Qubit Construction in 6D SCFTs

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Abstract

We consider a class of 6D superconformal field theories (SCFTs) which have a large $N$ limit and a semi-classical gravity dual description. Using the quiver-like structure of 6D SCFTs we study a subsector of operators protected from large operator mixing. These operators are characterized by degrees of freedom in a one-dimensional spin chain, and the associated states are generically highly entangled. This provides a concrete realization of qubit-like states in a strongly coupled quantum field theory. Renormalization group flows triggered by deformations of 6D UV fixed points translate to specific deformations of these one-dimensional spin chains. We also present a conjectural spin chain Hamiltonian which tracks the evolution of these states as a function of renormalization group flow, and study qubit manipulation in this setting. Similar considerations hold for theories without $AdS$ duals, such as 6D little string theories and 4D SCFTs obtained from compactification of the partial tensor branch theory on a $T^2$.

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1 Introduction

There are intriguing connections between between information theoretic structures and the emergence of spacetime. An undergirding assumption in this setup is that entangled “qubits” in a conformal field theory (CFT) serve to encode the structure of a bulk gravitational dual. There are by now a number of suggestive holographic structures in this setting as found in references [1–5]. There are also proposed connections to phenomena from condensed matter and quantum information systems such as references [6–17].

This also raises some additional questions. For example, to make further tests of these proposals it would be nice to have some explicit examples of qubits in interacting conformal field theories with genuine stringy holographic duals. Additionally, there is a suggestive geometric “lattice structure” built into many discussions connecting holography and tensor networks, but this also poses some puzzles for the role of the lattice suggested by a tensor network and its connection to spacetime locality.

Our aim in the present note will be to construct some explicit examples of qubit systems in superconformal field theories (SCFTs). Perhaps surprisingly, we find it simplest to construct these states in 6D SCFTs, as well as in their dimensional reduction on a $\mathbb{T}^2$. The class of theories we consider resemble generalized quiver gauge theories which take the form:

$$[G_0] - G_1 - \ldots - G_{N-1} - [G_N].$$

Here, each $G_i$ for $i = 1, \ldots, N$ denotes a gauge symmetry factor, with $G_0$ and $G_N$ flavor symmetry factors. The links between these gauge groups correspond to “conformal matter,” as in references [18,19].

The main class of 6D SCFTs we consider arise from the worldvolume of $N$ M5-branes filling $\mathbb{R}^{5,1}$ and probing the transverse geometry $\mathbb{R}_\perp \times \mathbb{C}^2/\Gamma_{ADE}$ where $\Gamma_{ADE} \subset SU(2)$ is a finite subgroup. The subscript here serves to remind us that there is an ADE classification of such finite subgroups, and these are in one to one correspondence with the ADE series of Lie algebras. Indeed, M-theory on $\mathbb{R}^{6,1} \times \mathbb{C}^2/\Gamma_{ADE}$ engineers 7D Super Yang-Mills theory with the corresponding gauge group. Placing M5-branes on top of an ADE singularity but separated from each other in the transverse $\mathbb{R}_\perp$ direction leads to a quiver-like structure as in line (1.1) with gauge group factors $G_i = G_{ADE}$, with the strength of each gauge coupling depending inversely on the separation length between neighboring M5-branes. The “matter” of the generalized quiver corresponds to degrees of freedom localized on each M5-brane. We reach a non-trivial conformal fixed point by making all M5-branes coincide, which corresponds to a point of strong coupling in the moduli space of the 6D field theory. In the large $N$ limit, we also achieve a holographic dual description in M-theory given by the spacetime $AdS_7 \times S^4/\Gamma_{ADE}$, with orbifold fixed points at the north and south poles of the $S^4$. These fixed points correspond to the left and right flavor symmetry factors $G_0$ and $G_N$. There are similar long quivers without a semi-classical gravity dual [20,21].
also produce 6D little string theories (LSTs) by gauging a diagonal subgroup of $G_0 \times G_N$, as in reference [22]. This last class of examples produces a non-AdS holographic dual with a linear dilaton background [23].

The main message of recent work in classifying 6D SCFTs (see [20, 21] and [24] for a review) is that the vast majority of such theories resemble quiver gauge theories, and in fact all known theories can be obtained through a process of fission and fusion [21, 25, 26] built from such quiver-like structures. So, lessons learned here apply to a broad class of 6D theories and their lower-dimensional descendants obtained from further compactification.

