Abstract. $D$-term inflation is one of the most interesting and versatile models of inflation. It is possible to implement naturally $D$-term inflation within high energy physics, as for example supersymmetry grand unified, supergravity, or string theories. $D$-term inflation avoids the $\eta$-problem, while in its standard form it always ends with the formation of cosmic strings. Given the recent three-year Wilkinson Microwave Anisotropy Probe data on the cosmic microwave background temperature anisotropies, we examine whether $D$-term inflation can be successfully implemented in non-minimal supergravity theories. We show that for all our choices of Kähler potential, there exists a parameter space for which the predictions of $D$-term inflation are in agreement with the measurements. The cosmic string contribution to the measured temperature anisotropies is always dominant, unless the superpotential coupling constant is fine-tuned; a result already obtained for $D$-term inflation within minimal supergravity. In conclusion, cosmic strings and their role in the angular power spectrum cannot be easily hidden by just considering a non-flat Kähler geometry.

Keywords: string theory and cosmology, inflation, physics of the early universe

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1. Introduction and motivations

Inflation offers a simple solution to the shortcomings of the standard hot big bang model. In addition, its predictions about the initial density perturbations, leading to the observed structure formation, are in a remarkable agreement with measurements of the cosmic microwave background (CMB) temperature anisotropies. Among the various inflationary models, one should select the ones which lead to a better agreement with data, with the additional requirement that such models should be naturally built within a fundamental theoretical framework. In spite of its enormous success, inflation remains still a paradigm in search of a model.

Despite the elegance of chaotic inflation [1, 2], this simple model faces a fine-tuning problem. Consistency, between predictions for the amplitude of CMB temperature anisotropies and measurements, requires a tiny coupling constant. To avoid this problem, Linde [3, 4] has proposed hybrid inflation, a model of inflation based on Einstein gravity, but driven by a false vacuum. In this model, the inflaton field rolls while another scalar field remains trapped in a false vacuum state. The false vacuum becomes unstable when the magnitude of the inflaton field falls below some critical value, leading to a phase transition to the true vacuum. The energy density is dominated by the false vacuum energy density so that the phase transition signals the end of hybrid inflation. The phase transition at the end of inflation leads to topological defect formation [5].

Theoretically motivated inflationary models can be built in a context of supersymmetry (SUSY) and supergravity (SUGRA) theories. $N = 1$ supersymmetric
models contain complex scalar fields, which often have flat directions in their potential, thus offering natural candidates for inflationary models. In this framework, hybrid inflation (driven by the $F$-terms or the $D$-terms) are the most standard models. Such inflationary models lead generically to cosmic string formation at the end of the inflationary era. Cosmic strings in supersymmetric theories may have new properties, as compared to their non-supersymmetric counterparts; we do not address this issue here.

A gauge symmetry can be broken spontaneously in $N = 1$ globally supersymmetric theories, either by adding $F$-terms to the superpotential or, in the Abelian case, by introducing Fayet–Iliopoulos (FI) $D$-terms. The Higgs mechanism leads generically [6] to Abrikosov–Nielsen–Olesen (ANO) strings. Depending whether they were formed at the end of $F$- or $D$-term inflation they are called $F$-term or $D$-term strings, respectively. $F$-term inflation is potentially plagued with the $\eta$-problem, while $D$-term inflation avoids it. This problem arises from the presence of large corrections (of the order of the Hubble parameter during inflation) to the inflaton mass, which spoil the required flatness of the inflaton potential. $D$-term inflation can be successfully implemented in the framework of SUGRA, while in addition, it can be easily accommodated within string theory models.

In the simplest models of $D$-term inflation within SUGRA, in which the constant FI term gets compensated by a single complex scalar field at the end of the inflationary era, the $D$-term strings formed at the end of inflation are topologically stable, since $\pi_1(M) \neq I$, with $M$ the vacuum manifold of the broken $U(1)$ symmetry. Our study concerns such models. However, they have been proposed [7]–[9] models where $D$-term strings can become unstable. For example, one can introduce additional matter multiplets so as to obtain a non-trivial global symmetry such as $SU(2)$, leading to a simply connected vacuum manifold and the production of semi-local strings. Alternatively, it has been suggested [9] that the waterfall Higgs fields are non-trivially charged under some other gauge symmetries $H$, such that the vacuum manifold, $[H \times U(1)]/U(1)$, is simply connected, leading to the formation of semi-local strings.

$D$-term inflation requires the existence of a non-zero constant FI term, which can be added to the Lagrangian only in the presence of a $U(1)$ gauge symmetry. This extra $U(1)$ symmetry can be of a different origin. Some models have been suggested for field-dependent FI terms, arising in the presence of a chiral superfield $\Phi$ shifting under $U(1)$. The imaginary part of the scalar part of the chiral superfield plays the role of an axion, and cancels the chiral anomaly by shifting under the $U(1)$ symmetry. Such a $U(1)$ symmetry is called anomalous, or pseudo-anomalous since the total anomaly vanishes. Here the FI term depends on the real part of the chiral superfield. In supersymmetry, models with anomalous FI terms have been developed [10] within heterotic string theory. In a cosmological set-up one has first to assume the stabilization of the chiral superfield $\Phi$; the role of $\Phi$ may be played by any modulus (a dilaton or any volume modulus). Only if this assumption holds the dilaton-dependent $D$-term can be considered as a constant FI term. However, the issue of dilaton and moduli stabilization in the heterotic string theory is far from being resolved. It is still not clear how to derive constant FI terms from string or M-theory and only field-dependent $D$-terms have been identified. As we will discuss, in absence of constant FI terms, local supersymmetry requires the superpotential to be invariant under the $U(1)$ gauge symmetry.

In the context of theories with large extra dimensions, brane inflation occurs in a similar way to hybrid inflation within supergravity, leading to cosmic string-like objects.
In string theories, D-brane–D-anti-brane annihilation leads generically to the production of lower dimensional D-branes, with D3- and D1-branes, which are D-strings, being predominant [11, 12] in IIB string theories. To illustrate brane inflation let us consider [13] a Dp–Dp system in IIB string theory. Six of the spatial dimensions are compactified on a torus, while the branes move relatively to each other in some directions. As the two branes approach, the open string modes between the branes develop a tachyon, thus an instability. Brane inflation [14] ends by a phase transition mediated by open string tachyons. Since the tachyonic vacuum has a non-trivial \( \pi_1 \) homotopy group, one concludes that there must exist stable tachyonic string solutions with \( p - 2 \) co-dimensions; they are stable BPS (Bogomol’nyi–Prasad–Sommerfield) D\((p - 2)\)-branes. Since all dimensions are compact these daughter branes are seen as one-dimensional objects for a four-dimensional observer; they are the D-strings.

The D-strings (D1-branes) have been identified [15] in the low energy supergravity with the \( D \)-term strings. The justification for this conjecture is that only \( D \)-term strings remain BPS states in \( N = 1, d = 4 \) supergravity. In supergravity, \( F \)-term strings break all the supersymmetries, whereas the \( D \)-term strings preserve half of it.

An interesting and successful brane inflationary model is the D3/D7 one [16], which has also an effective description as a \( D \)-term inflationary model. The flat direction of the inflaton potential is associated with the shift symmetry, which protects the inflaton field from acquiring a large mass that would spoil the required flatness of the potential. In its original version this model leads to the formation of topologically stable ANO BPS strings. In a later developed version [17] of the D3/D7 model, where in terms of an effective gauge theory the model has a local \( U(1) \) gauge symmetry and a global \( SU(2) \) symmetry, semi-local strings are formed. Such strings can unwind without any cost of potential energy. In what follows we concentrate on inflationary models leading to the formation of topologically stable strings.

Strings formed at the end of an inflationary era, contribute [18]–[24] in the spectrum of temperature anisotropies; their contribution is heavily constrained from CMB data. Compatibility between CMB measurements and theoretical predictions constrain [25, 26] the parameters space (mass scales and couplings) of the inflationary models. These constraints have been obtained [25, 26] for \( D \)-term inflation within minimal SUGRA. Here, we would like to investigate whether the constraints on the parameters space are a result of our choice of minimal supergravity, or whether they are a generic outcome of \( D \)-term hybrid inflation. We will therefore examine \( D \)-term inflation originated by different choices of the Kähler potential. The main motivation being that a minimal Kähler potential can be considered as a peculiar and unmotivated choice [5].

We base our study in a formulation of supergravity constructed from superconformal theory, since the standard formulation may be insufficient in the presence of constant Fayet–Iliopoulos terms [9].

To be able to constrain the parameters space of the models we should know the power spectrum of a cosmic strings network and the allowed upper limit on the cosmic string contribution to the measured temperature anisotropy spectrum. The upper limit imposed on the cosmic string contribution to the CMB data depends on the numerical simulation employed in order to calculate the cosmic string power spectrum. The upper limit found in the literature [27] is 7% or 11%, depending on the simulation, with 95% confidence level. There are some uncertainties in these results due to the cosmic string evolution.
codes they are based on and therefore in our calculations we use an average value for the upper bound equal to 9%. Note that none of the existing simulations take into account a non-trivial microstructure of the cosmic strings formed: however they have been shown to be superconducting in supersymmetric Abelian symmetry breaking through $F$- or $D$-terms [32].

We plan the rest of the paper as follows. In section 2 we address hybrid $D$-term inflation, first within the standard formulation of supergravity, and subsequently within the effective supergravity theory built upon superconformal field theory. In the rest of the paper we focus on the second approach. We briefly review the effective supergravity formulation because of its consequences for $D$-term inflation, often not taken into consideration. In section 3 we review $D$-term inflation in minimal supergravity. In section 4 we discuss inflation in a supergravity theory with shift symmetry. In section 5 we consider $D$-term inflation in supergravity models with higher order terms, including all corrections up to order $M_{Pl}^{-2}$. We round up our conclusions in section 6.

2. Hybrid inflation and supergravity formulations

We first review hybrid $D$-term inflation model in its standard supergravity formulation.

