The Impact of Foregrounds on Redshift Space Distortion Measurements With the Highly-Redshifted 21 cm Line

Jonathan C. Pober\textsuperscript{1,2}

\textsuperscript{1}Physics Department, University of Washington, Seattle, WA
\textsuperscript{2}National Science Foundation Astronomy and Astrophysics Postdoctoral Fellow

11 November 2014

ABSTRACT

The highly redshifted 21 cm line of neutral hydrogen has become recognized as a unique probe of cosmology from relatively low redshifts ($z \sim 1$) up through the Epoch of Reionization ($z \sim 8$) and even beyond. To date, most work has focused on recovering the spherically averaged power spectrum of the 21 cm signal, since this approach maximizes the signal-to-noise in the initial measurement. However, like galaxy surveys, the 21 cm signal is effected by redshift space distortion effects, and is inherently anisotropic between the line-of-sight and transverse directions. A full measurement of this anisotropy can yield unique cosmological information, potentially even isolating the matter power spectrum from astrophysical effects at high redshifts. However, foregrounds also have an anisotropic footprint between the line-of-sight and transverse directions: the so-called foreground “wedge”. Although techniques to subtract foregrounds are actively being developed, a “foreground avoidance” approach of simply ignoring contaminated modes has arguably proven most successful to date. In this work, we analyze the effect of this foreground anisotropy in recovering the redshift space distortion signature in 21 cm measurements using a foreground avoidance approach at both high and intermediate redshifts. We find the foreground wedge corrupts nearly all of the redshift space signal for even the largest proposed EoR experiments (HERA and the SKA), making cosmological information unrecoverable without foreground subtraction. The situation is somewhat improved at lower redshifts, where the redshift-dependent mapping from observed coordinates to cosmological coordinates significantly reduces the size of the wedge. Using only foreground avoidance, we find that a large experiment like CHIME can place non-trivial constraints on cosmological parameters.

Key words: dark ages, reionization, first stars — large scale structure of the Universe — cosmological parameters — techniques: interferometric

1 INTRODUCTION

The highly-redshifted 21 cm line of neutral hydrogen has become recognized as a unique probe of cosmology and astrophysics. Due to its small optical depth, 21 cm line observations are sensitive to emission from neutral hydrogen over nearly all of cosmic history. Depending on the frequencies at which observations are conducted, the 21 cm line has the potential to offer insight into the nature of dark energy at late times (cosmic redshift $z \sim 1 - 3$), the formation of the first galaxies during the Epoch of Reionization ($z \sim 6 - 13$), the birth of the first stars during “cosmic dawn” ($z \sim 15 - 30$), and possibly even the physics of inflation and the early universe through observations of primordial fluctuations during the cosmic “dark ages” ($z \sim 30 - 200$). For reviews of the 21 cm cosmology technique and the associated science drivers, see Furlanetto, Oh & Briggs (2006), Morales & Wyithe (2010), Pritchard & Loeb (2012) and Zaroubi (2013).

High-redshift 21 cm observations are not without their challenges. The combination of an inherently faint cosmological signal with extremely bright astrophysical foregrounds necessitates a dynamic range that has never been achieved with radio telescopes. In the search for fluctuations in the 21 cm signal during the Epoch of Reionization (as opposed to “global” experiments targeting the mean signal evolution), several experiments are currently conducting long observing campaigns, such as the LOw Frequency ARray (LOFAR; Yatawatta et al. 2013) and the

\textsuperscript{1}http://www.lofar.org
While this picture is certainly complicated by non-linear effects, the prospect of recovering the primordial density

power spectrum at \( z \sim 10 \) motivates continued efforts to understand and eventually recover the signal.

21 cm redshift space distortions are less studied during the \( z \sim 1 - 3 \) epoch, but since the 21 cm signal is expected to closely trace the matter power spectrum during this period, the results from studies of galaxy surveys are generally applicable (e.g. Fisher et al. 1994, Heavens & Taylor 1995, Pápai & Szapudi 2008 and Percival & White 2009). Measurements of redshift space distortions at these moderate redshifts probe the history of cosmic structure formation and have the potential to distinguish dark energy models from modified gravity theories (Song & Percival 2009).

