Optimal Facility Allocation and Determination of Demand Response Participation Rate Considering Uncertainties in Power Systems*

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It is necessary to install a large number of renewable energy (RE) systems such as photovoltaic systems (PVs) to solve environmental problems. However, an electricity grid with RE systems experiences problems such as power shortages and surpluses because of uncertainties in generating outputs. Demand response (DR) can restrain both shortages and surpluses caused by PV outputs, but also increase power shortages and surpluses due to the uncertainties in DR capabilities. This research employs energy storage systems (ESSs) to reduce both power shortages and surpluses, and formulates the optimization problem of facility allocation and determination of the DR participation rate to minimize the number of installed ESSs, as they are very costly. To solve this optimization problem, we propose “slow” and “fast” ESSs, which can quantify the impacts of the increase in participants of the DR. As a result, this study demonstrates that an optimal allocation of PVs and ESSs can be achieved that enables the minimization of installed ESSs, satisfies the PVs installation target, and satisfies the constraints of the probabilistic indices for shortage and surplus to evaluate the uncertainties of PV outputs and DR capabilities. Moreover, the DR participation rate is determined in the same optimization problem.

**Nomenclatures**

Indices and Sets:
- \( i \) Index of buses
- \( t \) Index of time
- \( g \) Index of generators
- \( G \) Set of buses with generators

Decision variables:
- \( P_{G_{i,t}} \) Output of generator in bus \( i \) at \( t \)
- \( C_{PV_i} \) Capacity of PVs
- \( C_{ESS_i} \) Capacity of slow ESSs
- \( C_{ESS_{FU}} \) Capacity of fast ESSs for shortages and surpluses

Parameters:
- \( N \) Number of all buses
- \( p_{PV} \) Price of PVs
- \( p_{ESS} \) Price of ESSs
- \( P_{V_{i,t}} \) Output of PV per unit in bus \( i \) at \( t \)
- \( P_{L_{i,t}} \) Electricity demand in bus \( i \) at \( t \)
- \( P_{DR_{i,t}} \) Capability of DR per unit in bus \( i \) at \( t \)
- \( B_{ij} \) Susceptance between buses \( i \) and \( j \)
- \( P_{max} \) Capacity of transmission line between buses \( i \) and \( j \)
- \( \alpha_{i,t}, \beta_{i,t} \) Upper limits of power shortages and surpluses for 1 h in bus \( i \) at \( t \)

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1. Introduction

It is essential to install several photovoltaic systems (PVs) in electricity grids to reduce the use of fossil fuels as energy resources and to ensure energy security. However, the uncertainty of PV output often causes both power shortages and surpluses. There are several methods to solve problems of power shortage and surplus; for example, the installation of energy storage systems (ESSs) and demand response (DR) systems. ESSs, such as batteries and pumped-storage power plants, can reduce power shortage and surplus by discharging and charging with high reliability. However, the number of ESSs installed should be minimized because of their high cost. As well as ESSs, DR systems can be one of the solutions for both power shortages and surpluses, the DR system can be cheaper than ESSs for reducing power shortages and surpluses, which encourages people to travel less. Although a DR system can be cheaper than ESSs for reducing power shortages and surpluses, the DR system capabilities have uncertainties because of the variability in participants' behavior. Consequently, we need to focus on both the uncertainties based on PV outputs and the capabilities of a DR system. Moreover, the excess of participants in DR can result in a larger number of installed ESSs being required to restrain power shortages and surpluses caused by the uncertainties. Therefore, it is necessary to determine the optimal DR participation rate that enables minimization of the number of ESSs installed in a grid.

Previous research [1–4] has discussed the optimal allocation of PVs and the operation of grids with renewable energy resources, as well as the optimal DR strategy, considering the uncertainty in the capabilities of RE and a DR system. Our review of the literature revealed that no studies have focused on both the uncertainties using PV outputs and the capabilities of DR to determine the optimal allocation of PVs and ESSs, especially in energy consumption. In addition, to our knowledge, no research has revealed the optimal DR participation rate, focusing on the uncertainties in the capabilities of DR from the viewpoint of minimizing the installation of ESSs.

This study determines not only the optimal allocation of PVs and minimized ESSs but also the optimal DR participation rate, while limiting the power shortages and surpluses caused by uncertainties in PV outputs and the capabilities of DR, and meeting a target level for the installation of PVs. To introduce limits to power shortages and surpluses under constraints, it is necessary to evaluate the power shortages and surpluses given the uncertainties in operating an electricity grid.

