xi \rho, \quad \sigma \mapsto \xi \sigma \\
  \kappa_{\text{NP}} &\mapsto e^{i\theta} \kappa_{\text{NP}}, \quad \rho \mapsto \rho, \quad \sigma \mapsto e^{2i\theta} \sigma
\end{align*}

where $\kappa_{\text{NP}}, \rho, \sigma$ are smooth functions on $\Delta$. Under (1), (2) and (3), $\kappa_{\text{NP}}, \rho, \sigma$ transform respectively as
\[ \kappa_{NP} \mapsto \kappa_{NP}, \ \rho \mapsto \rho - c\kappa_{NP}, \ \sigma \mapsto \sigma - \bar{c}\kappa_{NP}. \]  
\tag{7}

Since they transform homogeneously, their vanishing remains invariant under (1)–(3). However, under (4) they transform inhomogeneously:

\[ \kappa_{NP} \mapsto \bar{b}\rho - b\sigma + |b|^2\tau + 2\bar{b}^2\alpha + 2|b|^2\beta \]
\[ - 2\bar{b}[b]_G + 2\bar{b}\epsilon - \bar{b}[b]_G^2(\mu - \bar{\mu}) + \bar{b}^2|b|^2\nu \]
\[ + \bar{b}^2\pi - \bar{b}^2\lambda + D\bar{b} - b\bar{b} - \bar{b}^2\bar{b} + |b|^2\Delta\bar{b} \]
\[ \rho \mapsto \rho - b\tau - 2b\alpha + 2|b|^2\gamma - \bar{b}[b]_G^2 + \bar{b}^2\lambda + \bar{b}b - \bar{b}\Delta\bar{b} \]
\[ \sigma \mapsto \sigma - \bar{b}\tau - 2b^2\pi - \bar{b}^2\gamma + \bar{b}^2\mu + \bar{b}\bar{b} - \bar{b}\Delta\bar{b}, \]  
\tag{8}

where \( D = \nabla I, \Delta = \nabla_m, \delta = \nabla_m \) and \( \bar{\delta} = \nabla_{\bar{m}} \). Clearly, the NEH boundary conditions are satisfied if and only if \( b = 0 \).

The Lorentz matrices associated with the transformations (1)–(3) are, respectively,

\[ \Lambda_{ij} = -\xi_1 m_{ij} - \xi^{-1} n_{ij} + 2m_i(m_1\bar{m}_j), \]  
\tag{9}

\[ \Lambda_{ij} = -2\xi_1 m_{ij} + (e^{i\theta} m_1(m_1\bar{m}_j) + c.c.), \]  
\tag{10}

\[ \Lambda_{ij} = -i(n_{ij} - (n_{l} - cm_l - \bar{c}m_l + |c|^2 l_i)l_j + (m_l - \bar{c}l_i)m_j + (\bar{m}_l - c_l_i)m_j, \]  
\tag{11}

and the corresponding generators are, respectively,

\[ B_{ij} = (\partial \Lambda_{ij}/\partial \xi)_l = -2i[m_{ij}], \]
\[ R_{ij} = (\partial \Lambda_{ij}/\partial \bar{\xi})_l = 2im_{ij}\bar{m}_j, \]
\[ P_{ij} = (\partial \Lambda_{ij}/\partial \bar{c} l_i)_{c_{l_0}} = 2m_j l_{ij} + 2\bar{m}_j l_{ij}, \]
\[ Q_{ij} = (\partial \Lambda_{ij}/\partial \bar{c} l_i)_{c_{l_0}} = 2im_j l_{ij} - 2\bar{m}_j l_{ij}, \]
\tag{12-15}

where \( B, R \) generate (1) and (2), respectively, and \( P, Q \) generate (3). A straightforward calculation gives their Lie brackets

\[ [R, B] = 0, \quad [R, P] = Q, \quad [R, Q] = -P, \]
\[ [B, P] = P, \quad [B, Q] = Q, \quad [P, Q] = 0, \]  
\tag{16}

where \([R, B] = R_{ik}B^k_j - B_{ik}R^k_j \) and so on. This is the Lie algebra of \( ISO(2) \times \mathbb{R} \) where the symbol \( \times \) stands for the semidirect product; \( R, P, Q \) generate \( ISO(2) \) and \( B \) generates \( \mathbb{R} \).

