SOLVING A POSYNOMIAL GEOMETRIC PROGRAMMING PROBLEM WITH FULLY FUZZY APPROACH

Samira KAMAEI
Department of Mathematics, Faculty of Mathematical Sciences and Computer, Shahid Chamran University of Ahvaz, Ahvaz-Iran
samira.kamaee94@gmail.com

Sareh KAMAEI
Department of Mathematics, Faculty of Mathematical Sciences and Computer, Shahid Chamran University of Ahvaz, Ahvaz-Iran
sareh.kamai67@gmail.com

Mansour SARAJ
Corresponding author
Department of Mathematics, Faculty of Mathematical Sciences and Computer, Shahid Chamran University of Ahvaz, Ahvaz-Iran
msaraj@scu.ac.ir

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Abstract: In this paper we have investigated a class of geometric programming problems in which all the parameters are fuzzy numbers. In fact, due to impreciseness of the cost components and exponents in geometric programming with their inherently behavior as in economics and many other areas, we have used fuzzy parametric geometric programming. Transforming the primal problem of fuzzy geometric programming into its dual and using the Zadeh’s extension principle, we convert the dual form into a pair of mathematical programs. By applying the \( \alpha \)-cut on the objective function and \( r \)-cut on the constraints in dual form of geometric programming, we obtain an acceptable \((\alpha, r)\) optimal values. Then, we further calculate the lower and upper bounds of the fuzzy objective with emphasize on modification of a method presented in [14, 32]. Finally, we illustrate the methodology of the approach with a numerical example to clarify the idea by drawing the different steps of \( LR \) representation of \( Z_{\alpha r} \).

Keywords: Fuzzy logic, Posynomial, Geometric Programming, Optimization.
1. INTRODUCTION

The formulation of engineering design problems with specific types of non-linear optimization problems with flexible variables are known as geometric programming. Duffin et al. [9] proposed an excellent idea to solve application of engineering problems by developing basic theories of geometric programs. Since last few decades, we have seen a rapid development in geometric programming used in a variety of optimization problems involving digital circuit design [4, 5, 8], resource allocation in communication network systems [27], linear multi-objective geometric programming problems via reference point approach [2], and the problem of temperature-aware floor planning in which the parameters of the problem are often undetermined [36]. Therefore, in this paper, to clarify the subject, we consider geometric programming problems where the exponents of the variables, cost coefficients, and the constraint coefficients and their right-hand sides are all fuzzy numbers.

Due to uncertainty of the parameters of the real-world, Bellman and Zadeh investigated the problem of decision-making in a fuzzy environment and management science [3]. Fuzzy logic is a very powerful tool to handle the problem of system design in optimization of the solution of non-convex optimization problems in multiple-input multiple-output systems on using fuzzy predictive filters, which was investigated by Mendoça et al. [22]. A number of methods have been so far proposed to solve the fuzzy linear programming problems [1, 10, 12, 23], and proposing a new algorithm to solve fuzzy linear programming problems using the MOLP problem is a recent work done in [11]. Different models have been so far presented to deal with decision making problems where evaluations of alternatives are uncertain or affected by a fuzzy parameters [26]. A multi-objective problem with fuzzy parameters is being investigated by larbani [13] and Sakawa [30]. Ojha and Das [24] developed a solution procedure using geometric programming technique by splitting the coefficients and exponents with the help of binary numbers. Multi-objective geometric programming problem is worked out by Ojha et al. [25], in which they have proposed $\varepsilon$-constraint method that has been applied to find the non-inferior solution. In view of Rajgopal et al. [28], the problem of posynomial geometric programming has been studied via generalized linear program.

A lot of research works have been done in the area of risk management, inventory management and planning [29, 33]. Mahapatra and Mandal have discussed parametric functional form of an interval number and then solved the problem by geometric programming technique [17, 21]. They got optimal solution of the objective function directly without solving the equivalent transformed problem. They have also presented production inventory model with fuzzy coefficients using parametric geometric programming approach [18]. Mahapatra and Mahapatra [15] used fuzzy parametric geometric programming with cost constraint to find optimal reliability, and they have considered reliability
series system with limited system cost as a constraint function [16]. Mahapatra et al. [20, 19] investigated and developed the problem of economic production quantity model with demand dependent unit production cost under fuzzy environment. Sen and Pal [31] solved linear multi-objective fuzzy goal programming problem with interval weights. Chen and Tsai [7] studied different methodologies to derive weights or priorities of fuzzy goal programming. An essential book about fuzzy geometric programming is written by Cao in [6]. Yang and Cao [35] presented an outline of the applications of fuzzy geometric programming. Global optimization of signomial geometric programming problems is investigated by Xu [34].

