Zero Moment Point Estimation Based on Resonant Frequencies of Wheel Joint for Wheel-Legged Mobile Robot

Kenta Nagano*1) Member, Yasutaka Fujimoto** Senior Member

(Manuscript received February 22, 2021, revised November 13, 2021)

The zero moment point (ZMP) is a measure for realizing dynamic motion in legged robots. An important concern in ZMP-based motion control is the measurement of the ZMP itself. General humanoid robots use sensors attached to their feet to measure the reaction forces. However, this method is not applicable for wheel-legged mobile robots because it is difficult to attach force sensors inside the wheels. Therefore, this study proposes a method to determine the ZMP based on the relationship between the load and resonant frequency of the tires. The resonant frequency of the tires depends on the tire pressure. First, the relationship between the load and resonant frequency is confirmed by a frequency analysis and experiments on the wheel joints. Second, a method used to estimate the ZMP based on the relationship between the load and resonant frequency is described. Third, an online estimation technique for the ZMP is described. Finally, factors other than the vertical force that affect the frequency characteristics of the wheel are discussed. The average of the root-mean-square errors from the sensor value in the online-ZMP estimation results is 0.0361 m, which is sufficiently smaller than the typical stability margin of the support polygon of the robot.

Keywords: wheel-legged mobile robot, zero moment point, resonant frequency, load estimation.

1. Introduction

Robots that can perform unanticipated tasks and move by themselves in different environments are anticipated. A wheel-legged mobile robot has both leg and wheel mechanisms, and its modes of movement meet this expectation: it can use the wheel mechanism to move over flat surfaces and the leg mechanism to move over uneven terrain. Using the leg or wheel mechanism selectively, the robot can move efficiently in a variety of environments. Boston Dynamics has developed a wheel-legged mobile robot called Handle that realizes high-speed and effective motion in various environments using its two wheel-legged mechanisms (1). Moreover, wheel-legged mobile robots have been discussed in several related studies. For example, this type of robot has been studied for rescue applications (2), for movement in extreme environments (3), (4), for personal mobility (5), and for human-to-human support environments (6), (7).

Stability must be measured when the robot moves. Several static stability measures have been proposed for the robot (8)–(11), as well as dynamic stability measures (12), (13). However, the downside to these stability measurements is that the robot dynamics and the external forces must be known, and they need to be transformed into the measurement frame. To realize effective motion in various environments, this study focuses on motion control based on the zero moment point (ZMP), which is the center of pressure of the ground reaction force (14). The ZMP is simple to calculate when the external forces are known compared with other measures. ZMP-based methods are used for dynamic movement stabilization, thus allowing robots to perform unanticipated tasks and movements in different environments. For wheel-legged mobile robots, there have been several studies on ZMP-based wheeled locomotion (15)–(17). However, none of these studies have attempted dynamic motion stabilization using feedback of ZMP measured from reaction forces in a real environment.

Accurate measurement of the ZMP is necessary for ZMP-based motion control. However, measuring the ZMP with force sensors attached inside the feet, as in a conventional humanoid robot (18)–(22), is difficult in a wheel-legged mobile robot. Nevertheless, ZMP-based motion control has already been realized in a wheeled mobile robot by calculating the ZMP with a robot model (23). In addition, methods have been proposed for measuring the ZMP using force sensors attached between the body and the wheel joint (23)–(25). These methods were realized the effectiveness of motion control for the wheeled mobile robot based on the ZMP. However, it can be difficult to realize walking motion in a wheel-legged mobile robot using these methods because they do not use the ground reaction force at the contact point of the wheel. Therefore, it is necessary to consider a new technique in measuring the ZMP. Measuring the force at the contact point of the tire is necessary when calculating the ZMP of a wheel-legged mobile robot. Several methods have been proposed for obtaining this force in mobile robots and automobiles, including methods using mathematical models (26), (27), methods using observers in addition to mathematical models (28), (29), and a method using special sensors in addition to mathematical models (30).

* Correspondence to: k.nagano@ieee.org
** Department of Electrical Engineering, National Institute of Technology, Tokyo College
1220-2, Kunugita, Hachioji, Tokyo 193-0997, Japan
† Faculty of Engineering, Yokohama National University
79-5, Tokiwadai, Hodogaya-ku, Yokohama, Kanagawa 240-8501, Japan

© 2022 The Institute of Electrical Engineers of Japan.
The present paper considers a method for estimating the tire force by concentrating on changes in tire pressure. In most automobiles, tire pressure is measured using a tire-pressure monitoring system (TPMS)\(^{33}\). In an automobile, the resonant frequency of an inflated tire changes according to the tire pressure\(^{34,35}\). Therefore, the method proposed in this paper is used to estimate the ZMP using changes in the resonant frequency of the tire as detected at the wheel joint.

This paper is organized as follows. Section 2 shows how a frequency analysis and experiments were used to assess the dependence of the frequency characteristics of a wheel-legged mobile robot on the contact-point load. Section 3 describes how we used the changes in the frequency characteristics to estimate the load, which is then used to estimate the ZMP. The performance of the proposed method is assessed experimentally. Section 4 discusses the technique used for the online estimation of the ZMP. Section 5 discusses factors that affect the frequency characteristics of the wheel joint other than the vertical force. Finally, Sect. 6 presents our conclusions.

