Performance Analysis of BDS-3 FCB Estimated by Reference Station Networks over a Long Time

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Abstract: The stability and validity of the BDS-3 precise point positioning ambiguity solution (PPP-AR) is becoming more and more important along with the development of BDS-3 orbit and clock products over long durations. Satellite phase fractional cycle biases (FCBs) are key in PPP-AR, so it is important to ensure the validity and stability of FCBs over a long duration. In this study, we analyzed the validity and stability of BDS-3 phase FCBs by estimating them. The BDS-3 FCB experiments showed that BDS-3 FCBs have the same stability as GPS/GAL/BDS-2. BDS-3 widelane (WL) FCBs also have stable characteristics and the maximal fluctuation value of WL FCBs was found to be 0.2 cycles in a month. BDS-3 narrowlane (NL) FCBs were found to be unstable and the maximal fluctuation value of NL FCBs was more than 0.25 cycles over one day. Analyzing the posteriori residual errors of BDS-3 WL and NL ambiguities showed that the BDS-3 FCBs had the same accuracy as GPS/GAL/BDS-2. However, the ambiguity-fixed rate of BDS-3 was about 70%, which was less than GPS/GAL/BDS-2 in PPP-AR experiments. For this reason, we analyzed the quality of data and the accuracy of orbit and clock products by using different analysis center products. The results showed that the low accuracy of the BDS-3 orbit and clock products was the main reason for the low-ambiguity fixed rate.

Keywords: BDS-3; PPP-AR; FCB; stability; validity

MSC: 86-05

1. Introduction

The International GNSS Service (IGS) is an international organization established by the International Association of Geodesy to support geodetic and geodynamic research. In the mid-1990s, the IGS began to provide precise orbit and clock products for users around the world, which enabled the realization of PPP based on an undifferenced model. The authors of [1] realized centimeter-level static PPP with an ionosphere-free (IF) combined model, which used precise ephemeris and clock products provided by the IGS. It was verified through experiments that it is completely theoretically feasible to use undifferenced observations for PPP. The authors of [2] also obtained centimeter-level positioning accuracy by using a dual-frequency IF-combined model in PPP. The authors of [3] analyzed PPP models and discussed final and real-time PPP models based on networks. The authors of [4] proposed a University of Calgary (UofC) model, which significantly decreased the problem of large observation noise in an IF model and shortened the convergence time of PPP.

With the advantages of more redundant observations and better satellite spatial geometry, multi-GNSS combinations can shorten the convergence time and improve the positioning accuracy of PPP. Some research has shown that multi-GNSS combinations can effectively speed up the convergence in PPP by adding satellites of GLONASS; the
effect is obvious in improving the accuracy of static positioning, but the effect is not obvious in improving the accuracy of dynamic positioning [5–9]. In 2011, the IGS started the Multi-GNSS Experiment (MGEX) program with the purpose of observing, collecting, and analyzing Multi-GNSS data in order to provide precise products and services [10]. With the completion of BDS-3 in 2020, there are now four global satellite navigation systems (GPS, GLONASS, BDS, and Galileo) and two regional satellite navigation systems (QZSS and IRNSS). Multi-GNSS PPP-AR now embodies the development trend of precise positioning [11]. As an important part of GNSS, it is necessary to ensure the stability and validity of BDS-3 PPP-AR in the long term.

Traditional PPP technology still has the problems of long convergence time and low accuracy and also needs 15–20 min to obtain a convergence solution; the accuracy of solution is about 20 cm. A PPP-fixed solution can significantly improve positioning accuracy and shorten convergence time. The key to PPP-AR is to separate FCB phases between receivers and satellites and then recover the integer characteristics of ambiguity. The authors of [12] decomposed an IF-combined ambiguity into WL and NL ambiguities with integer characteristics and then they used the GPHASE function to separate FCBs from the ambiguities and tried to solve the WL and NL single-difference FCBs between satellites. However, due to the limitation of the precise ephemeris and clock products at that time, only the WL FCBs could be effectively separated. Thanks to the improved accuracy of precise IGS products, the authors of [13] used a single-difference model to estimate the WL and NL FCBs and they successfully fixed the single-difference ambiguities. The authors of [14] used the least squares method to estimate the undifferenced FCBs between receivers and satellites by constructing observation equations of the undifferenced FCBs. The authors of [15] proposed the use of an integer phase clock (IPC), i.e., an NL FCB is sucked into the satellite clock error to form the IPC, which users can use to fix ambiguities. The authors of [16] proposed the use of the decoupled clock (DC) method on the basis of an IPC to estimate the satellite pseudorange and phase clock errors at the same time. The authors of [17,18] proved the equivalence of the FCB, IPC, and DC through experiments. However, the IPC and DC methods limit users’ choice of clock error products. The FCB method has the advantages of simplicity, with easy algorithm programming, mature theory, and ease of use. This method can adapt to the orbit and clock error products of different analysis centers for users. Therefore, we used the FCB method to estimate the BDS-3 FCBs in this study.

