A Spectroscopic Method to Measure Macho Proper Motions

Dan Maoz
School of Physics & Astronomy and Wise Observatory
Tel-Aviv University, Tel-Aviv 69978, ISRAEL
E-mail: dani@wise.tau.ac.il

and

Andrew Gould
Department of Astronomy
Ohio State University, Columbus, OH 43210
E-Mail: gould@payne.mps.ohio-state.edu

submitted to The Astrophysical Journal Letters: January 3, 1994
Abstract

A Massive Compact Halo Object (Macho) that lenses a background star will magnify different parts of the rotating stellar disk by varying amounts. The differential magnification will cause a shift in the centroid of the star’s spectral lines during the lensing event. The shift is proportional to the ratio of the stellar radius to the projected separation of the Macho from the star. It therefore provides a direct measure of the Einstein ring radius, and so also a measure of the Macho’s proper motion (angular speed). This measurement can remove some of the degeneracy between mass, distance to the lens, and transverse velocity that exists in the interpretation of results from ongoing microlensing experiments, and is an independent test of the lensing nature of the event. We show that using the high precision attainable by stellar radial velocity measurements, it is possible to measure proper motions for $\sim 10\%$ of Machos that lens A-stars in the Large Magellanic Cloud (LMC), i.e. $\sim 7\%$ of the type of relatively high-magnification events that have been reported to date. If this proper-motion measurement were combined with a parallax measurement of the “reduced velocity”, then the Macho mass, distance, speed, and direction could each be separately determined. The shift can be measured for $\sim 20\%$ of the A-star events generated by Machos in the dark halo of the LMC. This in turn would provide a measurement of the fraction of LMC vs. Galactic Macho events.

Subject Headings: spectroscopy – dark matter – gravitational lensing – Magellanic Clouds
1 Introduction

Two independent groups have recently found a total of three candidate microlensing events, apparently caused by Massive Compact Halo Objects (Machos) along the line of sight toward the Large Magellanic Cloud (LMC) (Alcock et al. 1993; Aubourg et al. 1993). The events are achromatic, have maximum magnifications of $A_{\text{max}} = 6.8, 3.3, \text{ and } 2.5$, and characteristic times $\omega^{-1} = 17 \text{ days}, 30 \text{ days} \text{ and } 26 \text{ days}$. The light curves fit the form first predicted by Paczyński (1986) quite well:

$$A[x(t)] = \frac{x^2 + 2}{x(x^2 + 4)^{1/2}}, \quad x(t) = \sqrt{\beta^2 + \omega^2(t - t_0)^2}. \quad (1)$$

Here $t_0$ is the midpoint of the event, $\omega^{-1}$ is the characteristic time, and $\beta$ is the dimensionless impact parameter (normalized to the Einstein ring radius).

Of the three measurable parameters in equation (1), only the characteristic time, $\omega^{-1}$ yields any information about the Machos. It is related to the underlying physical parameters by

$$\omega^{-1} = \frac{\theta_* D_{\text{OL}}}{v} = \frac{[4GM D_{\text{OL}}(1 - D_{\text{OL}}/D_{\text{LMC}})]^{1/2}}{v_c}, \quad (2)$$

where $\theta_*$ is the Einstein ring radius, $M$ is the Macho mass, $v$ is its transverse velocity, and $D_{\text{OL}}$ is its distance from the observer. The distance of the lensed star is denoted $D_{\text{LMC}}$. For any given event, one cannot determine the mass, distance, and speed separately. For an ensemble of events, the typical speed and distance are expected to be $v \sim 200 \text{ km s}^{-1}$ and $D_{\text{OL}} \sim 10 \text{ kpc}$, respectively. Hence, $M \sim (70\omega \text{ days})^{-2}M_\odot$. If the first few events prove typical, it will imply a mass scale of $\sim 0.1 M_\odot$, near the hydrogen-burning limit.

One would like to determine as much as possible about the distribution of Machos, not just the mass scale. For example, one would like to know if the Machos actually lie in a halo as opposed to a disk or thick disk, whether the halo is spherical or
oblate, whether the halo is truncated or extends all the way to the LMC, whether it is rotating, whether Machos populate the LMC as well as the Galaxy, and what the distribution of Macho masses is. To extract these additional pieces of information, new methods of analyzing the data as well as new experiments are required.

