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New Bounds on Dark Energy Induced Fifth Forces

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We consider the gravitational Wilsonian effective action at low energy when all the particles of the standard model have decoupled. When the $R^2$ terms dominate, the theory is equivalent to a scalar-tensor theory with the universal coupling $\beta = 1/\sqrt{6}$ to matter for which we present strong lower and upper bounds on the scalaron mass $m$ obtained by using results from the Eötvös experiment on the modification of the inverse-square law, the observations of the hot gas of galaxy clusters and the Planck satellite data on the neutrino masses. In terms of the range of the scalar interaction mediated over a distance of order $m^{-1}$, this leads to the small interval $4\mu m \lesssim m^{-1} \lesssim 68 \mu m$ within reach of future experimental tests of deviations from Newton’s gravitational inverse-square law.
I. INTRODUCTION

The recent results by LIGO/VIRGO show that most of the self-acceleration models of dark energy and modified gravity lead to excluded speeds for the gravitational waves [1, 2]. This leaves only as a viable option a whole swath of dark energy models where the magnitude of the vacuum energy has to be tuned. A possibility which has not been explored so far is that gravity itself, seen as a low energy effect, could actively participate in the mechanism leading to the acceleration without new degrees of freedom being added in an ad hoc fashion (such as quintessence fields or additional metrics). This can be realized using the higher order corrections to the Einstein-Hilbert action, which contain higher derivative interactions. Indeed, the fundamental symmetries of General Relativity allow for the presence of these higher derivative contributions to the local and diffeomorphism-invariant Lagrangian. Even if they are classically set to zero, they are generated by quantum corrections as counter-terms to ultraviolet divergences [3, 4]. These corrections can be seen to generate new intrinsic degrees of freedom. Generically, these degrees of freedom are ghost-like and can only be tolerated at low energy when the suppression scale of all the higher order operators lies at the cut-off scale of the low energy description [5].

As the dark energy scale $\rho_{\text{vac}} = 3\Omega_{\Lambda} M_{\text{Pl}}^2 H_0^2$ is of the milli-electron-Volt mass order, $\rho_{\text{vac}}^{1/4} \simeq 2$ meV, we can also integrate out the electron and all heavier particles of the standard model and concentrate on the very low energy degrees of freedom when considering the dynamics of the Universe at late time. This sets the cut-off scale of the low-energy effective field theory to the mass of the electron as a typical order of magnitude. The remaining low-energy degrees of freedom are gravity itself and the neutrinos. Amongst the higher derivative interactions in the gravitational sector, the Ricci scalar squared $R^2$ invariant plays a special role as it does not give rise to ghosts and is the most relevant interaction at low energy amongst the higher order gravitational interaction terms. This motivates hierarchical scenarios where the scalaron associated with $R^2$ has a low energy mass while the ghost-like contributions are rejected at the cut-off scale, as they should to satisfy theoretical and observational constraints. Then, the quantum fluctuations of the scalaron could act as dark energy and lead to the acceleration of the expansion of the Universe, without invoking any new physics at low energy. We have explicitly presented in [6] such a model, where the scalaron provides the new light degree of freedom that sets the dark energy scale and keeps
a local quantum field description, while the ghost-like contributions only arise at the cut-off scale.

In this paper we relax this hypothesis and do not consider that dark energy is necessarily driven by the scalaron’s vacuum energy only. We also allow for the existence of other light and decoupled-from-matter scalar degrees of freedom, such as light bosonic dark matter, of masses smaller than 0.1 eV, which can also contribute to the dark energy. In their absence we predict that the scalaron’s mass must be close to the averaged neutrino mass $\bar{m}_\nu$ (see section V) of order 0.1 eV in order to compensate for the negative contribution of the neutrinos to the dark energy and to satisfy the bound on the vacuum energy density in galaxy clusters. In this case, as we explain, the scalaron induces a deviation from Newton’s law with a coupling strength $\beta = 1/\sqrt{6}$ at a distance of order 4 $\mu$m. On the other hand if decoupled light scalars of masses less than 0.1 eV are present, they must evade gravitational tests and their contributions to the dark energy together with the scalaron’s compensate for the neutrino’s. This implies strong bounds (18) on the scalaron mass $m$ close to the current sensitivity of the Eöt-Wash experiments [7, 8] and within reach of the new runs [9] which have been recently presented.

