Non-degenerate wavelength computational ghost imaging with thermal light

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ABSTRACT

Non-degenerate wavelength computational ghost imaging with thermal light source is studied theoretically and experimentally. The acquired computational ghost images are of high quality when the wavelength of computed light is different from the light detected by bucket detector. Compared to the necessary light of short wavelength in previous ghost imaging, the use of longer wavelength light is demonstrated to bring about ghost images with higher spatial resolution, in strong atmospheric turbulence.

1 Introduction

Ghost imaging is a transverse active-imaging technique that exploits the correlation between two light beams to image an object without spatially resolving measurements of the light beam that has undergone object interaction [1]. Over the past two decades, GI has been found to have some unique advantages over conventional optical imaging technologies, such as super-resolution [2,3], good imaging quality in harsh optical environment [4,5]. Recently, the experiments of GI with X-ray and electronics sources [6-8] have been reported, indicating that GI has become a powerful comprehensive tool in exploring and analyzing the internal characteristics of complex material, e.g. biomolecular structures and nanomaterials.

GI is considered to have broad applied prospects [9-13], especially in remote sensing and laser radar. One of the key reasons that many applications have not yet realized in practice is the complexity of the optical path structure of conventional GI. Fortunately, in 2008, Shapiro proposed a novel GI scheme-computational ghost imaging (CGI)[14], in which the image is reconstructed by correlating a calculated pattern with the signal of a bucket detector located behind the object illuminated by modulated light by spatial light modulator (SLM). Deterministic modulation of a laser beam with an SLM can provide the signal field used for target interrogation, while the on-target intensity pattern needed for the reference field can then be calculated via diffraction theory [15,16].

CGI is currently the most promising imaging solution for remote sensing and laser radar because it has only one optical path, which is different from the conventional GI essentially. Although GI can bring about good imaging quality in harsh optical environment, it is not negligible that the ghost image will be significantly degraded with strong turbulence and large propagation distance. 2011, Meyers et al proposed turbulence-free GI, which played a crucial role in applications [4]. Previous works have shown that the resolution of non-degenerate wavelength GI is higher than that of degenerate wavelength GI [17,18]. In addition, the source with shorter wavelength illuminating the object is effective for ghost images with higher resolution in atmospheric turbulence [19,20].

Non-degenerate wavelength CGI with thermal light source relies on a modification of the conventional CGI set-up (Fig.1). A binocular charge-coupled device (CCD) camera is employed to replace the bucket detector, compared to the conventional CGI scheme. The binocular CCD camera can not only output the bucket signal but also be used as a tool to measure the distance between the SLM and the object [21,22].

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2 Theory

In the following, we theoretically illustrate the concept of nondegenerate wavelength CGI with thermal light source. The collected light by the detector is named as signal light, while the calculated patterns by the SLM is named as reference light. To explore the properties of nondegenerate wavelength CGI, a continuous laser beam $E_s$ is projected to an SLM and the modulated light illuminates the object after propagating a distance of $z$ in free space. The light field detected by the binocular CCD camera can be expressed as \cite{5, 23}

$$
E_s(x_s) = \frac{-1}{\lambda_1 \sqrt{z_2}} \int dy dE'_r \left( \frac{\lambda}{2\pi} (y-u) \right) T(y) e^{\frac{2\pi}{\lambda_1 z_2} (x_s-y)},
$$

where $T(y)$ represents the information of object. The quantities $u$, $y$, $x$ represent the transverse coordinates of the SLM, the object plane and the bucket detector plane, respectively. $z_1$ is the distance from the SLM to the object. $z_2$ represents the distance from object to the binocular CCD camera.

