Constraints on flavor-changing $Z'$ models by $B_s$ mixing, $Z'$ production, and $B_s \to \mu^+ \mu^-$

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Certain string-inspired $Z'$ models have non-universal interactions to three families of fermions and induced tree-level flavor-changing couplings. We use recent results on $B_s$-$\bar{B}_s$ mixing to constrain the size of the flavor-changing couplings in the $b$-$s$ sector. In some highly predictive $Z'$ models, such a constraint on $b$-$s$ coupling can be translated into the flavor-diagonal couplings. Based on the $Z'$ production limits at the Tevatron, we obtain the limit on the leptonic couplings of $Z'$ and then make predictions for $B_s \to \mu^+ \mu^-$ branching ratios. We conclude that with the present constraints from $B_s$ mixing and $Z'$ production, the muonic decay of $B_s$ may not be observed at the Tevatron if the projected integrated luminosity is less than $O(5 - 10)$ fb.

Searches for flavor-changing neutral currents (FCNC) have been pursued for many years. So far, the sizes of FCNC in the $u$-$c$, $b$-$s$, $s$-$d$, and $d$-$b$ sectors, in general, agree with the Standard Model (SM) predictions, namely, those derived from the Cabibbo-Kobayashi-Maskawa (CKM) mechanism in higher order. The FCNC effect in $b$-$s$ sector was recently confirmed in the $B_s$ meson mixing observed by both CDF and DØ:

CDF : $\Delta M_s = 17.33^{+0.42}_{-0.21}$ (stat.) $\pm 0.07$ (syst.) ps$^{-1}$,
DØ : $\Delta M_s = 19.0 \pm 1.215$ ps$^{-1}$,

where we have converted the 90% C.L. bound $17 < \Delta M_s < 21$ ps$^{-1}$ of DØ into 1σ range assuming the error is Gaussian. We combine both results, again assuming Gaussian errors, and get

$$\Delta M_s^{\text{exp}} = 17.46^{+0.47}_{-0.30} \text{ ps}^{-1} \quad (1\sigma \text{ range}).$$ (1)

We re-evaluate the SM prediction $\Delta M_s^{\text{SM}} = 19.52 \pm 5.28$ ps$^{-1}$. (2)

It is important to use the best-fitted inputs without the new $B_s$ mixing data in order to determine if there is any discrepancy between the data and the SM prediction. Measurement of $B_s$ mixing is often used to determine the value of $|V_{ts}|$, but this is clearly inappropriate when the mixing has additional contribution from new physics. The SM prediction in Eq. (2) contains large uncertainty from the hadronic parameters, nevertheless, the data agrees fairly well with the SM value. Therefore, we can use the $B_s$ data to constrain new physics that may induce the $b$-$s$ transitions.

Another important channel to search for FCNC is the muonic $B_s$ decay, $B_s \to \mu^+ \mu^-$, which has the largest chance to be detected at hadronic machines. In the SM, this process is loop-suppressed. However, many extensions of the SM predict a branching ratio large enough to be seen at hadron colliders. We consider an FCNC $Z'$ model inspired by string theory in this letter. The $B_s$ mixing and $B_s \to \mu^+ \mu^-$ are highly correlated because from the $B_s$ data one can constrain the FCNC $b$-$s$-$Z'$ coupling, which is an essential element in the calculation of the muonic decay. One additional element is the $\mu$-$\mu$-$Z'$ coupling. In order to make a reliable prediction for the muonic decay branching ratio, we take into account the $\sigma(Z') \cdot B(Z' \to e^+ e^-)$ limits from the Tevatron. In the $Z'$ model considered here, the FCNC $b$-$s$-$Z'$ coupling is related to the flavor-diagonal couplings $qqZ'$ in a predictive way, which are then used to obtain the upper limits on the leptonic $\ell\ell Z'$ couplings. Therefore, we are able to predict the maximally allowed branching ratio for the muonic decay of $B_s$. The predicted branching ratio is always less than $9 \times 10^{-9}$ for $M_{Z'} = 200 - 900$ GeV. It implies that with the present constraints from $B_s$ mixing and $Z'$ production, the muonic decay of $B_s$ may not be observable at the Tevatron if the projected integrated luminosity is less than $O(5 - 10)$ fb.

