Larger domains of disoriented chiral condensate through annealing

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Abstract

Relativistic heavy ion collisions can generate metastable domains in which the chiral condensate is disoriented. Nucleus-sized domains can yield measurable fluctuations in the number of neutral and charged pions. We propose a scenario in which domains are ‘annealed’ by a dynamically evolving effective potential in the heavy ion system. Domains of sizes exceeding 3 fm are possible in this scenario.
The order of the chiral restoration phase transition in QCD for realistic values of the up, down and strange quark masses is currently unknown \[1, 2\]. If the transition is nearly second order, Rajagopal and Wilczek \[3\] have speculated that the nonequilibrium dynamics of a heavy ion collision can generate metastable domains in which the chiral condensate is disoriented. Such domains can provide a coherent source of pions that can exhibit novel “centauro-like” \[4\] fluctuations of neutral and charged pions \[5, 6, 7\]. However, the ability of the coming ion-ion experiments at the RHIC and LHC colliders to resolve such fluctuations critically depends on the size and energy content of these domains.

Numerical simulations \[8\] show that the zero-temperature linear sigma model advocated in \[3\] leads to domains that are roughly pion sized, too small to be resolved in experiments. Nevertheless, we illustrate in this Letter that finite temperature effects in a heavy ion collision can enhance the size of domains relative to the zero-temperature estimates.

Rajagopal and Wilczek propose a “quench” scenario in which the condensate is initially chirally symmetric as appropriate at high temperature, but its evolution is taken to follow classical equations of motion in the absence of a heat bath.\[1\] For this quench scenario to be realistic, the expansion rate of the heat bath needed to create the symmetric initial state must greatly exceed the rate at which the mean field evolves. The opposite is more likely the case. In \[3\] and \[8\], the $\sigma$ and $\vec{\pi}$ fields in the O(4) linear sigma model are taken to characterize the dynamics of the quark antiquark condensate in two-flavor QCD. These fields evolve over the time scale $\sim m_\sigma^{-1} \sim 1/3$ fm \[8\], where $m_\sigma = 600$ MeV is the zero-temperature $\sigma$ mass. On the other hand, model calculations \[9\] indicate that chiral symmetry is broken very late in RHIC-energy collisions, perhaps at proper times $\tau_c \sim 5 - 20$ fm, so that the expansion time scale $\sim \tau_c$ is much larger in comparison.

We present an alternative “annealing” scenario in which the sigma-model condensate evolves semiclassically in the presence of a bath of quasiparticle excitations. Fluctuations induced by the quasiparticles create an effective potential \[10\] in which a chirally symmetric state is initially stable (modulo finite quark-mass effects). As the system expands and rarefies, the effective potential develops a “wine-bottle” shape and slowly changes towards its free-space depth. The condensate then evolves in this changing potential.

Transient domains of disoriented $\Phi = (\sigma, \vec{\pi})$ condensate can develop because the $\Phi \approx 0$ initial state is unstable in the wine bottle potential. Consider first a quench scenario in which the potential,

$$V_0(\Phi) = \lambda(\Phi^2 - v^2)/4 - H\sigma,$$

is fixed. The system “rolls down” from the unstable local maximum of $V_0(\Phi)$ towards the nearly stable values with $|\Phi| = v$, since the symmetry breaking term $-H\sigma$ is relatively small. This process is analogous to spinodal decomposition in condensed matter physics \[11, 12\]. Field configurations with $\vec{\pi} \neq 0$ develop during the roll-down period. The field will eventually settle into stable oscillations about the unique vacuum $(f_\pi, 0)$ for $H \neq 0$, but

\[1\] This procedure is not the standard quench in condensed matter physics, in which the system remains in contact with a heat bath of zero temperature. We continue to use the term “quench” here, however, because a $T = 0$ heat bath would also effectively imply decoupling from the system since the interactions of pions effectively disappear at zero momentum due to the approximate chiral symmetry.
oscillations will continue until interactions eventually damp the motion. In the heavy-ion system a domain can radiate pions preferentially according to its isospin content.

Domain growth in the annealing scenario with an evolving effective potential differs from the quench as follows. Initially, \( V_{\text{eff}} \) is nearly flat for \( \sigma \approx \pi \approx 0 \) as shown in fig. 1. Therefore, the roll-down time scale can be very large at first, or infinite if the corresponding equilibrium transition is truly second order (as is certainly the case for \( H = 0 \)). Only as the potential approaches its free space shape does the roll-down become rapid. In both quenching and annealing, domains can grow as long as \( \sigma \) is substantially different from \( f_\pi \). In the annealing scenario, however, that time scale is limited by the time needed for \( V_{\text{eff}} \) to approach \( V_0 \). That time in turn depends on the global dynamics of the nuclear collision.

