T Violating Muon Polarization in $K^+ \rightarrow \mu^+ \nu \gamma$

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Abstract

We study the T violating transverse muon polarization in the decay of $K^+ \rightarrow \mu^+ \nu \gamma$ due to CP violation in theories beyond the standard model. We find that the polarization asymmetry could be large in some CP violation models and it may be detectable at the ongoing KEK experiment of E246 as well as the proposed BNL experiment.
1 Introduction

In the framework of local quantum field theories, with Lorentz invariance and the usual spin-statistics connection, time-reversal (T) violation implies CP violation (and vice versa), because of the CPT invariance of such theories. Experimentally, only CP violation has been observed so far and this only in the neutral kaon system. But the origin of this violation remains unclear. In the standard model, CP Violation arises from a unique physical phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix \([1]\). To ensure that this phase is indeed the source of CP violation or T violation, one needs to look for new processes, especially that outside \(K^0\) system. It would be particularly interesting if the time reversal symmetry is directly violated in these processes, rather than inferring it as a consequence of CPT invariance.

Within kaon physics, the most interesting search of T violation would be to look for the component of the muon polarization normal to the decay plane, called transverse muon polarization \((P_T)\), in the charged kaon decays such as \(K^+ \to \pi^0\mu^+\nu\) \((K_{\mu3}^+)\) \([2]\), \(K^+ \to \mu^+\nu\gamma\) \((K_{\mu2\gamma}^+)\) \([3]\) and \(K^+ \to \pi^+\mu^+\mu^-\) \([4]\). The polarization in \(K_{\mu3}^+\) as well as that in \(K_{\mu2\gamma}^+\) will be measured to a high accuracy at the ongoing KEK E246 experiment \([5]\) and at a recently proposed BNL experiment \([6]\), while for \(K^+ \to \pi^+\mu^+\mu^-\) that would be done in a future kaon factory \([7]\).

In this paper, we concentrate on the radiative \(K_{\mu2}^+\) decay. We will first give a general analysis on all components of the muon polarization and then present our estimations on the transverse component in various CP violation theories beyond the standard model. The transverse muon polarization in the decay of \(K^+ \to \mu^+\nu\gamma\) is related to the T odd triple correlation, i.e.,

\[ P_T \propto \bar{s}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\gamma), \tag{1} \]

where \(\bar{s}_\mu\) is the muon spin vector and \(\vec{p}_i\), \((i = \mu \text{ and } \gamma)\) the momenta of the muon and photon in the rest frame of \(K^+\), respectively. It is expected \([8]\) that the CKM phase does not induce the polarization in Eq. (1). Therefore measuring the polarization could be a signature of physics beyond the standard model. There are many different sources that might give rise to the polarization. The most exciting ones are the weak CP violation from some kinds of non-standard CP violation models. However, the electromagnetic interaction among the final state particle can also make a contribution, which is usually less interesting and it could even hide the signals from the weak CP violation. We shall
refer the final-state-interaction (FSI) contribution as a theoretical background of that by the weak CP violation. It has been estimated that $P_T \sim 10^{-3}$ and $10^{-6}$ in $K^{0}_{\mu 3}$ $(K^{0} \rightarrow \pi^{-}\mu^{+}\nu)$ [9] and $K^{+}_{\mu 3}$ $(K^{+} \rightarrow \pi^{0}\mu^{+}\nu)$ [10], due to the FSI effects at one and two-loop levels, respectively. For $K^{+} \rightarrow \mu^{+}\nu\gamma$, although there is only one charged final state particle like $K^{+}_{\mu 3}$, FSI arises at one-loop diagrams because of the existence of the photon in the final states [11, 12] and it is found that $P_{FSI}T(K^{+} \rightarrow \mu^{+}\nu\gamma) \sim 10^{-3}$ in most of the decay allowed phase space [13]. To distinguish the real CP violating effects from the FSIs, one has to explore various possible models with the polarization being larger than $10^{-3}$.

The paper is organized as follows. In Sec. 2, we carry out a general analysis of the muon polarization in $K^{+} \rightarrow \mu^{+}\nu\gamma$. In Sec. 3, we study the CP violating muon polarization effects in some extensions of the standard model. We give our concluding remarks in Sec. 4.

2 General Analysis for Muon Polarization

For a general investigation of the transverse muon polarization in $K^{+} \rightarrow \mu^{+}\nu\gamma$ decay including some new CP violating sources, we first carry out the most general four-Fermion interactions given by

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \sin \theta_c \bar{s} \gamma^\alpha (1 - \gamma_5) u \bar{\nu} \gamma_\alpha (1 - \gamma_5) \mu + G_S \bar{s} u \bar{\nu} (1 + \gamma_5) \mu + G_P \bar{s} \gamma_5 u \bar{\nu} (1 + \gamma_5) \mu + G_V \bar{s} \gamma^\alpha u \bar{\nu} \gamma_\alpha (1 - \gamma_5) \mu + h.c.,$$

where $G_F$ is the Fermi constant, $\theta_c$ is the Cabibbo mixing angle, and $G_S$, $G_P$, $G_V$, and $G_A$, arising from new physics, denote scalar, pseudoscalar, vector, and axial vector interactions, respectively. From the interactions in Eq. (2), we can write the amplitude of the decay $K^{+}_{\mu 2\gamma}$ in terms of “inner bremsstrahlung” (IB) and “structure-dependent” (SD) contributions, which can be written as [3, 14]

$$M = M_{IB} + M_{SD},$$

with

$$M_{IB} = i e \frac{G_F}{\sqrt{2}} \sin \theta_c f_{K} m_{\mu} \epsilon^{*}_{\alpha} K^\alpha,$$

$$M_{SD} = -i e \frac{G_F}{\sqrt{2}} \sin \theta_c \epsilon^{*}_{\mu} L_{\nu} H^{\mu \nu},$$

where

$$K^\alpha = \bar{u}(p_{\nu})(1 + \gamma_5) \left( \frac{p^{\alpha}}{p \cdot q} - \frac{2p^{\alpha} + q \gamma^{\alpha}}{2p_{\mu} \cdot q} \right) v(p_{\mu}, s_{\mu}),$$
\[ L_\nu = \bar{u}(p_\nu) \gamma_\nu (1 - \gamma_5) v(p_\mu, s_\mu), \]
\[ H^{\mu\nu} = \frac{F_A}{m_K} (-g^{\mu\nu} p \cdot q + p^\mu q^\nu) + i \frac{F_V}{m_K} \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta, \] (4)

\( \epsilon_\alpha \) is the photon polarization vector, \( p, p_\nu, p_\mu \), and \( q \) are the four-momenta of \( K^+, \nu, \mu^+, \) and \( \gamma \), respectively, \( s_\mu \) is the polarization vector of the muon, and \( f_K, F_V, \) and \( F_A \) are the form factors given by

\[
\begin{align*}
    f_K &= f^0_K (1 + \Delta_P + \Delta_A), \\
    F_A &= F^0_A (1 + \Delta_A), \\
    F_V &= F^0_V (1 - \Delta_V),
\end{align*}
\] (5)

with

\[
\Delta_{(P,A,V)} = \frac{\sqrt{2}}{G_F \sin \theta_c} \left( \frac{G_P m_K^2}{(m_s + m_u) m_\mu}, G_A, G_V \right). \] (6)

