MSSM with Soft SUSY Breaking Terms from $D7$–Branes with Fluxes

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Abstract

We discuss the structure of the soft supersymmetry breaking terms in a MSSM like model, which can be derived from $D7$–branes with chiral matter fields from 2–form $f$–fluxes and supersymmetry breaking from 3-form $G$–fluxes.
1. Introduction

Whether the minimal supersymmetric standard model (MSSM) or some of its ramifications will be experimentally discovered at the LHC is of burning interest also for theoretical particle physics. In the MSSM, supersymmetry breaking is usually parametrized by a set of soft SUSY breaking parameters, like gaugino, squark and slepton masses, which have the virtue that they do not spoil the good renormalization behaviour of supersymmetric field theories. But the MSSM does not offer any deeper microscopic explanation of the origin of the soft SUSY breaking parameters. Nevertheless there are some phenomenological constraints on the structure of the soft terms, e.g. the absence of flavor changing neutral currents strongly favors squark masses, which are universal for all squark flavors.

As is well known, a controllable way to obtain the soft supersymmetry breaking terms of the MSSM is provided by coupling the matter sector of the MSSM to local N=1 supergravity. Then spontaneous supersymmetry breaking by non-vanishing $F$– or $D$–terms induces soft supersymmetry breaking terms in the matter field action. Superstring theory offers a concrete, microscopic realization of soft SUSY breaking in N=1 supergravity: the effective low energy action of supersymmetric string compactifications to four space-time dimensions is given by the N=1 supergravity action of $[1]$. Furthermore, spontaneous supersymmetry breaking is due to $F$–terms of the gauge singlet scalar fields, namely the dilaton $S$ or the geometric moduli $M$, whose $F$–terms are called $F^S$ and $F^M$, respectively. Then supersymmetry breaking is transmitted from the gauge neutral sector to the charged sector of the MSSM by gravitational interactions. This scenario already allows for a fairly model independent analysis of the soft terms, which are all proportional to certain combinations of $F^S$ or $F^M$ $[2]$. In particular, the dilaton dominated scenario with $F^S \neq 0$, $F^M = 0$ possesses the feature of flavor universal soft scalar masses, which is usually spoiled by non-vanishing vevs for $F^M$. In more generic scenarios, in which both $F^S \neq 0$ and $F^M \neq 0$, the soft SUSY breaking terms can be nicely parametrized by a so-called goldstino angle $\tan \theta_g \sim F^S/F^T$ $[3]$, where $T$ is the overall volume modulus of the internal space.

The final step for a complete understanding of the soft-terms is undertaken by knowing (i) how the matter sector of the MSSM is microscopically built in string theory, and (ii) how the supersymmetry breaking auxiliary fields $F^S$, $F^M$ are induced, i.e. by knowing how a non-trivial effective superpotential for the fields $S$ and $M$ is generated. In this paper we are interested in compactifications of the type I strings, namely the so-called orientifold compactifications of the type IIA/B superstring. Let us first recall how point (i) can be realized in orientifold compactifications. Namely one very promising way to build the MSSM is to use intersecting D6–branes in type IIA orientifolds (for a review see $[4]$). The gauge degrees of freedom are due to open strings living on each of the various stacks of D6–branes, whereas the chiral matter fields are localized on the lower-dimensional intersection loci of
the $D6$–branes. More specifically, the $D6$–branes, all completely filling four-dimensional Minkowski space-time, are wrapped around supersymmetric 3-cycles in the internal space $X_6$, which generically intersect just on points in $X_6$. Note that the internal intersection numbers are normally larger than one, a fact, which offers a nice explanation for the family replication of the MSSM. In order to preserve $N=1$ space-time supersymmetry in the open string sectors on the intersecting $D$–branes, the intersection angles must obey certain conditions, and for consistent model building, all Ramond tadpoles must be cancelled.

Starting from the original work on non-supersymmetric models [5–9], several semirealistic MSSM-like models with intersecting $D6$–branes were constructed during the last years [10–13]. However, for practical reasons, when turning on the SUSY-breaking 3-form fluxes (see later), it is more convenient to use instead of the type $IIA$ orientifolds with intersecting $D6$–branes the mirror ($T$–dual) type $IIB$ orientifold description. Then, after an appropriate mirror transformation, the (supersymmetric) $D6$–branes are transformed into a system of $D3$–branes plus supersymmetric $D7$–branes, where the non-trivial intersection angles in type $IIA$ become open string 2-form gauge fluxes (magnetic $f$-field background fields) living on internal 4-cycles on the different $D7$–brane world volumes. Note that $f$–fluxes are required at least on some of the various stacks of $D7$–branes in order to obtain realistic models with more than one chiral generation of quarks and leptons. Hence for getting chiral fermions, some of the $D7$–branes possess mixed Dirichlet/Neumann boundary conditions in certain internal directions, which means that they are a kind of hybrid between $D3$– and $D7$–branes. This fact will become important for the structure of the soft terms for the matter fields on the $D3/D7$–brane world volumes.

Now coming to the second issue (ii) of spontaneous supersymmetry breaking we will consider the generation of an effective superpotential [14–17] for the dilaton $S$ and the moduli fields $M$ by flux compactifications [18,19] with non-vanishing, internal fluxes of the type $IIB$ 3-form $G_3 = F_3 - SH_3$, where $F_3$ and $H_3$ are the field strengths of the Ramond and the Neveu–Schwarz 2-form gauge potentials $C_R^2, B_{NS}^2$, respectively. As it was shown in [20,21], the 3-form fluxes in general contribute to the tadpole conditions, but still preserve $N=1$ supersymmetry, i.e. $F^S = F^M = 0$ in the vacuum, if $G_3$ is imaginary self-dual ($ISD$ flux) and of Hodge type $(2,1)$ on the internal Calabi-Yau space. However all complex structure moduli $U^i$ as well as the dilaton field $S$ are already fixed in a generic supersymmetric flux vacuum. On the other hand, if $G_3$ is an $ISD$ $(0,3)$-form, it corresponds to a non-vanishing auxiliary field $F^T$ of the overall Kähler modulus of $X_6$, and supersymmetry is spontaneously broken; if $G_3$ is an imaginary anti-self dual ($IASD$) $(3,0)$-form it is equivalent to an auxiliary field $F^S$; finally if $G_3$ contains some of the $IASD$ $(1,2)$ forms, this is described by non-vanishing auxiliary fields $F^{U^i}$ for the complex structure moduli $U^i$ in the effective field theory description. In the following, we will mainly concentrate on the two cases of $ISD$ $(0,3)$–flux and/or $IASD$ $(3,0)$–fluxes, since these are the generic 3-form fluxes for all $\mathbb{Z}_N \times \mathbb{Z}_M$ orientifold compactifications.
In order to derive the soft SUSY breaking parameters, one has to compute the couplings between the open string matter fields on the $D3/D7$–branes and the closed string 3–form field strengths $G_3$. Explicit type IIB orientifold models with ISD-fluxes have been already constructed in \[21,23,24\], with chirality in \[25,26,27\]. Recently, MSSM–like flux models with $D3/D7$–branes (including magnetized $D9$–branes) and complete cancellation of both $R$– and $NS$–tadpoles have been constructed in Ref. \[28\] (see also Ref. \[29\]). The four-dimensional $N=1$ effective action of orientifolds with $D3$– and/or $D7$-branes can be obtained by the calculation of the open/closed string scattering amplitudes \[30\] or by Kaluza-Klein reduction of the Dirac-Born-Infeld and Chern-Simons action \[31,32\]. Then the soft SUSY breaking terms can be derived either by studying the Born-Infeld action on the $D$–brane world volumes coupled to the flux $G_3$ as accomplished for $D3$–branes in \[33,34\], and for $D7$–branes in \[35,36\], or by coupling the effective action from open/closed string scattering amplitudes to the effective closed string action with 3–form fluxes turned on, as it was performed for $D3$– and $D7$–branes in \[27\] (see also the discussion of soft terms in intersecting brane world models in Ref. \[37\]). Note that in Ref. \[27\] also the open string 2–form $f$–flux on the $D7$–branes has been taken into account, which is crucial for realistic model building with chiral fermions. In any case, the results of the two different approaches \[27\] and \[33,36\] are completely consistent with each other and lead to identical results for vanishing $f$–flux. The results can be summarized as follows:

- **gaugino masses**: Since for $D3$–branes the gauge kinetic function is given as $f \sim S$, the $D3$–brane gaugino masses are sensitive to non-vanishing $ISD(3,0)$-flux with $F^S \neq 0$, but still vanish for non-trivial $ISD(0,3)$-flux. This situation is reversed for $D7$–branes with zero $f$–flux: their gauge kinetic function is proportional to the transversal Kähler moduli $T^i$, $f_i \sim T^i$, and hence the gaugino masses only feel the $ISD(0,3)$-flux with $F^T \neq 0$. So the role of $D3$–branes and pure $D7$–branes is reversed. Note however that for $D7$–branes with non-vanishing $f$–flux, i.e. with mixed $D/N$–boundary conditions, the gauge kinetic function contains both the dilaton $S$ as well as the Kähler moduli $T^i$. Therefore the corresponding gaugino masses get contributions both from the $(0,3)$ and also from the $(3,0)$-flux, as it will happen in realistic models with three chiral generations. Hence, for the $D7$–branes with mixed boundary conditions it will be convenient to parametrize the gaugino masses by the goldstino angle $\sin \theta_g$.

- **scalar masses**: the soft scalar masses follow a similar pattern as compared to the gaugino masses. For the scalars living on the $D3$–branes, a mass is only generated by the $(3,0)$-flux, while scalars on pure $D7$–branes get their masses partly also from $(0,3)$-flux. On the other hand, scalars on $D7$–branes with $f$–fluxes get mass contributions both from $(3,0)$- and $(0,3)$-fluxes. Most importantly, ‘chiral’ scalar fields, which correspond to twisted open string sectors, *i.e.* open strings which stretch between two $D7$–branes with different type of $f$–flux boundary conditions, get also masses from $(3,0)$- as well as from $(0,3)$-fluxes.
The outline of our work is the following: In the next section we shall recall the general structure of the 3-form flux induced soft terms for $D7$–branes with $f$–flux, following our previous work in Ref. [27]. In section three, we shall make an attempt to gather some generic information on the structure of the soft terms in MSSM–like orientifold constructions. Here our strategy is the following. Instead of considering compact models which satisfy all tadpole conditions, we will rather consider a locally supersymmetric $D7$–brane set-up, which precisely contains the open string matter fields with three generation MSSM quantum numbers. Specifically, a minimal way to build the MSSM via three stacks of intersecting $D6$–branes on a six-torus $T^6$ (or also on an orbifold) was proposed in [38]. We will use this type IIA setup, perform the mirror transformation to type IIB and will derive the equivalent brane configuration. The latter now consists of three different stacks of $D7$–branes, one being equipped with non-trivial open string 2-form $f$–flux. Via this rather simple construction we can finally compute all relevant soft terms. In section 4 we shall parametrize our results by the goldstino angle, while in section 5 we discuss the scales of the gravitino mass and soft–masses induced by non–vanishing $(0,3)$ and $(3,0)$–form fluxes. Finally, in section 6 we give some concluding remarks.

2. Soft terms for $D7$–branes with $f$–flux

In this chapter, we will recall the general structure of the soft terms for matter fields originating from $D3$– or $D7$–branes with $f$–fluxes. We will follow the approach of Ref. [27], where these terms have been determined by computing the effective action for the open string matter fields by a direct calculation of string scattering amplitudes, and subsequently coupling the matter fields to the 3–form flux induced superpotential. An alternative derivation of the soft terms for $D7$–branes without $f$–flux using the Born–Infeld action can be found in [35].

2.1. Three–form $G$–flux

Let us start by reviewing the main aspects of type $IIB$ 3-form fluxes and the corresponding superpotential. We concentrate on orientifolds of type $IIB$ compactified on the toroidal orbifold

$$X_6 = \frac{T^6}{Z_N \times Z_M}, \quad (2.1)$$

with the orbifold group $\Gamma = \mathbb{Z}_N \times \mathbb{Z}_M$. There are $h_{(1,1)}(X_6)$ Kähler moduli and $h_{(2,1)}(X_6)$ complex structure moduli, which split into twisted and untwisted moduli. In the following the dimension of the latter is denoted by $h^{\text{untw}}_{(1,1)}(X_6)$ and $h^{\text{untw}}_{(2,1)}(X_6)$, respectively. In addition, there is the complex dilaton field $S$. To obtain an $N=1$ (closed) string spectrum, one introduces an orientifold projection $\Omega_n$, with $\Omega$ describing a reversal of the orientation
of the closed string world–sheet and $I_n$ a reflection of $n$ internal coordinates. For $\Omega I_n$ to represent a symmetry of the original theory, $n$ has to be an even integer in type $IIB$. Generically, this projection produces orientifold fixed planes $[O(9-n)–\text{planes}]$, placed at the orbifold fixpoints of $T^6/I_n$. They have negative tension, which has to be balanced by introducing positive tension objects. Candidates for the latter may be collections of $D(9-n)$–branes and/or non–vanishing three–form fluxes $H_3$ and $C_3$. The orbifold group $\Gamma$ mixes with the orientifold group $\Omega I_n$. As a result, if the group $\Gamma$ contains $Z_2$–elements $\theta$, which leave one complex plane fixed, we obtain additional $O(9-|n-4|)$– or $O(3+|n-2|)$– planes from the element $\Omega I_n \theta$.

In the following, only the two cases of $n = 6$ ($O3$–plane) and $n = 2$ ($O7$–planes) will be relevant to us. Tadpoles may be completley cancelled by adding $D3$– or $D7$–branes, provided the orbifold twist $\Gamma$ is $Z_2 \times Z_2$, $Z_2 \times Z_3$, $Z_2 \times Z_6$, $Z_3 \times Z_3$, $Z_6 - I$, $Z_6 - I I$, $Z_3 \times Z_6$, $Z_6 \times Z_6$, $Z_7$ or $Z_{12} - I$ \[39,40\]. This is to be contrasted with type $IIA$ intersecting $D6$–brane constructions, where it has been recently shown that essentially all orbifold groups $\Gamma$ allow for tadpole cancellation due to the appearance of only untwisted and $Z_2$–twisted sector tadpoles \[11\].

Let us now give non–vanishing vevs to some of the (untwisted) flux components $H_{ijk}$ and $F_{ijk}$, with $F_3 = dC_2$, $H_3 = dB_2$. The two 3–forms $F_3, H_3$ are organized in the $SL(2,Z)_S$ covariant field:

$$G_3 = F_3 - SH_3 \ . \quad (2.2)$$

On the torus $T^6$, we would have $20+20$ independent internal components for $H_{ijk}$ and $F_{ijk}$. However, only a portion of them is invariant under the orbifold group $\Gamma$. More precisely, of the 20 complex (untwisted) components comprising the flux $G_3$, only $2h_{\text{untw.}}^{(2,1)}(X_6) + 2$ survive the orbifold twist. The orientifold action $\Omega(-1)^{F_L} I_6$ producing $O3$–planes does not give rise to any further restrictions. If the orbifold group $\Gamma$ contains $Z_2$–elements $\theta$, which leave the $j$–th complex plane fixed, we also encounter $O7_j$–planes transverse to the $j$–th plane. Since $I_2^J = I_6 \theta$, the orientifold generator $\Omega(-1)^{F_L} I_2^J$ does not put further restrictions on the $2h_{\text{untw.}}^{(2,1)}(X_6) + 2$ twist invariant components. Hence, the allowed flux components are most conveniently found in the complex basis, in which the orbifold group $\Gamma$ acts diagonally. In the following we shall concentrate\[4 on the type $IIB$ orientifold/orbifolds $T^6/(\Gamma + \Gamma \Omega I_6)$, with $\Gamma$ being one of the (consistent) orbifold twists encountered above. Note, that $O7$–planes appear, in the case, that the orbifold twist $\Gamma$ is of even order.

