Exotic plasmon modes of double layer graphene

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We derive the plasmon dispersion in doped double layer graphene (DLG), made of two parallel graphene mono-layers with carrier densities $n_1$ and $n_2$ respectively, and an inter-layer separation of $d$. The linear chiral gapless single particle energy dispersion of graphene leads to exotic DLG plasmon properties with several unexpected experimentally observable characteristic features such as a nontrivial influence of an undoped ($n_2 = 0$) layer on the DLG plasmon dispersion and a strange influence of the second layer even in the weak-coupling $d \to 0$ limit. Our predicted exotic DLG plasmon properties clearly distinguish graphene from the extensively studied usual parabolic 2D electron systems.

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Graphene, a two dimensional (2D) layer of carbon atoms arranged in a honeycomb lattice, has attracted a great deal of attention because of its unique electronic properties arising from its chiral, gapless, linear band dispersion [1]. Of particular interest and fundamental importance are those graphene properties which are qualitatively different from the behavior of electrons (or holes) in the extensively studied 2D semiconductor systems. Some of these unique graphene properties [1] are its ‘half-integral’ integer quantized Hall effect, a carrier mobility independent of carrier density away from the charge neutrality point (i.e. the Dirac point), and the finite minimum conductivity around the Dirac point without any obvious metal-insulator transition. In this Letter, we predict a new exotic collective property of graphene electrons, which distinguishes graphene qualitatively from the semiconductor-based parabolic 2D electron systems. We find the surprising result that the plasma modes of an interacting double layer graphene (DLG) system are completely different from the well studied double layer semiconductor quantum well plasmons [2]. Our finding is unexpected and interesting because the collective plasmon excitations arise from the long-range Coulomb interaction which is the same in graphene and parabolic 2D systems. The exotic DLG plasmon properties are experimentally observable and should have important consequences for the many-body effects in doped multi-layer graphene structures.

Plasma modes of 2D double layer structures have been extensively studied ever since the existence of an undamped acoustic plasmon mode was predicted [2] in semiconductor double quantum well systems. It is known that when the two 2D layers are put in close proximity with a high insulating barrier between them to prevent interlayer tunneling, the 2D plasmons are coupled by the interlayer Coulomb interaction leading to the formation [2] of two branches of longitudinal collective excitation spectra called the optical plasmon (OP, $\omega_+ \sim q^{1/2}$, where $q$ is the 2D wave vector) and the acoustic plasmon (AP, $\omega_- \sim q$), where the density fluctuations in each component oscillate in phase (OP) and out of phase (AP), respectively, relative to each other, assuming the two charge components to be the same. These collective modes of the double layer structures, which have been directly experimentally observed [3], play important roles in the many-body properties such as screening and drag [4, 5].

Here we consider the experimentally relevant issue of the collective mode dispersion in DLG formed by two parallel single layer graphene (SLG) separated by a distance $d$. The DLG is fundamentally different from well studied bilayer graphene [6] because there is no inter-layer tunneling, only inter-layer Coulomb interaction. Spatially separated two-component DLG can be fabricated by folding an SLG over a high insulating substrate [7].

We calculate the DLG plasmon modes and their loss functions (spectral strength). We find many intriguing and unexpected features of coupled plasmon modes and their Landau damping in DLG. Even though we recover in the long wavelength ($q \to 0$) limit the well-known optical ($\omega_+ \sim q^{1/2}$) and acoustic ($\omega_- \sim q$) plasmons, the density dependence is given by $\omega_+^2(q \to 0) \propto \sqrt{n_1 + \sqrt{n_2}}$ and $\omega_-^2(q \to 0) \propto \sqrt{n_1 n_2}/(\sqrt{n_1} + \sqrt{n_2})$ compared to $\omega_+^2 \propto N$ and $\omega_-^2 \propto n_1 n_2/N$ in the ordinary 2D system, where $N = n_1 + n_2$. When the interlayer Coulomb coupling is weak (i.e. for large separations, $k_F d \gg 1$) the undamped DLG modes ($\omega_{\pm}$) become degenerate and have the same frequency as the uncoupled SLG plasmon [8], but inside the interband Landau damping regime this degenerate mode splits into two coupled modes. In the strong coupling regime ($k_F d \ll 1$) the $\omega_-$ mode approaches the intraband electron-hole continuum (i.e. $\omega_- \rightarrow v_F q$) and is overdamped, but the $\omega_+$ mode is shifted to higher frequency and in the long wavelength limit becomes the SLG plasmon with a density $N = n_1 + n_2 + 2\sqrt{n_1 n_2}$ instead of the density $N = n_1 + n_2$ as happens in regular 2D double layer systems. We also find that the DLG plasmon modes are heavily damped even in the...
long wave length limit when the two layers have unequal density, even when one layer is undoped (i.e. \( n_2 = 0 \)). We believe that our predictions may be easily observable via inelastic light scattering spectroscopy \([4]\), frequency-domain far infrared (or microwave) spectroscopy \([10]\), or inelastic electron scattering spectroscopy \([11, 12]\).