Here \( f^0_K \) is the kaon decay constant and \( F^0_V(A) \) the vector (axial-vector) form factor, defined by

\[
\begin{align*}
    \langle 0| \bar{s} \gamma^\mu \gamma_5 u |K^+(p)\rangle &= -i f^0_K p^\mu, \\
    \int dxe^{iqx} \langle 0| T(J^\mu_{em}(x)\bar{s} \gamma^\nu \gamma_5 u(0)) |K^+(p)\rangle &= -f^0_K \left( g^{\mu\nu} + \frac{p^\mu (p - q) p^\nu}{p \cdot q} \right) + \frac{F^0_A}{m_K} (g^{\mu\nu} p \cdot q - p^\mu q^\nu), \\
    \int dxe^{iqx} \langle 0| T(J^\mu_{em}(x)\bar{s} u(0)) |K^+(p)\rangle &= i \frac{F^0_V}{m_K} \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta,
\end{align*}
\] (7)

and the hadronic matrix elements involving the scalar and pseudoscalar currents in Eq. (4) are given by [13]

\[
\begin{align*}
    \langle 0| \bar{s} \gamma_5 u |K^+(p)\rangle &= i f^0_K \frac{m^2_K}{m_s + m_u}, \\
    \int dxe^{iqx} \langle 0| T(J^\mu_{em}(x)\bar{s} \gamma_5 u(0)) |K^+(p)\rangle &= f^0_K \frac{m^2_K}{m_s + m_u} \frac{p^\mu}{p \cdot q}, \\
    \int dxe^{iqx} \langle 0| T(J^\mu_{em}(x)\bar{s} u(0)) |K^+(p)\rangle &= 0,
\end{align*}
\] (8)

where \( J^\mu_{em} \) is the electromagnetic current. Numerically, one has \( f^0_K = 0.16 \text{ GeV} \) from the experiment and \( F^0_V = -0.095 \) and \( F^0_A = -0.043 \) found in the chiral perturbation theory at one-loop level [14].

We use the standard techniques to calculate the probability of the process \( K^+ \to \mu^+ \nu \gamma \) as a function of the 4-momenta of the particles and the polarization 4-vector \( s_\mu \) of the
muon. We write the components of \( s_\mu \) in terms of \( \vec{\xi} \), a unit vector along the muon spin in its rest frame, as

\[
s_0 = \frac{\vec{p}_\mu \cdot \vec{\xi}}{m_\mu}, \quad \vec{s} = \vec{\xi} + \frac{s_0}{E_\mu + m_\mu} \vec{p}_\mu.
\]  

(9)

In the rest frame of \( K^+ \), the partial decay width is found to be

\[
d\Gamma = \frac{1}{2m_K} |M|^2 (2\pi)^4 \delta(p - p_\mu - p_\nu - q) \frac{d\vec{q}}{(2\pi)^3 2E_q} \frac{d\vec{p}_\mu}{(2\pi)^3 2E_\mu} \frac{d\vec{p}_\nu}{(2\pi)^3 2E_\nu},
\]

(10)

with

\[
|M|^2 = \rho_0(x,y) \left[ 1 + (P_L \vec{e}_L + P_N \vec{e}_N + P_T \vec{e}_T) \cdot \vec{\xi} \right]
\]

(11)

where \( \vec{e}_i \) \( (i = L, N, T) \) are the unit vectors along the longitudinal, normal and transverse components of the muon polarization, defined by

\[
\vec{e}_L = \frac{\vec{p}_\mu}{|\vec{p}_\mu|}, \quad \vec{e}_N = \frac{\vec{p}_\mu \times (\vec{q} \times \vec{p}_\mu)}{|\vec{p}_\mu \times (\vec{q} \times \vec{p}_\mu)|}, \quad \vec{e}_T = \frac{\vec{q} \times \vec{p}_\mu}{|\vec{q} \times \vec{p}_\mu|},
\]

(12)

respectively, and

\[
\rho_0(x,y) = \frac{1}{2} e^2 G_F^2 \sin^2 \theta_c (1 - \lambda) \left\{ \frac{4m_\mu^2 |f_K|^2}{\lambda x^2} \left[ x^2 + 2(1 - r_\mu) \left( 1 - x - \frac{r_\mu}{\lambda} \right) \right] 
+ m_K^4 x^2 \left[ |F_V + F_A|^2 \frac{\lambda^2}{1 - \lambda} \left( 1 - x - \frac{r_\mu}{\lambda} \right) + |F_V - F_A|^2 (y - \lambda) \right] 
- 4m_K m_\mu^2 \left[ \text{Re}[f_K (F_V + F_A)^*] \left( 1 - x - \frac{r_\mu}{\lambda} \right) \right.
\left. - \text{Re}[f_K (F_V - F_A)^*] \frac{1 - y + \lambda}{\lambda} \right] \right\}
\]

(13)

with \( \lambda = (x + y - 1 - r_\mu)/x \), \( r_\mu = m_\mu^2/M_K^2 \) and \( x = 2p \cdot q/p^2 = 2E_\gamma/m_K \) and \( y = 2p \cdot p_\mu/p^2 = 2E_\mu/m_K \) being normalized energies of the photon and muon, respectively. If we define the longitudinal, normal and transverse, muon polarization asymmetries by

\[
P_i(x,y) = \frac{d\Gamma(\vec{e}_i) - d\Gamma(-\vec{e}_i)}{d\Gamma(\vec{e}_i) + d\Gamma(-\vec{e}_i)}, \quad (i = L, N, T),
\]

(14)

we find that

\[
P_i(x,y) = \frac{\rho_i(x,y)}{\rho_0(x,y)}, \quad (i = L, N, T),
\]

(15)
Eq. (14). We rewrite 

$$\rho_L(x, y) = -e^2G_F^2\sin^2\theta_c \frac{(1-\lambda)}{2\lambda \sqrt{y^2 - 4r_\mu}} \left\{ \frac{4m^2_{\mu}|f_K|}{\lambda x^2} [x(\lambda y - 2r_\mu)(x + y - 2\lambda) - (y^2 - 4r_\mu)(\lambda x + 2r_\mu - 2\lambda)] - M^2_K\lambda x^2 \left[ |V + A|^2 \frac{\lambda}{1-\lambda} (\lambda y - 2r_\mu) \right] \right\}$$

$$\rho_N(x, y) = e^2G_F^2\sin^2\theta_c \frac{(1-\lambda)}{M_K\lambda \sqrt{y^2 - 4r_\mu}} \left\{ \frac{4m^2_{\mu}|f_K|^2}{\lambda x}(x + y - 2\lambda) \right\}$$

$$\rho_T(x, y) = -2e^2G_F^2\sin^2\theta_c m^2_K m_{\mu} \frac{1-\lambda}{\lambda} \sqrt{\lambda y - \lambda^2 - r_\mu} \left\{ Im[f_K(F_V + F_A)\ast] \frac{\lambda}{1-\lambda} \right\}$$

$$\times \left( 1 - x - \frac{r_\mu}{\lambda} \right) + Im[f_K(F_V - F_A)\ast] \right\}.$$  \quad (16)

