Rotating Charged Hairy Black Hole in (2+1) Dimensions and Particle Acceleration

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Abstract In this paper, we construct rotating charged hairy black hole in (2+1) dimensions for infinitesimal black hole charge and rotation parameters. Then we consider this black hole as particle accelerator and calculate the center-of-mass energy of two colliding test particles near the rotating charged hairy black hole in (2+1) dimensions. As we expected, the center-of-mass energy has infinite value.

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1 Introduction

Recently, charged black hole with a scalar hair in (2+1) dimensions studied in Ref. [1] and rotating hairy black hole in (2+1) dimensions studied in Ref. [2]. The (2+1)-dimensional theories are toy models to investigate some fundamental ideas to understand higher dimensional theories because they are easy to study.[3] Also, it is useful to study gauge/gravity dualities.[14–7] These kinds of black hole recover well known BTZ black hole in (2+1) dimensions.[8–12]

In this work we would like collect Refs. [1–2] to construct rotating charged hairy black hole in (2+1) dimensions. In that case we assume that the electric charge and rotational parameter are infinitesimal.

Then, the main goal of this paper is particle acceleration mechanism. It has been shown that free particles falling from rest at infinity outside a rotating black holes may collide with arbitrarily high center-of-mass (CM) energy and hence rotating black holes may be considered as a particle accelerator.[13–14] It is found that the CM energy of elastic and inelastic scattering of particles in the gravitational field of static and rotating Kerr black holes is limited for the static and is unlimited for the rotating black holes.[15] Several studies indicate that having infinite CM energy of colliding particles is a generic property of a rotating black holes.[16–27] Now, we verify this general property for the rotating charged hairy black hole in (2+1) dimensions.

In the next section we construct a black hole in (2+1) dimensions include electric charge, scalar charge and rotational parameter, and then in Sec. 3 we obtain field equations and discuss geometric properties of this black hole. In Sec. 4 we discuss about horizon structure of rotating charged hairy black hole and obtain event horizon for small radius limit. In Sec. 5 we study particle acceleration and obtain CM energy of two colliding particles near the black hole. In Sec. 6 we discuss about the effective potential, and finally in Sec. 7 we summarize our results and give conclusion.

2 Rotating Charged Hairy Black Hole in (2+1) Dimensions

We consider the (2+1)-dimensional gravity with a non-minimally coupled scalar field and self coupling potential \( V(\phi) \), which is described by the following action,[2]

\[
S = \frac{1}{2} \int d^3x \sqrt{-g} \left[ R - g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \xi \phi^2 - 2V(\phi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right],
\]

where \( \xi \) is a coupling constant between gravity and the scalar field which will be fixed as \( \xi = 1/8 \). The proposed metric for this black hole is similar to Ref. [2],

\[
d s^2 = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 (d\psi + \omega(r) dt)^2.
\]

Under assumption of infinitesimal \( a \) and \( Q \) we obtain,

\[
f(r) = 3\beta - \frac{Q^2}{4} + \left( 2\beta - \frac{Q^2}{9} \right) \frac{B}{r} - Q^2 \left( \frac{1}{2} + \frac{B}{3r} \right) \ln(r) + \frac{(3r + 2B)^2 a^2}{r^4} + \frac{r^2}{l^2} + O(a^2 Q^2).
\]

\( Q \) is the electric charge, \( a \) is a rotation parameter and is related to the angular momentum of the solution and \( l \) is related to the cosmological constant via \( \Lambda = -1/l^2 \). \( \beta \) is integration constant depends on the black hole charge and

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These yield to the following Ricci scalar,

\[ \beta = \frac{1}{3} \left( \frac{Q^2}{4} - M \right), \tag{4} \]

and scalar charge \( B \) related to the scalar field as,

\[ \phi(r) = \pm \sqrt{\frac{8B}{r + B}}. \tag{5} \]

