Determination of the vapor film thickness at steady superfluid helium film boiling

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Abstract. Heat mass transfer processes at superfluid helium boiling is considered for different conditions. Mathematical description is formulated with taking into account the peculiarities of superfluid helium. System of equations is based on the methods of continuum mechanics and molecular kinetic theory. Pressure distribution in the liquid is determined by the hydrostatic difference. At the vapor-liquid interface the Laplace formula is used. The vapor pressure in the film is determined by the non-equilibrium effects in the vapor near the interface. Heat transfer in a liquid is described by the Gorter-Mellink semi-empirical theory. Experimental data on the stationary boiling of superfluid helium under various conditions are interpreted based on the formulated mathematical model. For all cases in the considered range of parameters, the qualitative and in some cases quantitative agreement between the calculated and experimental values of the vapor film thickness were obtained.

1. Introduction

The investigation of the dynamic and thermal properties of superfluid helium is important for a number of applied problems, for example, in the development of cryostatting systems for superconducting devices to develop methods that ensure reliable cryostatting of superconducting magnets, cables and other devices. Further, the use of He-II for cooling superconductors has some advantages over other cryogenic liquids. Since the temperature of superfluid helium does not exceed 2.17K, when cooling superconductors it is possible to increase the critical current density and, consequently, the critical magnetic field. In the design of cryogenic systems, it is important to study the heat transfer processes in He-II, since the jumps in the heat flux in the superconductor can lead to emergency situations, such as overheating of a part of the winding, mechanical damage due to a sharp boiling of helium, and so on.

The exceptional importance of this scientific problem is connected with the study of the dynamics of the interface boundary and transfer processes without the influence of gravity. It is known that boiling of He-II is realized only in film mode, and the limitations of heat transfer are caused by processes on the vapour-liquid interface surface. Under the distorting influence of gravitational forces, the vapor structures change, which leads to the appearance of hard-to-control boiling regimes, for example, the noise boiling of superfluid helium at large immersion depths of the heater. This leads to the impossibility of accurately providing the desired process parameters. At microgravity the hydrostatic pressure is absence. The vapour film can grow unlimited and transport processes cannot be study on the vapor-liquid interface. In this regard, a porous structure is of interest, which, while leaving possibility for heat and mass transfer in the liquid, limits the uncontrolled growth of the vapor domain. This will allow us to study the dynamics of the interface and transport processes without the influence of gravity.
Furthermore, the study of the regularities of heat transfer in the film boiling of He-II under conditions of microgravity is of particular interest in connection with the possibility of obtaining information on the specifics of transport processes on the vapor-liquid interface. Superfluid helium has large heat transfer efficiency in liquid, as a result of which the dynamics of two-phase systems is mainly determined by nonequilibrium effects at the vapor-liquid interface. The obtained results can be used to solve the problem about optimization of operating modes and design of heat exchangers for special purposes.

2. Stationary boiling in large volume

The following problem statement of the superfluid helium steady-state film boiling in a large volume is considered (Fig.1). Cylindrical heater of radius $R_w$ is immersed in the superfluid helium at the depth $h$. As the heater flux $q_w$ is applied the smooth stationary vapor film of radius $R_1$ (thickness $\delta = R_1 - R_w$) is formed as coaxial heater. Above the free surface of the liquid constant pressure $P_b$ is maintained so the liquid is in equilibrium with the vapor $P_b = P_S(T_b)$, where $T_b$ is the temperature of liquid helium.

![Figure 1. Schematic diagram of the problem.](image)

As can be seen from the schematic diagram of the problem, the liquid is separated from the heater by the vapor film. Heat is supplied to the interface from the vapor side. Vapor film is closed from the outside space. Interface is nearly of cylindrical shape. Possible fluctuations of vapor film are not considered due to the experimental data [1, 2]. As the problem is stationary mass flux on the vapor-liquid interface is equal to zero.

The application of the model [3], which describes the method of calculating the recovery heat flux for superfluid helium boiling, to the considered problem, allows us to obtain a system of equations. Previously, this approach was successfully used to describe the stationary boiling of superfluid helium on a sphere [4].

The pressure in the fluid $P'$ is determined by the hydrostatic difference:

$$ P' = P_b + \rho'gh, \quad (1) $$

where $\rho'$ – liquid density, $g$ – gravity acceleration.

The vapor pressure $P''$ in the film is determined by the nonequilibrium boundary condition obtained by solving of the kinetic Boltzmann equation for evaporation-condensation problems [5]. The relationship between the actual vapor pressure $P''$, the saturation pressure $P_S(T_1)$ corresponding to vapour liquid interface temperature $T_1$ and the heat flux density at this surface $q_1$ is the following:

$$ P'' = P_S(T_1) + \frac{0.44q_1}{\sqrt{2R_hT_1}}, \quad (2) $$

where $R_h$ – individual gas constant for helium.

