Effectiveness Measurement of UAV Combat in Uncertain Environment

Bing He, Xianyang Zhang and Qingyong Li
Rocket Force University of Engineering in 2nd Gate, Tongxing Road, Baqiao District, Xi’an Shanxi, China.
Email: zhangxyang177@163.com

Abstract. This paper focuses on the combat effectiveness measurement of UAV for reconnaissance and combat. Based on the establishment of typical battlefield scenarios and UAV reconnaissance and strike probability models, it focuses on the analysis and measurement of combat effectiveness when a single UAV performs reconnaissance strikes. The three main factors affecting UAV combat effectiveness—the true target interpretation probability, sensor parameters, and the number of warheads—are subjected to numerical simulation analysis. Finally, the UAV design parameters for reconnaissance and strike integration with optimal combat effectiveness are obtained: number of warheads are 12, interpretation probability of true target is 0.84, Sensor ROC curve parameters are 10.

1. Introduction
With the universal application of the information winning mechanism, the leading role of combat effectiveness measurement and assessment in information warfare becomes more prominent. Scholars have conducted in-depth research on combat effectiveness measurement and evaluation in military operations such as land, sea, air, and space. Methods such as exponential method, ADC method [1], SEA method [2][3], fuzzy comprehensive evaluation method [4][5] and other methods of combat effectiveness measurement began to appear and become effective use. [6] [7] This paper will analyze and discuss the "fog" effect of war scenes caused by the nature of uncertain targets and the threat environment. In the battlefield uncertainty space model [8], we will study some typical war scenarios with multiple true targets/multiple false targets. Analyze and discuss the true target readability of sensors, sensor performance parameters and the impact of warheads on combat effectiveness [9], and then provide important reference for reconnaissance and combat integration of UAV related system design.

2. Typical Battlefield Scenarios and Sensor Modeling
At present, there are mainly two search modes for reconnaissance/combating UAVs in various battlefield scenarios [10]. One is a linear search mode, and the other is a concentric circular search mode for origin diffusion. For a UAV flying at a constant altitude, its area coverage is the product of the speed and the width of the search zone.

2.1. Typical Battlefield Scene Settings
The occurrence of operational UAV reconnaissance missions based on true targets and false targets cannot be accurately determined in battlefield scenarios. In order to simplify the complexity of the problem, it can be assumed that the true target and the false target obey a certain probability distribution. The probability distribution can be described using a uniform distribution, a Poisson
distribution, and a normal distribution. Since the number of true and false targets is uncertain, many situations in the battlespace environment can be set based on these probability distributions. In these battlefield environments, some intelligence information (the number of true and false targets, the distribution of the target's compliance, etc.) can be clearly defined. Certainly, some intelligence information cannot be set explicitly. The relevant literature introduced seven typical battlefield spatial scenarios. According to the characteristics of the battlefield space, two typical battlefield spaces, linear symmetry and circular, can be divided. Due to limitations in the length of the article, only one of the situations is discussed here. In a symmetrical symmetry battlefield space scenario, multiple real targets and multiple false targets are all subject to Poisson distribution.

2.2. Sensor Parameter Modeling and Simulation

UAV carries out the operational reconnaissance mission when the sensor mainly is the optical sensor, because of the performance reason of the sensor, cannot carry on the very accurate classification to the target, its sensor parameter will have the direct influence to the UAV to the target interpretation result. The following analysis of the parameters of the UAV equipped with sensors, and then simulation analysis, the impact of parameter changes on interpretation probability.

When the UAV carries a sensor payload to detect a target, it compares the detected target image with the saved baseline image to determine whether the target is a true target or a false target. If the target is a true target, it will attack it. In fact, the performance of the sensor is determined by its correctness of the judgment target. To this end, a binary ambiguity matrix is established and a symbolic description is made:

| Table 1. Binary ambiguity matrix |
|---------------------------------|
| **Target interpretation results** | **Target reality** |       |
| True goal | $P_{TR}$ | $1 - P_{FTR}$ |
| False target | $1 - P_{TR}$ | $P_{FTR}$ |

Among them, $P_{TR}$ indicates the true target interpretation probability, that is, when the true target appears, the sensor correctly judges the probability that this is a true target, and $P_{FTR}$ is the false target interpretation probability. The parameters $P_{TR}$ and $P_{FTR}$ are not independent and they satisfy the ROC curve model.