Here, the FCB method was used to estimate BDS-3 FCBs and the stability and validity of BDS-3 FCBs were analyzed. Due to inter-system biases (ISB) between BDS-2 and BDS-3, we think these two systems are different. Therefore, we estimate BDS-2 and BDS-3 FCB, respectively. We also calculate GPS and GAL FCB for comparison. In the second section of this paper, we analyze the PPP model and the error sources of BDS-3. In the third section, we introduce the estimation methods of BDS-3 WL and NL FCBs. In the fourth section, we present an analysis of the stability of the BDS-3 WL FCBs over 30 days and an evaluation of the accuracy of the WL FCBs. Then, we present an estimation of the single-day BDS-3 NL FCBs and an accuracy analysis. Finally, we conducted PPP-AR experiments by using the estimated FCBs, which proved the validity of the estimated FCBs. In the fifth section, we analyze the problem of the low-ambiguity fixed rate of BDS-3.

2. Method

2.1. BDS-3 PPP Model

The BDS-3 multi-frequency pseudorange and carrier phase observation equation can be expressed as:

\[ P_{r,i}^s = \rho_r^s + dt_r - dt_i^s + \gamma_i I_{r,1}^s + T_r^s + b_{r,i} - b_i^s + \epsilon_{r,i} \]  

\[ L_{r,i}^s = \rho_r^s + dt_r - dt_i^s - \gamma_i I_{r,1}^s + T_r^s + \lambda_i (N_{r,i}^s + \phi_{r,i} - \phi_i^s) + \delta_{r,i} \]  

(1)  

(2)
where $s$, $i$, and $r$ are the satellite system, frequency, and receiver, respectively; $P_{r,j}^s$ and $L_{r,j}^s$ are the pseudorange and carrier phase observation values (m), respectively; $\rho_r^s$ is the Euclidean distance between the satellite and receiver; $dt_r$ and $dt_i^s$ are receiver and satellite clock errors, respectively; $\gamma_i$ is the ionospheric mapping factor; $I_r^s$ is the ionospheric delay at the first frequency; $T_r^s$ is the tropospheric delay; $b_r,j$ and $b_i^s$ are the receiver and satellite pseudorange hardware delay, respectively; $N_r^s$, is the carrier phase integer ambiguity; $\varphi_r^s$ and $\varphi_i^s$ are the receiver and satellite phase delay, respectively; and $\epsilon_r^s$ and $\epsilon_i^s$ are the pseudorange and carrier observation noise errors, respectively. In the above equation, the antenna phase center correction, relativistic effect, tide loading correction (solid tide, extreme tide, and ocean tide), Sagnac effect, satellite antenna phase wind-up, and other corrections at the satellite and receiver are not included. These biases are corrected by the correction model in advance [19].

The dual-frequency IF combination commonly used in PPP eliminates the effect of first-order ionospheric delay. The pseudorange and carrier phase IF combination of frequencies $i$ and $j$ can be expressed as:

$$
\begin{align*}
    P_{r,IF}^s &= \alpha_{ij}P_{r,j}^s + \beta_{ij}P_{r,j}^s \\
    L_{r,IF}^s &= \alpha_{ij}L_{r,j}^s + \beta_{ij}L_{r,j}^s
\end{align*}
$$

where $\alpha_{ij}$ and $\beta_{ij}$ are the IF combination coefficients:

$$
\alpha_{ij} = \frac{f_i^2}{f_i^2 - f_j^2}, \quad \beta_{ij} = -\frac{f_j^2}{f_i^2 - f_j^2} \quad (4)
$$

Then, the IF observation model corresponding to Equations (3) and (4) can be written as:

$$
\begin{align*}
    P_{r,IF}^s &= \rho_r^s + dt_r - dt_i^s + T_r^s + b_r,IF - b_i^IF + \epsilon_r^IF \\
    L_{r,IF}^s &= \rho_r^s + dt_r - dt_i^s + T_r^s + \lambda_{IF}(N_r^s + \varphi_r,IF - \varphi_i^IF) + \delta_r^IF
\end{align*}
$$

Then,

$$
\begin{align*}
    b_r,IF &= \alpha_{ij}b_{r,j} + \beta_{ij}b_{r,j} \\
    b_i^IF &= \alpha_{ij}b_i^s + \beta_{ij}b_i^s \\
    N_r^s &= \left(\alpha_{ij}\lambda_iN_{r,j}^s + \beta_{ij}\lambda_iN_{r,j}^s\right)/\lambda_{IF} \\
    \varphi_r,IF &= \left(\alpha_{ij}\lambda_i\varphi_r,IF + \beta_{ij}\lambda_i\varphi_i^IF\right)/\lambda_{IF} \\
    \varphi_i^IF &= \left(\alpha_{ij}\lambda_i\varphi_i^IF + \beta_{ij}\lambda_i\varphi_i^IF\right)/\lambda_{IF}
\end{align*}
$$

where $b_r,IF$ and $b_i^IF$ comprise the IF combination of receiver and satellite pseudorange hardware delay, respectively; $N_r^s, IF$, $\varphi_r,IF$, and $\varphi_i^IF$ comprise the IF combination of carrier phase ambiguity, receiver, and satellite phase delay, respectively; $N_r,IF$ is the IF float ambiguity; and $\lambda_{IF}$ is the IF wavelength.

