ELKO as dark matter candidate

Bakul Agarwal
Department of Electrical Engineering, Indian Institute of Technology, Kanpur

Alekha C. Nayak and Ravinda K. Verma
Department of Physics, Indian Institute of Technology, Kanpur
Kanpur, India-208016

We study the implications of the ELKO fermions as a dark matter candidate. Such fermions arise in theories which are not symmetric under the full Lorentz group. Although they do not carry electric charge, ELKOs can couple to photons through a non-standard interaction. They also couple to Higgs but do not couple to other standard model particles. We impose limits on their coupling strength and the ELKO mass assuming that these particles give dominant contribution to the cosmological cold dark matter.

I. INTRODUCTION

Current cosmological observations indicate that cold dark matter contributes 23% of the energy density of the Universe. The nature of this matter is so far unknown. There are many theories proposed for dark matter [1–5]. In 2005, Ahluwalia and Grumiller proposed a spin half fermion with mass dimension 1 [6, 7]. The field is an eigenspinor of the charge conjugate operator, and is called ELKO in the literature. ELKO spinor fields arise in theories which do not obey the full Lorentz group but a subgroup, such as SIM(2) [8]. These theories propose the existence of a preferred axis [8–10], and hence break Lorentz and rotational invariance. In 2006, Cohen and Glashow [8] argued that "Many empirical successes of special relativity need not demand Lorentz invariance of the underlying framework." All the physical properties, except some discrete symmetries, can be explained by SIM(2), the largest subgroup of VSR [8]. Cosmological observations show some evidence for the existence of a preferred axis in the Universe [11, 12]. The mismatch of the mass dimension of ELKO and the standard model fermions restricts its interactions with the SM particles [9]. In particular, the field does not carry electric charge and hence may act as a dark matter candidate. Different properties of ELKO fields and its implication on cosmology, astrophysics and particle physics have been studied by several authors [13–24]. The interaction with Higgs field imply the possibility of detecting the signature of ELKO at LHC. It has a quadratic self-interaction with itself and an ELKO-photon and ELKO-Higgs doublet interaction.

In this paper, we consider the interaction of ELKO fermions to the electromagnetic field tensor \( F^{\mu \nu} \) and coupling of this field to the Higgs. We assume that ELKOs were in thermal equilibrium with the cosmic fluid at early times and were non-relativistic at the time of decoupling.

We determine the range of parameters for which this is possible. This range is further restricted by requiring that ELKOs give the dominant contribution to the cosmological cold dark matter.

This paper is organized as follow: In sections II we briefly review the ELKO field and its interaction. In section III we compute the pair annihilation cross-section of ELKO and its coupling with photon and Higgs using cosmological and dark matter data in section IV. In section the we discuss scattering of ELKO with proton and its implication for ELKO-photon coupling.

II. A BRIEF REVIEW OF ELKO FERMIOM AND ITS INTERACTIONS

The Fourier decomposition of the ELKO field may be written as [25]:

\[
\tilde{f}(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2mE(p)}} \sum_\alpha \left[ a_\alpha(p) \lambda^S_\alpha(p) \exp(-ip_\mu x^\mu) \\
+ b_\alpha(p) \lambda^A_\alpha(p) \exp(ip_\mu x^\mu) \right] \tag{1a}
\]

and its dual

\[
\tilde{\bar{f}}(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2mE(p)}} \sum_\alpha \left[ a^\dagger_\alpha(p) \lambda^S_\alpha(p) \exp(ip_\mu x^\mu) \\
+ b^\dagger_\alpha(p) \lambda^A_\alpha(p) \exp(-ip_\mu x^\mu) \right] \tag{1b}
\]

The creation and annihilation operators satisfy the following commutation relations

\[
\{ a_\alpha(p), a^\dagger_{\alpha'}(p') \} = (2\pi)^3 \delta^3(p - p') \delta_{\alpha\alpha'} \tag{2a}
\]

\[
\{ a_\alpha(p), a_{\alpha'}(p') \} = 0, \quad \{ a^\dagger_\alpha(p), a^\dagger_{\alpha'}(p') \} = 0 \tag{2b}
\]

with similar relations for \( b \)'s. The spinors, \( \lambda^S_\alpha \) and \( \lambda^A_\alpha \) are eigenstates of the charge conjugation operator, \( C \), such that

\[
C \lambda^S_\alpha = + \lambda^S_\alpha, \quad C \lambda^A_\alpha = - \lambda^A_\alpha \tag{3}
\]
The invariant amplitude for this process is given by

