Total and geometric phases, Majorana and Dirac neutrinos

ANTONIO CAPOLUPO

Dipartimento di Fisica E.R. Caianiello, Universita’ di Salerno, ITALY
INFN - Gruppo Collegato di Salerno, ITALY

The analysis of the total and geometric phases generated by the neutrino oscillation shows that these phases for Majorana neutrinos are depending on the representation of the mixing matrix and they are different from those of Dirac neutrinos.

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**Introduction**: The discovery of the neutrino oscillation [1]-[3] has definitively shown that the neutrino has a mass. It remains to determine the nature of the neutrino. Indeed, since the neutrinos are electrically neutral, they can be Dirac particles (fermions different from their antiparticles), or Majorana particles (fermions coinciding with their antiparticles). For Dirac neutrinos, the Lagrangian is invariant under $U(1)$ global transformation. Then the total lepton charge is conserved and, in the case of the mixing of $n$ fields, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix has $N_D$ physical phases given by $N_D = \frac{(n-1)(n-2)}{2}$. By contrast, for Majorana neutrinos, the Lagrangian breaks the $U(1)$ symmetry, then processes violating the lepton number (such as the neutrinoless double $\beta$ decay) are allowed and the mixing matrix contains $N_M$ physical phases given by $N_M = \frac{n(n-1)}{2}$. The $n - 1$ extra phases present in the mixing matrix of Majorana neutrinos are called Majorana phases. Many representations of Majorana mixing matrix can be achieved by the rephasing the charged fields in the charged current weak-interaction Lagrangian [4]. For example, in two flavor mixing, $N_M = 1$ and we can consider the following mixing matrices for Majorana neutrinos

$$U_1 = \begin{pmatrix} \cos \theta & \sin \theta \ e^{i\phi} \\ -\sin \theta & \cos \theta \ e^{i\phi} \end{pmatrix}, \quad \text{or} \quad U_2 = \begin{pmatrix} \cos \theta & \sin \theta \ e^{-i\phi} \\ -\sin \theta \ e^{i\phi} & \cos \theta \end{pmatrix}, \quad (1)$$

where $\theta$ is the mixing angle and $\phi$ is the Majorana phase. This phase can be removed for Dirac neutrinos. Notice that the Majorana phases have no effect in neutrino oscillation formulae that are the same for Dirac and for Majorana neutrinos [4]. Therefore, the oscillation formulae do not allow to reveal the neutrino nature. However, recently it has been shown that the phenomenon of the decoherence in neutrino evolution can lead to oscillation formulae for Majorana neutrinos different from those of Dirac neutrinos [5]. Moreover it has been shown that quantities such as the Leggett-Garg $K_3$ quantity [6] and the phases generated in the neutrino oscillation [7] can in principle discriminates between Dirac and Majorana neutrinos.

In this paper, we report the results presented in Ref.[7] and we show that the phases due to the transitions among different flavors states depend on the mixing matrix $U$ considered. Indeed, different choices of $U$ for Majorana neutrinos generate different values of the total and geometric phases. We show these differences by considering the two flavor neutrino mixing case and the matrix $U_1$ and $U_2$ in Eq.(1). We demonstrate that by using $U_2$, the values of the total and geometric phases of Majorana neutrinos are different from those obtained for Dirac neutrinos, while, by using $U_1$, all the phases are independent of $\phi$ and the results for Majorana and for Dirac neutrinos coincide. Thus, the total and geometric phase provide a tool to determine the choice of $U$ and to reveal the nature of the neutrino.

**Neutrino mixing and phases**: The study of the interferometry has attracted a great attention in the recent years and new progress on the detection of the geometric phase [8] has been made. The geometric phase appears in the evolution of any...
quantum state describing a physical system characterized by a Hamiltonian defined on a parameter space.

