Double-strangeness production in $\Lambda p \rightarrow K^+ X$ reaction

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We investigate $S = -2$ production from the $\Lambda p \rightarrow K^+ X$ reactions within the effective Lagrangian approach. The $\Lambda p \rightarrow K^+ \Lambda$ and $\Lambda p \rightarrow K^+ \Xi^-$ reactions are considered to find the lightest $S = -2$ system, which is $H$-dibaryon. We assume that the $H(2250) \rightarrow \Lambda \Lambda$, and $H(2270) \rightarrow \Xi^- p$ decays with the intrinsic decay width of 1 MeV. According to our calculations, the total cross-sections for $\Lambda p \rightarrow K^+ \Lambda$ and $\Lambda p \rightarrow K^+ \Xi^-$ reactions were found to be of the order of a few mb in the $\Lambda$ beam momentum range of up to 5 GeV/$c$. Furthermore, the direct access of information regarding the interference patterns between the $H$-dibaryon and non-resonant contributions was demonstrated.

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I. INTRODUCTION

Double-strangeness baryon systems involve an $H$-dibaryon, double hypernuclei, and possibly the inner core of neutron stars [1]. An observation of several double hypernuclei reveals that the $\Lambda\Lambda$ interaction is weakly attractive. However, the $\Xi^- N$ interaction was only studied in heavy-ion collisions, which indicates a strong, attractive interaction [2]. Recently, the $\Xi^- N$ coupling was determined to be weak based on an initial observation of a Coulomb-assisted bound state for the $\Xi^- N$ system [3], which was predicted to exist considering the evidence for a deeply-bound $\Xi^- N$ state reported in a hybrid emulsion experiment at KEK-PS [4]. While strangeness $S = -2$ baryon-baryon interactions provide critical information on exploring the smallest object ($H$-dibaryon), and the largest (the inner core of neutron stars), the experimental data is limited.

The lightest $S = -2$ system is the $H$-dibaryon, which can be decomposed into a compact 6-quark state, and two baryon states involving $\Lambda\Lambda$, $\Xi N$, and $\Sigma\Sigma$ components. The mass range of the $H$-dibaryon is strongly connected with the existence of double $\Lambda$ hypernuclei. Several $\Lambda$ hypernuclei have been reported: $^6\Lambda\Lambda$He [5], $^{10}\Lambda\Lambda$Be [6], and $^{13}\Lambda\Lambda$B [7]. Because the $\Lambda\Lambda \rightarrow H$ decay was not observed in the aforementioned studies, the $H$ must be heavier than $m_H > 2m_\Lambda + B_{\Lambda\Lambda} \approx 2.22$ GeV/$c^2$.

Recently, the HAL QCD collaboration has indicated that the $\Lambda\Lambda(1S_0)$ interaction is not sufficiently attractive to generate a bound or resonant state close to the $\Lambda\Lambda$ threshold, whereas the $\Xi N(1S_0)$ phase shift increases sharply just above the $\Xi N$ threshold [8]. Experimental confirmation of the $H$-dibaryon would be a significant accomplishment for a better understanding of hyperon interactions.

Enhanced $\Lambda\Lambda$ production close to the $\Lambda\Lambda$ threshold was reported in $^{12}\text{C}(K^-, K^+)$ reactions at $p_{K^-} = 1.65$ GeV/$c$ [9][10]. This threshold enhancement may provide insight for the possible existence of an $H$-dibaryon near the $\Lambda\Lambda$ or $\Xi^- p$ thresholds. A high-statistics experimental reconfirmation should be awaited until the dedicated $H$-dibaryon search experiment E42[11] is performed using a high-intensity $K^-$ beam at J-PARC.

The simplest method for producing the $H$-dibaryon is to employ the double-strangeness and double-charge exchange ($K^-, K^+$) reaction on a light nuclear target to retain two units of strangeness in a $^{12}\text{C}$ nucleus, similar to the J-PARC E42 with a diamond target at $p_{K^-} = 1.8$ GeV/$c$. Furthermore, the $H$-dibaryon is also available in other reactions, such as $p p$, $p \Lambda$, $p \pi$, and $p \Sigma$, most of which involve nuclear targets that contain at least two nucleons coupled to the $H$-dibaryon production; therefore, the overlap of wavefunctions for hyperons and intranuclear nucleons should be considered. A cross-section measurement for the $\Lambda\Lambda$ production was reported to be $6.7 \pm 1.5$ mb in a $p\text{Ta}$ reaction at $4$ GeV/$c$ [12]. Heavy-ion reactions can be used to produce $\Lambda$ and $\Xi^-$ hyperons copiously so that the coalescence of two of these particles into the $H$-dibaryon may be observed. However, the $H$-dibaryon should be observed in a high-multiplicity environment for high-energy heavy-ion collisions.

