Self-organized criticality and interacting soft gluons in deep-inelastic electron-proton scattering

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Abstract

It is suggested that the colorless systems of interacting soft-gluons in large-rapidity-gap events are open dynamical complex systems in which self-organized criticality and BTW-clusters play an important role. Theoretical arguments and experimental evidences supporting such a statistical approach to deep-inelastic scattering are presented.

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I. Interacting soft gluons in the small-$x_B$ region of DIS

A number of striking phenomena have been observed in recent deep-inelastic electron-proton scattering experiments in the small-$x_B$ region. In particular it is seen, that the contribution of the gluons dominates, and that large-rapidity-gap (LRG) events exist. The latter shows that the virtual photons in such processes may encounter colorless objects originating from the proton.

The existence of LRG events in these scattering processes have attracted much attention, and there has been much discussion on problems associated with the origin and/or the properties of such colorless objects. Reactions in which “exchange” of such objects dominate are known in the literature as “diffractive scattering processes”. While the concepts and methods used by different authors are in general very much different from one another, all the authors in describing such processes (experimentalists as well as theorists) seem to agree on the following (see also Refs. [2–4, 6–8]): (a) Interacting soft gluons play a dominating role in understanding the phenomena in the small-$x_B$ region in general, and in describing the properties of LRG events in particular. (b) Perturbative QCD should be, and can be, used to describe the LRG events associated with high transverse-momentum ($p_\perp$) jets which have been observed at HERA and at the Tevatron. Such events are, however, rather rare. For the description of the bulk of LRG events, concepts and methods beyond the perturbative QCD (for example, Pomeron Models based on Regge Phenomenology) are needed. The question, whether or how perturbative QCD plays a role in such non-perturbative approaches does not have an unique answer.

In a previous paper, we suggested that the observed dominance of interacting soft gluons and the existence of LRG events in the small-$x_B$ region are closely related to each other, and that the interacting soft gluons may form colored and colorless systems — which we called “gluon clusters”. Such gluon clusters have finite lifetimes which (in the small-$x_B$ region) can be of the same order as the interaction time $\tau_{int}$ — the time-interval in which the virtual photon $\gamma^*$ “sees” the cluster in the sense that it is absorbed by the charged
constituents of the latter. In Ref. [8] the lifetime of such a gluon-cluster was estimated by using uncertainty principle and kinematical considerations — without any dynamical input. In analogy to hadron structure function, a quantity $F_{2}^{c}$ which we called “the structure function of the gluon cluster $c_{0}$” was introduced, and then it was set to be a constant — in accordance with the purpose of that paper which is to discuss the kinematical aspects of a statistical approach.

After having seen what phase-space considerations can, and cannot do, we decided to go one step further, and study the dynamical aspects of the interacting soft-gluons in these scattering processes. In doing so, we realized that the system of interacting soft-gluons is extremely complicated. It is not only too complicated (at least for us) to take the details of local interactions into account (for example by describing the reaction mechanisms in terms of Feynman diagrams), but also too complicated to apply well-known concepts and methods in conventional equilibrium statistical mechanics. In fact, having the above-mentioned empirical facts about LRG events and the basic properties of gluons prescribed by the QCD-Lagrangian in mind, we are readily led to the following picture:

Such a system is an open dynamical system with many degrees of freedom, and it is in general far from equilibrium. This is because, once we accept that the colorless object (which the virtual photon encounters) is a system of soft gluons whose interactions are not negligible, we are also forced to accept that, in such a system, gluons can be emitted and absorbed by the members of the system as well as by gluons and/or quarks and antiquarks outside the system (we note in particular that, since the gluons are soft, their density in space is high, and the distances between the interacting gluons are in general not short, the “running-coupling-constant” can be very large). Furthermore, since in general more than one gluons can be emitted or absorbed by the members of the system, the system itself can remain to be a color-singlet. This means in particular that, in such a system, neither the number of gluons nor the energy of the system can be a conserved quantity.

Do we see comparable open, dynamical, complex systems in Nature? If yes, what are the characteristic features of such systems?
II. Characteristic features of open dynamical complex systems

Open dynamical complex systems are not difficult to find in Nature — at least not in the macroscopic world! Such systems have been studied, and in particular the following have been observed by Bak, Tang and Wiesenfeld (BTW) some time ago: Open dynamical systems with many degrees of freedom may evolve to self-organized critical states which lead to fluctuations extending over all length- and time-scales, and that such fluctuations manifest themselves in form of spatial and temporal power-law scaling behaviors showing properties associated with fractal structure and flicker noise respectively.

BTW and many other authors proposed, and demonstrated by numerical simulations, the following: Dynamical systems with local interacting degrees of freedom can evolve into self-organized structures of states which are barely stable. A local perturbation of a critical state may “propagate”, in the sense that it spreads to (some) nearest neighbors, and then to the next-nearest neighbors, and so on in a “domino effect” over all length scales, the size of such an “avalanche” can be as large as the entire system. Such a “domino effect” eventually terminates after a total time $T$, having reached a final amount of dissipative energy and having effected a total spatial extension $S$. The quantity $S$ is called by BTW the “size”, and the quantity $T$ the “lifetime” of the avalanche — named by BTW a “cluster” (hereafter referred to as BTW-cluster). As we shall see in more details later on, it is of considerable importance to note that a BTW-cluster cannot, and should not be identified with a cluster in the usual sense. It is an avalanche, not a static object with a fixed structure which remains unchanged until it decays after a time-interval (known as the lifetime in the usual sense).

It has been shown that the distribution ($D_S$) of the “size” (which is a measure of the dissipative energy, $S$) and the distribution ($D_T$) of the lifetime ($T$) of BTW-clusters in such open dynamical systems obey power-laws:

$$D_S(S) \sim S^{-\mu},$$

$$D_T(T) \sim T^{-\nu},$$

$$3$$
where \( \mu \) and \( \nu \) are positive real constants. In fact, such spatial and temporal power-law scaling behaviors can be, and have been, considered as the universal signals — the “fingerprints” — of the locally perturbed self-organized critical states in such systems. It is expected\(^9\)\(^{10}\)\(^{12}\)\(^{13}\)\(^{14}\)\(^{15}\) that the general concept of self-organized criticality (SOC), which is complementary to chaos, may be the underlying concept for temporal and spatial scaling in a wide class of open non-equilibrium systems — although it is not yet known how the exponents in such power law can be calculated analytically.

SOC has been observed in a large number of open dynamical complex systems in non-equilibrium\(^9\)\(^{10}\)\(^{12}\)\(^{13}\)\(^{14}\)\(^{15}\) among which the following examples are of particular interest, because they illuminate several aspects of SOC which are relevant for the discussion in this paper.

First, the well known Gutenberg-Richter law\(^11\)\(^{12}\) for earthquakes as a special case of Eq.(1): In this case, \( S \) stands for the released energy (the magnitude) of the earthquakes. \( D_S(S) \) is the number of earthquakes at which an energy \( S \) is released. Such a simple law is known to be valid for all earthquakes, large (up to 8 or 9 in Richter scale) or small! We note, the power-law behavior given by the Gutenberg-Richter law implies in particular the following. The question “How large is a typical earthquake?” does not make sense!