![Figure 1: Setup of the non-degenerate-wavelength computational ghost imaging with thermal light source. SLM: spatial light modulator, BBD: binocular bucket detector.](image)

In practice, $z_1$ and $z_2$ need to be measured in real time to get the calculated light field according to the diffraction principle. Fortunately, the distance from the object to detector can be accurately measured by the binocular parallax principle without adding other equipments \cite{21, 22}. For simplicity and practicality, we assume $z_1 = z_2 = L_1$. In order to obtain a nondegenerate-wavelength computational ghost image, the wavelength of the calculated light $E_2$ should be different from that of the detected light $E_s$, i.e., $\lambda_s \neq \lambda_r$. Furthermore, the propagation distance $L_2 = z_3$ of the calculated light is determined by the optimal imaging conditions of the non-degenerate GI ($\lambda_s L_1 = \lambda_r L_2$) \cite{17}. Thus, the distribution of the computed light is given by

$$
E_r(x_r) = \left( \frac{-i}{\lambda_r \sqrt{z_3}} \right)^{1/2} \int du E'_r \left( \lambda_r, u \right) e^{\frac{2\pi}{\lambda_r z_3} (u-x_r)}.
$$

In order to reconstruct the image, the calculated patterns at the object plane are cross-correlated with the signal output by the detector, i.e.,

$$
G(x_s, x_r) = \langle I_r \left( x_r \right) I_r \left( x_s \right) \rangle - \langle I_r \left( x_s \right) \rangle \langle I_r \left( x_r \right) \rangle \nonumber
$$

$$
= \langle E_s^* \left( x_s \right) E_s \left( x_s \right) E_r^* \left( x_r \right) E_r \left( x_r \right) \rangle - \langle E_s^* \left( x_s \right) E_s \left( x_s \right) \rangle \langle E_r^* \left( x_r \right) E_r \left( x_r \right) \rangle 
$$

$$
= \frac{1}{\lambda_r^2 \lambda_s z_1 2 z_2 z_3} \int du_1 du'_1 du_2 du'_2 dy dy' C \left( \frac{\lambda_s \lambda_r; u_1, u'_1, u_2, u'_2}{\lambda_r^3} \right) T(y) T^*(y) \nonumber
$$

$$
\times e^{\frac{2\pi}{\lambda_r z_2} \left( y-u \right)^2} e^{\frac{2\pi}{\lambda_r z_3} \left( x_s-y' \right)^2} e^{\frac{2\pi}{\lambda_r z_3} \left( x_r-y'' \right)^2} e^{\frac{2\pi}{\lambda_s z_1} \left( x_s-y'' \right)^2} e^{\frac{2\pi}{\lambda_s z_1} \left( x_r-y'' \right)^2}.
$$
We calculate the exponential function
\[ \exp \left( \frac{\omega^2}{2} \right) \]
where \( \omega \) is the transverse size of the laser beam and \( l_c \) is the correlation parameter of the random amplitude caused by the SLM. Substituting Eq.(5) into Eq.(3), we can thus write the ghost image of Eq.(3) as
\[ G(x_r) = \frac{\pi^2}{\lambda_s \lambda_r z_1 z_2 \sqrt{ABCD}} \int dyO(y) \exp \left( \frac{S^2}{4A} + \frac{P^2}{4B} + \frac{Q^2}{4C} + \frac{R^2}{4D} \right), \]
where,
\[
A = \frac{1}{4\omega^2} + \frac{1}{2l_c^2} - \frac{i\pi}{\lambda_s z_1}, \quad B = \frac{1}{4\omega^2} + \frac{1}{2l_c^2} + \frac{i\pi}{\lambda_s z_1},
\]
\[
C = \frac{1}{4\omega^2} + \frac{1}{2l_c^2} - \frac{i\pi}{\lambda_r z_3} - \frac{4Bz_1}{l_c^3}, \quad D = \frac{1}{4\omega^2} + \frac{1}{2l_c^2} + \frac{i\pi}{\lambda_r z_3} - \frac{1}{4Al_c^2},
\]
\[
S = \frac{2i\pi y}{z_1}, \quad P = \frac{2i\pi y}{z_1}, \quad Q = \frac{2i\pi x_2}{l_c^2 \lambda_s z_1}, \quad R = \frac{2i\pi x_2}{\lambda_r z_3} - \frac{i\pi y}{Al_c^2 \lambda_r z_1},
\]
\[ \langle T(y)T^*(y) \rangle = \lambda_s O(y) \delta \left( y - y^* \right). \]
From Eq.(6), we can see that the correlation function \( G(x_r) \) has the same form as that of two-color GI [5,17]. Therefore, the non-degenerate wavelength thermal CGI feasible in practice even when the difference of the wavelengths is very large. Different from the two-color GI, as the reference light field is completely calculated, the wavelength can be set arbitrarily.

