Possible interpretation of the $Z_b(10610)$ and $Z_b(10650)$ in a chiral quark model

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Abstract. Motivated by the two charged bottomonium-like resonances $Z_b(10610)$ and $Z_b(10650)$ newly observed by the Belle collaboration, the possible molecular states composed of a pair of heavy mesons, $B\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}$, $B_s\bar{B}$, etc (in S-wave), are investigated in the framework of chiral quark models by the Gaussian expansion method. The bound states $B\bar{B}^*$ and $B^*\bar{B}$ with quantum numbers $I(J^{PC}) = 1(1^{-+})$, which are good candidates for the $Z_b(10610)$ and $Z_b(10650)$ respectively, are obtained. Other three bound states $B\bar{B}^*$ with $I(J^{PC}) = 0(1^{++})$, $B^*\bar{B}$ with $I(J^{PC}) = 1(0^{++})$, $0(2^{++})$ are predicted. These states may be observed in open-bottom or hidden-bottom decay channel of highly excited $\Upsilon$. When extending directly the quark model to the hidden color channel of the multi-quark system, more deeply bound states are found. Future experimental search of those states will cast doubt on the validity of applying the chiral constituent quark model to the hidden color channel directly.

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1. Introduction

Very recently, the Belle collaboration observed two narrow peaks, which are named \(Z_b(10610)\) and \(Z_b(10650)\), in the \(\pi^+\Upsilon(nS)\) \((n = 1, 2, 3)\) and \(\pi^\pm h_b(mP)\) \((m = 1, 2)\) invariant mass spectra in the hidden-bottom decay channels of \(\Upsilon(5S)\) [1]. The measured masses and widths of the two structures are,

\[
M_{Z_b(10610)} = 10608.4 \pm 2.0 \text{ MeV}, \quad \Gamma = 15.6 \pm 2.5 \text{ MeV} \\
M_{Z_b(10650)} = 10653.2 \pm 1.5 \text{ MeV}, \quad \Gamma = 14.4 \pm 3.2 \text{ MeV}.
\]

Analysis favors quantum numbers of \(I^G(J^P) = 1^+(1^+)\) for both states. The \(Z_b(10610)\) and \(Z_b(10650)\) are both charged bottomonium-like resonances and the masses are very close to the thresholds of the open bottom channels \(B^*\bar{B}(10604.6 \text{ MeV})\) and \(B^*\bar{B}^*(10650.2 \text{ MeV})\), so the molecular states of S-wave \(B^*\bar{B}\) and \(B^*\bar{B}^*\) assignment are suggested by Belle collaboration.

In the hadron level, Nils A. Törnqvist investigated the deuteron-like meson-meson bound states by meson exchange model [2]. The study shows that the energy of isoscalars \(BB^*\) with \(J^{PC} = 0^{--}, 1^{++}\), \(B^*\bar{B}^*\) with \(J^{PC} = 0^{++}, 0^{--}, 1^{+-}, 2^{++}\) are about 50 MeV below the corresponding \(BB^*\) and \(B^*\bar{B}^*\) thresholds. No bound state, however, appears for isovectors. Recently, by taking the pseudoscalar, scalar and vector mesons exchange into account in the framework of the meson exchange model, Liu et al. found that the loosely bound states probably exists in S-wave \(BB^*\) [3, 4]. Very recently, Sun et al. believe that the \(Z_b(10610)\), \(Z_b(10650)\) are respectively \(B^*\bar{B}\) and \(B^*\bar{B}^*\) molecular state after considering S-wave and D-wave mixing [5, 6].

In the quark level, by solving the resonating group method equation, Liu et al. [7] also investigated the system composed of \([bq][b\bar{q}], [b\bar{q}][bq], [bq]^*[b\bar{q}^*]\) \((q = u, d, s)\) by two chiral quark models in which the pseudoscalar, scalar and vector mesons exchange are taken. The isoscalars \(BB, BB^*(C = +), B^*\bar{B}^*(J = 2)\) favor molecular states. Bondar et al. also discussed the heavy quark spin structure of the \(Z_b(10610)\) and \(Z_b(10650)\) assuming that these are molecular state \(B^*\bar{B}\) and \(B^*\bar{B}^*\) [8]. By considering the contribution from the intermediate \(Z_b(10610)\) and \(Z_b(10650)\) states to the \(\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-\) decay process, the anomalous \(\Upsilon(2S)\pi^+\pi^-\) production near the peak of \(\Upsilon(5S)\) at \(\sqrt{s} = 10.87 \text{ GeV}\) [9], observed by Belle collaboration, can be explained naturally [10]. The possibility of \(Z_b(10610)\) and \(Z_b(10650)\) being tetraquark states are discussed by the authors of Ref [11, 12]. The authors of Ref [13, 14] think the tetraquark and molecular structure both can interpret the \(Z_b(10610)\) and \(Z_b(10650)\) in the QCD sum rule calculation. Further theoretical efforts concern the decay and mass of the \(Z_b(10610)\) and \(Z_b(10650)\) states discussed in Refs. [15, 16, 17].

