Constraining anomalous gauge boson couplings in $e^+e^- \to W^+W^-$ using polarization asymmetries with polarized beams

Rafiqul Rahaman$^a$, Ritesh K. Singh$^b$

$^a$Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur, 741246, India
$^b$email:rr13rs033@iiserkol.ac.in

1 Introduction

The non-abelian gauge symmetry $SU(2) \times U(1)$ of the Standard Model (SM) allows the WWV ($V = \gamma, Z$) couplings in $e^+e^- \to W^+W^-$ using the complete set of polarization observables of $W$ boson with longitudinally polarized beams. We use most general Lorentz invariant form factors parametrization as well as $SU(2) \times U(1)$ invariant dimension-6 effective operators for the effective $W^+W^-V$ couplings. We estimate simultaneous limits on the anomalous couplings in both the parametrizations using cross section, forward backward asymmetry and polarization observables of $W$ boson with different kinematical cuts using Markov–Chain–Monte-Carlo (MCMC) method for an $e^+e^-$ collider running at centre of mass energy of $\sqrt{s} = 500$ GeV and $\mathcal{L} = 100 fb^{-1}$. The best limits on form factors are obtained to be $1 \sim 5 \times 10^{-2}$ for $e^-$ and $e^+$ polarization being $(+0.4, -0.4)$. For operator’s coefficients, the best limits are obtained to be $1 \sim 16$ TeV$^{-2}$.

Abstract

We study the anomalous $W^+W^-V$ ($V = \gamma, Z$) couplings in $e^+e^- \to W^+W^-$ using the complete set of polarization observables of $W$ boson with longitudinally polarized beams. We use most general Lorentz invariant form factors parametrization as well as $SU(2) \times U(1)$ invariant dimension-6 effective operators for the effective $W^+W^-V$ couplings. We estimate simultaneous limits on the anomalous couplings in both the parametrizations using cross section, forward backward asymmetry and polarization observables of $W$ boson with different kinematical cuts using Markov–Chain–Monte-Carlo (MCMC) method for an $e^+e^-$ collider running at centre of mass energy of $\sqrt{s} = 500$ GeV and $\mathcal{L} = 100 fb^{-1}$. The best limits on form factors are obtained to be $1 \sim 5 \times 10^{-2}$ for $e^-$ and $e^+$ polarization being $(+0.4, -0.4)$. For operator’s coefficients, the best limits are obtained to be $1 \sim 16$ TeV$^{-2}$.

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Effective operators which provide the WWV form factors after EWSB [9] and add to the SM Lagrangian as

$$\mathcal{L}_\text{eff} = \mathcal{L}_\text{SM} + \sum_i \frac{c_i^f}{\Lambda^2} \mathcal{O}_i,$$

Here $c_i^f$ are couplings of the dimension-six operators $\mathcal{O}_i$ and $\Lambda$ is the energy scale below which the theory is valid. To the lowest order (upto dimension-6) the operators contributing to WWV couplings are [10, 11]

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu}W_{\lambda\rho}W_{\alpha\beta}^\dagger],$$

$$\mathcal{O}_W = (D_\mu \Phi)^\dagger W^{\mu\nu}(D_\nu \Phi),$$

$$\mathcal{O}_B = (D_\mu \Phi)^\dagger B^{\mu\nu}(D_\nu \Phi),$$

$$\mathcal{O}_{WWV} = \text{Tr}[W_{\mu\nu}W_{\lambda\rho}W_{\alpha\beta}^\dagger],$$

$$\mathcal{O}_{VV} = (D_\mu \Phi)^\dagger W^{\mu\nu}(D_\nu \Phi),$$

where $\Phi$ is the Higgs doublet field and

$$D_\mu = \partial_\mu + \frac{i}{2} g \tau^I W^I_\mu + \frac{i}{2} g' B_\mu,$$

$$W^{\mu\nu} = \frac{i}{2} g \tau^I (\partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g e_{IJK} W^J_\mu W^K_\nu),$$

$$B^{\mu\nu} = \frac{i}{2} g' (\partial_\nu B_\mu - \partial_\mu B_\nu).$$

Here $g$ and $g'$ are $SU(2)$ and $U(1)$ couplings, respectively. Among these operators $\mathcal{O}_{WWW}$, $\mathcal{O}_W$ and $\mathcal{O}_B$ are CP conserving, while $\mathcal{O}_{WWV}$ and $\mathcal{O}_{VV}$ are CP violating. These effective operators in Eq. 2, after EWSB, also provides ZZV, HZV couplings which can be examine in various processes, e.g. ZV production, WZ production, HV production processes. The couplings in these processes may contains some other effective operator as well.

The other way to go beyond the SM WWV structure is to consider a most general Lorentz invariant structure in a model independent way. A Lagrangian corresponding to the most general Lorentz invariant set of form factors for the WWV couplings is given by [12]

$$\mathcal{L}_{WWV} = ig_{WWV} \left( g^I \left( W^{\mu}_{\mu} W_{\nu} - W^{\nu}_{\mu} W_{\nu} \right) V^V \right)$$
lous couplings to be the couplings of the operators in Eq. 2 as \([10, 11, 13]\) \(\Delta\) providing the weak mixing angle. In the SM \(\Delta g_1^V = 1, \Delta \kappa^V = 1\) and other couplings are zero. The anomalous part in \(\Delta \) conventions are defined as \(g_{WW\gamma} = -g \sin \theta_W\) and \(g_{WWZ} = -g \cos \theta_W\), \(\theta_W\) being the weak mixing angle. In the SM \(g_1^V = 1, \kappa^V = 1\) and other couplings are zero. The anomalous part in \(g_1^V, \kappa^V\) would be \(\Delta g_1^V = g_1^V - 1, \Delta \kappa^V = \kappa^V - 1\), respectively. The couplings \(g_1^V, \kappa^V\) and \(\lambda^V\) of Eq. 4 conserve CP (both C and P-even), while \(g_4^V\) (odd in C, even in P), \(\tilde{\kappa}^V\) and \(\bar{\lambda}^V\) (even in C, odd in P) violate CP. On the other hand \(g_3^V\) violates both C and P leaving it to CP conserving. We label these set of 14 anomalous couplings to be \(c_i^{\tilde{\kappa}}\) as given in Eq. A.2 in Appendix A for later uses.