The plasmon modes are given by the poles of the density-density correlation function, or equivalently by the zeros of the dynamical dielectric function. For a double layer system the collective modes are given by the zeros of the generalized dielectric tensor \( \epsilon_{lm} \) where \( l, m = 1 \) or 2 with 1.2 denoting the two layers. \([2]\) The dielectric function \( \epsilon_m(q, \omega) = \delta_{lm} - v_{lm} \Pi_m \) is obtained within the mean field random phase approximation (RPA) in our theory, and the two component determinantal equation becomes

\[
\begin{align*}
\epsilon(q, \omega) &= [1 - v_{11}(q)\Pi_1(q, \omega)] [1 - v_{22}(q)\Pi_2(q, \omega)] \\
& \quad - v_{12}(q)v_{21}(q)\Pi_1(q, \omega)\Pi_2(q, \omega),
\end{align*}
\]

where \( v_{il}(q) \) and \( v_{im}(q) \) are respectively the interlayer and intralayer Coulomb interaction matrix elements. In our model, \( v_{il}(q) = 2\pi e^2/(\kappa q) \); \( v_{im}(q) = \nu_i(q) \exp(-qd) \), with \( \kappa \) as the background lattice dielectric constant. Finally, \( \Pi_i(q, \omega) \) in Eq. \((1)\) is the noninteracting SLG polarizability function given by \([3]\):

\[
\Pi_i(q, \omega) = -\frac{g}{L^2} \sum_{k, s} \frac{f_{sk} - f_{sk'}}{\omega + \epsilon_{sk} - \epsilon_{sk'} + i\eta} F_{s, s'}(k, k'),
\]

where \( g = g_sg_v \) (with \( g_s = 2 \) and \( g_v = 2 \) being spin and valley degeneracy), \( k' = k + q, s, s' = \pm 1 \) denote the band indices, \( \epsilon_{sk} = sv_F|k| \) \((v_F \text{ being the Fermi velocity of graphene and } h = 1 \text{ throughout this paper})\), and \( F_{s, s'}(k, k') \) is the overlap of states and given by \( F_{s, s'}(k, k') = \frac{1}{2}(1 + ss' \cos \theta) \), where \( \theta = \frac{k - k'}{2} \) is the angle between \( k \) and \( k' \), and \( f_{sk} \) is the Fermi distribution function, \( f_{sk} = [\exp(\beta(\epsilon_{sk} - \mu_l)) + 1]^{-1} \), with \( \beta = 1/k_BT \) and \( \mu_l \) the chemical potential of \( l \)-the layer. In the long wavelength limit \((q \to 0)\) the polarizability becomes \([3]\):

\[
\Pi_i(q, \omega) = \frac{g k_{F_i}}{4\pi v_F} \frac{q^2}{\omega^2} \left[ 1 - \frac{\omega}{2E_{F_i}} \ln \frac{2E_{F_i} + \omega}{2E_{F_i} - \omega} \right],
\]

where \( k_{F_i} (E_{F_i} = v_F k_{F_i}) \) is the Fermi wave vector (Fermi energy) of the \( i \)-th layer.