We describe the evolution of the condensate in the context of a self consistent Hartree-like semiclassical approach. There, the condensate evolves as a classical field in the presence of a fluctuating quasiparticle bath, see e.g. \[13\]. The condensate fields \( \Phi = (\sigma, \pi) \) obey relativistic Ginzberg-Landau \[14\] equations:

\[
(\partial^2 / \partial t^2 - \nabla^2) \Phi = \lambda \left\{ v^2 - 3\langle \delta \Phi^2_\| \rangle - \langle \delta \Phi^2_\perp \rangle - |\Phi|^2 \right\} \Phi - H n_\sigma,
\]

to quadratic order in the fluctuations. The symmetry breaking field \( H \) is fixed in the sigma direction, \( n_\sigma = (1, 0) \). Both the condensate \( \Phi \) and the fluctuation fields \( \delta \Phi \) can instantaneously have any orientation in the O(4) internal space. When the \( T = 0 \) condensate \( \Phi = (f_\pi, 0) \) is realised, the fluctuations \( \delta \Phi_\| \) along \( \Phi \) can be identified with the sigma field with the familiar tree-level mass \( m_\sigma = (2\lambda f_\pi^2 + m_\pi^2)^{1/2} \). Similarly, the three transverse modes \( \delta \Phi_\perp \) are pion-like with the \( T = 0 \) mass \( m_\pi = (H/f_\pi)^{1/2} \).

The Hartree correction \( 3\langle \delta \Phi^2_\| \rangle + \langle \delta \Phi^2_\perp \rangle \) accounts for the interactions of sigma-like and pion-like quasiparticles with the condensate. To estimate this correction, we must understand the nonequilibrium dynamics of the quasiparticles in the heavy ion system — no trivial task! To simplify our exploratory treatment, we assume that the quasiparticles are localized excitations of momentum \( \vec{p} \) and energy \( E^\sigma,\pi_{\vec{p}} = \sqrt{\vec{p}^2 + (m^\sigma,\pi_{\text{eff}})^2} \), where \( m^\sigma,\pi_{\text{eff}} \) are the effective masses of the sigma- and pion-like quasiparticles. We can then write the Hartree term in terms of semiclassical phase space distributions \( f_{\sigma,\pi}(\vec{p}, \vec{r}, t) \):

\[
3\langle \delta \Phi^2_\| \rangle + \langle \delta \Phi^2_\perp \rangle = 3 \int d\Gamma_p \frac{f_\sigma(p)}{E^\sigma_p} + (N - 1) \int d\Gamma_p \frac{f_\pi(p)}{E^\pi_p}
\]

where \( d\Gamma_p = d^3p/(2\pi)^3 \) and \( N = 4 \). Note that for quasiparticles near local thermal equilibrium, the \( f_\sigma(p) \) are Bose distributions so that \( 3\langle \delta \Phi^2_\| \rangle + \langle \delta \Phi^2_\perp \rangle \approx T^2/2 \) for \( T \gg m^\sigma_{\text{eff}} \) and \( m^\pi_{\text{eff}} \). The equations of motion (2) with (3) then correspond to the more familiar effective potential \[10\] shown in fig. 1. The critical temperature at which the wine bottle shape of \( V_{\text{eff}} \) disappears is then \( T_c = \sqrt{2} f_\pi \approx 132 \text{ MeV} \). [Lattice simulations suggest that the condensate in equilibrium QCD substantially changes at temperatures that are somewhat higher than this O(4) model estimate \[1\] \[13\].]

In general, the quasiparticle distributions will satisfy Boltzmann-Vlasov kinetic equations in which \( m^\sigma,\pi_{\text{eff}} \) are self consistent functions of the field (see, e.g. \[10\]). Here we will neglect
the following: (i) the effect of collisions among quasiparticles, and (ii) the dynamical effect of the quasiparticle masses. It is reasonable to neglect collisions since, by the time $T < T_c$, the system is sufficiently dilute that the collision frequency is small compared to the expansion rate [14]. On the other hand, assumption (ii) is certainly wrong, since $m_{\text{eff}}^a$ must eventually change from $\sim m_\pi$ at $T_c$ to its free-space value $\approx 600$ MeV. We emphasize that these approximations, while crude, allow for the most rapid “quench” consistent with causality, since the quasiparticles then stream away, unimpeded by the collisions or the gain in mass.

