Like-sign dimuon charge asymmetry in Randall-Sundrum model

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We confirm that in order to account for the recent DØ result of large like-sign dimuon charge asymmetry, a considerable large new physics effect in $\Gamma_s^{12}$ is required in addition to a large CP violating phase in $B_s - \bar{B}_s$ mixing. In the Randall-Sundrum model of warped geometry, where the fermion fields reside in the bulk, new sources of flavor and CP violation are obtained. We analyze the like-sign dimuon asymmetry in this class of model as an example of the desired new physics. We show that the wrong-charge asymmetry, $a_{s_{\text{sl}}}$, which is related to the dimuon asymmetry, is significantly altered compared to the standard model value. However, experimental limits from $\Delta M_s$, $\Delta \Gamma_s$ as well as $K$ mixing and electroweak corrections constrain it to be greater than a $\sigma$ away from its experimental average value. This model cannot fully account for the DØ anomaly due to its inability to generate a sufficient new contribution to the width difference $\Gamma_s^{12}$, even though the model can generate large contribution to the mass difference $M_{12}^s$.

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I. INTRODUCTION

The B factories, BaBar and Belle, have firmly established the CKM mechanism as the leading order contributor to CP violating phenomena in the quark sector. New physics (NP) effects can add to the leading order term producing deviations from the standard model (SM) predictions. These deviations are expected to be more pronounced in rare FCNC processes, as they are suppressed in the SM. The Belle experiment is scheduled for an upgrade [1] which will result in very precise results in $B$ decays. LHCb is ready to take data and is expected to make many important measurements in $b$ quark decays. These measurements may reveal the presence of new physics.

In recent years, there have been several measurements of $B$ decays which differ from the predictions of the SM by $\sim 2\sigma$. For example, in $B \to \pi K$, the SM has difficulty accounting for all the experimental measurements [2]. The measured indirect (mixing-induced) CP asymmetry in several $b \to s$ penguin decays is not found to be identical to that in $B^0_d \to J/\psi K_s$ [3], counter to expectations of the SM and could be

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providing hints for new physics \([4]\). The large transverse polarization in some penguin dominated decays to light vector particles, like \(B \to \phi K^* \) \([2]\), are also somewhat difficult to understand in the SM where naively one expects the transverse polarization amplitudes to be suppressed. A further effect has recently been seen in the lepton sector: in the exclusive decay \(B_d^0 \to \bar{K}^{*0} \mu^+ \mu^- \), the forward-backward asymmetry has been found to deviate somewhat from the predictions of the SM \([4, 7]\). Although this disagreement is not statistically significant, the Belle experiment itself claims this measurement shows a clear hint of physics beyond the SM \([5]\), and the CDF \([10]\) and D0 \([11]\) Collaborations have measured indirect CP violation in \(B_s \to J/\psi \phi \). The first measurement is the phase of \(\Delta \beta_s \), and the “wrong-charge” asymmetries \(\Delta \gamma_5 \) between the two mass eigenstates have been measured. Two other measurements in the \(B_s\) system have generated enormous interest as they do not appear to agree with the SM predictions. The first measurement is the phase of \(B_s^0 - \bar{B}_s^0\) mixing which can be measured via indirect CP violation in \(B_s \to J/\psi \phi \). The CDF \([10]\) and D0 \([11]\) Collaborations have measured indirect CP violation in \(B_s \to J/\psi \phi \). The experiments measured \(S_{\psi \phi} = -2\beta_s\), and found \([3]\)

\[
\beta_s = 0.41^{+0.18}_{-0.15} \quad \text{or} \quad 1.16^{+0.15}_{-0.18} .
\]  

This disagrees with the SM prediction

\[
\beta_s^{\text{SM}} = 0.019 \pm 0.001.
\]  

Implications of this measurement for NP models have been analyzed \([12, 13]\).

The second measurement was made recently in the \(B_s^0 - \bar{B}_s^0\) system when the D0 Collaboration measured the like-sign dimuon charge asymmetry with 6.1 fb\(^{-1}\) of data \([14]\). The following result was reported:

\[
A_{\text{sl}}^b = -(9.57 \pm 2.51\, \text{(stat)} \pm 1.46\, \text{(syst)}) \times 10^{-3}.
\]  

The like-sign dimuon charge asymmetry \(A_{\text{sl}}^b\) for semileptonic decays of \(b\) hadrons produced in \(\bar{p}p\) collision is defined as

\[
A_{\text{sl}}^b = \frac{N_{b}^{++} - N_{b}^{--}}{N_{b}^{++} + N_{b}^{--}},
\]  

where \(N_{b}^{++}\) and \(N_{b}^{--}\) are the number of events containing two \(b\) hadrons that decay semileptonically into \(\mu^+\mu^+X\) and \(\mu^-\mu^-X\), respectively. The semileptonic decays of both \(B_d\) and \(B_s\) can contribute to \(A_{\text{sl}}^b\). The relation between \(A_{\text{sl}}^b\) and the ”wrong-charge” asymmetries \(a_{\text{sd}}^q\) and \(a_{\text{sl}}^q\) is given by \([15]\)

\[
A_{\text{sl}}^b = \left( \frac{f_d z_d}{f_d z_d + f_s z_s} \right) a_{\text{sd}}^q + \left( \frac{f_s z_s}{f_d z_d + f_s z_s} \right) a_{\text{sl}}^q,
\]  

where \(z_q = 1/(1 - y_q^2) = 1/(1 + x_q^2)\) \((q = d, s)\). Here \(f_d\) and \(f_s\) denote the production fraction of \(B_d\) and \(B_s\), and the quantities \(x_q\) and \(y_q\) are given as,

\[
x_q = \frac{\Delta M_q}{\Gamma_q}, \quad y_q = \frac{\Delta \Gamma_q}{2 \Gamma_q},
\]  

where \(\Delta M_q\) and \(\Delta \Gamma_q\) are the mass and width differences in the \(B_q^0 - \bar{B}_q^0\) system. The semileptonic wrong-charge asymmetry \(a_{\text{sl}}^q\) is defined as

\[
a_{\text{sl}}^q = \frac{\Gamma(B_q \to \mu^+X) - \Gamma(B_q \to \mu^-X)}{\Gamma(B_q \to \mu^+X) + \Gamma(B_q \to \mu^-X)}
\]
Using the known values \( f_d = 0.323 \pm 0.037, f_s = 0.118 \pm 0.015, x_d = 0.774 \pm 0.008, y_d \approx 0, x_s = 26.2 \pm 0.5 \) and \( y_s = 0.0046 \pm 0.027 \) [14, 16], one can rewrite Eq. (5) as
\[
A_s^b = (0.506 \pm 0.043)a_s^d + (0.49 \pm 0.043)a_s^s.
\]

The SM predictions for the charge asymmetries are [14]
\[
a_s^d = (-4.8^{+1.0}_{-1.2}) \times 10^{-4}, \quad a_s^s = (2.1 \pm 0.6) \times 10^{-5}.
\]

The SM result for \( A_s^b \) can be obtained using Eqs. (8) and (9) as
\[
A_s^b = (-2.3^{+0.5}_{-0.6}) \times 10^{-4},
\]
which is about 3.2 \( \sigma \) away from the value in Eq. (3).

The SM prediction of the charge symmetry \( a_s^d \) in Eq. (9) is consistent with the observed value \( a_s^d = 0.0047 \pm 0.0046 \) [3, 14], within errors. In order to obtain DO measurement for \( A_s^b \) in Eq. (3) using the measured \( a_s^d \), the value of the charge symmetry \( a_s^s \) needs to be [14]
\[
a_s^s = -(14.6 \pm 7.5) \times 10^{-3}.
\]

This value is much larger than its SM prediction in Eq. (9). The DØ direct measurement of \( a_s^s = -(1.7 \pm 9.1 \text{(stat)}^{+1.4}_{-1.5} \text{(sys)}) \times 10^{-3} \) [17], is consistent with the SM value in Eq. (9). An average value for \( a_s^s \) can be extracted by combining the DO and CDF [18] measurements as [19]
\[
(a_s^s)_{\text{avg}} \approx -(12.7 \pm 5.0) \times 10^{-3}.
\]

This average value of \( a_s^s \) is about 2.5\( \sigma \) away from its SM value in Eq. (9). A confirmation of this deviation would be an unambiguous evidence for new physics, and already interpretations of this result in terms of NP have been performed in various extensions of the SM [19, 22].

In this work we consider the warped extra dimension Randall-Sundrum (RS) model [23]. This model was proposed to solve the hierarchy problem in the SM and in this framework some of the flavor puzzles in the SM can be addressed in the split fermion scenario with the fermions located at different points in the extra dimension [24, 26].

