I. INTRODUCTION. FROM QUANTUM WIRES TO QUANTUM CROSSBARS

The behavior of electrons in arrays of 1D quantum wires was recognized as a challenging problem soon after the consistent theory of elementary excitations and correlations in a Luttinger liquid (LL) of interacting electrons in one dimension was formulated (see, e.g., Ref. 1 for a review). One of the fascinating challenges existing in this field is a search for LL features in higher dimensions. Although the Fermi liquid state seems to be rather robust for \( D > 1 \), the possible way to retain some 1D excitations modes in 2D and even 3D systems is to consider highly anisotropic objects, in which the electron motion is spatially confined in major part of the real space (e.g., it is confined to separate linear regions by potential relief). One may hope that in this case weak enough perturbation does not violate the generic long-wave properties of the LL state. Arrays of interacting quantum wires may be formed in organic materials and in striped phases of doped transition metal oxides. Artificially fabricated structures with controllable configurations of arrays and variable interactions are available now due to recent achievements in nanotechnology (see, e.g., Ref. 2).

We start with a discussion of an array of parallel quantum wires. The conventional LL regime in a 1D quantum wire is characterized by bosonic fields describing charge and spin modes. We confine our discussion to the charge sector (LL in the spin-gapped phase). The Hamiltonian of an isolated quantum wire may then be represented in a canonical form

\[
H = \frac{\hbar v}{2} \int_{-L/2}^{L/2} dx \left\{ g \pi^2(x) + \frac{1}{g} (\partial_x \theta(x))^2 \right\} \tag{1}
\]

Here \( L \) is the wire length, \( v \) is the Fermi velocity, \( \theta, \pi \) are the conventional canonically conjugate boson fields and \( g \) is the dimensionless parameter which describes the strength of the interaction within the chain (see, e.g., Ref. 2).

The interwire interaction may transform the LL state existing in isolated quantum wires into various phases of 2D quantum liquid. The most drastic transformation is caused by the interwire tunneling \( t_{\perp} \) in arrays of quantum wires with intrawire Coulomb repulsion. This coupling constant rescales towards higher values for strong interaction \( (g < 1/2) \), and the electrons in array transform into 2D Fermi liquid. The reason for this instability is the orthogonality catastrophe, i.e. the infrared divergence in the low-energy excitation spectrum that accompanies the interwire hopping processes.

Unlike interwire tunneling, the density-density or current-current interwire interactions do not modify the low-energy behavior of quantum arrays under certain conditions. In particular, it was shown recently \( 3,4,5 \) that an interaction of the type \( W(n - n') \), which depends on the distance between wires \( n \) and \( n' \) but does not contain current coordinates \( x, x' \), imparts the properties of a sliding phase to 2D array of 1D quantum wires. In this state an additional interwire coupling leaves the fixed-point action invariant under the "sliding" transformation \( \theta_n \rightarrow \theta_n + \alpha_n \) and \( \pi_n \rightarrow \pi_n + \alpha'_n \). The contribution of interwire coupling reduces to a renormalization of the parameters \( v \rightarrow v(q_\perp), g \rightarrow g(q_\perp) \) in the LL Hamiltonian \( 6 \), where \( q_\perp \) is a momentum perpendicular to the chain orientation. Such LL structure can be interpreted as a quantum analog of classical sliding phases of coupled XY chains. Recently, it was found \( 7,8 \) that a hierarchy of quantum Hall states emerges in sliding phases when a quantizing magnetic field is applied to an array.

