Current Density Distributions and a Supersymmetric Action for Interacting Brane Systems

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Abstract

We propose a method to obtain a manifestly supersymmetric action functional for interacting brane systems. It is based on the induced map of the worldvolume of low-dimensional branes into the worldvolume of the space-time filling brane ((D-1)-brane), which may be either dynamical or auxiliary, and implies an identification of Grassmann coordinate fields of lower dimensional branes with an image of the Grassmann coordinate fields of that (D-1)-brane. With this identification the covariant current distribution forms with support on the superbrane worldvolumes become invariant under the target space supersymmetry and can be used to write the coupled superbrane action as an integral over the D-dimensional manifolds ((D-1)-brane worldvolume). We compare the equations derived from this new (‘Goldstone fermion embedded’) action with the ones produced by a more straightforward generalization of the free brane actions based on the incorporation of the boundary terms with Lagrange multipliers (‘superspace embedded’ action). We find that both procedures produce the same equations of motion and thus justify each other. Both actions are presented explicitly for the coupled system of a $D = 10$ super-D3-brane and a fundamental superstring which ends on the super-D3-brane.

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Introduction

Intensive studies of interacting branes (intersecting branes and branes ending on branes) \[1\]–\[6\] were performed for the pure bosonic limit \[7, 1, 3\] or in the framework of the 'probe brane' approach \[2, 5\]. In spite of many successes achieved in this way, it is desirable to obtain a complete and manifestly supersymmetric description of the interacting brane systems at the level of (quasi)classical effective action.

Actually, the preservation of symmetries in the presence of boundaries (including the boundaries of open branes ending on other branes) requires a consideration of anomalies \[10, 2\], while at the classical level the boundary breaks at least half of the supersymmetry \[11, 12, 2\] (see Appendix A). So at that level one may search for an action for a coupled brane system, which includes manifestly supersymmetric bulk terms for all the branes and allows direct variations.

In this paper we propose a procedure to obtain such a supersymmetric action for an interacting brane system. It involves a (dynamical or auxiliary) space-time filling brane ('(D-1)-brane dominance') and uses the identification of all the Grassmann coordinate fields of lower dimensional branes $\tilde{\Theta}(\xi)$ with images of the (D-1)-brane Grassmann coordinate fields $\tilde{\Theta}(\xi) = \Theta(x(\xi))$. In this sense intersecting branes are considered as embedded into the Goldstone fermion theory \[8\] (which is just the space-time filling superbrane \[9\]) rather than into superspace. As a result we obtain less fermionic equations than expected. The equations can be split in separate ones for the open brane and the host brane, but with an indefinite source localized at the intersection.

To justify the above result we turn to a more straightforward generalization of the free superbrane actions to the coupled brane system. It produces the necessary identification of the supercoordinate function at the intersection by an incorporation of a bosonic vector 1-form and a Grassmann spinor 1-form Lagrange multipliers into the boundary term. (To our knowledge such a quite simple action was not considered before). We find that the Lagrange multipliers involve an ambiguity in the equations of motion. Due to that ambiguity the equations of motion obtained from those ('superspace embedded' or SSPE) action and ones derived from the above mentioned ('Goldstone fermion embedded' or GFE) action functional turns out to be equivalent. Thus the two approaches justify each other.

We find that an ambiguity in the sources localized at the intersection appears in the bosonic coordinate equations as well and, thus, have to be taken into account even in the pure bosonic limit of the coupled brane system.

We present an explicit form of both SSPE and GFE actions for the system of the closed super-D3-brane and the fundamental superstring ending on the super-D3-brane \[1\]. The latter is of particular interest for String/M-theory \[14\], its applications to gauge theory \[15\], as well as in the frame of the Maldacena conjecture \[16\]. To be concrete, we describe our approach just for this specific system.

1 An action with Lagrange multipliers

The actions of a free type $IIB$ superstring and a free super-D3-brane can be presented as integrals of a Lagrangian 2–form $L_{2}^{IIB}$ and 4-form $L_{4}^{D3}$

$$
S_{0}^{IIB} = \int_{M^{1+1}} L_{2}^{IIB}, \quad S_{0}^{D3} = \int_{M^{1+3}} L_{4}^{D3},
$$

over the the worldsheet $M^{1+1} = (\xi^{\mu}) = (\tau, \sigma)$ and the D3-brane worldvolume $M^{1+3} = \zeta^{m}$ ($m = 0, \ldots, 3$). They should be regarded as surfaces embedded into the $D = 10$ type $IIB$

\[1\] This system is a special one for string perturbation theory. Here D-branes are considered as (sub)manifolds where the open string endpoints live upon and can be described by open string states (see \[13\] and refs. therein). However, for the description in the language of brane effective action functionals, which is considered here, this system provides a quite generic example of interacting branes (brane democracy \[13\]).
The target superspace should have the same image on $\partial M$ functions (3), (4) which define the embeddings of the worldsheet and the worldvolume into the $M^{1+9\,32}$.

