Cosmology with interaction between phantom dark energy and dark matter and the coincidence problem

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Abstract. We study a cosmological model in which phantom dark energy is coupled to dark matter by phenomenologically introducing a coupled term to the equations of motion of dark energy and dark matter. This term is parameterized by a dimensionless coupling function $\delta$, the Hubble parameter and the energy density of dark matter, and it describes an energy flow between the dark energy and dark matter. We discuss two cases: one is the case where the equation of state $\omega_e$ of the dark energy is a constant; the other is that where the dimensionless coupling function $\delta$ is a constant. We investigate the effect of the interaction on the evolution of the universe, the total lifetime of the universe and the ratio of the period when the universe is in the coincidence state to its total lifetime. It turns out that the interaction will produce significant deviation from the case without the interaction.

Keywords: dark matter, dark energy theory

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Cosmology with interaction between phantom dark energy and dark matter and the coincidence problem

Contents

1. Introduction 2
2. Phantom dark energy interacting with dark matter 3
3. Cosmology with a constant equation of state of the phantom dark energy 5
4. Cosmology with a constant coupling parameter 10
5. Conclusion 14
   Acknowledgments 15
   References 16

1. Introduction

A lot of evidence of astronomical observations indicates that our universe is currently in accelerated expansion. This is first revealed by observing high red shift supernova Ia [1]. Cross-checks confirm this from the cosmic microwave background radiation [2] and large scale structure [3]. To explain this accelerated expansion, some proposals have been suggested recently in the literature, for instance, by modifying Einstein’s general relativity in the cosmic distance scale, employing the brane world scenario and so on.

In Einstein’s general relativity, however, in order to give an explanation for the accelerated expansion, one has to introduce an energy component to the energy density of the universe with a large negative pressure, which drives the universe to accelerated expansion. This energy component is dubbed dark energy in the literature. All astronomical observations indicate that our universe is flat and it consists of approximately 72% dark energy, 21% dark matter, 4.5% baryon matter and 0.5% other (radiation etc). A simple candidate for being the dark energy is a tiny positive cosmological constant, which was introduced by Einstein in 1917, two years after he established general relativity. If the dark energy is the cosmological constant, one has to answer the question of why the cosmological constant is so small, $\sim 10^{-122}(M_p)^4$, rather than $\sim (M_p)^4$, which is expected from local quantum field theory [4]. Here $M_p \sim 10^{19}$ GeV is the Planck mass scale. Although the small cosmological constant is consistent with all observational data obtained so far, recall that a slow roll scalar field can drive the universe to accelerated expansion in the inflation models; it is therefore feasible to use a dynamical field to mimic the behaviour of the dark energy. In particular, it is hoped that one can solve the so-called coincidence problem by employing a dynamical field. The model of scalar field(s) acting as the dark energy is called the quintessence model [5]. Following the $k$-inflation model [6], it is also natural to use a field with a noncanonical kinetic term to explain the currently accelerated expansion of the universe. Such models are referred to as $k$-essence models and have some interesting features [7]. Suppose that the dark energy has the equation of state $p_e = \omega_e \rho_e$, where $p_e$ and $\rho_e$ are the pressure and energy density, respectively. In order to drive the universe to accelerated expansion, one has to have $\omega_e < -1/3$. Note that for the cosmological constant, $\omega_e = -1$; for the quintessence model, $-1 < \omega_e < -1/3$; and
for the $k$-essence model, in general one may have $\omega_e > -1$ or $\omega_e < -1$, but it is physically implausible to cross $\omega_e = -1$ [8].

It is well known that if $\omega_e < -1$, the dark energy will violate all energy conditions [9]. However, such dark energy models [10] are still consistent with observation data ($-1.46 < \omega_e < -0.78$) [11]. The dark energy model with $\omega_e < -1$ is called the phantom dark energy model. One remarkable feature of the phantom model is that the universe will end with a ‘big rip’ (future singularity). That is, for a phantom dominated universe, its total lifetime is finite (see also [12, 13]). Before the death of the universe, the phantom dark energy will rip apart all bound structures like the Milky Way, solar system, Earth, and ultimately the molecules, atoms, nuclei and nucleons of which we are composed [14].

Usually it is assumed that the dark energy is coupled to other matter fields only through gravity. Since the first principle is still not available to discuss the nature of dark energy and dark matter, it is therefore conceivable to consider possible interaction between the dark energy and dark matter. Indeed there exists a lot of literature on this subject (see for example [15–19] and references therein). In this paper, we also consider an model of interaction between the dark energy and dark matter by phenomenologically introducing an interaction term into the equations of motion of dark energy and dark matter, which describes an energy flow between the dark energy and dark matter. We restrict ourselves to the case where the dark energy is a phantom one. Constraints from supernova type Ia data on such a coupled dark energy model have been investigated very recently [19] (see also [17, 18]). Here we are interested in how such an interaction between the phantom dark energy and dark matter affects the evolution and total lifetime of the universe.

