The Principle and Accuracy Analysis of Automatic Target-Scoring System Based on Acoustic Detection

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Abstract. This paper derived the principles of 3-station TDOA (time difference of arrival) location and 4-station TDOA location based on acoustic detection, and the theoretical accuracy analysis of the two kinds of TDOA location is carried out. Under the assumption that the time difference measurement error is linearly related to the distance of sound propagation, the influence of different station distribution modes on the location accuracy is analyzed. The results show that with reasonable distribution of measurement points, the automatic target-scoring system based on acoustic detection can meet the requirement of impact measurement in missile test range. Compared with traditional target-scoring system, the automatic target-scoring system based on acoustic detection has its unique advantages, such as unattended, fast, automatic and high-accuracy. The research has engineering reference and practical applied value.

Introduction

The attack accuracy has always been an important index for evaluating missile test. In missile test tasks, it is usually required that the range report the real-time measurement results of the landing point of missile. At present, many optical measuring equipments are deployed around the theoretical landing point, after the missile landed, the observation angle data is sent to the remote control station in real time to calculate the coordinates of the landing point. This measurement method consumes lots of time, labour and material resources. Therefore, it is necessary to research a new method of measuring the landing point to make up for the shortcomings of the existing methods. Multipoint passive TDOA location technology has been widely used in military, civil and other industries. Multipoint location technology has developed from basic three-station TDOA to multi-station and multi-station networking technology [1, 2]. The landing or airburst of missile will produce a huge sound, so the multi-station TDOA location based on acoustic detection can be considered for measuring the landing or airburst point.

The TDOA location requires three or more detection stations. The principles of TDOA location with three stations and four stations are deduced in this paper. The location accuracy is related to the location of the detection stations and the target. In this paper, the accuracies of 3-station TDOA and 4-station location are theoretically analyzed, and the location accuracies of several typical station distribution ways are simulated and analyzed. Finally, the ideal station distribution way is obtained, which has practical value.

Principle of Multi-station TDOA Location Based on Acoustic Detection

There are usually three stations, four stations or more stations in multi-station TDOA location system. In theory, more stations are distributed, the location accuracy will be higher, but the cost of station distribution will be higher, which also affect the calculation speed [3]. Three stations and four stations are two typical forms in engineering. This paper focuses on these two cases.
Principle of 2D 3-station TDOA Location

As shown in Fig. 1, assume that $P$ is the point to be measured. $O$, $A$ and $B$ are three acoustic detection points. $P$, $O$, $A$ and $B$ are all on the same plane, which corresponds to the landing situation of the missile. Without loss of generality, it is advisable to set $O$ as the origin of coordinates. Assume that the distances from $P$ to the three detection points are $r_0$, $r_1$, and $r_2$, respectively. The time difference between the sound reaches points $A$ and $O$ is $\Delta t_1$, which for points $B$ and $O$ is $\Delta t_2$, the sound velocity is $v$. According to the geometric relation, the following formulas can be obtained:

\[
\begin{align*}
\Delta r_1 &= v \cdot \Delta t_1 = r_1 - r_0 = \sqrt{(x-x_1)^2 + (y-y_1)^2} - \sqrt{x^2 + y^2}, \\
\Delta r_2 &= v \cdot \Delta t_2 = r_2 - r_0 = \sqrt{(x-x_2)^2 + (y-y_2)^2} - \sqrt{x^2 + y^2}.
\end{align*}
\]

(1)

Transform Eq. 1, we can get

\[
\begin{align*}
x_1 \cdot x + y_1 \cdot y &= \frac{1}{2} (x_1^2 + y_1^2 - \Delta r_1^2) - \Delta r_1 \cdot r_0, \\
x_2 \cdot x + y_2 \cdot y &= \frac{1}{2} (x_2^2 + y_2^2 - \Delta r_2^2) - \Delta r_2 \cdot r_0.
\end{align*}
\]

(2)

Let matrices

\[
A = \begin{bmatrix} x_1 & y_1 \\
                x_2 & y_2 \end{bmatrix},
\]

(3)

\[
X = \begin{bmatrix} x \\
y \end{bmatrix}^T,
\]

(4)

\[
B = \begin{bmatrix} \frac{1}{2} (x_1^2 + y_1^2 - \Delta r_1^2) - \Delta r_1 \cdot r_0 \\
                \frac{1}{2} (x_2^2 + y_2^2 - \Delta r_2^2) - \Delta r_2 \cdot r_0 \end{bmatrix} \triangleq \begin{bmatrix} k_1 - \Delta r_1 \cdot r_0 \\
k_2 - \Delta r_2 \cdot r_0 \end{bmatrix},
\]