\[ |\mathcal{M}|^2 = \frac{16g_1^2e^2q_f}{m^2q^4} \sum_{\alpha,\beta} (\bar{\lambda}^A(k')\sigma^{\mu\nu} \lambda^S_\beta(k))(\lambda^{S^+}_\beta(k)\sigma^{\nu\lambda} \bar{\lambda}^A_\alpha(k')) \times Tr[(\not{p} + m_e)\gamma_\nu(p' - m_e)\gamma_\mu] \]

where \( q \) is the momentum transformed in the process. Hence,

\[ |\mathcal{M}|^2 = \frac{128g_1^2e^2}{m^2q^4} (2E'^2 + 2m_e^2 + p_1^2 + p_2^2 + (p_2^2 - p_1^2) \cos 2\phi - 2p_1p_2\sin 2\phi) \]

where \( m_e \) is the electron mass, \( E' \) energy of electron. The scattering cross-section is given by

\[ d\sigma_v = \frac{1}{32\pi^2 s} \frac{1}{4} |\mathcal{M}|^2 d\Omega \]

where \( s = 4m^2 \). Thus we get

\[ <\sigma v> = \frac{5.33g_1^2\alpha(2m^2 + m_e^2)}{m^4} \]

where \( \alpha = \frac{e^2}{\pi} \) is the fine structure constant.

III. ELKO PAIR ANNIHILATION

We are interested in the process in which an ELKO \( f(k) \) annihilates with it’s anti-particle \( \bar{f}(k') \) producing an electron, \( e^- (p) \) and positron, \( e^+ (p') \),

\[ \bar{f}f \rightarrow e^- e^+ \] (8)

as shown in the Fig 1.

![Fig. 1. Annihilation process for ELKO](image)

The Feynman rule for the basic \( \bar{f}f\gamma \) vertex is

\[ \frac{2ieq}{m} \sigma^{\mu\nu} (q_\mu e^\nu_{\mu} - q_\nu e^\nu_{\mu}) \] where \( m \) is the mass of ELKO.

The invariant amplitude for this process is given by

\[ i\mathcal{M} = 4g_1\bar{\lambda}_\alpha^A(k')\sigma^{\mu\nu} \lambda^S_\beta(k) \frac{q_\mu q_\nu}{mq^2} i e\bar{u}(p)\gamma^\nu v(p') \] (9)

IV. COSMOLOGICAL BOUNDS

In this section, we discuss the cosmological bound on mass of ELKO particle and its coupling with photon and Higgs.

A. Coupling with photon

We first determine the range of parameters for which ELKO particles may be in thermal equilibrium with the cosmic plasma at early times, during the radiation dominant phase of the universe. Let \( T_f \) denote the freeze-out temperature \( 26 \), i.e. the temperature at which ELKO fermions decouple from the cosmic plasma. At the time of freeze out the interaction rate becomes equal to the expansion rate, i.e. \( \Gamma = H \). We assume that ELKOs form cold dark matter and hence decouple when they are non-relativistic. The interaction rate is \( \Gamma = n <\sigma v> \), where number density \( n \), in the non-relativistic limit, is given by, \( n = g_A \left( \frac{m_f}{2\pi} \right)^{3/2} e^{-m/T_f} \) where \( g_A \) is the degeneracy factor which is equal to 2 for ELKO. Furthermore, the Hubble constant, \( H(T_f) = 5.44 \left( \frac{T_f^2}{m_f^3} \right) \).
Using $<\sigma v>$ from [3], we obtain,

$$g_A \left( \frac{mT_f}{2\pi} \right)^\frac{3}{2} \exp[ -m/T_f ] \left( \frac{5.33g_1^2\alpha(2m^2 + m_e^2)}{m^4} \right) = 5.44 \frac{T_f^2}{m_{pl}}$$

where Planck mass $m_{pl} = 1.22 \times 10^{19}$ GeV. In Fig. 2 we plot this relationship between the coupling $g_f$ and mass $m$ for different values of decoupling temperature, $T_f = 0.1, 1, 10, 100$ and $1000$ GeV. We restrict the range of coupling values $g_f < 1$, for which perturbation theory is applicable.