Gould (1992, 1993, 1994a,b), Sackett & Gould (1993), and Gould, Miralda-Escudé & Bahcall (1994) have developed a number of different techniques for extracting additional parameters from lensing events. In particular, Gould (1994a) showed that the non-zero size of the lensed stellar disk modifies the light curve of the event in a manner that allows one to measure the Macho’s Einstein radius, \( \theta^* \), and hence its proper motion (angular speed), \( \omega \theta^* \). In practice, however, the effect is measurable only for a fraction of events during which the Macho passes over the line of sight to the face of the source star, \( (1/550)(M/0.1\, M_\odot)^{-1/2} \) for Machos in the Galactic halo and \( (1/70)(M/0.1\, M_\odot)^{-1/2} \) for Machos in the LMC.

In this Letter, we predict a second, spectroscopic, effect which also arises from the non-zero size of the source, combined with the source rotation. Differential magnification of the stellar disk during the event induces a shift in the stellar spectral lines. By measuring the shift as a function of time one can determine the Einstein ring radius and so the proper motion of the Macho. In contrast to the photometric effect analyzed by Gould (1994a), this spectroscopic effect can be measured even when the Macho is many stellar radii from the source. Hence, the proper-motion measurement can be made for a larger fraction of events. We estimate that with an 8m telescope the effect can be measured in \( \sim 7\% \) of photometrically detected relatively high-magnification \( (\beta \lesssim 0.5) \) Macho events. The effect can be measured in \( \sim 15\% \) of high-magnification events generated by Machos in the LMC.

Measurement of the proper motion of Galactic Machos would, in itself, remove some of the degeneracy among the three Macho parameters, mass, distance, and
transverse speed. However, if this measurement were combined with a parallax measurement of the “reduced velocity”, then the degeneracy could be completely broken and the three parameters plus the transverse direction could be separately determined.

The proper motion of Galactic Machos is \( \sim 15 \) times greater than that of LMC Machos. Hence, a proper-motion measurement clearly distinguishes between the two. By identifying even a handful of LMC Machos, one could determine the fraction of events generated by them. This would both provide direct information about the LMC halo and remove a major background to the primary Galactic signal (see Sackett & Gould 1993).

2 Spectral Shift in Lines of a Microlensed Star

The spectral line profile of an unlensed star is broadened by the effect of the star’s rotation. Different parts of a finite stellar disk are magnified by varying amounts during a Macho lensing event. In order to determine the time-dependent rotational line profile, we apply equation (1) to calculate the magnification at each point on the star. Expressing all angles in units of the Einstein radius, \( \theta_\ast \), the line profile is

\[
f(v, t) = C \int_{-\sqrt{\rho^2 - z_1^2}}^{\sqrt{\rho^2 - z_1^2}} d z_2 A(|x(t) + z|)S(z), \quad v \equiv \frac{z_1}{\rho} v_{\text{rot}} + v_0
\]

where \( \rho \) is the stellar radius, \( x(t) \) is the projected separation of the stellar center from the Macho, \( S(z) \) is the surface brightness as a function of position, \( z \), on the stellar disk, \( v_{\text{rot}} \) is the projected rotation speed of the star, \( v_0 \) is the velocity of the stellar center of mass, and \( C \) is a constant. The \( z_2 \) axis is defined to be aligned with the stellar rotation axis. The effect of limb-darkening can be approximated (e.g. Allen 1973) by the form:

\[
S(z) = 1 - u_2 - v_2 + u_2(1 - |z|^2/\rho^2)^{1/2} + v_2(1 - |z|^2/\rho^2),
\]

with \( u_2 = 0.99 \) and \( v_2 = -0.17 \) at 4500 Å. We assume the star is devoid of star-spots.
The magnitude of the separation function \( x(t) = \sqrt{\beta^2 + (\omega t)^2} \) is known from the overall light curve. One can integrate equation (3) numerically and compare the expected profiles with the rotational profiles observed in a series of measurements. It will generally be more practical to measure the shift in the centroid of the line, i.e. the star’s apparent radial velocity, as a function of time:

\[
\langle v(t) \rangle = \frac{\int dv f(v,t) v}{\int dv f(v,t)}.
\]  

Apart from the rotational broadening described by equation (3), stellar line profiles can be intrinsically broadened by thermal and turbulent motions, by the natural line width, and by pressure broadening. The intrinsic broadening does not, however, affect the line centroid.