2. Resonant Frequency of a Wheel Joint

We begin this section by describing the wheel joint of our robot. A photograph of our wheel-legged mobile robot is shown in Fig. 1. We then describe our frequency analysis and experiments to measure the frequency characteristics and to establish a relationship between the resonant frequency and the load. In this section, we confirm this relationship for tubular and solid tires.

The wheel joint of our robot is shown in Fig. 2. The output of a DC motor is connected to the wheel using a synchronous belt pulley and a reduction gear (harmonic drive). In addition, it is possible to change the tire in the wheel joint from a solid one to a tubular one.

2.1 Frequency Analysis of the Wheel

The frequency analysis used to establish the relationship between the resonant frequency and the load was performed using the SolidWorks modeling software. The wheel model comprises a tire and a steel wheel. Circumferentially, the tire has a cylindrical surface with a planar contact portion at the bottom. In addition, the tubular tire model has a hollow structure that contains an air model. The wheel model has constraints that permit circumferential motion of the tire and the steel wheel. A distributed load of 5.0 or 10.0 kg was applied to the steel wheel.

From the frequency analysis, the resonant frequencies of the solid and tubular tires for each mode order are shown in Fig. 3. In addition, the effective mass participation factor (EMPF) for each mode order for loads of 5.0 and 10.0 kg is shown in Fig. 4 for the solid and tubular tires. The EMPF is the percentage of the mass that contributes to a specific mode. Therefore, the larger the EMPF of a mode, the greater the influence of that mode on the dynamic response of the system. In this paper, the EMPF is calculated for the longitudinal, lateral, and normal axes of the tire.

For a solid tire, the results in Fig. 3 show that the resonant frequency increases with the load for modes higher than second order. The change in EMPF with load is the largest for the second- and third-order modes, as shown in Fig. 4(a). For a tubular tire, the results in Fig. 3 show the resonant frequency increasing with the load for modes higher than third order. The change in EMPF with load is the largest for the
ZMP Estimation for Wheel-Legged Mobile Robot (Kenta Nagano et al.)

Fig. 4. Effective mass participation factor (EMPF) for loads of 5.0 and 10.0 kg from the frequency analysis: (a) solid tire and (b) tubular tire.

Fig. 5. Experimental setup for measuring the frequency characteristics of the wheel joint.

third- and fourth-order modes, as shown in Fig. 4(b). Hence, we conclude from the above that the resonant frequency depends on the load on the tire.

2.2 Frequency Characteristics of the Wheel Joint

In this section, the frequency characteristics of the wheel joint are measured to verify the results from the frequency analysis. The frequency characteristics of the wheel in the first leg were measured five times when the load on the contact point was either 5.0 or 10.0 kg. This load was measured using a load meter. The experimental setup is shown in Fig. 5. The measurement conditions are listed in Table 1.

The measurement results for the frequency characteristics of the solid tire and the tubular tire are shown in Fig. 6. For the solid tire, from the measurement results in Fig. 6(a), the resonant frequency of the gain characteristic is seen to change from 50.0 to 60.0 Hz when the load is increased. In addition, the resonant frequency of the phase characteristic also changes from 45.0 to 50.0 Hz. As with the solid tire, the resonant frequency of the tubular tire changed with the load, as shown in Fig. 6(b). From the above, the dependence of the resonant frequency on the load has been confirmed, which agrees with the frequency analysis. Therefore, we now proceed to consider how to estimate the ZMP using the relationship between the load and the resonant frequency. The following discussion will consider only the solid tire because both tires have similar characteristics.

3. Method for Estimating the Zero Moment Point

From the results of the frequency analysis and the frequency characteristics of the wheel joints in section 2, the relationship between the load and the resonant frequency was confirmed. In this section, the method for estimating the ZMP using the relationship between the load on the contact point and the resonant frequency of the wheel is described. Firstly, the calibration data are measured and are used to create an approximation model of the gain and phase characteristics based on the resonant frequencies. Next, the load on each leg is estimated from the test data using the approximation model. Then, the ZMP for the robot is calculated from the estimated load on each leg.
Table 1: Measurement Conditions

| Subject                          | Section 2.2 | Section 3 | Section 4 (Calibration / Test) |
|----------------------------------|-------------|-----------|-------------------------------|
| Calculation type                 | Offline     | Offline   | Offline / Online              |
| Input signal for the frequency characteristics | Motor torque | Motor torque | Motor torque                  |
| Kind of input signal             | Swept sine wave from 0 to 250 Hz | White Gaussian noise | White Gaussian noise |
| Output signal for the frequency characteristics | Motor angular velocity | Motor angular velocity | Motor angular velocity |
| Sampling interval of input and output signal | 0.5 ms | 1.0 ms | 0.05 ms |
| Control interval of each joint   | —           | 1.0 ms    | 1.0 ms                        |
| Calculation interval of the frequency characteristics | — | — | — / 0.5 s |
| Sampling length of input and output signal | 5.0 s | 8.192 s | 3.2768 s / 0.4096 s |
| Smoothing of the frequency characteristics | Moving average filter | Moving average filter | Moving average filter |

† The motor angular velocity was calculated from the angle provided by the encoder attached to the motor.
†† Each joint other than the wheel joint was controlled by a joint space control using proportional-derivative control and a disturbance observer.