In this paper we work out the contribution to the parameters \( \Gamma_{12}^s \) and \( M_{12}^s \) in the \( B_s^0 - \bar{B}_s^0 \) system for the general case of NP with operators that are of the vector and/or axial vector types. The general formula that we derive can be used for several extensions of the SM. Taking the RS model as an example for new physics we use our general expressions to compute the contribution to \( \Gamma_{12}^s \) and \( M_{12}^s \).

The paper is organized as follows. In the first section, we present an overview of the phenomenology of the \( B_s^0 - \bar{B}_s^0 \) system including constraints on NP with present measurements. In the second section, we present the general expression for \( \Gamma_{12}^s \) and \( M_{12}^s \) for general new physics containing vector and/or axial vector operators. FCNC effects in the RS model with split fermions are discussed in the next section. The subsequent sections contain our numerical results and conclusions.

II. MODEL INDEPENDENT ANALYSIS OF \( B_s^0 - \bar{B}_s^0 \) MIXING

In this section we will briefly review the phenomenology of the \( B_s^0 - \bar{B}_s^0 \) system for \( q = s, d \). The formalism for \( B \) mixing is well known but we will review it here for completeness and study the constraints on NP imposed by measurements in this system.
The $B_q^0$ and $\bar{B}_q^0$ states can mix in the presence of weak interactions. The resulting mass eigenstates can differ in their masses and lifetimes. In the $B_q - \bar{B}_q$ system, the time evolution of the general state is governed by the Schrödinger equation

$$i \frac{d}{dt} \begin{pmatrix} B_q(t) \\ \bar{B}_q(t) \end{pmatrix} = \mathcal{H}_q \begin{pmatrix} B_q(t) \\ \bar{B}_q(t) \end{pmatrix},$$

(13)

where the Hamiltonian $\mathcal{H}_q$ is given in terms of the $2 \times 2$ Hermitian mass ($M_q$) and the decay width ($\Gamma_q$) matrices

$$\mathcal{H}_q = \left( M_q - \frac{i}{2} \Gamma_q \right) = \begin{bmatrix} M_{11}^q - \frac{i}{2} \Gamma_{11}^q & M_{12}^q - \frac{i}{2} \Gamma_{12}^q \\ M_{12}^q - \frac{i}{2} \Gamma_{12}^q & M_{11}^q - \frac{i}{2} \Gamma_{11}^q \end{bmatrix}.$$  

(14)

The mass eigenstates are the eigenvectors of $\mathcal{H}_q$. The eigenvectors with the lightest and heaviest mass eigenvalues can be written as

$$|B_q^L\rangle = p |B_q\rangle + q |\bar{B}_q^0\rangle, \quad |B_q^H\rangle = p |B_q\rangle - q |\bar{B}_q^0\rangle,$$

(15)

with $|p|^2 + |q|^2 = 1$. The masses and widths of these mass eigenstates are

$$M_q^{H,L} = M_{11}^q \pm Re\sqrt{\left(M_{12}^q - \frac{i}{2} \Gamma_{12}^q\right)^2 - \left(M_{12}^q - \frac{i}{2} \Gamma_{12}^q\right)^2},$$

$$\Gamma_q^{H,L} = \Gamma_{11}^q \mp 2Im\sqrt{\left(M_{12}^q - \frac{i}{2} \Gamma_{12}^q\right)^2 - \left(M_{12}^q - \frac{i}{2} \Gamma_{12}^q\right)^2}.$$  

(16)

One can now construct the following observables

$$M_q = \frac{M_q^H + M_q^L}{2} = M_{11}^q, \quad \Gamma_q = \frac{\Gamma_q^H + \Gamma_q^L}{2} = \Gamma_{11}^q,$$

$$\Delta M_q = M_q^H - M_q^L = 2Re\sqrt{\left(M_{12}^q - \frac{i}{2} \Gamma_{12}^q\right)^2 - \left(M_{12}^q - \frac{i}{2} \Gamma_{12}^q\right)^2},$$

$$\Delta \Gamma_q = \Gamma_q^L - \Gamma_q^H = 4Im\sqrt{\left(M_{12}^q - \frac{i}{2} \Gamma_{12}^q\right)^2 - \left(M_{12}^q - \frac{i}{2} \Gamma_{12}^q\right)^2}.$$  

(17)

The mass difference, the width difference and the parameters in the eigenvectors expression in Eq. (15) can be written as

$$\left(\Delta M_q\right)^2 - \frac{1}{4} (\Delta \Gamma_q)^2 = 4 \left(|M_{12}^q|^2 - \frac{1}{4} |\Gamma_{12}^q|^2\right),$$

$$\Delta M_q \Delta \Gamma_q = -4 Re\left[M_{12}^q \Gamma_{12}^q\right],$$

$$\left(\frac{q}{p}\right)_q = -\frac{2 M_{12}^q - i \Gamma_{12}^q}{\Delta M_q + \frac{1}{2} \Delta \Gamma_q}.$$  

(18)

One usually defines the two dimensionless quantities

$$x_{B_q} = \frac{\Delta M_q}{\Gamma_q}, \quad y_{B_q} = \frac{\Delta \Gamma_q}{2 \Gamma_q}.$$  

(19)

Measurements indicate $y_{B_q} \sim O(10^{-2})$ while $x_{B_q} \sim 1$. These results model independently imply

$$\Delta \Gamma_q \ll \Delta M_q.$$  

(20)
Thus to a good approximation
\[ \Delta M_q = 2|M_{12}^q|, \quad \Delta \Gamma_q = -2Re[M_{12}^q \Gamma_{12}^{q*}] = 2|\Gamma_{12}| \cos \phi_q, \quad \left( \frac{q}{p} \right) = -\frac{M_{12}^q}{|\Gamma_{12}|} \left( 1 - \frac{1}{2} Im \Gamma_{12}^{q*} \right), \] (21)
where
\[ \frac{M_{12}}{|\Gamma_{12}|} = -|M_{12}| e^{i \phi_q}, \quad \phi_q = arg \left( -\frac{M_{12}}{|\Gamma_{12}|} \right). \] (22)

The semileptonic wrong-charge asymmetry \( a_{sl}^q \) is defined as
\[ a_{sl}^q = \frac{\Gamma(\bar{B}_q \to \mu^+ X) - \Gamma(B_q \to \mu^- X)}{\Gamma(\bar{B}_q \to \mu^+ X) + \Gamma(B_q \to \mu^- X)} = Im[\Gamma_{12}^q/|\Gamma_{12}|], \]
\[ a_{sl}^q = \frac{|\Gamma_{12}|}{|\Gamma_{12}|} \sin \phi_q = \frac{\Delta \Gamma_q}{\Delta M_q} \tan \phi_q. \] (23)

The off-diagonal element \( M_{12}^q \) of the matrix \( M \) is related to the dispersive part of the \( \Delta B = 2 \) transition amplitude
\[ M_{12}^q = \frac{1}{2m_{B_q}} \langle \bar{B}_q | H_{eff}^{\Delta B = 2} | B_q \rangle, \] (24)
where \( m_{B_q} \) is the \( B_q \) meson mass. In the presence of NP contributions to \( M_{12}^q \), it can be written as
\[ M_{12}^q = M_{12}^{q,SM} + M_{12}^{q,NP} = M_{12}^{q,SM} R_M^q e^{i \delta_M^q}, \] (25)
where
\[ R_M^q = |1 + r_M^q e^{i \delta_M^q}| = \sqrt{1 + 2r_M^q \cos \delta_M^q + (r_M^q)^2}, \quad \delta_M^q = arg[1 + r_M^q e^{i \delta_M^q}], \] (26)
with \( r_M^q e^{i \delta_M^q} = M_{12}^{q,NP}/M_{12}^{q,SM} \). The mass difference in Eq. (21) is modified as
\[ \Delta M_q = \Delta M_{B_q}^S R_M^q. \] (27)

The experimental result of \( \Delta M_d \) is \( \Delta M_d = 0.507 \pm 0.004 \text{ ps}^{-1} \), which is consistent with the SM expectation with \( R_M^d \approx 1 \). This imposes a stringent constraint on any NP contribution to the \( B_d - \bar{B}_d \) mixing. One may estimate the SM contribution to \( \Delta M_d \) through the ratio \( \Delta M_{d,SM}/\Delta M_{d,SM} \), in order to minimize the hadronic uncertainties. For quark mixing angle \( \gamma = 67^\circ \), one finds \( \Delta M_{s,SM} \approx 19 \text{ ps}^{-1} \), which is in agreement with the latest measurements by CDF [27] and DO [28]:
\[ \Delta M_s = 17.77 \pm 0.10(\text{stat.}) \pm 0.07(\text{syst.}), \]
\[ \Delta M_s = 18.53 \pm 0.93(\text{stat.}) \pm 0.30(\text{syst.}). \] (28)