Second, the sandpile experiments\(^9\)\(^{10}\) which show the simple regularities mentioned in Eqs.(1) and (2): In this example, we see how local perturbation can be caused by the addition of one grain of sand (note that we are dealing with an open system!). Here, we can also see how the propagation of perturbation in form of “domino effect” takes place, and develops into avalanches of all possible sizes and durations. The size- and duration-distributions are given by Eqs.(1) and (2) respectively. This example is indeed a very attractive one, not only because such experiments can be, and have been performed in labs\(^9\)\(^{10}\), but also because they can be readily simulated on a PC\(^9\)\(^{10}\).

Furthermore, it has been pointed out, and demonstrated by simple models\(^9\)\(^{10}\)\(^{12}\)\(^{13}\)\(^{14}\)\(^{15}\), that the concept of SOC can also be applied to Biological Sciences. It is amazing to see how phenomena as complicated as Life and Evolution can be simulated by simple models such as the “Game of Life”\(^13\) and the “Evolution Model”\(^14\)\(^{15}\).
Having seen that systems of interacting soft-gluons are open dynamical complex systems, and that a wide class of open systems with many degrees of freedom in the macroscopic world evolve to self-organized critical states which lead to fluctuations extending over all length- and time-scales, it seems natural to ask the following: Can such states and such fluctuations also exist in the microscopic world — on the level of quarks and gluons?

III. Are gluon-clusters hadron-like?

How can we find out whether the general concept of self-organized criticality (mentioned in Section II) plays a role in diffractive deep-inelastic lepton-hadron scattering processes (discussed in Section I)? A simple and effective way of doing this, is to check whether the “fingerprints” mentioned in Eqs.(1) and (2), which can be considered as the necessary conditions for the existence of self-organized criticality, show up in the relevant experiments. For such a comparison, we need the spatial and the temporal distributions of the gluon-clusters. Hence, an important step in our quantitative study is to obtain these distributions directly from the experimental data — if possible, without any theoretical input. Having this goal in mind, we now try to express such cluster-distributions in terms of the measured “diffractive structure function” 

\[ F_2^{D(3)}(\beta, Q^2; x_P) \equiv \int dt F_2^{D(4)}(\beta, Q^2; x_P, t). \]

Here, we note that \( F_2^{D(4)}(\beta, Q^2; x_P, t) \) is related to the differential cross-section for large-rapidity-gap events

\[ \frac{d^4\sigma^D}{d\beta dQ^2 dx_P dt} = \frac{4\pi\alpha^2}{\beta Q^4} (1 - y + \frac{y^2}{2}) F_2^{D(4)}(\beta, Q^2; x_P, t), \]

in analogy to the relationship between the corresponding quantities [namely \( d^2\sigma/(dx_B dQ^2) \) and \( F_2(x_B, Q^2) \)] for normal deep-inelastic electron-proton scattering events

\[ \frac{d^2\sigma}{dx_B dQ^2} = \frac{4\pi\alpha^2}{x_B Q^4} (1 - y + \frac{y^2}{2}) F_2(x_B, Q^2). \]

The kinematical variables, in particular \( \beta, Q^2, x_P \) and \( x_B \) (in both cases) are directly measurable quantities, the definitions of which are shown in Fig.1 together with the corresponding diagrams of the scattering processes. We note that, although these variables are
Lorentz-invariants, it is sometimes convenient to interpret them in a “fast moving frame”, for example the electron-proton center-of-mass frame where the proton’s 3-momentum $\vec{P}$ is large (i.e. its magnitude $|\vec{P}|$ and thus the energy $P^0 \equiv (|\vec{P}|^2 + M^2)^{1/2}$ is much larger than the proton mass $M$). While $Q^2$ characterizes the virtuality of the space-like photon $\gamma^*$, $x_B$ can be interpreted, in such a “fast moving frame” (in the framework of the celebrated parton model), as the fraction of proton’s energy $P^0$ (or longitudinal momentum $|\vec{P}|$) carried by the struck charged constituent.

We recall, in the framework of the parton model, $F_2(x_B, Q^2)/x_B$ for “normal events” can be interpreted as the sum of the probability densities for the above-mentioned $\gamma^*$ to interact with such a charged constituent inside the proton. In analogy to this, the quantity $F_2^{D(3)}(\beta, Q^2; x_P)/\beta$ for LRG events can be interpreted as the sum of the probability densities for $\gamma^*$ to interact with a charged constituent which carries a fraction $\beta \equiv x_B/x_P$ of the energy (or longitudinal momentum) of the colorless object, under the condition that the colorless object (which we associate with a system of interacting soft gluons) carries a fraction $x_P$ of proton’s energy (or longitudinal momentum). We hereafter denote this charged-neutral and color-neutral gluon-system by $c_0^*$ (in Regge pole models this object is known as the “pomeron”). Hence, by comparing Eq. (3) with Eq. (4) and by comparing the two diagrams shown in Fig. 1(a) and Fig. 1(b), it is tempting to draw the following conclusions:

The diffractive process is nothing else but a process in which the virtual photon $\gamma^*$ encounters a $c_0^*$, and $\beta$ is nothing else but the Bjorken-variable with respect to $c_0^*$ (this is why it is called $x_{BC}$ in Ref.[8]). This means, a diffractive $e^-p$ scattering event can be envisaged as an event in which the virtual photon $\gamma^*$ collides with “a $c_0^*$-target” instead of “the proton-target”. Furthermore, since $c_0^*$ is charge-neutral, and a photon can only directly interact with an object which has electric charges and/or magnetic moments, it is tempting to assign $c_0^*$ an electromagnetic structure function $F_2^c(\beta, Q^2)$, and study the interactions between the virtual photon and the quark(s) and antiquark(s) inside $c_0^*$. In such a picture (which should be formally the same as that of Regge pole models, if we would replace the $c_0^*$’s by “pomerons”) we are confronted with the following two questions:
First, is it possible and meaningful to discuss the $x_P$-distributions of the $c_0^*$’s without knowing the intrinsic properties, in particular the electromagnetic structures, of such objects?

Second, are gluon-clusters hadron-like, such that their electromagnetic structures can be studied in the same way as those for ordinary hadrons?

We discuss the second question here, and leave the first question to the next section. We note, in order to be able to answer the second question in the affirmative, we need to know whether $F_{2D}(3)(\beta, Q^2; x_P)$ can be factorized in the form

$$
F_{2D}(3)(\beta, Q^2; x_P) = f_c(x_P)F_{2c}^c(\beta, Q^2). 
$$

(5)

Here, $f_c(x_P)$ plays the role of a “kinematical factor” associated with the “target $c_0^*$”, and $x_P$ is the fraction of proton’s energy (or longitudinal momentum) carried by $c_0^*$. [We could call $f_c(x_P)$ “the $c_0^*$-flux” — in exactly the same manner as in Regge pole models, where it is called “the pomeron flux”]. $F_{2c}^c(\beta, Q^2)$ is “the electromagnetic structure function of $c_0^*$” [the counterpart of $F_2(x_B, Q^2)$ of the proton] which — in analogy to proton (or any other hadron) — can be expressed as

$$
\frac{F_{2c}^c(\beta, Q^2)}{\beta} = \sum_i e_i^2[q_i^s(\beta, Q^2) + \bar{q}_i^s(\beta, Q^2)],
$$

(6)

where $q_i^s(\bar{q}_i^s)$ stands for the probability density for $\gamma^*$ to interact with a quark (antiquark) of flavor $i$ and electric charge $e_i$ which carries a fraction $\beta$ of the energy (or longitudinal momentum) of $c_0^*$. It is clear that Eq.(6) should be valid for all $x_P$-values in this kinematical region, that is, both the right- and the left-hand-side of Eq.(6) should be independent of the energy (momentum) carried by the “hadron” $c_0^*$.

