Forecasting of Non-Stationary Sales Time Series Using Deep Learning

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Abstract

The paper describes the deep learning approach for forecasting non-stationary time series with using time trend correction in a neural network model. Along with the layers for predicting sales values, the neural network model includes a subnetwork block for the prediction weight for a time trend term which is added to a predicted sales value. The time trend term is considered as a product of the predicted weight value and normalized time value. The results show that the forecasting accuracy can be essentially improved for non-stationary sales with time trends using the trend correction block in the deep learning model.

Keywords: Sales forecasting, non-stationary time series, deep learning, trend correction.

1 Introduction

Sales and demand forecasting are widely being used in business analytics [1, 2, 3]. Sales can be treated as time series. Different time series approaches are described in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Machine learning is widely used for forecasting different kinds of time series along with classical statistical methods like ARIMA, Holt-Winters, etc. Modern deep learning algorithms DeepAR [19, 20], N-BEATS [21], Temporal Fusion Transformers [22] show state-of-the-art results for time series forecasting. Sales prediction is more a regression problem than a time series problem. The use of regression approaches for sales forecasting can often give us better results compared to time series methods. Machine-learning algorithms make it possible to find patterns in the time series. Some of the most popular ones are tree-based machine-learning algorithms [23], e.g., Random Forest [24], Gradient Boosting Machine [25, 26]. The important time series features for their successful forecasting are their stationarity and sufficiently long time of historical observation to be able to capture intrinsic time series patterns. One of the main assumptions of regression methods is that the patterns in the past data will be repeated in future. There are some limitations of time series approaches for sales forecasting. Let us consider some of them. We need to have historical data for a long time period to capture seasonality. However, often we do not have historical data for a target variable, for example in case when a new product is launched. At the same time, we have sales time series for a similar product and
we can expect that our new product will have a similar sales pattern. Sales data can have a lot of outliers and missing data. We must clean those outliers and interpolate data before using a time series approach. We need to take into account a lot of exogenous factors which impact on sales. On the other hand, sales time series have their own specifics, e.g. their dynamics is caused by rather exogenous factors than intrinsic patterns, they are often highly non-stationary, we frequently face with short time sales observations, e.g. in the cases when new products or stores are just launched. Often non-stationarity is caused by a time trend. Sales trends can be different for different stores, e.g. some stores can have an ascending trend, while others have a descending one. Applying machine learning regression to non-stationary data, bias in sales prediction on validation dataset can appear. This bias can be corrected by additional linear regression on a validation dataset when a covariate stands for predicted sales and a target variable stands for real sales [27].

In [28], we studied linear models, machine learning, and probabilistic models for time series modeling. For probabilistic modeling, we considered the use of copulas and Bayesian inference approaches. In [29], we studied stacking approaches for time series forecasting and logistic regression with highly imbalanced data. In [27], we study the usage of machine-learning models for sales time series forecasting. In [30], we analyse sales time series using Q-learning from the perspective of the dynamic price and supply optimization. In [31], we study the Bayesian approach for stacking machine learning predictive models for sales time series forecasting.

In this case study, we consider the deep learning approach for forecasting non-stationary time series with the trend using a trend correction block in the deep learning model. Along with the layers for predicting sales values, the model includes a subnetwork block for the prediction weight for a trend term which is added to the predicted sales value. The trend term is considered as a product of the predicted weight value and normalized time value.

2 Sales time series with time trend

For the study, we have used the sales data which are based on the dataset from the 'Rossman Store Sales' Kaggle competition [32]. These data represent daily sales aggregated on the granularity level of customers and stores. As the main features, 'month', 'weekday', 'trendtype', 'Store', 'Customers', 'StoreType', 'StateHoliday', 'SchoolHoliday', 'CompetitionDistance' were considered. The 'trendtype' feature can be used in the case if the sales trend has different behavior types on different time periods. To study non-stationarity, arbitrary time trends were artificially added to the data grouped by stores. The calculations were conducted in the Python environment using the main packages pandas, sklearn, numpy, keras, matplotlib, seaborn. To conduct the analysis, Jupyter Notebook was used. Figures [1][3] show the arbitrary examples of aggregated sales time series with different time trends for different stores. Both training and validation datasets were received by splitting the dataset by date, so the validation time period is next with respect to the training period. Figure [4] shows the probability density function (PDF) for all stores for training and validation datasets. Figures [5][7] show the probability density function for sales in data samples, of specified stores which correspond to the stores time series shown in Figures [1][3]. For some stores, sales PDF are different for training and validation datasets due to non-stationarity caused by different sales trends for different stores.
Figure 1: Store sales time series

