The redefined SI and the electromagnetic quantities in detail – part II: resistance, conductance, charge, capacitance, inductance, power, magnetic flux density and magnetic flux

R P Landim, H R Carvalho and V C de Oliveira
Instituto Nacional de Metrologia, Qualidade e Tecnologia – Inmetro
E-mail: rplandim@inmetro.gov.br

Abstract. This paper describes the changes in the electromagnetic units due to the redefined SI and the details regarding how they are realized. It is divided into two parts. In this second part, we review the state-of-the-art performance, the physical principles, diagrams, electrical circuits, equations and uncertainties of the ohm, siemens, coulomb, farad, henry, watt, tesla and weber realizations.

1. Introduction
Part I of this review paper [1] presented the changes in the electromagnetic units due to the redefined SI and the details regarding how the ampere and volt units are realized. In this review paper (part II), we present the details regarding how the resistance, conductance, charge, capacitance, inductance, power, magnetic flux density and magnetic flux units are realized in the redefined SI. We discuss the physical principles, diagrams, electrical circuits, equations and uncertainties.

2. The Redefined SI and the Electrical Quantities

2.1. Mise en pratique (MeP) for the definition of the ampere and other electric units in the SI [2]

2.1.1. Practical realization of the ohm, Ω, SI derived unit of electric resistance and impedance. The ohm Ω can be realized as follows:

a) by using the quantum Hall effect in a manner consistent with the CCEM Guidelines [3] and the following value of the von Klitzing constant \( R_K \):

\[
R_K = \frac{h}{e^2} \approx 25 812.807 459 304 5 \ \Omega
\]

Similarly to the Josephson constant \( K_J \) case, \( R_K \) numerical representation to practical application has been calculated to 15 significant digits, which is in error by less than 1 part in \( 10^{15} \), therefore intended to be negligible in the vast majority of applications.

In 1980, von Klitzing observed the quantum Hall effect when a semiconductor device containing a two-dimensional electron gas was placed under a strong magnetic field at very low temperatures [4]. The semiconductor device may be MOSFET, gallium arsenide (GaAs/GaAlAs heterostructures) or, ultimately, graphene.
In the case of GaAs/GaAlAs heterostructures, gallium arsenide with one region which was doped with aluminum, the electric current passes only at the interface between the two regions. When this semiconductor is placed at high magnetic field and very low temperatures, the electrical resistance is no longer a linear function of the applied magnetic field (as in the classic Hall effect). In the graph of resistance versus magnetic field, as shown in figure 1, some plateaus appear. In each of these ranges of magnetic field, the resistance value is the same over each plateau. These values are given by \( R_H(i) = R_K/i \), where \( i \) is an integer number referring to the plateau index. At a plateau, the value of longitudinal voltage, \( V_L \) (measured between two points in the same device lateral) is zero and this is used to confirm the quantum regime on plateau.

![Figure 1: Graph \( R_H \) and \( V_L \) versus Magnetic Field.](image)

The measurement system commonly used for calibrations from quantum Hall resistances is the Cryogenic Current Comparator (CCC) [5]. Its circuit is shown in figure 2, where \( R_2 \) is a quantum Hall device and \( R_1 \) is a resistor under test.

![Figure 2: Cryogenic Current Comparator circuit.](image)

The coils 1 and 2 have turn numbers \( N_1 \) and \( N_2 \), respectively. Coils 1, 2 and B (this latter with \( N_B \) turns) generate magnetic flux whose net flux is detect by a superconductive quantum interference device, or SQUID (coil 1 is wound in the opposite direction relative to the others). Let us consider \( I_B \) current equals to zero, for a while. The current source of the master arm generates the current \( I_1 \) through the arm where the resistor \( R_1 \) is located while another source (slave) generates the current \( I_2 \) which flows through the resistor \( R_2 \). \( I_2 \) current is adjusted by the SQUID signal through digital control. This adjustment makes the balance \( I_1N_1 - I_2N_2 = 0 \). The ratio \( N_1/N_2 \) is chosen to be as close as possible to the nominal ratio \( R_1/R_2 \). The voltage measured between the resistors on the nanovoltmeter (“V”) is equal to \( R_1I_1 - I_2R_2 \).
A signal from the nanovoltmeter proportional to this imbalance (analog output, for example) is used to balance the bridge. Through digital control, this signal is used to make a CCC balance fine adjustment by $I_B$ current which flows through the coil B as shown in figure 2.