3.1 Measurement of Calibration Data This section describes the measurement of the calibration data for creating the approximation model. Figure 7 shows the experimental conditions for estimating the ZMP. The frequency characteristics of the wheel were measured five times using a load meter for contact-point loads of 5.0, 7.5, and 10.0 kg. In addition, the contact-point load was changed by moving the posture of the robot using a chain block. The chain block can move in the lateral and longitudinal directions relative to the robot, and it can lift the robot. Therefore, the experiment was conducted by static motion. This is because this paper focuses on the verification of the ZMP estimation from the relationship between the resonant frequency of the wheel and the load. The load was changed in the same way for the subsequent measurements. The measurement conditions are shown in Table 1. Here, we cannot obtain the resonant frequency only with the joint servo signal, and the swept sine wave cannot be measured while the joint is being driven. Therefore, white Gaussian noise is used because it has a uniform frequency component.

The measurement results for the frequency characteristics of the calibration data are shown in Fig. 8. Here, the frequency characteristics of the first leg are shown as an example. In addition, the resonant frequencies of the calibration data are shown in Fig. 9. Data are excluded if we cannot obtain the resonant frequency because of noise. From the results, the frequency characteristics become the noisy shapes because the white Gaussian noise was applied as the input signal. Therefore, the resonant frequency obtained after smoothing was varied. Nevertheless, the resonant frequency of each leg clearly depends on the load on the contact point.

3.2 Approximation Model for the Resonant Frequency The approximation model for each leg is created using the resonant frequency of the calibration data. The coefficients of the following bivariate function are obtained:

![Fig. 7. Experimental conditions for the ZMP estimation.](image1)

![Fig. 8. Frequency characteristics of the calibration data in the first leg. The solid lines and shaded areas indicate the mean and the standard deviation for five measurements.](image2)

![Fig. 9. Resonant frequencies of the calibration data. The dots and error bars indicate the mean and the standard deviation for five measurements. The solid lines show the fitting line of the mean values by the least-squares calculation.](image3)
The approximation models are obtained using least-squares calculations from the calibration data for each leg as follows:

\[ W_{\text{leg1}} = \omega_{G_{\text{cal}}^1} + 0.3762 \omega_{P_{\text{cal}}^1} - 27.1874 \cdot \cdots (5) \]

\[ W_{\text{leg2}} = 0.4265 \omega_{G_{\text{cal}}^2} + 0.3522 \omega_{P_{\text{cal}}^2} - 32.8957 \cdot \cdots (6) \]

\[ W_{\text{leg3}} = 0.3176 \omega_{G_{\text{cal}}^3} + 0.1450 \omega_{P_{\text{cal}}^3} - 18.8801 \cdot \cdots (7) \]

\[ W_{\text{leg4}} = 0.2319 \omega_{G_{\text{cal}}^4} + 0.3914 \omega_{P_{\text{cal}}^4} - 25.6484 \cdot \cdots (8) \]

where \( W_{\text{legi}} \), \( \omega_{G_{\text{cali}}} \), and \( \omega_{P_{\text{cali}}} \) are the load on the contact point, the resonant frequency of the gain characteristic, and the resonant frequency of the phase characteristic of the \( i \)th leg, respectively.

### 3.3 ZMP Estimation Using the Approximation Models

In this section, load estimation using the approximation models and the calculation of the ZMP are described. The frequency characteristics of the test data used for estimating the ZMP were measured under the same conditions as those for the calibration data. Here, the load was applied to each leg for the four cases where the ZMP of the robot moves to the front or rear and to the left or right, as shown in Table 2. Five measurements were recorded. The resonant frequencies of the test data are shown in Fig. 10. Then, the load on each leg was estimated using the approximation models and the resonant frequencies of the test data. The estimated loads are shown in Fig. 11. In addition, the root-mean-square errors (RMSEs) of the estimated loads are given in Table 3.

The frequency characteristics of the test data used for estimating the ZMP are described. In this section, load estimation using the approximation models and the calculation of the ZMP are described. The frequency characteristics of the test data used for estimating the ZMP were measured under the same conditions as those for the calibration data. Here, the load was applied to each leg for the four cases where the ZMP of the robot moves to the front or rear and to the left or right, as shown in Table 2. Five measurements were recorded. The resonant frequencies of the test data are shown in Fig. 10. Then, the load on each leg was estimated using the approximation models and the resonant frequencies of the test data. The estimated loads are shown in Fig. 11. In addition, the root-mean-square errors (RMSEs) of the estimated loads are given in Table 3.

From the results shown in Fig. 11, the estimated load on each leg is seen to depend on the applied load. However, the variation in the estimated load is large in Fig. 11 and Table 3 because the noise in the frequency characteristics affects the resonant frequency. In particular, the amount of change in the resonant frequencies of the gain and phase characteristics is small, as shown in Fig. 9. In addition, a load meter was used to measure the vertical force. However, the proposed method cannot be used to give an estimate by considering the reaction force in each direction. In the experiment, the center of gravity was moved toward the lateral and longitudinal directions.