Figure 2: Store sales time series

Figure 3: Store sales time series
Figure 4: Probability density function for sales in all stores

Figure 5: Probability density function for sales in specified store

Figure 6: Probability density function for sales in specified store
3 Deep learning model with time trend correction

Let us consider including a correction trend block into the deep learning model. Along with the layers for predicting sales values, the model will include a subnetwork block for the prediction weight for the time trend term which is added to the predicted sales value. The time trend term is considered as a product of the predicted weight value and normalized time value. The predicted sales values and the time trend term are combined in the loss function. As a result, one can receive an optimized trend correction for non-stationary sales for different groups of data with different trends. For modeling and deep learning case study, the Pytorch deep learning library \[33, 34\] was used. Categorical variables \textit{Store}, \textit{Customer} with a large number of unique values were coded using embedding layers separately for each variable, categorical variable with a small number of unique values \textit{StoreType}, \textit{Assortment} were represented using one-hot encoding. Figure 8 shows the parameters of neural network layers. Figure 9 shows the neural network structure.

```
modeltrend()
  (inputblock): inputblock(
    (emb_list): ModuleList(
      (0): Embedding(151, 76)
      (1): Embedding(2853, 128)
      (2): Embedding(13, 7)
      (3): Embedding(7, 4)
    )
    (linear1): Linear(in_features=225, out_features=256, bias=True)
    (drop1): Dropout(p=0.1, inplace=False)
  )
  (mainblock): mainblock(
    (linear2): Linear(in_features=256, out_features=64, bias=True)
    (drop2): Dropout(p=0.1, inplace=False)
  )
  (trendblock): trendblock(
    (linear1): Linear(in_features=256, out_features=32, bias=True)
    (drop1): Dropout(p=0.1, inplace=False)
    (linear2): Linear(in_features=32, out_features=8, bias=True)
    (drop2): Dropout(p=0.1, inplace=False)
    (linear3): Linear(in_features=8, out_features=1, bias=False)
  )
```

Figure 8: Parameters of neural network layers
Figure 9: Neural network structure
The tensors of the output of embedding layers and numerical input values are concatenated and connected with the linear layer with ReLU activation that forms an input block. The output of this input block is directed to the main block which consists of fully connected linear layers with ReLU activation and dropout layers for predicting sales values using the output linear layer. The output of the input block is also directed to the trend correction block which consists of fully connected layer with ReLU activation, dropout layer and output linear layer for the prediction of the trend weight. The time trend term which is a normalized time value multiplied by the trend weight is added to the predicted sales values. For the comparison, we have considered two cases of the model with and without trend correction block. Let us consider the results of model training and evaluation. Figure 10 shows learning rate changes with epochs. Figure 11 shows train and validation loss for the model without trend correction block, Figure 12 shows these losses for the model with the trend correction block. Figure 13 shows the features importance which has been received using the permutation approach.

Figure 10: Learning rate changes with epochs

Figure 11: Train and validation loss for model without trend correction block
Numerical features and target variables before feeding into the neural network were normalized by extracting their mean values from them and dividing them by their standard deviation. The forecasting score over all stores is $RMSE = 1076$, for the model without the trend correction block and $RMSE_{trend} = 943$ for the model with the trend correction block. We can see a small improvement in the accuracy on the validation set for the model with the trend correction block. Figures 14-16 show aggregated store sales time series on the validation dataset, which was predicted using the model without and with the trend correction block. These results of predicted sales correspond to time series which are shown in Figures 1-3. One can see that for some stores sales with time trend, the forecasting accuracy can be essentially improved using the trend correction block.
Figure 14: Store sales time series on validation dataset, predicted using model without (pred) and with (pred\_trend) trend correction block ($RMSE = 706$, $RMSE_{trend} = 646$)

Figure 15: Store sales time series on validation dataset, predicted using model without (pred) and with (pred\_trend) trend correction block ($RMSE = 3699$, $RMSE_{trend} = 1753$)
4 Conclusions

Applying of machine learning for non-stationary sales time series with time trend can cause a forecasting bias. The approach with the trend correction block in the deep learning model for sales forecasting has been considered. The model predicts sales values simultaneously with the weight for the time trend term. The time trend term is considered as a product of the predicted weight value and normalized time value and then added to predicted sales values. As a result, an optimized weight for the time trend for different groups of sales data can be received, e.g. for each store with the intrinsic time trend, the optimized weight for the time trend can be found. The results show that the forecasting accuracy can be essentially improved for non-stationary sales with time trends using the trend correction block in the deep learning model.

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