Covariance transformation method for polar integrated navigation

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Abstract. The aircraft flying the trans-arctic routes faces the problem that the changes of navigation frames will affect the navigation accuracy. To solve this problem, taking Inertial Navigation System/Global Navigation Satellite System (INS/GNSS) integrated navigation system as an example, an integrated navigation method based on covariance transformation is proposed. This method aims at finding the transformation relationship of the system error state and the covariance between different navigation frames. The experiment and semi-physical simulation results show that the covariance transformation algorithm can effectively solve the filtering instability caused by the changes of the navigation frame. Compared with the non-covariance transformation, the system state error is reduced by an order of magnitude.

1. Introduction

The polar region has received more and more attention because of its resources and geographical location. It now became obvious that great savings in flying time could be accomplished by flying the trans-arctic routes. The aircraft usually applies INS/GNSS integrated navigation system to provide high-precision navigation\(^1\). The traditional INS/GNSS integrated navigation algorithms are based on a north-oriented geographic frame. However, as the latitude increase, the traditional algorithms lose efficacy in polar regions because of the meridian convergence. To solve this problem, the aircrafts usually plan their routes based on the grid reference frame in the high latitude region\(^2\).

Usually, the INS/GNSS integrated navigation takes the local geographic frame as the navigation frame at low and middle latitudes, and turns to grid frame at the high latitudes. When the navigation frame is changed between different navigation frames, such as grid frame(G-frame) and local geographic frame (n-frame), the structure of the filter changes. If the consistency of the error state cannot be guaranteed, the navigation accuracy will decrease. However, the current research on polar region navigation mainly focuses on the design of integrated navigation algorithm within the polar region\(^3\),\(^ 5\), ignoring the problem of how to achieve global navigation. Literature \(^6\) designed an INS/CNS integrated navigation algorithm based on the local geographic frame and grid frame in the transpolar flight scene. The negligence of the filter structure change led to the fluctuation of filter parameters. In order to solve the global consistency problem of inertial navigation, literature \(^7\) designed an improved polar inertial navigation algorithm based on pseudo-earth INS mechanization, but it did not fundamentally solve the consistency problem in frames changing. Literature \(^8\) proposed the virtual sphere n-vector algorithm, which realized the global inertial navigation mechanization.
However, it completely changed the navigation frame of the current airborne inertial navigation system, which was not conducive to system upgrades.

In order to solve the problem of filter instability caused by the changes of navigation frame, this paper proposes a polar region airborne INS/GNSS integrated navigation method based on covariance transformation. The transformation relationship between the system error state in geographic frame and that in grid frame is deduced. Flight experiment and semi-physical simulation confirm the effectiveness of the covariance transformation method.

2. The Grid SINS

2.1. Grid Frame and Grid SINS Mechanization

The definition of the grid reference frame is shown in Figure 1. The grid plane is parallel to the Greenwich meridian, and its intersection with the tangent plane at the position of the aircraft is the grid north. The angle between geographic north and grid north is the grid angle, which is clockwise as positive. The up direction of the grid frame is the same as that of the local geographic frame, and forms an orthogonal right-handed frame with the grid east and grid north.

The grid angle $\sigma$ is expressed as:

$$
\sin \sigma = \sin L \sin \lambda \left(1 - \cos^2 L \sin^2 \lambda\right)^{0.5}
$$

$$
\cos \sigma = \cos \lambda \left(1 - \cos^2 L \sin^2 \lambda\right)^{0.5}
$$

The direction cosine matrix $C^G_e$ between the G-frame and the e-frame (earth frame) is:

$$
C^G_e = C^G_n C^n_e = \begin{bmatrix}
-cos \sigma \sin \lambda + \sin \sigma \sin L \cos \lambda & \cos \sigma \cos \lambda - \sin \sigma \sin L \sin \lambda & - \sin \sigma \cos L \\
- \sin \sigma \sin \lambda - \cos \sigma \sin L \cos \lambda & \sin \sigma \cos \lambda - \cos \sigma \sin L \sin \lambda & \cos \sigma \cos L \\
\cos L \cos \lambda & \cos L \sin \lambda & \sin L
\end{bmatrix}
$$

where $n$ refers to the local horizontal geographic frame.