### Table 2: Applied Load in the Test Data

| Case | 1st leg (kg) | 2nd leg (kg) | 3rd leg (kg) | 4th leg (kg) |
|------|-------------|-------------|-------------|-------------|
| Front | 9.0         | 6.0         | 9.0         | 6.0         |
| Rear  | 6.0         | 9.0         | 6.0         | 9.0         |
| Left  | 9.0         | 9.0         | 6.0         | 6.0         |
| Right | 6.0         | 6.0         | 9.0         | 9.0         |

### Table 3: RMSEs of the Estimated Loads

| Case     | 1st leg (kg) | 2nd leg (kg) | 3rd leg (kg) | 4th leg (kg) |
|----------|-------------|-------------|-------------|-------------|
| Front    | 0.8525      | 1.0632      | 0.2795      | 1.7361      |
| Rear     | 0.7282      | 1.5023      | 1.4435      | 1.1730      |
| Left     | 0.6703      | 1.1811      | 1.1367      | 1.0716      |
| Right    | 0.8114      | 0.9156      | 0.7589      | 0.6871      |
of the robot by altering the posture. Therefore, a load larger than the measured value was estimated because the external force is applied in multiple directions.

The position of the ZMP is calculated from the following equations using the estimated load:

\[ x_{\text{zmp}} = \frac{x_1 f_1 + x_2 f_2 + x_3 f_3 + x_4 f_4}{f_1 + f_2 + f_3 + f_4} \]  \hspace{1cm} (9)

\[ y_{\text{zmp}} = \frac{y_1 f_1 + y_2 f_2 + y_3 f_3 + y_4 f_4}{f_1 + f_2 + f_3 + f_4} \]  \hspace{1cm} (10)

where \( x_{\text{zmp}} \) and \( y_{\text{zmp}} \) are the ZMP positions in the x and y directions, respectively, \( x_i \) and \( y_i \) are the contact positions of the \( i \)th leg viewed from the center of the support polygon in the x and y directions, respectively, and \( f_i \) is the reaction force of the \( i \)th leg. Here, the reaction force is calculated from the estimated load. The relationships among the variables are shown in Fig. 12, and the ZMP positions are shown in Fig. 13. In addition, the RMSEs of the ZMP positions are given in Table 4.

From the results shown in Fig. 13 and Table 4, it is clearly possible to calculate the ZMP position using the estimated load. The errors in the ZMP position given in Table 4 are due to the large variation in the estimated load. However, a multi-legged wheel-legged mobile robot has a large support polygon. For our robot, the support polygon extends 0.620 and 0.665 m in the x and y directions, respectively. Therefore, an error of approximately 0.02 m is considered to be acceptable.

4. Online Estimation of the Zero Moment Point

The online ZMP estimation is required to actually to use for robot motion control based on the ZMP because the ZMP estimation results in section 3 were the offline estimation. Therefore, this section describes an approach for the online ZMP estimation. To estimate the ZMP online, the frequency characteristics need to be computed online. This method uses a fast Fourier transform (FFT), the resolution of which is determined by the data length and sampling frequency. Thus, obtaining a sufficient data length for the FFT in a short time is necessary for online ZMP estimation. To accomplish this, we use zero padding, a technique that increases the data length by adding a zero to the FFT data to increase the apparent FFT resolution (\( \omega_p \)). This results in a sufficient data length for the FFT in a short time. Therefore, the frequency characteristics for estimating the ZMP can be obtained in a short time using zero padding.

Next, the calculation of the online estimation is verified through an experiment. The calibration data are measured using the same conditions as those for the offline case. The measurement conditions are shown in Table 1. Here, the data length was 65,536 points. The approximation models were obtained as follows:

\[ W_{\text{leg}1} = 0.3786 \omega_{G_{\text{leg}1}} + 0.4270 \omega_{P_{\text{leg}1}} - 33.1656 \cdots (11) \]

\[ W_{\text{leg}2} = 0.3175 \omega_{G_{\text{leg}2}} + 0.3511 \omega_{P_{\text{leg}2}} - 25.6501 \cdots (12) \]

\[ W_{\text{leg}3} = 0.2057 \omega_{G_{\text{leg}3}} + 0.2072 \omega_{P_{\text{leg}3}} - 11.8049 \cdots (13) \]

\[ W_{\text{leg}4} = 0.2911 \omega_{G_{\text{leg}4}} + 0.2234 \omega_{P_{\text{leg}4}} - 18.2135 \cdots (14) \]

These approximation models are different from those of Eqs. (5) to (8) because the conditions for the FFT and the filter are different from the offline calculation. The online ZMP estimation experiment was conducted using these equations. The measurement conditions are shown in Table 1. Here, zero padding was used to obtain the FFT data in a short time. The sampling length was measured to be 0.4096 s using a sampling interval of 0.05 ms. The data length was 8,192 points, which further increased to 65,536 points after zero padding. Table 2 gives the load on each leg. In addition, a multi-link model-based ZMP calculation (\( \dagger \)) was also performed for comparison. The experimental results for the estimated load and ZMP are shown in Figs. 14 and 15, respectively. In addition, the RMSEs of the estimated load and the estimated ZMP are shown in Tables 5 and 6, respectively.

From the results shown in Fig. 14 and Table 5, the load can be estimated, though the variation is large. The cause of
Fig. 14. Online estimated loads of each leg in each direction. The dots and error bars indicate the mean and the standard deviation for five seconds measurements.

