A Quantum-Mechanical Explanation of the Collapse of the Wave Function

Benjamin Ross *

Abstract

The quantum field of a single particle is expressed as the sum of the particle’s ordinary wave function and the vacuum fluctuations. An exact quantum-field calculation shows that the squared amplitude of this field sums, at any time, to a δ function representing a discrete corpuscle at one point and zero everywhere else. The peak of the δ function is located at the point where the vacuum fluctuations interfere constructively with the ordinary wave function. Similarly, the collapsed wave function after a measurement of an observable is determined by interference between the initial wave function and vacuum fluctuations.

Introduction

The meaning of the basic concepts of quantum mechanics has long been debated. Probably the most difficult problem involves the collapse of the wave function when a measurement is made on a quantum system. Commentators have focused increasingly on wave function collapse as the kernel of the disputes over the fundamentals of quantum mechanics.1,2,3

As Bohr, von Neuman, and others interpreted quantum mechanics, the collapse is an actual change in the wave function caused by the measurement. To make sense of this interpretation, one is forced to define a class of objects (such as “intelligent observers” or “macroscopic systems”) with special properties. The wave function collapses when the system interacts with one of these special objects.

Quantum theory itself provides no rule for identifying the special objects. This makes the theory seem incomplete as a description of physical reality. For example, the wave equations predict that the wave packet representing a large object in free flight (say, a baseball) will spread so slowly that dispersion can be ignored. But why baseballs always start out in well-prepared wave packets is not easy to explain. Most often, this question has been answered by assumption — by choosing the special objects so that they are present in all circumstances where baseballs ordinarily find themselves.

In recent years, theorists have offered statistical explanations in which wave functions collapse when they interact with large objects that have many degrees of freedom. These theories have not been entirely satisfactory, if only because they are more complex than the solution to such a fundamental problem ought to be.

The conventional dichotomy between “classical” and “quantum” behavior offers little help in understanding the collapse of the wave function. The difficulty is the same whether the wave function represents electromagnetic radiation or a massive particle. For example, in the famous Gendankenexperiment of Schrödinger’s cat, the cat’s execution may be triggered by radioactive decay of either an alpha or a gamma emitter, and identical questions arise in both cases. Indeed, the experiment runs equally well with the radioactive source replaced by a very faint lamp illuminating a photomultiplier tube. The underlying problem is to explain why waves act like particles, and it matters little whether the wave function is A or ψ.

The objective of this paper is to make particle-like behavior — a particle does or doesn’t impinge on a detector — fall out of the wave equations. To the extent that this program succeeds, peculiar assumptions such as a special role for intelligent observers or an infinitely multiplying ensemble of equally real universes are no longer needed to explain quantum mechanics.

We suggest that the collapse of the wave function when a quantum system interacts with an apparatus, and consequently the “classical” behavior of macroscopic objects such as laboratory apparatus, is predicted by quantum field theory. Particle-like features are lost when the quantum field equations are approximated by Schrödinger’s or Maxwell’s Equations. Wave function collapse is an ad hoc insertion that recovers the behavior not predicted by these less exact equations.

Figures 1 and 2 show how the underlying logic of our approach differs from textbook derivations of basic quantum concepts. Figure 1 summarizes the traditional distinction between classical and quantum realms. Classical equations describe things that can be perceived directly. Quantum equations are obtained by quantizing the classical equations. The variables in the quantum equations are measured through their effect on classical variables but are distinct from them. An interpretation is needed to relate the two sets of variables and connect quantum theory to what human beings directly apprehend.

Figure 2 shows our reclassification of the ways of describing a physical system. The contrast between waves and particles replaces the classical-quantum distinction. The direction of inference proceeds by way of approxima-
Recent research in quantum optics has shown that shot noise in detectors, which has traditionally been understood as a manifestation of the particle nature of light, can be explained as the result of interference between the signal field and vacuum fluc-
tuations.\textsuperscript{4} (Similar but less rigorous results were obtained in the 1970s by the stochastic electrodynami-
ics school.\textsuperscript{5}) In this section, we show that the behavior of matter as localized particles rather than diffuse waves can be explained, like electromagnetic shot noise, as a result of interference between the quantum wave function and vacuum fluc-
tuations. When vacuum fluctuations are included in the field, its squared amplitude adds up to a discrete cor-
puscle at the one location where constructive interference occurs and nothing everywhere else.