Our aim is to calculate a lower bound and an upper bound for the objective function by applying \((\alpha, r)\)-cut on both fuzzy parameters of the objective function and the constraints which is based on Zadeh’s extension principle [37]. Here, we present \((\alpha, r)\) optimum value for fully fuzzy geometric programming problems in which the exponents of the variables, cost coefficients, and the constraint coefficients and the resources are all fuzzy numbers. This paper is organized as follows. We first introduce the fuzzy geometric programming problem and next we calculate the lower and upper bounds of the objective value at different \((\alpha, r)\)-levels. We draw the graph of the membership function of fuzzy objective value, and finally, the implementation of our proposed model is illustrated by a numerical example. A brief summary is presented in the conclusion.

2. MATHEMATICAL MODELLING

The general form of posynomial geometric programming problem is as follows

\[
Z = \min_t \sum_{k=1}^{l_l} c_{0k} \prod_{j=1}^{n} t_j^{a_{0kj}}
\]

Subject to

\[
\sum_{k=1}^{l_i} c_{ik} \prod_{j=1}^{n} t_j^{a_{ikj}} \leq b_i \quad i = 1, \ldots, m
\]

\[
t_j > 0 \quad j = 1, \ldots, n
\]

By the definition of posynomial all \(b_i, i = 1, 2, \ldots, m\), are positive real numbers and the exponents \(a_{ikj}, i = 0, 1, \ldots, m, j = 1, 2, \ldots, n\) and all the coefficients \(c_{ik}, i = 0, 1, \ldots, m, k = 1, 2, \ldots, l_i\), are positive. If at least one of the parameters \(a_{0kj}, a_{ikj}, b_i, c_{0k}\) or \(c_{ik}\) is fuzzy, then the objective value will be fuzzy as well. Let \(c_{0k}, c_{ik}, b_i, a_{0kj}\) and \(a_{ikj}\) be fuzzy numbers of the corresponding posynomial geometric program given by Model (1) that can be replaced by the convex fuzzy sets \(\tilde{C}_{0k}, \tilde{C}_{ik}, \tilde{B}_i, \tilde{A}_{0kj}\) and \(\tilde{A}_{ikj}\) respectively. Therefore (1) can be
reformulated as the following fuzzy geometric programming problem.

\[ \hat{Z} = \min \sum_{k=1}^{l_0} \tilde{C}_{0k} \prod_{j=1}^{n} \tilde{A}^{\hat{A}_{kj}} \]

Subject to

\[ \sum_{k=1}^{l_i} \tilde{C}_{ik} \prod_{j=1}^{n} \tilde{A}^{\hat{A}_{kj}} \leq \tilde{B}_i \quad i = 1, \ldots, m \]

\[ t_j > 0 \quad j = 1, \ldots, n \]

Since geometric programs are solved via their duals, so (2) can be written in the form of its dual as:

\[ \hat{Z} = \max \prod_{k=1}^{l_0} \left( \frac{\tilde{C}_{0k}}{\tilde{A}^{\hat{A}_{kj}}} \right) \prod_{i=1}^{m} \left( \sum_{k=1}^{l_i} \lambda_{ik} \right) \prod_{k=1}^{l_i} \left( \frac{\tilde{B}_i}{\tilde{A}_{ik}^{\hat{A}_{kj}}} \right) \]

Subject to

\[ \sum_{k=1}^{l_0} \lambda_{ik} = 1 \quad \text{(Normal Condition)} \]

\[ \sum_{i=0}^{m} \sum_{k=1}^{l_i} \tilde{A}_{kj} \lambda_{ik} = 0 \quad j = 1, \ldots, n \quad \text{(Orthogonal Conditions)} \]

\[ \lambda_{ik} \geq 0 \quad \forall i, k \]

Let \( \mu_{\tilde{C}_{0k}}, \mu_{\tilde{C}_{ik}}, \mu_{\tilde{B}_i}, \mu_{\tilde{A}_{ikj}} \) and \( \tilde{A}_{ikj} \) be membership functions of \( \tilde{C}_{0k}, \tilde{C}_{ik}, \tilde{B}_i, \tilde{A}_{ikj} \) and \( \tilde{A}_{ikj} \) respectively. Without loss of generality, all \( \tilde{C}_{0k}, \tilde{C}_{ik}, \tilde{B}_i, \tilde{A}_{ikj} \) and \( \tilde{A}_{ikj} \) \( \forall i, j, k \) in (3) are assumed to be convex fuzzy numbers. Therefore, the objective value \( \hat{Z} \) will be fuzzy as well. On applying the \( \alpha \)-cuts (\( \alpha \in [0, 1] \)) of \( \tilde{C}_{0k}, \tilde{C}_{ik}, \tilde{B}_i \), and \( r \)-cuts (\( r \in [0, 1] \)) of \( \tilde{A}_{ikj} \) \( \forall i, j, k \) and denoting them by \( \tilde{C}_{0k}^{\alpha}, \tilde{C}_{ik}^{\alpha}, \tilde{B}_i^{\alpha}, \tilde{A}_{ikj}^{\alpha} \) respectively and further, using Zadeh’s extension principle [37], we define the membership function \( \mu_{\hat{Z}} \) as follow

\[ \mu_{\hat{Z}}(z) = \sup_{a,b,c} \min \left\{ (C_{0k}^L)_\alpha \leq c_{0k} \leq (C_{0k}^U)_\alpha, (C_{ik}^L)_\alpha \leq c_{ik} \leq (C_{ik}^U)_\alpha, (B_i^L)_\alpha \leq b_i \leq (B_i^U)_\alpha, (A_{0kj}^L)_r \leq a_{0kj} \leq (A_{0kj}^U)_r, (A_{ikj}^L)_r \leq a_{ikj} \leq (A_{ikj}^U)_r, \quad \forall i,j,k \right\} \]