We employ two probabilistic indices to take into account the uncertainties of using PV outputs and the capabilities of DR: Expected Energy Not Supplied (EENS) [5] and Expected Energy Not Used (EENU) [6]. However, it is difficult to derive the optimal facility allocation that satisfies the constraints of EENS and EENU, because it is not ensured that the problem expressed in terms of EENS and EENU has feasible solutions. Therefore, this research proposes “slow” and “fast” ESSs to resolve the above issues, and applies these two types of ESSs to determine the optimal DR participation rate. This study demonstrates that the optimal facility allocation and DR participation rate can be achieved while satisfying the constraints of the supply reliability indices (EENS and EENU) and the PV installation targets.

2. Optimal Allocation Problem of PVs and ESSs

This section first describes a formulation for optimizing the allocation of PVs and ESSs without uncertainties. As a first step, the DR participation rate, \( \pi_{\text{DR},i} \), is fixed in this problem. This formulation is expanded from a multi-period optimal power flow [7], and can determine not only the grid operation, but also facility allocation. To simplify the problem and reduce the calculation time, the power system is approximated as a DC circuit in which each bus has the same voltage, because our target power system is a transmission system [8]. The time step for grid operation is assumed to be 1 h.
(QP)

\[
\begin{align*}
\min_{C, P} & \quad p_{\text{PV}} \sum_{i=1}^{N} C_{\text{PV}i} + p_{\text{ESS}} \sum_{i=1}^{N} C_{\text{ESS}i} \\
& + \sum_{t=1}^{24} \sum_{g \in G} f_{g}(P_{g,t}) \\
\text{s.t.} & \quad \sum_{i=1}^{N} C_{\text{PV}i} \geq C_{\text{PV}}^{\text{target}} \tag{2} \\
& \quad P_{N_i,t} = P_{G_i,t} + C_{\text{PV}i} \cdot \tilde{p}_{\text{unit}i,t} + P_{E_i,t} \\
& \quad - P_{B_i,t} - (1 + r_{\text{DRi}} \cdot \tilde{p}_{\text{unit}i,t}) P_{L_i,t} \tag{3} \\
& \quad - \alpha_{i,t} \leq P_{N_i,t} \leq \beta_{i,t} \tag{4} \\
& \quad P_{G_i}^{\text{min}} \leq P_{G_i,t} \leq P_{G_i}^{\text{max}} \tag{5} \\
& \quad |P_{G_i,t} - P_{G_i}^{\text{nom}}| \leq \Delta P_{G_i} \tag{6} \\
& \quad |B_{ij}(\theta_{i,t} - \theta_{j,t})| \leq |P_{B_j}| \tag{7} \\
& \quad P_{B_i,t} = \sum_{j \neq i}^{N} B_{ij}(\theta_{i,t} - \theta_{j,t}) \tag{8} \\
& \quad S_{i,t+1} - S_{i,t} = -P_{E_i,t} \tag{9} \\
& \quad 0 \leq S_{i,t} \leq C_{\text{ESS}i} \tag{10}
\end{align*}
\]

where \( C := [C_{\text{PV}}, C_{\text{ESS}}]^T \), and \( P := [P_N, P_G, P_E, P_B]^T \). Eq. (1) is the objective function for optimizing the allocation of PVs and ESSs and minimizing the costs of grid operation and facility allocation of the PVs and ESSs. The cost of grid operation includes the fuel cost of conventional power plants apart from PVs such as thermal power plants, which is usually modeled by a quadratic or linear function. The facility costs have much bigger impacts on the objective function than the fuel cost, because the order of the facility costs is relatively large. Eq. (2) is a constraint for the installation target of PVs in the grid. Here, Eqs. (3)–(10) are constraints related to grid operation, and apply for all \( i,t \). Eq. (3) is a constraint of supply-demand balance in bus \( i \) at \( t \) that considers the PV outputs and scheduling of ESSs, where negative and positive values of \( P_{N_i} \) denote power shortage and surplus, respectively. \( P_{N_i} \) is limited by eq. (4), considering both shortages and surpluses. Moreover, this research employs two assumptions when giving \( \alpha_{i,t}, \beta_{i,t} \): no power shortages and surpluses in buses without demands, and the same upper limits of power shortages and surpluses for all buses and times. Eqs. (5), (6) are constraints for conventional power plants, and represent the upper and lower limits of generator output and the upper and lower limits of the rate of change of generation, respectively. Eq. (7) places a constraint on the transmission capacity limits, and eq. (8) is the relationship between the phase angle in a bus and the transmission power using the DC approximation. Eqs. (9), (10) are constraints related to ESSs; the former is a relational expression for charging/discharging and state of charge over 1 h, and the latter is a constraint of limits of stage of charge.