The complexified Lorentz algebra is isomorphic with \( sl(2, \mathbb{C}) \), which is generated by three elements \([\sigma_3, \sigma_+ \] such that \([\sigma_3, \sigma_\pm] = \pm 2\sigma_\pm \) and \([\sigma_+, \sigma_-] = \sigma_3 \). Its Borel subalgebra is generated by \([\sigma_3, \sigma_+ \), which is isomorphic with (16). Explicitly, [16]

\[ P = i\sigma_+, \quad Q = \sigma_+, \]
\[ R = i\sigma_+ - \frac{1}{2}\sigma_3, \quad B = -\sigma_+ + \frac{1}{2}\sigma_3. \]  
\tag{17}

It is an elementary exercise to show that \( P, Q, R, B \), as defined by (17), are linearly independent in the field of real numbers. Clearly, the NEH boundary conditions are invariant only under this subgroup of the local Lorentz group. We should keep note of the fact that the group \( ISO(2) \times \mathbb{R} \) is non-semisimple; its Cartan–Killing metric \( K \) is doubly degenerate

\[ K = \begin{pmatrix} R & B & P & Q \\ -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \]  
\tag{18}
Let us consider the Palatini connection $\mathcal{A}_{J}$ and in the interior of the spacetime let us expand $\mathcal{A}_{J}$ in the internal Lorentz basis,

$$\mathcal{A}_{J} = -2Wl_{l}[n_{J}]+2Vm_{j}[\bar{m}_{J}]+2(\bar{U}_{l}[m_{J}]+c.c.)+2(\bar{U}_{l}[m_{J}]+c.c.),$$

where $\mathcal{W}$, $\mathcal{V}$, $\mathcal{N}$, $\mathcal{U}$ are connection 1-forms; as defined, $\mathcal{W}$ is real, $\mathcal{V}$ is imaginary and $\mathcal{N}$, $\mathcal{U}$ are complex (in all, there are six of them associated with the six generators). For the rest of our analysis, we will fix an internal Lorentz frame for which $l_{l}$, $n_{J}$, $m_{j}$, $\bar{m}_{J}$ are constants. However, our results will be unaffected by such a choice.

The pull-back of the Palatini connection to the NEH $\Delta$ is of the form

$$A_{IJ} \triangleq -2Wl_{l}[n_{J}]+2Vm_{j}[\bar{m}_{J}]+2(\bar{U}_{l}[m_{J}]+c.c.),$$

where $W$, $V$, $U$ are respectively the pull-backs of $\mathcal{W}$, $\mathcal{V}$, $\mathcal{U}$. Clearly, the 1-form $N$, which is the pull-back of $\mathcal{N}$, vanishes on $\Delta$ by the NEH boundary conditions. The proof is as follows. The simplest way to show this is to relate the connection 1-forms to the Newman–Penrose coefficients (the constant $l_{l}$, $n_{J}$, $m_{j}$, $\bar{m}_{J}$ bases simplify these relations):

$$\mathcal{W} = -(\gamma+\bar{\gamma})l-(\epsilon+\bar{\epsilon})n+(\alpha+\bar{\beta})m+(\bar{\alpha}+\beta)\bar{m},$$

$$\mathcal{V} = -(\gamma-\bar{\gamma})l-(\epsilon-\bar{\epsilon})n+(\alpha-\bar{\beta})m+(\bar{\alpha}-\beta)\bar{m},$$

$$\mathcal{U} = -\bar{\nu}l-\bar{\pi}n+\bar{\mu}m+\bar{\lambda}\bar{m},$$

$$\mathcal{N} = \tau l+k_{\mathcal{N}}n-\rho m-\sigma\bar{m}.$$

So only four independent connection 1-forms $W$, $V$, $U$ and $N$ survive on $\Delta$. This is consistent with our earlier result that the residual gauge group on $\Delta$ is $\text{ISO}(2) \ltimes \mathbb{R}$ that has only four generators. However, below we present an independent analysis for the connection to prove this.