Inspired by the new states \(Z_b(10610)\) and \(Z_b(10650)\) reported by Belle collaboration [1] and the related work, a systematical study of the possible S-wave \(BB, BB^*\) and \(B^*\bar{B}^*\) states is performed in this work. Here the \(B\) and \(\bar{B}\) stand for \((B^+, B^0, B^{*0}_s)\) and \((B^-, B^0, B^{*0}_s)\) triplets, respectively. It is worthwhile to investigate the intrinsic structure of the \(Z_b(10610)\), \(Z_b(10650)\) and other possible exotic states with \(b, \bar{b}\)
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quarks, especially in view of the great potential of finding new particles at Belle, BaBar, LHC and other collaborations.

To study the mass spectrum of above possible exotic states, two types of chiral quark models (ChQM) [18] are employed in this work. The numerical method, which is able to provide almost exact solutions, is very important in the study of few-body systems. Here, a high precision numerical method for few body system, which is different from the methods used in the previous work by other researchers, the Gaussian Expansion Method (GEM) is used. The detail of GEM can be found in Refs. [19] [20].

The paper is organized as follows. In the next section we introduce the Hamiltonian of the chiral quark models. Section 3 is devoted to discuss the wave function of possible molecular states $B\bar{B}$, $BB^*$ and $B^*\bar{B}^*$. In Section 4, we present and analyze the results obtained in our calculation. Finally, the summary of the present work is given in the last section.

2. The chiral constituent quark model

In the ChQM, the Hamiltonian usually includes Goldstone-boson exchange in addition to color confinement and one-gluon-exchange (OGE). The chiral partner, $\sigma$-meson, is also usually introduced, although its existence is still in controversy [21]. The Hamiltonian of the ChQM used here is given as follows,

$$H = \sum_{i=1}^{4} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1}^{4} (V_{ij}^C + V_{ij}^G + V_{ij}^\chi + V_{ij}^\sigma),$$  

where $\chi = \pi, K, \eta$, $T_{CM}$ is the kinetic energy operator of the center-of-mass motion of whole system.

The linear confining potential, which is suggested by lattice QCD calculation of $q\bar{q}$ system, can be written as

$$V_{ij}^C = \lambda_i^c \cdot \lambda_j^c \left( - a_c r_{ij} - \Delta \right).$$  

For one-gluon-exchange, the potential takes the form

$$V_{ij}^G = \alpha_s \frac{\lambda_i^c \cdot \lambda_j^c}{4} \left[ \frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j} \left( \sigma_i \cdot \sigma_j \right) \delta(r_{ij}) \right],$$  

where $\lambda, \sigma, \lambda$ are the SU(2) Pauli matrices and the SU(3) Gell-Mann matrices, respectively. The $\lambda$ should be replaced by $-\lambda^*$ for the antiquark. In the non-relativistic quark model, the delta function $\delta(r_{ij})$ should be regularized [22], because of the finite size of the constituent quark. The regulation is flavor dependent and reads [18] [23]

$$\delta(r_{ij}) = \frac{1}{4\pi r_{ij} r_0^2(\mu)} e^{-r_{ij}/r_0(\mu)},$$  

where $r_0(\mu) = r_0/\mu$ and $\mu$ is the reduced mass of quark-quark (or antiquark) system. The wide energy covered from light to heavy quark requires an effective scale-dependent strong coupling constant $\alpha_s$ in Eq. [3] that cannot be obtained from the usual one-loop
expression of the running coupling constant because it diverges when \( Q \rightarrow \Lambda_{QCD} \). Hence an effective scale-dependent strong coupling constant \([18]\) is taken as

\[
\alpha_s(\mu) = \frac{\alpha_0}{\ln \left[ \frac{\mu^2 + \mu_0^2}{\Lambda_0^2} \right]},
\]

where \( \mu_0 \) and \( \Lambda_0 \) are the free parameters.