The optimization problem (QP) is formulated as a quadratic programming problem, because the objective functions are quadratic and the constraints are linear based on the assumption of a DC circuit. Moreover, the PV outputs are considered to be deterministic in this step.

### 3. Probabilistic Indices

This research employs probabilistic indices to evaluate the power shortages and surpluses caused by uncertainties in PV outputs and DR capabilities. In this section, we define a probabilistic index and describe its application.

First, to simplify the calculation of indices, \( \bar{P} \) is set as shown in eq. (11), which is derived from eq. (3) with uncertainties.

\[
\bar{P}_{i,t} = \bar{P}_{N_i,t} + P_{L_i,t} = P_{G_i,t} + C_{\text{PV}i} \cdot \tilde{p}_{\text{unit}i,t} + P_{E_i,t} - P_{B_i,t} - r_{\text{DRi}} \cdot \tilde{p}_{\text{unit}i,t} P_{L_i,t} \tag{11}
\]

This stochastic variable includes both uncertainties in PV outputs and DR capabilities. We denote the mean and standard deviation of \( \bar{P} \) as \( \bar{\sigma}_P \) and \( \sigma_P \), respectively.

Second, we assume that the PV outputs and DR capabilities for 1 h follow a Gaussian distribution. In fact, this assumption is not perfect but we employ this because Gaussian distribution has often been used in other research related to RE and demands in order to simplify the formulation, including uncertainties[9–12]. The use of other kinds of probability distributions is reserved for future work.

Third, we assume that the PV outputs and DR capabilities are not correlated.

These assumptions model \( \bar{P} \) and \( \sigma_P \), as shown in eqs. (12), (13).

\[
\begin{align*}
\bar{P}_{i,t} &= \bar{P}_{N_i,t} + P_{L_i,t} = P_{G_i,t} + C_{\text{PV}i} \cdot \tilde{p}_{\text{unit}i,t} + P_{E_i,t} - P_{B_i,t} - r_{\text{DRi}} \cdot \tilde{p}_{\text{unit}i,t} P_{L_i,t} \tag{12} \\
\sigma_{P_{i,t}} &= \sqrt{\sigma_{P_{\text{PV}i},t}^2 + \sigma_{\text{DRi}}^2} \\
&= \sqrt{(C_{\text{PV}i} \cdot \tilde{p}_{\text{unit}i,t})^2 + (r_{\text{DRi}} \cdot \tilde{p}_{\text{unit}i,t})^2} \tag{13}
\end{align*}
\]

Moreover, the probability density function can be expressed as

\[
\begin{align*}
d_P(x, \bar{P}, \sigma_P) &= \frac{1}{\sigma_P \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \bar{P}}{\sigma_P} \right)^2 \right]. \tag{14}
\end{align*}
\]

Using this assumption for the probability density function, two probabilistic indices are formulated: EENS[5] and EENU[6]. The former measures the incidence of power shortages, and the latter is a metric for power surpluses. Based on the probability density function of \( \bar{P} \), Fig. 1 shows the amount of shortages and surpluses, where the probability of shortages and surpluses is indicated in blue and red, respectively.

Eqs. (15), (16) describe the formulation of EENS
and EENU, which are defined as the expectations of power shortages or surpluses given the inherent uncertainties, respectively. These indices are introduced in the constraints of our optimization problem with uncertainties.

\[ e_{S_{i,t}}(\bar{P}_{i,t}, \sigma_{P_{i,t}}, P_{Li,t}) = \int_{-\infty}^{\bar{P}_{i,t}} (\bar{P}_{i,t} - x) dP(x, \bar{P}_{i,t}, \sigma_{P_{i,t}}) dx \]

\[ = \frac{\sigma_{P_{i,t}}}{\sqrt{2}} \left[ z_{i,t}(\text{erf}(z_{i,t}) - 1) + \frac{1}{\sqrt{\pi}} \exp(-z_{i,t}^2) \right] \]  
(15)

\[ e_{U_{i,t}}(\bar{P}_{i,t}, \sigma_{P_{i,t}}, P_{Li,t}) = \int_{\bar{P}_{i,t}}^{\infty} (x - \bar{P}_{i,t}) dP(x, \bar{P}_{i,t}, \sigma_{P_{i,t}}) dx \]

\[ = \frac{\sigma_{P_{i,t}}}{\sqrt{2}} \left[ z_{i,t}(\text{erf}(z_{i,t}) + 1) + \frac{1}{\sqrt{\pi}} \exp(-z_{i,t}^2) \right] \]  
(16)

Here, \( z_{i,t} \) can be expressed as eq. (17), and \( \text{erf}(\cdot) \) denotes the error function, as shown in eq. (18).