We treat the ordinary case in elementary quantum mechanics, a particle with a wave function $f_0(x)$ in a space of finite volume $V$. We construct a complete set of basis functions $f_i$ which includes $f_0$.

To take into account interference with vacuum fluctuations, we write the wave function (with the time dependence suppressed) as

$$\psi(x) = b_0^\dagger f_0(x) + \sum_{i \neq 0} b_i f_i(x) \tag{1}$$

where the $b_i$ are annihilation operators. We interpret $\psi^\dagger \psi$ as the numerical density of particles in space. It must be emphasized that $\psi^\dagger \psi$ is not interpreted here as a probability; it is the actual amount of matter (of a particular kind) at a location. The number of particles in a volume $v < V$ is

$$N_v = \int_v \psi^\dagger(x)\psi(x)dx \tag{2}$$

which can be written out as

$$N_v = \int_v \left[ b_0^\dagger b_0 f_0^\star(x)f_0(x) + \sum_{i \neq 0} b_i^\dagger f_i^\star(x)b_i f_0(x) + \sum_{i,j \neq 0} b_i^\dagger b_j f_i^\star(x)f_j(x) \right] dx \tag{3}$$

Now, the vector $|0\rangle$ is the joint probability density function of the complex amplitudes of the vacuum fluctuations in all the modes. The average value of an operator such as $N_v$ is obtained by operating on $|0\rangle$ to the left and right, so that

$$\langle N_v \rangle = \int_v \langle 0 | \psi^\dagger(x)\psi(x) | 0 \rangle \, dx \tag{4}$$
Equation (4) is very much like the usual expression in quantum field theory where the field operator is \( \psi = \sum b_i f_i \) and the state vector is \( b_0^\dagger |0\rangle \). Formally, we have done little more than move the creation operator \( b_0^\dagger \) into the field operator. But the effect of this rearrangement is that the operator rather than the ket contains the physical description of the system. It now becomes possible to inquire into the instantaneous values of objects like \( N_v \), as well as into their statistical properties.

The \( b_i \) and \( b_i^\dagger \) operate on the phase space of the amplitudes of the individual modes, not on configuration space, and therefore commute with the \( f_i \). Because by hypothesis there is exactly one particle in state 0, these operators must act like fermion field operators:

\[
\begin{align*}
    b_i |0\rangle &= 0 \tag{5} \\
    b_i b_j^\dagger |0\rangle &= \delta_{ij} |0\rangle \tag{6} \\
    b_i b_j^\dagger + b_j b_i^\dagger &= \delta_{ij} \tag{7} \\
    b_i b_j + b_j b_i &= 0 \tag{8}
\end{align*}
\]

It follows immediately from (3), (5), and (6) that

\[
\langle N_v \rangle = \int_v f_0^*(x) f_0(x) dx = m \tag{9}
\]

in agreement with ordinary quantum mechanics. Nothing in this result tells us whether, in individual realizations of the random process, \( N_v \) takes integral or fractional values. In other words, we do not yet know whether the particle is, at a given time, all in one place. This we investigate next.

To determine how compact the particle is, we examine higher moments of \( N_v \). The second moment is written, taking advantage of (5) and its hermitian conjugate, as follows:

\[
\langle N_v^2 \rangle = \langle 0 | \int_v \int_v \left[ b_0 b_0^\dagger b_0 b_0^\dagger f_0^*(x)f_0(x)f_0^*(z)f_0(z) \right] |0\rangle \tag{10}
\]

Using (7) and (8) gives

\[
\langle N_v^2 \rangle = \langle 0 | \int_v \int_v \left[ b_0 b_0^\dagger b_0 b_0^\dagger f_0^*(x)f_0(x)f_0^*(z)f_0(z) \right] |0\rangle \tag{11}
\]

The two terms can be combined to give

\[
\langle N_v^2 \rangle = \langle 0 | \int_v \int_v b_0 b_0^\dagger f_0^*(x)f_0(x) \sum_i b_i b_i^\dagger f_i(x)f_i^*(z) |0\rangle \tag{12}
\]