Since a fuzzy number is uniquely represented by its \( \alpha \)-cut, which is a closed interval for all \( \alpha \), this enables us to define arithmetic operations on fuzzy number in term
of their \((\alpha, r)\)-cuts.

\[
Z_{\alpha r} = \max_{\lambda} \prod_{k=1}^{I_0} \left( \frac{\left( (C_{0k})_{\alpha}^L, (C_{0k})_\alpha^U \right)_{\lambda_{0k}}}{\lambda_{0k}} \right) \prod_{i=1}^{I_1} \left( \sum_{k=1}^{I_1} \lambda_{ik} \right)^{\lambda_{ik}} \prod_{k=1}^{I_1} \left( \frac{\left( (C_{ik})_{\alpha}^L, (C_{ik})_\alpha^U \right)_{(B_i)_\alpha^L, (B_i)_\alpha^U} \lambda_{ik}}{} \right)_{\lambda_{ik}}
\]

Subject to

\[
\sum_{k=1}^{I_0} \lambda_{0k} = 1 \quad \text{(Normal Condition)}
\]

\[
\sum_{i=0}^{M} \sum_{k=1}^{I_1} a_{ikj} \lambda_{ik} = 0 \quad j = 1, \ldots, n \quad \text{(Orthogonal Conditions)}
\]

\[
(A_{ikj})_r^L \leq a_{ikj} \leq (A_{ikj})_r^U \quad \lambda_{ik} \geq 0 \quad \forall i, k, j
\]

(5)

In fact, calculation of \(\mu_{\tilde{Z}}\) of the form (4) is difficult. To obtain the membership function of objective value, we need to find the left shape and right shape functions of \(\mu_{\tilde{Z}}\), which is equivalent to finding the upper and lower bounds of objective value \(\tilde{Z}\) at different \((\alpha, r)\) level possibility.

3. SOLUTION METHODOLOGY

For fuzzy numbers, \(\tilde{A} = [A_{\alpha}^L, A_{\alpha}^U]\) and \(\tilde{B} = [B_{\alpha}^L, B_{\alpha}^U]\) in which \(\tilde{A} \in F(R^\geq 0)\) and \(\tilde{B} \in F(R^> 0)\), we have:

\[
\begin{pmatrix}
\tilde{A} \\
\tilde{B}
\end{pmatrix}_\alpha = \begin{pmatrix}
A_{\alpha}^L & A_{\alpha}^U \\
B_{\alpha}^L & B_{\alpha}^U
\end{pmatrix} \quad \forall \alpha \in [0, 1].
\]

Therefore, to find the lower bound of the objective value, we choose \((C_{0k})_{\alpha}^L\) as the lower bound of the interval \([(C_{0k})_{\alpha}^L, (C_{0k})_{\alpha}^U]\) and in the same manner we choose \([C_{ik}]_{\alpha}^L\) as the lower bound of \([C_{ik}]_{\alpha}^L, (C_{ik})_{\alpha}^U]\), which converts (5) in the form
of (6) as below:

\[
Z_{\alpha r}^L = \max_{\lambda} \prod_{k=1}^{l_0} \left( \frac{(C_{0k})_{\alpha}}{\lambda_{0k}} \right)^{\lambda_{0k}} \prod_{i=1}^{l_i} \left( \sum_{k=1}^{l_{ik}} \lambda_{ik} \right)^{\sum_{k=1}^{l_{ik}} \lambda_{ik}} \prod_{k=1}^{l_k} \left( \frac{(C_{ik})_{\alpha}}{(B_{ik})_{\alpha}} \right)^{\lambda_{ik}}
\]

\text{Subject to}

\[
\sum_{k=1}^{l_0} \lambda_{0k} = 1
\]

\[
\sum_{i=0}^{m} \sum_{k=1}^{l_i} a_{ikj} \lambda_{ik} = 0 \quad j = 1, \ldots, n
\]

\[
(A_{ikj})_{\alpha}^L \leq a_{ikj} \leq (A_{ikj})_{\alpha}^U
\]

\[
\lambda_{ik} \geq 0 \quad \forall i, k, j
\]

(6)

Also, to obtain the upper bound of the objective value, we choose \((C_{0k})_{\alpha}^U\) as the upper bound of the interval \([((C_{0k})_{\alpha}^L, (C_{0k})_{\alpha}^U]\) and in the same manner \([((C_{ik})_{\alpha}^L, (C_{ik})_{\alpha}^U]\) through which (5) can be reformulated as (7).