\[ z_{i,t} = \frac{\bar{P}_{i,t} - P_{Li,t}}{\sqrt{2} \sigma_{P_{i,t}}} = \frac{\bar{P}_{Ni,t}}{\sqrt{2} \sigma_{P_{i,t}}} \]  
(17)

\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-y^2) dy \]  
(18)

4. Optimization Problem Considering Uncertainties

This section describes an optimization problem to determine both the facility allocation and the DR participation rate, including uncertainties in PV outputs and DR capabilities. To simplify the expressions, \( F_{CV}, F_{CE}, F_P, \) and \( F_R \) are defined as eqs. (19)–(22).

\[ F_{CV} = p_{PV} \sum_{i=1}^{N} C_{PV_i} \]  
(19)

\[ F_{CE} = p_{ESS} \sum_{i=1}^{N} C_{ESS_i} \]  
(20)

\[ F_P = \sum_{t=1}^{24} \sum_{g \in G} f_{G_g}(P_{G_g,t}) \]  
(21)

\[ F_R = \rho \sum_{t=1}^{24} \sum_{i=1}^{N} (\alpha_{i,t}^r + \beta_{i,t}^r) \]  
(22)

The formulation can be modeled as a two-stage stochastic program with recourse (SP).

\[ \min_{C, \rho_{DR}, \alpha^r, \beta^r} \left[ F_{CV} + F_{CE} + F_{PR}^\min (P) \right] \]  
(23)

s.t. \eqref{eq:2}, \eqref{eq:25}

\[ 0 \leq r_{DR} \leq 1 \]  
(24)

\[ F_{PR}^\min (P) = \min_{P, \alpha^r, \beta^r} \]  
(25)

s.t. \( \bar{P}_{Ni,t} = P_{Gi,t} + C_{PV_i} \cdot \bar{P}_{V_{i,t}} + P_{Ei,t} \)

\[ -P_{Bi,t} - (1 + r_{DR} \cdot \bar{P}_{DR_{i,t}}) P_{Li,t} \leq \alpha_{i,t} + \alpha_{i,t}^L \]  
(26)

\[ e_{S_{i,t}}(\bar{P}_{Ni,t}, \sigma_{P_{i,t}}) \leq \beta_{i,t} + \beta_{i,t}^L \]  
(27)

\[ e_{U_{i,t}}(\bar{P}_{Ni,t}, \sigma_{P_{i,t}}) \leq \beta_{i,t} + \beta_{i,t}^L \]  
(28)

eqs. (5)–(10)

Eq. (23) minimizes the total cost of grid operation and facility allocation, considering the given uncertainties. Eq. (24) is a constraint on the DR participation rate. As we are accounting for the uncertainties, the power flow constraint changes from eq. (3) to eq. (26), which includes probabilistic variables. Moreover, EENS and EENU are employed to formulate a constraint on the power shortages and surpluses, as shown in eqs. (27), (28). \( \alpha_{i,t}^r \) and \( \beta_{i,t}^r \) are introduced to ensure that the shortage and surplus constraints have feasible solutions in all cases. These variables play the role of relaxing EENS and EENU, and are minimized in eq. (23) as a penalty function.

Although we have formulated this optimization problem with stochastic variables, it can be reformulated without stochastic variables because the EENS and EENU constraints are deterministic in terms of \( \bar{P} \) and \( \sigma_{PV} \), based on eqs. (15), (16).

Therefore, the optimization problem can be written as a one-stage problem as follows:

\[ \min_{C, \rho_{DR}, \alpha^r, \beta^r} \left[ F_{CV} + F_{CE} + F_P + F_R \right] \]  
(29)

s.t. \eqref{eq:2}, \eqref{eq:4}–\eqref{eq:10}, \eqref{eq:24}, \eqref{eq:26}, \eqref{eq:27}, \eqref{eq:28}

As shown in eqs. (27)–(29), \( \alpha_{i,t}^r \) and \( \beta_{i,t}^r \) are also necessary in the one-stage problem. This is because problem CSP could not have feasible solutions when both constraints of EENS and EENU are introduced in the problem. Moreover, it is difficult to solve problem CSP not only because this nonlinear constraints including an exponential function but also because the scale of CSP is so large. To solve problem CSP, we use a “Block Coordinate Descent Method”. We derive a two-stage optimization problem in (CSP-1)
and (CSP-2), which are solved alternately. The optimized variables derived from the first stage are used as parameters in the second stage, and vice versa.

