Optimization of electrical networks modes by transformer ratios

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Abstract. The problem of optimization the modes of electrical networks provides determination the optimal values of reactive power of sources, voltages of reference nodes and transformation ratios of controlled transformers. At present, methods and algorithms for optimization of reactive power and node voltages are sufficiently developed. However, the development of new efficient optimization algorithms of transformation ratios remains as an important topic. In this case, the issues of optimization of transformation ratios of transformers included in closed networks have a particular importance. This paper provides an effective algorithm for optimization the modes of electrical networks on transformation ratios of controlled transformers. The results of research of its efficiency in the presence of transformers with phase-rotary and longitudinal control in the circuit are presented. It is shown that minimization of losses in closed electrical networks when optimizing transformation ratios occurs due to the rational distribution of power flows in the branches.

1. Introduction

The problem of complex optimization of the electric power system (EPS) mode on all adjustable parameters is a complex problem of nonlinear mathematical programming with a large number of different-scale variables, various constraints in the form of equalities and inequalities. To overcome the computational difficulties in solving this problem, in many approaches, it is decomposed into two main problems - optimization of power system modes and optimization of power network modes [1-4]. To obtain a solution of the original problem of complex optimization, under conditions of sufficiency of the necessary initial information, when using such decomposition, calculations should be performed by iterative method, at each step of which the first and second problems are sequentially solved. When solving one problem, the parameters of other are taken as given. However, it has been established that at present, a single sequential solution of these two problems allows to obtain an optimal solution of the original problem with accuracy sufficient for practical purposes [2, 3]. When solving the first problem, the parameters of the second should be taken from measurements or calculation results for the characteristic intervals of the previous characteristic days. This paper deals with the issues of solving the second problem - the problem of optimization the modes of electrical networks by regulating the transformation ratios. In it, the objective function is represented as a function of the total losses of active power in electrical networks, which is minimized. Solution of the problem of optimization of electrical network mode for any interval of the considered short-term planning period is to determine the optimal values of the controlled parameters - reactive powers of stations and compensators, voltages of reference nodes and transformer ratios, at which the
given constraints are fulfilled and at the same time the function of total active power losses in the electrical network has minimum value. In recent years, there have been many works devoted to the optimization of electrical network modes, such as [5-11], which recommend using of various optimization methods and algorithms such as classical, heuristic, evolutionary, artificial intelligence, etc. They deal mainly with the optimization of reactive power and node voltages. In [9-11] methods and algorithms for optimization the modes of distribution electrical networks by compensating for reactive power and choosing the optimal places for installing compensators are proposed. Therefore, it can be considered that the methods and algorithms for optimization the modes of electrical networks for these parameters are rather developed.

Optimization of modes of electrical networks in terms of transformation ratios of controlled transformers has not been studied enough. These include works [12-15]. According to the algorithm proposed in [12], the transformation ratio of the transformer is optimized based on the choice of optimal voltages on the secondary side of the transformer. In our opinion, the use of this interesting algorithm in the presence of many functional restrictions in the form of inequalities on limiting power flow or current in branches can lead to some calculation difficulties. In [13] an algorithm for minimization of losses in closed electrical networks, based on opening the circuits and compensating for reactive power is given. In [14] an algorithm for optimization the electrical network mode by particle swarm optimization method is proposed. However, it is not clear how functional constraints in the form of inequalities are taken into account. The algorithm described in [15] provides for the optimization of the transformation ratios of transformers in terms of ensuring admissible voltage levels on the secondary side.

In recent years, in many countries, in order to ensure reliable, high-quality and economical modes of operation of electrical networks, FACTS and STATCOM devices have begun to be used, which automatically, in real time, provide rational power flows in electrical networks [16-18]. Ensuring of optimal operating modes of electrical networks by these devices provides for preliminary planning of optimal values of the controlled parameters, in particular the transformation ratios.

Below, an algorithm for optimization the modes of electrical networks on transformation ratios based on the gradient method and effective modeling of a transformer branch in a closed circuit is proposed.