It can be seen from Equations (5) and (6) that the IF ambiguity does not have integer characteristics. In order to obtain a fixed solution by using the IF model in PPP, the IF float ambiguity is generally decomposed into a linear combination of WL and NL integer ambiguity ($i = 1, j = 2$):

$$
N_{r,IF}^s = \left(\frac{c f_2}{f_1^2 - f_2^2}N_{r,IF}^s + \frac{c}{f_1 + f_2}N_{r,IF}^NL\right)/\lambda_{IF}
$$

where $N_{r,IF}^s$ is the WL integer ambiguity. The WL float ambiguity is generally calculated with the Melbourne-Wubenna MW combination [20,21].
When the WL ambiguity is fixed, the NL ambiguity can be expressed as the combination of $R_s$ where

$$N_{r, WL} = \left[ \left( f_1 L_{r,1}^s - f_2 L_{r,2}^s \right) / (f_1 - f_2) - \left( f_1 P_{r,1}^s - f_2 P_{r,2}^s \right) / (f_1 + f_2) \right] / \lambda_{IF}$$

$$= N_{r, WL}^s + \varphi_{r, WL} - \varphi_{r, WL}^s$$

where $N_{r, WL}$ is the WL float ambiguity and $\lambda_{WL}$ is the WL wavelength. It can be seen from Equation (8) that the WL ambiguity $N_{r, WL}^s$ can be fixed by eliminating $\varphi_{r, WL}$ and $\varphi_{r, WL}^s$. When the WL ambiguity is fixed, the NL ambiguity can be expressed as the combination of IF and WL ambiguities:

$$N_{r, NL}^s = \frac{\lambda_{IF}(f_1 + f_2)N_{r, IF}}{c} - f_2(f_1 - f_2)N_{r, WL}^s$$

$$= N_{r, 1}^s + \varphi_{r, NL} - \varphi_{r, NL}^s$$

$$\varphi_{r, NL}^s = \frac{f_1 + f_2}{c(\varphi_{r, IF}^{\lambda_{IF}} - b_{r, 1})}$$

$$= f_1(f_1 - f_2)(\varphi_{r, 1} - b_{r, 1}/\lambda_1) - f_2(f_1 - f_2)(\varphi_{r, 2} - b_{r, 2}/\lambda_2)$$

$$\varphi_{r, NL} = \frac{f_1 + f_2}{c(\varphi_{r, IF}^{\lambda_{IF}} - b_{r, IF})}$$

$$= f_1(f_1 - f_2)(\varphi_{r, 1} - b_{r, 1}/\lambda_1) - f_2(f_1 - f_2)(\varphi_{r, 2} - b_{r, 2}/\lambda_2)$$

We can obtain the IF ambiguity by recombining the fixed WL ambiguity and the NL ambiguity. The IF ambiguity should have the same accuracy with integer ambiguity. Then, PPP-AR can be achieved by using the IF model. Therefore, the IF model is the key for successfully fixing the WL and NL ambiguities.

2.2. BDS-3 WL FCB Estimation Method

From Equation (8), we can see that the WL float ambiguity can be obtained via the MW combination, which can be written as:

$$N_{r, WL}^s = N_{r, WL}^s + \varphi_{r, WL} - \varphi_{r, WL}^s$$

where $\varphi_{r, WL}$ and $\varphi_{r, WL}^s$ are the receiver and satellite FCBs, respectively. Generally, in PPP-AR, the ambiguity is fixed by the single-difference between satellites and $\varphi_{r, WL}$ can be eliminated by a single-difference model. However, the authors of [14] used the least squares method to estimate the undifferenced FCBs between receivers and satellites by constructing observation equations of the undifferenced FCBs. The single-difference model of FCBs is the same with undifferenced model FCBs in theory. Therefore, we use the undifferenced method to estimate the satellite and receiver FCBs in this paper.

Accordingly, Equation (14) can be rewritten as:

$$R_{r}^s = N_{r, WL} - \text{int}[N_{r, WL}^s] = \varphi_{r, WL} - \varphi_{r, WL}^s$$

where $R_{r}^s$ is the fractional part of the WL float ambiguity and $\text{int}[\ ]$ is used to round up to the nearest integer (e.g., $\text{int}[-5.6]$ is $-6$). Assuming that $m$ satellites are observed by $n$ stations, the float ambiguities of the continuous arc of each station–satellite can be combined into the following equation:
Because of the linear correlation between the WL FCBs at the receiver and satellite in Equation (16), the equation system is rank deficient and the rank-deficient number is 1. There are three commonly used methods to solve the rank deficiency:

1. set a receiver FCB to 0;
2. set a satellite FCB to 0; or
3. use a satellite FCB center of gravity reference, i.e., set the sum of all satellite FCBs to 0. The three benchmarks are theoretically equivalent. In this study, we selected 3. Then:

\[ \varphi_1 + \varphi_2 + \cdots + \varphi_m = 0 \]  

After combining Equations (16) and (17), they can be written in the matrix form as:

\[
\begin{bmatrix}
R_0 \\
0
\end{bmatrix}
= \begin{bmatrix}
A_0 \\
A_1
\end{bmatrix} X
\]  

(18)

where \( A \) is the design matrix. Then, the receiver and satellite WL FCBs can be calculated via the least squares method:

\[ X = \left( A^T PA \right)^{-1} A^T PR \]  

(19)

In Equation (19), the weight matrix \( P \) can be determined by the variance in the float WL ambiguity in continuous arcs. In order to improve the robustness and accuracy of the solution, the IGG3 method can be used for iterative calculations [22]. The following points should be noted when calculating WL FCBs: the observation time of the WL float ambiguity is not less than 45 min in a continuous arc; the WL float ambiguity in the arc can be averaged to reduce the influence of observation noise and multipath; and for the ambiguity whose absolute residual error value is greater than 0.5 cycles, the corresponding integer ambiguity can be adjusted to perform a \( \pm 1 \) cycles operation.

