Fractional order $\beta$-Laplace integral transform

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Abstract. We introduce a (new) generalized form of $\beta$-Laplace integral transform, fractional $\beta$-Laplace integral transform, or $\beta$-Laplace integral transform of fractional order. The $\beta$-Laplace integral transform of fractional order can be applied to functions which are fractional differentiable but are not differentiable. After review some literature on fractional analysis based on the modified Riemann-Liouville derivative, we define the fractional order $\beta$-Laplace integral transform, obtain some main properties, convolution property and formula for the inverse of fractional order $\beta$-Laplace integral transform.

1. INTRODUCTION
The classical Laplace transform of a given function $\phi(t)$, which is defined on non-negative real line ($t \geq 0$) is given by

$$\mathcal{L}\{\phi(t)\}(s) = \int_{0}^{\infty} e^{-s\zeta} \phi(\zeta) d\zeta, \quad \Re(s) > 0 \quad (1.1)$$

When it converges. It has numerous applications in the field of applied mathematics and engineering sciences [1–3]. Recently Gaur et. al. [4], has introduced $\beta$-Laplace integral transform which is a new form of exponential kernel type generalization of Laplace transform and this new $\beta$-Laplace integral transform is defined by [4].

$$\mathcal{L}_\beta\{\phi(t)\}(s) = \int_{0}^{\infty} e^{-s\zeta} \phi(\zeta) d\zeta, \quad \beta > 1, \quad \Re(s) > 0. \quad (1.2)$$

and this new generalization shows many special properties over Laplace integral transform [4,5]. Beauty of new generalized transform is that many of the recently introduced exponential kernel type linear integral transforms are particular case of the $\beta$-Laplace integral transform, for detail see [6].

Mittag-Leffler function $E_\alpha(z)$ is given by (one parameter)

$$E_\alpha(s) = \sum_{n=0}^{∞} \frac{s^n}{\Gamma(1+\alpha k)}, \quad s \in \mathbb{C} \quad (1.3)$$

Mittag-Leffler function has great importance because of it is natural generalization of most of the elementary function such as exponential, trigonometric and hyperbolic functions, since
\[ E_1(s) = e^s, \quad E_2((-s)^2) = \cos(s), \quad E_2(s^2) = \cosh(s) \]

Usually, involved function in Eq.(1.2) is continuous and continuously differentiable. But here a question arises what happen when involved function in Eq.(1.2) is a continuous function with a fractional-order derivative but not necessarily has a derivative. Here Two main cases arises:

(i) \( \phi(t) \) has both a continuous and fractional order derivative.
(ii) \( \phi(t) \) has a fractional order derivative but no derivative.

in first case the equation (1.2) is quite mathematically valid but in the second case, equation (1.2) cannot be applied and to counter the problem we need an alternative. To present the alternative of this problem is the main objective of the present paper by introducing a fractional order \( \beta \)-Laplace integral transform.

2. DEFINITIONS AND PREPOSITIONS

2.1. Fractional order difference

**Definition 2.1.** Let \( \phi(t) \) be a continuous function defined on non-negative real numbers, and the forward operator \( E_{(d)} \) is defined by

\[
E_{(d)}\phi(t) = \phi(t + d), \quad d > 0
\]  

then the fractional order difference [7–12] is given by

\[
\Delta^n \phi(t) = (E_{(d)} - 1)^n \phi(t) = \sum_{j=0}^{\infty} (-1)^j \left( \frac{\alpha}{j} \right) \phi[t + (\alpha - j)d], \quad 0 < \alpha < 1
\]  

2.2. Fractional order derivative

**Definition 2.2.** Fractional-order derivative of the defined function (2.1) of order \( \alpha, \quad 0 < \alpha < 1 \) is in terms of fractional order difference is given by

\[
\phi^{(\alpha)}(t) = \lim_{d \to 0} \frac{\Delta^\alpha \phi(t)}{d^\alpha}
\]