Measurements of redshift space distortions with the 21 cm line therefore offer unique cosmological information. Several studies have found that such measurements are not beyond the realm of possibility for next-generation and even current 21 cm experiments (Mao et al. 2008; Jensen et al. 2013). To date, however, the anisotropy of foreground emission between the transverse and line of sight directions has not been considered in conjunction with redshift space distortion measurements. While foreground emission does not actually “live” in the cosmological Fourier space where the power spectrum is measured, the same analysis used to convert redshifted 21 cm line frequencies to cosmological spectral (and therefore spherical) Fourier modes is contaminated by foregrounds. This mapping is complicated by the “mode-mixing” effects of the interstellar hydrogen emission, the effect of the wedge is to limit the range of \( \mu \) that can be measured by 21 cm experiments, potentially spoiling their ability to measure the redshift space signatures of interest. The goal of this paper is to therefore determine the impact of the wedge on potential redshift space distortion measurements with 21 cm experiments at both high \( z \sim 6 - 10 \) and moderate \( z \sim 1 - 3 \) measurements.

The structure of this paper is as follows. We first consider Epoch of Reionization redshifts (\( z \sim 6 - 10 \)) in §2. Specifically, we describe the redshift space distortion signature of the 21 cm signal and its power spectrum (Bharadwaj & Srikant 2004, Barkana & Loeb 2005, McQuinn et al. 2006, Lidz et al. 2007, Mao et al. 2008, 2012, Majumdar, Bharadwaj & Choudhury 2013, Jensen et al. 2013). One of the most exciting results of these studies is that the contributions of various components to the 21 cm power spectrum during the Epoch of Reionization — the detailed shape of which is determined by a complicated combination of ionization fluctuations, density fluctuations, and their cross-correlations — may be separable using the distinct redshift space signatures of each term. While this picture is certainly complicated by non-linear effects, the prospect of recovering the primordial density

power spectrum at \( z \sim 10 \) motivates continued efforts to understand and eventually recover the signal.

21 cm redshift space distortions are less studied during the \( z \sim 1 - 3 \) epoch, but since the 21 cm signal is expected to closely trace the matter power spectrum during this period, the results from studies of galaxy surveys are generally applicable (e.g. Fisher et al. 1994, Heavens & Taylor 1995, Pápai & Szapudi 2008 and Percival & White 2009). Measurements of redshift space distortions at these moderate redshifts probe the history of cosmic structure formation and have the potential to distinguish dark energy models from modified gravity theories (Song & Percival 2009).

Measurements of redshift space distortions with the 21 cm line therefore offer unique cosmological information. Several studies have found that such measurements are not beyond the realm of possibility for next-generation and even current 21 cm experiments (Mao et al. 2008; Jensen et al. 2013). To date, however, the anisotropy of foreground emission between the transverse and line of sight directions has not been considered in conjunction with redshift space distortion measurements. While foreground emission does not actually “live” in the cosmological Fourier space where the power spectrum is measured, the same analysis used to convert redshifted 21 cm line frequencies to cosmological distances provides a methodology for determining which Fourier modes are contaminated by foregrounds. This mapping is complicated by the “mode-mixing” effects of the interstellar hydrogen emission, the effect of the wedge is to limit the range of \( \mu \) that can be measured by 21 cm experiments, potentially spoiling their ability to measure the redshift space signatures of interest. The goal of this paper is to therefore determine the impact of the wedge on potential redshift space distortion measurements with 21 cm experiments at both high \( z \sim 6 - 10 \) and moderate \( z \sim 1 - 3 \) measurements.

The structure of this paper is as follows. We first consider Epoch of Reionization redshifts (\( z \sim 6 - 10 \)) in §2. Specifically, we describe the redshift space distortion signature of the 21 cm signal and its power spectrum (Bharadwaj & Srikant 2004, Barkana & Loeb 2005, McQuinn et al. 2006, Lidz et al. 2007, Mao et al. 2008, 2012, Majumdar, Bharadwaj & Choudhury 2013, Jensen et al. 2013). One of the most exciting results of these studies is that the contributions of various components to the 21 cm power spectrum during the Epoch of Reionization — the detailed shape of which is determined by a complicated combination of ionization fluctuations, density fluctuations, and their cross-correlations — may be separable using the distinct redshift space signatures of each term. While this picture is certainly complicated by non-linear effects, the prospect of recovering the primordial density

power spectrum at \( z \sim 10 \) motivates continued efforts to understand and eventually recover the signal.
wise stated, all calculations assume a closed ΛCDM universe with $\Omega_\text{m} = 0.27$, $\Omega_\Lambda = 0.73$, and $h = 0.7$.