II. THE LOW ENERGY EFFECTIVE $R^2$ MODEL

Our metric has signature $(- + + +)$. At low energy, below a cut-off scale $M$, the leading correction to the Einstein-Hilbert action reads $S + \delta S$, where

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R - \rho_\Lambda(\mu) + c_0(\mu)R^2 + c_2(\mu)(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2) \right],$$

and $\delta S$ contains all the higher order terms in the curvature invariants,

$$\delta S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} \sum_{n \geq 3} \alpha_n(\mu)M^2 \left( \frac{R}{M^2} \right)^n.$$  

Here $\alpha_n(\mu)$ are dimensionless coefficient of order $\mathcal{O}(1)$ and $R$ stands for the various Riemann tensor components $R_{\mu\nu\rho\sigma}$. The scale $M$ plays the role of the cut-off scale of the effective gravitational field theory, which is valid for $R \ll M^2 \ll M_{\text{Pl}}^2$. We are interested in a low-energy effective action in the energy range of the dark energy scale, below the electron mass, $\mu \ll m_e$. In this regime the typical curvature is tiny on cosmological scales [10]. As an effective field theory with higher order derivatives, there are new degrees of freedom per power of the Riemann tensor. They are generically ghost-like with a mass of order the Ultra-Violet cut-off scale $\mathcal{O}(M)$. Therefore, they do not play a role at low energy below $M$. 

It has been shown in [11, 12] that the coefficient $c_2(\mu)$ is always asymptotically free, since $dc_2(\mu)/d\log \mu^2 > 0$, whereas $c_0(\mu)$ is asymptotically safe, $dc_0(\mu)/d\log \mu^2 < 0$. For the non-tachyonic case $c_0(\mu) > 0$, which is the case of interest in this paper. Therefore, at low-energy $c_2(\mu)$ tends to zero whereas $c_0(\mu)$ grows, leading to the hierarchy $c_0(\mu) \gg c_2(\mu)$ at very low energy. Hence the quadratic Ricci scalar term is enhanced as compared with other quadratic and higher order contributions.

We will then work with the low-energy effective action

$$S_\mu = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \rho_\Lambda(\mu) + c_0(\mu)R^2 \right] + S_{\text{matter}},$$

obtained after integrating out all the massive particles of the standard model of masses above the electron mass, i.e. the matter action only involves the light matter fields of masses less than the electron mass.

### III. THE MINIMAL SCALARON MODEL

The $R^2$ theories are equivalent to scalar field models as reviewed in [13]. As such, they also correspond to the scalar-tensor theories

$$S_{\mu,\phi} = \int d^4x \sqrt{-g} \left( \frac{M_{Pl}^2}{2} R - \rho_\Lambda(\mu) - \frac{(\partial \phi)^2}{2} - \frac{m^2}{2} \phi^2 \right) + S_{\text{matter}}(\psi_1, e^{2\beta \phi/M_{Pl}} g_{\mu\nu}).$$

The massive scalar couples to the matter stress-energy tensor with the universal strength $\beta = 1/\sqrt{6}$ and the mass of the scalaron is

$$m(\mu)^2 = \frac{\beta^2 M_{Pl}^2}{2c_0(\mu)} \quad \text{with} \quad \beta = \frac{1}{\sqrt{6}},$$

and $S_{\text{matter}}$ is the matter action depending on the matter fields $\psi_i$. The scalar potential has been expanded to lowest order in $\phi/M_{Pl}$ as we are interested in the regime where $c_0 R \ll M_{Pl}^2$, and the standard Einstein-Hilbert term dominates so as to ensure convergence to General Relativity in the very low curvature regime. The scalaron self-interactions are negligible, being suppressed by powers of $m$ which is very small in Planck mass units.