Because the $H$-dibaryon can be formed directly via $\Xi N$ and $\Lambda\Lambda$ interactions, the production reaction involving the minimum number of vertices is the $\Xi^- p \rightarrow H$ reaction with a proton target. However, in this case, the mass range of the $H$-dibaryon is accessible only above the $\Xi^- p$ mass threshold. Because a $\Lambda\Lambda$ scattering experiment is unavailable, the second-best choice is a $\Lambda p \rightarrow H K^+$ reaction via a strangeness-exchange process, with which the $H$-dibaryon can be observed in the mass range below the $\Lambda\Lambda$ threshold to a higher mass region.

A $\Lambda$ beam is available via photoproduction and $\pi^-$-induced reactions by tagging $K^+$, $K^0$, or $K^{(892)}$ in the final state. For example, the $\pi^-$-induced reactions can either be a $\pi^- p \rightarrow K^0 \Lambda$ or $\pi^- p \rightarrow K^{(892)} \Lambda$ reaction. As the detection of a $K^{(892)}$ decay triggers the production of both $S = +1 K^0$ and $S = -1 K^-$ with nearly equal probability, the production of $\Lambda$ particles cannot be uniquely tagged. Therefore, the $\pi^- p \rightarrow K^{(892)} \Lambda$ reaction is selected as a primary reaction for the $\Lambda p$ elastic scattering measurement using an $8$ GeV/$c$ $\pi^-$ beam at J-PARC [13]. In this case, the $\Lambda$ beam is available in the momentum ranging from $0.2$ to $2.0$ GeV/$c$, and it is unavailable for double-strangeness production above the threshold $\Lambda$ momentum of $2.6$ GeV/$c$. 


However, the $\gamma\rho \rightarrow K^+\Lambda$ reaction facilitates the production of a high-momentum $\Lambda$ with $\Lambda$ polarization in the photon beam energy region above 2.5 GeV. The measurement of $\Lambda p \rightarrow K^+\Lambda\Lambda$ and $\Lambda p \rightarrow K^+\Xi^- p$ is viable with the CLAS data [13] and the upcoming LEPS data [15]. This $(\Lambda, K^+)$ reaction measurement leads to a decisive conclusion regarding the existence of the $H$-dibaryon near the $\Lambda\Lambda$ and $\Xi^- p$ thresholds. Moreover, possible interference effects among the $K^+\Lambda\Lambda$ and $K^+\Xi^- p$ channels are noteworthy.

In this study, numerical calculation results for the $\Lambda p \rightarrow K^+\Lambda\Lambda$ and $\Lambda p \rightarrow K^+\Xi^- p$ reactions within the effective Lagrangian approach have been reported. We calculate the Dalitz plot densities $(d^2\sigma/dM_{\Lambda\Lambda}dM_{K^+})$ for the $\Lambda p \rightarrow K^+\Lambda\Lambda$ reaction and $(d^2\sigma/dM_{\Xi^-}dM_{K^+})$ for the $\Lambda p \rightarrow K^+\Xi^- p$ reaction. The $H$-dibaryon states are assumed to appear at 2.25 GeV/$c^2$ and 2.27 GeV/$c^2$ in the $\Lambda\Lambda$ and $\Xi^- p$ channels, respectively. The intrinsic width of the $H$-dibaryon was chosen to be $1\ MeV$. Based on calculations, the total cross-sections for the $\Lambda p \rightarrow K^+\Lambda\Lambda$ and $\Lambda p \rightarrow K^+\Xi^- p$ reactions were determined to be within the order of a few $\mu b$ in the $\Lambda$ beam momentum of up to 5 GeV/$c$. Furthermore, we demonstrated that information regarding the interference patterns between the $H$-dibaryon and non-resonant contributions can be directly accessed.