The Laplace formula is written on the vapour liquid boundary, taking into account the cylindrical geometry:
where \( \sigma \) – surface tension coefficient.

Heat transfer in He-II is described on the basis of the Gorter-Mellink semi-empirical theory. [6]:

\[
q_1^3 = \frac{2}{f(T)R_i}(T_1 - T_b),
\]

where \( f(T) \) – is the Gorter-Mellink constant average in the temperature range \( T_b + T_1 \).

The heat flux at the interface is determined based on the next correlation:

\[
q_1R_i = q_wR_w,
\]

Equation from (2) to (4) is transformed using Clapeyron-Clausius equation:

\[
P_3(T_1) - P_b = P_3(T_b) - P_3(T_b) = \frac{f(T)}{2} \frac{q_w^3R_w^3}{R_i^2} \frac{r \cdot P_b}{R_b T_b^2},
\]

where \( r \) is the latent evaporation heat of helium.

The system of equations (1) - (6) is reduced to the following equation for \( R_1 \):

\[
\frac{f(T)}{2} \frac{q_w^3R_w^3}{R_i^2} \frac{r \cdot P_b}{R_b T_b^2} + \frac{0.44q_wR_w}{R_1} = \rho'g h + \frac{\sigma}{R_i},
\]

If we accept that the change in temperature of superfluid helium can be neglected, then equation (7) is reduced to the next form:

\[
R_i = \frac{1}{\rho'g h} \left( \frac{0.44q_wR_w}{\sqrt{2R_b T_b^2}} - \sigma \right),
\]

The differences in the calculations results of the vapor film thickness \( \delta \) according to equations (7) and (8) is a few percents (1-3%), therefore, the formula (8) can be recommended for practical use.

The calculations results of the vapor film thickness \( \delta \) for the experimental data [1] with different heater sizes \( D_w \) and different heat fluxes \( q_w \) show that agreement is achieved quite satisfactory (Table 1). Differences vary in the range of 10–35% for different points. The range of the experimental parameters is wide as for heater diameter \( D_w \) as for heat flux \( q_w \) and immersion depth \( h \). The limitations of the experimental data do not allow us to establish the nature of the dependence of the vapor film thickness \( \delta \) on any particular parameter. But it is obvious, that presented model gives the possibility to determine the value of vapor film thickness \( \delta \) at steady superfluid helium film boiling.

Table 1. Comparison of experimental and calculation data.

| Number | 1    | 2    | 3    | 4    | 5    |
|--------|------|------|------|------|------|
| Heater | tube | wire | wire | wire | wire |
| Heater diameter \( D_w \), mm | 2.00 | 0.80 | 0.12 | 0.10 | 0.19 |
| Immersion depth \( h \), cm   | 10.5 | 10.7 | 20.3 | 8.5  | 2.8  |
| Temperature \( T_b \), K     | 1.94 | 1.92 | 1.98 | 1.98 | 1.57 |
| Heat flux \( q_w \), kW/m²   | 3.62 | 4.07 | 18.1 | 7.92 | 1.81 |
| Experimental vapor film thickness \( \delta_e \), mm | 0.16 | 0.11 | 0.09 | 0.12 | 0.11 |
| Calculation vapor film thickness \( \delta_c \), mm | 0.178 | 0.121 | 0.120 | 0.105 | 0.132 |
| Deviation, %                  | 11   | 10   | 34   | 12   | 20   |
3. Boiling of superfluid helium in microgravity

In experiments [7], the conditions of microgravity are ensured by the free fall of the container in a pipe with a height of 122 m, the pressure in which is about 50 Pa. The experiment lasts approximately 4.7 s, during which the level of microgravity is \( g_0 = 10^{-5} g \).

To create a single bubble in helium-II, a microheater of manganine wire with an outer diameter of 0.05 mm and a length of 1.88 mm is placed in the cryostat. Heater is fixed on both sides by two superconducting monofilaments. The heat load of the heater was measured by a four-wire circuit. Most of the heat was released in the manganin part, even when the superconducting wires were partially in the vapor bubble. The direct current is switched on simultaneously with the onset of free fall and remains on during the entire time of microgravity (until the start of braking).

During the experiment, a single vapor bubble was formed on the heater surface, which grew to a size of \((6 \div 10) \text{ mm}\) in diameter at a bath temperature of 1.9K. The size dependence of the vapor film on the time was calculated by the area occupied by the vapor in the frames of the video.

The problem of superfluid helium boiling in microgravity corresponds to Fig. 1. However, vapor film on the heater surface is formed in ideal spherical shape as shown by experimental data [7]. Therefore, equations (4) - (6) are transformed as follows in the mathematical description

\[
\frac{\sigma^*}{R_i'} = \frac{2\sigma}{R_i}, \tag{9}
\]

\[
q_i^3 = \frac{5}{f(T)R_i}(T_i - T_b), \tag{10}
\]

\[
q_i 4\pi R_i^2 = q_w \pi dL = Q, \tag{11}
\]

where \( Q \) – heater power.