2.1. Standard formulation of supergravity

Following the standard formulation of supergravity [33,34], the general SUGRA Lagrangian for chiral superfields $\Phi_i$, and a vector superfield, depends on three generic functions: the Kähler potential $K(\Phi_i, \bar{\Phi}_i)$, the superpotential $W(\Phi_i)$, and the kinetic function $f_{ab}(\Phi_i)$ for the vector multiplets. It can be expressed as an integral over the Grassmann variables $\theta$ and $\bar{\theta}$ (over superspace), as [33]:

$$\mathcal{L}_{\text{SUGRA}} = \int d^2 \theta d^2 \bar{\theta} K(\Phi_i^* e^{2\phi} V, \Phi_i) + \int d^2 \theta [W(\Phi_i) + \text{H.c.}] + \int d^2 \theta [f_{ab}(\Phi_i) W_a^\alpha W_{ab} + \text{H.c.}],$$

where H.c. stands for Hermitian conjugate. It turns out that the effective Lagrangian obtained from equation (1) depends only on one combination of the Kähler potential $K(\Phi_i, \bar{\Phi}_i)$ and the superpotential $W(\Phi_i)$, the following:

$$G(\Phi_i, \bar{\Phi}_i) = \frac{K(\Phi_i, \bar{\Phi}_i)}{M_{Pl}^2} + \ln \frac{|W(\Phi_i)|^2}{M_{Pl}^6}. \quad (2)$$

A priori, the only restriction for the choice of the Kähler potential $K$ and the superpotential $W$ is that $K$ must be a real function and $W$ must be a holomorphic function. However, there exists a degeneracy in the choices of $K$ and $W$, while the
Lagrangian is invariant under the K"ahler transformation
\[
K(\Phi_i, \bar{\Phi}_i) \rightarrow K(\Phi_i, \bar{\Phi}_i) + h(\Phi_i) + h^*(\bar{\Phi}_i)
\]
\[
W(\Phi_i) \rightarrow e^{-h}W(\Phi_i).
\]
Assuming that the Lagrangian must be invariant also under a gauge symmetry, \(W\) and \(K\) must be invariant, at least up to a K"ahler transformation. The gauge kinetic function \(f_{ab}(\Phi_i)\) must be holomorphic and covariant under gauge symmetry. Choosing for example the minimal K"ahler potential
\[
K_{\text{min}}(\Phi_i, \bar{\Phi}_i) = \sum_i |\Phi_i|^2,
\]
the kinetic terms for the scalar parts of the \(\Phi_i\)’s are simply
\[
K_i^j D_\mu \phi_i D^\mu \phi_j^* = D_\mu \phi_i D^\mu \phi_i^*.
\]
In minimal SUGRA, we also set the gauge kinetic function \(f_{ab}\) equal to \(\delta_{ab}\).

Within this framework of supergravity, and assuming that there is a gauge symmetry, the scalar potential \(V\) for the scalar components \(\phi_i\) of the superfields \(\Phi_i\) reads
\[
V = \frac{e^G}{M_{\text{Pl}}^2} [G_i(G^{-1})_j G^j - 3] + \frac{1}{2} [\text{Re} f_{ab}(\Phi_i)]^{-1} \sum_a g_a^2 D_a^2;
\]
the first term in the rhs of equation (6) is called \(F\)-term and the second one \(D\)-term. While the \(F\)-term has in general positive and negative contributions, the \(D\)-term is positive definite. \(D_a\) is given by
\[
D_a = \phi_i(T_a)_i^j K_j^i + \xi_a,
\]
where the Fayet–Iliopoulos term, \(\xi_a\), is non-zero only if the gauge symmetry is Abelian; \(g_a\) is the gauge coupling of the symmetry generated by \(T_a\). Assuming a constant FI term, \(D\)-term potential may lead to de Sitter type of solutions, which is particularly interested for building an inflationary model within supergravity.

The standard \(D\)-term hybrid inflation model is based on the superpotential \([36, 35]\]
\[
W = \lambda S\Phi_+\Phi_-,
\]
where \(S, \Phi_+, \Phi_-\) are three chiral superfields and \(\lambda\) is the superpotential coupling. This model assumes an invariance under an Abelian gauge group \(U(1)_\xi\), under which the three superfields, \(S, \Phi_+, \Phi_-\), have charges 0, +1 and −1, respectively. This model also assumes the existence of a constant FI term\(^5\).

However, the above summarized supergravity formulation (called hereafter standard) is inappropriate to describe \(D\)-term inflation \([9]\). Indeed, in \(D\)-term inflation the superpotential vanishes at the unstable de Sitter vacuum, as it also vanishes in the absolute Minkowski vacuum; anywhere else the superpotential is non-zero. Thus, the standard formulation of supergravity, where the Lagrangian depends on \(K\) and \(W\) only through the combination given in equation (2) is inappropriate, since the theory is ill defined at \(W = 0\). In conclusion, \(D\)-term inflation in supergravity must be described with a non-singular formulation of SUGRA when the superpotential vanishes.

\(^4\) We use the notations of \([33]\) for indices.

\(^5\) A supersymmetric description of the standard \(D\)-term inflation is insufficient, the reason being that the inflaton field reaches values of the order of the Planck mass, or above it, even if one concentrates only around the last 60 e-folds of inflation \([26, 25]\). The correct analysis is indeed in the context of supergravity \([26, 25]\).
2.2. Effective supergravity from superconformal field theory

2.2.1. Superconformal Lagrangian. Various formulations of effective supergravity can be constructed from the superconformal field theory (see [37] for a recent review). The main idea is, in a first stage, to build a Lagrangian with full superconformal theory. In a second stage, the gauge symmetries that are absent in Poincaré supergravity (e.g., local dilatations, local chiral $U(1)$ symmetry and local $S$ supersymmetry) are gauge fixed. Starting from this framework in order to derive the effective supergravity theory, one can construct a non-singular theory at $W = 0$, where the action depends on all three functions, $K, W$ and $f_{\alpha\beta}$.

The superconformal field theory (SCFT) is based on the $SU(2, 2 | 1)$ symmetry. The superconformal Lagrangian contains three parts, each of them being separately conformally invariant. For $n + 1$ chiral multiplets $X_I$ (with $(X_I)^* \equiv X^I$) and some vector multiplets $\lambda^\alpha$ superconformally coupled to supergravity, the superconformal Lagrangian reads [9]

$$\mathcal{L}_{\text{SCFT}} = \left[ N(X, X^*)\right]_D + \left[ W(X)\right]_F + \left[ f_{\alpha\beta}(X) \bar{\lambda}^\alpha \lambda^\beta \right]_F. \tag{9}$$

The homogeneous function $N(X, X^*)$ and the holomorphic functions $W(X), f_{\alpha\beta}(X)$ encode the Kähler potential, the superpotential and gauge kinetic function, respectively, once the extra gauge symmetries have been gauge fixed.

After constructing the Lagrangian, we fix the extra symmetries. Fixing local dilatation, the number of chiral scalars is decreased by one: there will be only $n$ physical scalar fields in supergravity. Fixing $S$ supersymmetry makes a free fermion field to be removed. One thus makes a transformation from the $n + 1$ variables $X_I$ to the conformon scalar $Y$ and $n$ physical scalars $z_i$, the $n$ chiral superfields of standard supergravity. More precisely,

$$X_I = Y x_I(z_i), \tag{10}$$

where $x_I$ are a set of holomorphic functions. The Kähler potential $K$ is related to the function $N(X, X^*)$, appearing in the superconformal Lagrangian, and the conformon superfield $Y$ through

$$K(z, z^*) = -3 \ln \left( -\frac{1}{3} N/YY^* \right). \tag{11}$$

Chiral and dilatation symmetry imply that the holomorphic function $W$ is

$$W(Y, z) = Y^3 M^{-3}_P W(z). \tag{12}$$

Apart the local $SU(2, 2 | 1)$ symmetry, the Lagrangian may also have some Yang–Mills gauge symmetries, which commute with local superconformal symmetries. The superconformal functions $N, W$ are invariant, while the superconformal function $f_{\alpha\beta}$ is covariant under the Yang–Mills gauge symmetries. The Yang–Mills transformations of all chiral superfields in the superconformal action are

$$\delta_{\alpha} Y = Y r_{\alpha}(z) \quad \text{and} \quad \delta_{\alpha} z_i = \eta_{\alpha i}(z), \tag{13}$$

$r_{\alpha}(z)$ and $\eta_{\alpha i}(z)$ are $n + 1$ holomorphic functions for every symmetry. Note that the splitting given in equation (13) is not unique; the action invariance under Kähler
transformation, equation (3), has its origin exactly in this remark. The meaning of the
value of \( r_\alpha \) is the transformation of the conformon field \( Y \).

The invariance of \( \mathcal{N} \) leads to [9]

\[
0 = \mathcal{N} \{ r_\alpha(z) + r_\alpha^*(z^*) - \frac{1}{3} [\eta_{\alpha i} \partial^i \mathcal{K}(z, z^*) + \eta^*_{\alpha i} \partial^i \mathcal{K}(z, z^*)] \},
\]

implying that \( r_\alpha(z) \) describes the non-invariance of the Kähler potential \( \mathcal{K}(z, z^*) \) given by

\[
\delta_\alpha \mathcal{K} = \partial^i \delta_\alpha z_i + \partial_i \mathcal{K} \delta_\alpha z_i.
\]

Note that derivatives \( \partial^i \) (and \( \partial_i \)) stand for derivatives w.r.t. \( z_i \) (and \( z_i^* \), respectively). Assuming that the transformation of the conformon superfield \( Y \) is given by imaginary constants, then

\[
r_\alpha = i \frac{g \xi_\alpha}{3 M_{Pl}^2} \quad \text{and} \quad \partial_i \xi_\alpha = 0;
\]

\( g \) is the gauge coupling constant. If \( r_\alpha(z) \neq 0 \), the conformon superfield \( Y \) transforms under a \( U(1) \). Only if \( r_\alpha(z) = 0 \) the symmetry is preserved without corrections of the superconformal \( U(1) \). Equation (15) shows the superconformal origin of FI terms via the gauge transformations of the conformon field. Equation (14) implies that the Kähler potential is invariant [9]

\[
\delta_\alpha \mathcal{K} = 0.
\]

The function \( W \) should be invariant under Yang–Mills transformations,

\[
\delta_\alpha W = 0,
\]

implying

\[
\delta_\alpha W \equiv \eta_{\alpha i} \partial^i W = -3 r_\alpha W.
\]

Since the rhs of the above equation, equation (18), is non-zero when FI terms are present, one concludes that the superpotential \( W \) cannot be gauge invariant, if the conformon multiplet \( Y \) transforms under gauge transformations. Assuming that \( r_\alpha \) is given by equation (15), which corresponds to a constant FI term in supergravity, one gets [9]

\[
\delta_\alpha W = \eta_{\alpha i} \partial^i W = -i \frac{g \xi_\alpha}{M_{Pl}^2} W.
\]

To construct a formulation of supergravity with constant FI terms from superconformal theory, one finds [9] that under \( U(1) \) gauge transformations in the directions in which there are constant FI terms \( \xi_\alpha \), the superpotential \( W \) must transform as in equation (19). As a consequence, we cannot keep any longer the charge assignment described in the case of standard supergravity.