In some string-inspired models, the three generations of SM fermions are constructed differently and may result in family non-universal couplings to an extra $U(1)$ gauge boson, $Z'$. Without loss of generality, we consider the case that the $Z'$ couples with a different strength to the third generation, as motivated by a particular class of string models [2]. Once we do a unitary rotation from the interaction basis to mass eigenbasis, tree-level FCNCs are induced naturally. Several works have recently been done regarding the FCNCs in the down-quark sector [1, 3, 4]. In order to increase the predictive power, we assume that the left-handed (LH) up-type sector is already in diagonal form, such that $V_{\text{CKM}} = V_{\text{DL}}$, where $V_{\text{DL}}$ is the LH down-type sector unitary rotation matrix. Since we do not have much information about either the right-handed (RH) up-type or the RH down-type sectors, we simply assume that their interactions with $Z'$ are family-universal and flavor-diagonal in the interaction basis. In this case, unitary rotations keep the RH couplings flavor-diagonal.
Therefore, the FCNCs only arise in the LH d-s-b sector. The couplings depend on the CKM matrix elements and one additional parameter \( x \), which denotes the strength of \( Z' \) coupling to the third generation LH quarks relative to the first two generations. Consequently, if \( x \) is an \( O(1) \) parameter but not exactly equal to 1, the b-s-Z' coupling will induce a significant FCNC effect.

We follow closely the formalism given in Ref. [5]. We assume for simplicity that there is no mixing between \( Z \) and \( Z' \), as favored by the precision data. The current associated with the additional \( U(1) \) gauge symmetry is

\[
J^{(2)}_{\mu} = \sum_{i,j} \overline{\psi}_{L,i} \gamma_{\mu} \left[ \varepsilon^{(2)}_{\psi_{L,j}} P_L + \varepsilon^{(2)}_{\psi_{R,j}} P_R \right] \psi_j , \tag{3}
\]

where \( \varepsilon^{(2)}_{\psi_{L,R,i,j}} \) is the chiral coupling of \( Z' \) with fermions \( i \) and \( j \) running over all quarks and leptons. The \( Z' \) couplings to the leptons and up-type quarks are assumed flavor-diagonal and family-universal:

\[
\varepsilon^{(2)}_{L,R} = Q^{(2)}_{L,R} \mathbb{1}, \quad \varepsilon^{(2)}_{L,R} = Q^{(2)}_{L',R} \mathbb{1} \quad \text{and} \quad \varepsilon^{(2)}_{R} = Q^{(2)}_{R} \mathbb{1}
\]

where \( \mathbb{1} \) is the 3 x 3 identity matrix in the generation space and \( Q^{(2)}_{L',R} \) and \( Q^{(2)}_{R} \) are the chiral charges. On the other hand, the interaction of \( Z' \) with the down-type quarks is

\[
\mathcal{L}^{(2)}_{NC} = -g_2 Z_{d}^I \left( \bar{d} \gamma_{\mu} \bar{s} \gamma_{\mu} \right) \varepsilon^{(2)}_{L,R} P_L \varepsilon^{(2)}_{R} P_R \left( \begin{array}{c} d \\ s \end{array} \right) I, \tag{4}
\]

where the subscript \( I \) denotes the interaction basis. For definiteness in our predictions, we assume

\[
\varepsilon^{(2)}_{L} = Q^{(2)}_{L} \left( \begin{array}{ccc} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & x \end{array} \right), \quad \varepsilon^{(2)}_{R} = Q^{(2)}_{R} \left( \begin{array}{ccc} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \end{array} \right) . \tag{5}
\]

The deviation from family universality and thus the magnitude of FCNC are characterized by the parameter \( x \) in the \( \overline{b}\tau, bL-Z' \) entry. The chiral charges have to be specified by the \( Z' \) model of interest.

When diagonalizing the down-type Yukawa matrix, we rotate the LH and RH fields by \( V_{dl} \) and \( V_{dr} \), respectively. With the form of \( \varepsilon^{(2)}_{L,R} \) assumed as in Eq. (4), the RH sector remains flavor-diagonal in the mass eigenbasis but the LH sector \( V_{dl}^\dagger \varepsilon^{(2)}_{L} V_{dl} \), is in general non-diagonal. No mixing in up-quark sector, we have \( V_{CKM} = V_{dl} \), making the model very predictive. Explicitly,

\[
B^{d}_{L} \equiv V_{dl}^\dagger \overline{Q}^{d}_{L} V_{dl} = V_{CKM}^d \overline{Q}^{d}_{L} \overline{V}^{d}_{CKM}
\]

\[
\approx Q^{d}_{L} \left( \begin{array}{ccc} 1 \\
(x - 1)V_{td} V_{td}^* \\
(x - 1)V_{td} V_{td}^* \end{array} \right) \left( \begin{array}{ccc} (x - 1) V_{tb} V_{bs} \\
(x - 1) V_{tb} V_{bs} \\
(x - 1) V_{tb} V_{bs} \end{array} \right) x
\]

where we have made simplifications using the unitarity of \( V_{CKM} \). It is interesting to note that the sizes of the flavor-changing couplings satisfy the hierarchy: \( |B^{bs}_{L}| > |B^{bd}_{L}| > |B^{bd}_{L}| \).