In the present paper we concentrate on another aspect of the problem of interacting quantum wires. Instead of studying the conditions under which the LL behavior is preserved in spite of interwire interaction, we consider situations where the dimensional crossover from 1D to 2D occurs. In other words, we investigate regimes, where the excitations in quantum array demonstrate either 1D or 2D behavior in different parts of phase space. The most promising type of artificial structures where this effect may be expected is a periodic 2D system of two arrays of parallel quantum wires crossing each other at an angle \( \varphi \). We call it "quantum crossbars" (QCB). The square grids of this type were considered in various physical contexts in early papers \( 9,10 \). In Ref. 11, it was shown that the fragility of the LL state against interwire tunneling in the crossing areas

\[
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\]

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of QCB was studied. It was found that a new periodicity imposed by the interwire hopping term results in the appearance of a low-energy cutoff $\Delta \sim \hbar v t / a$ where $a$ is a period of the quantum grid. Below this energy, the system is “frozen” in its lowest one-electron state. As a result, the LL state remains robust against orthogonality catastrophe, and the Fermi surface conserves its 1D character in the corresponding parts of the 2D Brillouin zone. This cutoff energy tends to zero at the points where the one-electron energies for two perpendicular arrays $\epsilon_{k_1}$ and $\epsilon_{k_2}$ become degenerate. As a result, a dimensional crossover from 1D to 2D Fermi surface (or from LL to FL behavior) arises around the points $\epsilon_{F_1} = \epsilon_{F_2}$.

We study this dimensional crossover for Bose excitations (plasmons) described by canonical variables $\theta, \pi$ in QCB. In order to unravel the pertinent physics we consider a grid with short-range capacitive inter-wire interaction. This approximation seems natural for 2D grids of carbon nanotubes or artificially fabricated bars of quantum wires with grid periods $a_{1,2}$ which exceed the lattice spacing of a single wire or the diameter of a nanotube. It will be shown below that this interaction can be made effectively weak. Therefore, QCB retains the 1D LL character for motion along the wires similarly to the case considered in Ref.\(^1\). At the same time, the boson mode propagation along some resonant directions is also feasible. This is essentially a 2D process in the 2D Brillouin zone of the reciprocal space.

II. QUANTUM CROSSBARS: BASIC NOTIONS

![FIG. 1. 2D quantum bar formed by two interacting arrays of parallel quantum wires. Here $\mathbf{e}_1, \mathbf{e}_2$ are the unit vectors of the superlattice, $a_1, a_2$ are the superlattice periods and $d$ is the vertical interarray distance.](image)

A quantum crossbars may be defined as a 2D periodic grid, i.e., two periodically crossed arrays of 1D quantum wires. In fact these arrays are placed on two parallel planes separated by an inter-plane distance $d$, but in this section we consider QCB as a genuine 2D system. We assume that all wires of the $j$-th array, $j = 1, 2$, have the same length $L_j$, Fermi velocity $v_j$ and Luttinger parameter $g_j$. They are oriented along unit vectors $\mathbf{e}_{1,2}$ with an angle $\varphi$ between them. Thus, the QCB periods along these directions are $a_1$ and $a_2$, and the corresponding QCB basic vectors are $\mathbf{a}_j = a_j \mathbf{e}_j$ (Fig. 1). The interaction between the excitations in different wires is assumed to be concentrated near the crossing points with coordinates $n_1 a_1 + n_2 a_2 \equiv (n_1, n_2)$. The integers $n_j$ enumerate the wires within the $j$-th array. Such interaction imposes a superperiodicity on the energy spectrum of initially one dimensional quantum wires, and the eigenstates of this superlattice are characterized by a 2D quasimomentum $\mathbf{q} = q_1 \mathbf{g}_1 + q_2 \mathbf{g}_2 \equiv (q_1, q_2)$. Here $\mathbf{g}_{1,2}$ are the unit vectors of the reciprocal superlattice satisfying the standard orthogonality relations $\mathbf{e}_i \cdot \mathbf{g}_j = \delta_{ij}$. The corresponding basic vectors of the reciprocal superlattice have the form $(m_1 \mathbf{Q}_1, m_2 \mathbf{Q}_2)$, where $Q_j = 2\pi / a_j$ and $m_{1,2}$ are integers.