On the other hand, one important aspect of the dark energy problem is the so-called coincidence problem. Roughly speaking, the question is why the energy densities of dark energy and dark matter are of the same order just now. In other words, we live in a very special epoch when the dark energy and dark matter densities are comparable. Most recently, developing the idea proposed by McInnes [20], Scherrer [21] has attacked this coincidence problem for a phantom dominated universe. Since the total lifetime of the phantom universe is finite, it is possible to calculate the fraction of the total lifetime of the universe for which the dark energy and dark matter densities are roughly comparable. It has been found that the coincidence problem can be significantly ameliorated in such a phantom dominated universe in the sense that the fraction of the total lifetime is not negligibly small. In this paper we will also study the effect of the interaction on the ratio of the period to its total lifetime when the universe is in the coincidence state.

The organization of this paper is as follows. In the next section, we first introduce the coupled dark energy model. In section 3 we discuss the case where the phantom dark energy has a constant equation of state $\omega_e$. In section 4 we study the case with a constant coupling function $\delta$ introduced in section 2. In this case, the equation of state $\omega_e$ will no longer be a constant. The conclusion and discussion are presented in section 5.

2. Phantom dark energy interacting with dark matter

Let us consider a cosmological model which only contains dark matter and dark energy (generalizing to include the baryon matter and radiation is straightforward). A phenomenological model of the interaction between the dark matter and dark energy is
assumed, through an energy exchange between them. Then the equations of motion of dark matter and dark energy in a flat FRW metric with a scale factor $a$ can be written as

\[ \dot{\rho}_m + 3H(\rho_m + p_m) = \delta H \rho_m, \quad (2.1) \]

\[ \dot{\rho}_e + 3H(\rho_e + p_e) = -\delta H \rho_m, \quad (2.2) \]

where $\rho_m$ and $p_m$ are the energy density and pressure of dark matter, while $\rho_e$ and $p_e$ are those for dark energy, $H \equiv \dot{a}/a$ is the Hubble parameter and $\delta$ is a dimensionless coupling function. Suppose that the dark matter has $p_m = 0$ and the dark energy has the equation of state $p_e = \omega_e \rho_e$. Note that in general $\omega_e$ is a function of time, rather than a constant. Clearly the total energy density of the universe, $\rho_t = \rho_m + \rho_e$, obeys the usual continuity equation

\[ \dot{\rho}_t + 3H(\rho_t + p_t) = 0, \quad (2.3) \]

with the total pressure $p_t = p_e$. The Friedmann equation is

\[ H^2 = \frac{8\pi G}{3} \rho_t, \quad (2.4) \]

and the acceleration of the scale factor is determined from the equation

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_t + 3p_t), \quad (2.5) \]

where $G$ is the Newton gravitational constant.

In general the coupling function $\delta$ may depend on all degrees of freedom of dark matter and dark energy. However, if $\delta$ is dependent on the scale factor only, one then can integrate (2.1) and obtain

\[ \rho_m = \rho_{m,0} a^{-3} e^{\int \delta \, \mathrm{d}x}, \quad (2.6) \]

where $\alpha = \log a$ and $\rho_{m,0}$ is an integration constant. Substituting this into (2.2), in order to get the relation between the energy density of the dark energy and the scale factor, one has to first be given the relation of the pressure to the energy density of the dark energy, namely the equation of state $\omega_e$. Here we follow another approach to study the cosmological model by assuming a relation between the energy density of the dark energy and that of the dark matter as follows [19]:

\[ r = \frac{\rho_e}{\rho_m} = \frac{\rho_{e,0}}{\rho_{m,0}} \left( \frac{a}{a_0} \right)^{\xi}, \quad (2.7) \]

where $\rho_{e,0}$, $a_0$ and $\xi$ are three constants. Setting the current value of the scale factor to one, that is $a_0 = 1$, $\rho_{e,0}$ and $\rho_{m,0}$ can be interpreted as the current dark energy density and dark matter energy density, respectively.

In this paper we will consider two special cases. One is the case where $\omega_e$ is kept as a constant. The other is the case where the coupling function $\delta$ is a constant.
3. Cosmology with a constant equation of state of the phantom dark energy

In this section we consider the case with a constant \( \omega_e \). From the relation (2.7) we have

\[
\rho_e = \frac{Aa^\xi}{1 + Aa^\xi}\rho_t, \quad \rho_m = \frac{1}{1 + Aa^\xi}\rho_t, \tag{3.1}
\]

where the constant \( A = \rho_{e,0}/\rho_{m,0} = \Omega_{e,0}/\Omega_{m,0} \). \( \Omega_{e,0} \) and \( \Omega_{m,0} \) are the fractions of the energy densities of dark energy and dark matter at present, respectively. The total energy density satisfies

\[
\frac{d\rho_t}{da} + \frac{3(1 + \omega_e)Aa^\xi}{1 + Aa^\xi}\rho_t = 0. \tag{3.2}
\]