\[
1 - P_{FTR} = \frac{P_{TR}}{(1-q)P_{TR} + q}
\]

Where $q \in [1, \infty]$ is determined by the sensor and data processing algorithm. When $q$ increases, the ROC curve also increases. When $q \to \infty$, the area under the curve approaches unity, indicating that the reconnaissance efficiency is the highest. The following figure shows two ROC curves when $q$ takes different values.
Because of the many scenes and complex problems in the battlefield, due to the limited space of the paper, this paper only deeply analyzes the situation in which multiple true targets and multiple false targets obey the Poisson distribution in the battlefield space with linear symmetry. Other typical battlefield scenarios the analytical calculation process is similar to this.

### 3.1. Scene Analysis and Parameter Symbol Description

At a certain flying height, it is flying at a rate of $V$ in a band area $W$ with a width of $A_s$, as shown in the figure 2.

If $T$ is used to indicate the UAV's total time in the battlefield area, use $\tau$ to indicate the UAV reconnaissance area $A$. It is easy to get the UAV flying speed and the rectangular boundary of the battlefield area.

$$A_s = WVT, \quad A = WV\tau$$

In the same way, the areas being investigated include
In order to effectively reduce the complexity of the problem, the standardized time is set, that is, the time is normalized and expressed by

\[ x = \frac{\tau}{T} \]  

(4)

According to the above analysis, \( 0 \leq x \leq 1 \) can be obtained; similarly, \( x \) can also be used to indicate the proportion of the reconnaissance area occupying the battle space in the battle field that is

\[ x = \frac{A_s}{A} \]  

(5)

In order to facilitate calculation and unified analysis, here are some of the symbols in the following definitions. \( \alpha \) is distribution density of false targets in the reconnaissance area \( A_s (1/km^2) \); \( \beta \) is distribution density of true targets in the reconnaissance area \( A_s (1/km^2) \); \( \lambda_r \) is parameters of the false target where the reconnaissance area \( A_s \) obeys Poisson distribution, ie \( \lambda_r = \alpha A_s \); \( \lambda_t \) is parameters of the reconnaissance area obeys the parameters of the Poisson distribution’s true target, that is, \( \lambda_t = \beta A_s \); \( M \) is the event of discovering a true target and attacking; \( F \) is the event of finding a false target but being interpreted as a true target and performing an attack; \( M_{t,A} \) finds \( t \) true targets and attacks; \( F_{t,f} \) is finds \( f \) false targets but is interpreted as true and attacks.

For the sake of convenience, an event occurring in a certain area must be marked together with the area as a subscript of the symbol of the event. For example, \( M_{t,A} \) indicates that \( A \) real targets in area \( t \) are attacked.

3.2. Reconnaissance and Combat UAV Combat Effectiveness Measurement

In this article, to intuitively sense the significance of combat effectiveness, the UAV’s combat effectiveness is understood as the number of UAV attack targets in a certain battlefield scenario. Due to the fact that the UAV is equipped with sensors and the probability distribution of true and false targets in the battlefield scene, it is impossible to accurately give the true target number. Through probability calculation, it can be converted into a probability that at least a certain number of targets are attacked \([6][7]\).

To achieve the above calculations, we must first know the probability that a certain number of true targets will be attacked. Since all UAVs will be consumed if all the warheads carried out an attack task before a certain time, the UAV will be unable to perform operations after that time. Attack task. Therefore, for the UAV to perform a reconnaissance strike task, the calculation of the determined number of true target attack probabilities should be strictly distinguished from the two cases of carrying out warhead exhaustion and remaining warheads at this time for classification discussion. Due to space limitations, the results are given directly here.

(1) Shootout exhausted \((t + f = w)\)

Through the extension, it can be obtained that in the area \( A_s \) there are \( t \) true targets and \( f \) false targets subjected to an attack, and the probability that the warheads are all consumed \( P_{t,f,w}^{s/w} (A_s) \) can be obtained in the area \( A_s \), attacking \( t \) true targets and \( f \) false targets. The probability is

\[ P_{t,f,w}^{s/w} (A_s) = \frac{\lambda_t^t \lambda_r^f}{(t)! (f)!} \int_0^w \left( e^{-(\lambda_r + \lambda_t) x} x^{w-1} \right) dx \]  

(7)

(2) There is still warhead remaining \((t + f < w)\). In the area, the probability of attacking a false target is
4. Combat Effectiveness Measurement Simulation Analysis

In the previous section, the UAV’s probability of attacking multiple true and false targets in the designated combat area $A_S$ and its mathematical expectation were carefully deduced. This paper will conduct numerical simulation analysis on the application of the deduced formula to improve the performance of the UAV Combat effectiveness.

