Autocorrelation and limited sampling algorithm with fixed packet length in period measurement

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Abstract. This paper proposed a real time autocorrelation and limited sampling algorithm with fixed packet length based on Field Programmable Gate Array (FPGA) for the period estimation and significant data transmission for arbitrary periodic signal, which contributes to information transfer in the Internet of things for smart sensors, and has a wide applications such as radar, communication, sensors network and measurement. Firstly, an explanation of the algorithm is shown. Secondly, several simulation results are analyzed. Finally, experimental and hardware implementation results are shown.

1. Introduction

Significant data transmission and period measurement of arbitrary periodic signal has received much attention in the literature because of its wide application in numerous engineering applications, such as radar, communication, power systems, measurement [1-6], sensors network and Internet of things (IoT). Sensors network, the basic part of Internet of Things, has played an important role in data acquisition and transmission. [7-8] Here are some previous work about significant data transmission based on period measurement:

In [11-14], a compressive sensing algorithm is used to estimate the period of signal which takes casually a fixed number of samples from a signal. However, compressive sensing requires keeping memory, which could be very large and not very suitable for real-time transmission.

In [7,10], a real time segmentation and labeling algorithm is proposed for period measurement to decrease the amount of data without loss of information content. This technique only saves some important samples of the acquired signal, which greatly reduces the computational burden. But due to the constant sampling time of the segmentation step, oversampling will still cause a waste of storage, and furthermore, different transmission packet length will increase the encoding and decoding burden of the receiving system and other systems afterwards.

2. Real Time Autocorrelation and Limited Sampling Algorithm

The advantage of this algorithm is based on the need of describing the signal, aiming to extract the most important samples and standardized them into a fixed length packet for transmission. In this manner the acquired signal can be fully described by packets consist of period and significant data. With the period of the acquired signal estimated in advance, which changing the sampling rate to prevent oversampling and maintain a fixed packet length, the sampling time limitation and packet formation is set without loss of important data content. Moreover, it is possible to extract simple features from the packets standardized by the algorithm, which gives information on the acquired
signal, and analyzes the arbitrary periodic signal with low computational burden. The algorithm can be divided into two main phases: the autocorrelation phase, and the limited sampling phase (called A&L algorithm in the rest of the paper).

2.1. Autocorrelation
Starting from the acquired samples, acquired in a default sampling rate \( f_s \), the main task of the first phase is to calculate the autocorrelation of the time interval series, and estimate the period of the acquired signal. The acquired signal is equally divided into two paths (in the following called path \( a \) and path \( b \)). Since the impedance of each path are equal, the period and the amplitude in each path is equal to the acquired signal, and the power of each path accounts for half of the acquired signal. As shown in Figure 1, the signal in path \( a \) becomes a square wave (called reshaped signal in the following passage) after passing through the high-frequency hysteresis comparator TLV3501 with a low latency of 4.5ns.

Once a rising edge of the reshaped signal is detected, it calculates the time interval between two adjacent rising edges immediately and stored in a discrete sequence \( \{ T_n \} = 0, 1, 2, \ldots \) using the autocorrelation function to calculate the autocorrelation value of the discrete sequence \( \{ T_n \} \), the calculation process is as follows:

\[
R_{x,x}(s) = \left( \left\lfloor 2f_s \right\rfloor \right)^{1} \sum_{i=0}^{\left\lfloor 2f_s \right\rfloor - 1} T_i \times T_{i+s}
\]

(1)

Where integer \( s \) is the offset variable of the discrete sequence, which represents the interval between two discrete values in a sequence \( s \in \{0, 1, 2, \ldots, N-1\} \). Find the adjacent maxima of its autocorrelation as \( R_{x,x}(m), R_{x,x}(m+k) \). The process is as shown in the Figure 2. Where \( k \) is used to represent the period of \( R_{x,x}(s) \). Since the period of \( R_{x,x}(s) \) is the same as that of \( \{ T_n \} \), the period of the reshaped signal \( S_e = k \), and the period of the acquired signal can be estimated as followed:

\[
T_e = \left( \left\lfloor 2f_s \right\rfloor - k \right)^{1} \sum_{i=0}^{\left\lfloor 2f_s \right\rfloor - k} \sum_{s=0}^{S_e} T_{i+s}
\]

(2)

However, \( k \) is not a fixed value when the period of acquired signal changes, and since \( T_e \) changes with \( k \), it will affect the operation of the next phase accordingly. Details will be described in the next section.