\[
Z_{\alpha}^U = \max_{\lambda} \prod_{k=1}^{l_0} \left( \frac{(C_{0k})_{\alpha}}{\lambda_{0k}} \right)^{\lambda_{0k}} \prod_{i=1}^{l_i} \left( \sum_{k=1}^{l_{ik}} \lambda_{ik} \right)^{\sum_{k=1}^{l_{ik}} \lambda_{ik}} \prod_{k=1}^{l_k} \left( \frac{(C_{ik})_{\alpha}}{(B_{ik})_{\alpha}} \right)^{\lambda_{ik}}
\]

\text{Subject to}

\[
\sum_{k=1}^{l_0} \lambda_{0k} = 1
\]

\[
\sum_{i=0}^{m} \sum_{k=1}^{l_i} a_{ikj} \lambda_{ik} = 0 \quad j = 1, \ldots, n
\]

\[
(A_{ikj})_{\alpha}^L \leq a_{ikj} \leq (A_{ikj})_{\alpha}^U
\]

\[
\lambda_{ik} \geq 0 \quad \forall i, k, j
\]

(7)

From the \((\alpha, r)\) acceptable value of \(\tilde{Z}\) for different values of \(r\), we can obtain the crisp interval \([Z_{\alpha r}^L, Z_{\alpha r}^U]\) from (6) and (7) respectively.

The feasible regions defined by \(\alpha_1\) in (6) and (7) are smaller than those defined by \(\alpha_2\) with regards to \(0 \leq \alpha_2 < \alpha_1 \leq 1\) for two possibility levels \(\alpha_1\) and \(\alpha_2\) which results \(Z_{\alpha_1 r}^L \geq Z_{\alpha_2 r}^L\) and \(Z_{\alpha_1 r}^U \leq Z_{\alpha_2 r}^U\). According to nondecreasing left shape function \(L(Z) = [Z_{\alpha r}^L]^{-1}\) and nonincreasing
right shape function $R(Z) = [Z^U_{\alpha r}]^{-1}$, the membership function $\mu_{\tilde{Z}}$ for $L(Z)$ and $R(Z)$ is constructed as:

$$\mu_{\tilde{Z}} = \begin{cases} 
L(Z) & \text{if } Z^L_{(\alpha=0)r} \leq z \leq Z^L_{(\alpha=1)r} \\
1 & \text{if } Z^L_{(\alpha=1)r} \leq z \leq Z^U_{(\alpha=1)r} \\
R(Z) & \text{if } Z^U_{(\alpha=1)r} \leq z \leq Z^U_{(\alpha=0)r}
\end{cases}$$

4. NUMERICAL EXAMPLE

Consider the following geometric programming problem with fuzzy exponents in the objective function and the constraints.

$$\min_t (36, 40, 42) t_1^{-1} t_2^{-1} t_3^{-1} + 20 t_1 t_2 t_4$$

Subject to

$$t_1^t (0.7, 0.75, 0.8) t_3 + (3, 4, 5) \tilde{t}_2^{0.5} t_4^{(-2.2, -2, -1.8)} \leq (2, 3, 5)$$

$$8 t_1^{-1.2, -1, -0.8} t_2^{-1} t_3 t_4 \leq 1$$

$$t_j > 0 \quad j = 1, \ldots, 4$$  \hspace{1cm} (8)

The dual form of (8) is as follows:

$$\tilde{Z} = \max_{\lambda} \left( \begin{array}{c}
(36, 40, 42) \\
\lambda_{01}
\end{array} \right) \lambda_{02} \left( \begin{array}{c}
20 \\
\lambda_{02}
\end{array} \right) \lambda_{11} \left( \begin{array}{c}
1 \\
(2, 3, 5) \lambda_{31}
\end{array} \right) \lambda_{12} \lambda_{121} \left( \begin{array}{c}
(3, 4, 5) \\
(2, 4, 5) \lambda_{31}
\end{array} \right)$$

Subject to

$$-\lambda_{01} + \lambda_{02} + 3 \lambda_{11} + (-1.2, -1, -0.8) \lambda_{21} = 0$$

$$(-0.6, -0.5, -0.4) \lambda_{01} + \lambda_{02} + (0.7, 0.75, 0.8) \lambda_{11} + 0.5 \lambda_{12} - \lambda_{21} = 0$$

$$-\lambda_{01} + \lambda_{11} + \lambda_{21} = 0$$

$$\lambda_{02} + (-2.2, -2, -1.8) \lambda_{12} + \lambda_{21} = 0$$

$$\lambda_{01} + \lambda_{02} = 1$$

$$\lambda_{ik} \geq 0 \quad \forall i, k$$
The $Z_L^{\alpha r}$ can be calculated by performing the Model (6) and $Z_U^{\alpha r}$ by the Model (7) as follows:

$$Z_L^{\alpha r} = \max_{\lambda} \left( \left( \frac{36 + 4\alpha}{\lambda_{01}} \right)^{\lambda_{01}} \left( \frac{20}{\lambda_{02}} \right)^{\lambda_{02}} \left( \frac{1}{(5 - 2\alpha)\lambda_{11}} \right)^{\lambda_{11}} \left( \frac{(3 + \alpha)}{(5 - 2\alpha)\lambda_{12}} \right)^{\lambda_{12}} \right) \lambda_{21} \left( \lambda_{11} + \lambda_{12} \right)^{(\lambda_{11} + \lambda_{12})}$$