Table 5: RMSEs of the Online Estimated Load

| Case    | 1st leg (kg) | 2nd leg (kg) | 3rd leg (kg) | 4th leg (kg) |
|---------|--------------|--------------|--------------|--------------|
| Front   | 1.9725       | 1.6013       | 0.5551       | 1.7212       |
| Rear    | 1.3111       | 2.1615       | 0.6283       | 1.9572       |
| Left    | 1.2144       | 3.0812       | 0.3932       | 1.8120       |
| Right   | 3.0726       | 1.8057       | 1.2169       | 1.3908       |

this variation is that the frequency component was not sufficiently large because the original FFT signal was too short. From the ZMP results in Fig. 15 and Table 6, there was an estimation error in the multi-link model-based ZMP calculation. This method is affected by the change in posture and the initial conditions because the ZMP is calculated using the joint angle. In these experiments, the reaction force has been changed because of the external force, which is affected by the chain block attached to the robot. The multi-link model-based ZMP calculation cannot take the influence of the reaction force due to such an external force into account, and there is a large error. The ZMP estimated using the proposed method tends to be close to the measured ZMP because the influence of the reaction force at the contact point can be considered as the estimated load; however, the estimated ZMP has variations caused by the variation in the estimated load. Comparing the totals from the x- and y-axes, the average of the RMSEs for the proposed method is 0.0361 m, and for the multi-link model-based ZMP calculation, it is 0.0443 m. The proposed method reduced the ZMP error to about 82% of the multi-link model-based ZMP calculation. The accuracy of the estimated ZMP is evaluated to compare with the case of assuming the force sensor used in general ZMP measurement. Here the stability room is used for the evaluation measure. This measure is the minimal distance from ZMP to the boundary of the stable region. In this paper, the stable region was calculated by subtracting the error as the margin from the support polygon. The position of the ZMP is assumed at the most stable point which is the center of the support polygon, and the stability room was 0.274 m when the average of the RMSE of the online ZMP estimation is considered as the margin. Next, the stability room is obtained under the assumption of the force sensor. The weight of our robot is about 55 kg, therefore, the allowable load in the vertical direction is 1000 N to have a sufficient margin. When assuming the allowable load in the vertical direction is 1000 N, the resolution of the force sensor is 0.125 N to 0.5 N, and the median is 0.313 N. Then, the error of ZMP caused by the resolution of the force sensor in the case of the load in Table 2 applied on each leg is 0.00132 m in the x direction, 0.00142 m in the y direction, and 0.00137 m on average. The stability room considering the error from the resolution of the force sensor is 0.309 m. From the above, the proposed method can achieve an accuracy of 88.7% when assuming the use of the force sensor.

Finally, we considered the uncertainty in the experimental results, which is shown in Table 7. The uncertainties in the measurement by the load meter, the proposed method, and the multi-link model-based ZMP calculation were most affected by the accuracy of the load meter, the measurement repeatability, and the resolution of the encoder, respectively.

Table 6: RMSEs of the Online Estimated ZMP

| Method           | Case      | Estimated ZMP |  x_{zmp} (m) |  y_{zmp} (m) |
|------------------|-----------|---------------|--------------|--------------|
| Front            | Proposed  | 0.0361        | 0.0457       | 0.0389       |
| Rear             | Proposed  | 0.0394        | 0.0376       | 0.0394       |
| Left             | Proposed  | 0.0355        | 0.0303       | 0.0319       |
| Right            | Proposed  | 0.0575        | 0.0611       | 0.0447       |
| Rear             | Multi-link| 0.0593        | 0.0012       | 0.0420       |
| Left             | Multi-link| 0.0014        | 0.0704       | 0.0498       |
| Right            | Multi-link| 0.0020        | 0.0631       | 0.0447       |

n_{x_{zmp}} is the RMSE that combines x and y directions.
From Table 7, the estimated ZMP of the proposed method was closer to the value measured by the load meter than the value from the multi-link model-based ZMP calculation. In addition, the expanded uncertainty of the proposed method is larger than that of the multi-link model-based ZMP calculation; however, the uncertainty is acceptable because the expanded uncertainty is sufficiently smaller than the support polygon of the robot. From the above, the ZMP of a wheel-legged mobile robot is possible to estimate by the proposed method. The proposed method estimates the ZMP based on the relationship between the resonant frequency of the wheel and the load. This relationship is held on the wheel joint of other mobile systems that have the active wheel joint. Therefore, the proposed method can be applied to the active-wheeled mobile system other than the wheel-legged mobile robot and can be used for its motion control.

5. Discussion

The only vertical force was considered by the offline and online ZMP estimation in the previous sections. In a real environment, several factors affect the resonant frequency of the tire other than the vertical force. These factors include that the force is in multiple directions (multiple-direction force) as well as the vertical direction, temperature changes, and the stiffness and roughness of the ground. The effect of each factor was examined experimentally. The measurement conditions for the frequency characteristics were the same as those in Sect. 2.2. The load at the contact point was changed by adjusting the posture at the chain block.

5.1 Effect of the Multiple-Direction Force

The effects of the lateral and longitudinal forces on the wheel joint were then explored. The frequency characteristics were measured when the external force in the lateral and longitudinal directions was applied as shown in Fig. 16. Each applied load weighed 2.5 kg. The resonant frequencies of the vertical force alone and with the addition of the external force in the lateral and longitudinal directions are shown in Fig. 17. The first resonant frequencies in Fig. 17 correspond to the resonant frequencies seen in the frequency characteristics. A quartic model was considered for the multiple-direction force. This model included the calibration data for the lateral and longitudinal forces. The number of degrees of freedom of the model was increased by including another resonant frequency between 20.0 and 40.0 Hz. By considering the two resonant frequencies, the following quartic equation was obtained through an expansion of Eq. (1):

\[ W = b_1 \omega_{G1} + b_2 \omega_{P1} + b_3 \omega_{G2} + b_4 \omega_{P2} + b_5 \cdots \cdots \cdots \cdots (15) \]

where \( \omega_{G1} \) and \( \omega_{G2} \) are the resonant frequencies of the first and second gain characteristics, \( \omega_{P1} \) and \( \omega_{P2} \) are the resonant frequencies of the first and second phase characteristics. The coefficients of the quartic equation are \( b_1, b_2, b_3, b_4, \) and \( b_5 \). Equation (15) was created by including the resonant frequencies from Fig. 17 as follows:

\[ W = -0.1819 \omega_{G1} + 0.5181 \omega_{P1} + 0.2338 \omega_{G2} - 0.0473 \omega_{P2} - 11.9825 \cdots \cdots \cdots (16) \]

