H∞ Control Design of Power System Based on Generalized Model

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Abstract. The generalized model power system is a complex interconnected system with complex structure and many external disturbance factors, which makes the stability of the system difficult to control. In order to guarantee the stability of the generalized model power system under the condition of external disturbance, more and more effective stability controller design methods are sought. Based on ricarty method and LMI method, the design conditions of H ∞ controller for the generalized power system composed of N subsystems are studied. At the same time, it also provides an effective way and method for analyzing the stability of similar systems.

1. Introduction
Conventional Power Systems consist of power plants, substations, transmission lines, and load centers. With the development of power industry, regional interconnection is gradually formed and generalized model power system is formed. The generalized model power system is often composed of many subsystems which are interrelated. The dynamic stability problem easily occurs after the interconnection, which seriously affects the whole operation reliability of power system. Therefore, for the generalized model power system, improving the stability of the whole system is the basic condition to ensure the normal operation of the system.

In recent years, the application of h ∞ control theory in time-varying systems[1], Nonlinear Systems[2], stochastic systems [3] and discrete systems[4] has become one of the research hotspots. Among h ∞ control methods, Algebraic Riccati equation Method [5] and Lmi method [6] are easy to grasp and widely used. So far, the domestic and foreign literatures mainly focus on the general power system model to analyze the system stability [7-10]. By using the Lyapunov equation and Riccati equation of singular systems, the conditions for the existence of decentralized state feedback to stabilize the closed-loop system structure with certain robust stability margin are given in reference [11]. Literature [12] applies sliding mode variable structure method and improved particle swarm optimization algorithm to load power control in interconnected power systems. For the problem of low frequency oscillation in wide area interconnected power systems, a decentralized damping controller is designed for the linearized model of interconnected power systems to improve the stability of the systems. The above literatures only focus on the stability analysis and controller design of general power system. Lmi Method is used to realize the mean square asymptotic stability analysis and controller design for General Power System Model in reference [14]. In this paper, according to the
characteristics of the generalized model power system, the H \infty controller design of the generalized model power system is studied by establishing a reasonable mathematical model and using Riccati method and LMI method respectively, the correctness of the two methods is verified by the same number of examples.

2. Problem description

Generalized Model Power System is a class of generalized systems, which is composed of several generalized subsystems. The system model can be expressed as

\[
\frac{dx}{dt} = A_j x_j + B_j u_j + \sum_{j=1,j\neq j}^{N} A_{ij} x_i + G_i \omega_{ij}
\]

\[
y_j = C_{ij} x_j + D_{ij} \omega_{ij}
\]

\[
z_j = C_{ij} x_j + D_{ij} u_j
\]

Compared with the general generalized system, the model of the system is more complex because of the addition of interconnected subsystems. To stabilize the closed-loop system, the state controller is constructed as follows:

\[u_i = -K_i x_i\]

There are

\[x = \begin{bmatrix} x_1^T & x_2^T & \cdots & x_N^T \end{bmatrix}^T, u = \begin{bmatrix} u_1^T & u_2^T & \cdots & u_N^T \end{bmatrix}^T, y = \begin{bmatrix} y_1^T & y_2^T & \cdots & y_N^T \end{bmatrix}^T, z = \begin{bmatrix} z_1^T & z_2^T & \cdots & z_N^T \end{bmatrix}^T,\]

\[e_1 = \begin{bmatrix} e_{11}^T & e_{12}^T & \cdots & e_{1N}^T \end{bmatrix}^T, e_2 = \begin{bmatrix} e_{21}^T & e_{22}^T & \cdots & e_{2N}^T \end{bmatrix}^T, E = \text{diag}[E_1 E_2 \cdots E_N], B = \text{diag}[B_1 B_2 \cdots B_N],\]

\[C_1 = \text{diag}[C_{11} C_{12} \cdots C_{1N}], C_2 = \text{diag}[C_{21} C_{22} \cdots C_{2N}], D_{21} = \text{diag}[D_{211} D_{212} \cdots D_{21N}],\]

\[D_{22} = \text{diag}[D_{221} D_{222} \cdots D_{2N1}], G_i = \text{diag}[G_1 G_2 \cdots G_N].\]