![FIG. 2. Fermi surface of 2D metallic quantum bar in the absence of charge transfer between wires. $\mathbf{g}_1, \mathbf{g}_2$ are the unit vectors of the reciprocal superlattice](image)

In conventional 2D systems, forbidden states in the inverse space arise due to Bragg diffraction in a periodic potential, whereas the whole plane is allowed for wave propagation in real space, at least till the periodic potential is weak enough. In strongly anisotropic QCB, most of the real space is forbidden for electron and plasmon propagation, whereas the Bragg conditions for the wave vectors are still the same as in conventional 2D plane modulated by a periodic potential. The excitation motion in QCB is one-dimensional in major part of the 2D plane, and the anisotropy in real space imposes restrictions on the possible values of the 2D coordinates $x_1, x_2$ ($r = x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2$). At least one of them, e.g., $x_2$ ($x_1$) should be an integer multiple of the corresponding array period $a_2$ ($a_1$), so that the vector $r = (x_1, n_2 a_2)$ ($r = (n_1 a_1, x_2)$) characterizes a point with a 1D coordinate $x_1$ ($x_2$) lying at the $n_2$-th ($n_1$-th) wire of the first
We confine ourselves with the first BZ of a superchain, an eigenstate with a given wave vector \( \mathbf{k} \) in the BZ because of the kinematic restrictions mentioned above. Even in non-interacting arrays of quantum wires the 2D basis is two sets of 1D waves. These are 1D excitations propagating along each wire of array 1 characterized by a unit vector \( k_1 \mathbf{g}_1 \) with a phase shift \( a_2 k_2 \) between adjacent wires, and the same for array 2. The states of equal energy obtained by means of this procedure form straight lines in the 2D BZ. Respectively, the Fermi sea is not a circle with radius \( k_F \) like in the case of free 2D gas, but a cross in the \( k \) plane bounded by these four lines (see Fig. 2).

Due to the weak inter-wire interaction, the excitations in the 2D BZ depicted in Fig. 2 acquire two-dimensionality characterized by the quasimomentum \( q = (q_1, q_2) \). However, in case of interaction, the 2D waves constructed from the 1D plane waves in accordance with the above procedure form an appropriate basis for the description of elementary excitations in QCB, in close analogy with the nearly free electron approximation in conventional crystalline lattices. It is easy to believe that the inter-wire interaction does not completely destroy the above quasimomentum classification of eigenstates, and the 2D reconstruction of the spectrum may be described in terms of wave mixing similarly to the standard Bragg diffraction in a weak periodic potential. Moreover, the classification of eigenstates of non-interacting crossed arrays of 1D wires ("empty superlattice") may be effectively used for the classification of energy bands in a real QB superlattice. Our next task is to construct a complete 2D basis for this empty superlattice.

Excitations in a given wire are described as plane waves \( e^{i k x} \) with wave number \( k \) and initial dispersion law \( \omega(k) = v|k| \) (the array number is temporarily omitted). Each excitation in an "empty superchain" is described by its quasi wavenumber \( q \) and a band number \( s \) \( (s = 1, 2, \ldots) \). Its wave function has the Bloch-type structure,

\[
\psi_{s,q}(x) = \frac{1}{\sqrt{L}} e^{i q x} u_{s,q}(x). \tag{2}
\]

We confine ourselves with the first BZ of a superchain, \( |q| \leq Q/2 \), where the Bloch amplitude \( u_{s,q} \) and dispersion law \( \omega_s \) have the following form:

\[
u_{s,q}(x) = \exp \left\{ i Q x \frac{(-1)^{s-1} q}{Q} \right\}, \tag{3}
\]

\[
\omega_s(q) = v Q \left( \frac{q}{2} + (-1)^{s-1} \frac{|q|}{Q} \right). \tag{4}
\]

To write down these formulas for a specific array, one should add the array index \( j \) to the wave function \( \psi \), Bloch amplitude \( u \), coordinate \( x \), quasimomentum \( q \), periods \( a \) and \( Q \) of the 1D lattice in real and reciprocal space.