To construct some explicit qubits, we make use of the recent work of reference [27] which found that for operators with large R-charge, there are nearly protected operator subsectors which only mix with themselves. That this is possible at strong coupling follows from the fact that at large R-charge, there is a further suppression in operator mixing which goes inversely in the R-charge, much as in references [28, 29].

Using this fact, reference [27] identified a class of gauge invariant local operators of the form:

$$O_{m_1...m_N} = X_{i_1}^{(m_1)}...X_{i_i}^{(m_i)}...X_{i_N}^{(m_N)},$$

(1.2)

where we view the $X_{i_i}^{(m_i)}$ as bifundamental operators between neighboring pairs of groups:

$$G_{i-1} \rightarrow G_i.$$  

(1.3)

We note that our indexing convention here is slightly different from [27]. These operators are constructed on the partial tensor branch of the 6D SCFT, and to reach the conformal fixed point one must take a further decoupling limit in which momentum transverse to the stack of M5-branes is set to zero. This imposes a mild condition on the spectrum of quasi-particle excitations in the 1D lattice of spins, and for the most part we will keep this point implicit in what follows. See reference [27] for details.

Treating the $G_{i-1}$ and $G_i$ as flavor symmetries, the $X_i$ define “conformal matter” operators which have have non-trivial scaling dimension $\Delta_X$ and transform in a spin $s_X$ representation of the $SU(2)_R$ R-symmetry. In the special case where all the $G_i = SU(K)$, $s_X = 1/2$ but for the D- and E-type theories, the spin is higher [19,27]:

|        | $A_K$ | $D_K$ | $E_6$ | $E_7$ | $E_8$ |
|--------|-------|-------|-------|-------|-------|
| $s_X$  | 1/2   | 1     | 3/2   | 2     | 3     |

(1.4)

The key feature found in [27] is that for the composite gauge invariant operators $O_{m_1...m_N}$, the action of the one-loop Dilatation operator on the $O_{m_1...m_N}$ is simply that of a 1D spin chain Hamiltonian. In the case where we have A-type gauge groups with $G_i = SU(K)$, the
one-loop Dilatation operator takes the form:

\[
\Delta = E^{(0)} - \lambda A \sum_{i=1}^{N} 2 \vec{S}_i \cdot \vec{S}_{i+1}.
\]  (1.5)

Here, the \( \vec{S}_i \) denote the usual angular momentum operators in the spin 1/2 representation and we have set \( \vec{S}_{N+1} = 0 \). For a spin chain with periodic boundary conditions we would instead set \( \vec{S}_{N+1} = \vec{S}_1 \). The constants \( E^{(0)} \) and \( \lambda_A > 0 \) were computed in [27]. We will not need their explicit values in what follows. The expression (1.5) is the Hamiltonian for the celebrated ferromagnetic XXX\(_{s=1/2}\) Heisenberg spin chain [30]. It also shows up prominently in the context of \( N = 4 \) Super Yang-Mills theory (see e.g. [31–33]) which served as a motivation for reference [27].

In the case of the D- and E-type spin chains, similar considerations hold, but we instead get a Hamiltonian constructed from a polynomial in the \( \vec{S}_i \cdot \vec{S}_{i+1} \). The precise form of this polynomial can be fully fixed by assuming that the integrable structure present in the A-type theories persists in this broader setting. The spectrum of excitations can now be studied using methods such as the algebraic Bethe ansatz (see [34,35] and [36] for a review). One can in principle also extend this to more general excitations of the full superconformal algebra \( \text{osp}(8^*|1) \). Another generalization has to do with taking more general closed loops and “operator impurity insertions.” All of these cases produce similarly rich spin chain systems which are also amenable to the same sort of analysis.

From this starting point, we can now see the emergence of a natural system of qubits. Our plan in this note will be to use this to build up a system of protected qubits in a higher-dimensional SCFT. Additionally, because of the explicit form of these qubits, we can perform computations of entanglement entropy for these states. We can also use this same setup to track the effects of entanglement of states under 6D renormalization group flow as well as to construct a protocol for qubit manipulation. The picture we arrive at is reminiscent of notions appearing in the holographic tensor network literature though we shall not attempt to make any exact correspondences with the statements found there.