2 EPOCH OF REIONIZATION MEASUREMENTS

2.1 The Redshift Space Distortion Signal

During the epoch of reionization, the 21 cm power spectrum receives contributions from both density and ionization fluctuations. Generally, the ionization fluctuations dominate the total power, but they are unaffected by redshift space effects because only the density perturbations source gravitational infall. Therefore, the density fluctuation power spectrum will vary with $\mu$, while the ionization power spectrum will remain isotropic. Barkana & Loeb (2005) propose using this distinct angular dependence to separate the two components (and their cross-correlation) to potentially measure the density power spectrum alone. Mao et al. (2012) present a “quasi-linear” extension (like Barkana & Loeb (2005), they use linear theory for treating the density and velocity fluctuations, but include non-linear effects in their calculations of ionization fluctuations) to this analysis. In their approximation, the power spectrum of 21 cm brightness fluctuations can be written as:

$$P_{21}(k, \mu) = P_{\mu^0}(k) + P_{\mu^2}(k)\mu^2 + P_{\mu^4}(k)\mu^4,$$

where the three moments are:

$$P_{\mu^0} = \delta T_b^2 P_{\delta_{\text{HI}}\delta_{\text{HI}}}(k),$$

$$P_{\mu^2} = 2\delta T_b^2 P_{\delta_{\text{HI}}\delta_{\text{HI}}}(k),$$

$$P_{\mu^4} = 3\delta T_b^2 P_{\delta_{\text{HI}}\delta_{\text{HI}}}(k),$$

where $\delta T_b$ is the mean brightness temperature of the 21 cm signal relative to the CMB, and $\delta_{\text{HI}}$ and $\delta_{\text{HI}}$ are the fractional overdensities of neutral hydrogen and ionized hydrogen relative to the cosmic mean, respectively.

To produce a simulated signal for our calculations, we use the publicly available 21cmFAST\(^9\) version 1.01 code (Mesinger & Furlanetto 2007). 21cmFAST is a semi-numerical code that provides three dimensional simulations of the 21 cm signal over relatively large volumes (400 Mpc in the simulations used here). We use all the fiducial values of the 21 cm code (see Mesinger, Furlanetto & Cen 2011 for a description of the relevant parameters) and assume that $T_{\text{spin}} \gg T_{\text{CMB}}$ for the entirety of the simulation. Rather than use the simulated 21 cm brightness temperature cubes, we use the separate ionization and density fluctuation output cubes to construct a $P(k, \mu)$ power spectrum using the quasi-linear approximation of Equation (1). As several authors have found, this quasi-linear formula only provides a good approximation to the 21 cm power spectrum at relatively high neutral fractions ($x_{\text{HI}} \gtrsim 0.3$). We use a fiducial 21 cm power spectrum with an neutral fraction of $x_{\text{HI}}$ at $z \sim 0.5$.

We have confirmed that at this neutral fraction, the quasi-linear approximation produces a 21 cm power spectrum that is $\sim 25\%$ higher than the full non-linear calculation done by 21cmFAST. Since our predicted sensitivities in Figures 2-3 are quite poor, using the quasi-linear approximation has the effect of being a conservative error and has little effect on our conclusions.

Figure 1 shows a spherically averaged version of our fiducial model constructed using the quasi-linear approximation of Equation (1) (solid blue). The individual components are shown as thinner lines. The combined quasi-linear power spectrum is less than that of the ionization power spectrum because the correlation between the ionization and density fields, $P_{\delta_{\text{HI}}\delta_{\text{HI}}}(k)$, (dashed cyan line) is negative; its absolute value is plotted here. This model was produced with simulations from 21cmFAST.

Figure 1. The spherically averaged dimensionless power spectrum of our fiducial model constructed using the quasi-linear approximation of Equation (1) (solid blue). The individual components are shown as thinner lines. The combined quasi-linear power spectrum is less than that of the ionization power spectrum because the correlation between the ionization and density fields, $P_{\delta_{\text{HI}}\delta_{\text{HI}}}(k)$, (dashed cyan line) is negative; its absolute value is plotted here. This model was produced with simulations from 21cmFAST.