We calculate the exponential function \( \exp \left( \frac{S^2}{4A} + \frac{P^2}{4B} + \frac{Q^2}{4C} + \frac{R^2}{4D} \right) \) on the right of Eq.(6), and then give the standard form of GI. This calculation process is simple but cumbersome, so the result is directly given by
\[ G(x_r) = \frac{\pi^2}{\lambda_s \lambda_r z_1 z_2 \sqrt{ABCD}} \exp \left( -\frac{x_r^2}{2W_{fov}^2} \right) \int dyO(y) \exp \left( -\frac{(y - mx_r)^2}{2W_{psf}^2} \right), \]
where \( W_{fov} \) and \( m \) represent the view of field and magnification factor [5], respectively.
\[ W_{psf} = Re \sqrt{\frac{1}{2 \left( K_1^2 + K_2^2 + K_3^2 + K_4^2 \right)}} \]
is the width of point spread function (PSF).
\[
K_1 = \frac{\pi}{\sqrt{A\lambda_s z_1}}, \quad K_2 = \frac{\pi}{\sqrt{B\lambda_s z_1}}, \quad K_3 = \frac{\pi}{2B\sqrt{C\lambda_s z_1 l_c}}, \quad K_4 = \frac{\pi}{2\sqrt{D\lambda_r z_1}} \left( \frac{1}{Al_c^2} \right).
\]
Equation (8) is plotted in Fig.2 as a function of variable \( \lambda_s \), \( \lambda_r \), and \( L_1 = z_1 \). It is seen that the width of the PSF attains its smallest value when \( \lambda_s L_1 = \lambda_r L_2 \), in previous work[15]. Fig.2(a) compares the resolution of conventional CGI with single wavelength light source (blue dot line) and nondegenerate wavelength CGI (other kinds of lines), from which we found that there is no significant difference between them (minimum value of PSF). Compared to this one, the spatial resolution changes significantly as the wavelength of signal light changes. This result has illustrated the
results that the spatial resolution of non-degenerate wavelength CGI depends more strongly on signal light \( \lambda_s \) than reference light \( \lambda_r \) (Fig. 2(b)).

In the case of atmospheric turbulence, based on the extended Huygens-Fresnel integral, the propagation function from the source plane to the detector plane is given by [5, 23]

\[
E_s(x_s) = \frac{-1}{\lambda_s \sqrt{z_1 z_2}} \int du dy E'_s(\lambda_s, u_1) e^{\frac{i\pi}{\lambda_s}(y-u_a)} \times \exp [\phi_1(y, u)] T(y) \exp [\phi_2(x, y)] T(y) e^{\frac{i\pi}{\lambda_s z_2}(x_s - y)},
\]

where \( \phi_1 \) and \( \phi_2 \) characterize the atmospheric turbulence effects in the SLM-to-object path and the object-to-bucket-detector path, respectively. Because the reference arm can be computed by the simulation program according to the diffraction theory, the computed light field is not affected by atmospheric turbulence. Thus, the imaging function of nondegenerate-wavelength CGI with atmospheric turbulence can be expressed as

\[
G(x_r) = \frac{\pi^2}{\lambda_s \lambda_r z_1 z_3 \sqrt{A'B'C'D'}} \exp \left( -\frac{x_r^2}{2W_{fov}^2} \right) \times \int dy O(y) \exp \left( -\frac{(y - mx_2)^2}{2W_{psf}^2} \right).
\]