II. THEORETICAL FRAMEWORK

In this section, we introduce the theoretical framework to investigate the $\Lambda p \rightarrow K^+X$ reactions within the effective Lagrangian approach. We consider the $\Lambda p \rightarrow K^+\Lambda\Lambda$ and $\Lambda p \rightarrow K^+\Xi^- p$ reactions. The relevant Feynman diagrams for the reactions are illustrated in Figs. 1 and 2. Diagrams (a) and (b) indicate an $H$-dibaryon-pole with $\Lambda$ and $\Xi$ exchanges, respectively. The other diagrams (c–f) denote various baryon-pole contributions with $t$-channel meson exchanges. The effective Lagrangians for the Yukawa vertices are defined as follows:

$$\mathcal{L}_{BBH} = -g_{BBH} B^i B^j H + \text{h.c.},$$
$$\mathcal{L}_{BBH} = g_{BBB} B^i B^j \Gamma B + \text{h.c.},$$
$$\mathcal{L}_{BBB} = -ig_{BBB} B \Gamma B + \text{h.c.},$$
$$\mathcal{L}_{PPB} = -ig_{PBB} P \Gamma B + \text{h.c.},$$
$$\mathcal{L}_{PPB} = g_{PPB} \frac{V}{\sqrt{2}} P \Gamma B + \text{h.c.},$$
$$\mathcal{L}_{VBB} = g_{VBB} V B + \text{h.c.},$$

where $B, B', H, S, P,$ and $V$ denote the fields for the 1/2$^+$ baryon, 1/2$^-$ baryon, $S = -2$ isoscalar-scalar $H$ dibaryon [16], scalar meson, pseudoscalar meson, and vector meson, respectively. The coupling constant $g_{BBH}$ is given by $g_{BBH} = g_{\Lambda\Lambda H} = g_{BBH}/\sqrt{8}$ and $g_{BBH} = g_{BBH}/\sqrt{2}$, where $g \approx 2.4/\sqrt{\text{MeV}}$ is used to reproduce the HAL-QCD collaboration results in the flavor SU(3) limit [16 17]. The values of $g_{(S,P,Y)BB}$ are obtained from the Nijmegen soft-core model (NSC) [18], whereas those for $g_{PPB}$ are determined by the experimental results [19] and the SU(3) relations. All the relevant couplings for $\Lambda p \rightarrow K^+\Lambda\Lambda$ and $\Lambda p \rightarrow K^+\Xi^- p$ are listed in Table I.

Diagrams (a) and (b) for the $\Lambda p \rightarrow K^+\Lambda\Lambda$ ($\Lambda\Lambda$ channel in the following) can be computed using these Lagrangians.

![FIG. 1. (Color online) Relevant Feynman diagrams for the present reaction processes of $\Lambda p \rightarrow K^+\Lambda\Lambda$ at the tree level. Diagrams (a) and (b) indicate the $H$-dibaryon-pole contributions with the $\Lambda$ and $\Xi$ exchanges. Diagrams (c–f) denote the various baryon-pole contributions with the strange and nonstrange meson exchanges in the $t$-channel.]

![FIG. 2. (Color online) Corresponding Feynman diagrams for the reaction processes of $\Lambda p \rightarrow K^+\Xi^- p$ at the tree level. Diagrams (a) and (b) indicate the $H$-dibaryon-pole contributions with the $\Lambda$ and $\Xi$ exchanges. Diagrams (c–f) denote the various baryon-pole contributions with the strange and nonstrange meson exchanges in the $t$-channel.]

| $h$ | $\sigma$ | $\eta$ | $\omega$ | $\kappa$ | $K$ |
|-----|--------|------|------|-------|----|
| $\Gamma_h [\text{MeV}]$ | 550    | 1.31 | 8.49 | 478   | 0  |
| $h$ | $K^+$ | $(N, \Lambda, \Xi)$ | $N^*(1650)$ |
| $\Gamma_h [\text{MeV}]$ | 50.8  | 0    | 158.2 |

TABLE I. Values of the full decay widths for the relevant hadrons from Refs. [19].
TABLE II. Relevant coupling constants for the present reaction process. These values are obtained from Refs. [16–18, 20, 22]. Here, $\sigma$ and $N^*$ denote $f_0(500,0^+)$ and $N^*(1650, 1/2^-)$, which is the most important contribution in the vicinity of the production threshold.