On restricting to the SU(2) \(\times U(1)\) gauge, the coupling \((c_i^{\tilde{\kappa}})\) of the Lagrangian in Eq. 4 can be written in terms of the couplings of the operators in Eq. 2 as [10, 11, 13]

\[
\Delta g_1^V = c_W \frac{M_Z^2}{2\Lambda^2},
\]

\[
g_4^V = g_5^V = g_7^V = 0.
\]

\[
\lambda_f = \lambda_Z = c_{\text{WW}} \frac{3g_2^2 M_W^2}{2\Lambda^2},
\]

\[ \tilde{\lambda}_f = \tilde{\lambda}_Z = \hat{c}_{\text{WW}} \frac{3g_2^2 M_W^2}{2\Lambda^2}, \]

\[ \Delta \kappa_f = \{c_W + c_B\} \frac{M_Z^2}{2\Lambda^2}, \]

\[ \Delta \kappa_f = \{c_W - c_B \tan^2 \theta_W\} \frac{M_Z^2}{2\Lambda^2}, \]

\[ \tilde{\kappa}_f = c_W \frac{M_Z^2}{2\Lambda^2}, \]

\[ \Delta \kappa_f = -c_W \tan^2 \theta_W \frac{M_Z^2}{2\Lambda^2}. \]

In this case some of the Lagrangian couplings become dependent to each others and they are

\[
\Delta g_1^V = \Delta \kappa_f + \tan^2 \theta_W \Delta \kappa_f,
\]

\[
\tilde{\kappa}_f + \tan^2 \theta_W \kappa_f = 0. \tag{6}
\]

We label the non-vanishing 9 couplings in \(SU(2) \times U(1)\) gauge as \(c_i^{\tilde{\kappa}}\) given in Eq. A.3 in Appendix A for later uses.

The anomalous WWV couplings has been studied in the effective operators formalism as well as in the effective vertex factor approach given in the Lagrangian \(Z_{\text{WWV}}\) (Eq. 4) in \(SU(2) \times U(1)\) gauge for \(e^+e^-\) linear collider [12, 14–24], Large Hadron electron collider (LHeC) [25, 26], \(e^+\gamma\) collider [27], hadron collider (LHC) [21, 22, 28–34]. These couplings has also been addressed from loop level contribution [16] and Georgi-Machacek model [35]. Some CP-violating WWV couplings has been studied in Refs [24, 34].

On the experimental side the anomalous WWV couplings have been explored and stringent limits on them have been obtained in different process (\(W^+W^-, W^+\bar{W}^-\) production) and different channel (\(eeJ, qgvl\)) in the LEP [3, 36–38], Tevatron [39, 40], LHC [41–52], Tevatron-LHC [53]. The tightest one parameter limit observed on the anomalous couplings from experiments are given in Table 1. The tightest limit on operator couplings \((c_i^{\tilde{\kappa}})\) are obtained in Ref. [42] for CP-even ones and in Ref. [43] for CP-odd ones. The limits on the couplings of the Lagrangian in Eq. 4 are tighter when \(SU(2) \times U(1)\) symmetry is assumed and the tightest ones are obtained in Ref. [42] for CP-even and in Ref. [37, 43] for CP-odd parameters. These limits on \(c_i^{\tilde{\kappa}}\) are actually translated from the limits of the operator couplings \(c_i^{\tilde{\kappa}}\). The tightest limits on the couplings, which are zero when \(SU(2) \times U(1)\) symmetry is assumed (see Eq. 5), are obtained in Ref. [36, 37] considering the Lagrangian in Eq. 4.

The process \(e^+e^- \rightarrow W^+W^-\) will be one of the important process which will be studied at the future International Linear Collider (ILC) [54–56] for precision test [57] as well as for BSM physics. This process has been studied earlier for SM phenomenology as well as for various BSM physics with and without beam polarizations [12, 58–62]. Here we intend to study WWV anomalous couplings in \(e^+e^- \rightarrow W^+W^-\) at \(\sqrt{s} = 500\) GeV, \(\mathcal{L} = 100\) fb\(^{-1}\) using the cross section, forward backward asymmetry and 8 polarizations asymmetries of \(W^-\) with longitudinally polarized \(e^+\) and \(e^-\) beams. Here, first we study the anomalous couplings \((c_i^{\tilde{\kappa}})\) in the Lagrangian \(Z_{\text{WWV}}\) (Eq. 4) and estimate simultaneous limits on all 14 couplings. Next we consider the \(SU(2) \times U(1)\) effective operators (given in Eq. 2) contributing to the anomalous couplings and obtain simultaneous limit on the corresponding couplings \((c_i^{\tilde{\kappa}})\) given in Eq. A.1. The translated limit on the reaming couplings \((c_i^{\tilde{\kappa}})\) given in Eq. A.3 of the Lagrangian in Eq. 4 has also been obtained.

The rest of the paper is arranged in the following way. In Sect. 2 we introduce the compete set polarization observables of a spin-1 particle along with the forward backward asymmetry and study the effect of beam polarizations on the observables. In Sect. 3 we use the vertex form factors for the Lagrangian in Eq. 4 and obtained expressions for all the observables. In this section we cross check analytical results against the numerical result from HadGraph5 [63] for sanity check. We also study the \(\cos \theta\) (of \(W\) ) dependences of the observables and study their sensitivity on the anomalous couplings. In this section we also estimates simultaneous limits on \(c_i^{\tilde{\kappa}}, c_i^{\tilde{\kappa}}\) and the translated limits on \(c_i^{\tilde{\kappa}}\). Next in Sect. 4 we give insight on the choice of beam polarizations in this process. We conclude in Sect. 5.
Table 1 The list of tightest limits observed on anomalous couplings of Eq (2) (dimension-6 operators), Eq. (4) (effective Lagrangian) in \( S(2) \times U(1) \) (except \( g_2^Z \) and \( g_2^Z \)) at 95% C.L. from experiments

| \( c^0_i \) | Limits (TeV\(^{-2}\)) | Remark |
|------------|------------------|--------|
| \( \frac{c_{BBMK}}{\lambda^1} \) | \([-2.7, +2.7]\) | CMS \( \sqrt{s} = 8 \) TeV, \( \mathcal{L} = 19 \) fb\(^{-1}\), \( SU(2) \times U(1) \) [42] |
| \( \frac{c_{BBMK}}{\lambda^2} \) | \([-2.0, +5.7]\) | CMS [42] |
| \( \frac{\lambda}{\lambda^2} \) | \([-14, +17]\) | CMS [42] |
| \( \frac{c_{BBMK}}{\lambda^2} \) | \([-11, +11]\) | ATLAS \( \sqrt{s} = 7(8) \) TeV, \( \mathcal{L} = 4.7(20.2) \) fb\(^{-1}\) [43] |
| \( c_{\tilde{g}}^Z \) | \([-580, 580]\) | ATLAS [43] |