In the following, for simplicity, we skip the index $j$ (connected components of the superstring worldsheet boundary). For the case of one open superstring ending on the D3-brane $j = 1, 2$. Each worldline can be parametrized by the proper time $\tau_j$. Thus the manifold $\partial M^{1+1}$ can be defined parametrically as a submanifold of the worldsheet $M^{1+1}$

$$\partial M^{1+1} \in M^{1+1}: \quad \xi^\mu = \xi^\mu(\tau_j),$$

and as a submanifold of the worldvolume $M^{1+3}$

$$\partial M^{1+1} \in M^{1+3}: \quad \zeta^m = \zeta^m(\tau_j).$$

In the following, for simplicity, we skip the index $j$. The bosonic and fermionic coordinate functions (3), (4) which define the embeddings of the worldsheet and the worldvolume into the target superspace should have the same image on $\partial M^{1+1}$ (i.e. should coincide when restricted to $\partial M^{1+1}$)

$$\dot{X}^m(\xi^\mu(\tau)) = \dot{X}^m(\zeta^m(\tau)) \equiv \dot{X}^m(\tau), \quad \dot{\Theta}^\mu_\mu(\xi^\mu(\tau)) = \dot{\Theta}^\mu_\mu(\zeta^m(\tau)) \equiv \dot{\Theta}^\mu_\mu(\tau).$$

The above mentioned problem with the action (5) appears because, due to the identification (3), the variations $\delta \dot{X}^m(\xi^\mu)$, $\delta \dot{\Theta}^\mu_\mu(\xi^\mu)$ and $\delta \dot{X}^m(\zeta^m)$, $\delta \dot{\Theta}^\mu_\mu(\zeta^m)$ may not be regarded as completely independent ones.

The straightforward way to take Eqs. (3) into account is to incorporate them into the action (5) by means of a bosonic and a fermionic Lagrangian multiplier 1-form $\dot{P}_{1m} = d\tau \dot{\pi}_m$ and $\dot{\pi}_m = d\tau \dot{\pi}_m$

$$S^\pi = \int_{M^{1+1}} L_{IB}^2 + \int_{M^{1+3}} L_{D3}^4 + \int_{\partial M^{1+1}} A +$$

$$+ \int_{\partial M^{1+1}} \dot{P}_{1m} \left( \dot{X}^m(\xi^\mu(\tau)) - \dot{X}^m(\zeta^m(\tau)) \right) + \int_{\partial M^{1+1}} i \dot{\pi}_m \left( \dot{\Theta}^\mu_\mu(\xi^\mu(\tau)) - \dot{\Theta}^\mu_\mu(\zeta^m(\tau)) \right).$$

However, as we will see below, these Lagrange multipliers cannot be determined from the equations of motion. Because, in addition, their nature may seem unclear, doubts could arise whether the Lagrange multiplier method is applicable at all here. An example of a system where this method indeed fails is provided by self-dual gauge fields, whose covariant description at the level of the action functional required the development of a special (PST) approach [18].
Thus to justify the applicability of the action (10) it is useful to describe the interacting branes in a different manner.

In fact, in order to be able to vary the action (5) directly, one could try (instead of using Lagrange multipliers (10)) to find a supersymmetric way to write all the terms as integrals over a manifold containing both the super-D3-brane worldvolume and the superstring worldsheet.

It turns out that this is indeed possible. Moreover, the dynamical system then may be extended possibly by inclusion of an action for supergravity.

2 Space-time filling branes and induced embeddings

We find it useful to first extend our system by inclusion of the super-D9-brane, which is a space-time filling brane of the $D = 10$ type IIB superspace

$$S = \int_{M^{1+9}} L_{10} + \int_{M^{1+1}} L_{IIB}^2 + \int_{M^{1+3}} L_{D3}^4 + \int_{\partial M^{1+1}} A. \tag{11}$$

Here $L_{10}$ is the super-D9-brane Lagrangian form (see e.g. [19]). The essential point is that the super-D9-brane implies the existence of the map of a $d = 10$ dimensional bosonic surface $M^{1+9} = \{ x^\bar{m} \}$ ($\bar{m} = 0, \ldots, 9$) into type IIB superspace

$$M^{1+9} \rightarrow M^{(1+9|32)} : \quad \hat{X}^\bar{m} = \hat{X}^\bar{m}(\hat{x}^\bar{m}), \quad \bar{\Theta}^{I\mu} = \bar{\Theta}^{I\mu}(\hat{x}^\bar{m}), \tag{12}$$

with an invertible function $X^\bar{m} = \hat{X}^\bar{m}(\hat{x}^\bar{m})$. This allows the definition of an induced embedding of the superstring worldsheet and the super-D3-brane worldvolume into the bosonic surface $M^{1+9}$ (D9-brane worldvolume)

$$x^\bar{m} = \hat{x}^\bar{m}(\xi) \quad \leftrightarrow \quad \hat{X}^\bar{m}(\xi) = \hat{X}^\bar{m}((\hat{x}^\bar{m}(\xi)), \tag{13}$$

$$x^\bar{m} = \bar{x}^\bar{m}(\zeta) \quad \leftrightarrow \quad \hat{X}^\bar{m}(\zeta) = \hat{X}^\bar{m}((\hat{x}^\bar{m}(\zeta)), \tag{14}$$

and to consider the superstring and super-D3-brane coordinate functions as images of the functions defined on $M^{1+9}$ on the worldsheet and on the worldvolume, respectively:

$$\hat{X}^\bar{m}(\xi) = \hat{X}^\bar{m}(\hat{x}^\bar{m}(\xi)), \quad \hat{\Theta}^{I\mu}(\xi) = \hat{\Theta}^{I\mu}(\hat{x}^\bar{m}(\xi)), \tag{15}$$

$$\hat{X}^\bar{m}(\zeta) = \hat{X}^\bar{m}(\hat{x}^\bar{m}(\zeta)), \quad \hat{\Theta}^{I\mu}(\zeta) = \hat{\Theta}^{I\mu}(\hat{x}^\bar{m}(\zeta)). \tag{16}$$

Actually, what we need are the induced embeddings (13), (16). Below we will treat the 9-brane as auxiliary and drop the Lagrangian $L_{10}$ altogether. The study of the interaction of the fundamental string with this super-D9-brane in the framework of the present approach is the subject of another paper [20].