FIG. 3. Two dimensional Brillouin zone of QB. Four polygonal lines along which the dispersion of Bose excitations is calculated in Section V are marked by \( AOA' \), \( FCF' \), and \( ODE \).

The 2D basis of periodic Bloch functions for an empty superlattice is constructed in terms of 1D Bloch functions \( \Phi \), \( \Phi' \)

\[
\Psi_{s,s',q}(x) = \psi_{1,s,q_1}(x_1) \psi_{2,s',q_2}(x_2). \tag{5}
\]

Here the 2D quasimomentum \( q = (q_1, q_2) \) belongs to the first BZ, \( |q_j| \leq Q_j/2 \). The corresponding eigenfrequencies are

\[
\omega_{s,s'}(q) = \omega_{1,s}(q_1) + \omega_{2,s'}(q_2). \tag{6}
\]

We will use this basis in the next section when constructing the excitation spectrum of QB within the reduced band scheme.

The full Hamiltonian of the QB is,

\[
H = H_1 + H_2 + H_{\text{int}}, \tag{7}
\]

where \( H_j \) describes the 1D boson field characterised by the parameters \( v_j, q_j \) in the \( j \)-th array (see eq. (1)), and \( H_{\text{int}} \) is the interwire interaction. One may neglect interwire tunneling and restrict oneself by the capacitive interaction only, provided the vertical distance \( d \) between the wires is substantially larger than the screening radii \( r_j \) within the wires. Then
\[ H_{int} = V_0 \sum_{n_1,n_2} \int dx_1 dx_2 \Phi \left( \frac{x_1 - n_1 a_1}{r_1}, \frac{n_2 a_2 - x_2}{r_2} \right) \times \partial_x \theta_1 (x_1, n_2 a_2) \partial_x \theta_2 (n_1 a_1, x_2). \]  

(8)

It stems from the Coulomb interaction between intrawire charge fluctuations within the crossing area. The size of intrawire fluctuations is determined by \( r_j \). The coupling strength is \( V_0 = 2e^2/d \), and the function \( \Phi(\xi_1, \xi_2) \) is

\[ \Phi(\xi_1, \xi_2) = \frac{\zeta_1(\xi_1)\zeta_2(\xi_2)}{\sqrt{1 + |r_{12}|^2/d^2}} \approx \zeta_1(\xi_1)\zeta_2(\xi_2), \]  

(9)

provided \( |r_{12}|^2/d^2 \ll 1 \). Thus the interaction is separable in this limit.

The above approximation looks realistic for QCB fabricated from carbon nanotubes. In this case the Coulomb interaction is screened at a distance of the order of the nanotube radius, \( R_0 \), therefore \( r_{1,2} \approx R_0 \). The minimal radius of a single-walled carbon nanotube is about \( R_0 = 0.35 \pm 0.4 \text{nm} \) (see \( \text{eq.} \). The vertical distance \( d \) in artificially produced nanotube networks is estimated as \( d \approx 2 \text{nm} \). Therefore the ratio \( r_0^2/d^2 \approx 0.04 \) is really small.

In the quasimomentum representation, the full Hamiltonian (7) acquires the form,

\[ H = \frac{\hbar v g}{2} \sum_{s,q}^{2} \sum_{j=1}^{2} \pi_{j,s,q}^{\dagger} \pi_{j,s,q} + \frac{\hbar^2}{2v g} \sum_{j,j'}^{2} \sum_{s,s',q} W_{j,s}^{j',s'} q^{j,s} q^{j',s'} q \]  