2.3. Modified Riemann–Liouville fractional-order derivative [9]

**Definition 2.3.** Let \( \phi(t) \) be a continuous function defined on non-negative real numbers then

(i) If \( \phi(t) = K \) (constant), then its fractional-order derivative

\[
D_t^\alpha K = \frac{K}{\Gamma(1 - \alpha)t^{\alpha}}, \quad \text{if} \quad \alpha \leq 0,
\]

\[
= 0, \quad \text{otherwise.}
\]

(ii) If \( \phi(t) \) is not a constant function, then

\[
\phi(t) = \phi(0) + \{\phi(t) - \phi(0)\}
\]

and its fractional-order derivative

\[
\phi^{(\alpha)}(t) = D_t^\alpha \phi(0) + D_t^\alpha \{\phi(t) - \phi(0)\},
\]
for negative $\alpha (< 0)$, we have
\[
D^\alpha_t (\phi(t) - \phi(0)) = \frac{1}{\Gamma(-\alpha)} \int_0^t (t - \zeta)^{-(\alpha+1)} \phi(\zeta) d\zeta,
\] (2.6)

for positive $\alpha (> 0)$, we have
\[
D^\alpha_t (\phi(t) - \phi(0)) = D^\alpha_t (\phi(t)) = D_t (\phi^{(\alpha-1)}(t))
\] (2.7)

when $k \leq \alpha < k + 1$, then
\[
\phi^{(\alpha)}(t) = \left( \phi^{(\alpha-n)}(t) \right)^{(n)}, \quad k \leq \alpha < k + 1, k \geq 1.
\] (2.8)

### 2.4. Integration with respect to $(dt)^\alpha$

**Definition 2.4.** Consider the fractional differential equation
\[
dz = \phi(t)(dt)^\alpha, \quad t \geq 0, \ z(0) = 0
\] (2.9)

And solution is given by
\[
z = \int_0^t \phi(\zeta)(d\zeta)^\alpha
\] (2.10)
\[
= \alpha \int_0^t \zeta^{(\alpha-1)} \phi(\zeta) d\zeta \quad 0 < \alpha < 1
\] (2.11)

To understand above results and for detail review on fractional calculus, see [13–29].

### 2.5. Fractional-order Dirac’s delta function

**Definition 2.5.** Fractional-order Dirac’s delta function is defined by [13]
\[
\delta_{\alpha}(t) = \lim_{\epsilon \to 0} \left\{ \begin{array}{ll}
0 & x \notin [0, \epsilon] \\
\epsilon^{-\alpha} & 0 < t \leq \epsilon
\end{array} \right.
\] (2.12)

And also, can be defined by equality
\[
\int_{-\infty}^{+\infty} \phi(t)\delta_{\alpha}(t)(dt)^\alpha = \alpha \phi(0)
\] (2.13)

**Lemma 2.1.** The following result holds:
\[
D_t^\alpha \int_0^t \phi(\zeta)(d\zeta)^\alpha = \Gamma(\alpha + 1) \phi(t)
\] (2.14)

**Proof.** This result can be obtained directly from the definition 2.3

**Lemma 2.2.** ([13]) The following result holds:
\[
\int_0^\infty E_{\alpha}(-(st)^\alpha)E_{\alpha}(-(ct)^\alpha)\phi(t)(dt)^\alpha = \int_0^\infty E_{\alpha}(-(s + c)t)^\alpha)\phi(t)(dt)^\alpha
\] (2.15)

where $c, s \in \mathbb{C}$
Lemma 2.3. If $M_\alpha$ denotes the period of Mittag-Leffler function where period is defined by

$$E_\alpha(i(M_\alpha)^\alpha) = 1$$

then the following result holds:

$$\frac{\alpha(\ln \beta)^\alpha}{(M_\alpha)^\alpha} \int_{-\infty}^{+\infty} E_\alpha(i(-\omega \ln \beta)^\alpha)(d\omega)^\alpha = \delta_\alpha(t)$$\hspace{1cm}(2.16)

Proof. We check that Eq. (2.16) is consistent with

$$\alpha = \int_{-\infty}^{+\infty} E_\alpha(i(\omega \ln \beta t)^\alpha) \delta_\alpha(t)(dt)^\alpha$$