For the mesons exchange, potential takes the form

\[
V_{ij}^\pi = C(g_{ch}, \Lambda, m_{\pi}) \frac{m_{\pi}^2}{12m_i m_j} H_1(m_{\pi}, \Lambda, r_{ij})(\sigma_i \cdot \sigma_j) \sum_{a=1}^{3} \lambda_i^a \cdot \lambda_j^a,
\]

\[
V_{ij}^K = C(g_{ch}, \Lambda, m_K) \frac{m_K^2}{12m_i m_j} H_1(m_K, \Lambda, r_{ij})(\sigma_i \cdot \sigma_j) \sum_{a=4}^{7} \lambda_i^a \cdot \lambda_j^a,
\]

\[
V_{ij}^\eta = C(g_{ch}, \Lambda, m_{\eta}) \frac{m_{\eta}^2}{12m_i m_j} H_1(m_{\eta}, \Lambda, r_{ij})(\sigma_i \cdot \sigma_j)
\times \left[ \cos \theta_P(\lambda_i^8 \cdot \lambda_j^8) - \sin \theta_P(\lambda_i^0 \cdot \lambda_j^0) \right],
\]

\[
V_{ij}^\sigma = -C(g_{ch}, \Lambda, m_{\sigma}) H_2(m_{\sigma}, \Lambda, r_{ij})
\]

\[
H_1(m, \Lambda, r) = \left[ Y(m r) - \frac{\Lambda}{m^3} Y(\Lambda r) \right]
\]

\[
H_2(m, \Lambda, r) = \left[ Y(m r) - \frac{\Lambda}{m} Y(\Lambda r) \right]
\]

\[
C(g_{ch}, \Lambda, m) = \frac{g_{ch}^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2} m
\]

where the \( \sigma \) exchange only occurs between the lightest quarks (\( u \)- or \( d \)-quark) which is different from Ref. \([18]\) due to its non-strange nature. The strange scalar meson (with large mass) exchange is not taken into account in the present work because of its small effect. We adopt \( \lambda^0 = \sqrt{\frac{2}{3}} I \) due to the normalization of SU(3) matrix. \( Y(x) \) is the standard Yukawa function defined by \( Y(x) = e^{-x}/x \) and the rest symbols have their usual meaning. The chiral coupling constant \( g_{ch} \) is determined from the \( \pi NN \) coupling constant through

\[
g_{ch}^2 = \left( \frac{3}{5} \right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2},
\]

and flavor SU(3) symmetry is assumed. The tensor term and the spin-orbital term have been omitted in the potentials since we consider only S-wave states.

The above model is denoted as ChQM1. To testing the effect of \( \sigma \)-exchange between the lightest and strange quark or strange quark pairs, and long-range color screening on the binding energy of the molecular states, the Salamanca version of the chiral quark model \([18]\), which is referred as ChQM2, is also employed here. The screened confinement interaction in this model is

\[
V_{ij}^C = \lambda_i^c \cdot \lambda_j^c \left\{ -a_c(1 - e^{-\mu_c r_{ij}}) + \Delta \right\},
\]

where \( \mu_c \) is a color screening parameter. The other potentials are the same as the above with the exception that the \( \sigma \)-meson is exchanged between all the light quarks, \( u, d, s \).
3. Wave function

The total wave function of multi-quark system can be written as,

$$\Psi_{I,I_z}^{I_z, I_z} = |\xi\rangle |\eta\rangle |\chi\rangle \Phi_{JJ_z}$$

where $|\xi\rangle$, $|\eta\rangle$, $|\chi\rangle$, $\Phi_{JJ_z}$ represent color singlet, isospin with $I$, spin with $S$ and spacial wave function with angular momentum $L_T$, respectively.

All possible molecule structures composed of S-wave $B$ and $\bar{B}$, which stand for $(B^+, B^0, B^0)$ and $(B^-, \bar{B}^0, \bar{B}^0)$ triplets, respectively, are investigated in this work. According to the total isospin, the $P\bar{P}$ (pseudoscalar meson) and $VV$ (vector meson) flavor wave functions are listed in Table 1. Another possible molecule structure for four-quark system is bottomonium+light meson. In this case, there is no interaction between colorless bottomonium and light meson (color dependent interaction is zero between two colorless cluster if no exchange term exists and there is also no Goldstone-boson exchange between heavy and light quarks), so no bound state can be formed in this case. Therefore, we do not take into account of this case in the present work.