The mass of the scalaron is affected by renormalisation effects. In the Jordan frame, where the particles of the standard models are coupled to $e^{2\beta \phi/M_{Pl}} g_{\mu\nu}$, the quantum fluctuations due to massive particles and the phase transitions in the matter sector are all scalar-independent. It is only when changing to the Einstein frame that the potential term of the $R^2$ model is
corrected by a term $e^{4\beta\phi/M_{Pl}}\rho_{\Lambda}(\mu)$ coming from the coupling of the energy density to matter. Expanding $e^{4\beta\phi/M_{Pl}}$ in powers of $\phi/M_{Pl}$ leads to the $\mu$ dependent mass

$$m^2(\mu) = m^2 + 16\beta^2 \rho_{\Lambda}(\mu) M_{Pl}^2. \quad (6)$$

At low energy below the electron mass, this correction is negligible as we shall confirm below. This implies that the coefficient $c_0$ in the low-energy $R^2$ action is also independent of renormalisation effects associated with the massive fields of the standard model.

To sum-up, the low-energy degrees of freedom below the electron mass are the neutrinos and the scalaron of mass $m \ll m_e$, associated with the Ricci scalar $R^2$ term. Notice that the new cut-off of this effective action is $m_e$.

IV. COSMOLOGICAL CONSTANT AND VACUUM ENERGY

We consider this effective field theory to be valid at a scale $\mu$ much lower than the electron mass, and to describe the late-time acceleration of the expansion of the Universe. At this energy scale, the only fields which have not been integrated out yet, when less massive than $\mu$, are the three massive neutrinos and the scalaron $\phi$. The vacuum energy $\rho_{\Lambda}(\mu)$ corresponds to the combined effect of all the quantum corrections associated to massive particles that have been integrated out, the various phase transitions including the QCD and electroweak ones $\rho_{SM}^{\Lambda}(\mu)$, and the bare cosmological constant $\rho_0^{\Lambda}$ seen as finite counter-term after renormalisation,

$$\rho_{\Lambda}(\mu) = \rho_{SM}^{\Lambda}(\mu) + \rho^0_{\Lambda}. \quad (7)$$

We shall work in the minimal decoupling subtraction scheme $\mathcal{DS}$ [5, §4.1.4], where the parameters of the Wilsonian action can be derived by integrating the renormalisation group equations obtained by taking into account in the $\beta$ functions only the particles that have not been integrated out yet. We refer to [5] for a lucid presentation of the Wilsonian effective actions and [14] for a review of various renormalisation group approaches to the cosmological constant question. Thus, at energies $\mu < m_e$, at one-loop order the vacuum energy receives quantum corrections due to the scalar field $\phi$ and the neutrinos only. This leads to the renormalisation group equation

$$\frac{d\rho_{\Lambda}(\mu)}{d\log \mu^2} = -\frac{m^4}{64\pi^2} \theta(\mu > m) + 2 \sum_{f=1}^3 \frac{m_f^4}{64\pi^2} \theta(\mu > m_f), \quad (8)$$
which includes the contribution of the three neutrinos, see the appendix A of [15] for the one-loop evaluation. The Heaviside factors ensure that the scalaron and the neutrinos no longer contribute to the running at energies below their mass threshold, as they decouple [5]. There are no contributions from photons or gluons in this very low energy regime. A similar equation has already been considered in [16] without the scalaron contribution.

At low energy, \( \max(m, m_f) < \mu < m_e \), before the scalaron and the neutrinos decouple, the vacuum energy \( \rho_\Lambda(\mu) \) is given by

\[
\rho_\Lambda(\mu) = \rho_\Lambda(m_e) + \left( \frac{m^4}{64\pi^2} - 2 \sum_{f=1}^{3} \frac{m_f^4}{64\pi^2} \right) \ln \frac{m_e^2}{\mu^2}. 
\]