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\[ As an example, we may take the orientifold with orbifold group $\Gamma = Z_2 \times Z_2$, discussed in Ref. \[27\]. For this compactification we have $h_{\text{untw.}}^{(2,1)}(X_6) = 3$. Hence we have $8 + 8$ untwisted flux components $H_{ijk}$ and $F_{ijk}$. 

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5
The most general (untwisted) 3–form flux $G_3$ may be written as linear combination of the complex cohomology group $H^3(X_6, \mathbb{C})$

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = \sum_{i=0}^{3} (A^i \omega_{A_i} + B^i \omega_{B_i}) ,$$

with a basis of $H^3 = H^{(3,0)} \oplus H^{(2,1)} \oplus H^{(1,2)} \oplus H^{(0,3)}$:

$$\omega_{A_0} = dz^1 \wedge dz^2 \wedge dz^3, \quad \omega_{A_1} = dz^1 \wedge dz^2 \wedge \bar{dz}^3,$$
$$\omega_{A_2} = dz^1 \wedge \bar{dz}^2 \wedge dz^3, \quad \omega_{A_3} = dz^1 \wedge \bar{dz}^2 \wedge \bar{dz}^3,$$
$$\omega_{B_0} = d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3, \quad \omega_{B_1} = d\bar{z}^1 \wedge d\bar{z}^2 \wedge dz^3,$$
$$\omega_{B_2} = d\bar{z}^1 \wedge d\bar{z}^2 \wedge \bar{dz}^3, \quad \omega_{B_3} = d\bar{z}^1 \wedge d\bar{z}^2 \wedge dz^3 .$$

The expansion (2.3) is to be understood such, that according to the discussion from above only the twist invariant 3–forms contribute in the sum. In the form (2.4) the cohomology structure of $G_3$ is manifest, but the $SL(2, \mathbb{Z})_S$–covariance is not. The form $\omega_{A_0}$ corresponds to the $(0,3)$–part of the flux, the $\omega_{A_i}$, $i = 1, 2, 3$, correspond to the $(2,1)$–part, $\omega_{B_0}$ comprises the $(3,0)$–part and the $\omega_{B_i}$, $i = 1, 2, 3$ the $(3,0)$–part. All twist–invariant fluxes fulfill the primitivity condition $G_3 \wedge J = 0$, with $J$ the Kähler form (for more details we refer to Ref. [42]).

In order to impose flux quantization on $G_3$, one has to transform the forms (2.4) into a real basis of 3–forms $H^3(T^6, \mathbb{Z})$ on $T^6$:

$$\alpha_0 = dx^1 \wedge dx^2 \wedge dx^3 , \quad \beta^0 = dy^1 \wedge dy^2 \wedge dy^3 ,$$
$$\alpha_1 = dy^1 \wedge dx^2 \wedge dx^3 , \quad \beta^1 = -dx^1 \wedge dy^2 \wedge dy^3 ,$$
$$\alpha_2 = dx^1 \wedge dy^2 \wedge dx^3 , \quad \beta^2 = -dy^1 \wedge dx^2 \wedge dy^3 ,$$
$$\alpha_3 = dx^1 \wedge dx^2 \wedge dy^3 , \quad \beta^3 = -dy^1 \wedge dy^2 \wedge dx^3 .$$

with the six real periodic coordinates $x^i, y^i$ on the torus $T^6$, i.e. $x^i \cong x^i + 1$ and $y^i \cong y^i + 1$. This is achieved through introducing complex structures:

$$dz^j = \sum_{i=1}^{3} \rho_i^j \ dx^i + \tau_i^j \ dy^i , \quad j = 1, 2, 3 .$$

Most of the parameters $\rho_i^j$ and $\tau_i^j$ are fixed through the orbifold twist $\Gamma$, with only those remaining undetermined, which correspond to the $\mathbb{Z}_2$–elements of $\Gamma$. The latter are eventually fixed through the flux quantization condition. For further details see Ref. [12]. The
basis (2.3) has the property \( \int_{X_6} \alpha_i \wedge \beta^j = \delta^j_i \). Expressed in this basis, the \( G_3 \)-flux (2.2) takes the following form:

\[
\frac{1}{(2\pi)^2} \alpha^j \gamma^i \alpha^i G_3 = \frac{3}{(2\pi)^2} \sum_{i=0}^{3} [(a^i - Sc^i) \alpha^i + (b_i - Sd_i) \beta^i].
\] (2.7)

In this basis, the \( SL(2, \mathbb{Z}) \)-covariance of \( G_3 \) is manifest. The coefficients \( a^i, b_i \) refer to the Ramond part of \( G_3 \), whereas the coefficients \( c^i, d_i \) refer to the Neveu-Schwarz part.

As described above, not all of the eight flux components in (2.7) or (2.3) survive the orbifold projection. In addition, some or all complex structure moduli are frozen to discrete values by the \( \mathbb{Z}_N \times \mathbb{Z}_M \) modding (see Ref. [42] for more details). On the other hand, in the \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold all eight flux components survive and all three complex structure moduli \( U_j \), \( j = 1, 2, 3 \) remain unfixed. However e.g. in the \( \mathbb{Z}_3 \) orbifold, only the components \( G_{(3,0)} \) and \( G_{(0,3)} \) are allowed, and all \( U^i \) are frozen to \( U^i = \rho := \frac{1}{2} + \frac{i}{2} \sqrt{3} \). Only the IASD–flux \( G_{(3,0)} \) and the ISD–flux \( G_{(0,3)} \) are generic flux components being invariant under all possible orbifold groups [42]. Let us remark, that due to the absence of the ISD (2, 1) 3–form fluxes in most of the \( \mathbb{Z}_N \)–orbifold models, supersymmetric flux solutions do not exist for these cases. Therefore, we shall concentrate in the following discussion on these two complex fluxes, which are parametrized by four real coefficients. Expressed in terms of the complex basis (2.3), the \( G_{(3,0)} \) and \( G_{(0,3)} \) fluxes take the following form:

\[
\frac{1}{(2\pi)^2} \alpha^j \gamma^i \alpha^i G_{03} = A_0 \omega_{A0} = A_0 \left( dz^1 \wedge dz^2 \wedge dz^3 \right),
\]

\[
\frac{1}{(2\pi)^2} \alpha^j \gamma^i \alpha^i G_{30} = B_0 \omega_{B0} = B_0 \left( d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3 \right).
\] (2.8)

2.2. Closed string low–energy effective action

Now we shall consider the low-energy effective action of the closed string moduli fields \( M \) for non–vanishing \( G_{(3,0)} \)– or \( G_{(0,3)} \)–flux, bearing in mind that some or all of the complex structure moduli may be frozen to specific values in many of the orbifold compactifications. Here, \( M \) collectively accounts for the closed string moduli fields \( S, T^j, U^j \). The kinetic energy terms of these bulk fields are derived from the Kähler potential \( \hat{K} \) given by [43,30]:

\[
\kappa_4^2 \hat{K}(M, \overline{M}) = - \ln(S - \overline{S}) - \sum_{j=1}^{h^{un-tw.}_{1(1,1)}} \ln(T^j - \overline{T}^j) - \sum_{j=1}^{h^{un-tw.}_{2(2,1)}} \ln(U^j - \overline{U}^j).
\] (2.9)

The moduli fields \( M \) refer to complex scalars of N=1 chiral multiplets. These fields \( M \) have a functional dependence on the moduli fields \( \mathcal{M} \) one uses in string–theory. The latter, which will be introduced in Ref. [42] through their geometric meaning refer to the vertex operators following from the \( \sigma \)–model and are used to study duality symmetries.

7
For toroidal type IIB orientifolds with D3– and D7–branes and \( h_{(1,1)}^{\text{untw.}} = 3 \) we have the following relations

\[
T^j = a^j + i \frac{e^{-\phi_4}}{2\pi \alpha'/2} \sqrt{\frac{\text{Im} T^k \text{Im} T^l}{\text{Im} T^j}},
\]

\[
S = C_0 + i \frac{e^{-\phi_4}}{2\pi \sqrt{\text{Im} T^1 \text{Im} T^2 \text{Im} T^3}},
\]

\[
U^j = \mathcal{U}^j, \quad j = 1, 2, 3,
\]

with the geometric (untwisted) Kähler moduli \( T^j \) and complex structure moduli \( U^j \) to be specified in [42]. Here the axion follows from integrating the Ramond 4–form over a 4–cycle: \( a^j = \int_{T^2, k \times T^2, l} C_4 \). The Kähler potential (2.9) is quite generic for all type II toroidal orbifolds, which essentially only differ by their number \( h_{(1,1)}^{\text{untw.}} \) of Kähler \( T^j \) and their number \( h_{(2,1)}^{\text{untw.}} \) of complex structure moduli \( U^j \).

The effective superpotential \( \widehat{W} \) arising for non-vanishing 3-form fluxes takes the following form [13]:

\[
\widehat{W} = \frac{\lambda}{(2\pi)^2 \alpha'} \int_{X_6} G_3 \wedge \Omega,
\]

where \( \lambda \) serves to fix the mass dimension to the correct value of 3. The superpotential gives rise to the standard \( F \)–term scalar potential of N=1 supergravity:

\[
\widehat{V} = \hat{K}_{T^6} F^T \hat{F} - 3 \text{e}^{\kappa_2 \hat{K}} \kappa_4^2 |\hat{W}|^2.
\]

This is positive semidefinite since the negative contribution in \( \widehat{V} \) is cancelled by the contribution of the Kähler moduli \( T^i \). The integral (2.11) has been worked out for the orbifold compactifications \( X_6 \) we are discussing here in Ref. [27]:

\[
\frac{1}{\lambda} \widehat{W} = (a^0 - Sc^0) U^1 U^2 U^3 - \left\{ (a^1 - Sc^1) U^2 U^3 + (a^2 - Sc^2) U^1 U^3 \right\} + (a^3 - Sc^3) U^1 U^2 - \sum_{i=1}^3 (b_i - Sd_i) U^i - (b_0 - Sd_0).
\]

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2 Note, that \( h_{(1,1)}^{\text{untw.}} = 3 \) for almost all \( \mathbb{Z}_N \times \mathbb{Z}_M \)–orbifolds, except: \( h_{(1,1)}^{\text{untw.}} (T^6/\mathbb{Z}_3) = 9 \) and \( h_{(1,1)}^{\text{untw.}} (T^6/\mathbb{Z}_6 - I) = 5. \) In these two special cases the Kähler potential (2.10) describes only the three diagonal Kähler moduli. Furthermore, \( h_{(2,1)}^{\text{untw.}} \leq 1, \) except \( h_{(2,1)}^{\text{untw.}} (T^6/\mathbb{Z}_2 \times \mathbb{Z}_2) = 3. \) For further information see Ref. [12].
For our purposes, only the supersymmetry breaking $F$–terms $F^S$ and $F^T^i$, which are proportional to $G_{(3,0)}$ or $G_{(0,3)}$, respectively are relevant [27]:

$$F^S = \left( S - \overline{S} \right)^{1/2} \prod_{i=1}^{3} (T^i - \overline{T}^i)^{-1/2} \prod_{i=1}^{3} (U^i - \overline{U}^i)^{-1/2} \frac{\kappa_2}{(2\pi)^{2}\alpha'} \lambda \int T_3 \wedge \Omega$$

$$= \lambda \kappa_4^2 \left( S - \overline{S} \right)^{1/2} \prod_{i=1}^{3} (T^i - \overline{T}^i)^{-1/2} \prod_{i=1}^{3} (U^i - \overline{U}^i)^{-1/2}$$

$$\times \{ (a^0 - \overline{S}c^0) U^1 U^2 U^3 - [(a^1 - \overline{S}c^1) U^2 U^3 + (a^2 - \overline{S}c^2) U^1 U^3$$

$$+ (a^3 - \overline{S}c^3) U^1 U^2] - \sum_{i=1}^{3} (b_i - \overline{S}d_i) U^i - (b_0 - \overline{S}d_0) \} ,$$

$$F^T^i = \left( S - \overline{S} \right)^{-1/2} (T^i - \overline{T}^i)^{1/2} (T^j - \overline{T}^j)^{-1/2} \prod_{j=1}^{3} (U^j - \overline{U}^j)^{-1/2} \frac{\kappa_4^2}{(2\pi)^{2}\alpha'} \lambda \overline{W} .$$

(2.14)

2.3. $D7$–branes with f–flux

Now we will include $D7$–branes together with their open string sectors. To obtain a chiral spectrum, we must introduce (magnetic) two–form fluxes $F^j dx^j \wedge dy^j$ on the internal part of the $D7$–brane world volume. Together with the internal $NS$ $B$–field $b^j$, we have the complete 2–form flux $F = \sum_{j=1}^{3} F^j := \sum_{j=1}^{3} (b^j + 2\pi \alpha' F^j) dx^j \wedge dy^j$. The latter gives rise to the total internal antisymmetric background

$$\begin{pmatrix} 0 & f^j \\ -f^j & 0 \end{pmatrix} , \quad f^j = \frac{1}{(2\pi)^2} \int_{T^2,j} F^j ,$$

w.r.t. the $j$–th internal plane. The 2–form fluxes $F^j$ have to obey the quantization rule:

$$m^j \frac{1}{(2\pi)^2 \alpha'} \int_{T^2,j} F^j = n^j , \quad n \in \mathbb{Z} ,$$

(2.15)

i.e. $f^j = \alpha' \frac{n^j}{m^j}$. This setup is $T$–dual to intersecting $D6$–branes in type $IIA$ orientifold compactifications. In a compact model, all tadpoles arising from the Ramond forms $C^4$ and $C_8$ must be cancelled by the $D$–branes or/and by the 3–form fluxes. More concretely, the cancellation condition for the tadpole arising from the $RR$ 4–form $C_4$ is

$$N_{flux} + 2 \sum_{a} N_a n^1_a n^2_a n^3_a = 32 ,$$

(2.17)

where $N_{flux}$ is given by

$$N_{flux} = \frac{1}{(2\pi)^4 \alpha'^2} \int_{X_6} H_3 \wedge F_3 .$$

(2.18)
For the ISD \((0,3)\)-flux, one finds
\[
N_{\text{flux}} = 4 |A_0|^2 \geq 0,
\]
and in the case of the \((3,0)\)-flux,
\[
N_{\text{flux}} = -4 |B_0|^2 \leq 0.
\]
Furthermore, the cancellation conditions for the 8–form tadpoles yield:
\[
\begin{align*}
2 \sum_a N_a m_a^1 m_a^2 n_a^3 & = -32, \\
2 \sum_a N_a m_a^1 m_a^3 n_a^2 & = -32, \\
2 \sum_a N_a m_a^2 m_a^3 n_a^1 & = -32.
\end{align*}
\]
The requirement that a branes \(a\) with internal 2–form fluxes \(f_a^j\) is supersymmetric has the form:
\[
\sum_{j=1}^{3} \arctan \left( \frac{f_a^j}{\text{Im}(T^j)} \right) = 0.
\]
Furthermore, the condition, that branes \(a\) with 2–form fluxes \(f_a^j\) and \(b\) with 2–form fluxes \(f_b^j\) are mutually supersymmetric is
\[
\sum_{j=1}^{3} \theta_{ab}^j = 0 \mod 2,
\]
with the relative “flux” \(\theta_{ab}^j\):
\[
\theta_{ab}^j = \frac{1}{\pi} \left[ \arctan \left( \frac{f_b^j}{\text{Im}(T^j)} \right) - \arctan \left( \frac{f_a^j}{\text{Im}(T^j)} \right) \right].
\]
These conditions will fix some of the Kähler moduli \(T^j\).