Obviously the components $P\bar{V}$ and $V\bar{P}$ do not have definite $C$ parity, one can get $C$ parity $= \pm |P\bar{V} \pm \tilde{C} (PV)|/\sqrt{2}$ and $|V\bar{P} \pm \tilde{C} (V\bar{P})|/\sqrt{2}$ for the neutral states $[2]$ such as $(I, I_z) = (1, 0), \ (0, 0)$ shown in Table 1 (All the orbital angular momenta are set to zero because we concentrate on ground states). Hence, the coefficient $C = \pm 1$ represent $C$-even and -odd parity respectively, which are different from that of Ref. $[4,3,24]$, since we use normal convention of PDG $[25]$ i.e. $B^0 = \bar{d}b$ and $\bar{B}^0 = \bar{b}d$. One can easy find that $G$ parity $= \mp$ are corresponding to $C$ parity $= \pm$ for these states with $(I, I_z) = (1, 0)$. However, there is no interaction depending on $C$ and $G$ parity in the model Hamiltonian Eq.(11), so these two states with $\pm C$ and $G$ parity must be degenerate in our calculation. The two states separated by comma in each row of Table 1 will be coupled in the calculation.

The spatial structures of molecular states are pictured in Fig. 1. The relative coordinates are defined as following,

$$r = r_1 - r_2, \quad R = r_3 - r_4,$$

$$\rho = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} - \frac{m_3 r_3 + m_4 r_4}{m_3 + m_4},$$

and the coordinate of the mass-center is

$$R_{cm} = \frac{\sum_{i=1}^{4} m_i r_i}{\sum_{i=1}^{4} m_i},$$

where $m_i$ is the mass of the $i$th quark.

Then the outer product of space and spin wave functions is

$$\Phi_{JJ_z} = [\phi^G_{\beta\gamma}(\rho) \chi_{s_1 m_1}] J_1 M_1 [\psi^G_{LM}(R) \chi_{s_2 m_2}] J_2 M_2 ] J_{12} M_{12} \phi^G_{\beta\gamma}(\rho)] JJ_z.$$
Table 1. The flavor wave functions of the $\bar{B}B$, $\bar{B}B^*$ systems. “C” is the charge parity.

| Isospin | $\bar{B}B$          | $\bar{B}B^*$         |
|---------|----------------------|-----------------------|
| $I=\frac{1}{2}$ | $B^+\bar{B}^0$, $B^+\bar{B}_s^*$ | $B^{*+}\bar{B}^0$, $B^{*+}\bar{B}_s^*$ |
| | $B^0\bar{B}^0$ | $B^{*0}\bar{B}_s^0$, $B^{*0}\bar{B}_s^*$ |
| | $B^0\bar{B}^-$ | $B^{*0}\bar{B}^-$, $B^{*0}\bar{B}_s^-$ |

For $I=1$:

| | $\frac{1}{\sqrt{2}}(B^+B^- - B^0\bar{B}^0)$ | $\frac{1}{\sqrt{2}}[(B^{*+}B^- - B^{*0}\bar{B}^0) + C(B^{*+}B^- + B^{*0}\bar{B}^0)]$, $B^{*0}B^-$, $B^0B^*$ |
| | | $\frac{1}{2}[(B^{*+}B^- - B^{*0}\bar{B}^0) + C(B^-B^{*+} - B^{*0}B^0)]$, $B^{*0}B^-$, $B^0B^*$ |

For $I=0$: $l=1$:

| | $\frac{1}{\sqrt{2}}(B^+B^- + B^0\bar{B}^0)$ | $\frac{1}{2}[(B^{*+}B^- + B^{*0}\bar{B}^0) + C(B^{*+}B^- + B^{*0}\bar{B}^0)]$, $B^{*0}B^-$, $B^0B^*$ |
| | | $\frac{1}{2}[(B^{*+}B^- + B^{*0}\bar{B}^0) + C(B^-B^{*+} + B^{*0}B^0)]$, $B^{*0}B^-$, $B^0B^*$ |

For $I=0$ $s=1$:

| $B^0\bar{B}_s^0$ | $\frac{1}{\sqrt{2}}(B^{*0}\bar{B}_s^0 + C\bar{B}_s^0\bar{B}_s^0)$, $\frac{1}{\sqrt{2}}(B^0\bar{B}_s^0 + \bar{B}_s^0\bar{B}_s^0)$ |

Figure 1. The relative coordinate for a meson and antimeson system. Solid and hollow circles represent quarks and antiquarks, respectively.

In GEM, three relative motion wave functions are written as,

$\phi_{Glm}^{l}(r) = \sum_{n=1}^{n_{max}} c_{n}N_{nl}r^{l}e^{-\nu_{n}r^{2}}Y_{lm}(\hat{r}) \quad (20)$

$\psi_{GLM}^{N}(R) = \sum_{N=1}^{N_{max}} c_{N}N_{NL}R^{L}e^{-\zeta_{N}R^{2}}Y_{LM}(\hat{R}) \quad (21)$

$\varphi_{G\beta\gamma}(\rho) = \sum_{a=1}^{\alpha_{max}} c_{a}N_{\alpha\beta}\rho^{\beta}e^{-\omega_{\alpha}\rho^{2}}Y_{\beta\gamma}(\hat{\rho}) \quad (22)$

Gaussian size parameters are taken as geometric progression

$\nu_{n} = \frac{1}{r_{n}}, r_{n} = r_{1}a^{n-1}, a = \left(\frac{r_{n_{max}}}{r_{1}}\right)^{1/n_{max}-1} \quad (23)$

The expression of $\zeta_{N}, R_{N}, \omega_{\alpha}, \rho_{\alpha}$ in Eqs. (21) - (22) are similar to Eq. (23).