2. Methods

Soil The problem of optimization the modes of electrical networks for the interval of the planning period can be presented in the following form:

\[ \pi = P_1 + P_2 + P_3 + \ldots + P_n \rightarrow \min \]  

subject to constraints in the form of equality

\[ W_i = P_i - P_i^g = 0, \ i \in N; \]  
\[ W_i = Q_i - Q_i^g = 0, \ i \in N \cup G_v; \]  

and inequalities

\[ P_{0_{\min}} \leq P_0 \leq P_{0_{\max}}; \]  
\[ Q_{i_{\min}} \leq Q_i \leq Q_{i_{\max}}, \ i \in G + G_q; \]  
\[ V_{i_{\min}} \leq V_i \leq V_{i_{\max}}, \ i \in N; \]  
\[ k'_{l_{\min}} \leq k'_{l} \leq k'_{l_{\max}}; \ k''_{l_{\min}} \leq k''_{l} \leq k''_{l_{\max}}, \ l \in T_k; \]  
\[ P_{l_{\min}} \leq P_l \leq P_{l_{\max}}, \ l \in L_p; \]  
\[ I_{l_{\min}} \leq I_l \leq I_{l_{\max}}, \ l \in L_i; \]
where \( n \) is the number of nodes in the electrical network (including the balancing node); \( N \) is the set of all nodes in EPS (except for the balancing node); \( G_q \) - a set of nodes with controlled reactive power; \( G_v \) is a set of reference nodes with specified or optimized voltage modules; \( P, Q, P_{iq}, Q_{ig} \) - calculated and specified active and reactive powers of the \( i \)th node; \( T_k \) is a set of adjustable transformers; \( L_p, L_I \) - set of branches in which active power flows and currents are controlled; \( P, Q, V, P_{min}, Q_{min}, V_{min}, P_{max}, Q_{max} \) - calculated and specified limit values of active and reactive power, as well as voltage of the \( i \)th node; \( K^*, K'^*, K_{min}, K_{max} \) - calculated and specified limit values of real and imaginary components of the adjustable transformation ratio of the transformer in the \( i \)th branch; \( P, I, P_{min}, I_{min}, P_{max}, I_{max} \) - calculated and specified limiting values of active power flow and current of the \( i \)th controlled branch.

The active and reactive powers of nodes \( P, Q \) are expressed through the elements of matrix of intrinsic and mutual conductivities \( g_{ii}, b_{ii}, g_{ij}, b_{ij} \) modules and phase angles of complex voltages \( V, V_i, \delta, \delta_j \) as follows:

\[
P_i = \sum_{j \in I_i} P_{ij} = g_{ii}V_i^2 - V_i \sum_{j \in I_i} V_j (g_{ij} \cos \delta_{ij} - b_{ij} \sin \delta_{ij})
\]

\[
Q_i = \sum_{j \in I_i} P_{ij} = g_{ii}V_i^2 - V_i \sum_{j \in I_i} V_j (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij})
\]

where \( \delta_{ij} = \delta_i - \delta_j \).

Here, the function of total active power losses (1) is presented as a algebraic sum of active powers of all \( n \) nodes in electrical network (including the balancing node). In turn, the power of each \( i \)th node is presented as the sum of active power flows along the branches outgoing from it.

When it comes to optimization the transformation ratio, we mean a variable transformer that is located in a branch that makes up a closed circuit. Since the regulation of the transformation ratio of a transformer in an open network is equivalent to regulation of the voltage on its secondary side, it is advisable to reduce the problem of optimization the mode of such network on transformation ratio to optimization on node voltage.
Minimization of active power losses in closed electrical networks is achieved by forced optimal distribution of power flows in them through regulating the transformation ratios of loop transformers. The proposed algorithm provides for the representation of a branch between nodes \( p \) and \( q \) (branch \( p-q \)) with a controlled transformer in the form shown in Figure 1. In it \( \tilde{Y}_{pq} \), \( Y_p \) - the longitudinal and transverse conductivity of the transformer branch. The trans- former core loss is accounted through the transverse conductance (or the correspond- ing load) at node \( p \).