An interesting alternative for future study would be to consider $L_{10}$ in (11) as a Lagrangian form of a counterpart of the group manifold action [21] for $D = 10$ type IIB supergravity, which assumes the map (12) as well. This provides the possibility to generalize our consideration for the case of curved superspace. Thus the construction of such group manifold action for type IIB supergravity on the basis of the PST action [22] seems to be another promising problem, which we however do not address here.

3 Current form distributions and supersymmetry

3.1 Covariant current distribution forms

To write the action for the coupled system (11) in a unique form, let us define first the 10-dimensional manifestly covariant current densities with support on the superstring worldsheet
and on the super-D3-brane worldvolume respectively (see \cite{22} for the bosonic M2-brane and M5-brane \cite{24} interacting with D=11 supergravity)

\[ J_8 = (dx)^\wedge 8_{\bar{m}m} = (dx)^\wedge 8_{\bar{m}m} \int_{M^{1+9}} d\tilde{x}^\bar{m}(x) \wedge d\tilde{x}^m(x) \delta^{10}(x - \hat{x}(\xi)), \]  

(17)

\[ J_6 = (dx)^\wedge 6_{\bar{m}_1 \ldots \bar{m}_4} = (dx)^\wedge 6_{\bar{m}_1 \ldots \bar{m}_4} \int_{M^{1+3}} d\tilde{x}^\bar{m}_1(\xi) \wedge \ldots \wedge d\tilde{x}^\bar{m}_4(\xi) \delta^{10}(x - \hat{x}(\xi)), \]  

(18)

with

\[ (dx)^\wedge n_{\bar{m}_1 \ldots \bar{m}_{10-n}} = \frac{1}{n!(10-n)!} \epsilon_{\bar{m}_1 \ldots \bar{m}_{10-n} \bar{m}_n} dx^\bar{m}_1 \wedge \ldots \wedge dx^\bar{m}_n \]  

(19)

Their main properties are

\[ \int_{M^{1+9}} J_8 \wedge L_2 = \int_{M^{1+9}} \hat{L}_2, \quad \int_{M^{1+9}} J_6 \wedge L_4 = \int_{M^{1+9}} \hat{L}_4, \]  

(20)

where

\[ L_2 = \frac{1}{2} dx^\bar{m} \wedge dx^m L_{\bar{m}m}(x), \quad L_4 = \frac{1}{4!} dx^\bar{m}_1 \wedge \ldots \wedge dx^\bar{m}_4 L_{\bar{m}_4 \ldots \bar{m}_1}(x) \]  

(21)

are arbitrary 2-form and 4-forms on the bosonic surface \( M^{1+9} \) and

\[ \hat{L}_2 = \frac{1}{2} dx^\bar{m}(\xi) \wedge dx^m(\xi) L_{\bar{m}m}(x), \quad \hat{L}_4 = \frac{1}{4!} dx^\bar{m}_1(\xi) \wedge \ldots \wedge dx^\bar{m}_4(\xi) L_{\bar{m}_4 \ldots \bar{m}_1}(x) \]  

(22)

are their pull-backs onto the worldsheet and the worldvolume, respectively.

### 3.2 Brane boundary and current (non)conservation

If we assume that the worldvolume of a brane, say super-D3-brane, is closed \( \partial M^{1+3} = 0 \), then the brane current \( J^{\bar{m}_1 \ldots \bar{m}_4} \) is conserved, \( \partial \bar{m}_4 J^{\bar{m}_1 \ldots \bar{m}_4} = 0 \), and thus the current density form \( J_6 \) is a closed form \( dJ_6 = 0 \).

This is not true for open branes. Here we are interested in the open superstring case \( \partial M^{1+1} = \{ \tau \} \neq 0 \). Substituting the closed two form, say \( dA \), instead of \( L_2 \) into \( J_8 \) and using Stokes' theorem one arrives at

\[ \int_{\partial M^{1+1}} A = \int_{M^{1+1}} dA = \int_{M^{1+9}} J_8 \wedge dA = \int_{M^{1+9}} dJ_8 \wedge A. \]  

(23)

Eq. (23) demonstrates that the form \( dJ_8 \) has a support localized at the boundary of the worldsheet, i.e., on the worldline of the string endpoints parametrized by the proper time \( \tau \). This again can be justified by an explicit calculation with Eqs. (17), which results in

\[ dJ_8 = -(dx)^\wedge 9_{\bar{m}} \int_{\partial M^{1+1}} d\tilde{x}^m(\tau) \delta^{10}(x - \hat{x}(\tau)). \]  

(24)

For the description of the above situation it is useful to introduce also superstring and super-D3-brane current form distributions \( j_1 \) and \( j_3 \) with support on the boundary of the superstring worldsheet (3)

\[ j_1 = dx^\mu \epsilon_{\mu
u} \int_{\partial M^{1+1}} d\tilde{x}^\nu(\tau) \delta^2(\xi - \tilde{\xi}(\tau)), \quad \xi^\mu = (\tau, \sigma) \]  