The physical state must be in color singlet, which can be constructed in two ways:
color-singlet and color octet,
\[ |\xi_1\rangle = |1_{12} \otimes 1_{34}\rangle, \quad |\xi_2\rangle = |8_{12} \otimes 8_{34}\rangle. \]  (24)

The state in color octet channel is called hidden color states by analogy to states which appear in the nucleon-nucleon problem [26].

The total spin of \( PP \) and \( PV \) and \( VP \) system is only 0, 1 respectively. However, the \( V\bar{V} \) system can coupling to total spin 0, 1 and 2.

4. Numerical results and discussion

Solving the Schrödinger equation
\[ (H - E) \Psi_{I,I_z} = 0 \]  (25)
with Rayleigh-Ritz variational principle, the energies of normal mesons, \( B\bar{B}, B\bar{B}^* \) and \( B^*\bar{B}^* \) systems can be obtained by using different total wave functions, respectively.

To determine if the \( B\bar{B}, B\bar{B}^* \) and \( B^*\bar{B}^* \) systems are bound or not, the threshold of the system should be fixed. Clearly the threshold is governed by two corresponding meson masses. So one believes that a good fit of meson spectra, with the same parameters used in four-quark calculations, must be the most important criterium [23, 27, 28, 29, 30, 31]. Of course, there is another possible threshold for four-quark system, bottomonium+light meson, e.g., \( \Upsilon(1S) + \rho \) for \( IJ^P = 11^+ \) channel. Generally the threshold in this case is lower, so the bound state \( B^{(*)}\bar{B}^{(*)} \) will become a resonance in this channel. Since the transition from \( B^{(*)}\bar{B}^{(*)} \) to bottomonium+light meson involves string rearrangement, we leave this for the future work.

In GEM, the calculated results of normal meson spectra (or the spectra of \( B\bar{B}, B\bar{B}^* \) and \( B^*\bar{B}^* \) systems) are converged with the number of gaussians \( n_{\text{max}} = 7 \) (\( n_{\text{max}} = 7, N_{\text{max}} = 7, \alpha_{\text{max}} = 12 \), and the size parameter \( r_n \) \( (r_n, R_N, \rho_a) \) running from 0.1 to 2 (2, 2, 6) fm. The convergence properties of the energies have been discussed in detail in Ref. [20]. The parameters and the normal meson spectra in two types of ChQM are listed in Table 2 and 3 respectively.

By solving the equation (25), the energy of the \( B\bar{B}, B\bar{B}^* \) and \( B^*\bar{B}^* \) systems can be obtained. If the binding energy, \( \Delta E = M_{\text{system}} - M_{b\bar{q}} - M_{\bar{b}q} \) \((q = u, d, s)\), is negative, then the system would be bound. According to the Table. 3 the thresholds of possible molecular states \( B\bar{B}, B\bar{B}^*, B^*\bar{B}^* \) are easily listed in Table 4.

The color-singlet, color-octet channel, and channel coupling calculation of \( B\bar{B}, B\bar{B}^*, B^*\bar{B}^* \) systems are done in the two types of ChQM, and the results are presented in Table 5-7 in which the results are denoted by “1\( \otimes 1 \)”,”8\( \otimes 8 \)” and “channel coupling”, respectively. Due to the strong interaction is invariant under the rotation of isospin, the different states corresponding to the different components of isospin \( I \) are degenerate, so we present the results for each total isospin \( I \). From Table 5-7 we can see two models give very similar results.