Hence, to find out experimentally whether the second question can be answered in the affirmative, we only need to check whether the data are in agreement with the assumption that $F_2(\beta, Q^2)$ prescribed by Eqs.(5) and (6) exists. For such a test, we take the existing data and plot $\log[F_{2D}(3)(\beta, Q^2; x_P)/\beta]$ against $\log \beta$ for different $x_P$-values. We note, under the assumption that the factorization shown in Eq.(5) is valid, the $\beta$-dependence for a
given $Q^2$ in such a plot should have exactly the same form as that in the corresponding
\[ \log[F_2^c(\beta, Q^2)/\beta] \] vs $\log \beta$ plot; and that the latter is the analog of $\log[F_2(x_B, Q^2)/x_B]$ vs $\log x_B$ plot for normal events. In Fig.2 we show the result of such plots for three fixed $Q^2$-values (3.5, 20 and 65 GeV$^2$, as representatives of three different ranges in $Q^2$). Our goal is to examine whether or how the $\beta$-dependence of the function given in Eq.(6) changes with $x_P$. In principle, if there were enough data points, we should, and we could, do such a plot for the data-sets associated with every $x_P$-value. But, unfortunately there are not so much data at present. What we can do, however, is to consider the $\beta$-distributions in different $x_P$-bins, and to vary the bin-size of $x_P$, so that we can explicitly see whether/how the shapes of the $\beta$-distributions change. The results are shown in Fig.2. The $\beta$-distribution in the first row, corresponds to the integrated value $\tilde{F}_2^D(\beta, Q^2)$ shown in the literature. Those in the second and in the third row are obtained by considering different bins and/or by varying the sizes of the bins. By joining the points associated with a given $x_P$-interval in a plot for a given $Q^2$, we obtain the $\beta$-distribution for a $c_0^*$ carrying approximately the amount of energy $x_PP^0$, encountered by a photon of virtuality $Q^2$. Taken together with Eq.(6) we can then extract the distributions $q_i^c(\beta, Q^2)$ and $\bar{q}_i^c(\beta, Q^2)$ for this $Q^2$-value, provided that $F_2^c(\beta, Q^2)/\beta$ is independent of $x_P$. But, as we can see in Fig.2, the existing data show that the $x_P$-dependence of this function is far from being negligible! Note in particular that according to Eq.(5), by choosing a suitable $f_P(x_P)$ we can shift the curves for different $x_P$-values in the vertical direction (in this log-log plot); but we can never change the shapes of the $\beta$-distributions which are different for different $x_P$-values!

In order to see, and to realize, the meaning of the $x_P$-dependence of the distributions of the charged constituents of $c_0^*$ expressed in terms of $F_2^c(\beta, Q^2)/\beta$ in LRG events [see Eqs.(5) and (6)], let us, for a moment, consider normal deep-inelastic scattering events in the $x_B$-region where quarks dominate ($x_B > 0.1$, say). Here we can plot the data for $\log[F_2(x_B, Q^2)/x_B]$ as a function of $\log x_B$ obtained at different incident energies ($P^0$'s) of the proton. Suppose we see, that at a given $Q^2$, the data for $x_B$-distributions taken at different values of $P^0$ are very much different. Would it still be possible to introduce
\(F_2(x_B, Q^2)\) as “the electromagnetic structure function” of the proton, from which we can extract the \(x_B\)-distribution of the quarks \(q_i(x_B, Q^2)\) at a given \(Q^2\)?

IV. Distributions of the gluon-clusters

After having seen that the existing data are not in agreement with the picture in which the colorless gluon-clusters (\(c_0^*\)’s) are hadron-like, we now come back to the first question in Section III, and try to find out whether it is never-the-less possible and meaningful to talk about the \(x_P\)-distribution of \(c_0^*\). We shall see in this section, the answer to this question is Yes! Furthermore, we shall also see, in order to answer this question in the affirmative, we do not need the factorization mentioned in Eq.(5); and we do not need to know whether the gluon-clusters are hadron-like. But, as we shall show later on, it is of considerable importance to discuss the second question in understanding the nature of the \(c_0^*\)’s.

In view of the fact that we do use the concept “distributions of gluons” in deep-inelastic lepton-hadron scattering, although the gluons do not directly interact with the virtual photons, we shall try to introduce the notion “distribution of gluon-clusters” in a similar manner. In order to see what we should do for the introduction of such distributions, let us recall the following:

For normal deep-inelastic \(e^- p\) collision events, the structure function \(F_2(x_B, Q^2)\) can be expressed in term of the distributions of partons, where the partons are not only quarks and antiquarks, but also gluons which can contribute to the structure function by quark-antiquark pair creation and annihilation. In fact, in order to satisfy energy-momentum-conservation (in the electron-proton system), the contribution of the gluons \(x_g g(x_g, Q^2)\) has to be taken into account in the energy-momentum sum rule for all measured \(Q^2\)-values. Here, we denote by \(g(x_g, Q^2)\) the probability density for the virtual photon \(\gamma^*\) (with virtuality \(Q^2\)) to meet a gluon which carries the energy (momentum) fraction \(x_g\) of the proton, analogous to \(q_i(x_B, Q^2)\) [or \(\bar{q}_i(x_B, Q^2)\)] which stands for the probability density for this \(\gamma^*\) to interact with a quark (or an antiquark) of flavor \(i\) and electric charge \(e_i\) which carries the energy
(momentum) fraction $x_B$ of the proton. We note, while both $x_B$ and $x_g$ stand for energy (or longitudinal momentum) fractions carried by partons, the former can be, but the latter cannot be directly measured.

Having these, in particular the energy-momentum sum rule in mind, we immediately see the following: In a given kinematical region in which the contributions of only one category of partons (for example quarks for $x_B > 0.1$ or gluons for $x_B < 10^{-2}$) dominate, the structure function $F_2(x_B, Q^2)$ can approximately be related to the distributions of that particular kind of partons in a very simply manner. In fact, the expressions below can be, and have been, interpreted as the probability-densities for the virtual photon $\gamma^*$ (with virtuality $Q^2$) to meet a quark or a gluon which carries the energy (momentum) fraction $x_B$ or $x_g$ respectively.

$$\frac{F_2(x_B, Q^2)}{x_B} \approx \sum_i e_i^2 q_i(x_B, Q^2) \quad \text{or} \quad \frac{F_2(x_B, Q^2)}{x_g} \approx g(x_g, Q^2). \quad (7)$$

The relationship between $q_i(x_B, Q^2)$, $g(x_g, Q^2)$ and $F_2(x_B, Q^2)$ as they stand in Eq.(7) are general and formal (this is the case especially for that between $g$ and $F_2$) in the following sense: Both $q_i(x_B, Q^2)$ and $g(x_g, Q^2)$ contribute to the energy-momentum sum rule and both of them are in accordance with the assumption that partons of a given category (quarks or gluons) dominate a given kinematical region (here $x_B > 0.1$ and $x_B < 10^{-2}$ respectively). But, neither the dynamics which leads to the observed $Q^2$-dependence nor the relationship between $x_g$ and $x_B$ are given. This means, without further theoretical inputs, the simple expression for $g(x_g, Q^2)$ as given by Eq.(7) is practically useless!

