Synthesis and networking of tetrahedral deployable mechanism based on screw theory

Zhirong Huang¹, Xiaofei Ma¹,⁴, Fei Hu¹, Jialong Zhu¹,² and Yundou Xu³

¹ China Academy of Space Technology (Xi’an), Xi’an 710100, China; ² School of Mechanical and Electrical Engineering, Harbin Institute of Technology, Harbin 150001, China; ³ Parallel Robot and Mechatronic System Laboratory of Hebei Province, Yanshan University, Qinhuangdao 066004, China

Email: maxf041600@sina.com

Abstract. Aiming at the synchronous motion of tetrahedral deployable unit, the kinematic pair is designed and the coupling motion branch is synthesized based on screw theory. The optimal deployable unit is determined by the application. According to the networking requirements, a networking mechanism composed of multiple units with coupling degree of freedom (DOF) is synthesized. The application of the screw theory in the configuration synthesis of deployable mechanism is realized in this paper. The mechanism is suitable for the deployable mechanism of satellite antenna reflector or other space surface deployable support mechanism, which has broad application prospects.

1. Introduction
The networking deployable mechanism is a type of mechanism with deploying and folding applications for aerospace. Generally, it is composed of multiple basic deployable units. And the whole mechanism can be synchronously deployed or folded, which has an excellent strength and rigidity in the deployable state. Many research institutions and scholars have carried out in-depth research on the unit-networking deployable mechanism, because it can realize the flexible application-oriented structure by changing the number of basic truss units or adopting different connection modes. Johnson Space Center (JSC) developed a 7m-diameter deployable antenna which was successfully applied to the “Kondor” spacecraft [1]. Herr proposed a deployable truss reflector. It was formed by a group of tetrahedral deployable units which were consistent with the mesh division of reflector. It had better surface adaptability and good overall stiffness of multi-loop redundant mechanism [2]. Fang carried out the application research of tetrahedral deployable unit, and realized the application of deployable antenna based on shape memory composite hinge [3]. Liu developed the design of multi-module deployable mechanism with paraboloid [4], and used complex number method for kinematic analysis and driving design [5]. In addition, he carried out the synthesis and analysis of topological structure of quadrangular pyramid element for satellite SAR antenna [6]. Ding proposed a new type of triangular prism deployable structure [7]. Chen proposed a six-prism deployable unit driven by a scalable rod, and designed a large truss parabolic antenna [8]. Taking the technical index of structural design and the set of all components as the system elements, Yang designed a tetrahedral deployable truss parabolic antenna [9]. At present, the configuration synthesis of deployable mechanism for networking is mostly confined to planar mechanism or unit-networking mechanism without coupling
degree of freedom (DOF). Moreover, the application of curved surface support mechanism for truss antenna is mostly focused on the DOF analysis and structural design, especially the method of configuration synthesis is less.

In order to get an antenna with large storage ratio, the unit configuration synthesis and networking are carried out. In this paper, the configuration synthesis method of the tetrahedral unit for curved surface support structure is studied, and a series of tetrahedral deployable units are obtained. Combined with the unit characteristics, the networking mechanism is integrated and obtained. The method of configuration synthesis based on screw theory provides a reference for the design of unit-networking deployable mechanism represented by the truss reflector with coupling DOF.

2. Synthesis of deployable unit

The tetrahedral deployable unit is composed of four motion components A, B, C and D, and a number of motion pairs and links between them, as shown in Figure 1. The mechanism has the DOF of deployment. It takes D as the reference component (fixed platform) and A, B, C as the motion output components (moving platforms), which can realize the synchronous approach of components A, B and C to the geometric center of three points. At the same time, it can realize the volume deployment and collection of this tetrahedral deployable mechanism.

![Figure 1. Motion description of the tetrahedral deployable unit.](image)

2.1. Synthesis of synchronous motion branch

The tetrahedral edges adopt RR swing mechanism branch to realize the common motion as shown in Figure 2, node A is moved towards the axis by swinging motion. And 3-RR mechanism can be obtained from the three equal-length edges as shown in Figure 3. The 3-RR mechanism realizes the deployable motion of three nodes to the target axis through six drivers. The motions are independent of each other.