2.2. Limited Sampling
This phase reassigns the most suitable sampling frequency \( f_{ms} \) based on \( T_e \) to resample the acquired signal, and pack the collected data in a specific style. During the autocorrelation phase, \( T_e \), known as
the predicted period of the acquired signal, is determined, and the frequency of the acquired signal \( f_x \) can be predicted accordingly. Set \( f_{ms} = f_x \times M \) and resample the acquired signal at a sampling rate of \( f_{ms} \). Setting a proper parameter \( M \) will minimize the reconstruction distortion, thus improving the accuracy. Normally, \( M \) is preset to 50 to achieve the best performance for arbitrary periodic signal. The Limited-Sampling phase is shown in Figure 3.

The collected data is packed in a specific style, which can describe the important characteristics of the acquired signal. It stored the period \( T_e \) at the top of data pack, while other significant data is sequentially stored in the rest of data pack. This means that even if the input signal changes, the length of the packet remains the same. This specific packaging method specifies a fixed format for the entire sensor network, reduces the burden on the network due to packet cutting or splicing, and can determine whether transmission errors occur through the packet length, which greatly improves error detection capability of the entire network. This is enough to make up for the estimation time of the algorithm due to its complexity. The format of the data pack is as shown in the Figure 4.

![Figure 3. Evaluation of the collected data by real time autocorrelation and limited sampling.](image)

![Figure 4. Data packet format diagram.](image)

### 3. Simulation and Results

Simulations of the algorithm are implemented in MATLAB® to evaluate the uncertainty in algorithm's measurement process and relative error of period estimation, with reference to multiple waveforms as sine, square, triangular wave or multi-tone.

Based on A&L algorithm, a Simulink simulation model was built to test the performance of the algorithm. As is shown in Figure 5(a)(c), the input signal is obtained by adding the sinusoidal signal of 25Hz 2Vpp to the sinusoidal signal of 60Hz2Vpp under two different SNR condition. And in Figure 5(b)(d), and the autocorrelation function at the length of 120 is shown in Figure(b)(d) respectively. According to the autocorrelation function, it can be inferred that in the high SNR(30dB) environment, the occurrence position of the maximum shows a good periodicity, that is, an accurate period estimation value can be obtained, and in the low SNR(10dB) environment, although the autocorrelation function looks irregular, the occurrence positions of maximum still have a certain periodicity. When a proper maximum resolution threshold is set, great periodicity can still be obtained. According to the periodicity of the occurrence position of the autocorrelation function maximum, the estimated period of the input signal can be calculated using equation (2) appears above. The calculation results are compared with the input signal period, and the relative errors estimated by the A&L algorithm are shown in table 1, where the period of the input signal is preset to 100kHz(10us) by MATLAB® & Simulink. It can be concluded from the simulation data that the algorithm has certain anti-noise ability. Under the condition that SNR=30dB, the relative error of the four periodic signals of 100kHz is kept at around 1%, and the relative error of the sinusoidal signal is as low as 0.5%. Under the condition that SNR=10dB, the period relative error remains below 2.7%. Therefore, it can be
inferred that the algorithm has great periodic estimation ability in a variety of noise testing environments.

Figure 5. Autocorrelation diagram of interval in multi-tone signal in different SNR condition. Input waveform is the sum of 25Hz2Vpp and 60Hz2Vpp sinusoidal signal (a) waveform SNR=30dB (b) autocorrelation diagram SNR=30dB (c) waveform SNR=10dB (d) autocorrelation diagram SNR=10dB.