| \( \frac{\Delta}{\lambda^1} \) | Limits (\( 10^{-2}\)) | Remark |
|-------------------|-----------------|--------|
| \( \lambda^V \) | \([-1.1, +1.1]\) | CMS [42] |
| \( \Delta \lambda^z \) | \([-4.4, +6.3]\) | CMS [42] |
| \( \Delta \kappa^z \) | \([-0.87, +2.4]\) | CMS [42] |
| \( \Delta \kappa^z \) | \([-0.5, +0.4]\) | CMS [42] |
| \( \lambda^V \) | \([-4.7, +4.6]\) | ATLAS [43] |
| \( \Delta \kappa^z \) | \([-14, -1]\) | DELPHI (LEP2), \( \sqrt{s} = 189-209 \) GeV, \( \mathcal{L} = 520 \) pb\(^{-1}\) [37] |

2 Observables and effect of beam polarizations on them

We study \( W^+W^- \) production at ILC running at \( \sqrt{s} = 500 \) GeV and integrated luminosity \( \mathcal{L} = 100 \) fb\(^{-1}\) using longitudinal polarization of \( e^- \) and \( e^+ \) beams. The Feynman diagram for the process is shown in Fig. 1 where Fig. 1a corresponds to the \( V \) mediated \( t \)-channel diagram and the Fig. 1b corresponds to the \( V \) (Z, \( \gamma \)) mediated \( s \)-channel diagram. The decay mode is chosen to be

\[
W^+ \to q_u \bar{q}_d, \quad W^- \to l^- \bar{\nu}_l, \tag{7}
\]

where \( q_u \) and \( q_d \) are up-type and down-type quarks, respectively. We use complete set of eight spin-1 observables of \( W^- \) boson [6, 7].

The \( W \) boson being a spin-1 particle, its normalised production density matrix in the spin basis can be written as [2, 5]

\[
\rho(\lambda, \lambda') = \frac{1}{3} \left[ I_{3\times3} + \frac{3}{2} p.S + \sqrt{\frac{3}{2}} T_{ij} (S_i S_j + S_j S_i) \right], \tag{8}
\]

where \( p = \{p_x, p_y, p_z\} \) is the vector polarization of a spin-1 particle, \( S = \{S_x, S_y, S_z\} \) are the spin basis and \( T_{ij} \) is the 2nd-rank symmetric traceless tensor, \( \lambda \) and \( \lambda' \) are helicities of the particle. The tensor \( T_{ij} \) has 5 independent elements, which are \( T_{xy}, T_{yx}, T_{xx} - T_{yy} \) and \( T_{zz} \). Combining the \( \rho(\lambda, \lambda') \) with normalised decay density matrix of the particle to a pair of fermion \( f \), the differential cross section would be [5]

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega_f} = \frac{3}{8\pi} \left( \frac{2}{3} \left( 1 - 3\delta \right) \frac{T_{zz}}{\sqrt{6}} + \alpha p_c \cos \theta_f + \sqrt{\frac{2}{3} \left( 1 - 3\delta \right) T_{zz}} \cos^2 \theta_f \right. \\
\left. \left. + \sqrt{\frac{2}{3} \left( 1 - 3\delta \right) T_{xx}} \sin \theta_f \cos \phi_f + \sqrt{\frac{2}{3} \left( 1 - 3\delta \right) T_{yy}} \sin \theta_f \sin \phi_f \right. \\
\left. \left. + (1 - 3\delta) \left( \frac{T_{xx} - T_{yy}}{\sqrt{6}} \right) \sin^2 \theta_f \cos^2 \phi_f \right) + \sqrt{\frac{2}{3} \left( 1 - 3\delta \right) T_{xy}} \sin^2 \theta_f \sin(2\phi_f) \right]. \tag{9}
\]
Here $\theta_f$, $\phi_f$ are the polar and the azimuthal orientation of the fermion $f$, in the rest frame of the particle ($W$) with its would be momentum along z-direction. In this case $\alpha = -1$ and $\delta = 0$. The vector polarizations $p$ and independent tensor polarizations $T_{ij}$ are calculable from the asymmetries constructed from the decay angular information of lepton (here $l^-$). For example $p_z$ can be calculated from the asymmetry $A_k$ as
\[ A_k = \frac{\sigma(\cos \phi > 0) - \sigma(\cos \phi < 0)}{\sigma(\cos \phi > 0) + \sigma(\cos \phi < 0)} \equiv 3\theta p_z. \] (10)

The asymmetries corresponding to all other polarizations, vector polarizations $p_y$, $p_z$ and independent tensor polarizations $T_{ij}$ are $A_y$, $A_z$, $A_{xy}$, $A_{yz}$, $A_{xz}$, $A_{y-x^2}$, $A_{z-x^2}$ (see Ref. [7] for details).

Owing to the $t$-channel process (Fig. 1a) and not a $u$-channel process like in ZV production [7, 64], the $W^\pm$ produced are not forward backward symmetric. We add forward-backward asymmetry defined as
\[ A_{fb} = \frac{1}{\sigma_{W^+W^-}} \left[ \int_0^1 \frac{d\sigma_{W^+W^-}}{d\cos \theta} - \int_{-1}^0 \frac{d\sigma_{W^+W^-}}{d\cos \theta} \right] \] (11)
of the $W$ to the set of observables making total of ten observables including the cross section. Here $\cos \theta$ is the production angle of the $W^-$ w.r.t. the $e^-$ beam direction and $\sigma_{W^+W^-}$ is the production cross section.

These asymmetries can be measured in a real collider from the final state lepton $l^-$. One has to calculate the asymmetries at the rest frame of $W^-$ which require the missing $\bar{\nu}_l$ momenta to be reconstructed. At an $e^+e^-$ collider, as studied here, reconstructing the missing $\bar{\nu}_l$ is possible because only one missing particle is involved and no parton distribution function (PDF) is involved, i.e., initial momenta are known. But for a collider where PDF is involved, reconstructing the actual missing momenta may not be possible.

