Erratum: Numerical values of the $f^F, f^D$ and $f^S$ coupling constants in SU(3) invariant Lagrangian of the interaction of the vector-meson nonets with $1/2^+$ octet baryons [Phys. Rev. C93, 055208 (2016)]

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Some aspects of the presented determination of numerical values of the \( f^F, f^D \) and \( f^S \) coupling constants in the SU(3) invariant interaction Lagrangian

\[
L_{VVBB} = \frac{i}{\sqrt{2}} f^F [\bar{B}^\alpha \gamma_\mu B^\beta_\gamma - \bar{B}^\beta \gamma_\mu B^\alpha_\gamma] (V^\mu)_{\gamma\alpha} + \\
+ \frac{i}{\sqrt{2}} f^D [\bar{B}^\alpha \gamma_\mu B^\beta_\gamma + \bar{B}^\beta \gamma_\mu B^\alpha_\gamma] (V^\mu)_{\gamma\alpha} + \\
+ \frac{i}{\sqrt{2}} f^S \bar{B}^\alpha \gamma_\mu B^\beta \omega^\mu
\]

of the vector-meson nonet with \(1/2^+\) octet baryons in [1] were not correct.

The crucial ingredient in that determination is an application of the \( \omega - \phi \) mixing, which has been entered into the procedure in [1] two times. First in a derivation of expressions (51), (52), (55) and (56) for coupling constants of vector-mesons with nucleons from the above-mentioned Lagrangian, and so, also in a derivation of the reverse expressions (66), (67), (68) and (69), and then in a determination of the signs of the universal vector-meson coupling constants \( f^\rho, f^\omega \) and \( f^\phi \).

Generally there are in literature the following four different physically acceptable forms of the \( \omega - \phi \) mixing

1. \( \phi = \omega_8 \cos \theta - \omega_0 \sin \theta \)
   \( \omega = \omega_8 \sin \theta + \omega_0 \cos \theta \) \hspace{1cm} (1)

2. \( \phi = -\omega_8 \cos \theta + \omega_0 \sin \theta \)
   \( \omega = \omega_8 \sin \theta + \omega_0 \cos \theta \) \hspace{1cm} (2)

3. \( \phi = -\omega_8 \cos \theta + \omega_0 \sin \theta \)
   \( \omega = -\omega_8 \sin \theta - \omega_0 \cos \theta \) \hspace{1cm} (3)

4. \( \phi = \omega_8 \cos \theta - \omega_0 \sin \theta \)
   \( \omega = -\omega_8 \sin \theta - \omega_0 \cos \theta \) \hspace{1cm} (4)

which manifest themselves in the four different forms of expressions for \( f^F, f^D, f^S \)
1. \( F^F = \frac{1}{2} \left[ f_{\rho NN} + \sqrt{3}(f_{\phi NN} \cos \theta + f_{\omega NN} \sin \theta) \right] \)
\( F^D = \frac{1}{2} \left[ 3f_{\rho NN} - \sqrt{3}(f_{\phi NN} \cos \theta + f_{\omega NN} \sin \theta) \right] \)
\( F^S = \sqrt{2}(f_{\omega NN} \cos \theta - f_{\phi NN} \sin \theta) \) (5)

2. \( F^F = \frac{1}{2} \left[ f_{\rho NN} + \sqrt{3}(-f_{\Phi NN} \cos \theta + f_{\omega NN} \sin \theta) \right] \)
\( F^D = \frac{1}{2} \left[ 3f_{\rho NN} - \sqrt{3}(-f_{\Phi NN} \cos \theta + f_{\omega NN} \sin \theta) \right] \)
\( F^S = \sqrt{2}(f_{\omega NN} \cos \theta + f_{\Phi NN} \sin \theta) \) (6)

3. \( F^F = \frac{1}{2} \left[ f_{\rho NN} - \sqrt{3}(f_{\Phi NN} \cos \theta + f_{\omega NN} \sin \theta) \right] \)
\( F^D = \frac{1}{2} \left[ 3f_{\rho NN} + \sqrt{3}(f_{\Phi NN} \cos \theta + f_{\omega NN} \sin \theta) \right] \)
\( F^S = -\sqrt{2}(f_{\omega NN} \cos \theta - f_{\Phi NN} \sin \theta) \) (7)