2.2 The Foreground Footprint

To model the effects of foregrounds in $(k_1, k_2)$ space, we use the approach of Pober et al. (2013b) and Pober et al. (2014), where modes which fall inside the "wedge" are deemed contaminated and considered as if they were not measured. We consider the wedge as extending to the horizon limit (Parsons et al. 2012b), but do not exclude any further modes outside the horizon. The mapping of the horizon limit for a given baseline (which is equivalent to the maximum delay between the two antennas measured in light-travel time) to cosmological $k_1$ modes is given in Parsons et al. (2012b) and
where $\nu$ is observing frequency, and $X$ and $Y$ are cosmological scalars for converting observed bandwidths and solid angles to $h \text{Mpc}^{-1}$, respectively, defined in Parsons et al. (2012a) and Furlanetto, Oh & Briggs (2006). The cosmological scalars $X$ and $Y$ are of particular importance here; they depend on the angular diameter distance and Hubble constant at the redshift of the measurement, and can change significantly between the EoR experiments and the intensity mapping experiments discussed in [4]. At the redshift of our fiducial power spectrum $z = 9.5$ the horizon limit gives a slope of $k_{\text{hor}} = 3.73$.

The steepness of this slope is evident in Figure 2 which reproduces the wedge obtained from PAPER observations in [5]. Unlike Pober et al. (2013b), here we plot the $k_{\parallel}$ and $k_{\perp}$ axes on the same scale to highlight another important feature of EoR 21 cm experiments: a mismatch between $k_{\parallel}$ and $k_{\perp}$ scales. The $k_{\perp}$ axis extends to $k_{\perp} = 0.12 h \text{Mpc}^{-1}$, which corresponds to the longest baseline in the PAPER array of $\sim 300$ m. The $k_{\parallel}$ axis, however, is truncated; PAPER has a frequency resolution of 48.8 kHz, corresponding to a maximum $k_{\parallel}$ of $10.4 h \text{Mpc}^{-1}$. The white lines show contours of constant $|k|$; even ignoring the wedge, a full range of $|k|$ is measured for only very small values of $|k|$. If modes within the wedge are considered contaminated, this loss of modes serves to set a minimum $\mu$ below which modes cannot be measured. For EoR observations at $z = 9.5$, this value is $\mu_{\text{min}} = 0.966$. This large value of $\mu_{\text{min}}$ is quite discouraging for EoR experiments looking to measure the effects of redshift space distortions: only with foreground removal working well into the wedge can a reasonable range of $\mu$ be recovered.

### 2.3 Sensitivity Calculations

Despite the discouragingly large value of $\mu_{\text{min}}$ when modes inside the wedge are excluded, there is still a small hope for making a redshift space distortion measurement using only a foreground avoidance approach. As Equation 1 shows, the density power spectrum enters as $\mu^4$. Therefore, even though the measurable range of $\mu$ is extremely limited, it is at high values of $\mu$, where the signal is changing most rapidly. It is therefore conceivable that for even measuring only modes outside the wedge, an EoR experiment could pick out the component of the power spectrum with a $\mu^4$ dependence with reasonable significance.

In this section, we look at the potential for 21 cm experiments to detect redshift space distortion effects in our fiducial power spectrum. To do so, we use a version of the 21cmSense code [10] (Pober et al. 2013b, 2014) which has been modified to retain 2D $(k_{\perp}, k_{\parallel})$ information (as opposed to the standard package, which performs a spherical average to 1D). We then fit a quartic polynomial in the form of Equation 1 to $P(k, \mu)$ for bins of constant $|k|$. The constant term therefore recovers the isotropic ionization power spectrum at $|k|$, the quadratic term the ionization-density cross power spectrum, and the quartic term the density power spectrum. Given the difficult nature of the measurement, we consider two proposed next-generation experiments: HERA (Pober et al. 2014) and the core of Phase 1 of the SKA-Lo (following the design specifications of Dewdney et al. 2013). We summarize the properties of these arrays in Table 1. We consider observations spanning 1080 hours using the observing strategy described in [11]. Figure 3 plots the power spectrum as a function of $\mu$ in one annulus of $|k| = 0.18 h \text{Mpc}^{-1}$. The green curve shows the value of the isotropic real-space power spectrum, while the blue curves show the effect of redshift space distortions as decreasing the power at high $\mu$, the result of the negative sign of the density-ionization fluctuation cross power spectrum. At the $\mu$ values probed by EoR experiments, the decrease in power is of order a factor of 2. A more optimistic

\[ k_{\text{hor}} = \left( \frac{1}{\sqrt{X}} \right) k_{\perp}, \]
Table 1. 21 cm Epoch of Reionization Experiment Properties

| Instrument | Number of Elements | Element Size (m²) | Collecting Area (m²) | Configuration |
|------------|-------------------|------------------|----------------------|---------------|
| HERA       | 547               | 154              | 84,238               | Filled 200 m hexagon |
| SKA1-Low   | 866               | 962              | 833,190              | Filled 270 m core with Gaussian distribution beyond |