2.2.2. Supergravity formulations from gauge fixing. As was explicitly shown in [9], the gauge fixing of the local dilatational invariance introduces the mass scale \( M_{Pl} \), which we set \( M_{Pl}^2 = -\frac{1}{3} \mathcal{N} \), and fixes the value of the conformon superfield in terms of the Kähler potential \( \mathcal{K}(z, z^*) \), which depends only on the physical scalars \( z \) and \( z^* \), namely

\[
|Y|^2 = M_{Pl}^2 \exp(\mathcal{K}(z, z^*)/3).
\]

The value \( |Y| \) being fixed, we then fix the phase of \( Y \). The superconformal Lagrangian, equation (9), is invariant under the redefinition

\[
Y \rightarrow Y e^{\lambda_Y(z)/3},
\]
for an arbitrary holomorphic function $\Lambda_Y(z)$, since this redefinition can be absorbed in a redefinition of $z$. We can check that the effect of this redefinition on $\mathcal{K}$ and $W$ is precisely the Kähler transformation introduced in equation (3).

Two choices have been proposed [9, 37] in order to fix the phase of $Y$. Either

$$\mathcal{W} = \mathcal{W}^*,$$  \hfill (22)

leading to the standard formulation of SUGRA, described previously, or alternatively

$$Y = Y^*.$$  \hfill (23)

The first choice, equation (22), makes sense only for $\mathcal{W} \neq 0$, while the second one, equation (23), is appropriate for cases where $\mathcal{W}$ can become zero. Since in what follows we focus on $D$-term inflation, where the superpotential vanishes during inflation as well as at the end of the inflationary era, we adopt the second choice of gauge fixing. In this case, the form of the Lagrangian is explicitly given in [9, 37]. We give below several quantities which are of interest for the purpose of our study.

To switch to dimensional Kähler potential and fields, we redefine $z_i$ and $\mathcal{K}$, as [9]

$$z_i = z_i^0 + \frac{\Phi_i}{M_{Pl}} \quad \text{and} \quad \mathcal{K} = M_{Pl}^{-2}K(\Phi_1, \Phi^*_1, M_{Pl}^{-1});$$  \hfill (24)

$K(\Phi_1, \Phi^*_1, M_{Pl}^{-1})$ is regular at $M_{Pl}^{-1} = 0$. We denote the scalar part of the superfield $\Phi_i$ by $\phi_i$. From now on derivatives $\partial^i$ stand for derivatives w.r.t. $\phi_i$; we thus use the fields $\phi_i$ and their complex conjugates $\phi^*_i$ to indicate scalar fields. The previous choice of conformon transformation, equation (15), now reads $\tilde{r}_\alpha(\phi) = ig\xi_\alpha/3$. The supergravity Lagrangian depends on $W(\phi), K(\phi, \phi^*)$ and $f_{ab}(\phi)$. Constant FI terms can be introduced for some of the $U(1)$ gauge groups.

We write below the bosonic and fermionic parts of the Lagrangian for the scalar fields $\phi_i$, which are relevant for $D$-term inflation in the context of supergravity formulated using superconformal theory. The part of the bosonic sector which interests us, is

$$e^{-1}\mathcal{L}_{bos} = -g_i^j(\partial_\mu \phi^i)(\partial^\mu \phi_j) - V_F - V_D + \cdots,$$  \hfill (25)

with

$$V_F = e^{K/M_{Pl}^2} \left[ (D^i W)(g^{-1})^i_j (D_j W^*) - 3\frac{|W|^2}{M_{Pl}^2} \right],$$

$$V_D = \frac{1}{2} \left[ (\text{Re } f)^{-1} \right]^{\alpha \beta} \mathcal{P}_\alpha \mathcal{P}_\beta,$$  \hfill (26)

where

$$\mathcal{P}_\alpha(\phi, \phi^*, M_{Pl}^{-1}) = i[(\delta_\alpha \phi_i)\partial^i K(\phi, \phi^*, M_{Pl}^{-1}) - 3\tilde{r}_\alpha(\phi, M_{Pl}^{-1})]$$  \hfill (27)

and

$$D^i W = \partial^i W + M_{Pl}^{-2}(\partial^i K)W.$$  \hfill (28)

The fermionic mass terms for fermions $\chi_i$ of chiral supermultiplets $\phi_i$ are given by the fermionic sector of the Lagrangian, namely [9]

$$e^{-1}\mathcal{L}_{ferm} = -g_i^j \left[ \bar{\chi}_j D^i_\chi^j + \bar{\chi}^i D_j \chi_j \right] - m_{ij} \bar{\chi}_i \chi_j - m_{ij} \bar{\chi}^i \chi^j + e^{-1}\mathcal{L}_{mix},$$  \hfill (29)
where \( m \equiv e^{K/2} W \) and

\[
m_{ij} \equiv D^i D^j m = \left( \partial^i + \frac{1}{2} (\partial^j K) \right) m^j - \Gamma_{ij}^k m^k \\
m_i \equiv D_i m = e^{K/2} D^j W = \partial_i m + \frac{1}{2} (\partial_i K) m
\]

and where the Kähler metric and Kähler connection are defined by

\[
(g)_{ij} \equiv \partial_i \partial_j K \\
\Gamma_{ij}^k \equiv (g^{-1})^l_j \partial_j g_{il}.
\]

In addition to the standard mass terms proportional to \( m_{ij} \), some mixing mass terms between the chiral fermions \( \chi_i \) and the gaugino or the gravitino are contained in \( L_{\text{mix}} \). They can potentially contribute to the fermionic mass matrix [9]:

\[
e^{-1} L_{\text{mix}} = -2 m_{\alpha i} \overline{\chi^i} \lambda^\alpha - 2 m^{\alpha i} \overline{\chi_i} \lambda^\alpha + \left[ g_{ij} \overline{\psi}_{R} (\partial \phi^j) \gamma^\mu \xi^L_i + \overline{\psi}_R \cdot \gamma v^L_i + \text{H.c.} \right],
\]

where

\[
m_{\alpha i} = -i \left[ \partial_i \mathcal{P}_\alpha - \frac{1}{4} (\text{Ref})^{-1} \beta \gamma \mathcal{P}_\beta f_{\gamma \alpha i} \right] \\
v^L_i = \frac{1}{2} i \mathcal{P}_\alpha \lambda^\alpha + m^i \chi_i.
\]

In the case of non-vanishing \( L_{\text{mix}} \), the masses of are obtained by diagonalizing the fermionic mass matrix.

2.2.3. Consequences for D-term inflation in SUGRA. Let us investigate the consequences of a formulation of supergravity constructed from superconformal theory for D-term inflation. First, the transformation of the SUGRA superpotential given by equation (19) imposes a modification of the charge assignment of section 2.1. This charge assignment holds for the global supersymmetric limit. Since the charge of the superpotential is given by the sum of the charges of the fields \( S, \Phi_\pm \), we must impose [9]

\[
q(S) = - \frac{\xi}{M_{\text{Pl}}} \rho_S, \quad q(\Phi_\pm) = \pm 1 - \frac{\xi}{M_{\text{Pl}}^2} \rho_\pm, \quad \text{where} \quad \sum_{i=S,\pm} \rho_i = 1.
\]

Note that as a consequence, we can relate the FI amplitude to the anomaly of the symmetry \( U(1) \)

\[
\frac{\xi}{M_{\text{Pl}}^2} = - \text{Tr} Q = \sum_{i=S,\pm} q_i,
\]

similarly to the generation of FI terms with anomalous \( U(1) \) symmetry within weakly coupled string theories.

In [9] the authors suggest a way to solve the anomalous \( U(1) \) symmetry. More precisely, the authors introduce new matter superfields, which are not involved in inflation, with appropriate charge assignments so that the anomaly gets cancelled\(^6\). In order to

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\(^6\) Cancellation conditions for gauge anomalies within \( N = 1 \) four-dimensional supergravity with Fayet–Iliopoulos couplings has been recently addressed in [38, 39].
avoid an $\eta$-problem we set $\rho_S = 0$. This modification of the charge assignment induces a modification of the $D$-terms of the scalar potential. Since the function $K$ is real and $W$ is holomorphic, we can check that the expression of the $F$-terms, equation (26), is identical to the standard formulation of supergravity.

Concerning the radiative corrections, the modification of the charges induces a modification of the mass terms for the scalar components of the chiral superfields, since part of their mass terms comes from the $D$-terms. The fermionic Lagrangian, equation (29), is identical to the expression obtained in the context of standard supergravity given in [33,34]. Note that in the case of interest, there are no additional contributions to the chiral fermion mass terms from $L_{\text{mix}}$ given in equation (33).