To avoid technical complications, we mainly work with the special case of M5-branes probing an A-type singularity. Similar considerations hold for all the D- and E-type singularities, but with the mild caveat that we are then dealing with higher spin representations. In what follows, we will also not dwell on the distinction between spin chains with open boundary conditions and those with periodic boundary conditions since we will work in a thermodynamic limit where \( N \) is quite large. The case of periodic boundary conditions occurs anyway in the study of 6D LSTs [27].
2 Qubits in 6D SCFTs

We now proceed to build a system of qubits in a 6D SCFT. As already mentioned, we are interested in the class of operators $O_{m_1...m_N}$ where $m_i = \pm 1/2$. Working with the radially quantized SCFT, we see that each local operator specifies a state in the Hilbert space:

$$O_{m_1...m_N}(0) |\text{GND}\rangle = |O_{m_1...m_N}\rangle \in \mathcal{H}_{6D}. \quad (2.1)$$

On the other hand, the $m_i$ also specify a state in a 1D spin chain Hilbert space:

$$|m_1...m_N\rangle \in \mathcal{H}_{1D}. \quad (2.2)$$

From all that we have said, $\mathcal{H}_{1D}$ defines a protected subsector of states in the 6D SCFT. This is the qubit system we wish to study.

The spatial direction of the spin chain is clearly related to the $\mathbb{R}_\perp$ direction of the M5-brane probe theory. Note that in the holographic dual, the spin chain direction corresponds to a great arc passing from the north pole to the south pole of $S^4/\Gamma_{ADE}$.

Returning to our spin chain Hamiltonian:

$$\Delta = E^{(0)} - \lambda_A \sum_{i=1}^{N} 2 \vec{S}_i \cdot \vec{S}_{i+1}, \quad (2.3)$$

we observe that the lowest energy states actually have a large degeneracy. To see this, observe that the total angular momentum operator:

$$\vec{S} = \sum_{i=1}^{N} \vec{S}_i \quad (2.4)$$

commutes with $\Delta$, namely $[\vec{S}, \Delta] = 0$. So, we can organize our energy eigenstates into representations of $\vec{S}$. The system is also gapless in the sense that it costs very little energy to produce an excitation above the lowest energy states. Note also that there is a non-relativistic dispersion relation with energy $\epsilon(p) \sim p^2$, so we get a scale invariant but non-Lorentz invariant system. Some examples of entanglement entropy calculations for Heisenberg spin chains and deformations thereof have been carried out in references [37–39].

Let us now discuss in more detail the lowest energy states of the system. To begin, take the state with all $m_i = 1/2$. This is the highest weight state of a spin $N/2$ representation. Introducing $S^\pm = S^x \pm iS^y$, we reach the other states with the same scaling dimension by successive applications of the lowering operator $S^-$. The resulting form of the states obtained
from $M$ such spin flips are discussed in [39] and are given by:

$$|N, M\rangle = \frac{1}{\sqrt{C_{M,N}}} \sum_{\sigma} |m_{\sigma(1)}...m_{\sigma(N)}\rangle,$$

(2.5)

where we sum over all permutations of $M$ down spins on $N$ sites. Here, we have also introduced the combinatorial factor:

$$C_{M,N} = \frac{N!}{M!(N-M)!}.$$

(2.6)

We claim that the resulting qubits of this ground state are highly entangled states, and similar considerations hold for quasi-particle excitations of the spin chain. Indeed, introducing the pure state:

$$\rho_{M,N} = |N, M\rangle \langle N, M|,$$

(2.7)

we get a mixed state by performing a partial trace of $N-n$ spins, not necessarily in a single contiguous block. We then get the reduced density matrix:

$$\rho_{M,N}^{(n)} = \text{Tr}_{(N-n)} \rho_{M,N}.$$

(2.8)

The entanglement entropy for this was computed in [39] for periodic boundary conditions. In the thermodynamic limit where $N \to \infty$ with $M/N = p$ held fixed and $n/N \leq 1/2$ also fixed, this takes the form:

$$S^{(n)} = -\text{Tr}\rho_{M,N}^{(n)} \log \rho_{M,N}^{(n)} \approx \frac{1}{2} \log n + \frac{1}{2} \log(2\pi e p q),$$

(2.9)

with $p + q = 1$.

What is the interpretation of this in the original M-theory picture? We can consider starting with our stack of $N$ M5-branes, and can perform a partial trace over all but $n$ of them. Doing so, we get a highly entangled state. Indeed, the proof of this is that in our 1D system we have an entanglement entropy proportional to $\log n$.