The power flows along the transformer branch \( S_{pq} \), \( S_{qp} \) and their components (active power flows) \( P_{pq} \) \( P_{qp} \) are expressed through the branch conductivity \( \tilde{Y}_{pq} = g_{pq} + jb_{pq} \), node voltages \( V_p \), \( V_q \), phase angles of voltages of these nodes \( \delta_{pq}, \delta_q \) and the transfor- mation ratio (in the general case, complex) of transformer \( K_{pq} = K_{pq}^* + jK_{pq}'' \).

The algorithm is based on the following provisions.

At the next optimization step, the transformation ratio of the transformer changes by \( \Delta K_{pq} = \Delta K_{pq}^* + j\Delta K_{pq}'' \), as a result of which the power flows \( P_{pq}, Q_{pq}, P_{qp}, Q_{qp} \) change by values \( \Delta P_{pq}, \Delta Q_{pq}, \Delta P_{qp}, \Delta Q_{qp} \), that are expressed as

\[
\Delta P_{pq} = -V_p V_q \left[ \Delta K_{pq}^* (g_{pq} \cos \delta_{pq} + b_{pq} \sin \delta_{pq}) + \Delta K_{pq}'' (g_{pq} \sin \delta_{pq} - b_{pq} \cos \delta_{pq}) \right],
\]

\[
\Delta P_{qp} = g_{pq} V_q^2 \left[ (\Delta K_{pq}^*)^2 + (\Delta K_{pq}'')^2 \right] + 2(\Delta K_{pq}^* \Delta K_{pq}'') \Delta K_{pq}''^2 + \Delta K_{pq}'' (g_{pq} \sin \delta_{pq} - b_{pq} \cos \delta_{pq}) \]  \( \Delta K_{pq}'' \) \( \cdot \) \[ \Delta P_{pq} = \Delta \]  \( \Delta Q_{pq} = \Delta K_{pq}'' \Delta K_{pq}'' - \Delta K_{pq}'' \Delta K_{pq}'' \]

The total losses of active power when changing the transformation ratio of transformer to \( \Delta K_{pq} \) is expressed in the following form:

\[
\pi = P_1 + P_2 + P_3 + ... + P_n + \Delta P_{pq} + \Delta P_{qp}
\]  \( \Delta P_{pq} \) \( \Delta P_{qp} \)

At each step of iterative optimization process the objective function (14) is minimized taking into account the constraints (2) - (9).

Solution of the obtained optimization problem can be reduced to finding the transfor mation ratio that provides the minimum of function

\[
F = \pi + PF + \sum_{I \in G + N} \mu_i W_i' + \sum_{I \in G + N \Rightarrow \tilde{G}} \tilde{\mu}_i W_i' \]

where:

\[
F = \pi + PF_0 + \sum_{I \in \tilde{G}} PF_{Q_i} + \sum_{I \in G + N} PF_{V_i} + \sum_{I \in L_p} PF_{p1} + \sum_{I \in L_l} PF_{l1}
\]

total penalty function taking into account constraints (4), (5), (6), (8) and (9), respectively; \( \mu_i, \tilde{\mu}_i \) are indefinite Lagrange multipliers.

At the point of optimal solution, where the function \( F \) has a minimum value, the first partial derivatives with respect to all variables are equal to zero. In principle, the joint solution of the resulting system of nonlinear equations allows you to simultaneously determine the optimal transformation ratios. However, the solution of such system, taking into account a large number of simple and functional constraints in the form of inequalities, as well as variables of different physical nature for complex electrical networks, is a difficult problem. Moreover, the convergence of the iterative process is unreliable in this calculation.