Now the closure relation\(^6\) states that if and only if the \( f_i \) are a complete set of basis functions, the following equation holds:

\[
\sum_n f_n^*(z)f_n(x) = \delta(z-x) \tag{13}
\]

Inserting this into (12) after simplifying with (6) yields

\[
\langle N_v^2 \rangle = \int_v \int_v f_0^*(x)f_0(z) \delta(x - z) \tag{14}
\]

The third moment is, using (5) to eliminate terms,

\[
\langle N_v^3 \rangle = \int_v \int_v \int_v f_0^*(x)f_0(y)f_0(z) \delta(x - y) \delta(y - z) \tag{15}
\]

Applying (6) and the anticommutation relations (7) and (8) gives

\[
\langle N_v^3 \rangle = \int_v \int_v \int_v f_0^*(x)f_0(y)f_0(z) \tag{16}
\]

Applying the closure relation twice yields

\[
\langle N_v^3 \rangle = \int_v \int_v \int_v f_0^*(x)f_0(x) \delta(x - y) \delta(y - z) \tag{18}
\]

A similar calculation for the fourth moment is summarized in Table 1. The result is \( \langle N_v^4 \rangle = m \).

The series \( \langle N_v \rangle = \langle N_v^2 \rangle = \langle N_v^3 \rangle = \langle N_v^4 \rangle = m \) gives the moments of a bimodal distribution in which \( N_v \) takes the
value 1 with probability $m$ and the value 0 with probability $1 - m$. This distribution holds for all subspaces $v$ if and only if $\psi^\dagger\psi$ is equal to a $\delta$ function with its peak at one point in space. Physically, this point is the place where there is constructive interference between the ordinary wave function $f_0(x)$ and the vacuum fluctuations.

If the anticommutation relations (7) and (8) are replaced by commutation relations, a calculation very similar to that presented above shows that the first three moments of $N_v$ take the values for Bose-Einstein statistics. If $b_{0}$ is an ordinary complex number and the remaining $b_{i}$ are operators, the moments of $N_{v}$ correspond to a Poisson distribution. (This last calculation is formally identical to the calculation that Henry and Kazarinov\[^{4}\] use to demonstrate that shot noise in a photodetector is due to interference with vacuum fluctuations.) In both of these cases, each particle is still localized as a discrete corpuscle, but more than one particle can now be present.

A measurement of the position of a particle whose wave function is $f_0(x)$ does not change the wave function but merely identifies the point where constructive interference occurs. What is referred to in wave mechanics as the post-measurement wave function can be thought of as the sum of the pre-measurement wave function and the measured components of the vacuum fluctuations.

One may ask how a particle whose location has been fixed at a point can continue to move with the indeterminacy of quantum mechanics, if the vacuum fluctuations have been measured well enough to determine that they cancel its wave function almost everywhere. The resolution of this apparent paradox relies on the existence of infinitely many modes of vacuum fluctuation. An experiment which measures the particle position at one time imposes an algebraic constraint, thus removing one (three-dimensional) degree of freedom from a system with an infinite number of degrees of freedom. After the measurement, the system retains its indeterminate character, with only an infinitesimal reduction in the degree of uncertainty.

### Measurement

Suppose we want to measure the value of some observable $O$ for the particle with wave function $f_0(x)$. The eigenfunctions of $O$, which for simplicity we assume nondegenerate, are $g_n(x)$.

In principle, to determine whether the value of $O$ is $n$, the wave function should be passed through a filter that passes only $g_n$. For example, in a beam the wave functions corresponding to different components of spin are separated magnetically, and a screen with a hole where one spin component is expected is placed in the beam. After the wave function is filtered, the amount of matter that remains is measured.

For simplicity, we again consider a particle in a box, with the filtering process occurring at the same instant throughout the box. From (1), the filtered wave function can be written as

$$\psi_n = b_{0}^\dagger f_{0n} + \sum_{i \neq 0} b_{i} f_{in}$$

where

$$f_{in} = \int g_{n}^*(x)f_{i}(x)dx$$

The average amount of matter detected is

$$\langle 0|\psi_n^\dagger\psi_n|0 \rangle = f_{0n}^*f_{0n}$$

Higher moments of $\psi^\dagger\psi$ can be calculated as in the previous section, using in place of (13) the relation

$$\sum_{i} f_{in}^*f_{in} = 1$$

which holds when $O$ is an observable. The moments are all equal, showing that either one particle is detected or none. Hence a measurement of an observable always yields an eigenvalue.