It matches with the vacuum energy \( \rho_\Lambda(m_e) \) at the energy scale \( \mu = m_e \), coming from the evolution of \( \rho_\Lambda(\mu) \) at energies \( \mu > m_e \) as required by the decoupling [17] at the scale the electron mass. On the other hand, when considering dark energy, as the Hubble scale today is \( H_0 \sim 10^{-42} \text{GeV} \), we are interested in the very low energy Wilsonian action for \( \mu \) much lower than the neutrino masses and the scalaron mass. In this regime, the neutrinos and the scalaron have decoupled and the vacuum energy becomes a constant corresponding to the 1PI vacuum energy which appears in the classical equations of motion of the theory. This is the dark energy density \( \rho_{\text{vac}} \) measured by cosmological probes,

\[
\rho_{\text{vac}} \simeq 2.7 \times 10^{-11} \text{eV}^4. 
\]

Therefore, integrating (8) from \( m_e \) down to \( \mu < \min(m, m_f) \), taking into account the jump of the vacuum energy \( \beta \) function at the masses of the scalaron and the neutrinos, we get

\[
\rho_{\text{vac}} = \rho_\Lambda(m_e) + \frac{m^4}{64\pi^2} \ln \frac{m_e^2}{m^2} - 2 \sum_{f=1}^{3} \frac{m_f^4}{64\pi^2} \ln \frac{m_e^2}{m_f^2}. 
\]

The energy density \( \rho_{\text{vac}} \) is much lower than the order of magnitude of particle physics scales, e.g. early Universe phase transitions and quantum fluctuations of very massive fields. These contributions to the vacuum energy density have all been subsumed in \( \rho_\Lambda(m_e) \), which contains all the physical effects at energies higher than \( m_e \) and contributing to the renormalised energy density, where the bare cosmological constant has been used as a counter-term in the renormalisation process.
V. NEUTRINO CONTRIBUTIONS

We note that the neutrino contributions to $\rho_{\text{vac}}$ are strongly constrained by cosmological and astrophysical measurements. The Planck results [18–21] for the cosmic microwave background provide the upper bound $m_1 + m_2 + m_3 < 0.12$ eV for the sum of the neutrino masses. The oscillations of the solar neutrinos yield the squared mass difference $m_2^2 - m_1^2 = 7.5 \times 10^{-5}$ eV$^2$. For the case of normal ordering of neutrino masses [22], $m_3^2 - m_1^2 = 2.524 \times 10^{-3}$ eV$^2$, whereas for the case of inverse ordering, $m_3^2 - m_2^2 = 2.514 \times 10^{-3}$ eV$^2$. For both orderings the neutrino contribution is bounded,

$$10^4 \rho_{\text{vac}} \leq \sum_{j=1}^{3} \frac{m_j^4}{64 \pi^2} \log \left( \frac{m_j^2}{m_j^2} \right) \leq 2 \times 10^4 \rho_{\text{vac}} \cdot$$

(12)

This is of course a remnant of the usual cosmological constant problem, i.e. the overestimate of the vacuum energy density by particle physics expectations. It implies that either the vacuum energy $\rho_{\Lambda}(m_e)$, the scalaron contribution in $m^4$, or their sum, must compensate the neutrino contribution

$$- \frac{\bar{m}_\nu^4}{32 \pi^2} \ln(m_e^2/\bar{m}_\nu^2) = - \sum_{j=1}^{3} \frac{m_j^4}{32 \pi^2} \ln(m_e^2/m_j^2) \quad \text{(13)}$$

in (11).

VI. BOUNDS ON THE SCALARON MASS

Lower bound from Eötvös experiments. The scalar curvature square term $R^2$ induces a modification of the large-distance gravitational potential from objects of mass $M$ [23],

$$V(r) = - \frac{GM}{r} \left( 1 + \frac{1}{3} e^{-mr} \right).$$

(14)

The absence of evidence for short range forces in the Eötvös experiment [7, 8] provides a strong upper bound on the range of such fifth forces,

$$m^{-1} \lesssim 68 \mu m \cdot$$

(15)

This also reads

$$m \gtrsim 2.8 \times 10^{-3} \text{ eV} \simeq 1.22 \rho_{\text{vac}}^{1/4}. \quad \text{(16)}$$

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1 We would like to thank Sunny Vagnozzi for communications about the most recent bound on the neutrino masses from Planck data.
which happens to be of the same order as $\rho_{\text{vac}}^{1/4}$. As pointed out in [6], it is thus possible that the scalaron would be responsible for the vacuum energy density observed today in (11), in which case its mass would be close to current Eötvös-Wash experimental bounds. However, in this paper we do not assume that this is the case and relax the link between the scalaron’s quantum fluctuations and the vacuum energy.