Under the local Lorentz transformations (1)–(4), the Palatini connection (19) transforms as

$$\mathcal{A}_{J} \mapsto \Lambda^{K}_{J}\mathcal{A}_{K} + \Lambda_{JK}d\Lambda^{K},$$

where $\Lambda_{J}$ are the associated Lorentz matrices (9)–(11) for (1)–(3) and for (4)

$$\Lambda_{IJ} = -(l_{l}-bm_{J}+\bar{b}m_{j}+\bar{b}n_{J})m_{l}-n_{l}l_{J}+(m_{l}-bm_{J})\bar{m}_{J}+(\bar{m}_{l}-bm_{j})m_{J}.$$  

A lengthy but straightforward calculation shows that under the Lorentz transformations (9)–(11), the connection 1-forms transform as

$$\mathcal{W} \mapsto \mathcal{W}-d\ln \xi, \mathcal{V} \mapsto \mathcal{V}, \mathcal{U} \mapsto \mathcal{U}, \mathcal{N} \mapsto \mathcal{N}^{-1},$$

$$\mathcal{W} \mapsto \mathcal{W}, \mathcal{V} \mapsto \mathcal{V}-id\theta, \mathcal{U} \mapsto e^{-i\theta}U, \mathcal{N} \mapsto e^{-i\theta}N,$$

$$\mathcal{W} \mapsto \mathcal{W}-\xi N_{l}-\bar{\xi}\bar{N}_{l}, \mathcal{V} \mapsto \mathcal{V}-\xi N+\bar{\xi}\bar{N},$$

$$\mathcal{U} \mapsto \mathcal{U}-d\bar{\xi}+\bar{\xi}(\mathcal{W}-\mathcal{V})-\bar{\xi}^{2}\bar{N}, \mathcal{N} \mapsto \mathcal{N}.$$  

Since $\mathcal{N}$ transforms homogeneously, its pull-back $N \triangleq 0$ in one frame implies that it vanishes in all Lorentz frames related by (9)–(11). However, under (26), the connection 1-forms transform as

$$\mathcal{W} \mapsto \mathcal{W}+bU+b\bar{U}, \mathcal{V} \mapsto \mathcal{V}-bU+b\bar{U},$$

$$\mathcal{U} \mapsto \mathcal{U}, \mathcal{N} \mapsto \mathcal{N}+db-b(\mathcal{W}+\mathcal{V})-b^{2}\bar{U}.$$
Clearly, in this case $N \equiv 0$ if and only if $b$ satisfies the equation $db = b(W - V + bU) =: bY$, where $Y$ is a 1-form. This equation has a nontrivial solution if and only if $Y$ is a closed 1-form. However, we show that the equation admits only the trivial solution, $b = 0$. The proof is as follows. Since $b$ is a constant in the phase space of an NEH, it is sufficient to show that $Y$ is not closed for one specific NEH. Consider for example the event horizon of the Schwarzschild solution. In units $G = 1$ and in advanced Eddington–Finkelstein coordinates

$$W = \frac{1}{4M} dv, \quad U = -\frac{1}{\sqrt{2}}(d\theta + i \sin \theta d\phi),$$