Fig. 1 shows the allowed ranges for \( \delta_M^q - r_M^q \) where we have neglected the SM phase in \( M_{12}^q \). The green scatter points satisfy the combined result of CDF and DO in Eq. (28) within the 1\( \sigma \) limit. The phase \( \delta_M^q \) is varied in the range \( |0, 2\pi| \), and it is not constrained below \( r_M^q \lesssim 0.4 \). Also, one can conclude that \( R_M^q \) is limited by:
\[ 0.7 \lesssim R_M^q \lesssim 1.4. \] (29)

The off-diagonal element \( \Gamma_{12}^q \) of the matrix \( \Gamma \) is related to the absorptive part of the transition amplitude from \( B_q \) to \( \bar{B}_q \). \( \Gamma_{12}^q \) can be written as
\[ \Gamma_{12}^q = \frac{1}{2m_{B_q}} \langle \bar{B}_q | \mathcal{T} | B_q \rangle, \] (30)
where the transition operator $T$ is defined by

$$ T = \text{Im} \left[ i \int d^4x T H_{eB}^{-1}(x) H_{eB}^{-1}(0) \right]. $$

(31)

In the presence of NP contributions to $\Gamma_{12}$, it can be written as

$$ \Gamma_{12} = \Gamma_{12}^{SM} + \Gamma_{12}^{NP} = \Gamma_{12}^{SM} R_{q}^{e,i\gamma} e^{i\phi_{q}^{SM}}. $$

(32)

where

$$ R_{q}^{e,i\gamma} = |1 + r_{q}^{e,i\gamma}| = \sqrt{1 + 2 r_{q}^{e,i\gamma} \cos \delta_{q}^{SM} + (r_{q}^{e,i\gamma})^{2}}, \quad \phi_{q} = \text{arg}[1 + r_{q}^{e,i\gamma}]. $$

(33)

with $r_{q}^{e,i\gamma} = \Gamma_{12}^{NP}/\Gamma_{12}^{SM}$. The width difference in Eq. (21) is modified as

$$ \Delta \Gamma_{q} = 2 |\Gamma_{12}^{SM} R_{q}^{e,i\gamma} e^{i\phi_{q}^{SM}}|. $$

(34)

The decay width difference $\Delta \Gamma_{s}$ has been measured independently. The angular analysis of $\bar{B}_{s} \to J/\psi \phi$ gives [3, 29, 30]

$$ \Delta \Gamma_{s} = \pm (0.154^{+0.054}_{-0.070}) \text{ ps}^{-1}, $$

(35)

to be compared with the SM prediction [31]

$$ \Delta \Gamma_{SM}^{s} = (0.096 \pm 0.039) \text{ ps}^{-1}. $$

(36)

The measurement of $\Delta \Gamma_{s}$, in principle, constrains NP contributions to $\Gamma_{12}^{s}$. Note that the present theory predictions are consistent with experimental measurements though it should be kept in mind that theory predictions for $\Delta \Gamma_{s}$ can contain hadronic uncertainties and constraints on NP from this measurement is not that strong.

In the presence of NP contributions to both $M_{12}^{q}$ and $\Gamma_{12}^{q}$, the charge asymmetry in Eq. (23) can be rewritten using Eq. (25) and Eq. (32) as

$$ a_{q}^{s} = \frac{R_{q}^{q,s}}{R_{M}^{q,s}} \left| \frac{\Gamma_{12}^{q,SM}}{M_{12}^{q,SM}} \right| \sin(\phi_{q}^{SM} + \phi_{M}^{q} - \phi_{T}^{s}). $$

(37)

If one neglects the NP effects to the $\Delta B = 1$ effective Hamiltonian, i.e., $\Gamma_{12}^{s} = \Gamma_{12}^{s,SM}$ ($R_{T}^{s} = 0, \phi_{T}^{s} = 0$), then using Eq. (37), the charged asymmetry $a_{q}^{s}$ is given by

$$ a_{q}^{s} = \frac{1}{R_{M}^{s}} \left| \frac{\Gamma_{12}^{s,SM}}{M_{12}^{s,SM}} \right| \sin(\phi_{M}^{s}), $$

(38)

FIG. 1: The allowed ranges for $\delta_{M}^{s} - r_{M}^{s}$ from the combined result of CDF and DO in Eq. (28) are shown.
where we have neglected the SM phase $\phi_{SM}^s = 2\beta_{SM}^s$. As indicated earlier, using the experimental value for $a_{sl}^d = -0.0047 \pm 0.0046$, one finds that in order to account for the DO results, $a_{sl}^s$ must be given by

$$a_{sl}^s = (-14.6 \pm 7.5) \times 10^{-3}. \tag{39}$$

This implies that

$$\sin \phi_M^s = -(2.9 \pm 1.5)|R_M^s|. \tag{40}$$

Using the fact that $|R_M| \simeq 1$, one finds that $\sin \phi_M^s \gg 1$, which is unphysical. Therefore, one concludes that new physics contribution to $\Gamma_{s12}^s$ is necessary to explain the observed CP charge asymmetry $a_{sl}^s$. This conclusion was also discussed earlier in Refs. [19–22]. The measured $\Delta M_s$, $\Delta \Gamma_s$, and $(a_{sl}^s)_{(avg)}$ in Eqs. (25) and (32) can be used to determine model independently the ranges for the NP quantities involved in Eq. (25). In Fig. 2 we show the possible ranges for $\delta_{\Gamma}^s$, $\delta_{M}^s$, $\sin (\phi_M^s - \phi_\Gamma^s) - R_M^s$, and $\sin (\phi_M^s - \phi_\Gamma^s) - R_M^s$. The green scatter points satisfy the measured $\Delta M_s$, $\Delta \Gamma_s$, and $(a_{sl}^s)_{(avg)}$ within the 1\(\sigma\) limit. The allowed ranges for $\delta_{\Gamma}^s$, $\delta_{M}^s$, and $R_M^s$ remain the same as shown earlier. The phase $\delta_{\Gamma}^s$ is varied in the range $[0, 2\pi]$, and it is highly constrained below $r_M^s \geq 0.4$. Also, one can see that the allowed ranges for $r_M^s \geq 1.4$. These results indicate that a considerably large NP effect in $\Gamma_{s12}^s$ is required to address the observed charge asymmetry $a_{sl}^s$. We note that similar plots and similar conclusions can be found in recent literature [22].

![Diagram](image)

**FIG. 2:** The allowed ranges for $\delta_{\Gamma}^s$, $\delta_{M}^s$, $\sin (\phi_M^s - \phi_\Gamma^s) - R_M^s$, and $\sin (\phi_M^s - \phi_\Gamma^s) - R_M^s$. The scatter points satisfy the measured $\Delta M_s$, $\Delta \Gamma_s$, and $(a_{sl}^s)_{(avg)}$ within the 1\(\sigma\) limit.

### III. EFFECTS OF NEW PHYSICS: GENERAL CASE

#### A. NP contribution to the decay width $\Gamma_{12}^s$

In this section we present the SM and NP calculations of the off-diagonal element $\Gamma_{12}^s$ of the matrix $\Gamma_s$ defined in Eq. (20). The results for $\Gamma_{12}^s$ including general NP are new to the best of our knowledge. We will
only consider vector/axial vector operators in the NP Hamiltonian. In the $B_s^0$ to $\overline{B_s}^0$ transition amplitude

\[
\mathcal{H}_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \sum_{n=1}^6 (C_n Q_n + h.c),
\]

where the operators are

\[
\begin{align*}
Q_1 &= (\bar{b}_i c_j)_{V-A}(\bar{c}_j s_i)_{V-A}, & Q_2 &= (\bar{b}_i c_i)_{V-A}(\bar{c}_j s_j)_{V-A}, \\
Q_3 &= (\bar{b}_i s_i)_{V-A}(\bar{q}_j g_j)_{V-A}, & Q_4 &= (\bar{b}_i s_j)_{V-A}(\bar{q}_j g_i)_{V-A}, \\
Q_5 &= (\bar{b}_i s_j)_{V-A}(\bar{q}_j g_j)_{V+A}, & Q_6 &= (\bar{b}_i s_j)_{V-A}(\bar{q}_j g_i)_{V+A}.
\end{align*}
\]

Here the $i$ and $j$ denote color indices and a summation over these color indices is implied. The notation $(\bar{f}_1 f_2)_{V\pm A}$ means $\bar{f}_1 \gamma^\mu (1 \pm \gamma_5) f_2$ while $q = u, d, s, c, b$. $Q_1$, $Q_2$ are color suppressed and color allowed tree-level operators, respectively, and $Q_3...Q_6$ are QCD penguin operators. The Wilson coefficients of these operators are denoted by $C_1...C_6$, respectively. $\Gamma_{12}$ is determined up to next-to-leading order in both $\Lambda/m_b$ and $\alpha_s(m_b)$ in \cite{31, 32, 33, 34, 35}. In the rest of this section, we summarize the leading order results of $\Gamma_{12}^s$ in the $1/m_b$ expansion and introduce the effects of new physics to $\Gamma_{12}^s$. We point out that in our numerical analysis, we used the update SM value of $\Delta \Gamma_{12}^{SM}$ in Eq. \eqref{eq:deltaGammaSM}.