(25)

\[ j_3 = dx^m \int_{\partial M^{1+1}} d\tilde{x}^m(\tau) \delta^4(\zeta - \tilde{\zeta}(\tau)), \quad \zeta^m = (\zeta^0, \ldots, \zeta^3) \]  

(26)

with the properties

\[ \int_{M^{1+1}} j_1 \wedge \hat{A} = \int_{\partial M^{1+1}} \hat{A}, \quad \int_{M^{1+3}} j_3 \wedge A = \int_{\partial M^{1+1}} A. \]  

(27)
Collecting Eqs. [27] and [24]

\[ \int_{M^{1+1}} d\tilde{A} = \int_{M^{1+9}} dJ_8 \wedge A = \int_{M^{1+3}} j_3 \wedge \tilde{A} = \int_{M^{1+1}} j_1 \wedge \tilde{A} \]  

(28)

one can write down formal relations between current distribution forms

\[ dJ_8 = j_1 \wedge J_8 = j_3 \wedge J_6 \]  

(29)

Here formal extrapolations of the relations (20) to arbitrary worldvolume forms have been used.

Eq. (29) represents in a compact and transparent manner the nonconservation of the superstring current form due to the presence of the worldsheet boundary. Let us stress, however, that it is really true in the sense of the integrated equations (28) with a test 1-form \( A \).

### 3.3 Supersymmetric invariance of distribution forms

Performing a general coordinate transformation, the current densities can be expressed in terms of the coordinate fields \( X^m \) as, e.g.

\[ J_8 = (dX)^{8\mu} J^{\mu \nu}(X) = (dX)^{8\mu} \int_{M^{1+1}} d\tilde{X}^m(\xi) \wedge d\tilde{X}^\mu(\xi) \delta^{10} \left( X - \tilde{X}(\xi) \right) . \]  

(30)

In (30) one recognizes the current densities used for the description of intersection of the bosonic branes [23]. It does not include the Grassmann coordinate fields \( \Theta^I, \hat{\Theta}^I \) and this may lead to doubts concerning its invariance under supersymmetry, which, however, holds after the identification (15), (16), as will be seen below.

The variation of the form (30) can be written as

\[ \delta J_8 = 3(dX)^{8\mu} \partial_\nu \int_{M^{1+1}} d\tilde{X}^m(\xi) \wedge d\tilde{X}^\mu(\xi) \left( \delta X^\mu - \delta \tilde{X}^\mu(\xi) \right) \delta^{10} \left( X - \tilde{X}(\xi) \right) - 2(dX)^{8\mu} \int_{\partial M^{1+1}} d\tilde{X}^m(\tau) \left( \delta X^\mu - \delta \tilde{X}^\mu(\tau) \right) \delta^{10} \left( X - \tilde{X}(\tau) \right) . \]  

(31)

The target superspace supersymmetry transformations

\[ \delta X^m = \Theta^I \sigma^I e^I, \quad \delta \Theta^I = e^I \]  

(32)

imply the transformations of the superstring coordinate fields

\[ \delta \tilde{X}^m(\xi) = \tilde{\Theta}^I(\xi) \sigma^I e^I, \quad \delta \tilde{\Theta}^I(\xi) = e^I, \]  

(33)

and the ones of the (auxiliary) 9-brane

\[ \delta X^m(x) = \Theta^I(x) \sigma^I e^I, \quad \delta \Theta^I(x) = e^I. \]  

(34)

In the parametrization of the 9-brane worldvolume by \( X^m \) coordinates, which is possible due to the invertibility (12) of the embedding function \( X(x) \), the transformation (34) coincides with the Goldstone fermion realization [8] of the type IIB supersymmetry

\[ \delta X^m(X) = \Theta^I(X) \sigma^I e^I, \quad \delta \Theta^I(X) = e^I. \]  

(35)

The variation of the current form (30) under the transformations (33) (cf. (31)) becomes

\[ \delta J_8 = 3(dX)^{8\mu} \partial_\nu \int_{M^{1+1}} d\tilde{X}^m(\xi) \wedge d\tilde{X}^\mu(\xi) \left( \Theta^I(X) - \tilde{\Theta}^I(\xi) \right) \sigma^I e^I \delta^{10} \left( X - \tilde{X}(\xi) \right) - (36) \]
\[ -2(dX)^{\wedge 8}_{mn} \int_{\partial M^{1+1}} d\tilde{X}^{m}(\tau) \left( \Theta^{I}(X) - \tilde{\Theta}^{I}(\tau) \right) \sigma^{m} \sigma^{n} \delta^{10} \left( X - \tilde{X}(\tau) \right). \]

The key observation is that if one identifies the Grassmann coordinates fields of the lower dimensional branes $\tilde{\Theta}^{I}(\xi)$, $\tilde{\Theta}^{I}(\zeta)$ with the images of the 9-brane Grassmann coordinate fields on the worldvolumes (Goldstone fermions [8]) $\Theta^{I}(X)$

\[ \hat{\Theta}^{I}(\xi) = \Theta^{I}(\hat{X}(\xi)), \quad \hat{\Theta}^{I}(\zeta) = \tilde{\Theta}^{I}(\tilde{X}(\zeta)) \]  

(37)

one finds that the current forms $J_{8}$ and $J_{6}$ are supersymmetric invariant!