Once the CCC bridge is balanced, with the coil B effect in the circuit, the following equations hold:

$$\begin{align*}
N_1 I_1 - N_2 I_2 - N_B I_B &= 0 \\
R_1 I_1 - R_2 I_2 &= 0
\end{align*}$$

From the equation system above, one can get:

$$\frac{R_1}{R_2} = \frac{N_1}{N_2} \left( 1 - \frac{I_B N_B}{I_1 N_1} \right)$$

Equation (3) shows the ratio of the two resistors involved. So, if one of them is known (standard), the other can be determined.

b) by comparing an unknown resistance to the impedance of a known capacitance using, for example, a quadrature bridge, where, for example, the capacitance has been determined by means of a calculable capacitor and the value of the electric constant given by equation (12) in section 2.1.9 below.

The SI capacitance unit (farad) can be realized using a calculable capacitor (see Sec. 2.1.4 (b)), whose impedance can be directly compared to the unknown resistance using a quadrature bridge. For instance, a 1 pF calculable capacitor is used as a reference for a 10 nF capacitor, through capacitance bridges. When the quadrature bridge is balanced, the capacitive reactance of the 1 nF capacitor is equal to the resistance of the unknown standard resistor, and the following equation is valid (figure 3):

$$R = \frac{1}{2 \pi f C}$$

where $R$ is the resistance (around 10 kΩ); $f$ is the frequency (around 1.59 kHz) of the applied ac voltage; $C$ is the capacitance (around 10 nF).

A fine adjustment of $f$ will allow the capacitive reactance to be as close as possible (in module) of the resistance.

![Figure 3: From the farad to the ohm and vice-versa [6].](image-url)
2.1.2. Practical realization of the siemens, S, SI derived unit of electric conductance

The siemens S can be realized from a realization of the ohm (see Sec. 2.1.1) since S is related to Ω by the unit relation S = Ω⁻¹.

2.1.3. Practical realization of the coulomb, C, SI derived unit of electric charge, Q

The coulomb C can be realized as follows:

a) by measuring the duration in terms of the SI unit of time, the second s, of the flow of an electric current known in terms of the ampere realized as indicated in [1], [2], according to equation (5).

\[ Q = \int i \, dt \]  \hspace{1cm} (5)

b) “by determining the amount of charge placed on a capacitance known in terms of the farad F realized by any of the methods in Sec. 2.1.4, using the relation \( Q = C \cdot V_{\text{CAP}} \) and its unit relation \( C = F \cdot V \)” (where C is coulomb, the unit of electric charge \( Q \); F is farad, the unit of capacitance \( C \); and V is volt, the unit of capacitance voltage \( V_{\text{CAP}} \)). By measuring the voltage across the capacitance in terms of the volt V as realized by the Josephson effect and the value of the Josephson constant given in [1], [2], one can get the electric charge \( Q \) (figure 4, equation (6)).

![Figure 4: Simplified electrical diagram for electric charge measurement in a capacitor using a Josephson voltage standard (JVS).](image)

\[ Q = C \times (V_{\text{PJVS}} - V_{\text{DVM}}) \]  \hspace{1cm} (6)

c) “by using a SET or similar device to transfer a known amount of charge based on the value of \( e \), given in the definition of the ampere, onto a suitable circuit element”.

By definition, \( e = 1.602 \, 176 \, 634 \times 10^{-19} \) C. Since single-electron tunneling transistors (SETT) are capable of controlling the passage of one single electron [1], [2], they are natural electrometers [7]. SETT electrometers can detect \( \sim 10^{-5} \) e (that means \( \sim 1.6 \times 10^{-24} \) C) in a 1 Hz bandwidth [8].

2.1.4. Practical realization of the farad, F, SI derived unit of capacitance

The farad F can be realized as follows:

a) by comparing the impedance of a known resistance obtained using the quantum Hall effect and the value of the von Klitzing constant given in equation (1) (see Sec. 2.1.1), including a quantized Hall resistance itself, to the impedance of an unknown capacitance using, for example, a quadrature bridge. In this case, it is taken the opposite direction followed in Sec. 2.1.1 (b). See figure 3.

b) by using a calculable capacitor and the value of the electric constant \( \varepsilon_0 \) given by equation (12).

