Understanding Popper’s experiment

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An experiment proposed by Karl Popper is considered by many to be a crucial test of quantum mechanics. Although many loopholes in the original proposal have been pointed out, they are not crucial to the test. We use only the standard interpretation of quantum mechanics to point out what is fundamentally wrong with the proposal, and demonstrate that Popper’s basic premise was faulty.

I. INTRODUCTION

Quantum theory is a tremendously successful theory when it comes to explaining or predicting physical phenomena. However, there is no consensus on how it is to be interpreted. For example, it is not clear whether the wave function is to be considered a real object or just a mathematical tool for calculating probabilities. However, these debates do not seem to have any bearing on the predictions for the outcomes of experiments based on quantum theory. Thus, most scientists continue to use quantum mechanics as a tool, leaving the debate on its meaning to others.

Karl Popper, a philosopher of science, has proposed an experiment to test the standard interpretation of quantum theory. Popper’s experiment is of much interest because the outcome depends on the interpretation of quantum theory. Ideas that used to fall under the realm of philosophy appeared to be testable. New interest was generated by its experimental realization by Kim and Shih and by claims that it proved the absence of quantum nonlocality. At the heart of Popper’s proposal is the concept of entanglement, which is a unique quantum phenomenon. Spatially separated, entangled particles, seem to depend on each other, even though there is no physical interaction. The implications of entangled states were discussed by Einstein, Podolsky and Rosen (EPR) in their famous paper. Such states are now commonly referred to as EPR states.

II. POPPER’S PROPOSED EXPERIMENT

Popper’s proposed experiment consists of a source $S$ that can generate pairs of particles traveling to the left and to the right along the $x$-axis. The momentum along the $y$-direction of the two particles is entangled in such a way so as to conserve the initial momentum at the source, which is zero. There are two slits, one each in the paths of the two particles. Behind the slits are semicircular arrays of detectors which can detect the particles after they pass through the slits (see Fig. 1).

Being entangled in momentum space implies that in the absence of the two slits, if a particle on the left is measured to have a momentum $p$, the particle on the right will necessarily be found to have a momentum $-p$. One can imagine a state similar to the EPR state $\psi(y_1, y_2) = \int_{-\infty}^{\infty} e^{ipy_1/\hbar} e^{-ipy_2/\hbar} dp$. As we can see, this state also implies that if a particle on the left is detected at a distance $y$ from the horizontal line, the particle on the right will necessarily be found at the same distance $y$ from the horizontal line. A tacit assumption in Popper’s setup is that the initial spread in momentum of the two particles is not very large. Popper argued that because the slits localize the particles to a narrow region along the $y$-axis, they experience large uncertainties in the $y$-components of their momenta. This larger spread in momentum will show up as particles being detected even at positions that lie outside the regions where particles would normally reach based on their initial momentum spread. The momentum spread, because of a real slit, is expected.

Popper suggested that slit B be made very large (in effect, removed). In this situation, Popper argued that when particle 1 passes through slit A, it is localized to within the width of the slit. He further argued that the standard interpretation of quantum mechanics tells us that if particle 1 is localized in a small region of space, particle 2 should become similarly localized, because of entanglement. In fact, when this experiment is done without the slits, the correlation in the detected positions of particles 1 and 2, is an example of such a localization. Popper completed his argument by saying that if particle

![Schematic diagram of Popper’s thought experiment.](image)
2 is localized in a narrow region of space, its momentum spread also will increase, causing other detectors to register:

“We thus obtain fairly precise ‘knowledge’ about \( y(B) \) — we have ‘measured’ it indirectly. And since it is, according to the Copenhagen interpretation, our knowledge which is described by the theory — and especially by the Heisenberg relations — we should expect that the momentum . . . of the beam that passes through slit B scatters as much as that of the beam that passes through slit A, even though the slit A is much narrower . . . If the Copenhagen interpretation is correct, then such counters on the far side of slit B that are indicative of a wide scatter . . . should now count coincidences; counters that did not count any particles before the slit A was narrowed . . .”

Popper had reasons to believe that if one were to actually carry out the experiment, particle 2 would not show any additional momentum spread. He argued that this absence of additional momentum spread would prove that the standard interpretation of quantum mechanics was wrong.