Such an invariance is quite evident in the representation (17), (18), as the coordinates $x^{n}$ (12) are inert under the target space supersymmetry. The identification (37) (see (13), (14))

\[ \hat{\Theta}^{I}(\xi) = \Theta^{I}(\hat{X}(\xi)), \quad \tilde{\Theta}^{I}(\zeta) = \tilde{\Theta}^{I}(\tilde{X}(\zeta)) \]  

(38)

is implied here by the assumption that it is possibile to lift the complete superbrane actions to the 10-dimensional integral form using the relations (20). The manifestly supersymmetric form of the current densities appears after passing to the supersymmetric basis of the space tangent to $M^{1+9}$

\[ \Pi^{m} = dx^{m} \Pi^{m}_{m} = dX^{m} - id\Theta^{I} \sigma^{m} \Theta^{I}, \quad \Pi^{m}_{n} = \partial_{n}X^{m} - i\partial_{n}\Theta^{I} \sigma^{m} \Theta^{I}. \]  

(39)

We arrive at

\[ J_{8} = (\Pi)^{\wedge 8}_{mn} \frac{1}{\det(\Pi^{\wedge 2})} \int_{M^{1+1}} \tilde{\Pi}^{m} \wedge \tilde{\Pi}^{m}_{n} \delta^{10} (x - \hat{X}(\xi)), \]  

(40)

\[ J_{6} = (\Pi)^{\wedge 6}_{m_{1}m_{2}...m_{4}} \frac{1}{\det(\Pi^{\wedge 2})} \int_{M^{1+1}} \tilde{\Pi}^{m_{1}} \wedge ... \wedge \tilde{\Pi}^{m_{4}} \delta^{10} (x - \tilde{X}(\zeta)). \]  

(41)

4 Lagrangian forms and action for the interacting system

Thus, if we assume that the Lagrangian form for superstring $L_{IIB}^{2}$ and super-D3-brane $L_{D}^{D3}$ can be presented as the pull-back of some 10-dimensional 2-form and 4-form living on the bosonic surface $M^{1+9}$ (see [21], [22]), we can write the action for the coupled system [8] in a way which allows direct variation

\[ S^{G} = \int_{M^{1+9}} \left[ J_{8} \wedge L_{IIB}^{2} + J_{6} \wedge L_{D}^{D3} + dJ_{8} \wedge A \right] \]  

(42)

The above requirement is not satisfied by the leading (kinetic) terms of the standard actions [11, 17]

\[ L_{IIB}^{11} = d^{2}\xi \sqrt{\det(\hat{g}_{\mu\nu})} - B_{2}, \]  

(43)

\[ L_{D}^{D3} = d^{4}\xi \sqrt{\det(g_{mn} + F_{mn})} + e^{\mathcal{F}} \wedge C \big|_{4} \]  

(44)

where

\[ \hat{g}_{\mu\nu} = \hat{\Pi}^{m}_{\mu} \hat{\Pi}^{m}_{\nu}, \quad \hat{\Pi}^{m} = d\xi^{m} \hat{\Pi}^{m}, \quad g_{mn} = \Pi^{m}_{m} \Pi^{n}_{n}, \quad \Pi^{m} = d\zeta^{m} \Pi^{m} \]  

(45)

are the superstring and the super–D3–brane induced metrics and $\mathcal{F}$ is the generalized field strength of the gauge field $A$

\[ \mathcal{F} = dA - B_{2} \]  

(46)

The D3–brane Wess-Zumino term is defined by [17]

\[ e^{\mathcal{F}} \wedge C \big|_{4} = C_{4} + C_{2} \wedge \mathcal{F} + C_{0} \wedge \mathcal{F} \wedge \mathcal{F}. \]  

(47)
Here $C_{2k}$ are RR gauge fields of type IIB supergravity with a flat superspace field strength
\[ R = \bigoplus_{n=0}^{5} R_{2n+1} = e^{-F} \wedge d(e^{-F} \wedge C) = 2i d\Theta^{2\mu} \wedge d\Theta^{1\nu} \wedge \bigoplus_{n=0}^{4} \sigma^{(2n+1)}, \] (48)
while $B_2$, entering (43), (46), is the NS-NS gauge field, whose flat superspace value is
\[ B_2 = i \Pi^m \wedge \left( d\Theta^1 \sigma_m \Theta^1 - d\Theta^2 \sigma_m \Theta^2 \right) + d\Theta^1 \sigma^m \Theta^1 \wedge d\Theta^2 \sigma^m \Theta^2 \] (49)
\[ H_3 = dB_2 = i \Pi^m \wedge \left( d\Theta^1 \sigma_m \wedge d\Theta^1 - d\Theta^2 \sigma_m \wedge d\Theta^2 \right). \] (50)

One can actually consider the action (42) with Lagrangian form (43), (44), extending formally the relations (20) to arbitrary forms on the worldsheet and worldvolume respectively. However, a more rigorous procedure (which actually could motivate formal manipulations of this type also in another context) consists in searching for an equivalent representation of the superstring and superbrane actions, whose Lagrangian form can be considered as a pull–back of the 10-dimensional forms. Fortunately such actions do exist. They were proposed in the frame of the Lorentz harmonic approach for superstrings [25] (see also [28, 29, 31]) and super-Dp-branes [26, 31, 19] respectively.