Note that in the locally supersymmetric MSSM, being discussed in the next chapter, the Ramond tadpole conditions are not satisfied, and hence a model-dependent hidden sector will always be required to fulfill these conditions.

\[3\] These equations are to be understood, that the numbers \(m_a, n_a\) come in orbits of the orbifold group \(\mathbb{Z}_N\) or \(\mathbb{Z}_N \times \mathbb{Z}_M\).
2.4. Open string low-energy effective action and soft terms

The low–energy effective action for the massless open string sector of the $D3/D7$–branes was computed by calculating string scattering amplitudes among open string matter fields on the $D$-branes and bulk moduli fields \[eqref{string_scattering_amplitudes} \]. Specifically the charged matter fields $C$ enter the K"ahler potential at quadratic order as (for large K"ahler moduli, which corresponds to the supergravity approximation under consideration):

$$
K(M, \overline{M}, C, \overline{C}) = \hat{K}(M, \overline{M}) + \sum_{a} \sum_{j=1}^{3} \sum_{i=1}^{3} G_{C^7_{a,j} C^7_{a,j}}(M, \overline{M}) C^7_{a,j} \overline{C^7_{a,j}} + \sum_{a \neq b} G_{C^7_{a,b} C^7_{a,b}}(M, \overline{M}) C^7_{a,b} \overline{C^7_{a,b}} + O(C^4). \tag{2.23}
$$

Here, $\hat{K}(M, \overline{M})$ is the closed string moduli K"ahler potential \[eqref{closed_string_kahler_potential} \], discussed before. The open string moduli fields $C$ summarize both untwisted $D7$–brane moduli $C^7_{a,j}$ and twisted matter fields $C^7_{a,b}$. The fields $C^7_{a,j}$ account for the transverse $D7$–brane positions $C^7_{a,j}$ on the $j$–th subplane and for the Wilson line moduli $C^7_{a,j}$, $i \neq j$ on the $D7$–brane world volume. On the other hand, the fields $C^7_{a,b}$ represent twisted matter fields originating from strings stretched between two stacks of $D7$–branes $a$ and $b$. We have only displayed the $D7$–brane sector, as the $D3$–brane sector follows from the latter by taking the limits $f^j \to \infty$. Furthermore, the holomorphic superpotential $W$ takes the form:

$$
W(M, C) = \hat{W}(M) + \sum_{a=1}^{3} C^7_{a} C^7_{2} C^7_{3} + \sum_{a,b,j} d_{abj} C^7_{a} C^7_{a} C^7_{b} + \sum_{I,J,K} Y_{IJ,K}(U^I) C^I C^J C^K + O(C^4). \tag{2.24}
$$

Again, $\hat{W}(M)$ is the closed string superpotential \[eqref{closed_string_superpotential} \], discussed before. Finally, the coupling of the (closed string) moduli to the gauge fields is described by the gauge kinetic functions. For the gauge fields living on the $D7$–branes, wrapped around the 4–cycle $T^{2,k} \times T^{2,l}$, these functions are given by \[eqref{gauge_kinetic_functions} \]

$$
f_{D7_j}(S, T^j) = |m^k m^l| (T^j - \alpha' f^k f^l S), \quad (j, k, l) = (1, 2, 3), \tag{2.25}
$$
m^k, m^l being the wrapping numbers.

As we will see in the next chapter, the MSSM–like model entirely consists of three stacks of $D7$–branes, one being equipped with non-trivial $f$–flux. The MSSM matter fields correspond to twisted open string sectors which preserve $N=1$ supersymmetry (1/4 BPS sectors). For those two stacks of $D7$–branes $a$ and $b$, which wrap different 4–cycles, in each plane there is always a non–vanishing relative “flux” $\theta^{j}_{ab}$, given in Eq. \[eqref{theta_flux} \].
In that case the matter field Kähler metric describing a $1/4$ BPS sector is given by the following expression:

$$G_{\gamma^a \gamma^b \bar{c} \bar{c}} = \kappa_4^{-2} (S - \bar{S})^{-\frac{1}{2} + \frac{3\beta + \gamma}{2}} \prod_{j=1}^{3} (T^{j} - \bar{T}^{j})^{-\frac{1}{2} - \frac{\gamma(1-\theta^{j})}{2}} (U^{j} - \bar{U}^{j})^{-\theta^{j}} \sqrt{\frac{\Gamma(\theta^{j}_{ab})}{\Gamma(1 - \theta^{j}_{ab})}}. \quad (2.26)$$

On the other hand, for twisted open string states from the $1/2$ BPS sector, the metric takes a different form:

$$G_{\gamma^a \gamma^b \bar{c} \bar{c}} = -\kappa_4^{-2} \frac{1}{(S - \bar{S})^{1/2}(T^{1} - \bar{T}^{1})^{1/2}(U^{2} - \bar{U}^{2})^{1/2}(U^{3} - \bar{U}^{3})^{1/2}}. \quad (2.27)$$

The metric for the untwisted matter fields living on the same stack of $D7$–branes is the following:

$$G_{\gamma^i \gamma^j \bar{c} \bar{c}} = \frac{-\kappa_4^{-2}}{(U^{i} - \bar{U}^{i})(T^{k} - \bar{T}^{k})} \frac{|1 + i \bar{f}^{k}|}{|1 + if^{i}|},$$

$$G_{\gamma^i \gamma^j \bar{c} \bar{c}} = \frac{-\kappa_4^{-2}}{(U^{j} - \bar{U}^{j})(S - \bar{S})} |1 - \bar{f}^{i} \bar{f}^{k}|, \quad i \neq k \neq j. \quad (2.28)$$

Once we have calculated the corresponding Riemann tensors, we are ready to write down the scalar mass terms. For Kähler manifolds the components of the Riemann curvature tensor are given as follows:

$$R_{MNPij} = K_{C_{\gamma}C_{\gamma}MN} - K_{C_{\gamma}C_{C}k} G^{\gamma \gamma} C^{k} K_{C_{\gamma}N\gamma^{k}}. \quad (2.29)$$

For the untwisted matter fields of the stacks without $f$-flux, the curvature tensors take a particularly simple form, but as the expressions are more cumbersome for stack 1 and for the twisted case, the reader is referred to appendix A for details.

\footnote{For $\beta, \gamma = 0$ this expression agrees with the two results Eqs. (5.22) and (5.25) of [30] after transforming the latter into the Einstein frame. The latter have been extracted from a certain three–point and four–point amplitude in Type IIA. However, to completely fix the moduli dependence on $T^{i}$ ($U^{i}$ in type IIA), \textit{i.e.} to fix the constants $\beta, \gamma$, one has to calculate a four–point disk amplitude involving two twisted matter fields and two Kähler moduli $T^{i}$ (two complex structure moduli $U^{i}$ in type IIA) [44]. Note, that as in the heterotic case, these constants cannot be determined from factorizing the four–twist correlator on the disc. Moreover, results from the heterotic string suggest, that $\beta, \gamma \neq 0$. Recently, in Refs. [45] soft–terms have been calculated with assuming $\beta, \gamma = 0$.}
The general formula for the scalar mass terms is the following:

\[
\begin{align*}
(m_{N}^{7,j})^2 &= \kappa_4^2 \left[ \left( \frac{m_{3/2}}{2} + \kappa_4^2 \tilde{V} \right) G_{C_7^j,C_7^i} - \sum_{M,N} F_M \tilde{F}_N R_{M \tilde{N}}^{7,j} \right], \\
(m_{a7b}^{7})^2 &= \kappa_4^2 \left[ \left( \frac{m_{3/2}}{2} + \kappa_4^2 \tilde{V} \right) G_{C_7^a,C_7^b} - \sum_{M,N} F_M \tilde{F}_N R_{M \tilde{N}}^{7,a7b} \right],
\end{align*}
\]

(2.30)

where \( M, N \) run over \( S, T^i, U^i \). The trilinear coupling is

\[
A_{IJK} = i \prod_M (M - \overline{M})^{-1} \frac{\kappa_4^2 \lambda}{(2\pi)^2 \alpha'} \left[ Y_{IJK} \int G_3 \wedge \overline{\Omega} + 3 Y_{IJK} \int \overline{G}_3 \wedge \overline{\Omega} \\
+ \sum_i \left[ \int \overline{G}_3 \wedge \overline{\omega}_{A_i} \left( Y_{IJK} - (U^i - \overline{U}^i) \partial_{U^i} Y_{IJK} \right) \right] \\
- i \prod_M (M - \overline{M})^{-1/2} F^N G^{C_i C_1} \partial_N G_{C_i(\overline{C}_1 Y_{IJK})I} \right].
\]

(2.31)

The term \( - \sum_i \int \overline{G}_3 \wedge \overline{\omega}_{A_i} (U^i - \overline{U}^i) \partial_{U^i} Y_{IJK} \) appears, because general \( Y_{IJK} \) may depend on the complex structure moduli. For the gaugino masses we need the gauge kinetic functions (2.25). Through them, we obtain the gaugino masses:

\[
m_{g,D7j} = F^S \frac{-\alpha'^{-2} f^k f^l}{(T^j - \overline{T}^j) - \alpha'^{-2} f^k f^l (S - \overline{S})} + F^{T^j} \frac{1}{(T^j - \overline{T}^j) - \alpha'^{-2} f^k f^l (S - \overline{S})}.
\]

(2.32)

3. Soft terms for the MSSM from a local \( D7 \)-brane construction

3.1. Local MSSM construction with three generations

The locally supersymmetric MSSM-model was originally formulated \[38\] in terms of three stacks of intersecting \( D6 \)-branes in type IIA compactifications. The non–trivial intersection angles are necessary in order to obtain three generations of chiral quark and lepton superfields. As emphasized before, we are discussing supersymmetry breaking in the context of type \( IIB \) orientifold compactifications. Hence we have to consider the \( T \)-dual version of the MSSM \( D6 \)-brane configuration of \[38\]. This \( T \)-duality transformation is very easy to find, one has to perform three \( T \)-duality transformations with respect to either an \( x \)- or an \( y \)-direction in each of the three two–dimensional subtori \( T^{2,i} \). After the \( T \)-duality transformation, all three stacks become \( D7 \)-branes, which are wrapped around the 4–cycles \( T^{2,1} \times T^{2,2}, T^{2,1} \times T^{2,3} \) or \( T^{2,2} \times T^{2,3} \), \textit{i.e.} being transversal to \( T^{2,3}, T^{2,2}, T^{2,1} \), respectively. The last stack is equipped with non-trivial \( f \)-flux, which is required for a realistic model with three generations of chiral matter fields. Specifically, the locally
supersymmetric MSSM is built by three stacks of $D7$–branes with the $f$–flux quantum numbers $(f^j = \alpha' n_j^i / m_j)$ displayed in Table 1.

| Stack | Gauge group | $(m_1, n_1)$ | $(m_2, n_2)$ | $(m_3, n_3)$ | $N_a$ |
|-------|-------------|--------------|--------------|--------------|-------|
| 1     | $U(4)$      | $-$          | $(1, g)$     | $(-1, g)$    | 4     |
| 2     | $SU(2)$     | $(1, 0)$     | $-$          | $(-1, 0)$    | 2     |
| 3     | $SU(2)$     | $(1, 0)$     | $(-1, 0)$    | $-$          | 2     |

Table 1: MSSM $D7$–brane configuration with $f$–flux numbers $(m^i, n^j)$.

The corresponding gauge group is $G = U(4) \times U(2) \times U(2)$. Only stack 1 carries non-trivial $f$–flux. For $g = 3$, it contains three chiral generations of supersymmetric MSSM matter fields, namely left-handed matter fields in the representations $3(4, 2, 1)$ from open strings stretching between the (12)-branes, 3 right-handed matter fields in the representations $3(4, 1, 2)$ from the (13) open string sector and a Higgs multiplet in the representations $(1, 2, 2)$ from the (23)-sector. By pulling apart the first stack of branes into a stack of 3 $D$–branes plus one $D$–brane, the $SU(4)$ gauge group is Higgsed to $SU(3) \times U(1)_{B–L}$, and the matter fields decompose into the known SM representations of quarks and leptons.

The supersymmetry condition (2.20) applied on stack 1 yields the following requirement for the Kähler moduli:

$$T^2 = T^3 \equiv T.$$  \hfill (3.1)

For stacks 2 and 3, it is fulfilled trivially, as those stacks do not carry $f$-flux. These three stacks of $D7$–branes will be a subsector in any concrete global model that satisfies the Ramond tadpole conditions (2.17) and (2.19) by the addition of fluxes and some additional hidden sectors (see e.g. the model of [28] which includes also supersymmetric or non-supersymmetric 3–form fluxes). However, as already emphasised, these three stacks of $D7$–branes alone do not satisfy the tadpole conditions. Plugging in the $f$–flux numbers of the MSSM-branes into equations (2.17) and (2.19), the so far uncancelled tadpoles must be eliminated by the hidden sector branes together with the 3–form fluxes. For $N_1 = 4$, $N_2 = N_3 = 2$ (see also the previous footnote), the hidden $D$–branes must satisfy the

\footnote{Note that in some orbifold models, the $N_a$ will take values different from those in Table 1, if the $D$–branes are fixed under the orbifold group $\mathbb{Z}_N \times \mathbb{Z}_N$ and $\Omega I_n$; e.g. for the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold, $N_1 = 8$ because the corresponding gauge group is broken to $U(N_1/2)$ by the orbifold symmetry (see e.g. [28]).}
following Ramond tadpole conditions conditions \((b \text{ runs over all hidden branes})\):

\[
N_{\text{flux}} + 2 \sum_a N^h_a n^1_b n^2_b n^3_b = -40 ,
\]

\[
2 \sum_b N^h_b m^1_b m^2_b n^3_b = -28 ,
\]

\[
2 \sum_b N^h_b m^1_b m^3_b n^2_b = -28 ,
\]

\[
2 \sum_b N^h_b m^2_b m^3_b n^1_b = -24 .
\]

(3.2)

How these conditions will be eventually met depends on the concrete compact model. Generically, as it has been recently emphasized in Ref. [28], a setup of only \(D3\)- and \(D7\)-branes is not enough. One should add magnetized \(D9\)-\(D9\)-branes.

3.2. Soft terms for the local MSSM construction

As we have seen in the last section, the only soft terms that are generated by turning on 3–form flux in our brane setup are the scalar mass terms and the trilinear couplings, plus gravitino and gaugino masses. Due to the fact that we break supersymmetry from \(N = 1\), no fermionic mass terms and no \(B\)-terms appear.

The open strings representing the untwisted matter fields are those which have both ends on the same brane stack and originate from dimensional reduction of the \(D = 10\) gauge field. In the case of a \(D7\)-brane, we have two complex Wilson line moduli and one complex scalar which describes the transverse position of the brane. These fields do not correspond to any MSSM–fields and must, for the model to be realistic, acquire large masses by some additional effect.

The open strings which are interesting for us from the Standard Model point of view come from the twisted sector, which consists of fields living at the intersections of the three brane stacks. The massless twisted \(R\)-sector gives rise to chiral fermions in the bifundamental representation, while the massless scalar matter fields stem from the twisted \(NS\)-sector. The matter fields living at the \((12)\)-intersection form the left-handed part of the spectrum, the fields living at the \((13)\)-intersection form the right-handed part of the spectrum, while the fields living at the \((23)\)-intersection form the Higgs multiplet. Expressed in \(N = 1\) language, the above matter fields form chiral multiplets, consisting of a complex scalar, a spinor and an auxiliary scalar. It is the scalar component, that acquires mass through SUSY-breaking. So what we calculate in the twisted sector are squark and slepton masses, as well as the mass that appears in the Higgs potential. The contribution to the soft terms that is sensitive to the \(f\)-fluxes comes from the open string matter metrics, which in turn receive their \(f\)-form flux dependence from the mixed boundary conditions.
In our specific setup, we have to deal with the metric for the untwisted $D7$-brane matter fields and the metric for the twisted $D7$-brane matter fields, where the fields come from two different stacks of branes. We use the metrics that were computed in [27] and plug in the specific values of $f^3 = \alpha'_m/m^3$ of our model as given in Table 1.