To study the behavior of the velocity shift, we first make the simplifying assumptions that \( \rho \ll x \ll 1 \) and \( S(z) = \text{constant} \). The integral for the line profile (eq. 3) can then be approximated as

\[
f(v,t) = C v_{\text{rot}} \rho \sqrt{v_{\text{rot}}^2 - (v - v_0)^2} \left[ 1 + \frac{v - v_0}{v_{\text{rot}}^2} \frac{\beta \cos \alpha + \omega t \sin \alpha}{\beta^2 + (\omega t)^2} \right],
\]

and the line centroid (eq. 5) becomes

\[
\langle v(t) \rangle = v_0 + \xi \frac{v_{\text{rot}}}{4} \rho \frac{\beta \cos \alpha + \omega t \sin \alpha}{\beta^2 + (\omega t)^2},
\]  

where \( \xi = 1 \) and \( \alpha \) is the angle between the Macho velocity \( dx/dt \) and the star’s projected rotation axis. For a limb-darkened stellar profile as described by equation (4), equation (7) remains valid with \( \xi = (30 - 14u_2 - 20v_2)/(30 - 10u_2 - 15v_2) \), e.g., \( \xi = 0.86 \) at 4500Å. We find numerically that for \( \rho \ll x \ll 1 \), the velocity shift in equation (7) should be reduced by a factor \([1 - 0.2(\rho/x)^2]\).

Figure 1 shows the shift in the line centroid, \( \Delta \langle v(t) \rangle \equiv \langle v(t) \rangle - v_0 \) as a function of time for an event with timescale \( \omega^{-1} = 17 \) days and a star with \( v_{\text{rot}} = 100 \text{ km s}^{-1} \). The dependence of the line-shift curve on the parameters \( \rho, \beta, \) and \( \alpha \), the angle between
the stellar axis and the Macho trajectory, is illustrated in the separate panels. As
reference, for a $0.1M_\odot$ Macho in the Galactic halo at a distance of 10 kpc lensing
an A star of radius $2.5R_\odot$ in the LMC, $\rho = 0.001$. Such a Macho in the LMC halo,
lensing an LMC star 2.5 kpc behind it, will have $\rho = 0.01$. The fraction of lensing
events having dimensionless impact parameter of $\beta$ or less (and hence maximum
magnification $\beta^{-1}$ or greater) is simply $\beta$. All values of the angle $\alpha$ are equally likely.

As can be seen from equation (7) and Figure 1, varying the value of $\rho$, the ratio
of the apparent stellar radius to the Einstein radius, affects only the amplitude of
the line-shift curve. Changes in the impact parameter $\beta$ affect both the amplitude of
the curve and its shift in time relative to the time of maximum total magnification.
Changes in the projected angle $\alpha$ between the stellar spin-axis and the Macho tra-
jectory affect the amplitude, the time-shift, and the shape of the curve. The impact
parameter $\beta$ can be fit independently to the photometric light curve. The parameters
$\rho$ and $\alpha$ can then be fit unambiguously to an event, given enough signal to noise, and
the one interesting parameter, $\rho$, measured. Since the physical stellar radius is known
from the star’s spectral type, as is the distance to the LMC, $\rho$ provides a measure of
the Macho Einstein ring radius, $\theta_*$ and the Macho proper motion $\omega \theta_*$. Knowledge of
the proper motion removes one part of the degeneracy between Macho mass, distance,
and transverse velocity in the interpretation of the photometric light curve (see eq. 2).
For example, for an assumed velocity $v$, the distance $D_{OL}$ to the lens can be derived,
and then from this distance and $\theta_*$, the mass $M$ can be calculated.

From equation 7 and the relevant range of parameters, a velocity resolution of
$\delta v / v_{\text{rot}} \sim \rho \sim 0.01$–$0.001$ is required in order to measure $\rho$ in some fraction of lensing
events. A temporal resolution of order hours is needed. We examine the observational
aspects more carefully below.