| $\Lambda\Lambda H$ | $N\Xi H$ | $(\kappa, K', K^*)\Lambda\Lambda$ | $(\kappa, K', K^*)\Lambda\Xi$ | $(\sigma, \eta, \omega)\Lambda N$ | $(\sigma, \eta, \omega)\Lambda\Lambda$ | $(\sigma, \eta, \omega)\Xi\Xi$ |
|-------------------|-----------|----------------------------------|--------------------------|---------------------------------|---------------------------------|---------------------------------|
| $-\frac{1}{\sqrt{2}}$ | $\frac{2\Lambda}{\sqrt{2}MeV}$ | $(\kappa, K', K^*)\Lambda\Lambda$ | $(\kappa, K', K^*)\Lambda\Xi$ | $(\sigma, \eta, \omega)\Lambda\Lambda$ | $(\sigma, \eta, \omega)\Xi\Xi$ |
| $-\frac{1}{2} \frac{1}{\sqrt{2}}$ | $\frac{3}{2} \frac{1}{\sqrt{2}}$ | $-\frac{3}{2} \frac{1}{\sqrt{2}}$ | $-\frac{1}{2} \frac{1}{\sqrt{2}}$ | $-\frac{1}{2} \frac{1}{\sqrt{2}}$ | $\frac{1}{2} \frac{1}{\sqrt{2}}$ | $-\frac{1}{2} \frac{1}{\sqrt{2}}$ |
| $0.53$ | $0.35$ | $(\kappa, K', K^*)\Lambda\Lambda$ | $(\kappa, K', K^*)\Lambda\Xi$ | $(\sigma, \eta, \omega)\Lambda\Lambda$ | $(\sigma, \eta, \omega)\Xi\Xi$ |

resulting in the following invariant amplitudes:

$$iM^{\Lambda\Lambda}_{(c)} = \frac{g_{kn}g_{\Lambda\Lambda}g_{\Lambda\Lambda}u^\dagger_s u^\dagger_u D^0_H(q_{4s+5})u_1 D^{1/2}_2(q_{2-3})\gamma_5 u_2 - (4 \leftrightarrow 5)},$$  

$$iM^{\Lambda\lambda}_{(b)} = \frac{1}{\sqrt{2}} g_{kn}g_{\Lambda\Lambda}g_{\Lambda\Lambda}u^\dagger_s u^\dagger_u D^0_H(q_{4s+5})u_2 D^{1/2}_2(q_{1-3})\gamma_5 u_1 - (4 \leftrightarrow 5),$$

where $q_{i+j} \equiv k_i \pm k_j$ and $D^0_h$ indicate the dressed propagator for a hadron $h$ with spin $s$. Its explicit form in the present work is as follows:

$$D^0_h(q) = \frac{F_h(q^2)}{q^2 - M^2_h - i\Gamma_h M_h},$$

$$D^{1/2}_h(q) = \frac{1}{q^2 - M^2_h - i\Gamma_h M_h} \left( M^2_h - q^2 q^\nu q^\nu - \frac{\Gamma^2_h}{4} \right),$$

where $q_{i+j} \equiv k_i \pm k_j$ and $D^0_h$ indicate the dressed propagator for a hadron $h$ with spin $s$. Its explicit form in the present work is as follows:

$$F_h(q^2) = \left( \frac{\Lambda^4_h}{\lambda^4_h + (M^2_h - q^2)^2} \right).$$

The cut-off mass $\Lambda_h$ is determined from other experimental data in the next section. Notably, the interchange of the $\Lambda$ baryons in the final state $(4 \leftrightarrow 5)$ in Eq. (2) gives a negative sign, owing to the Fermi-Dirac statistics. All the relevant meson-baryon couplings are obtained from the Nijmegen soft-core potential model [18], as listed in Table II. The invariant amplitudes for the diagram (c) can be evaluated as follows:

$$iM^{\eta\rho}_{(c)} = -g_{\sigma\alpha\lambda}g_{k\eta\lambda}g_{\alpha\eta\lambda}N N[f_{S\gamma_5}D^{1/2}_p(q_{3s+5})u_2]D^0_{\sigma\eta}(q_{1-4})[\bar{u}_4 u_1] - (4 \leftrightarrow 5),$$

$$iM^{\eta\nu}_{(c)} = -g_{\eta\alpha\lambda}g_{k\eta\lambda}g_{\alpha\eta\lambda}N N[f_{S\gamma_5}D^{1/2}_p(q_{3s+5})\gamma_\mu u_2]D^0_{\eta\lambda}(q_{1-4})[\bar{u}_4 \gamma^\mu u_1] - (4 \leftrightarrow 5),$$

$$iM^{\rho\nu}_{(c)} = -g_{\rho\omega\lambda}g_{k\rho\lambda}g_{\omega\eta\lambda}N N[f_{S\gamma_5}D^{1/2}_p(q_{3s+5})\gamma_\nu u_2]D^0_{\rho\lambda}(q_{1-4})[\bar{u}_4 \gamma^\nu u_1] - (4 \leftrightarrow 5),$$

for the $(\sigma, \eta, \omega)$ meson exchange in the $t$-channel. The superscripts in $iM_{h}^{(c)}$ denote the intermediate hadrons as shown in Fig. 1. Regarding the nucleon-resonance and $\Lambda$-baryon contributions, we only consider the couplings to the $\eta$ meson to avoid theoretical uncertainties.