2.3. BDS-3 NL FCB Estimation Method

From Equation (11), the NL float ambiguity can be expressed as:

\[ N_{r,NL} = N_{r,1} + \varphi_{r,NL} - \varphi_{NL} \]  

(20)

The estimation method of BDS-3 NL FCBs is similar to that of WL FCBs, as it can be estimated with Equations (16)–(19). Since the NL float ambiguity is the combination of the IF float ambiguity and the WL integer ambiguity, the variance in NL FCBs is:

\[ \sigma_{NL} = \frac{f_1 + f_2}{f_1} \sigma_{IF} \]  

(21)

where \( \sigma_{IF} \) is the variance in IF float ambiguity, which can be used to determine the weight of NL FCBs with Equation (21). In order to improve the robustness and accuracy of the
solution, the IGG3 method can be used for iterative calculations. The following points should be noted when calculating NL FCBs.

Due to the short wavelength of the NL ambiguity, NL FCBs are susceptible to other errors. The stability of NL FCBs can be improved by providing the initial value. The method that can be used to estimate the initial value of the NL FCB is described below: First, a station with the largest number of satellite observations is selected as the reference station and the receiver FCB of the station is set to 0. Then, the satellite FCBs of this station can be obtained. For the next station with common-view satellites, the ambiguities of common-view satellites can be corrected by using the FCB of a common-view satellite and these ambiguities should have a close fractional part. The receiver FCBs can be obtained by averaging common-view ambiguities. The uncommon-view satellite FCBs can be obtained by correcting the receiver FCBs in this station. Finally, this method can be used to traverse all stations to obtain all satellite FCBs.

NL FCBs are unstable over a single day, so we estimated them every 15 min. This method can be used to improve the accuracy of IF float ambiguity with fixed station coordinates in PPP and it can also be used to improve the accuracy of the NL float ambiguity.

The BDS-3 WL and NL FCBs can be successfully estimated by using the abovementioned method. A specific flow chart of the estimation method is shown in Figure 1.

![Flow chart of BDS-3 FCB estimation](image)

**Figure 1.** Flow chart of BDS-3 FCB estimation.

3. BDS-3 Experiment Analysis

In order to analyze the stability and validity of BDS-3 FCBs, we adopted the observation data of the MGEX station in 2021 to estimate BDS-3 WL and NL FCBs. We also estimated GPS/ GAL/BDS-2 WL and NL FCBs for comparison. The distribution of MGEX sites is shown in the Figure 2.

![Distribution of MGEX sites](image)

**Figure 2.** Distribution of MGEX sites.
Since the IF float ambiguity is required when NL FCBs are estimated, it was necessary to perform PPP calculations for each MGEX station. The data processing model used for PPP in this paper is shown in the Table 1.

Table 1. PPP data processing model.

| Processing Type                          | Correction Model                                      |
|-----------------------------------------|-------------------------------------------------------|
| Satellite orbit error                   | Precise ephemeris products (COD)                      |
| Satellite orbit error                   | Precise ephemeris products (COD)                      |
| Error caused by the rotation of the Earth| Erp products (COD)                                    |
| DCB                                     | DCB product (COD)                                     |
| Tropospheric delay                      | Saastamoinen + GPT2w + Estimate [23]                  |
| Ionospheric delay                       | IF model                                              |
| PCO/PCV                                 | IGS14 atx                                             |
| PCO/PCV                                 | IGS14 atx                                             |
| Receiver clock error                    | estimate                                              |
| Phase wind-up                           | Model correction [24]                                 |
| Solid tide, extreme tide and ocean tide | Model correction [24]                                 |
| Elevation mask angle                    | 7                                                     |
| Stochastic model                        | Elevation model                                       |
| Parameter estimation method             | Kalman filter (constrained station coordinates) [25]  |

3.1. WL FCB Evaluation

As WL FCBs are relatively stable, only one value per day can be estimated. In this paper, we used the MW combination to calculate GPS, GAL, and BDS-2 and BDS-3 WL FCBs. The 30-day WL FCBs with the Doy of 182–213 are shown in the following figures.

It can be seen from Figures 3–6 that the WL FCBs of all systems maintained stable characteristics over 30 days. Most satellites had a fluctuation range of no more than 0.1 cycles in a single day. Only G03 had a fluctuation of about 0.12 cycles at day 191 and C09 had a fluctuation of about 0.2 cycles at day 198, which may have been caused by the restart of the satellite. Since G11 and G28 are marked as unhealthy satellites in the GPS navigation ephemeris, the WL FCBs were not estimated. The GEO orbit accuracy of BDS-2 and BDS-3 was bad and the WL FCBs were not estimated.