The stars being monitored by the MACHO and EROS collaborations are $\sim 1/3$
main-sequence stars, mostly A stars, and \( \sim 2/3 \) red giants (C. Alcock 1993, private communication). The projected rotation speeds of A stars have a broad distribution with a mean \( v_{\text{rot}} = 150 \text{ km s}^{-1} \) for A0 types, decreasing to \( v_{\text{rot}} = 80 \text{ km s}^{-1} \) for F0 (e.g. Allen 1973). Giants bluer than about G2 have similar rotation properties, with mean \( v_{\text{rot}} \sim 75 - 55 \text{ km s}^{-1} \) for F0 III to G0 III. A “rotation boundary” exists for giants at spectral type G3 (Gray 1989; a similar boundary exists in the main sequence at F5). Giants redder than G3 rotate very slowly, with mean velocities of \( v_{\text{rot}} \sim 5 - 2 \text{ km s}^{-1} \) for G3 III to K3 III. Giants later than G3 rarely or never rotate faster than 9 km s\(^{-1}\). Furthermore, their line broadening is dominated by photospheric macroturbulence of order \( \sim 4 - 8 \text{ km s}^{-1} \) (Gray 1989), which affects the accuracy with which their \( v_{\text{rot}} \) can be determined. (Below, in § 3, we will show that the accuracy of the proper-motion measurement is fundamentally limited by how well \( v_{\text{rot}} \) is known.) A precision \( \delta v \sim 100 \text{ m s}^{-1} \) is required for A stars and \( \sim 5 \text{ m s}^{-1} \) for red giants.

Such precisions can be obtained today in stellar radial-velocity measurements. For example, the stellar radial-velocity survey at the CfA achieves accuracies \( \sim 1 \text{ km s}^{-1} \) for 12 mag A stars and \( \sim 200 \text{ m s}^{-1} \) for G-stars in 20-minute exposures on a 1.5m telescope (Latham 1992; T. Mazeh 1993, private communication). The survey uses an intensified reticon detector, which can utilize only one echelle order. Since the survey is not optimized for A stars, the one order includes only weak metal lines, and none of the Balmer lines which are strong in A stars. Modern CCD detectors can have at least 5 times higher quantum efficiency. CCDs can simultaneously record many echelle orders and so increase the observed number of lines and hence the precision. The precision of radial velocity measurements increases as \((\text{No.photons} \times \text{No.lines})^{1/2}\), but is ultimately limited by systematic effects, such as spectrograph stability. It is plausible that a 17 mag A star in the LMC could be measured to \( \sim 100 \text{ m s}^{-1} \) in a one-hour exposure with a CCD on an 8m telescope with a spectroscopic setup.
optimized for A stars. (Recall that the stars being monitored, with typical unlensed magnitudes of 19-20, are in the process of being magnified by several magnitudes.) While improved velocity resolution is possible for red giants because of their numerous and strong lines, techniques yielding resolutions of $< 10 \text{ m s}^{-1}$ have been applied only to very bright ($V \sim 5 \text{ mag}$) stars (e.g. Walker 1992), and may not be feasible here. Because of this difficulty as well as the previously mentioned problem of determining the rotation speed of red giants, it may not be possible to make a spectroscopic determination of the proper motion of Machos which lens this class of star. When we estimate the fraction of measurable events below, we therefore consider only A stars.

3 Error Estimates

We examine now the errors that would be involved in an actual measurement. Parameterizing the separation $x$ by an angular variable $\theta$,

$$x \equiv \beta \sec \theta,$$

(8)

equation (7) becomes

$$\langle v \rangle = v_0 + \frac{v_{\text{rot}} \xi}{4 \beta} \rho \cos \alpha \cos^2 \theta + \frac{v_{\text{rot}} \xi}{4 \beta} \rho \sin \alpha \cos \theta \sin \theta = \sum_{i=0}^{2} a_i f_i(\theta).$$

(9)

where

$$a_0 \equiv \frac{v_0}{Q}, \quad a_1 \equiv \rho \sin \alpha, \quad a_2 \equiv \rho \cos \alpha,$$

(10)

$$f_0(\theta) \equiv Q, \quad f_1(\theta) \equiv Q \sin \theta \cos \theta, \quad f_2(\theta) \equiv Q \cos^2 \theta,$$

(11)

and $Q \equiv \frac{v_{\text{rot}} \xi}{4 \beta}$. (Note that by including $Q$ in the trial functions, $f_i(\theta)$, we have implicitly assumed that $Q$ is known. If there is a fractional uncertainty in $Q$ – due for example to an uncertainty in $v_{\text{rot}}$ – this will induce a similar uncertainty in $\rho$ over and above those analyzed below.)
We now suppose that a series of measurements are made of the velocity centroid and that the errors in these measurements scale inversely as the square root of the product of the exposure time and the luminosity. We normalize these to the error of a one hour observation at maximum magnification, which we take to be \( \sigma_0 = \delta v \).