Here, we analyze in particular, the total phase and the non–cyclic geometric phase \[9\] (which generalizes the Berry phase to the case of not cyclic, not adiabatic evolution) for neutrinos propagating in vacuum and through the matter. We use the mixing matrix \(U_2\) to show the differences between Majorana and Dirac neutrinos. We also compare the results obtained with those achieved by using the matrix \(U_1\). We remind that the matter effects are described by replacing \(\Delta m^2\) with \(\Delta m^2_{\text{m}} = \Delta m^2 R_\pm\), and \(\sin 2\theta\) with \(\sin 2\theta_m = \sin 2\theta/R_\pm\), where \(R_\pm = \sqrt{\left(\frac{\cos 2\theta \pm \frac{2\sqrt{2}G_F m_N}{\Delta m^2 R_\pm}}{2}\right)^2 + \sin^2 2\theta}\), with + for antineutrinos and – for neutrinos \[10\]. Then, the results obtained for propagation in vacuum and through the matter are formally the same. In the following we consider the propagation in a medium and we use the quantities \(\Delta m^2_{\text{m}}\) and \(\sin 2\theta_m\).

The geometric phase for a quantum system with state vector \(|\psi(s)\rangle\), is given by the difference between the total phase \(\phi^\text{tot}_\psi = \arg \langle \psi(s_1)|\psi(s_2)\rangle\) and the dynamic phase \(\phi^\text{dyn}_\psi = \Im \int_{s_1}^{s_2} \langle \psi(s)|\dot{\psi}(s)\rangle ds\), i.e. \(\phi^g = \phi^\text{tot}_\psi - \phi^\text{dyn}_\psi\), where \(s\) is a real parameter such that \(s \in [s_1, s_2]\), and the dot denotes the derivative with respect to \(s\). We find that the geometric phase for electron neutrino is independent on the Majorana phase, therefore it is the same for Majorana and for Dirac neutrinos. It is given by

\[
\Phi^g_{\nu_e}(z) = \arg \left[ \langle \nu_e(0)|\nu_e(z)\rangle \right] - \Im \int_0^z \langle \nu_e(z')|\dot{\nu}_e(z')\rangle dz',
\]

(2)

Similar result is obtained for muon neutrino, indeed we have \(\Phi^g_{\nu_\mu}(z) = -\Phi^g_{\nu_e}(z)\). Let us consider now the following phases due to the transitions \(\nu_e \rightarrow \nu_\mu\) and \(\nu_\mu \rightarrow \nu_e\)

\[
\Phi_{\nu_e \rightarrow \nu_\mu}(z) = \arg \left[ \langle \nu_e(0)|\nu_\mu(z)\rangle \right] - \Im \int_0^z \langle \nu_\mu(z')|\dot{\nu}_e(z')\rangle dz',
\]

(3)

\[
\Phi_{\nu_\mu \rightarrow \nu_e}(z) = \arg \left[ \langle \nu_\mu(0)|\nu_e(z)\rangle \right] - \Im \int_0^z \langle \nu_e(z')|\dot{\nu}_\mu(z')\rangle dz'.
\]

(4)

They are gauge invariant and reparametrization invariant, therefore they are geometric phases. By using the Majorana neutrino states obtained by the \(U_2\) mixing matrix, we have \(\Phi_{\nu_e \rightarrow \nu_\mu} \neq \Phi_{\nu_\mu \rightarrow \nu_e}\), indeed

\[
\Phi_{\nu_e \rightarrow \nu_\mu}(z) = \frac{3\pi}{2} + \phi + \left( \frac{\Delta m^2_{\text{m}}}{4E} \sin 2\theta_m \cos \phi \right) z,
\]

(5)

\[
\Phi_{\nu_\mu \rightarrow \nu_e}(z) = \frac{3\pi}{2} - \phi + \left( \frac{\Delta m^2_{\text{m}}}{4E} \sin 2\theta_m \cos \phi \right) z.
\]

(6)
Moreover, for the total phases, we have \( \Phi_{\nu_e \rightarrow \nu_\mu}^{\text{tot}} = \frac{3\pi}{2} + \phi \) and \( \Phi_{\nu_\mu \rightarrow \nu_e}^{\text{tot}} = \frac{3\pi}{2} - \phi \). On the contrary, for Dirac neutrinos we obtain

\[
\Phi_{\nu_e \rightarrow \nu_\mu}(z) = \Phi_{\nu_\mu \rightarrow \nu_e}(z) = \frac{3\pi}{2} + \left( \frac{\Delta m^2_e}{4E} \sin 2\theta_m \right) z,
\]

and the total phases become \( \Phi_{\nu_e \rightarrow \nu_\mu}^{\text{tot}}(z) = \Phi_{\nu_\mu \rightarrow \nu_e}^{\text{tot}}(z) = \frac{3\pi}{2} \). Notice that \( \Phi_{\nu_e \rightarrow \nu_\mu}, \Phi_{\nu_\mu \rightarrow \nu_e} \) and the total phases depend on the choice of the mixing matrix. Indeed, by using the mixing matrix \( U_1 \), we derive the result of Eq.(7) also for Majorana neutrinos.