The abundance of ELKO fermions in the present universe is given by [27]

$$\Omega_s = \frac{74.7S_0 m}{2\pi^2m_{pl}\sqrt{\pi T_f}\rho_c(sv)_f}$$

where $S_0 = 2.97 \times 10^3 cm^{-3}$ is the present value of entropy density, $\rho_c = 1.05 \times 10^2 h^2 GeV/cm^3$ is the critical density of the universe and we have assumed that $g_\ast = 106.75$, corresponding to the relativistic degrees of freedom at the time of decoupling. Assuming that the relic density of ELKO fermions is equal to the dark matter density implies that, $\Omega_s \approx 0.3$. Using equations [13] and [15] we have

$$m^5 = 6.86 \times 10^8 T_f g_1^2(2m^2 + m_e^2) GeV^2$$

This relationship between $m$ and $g_f$ is also shown in Fig. 2 for various values of $T_f$. The points where these lines cross the plots corresponding to the decoupling condition, represent parameter values for which ELKO may yield dominant contribution to the cosmological cold dark matter. The solid line in Fig. 2 which joins these points, therefore represents the range of allowed parameters satisfying these conditions. Parameter values lying below this black line are ruled out since they will lead to cold dark matter density larger than observed.

The region above the solid dark line is allowed in Fig. 2. Note that at decoupling temperature($T_f$) about 1000 GeV, there is no point of intersection between relation Eq. [14] and Eq. [16] for $g_f \leq 1$. So, ELKO and anti-ELKO annihilation to electron and positron pair interaction process gives upper bound on ELKO mass nearly $10^3$GeV to consider ELKO as a cold dark matter candidate. However we shall see in the next section that direct dark matter searches impose more stringent constraints on these parameters.

### B. Coupling to Higgs

#### 1. Scattering with Higgs

An ELKO $f(k)$ scatters with Higgs $H(p)$ into $f(k')$ and $H(p')$ via $-g_{\phi f}(x)\phi^\dagger(x)\phi(x)$ interaction as shown in the Fig. 3. This interaction is a point interaction as the mass dimension of ELKO field is one.

![Fig. 3. ELKO-Higgs Scattering](image)

The amplitude for this process is given by

$$i\mathcal{M} = \frac{g_\phi}{m} \lambda_\alpha(k') \lambda_\beta(k)$$

This leads to,

$$|\mathcal{M}|^2 = \frac{g_\phi^2}{m^2} 4(EE' - kk'\cos(\theta - \theta'))(1 + \cos(\phi - \phi'))$$

The thermal averaged cross section for this process is

$$<\sigma v> = \frac{g_\phi^2}{32\pi^2m^2s} \frac{1}{2} 4\pi(4EE' - \pi kk'\sin\theta)$$

where $E, E'$ are initial and final energy of ELKO respectively. Assuming the initial ELKO coming from all possible direction and scatter with Higgs particle, then integrating over $\theta$, the second term inside the bracket vanishes. In the non-relativistic limit, the thermal averaged cross section is

$$<\sigma v> = \frac{g_\phi^2}{2\pi(m_H + m)^2}$$

Where $m_H = 125$ GeV is the mass of Higgs and $g_\phi$ the ELKO-Higgs coupling.
2. Annihilation of ELKO’s to Higgs pair

An ELKO \( f(k) \) also annihilates with its anti-particle \( \bar{f}(k') \) to Higgs pair as shown in Fig. 4.

\[
iM = \frac{g_{f\phi}}{m} \lambda'_{\alpha'}(k') \lambda^\beta_S(k)
\]

(21)

The square of the invariant amplitude is

\[
|M|^2 = \frac{8g_{f\phi}^2}{m^2} (m^2 + 2p^2)
\]

(22)

In the non-relativistic limit, the thermal averaged cross section is

\[
<\sigma v> = \frac{g_{f\phi}^2}{16\pi m^2}
\]

(23)

Both scattering of ELKO with Higgs and annihilation of ELKO’s to Higgs will contribute to the total thermal averaged cross section at the time of decoupling of ELKO from cosmic plasma. The decoupling temperature is given by, \( \Gamma = H \), which implies,

\[
g_A \left( \frac{mT_f}{2\pi} \right)^2 \exp[-m/T_f] \left( \frac{g_{f\phi}^2}{2\pi(m_H + m)^2} + \frac{g_{f\phi}^2}{16\pi m^2} \right) = \frac{5.44 T_f^2}{m_{pl}}
\]

(24)

In Fig 4, we plot mass of ELKO vs coupling with Higgs for different range of decoupling temperature. We also restrict the coupling constant value to be less than 1 so that perturbation theory is applicable. Here, the lower mass range of ELKO is 100GeV because Higgs decouples from the cosmic plasma temperature around 80GeV. Hence below this temperature, ELKOs can’t maintain equilibrium with the cosmic plasma due to their interaction with the Higgs. Assuming the relic density of ELKO fermions equal to the dark matter density, i.e. \( \Omega_s \approx 0.3 \), we get

\[
m^3(m_H + m)^2 = 3.37 \times 10^8 (8m^2 + (m_H + m)^2) T_f g_{f\phi}^2 GeV^2
\]