Integrating this yields

\[
\rho_t = \rho_{t,0}a^{-3}[1 - \Omega_{e,0}(1 - a^\xi)]^{-3\omega_e/\xi}, \tag{3.3}
\]

where the constant \( \rho_{t,0} = \rho_{e,0} + \rho_{m,0} \). Therefore the Friedmann equation can be written down as

\[
H^2 = H_0^2a^{-3}[1 - \Omega_{e,0}(1 - a^\xi)]^{-3\omega_e/\xi}, \tag{3.4}
\]

with \( H_0 \) being the present Hubble parameter. By use of (3.1) and (3.3), one can get the coupling function \( \delta \) from (2.1),

\[
\delta = 3 + \frac{\rho_m}{H\rho_0} = -\frac{(\xi + 3\omega_e)Aa^\xi}{1 + Aa^\xi} = -\frac{\xi + 3\omega_e}{\rho_t}\rho_t, \tag{3.5}
\]

where an overdot denotes the derivative with respect to the cosmic time \( t \). This can be expressed further as

\[
\delta = \frac{\delta_0}{\Omega_{e,0} + (1 - \Omega_{e,0})a^{-\xi}}, \tag{3.6}
\]

where \( \delta_0 = -\Omega_{e,0}(\xi + 3\omega_e) \). We have \( \delta(a \rightarrow 1) = \delta_0 \) and \( \delta(a \rightarrow \infty) = \delta_0/\Omega_{e,0} \). Therefore we see that when \( \xi > -3\omega_e \), \( \delta < 0 \), which implies that the energy flow is from the dark matter to the dark energy. In contrast, when \( 0 < \xi < -3\omega_e \), the energy flow is from the phantom dark energy to dark matter. This can also be understood from the equations of motion (2.1) and (2.2). Further, we can see from (3.5) that there is no coupling between the dark energy and dark matter as \( \xi = -3\omega_e \). Of course this is true only for the case where \( \omega_e \) is a constant. In addition, we can see from (3.1) and (3.3) that, in this model, the universe is dominated by the dark matter at early times, while it is dominated by the phantom dark energy at later times.

The deceleration parameter \( q \) is

\[
q \equiv -\frac{a\ddot{a}}{a^2} = -1 + \frac{\ddot{H}}{H^2} = -1 + \frac{3(1 + \omega_e)/(2\Omega_{e,0})}{2(1 - \Omega_{e,0}(1 - a^\xi))}. \tag{3.7}
\]

Note that \( q(a \rightarrow \infty) = -1 + 3(1 + \omega_e)/(2\Omega_{e,0}) \) and \( q(a \rightarrow 1) = -1 + 3\omega_e\Omega_{e,0}/2 \); they are always negative because \( \omega_e < 0 \) and \( q(a \rightarrow 1) < q(a \rightarrow \infty) \). In figures 1–3 we plot the relation of the deceleration parameter to the red shift defined by \( z = 1/a - 1 \) for the different \( \omega_e \) and \( \xi \). In the plots we take the fraction of the dark energy \( \Omega_{e,0} = 0.72 \). From the figures we can see that for the case with a fixed \( \omega_e \), a larger \( \xi \) leads to a smaller red
shift when the universe transits from the deceleration phase to the acceleration phase. On the other hand, for the case with a fixed $\xi$, a larger $\omega_e$ has a smaller red shift for that transition from the deceleration to acceleration phase.

The total lifetime of the universe can be obtained by integrating the Friedmann equation (3.4). It is

$$t_U = H_0^{-1} \int_0^\infty da \frac{a^{1/2}}{[1 - \Omega_{e,0}(1 - a^\xi)]^{3\omega_e/2\xi}}. \quad (3.8)$$

Here we are interested in the change of the lifetime due to the interaction between the dark energy and dark matter. Note that when $\xi = -3\omega_e$, the interaction disappears.
Denote the total lifetime by $t_T$ for this case; one has

$$t_T = H_0^{-1} \int_0^\infty da \, a^{1/2} \left[1 - \Omega_{e,0}(1 - a^{-3\omega_e})\right]^{-1/2}. \quad (3.9)$$

Denote the ratio of the lifetimes $t_U$ to $t_T$ by $g$:

$$g \equiv \frac{t_U}{t_T} = \frac{\Omega_{e,0}^{-3/2\xi}(1 - \Omega_{e,0})^{3(1+\omega_e)/2\xi} \int_0^\infty \Omega_e^{1/2\omega} (1 - \Omega_{e,0})^{-(-1+\omega_e)/2\omega - 1/2\omega_e - 1/2} \, dr}{\Omega_{e,0}^{1/2\omega} (1 - \Omega_{e,0})^{-(-1+\omega_e)/2\omega - r - 1/2\omega_e - 1/2} \, dr}. \quad (3.10)$$

