Circularly polarized states spawning from bound states in the continuum

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Bound states in the continuum in periodic photonic systems like photonic crystal slabs are proved to be accompanied by vortex polarization singularities on the photonic bands in the momentum space. The winding structures of polarization states not only widen the field of topological physics but also show great potential that such systems could be applied in polarization manipulating. In this work, we report the phenomenon that by in-plane inversion (C\textsubscript{2}) symmetry breaking, pairs of circularly polarized states could spawn from the eliminated Bound states in the continuum. Along with the appearance of the circularly polarized states as the two poles of the Poincaré sphere together with linearly polarized states covering the equator, full coverage on the Poincaré sphere could be realized. As an application, ellipticity modulation of linear polarization is demonstrated in the visible frequency range. This phenomenon provides new degree of freedom in modulating polarization.

Polarization is one of electromagnetic wave’s most essential properties. Controlling the polarization is found very important in a lot of fields, such as 3d imaging \cite{1}, optical communication \cite{2}, and quantum optics \cite{3}. In recent years, great attention has been paid to modulate polarization of light with compact structures such as metasurfaces \cite{4,7} instead of classical wave plates and polarizers, which are more applicable in on-chip devices. On the other hand, application of another kind of structure, photonic crystal (PhC) slab, in modulating polarization, is capturing interest nowadays \cite{8,11}. Their simplicity in fabrication, designable band structures, and complex polarization features in the momentum space which are topologically linked with bound states in the continuum \cite{10,26} are favorable for polarization modulation. However, the reported PhC slabs only support nearly linearly polarized resonances which only cover a small area on the Poincaré sphere. The coverage on the Poincaré sphere characterized the polarization properties of the system. Small coverage with the two poles (circular states of polarization) missing indicates its limited capability in full Stokes polarization modulating. In this paper, we report that, by breaking the in-plane inversion (C\textsubscript{2}) symmetry of a PhC slab, the at-Γ BICs also known as vortex polarization singularities (V-points) \cite{12,13,15,17,21,23,27} on photonic bands will be eliminated \cite{22}. As the singularities are broken, the winding of the main axis of the polarization states are preserved, leading to generation of pairs of circularly polarized states (C-points) near the Γ point. With a line of linearly polarized states (L-line) enclosing the position of the original BIC, the generation of C-points would even enable full coverage on the Poincaré sphere. We then demonstrated the application of this phenomenon in ellipticity modulation of light.

For a 2D PhC slab, there would be a series of Bloch resonant modes with different frequencies \( f \) and wave vectors \( k \) forming photonic bands. Radiation polarization states of these modes on an arbitrary band could be projected into the structure plane and mapped onto the Brillouin zone (BZ), which defines a polarization field in the momentum space \cite{12,18,15,17,21,23,27}. These Bloch modes are mostly radiative, unless destructive interference or symmetry mismatch makes them non-radiative, i.e., BICs. The BICs appear as vortex singularities (V-points) in the polarization field. To characterize the po-

FIG. 1. Schematic view of how circularly polarized states (C-points) spawn from a bound state in the continuum when in-plane inversion (C\textsubscript{2}) symmetry is broken. C-points and lines of linearly polarized states (L-lines) could be found near Γ point. The asymmetric field of the mode in the C\textsubscript{2} symmetry broken structure induces extra in-plane multipole moments. L(R)CP: left(right)-handed circular polarization; L(R)H: left(right)-handed.
Polarization properties of the system, we can map the polarization states of the Bloch modes onto the Poincaré sphere of which the coordinates are specified by Stokes parameters $S_0$, $S_1$, $S_2$ and $S_3$. In this language, those non-radiative singularities will have zero values of Stokes parameters. Thus, they are also singularities in the normalized Stokes parametric space, shown as the left panel of Fig. 1. Proved by C. W. Hsu et al. [9], the polarization states are represented by ellipses of which the red color corresponds to left-handed states and the blue color corresponds to right-handed ones. The data points are sampled per 10 degrees on every loop in the BZ.

The results could be found in the Supplemental Material (SM) [38]. Subsequently, we obtained the projected ratio of the reduced topline length $2\Delta \alpha / L$ to the baseline length $L$, shown in Fig. 2(d). Increasing $\alpha$ from 0 to 1 would transform the holes from squares to isosceles triangles, while $L = \sqrt{2S_0/(2 - \alpha)}$.

We first focus on the BIC at the Γ point of the band TE2 for example. Applying finite element method calculation, we firstly confirmed that the quality factors of the states at the Γ point on band TE2 obey the law $\alpha^{-2}$ when the perturbation is small, which is proved in [22]. The results could be found in the Supplemental Material (SM) [38]. Subsequently, we obtained the projected radiation polarization states $(d_x, d_y)$ on band TE2 to see the generation of C-points. The band structures and polarization maps near the Γ point of systems with different asymmetry parameters $\alpha = 0, 0.1, 1$ are plotted in
analyzed 2D PhC slab with $\alpha = 1$, finding the phenomenon not only common but also giving different topological polarization configurations of C-points near the $\Gamma$ point \cite{8, 33}. A ‘lemon’ is found on TE2, a ‘star’ is found on TM2, and a ‘monstar’ is found on TM5. The band structures along $\Gamma$-X and $\Gamma$-X’ are plotted in Fig. 3(a), while the polarization distributions of the different C-points are plotted in Fig. 3(b). The different configurations on different bands give us much more freedom in designing the eigenmodes with desired polarization states at desired incident or radiating directions.

