On the possible virtual state nature of the LHCb $P_c(4312)^+$ signal

J. A. Silva-Castro
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Ciudad de México 04510, México
E-mail: jorge.silva@correo.nucleares.unam.mx

Abstract. In this contribution we study the nature of the new $P_c(4312)^+$ signal reported by LHCb collaboration in the $J/\psi p$ spectrum. We use $S$-matrix principles to perform a minimum-bias analysis of the data, focusing on the analytic properties that can be related to the microscopic origin of the $P_c(4312)^+$ peak. Using the scattering length approximation we find evidence for interpretation of the signal as a virtual state generated by the attractive effect of the $\Sigma_c^+ D^0$ channel opening.

1. Introduction
Searching for and understanding of exotic hadron states, say, those that go beyond the minimal quark model combination of three quarks for baryons and a quark-antiquark pair for mesons, has become a relevant topic in physics over the recent years, because of the insight that these states can provide on the strong interaction in the hadronic energy regime [1, 2, 3]. These states are allowed within Quantum Chromodynamics (QCD), as the fundamental theory of the strong interaction only requires that hadrons are color singlets without limiting the number of quarks. In this way, baryons that go beyond the three quark picture are not precluded by QCD.

In the baryon sector, the only exotic candidates so far are the hidden-charm pentaquarks recently found by the LHCb collaboration in the $\Lambda_b \rightarrow J/\psi p K^-$ decay [4, 5, 6], labeled as $P_c(4312)^+$, $P_c(4380)^+$, $P_c(4440)^+$, and $P_c(4457)^+$. In this work we focus our attention in the $P_c(4312)^+$ candidate [6], which is approximately 5 MeV below the $\Sigma_c^+ D^0$ threshold. There are several interpretations of this signal, such as a $\Sigma_c^+ D^0$ molecule [7, 8, 9, 10, 11, 12, 13, 14, 15], a compact pentaquark [16, 17, 18, 19], and a virtual state as we suggest [20].

To reach the conclusion that the virtual state interpretation was a plausible one, we used a data-driven approach based on $S$-matrix theory and minimally biased methods as shown in the next section. The detailed description of the method and the full results can be found in [20].

2. Analysis of the $P_c(4312)^+$ region
A thoroughgoing analysis of the $P_c(4312)^+$ signal, for example, to determine its quantum numbers would require a full six-dimensional amplitude analysis fitting both the energy and angular dependencies. Nonetheless, the manifestation of the this signal in the experimental

\footnote{A virtual state is produced for example by an attractive interaction which is not strong enough to bind a state, as in the neutron-neutron scattering [21, 22].}
data allows a one dimensional analysis because it stands out as a narrow peak \((\sim 10 \text{ MeV})\) in what otherwise appears to be a smooth background. The closeness of the \(\Sigma_c^+\bar{D}^0\) channel opening indicates that it could be the driving effect behind the appearance of the peak. If it is so, the \(P_c(4312)^+\) would likely be either a molecule or a virtual state.

Thereby, we consider a coupled channel amplitude between the \(J/\psi p\) and \(\Sigma_c^+\bar{D}^0\) channels and restrict the analysis to the 4250-4380 MeV region where the \(P_c(4312)^+\) signal is found. We assume that the \(P_c(4312)^+\) signal has a well defined spin i.e., it appears in a single partial wave \(F(s)\), furthermore, the background from all other partial waves \(\mathcal{B}(s)\) is added incoherently and parametrized as a linear polynomial \(\mathcal{B}(s) = b_0 + b_1 s\), which also encodes the contribution from further singularities.\(^2\)

Hence, the events distribution is given by:

\[
\frac{dN}{ds} = \rho(s) \left[ |F(s)|^2 + \mathcal{B}(s) \right] = \rho(s) \left[ |P_1(s)T_{11}(s)|^2 + b_0 + b_1 s \right],
\]

where \(\rho(s)\) is the phase space factor for the decay \(\Lambda_c^0 \rightarrow J/\psi p K^-\), given by \(\rho(s) = m_{\Lambda_c} p q\) with \(p = \lambda^{1/2}(s, m_{\Lambda_c}^2, m_K^2)/2m_{\Lambda_c}\) and \(q = \lambda^{1/2}(s, m_{\Lambda_c}^2, m_{D^0}^2)/2\sqrt{s}\), where \(\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz\) is the Källén function. This expression assumes that the signal is on the \(S\)-wave, and remains valid even if the signal is on another \(\ell\) wave. This would imply adding a term \(q^\ell\) in front of \(F(s)\), which in practice remains constant due to \(q\) does not change on the energy range considered.