(25)

Plotting Eq 24 and Eq 25 between \( m \) and \( g_{f\phi} \) for different \( T_f \), the point of intersection gives the allowed range of parameter for mass and coupling with Higgs. This is shown in Fig 5. The upper region of the dark line gives the parameter range for mass of ELKO and its coupling with Higgs for which it acts like a cold dark matter candidate.

V. LIMIT ON ELKO DIRECT DARK MATTER SEARCHES

We consider the scattering of ELKO with proton in non-relativistic limit. Using the CDMS II [28] results, we impose further constraints on the parameter range of mass of ELKO and its coupling with photon in comparison to that obtained from ELKO pair annihilation to electron and positron pair in section IV.A. The t-channel process for \( f(k)p(p) \rightarrow f(k')p(p') \) is shown in Fig 6.

\[
\begin{align*}
\end{align*}
\]

\[
\begin{align*}
\end{align*}
\]

FIG. 4. Annihilation of ELKO to Higgs

FIG. 5. The curved, vertical lines show the decoupling at different freeze-out temperature. The slanted, parallel lines are obtained by imposing the condition \( \Omega_s = 0.3 \). The upper region of dark line is allowed.

FIG. 6. Elko-Proton Scattering.
The invariant amplitude for this process is given by
\[
i\mathcal{M} = 4\i g_f (\bar{\lambda}_{\alpha}^{S}(k')\sigma^{\mu\nu} \lambda_{\beta}^{S}(k)) \frac{q_{\mu}q_{\nu}a}{m q^2} \times \\
i e\bar{u}_{\nu}(p') [F_1 \gamma^\sigma + \frac{k}{2m_p} F_2 i\sigma^\alpha q_\alpha] u(p)
\]
where \(F_1(q^2)\), \(F_2(q^2)\) are the proton form factors and \(k\) is the anomalous magnetic moment. The momentum transfer in the process is \(q = p' - p\). The amplitude squared becomes
\[
|\mathcal{M}|^2 = \frac{16g_f^2 e^2q_{\mu}q_{\nu}}{m^2q^4} (\bar{\lambda}_{\alpha}^{S}\sigma^{\mu\nu} \lambda_{\beta}^{S})(\bar{\lambda}_{\beta}^{S}\gamma^\sigma \lambda_{\alpha}^{S}) \times Tr[(\bar{p}' + m_p)(F_1 \gamma^\nu + \frac{k}{2m_p} F_2 i\sigma^\alpha q_\alpha)] \\
\quad \times \bar{(\bar{\phi} + m_p)}(F_1 \gamma^\nu - \frac{k}{2m_p} F_2 i\sigma^\alpha q_\alpha)
\]
Since ELKOs are dark matter candidates, we assume that they are moving in random directions with respect to the Milky Way center. We consider an incoming proton, moving in the z-direction i.e. \(p_z = (E_p, 0, 0, -p_3)\), with velocity \(v = 232\, km/s\), which is equal to the speed of Sun around the galactic center. We consider its scattering with an ELKO at rest. The proton recoil energy range turns out to be approximately 10keV. In the non-relativistic limit, \(F_1(q^2 \approx 0) = 1\), \(F_2(q^2 \approx 0) = 1\). The scattering cross section in the non-relativistic limit is determined as,
\[
\sigma = \frac{2.212 \times 10^{13} g_f^2(1.958 + 1.305 \cos(\phi - \phi'))}{(m_p + m)^2}
\]
Integrating over \(\phi'\), the cross section becomes
\[
\sigma = \frac{2.72 \times 10^{14} g_f^2}{(m_p + m)^2}
\]
Scattering of WIMP with silicon detector in CDMS II [28] gives the WIMP-nucleon scattering cross section \(\sigma\) less than \(1.9 \times 10^{-41}\, cm^2\). Using this value in Eq.29 the coupling vs mass is shown in the Fig.7.