We explore the dependence of the cross section and asymmetries on longitudinal polarization $\eta_3$ of $e^-$ and $\xi_3$ of $e^+$. In Fig. 2 we show the production cross section $\sigma_{W^+W^-}$ and $A_k$ as a function of beam polarization as an example. The cross section decreases along $\eta_3 = -\xi_3$ path from 20 pb on the left-top corner to 7.2 pb at unpolarized point and further to 1 pb in the right-bottom corner. This is because the $W^\pm$ couples to left chiral $e^-$ i.e., it requires $e^-$ to be negatively polarized and $e^+$ to be positively polarized for higher cross section. The variation of $A_{fb}$ (not shown) with beam polarization is same as cross section but the variation is very slow above the line $\eta_3 = \xi_3$. From this we can expect that a positive $\eta_3$ and a negative $\xi_3$ will reduce the SM values of observables increasing the $S/\sqrt{B}$ ratio ($S =$ signal, $B =$ background). Some other asymmetries like $A_k$ has opposite dependence on the beam polarizations compared to the cross section, its modulus reduces for negative $\eta_3$ and positive $\xi_3$. So, some beam polarization in between ($\pm 0.8, \mp 0.8$) may come out to be a good choice for obtaining best simultaneous limits on anomalous couplings as will be explored in the next section.

3 Probe to the Anomalous Lagrangian

The $W^+W^-V$ vertex (Fig. 3) for the Lagrangian in Eq. 4 for on-shell $W$s would be $i g_{WWV} \Gamma_V^{\mu \alpha \beta}$ [12, 14] and it is given by
\[ \Gamma_V^{\mu \alpha \beta} = f_1^V (q - \bar{q})^\mu g_{\alpha \beta} - \frac{f_2^V}{M_W} (q - \bar{q})^\mu p^{\alpha} p^{\beta} \]
\[ + f_3^V (p^{\alpha} g_{\mu \beta} - p^{\beta} g_{\mu \alpha}) + i f_4^V (p^{\alpha} g_{\mu \beta} + p^{\beta} g_{\mu \alpha}) \]
The momentum \( P \) is incoming to the vertex while, \( q, \bar{q} \) are outgoing from the vertex

\[ +i \frac{g}{2M_W} \epsilon^{\mu \nu \rho \sigma} (q - \bar{q})_{\rho} - \frac{g}{M_W} \epsilon^{\mu \alpha \beta \rho} p_{\rho} \]

\[ + \frac{g}{M_W} \left( q^2 \epsilon^{\mu \nu \rho \sigma} + q^{\beta} \epsilon^{\mu \alpha \rho \sigma} \right) q_{\rho} \bar{q}_{\sigma}, \tag{12} \]

where \( P, q, \bar{q} \) are the four-momenta of \( V, W^-, W^+ \), respectively. The momentum conventions are shown in Fig. 3. The blob in the vertex of Figs. 1 & 3 represent the presence of anomalous contribution. The form factor \( f_{is} \) has been obtained from the Lagrangian in Eq. 4 using FeynRULES [65] to be

\[ f_1^V = g_1^V + \frac{\delta}{2M_W} \lambda^V, \quad f_2^V = \lambda^V, \quad f_3^V = g_3^V + \lambda^V, \]

\[ f_4^V = g_4^V, \quad f_5^V = g_5^V, \quad f_6^V = \lambda^V + \left( 1 - \frac{\delta}{2M_W} \right) \lambda^V, \]

\[ f_7^V = \lambda^V. \tag{13} \]

We use the vertex factor in Eq. 12 for the analytical calculation of our observables and cross validate them numerically with MadGraph5 [63] implementation of Eq. 4. As an example, we present two observables \( \sigma_1 \) and \( A_\gamma \) for the SM \((c_\gamma^V = 0.0)\) and for a chosen couplings point \( c_\gamma^V = 0.05 \), in Fig. 4. The agreement between the analytical and the numerical calculations over a range of \( \sqrt{s} \) indicates the validity of relations in Eq. 13, specially the \( s \) dependence of \( f_1^V \) and \( f_6^V \).

Analytical expressions of all the observables has been obtained and their dependence on the anomalous couplings \( c_\gamma^V \) are given in Table 6 in Appendix A. The CP-even couplings in CP-even observables \( \sigma, A_\gamma, A_{\gamma \gamma}, A_{\gamma \gamma \gamma, A_{\gamma \gamma \gamma}} \) appear in linear as well as in quadratic form but do not appear in the CP-odd observables \( A_\gamma, A_{\gamma \gamma}, A_{\gamma \gamma \gamma} \). On the other hand CP-odd couplings appears linearly in CP-odd observables and quadratically in CP-even observables. Thus the CP-even couplings may have double patch in their confidence interval leading to asymmetric limits which will be discussed in Sect. 3.1. On the other hand the CP-odd couplings will single patch in their confidence interval and will poses symmetric limits. To this end we discuss sensitivity and limits on the anomalous couplings in the next subsection.

### 3.1 Sensitivity of observables on anomalous couplings and their binning

Sensitivity of an observables \( \mathcal{O} \) depending on anomalous couplings \( \Gamma \) with beam polarization \( \eta_1, \xi_3 \) is given by

\[ \delta \mathcal{O}(\Gamma, \eta_1, \xi_3) = \left| \frac{\partial \mathcal{O} (\Gamma, \eta_1, \xi_3)}{\partial \mathcal{O}(\Gamma, \eta_1, \xi_3)} \right|, \]

where \( \delta \mathcal{O} = \sqrt{\left( \mathcal{O}_{\text{stat}} \right)^2 + (\mathcal{O}_{\text{sys}})^2} \) is the estimated error in \( \mathcal{O} \). The error for cross-section would be,

\[ \delta \sigma(\eta_1, \xi_3) = \sqrt{\sigma(\eta_1, \xi_3) \epsilon_\sigma^2} + \epsilon_\sigma^2 \sigma(\eta_1, \xi_3)^2, \]

where as the estimated error in asymmetries would be,

\[ \delta A(\eta_1, \xi_3) = \frac{1 - A(\eta_1, \xi_3)^2}{\mathcal{L} \sigma(\eta_1, \xi_3)} + \epsilon_A^2. \]
Here $\mathcal{L}$ is the integrated luminosity, $\varepsilon_{\sigma}$ and $\varepsilon_{A}$ are the systematic fractional error in cross-section and asymmetries respectively, we take $\mathcal{L} = 100 \text{ fb}^{-1}$, $\varepsilon_{\sigma} = 0.02$ and $\varepsilon_{A} = 0.01$ as a benchmark scenario for the present analyses.