Popper believed that quantum mechanics could be interpreted “realistically,” so that we could talk of the position and momentum of a particle at the same time. He also did not like the notion, which is central to the Copenhagen interpretation of quantum mechanics, that the knowledge gained about particle 1 could have any influence on particle 2. Thus, he intended to demonstrate by this experiment that a position measurement on particle 1 would have no effect on the momentum spread of particle 2.

### III. OBJECTIONS TO POPPER’S EXPERIMENT

In 1985, Sudbery pointed out that the EPR state already contained an infinite spread in momenta, so no further spread could be seen by localizing one particle. Sudbery further stated that collimating the original beam, so as to reduce the momentum spread, would destroy the correlations between particles 1 and 2. We will show that having a reduced momentum spread doesn’t completely destroy the correlations. The presence of correlations despite a reduced momentum spread, is also seen in the experimentally observed spontaneous parametric down-conversion (SPDC) photon pairs.

In 1987 there came a major objection to Popper’s proposal from Collet and Loudon. They pointed out that because the particle pairs originating from the source had a zero total momentum, the source could not have a sharply defined position. They showed that once the uncertainty in the position of the source is taken into account, the blurring introduced washes out the Popper effect. However, it has been demonstrated that a point source is not crucial for Popper’s experiment, and a broad SPDC source can be set up to give a strong correlation between the photon pairs.

Redhead analyzed Popper’s experiment with a broad source and concluded that it could not yield the effect Popper that was seeking. However, a modified setup using a broad source and a converging lens has been shown to lead to a localizing effect.

Popper’s experiment was realized in 1999 by Kim and Shih using a SPDC photon source. They did not observe an extra spread in the momentum of particle 2 due to particle 1 passing through a narrow slit. In fact, the observed momentum spread was narrower than that contained in the original beam. This observation seemed to imply that Popper was right. Short has criticized Kim and Shih’s experiment, arguing that because of the finite size of the source, the localization of particle 2 is imperfect, which leads to a smaller momentum spread than expected. However, Short’s argument implies that if the source were improved, we should see a spread in the momentum of particle 2.

We have analyzed Popper’s proposal and showed that the mere presence of slit A doesn’t lead to a reduction of the wavefunction. So, we should not expect any effect of slit A on particle 2. We concluded that in the original Popper’s proposal and in Kim and Shih’s realization, we would not see any spread in the momentum of particle 2, just due to the presence of slit A in the path of particle 1. This conclusion was based on the standard interpretation of quantum mechanics. Our conclusion also implied that even if the source is improved to give a better correlation of photons, we would not see any spread in the momentum of particle 2. So, Popper may have been right in saying that there would be no spread, but for the wrong reasons.

### IV. WHAT IS WRONG WITH POPPER’S PROPOSAL?

It is easy to see that our earlier objection to Popper’s experiment can be remedied by putting a detector immediately behind slit A, such that a photon passing through the slit is detected immediately. In this case, as soon as the particle passes through slit A, we acquire the information that causes a reduction of the wavefunction because of the detector. The question we now ask is will we see any extra spread in the momentum of particle 2? After all, we can make the slit A as narrow as we want, and the resultant localization of particle 2 should lead to an increasing momentum spread.

Let us investigate this scenario rigorously. From practical considerations, the initial momentum spread has to be finite. Let us assume an initial wavefunction of the
where $\psi$ could be an entangled state. We first calculate the uncertainty in the momentum of, say, particle 2. The wavefunction defined by Eq. (1) after integrating over $p$, also can be written as:

$$\psi(y_1, y_2) = 2A\sqrt{\pi}\sigma e^{-(y_1 - y_2)^2/\hbar^2} e^{-(y_1 + y_2)^2/16\sigma_0^2}.$$  

Because $\langle p_{2y} \rangle = 0$, the uncertainty in $p_{2y}$ is given by

$$\Delta p_{2y} = \left[ |A|^2 4\pi\sigma^2 \int_{-\infty}^{\infty} dy_1 \int_{-\infty}^{\infty} dy_2 e^{-\frac{(y_1 - y_2)^2}{\hbar^2}} e^{-\frac{(y_1 + y_2)^2}{16\sigma_0^2}} \right]^{1/2} - \hbar^2 \frac{d^2}{dy_2^2} e^{-\frac{(y_1 - y_2)^2}{\hbar^2}} e^{-\frac{(y_1 + y_2)^2}{16\sigma_0^2}} \right]^{1/2}$$

$$= [\sigma^2 + \frac{\hbar^2}{16\sigma_0^2}]^{1/2}. \tag{4}$$

Because the state (1) is symmetric in $y_1$ and $y_2$, the uncertainty in $p_{1y}$ is also the same as that for $p_{2y}$. The position uncertainty of the two particles is $\Delta y_1 = \Delta y_2 = \sqrt{\Omega_0^2 + \hbar^2/16\sigma^2}$.

