Remote Sensing Image Restoration Based on an improved Landweber Iterative Method for Forest Monitoring and Management

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Abstract. As an advanced space exploration technologies, remote sensing technology has been widely used in forest monitoring and management. Forest current conditions could be reflected in real-time remote sensing images, but due to various imaging system and its environmental constraints, the original remote sensing images is often blurred. In this paper, two common remote-sensing imaging blur, named motion blur and atmospheric turbulence blur, were studied and discussed the mechanism of the image blurred, and proposed a remote sensing image restoration method based on an improved Landweber iteration method which expedites the convergence only in the signal domain. As a result, we can still improve the image restoration accuracy of results at the same time of speeding up convergences.

1. Introduction

Forest monitoring and management are important means for understanding the status of forest health and managing the forest resources scientifically [1]. By the analysis and evaluation of spatial data or images captured by remote sensing imaging technology, and as a complete forest observation systems combined with GPS positioning system, it is very effective to implement the monitoring and management of forest health [2]. From the previous literatures, we can well conclude that remote sensing imaging technology mainly involves three aspects, i.e. (i) Use remote sensing to detect forest growth rates and insect defoliation; (ii) Use of remote sending in forestry for fire condition monitoring can be found in many former researches; (iii) Measure the biochemical ingredient using hyperspectral remote sensing imaging system. In the process of analyzing remote sensing data, the quality of the image is especially critical to the results. However, due to various imaging system and its environmental constraints, the original remote sensing images is often blurred.

This paper focuses on two common remote sensing image deteriorations named motion blur and atmospheric turbulence blur separately, and proposed an improved landweber iteration method which can more rapidly converges to more accurate solution.

2. Image degradation

Image degradation is compared to an ideal or perfect image and represents a lower imaging quality, which is commonly introduced by the process of image formation and the process of image recording.

The degradation due to the process of image formation is usually denoted by blurring. In the case of remote sensing imaging, the blurring is often due to relative motion between the camera and the
ground or atmospheric turbulence interference before the light goes into the sensor of imaging system. The former is referred to as motion blur, and the latter is called atmospheric turbulence blur.

The degradation introduced by the recording process is usually denoted by noise, which is a statistical process. In the case of remote sensing imaging, the noise registers as Gaussian distribution, so it’s also called Gaussian noise. For improving the quality of blurred and noisy image, an effective method must be proposed to get a restoration image closed to the initial object.

3. Materials and Methods

Image restoration is often related to solving large linear systems shaped $g = A \ast f$, that is to estimate the discrete image $f$ representing the real objects according to the given discrete noise image $g$, in which matrix $A$ associated with the point spread function $K(x)$, defined as

$$
(\mathbf{A}f)_{j,m} = \sum_{k=0}^{n-1} K_{m,n} \ast f_{k,j}.
$$

Matrix $A$ can be seen as a discrete form of the point spread function $K(x)$.

Image restoration is often accompanied by ill-posedness, it is to say that solving of the equation $g = A \ast f$ is an ill-posed problem. All sorts of regularization methods (Tikhonov regularization, SVD truncate the regularization, Landweber iterative regularization, etc.) are introduced to solve this problem. The iterative method has remarkable predominance in solving large linear systems in both time and space, so the main choice for large scale problems is the iterative regularization method [3].

Landweber iteration method is a simple iterative regularization method, which has been widely applied for many fields because of its simple and good regularization effect. However, the drawback of this method is slow convergence, then it always requires many iterations to converge to the appropriate solution.

Strand presents a simple preconditioning technique, greatly improving the convergence speed, but this process has accelerated the propagation of noise, which reduces the image restoration accuracy [4].

In order to tackle the problems of Landweber iterative method, we first guarantee to improve the convergence speed of Landweber iterative method, meanwhile, we analyze the reasonable factors that affect the accuracy of image restoration. By controlling and regulating these factors, we not only could substantially increase the convergence rate but also strongly inhibits the propagation of noise, so as to improve the restoration accuracy of results at the same time of speeding up convergences.

3.1. Landweber iterative method

The Landweber method is the following iterative method for solving the Least squares equation $A' Af = A' g$, where $A$ named a Square integrable linear operator, $A'$ named adjoint operator of $A$. Let $f_0$ be an arbitrarily chosen initial guess, for $k = 0, 1, 2, 3, \ldots$, we compute the iteration

$$
f_{k+1} = f_k + \tau (A' g - A' Af_k),
$$

where $f_k$ represents the results obtained by $k$ iterations, $\tau$ is so-called Relaxation parameter.

