Magnetism and superconductivity within extended Hubbard model for a dimer.

Exact results

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February 8, 2020

Abstract

We consider the extended Hubbard model containing intrasite and intersite Coloumb interactions. This model we apply to a dimer as an interesting and nontrivial example of two interacting ions which possesses exact, analytical solution. We find the eigenvalues $E_\alpha$ and eigenvectors $|E_\alpha\rangle$ of the dimer and we represent each part $E_\alpha |E_\alpha\rangle \langle E_\alpha|$ of the dimer Hamiltonian ($\alpha = 1, 2, ..., 16$) in the second quantizations with the use of the Hubbard and spin operators. This procedure gives a review of all competitive, intrinsic interactions together with their exactly calculated coupling constants expressed by the model parameters. These competitive interactions, deeply hidden in the original form of the extended Hubbard model, make an evidence that the model is extremely complex because it describes a competition between magnetism and superconductivity. Among competitive interactions we can find ferromagnetic and antiferromagnetic interactions (also in the form appearing in the $t - J$ model), hopping of the Cooper pairs between different dimer lattice sites (similar as in Kulik-Pedan, Penson-Kolb models), as well as, intersite Cooper pair interactions. We plot several coupling constants of these interactions vs Coulomb intrasite interaction $U$ to show that the competition between them strongly depends on the model parameters. The thermodynamical activity of each particular interaction, belonging to a given energy level, depends, however, on the occupation of this level. When e.g. a given dimer energy level is empty the corresponding intrinsic interaction belonging to this level is completely passive (irrelevant). From the presented review of interactions it is evident that the extended Hubbard model is capable to describe the properties of superconductors with magnetic ordering (including also high-$T_C$ superconductors) where a strong competition between magnetism and

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1 Introduction

It is commonly believed that the Hubbard model and its extensions can widely be used to explain many of physical phenomena observed in quite different areas of the solid state physics: magnetic and transport properties of transition metals, their compounds and alloys, including insulator-metal transitions (cf e.g. Refs [1]-[13] and papers cited therein), superconductors (magnetic superconductors (see e.g. Refs [14], [15]), high-\(T_C\) superconductors (negative \(U\) models, cf. e.g. Ref. [16]; \(t - J\) based models (cf Refs [17]-[26])), fluctuating valence systems (Anderson-like models, Ref. [27], cf also e.g. Refs [11], [28]), liquid \(\text{He}^3\) (cf e.g. Refs [29]-[31]), fullerenes (cf e.g. Refs [32]-[34]), etc..

To demonstrate a high degree of complexity of the extended Hubbard model we consider one-band electronic system described by the Hamiltonian

\[
H = \sum_{i \neq j, \sigma} t_{i,j} c_{i,\sigma}^+ c_{j,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow} + \frac{1}{2} \sum_{i \neq j, \sigma} J^{(1)}_{ij} n_{i,\sigma} n_{j,\sigma} + \frac{1}{2} \sum_{i \neq j, \sigma} J^{(2)}_{ij} n_{i,\sigma} n_{j,-\sigma}.
\]

(1)

The indices \((i, j)\) enumerate the lattice points \((\mathbf{R}_i, \mathbf{R}_j)\), \(t_{i,j}\) is the hopping integral, \(U\) (positive or negative) denotes the effective intrasite Coulomb interaction, \(J^{(1)}\) and \(J^{(2)}\) (generally, not necessary equal) describe the effective intersite interactions, resulting from the original Coulomb repulsion modified by the polaronic effects (see e.g. Ref. [16] for details). The operators \(c_{i,\sigma}(c_{i,\sigma}^+)\) are the electronic annihilation (creation) operators in the lattice site \(i\) with spin index \(\sigma = \uparrow, \downarrow\) and \(n_{i,\sigma} = c_{i,\sigma}^+ c_{i,\sigma}\). The model (1) cannot be solved exactly in a general case. There is, however, a special but nontrivial case of two interacting ions (a dimer problem) which possesses exact, analytical solution. The Hamiltonian of the dimer has the form (see (1))

\[
H_D = -t \sum (c_{1,\sigma}^+ c_{2,\sigma} + c_{2,\sigma}^+ c_{1,\sigma}) + U (n_{1,\uparrow} n_{1,\downarrow} + n_{2,\uparrow} n_{2,\downarrow}) + J^{(1)} \sum_{\sigma} n_{1,\sigma} n_{2,\sigma} + J^{(2)} \sum_{\sigma} n_{1,\sigma} n_{2,-\sigma}
\]