The sensitivity of all 10 observables have been studied on the all 14 couplings of the Lagrangian in Eq. 4 with the chosen $\sqrt{s}$, $\mathcal{L}$ and systematic uncertainties. The sensitivity of all observables on $g_{Z}^{\epsilon}$ and $\Delta \kappa^{\gamma}$ are shown in Fig. 5 as representative. Being CP-odd (either only linear or only quadratic terms present) $g_{Z}^{\epsilon}$ has single patch in the confidence interval, while the $\Delta \kappa^{\gamma}$ being CP-even (linear and quadratic terms present), it has two patches in the sensitivity
curve. The CP-odd observable $A_t$ provides the tightest one parameter limit on $g_2^Z$. The tightest 1σ limit on $\Delta \kappa^Z$ is obtained using $A_{fb}$, while at 2σ level a combination of $A_{fb}$ and $A_t$ provides the tightest limit.

Here, we have a total of 14 different anomalous couplings to measure, while we only have 10 observables. A certain combination of large couplings may mimic the SM within the statistical errors. To avoid these we need more number of observables to be included in the analysis. To this end we divide $\cos \theta$ (production angle of $W$) into eight bins and calculate the cross section and polarization asymmetries in all of them. The cross section and the polarization asymmetries $A_x$, $A_z$, and $A_y$ as a function of $\cos \theta$ are shown in Fig. 6 for the SM and aTGC. The SM values for unpolarized case is shown in dotted (blue), SM with polarization of $(\eta, \xi) = (+0.8, -0.8)$ is shown in dashed (black) lines. The solid (red) lines corresponds to unpolarized aTGC values while dashed-dotted (green) lines represent polarized aTGC values of observables. For the cross section (left-top panel) we take $\Delta g_2^Z$ to be 0.1 and all other couplings to be zero for both the polarized and unpolarized case. We see that the fractional deviation from the SM values is larger in the the most backward bin $\cos \theta \in (-1.0, -0.75)$ and gradually reduces in the forward direction. The deviation is even larger in case of beam polarization. Sensitivity of cross section on $\Delta g_2^Z$ is thus expected to be high in the most backward bin. In case of asymmetries $A_x$ (right-top panel), $A_z$ (left-bottom panel) and $A_y$ (right-bottom panel) the aTGC is assumed to be $\Delta \kappa^Z = 0.05$, $\lambda^Z = 0.05$ and $g_4^Z = 0.05$, respectively, while others are kept at zero. The change in the asymmetries due to aTGC is larger in the backward bin for both the beam polarizations. The value of asymmetries in each bin are comparable to the total values. The asymmetries may not have highest sensitivity in the most backward bin but in other bin. So we can not ignore observables in any bin, we will use all 9 observables ($A_{fb}$ excluded) in 8 bin totalling 72 observables in our analysis.

One parameter sensitivity of the set of 9 observables in 8 bin has been studied. We show sensitivity of $A_t$ on $g_2^Z$ and of $A_x$ on $\Delta \kappa^Z$ in the 8 bin in Fig. 7. We can see the tightest limits based on sensitivity got much tighter (tighten by a factor of 2, roughly) compared to the unbin case in Fig. 5. Thus we expect simultaneous limits on all the couplings to be tighter when observables get binned.

We perform a set of MCMC analyses with different set of observables for different kinematical cuts with unpolarized beams to understand their roles in providing limits on the anomalous couplings. These analyses are listed in Table 2. The corresponding 14 dimensional rectangular volume made out of 95% Bayesian confidence interval (BCI) on the anomalous couplings are also listed in Table 2 in the last column. The simplest analysis would be to consider only cross section in the full $\cos \theta$ domain and perform MCMC analysis which is named as $\sigma$-ubinned. The typical 95% limits on the parameters range from $\sim \pm 0.04$

### Table 2

| Analysis name | Set of observables | Kinematical cut on $\cos \theta$ | Volume of Limits |
|---------------|--------------------|----------------------------------|------------------|
| $\sigma$-ubinned | $\sigma$ | $\cos \theta \in [-1.0, 1.0]$ | $4.4 \times 10^{-11}$ |
| Unbinned | $\sigma, A_{fb}, A_t$ | $\cos \theta \in [-1.0, 1.0]$ | $3.1 \times 10^{-12}$ |
| Backward | $\sigma, A_t$ | $\cos \theta \in [-1.0, 0.0]$ | $1.3 \times 10^{-13}$ |
| $\sigma$-binned | $\sigma$ | $\cos \theta \in \left[\frac{\pi}{m}, \frac{\pi}{m-1}\right], m = 1, 2, 3, \ldots, 8$ | $5.4 \times 10^{-12}$ |
| Binned | $\sigma, A_t$ | | $1.2 \times 10^{-16}$ |
Table 3 List of Posterior 95% BCI of anomalous couplings $c_i^\ell$ (10$^{-2}$) of the Lagrangian in Eq. 4 in $e^+e^- \rightarrow W^+W^-$ at $\sqrt{s} = 500$ GeV, $\mathcal{L} = 100$ fb$^{-1}$ for unbinned and binned case for 5 chosen set longitudinal beam polarizations $\eta_3$ and $\xi_3$ from MCMC. The pictorial visualisation for these 95% BCI of $c_i^\ell$ is shown in Fig. 8. The rectangular volume of couplings at 95% BCI is given in the last row. The one parameter limits (10$^{-2}$) at 2$\sigma$ level with unpolarized beams are given in the last column. The notation used here is $\ell \equiv \text{low}$ with low being lower limit and high being upper limit.

| $c_i^\ell$ | Unbinned | Binned | Unbinned | Binned | Unbinned | Binned | Unbinned | Binned | Unbinned | Binned | Unbinned | Binned |
|------------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|
| $\Delta g_1^\ell$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ |
| $\Delta g_2^\ell$ | $-33$ | $-33$ | $-33$ | $-33$ | $-33$ | $-33$ | $-33$ | $-33$ | $-33$ | $-33$ | $-33$ | $-33$ |
| $\Delta g_3^\ell$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ | $-38$ |
| $\Delta g_4^\ell$ | $-34$ | $-34$ | $-34$ | $-34$ | $-34$ | $-34$ | $-34$ | $-34$ | $-34$ | $-34$ | $-34$ | $-34$ |
| $\Delta g_5^\ell$ | $-39$ | $-39$ | $-39$ | $-39$ | $-39$ | $-39$ | $-39$ | $-39$ | $-39$ | $-39$ | $-39$ | $-39$ |
| $\Delta g_6^\ell$ | $-32$ | $-32$ | $-32$ | $-32$ | $-32$ | $-32$ | $-32$ | $-32$ | $-32$ | $-32$ | $-32$ | $-32$ |