Table 2. First three columns: Detection significance (i.e. “number of sigmas”) for each of the three $\mu$ moments of the 21 cm power spectrum. Right hand column: Detection significance of the spherically averaged power spectrum. Despite very high SNR spherically averaged power spectrum measurements, the $\mu$ dependence cannot be recovered with any significance.

| Instrument | Constant | Quadratic | Quartic | Spherically Avg. |
|------------|----------|-----------|---------|------------------|
| HERA       | 0.07     | 0.03      | 0.02    | 108.1 |
| SKA        | 0.33     | 0.16      | 0.09    | 95.6  |

Figure 3. Potential measurements of HERA (left) and the SKA (right) of the 21 cm power spectrum as a function of $\mu$ for $|k| = 0.18 \, \text{hMpc}^{-1}$. The blue line shows our fiducial 21 cm power spectrum including redshift space distortion effects, while the green dashed line contains only isotropic monopole term. No binning of the measurements has been performed; their spacing is set by the range of $k_{\perp}$ and $k_{\parallel}$ values probed by the instruments. Only one value of $|k|$ is plotted, but the results are generic for all $|k|$s: the foreground wedge limits measurements to $\mu > 0.97$, severely hampering attempts to measure the redshift-space distortion signal (i.e. to detect any $\mu$-dependence in the power spectrum).

Table 2 shows the “number of sigmas” at which each experiment constrains the three components (constant, quadratic, and quartic) of the redshift space power spectrum; while Figure 3 shows the measurements only in one $|k|$ bin, these numbers are for the cumulative measurement over all $|k|$s probed. Also listed are the total significance values for measurements of the spherically averaged 1D power spectrum. While the spherically averaged power spectrum is measured with very high significance, it is clear that even the next generation of 21 cm EoR experiments will not make significant measurements of the redshift space distortion effects without a foreground subtraction technique that allows recovery of modes well inside the wedge. It should also be noted that without a full range of $\mu$ measurements, it will be difficult to separate the decrement of the power spectrum amplitude at high $\mu$ from the isotropic amplitude. This error will potentially bias the interpretation of initial spherically averaged power spectrum measurements, but attempting to quantify this effect is outside the scope of this present work.

3 INTENSITY MAPPING EXPERIMENTS

3.1 The Redshift Space Distortion Signal

The redshift space distortion signal is relatively simpler at the $z \sim 1 – 3$ measurements of intensity mapping experiments. At these redshifts, all the neutral hydrogen resides in self-shielded halos, which trace the matter power spectrum. Since the entirety of the 21 cm signal comes from only density fluctuations, there are no cross-correlation terms between different components. Furthermore, since 21 cm experiments are probing mainly large scales that have not gone non-linear these redshifts, we use the Kaiser approximation to model redshift space distortions (Kaiser 1987):

$$P(k, \mu) = (1 + \beta \mu^2) P(k),$$

where $\beta \equiv f/b$ (where $f$ is the logarithmic growth rate of structure $f \equiv d \ln D/d \ln a$ and $b$ is the bias of neutral hydrogen containing halos). A principal goal of redshift space distortions measurements at these redshifts is to constrain $\beta$, which has been measured to be $\approx \Omega_m^{0.6}$. (Fisher et al.)
a significantly shallower slope than at EoR redshifts. This power spectrum at \( \mu \) by measuring the respect to the cosmic baryon content, which can be broken late time acceleration in cosmic expansion (Percival & White 1994; Heavens & Taylor 1995; Pápai & Szapudi 2008; Per-

Figure 4. The spherically averaged 21 cm power spectrum at \( z = 1.19 \) including redshift space distortion effects.

1994, 1995, 2008, 2009]. A detailed history of \( \beta \) as a function of redshift can be used to trace the growth of structure over cosmic time and potentially even distinguish between dark energy and modified gravity as the cause of the observed late time acceleration in cosmic expansion (Percival & White 2000; Song & Percival 2009). In 21 cm measurements, there is also a degeneracy in the power spectrum amplitude between \( b \) and \( f_{\text{HI}} \), the mass fraction of neutral hydrogen with respect to the cosmic baryon content, which can be broken by measuring the \( \mu \) dependence of the power spectrum.