4. \( F^F = \frac{1}{2} \left[ f_{\rho NN} + \sqrt{3}(f_{\Phi NN} \cos \theta - f_{\omega NN} \sin \theta) \right] \)
\( F^D = \frac{1}{2} \left[ 3f_{\rho NN} - \sqrt{3}(f_{\Phi NN} \cos \theta - f_{\omega NN} \sin \theta) \right] \)
\( F^S = -\sqrt{2}(f_{\omega NN} \cos \theta + f_{\Phi NN} \sin \theta). \) (8)

On the other hand an application of the same \( \omega - \phi \) mixing configurations (1), (2), (3), (4) leads to the rates of the overthrown values of the universal vector-meson coupling constants with different signs as follows

1. \( \frac{1}{f_{\rho}} : \frac{1}{f_{\omega}} : \frac{1}{f_{\phi}} = \sqrt{3} : \sin \theta : \cos \theta, \) (9)

which can be found e.g. in ref. [2] on p. 52 relation (A)

2. \( \frac{1}{f_{\rho}} : \frac{1}{f_{\omega}} : \frac{1}{f_{\phi}} = \sqrt{3} : \sin \theta : -\cos \theta \) (10)

to be found in ref. [3] on p. 539

3. \( \frac{1}{f_{\rho}} : \frac{1}{f_{\omega}} : \frac{1}{f_{\phi}} = \sqrt{3} : -\sin \theta : -\cos \theta \) (11)

4. \( \frac{1}{f_{\rho}} : \frac{1}{f_{\omega}} : \frac{1}{f_{\phi}} = \sqrt{3} : -\sin \theta : \cos \theta, \) (12)

and this last case can be found e.g. in ref. [4] on p. 446.

All these relations can be explained by the following considerations.
Starting e.g. from the $\omega - \phi$ mixing configuration (11) and substituting explicitly

\[
\omega_8 = \frac{1}{\sqrt{6}} (\bar{u}u + \bar{d}d - 2\bar{s}s)
\]

\[
\omega_0 = \frac{1}{\sqrt{3}} (\bar{u}u + \bar{d}d + \bar{s}s)
\]

and for the ideal mixing angle $\theta = 35.3^\circ$

\[
\sin \theta = \sqrt{\frac{1}{3}}; \quad \cos \theta = \sqrt{\frac{2}{3}},
\]

one obtains

\[
\phi = -\bar{s}s
\]

and

\[
\omega = + \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d).
\]

On the other hand the hadronic electromagnetic (EM) current

\[
J^h_\mu = \frac{2}{3} \bar{u}\gamma_\mu u - \frac{1}{3} \bar{d}\gamma_\mu d - \frac{1}{3} \bar{s}\gamma_\mu s,
\]

can be formally arranged to the shape

\[
J^h_\mu = \frac{1}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \frac{1}{6} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) - \frac{1}{3} (\bar{s}\gamma_\mu s),
\]

and because

\[
\frac{1}{\sqrt{2}} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) = J^\rho_\mu
\]

\[
+ \frac{1}{\sqrt{2}} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) = J^\omega_\mu \text{ due to the sign } "+" \text{ in } (16)
\]

\[-\bar{s}\gamma_\mu s = J^\phi_\mu \text{ due to the sign } "-" \text{ in } (15),
\]

are the $\rho^0-, \omega-, \phi-$ meson EM currents, respectively, the hadronic EM current acquires the following form

\[
J^h_\mu = \frac{1}{\sqrt{2}} J^\rho_\mu + \frac{1}{3\sqrt{2}} J^\omega_\mu + \frac{1}{3} J^\phi_\mu.
\]

Now, if the results of the Kroll-Lee-Zumino paper [5], that a linear combination of the neutral vector-meson fields $\rho^0_\mu, \omega_\mu, \phi_\mu$

\[
J^h_\mu = \frac{m^2}{f_\rho} \rho^0_\mu + \frac{m^2}{f_\omega} \omega_\mu + \frac{m^2}{f_\phi} \phi_\mu,
\]

with the universal vector-meson coupling constants $f_\rho, f_\omega, f_\phi$, is proportional by some real constant $A$ to the hadronic EM current (18), are taken into account, considering a dimension
of the Dirac quark fields $u, d, s$ in (18), in the framework of the natural units $\hbar = c = 1$, to be $m^3$ and a dimension of the vector-meson fields in (19) to be $m^1$, the relations

$$\frac{1}{f_\rho} = A \frac{1}{\sqrt{2}}; \quad \frac{1}{f_\omega} = +A \frac{1}{3\sqrt{2}}; \quad \frac{1}{f_\phi} = +A \frac{1}{3};$$

(20)

are found.

