Suk, Andrew; Tomon, István
Hasse diagrams with large chromatic number. (English) Zbl 1470.05062
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Summary: For every positive integer $n$, we construct a Hasse diagram with $n$ vertices and independence number $O(n^{3/4})$. Such graphs have chromatic number $\Omega(n^{1/4})$, which significantly improves the previously best-known constructions of Hasse diagrams having chromatic number $\Theta(\log n)$. In addition, if we also require girth of at least $k \geq 5$, we construct such Hasse diagrams with independence number at most $n - \frac{k}{n(1)}$. The proofs are based on the existence of point-line arrangements in the plane with many incidences and avoids certain forbidden subconfigurations, which we find of independent interest.

These results also have the following surprising geometric consequence. They imply the existence of a family $C$ of $n$ curves in the plane such that the disjointness graph $G$ of $C$ is triangle-free (or has high girth), but the chromatic number of $G$ is polynomial in $n$. Again, the previously best-known construction, due to J. Pach et al. [LIPIcs – Leibniz Proc. Inform. 77, Article 59, 15 p. (2017; Zbl 1432.68365)], had only logarithmic chromatic number.

MSC:
05C15 Coloring of graphs and hypergraphs
05D10 Ramsey theory
52C10 Erdős problems and related topics of discrete geometry

Keywords:
disjointness graph; triangle-free graph; clique number; symmetric Ramsey number

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