The analytical formula for the long-wavelength plasmon dispersion can be obtained in the general situation where the two layer have different Fermi wave vectors and Fermi energies by virtue of having different densities \( n_1, n_2 \). From Eqs. \((1)\) and \((3)\) it is possible to obtain the following long wavelength plasma modes of the coupled DLG:

\[
\begin{align*}
\omega_1^2(q \to 0) &= 2r_s v_F^2 (k_{F_1} + k_{F_2}) q, \tag{4a} \\
\omega_2^2(q \to 0) &= \frac{4r_s v_F^2 k_{F_1} k_{F_2} d}{k_{F_1} + k_{F_2}} q^2, \tag{4b}
\end{align*}
\]

where \( r_s = e^2/\kappa v_F \) is the effective graphene fine structure constant. Thus, one recovers in a straightforward fashion the well-known optical \((\omega_+ \sim q^{1/2})\) and acoustic \((\omega_- \sim q)\) plasmons of a double layer system. In addition we have the density dependence of DLG plasmons as \( \omega_1^2(q \to 0) \propto \sqrt{n_1 + n_2} \) and \( \omega_2^2(q \to 0) \propto \sqrt{n_1 n_2}/(\sqrt{n_1} + \sqrt{n_2}) \), which are different from the density dependence of the corresponding plasmon dispersion in ordinary 2D systems \([2]\) where the \( \omega_2^2 \) depends on the total 2D electron density \( N = n_1 + n_2 \), i.e. \( \omega_+ \propto N \), and \( \omega_- \propto n_1 n_2/N \).

In Fig. \(\text{I}\) we show the calculated DLG plasmon dispersions \((\omega_\pm)\) for equal densities of \( n_1 = n_2 = 10^{12} \text{cm}^{-2} \) and several layer separations. The plasmon mode dispersion of the uncoupled SLG with the same density is shown as the dot-dashed line. In Fig. \(\text{I}\) we also show the electron-hole continua or single-particle excitation (SPE) region, which determines the decay (Landau damping) of the plasmon at a given frequency and wave vector. If the collective mode lies inside the SPE continua, we expect the mode to be Landau damped. The SPE continuum is defined by the nonzero value of the imaginary part of the total dielectric function, \( \text{Im} [\epsilon(q, \omega)] \neq 0 \), because the damping of the plasmon by emitting an electron-hole single pair excitation is allowed in the region of nonzero \( \text{Im} [\Pi(q, \omega)] \).

For graphene, both intraband and interband SPE transitions are possible \([8]\), and the boundaries (dashed lines) are given in Fig. \(\text{I}\). Due to the phase-
space restriction, the interband SPE continuum opens a gap in the long wavelength region satisfying the following condition: \( v_F q < \omega < 2E_F - v_F q \), in which the undamped plasmon modes exist. Since the normalized plasmon mode dispersion in \( E_F \) scales with normalized Fermi wave vector, our calculated results of Fig. 1 apply also for other densities.

As shown in Fig. 1(a), for a small spatial separation \((k_F d < 1)\), the frequency of the acoustic mode \( \omega_- \) decreases compared with the uncoupled mode and approaches the upper boundary of intraband SPE (i.e. \( \omega = v_F q \)) while the optical mode \( \omega_+ \) shifts to higher energy. Especially, as \( d \to 0 \), the \( \omega_- \) mode becomes degenerate with the electron-hole continuum and loses its identity (i.e. \( \omega_- \to v_F q \)). On the other hand, the \( \omega_+ \) mode becomes the SLG plasmon with a density \( N = n_1 + n_2 + 2\sqrt{n_1 n_2} \) in the long wavelength limit. This is very interesting because, when the two layers with densities \( n_1, n_2 \) become one combined system \((d \to 0)\), the optical plasmon mode \( (\omega_+) \) should correspond to the SLG plasmon of total density \( N = n_1 + n_2 \), which is precisely what happens in regular 2D double layer system. As the spatial separation increases the optical (acoustic) plasmon frequency decreases (increases) slowly and the mode coupling occurs at the boundary of interband SPE (see Fig. 1(b)). As \( d \) increases further the two modes become degenerate below the SPE \( \text{inter} \) (i.e. in the undamped region), but the degenerate mode is divided into two coupled modes inside interband SPE (see Fig. 1(c) and (d)). For infinite separation, the two modes become uncoupled, and have the dispersion of SLG plasmon with the density \( n = n_1 = n_2 \).