![Figure 3. GPS WL FCB (reference satellite is G01).](image)

The residual error distribution of FCBs is another standard for testing the quality of FCB products. In order to analyze the estimation accuracy of the WL FCBs, the posteriori residual errors of the WL float ambiguities are shown in the following figures (using doy 185 as an example).
The RMS values of GPS, GAL, BDS-2, and BDS-3 were 0.083 cycles, 0.061 cycles, 0.101 cycles, and 0.098 cycles, respectively. The accuracy of WL FCBs was high, mainly because the WL ambiguity had a long wavelength and was not susceptible to various residual errors and observation noises. By analyzing the posteriori residual error figures of Figures 7 and 8, it can be seen that the percentages of GPS, GAL, and BDS-2 and BDS-3 WL float ambiguity posteriori residual errors less than 0.15 cycles were: 93.12%, 97.38%, 89.15%, and 91.73%, respectively. The percentages of less than 0.25 cycles were: 98.51%, 99.31%, 95.64%, and 96.27%, respectively. In addition, the residual distribution of each system was even and symmetrical and the mean of residual errors was close to 0, which further verified the precision of the Multi-GNSS WL FCBs.
3.2. NL FCB Evaluation

It can be seen that we could obtain the NL float ambiguity by using Equation (11) when we fixed the WL ambiguity by using estimated WL FCBs. The IF float ambiguity needed to be obtained by PPP before calculating the NL float ambiguity. In order to improve the accuracy and stability of the IF float ambiguity, we used tight constraints in PPP by using the station coordinates.

Similarly, we used the data described in Section 3.1 to obtain the PPP solution and we fixed the BDS-3 WL ambiguity by WL FCBs that was calculated in Section 3.1. Then, we calculated the BDS-3 NL float ambiguity. We estimated NL FCBs every 15 min, because the NL FCBs are unstable over one day. The BDS-3 NL FCBs for doy 185 in 2021 are shown in the following figures.

It can be seen from Figures 9–12 that the stability of NL FCBs was poorer than that of WL FCBs. Among the studied FCBs, G12 NL FCBs had the largest single-day change of 0.20 cycles in GPS, E02 NL FCBs had the largest single-day change of 0.15 cycles in GAL, C12 NL FCBs had the largest single-day change of 0.30 cycles in BDS-2, and C44 NL FCBs had the largest single-day change of 0.38 cycles in BDS-3. Due to the short wavelength of the NL ambiguity, it was susceptible to other errors. BDS-3 has just been built and its error models (such as light pressure models) are imperfect, which significantly affects the stability of BDS-3 NL FCBs.

Assessing the residual error distribution of FCBs is another standard for testing the quality of FCB products. In order to analyze the estimation accuracy of the BSD-3 NL FCBs, the posteriori residual errors of the NL float ambiguities are shown in the following figures (using doy 185 as an example).
Figure 9. GPS NL FCB (reference satellite is G01).

Figure 10. GAL NL FCB (reference satellite is E01).

Figure 11. BDS-2 NL FCB (reference satellite is C08).

Figure 12. BDS-3 NL FCB (reference satellite is C19).

The RMS statistics of GPS, GAL, BDS-2, and BDS-3 were 0.088 cycles, 0.073 cycles, 0.087 cycles, and 0.097 cycles, respectively. The accuracy of NL FCBs was high, mainly because the IF ambiguity was relatively stable after fixing the coordinates in PPP. By analyzing the posteriori residuals of Figures 13 and 14, it can be seen that the percentages...
of GPS, GAL, BDS-2, and BDS-3 NL float ambiguity posteriori residual errors of less than 0.15 cycles were: 91.73%, 94.93%, 91.80%, and 89.73%, respectively. The percentages of less than 0.25 cycles were: 97.81%, 98.90%, 98.10%, and 96.67%, respectively. In addition, the residual distribution of each system was even and symmetrical and the mean residual error was close to 0, which further verified the precision of the Multi-GNSS NL FCBs.

![Figure 13. Distribution of NL float ambiguity posteriori residual errors (2021, Doy 185) as: (a) GPS NL float ambiguity posteriori residual errors; (b) GAL NL float ambiguity posteriori residual errors.](image)

![Figure 14. Distribution of NL float ambiguity posteriori residual errors (2021, Doy 185) as: (a) BDS-2 NL float ambiguity posteriori residual errors; (b) BDS-3 NL float ambiguity posteriori residual errors.](image)

By comparing the residual parts of each system, it was found that the accuracy of BDS-3 NL FCBs was low. The reasons are as follows: (1) the stations are unevenly distributed, so BDS-3 satellites could be observed; (2) the BDS-3 error-correction models (such as light pressure models) are not perfect; and (3) the accuracy of BDS-3 orbit and clock products still needs to be improved. We will discuss this problem in the fourth section.

### 3.3. PPP-AR Experiment

1. **PPP-AR experiment over one day**

   In order to verify the correctness of the Multi-GNSS WL and NL FCBs estimated in Sections 3.1 and 3.2, we used the MGEX station that did not participate in the FCB estimation for doy 185 in 2021 to conduct a static PPP experiment. To ensure that BDS-2 could perform PPP-AR alone, the reference stations selected in this section are in the Asia-Pacific region. Taking the coordinates in the SNX products as the reference coordinates, the positioning bias of each PPP-AR system is shown in the following figures.