Let us suppose that a measurement is performed on particle 1 at slit A such that the wavefunction of particle 1 is reduced to

$$\phi_1(y_1) = \frac{1}{(\epsilon^2 2\pi)^{1/4}} e^{-y_1^2/4\sigma^2}. \tag{5}$$

In this state, the uncertainty in $y_1$ is given by

$$\Delta y_1 = \sqrt{\langle \phi_1 | (\hat{y}_1 - \langle \hat{y}_1 \rangle)^2 | \phi_1 \rangle} = \epsilon. \tag{6}$$

After the measurement, the particles are disentangled, and the subsequent evolution of one is independent of the other in the sense that they are governed by different wavefunctions. The wavefunction of particle 2 is now reduced to:

$$\psi(y_1, y_2) = \int_{-\infty}^{\infty} \psi(y_1, y_2) \phi_1(y_1) dy_1 = \frac{2A\sqrt{\pi}\sigma}{(\epsilon^2 2\pi)^{1/4}} \int_{-\infty}^{\infty} e^{-\frac{(y_1 - y_2)^2}{\hbar^2}} e^{-\frac{(y_1 + y_2)^2}{16\sigma_0^2}} d\frac{y_2}{2\Delta y_1} = \frac{2A\sqrt{\pi}\sigma}{(\epsilon^2 2\pi)^{1/4}} \int_{-\infty}^{\infty} e^{-\frac{(y_1 - y_2)^2}{\hbar^2}} e^{-\frac{(y_1 + y_2)^2}{16\sigma_0^2}} d\frac{y_2}{2\Delta y_1}$$

where $\alpha = \frac{\sigma^2}{\hbar^2} + \frac{1}{16\sigma_0^2} + \frac{1}{4\epsilon^2}$, and

$$\Omega = \sqrt{\frac{\epsilon^2 (1 + \frac{\hbar^2}{16\sigma_0^2}) + \hbar^2/4\epsilon^2}{1 + \frac{\epsilon^2}{16\sigma_0^2} + \frac{\hbar^2}{16\sigma_0^2}}} \tag{8}$$

From Eq. (4) it follows that the uncertainty in the position of particle 2 is given by:

$$\Delta y_2 = \sqrt{\frac{\epsilon^2 (1 + \frac{\hbar^2}{16\sigma_0^2}) + \hbar^2/4\epsilon^2}{1 + \frac{\epsilon^2}{16\sigma_0^2} + \frac{\hbar^2}{16\sigma_0^2}}} \tag{9}$$

Equation (9) implies that when a measurement is performed on particle 1, so as to localize it within a spatial region $\epsilon$, particle 2 becomes localized in a region $\Delta y_2$ given by Eq. (9). Once particle 2 is localized to a narrow region in space, its subsequent evolution should show the momentum spread dictated by the uncertainty principle. The uncertainty in the momentum of particle 2 is now given by

$$\Delta p_{2y} = \frac{\hbar}{2\Delta y_2} = \sqrt{\frac{\sigma^2 (1 + \frac{\epsilon^2}{\Omega_0}) + \hbar^2/16\sigma_0^2}{\epsilon^2 (1 + 4\epsilon^2 (\sigma^2/\hbar^2 + 1/16\sigma_0^2))}} \tag{10}$$

Now we have all the results needed to examine what happens in Popper’s experiment. Let us look for the maximum possible scatter in the momentum of particle 2. To do so we have to localize particle 1 in a very narrow region, which is what Popper wanted to achieve by narrowing slit A. Let us look at the momentum uncertainty of particle 2 in the limit $\epsilon \to 0$:

$$\lim_{\epsilon \to 0} \Delta p_{2y} = \sqrt{\sigma^2 + \hbar^2/16\sigma_0^2} \tag{11}$$

But the right-hand side of Eq. (11) is exactly the uncertainty in the momentum of particle 2 in the initial state (1), before particle 1 entered the slit (see Eq. (1)). So, even in the best case, there is no extra spread in the momentum spread in particle 2, as it is at variance with what Popper had concluded regarding the standard interpretation of quantum mechanics. On the other hand, the momentum spread of particle 1 after the measurement is given by

$$\Delta p_{1y} = \frac{\hbar}{2\Delta y_1} = \frac{\hbar}{2\epsilon} \tag{12}$$
which, for $\epsilon \to 0$ will become infinite.