**Subject to**

- $-\lambda_{01} + \lambda_{02} + 3\lambda_{11} + a_{211}\lambda_{21} = 0$
- $a_{012}\lambda_{01} + \lambda_{02} + a_{112}\lambda_{11} + 0.5\lambda_{12} - \lambda_{21} = 0$
- $-\lambda_{01} + \lambda_{11} + \lambda_{21} = 0$
- $\lambda_{02} + a_{124}\lambda_{12} + \lambda_{21} = 0$
- $\lambda_{01} + \lambda_{02} = 1$
- $(-1.2 + 0.2r) \leq a_{211} \leq (-0.8 - 0.2r)$
- $(-0.6 + 0.1r) \leq a_{012} \leq (-0.4 - 0.1r)$
- $(0.7 + 0.05r) \leq a_{112} \leq (0.8 + 0.05r)$
- $(-2.2 + 0.2r) \leq a_{124} \leq (-1.8 - 0.2r)$
- $\lambda_{ik} \geq 0 \quad \forall i, k$
| ↓ r / α → | 0.00  | 0.25  | 0.50  | 0.75  | 1.00  |
|-----------|-------|-------|-------|-------|-------|
| **state (1)** |       |       |       |       |       |
| 0.00       | 105.6194 | 117.3167 | 130.9025 | 147.0518 | 166.8195 |
| 0.25       | 107.1390 | 119.2289 | 133.2972 | 150.0532 | 170.6070 |
| 0.50       | 108.6877 | 121.1890 | 135.7644 | 153.1601 | 174.5459 |
| 0.75       | 110.2633 | 123.1957 | 138.3044 | 156.3750 | 178.6415 |
| 1.00       | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| **state (2)** |       |       |       |       |       |
| 0.00       | 105.7386 | 119.2822 | 135.2639 | 154.5846 | 178.6717 |
| 0.25       | 107.1984 | 120.6836 | 136.5628 | 155.7162 | 179.5354 |
| 0.50       | 108.7035 | 122.1429 | 137.9365 | 156.9454 | 180.5287 |
| 0.75       | 110.2572 | 123.6631 | 139.3870 | 158.2728 | 181.6501 |
| 1.00       | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| **state (3)** |       |       |       |       |       |
| 0.00       | 105.9267 | 117.7360 | 131.4705 | 147.8196 | 178.8761 |
| 0.25       | 107.3523 | 119.5248 | 133.7033 | 150.6084 | 171.3686 |
| 0.50       | 108.8167 | 121.3716 | 135.2639 | 154.5846 | 178.6717 |
| 0.75       | 110.3201 | 123.2785 | 138.4225 | 156.5417 | 180.5287 |
| 1.00       | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| **state (4)** |       |       |       |       |       |
| 0.00       | 105.9698 | 119.5997 | 135.7014 | 155.1904 | 179.5181 |
| 0.25       | 107.3642 | 120.9166 | 136.8890 | 156.1726 | 180.1775 |
| 0.50       | 108.8090 | 122.2949 | 138.1527 | 157.2511 | 180.9616 |
| 0.75       | 110.3075 | 123.7375 | 139.9496 | 158.4264 | 181.8691 |
| 1.00       | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| **state (5)** |       |       |       |       |       |
| 0.00       | 114.3829 | 126.7323 | 140.9667 | 157.7490 | 178.1094 |
| 0.25       | 114.2248 | 126.8822 | 141.5279 | 158.8661 | 179.9943 |
| 0.50       | 113.7688 | 126.7052 | 141.7319 | 159.5948 | 181.4597 |
| 0.75       | 112.9915 | 126.1725 | 141.5433 | 159.8913 | 182.4510 |
| 1.00       | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| **state (6)** |       |       |       |       |       |
| 0.00       | 114.7284 | 129.2709 | 146.3344 | 166.8364 | 192.2328 |
| 0.25       | 114.4116 | 128.6669 | 145.3769 | 165.4336 | 190.2420 |
| 0.50       | 113.8421 | 127.8104 | 144.1726 | 163.7975 | 188.0527 |
| 0.75       | 113.0002 | 126.6791 | 142.6957 | 161.8978 | 185.6189 |
| 1.00       | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| **state (7)** |       |       |       |       |       |
| 0.00       | 115.2110 | 127.7051 | 142.1192 | 159.1294 | 179.7869 |
| 0.25       | 114.7814 | 127.5464 | 142.3277 | 159.8402 | 181.1987 |
| 0.50       | 114.0739 | 127.0784 | 142.1927 | 160.1700 | 182.1883 |
| 0.75       | 113.0987 | 126.3101 | 141.7211 | 160.1229 | 182.7563 |
| 1.00       | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| $r / \alpha$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 |
|-------------|------|------|------|------|------|
| state(8)    |      |      |      |      |      |
| 0.00        | 115.5574 | 130.2476 | 147.4976 | 168.2408 | 193.9493 |
| 0.25        | 114.9536 | 129.3140 | 146.1584 | 166.3907 | 191.4352 |
| 0.50        | 114.1338 | 128.1669 | 144.6135 | 164.3502 | 188.7574 |
| 0.75        | 113.1022 | 126.8101 | 142.8654 | 162.1197 | 185.9130 |
| 1.00        | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| state(9)    |      |      |      |      |      |
| 0.00        | 104.9160 | 117.0065 | 131.1632 | 148.1343 | 169.0964 |
| 0.25        | 106.9428 | 119.3579 | 133.8905 | 151.3073 | 172.8137 |
| 0.50        | 108.7790 | 121.5175 | 136.4268 | 154.2929 | 176.3517 |
| 0.75        | 110.4205 | 123.4818 | 138.7696 | 157.0905 | 179.7126 |
| 1.00        | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| state(10)   |      |      |      |      |      |
| 0.00        | 103.7664 | 115.8436 | 130.0133 | 147.0359 | 168.1092 |
| 0.25        | 106.6384 | 120.3176 | 136.5065 | 156.1367 | 180.6869 |
| 0.50        | 108.6298 | 122.2499 | 138.3111 | 157.7123 | 181.8758 |
| 0.75        | 110.3721 | 123.8932 | 139.7803 | 158.8977 | 182.6088 |
| 1.00        | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| state(11)   |      |      |      |      |      |
| 0.00        | 102.9904 | 116.5854 | 132.7592 | 154.4804 | 179.2931 |
| 0.25        | 105.9644 | 119.6202 | 135.8004 | 155.4447 | 180.0453 |
| 0.50        | 108.5921 | 121.3571 | 136.3098 | 154.2431 | 176.4052 |
| 0.75        | 110.4038 | 123.4847 | 138.8011 | 157.1632 | 179.8456 |
| 1.00        | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| state(12)   |      |      |      |      |      |
| 0.00        | 102.7248 | 116.5854 | 132.7592 | 154.4804 | 179.2931 |
| 0.25        | 105.9644 | 119.6202 | 135.8004 | 155.4447 | 180.0453 |
| 0.50        | 108.5921 | 121.3571 | 136.3098 | 154.2431 | 176.4052 |
| 0.75        | 110.4038 | 123.4847 | 138.8011 | 157.1632 | 179.8456 |
| 1.00        | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| state(13)   |      |      |      |      |      |
| 0.00        | 118.7248 | 132.0202 | 147.4587 | 165.8024 | 188.2415 |
| 0.25        | 116.9060 | 130.2101 | 145.6895 | 164.1214 | 186.7215 |
| 0.50        | 115.1586 | 128.4802 | 144.0114 | 162.5453 | 185.3248 |
| 0.75        | 113.4788 | 126.8270 | 142.4214 | 161.0720 | 184.0508 |
| 1.00        | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
| state(14)   |      |      |      |      |      |
| 0.00        | 118.4241 | 133.7704 | 151.8835 | 173.7824 | 201.0800 |
| 0.25        | 116.7268 | 131.5398 | 148.9852 | 170.0278 | 196.1930 |
| 0.50        | 115.0668 | 129.3764 | 146.1939 | 166.4348 | 191.5437 |
| 0.75        | 113.4453 | 127.2793 | 143.5057 | 162.9947 | 187.1171 |
| 1.00        | 111.8630 | 125.2472 | 140.9166 | 159.6996 | 182.8989 |
Figure 1: Lower bounds $Z^L_{ar}$ for the objective value

