1. Introduction

In order to meet increasing demands on increased productivity and improved as well as optimum quality for each customer, much effort is spent on improving the control of secondary refining operations. Therefore there is a growing interest in creating mathematical models that can describe secondary refining operations such as heating. Models of stirred ladles can be used to predict fundamental fluid-flow parameters such as steel velocities and turbulent kinetic energy. If information of the heat transfer from the electrodes to the steel bath is available it is also possible to predict the heat flow in ladle furnaces. Finally, it is also important to verify the model predictions with experimental data in order to make sure that the models are reliable.

Improving modeling of heating efficiency in ladle furnaces requires a fundamental understanding of the physics involved in the process, i.e. heat transfer, mass transfer, fluid flow and the electromagnetic phenomena. To achieve this the use of CFD models has become increasingly popular. Arcs have been modeled\(^1\text{–}^5\) and models of inductively stirred steel ladles have been created\(^6\text{–}^{10}\). However, no models have been reported as being able to predict the transient effect of heating by electrodes. This made it possible to predict temperature gradients in the steel during the heating process.

The first part of the paper briefly discusses the electric arc model. Emphasis is put on the different energy transport mechanisms that are responsible for the transfer of heat to the steel. Models of stirred ladles can be used to predict fundamental fluid-flow parameters such as steel velocities and turbulent kinetic energy. If information of the heat transfer from the electrodes to the steel bath is available it is also possible to predict the heat flow in ladle furnaces. Finally, it is also important to verify the model predictions with experimental data in order to make sure that the models are reliable.

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2. Theoretical Model of an Electric Arc

2.1. The Modeling of the Arc

In this model, the arc is treated as a fluid\(^6\text{–}^{15}\) with temperature-dependent properties.\(^16\text{–}^{17}\) The coupled equations for conservation of energy, mass and momentum, which define the plasma temperature, pressure and velocity are solved. In this report only the assumptions and the region of integration are described. Detailed information regarding the mathematical model of the arc can be found in a previous publication.\(^14\)

The following assumptions were made in the statement of the mathematical model of the electric arc.

- The arc is axis-symmetric.
The system is steady state, i.e. the solution is independent of time (DC).
The arc is in local thermal equilibrium (LTE).\(^{19}\)
The anode (border to the steel bath) is assumed to be flat and the bath itself is neglected.
MHD approximation\(^{20}\) is applicable.

According to the assumptions above, the arc was described with a two-dimensional model. A schematic representation of the arc model is given in Fig. 1. The model consists of the cathode (graphite electrode), the arc column, and the anode (steel bath). The calculation domain is defined in such a way that entrainment and general interaction with the surrounding gases are allowed. When a current is passed from the anode to the cathode, the resultant heating of the gas, together with the Lorentz force, leads to the ionization of gas in the arc areas. The rate of heat generation as well as the convective and radiative heat transfer determines the temperature distribution in the arc areas.

### 2.2. Heat Transfer from the Arc

The four different mechanisms responsible for heat transfer from the arc to the steel considered in theoretical model are:

- convective heat transfer
- radiative heat transfer
- the Thompson effect
- the condensation of electrons (work function and anode fall)

#### 2.2.1. Convective Heat Transfer

The convective heat transfer from the partly ionized gas to the molten steel, \(Q_{\text{conv}}\), is described by the following equation\(^{2}\):

\[
Q_{\text{conv}} = \frac{0.915}{\sigma_{\text{Tw}}} \left( \frac{\rho_u \mu_{u}^2}{\sigma_{\text{Tw}}} \right)^{0.43} \left( \frac{\rho_u \mu_{u} \gamma_{u}^2}{\tau} \right)^{0.5} (h_b - h_w) \quad \ldots (1)
\]

where \(\mu_u\) is the molecular dynamic viscosity, \(\rho_u\) is the density, \(\sigma\) is the thermal conductivity, \(\gamma\) is the radial component of the velocity and \(h\) is the gas enthalpy. The subscripts \(w\) and \(b\) denote the values of the bath surface and the boundary layer, respectively.

#### 2.2.2. Radiative Heat Transfer

The radiative heat transfer to the liquid steel surface, \(Q_r\), is calculated by means of approximate view factors as follows:

\[
Q_r = \sum \frac{S_{A,i,j}}{4\pi d_{i,j}^2} \cos(\psi) \Delta V_j \quad \ldots (2)
\]

where \(S_{A,i,j}\) is the radiation loss from volume element \(\Delta V_j\) in the arc column to the steel surface \(i\), \(d_{i,j}\) is the vector joining the surface \(i\) and the volume \(j\) and \(\psi\) is the angle between \(d\) and the normal vector of the surface. In these calculations the net heat losses are used to predict radiative heat transfer to the anode.\(^{19,21}\)

#### 2.2.3. Thompson Effect

The transport of thermal energy by electrons, \(Q_e\), (the Thompson effect) is described by the following equation\(^{3}\):

\[
Q_e = \frac{5J_A}{2e} k_b (T_{A,g} - T_{\text{Anode}}) \quad \ldots (3)
\]

where \(J_A\) is the current density at the anode, \(k_b\) is the Boltzmann constant, \(e\) is the electron charge, \(T_{A,g}\) is the temperature at a distance of 0.1 mm from the anode\(^{21}\) and \(T_{\text{Anode}}\) is the temperature at the anode. In this study \(T_{A,g}\) is taken as the temperature corresponding to the enthalpy at the cell closest to the anode. The distance from this cell to the anode surface is 0.5 mm.