(10)

where \( v = \sqrt{v_1 v_2} \), \( g = \sqrt{g_1 g_2} \), while \( \sqrt{v g}/v_j g_j \theta_{j,s,q} \) and \( \sqrt{v_j g_j v g} \pi_{j,s,q} \) are the Fourier components of the boson fields \( \theta_j \) and \( \pi_j \). The matrix elements for interwire coupling are given by:

\[ W_{j,s}^{j',s'} q = \omega_{j,s}(q) \omega_{j',s'}(q) \left[ \delta_{jj'} \delta_{ss'} + \phi_{j,s}^{j',s'}(q) (1 - \delta_{jj'}) \right]. \]

Here \( \omega_{j,s}(q) \) are the eigenfrequencies (4) of the “unperturbed” 1D mode pertaining to an array \( j \). The coefficients

\[ \phi_{s + s'} = (-1)^{s + s'} \text{sign}(q_1 q_2) \Phi_{s + s'} q \], \[ \phi = \frac{g V_0 r_0^2}{\hbar v g} \]  

(11)

are proportional to the dimensionless Fourier component of the interaction strengths

\[ \Phi_{s + s'} q = \int \xi_1 d\xi_2 \Phi(\xi_1, \xi_2) e^{-(r_1 q_1 \xi_1 + r_2 q_2 \xi_2)} \times \]  

\[ u_{1, s, q_1}^{\dagger} (r_1 \xi_1) u_{2, s', q_2}^{\dagger} (r_2 \xi_2). \]  

(12)

The diagonalization procedure is cumbersome in the general case due to mixing of states belonging to different bands and arrays. However, in the case of separable interwire potential (4) one easily comes to a compact secular equation for the eigenfrequencies of QCB:

\[ F_{1, q_1}(\omega^2) F_{2, q_2}(\omega^2) = \frac{1}{\varepsilon}, \]  

(13)

where

\[ F_{q}(\omega^2) = \frac{r_j}{a_j} \sum_{s}^{2} \phi_{s}^{q}(q) \omega_{s}^{q}(q) - \omega^2, \]  

(14)

and the dimensionless coupling constant \( \varepsilon \) can be written as

\[ \varepsilon = \left( \frac{a}{r_0} \right)^2 \left( \frac{g V_0 r_0}{\hbar v} \right)^2 = \left( \frac{2 R_0 g e^2}{d \hbar v} \right)^2. \]  

(15)

For nanotube QCB, the first factor within parentheses is about 0.35. The second one, that is nothing but the corresponding QCB “fine structure” constant, can be estimated as 0.9 (we used the values of \( g = 1/3 \) and \( v = 8 \times 10^7 \text{cm/sec} \), see Ref.\( \text{[1]} \)). Therefore \( \varepsilon \approx 0.1 \), and the coupling is really weak.

### III. ENERGY SPECTRUM

Due to weakness of the interaction, the systematics of unperturbed plasmon levels and states is grossly conserved, at least in the low energy region corresponding to the first few bands. This means that perturbed eigenstates could be described by the same quantum numbers as the unperturbed ones. The interband mixing is significant only along the high symmetry directions in the first BZ (BZ boundaries and lines \( q_j = 0 \)). In zeroth approximation with respect to the weak interaction, these lines are determined by the Bragg conditions. Inter-array mixing within the same energy band is strong only for waves with quasimomenta close to the resonant lines in the BZ. In zeroth approximation with respect to the interaction, these lines are determined by the conditions \( \omega_{s}^{q}(q_1) = \omega_{s'}^{q}(q_2) \) with all possible positive integers \( s, s' \). In the rest of the BZ, the initial systematics can be used.