For our fiducial signal, we use a simulation of the matter power spectrum at \( z = 1.19 \) from CAMB (Lewis, Challinor & Lasenby 2000) [14] multiplied by a scalar converting the matter power spectrum to 21 cm brightness temperature (Madau, Meiksin & Rees 1997; Barkana & Loeb 2007; Ansari et al. 2012; Pober et al. 2013b):

\[
P(k) = \left( \tilde{T}_{21}(z) \right)^2 b^2 P_{\text{m}}(k),
\]

where \( \tilde{T}_{21}(z) \) is the mean 21 cm brightness temperature at redshift \( z \), \( P_{\text{m}}(k) \) is the matter power spectrum, \( \Omega_A \) is the cosmological constant, and \( \Omega_m \), \( \Omega_B \) are the matter and baryon density in units of the critical density, respectively. Our fiducial value for \( b \) is 1.5 (Chang et al. 2010). We plot a 1D spherical average of our fiducial power spectrum, including the redshift space effects of Equation (6) in Figure 4.

3.2 The Foreground Footprint

Evaluating Equation 5 for \( z = 1.19 \) (650 MHz in the 21 cm line, near the center of band for a number of intensity mapping experiments) gives a horizon slope of \( k_{\text{hor}} = 0.77k_\perp \), a significantly shallower slope than at EoR redshifts. This fact mainly stems from the large evolution in the angular diameter distance and Hubble parameter between \( z \sim 8 \) and \( z \sim 1 \). The corresponding \( \mu_{\text{min}} \) for this horizon slope is 0.61; although a large range of \( \mu \) is still excluded by the wedge, there remain enough high \( \mu \) values that measuring redshift space distortions at \( z \sim 1 \) with a foreground avoidance strategy becomes a feasible proposition.

3.3 Sensitivity Calculations

Following the procedure described in [2,3] we calculate the sensitivities of three intensity mapping array concept designs to redshift space distortion signatures in our fiducial \( z = 1.19 \) power spectrum: a 144-element BAOBAB-like array (Pober et al. 2013b), a 4006-element CHIME-like array (Shaw et al. 2014), and an SKA-Mid concept array meeting the design specifications of Dewdney et al. (2013) (although for simplicity the 64 MeerKAT dishes are assumed to be identical to the 190 SKA dishes).

The configuration details of these arrays are described in Table 3 and their achievable constraints on the redshift space distortion signal are presented in Table 4. For each array, we calculate the sensitivities for a 100 MHz band centered on 650 MHz (\( z = 1.19 \)); most of these experiments have wider bandwidths (e.g. 400 – 800 MHz for CHIME), and so will be able to deliver measurements over a range of redshifts simultaneously.

We see that the relatively smaller BAOBAB-like array cannot make a significant measurement of the redshift space distortion terms, while the CHIME-like instrument can reach moderate significances. The SKA design does poorly, despite its large collecting area; this is because the minimum measurable \( k_\perp \) for the 15 m dish of the SKA is 0.07 hMpc\(^{-1}\). This scale is just beyond the peak of the power spectrum, and the vast majority baselines are significantly longer. Put another way, the SKA Mid design is not tuned for measuring the large-scale structure of the neutral hydrogen power spectrum. Example measurements at \( k = 0.11 \) hMpc\(^{-1}\) are shown for the BAOBAB-like, CHIME-like, and SKA arrays in Figure 5. As noted, the SKA has very few measurements at this small value of \( |k| \). On the other hand, the BAOBAB-like array does not have measurements reaching down to \( \mu_{\text{min}} \) because it does not have enough long baselines to measure \( |k| = 0.11 \) hMpc\(^{-1}\) modes with a significant transverse component. At smaller values of \( |k| \) than plotted, the full accessible range of \( \mu \) is recovered.

In Table 4 we also present the achievable fractional error on \( \beta \), calculated by taking ratios of different power spectrum moments to cancel out uncertainties in other parameters and then propagating theoretical uncertainties. Although more sophisticated analyses techniques now exist for redshift-space distortion measurements (e.g. Percival & White 2009), we present constraints using simple calculations as illustrative of the instrument sensitivities. Presumably, more advanced analyses could be used to improve these measurements. Under this framework, a CHIME-like experiment can yield 15% errors on \( \beta \), comparable to the last generation of galaxy surveys (Percival et al. 2004, Ross et al. 2007), while neither the smaller BAOBAB-like instrument nor the SKA with its very long baselines can make a significant measurement. With significant constraints on \( \beta \), an