(2)

where \(t = -t_{1,2} = -t_{2,1}\). We can exactly show that \(H_D\) incorporates many competitive interactions, deeply hidden in its original form. To bring them all into light we will follow the method, applied earlier to the Hubbard dimer alone (see Ref. [35]). The application of this method to the extended Hubbard model for a dimer seems to be very important because it gives a review of all intrinsic interactions, possible to appear in this more general case. According to the Ref. [35] we first find the exact solution of the dimer problem (see Sect. 2). This solution consists of 16 energy levels \(E_\alpha\) and corresponding eigenvectors \(|E_\alpha\rangle\) \((\alpha = 1, 2, ..., 16)\). In the next step we use the equivalent expression for the
dimer Hamiltonian

\[ H_D = \sum_{\alpha=1}^{16} E_\alpha P_\alpha \]  

where \( P_\alpha = |E_\alpha\rangle\langle E_\alpha| \). For each \( \alpha = 1, 2, \ldots, 16 \) the operator \( P_\alpha \) can be represented in the second quantization where we introduce Hubbard and spin operators. When using the relation (3) we can decompose the dimer Hamiltonian (2) into 10 parts collecting the terms belonging to the same energy level. Such decomposition according to the exact dimer energy levels possesses two main advantages. First, we bring into light all competitive interactions, possible to appear within the exactly solvable dimer. We find ferromagnetic and antiferromagnetic interactions, Cooper pair hopping terms (similar as in Kulik-Pedan, Penson-Kolb models (cf. Refs [36], [37])) with positive and negative coupling constants, as well as, intersite Cooper pair interactions (see e.g. Ref [16]). In this way we exactly show that the extended Hubbard model for a dimer is, in fact, a mixture of different intrinsic interactions leading to magnetic and superconducting properties of the model which compete together. Second, the presence of each intrinsic interaction is strictly ascribed to a given energy level which can be occupied or empty (it depends on the model parameters and assumed average occupation number of electrons \( n \in [0, 4] \)). In other words, we can foresee which intrinsic interaction can be "thermodynamically" active or not (it depends on the position of the chemical potential \( \mu \) with respect to a given energy level). Coupling constants connected with each type of intrinsic interaction are calculated exactly and several of them we plot as a functions of the intrasite Coulomb interaction \( U \) to visualize the competition between them. The method applied in this paper is exact in comparison with approximative approaches (perturbation expansion or canonical transformations) used in the literature till now (see Refs [38]-[53] and [11] and [12] for a review). The knowledge about possible intrinsic interactions within the extended Hubbard model seems to be very important because this model is widely used in different areas of the solid state physics. The presence of ferromagnetic and antiferromagnetic interactions competing with different types of interactions leading to superconductivity unambiguously suggests the use of the model to magnetic superconductors (cf e.g. Refs [14], [15] and papers cited therein). There is, however, one important and unsolved problem which arises from our exact calculations for a dimer. When apply the model to a real lattice (real materials) we are always forced to make approximations which can destroy a delicate harmony of the model leading to the overestimation or underestimation of some important competitive interactions. Thus, the problem is how to formulate a resonable approximation which treats all of them on equal footing. Besides, a high degree of complexity of the extended Hubbard model, explicitly shown here for a dimer, can also be a reason why the understanding of pairing in the high-\( T_C \) superconductors is such a difficult problem till now. It seems to be evident that the \( t-J \) model alone (also present in our approach), introduced in Refs [17]-[26] (see also Ref. [53] for a review), does not contain enough ingredients to describe high-\( T_C \) superconductors because we additionally have a competition between ferromagnetism and antiferromagnetism accompanied by
2 Intrinsic problem for the dimer Hamiltonian (2) can be solved exactly when start from the vectors \(|n_1,\sigma;n_2,\tau\rangle\) \((n_i,\sigma = 0,1; i = 1,2; \sigma = \uparrow,\downarrow)\) forming the Fock basis (cf. also Refs [35, [54], [55])