Replace $\delta_\alpha(t)$ by eq.(2.16) to obtain

$$\alpha = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\alpha(\ln \beta)^\alpha}{(M_\alpha)^\alpha} E_\alpha(i(\omega - \zeta) \ln \beta)^\alpha)(d\zeta)^\alpha(dt)^\alpha$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\alpha(\ln \beta)^\alpha}{(M_\alpha)^\alpha} E_\alpha(i(-\omega \ln \beta)^\alpha)(dv)^\alpha(dt)^\alpha$$

$$= \int_{-\infty}^{+\infty} \delta_\alpha(t)(dt)^\alpha$$

\[\square\]

3. FRACTIONAL-ORDER $\beta$-LAPLACE INTEGRAL TRANSFORM

Definition 3.1. Let $\phi(t)$ be a function defined on non-negative real numbers, its $\beta$-Laplace integral transform $L^\alpha_\beta\{\phi(t)\}_{(s)}$ of order $\alpha$ is defined by:

$$L^\alpha_\beta\{\phi(t)\}_{(s)} = \int_0^\infty E_\alpha(-(s\ln \beta)^\alpha)\phi(\zeta)(d\zeta)^\alpha \hspace{1cm} \beta > 1, s \in \mathbb{C}.$$ \hspace{1cm}(3.1)

when it converges.

4. PROPERTIES OF $\beta$-LAPLACE INTEGRAL TRANSFORM OF FRACTIONAL ORDER

4.1. Linearity Property

Theorem 4.1. Let $\phi_1, \phi_2 : [0, \infty) \to \mathbb{R}$ be continuous functions and $c_1, c_2 \in \mathbb{C}$ then

$$L^\alpha_\beta\{c_1\phi_1(t) + c_2\phi_2(t)\}_{(s)} = c_1 L^\alpha_\beta\{\phi_1(t)\}_{(s)} + c_2 L^\alpha_\beta\{\phi_2(t)\}_{(s)}$$ \hspace{1cm}(4.1)

Proof. By the definition of Fractional order $\beta$-Laplace integral transform

$$L^\alpha_\beta\{c_1\phi_1(t) + c_2\phi_2(t)\}_{(s)} = \int_0^\infty E_\alpha(-(s\ln \beta)^\alpha)\{c_1\phi_1(t) + c_2\phi_2(t)\}(dt)^\alpha$$

$$= c_1 \int_0^\infty E_\alpha(-(s\ln \beta)^\alpha)\phi_1(t)(dt)^\alpha$$

$$+ c_2 \int_0^\infty E_\alpha(-(s\ln \beta)^\alpha)\phi_2(t)(dt)^\alpha$$

$$= c_1 L^\alpha_\beta\{\phi_1(t)\}_{(s)} + c_2 L^\alpha_\beta\{\phi_2(t)\}_{(s)}$$

\[\square\]
4.2. Shifting Property

**Theorem 4.2.** Let \( \phi : [0, \infty) \rightarrow \mathcal{R} \) be continuous function then following result holds:

\[
\mathcal{L}_\alpha^\beta \{ \phi(t-b) \}_{(s)} = E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\mathcal{L}_\alpha^\beta \{ \phi(t) \}_{(s)} - \Gamma(1 + \alpha)\phi(0)
\]

(4.2)

\[
\mathcal{L}_\alpha^\beta \{ E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\phi(t-b) \}_{(s)} = \mathcal{L}_\alpha^\beta \{ \phi(t) \}_{(s+c)}
\]

(4.3)

**Proof.** We can simply obtain by using the definition (3.1)

\[
\mathcal{L}_\alpha^\beta \{ \phi(t-b) \}_{(s)} = \int_0^\infty E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\phi(t-b)(dt)^\alpha
\]

after substituting \( t-b = u \), and by the Eq.(2.15) we get our desired result

\[
\mathcal{L}_\alpha^\beta \{ \phi(t-b) \}_{(s)} = E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\mathcal{L}_\alpha^\beta \{ \phi(t) \}_{(s)}
\]

\( \square \)

4.3. Scaling Property

**Theorem 4.3.** Let \( \phi : [0, \infty) \rightarrow \mathcal{R} \) be continuous function then following result holds:

\[
\mathcal{L}_\alpha^\beta \{ \phi(at) \}_{(s)} = \left(\frac{1}{a}\right)^\alpha \mathcal{L}_\alpha^\beta \{ \phi(t) \}_{\left(\frac{\xi}{a}\right)}
\]

(4.4)

where \( a \neq 0 \) is any constant.