14 http://camb.info
Table 3. 21 cm Intensity Mapping Experiment Properties

| Instrument | Number of Elements | Element Size (m²) | Collecting Area (m²) | Configuration                      |
|------------|--------------------|-------------------|----------------------|-----------------------------------|
| BAOBAB     | 144                | 6.25              | 900                  | Filled 12 x 12 square             |
| CHIME      | 4096               | 2.25              | 9,216                | Filled 64 x 64 square             |
| SKA-Mid    | 254                | 176.6             | 44,863               | Random locations with power-law baseline distribution |

Figure 5. Potential measurements of BAOBAB (left), CHIME (center), and the SKA (right) of the 21 cm power spectrum as a function of $\mu$ for $|k| = 0.11$ hMpc$^{-1}$ (BAOBAB and CHIME) and $|k| = 0.27$ hMpc$^{-1}$ (SKA). The different $k$ mode and scale for the SKA results from the fact that it cannot probe the shorter $k = 0.11$ hMpc$^{-1}$ mode at 600 MHz. The blue line shows our fiducial 21 cm power spectrum including redshift space distortion effects, while the green dashed line contains only isotropic monopole term. No binning of the measurements has been performed; their spacing is set by the range of $k_\parallel$ and $k_\perp$ values probed by the instruments. Only one value of $|k|$ is plotted, but the results are generic for all $|k|$s: low values of $\mu$ cannot be measured due to foregrounds, but enough measurements are possible to see the $\mu$ dependence introduced by redshift space distortions. Only CHIME has enough SNR to fit the functional form of the power spectrum and recover cosmological information.

The situation is somewhat more promising for lower redshift “intensity mapping” experiments. While foregrounds still prevent recovery of low $\mu$ modes, the window for making 21 cm measurements is much larger. This difference is due to the large evolution in the Hubble parameter and angular diameter distance between redshifts $z \sim 8$ and $z \sim 1$. These cosmological parameters determine the mapping of angular and frequency values in data to $k_\perp, k_\parallel$ coordinates, with the effect of making the wedge much smaller at lower redshifts. In our calculations at $z = 1.19$ over half of $\mu$ modes still fall within the foreground wedge; however, since the redshift space distortion signals are (to first approximation) quadratic and quartic in $\mu$, larger values of $\mu$ are better for distinguishing the signal from the $\mu$-independent monopole. For a large 21 cm experiment optimized for intensity mapping like CHIME, foreground avoidance will still allow for the recovery of cosmological information from the redshift space signal. Smaller experiments like BAOBAB and less-optimized experiments like the SKA, however, still cannot recover the signal.

While this work has primarily considered a foreground avoidance technique, a “foreground-free” scenario was shown to increase intensity mapping constraints by as much as $\sim 7$. While a no foreground scenario is clearly implausible, further development of foreground removal algorithms should...
allow for additional sensitivity in redshift space distortion measurements with the 21 cm line. Even if the end result of foreground subtraction is only to push the wedge back from the horizon limit, this will have the effect of lowering the minimum measurable $\mu$ mode, and so further open the window on 21 cm redshift space distortion measurements.

**ACKNOWLEDGEMENTS**

JCP is supported by an NSF Astronomy and Astrophysics Fellowship under award AST-1302774. We thank James Aguirre, Daniel Jacobs, Adrian Liu and Matt McQuinn for helpful conversations. The data shown in Figure 2 is the result of the hard work of the entire PAPER collaboration.

**REFERENCES**

Ansari R., Campagne J.-E., Colom P., Magneville C., Martin J.-M., Moniez M., Rich J., Yèche C., 2012, Comptes Rendus Physique, 13, 46

Barkana R., Loeb A., 2005, ApJL, 624, L65

Barkana R., Loeb A., 2007, Reports on Progress in Physics, 70, 627

Bharadwaj S., Srikant P. S., 2004, Journal of Astrophysics and Astronomy, 25, 67

Bowman J. D. et al., 2013, Publications of the Astronomical Society of Australia, 30, 31

Chang T.-C., Pen U.-L., Bandura K., Peterson J. B., 2010, Nature, 466, 463

Datta A., Bowman J. D., Carilli C. L., 2010, ApJ, 724, 526

Dewdney P. E., Turner W., Millenaar R., McCool R., Lazio J., Cornwell T. J., 2013, SKA1 System Baseline Design. Tech. rep., SKA Program Development Office, Document Number: SKA-TEL-SKO-DD-001

Fisher K. B., Davis M., Strauss M. A., Yahil A., Huchra J. P., 1994, MNRAS, 267, 927