\[
\begin{align*}
|0\rangle &= |0,0;0,0\rangle, & |21\rangle &= |1,1;0,0\rangle, & |31\rangle &= |0,1;1,1\rangle, \\
|11\rangle &= |1,0;0,0\rangle, & |22\rangle &= |1,0;1,0\rangle, & |32\rangle &= |1,0;1,1\rangle, \\
|12\rangle &= |0,1;0,0\rangle, & |24\rangle &= |0,1;1,0\rangle, & |34\rangle &= |1,1;1,0\rangle, \\
|13\rangle &= |0,0;1,0\rangle, & |25\rangle &= |0,1;0,1\rangle, \\
|14\rangle &= |0,0;0,1\rangle, & |26\rangle &= |0,0;1,1\rangle, & |4\rangle &= |1,1;1,1\rangle.
\end{align*}
\]

The basis vectors (4) have the form \(|n_\beta\rangle\) where \(n = \sum_{i,\sigma} n_{i,\sigma} = (0,1,2,3,4)\). The second index \(\beta\) (if necessary) enumerates the vectors belonging to the subspace of a given \(n\). The exact solution of the dimer eigenvalue problem \(H_D|E_\alpha\rangle = E_\alpha|E_\alpha\rangle\) can be obtained with the use of a standard procedure. We obtain \(E_1 = 0; |E_1\rangle = |0\rangle\),

\[
\begin{align*}
E_2 &= -t; & |E_2\rangle &= \frac{1}{\sqrt{2}}(|11\rangle + |13\rangle), \\
E_3 &= t; & |E_3\rangle &= \frac{1}{\sqrt{2}}(|11\rangle - |13\rangle), \\
E_4 &= -t; & |E_4\rangle &= \frac{1}{\sqrt{2}}(|12\rangle + |14\rangle), \\
E_5 &= t; & |E_5\rangle &= \frac{1}{\sqrt{2}}(|12\rangle - |14\rangle), \quad (5)
\end{align*}
\]

\[
\begin{align*}
E_6 &= J^{(2)}; & |E_6\rangle &= \frac{1}{\sqrt{2}}(|23\rangle + |24\rangle), \\
E_7 &= U; & |E_7\rangle &= \frac{1}{\sqrt{2}}(|21\rangle - |26\rangle), \\
E_8 &= C + \frac{U+J^{(2)}}{2}; & |E_8\rangle &= a_1(|21\rangle + |26\rangle) - a_2(|23\rangle - |24\rangle), \\
E_9 &= -C + \frac{U+J^{(2)}}{2}; & |E_9\rangle &= a_2(|21\rangle + |26\rangle) + a_1(|23\rangle - |24\rangle), \\
E_{10} &= J^{(1)}; & |E_{10}\rangle &= |22\rangle, \\
E_{11} &= J^{(1)}; & |E_{11}\rangle &= |25\rangle, \\
E_{12} &= t + U + J^{(1)} + J^{(2)}; & |E_{12}\rangle &= \frac{1}{\sqrt{2}}(|31\rangle + |33\rangle), \\
E_{13} &= -t + U + J^{(1)} + J^{(2)}; & |E_{13}\rangle &= \frac{1}{\sqrt{2}}(|31\rangle - |33\rangle), \\
E_{14} &= t + U + J^{(1)} + J^{(2)}; & |E_{14}\rangle &= \frac{1}{\sqrt{2}}(|32\rangle + |34\rangle), \\
E_{15} &= -t + U + J^{(1)} + J^{(2)}; & |E_{15}\rangle &= \frac{1}{\sqrt{2}}(|32\rangle - |34\rangle), \\
E_{16} &= 2(U + J^{(1)} + J^{(2)}); & |E_{16}\rangle &= |4\rangle
\end{align*}
\]
where

\[ C = \sqrt{\left(\frac{U - J(2)}{2}\right)^2 + 4t^2}, \]  