$$Z^U_{ar} = \max _\lambda \left( \frac{(42-2\alpha)}{\lambda_{10}} \right)^{\lambda_{01}} \left( \frac{20}{\lambda_{02}} \right)^{\lambda_{02}} \left( \frac{1}{(2+\alpha)\lambda_{11}} \right)^{\lambda_{11}} \left( \frac{(5-\alpha)}{(2+\alpha)\lambda_{12}} \right)^{\lambda_{12}} \lambda_{21} (\lambda_{11} + \lambda_{12})^{(\lambda_{11}+\lambda_{12})}$$

subject to

$-\lambda_{01} + \lambda_{02} + 3\lambda_{11} + a_{211} \lambda_{21} = 0$

$a_{012} \lambda_{01} + \lambda_{02} + a_{112} \lambda_{11} + 0.5 \lambda_{12} - \lambda_{21} = 0$

$-\lambda_{01} + \lambda_{11} + \lambda_{21} = 0$

$\lambda_{02} + a_{124} \lambda_{12} + \lambda_{21} = 0$

$\lambda_{01} + \lambda_{02} = 1$

$(-1.2 + 0.2r) \leq a_{211} \leq (-0.8 - 0.2r)$

$(-0.6 + 0.1r) \leq a_{012} \leq (-0.4 - 0.1r)$

$(0.7 + 0.05r) \leq a_{112} \leq (0.8 - 0.05r)$

$(-2.2 + 0.2r) \leq a_{124} \leq (-1.8 - 0.2r)$

$\lambda_{ik} \geq 0 \ \forall i, k$
| ↓ \( r / \alpha \) → | 0.00  | 0.25  | 0.50  | 0.75  | 1.00  |
|----------------|------|------|------|------|------|
| **state(1)**   |      |      |      |      |      |
| 0.00            | 234.4367 | 213.6795 | 195.9357 | 180.4844 | 166.8195 |
| 0.25            | 241.2236 | 219.5022 | 200.9633 | 184.8432 | 170.6070 |
| 0.50            | 248.3574 | 225.6060 | 206.2196 | 189.3884 | 174.5459 |
| 0.75            | 255.8590 | 232.0064 | 211.7160 | 194.1280 | 178.6415 |
| 1.00            | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| **state(2)**   |      |      |      |      |      |
| 0.00            | 264.5794 | 237.7494 | 215.1239 | 195.6702 | 178.6717 |
| 0.25            | 263.9667 | 237.6592 | 215.4331 | 196.2899 | 179.5354 |
| 0.50            | 263.6334 | 238.7969 | 215.9301 | 197.0652 | 180.5287 |
| 0.75            | 263.5654 | 238.1530 | 216.6086 | 197.9927 | 181.6501 |
| 1.00            | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| **state(3)**   |      |      |      |      |      |
| 0.00            | 236.4063 | 215.3328 | 197.3399 | 181.6888 | 167.8621 |
| 0.25            | 242.6814 | 220.7212 | 201.9949 | 185.7254 | 171.3686 |
| 0.50            | 249.3105 | 226.3996 | 206.8886 | 189.9584 | 175.0364 |
| 0.75            | 256.3299 | 232.3907 | 212.0384 | 194.4016 | 185.8761 |
| 1.00            | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| **state(4)**   |      |      |      |      |      |
| 0.00            | 266.2284 | 239.1141 | 216.2713 | 196.6489 | 179.5181 |
| 0.25            | 265.2308 | 238.7038 | 216.3094 | 197.0351 | 180.1775 |
| 0.50            | 264.4936 | 238.5068 | 216.5245 | 197.5693 | 180.9616 |
| 0.75            | 264.0040 | 238.5146 | 216.9108 | 198.2484 | 181.8691 |
| 1.00            | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| **state(5)**   |      |      |      |      |      |
| 0.00            | 247.7874 | 226.5879 | 208.3391 | 192.3436 | 178.1094 |
| 0.25            | 252.6213 | 230.4286 | 211.3893 | 194.7536 | 179.9943 |
| 0.50            | 256.9593 | 233.7884 | 213.9771 | 196.7221 | 181.4597 |
| 0.75            | 260.7087 | 236.5864 | 216.0320 | 198.1871 | 182.4510 |
| 1.00            | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| **state(6)**   |      |      |      |      |      |
| 0.00            | 283.0472 | 254.8707 | 230.9833 | 210.3430 | 192.2328 |
| 0.25            | 278.3573 | 251.0478 | 227.8776 | 207.8424 | 190.2420 |
| 0.50            | 273.6233 | 247.1202 | 224.6223 | 205.1588 | 188.0527 |
| 0.75            | 268.7782 | 243.0312 | 221.1689 | 202.2502 | 186.6189 |
