Observational constraint on spherical inhomogeneity with CMB and local Hubble parameter

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Abstract. We derive an observational constraint on a spherical inhomogeneity of the void centered at our position from the angular power spectrum of the cosmic microwave background (CMB) and local measurements of the Hubble parameter. The late time behaviour of the void is assumed to be well described by the so-called $\Lambda$-Lemaître-Tolman-Bondi (ΛLTB) solution. Then, we restrict the models to the asymptotically homogeneous models each of which is approximated by a flat Friedmann-Lemaître-Robertson-Walker model. The late time ΛLTB models are parametrized by four parameters including the value of the cosmological constant and the local Hubble parameter. The other two parameters are used to parametrize the observed distance-redshift relation. Then, the ΛLTB models are constructed so that they are compatible with the given distance-redshift relation. Including conventional parameters for the CMB analysis, we characterize our models by seven parameters in total. The local Hubble measurements are reflected in the prior distribution of the local Hubble parameter. As a result of a Markov-Chains-Monte-Carlo analysis for the CMB temperature and polarization anisotropies, we found that the inhomogeneous universe models with vanishing cosmological constant are ruled out as is expected. However, a significant under-density around us is still compatible with the angular power spectrum of CMB and the local Hubble parameter.

Keywords: cosmic flows, cosmological parameters from CMBR, dark energy theory, supernova type Ia - standard candles

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1 Introduction

In observational cosmology, the global homogeneity and isotropy is a commonly unquestioned hypothesis, which is therefore called the cosmological principle. Actually, homogeneous and isotropic universe models have achieved great success to explain observational data and describe our universe. Nevertheless, it is interesting to ask how large magnitude of cosmological scale inhomogeneity can be compatible with the current cosmological observations. The observational test of the cosmological principle may be one of the most fundamental issues in cosmology just like old times. From this viewpoint, here we consider an observational constraint on cosmological scale inhomogeneity with the cosmic microwave background (CMB) and the local Hubble parameter. Since the isotropy of the universe is strongly supported by the isotropy of the CMB temperature, we focus on spherically symmetric inhomogeneous universe models.

Once we are allowed to be at the center of the universe with a spherical inhomogeneity, since most observables are limited on our past lightcone, the spatial inhomogeneity and temporal dependence may degenerate with each other. This fact gives one of the main difficulties in analyses of inhomogeneous universe models differently from homogeneous and isotropic universe models. Therefore, careful evaluation of observables and multi-directional analyses are important for the observational test of spherical inhomogeneity of our universe. We make a contribution to this issue from one direction in this paper.

Spherically symmetric dust universe models, so called the Lemaître-Tolman-Bondi (LTB) models [1–3], have been extensively studied in the last decade. The LTB models have been attracted much attention mainly as an alternative scenario to explain the apparent accelerated expansion of our universe without dark energy [4–6]. Actually, it is known that there exist the LTB models which can explain the observed luminosity distance redshift relation without a cosmological constant $\Lambda$ [5, 7, 8]. Especially, void-type inhomogeneity composed of growing modes has been actively studied because it can be compatible with the inflationary paradigm and the apparent accelerated expansion. Eventually, it has been revealed that the apparent accelerated expansion cannot be explained only by the radial inhomogeneity without a cosmological constant if we assume the standard cosmological history before the last scattering surface of CMB photons (see, e.g. refs. [9, 10] for a detailed analysis).
In contrast, the models with a non-vanishing cosmological constant, namely, ALTB models, have been studied not very often although we can find several restricted analyses [10–13]. In this paper, we assume that the late time behaviour of our universe is well described by a ALTB model. One remarkable feature in our approach is that we specify the spherically symmetric inhomogeneity by using the so-called inverse construction from a distance redshift relation. The same procedure is adopted in ref. [14]. The inverse construction is a method to construct the ALTB model in which the distance redshift relation for the central observer agrees with the designated one. In the case of ALTB models, as is shown in ref. [15], once the value of the cosmological constant is fixed and the angular diameter distance is specified as a function of the redshift, we can uniquely determine the ALTB model. Therefore, the parameters to specify a ALTB model are equivalent to the parameters contained in the distance redshift relation beside the value of the cosmological constant. In our analysis, the distance redshift relation is assumed to be given by the same form as the distance in the dust-dominated Friedmann-Lemaître-Robertson-Walker (FLRW) universe and parametrized by the Hubble constant $H_0$ and two fictitious cosmological parameters $\Omega_{\text{dis}m0}$ and $\Omega_{\text{dis}\Lambda0}$. It should be emphasized that $\Omega_{\text{dis}m0}$ and $\Omega_{\text{dis}\Lambda0}$ are not necessarily related to the real matter density and the cosmological constant $\Lambda$ but just parameters to specify a distance redshift relation. This procedure is different from conventional methods of artificial direct parametrization of LTB models adopted in previous works (see, e.g. [10, 16]). Therefore, it could be possible to extract unknown effects of the spherical inhomogeneity around us.

