On centenary climate evolution in the circumpolar region from the bifurcation analysis viewpoint

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Abstract. The paper presents the bifurcation analysis viewpoint on the local climate dynamics by the example of the data observed in the circumpolar region of the European part of Russia. With this purpose we turn to the integrating analytics in order to realize the idea, in accordance to which the notion of a “stability margin” is used as a distance to a bifurcation boundary under adjusted conditions. The peculiarity of the integrating analytics consists in building various modified bifurcation diagrams which are originally intended to integrate traditionally incompatible spaces and images without losses and distortions of the necessary cause-effect relations. So, such modified bifurcation diagrams are free from the restrictions inherent to the classical bifurcation diagrams. Following the stability margin idea, we show that images of dynamics evolution can be conflict-free estimated in relation to images of the boundaries, about which the prerequisites of qualitative changes of the habitual dynamics can appear. Since this method opens fundamentally other viewpoint on the local evolution, then we believe that the paper could be interesting for scientists and specialists, activity areas of which relate to the contemporary challenges connected with climate changes.

1. Introduction

The fact of global climate changes is incontestable; however estimations of the coming consequences are quite various [1, 2, 3, 4, 5, 6]. System full-scale investigations of ecosystem changes are usually quite long and labour-intensive. Moreover, results and conclusions of such investigations can even be retarded from too fast changing reality and have never before expected or imagined due to complex cause-effect relations between objective and subjective factors: objective reasons concern restrictions of scientific knowledge, deficiency in necessary data, and so on; subjective reasons concern goal positions of desirable economic profits, habitual thinking, and so on. One of the illustrative examples relates to Trans-Arctic shipping which was only discussed at the beginning of the XXI-st century; this year, the North Sea route is already opened. Due to the heightened ecological vulnerability together with the inevitable necessity of international cooperation, Arctic region seems to be very sensitive detector to show in what extent the coming climate changes are comprehended. Traditionally, land surface air temperature is one of the main variables which are used in order to estimate both rate and scale of climate changes.

The peculiarity of our viewpoint consists in the following: we apply the bifurcation analysis conceptions to analysis of temperature evolution. This seems to be important because local climate
systems represent nonlinear systems. This fact was physically-based described by the conception model proposed by Lorenz [7], and many nonlinear effects are shown in various time and space scales, for example [8, 9, 10, 11, 12]. The bifurcation analysis represents the main “tool” intended to ground existence/absence of evolitional changes in dynamics of nonlinear systems. So, it is the paradox that the bifurcation analysis remains to be ignored by the contemporary official reports concerning estimations on local, regional and global climate dynamics, for example [2, 3, 13, 14, 15]. Attempts to consider the bifurcation analysis in application to weather and climate dynamics remain comparatively complex, for example [10, 16, 17, 18, 19]. Practical usage of these results in order to forecast emergencies in local climate dynamics evolution seem to be restricted. First of all, it is a point on how to sew the deep gap between weather and climate scales [20, 21, 22, 23, 24]? In other words, it is necessary to have a dynamical system which can conceptually describe why does the interannual variability occur? Such conceptual model should analyze rough weather details (i.e. daily means) in a short-term climate scale (i.e. several decades) at least.

At present, there is only one conceptual model (so-called HDS-model) which allows not only to sew weather and climate scales without distortions and losses of the practically important information, but also to build the bifurcation diagrams in order to demonstrate the evolution of local climate dynamics in terms of last 135 years. This model was proposed and verified by the results obtained by processing the data of meteorological observations, for example [12, 18, 24]. Here we turn to the integrating analytics based on the bifurcation analysis conceptions in order to realize the idea, in accordance to which the notion of a “stability margin” is used as a distance to a bifurcation boundary under adjusted conditions. We restrict analytics by the daily mean land surface air temperature observations over the last century in the circumpolar region of Russia (from open-access sources available at www.meteo.ru). Yes, there are more long historical and indirect data of meteorological reconstructions, for example [10, 17, 25, 26]; however, such data are too aggregative or/and irregular in order to be used in practice-oriented applications [27].

Since the discussion concerns the interdisciplinary interrelations between the HDS-model, the interannual variability, the bifurcation analysis, and the integrating analytics, then section 2 introduces preliminary remarks to the problem statement. Section 3 presents the proposed viewpoint aimed at conflict-free estimations of dynamics evolution images in relation to the boundaries of atypical local climate dynamics. Concluding remarks and future outlook are presented in Section 4.