In the SM, $\Gamma_{12}^s$ for the tree-level operators $Q_1$ and $Q_2$ is obtained by computing the related matrix elements in Eq. \eqref{eq:GammaSM}. The corresponding Feynman diagram is illustrated in Fig. \ref{fig:Gamma12}. $\Gamma_{12}^s$ can be written, to leading order in the $1/m_b$ expansion, as \cite{32, 33}.\[\Gamma_{12}^s = -\frac{G_F^2 m_b^2}{12\pi(2M_{B_s})} (V_{cb}^* V_{cs})^2 \left( G_0(z) < Q_- > + G_{S0} < Q_S^- > \right),\]

where $z = m^2_\ell/m_b^2$ and the functions $G_0(z)$ and $G_{S0}$ are given by

\[
\begin{align*}
G_0(z) &= \sqrt{1-4z}\left((1-z)C_A + \frac{1}{2}(1-4z)C_B\right), \\
G_{S0}(z) &= \sqrt{1-4z}(1+2z)(C_A-C_B).
\end{align*}
\]

The combinations of Wilson coefficients $C_A$ and $C_B$ are given by

\[
C_A = \left(N_c C_1^2 + 2C_1 C_2\right), \quad C_B = C_2^2.
\]
where \( N_C = 3 \). The two \( \Delta B = 2 \) operators \( Q_- \) and \( Q_{S-} \) in Eq. (43) are given by

\[
Q_- = (\bar{b}_i \gamma^\mu (1 - \gamma_5) s_i)(\bar{b}_j \gamma_\mu (1 - \gamma_5) s_j), \quad Q_{S-} = (\bar{b}_i (1 - \gamma_5) s_i)(\bar{b}_j (1 - \gamma_5) s_j).
\] (46)

The matrix elements \( <Q_- > \) and \( <Q_{S-} > \) can be found using a vacuum insertion method as \[33\]

\[
\langle <Q_- > \rangle = \langle \bar{B}_s | Q_- | B_s \rangle = \frac{8}{3} m_{B_s} f_{B_s} B_1, \quad \langle <Q_{S-} > \rangle = \langle \bar{B}_s | Q_{S-} | B_s \rangle = -\frac{5}{3} m_{B_s} f_{B_s} r_\chi^s B_2,
\] (47)

where \( B_1 \) and \( B_2 \) are bag parameters, and the quantity \( r_\chi \) is defined by

\[
r_\chi = \frac{m_{B_s}}{(m_b(m_b) + m_s(m_b))^2}.
\] (48)

The contributions of the all six operators \( Q_i(i = 1..6) \) in Eq. (42) to \( \Gamma_{12}^s \) are given by

\[
\Gamma_{12}^{s,SM} = -\frac{G_F^2 m_s^2}{12\pi (2M_{B_s})} (V^*_{cb} V_{cs})^2 \left( G_0'(z) <Q_- > + G_{S0}'(z) <Q_{S-} > \right),
\] (49)

where the functions \( G_0'(z) \) and \( G_{S0}'(z) \) are given by \[33\]

\[
G_0'(z) = \sqrt{1 - 4z} \left( (1 - z) C_A' + \frac{1}{2} (1 - 4z) C_B' + 3z C_C' \right),
\]

\[
G_{S0}'(z) = \sqrt{1 - 4z(1 + 2z)} \left( (C_A' - C_B') \right).
\] (50)

The combinations of Wilson coefficients \( C_A' \) and \( C_B' \) are given by

\[
C_A' = \left( N_C(C_1 + C_3)^2 + 2(C_1 + C_3)(C_2 + C_4) + 2C_5 C_6 + N_C C_5^2 \right),
\]

\[
C_B' = \left( (C_2 + C_4)^2 + C_5^2 \right),
\]

\[
C_C' = 2 \left( N_C(C_1 + C_3) C_5 + (C_1 + C_3) C_6 + (C_2 + C_4) C_5 + (C_2 + C_4) C_6 \right).
\] (51)

Setting \( C_{i=3..6} = 0 \) in Eq. (49) one recovers Eq. (43). The effects of \( C_{i=3..6} \) on \( \Gamma_{21}^{s,SM} \) are small.

Next we consider the contributions to \( \Gamma_{12}^s \) from NP operators involving \( b \to \bar{c}c\bar{s} \) transitions. Note that these operators can also contribute to the lifetimes of the \( b \) hadrons and we will consider constraints on these operators from lifetime measurements in our numerical analysis. In general NP can also contribute to \( b \to q\bar{q}s \) transitions with \( q = u, d, s \). However, we note that in the SM these transitions are suppressed and measurements in decays like \( B \to K\pi, B \to \phi K \) etc. already constrain these NP contributions. In other words, an NP contribution to \( b \to q\bar{q}s \) with \( q = u, d, s \) cannot significantly affect \( \Gamma_{12}^s \) and hence we ignore these contributions. The \( b \to \bar{c}c\bar{s} \) transitions, on the other hand, are tree level in the SM and measurements of decays with the underlying \( b \to \bar{c}c\bar{s} \) transitions, such as \( B_d \to D_s D, J/\psi K_s \) etc. might still allow for NP in \( b \to \bar{c}c\bar{s} \) transitions that are not small. In this section, we only present model independent results, and for the numerics in the following section we consider the RS model with split fermions. The RS model with split fermions can also generate NP contributions to \( b \to s\tau^+\tau^- \) which are not that well constrained from experiments. However, these transitions are generated by the exchange of KK electroweak bosons and hence they are much smaller than the \( b \to \bar{c}c\bar{s} \) transitions that are generated by KK gluon exchange.

The \( \Delta B = 1 \) effective weak Hamiltonian for NP operators is written as

\[
\mathcal{H}_{eff}^{NP} = \left( \lambda_{LL} Q_{LL} + \lambda'_{LL} Q'_{LL} + \lambda_{LR} Q_{LR} + \lambda'_{LR} Q'_{LR} + \lambda_{RR} Q_{RR} + \lambda'_{RR} Q'_{RR} + \lambda_{RL} Q_{RL} + \lambda'_{RL} Q'_{RL} \right),
\] (52)
where

\[ Q_{LL} = (\bar{b}_i s_i)_{V-A}(\bar{c}_j c_j)_{V-A}, \quad Q'_{LL} = (\bar{b}_i s_j)_{V-A}(\bar{c}_j c_i)_{V-A}, \]
\[ Q_{LR} = (\bar{b}_i s_i)_{V-A}(\bar{c}_j c_j)_{V+A}, \quad Q'_{LR} = (\bar{b}_i s_j)_{V-A}(\bar{c}_j c_i)_{V+A}, \]
\[ Q_{RR} = (\bar{b}_i s_i)_{V+A}(\bar{c}_j c_j)_{V+A}, \quad Q'_{RR} = (\bar{b}_i s_j)_{V+A}(\bar{c}_j c_i)_{V+A}, \]
\[ Q_{RL} = (\bar{b}_i s_i)_{V+A}(\bar{c}_j c_j)_{V-A}, \quad Q'_{RL} = (\bar{b}_i s_j)_{V+A}(\bar{c}_j c_i)_{V-A}. \] (53)

The eight couplings \( \lambda_{AB} \), and \( \chi'_{AB} \) (A, B = L, R) are in general complex and can be determined from specific models. Thus, the total effective Hamiltonian can be written as

\[ H_{eff} = H_{eff}^{SM} + H_{eff}^{NP}, \]
\[ H_{eff} = \frac{G_F}{\sqrt{2}} (V_{cb}^* V_{cs}) \left( (C_1 + C_3 + C_5^{LL}) Q_1 + (C_2 + C_4 + C_5^{LR}) Q_2 + (C_5^{LL} + C_5^{RR}) Q_5 + (C_6 + C_6^{LR}) Q_6 \right. \]
\[ \left. + C_3^{RR} Q_{RR} + C_4^{RR} Q'_{RR} + Q_{RL}^{RL} Q_{RL} + C_6^{RL} Q_{RL}' \right), \] (54)

where the new Wilson coefficients (A, B = L, R) are

\[ C_3^{AA} = \frac{\sqrt{2} \lambda_{AA}}{G_F V_{cb}^* V_{cs}}, \quad C_4^{AA} = \frac{\sqrt{2} \lambda'_{AA}}{G_F V_{cb}^* V_{cs}}, \]
\[ C_5^{AB} = \frac{\sqrt{2} \lambda_{AB}}{G_F V_{cb}^* V_{cs}}, \quad C_6^{AB} = \frac{\sqrt{2} \lambda'_{AB}}{G_F V_{cb}^* V_{cs}} \quad (A \neq B). \] (55)