From Eq. (14), it is easily seen that the asymmetries of \(P_L\) and \(P_N\) are even quantities under time-reversal transformation, while \(P_T\) is an odd one. Since we are interested in CP violation, we will concentrated on the transverse part of the polarization shown in Eq. (14). We rewrite \(P_L(x, y)\) as

$$P_T(x, y) = P_T^V(x, y) + P_T^A(x, y) \quad (17)$$

with

$$P_T^V(x, y) = \sigma_V(x, y) [Im(\Delta_A + \Delta_V)],$$

$$P_T^A(x, y) = [\sigma_V(x, y) - \sigma_A(x, y)] Im(\Delta_P), \quad (18)$$

where

$$\sigma_V(x, y) = 2e^2G_F^2\sin^2\theta_c m^2_K m_{\mu} f^0_K F^0_V \sqrt{\frac{\lambda y - \lambda^2 - r_\mu}{\rho_0(x, y)}} \left( \frac{-1 + \lambda}{\lambda} - \left( 1 - x - \frac{r_\mu}{\lambda} \right) \right),$$

$$\sigma_A(x, y) = 2e^2G_F^2\sin^2\theta_c m^2_K m_{\mu} f^0_K F^0_A \sqrt{\frac{\lambda y - \lambda^2 - r_\mu}{\rho_0(x, y)}} \left( \frac{-1 + \lambda}{\lambda} + \left( 1 - x - \frac{r_\mu}{\lambda} \right) \right). \quad (19)$$
Clearly, to have CP violating transverse muon polarization of $P_T$ in Eq. (18), at least one of the couplings $G_i \ (i = P, A, V)$ in Eq. (2) has to exist and be complex.

In the standard model, from Eqs. (17) and (18), we see that the transverse muon polarization is zero since $f_K = f_K^0$ and $F_{V(A)} = F_{V(A)}^0$ which are both real due to $G_i = 0$ and $\Delta_i = 0 \ (i = P, V, A)$. However, the longitudinal and normal muon polarizations are non-vanishing. In Figs. 1-3, we show the Dalitz plots for $\rho_0$, $P_L$, and $P_N$, respectively. In the non-standard models, $P_T$ could be non-zero if the new physics couplings $G_i \ (i = P, A, V)$ have some phases. In Figs. 4 and 5, we display the Dalitz plots of $\sigma_V(x, y)$ and $\sigma_V(x, y) - \sigma_A(x, y)$, respectively. From the figures, we see that they are all in the order of $10^{-1}$ in most of the allowed parameter space. We shall use $\sigma_V$ and $\sigma_V - \sigma_A$ as 0.1 in our numerical estimations of the next section. However, it is interesting to note that there is no contribution to the transverse muon polarization if the interaction beyond the standard model contains only left-handed vector current, i.e., $G_V = -G_A$, because of the zero relative phase between the amplitudes of $M_{IB}$ and $M_{SD}$.

3 Transverse Muon Polarization in Various Models

Since $P_T(K_{\mu2\gamma}^+) = 0$ at tree level in the standard model, a nonzero value of the transverse muon polarization provides an evidence for new CP violating source outside the CKM mechanism after taking care of the theoretical background. In such case, $P_T$ may arise from the interference between the tree level amplitude in the standard model and the new CP violating amplitude. In the following we will study various CP violation models such as the left-right symmetric, two-Higgs-doublets, supersymmetry (SUSY), and leptoquark models and discuss the possibilities of having large $P_T$ in these theories.

3.1 Left-Right symmetric models

In this subsection we study the prediction of T-violating muon polarization for $K_{\mu2\gamma}^+$ decay in models with left-right symmetric gauge symmetries $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [10]. In these models, the minimal set of Higgs multiplets to break the gauge symmetry down to $U(1)_{em}$ is: one doublet $\phi$ and two triplets $\Delta_{L,R}$. The transformations of the Higgs bosons under $SU(2)_L \times SU(2)_R$ are given by

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ -\phi_1^- & \phi_2^+ \end{pmatrix} \rightarrow U_L \phi U_R^\dagger,$$

$$\Delta_{L,R} = \begin{pmatrix} \delta^+ / \sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+ / \sqrt{2} \end{pmatrix}_{L,R} \rightarrow U_{L,R} \Delta_{L,R} U_{L,R}^\dagger, \quad (20)$$
with the vacuum expectation values (VEVs) being

\[
\langle \phi \rangle = e^{i\alpha} \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}
\]

\[
\langle \Delta_{L,R} \rangle = e^{i\theta_{L,R}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix},
\]

(21)

where \(\alpha\) and \(\theta_{L,R}\) are the CP violating phases. After the spontaneous symmetry breaking (SSB), the masses of fermions can be generated by the Yukawa coupling terms

\[
\mathcal{L}_Y = \bar{Q}_L F \phi Q_R + \bar{Q}_L \tilde{G} \phi Q_R + h.c.,
\]

(22)

where the generation indices have been suppressed, \(Q_{L,R}\) stand for the left- and right-handed fermions doublets, \(F\) and \(G\) correspond to the \(N \times N\) mass matrices for \(N\) generations, and \(\phi \equiv \tau_2 \phi^* \tau_2\). In addition, the eigenstates of both \(W_{L,R}\) bosons related to weak eigenstates can be written as

\[
W_1 = \cos \xi W_L + \sin \xi W_R,
\]

\[
W_2 = -\sin \xi W_L + \cos \xi W_R,
\]

(23)

where \(\xi\) is the left-right mixing angle. In any event, due to the \(W_R\) gauge boson, we have new mixing matrix, called right-handed CKM (RCKM) matrix, coming from the diagonalization of right-handed quarks. The charged current interactions are given by

\[
\mathcal{L}_{CC} = \frac{g_L}{\sqrt{2}} W^\mu_L \bar{u} \gamma_\mu K^L P_L D + \frac{g_R}{\sqrt{2}} W^\mu_R \bar{u} \gamma_\mu K^R P_R D + h.c.,
\]

(24)

where \(g_L\) and \(g_R\) are coupling constants for \(SU(2)_L\) and \(SU(2)_R\), \(U^T = (u,c,t)\) and \(D^T = (d,s,b)\) are the physical states of up-type quarks and down-type quarks, \(K^L\) and \(K^R\) are the CKM and RCKM matrices, and \(P_{L,R} = (1 \mp \gamma_5)/2\), respectively.