Then the system (1) can be represented as a compact system (2)

\[
E \dot{x} = Ax + Bu + G \omega_h
\]

\[
z = C_1 x + D_{12} u
\]

\[
y = C_2 x + D_{21} \omega_2
\]

\[
(2)
\]

Definition 1 Let \(A, E, R, M \in R^{n \times n}, E\) be a singular Matrix, and \(R = R^T, M = M^T\), then the \(R\) and \(M\) are symmetric. Here is a quadratic matrix equation for an unknown \(X\) Matrix, then

\[A^T X + X A - X^T RX + M = 0\]

\[E^T X = X^T E \geq 0\]

(3)

It's called Riccati equation. There is

\[H = \begin{bmatrix} A & -R \\ -M & -A^T \end{bmatrix}\]

Definition Matrix \(H\) as Hamiltonian Matrix, it related to (3).

Definition 2 in the formula (3), if \(R = -\gamma^{-2} BB^T, M = C^T C\), and \(\gamma > 0\) is given, then the formula is
\begin{align*}
A^T X + X^T A + \gamma^2 X^T B B^T X + C^T C &= 0 \\
E^T X &= X^T E \geq 0
\end{align*}

(4)

called the standard $H_\infty$ Riccati equation, write it down as $H_\infty - \text{GARE}$.

Theorem 1 For the Generalized System (3) and its transfer function $G(s)$, the following propositions are equivalent:

1. $(E, A)$ is permissible, and $\|G(s)\|_\infty < \gamma$.

2. $(E, A)$ is permissible, $\|C_2 B_2\|_\infty < \gamma$, and $(\hat{E}, H)$ are no eigenvalues on the imaginary axis.

3. There is a real matrix $X$ when the $H_\infty - \text{GARE}$ equation, and $(E, A + \gamma^{-2} B B^T X)$ is permissible.

Lemma 1 If $Q \in \mathbb{R}^{n \times n}, L \in \mathbb{R}^{r \times n}$, and $Q = Q^T, \text{rank}(L) = r$, $Lx = 0$ for arbitrary nonzero vector $x \in \mathbb{R}^n$, then $x^T Q x < 0$ is true. There is a positive number $\mu_0 > 0$ make the $Q - \mu L^T L < 0$, it is true for all of $\mu \geq \mu_0$.

3. $H_\infty$ controller design based on Riccati method

3.1. Controller design
Riccati method is using the standard tools to solve the Riccati algebraic inequality equations. Getting the $H_\infty$ controller.

For the system (5)

\begin{align*}
E \dot{x} &= A x + B u + G \omega_l \\
z &= C_1 x + D_{12} u
\end{align*}

(5)

The state feedback controller is selected

\begin{align*}
u &= -K x
\end{align*}

(6)

replace (6) with (5)

\begin{align*}
E \dot{x} &= A x + B (K x) + G \omega_l = (A + BK) x + G \omega_l = A_c x + G \omega_l \\
z &= C_1 x + D_{12} (K x) = (C_1 + D_{12} K) x = C_c x
\end{align*}

(7)

Here are $A_c = A + BK, C_c = C + DK$.

For the Generalized System (5) and the positive number $\gamma > 0$, order $T_{zw}(s)$, and from $\omega$ to $z$ the Closed loop transfer function is $T_{zw}(s)$, and $\|T_{zw}(s)\|_\infty < \gamma$.

There are some hypothesis:

(a) $(E, A, B)$ is stable and pulse-controlled.

(b) $D$ is full rank.

(c) $(E, A, B)$ is stable $(E, A, C_1)$ is detected.

(d) $D^T \begin{bmatrix} C_1 & D \end{bmatrix} = [0 \quad I]$, is the orthogonality condition.
Theorem 2 If there are hypothesis (c) and (d) in system (5), the following propositions are equivalent:

1. There are state feedback matrix $K$, then the generalized System (7) is permissible, and
   $$\|T_{zw}(s)\|_\infty < \gamma .$$
2. There are invertible matrix $X$ and generalized Riccati equation (8)
   $$A^T X + X^T A + X^T (\gamma^2 G^T - BB^T) X + C_1^T C_1 = 0$$
   $$E^T X = X^T E \geq 0$$

If 2 found, then an allowable state feedback matrix is obtained
   $$K = -B^T X$$