In the S-wave \( PP \) system, the quantum numbers \( J^P \) are 0\(^+\). Apart from scalar meson \( \sigma \), the pseudoscalar mesons e.g. \( \pi, K, \eta \) cannot be exchange in \( B\bar{B} \) system because
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Table 2. Parameters of two quark models. The masses of $\pi, \eta$ in Eqs.(6)-(8) are got from the experimental data which are $m_\pi = 0.7$ fm$^{-1}$, $m_\eta = 2.77$ fm$^{-1}$, respectively; $m_\sigma, \Lambda_\pi$, $\Lambda_\eta$, $\theta_p$ are taken the same values as Ref.[18], namely $m_\sigma = 3.42$ fm$^{-1}$, $\Lambda_\pi = 4.2$ fm$^{-1}$, $\Lambda_\eta = 5.2$ fm$^{-1}$, $\theta_p = -15^\circ$, $g_{ch}^2/4\pi=0.54$

| Parameters | ChQM1 | ChQM2 |
|------------|-------|-------|
| $m_u,d$ (MeV) | 313 | 313 |
| $m_s$ (MeV) | 525 | 555 |
| $m_c$ (MeV) | 1731 | 1752 |
| $m_b$ (MeV) | 5100 | 5100 |
| $a_c$ (MeV fm$^{-1}$) | 160 | 430 |
| $\Delta$ (MeV) | -131.1 | 181.1 |
| $\mu_c$ (fm$^{-1}$) | 0.7 | 0.7 |
| $\alpha_0$ | 2.65 | 2.118 |
| $r_0$ (MeV fm) | 28.17 | 28.17 |
| $\mu_0$ (MeV) | 36.976 | 36.976 |
| $\Lambda_0$ (fm) | 0.075 | 0.113 |

Table 3. Numerical results of normal meson spectrum(unit: MeV) in the two quark models. The experimental data marked “Exp.” takes from the latest Particle Data Group[25], and the ground state of bottom $\eta_b(1s)$ is observed by BABAR Collaboration from the radiative transition $\Upsilon(3S) \rightarrow \gamma \eta_b$[32]

| Meson | ChQM1 | ChQM2 | Exp. | Meson | ChQM1 | ChQM2 | Exp. |
|-------|-------|-------|------|-------|-------|-------|------|
| $\pi$ | 140.1 | 153.2 | 139.57±0.00035 | $D_s$ | 1966.6 | 1991.8 | 1968.49±0.34 |
| K | 496.2 | 484.9 | 493.67±0.016 | $D_s^*$ | 2091.1 | 2094.1 | 2112.3±0.5 |
| $\rho(770)$ | 775.3 | 773.1 | 775.49±0.34 | $B_s^\pm$ | 5284.7 | 5277.9 | 5279.15±0.31 |
| $K'(892)$ | 917.9 | 907.7 | 896.00±0.25 | $B_s^0$ | 5284.7 | 5277.9 | 5279.53±0.33 |
| $\omega(782)$ | 703.7 | 696.6 | 782.65±0.12 | $B_s^*$ | 5324.3 | 5318.8 | 5325.1±0.5 |
| $\phi(1020)$ | 1016.8 | 1011.9 | 1019.42±0.02 | $B_s^0$ | 5360.6 | 5355.8 | 5366.3±0.6 |
| $\eta_b(1s)$ | 2995.7 | 2999.8 | 2980.3±1.2 | $B_s^*$ | 5403.6 | 5400.5 | 5412.8±1.3 |
| $J/\psi(1s)$ | 3097.6 | 3096.7 | 3096.916±0.011 | $\eta_b(1s)$ | 9384.6 | 9467.9 | 9388.9±3.1 (stat) |
| $D^0$ | 1882.2 | 1898.4 | 1864.84±0.17 | $\Upsilon(1s)$ | 9462.4 | 9504.7 | 9460.30±0.26 |
| $D^*$ | 2000.2 | 2017.3 | 2006.97±0.19 | |

of the parity conservation. One can find in the Table 5 that the $\sigma$ meson exchange do not contribute enough attraction to bind the $B\bar{B}$ system in color-singlet channel in the ChQM1. Due to the $\sigma$-exchange also occurs between $ss$ pair and $us$ or $ds$ pairs, loosely bound states of $B\bar{B}$ with $I = 0(\bar{s})$, or $1/2$ are obtained in the ChQM2. Noteworthily, there is no one-gluon-exchange between two separate mesons just in this channel. More bound states are formed if we take the coupling of color-singlet and color-octet channels into account. Obviously, in the color octet channel, the color-magnetic terms of OGE between two separate colorful mesons contribute attraction to $B\bar{B}$ system, because of
the requirement of total color-singlet of the state. In this case, due to the masses of $u$, $d$ quarks are much smaller than the mass of $b$ quark, the cross matrix between color-singlet and -octet channels of the color-magnetic interaction, which is in proportional to $1/(m_im_j)$, is very large. So the energy of each $B\bar{B}$ system is depressed by it, which was discussed in detail in Ref. [23, 33].