**Proof.** We can obtain by the definition of fractional order \( \beta \)-Laplace Integral transform

\[
\mathcal{L}_\alpha^\beta \{ \phi(at) \}_{(s)} = \int_0^\infty E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\phi(at)(dt)^\alpha
\]

after substituting \( at = u \), we get our desired results

\[
\mathcal{L}_\alpha^\beta \{ \phi(at) \}_{(s)} = \left(\frac{1}{a}\right)^\alpha \mathcal{L}_\alpha^\beta \{ \phi(t) \}_{\left(\frac{\xi}{a}\right)}
\]

\( \square \)

4.4. Derivative Property

**Theorem 4.4.** Let \( \phi(t) \) be continuous real valued function defined on non-negative real number has a fractional-order derivative then

\[
\mathcal{L}_\alpha^\beta \{ \phi^{(\alpha)}(t) \}_{(s)} = s^\alpha \mathcal{L}_\alpha^\beta \{ \phi(t) \}_{(s)} - \Gamma(1 + \alpha)\phi(0)
\]

(4.5)

**Proof.** The equality

\[
(uv)^\alpha = u^\alpha v + uv^\alpha
\]

(4.6)

yields

\[
D_\alpha^\beta E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\phi^{(\alpha)}(t) = -(s\ln \beta)^\alpha E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\phi(t) + E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\phi^{(\alpha)}(t)
\]

integrating \( (D^{-\alpha}) \) both sides and using eq. (4.6), we obtain

\[
D^{(-\alpha)}\phi(t) = \frac{1}{\Gamma(1 + \alpha)} \int_0^t \phi(\xi)(d\xi)^\alpha
\]

then, we have

\[
\frac{1}{\Gamma(1 + \alpha)} \lim_{t \to \infty} E_\alpha\left(-\left(s\ln \beta\right)^\alpha\right)\phi(t) - \phi(0) = -(s\ln \beta)^\alpha \mathcal{L}_\alpha^\beta \{ \phi(t) \}_{(s)} + \mathcal{L}_\alpha^\beta \{ \phi^{(\alpha)}(t) \}_{(s)}
\]

\[
\mathcal{L}_\alpha^\beta \{ \phi^{(\alpha)}(t) \}_{(s)} = s^\alpha \mathcal{L}_\alpha^\beta \{ \phi(t) \}_{(s)} - \Gamma(1 + \alpha)\phi(0)
\]

\( \square \)
4.5. Integral Property

**Theorem 4.5.** Let \( \phi(t) \) be continuous real valued function defined on non-negative real number \( t \geq 0 \) has a fractional-order derivative and \( \int_0^\infty f(u)(du)^\alpha \) converges then following result holds:

\[
\mathcal{L}^\beta_\alpha \left\{ \int_0^t \phi(u)(du)^\alpha \right\}(s) = \frac{\Gamma(1 + \alpha)}{(s \ln \beta)^\alpha} \mathcal{L}^\beta_\alpha \{ \phi(t) \}(s) \tag{4.7}
\]

**Proof.** By using the derivative property

\[
\mathcal{L}^\beta_\alpha \left\{ D^\alpha_x \int_0^t \phi(u)(du)^\alpha \right\}(s) = (s \ln \beta)^\alpha \mathcal{L}^\beta_\alpha \left\{ \int_0^t \phi(t)(du)^\alpha \right\}(s)
\]

by using the equality

\[
D^\alpha_x \int_0^t \phi(u)(du)^\alpha = \Gamma(\alpha + 1)\phi(t)
\]

yields the result

\[
\mathcal{L}^\beta_\alpha \left\{ \int_0^t \phi(\zeta)(d\zeta)^\alpha \right\}(s) = \frac{\Gamma(1 + \alpha)}{(s \ln \beta)^\alpha} \mathcal{L}^\beta_\alpha \{ \phi(t) \}(s)
\]

\[\square\]