Next, the effect was confirmed by estimating the load using Eq. (16), and the resonant frequencies from another data set were used as a test. The estimated load both without external forces and with an external force with a lateral or longitudinal component is shown in Fig. 18. The load was estimated using multiple resonant frequencies, even when the external

### Table 7: Uncertainty of the Online Estimated ZMP

| Method                  | Case   | \( z_{\text{mp}} \) (mm) | \( z_{\text{mp}} \) (mm) |
|-------------------------|--------|--------------------------|--------------------------|
| Measured by load meter  | Rear   | -60.9 ± 2.7              | 1.22 ± 2.9               |
|                         | Left   | 0.517 ± 2.7              | 7.04 ± 2.9               |
|                         | Right  | -0.982 ± 2.7             | -63.1 ± 2.9              |
| Proposed                | Rear   | 38.4 ± 15                | -35.5 ± 18               |
|                         | Left   | -9.56 ± 26               | 48.8 ± 20                |
|                         | Right  | -7.30 ± 22               | -64.6 ± 20               |
| Multi-link model-based  | Rear   | 0.951 ± 1.3              | 0.0249 ± 0.87            |
|                         | Left   | 0.582 ± 1.2              | 0.00314 ± 0.87           |

The first term of each value indicates the average. The second term shows the expanded uncertainty when the coverage factor is 2.0.
force was affected. From Fig. 18, it can be seen that a correlation for the external force is possible by preparing a model in advance.

5.2 Effect of Temperature

The effect of a temperature change was confirmed by measuring the frequency characteristics while changing the temperature of the room. Measurements were conducted when the surface temperature of the wheel was 18.0°C, 21.0°C, 24.0°C, 27.0°C, 29.0°C, and 31.0°C. The resonant frequencies at each temperature are shown in Fig. 19. From the results, a change in the resonant frequency corresponding to a change in temperature can be confirmed. This change is caused by the change in the hardness of the rubber due to temperature. Moreover, the change in the resonant frequency corresponding to the load is constant at each temperature. This means that the reference value for the resonant frequency can be modeled. The relationship between temperature and the resonant frequency can be modeled almost linearly, although there were slight variations, as shown in Fig. 19. Accounting for this relation using a linear model has been examined in a TPMS of automotive applications. Therefore, the effect of a temperature change can be accounted for by measuring it using a sensor and applying an offset to the resonant frequency corresponding to the measured temperature.

5.3 Effect of Contact Plane

The effect of the contact plane was also considered. Frequency characteristic measurements were conducted on sand and gravel, as shown in Fig. 20. The resonant frequencies for a flat plane, sand, and gravel are shown in Fig. 21. The first resonant frequencies in Fig. 21 also correspond to the resonant frequencies used to estimate the ZMP in the previous section. From the results, it can be seen that the resonant frequency increased for sand and gravel. This is caused by the apparent weight of the contact plane decreasing through small slips on the sand and gravel.

Here, we considered increasing the number of degrees of freedom using multiple resonant frequencies, like the method used in Sect. 5.1, to account for the effect of the contact plane. The resonant frequencies for the entire contact plane from 20.0 to 40.0 Hz, as shown in Fig. 21, were used to create the following equation:

\[
W = -0.1690\omega_{G1} + 0.1954\omega_{P1} + 0.0752\omega_{G2} + 0.4593\omega_{P2} - 5.2420 \\
\]

Next, the load was estimated using Eq. (17), and the resonant frequency for the test data was measured to confirm the effect. The estimated loads for the flat plane, sand, and gravel are shown in Fig. 22. The load was estimated using multiple resonant frequencies even when the environment of the contact plane varied. This shows that the effect of the environ-
ment can be calculated by preparing a model of the contact plane in advance.

5.4 Effect of Multiple Factors The previous sections discussed the effects of the force acting in multiple directions as well as the vertical direction, and the effects of a temperature change, and the stiffness and roughness of the ground. These effects occur simultaneously during actual motion. Therefore, we considered the composite effects. Firstly, the effect of temperature can be solved independently based on the temperature measured by an additional sensor because this effect can be approximated as a linear model. Next, the resonant frequencies of the tire are below 100 Hz from the results of Sect. 2. Therefore, the model used to account for the effects of the multiple-direction force and the contact plane has about four degrees of freedom. In addition, the effects of the multiple-direction force and the contact plane can be solved using the respective effects simultaneously for the calibration data because the change in the resonant frequencies was different, as well as Figs. 17 and 21. From the above, the estimation is possible even when there are multiple simultaneous factors.