The PSF has the same form as Eq. 8, i.e.,

\[
W_{psf} = \text{Re} \left( \sqrt{\frac{1}{2 (K_1^2 + K_2^2 + K_3^2 + K_4^2)}} \right).
\]

with

\[
A' = \frac{1}{4\omega^2} + \frac{1}{2l_c^2} + \frac{1}{2\rho^2} - \frac{i\pi}{\lambda_s z_1}, \quad B' = \frac{1}{4\omega^2} + \frac{1}{2l_c^2} + \frac{1}{2\rho^2} + \frac{i\pi}{\lambda_s z_1} - \frac{1}{4A' \rho^4},
\]

\[
C' = \frac{1}{4\omega^2} + \frac{1}{2l_c^2} + \frac{1}{2\rho^2} - \frac{i\pi}{\lambda_r z_3} - \frac{1}{4B' l_c^2},
\]

\[
D' = \frac{1}{4\omega^2} + \frac{1}{2l_c^2} + \frac{i\pi}{\lambda_r z_3} - \frac{1}{4A' l_c^2} - \frac{1}{16A'^2 B'^4 \rho^4} - \frac{1}{64A'^2 B'^2 C'^8 \rho^4}.
\]
\[ K_1 = \frac{\pi}{\sqrt{A' \lambda_s z_1}}, \quad K_2 = \frac{\pi}{\sqrt{B' \lambda_s z_1}} \left( 1 - \frac{1}{2A' \rho^2} \right), \]
\[ K_3 = \frac{\pi}{2B' \sqrt{C' \lambda_s z_1 l_c^2}} \left( 1 - \frac{1}{2A' \rho^2} \right), \]
\[ K_4 = \frac{\pi}{2\sqrt{D' \lambda_s z_1}} \left[ -\frac{1}{A' l_c^2} + \frac{1}{B' l_c^2} \left( 1 - \frac{1}{2A' \rho^2} \right) \right. \]
\[ \times \left. \left( \frac{1}{2A' \rho^2} + \frac{1}{8A' B' C' l_c^4 \rho^2} \right) \right], \]

where \( \rho = \left( 0.55C_{n,s}^2 k_s^2 z_0 \right)^{-\frac{1}{2}} \), \( C_{n,s}^2 \) is the turbulence strength.

Equation (11) is plotted in Fig.3 as a function of \( \lambda_r \) and \( \lambda_s \). We test the resolution of nondegenerate wavelength thermal CGI with the wavelength \( \lambda_s \) fixed and the wavelength \( \lambda_r \) changed. The results are shown in Fig.3(a)-3(d). The results show that the spatial resolution of non-degenerate wavelength CGI with thermal light is equal to that of conventional CGI in turbulence. The spatial resolution will be reduced with the increase of turbulence. We fix the \( \lambda_r \) and change \( \lambda_s \), the results are shown in Fig.3(e)-3(h). In weak turbulence (\( C_{n,s}^2 = 1 \times 10^{-15} \)), ghost images with higher resolution can be obtained when the light of signal arm carries shorter wavelength, which is the same as the previous works [5,16,19]. However, in strong turbulence (\( C_{n,s}^2 = 1 \times 10^{-12} \)), ghost images with higher resolution can be obtained when the light of signal arm carries longer wavelength, which is different from previous works [5,16,19].
3 Experiments

To experimentally demonstrate the non-degenerate wavelength CGI, we used the imaging setup described in the previous section to image a Rubik’s Cube. The setup is based on a two-dimensional amplitude-only ferroelectric liquid crystal spatial light modulator (FLC-SLM, Meadowlark Optics A512-450-850), with 512 × 512 addressable 15µm × 15µm pixels. The light sources are CW solid-state lasers with wavelength 532nm and 635 nm respectively. The bucket detector is replaced by a binocular camera (CCD) with 640 × 480 addressable 3.0µm × 3.0µm. A 5 × 5 × 5cm³ object (Rubik’s Cube) located at a distance $L_1 = 100cm$ from the SLM, and a lens collects the reflected light onto the
detector. In each realization a grayscale with $512 \times 512$ is sent to the SLM. The distance between the two detectors of the binocular camera is 12 cm. The measurement error of binocular ranging is about 1%. The accuracy of the measurement can be higher by optimizing the algorithm and calibration.

