The Influence of Different Constitutive Model Relations on Mechanical Behavior of Crack Tip Plastic Zone

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Abstract For ductile materials with cracks, when the stress at crack tip exceeds the yield strength, a plastic zone will be formed around crack tip. In this paper, the extended finite element method (XFEM) in ABAQUS was used to simulate the crack, and the mechanical behaviors of crack tip plastic zone under different constitutive models were studied by taking the three-point bending specimen as the research object. The COD experiment and theoretical verification of three-point bending demonstrate the effectiveness of this method. The effect of the plastic modulus on size of crack tip plastic zone under the double broken line constitutive model is analyzed, and the effects of the load on plastic zone size and stress distribution near crack tip under the true stress-strain constitutive model is discussed. Finally, the difference of stress distributions around crack tip obtained from two constitutive models is compared.

1. Introduction
Ductile materials are widely used in engineering structures, but there are often defects similar to cracks in structures due to manufacturing and other reasons. The stress-strain field at crack tip obtained by the linear theory of elasticity is singularity, and it does not exist in reality [1,2]. For most metallic materials, when subjected to load, the plastic zone at the crack tip exists, and plays a decisive role in the initiation, expansion and instability of the crack [3]. Irwin [4] considered the stress release in the plastic zone, and gave the elastic solution of the plastic zone size of an infinite plate with a central penetrating crack. Rise, Rosengren and Hutchinson independently obtained the asymptotic solutions of the stress and strain fields around the crack tip of hardened materials [5,6].

It is difficult to study the mechanical behavior around crack tip plastic zone by using experimental method, and the size of the plastic zone is not easy to observe. In this paper, the numerical simulation technology is used to simulate and analyze the mechanical behaviors of three-point bending specimens with cracks. The accuracy of the simulation was illustrated by comparing the obtained results with the parameters measured by experiments.

In this paper, different models of constitutive relationship of 4130X steel are selected, and the stress-strain behaviors of crack tip are simulated by ABAQUS. Simulation research is mainly divided into three aspects: (1) The change rule of stress field around crack tip plastic zone when the plastic modulus of double broken line constitutive model changes. (2) The trend of the plastic zone around the crack tip...
under different load by taking the true stress-strain curve of 4130X steel obtained from experiments as the constitutive model. (3) The effect of constitutive model on stress field of crack tip with the loads of 20kN.

2. Numerical simulation model

2.1 Model Size
In order to better study the stress-strain field properties of crack tip and eliminate the influence of shape, size and other factors on crack tip properties, this paper applied the standard specimen recommended in GB/T 21143-2014 *Unified Test Method for Quasi-static Fracture Toughness of Metal Materials* [7] to conduct numerical simulation research. The size of the standard three-point bending specimen selected is 120mm × 15mm × 30mm, and the crack length is 14mm, as shown in Figure 1. The extended finite element method (XFEM) in ABAQUS is used for modeling to add line cracks, which is closer to the actual situation of sharp cracks.

![Fig.1 Diagram of three-point bending specimen](image)

2.2 Material properties

2.2.1 Double broken line constitutive model of 4130X steel
This paper is based on the basic mechanical properties of 4130X steel, as shown in Table 1. To study the effect of plastic parameters on mechanical behavior of crack tip, this paper set up different plastic modulus respectively without changing the elastic modulus, yield strength and tensile strength. The different plastic modulus are 4GPa, 8GPa, 12GPa, 16GPa and ideal elastic-plastic, corresponding to different simplified engineering stress-strain curve of plastic modulus (double broken line model), as shown in figure 2.

| Modulus of elasticity | Poisson ratio | Yield strength | Tensile strength |
|-----------------------|--------------|----------------|-----------------|
| 2e5MPa                | 0.3          | 624MPa         | 893MPa          |

![Fig.2 4130X double broken line constitutive model](image)
2.2.2 True stress-strain constitutive model of 4130X steel

When the specimen subjected to uniaxial tension load, the volume after stretching is equal to the original volume according to the principle of plastic volume invariance, \( A_{0}l_{0} = \overline{A}l \), so \( \overline{A} = \frac{l_{0}}{l_{0} + \Delta l} A_{0} \). That is to say:

\[
\text{true stress } \overline{\sigma} = \frac{F}{A} = \frac{l_{0} + \Delta l}{l_{0}} \frac{F}{A_{0}} = \sigma(1 + \varepsilon)
\]

\[
\text{true strain } \overline{\varepsilon} = \int_{l_{0}}^{l_{0} + \Delta l} \frac{dl}{l_{0}} = \ln \frac{l}{l_{0}} = \ln(1 + \varepsilon)
\]

The true stress-strain curve of 4130X steel can be obtained by processing the stress-strain curve data of the tensile test of 4130X steel according to the above equation [8], and the simplified true stress-strain constitutive model is shown in Fig. 3.

