Form Factors in $D$ Meson Decays

Dae Sung Hwang$^{(a)}$ and Do-Won Kim$^{(b)}$

$^a$: Department of Physics, Sejong University, Seoul 143–747, Korea

$^b$: Department of Physics, Kangnung National University, Kangnung 210-702, Korea

Abstract

We study the $d\Gamma/dq^2$ spectra and the branching fractions of the $D$ meson exclusive semileptonic decays with the lepton mass effects into consideration. We investigate their sensitivity to form factor models, and find that the decays to a pseudoscalar meson and a lepton pair are sensitive to the property of the form factor $F_1(q^2)$, and those to a vector meson and a lepton pair to the form factor $A_1(q^2)$. We also analyze the experimental results of the branching fractions $B(D^0 \to K^-(or\ \pi^-)\pi^+)$ and $B(D^0 \to K^-(or\ \pi^-)e^+\nu)$, and show that it is implied that $F_1(q^2)$ is of dipole type, instead of simple pole type which is commonly assumed.

PACS codes: 13.20.Fc, 13.20.-v, 13.25.Ft, 14.40.Lb

Key words: $D$ Meson, Semileptonic Decay, Lepton Mass Effect, Form Factor

e-mail: dshwang@kunja.sejong.ac.kr

e-mail: dwkim@phys1.kangnung.ac.kr
1. Introduction

The CP-violation phenomenon is discovered only in the $K_L \to \pi\pi$ decay and the charge asymmetry in the decay $K_L \to \pi^\pm l^\mp \nu$ for more than 30 years. The B-factories at KEK and SLAC are under construction for the discovery of CP-violation in the $B$ meson system. The mechanism of CP-violation through the complex phase of the CKM three family mixing matrix [1] is presently considered standard for the CP-violation. In order to measure the CKM matrix elements accurately, it is important to know the hadronic form factors of the transition matrix elements reliably. For the heavy to heavy transitions the heavy quark effective theory provides good informations for the form factors. However, for the heavy to light transitions the understanding of the form factors is still limited and this fact hinders the extractions of the CKM matrix elements from experimental results significantly. At the same time, we will be able to have important clues for the internal structures of hadrons by knowing these form factors well. Semileptonic decay processes are good sources for the knowledge of the form factors both experimentally and theoretically, and the lepton mass effects in heavy meson exclusive semileptonic decays were studied by Körner and Schuler [2].

We derive the formulas for $d\Gamma/dq^2$ with non-zero lepton mass in the forms which are efficient to study the form factor dependences. This formula for the pseudoscalar to pseudoscalar transition was also given by Khodjamirian et al. [3]. By using these formulas we study the $d\Gamma/dq^2$ spectra and branching fractions of the exclusive semileptonic $D$ meson decays: $D^0 \to K^{(*)-}e^+\nu$, $D^0 \to K^{(*)-}\mu^+\nu$, $D^0 \to \pi^-(or \, \rho^-) \, e^+\nu$ and $D^0 \to \pi^-(or \, \rho^-) \, \mu^+\nu$. In this analysis we employ three models of form factors and show how the results are influenced by the difference of form factors. For the decays to a pseudoscalar meson and a lepton pair, the results are sensitive to the property of the form factor $F_1(q^2)$. For the decays to a vector meson and a lepton pair, the results are sensitive to the form factor...
$A_1(q^2)$, and not to the other ones ($A_0(q^2)$, $A_2(q^2)$ and $V(q^2)$). We also analyze the experimental results of the branching fractions $\mathcal{B}(D^0 \to K^-(or\ \pi^-)\ \pi^+)$ and $\mathcal{B}(D^0 \to K^-(or\ \pi^-)\ e^+\nu)$, and show that it is implied that the form factor $F_1(q^2)$ of the $D$ to $K$ and the $D$ to $\pi$ transitions are of dipole type, instead of simple pole type which is commonly assumed in the studies of the $D$ meson decays.

2. Semileptonic Decays of Heavy Mesons

From Lorentz invariance one finds the decomposition of the hadronic matrix element for pseudoscalar to pseudoscalar meson transition in terms of hadronic form factors:

\[
< P(p)|J\mu|P(P) > = (P+p)\mu f_+(q^2) + (P-p)\mu f_-(q^2)
\]
\[
= \left( (P+p)\mu - \frac{M^2-m^2}{q^2} q\mu \right) F_1(q^2) + \frac{M^2-m^2}{q^2} q\mu \ F_0(q^2),
\]
where $J_\mu = \bar{q}' \gamma_\mu (1-\gamma_5) q$. We use the following notations: $M$ represents initial meson mass, $m$ final meson mass, $m_l$ lepton mass, $P$ initial meson momentum, $p$ final meson momentum, and $q_\mu = (P-p)_\mu$. The form factors $F_1(q^2)$ and $F_0(q^2)$ correspond to $1^-$ and $0^+$ exchanges, respectively. At $q^2 = 0$ we have the constraint $F_1(0) = F_0(0)$, since the hadronic matrix element in (1) is nonsingular at this kinematic point.