According to the expected motion, the configuration of the relative branch between any two nodes is established to achieve synchronous motion [10]. In Figure 3, the top of the swing rods are intersected with one point, which is the origin of the coordinate system, the z-axis is taken as the direction of the folding axis, the x-axis is the direction of the node A pointing to node B, and the y-axis is determined by the right-hand rule. The length of the swing rod is \( I \), and the swing angle between swing rods is \( \theta \).

![Figure 2. RR swing mechanism.](image)  ![Figure 3. Motion analysis of 3RR mechanism.](image)
According to the screw theory \[11,12\], the motion pair of each node is described as
\[
\begin{align*}
S_1 &= \begin{bmatrix} \cos \alpha & -\sin \alpha & 0; 0 & 0 \end{bmatrix}^T \\
S_2 &= \begin{bmatrix} \cos \alpha & -\sin \alpha & 0; -l \sin \alpha \cos \theta & -l \cos \alpha \cos \theta & l \sin \theta \end{bmatrix}^T \\
S_3 &= \begin{bmatrix} \cos \alpha & \sin \alpha & 0; 0 & 0 \end{bmatrix}^T \\
S_4 &= \begin{bmatrix} \cos \alpha & \sin \alpha & 0; l \sin \alpha \cos \theta & l \cos \alpha \cos \theta & l \sin \theta \end{bmatrix}^T \\
S_5 &= \begin{bmatrix} -\sin \beta & \cos \beta & 0; 0 & 0 \end{bmatrix}^T \\
S_6 &= \begin{bmatrix} -\sin \beta & \cos \beta & 0; l \cos \beta \cos \theta & l \sin \beta \cos \theta & l \sin \theta \end{bmatrix}^T
\end{align*}
\tag{1}
\]

Since RR branch is a series branch, the output motion of its end is a linear combination of joint motion. The two revolute pairs of the swing rod realize the swing and the attitude compensation, so as to ensure the node attitude remains unchanged. If the driving speeds of \( R_1, R_3 \) and \( R_5 \) are \( \omega \), then the driving speeds of \( R_2, R_4 \) and \( R_6 \) are \(-\omega\), the motion screws of nodes are
\[
\begin{align*}
S'_a &= \omega S_1 - \omega S_2 \\
S'_b &= \omega S_3 - \omega S_4 \\
S'_c &= \omega S_5 - \omega S_6
\end{align*}
\tag{4}
\]

Further, the relative motion screws between two nodes are written in the form of unit screw
\[
\begin{align*}
S'_{ab} &= S'_a - S'_b = \begin{bmatrix} 0 & 0 & 0; 1 & 0 & 0 \end{bmatrix} \\
S'_{bc} &= S'_b - S'_c = \begin{bmatrix} 0 & 0 & 0; -\sin \alpha + \cos \beta \cos \alpha + \sin \beta & 0 \end{bmatrix} \\
S'_{ca} &= S'_c - S'_a = \begin{bmatrix} 0 & 0 & 0; \cos \beta + \sin \alpha \sin \beta + \cos \alpha & 0 \end{bmatrix}
\end{align*}
\tag{5}
\]

Equation (5) indicates the three screws are movements along the line of two nodes. It is consistent with the expected motion of the node, and is a sufficient condition for the overall deployment of the unit. Therefore, the same constraint branch can be constructed between two nodes. Taking A and B nodes as examples, then solve the inverse screws of \( S'_{ab} \), the constrained screws of branch AB are
\[
\begin{align*}
S'_r &= \begin{bmatrix} 0 & 1 & 0; 0 & 0 \end{bmatrix} \\
S'_s &= \begin{bmatrix} 0 & 0 & 1; 0 & 0 \end{bmatrix} \\
S'_t &= \begin{bmatrix} 0 & 0 & 0; 1 & 0 \end{bmatrix} \\
S'_u &= \begin{bmatrix} 0 & 0 & 0; 0 & 1 \end{bmatrix}
\end{align*}
\tag{6}
\]

where \( S'_r, S'_s \) represent the constrain couples around the y-axis and along the z-axis respectively; \( S'_t, S'_u \) represent the constraint forces along x-axis, y-axis and z-axis respectively.

The constraints of node A are analyzed. When a branch is added between nodes A and B, a closed-loop constrained branch is formed with RR branch of node B, as shown in Figure 4.