The NP contributions to \( \Gamma_{21} \) are obtained by computing the related matrix elements in Eq. \( 51 \) to the leading order in the \( 1/m_b \) expansion using the \( \overline{MS} \) scheme. The NP contributions contain both pure NP and SM-NP interference terms. In general the latter dominate NP contributions due to the large SM Wilson coefficients. We obtain NP contributions to \( \Gamma_{21} \) as

\[ \Gamma_{12}^{s,NP} = \Gamma_{12}^{s,LL} + \Gamma_{12}^{s,RR} + \Gamma_{12}^{s,mix}, \] (56)

where \( \Gamma_{12}^{s,LL}, \Gamma_{12}^{s,RR}, \) and \( \Gamma_{12}^{s,mix} \) contain the contributions from LL and LR, RR and RL, and all possible type of operators, respectively. They can be expressed in terms of the matrix elements of eight \( \Delta B = 2 \) operators

\[ Q_- = (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}, \quad Q_{S_-} = (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}, \]
\[ Q_+ = (\bar{b}_i s_i)_{V+A}(\bar{b}_j s_j)_{V+A}, \quad Q_{S_+} = (\bar{b}_i s_i)_{S+P}(\bar{b}_j s_j)_{S+P}, \]
\[ Q_{\pm} = (\bar{b}_i s_i)_{V+A}(\bar{b}_j s_j)_{V-A}, \quad Q_{S_{\pm}} = (\bar{b}_i s_i)_{S+P}(\bar{b}_j s_j)_{S-P}, \] (57)

where \( S_{\pm} P = 1 \pm \gamma_5 \). The explicit forms of \( \Gamma_{12}^{s,LL}, \Gamma_{12}^{s,RR}, \) and \( \Gamma_{12}^{s,mix} \) are

\[ \Gamma_{12}^{s,LL} = -\frac{G_F^2 m_b^2}{12 \pi (2 M_{B_s})} (V_{cb}^* V_{cs})^2 \left( G_0''(z) < Q_- > + G_0'' S_0(z) < Q_{S-} > \right), \]
\[ \Gamma_{12}^{s,RR} = -\frac{G_F^2 m_b^2}{12 \pi (2 M_{B_s})} (V_{cb}^* V_{cs})^2 \left( \tilde{G}_0''(z) < Q_+ > + \tilde{G}_0'' S_0(z) < Q_{S+} > \right), \]
\[ \Gamma_{12}^{s,mix} = -\frac{G_F^2 m_b^2}{12 \pi (2 M_{B_s})} (V_{cb}^* V_{cs})^2 \left( \tilde{G}_1(z) + \tilde{G}_2(z) + \tilde{G}_3(z) + \tilde{G}_4(z) \right). \] (58)
where the G functions are

\[ \begin{align*}
G''_0(z) &= \sqrt{1 - 4z} (1 - z)C''_A + \frac{1}{2} (1 - 4z)C''_B + 3zC''_C, \\
G''_S(z) &= \sqrt{1 - 4z} (1 + 2z) (C''_A - C''_B), \\
\tilde{G}_0(z) &= \sqrt{1 - 4z} (1 - z) \tilde{C}_A + \frac{1}{2} (1 - 4z) \tilde{C}_B + 3z \tilde{C}_C, \\
\tilde{G}_S(z) &= \sqrt{1 - 4z} (1 + 2z) (\tilde{C}_A - \tilde{C}_B), \\
G_1(z) &= 2\sqrt{1 - 4z} (C_1 + C_3 + C_3^{LL}) \left[ \left( 1 - z \right) (N_C C_5^{LL} + C_6^{RL}) + 3z (N_C C_3^{RR} + C_4^{RR}) \right] < Q> + (1 + 2z) (N_C C_5^{RL} + C_6^{RL}) < Q_S >, \\
G_2(z) &= 2\sqrt{1 - 4z} (C_2 + C_4 + C_4^{LL}) \left[ 3z C_3^{RR} + (1 - z) C_5^{RL} \right] < Q > + (1 + 2z) C_5^{RL} < Q_S >, \\
G_3(z) &= 2\sqrt{1 - 4z} (C_5 + C_5^{LL}) \left[ (1 - z) (N_C C_3^{RR} + C_4^{RR}) + 3z (N_C C_5^{RL} + C_6^{RL}) \right] < Q > + (1 + 2z) (N_C C_3^{RR} + C_4^{RL}) < Q_S >, \\
G_4(z) &= 2\sqrt{1 - 4z} (C_6 + C_6^{LL}) \left[ (1 - z) C_3^{RR} + 3z C_6^{RL} \right] < Q > + (1 + 2z) C_3^{RR} < Q_S > + 3z (-C_4^{RR} < Q > + 2C_6^{RL} < Q_S >). \tag{59}
\end{align*} \]

The combinations of Wilson coefficients are

\[ \begin{align*}
C'_A &= \left[ N_C \left( (C_3^{LL})^2 + (C_5^{LR})^2 \right) + 2C_3^{LL} C_4^{LL} + 2C_3^{LR} C_5^{LR} + 2C_3^{LL} (N_C (C_1 + C_3) + C_2 + C_4) \\
&+ 2C_4^{LL} (C_1 + C_3) + 2C_5^{LR} (N_C C_5 + C_6) + 2C_6^{LR} C_5 \right], \\
C'_B &= \left[ (C_5^{LL})^2 + (C_6^{LR})^2 + 2C_5^{LL} (C_2 + C_4) + 2C_6^{LR} C_6 \right], \\
C'_C &= \left[ 2C_3^{LL} (N_C C_5^{RR} + C_6^{LR}) + 2C_4^{LL} (C_5^{RR} + C_6^{LR}) + 2C_3^{LL} (N_C C_5^{RR} + C_6^{LR}) \\
&+ 2C_4^{LL} (C_5 + C_6) + 2C_5^{LR} (N_C (C_1 + C_3) + C_2 + C_4) + 2C_6^{LR} (C_1 + C_2 + C_3 + C_4) \right], \\
\tilde{C}_A &= \left[ N_C \left( (C_3^{RR})^2 + (C_5^{RL})^2 \right) + 2C_3^{RR} C_4^{RR} \right], \\
\tilde{C}_B &= \left[ (C_4^{RR})^2 + (C_6^{RL})^2 \right], \\
\tilde{C}_C &= \left[ 2(N_C C_3^{RR} C_5^{RL} + C_3^{RR} C_6^{RL} + 2C_4^{RR} C_5^{RR} + 2C_4^{RR} C_6^{RL}) \right]. \tag{60} \end{align*} \]

The matrix elements \( < Q > = \langle \bar{B}_s | Q | B_s \rangle \) of the operators in Eq. (57) are given by

\[ \begin{align*}
< Q > &= < Q_+ >= \frac{8}{3} m_B^2 f_B r_s^s B_1, \\
< Q_{-} > &= < Q_{S_-} >= -\frac{5}{3} m_B^2 f_B r_s^s B_2, \\
< Q_{S_+} > &= < Q_{S\pm} >= 2m_B^2 f_B r_s^s B_4, \\
< Q_{S\mp} > &= < Q_{\pm} >= -\frac{4}{3} m_B^2 f_B r_s^s B_5. \tag{61} \end{align*} \]

where \( B_i \)'s are the bag parameters and their numerical values for \( B_0^0 - \bar{B}_0^0 \) system in the \( \overline{MS} \)-NDR scheme at \( m_b = 4.6 \) GeV can be found in \( \text{[37]} \).
B. NP contribution to the mass parameter $M_{12}^*$

In this section we present the SM and NP calculations of the off-diagonal element $M_{12}^*$ of the matrix $M_q$ defined in Eq. (24). The effective Hamiltonian for $\Delta B = 2$ transition that generates $B^0_s \rightarrow \bar{B}^0_s$ mixing in the SM can be written as

$$H_{\text{eff}}^{\Delta B=2, SM} = \frac{G_F}{16\pi^2} M_W^2 (V_{tb} V_{ts}^*)^2 \eta_B, S_0(x_t) (\bar{b}s) V^- (\bar{b}s) V^-,$$

where $\eta_B \simeq 0.551$ [3] is the QCD correction, and the loop function $S_0(x_t)$ ($x_t = m_t^2/M_W^2$) is given by [40]

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^2\ln(x_t)}{2(1-x_t)^3}.$$  

(62)

The SM contribution to $M_{12}^*$ can be obtained using Eq. (24) as

$$M_{12}^{*, SM} = \frac{G_F^2}{12\pi^2} M_W^2 (V_{tb} V_{ts}^*)^2 \eta_B, S_0(x_t) m_B, f_B^2, B_1.$$  

(64)