The $q^2$ distribution of the semileptonic decay $D^0 \to K^-l^+\nu$ is given in terms of the hadronic form factors $F_1(q^2)$ and $F_0(q^2)$ as:

\[
\frac{d\Gamma(D^0 \to K^-l^+\nu)}{dq^2} = \frac{G_F^2}{12\pi^3} |V_{cs}|^2 \frac{1}{2} \left( M^2 + m^2 - q^2 \right)^2 \left( 1 - \frac{m_l^2}{q^2} \right)^2 \times
\]
\[
\left[ (K(q^2))^2 \left( 1 + \frac{1}{2} \frac{m_l^2}{q^2} \right) |F_1(q^2)|^2 + M^2 \left( 1 - \frac{m_l^2}{M^2} \right)^2 \frac{3}{8} \frac{m_l^2}{q^2} |F_0(q^2)|^2 \right],
\]
where $K(q^2)$, momentum of the final meson in the $D$ meson rest frame, is given by

\[
K(q^2) = \frac{1}{2M} \left( (M^2 + m^2 - q^2)^2 - 4M^2m^2 \right)^{1/2},
\]
and the physically allowed range of $q^2$ is given by

$$m_l^2 \leq q^2 \leq (M - m)^2.$$  \hspace{1cm} (4)

For $m_l = 0$, (2) is reduced to the commonly used well-known formula:

$$\frac{d\Gamma(D^0 \to K^- l^+ \nu)}{dq^2} = \frac{G_F^2}{2\pi^3} |V_{cs}|^2 (K(q^2))^3 |F_1(q^2)|^2,$$  \hspace{1cm} (5)

and $0 \leq q^2 \leq (M - m)^2$. We note in (5) that only $F_1(q^2)$ contributes for $m_l = 0$, however, for $m_l \neq 0$ $F_0(q^2)$ also contributes as we can see in (2).

From Lorentz invariance one finds the decomposition of the hadronic matrix element for pseudoscalar to vector meson transition in terms of hadronic form factors:

$$< V(p) | J_\mu | P(P) > = \varepsilon^\nu(p) \left( (M + m)g_{\mu\nu}A_1(q^2) - 2 \frac{P_\mu P_\nu}{M + m} A_2(q^2) + \frac{q_\mu P_\nu}{M + m} A_3(q^2) \right.$$

$$+ i\varepsilon_{\mu\rho\sigma} \frac{P_\rho P_\sigma}{M + m} V(q^2) \left. \right),$$  \hspace{1cm} (6)

where $\varepsilon_{0123} = 1$ and

$$2mA_0(q^2) = (M + m)A_1(q^2) - \frac{M^2 - m^2 + q^2}{M + m} A_2(q^2) + \frac{q^2}{M + m} A_3(q^2).$$  \hspace{1cm} (7)

The form factors $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$ and $A_0(q^2)$ correspond to $1^-, 1^+, 1^+$ and $0^-$ exchanges, respectively. At $q^2 = 0$ we have the constraint $2mA_0(0) = (M + m)A_1(0) - (M - m)A_2(0)$, since the hadronic matrix element in (16) is nonsingular at this kinematic point.

After a rather lengthy calculation, the $q^2$ distribution of the semileptonic decay $D^0 \to K^{*-} l^+ \nu$ is given in terms of the hadronic form factors $A_1(q^2)$, $A_2(q^2)$, $A_3(q^2)$ and $V(q^2)$ as [1]:

$$\frac{d\Gamma(D^0 \to K^{*-} l^+ \nu)}{dq^2} = \frac{G_F^2}{32\pi^3} |V_{cs}|^2 \frac{1}{M^2} K(q^2) \left( 1 - \frac{m_l^2}{q^2} \right)^2 \times$$

$$\{ |A_1(q^2)|^2 \left( \frac{M + m}{m^2} \right)^2 \frac{1}{3} (MK)^2 (1 - \frac{m_l^2}{q^2}) + q^2 m^2 + (MK)^2 \frac{m_l^2}{q^2} + \frac{1}{2} m^2 m_l^2 \right\} \hspace{1cm} \text{(8)}$$
When we use the relations (9), it agrees with the formula for $m_l = 0$ given in Refs. 3, 8:

$$\frac{d \Gamma(D^0 \rightarrow K^{*-} l^+ \nu)}{dq^2} = \frac{G_F^2}{96 \pi^3} |V_{cs}|^2 \frac{q^2}{M^2} K(q^2) (|H^+(q^2)|^2 + |H^-(q^2)|^2 + |H^0(q^2)|^2),$$