In order to keep the predictability of the CMB anisotropy, we restrict our attention to asymptotically homogeneous models, and gradually connect each of the ALTB models to a flat FLRW universe model. The connection is performed in the redshift interval $2 < z < 15$, and the models are described by the standard homogeneous and isotropic universe models including relativistic energy components before $z = 15$. The late time ALTB models are parametrized by four parameters including the value of the cosmological constant and the local Hubble parameter. Including conventional parameters for the CMB analysis, we characterize our models by seven parameters in total. For these seven parameters, we perform a Markov Chain Monte Carlo (MCMC) analysis by modifying the package CosmoMC. The local Hubble measurements are reflected in the prior distribution of the local Hubble parameter.

This paper is organized as follows. In section 2, we briefly review the inverse construction method reported in ref. [15] and how to construct a universe model in the late time domain. The method to calculate the CMB angular power spectrum and the parameter set for the MCMC analysis is summarized in section 3. In section 4, we show contour maps of the allowed regions for substantial parameters including the amplitude of the under-density. Section 5 is devoted to a summary and discussion.

In this paper, we use geometrized units in which the speed of light and Newton’s gravitational constant are one, respectively.

## 2 Late time model construction

The LTB solution is the solution for the Einstein equations of the spherically symmetric dust fluid system. A line element of the LTB solution is written in the form:

$$\text{d}s^2 = -\text{d}t^2 + \frac{(\partial_t R(t, r))^2}{1 - k(r)r^2}\text{d}r^2 + R^2(t, r)d\Omega^2,$$  \hspace{1cm} (2.1)
where $R(t,r)$ is the areal radius and $k(r)$ is the function of the radial coordinate $r$ called the curvature function. From the Einstein equations with the cosmological constant $\Lambda$, we obtain the following equation:

$$(\partial_t R)^2 = -k(r)r^2 + \frac{m(r)}{3R} + \frac{1}{3}\Lambda R^2 := f(r,R),$$

where $m(r)$ is an arbitrary function of $r$. The comoving energy density $\rho$ is given by

$$\rho(t,r) = \frac{1}{4\pi} \frac{\partial_r M(r)}{R^2 \partial_r R},$$

with $M(r) = m(r)r^3/6$. We can formally integrate eq. (2.2) as

$$t - t_B(r) = \int_0^R \frac{dX}{\sqrt{f(r,X)}},$$

where $t_B(r)$ is the function of $r$ which gives the bigbang time. The LTB solution has three arbitrary functions $k(r)$, $m(r)$ and $t_B(r)$. Since the inhomogeneity associated with $t_B(r)$ corresponds to decaying modes, we simply assume $t_B = \text{const}$. in this paper, where the constant value can be set to zero by shifting the origin of the time. In addition, by using the gauge degree of freedom to choose the radial coordinate $r$, we set $m = \text{const}$. In ref. [15], it is shown that, for $t_B = \text{const}$, $k(r)$ and the value of $m$ are uniquely determined, once the Hubble parameter $H_0$ and the normalized cosmological constant $\Omega_{\Lambda_0}$ are fixed and the cosmological distance $D(z)$ is given as a function of the redshift $z$. In this paper, we use the same functional form of $D(z)$ as that in the matter dominated homogeneous and isotropic universe models:

$$D(z) = D_{\Lambda\text{CDM}}(z; \Omega_{m \text{dis}}^{\text{dis}}, \Omega_{\Lambda \text{dis}}^{\text{dis}}, H_0),$$

where $\Omega_{m \text{dis}}^{\text{dis}}$ and $\Omega_{\Lambda \text{dis}}^{\text{dis}}$ are the normalized matter density and the cosmological constant for the reference homogeneous and isotropic universe. It should be noted that $\Omega_{m \text{dis}}^{\text{dis}}$ and $\Omega_{\Lambda \text{dis}}^{\text{dis}}$ are not necessarily related to the real matter density and the cosmological constant $\Lambda$ but just parameters to specify the distance-redshift relation. For later convenience, we define $R_{\Lambda}$ as