We will first examine the untwisted matter metrics. Only stack 1 carries non-trivial $f$-flux

$$G^7_{C_1 C_1} = \frac{-\kappa_4^{-2}}{(U^1 - \overline{U}^1)(S - \overline{S})} |1 + a_2 a_3| ,$$

$$G^7_{C_2 C_2} = \frac{-\kappa_4^{-2}}{(U^2 - \overline{U}^2)(T^3 - \overline{T}^3)} |1 - i a_3| ,$$

$$G^7_{C_3 C_3} = \frac{-\kappa_4^{-2}}{(U^3 - \overline{U}^3)(T^2 - \overline{T}^2)} |1 - i a_2| ,$$

(3.3)

where we define $a_2 = \frac{\alpha'_q}{\alpha_1 m_f}$, $a_3 = \frac{\alpha'_q}{\alpha_1 m_f}$. The other two brane stacks have vanishing $f$-flux and the metrics reduce to the simple form (e.g. stack 2):

$$G^7_{C_1 C_1} = \frac{-\kappa_4^{-2}}{(U^1 - \overline{U}^1)(T^3 - \overline{T}^3)} |1 - i a_3| ,$$

$$G^7_{C_2 C_2} = \frac{-\kappa_4^{-2}}{(U^2 - \overline{U}^2)(S - \overline{S})} |1 + a_2 a_3| ,$$

$$G^7_{C_3 C_3} = \frac{-\kappa_4^{-2}}{(U^3 - \overline{U}^3)(T^2 - \overline{T}^2)} |1 - i a_2| ,$$

(3.4)

For stack 3, we have the same form with indices 2 and 3 interchanged. Now we turn to the twisted matter metrics. The matter fields between stacks 1 and 2 and between stack 1 and 3 form a $1/4$ BPS sector. Their metrics are:

$$G^7_{C_1 C_2 \overline{C}^7_{\overline{1}}} = 2i \kappa_4^{-2} (S - \overline{S})^{-\frac{1}{2} + \frac{3q}{2} + \gamma} \frac{\Gamma[\frac{1}{2} - \frac{i}{\pi} \arctan(a_2)]}{\pi^{1/2}(1 + a_2^2)^{1/4}} \frac{\Gamma[\frac{1}{2} \arctan(a_3)]}{(\frac{\pi}{a_2^2} + 1 + a_3^2)^{1/2}}$$

$$\times (T^1 - \overline{T}^1)^{-\frac{1}{2} - \frac{\gamma}{2}} (T^2 - \overline{T}^2)^{-\frac{1}{2} - \frac{\gamma}{2} - \frac{\pi}{2} \arctan(a_2)} (T^3 - \overline{T}^3)^{-\frac{1}{2} - \frac{\gamma}{2} + \frac{\pi}{2} \arctan(a_3)}$$

$$\times (U^1 - \overline{U}^1)^{1/2} (U^2 - \overline{U}^2)^{-\frac{1}{2} - \frac{\gamma}{2} + \frac{\pi}{2} \arctan(a_2)} (U^3 - \overline{U}^3)^{-\frac{1}{2} - \frac{\gamma}{2} - \frac{\pi}{2} \arctan(a_3)} .$$

(3.5)
The matter fields between stacks 2 and 3, which do not carry fluxes are 1/2 BPS states and the metric has a different form, as given in (2.27).

For simplicity, and as these fluxes are generic for all orbifold groups, only (0,3)- and (3,0)-fluxes are turned on.

(i) Scalar mass terms

First, we examine the untwisted case. With the curvature tensors given in appendix A and

\[
Y = (S - \overline{S}) \prod_{j=1}^{3} (T^j - \overline{T}^j) (U^j - \overline{U}^j),
\]

we come to the following result:

Stack 1:

\[
(m_{1T}^{7,1})^2 = \frac{\lambda^2 \kappa_4^6}{(2\pi)^4 \alpha'^2} \frac{G_{C_1 \overline{C}_1}^{7,1}}{|Y|} \times \left\{ \left( 1 - \frac{\text{Im}T^2 \text{Im}T^3 (2g^2 \alpha'^2 + \text{Im}T^2 \text{Im}T^3)}{(g^2 \alpha'^2 + \text{Im}T^2 \text{Im}T^3)^2} \right) |\int \overline{G}_3 \wedge \Omega|^2 \right. \\
+ \frac{g^2 \alpha'^2 \text{Im}T^2 \text{Im}T^3}{(g^2 \alpha'^2 + \text{Im}T^2 \text{Im}T^3)^2} \left( \int \overline{G}_3 \wedge \Omega \int \overline{G}_3 \wedge \overline{\Omega} + \text{c.c.} \right) \\
\left. + \left( 1 - \frac{g^2 \alpha'^2 (2g^2 \alpha'^2 + 2\text{Im}T^2 \text{Im}T^3)}{(g^2 \alpha'^2 + \text{Im}T^2 \text{Im}T^3)^2} \right) |\int G_3 \wedge \Omega|^2 \right\},
\]

Stack 2:

\[
(m_{2T}^{7,1})^2 = \frac{\lambda^2 \kappa_4^6}{(2\pi)^4 \alpha'^2} \frac{G_{C_2 \overline{C}_2}^{7,1}}{|Y|} \times \left\{ \left( 1 + \frac{g^4 \alpha'^4 [(\text{Im}T^2)^2 - (\text{Im}T^3)^2]}{2 [g^2 \alpha'^2 + (\text{Im}T^2)^2]^2 [g^2 \alpha'^2 + (\text{Im}T^3)^2]^2} \right) |\int \overline{G}_3 \wedge \Omega|^2 \right. \\
- \frac{1}{2} g^2 \alpha'^2 \left( \frac{\text{Im}T^2}{[g^2 \alpha'^2 + (\text{Im}T^2)^2]^2} - \frac{(\text{Im}T^3)^2}{[g^2 \alpha'^2 + (\text{Im}T^3)^2]^2} \right) \left( \int \overline{G}_3 \wedge \Omega \int \overline{G}_3 \wedge \overline{\Omega} + \text{c.c.} \right) \\
\left. + \left( \frac{1}{2} - \frac{(\text{Im}T^2)^4}{2 [g^2 \alpha'^2 + (\text{Im}T^2)^2]^2} - \frac{g^4 \alpha'^4 + 2g^2 \alpha'^2 (\text{Im}T^3)^2}{2 [g^2 \alpha'^2 + (\text{Im}T^3)^2]^2} \right) |\int G_3 \wedge \Omega|^2 \right\}.
\]

The mass \((m_{33}^{7,1})^2\) is obtained by using \(G_{C_3 \overline{C}_3}^{7,1}\) instead of \(G_{C_2 \overline{C}_2}^{7,1}\) and interchanging \(\text{Im}T^2\) and \(\text{Im}T^3\), otherwise is has the same structure as \((m_{2T}^{7,1})^2\).
Stack 2:

\[
(m_{11}^\tau)^2 = \frac{\lambda^2 m_{11}^\tau}{(2\pi)^4 \alpha'^2} \left| \frac{G^7.2_{C_3 \bar{C}_3}}{|Y|} \right| \int \overline{G}_3 \wedge \Omega \right|^2,
\]

\[
(m_{22}^\tau)^2 = \frac{\lambda^2 m_{22}^\tau}{(2\pi)^4 \alpha'^2} \left| \frac{G^7.2_{C_2 \bar{C}_2}}{|Y|} \right| \int \overline{G}_3 \wedge \Omega \right|^2,
\]

\[
(m_{33}^\tau)^2 = \frac{\lambda^2 m_{33}^\tau}{(2\pi)^4 \alpha'^2} \left| \frac{G^7.2_{C_3 \bar{C}_3}}{|Y|} \right| \int \overline{G}_3 \wedge \Omega \right|^2.
\]

The mass terms for stack 3 are obtained by interchanging the indices 2 and 3.

In stack 1, which is the stack carrying \(f\)-fluxes, all the flux components appear, they even mix. For the stacks without \(f\)-flux, the general formula (2.30) simplifies drastically. Remarkably, in stacks 2 and 3 the mass term concerning the two-cycle which is not wrapped by the respective stack of 7-branes differs from the others in its dependence on the 3-form flux: While the other mass terms contain the (3, 0)-flux piece, \(m_{33}^7\) and \(m_{22}^7\) contain the (0, 3)-flux piece.

Now, we look at the twisted case. The values of \(\theta_{ij}^j\) (which is the relative angle between the brane stacks in the \(T\)-dual picture) are the following for our specific model:

\[
\begin{align*}
\theta_{12}^1 &= -\frac{1}{2}, & \theta_{12}^2 &= \frac{1}{2} - \frac{1}{\pi} \arctan(a_2), & \theta_{12}^3 &= \frac{1}{\pi} \arctan(a_3), \\
\theta_{13}^1 &= -\frac{1}{2}, & \theta_{13}^2 &= -\frac{1}{\pi} \arctan(a_2), & \theta_{13}^3 &= \frac{1}{2} + \frac{1}{\pi} \arctan(a_3), \\
\theta_{23}^1 &= 0, & \theta_{23}^2 &= -\frac{1}{2}, & \theta_{23}^3 &= \frac{1}{2}.
\end{align*}
\]

Again, we get a simple form for the (23)-sector, whereas for the (12)- and (13)-sectors with
nontrivial \( \theta_{ab} \), the case is more complicated (see appendix A for details):

\[
(m^{7_37_3})^2 = \frac{\lambda^2 \kappa_4^6}{(2\pi)^4 \alpha'^2} \frac{G_{C^{7_37_3}C^{7_37_3}}}{|Y|} \times \left[ \left( \frac{3}{4} + \frac{3}{2} \beta + \gamma \right) \left| \int G_3 \wedge \Omega \right|^2 + \left( \frac{1}{4} - \frac{3}{2} \beta - 3 \gamma + \gamma \sum_j \theta_j^i \right) \left| \int G_3 \wedge \Omega \right|^2 \right. \\
+ \frac{\gamma}{2\pi} \sum_j s_j \frac{\alpha' g \text{Im} T^j}{(\alpha' g)^2 + (\text{Im} T^j)^2} \left( -2 \left| \int G_3 \wedge \Omega \right|^2 + \left( \int \overline{G}_3 \wedge \Omega \int G_3 \wedge \overline{\Omega} + c.c. \right) \right) \\
- \frac{1}{4} \sum_{i=2,3} \pi \left( (g\alpha')^2 + (\text{Im} T^i)^2 \right) \\
\times \left\{ s_i \left( \gamma \ln(T^i \overline{-T^i}) - \ln(U^i - \overline{U}^i) + \frac{1}{2} \left[ \psi_0(\theta_{ab}) + \psi_0(1 - \theta_{ab}) \right] \right) \right. \\
\times \left[ (\text{Im} T^i)^3 \left( \left| \int \overline{G}_3 \wedge \Omega \right|^2 - 3 \left| \int G_3 \wedge \Omega \right|^2 + \left( \int \overline{G}_3 \wedge \Omega \int \overline{G}_3 \wedge \overline{\Omega} + c.c. \right) \right) \\
+ (\alpha' g)^2 \text{Im} T^i \left( 3 \left| \int \overline{G}_3 \wedge \Omega \right|^2 - \left| \int G_3 \wedge \Omega \right|^2 - \left( \int \overline{G}_3 \wedge \Omega \int \overline{G}_3 \wedge \overline{\Omega} + c.c. \right) \right) \right. \\
\left. + \frac{1}{2} \frac{\alpha' g}{\pi} (\text{Im} T^i)^2 \left[ \psi_1(\theta_{ab}) - \psi_1(1 - \theta_{ab}) \right] \right. \\
\times \left( \left| \int \overline{G}_3 \wedge \Omega \right|^2 + \left| \int G_3 \wedge \Omega \right|^2 - \left( \int \overline{G}_3 \wedge \Omega \int \overline{G}_3 \wedge \overline{\Omega} + c.c. \right) \right) \right\} . \tag{3.10}
\]

with \((a, b) = (1, 2)\) or \((1, 3)\). In this case, the 2-form flux dependence is very complicated, as the appearance of the Gamma–function in the original metric already suggested. The 3-form flux appears again with a \((3, 0)\)-part, a \((0, 3)\)-part and a combination of the two.

For \((a, b) = (2, 3)\), the \(1/2\) \textit{BPS}–case that corresponds to the Higgs multiplet, we get

\[
(m^{7_37_3})^2 = \frac{\lambda^2 \kappa_4^6}{(2\pi)^4 \alpha'^2} \frac{G_{C^{7_37_3}C^{7_37_3}}}{2|Y|} \left( \left| \int G_3 \wedge \Omega \right|^2 + \left| \int \overline{G}_3 \wedge \Omega \right|^2 \right) \tag{3.11},
\]

which is a lot simpler as the world–volume 2–form \(f\)–flux does not enter.

\textbf{\(\text{(ii) Trilinear couplings}\)}

Now, we will examine the trilinear coupling, at least in the example of the untwisted matter fields. In the case of all three fields living on the same stack of branes, the general
formula (2.31) simplifies considerably, as \( Y_{ijk} = \epsilon_{ijk} \):

\[
A_{ijk}^{7,1} = \epsilon_{ijk} \prod |M - \overline{M}|^{-1} \frac{\kappa^2 \lambda}{(2\pi)^2 \alpha'} \left\{ \frac{-(\alpha'g)^2}{(\alpha'g)^2 + \text{Im}T^2\text{Im}T^3} - \frac{(\alpha'g)^2}{(\alpha'g)^2 + (\text{Im}T^2)^2} \right. \\
- \frac{(\alpha'g)^2}{(\alpha'g)^2 + (\text{Im}T^2)^2} \left. \right\} \int G_3 \wedge \overline{\Omega} + \left[ 1 + \frac{(\alpha'g)^2}{(\alpha'g)^2 + \text{Im}T^2\text{Im}T^3} \left. \frac{(\alpha'g)^2}{(\alpha'g)^2 + (\text{Im}T^2)^2} \right\} \int G_3 \wedge \overline{\Omega} \right),
\]

\[
A_{ijk}^{7,2} = A_{ijk}^{7,3} = \epsilon_{ijk} \prod |M - \overline{M}|^{-1} \frac{\kappa^2 \lambda}{(2\pi)^2 \alpha'} \int G_3 \wedge \overline{\Omega}.
\]

For the stacks with vanishing 2-form flux, only the \((0,3)\)-part contributes, as opposed to the result obtained for untwisted matter fields on a \(D3\)-brane, where only the \((3,0)\)-part contributes. The coupling for stack 1 remains complicated due to the non-trivial moduli dependence of the metric.