If we use Fourier transform to equation (1), we get

$$
\hat{f}_{k+1}(\omega) = (1 - \tau \hat{K}(\omega))^{-1} \hat{f}_k(\omega) + \frac{1}{\tau} \hat{g}(\omega) \hat{K}(\omega)^{-1} \hat{f}_k(\omega),
$$

where $\hat{f}_k(\omega)$, $\hat{K}(\omega)$, $\hat{g}(\omega)$ denote Fourier transform form of $f_k$, $A$, $g$ respectively.

In equation (2), $R^{k+1}g$ represents the approximate solution $f_k$ after $k$ iterations, and $f_k - f^{(0)}$ represents the error about iterative solution, then we get

$$
f_k - f^{(0)} = R^{k+1}g - f^{(0)} = R^{k+1}(Af^{(0)} + w) - f^{(0)}
$$

$$
= (R^{k+1}Af^{(0)} - f^{(0)}) + R^{k+1}w
$$

where $f^{(0)}$ is the initial precise value, $w$ is the additive noise.

As known from equation (3), the error of the iterative solution is mainly affected by the terms denoted $R^{k+1}Af^{(0)}$ and $R^{k+1}w$, which are given as follows
\[
\|R^k A f^{(n)} - f^{(n)}\| = \frac{1}{(2\pi)^2} \int \left|K^{(n)}(\omega) - \frac{1}{i\omega} \hat{f}^{(n)}(\omega)\right|^2 d\omega \\
= \frac{1}{(2\pi)^2} \int \left|\left(1 - i\tau K^{(n)}(\omega)\right)^2 \hat{f}^{(n)}(\omega)\right|^2 d\omega \\
\|R^k w\| = \frac{1}{(2\pi)^2} \int \left|\left(1 - i\tau K^{(n)}(\omega)\right)^2 \hat{g}(\omega)\right|^2 d\omega 
\]

(4)

(5)

Obviously, equation (4) is a decreasing function of \( k \), called approximation error, and equation (5) is an increasing function of \( k \), called noise-propagation error. Above meaning implies that, with the increase in iterations, the approximation error gets smaller while noise-propagation error gets larger. As a result, it is inevitable that restoration process shows the nature of semi-convergence. That is, when we increase the number of iterations, the iterates first approach \( f^{(0)} \), then go away. Therefore there exists an optimum value of the number of iterations, \( k_{\text{opt}} \), corresponding to an iterate which has minimal distance from \( f^{(0)} \).

### 3.2. Choice of preconditioners

In the formula (3), we need many iterations to get the solution of least squares problem. This shows that a remarkable drawback of the landweber method is slow convergence speed. Strand has proposed a method to overcome this shortcoming, following this method, the linear system \( A'Af = A'g \) is replaced by an algebraic equivalent system [5].

\[
DA' Af = DA' g 
\]

where the matrix D is the preconditioner which approximates the generalized inverse of \( A'A \).

The original problem can be solved more easily by using the preconditioner, and we call this process as preconditioning [6].

Applying the Landweber method to solve the equation (6), then we get

\[
\hat{f}_0 = f_0 + \tau D(A'g - A'f_0) \quad \text{where} \quad 0 < \tau < 2 
\]

(7)

Not to lose generalization, let \( f_0 = 0 \), after Fourier transform to equation (7), we get the following formula

\[
\hat{f}_0(\omega) = 1 - \left(1 - \tau D(\omega)\right) K(\omega)^{-1} \hat{g}(\omega) \\
= \frac{\gamma}{\|K(\omega)\|^2 + 1} \hat{g}(\omega) \\
= \hat{s}(\omega) + \hat{n}(\omega) 
\]

(8)

Let \( \hat{b}(\omega) = \hat{K}(\omega) + \gamma^{-1} \hat{g}(\omega) \) ( \( \gamma \) is a constant), and let \( \tau = 1 \), we have

\[
\hat{f}_1(\omega) = 1 - \left(1 - \gamma \hat{K}(\omega)\right) \hat{g}(\omega) \\
= 1 - \left(1 - \gamma \hat{K}(\omega)\right)^{-1} \hat{g}(\omega) \\
= \hat{s}_1(\omega) + \hat{n}_1(\omega) 
\]