Table 5. The binding energy (unit: MeV) of $B\bar{B}$. The "1 $\otimes$ 1", "8 $\otimes$ 8" and "channel coupling" represent $B\bar{B}$ in color-singlet, color-octet and coupling of color-singlet and color-octet channel, respectively.

| Isospin | ChQM1 | ChQM2 |
|---------|-------|-------|
|         | 1 $\otimes$ 1 | 8 $\otimes$ 8 | channel coupling | 1 $\otimes$ 1 | 8 $\otimes$ 8 | channel coupling |
| $I=\frac{1}{2}$ | 0.5 | 51.5 | -17.5 | -0.2 | 97.7 | -2.6 |
| $I=1$ | 0 | 0.1 | -72.6 | 0 | 45.6 | -29.9 |
| $I=0(l)$ | 0 | 0.1 | -72.6 | 0 | 45.6 | -29.9 |
| $I=0(s)$ | 0.3 | 77.4 | 0.2 | -0.7 | 137.5 | -1.5 |

For the $B\bar{B}^*$ system, the calculation results are listed in Table 6. The $\sigma$, $\pi$, $\eta$ mesons can all be exchanged in such systems. Two bound states are both found in two quark models in color-singlet single channel. For $I = 1$ state, the binding energy are both about $-1$ MeV with regard to $B\bar{B}^*$ threshold in two quark models, and the distance of each pair quarks in ChQM1 are

$$\sqrt{\langle r_{12}^2 \rangle} = 0.63 \text{ fm}, \quad \sqrt{\langle r_{34}^2 \rangle} = 0.63 \text{ fm},$$

$$\sqrt{\langle r_{13}^2 \rangle} = 2.1 \text{ fm}, \quad \sqrt{\langle r_{24}^2 \rangle} = 2.1 \text{ fm},$$

$$\sqrt{\langle r_{14}^2 \rangle} = 2.02 \text{ fm}, \quad \sqrt{\langle r_{23}^2 \rangle} = 2.18 \text{ fm}.$$
by Belle collaboration as the molecular state $B\bar{B}^*$ with $I(J^{PC}) = 1(1^{-+})$. For this state, there are also other thresholds, $\Upsilon(1S)\rho$ and $h_\nu\pi$ [15]. The energy of the state is a little higher than these thresholds. Because of the different color structures the states $B\bar{B}^*$ and $\Upsilon(1S)\rho$, $h_\nu\pi$ have, the transition involves the color structure rearrangement, $B\bar{B}^*$ may appear as a resonance in the $\Upsilon(1S)\rho$ and $h_\nu\pi$ channels [31]. The calculation of the transition, which is out of the scope of the present work, is left for future work. In our calculation, the $I = 0$ state without strange quark is also a bound state with binding energy $-12.1$ MeV in two quark models. In the ChQM2, in addition to the above two states, the $I = \frac{1}{2}$ and $0(s)$, also form a bound state for the $\sigma$-exchange contributes to these channels. The color-singlet singlet and hidden color channel coupling leads to that all the states are bound.

| $J^P$ | Isospin | ChQM1 | ChQM2 |
|-------|---------|-------|-------|
|       |         | \(1 \otimes 1\) | \(8 \otimes \bar{8}\) | channel coupling | \(1 \otimes 1\) | \(8 \otimes \bar{8}\) | channel coupling |
| \(0^+\) | \(I=\frac{1}{2}\) | 0.4 | -111.8 | -172.9 | -0.5 | -56.6 | -108.4 |
|       | \(I=1\) | -3.4 | -194.4 | -266.2 | -3.0 | -132.3 | -195.5 |
|       | \(I=0(\ell)\) | 0.4 | -104.9 | -175.9 | 0.4 | -49.8 | -115.3 |
|       | \(I=0(s)\) | 0.3 | -58.9 | -112.2 | 0.0 | 7.6 | -37.7 |
| \(1^+\) | \(I=\frac{1}{2}\) | 0.4 | -71.4 | -129.0 | -0.3 | -22.5 | -73.9 |
|       | \(I=1\) | -1.2 | -135.7 | -201.5 | -0.9 | -83.2 | -144.6 |
|       | \(I=0(\ell)\) | 0.4 | -94.9 | -160.1 | 0.5 | -44.9 | -107.6 |
|       | \(I=0(s)\) | 0.3 | -34.9 | -86.0 | -0.2 | 26.5 | -21.2 |
| \(2^+\) | \(I=\frac{1}{2}\) | 0.4 | -2.6 | -56.4 | 0.0 | 36.1 | -19.3 |
|       | \(I=1\) | 0.3 | -37.6 | -97.0 | 0.4 | -1.9 | -62.6 |
|       | \(I=0(\ell)\) | -11.1 | -74.3 | -133.0 | -11.0 | -35.7 | -95.1 |
|       | \(I=0(s)\) | 0.3 | 8.4 | -41.0 | -1.1 | 61.1 | -5.7 |