Having learned this, we now discuss what happens if we assume, in diffractive lepton-nucleon scattering, the colorless gluon-clusters ($c_0^*$’s) dominate the small-$x_B$ region ($x_B < 10^{-2}$, say). In this simple picture, we are assuming that the following is approximately true: The gluons in this region appear predominately in form of gluon clusters. The interaction between the struck $c_0^*$ and the rest of the proton can be neglected during the $\gamma-c_0^*$ collision such that we can apply impuls-approximation to the $c_0^*$’s in this kinematical region. That is, here we can introduce — in the same manner as we do for other partons (see Eq.4), a probability density $D_S(x_P|\beta, Q^2)$ for $\gamma^*$ in the diffractive scattering process to “meet” a
which carries the fraction \( x_P \) of the proton’s energy \( P^0 = (|\vec{P}|^2 + M^2)^{1/2} \approx |\vec{P}| \) (where \( \vec{P} \) is the momentum and \( M \) is the mass of the proton). In other words, in diffractive scattering events for processes in the kinematical region \( x_B < 10^{-2} \), we should have, instead of \( g(x_g, Q^2) \), the following:

\[
\frac{F_2^{D(3)}(\beta, Q^2; x_P)}{x_P} \approx D_S(x_P | \beta, Q^2).
\]  

(8)

Here, \( x_P P^0 \) is the energy carried by \( c_0^* \), and \( \beta \) indicates the corresponding fraction carried by the struck charged constituent in \( c_0^* \). In connection with the similarities and the differences between \( q_i(x_B, Q^2) \), \( g(x_B, Q^2) \) in (7) and \( D_S(x_P | \beta, Q^2) \) in (8), it is useful to note in particular the significant difference between \( x_g \) and \( x_P \), and thus that between the \( x_g \)-distribution \( g(x_g, Q^2) \) of the gluons and the \( x_P \)-distribution \( D_S(x_P | \beta, Q^2) \) of the \( c_0^* \)’s: Both \( x_g \) and \( x_P \) are energy (longitudinal momentum) fractions of charge-neutral objects, with which \( \gamma^* \) cannot directly interact. But, in contrast to \( x_g \), \( x_P \) can be directly measured in experiments, namely by making use of the kinematical relation

\[
x_P \approx \frac{Q^2 + M_x^2}{Q^2 + W^2},
\]  

(9)

and by measuring the quantities \( Q^2, M_x^2 \) and \( W^2 \) in every collision event. Here, \( Q, M_x \) and \( W \) stand respectively for the invariant momentum-transfer from the incident electron, the invariant-mass of the final hadronic state after the \( \gamma^* - c_0^* \) collision, and the invariant mass of the entire hadronic system in the collision between \( \gamma^* \) and the proton. Note that \( x_B \equiv \beta x_P \), hence \( \beta \) is also measurable. This means, in sharp contrast to \( g(x_g, Q^2) \), experimental information on \( D_S(x_P | \beta, Q^2) \) in particular its \( x_P \)-dependence can be obtained — without further theoretical inputs!

V. The first SOC-fingerprint: Spatial scaling

We mentioned at the beginning of Section III, that in order to find out whether the concept of SOC indeed plays a role in diffractive DIS we need to check the fingerprints of SOC
shown in Section II, and that such tests can be made by examining the corresponding cluster-distributions obtained from experimental data. We are now ready to do this, because we have learned in Sections III and IV, that it is not only meaningful but also possible to extract $x_P$-distributions from the measured diffractive structure functions, although the gluon-clusters cannot be treated as hadrons. In fact, as we can explicitly see in Eqs.(8) and (9), in order to extract the $x_P$-dependence of the gluon-clusters from the data, detailed knowledge about the intrinsic structure of the clusters are not necessary.

Having these in mind, we now consider $D_S$ as a function of $x_P$ for given values of $\beta$ and $Q^2$, and plot $F_2^{D(3)}(x_P; \beta, Q^2)/x_P$ against $x_P$ for different sets of $\beta$ and $Q^2$. The results of such log-log plots are shown in Fig. 3. As we can see, the data suggest that the probability-density for the virtual photon $\gamma^*$ to meet a color-neutral and charged-neutral object $c_0^*$ with energy (longitudinal momentum) fraction $x_P$ has a power-law behavior in $x_P$, and the exponent of this power-law depends very little on $Q^2$ and $\beta$. This is to be compared with $D_S(S)$ in Eq.(4), where $S$, the dissipative energy (the size of the BTW-cluster) corresponds to the energy of the system $c_0^*$. The latter is $x_P P^0$, where $P^0$ is the total energy of the proton.

It means, the existing data show that $D_S(x_P|\beta, Q^2)$ exhibits the same kind of power-law behavior as the size-distribution of BTW-clusters. This result is in accordance with the expectation that self-organized critical phenomena may exist in the colorless systems of interacting soft gluons in diffractive deep-inelastic electron-proton scattering processes.

We note, up to now, we have only argued (in Section I) that such gluon-systems are open, dynamical, complex systems in which SOC may occur, and we have mentioned (in Section II) the ubiquitousness of SOC in Nature. Having seen the first piece of experimental evidence that one of the necessary conditions for the existence of SOC is satisfied, let us now take a second look at the colorless gluon-systems from a theoretical point of view: Viewed from a “fast moving frame” which can for example be the electron-proton c.m.s. frame, such colorless systems of interacting soft gluons are part of the proton (although, as color-singlets, they can also be outside the confinement region). Soft gluons can be intermittently emitted
or absorbed by gluons or by gluons, quarks and antiquarks outside
the system. The emission- and absorption-processes are due to local interactions prescribed
by the well-known QCD-Lagrangian (here “the running coupling constants” are in general
large, because the distances between the interacting colored objects cannot be considered
as “short”; remember that the spatial dimension of a $c^0_*$ can be much larger than that of
a hadron!). In this connection, it is however very useful to keep in mind that, due to the
complexity of the system, details about the local interactions may be relatively unimportant,
while general and/or global features — for example energy-flow between different parts
(neighbors and neighbor’s neighbors . . .) of the system — may play an important role.