The scalar meson $\sigma$ represents $f_0(500,0^+)$ (19). For the production of $H$-dibaryon near the threshold ($\sqrt{s} \approx 2725 MeV$), only the nucleon resonance $N'(1650, 1/2^-)$ becomes relevant to the amplitude, $iM^{\eta\rho}_{(c)}$. The strong coupling constants corresponding to $N'(1650, 1/2^-)$ are also obtained from the chiral coupled-channel method [20], as listed in Table II. Similarly, the $(K, K', K^*)$ meson-exchange contributions are as follows:
where the strange scalar meson denotes $\kappa^-(800)$ [19].
momentum \( q \) can be regularized simply using the dimensionalregularization method \( [21] \) as follows:

\[
G_{B_1B_2}^{\text{PV,OnF}} \approx i \int \frac{d^4q}{(2\pi)^4} \frac{M_a(M_{4s5} - M_a + M_b)}{[(q^2 - M_a^2)(q^2 - M_b^2) - i\epsilon]} \\
\times \left[ \ln \frac{M_b^2}{\mu^2} + \ln \frac{M_a^2 + M_2^2 + M_{4s5}^2}{2M_{4s5}^2} \ln \frac{M_a^2}{M_b^2} \right. \\
+ \left. \frac{\eta}{M_{4s5}} (L_{++} + L_{--} - L_{+-} - L_{-+}) \right]
\]  

(25)

where \( G_{1,2,3} \) indicates \( G_{\Lambda\Lambda, N\Xi, \Sigma\Sigma} \). Finally, considering all the factors previously indicated, we obtain the FSI-corrected total amplitude for the \( \Lambda\Lambda \) channel:

\[
i\hat{M}_{\Lambda\Lambda->\Lambda\Lambda}^{\text{cc,OnF}} = -\frac{\lambda_1[16 - 6i\lambda_1(4G_2 + 3G_3) - 15\lambda_2^2G_2G_3]}{128i + 16\lambda_1(G_1 + 4G_2 + 3G_3) + 6i\lambda_1^2(4G_1G_2 + 3G_1G_3 + 12G_2G_3) - 15\lambda_2^3G_1G_2G_3},
\]

(29)

follows:

\[
iM_{\text{tree+FSI}}^{\Xi^-p,OnF} = iM_{\text{tree}}^{\Xi^-p} \left[ 1 + \frac{1}{\sqrt{2}} (G_{\Xi^-p,OnF}) (i\hat{M}_{\Lambda\Lambda->\Lambda\Lambda}^{\text{cc,OnF}} \hat{M}_{\Xi^-p->\Xi^-p}) \right]
\]

(31)

where

\[
i\hat{M}_{\Xi^-p->\Xi^-p}^{\Lambda\Lambda->\Lambda\Lambda} = -\frac{\lambda_1[64 + 24i\lambda_1(4G_2 + 3G_3) - 15\lambda_2^2G_2G_3]}{128i + 16\lambda_1(G_1 + 4G_2 + 3G_3) + 6i\lambda_1^2(4G_1G_2 + 3G_1G_3 + 12G_2G_3) - 15\lambda_2^3G_1G_2G_3}.
\]

(32)

### III. Numerical Results and Discussions

In this section, we provide the numerical calculation results with details regarding the \( \Lambda p \rightarrow K^+\Lambda\Lambda \) (\( \Lambda\Lambda \) channel) and \( \Lambda p \rightarrow K^+\Xi^- p \) (\( \Xi^- p \) channel) reaction processes. In this calculation, the \( H \)-dibaryon is assumed to be unbound above the \( \Lambda\Lambda \) threshold. The mass range of the \( H \)-dibaryon is strongly connected with the observation of the double \( \Lambda \) hypernuclei, which imposes that the \( H \)-dibaryon mass should be larger than \( 2.22 \text{ GeV}/c^2 \). Recent Lattice QCD calculation results indicate that the mass ranges between \( \Lambda\Lambda \) and \( \Xi^- p \) thresholds \[8 \][16] \[17] \[25] \[26] . Two \( H \)-dibaryon states, below and above the \( \Xi^- p \) threshold, were chosen considering the \( H(2250) \rightarrow \Lambda\Lambda \) and \( H(2270) \rightarrow \Xi^- p \) decays.