where

$$H^0(q^2) = \frac{-1}{2m \sqrt{q^2}} ((M^2 - m^2 - q^2)(M + m)A_1(q^2) - \frac{4M^2K^2}{M + m}A_2(q^2)),$$

$$H^\pm(q^2) = -\left((M + m)A_1(q^2) \mp \frac{2MK}{M + m}V(q^2)\right).$$

In the case of the $B$ to $D$ meson (heavy to heavy) transition, the heavy quark effective theory (HQET) gives the useful relations between the relevant form factors [4]:

$$F_1(q^2) = V(q^2) = A_0(q^2) = A_2(q^2) = \frac{M + m}{2\sqrt{Mm}} \mathcal{F}(y),$$

$$F_0(q^2) = A_1(q^2) = \frac{2\sqrt{Mm}}{M + m} \frac{y + 1}{2} \mathcal{F}(y),$$

where $y = (M^2 + m^2 - q^2)/(2Mm) = E_{D^{(*)}}/m$ ($E_{D^{(*)}}$ is the energy of $D^{(*)}$ meson in the $B$ meson rest frame), and $\mathcal{F}(y)$ is a form factor which becomes the Isgur-Wise function in the infinite heavy quark mass limit. When we use the relations
for $m_l = 0$ the formula (8) becomes the well-known formula for the $B$ to $D^*$ transition:

$$
\frac{d\Gamma(B^0 \rightarrow D^{*+}l^-\bar{\nu})}{dq^2} = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m^3 (M - m)^2 \sqrt{y^2 - 1} \frac{1}{(y + 1)^2} \{1 + \frac{4y}{y + 1} \frac{1 - 2yr + r^2}{(1-r)^2}\} (F_{D^*}(y))^2,
$$

(12)

where $r = m/M$.

3. $D^0 \rightarrow K^{(*)-}l^+\nu$

For the form factors concerned with the exclusive semileptonic decays of $D$ meson, we cannot use the relations (11) of the HQET. Therefore, in the study of $D$ meson decays we use models for form factors. The pole-dominance idea suggests the following $q^2$ dependence of the form factors [8]:

$$
f_i(q^2) = f_i(0) \frac{1}{(1 - \frac{q^2}{m_{f_i}^2})^{n_{f_i}}},
$$

(13)

where $n_{f_i}$ and $m_{f_i}$ are corresponding power and pole mass of the form factors $f_i(q^2)$, respectively. The WSB model [8] adopts $n_{f_i} = 1$. However, the exact values of $n_{f_i}$ are not known. The relations (11) of the HQET gives the following approximate relation among the powers of the form factors for the heavy to heavy transitions:

$$
n_{F_1} = n_V = n_{A_0} = n_{A_2} = n_{F_0} + 1 = n_{A_1} + 1.
$$

(14)

Non-perturbative analysis of QCD [9] suggests the same relation as (14) for the form factors of the heavy to light transitions. The lattice calculations also show that the form factors $F_1$, $V$ and $A_0$ are more rapidly increasing functions of $q^2$ than the form factors $F_0$ and $A_1$ [3, 10], which favors the relation (14). Therefore, we will adopt two other models incorporating the relation (14), as well as the WBS model which assumes $n_{f_i} = 1$ [8], for the study of the exclusive semileptonic decays of $D$ meson. We organize in Table 1 the values of the powers $n_{f_i}$ of the
three models which we use in this work. For the values of the pole masses and 
those of the form factors at $q^2 = 0$, we use the values organized in Table 2 and 
3, which were given by Wirbel, Stech and Bauer [8]. Their precise values are not 
known and they should be different for each of the three models. However, their 
exact values are not crucial for our work of clarifying the model dependences of 
the $d\Gamma/dq^2$ spectra and the branching fractions.

For $D^0 \to K^- l^+\nu$, we use the formula (2) with non-zero lepton mass, instead of 
the commonly used formula (5) which is true for zero lepton mass. The obtained 
$d\Gamma (D^0 \to K^- l^+\nu)/dq^2$ spectrum and branching fractions are presented in Figure 
1 and Table 4, for each of the three models we adopt in this work: WSB, Model I 
and Model II explained in Table 1. We find that the shape of spectrum of Model 
II is different from those of WSB and Model I in Figure 1. That is, the value 
of $n_{F_1}$ determines the shape of spectrum. The experimental result of the E687 
Collaboration for this spectrum was given in Fig. 3 (a) of Ref. [13], and the shape of 
their spectrum favors Model II better than WSB and Model I. In the experimental 
extraction of the value of $F_{1DK}^1(0)$, the simple pole of the form factors has been 
commonly assumed [11, 12]. Under this assumption, $F_{1DK}^1(0) = 0.75 \pm 0.02 \pm 0.02$ 
was extracted [11] from the experimentally measured branching fraction $B(D^0 \to 
K^- e^+\nu) = (3.68 \pm 0.21) \times 10^{-2}$. (In Ref. [12], $F_{1DK}^1(0) = 0.76 \pm 0.03$ was presented.) 
However, if we assume in this extraction Model II which has the dipole form factor 
for $F_1(q^2)$, we would get

$$F_{1DK}^1(0) = 0.75 \times \sqrt{\frac{3.49 \times 10^{-2}}{4.78 \times 10^{-2}}} = 0.75 \times 0.85 = 0.64$$