\[ \text{(CSP-1)} \]
\[
\begin{align*}
\text{min} & \quad c_{\text{ESS}, \mathbf{P}} [F_{CE} + F_P + F_{CV} + F_{R}] \\
\text{s.t.} & \quad P_{N_i,t} = P_{G_{i,t}} + C_{PV_{i,t}} + P_{\text{unit}_{i,t}} + P_{E_{i,t}} \\
& \quad -P_{B_{i,t}} - (1 + r_{\text{DR}_{i,t}}) P_{\text{unit}_{i,t}} \\
& \quad e_{S_{i,t}}(P_{N_i,t}, \sigma_{P_{i,t}}) \leq -\alpha_{i,t} + \alpha_{i,t}^* \\
& \quad e_{U_{i,t}}(P_{N_i,t}, \sigma_{P_{i,t}}) \leq \beta_{i,t} + \beta_{i,t}^* \\
& \quad \text{eqs. (5)-(10)}
\end{align*}
\]

\[ \text{(CSP-2)} \]
\[
\begin{align*}
\text{min} & \quad c_{\text{PV}, t, \alpha^*, \beta^*} [F_{CV} + F_R + F_{CE} + F_P'] \\
\text{s.t.} & \quad P_{G_{i,t}} = P^*_{G_{i,t}} + C_{PV_{i,t}} + P_{\text{unit}_{i,t}} + P_{E_{i,t}} \\
& \quad -P^*_{B_{i,t}} - (1 + r^*_{\text{DR}_{i,t}}) P_{\text{unit}_{i,t}} \\
& \quad \text{eqs. (2), (24), (27), (28)}
\end{align*}
\]

To accelerate the process of solving (CSP-1), some constraints should be reformulated. In particular, eqs. (27), (28) should be converted from nonlinear constraints, including exponential functions, to linear constraints using the inverse functions of EENS and EENU, as shown in eq. (37).

\[
e_{S_{i,t}}(\sigma_{P_{i,t}}^*, \alpha^*) \leq P_{N_{i,t}} \leq e_{U_{i,t}}(\sigma_{P_{i,t}}^*, \beta^*) (37)
\]

This is because \( e_{S_{i,t}}(\sigma_{P_{i,t}}^*, \alpha^*) \) and \( e_{U_{i,t}}(\sigma_{P_{i,t}}^*, \beta^*) \) are used as parameters when \( C_{PV_{i,t}} \) and \( r_{\text{DR}_{i,t}} \) are determined. Moreover, \( \alpha_{i,t}^* \) and \( \beta_{i,t}^* \) ensure that \( P_{N_{i,t}} \) has feasible solutions, as well as ensuring the feasibility in eqs. (27), (28).

Therefore, (CSP-1) can be converted to (CSP-1’), a quadratic programming problem, as follows:

\[ \text{(CSP-1')} \]
\[
\begin{align*}
\text{min} & \quad c_{\text{ESS}, \mathbf{P}} [F_{CE} + F_P + F_{CV} + F_{R}] \\
\text{s.t.} & \quad \text{eqs. (5)-(10), (32), (37)}
\end{align*}
\]

As the objective function is convex, the convergence of (CSP) is guaranteed by the results of [13].

When we solve these problems (CSP-1’) and (CSP-2), the former should be solved in the first step because it is comparatively easy to decide the initial values of \( C_{PV_{i,t}}, r_{\text{DR}_{i,t}}, \alpha_{i,t}^*, \) and \( \beta_{i,t}^* \) in (CSP-1’). In detail, the initial value of \( C_{PV_{i,t}} \) was set in such a way that the sum of PVs is beyond the target amount sufficiently and that the PVs are allocated evenly in each bus. As for the others, the initial value of \( r_{\text{DR}_{i,t}} \) was set to 0 %, and sufficiently larger values were set as the initial values of \( \alpha_{i,t}^* \) and \( \beta_{i,t}^* \).

Based on the above optimization method, especially for problem (CSP-2), it is possible to minimize the amount of relaxed EENS and EENU, but it is impossible to enable the facility allocation that satisfies the constraints of power shortages and surpluses, which is necessary in the actual operation of electric power grids. Therefore, this research proposes introducing another class of ESSs, defined as “fast” ESSs instead of relaxing EENS and EENU.