(iii) Gaugino masses

Last, but not least, we also give the gaugino masses for our specific model. The gaugino masses are derived from the gauge kinetic function given in (2.25) via \( m_{g,j} = F^M \partial_M \log(\text{Im}f_{D7,j}) \). For our model, we have:

\[
f_1 = T^1 + g^2 S, \\
f_j = T^j, \quad j = 2, 3.
\]

This gives us

\[
m_{g,1} = \frac{F^{T^1} + g^2 F^S}{(T^1 - \overline{T^1}) + g^2 (S - \overline{S})},
\]

\[
m_{g,j} = \frac{F^{T^j}}{(T^j - \overline{T^j})}, \quad j = 2, 3.
\]

3.3. Concrete Example

We will get even more specific now. We will look at the soft terms for our MSSM-model with the SUSY-condition (3.1) enforced, which leads to \( T^2 = T^3 = \overline{T} \). Thanks to the supersymmetry condition, we are able to eliminate the string basis moduli completely in the following. Furthermore, we turn on a specific 3-form flux consisting of a \((3,0)\)-part and a \((0,3)\)-part obtained for \( U^1 = U^2 = U^3 = S = i \) and take \( g = 3 \), which results in three fermion generations.
A flux solution with \((0,3)\)- and \((3,0)\)-component for \(U^1 = U^2 = U^3 = S = i\) is

\[
\frac{1}{(2\pi)^2\alpha'} G_{(0,3)+(3,0)} = \left( b_3 - id_3 \right) \alpha_0 + \left( -b_0 + id_0 \right) \alpha_1 + \left( -b_0 + id_0 \right) \alpha_2 + \left( -b_0 + id_0 \right) \alpha_3 \\
+ \left( b_0 - id_0 \right) \beta_0 + \left( b_3 - id_3 \right) \beta_1 + \left( b_3 - id_3 \right) \beta_2 + \left( b_3 - id_3 \right) \beta_3 .
\]  

\( (3.15) \)

For the real coefficients \(b_0, b_3, d_0, d_3\), any integer number can be chosen. To avoid possible complications with flux quantization, we take the coefficients to be multiples of 8, though. Expressed in the complex basis, this flux reads

\[
\frac{1}{(2\pi)^2\alpha'} G_{(0,3)+(3,0)} = \frac{1}{2} \left( b_3 + d_0 + i(b_0 - d_3) \right) \omega_A + \frac{1}{2} \left( b_3 - d_0 - i(b_0 + d_3) \right) \omega_B .
\]  

\( (3.16) \)

We will need the following flux integrals:

\[
\frac{1}{(2\pi)^4\alpha'^2} \left| \int G_3 \wedge \Omega \right|^2 = 16 \left[ (b_3 + d_0)^2 + (b_0 - d_3)^2 \right] ,
\]

\[
\frac{1}{(2\pi)^4\alpha'^2} \left| \int \bar{G}_3 \wedge \Omega \right|^2 = 16 \left[ (b_3 - d_0)^2 + (b_0 + d_3)^2 \right] ,
\]

\[
\frac{1}{(2\pi)^4\alpha'^2} \int G_3 \wedge \Omega \times \int G_3 \wedge \bar{\Omega} = -16 \left[ b_0^2 + b_3^2 - d_0^2 - d_3^2 - 2i(b_0d_0 + b_3d_3) \right] ,
\]  

\( (3.17) \)

\[
\frac{1}{(2\pi)^2\alpha'} \int \bar{G}_3 \wedge \bar{\Omega} = 4 \left[ b_0 - d_3 + i(b_3 + d_0) \right] ,
\]

\[
\frac{1}{(2\pi)^2\alpha'} \int G_3 \wedge \bar{\Omega} = -4 \left[ b_0 + d_3 + i(b_3 - d_0) \right] .
\]

We will again first examine the metrics. The untwisted matter metric for stack 1 simplifies considerably as the SUSY-condition leads to \(a_2 = a_3 \equiv a\):

\[
G^{7,1}_{C_1\bar{C}_1} = \frac{i}{2} \kappa_4^{-2} \left( \frac{1}{S - \bar{S}} + \frac{9}{T^1 - \bar{T}^1} \right) ,
\]

\[
G^{7,1}_{C_2\bar{C}_2} = G^{7,1}_{C_3\bar{C}_3} = \frac{i}{2} \frac{\kappa_4^{-2}}{(\bar{T} - T)} .
\]  

\( (3.18) \)

Stack 2 has the following metric:

\[
G^{7,2}_{C_1\bar{C}_1} = \frac{i}{2} \frac{\kappa_4^{-2}}{(\bar{T} - T)} ,
\]

\[
G^{7,2}_{C_2\bar{C}_2} = \frac{i}{2} \frac{\kappa_4^{-2}}{S - \bar{S}} ,
\]

\[
G^{7,2}_{C_3\bar{C}_3} = \frac{i}{2} \frac{\kappa_4^{-2}}{(T^1 - \bar{T}^1)} .
\]  

\( (3.19) \)
The metric for stack 3 can be obtained as usual by interchanging the indices 2 and 3. The twisted matter metrics simplify as well. With \( a = 3\sqrt{\frac{s-3}{T^1-T^3}} \), we obtain:

\[
G_{\tilde{C}\tilde{t}\tilde{t}2\tilde{t}2} = 2i \kappa^2 \left( S - \tilde{S} \right)^{-\frac{1}{2} + \frac{i}{2} \beta + \gamma} \left( T^1 - \tilde{T}^1 \right)^{-\frac{1}{2} - \frac{i}{2} \beta - \gamma} ( \tilde{T} - \tilde{T})^{-\frac{1}{2} - \frac{i}{2} \gamma - \frac{i}{2} \beta - \frac{i}{2} \gamma} \\
\times \frac{1}{\pi} \sqrt{\frac{a}{1 + a^2}} \Gamma \left[ \frac{1}{2} - \frac{1}{\pi} \arctan(a) \right] \Gamma \left[ \frac{1}{\pi} \arctan(a) \right],
\]

\[
G_{\tilde{C}\tilde{t}\tilde{t}3\tilde{t}3} = -2 \kappa^2 \left( S - \tilde{S} \right)^{-\frac{1}{2} + \frac{i}{2} \beta + \gamma} \left( T^1 - \tilde{T}^1 \right)^{-\frac{1}{2} - \frac{i}{2} \beta - \frac{i}{2} \gamma} ( \tilde{T} - \tilde{T})^{-\frac{1}{2} - \frac{i}{2} \gamma - \frac{i}{2} \beta - \frac{i}{2} \gamma} \\
\times \frac{1}{\pi} \sqrt{\frac{a}{1 + a^2}} \Gamma \left[ \frac{1}{2} + \frac{1}{\pi} \arctan(a) \right] \Gamma \left[ -\frac{1}{\pi} \arctan(a) \right].
\]

\[
G_{\tilde{C}\tilde{t}\tilde{t}3\tilde{t}3} = -\kappa^2 \left( 2i \right)^{-3/2} \frac{1}{\left( T^1 - \tilde{T}^1 \right)^{1/2}}.
\]

(i) Scalar mass terms

For the untwisted matter fields, the only non-trivial case for the Riemann tensor is \( R_{\tilde{S}\tilde{t}\tilde{t}1} \), see appendix A. The scalar masses simplify as follows:

Stack 1:

\[
(m_{1T}^{7,1})^2 = 2 \lambda^2 \kappa^6 \frac{G_{C1\tilde{t}1}}{(S - \tilde{S})(\tilde{T} - \tilde{T})^2(T^1 - \tilde{T}^1)} \\
\times \left\{ \left( 1 + \frac{(T^1 - \tilde{T}^1)}{(S - \tilde{S})^2} \left[ 18 \left( S - \tilde{S} \right) + T^1 - \tilde{T}^1 \right] \right) \left[ (b_3 - d_0)^2 + (b_0 + d_3)^2 \right] \\
+ \frac{18}{9 \left( S - \tilde{S} \right) + T^1 - \tilde{T}^1} \left( b_0^2 + b_3^2 - d_0^2 - d_3^2 \right) \\
+ \left( 1 + \frac{9 \left( S - \tilde{S} \right)}{(T^1 - \tilde{T}^1)^2 \left[ 9 \left( S - \tilde{S} \right) + 2 \left( T^1 - \tilde{T}^1 \right) \right]} \right) \left[ (b_3 + d_0)^2 + (b_0 - d_3)^2 \right] \right\},
\]

\[
(m_{2T}^{7,1})^2 = 2 \lambda^2 \kappa^6 \frac{G_{C2\tilde{t}2}}{(S - \tilde{S})(\tilde{T} - \tilde{T})^2(T^1 - \tilde{T}^1)} \left[ (b_3 - d_0)^2 + (b_0 + d_3)^2 \right],
\]

\[
(m_{3T}^{7,1})^2 = 2 \lambda^2 \kappa^6 \frac{G_{C3\tilde{t}3}}{(S - \tilde{S})(\tilde{T} - \tilde{T})^2(T^1 - \tilde{T}^1)} \left[ (b_3 - d_0)^2 + (b_0 + d_3)^2 \right].
\]

As apparent already in equation (3.7), as well as from the metrics (3.18), the 2-form flux dependence drops completely out of \( (m_{2T}^{7,1})^2 \) and \( (m_{3T}^{7,1})^2 \) for \( \text{Im} T^2 = \text{Im} T^3 \).
Stack 2:

\[
(m_{1T}^{7,2})^2 = 2 \lambda^2 \kappa_4^6 \left( \frac{G_{C_1 \tilde{C}_1}}{(S - \overline{S})(T - \overline{T})^2(T^1 - \overline{T^1})} \right) [(b_3 - d_0)^2 + (b_0 + d_3)^2],
\]

\[
(m_{2T}^{7,2})^2 = 2 \lambda^2 \kappa_4^6 \left( \frac{G_{C_2 \tilde{C}_2}}{(S - \overline{S})(T - \overline{T})^2(T^1 - \overline{T^1})} \right) [(b_3 + d_0)^2 + (b_0 - d_3)^2],
\]

\[
(m_{3T}^{7,2})^2 = 2 \lambda^2 \kappa_4^6 \left( \frac{G_{C_3 \tilde{C}_3}}{(S - \overline{S})(T - \overline{T})^2(T^1 - \overline{T^1})} \right) [(b_3 - d_0)^2 + (b_0 + d_3)^2].
\]

The result for stack 3 is obtained by changing the indices.

From the above, we immediately see that we are left with a number of unfixed parameters: the imaginary parts of the two Kähler moduli \(T^1\) and \(\tilde{T}\) which are left unfixed, and the four real parameters describing the 3-form flux. The 2-form flux is fixed by the requirement that we want to obtain 3 particle generations as in the Standard Model.

In the twisted case, we get the following mass terms for the 1/4 BPS-states \((b = 2, 3)\)

\[
(m_{7^b \bar{7}^b}^{7})^2 = \lambda^2 \kappa_4^6 \left( \frac{G_{C_7 \bar{C}_7 \tilde{C}_7 \bar{C}_7}}{(S - \overline{S})(T - \overline{T})^2(T^1 - \overline{T^1})} \right) \left( \frac{1}{[9 (S - \overline{S}) + T^1 - \overline{T^1}]^2} \right)
\]

\[
\times \left\{ \left( m_{SS} - [27 (S - \overline{S}) + (T^1 - \overline{T^1})] \psi_0^{(0)}(a) + \psi_1^{(1)}(a) \right) [(b_3 - d_0)^2 + (b_0 + d_3)^2]
\]

\[
+ \left( m_{TT} + [9 (S - \overline{S}) + 3 (T^1 - \overline{T^1})] \psi_0^{(0)}(a) + \psi_1^{(1)}(a) \right) [(b_3 + d_0)^2 + (b_0 - d_3)^2]
\]

\[
- \left( m_{ST} + [9 (S - \overline{S}) - (T^1 - \overline{T^1})] \psi_0^{(0)}(a) - \psi_1^{(1)}(a) \right) (b_0^2 + b_3^2 - d_0^2 - d_3^2) \right\},
\]

with:

\[
\psi_0^{(0)}(a) = \frac{3}{2\pi} \left( S - \overline{S} \right)^{1/2} \left( T^1 - \overline{T^1} \right)^{1/2} \left[ \psi_0 \left( \frac{a}{\pi} \right) - \psi_0 \left( \frac{1}{2} - \frac{a}{\pi} \right) \right],
\]

\[
\psi_1^{(1)}(a) = \frac{9}{2\pi^2} \left( S - \overline{S} \right) \left( T^1 - \overline{T^1} \right) \left[ \psi_1 \left( \frac{a}{\pi} \right) + \psi_1 \left( \frac{1}{2} - \frac{a}{\pi} \right) \right],
\]

\[
\psi_0^{(0)}(a) = -\frac{3}{2\pi} \left( S - \overline{S} \right)^{1/2} \left( T^1 - \overline{T^1} \right)^{1/2} \left[ \psi_0 \left( -\frac{a}{\pi} \right) - \psi_0 \left( \frac{1}{2} + \frac{a}{\pi} \right) \right],
\]

\[
\psi_1^{(1)}(a) = \frac{9}{2\pi^2} \left( S - \overline{S} \right) \left( T^1 - \overline{T^1} \right) \left[ \psi_1 \left( -\frac{a}{\pi} \right) + \psi_1 \left( \frac{1}{2} + \frac{a}{\pi} \right) \right],
\]

\[
m_{SS} = 81 (3 - 3\beta - 2\gamma) (S - \overline{S})^2 + (2 - 3\beta - 2\gamma) (T^1 - \overline{T^1}) \left[ 18 (S - \overline{S}) + (T^1 - \overline{T^1}) \right],
\]

\[
m_{TT} = (4 + 3\beta + 6\gamma) (T^1 - \overline{T^1})^2 + 27 (1 + \beta + 2\gamma) (S - \overline{S}) \left[ 9 (S - \overline{S}) + 2 (T^1 - \overline{T^1}) \right],
\]

\[
m_{ST} = 9 (S - \overline{S}) (T^1 - \overline{T^1}).
\]

(3.24)
The 1/2 BPS mass is the following:

\[ (m_{7/2}^2)^2 = 2 \lambda^2 \kappa_4^6 \frac{G_{C7/27/2}}{(S - \bar{S}) (\bar{T} - T)^2 (T^1 - \bar{T}^1)} \left( b_0^2 + b_3^2 + d_0^2 + d_3^2 \right). \]  

(3.25)

(ii) Trilinear couplings

\[ A_{ij}^{7,1} = \epsilon_{ijk} \frac{1}{|T^1 - \bar{T}^1|} \frac{1}{|T - \bar{T}|^2} \frac{\kappa_4^2 \lambda}{(2\pi)^2 \alpha'} \left\{ \frac{3(\alpha' g)^2}{(\alpha' g)^2 + (e^{\phi \pi})^4(T^1 - \bar{T}^1)(\bar{T} - T)^2} \right\} [b_0 + d_3 + i(b_3 - d_0)] + \left[ 1 + \frac{3(\alpha' g)^2}{(\alpha' g)^2 + (e^{\phi \pi})^4(T^1 - \bar{T}^1)(\bar{T} - T)^2} \right] [b_0 - d_3 + i(b_3 + d_0)] \right\}, \]

\[ A_{ij}^{7,2} = A_{ij}^{7,3} = \epsilon_{ijk} \frac{1}{4 |T^1 - \bar{T}^1|} \frac{1}{|T - \bar{T}|^2} \frac{\kappa_4^2 \lambda}{(2\pi)^2 \alpha'} [b_0 - d_3 + i(b_3 + d_0)]. \]  

(3.26)

(iii) Gaugino masses

The gaugino masses (3.14) have the following form:

\[ m_{g,1} = \frac{1}{2} \frac{\kappa_4^2 \lambda}{(S - \bar{S})^{1/2}(T^1 - \bar{T}^1)^{1/2}(\bar{T} - T)} \times \frac{(T^1 - \bar{T}^1)[b_0 - d_3 + i(b_3 + d_0)] - 9(S - \bar{S})[b_0 + d_3 + i(b_3 - d_0)]}{(T^1 - \bar{T}^1) + 9(S - \bar{S})}, \]  

(3.27)

\[ m_{g,2} = m_{g,3} = \frac{1}{2} \frac{\kappa_4^2 \lambda}{(S - \bar{S})^{1/2}(T^1 - \bar{T}^1)^{1/2}(\bar{T} - T)} \left[ b_0 - d_3 + i(b_3 + d_0) \right]. \]

4. Goldstino angle and structure of soft–terms

In this section, we want to rewrite our results from the last section in terms of the so–called goldstino angle, for which an isotropic compactification is assumed, i.e. \( T^1 = T^2 = T^3 \equiv T \), or \( T^1 = T^2 \equiv T^3 \equiv T \), respectively and \( U^j = i \). Note, that this requirement automatically fulfills the supersymmetry condition (3.1). For an isotropic compactification, the Kähler potential (2.3) boils down to

\[ \kappa_4^2 \tilde{K} = - \ln(S - \bar{S}) - 3 \ln(T - \bar{T}) - 3 \ln(2i). \]  

(4.1)
With this we get the two $F$–terms $F^S$, $F^T$, where $F^T$ refers to the overall Kähler modulus $T$ (cf. section 3):

\[
F^S = (2i)^{-3/2} (S - \overline{S})^{1/2} (T - \overline{T})^{-3/2} \kappa_4^2 \frac{\lambda}{(2\pi)^2 \alpha'} \int G_3 \wedge \Omega ,
\]

\[
F^T = (2i)^{-3/2} (S - \overline{S})^{-1/2} (T - \overline{T})^{-1/2} \kappa_4^2 \frac{\lambda}{(2\pi)^2 \alpha'} \int G_3 \wedge \Omega .
\]

(4.2)