Hence for a general observation of length \( \delta t \) and position \( \theta \),

\[
\sigma^2(\theta) = (\delta v)^2 \frac{hr}{\delta t} \sec \theta. \tag{12}
\]

We note that during the time interval \( dt \), the lens moves by \( d\theta = \cos^2 \theta \omega/\beta dt \), where \( \omega^{-1} \) is the time required to cross an Einstein ring radius. According to standard linear theory (Press et al. 1989), the inverse covariance matrix \( b_{ij} \) for the three parameters \( a_i \) is given by

\[
b_{ij} = \sum_k \frac{f_i(\theta_k)f_j(\theta_k)}{\sigma^2(\theta_k)}. \tag{13}
\]

For purposes of illustration, we assume that the observations are conducted continuously between \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \). Then, the above sum can be converted into an integral

\[
b_{ij} = \Delta^{-2} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} d\theta \left( \begin{array}{ccc}
\sec \theta & \sin \theta & \cos \theta \\
\sin \theta & \sin^2 \theta \cos \theta & \sin \theta \cos^2 \theta \\
\cos \theta & \sin \theta \cos^2 \theta & \cos^3 \theta
\end{array} \right), \tag{14}
\]

where

\[
\Delta \equiv \frac{4\delta v}{v_{\text{rot}}\xi} \sqrt{\beta \omega hr}. \tag{15}
\]

For example, for an event with peak magnification of 5, \( \omega = (17\text{days})^{-1} \), and 8 hours observations per day, \( \Delta = 4\delta v/(v_{\text{rot}}\xi) \sqrt{0.2(17)^{-1}/8} \).

For simplicity, we consider the case where the observations are made symmetrically around the peak of the light curve, \( \theta_{\text{max}} = -\theta_{\text{min}} = \theta_0 \). The total length of the observations is then \( T = 2(\beta/\omega) \tan \theta_0 \). For this case, the even and odd components decouple and \( b_{ij} \) takes a relatively simple form. We invert \( b \) to find the covariance matrix \( c \equiv b^{-1} \). The relevant terms are

\[
c_{11} = \frac{3}{2} \csc^3 \theta_0 \Delta^2, \quad c_{22} = \frac{1}{2} \left( \sin \theta_0 - \frac{\sin^3 \theta_0}{3} - \frac{\sin^2 \theta_0}{\ln \tan(\theta_0/2 + \pi/4)} \right)^{-1} \Delta^2, \tag{16}
\]
for the variances of \( a_1 \equiv \rho \sin \alpha \) and \( a_2 \equiv \rho \cos \alpha \), respectively. Note that the quantity of interest is \( \rho = \sqrt{a_1^2 + a_2^2} \).

In the above formal treatment, we have implicitly assumed that the source star is moving with constant velocity relative to the Earth. However, in general there will be some relative acceleration due to the motion of the Earth about the Sun and to the possible motion of the source about an unseen companion. The Earth’s motion is known and can be taken out. The motion of the source could likewise be taken out if it were measured by long-term observation of the source radial velocity. However, it is much easier to measure the source velocity when the source is highly magnified, so the best constraint on the acceleration of the source may come from the observations during the lensing event. This acceleration will be very nearly linear provided that the orbital period is much larger than the span of observations. We assume that this is the case.

The effect of an acceleration, \( g \), is to add a term which is linear in time, i.e. \( \propto \tan \theta \). Formally we write \( f_3(\theta) = Q \tan \theta; \quad a_3 = g/Q \). Once this additional degree of freedom is allowed, the covariance matrix becomes four-dimensional. However, under the assumption of a symmetric interval of observations, the even and odd components decouple. We can therefore examine two smaller submatrices, \( b^+ \) for components 0 and 2, and \( b^- \) for components 1 and 3. For the odd terms, we find

\[
b_{ij}^- = \Delta_2^{-2} \int_{-\theta_0}^{\theta_0} d\theta \begin{pmatrix} \sin^2 \theta \cos \theta & \sin \theta \tan \theta \\ \sin \theta \tan \theta & \tan^2 \theta \sec \theta \end{pmatrix},
\]

which implies a variance of \( a_1 = \rho \sin \alpha \) of

\[
c_{11}^+ = \frac{1}{2} \left( \frac{\sin^3 \theta_0}{3} - 2 \frac{[\ln \tan(\theta_0/2 + \pi/4) - \sin \theta_0]^2}{\tan \theta_0 \sec \theta_0 - \ln \tan(\theta_0/2 + \pi/4)} \right)^{-1} \Delta^2.
\]