For an experimental observation of this phenomenon in the visible frequency range, we fabricated the simulated PhC slabs. The designed hole array is etched on a freestanding silicon nitride layer applying reactive-ion etching with the assistance of the mask poly(methyl...
methacrylate) layer etched by the electron beam lithography. For optical measurements, we applied our homemade momentum-space imaging spectroscopy system \cite{10} to obtain their polarization-dependent angle-resolved transmittance spectra. Under LCP (and RCP) incidence, the spectra [Fig. 3(c)] of the $\alpha = 0$ sample shows no diminishing point other than the non-excited BICs at the $\Gamma$ point. In contrast, the $a$- and $\Gamma$-states of the $\alpha = 1$ sample [Fig. 3(a)] are radiative and some regions near the $\Gamma$ point on the bands are diminished instead under LCP incidence. The non-excited states in these regions are hence RH polarized with C-points at the center. Changing the incidence to RCP, the diminished regions of the $\alpha = 1$ sample switch to the other side of the BZ, meaning another C-point with opposite chirality [Fig. 3(b)]. The zoomed-in plots of the diminished regions are shown in Fig. 4(d), with the detailed RCP spectra showing the C-point on band TE2 included in the SM \cite{38}. The existence of the C-points are proved and agree with the simulations above, proving the phenomenon.

Basing on the angle-resolved polarization dependent spectra we measured, we mapped the transmittance along the band TE2 as a function of $f$ and $k$ under LCP and RCP incidence in order to observe the C-points and the coverage on Poincaré sphere more clearly, plotted in Fig. 5(a). Note that, the maps not only cover the BZ, but also span a spectral range. Changing the period to 400 nm, the bottom line length of the triangle to 300 nm, the thickness of the slab to 215 nm, we were able to suppress the effect of Fano resonances, and the reflectance ($1 - \text{transmittance}$) in such a case shall approximately correspond to the inner product of the projected polarization state and the incident circular polarization [detailed in the SM \cite{38}]. On either map, we can see one spot where the transmittance is close to 1, indicating a C-point with which the incident light cannot couple. From the map, the distribution of polarization states is clearly exhibited, and we can see that the coverage approaching full Poincaré sphere is directly observed in a rather small range of the BZ.

The coverage approaching full Poincaré sphere gives us wider choices in modulating polarization with PhC slabs. Here in the visible frequency range, we demonstrate the application in modulating the ellipticity $\rho_c = S_3/S_0$. ($S_0, S_3$ are the first and the fourth Stokes parameters.) In Fig. 5(b), we plotted the ellipticity map of the completely polarized transmitted light on resonances of the band TE2, applying the sample measured in Fig. 5(a). $\rho_c$ here is calculated from the on-resonance transmittance measured with $p$, $s$- and $\pm 45^\circ$-polarized analyzers. As we can see from Fig. 5(b), the linearly $p$-polarized incident light ($\rho_c = 0$) is transformed to elliptically polarized light, of which $\rho_c$ reaches 0.7 at most. The results of experiments (right panel) match up with rigorous coupled wave algorithm (RCWA) simulations (left panel) well. Choosing the right parameters and with higher fabrication precision, the linearly polarized light can be perfectly transformed into circularly polarized light in theory. An example is detailed in the SM \cite{38}. To be noted that, this effect can be viewed as a result of strong orbit-spin coupling induced by $C_2$ symmetry breaking, and similar phenomenon in a mono-chromatic beam also induced by symmetry breaking has been shown previously \cite{39,40}.

In conclusion, we reported the phenomenon that by breaking the in-plane inversion symmetry ($C_2$) of a 2D PhC slab, the non-radiative vortex singularities at the $\Gamma$ point will be eliminated. As a consequence, C-points with different polarization configuration could be found near the $\Gamma$ point. Together with an $L$-line looping around, the full Poincaré sphere coverage can be realized. We theoretically verified and experimentally observed the generation of C-points and the coverage approaching full Poincaré sphere. This phenomenon enriches the knowledge on polarization properties of 2D PhC slabs in the momentum space. The coverage on the Poincaré sphere offers us new latitude to modulate polarization with 2D PhC slabs, which may be applied in vector beam generating or quantum optics.

We thank Dr. Ang Chen, Prof. Chao Peng, Prof. Chia Wei Hsu, Prof. Dezhuang Han, Prof. Ling Lu, Prof. Meng Xiao, Prof. Haiwei Yin and Prof. Shaoyu Yin for helpful discussions. The work was supported by 973 Program and China National Key Basic Research Program (2015CB659400, 2016YFA0301100,
2016YFA0302000 and 2018YFA0306201) and National Science Foundation of China (11774063, 11727811, 91750102 and 11604355). The research of L. S. was further supported by Science and Technology Commission of Shanghai Municipality (17ZR1442300, 17142200100).

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