We note that if $\epsilon << \Omega_0$, the position spread of particle 2 becomes smaller as a result of the measurement performed on particle 1. However, the momentum spread of particle 2 as given in Eq. (11) also is smaller than the original spread given by Eq. (9). This smaller momentum spread is possible because the original state is not a minimum uncertainty state. The spread in both conjugate variables can thus be reduced at the same time, within limits of course. Because the momentum of particle 2 cannot show any additional spread for a minimum uncertainty initial state, the position spread of particle 2 also should remain unchanged during the measurement performed on particle 1. Indeed, a calculation confirms an interesting scenario. For $\Omega_0 = \frac{h}{2\sigma}$, the initial state is a minimum uncertainty state for particles 1 and 2. With this choice of $\Omega_0$, Eqs. (11) and (1) are identical and equal to $\sqrt{2}\sigma$. In this case, the initial position spread of particle 2 is $\Delta y_2 = h/(2\sqrt{2}\sigma)$, which is the identical result given by Eq. (9). This result implies that a position measurement on particle 1 has absolutely no effect on particle 2. This conclusion might look very surprising, but we can check that the initial state (3) becomes disentangled for $\Omega_0 = h/4\sigma$.

We see that the fundamental mistake made by Popper was to assume that according to the standard interpretation and a finite initial momentum spread, localizing particle 1 to a narrow region will lead to the localization of particle 2 in a region as narrow. In contrast, we have seen that if particle 1 is localized to a region of size $\epsilon$, particle 2 is localized to $\frac{\sqrt{\epsilon^2(1+h^2/16\epsilon^2\sigma^2)+h^2/4\epsilon^2}}{1+\epsilon^2/16\sigma^2+h^2/16\epsilon^2\sigma^2}$. Only in the limits $\sigma \to \infty$ and $\Omega_0 \to \infty$, does the latter reduce to $\epsilon$. But in that case, the initial momentum spread is already infinite. Short’s argument regarding the finite size source leading to imperfect localization of the second particle is fully consistent with the general analysis presented here.

Finally, we verify if quantum mechanics shows what, in Popper’s view, would constitute an effect of the position measurement of particle 1 on particle 2. Popper states: “To sum up: if the Copenhagen interpretation is correct, then any increase in the precision in the measurement of our mere knowledge of the particles going through slit B should increase their scatter. This view is less stringent – it does not demand that the localization of particle 2 be as much as that of particle 1. It just says that if the (indirect) localization of particle 2 is made more precise, the momentum spread should show an increase.

The momentum spread of particle 2 in Eq. (10) in the limit in which the correlation between the two particles is expected to be stronger, namely $\sigma \gg h/4\Omega_0$, $\epsilon/\Omega_0 \ll 1$, is

$$\Delta p_{2y} \approx \frac{h}{\sqrt{h^2/\sigma^2 + 4\epsilon^2}}$$

Clearly, if $\epsilon$ is decreased, $\Delta p_{2y}$ increases. While analyzing Popper’s experiment, Krips had predicted that narrowing slit A would lead to momentum spread increasing at slit B, which is the same as our conclusion. Our result relies only on the mathematics of quantum mechanics. So, we conclude that Krips’s prediction was correct. Krips also had correctly argued that this conclusion can be justified using the formalism of quantum theory, independent of any particular interpretation.

We deduce that in a real experiment, for a general initial state, the approximate position localization of particle 1 would lead to a somewhat reduced momentum spread of particle 2. This conclusion is in contradiction with what Popper thought the Copenhagen interpretation implies and what many defenders of the Copenhagen interpretation probably imagined. The measurement on particle 1 does have an influence on particle 2, although not of the form one might have naively expected. So, there is no escape from quantum nonlocality. If Popper had imagined that the Copenhagen interpretation implies that a measurement on particle 1 would lead to additional scatter in the momentum of the second particle, his discomfort with it was justified. Quantum mechanics doesn’t have that kind of nonlocal influence – the non-locality is only at the level of correlations. The lesson is that quantum mechanics is full of surprises, and we should be careful when analyzing thought experiments in quantum physics.

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