(15)

for the mean value. In [13] we used our results in Table 4 of the branching fraction 
$B(D^0 \to K^- e^+\nu)$ for WSB ($n_{F_1} = 1$) and Model II ($n_{F_1} = 2$), which were obtained 
by using the same value of $F_{1DK}^1(0)$. That is, the experimentally extracted value 
of $F_{1DK}^1(0)$ is much dependent on the form factor models used in the analysis. In 
reality, it is not yet established which type of form factors is the right one.

For $D^0 \to K^+ l^+\nu$, we use the formula (8) with non-zero lepton mass. The
obtained spectra and branching fractions are presented in Figure 2 and Table 5. We find that the results of WSB and Model II are almost the same, and they are significantly different from the results of Model I. This fact implies that the value of $n_{A_1}$ mainly determines the spectra and branching fractions of $D^0 \to K^{*-} l^+ \nu$.

4. $D^0 \to \pi^- (\text{or } \rho^-) l^+ \nu$

For $D^0 \to \pi^- l^+ \nu$, we use the formula (2) with the replacement of $V_{cs}$ by $V_{cd}$. The obtained spectra and branching fractions are presented in Fig. 3 and Table 6. We find that the branching fractions of Model II in Fig. 3 are about twice those of WSB and Model I. Therefore, in case that we determine the value of $F_D^{D\pi}(0)$ from an experimentally measured branching fraction of $D^0 \to \pi^- l^+ \nu$, the value of $F_D^{D\pi}(0)$ determined with Model II will be about $1/\sqrt{2}$ times its value determined with WSB or Model I. From Table 4 and 6, we also find that the ratio $\mathcal{B}(D^0 \to \pi^- l^+ \nu)/\mathcal{B}(D^0 \to K^- l^+ \nu)$ from Model II is pretty bigger than that from WSB or Model I, but its present experimental result $0.101 \pm 0.020 \pm 0.003$ [14] can not discriminate them yet.

For $D^0 \to \rho^- l^+ \nu$, we use the formula (8) with the replacement of $V_{cs}$ by $V_{cd}$. The obtained spectra and branching fractions are presented in Fig. 4 and Table 7. We find in Fig. 4 and Table 7 that the results of WSB and Model II are almost the same, and they are much different from the results of Model I. Therefore, like the $D^0 \to K^{*-} l^+ \nu$ case, the property of the form factor $A_1(q^2)$ mainly determines the spectra and branching fractions, and the results are not sensitive to the other form factors ($A_0(q^2)$, $A_2(q^2)$ and $V(q^2)$).

5. Implications of Experimental Results

In this section we compare the exclusive semileptonic decays and the two-body
hadronic decays. We start by recalling the relevant effective weak Hamiltonian for the two-body hadronic decay $D^0 \rightarrow K^-\pi^+$:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} [C_1(\mu) \mathcal{O}_1 + C_2(\mu) \mathcal{O}_2] + \text{H.C.},$$

(16)

where $G_F$ is the Fermi coupling constant, $V_{cs}$ and $V_{ud}$ are corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and $\Gamma_\rho = \gamma_\rho (1 - \gamma_5)$. The Wilson coefficients $C_1(\mu)$ and $C_2(\mu)$ incorporate the short-distance effects arising from the renormalization of $\mathcal{H}_{\text{eff}}$ from $\mu = m_W$ to $\mu = O(m_c)$. By using the Fierz transformation under which $V^-A$ currents remain $V^-A$ currents, we get the following equivalent forms:

$$C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 = (C_1 + \frac{1}{N_c} C_2) \mathcal{O}_1 + C_2 (s\bar{\Gamma}_\rho T^a d)(\bar{u}\Gamma_\rho T^a c)$$

$$= (C_2 + \frac{1}{N_c} C_1) \mathcal{O}_2 + C_1 (\bar{u}\Gamma_\rho T^a d)(s\bar{\Gamma}_\rho T^a c),$$

(18)

where $N_c = 3$ is the number of colors and $T^a$’s are $SU(3)$ color generators. The second terms in (18) involve color-octet currents. In the factorization assumption, these terms are neglected and $\mathcal{H}_{\text{eff}}$ is rewritten in terms of “factorized hadron operators” [8]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[ a_1 (\bar{u}\Gamma_\rho d)_{H} [s\bar{\Gamma}_\rho c]_{H} + a_2 (s\bar{\Gamma}_\rho d)_{H} [\bar{u}\Gamma_\rho c]_{H} \right] + \text{H.C.},$$