5. CONVOLUTION PROPERTY

**Theorem 5.1.** Let \( \phi_1, \phi_2 : [0, \infty) \rightarrow \mathbb{R} \) be two continuous functions and if the convolution of fractional-order \( \alpha \) is defined by

\[
\left( \phi_1(t) * \phi_2(t) \right)_\alpha = \int_0^\infty \phi_1(t - \zeta)\phi_2(\zeta)(d\zeta)^\alpha \tag{5.1}
\]

then,

\[
\mathcal{L}^\beta_\alpha \{ (\phi_1(t) * \phi_2(t))_\alpha \}(s) = \mathcal{L}^\beta_\alpha \{ \phi_1(t) \}(s) \mathcal{L}^\beta_\alpha \{ \phi_2(t) \}(s) \tag{5.2}
\]

**Proof.** By the definition

\[
\mathcal{L}^\beta_\alpha \{ (\phi_1(t) * \phi_2(t))_\alpha \}(s) = \int_0^\infty E_\alpha(-s t \ln \beta)^\alpha \left( \phi_1(t) * \phi_2(t) \right)_\alpha (dt)^\alpha
\]

\[
= \int_0^\infty E_\alpha(-s t \ln \beta)^\alpha \left\{ \int_0^\infty \phi_1(t - \zeta)\phi_2(\zeta)(d\zeta)^\alpha \right\}(dt)^\alpha
\]

substitute \( y = t - \zeta, v = \zeta \)

\[
\mathcal{L}^\beta_\alpha \{ (\phi_1(t) * \phi_2(t))_\alpha \}(s) = \int_0^\infty \int_0^\infty E_\alpha(-s y \ln \beta)^\alpha E_\alpha(-s v \ln \beta)^\alpha \phi_1(y)\phi_2(v)(dy)^\alpha(dv)^\alpha
\]

\[
= \mathcal{L}^\beta_\alpha \{ \phi_1(t) \}(s) \mathcal{L}^\beta_\alpha \{ \phi_2(t) \}(s)
\]

\[\square\]
6. FORMULA FOR INVERSE FRACTIONAL-ORDER $\beta$-LAPLACE INTEGRAL TRANSFORM

**Theorem 6.1.** Let $\phi(t)$ be a function and $\mathcal{L}_\beta^{\alpha}\{\phi(t)\}_{(s)}$ be the Fractional order $\beta$-Laplace integral transform then the inverse $\beta$-Laplace transform is given by the formula

$$
\phi(t) = \frac{(\ln \beta)^\alpha}{(M_\alpha)^\alpha} \int_{-\infty}^{+\infty} E_\alpha\left((st \ln \beta)^\alpha\right) \mathcal{L}_\beta^{\alpha}\{\phi(t)\}_{(s)} (ds)^\alpha \tag{6.1}
$$

**Proof.** After substituting Eq.(3.1) into Eq.(6.1), we obtain

$$
\frac{(\ln \beta)^\alpha}{(M_\alpha)^\alpha} \int_{-\infty}^{+\infty} E_\alpha\left((st \ln \beta)^\alpha\right) (ds)^\alpha \int_0^{\infty} E_\alpha\left(-(s\zeta \ln \beta)^\alpha\right) \phi(\zeta) (d\zeta)^\alpha
\Rightarrow \frac{1}{\alpha} \int_{\mathbb{R}} \phi(\zeta) \delta_\alpha (t - \zeta) (d\zeta)^\alpha = \phi(t)
$$

7. CONCLUSION

Proposed new form of generalization is mathematical valid for an involved function whether it has continuous derivative or continuous fractional derivative.

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