![Fig.3 True stress-strain constitutive of 4130X](image)

2.3 Load and boundary conditions

According to the actual load condition of three-point bending test, the line load was applied at the position opposite the crack on the top of the specimen. For the support part at the bottom of the specimen, the movement in the vertical (y) direction and the rotation in the x and y directions are constrained.

2.4 Grid division

In order to make the calculation results more accurate, the mesh control adopts the hexahedral sweep division technology, and the element type is C3D8R (8-node hexahedral linear reduction integral cell), the overall seed size is 1mm. The element within the 4mm \( \times \) 4mm area around the crack tip was refined with a mesh size of 0.03mm. The refining results are shown in Fig. 4.

![Fig.4 Mesh division and local refinement](image)

3. Numerical simulation results

3.1 The influence of plastic modulus on crack tip plastic zone under 4130X double broken line model

When the applied load is 20kN, the equivalent stress in plastic zone around crack tip obtained from four double-broken line models with different plastic modulus was shown in Fig. 5.
3.2 Influence of load on plastic zone around crack tip under 4130X true stress-strain constitutive condition

Since the yield strength of 4130X steel is 624MPa, in this paper, the crack tip area is divided into elastic zone and plastic zone with 624MPa as the boundary. Under the 4130X true stress-strain constitutive model, the distribution of the plastic zone at crack tip under loads of 10kN, 20kN, 22kN, 24kN, 26kN and 28kN is shown in Fig. 6. The distribution of equivalent stress around crack tip under different loads is shown in Fig. 7.
4. Results discussion

4.1 The influence of plastic modulus on crack tip plastic zone under 4130X double broken line model

When the yield limit and strength limit of a material are constant, the magnitude of plastic modulus is closely related to elongation, which is a key plastic parameter of a material. The smaller the plastic modulus, the larger the plastic deformation of a material, and the greater the elongation. It is difficult to control the plastic modulus by experimental method. By changing the plastic modulus through ABAQUS for numerical calculation, the stress-strain field in plastic zone around crack tip under different plastic modulus can be obtained intuitively [9].

\[
\rho = \frac{1}{2\pi} \left( \frac{K}{\sigma_y} \right)^2
\]

In the formula: \( K \) -- stress intensity factor; \( \sigma_y \) --- yield stress

Equation (1) is the size of the plastic zone proposed by Irwin, and this equation is based on the ideal elastoplastic model. It can be seen from the equation that Irwin thinks the size of the plastic zone is mainly affected by the yield stress of the material and the stress intensity factor \( K \) at the crack tip [4,11].

As can be seen from Fig.5, when the load is constant, no matter what the constitutive model is, elastoplastic or double broken line, an obvious plastic zone is generated around crack tip. However, the size of the plastic zone changes slightly with the change of plastic modulus. The size of the plastic zone under ideal elastoplastic condition is 19% larger than that under plastic modulus of 4GPa, and the size of the plastic zone under 4GPa is 3.27% larger than that under 16GPa. This is because when the plastic modulus is smaller, the material is softer and the stress relaxation effect makes the plastic zone larger. The distribution of the stress-strain field inside the plastic zone changes with the plastic modulus, and the equivalent stress at the same position away from the crack tip increases with the increase of the plastic modulus. It can also be seen that the singularity of crack tip is different when the plastic modulus is changed, and the singularity of crack tip decreases with the increase of the plastic modulus.

4.2 Influence of load on plastic zone around crack tip under 4130X true stress-strain constitutive condition

![three-point bending test](image)

![Stress distribution contour around crack tip](image)

The plastic deformation occurs due to the stress concentration around crack tip, it will lead to the passivation of crack tip, and the crack surface will open accordingly. The crack opening displacement (COD) [10] can be used as the deformation parameter to represent the comprehensive effect of the stress and strain field around crack tip. In order to illustrate the accuracy of the simulation, a three-point bending was loaded and the crack opening displacement was measured by a clip gauge.