The goldstino angle $\theta_g$ describes where the source of supersymmetry breaking originates: If $\theta_g = \frac{\pi}{3}, \frac{2\pi}{3}, \ldots$, the breaking is due to an $F_S$–term from $(3, 0)$–form fluxes only (dilaton–dominated), whereas in the case $\theta_g = 0, \pi, \ldots$, the breaking is entirely due to an $F_T$–term from $(0, 3)$–form fluxes. Hence, the ratio between $F^S$ and $F^T$ can be used to define the goldstino angle

\[
\tan \theta_g = e^{i(\alpha_T - \alpha_S)} \frac{\hat{K}^{1/2}_{SS} F^S}{\hat{K}^{1/2}_{TT} F^T} = \frac{1}{\sqrt{3}} e^{i(\alpha_T - \alpha_S)} \frac{\int G_3 \wedge \overline{\Omega}}{\int G_3 \wedge \Omega} ,
\]

(4.3)

with

\[
\hat{K}^{1/2}_{SS} F^S = \sqrt{3} C m_{3/2} e^{i\alpha_S} \sin \theta_g ,
\]

\[
\hat{K}^{1/2}_{TT} F^T = \sqrt{3} C m_{3/2} e^{i\alpha_T} \cos \theta_g ,
\]

(4.4)

and $\alpha_S$, $\alpha_T$ are the phases of the respective $F$–terms. The real constant $C$ follows from the relation

\[
\hat{K}_{SS} |F^S|^2 + \hat{K}_{TT} |F^T|^2 = 3 C^2 |m_{3/2}|^2 = 3 |m_{3/2}|^2 + \hat{V} ,
\]

with:

\[
|m_{3/2}|^2 = \frac{1}{3} \hat{K}_{TT} |F^T|^2 ,
\]

\[
\hat{V} = \hat{K}_{SS} |F^S|^2 .
\]

(4.5)

Hence, we have:

\[
C^2 = 1 + \frac{\hat{V}}{3|m_{3/2}|^2} .
\]

(4.6)

We have $C = 1$ for vanishing cosmological constant $\hat{V}$ and non–vanishing gravitino mass $m_{3/2}$. From Eqs. (4.1), (4.2) and (4.4) we obtain:

\[
\kappa_4^2 \frac{\lambda^2}{(2\pi)^4 \alpha'^2} \left| \int_{X_6} G_3 \wedge \Omega \right|^2 = 2^3 C^2 |m_{3/2}|^2 \cos^2 \theta_g (T - \overline{T})^3 (S - \overline{S}) ,
\]

\[
\kappa_4^2 \frac{\lambda^2}{(2\pi)^4 \alpha'^2} \left| \int_{X_6} \overline{G}_3 \wedge \Omega \right|^2 = 3 \cdot 2^3 C^2 |m_{3/2}|^2 \sin^2 \theta_g (T - \overline{T})^3 (S - \overline{S}) ,
\]

\[
\kappa_4^2 \frac{\lambda^2}{(2\pi)^4 \alpha'^2} \int_{X_6} \overline{G}_3 \wedge \Omega \int_{X_6} \overline{G}_3 \wedge \Omega = \sqrt{3} \cdot 2^3 C^2 |m_{3/2}|^2 \sin \theta_g \cos \theta_g e^{i\alpha_T - i\alpha_S} (T - \overline{T})^3 (S - \overline{S}) .
\]

(4.7)
In the following, we shall parametrize the results of the last section with the goldstino angle.

(i) Untwisted scalar mass terms

Stack 1:

\[(m_{11}^{7,1})^2 = C^2 |m_{3/2}|^2 \kappa_4^2 G_{C_1}\bar{C}_1 1 \bigg\{ 3 \left( 1 + \frac{(T - \overline{T}) \[18 (S - \bar{S}) + T - \overline{T}\]}{(S - \bar{S})^2 \left[9 (S - \bar{S}) + T - \overline{T}\right]^2} \right) \sin^2 \theta_g \\
+ \left( 1 + \frac{9 (S - \bar{S}) [9 (S - \bar{S}) + 2 \left( T - \overline{T}\right)]}{\left( T - \overline{T}\right)^2 \left[9 (S - \bar{S}) + T - \overline{T}\right]^2} \right) \cos^2 \theta_g \\
- \frac{18 \sqrt{3}}{9 (S - \bar{S}) + T - \overline{T}} \sin \theta_g \cos \theta_g \cos(\alpha_S - \alpha_T) \bigg\},\]

\[(m_{22}^{7,1})^2 = 3 \kappa_4^2 G_{C_2}\bar{C}_2 C^2 |m_{3/2}|^2 \sin^2 \theta_g = \frac{3}{2} \frac{1}{|T - \overline{T}|} C^2 |m_{3/2}|^2 \sin^2 \theta_g ,\]

\[(m_{33}^{7,1})^2 = 3 \kappa_4^2 G_{C_3}\bar{C}_3 C^2 |m_{3/2}|^2 \sin^2 \theta_g = \frac{3}{2} \frac{1}{|T - \overline{T}|} C^2 |m_{3/2}|^2 \sin^2 \theta_g .\] 

(4.8)

Stack 2:

\[(m_{11}^{7,2})^2 = 3 \kappa_4^2 C^2 |m_{3/2}|^2 \sin^2 \theta_g G_{C_1}\bar{C}_1^{7,2} = \frac{3}{2} \frac{1}{|T - \overline{T}|} C^2 |m_{3/2}|^2 \sin^2 \theta_g ,\]

\[(m_{22}^{7,2})^2 = \kappa_4^2 C^2 |m_{3/2}|^2 \cos^2 \theta_g G_{C_2}\bar{C}_2^{7,2} = \frac{1}{4} C^2 |m_{3/2}|^2 \cos^2 \theta_g ,\]

\[(m_{33}^{7,2})^2 = 3 \kappa_4^2 C^2 |m_{3/2}|^2 \sin^2 \theta_g G_{C_3}\bar{C}_3^{7,2} = \frac{3}{2} \frac{1}{|T - \overline{T}|} C^2 |m_{3/2}|^2 \sin^2 \theta_g .\] 

(4.9)

(ii) Twisted scalar mass terms

For \( T^i = T \), the twisted scalar masses \([3:20]\) become \( a = 3 \sqrt{\frac{S - \bar{S}}{T - \overline{T}}} \):

\[G^{\tau_1 \tau_2 \bar{C}^{\tau_1 \tau_2}} = 2i \kappa_4^{-2} (S - \bar{S})^{-\frac{3 + 2 \beta + \gamma}{2}} (T - \overline{T})^{-\frac{3 - \beta}{2} - \gamma} \]
\[\times \frac{1}{\pi} \sqrt{\frac{a}{1 + a^2}} \Gamma \left[ \frac{1}{2} - \frac{1}{\pi} \arctan(a) \right] \Gamma \left[ \frac{1}{\pi} \arctan(a) \right] ,\]

\[G^{\tau_2 \tau_3 \bar{C}^{\tau_2 \tau_3}} = -2 \kappa_4^{-2} (S - \bar{S})^{-\frac{3 + 2 \beta + \gamma}{2}} (T - \overline{T})^{-\frac{3 - \beta}{2} - \gamma} \]
\[\times \frac{1}{\pi} \sqrt{\frac{a}{1 + a^2}} \Gamma \left[ \frac{1}{2} + \frac{1}{\pi} \arctan(a) \right] \Gamma \left[ -\frac{1}{\pi} \arctan(a) \right] ,\]

\[G^{\tau_2 \tau_3 \bar{C}^{\tau_2 \tau_3}} = -\kappa_4^{-2} (2i)^{-3/2} \frac{1}{(T - \overline{T})^{1/2}} .\] 

(4.10)
\[
\begin{align*}
(m_3^7)^2 &= -\kappa^2_4 C^2 |m_{3/2}|^2 \frac{G_{\gamma_1 \gamma_6 \bar{\gamma}^2 \gamma_7}}{[9 (S - \bar{S}) + T - \bar{T}]^2} \\
&\times \left\{ 3 \left( m_{SS} - [27 (S - \bar{S}) + (T - \bar{T})] \Psi^{(0)}_b(a) + \Psi^{(1)}_b(a) \right) \sin^2 \theta_g \\
&+ \left( m_{TT} + [9 (S - \bar{S}) + 3 (T - \bar{T})] \Psi^{(0)}_b(a) + \Psi^{(1)}_b(a) \right) \cos^2 \theta_g \\
&+ 2\sqrt{3} \left( m_{ST} + [9 (S - \bar{S}) - (T - \bar{T})] \Psi^{(0)}_b(a) - \Psi^{(1)}_b(a) \right) \sin \theta_g \cos \theta_g \cos(\alpha_S - \alpha_T) \right\},
\end{align*}
\]

with
\[
m_{SS} = -81 (1 + 3\beta + 2\gamma) (S - \bar{S})^2 - (2 + 3\beta + 2\gamma) (T - \bar{T}) [18 (S - \bar{S}) + (T - \bar{T})],
\]
\[
m_{TT} = 3 (\beta + 2\gamma) (T - \bar{T})^2 - 9 (1 - 3\beta - 6\gamma) (S - \bar{S}) [9 (S - \bar{S}) + 2 (T - \bar{T})]
\]

and the expressions \(\Psi^{(n)}_b\), \(m_{ST}\) defined in (3.24), subject to the replacement \(T^1 \rightarrow T^1/2\) BPS:

\[
(m^7)^2 = \frac{1}{2} C^2 |m_{3/2}|^2 \frac{G_{\gamma_1 \gamma_6 \bar{\gamma}^2 \gamma_7}}{[9 (S - \bar{S}) + T - \bar{T}]^2} \sin^2 \theta_g \cos \theta_g \cos(\alpha_S - \alpha_T).
\]

(iii) Trilinear couplings

\[
A_{ijk}^{7,1} = \frac{1}{4} \epsilon_{ijk} \frac{C m_{3/2}}{(T - \bar{T})^{3/2}} \left\{ -\frac{27\sqrt{3} (S - \bar{S})}{9 (S - \bar{S}) + (T - \bar{T})} \sin \theta_g e^{i\alpha_S} \\
+ \left[ 1 + \frac{27(S - \bar{S})}{9 (S - \bar{S}) + (T - \bar{T})} \right] \cos \theta_g e^{i\alpha_T} \right\},
\]

\[
A_{ijk}^{7,2} = A_{ijk}^{7,3} = \frac{1}{4} \epsilon_{ijk} \frac{C m_{3/2}}{(T - \bar{T})^{3/2}} \cos \theta_g e^{i\alpha_T}.
\]

(iv) Gaugino mass terms

\[
m_{g,1} = C m_{3/2} \frac{(T - \bar{T}) \cos \theta_g e^{i\alpha_T} + 9(S - \bar{S})\sqrt{3} \sin \theta_g e^{i\alpha_S}}{(T - \bar{T}) + 9(S - \bar{S})},
\]

\[
m_{g,2} = m_{g,3} = C m_{3/2} \cos \theta_g e^{i\alpha_T}.
\]

5. Gravitino mass and scales: scalar and gaugino masses

In this section, we shall discuss the scales of the soft–supersymmetry breaking terms. The key quantity entering all formulae is the product \(C := C m_{3/2}\), with \(C\) introduced in
Eq. (1.6) and the gravitino mass \( m_{3/2} = |\hat{W}| \kappa_4^2 e^{\frac{1}{2} \kappa_4^2 \hat{K}} \). Here, the Kähler potential \( \hat{K} \) is given in Eq. (1.1), and the superpotential \( \hat{W} \) in Eq. (2.11). In the general case of turning on both \((3,0)-\) and \((0,3)-\) flux components, \( i.e. \) generic goldstino angle \( \theta_g \neq 0, \frac{\pi}{2}, \ldots \) we have

\[
C = \sqrt{m_{3/2}^2 + \frac{1}{3} \hat{V}},
\]

which boils down to

\[
C = \begin{cases} 
  m_{3/2}, & (0,3)-\text{flux}, \\
  \frac{1}{\sqrt{3}} \hat{V}^{1/2}, & (3,0)-\text{flux}
\end{cases}
\]

for the two special cases \( \theta_g = 0 \) and \( \theta_g = \frac{\pi}{2} \), respectively. Recall, that only for \( IASD-\) fluxes the cosmological constant \( \hat{V} \) is non–vanishing (to lowest order), while only a \((0,3)-\) flux component \( G_3 \) gives rise to a non–vanishing gravitino mass \( m_{3/2} \). Hence, in either case the quantity \( C \) is non–vanishing and generically allows for non–vanishing soft–masses in the following. The gravitino mass may be written

\[
m_{3/2} = \frac{1}{\sqrt{2}} \frac{(2\pi)^{6}}{M_{\text{string}}^4} \left| \frac{\prod_{j=1}^{3} \text{Im}(U_j)^{1/2}}{\text{Vol}(X_6)} \int_{X_6} G_3 \wedge \Omega \right| \left( \int_{X_6} G_3 \wedge \Omega \right)^{-1/2}
\]

\[
= \frac{g_{\text{string}}^2}{\sqrt{2}} \frac{(2\pi)^{4}}{M_{\text{string}}^4} \frac{3}{M_{\text{Planck}}^2} \left| \frac{\prod_{j=1}^{3} \text{Im}(U_j)^{-1/2}}{\text{Vol}(X_6)} \int_{X_6} G_3 \wedge \Omega \right|,
\]

with the type \( IIB \) string coupling constant \( g_{\text{string}} = e^{\phi_{10}} = (2\pi \text{ Im} S)^{-1} \). The latter is assumed to be small in order to justify a perturbative orientifold construction. The factor \( \text{Vol}(X_6) = \text{Im}(T^1) \text{Im}(T^2) \text{Im}(T^3) \) is the volume of the six–dimensional compactification manifold \( X_6 \), measured in string units \( \alpha'^3 \). The relation between the string scale \( \alpha' = M_{\text{string}}^{-2} \) and the four–dimensional Planck mass \( M_{\text{Planck}} \) is given by:

\[
M_{\text{Planck}} = 2^{3/2} \frac{g_{\text{string}}^{-1} M_{\text{string}}^4}{\sqrt{\text{Vol}(X_6)}}.
\]

Qualitatively, the integral \( \left| \int_{X_6} G_3 \wedge \Omega \right| \) is of order \( \frac{M_{\text{Planck}}^2}{M_{\text{string}}^2} \). Since the moduli fields \( S, T^j, U^j \) are dimensionsless we deduce from the first line of (5.3): \( m_{3/2} \sim \frac{M_{\text{string}}^2}{M_{\text{Planck}}} \).

In the following, as in the previous section, let us assume an isotropic compactification of radius \( R \), \( i.e. \) \( \text{Vol}(X_6) = R^6 \) and \( U^j = i \). The latter clearly obeys the supersymmetry

\[\text{for more details. Besides, in (2.11) we have chosen } \lambda^{-1} = 16\pi^2 \alpha'^3, \text{ such that } \kappa_{10}^{-2} = \frac{1}{(2\pi)^2 \alpha'^3}.\]
The flux quantization condition \( \frac{1}{(2\pi)^2 \alpha'} \int_{C_3} G_{3}^{(0,3)} = \xi_1 \in \mathbb{Z} \) for a \((0,3)\)-form flux component of \( G_3 \) essentially yields the estimate:

\[
G_{3}^{(0,3)} \sim (2\pi)^2 \frac{\xi_1 \alpha'}{R^3}.
\] (5.5)

With this information and

\[
\left| \int_{X_6} G_3 \wedge \Omega \right| = (2\pi)^8 \xi_1 \alpha' R^3 = 2^{-3/2} (2\pi)^8 g_{\text{string}} \frac{M_{\text{Planck}}}{M_{\text{string}}^6} \xi_1,
\]

we obtain for the gravitino mass \( m_{3/2} \):

\[
m_{3/2} = \pi^2 \frac{1}{\text{Im}(S)^{1/2} \text{Im}(T)^{3/2}} \frac{M_{\text{string}}^2}{M_{\text{Planck}}} \xi_1.
\] (5.6)

Since the physical moduli fields \( T \) are dimensionless, we have:

\[
m_{3/2} \sim g_{\text{string}}^{1/2} \frac{M_{\text{string}}^2}{M_{\text{Planck}}} \xi_1.
\] (5.7)

Hence, for the model introduced in subsection 3.3, with \( \text{Im}(S) = 1 \) as a result from the flux quantization condition, we obtain:

\[
m_{3/2} = \pi^2 \text{Im}(T)^{-3/2} M_{\text{string}} \left( \frac{M_{\text{string}}}{M_{\text{Planck}}} \right) \xi_1.
\] (5.8)

To relate the flux density \( \xi_1 \) to the goldstino angle and the quantity \( C \), introduced in the previous section, we also define the density \( \xi_2 \) for a pure \((3,0)\)-flux component:

\[
\frac{1}{(2\pi)^2 \alpha'} \int_{C_3} G_{3}^{(3,0)} = \xi_2 \in \mathbb{Z}, \quad \text{i.e.} \quad G_{3}^{(3,0)} \sim (2\pi)^2 \frac{\xi_2 \alpha'}{R^3}.
\]

Then, we obtain the following relations:

\[
\frac{\xi_1}{\xi_2} = \frac{m_{3/2}}{\sqrt{V}},
\]

\[
\xi_2 = \xi_1 \sqrt{3} \tan \theta_g.
\] (5.9)

Besides, we have:

\[
\xi_2 = \pi \left( \frac{M_{\text{Planck}}}{M_{\text{string}}} \right)^3 M_{\text{string}}^{-1} \sqrt{V}.
\] (5.10)

Hence, in the following, whenever only the density \( \xi_1 \) appears, the relations (5.9) allow us to replace \( \xi_1 \) by \( \xi_2 \) through the goldstino angle.