The update equations of the attitude, the velocity and the position in the grid frame are expressed as:

$$
C^G_b = C^G_b \left[ \omega_b^b \times \right] - \left[ \omega_b^G \times \right] C^G_b
$$

$$
\dot{v}^G = C^G_e f^b - \left(2 \omega_e^G + \omega_e^G \times v^G\right) \times v^G + g^G
$$

$$
\dot{R}^e = C^G_e v^G
$$

where
\[ \mathbf{\omega}_G = \mathbf{\omega}_e^G + \mathbf{\omega}_{eg}^G = C_e^G \mathbf{\omega}_e + \mathbf{\omega}_{eg}^G \]

\[ \mathbf{\omega}_e^G = \begin{bmatrix} -\omega_{ie} \sin \sigma \cos L \\ \omega_{ie} \cos \sigma \cos L \\ \omega_{ie} \sin L \end{bmatrix} \]

where \( R_e \) is the radius of curvature of the grid east, \( R_i \) is the radius of curvature of the grid north, \( \tau_f \) is the distorted radius.

Since the meridian convergence rapidly in the polar region, the position of the aircraft in the polar region is usually expressed in the ECEF frame. The relationship between the coordinates \( x, y, z \) and the latitude \( L \) and longitude \( \lambda \) is:

\[ \begin{align*}
  x &= (R_N + h) \cos L \cos \lambda \\
  y &= (R_N + h) \cos L \sin \lambda \\
  z &= \left[ R_N \left(1 - f^2\right) + h\right] \sin L
\]  

where \( R_N \) represent the radiiuses of curvature in prime vertical, \( f \) is the ellipse flatness, \( h \) is the height.

2.2. Dynamic Model of the Grid SINS

The mechanization of grid SINS is accomplished in 2.1. Next, the Kalman filter based on G-frame needs to be designed. In order to design the Kalman filter, the dynamic model of G-frame including three differential equations is given below \([6]\).

The attitude error equation is expressed by:

\[ \dot{\mathbf{\phi}} = -\mathbf{\omega}_G^e \times \mathbf{\phi} + \mathbf{\delta \omega}_G^e - C_b^G \mathbf{\delta \omega}_b 
\]

The velocity error equation is expressed by:

\[ \dot{\mathbf{v}}^G = f^G \times \mathbf{\phi} + \mathbf{v}^G + (2 \mathbf{\delta \omega}_e^G + \mathbf{\delta \omega}_G^e) - (2 \mathbf{\omega}_e^G + \mathbf{\omega}_G^e) \times \mathbf{\delta v}^G + C_b^G \mathbf{\delta f}^b 
\]

The position error equation is as followed:

\[ \delta \mathbf{R}^C = C^G_b \delta \mathbf{v}^G + \delta C^G_v \mathbf{v}^G 
\]

3. Design of INS/GNSS Integrated Navigation Filter Model with Covariance Transformation

When the aircraft flies to polar region, it is needed to changed navigation frame from n-frame to G-frame. In addition to the tranformation of navigation parameters, the integrated navigation filter is also needs to be transformed. The Kalman filter includes the state equation and the observation equation which is determined by the dynamic model. Different Kalman filters designed on different navigation frames have different filter states \( \mathbf{x} \) and covariance matrices \( \mathbf{P} \), which need to be transformed.

The filtering state at low and middle latitudes is expressed by:

\[ \mathbf{x}^l(t) = [\phi^b_E, \phi^b_N, \phi^b_U, \delta \mathbf{v}^b_E, \delta \mathbf{v}^b_N, \delta \mathbf{v}^b_U, \delta L, \delta \lambda, \delta h, \epsilon^b_x, \epsilon^b_y, \epsilon^b_z, \nabla^b_x, \nabla^b_y, \nabla^b_z]^T 
\]

At the high latitude, the intergrated filter is designed in the grid frame. The filtering state is expressed by:

\[ \mathbf{x}^G(t) = [\phi^G_E, \phi^G_N, \phi^G_U, \delta \mathbf{v}^G_E, \delta \mathbf{v}^G_N, \delta \mathbf{v}^G_U, \delta x, \delta y, \delta z, \epsilon^b_x, \epsilon^b_y, \epsilon^b_z, \nabla^b_x, \nabla^b_y, \nabla^b_z]^T 
\]
Then, the transformation of the filtering state and the covariance matrix need to be deduced.
Comparing (11) and (12), it can be found that the states that remain unchanged before and after
the navigation frame changes are: the bias of the gyroscope $b\boldsymbol{\varepsilon}$ and the accelerometer $b\nabla$.
Therefore, it is sufficient to establish a transformation relationship between the attitude error $\phi$,
the velocity error $\delta v$ and the position error $\delta p$.