The last consequence of the superconformal origin of supergravity concerns the form of the superpotential. We first remark that the transformation of the superpotential, equation (19), combined with the requirement that the inflaton field should remain uncharged in order to avoid the $\eta$-problem, protect the form of the superpotential

$$W = \lambda S\Phi_+\Phi_-;$$

all non-renormalizable terms of the form $S(\Phi_+\Phi_-)^n/M_{\text{Pl}}^{2n-2}$ are forbidden. Secondly, if we assume in addition that the superfields and the superpotential transform under an $R$ symmetry as

$$\Phi_+ \to e^{i\beta} \Phi_+, \quad \Phi_- \to e^{-i\beta} \Phi_-,$$

$$S \to e^{i\alpha} S, \quad W \to e^{i\alpha} W,$$

then all terms proportional to $S^n$, with $n > 1$, and all terms of the form $f(S)g(\Phi_+)$ or $f(S)g(\Phi_-)$ are forbidden. We end up with a $D$-term inflation that is precisely described by the minimal superpotential, equation (37), even if in supergravity non-renormalizable terms can be present. This motivates our choice to consider in this work, the standard form of superpotential.

### 3. $D$-term inflation in minimal SUGRA

The minimal supergravity description is based on the minimal Kähler potential

$$K_{\text{min}} = \sum_i |\Phi_i|^2 = |\Phi_-|^2 + |\Phi_+|^2 + |S|^2$$

and the minimal structure for $f_{ab}(\Phi_i)$, namely $f_{ab}(\Phi_i) = \delta_{ab}$. The tree level scalar potential reads\(^7\) [9]

$$V_{\text{SUGRA}} = \lambda^2 \exp \left( \frac{|\phi_-|^2 + |\phi_+|^2 + |S|^2}{M_{\text{Pl}}^2} \right)$$

$$\times \left[ |\phi_+\phi_-|^2 \left( 1 + \frac{|S|^4}{M_{\text{Pl}}^4} \right) + |\phi_+S|^2 \left( 1 + \frac{|\phi_-|^4}{M_{\text{Pl}}^4} \right) + |\phi_-S|^2 \left( 1 + \frac{|\phi_+|^4}{M_{\text{Pl}}^4} \right) + 3\frac{|\phi_-\phi_+S|^2}{M_{\text{Pl}}^2} \right]$$

$$+ \frac{g^2}{2} (q_+|\phi_+|^2 + q_-|\phi_-|^2 + \xi)^2.$$

\(^7\) This formula differs slightly from the one in [9]. In the last $F$-term there is a factor 3 instead of 6.
The next step is the calculation of the masses of the components of the superfields $\Phi_\pm$. The scalar mass squared can be read directly from the scalar potential [9], namely

$$m_\pm^2 = \lambda^2 e^{S^2/M_{Pl}^2} |S|^2 + g^2 q_\pm \xi.$$

(41)

Strictly speaking, one must use the charge assignment given by equation (35), for example,

$$q_\pm = \pm 1 - \frac{1}{2} \frac{\xi}{M_{Pl}^2}.$$

(42)

Hereafter we assume $\xi/M_{Pl}^2 \ll 1$ and therefore the expression for the scalar masses, equation (41), can be approximated as

$$m_\pm^2 \simeq \lambda^2 e^{S^2/M_{Pl}^2} |S|^2 \mp g^2 \xi.$$

(43)

In the inflationary valley, where $\phi_\pm = 0$, the associated Dirac fermions have a mass squared

$$m_f^2 = \lambda^2 e^{S^2/M_{Pl}^2} |S|^2,$$

(44)

unchanged compared to the case of standard minimal supergravity [25, 26].

Using the Coleman–Weinberg formula, we can then compute the effective inflationary potential, taking into account the tree level and the one-loop radiative corrections due to the mass splitting between components of the chiral superfields $\Phi_\pm$. The effective potential reads [25, 26]

$$V_{\text{eff}}(|S|) = \frac{g^2 \xi^2}{2} \left( 1 + \frac{g^2}{16 \pi^2} \left[ 2 \ln \left( \frac{g^2 \xi}{\Lambda^2} \right) + f_V(z) \right] \right),$$

(45)

where

$$f_V(z) = (z + 1)^2 \ln \left( 1 + \frac{1}{z} \right) + (z - 1)^2 \ln \left( 1 - \frac{1}{z} \right)$$

(46)

and

$$z \equiv \frac{\lambda^2}{g^2 \xi} |S|^2 \exp \left( \frac{|S|^2}{M_{Pl}^2} \right).$$

(47)

$D$-term inflation leads to cosmic strings formation at the end of the inflationary era. Consistency between CMB measurements and theoretical predictions constrain the parameters space. The constrains are imposed on the couplings and mass scales. They are shown in figure 1 and can be summarized as [25, 26]

$$g \lesssim 2 \times 10^{-2} \quad \text{and} \quad \lambda \lesssim 3 \times 10^{-5}.$$

(48)

The above constraints, equation (48), can be expressed [25, 26] as a single constraint on the Fayet–Iliopoulos term $\xi$, namely,

$$\sqrt{\xi} \lesssim 2 \times 10^{15} \text{ GeV}.$$

(49)

We can therefore see that our assumption that $\xi/M_{Pl}^2 \sim 10^{-6} \ll 1$ in equation (43) is justified [40]–[42], which implies that for the purpose of our study and for minimal $D$-term inflation, we can neglect the corrections introduced by the superconformal origin of supergravity. Moreover, since the tree level potential is given by $V_0 \propto \xi^2$, the limit
$D$-term inflation in non-minimal supergravity

Figure 1. In the framework of standard $D$-term inflation in minimal SUGRA, cosmic string contribution to CMB quadrupole anisotropies, for various values of the gauge coupling $g$, as a function of the superpotential coupling constant $\lambda$.

$\xi/M_{Pl}^2 \ll 1$ should hold since otherwise the energy density would become trans-Planckian and the quantum gravity corrections, which have been so far neglected \cite{25}, would become important \cite{2}.

In what follows we address whether the restrictions found in the framework of minimal SUGRA, are still qualitatively valid for a non-minimal SUGRA theory. We will thus study $D$-term hybrid inflation with a superpotential $W$ defined by equation (8) and different choices for the form of the Kähler potential.

4. Inflation with shift symmetry

We first study hybrid inflation in the context of supergravity, with a Kähler potential obeying a shift symmetry. This symmetry can be used for model building in the framework of supergravity, while it is also motivated from string theory \cite{43}. More precisely, the shift symmetry,

$$S \rightarrow S + iC,$$

with $C$ a real constant, has been used in order to obtain flat potentials from $F$-terms in SUGRA, so that the $\eta$-problem is cured, or to make compatible chaotic inflation, which requires an inflaton field larger than the Planck scale, and supergravity \cite{44}.

Thus, we consider the following form for the Kähler potential

$$K_1 = \frac{1}{2}(S + \bar{S})^2 + |\phi_+|^2 + |\phi_-|^2$$

for the Kähler potential, equation (39), $K_1$ has two additional terms, $S^2/2$ and $\bar{S}^2/2$, which do not affect the kinetic terms.

Using the standard expression for the scalar potential, equation (6), we obtain,

$$V = V_F + V_D,$$

where the $D$-terms are not modified as compared to the minimal SUGRA case. Thus, assuming $\xi/M_{Pl}^2 \ll 1$,

$$V_D \approx \frac{g^2}{2}(|\phi_+|^2 - |\phi_-|^2 + \xi)^2$$
and the $F$-terms are given by

$$V_F = \lambda^2 \exp \left( \frac{|\phi_-|^2 + |\phi_+|^2}{M_{Pl}^2} \right) \exp \left( \frac{(S + \bar{S})^2}{2M_{Pl}^2} \right) \times \left[ |\phi_+\phi_-|^2 \left( 1 + \frac{S^2 + \bar{S}^2}{M_{Pl}^2} + \frac{|S|^2|S + \bar{S}|^2}{M_{Pl}^4} \right) + |\phi_+S|^2 \left( 1 + \frac{|\phi_-|^4}{M_{Pl}^4} \right) \right] + |\phi_-S|^2 \left( 1 + \frac{|\phi_+|^4}{M_{Pl}^4} + 3\frac{|\phi_+S|^2}{M_{Pl}^2} \right).$$

As in the case of $D$-term inflation studied within minimal supergravity, the potential has a global minimum

$$V = 0 \quad \text{for } \langle S \rangle = 0, \quad \langle \Phi_+ \rangle = 0, \quad \langle \Phi_- \rangle = \sqrt{\xi}. \quad (55)$$

There is also a local minimum for large values of $S$,

$$V = \frac{g^2 \xi^2}{2} \quad \text{for } \langle S \rangle \gg S_c, \quad \langle \Phi_\pm \rangle = 0. \quad (56)$$

Comparing with the scalar potential obtained in minimal SUGRA, we can identify two differences. The first one is that there are several new terms proportional to $|\Phi_+\Phi_-|^2$. These terms will not affect the effective mass terms of the $\Phi_\pm$ fields, thus they will not affect the inflationary potential, even when one-loop radiative corrections have been taken into account. The second one is that the exponential factor $e^{S^2/2}$ in minimal SUGRA, has been here replaced by $e^{(S+\bar{S})^2/2}$. Writing $S = \eta + i\phi_0$, we get $e^{(S+\bar{S})^2/2} = e^{\eta^2}$. If we identify the inflaton field with the real part of $S$, then we obtain exactly the same potential as in the minimal case. However, with the present choice for the Kähler potential, $K_1$, it would be more logical to identify the inflaton field with the imaginary part of $S$, namely $\phi_0$. The reason being that with this choice the exponential term is constant during inflation, thus it cannot spoil the slow roll conditions. Then we can check that the inflationary potential we get is identical to the usual $D$-term inflation within the global SUSY framework. This model has been studied in [25]. The result obtained in [25] is reminded in figure 2. Clearly, in such a model the cosmic string contribution to the

**Figure 2.** Cosmic string contribution to the CMB temperature anisotropies as a function of the superpotential coupling constant $\lambda$, in the case of the $D$-term inflation model in the framework of global SUSY. Figure taken from [25].
CMB anisotropies is dominant, in contradiction with the CMB measurements, unless the superpotential coupling is constrained to be

$$\lambda \lesssim 3 \times 10^{-5}. \quad (57)$$

Concluding, we state that simply imposing a shift symmetry to the Kähler potential is not enough to escape the cosmic string problem of D-term inflation.