In figure 4 we plot the ratio $g$ for three different equations of state $\omega_e = -1.5, -1.3$ and $-1.1$. Clearly, for a fixed $\omega_e$, the universe with a larger $\xi$ has a longer lifetime, while for a fixed $\xi$, a larger $\omega_e$ leads to a longer lifetime. Note that in figure 4 the three points where the three curves cross the $\xi$ axis correspond to the situation ($\xi = -3\omega_e$) without interaction between the dark energy and dark matter. In figure 5 the ratio $g$ is plotted versus the parameters $\xi$ and $\omega_e$. Note that for a constant equation of state $\omega_e$, the total lifetime of the universe is approximately [21]

$$t_T = \frac{\omega_e}{1 + \omega_e} \frac{t_m}{1 + \omega_e},$$

where $t_m$ is the age of the universe when the matter and phantom dark energy densities are equal. When $\omega_e = -1.5, -1.3$ and $-1.1$, one has $t_T = 3t_m, 4.3t_m$ and $11t_m$, respectively.

In order to satisfy the observation data, $\xi$ has to be chosen so that at least $g > 1/3, 1/4.3$ and $1/11$, respectively. We can see from figure 4 that the constraint of the age of the universe on the model is very weak; the parameter $\xi$ can be as small as 1. In contrast, if one requires that the transition of the universe from the deceleration phase to the acceleration phase happens around the red shift $z \leq 1$, one can see from figures 1–3 that one needs to take $\xi \geq 3$. Note that the best fit of the $\Lambda$CDM model indicates that
Cosmology with interaction between phantom dark energy and dark matter and the coincidence problem

![Figure 4](image_url)

**Figure 4.** The ratio $g$ of total lifetimes versus the parameter $\xi$ for the case of $\Omega_{e,0} = 0.72$. The three curves from bottom to top correspond to the cases $\omega_e = -1.5$, $-1.3$ and $-1.1$, respectively.

![Figure 5](image_url)

**Figure 5.** The ratio $g$ of total lifetimes versus the parameters $\xi$ and $\omega_e$ in the case of $\Omega_{e,0} = 0.72$.

this transition happens at $z \sim 0.5$. However, it is allowed that the transition happens at $z \in (0.3–1)$ in the different models. Clearly it is quite necessary to make a detailed numerical analysis and to give constraints on the parameters of the coupled dark energy model by using of the data from the supernova, cosmic microwave background radiation and large scale structure. Here let us just mention that a partial analysis of the model according to the data from the supernova has been carried out more recently [19]. The results shows that the supernova data favour a negatively coupled phantom dark energy with $\omega_e < -1$ and $-10 < \delta_0 < -1$. 
Next we turn to the coincidence problem. Following [21], we calculate the ratio of the period when the universe is in the coincidence state to the total lifetime of the universe. That is, we will calculate the quantity

\[ f = \frac{t_c}{t_U}, \]

where \( t_c \) is defined by

\[ t_c = H_0^{-1} \int_{a_1}^{a_2} da \frac{a^{1/2} [1 - \Omega_{e,0} (1 - a^\xi)]^{3\omega_e/2\xi}}{r}. \]

During the coincidence state time, the energy density of dark energy is comparable to that of dark matter and the scale factor evolves from \( a_1 \) to \( a_2 \). What is the exact meaning of the term ‘comparable’? This is not a well-defined point for determining the scale factors \( a_1 \) and \( a_2 \). In [21], Scherrer defined a scale of the energy density ratio \( r_0 \) such that the dark energy and dark matter densities are regarded as being comparable if they differ by less than the ratio \( r_0 \) in either direction. He found that the ratio varies from \( 1/3 \) to \( 1/8 \) as \( \omega_e \) varies from \(-1.5\) to \(-1.1\) if \( r_0 = 10 \) in a phantom dark energy model without interaction between the dark energy and dark matter. In this sense, the coincidence problem is indeed significantly ameliorated in the phantom model because the ratio is not so small. As a result it is not so strange that we live in the epoch when the energy densities of dark energy and dark matter are of the same order. Now we want to see how the fraction varies when the phenomenological interaction is introduced.

The fraction of the total lifetime of the universe for which the universe is in the coincidence state turns out to be

\[ f = \frac{\int_{1/r_0}^{r_0} r^{3/2\xi-1} (1 + r)^{3\omega_e/2\xi} dr}{\int_0^\infty r^{3/2\xi-1} (1 + r)^{3\omega_e/2\xi} dr} = \frac{2}{3} \left[ \frac{3}{2\xi}, -\frac{3\omega_e}{2\xi}, 1 + \frac{3}{2\xi}, -r_0 \right] - \frac{r_0^{-3/2\xi}}{3} \left[ \frac{3}{2\xi}, -\frac{3\omega_e}{2\xi}, 1 + \frac{3}{2\xi}, -\frac{1}{r_0} \right] \]

\[ \frac{\Gamma[3/2\xi] \Gamma[-3(1 + \omega_e)/2\xi]}{\Gamma[-3\omega_e/2\xi]} \]