(6)

\[ a_1 = \frac{1}{2} \sqrt{1 + \frac{(U - J(2))}{2C}}, \]  

(7)

\[ a_2 = \frac{1}{2} \sqrt{1 - \frac{(U - J(2))}{2C}}. \]  

(8)

Let us introduce the Hubbard operators

\[ a_{i,\sigma} = c_{i,\sigma} (1 - n_{i,-\sigma}), \]  

(9)

\[ b_{i,\sigma} = c_{i,\sigma} n_{i,-\sigma} \]  

(10)

and spin operators

\[ S_z^i = \frac{1}{2} (n_{i,\uparrow} - n_{i,\downarrow}) = \frac{1}{2} (n_{i,\uparrow}^a - n_{i,\downarrow}^a), \]  

(11)

\[ S_i^+ = c_{i,\uparrow}^+ c_{i,\downarrow} = a_{i,\uparrow}^+ a_{i,\downarrow}, \]  

(12)

\[ S_i^- = c_{i,\downarrow}^+ c_{i,\uparrow}^+ = a_{i,\downarrow}^+ a_{i,\uparrow} \]  

(13)

where \( n_{i,\sigma}^a = a_{i,\sigma}^+ a_{i,\sigma} (i = 1, 2; \sigma = \uparrow, \downarrow). \)

To obtain all intrinsic interactions, deeply hidden in the original form of the extended Hubbard model for a dimer (2) we apply the equivalent form the dimer Hamiltonian (3). Each product \( E_\alpha P_\alpha \) in the formula (3) where we insert \( E_\alpha \) and \( |E_\alpha\rangle \) from the formulae (5) can be represented as a linear combination of the basis vectors (4) which, in turn, with the use of the Hubbard and spin operators (9)-(13) can be rewritten in the second quantization (see Refs. [35], [54], [55]).

We collect all the terms together which correspond to the same energy level (we take into account the degeneration of the levels as e.g. \( E_4 = E_2, E_5 = E_3, E_{11} = E_{10}, E_{14} = E_{12}, E_{15} = E_{13} \)). In this way we can split the Hamiltonian (2) written in the form of the formula (3) into 10 terms, corresponding to 10 different dimer energy levels (see (5)). We obtain

\[ H_D = \sum_{i=1}^{10} H_d^{(i)} \]  

(14)

where
\[ H_D^{(1)} = E_2 P_2 + E_4 P_4 = -\frac{t}{2} [n_1^b (1 - n_1^a - \frac{n_2^b}{2}) + n_2^b (1 - n_1^a - \frac{n_1^b}{2}) - \frac{t}{2} \sum_\sigma [a_{1,\sigma}^+ a_{2,\sigma} + a_{2,\sigma}^+ a_{1,\sigma}], \tag{15} \]

\[ H_D^{(2)} = E_3 P_3 + E_5 P_5 = \frac{t}{2} [n_1^b (1 - n_2^a - \frac{n_2^b}{2}) + n_2^a (1 - n_1^a - \frac{n_1^b}{2}) - \frac{t}{2} \sum_\sigma [a_{1,\sigma}^+ a_{2,\sigma} + a_{2,\sigma}^+ a_{1,\sigma}], \tag{16} \]

\[ H_D^{(3)} = E_6 P_6 = -J^{(2)} [S_1^z \cdot S_2^z - \frac{n_1^a n_2^a}{4}] + \frac{J^{(2)}}{2} (S_1^+ \cdot S_2^- + S_1^- \cdot S_2^+) \tag{17} \]

\[ H_D^{(4)} = E_7 P_7 = \frac{U}{4} [n_1^b (1 - n_2^a - \frac{n_2^b}{2}) + n_2^b (1 - n_1^a - \frac{n_1^b}{2}) - \frac{t}{2} \sum_\sigma b_{1,\sigma}^+ a_{1,\sigma} + b_{2,\sigma}^+ a_{2,\sigma} + b_{1,\sigma}^+ a_{1,\sigma} b_{1,\sigma}], \tag{18} \]