When the load is 10kN and the plastic modulus is 4GPa, the simulation value of crack opening displacement is 0.0823mm under the double broken line constitutive model, and the measured value is 0.108mm. The relative error between the two is 23.8%. The results obtained under double broken line constitutive model cannot fully represent the actual stress-strain condition around crack tip of 4130X steel. Under the true stress-strain constitutive model, the simulated crack opening displacement is 0.0987mm. The relative error between the simulated crack opening displacement and the measured value is only 8.61%. It is considered that the stress-strain field around crack tip simulated by the true
The stress-strain model is in good agreement with the actual distribution under the premise of the same crack opening displacement.

It can be seen from Fig. 7 that the size of the plastic zone around crack tip is related to load. The stress intensity factor $K$ represents the degree of load and deformation at crack tip \cite{11}, it is a function of the component geometry, the crack size and the external load. For the three-point bending specimen, the calculation formula is shown as (2).

$$K_1 = \frac{F \cdot S}{B(W)^{1/2}} g_1\left(\frac{a}{W}\right)$$  \hspace{1cm} (2)

Where, $F$ is the load, $S$ is the specimen span, $B$ is the specimen thickness, $W$ is the specimen height, $g_1\left(\frac{a}{W}\right)$ is the shape coefficient.

As the load $F$ increases, it can be seen from equation (2) that the stress intensity factor $K$ around crack tip increases accordingly. According to equation (1), when the yield strength is constant, the size of the plastic zone is directly related to $K$ and increases with the increase of $K$. In the plastic zone around crack tip, the equivalent stress decreases rapidly along the crack propagation direction and gradually approaches to the yield strength value. The larger the load, the larger the stress value at the same position at the crack tip. By observing the downward slope of the elastic zone along the direction of the crack in Fig. 7, it can be seen that the slope of the stress-strain curve in the elastic zone becomes smaller with the increase of load. As we can see, the curves of 26kN and 28kN almost coincide. This is because the value of these two kinds of loads are both large enough, so that the plastic zone of the loading area and the crack tip plastic zone intersect into one, as shown in Fig.6.

4.3 Stress field around crack tip plastic zone under different constitutive model at 20kN

Fig. 10 shows the distribution of stress field of the three-point bending specimen under the 4130X steel double broken line constitutive model and true stress-strain constitutive model at 20kN. It can be seen from the figure that the size of the plastic zone around crack tip and the stress distribution within the plastic zone are both related to the selection of the constitutive model.

We can see that the plastic zone size of true stress-strain constitutive model is larger than that of double broken line constitutive model from Fig.10. This is because true stress-strain constitutive model shows more plastic deformation than double broken line constitutive model. The maximum strain of true stress-strain constitutive model is about 0.18, which is two times larger than that of double broken line constitutive model (ultimate strength). The strain energy generated by the extra high strain zone releases, which makes the size of plastic zone become larger.

For the equivalent stress at crack tip, the stress value under the true stress-strain constitutive is greater than that under the double broken line constitutive at the same position of crack tip in the plastic zone. The difference of them is not obvious in the distance from the crack tip, but the closer to the crack tip, the larger the difference between them. At about 60μm from the crack tip, the equivalent stress calculated under the true stress-strain constitutive is about 1050MPa, which has exceeded the strength limit point.
in the engineering stress-strain curve. The true stress-strain constitutive can better reflect the high stress in the local region of crack tip.

5. Conclusions
In this paper, the three-point bending specimen was simulated to study the influence of different constitutive models on the plastic behavior of crack tip, and we get the following conclusions:

(1) When the load is constant, no matter what the constitutive model is, elastoplastic or double broken line, an obvious plastic zone is generated at crack tip. The size of the plastic zone changes slightly with the change of plastic modulus. Inside the plastic zone, the equivalent stress at the same position from crack tip increases with the increase of the plastic modulus. When the plastic modulus is changed, the singularity of crack tip is different, the singularity of the crack tip is reduced with the increase of the plastic modulus.

(2) The size of plastic zone around the crack tip increases with the increase of load. In the plastic zone, the equivalent stress decreases rapidly along the crack propagation direction and gradually approaches to the yield strength value. The larger the load, the larger the stress value at the same position from crack tip.

(3) The size of the plastic zone and the stress distribution in plastic zone are related to the selection of the specimen constitutive model. The size of the plastic zone under the true stress-strain constitutive model is larger than that obtained under the double broken line constitutive model. Within the plastic zone, the stress value under the true stress-strain constitutive model is larger than that under the double broken line constitutive model when the location is the same. The difference between them is not obvious at a distance from crack tip, but the closer to crack tip, the greater the difference between them. The true stress-strain constitutive model can better reflect the high stress in the local region of crack tip.

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