Note that this ratio is independent of the current density parameter \( \Omega_{e,0} \). In figures 6 and 7 we plot the ratio \( f \) versus the scale \( r_0 \) for different parameters \( \omega_e \) and \( \xi \). Clearly for the case with fixed \( \omega_e \) and \( \xi \), a larger \( r_0 \) leads to a larger ratio \( f \). On the other hand, for the case with fixed \( r_0 \) and \( \omega_e \), a smaller \( \xi \) gives us a larger ratio. For example, we see from figure 6 that when \( r_0 = 10 \) and \( \omega_e = -1.5 \), the ratio \( f \) is about 0.45 for \( \xi = 3 \), larger than for the case \( (f = 1/3) \) without the interaction. Note that the middle curve in figure 6 corresponds to the case without the interaction \( (\xi = -3\omega_e) \). In figure 8, we plot the ratio \( f \) versus the parameters \( \xi \) and \( r_0 \) for a fixed \( \omega_e = -1.3 \). From figures 6–8, one can see that the period when the universe is in the coincidence state is indeed comparable to its total lifetime. In addition, we note from (2.7) that a larger \( \xi \) means that the universe will be dominated more quickly by the phantom dark energy; this corresponds to having a longer total lifetime of the universe, which can be seen from figures 4 and 5. Thus it is natural to have a smaller fraction \( f \) than for the case with a smaller \( \xi \).
4. Cosmology with a constant coupling parameter

In this section we consider the case with a constant coupling function $\delta$. In this case, we have the energy density of dark matter

$$\rho_m = \rho_m,0 a^{-3+\delta}. \quad (4.1)$$

And then the dark energy density has the relation to the scale factor

$$\rho_e = \rho_e,0 a^{-3+\delta+\xi}. \quad (4.2)$$

The Friedmann equation turns out to be

$$H^2 = H_0^2(\Omega_m,0 a^{-3+\delta} + \Omega_e,0 a^{-3+\delta+\xi}). \quad (4.3)$$
Cosmology with interaction between phantom dark energy and dark matter and the coincidence problem

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{The ratio $f$ versus the parameters $r_0$ and $\xi$ for the case of $\omega_e = -1.3$.}
\end{figure}

In this case, the equation of state $\omega_e$ of the dark energy will depend on time (scale factor). From (2.2), we can obtain

$$\omega_e = \frac{\delta + \xi}{3} - \frac{\delta \Omega_{m,0}}{3 \Omega_{e,0}} a^{-\xi}. \quad (4.4)$$

When $\delta = 0$, one has $\xi = -3\omega_e$. This situation is just the case without the interaction. From (4.2) and (4.4), one can see that in order for the dark energy to be phantom, $\delta + \xi > 3$ has to be satisfied so that the dark energy density increases with the scale factor. When $\delta + \xi = 3$, although the dark energy density keeps constant, it does not act as a cosmological constant due to the interaction between the dark energy and dark matter. We see from (4.4) that

$$\omega_e(a \to 0) = -\text{sgn}(\delta) \cdot \infty, \quad \omega_e(a \to 1) = -\frac{\delta + \xi}{3} \frac{\delta \Omega_{m,0}}{3 \Omega_{e,0}}, \quad (4.5)$$

$$\omega_e(a \to \infty) = -\frac{\delta + \xi}{3}.$$  

It is interesting to note that $\omega_e$ diverges as $a \to 0$. This is due to the fact that the dark energy density (4.2) approaches zero very quickly, while the energy density of dark matter (4.1) goes to infinity as $a \to 0$; in order for equation (2.2) to hold, the parameter $\omega_e$ has to go to infinity. In fact, the dark energy does not play any role at early times, which can be seen from the Friedmann equation (4.3). Therefore here the divergence of $\omega_e$ does not make any sense in physics.

The deceleration parameter is found to be

$$q = -1 + \frac{1}{2} \frac{(3 - \delta) \Omega_{m,0} + (3 - \delta - \xi) \Omega_{e,0} a^\xi}{\Omega_{m,0} + \Omega_{e,0} a^\xi}, \quad (4.6)$$
which has $q = -1 + (3 - \delta - \xi \Omega_{e,0})/2$ when $a = 1$ and $q = (1 - \delta - \xi)/2$ when $a \to \infty$. In figures 9 and 10 we plot the deceleration parameter versus the red shift for the case with a fixed $\xi = 4$, different $\delta$ and the case with a fixed $\delta = 0.3$, different $\xi$, respectively. Note that the coupling parameter $\delta$ will affect the expansion law (4.1) of dark matter density. We do not expect that the usual behaviour ($\rho_m \sim a^{-3}$) will be changed much. So we take the value of $\delta$ so that the exponent is changed within 10%. That is, $\delta$ is taken to be in the range of $(-0.3, 0.3)$. Of course, in order to get a correct constraint on the parameter $\delta$ from the observation data, a detailed numerical analysis has to be done. We see from figure 9 that for the case with a given $\xi$, a larger $\delta$ leads to a smaller red shift when the universe transits from a deceleration phase to an acceleration phase, while figure 10 tells us that for the case with a fixed $\delta$, a larger $\xi$ gives us a smaller red shift.
From (4.3) we can get the total lifetime of the universe

$$t_U = H_0^{-1} \int_0^\infty da a^{-1}(\Omega_{m,0}a^{-3+\delta} + \Omega_{e,0}a^{-3+\delta+\xi})^{-1/2}. \quad (4.7)$$