The three next figures illustrate the main features of the excitation spectrum. In Fig.3 the dispersion curves, corresponding to quasi momenta changing along the line \( AOA' \) of Fig.2 are plotted in comparison with those for non interacting arrays. (In all figures within this section we use units \( \hbar = Q_2 = v_2 = 1 \), and \( v_1 Q_1 = 1.4 \). In what follows we use \( (j, s) \) notations for the unperturbed boson propagating along the \( j \)-th array in the \( s \)-th band. Then the lowest curve in the left part of Fig.3 (line \( AO \) in...
Fig. 3) is, in fact, the slightly renormalized dispersion of a (2, 1) plasmon, the middle curve describes (1, 1) plasmon, and the upper curve is the dispersion of a (1, 2) plasmon. The fourth frequency, corresponding to a (2, 2) plasmon, is far above and is not displayed in this part of the figure. The right part of Fig. 4 describes (1, 1) plasmon (lowest curve), (2, 1) plasmon (middle curve) and (2, 2) plasmon (upper curve). It is seen that the dispersion remains linear along the whole line $AOA'$ except at a nearest vicinity of the BZ boundary (see insets in Fig. 4). It is clearly seen that the plasmon preserve their 1D character along these lines, and small deviation from linearity is observed only near the boundaries of the BZ. The interband hybridization gaps for bosons propagating along the $j$-th array can be estimated as $\Delta \omega_{12}^j \sim v_j Q_j \varepsilon r_0 / a$. 

More pronounced effects of wave mixing are seen in Fig. 5 where the dispersion curves corresponding to the line $FCF'$ (Fig. 3) along the boundary of the BZ are plotted. Again, the dispersion laws retain their 1D character along the major part of the boundary. The interaction opens the gap in the 1D bands for arrays 1 and 2 along the lines $FC$ and $CF'$ respectively. Odd (u) and even (g) combinations of two waves are formed as a result of wave mixing. Strong 2D effects are observed around the points $D$ and $E$. As a result we observe the dimensional crossover $1D \to 2D$ when moving along the boundary of the Z.

The strongest wave-mixing effects are observed along the line $ODE$ in Fig. 6. Here the plasmons belonging to arrays 1 and 2 are mixed along the whole line. They form odd and even combinations but the dispersion is nearly linear everywhere except at the vicinity of the points $D$ and $E$ where three-wave mixing takes place. In a square QCB the strong wave-mixing occurs in the vicinity of the diagonals of the BZ.

The low-energy part of the spectrum along most part of the line $OD$ is described by the secular equation

$$\prod_{j=1}^{2} \left( \frac{\varphi_j^2(q_j) \omega^2}{\omega_j^2(q_j) - \omega^2} + F_j \right) = \frac{1}{\varepsilon}, \quad (16)$$
which follows from the general equation\((14)\). Its solution gives two nearly linear plasmon bands which conserve their LL character in spite of the 2D wave mixing. Just this solution is described by eq. (3.10) of Ref.\(^8\). So, our exact procedure confirms the conclusion of renormalization approach of this paper that the sliding LL phase may exist in two-dimensional QCB, and the inter-array density-density interaction is irrelevant for the LL fixed point.

Finally we show the lines of equal frequency for Bose excitations (Figs. 7,8). These lines should be compared with the Fermi surface "cross" shown in Fig. 2. Their rounding near the broken line \(ODE\) is a manifestation of \(1D \rightarrow 2D\) crossover. Similar rounding of the 1D Fermi surface due to inter-array tunneling was discussed in Ref.\(^14\).

**IV. CORRELATIONS AND OBSERVABLES**

The correlation functions of QCB in the infrared limit are usually discussed in a framework of the theory of sliding LL phases\(^8\). These are the Drude peak in the optical conductivity, \(\sigma(\omega) = D(T)\delta(\omega)\), the power-law temperature dependence of resistivity, and the crossover from isotropic to anisotropic conductivity at a certain length scale, when the current is inserted at a point on array 1 and extracted at another point on array 2. All these features are reproduced by our exact solution which generates the LL thermodynamics and transport as an intrinsic property of QCB Hamiltonian \((10)\).