Volume $5.1 \times 10^{-4}$ $1.6 \times 10^{-8}$ $2.9 \times 10^{-8}$ $3.0 \times 10^{-13}$ $3.0 \times 10^{-12}$ $1.0 \times 10^{-16}$ $1.1 \times 10^{-13}$ $5.0 \times 10^{-18}$ $1.0 \times 10^{-8}$ $6.0 \times 10^{-13}$ $9.4 \times 10^{-25}$

As we have polarizations asymmetries, the straight forward analysis would be to consider all observables for full domain of $\cos \theta$. This analysis is named Unbinned where limits on anomalous couplings get constrained better reducing the volume of limits by a factor of 10 compared to the $\sigma$-unbinned. Motivated by $\cos \theta$ dependence of the observables (in Fig. 6) we perform the analysis Backward using all the observables ($A_{FB}$ is no longer observables here) in the backward direction, i.e. $\cos \theta \in [-1.0,0.0]$. In the Backward case the limits get improved further, the volume of limit reduces by a factor of 10 compared to the Unbinned case. To see how binning improve the limits we perform a analysis named $\sigma$-binned using only cross section in 8 bin. We see the analysis $\sigma$-binned is better than $\sigma$-unbinned and comparable to the analysis Unbinned but not better than the analysis Backward. The most natural and complete analysis would be to consider all the observables after binning. The analysis is named as Binned which has limits much better than any analysis. We also calculate one parameters limit on all the 14 couplings at 2$\sigma$ sensitivity ($\chi^2 = 4$) considering all the binned observables for unpolarized beams. The one parameter limits are presented in Table 3 in the last column for comparison. Although the Binned analysis provides best limits it is natural to perform Unbinned analysis also for comparison. We perform these two analysis for a set of beam polarization in the next subsection to obtain better limits that the one provided by unpolarized beams.

3.2 Limits on the Lagrangian couplings $c_i^\ell$

We estimate simultaneous limit on all 14 (independent) anomalous couplings of the Lagrangian in Eq. 4 using MCMC method for Unbinned and Binned case. The 95% simultaneous limits on anomalous couplings are shown in Table 3 for five different set of chosen beam polarization ($\eta_3$, $\xi_3$) namely $(-0.8,+0.8)$, $(-0.4,+0.4)$, $(0.0,0.0)$, $(+0.4,-0.4)$ and $(+0.8,-0.8)$, which are along the cross-diagonal of $\eta_3$, $\xi_3$ plane. The choice of beam polarizations is made motivated by the result in Ref. [64] where best choice of beam polarization are along the $\eta_3 = -\xi_3$ line. While presenting limits the notation used is following $\ell \equiv \text{low}$ with low being lower limit and high being upper limit.

We observe that the limits on anomalous couplings are tightest for the polarization $(+0.4,-0.4)$ in both Unbinned and Binned case and in each case of polarization the Binned limits are roughly twice better than the Unbinned limits on average. We estimate simultaneous limit on couplings to $\pm 0.25$ giving the volume of limits to be $4.4 \times 10^{-11}$. As we have polarizations asymmetries, the straight forward analysis would be to consider all observables for full domain of $\cos \theta$. This analysis is named Unbinned where limits on anomalous couplings get constrained better reducing the volume of limits by a factor of 10 compared to the $\sigma$-unbinned. Motivated by $\cos \theta$ dependence of the observables (in Fig. 6) we perform the analysis Backward using all the observables ($A_{FB}$ is no longer observables here) in the backward direction, i.e. $\cos \theta \in [-1.0,0.0]$. In the Backward case the limits get improved further, the volume of limit reduces by a factor of 10 compared to the Unbinned case. To see how binning improve the limits we perform a analysis named $\sigma$-binned using only cross section in 8 bin. We see the analysis $\sigma$-binned is better than $\sigma$-unbinned and comparable to the analysis Unbinned but not better than the analysis Backward. The most natural and complete analysis would be to consider all the observables after binning. The analysis is named as Binned which has limits much better than any analysis. We also calculate one parameters limit on all the 14 couplings at 2$\sigma$ sensitivity ($\chi^2 = 4$) considering all the binned observables for unpolarized beams. The one parameter limits are presented in Table 3 in the last column for comparison. Although the Binned analysis provides best limits it is natural to perform Unbinned analysis also for comparison. We perform these two analysis for a set of beam polarization in the next subsection to obtain better limits that the one provided by unpolarized beams.
on several other polarization point along $\eta_3 = -\xi_3$ direction and find the $(+0.4, -0.4)$ polarization to be the best to provide tightest limit. To check that $(+0.4, -0.4)$ to be best polarization we make a finer grid with 9 polarization points around it as

$$(\eta_3, \xi_3) = \{(+0.35, +0.40, +0.45), (-0.35, -0.40, -0.45)\}$$

and find simultaneous limits on them. We find that the limits on the couplings in these points are roughly same with slight variation and $(+0.4, -0.4)$ is best among them in both Unbinned and Binned case. The lowest row in the Table 3 shows the volume of the rectangular box that is formed by the limits of the couplings at the 95% BCI. The volume is smallest for Binned with polarization $(+0.4, -0.4)$ as discussed above. A pictorial representation of the limits on couplings given in Table 3 is shown in Fig. 8 for the easy comparisons.