Eqs. (39), (40) show that two terms can be separately extracted from the inverse functions of EENS and EENU: the terms of the upper limits of EENS and EENU and terms of the relaxed EENS and EENU.

\[
e_{S_{i,t}}(\sigma_{P_{i,t}}^*, \alpha^*) = e_{S_{i,t}}(\sigma_{P_{i,t}}^*, \lambda) - P^*_{E_{i,t}} (39)
\]

\[
e_{U_{i,t}}(\sigma_{P_{i,t}}^*, \beta^*) = e_{U_{i,t}}(\sigma_{P_{i,t}}^*, \beta) - P^*_{E_{i,t}} (40)
\]

Since \( \lambda_{i,t}^* \) and \( \beta_{i,t}^* \) are required due to the term of \( \sigma_P \), the new variables, \( P^*_{E_{i,t}} \) and \( P^*_{U_{i,t}} \), play the role of restricting power shortages and surpluses caused by sudden changes in power supply and demand. Thus, these variables can be regarded as “fast” discharging and charging in ESSs, respectively, and we call these “fast” ESSs for shortages (\( P^*_{E_{i,t}} \)) and for surpluses (\( P^*_{U_{i,t}} \)).

To simplify the problem, we assume that fast ESSs for shortages and surpluses can only discharge and charge, respectively. The required capacities of fast ESSs can be easily determined given by eqs. (41), (42) based on this assumption.

\[
C^*_{E_{ESS}} = \sum_{t=1}^{24} P^*_{E_{i,t}} (41)
\]

\[
C^*_{U_{ESS}} = \sum_{t=1}^{24} (-P^*_{U_{i,t}}) (42)
\]

Therefore, the penalty term for the amount of violated EENS and EENU, \( \rho \) in eq. (22), should be the same as \( p_{E_{ESS}} \), the price of ESSs.

To distinguish conventional ESSs (\( P^*_{E_{i,t}}, C^*_{ESS} \)) from fast ESSs, we refer to those that are included in the case that does not consider uncertainties as “slow” ESSs because slow ESSs can decrease the mean mismatch of power demand and supply, \( P_N \), as shown in eq. (30).

As we described, these two types of ESSs play different roles in restraining power shortages and surpluses based on uncertainties, and this difference can be used to determine not only the facility allocation but also the DR participation rate.

Note that it is necessary to install the total capacities of three types of ESSs, i.e., a slow ESS and fast ESSs for both shortages and surpluses, because each ESS has a different role in the corresponding day, as shown in Fig. 3. Moreover, we need to secure space for charging in an allocated ESS at the beginning of the corresponding day to charge surplus power sufficiently.

5. Simulation

This section presents the simulation results for the optimization problems developed above. We conducted simulations for an OPF using Matpower[14].
Fig. 2 IEEE 14 bus system [15]

For charging in case of surplus
For discharging in case of shortage
For charging/discharging

Total ESS to be allocated:

\[ C_{\text{ESS}}^{\text{total}} = C_{\text{ESS}}^{\text{in}} + C_{\text{ESS}}^{\text{out}} \]

Fig. 3 The capacity of allocated ESS and its usage

Table 1 Peak demand in IEEE 14 bus system [15]

| Bus | Demand [MW] |
|-----|-------------|
| 2   | 21.7        |
| 3   | 94.2        |
| 4   | 47.8        |
| 5   | 7.6         |
| 6   | 11.2        |
| 9   | 29.5        |
| 10  | 9           |
| 11  | 3.6         |
| 12  | 6.1         |
| 13  | 13.5        |
| 14  | 14.9        |
| Total | 259.0      |

5.1 Modeling and Assumptions

5.1.1 Power System Model Assumptions

This research considers the IEEE 14 bus system [15] shown in Fig. 2 as a power system to operate and allocate facilities, and Table 1 shows the peak demand in each bus of the IEEE 14 bus system for 1 year.

In addition, we use the IEEE Reliability Test System (RTS) [16] because this contains data for the annual electricity demand load, which are required when considering changes across seasons and days of the week. Thus, we can obtain the allocation of PVs and ESSs, considering time-series changes in electricity demand.

5.1.2 PV Model Assumptions

This research models PV outputs using actual data from March 1, 2013, to February 28, 2014, published by California ISO [17]. However, these data include increases in the number of PVs installed in California; hence, it was necessary to modify the data using the installation capacities of the PVs in order to determine the actual PV outputs per unit capacity at a certain time. This research compensates for the PV outputs using data from California Solar Statics [18].

5.1.3 DR Model Assumptions

This research considers both saving and consumption of energy because both capabilities are important when a large number of PVs are installed in a power grid. The former is for the case of power shortages by increasing the price of electricity, and the latter is for the case of power surpluses by reducing the price.