The number of physical CP violating phases in CKM and RCKM matrices with \(N\) generations are \(N_L = (N-1)(N-2)/2\) and \(N_R = N(N+1)/2\), respectively. For example, we have three CP violating phases from \(K^R\) for the case of two generations. Here, we do not assume parity invariant as well as any special relation in the matrices so that, in general, the gauge coupling constant \(g_L\) is not equal to \(g_R\) and the matrix elements of RCKM are free parameters. Including the left-right mixing parameter \(\xi\) we can generate \(P_{T}(K_{\mu 2\gamma}^+)\) from the tree diagram as shown in Fig. 6. The quark level four-Fermion interaction which contributes to \(K_{\mu 2\gamma}^+\) decay is given by

\[
\mathcal{L}_{RL} = -2\sqrt{2} G_F \frac{g_R}{g_L} K_{us}^R \xi \bar{s} \gamma_\mu P_R u \bar{\nu} \gamma_\mu P_L \mu.
\]

(25)
Compare Eq. (25) with Eq. (2), we get
\[ G_V = G_A = -\frac{G_F g_R}{\sqrt{2} g_L} K^R_{us} \xi, \] (26)

which leads to
\[ \Delta_A = \Delta_V = -\frac{\xi K^R_{us} g_R}{\sin \theta_c g_L}, \] (27)
in terms of Eq. (2). From the expression of \( \delta_{AV} \) in Eq. (27) and the definition of the muon transverse polarization in Eq. (17), we obtain
\[ P_T = 2\sigma_V \frac{\xi g_R}{g_L} \text{Im}(K^R_{us}). \] (28)

To illustrate the prediction of the transverse polarization, we consider two-generation case. The RCKM matrix which contains three physical phases can be parametrized as
\[ U_R = e^{i\gamma} \begin{bmatrix} e^{-i\delta_2 \cos \theta_c} & e^{-i\delta_1 \sin \theta_c} \\ -e^{i\delta_1 \sin \theta_c} & e^{i\delta_2 \cos \theta_c} \end{bmatrix}, \] (29)

\( \delta_{1,2} \) and \( \gamma \) are the three physical CP violating phases, and \( \theta_c \) is the Cabibbo angle. From the mixing matrix in Eq. (29), we find that
\[ \text{Im}(K^R_{us}) = \sin \theta_c \sin(\gamma - \delta_1) \leq \sin \theta_c. \] (30)

In the typical left-right symmetric models as shown in Ref. [17], one generally has
\[ \xi \frac{g_R}{g_L} < 4.0 \times 10^{-3}, \] (31)

and thus one gets
\[ P_T < 8.0 \times 10^{-4} \] (32)

with choosing that \( \sigma_V \sim 0.1 \) and \( \text{Im}(K^R_{us}) \sim \sin \theta_c \). However, in a class of the specific models studied in Ref. [18], it is found that
\[ \xi \frac{g_R}{g_L} < 3.3 \times 10^{-2} \] (33)

for \( M_R > 549 \text{ GeV} \). In such models, with the same set of the parameters \( \sigma_V \) and \( \text{Im}(K^R_{us}) \), one gets
\[ P_T < 6.6 \times 10^{-3}, \] (34)

which is within the experimental detecting range. The bounds in Eqs. (32) and (34) can be even larger if one uses a larger value of \( \sigma_V \). We remark that the muon transverse polarization for the decay of \( K^+ \rightarrow \pi^0 \mu^+ \nu \), i.e., \( P_T(K^+_{\mu^3}) \), vanishes in the left-right symmetric models as shown in Ref. [19]. This is because the photon is a vector particle while the pion is a pseudo-scalar one.
3.2 Two-Higgs-Doublet Models with FCNC

In a two-Higgs doublet model (THDM) without introducing global symmetry [20], the up and down-type quarks will couple to the both two Higgs doublets. However, the up and down-type quarks mass matrices cannot be simultaneously diagonalized. Therefore, flavor changing neutral current (FCNC) is induced at tree level. To suppress the effects of the FCNC, one can impose a discrete symmetry. However, the discrete symmetry normally also constrains the scalar potential such that spontaneous CP violation (SCPV) may not occur. In this section, instead of having a discrete symmetry, we assume that the couplings related to the FCNC are small. The various possible theories with naturally small couplings have been explored in Refs. [21, 22].

The Yukawa coupling terms in the weak eigenstate can be written by

\[
L_Y = \eta_{ij}^D \bar{Q}_L \phi_1 D_{Rj} + \eta_{ij}^U \bar{Q}_L \tilde{\phi}_1 U_{Rj} + \xi_{ij}^D \bar{Q}_L \phi_2 D_{Rj} + \xi_{ij}^U \bar{Q}_L \tilde{\phi}_2 U_{Rj} +
\]

\[
\eta_{ij}^E \bar{L}_i \phi_1 E_{Rj} + \xi_{ij}^E \bar{L}_i \tilde{\phi}_2 U_{Rj} + h.c.,
\]

(35)

where \( \eta_{ij}^{U,D,E} \) and \( \xi_{ij}^{U,D,E} \) are dimensionless and real parameters, \( Q_L \) and \( L \) denote the left-handed quark and lepton doublets, \( D_R, U_R, \) and \( E_R \) are the right-handed down-type quarks, up-type quarks, and leptons, \( \phi_1 \) and \( \phi_2 \) are the two Higgs doublets with \( \tilde{\phi} \equiv i\sigma_2\phi^* \) respectively. The VEVs of the Higgs doublets are given by \( \langle \phi_1 \rangle = v_1 \) and \( \langle \phi_2 \rangle = e^{i\theta}v_2, \) where \( \theta \) is the CP violating phase. From Eq. (35) the Yukawa interactions of quarks and leptons with neutral and charged Higgs bosons can be expressed by

\[
L_{NH} = \frac{1}{\sqrt{2}} \bar{U}_L \tilde{\xi}_U U_R (h^0 - iA^0) + \frac{1}{\sqrt{2}} \bar{D}_L \tilde{\xi}_D D_R (h^0 + iA^0) + h.c.,
\]

\[
L_{CH} = -\bar{U}_R \xi_U^\dagger K^L D_L H^\dagger + \bar{U}_L K^L \xi_D^\dagger D_R H^\dagger + \bar{N}_L K^\dagger \xi_E^\dagger E_R E^\dagger H^\dagger + h.c.,
\]

(36)

respectively, with

\[
\tilde{\xi}_U \equiv V_L^U (\eta^U \sin \beta + e^{-i\theta} \xi^U \cos \beta) V_R^U, \\
\tilde{\xi}_D \equiv V_L^D (\eta^D \sin \beta + e^{i\theta} \xi^D \cos \beta) V_R^D, \\
\tilde{\xi}_E \equiv (\eta^E \sin \beta + e^{i\theta} \xi^E \cos \beta) V_R^E, 
\]