The pure states $\rho_{M,N}$ realize explicit examples of $N$ qubit W-states as well as generalizations thereof, as opposed to GHZ states. For example, in a system with $N$ qubits, we can introduce the pure states:

1

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\ldots\uparrow\rangle + |\downarrow\ldots\downarrow\rangle)$$

(2.10)

$$|W\rangle = \frac{1}{\sqrt{N}} (|\downarrow\uparrow\ldots\uparrow\rangle + \ldots + |\uparrow\uparrow\ldots\downarrow\rangle),$$

(2.11)

As mentioned before, we are ignoring the zero momentum constraint on quasi-particle excitations of the spin chain. We expect this to be a subleading effect so we neglect it in the discussion which follows.

The original terminology applies to the three qubit case [40], but it clearly extends, with the caveat that there are many ways to entangle four or more qubits [41].
and then form the corresponding density matrices. In both cases, performing a partial trace over a single qubit leads to a non-reduced density matrix, but we observe that after doing this, the resulting density matrix for the GHZ state is separable (no further entanglement between qubits after performing additional partial traces) whereas in the case of the W-state entanglement persists after performing a further partial trace. It is in this sense that the qubits we have constructed are “highly entangled”.

3 Entanglement and 6D RG Flows

Starting from a conformal field theory, we can consider deformations which trigger a flow to another conformal fixed point in the infrared (IR). In the case of 6D SCFTs, the available options for supersymmetry preserving renormalization group flows are quite limited. These are always specified by background operator vacuum expectation values (vevs) and are referred to as tensor branch flows and Higgs branch flows [42, 43] (see also [44]). We consider both sorts of flows.

3.1 Tensor Branch Flows

Consider first tensor branch flows. Some examples of tensor branch flows arise from just separating the M5-branes from one another. In the case of M5-branes probing an A-type singularity, this turns out to be the only possibility. For more general 6D SCFTs other tensor branch deformations are possible and they are all classified by suitable Kähler deformations of the associated F-theory model [20, 21].

Sticking to the simplest case where we separate our M5-branes into two stacks \( N_1 \) and \( N_2 \) such that \( N_1 + N_2 = N \), we can clearly see that in the deep infrared, our single spin chain has broken up into two independent spin chains:

\[
\begin{align*}
[G_0] - G_1 - \ldots - G_{N_1-1} &\oplus [G_{N_1}] - G_{N_1+1} - \ldots - G_{N-1} \oplus [G_N] 
\end{align*}
\]  

(3.1)

where we have indicated how the gauge group \( G_{N_1} \) has become a flavor symmetry. In the deep infrared, this flavor symmetry acts independently on the two decoupled SCFTs (see figure 1).

Now, we can ask about the structure of the Hilbert space of the 6D SCFT in this limit. Clearly, we expect a split into two “decoupled” SCFTs in the infrared, so we can write:

\[
\mathcal{H}_{IR} = \mathcal{H}_{N_1} \otimes \mathcal{H}_{mix} \otimes \mathcal{H}_{N_2},
\]  

(3.2)

where here, \( \mathcal{H}_{mix} \) denotes the Hilbert space of a TQFT coupled to some free fields (see e.g. [45]).

In the spin chain, we can visualize this process by working with a slightly more general
Figure 1: Depiction of a tensor branch flow from the UV to the IR. In the M5-brane picture (left) this involves separating a stack of $N = N_1 + N_2$ M5-branes into two stacks. In the associated spin chain (right), the resulting spins separate into two decoupled sectors. There can still be significant entanglement between the two sectors.

Dilatation operator / spin chain Hamiltonian:

$$\Delta_{\text{IR}} = E_{\text{IR}}^{(0)} - \sum_{i=1}^{N} 2\lambda_i \vec{S}_i \cdot \vec{S}_{i+1},$$

where the couplings can now be position dependent. The limit we are discussing amounts to setting:

$$\lambda_1 = \lambda_2 = \ldots = \lambda_{N_1-1},$$

$$\lambda_{N_1} = 0,$$

$$\lambda_{N_1+1} = \lambda_{N_1+2} = \ldots = \lambda_{N-1}.$$  

We note that the computation of the spin chain couplings performed in reference [27] displays a non-trivial dependence on $N$, so in particular when $N_1 \neq N_2$, we do not expect the couplings on the left and righthand sides of the decoupled spin chains to be the same in the deep IR.