| 1.00            | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| **state(7)**   |      |      |      |      |      |
| 0.00            | 250.4670 | 228.9409 | 210.4255 | 194.2082 | 179.7869 |
| 0.25            | 254.6041 | 232.1547 | 212.9077 | 196.1009 | 181.1987 |
| 0.50            | 258.2074 | 234.8628 | 214.9125 | 197.5442 | 182.1883 |
| 0.75            | 261.2633 | 237.0561 | 216.4347 | 198.5361 | 182.7563 |
| 1.00            | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| $r / \alpha$ | 0.00  | 0.25  | 0.50  | 0.75  | 1.00  |
|------------|-------|-------|-------|-------|-------|
| state(8)   |       |       |       |       |       |
| 0.00       | 285.8771 | 257.3336 | 233.1513 | 212.2695 | 193.9493 |
| 0.25       | 280.3570 | 252.7772 | 229.3912 | 209.1805 | 191.4352 |
| 0.50       | 274.8447 | 248.1666 | 225.5301 | 205.9549 | 188.7574 |
| 0.75       | 269.3164 | 243.4856 | 221.5755 | 202.5866 | 185.9130 |
| 1.00       | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| state(9)   |       |       |       |       |       |
| 0.00       | 240.7453 | 218.5521 | 199.7133 | 183.4169 | 169.0964 |
| 0.25       | 246.6418 | 223.7829 | 204.3729 | 187.5774 | 172.8137 |
| 0.50       | 252.4309 | 228.8802 | 208.8796 | 191.5700 | 176.3517 |
| 0.75       | 258.1283 | 233.8553 | 213.2407 | 195.3992 | 179.7126 |
| 1.00       | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| state(10)  |       |       |       |       |       |
| 0.00       | 267.4016 | 239.6046 | 216.2949 | 196.3590 | 179.0272 |
| 0.25       | 267.5801 | 240.3064 | 217.4246 | 197.7975 | 180.6869 |
| 0.50       | 266.9079 | 240.3716 | 217.9796 | 198.7161 | 181.8758 |
| 0.75       | 265.6431 | 239.8330 | 217.9522 | 199.1345 | 182.6088 |
| 1.00       | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| state(11)  |       |       |       |       |       |
| 0.00       | 240.1346 | 217.7751 | 198.8280 | 182.4652 | 168.1092 |
| 0.25       | 246.7571 | 223.7357 | 204.2109 | 187.3349 | 172.5165 |
| 0.50       | 252.8307 | 229.1515 | 209.0559 | 191.6757 | 176.4052 |
| 0.75       | 258.4634 | 234.1180 | 213.4485 | 195.5348 | 179.8456 |
| 1.00       | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| state(12)  |       |       |       |       |       |
| 0.00       | 265.4099 | 237.6435 | 214.3930 | 194.5348 | 177.2931 |
| 0.25       | 267.0008 | 239.7216 | 216.7982 | 197.1545 | 180.0453 |
| 0.50       | 267.9929 | 240.4556 | 217.9929 | 198.6805 | 181.8072 |
| 0.75       | 265.9384 | 240.0606 | 218.1621 | 199.2730 | 182.7183 |
| 1.00       | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| state(13)  |       |       |       |       |       |
| 0.00       | 264.9264 | 241.4013 | 221.2802 | 203.7503 | 188.2415 |
| 0.25       | 264.3186 | 240.4605 | 220.0902 | 202.3724 | 186.7215 |
| 0.50       | 263.9157 | 239.6973 | 219.0563 | 201.1328 | 185.3248 |
| 0.75       | 263.7241 | 239.1156 | 218.1801 | 200.0316 | 184.0508 |
| 1.00       | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
| state(14)  |       |       |       |       |       |
| 0.00       | 298.3973 | 268.0098 | 242.3793 | 220.3392 | 201.0800 |
| 0.25       | 288.9065 | 260.0235 | 235.6173 | 214.5939 | 196.1930 |
| 0.50       | 280.0005 | 252.5039 | 229.2290 | 209.1476 | 191.5437 |
| 0.75       | 271.6300 | 245.4138 | 223.1864 | 203.9792 | 187.1171 |
| 1.00       | 263.7508 | 238.7196 | 217.4640 | 199.0698 | 182.8989 |
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Figure 2: Upper bounds $Z^U_{\alpha r}$ for the objective value