As indicated in Section II, we employ the coupling constants for the dibaryon \( g_{BB'H} \) from the bare \( H \)-dibaryon model, in...
which the values were determined to fit the flavor SU(3) symmetric HAL-QCD data [16]. Therefore, the values of \( g_{BB-H} \) may be different from reality, where the flavor symmetry is heavily broken. Nonetheless, as guidance for the present theoretical calculations, these symmetric values were adopted as a trial. In Ref. [17], the full decay width of the dibaryon was \( \Gamma_H = 2.7 \text{ MeV} \) at the physical point.

First, the cutoff mass was fixed in the form factors in Eq. (7). In Ref. [27], a few events of the \( ^{12}\text{C}(\Xi^-, \Lambda\Lambda)X \) reaction were reported. Using the eikonal approximation, the total cross-section of the \( \Xi \) and \( \Lambda \) mass is determined to reproduce the data point \( \Xi \) and \( \Lambda \) were shown in Fig. 4. Thus, the cutoff masses for the three meson exchanges were chosen to be the same for brevity. The relevant invariant amplitude is then obtained as follows:

\[
iM_{\Xi^{-} p \rightarrow \Lambda\Lambda} = \sum_{\phi=\kappa, K, K'} iM_{\phi} - (c \leftrightarrow d),
\]

(33)

where

\[
iM_{\phi} = \frac{i g_{\phi} \Sigma_{\phi} g_{N\Lambda}(\bar{u}_{\phi} \Gamma_{\phi} u_{\phi})(\bar{u}_{\phi} \Gamma_{\phi} u_{\phi})F_{\phi}(t)}{t - M_{\phi}^2 - i\Gamma_{\phi} M_{\phi}},
\]

\( \Gamma_{\kappa, K, K'} = (1_{\kappa	imes 4}, \gamma_5, \gamma_\mu). \)

(34)

All relevant inputs are listed in Tables I and II. Thus, the cutoff mass is determined to be \( \Lambda = 435 \text{ GeV} \). The numerical results are shown in Fig. 4.

In Fig. 5, the numerical results for the \( \Xi^{-} p \rightarrow \Lambda\Lambda \) (solid) and \( \Xi^{-} p \rightarrow \Xi^{-} p \) (dotted) using Eqs. (33) and (34) with the cutoff mass \( \Lambda_{\Lambda\Lambda} = 435 \text{ MeV} \) and \( \Lambda_{\Xi^{-} p} = 535 \text{ MeV} \), respectively. \( \Lambda_{\Lambda\Lambda} \) is determined to reproduce the data point \( \sigma = 4.3^{+6.3}_{-2.7} \) extracted from \( ^{12}\text{C}(\Xi^-, \Lambda\Lambda)X \) data [27].

In the left panel of Fig. 5, the numerical results for the total cross-sections of the \( \Lambda\Lambda \) (square) and \( \Xi^{-} p \) (circle) channels are presented for the total (thick) and \( H \)-dibaryon-only (thin) contributions as functions of the \( \Lambda \) beam momentum \( p_{\text{lab}} \). The total cross-sections from the \( \Lambda\Lambda \) channel are approximately twice as large as that of the \( \Xi^{-} p \) because there are more possible contributions, as shown in the relevant Feynman diagrams in Fig. 1 in addition to the larger Nijmegen coupling constants. On the contrary, if we only consider the \( H \)-dibaryon, the order of the cross-sections is reversed, owing to the value of \( g_{H\Lambda\Lambda} \) being smaller than \( g_{H\Xi^{-} p} \) by a factor of two considering the isospin factor. Note, the production cross-section for the \( H \) dibaryon is a few tens of nanobarn. As shown in the right panel of Fig 5 to test the \( H \)-dibaryon mass dependence of the total cross-sections, they are depicted with \( M_H = (2.25 \sim 2.29) \text{ GeV}/c^2 \) for the two reaction channels. The effects from the mass changes are unapparent, while considerable difference can be observed for the \( \Xi^{-} p \) channel with \( M_H = 2.25 \text{ GeV}/c^2 \).