We can also see that there is a great deal of entanglement between the two separated M5-brane sectors of line (3.2). Indeed, from our discussion in the previous section, we know that even in our spin chain subsector this scales as $\log N_1$ (in the case where $N_1 < N_2$). This provides evidence for the existence of a non-trivial TQFT which couples these two sectors, in accord with the general considerations presented in [46] where 6D SCFTs were visualized.
as “edge modes” of a bulk 7D theory (see also [47–49]).

We can generalize this to multiple boundaries by considering other partitions of $N$:

$$N = N_1 + ... + N_k.$$  \hfill (3.7)

In this way, we can build multi-party entangled qubits. See figure 2 for a depiction. It would be interesting to see whether this provides a higher-dimensional analog of the situation considered in references [12,13,16,17].

### 3.2 Higgs Branch Flows

Consider next the case of Higgs branch flows. The distinguishing feature here is that the $SU(2)_{\mathbb{R}}$ R-symmetry is broken along the flow, but a new R-symmetry emerges in the deep infrared. The resulting class of theories which can be achieved in these cases again resemble quiver gauge theories, but in which there can now be different ranks of gauge groups in the generalized quiver, as well as possible decorations by conformal matter on the left and righthand sides [21,25,26]. Importantly, in the vast majority of Higgs branch flows, the actual number of gauge group factors again remains of order $N$. This means that even in the associated spin chain generated in the deep IR, we again have the same number of spins, but can now have more general position dependence:

$$\Delta_{\text{IR}} = E^{(0)}_{\text{IR}} - \sum_{i=1}^{N} 2\lambda_i \vec{S}_i \cdot \vec{S}_{i+1}.$$  \hfill (3.8)

Indeed, in such situations, we can ask about the structure of the “ground states” found for the ultraviolet (UV) Hamiltonian. Note, however, that $\vec{S} = \vec{S}_1 + ... + \vec{S}_N$ still commutes with $H_{\text{IR}}$. So, we again have a large degeneracy in the ground state of the spin chain. Moreover, even though the spectrum of excitations has moved around, the impact on the structure of the spin chain Hilbert space is relatively mild.

### 4 Interpolation

At this point, it is interesting to ask about how to interpret the structure of the spin chain and its Hamiltonian as we proceed from a perturbation of the ultraviolet fixed point to a new one in the deep infrared. Here we discuss some speculative comments in this direction.

First of all, we note that in the 6D SCFT, the spin chain Hamiltonian has the interpretation as the one-loop Dilatation operator. Once we break conformal symmetry, our interpretation must also be suitably loosened. That being said, it is also clear that we can still take a state and ask how it evolves as a function of scale. In the holographic dual setup, this corresponds to motion from the “UV brane” to the “IR brane”. Observe that at least for
Figure 2: Depiction of a multi-throat spacetime generated by pulling $N = N_1 + ... + N_k$ M5-branes apart into separate stacks. Starting from a configuration of spins in the parent theory, we get a multi-party entangled state in the IR theory.

tensor branch flows, this is immediately realized in terms of conventional branes: We simply take some number of M5-branes and pass them down the throat of the AdS geometry (see figure 2).

As a first generalization, then, we can consider a family of spin chain Hamiltonians, one for each step in the RG direction. Labelling this family as $\Delta(z)$ such that $z_{UV}$ corresponds to the UV and $z_{IR}$ corresponds to motion into the IR, we now allow our nearest neighbor interactions to depend on RG time, writing $\lambda_i(z)$. In this context, it is appropriate to also permit our spin operators to be $z$ dependent as well, so we write $\vec{S}_i(z)$ to reflect this fact. At a given RG time slice, we now can write:

$$\Delta(z) = E^{(0)}(z) - \sum_{i=1}^{N} 2\lambda_i(z) \vec{S}_i(z) \cdot \vec{S}_{i+1}(z) + ... \quad (4.1)$$

We can thus consider a slightly broader class of “time dependent” spin chains in which we evolve from $\Delta(z_{UV})$ to $\Delta(z_{IR})$.

