Synthesis of the suboptimal control algorithm for electroplating processes under conditions of uncertainty in the range of processed products

D S Solovjev¹², I A Solovjeva² and Yu V Litovka²

¹ Tambov State University named after G.R. Derzhavin, 33, Internacional’naya, Tambov, 392036, Russia
² Tambov State Technical University, 106, Sovetskaya, Tambov, 392000, Russia
* solovjevdenis@mail.ru

Abstract. The paper discusses the main difficulties in finding the optimal control for obtaining high-quality electroplating in a rapidly changing product range. A suboptimal control search algorithm is proposed for solving this problem. This algorithm is reduced to finding the geometrically closest product to the existing ones, for which the optimal solution in the control matrix is known. The results of the suboptimal control search algorithm are given on the example of the zinc coating of a complex shaped product. Recommendations are given to improve the performance of the algorithm and the quality of the found suboptimal control.

1. Introduction
Electroplating coatings are used to protect metal products from corrosion and weathering [1]. Obtaining high-quality coating on the product surface is a prerequisite for achieving a long period of it protection. Uniformity is one of the indirect indicators of electroplated coatings that determine corrosion resistance [2]. Optimization of process parameters for the purpose of subsequent control of electrolysis is used to improve the coating uniformity. The distance between the anode and cathode is selected [3,4], and the shape of the anode is found [5]. Technological modes of electrolysis are calculated, such as: cathode current density [6], temperature [7], acidity [8] and concentration of electrolyte components [9]. It should be noted that mathematical optimization of the process is impossible without a large amount of experimental data and the use of numerical methods for solving partial differential equations, which the process model contains [10]. These circumstances make operational high-quality electroplating processing difficult for a rapidly changing product range. Therefore, electroplating processing becomes the bottleneck of the entire production process. Synthesis of the suboptimal control algorithm is a way out of the current situation.

Synthesis of the suboptimal control algorithm for electroplating processes under uncertainty conditions of the range of processed products is the article purpose.

2. Suboptimal control of electroplating processes under uncertainty conditions
Consider the electroplating process as a control object to solve the problem.

The following set of coordinates is input to the object:
where $S_m$ is the configuration of the electroplating bath (length, width, height); $S_c$ is the surface of the product (cathode); $k$ is the electrochemical equivalent of the coating metal; $\rho$ is the density of the coating metal; $Ra$ is the cathode surface roughness.

Information about the coating thickness $\delta$ and the process duration $T$ is collected from the output of the control object:

$$
\{ \delta, T \} = Y 
$$

The following set of coordinates is the control vector:

$$
\{ L, S_c, d, i_c, t, pH, C_1, C_2, \ldots \} = U
$$

where $L$ is the electrolyte level in the bath; $S_c$ is the surface of the anode; $d$ is the distance between the anode and cathode; $i_c$ is cathode current density; $t$ is the temperature of the electrolyte; $pH$ is acidity of the electrolyte; $C_1, C_2, \ldots$ are the concentrations of electrolyte components.

The problem of search the optimal control is set as follows. For electroplating treatment of the product defined by the vector (1), it is necessary to search the components values of the vector (3), which provide extreme components values of the vector (2).

Let all information about optimal control for $N$ products, which are processed at the enterprise, is collected in the form of (1) - (3) in the control matrix:

$$
\mathbf{M} = \bigcup_{i=1}^{N} U_i : \mathbf{X} \rightarrow \mathbf{Y}_i 
$$

Then the presence of the existing solution (3) for the vector (1) is checked in the generated control matrix (4) for the product received for electroplating processing:

$$
\exists \mathbf{X}_i = \mathbf{X} \Rightarrow \exists U_i : \mathbf{X} \rightarrow \mathbf{Y}_i 
$$

However, a pre-found optimal solution does not exist for a product that is processed for the first time, therefore there is no row in the matrix (4) to satisfy condition (5). In this case, information about previously found optimal controls can be applied in the problem of searching a suboptimal control as follows. Problem (5) using the matrix (4) transforms to the following form:

$$
\exists \mathbf{X}_i - \mathbf{X} \rightarrow 0 = \left( \| U_i - U \| \rightarrow 0 \right) \wedge \left( \| Y_i - Y \| \rightarrow 0 \right) 
$$

The expression (6) means that the closer the components values of the $\mathbf{X}_i$ vector to the components of the $\mathbf{X}$ vector are for the product being processed for the first time, the closer the previously found control $U_i$ and output $Y_i$ vectors to the optimal values $U$ and $Y$, respectively. The solution to problem (6) is to search the closest $\mathbf{X}_i$ vector from control matrix (4) to $\mathbf{X}$ by values.