Then from these last relations the rates

$$\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = \frac{1}{\sqrt{2}} : \frac{1}{3\sqrt{2}} : \frac{1}{3}$$

$$= \frac{\sqrt{6}}{\sqrt{2}} : \frac{\sqrt{6}}{3\sqrt{2}} : \frac{\sqrt{6}}{3}$$

$$= \sqrt{3} : \frac{1}{\sqrt{3}} : \frac{\sqrt{2}}{3}$$

$$= \sqrt{3} : \sin \theta : \cos \theta,$$

are obtained, giving the signs of the universal vector-meson coupling constants $+f_\rho, +f_\omega, +f_\phi$.

As a matter of fact the signs of the universal vector-meson coupling constants are specified already from the relations (20), however, a community of physicists prefers the relations (9)-(12), in which the universal vector-meson coupling constants are related to $\sin \theta$ and $\cos \theta$ where the angle $\theta$ is determined from the quadratic Gell-Mann-Okubo vector-meson mass formula, which provide more realistic values of the latter to be in fair agreement with experimental evaluations. Therefore we also favor a presentation of the universal vector-meson coupling constants signs in the form (9)-(12).

In a like manner, starting from the $\omega - \phi$ mixing configuration (2), and relations (13) and (14), one obtains

$$\phi = +\bar{s}s$$

(21)

and

$$\omega = +\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d).$$

(22)

Then comparing the hadronic EM current to be multiplied by the real constant $A$

$$J^h_\mu = \frac{1}{\sqrt{2}}J^\rho_\mu + \frac{1}{3\sqrt{2}}J^\omega_\mu - \frac{1}{3}J^\phi_\mu,$$

(23)
where

\[ J_\mu^\rho = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) \]

\[ J_\mu^\omega = +\frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \]

due to the sign ”+” in (22)

\[ J_\mu^\phi = +\bar{s}\gamma_\mu s \]

due to the sign ”+” in (21),

with (19), one obtains relations

\[ \frac{1}{f_\rho} = A\frac{1}{\sqrt{2}}; \quad \frac{1}{f_\omega} = +A\frac{1}{3\sqrt{2}}; \quad \frac{1}{f_\phi} = -A\frac{1}{3} \]  

(24)

and from them the rates (10), giving the following signs of universal vector-meson coupling constants +f_\rho, +f_\omega, -f_\phi.

Again, starting from the \( \omega - \phi \) mixing configuration (3), and relations (13) and (14), one obtains

\[ \phi = +\bar{s}s \]  

(25)

and

\[ \omega = -\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d). \]  

(26)

Then comparing the hadronic EM current to be multiplied by the real constant A

\[ J_\mu^h = \frac{1}{\sqrt{2}}J_\mu^\rho - \frac{1}{3\sqrt{2}}J_\mu^\omega - \frac{1}{3}J_\mu^\phi, \]  

(27)

where

\[ J_\mu^\rho = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) \]

\[ J_\mu^\omega = +\frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \]

due to the sign ”-” in (26)

\[ J_\mu^\phi = +\bar{s}\gamma_\mu s \]

due to the sign in (25),

with (19), one obtains relations

\[ \frac{1}{f_\rho} = A\frac{1}{\sqrt{2}}; \quad \frac{1}{f_\omega} = -A\frac{1}{3\sqrt{2}}; \quad \frac{1}{f_\phi} = -A\frac{1}{3} \]  

(28)

and from them the rates (11), giving the following signs of universal vector-meson coupling constants +f_\rho, -f_\omega, -f_\phi.