$$V = -i \cos \theta d\phi. \quad (31)$$

As a result, $dV$ and $dU$ are proportional to the 2-sphere area 2-form and $dW = 0$. However, since $Y$ depends on $b$, one may ask whether there is any $b$ for which $dY \equiv 0$. The answer is explicitly verifiable and one easily finds that $dY \equiv 0$ if and only if $b = 0$. Since $Y$ is not closed, acting $d$ once more on the equation $db = bY$, one obtains

$$0 = db \wedge Y + bdY = bY \wedge Y + bdY = bdY, \quad (32)$$

which yields the unique solution $b = 0$. This shows that the connection (20) is indeed an $ISO(2) \ltimes \mathbb{R}$ connection. Here, we wish to remark that one could also arrive at (27)–(30) directly using relations (21)–(24) and the appropriate Lorentz transformations of the Newman–Penrose coefficients [15].

It is to be noted that unlike the Palatini connection, the H{"o}lst connection $\mathbb{H}_{ij} := \delta_{ij} - \frac{1}{2} \gamma_{(i\ell} R_{j)\ell}, \delta_{ij}$, where $\gamma_{\ell}$ is the Barbero–Immirzi parameter, does not transform as a connection under any of the local Lorentz transformations (1)–(4). For later convenience, we expand (20) in the bases (12)–(15) of the Lie algebra $\mathfrak{iso}(2) \ltimes \mathbb{R}$:

$$A_{ij} = 2A_B B_{ij} + 2A_K R_{ij} + 2A_P P_{ij} + 2A_Q Q_{ij}, \quad (33)$$

where $2A_B = W$, $2A_K = -iV$, $2A_P = -\text{Re} U$ and $2A_Q = \text{Im} U$. The connection 1-forms $A_B, A_K, A_P, A_Q$ will turn out to be more useful in the context of an effective theory on the horizon.

Let us now turn our attention to the symplectic structures. The H{"o}lst action [17] gives rise to the symplectic current 3-form (in units of $4\pi G \gamma_B = 1$ and $E^j$ is the spacetime tetrad 1-form):

$$J(\delta_1, \delta_2) = -\frac{1}{2} \text{Tr}(\delta_1 (E \wedge \bar{E}) \wedge \delta_2 \bar{E} - (1 \leftrightarrow 2)). \quad (34)$$

Here the trace involves the $sl(2, \mathbb{C})$ Cartan–Killing metric.

The expansion of the tetrad in the null tetrad basis is $E^j = -n^j - ln^j + \bar{m}n^j + \bar{m} \bar{m}^j$. So the 2-form $E^j \wedge E^j$ pulled back onto $\Delta$ and expanded in the $\mathfrak{iso}(2) \ltimes \mathbb{R}$ basis is given by

$$E^j \wedge E^j \equiv \bar{\epsilon} E^j \wedge \bar{E}^j + \text{Re}(n \wedge m) P^{j\ell} - \text{Im}(n \wedge m) Q^{j\ell}, \quad (35)$$

where $\bar{\epsilon} = im - \bar{m}$. Now the symplectic current (34) is a closed spacetime 3-form $dJ = 0$. Integrating $dJ$ over $\mathcal{M}$, we find that the sum-total contribution of the symplectic current from the boundaries of $\mathcal{M}$ must vanish (figure 1):

$$\int_{\mathcal{M} \cup \partial \mathcal{M}} J(\delta_1, \delta_2) = 0. \quad (36)$$

We assume that the boundary conditions at infinity are such that the contribution of $\delta_1$ to the integral (36) vanishes. We must also ensure that the symplectic structure is independent of the choice of our foliation by the partial Cauchy slices. Using (33) and (35) and the fact that the trace in (34) is taken over a degenerate Killing metric (18), the pull-back of the symplectic current (34) is

$$J(\delta_1, \delta_2) \equiv \frac{1}{2} \delta_1^2 \bar{\epsilon} \wedge \delta_2 (V + \gamma_B W) - (1 \leftrightarrow 2). \quad (37)$$

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It is easy to see why only the combination $iV + \gamma W$ survives the pull-back: the pull-back of the connection $A$, hence also of $H$, has all the $\text{iso}(2) \times R$ components. However, the pull-back $E^1 \wedge E^2$ is only $\text{iso}(2)$-valued, as is obvious from (35). Furthermore, only the $RR$ and $BB$ components survive the tracing because of the degeneracy of the metric (18). Since $E^1 \wedge E^2$ has no $B$ component, only the $RR$ components survive in the symplectic current, which gives rise to the combination in (37).