$$R_{\Lambda} := \Omega_{\Lambda_0}/\Omega_{\Lambda \text{dis}}^{\text{dis}},$$

where $\Omega_{\Lambda_0} := \Lambda / (3H_0^2)$. Then, the ALTB models are parametrized by the four parameters: $H_0$, $R_{\Lambda}$, $\Omega_{m \text{dis}}^{\text{dis}}$ and $\Omega_{\Lambda \text{dis}}^{\text{dis}}$. Readers may refer to ref. [15] for details of the construction method. Here we note that, by using above procedure, we can obtain the curvature function $k$ and the redshift $z$ as functions of the radial coordinate $r$.

As is mentioned in section 1, we focus on the models each of which asymptotically coincides with a flat FLRW model. For this purpose, we gradually connect each LTB model to a flat FLRW universe model through the redshift domain $2 < z < 15$, where we do not have significant observational constraints. Specifically, for $z > 2$, we assume the following form of the curvature function $k$:

$$k(r) \rightarrow k_{\text{mod}}(r) = \beta(z(r)) \left[ k_{z=2} + \left( \frac{dk}{dz} \right)_{z=2} (z(r) - 2) \right],$$

where $\beta(z(r))$ is a function of the redshift $z(r)$.
where $z(r)$ is the redshift as a function of $r$ given for a $\Lambda$LTB model specified by the four parameters: $H_0$, $R_\Lambda$, $\Omega_{\text{dis}}^m$ and $\Omega_{\text{dis}}^\Lambda$.

\[
\beta(z) = \frac{1}{1 + \exp[3(z - z_b/2)/2]} \tag{2.8}
\]

with $z_b = 15$ (see figure 1 for the functional form of $\beta(z)$). Examples of the curvature function $k(r)$ and $k_{\text{mod}}(r)$ are shown in figure 2.

In the higher redshift region, for accurate calculation of the CMB spectrum, we need to describe our universe taking the contribution of radiation components into account (figure 3). In this paper, we simply add the radiation components with the density $\rho_{rb} = 7.804 \times 10^{-34}(1 + z_b)^4 \text{g cm}^{-3}$ at $z_b$. Since the radiation effect for the dynamics of the universe is negligible for $z < z_b = 15$, the gap of the Hubble expansion rate at $z = z_b$ is negligible. Then the discontinuity of this procedure does not significantly affect the final results. Actually, we have confirmed that the results do not depend on the value of $z_b$ for $2 < z_b < 15$. 

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**Figure 1.** Functional form of $\beta(z)$. 

**Figure 2.** Curvature functions $k(r)$ and $k_{\text{mod}}(r)$ for $\Omega_{\text{dis}}^m = 0.3$ and $\Omega_{\text{dis}}^\Lambda = 0.7$, and $R_\Lambda = 0.4$ and 0.8.
3 CMB anisotropy and the MCMC analysis

3.1 Angular power spectrum

For the calculation of the CMB temperature anisotropy, we use the open code CAMB.\(^2\) Since homogeneous and isotropic universe models are supposed in this code, we need to appropriately modify input parameters and the output temperature anisotropy for our purpose. In this paper, we mainly focus on the primary effects on the CMB anisotropy, that is, we consider the temperature anisotropy that originates from inhomogeneity of the gravitational potential on the LSS. Inhomogeneity in our models are composed of growing modes, and it may significantly affect the secondary effects on the CMB anisotropy in low $\ell$ domain. Therefore, in our analysis, we simply ignore the angular power spectrum $C_\ell$ for $\ell < 20$.

In order to calculate $C_\ell$ observed at the center, we consider the fictitious flat FLRW universe which shares the same LSS with the inhomogeneous universe of our interest (see figure 3). We define the cosmological parameters, the angular diameter distance to LSS and the angular power spectrum in the fictitious FLRW model as $\Omega_{X0}$, $D_A$ and $\bar{C}_\ell$, respectively. The cosmological parameters are fixed when we connect a late time LTB universe model to the corresponding flat FLRW universe model at $z = z_h$. Then, the primary effects on the temperature anisotropy are given by those in the fictitious FLRW model, while the late time behaviour of the universe model is different from the homogeneous universe. Therefore we need to take the difference of the angular diameter distance between these models into account. Since we focus on $\ell > 20$, in this range, the flat-sky approximation is valid. In the flat-sky approximation, $C_\ell$ is given by the following form (see, e.g., [17])

\[
C_\ell = \left( \frac{D_A}{D_A^{\text{dis}}} \right)^2 \bar{C}_\ell, \tag{3.1}
\]

\(^2\)http://camb.info.
where $\bar{\ell} = \ell D_A^{\text{las}} / D_A^{\text{ls}}$. The flat-sky approximation is valid in the accuracy of 1% for $l > 20$ [18].