Since the RR branch has provided four constraint screws to node A, the non-redundant constraint of the closed-loop branch composed of the synchronous branch and RR branch of node B is up to 1. Therefore, the number of constraints of synchronous branches is less than 5, and the constraint screws provided by Equation (6) contain redundant constraints. By solving the inverse screw of Equation (1), the constraint screws of the RR branch of node A are obtained as follows.
Similarly, the constraint screws of the RR branch of node B are
\[
\begin{align*}
S_{d1}^B &= [0 0 0; 0 0 1]^T \\
S_{d2}^B &= [0 0 0; \sin \alpha \cos \alpha 0]^T \\
S_{d3}^B &= [\cos \alpha -\sin \alpha 0; 0 0 0]^T \\
S_{d4}^B &= [\sin \theta \sin \alpha \sin \theta \cos \alpha \cos \theta; 0 1 0]^T
\end{align*}
\] (8)

To keep the attitude of node A unchanged in the course of motion, it can be seen from Equation (7) and Equation (8) that the newly constructed synchronous branch is subject to a constraint couple that is not parallel to the z-axis, which can satisfy the constraint requirements of node A. The constraint couple can be expressed as
\[
S_{4B} = [0 0 0; P \sin \alpha -P \cos \alpha Q]^T \tag{9}
\]
where P and Q are arbitrary parameters, and P≠0.

By configuring the different types of synchronous branch between two nodes, a variety of typical deployable units can be obtained, such as 3RR-3URU, 3RR-3PPP, 3RR-3RRR, 3RR-3RP, 3RR-3P, 3RR-3(4R1P). Among them, 3RR-3RRR, 3RR-3P and 3RR-3(4R1P) mechanisms are shown in the Figure 5. Among them, R represents revolute pair, U represents universal pair, P represents prismatic pair.

Figure 4. Synchronous branch between node A and B.
Figure 5. Configurations of deployable unit.

From Equation (6)~ Equation (9), the configurations synthesized above constrain the rotational DOF of the node and the movement in some directions, and retain the movement to the center. Therefore, the synchronous deployable units can be realized only by a single drive.

2.2. Optimization selection of unit configurations
Considering the realizability of space mechanism design, the following two indexes are the preferred principles: (1) High storage ratio; (2) Good adaptability to high and low temperature environment in space.

The high storage ratio is a constraint on the volume ratio of deploying and folding states. The full revolute pairs mechanism has a higher storage ratio. The impact of high and low temperature environments is a constraint on the choice of hinges. Due to the direct relationship between thermal deformation and linear dimension, the prismatic pair is greatly affected by the space temperature, while the influence on the revolute pair is relatively small. Therefore, the preferred principle can be summarized as the fewer the revolute pairs and the fewer the number of components. The characteristic analysis of typical unit synthesized is shown in Table 1.
Table 1. Configuration characteristics of unit

| Configuration   | Number of components | Number of prismatic pairs |
|-----------------|----------------------|---------------------------|
| 3RR-3URU        | 13                   | 0                         |
| 3RR-3PPP        | 13                   | 9                         |
| 3RR-3RRR        | 13                   | 0                         |
| 3RR-3RPR        | 13                   | 3                         |
| 3RR-3RP         | 10                   | 3                         |
| 3RR-3P          | 7                    | 3                         |
| 3RR-3(4R1P)     | 22                   | 12                        |

According to a preferred principle, the 3RR-3RRR mechanism of the 13 rods without prismatic pair is determined as the optimum tetrahedral unit. The DOF of the mechanism is one and continuous.

3. Configuration of networking

The networking mechanism can be used to support the space structure. According to the principle of space surface partition, the combination mechanism 3(3RR-3RRR) shown in Figure 6 is taken as the research object, in which the bottom node component is located on the space surface.

In the process of folding, the attitude of each element's axis remains unchanged as shown in Figure 7, and the top node components of each element are located on the folding axis, so there is a certain distance between the top node. The mechanism cannot achieve the complete closure, which reduces the storage ratio of the whole networking mechanism, seriously affecting its applicability.

According to the previous analysis, in order to ensure the maximum storage ratio, the relative motion of the swing rod is three rotations, forming a spherical joint, and the corresponding mechanism is changed to 3 (3SR-3RRR) mechanism, as shown in Figure 8. S represents spherical pair, which can be composed of three revolute pairs.