Accordingly, equation (7) is transformed in the next form:

\[
\frac{f(T)R_i}{5} \left( \frac{Q}{4\pi R_i^2} \right) \frac{r \cdot P_{A}}{R_i T_b} + \frac{0.44Q}{4\pi R_i^2} \sqrt{2R_b \left( \frac{T_b}{f(T)R_i} \left( \frac{Q}{4\pi R_i^2} \right) \right)} = \rho' g_0 h + \frac{2\sigma}{R_i}, \tag{12}
\]

The level of microgravity in experiments [7] was \( g_0 = 10^{-5} g \). The estimate shows that the hydrostatic pressure difference can be neglected \( \rho' g_0 h = 7 \cdot 10^{-4} \text{ Pa} \). The capillary pressure difference is \( \frac{2\sigma}{R_i} = 0.124 \text{ Pa} \) at the maximum radius of the film \( R_i = 5 \text{ mm} \). Then equation (12) with the assumption of neglected temperature difference in the liquid is transformed to the following

\[
R_i = \frac{0.44Q}{8\pi\sigma \sqrt{2R_b T_b}}, \tag{13}
\]

The results of the comparison of the calculated and experimental data are presented in Fig. 2. The dependencies of vapour bubble diameter \( D_1 \) on heat flux \( q_w \) and heater load \( Q \) are plotted for calculation (lines) and experimental (points) data for the same bath temperature \( T_b \) as mentioned above. As noted in [8], the heat load loss is 0.2\( Q \). At this we taken into account this loss of heat power in (13). As we can see from the picture the agreement between data is good enough. The differences in the values of the vapor film stationary size \( D_1 \) are 2-7% for experimental and calculative data. In fig. 3, the abscissa axis also shows the value of thermal power \( Q \), taking into account losses.

The simple analysis of equation (13) and corresponding dependencies show that steady diameter of vapour film \( D_1 \) is determined by the balance between capillary forces and non-equilibrium effect near vapour-liquid interface.
Figure 2. Comparison of experimental and calculation data for microgravity:
   a) the dependencies of vapor film diameter \(D_1\) on heater load \(Q\);
   b) the dependencies of vapor film diameter \(D_1\) on heat flux \(q_w\).

4. Superfluid helium boiling in confined volume
The superfluid helium boiling on the cylindrical heater inside porous shell can be described as presented in [9]. At this the radius of vapor film \(R_1\) is depend on permeability of porous structure \(k_p\)

\[
R_1 = \frac{0.44 q_w R_w}{k_p \sigma} - \frac{\eta' q_w R_w}{2RT_b} \ln \left( \frac{R_b + L}{R_0} \right) + \rho' g h,
\]

(14)

where \(\eta'\) – viscosity of normal component, \(S\) – entropy of liquid, \(R_0\) – inner radius of porous tube, \(L\) – the thickness of porous layer.

The behavior of superfluid helium inside porous body differs from the free volume [10]. At this the corresponding comparison between free volume and confined state is presented on fig. 3. The heater radius was \(R_w = 2\) mm, permeability \(k_p = 10^{-12}\) m\(^2\), temperature \(T_b = 2\)K. The dependencies of vapor film thickness \(\delta\) on immersion depth \(h\) are plotted for two heat flux densities \(q_w\).

Figure 3. The dependence of vapor film thickness on immersion depth:
   inside porous shell 1 – \(q_w = 40\) kW/m\(^2\), 2 - \(q_w = 50\) kW/m\(^2\),
   for free volume 3– \(q_w = 40\) kW/m\(^2\), 4 – \(q_w = 50\) kW/m\(^2\).
As we can see from the fig. 3 vapor film thickness $\delta$ for free volume differs from confined conditions moreover as the permeability is less, as the vapor size is less too. But at the permeability $k_p = 10^{-11}$ m$^2$ and higher the difference between free volume and confined condition is neglected. At the higher heat flux the vapor film is thicker.

**Conclusion**
The calculation method for vapor film thickness at stationary film boiling of superfluid helium is developed based on continuum mechanics and molecular kinetic theory. The peculiarity of the model is taking into account the surface tension. The different experimental data were analyzed. The deviation of vapor film thickness at the boiling on the cylindrical heater is 10-35% at free volume and 2-7% at the microgravity. At this, suggested approach describe satisfactory the known experimental data. The method of vapor film thickness calculation at the superfluid helium boiling inside porous structure, taking into account peculiarities of heat transfer in porous shell, is presented. Influence of the porous structure permeability on the vapor film size is demonstrated.

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