From the CDMS II [28] results of scattering cross section of ELKO with proton, the lower region of blue line is the allowed range of parameter for ELKO mass and coupling with photon. It is clear that the CDMS II result rules out the entire range of coupling for which ELKO may act as a cosmological dark matter candidate through its coupling with photon. Hence ELKO can act as a dark matter candidate only through its interaction with the Higgs.

\[\text{VI. CONCLUSION}\]

The ELKO fermion with mass dimension one couple to photon via non-standard \(g_f \bar{f}(x)[\gamma_{\mu}, \gamma_{\nu}]f(x)F^{\mu\nu}(x)\) interaction. Pair annihilation of ELKO to electron and positron gives a parameter range for ELKO mass and its coupling to photon \((g_f)\), however, this range is ruled out by direct dark matter search experiments, such as CDMS II. Since the mass dimension of ELKO field is one, it has a point-like interaction with the Higgs. We assume that perturbation theory is applicable and impose an upper bound on Higgs-ELKO coupling \((g_{\phi})\) to be less than one. From ELKO-Higgs scattering cross section and cosmological constraints, we obtain the upper bound on ELKO mass \(\approx 9000\, GeV\) and lower bound on Higgs-ELKO coupling \((g_{\phi})\) \(\approx 0.01\), for which ELKO can be considered as a cold dark matter candidate.

\[\text{VII. ACKNOWLEDGMENT}\]

We thank Prof. Pankaj Jain and Gopal Kasyap for useful discussions.

[1] G. Bertone, D. Hooper, and J. Silk, Phys.Rept. 405, 279 (2005), hep-ph/0404175.
[2] G. Jungman, M. Kamionkowski, and K. Griest, Phys.Rept. 267, 195 (1996), hep-ph/9506380.
[3] L. Bergstrom, Rept.Prog.Phys. 63, 793 (2000), hep-ph/0002126.
[4] J. L. Feng, Ann.Rev.Astron.Astrophys. 48, 495 (2010), 1003.0904.
[5] G. Bertone, Particle Dark Matter: Observations, Models and Searches (Cambridge University Press, Cambridge, England, 2010).
[6] D. V. Ahluwalia and D. Grumiller, JCAP 0507, 012 (2005), hep-th/0412080.
[7] D. V. Ahluwalia and D. Grumiller, Phys.Rev. D72, 067701 (2005), hep-th/0410192.
[8] A. G. Cohen and S. L. Glashow, Phys.Rev.Lett. 97, 021601 (2006), hep-ph/0601236.
[9] D. Ahluwalia and S. Horvath, JHEP 1011, 078 (2010), 1008.0436.
[10] D. Ahluwalia, C.-Y. Lee, D. Schritt, and T. Watson, Phys.Lett. B687, 248 (2010), 0804.1854.
[11] J. P. Ralston and P. Jain, Int.J.Mod.Phys. D13, 1857 (2004), astro-ph/0311430.
[12] P. Ade et al. (Planck Collaboration) (2013), 1303.5083.
[13] R. da Rocha, A. E. Bernardini, and J. Hoff da Silva, JHEP 1104, 110 (2011), 1103.4759.
[14] C. Boehmer, Annalen Phys. 16, 325 (2007), gr-qc/0701087.
[15] C. G. Boehmer, Annalen Phys. 16, 38 (2007), gr-qc/0607088.
[16] C. G. Boehmer, Phys.Rev. D77, 123535 (2008), 0804.0616.
[17] L. Fabbri, Phys.Lett. B704, 255 (2011), 1011.1637.
[18] L. Fabbri, Gen.Rel.Grav. 43, 1607 (2011), 1008.0334.
[19] L. Fabbri, Mod.Phys.Lett. A25, 2483 (2010), 0911.5304.
[20] A. Basak, J. R. Bhatt, S. Shankaranarayanan, and K. Prasantha Varma, JCAP 1304, 025 (2013), 1212.3445.
[21] A. Bernardini and R. da Rocha, Phys.Lett. B717, 238 (2012), 1203.1049.
[22] L. Fabbri, Phys.Rev. D85, 047502 (2012), 1101.2566.
[23] M. Dias, F. de Campos, and J. Hoff da Silva, Phys.Lett. B706, 352 (2012), 1012.4642.
[24] A. Alves, F. de Campos, M. Dias, and J. M. Hoff da Silva (2014), 1401.1127.
[25] D. V. Ahluwalia (2013), 1305.7509.
[26] T.Padmanabhan, Structure formation in the universe (Manning Publications Co., Connecticut, USA, 2010).
[27] E. W. Kolb and M. S. Turner, Front.Phys. 69, 1 (1990).
[28] R. Agnese et al. (CDMS Collaboration), Phys.Rev.Lett. 111, 251301 (2013), 1304.4279.