Can Thorne-Żytkow Objects source GW190814-type events?

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The LIGO-Virgo collaboration reported in their third run the coalescence event GW190814 involving a 2.6 $M_\odot$ object with a 23 $M_\odot$ black hole. In this letter we study the conditions under which Thorne-Żytkow objects (TZO) can be connected to that type of events. We evaluate first the rate of appearance of TZO in the local Universe. Under the assumption that TZO eventually become low mass gap black holes we evaluate how those black holes end up in binaries with other stellar mass black holes and compare to the reported rate for GW190814-type of events (1-23 Gpc$^{-3}$ yr$^{-1}$).

We find that TZO in dense stellar clusters cannot explain the LIGO-Virgo rate without a TZO population in the field providing a dominant contribution. We also find that TZO formed within hierarchical triple systems in the field with the third more distant star being the progenitor of a stellar mass black hole, may be able to give a rate comparable to that of GW190814-type events. In that case, future observations should discover mergers between stellar mass and low mass gap black holes, with the lower mass spanning the entire low mass gap range.

Introduction: The recent detection of the LIGO-Virgo coalescence event GW190814, where a 2.6 $M_\odot$ object merged with a 23 $M_\odot$ black hole represents a new class of gravitational wave (GW) events, involving compact objects with mass in the low mass gap range of 2.5-5 $M_\odot$. The 2.6 $M_\odot$ object may be the lightest black hole (BH), or the most massive neutron star (NS) ever observed [1-9]. A detection may suggest that objects with masses in such range are more commonly found than previously suggested. Potential pathways to objects with masses in the low mass gap range may involve objects that are formed from NSs by accretion of matter (see e.g. [10]). One of these pathways can be Thorne-Żytkow objects (TZO) that contain a NS inside a red giant [11, 12]. The final object may be a BH in the low mass gap range. Given that TZO require tight binary systems as their starting point we expect that such objects are mostly found in regions rich in stars.

In this letter we examine the conditions under which low mass gap BHs sourced from TZO can form binaries with regular stellar mass range BHs (i.e. $\geq 5 M_\odot$) that are tight enough to merge within a Hubble time and at a rate similar to that of the GW190814 class, evaluated by the LIGO-Virgo collaboration to be $1-23$ Gpc$^{-3}$ yr$^{-1}$ [13]. We find that while dense stellar environments can not contribute significantly to the observed rate, hierarchical triple systems in the field may be able to explain events as the GW190814.

An upper bound estimation of TZO becoming low mass gap BHs: Using massive X-ray binaries Ref. [14] estimated that in a galaxy of the size of the Milky Way, the emergence rate of TZO is $1-2 \times 10^{-4}$ yr$^{-1}$. More recent estimates suggest a rate of at least $0.9 \times 10^{-4}$ yr$^{-1}$ and $1.5 \times 10^{-4}$ yr$^{-1}$ [15, 16]. We will use from here on that TZO may emerge with a rate of $1.5 \times 10^{-4}$ yr$^{-1}$ per Milky Way-like galaxy [7].

The Sloan Digital Sky Survey (SDSS) has measured the number density of galaxies in the redshift range of 0.02-0.06 [18]. The number density of galaxies in a mass bin $dM_*$ follows a Schechter mass function $\Phi$,

$$n_{gal} = \Phi(M_*) dM_* = \Phi^* e^{-M_*/M_{\odot}} \left( \frac{M_*}{M_{\odot}} \right) \alpha dM_*.$$  

$\Phi^*$ is the overall normalization, $\alpha$ is the mass function power-law and $M_{\odot}$ sets an exponential suppression at high masses. Late and early type galaxies have separate mass functions. Their combined mass function is (ex-
pressed in log\(M^*\) \cite{18}.

\[
\Phi \log_{10} M^* = \ln(10) e^{-M^*/M_\odot^*} \left\{ \Phi_1^* \left( \frac{M^*}{M_\odot^*} \right)^{\alpha_1 + 1} + \Phi_2^* \left( \frac{M^*}{M_\odot^*} \right)^{\alpha_2 + 1} \right\} \log_{10} M^*. \tag{3}
\]

The appropriate normalizations are \(\Phi_1^* = h^3 10^{-3.31} \text{Mpc}^{-3}\), \(\Phi_2^* = h^3 10^{-2.01} \text{Mpc}^{-3}\) with power-laws \(\alpha_1 = -1.69\) and \(\alpha_2 = -0.79\) for \(M_\odot^* = 10^{10.79} M_\odot\) and \(h = 0.7\) \cite{18}.