(37)

where flavor indices are suppressed, \( V_{L,R}^{U,D,E} \) are the unitary matrices which transform the fermionic weak-eigenstates to the mass-eigenstates, and \( \tan \beta = v_2/v_1. \) However, for simplicity we can reparameterize the diagonal parts of matrix \( \xi^E \) to be \( (\xi^E)_{ii} \equiv zm_l, \) where \( z \) is unknown parameter and \( m_l \) is the lepton mass. The parameter \( z \) can be bounded by the \( \mu-e \) universality in tau decay.
From Eq. (36), we find the following four-Fermion interaction
\[
\mathcal{L}_{K_{\mu 2}^+} = \frac{z m_{\mu}}{M_H^2} s \left( - \sum_j K_j^{L*} \bar{\xi}_{ij} P_R + \sum_i \bar{\xi}_{i2}^{D*} K_{ii}^{L*} P_L \right) u \bar{\nu} P_R \mu ,
\] (38)
where \( M_H \) is the mass of charged Higgs particle. From Eq. (36), we clearly see that the off-diagonal elements \( \bar{\xi}_{ij}^{U,D} \) \((i \neq j)\) are related to the FCNC at tree level. To illustrate the polarization effect, we use the ansatz in Ref. [23] by Cheng and Sher, in which the couplings \( \bar{\xi}_{ij}^{U,D} \) in Eq. (38) are taken to be
\[
\bar{\xi}_{ij}^{U,D} = \lambda_{ij} \frac{\sqrt{m_i^{U,D} m_j^{U,D}}}{v} \quad \text{for } i \neq j,
\] (39)
where \( \lambda_{ij} \) are undetermined parameters, \( v = \sqrt{v_1^2 + v_2^2} = (2\sqrt{2}G_F)^{-1/2} \), and \( m_i^{U(D)} \) the masses of up (down)-type quarks. We now simplify Eq. (38) to
\[
\mathcal{L}_{K_{\mu 2}^+} = \frac{z m_{\mu}}{M_H^2} s K_{us}^{L*} (\bar{\xi}_{12}^{U} P_R + \bar{\xi}_{22}^{D*} P_L) u \bar{\nu} P_R \mu ,
\] (40)
by using Eq. (39). We consider the constraints on \( \bar{\xi}_{11}^{U} \) and \( \bar{\xi}_{22}^{D*} \) from the \( K^0 - \bar{K}^0 \) mixing, arising from the box diagrams with the \( \mu \)-quark as the internal fermion as shown in Fig. 7. From the figures, we find that
\[
<K^0 | M_{HH}^{box} | \bar{K}^0> = -\frac{f_c^2 m_K}{6(4\pi)^2 M_H^2} (K_{us}^{L*} K_{us}^{L})^2 (\bar{\xi}_{12}^{U} \bar{\xi}_{22}^{D*} - \bar{\xi}_{11}^{U} \bar{\xi}_{11}^{D*})^2,
\]
\[
<K^0 | M_{HW}^{box} | \bar{K}^0> = \frac{4\sqrt{2}G_F f_c^2 m_K}{(4\pi)^2} \frac{m_\mu^2}{3 m_s^2} (K_{us}^{L*} K_{us}^{L})^2 \bar{\xi}_{11}^{D*} \bar{\xi}_{22}^{D*} D_{HW} \left( \frac{M_H^2}{M_W^2}, \frac{m_\mu^2}{M_W^2} \right),
\] (41)
with
\[
D_{HW}(x, y) = \frac{\ln x}{2x} + \frac{\ln y}{2y} + \frac{\ln(xy)}{2xy}.
\] (42)
Using the experimental value on the \( K^0 - \bar{K}^0 \) mixing, we obtain
\[
|\bar{\xi}_{11}^{D*} \bar{\xi}_{22}^{D*} - \bar{\xi}_{11}^{U} \bar{\xi}_{11}^{D*}| < 7.3 \times 10^{-5} M_H \text{ GeV}^{-1},
\]
\[
|\bar{\xi}_{11}^{D*} \bar{\xi}_{22}^{D*}| < 1.3 \times 10^{-2} m_s \frac{\text{GeV}^{-1}}{\sqrt{D_{HW}(M_H^2/M_W^2, m_\mu^2/M_W^2)}}.
\] (43)
For \( M_H \sim 200 \text{ GeV} \), we find that \( |\bar{\xi}_{11}^{D*} \bar{\xi}_{22}^{D*}| < 5.6 \times 10^{-3} \) and thus,
\[
|\bar{\xi}_{11}^{U}| < 8.6 \times 10^{-3} \sqrt{M_H}.
\] (44)
The strict constraint on \( z \), as pointed out by Grossman [24], is from the \( \mu \)-e universality in \( \tau \) decay as well as the perturbativity bound, and is given by
\[
|z| < \min(1.1 \times 10^{-2} M_H \text{ GeV}^{-2}, 2.0 \text{ GeV}^{-1}).
\] (45)
One notes that if \( M_H > 175 \, \text{GeV} \), the bound on \( z \) mainly comes from the perturbativity as shown in Eq. (43). From Eqs. (2), (6) and (40), we get

\[
G_P = -\frac{z m_\mu}{4 M_H^2} \frac{4}{K^*_{us}} (\tilde{\xi}^U_{11} + \tilde{\xi}^D_{22}), \tag{46}
\]

and

\[
\Delta_P = -\frac{m_K^2}{m_s + m_u} \frac{\sqrt{2} z}{4 G_F M_H^2} (\tilde{\xi}^U_{11} + \tilde{\xi}^D_{22}), \tag{47}
\]

which leads to the muon transverse polarization as

\[
P_T = (\sigma_V - \sigma_A) \frac{m_K^2}{m_s + m_u} \frac{\sqrt{2} z}{4 G_F M_H^2} \text{Im}(\tilde{\xi}^U_{11} + \tilde{\xi}^D_{22}). \tag{48}
\]

Using the constraints in Eqs. (44) and (45), we obtain

\[
P_T \leq 0.048 \tag{49}
\]

for \( |\sigma_V - \sigma_A| \sim 0.1 \) and \( M_H \sim 100 \, \text{GeV} \).

Using the interaction in Eq. (40) by neglecting the contribution of \( \tilde{\xi}^D_{22} \), the similar estimation for the transverse muon polarization in \( K^+ \to \pi^0 \mu^+ \nu \) can be done. By taking the same values of parameters as in the decay of \( K_{\mu2}\gamma \), we find that \( P_T(K_{\mu3}^+) \) could be as large as the current experimental limit, \( i.e., \, 1 \times 10^{-2} \). Therefore, \( P_T \) in both decays of \( K_{\mu2}\gamma \) and \( K_{\mu3}^+ \) can be very large in the Higgs models with FCNC. We note that the muon polarization effects of the two modes in the three Higgs-doublet models with NFC could be also large as studied in Refs. \[15, 19, 25\].