6. Conclusions

This paper focused on the relationship between the wheel load and resonant frequency to obtain the reaction force at the contact point. Firstly, frequency analyses and experiments were conducted to confirm the relationship between the load and resonant frequency in the wheel joint of a wheel-legged mobile robot. Next, a method used to estimate the ZMP using a model based on the relation between the load and the resonant frequency was proposed. Then, the effectiveness of the proposed method was confirmed by an experiment with online estimation. From the experimental results of the online estimation, the average of the RMSEs in the x and y directions was 0.0361 m. This value is acceptable because it is sufficiently smaller than the support polygon of the robot. Furthermore, the factors that affected the resonant frequency of the wheel joint were taken into account. From the above, we can estimate the ZMP of a wheel-legged mobile robot.

References

(1) Boston Dynamics: “Handle–Boston Dynamics.” [Online]. Available: https://www.bostondynamics.com/handle. Accessed on: Feb. 05 (2021)
(2) M. Schwartz et al.: “ NimRo Rescue: Solving disaster-response tasks with the mobile manipulation robot Momoar”, J. of Field Robotics, Vol. 34, No. 2, pp. 400–425 (2017)
(3) L. X. T. Pnong, I. Sharf, and B. Beckman: “Implementation of a leader-follow controller for a skid-steering wheel-legged robot”, Proc. IEEE Int. Workshop on Advanced Motion Control, pp. 304–309 (2016)
(4) A. Bouton, C. Grand, and F. Benamar: “Obstacle negotiation learning for a compliant wheel-on-leg robot”, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 2420–2425 (2017)
(5) S. Nakajima: “A New Personal Mobility Vehicle for Daily Life: Improvements on a New RT-Mover that Enable Greater Mobility are Showcased at the Cybathlon”, IEEE Robotics Automation Magazine, Vol. 24, No. 4, pp. 37–48 (2017)
(6) N. Koyachi et al.: “Step over motion of four wheeled and four legged flexible personal robot”, Proc. IEEE Int. Conf. on Robotics and Biomimetics, pp. 616–621 (2009)
(7) D. Leidner, A. Dietrich, M. Beetz, and A. Albu-Schaffer: “Knowledge-enabled parameterization of whole-body control strategies for compliant service robots”, Autonomous Robots, Vol. 40, No. 3, pp. 519–536 (2016)
(8) R. B. McGhee and A. A. Frank: “On the stability properties of quadruped creeping gaits”, Math. Biosciences, Vol. 3, pp. 331–351 (1968)
(9) C. D. Zhang and S. M. Song: “Gaits and geometry of a walking chair for the disabled”, J. of Terramechanics, Vol. 26, No. 3, pp. 211–233 (1989)
(10) D. A. Messuri and C. A. Klein: “Automatic Body Regulation for Maintaining Stability of a Legged Vehicle During Rough-Terrain Locomotion”, IEEE J. of Robotics and Automation, Vol. 1, No. 3, pp. 132–141 (1985)
(11) A. Ghasempoor and N. Sepehri: “A Measure of Machine Stability for Moving Base Manipulators”, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 2249–2254 (1995)
(12) E. G. Papadopoulos and D. A. Rey: “A new measure of tipover stability margin for mobile manipulators”, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 3111–3116 (1996)
(13) S. A. A. Moosavian and K. Alipour: “Stability evaluation of mobile robotic systems using moment-height measure”, Proc. IEEE Conf. on Robotics, Automation and Mechatronics, pp. 1–6 (2006)
(14) S. Kajita et al.: “Biped walking pattern generation by using preview control of zero-moment point”, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 1620–1626 (2003)
(15) S. I. An, Y. Oh, and D. S. Kwon: “Zero-moment point based balance control of leg-wheel hybrid structures with inequality constraints of dynamic behavior”, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 2365–2370 (2012)
(16) A. Surumura and Y. Fujimoto: “Real-time motion generation and control systems for high wheel-legged robot mobility”, IEEE Trans. on Industrial Electronics, Vol. 61, No. 7, pp. 3648–3659 (2014)
(17) M. Bjelonic et al.: “Keep Rollin—Whole-Body Motion Control and Planning for Wheeled Quadrupedal Robots”, IEEE Robotics and Automation Letters, Vol. 4, No. 2, pp. 2116–2123 (2019)
(18) K. Kaneko et al.: “Design of advanced leg module for humanoids project of METT”, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 38–45 (2002)
(19) I. W. Park, J. Y. Kim, J. Lee, and J. H. Ohk: “Mechanical design of the humanoid robot platform, HUBO”, J. of Advanced Robotics Systems, Vol. 21, No. 11, pp. 1305–1322 (2007)
(20) M. Hirose and K. Ogawa: “Honda humanoid robots development”, Philosophical Trans. of the Royal Society A: Mathematical, Physical and Engineering Sciences, Vol. 365, No. 1850, pp. 11–19 (2007)
(21) S. Lohmeyer, T. Buschmann, and H. Ubiicht: “System design and control of anthropomorphic walking robot LOLA”, IEEE/ASME Trans. on Mechatronics, Vol. 14, No. 6, pp. 658–666 (2009)
(22) N. G. Tsagarakis, Z. Li, J. Saglia, and D. G. Caldwell: “The design of the lower body of the compliant humanoid robot "Cub"”, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 2035–2040 (2011)
(23) N. Hirose, R. Tajima, K. Sugikura, and M. Tanaka: “Personal robot assisting transportation to support active human life—Reference generation based on model predictive control for robust quick turning—”, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 2223–2230 (2014)
(24) D. Choi, M. Kim, and J. H. Ohk: “Development of a rapid mobile robot with a multi-degree-of-freedom inverted pendulum using the model-based zero-moment point stabilization method”, J. of Advanced Robotics, Vol. 26, No. 5–6, pp. 515–535 (2012)
(25) I. W. Park, J. O. Kim, M. H. Oh, and W. Yang: “Realization of stabilization using feed-forward and feedback controller composition method for a mobile robot”, Int. J. of Control, Automation, and Systems, Vol. 13, No. 5, pp. 1201–1211 (2015)