\[ H_D^{(5)} = E_8 P_8 = \left\{ \frac{J^{(2)}}{2} - \left[ \frac{J^{(2)} (U - J^{(2)})}{4C} - \frac{2t^2}{C} \right] \right\} [S_1^+ \cdot S_2^- - \frac{n_1^a n_2^a}{4}] \\
+ \frac{1}{4} \left[ U + C + \frac{(U^2 - (J^{(2)})^2)}{4C} \right] \left[ b_{1,\sigma}^+ a_{1,\sigma} b_{2,\sigma} + b_{2,\sigma}^+ a_{2,\sigma} b_{1,\sigma} \right] \\
+ \left\{ \frac{U}{8} + \frac{1}{8} \left[ C + \frac{(U^2 - (J^{(2)})^2)}{4C} \right] \right\} [n_1^b (1 - n_2^a - \frac{n_2^b}{2}) + n_2^b (1 - n_1^a - \frac{n_1^b}{2}) - \frac{t}{2} \sum_\sigma [a_{1,\sigma}^+ b_{1,\sigma}^+ + b_{1,\sigma}^+ a_{1,\sigma}], \tag{19} \]

\[ H_D^{(6)} = E_9 P_9 = \left\{ \frac{J^{(2)}}{2} - \left[ \frac{J^{(2)} (U - J^{(2)})}{4C} - \frac{2t^2}{C} \right] \right\} [S_1^+ \cdot S_2^- - \frac{n_1^a n_2^a}{4}] \\
+ \frac{1}{4} \left[ U - C + \frac{(U^2 - (J^{(2)})^2)}{4C} \right] \left[ b_{1,\sigma}^+ a_{1,\sigma} b_{2,\sigma} + b_{2,\sigma}^+ a_{2,\sigma} b_{1,\sigma} \right] \\
+ \left\{ \frac{U}{8} + \frac{1}{8} \left[ C + \frac{(U^2 - (J^{(2)})^2)}{4C} \right] \right\} [n_1^b (1 - n_2^a - \frac{n_2^b}{2}) + n_2^b (1 - n_1^a - \frac{n_1^b}{2}) - \frac{t}{2} \sum_\sigma [a_{1,\sigma}^+ b_{1,\sigma}^+ + b_{1,\sigma}^+ a_{1,\sigma}], \tag{20} \]

\[ H_D^{(7)} = E_{10} P_{10} + E_{11} P_{11} = 2J^{(1)} [S_1^z \cdot S_2^z - \frac{n_1^a n_2^a}{4}] \tag{21} \]
\[ H_D^{(8)} = E_{12}P_{12} + E_{14}P_{14} = \frac{(t + U + J^{(1)} + J^{(2)})}{4} [n_1^a n_2^b + n_2^a n_1^b] \]
\[ - \frac{(t + U + J^{(1)} + J^{(2)})}{2} \sum_\sigma [b_{1,\sigma}^+ b_{2,\sigma} + b_{2,\sigma}^+ b_{1,\sigma}], \quad (22) \]

\[ H_D^{(9)} = E_{13}P_{13} + E_{15}P_{15} = \frac{(-t + U + J^{(1)} + J^{(2)})}{4} [n_1^a n_2^b + n_2^a n_1^b] \]
\[ + \frac{(-t + U + J^{(1)} + J^{(2)})}{2} \sum_\sigma [b_{1,\sigma}^+ b_{2,\sigma} + b_{2,\sigma}^+ b_{1,\sigma}], \quad (23) \]

\[ H_D^{(10)} = E_{16}P_{16} = \frac{(U + J^{(1)} + J^{(2)})}{2} n_1^b n_2^b, \quad (24) \]