(19)

where the subscript $H$ stands for hadronic implying that the Dirac bilinears inside the brackets be treated as interpolating fields for the mesons and no further Fierz-reordering need be done. The phenomenological parameters $a_1$ and $a_2$ are related to $C_1$ and $C_2$ by $a_1 = C_1 + \frac{1}{N_c} C_2$ and $a_2 = C_2 + \frac{1}{N_c} C_1$. The numerical values of $a_1$ and $a_2$ for $D$ meson decays are given by [15]

$$a_1 = 1.10 \pm 0.05, \quad a_2 = -0.49 \pm 0.04.$$

(20)

For the two body decay, in the rest frame of initial meson the differential decay rate is given by

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|P_1|}{M^2} d\Omega,$$

(21)
where $M$ is the initial meson mass, $m_1$ and $m_2$ the final meson masses, and $\mathbf{p}_1$ the momentum of one of the final mesons in the initial meson rest frame. By using (1), (19) and $<0|\Gamma_{\mu}|\pi^-(q)>= i q_{\mu} f_{\pi^-}$, (21) gives the following formula for the branching ratio of the process $D^0 \rightarrow K^-\pi^+$:

$$B(D^0 \rightarrow K^-\pi^+) = (\frac{G_F m_D^2}{\sqrt{2}})^2 |V_{ud}|^2 \frac{m_D}{8\pi} \frac{f_{\pi}^2}{m_D^2} |V_{cs} F_{0}^{DK}(m_{\pi}^2)|^2 \times \left(1 - \frac{m_K^2}{m_D^2}\right)^2 \times \left(1 - \frac{(m_K + m_{\pi})^2}{m_D^2}\right) \left(1 - \frac{(m_K - m_{\pi})^2}{m_D^2}\right)^{\frac{1}{2}}.$$  \(\text{(23)}\)

On the other hand, from (5) and (13) the branching ratio $B(B^0 \rightarrow K^-e^+\nu)$ is given by

$$B(B^0 \rightarrow K^-e^+\nu) = (\frac{G_F m_D^2}{\sqrt{2}})^2 \frac{m_D}{8\pi} \frac{2}{192\pi^3} |V_{cs} F_{1}^{DK}(0)|^2 \times I_{DK},$$  \(\text{(24)}\)

where the dimensionless integral $I_{DK}$ is given by

$$I_{DK} = \int_0^{(1 - \frac{m_K^2}{m_D^2})^2} dx \frac{x^2 - 4 \frac{m_K^2}{m_D^2} x}{\left(1 - \frac{m_D^2}{m_D^2} x\right)^{2n_{D1}}}.$$  \(\text{(25)}\)

In the above, we neglected the electron mass. From (23) and (24) we have

$$\frac{B(D^0 \rightarrow K^-\pi^+)}{B(D^0 \rightarrow K^-e^+\nu)} = 6\pi^2 |V_{ud}|^2 \frac{f_{\pi}^2}{m_{\pi}^2} \left(1 - \frac{m_K^2}{m_D^2}\right)^2 \times \left(1 - \frac{(m_K + m_{\pi})^2}{m_D^2}\right) \left(1 - \frac{(m_K - m_{\pi})^2}{m_D^2}\right)^{\frac{1}{2}} |V_{cs} F_{0}^{DK}(m_{\pi}^2)|^2 \frac{a_1^2}{I_{DK}},$$  \(\text{(26)}\)

where we used the fact $F_{0}^{DK}(m_{\pi}^2) \simeq F_{0}^{DK}(0)$ and the following experimentally values:

1. $m_D = m_{D^0} = 1.8646 \pm 0.0005$ GeV, $m_K = m_{K^-} = 493.677 \pm 0.013$ MeV, $m_{\pi} = m_{\pi^+} = 139.56995 \pm 0.00035$ MeV, $f_{\pi} = f_{\pi^+} = 131.74 \pm 0.15$ MeV and $V_{ud} = 0.9753 \pm 0.0008$. 

