DOA Estimation for Coherent Signals Based on Modified Matrix Reconstruction

Wei Zhang*, Yong Han

1 School of Electrical & Information Engineering, North Minzu University, Yinchuan, Ningxia, 750021, China
2 School of Information Engineering, Harbin Institute of Technology at Weihai, Weihai, Shandong, 264209, China
*Corresponding author’s e-mail: zv45@163.com

Abstract. In this paper, we propose a modified ESPRIT-like method for direction-of-arrival estimation of coherent signals. Any two rows of the output covariance matrix is adopted to reconstruct two Toeplitz matrices, after Hermitian transposition and forward and backward smoothing, the equivalent data matrix is utilized for coherent signals DOA estimation combined subspace-based algorithm. The presented method solves the problem that the equivalent covariance matrix of ESPRIT-like method is not Hermitian matrix, which leads to the incomplete information utilization. The simulation results show the effectiveness of the proposed method.

1. Introduction
Direction-of-arrival (DOA) estimation[1-4] of array signal processing, as one of the important research directions, has broad application prospects in many applications. Typical high resolution subspace algorithms such as MUSIC[5] and ESPRIT[6], have good estimation performance in the case of uncorrelated sources. However, in the presence of a large number of coherent sources due to multipath propagation and co-channel interference, the estimation performance is drastically degraded or even ineffective. Classical de-coherence methods include spatial smoothing algorithms, such as forward spatial smoothing(SS)[7], forward and backward spatial smoothing(FBSS)[8], which divide the array into several subarrays to restore the rank of the source covariance matrix. The implementation is more complex, and the number of smoothing is determined by the known sources number.

Another efficient algorithms based on matrix reconstruction are rearranged the covariance matrix elements of the received signals to reconstruct one or more Toeplitz matrices. Its rank is only related to the DOAs of the signal, and is not affected by signal correlation. Han proposes an ESPRIT-like[9] method. The Toeplitz matrix is reconstructed by choosing any row of the covariance matrix of the received data. However, the equivalent data covariance matrix is not Hermitian matrix, and its information utilization is lost. Moreover, only one row of covariance matrix is used to reconstruct that leads to the incomplete information utilization, which affects the estimation performance. In [10], ESPRIT-like algorithm based on Uniform Linear Array (ULA) is extended to two-dimensional uniform rectangular arrays. In this paper, we propose an improved method, which uses any two rows of the covariance matrix to reconstruct the equivalent Hermitian covariance matrix. Computer simulation verifies that our method improves the estimation performance and resolution probability compared with ESPRIT-like method. The symbols “E[·]”, “(·)’”, “(·)’” and “(·)’’ are used to
express mathematical expectation, conjugate, transposition and conjugate transposition respectively. \( I_m \) represents an \( m \times m \) dimension unit matrix and \( \text{diag}(\cdot) \) denotes diagonal matrix.

2. Data model

We consider a symmetrical uniform linear array (SULA) (See ref. [9]) with the distance between adjacent sensors \( d = \lambda/2 \), where \( \lambda \) is the carrier wavelength. Supposing there are \( P (P \leq M) \) narrowband far-field signals from directions \( \theta_i, i = 1,2,...,P \) respectively, the former \( L \) signals are coherent, the latter \( P - L \) signals are uncorrelated and independent of the former \( L \) signals. Let the index of the central array element be 0, and the \( (2M + 1) \times 1 \) receiving vector of the array is

\[
X(t) = AS(t) + N(t) = [x_{-M}(t), \ldots, x_{-M_1}(t), \ldots, x_M(t)]^T
\]

(1)

where \( S(t) = [s_1(t), \ldots, s_p(t)]^T \) is the \( P \times 1 \) source vector, \( N(t) = [n_{-M}(t), \ldots, n_{-M_1}(t), \ldots, n_M(t)]^T \) is the \( (2M + 1) \times 1 \) white noise vector assumed to be uncorrelated to signals with zero mean and variance \( \sigma_n^2 \).

\[
A = [a(\theta_1), \ldots, a(\theta_P)]
\]

is the \( (2M+1) \times P \) steering matrix with \( a(\theta_p) = [e^{-j2\pi \lambda M d \sin \theta_1}, \ldots, e^{-j2\pi \lambda P d \sin \theta_p}]^T \). Hence, the \( k \)th output of the array is given by

\[
x_k(t) = s_k(t) + \sum_{i=1}^{P} \beta_i e^{-j2\pi \lambda M d \sin \theta_i} + n_k(t), \quad (k = -M, \ldots, 0, \ldots, M)
\]