In Fig 6, the numerical results for the differential cross-sections of the \( \Lambda\Lambda \) (left) and \( \Xi^{-} p \) (right) channels are presented...
as the function of the scattering angle of $K^+$ in the center-of-mass frame (cm) $\theta$. We also analyzed the differential cross-sections in the energy range of $E_{cm} = 2.8-3.0$ GeV. The thick and thin lines denote the cases with and without the $H$-dibaryon contributions, respectively. The angular dependence for the two channels is relatively flat at a low energy and forward scattering as the energy increases. Note, the angular dependence of the $H$-dibaryon production is nearly flat at high energies, indicating the $S$-wave nature of the particle.

To investigate the production mechanisms more carefully, we present the numerical results for the differential cross-sections for each contribution individually at $E_{cm} = 2.8$ GeV ($p_{lab} = 2.83$ GeV/$c$), in the same manner as presented in Fig. 7. Regarding the $\Lambda\Lambda$ channel, the $\omega$ and $K^-$ exchanges in the proton-pole diagrams ($c$ and $d$) are predominant owing to the combinations of the larger Nijmegen coupling constants. Moreover, the $H$-dibaryon production diagram with the $\Lambda$ pole ($a$) is significantly larger than that of the $\Xi^-p$-pole diagram ($b$). Regarding the $\Xi^-p$ channel, the $H$-dibaryon production diagrams are considerably larger than others, and the $\kappa$ exchange in the $t$ channel ($e$) provides a meaningful contribution. Generally, we determined that the $H$-dibaryon production diagram ($a$) with the Mandelstam variable $t = (k_{\Lambda} - k_{K^+})^2$ enhances forward scattering, and vice versa for the diagram ($b$) with $u = (k_p - k_{K^-})^2$, as expected.

The Dalitz plot for the $\Lambda p \rightarrow K^+\Lambda\Lambda$ and $\Lambda p \rightarrow K^+\Xi^-p$ reactions are plotted in Fig. 8 for a $\Lambda$ beam momentum of 2.83 GeV/$c$. Because no background processes form structure in the Dalitz plots, the $H$-dibaryon band appears predominant. The numerical results for the invariant-mass plots are provided in Fig. 9 with $M_H = 2.25$ GeV/$c^2$ and 2.27 GeV/$c^2$ for the $\Lambda\Lambda$ (left) and $\Xi^-p$ (right) channels, respectively, at $E_{cm} = 2.8$ GeV. The width of the $H$ dibaryon is assumed to be 1 MeV, whereas the phase angle $\phi$ is tested for 0 (thick) and $\pi$ (thin). The shaded areas indicate the cases without the $H$ dibaryon. The light and heavy shared areas indicate the cases without and only with the $H$ dibaryon. We observed that the $H$-dibaryon production rates are larger for the $\Xi^-p$ channel by a factor of two, than that
FIG. 8. (Color online) Dalitz plots for (a) the $\Lambda p \rightarrow K^+\Lambda\Lambda$ and (b) $\Lambda p \rightarrow K^+\Xi^- p$ reactions at $p_\Lambda = 2.83$ GeV/c, respectively. Both contain the relative phase angle $\phi = 0$. The Dalitz plots are projected onto the $\Lambda\Lambda / \Xi^- p$ and $K^+ / K^+\Xi^-$ mass axes and plotted as histograms on the top and right sides, respectively.

for the $\Lambda\Lambda$ channel, and vice versa for the total background contributions, as shown in Fig. 9. The signal-to-background ratio is approximately 0.3 for the $\Lambda\Lambda$ channel, whereas the larger value of 1.6 is for the $\Xi^- p$ channel. Therefore, the $\Xi^- p$ channel enables us to search for the $H$ dibaryon significantly easier than the $\Lambda\Lambda$ channel. The production cross-sections for $H(2250) \rightarrow \Lambda\Lambda$ and $H(2270) \rightarrow \Xi^- p$ are approximately 40 nb and 38 nb, respectively. Significant changes are obtained by the different phase factors, clearly shown in the $\Xi^- p$ channel, owing to the smaller interference with the background processes. Furthermore, we note that the channel opening effects from the final-state interactions were small, resulting in cusp-like structures being hardly observed.