#### 2.2.4. Condensation of Electrons

The contribution from the condensation heat flow created by the electrons passing the anode-fall and entering the liquid steel, \(Q_A\), can be expressed as follows\(^{10}\):

\[
Q_A = J_A V_A (V_A + q_A) \quad \ldots (4)
\]

where \(V_A\) is the anode fall voltage and \(q_A\) is the work function (required anode fall to release an electron from the anode). In this investigation \(V_A\) and \(q_A\) were both assumed to be 4 V\(^{3}\).

The contributions to heat transfer from convection, radiation, and condensation were implemented as source terms in the first cell closest to the anode. The Thompson effect was included at the anode as well as in the rest of the calculation domain.

### 3. Theoretical Model of an Induction-stirred Ladle

A three-dimensional model of an inductively stirred ladle was used in the simulations of simultaneous heating and induction stirring. Here, only a brief description of the most important assumptions, transport equations and the energy equation is provided. Detailed information regarding the transport and turbulence equations is presented in earlier publications\(^{23,24}\). In these references the validation of the model predictions is also discussed.

The main assumptions made in the statement of the mathematical model\(^{23,24}\) of an induction-stirred ladle are that:

- the free surface at the liquid/air interface is frictionless.
- a pressure-correction expression can be used for recursive calculation of the shape of the surface.

The following governing transport equations for the induction-stirred ladle are solved:

- conservation of mass
- conservation of momentum
- conservation of thermal energy
- the \(k-\varepsilon\) model for turbulence

Conservation of Thermal Energy
The enthalpy, where at a value of 12.5 kW/m².

5.1. Prediction of Heat Transfer from the Arc to the Steel Bath

The heat transfer from the arc to the anode is described by Eqs. (1)–(4). They require the current density, temperature and velocity at the anode, as well as the temperature in the arc to predict the rate of heat transfer between the arc region and the steel. To determine these parameters, a realistic description of the arc was needed. Presented below are results from the arc model used in this study for cases with a current of 36 kA and four different arc lengths (ranging from 15–30 cm) using air as the conducting gas.

The current density in the partly ionized gas determines the shape of the arc, since most of the current in the ionized gas is transported in a narrow arc column. For a constant current, this affected the current distribution at the anode, as shown by the results presented in Fig. 2. It is apparent from this figure that a longer arc results in a somewhat wider distribution of the current and a lower maximum current density.

The Lorentz force is proportional to the current density and causes a flow of mass and a flux of momentum from the cathode towards the anode. The resulting radial gas velocity at the anode influences the convective rate of heat transfer, as indicated by Eq. (1). A plot of gas velocities at the anode is shown in Fig. 3 for calculations using a 36 kA current in which it can be seen that the gas velocity increases with a decreased arc length at the center of the arc region.

Table 1. Boundary conditions in the ladle.

| Surface         | \( \frac{\partial u}{\partial x} = 0 \) | \( \frac{\partial v}{\partial x} = 0 \) | \( \frac{\partial w}{\partial x} = 0 \) | \( \frac{\partial k}{\partial x} = 0 \) | \( \frac{\partial \varepsilon}{\partial x} = 0 \) | \( Q_{\text{loss}} \) |
|-----------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|-----------------|
| Bottom          | 0                                      | 0                                      | 0                                      | 0                                      | 0                                      | Q = 12.5 kW/m²   |
| Wall            | 0                                      | 0                                      | 0                                      | 0                                      | 0                                      | Q = 12.5 kW/m²   |

where \( Q_{\text{loss}} \) is the heat loss, \( \sigma \) is the Stefan–Bolzmann constant, \( \alpha \) is the emissivity, \( T_{\text{steel}} \) is the steel temperature at the surface and \( T_{\text{lid}} \) is the temperature at the lid. In this work \( T_{\text{lid}} \) is assumed to be constant at a temperature of 1 016 K, based on measurements at Ovako Steel. The emissivity is assumed to be constant at 0.95.

4.2. Heating with AC Electrodes

As a means of adapting the model to an ASEA-SKF ladle furnace, the described mechanisms of heat transfer are introduced as boundary conditions at the locations of the three electrodes. The current in the DC arc calculated with the arc model is approximately equal to the effective current in an AC arc. Since this work was considered a first attempt to model heating with AC electrodes, this simplified approach was taken. It was also assumed that the currents inside the liquid phase were negligible since the resistivity of the molten steel is low (1.39×10⁻⁷ Ωm), which results in very low heat generation from the current in the liquid phase compared to the heat transferred from the arcs.

4.3. Method of Solution

The solutions of the governing equations, boundary conditions, and source terms are obtained using the commercial finite difference code Phoenix. During a calculation, the finite difference equations are solved by iteration until the residuals are less than 1%. The calculations carried out in this study were performed using the transient solution mode with a time step of 0.1 seconds. A typical calculation used 5 sweeps for each time step using a 20×28×20 non-uniform mesh.