The transformation relationship between the attitude error $\phi^n$ and $\phi^G$ is determined as follows:
According to the definition of $\delta C_b^G$,
$$\delta C_b^G = -[\phi^G \times C_b^G]$$  \hspace{1cm} \text{(13)}

From equation $C_b^G = C_n^G C_b^n$, $\delta C_b^G$ can be expressed as:
$$\delta C_b^G = \delta C_n^G C_b^n + C_n^G \delta C_b^n = -[\phi_n^G \times C_b^n] - C_n^G [\phi^n \times C_b^n]$$  \hspace{1cm} \text{(14)}

Substituting for $\delta C_b^G$ from equation (13), $\phi^G$ can be described as:
$$\phi^G = C_n^G \phi^n + \phi_n^G$$  \hspace{1cm} \text{(15)}

where $\phi_n^G$ is the error angle vector of $C_n^G$:
$$\delta C_n^G = \bar{C}_n^G - C_n^G = -[\phi_n^G \times C_n^G]$$  \hspace{1cm} \text{(16)}
$$\phi_n^G = [0 \ 0 \ -\delta \sigma]^T$$  \hspace{1cm} \text{(17)}
$$\delta \sigma = \frac{\sin \sigma \cos \sigma}{\sin L} \cos L + \frac{1 - \cos^2 \sigma \cos^2 L}{\sin L} \delta \lambda$$  \hspace{1cm} \text{(18)}

The transformation relationship between the velocity error $\delta v^n$ and $\delta v^G$ is determined as follows:
$$\delta v^G = C_n^G \delta v^n + \delta C_n^G v^n = C_n^G \delta v^n - [\phi_n^G \times C_n^G v^n]$$  \hspace{1cm} \text{(19)}

From equation (7), the position error can be written as:
$$\begin{bmatrix}
\delta x \\
\delta y \\
\delta z
\end{bmatrix} = - (R_N + h) \sin L \cos \lambda - (R_N + h) \cos L \sin \lambda \cos L \cos \lambda \begin{bmatrix}
\delta L \\
\delta \lambda \\
\delta h
\end{bmatrix}$$  \hspace{1cm} \text{(20)}

To sum up, the transformation relationship between the system error state $x^n(t)$ and $x^G(t)$ is as follows:
$$x^G(t) = \Phi x^n(t)$$  \hspace{1cm} \text{(21)}

where $\Phi$ is determined by the equation (15), (19) and (20).

The transformation relation of the covariance matrix is as follows:
$$P^G(t) = \Phi P^n(t) \Phi^T$$  \hspace{1cm} \text{(22)}

When the aircraft flies out of the polar region, $x^G$ and $P^G$ are converted to $x^n$ and $P^n$, which can
be described as:
\[ x^n(t) = \Phi^{-1} x^G(t) \]
\[ P^n(t) = \Phi^{-1} P^G(t) \Phi^{-T} \]  

(23)

4. Experimental Results and Discussions

4.1. Flight experiment

The laser gyro inertial navigation system was used to conduct flight experiments. The laser gyro bias stability was less than 0.01 °/h. The accelerometer bias stability was less than 20 ug. The GNSS positioning error is less than 10 m, which is used as the position reference. The update frequency of gyro and accelerometer is 200 Hz while the update frequency of GNSS is 1 Hz. Experiments were carried out at middle latitudes. The alignment time was 0.5 h. Then, the aircraft took off. Firstly, the navigation frame is local geographic frame. After flying for half an hour, the navigation frame was changed to the grid frame until the end of the flight.

Since the flight experiment was carried out at middle latitudes, INS/GNSS integrated navigation result based on the local geographic frame without navigation frame change have high accuracy. Thus, it can be used as a reference for comparison. The navigation results based on the covariance transformation and non-covariance transformation are converted to the local geographic frame for comparison, which are shown as Figure 2 to Figure 5.

As shown in Figure 2, the change of the filter structure result in fluctuation of the relative attitude error. Among the three attitude errors, the relative yaw error reaches the maximum value of 2.2 °.
without covariance transformation. As a comparison, the covariance transformation method reduces this error to 0.3 '.