And the top node component should also be connected with the swing rod as a spherical pair, that is, a 3 (3SS-3RRR) -3RRR mechanism, as shown in Figure 9. Since the additional back RRR branch is only a synchronous deployable rod in the folding process without introducing redundancy constraints, the obtained 3 (3SS-RRR) -3RRR mechanism still has only four DOFs. According to the scale requirement, the tetrahedral deployable mechanism can be networked by \( n(3SS-3RRR)-mRRR \) (\( m, n \) are integers) configuration.
4. Deployable antenna mechanism simulation
Taking the application of a space antenna reflector as an example, the $27(3SS-3RRR)-54RRR$ tetrahedral deployable networking mechanism is obtained according to the surface parameters and mesh partition results, as shown in Figure 10. The dynamic simulation model of the networking mechanism is further established, and the driving simulation is added to analyze the folding process of the networking mechanism as shown in Figure 11.

![Figure 10. Model of 27(3SS-RRR) -54RRR mechanism.](image1)

![Figure 11. Folding process simulation.](image2)

The dynamic simulation process confirms that the node of the networking mechanism are located on the space surface; the mechanism is coordinated in the process of deployment; the upper and lower nodes of the folding state are flat, and the members of each unit are close together without angle. The kinematic process of the mechanism is consistent with the expectation, which shows the correctness of the configuration method.

5. Conclusions
A series of typical tetrahedral deployable units are constructed, and the 3RR-3RRR unit with higher reception is selected. The proposed method of unit configuration synthesis provides a way for the multiple motion output components. And a kind of $n(3SS-3RRR) -mRRR$ networking deployable mechanism is obtained. The obtained mechanism has the characteristics of synchronous motion and high storage ratio. It is suitable for the deployable mechanism of satellite antenna reflector or other space surface deployable support mechanism. The method of deployable mechanism configuration with coupling DOF can provide a reference for the design of deployable mechanism.

Acknowledgment
This work was supported by the National Natural Science Foundation of China (NNSFC) under Grant NO. U1537213.

References
[1] Chebotarev A S, Panteleev V A, Feyzulla N M, et al 2014 Truss-type deployable reflector antenna systems for synthetic aperture radar mounted on a small spacecraft[C] Microwave & Telecommunication Technology 521-522
[2] Hree R W, Horner G C 1980 Deployment test of a 36-element tetrahedral truss module[R]. Dallas Texas USA: Vought Corp
[3] Fang H , Shook L , Lin J , et al 2013 A large and high radio frequency deployable reflector[C]. Aiaa/asme/asce/ahs/asce Structures, Structural Dynamics & Materials Conference Aiaa/asme/ahs Adaptive Structures Conference Aiaa 1838-1842
[4] Chu Z, Deng Z, Qi X, et al 2014 Modeling and analysis of a large deployable antenna structure[J]. Acta Astronautica 95 51-60
[5] Liu R, Jin G, Liu Z, et al 2014 Kinematics analysis and driving design of multi-module deployable structure[J]. Infrared and Laser Engineering 43 65-71
[6] Wang Y, Deng Z, Liu R, et al 2014 Topology structure synthesis and analysis of spatial pyramid deployable truss structures for satellite SAR antenna[J] *Chinese Journal of Mechanical Engineering* **27**(4) 683-692

[7] Ding X, Yang Y, Dai J 2013 Design and kinematic analysis of a novel prism deployable mechanism[J] *Mechanism and Machine Theory* **63** 39-49

[8] Chen X, Guan F 2001 A large deployable hexapod paraboloid antenna[J] *Journal of Astronautics* **22**(1) 75-78

[9] Yang Y, Guan F, Hou G, et al 2009 Deployable tetrahedral truss antenna initial structural design based on interpretative structural model analysis [J] *China Mechanical Engineering* **20**(16) 1969-1973

[10] Xu Y, Yao J, Zhao Y 2011 Type synthesis of forging manipulators based on screw theory[J] *China Mechanical Engineering* **22**(13) 1540-1545

[11] Huang Z, Li Q 2003 Type synthesis principle of lower-mobility parallel mechanisms[J] *Science in China(Series E)* **33**(9) 813-819

[12] Ball R S 1900 A Treatise on the Theory of Screws[M] *Cambridge, United Kingdom: Cambridge University Press*