Thus in the Lorentz harmonic approach the action (42) of the interacting system of the super-D3-brane and the fundamental superstring ending on the super–D3–brane becomes
\[ S^G = \int_{M^{1+9}} J_6 \wedge \left[ E^{A \wedge 4} \sqrt{-\det(\eta_{ab} + F_{ab})} + Q_2 \wedge \left( dA - B_2 - \frac{1}{2} E^a \wedge E^b F_{ba} \right) + e^{-F} \wedge C \right] + \right. \] \[ + \int_{M^{10}} J_8 \wedge \left( \frac{1}{2} E^{++} \wedge E^{--} - B_2 \right) + \int_{M^{1+9}} dJ_8 \wedge A. \] (51)

Here $Q_2$ is a 2-form Lagrange multiplier, $F_{ab} = -F_{ba}$ is an auxiliary $d = 4$ antisymmetric tensor field and
\[ E^a = \Pi^m u^a_m, \quad E^{\pm\pm} = \Pi^m U^{\pm\pm}_m, \] (52)
where
\[ u^a_m(\xi) \equiv (u^a_m, u^i_m) \in SO(1, D - 1) \] (53)
\[ \Rightarrow u^a_m u^{am} = \eta^{ab}, \quad u^a_m u^{im} = 0, \ldots, \quad i = 1, \ldots, 6 \] (54)
\[ U^a_m(\xi) \equiv (U_+^m, U_-^m, U^+_m) \in SO(1, D - 1) \] (55)
\[ \Rightarrow U_+^m U_+^{+m} = 0, \quad U_-^m U_-^{-m} = 0, \quad U^+_m U^{\pm\pm}_m = 0, \quad \hat{U}_i^m U^{\pm\pm}_m = 0, \quad i = 1, \ldots, 8 \] (56)
are auxiliary Lorentz group valued matrix fields (Lorentz harmonics, see [27, 25, 28, 29] and refs. in [28]).

5 Properties of the equations of motion for coupled branes

Our study so far relies on the embedding of the branes into the Goldstone fermion theory (dynamical or auxiliary (D-1)–brane) rather than into the superspace. Thus we clearly have less fermionic equations for the coupled branes than can be expected. Nevertheless, as we will see, this is not a drawback of just the GFE action (42), (51), as the equations obtained from the SSPE action (10) are actually equivalent.

For the SSPE action (10) we have twice as many fermionic variables as in the case above. However it includes the Lagrange multiplier 1-forms $P_1$, $\pi_1$ which remain indefinite and appear in the equations of motion just inside a source localized at the intersection. As we clarify below,
these equations with indefinite source are equivalent to the fermionic equations following from the GFE action.

We first consider the SSPE action \[ L_{11B} = \frac{1}{2} E^{++} \wedge E^{--} - B_2 \] (55)

\[ L_{D3} = E^{11} \wedge \left( \text{det}(\eta_{ab} + F_{ab}) + Q_2 \wedge \left( dA - B_2 - \frac{1}{2} E^{a} \wedge E^{b} F_{ba} \right) + e^F \wedge C \right) \] (56)

(29, 26) (see (51)–(54)). To obtain the simplest form of the equations of motion it is convenient to pass from the ‘holonomic’ basis in the space of variations \( \delta X, \delta \Theta^I, \delta A, \ldots \) to the ‘supersymmetric’ one (cf. (39), (46), (49), (52), (53), (54))

\[ i_{\delta} \Pi^m = \delta X^m - i \delta \Theta^I \sigma^m \Theta^I, \quad i_{\delta} F = \delta A - i_{\delta} B_2 + E^b F_{ba} i_{\delta} E^a, \ldots \] (57)

Then the variation with respect to coordinate fields \( \hat{X}(\xi), \tilde{\Theta}^I(\xi) \) and \( \hat{X}(\zeta), \tilde{\Theta}^I(\zeta) \) becomes

\[ \delta S = \int_{M^{1+1}} \left( (\hat{M}_{2m} - j_1 \wedge P_{1m} + j_1 \wedge \hat{M}_{1m}) i_{\delta} \hat{\Pi}^m + i(\hat{\Psi}_{2\mu}^I - j_1 \wedge \psi_{1\mu}^I i_{\delta} \hat{\Theta}^I) \right) + \int_{M^{1+3}} \left( (\tilde{M}_{4m} + j_3 \wedge P_{1m}) i_{\delta} \tilde{\Pi}^m + i(\tilde{\Psi}_{4\mu}^I + j_1 \wedge \psi_{1\mu}^I i_{\delta} \tilde{\Theta}^I) \right), \]

where we used the density 1–forms \( j_1, j_3 \) to lift the boundary inputs to the worldsheet and the worldvolume, respectively, and abbreviate

\[ \psi_{1\mu}^I = \pi_{1\mu}^I - i P_{1m} (\sigma^m \Theta^I)_\mu. \] (59)

The expressions \( \hat{M}_{2m}, \tilde{M}_{4m} \) and \( \hat{\Psi}_{2\mu}^I, \tilde{\Psi}_{4\mu}^I \) denote the l.h.s.-s. of the bosonic and fermionic equations for the free (closed) type \( 11B \) superstring

\[ \partial M^{1+1} = 0 \Rightarrow \hat{M}_{2m} = 0, \quad \hat{\Psi}_{2\mu}^I = 0 \] (60)

and the free super–D3-brane

\[ \partial M^{1+1} = 0 \Rightarrow \tilde{M}_{4m} = 0, \quad \tilde{\Psi}_{4\mu}^I = 0, \] (61)

written in terms of differential forms \( [28, 26, 31, 19] \) (we will not need their explicit expressions below), while \( \hat{M}_{1m} \) denotes the coordinate variation localized at the boundary, which appears due to the integration by part in the ‘bulk’ superstring action \([1]\). We should stress that in the basis \([27]\) no boundary input with the variation \( \delta \Theta^I \) appears (see Appendix A, and \([20]\) for details). Note also that we use the Lorentz harmonic formulations of superstring and super-D3-brane \([23, 28, 31]\) as here the free equations of motion appear in the form which allows a lifting to the 10-dimensional space, while the standard formulations \([13–50]\) can be considered in such a way only formally.