The Thompson-Lampard theorem [9] is used to construct calculable capacitors. Such capacitors’ materials, dimensions and parameters are designed in order to get a direct relationship between one accurately variable (distance, traceable to the SI) and the capacitance. For instance, in a cylindrical cross-capacitor [10], [11], four cylinders compose two (crossed) capacitors (\( C'_{1} \) and \( C'_{2} \), figure 5). These capacitors can be calculated by equation (7) below:
\[
C_1' = C_2' = C = \varepsilon_0 \frac{\ln 2}{\pi} L
\]  

(7)

where \(\varepsilon_0\) is the electric constant given by equation (12) and \(L\) is the length measured by a laser interferometer.

![Cylindrical calculable capacitor](image1)

![Capacitances](image2)

Figure 5: (a) Cylindrical calculable capacitor (b) Capacitances [11].

2.1.5. Practical realization of the henry, \(H\), SI derived unit of inductance

The henry \(H\) can be realized as follows:

a) “by comparing the impedance of an unknown inductance to the impedance of a known capacitance with the aid of known resistances using, for example, a Maxwell-Wien bridge, where the known capacitance and resistances have been determined, for example, from the quantum Hall effect and the value of \(R_K\) given in equation (1)” (see Secs. 2.1.1 and 2.1.4).”

When the Maxwell-Wien bridge (figure 6) is balanced, the following equation is valid:

\[
L_X = C_3 R_1 R_2
\]  

(8)

![Maxwell-Wien bridge](image3)

Figure 6: Maxwell-Wien bridge [12].

b) “by using a calculable inductor of, for example, the Campbell type of mutual inductor and the value of the magnetic constant \(\mu_0\) given by equation (15).”.

Campbell calculable mutual inductance standards are based on a coaxial set of coils: two primary coils and a secondary one, arranged according to figure 7 [12], [13]. The calculated mutual inductance of this design only requires five accurate dimensional measurements (such as the mean...
primary coil diameter and pitch length, and secondary coil diameter, width and depth), bringing more accuracy (at that time) than the usual self-inductance standards (which required a larger number of dimensional measurements) [12].

![Figure 7: A Campbell mutual inductor [12], [13].](image)

2.1.6. Practical realization of the watt, $W$, SI derived unit of power

“The watt $W$ can be realized using electrical units by using the fact that electric power is equal to current times voltage, the unit relation based on Ohm’s law, $W = V^2/\Omega$, and realizations of the volt and ohm using the Josephson and quantum Hall effects and the values of the Josephson and von Klitzing constants given in [1] and in equation (1)” (see Sec. 2.1.1).

2.1.7. Practical realization of the tesla, $T$, SI derived unit of magnetic flux density

The tesla $T$ can be realized as follows:

a) “by using a solenoid, Helmholtz coil or other configuration of conductors of known dimensions carrying an electric current determined in terms of the ampere realized as discussed in [1], and the value of the magnetic constant $\mu_0$ given in equation (15) in the calculation of the magnetic flux density generated by the current carrying conductors”.

Helmholtz coil is based on the Biot-Savart law, which states that the magnetic field $\vec{B}$ from a wire length $ds$, carrying a steady current $I$ is given by:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I ds \times \vec{r}}{r^3}$$

where $\mu_0$ given in equation (15) and $\vec{r}$ is the displacement vector from the current element $I ds$ to a point P where one wish to evaluate the magnetic field.

The magnitude of the magnetic field $B$ at the center of a circular loop of radius $R$, carrying steady current $I$ through a coil of $N$ turns can be expressed as

$$B = \frac{\mu_0 I N}{2 R}$$

One can get a stronger and more uniform magnetic field by using two identical current loops, aligned along their axis and separated by a distance $R$ (identical to the radius), each carrying equal steady currents in the same direction. This is known as “Helmholtz coil” (figure 8) and the magnetic field at the center O of this configuration is given by [14], [15]:

$$B = \frac{8}{5\sqrt{5}} \frac{\mu_0 I N}{R}$$
Nuclear magnetic resonance (NMR) is a technique which uses the magnetic resonance principle: a collection of nuclei of atoms with magnetic properties is distributed onto various energy levels defined by the orientation of their magnetic moments with respect to an external magnetic field. After reaching the so-called thermal equilibrium, nuclei are irradiated by a second weak radiofrequency field. The excited nuclei give back their excess energy and return to low energy levels.” [17].