   It can be seen from Figures 15–17 that FCBs effectively improved the convergence speed and positioning accuracy of PPP, which was most obvious in the E direction. Table 2 shows statistics of the mean time to first fix (TTFF), the WL mean ambiguity-fixed rate, the NL mean ambiguity-fixed rate, and the ENU mean direction convergence accuracy of three stations with GPS, GAL, BDS-2, and BDS-3 systems in Table 2.
3.3. PPP-AR Experiment

The performance of PPP-AR systems is also affected by the number of satellites and the data processing approach. The BDS-3 system is still not fully operational, and the satellite constellation density is relatively low. Therefore, the convergence accuracy of BDS-3 is not as good as GPS and GAL. However, the BDS-3 system provides more resilient and stable navigation signals and better service availability, which are significant advantages in practical applications. The BDS-3 system is expected to further improve its performance in the future.


dd

| GPS       | GAL       | BDS-2     | BDS-3     |
|-----------|-----------|-----------|-----------|
| TTFF      | 23.5 min  | 34 min    | 21.5 min  | 26 min    |
| WL ambiguity fixed rate | 93.63%    | 97.75%    | 97.73%    | 98.19%    |
| NL ambiguity fixed rate | 90.12%    | 95.37%    | 81.65%    | 75.81%    |
| E direction convergence accuracy (cm) | 0.02      | 0.06      | 0.23      | 0.28      |
| N direction convergence accuracy (cm) | 0.63      | 0.83      | 0.75      | 0.59      |
| U direction convergence accuracy (cm) | 1.21      | 1.19      | 1.32      | 1.78      |
| point accuracy (cm) | 1.36      | 1.45      | 1.53      | 1.89      |
Table 2 shows that the mean TTFF of BDS-2 was the shortest and that the NL ambiguity was fixed at 21.5 min, because these stations are located in the Asia–Pacific region and BDS-2 has a large number of visible satellites. The WL mean ambiguity-fixed rates of three stations of GPS, GAL, BDS-2, and BDS-3 were above 95%. The GPS WL mean ambiguity-fixed rate was the lowest because G11 and G28 were marked as unavailable and the number of observable satellites was reduced, which reduced the WL ambiguity-fixed rate. The BDS-3 NL mean ambiguity-fixed rate was the lowest, mainly due to the low accuracy of orbit and clock products and its imperfect error-correction model. Based on the ENU mean convergence accuracy, it is obvious that all systems had the highest convergence accuracy in the E direction, followed by the N direction, and the U direction is the worst. The GPS mean point accuracy was the highest and the BDS-3 mean point accuracy was the lowest, which was directly related to the accuracy of the orbit and clock products.

(2) PPP-AR experiment over 30 days

In order to ensure the long-time validity of GPS, GAL, BDS-2 WL, BDS-3 WL, and NL FCBs, we used the observation data of Asia–Pacific MGEX stations (CUSV, JOZ2 and PTGG) in 2021 (Doy 182–211) to perform the PPP-AR experiment. The WL and NL ambiguity-fixed rates are shown in the following figures.

From Figures 18 and 19, we can see that GPS, GAL, BDS-2, and BDS-3 had relatively stable WL and NL ambiguity-fixed rates over 30 days, fully demonstrating the stability and validity of WL and NL FCBs. Among them, because the G11 and G28 satellites were continuously marked as unavailable in the ephemeris, the GPS WL ambiguity-fixed rate of three stations was about 95% and the NL ambiguity-fixed rate was about 90%. The GAL WL and NL ambiguity-fixed rates were the highest, the WL ambiguity-fixed rate was about 97%, and the NL ambiguity-fixed rate was about 92%. The BDS-2 WL ambiguity-fixed rate was about 95% and the NL ambiguity-fixed rate was about 80%. The BDS-3 WL ambiguity-fixed rate was high, at about 96%, and the NL ambiguity-fixed rate was more than 75%.

Figure 18. WL ambiguity-fixed rate of each system over doy 182–211 of MGEX stations in 2021.

Due to the long wavelength of WL ambiguity, we can see from the figures that GPS, GAL, BDS-2, and BDS-3 had high ambiguity-fixed rates. The GPS, GAL, BDS-2, and BDS-3 30-day average fixed rates were 96.41%, 98.21%, 95.83%, and 97.93%, respectively. The NL ambiguity had the shortest wavelength and was susceptible to other errors. The NL ambiguity-fixed rate was lower than that of WL. The GPS, GAL, BDS-2, and BDS-3 30-day average fixed rates were 92.28%, 93.92%, 82.15%, and 77.41%, respectively. It can be seen that BDS-3 had the lowest ambiguity fixation rate due to orbital clock errors and unmodeled errors and there is still room for development. We will discuss this problem in the next section.
number of observable satellites was reduced, which reduced the WL ambiguity fixed rate of each system over doy 182–211 of MGEX stations in 2021.

4. Discussion

In view of the low fixed rate of the BDS-3 ambiguity in the above-described experiment, we analyzed the BDS-3 signal-to-noise ratio (SNR), orbit error accuracy, and clock error accuracy over 185 days and then we compared them with those of GPS. The results are shown in the following figures.