and \( n_{i,b} = \sum_\sigma n_{i,\sigma}^a, \quad n_{i,\sigma}^b = b_{i,\sigma}^+ b_{i,\sigma} = n_{i,\sigma} n_{i,-\sigma} \quad (i = 1, 2), \quad \bar{t} = 1 \) when \( i = 2 \) and \( \bar{t} = 2 \) when \( i = 1 \). As a test for the correctness of the decomposition (14) we can sum up all the terms (15)-(24) and we obtain the dimer Hamiltonian in its original form (2). The decomposition of the dimer Hamiltonian (14) according to the dimer energy levels where each part is given by the expressions (15)-(24) has, however, very important advantages. First of all it visualizes in an explicit and exact way all possible intrinsic interactions with corresponding coupling constants. Second, due to the decomposition according to the dimer energy levels we can easily see which interaction can be active or passive. It is simply connected with the occupation of a given level. If a dimer level is occupied by the electrons the intrinsic interaction ascribed to this level will be active, otherwise passive (it can be neglected). The occupation, in turn, depends on the model parameters, temperature and the assumed average number of electrons \( n = \sum_{i,\sigma} \langle n_{i,\sigma} \rangle \) which determines the position of the chemical potential and decides which energy levels will be occupied or empty.

The most important intrinsic interactions (leading to magnetism or superconductivity), presented in the expressions (15)-(24) can be divided into two classes. First of them belongs to magnetic interactions (ferromagnetic, antiferromagnetic - it depends on the sign of the parameters \( J^{(1)}, J^{(2)} \)). Such interactions are present in the formulae (17) and (21) and describe Ising type interactions with coupling constants generated by \( J^{(1)} \) and \( J^{(2)} \). The formula (17) contains also the transverse interaction between spins (see also later). The Heisenberg type magnetic interactions can be seen in the first terms of the formulae (19) and (20), generated by a more complex coupling constants, expressed by the model parameters \( J^{(2)}, U \) and \( t \). It is interesting to note that when \( J^{(2)} = 0 \) the first term in the formulae (19) and (20) describes ferromagnetic or antiferromagnetic interactions (it depends on the sign of \( U \), similar to well-known \( t - J \) model because the coefficient \( \frac{4t^2}{U} \approx \frac{4t^2}{|U|} \) for large \( |U| \) (see the formula (6) when \( J^{(2)} = 0 \)). The first terms in the expressions (19) and (20) can also be considered
Figure 1: Plot of the coupling constant $A_{19}$ (the coefficient at the first term in the formula (19)) vs Coulomb interaction $U$ for several values of $J^{(2)}$. The parameter $t = 1/6$ eV for all Figs.

as generalized $t - J$ interactions, valid in a more general case of the extended Hubbard model. The coefficients $A_{19} = -\frac{J^{(2)}}{2} + \left[\frac{J^{(2)}(U - J^{(2)})}{4C} - \frac{t^2}{C}\right]$ (the coupling constant at the first term in (19)) and $A_{20} = -\frac{J^{(2)}}{2} - \left[\frac{J^{(2)}(U - J^{(2)})}{4C} - \frac{t^2}{C}\right]$ (the coupling constant at the first term in (20)) are depicted in Fig.1 and Fig.2, respectively, as functions of the intrasite Coulomb interaction $U$ for several values of the parameter $J^{(2)}$. We see that $A_{19}$ is negative and $A_{20}$ is positive.

Thus, the first terms in (19) and (20) describe competitive ferromagnetic and antiferromagnetic Heisenberg interactions.

Let us note that the terms like $b_{\uparrow}^1 a_{\downarrow}^1 a_{\uparrow}^2 b_{\uparrow} \rightarrow c_{\uparrow}^1 c_{\downarrow}^1 c_{\uparrow}^2 c_{\downarrow}^2$ describe the hopping of the Cooper pairs (see the formulae (9) and (10)). Such a competitive hopping is present in the second terms in (18), (19) and (20). The plots of the corresponding coupling constants $B_{19} = \frac{1}{4}U + C + \left[\frac{(U^2 - (J^{(2)})^2)}{4C}\right]$ (the second term in the formulae (19)) and $B_{20} = \frac{1}{4}[U - C - \left\{\frac{(U^2 - (J^{(2)})^2)}{4C}\right\}]$ (the second term in the formulae (20)) are depicted in Fig.3 and Fig.4, respectively. The coupling constant $B_{19}$ is positive and $B_{20}$ negative. Thus, the second terms in the formulae (19) and (20) describe the hopping of the Cooper pairs with positive and negative coupling constants. Such terms are present in the Kulik-Pedan, Penson-Kolb superconductivity models (cf. Refs [36], [37]).