There are good reasons to generalize this slightly further. For one thing, we note that as written, this Hamiltonian still preserves $SU(2)_R$ R-symmetry. On the other hand, we also know that at least in Higgs branch flows, we expect the R-symmetry to be broken, only to reemerge deep in the IR. As a further generalization, we therefore allow various sorts of R-symmetry breaking as generated by vevs of operators in the parent UV theory.
In the 1D spin chain, this suggests a further generalization where we allow $SU(2)_R$ R-symmetry breaking terms. One option is to just include some background magnetic field terms. We can also allow various $SU(2)_R$ braking terms of the sort appearing in integrable $XYZ$ models. Including both sorts of terms, we get, in the obvious notation:

$$\Delta(z) = E(z) - \sum_{i=1}^{N} \sum_{a=1}^{3} \left( 2\lambda_{i}^{(a)}(z) S_{i}^{(a)}(z) \cdot S_{i+1}^{(a)}(z) + h_{i}^{(a)}(z) \cdot S_{i}^{(a)}(z) \right). \tag{4.2}$$

The main condition we need to impose is that once we reach the IR where we recover a 6D SCFT, all $SU(2)_R$ breaking terms go to zero as $z \rightarrow z_{IR}$.

## 5 Qubit Manipulation

Given the suggestive form of our spin chain system, it is of course interesting to ask whether we can directly manipulate the associated qubits, and using this, probe additional structure in these systems.\(^3\) From the perspective of the 6D SCFT, the natural operations on states include acting with the symmetry generators of the conformal field theory, including the Dilatation and R-symmetry operators. In particular, the Dilatation operator corresponds to the Hamiltonian of the spin chain, governing time evolution as a function of renormalization group scale in the 6D SCFT. At first pass, the use of the R-symmetry generators provides a way for us to rotate qubits, but in the interacting SCFT, we really have only the operator:

$$\vec{S} = \sum_{i=1}^{N} \vec{S}_{i}, \tag{5.1}$$

which would simultaneously manipulate many qubits all at once.

Using the M5-brane picture, however, we can see how to build a general protocol for qubit manipulation. As a warmup, consider the 4D SCFT obtained by compactifying the tensor branch theory on a $T^2$. This system has basically the same qubit structure as the 6D theory, but there is no zero momentum constraint \([27]\). To manipulate an individual qubit, we consider a deformation which involves separating a single M5-brane from all of its neighbors in the quiver. In this limit, a link between $G_{i-1}$ and $G_i$ is isolated from the rest of the system, and it has its own emergent R-symmetry in the infrared. So, we can apply a unitary $SU(2)_R$ transformation along with an overall phase factor $\exp(i\gamma) \exp(-i\vec{\theta} \cdot \vec{S})$. Doing so, we can generate the standard single qubit operations, visualized as rotations on the Bloch sphere. Some simple examples include the bit flip operation, or Pauli $X$-gate:

$$\sigma_X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \exp(i\pi/2) \exp(-i\pi S^{(x)}). \tag{5.2}$$

\(^3\)We thank A. Kar for some comments which prompted us to consider this possibility.
Figure 3: Example of qubit manipulation as a function of RG time / trajectory time in a moduli space flow with local coordinate $z$. The starting point is to separate M5-branes from one another. This is followed by a general Bloch sphere / $SU(2)_{\mathcal{R}}$ rotation. After this, the M5-branes are recombined. In the case of 6D SCFTs this is followed by a projection onto the zero momentum sector of the 1D spin chain Hilbert space.

Another example is the Hadamard gate:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \exp(i\pi/2) \exp\left(-i \frac{\pi}{\sqrt{2}} (S^{(x)} + S^{(z)}) \right). \quad (5.3)$$

Now we can see a general way to start manipulating individual qubits: We begin by pulling all the M5-branes away from each other. In the spin chain Hamiltonian this corresponds to specific $z$-dependent behavior for the nearest neighbor interactions. After they are well separated, they each have their own emergent $SU(2)_{\mathcal{R}}$ R-symmetry and we can manipulate their qubits individually. After this, we can bring the M5-branes back together, and evolve further with the Dilatation operator. Note that this is a flow in moduli space with the local coordinate of the flow playing the role of time evolution in the qubit system.

Similar considerations clearly hold for the 6D SCFT system, with the mild caveat that we need to impose the zero momentum constraint on possible excitations. This means, for example, that after manipulating individual qubits and bringing the stack of M5-branes back together that we need to perform a further projection onto the zero momentum sector of the 1D spin chain Hilbert space. So, in an actual quantum computation we would perform this operation at the very end.

As a final generalization, we can now see how to build far more involved qubit operations.
In this case we consider separating a stack of $M$ M5-branes from the rest of the system via a tensor branch flow, manipulate that individual set of qubits, and then bring it back to the rest of the configuration. An example of this sort of qubit manipulation is depicted in figure 3. It would be interesting to study further the class of qubit operations which can be engineered in this way.

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