We now consider the effect of the interaction on the total lifetime. Note that the total lifetime of the universe without the interaction is

$$t_T = H_0^{-1} \int_0^\infty da a^{-1}(\Omega_{m,0}a^{-3} + \Omega_{e,0}a^{-3+\xi})^{-1/2}. \quad (4.8)$$

Denote the ratio $t_U/t_T$ by $g$; we can express this as

$$g = \frac{\int_0^\infty \frac{dr}{r} ((1 - \Omega_{e,0})/\Omega_{e,0})^{(3-\delta)/2} r^{(3-\delta-2\xi)/2}(1 + r)^{-1/2}}{\int_0^\infty \frac{dr}{r} ((1 - \Omega_{e,0})/\Omega_{e,0})^{3/2} r^{(3-2\xi)/2}(1 + r)^{-1/2}}. \quad (4.9)$$

In figure 11, the ratio $g$ is plotted versus the parameters $\xi$ and $\delta$ for the case $\Omega_{e,0} = 0.72$. We see that the case $\delta > 0$ is quite different from the case of $\delta < 0$. For the case with a fixed $\delta > 0$, a larger $\xi$ leads to a longer lifetime of the universe. In contrast, for the case with a fixed $\delta < 0$, a smaller $\xi$ gives us a longer lifetime. Further, for the case with a fixed $\xi$, a smaller $\delta$ has a longer lifetime of the universe.

Finally we consider the fraction of the total lifetime for which the universe is in the coincidence state. As in the case with a constant $\omega_e$ considered in the previous section, we calculate the following ratio:

$$f = \frac{\int_{r_0}^{r_f} \frac{dr}{r} (3-\delta-2\xi)/2(1 + r)^{-1/2}}{\int_0^{r_f} \frac{dr}{r} (3-\delta-2\xi)/2(1 + r)^{-1/2}}. \quad (4.10)$$

In figure 12 we plot the ratio $f$ versus the scale $r_0$ for a fixed $\xi$, but different $\delta$. This shows that a larger $\delta$ gives a larger ratio for a fixed $r_0$. On the other hand, we plot the ratio $f$...
Cosmology with interaction between phantom dark energy and dark matter and the coincidence problem

**Figure 12.** The ratio $f$ versus the parameter $r_0$ for a fixed $\xi = 4$. The three curves from top to bottom correspond to the cases of $\delta = 0.3, 0$ and $-0.3$, and the corresponding equations of state are $\omega_{\psi,0} = -1.47, -1.33$ and $-1.19$, respectively.

**Figure 13.** The ratio $f$ versus the parameter $r_0$ for a fixed $\delta = 0.3$. The three curves from top to bottom correspond to the cases of $\xi = 5, 4$ and $3$, and the corresponding equations of state are $\omega_{\psi,0} = -1.80, -1.47$ and $-1.13$, respectively.

in figure 13 versus the scale $r_0$ for a fixed $\delta$, but different $\xi$, which shows that a larger $\xi$ gives us a larger ratio for a fixed $r_0$. Note that the case with $\omega_{\psi,0} = -1.80$ is already ruled out [11]. Here we show this case just for illustration. Figure 14 shows the relation of the ratio $f$ versus the parameters $\xi$ and $\delta$ for a fixed scale $r_0 = 5$.

5. Conclusion

In summary, we discuss a cosmological model in which phantom dark energy has an interaction with dark matter. The interaction is introduced phenomenologically by considering an additional term (see (2.1) and (2.2)) in the equations of motion of dark
energy and dark matter. This term is parameterized by a product of a dimensionless coupling function $\delta$, the Hubble parameter and the energy density of dark matter, and it describes an energy flow between the dark energy and dark matter. We discuss two cases: one is the case where the equation of state $\omega_e = p_e/\rho_e$ of the dark energy is kept constant; the other corresponds to the case with a constant coupling function $\delta$. We investigate the effect of the interaction on the evolution of the universe, the total lifetime of the universe and the fraction of the total lifetime of the universe for which the universe is in the coincidence state, where the energy densities of the dark energy and dark matter are comparable. We find that the interaction has rich and significant consequences for these issues. For example, the fraction of the total lifetime of the universe for which the universe is in the coincidence state can approximately reach 0.45 if we take $\omega_e = -1.5$, $r_0 = 10$ and $\xi = 3$. This means that the period when the energy densities of the dark energy and dark matter are comparable is significantly long, compared to its total lifetime. Thus, it is not so strange that we now live in the coincidence state of the universe. In this sense the coincidence problem can indeed be significantly ameliorated in such an interacting phantom dark energy model. Finally let us stress that the constraints on the parameters of the coupled dark energy model from the supernova Ia data have been analysed recently [19] (see also [17,18]); the values of parameters we take in this paper are all in the allowed region. Certainly it is not enough to consider the constraints from the supernova data only. We expect that the data from the cosmic microwave background radiation and, in particular, from the large scale structure will give more restrictive constraints on this coupled dark energy model. This issue is currently under investigation.