Also one can obtain,

\[ \omega(r) = - \frac{(3r + 2B)a}{r^3}. \tag{6} \]

\[ V(\phi) = \frac{2}{r^2} + \frac{1}{512} \left[ \frac{1}{r^2} + \frac{\beta}{B^2} + \frac{Q^2}{9B^2} \left( 1 - \frac{3}{2} \ln \left( \frac{8B}{\phi^2} \right) \right) \right] \phi^6 \]

\[ \Gamma^t_{tt} = -\Gamma^r_{tr} = \frac{36r^3 - 9t^2Q^2r + 6Bt^2Q^2 \ln(r) - 4Bt^2Q^2 - 36\beta Bt^2}{36t^2r^2}, \]

\[ \Gamma^r_{tr} = \Gamma^r_{r\psi} = -\Gamma^\psi_{r\psi} = \frac{aB}{r^2}, \quad \Gamma^\psi_{\psi r} = -\Gamma^r_{\psi r} = r, \quad \Gamma^r_{rr} = \frac{A}{B^2}, \tag{8} \]

where,

\[ A = -36r^6 + 9t^2Q^2r^4 + 4Bt^2(9\beta + Q^2)r^3 + 324\beta^2a^2r^2 + 648B^2a^2r + 288\beta^2a^2B^2 - 6B^2Q^2 \ln(r)r^3, \]

\[ B = 36r^6 + 9t^2(12\beta - Q^2)r^4 + 4Bt^2(18\beta - Q^2)r^3 + 324\beta^2a^2r^2 + 432\beta^2a^2B^2 + 144\beta^2a^2B^2 - 12B^2Q^2 \ln(r)r^3 - 18B^2Q^2 \ln(r)^2 \cdot r^4. \tag{9} \]

These yield to the following Ricci scalar,

\[ R = \frac{-36r^6 - 3t^2Q^2r^4 + 2Bt^2Q^2r^3 + 216B^2a^2r + 180\beta^2a^2B^2}{6t^2r^6}, \tag{10} \]

which is singular at \( r = 0 \), and yields to \( R = -6t^2 \) for \( Q = 0 \) and \( a = 0 \). The Riemann and Ricci tensors also are singular at \( r = 0 \) for \( Q \neq 0 \) and \( a \neq 0 \). One of the non-vanishing components of Cotton tensor obtained as the following,

\[ C_{\psi r \psi} = \left( \frac{3B}{4r^2} - \frac{B \ln(r)}{2r^2} + \frac{1}{4r} \right) Q^2 + \left( \frac{54}{r^3} + \frac{180B}{r^4} + \frac{120B^2}{r^5} \right) a^2 + \frac{3\beta B}{r^2}. \tag{11} \]

In order to find event horizon we decompose Eq. (12) as the following,

\[ x = (r^2 + c_1)(r^2 - c_2)(r^2 + c_3)(r^2 - c_4), \tag{13} \]

where \( r = \pm 1 \sqrt{c_1} \) are imaginary, \( r = -\sqrt{c_2} \) and \( r = -\sqrt{c_3} \) are negative solutions. Only physical solutions are \( r_+ = \sqrt{c_2} \) and \( r_- = c_4 \), which will be interpreted as outer and inner horizons respectively and obtained as the following,

\[ c_2 = \frac{B}{3Q^2} \left( \frac{7}{6}Q^2 - 2M \right) \times \left( -1 + \sqrt{1 + \frac{216a^2Q^2}{B((7/6)Q^2 - 2M)^2}} \right), \tag{14} \]

\[ c_4 = \frac{Q^2}{4} \left( -1 + \sqrt{1 + \frac{8B}{3Q^2}} \right). \tag{15} \]

It is clear that the \( 7Q^2 \geq 12M \) is crucial condition.

### 5 Particle Acceleration

In order to obtain the CM energy, we should calculate the velocities of the particles, which will be obtained as the following,

\[ i = \frac{\omega(r)L - E}{f(r)}, \]
\[ \dot{r} = \sqrt{f(r)} \left( 1 + \frac{E^2}{f(r)} + \frac{\omega(r)^2 r^2 - f(r) L^2}{f(r) r^2} - \frac{2 \omega(r) E L}{f(r)} \right), \]
\[ \dot{\phi} = \frac{\omega(r) (\omega(r) L - E) r^2 - f(r) L}{f(r) r^2}, \]
where \( E \) denotes the test particle energy per unit mass and \( L \) denotes the angular momentum per unit mass. We use velocity components (16) to obtain CM energy of the two-particle collision in the background rotating charged hairy \((2+1)\)-dimensional black hole. It is assumed that two particles have the angular momentum per unit mass \( L_1 \) and \( L_2 \), energy per unit mass \( E_1 \) and \( E_2 \). Moreover we take \( m_0 \) as the rest mass of both particles. Then, by using the following relation,
\[ E_{CM} = \sqrt{2m_0 \sqrt{1 + g_{\mu\nu} v_i^\mu v_i^\nu}}, \]
where \( v_i = (\dot{t}_i, \dot{r}_i, \dot{\phi}_i) \), we can find the CM energy of two-particle collision as the following expression,
\[ E_{CM} = \frac{1}{f(r) r^2} (f(r) r^2 + E_1 E_2 r^2 - L_1 L_2 (f(r) - \omega(r)^2 r^2) - \omega(r) r^2 (E_1 L_2 + E_2 L_1) - H_1 H_2), \]
where \( E_{CM} \equiv E_{CM}^2/2m_0^2 \), and,
\[ H_i = \sqrt{f(r) r^2 + E_i^2 r^2 - (f(r) - \omega(r)^2 r^2) L_i^2 - 2 \omega(r) r^2 E_i L_i}, \]
with \( i = 1, 2 \). Using relations (3) and (6) in Eq. (18) we draw typical behavior of CM energy in Fig. 1 which shows that near the black hole horizon the CM energy takes large value which is expected for the rotational black hole. Also dotted line of Fig. 1, which is corresponding to static black hole \((a = 0)\) shows that CM energy has finite value which is also expected.