(5)

then Eq. 2 can be expressed as

\[
AX = B.
\]

(6)

When points $O$, $A$ and $B$ are not collinear, the matrix $A$ is invertible, thus $X = A^{-1}B$. Let

\[
A^{-1} = \begin{bmatrix} a_{11} & a_{12} \\
                       a_{21} & a_{22} \end{bmatrix},
\]

(7)
then

\[
\begin{align*}
\{ & x = a_1 \cdot k_1 + \Delta a_1 \cdot \Delta r_1 + a_2 \cdot \Delta r_2 \cdot r_0 \triangleq b_1 \cdot b_2 \cdot r_0, \\
& y = a_2 \cdot k_1 + \Delta a_2 \cdot \Delta r_1 + a_2 \cdot \Delta r_2 \cdot r_0 \triangleq b_2 \cdot b_2 \cdot r_0. \\
\}
\]

(8)

Bring Eq. 8 into the formula 

\[
r_0 = \sqrt{x^2 + y^2} ,
\]

we can obtain

\[
0 = (b_2^2 + b_2^2 - 1) \cdot r_0^2 - 2(b_1 \cdot b_2 + b_2 \cdot b_2 \cdot r_0) + (b_1^2 + b_2^2) \triangleq a \cdot r_0^2 - b \cdot r_0 + c .
\]

(9)

By solving the above quadratic equation, the value of \( r_0 \) can be obtained, and \( r_0 \geq 0 \). For quadratic Eq.9, when \( b^2 - 4ac < 0 \), there is no solution, it is due to the acoustic detection errors. When \( b^2 - 4ac \geq 0 \), there are two solutions, if the two solutions are both negative, this is obviously impossible and which is also due to the acoustic detection errors. If one of the two solutions is positive and the other is negative, then bring the positive one in Eq. 8 and the coordinates of the target point P can be obtained. If the two solutions are both positive, then a fuzzy solution appears, the correct solution can be obtained by adding an acoustic detection station or adding the azimuth detection function of the acoustic detection station.

**Principle of 3D 4-station TDOA Location**

As shown in Fig. 2, assume that \( P \) is the point to be measured. \( O, A, B \) and \( C \) are four acoustic detection points. \( P, O, A, B \) and \( C \) are distributed in 3D (three-dimensional) space, which corresponds to the airburst situation of the missile. Similar to the above, set \( O \) as the origin of coordinates. Assume that the distances from \( P \) to the four detection points are \( r_0, r_1, r_2 \) and \( r_3 \), respectively. The time difference between the sound reaches points \( A \) and \( O \) is \( \Delta t_1 \), which for points \( B \) and \( O \) is \( \Delta t_2 \), which for points \( C \) and \( O \) is \( \Delta t_3 \), the sound velocity is \( v \). Similar to the above deductions, Let matrixes

\[
A = \begin{bmatrix}
\begin{array}{c}
 x_1 \\
y_1 \\
z_1 \\
 x_2 \\
y_2 \\
z_2 \\
 x_3 \\
y_3 \\
z_3 \\
\end{array}
\end{bmatrix},
\]

(10)

\[
X = \begin{bmatrix}
 x \\
y \\
z \\
\end{bmatrix}^T,
\]

(11)

Figure 2. Diagram of 3D 4-station TDOA Location.
\[
B = \begin{bmatrix}
\frac{1}{2}(x^2_1 + y^2_1 + z^2_1 - \Delta r^2_1) - \Delta r_1 \cdot r_0 \\
\frac{1}{2}(x^2_2 + y^2_2 + z^2_2 - \Delta r^2_2) - \Delta r_2 \cdot r_0 \\
\frac{1}{2}(x^2_3 + y^2_3 + z^2_3 - \Delta r^2_3) - \Delta r_3 \cdot r_0
\end{bmatrix}
\]
\[
\mathbf{A}^{-1} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]
we can obtain
\[
AX = B.
\]
When \(O, A, B\) and \(C\) are not coplanar, the matrix \(A\) is invertible, thus \(X = A^{-1}B\). Let