The function \(F(s)\) is a product of a function \(P_1(s)\) which provides the production of \(J/\psi p K^-\) and also takes into account the effect of other signals projected onto the same partial wave of the \(P_c(4312)^+\), and the \(T_{11}(s)\) amplitude, which describes the \(J/\psi p \rightarrow J/\psi p\) scattering, where the \(P_c\) is.

Near the \(\Sigma_c^+\bar{D}^0\) threshold the two coupled channel \(T\) matrix can be written as \(^{21}\):

\[
\begin{align*}
T_{11}(s) &= \frac{M_{22} - ik_2}{(M_{11} - ik_1)(M_{22} - ik_2) - M_{12}^2}, \\
T_{12}(s) &= \frac{-M_{12}}{(M_{11} - ik_1)(M_{22} - ik_2) - M_{12}^2}, \\
T_{22}(s) &= \frac{M_{11} - ik_1}{(M_{11} - ik_1)(M_{22} - ik_2) - M_{12}^2},
\end{align*}
\]

where \(k_i = \sqrt{s - s_i}\), and \(s_1 = (m_{\psi} + m_p)^2\), \(s_2 = (m_{\Sigma_c^+} + m_{D^0})^2\) are the thresholds of the two channels. In this way, \(T_{11}(s)\) represents the reaction \(J/\psi p \rightarrow J/\psi p\), \(T_{22}(s)\) the reaction \(\Sigma_c^+\bar{D}^0 \rightarrow \Sigma_c^+\bar{D}^0\) and the off diagonal terms the channels \(T_{12}(s)\) \(J/\psi p \rightarrow \Sigma_c^+\bar{D}^0\) and \(T_{21}\) \(\Sigma_c^+\bar{D}^0 \rightarrow J/\psi p\).

Due to the unitarity condition, the elements of the real symmetric \(2 \times 2\) matrix \(M(s)\) are singularity free and can be Taylor expanded. In this work we focus in the scattering length approximation which results from keeping only the first term. The first-order effective range expansion, say \(M_{ij}(s) = m_{ij} - c_{ij} s\), is discussed in \(^{20}\). The function \(P_1(s)\) is analytic in the data region, and, given the small mass range considered, it can be parametrized with a first order polynomial \(P_1(s) = p_0 + p_1 s\). The \(M_{12}\) parameter is linked to the channel coupling. We stress that, since the \(J/\psi p\) threshold is far away from the region of interest, this channel can effectively absorb all the other channels with distant thresholds. In principle we should add\(^2\) more formally, an analytical function can be expanded into a Laurent series around a singularity in \(a\) as

\[
f(z) = \frac{b}{z^a} + c_0 + c_1 z + c_2 z^2 + \cdots,
\]

where \(b\) is the residue of the pole and \(c_i\) coefficients, far away from the singularity only the polynomial part contributes to the signal. With a linear polynomial in our analysis we have \(\chi^2/d.o.f. = 0.8\) with no improvements to higher orders.

\(^2\) More formally, an analytical function can be expanded into a Laurent series around a singularity in \(a\) as
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Figure 1. (a) Fit to the cos $\theta_{Pc}$-weighted $J/\psi p$ mass distribution from LHCb [6] in the scattering length approximation, (equations (1) and (2a)). The solid line and blue band show the result of the fit and the $1\sigma$ confidence level provided by the bootstrap analysis respectively. (b) Poles obtained from the $10^4$ bootstrap fits in the scattering length approximation. The physical region is highlighted with a pink band. For each bootstrap fit only one pole appears in this region and the blue ellipse accounts the 68% of the cluster concentrating above threshold.