Also we can obtain \( E_{CM} \) near the horizon analytically. In that case we should expand \( E_{CM} \) for \( r \rightarrow r_+ \) to obtain,
\[ E_{CM} = \frac{r_+^2 ((3r_+^2 + 2B)(L_1 + L_2) a/r_+^3 + E_1 + E_2)^2 - (L_1 E_2 - E_1 L_2)^2}{2r_+^2 ((3r_+^2 + 2B)L_1 a/r_+^3 - E_1)((3r_+^2 + 2B)L_2 a/r_+^3 - E_2)}, \]
It yields to a critical angular momentum where CM energy will be infinite,
\[ L_{ci} = \frac{E_1 r_+^3}{(3r_+^2 + 2B)a}. \]
It means that the particles with the critical angular momentum \( L_{ci} \) can collide with arbitrary high CM energy near the horizon. in the special case of vanishing scalar charge \((B = 0)\) we can see that \( L_{ci} \propto r_+^2 \), which agrees with the results of Refs. [26–27]. In Fig. 2 we draw critical angular momentum in terms of rotational parameter \( a \).

![Fig. 1](image1.png)

**Fig. 1** Plots of \( \dot{E}_{CM} \) in terms of \( r \) by choosing \( E_1 = E_2 = 10, L_1 = -L_2 = -10, \) and \( l = 1 \). We fix black hole parameters as \( M = 5, Q = 5, B = 5, \) and \( a = 1 \) (solid line), \( M = 1, Q = 10, B = 3, \) and \( a = 1 \) (dashed line), and \( M = 1, Q = 10, B = 3, \) and \( a = 0 \) (dotted line).

![Fig. 2](image2.png)

**Fig. 2** Plots of critical angular momentum \( L_{ci} \) in terms of \( a \) by choosing \( E_i = 10, M = 5, Q = 5, B = 5, \) and \( l = 1 \).

### 6 Effective Potential

The effective potential is given by the following relation,
\[ V_{eff} = \frac{E^2 - R}{2}, \]
where \( R = \dot{r}^2 \) obtained by using the relation (16). In Fig. 3 we draw effective potential (22) for various values of the black hole parameters. Figures 3(a) and 3(b) show that there is a critical radius \( r_c \) so for the \( r > r_c \) electric charge \( Q \) and scalar charge \( B \) decreased the effective potential but for the \( r < r_c \) the effective potential takes infinite value. Indeed, this region is near the black hole horizon and infinite value of the energy is expectable. In
the special case of \( r = r_c \) variation of \( Q \) and \( B \) have not any effects on the effective potential. In Fig. 3(c) we learn that the rotational parameter \( a \) reduced the effective potential.

The parameter \( R \) is also useful for another reason. The particle with the critical angular momentum may have an orbit outside the outer horizon if,

\[
O = \frac{dR}{dr} \bigg|_{r=r_c} > 0 . \tag{23}
\]

In Fig. 4 we draw \( O \) in terms of the black hole parameters. Figure 4(a) tells that for the case of \( a = 1 \) and \( B = 2 \), the electric charge restricted as \( Q \geq 5 \). Figure 4(b) tells that for the case of \( a = 1 \) and \( Q = 5 \) the scalar charge restricted as \( 1.5 < B < 26 \). Figure 4(c) tells that for the case of \( Q = 5 \) and \( B = 2 \) the rotational parameter restricted as \( a \leq 1 \). In summary we conclude that the condition of having an orbit outside the outer horizon is choosing small \( a \) and large \( Q \) with \( B_{\min} < B < B_{\max} \), which values of \( B_{\min} \) and \( B_{\max} \) are depend on values of \( Q \) and \( a \). In another word the electric charge increases \( O \) but rotational parameter decreases \( O \), while scalar charge may increase or decrease \( O \).