When designing the UAV for reconnaissance and combat, it is mainly necessary to analyze the parameters that affect the combat effectiveness, such as sensor parameters and the number of warheads. Improving the operational effectiveness of UAVs in reconnaissance and combat is to study how to adjust the parameters so that they can attack as many real targets as possible, and at the same time, reduce the attack on the false targets (collateral damage).

In order to ensure that the UAV is capable of performing an attack on the combat target during the execution of the attack mission, the warhead validity $P_k$ parameter is defined. Warhead Effectiveness $P_k$ indicates the probability of a UAV attacking a target and successfully destroying the combat target. Definition $E(t_a)$ represents the mathematical expectation of hitting the true target and successfully destroying the attack target, then $E(t_a) = P_kE(e)$.

![Figure 3](image-url)

**Figure 3.** The curve of $E(t_a)$ changing with $P_{tr}$
As shown in Fig. 4 below, we set the relevant parameter to $w=10, \lambda_T=20, \lambda_r=10, P_r=0.8, x=1$, and study the relationship between the true target interpretation probability $q$ and $P_{te}$ when the parameter $E(t_i)$ of the optical sensor ROC curve carried by the UAV takes 10 and 18, respectively. From the simulation graph, for the $q=18$ sensor, the probability of attacking a false target more than $P_{te}$ increases faster than the probability of a target attack; if for some reason limiting $P_{te} \leq 0.7$, we choose the cheap sensor with $q=10$.

In the simulation of $P_r - E(t_i)$, was set $w=10$. The same can be obtained when $w=18$ and $P_{te}=0.92$, other parameter settings remain unchanged, the expected number of killing $E(t_i)$ best.
We set $P_{tr} = 0.92$ in the subsequent simulation to study the relationship between $w$ and the expected number of killing $x$ when the warhead number $E(t)$ remains constant. We set the relevant parameter $q=18, \lambda = 20, P_0$, to simulate the analysis of the relationship between $x$ and $E(t)$, and get the numerical simulation figure 6, which can be easily obtained from the figure. When $w=14$, the expected number of kill target $E(t)$ is relatively optimal.

![Figure 6](image)

Figure 6. The curve of $E(t)$ and $E(f)$ changing with $P_{tr}$

In the above analysis, only the effects of $P_{tr}$, $q$, and $w$ on the expected number of killing true targets $E(t)$ were considered, and the false killing of false targets was not considered. However, in practical applications, false killing and injury of false targets is equivalent to collateral damage, and it has no effect on improving combat effectiveness of reconnaissance and combat integrated UAVs. Therefore, in order to effectively improve UAV's combat effectiveness, it is also necessary to consider minimizing the killing of false targets or reducing the number of false target kills.

In order to study the effect of false target killing on combat effectiveness, a false target killing threshold $E[f]$ was introduced, which can make the number of kills of false targets within a certain range. Under the specified conditions, set the correlation parameters between $q=18, \lambda = 20, \lambda = 10, P_t = 0.8, E[f] \leq 0.1$, the numerical simulation $E(t)$, $E(f)$and the true target reading probability $P_{tr}$, adjust the number of warheads $w(w=2, 20)$, and perform numerical simulation to obtain Figure 7.