5. D-term inflation with higher order terms

In this section, we consider another choice of Kähler potential that contains non-renormalizable terms. More precisely, we consider the following form

$$K_2 = |S|^2 + |\Phi^+|^2 + |\Phi^-|^2 + f_+ \left( \frac{|S|^2}{M_{Pl}^2} \right) |\Phi^+|^2 + f_- \left( \frac{|S|^2}{M_{Pl}^2} \right) |\Phi^-|^2 + b \frac{|S|^4}{M_{Pl}^4}, \quad (58)$$

where $f_\pm$ are arbitrary functions of $(|S|^2/M_{Pl}^2)$. Contrary to the previous section, this new form should modify the kinetic terms. The motivation for this choice is that it contains the next-to-minimal Kähler potential, with all terms up to order $M_{Pl}^{-2}$. We consider in a first place a reduced version of this potential and then we proceed with the full expression. We remind to the reader that our primary goal is not to build a new model for D-term inflation in the context of supergravity, but to study the robustness of our prediction that there is a dominant contribution of cosmic strings to the CMB temperature anisotropies, unless we fine-tune the superpotential coupling constant $\lambda$.

5.1. Simplified case: $b = 0$

We first consider the case of the previous Kähler potential, equation (58), setting $b = 0$, implying

$$K_3 = |S|^2 + |\Phi^+|^2 + |\Phi^-|^2 + f_+ \left( \frac{|S|^2}{M_{Pl}^2} \right) |\Phi^+|^2 + f_- \left( \frac{|S|^2}{M_{Pl}^2} \right) |\Phi^-|^2, \quad (59)$$

where $f_\pm$ are arbitrary functions of $(|S|^2/M_{Pl}^2)$, while the superpotential is given in equation (8). One can argue that the Kähler potential given in equation (59) is quite general, since during inflation the Higgs fields are small. However, one can also criticize this choice for the following reason: even though $|\phi_\pm|^4$-terms are indeed negligible as compared to the $|\phi_\pm|^2$-terms (the $|\phi_\pm|$-terms are small during the inflationary era) there is no reason for neglecting $|S|^4/M_{Pl}^4$-terms. They will be taken into account in next section.

In this model, the scalar potential can be calculated using equation (6) and reads

$$V(|S|) = V_F + V_D, \quad (60)$$
where the \( F \)-part is\(^8\)

\[
V_F = \lambda^2 \frac{e^K}{\det K^{i\ell}} \left\{ (1 + f_+)(1 + f_-) \left[ 1 + \frac{|S|^2}{M_{Pl}^2} \left( 1 + f'_+ \frac{\phi_+^2}{M_{Pl}^2} + f'_- \frac{\phi_-^2}{M_{Pl}^2} \right) \right] \right\} \frac{d^2 \phi_+}{M_{Pl}^2} \frac{d^2 \phi_-}{M_{Pl}^2} \frac{\phi_+ \phi_-}{M_{Pl}^2}

+ \left\{ \left[ (1 + f_+)(1 + \Delta) - f_+ \frac{2\phi_+^2}{M_{Pl}^2} \right] \left[ 1 + \frac{\phi_-^2}{M_{Pl}^2} (1 + f_-) \right] \right\} \frac{|\phi_+|^2}{M_{Pl}^2}

+ \left\{ \left[ (1 + f_-)(1 + \Delta) - f_- \frac{2\phi_-^2}{M_{Pl}^2} \right] \left[ 1 + \frac{\phi_+^2}{M_{Pl}^2} (1 + f_+) \right] \right\} \frac{|\phi_-|^2}{M_{Pl}^2}

- 2 \left\{ \left[ 1 + \frac{|S|^2}{M_{Pl}^2} \left( 1 + f'_+ \frac{\phi_+^2}{M_{Pl}^2} + f'_- \frac{\phi_-^2}{M_{Pl}^2} \right) \right] \right\} \frac{|\phi_+ \phi_-|^2}{M_{Pl}^2} \right\\} \frac{d^2 \phi_+}{M_{Pl}^2} \frac{d^2 \phi_-}{M_{Pl}^2} \frac{\phi_+ \phi_-}{M_{Pl}^2} \]

(61)

and the \( D \)-part is

\[
V_D = \frac{g^2}{2} [g_+ (1 + f_+)|\phi_+|^2 + g_- (1 + f_-)|\phi_-|^2 + \xi^2].
\]

We have used the following notations

\[
f_\pm \equiv f_\pm(1 + f_+) \frac{|\phi_+|^2}{M_{Pl}^2}, \quad f'_\pm \equiv \frac{df_\pm(x)}{dx} \bigg|_{x=|S|^2/M_{Pl}^2}, \quad f''_\pm \equiv \frac{d^2f_\pm(x)}{dx^2} \bigg|_{x=|S|^2/M_{Pl}^2},
\]

\[
\Delta \equiv \left( f'_+ + f''_+ \frac{|S|^2}{M_{Pl}^2} \right) \frac{\phi_+^2}{M_{Pl}^2} + \left( f'_- + f''_- \frac{|S|^2}{M_{Pl}^2} \right) \frac{\phi_-^2}{M_{Pl}^2}
\]

(63)

and

\[
\det K_{i\ell} = (1 + \Delta)(1 + f_+)(1 + f_-) - (1 + f_+) f'_- \frac{2\phi_-^2}{M_{Pl}^2} - (1 + f_-) f'_+ \frac{2\phi_+^2}{M_{Pl}^2}.
\]

(64)

As in minimal supergravity, the tree level inflationary potential is constant and equal to

\[
V_0 = \frac{1}{2} g^2 \xi^2,
\]

(65)

while supersymmetry is broken. The spontaneous symmetry breaking of supersymmetry in the inflationary valley introduces a splitting in the masses of the components of the

\(^8\) We disagree with the expression given in [45]. We believe that in [45] the authors made some unjustified assumptions under which our expression for \( V_F \), given in equation (61), can be reduced to the expression given in [45]. Even though this disagreement has no implications for the rest of our study, we would like to bring to the attention of the reader that the correct expression for the \( F \)-contribution to the scalar potential, \( V_F \), is indeed given in equation (61).
chiral superfields $\Phi_{\pm}$. As a result, one obtains two scalars with squared masses

$$m_{\pm}^2 = \lambda^2 |S|^2 \exp \left( \frac{|S|^2}{M_{\text{Pl}}^2} \right) \frac{1}{(1 + f_+)(1 + f_-)} \pm g^2 q_{\pm} \xi$$

and a Dirac fermion with squared mass

$$m_{\text{fermion}}^2 = \lambda^2 |S|^2 \exp \left( \frac{|S|^2}{M_{\text{Pl}}^2} \right) \frac{1}{(1 + f_+)(1 + f_-)}.$$  \hfill (67)

The one-loop radiative corrections to the inflationary potential can be calculated using the Coleman–Weinberg formula. Taking also into account the tree level contribution, the effective scalar potential for the considered $D$-term inflationary model reads \cite{45}

$$V_{\text{eff}}(|S|) = \frac{g^2 \xi^2}{2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[ 2 \ln \left( z \frac{g^2 \xi}{\Lambda^2} \right) + f_V(z) \right] \right\},$$

with

$$f_V(z) = (z + 1)^2 \ln \left( 1 + \frac{1}{z} \right) + (z - 1)^2 \ln \left( 1 - \frac{1}{z} \right)$$

and

$$z \equiv \frac{\lambda^2 |S|^2}{g^2 \xi^2} \exp \left( \frac{|S|^2}{M_{\text{Pl}}^2} \right) \frac{1}{(1 + f_+)(1 + f_-)}. \hfill (70)$$

Strictly speaking the above expression holds for the limit where $\xi/M_{\text{Pl}}^2 \ll 1$, which is indeed the case as one can confirm in the end of this section.

Note that the expression for the effective scalar potential is identical to that of minimal supergravity, except for the expression of $z$. The first derivative of the potential is equal to

$$V'_{\text{eff}}(|S|) \equiv \frac{dV_{\text{eff}}}{d|S|} = \frac{g^4 \xi^2}{16\pi^2} z f_z(|S|) f_V(z), \hfill (71)$$

where

$$f_V(z) \equiv (z + 1) \ln \left( 1 + \frac{1}{z} \right) + (z - 1) \ln \left( 1 - \frac{1}{z} \right), \hfill (72)$$

$$f_z(|S|) \equiv 2 |S| \left[ \frac{1}{M_{\text{Pl}}^2} + \frac{1}{|S|^2} - \frac{f'_+}{(1 + f_+)} - \frac{f'_-}{(1 + f_-)} \right]; \hfill (73)$$

$f'_\pm$ denote the first derivative of $f_\pm$ with respect to $|S|^2$. A choice which we will later consider for $f_\pm$ is

$$f_\pm \left( \frac{|S|^2}{M_{\text{Pl}}^2} \right) = c_\pm \frac{|S|^2}{M_{\text{Pl}}^2}, \quad \text{therefore } f'_\pm = c_\pm \frac{1}{M_{\text{Pl}}^2}. \hfill (74)$$

In the large $|S|$-limit the effective potential and its first derivative with respect to $|S|$ reduce to

$$V_{\text{eff}}(|S|) \simeq \frac{g^4 \xi^2}{16\pi^2} \left[ \ln \left( \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)\Lambda^2} \right) + \frac{|S|^2}{M_{\text{Pl}}^2} \right], \hfill (75)$$
Figure 3. Cosmic string contribution to the CMB quadrupole anisotropies as a function of the superpotential coupling constant $\lambda$ with partial (dashed line) or full (continuous line) scalar potential.

and

$$\frac{dV_{\text{eff}}}{d|S|} \equiv V_{\text{eff}}'(|S|) \simeq \frac{g^4 \xi^2}{16\pi^2} f_z(|S|), \quad (76)$$

respectively.