10
When we use the experimental results $\mathcal{B}(D^0 \to K^-\pi^+) = 3.85 \pm 0.09 \%$ and $\mathcal{B}(D^0 \to K^-e^+\nu) = 3.66 \pm 0.18 \%$, (26) gives

\[
I_{DK}^{\text{Expt.}} = 0.213 \ (1 \pm 0.054) \ a_1^2 = 0.258 \ (1 \pm 0.054) \ (1 \pm 0.091)
\]

\[
= 0.258 \ (1 \pm 0.106) = 0.231 \sim 0.286,
\]

(27)

where we used the value of $a_1$ given in (20). On the other hand, when we calculate $I_{DK}$ directly from (25) with $m_{F_1} = 2.11$ GeV given in Table 2, we obtain the following results:

\[
I_{DK}[n_{F_1} = 1] = 0.195 \quad \text{for } n_{F_1} = 1,
\]

\[
I_{DK}[n_{F_1} = 2] = 0.267 \quad \text{for } n_{F_1} = 2.
\]

(28)

From (27) and (28), we find that the experimental results of $\mathcal{B}(D^0 \to K^-\pi^+)$ and $\mathcal{B}(D^0 \to K^-e^+\nu)$ imply $n_{F_1} = 2$.

In the same way as the above, for the $D$ to $\pi$ transition we get the formula

\[
\frac{\mathcal{B}(D^0 \to \pi^-\pi^+)}{\mathcal{B}(D^0 \to \pi^-e^+\nu)} = 6\pi^2 |V_{ud}|^2 \frac{f_\pi^2}{m_D^2} \left(1 - \frac{m_\pi^2}{m_D^2}\right)^2
\]

\[
\times \left[1 - \left(\frac{m_\pi + m_\pi}{m_D}\right)^2\right]^2 \frac{|V_{ud} F_0^{DK}(m_\pi^2)|^2}{|V_{ud} F_0^{DK}(0)|^2} \frac{a_1^2}{I_{D\pi}}
\]

\[
= 0.275 \times \frac{a_1^2}{I_{D\pi}},
\]

(29)

where we used the fact $F_0^{DK}(m_\pi^2) \simeq F_0^{DK}(0)$, and the dimensionless integral $I_{D\pi}$ is given by

\[
I_{D\pi} = \int_0^{(1 - \frac{m_\pi}{m_D})^2} dx \left(1 + \frac{m_\pi^2}{m_D^2} - x\right)^2 - \frac{4 m_\pi^2}{m_D^2} x \right)^2 \left(1 - \frac{m_\pi^2}{m_D^2} x\right)^{2 n_{F_1}}.
\]

(30)

When we use the experimental results $\mathcal{B}(D^0 \to \pi^-\pi^+) = (1.53 \pm 0.09) \times 10^{-3}$ and $\mathcal{B}(D^0 \to \pi^-e^+\nu) = (3.7 \pm 0.6) \times 10^{-3}$, (29) gives

\[
I_{D\pi}^{\text{Expt.}} = 0.665 \ (1 \pm 0.173) \ a_1^2 = 0.804 \ (1 \pm 0.173) \ (1 \pm 0.091)
\]

\[
= 0.804 \ (1 \pm 0.195) = 0.648 \sim 0.961,
\]

(31)
where we used again the value of $a_1$ given in (20). When we calculate $I^{D\pi}$ directly from (25) with $m_{F_1} = 2.01$ GeV given in Table 2, we obtain the following results:

\[
I^{D\pi}[n_{F_1} = 1] = 0.385 \quad \text{for } n_{F_1} = 1,
\]
\[
I^{D\pi}[n_{F_1} = 2] = 0.783 \quad \text{for } n_{F_1} = 2.
\]

The results in (32) show that the value of $I^{D\pi}$ is more sensitive to $n_{F_1}$ than that of $I^{DK}$. From (31) and (32), we find that the experimental results of $B(D^0 \to \pi^- \pi^+)$ and $B(D^0 \to \pi^- e^+ \nu)$ imply $n_{F_1} = 2$.

6. Conclusion

We studied the $D$ meson exclusive semileptonic decays with the lepton mass effects into consideration, and investigated their sensitivity to form factor models. The results show that the decays to a pseudoscalar meson and a lepton pair are sensitive to the property of the form factor $F_1(q^2)$, and those to a vector meson and a lepton pair are sensitive to the form factor $A_1(q^2)$ and not to the other ones ($A_0(q^2)$, $A_2(q^2)$ and $V(q^2)$). Concerned with the experimental extraction of the form factor values at $q^2 = 0$, $F_1^{DK}(0) = 0.75 \pm 0.02 \pm 0.02$ has been obtained from the experimental result $B(D^0 \to K^- e^+ \nu) = (3.68 \pm 0.21) \times 10^{-2}$ by assuming $n_{F_1} = 1$ \cite{13}, however, its mean value is modified to 0.64 if $n_{F_1} = 2$ is adopted. In reality, the value of $n_{F_1}$ is not known presently. The experimental extraction of the reliable value of $F_1^{DK}(0)$ is very important for the determinations of the fundamental parameters, and also for the reason that the experimentally extracted value of $F_1^{DK}(0)$ is compared with its values obtained by different theoretical calculations such as quark models, lattice QCD, and QCD sum rules \cite{13, 14}. We emphasized that this extraction is very much dependent on the value of $n_{F_1}$. In this context, we also analyzed the experimental results of the branching fractions $B(D^0 \to K^-(or \, \pi^-) \, \pi^+)$ and $B(D^0 \to K^-(or \, \pi^-) \, e^+ \nu)$, and showed that it is implied that the form factor $F_1(q^2)$ of the $D$ to $K$ and that of the $D$ to $\pi$ tran-
sitions are of dipole type \((n_F = 2)\), instead of simple pole type \((n_F = 1)\) which is commonly assumed in the studies of the \(D\) meson decays. We think that careful studies on the form factors are important not only for the extractions of the CKM matrix elements but also for the understanding of the internal structures of hadrons.