### 3.3 Supersymmetric Models

In this subsection we consider the effects on \( P_T \) in theories with SUSY. It is known that, in general, SUSY theories would contain couplings with the violation of baryon or/and lepton numbers, that could induce the rapid proton decay. To avoid such couplings, one usually assigns R-parity, defined by \( R \equiv (-1)^{3B+L+2S} \) to each field \[26\], where \( B(L) \) and \( S \) denote the baryon (lepton) number and the spin, respectively. Thus, the R-parity can be used to distinguish the particle (R=+1) from its superpartner (R=−1). Recently, Ng and Wu \[27\] have investigated the \( T \) violating \( P_T \) for \( K_{\mu2}\gamma \) in SUSY models with R-parity and they find that when the squark family mixings are taken into account, large enhancement effects would appear due to the heavy quark masses and large \( \tan \beta = v_2/v_1 \). In this paper, we will concentrate on the SUSY models without R-parity. To evade the stringent constraint from proton decay, we can simply require that B violating couplings
do not coexist with the L violating ones. In the following we consider the theories with 
the violation of the R-parity and the lepton number. In such cases, the superpotential is 
given by

$$W = \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c,$$

where the subscript $ijk$ are the generation indices, $L$ and $E^c$ denote the chiral superfields 
of lepton doublets and singlets, and $Q$ and $D^c$ are the chiral superfields of quark doublets 
and down-type quark singlets, respectively. We note that the first two generation indices 
of $\lambda_{ijk}$ are antisymmetry, i.e., $\lambda_{ijk} = -\lambda_{jik}$. The corresponding Lagrangian is

$$L_{RV} = -\frac{\lambda^*_{22} \lambda'_{112}}{M^2_{\tilde{e}_{Li}}} 8 P_L u \tilde{\nu} P_R \mu,$$

From Eq. (52), we find that the four-Fermion interaction of $\bar{s}u \rightarrow \mu \bar{\nu}$ with the slepton as the 
intermediate state shown in Fig. 8 can be written as

$$L_{RV} = -\frac{\lambda^*_{22} \lambda'_{112}}{M^2_{\tilde{e}_{Li}}} 8 P_L u \tilde{\nu} P_R \mu,$$

for $i = 1, 3$. We therefore obtain the transverse muon polarization of $K_{\mu 2\gamma}$ as

$$P_T = -(\sigma_V - \sigma_A) \frac{\sqrt{2}}{4 G_F \sin \theta_C} \frac{m^2_K}{(m_s + m_a) m_{\mu}} \frac{\lambda^*_{22} \lambda'_{112}}{M^2_{\tilde{e}_{Li}}}.$$

In order to give the bounds on the R-parity violating couplings, we need to examine the various 
processes induced by the FCNC. We first study the decay of $\mu \rightarrow e \gamma$. Based on the 
analysis in Ref. [28], the total branching ratio for $\mu \rightarrow e \gamma$ decay can be expressed by

$$Br(\mu \rightarrow e \gamma) = \frac{12 \pi^2}{G_F} (|A_{LR}|^2 + |A_{RL}|^2),$$

where

$$A_i = A^\lambda_i + A^\lambda'_{i} + A^\Delta_{im}.$$
with \( i = LR \) and \( RL \). In Eq. (57), the amplitudes \( A_i^{\Delta m} \) stand for the neutralino- and slepton-mediated contributions and \( \Delta m \) is the soft breaking mass. For convenience, we only consider \( A_i^\lambda \) contributions. Using the results in Ref. [28], we get

\[
A_{\lambda}^{LR} = \frac{e}{96\pi^2} \sum_{i,k=1}^{3} \lambda_{i1k}\lambda_{i2k} \left( \frac{1}{M_{\tilde{\nu}_i}} - \frac{1}{2 M_{\tilde{e}_{iR}}^2} \right),
\]

\[
A_{\lambda}^{RL} = \frac{e}{96\pi^2} \sum_{i,j=1}^{3} \lambda_{ij1}\lambda_{ij2} \left( \frac{1}{M_{\tilde{\nu}_i}} - \frac{1}{2 M_{\tilde{e}_{iL}}^2} \right).
\]

(58)

Therefore, the bounds on R-parity violating couplings \( \lambda \) are given by

\[
\frac{|\lambda_{31k}\lambda_{32k}|}{M^2} < 4.6 \times 10^{-8} \frac{1}{GeV^2}, \quad \text{for } k = 1, 2;
\]

\[
\frac{|\lambda_{ij1}\lambda_{ij2}|}{M^2} < 2.3 \times 10^{-8} \frac{1}{GeV^2}, \quad \text{for } i, j = 1, 2,
\]

where we have assumed that \( M_{\tilde{\nu}_r} \approx M_{\tilde{e}_{iR}} \approx M_{\tilde{e}_{iL}} \approx M \). For simplicity, we take \( |\lambda_{31k}| \sim |\lambda_{32k}| \) and \( |\lambda_{211}| \sim |\lambda_{212}| \), and we have

\[
\frac{|\lambda_{322}|}{M} < 2.1 \times 10^{-4} \frac{1}{GeV};
\]

\[
\frac{|\lambda_{212}|}{M} < 1.5 \times 10^{-4} \frac{1}{GeV}.
\]

(60)

The bounds on \( \lambda' \) can be extracted from the experimental limit on \( K^+ \rightarrow \pi^0 \mu^+\nu \) as shown in Ref. [29]. One finds that

\[
\frac{|\lambda'_{ijk}|}{M_{d_{jk}}} < 1.2 \times 10^{-4} \frac{1}{GeV}, \quad \text{for } j = 1, 2,
\]

where \( M_{d_{jk}} \approx M \) is the sdown-quark mass.

From Eq. (53) and the bounds in Eqs. (59) and (61), we find

\[
P_T(K_{\mu 2\gamma}^+) \leq 0.01
\]

(62)

for \( |\sigma_V - \sigma_A| \sim 0.1 \).

The interaction in Eq. (52) could also yield transverse muon polarization in \( K^+ \rightarrow \pi^0 \mu^+\nu \). We find [13] that \( P_T(K_{\mu 3}^+) < 10^{-2} \) by using the same parameter values as in the case of \( K_{\mu 2\gamma}^+ \). Thus, in R-parity violation SUSY models with R-parity violation, one can also get a large prediction of \( P_T(K_{\mu 3}^+) \).