At this point, we would like to note that $D$-term inflation can be realized with the last 60 e-folds being very close to the critical value $z_{\text{end}} = 1$, implying that the above reduced formulae, equations (75), (76), cannot be used to compute the predictions of the model regarding the CMB temperature anisotropies. Therefore, we disagree with the approach of [45], where the authors have used the reduced formulae for the effective potential and its first derivative, to compute the inflaton contribution to the CMB temperature anisotropies.

To illustrate the above remark, we represent in figure 3 the error made by this assumption. The cosmic string contribution is drawn as a function of the superpotential coupling constant $\lambda$, considering either the partial or the full scalar potential. One can realize that the error on the cosmic string contribution can reach 50%, leading to a considerable error in the induced constraints on the parameters space.

The number of e-foldings of inflation between the initial and final value of the inflaton field is

$$N_Q \equiv \ln \left( \frac{a_{\text{end}}}{a_Q} \right) = \frac{8\pi^2}{g^2 M_{\text{Pl}}^2} \int_1^{z_Q} \frac{dz}{z^2 f_z^2(|S(z)|) f_{V'}(z)}, \quad (77)$$

where we note that the index $Q$ denotes the scale responsible for the quadrupole anisotropy in the CMB. For the integral appearing in the above expression for $N_Q$ to be correctly defined, one must assume that the function

$$z \equiv \frac{\lambda^2}{g^2 \xi^2} |S|^2 \exp \left( \frac{|S|^2}{M_{\text{Pl}}^2} \right)$$
can be inverted. Since it is a monotonic and increasing function, one can verify, at least numerically, that the function $z(|S|)$ can be inverted. We denote the inverted function by $S(z)$.

The above expression for $N_{Q}$, equation (77), will enable us to write a relation between $\xi$ and $z_{Q}$. Setting $N_{Q} = 60$, we can fix the value of the inflaton field $S_{Q}$, responsible for the quadrupole anisotropy of the CMB.

5.1.1. Normalization to COBE. D-term inflation leads generically to cosmic strings formation, which also contribute to the CMB temperature anisotropies. The total contribution, from the inflaton field as well as from the cosmic strings, has to be normalized to the value measured by the COBE-DMR experiment. This normalization will fix the mass scale $\sqrt{\xi}$ as a function of the superpotential coupling $\lambda$ and the gauge coupling $g$, which are considered here as parameters. Thus, we have to calculate both, the cosmic strings contribution and the inflaton field one, and normalize their sum to COBE-DMR. Here we emphasize that different approaches can be found in the literature where the authors do NOT normalize the sum of the two contributions to the data. Clearly not to normalize the sum to the data is a simplification, which may lead to an important error for the result. Normalizing only the inflationary contribution to the quadrupole absolute value can be considered as an assumption where the cosmic string contribution is neglected. This assumption could be allowed only if one wants to argue that the cosmic strings are sub-dominant. However, since we already know that cosmic strings are indeed sub-dominant and our aim is to constrain the parameters space and calculate exactly how sub-dominant the strings are, this simplification in the normalization implies important errors in the calculated cosmic string contribution.

From the scalar potential given in equation (60), one can see that the spontaneous symmetry breaking is generated when $\phi_{\pm}$ takes a vacuum expectation value $\sqrt{\xi}$. Thus, the quadrupole contribution to the CMB temperature anisotropies from the cosmic strings formed at the end of hybrid inflation, which is

$$
\left( \frac{\delta T}{T} \right)_{Q-CS} \simeq (9-10)G\mu \quad \text{with} \quad \mu = 2\pi (\mathcal{X})^{2},
$$

where $\mathcal{X}$ is the vacuum expectation value of the Higgs field responsible for the formation of cosmic strings, is approximately equal to

$$
\left( \frac{\delta T}{T} \right)_{Q-CS} \simeq \frac{9}{4} \xi.
$$

For the contribution from the inflaton field, we evaluate the Sachs–Wolfe term split into the scalar and tensor parts, using

$$
\left( \frac{\delta T}{T} \right)_{Q-scal} \simeq \frac{1}{4\sqrt{45\pi}} \frac{V^{3/2}(S_{Q})}{M_{Pl}^{3} V'(S_{Q})}
$$

and

$$
\left( \frac{\delta T}{T} \right)_{Q-tens} \simeq \frac{(0.77) V^{1/2}(S_{Q})}{(8\pi)} \frac{1}{M_{Pl}^{2}}.
$$
From equations (65), (71), (80) and (81) we get
\[
\left( \frac{\delta T}{T} \right)_{Q-\text{scal}} \approx \frac{\sqrt{2\pi} \xi}{\sqrt{45} g M_{Pl}^3} z_Q^{-1} f_{V'}^{-1}(z_Q) f_{z}^{-1}(S_Q) \quad (82)
\]
and
\[
\left( \frac{\delta T}{T} \right)_{Q-\text{tens}} \approx \frac{0.77}{8\sqrt{2\pi} M_{Pl}^2} g \xi. \quad (83)
\]
We note that the ratio of the tensor part of the inflaton field contribution to the cosmic strings one is constant, and that the tensor part contribution can be neglected. The tensor over scalar ratio, \( r_{\text{infl}} \), is less straightforward and it is given by
\[
r_{\text{infl}} = \frac{0.77 \sqrt{45}}{16\pi^2} g^2 z_Q M_{Pl} f_{V'}(z_Q) f_{z}(S_Q). \quad (84)
\]
We then proceed with the cosmic string contribution to the quadruple CMB temperature anisotropy. This contribution in computed as a function of the superpotential coupling \( \lambda \), for various values of \( g \) and \( c = c_\pm \), which are considered as parameters. The results are plotted in figure 4.

To show the dependence of the cosmic string contribution on the gauge coupling \( g \), we draw in figure 5 the cosmic string contribution for \( g = 10^{-1} \) and \( c = c_\pm = 0, 1, 2, 5 \). Clearly, this case \( (g = 10^{-1}) \) is excluded since the cosmic string contribution is above the allowed one. We note that we do not take \( c \) higher than 5, since positivity condition \( V'(S) > 0 \) requires \( c < 3 + 2\sqrt{2} \) [45].

We quantify below the constraints on the parameters space imposed from the three-year Wilkinson Microwave Anisotropy Probe (WMAP) [46] measurements. We want to set precise constraints on the free parameters since this can be of importance for concluding of whether \( D \)-term inflation remains in agreement with CMB data once we also include constraints imposed from the allowed value of the spectral index [47].
Figure 5. The dashed line corresponds to the minimal SUGRA case. The other lines correspond to various values of the parameter $c = c_\pm = 1, 2, 5$ from the top to the bottom, respectively. This is for $g = 10^{-1}$.

For $g \gtrsim 10^{-1}$, there is no parameters space in agreement with measurements. In the range $g \in [2 \times 10^{-2}, 10^{-1}]$, the parameters space is extremely small, around $\lambda \sim 2 \times 10^{-5}$; the presence of $c$ enlarges slightly the allowed window. For $g = 10^{-2}$, the 9% upper limit on the allowed cosmic string contribution to the CMB imposes, at 95% of confidence level, the following constraints:

$$
\begin{align*}
&3 \times 10^{-8} \\
&5 \times 10^{-8} \\
&9 \times 10^{-8} \\
&2 \times 10^{-7}
\end{align*}
\leq \lambda \leq \begin{cases} 
2.5 \times 10^{-5} & \text{for } c = 0 \\
3.5 \times 10^{-5} & \text{for } c = 1 \\
4.0 \times 10^{-5} & \text{for } c = 2 \\
5.3 \times 10^{-5} & \text{for } c = 5
\end{cases} \quad (85)
$$

or, equivalently,

$$
\sqrt{\xi} \leq 2.2 \times 10^{15} \text{ GeV} \iff G\mu \leq 8.4 \times 10^{-7}. \quad (86)
$$

Clearly the new degree of freedom, namely $c = c_\pm$, allows a slightly higher upper bound on the coupling, which is however at best higher by only a factor of 2, thus concluding that fine-tuning is still required. All constraints are equivalent to a single constraint on $\sqrt{\xi}$, or $G\mu$, as already stated in [25, 26]. Therefore, there is a bijection between the cosmic string contribution and the mass scale of inflation, which nevertheless does not mean that one can normalize only the inflaton contribution to the CMB data. Clearly, the error made by this assumption is big when cosmic strings have an important weight, while it is small if the cosmic string contribution is small, of the order the one found here. With the current upper limit of 9% on the allowed cosmic string contribution to the temperature anisotropies, the relative error made on the string contribution, by not normalizing the sum, is of the order of 10%.

As one can see from figures 4, 5 the curve which showing the cosmic string contribution becomes singular at its minimum, for large values of $c_\pm$. The reason for this behaviour is that the function $F(\xi)$, denoting the sum of the cosmic strings and inflaton contributions as a function of $\xi$ becomes non-bijective for certain values of $\lambda$. This implies that there
are more than one solutions for the function $F(\xi)$ normalized to the COBE data, for a tiny window of the parameter $\lambda$ around the value $10^{-5}$. We illustrate this in figure 6. We have checked that this degeneracy does not influence the validity of our results since the different solutions for the normalization to COBE are of the same order of magnitude and this behaviour is observed for a tiny parameters space.

As a final remark, we would like to note that there is also another point where we disagree with [45]. In their model specified by the choice of the Kähler potential and with the simplifications made for the scalar potential, the authors obtain [45] a value for $\sqrt{\xi}$ required to generate the appropriate magnitude of density perturbations. The authors argue that this value is consistent with the upper limit imposed on $\xi$ so that the cosmic string contribution is within the allowed window, following the results of [48]. However, the analysis of [48] was done employing the curvaton mechanism; a contribution which the authors of [45] have not considered at all.