How far can one go in neglecting dynamical details when one deals with such open
complex systems? In order to see this, let us recall how Bak and Sneppen\textsuperscript{14} succeeded
in modelling some of the essential aspects of The Evolution in Nature. They consider
the “fitness” of different “species”, related to one another through a “food chain”, and
assumed that the species with the lowest fitness is most likely to disappear or mutate at
the next time-step in their computer simulations. The crucial step in their simulations
that \textit{drives} evolution is the adaption of the individual species to its present \textit{environment}
(neighborhood) through mutation and selection of a fitter variant. Other interacting species
form part of the \textit{environment}. This means, the neighbors will be influenced by every time-
step. The result these authors obtained strongly suggests that the process of evolution is a
self-organized critical phenomenon. One of the essential simplifications they made in their
evolution models\textsuperscript{14,15} is the following: Instead of the explicit connection between the fitness
and the configuration of the genetic codes, they use random numbers for the fitness of the
species. Furthermore, as they have pointed out in their papers, they could in principle have
chosen to model evolution on a less coarse-grained scale by considering mutations at the
individual level rather than on the level of species, but that would make the computation
prohibitively difficult.

Having these in mind, we are naturally led to the questions: Can we consider the creation
and annihilation processes of colorless systems of interacting soft gluons associated with a
proton as “evolution” in a microscopic world? Before we try to build models for a quantitative description of the data, can we simply apply the existing evolution models\textsuperscript{14,15} to such open, dynamical, complex systems of interacting soft-gluons, and check whether some of the essential features of such systems can be reproduced?

To answer these questions, we now report on the result of our first trial in this direction: Based on the fact that we know very little about the detailed reaction mechanisms in such gluon-systems and practically nothing about their structures, we simply ignore them, and assume that they are self-similar in space (this means, colorless gluon-clusters can be considered as clusters of colorless gluon-clusters and so on). Next, we divide them in an arbitrary given number of subsystems \( s_i \) (which may or may not have the same size). Such a system is open, in the sense that neither its energy \( \varepsilon_i \), nor its gluon-number \( n_i \) has a fixed value. Since we do not know, in particular, how large the \( \varepsilon_i \)’s are, we use random numbers. As far the \( n_i \)’s are concerned, since we do not know how these numbers are associated with the energies in the subsystems \( s_i \), except that they are not conserved quantities, we just ignore them, and consider only the \( \varepsilon_i \)’s. As in Ref.\textsuperscript{14} or in Ref.\textsuperscript{15}, the random number of this subsystem as well as those of the fixed\textsuperscript{14} or random (see the first paper of Ref.\textsuperscript{15}) neighbors will be changed at every time-step. Note, this is how we simulate the processes of energy flow due to exchange of gluons between the subsystems, as well as those with gluons/quarks/antiquarks outside the system. In other words, in the spirit of Bak and Sneppen\textsuperscript{14} we neglecting the dynamical details totally. Having in mind that, in such systems, the gluons as well as the subsystems (\( s_i \)’s) of gluons are virtual (space-like), we can ask: “How long can such a colorless subsystem \( s_i \) of interacting soft gluons exist, which carries energy \( \varepsilon_i \)?” According to the uncertainty principle, the answer should be: “The time interval in which the subsystem \( s_i \) can exist is proportional to \( 1/\varepsilon_i \), and this quantity can be considered as the lifetime \( \tau_i \) of \( s_i \).” In this sense, the subsystems of colorless gluons are expected to have larger probabilities to mutate because they are associated with higher energies, and thus shorter “lifetimes”. Note that the basic local interaction in this self-organized evolution process is the emission (or absorption) of gluons by gluons prescribed...
by the QCD-Lagrangian — although the detailed mechanisms (which can in principle be explicitly written down by using the QCD-Lagrangian) do not play a significant role.

In terms of the evolution model\cite{14,15}, we now call \( s_i \) the “species” and identify the corresponding lifetime \( \tau_i \) as the “fitness of \( s_i \)”. Because of the one-to-one correspondence between \( \tau_i \) and \( \varepsilon_i \), where the latter is a random number, we can also directly assign random numbers to the \( \tau_i \)’s instead. From now we can adopt the evolution model\cite{14,15} and note that, at the start of such a process (a simulation), the fitness on average grow, because the least fit are always eliminated. Eventually the fitness do not grow any further on average. All gluons have a fitness above some threshold. At the next step, the least fit species (i.e. the most energetic subsystem \( s_i \) of interacting soft gluons), which would be right at the threshold, will be “replaced” and starts an avalanche (or punctuation of mutation events), which is causally connected with this triggering “replacement”. After a while, the avalanche will stop, when all the fitnesses again will be over that threshold. In this sense, the evolution goes on, and on, and on. As in Refs.\cite{14} and \cite{15}, we can monitor the duration of every avalanche, that is the total number of mutation events in everyone of them, and count how many avalanches of each size are observed. The avalanches mentioned here are special cases of those discussed in Section II. Their size- and lifetime-distributions are given by Eq.\( (1) \) and Eq.\( (2) \) respectively. Note in particular that the avalanches in the Bak-Sneppen model correspond to sets of subsystems \( s_i \), the energies \( (\varepsilon_i) \) of which are too high “to be fit for the colorless systems of low-energy gluons”. It means, in the proposed picture, what the virtual photon in deep-inelastic electron-proton scattering “meet” are those “less fit” one — those who carry “too much” energy. In a geometrical picture this means, it is more probable for such “relatively energetic” colorless gluons-clusters to be spatially further away from the (confinement region of) the proton.

There exists, in the mean time, already several versions of evolution models\cite{10,14,15} based on the original idea of Bak and Sneppen\cite{14}. Although SOC phenomena have been observed in all these cases\cite{10,14,15}, the slopes of the power-law distributions for the avalanches are different in different models — depending on the rules applied to the mutations. The values range
from approximately $-1$ to approximately $-2$. Furthermore, these models seem to show that neither the size nor the dimension of the system used for the computer simulation plays a significant role.

Hence, if we identify the colorless charge-neutral object $c^*_0$ encountered by the virtual photon $\gamma^*$ with such an avalanche, we are identifying the lifetime of $c^*_0$ with $T$, and the “size” (that is the total amount of dissipative energy in this “avalanche”) with the total amount of energy of $c^*_0$. Note that the latter is nothing else but $x_P P^0$, where $P^0$ is the total energy of the proton. This is how and why the $S$-distribution in Eq. (1) and the $x_P$-distribution of $D_S(x_P|\beta, Q^2)$ in Eq.(8) are related to each other.

VI. The second fingerprint: Temporal scaling

In this section we discuss in more detail the effects associated with the time-degree-of-freedom. In connection with the two questions raised in Section III, one may wish to know why the parton-picture does not always work when we apply it in a straightforward manner — not only to hadrons but also to gluon-clusters. The answer is very simple. The time-degree of freedom cannot be ignored when we wish to find out whether impulse-approximation is applicable, and the applicability of the latter is the basis of the parton-model. We recall that, when we apply this model to stable hadrons, the quarks, antiquarks and gluons are considered as free and stable objects, while the virtual photon $\gamma^*$ is associated with a given interaction-time $\tau_{\text{int}}(Q^2, x_B)$ characterized by the values $Q^2$ and $x_B$ of such scattering processes. We note however that, this is possible only when the interaction-time $\tau_{\text{int}}$ is much shorter than the corresponding time-scales (in particular the average propagation-time of color-interactions in hadron). Having these in mind, we see that, we are confronted with the following questions when we deal with gluon-clusters associated with finite lifetimes: Can we consider the $c^*_0$’s as “free” and “stable” particles when their lifetimes are shorter than the interaction-time $\tau_{\text{int}}(Q^2, x_B)$? Can we say that a $\gamma^* - c^*_0$ collision process takes place, in which the incident $\gamma^*$ is absorbed by one a or a system of the charged constituents of $c^*_0$, 

16
when the lifetime $T$ of $c^*_0$ is shorter than $\tau_{\text{int}}(Q^2, x_B)$?