Acknowledgements
This work was supported by Non-Directed-Research-Fund, Korea Research Foundation 1997, by the Basic Science Research Institute Program, Ministry of Education, Project No. BSRI-97-2414, by the Korea Science and Engineering Foundation, Grant 985-0200-002-2, and by the Research Fund of Kangnung National University 1997.
References

[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[2] J.G. Körner and G.A. Schuler, Phys. Lett. B 231, 306 (1989); Z. Phys. C 46, 93 (1990).

[3] A. Khodjamirian, R. Rückl and C.W. Winhart, Phys. Rev. D 58, 054013 (1998).

[4] D.S. Hwang and D.-W. Kim, hep-ph/9806242 (1998).

[5] F.J. Gilman and R.L. Singleton, Jr., Phys. Rev. D D41, 142 (1990).

[6] UKQCD Collaboration, K.C. Bowler et al., Phys. Rev. D D51, 4905 (1995).

[7] N. Isgur and M.B. Wise, Phys. Lett. B 232, 113 (1989); 237, 527 (1990).

[8] M. Wirbel, B. Stech and M. Bauer, Z. Phys. C 29, 637 (1985); 34, 103 (1987); M. Bauer and M. Wirbel, Z. Phys. C 42, 671 (1989).

[9] D.S. Hwang and B.-H. Lee, hep-ph/9801421 (1998), to appear in Eur. Phys. J. C.

[10] UKQCD Collaboration, D.R. Burford et al., Nucl. Phys. B 447, 425 (1995); L.D. Debbio, Phys. Lett. B 416, 392 (1998).

[11] R.J. Morrison and J.D. Richman, Note on semileptonic decays of D and B mesons, Part I (in Particle Data Group 1994), Phys. Rev. D50, 1565 (1994).

[12] A. Ryd, Review of Form Factor Measurements, in Seventh International Symposium on heavy Flavor Physics, Santa Barbara, July, 1997.

[13] E687 Collaboration, P.L. Frabetti et al., Phys. Lett. B 364, 127 (1995).
[14] E687 Collaboration, P.L. Frabetti et al., Phys. Lett. B 382, 312 (1996).

[15] M. Neubert and B. Stech, hep-ph/9705292, to appear in second edition of *Heavy Flavours*, ed. by A.J. Buras and M. Lindner, World Scientific, Singapore.

[16] Review of Particle Physics 1998, Eur. Phys. J. C3, 1 (1998).
Table 1: The values of the power of pole for the three models used in this paper.

| Model                  | $n_{F_1}$ | $n_{F_0}$ | $n_V$ | $n_{A_1}$ | $n_{A_2}$ | $n_{A_0}$ |
|------------------------|-----------|-----------|-------|-----------|-----------|-----------|
| WSB(pole/pole)         | 1         | 1         | 1     | 1         | 1         | 1         |
| Model I(pole/const.)   | 1         | 0         | 1     | 0         | 1         | 1         |
| Model II(dipole/pole)  | 2         | 1         | 2     | 1         | 2         | 2         |

Table 2: The values of pole masses (GeV) used in numerical calculations.

| Current Relevant Form Factors | $m(0^-)$ | $m(1^-)$ | $m(0^+)$ | $m(1^+)$ |
|------------------------------|----------|----------|----------|----------|
| $\bar{s}c$                  | 1.97     | 2.11     | 2.60     | 2.53     |
| $\bar{d}c$                  | 1.87     | 2.01     | 2.47     | 2.42     |

Table 3: The values of the form factors at $q^2 = 0$ used in numerical calculations.

| Current Form Factors | $F_1(0) = F_0(0)$ | $V(0)$ | $A_1(0)$ | $A_2(0)$ | $A_0(0)$ |
|---------------------|--------------------|--------|----------|----------|----------|
| $D \to K^{(*)}$     | 0.762              | 1.226  | 0.880    | 1.147    | 0.733    |
| $D \to \pi(\rho)$   | 0.692              | 1.225  | 0.775    | 0.923    | 0.669    |
Table 4: The obtained branching fractions and their ratios for $D^0 \to K^- l^+ \nu$.