**Constraints from galaxy clusters.** The X-ray emitting gas of a galaxy cluster has a typical temperature of $T_X \sim 1$ keV, in regions of total baryonic and dark matter density of about 500 times the mean density of the Universe, i.e. $200\rho_{\text{vac}}$. These systems typically appeared at a redshift $z \gtrsim 0.1$ and already have a lifetime of the order of the age of the Universe. In such clusters the scalaron and the neutrinos, coming either from the early Universe with an energy of order $10^{-4}$ eV or from astrophysical processes such as the burning of stars with an energy around $100$ keV, have a very small cross section with matter $\sigma \approx \beta^2/M_{\text{Pl}}^2$ and $\sigma \approx m_{\nu}^2/M_{Z}^2$, where $M_{Z} \approx 10^2$ GeV is the mass of the $Z$ boson, respectively. This implies that both the neutrinos and the scalaron decouple from the physics inside the clusters, which can then be described by non-relativistic matter particles (such as electrons and protons) and General Relativity augmented with a vacuum energy. We make the strong assumption that the latter only takes into account all the physics for energy scales greater than $T_X$, i.e. $\rho_{\Lambda}(m_{\nu})$, which excludes the quantum fluctuations of the scalarons and the neutrinos which have decoupled from the plasma. This implies strong constraints on the scalaron mass. Indeed, if the vacuum energy $|\rho_{\Lambda}(m_{\nu})|$ were greater than the local matter density within the virial radius, where the gas has been shocked to $T_X \sim 1$ keV, it would significantly affect the dynamics within the cluster. In a spherical approximation, the cluster would behave as a separate universe [24], with its own vacuum energy $\rho_{\Lambda}(m_{\nu})$. The formation of the cluster, from the turn-around time until virialization, would proceed in the same manner, but the later stages when the gas reaches high temperatures would be such that the hydrostatic equilibrium would be displaced or beyond reach. To ensure small dynamical effects, we have the conservative bound

$$|\rho_{\Lambda}(m_{\nu})| \lesssim 200 \rho_{\text{vac}},$$

as the hot gas is typically measured in X-ray clusters at density contrasts of 500 compared to the cosmological background. This reasoning makes use of the presence at low redshift of hot high-density structures within the cooler and lower-density cosmological background.
Combining (17) with the neutrino bounds (12), we obtain from (11) the numerical estimate \( m \simeq \bar{m}_\nu \simeq 0.05 \) eV, hence we get the noticeably short range \( m^{-1} \simeq 4 \mu m \), which is compatible with the Eötvös-Wash bound (15).

**The extended scalaron model** So far we have assumed that only the scalaron and the neutrinos have a mass smaller than the electron mass. Other particles such as light bosonic dark matter candidates [25] could also be present. In this extended scenario, the bounds (17) could be satisfied as long as the scalaron’s and the other scalar fields’ contributions to the vacuum energy almost compensate the one of the neutrinos. In this case the mass of the scalaron is still bounded from above by \( \bar{m}_\nu \) whilst being bounded from below by the Eötvös-Wash bound \( 1.22 \rho_{\text{vac}}^{1/4} \) leading to a range

\[
4 \mu m \lesssim m^{-1} \lesssim 68 \mu m.
\]  

for the scalaron-mediated interaction.