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Note added. After this article was submitted, we were informed that a similar idea has also been considered by Z K Guo and Y Z Zhang. In addition, we have noted that following [21], the coincidence problem has been discussed in a scalar field dark energy model with a linear effective potential [22].

References

[1] Riess A G et al (Supernova Search Team Collaboration), 1998 Astron. J. 116 1009 [SPIRES] [astro-ph/9805012]
Perlmutter S et al (Supernova Cosmology Project Collaboration), 1999 Astrophys. J. 517 565 [SPIRES] [astro-ph/9812133]
Riess A G et al (Supernova Search Team Collaboration), 2004 Astrophys. J. 607 665 [SPIRES] [astro-ph/0402512]

[2] Bennett C L et al, 2003 Astrophys. J. Suppl. 148 1 [astro-ph/0302207]
Spergel D N et al (WMAP Collaboration), 2003 Astrophys. J. Suppl. 148 175 [astro-ph/0302209]

[3] Tegmark M et al (SDSS Collaboration), 2004 Phys. Rev. D 69 103501 [SPIRES] [astro-ph/0310723]
Abazajian K et al, 2004 Preprint astro-ph/0410239
Abazajian K et al (SDSS Collaboration), 2004 Astron. J. 128 502 [SPIRES] [astro-ph/0403325]
Abazajian K et al (SDSS Collaboration), 2003 Astron. J. 126 2081 [SPIRES] [astro-ph/0305492]

Hawkins E et al, 2003 Mon. Not. R. Astron. Soc. 346 78 [astro-ph/0212375]
Verde L et al, 2002 Mon. Not. R. Astron. Soc. 335 432 [astro-ph/0112161]

[4] Weinberg S, 1989 Rev. Mod. Phys. 61 1 [SPIRES]
Peebles P J E and Ratra B, 2003 Rev. Mod. Phys. 75 559 [SPIRES] [astro-ph/0207347]
Padmanabhan T, 2003 Phys. Rep. 380 235 [SPIRES] [hep-th/0212290]

[5] Wetterich C, 1988 Nucl. Phys. B 302 668 [SPIRES]
Peebles P J E and Ratra B, 1988 Astrophys. J. 325 L17 [SPIRES]
Ratra B and Peebles P J E, 1988 Phys. Rev. D 37 3406 [SPIRES]
Friedmann J A, Hill C T, Stebbins A and Waga I, 1995 Phys. Rev. Lett. 75 2077 [SPIRES] [astro-ph/9505060]
Turner M S and White M J, 1997 Phys. Rev. D 56 4439 [SPIRES] [astro-ph/9701138]
Calderwood R R, Dave R and Steinhardt P J, 1998 Phys. Rev. Lett. 80 1582 [SPIRES] [astro-ph/9708069]
Liddle A R and Scherrer R J, 1999 Phys. Rev. D 59 023509 [SPIRES] [astro-ph/9809272]
Steinhardt P J, Wang L M and Zlatev I, 1999 Phys. Rev. D 59 123504 [SPIRES] [astro-ph/9812313]

[6] Armendariz-Picon C, Damour T and Mukhanov V, 1999 Phys. Lett. B 458 209 [SPIRES] [hep-th/9904075]
Garriga J and Mukhanov V F, 1999 Phys. Lett. B 458 219 [SPIRES] [hep-th/9904176]

[7] Chiba T, Okabe T and Yamaguchi M, 2000 Phys. Rev. D 62 023511 [SPIRES] [astro-ph/9912463]
Armendariz-Picon C, Mukhanov V and Steinhardt P J, 2000 Phys. Rev. Lett. 85 4438 [SPIRES] [astro-ph/0004134]
Armendariz-Picon C, Mukhanov V and Steinhardt P J, 2001 Phys. Rev. D 63 103510 [SPIRES] [astro-ph/0006373]

[8] Vikman A, 2004 Preprint astro-ph/0407107

[9] Wald R M, 1984 General Relativity (Chicago, IL: Chicago University Press)

[10] Caldwell R R, 2002 Phys. Lett. B 545 23 [SPIRES] [astro-ph/9908168]

[11] Knop R A et al, 2003 Astrophys. J. 598 102 [SPIRES] [astro-ph/0309368]
Riess A G et al (Supernova Search Team Collaboration), 2004 Astrophys. J. 607 665 [SPIRES] [astro-ph/0402512]

[12] Gonzalez-Diaz P F, 2003 Phys. Rev. D 68 021303 [SPIRES] [astro-ph/0305559]
Nojiri S and Odintsov S D, 2004 Phys. Lett. B 595 1 [SPIRES] [hep-th/0405078]
Nojiri S and Odintsov S D, 2004 Preprint hep-th/0408170
Elizalde E, Nojiri S and Odintsov S D, 2004 Phys. Rev. D 70 043539 [SPIRES] [hep-th/0405034]