Since the notion “stable objects” or “unstable objects” depends on the scale which is used in the measurement, the question whether a $c^*_0$ can be considered as a parton (in the sense that it can be considered as a free “stable object” during the $\gamma^*-c^*_0$ interaction) depends very much on on the interaction-time $\tau_{\text{int}}(Q^2, x_B)$. Here, for given values of $Q^2$, $x_B$, and thus $\tau_{\text{int}}(Q^2, x_B)$, only those $c^*_0$’s whose lifetime ($T$’s) are greater than $\tau_{\text{int}}(Q^2, x_B)$ can absorb the corresponding $\gamma^*$. That is to say, when we consider diffractive electron-proton scattering in kinematical regions in which $c^*_0$’s dominate, we must keep in mind that the measured cross-sections (and thus the diffractive structure function $F_{2D(3)}$) only include contributions from collision-events in which the condition $T > \tau_{\text{int}}(Q^2, x_B)$ is satisfied!

We note that $\tau_{\text{int}}$ can be estimated by making use of the uncertainty principle. In fact, by calculating $1/q^0$ in the above-mentioned reference frame, we obtain

$$\tau_{\text{int}} = \frac{4|\vec{P}|}{Q^2} \frac{x_B}{1-x_B},$$

which implies that, for given $|\vec{P}|$ and $Q^2$ values,

$$\tau_{\text{int}} \propto x_B, \quad \text{for } x_B \ll 1.$$  

This means, for diffractive $e^-p$ scattering events in the small-$x_B$ region at given $|\vec{P}|$ and $Q^2$ values, $x_B$ is directly proportional to the interaction time $\tau_{\text{int}}$. Taken together with the relationship between $\tau_{\text{int}}$ and the minimum lifetime $T(\text{min})$ of the $c^*_0$’s mentioned above, we reach the following conclusion: The distribution of this minimum value, $T(\text{min})$ of the $c^*_0$’s which dominate the small-$x_B$ ($x_B < 10^{-2}$, say) region can be obtained by examining the $x_B$-dependence of $F_{2D(3)}(\beta, Q^2; x_P)/\beta$ discussed in Eqs. (5), (6) and in Fig. 2. This is because, due to the fact that this function is proportional to the quark (antiquark) distributions $q^c_i(\vec{q}^c_i)$ which can be directly probed by the incident virtual photon $\gamma^*$, by measuring $F_{2D(3)}(\beta, Q^2, x_P)/\beta$ as a function of $x_B \equiv \beta x_P$, we are in fact asking the following questions: Do the distributions of the charged constituents of $c^*_0$ depend on the interaction time $\tau_{\text{int}}$, and thus on the minimum lifetime $T(\text{min})$ of the to be detected gluon-clusters? We use
the identity $x_B \equiv \beta x_P$ and plot the quantity $F^{D(3)}_2(\beta, Q^2; x_P)/\beta$ against the variable $x_B$ for fixed values of $\beta$ and $Q^2$. The result of such a log-log plot is given in Fig.4. It shows not only how the dependence on the time-degree-of-freedom can be extracted from the existing data, but also that, for all the measured values of $\beta$ and $Q^2$, the quantity

$$p(x_B|\beta, Q^2) \equiv \frac{F^{D(3)}_2(\beta, Q^2; x_B/\beta)}{\beta}$$

(12)

is approximately independent of $\beta$, and independent of $Q^2$. This strongly suggests that the quantity given in Eq.(12) is associated with some *global* features of $c_0^*$ — consistent with the observation made in Section III which shows that it cannot be used to describe the structure of $c_0^*$. This piece of empirical fact can be expressed by setting $p(x_B|\beta, Q^2) \approx p(x_B)$.

By taking a closer look at this log-log plot, as well as the corresponding plots for different sets of fixed $\beta$- and $Q^2$-values (such plots are not shown here, they are similar to those in Fig.3), we see that they are straight lines indicating that $p(x_B)$ obeys a power-law. What does this piece of experimental fact tell us? What can we learn from the distribution of the lower limit of the lifetimes (of the gluon-systems $c_0^*$’s)?

In order to answer these questions, let us, for a moment, assume that we know the lifetime-distribution $D_T(T)$ of the $c_0^*$’s. In such a case, we can readily evaluate the integral

$$I[\tau_{int}(x_B)] \equiv \int_{\tau_{int}(x_B)}^{\infty} D_T(T) dT,$$

(13)

and thus obtain the number density of all those clusters which live longer than the interaction time $\tau_{int}(x_B)$. Hence, under the statistical assumption that the chance for a $\gamma^*$ to be absorbed by one of those $c_0^*$’s of lifetime $T$ is proportional to $D_T(T)$ (provided that $\tau_{int}(Q^2, x_B) \leq T$, otherwise this chance is zero), we can then interpret the integral in Eq.(13) as follows: $I[\tau_{int}(Q^2, x_B)] \propto p(x_B)$ is the probability density for $\gamma^*$ [associated with the interaction-time $\tau_{int}(x_B)$] to be absorbed by $c_0^*$’s. Hence,

$$D_T(x_B) \propto \frac{d}{dx_B} p(x_B).$$

(14)

This means in particular, the fact that $p(x_B)$ obeys a power-law in $x_B$ implies that $D_T(T)$ obeys a power-law in $T$. Such a behavior is similar to that shown in Eq.(2). In order to see
the quality of this power-law behavior of $D_T$, and the quality of its independence of $Q^2$ and $\beta$, we compare the above-mentioned behavior with the existing data. In Fig.5, we show the log-log plots of $d/dx_B[p(x_B)]$ against $x_B$. We note that $d/dx_B[p(x_B)]$ is approximately $F_2^{D(3)}(\beta, Q^2; x_B/\beta)/(\beta x_B)$. The quality of the power-law behavior of $D_T$ is explicitly shown in Fig.5.

VII. $Q^2$-dependent exponents in the power-laws?

We have seen, in Sections V and VI, that in diffractive deep-inelastic electron-proton scattering, the size- and the lifetime-distributions of the gluon-clusters obey power-laws, and that the exponents depend very little on the variables $\beta$ and $Q^2$. We interpreted the power-law behaviors as the fingerprints of SOC in the formation processes of such clusters. Can such approximately independence (or weak dependence) of the exponents on $Q^2$ and $\beta$ be understood in a physical picture based on SOC? In particular, what do we expect to see in photoproduction processes where the associated value for $Q^2$ is zero?