To illustrate how the fields and quasiparticles evolve with time, we restrict our attention for the moment to a single domain in the interior of the collision volume. An expanding system created in a central Au+Au collision at RHIC will reach temperatures below $T_c \approx 132$ MeV only after a time $\sim 5 - 20$ fm comparable to the Au transverse radius $R_A \sim 7$ fm. We take the quasiparticle flow to be roughly 3-dimensional and homologous, allowing for the fastest (causal) flow. We assume that at this late time in the evolution of the reaction, the condensate is invariant under radial boosts, so that it is a function only of the proper time $\tau \equiv (t^2 - r^2)^{1/2}$ for radial distances $r < R_A$ and late times $t > R_A$. The left side of (2) is then

$$\left( \frac{\partial^2}{\partial \tau^2} - \nabla^2 \right) \Phi = \left( \frac{\partial^2}{\partial \tau^2} + \frac{3}{\tau} \frac{\partial}{\partial \tau} \right) \Phi.$$  \hspace{1cm} (4)

Note that for the one-dimensional Bjorken expansion valid for $r, t < R_A$, one would replace $\tau$ by $(t^2 - z^2)^{1/2}$ and the “3” multiplying the first derivative by “1”. However, we stress that chiral symmetry is unbroken within that space-time region in nuclear collisions.

Similarly, the quasiparticle distribution functions $f_a(\vec{p}, \vec{r}, t)$ for $a = \sigma, \pi$ are functions only of $p' \equiv (pr - E_{\vec{p}}) / \tau$ and $\tau$. Approximation (i) implies that $f(p)$ satisfies a collisionless Boltzmann equation (see [17] and refs. therein) describing the free-streaming evolution of the phase-space distribution. We obtain $f_a(\vec{p}, \vec{r}, t) = f^{c}_a(p' \tau / \tau_c)$, where the initial distribution $f^c_a(p)$ at the time $\tau_c$ is assumed to have the thermal equilibrium form $\exp(-E_a p / T_c - 1)$. Neglecting quasiparticle collisions, as done here, this form applies even if effective masses are functions of the condensate $\Phi(\tau)$. This solution also applies in quasiparticle-collision dominated regime where the expansion is adiabatic, provided that the $m_{\text{eff}}^a$ can be neglected. The Hartree term (3) is then a function of $\tau$ alone. With (ii), we estimate

$$\langle 3\langle \delta \Phi^2 \rangle + \langle \delta \Phi^2 \rangle \rangle(\tau) \approx \langle 3\langle \delta \Phi^2 \rangle + \langle \delta \Phi^2 \rangle \rangle_c(\tau_c / \tau)^2,$$  \hspace{1cm} (5)

an approximation that is adequate provided that we focus only on the evolution of the condensate.

We determine the average field in a single domain $\Phi = (\sigma, \vec{\pi})$ as a function of $\tau$ by combining (4), (3) and (5) to find

$$\left( \frac{\partial^2}{\partial \tau^2} + \frac{3}{\tau} \frac{\partial}{\partial \tau} \right) \Phi = \lambda \nabla^2 \left[ 1 - \epsilon \left( \frac{\tau_c}{\tau} \right)^2 \right] \Phi - \lambda |\Phi|^2 \Phi - H n_\sigma.$$  \hspace{1cm} (6)

By neglecting the time evolution of masses, we avoid the complicated treatment in [11] that ensures that all quasiparticle modes have effective masses satisfying $m^2 > 0$. The present approach is adequate, since we focus on the behavior of the condensate, and not the quasiparticles.
The standard parameters $\lambda = 20$, $v = 87.4$ MeV, and $H = (119 \text{ MeV})^3$ are consistent with the values of the pion decay constant $f_\pi = 92.5$ MeV and the zero temperature meson masses $m_\pi = 140$ MeV and $m_\sigma = 600$ MeV. The annealing parameter,

$$
\epsilon \equiv v^{-2} \left( 3 \langle \delta \Phi^2 \rangle_c + \langle (\delta \Phi^2) \rangle_c \right),
$$
determines the strength of the Hartree term. A value $\epsilon = 1$ applies for $m^{\sigma,\pi}_{\text{eff}}(T_c) \ll T_c$. For $m^{\sigma,\pi}_{\text{eff}} \sim 140$ MeV at $T_c \approx 132$ MeV, we find $\epsilon \approx 0.5$. The value $\epsilon = 1$ applies if chiral restoration in an equilibrium system occurs as a true second order phase transition. On the other hand, a consistent treatment at the Hartree (tadpole) level for the O(4) model with $H \neq 0$ suggests $m^{\sigma,\pi}_{\text{eff}} > 0$ and a gradual, continuous chiral restoration, rather than a phase transition. As mentioned earlier, the case of QCD applying to the real world is currently ambiguous.