In Fig.1 we plot the total and geometric phases for electron neutrino propagating through the matter. We consider the values of the parameters of RENO experiment [2]: neutrino energy \( E \in [2 - 8] \text{MeV} \), electron earth density \( n_e = 10^{24} \text{cm}^{-3} \), \( \Delta m^2 = 7.6 \times 10^{-3} \text{eV}^2 \) and distance \( z = 100 \text{km} \). In Fig.2 we plot \( \Phi_{\nu_e \rightarrow \nu_\mu} \) and \( \Phi_{\nu_\mu \rightarrow \nu_e} \) presented in Eqs.(5) and (6), by considering \( \phi = 0.3 \) and the values of \( n_e \) and \( \Delta m^2 \) of Fig.1. We use the values of the parameters of T2K experiment [3]: \( E \sim 1 \text{GeV} \) and \( z = 300 \text{km} \).

- Conclusions: We have shown that, for Majorana neutrinos, the total and the geometric phases due to the oscillation assume different values depending on the representation of the mixing matrix. In particular, we have proved that the total and the geometric phases of Majorana neutrino can be different from those of Dirac neutrino. Therefore these phases can represent a new tool to study the nature of neutrinos. We have presented also a numerical analysis and plotted the geometric phases by using the characteristic parameters of RENO and T2K experiments.

References

[1] F. P. An et al. [Daya-Bay Collaboration], Phys. Rev. Lett. 108, 171803 (2012); P. Adamson et al. [MINOS Collaboration], Phys. Rev. Lett. 107, 181802 (2011).
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Figure 2: Plot of the phases $\Phi_{\nu_e \rightarrow \nu_\mu}$ (the blue dashed line) and $\Phi_{\nu_\mu \rightarrow \nu_e}$ (the red dot dashed line) for Majorana neutrinos as a function of $E$, for $z = 300\text{ km}$. The phases $\Phi_{\nu_e \rightarrow \nu_\mu} = \Phi_{\nu_\mu \rightarrow \nu_e}$ for Dirac neutrinos is represented by the black solid line.

[2] J. K. Ahn et al. [RENO Collaboration], Experiment, Phys. Rev. Lett. 108, 191802 (2012).

[3] K. Abe et al. [T2K Collaboration], Phys. Rev. Lett. 107, 041801 (2011).

[4] C. Giunti, Phys. Lett. B 686 41, (2010).

[5] A. Capolupo, S. M. Giampaolo and G. Lambiase, Phys. Lett. B 792 298303, (2019).

[6] M. Richter, B. Dziewit and J. Dajka, Phys. Rev. D 96, no. 7, 076008 (2017).

[7] A. Capolupo, S. M. Giampaolo, B. C. Hiesmayr and G. Vitiello, Phys. Lett. B 780, 216 (2018).

[8] M. V. Berry, Proc. Roy. Soc. Lond. A 392, 45 (1984); J. Samuel and R. Bhandari, Phys. Rev. Lett. 60, 2339 (1988); S. Pancharatnam, Proc. Indian Acad. Sci. A 44, 1225 (1956); A. Shapere and F. Wilczek, Geometric Phases in Physics, World Scientific, Singapore, 1989; A. K. Pati, Phys. Rev. A 52, 2576 (1995).

[9] N. Mukunda and R. Simon, Ann. Phys.(N.Y) 228, 205 (1993).

[10] S. P. Mikheev, A. Yu. Smirnov, Sov. J. Nuc. Phys. 42 (6): 913917, (1985); L. Wolfenstein Phys. Rev. D 17 (9): 2369, (1978).