### 3.4 Leptoquark Model

There exist three scalar leptoquark models contributing to the decay \( K_{\mu 2\gamma}^+ \) through the tree diagrams which is similar to the cases for \( K_{\mu 3}^+ \) shown in Ref. [19]. The quantum
numbers of the leptoquarks under the standard group $SU(3)_C \times SU(2)_L \times U(1)_Y$ are
\[19, 30, 31\]

\[
\phi_1 = (3, 2, \frac{7}{3}), \text{ (Model I)},
\]
\[
\phi_2 = (3, 1, -\frac{2}{3}), \text{ (Model II)},
\]
\[
\phi_3 = (3, 3, -\frac{2}{3}), \text{ (Model III)},
\]

respectively. The general couplings involving these leptoquarks are given by \[31\]

\[
\mathcal{L}^I = (\lambda_1 \bar{Q}_L e_R + \lambda_1' \bar{u}_R L_L)\phi_1 + \text{h.c.},
\]
\[
\mathcal{L}^{II} = (\lambda_2 \bar{Q}_L L^c_R + \lambda_2' \bar{u}_R e_R)\phi_2 + \text{H.C.},
\]
\[
\mathcal{L}^{III} = \lambda_3 \bar{Q}_L L^c_R \phi_3 + \text{H.C.},
\]

where $Q = \begin{pmatrix} u \\ d \end{pmatrix}$ and $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$. Here the coupling constants $\lambda_k$ ($k = 1, \ldots, 3$) are complex and thus CP violation could arise from the Yukawa interactions in Eq. \(64\). We assume that CP violation in $K \to \pi \pi$ decays can be accounted for by the non-vanishing KM phase, and investigate the effect on the muon polarization of adding another CP violation mechanism in Eq. \(64\).

In terms of each charge components of the leptoquarks we rewrite Eq. \(64\) as

\[
\mathcal{L}^I = \sum_{i,j} \left\{ \right. \\
\left. [\lambda^i_2 \bar{d}_i \frac{1}{2} (1 + \gamma_5) \nu_j + \lambda^i_2' \bar{u}_i \frac{1}{2} (1 - \gamma_5) e_j] \phi_1(\frac{2}{3}) + \\
[\lambda^i_1 \bar{Q}_L \frac{1}{2} (1 + \gamma_5) e_j + \lambda^i_1' \bar{u}_i \frac{1}{2} (1 - \gamma_5) \nu_j] \phi_1(\frac{2}{3}) \right. \} + \text{H.C.},
\]
\[
\mathcal{L}^{II} = \sum_{i,j} \left\{ \lambda^i_2 \left[ -\bar{u}_i \frac{1}{2} (1 + \gamma_5) e_j + \lambda^i_2' \bar{u}_i \frac{1}{2} (1 - \gamma_5) \nu_j \right] \phi_2(\frac{2}{3}) + \\
\bar{d}_i \frac{1}{2} (1 + \gamma_5) \nu_j + \lambda^i_2' \bar{u}_i \frac{1}{2} (1 - \gamma_5) e_j \right\} \phi_2(\frac{2}{3}) + \text{H.C.},
\]
\[
\mathcal{L}^{III} = \sum_{i,j} \left\{ \lambda^i_3 \left[ \bar{u}_i \frac{1}{2} (1 + \gamma_5) \nu_j \phi_3(\frac{2}{3}) + [\bar{u}_i \frac{1}{2} (1 + \gamma_5) e_j \phi_3(\frac{2}{3}) + \\
\bar{d}_i \frac{1}{2} (1 + \gamma_5) \nu_j \phi_3(\frac{2}{3}) + \bar{d}_i \frac{1}{2} (1 + \gamma_5) e_j \phi_3(\frac{2}{3}) \right] + \text{H.C.},\right. \}
\]

where $i, j$ are family indices and $Q_e$ in $\phi_e(Q_e)$ are the electric charges. From Eq. \(65\), we see that the relevant terms for the process $K^{+}_{\mu2\gamma}$ are the ones involving $\phi_1(\frac{2}{3})$, $\phi_2(\frac{-2}{3})$ and $\phi_3(\frac{-2}{3})$ couplings, respectively. We will concentrate on these terms in our discussions. The effective interactions from these leptoquark exchanges are

\[
\mathcal{L}_{\text{eff}}^I = \frac{\lambda_{12}^2 (\lambda_{11}^I)^*}{4 M_{\phi_1}^2} \bar{s}(1 + \gamma_5) \mu \bar{v}(1 + \gamma_5) u + \text{H.c.},
\]
\[ \mathcal{L}_{\text{eff}}^{I} = \frac{1}{4M_{\phi_2}} \left[ -\lambda_2^2 (\lambda_2^1) \bar{s}(1 + \gamma_5) \nu_i \bar{\mu}^c (1 - \gamma_5) u + \lambda_2^2 (\lambda_2^1)^* \bar{s}(1 + \gamma_5) \nu_i \bar{\mu}^c (1 + \gamma_5) u \right] + \text{H.C.} \]  

\[ \mathcal{L}_{\text{eff}}^{II} = \frac{\lambda_2^2 (\lambda_3^1)^*}{4M_{\phi_3}} \bar{s}(1 + \gamma_5) \nu_i \bar{\mu}^c (1 - \gamma_5) u + \text{H.C.} \]  

where \( M_{\phi_k} (k = 1, \ldots, 3) \) are the masses of \( \phi_1^{(2)} \), \( \phi_2^{(-\frac{1}{2})} \) and \( \phi_3^{(-\frac{1}{2})} \), respectively. Using the Fierz transformations and Eq. (6), we have

\[ \Delta^I_P = -\sqrt{2} m_k^2 G_F \sin \theta_c (m_s + m_u) m_\mu \frac{\lambda_2^2 (\lambda_1^{n1})^*}{8 M_{\phi_1}^2}, \]

\[ \Delta^{II}_P = -\sqrt{2} m_k^2 G_F \sin \theta_c (m_s + m_u) m_\mu \frac{\lambda_2^2 (\lambda_2^{n2})^*}{8 M_{\phi_2}^2}, \]  

which give rise to

\[ [P^I_T, P^{II}_T] = (\sigma_V - \sigma_A) \frac{\sqrt{2}}{G_F \sin \theta_c (m_s + m_u) m_\mu} \frac{m_k^2}{M_{\phi_k}^2} \left[ \frac{\lambda_2^{22} (\lambda_1^{n1})^*}{8 M_{\phi_1}^2}, \frac{\lambda_2^{2i} (\lambda_2^{n2})^*}{8 M_{\phi_2}^2} \right], \]  

respectively, where we have neglected the tensor interactions and the contribution of \( \mathcal{L}^{III} \) because it contains only left-handed vector current interactions which have no relative phases between \( M_{IB} \) and \( M_{SD} \).

For Model I, since the leptoquarks model can give a large contribution to the muon transverse polarization in \( K^+_{\mu 3} \) [19], we may use the present experimental bound on \( P_T(K^+_{\mu 3}) \) to constrain the CP violating parameters in Eq. (67). Explicitly, we find

\[ \left| \frac{\lambda_2^{22} (\lambda_1^{n1})^*}{M_{\phi_1}^2} \right| < 4.4 \times 10^{-8}, \]  

and therefore we get

\[ |P^I_T(K^+_{\mu 2\gamma})| < 4.8 \times 10^{-3} \]  

for \( |\sigma_V - \sigma_A| \sim 0.1 \). Similar results can be obtained in Model II.