Eqs. \ref{eq:1} give a local \((z \geq 0.06)\) TZO emergence rate density \(\Phi\),

\[
R_{\mathrm{TZO}}^T(z < 0.1) = \int_{M_{\min}^*}^{M_{\max}^*} dM^* \mathcal{G}_{\mathrm{gal}}^T \left( M^* \right) \Phi(M^*) \frac{M^*}{M_\odot^*}. \tag{4}
\]

Taking \(M_{\min}^* = 10^3 M_\odot\) \((M_{\min}^* = 10^{10} M_\odot)\) and \(M_{\max}^* = 10^{11.5} M_\odot\) we get a local TZO emergence rate density of \(1.2 \times 10^3 \text{yr}^{-1}\) for galaxies with stellar mass of \(M^* \geq 10^9 M_\odot\) and \(M_{\max}^* \geq 10^{10} M_\odot\).

As all TZOs composed by a NS and a massive red giant are going to end up in a BH, we can calculate the rate by which BHs are created just from these objects \cite{2}. To distinguish these BHs from those that originated directly from regular core-collapse we will denote these as BH\textsuperscript{TZO}. Their rate is,

\[
R_{\mathrm{BH}TZO}(z) = \int_0^z dz' R_{\mathrm{TZO}}^T(z') \frac{dV_c}{dz'} (1 + z')^{-1}, \tag{5}
\]

where \(dV_c/dz\) is the comoving volume element. Integrating to redshift of 0.1 we get \(R_{\mathrm{BH}TZO}(0.1) = 3.8 \times 10^2 \text{yr}^{-1}\) from galaxies with stellar mass of \(M^* \geq 10^9 M_\odot\) and \(R_{\mathrm{BH}TZO}(0.1) = 3.3 \times 10^2 \text{yr}^{-1}\) from galaxies with stellar mass of \(M^* \geq 10^{10} M_\odot\), making our estimates insensitive to the low-mass end of the galaxies mass-function.

Only the BH\textsuperscript{TZO} falling in the low mass gap are of importance here. These will be created if the initial NS accretes at least \(1 M_\odot\) of mass \cite{19,20}. Assuming that the neutron star of the TZO accreted 1/5th of the mass of the red giant, only stars with initial mass between 6 and 18 \(M_\odot\) will give BH\textsuperscript{TZO} inside the low mass gap. Relying on the Kroupa initial mass function \cite{21}, with its uncertainties we find that 60-90\% of the TZOs will lead to a such a low mass BH\textsuperscript{TZO}.

Using the central values of \cite{21} \cite{3} we get a local formation rate density of BH\textsuperscript{TZO} with mass in the low mass gap of \(9 \times 10^2 \text{Gpc}^{-3} \text{yr}^{-1}\). That rate is based on active star formation regions and may have been even larger in past epochs. Our rate is insensitive to the exact fraction of the mass of the red giant that is being absorbed. As an example, if the NS accretes 1/3rd of the total mass of the giant, the birth rate density varies by only \(\approx 5\%\).

Of the Milky Way’s stellar mass about 1/3rd is in the bulge \cite{22,24}. For small elliptical galaxies that are formed in a single epoch of collapse of gas as the much smaller in mass globular clusters have, the fraction of the stellar mass in dense regions may be even higher than for barred spiral galaxies (as the Milky Way). Also for the massive elliptical galaxies that are the result of mergers of smaller galaxies the relevant fraction will be as large as their progenitor galaxies. As a result of the local birth rate density of BH\textsuperscript{TZO} with mass in the low mass gap that we estimated to be \(9 \times 10^2 \text{Gpc}^{-3} \text{yr}^{-1}\), \(3 \times 10^2 \text{Gpc}^{-3} \text{yr}^{-1}\) will be in dense stellar environments where the BH\textsuperscript{TZO} can interact and subsequently merge with other massive BHs.

\begin{itemize}
\item \textbf{Forming binaries of a low mass gap and a regular stellar mass BH:} We consider two distinct paths for the formation of binaries composed of a BH\textsuperscript{TZO} within the low mass gap and a BH of \(5 M_\odot\) or larger. In the first one, the BH\textsuperscript{TZO} is initially in no binary and only after dynamical interactions forms a binary system with another BH.
\end{itemize}