So far, we have not specified the RH chiral couplings of the down sector nor the chiral couplings of the up sector. In order to obtain constraints from \( Z' \) production at the Tevatron, we take the following assumptions

\[
|Q^{d}_{R}| = |Q^{d}_{L}|, \quad |Q^{d}_{L,R}| = |Q^{d}_{L,R}| . \tag{7}
\]

Such assumptions are reasonable; many \( Z' \) models predict chiral couplings to be of a similar order (e.g., the \( Z_{\psi} \) model has all chiral couplings equal to \( 1/\sqrt{2} \)). Moreover, the prediction of \( Z' \) production depends on the factors \( Q^{d}_{L,2} + Q^{d}_{R,2} \). Thus, changing to another \( Z' \) model would not affect the limits significantly.

Within the SM the mass difference in the \( B_s \) system is

\[
\Delta M^{SM}_{B_s} = \frac{G^2_F}{6\pi^2} M^0_{W} m_{B_s} J^2_B \left( V_{tb} V_{ts} \right)^2 \eta_{SB} S_0(x_t) \times \left[ \frac{\alpha_s(m_b)}{4\pi} \right]^{6/23} \left[ 1 + \frac{\alpha_s(m_b)}{4\pi} J_5 \right] B_{B_s}(m_b) , \tag{8}
\]

where \( S_0(x_t) = 2.463 \) and the NLO short-distance QCD corrections are encoded in the parameters \( \eta_{SB} \approx 0.551 \) and \( J_5 \approx 1.627 \). We have taken \( M_W = 5.3696 \pm 0.0024 \) GeV, and \( \tau_{B_s} = 1.466 \pm 0.059 \) ps\(^{-1} \). For the Wolfenstein parameters, we use the CKMfitter results after EPS 2005: \( \lambda = 0.2262 \pm 0.00100 \) and \( A = 0.825^{+0.011}_{-0.009} \). The hadronic parameter \( f_{B_s} B_{B_s} = 0.262 \pm 0.035 \) is taken from the lattice calculation \( 2 \). After taking the mean for asymmetric errors, we obtain the SM prediction \(^4\) as in Eq. (8). The effect of LH FCNC induced by \( Z' \) is given by

\[
\frac{\Delta M_{exp}}{\Delta M_{SM}} = \left[ 1 + 3.57 \times 10^5 \left( \rho^b_{L} \right)^2 e^{2\phi^b_{L}} \right] = 0.894 \pm 0.243 , \tag{9}
\]

where \( \phi^b_L \) is the weak phase associated with the coupling \( B_s^{b} \). In the model that we consider

\[
\rho^b_{L} = \frac{g_2 M_Z}{g_{1} M_{Z'}} B_{s}^{b} = \left| \frac{g_2 M_Z}{g_{1} M_{Z'}} (x - 1) Q^{d}_{L} V_{tb} V_{ts} \right| , \tag{10}
\]

and \( \phi^b_{L} = 180^\circ \). We have combined the relative errors of \( \Delta M_{exp} \) and \( \Delta M_{SM} \) in quadrature in Eq. (9).

We show the allowed parameter space of \( (\rho^b_{L}, \phi^b_{L}) \) in Fig. 1. For \( \phi^b_{L} = 0 \) or \( 180^\circ \), \( \rho^b_{L} \) is constrained to be less than \( 6.20 \times 10^{-4} \). In more general models, \( \phi^b_{L} \) may have a different value. For example, if \( \phi^b_{L} = 90^\circ \), \( \rho^b_{L} \) is constrained to be less than \( 9.87 \times 10^{-4} \). Note that there are regions with \( \rho^b_{L} > 9.87 \times 10^{-4} \) also allowed by the current

\(^1\) For consistency, this SM value is obtained without referring to the measured \( \Delta M_b \) because \( Z' \) can also have contributions in the \( B_d \) system, even though the uncertainty in Eq. (9) could have been much smaller by doing so.
ΔM_s constraint, e.g., 2.15 × 10^{-3} ≤ ρ_s^b ≤ 2.45 × 10^{-3} for φ^b = 90°. However, some of these regions correspond to Z′ contributions larger than the SM contributions. Although not completely impossible, we think it is unlikely and thus leave it out from the discussions in the rest of the paper.