Next we consider NP effects in the $\Delta B = 2$ transition. The effective Hamiltonian for this transition is written as

$$H_{\text{eff}}^{\Delta B=2, NP} = \left( \delta_{LL}^2 Q^- + \delta_{LL}^2 Q'^- + \delta_{LR}^2 Q^+ + \delta_{LR}^2 Q^+ \right) + \delta_{RR} Q^+ + \delta_{RL} Q^+ + \delta_{RL} Q^+,$$

(65)

where $\delta_{AB}$ (A, B = L, R) are NP couplings. The operators $Q^-, Q^+, Q^z$, and $Q^\pm$ are given in Eq. (57) and their color suppressed counterpart operators can be written as

$$Q^- = (\bar{b}_i s_j) V^- (\bar{b}_j s_i) V^-; \quad Q'^- = (\bar{b}_i s_j) V^+ (\bar{b}_j s_i) V^+; \quad Q^+ = (\bar{b}_i s_j) V^- (\bar{b}_j s_i) V^-; \quad Q^\pm = (\bar{b}_i s_j) V^+ (\bar{b}_j s_i) V^-.$$  

(66)

Using Eq. (24) and applying the Fierz transformation, one can obtain the contribution of these NP operators to $M_{12}^*$ as

$$M_{12, NP}^* = \frac{1}{2m_B} \left( \left( \delta_{LL}^2 + \delta_{LL}^2 \right) < Q^- > + \left( \delta_{RR}^2 + \delta_{RR}^2 \right) < Q^- > + \left( \delta_{LR}^2 + \delta_{RL}^2 \right) < Q^+ > - 2 \left( \delta_{LL}^2 + \delta_{RR}^2 \right) < Q^\pm > \right).$$  

(67)

For certain classes of models including the RS model with split fermions the following relations hold:

$$\delta_{LL} = -1/3\delta_{LL}^2, \quad \delta_{RR} = -1/3\delta_{RR}^2, \quad \delta_{LR} = -1/3\delta_{LR}^2, \quad \delta_{RL} = -1/3\delta_{RL}^2, \quad \delta_{LR} = \delta_{RL}.$$  

(68)

One can then obtain

$$M_{12, NP}^* = \frac{4}{3} m_B, f_B^2 \left[ \frac{2}{3} \left( \delta_{LL}^2 + \delta_{RR}^2 \right) B_1 + \delta_{LR}^2 \left( \frac{1}{3} r^* B_5 - 3r^* B_4 \right) \right],$$

(69)

where the bag parameters are defined in Eq. (61).

IV. NEW SOURCE OF FLAVOR AND CP VIOLATION IN THE RS MODEL

The warped extra dimension model has been proposed as a solution of the hierarchy problem. In the original RS model, the SM fields are localized to one of the boundaries and gravity is allowed to propagate
in the bulk. However, it was realized that scenarios with SM gauge bosons and fermions in the bulk may lead to a new geometrical interpretation for the hierarchy of quark and lepton masses. The Higgs field has to be confined to the TeV brane in order to obtain the observable masses of the $W$ and $Z$ gauge bosons.

We will consider the scenario of Ref. [23], based on the metric

$$
\begin{align*}
\frac{ds^2}{13} &= e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,
\end{align*}
$$

(70)

where $\sigma(y) = \kappa |y|$ and $\kappa \sim M_P$ is the curvature scale determined by the negative cosmological constant in the five dimensional bulk. The fermion fields reside in the bulk of this nonfactorizable geometry and can be decomposed as

$$
\Psi(x,y) = \frac{1}{\sqrt{2\pi r_c}} \sum_{n=0}^{\infty} \psi^{(n)}(x) e^{2\sigma(y)} f^{(n)}(y).
$$

(71)

Here $r_c$ is the radius of the compactified fifth dimension on an orbifold $S_1/Z_2$ so that the bulk is a slice of $AdS_5$ space between two four dimensional boundaries. The left-handed zero mode wave function is given by

$$
\begin{align*}
J_0^{(0)}(c_{f_A}, y) &= \frac{e^{-c_{f_A} \sigma(y)}}{N_0}.
\end{align*}
$$

(72)

where $c_{f_A} = m_{f_A}/\kappa$ and $m_{f_A}$ is the bulk mass term. Using the orthonormal condition:

$$
\frac{1}{2\pi r_c} \int_{-\pi r_c}^{\pi r_c} dy e^{\sigma(y)} J_0^{(0)}(c_{f_A}, y) J_0^{(0)}(c_{f_A}, y) = 1,
$$

(73)

one finds that $N_0$ is given by

$$
N_0 = \sqrt{\frac{e^{\pi r_c(1-2c_{f_A})} - 1}{\pi r_c(1-2c_{f_A})}}.
$$

(74)

The right-handed zero mode wave function can be obtained from

$$
\begin{align*}
J_0^{(0)}(c_{f_A}, y) &= J_0^{(0)}(-c_{f_A}, y).
\end{align*}
$$

(75)

The tower of fermion KK excited states is not relevant to our discussion here. Note that fermions with $c_f > 1/2$ are localized near the Planck brane at $y = 0$ and fermions with $c_f < 1/2$ are localized near the TeV brane at $y = \pi r_c$.

The massless gauge fields that propagate in this curved background can be decomposed as

$$
A_\mu(x,y) = \frac{1}{\sqrt{2\pi r_c}} \sum_{n=0}^{\infty} A_\mu^{(n)}(x) f_\mu^{(n)}(y),
$$

(66)

with $f_\mu^{(n)}(y)$ is dimensionless. The n-th $(n > 0)$ mode function is given as

$$
\begin{align*}
f_\mu^{(n)}(y) &= \frac{e^{\sigma(y)}}{N_n} \left[ J_1 \left( m_{A}^{(n)} \sigma(y) \right) + b_A(m_{A}^{(n)}) Y_1 \left( m_{A}^{(n)} \sigma(y) \right) \right],
\end{align*}
$$

(77)

where $J_1$ and $Y_1$ are the J- and Y-type Bessel functions of order one and

$$
\begin{align*}
b_A(m_{A}^{(n)}) &= \frac{J_0(m_{A}^{(n)}/\kappa)}{Y_0(m_{A}^{(n)}/\kappa)}.
\end{align*}
$$

(78)
The coupling of the gauge KK modes to the fermion at the vertex \( q_{L(R)}^n \bar{q}_{L(R)}^\beta g^{(n)} \) is given by

\[
g^{(n)}(c_{f_\alpha})_{L(R)} = \frac{g^{(5)}}{(2\pi r)^{3/2}} \int_{-\pi r}^{\pi r} e^{\alpha(y)} f^{(0)}_{L(R)}(c_{f_\alpha}, y) f^{(0)}_{L(R)}(c_{f_\alpha}, y) f^{(n)}_{A}(y) dy.
\]  

(79)

The nonuniversal parameters \( c_{f_\alpha} \) lead to nonuniversal couplings to the KK state of the gluon. In the basis of mass eigenstates we have the following flavor dependent couplings:

\[
(U^{u,d(n)}_{L(R)})_{\alpha\beta} = (V^{u,d+}_{L(R)})_{\alpha\gamma} g^{(n)}_{L(R)}(c_{f_\alpha}) (V^{u,d}_{L(R)})_{\gamma\beta},
\]

(80)

where the \( g^{(1)}_{L(R)} \) is given by

\[
g^{(1)}_{L(R)}(c_{f_\alpha}) = g \left( \frac{1 - 2c_{f_\alpha}}{e^{\pi r}(1 - 2c_{f_\alpha}) - 1} \right) \frac{\kappa}{N_0} \int_{0}^{\pi r} e^{(1-2c_{f_\alpha})\kappa y} \left[ J_1 \left( \frac{m_A^{(1)}}{\kappa} e^{\kappa y} \right) + b_A(m_A^{(1)}) Y_1 \left( \frac{m_A^{(1)}}{\kappa} e^{\kappa y} \right) \right].
\]

(81)

The tree-level relation between the 5D and 4D QCD couplings \( g_5 \) and \( g \) is \( g = g_5/\sqrt{2\pi r} \). However, at the one loop level the relation between these two couplings also depends on the value of the brane-localized kinetic terms for the bulk gauge fields, and there can be significant corrections to the tree level relation. A detailed discussion on this topic can be found in Ref. [41]. Finally, the unitary matrices \( V_{L(R)}^{u,d} \) diagonalize the up/down quark mass matrix \( M_{u,d}^{(\alpha\beta)} = (v/\sqrt{2}) Y_{\alpha\beta}^{u,d} \), which is given in the model as

\[
Y_{\alpha\beta}^{u,d} = \frac{l_{\alpha\beta}}{\kappa r} f^{(0)}_{L}(c_{Q_\gamma} y = \pi r_c) f^{(0)}_{R}(c_{u,d} y = \pi r_c).
\]