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\item \textbf{Forming binaries inside globular clusters:} In globular clusters all stars enter the main sequence at approximately the same moment. Thus the massive red giants of 6 to 18 \(M_\odot\) are present only for a short amount of time early in the history of these systems. Relying on Eq. \ref{eq:4} the formation rate of BH\textsuperscript{TZO}s with mass of 2.5 to 5 \(M_\odot\) is,
\end{itemize}

\[
\Gamma_{\mathrm{sc}}^T(M_{\sc}, t) = (0.6 - 0.9) \times 1.5 \times 10^{-4} \left( \frac{M_{\sc}}{10^{10.79} M_\odot} \right) \times H(t - T_1)H(T_2 - t) \text{yr}^{-1}. \tag{6}
\]

\(M_{\sc}\) is the mass in stars within a given stellar cluster, i.e. \(M_{\sc} \sim 10^5 M_\odot\) for a massive globular cluster. The source term at the globular clusters is taken for simplicity to be constant in time \(t\) between \(T_1\) and \(T_2\), though the product of two Heaviside functions \(H(t - T_1)H(T_2 - t)\). \(T_1\) and \(T_2\) are the timescales of collapse of a 18 \(M_\odot\) (for \(T_1\)) and a 6 \(M_\odot\) (for \(T_2\)) star. Approximately, \(T_1 = 10 \text{Myr}\) and \(T_2 = 130 \text{Myr}\). We remind that our estimate relies on current star forming regions and may be different for the early stages of globular clusters.

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2 TZOs where the original companion of the NS was a low mass star may still give a NS as a final product. All our rates here rely on the observational constraints where the companion will become a massive red giant.

3 Assuming a Kroupa initial mass function that scales with the mass of the star \(m_{\text{star}}\) as \(\propto m_{\text{star}}^{-2.3}\).

4 Time of collapse or full loss of envelope is taken to be \(\sim 1.2\times\) the main sequence lifetime.
The rate $\Gamma_{\text{BH}}^{TZO}(M_\star)$ of Eq. 6 can be used as a source term of low mass gap BHs inside clusters. In Ref. 25, a numerical scheme was developed to model the dynamical interactions of low mass gap objects (as our BH$^{TZO}$s) with regular mass stellar BHs, massive second generation BHs and stars. Once formed, the BH$^{TZO}$s will first bind in binaries with stars. Those first binaries will then have exchange interactions with other BH$^{TZO}$s and more massive BHs, creating binaries composed solely by compact objects. At the same time since these new binaries are surrounded by stars, binary-single star interactions will take place resulting in the loose binaries breaking up and the hard ones becoming even tighter. In 25, all those interactions are included for a sequence of the observed globular cluster systems, taking into account their environmental parameters (densities and velocity distribution profiles). We implement the same code in this work taking all the observed Milky Way clusters as reference in order to evaluate a merger rate between low mass gap BH$^{TZO}$ and BHs in the regular mass range. In Figure 1 we show the total rate of mergers from all Milky Way globular clusters $R_{\text{GW190814}}^{\text{gcs in gal}}$. From our results that rate can be taken to be roughly constant up to redshift of 1.

$$R_{\text{GW190814}}^{\text{gcs in gal}}(M_\star) = 2 \times 10^{-11} \left( \frac{M_\star}{10^{10.79} M_\odot} \right) \text{yr}^{-1}. \quad (7)$$

Combining Eq. 7 with Eq. 4 where in the place of $R_{\text{gal}}^{\text{GW190814}}$ we substitute with $R_{\text{gcs in gal}}^{\text{GW190814}}$, we get that the local GW190814-like event rate density of,

$$R_{\text{in gcs}}^{\text{GW190814}}(z < 0.1) = \int_{M_{\text{min}}}^{M_{\text{max}}} dM_\star R_{\text{gcs in gal}}^{\text{GW190814}}(M_\star) \frac{\Phi(M_\star)}{M_\star}. \quad (8)$$

This gives $R_{\text{in gcs}}^{\text{GW190814}}(z < 0.1) = 1.6 \times 10^{-4}$ Gpc$^{-3}$ yr$^{-1}$. This result is about four orders of magnitude smaller than the reported GW190814 rate density. Thus, globular clusters are excluded as the only environment where TZO can create a low mass gap black hole that subsequently via dynamical interactions, will form a binary with a BH and merge giving GW190814-type events.