Furlanetto S. R., Oh S. P., Briggs F. H., 2006, Phys. Rep., 433, 181

Heavens A. F., Taylor A. N., 1995, MNRAS, 275, 483

Jensen H. et al., 2013, MNRAS, 435, 460

Kaiser N., 1987, MNRAS, 227, 1

Lewis A., Challinor A., Lasenby A., 2000, ApJ, 538, 473

Lidz A., Zahn O., McQuinn M., Zaldarriaga M., Dutta S., Hernquist L., 2007, ApJ, 659, 865

Lonsdale C. J. et al., 2009, IEEE Proceedings, 97, 1497

Madau P., Meiksin A., Rees M. J., 1997, ApJ, 475, 429

Majumdar D., Bharadwaj S., Choudhury T. R., 2013, MNRAS, 434, 1978

Mao Y., Shapiro P. R., Mellem G., Iliiev I. T., Koda J., Ahn K., 2012, MNRAS, 422, 926

Mao Y., Tegmark M., McQuinn M., Zaldarriaga M., Zahn O., 2008, Phys. Rev. D, 78, 023529

McQuinn M., Zahn O., Zaldarriaga M., Hernquist L., Furlanetto S. R., 2006, ApJ, 653, 815

Mellemma G. et al., 2013, Experimental Astronomy, 36, 235

Mesinger A., Furlanetto S., 2007, ApJ, 669, 663

Mesinger A., Furlanetto S., Cen R., 2011, MNRAS, 411, 955

Morales M. F., Hazelton B., Sullivan I., Beardsley A., 2012, ApJ, 752, 137

Morales M. F., Wyithe J. S. B., 2010, ARA&A, 48, 127

Papai P., Szapudi I., 2008, MNRAS, 389, 292

Parsons A., Pober J., McQuinn M., Jacobs D., Aguirre J., 2012a, ApJ, 753, 81

Parsons A. R. et al., 2010, AJ, 139, 1468

Parsons A. R. et al., 2014, ApJ, 788, 106

Parsons A. R., Poer J. C., Aguirre J. E., Carilli C. L., Jacobs D. C., Moore D. F., 2012b, ApJ, 756, 165

Percival W. J. et al., 2004, MNRAS, 353, 1201

Percival W. J., White M., 2009, MNRAS, 393, 297

Pober J. C. et al., 2014, ApJ, 782, 66

Pober J. C. et al., 2013a, ApJL, 768, L36

Pober J. C. et al., 2013b, AJ, 145, 65

Pritchard J. R., Loeb A., 2012, Reports on Progress in Physics, 75, 086901

Ross N. P. et al., 2007, MNRAS, 381, 573

Shaw J. R., Sigurdson K., Pen U.-L., Stebbins A., Sitwell M., 2014, ApJ, 781, 57

Song Y.-S., Percival W. J., 2009, Journal of Cosmology and Astroparticle Physics, 10, 4

Thyagarajan N. et al., 2013, ApJ, 776, 6

Tingay S. J. et al., 2013, Publications of the Astronomical Society of Australia, 30, 7

Trott C. M., Wayth R. B., Tingay S. J., 2012, ApJ, 757, 101

van Haarlem M. P. et al., 2013, A&A, 556, A2

Vedantham H., Udaya Shankar N., Subrahmanyan R., 2012, ApJ, 745, 176

Xu Y., Wang X., Chen X., 2014, ArXiv e-prints

Yatawatta S. et al., 2013, A&A, 550, A136

Zaroubi S., 2013, in Astrophysics and Space Science Library, Vol. 396, Astrophysics and Space Science Library, Wilkind T., Mobasher B., Bromm V., eds., p. 45

Table 4. Detection significance (i.e. “number of sigmas”) for each of the three $\mu$ moments of the 21 cm power spectrum, followed by the total detection significance of the spherically averaged 21 cm power spectrum. The right-hand column shows the achievable fractional error on $\beta$. Only CHIME produces measurements that are of any cosmological significance.

| Instrument | Constant | Quadratic | Quartic | Spherically Avg. | $\beta$ (frac. err.) |
|------------|----------|-----------|---------|-----------------|---------------------|
| BAOBAB     | 0.4      | 0.3       | 0.26    | 23.1            | 1.99                |
| CHIME      | 5.7      | 4.3       | 3.4     | 205.8           | 0.15                |
| SKA        | 1.8      | 1.4       | 1.1     | 65.66           | 0.47                |