In Fig. 7, the point at which the maximum value on the $P_{tr} - E(t)$ curve is marked. The simulation results show that as the number of warheads $w$ increases, the value of $E(t)$ increases. However, in actual battlefield environments, the number of false targets is much higher than the number of true targets, so warheads are given. Each curve of the number $w$ will have a maximum point. As the $P_{tr}$ continues to increase, the number of killing targets will increase dramatically, resulting in a $P_{tr} - E(f)$ change curve.
Figure 7. The curve of $E(t_1)$ and $\sigma$, changing with $P_{tr}$

By setting the target number threshold for killing and deterrence, i.e., $E[f] \leq 0.1$, the vertical diagonal line of each curve in the $P_{tr} - E(t_1)$ curve variation diagram can be obtained, which represents the constraint of $E[f] \leq 0.1$ on the false target killing. The variation curve on the left side of the vertical diagonal line does not satisfy the threshold constraint condition. Under the condition that the constraint is satisfied, $P_{tr}$ continues to increase, resulting in a decrease in $E(t_1)$. Especially when the number of warheads is $w > 12$, the increase in the number of warheads $w$ has less effect on the increase in $E(t_1)$, so when designing a UAV that is capable of reconnaissance and combat, UAVs can be allowed to carry up to 12 warheads.

In addition, in order to ensure the credibility of the above research, the variance $\sigma$ of the expected value $E(t_1)$ of the true target killing quantity and the variance $\sigma_f$ of the expected value of the false target killing quantity are introduced. Study the curve between the $E(t_1)$, $\sigma$, and the true target interpretation probability $P_{tr}$ under the same parameter setting conditions to determine the stability of the above research results. Set $q=18, \lambda_{tr}=20, \lambda_c=10, P_{tr}=0.8, E[f] \leq 0.1$ increase the expected threshold $E[f] + \sigma_f \leq 0.1$ of the number of false targets, and adjust the number of warheads $w (w = 2: 20)$ uniformly.

The numerical simulation results in $P_{tr} - E(t_1)$ $P_{tr} - E(f)$ the curve changes to Figure 8.

In Fig. 8, due to the addition of a variance constraint condition involving the expected number of false target kills, the same method as described above can be used to obtain two constraint lines, one for the $E[f] \leq 0.1$ constraint and the other for the $E[f] + \sigma_f \leq 0.1$. 
The numerical simulation did not simulate the change curve of $P_{tr}, E[f]$. To meet the threshold limit of $E[f]$, the parameters of the sensor DDROC curve need to be adjusted, but due to the reduction of $P_{tr}$, a significant decrease in $E(t_i)$ will be caused. In order to consider the influence of variance, calculate $E(t_i) - \sigma$ for any given $P_{tr}$, $w$ value, and simulate the given $E(t_i) - \sigma$ value and warhead number $w$ to obtain the result shown in Figure 8 below.

In Fig.8, the curve change graph can more directly show the influence of the two constraint line pair $E(t_i) - \sigma$. It can be seen that when $w = 12$ is obtained, $E(t_i)$ is the largest, and the combat effectiveness is relatively optimal at this time. If the number of warheads continues to increase, $w$ has little effect on the improvement of combat effectiveness. It can be seen that $w = 12$ is the optimal number of warhead designs.

In order to verify the validity of this method, we compare the method with Vague Comprehensive Evaluation method and SEA method. We set the warhead number $w = 12$, true Target interpretation probability $P_{tr} = 0.84$, sensor ROC curve parameter $q = 10$. Under these conditions, the effectiveness evaluation results of the three methods are as follows.

**Table 2.** The effectiveness evaluation results of the three methods

| Conditions setting | The method in this paper | Vague Comprehensive Evaluation Method | SEA |
|--------------------|--------------------------|--------------------------------------|-----|
| $E(t_i)$           | $w = 12$                 | 0.97                                 | 0.93| 0.88|
|                    | $P_{tr} = 0.84$          | 7.5                                  | 6.9 | 7.4 |
|                    | $q = 10$                 | 4.88                                 | 4.68| 4.66|

From the above table, the UAV combat effectiveness measurement method in the uncertain environment of this study is more effective than the fuzzy comprehensive evaluation method and the SEA method.

5. Conclusion
This paper studied the operational effectiveness of reconnaissance and combat UAVs. We analysed the operational effectiveness and main influencing factors of a single UAV in reconnaissance and strike missions. Based on the expectations formula, we selected three factors affecting UAV combat
effectiveness. They are the true target interpretation probability, sensor parameters, and the number of warheads. By performing numerical simulation analysis, we get their values when UAV combat effectiveness is optimal: the warhead number \( w = 12 \), true Target interpretation probability \( P_{TR} = 0.84 \), sensor ROC curve parameter \( q = 10 \). The effectiveness of this method is also illustrated by comparison.

6. Reference

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