### 4 Conclusions

We have studied the transverse muon polarization in the decay of \( K^+ \rightarrow \mu^+ \nu \gamma \) in various CP violation theories. We have explicitly demonstrated that the polarization effect can be large in models with the left-right symmetry, multi-Higgs bosons, SUSY, and leptoquarks, respectively. These results as well as that from other CP violation models are summarized in Table 1. The estimations for the transverse muon polarization in \( K^+_{\mu 3} \) are also presented.
in Table 1. It is interesting to see that a large $P_T(K_{\mu 2\gamma}^+)$ corresponds a large $P_T(K_{\mu 3}^+)$ in the most CP violation models shown in the table except the left-right symmetric theories, in which $P_T(K_{\mu 3}^+)=0$. Therefore, the decay of $K_{\mu 2\gamma}^+$ has a comparable or even more sensitivity with that of $K_{\mu 3}^+$ to the new CP violation mechanism.

In conclusion, the transverse muon polarization of $K^+ \rightarrow \mu^+ \nu \gamma$ could be at the level of $10^{-2}$ in the non-standard CP violation theories, which may be detectable at the ongoing KEK experiment of E246 as well as the proposed BNL experiment. The measurement of such effect is a clean signature of CP violation beyond the standard model since that from the theoretical background, i.e., FSI, is $\leq 10^{-3}$.

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References

[1] N. Cabbibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).

[2] J. J. Sakurai, *Phys. Rev.* **109**, 980 (1958).

[3] D. A. Bryman et al., Phys. Rep. **88**, 151 (1982).

[4] P. Agrawal, J. N. Ng, G. Bélanger and C. Q. Geng, *Phys. Rev. Lett.* **67**, 537 (1991); *Phys. Rev.* **D45**, 2383 (1992).

[5] KEK E246 Collaboration, J. Imazato et al., *KEK-PS Research Proposal, KEK-91-8*; Y. Kuno, Nucl. Phys. **B** (Proc. Suppl.), 37A, 87 (1994).

[6] M. V. Diwan et al., AGS Proposal - Search for T Violating Muon Polarization in $K^+ \rightarrow \pi^0\mu^+\nu$ Decay, July 1996.

[7] Y. Kuno, Talk presented at the *Workshop on Science at the Kaon Factory*, Vancouver, 1990.

[8] C. Q. Geng, Nucl. Phys. **B** (Proc. Suppl.), 37A, 59 (1994).

[9] N. Byers, S. W. MacDowell and C. N. Yang, in *Proceedings of the International Seminar in High Energy Physics and Elementary Particles*, Trieste, Italy, 1965 (International Atomic Energy Agency, Vienna, Austria, 1965), p. 953; L. B. Okun and I. B. Khriplovich, *Yad. Fiz.* **6**, 821 (1967).

[10] A. R. Zhitniskii, Yad. Fiz **31**, 1024 (1980) [Sov. J. Nucl. Phys. **31**, 529 (1980)].

[11] W. Marciano, *BNL Reprot No. BNL-48760*.

[12] C. Q. Geng and S. K. Lee, Phys. Rev. **D51**, 99 (1995).

[13] C. Q. Geng, I. J. Hsu and C. C. Lih, *in preparation*.

[14] J. Bijnens, G.Ecker, and J. Gasser, Nucl. Phys. **B** **396**, 81 (1993).

[15] M. Kobayashi, T.-T. Lin, and Y. Okada, Prog. Theor. Phys. **95**, 361 (1996).

[16] J.C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1975); R.N. Mohapatra and J.C. Pati, Phys. Rev. **D11**, 566 (1975); G. Senjonovic and R.N. Mohapatra, Phys. Rev. **D12**, 1502 (1975).
[17] J.F. Donoghue and B. Holstein, Phys. Lett. B113, 382 (1982); L. Wolfenstein, Phys. Rev. D29, 2130 (1984).

[18] G. Barenboim, J. Bernabéu, J. Prades, and M. Raidal, Phys. Rev. D55, 4213 (1997).

[19] Bélanger and C. Q. Geng, Phys. Rev. D44, 2789 (1991).

[20] T.D. Lee, Phys. Rev. D8, 1226 (1973).

[21] Y.L. Wu and L. Wolfenstein, Phys. Rev. Lett. 73, 1762 (1994).

[22] L.J. Hall and S. Weinberg, Phys. Rev. D48, 979 (1993).

[23] T.P Cheng and M. Sher, Phys. Rev. D35, 3484 (1987).

[24] Y. Grossman, Nucl. Phys. B426, 385 (1994).

[25] R. Garisto and G. Kane, Phys. Rev. D44, 2038 (1991).

[26] A. Salam and J. Srathdee, Nucl. Phys. B87, 85 (1975); P. Fayet, Nucl. Phys. B90, 104 (1975).

[27] Guo-Hong Wu and John N. Ng, hep-ph/9610533.

[28] B. de Carlos and P.L. White, Phys. Rev. D54, 3427 (1996).

[29] K. Agashe and M. Graesser, Phys. Rev. D54, 4445 (1996).

[30] J.F. Nieves, Nucl. Phys. B189, 382 (1981).

[31] A.J. Davies and X.G. He, Phys. Rev. D43, 225 (1991).
**Table 1.** Summary of the upper values of (1) $P_T(K_{\mu3}^+)$ and (2) $P_T(K_{\mu2\gamma}^+)$ for (a) current experimental limits, (b) sensitivities in KEK-PS E246, (c) theoretical background (FSI), (d) standard CKM model, (e) left-right symmetric models, (f) multi-Higgs models with NFC, (g) multi-Higgs models without NFC, (h) SUSY with R-parity, (i) SUSY without R-parity, and (j) leptoquark models.

|   | a   | b       | c       | d   | e   | f       | g       | h       | i       | j       |
|---|-----|---------|---------|-----|-----|---------|---------|---------|---------|---------|
| (1) | 10^{-2} | 5 \cdot 10^{-4} | 10^{-6} | 0   | 0   | 10^{-2} | 10^{-2} | 10^{-2} | 10^{-2} | 10^{-2} |
| (2) | 10^{-3} | 10^{-3} | 0       | 7 \cdot 10^{-3} | 5 \cdot 10^{-3} | 5 \cdot 10^{-2} | 2 \cdot 10^{-2} | 1 \cdot 10^{-2} | 5 \cdot 10^{-3} |
Fig. 1. Dalitz plot of $\rho_0(x, y)$.

Fig. 2. Dalitz plot of $P_L(x, y)$. 
Fig. 3. Dalitz plot of $P_N(x, y)$.

Fig. 4. Dalitz plot of $\sigma_V(x, y)$. 
Fig. 5. Dalitz plot of $\sigma_V(x,y) - \sigma_A(x,y)$.

Fig. 6. Tree diagram to $s u \rightarrow \mu \bar{\nu}$ induced by the left-right gauge boson mixing.
Fig. 7. Box diagrams to $K^0 - \bar{K}^0$ mixing with the $u$-quark as the internal fermion.

Fig. 8. Tree diagram to $\bar{s}u \rightarrow \mu \bar{\nu}$ with the slepton as the intermediate state.