3.3 Limits on operator couplings $c^\ell_i$ and their translation to couplings $\lambda_i^T$ in $SU(2) \times U(1)$ gauge

We also study the anomalous charge gauge boson couplings in $e^+ e^- \rightarrow W^+ W^-$ in the framework of effective higher dimensional operator in Eq. 2 in the $SU(2) \times U(1)$ gauge.
Table 5 The list of posterior 95% BCI of anomalous couplings $c_{ij}^{f}$ (10^{-2}) translated from $c_{ij}^{f}$ (using Eq. 5) in SU(2) × U(1) gauge in $e^+ e^- \rightarrow W^+ W^-$ at $\sqrt{s} = 500$ GeV, $L = 100$ fb^{-1}. Other details are same as in Table 3.

| $(\eta_1, \xi_1)$ | (−0.80, +0.80) | (−0.40, +0.40) | (0.0, 0.0) | (+0.40, −0.40) | (+0.80, −0.80) | (0.0, 0.0) |
|------------------|---------------|---------------|----------|---------------|---------------|----------|
| $c_{ij}^{f}$     | Unbinned      | Binned        | Unbinned | Binned        | Unbinned      | Binned    |
| $\lambda_V$     | +2.5          | +0.4          | +2.7     | +0.4          | +2.7          | +0.5     |
| $\lambda_\kappa$| −2.1          | −0.1          | −2.0     | −0.5          | −2.0          | −0.8     |
| $\Lambda V$     | +2.6          | +0.3          | +2.5     | +0.4          | +2.3          | +0.6     |
| $\Lambda \kappa$| −2.5          | −0.3          | −2.5     | −0.4          | −2.3          | −0.6     |
| $\Delta \kappa^f$| +11.0        | +3.2          | +4.6     | +1.7          | +1.5          | +0.5     |
| $\kappa^f$      | −47.0         | −3.9          | −14.9    | −5.3          | −8.7          | −6.3     |
| $\Delta \kappa^g$| +14.9        | +1.5          | +4.1     | +2.1          | +2.3          | +2.1     |
| $\kappa^g$      | −52.2         | −1.6          | −3.5     | −0.9          | −2.7          | −0.6     |
| $\Delta \kappa^d$| +28.3        | +2.7          | +7.7     | +3.6          | +4.0          | +3.6     |
| $\kappa^d$      | −8.2          | −2.6          | −4.4     | −1.4          | −2.3          | −0.7     |
| $\Delta \kappa^c$| +5.3          | +0.9          | +4.5     | +1.0          | +3.7          | +1.2     |
| $\kappa^c$      | −5.3          | −0.9          | −4.5     | −1.0          | −3.7          | −1.2     |

Fig. 9 The pictorial visualisation of simultaneous limits on anomalous couplings $c_{ij}^{f}$ of effective operators (in Eq. 2) on top row and translated limits on $c_{ij}^{f}$ of the Lagrangian (using Eq. 5) on bottom row in SU(2) × U(1) gauge at 95% C.L. in $e^+ e^- \rightarrow W^+ W^-$ for $\sqrt{s} = 500$ GeV, $L = 100$ fb^{-1} for Unbinned case (left column) and Binned case (right column). The numerical values of the limits can be read off in Tables 4 and 5.

Similar to the case of effective vertex formalism, we calculate one parameter limit on the couplings at 2σ sensitivity ($\chi^2 = 4$) in this scenario by varying one parameter at a time taking all the observables in Binned case with unpo-
4 On the choice of beam polarizations

In the previous section, we used the volume of the 95% limits as a measure of goodness of the combined limits to obtain the best choice of beam polarization. Here we discuss the average likelihood or the weighted volume of the parameter space define as [64]

\[
L(V_t, \{ \theta_i \}; \eta_3, \xi_3) = \int_{V_t} \exp \left[ - \frac{1}{2} \sum_i \mathcal{J}(\theta_i(f, \eta_3, \xi_3))^2 \right] df
\]

\[
= \int_{V_t} \mathcal{J}(f, \{ \theta_i \}; \eta_3, \xi_3) df,
\]

(17)

to cross-examine the beam polarization choices made in the previous section. Here \( f \) are the couplings and \( V_t \) is the volume of parameters space over which the average is done and has to be well above the volume of limits. One naively expects the limits to be tightest when \( L(V_t, \{ \theta_i \}; \eta_3, \xi_3) \) is minimum. We calculate the above quantity as a function of \((\eta_3, \xi_3)\) along \( \eta_3 = -\xi_3 \) for both the Unbinned and Binned case in the effective vertex formalism given in Lagrangian in Eq. 4. The normalized (normalized to 1) averaged likelihood as a function \( \eta_3(-\xi_3) \) is shown in Fig. 10. We observe that the averaged likelihood curve does not follow the variation of limits over beam polarization presented in Table 3 and Fig. 8, also it does not have minima where limits are tightest. This is contrary to the naive expectations. This is because the region of the 14 dimensional parameter space with \( \mathcal{J}(f, \{ \theta_i \}; \eta_3, \xi_3) > e^{-25/2} \) (say the blind region) is not a convex hull, i.e. the region of parameter space consistent with the SM at 5\( \sigma \) has a hole in it, like a 14 dimensional hollow (or broken) ellipsoid, for \( \eta_3 > 0 \). As a result, the weighted volume of the blind region can become small while its size, the 1-dimensional projections, remain large. Even in the two dimensional projection of the blind region we see disconnected regions. This is shown in the two dimensional posterior 95% contour in \( \Delta \kappa^Z - \Delta \kappa^Z \) plane in the Binned case for five chosen polarizations in Fig. 11. We see that an elliptical contour for beam polarization of \((-0.8,0.8)\) (dotted/black) breaks into two disconnected regions for \((0.4,-0.4)\) (solid/green) and then these regions grow in size for \((0.8,-0.8)\) (dashed/purple).

To further illustrate the non-convex nature of the blind region we look for regions where couplings \( c_{10}^{Z} \) are large but \( \chi^2 \) is low in both Unbinned and Binned cases and show the variation of \( \chi^2 \) along the line joining the SM point and the point \( c_{10}^{Z} \). The couplings along this line are parametrized as:

\[
c_i^{Z} = t \times c_{10}^{Z}, \quad i = 1,2,...14,
\]

(18)
giving us the SM point for \( t = 0 \) and the point \( c_i^{Z} \) for \( t = 1 \). The variation of \( \chi^2 \) as a function of \( t \) is shown in Fig. 12 for Unbinned and Binned cases at \((0.8,-0.8)\) beam polarization. We see that, for the Unbinned case (Fig. 12, top),

Fig. 10 The normalized averaged Likelihood \( L_{\text{Av}} = L(V_t, \{ \theta_i \}; \eta_3, \xi_3) \) (normalized to 1) in log scale as a function of \( \eta_3(-\xi_3) \) in \( e^+e^- \rightarrow W^+W^- \) in the Lagrangian approach for \( \sqrt{s} = 500 \text{ GeV}, \mathcal{L} = 100 \text{ fb}^{-1} \). The solid (blue) line and dashed (green) line represent the Unbinned and Binned case, respectively.
the regions with \( t \in [0.11, 0.88] \) has \( \chi^2 > 25 \) and hence outside the blind region. That is the blind region for the Unbinned case is not a convex hull. Similarly, for the Binned case (Fig. 12, bottom) the region with \( t \in [0.04, 0.97] \) are outside the blind region. This non-convex shape of the blind region leads to a small value of \( L(V_t, \{ \theta^i \}; \eta_2, \xi_3) \) while the size of the blind region remains large.

\[
\begin{align*}
\Delta \kappa^Z & = (0.4, -0.8) \\
\Delta \kappa^\gamma & = (0.4, -0.4) \\
\Delta \kappa & = (0.8, -0.8)
\end{align*}
\]

\[\text{Fig. 11} \] Two dimensional posterior 95\% BCI contour of \( \Delta \kappa^\gamma \) and \( \Delta \kappa^Z \) in \( e^+e^- \to W^+W^- \) for \( \sqrt{s} = 500 \text{ GeV} \) and integrated luminosity of \( \mathcal{L} = 100 \text{ fb}^{-1} \) for 5 chosen set of longitudinal beam polarizations of \( e^- \) and \( e^+ \).