![Fig. 3](image1)

**Fig. 3** Plot of Effective potential in terms of \( r \) for \( E = 5, L = 10, M = 5 \), and \( l = 1 \). (a) \( B = 1, a = 1, Q = 0 \) (dashed line), \( Q = 5 \) (solid line) and \( Q = 10 \) (dashed line). (b) \( Q = 5, a = 1, B = 0 \) (dashed line), \( B = 1 \) (solid line) and \( B = 5 \) (dashed line). (c) \( Q = 5, B = 1, a = 0 \) (dashed line), \( a = 1 \) (solid line) and \( a = 2 \) (dashed line).

![Fig. 4](image2)

**Fig. 4** Plot of \( O \) in terms of (a) \( Q \), (b) \( B \) and (c) \( a \) for \( E = 10, M = 5 \) and \( l = 1 \).

Particles on a circle orbit have the following angular momentum,

\[
L_{\text{col}} = \frac{\omega(r)E r^2 + r \sqrt{f(r)} \left( \frac{E^2 + \omega(r)^2 r^2 - f(r)}{\omega(r)^2 r^2 - f(r)} + \frac{\omega(r)^2 r^2 + f(r)}{\omega(r)^2 r^2 - f(r)} \right)}{\omega(r)^2 r^2 - f(r)}, \quad L_{\text{col}2} = \frac{\omega(r)E r^2 - r \sqrt{f(r)} \left( \frac{E^2 + \omega(r)^2 r^2 - f(r)}{\omega(r)^2 r^2 - f(r)} - \frac{\omega(r)^2 r^2 + f(r)}{\omega(r)^2 r^2 - f(r)} \right)}{\omega(r)^2 r^2 - f(r)} . \tag{24}
\]

It is easy to check that at \( r = r_+ \) where \( f(r_+) = 0 \) we have \( L_{\text{col}} = L_{\text{col}2} \). This is also happened if \( E^2 \geq f(r) - \omega(r)^2 r^2 \). In these cases there is no circle orbit. Existing the circle orbit needs the angular momentum be in the interval \( L \in [L_{\text{col}2}, L_{\text{col}}] \). If we set \( L_{\text{col}} = L_1 \), and \( L_{\text{col}2} = L_2 - \delta \), where \( 0 \leq \delta \leq L_{\text{col}1} - L_{\text{col}2} \), then the CM energy in the circle orbit obtained as the following,

\[
E_{\text{CM}} = 1 + \frac{E_1 E_2 + \sqrt{(E_1^2 + \omega(r)^2 r^2 - f(r))(E_2^2 + \omega(r)^2 r^2 - f(r))} - \sqrt{E_1^2 + \omega(r)^2 r^2 - f(r)}}{\sqrt{f(r)r}} - \delta + O(\delta^2) . \tag{25}
\]
It means that the first particle is a target and the second one on the circle orbit collide with the target. $\delta$ is the small parameter and interpreted as the drift of the second particle from the circle orbit.

7 Conclusions

In this work we constructed rotating charged black hole in $(2+1)$ dimensions with a scalar hair and extended recent works of Xu et al. \cite{1-2}. We obtained event horizon for infinitesimal black hole parameters and found that electric charge and scalar charge increase size of event horizon which is agree with the results of the Ref. \cite{1}.

The main part of our paper is consideration of rotating charged hairy black hole in $(2+1)$ dimensions as particle accelerator. We confirmed that having arbitrary high CM energy of two colliding test particle near the rotating black holes is universal property. Also we found that CM energy will be finite for static black holes as expected. We found that the angular momentum corresponding to collision near the black hole is negative and decreased by rotational parameter $a$. We also discussed about effective potential and circle orbit near the black hole and found effect of black hole parameters.

There are also several open problems related to these new solutions. In Refs. \cite{28-29} thermodynamics of charged and rotating hairy black holes studied separately. Now, it is interesting to have similar studies to Refs. \cite{30-32} for present background and investigate statistical and thermodynamical quantities.

It is interesting to obtain quasi-normal modes of rotating charged hairy black hole in $(2+1)$ dimensions.

As we mentioned in the introduction there are some papers which studied AdS/CFT correspondence in 3D hairy black hole. If this is the case and there is a dual CFT for this background, then an important problem will be calculated of some parameters such as drag force and jet-quenching. \cite{33-40}

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