**VII. LABORATORY EXPERIMENTS**

Different types of experiments can in principle test new interactions in the range of a few micrometers. Preliminary results of new runs of this experiment [9] indicate a possible new upper bound of 40 \( \mu m \), therefore reducing the range of allowed mass almost by half. Another experiment which could test the presence of a scalaron is the measurement of the energy levels of the neutron over a horizontal mirror at \( z = 0 \) in the terrestrial gravitational field [26]. The presence of the scalaron would shift the \( n \)-th energy level \( |n\rangle \) by an amount

\[
\delta E_n = -\alpha_n \frac{\beta^2 m_N \rho}{M_{\text{Pl}} m_N^2} e^{-m_N z_0}
\]

where \( m_N \) is the neutron mass, \( \rho \) the density of the mirror and the \( O(1) \) number \( \alpha_n \) is such that \( \langle n|e^{-m_N^2}|n\rangle = \alpha_n e^{-m_N^2} \) where \( z_0 = (\hbar^2/2m_N^2 g)^{1/3} \simeq 6 \mu m \). Detecting a scalaron of mass \( m \lesssim z_0^{-1} \) for \( \rho \simeq 10 \) g/cm\(^3\) would require to have a sensitivity on the energy levels of order \( 10^{-22} \) eV. This is much below the present sensitivity of order \( 10^{-14} \) eV [27], and even below the best sensitivity achievable by such an experiment which is given by the inverse of the neutron life-time thanks to the uncertainty relation \( \Delta E \simeq 10^{-19} \) eV. Hence this type of experiment will never be sensitive to the scalaron. Casimir force experiments could lead to strong constraints on Yukawa exponential corrections \( \alpha e^{-r/\lambda}/r \) to the Newton potential [28]. The scale (18) would be eventually tested if Yukawa interactions were probed.
in the 10 micrometer ballpark. This would correspond to future Casimir experiments such as CANNEX [29] but their expected sensitivity at the 0.1 pN/cm² would not be low enough to compete with direct searches for gravitational interactions. Indeed the scalar pressure between two plates separated by a distance \( d \) is given by

\[
\frac{F}{A} = \frac{\beta^2 \rho^2}{2M^2_{\text{Pl}} m^2} e^{-md}.
\]  

(20)

For a scalaron of mass given by the Eötvös-Wash bound and a distance of 10 microns, this would require a sensitivity of around \( 10^{-2} \) pN/cm² which is one order of magnitude below the CANNEX expected sensitivity. For larger masses corresponding to short ranges for the scalaron interaction, the sensitivity would have to be even better. Finally one extremely promising possibility which would overcome some of the shortcomings of ground-based experiments would be to have a torsion pendulum experiment of the Eötvös-Wash type aboard a satellite. Such a project has already been considered [30] with a target of force ranges around 10 micrometers which would be sensitive to coupling of order one or below such as \( \beta = 1/\sqrt{6} \) [31]. Of course such a future experiment would have the power to vindicate or exclude the scalaron that we have considered in this work as the torque between two parallel and rotating plates of common surface area \( A(\theta) \) depending on the rotation angle \( \theta \) is given by

\[
T = \frac{dA(\theta)}{d\theta} \frac{\beta^2 \rho^2}{2M^2_{\text{Pl}} m^3} e^{-md},
\]  

(21)

where \( \rho \) is their common density and \( d \) their separation. Measurements of the Yukawa decrease and the amplitude would give access to both \( m \) and \( \beta \).

VIII. DISCUSSION

Upon the hypothesis that inside clusters of galaxies the relevant vacuum energy density is given by \( \rho_\Lambda(m_e) \), we have bounded the mass of the scalaron giving rise to a modification of the Newtonian potential (14) with a range within reach of the new runs [9] of the Eötvös-Wash experiment. The measurement of the value of coupling to matter \( \beta \) compatible with \( 1/\sqrt{6} \) would point towards a gravitational theory \( f(R) \) with \( R^2 \) being the leading contribution [6]. An altogether different value would indicate that the detected scalar is not the one generated by gravitational corrections to General Relativity and would come from some new and unknown physics at low energy. If no signal in laboratory experiments searching for fifth forces were found in this small mass range, this would also signify that the scalaron has a
much larger mass with a much smaller coefficient $c_0$ closer to the one of the other curvature squared terms. This would eventually imply that the hierarchy $c_0 \gg c_2$ is not realised and that new light degrees of freedom as suggested in [32, 33] must be present at low energy. Such degrees of freedom would have to be very weakly coupled to have escaped direct detection. In this case these light degrees of freedom would have their masses bounded from above by the averaged neutrino mass $\bar{m}_\nu \simeq 0.05$ eV.

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