The production cross section of Z′ followed by the leptonic decay is given by

\[ σ(p\bar{p} \to Z' \to ℓ^+ ℓ^-) = \frac{g_2^4}{144π} \frac{M_{Z'}}{η_{Z'}} (Q_L^2 + Q_R^2) \]

× \sum_{q=u,d,s,c} \left( Q_L^q + Q_R^q \right) \int_0^1 \frac{dx}{x} f_2(x) f_3 \left( \frac{1}{x} \right) \]

where \( \sqrt{s} = 1960 \text{ GeV} \), \( r = M_{Z'}^2/s \) and \( Γ_{Z'} \) is the total width. The partial width \( Z' \to ℓ^+ ℓ^- \) is

\[ Γ(Z' → ℓ^+ ℓ^-) = \frac{N_f g_2^2 M_{Z'}}{48π} \sqrt{1 - 4μ} \left( |Q_L|^2 + |Q_R|^2 \right) \]

**TABLE I:** The 95% C.L. limits on \( σ(Z') \cdot B(Z' \to ℓ^+ ℓ^-) \) given by the preliminary CDF result in Ref. [10] as a function of \( M_{Z'} \).

| \( M_{Z'} \) (GeV) | \( σ \cdot B^{5σ} \) (pb) | \( M_{Z'} \) (GeV) | \( σ \cdot B^{5σ} \) (pb) |
|------------------|-----------------|------------------|-----------------|
| 200              | 0.0505          | 600              | 0.0132          |
| 250              | 0.0743          | 650              | 0.0136          |
| 300              | 0.0289          | 700              | 0.0134          |
| 350              | 0.0404          | 750              | 0.0126          |
| 400              | 0.0261          | 800              | 0.0171          |
| 450              | 0.0259          | 850              | 0.0172          |
| 500              | 0.0172          | 900              | 0.0215          |
| 550              | 0.0138          | 950              | 0.0246          |

FIG. 1: The allowed parameter space in the Z′ model with FCNC only in the LH sector. The shaded area corresponds to the 1σ C.L. limits of \( ΔM_s^{exp}/ΔM_s^{SM} = 0.894 ± 0.243 \). The central value 0.894 corresponding to \( ΔM_s^{exp} = 17.46 \text{ ps}^{-1} \) is also indicated.

FIG. 2: The upper limits on \( g_2\sqrt{(Q_L^2 + Q_R^2)/2} \) obtained using the CDF 95% C.L. limits of \( σ(Z') \cdot B(Z' \to ℓ^+ ℓ^-) \) tabulated in Table I. The constrained value of \( ρ_s^b = 6.20 × 10^{-4} \) is used.

\[ (1 - μ) + 12μ Q_L^f Q_R^f \]

where \( N_f = 3(1) \) for quark (lepton) and \( μ = m_f^2/M_{Z'}^2 \).

We have included all leptonic and hadronic modes in the total width. Note that the FCNC contributions are negligible and \( Z → W^+W^- \) is highly suppressed by the \( Z-Z' \) mixing angle, which is severely constrained by electroweak precision data [11].

The most recent (though preliminary) upper limits on the Z′ search was performed by CDF with an integrated luminosity 819 pb\(^{-1}\) [10]. We read off the limits of \( σ \cdot B(Z' → ℓ^+ ℓ^-) \) from their figure, and tabulated in Table I. We use the constrained value of \( ρ_s^b \) to obtain the value for \( g_2 Q_{L,R}^b \), which is in turned related to \( g_2 Q_{L,R}^f \) by Eq. (10). With our assumptions in Eq. (7), we can then obtain the upper limits on \( g_2\sqrt{(Q_L^2 + Q_R^2)/2} \) from the CDF 95% C.L. limits of \( σ(Z') \cdot B(Z' → ℓ^+ ℓ^-) \).

We show the limits of \( g_2\sqrt{(Q_L^2 + Q_R^2)/2} \) with the constrained value of \( ρ_s^b = 6.20 × 10^{-4} \) in Fig. 2. At smaller \( M_{Z'} \) the limits are insensitive to the value of \( x \), while the difference is more visible at larger \( M_{Z'} \). When \( x \) gets larger toward 1, the allowed value for \( Q_L^f \) from \( ρ_s^b \) increases.