The variance \( c_{22}^- \) of \( a_2 = \rho \cos \alpha \) remains as in equation (16). The rms uncertainty in \( \rho \) (averaged over all angles \( \alpha \)) is \( \delta \rho = [(c_{11} + c_{22})/2]^{1/2} \).
Figure 2 shows $\delta \rho/\rho$ as a function of the total time span of the observations $T$, where we have assumed $\omega^{-1} = 17$ days and $\delta v/v_{\text{rot}} \xi = 10^{-3}$ per one-hour observation at maximum magnification. The two panels show the errors in $\rho$ for $\rho = 0.001$ (typical Galactic Machos) and for $\rho = 0.01$ (typical Machos in the LMC) for various values of the impact parameter $\beta$. We have assumed 8 hours of observation per day in the Galactic Macho panel ($\rho = 0.001$), and just one hour per day for the LMC halo Machos (or, equivalently, a longer observation on a smaller telescope, achieving the same $\delta v/v_{\text{rot}}$). Assuming one is interested in $< 30\%$ error in $\rho$ (i.e., a $3\sigma$ detection) and one can dedicate $< 8$ days of telescope time to the monitoring of the Macho event, we see that $\beta < 0.1$ is required for typical Galactic events. One measurement per night for 8 days would be sufficient to detect LMC Machos with $\beta < 0.2$. If the LMC has a dark halo, $\sim 10\%$ of all events are expected to be LMC events (Gould 1993). Hence $\sim 10\%$ of all A-star events can be detected at the $3\sigma$ level. Note that the detection capabilities of the MACHO and EROS experiments are biased toward low-$\beta$ events, and that all their detected events so far have magnification $> 2$ (i.e. $\beta < 0.5$). The velocity shift is therefore measurable in $\sim 20\%$ of the A-star events of the type that is being detected, or $\sim 7\%$ of the events being detected on all stars.

4 Discussion and Conclusions

We predict the existence of a spectroscopic line-shift, resulting from differential magnification of a rotating stellar disk, during microlensing events in the ongoing MACHO and EROS experiments. The effect can be measured and monitored on large telescopes for a fraction of the A-star lensing events (A-stars should supply $\sim 1/3$ of the events), provided that the spectroscopic observations can be initiated fast enough to catch the event before maximum light. The spectroscopic signature can serve as an additional test of the lensing (as opposed to variable-star) nature of the event. More
importantly, the Einstein ring radius can be measured from the temporal behavior of the line-shift. This technique can therefore remove part of the degeneracy in the derivation of Macho parameters in the ongoing experiments.

The spectral shift can be measured with relatively smaller effort (one measurement per night) in a large fraction of microlensing events involving Machos in the halo of the LMC itself, which can then be recognized as such. If, for example, 10% of the events are due to LMC Machos, spectroscopic measurements of 50 A-star events with $\beta < 0.2$ for one hour per night for a week will reveal the effect in $\sim 5$ cases, and would provide a direct measurement of the LMC fraction of Macho events. The LMC events can be distinguished from any Galactic “foreground” events by their much lower proper motion (Gould 1994a).

With a more ambitious program in which events are followed for a week with eight measurements per night (likely requiring an 8m telescope), the proper motion can also be measured for the A-star + Galactic Macho events with $\beta \lesssim 0.10$. While this is a lot of telescope time the potential payoff is very great, particularly if the “reduced velocity” can be measured from parallax observations (Gould 1992, 1994b; Gould et al. 1994). The reduced speed is given by $(D_{\text{OL}}^{-1} - D_{\text{LMC}}^{-1})^{-1} \omega \theta_\ast$. Hence, by measuring the proper motion, $\omega \theta_\ast$ one would find the distance to the Macho, $D_{\text{OL}}$. The distance and the Einstein radius, $\theta_\ast$, yield the mass, $M$. Combining these with $\omega$ gives the transverse speed. In general, measurement of the reduced velocity requires observation from two small satellites in solar orbit. It is likely that the reduced speed could be obtained from only one such satellite (Gould 1994b). Gould et al. (1994) show that it is generally possible to measure one component of the reduced velocity from ground-based observations provided that the Machos are going of order the Earth’s orbital speed of $30 \text{ km s}^{-1}$. This would be extremely rare for Machos in the halo. However, the measurement proposed by Gould et al. actually becomes much
more sensitive for very high-magnification events \((\beta \lesssim 0.1)\) such as those for which proper-motion determinations can be made. For these events, we conjecture that one or perhaps even both components of the transverse velocity could be measured from the ground, even for Machos traveling at halo speeds, \(\sim 200 \text{ km s}^{-1}\).