Finally, considering the decay-angle distribution of the $H$-dibaryon, the decay angle distribution of the $S$-wave $H$-dibaryon is isotropic at the rest frame of the $H$-dibaryon. We define the double differential cross-section as $d^2\sigma / d\cos \theta_{cm} d\cos \theta_{rest}$, where the angle $\theta_{cm}$ denotes that of the outgoing $K^+$ in the cm frame. The angle $\theta_{rest}$ is defined by

$$\theta_{rest} = \theta_{fr} - \theta_{fr}^{rest}$$ (35)

where $\vec{k}_{fr}$ and $\vec{k}_{fr}^{rest}$ indicate the three momenta of the one decaying baryon in the final state at the $H$-dibaryon rest frame and the outgoing $K^+$ in the cm frame, respectively. In Fig. 10 we depict the numerical results for the double differential cross-sections as a function of $\cos \theta_{cm}$ and $\cos \theta_{rest}$ with the $H$-dibaryon contribution only. As expected, we clearly observe
that the decay-angle distribution, i.e., the double differential cross-sections, are nearly flat for the various $\cos \theta_{\text{cm}}$ values.

\[
\Lambda p \rightarrow K^+ \Lambda \Lambda \text{ at } \sqrt{s} = 2.8 \text{ GeV}
\]

\[
\Lambda p \rightarrow K^+ \Xi^- p \text{ at } \sqrt{s} = 2.8 \text{ GeV}
\]

**FIG. 10.** (Coloronline) Double differential cross-section as a function of $\cos \theta_{\text{cm}}$ and $\cos \theta_{\text{rest}}$ for (a) the $\Lambda \Lambda$ and (b) $\Xi^- p$ (right) channels. See the text for details.

**IV. SUMMARY**

In this study, we investigated the $H(I = 0, J = 0)$-dibaryon production via $\Lambda p \rightarrow \Lambda \Lambda K^+$ theoretically. Thus, we employed the effective Lagrangian approach at the tree-level Born approximation. We considered the mass and decay width of the dibaryon as the theoretical input parameters, and they were chosen by considering presently available theoretical and experimental results, such as the lattice-QCD data analyses with the flavor SU(3) breaking effects: $2M_H \leq M_H \leq (M_{\Xi^-} + M_p)$ and $\Gamma_H = (1 \sim 10)$ MeV. The critical observations made in this study are as follows:

- The total cross-sections for the $\Lambda \Lambda$ and $\Xi^- p$ channels are determined to be within the order of a few $\mu b$ in the $\Lambda$ beam momentum of up to 5 GeV/c, while the production cross-section for the $H$-dibaryon is approximately 100 nb. Here, we determined our model parameters such as the cutoff masses for the form factors, based on the experimental data for the $\Xi^- p$ elastic and $\Xi^- p \rightarrow \Lambda \Lambda$ scattering cross-sections.

- The total cross-sections do not change significantly with the $H$-dibaryon mass from 2.25 GeV/$c^2$ to 2.27 GeV/$c^2$. Because the $\Lambda \Lambda$ production processes than the $\Xi^- p$ channel, by ignoring the channel via an exotic pentaquark-state, the $H$-dibaryon contribution appears to be relatively large in the $\Xi^- p$ channel.

- We observed that the differential cross-sections for the $\Lambda p \rightarrow K^+ \Lambda \Lambda$ and $\Lambda p \rightarrow K^+ \Xi^- p$ channels peak at the forward $K^+$ angles in the cm frame, owing to the $t$-channel meson and baryon exchange processes. The $H$-dibaryon-pole contributions are significant near the threshold and depend minimally on the $K^+$ angle.

- From the invariant mass distributions, the signal-to-background ratios are approximately 0.3 and 1.6 for the $\Lambda \Lambda$ and $\Xi^- p$ channels, respectively, owing to the smaller background contributions in the $\Xi^- p$ channel. Note, the $H$-dibaryon peak areas yield 40 nb and 38 nb for the $\Lambda \Lambda$ and $\Xi^- p$ channels, respectively.

- We also explored the change in the interference patterns between the $H$-dibaryon and background amplitudes with the relative phase angle for the $\Xi^- p$ channel. The channel opening effects from the final-state interactions were small; therefore, cusp-like structures were hardly observed.

- Lastly, we calculated the decay angular distributions of the $H \rightarrow \Lambda \Lambda$ and $H \rightarrow \Xi^- p$ decays in the helicity frame in which the quantization axis is in the opposite direction of $K^+$ in the $H$-dibaryon rest frame. The angular distributions are flat over the $H$-dibaryon mass region, as expected for the $S$-wave resonance.

Considering the aforementioned factors, we conclude that the $H$-dibaryon could be clearly identified in the $\Lambda p \rightarrow K^+ \Lambda \Lambda$ and $\Lambda p \rightarrow K^+ \Xi^- p$ reactions close to the production threshold, if it exists close to the $\Xi^- p$ threshold. Further studies related to other $H$-dibaryon production reactions are in progress and will appear elsewhere.

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**APPENDIX**

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