In Fig. 2a we show numerical solutions of (2) for $\tau_c = 10$ fm, $\pi^0(\tau_c) = 48$ MeV, $d\pi^1/d\tau|_{\tau_c} = 95$ MeV/fm, while in Fig. 2b we show the case $\pi^0(\tau_c) = 5$ MeV, $d\pi^1/d\tau|_{\tau_c} = 0$. The dark lines and light lines correspond to calculations for $\epsilon = 1$ and 0.5, respectively. In Fig 2a, we see that the time $\tau_f$ needed for $\sigma$ to begin to oscillate about $\sigma = f_\pi$ is roughly 2.5 $\tau_c$ for both values of $\epsilon$. As expected, $\tau_f$ is essentially the time needed for $V_{\text{eff}}$ to fall within roughly 20% of the $T = 0$ potential. For $\epsilon = 0.5$, the solution for the more quiescent initial condition shows a similar structure, but reaches $\sigma \sim f_\pi$ sooner, at $\tau_f \approx 1.3 \tau_c$.

To study the size of domains, we now shift our focus from the average $\Phi(\tau)$ in a single domain to the spatial variation of the condensate due to fluctuations in the initial fields. Domains arise because a spinodal instability enables small fluctuations in the initial field configuration to grow (12). Neglecting the expansion of the system for the moment, we see that small fluctuations about $\Phi = 0$ in (2) “run away” as $e^{\delta t}$ where $\Omega^2 \equiv \tau_{sp}^{-2} - k^2$, for a wave number $k < \tau_{sp}^{-1}$. The $k = 0$ mode grows the fastest, with a time scale

$$
\tau_{sp} = \left[ \lambda (v^2 - 3 \langle \delta \Phi^2 \rangle_c - \langle (\delta \Phi^2) \rangle_c) \right]^{-1/2}, \quad \text{(8)}
$$

which is a function of time $\tau$ itself. In the heavy-ion system domains grow due to the macroscopic expansion as well as the microscopic instability of the infrared modes of the coherent chiral condensate. We use (2) to find that small fluctuations follow

$$
\left\{ \frac{\partial^2}{\partial \tau^2} + 3 \frac{\partial}{\tau \partial \tau} - \frac{1}{\tau_{sp}^2} + \frac{\tau_c}{\tau} k^2 \right\} \Phi = 0 \quad \text{(9)}
$$

for initial conditions specified at $\tau = \tau_c$. The factor $(\tau_c/\tau)^2$ in front of $k^2$ in (2) describes the homologous expansion of domains; an analogous term enters in descriptions of the smoothing out of fluctuations on a local scale due to the expansion of scales in inflationary cosmology (13).

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3 In principle the initial conditions are determined by microscopic fluctuations. The initial spread in $d\Phi/d\tau$ and $\Phi$ can be quite large compared to $v$. We estimate $(d\Phi/d\tau)^2 = \langle d\Phi^2 \rangle_c$, where $\langle \Phi^2 \rangle_c \approx \sum_a \int d\Gamma(E_a/p) f^0_c(p) \approx (95 \text{ MeV/fm})^2$ while $\langle (\Phi^2) \rangle_c \approx \sum_a \int d\Gamma(E_a/p) f^0_c(p) \approx (48 \text{ MeV})^2$. 

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4
To obtain a rough estimate of the growth of the domain size in the comoving frame, we follow Boyanovsky et al. [11] and calculate the contribution of the growing fluctuations to the correlation function

\[ \langle \pi^0(\vec{r}, t)\pi^0(\vec{0}, t) \rangle = \int \frac{d^3k}{2\pi^3} e^{i\vec{k} \cdot \vec{r}} \langle \pi^0(\vec{k}, t)\pi^0(-\vec{k}, t) \rangle, \]

where \( \langle \ldots \rangle \) represents an average over the fluctuating initial conditions. For definiteness, we choose the \( \pi^0 \) direction in isospin, but all cartesian pion directions are equivalent by isospin invariance. The initial fluctuation spectrum \( \langle \pi^0(\vec{k}, \tau_c)\pi^0(-\vec{k}, \tau_c) \rangle \) is thermal to a high degree of accuracy, yielding spatial correlations over characteristic distances of a thermal wavelength \( \langle \pi T \rangle^{-1} \), i.e. \( \langle \pi^0(\vec{r}, t)\pi^0(\vec{0}, t) \rangle \sim \exp(-\pi T|\vec{r}|) \). For \( \langle \pi T \rangle^{-2} < r^2 < \int \tau_{sp}(\tau_c/\tau)^2 d\tau, \) we can then evaluate (10) using a WKB approximation and integrate (11) applying the saddle-point method. We find