Surfaces comparison of products with each other is the most difficult moment in searching a solution to problem (6), because all components of vector (1), except for the surface of the product, are numerical values. There is a difference between $S_c$ and $S_{ci}$ from the control matrix (4), since $S_c$ is the surface of the product being processed for the first time:

$$
\Delta S_{\alpha} = |S_c - S_{ci}|. 
$$

Then the solution to the problem of searching a suboptimal control (6) is reduced to searching such $\mathbf{X}_i$ from the control matrix (4), whose value (7) is minimal at the remaining values of the components that coincide with the $\mathbf{X}$ vector:

$$
i^* = \arg \min_{i=1}^{N} \Delta S_{\alpha}.
$$

The value of expression (7) must be calculated when solving problem (8). The product surface is represented as the union of its three projections to solve problem (8):

$$
S_c = pr_{xz}(S_c) \cup pr_{xy}(S_c) \cup pr_{yz}(S_c),
$$

where $pr_{xz}, pr_{xy}, pr_{yz}$ are the projections of the product surface on the $xz, xy, yz$ axis, respectively.

Expression (7) is represented according to expression (9) in the following form:
\[ \Delta S_y = \alpha \cdot K^{\alpha} (\Delta S_y) + \beta \cdot K^{\beta} (\Delta S_y) + \gamma \cdot K^{\gamma} (\Delta S_y), \]  
where \( K^{\alpha}, K^{\beta}, K^{\gamma} \) are the difference coefficients of the projections on the \( xy, xz, yz \) axes, respectively; \( \alpha, \beta, \gamma \) are the weight coefficients.

The difference coefficients in (10) are defined as the ratio of the figure integral obtained as a result of the intersection of the corresponding projections \( S_y \) and \( S'_y \) to the figure integral obtained by union the corresponding projections \( S_y \) and \( S'_y \):

\[
\begin{align*}
K^{\alpha} (\Delta S_y) &= 1 - \frac{I^{\alpha} \cap}{I^{\alpha} \cup}, \\
K^{\beta} (\Delta S_y) &= 1 - \frac{I^{\beta} \cap}{I^{\beta} \cup}, \\
K^{\gamma} (\Delta S_y) &= 1 - \frac{I^{\gamma} \cap}{I^{\gamma} \cup},
\end{align*}
\]  

where \( I^{\alpha}, I^{\beta}, I^{\gamma} \) are the area values of the figure as a result of intersection and union of projections on the corresponding axes.

The weight coefficients are calculated as follows:

\[
\begin{align*}
\alpha + \beta + \gamma &= 1, \\
\alpha, \beta, \gamma &> 0,
\end{align*}
\]  

and allow the decision maker to set the importance of the electroplating processing quality for the corresponding projection of the product \( S_y \) surface.

3. Materials and methods

The work of the search algorithm for the suboptimal control of the electroplating process will be considered in the following example. There is a vector \( \textbf{X} \) input coordinates, which describes the product for zinc coating, with the following component values: \( S_{\text{as}} = 200 \times 300 \times 200 \text{ mm} \); the surface projections of the product \( S_y \) on the \( xy, xz, yz \) axis are shown in figure 1a; \( k = 1.22 \text{ g}/(\text{A} \cdot \text{h}) \); \( \rho = 7.13 \text{ g/cm}^3 \); \( Ra = 5 \text{ μm} \). Suppose that in solving the suboptimal control problem (6) for the numerical coordinates of the vector \( \textbf{X} \), there coincided \( N = 3 \) vectors \( \textbf{X}_1, \textbf{X}_2, \textbf{X}_3 \) from the control matrix (4).

For \( \textbf{X}_1, \textbf{X}_2, \textbf{X}_3 \) the values of the control vectors \( U_1, U_2, U_3 \) and the output coordinates \( Y_1, Y_2, Y_3 \) are known. Vectors \( \textbf{X}_1, \textbf{X}_2, \textbf{X}_3 \) contain product surfaces \( S_{\gamma}, S_{\alpha}, S_{\beta}, S_{\delta}, S_{\epsilon} \), the projections of which on the \( xy, xz, yz \) axes are presented in figure 1b, 1c and 1d, respectively.