The proposed algorithm provides for solving the resulting system of nonlinear equa- tions by dividing into several systems. The solution of the first system of equations obtained from the conditions \( \partial F / \partial l = 0, \partial F / \partial \lambda = 0, \) in practice, is the calculation of the steady state of the electrical network. Then, by solving the system of equations ob- tained from the conditions \( \partial F / \partial \lambda = 0, \partial F / \partial V_i = 0, \) the indefinite Lagrange multipliers \( \mu_i, \tilde{\mu}_i \) are determined. Based on the results obtained for the successive solution of these two systems, the components of the gradient of function (15) \( \partial F / \partial \Delta K \), \( \partial F / \partial \Delta K \) are determined. Formulas for calculation the components of optimal complex transformation ratio of transformer in branch \( l \) at the \( k \)th iteration are as follows:
\[ K'(k) = K'(k-1) + h'(k) \frac{\partial F(k-1)}{\partial \Delta K'}, \]  
(16)  
\[ K''(k) = K''(k-1) + h''(k) \frac{\partial F(k-1)}{\partial \Delta K''}, \]  
(17)  
where, \( h'(k), h''(k) \), are optimization steps determined at each iteration by the condition (for example, for \( h'(k) \))

\[ h'(k) = \lambda_1 h'(k-1) \text{ if } \frac{\partial F(k-1)}{\partial \Delta K'} \frac{\partial F(k-2)}{\partial \Delta K'} > 0, \]
\[ h'(k) = \lambda_2 h'(k-1) \text{ if } \frac{\partial F(k-1)}{\partial \Delta K'} \frac{\partial F(k-2)}{\partial \Delta K'} < 0. \]

In order to ensure reliable convergence of the iterative process, the coefficients \( \lambda_1 \) and \( \lambda_2 \) are selected within \( 0 < \lambda_1 < 1, 1 < \lambda_2 < 2 \).

The described algorithm can also be effectively used in conditions of probabilistic nature and partial uncertainty of initial information based on application of techniques described in [19, 20].

3. Results and Discussion

![Diagram of the electrical network](image)

**Table 1. Parameters of initial mode of the electrical network.**

| Number of node | \( V_i, kV \) | \( \delta_i, \text{rad.} \) | \( P_i, MW \) | \( Q_i, \text{MVAR} \) |
|---------------|-------------|-----------------|-------|------------|
| 1             | 517.02      | -0.0181         | -34.15 | -18.00    |
| 2             | 236.88      | -0.0099         | -255.00 | -5.00     |
| 3             | 214.58      | -0.1429         | 120.00 | 90.00     |
| 4             | 217.07      | -0.1505         | 11.00  | 5.00      |
| 5             | 222.08      | -0.1563         | 70.00  | 0.00      |
| 6             | 497.94      | -0.0872         | 0.00   | 0.00      |
| 7             | 497.08      | -0.0513         | -550.00 | -140.00  |
| 8             | 227.37      | -0.0933         | 250.00 | 25.00     |
| 9             | 22305       | -0.1555         | 35.00  | 35.00     |
| 10            | 222.12      | -0.2503         | -150.00 | -80.00   |
| 11            | 220.11      | -0.2618         | 170.00 | 100.00    |
| 12            | 210.56      | -0.2901         | 180.00 | 70.00     |
| 13            | 214.77      | -0.2096         | 280.00 | 150.00    |
| 14            | 520.00      | 0.00            | -145.82 | -33.96   |

Total active power losses: \( \pi = 18.972 \text{ MW} \)

Power flows through transformer branches: \( \dot{S}_{13} = -117.48 + j57.01 \text{ MVA}, \dot{S}_{24} = 355.97 + j39.78 \text{ MVA} \)
The efficiency of the described algorithm was studied using the example of optimization of electrical network mode, the diagram of which is shown in Figure 2, on transformation ratios of transformers in branches 1-2 and 7-8. The parameters of the equivalent circuit of the electric network are given in [21, 22].

To compare the results and evaluate the effectiveness of optimization on transformation ratios of transformers, Table 1 shows the initial operating parameters.

To assess the dependence of the optimal power flows in closed networks with transformation ratios, we first considered the optimization problem only for the transformation ratio of the branch 1-2. The optimal power flow along this branch is determined by the simple selection method. For this, an opening was performed in this branch, and in the new nodes obtained, were supplied power flows as a load and generation as in Figure 3.