It can be seen from Figure 20 that the SNR of B1I/B3I was close to L1 and slightly higher than L2, indicating that the overall signal strength of BDS-3 was higher than GPS. This means that the BDS-3 observation had a lower noise and a higher accuracy. This also shows that the observation accuracy was not the reason for the low ambiguity-fixed rate of BDS-3. Accordingly, we calculated the orbit and clock error difference RMS of GPS and BDS-3 by using different IGS analysis center precise products.

At present, some IGS analysis centers (AC) can provide BDS-3 precise products, i.e., CODE, GFZ, SHA, WHU, and IAC. IAC is the Information and Analysis Center; therefore, we compare other Ac products with IAC. As such, we calculated orbit and clock error difference RMS values between different Ac and IAC.

Figures 21 and 22 show that the GPS orbit difference RMS values of CODE-IAC, GFZ-IAC, WHU-IAC, and SHA-IAC were about 1.64 cm, 1.73 cm, 5.15 cm, and 5.93 cm, respectively; the clock error difference RMS of CODE-IAC, GFZ-IAC, WHU-IAC, and SHA-IAC was about 0.11 m, 0.09 m, 0.10 m, and 0.15 m. The orbit and clock error difference RMS between the different ACs was relatively small. This shows that the GPS orbit and clock error estimation accuracy were high and there was little difference between different ACs. The BDS-3 orbit difference RMS values of CODE-IAC, GFZ-IAC, WHU-IAC, and SHA-IAC were about 4.05 cm, 3.59 cm, 10.85 cm, and 18.18 cm, respectively. The orbit difference RMS values of three Inclined GeoSynchronous Orbit (IGSO) satellites (C38, C39,
and C40) were higher at about 5~40 cm between different ACs, respectively. The BDS-3 clock error difference RMS of CODE-IAC, GFZ-IAC, WHU-IAC, and SHA-IAC was about 0.97 m, 0.55 m, 0.50 m, and 0.50 m. The BDS-3 orbit and clock error difference RMS values between the different ACs were relatively large, indicating that the consistency of the BDS-3 orbit and clock error products was poor and that there were significant differences between different ACs, especially the IGSO satellite and Medium orbit earth (MEO) satellite (C45 and C46).

In conclusion, we analyzed the reason for the low ambiguity-fixed rate of BDS-3. The results showed that the consistency of BDS-3 orbit and clock products was low, especially BDS-3 IGSO satellites. The BDS-3 orbit and clock error difference RMS values showed that the consistency of BDS-3 orbit and clock products was poor and that there were significant differences between different ACs, especially the IGSO satellite and Medium orbit earth (MEO) satellite (C45 and C46).

Figure 21. BDS-3 and GPS orbit difference RMS.

Figure 22. BDS-3 and GPS clock error difference RMS.

The experiment showed that the observation accuracy of BDS-3 was equivalent to GPS but the consistency of the orbit and clock error of BDS-3 was relatively poor, which is also the main reason for the low ambiguity-fixed rate of BDS-3. The BDS-3 IGSO satellite had the worst orbit and clock error accuracy.

5. Conclusions

BDS-3 WL FCBS were found to be stable over 30 days, with a change of no more than 0.1 cycles. The estimated FCB accuracy was high, with 90% residual errors of less than 0.15 cycles and 95% residual errors of less than 0.25 cycles. The BDS-3 NL FCBS were unstable over one day, thus, confirming the unstable characteristics of NL FCBS. The fluctuation in BDS-3 NL FCBS was shown to be about 0.32 cycles.

The PPP-AR BDS-3 experiments showed that FCB products could effectively improve the convergence time and accuracy of PPP, especially in the E direction. Of the studied products, GPS had the best positioning accuracy and BDS-3 had the worst positioning accuracy. The 30-day PPP-AR experiment showed that the BDS-3 WL and NL FCBS estimated in this study had high stability and validity. Furthermore, the NL ambiguity-fixed rate of BDS-3 was only about 70%.
In conclusion, we analyzed the reason for the low ambiguity-fixed rate of BDS-3. The results showed that the consistency of BDS-3 orbit and clock products was low, especially those of the IGSO satellite. Therefore, we suggest avoiding the BDS-3 IGSO satellite when selecting a reference satellite for PPP-AR.

As the number of global MGEX stations that can observe BDS-3 satellites is increasing, the accuracy of BDS-3 orbit, clock offset, and other products is increasing and the error-correction model is becoming more reliable. The accuracy of BDS-3 NL FCBs is expected to be further improved. Furthermore, as the number of receivers that can receive new B1C/B2A frequency signals increases, using more accurate observations can further improve the accuracy of BDS-3 NL FCBs.

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**References**

1. Zumberge, J.; Heflin, M.; Jefferson, D.; Watkins, M.; Webb, F. Precise point positioning for the efficient and robust analysis of GPS data from large networks. *J. Geophys. Res.* [**1997**], 102, 5005–5017. [CrossRef]

2. Héroux, P.; Kouba, J. GPS precise point positioning using IGS orbit products. *Phys. Chem. Earth Part A Solid Earth Geod.* [**2001**], 26, 573–578. [CrossRef]

3. Ye, S. Theory and Realization of GPS Precise Point Positioning Using Un-Differenced Phase Observation; Wuhan University: Wuhan, China, 2002.