Let us apply these ideas to the classic Gedankenexperiment of Schrödinger’s cat. We consider a slight variation of this experiment in which there is a radioactive alpha source consisting of one nucleus with a half-life much shorter than the duration of the experiment. A detector is placed on one side, covering a solid angle of $2\pi$. As in the traditional version, the detector is connected to a hammer which breaks a vial of prussic acid, causing the cat to make a state transition from $|A\rangle$ (alive) to $|D\rangle$ (dead). Ordinary quantum mechanics instructs us to write the wave function of the cat at the end of the experiment as $2^{-1/2}|A\rangle + 2^{-1/2}|D\rangle$.

The alpha particle has a spherically symmetric wave function which dies off exponentially with time. In the view propounded here, the vacuum fluctuations interfere constructively with the particle wave function at some particular time along some path leading away from the source.

| Terms Like | Number of Terms | $m$ | $m^2$ | $m^3$ | $m^4$ |
|------------|----------------|-----|-------|-------|-------|
| $b_{n}b_{i}^\dagger b_{n}b_{i}^\dagger b_{n}b_{n}^\dagger$ | 1 | 1 | | | |
| $b_{n}b_{i}^\dagger b_{n}b_{i}^\dagger b_{n}b_{i}^\dagger b_{n}^\dagger$ | 3 | 3 | -3 | | |
| $b_{n}b_{i}^\dagger b_{j}^\dagger b_{n}b_{i}^\dagger b_{n}b_{i}^\dagger b_{n}^\dagger$ | 2 | 2 | -4 | 2 | |
| $b_{n}b_{i}^\dagger b_{i}^\dagger b_{n}b_{j}^\dagger b_{i}^\dagger b_{n}^\dagger$ | 1 | 1 | -2 | 1 | |
| $b_{n}b_{i}^\dagger b_{j}^\dagger b_{k}b_{i}^\dagger b_{n}^\dagger$ | 1 | 1 | -3 | 3 | -1 |
| **Total** | 1 | 0 | 0 | 0 | |
Schiff\textsuperscript{7} shows that in a given experiment, if the alpha particle is detected at a particular time and place, it will also be detected at immediately subsequent times near a line leading directly away from the detector. At other times and places, it will not be detected. If the path of constructive interference leads to the detector, the cat makes the transition to state $|D\rangle$; otherwise it remains in state $|A\rangle$. Once killed, the cat remains indefinitely in $|D\rangle$ because the barrier between the two states is large enough (as compared to Planck's constant) that the chance of a spontaneous transition to $|A\rangle$ is extremely small.

At any time, the state of the cat is either $|A\rangle$ or $|D\rangle$. The complex amplitudes of the vacuum fluctuations determine when and where the spherical wave function of the alpha particle collects itself in the form of an impulse, and consequently they determine whether it kills the cat. This gives the vacuum amplitudes a role resembling the hidden variables suggested by de Broglie, Bohm, and others.

Because the wave function of the vacuum fluctuations contains spatial correlations when more than one particle is present, this explanation is non-local. It thus is allowed by Bell's theorem\textsuperscript{8} which forbids any purely local theory of mechanics to give the same predictions as quantum mechanics.

**Discussion**

In previous writings the hidden variables have been supposed to be determined at the start of an experiment. The vacuum field invoked here is itself a quantum object, and it only takes a definite value when it is measured. One could argue that little has been accomplished in explaining quantum measurement, if the collapse of the particle wave function has merely been replaced by the collapse of the vacuum fluctuation wave function. But the topic is sufficiently difficult and controversial that even a partial explanation of how the wave function chooses the value to which it collapses is helpful.