![Figure 3. Representation of the transformer branch to determine the optimal power flow](image)

The optimal power flow \( \hat{S}_{12}^{\text{opt}} = P_{12}^{\text{opt}} + jQ_{12}^{\text{opt}} \), at which the minimum active power losses in the electrical network are ensured, is determined by the coordinate descent method, i.e. by simple alternating variation of its components, until the lowest total active power losses are obtained. As a result of this calculation, the optimal power flow \( \hat{S}_{12}^{\text{opt}} = P_{12}^{\text{opt}} + jQ_{12}^{\text{opt}} = -156.66 + j78.53 \) MVA was obtained, at which the total losses of active power is \( \pi = 17.376 \) MW. Table 2 shows the results of optimization of the electrical network mode on transformation ratio of transformer in branch 1-2 by the algorithm given in this work.

| Table 2. Optimization results by transformation ratio of transformer branch 1-2 |
|---------------------------------|------|----------------|
| \( K_{12}^{\text{opt}} = K_{12}^{\text{opt}} + jK_{12}^{\text{opt}} \) | \( \pi \), MW | \( \hat{S}_{12} = P_{12} + jQ_{12} \), MVA |
| 2.052+j0.251 | 17.376 | -156.66+j78.53 |

Comparison the results of optimization of power flow and transformation ratio of transformer in branch 1-2 (Table 2) gives reason to conclude that the optimal transformation ratio of a branch transformer in a closed network will ensure the minimum total losses of active power by creating an optimal EMF in circuit and the corresponding optimal power flows.

The paper also explored the possibilities of optimization the electrical network mode by transformation ratio of transformer in branch 7-8 and simultaneously by the transformation ratios of transformers in both branches. Table 3 shows the results of these optimization calculations.

Comparing the results shown in Table 3, we will make sure that with increasing of number of regulated transformers in electrical networks, the possibility of further reducing the total losses of active power is increases.

In the absence of a phase-rotary transformer in the branch, the transformation ratio is a real number. In such conditions, according to the algorithm described above, only the real components of
transformation ratios $K'_l$ are optimized at $K''_l = 0$. Table 4 shows the results of optimization of modes of considered electrical network on real transformation ratios of transformers of individual branches and simultaneously of both branches.

Table 3. Optimization results on transformation ratios of transformers of individual branches.

| Optimized transformation ratios | $K'_{12}^{\text{opt}}$ = $K'_{12}^{\text{opt}}$ + j$K''_{12}^{\text{opt}}$ | $K'_{78}^{\text{opt}}$ = $K'_{78}^{\text{opt}}$ + j$K''_{78}^{\text{opt}}$ | $\pi$, MW |
|-------------------------------|---------------------------------|---------------------------------|------------|
| $K'_{78}$                     | 2.174                           | 1.980 + j0.056                  | 18.352     |
| $K'_{12}$ and $K'_{78}$       | 2.053 + j0.255                  | 1.98 + j0.078                   | 16.740     |

Table 4. Optimization results by real transformation ratios of transformers.

| Optimized transformation ratios | $K'_{12}$ | $K'_{78}$ | $\pi$, MW |
|-------------------------------|-----------|-----------|------------|
| $K'_{12}$                     | 2.066     | 2.174     | 18.721     |
| $K'_{78}$                     | 2.174     | 1.978     | 18.397     |
| $K'_{12}$ and $K'_{78}$       | 2.066     | 1.978     | 18.155     |

4. Conclusions
1. The algorithm of optimization of modes of electrical networks by transformation ratios of controlled transformers is proposed.
2. Optimization of transformation ratios of transformers in the branches of closed electrical networks, in general case, can significantly reduce the loss of active power and energy by ensuring the optimal power flow distribution.
3. The distribution of power flows in closed electrical networks strongly depends on the phase rotation of transformers. Therefore, to achieve a significant reduction in energy losses in such networks, it is recommended to install phase-rotary transformers.
4. The proposed algorithm can be effectively used by dispatching services in planning of short-term modes of electrical networks.

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