In this section we discuss in brief the correlation properties which allow one to reveal specific 2D features of QCB at finite frequency and momentum. One of the main effects specific for a QCB is the appearance of non-zero transverse momentum–momentum correlation function. In space-time coordinates \((x, t)\) it reads,

\[
G_{12}(x; t) = \langle \{\pi_1(x_1, 0; t), \pi_2(0, x_2; 0)\} \rangle.
\]

This function describes the momentum response at the point \((0, x_2)\) of the second array at time \(t\) caused by initial \((t = 0)\) perturbation at the point \((x_1, 0)\) of the first array. Standard calculations lead to the following expression,

\[
G_{12}(x; t) = -\frac{\hbar V_0 r_0^2}{4\pi^2 h} \int_{-\infty}^{\infty} dk_1 dk_2 \phi_1(k_1) \phi_2(k_2) k_1 k_2 \times \\
\times \sin(k_1 x_1) \sin(k_2 x_2) \frac{v_2 k_2^2 \sin(v_2 k_2 t) - v_1 k_1 \sin(v_1 k_1 t)}{v_2 k_2^2 - v_1 k_1^2},
\]

where \(\phi_j(k)\) is the Fourier component \((14)\) written in the extended BZ.

This correlator is shown in Fig. 3. Here the non-zero response corresponds to the peak located at the line determined by the obvious kinematic condition \(|x_1| + |x_2| = vt\). The finiteness of the interaction radius slightly spreads this peak and changes its profile.
Further manifestation of the 2D character of QCB is related to a periodic energy transfer between the two arrays of wires. Consider an initial perturbation which, in the system of non-interacting arrays, excites a plane wave propagating within the first array along the $e_1$ direction, 

$$
\langle \theta_1(x_1,n_2a_2; t) \rangle = \frac{\rho_0}{\sqrt{2|q_1|}} \sin(q_1x_1 + q_2n_2a_2 - v_1|q_1|t), \\
\langle \theta_2(n_1a_1,x_2; t) \rangle = 0, 
$$

(\rho_0 is the charge density amplitude). If the wave vector $q$, satisfying the condition $|q| << Q_{1,2}/2$, is not close to the resonant line of the first BZ, weak interwire interaction $\phi = \varepsilon r_0/a$ slightly changes the $\langle \theta_1 \rangle$ component and leads to the appearance of a small $\langle \theta_2 \rangle \sim \phi$ component. But for $q$ lying on the resonant line $(v_1|q_1| = v_2|q_2| \equiv \omega_q)$, both components within the main approximation have the same order of magnitude

$$
\theta_1(x_1,n_2a_2; t) = \frac{\rho_0}{\sqrt{2|q_1|}} \cos\left(\frac{1}{2}\phi_q \omega_q t^\prime\right) \times \\
\sin(q_1x_1 + q_2n_2a_2 - \omega_q t), \\
\theta_2(n_1a_1,x_2; t) = \frac{\rho_0}{\sqrt{2|q_1|}} \sin\left(\frac{1}{2}\phi_q \omega_q t^\prime\right) \times \\
\cos(q_1n_1a_1 + q_2x_2 - \omega_q t). 
$$

Here $\phi_q = \phi_{121q}$ (see Eq. 12). This corresponds to a 2D propagation of a plane wave with wave vector $q$, modulated by a “slow” frequency $\sim \phi \omega$. As a result, an energy is periodically transferred from one array to another during a long period $T \sim (\phi \omega)^{-1}$ (see Fig. 9). These peculiar “Rabi oscillations” may be considered as one of the fingerprints of the physics exposed in QCB systems.

\section{V. Conclusion}

We have shown that the bosonization procedure may be applied to the Hamiltonian of 2D quantum grids at least in the first few Brillouin zones. The energy spectrum of QCB shows the characteristic properties of LL at $|q|, \omega \rightarrow 0$, but at finite $q$, the density and momentum waves may have either 1D or 2D character depending on the direction of the wave vector. Due to interwire interaction, unperturbed states, propagating along the two arrays are always mixed, and transverse components of correlation functions do not vanish. For quasi-momenta near the diagonal of the BZ, such mixing is strong, and the transverse correlators possess specific dynamical properties.

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