2. Preliminary remarks to the problem statement

Let us introduce several preliminary remarks in order to clarify the discussion. First, the mathematical view of the HDS-model represents a dynamical system with the variable structure, where the natural competition between the amplitude quantization (restricted by the temperature Hysteresis) and time quantization (the Double Synchronization) occurs. The corresponding control algorithm is formalized by the commutation function ($K_F$):

$$
K_F(t) = \begin{cases} 
1, & \text{x(t) > H / 2} \\
0, & \text{x(t) < -H / 2} \\
0 \rightarrow 1, & t = j \cdot T_S \quad \text{or} \quad x(t) = H / 2 \\
1 \rightarrow 0, & t = (j + 0.5) \cdot T_S \quad \text{or} \quad x(t) = -H / 2
\end{cases}
$$

(1)

where $x$ is a phase variable to establish correlations between local daily mean temperature ($T$) and the regulative restriction on a local temperature reference ($T_{ref}$), $x = T_{ref} - \beta T$; $\beta$ is an amplification coefficient and hereafter $\beta = 1$; $H$ is a local temperature hysteresis which restricts amplitude pulsations about an average; $T_S$ is a synchronization period (1 year); $j=0,1,2,…$ is a number of a period. The simplest view of the dynamical system is the following:
Figure 1. Diagram to illustrate how the periodical processes of the HDS-model are realized in accordance with the commutation function (a); schemes to the interannual variability (b) and to the ensemble of four elemental processes (c); the factual locations of temperature minimums (blue circled points) and maximums (red circled points) made by processing the temperature observations in Teriberka, WMO-index 22028 (d) and Arkhangelsk, WMO-index 22550 (e).

\[
\frac{dx}{dt} = ax + b(K_F) + c,
\]

where \(a, b, c\) are parameters. So, \(K_F\)-function determines the moments of structural changes of a phase trajectory in accordance with four surfaces (Fig.1a): two surfaces (denoted by 1 and 3) sew the phase trajectories by level; two surfaces (denoted by 2 and 4) sew the phase trajectories by time. Each of the moments of structural changes is a result of the natural competition between the amplitude quantization (the temperature Hysteresis) and time quantization (the Double Synchronization).
Third, in accordance with HDS-hypothesis proposed in [12, 18, 24, 27], local climate dynamics is described conceptually by this model. Thus two synchronization sequences (F1- and F2-sequences, Fig.1a) of the equal frequency (1/T2) with T2/2-shift in time restrict durations of warming and cooling stages. In physical sense, such time synchronization seems to be concordant with maximal and minimal distance between the Earth and the Sun (aphelion and perihelion correspondingly); a temperature hysteresis seems to be concordant with the local Earth surface heat budget in sense used in [28, 29]. From the control theory viewpoint, the time-and-amplitude competition provides the heightened stability of energy conversion processes with T5-periodicity at the expense of regular particular modifications of phase patterns. From the nonlinear dynamics viewpoint, these modifications is mainly described by three elementary processes with the same periodicity (T2) and the different orders of the structural changes (Fig.1b): R-process (right-process through 3-2-4-3 surfaces) with k > 0.5; C-process (central-process through 2-4-2-surfaces) with k = 0.5; L-process (left-process through 2-1-4-2 surfaces) with k < 0.5. Here k is the relative duration of the warming stage which is considered as the main bifurcation parameter. A transition from one elementary process to other one is realized through a border-collision bifurcation (such bifurcation is also denoted by C-bifurcation).

The HDS-hypothesis was proposed because of the specific regularities of HDS-model dynamics are clearly concordant with the specific regularities of the observed local climate dynamics, for example [12, 18], including simple and logically consistent explanation of the interannual variability phenomenon. This phenomenon can be simply reduced to the bimodality of the statistical distributions calculated for the minimums and maximums of the annual temperature variation. In other words, during winter and during summer there are two main centerlines, about which the external values are concentrated (Fig.1b); so, the annual warming-cooling cycles can be “compressed” or “stretched” in comparison with the regular calendar year. As a result, formally, there can be four elementary cases similar to HDS-model dynamics (Fig.1c): k>0.5 (denoted by blue), k<0.5 (denoted by red), and two cases with k=0.5 (denoted by solid and dotted black lines). In the conventional practice, each actual local climate norm is associated with only one of these cases. In particular, in accordance with the actual WMO-convention (WMO-No.1137 2014, Heidelberg, Germany, 3–8 July), the climate norms are calculated over 1961-1990 years and/or over 1981-2010 years, where monthly means are taken into account.