As shown in Figure 3, the relative position error is less than 10 m regardless of whether the covariance transformation is used. The INS/GNSS integrated navigation filter uses the position information provided by GNSS as observations to correct the INS results, resulting in less position error.

As shown in Figure 4 and 5, the maximum bias error of the gyroscope with and without covariance transformation reach 0.003 °/h and 0.01 °/h respectively. The maximum bias error of accelerometer with and without covariance transformation reaches 6 ug and 50 ug respectively. Because of the non-zero values of non-diagonal elements in the covariance matrix, the bias estimates of the gyroscope and accelerometer are affected by the cross-coupling of other error states. As a result, the bias estimates of the gyroscope and accelerometer also show instability.

To sum up, when the navigation frame changes directly, the integrated navigation result showed severe fluctuation, taking more than an hour to reach stability again. The lower observability of the error state is, the larger the error amplitude is. The integrated navigation based on the covariance transformation method don’t fluctuate during the change of the navigation frame, which are consistent with the reference results. Experimental results confirm the effectiveness of the algorithm.

4.2. Semi-physical simulation experiment

In order to verify the applicability of the algorithm at high latitudes, the latitude incremental method is used to convert the measured aviation data to 80° latitude to obtain the semi-physical simulation data. At high latitudes, the accuracy of the navigation based on grid frame is better than that based on local geography frame. Though, the navigation result based on the grid frame is used as a reference. Other experimental conditions are consistent with the flight experiment in 3.1. The comparison of navigation results with and without covariance transformation is shown in Figure 6 to Figure 9.

Fig. 6 Relative attitude error in simulation.

Fig. 7 Relative position error in simulation.
As shown in Figure 6, among the three attitude errors, the relative yaw error is the largest. Without covariance transformation, the relative yaw angle error reaches 5’. The integrated navigation result with covariance transformation has a less relative yaw error of 0.2’.

As shown in Figure 7, the relative position error is 12 m without covariance transformation. The integrated navigation result with covariance transformation shows better stability and smaller relative position error of 8 m.

As shown in Figure 8 and 9, the maximum bias error of the gyroscope with and without covariance transformation reach 0.001 °/h and 0.02 °/h respectively. The maximum bias error of accelerometer with and without covariance transformation reaches 0.1 ug and 25 ug respectively.

At high latitudes, the changes of navigation frames causes more severe error fluctuation. In this case, covariance transformation method still makes a smooth tranformation of the navigation frames, which is proved to be effective.

5. Conclusion
Covariance transformation method fundamentally solves the problem of filter fluctuation caused by the change of navigation frame. The advantage of this method is that it does not change the existing navigation algorithm based on n-frame and G-frame, and improves the navigation accuracy when crossing the polar region. The results of flight experiment and semi-physical simulation show that the covariance transformation method is effective at any latitude.

References
[1] Gan, X., Li, W., Yang, L. (2020) State-Space Measurement Update for GNSS/INS Integrated Navigation. Mathematical Problems in Engineering.
[2] Zhao, L., Kang, Y., Cheng, J. (2019) A Fault-Tolerant Polar Grid SINS/DVL/USBL Integrated Navigation Algorithm Based on the Centralized Filter and Relative Position Measurement. Sensors, 19(18).
[3] Babich O A. (2019) Extension of the Basic Strapdown INS Algorithms to Solve Polar Navigation Problems. Gyroscopy and Navigation 10(4) 330-338.
[4] Fang, T., Huang, W., Luo, L. (2019) Damping Rotating Grid Sins Based on a Kalman Filter for Shipborne Application. IEEE Access, PP(99) 1-1.
[5] Huang, L., Xu, X., Zhao, H. (2020) Transverse SINS/DVL Integrated Polar Navigation Algorithm Based on Virtual Sphere Model. Mathematical Problems in Engineering.
[6] Zhou, Q. (2012) All-Earth Inertial Navigation Algorithm for Large Aircraft. Northwestern Polytechnical University, Xian.
[7] Liu, M., Li, G., Gao, Y. (2018) Improved Polar Inertial Navigation Algorithm Based on Pseudo INS Mechanization. Aerospace science and technology, 77 105-116.
[8] Liu, C., Wu, W., Feng, G. (2020) Polar Navigation Algorithm for INS Based on Virtual Sphere n-vector. Journal of Chinese Inertial Technology, 28(04) 421-428.