Prove:
1 $\rightarrow$ 2

If there is a feedback matrix $K$, and $(E, A + BK)$ is permissible, $\|T_{zw}(s)\|_\infty < \gamma$. According to the theorem 1, there is a matrix $X$, satisfying equation (10):
   $$(A + BK)^T X + X^T (A + BK) + \gamma^2 X^T G G^T X + (C_1 + D K)^T (C_1 + D K) = 0$$
   $$E^T X = X^T E \geq 0$$

then
   $$(A + BK)^T X + X^T (A + BK) + \gamma^2 X^T G G^T X + (C_1 + D K)^T (C_1 + D K)$$
   $$= A^T X + K^T B^T X + X^T A + X^T BK + \gamma^2 X^T G G^T X + C_1^T C_1 + C_1^T D K + K^T D^T C_1 + K^T D^T DK$$
   $$= \Phi + (K^T B^T X + X^T BK + C_1^T D K + K^T D^T C_1 + K^T D^T DK)$$
   $$= \Phi - (X^T B + C_1^T D)(D^T D)^{-1}(B^T X + D^T C_1) + M^T M = 0$$

Here is $\Phi = A^T X + X^T (A + \gamma^2 X^T G G^T X + C_1^T C_1, M = D(K + B^T X)$

If $D = 0$, then (11) equals $\Phi - X^T B B^T X = 0$

From $\Phi$ to (11), then
   $$A^T X + X^T A + \gamma^2 X^T G G^T X + C_1^T C_1 - X^T B B^T X$$
   $$= A^T X + C_1^T C_1 + X^T (\gamma^2 G G^T - BB^T) X$$
   $$= 0$$

So 2 is true.
2 $\rightarrow$ 1

If there is an invertible matrix $X$ make (8) is true, and $K$ is given by (9), then $M = 0$ and (10) is true. According to the theorem 1, the system (7) is permissible, and
   $$\|T_{zw}(s)\|_\infty < \gamma .$$

Theorem 2 is true.
3.2. \textit{An example analysis of Riccati method}

In order to prove the effectiveness of this method, a generalized power system composed of second-order subsystems is given, with the following parameters:

\[
E_1 = E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 1 & 1 \\ 0 & 2.1 \end{bmatrix},
\]

\[
A_{21} = \begin{bmatrix} -1 & 0 \\ -1 & -2.1 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix},
\]

\[
B_1 = B_2 = \begin{bmatrix} 1,1 \\ 3 \end{bmatrix}, \quad G_1 = G_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},
\]

\[
C_{21} = C_{22} = \begin{bmatrix} 2 & 4 \end{bmatrix}, \quad D_{121} = D_{122} = D_{211} = D_{212} = 1
\]

\( \gamma = 2 \cdot (E, A, C) \) can be detected and pulse view, According to (2) and (3), LMI is used to solve the problem. The characteristic roots of the system are all in the left half plane of the complex plane such as \(-13.735, -3.319+1.4639i, -3.319-1.4639i, -3.0698\). The feedback matrix of the corresponding controller is \( K = \begin{bmatrix} 3.1849 & 4.4444 & -2.5552 & -2.823 \end{bmatrix} \) the closed-loop generalized power system is permissible and \( H_\infty \) norm of the transfer function less than 2, \( H_\infty \) controller is \( u(t) = \begin{bmatrix} -3.1849 & -4.4444 & 2.5552 & 2.823 \end{bmatrix} x(t) \) then the design is completed. The system state change curve of the state variable under the controller is shown in the fig 1. It can be seen that under the feasible solution state of this method, the system does not fluctuate greatly. And the method can make the system reach a stable state quickly with good control effect.

4. \( H_\infty \) Controller design based on LMI method

Lyapunov matrix inequality is a linear matrix inequality (LMI), and Riccati matrix inequalities and quadratic matrix inequalities contain quadratic terms, they are not linear matrix inequalities. Using LMI method to transform the Lyapunov matrix inequality and Riccati matrix inequality into
appropriate matrix equations. And then to solve Lyapunov matrix inequalities and Riccati matrix inequalities by Lyapunov equation and Riccati equation.

4.1. Controller design

Theorem 3 For system (5) satisfying hypothesis (a), the following propositions are equivalent:

① There is a state feedback matrix $K$, such that the system (7) is admissible, and $\|F_{zw}(s)\|_{\infty} < \gamma$.