|   | 0.00       | 0.00       | 0.25       | 0.50       | 0.75       | 1.00       |
|---|------------|------------|------------|------------|------------|------------|
| 0.00 | 299.6849  | 269.0237  | 243.1893  | 220.9960  | 201.6211  |
| 0.25 | 289.9987  | 260.8982  | 236.3279  | 215.1793  | 196.6821  |
| 0.50 | 280.7968  | 253.1492  | 229.7594  | 209.5891  | 191.9162  |
| 0.75 | 272.0555  | 245.7617  | 223.4748  | 204.2212  | 187.3226  |
| 1.00 | 263.7508  | 238.7196  | 217.4640  | 199.0698  | 182.8989  |
The upper and lower bounds of the objective value for different levels of \((\alpha, r)\)-values are obtained and illustrated in the Figure 3.

The Figures 4 and 5 represent the membership function of \(Z_{\alpha r}^L\) and \(Z_{\alpha r}^U\).
5. CONCLUSION

Due to uncertainty of design parameters and the closeness of fuzzy logic concept to such problems, which have many applications in engineering design, economics and management, we decided to study geometric programming with full fuzziness in exponents and coefficients of objective function and constraints as well. In fact, the full fuzziness in geometric programming helps us to get the result that is much closer to the real optimal solution of the problem due to uncertainty of the parameters in the real physical world.

A very clear representation of fuzzy behavior of the objective function and membership values is given for different steps of LR fuzzy types in Figures 1 to 4. We compared our results with (Liu 2007) and got much more accurate result for optimum value of the problem. The extension of this problem can be applied to interval valued geometric programming and fractional geometric programming, too.

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