**Acknowledgements:** We would like to thank D. Goldberg, T. Mazeh, and C. Pryor for helpful discussions. D.M. acknowledges support through an Alon Fellowship.
References

Alcock, C., et al. 1993, Nature, 365, 621
Allen, C.W. 1973, Astrophysical Quantities (London: Athlone)
Aubourg, E., et al. 1993, Nature, 365, 623
Gould, A. 1992, ApJ, 392, 442
Gould, A. 1993, ApJ, 404, 451
Gould, A. 1994a, ApJ Letters, 421, in press
Gould, A. 1994b, ApJ Letters, 421, in press
Gould, A., Miralda-Escudé, J. & Bahcall, J. N. 1994, ApJ Letters, in press
Gray, D.F. 1989, ApJ, 347, 1021
Latham, D.W. 1992, in Complementary Approaches to Double and Multiple Star
    Research–ASP Conference Series Vol. 32, eds. H.A. McAlister and W.I. Hartkopf,
    (San Francisco: ASP), p. 110
Paczyński, B. 1986, ApJ, 304, 1
Press, W. H., Flannery, B. P., Teukolsky, S. A., & Vetterling, W. T. 1989, Numerical
    Recipes (Cambridge Univ. Press)
Sackett, P. D. & Gould, A. 1993, ApJ, 419, 648
Walker, G.A.H. 1992, in Complementary Approaches to Double and Multiple Star
    Research–ASP Conference Series Vol. 32, eds. H.A. McAlister and W.I. Hartkopf,
    (San Francisco: ASP), p. 67
Figure Captions

**Figure 1** The line centroid shift $\Delta v$ in a microlensed star as a function of time. The plots are for an Einstein-radius crossing time $\omega^{-1} = 17$ days, and a projected stellar rotation speed $v_{\text{rot}} = 100$ km s$^{-1}$. (The horizontal and vertical axes can be scaled for any $\omega^{-1}$ and $v_{\text{rot}}$). The separate panels show the dependence of the velocity curve on the three parameters: dependence (a) on $\rho$, the ratio of the stellar angular radius to the Einstein radius; (b) on $\beta$, the ratio of the impact parameter to the Einstein radius; (c) on $\alpha$, the angle between the Macho velocity vector and the projected stellar rotation axis. $\alpha$ can be deduced from the shape of the velocity curve, and $\beta$ from the total photometric magnification of the event. The amplitude of the velocity curve then yields $\rho$, or, since the stellar radius is known, the Einstein radius of the Macho.

**Figure 2** The expected relative error in $\rho$, the ratio of the stellar angular radius to the Einstein radius, as a function of the time span of the observations, $T$ in days. The two panels are for $\rho = 0.001$ (typical for $0.1M_\odot$ Machos in the halo of the Galaxy) and for $\rho = 0.01$ (typical for Machos of this mass in the LMC halo). The various curves give the dependence of the error on the dimensionless impact parameter $\beta$. Since a fraction $\beta$ of lensing events have impact parameter $< \beta$, the fraction of lensing events where $\delta \rho/\rho$ can be measured to a specified accuracy given a stretch of telescope time $T$ can be read off the plots. It is assumed that the timescale of the event is $\omega^{-1} = 17$ days and the precision of the radial velocity measurement that can be obtained in 1 hour at the time of maximum magnification is $\delta v/v_{\text{rot}} = 10^{-3}$. Eight hours of observation per day are assumed in the $\rho = 0.001$ panel and just one hour (or one measurement with equivalent precision) per day in the $\rho = 0.01$ panel. About 10% of Galactic Macho events and 20% of LMC Macho events involving A stars can be measured to better than 30% accuracy in 8 days of observing per event.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9401020v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9401020v1