[13] Wu P X and Yu H W, 2004 Preprint astro-ph/0407424
Gao Z K, Piao Y S, Zhang X M and Zhang Y Z, 2004 Preprint astro-ph/0410654
Feng B, Li M, Piao Y S and Zhang X, 2004 Preprint astro-ph/0407432

[14] Onemli V K and Woodard R P, 2002 Class. Quantum Grav. 19 4607 [SPIRES] [gr-qc/0204065]
Onemli V K and Woodard R P, 2004 Preprint gr-qc/0406008
Cosmology with interaction between phantom dark energy and dark matter and the coincidence problem

Brunier T, Onemli V K and Woodard R P, 2004 Preprint gr-qc/0408080
Boisseau B, Esposito-Farese G, Polarski D and Starobinsky A A, 2000 Phys. Rev. Lett. 85 2236 [SPIRES] [gr-qc/0001066]
[14] Caldwell R R, Kamionkowski M and Weinberg N N, 2003 Phys. Rev. Lett. 91 071301 [SPIRES] [astro-ph/0302506]
Nesseris S and Perivolaropoulos L, 2004 Preprint astro-ph/0410309
[15] Amendola L, 2000 Phys. Rev. D 62 043511 [SPIRES] [astro-ph/9908023]
Chen X I and Kamionkowski M, 1999 Phys. Rev. D 60 104036 [SPIRES] [astro-ph/9905368]
Uzan J P, 1999 Phys. Rev. D 59 123510 [SPIRES] [gr-qc/9903004]
Perrotta F, Baccigalupi C and Matarrese S, 2000 Phys. Rev. D 61 023507 [SPIRES] [astro-ph/9906066]
Chiba T, 1999 Phys. Rev. D 60 083508 [SPIRES] [gr-qc/9903094]
Billyard A P and Coley A A, 2000 Phys. Rev. D 61 083503 [SPIRES] [astro-ph/9908224]
Faraoni V, 2000 Phys. Rev. D 62 023504 [SPIRES] [gr-qc/0002091]
Chimento L P, Jakubi A S and Pavon D, 2000 Phys. Rev. D 62 063508 [SPIRES] [astro-ph/0005070]
Batista A B, Fabris J C and de Sa Ribeiro R, 2001 Gen. Rel. Grav. 33 1237 [SPIRES] [gr-qc/0001055]
Bean R and Mageejo J, 2001 Phys. Lett. B 517 177 [SPIRES] [astro-ph/0007199]
Sen A A and Sen S, 2001 Mod. Phys. Lett. A 16 1303 [SPIRES] [gr-qc/0103098]
Zimdahl W and Pavon D, 2001 Phys. Lett. B 521 133 [SPIRES] [astro-ph/0105479]
Chiba T, 2001 Phys. Rev. D 64 103503 [SPIRES] [astro-ph/0106550]
Esposito-Farese G and Polarski D, 2001 Phys. Rev. D 63 063504 [SPIRES] [gr-qc/0009034]
[16] Bonanno A and Reuter M, 2002 Phys. Lett. B 527 9 [SPIRES] [astro-ph/0106468]
Gromov A, Baryshev Y and Teerikorpi P, 2002 Preprint astro-ph/0209458
Hoffman M B, 2003 Preprint astro-ph/0307350
Franca U and Rosenfeld R, 2004 Phys. Rev. D 69 063517 [SPIRES] [astro-ph/0308149]
Axenides M and Dimopoulos K, 2004 J. Cosmol. Astropart. Phys. JCAP07(2004)010 [SPIRES] [hep-ph/0401238]
Biswas T and Maziumdar A, 2004 Preprint hep-th/0408026
Piazza F and Tsujikawa S, 2004 J. Cosmol. Astropart. Phys. JCAP07(2004)004 [SPIRES] [hep-th/0405054]
Zimdahl W and Pavon D, 2003 Gen. Rel. Grav. 35 413 [SPIRES] [astro-ph/0210484]
Zimdahl W and Pavon D, 2004 Gen. Rel. Grav. 36 1483 [SPIRES] [gr-qc/0311067]
Pavon D, Sen S and Zimdahl W, 2004 J. Cosmol. Astropart. Phys. JCAP05(2004)009 [SPIRES] [astro-ph/0402067]
[17] Dalal N, Abazajian K, Jenkins E and Manohar A V, 2001 Phys. Rev. Lett. 87 141302 [SPIRES] [astro-ph/0105317]
Amendola L, Gasperini M and Piazza F, 2004 Preprint astro-ph/0407573
Majerotto E, Sapone D and Amendola L, 2004 Preprint astro-ph/0410543
McInnes B, 2002 Preprint astro-ph/0210321
Scherrer R J, 2004 Preprint astro-ph/0410508
Avelino P P, 2004 Preprint astro-ph/0411033