From the relations (5.7) or (5.8) we see, that for \( M_{\text{string}} \sim M_{\text{Planck}} \), the flux density \( \xi_1 \) has to be very small, in order to arrive at a small gravitino mass. In other words, the flux has to be largely thinned out over space–time. The latter effect may be achieved with a large warping suppression. On the other hand, if the string scale \( M_{\text{string}} \) is sufficiently low
(e.g. $M_{\text{string}} \sim 10^{11}$ GeV and $R \sim 10^{-9}$ GeV$^{-1}$), reasonable values for $\xi_1$ may be chosen. Similar conclusions apply for the soft masses, which have been derived in the previous two sections. We shall discuss their scales in the following.

With the assumption of an isotropic compactification\footnote{The closed string moduli fields, introduced in Eqs. (2.10), may be written in terms of $g_{\text{string}}$ as:
\begin{equation}
T^j = a^j + i \frac{g_{\text{string}}}{2\pi \alpha'^2} \text{Im} T^k \text{Im} T^l, \quad S = C_0 + i \frac{g_{\text{string}}^{-1}}{2\pi}.
\end{equation}
Hence, for an isotropic compactification, which respects the supersymmetry constraint (3.1), we have $\text{Im} T = \text{Im} T^j = \frac{g_{\text{string}}^{-1}}{2\pi \alpha'^2} R^4$.} we obtain\footnote{For the masses of the fields $C_7^{-1} T_3$, given in (4.11), we shall only show their power series w.r.t. $M_{\text{string}}/M_{\text{Planck}}$.} for the (untwisted sector) scalar masses (4.8) and (4.9):

\begin{align*}
m_{22}^7 &= m_{33}^7 = m_{11}^7 = m_{33}^7 = \left(\frac{3\pi}{2}\right)^{1/2} g_{\text{string}}^{1/2} \frac{\alpha'}{R^2} C \sin \theta_g \\
&= \sqrt{6} \pi g_{\text{string}}^{-1/6} \left(\frac{M_{\text{string}}}{M_{\text{Planck}}}\right)^{2/3} C \sin \theta_g, \\
m_{22}^7 &= m_{33}^7 = \frac{1}{2} C \cos \theta_g, \\
m_{11}^7 &= \left(1 + \frac{\alpha'^2}{R^4}\right)^{-1/2}
\times \left\{13 + \frac{234 \alpha'^2}{R^4} - 18 \frac{\alpha'^6}{R^{12}} - 81 \frac{\alpha'^8}{R^{16}} - \left(5 + 90 \frac{\alpha'^2}{R^4} + 648 \frac{\alpha'^4}{R^8} + 18 \frac{\alpha'^6}{R^{12}} + 81 \frac{\alpha'^8}{R^{16}}\right) \cos(2\theta_g) \right. \\
&\left. + 18 \frac{\alpha'^4}{R^8} \left[72 + \sqrt{3} \sin(2\theta_g)\right]\right\}^{1/2}.
\end{align*}

Furthermore, the soft mass for the (1/2 BPS) twisted matter field $C_7^{-1} T_3$ takes the form:

\begin{align}
m_{7273} &= 2^{-5/4} \pi^{1/4} g_{\text{string}}^{1/4} \frac{\alpha'^2}{R^4} - \frac{2}{3} \frac{\alpha'^6}{R^{12}} - \frac{81}{R^{16}} \left(5 + 90 \frac{\alpha'^2}{R^4} + 648 \frac{\alpha'^4}{R^8} + 18 \frac{\alpha'^6}{R^{12}} + 81 \frac{\alpha'^8}{R^{16}}\right) \cos(2\theta_g) \\
&\times \left\{13 + \frac{234 \alpha'^2}{R^4} - 18 \frac{\alpha'^6}{R^{12}} - 81 \frac{\alpha'^8}{R^{16}} - \left(5 + 90 \frac{\alpha'^2}{R^4} + 648 \frac{\alpha'^4}{R^8} + 18 \frac{\alpha'^6}{R^{12}} + 81 \frac{\alpha'^8}{R^{16}}\right) \cos(2\theta_g) \right. \\
&\left. + 18 \frac{\alpha'^4}{R^8} \left[72 + \sqrt{3} \sin(2\theta_g)\right]\right\}^{1/2}.
\end{align}

From the above equations (5.12) and (5.13), we deduce that the soft–masses for the untwisted matter fields and $m_{7273}$ are roughly of the same order $O(m_{3/2})$ for the case of $M_{\text{string}} \sim M_{\text{Planck}}$, $g_{\text{string}} \sim (2\pi)^{-1}$ and a goldstino angle $\theta_g \not= 0, \frac{\pi}{2}, \ldots$. The masses
$m_{2_{27}}^{7,1}$, $m_{3_{33}}^{7,1}$, $m_{1_{1T}}^{7,2}$, $m_{3_{33}}^{7,2}$ of the Wilson line moduli from the $D7$–branes without 2–form fluxes stay massless in the case of a pure $(0,3)$–form flux, i.e. $\theta_g \sim 0$. Contrarily, the moduli $C_{2}^{7,2}$, $C_{3}^{7,3}$ describing the positions of the second and third stack of $D7$–branes become massive in the case of a $(0,3)$–form flux. Furthermore, the modulus $C_{1}^{7,1}$ describing the position of the first stack of $D7$–branes with non–vanishing 2–form flux becomes massive through the combined effect of a $(0,3)$–form flux and the 2–form flux. The mixing with an additional $(3,0)$–form flux is described by the goldstino angle. There is a non–universality in the masses $m_{2_{27}}^{7,2}$, $m_{3_{33}}^{7,3}$ and $m_{1_{1T}}^{7,1}$. This effect is increased by the goldstino angle. However, as we shall see in a moment, this universality disappears for string scales $M_{\text{string}} \ll M_{\text{Planck}}$.

In the following, the cosmological constant $\tilde{V}$ is assumed to be small. According to (5.1), we may choose $\mathcal{C} \sim m_{3/2}$ in the above equations. Moreover, we expand the soft–masses w.r.t. the ratio $M_{\text{string}}/M_{\text{Planck}} \ll 1$ (cf. the discussion above). This leads to the following estimates for the scalar masses of the untwisted sector:

$$m_{2_{27}}^{7,1} = m_{3_{33}}^{7,1} = m_{1_{1T}}^{7,2} = m_{3_{33}}^{7,2} = (2\pi)^{2/3} \sqrt{3} m_{3/2} \left( \frac{M_{\text{string}}}{M_{\text{Planck}}} \right)^{2/3} \sin \theta_g ,$$

$$m_{2_{27}}^{7,2} = m_{3_{33}}^{7,3} = \frac{1}{2} m_{3/2} \cos \theta_g ,$$

$$m_{1_{1T}}^{7,1} = 2^{-5/2} m_{3/2} \sqrt{13 - 5 \cos(2\theta_g)} .$$ (5.14)

In order to keep the cosmological constant small, one should aim for a small goldstino angle $\theta_g$, in which case supersymmetry breaking is mainly due to $(0,3)$–flux components. In the regime of small ratio $M_{\text{string}}/M_{\text{Planck}} \ll 1$, we observe a universality in the untwisted sector masses:

The scalar masses $m_{2_{27}}^{7,1}$, $m_{3_{33}}^{7,1}$, $m_{1_{1T}}^{7,2}$, $m_{3_{33}}^{7,2}$ referring to the Wilson line moduli vanish for a small goldstino angle, while the masses $m_{1_{1T}}^{7,1}$, $m_{2_{27}}^{7,2}$, $m_{3_{33}}^{7,3}$ referring to the $D7$–brane positions become equal for $\theta_g = 0$. On the other hand, the latter vanish for $\theta_g \sim \frac{\pi}{2}$. Of course, this observation is just in lines with the fact that a pure $(3,0)$–form flux gives rise to the scalar masses of the Wilson line moduli only, while a pure $(0,3)$–form flux gives masses only to the transverse $D7$–brane position moduli. Note, that the 2–form flux dependence of $m_{1_{1T}}^{7,1}$ has completely disappeared in the limit $M_{\text{string}}/M_{\text{Planck}} \ll 1$. Hence, the universality is independent of the 2–form flux turned on, at least to lowest order in $M_{\text{string}}/M_{\text{Planck}}$.

Let us now turn to the expansion of the scalar masses of the twisted sector, given in Eqs. (4.13) and (4.11):

$$m_{2_{27}}^{7,2} = 2^{-2/3} \pi^{1/3} m_{3/2} \left( \frac{M_{\text{string}}}{M_{\text{Planck}}} \right)^{1/3} \sqrt{2 - \cos(2\theta_g)} ,$$

$$m_{1_{1T}}^{7,1} = 3^{-1/4} \frac{2^{1/2} + 2\beta + 4\gamma}{\pi^{7/2} + \beta + 2\gamma} m_{3/2} \left( \frac{M_{\text{string}}}{M_{\text{Planck}}} \right)^{1/2 + \beta + 2\gamma} \times \sqrt{2 + 3\beta - (1 + 6\beta + 6\gamma) \cos(2\theta_g)} .$$ (5.15)
Let us compare the $1/4$ BPS sector masses $m^{7_17_2}, m^{7_17_3}$ with the Higgs mass $m^{7_27_3}$. We obtain the following ratio

$$m^{7_17_2} / m^{7_27_3} = m^{7_17_3} / m^{7_27_3} = 3^{-1/4} \left( 2^{1 + \frac{1}{2} \beta + 4 \gamma} \pi^{\frac{1}{4} + \beta + 2 \gamma} \left( \frac{M_{\text{string}}}{M_{\text{Planck}}} \right)^{\beta + 2 \gamma} \right) \sqrt{\frac{2 + 3 \beta - (1 + 6 \beta + 6 \gamma) \cos(2 \theta_g)}{2 - \cos(2 \theta_g)}},$$

which becomes

$$m^{7_17_2} / m^{7_27_3} = m^{7_17_3} / m^{7_27_3} = 3^{-1/4} \left( 2^{1 + \frac{1}{2} \beta + 4 \gamma} \pi^{\frac{1}{4} + \beta + 2 \gamma} \left( \frac{M_{\text{string}}}{M_{\text{Planck}}} \right)^{\beta + 2 \gamma} \sqrt{1 - 3 \beta - 6 \gamma} \right)$$

for $\theta_g \to 0$. Hence, the ratio is very sensitive to the constants $\beta, \gamma$ to be determined in [44]. For $\beta, \gamma \neq 0$ the ratios (5.17) are in lines of a split–SUSY scenario [46].

Finally, let us discuss the gaugino masses, which have been presented in Eq. (4.15):

$$m_{g,2} = m_{g,3} = C \cos \theta_g \ e^{i \alpha_T},$$

$$m_{g,1} = C \left( 1 + 9 \frac{\alpha'^2}{R^4} \right)^{-1} (e^{i \alpha_T} \cos \theta_g + 9 \sqrt{3} \ e^{i \alpha_s} \frac{\alpha'^2}{R^4} \sin \theta_g).$$

As already anticipated in the introduction, the gaugino masses referring to $D7$–brane stacks without 2–form fluxes are only sensitive to $(0, 3)$–flux components, i.e. their masses are proportional to $\cos \theta_g$. On the other hand, $D7$–branes with non–vanishing 2–form fluxes on their internal world–volume lead to gaugino masses, which feel both $(0, 3)$– and $(3, 0)$–flux components. In other words, $m_{g,1}$ is generically non–vanishing. However, the dependence of $m_{g,1}$ on the $(3, 0)$–flux component is sub–leading in $\alpha'$. In particular, all three stacks give rise to gaugino masses with the same leading power behaviour w.r.t. $M_{\text{string}}/M_{\text{Planck}}$:

$$m_{g,1} = m_{g,2} = m_{g,3} = m_{3/2} \cos \theta_g \ e^{i \alpha_T}. \quad (5.19)$$

Hence, like the scalar masses the gaugino masses are universal for $M_{\text{string}}/M_{\text{Planck}} \ll 1$.

6. Concluding remarks

In this paper we have computed the $G$–flux induced soft supersymmetry breaking terms in semirealistic $D$–brane models, in which the gauge/matter sector of the MSSM originated from open strings on $D7$–branes with $f$–flux. Specifically, the matter fields, namely quarks, lepton, Higgs fields and their N=1 superpartners correspond to twisted open string sectors ending on $D7$–branes with different $f$–flux boundary conditions. The analysis was performed in a local $D7$–brane model whose twisted spectrum is just the one
of the MSSM. Tadpole cancellation on a compact orbifold will require additional hidden sector $D7$–branes with $f$–flux. The soft masses were computed as a function of the Kähler moduli, being still partially unfixed despite the supersymmetry conditions, and also as a function of two 3–form flux components, the $(0, 3)$–flux $G_{(0,3)}$ and the $(3, 0)$–flux $G_{(3,0)}$. In order to obtain a tiny cosmological constant together with a non-vanishing gravitino mass, the $G_{(3,0)}$ flux component must be much smaller compared to $G_{(0,3)}$. In other words, the goldstino angle $\theta_g$, introduced in subsection 3.4, has to be very small. Moreover, in order to keep $m_{3/2}$ much below $M_{\text{Planck}}$–namely in the $TeV$–region, the string scale must be sufficiently low. Then the gaugino masses are also of the order of $m_{3/2}$ (cf. section 4).

The squark and slepton masses exhibit a more complicated moduli dependence and are in general non-universal, as they depend on the $f$–fluxes of the involved $D7$–branes (the intersection angles in type $IIA$ language with intersecting $D6$–branes). As a result of our analysis in section 5, it turns out that for a low string scale, the squark and slepton masses are considerably different than the SUSY–breaking mass contribution to the Higgs field (cf. the ratio (5.17)). This may be in favor for a split–SUSY scenario [46]. The soft masses of squark and slepton fields in different families, i.e. open strings sitting at different ‘intersection angles’, are the same, as long as the ‘intersection angles’ of the different families agree. This will usually be the case in concrete MSSM–like models. Finally, the gaugino masses are typically of the same order as $m_{3/2}$ (cf. Eq. (5.13)).

The orientifold models on orbifold backgrounds considered here, so far suffer at least one serious phenomenological problem: as we discussed, there will be also matter fields from untwisted open string sectors (N=4 sectors) with open strings ending on the same $D7$–branes. The corresponding scalar and fermion fields transform in the adjoint representation of the gauge group factors, and they correspond to Wilson line fields or scalars, that describe the locations of the $D7$–branes, commonly denoted by $D7$–brane moduli. Without any other 3-form fluxes as $G_{(0,3)}$ and $G_{(3,0)}$ turned on, the adjoint fermions will stay massless and the adjoint scalar components will get a soft mass, shown in Eqs. (5.12) and (5.14). In the MSSM sector, this is clearly unacceptable since these states are not observed. But also in the hidden sectors they cause a serious problem, because they lead to a negative (non-asymtotically free) $\beta$-function, which forbids a non-perturbative superponential, e.g. by gaugino condensation. Therefore, additional effects are required in order to give these adjoint multiplets a large supersymmetry preserving mass.