② There exists an invertible matrix $X \in \mathbb{R}^{n \times n}$ and the matrix $Y \in \mathbb{R}^{m \times n}$ satisfies the following matrix inequality:

$$
\Psi(\gamma, X) = 
\begin{bmatrix}
AX + B_2Y + (AX + B_2Y)^T & B_1 & (CX + DY)^T \\
B_1^T & -\gamma^2 I & 0 \\
CX + DY & 0 & -I
\end{bmatrix} < 0
$$

$$
E^T X = X^T E \geq 0
$$

(12)

If the proposition ② is true, then the permissible state feedback matrix is

$$
K = YX^{-1}
$$

(13)

Prove

① $\rightarrow$ ②

If there is a feedback matrix $K$, then $(E, A + B_2K)$ is admissible. According to theorem 1, there exists an invertible matrix $V$ that satisfies the following inequality:

$$
\begin{bmatrix}
(A + B_2K)^T V + V^T (A + B_2K) & V^T B_1 & (C + DK)^T \\
B_1^T V & -\gamma^2 I & 0 \\
C + DK & 0 & -I
\end{bmatrix} < 0
$$

$$
E^T V = V^T E \geq 0
$$

(14)

For equation (14), left-multiply matrix $T^T$ and right-multiply matrix $T$, where

$$
T = 
\begin{bmatrix}
V^{-1} & 0 & 0 \\
0 & I & 0 \\
0 & 0 & I
\end{bmatrix}
$$

There $X = V^{-1}, Y = KX$, from (14) to (12) is true.

② $\rightarrow$ ①

If there is invertible matrix $X$ and matrix $Y$ satisfy the inequality (12). And $K$ is given by equation (13). The inverse process of ①$\rightarrow$ ② can obtain equation (14), and then according to theorem 1, the system (7) is admissible, and $\|F_{zw}(s)\|_{\infty} < \gamma$. Theorem 3 is true.

4.2. Example simulation of LMI method

In order to prove the effectiveness of this method, a generalized power system composed of second-order subsystems is given, with the following parameters the same to section 3.2, and $\gamma = 2, (E, A, C_2)$, the interconnected second-order system is sorted out into equation (15). According to equation (12), LMI is used to solve (15)
There are the results

\[
X = \begin{bmatrix}
0.9494 & -0.1943 & 0.1427 & 0.1268 \\
-0.1943 & 0.6268 & 0.0719 & 0.0809 \\
0.1427 & 0.0719 & 0.7290 & -0.4314 \\
0.1268 & 0.0809 & -0.4314 & 0.4174
\end{bmatrix}, \quad Y = \begin{bmatrix}
-1.0405 & -1.1241 & 0.0518 & -0.3476
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
-2.8980 & -3.6161 & 3.3364 & 4.1968
\end{bmatrix}
\]

Through verification and calculation, under the condition of the closed-loop system to allow, $H_\infty$ norm is less than 2 and $H_\infty$ controller is $u(t) = \begin{bmatrix} 2.8980 & 3.6161 & -3.3364 & -4.1968 \end{bmatrix} x(t)$

design completed. The curve of system state variation under the controller is shown in fig 2:

![Fig 2 LMI method state variable graph](image)

It can be seen from the state variable graph that under the feasible solution of reorganization, the disturbance generated by the system is relatively obvious, but the LMI method can still make the system reach the stable state quickly, and the control effect is better.

The above examples prove that the method is feasible. Through the comparison of the above two methods, both methods can be used for the design of $H_\infty$ controller. However, the answers obtained by different methods under different data conditions are not the same. However, both methods can achieve the design of $H_\infty$ controller for generalized model power system.
5. Conclusion

By using Riccati equation method and LMI method respectively, the $H_\infty$ control problem of generalized model power system is studied. These conclusions are established under certain specific conditions. When designing the controller, these conclusions can be applied as long as the actual system basically meets these specific conditions. This section verifies the two methods from the theoretical and numerical simulation aspects to illustrate the respective reliability of the two methods. In order to further popularize and apply the problem, we can make the condition of the problem more general form, and provide the basis for the following optimal control of interconnected systems under general conditions.

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