(9)

The whole frequency domain space can be divided into signal subspace \( S \) and noise subspace \( N \) according to the formula (9). Based on this fact that \( \|\hat{K}(\omega)\| < |K(\omega)| \) ( We use the Gaussian low pass filter in the paper), if \( \gamma = \|\hat{K}(\omega)\|^{-1} |K(\omega)| \), the following inequality holds

\[
\frac{\gamma}{\|\hat{K}(\omega)\|^2 + \gamma} < \frac{1}{2} \frac{\gamma}{|K(\omega)|^2 + \gamma} 
\]

(10)

In accordance with the equation (10), the convergence speed in the signal subspace is better than the speed in the noise subspace, which has indeed played a positive role in keeping down the propagation of noise. However, the convergence speed in the noise subspace is still accelerating after all. We would have gotten a very good restoration result if we have only preconditioned the signal subspace through separating the signal subspace and noise subspace.
4. Results and discussion
In this section we demonstrate the effect of the improved iterative methods. We use Normalized Mean Square Error (NMSE) to measure image restoration effects. Normalized Mean Square Error is defined as follows:

The experimental results of three methods directed against the motion blur are shown in Table 1.

| Preconditioners       | k   | Error(k=40) | k_{opt} | Error(k= k_{opt}) |
|-----------------------|-----|-------------|---------|-------------------|
| No preconditioner     | 40  | 21.4%       | 464     | 10.4%             |
| Strand’s preconditioner| 40  | 16.2%       | 72      | 14.1%             |
| Our preconditioner    | 40  | 10.1%       | 145     | 5.6%              |

Fig. 1 shows that comparison of restoration error of motion blur image about three methods in the same coordinate system. As is depicted in the figure, we can intuitively see that Landweber method with preconditioner based on signal and noise separation has a remarkable advantage both in accelerating convergence and in improving the accuracy of image restoration.

| Preconditioners       | k   | Error(k=40) | k_{opt} | Error(k= k_{opt}) |
|-----------------------|-----|-------------|---------|-------------------|
| No preconditioner     | 40  | 36.1%       | 432     | 16.8%             |
| Strand’s preconditioner| 40  | 22.4%       | 58      | 18.3%             |
| Our preconditioner    | 40  | 14.7%       | 117     | 8.7%              |

Fig. 2 shows that comparison of restoration error of atmospheric turbulence blur image about three methods in the same coordinate system. As shown in the figure, the improved Landweber iterative method is indeed high performance both in timing and precision.
Fig. 2 Comparison of restoration error of atmospheric turbulence blur image about three methods
(Horizontal axis represents iteration number, and Vertical axis represents restoration error)

Fig. 3 shows the comparison of restoration error between the different blur types with the improved iterative method. Under the same iterative method, minimum restoration error of motion blur image is 5.6%, and 8.7% to atmospheric turbulence blur image. It’s also very obvious by comparing restoration image. So, whether in visual view, or through the error analysis, motion-image blur restoration effect has a distinct advantage.

Fig. 3 Comparison of restoration error between the different blur types with the improved iterative method

5. Conclusion
The paper focuses on two kinds of image degradation named motion blur and atmosphere turbulence blur which can be commonly found in forest remote sensing imaging. A mathematical image degradation model was built using discrete point spread function which is a two-dimensional matrix, and applied to a clear image to simulate the image degradation so that some experiment data can be given.

Forest remote sensing images contain large amounts of spatial information and are often degraded. It’s a bottleneck of remote sensing imaging technology development in various fields of forest monitoring and management that how to get the distinct images from mass image data. This drives two requirements on an appropriate image restoration algorithm. In this instance, a preconditioning Landweber iterative algorithm based on signal and noise separation is presented. The main idea is that we only perform preconditioning in the signal domain by separating the signal and noise. The theoretical analysis and experimental results shows the method’s advantage: (1) faster convergence rate; (2) lower restoration error. The experimental results demonstrate that our proposed method gets the smallest restoration error in the same number of iterations. Therefore, this approach can guarantee that the degraded images can obtain a better solution with fast convergence.
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