4. Gao, Y.; Shen, X. Improving ambiguity convergence in carrier phase-based precise point positioning. In Proceedings of the 14th International Technical Meeting of the Satellite Division of the Institute of Navigation (ION GPS 2001), Salt Lake City, Utah, 11–14 September 2001; pp. 1532–1539.

5. Church, C.; Gupta, I. Performance Analysis of Precise Point Positioning Based on Combined GPS and GLONASS. In Proceedings of the 20th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2007), Fort Worth, TX, USA, 25–28 September 2007; pp. 858–865.

6. Cai, C.; Gao, Y. Precise point positioning using combined GPS and GLONASS observations. *Positioning* [**2007**], 1, 13–22. [CrossRef]

7. Cai, C.; Gao, Y. Modeling and assessment of combined GPS/GLONASS precise point positioning. *GPS Solut.* [**2013**], 17, 223–236. [CrossRef]

8. Martin, A.; Anquela, A.; Capilla, R.; Bémé, J. PPP technique analysis based on time convergence, repeatability, IGS products, different software processing, and GPS+ GLONASS constellation. *J. Surv. Eng.* [**2011**], 137, 99–108. [CrossRef]

9. Li, P.; Zhang, X. Integrating GPS and GLONASS to accelerate convergence and initialization times of precise point positioning. *GPS Solut.* [**2014**], 18, 461–471. [CrossRef]

10. Montenbruck, O.; Steigenberger, P.; Khachikyan, R.; Weber, G.; Langley, R.; Mervart, L.; Hugentobler, U. IGS-MGEX: Preparing the ground for multi-constellation GNSS science. *Inside GNSS* [**2014**], 9, 42–49.

11. Xiaohong, Z.; Xingxing, L.; Pan, L. Review of GNSS PPP and its application. *Acta Geod. Et Cartogr. Sin.* [**2017**], 46, 1399. [CrossRef]

12. Gabor, M.J.; Nerem, R.S. GPS carrier phase ambiguity resolution using satellite-satellite single differences. In Proceedings of the 12th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 1999), Nashville, TN, USA, 14–17 September 1999; pp. 1569–1578.

13. Ge, M.; Gendt, G.; Rothacher, M.A.; Shi, C.; Liu, J. Resolution of GPS carrier-phase ambiguities in precise point positioning (PPP) with daily observations. *J. Geod.* [**2008**], 82, 389–399. [CrossRef]

14. Zhang, X.; Li, P.; Guo, F. Ambiguity resolution in precise point positioning with hourly data for global single receiver. *Adv. Space Res.* [**2013**], 51, 153–161. [CrossRef]

15. Laurichesse, D.; Mercier, F.; BERTHIAS, J.P.; Broca, P.; Cerri, L. Integer ambiguity resolution on undifferenced GPS phase measurements and its application to PPP and satellite precise orbit determination. *Navigation* [**2009**], 56, 135–149. [CrossRef]

16. Collins, P.; Bisnath, S.; Lahaye, F.; Héroux, P. Undifferenced GPS ambiguity resolution using the decoupled clock model and ambiguity datum fixing. *Navig. J. Inst. Navig.* [**2010**], 57, 123–135. [CrossRef]
17. Geng, J.; Meng, X.; Dodson, A.H.; Teferle, F.N. Integer ambiguity resolution in precise point positioning: Method comparison. *J. Geod.* 2010, 84, 569–581. [CrossRef]
18. Shi, J.; Gao, Y. A comparison of three PPP integer ambiguity resolution methods. *GPS Solut.* 2014, 18, 519–528. [CrossRef]
19. Kouba, J. A Guide to Using International GNSS Service (IGS) Products. 2009. Available online: https://kb.igs.org/hc/en-us/articles/201271873-A-Guide-to-Using-the-IGS-Products (accessed on 1 May 2009).
20. Hatch, R. The synergyism of GPS code and carrier measurements. In Proceedings of the International Geodetic Symposium on Satellite Doppler Positioning, Las Cruces, NM, USA, 8–12 February 1983; pp. 1213–1231.
21. Melbourne, W.G. The case for ranging in GPS-based geodetic systems. In Proceedings of the the First International Symposium on Precise Positioning with the Global Positioning System, Rockville, MD, USA, 15–19 April 1985; pp. 373–386.
22. Yuanxi, Y. Robust estimation for dependent observations. *Manuscr. Geod.* 1994, 19, 10–17.
23. Wang, J.; Zhang, Q.; Huang, G. Estimation of fractional cycle bias for GPS/BDS-2/Galileo based on international GNSS monitoring and assessment system observations using the uncombined PPP model. *Satell. Navig.* 2021, 2, 1–11. [CrossRef]
24. Du, Z.; Chai, H.; Xiao, G.; Yin, X.; Wang, M.; Xiang, M. Analyzing the contributions of multi-GNSS and INS to the PPP-AR outage re-fixing. *GPS Solut.* 2021, 25, 1–12. [CrossRef]
25. Qi, W.; Chai, H.; Kun, X.; Min, W.; Yang, C. Influence of Different ISB Processing Strategies on the Accuracy of Undifferenced FCBs and PPP-AR Positioning. In Proceedings of the China Satellite Navigation Conference (CSNC 2021) Proceedings, Singapore, 11 June 2021; Springer: Singapore, 2021; pp. 256–269.