3. Results and discussion

Due to the square roots in the denominator, the amplitude in Eq. (2a) has branch cuts opening at the two thresholds. It turns out that there are four Riemann sheets, and in the scattering length approximation any pole can only appear on either the II or the IV sheets [21]. This parameterization allows for the description of bound molecules and unbound virtual states. When we turn off the coupling between the two channels, i.e. $M_{12} \rightarrow 0$, a molecular interpretation occurs if the pole moves to the real axis of the physical sheet below the heavier threshold, and a virtual state occurs if the pole moves onto the real axis of the unphysical sheet (see also Fig. 2 of Ref. [23] and the corresponding description). We note that the pole movement is model dependent, as the parameters are not independent but related by the underlying QCD dynamics. This is a problem common to every model or effective hadron theory.

We fit the cos $\theta_{Pc}$-weighted spectrum $dN/d\sqrt{s}$ measured in [6], with $\sqrt{s}$ being the $J/\psi p$ invariant mass, using MINUIT [24], considering the experimental resolution reported in [6]. This spectrum is obtained by applying cos $\theta_{Pc}$-dependent weights to each candidate to enhance the $P_c^+$ signal, where $\theta_{Pc}$ is the angle between the $K^-$ and $J/\psi$ in the $P_c^+$ rest frame (the $P_c$ helicity angle [6, 25]).

To estimate the sensitivity of the pole positions to the uncertainties in the data, we use the bootstrap technique [26, 27]; i.e. we generate $10^4$ pseudodata sets and fit each one of them. The statistical fluctuations in data reflect into the the uncertainty band plotted in Fig. 1(a).

Moreover, for each of these fits, we determine the pole positions, as shown in Fig. 1(b).

In this analysis it is possible to identify a cluster of virtual state poles across the II and IV sheet above the $\Sigma_c^+ D^0$ threshold (see also the discussion in Ref. [25]). If we use the customary definition of mass and width, $M_P = \text{Re} \sqrt{s_P}$, $\Gamma_P = -2 \text{Im} \sqrt{s_P}$ the main cluster has $M_P = 4319.7 \pm 1.6$ MeV, $\Gamma_P = -0.8 \pm 2.4$ MeV, where positive or negative values of the
width correspond to II or IV sheet poles, respectively. To establish the nature of this singularity, we track down the movement of the poles as the coupling between the two channels is reduced. By taking $m_{12} \rightarrow 0$, we can see how the cluster moves over to the upper side of the IV sheet and ends up on the real axis below the $\Sigma_c^+ \bar{D}^0$ threshold \[29\]. The fraction of poles that reach the real axis from the lower side of the II sheet is 0.7% only, and thus not significant. This result reinforces the interpretation of the pole as an unbound virtual state, meaning that the binding between the $\Sigma_c^+$ baryon and the $\bar{D}^0$ meson is enough to generate a signal, but insufficient to form a bound molecule.

As a cross-check we also analyze the unweighted $J/\psi p$ spectrum in the same region, both with and without the $m_{Kp} > 1.9$ GeV cut. All the results are consistent.

4. Conclusions

We have studied the $P_c(4312)^+$ signal reported by LHCb in the $J/\psi p$ spectrum by considering a reaction amplitude which satisfies the general principles of $S$-matrix theory, restricting the analysis to the scattering length approximation. The analytic properties of the amplitudes can be related to the microscopic origin of the signal. We fitted the LHCb mass spectrum in the 4312 MeV mass region including the experimental resolution. The statistical uncertainties in the data were propagated to the extracted poles using the bootstrap technique. We do not find support for a bound molecule, we conclude that the most likely interpretation of the $P_c(4312)^+$ peak is a virtual (unbound) state.

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[29] See Supplemental Material and http://cgl.soic.indiana.edu/jpac/pc4312.php for fit parameters, alternative parameterizations, and animations of the pole movements described in the text.