As described in Section 3., it is assumed that the uncertainty of DR capabilities per one unit, \( r_{\text{DR},i,t} \), is given for each hour. To give the average and standard deviation of DR capabilities \( R_{\text{DR},i,t} \), this research assumes that the value \( \mu + 2\sigma \) is regarded as the maximum DR capability and that the minimum capability is zero. Fig. 4 shows the maximum DR capabilities for each hour, using the relative values against the maximum demand in each bus. The average and standard deviation of the DR capabilities are calculated using these values. Here, these DR capabilities are common in both energy saving and consumption effects, and we assume that \( r_{\text{DR},i,t} \) has the same rate in all buses.

This research simply modeled the capabilities of DR because we are focusing not on the method to control DR capabilities, but on facility allocation to reduce the impacts on grids by DR capabilities. In addition, the maximum DR capabilities are relatively high in our model to make it easier to determine the impacts of DR. The detailed modeling of DR capabilities will be done in future work.

5.1.4 Date for Considering Grid Operation

This research determines the allocation of installed PVs and ESSs for a year; hence, it is necessary to consider long-term changes in electricity demand and PVs outputs. Moreover, it is important to consider the most severe cases of shortages and surpluses. To determine the allocation of facilities, we focus on grid operation for a day when the most surplus occurred, the demand was low, and PV output was high, because ESSs are necessary mainly to restrain power surpluses. We used data from the demand on the
Table 2 Coefficients in fuel cost function of generators in IEEE 14 bus system [15]

| Bus | $c_{2,k}/[$/MW$^2$] | $c_{1,k} [/$/MW] | $c_{0,k} [/$/] |
|-----|----------------------|-----------------|----------------|
| 1   | 0.043                | 20              | 0              |
| 2   | 0.250                | 20              | 0              |
| 3   | 0.010                | 40              | 0              |
| 6   | 0.010                | 40              | 0              |
| 8   | 0.010                | 40              | 0              |

266th day in IEEE RTS as the minimum demand, and
the PV output on September 27, 2013 as the maximum
PV output in September, by analyzing both annual
data sets. Figs. 5, 6 show the mean value, $P_{V,t}$,
and standard deviation of PV output per unit on the
date we focus on.

5.2 Simulation Results

This section explains the simulation results for the
optimal allocation of PVs and ESSs, and the optimal
DR participation rate, and compares the results with
those of the conventional method.

5.2.1 Parameter Settings

The installation target for PVs $C_{PV}^{\text{target}}$ was set to
250 MW because the total demand at peak time in the
IEEE 14 bus system is 250 MW, as shown in Table 1, in order to completely satisfy the demand from the RE system. Although the upper limits of EENS and EENU, $\sum_{t=1}^{T} \sum_{i=1}^{M} \alpha_{i,t}$ and $\sum_{t=1}^{T} \sum_{i=1}^{M} \beta_{i,t}$, can be freely determined by the power grid operator when performing a simulation, it is difficult to designate the limits because the requirements of the EENS and EENU are dependent on each area and its operator. Here, the upper limits of both EENS and EENU were set to 20 MWh/day. The prices of the PV and ESS, $p_{PV}$ and $p_{ESS}$, were 1.77 million $/MWh$[19] and 0.50 million $/MWh$[20], respectively. Table 2 shows the coefficients in a fuel cost function of generators in the IEEE 14 bus system[15], and refer to[15] to access other parameters related to a power system as follows: $B_{ij}, P_{Gt}^{\min}, P_{Gt}^{\max}$, and $\Delta P_{Gt}$.

As described in Section 4., it is necessary to set the initial values of variables that are optimized from (CSP-2). Based on the above parameter settings, $C_{PV}$, was set to 20 MW at all i, and both $\alpha_{i,t}^{*}$ and $\beta_{i,t}^{*}$ were set to 10 MWh/day.

In addition, the stopping criteria of the nonlinear optimization were as follows: the number of maximum iterations was $10^4$ and the tolerances of the step and objective functions were $10^{-10}$ and $10^{-6}$, respectively. These values were employed not only in the nonlinear programing for (CSP) and (CSP-2), but also in the Block Coordinate Descent, which is the repetitive optimization between (CSP-1') and (CSP-2).

5.2.2 Results by Proposed Methods

First, the derived DR participation rate $r_{DRi}$ was 29.8 %, which can minimize the number of installed ESSs, including both slow and fast ESSs by satisfying the above stopping criteria. As described later, this optimized rate has several impacts on the facility allocation. Second, the constraints of EENS and EENU were satisfied, owing to the installation of slow and fast ESSs and capabilities of DR.

Fig. 7 shows the allocation of PVs, $C_{PV}$, The total capacity of PVs, $\sum_{i=1}^{N} C_{PV,i}$, was 250 MW, satisfying the target. The PVs were almost evenly allocated to each bus except buses 1, 7, and 8 because no shortages and surpluses are allowed in these buses, which do not have electricity demands. The differences between buses with electricity demands were caused by the capabilities of the DR. For example, more PVs were allocated to bus 3 than other buses because more surpluses were curtailed due to the size of demands in bus 3, which has the biggest demands in the IEEE 14 bus system.