\[
\begin{aligned}
x &= a_{11} \cdot k_1 + a_{12} \cdot k_2 + a_{13} \cdot k_3 - (a_{11} \cdot \Delta r_1 + a_{12} \cdot \Delta r_2 + a_{13} \cdot \Delta r_3) \cdot r_0 \triangleq b_{11} - b_{12} \cdot r_0 \\
y &= a_{21} \cdot k_1 + a_{22} \cdot k_2 + a_{23} \cdot k_3 - (a_{21} \cdot \Delta r_1 + a_{22} \cdot \Delta r_2 + a_{23} \cdot \Delta r_3) \cdot r_0 \triangleq b_{21} - b_{22} \cdot r_0 \\
z &= a_{31} \cdot k_1 + a_{32} \cdot k_2 + a_{33} \cdot k_3 - (a_{31} \cdot \Delta r_1 + a_{32} \cdot \Delta r_2 + a_{33} \cdot \Delta r_3) \cdot r_0 \triangleq b_{31} - b_{32} \cdot r_0
\end{aligned}
\]
Bring Eq. 15 into the formula \(r_0 = \sqrt{x^2 + y^2 + z^2}\), we can get
\[
0 = (b_{12}^2 + b_{22}^2 + b_{32}^2 - 1) \cdot r_0^2 - 2(b_{11}b_{12} + b_{21}b_{22} + b_{31}b_{32}) \cdot r_0 + (b_{11}^2 + b_{21}^2 + b_{31}^2)
\triangleq a \cdot r_0^2 - b \cdot r_0 + c.
\]
Similarly, by solving the above quadratic equation, the value of \(r_0\) can be obtained, then the coordinates of point \(P\) are obtained. In the same way, the ambiguity can be eliminated by adding an acoustic detection station or adding the azimuth detection function of the acoustic detection station.

**Accuracy Analysis of Multi-station TDOA Location**

**Accuracy Analysis of 2D 3-station TDOA Location**

By using differential method on Eq. 1, the following formulas can be obtained:

\[
\begin{aligned}
v \cdot d(\Delta t_1) &= \left(\frac{x - x_1}{r_1} - \frac{x}{r_0}\right)dx + \left(\frac{y - y_1}{r_1} - \frac{y}{r_0}\right)dy \\
v \cdot d(\Delta t_2) &= \left(\frac{x - x_2}{r_2} - \frac{x}{r_0}\right)dx + \left(\frac{y - y_2}{r_2} - \frac{y}{r_0}\right)dy
\end{aligned}
\]

Let
\[
\begin{aligned}
dT &= \begin{bmatrix} d(\Delta t_1) & d(\Delta t_2) \end{bmatrix}^T, \\
dX &= \begin{bmatrix} dx & dy \end{bmatrix}^T.
\end{aligned}
\]
\[
C = \begin{bmatrix}
\frac{x-x_1 - x}{r_1} & \frac{y-y_1 - y}{r_1} \\
\frac{x-x_2 - x}{r_2} & \frac{y-y_2 - y}{r_2} \\
\frac{x-x_3 - x}{r_3} & \frac{y-y_3 - y}{r_3}
\end{bmatrix}
\]

then Eq. 17 can be expressed as

\[
C \cdot dX = v \cdot dT.
\] (20)

Thus

\[
dX = v \cdot C^+ \cdot dT.
\] (21)

Where \( C^+ \) is the generalized inverse of \( C \). It can be seen from Eq. 22 that the location error \((dx, dy)\) is correlated with the sound velocity \(v\), the time difference measurement error \(dT\), the locations of the acoustic detection stations and the positions of point \(P\) relative to the acoustic detection stations. The location accuracy is

\[
\sigma = \sqrt{dx^2 + dy^2}.
\] (23)

### Accuracy Analysis of 3D 4-station TDOA Location

Similar to the above deduction, let

\[
dT = [d(\Delta t_1) \ d(\Delta t_2) \ d(\Delta t_3)]^T,
\] (24)

\[
dX = [dx \ dy \ dz]^T,
\] (25)

\[
C = \begin{bmatrix}
\frac{x-x_1 - x}{r_1} & \frac{y-y_1 - y}{r_1} & \frac{z-z_1 - z}{r_1} \\
\frac{x-x_2 - x}{r_2} & \frac{y-y_2 - y}{r_2} & \frac{z-z_2 - z}{r_2} \\
\frac{x-x_3 - x}{r_3} & \frac{y-y_3 - y}{r_3} & \frac{z-z_3 - z}{r_3}
\end{bmatrix}
\]

the following formula can be obtained:

\[
dX = v \cdot C^+ \cdot dT.
\] (26)

Where \( C^+ \) is the generalized inverse of \( C \). It can be seen from Eq. 27 that the location error \((dx, dy, dz)\) is correlated with the sound velocity \(v\), the time difference measurement error \(dT\), the locations of the acoustic detection stations and the positions of point \(P\) relative to the acoustic detection stations. The location accuracy is

\[
\sigma = \sqrt{dx^2 + dy^2 + dz^2}.
\] (27)