(2)

where \( s_k(t) \) denotes the complex envelope of the \( k \)th signal. \( \beta_l = \rho_l e^{j\Delta \phi} \), \( l = 1,2,...,L \) is the complex amplitude fading coefficient of the \( l \)th coherent signal, \( \rho_l \) is the complex fading factor and \( \Delta \phi_l \) is the phase change of \( s_k(t) \) related to \( s_k(t) \). The output covariance matrix can be formulated as,

\[
R = E\left[ X(t) X^H(t) \right] = AR^H \sigma_n^2 + \sigma_e^2 I
\]

(3)

then we construct a Toeplitz matrix by using any row of \( R \) as follows,

\[
R(m) = \begin{bmatrix}
r(m,0) & r(m,1) & \cdots & r(m,M) \\
r(m,-1) & r(m,0) & \cdots & r(m,M-1) \\
\vdots & \vdots & & \vdots \\
r(m,-M) & r(m,-M+1) & \cdots & r(m,0) \\
\end{bmatrix}
\]

(4)

where \( D(m) = \text{diag}(d_{m,1},d_{m,2},\ldots,d_{m,P}) \), and \( A_m = [a(\theta_1), \ldots, a(\theta_P)] \) with \( a(\theta_p) = [1, e^{-j2\pi \lambda M d \sin \theta_1}, \ldots, e^{-j2\pi \lambda P d \sin \theta_p}]^T \).

According to the results in [4], the \((m,k)\) entry of \( R \) can be written as

\[
r(m,k) = \sum_{i=1}^{P} d_{m,i} e^{-j2\pi \lambda M d \sin \theta_i} + \sigma_e^2 \delta_{m,k},
\]

(5)

with

\[
d_{m,i} = \begin{cases}
P_i \beta_i e^{-j2\pi \lambda M d \sin \theta_i}, & i = 1,\ldots,L \\
P_i e^{-j2\pi \lambda L d \sin \theta_i}, & i = L+1,\ldots,P
\end{cases}
\]

(6)

here \( P_i = E[s_i(t) s_i^*(t)] \), \( l,i = 1,L+1,\ldots,P \), and \( \delta_{m,k} = \begin{cases} 1 & k = m \\ 0 & k \neq m \end{cases} \).
From (4) we deduce that $R$ is the Toeplitz matrix, but not Hermitian matrix, that means the eigenvalues in $R$ are complex numbers which leads to loss of the information. Another problem is that only one row of the covariance matrix is used and the information is not fully utilized.

Hence, we select any two rows to reconstruct two Toeplitz matrices, and another Toeplitz matrix is denoted similar to (4),

$$R(n) = A D(n) A^H + \sigma^2_s I_{M+n,s}$$

where $D(n) = \text{diag}(d_{n,1}, d_{n,2}, \ldots, d_{n,p})$. In the absence of noise, we add the two matrices by multiplying the conjugates themselves,

$$R_s = R(m) R^H(m) + R(n) R^H(n)$$

After the forward and backward smoothing averaging, the equivalent source covariance matrix is given by

$$R_j = (R_s + J R_j J) / 2$$

where $J$ denotes a permutation matrix with ones on its anti-diagonal and zeros elsewhere. Combined with subspace-based algorithms, the DOAs of coherent signals can be estimated.

3. Numerical results

In order to verify the effectiveness of our method, we test the resolution probability and estimation accuracy of the proposed algorithm compared with those of SS$^7$, FBSS$^8$ and ESPRIT-like algorithm$^9$. In this paper, our method is combined the ESPRIT same as ESPRIT-like algorithm and root-MUSIC is used in SS and FBSS estimators with the smoothing number being 3 and 2. Here we define the Root Mean Squared Error (RMSE) as

$$\text{RMSE}_p = \sqrt{\frac{1}{K P} \sum_{p=1}^{P} \sum_{i=1}^{K} (\hat{\theta}_{i,p} - \theta)^2}$$

where $K$ is the number of independent Monte-Carlo trails and $P$ is the number of source signals. Defining the absolute value of the difference between the angle estimate and the true value is less than $1^\circ$ as a successful resolution, and the probability of resolution is the ratio of the number of successful estimates to the number of Monte Carlo experiments. We consider three equal-power narrow-band incident signals with a center frequency of 9 MHz impinging on a nine-element half-wavelength SULA from -28°, 0° and 15°, there the first signal are uncorrelated and the coherent with two other signals. The sampling frequency is set to 1 kHz.