In the derivation of the symplectic current it is sufficient to assume that the spherical cross-section foliates $\Delta$ and is not necessarily a geometric 2-sphere. However, for the rest of our analysis we will restrict ourselves to the unique foliation of $\Delta$ in which each leaf is a geometric 2-sphere; this is possible if and only if the horizon $\Delta$ is spherically symmetric.

For such a horizon with a fixed area $A = \int \epsilon_2$, where $\epsilon_2$ is the area 2-form of some spherical cross-section of $\Delta$, the 1-form $W$ is closed and $dV$ is proportional to $2\epsilon [18, 19]$

$$\text{d}W \equiv 0, \quad dV \equiv \frac{4\pi i_2}{A} \epsilon,$$  

where $d$ is the exterior derivative intrinsic to $\Delta$. Using (38) we find that the symplectic current 3-form is exact on $\Delta$:

$$j(\delta_1, \delta_2) = \frac{A}{8\pi} \delta_1 (iV + \gamma W) \wedge \delta_2 (iV + \gamma W).$$  

This gives a foliation-independent symplectic structure, whose boundary part is given by

$$\Omega(\delta_1, \delta_2) = \frac{A}{8\pi^2 G_{\gamma R}} \int_S \delta_1 A_{CS} \wedge \delta_2 A_{CS},$$  

where $S$ is the unique spherical cross-section of $\Delta$ and $A_{CS} = A_R - \gamma AB$.

The form (40) suggests that on a spherically symmetric NEH one can take the effective boundary theory as a $U(1)$ Chern–Simons theory. Two distinct cases of $U(1)$ arise: (i) if either the pull-back of $A_R$ vanishes on $S$ [20] or one restricts the gauge freedom (1) to a constant class ($\xi =$ constant, as has been the original choice [6]), then one obtains a compact $U(1)$; (ii) in general, if no restrictions are imposed, then one obtains a noncompact $U(1)$. 

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**Figure 1.** $M_\pm$ are two partial Cauchy surfaces enclosing a spacetime region $\mathcal{M}$ and intersecting $\Delta$ at the two 2-spheres $S_\pm$, respectively, such that they extend to spatial infinity $i^\circ$. $M$ is any intermediate partial Cauchy slice that intersects $\Delta$ at $S_\Delta$. 

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We summarize our conclusions as follows.

(1) Starting from first order, locally $SL(2,\mathbb{C})$ invariant theory of gravity, we find that the gauge invariance on an NEH reduces to the subgroup $ISO(2) \ltimes \mathbb{R}$. The gauge invariance in the bulk remains $SL(2,\mathbb{C})$; however, the non-$ISO(2) \ltimes \mathbb{R}$ transformations are realized trivially on the horizon.

(2) Because the Cartan–Killing form of residual subalgebra $iso(2) \ltimes \mathbb{R}$ is degenerate, even an $ISO(2) \ltimes \mathbb{R}$ Chern–Simons theory on the horizon can manifest only a $U(1) \times U(1)$ invariance. Here however, only a single $U(1)$ survives for a more subtle reason that has been discussed in the paragraph following (37) and the effective $U(1)$ is either compact or noncompact, depending on some choices.

(3) Although the $ISO(2) \ltimes \mathbb{R}$ gauge invariance is not manifested at the level of symplectic structure (40), it may still play a role in the quantum theory of the horizon.

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