Moreover, the quantum field (1) can be approximated very well by a field

$$\psi(x) = f_0(x) + \sum_i a_i f_i(x) \quad (23)$$

where the $a_i$ are ordinary random numbers that are constant through any one experiment. The $a_i$ can either be Gaussian random variables or have a fixed magnitude and random phase; in either case their mean square absolute value is $\langle a_i^2 a_i \rangle = \frac{1}{2}$. With this definition of $\psi(x)$, the matter density (3) becomes an ordinary function rather than an operator. As has been shown in detail for electromagnetic radiation,\textsuperscript{5} the matter density still adds up to corpuses concentrated at points. The result of any quantum measurement is determined by the well-defined numbers $a_i$, whose values were already fixed at the time the experimental system was prepared.

This approximation opens an escape hatch for interpreters who prefer a deterministic understanding of quantum mechanics. In quantum field theory, the vacuum-field wave functions are themselves harmonic oscillator solutions that can be put into the form (1). If the collapse of the ordinary wave function can be explained by invoking a second quantization, the collapse of the wave function of the vacuum fluctuations can equally well be derived from a third quantization. The third quantization would reflect a fourth, and so on \textit{ad infinitum}.

This series of approximations alternates deterministic explanations of apparently random events (the complex amplitudes of the vacuum fluctuations that determine the value to which the wave function collapses) with the introduction of new elements of probability (the more exact quantum description of the collapse of the vacuum fluctuation wave function). Infinite regression leaves unresolved the philosophical choice between chance or determinism as the basis of quantum mechanics. The physics is, as far as can be seen from this vantage point, compatible with either philosophical belief.

The absence of any ultimate end to this series is a disappointment, to be sure. But other series of sequential explanations in physics, such as matter made of atoms containing bosons made of quarks, also lack a known end. No physicist would forego use of atomic theory while awaiting the discovery of particles that are truly indivisible.

**Conclusions**

Quantum mechanical theory can be connected directly to experiment by interpreting the wave function as matter itself rather than an instruction about where matter will be found. The sum of the single-particle wave function and the vacuum fluctuations is a discrete corpuscle at one point and zero everywhere else. The corpuscle is located at a location which is determined by the random complex amplitudes of the vacuum fluctuations, which play a role like that of hidden variables by supplementing the wave function to determine where a particle is observed. The principle that the probability that a particle will appear at any point is the squared amplitude of its wave function follows from the probability density function of the vacuum fluctuations and need not be assumed.

When the value of an observable is measured, interference with vacuum fluctuations ensures that the result is an eigenvalue. At the time of the measurement, the sum of wave function and vacuum fluctuations is the eigenfunction that corresponds to the measured eigenvalue. Thus the apparent effect of the measurement is to “collapse” the wave function into an eigenfunction.

The amplitudes of the vacuum fluctuations are themselves quantum variables that have wave functions rather than fixed values. The “hidden variables” can thus be un-
derstood as being intrinsically indeterminate themselves.

Alternatively, the vacuum fluctuation amplitudes can be approximated as random numbers that have well defined fixed values during any experiment. In this approximation, the value to which a wave function will collapse is determined before an experiment starts by the initial values of the vacuum fluctuation amplitudes. If one improves on this description by treating these amplitudes as quantum variables whose wave functions collapse at the time of a measurement, the collapse can be explained as interference with the vacuum fluctuations of a “third quantization”. These, in turn, collapse due to interference with vacuum fluctuations of yet higher order. In the end, how a quantum wave function chooses the value to which it collapses is described by an infinite series of successive approximations, in which deterministic and random descriptions of reality alternate.

Notes

1 J. S. Bell, in A. I. Miller, ed., *62 Years of Uncertainty*, Plenum, New York, 1990, 17.

2 J. G. Cramer, Rev. Mod. Phys. **58**, 647 (1986).

3 D. Bohm and B. J. Hiley, *The Undivided Universe*, Routledge, London and New York, 1993.

4 C. H. Henry and R. F. Kazarinov, Rev. Mod. Phys. **68**, 801 (1996).

5 See, e. g., T. H. Boyer, Phys. Rev. **D11**, 809 (1975).

6 A. Messiah, *Quantum Mechanics*, North-Holland, Amsterdam, 1965, Eq. (V.37).

7 L. I. Schiff, *Quantum Mechanics*, 3rd edn., McGraw-Hill, New York, 1968.

8 J. S. Bell, Physics **1**, 195 (1964).