(82)

The dimensionless parameters \( l_{\alpha\beta} \) are defined as

\[
l_{\alpha\beta} = \lambda_{\alpha\beta}^{(5)} \sqrt{\kappa},
\]

(83)

where \( \lambda_{\alpha\beta}^{(5)} \) are the 5D Yukawa couplings which are free parameters to be fixed by the observable masses and mixing. In Table I we present an example of the \( c_f \) parameters that leads to the correct quark masses and mixing with \( \lambda_{\alpha\beta}^{(5)} \sim \mathcal{O}(1) \). The corresponding first-KK gluon coupling constants are given by

\[
\begin{align*}
 g^{(1)}_{L}(c_{Q_1}) &= -0.199, & g^{(1)}_{L}(c_{Q_2}) &= -0.198, & g^{(1)}_{L}(c_{Q_3}) &= 1.496, \\
 g^{(1)}_{R}(c_{D_1}) &= -0.199, & g^{(1)}_{R}(c_{D_2}) &= -0.191, & g^{(1)}_{R}(c_{D_3}) &= -0.198, \\
 g^{(1)}_{R}(c_{U_1}) &= -0.199, & g^{(1)}_{R}(c_{U_2}) &= -0.195, & g^{(1)}_{R}(c_{U_3}) &= 3.38.
\end{align*}
\]

(84)

We note that the value of \( c_{D_3} \) in Table I is larger than for the 1st/2nd generation. Such a choice of \( c \)'s is not consistent with anarchic 5D Yukawa couplings discussed in Ref [26]. We also point out that for simplicity, we have used here the tree-level relation between the 5D and 4D QCD couplings. As can be seen from these values, \( g^{(1)} \) is of order one. In fact, this general conclusion can be obtained with any other values of \( c_f \). One can easily show that for \( c_f > 1/2 \) the coupling \( g^{(1)} \) approaches zero, while for \( c_f < 1/2 \) one finds \( g^{(1)} \leq 4 \). We expect the general features about \( g^{(1)} \) to remain true even when the one loop matching of the 5D and 4D QCD couplings is used.

In the RS model, the effective Hamiltonian for \( \Delta B = 2 \) transition can be generated at tree-level via the exchange of a KK gluon, as shown in Fig. 3. The \( \Delta B = 2 \) effective Hamiltonian is given by

\[
H_{c_{e_{B=2,KK}}}^{\Delta} = \sum_{A,B} \frac{1}{4m_{KK}^2} (U^{d(1)}_{A})_{32} (U^{d(1)}_{B})_{32} \left[ (b_i\gamma^\mu P A_{s_j}) (b_j\gamma^\mu P B_{s_i}) - \frac{1}{3} (b_i\gamma^\mu P A_{s_i}) (b_j\gamma^\mu P B_{s_j}) \right],
\]

(85)
The corresponding Wilson coefficients in the RS Model can be obtained from Eq. (55). The contribution to $M_{12}^s$ in the RS model can be obtained by comparing Eq. (85) with Eq. (65), and the couplings can be expressed as,

$$
\delta'_LL = \frac{1}{16m_{KK}^2}(U_{L}^{d(1)})_{32}(U_{L}^{d(1)})_{32}, \quad \delta'_RR = \frac{1}{16m_{KK}^2}(U_{R}^{d(1)})_{32}(U_{R}^{d(1)})_{32},
$$

$$
\delta'_LR = \frac{1}{16m_{KK}^2}(U_{L}^{d(1)})_{32}(U_{R}^{d(1)})_{32}, \quad \delta'_RL = \frac{1}{16m_{KK}^2}(U_{R}^{d(1)})_{32}(U_{L}^{d(1)})_{32}.
$$

Thus, Eq. (69) reduces to

$$
M_{12}^{s,KK} = \frac{1}{12m_{KK}^2}m_{B_J}^2 f_{B_J}^2 \left[ \frac{2}{3}(U_{L}^{d(1)})_{32}^2 + (U_{L}^{d(1)})_{32}(U_{R}^{d(1)})_{32} \right] B_1 + (U_{L}^{d(1)})_{32}(U_{R}^{d(1)})_{32} \left( \frac{1}{3}r_s B_5 - 3r_s B_4 \right). \tag{87}
$$

This result agrees with those in [38]. If $M_{12}^{s,KK}$ is dominated by the first term, then the ratio $r_{Ms}$ in Eq. (20) can be obtained for the RS model as

$$
r_{Ms}^{KK} = \frac{|M_{12}^{s,KK}|}{M_{12}^{s,SM}} = \frac{2\pi^2}{3m_{KK}^2 G_F M_W^2 V_{tb} V_{ts}^* s_{0}(x_t)} \approx \left( \frac{2415}{m_{KK}^2} \right)^2 \left( \frac{(U_{L}^{d(1)})_{32}}{V_{tb} V_{ts}^*} \right)^2. \tag{88}
$$

Therefore, $r_{Ms}^{KK} \sim O(1)$ requires $m_{KK} \sim 2.4$ TeV if we assume $(U_{L}^{d(1)})_{32} \sim V_{tb} V_{ts}^*$. If $(U_{L}^{d(1)})_{32} \sim O(1)$, the experimental limits on $\Delta M_{B_J}$ implies that $m_{KK} \gtrsim 10$ TeV, which imposes stringent constraint on the associated compactification scale.

The $\Delta B = 1$ effective Hamiltonian for the $b \to s\bar{c}c$ transition in the RS Model can be written as

$$
\mathcal{H}_{eff}^{\Delta B=1,KK} = \frac{1}{4m_{KK}^2}(U_{L}^{d(1)})_{32}(U_{B}^{u(1)})_{22} \left[ \bar{b}_i \gamma^\mu P_{A S i} (\bar{c}_j \gamma_\mu P_{B C i}) - \frac{1}{3}(\bar{b}_i \gamma^\mu P_{A S i})(\bar{c}_j \gamma_\mu P_{B C j}) \right]. \tag{89}
$$

The contribution of the RS model to $\Gamma_{12}^s$ can be obtained using Eq. (55) with the couplings

$$
\delta'_{LL} = \frac{1}{16m_{KK}^2}(U_{L}^{d(1)})_{32}(U_{L}^{u(1)})_{22}, \quad \delta_{LL} = \frac{1}{3}\delta'_{LL},
$$

$$
\delta'_{LR} = \frac{1}{16m_{KK}^2}(U_{L}^{d(1)})_{32}(U_{R}^{u(1)})_{22}, \quad \delta_{LR} = -\frac{1}{3}\delta'_{LR},
$$

$$
\delta'_{RR} = \frac{1}{16m_{KK}^2}(U_{R}^{d(1)})_{32}(U_{R}^{u(1)})_{22}, \quad \delta_{RR} = -\frac{1}{3}\delta'_{RR},
$$

$$
\delta'_{RL} = \frac{1}{16m_{KK}^2}(U_{R}^{d(1)})_{32}(U_{L}^{u(1)})_{22}, \quad \delta_{RL} = -\frac{1}{3}\delta'_{RL}. \tag{90}
$$

The corresponding Wilson coefficients in the RS Model can be obtained from Eq. (55).

---

**FIG. 4**: The gluon KK contribution to $B_s^0 - \bar{B}_s^0$ mixing is shown.
V. NUMERICAL RESULTS

The numerical inputs for the parameters in the SM [16] and RS model are summarized in Table. I. The values of the bag parameters in the \(\delta\)NDR scheme can be found in \([37]\), and the decay constant of \(B_s\) is from \([39]\). The relevant CKM matrix elements are obtained from the CKMfit collaboration \([43]\). The SM Wilson coefficients for the quark level \(b \to sq\bar{q}\) transition at next-to-leading order in the NDR scheme are obtained from \([44]\).