In order to answer these questions, let us recall the space-time aspects of the collision processes which are closely related to the above-mentioned power-law behaviors. Viewed in a fast moving frame (e.g. the c.m.s. of the colliding electron and proton), the states of the interacting soft gluons originating from the proton are self-organized. The colorless gluon-clusters caused by local perturbations and developed through “domino effects” are BTW-clusters. That is, they are avalanches (see Sections I and V), the size-distribution of which [see Eqs. (8) and (1)] are given by Fig.3. This explicitly shows that there are gluon-clusters of all sizes, because a power-law size-distribution implies that there is no scale in size. Recall that, since such clusters are color-singlets, their spatial extensions can be much larger than that of the proton, and thus they can be “seen” also outside the proton by a virtual photon originating from the electron. In other words, what the virtual photon encounters is a cloud of colorless gluon-clusters spatially extended in- and outside the proton.

The virtual photon, when it encounters a colorless gluon-cluster, will be absorbed by
the charged constituents (quarks and antiquarks due to fluctuation of the gluons) of the gluon-system. Here it is useful to recall that in such a space-time picture, $Q^2$ is inversely proportional to the transverse size, and $x_B$ is a measure of the interaction time [See Eqs. (10) and (11) in Section VI] of the virtual photon. It is conceivable, that the values for the cross-sections for virtual photons (associated with a given $Q^2$ and a given $x_B$) to collide with gluon-clusters (of a given size and a given lifetime) may depend on these variables. But, since the processes of self-organization (which produce such gluon-clusters) take place independent of the virtual photon (which originates from the incident electron and enters “the cloud” to look for suitable partners), the power-law behaviors of the size- and lifetime-distributions of the gluon-clusters are expected to be independent of the properties associated with the virtual photon. This means, by using $\gamma^*$’s associated with different values of $Q^2$ to detect clusters of various sizes, we are moving up or down on the straight lines in the log-log plots for the size- and lifetime distributions, the slopes of which do not change. In other words, the approximative $Q^2$-independence of the slope is a natural consequence of the SOC picture.

As far as the $\beta$-dependence is concerned, we recall the results obtained in Sections III and IV, which explicitly show the following: The gluon-clusters ($c_0^*$’s) can not be considered as hadrons. In particular, it is neither possible nor meaningful to talk about “the electromagnetic structure of the gluon-cluster”. This suggests, by studying the $\beta$-dependence of the “diffractive structure functions” we cannot expect to gain further information about the structure of the gluon-clusters or further insight about the reaction mechanisms.

Having seen these, we try to look for measurable quantities in which the integrations over $\beta$ have already been carried out. A suitable candidate for this purpose is the differential cross-section

$$\frac{d^2\sigma}{dQ^2 dx_P} = \int d\beta \frac{4\pi \alpha^2}{\beta Q^4} \left( 1 - \frac{y^2}{2} \right) \frac{F_2^{D(3)}(\beta, Q^2; x_P)}{x_P} \approx \int d\beta \frac{4\pi \alpha^2}{\beta Q^4} \left( 1 - \frac{y^2}{2} \right) D_S(x_P|\beta, Q^2)$$

(15)

Together with Eqs.(3) and (8), we see that this cross-section is nothing else but the effective $\beta$-weighted $x_P$-distribution $D_S(x_P|Q^2, \beta)$ of the gluon-clusters. Note that the weighting
factors shown on the right-hand-side of Eq.(15) are simply results of QED! Next, we use the data for $F_2^{D(3)}$ which are available at present, to do a log-log plot for the integrand of the expression in Eq.(15) as a function of $x_P$ for different values of $\beta$ and $Q^2$. This is shown in Fig.6a. Since the absolute values of this quantity depend very much, but the slope of the curves very little on $\beta$, we carry out the integration as follows: We first fit every set of the data separately. Having obtained the slopes and the intersection points, we use the obtained fits to perform the integration over $\beta$. The results are shown in the

$$\log \left( \frac{1}{x_P} \frac{d^2\sigma^D}{dQ^2 dx_P} \right) \text{ versus } \log (x_P)$$

plots of Fig.6b. These results show the $Q^2$-dependence of the slopes is practically negligible, and that the slope is approximately $-1.95$ for all values of $Q^2$.

Furthermore, in order to see whether the quantity introduced in Eq.(15) is indeed useful, and in order to perform a decisive test of the $Q^2$-independence of the slope in the power-law behavior of the above-mentioned size-distributions, we now compare the results in deep-inelastic scattering with those obtained in photoproduction, where LRG events have also be observed. This means, as in diffractive deep-inelastic scattering, we again associate the observed effects with colorless objects which are interpreted as system of interacting soft gluons originating from the proton. In order to find out whether it is the same kind of gluon-clusters as in deep-inelastic scattering, and whether they “look” very much different when we probe them with real ($Q^2 = 0$) photons, we replot the existing $d\sigma/dM_X^2$ data for photoproduction experiments performed at different total energies, and note the kinematical relationship between $M_X^2$, $W^2$ and $x_P$ for $Q^2 \ll M^2$ and $|t| \ll M_X^2$:

$$x_P \approx \frac{M_X^2 + t}{W^2 - M^2} \approx \frac{M_X^2}{W^2}$$  \hspace{1cm} (16)

The result of the corresponding

$$\log \left( \frac{1}{x_P} \frac{d\sigma}{dx_P} \right) \text{ versus } \log (x_P)$$

plot is shown in Fig.7. The slope obtained from a least-square fit to the existing data is $-1.98 \pm 0.07$. 

21
The results obtained in diffractive deep-inelastic electron-proton scattering and that for
diffractive photoproduction strongly suggest the following: The formation processes of gluon-
clusters in the proton is due to self-organized criticality, and thus the spatial distributions
of such clusters — represented by the $x_p$-distribution — obey power-laws. The exponents
of such power-laws are independent of $Q^2$. Since $1/Q^2$ can be interpreted, in a geometrical
picture, as a measure for the transverse size of the incident virtual photon, the observed
$Q^2$-independence of the exponents can be considered as further evidence for SOC — in the
sense that the self-organized gluon-cluster formation processes take place independent of the
virtual photon (which is “sent in” to detect the clusters).

VIII Concluding remarks

The existence of large rapidity gap (LRG) events in deep-inelastic electron-proton scatter-
ing is one of the most striking features, if not the most striking feature of the experimental
data obtained in the small-$x_B$ ($x_B < 10^{-2}$, say) region. Taken together with the empirical
facts that gluons dominate in this kinematical region and that their interactions are not
negligible, it seems quite natural to think, that such events are due to collisions between the
virtual photons originated from the lepton and colorless gluon-systems originating from the
proton.

What we propose in the present paper is a statistical approach to study such colorless
 gluon-systems. The reasons, why we think such an approach is useful, can be summarized
as follows:

First, a number of theoretical arguments and experimental indications suggest that such
a system of interacting soft-gluons is a system with the following properties: (a) It is a
complex system with many degrees of freedom, because in general it has a large — unknown
— number of gluons. (b) It is an open system. This is because the members of a colorless
gluon-system may interact (through emission and/or absorption of soft gluons) not only
with one another, but also with gluons and/or quarks and antiquarks outside the system.
Thus, due to such interactions, neither the gluon-number nor the energy of this system can remain constant. (c) It is neither in chemical nor in thermal equilibrium. This is because, as we can for example see in the analysis shown in Section III, it is not possible to consider the colorless gluon-cluster \( c_0 \) as a hadron-like object which has a given structure. In this sense, we are forced to consider it as a dynamical system — probably very far from thermal and chemical equilibria. (d) The basic interactions between the members of the system, as well as those between a member and quarks or gluons outside the system, are local. In fact, they are explicitly given by the well-known QCD-Lagrangian. But, as it is often the case in complex systems, whether the local dynamical details or the general global features of the system plays a more significant role is a different question.