For the coupled system one can expect some set of equations with sources localized at the intersection instead of (60), (61).

However, the fermionic equations which follows from (58)

\[ \hat{\Psi}_{2\mu}^I = j_1 \wedge \psi_{1\mu}^I, \quad \tilde{\Psi}_{4\mu}^I = -j_3 \wedge \psi_{1\mu}^I, \] (62)

include a source localized at the boundary and expressed through the the Lagrange multiplier 1–forms by \([54]\). This source is indefinite, as the Lagrange multipliers are not determined by the equations of motion.
In the above notations the variation of the GFE action (42), (51) with respect to coordinate fields $X^m(x)$ and $\Theta^I\mu(x)$ reads as

$$\delta S^G = \int_{M^{1+9}} \left( J_8 \wedge \hat{M}_{2m} + J_6 \wedge \hat{M}_{4m} + dJ_8 \wedge \hat{M}_{1m} \right) i_\delta \Pi^m + \int_{M^{1+9}} \left( iJ_8 \wedge \hat{M}^I_{2\mu} + iJ_6 \wedge \hat{M}^I_{4\mu} \right) \delta \Theta^I\mu$$

(63)

where the forms $\hat{M}_{2m}, \hat{M}_{4m}, \hat{M}_{1m}$, entering Eq. (58) are the pull-backs of the 10-dimensional forms $M_{2m}, M_{4m}, M_{1m}$ from (63). Due to the identification (13), (16) only one set of independent fermionic variations $\delta \Theta^I\mu(x)$ is included into (63) and, thus, the GFE action (12) produces only one set of fermionic equations

$$J_8 \wedge \hat{M}^I_{2\mu} + J_6 \wedge \hat{M}^I_{4\mu} = 0.$$

(64)

However, as the only intersection of the worldsheet with the super-D-brane worldvolume is assumed to be just the boundary of the worldsheet $M^{1+3} \cap M^{1+1} = \partial M^{1+1}$ (3), Eq. (64) allows the statement that the pair of the fermionic equations (62) with indefinite source $\psi_{1\mu}$ appears. Thus both methods of the description of the interacting superbranes produce equivalent fermionic equations with an indefinite source localized at the intersection. Actually some restrictions for the sources can be obtained using the explicit expressions for the l.h.s.-s of the fermionic equations [20], but an ambiguity remains.

Note that a similar ambiguity appears in the bosonic equations and, thus, cannot be removed by passing to the pure bosonic limit. Indeed, in accordance with (58) the bosonic equations

$$\hat{M}_{2m} = j_1 \wedge \hat{M}_{1m} - j_1 \wedge P_{1m}, \quad \hat{M}_{4m} = -j_3 \wedge P_{1m}$$

(65)

involve an indefinite Lagrange multiplier 1-form $P_{1m}$. The choice $P_{1m} = 0$ corresponds to a sourceless equation for the host brane, which is the super-D3-brane in our case. Note that a definite source localized at the intersection is present in the supersymmetrized Born-Infeld equations, i.e. in the gauge field equations for the host brane.

Conclusion and outlook

In this note we propose two ways to obtain a supersymmetric action for interacting superbrane systems and present an explicit form of the actions for an open superstring ending on a super-D3-brane: (10), (23), (24) and (12), (51). They allow to obtain a manifestly supersymmetric (see Appendix A) set of equations of motion by straightforward variation. One of the actions (10) uses the Lagrange multiplier method to incorporate the necessary identification of the coordinate fields at the intersection, while the other ((12), (51)) implies an identification of the Grassmann coordinate of intersecting branes with an image of the $D(=10)$-dimensional Goldstone fermion field. Thus such an action actually assumes the presence of an auxiliary or dynamical space-time filling brane and, hence, can be called ’(D-1)-brane dominance’ model. An action for the space-time filling brane (in our case super-D9-brane) can be easily included in the action for an interacting low dimensional brane system like (11). On the other hand, it opens the possibility to include the supergravity into the coupled brane system: in a complete action for a coupled brane system like (11) the group-manifold action for $D$-dimensional supergravity may replace the free action for the dynamical space-time filling brane.

Inclusion of the (auxiliary or dynamical) space-time filling brane or of supergravity requires the use of the moving frame (Lorentz harmonic) actions [25, 28, 29, 31] for low dimensional open branes and host branes. The reason is that their Lagrangian forms (in distinction to the ones of the standard actions) can be regarded as pull–backs of some D-dimensional differential
(p+1)-forms and, thus, the moving frame actions for free branes can be written easily in the form of integrals over a D-dimensional manifold by means of the current densities presented here (see [23] for bosonic branes). Just the existence of the moving frame formulation may motivate the formal lifting of the Lagrangian forms of the standard actions to D dimensions and their use for the description of the interaction with space–time filling branes and/or supergravity.

We studied the general structure of the equations of motion and found that for both approaches we arrive at an ambiguity in the source terms, which can be fixed only partially. Such an ambiguity actually appears as a result of the identification of the coordinate fields of the open brane and the host brane at the intersection. It is inherent not only for the supersymmetric case, but for the pure bosonic limit of intersecting branes as well.