Table 1. Estimated Period in Simulation Using A&L Algorithm.

| Input Freq | Sampling Freq | Waveform | SNR  | Estimated Period[μs] | Relative Error |
|------------|---------------|----------|------|----------------------|----------------|
| 100kHz     | 2MHz          | Sine     | 30dB | 10.05                | 0.5%           |
|            |                |          | 10dB | 10.13                | 1.3%           |
|            |                | Square   | 30dB | 9.92                 | 0.8%           |
|            |                |          | 10dB | 10.16                | 1.6%           |
|            |                | Triangular | 30dB | 10.11               | 1.1%           |
|            |                |          | 10dB | 9.77                 | 2.3%           |
|            |                | Multi-tone | 30dB | 10.13               | 1.3%           |
|            |                |          | 10dB | 10.27               | 2.7%           |

4. Hardware Implementation

Experimental results in testing the performance of A&L algorithm with FPGA platform are shown in this section. A development board from Terasic for a DE10-Lite is used for testing the algorithm, with a RIGOL DG1032 signal generator and a RIGOL DS2202A digital oscilloscope. In this section, we mainly do two experiments. One is to use FPGA to estimate the period of the signal generated by the signal generator. The second is to reconstruct the significant data to analog signals to evaluate the reliability of the A&L algorithm.
4.1. Period Estimation
In this part, we use the FPGA platform to test the period estimation ability of A&L algorithm. The input signal is generated by signal generator, in 4 different waveforms, with the period of 100kHz. The results, in table 2, are reported, compared with the period measured by the RIGOL DS2202A digital oscilloscope, and the estimated period is acquired in Quartus 17.0 SignalTap, which reserved four significant digits.

Table 2. Testing Results Using A&L Algorithm.

| Input Freq | Sampling Freq | Waveform | Estimated Period[μs] | Relative Error |
|------------|----------------|----------|----------------------|---------------|
| 100kHz     | 2MHz           | Sine     | 10.10                | 1.0%          |
|            |                | Square   | 10.19                | 1.9%          |
|            |                | Triangular | 10.14                | 1.4%          |
|            |                | Multi-tone | 9.740                 | 2.6%          |
| 2MHz       | 40MHz          | Sine     | 0.517                | 3.4%          |
|            |                | Square   | 0.746                | 4.8%          |
|            |                | Triangular | 0.516                | 3.2%          |
|            |                | Multi-tone | 10.10                 | 1.0%          |

4.2. Significant Data Reconstruction
In this part, significant data is used to reconstruct the signals, and the degree of waveform reconstruction reflects the reliability of acquired significant data. In this experiment, the FPGA was used to control the ADC and DAC to acquire and reconstruct the significant data, and the relative amplitude error between the reconstructed signal and the input signal was used to represent the degree of waveform reconstruction and reliability of significant data. The reliability of waveform data transmission increases with the reliability of significant data. The test results for the reconstruction of significant data are shown in Figure 6. It can be seen from the test results that when the input amplitude is less than 4V, the relative error of the reconstructed amplitude can be stabilized within 1%. When the amplitude is greater than 4V or less than 1V, the relative error has an upward trend. This is because the upper limit of the input voltage and the signal-to-noise ratio (SNR) of the channel are limited. When the input signal amplitude is too large, it will exceed the ADC acquisition range, which will cause errors in the measurement and cause damage to the device. When the input amplitude is too small, the SNR will drop sharply, causing an extreme loss of input signal clarity. Our recommended input signal amplitude is from 1.5V to 3.5V.

![Figure 6. Reconstruction relative amplitude error.](image)

With this algorithm, it will be possible to estimate the period from the input signal, and realize fixed packet length transmission of significant data. If we put this proposed algorithm in front of the combined sensor system, the algorithm can achieve high period measurement accuracy while reducing
the computational burden or error caused by the different packet lengths of all the subsystems in the subsequent stages. When designing a sensor network, if there are no excessive real-time requirements, placing the system in the front stage will simplify the hardware or software complexity of all subsystems in the rear stage and improve error detection capability of the sensor system.

5. Conclusion
This paper proposed an autocorrelation and limited sampling algorithm to realize the period estimation of high frequency arbitrary periodic signal (<10MHz) and significant data transmission in adaptive sampling rate, which reduced the incidence of oversampling and furthermore, creating a specific format with fixed packet length, thus unifying all significant data after this system into a format with fixed packet length to simplify the operation of the system on the package and improve the performance of the entire system. It shows the capability to implement it on FPGA device, and confirms the high reliability of the significant data generated by the A&L algorithm.

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