The projection areas of products \( S_y, S_{\gamma}, S_{\alpha}, S_{\beta}, S_{\delta}, S_{\epsilon} \) on the \( xy, xz, yz \) axis are given in table 1.

| Product | \( xy \) | \( xz \) | \( yz \) |
|---------|---------|---------|---------|
| \( S_y \) | 1180.3 | 966 | 1341.2 |
| \( S_{\gamma} \) | 2321.4 | 352 | 2321.4 |
| \( S_{\alpha} \) | 2321.4 | 236 | 2321.4 |
| \( S_{\beta} \) | 2360.7 | 236 | 2321.4 |
The calculation of the projection areas of products was carried out using the computer-aided design system Kompas-3D.

4. Results and discussion
The difference coefficients of the surface projections of products \( S_c \), \( S_{c1} \), \( S_{c2} \) and \( S_{c3} \) are calculated according to expression (11). The results are presented in tables 2, 3.

**Table 2.** The calculation results for the areas of intersection and union of the figure projections

| Product | \( I_{xy}^{pr} \), mm² | \( I_{xz}^{pr} \), mm² | \( I_{yz}^{pr} \), mm² | \( I_{xy}^{pr} \), mm² | \( I_{xz}^{pr} \), mm² | \( I_{yz}^{pr} \), mm² |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( S_{c1} \) | 2390.3 | 1140.6 | 1142.6 | 184.7 | 2308.8 | 1339.7 |
| \( S_{c2} \) | 2349.5 | 1131.1 | 1072.3 | 128.7 | 2241.3 | 1339.7 |
| \( S_{c3} \) | 2354.2 | 1179.4 | 1172.8 | 28 | 2308.8 | 1339.7 |

**Figure 1.** Surface projections \( S_c \) (a), \( S_{c1} \) (b), \( S_{c2} \) (c) and \( S_{c3} \) (d) on the \( xy \), \( xz \), \( yz \) axis
Table 3. The calculation results of the difference coefficients

| Difference coefficients | $xy$  | $xz$  | $yz$  |
|-------------------------|-------|-------|-------|
| $K_1$                   | 0.52282 | 0.83835 | 0.41974 |
| $K_2$                   | 0.51857 | 0.87997 | 0.40226 |
| $K_3$                   | 0.49902 | 0.97612 | 0.41974 |

The change dependence in index $i$ of vector $X_i$ as a result of solving a suboptimal control problem for different values of weight coefficients $\alpha, \beta, \gamma$ according to (12) is presented in figure 2.

![Figure 2](image.png)

Data analysis on figure 2 allows us to conclude that vector $X_1$ is a solution to the problem of searching suboptimal control in 56% of cases and corresponds to the following value sets of weight coefficients: $(\alpha > \beta > \gamma) \cap (\alpha > \beta + \gamma)$, $(\beta > \alpha > \gamma) \cap (\beta > \alpha + \gamma)$, $(\gamma > \beta > \alpha) \cap (\gamma > \alpha + \beta)$, $(\beta > \alpha, \gamma) \cap (\beta > 3(\alpha + \gamma))$. Vector $X_2$ is a solution to the problem in 33% of cases and corresponds to the following value sets of weight coefficients: $(\alpha > \gamma > \beta) \cap (\alpha > \beta + \gamma)$, $(\gamma > \alpha > \beta) \cap (\gamma > \alpha + \beta)$, $(\gamma > \alpha, \beta) \cap (\gamma > 3(\alpha + \beta))$. Vector $X_3$ is a solution to the problem in 11% of cases and corresponds to a set of values of weight coefficients $(\alpha > \beta, \gamma) \cap (\alpha > 3(\beta + \gamma))$.

Then the preferences in choosing control vectors as suboptimal for vector $X$ are as follows: $U_1 > U_2 > U_3$.

5. Conclusions
An extensive range of products that are subjected to electroplating processing makes it difficult to search the optimal control using traditional mathematical models in a reasonable time. The search for suboptimal control is a promising direction for solving this problem in terms of the speed of obtaining a solution. The search for suboptimal control in the developed algorithm is reduced to searching the geometrically closest product to the existing ones, for which the optimal solution in the control matrix is known. It should be noted that a restriction may be added to the suboptimal control being searched, for example, each of the product difference coefficients must be no greater than a threshold value. This
restriction will improve the quality of the found suboptimal control. On the one hand, the larger the control matrix, the closer the suboptimal control is to the optimal one. On the other hand, the smaller the control matrix, the faster the suboptimal control is calculated. Consequently, the search for a solution using the brute force method can be quite lengthy. Therefore, it is proposed to cluster the data in the control matrix, and search for suboptimal control only in the corresponding cluster. Since the solutions in the control matrix are independent, it is recommended to use the implementation of the developed algorithm in conjunction with parallel computing to increase search speed.

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