### Simulation Analysis of Station Distribution

#### Simulation Analysis of 3-station TDOA Location

Three-station TDOA location can realize 2D plane location, which is corresponding to the case of missile landing. In the following, let \(OA=OB=1000m\), the location accuracy is simulated by changing the angle between \(OA\) and \(OB\). Assuming that the measurement error of TDOA is linearly related to
the distance of sound propagation, \(d(\Delta t) = (r - r_0)/340 \times 0.001\), that is to say, the average error of sound propagation per second is 1 millisecond. Fig. 3 shows the distribution areas of location accuracy when the angle \(\angle AOB\) is 30°, 60°, 90°, 120° and 150°. The deeper the color, the higher the location accuracy. The specific statistical results are shown in Table 1. The numerical value represents the proportion of the area where the corresponding accuracy is located to the total area. The total area is the square area with a side length of 6000 meters as shown in the figure. As can be seen from the figure and the table, when the angle \(\angle AOB\) is 150°, the area of high-accuracy (less than 10 meters) is relatively large.

Table 1. Simulation Results of 3-station Location Accuracy.

| Angle(°) | 30  | 60  | 90  | 120 | 150 |
|---------|-----|-----|-----|-----|-----|
| Accuracy(m) | 0.023 | 0.026 | 0.034 | 0.050 | 0.085 |
| 1       | 0.062 | 0.072 | 0.096 | 0.165 | 0.412 |
| 5       | 0.104 | 0.113 | 0.158 | 0.294 | 0.628 |
| 10      | 0.187 | 0.191 | 0.278 | 0.469 | 0.771 |
| 20      | 0.353 | 0.349 | 0.453 | 0.604 | 0.808 |
| 40      |       |       |       |       |      |

Figure 3. Simulation Analysis Chart of 3-station Location Accuracy.

Simulation Analysis of 4-station TDOA Location

Four-station TDOA location can realize 3D spatial location, which is corresponding to the case of missile airburst. According to the actual situation, it is easy to build a tower less than 20 meters high near the landing point of the missile, and one of the acoustic detection stations can be placed on the top of the tower. Therefore, let the coordinates of point \(C\) be (0, -20, 20) and \(OA = OB = 1000\) m, similarly, the location accuracy is simulated by changing the angle between \(OA\) and \(OB\). Assuming that the measurement error of TDOA is linearly related to the distance of sound propagation, \(d(\Delta t) = (r - r_0)/340 \times 0.001\), that is to say, the average error of sound propagation per second is 1 millisecond. Fig. 4 shows the distribution areas of location accuracy when the angle \(\angle AOB\) is 30°, 60°, 90°, 120° and 150°. Different colors represent different accuracies, the larger the volume, the lower the location accuracy. The specific statistical results are shown in Table 2. The numerical
values represent the proportion of the volume of the region where the corresponding accuracy is located to the total volume. The total volume is the volume of the cube with a side length of 6000 meters as shown in the figure. As can be seen from the figure and the table, when the angle \( \angle AOB \) is 120°, the region of high-accuracy (less than 10 meters) is relatively large.

Table 2. Simulation Results of 4-station Location Accuracy.

| Angle (°) | Accuracy (m) |
|-----------|--------------|
|           | 30           | 60           | 90           | 120          | 150          |
| 1         | 0.013        | 0.019        | 0.019        | 0.000        | 0.000        |
| 5         | 0.086        | 0.131        | 0.243        | 0.312        | 0.066        |
| 10        | 0.174        | 0.232        | 0.348        | 0.533        | 0.348        |
| 20        | 0.309        | 0.368        | 0.474        | 0.651        | 0.846        |
| 40        | 0.489        | 0.530        | 0.619        | 0.755        | 0.941        |

![Figure 4. Simulation Analysis Chart of 4-station Location Accuracy.](image)

**Conclusion**

In this paper, the accuracy of the automatic target-scoring system based on acoustic detection is analyzed through theoretical deduction and simulation calculation. From the analysis results, it can be seen that for 2D 3-station TDOA location, when the distribution of the acoustic detection stations is isosceles triangle and the apex angle is 150°, which can cover relatively large area with high-accuracy (within 10 meters). For 3D 4-station TDOA location, when one of the acoustic detection stations is fixed on a tower of 20 meters high, the other three stations form an isosceles triangle on the ground and the top angle is 120°, which can cover relatively large space with high-accuracy (within 10 meters). The simulation results can be used for engineering reference. In practical application, more ground acoustic detection stations can be distributed to cover larger area with high-accuracy and eliminate location ambiguity.
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