Finally, starting from the \( \omega - \phi \) mixing configuration (4), and relations (13) and (14), one obtains

\[ \phi = -\bar{s}s \]  

(29)

and

\[ \omega = -\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d). \]  

(30)
Then comparing the hadronic EM current to be multiplied by the real constant $A$

$$J^h_\mu = \frac{1}{\sqrt{2}} J^\rho_\mu - \frac{1}{3\sqrt{2}} J^\omega_\mu + \frac{1}{3} J^\phi_\mu,$$  \(31\)

where

$$J^\rho_\mu = \frac{1}{\sqrt{2}} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d)$$

$$J^\omega_\mu = -\frac{1}{\sqrt{2}} (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d)$$

due to the sign "-" in (30)

$$J^\phi_\mu = -\bar{s} \gamma_\mu s$$

due to the sign "-" in (29),

with (19), one obtains relations

$$\frac{1}{f^\rho} = A \frac{1}{\sqrt{2}}; \frac{1}{f^\omega} = -A \frac{1}{3\sqrt{2}}; \frac{1}{f^\phi} = +A \frac{1}{3}$$  \(32\)

and from them the rates (12), giving the following signs of universal vector-meson coupling constants $+f^\rho, -f^\omega, +f^\phi$.

The signs of the universal vector-meson coupling constants $f^\rho, f^\omega, f^\phi$ are very important to be known, as the numerical values of these constants are regularly estimated from the experimental values [6] of the vector-meson lepton widths by means of the formula

$$\Gamma(V \rightarrow e^+e^-) = \frac{\alpha^2 m_V}{3} \left(\frac{f^2_V}{4\pi}\right)^{-1},$$  \(33\)

in which $f_V$ is contained in a quadratic form.

From (5), (6), (7), (8) it seems at first sight that numerical values of the coupling constants $f^F, f^D, f^S$ have to depend on the choice of the $\omega - \phi$ mixing version.

However, if in (5), (6), (7), (8) the coupling constants of vector-mesons with nucleons are determined e.g from $(f^\rho_{NN}/f^\rho), (f^\omega_{NN}/f^\omega), (f^\phi_{NN}/f^\phi)$, to be found in a fitting procedure of all existing data on nucleon EM structure, by means of the signs of $f^\rho, f^\omega, f^\phi$ following from (9), (10), (11), (12), as it is demonstrated above, then from all four different expressions in (5), (6), (7), (8) one obtains the same numerical values for $f^F_1, f^D_1, f^S_1$ as follows

$$f^F_1 = 5.414; \quad f^D_1 = -1.699; \quad f^S_1 = 42.916.$$  \(34\)

By means of a similar procedure one can find also numerical values of all other coupling constants under consideration

$$f^F_2 = 7.626; \quad f^D_2 = 21.088; \quad f^S_2 = -7.111;$$

$$f^F'_1 = 8.343; \quad f^D'_1 = 12.498; \quad f^S'_1 = -7.858;$$

$$f^F'_2 = -30.450; \quad f^D'_2 = -5.271; \quad f^S'_2 = -18.614.$$  \(35\)
In the paper [1] for the numerical evaluation of the vector-meson-nucleon coupling constants from the \((f_{\rho NN}/f_{\rho}), (f_{\omega NN}/f_{\omega}), (f_{\phi NN}/f_{\phi})\) we have applied the signs of the universal vector-meson coupling constants (10) (to be inspired by Close and Cottingham in [3]), which moreover have been combined with expressions for \(f^F, f^D, f^S\)

\[
\begin{align*}
 f^F &= \frac{1}{2} \left[ f_{\rho NN} + \sqrt{3} (f_{\phi NN} \cos \theta - f_{\omega NN} \sin \theta) \right] \\
 f^D &= \frac{1}{2} \left[ 3f_{\rho NN} - \sqrt{3} (f_{\phi NN} \cos \theta - f_{\omega NN} \sin \theta) \right] \\
 f^S &= \sqrt{2} \left( f_{\omega NN} \cos \theta + f_{\phi NN} \sin \theta \right)
\end{align*}
\]

to be generated by the physically non-acceptable form of the \(\omega - \phi\) mixing

\[
\begin{align*}
 \phi &= \omega_8 \cos \theta + \omega_0 \sin \theta \\
 \omega &= -\omega_8 \sin \theta + \omega_0 \cos \theta
\end{align*}
\]

used by Gasiorowicz [4] on p.325, leading to the numerical values of \(f^F, f^D, f^S\) differing from the correct values presented in this Erratum.

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