|      | $\mathcal{B}(D^0 \to K^- e^+ \nu)$ | $\mathcal{B}(D^0 \to K^- \mu^+ \nu)$ | $\mathcal{B}(e^+) : \mathcal{B}(\mu^+)$ |
|------|-----------------------------------|-----------------------------------|----------------------------------|
| WSB  | $3.49 \times 10^{-2}$              | $3.41 \times 10^{-2}$             | $1 : 0.98$                       |
| Model I | $3.49 \times 10^{-2}$              | $3.38 \times 10^{-2}$             | $1 : 0.97$                       |
| Model II | $4.78 \times 10^{-2}$              | $4.67 \times 10^{-2}$             | $1 : 0.98$                       |

Table 5: The obtained branching fractions and their ratios for $D^0 \to K^{*-} l^+ \nu$.

|      | $\mathcal{B}(D^0 \to K^{*-} e^+ \nu)$ | $\mathcal{B}(D^0 \to K^{*-} \mu^+ \nu)$ | $\mathcal{B}(e^+) : \mathcal{B}(\mu^+)$ |
|------|-----------------------------------|-----------------------------------|----------------------------------|
| WSB  | $3.88 \times 10^{-2}$              | $3.68 \times 10^{-2}$             | $1 : 0.95$                       |
| Model I | $3.28 \times 10^{-2}$              | $3.10 \times 10^{-2}$             | $1 : 0.95$                       |
| Model II | $3.85 \times 10^{-2}$              | $3.66 \times 10^{-2}$             | $1 : 0.95$                       |

Table 6: The obtained branching fractions and their ratios for $D^0 \to \pi^{-} l^+ \nu$.

|      | $\mathcal{B}(D^0 \to \pi^{-} e^+ \nu)$ | $\mathcal{B}(D^0 \to \pi^{-} \mu^+ \nu)$ | $\mathcal{B}(e^+) : \mathcal{B}(\mu^+)$ |
|------|-----------------------------------|-----------------------------------|----------------------------------|
| WSB  | $2.92 \times 10^{-3}$              | $2.88 \times 10^{-3}$             | $1 : 0.99$                       |
| Model I | $2.92 \times 10^{-3}$              | $2.86 \times 10^{-3}$             | $1 : 0.99$                       |
| Model II | $5.94 \times 10^{-3}$              | $5.86 \times 10^{-3}$             | $1 : 0.99$                       |

Table 7: The obtained branching fractions and their ratios for $D^0 \to \rho^{-} l^+ \nu$.

|      | $\mathcal{B}(D^0 \to \rho^{-} e^+ \nu)$ | $\mathcal{B}(D^0 \to \rho^{-} \mu^+ \nu)$ | $\mathcal{B}(e^+) : \mathcal{B}(\mu^+)$ |
|------|-----------------------------------|-----------------------------------|----------------------------------|
| WSB  | $2.77 \times 10^{-3}$              | $2.66 \times 10^{-3}$             | $1 : 0.96$                       |
| Model I | $2.19 \times 10^{-3}$              | $2.10 \times 10^{-3}$             | $1 : 0.96$                       |
| Model II | $2.77 \times 10^{-3}$              | $2.66 \times 10^{-3}$             | $1 : 0.96$                       |
Figure Captions

Fig. 1. $\left(1/\Gamma_{\text{tot}}\right)(d\Gamma/dq^2)$ of (a) $D^0 \to K^- e^+ \nu$ and (b) $D^0 \to K^- \mu^+ \nu$ for three models: solid line for WSB, dashed for Model I, and dotted for Model II. Solid and dashed lines almost overlap.

Fig. 2. $\left(1/\Gamma_{\text{tot}}\right)(d\Gamma/dq^2)$ of $D^0 \to K^*^- e^+ \nu$ and $D^0 \to K^*^- \mu^+ \nu$ for three models: solid line for WSB, dashed for Model I, and dotted for Model II. Solid and dotted lines almost overlap.

Fig. 3. $\left(1/\Gamma_{\text{tot}}\right)(d\Gamma/dq^2)$ of (a) $D^0 \to \pi^- e^+ \nu$ and (b) $D^0 \to \pi^- \mu^+ \nu$ for three models: solid line for WSB, dashed for Model I, and dotted for Model II. Solid and dashed lines almost overlap.

Fig. 4. $\left(1/\Gamma_{\text{tot}}\right)(d\Gamma/dq^2)$ of $D^0 \to \rho^- e^+ \nu$ and $D^0 \to \rho^- \mu^+ \nu$ for three models: solid line for WSB, dashed for Model I, and dotted for Model II. Solid and dotted lines almost overlap.
\[ \frac{1}{N} \frac{d\sigma}{dq^2} \]

**Fig. 4(a)**

\[ D \rightarrow \rho e \nu \]

**Fig. 4(b)**

\[ D \rightarrow \rho \mu \nu \]