5.2. General case

We proceed with the general case, namely we include all terms of the next to leading order

$$K_2 = |S|^2 + |\Phi_+|^2 + |\Phi_-|^2 + f_+ \left( \frac{|S|^2}{M^2_{Pl}} \right) |\Phi_+|^2 + f_- \left( \frac{|S|^2}{M^2_{Pl}} \right) |\Phi_-|^2 + b \frac{|S|^4}{M^2_{Pl}},$$

(87)

where the function $f_\pm$ is just

$$f_\pm \left( \frac{|S|^2}{M^2_{Pl}} \right) = c_\pm \frac{|S|^2}{M^2_{Pl}}.$$  

(88)

The motivation for such a choice is that beyond the minimal supergravity part of the Kähler potential, the leading order is $M^2_{Pl}$, implying that one should consider all corrections up to this order. During inflation, the charged fields $\Phi_\pm$ vanish, meaning
that terms of the form $|\Phi_{\pm}|^4/M_{P1}^2$ or $|\Phi_+|^2|\Phi_-|^2/M_{P1}^2$ can be neglected compared to $|\Phi_{\pm}|^2$-terms, which are in $K_{\text{min}} = |S|^2 + |\Phi_+|^2 + |\Phi_-|^2$. Thus, the next-to-minimal $D$-term inflationary model should be constructed with Kähler potential $K_2$. We again consider the superpotential

$$W = \lambda S\Phi_+\Phi_-,$$

(89)

involving the superfields $S$ and $\Phi_{\pm}$ with charges 0 and $q_{\pm}$, respectively, under the $U(1)_\xi$ symmetry of the Lagrangian. The charges $q_{\pm}$ must satisfy the constraint induced by the superconformal origin of the FI term, as discussed in section 2.2.3. The $D$-term part of the scalar potential is thus unchanged as compared the previous section and given by equation (62).

We first compute the inverse Kähler metric $g^{-1}$, defined by $g^{ij}(g^{-1})^k_j = \delta^i_k$:

$$(g^{-1})^i_j = \frac{1}{g} \times \mathcal{M},$$

(90)

where $\mathcal{M}$ is equal to

$$
\begin{pmatrix}
(1 + f_+)(1 + f_-) & -c_+ S^*\phi_+(1 + f_-) & -c_- S^*\phi_-(1 + f_+)

-c_+ S\phi_+^* (1 + f_-) & (1 + \Delta)(1 + f_-) - c_2 |S|^2 |\phi_-|^2/M_{P1}^2 & c_- S^*\phi_-(1 + f_+)

-c_+ S\phi_+^* (1 + f_+) & c_+ c_- |S|^2 |\phi_+\phi_-|^2/M_{P1}^2 & (1 + \Delta)(1 + f_+) - c_+ c_- |S|^2 |\phi_+|^2/M_{P1}^2
\end{pmatrix}
$$

(91)

and we have used equation (88) and the definitions

$$\Delta \equiv c_+ |\phi_+|^2 + c_- |\phi_-|^2 + 2b |S|^2/M_{P1}^2;$$

(92)

g \equiv \det(g)^i_j = (1 + \Delta)(1 + f_+)(1 + f_-) - (1 + f_+) c_2 |S|^2 |\phi_-|^2/M_{P1}^2 - (1 + f_-) c_+ |S|^2 |\phi_+|^2/M_{P1}^2.

(93)

Using equations (26) and (90) we calculate the $F$-term of the scalar potential:

$$V_F = \lambda^2 \frac{K/M_{P1}^2}{g} \left\{(1 + f_+)(1 + f_-)|\phi_+\phi_-|^2 \left[1 + \frac{|S|^2/\lambda}{M_{P1}^2}(1 + \Delta)\right]^2ight.\right.

+ (1 + \Delta)(1 + f_-)|S\phi_-|^2 \left[1 + \frac{|\phi_+|^2}{M_{P1}^2}(1 + f_+)^2\right]

- c_2 \frac{|S\phi_-|^4}{M_{P1}^2} \left[1 + \frac{|\phi_+|^2}{M_{P1}^2}(1 + f_+)^2\right]

+ (1 + \Delta)(1 + f_+)|S\phi_+|^2 \left[1 + \frac{|\phi_-|^2}{M_{P1}^2}(1 + f_-)^2\right]

- c_+ \frac{|S\phi_+|^4}{M_{P1}^2} \left[1 + \frac{|\phi_-|^2}{M_{P1}^2}(1 + f_-)^2\right]

- 2c_+(1 + f_+) \frac{|S\phi_+\phi_-|^2}{M_{P1}^2}.$$
whereas from equation (29) we get the mass squared for their fermionic partners:

\[ m^2_{\text{fermion}} = \exp \left( |S|^2 \right) + b \frac{|S|^4}{M^2_{\text{Pl}}} \]  
\[ \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)} + g^2 \phi^2. \]  

(96)

Following the same procedure as in the previous sections, we obtain the effective potential

\[ V_{\text{eff}}(|S|) = \frac{g^2 \phi^2}{2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[ 2 \ln \left( \frac{g^2 \phi^2}{\Lambda^2} \right) + f_V(z) \right] \right\}. \]  

(97)

where \( f_V(z) \) is the same as in section 5, equation (69); the only difference is the expression for \( z \), namely here

\[ z \equiv \frac{\lambda^2 |S|^2}{g^2 \phi^2} \exp \left( |S|^2 \right) + b \frac{|S|^4}{M^2_{\text{Pl}}} \frac{1}{(1 + f_+)(1 + f_-)}. \]  

(98)

The first derivative of the scalar potential with respect to \(|S|\) reads

\[ V'_{\text{eff}}(|S|) = \frac{dV_{\text{eff}}}{d|S|} = \frac{g^4 \phi^4}{16\pi^2} z f_V'(z) f_z(|S|), \]  

(99)

From the \( F \)-part of the scalar potential, different limits can be taken to recover known results. For example, the limit \( c_+ = c_- = b = 0 \) allows one to recover the standard minimal \( D \)-term inflation as discussed in [35]. The limit \( b = 0 \), leads to the potential analysed in section 5 and first studied in [45]. Finally, the limit \( c_+ = c_- = 0 \), gives the case of the scalar potential studied also in [45] with the choice \( h(|S|^2) = |S|^2 + |S|^4 \).

Equation (94) implies that during inflation \( V_F \) is minimized for \( \langle \phi_+ \rangle = \langle \phi_+ \rangle = 0 \), thus \( V_F = 0 \) and inflation is driven by the \( D \)-term. The total scalar potential \( V \) is constant at tree level and equal to \( V_0 = g^2 \phi^2/2 \). Having the scalar potential we calculate the masses of the canonically normalized scalar components of the superfields \( \Phi_\pm \):

\[ m^2_\pm = \exp \left( \frac{|S|^2}{M^2_{\text{Pl}}} + b \frac{|S|^4}{M^2_{\text{Pl}}} \right) \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)} + g^2 \phi^2, \]  

(95)

whereas from equation (29) we get the mass squared for their fermionic partners:
Figure 7. Cosmic string contribution to the CMB temperature anisotropies as a function of the superpotential coupling constant $\lambda$, in the next-to-minimal $D$-term inflationary model. The value of the gauge coupling constant is fixed and equal to $g = 10^{-2}$; there are no cross terms between $S$ and $\Phi^{\pm}$, i.e., $c^{\pm} = 0$. The minimal SUGRA for $b = 0$ is represented by the dashed line, while the different plain lines are calculated for $b = 0.5, 1, 2$, going from the bottom to the top. A fine-tuning of the superpotential coupling $\lambda$ is still required, in order to avoid a dominant contribution of cosmic strings.

where $f_{V'}$ is the same as in section 5, equation (72), but $f_z(|S|)$ is in this case given by

$$f_z(|S|) = 2|S| \left[ \frac{1}{M_{Pl}^2} + \frac{2b|S|^2}{M_{Pl}^4} + \frac{1}{|S|^2} - \frac{c_+}{(1 + f_+)M_{Pl}^2} - \frac{c_-}{(1 + f_-)M_{Pl}^2} \right].$$

The number of e-folds and the inflationary contribution to the CMB quadrupole anisotropy are still calculated using equations (77) and (82), respectively, with $f_z(|S|)$ given by equation (100) above. The cosmic strings contribution to the CMB temperature anisotropies is the same as in the minimal $D$-term SUGRA case. The contribution of cosmic strings to the CMB anisotropies in the whole parameters space is represented in figures 7–9.

Studying the above figures one easily concludes that considering a more general form for the Kähler potential does not solve the fine-tuning problem of $D$-term inflation. The new term in the Kähler potential, whose weight is given by the parameter $b$, induces an enhancement of the cosmic string contribution at low $\lambda$. We still observe a dominant contribution of cosmic strings if the superpotential coupling $\lambda$ is close to unity. Therefore, the constraints on $\lambda$ found on the previous section remain unchanged as given in equation (85).

6. Conclusions

$D$-term hybrid inflation is a successful and interesting model. In the context of supergravity, $D$-term inflation avoids the Hubble-induced mass problem, which plagues $F$-term hybrid inflation, while it can easily be implemented in string theory. In a standard formulation of $D$-term inflation, where the constant FI term gets compensated by a single complex scalar field, we do not add an additional discrete symmetry, and do not consider
Figure 8. Cosmic string contribution to the CMB temperature anisotropies as a function of the superpotential coupling constant $\lambda$, in the next-to-minimal $D$-term inflation model. The parameter $b$ is set equal to $b = 1$; there are no cross terms between $S$ and $\Phi_{\pm}$, i.e., $c_{\pm} = 0$. The different plain lines are calculated for $g = 10^{-3}, 10^{-2}, 10^{-1}$, going from the bottom to the top. A fine-tuning of the superpotential coupling $\lambda$ is still required to avoid a dominant contribution of cosmic strings.

Figure 9. Cosmic string contribution to the CMB temperature anisotropies as a function of the superpotential coupling constant $\lambda$, in the next-to-minimal $D$-term inflation model. The parameter $b$ is set equal to $b = 1$; there are no cross terms between $S$ and $\Phi_{\pm}$, i.e., $c_{\pm} = 0$. The different plain lines are calculated for $g = 10^{-3}, 10^{-2}, 10^{-1}$, going from the bottom to the top. A fine-tuning of the superpotential coupling $\lambda$ is still required, in order to avoid a dominant contribution of cosmic strings.

non-renormalizable terms in the potential, $D$-term strings are formed at the end of the phase transitions which signals the end of inflation. These strings are analogous to the D-strings formed at the end of brane inflation, which is the result of brane collisions.