**Forming binaries in nuclear star clusters:** Nuclear star clusters come with a variety of masses between galaxies, however their stellar mass is related to the mass of the host galaxy 26,

$$\log_{10}(M_{\text{NSC}}) = 1.094 \cdot \log_{10} \left( \frac{M_\star}{10^9 M_\odot} \right) + 2.881. \quad (9)$$

Following 26, we can also evaluate the tidal radius $r_t$ of nuclear star clusters hosted by galaxies of given stellar mass. Taking all these clusters to have the same concentration parameter $c$ of 0.7, where $c = \log_{10}(r_t/r_c)$ and $r_c$ is the core radius where all BHs reside, and using the numerical code of 25 we can evaluate the rate by which BH-BH$^{TZO}$ merger events take place in a nuclear star cluster hosted by a galaxy of given stellar mass. Properly weighting with the Schechter mass function for local galaxies in Eq. 3 we get that the rate of mergers from all nuclear star clusters in galaxies $\dot{N}_{\text{gcs in gal}}^{\text{GW190814}}$ is approximately described as,

$$\dot{N}_{\text{in gcs}}^{\text{GW190814}}(M_\star) = \begin{cases} 10^{-15} \text{yr}^{-1} & \text{for } M_\star < 10^7 M_\odot, \\ 10^{-15} \left( \frac{M_\star}{10^9 M_\odot} \right) \text{yr}^{-1} & \text{for } M_\star > 10^7 M_\odot. \end{cases} \quad (10)$$

In turn that gives us a rate density from nuclear star clusters that is $R_{\text{in gcs}}^{\text{GW190814}}(z < 0.1) = 1 \times 10^{-8}$ Gpc$^{-3}$ yr$^{-1}$. Nuclear star clusters are very dense environments and as a result all BH-BH$^{TZO}$ binaries even when formed will not remain binaries for long enough to merge via gravitational wave emission. Instead, in such dense environments the merging BH binaries will contain approximately equal mass members. As an example, in a nuclear star cluster with size of $10^8 M_\odot$, typically hosted by Milky Way sized galaxies, over a course of 10 Gyr there will be $\simeq 2 \times 10^3$ BH$^{TZO}$ objects formed. Yet, the probability of even one of them merging with a stellar mass BH is estimated to be only $10^{-4}$.

We note that nuclear star clusters do not have a clearly observed concentration-mass relation. Thus we have treated the concentration parameter $c$ as a free parameter. The value of $c = 0.7$ represents a low estimate of the allowed concentration in nuclear star cluster environments. That choice makes the BH-BH$^{TZO}$ merger rates higher. Higher values of $c$ will only further suppress significantly the estimated rate of Eq. 10.

Inside nuclear star clusters there is also the possibility of direct capture events between stellar mass BHs and BH$^{TZO}$. Such events will create very hard binaries with high eccentricities that will rapidly merge via gravitational wave emission and may not undergo any exchange interactions 27. For the direct captures higher concentrations enhance the creation of tight BH-BH$^{TZO}$ binaries. However, even for a very high value of concentration $c = 2$, we get a rate density of $3 \times 10^{-6}$ Gpc$^{-3}$ yr$^{-1}$ from direct capture events, that still is orders of magnitude smaller than the claimed rate for GW190814 type of events.
Hierarchical triples in the field: As we described most TZO's and in turn BH^TZO's are created in the field with a rate density of 6 × 10^2 Gpc^{-3} yr^{-1}. However, we need to include the probability that the BH^TZO will merge in a Hubble time with a stellar mass BH. Given the very low density of BHs, the only possibility for BH-BH^TZO mergers in the field is that the final BH-BH^TZO binary originated from a hierarchical triple of a specific configuration. We denote m_1, m_2 and m_3 the Zero Age Main Sequence (ZAMS) star masses of the three initial stars, with m_1 > m_2 > m_3. The relevant configuration leading to a GW190814-like event has to be such that in the triple at its creation, the inner binary contained the stars of masses m_2 and m_3 in an orbit with semi-major axis a_{in} and eccentricity e_{in}. The star with ZAMS mass m_2 has to be between 8 and up to 25 M⊙, so that when it had its supernova (SN) explosion it created a NS. Instead, the least massive star with mass m_3 has to be between 6 and 18 M⊙ giving us the red giant that with the NS will create the TZO. Finally, m_1 has to be massive enough that it will give a BH of mass 10-30 M⊙, i.e. m_3 has to be at least m_3 > 30 M⊙. That star was on an initial outer orbit with respect to the m_2 - m_3 binary with semi-major axis a_{out} and eccentricity e_{out}.