The explicit form of the equations of motion, the analysis of their properties and the study of $\kappa$–symmetry and supersymmetry issues for the action of interacting superbranes will be the subject of a forthcoming paper [20].

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**Appendix A: On boundaries and supersymmetry**

Our aim was to find the action which includes manifestly $N = 2, D = 10$ supersymmetric 'bulk' terms, allows direct variations and, hence, leads to equations of motion with manifestly supersymmetric l.h.s.-s. As it is well known, the presence of a boundary breaks the $(N = 2)$ supersymmetry of the classical action. For our system in the Lorentz harmonic formulation (51), (53), (56), (52), (54) the relevant boundary variation has the form

$$\delta S_{\text{boundary}} = \int_{\partial M^{1+1}} \left( \frac{1}{2} E^{++} U^{++} - \frac{1}{2} E^{--} U^{--} - E^b F_{ba} u^b_m \right) i_\delta \Pi^m,$$

(A.1)

where we use the basis (57) and put $i_\delta (F - F) \equiv \delta A - \epsilon_{1\delta} B_2 + E^b i_{1\delta} F_{ba} E^a = 0$, which corresponds, in particular to the supersymmetry transformations of the gauge field $A$ (see [17]) which is chosen to make the super–D3–brane action supersymmetric. As mentioned in Section 5, no boundary input with the variation $\delta \Theta^I$ appears. This does not contradict the well–known fact that the presence of a worldsheet boundary breaks at least a half of the target space $N = 2$ supersymmetry. Indeed, for the supersymmetry transformations (52) the variation $i_\delta \Pi^m$ is nonvanishing and has the form

$$i_\delta \Pi^m = 2\delta X^m = 2\Theta^I \sigma^m \epsilon^I.$$

(A.2)

Imposing the boundary conditions $\hat{\Theta}^{12}(\xi(\tau)) = \hat{\Theta}^{21}(\xi(\tau))$ one arrives at the conservation of $N = 1$ supersymmetry whose embedding into the type IIB supersymmetry group is defined by $\epsilon_{12} = -\epsilon_{21}$. Actually these conditions provide $i_\delta \Pi^m(\xi(\tau)) = 0$ and, as a consequence, the vanishing of the variation (A.1).

The above consideration in the frame of the Lorentz harmonic approach results in an interesting observation that the supersymmetry breaking by boundary is related to the 'classical
reparametrization anomaly': indeed the variation (A.1), which produces the nonvanishing variation under $N = 2$ supersymmetry transformation with (A.2), contains only the variations $i \hat{\Pi}^{\mu}_{m} U^{\pm \pm}_{+}$ and $i \hat{\Pi}^{\mu}_{m} U^{\pm \pm}_{-}$, which correspond to reparametrization gauge symmetry of the free superstring and free super–D3–brane, respectively.

There exists a straightforward way to keep half of the rigid target space supersymmetry of the superstring–super-D$p$-brane system by incorporation of the additional boundary term $\int_{\partial M_{1+1}} \hat{\phi}_{1\mu} \left( \hat{\Theta}_{1\mu}^{\nu}(\xi(\tau)) - \hat{\Theta}_{1\mu}^{\nu}(\xi(\tau)) \right)$ with a Grassmann Lagrange multiplier one form $\hat{\phi}_{1\mu}$. This involves an additional arbitrariness in the first set of the fermionic equations (32), which now read $\hat{\Psi}^{m}_{\mu} = j_{I} \hat{S}_{\mu}^{m} \sigma^{m} \Theta^{I} \Theta(X)$ . However, following [3, 5], we accept in this paper the ‘soft’ breaking of the supersymmetry by boundaries at the classical level (see [11, 2] for symmetry restoration by anomalies). We expect that the BPS states preserving part of the target space supersymmetry will appear as particular solutions of the coupled superbrane equations following from our actions.

Appendix B

In the search for a hypothetical generalization of our GFE action (42), (51) the following completely supersymmetric counterpart of the current form (30) can be useful

$$ J_{S} = \hat{\Pi}_{m}^{\nu} \int_{M^{1+1}} \hat{V}_{2}^{mn} \delta^{10}(\hat{S}) . $$

(B.1)

In this equation

$$ \hat{S}_{m} = X^{m} - X^{m}(\xi) - i \Theta^{I}(X) \sigma^{m} \Theta^{I}(\xi) $$

(B.2)

is the supersymmetric invariant interval introduced in [20] for $D = 4$. The measure $\hat{V}_{2}^{mn}$ can be constructed from supersymmetric invariant forms $\hat{\Pi}^{\mu}_{m}$ and

$$ \hat{dS}_{m} = -dX^{m} + i \Theta^{I}(\xi) \sigma^{m} \Theta^{I}(X) + i \Theta^{2}(\xi) \sigma^{m} \Theta^{2}(X) $$

$$ \hat{V}_{2}^{mn1} = \hat{dS}_{m} \wedge \hat{dS}_{n} , \quad \hat{V}_{2}^{mn2} = \hat{\Pi}^{\mu}_{m} \wedge \hat{\Pi}^{\mu}_{n} , \quad \hat{V}_{2}^{mn3} = \hat{\Pi}^{\mu}_{m} \wedge \hat{\Pi}^{\nu} \wedge \hat{dS}_{n} , \quad \ldots $$

(B.3)

The current form (B.1) is invariant under the flat target space supersymmetry (12) without the identification (13), but assumes, nevertheless the presence of a space-time filling brane. However, an evident problem following this direction is the lack of a curved superspace generalization of the supersymmetric interval (B.2).

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