The S-wave $B^*\bar{B}^*$ systems have quantum numbers $J^{PC} = 0^{++}$, $1^{+-}$, and $2^{++}$ for the neutral states. The $\pi$, $\eta$, $\sigma$ mesons can all be exchanged for $I=0(\ell)$ and $I=1$, while only $\eta$ is exchangeable for $I=0(s)$ and $\frac{1}{2}$ states. The $\sigma$ interaction is always attractive between the lightest $u$, $d$ quarks. According to Eq. [33], the $\pi$-exchange is attractive for the states with $I(J^P) = 1(0^+)$, $1(1^+)$, $0(2^+)$ and makes these states are all bound.
states, which are shown in Table 7 in the color-singlet single channel calculation. The $B^*\bar{B}^*$ with $I(J^P) = 1(1^+)$ has binding energy about -1 MeV in two quark models, and the distance of each pair quarks are

\[ \sqrt{\langle r^2_{12} \rangle} = 0.64 \text{ fm}, \quad \sqrt{\langle r^2_{34} \rangle} = 0.64 \text{ fm}, \]
\[ \sqrt{\langle r^2_{34} \rangle} = 2.2 \text{ fm}, \quad \langle \langle r^2_{24} \rangle \rangle = 2.2 \text{ fm}, \]
\[ \sqrt{\langle r^2_{23} \rangle} = 2.11 \text{ fm}, \quad \langle \langle r^2_{23} \rangle \rangle = 2.27 \text{ fm}. \]

The assignment of the newly observed state $Z_b(10650)$ to a molecular state $B^*\bar{B}^*$ with $I(J^{PC}) = 1(1^{-+})$ is favored. In the hidden color channel, almost all the states, except the one with $IJP = 02^+$ and hidden strange, are become bound. Again the channel coupling between color-singlet and hidden color channels makes all the states bound. In the present work, the $S$-$D$ mixing of $Z_b$ is not taken into account. The mixing will be important for states with energy on the threshold. From the calculation of deuteron, we estimate the $S$-$D$ mixing will increase the binding energy of $Z_b$ about 2 MeV.

5. Summary

In the framework of chiral quark model, a systematical study of the mass spectra of $B\bar{B}$, $B\bar{B}^*$ and $B^*\bar{B}^*$ systems is performed. The states $B\bar{B}^*$ and $B^*\bar{B}^*$ with quantum numbers $I(J^{PC}) = 1(1^+)$ are shown to be bound, which are respectively good candidates for the charged bottomonium-like resonances $Z_b(10610)$ and $Z_b(10650)$ newly observed by Belle collaboration. The color-singlet single channel calculation also shows that the states $B\bar{B}^*$ with $I(J^{PC}) = 0(1^{++})$, $B^*\bar{B}^*$ with $I(J^{PC}) = 1(0^{++})$, $0(2^{++})$ are bound states with a few MeV binding energy.

Recently Belle collaboration reported their high precision measurement of bottomonium mass: $M[\Upsilon(5S)] = 10.87 \text{ GeV}$ [9]. If the molecular states $B^*\bar{B}^*$ with $I(J^P) = 1(0^+)$ really exist, it could be observed in final state $\Upsilon(1S)\rho$ at the Belle, BaBar, LHC and other collaborations. Due to the phase space limitation, the isoscalar states $B\bar{B}^*(J^P = 1^+)$ and $B^*\bar{B}^*(J^P = 2^+)$ may be observed in decays of excited bottomonium which above the $\Upsilon(5S)$.

The $\sigma$-exchange plays important role for binding the $B\bar{B}$, $B\bar{B}^*$ and $B^*\bar{B}^*$ with $I = \frac{1}{2}$ and 0(s) in the ChQm2. To search these molecular states in the future experiment will test the contribution of the $\sigma$-exchange in the chiral constituent quark model.

The hidden color channel effect is complicated in multi-quark systems. Here we extend directly the quark model for colorless cluster to the colorful cluster in the study of $B\bar{B}$, $B\bar{B}^*$ and $B^*\bar{B}^*$ systems. In our calculation, the color-octet channel plays a dominate role in producing deeply bound states. If the quark-antiquark interaction in color singlet can be extended directly to color-octet by Casimir scaling [35], then the OGE interaction will be attractive between two color-octet cluster in some multi-quark systems. So it is inevitably to produce deeply bound states for hidden-bottom states because of the too small kinetic energy. More experimental data on bottomonium-like
resonances are needed to check the Casimir scaling, and cast doubt on the validity of applying the chiral constituent quark model to the hidden color channel directly.

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