Fig. 8 shows the allocation of slow ESSs, $C_{ESS_s}$, and the total size of the installed slow ESSs, $\sum_{i=1}^{N} C_{ESS_s}$ was 40.5 MWh. The slow ESSs were allocated to each bus evenly because the power shortages and surpluses caused by the PVs were allocated to each bus evenly in order to reduce the size of the installed ESSs. There are two reasons why power shortages and surpluses were allocated to each bus evenly when the size of ESSs was reduced. One reason is that the upper limit of power transmission in each line is relatively moderate and the other is that transmission losses are not calculated in the optimization problem.

Fig. 9 shows the allocation of fast ESSs for shortages and surpluses, $C_{ESS_f}$, and the total sizes of fast ESSs were 123.7 MWh and 466.0 MWh, $\sum_{i=1}^{N} C_{ESS_f}$, respectively. In contrast, both fast ESSs for shortages and surpluses were allocated to buses with electricity demands, as well as the allocation of PVs. Moreover, the fast ESSs were allocated more unevenly rather than the PVs because more surpluses could result in buses with higher demands by the integration of both uncertainties in PV outputs and DR capabilities.
In order to verify the relationship between the DR participation rate and the capacity of ESS, as well as the accuracy of the optimized solution, Table 3 shows the results of slow and fast ESSs in three cases with different DR participation rates: the case of 25%, 29.8% (the optimized rate), and 35%. To compare the amounts of installed ESSs with different DR participation rates, we performed the simulation with the result of PV allocation as shown in Fig. 7, and fixed DR participation rates, 25% and 35% by the same optimization method. All cases employed the same conditions, such as the PV installation target, and all constraints were satisfied. According to Table 3, as the DR participation rate increased, the size of required slow ESSs decreased and the size of fast ESSs increased. The reason why the former decreased and the latter increased is that the higher DR capabilities had both a positive impact on the mean value of power shortages/surpluses and a negative impact on the deviation of power. Thus, the two types of ESSs have a trade-off relationship, and we verified that the

| DR participation rate | Slow ESS | Fast ESS for shortage | Fast ESS for surplus | Total ESS |
|-----------------------|----------|----------------------|---------------------|-----------|
| 25 %                  | 57.3     | 117.2                | 457.4               | 631.9     |
| 29.8% (optimal rate by simulation) | 40.5     | 123.7                | 466.0               | 630.2     |
| 35 %                  | 27.1     | 139.6                | 469.1               | 635.8     |
DR participation rate was optimized in terms of minimizing the total number of ESSs.

5.2.3 Results by Comparison Methods

To verify the advantage of the proposed method, the results of a comparison method are also considered, which is derived by solving the CSP problem directly.

First of all, no stopping criteria were satisfied in the comparison method because of the scale of the CSP problem with nonlinear and complicated constraints.

The derived DR participation rate \( r_{DR} \) was 0%. Figs. 10–12 show the facility allocations. The allocations were roughly similar to the results by the proposed method, but the allocation of PV was less effective compared to that by the proposed approach. Because of the different rates of DR and the ineffective PV allocation, the total capacity of ESSs, including slow and fast ESSs, was 673.2 MWh, which was larger than the derived capacity by the proposed solution.

These results show the disadvantage in the convergence of the optimization problem, and they indicated the advantage of the proposed method because we succeeded in scaling down the optimization problem by using the Block Coordinate Descent.

6. Conclusions

This research formulated the optimal allocation problem of PVs and ESSs in a power grid with uncertain PV outputs and variable DR capabilities to reduce the requirement of installed ESSs. Moreover, this formulation can determine the optimal DR participation rate, which can minimize the installed ESSs. In order to solve this problem, we proposed the concept of “slow” and “fast” ESSs, which can play different roles for the uncertainties. As a result, the characteristics of slow and fast ESSs can be used to quantify the positive and negative impacts by the increase in DR participation rate, and the optimal DR participation rate was determined. Comparing the results with the conventional method, we showed the advantage of our proposed method in terms of the convergence of the optimization.

Our future work will investigate the actual operation of slow and fast ESSs in electric power grids, and model DR capabilities in detail, considering demand behavior. Moreover, our future work will be to reduce the capacity of fast ESS because this capacity might include what can alternate with other measures such as transmission of power from/to a bus. Furthermore, other types of probability distribution will be considered when evaluating the probabilistic indices.

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