One possibility to acquire this kind of desired mass terms is to consider $F$–theory with 4–form flux $G_4$ turned on, which gives rise to a flux superpotential of the form $W \sim \int G_4 \wedge \Omega_4$. Here, the $D7$–brane moduli represent complex structure moduli of the fourfold and therefore naturally enter the superpotential. A concrete example of this type, namely $F$–theory on $K3 \times K3$, was recently discussed in Ref. [23], where a flux induced supersymmetric mass term for the $D7$–brane moduli was indeed generated. The latter result agrees with computations of gauged supergravity [47]. However, the type
The interpretation of this mass term is still somewhat unclear, since in the type *II*\(\text{B}\) language, a mass for the open string moduli has to be generated, whereas the 3–form flux induced superpotential (2.11) a priori only depends on the closed string moduli fields. Nevertheless, this mass term for the \(D_7\)–brane moduli will arise in type *II*\(\text{B}\) orientifolds with \(N=2\) sectors after turning on a suitable supersymmetric \((2,1)\)–flux component, as we will show in Ref. [48].

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**Appendix A. Curvature tensors for untwisted and twisted \(D_7\)–brane fields**

For the untwisted matter fields of the stacks without \(f\)-flux, the curvature tensors take a particularly simple form. We get only the following non-zero elements:

\[
\begin{align*}
R_{U_iU_ji}^{7,2} &= -\frac{1}{(U_i - U_j)^2} G_{C_iC_j}^{7,2}, \quad i = 1, 2, 3, \\
R_{T^iT^j1\Omega}^{7,2} &= -\frac{1}{(T^i - T^j)^2} G_{C_1C_1}^{7,2}, \\
R_{S^1S^22\Omega}^{7,2} &= -\frac{1}{(S - S)^2} G_{C_2C_2}^{7,2}, \\
R_{T^iT^j3\Omega}^{7,2} &= -\frac{1}{(T^i - T^j)^2} G_{C_3C_3}^{7,2}.
\end{align*}
\]

For stack 3, we get the same result with indices 2 and 3 interchanged.

From now on, we will take \(M, N\) to run over \(S, T^i\) only as we consider here the case with \((3,0)\)– and \((0,3)\)–fluxes only.

Due to the non-vanishing \(f\)-flux on stack 1, the form of the curvature tensor is here much more involved, the components with mixed moduli are no longer zero and the expression is long and ugly, the reason for which being the dependence of the \(T^i\) on all the \(T^i\) and on \(S\):

\[
\begin{align*}
\text{Im}T^1 &= \alpha' \left( \frac{T^2_2 T^3_2}{S^2_2 T^2_2} \right)^{1/2}, \quad \text{Im}T^2 = \alpha' \left( \frac{T^1_2 T^3_2}{S^2_2 T^2_2} \right)^{1/2}, \\
\text{Im}T^3 &= \alpha' \left( \frac{T^1_2 T^2_2}{S^2_2 T^3_2} \right)^{1/2}, \quad e^{-\phi_4} = 2\pi \left( S^2_2 T^1_2 T^2_2 T^3_2 \right)^{1/4}.
\end{align*}
\]

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Here, we need to know the \( \partial_M (\text{Im} T^j) \):

\[
\begin{align*}
\frac{\partial (\text{Im} T^j)}{\partial S} &= \frac{i}{4} \frac{\text{Im} T^j}{\text{Im} S}, \\
\frac{\partial (\text{Im} T^j)}{\partial T^j} &= \frac{i}{4} \frac{\text{Im} T^j}{\text{Im} T^j}, \\
\frac{\partial (\text{Im} T^j)}{\partial T^k} &= -\frac{i}{4} \frac{\text{Im} T^j}{\text{Im} T^k}, \quad j \neq k. 
\end{align*}
\]

\( (A.2) \)

We will also need the \( \partial_M \partial_N (\text{Im} T^j) \):

\[
\begin{align*}
\frac{\partial (\text{Im} T^j)}{\partial S \partial S} &= \frac{3}{16} \frac{(\text{Im} T^j)^2}{(\text{Im} S)^2}, \\
\frac{\partial (\text{Im} T^j)}{\partial T^j \partial S} &= \frac{1}{16} \frac{(\text{Im} T^j)}{(\text{Im} S)(\text{Im} T^j)}, \\
\frac{\partial (\text{Im} T^j)}{\partial T^k \partial S} &= -\frac{1}{16} \frac{(\text{Im} T^j)}{(\text{Im} S)(\text{Im} T^j)}, \\
\frac{\partial (\text{Im} T^j)}{\partial T^k \partial T^j} &= \frac{1}{16} \frac{(\text{Im} T^j)}{(\text{Im} T^j)^2}, \\
\frac{\partial (\text{Im} T^j)}{\partial T^j \partial T^k} &= \frac{3}{16} \frac{(\text{Im} T^j)}{(\text{Im} T^j)^2}, \quad i \neq j \neq k.
\end{align*}
\]

\( (A.3) \)

where \( M, N \) run over \( S, T^i \).

After some algebra we get the following results:

\[
R^{7,1}_{M N 22} = \frac{G^{7,1}_{C_2 \overline{C}_2} (g \alpha')^2}{\text{Im} M \text{ Im} N} \left[ \frac{(g \alpha')^2}{\text{Im} T^2} \left( \frac{(g \alpha')^2}{\text{Im} T^2} \right)^2 \right] \\
\times \left\{ (g \alpha')^6 \left( \beta_2(M, N) - \beta_3(M, N) - \alpha_2(M, N) + \alpha_3(M, N) \right) \\
+ (g \alpha')^4 \left[ (\beta_2(M, N) - 2\beta_3(M, N) - 3\alpha_2(M, N) + 2\alpha_3(M, N)) (\text{Im} T^2)^2 \\
+ (2\beta_2(M, N) - 2\beta_3(M, N) - 2\alpha_2(M, N) + 2\alpha_3(M, N) + 3\alpha_3(M, N)) (\text{Im} T^3)^2 \right] \\
+ (g \alpha')^2 \left[ (-\beta_3(M, N) + \alpha_3(M, N)) (\text{Im} T^2)^4 + (\beta_2(M, N) - \alpha_2(M, N)) (\text{Im} T^3)^4 \right. \\
+ 2 \left( \beta_2(M, N) - \beta_3(M, N) - 3\alpha_2(M, N) + 3\alpha_3(M, N) \right) (\text{Im} T^2)^2 (\text{Im} T^3)^2 \right] \\
+ \left( \beta_2(M, N) - 3\alpha_2(M, N) \right) (\text{Im} T^3)^2 \right\}, \quad M, N \neq T^3,
\]

\[
R^{7,1}_{T^3 T^3 22} = \frac{G^{7,1}_{C_2 \overline{C}_2} (g \alpha')^2}{4 \left( \text{Im} T^3 \right)^2} \left\{ 1 - \frac{1}{2} \frac{(g \alpha')^2}{\left( \text{Im} T^2 \right)^2} \left[ \frac{(g \alpha')^2}{\text{Im} T^2} + 2 \left( \text{Im} T^2 \right)^2 \right] - \frac{1}{2} \frac{(g \alpha')^4}{\left( \text{Im} T^2 \right)^2} \right\}, \\
\quad (A.4)
\]

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The constants $\alpha_{2,3}$, $\beta_{2,3}$ and $\gamma_{2,3}$ are summarized in tables A.1 and A.2.

Now we will look at the twisted case. The curvature tensor for the sector (23) must be treated separately from the one for (12), (13), as these sectors do not possess the same amount of supersymmetry and have different metrics. We first examine the $1/4$ BPS sector. The non-zero components of the curvature are:

$$R_{MN}^{\alpha_1,\alpha_2} = \frac{- (g\alpha')^2 \, C_{\alpha_1, \alpha_2}}{[(g\alpha')^2 + \text{Im} T^2 \text{Im} T^3]^2 \, \text{Im} M \text{Im} N} \left\{ - [(g\alpha')^2 + 2 \text{Im} T^2 \text{Im} T^3] \, [\alpha_3(M, N) + \alpha_2(M, N)] 
+ [(g\alpha')^2 + \text{Im} T^2 \text{Im} T^3] \, [\beta_3(M, N) + \beta_2(M, N)] 
- \text{Im} T^2 \text{Im} T^3 [\gamma_3(M) \gamma_2(N) + \gamma_2(M) \gamma_3(N)] \right\}, \quad M, N \neq S,$$

$$R_{SS}^{\alpha_1,\alpha_2} = \frac{G_{C_{\alpha_1, \alpha_2}}}{4 (\text{Im} S)^2} \left\{ 1 - \frac{(g\alpha')^4}{[(g\alpha')^2 + \text{Im} T^2 \text{Im} T^3]^2} \right\}. \quad (A.5)$$

The constant $\alpha_{2,3}$, $\beta_{2,3}$ and $\gamma_{2,3}$ are summarized in tables A.1 and A.2.
with \((a, b) = (1, 2)\) or \((1, 3)\) and where \(\psi_n\) is the \(n\)'th Polygamma function, with \(\psi_0 \equiv \psi\). Obviously, \(\partial_M \theta_{12}^2 = \partial_M \theta_{13}^3 = 0\), and

\[
\partial_M \theta_{12}^2 = \partial_M \theta_{13}^3 = \frac{1}{\pi} \frac{\alpha' g}{(\alpha' g)^2 + (\text{Im} T^2)^2} \partial_M (\text{Im} T^2) ,
\]

\[
\partial_M \theta_{13}^3 = \partial_M \theta_{13}^3 = -\frac{1}{\pi} \frac{\alpha' g}{(\alpha' g)^2 + (\text{Im} T^3)^2} \partial_M (\text{Im} T^3) ,
\]

\[
\partial_M \partial_N \theta_{12}^2 = \partial_M \partial_N \theta_{13}^3 = \frac{1}{\pi} \frac{\alpha' g}{[(\alpha' g)^2 + (\text{Im} T^2)^2]^2} \left[ -2 \left( \text{Im} T^2 \right) \partial_M (\text{Im} T^2) \partial_N (\text{Im} T^2) \right.
\]

\[
\left. + \{(\alpha' g)^2 + (\text{Im} T^2)^2\} \partial_M \partial_N (\text{Im} T^2) \right] ,
\]

\[
\partial_M \partial_N \theta_{13}^3 = \partial_M \partial_N \theta_{13}^3 = -\frac{1}{\pi} \frac{\alpha' g}{[(\alpha' g)^2 + (\text{Im} T^3)^2]^2} \left[ -2 \left( \text{Im} T^3 \right) \partial_M (\text{Im} T^3) \partial_N (\text{Im} T^3) \right.
\]

\[
\left. + \{(\alpha' g)^2 + (\text{Im} T^3)^2\} \partial_M \partial_N (\text{Im} T^3) \right] .
\]

(A.7)

After substituting equations (A.2), (A.3) and (A.7) back into (A.6), we find after some algebra:

\[
R_{MN}^{7_17_2} = \frac{G_{C^{7_17_2}C^{7_17_2}}}{\text{Im} M \text{Im} N} \left[ X_{MN} + \sum_{i=2,3} \frac{g \alpha'}{\pi} \left[ (g \alpha')^2 + (\text{Im} T^i)^2 \right]^2 \right]
\]

\[
\times \left\{ s_i \left( \gamma \ln(T^i - \bar{T}^i) - \ln(U^i - \bar{U}^i) + \frac{1}{2} \left[ \psi_0(\theta_{ab}^i) + \psi_0(1 - \theta_{ab}^i) \right] \right) \right.
\]

\[
\times \left( [\beta_i(M, N) - 2 \alpha_i(M, N)] (\text{Im} T^i)^3 + \beta_i(M, N) (\alpha' g)^2 \text{Im} T^i \right)
\]

\[
+ \frac{1}{2} \frac{\alpha' g}{\pi} \left[ \psi_1(\theta_{ab}^i) - \psi_1(1 - \theta_{ab}^i) \right] \alpha_i(M, N) (\text{Im} T^i)^2 \right\} .
\]

(A.8)

Here, the \(\psi_n\) are the \(n\)'th Polygamma functions. In addition, we have introduced the factor \(s_i\), with \(s_1 = 0, s_2 = 1, s_3 = -1\) and:

\[
X_{S3S} = \frac{1}{16} \left( 1 - 6\beta - 4\gamma \right) ,
\]

\[
X_{ST^i} = -\frac{s_j}{8\pi} \gamma \frac{\alpha' g}{(\alpha' g)^2 + (\text{Im} T^j)^2} \text{Im} T^j ,
\]

\[
X_{T^iS} = \frac{1}{16} \left[ 1 + 2\beta + 4\gamma (1 - \theta_{ab}^j) \right] - \frac{s_j}{4\pi} \gamma \frac{\alpha' g}{(\alpha' g)^2 + (\text{Im} T^j)^2} \text{Im} T^j ,
\]

\[
X_{T^iT^j} = \gamma \frac{\alpha' g}{8\pi} \left[ s_i \frac{\text{Im} T^i}{(\alpha' g)^2 + (\text{Im} T^i)^2} + s_j \frac{\text{Im} T^j}{(\alpha' g)^2 + (\text{Im} T^j)^2} \right] , \quad i \neq j .
\]

(A.9)

For the 1/2 BPS sector (23), the calculation is simpler and leads to:

\[
R_{MM}^{7_27_3} = \frac{-1}{2} \frac{1}{(M - M)^2} G_{C^{7_27_3}C^{7_27_3}}
\]

(A.10)
for $M = S, T^1, U^2, U^3$, for the other moduli, the components are zero.

Finally, Table A.1 shows the quantities

$$\beta_j(M, N) = \frac{\text{Im}M \text{Im}N}{\text{Im}T^j} \frac{\partial^2 \text{Im}T^j}{\partial M \partial N}, \quad \alpha_j(M, N) = \frac{\text{Im}M \text{Im}N}{(\text{Im}T^j)^2} \frac{\partial \text{Im}T^j}{\partial M} \frac{\partial \text{Im}T^j}{\partial N},$$

| $(M, N)$ | $\beta_2$ | $\alpha_2$ | $\beta_3$ | $\alpha_3$ |
|-----------|-----------|-----------|-----------|-----------|
| $(S, S)$  | $\frac{1}{16}$ | $\frac{-1}{16}$ | $\frac{1}{16}$ | $\frac{-1}{16}$ |
| $(S, T^1)$| $\frac{1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ | $\frac{1}{16}$ |
| $(S, T^2)$| $\frac{-1}{16}$ | $\frac{1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ |
| $(S, T^3)$| $\frac{1}{16}$ | $\frac{-1}{16}$ | $\frac{1}{16}$ | $\frac{-1}{16}$ |
| $(T^1, T^1)$| $\frac{-1}{16}$ | $\frac{1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ |
| $(T^1, T^2)$| $\frac{-1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ |
| $(T^1, T^3)$| $\frac{1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ |
| $(T^2, T^2)$| $\frac{-1}{16}$ | $\frac{1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ |
| $(T^2, T^3)$| $\frac{-1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ |
| $(T^3, T^3)$| $\frac{-1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ | $\frac{-1}{16}$ |

Table A.1

while Table A.2 displays

$$\gamma_j(M) = \frac{\text{Im}M}{\text{Im}T^j} \frac{\partial \text{Im}T^j}{\partial M}$$

| $M$ | $\gamma_2$ | $\gamma_3$ |
|-----|-----------|-----------|
| $S$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $T^1$ | $\frac{-1}{4}$ | $\frac{-1}{4}$ |
| $T^2$ | $\frac{-1}{4}$ | $\frac{1}{4}$ |
| $T^3$ | $\frac{-1}{4}$ | $\frac{1}{4}$ |

Table A.2
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