Figure 4: The nondegenerate-wavelength computational ghost image with 500000 realizations. (a,e) object, (b-d) Parameters used are $\lambda_s = 532$ nm, $\lambda_r = 400$ nm, $532$ nm, 635 nm. (f-h) The corresponding results with $\lambda_s = 635$ nm, $\lambda_r = 400$ nm, 532 nm, 635 nm.

Figure 4 compares the nondegenerate-wavelength CGI with different wavelengths of signal light and reference light. The experiment results show that a high-quality ghost image can be obtained even when the wavelengths of light used in the signal and reference arms are very different. Furthermore, changing the wavelength of the reference light does not significantly improve the resolution of ghost image. The color of non-degenerate wavelength ghost image depends on the light detected by the bucket detector [24, 25]. It is noteworthy that the red color in Fig.4(a)-4(d) is produced by laser-activated coatings, which does not affect the experimental results.

Figure 5: The nondegenerate-wavelength computational ghost image with 500000 realizations. (a,b) object, (c) Parameters used are $\lambda_r = \lambda_s = 532$ nm. (d) The corresponding results with $\lambda_r = \lambda_s = 635$ nm.

To verify the effect of strong atmospheric turbulence on non-degenerate wavelength CGI, turbulence is introduced by adding heating elements at 550°C underneath optical path [4]. Heating of the air causes temporal and spatial fluctuations on its index of refraction that makes the classical image of the object jitter about randomly on the image plane causing a "blurred" picture. Fig.5 shows that the spatial resolution of ghost image obtained by 635 nm (Fig.5(d)) is better than that of 532 nm (Fig.5(c)). The reason for this phenomenon is that strong turbulence has less effect on long wavelength laser than that on short wavelength laser [26]. Further studies have shown that this conclusion applies to all types of ghost imaging.

4 Discussions and conclusion

Shapiro creatively proposed the concept of CGI based on the modulation of light field by SLM. Compared with conventional GI, the CGI is more suitable for laser radar, remote sensing and other fields. Chan et al. proposed two-color GI based on the modulation effect of spatial light modulator on different wavelength light, and proved that
the resolution of ghost image depends on each of these wavelengths in non-degenerate wavelength GI with thermal light [17]. Actually, two-color GI is still a conventional GI rather than CGI. Here, our results show that the resolution of non-degenerate wavelength CGI with thermal light depends primarily on the wavelength used to illuminate the object, but not on the calculated light. It is not difficult to understand that the correlation function of non-degenerate wavelength CGI is the same as that of two-color GI, because they are all based on the modulation of light field by spatial light modulator. Thus, the spatial intensity pattern of the light on the signal arm and computed (reference) arm is the same, which are similar to that of conventional GI [27].

In conclusion, we have shown that nondegenerate wavelength CGI with thermal light source is not only feasible but can also give a ghost image with high quality. The key to nondegenerate wavelength CGI is to use the binocular ranging principle to measure the distance of object in real time to obtain the calculated light field. Moreover, there is no fundamental limit in the extent for the wavelength in the reference arm as the corresponding field is completely calculated. It is out of the restriction of the spatial light modulator, which is better than the conventional two-color GI. More important, the results show that the longer wavelength laser is sent to illuminate the object in strong atmospheric turbulence, the higher resolution of ghost image can be obtained. In addition, this scheme may have potential imaging applications for moving objects, which will be discussed in detail in our future works.

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