(26) C. C. Ward and K. Iagnemma: “A dynamic-model-based wheel slip detector for mobile robots on outdoor terrain”, IEEE Trans. on Robotics, Vol. 24, No. 4, pp. 821–831 (2008)

(27) S. J. Kwon, S. Kim, and J. Yu: “Tilting-type balancing mobile robot platform for enhancing lateral stability”, IEEE/ASME Trans. on Mechatronics, Vol. 20, No. 3, pp. 1470–1481 (2015)

(28) L. R. Ray: “Nonlinear state and tire force estimation for advanced vehicle control”, IEEE Trans. on Control Systems Technology, Vol. 3, No. 1, pp. 117–124 (1995)

(29) R. Rajmani, G. Phanomchoeng, D. Piyahtongkarn, and J. Y. Lew: “Algorithms for real-time estimation of individual wheel tire-road friction coefficients”, IEEE/ASME Trans. on Mechatronics, Vol. 17, No. 6, pp. 1183–1185 (2012)

(30) G. Phanomchoeng and R. Rajmani: “Real-time estimation of rollover index for tripped rollovers with a novel unknown input nonlinear observer”, IEEE/ASME Trans. on Mechatronics, Vol. 19, No. 2, pp. 743–754 (2014)

(31) N. Mutoh: “Driving and braking torque distribution method for front- and rear-wheel-independent drive-type electric vehicles on roads with low friction coefficient”, IEEE Trans. on Industrial Electronics, Vol. 59, No.10, pp. 3919–3933 (2012)

(32) K. Nam, S. Oh, H. Fujimoto, and Y. Hori: “Estimation of sideslip and roll angles of electric vehicles using lateral tire force sensors through RLS and Kalman filter approaches”, IEEE Trans. on Industrial Electronics, Vol. 60, No. 3, pp. 988–1000 (2013)

(33) S. Velupillai and L. Gurven: “Tire pressure monitoring”, IEEE Control Systems Magazine, Vol. 27, No.6, pp. 22–25 (2007)

(34) M. Yonetani, K. Ohashi, T. Umeno, and Y. Inoue: “Development of tire pressure monitoring system using wheel-speed sensor signal”, Proc. Int. Technical Conf. on the Enhanced Safety of Vehicles, pp. 492–496 (1998)

(35) Q. Zhang, B. Liu, and G. Liu: “Design of tire pressure monitoring system based on resonance frequency method”, Proc. IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics, pp. 781–785 (2009)

(36) A. Mertins: Signal analysis: wavelets, filter banks, time-frequency transforms and applications, John Wiley & Sons, Chichester, England (1999)

(37) S. Sugano, Q. Huang, and I. Kato: “Stability Criteria in Controlling Mobile Robotic Systems”, Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Vol. 2, pp. 832–838 (1993)

(38) ATI Industrial Automation: “F/T Sensor: Nano25.” [Online]. Available: https://www.ati-ia.com/products/ft/ft_models.aspx?id=Nano25, Accessed on: Oct. 30 (2021)

(39) SAN-E TEC: “6-axis Force Torque Sensors.” [Online]. Available: http://www.san-e.co.jp/sanetec_e/products/index_15.html, Accessed on: Oct. 30 (2021)

(40) Hypersen Technologies: “6-Axis Force Torque Sensor.” [Online]. Available: https://en.hypersen.com/product/detail/24.html, Accessed on: Oct. 30 (2021)

Kenta Nagano (Member) received a B.E. degree in symbiotic systems science from Fukushima University in Fukushima, Japan, in 2013 and M.E. and Ph.D. degrees in electrical and computer engineering from Yokohama National University in Yokohama, Japan, in 2015 and 2018, respectively. From 2018 to 2020, he was a Postdoctoral Fellow at Yokohama National University. Since 2020, he has been with the Department of Electrical Engineering at National Institute of Technology, Tokyo College in Japan, where he is currently an assistant professor. His research interests include robotics, mechatronics, and motion control. He is a member of IEEE Japan, IEEE, and the Robotics Society of Japan.

Yasutaka Fujimoto (Senior Member) received B.E., M.E., and Ph.D. degrees in electrical and computer engineering from Keio University in Yokohama, Japan, in 1993, 1995, and 1998, respectively. In 1998, he joined the Department of Electrical Engineering at Keio University in Yokohama, Japan, as a research associate. Since 1999, he has been with the Department of Electrical and Computer Engineering at Yokohama National University in Japan, where he is currently a professor. His research interests include motion control and actuators, in particular, the modeling and control of mobile/legged robots and direct-drive actuators with high-thrust force density and high back-drivability. He is a senior member of IEEE and a member of the Robotics Society of Japan, SICE, JSAE, IEICE, and INFORMS.