Second, it has been proposed by Bak, Tang and Wiesenfeld\(^9\),\(^10\) some time ago, that a wide class of open dynamical complex systems far from equilibrium may evolve in a self-organized manner to critical states, which give rise to spatial and temporal power-law scaling behaviors. Such scaling behaviors are universal and robust, in fact they can be considered as the “fingerprints” of self-organized criticality (SOC). In the macroscopic world, there are many open dynamical complex systems which show this kind of scaling behavior.\(^9\),\(^10\). Under the condition (see above) that the colorless system of interacting gluons can indeed be considered as an open, dynamical, complex system, it would be of considerable interest to see whether there can be self-organized criticality also in the microscopic world — at the level of gluons and quarks.

Third, by using the existing data for deep inelastic electron-proton scattering\(^1\) and those for photoproduction\(^2\), where colorless systems of interacting soft-gluons are expected to play a dominating role, we checked the above-mentioned fingerprints. The obtained results show that the above-mentioned characteristic features for SOC indeed exist. Furthermore, it is seen that the relevant exponents in such power-laws are the same for different reactions. The existence of SOC in systems of interacting soft gluons in such reactions has a number of consequences. It seems worthwhile to study them in more detail. In particular, it would be very helpful to build realistic models and/or cellular automata to do quantitative
Fourth, based on the obtained results in particular the validity of the power-law behaviors, the physical picture for a colorless gluon-cluster should be as follows: It is not a hadron with a given structure. It has neither a typical size, nor a typical lifetime, and its structure is changing all the time. In fact, it has much in common with an earthquake or an avalanche (mentioned in more detail in Sections II, IV and V). Can we learn more about these objects by studying other reactions? Can we use the same concepts and methods to treat hadron-hadron and hadron-nucleus collision processes? It is known that “the exchange of colorless objects” plays an important role also in diffractive hadron-hadron collisions. Shall we see this kind of power-law behaviors also in diffractive inelastic hadron-hadron scattering processes? Studies along this line are in progress. The results will be published elsewhere, when they are ready.

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FIGURES

Fig. 1. The well-known Feynman diagrams (a) for diffractive and (b) for normal deep-inelastic electron-proton scattering are shown together with the relevant kinematical variables which describe such processes.

Fig. 2. \( F_2^{D(3)}(\beta, Q^2; x_P)/\beta \) is plotted as a function of \( \beta \) for given \( x_P \)-intervals and for fixed \( Q^2 \)-values. The data are taken from Ref. [3]. The lines are only to guide the eye.

Fig. 3. \( F_2^{D(3)}(\beta, Q^2; x_P)/x_P \) is plotted as a function of \( x_P \) for different values of \( \beta \) and \( Q^2 \). The data are taken from Ref. [3].

Fig. 4. \( F_2^{D(3)}(\beta, Q^2; x_B)/\beta \) is plotted as a function of \( x_B \) in the indicated \( \beta \)- and \( Q^2 \)-ranges. The data are taken from Ref. [3].

Fig. 5. \( F_2^{D(3)}(\beta, Q^2; x_B/\beta)/(\beta x_B) \) is plotted as a function of \( x_B \) for fixed \( \beta \)- and \( Q^2 \)-values. The data are taken from Ref. [3].

Fig. 6. Figure 6: (a) \((1/x_P) d^3\sigma^D/d\beta dQ^2 dx_P\) is plotted as a function of \( x_P \) in different bins of \( \beta \) and \( Q^2 \). The data are taken from Ref. [3]. (b) \((1/x_P)d^2\sigma^D/dQ^2 dx_P\) is plotted as a function of \( x_P \) in different bins of \( Q^2 \). The data are taken from Ref. [3].

Fig. 7. Figure 7: \((1/x_P)d\sigma/dx_P\) for photoproduction \( \gamma + p \to X + p \) is plotted as a function of \( x_P \). The data are taken from Ref. [16]. Note that the data in the second paper are given in terms of relative cross sections. Note also that the slopes of the straight-lines are the same. The two dashed lines indicate the lower and the upper limits of the results obtained by multiplying the lower solid line by \( \sigma_{tot} = 154 \pm 16(\text{stat.}) \pm 32(\text{syst.}) \mu b \). This value is taken from the third paper in Ref. [16].
Fig. 1 (a)

\[ q = k - k' \]
\[ P_x(M_x) \]
\[ P_y(M_y) \]

\[ Q^2 = -q^2 \]
\[ x_B = \frac{-q^2}{2Pq} \]
\[ y = \frac{qP}{kP} \]
\[ W^2 = (q+P)^2 \]
\[ t = -q_c^2 \]
\[ \beta = \frac{-q^2}{2q_cq} \]
\[ x_P = \frac{qq_c}{qP} = \frac{x_B}{\beta} \]
\[ q = k - k' \]
\[ W^2 = (q + P)^2 \]
\[ Q^2 = -q^2 \]
\[ x_B = \frac{-q^2}{2Pq} \]
\[ y = \frac{qP}{kP} \]
\[ F_2^{D(3)}(\beta, Q^2, x_p)/\beta \]

- \( Q^2 = 3.5 \text{ GeV}^2 \)
- \( Q^2 = 20 \text{ GeV}^2 \)
- \( Q^2 = 65 \text{ GeV}^2 \)

- \( 10^{-4} < x_p < 0.05 \)
- \( 10^{-3} < x_p < 10^{-2} \)
- \( 10^{-2} < x_p < 0.05 \)
- \( 10^{3.75} < x_p < 10^{3.50} \)
- \( 10^{3.50} < x_p < 10^{3.00} \)
- \( 10^{3.00} < x_p < 10^{2.75} \)
- \( 10^{2.75} < x_p < 10^{2.50} \)
- \( 10^{2.50} < x_p < 10^{2.25} \)
- \( 10^{2.25} < x_p < 10^{1.75} \)

\( \beta \)
Fig. 3
Fig. 4
$F_2(D(3))(\beta, Q^2, x_p = x_B) / (\beta x_B)$

Fig. 5
Slope: -1.954 ± 0.12

Fig. 6a
Fits to the data shown in Fig.6a integrated over $\beta$ for fixed $Q^2$
Slope: -1.954

Fig. 6b
Fig. 7

**H1 data**
- $<W> = 187$ GeV, $M_Y < 1.6$ GeV
- $<W> = 187$ GeV, $M_Y > 1.6$ GeV
- $<W> = 231$ GeV, $M_Y < 1.6$ GeV
- $<W> = 231$ GeV, $M_Y > 1.6$ GeV

**ZEUS data** (normalized to $\sigma_{tot}$)
- $<W> = 200$ GeV

$\frac{1}{x_P} \frac{d\sigma}{dx_P}$

$\text{power}=1.98 \pm 0.07$

$x_P = \frac{M_X^2}{W^2}$