5. Results and Discussion

5.1. Prediction of Heat Transfer from the Arc to the Steel Bath

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and decreases with a decreased arc length in the outer part of the arc.

The arc temperature is needed to determine the heat transfer due to radiation as described by Eq. (2). The temperature contours for a 36 kA arc with an arc length of 25 cm are plotted in Fig. 4. As illustrated, the highest arc temperature is found at a central location close to the cathode. This is where the ionization most likely takes place, causing a high current density, which in turn leads to substantial joule heating. Consequently, the radiated heat transfer is also greater closer to the cathode.

The arc temperature also influences the temperature at the anode, which subsequently affects the heat transferred to the steel by convection and the Thompson effect. In Fig. 5, radial temperature profiles computed at the anode are presented for different arc lengths at the same current of 36 kA. The figure shows that at a distance smaller than 0.03 m from the center of the arc, the temperature increases with a decreased arc length. At larger distances, the radial temperature decreases with a decreased arc length, as would be expected for a constant current.

In Fig. 6 the contributions of the different heat-transfer mechanisms to the total heat transfer to the steel are shown for the sake of comparison. Predictions were made using a 36 kA current and a 25 cm arc length. For this particular arc length and current, most of the heat is predicted to be transferred as a result of the anode fall at a distance of less than 0.07 m from the arc center. At greater distances the majority of the heat transfer is due to radiation and convection.

In Fig. 7, the total heat flux from the arcs to the steel for different arc lengths is compared. It can be seen that the heat flux increases with an increase in arc length, as is expected when the current is kept at a constant value. The mechanism mainly responsible for an increased heat flux for longer arc lengths is most likely increased radiation.27)
5.2. Predictions of Simultaneous Heating and Induction Stirring in a Ladle

Predictions were made using data for a ladle equipped with an ABB ORB 60 induction stirrer with an operating current of 1 000 A stirring in the upward direction. Data on the ladle and stirrer are provided in Table 2. All calculations were performed under the assumption that the initial steel temperature was uniform at 1 846 K (1573°C).

The simulation of the heating operation was conducted such that induction stirring was first employed without heating. With the model this was achieved by simulating stirring at a constant temperature. The results were then used as initial conditions for velocities and turbulent data when including the simulation of heating in the ladle model.

In order to validate the model a comparison between predicted and measured temperatures as a function of time was made. The measurements were performed in an inductively stirred ASEA-SKF ladle furnace at Ovako Steel AB. The ladle was heated with a 36 kA current and the temperature at the start of heating was 1 846 K. The temperature of the inside of the lid was measured by a pyrometer and was found to be 1 016 K during heating. This temperature was used in the radiation-loss boundary condition to determine the heat flow from the steel surface to the lid. The steel temperature was measured using the automatic sampling system at a location 200 mm under the slag/steel interface and 0.4 m from the ladle wall.

Figure 8 shows the measured temperature at the sampling location during the studied treatment time. In order to see which heat-flux input rendered the best prediction of temperature, model calculations were made using the data for the different arc lengths shown in Fig. 7. The results showed that the best agreement with the experimental values was obtained for data for a 25 cm arc length. In Fig. 8 it can also be seen that the steel temperature increases from 1.573 to 1.592°C during 8 min of combined heating and stirring.

When the steel bath is heated using electrodes, temperature gradients are created in the steel. These temperature gradients are a result of heat transfer from the arcs together with heat loss from the wall, bottom and steel surface. A contour plot of the heat transfer to the steel surface from the electrodes is shown in Fig. 9. The plot shows the highest heat transfer to the steel melt occurring directly under the electrodes, which is expected. Note that only a rather small area of the steel surface is actually heated. In Fig. 10 contours of temperature gradients in a three-dimensional slice of the ladle are shown at different treatment times. The heated steel at the surface at a position opposite the induction stirrer is transported downwards in the ladle when the stirrer is operated in the upward direction. The figure indicates that the steel gets warmer during heating but that the temperature gradients do not become steeper. Thus, there may be no need for longer stirring times after heating to equalize the temperature.

6. Conclusions

A model coupling the heating and inductive stirring of the steel in an ASEA-SKF ladle furnace was developed. A separate model for the arc was first used to predict the arc parameters. The output data from these predictions was then used as boundary condition input in a model of an inductively stirred ladle. As a first attempt to validate the ladle model, predicted steel temperatures were compared with measured temperatures. Since it was shown (Fig. 7) that the total heat flux from the electrodes to the steel bath varied with arc length, simulations were made using heat-flux data for different arc lengths as input for the ladle model. Predicted and measured temperature values of the sampling region agreed best when the heat-flux data for a 25 cm arc length was used. Thus, it is concluded that for these particular conditions data corresponding to a 25 cm arc length should be used as boundary condition input when predicting the combined effect of heating and induction stirring during ladle treatment. Further in-depth comparisons are deemed necessary in order to ensure that the predictions produced using the model are fully reliable.
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