In Fig. 2 we show the calculated DLG loss function (i.e. \(-\text{Im}[\epsilon(q, \omega)^{-1}]\)) plotted in \( q-\omega \) space. The loss function is related to the dynamical structure factor \( S(q, \omega) \) by \( S(q, \omega) \propto -\text{Im}[\epsilon(q, \omega)^{-1}] \). The dynamical structure factor gives a direct measure of the spectral strength of the various elementary excitations. Thus, our calculated loss function can be measured in experiments such as inelastic electron and Raman scattering spectroscopies. When both \( \text{Re}[\epsilon] \) and \( \text{Im}[\epsilon] \) become zero (i.e. \( \epsilon(q, \omega) = 0 \), which defines the plasmon mode), the imaginary part of the inverse dielectric function, \( \text{Im}[\epsilon(q, \omega)^{-1}] \), is a \( \delta \)-function with the strength \( W(q) = \pi |\partial \text{Re}[\epsilon(q, \omega)]/\partial \omega|_{\omega=\omega_p(q)}^{-1} \), where \( \omega_p(q) \) is the plasmon dispersion. Thus the undamped plasmon shows up as well defined \( \delta \)-function peak in the loss function. The damped plasmon, however, corresponds to a broadened peak in the loss function – for larger broadening, the plasmon is overdamped, and there is no peak in the loss function.

In Fig. 2 we show the \(-\text{Im}[\epsilon(q, \omega)^{-1}]\) in density plots where darkness represents the mode spectral strength. The solid lines indicate the \( \delta \)-function peaks and correspond to the well-defined undamped plasmon modes. The intraband electron-hole SPE continuum shows up as weak broad (incoherent) structure in Fig. 2 and carries small spectral weight. For a small spatial separation only the in-phase mode \( \omega_+ \) carries any significant spectral strength. As \( d \) increases the out-of-phase mode carries substantial spectral weight even at long wavelengths \((q \to 0)\). Both \( \omega_{\pm} \) modes in general carry finite spectral weights and should be observable in experiments [9, 10, 11, 12]. Since both \( \omega_{\pm}(q) \) are greater than \( \omega = v_F q \) for all \( q \), which is the upper boundary of the intraband electron-hole pair continuum, we expect these modes not to decay to intraband electron-hole pairs. But the modes enter into interband SPE at finite \( q_c \), which is given by the condition, \( \omega_{\pm}(q_c) = 2E_F - v_F q_c \). If \( q > q_c \), \( \epsilon(q, \omega_{\pm}) \) has a finite imaginary part and the plasmon modes become damped consequently (Landau damping). The plasmon mode inside the Landau damping region decays by emitting interband electron-hole pairs which is now allowed by energy-momentum conservation. Near the boundary of the interband SPE the mode damping is not significant.

In Fig. 3 we show the calculated DLG plasmon mode dispersion and the corresponding loss function \((-\text{Im}[\epsilon(q, \omega)^{-1}]\)) for unequal layer densities at fixed layer separation \((d = 100 \text{ Å})\). As the layer densities are different the undamped plasmon region becomes smaller in \( q-\omega \) space. As \( q \) increases the modes first enter SPE \( \text{inter} \) at smaller \( q_c \) and decay by producing interband electron-hole pairs in the layer 2. As \( q \) increases further the

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FIG. 2: The density plots of DLG loss function \((-\text{Im}[\epsilon(q, \omega)^{-1}]\)) in \((q, \omega)\) space for fixed densities of \( n_1 = n_2 = 10^{12} \text{cm}^{-2} \) and several layer separations \( d = 20, 100, 300, 500 \text{ Å} \). The dashed lines indicate the boundaries of SPEs and the solid lines represent the undamped plasmon modes (\( \delta \)-function peaks in the loss function).
and for different layer densities. Here we use $n^{-}-n^{+}$ very broad peak for high wave vectors. As the separation is finite (i.e. $d = 0$), the two plasmons merge at $q/k = 0$ and $m_{+}$ never meet. In Fig. 4(b)-(d) we show the loss function corresponding to the Fig. 4(a). Since all modes are inside of SPE, there is no undamped Landau region, so the modes are damped by producing interband electron-hole pairs in layer 2 even in the long wavelength limit. The loss function shows that the mode $\omega_{-}$ does not carry any spectral weight, and all spectral weight is carried by $\omega_{+}$. For $d = 0$, the $\omega_{+}$ mode becomes a well defined peak only in the long wavelength limit and appears as a very broad peak for high wave vectors. As the separation increases, $\omega_{+}$ is well defined below $q_{m}$ even inside SPE, and manifests well defined spectral peaks. Note that there is no significant spectral weight for $q > q_{m}$.

In conclusion, we investigate theoretically the dynamical response of DLG. We find interesting novel plasmon features unique to DLG which do not exist in ordinary 2D double quantum well structures.

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