In the first example, we test the RMSE performance and probability of resolution versus SNR with 100 snapshots via 1000 independent trials for each SNR. Fig. 1 shows that with the increase of SNR, the estimation accuracy of our proposed method is significantly better than that of ESPRIT-like and SS algorithm. When in the low/middle SNR (SNR<10 dB) our method has better estimation accuracy than FBSS, and slightly lower than FBSS at high SNR. The result of Fig. 2 shows that the successful resolution of our algorithm is slightly lower than FBSS under the whole SNR regime, which is always higher than the other two algorithms, especially when the SNR=20 dB, the success probability of our algorithm is close to 100%, and ESPRIT-like is less than 60%.

In the second example, we study the RMSE and resolution probability versus the number of snapshots. The SNR is fixed at 0 dB, and the number of snapshots increased from 10 to 800. 1000 independent Monte Carlo experiments were carried out respectively. The other simulation conditions are same as the first example. It is seen from Fig. 3 that the RMSE of our method has higher estimation performance than the other three algorithms when the number of snapshots is small (less
than 10). With an accumulation of snapshot numbers, our algorithm has better estimation performance than the SS and ESPRIT-like algorithm, slightly inferior to the FBSS algorithm. Fig. 4 shows that the successful resolution of our method is slightly lower than the FBSS algorithm for all values of snapshots, which is always higher than the SS algorithm and ESPRIT-like algorithm, especially under small snapshots (less than 100).

4. Conclusions
In this paper, an improved algorithm for DOA estimation of coherent signal is proposed. The equivalent data covariance matrix with Hermitian structure is obtained based on two reconstructed Toeplitz matrices, which solve the problems for the incomplete information utilization of ESPRIT-like algorithm with only one row of the covariance matrix, and the equivalent data covariance matrix being non-Hermitian matrix. The simulation experiments verify that the proposed algorithm has better estimation accuracy and successful resolution than ESPRIT-like and SS methods.

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References
[1] Li, J., Li, Y. and Zhang, X. (2019) Direction of arrival estimation using combined coprime and nested array. Electronics Letters, 55(8): 487–489.
[2] Zhang, W., Han, Y., etc. (2019) Multiple-Toeplitz matrices reconstruction algorithm for DOA estimation of coherent signals. IEEE Access, 7: 49504–49512.
[3] Qin, G., Amin, M. and Zhang, Y. (2019) DOA estimation exploiting sparse array motions. IEEE Transactions on Signal Processing, 67(11): 3013–3027.
[4] Wang, M., Zhang, Z. and Nehorai, A. (2019) Grid-less DOA estimation using sparse linear arrays based on Wasserstein distance. IEEE Signal Processing Letters, 26(6): 838–842.
[5] Zhang, X., Chen, W., etc. (2018) Localization of near-field sources: A reduced-dimension MUSIC algorithm. IEEE Communications Letters, 22(7): 1422–1425.
[6] Herzog, A., Habets, E. (2019) Eigenbeam-ESPRIT for DOA-vector estimation. IEEE Signal Processing Letters, 26(4): 572–576.
[7] Shan, T. J., Wax, M. (1985) On spatial smoothing for direction-of-arrival estimation of coherent signals. IEEE Transactions on Acoustics, Speech, and Signal Processing, 33(4): 806–811.
[8] Pillai, S. U., Kwon, B. H. (1989) Forward/backward spatial smoothing techniques for coherent signal identification. IEEE Transactions on Acoustics, Speech, and Signal Processing, 37(1): 8–15.
[9] Han, F. M., Zhang, X. D. (2005) An ESPRIT-like algorithm for coherent DOA estimation. IEEE Antennas and Wireless Propagation Letters, 4(1): 443–446.
[10] Ren, S., Ma X., etc. (2013) 2-D unitary ESPRIT-like direction-of-arrival (DOA) estimation for coherent signals with a uniform rectangular array. Sensors, 13(4):4272–4228.