Thus, the corresponding upper limits on \( Q_{L,R}^f \) have to be smaller. That is why at each \( M_{Z'} \) the chosen values of \( x = 0.1 \) to 0.9 go from top to bottom. The general trend for increasing \( M_{Z'} \) is the increase in the upper limit of \( Q_{L,R}^f \). This is easy to understand because at large \( M_{Z'} \) the dominant factor in the production cross section is the parton density, which becomes very small at large momentum fractions.

We are ready to compute the maximally allowed decay rate for \( B_s \to μ^+ μ^- \). We ignore the RG running effect at the b-s-Z′ vertex, which is good enough for an order-of-
different sets of lines in the drawing correspond to different

\[ \rho_L^{b} \rho_L^{\mu} = \rho_R^{\mu} \]

plane using the current upper bounds on \( B(B_s \to \mu^+ \mu^-) \):

\[ 1 \times 10^{-7} \] (CDF) and \[ 2.3 \times 10^{-7} \] (DO). The upper limit of

\[ \rho_L^{b} \] for \( M_{Z'} = 0^\circ \) and \( 90^\circ \) are also indicated using the blue
dashed and red dashed lines, respectively.

The branching ratio of \( B_s \to \mu^+ \mu^- \) is given by

\[ B(B_s \to \mu^+ \mu^-) = \tau(B_s) \frac{G_F^2 f_B^2 m_s^2 m_{B_s}}{4 \pi} \left( 1 - \frac{4 m_{\mu}^2}{m_{B_s}^2} \right) \left| V_{ts}^* V_{tb} \right|^2 \]

\[ \times \left\{ \alpha \left( \frac{m_t^2}{M_W^2} \right) + \left( \frac{\rho_L^{b} \rho_L^{\mu} \rho_L^{\mu} \rho_L^{b}}{V_{tb}^* V_{ts}} \right)^2 + \left( \frac{2 \rho_L^{b} \rho_L^{\mu}}{V_{tb}^* V_{ts}} \right)^2 \right\} , \tag{13} \]

where

\[ \rho_{L,R}^{\mu} = \frac{g_2 M_{Z'}}{g_1 M_{Z'}} Q_{L,R}^e \tag{14} \]
at the weak scale. One can find the definition of

\[ Y(m_t^2/M_W^2) \] in the SM part in Ref. [4]; its value is

about 1.05 here. Using the central value of the averaged \( B_s \) lifetime \( \tau_{B_s} = 1.461 \) ps, corresponding to

\( \Gamma_{B_s} = (4.49 \pm 0.18) \times 10^{-13} \) GeV and \( f_{B_s} = 230 \) MeV,

we obtain a SM branching ratio of about \( 4.2 \times 10^{-9} \). The current upper limits on \( B(B_s \to \mu^+ \mu^-) \) from CDF and

DO based on 780 and 700 pb\(^{-1}\) data, respectively, are

\[ B(B_s \to \mu^+ \mu^-) < 1.0 \times 10^{-7} \quad \text{(CDF)} \]

\[ B(B_s \to \mu^+ \mu^-) < 2.3 \times 10^{-7} \quad \text{(DO)} . \]

Fig. 3. The allowed parameter space on the \( \rho_L^{b} \rho_L^{\mu} = \rho_R^{\mu} \) plane using the current upper bounds on \( B(B_s \to \mu^+ \mu^-) \):

\[ 1 \times 10^{-7} \] (CDF) and \[ 2.3 \times 10^{-7} \] (DO). The upper limit of

\[ \rho_L^{b} \] for \( M_{Z'} = 0^\circ \) and \( 90^\circ \) are also indicated using the blue
dashed and red dashed lines, respectively.

Fig. 4. Dependence of the upper limits of \( B(B_s \to \mu^+ \mu^-) \) on \( M_{Z'} \), where different sets of lines correspond to different
values of the parameter \( x \) in our model, ranging from 0.1 to 0.9. We have set \( \rho_R^{\mu} = \rho_L^{\mu} \).

We conclude that with the present constraints from

\( B_s \) mixing and \( Z' \) production, the muonic decay of \( B_s \) may not be observed at the Tevatron if the projected
integrated luminosity is less than \( O(5 - 10) \) fb. However, at LHCb, with anticipated production of \( 10^{12} \) \( b \bar{b} \) per year,
the expected branching ratio of order \( 10^{-9} \) is observable.

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