\[ \langle \pi^0(\vec{r}, t)\pi^0(\vec{0}, t) \rangle \approx \langle \pi^0(0, t)^2 \rangle \exp \left\{ -r^2 \left[ 8 \int (\tau_c/\tau)^2 \tau_{sp}(\tau) d\tau \right]^{-1} \right\}. \]

This result suggests that domain size \( R_D \) increases roughly as

\[ R_D(t)^2 \sim 8 \int_0^t (\tau_c/\tau)^2 \tau_{sp}(\tau) d\tau \approx 8 \frac{\tau_{sp}\tau_c}{\sqrt{\epsilon}} \left[ \cos^{-1}\left( \frac{\tau_c}{\tau} \frac{\sqrt{\epsilon}}{t} \right) - \cos^{-1}\left( \sqrt{\epsilon} \right) \right], \]

where \( \tau_{sp} = (\lambda v)^{-1/2} \approx 0.5 \text{ fm.} \)

Domains can grow as long as the system is unstable, i.e. until the \( \sigma \) field begins to oscillate about the value \( \sigma \sim f_\sigma \). For a nearly critical system with \( \epsilon = 1, \) we estimate this time to be \( \tau_f - \tau_c \approx 1.5 \tau_c, \) see figs. 2. Such values imply that the domain size can reach \( R_D \approx 7 \text{ fm} \) — very large indeed! We obtain a more conservative estimate \( R_D \sim 3 - 4 \text{ fm} \) by taking \( \epsilon = 0.5 \) and supposing that domain growth stops after \( \sigma \) reaches \( f_\sigma \) for the first time at \( \tau_f - \tau_c \approx (0.3 - 0.5)\tau_c. \) Note that the numerical effect of the inflation factor \( (\tau_c/\tau)^2 \) on (12) is insignificant for the parameter values considered. Admittedly, numerical simulations for the annealing scenario are needed to provide more concrete estimates. Such simulations are in progress [19]. However, we point out that similar estimates worked surprisingly well in describing the results of quench simulations in [8].

To summarize, we have shown that the slow expansion of the matter in a relativistic nuclear collision forms disoriented chiral condensates through “annealing,” rather than “quenching” [3]. Our methods are schematic and intended only to illustrate the underlying physics. Nevertheless, we argue that annealing can lead to domains significantly larger than those expected from a quench [8]. Our results therefore strengthen the suggestion [3] that disoriented chiral condensates can be observed in heavy ion collisions at RHIC.

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Figure Caption

Figure 1: The Hartree effective potential as a function of $\Phi = (\sigma, \vec{\pi})$ for the temperatures $T_c = \sqrt{2}v$ and $T_c/3$ compared to the $T = 0$ potential $V_0 = \lambda(\Phi^2 - v^2)/4$.

Figure 2: The behavior of the field for initial conditions (a) $\pi^0(\tau_c) = 48$ MeV, $d\pi^1/d\tau(\tau_c) = 95$ MeV/fm and (b) $\pi^0(\tau_c) = 5$ MeV, $d\pi^1/d\tau(\tau_c) = 0$. The time at which the system first drops below $T_c$ is $\tau_c = 10$ fm. Dark curves are calculated for $\epsilon = 1$ and light curves for $\epsilon = 0.5$. 
\[ V_{\text{eff}}(\Phi) \]

\[ \Phi/f_{\pi} \]

- \[ T = T_c \]
- \[ T = T_c/3 \]
- \[ T = 0 \]
The diagram illustrates the behavior of the field $\text{field/f}_{\pi}$ as a function of $\tau/\tau_C$ for two different values of $\epsilon$: $\epsilon = 1$ and $\epsilon = 0.5$.

- For $\epsilon = 1$, the field is shown as a solid black line.
- For $\epsilon = 0.5$, the field is shown as a gray dashed line.

Key points:
- $\sigma = 0, \pi^0 = 48 \text{ MeV}$
- $d\pi^1/d\tau = 95 \text{ MeV}$
\[ \varepsilon = 1, \varepsilon = 0.5 \]

\[ \sigma = 0, \pi^0 = 5 \text{ MeV} \]