The transversal products $S_1^+ S_2^- \ (S_1^- S_2^+)$ are present in the second term in the formulae (17), as well as, in the scalar products of spin operators $S_1^z S_2^z + \frac{1}{2}(S_1^+ S_2^- + S_1^- S_2^+)$ in the first terms of the formulae (19) and (20). When use the second quantization representation we obtain $S_1^z S_2^z = -c_{\uparrow}^1 c_{\downarrow}^1 c_{\uparrow}^2 c_{\downarrow}^2$. 

8
Figure 2: Plot of the coupling constant $A_{20}$ (the coefficient at the first term in the formula (20)) vs Coulomb interaction $U$ for several values of $J^{(2)}$.

$(S_1^+S_2^+ = -c_{2,1}^+c_{1,1}^+c_{2,1}c_{1,1}^\uparrow)$ and we can see that such products of spin operators describe the interaction of the intersite Cooper pairs because we can write $S_1^+S_2^- = -d_{2,1}^+d_{1,2}^-$ ($S_1^-S_2^+ = -d_{1,2}^+d_{2,1}^-$) where $d_{1,2} = c_{1,1}c_{2,1}^\uparrow$ ($d_{2,1} = c_{2,1}c_{1,1}^\uparrow$). Thus, the second term in the formulae (17) and the transversal parts in the first terms of the formulae (19) and (20) describe, in fact, the competitive intersite Cooper pair interactions leading to superconductivity (cf. e.g. Ref [16] and papers cited therein). From the other side the application of the resonating valence bond approach (cf. Refs [17]-[26]) allows to rewrite the terms like $(S_1^\uparrow\cdot S_2^- - \frac{n_1^+n_2^-}{4})$, present in the first terms of the expressions (19) and (20), when introducing the following operators $f_{2,1} = \frac{1}{\sqrt{2}}(c_{2,1}c_{1,1}^- - c_{2,1}^\uparrow c_{1,1})$ and $f_{2,1}^+ = \frac{1}{\sqrt{2}}(c_{1,1}^\uparrow c_{2,1}^- - c_{1,1}c_{2,1}^\uparrow)$. For example, in the case of half-filling (cf. Ref. [21]) we can write $(S_1^\uparrow\cdot S_2^- - \frac{1}{4}) = -f_{2,1}^+f_{2,1}$.

3 Conclusions

Using the exact solution of the extended Hubbard model for a dimer we have reviewed the main, important interactions leading to magnetic and superconducting properties of the model. In this way we have demonstrated that the relative simplicity of the extended Hubbard model is only an illusion. We have shown in an exact way that the magnetic properties (ferromagnetism, antiferromagnetism) compete together. There is also additionally a challenge between two types of Cooper pair interactions (Cooper pair hopping and intersite Cooper pair interactions). Thus, at last, magnetic and superconducting tendencies of the model compete together. The final, thermodynamical properties of the ex-
tended Hubbard model are then a result of this competition. Which interaction wins in such a rally strongly depends on temperature (there is possible a change of the leader with varying temperature), model parameters and on the value of the assumed average number of electrons, determining the chemical potential of the system. Such indications can certainly be helpful when use the extended Hubbard model for real lattices. We have also indicated that the extended single-band Hubbard model is, in fact, capable to describe properties of such superconductors where the competition between magnetism and superconductivity can explain many astonishing properties of these materials (cf e.g. Refs [14]-[25]), including also high-$T_C$ materials and recently synthetized $CoO_2$ layer based systems (cf Ref. [26] and papers cited therein).

The nonperturbative formalism presented in this paper is, in principle, applicable to more complicated clusters consisting e.g. of one central ion and its nearest neighbours. We, however, know (see e.g. Refs [56]-[59] and papers cited therein) that the mathematical problems in this case exponentially increase with the size of the considered cluster. Another examples of a cluster approach to Hubbard-like models are given in Refs [60]-[65].

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Figure 4: Plot of the coupling constant $B_{20}$ (the coefficient at the second term in the formula (20)) vs Coulomb interaction $U$. The values of $J^{(2)}$ (as in Figs 1 and 2) have no significant influence on the plot in this scale.

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