5 Conclusion

In conclusion, we studied anomalous triple gauge boson couplings in \( e^+e^- \to W^+W^- \) with polarization observables of \( W \) boson together with the total cross section and the forward backward asymmetry for \( \sqrt{s} = 500 \text{ GeV} \) and integrated luminosity of \( \mathcal{L} = 100 \text{ fb}^{-1} \) with polarization of \( e^- \) and \( e^+ \) beams. We have 14 anomalous couplings, where as we have only 10 observables to measure them. So we Binned all the observables (\( A_{FB} \) excluded) in 8 regions of the \( \cos \theta \) of \( W \) to increase the number of observables to measure the couplings. We estimated simultaneous limit on all the couplings in Unbinned case as well as in Binned case for several chosen set of beam polarization. We find \((0.4, -0.4)\) to be the best beam polarization to obtain tightest constrain on the anomalous couplings \( c_{ij}^\gamma \) in both Unbinned and Binned cases and Binned limits are tighter (roughly twice) than the limits in Unbinned in each polarization in the effective vertex factor formalism given in the Lagrangian in Eq. 4. We further estimated limits on anomalous couplings \( c_{ij}^\gamma \) from \( SU(2) \times U(1) \) higher dimension effective operators in both Unbinned and Binned cases for the same set of chosen beam polarizations and the translated limits on the non-vanishing Lagrangian couplings \( c_{ij}^\gamma \). The limits on Lagrangian couplings in \( SU(2) \times U(1) \) are tighter than the limit when all 14 couplings considered. We show the inconsistency between best choice of beam polarizations and minimum likelihood averaged over the anomalous couplings. This is because the blind region in non-convex resulting in small weighted volume, while the rectangular volume of parameters space from the limits is large. The best choice of beam polarization will be governed solely based on the MCMC posterior limits on anomalous couplings, which is \((0.4, -0.4)\) in the case of Lagrangian approach (can be seen in Table 3 and Fig. 8). For effective operator formalism the best choice is \((-0.8, 0.8)\) on the basis of volume of rectangle formed by the limits can be seen in Tables 4, 5. All the couplings do not poses tighter limits at this couplings, some of them poses tighter limit on other choice of beam polarizations. Our one parameter limits on \( g_4^\gamma, g_5^\gamma \) from Table 3 and others from Tables 4, 5 are much better than the available tight-
est one parameter limits from experiments in Table 1, while simultaneous limits are comparable.

In the $W^±Z$ production at LHC, only the $W^+W^-Z$ couplings appear limiting the number of anomalous couplings to 7. With small number of couplings multivaluedness may be avoided and hence tighter limit on anomalous couplings is expected at high energy. The work on anomalous couplings in $W^±Z$ production at LHC is underway and will be presented elsewhere.

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Appendix A: Labelling of anomalous gauge boson couplings and the dependences of observables on
anomalous gauge boson couplings $c_i^V$ in $e^+e^-\to W^+W^−$.

The anomalous gauge boson couplings $c_i^V$ of effective operator in Eq. 2 and the couplings $c_i^\mu$ of the Lagrangian in Eq. 4 and the couplings $c_i^\nu$ of Lagrangian in $SU(2)\times U(1)$ (given in Eq. 5) gauge are labelled as

\[ c_i^V = \{c_{WW}^V, c_{WZ}^V, c_{WW^\pm}^V, c_{WW^\mp}^V\} \]  
\[ c_i^\mu = \{g_1^V, g_2^V, \lambda^V, \xi^V, \kappa^V, \tilde{\kappa}^V\}, \quad V = g, Z \]  
\[ c_i^\nu = \{\lambda^V, \lambda^\mu, \Delta \xi^V, \tilde{\xi}^V, \Delta \kappa^V, \Delta \tilde{\kappa}^V\} \]

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Table 6 Dependences of observables (numerators) on anomalous couplings in the form of $c_i^L$ (linear), $(c_i^L)^2$ (quadratic) and $c_i^L c_j^L$, $i \neq j$ (interference) in the process $e^+ e^- \rightarrow W^+ W^-$. For the linear and quadratic terms $V = \gamma / Z$ and for cross interference terms $V = \gamma Z$. The “✓” (checkmark) represents the presence and “—” (big-dash) corresponds to absence of the term in the first column in the same row.

| Parameters | $\sigma$ | $\sigma \times A_x$ | $\sigma \times A_y$ | $\sigma \times A_z$ | $\sigma \times A_{xy}$ | $\sigma \times A_{xz}$ | $\sigma \times A_{yz}$ | $\sigma \times A_{x2+y2}$ | $\sigma \times A_{y2}$ | $\sigma \times A_{y2}$ |
|------------|---------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $\Delta g_V^1$ | ✓ ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\gamma_V$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\gamma_V^2$ | ✓ ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\lambda_V^2$ | ✓ ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\Delta \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $(\Delta g_V^1)^2$ | ✓ ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $(\gamma_V^1)^2$ | ✓ ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $(\sigma_V^1)^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $(\Delta \kappa_V^1)^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $(\kappa_V^1)^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\Delta g_V^1 \sigma_V^4$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\gamma_V^1 \sigma_V^4$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\gamma_V^1 \sigma_V^4$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\Delta \kappa_V^1 \Delta \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\Delta \kappa_V^1 \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\Delta \kappa_V^1 \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\lambda_V \lambda_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\lambda_V \lambda_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\lambda_V \Delta \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\lambda_V \Delta \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\lambda_V \Delta \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\lambda_V \Delta \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\Delta \kappa_V^1 \kappa_V^2$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
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