Taking the opposite path, one can measure the resonance frequency of protons (hydrogen nuclei) in the magnetic field and get its estimated value. Since the resonance frequency depends only on atomic constants and the strength of the ambient magnetic field, the accuracy of this type of measurement can reach 1 μT/T (an ac component of 100 nT at a right angle to the main field will produce only 50 pT offset in the field of 50,000 nT) [18].

2.1.8. Practical realization of the weber, Wb, SI derived unit of magnetic flux

The weber Wb can be realized from the tesla based on the unit relation Wb = T m² or from the volt based on the unit relation Wb = V s. Use can also be made of the fact that the magnetic flux quantum Φ₀, which characterizes the magnetic properties of superconductors, is related to h and e as given in [2] by the exact relation Φ₀ = ℏ/2e = 1/K₄ ≈ 2.067 833 848 461 93 × 10⁻¹⁵ Wb.

2.1.9. Magnetic constant μ₀ and related quantities

“The new definitions of the kilogram, ampere, kelvin, and mole do not alter the relationships among the magnetic constant (permeability of vacuum) μ₀, electric constant (permittivity of vacuum) ε₀, characteristic impedance of vacuum Z₀, admittance of vacuum Y₀, and speed of light in vacuum c. Moreover, they do not change the exact value of c, which is explicit in the definition of the SI base unit of length, the metre, m. The relationships among these constants are”:

\[ \varepsilon_0 = \frac{1}{\mu_0 c^2} \]  
\[ Z_0 = \mu_0 c = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2} \]  
\[ Y_0 = \frac{1}{\mu_0 c} = \frac{\varepsilon_0 c}{\mu_0} = \left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2} = \frac{1}{Z_0} \]

“However, the new definitions do affect the value of μ₀, and hence the values of ε₀, Z₀, and Y₀. In particular, μ₀ no longer has the exact value 4π × 10⁻⁷ N A⁻² and must be determined experimentally. The value of μ₀ can be obtained with a relative standard uncertainty, u₀, identical to that of the fine structure constant α from the exact relation”:
\[ \mu_0 = \frac{\alpha_0 2 h}{c e^2} \]  

Since \( h, c, \) and \( e \) have fixed numerical values, and considering equations (12)-(15):

\[ u_r(h) = u_r(c) = u_r(e) = 0 \]  
\[ u_r(Y_0) = u_r(Z_0) = u_r(c_0) = u_r(\mu_0) = u_r(\alpha) \]

“The recommended values of \( h, e, k, \) and \( N_\lambda \) resulting from the 2017 CODATA special least-squares adjustment of the values of the fundamental constants [19] were the basis of the exact values used for these four constants in the new definitions of the kilogram, ampere, kelvin, and mole adopted by the 26th CGPM [20]. The 2017 special adjustment but with \( h, e, k, \) and \( N_\lambda \) taken to have the exact values used in the new definitions, yields the following currently recommended value of the magnetic constant”:

\[ \mu_0 = 12.566 \, 370 \, 6169(29) \times 10^{-7} \, \text{N} \, \text{A}^{-2} \]

“However, users should always compute the value from the most recent CODATA adjustment [16]. The values and uncertainties of the electric constant, characteristic impedance of vacuum and characteristic admittance of vacuum may always be obtained from the relationships of equations (12)-(15)”.

“It should be recognized that the recommended values for \( \mu_0, c_0, Z_0, \) and \( Y_0 \) are expected to change slightly from one future CODATA adjustment to the next, as new data that influence the value of \( \alpha \) become available. Of course, the values of \( h, e, k, \) and \( N_\lambda \) fixed by the new definitions will be unchanged from one adjustment to the next”.

3. Conclusions

This review paper (part I and part II) presented the changes in the electromagnetic units due to the redefined SI and the details regarding how they are realized. The physical principles, diagrams, electrical circuits, equations and uncertainties were discussed as well. Although some research in electrical current realizations are still needed, the redefined SI and the state-of-the art in precision electromagnetic measurements assure extremely low uncertainties, far beyond the most demanding needs in industry, scientific research and for the development of new technologies and products. Also, the redefinition of the SI could not be noticed by the public, since it had negligible numeric effect, hence not jeopardizing world trade relations.

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