So, let us pay attention to reliefs of these norms from the viewpoint of the annual variability. Let us illustrate typical results of the calculations by the examples of the temperature data observed in Arkhangelsk (WMO-index 22550) and Teriberka (WMO-index 22028). Namely, both climate norms in Arkhangelsk are characterized by the same relief (with k=0.5): the minimal temperature occurs in January, the maximal temperature occurs in July. The climate norms in Teriberka are characterized by other relief (with k<0.5): the minimal temperature occurs in February, the maximal temperature occurs in July; at the same time, minimal values in January and February over 1961-1990 years are practically the same, i.e. there is rather the minimal level over two months than the minimal peak over one month. However, if the factual locations of temperature minimums and maximums are determined with daily resolution, then all variants of realizations presented in Fig.1c can be observed in real temperature dynamics (Fig.2d,e). Moreover, such variety of annual temperature reliefs seems to be natural property of local climate dynamics over the whole period of the meteorological observations. So, effects of the interannual variability in climate systems look like the effects of transitions between elementary processes in HDS-model dynamics.

3. Evolutinal estimations from the bifurcation analysis

From the similarities in dynamics above mentioned and discussed in [12, 18], it seems to be legitimate to suppose that the HDS-model can be considered as a conceptual model to describe local climate dynamics. Then the interannual variability is treated as alternations between several elementary processes with the same periodicity and different structures of phase trajectories. Simply, a local climate system can “choose” one of several elementary behaviors and the “rule”, in accordance to which such choice occurs, is described by the HDS-regularity law. Thus, from the HDS-hypothesis,
the interannual variability was proposed to excuse by alternations between the elementary periodical processes (i.e. different variants of 1-year periodical processes) instead of chaotic fluctuations about average annual extremes in accordance with the probable conception, for example [3, 13, 14, 15], and instead of changes of periodicity in accordance with the periodical conception based on the processing the paleo-climate data, for example [8, 9, 10]. In other words, qualitative changes in local dynamics in a centennial term are supposed as trivial events. Let us demonstrate which moments concerning local climate dynamics evolution should be supposed from the bifurcation analysis in this case.

So, a nonlinear system can demonstrate a qualitative change of the actual behavior (i.e. a bifurcation); boundaries between different behaviors (i.e. bifurcation boundaries between different dynamical processes) are visualized in a parametrical space by means of so-called bifurcation diagrams (Fig.2a). The image of the actual stationary state (a point in the parametrical space, Fig.2a) can be estimated in relation to the bifurcation boundaries. In particular, the notion of a stability margin is used in the context of the practice-oriented bifurcation analysis in order to estimate some adjusted distance to a bifurcation boundary in the parametrical space (Fig.2a). In spite of the apparent simplicity, general realizations of the stability margin idea in application to real nonlinear systems meet many troubles, for example: a bifurcation boundary determined for a real nonlinear system (not for a mathematical model) is usually fuzzy (i.e. it is rather a bifurcation boundary domain than a bifurcation boundary curve); domains of dynamical processes can overlap in the parametrical space; transients from one periodical process to other one can not be shown in the parametrical space; the notion of emergency can be associated not only with the stability loss, but also with characteristics which can not be shown in the parametrical space; and so on.

In general, the integrating analytics was proposed in order to resolve these troubles for systems with variable structures by means of resolution of conflict-of-units [27]. In particular, so-called modified bifurcation diagrams are widely used within this conception in order to integrate traditionally incompatible spaces and objects without losses and distortions of the necessary cause-effect relations. Hereafter, we originate from the following considerations. Dynamics of systems with variable structure is conceptually similar, but differs essentially from the dynamics of systems with constant structures. So, it is proposed to translate the integrating analytics for similar problems (i.e. for emergency estimating) in application to the same kind of nonlinear systems (i.e. to a system with a variable structure) which is just natural (not technical). Such translation can not be extraordinary, since applications of “solutions” proposed by Nature in human practice are traditional at all times. Thus, we sew the causes (stability evolution) and effects (changes in the observed characteristics) in order to estimate the observed tendencies in relation to the preliminary built bifurcation boundaries for each local climate system.