The rate density of 6 × 10^2 Gpc^{-3} yr^{-1} for BH^TZO's already accounts for the fact that the inner binary will survive the SN explosion of m_2 and that a_{in} is small enough for the TZO to form. To connect that rate density to a BH-BH^TZO merger rate density we need to include, first the fact that the TZO will be in a triple, second that the third object is the most massive member with a ZAMS mass of at least 30 M⊙, third that the outer binary will survive both the SN kicks of m_1 and m_2, and fourth that once the BH-BH^TZO binary is created it will merge within a Hubble time. In the following we address the first three points.

In [28] it is noted that about 10% of low mass stars are in triples, with that fraction rising to about 50% for spectral type B stars. Given the high value of the m_2 mass we take that about 30% of systems that give rise to TZO's start out as triple systems. Using the Kroupa initial stellar mass function we estimate that if the m_1 mass is independent of the masses of the other stars in the hierarchical triple system then only in 0.1-2 % of the triple systems, the m_1 mass will be > 30 M⊙. That is a very dominant suppression and at face value would bring the BH-BH^TZO merger rate density down to 0.2-4 Gpc^{-3} yr^{-1} without including the third and fourth conditions. However, that suppression may be significantly mitigated by the fact that star systems likely form due to fragmentation processes. It is unlikely that the masses of the three stars are independent of each other. And since by having a TZO the mass m_2 is already massive enough, there may be an enhanced probability that the outer star is massive as well enhancing the BH-BH^TZO merger rates.

For these triples the mass ratio is m_1/(m_2 + m_3) ≃ 1. Since we have already included the suppression factor of hierarchical triples, stability arguments give that the semi-major axes ratio of the outer orbit to the inner orbit a_{out}/a_{in} is between 5 and 20 [29], a result that we also confirmed by semi-analytical calculations and in agreement with numerical simulations and observations of triple systems [30]. The initial (pre-SN explosions) eccentricity of the outer orbit of these triple systems follows a thermal distribution. Systems where the inner orbit will give a TZO are already very tight. Relying on observed orbital properties of binary systems [31], we find that the equivalent triple system will not break up by the SN kicks of either the m_1 or m_2 stars as the typical SN kicks are only ∼ 100 km/s [32]. In fact, the first natal kick of the m_1 SN explosion will marginally affect the system’s binding energy decreasing the semi-major axis of the outer binary by a factor of ∼ 3, which equals the fraction of the m_1 mass to that of its resulting BH remnant. We also find that the eccentricity distribution even after the SN explosion of m_1 is still going to be a thermal one. Thus, we find that effectively all triples where the inner binary will give a TZO survive the SN kicks of m_1 and m_2.

Finally, we want to know the probability that the BH-BH^TZO will merge within a Hubble time. That binary has to be sufficiently tight. As noted in [33] in binary BH mergers from binary star systems, the objects get close to each other through a common envelope phase preceding the formation of the second black hole. This would require a reliable estimate of the probability that the common envelope scenario takes place for our case, which in turn requires specialized codes that take into account both stellar evolution and orbital dynamics [28, 33]. Additional modifications to take into account the formation of TZO and BH^TZO would be necessary to such codes. That is beyond the scope of the present work. If all BH-BH^TZO merge within a Hubble time the merger rate density in the field from hierarchical triples can be 0.2-4 Gpc^{-3} yr^{-1} which is within the claimed LIGO-Virgo rate for GW190814-like events. Thus we find that it is important such modifications to include TZO's are performed in the future. Until then we are not yet able to rule this scenario out.

Observational perspectives: As a final note, if the observed rate of GW190814-like events is ∼ 2 Gpc^{-3} yr^{-1} then with the enhanced LIGO-Virgo-KAGRA sensitivity we expect a gradual filling up of the low mass gap range. The BH^TZO's will cover the entire low mass gap range, as we show in Figure 2. However, the most likely BH^TZO's are the lower ∼ 2.5M⊙ as their mass comes from the NS accreting a fraction of the red giant’s mass. That makes the observed 2.6M⊙ mass of GW190814 a quite likely outcome.

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5 The 2% fraction comes from taking into account the Kroupa mass function parametrization uncertainties such that more stars are predicted at its massive end.
FIG. 2. Number of BH-BH\textsuperscript{TZO} events at S/N >8 at design sensitivity after 5 years. We take the rate density of these mergers to be 2 Gpc\textsuperscript{−3} yr\textsuperscript{−1}.

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