$D$-term inflation cannot be studied in the standard formulation of supergravity, which is ill defined whenever the superpotential vanishes and there are present constant Fayet
Iliopoulos terms. Following an effective supergravity formulation based on superconformal theory, the superpotential transforms under the $U(1)$ symmetry along the directions where the FI terms are constant. This transformation defines the charge assignments of the superfields. $D$-term inflation has to be studied within this new formulation of supergravity, which is well defined when the superpotential vanishes.

Cosmic strings contribute to the cosmic microwave background temperature anisotropies. All current measurements put severe constraints on the allowed cosmic string contribution. To achieve a compatibility between measurements and theoretical predictions one should fine-tune the couplings. This was already found in the case of minimal supergravity. It was therefore natural to ask the question of whether this result still holds in non-minimal supergravity. It was previously claimed in the literature that higher order Kähler potentials suppress the cosmic strings contribution. Studying a case of non-minimal supergravity, where we include higher order corrections in the Kähler potential, as well as supergravity with shift symmetry we conclude that cosmic string contribution will be dominant unless the couplings are fine-tuned. We also find that, as in the minimal case [25, 26], the 9% constraint on the cosmic string contribution is equivalent to the constraint

$$\sqrt{\xi} \leq 2.2 \times 10^{15} \text{ GeV} \iff G\mu \leq 8.4 \times 10^{-7}.$$  \hspace{1cm} (101)

We would also like to emphasize that if $\sqrt{\xi}$ is higher by a factor of 2, the cosmic string contribution is of the order of 100%, as show in figure 7 of [25].

In conclusion, we definitely disagree with the statement that non-minimal Kähler potentials avoid the cosmic strings problem, which should imply that fine-tuning is not necessary. Even though we have not studied a large number of Kähler potentials, our current findings indicate that the problem of fine-tuning is unavoidable unless one considers a more complicated model, for example where strings become topologically unstable, namely semi-local strings. We have not yet studied [47] the spectral index in our models to check the consistency with the limits imposed by the recent three-year WMAP data, which however have been given only for purely adiabatic models. If the spectral index is higher than the one preferred from the measurements and since, in addition, the requirement for very small couplings seems more difficult to be satisfied in string theory, one should then look for mechanisms to suppress the role of strings, for example by making them unstable, along the lines of [9, 8], by adding new terms in the superpotential [49], or by considering the curvaton mechanism [48, 26].

As we were completing this work, [50] came to our attention. In that study, the authors find, in the context of minimal supergravity, slightly higher upper bounds for the parameters $\lambda$ and $g$, whereas the obtain the same with us upper bound on the FI term. This small discrepancy originates from the different analysis followed in [50]; the authors perform a full Markov chain Monte Carlo analysis and consider one more parameter, namely the spectral index.

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References

[1] Linde A, 1983 Phys. Lett. B 129 177 [SPIRES]
[2] Linde A, 1990 Particle Physics and Inflationary Cosmology (Contemporary Concepts in Physics vol 5) (New York: Harwood Academic)
[3] Linde A, 1991 Phys. Rev. Lett. B 259 38 [SPIRES]
[4] Linde A, 1994 Phys. Rev. D 49 748 [SPIRES] [astro-ph/9307002]
[5] Copeland E J, Liddle A R, Lyth D H, Stewart E D and Wands D, 1994 Phys. Rev. D 49 6410 [SPIRES] [astro-ph/9401011]
[6] Jeannerot R, Rocher J and Sakellariadou M, 2003 Phys. Rev. D 68 103514 [SPIRES] [hep-ph/0308134]
[7] Vachaspati T and Achucarro A, 1991 Phys. Rev. D 44 3067 [SPIRES]
[8] Urrestilla J, Achucarro A and Davis A C, 2004 Phys. Rev. Lett. 92 251302 [SPIRES] [hep-th/0402032]
[9] Binetruy P, Dvali G, Kallosh R and Van Proeyen A, 2004 Class. Quantum Grav. 21 3137 [SPIRES] [hep-th/0402046]
[10] Dine M, Seiberg N and Witten E, 1987 Nucl. Phys. B 289 585 [SPIRES]
[11] Majumdar M and Davis A C, 2002 J. High Energy Phys. JHEP03(2002)056 [SPIRES] [hep-th/0201408]
[12] Durrer R, Kunz M and Sakellariadou M, 2005 Phys. Lett. B 614 12 [SPIRES] [hep-th/0501163]
[13] Sen A, 1998 J. High Energy Phys. JHEP08(1998)010 [SPIRES] [hep-th/9805019]
[14] Dvali G R and Tye S-H, 1999 Phys. Lett. B 450 72 [SPIRES] [hep-th/9812483]
[15] Dvali G, Kallosh R and Van Proyen A, 2004 J. High Energy Phys. JHEP01(2004)035 [SPIRES] [hep-th/0312005]
[16] Dasgupta K, Herdeiro C, Hirano S and Kallosh R, 2002 Phys. Rev. D 65 126002 [SPIRES] [hep-th/0203019]
[17] Dasgupta K, Hsu J P, Kallosh R, Linde A and Zagermann M, 2004 J. High Energy Phys. JHEP08(2004)030 [SPIRES] [hep-th/0405247]
[18] Jeannerot R, 1997 Phys. Rev. D 56 6205 [SPIRES]
[19] Kofman L A and Linde A D, 1987 Nucl. Phys. B 322 555 [SPIRES]
[20] Linde A D and Riotto A, 1997 Phys. Rev. D 56 1841 [SPIRES]
[21] Lyth D H and Riotto A, 1999 Phys. Rep. 314 1 [SPIRES]
[22] Contaldi C, Hindmarsh M and Magueijo J, 1999 Phys. Rev. Lett. 82 2034 [SPIRES]
[23] Battye R A and Weller J, 2000 Phys. Rev. D 61 043501 [SPIRES]
[24] Bouchet F R, Peter P, Riazuelo A and Sakellariadou M, 2002 Phys. Rev. D 65 021301 [SPIRES] [astro-ph/0205022]
[25] Rocher J and Sakellariadou M, 2005 J. Cosmol. Astropart. Phys. JCAP03(2005)004 [SPIRES] [hep-th/0406120]
[26] Rocher J and Sakellariadou M, 2005 Phys. Rev. Lett. 94 011303 [SPIRES] [hep-ph/0412143]
[27] Fraisse A A, Limits on SUSY GUTs and defects formation in hybrid inflationary models with three-year WMAP observations, 2006 Preprint astro-ph/0603589
[28] Battye R A, Robinson J and Albrecht A, 1998 Phys. Rev. Lett. 80 4847 [SPIRES] [astro-ph/9711336]
[29] Pogosian L, Tye S H, Wasserman I and Wyman M, 2003 Phys. Rev. D 68 023506 [SPIRES] [hep-th/0304188]
[30] Pogosian L, Tye S H, Wasserman I and Wyman M, 2006 Phys. Rev. D 73 089904 [SPIRES] [astro-ph/0604141] (erratum)
[31] Bevis N, Hindmarsh M, Kunz M and Urrestilla J, CMB power spectrum contribution from cosmic strings using field-evolution simulations of the Abelian Higgs model, 2006 Preprint astro-ph/0605018
[32] Davis S C, Davis A-C and Trodden M, 1997 Phys. Lett. B 405 257 [SPIRES] [hep-ph/9702360]
[33] Bailin D and Love A, 2003 Supersymmetric Gauge Field Theory and String Theory (Bristol: Institute of Physics Publishing)
[34] Nilles H P, 1984 Phys. Rep. 110 1 [SPIRES]
[35] Binétruy P and Dvali G, 1996 Phys. Lett. B 388 241 [SPIRES] [hep-ph/9606342]
[36] Halyo E, 1996 Phys. Lett. B 387 43 [SPIRES] [hep-ph/9606423]
[37] Kallosh R, Kofman L, Linde A D and Van Proeyen A, 2000 Class. Quantum Grav. 17 4269 [SPIRES] [hep-th/0006179]
[38] Freedman D Z and Kors B, Kähler anomalies in supergravity and flux vacua, 2005 Preprint hep-th/0509217
[39] Elvang H, Freedman D Z and Kors B, Anomaly cancellation in supergravity with Fayet–Iliopoulos couplings, 2006 Preprint hep-th/0606012
[40] Sakellariadou M, 2006 Ann. Phys., NY 15 264 [SPIRES] [hep-th/0510227]
[41] Sakellariadou M, 2006 Cosmic Strings (Springer Lecture Notes in Physics (LNP)) (Berlin: Springer) [hep-th/0602276]
[42] Rocher J, Constraining SUSY GUTs and inflation with cosmology, Proc. A. Einstein’s Century Conf. [hep-ph/0603169]

[43] Linde A, Inflation and string cosmology, 2005 Preprint hep-th/0503195

[44] Kawasaki M, Yamaguchi M and Yanagida T, 2001 Phys. Rev. D 63 103514 [SPIRES] [hep-ph/0011104]

[45] Seto O and Yokoyama J, 2006 Phys. Rev. D 73 023508 [SPIRES] [hep-ph/0508172]

[46] Spergel D N et al, Wilkinson Microwave Anisotropy Probe (WMAP) three-year results: implications for cosmology, 2006 Preprint astro-ph/0603449

[47] In preparation

[48] Endo M, Kawasaki M and Moroi T, 2003 Phys. Lett. B 569 73 [SPIRES] [hep-ph/0304126]

[49] Lin C-M and McDonald J, Supergravity modification of D-term hybrid inflation: Solving the cosmic string and spectral index problems via a right-handed sneutrino, 2006 Preprint hep-ph/0604245

[50] Battye R A, Garbrecht B and Moss A, Constraints on supersymmetric hybrid inflation models, 2006 Preprint astro-ph/0607339