Researches of nonlinear dynamics of HDS-regulators, for example [30], shown that \((H/a, k)\)-space is basic to dynamics evolution due to \(R-, C-, L\)-processes occupy practically fully this space on the right of \(H/a=1\) and are symmetrically disposed in relation to \(k=0.5\) (Fig.2b). Here \(a\) means an annual temperature oscillation. Results of the analytics on the characteristics of the observed local climate behaviours, for example [12, 18, 24, 27], allow to conclude that typical behaviours of local climate systems (i.e. behaviours which are habitual for human being) represent alternations between \(R-, C-, L\)-processes existing mainly within the following limits: from \(H/a=1.1\) to \(H/a=1.7\), and from \(k=0.3\) to \(k=0.7\). So, let us consider the dynamics evolution in Teriberka and Arkhangelsk from these positions. If the evolution is directly imaged in \((H/a, k)\)-space, then only some tangled trajectories occur (Fig.2c,d), where directions of vectors from the first to the last state (denoted by red arrows in Fig.2c,d) can depend on the particular happy or unhappy cases.

In accordance with the integrating analytics, let us decompose evolution into each of the axes separately. Then the vertical direction relates to regularities of alternations between the elementary processes (i.e. between \(R-, C-, L\)-processes). However, such alternations mean so-called intermittency, where transitional constituents distort essentially the clear images of stationary states. Thus, the classic bifurcation diagrams are not originally intended to analyse such phenomena. In this connection, the modified bifurcation diagrams were proposed [12], where evolution was proposed to be shown in a
probable sense as comparative activities of the potential constituents over time. So, these constituents over time are denoted as R-, L-, and C-activities, for example Fig.2e,f. The main conclusions based on the modified bifurcation diagrams consist in the following: C-activity dominates in all the cases; tendencies towards warming and cooling are grounded by peculiarities of local competing between L- and R-activities; prevalence of L-activity means more intensive warming; prevalence of R-activity means more intensive cooling.

**Figure 2.** Schemes to the notion of the stability margin in the parametrical space (a) and to the basic parametrical space of the HDS-model (b). Results of analytical estimations made for Teriberka, WMO-index 22028 (left column) and Arkhangelsk, WMO-index 22550 (right column): tangled evolution of local states in the parametrical space (c,d); decomposition of this evolution into the tendencies of RLC-activities (e,f) and the tendencies towards potential emergencies (g,h).

The horizontal direction of the parametrical space relates to oscillations between the adjusted boundaries of typical behaviors: changes of periodicity (1-year) can be realized with H/a<1; too much differences in annual average temperature can be realized with H/a>2. For example, H/a-time series built for two considered local climate systems are shown in Fig.2g,h. Two main conclusions can be made by the obtained results. First, polar territories are usually more sensitive to impacts of global warming than even circumpolar ones (for example, L-activity identified in Teriberka is more intensive
than L-activity identified in Arkhangelsk, Fig. 2e,f). Second, evolitional peculiarities of local climate systems are quite various (for example, jumping states to/from the adjusted boundary identified in Teriberka in comparison with rather clear tendency in Arkhangelsk, Fig. 2g.h). Perhaps, passes the adjusted boundaries will represent emergencies; at least, these passes will lead to unusual annual temperature patterns.

4. Conclusion
At present, the HDS-hypothesis is grounded quite enough in order to be used at least conceptually, for example [12, 18, 24, 27]. From this assumption, we discussed how the idea of the stability margin used by the integrating analytics in application to the systems of variable structure in technical realizations can be projected to the similar problem for the similar systems but in natural realizations. We demonstrated that fundamental restrictions on such applications are absent. The obtained results are concordant to the opinion that Arctic region seems to be very sensitive detector to show in what extent the coming climate changes are comprehended. In particular, the corresponding local intrinsic responses on the global warming can be visualized. However, some expert part retains in the context of this analytics due to several interdisciplinary demands (first of all, from the fields of the nonlinear dynamics, the experimental bifurcation analysis, the climate theory, the nonlinear control theory) should be satisfied. Nevertheless, we believe that two circumstances are important and positive in order to comprehend deep mechanisms of the coming events. First, the discussion on evolitional tendencies is opened in the context of quite clear and sample terms known from the practice of the bifurcation analysis. Second, the discussion is carried out on the basis of processing the direct data of the instrumental observations in the “comfortable” time-and-space scale: centennial horizons with daily resolution in a local geographic point.

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