In the RS model, the matrix elements \(M_s^{12,KK}\) and \(\Gamma_s^{12,KK}\) depend on the four couplings \((U_{L(R)}^{d(1)})_{32}\) and \((U_{L(R)}^{u(1)})_{22}\) [see Eq. (80)]. Writing these couplings explicitly yield

\[
(U_{L}^{d(1)})_{32} = V_{L(32)}^{d(1)} V_{L(22)}^{d(1)} [g_{L(1)}^{d(1)} (c_{Q_1}) - g_{L(1)}^{d(1)} (c_{Q_1})] + V_{L(33)}^{d(1)} V_{L(32)}^{d(1)} [g_{L(1)}^{d(1)} (c_{Q_1}) - g_{L(1)}^{d(1)} (c_{Q_1})],
\]

\[
(U_{R}^{d(1)})_{32} = V_{R(32)}^{d(1)} V_{R(22)}^{d(1)} [g_{R(1)}^{d(1)} (c_{D_2}) - g_{R(1)}^{d(1)} (c_{D_2})] + V_{R(33)}^{d(1)} V_{R(32)}^{d(1)} [g_{R(1)}^{d(1)} (c_{D_2}) - g_{R(1)}^{d(1)} (c_{D_2})],
\]

\[
(U_{L}^{u(1)})_{22} = V_{L(21)}^{u(1)} V_{L(12)}^{u(1)} [g_{L(1)}^{u(1)} (c_{Q_1}) - g_{L(1)}^{u(1)} (c_{Q_1})] + V_{L(22)}^{u(1)} V_{L(22)}^{u(1)} [g_{L(1)}^{u(1)} (c_{Q_1}) - g_{L(1)}^{u(1)} (c_{Q_1})],
\]

\[
(U_{R}^{u(1)})_{22} = V_{R(21)}^{u(1)} V_{R(12)}^{u(1)} [g_{R(1)}^{u(1)} (c_{D_2}) - g_{R(1)}^{u(1)} (c_{D_2})] + V_{R(22)}^{u(1)} V_{R(22)}^{u(1)} [g_{R(1)}^{u(1)} (c_{D_2}) - g_{R(1)}^{u(1)} (c_{D_2})],
\]

where the unitarity of the \(V_{L(R)}^{d(1)}\) is used. The bulk parameters \(c_{f_s}\) specify the position of the fermion’s localized wavefunction in the bulk. The specific choice of \(c_s\) are model dependent and they should generate the 4-d Yukawa hierarchy of the quarks as well as the CKM mixing in the left-handed sector. In our numerical analysis, we do not consider any specific values for \(c_s\). We scan the allowed values for the couplings \((U_{L(R)}^{d(1)})_{32}\) and \((U_{L(R)}^{u(1)})_{22}\) after imposing the constraints from the experimental measurements of \(\Delta M_s\) and \(\Delta \Gamma_s\). Assuming the matrices \(V_{L(R)}^{d}\) to be CKM-like and using \(g_{L(1)}^{d(1)} \lesssim 4\) one notes from Eq. (81), that the coupling \((U_{L(R)}^{d(1)})_{32}\) is constrained to \((U_{L(R)}^{d(1)})_{32} \lesssim 0.1\), while \((U_{L(R)}^{u(1)})_{22}\) is of order one. Therefore, we vary the four KK couplings as: \(|U_{L(R)}^{d(1)}| \lesssim 0.1\), \(|U_{L(R)}^{u(1)}| \lesssim 1\), and \(|U_{L(R)}^{u(1)}| \lesssim 3\), and their phases in the range \([0, 2\pi]\). Note that this assumption of \(U_{L(R)}^{d}\) and \(V_{L(R)}^{u}\) being CKM-like is consistent with the scenario of anarchic 5D Yukawa couplings that has been considered in Ref. [26]. Our choice for the right-handed mixing angles are not quite consistent with the scenario of anarchic 5D Yukawa couplings but our assumption is that with proper tuning of the 5D Yukawa couplings we can have the mixing angles in the range considered in this analysis.

In the fit, \(\Delta M_s\) and \(\Delta \Gamma_s\) are constrained by their experimental results within 1\(\sigma\) errors. All SM input parameters are uniformly varied within their errors and the SM Wilson coefficients are evaluated at \(\mu_B = 4.2\) GeV. The bag parameters are kept at their central values and \(m_{KK}\) is varied in the range \([1.2, 10.0]\) TeV. Note that strong constraints on \(m_{KK}\) are obtained from measurements in \(K\) mixing \([45]\) but there are still regions of parameter space \([38, 41, 46]\) which allow values for \(m_{KK}\) considered in this work. Note that the lower end of the KK scale which is scanned here (a couple of TeV) would require some amount of tuning in order to satisfy the constraint from \(\epsilon_K\) (even with Higgs in the bulk) \([38]\). Similarly, the KK scale of 1.2 to 2 TeV used in the scan is generally disfavored by electroweak precision tests \([21, 47]\). Such a low KK scale might be allowed if the 1st and 2nd generation fermions are all chosen to have a (very) close-to-flat profile \([21]\), but then one loses the explanation of fermion mass hierarchies based solely on profiles.

We observe that \(\Delta M_s (\exp)\) constrains \(|U_{L(32)}^{d(1)}| \lesssim 10^{-2}\), and allows \(r_s^{s, KK} = |M_{12}^{s, KK} / M_{12}^{s, SM}| \lesssim 1\). Note that similar conclusion about \(r_s^{s, KK}\) was also reached in Ref. [38]. The fit results allow the ratio \(r_s^{s, KK} = |G_{s, KK}^{s, KK} / G_{s, KK}^{s, SM}| \lesssim 10\%\) for values of \(m_{KK} < 1.5\) TeV and this ratio falls quite quickly as \(m_{KK}\) is increased beyond 1.5 TeV. As indicated earlier, NP in \(b \to c \bar{s} s\) transitions will also contribute to the lifetimes of \(b\) hadrons. We expect the corrections to the total widths of the \(b\) hadrons to be also \(\lesssim 10\%\) which cannot be detected experimentally given the hadronic uncertainties in calculating the total widths \([48]\). The lifetime ratio of the \(B_s\) meson to the \(B_d\) meson, \(\frac{\tau_{B_s}}{\tau_{B_d}}\), has a tiny theoretical uncertainty in the SM \([48]\). New physics
TABLE I: Numerical values of the theoretical quantities used in the numerical analysis are shown.

| Parameter | Value          |
|-----------|----------------|
| $m_{B_s}$ | 5.366 GeV      |
| $m_{B_s}(m_c)$ | 4.19(±0.18)(-0.06) GeV |
| $m_c(m_b)$ | 1.27(±0.07)(-0.09) GeV |
| $m_s(2$ GeV) | 0.084 GeV      |
| $\tau_{B_s}$ | 1.425 (0.041) ps |
| $|V_{cb}|$ | 0.04128         |
| $|V_{cs}|$ | 0.97342         |
| $|V_{td}|$ | 0.999141        |
| $|V_{ts}|$ | 0.04054         |
| $C_1(m_b)$ | -0.1903        |
| $C_2(m_b)$ | 1.081          |
| $C_3(m_b)$ | 0.0137         |
| $C_4(m_b)$ | -0.036         |
| $C_5(m_b)$ | 0.009          |
| $C_6(m_b)$ | -0.042         |
| $f_{B_s}$ | 238(9.5) MeV    |
| $B_1$ | 0.87           |
| $B_2$ | 0.84           |
| $B_3$ | 0.91           |
| $B_4$ | 1.16           |
| $B_5$ | 1.75           |
| $B_6$ | 1.75           |

FIG. 5: The $a_{s1} - \sin(\phi_M^s - \phi_T^s)$ (left panel) and $a_{s1} - m_{KK}$ (right panel) correlation plots in the RS model. The gray band indicates the 1σ experimental allowed ranges for $a_{s1}^{(avg)}$. The green line indicates the SM prediction for $a_{s1}^{(avg)}$. As one can see from these figures, the fit results allow $a_{s1}^{(avg)} \sim -0.00498$, which is 1.54σ away from its experimental average value in Eq.(12). The value of the corresponding $\sin(\phi_M^s - \phi_T^s)$ is $-0.76$. Hence this model cannot fully account for the experimental results and this is due to the fact that this model cannot generate enough correction to the width difference $\Delta \Gamma_{12}$. Note that, as $m_{KK}$ is increased the suppression to $a_{s1}^s$ due to $m_{KK}$ is partially compensated by larger mixing angles which are within the considered ranges.
in this work and are consistent with experimental measurements of $\Delta M_s$ and $\Delta \Gamma_s$. However, after a certain point the suppression due to $m_{KK}$ cannot be compensated and $a^s_{2l}$ decreases with increasing $m_{KK}$ mass.

VI. CONCLUSIONS

In the past few years several measurements in rare B decays involving $b \rightarrow s$ transitions have been somewhat difficult to understand in the SM. This has put the focus on measurements in the $B^0_s - \bar{B}^0_s$ system. The measurements of the phase in $B^0_s - \bar{B}^0_s$ mixing and more recently the like-sign dimuon asymmetry have generated an enormous interest in the community as these measurements indicate possible deviations from the SM predictions. In this work, we presented calculations for general new physics corrections to the parameters in $B^0_s - \bar{B}^0_s$ mixing. Taking the Randall-Sundrum model as an example of new physics we calculated the wrong-charge asymmetry, $a^s_{2l}$, as well as other parameters in $B^0_s - \bar{B}^0_s$ mixing. Our calculations indicate that while the RS model can cause deviations from the SM predictions for the wrong-charge asymmetry, it cannot explain the present experimental average value within the 1$\sigma$ range. This is due its inability to generate sufficient new contribution to the width difference $\Delta \Gamma^s_{12}$, even though the model can generate large contribution to the mass difference $\Delta M^s_{12}$.

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