3.2 Parameter set for MCMC and calculation of $C_\ell$

In order to describe the CMB anisotropy, in addition to the cosmological parameters for the late time universe model, we introduce the following three parameters: the scalar spectral index $n_s$, amplitude of primordial fluctuation $A$, and baryon to matter ratio $\alpha := \Omega_{b0} / \Omega_{m0}$. In summary, we have the following seven free parameters: $\{\Omega_{m0}^{\text{dis}}, \Omega_{\Lambda 0}^{\text{dis}}, H_0, R_\Lambda, n_s, A, \alpha\}$. In our analysis, we fix the optical depth as $\tau = 0.1$ because the analysis does not use the low $\ell$ angular power spectrum, in which $\tau$ dependence is significant. We set a prior distribution for the Hubble parameter $H_0$ which is consistent with the observational value given in ref. [19]. That is, we restrict the value of $H_0$ by using the Gaussian prior with $72.5 \pm 2.5$ [km s$^{-1}$ Mpc$^{-1}$], where the interval is the 1$\sigma$ range.

Our procedure to calculate $C_\ell$ is summarized in figure 4. The calculation can be summarized in the following 4 steps.

• Step 1
  First, we solve the inverse problem to obtain the $\Lambda$LTB model in $z \leq 2$ as is discussed in section 2 by using the four parameters: $\{\Omega_{m0}^{\text{dis}}, \Omega_{\Lambda 0}^{\text{dis}}, H_0\}$. For $2 < z \leq 15$, we give the functional form of $k(z)$ by eq. (2.7). Then, we can calculate $D_A$ and determine $\Omega_m^{0}$, $H_0$ and $D_{\text{ls}} A$.

• Step 2
  Second, we fix the normalized baryon density $\Omega_{b0}$ and the dark matter density $\Omega_{c0}$ in the fictitious FLRW model as follows:

  \[
  \Omega_{b0} = \alpha \Omega_{m0}, \quad \Omega_{c0} = \Omega_{m0} - \Omega_{b0},
  \]

  (3.2)

  (3.3)

• Step 3
  Then, we calculate $C_\ell$ which is the angular power spectrum observed at $z = 0$ in the fictitious flat FLRW model by inputting the parameters: $\Omega_{b0}, \Omega_{c0}, H_0, n_s$ and $A$ to CAMB.

• Step 4
  Finally, we perform the correction given in eq. (3.1) to get $C_\ell$.

Because of the specification of CosmoMC, in the actual analysis, $\Omega_{m0}^{\text{dis}}, \Omega_{\Lambda 0}^{\text{dis}}, H_0$ and $\alpha = \Omega_{b0}^{\text{dis}} / \Omega_{m0}^{\text{dis}}$ are derived from another four parameters: $\Omega_{b0}^{\text{dis}} h^2$, $\Omega_{c0}^{\text{dis}} h^2$, $\Omega_{\Lambda 0}^{\text{dis}} h^2$ and the ratio $\theta$ between the sound horizon and the angular diameter distance, where $\Omega_{b0}^{\text{dis}} + \Omega_{c0}^{\text{dis}} = \Omega_{m0}^{\text{dis}}$ and $h$ is the dimensionless Hubble parameter defined as $h = H_0/100$.

4 Results

First, we show posterior distributions and contour maps of $\Omega_{\Lambda 0}^{\text{dis}} := 1 - \Omega_{m0}^{\text{dis}} - \Omega_{\Lambda 0}^{\text{dis}}, R_\Lambda, H_0, \Omega_{\Lambda 0}^{\text{dis}}, \Omega_{m0}^{\text{dis}}$ and $\Delta_0$ in figure 5, where $\Delta_0$ is the void depth defined by

\[
\Delta_0 := \frac{\rho(t_0, 0) - \rho(t_0, r_k(z_i))}{\rho(t_0, r_k(z_{1b}))},
\]

(4.1)
Figure 4. Parameter set for MCMC method and procedure to calculate $C_\ell$.

Figure 5. The posterior probability distribution and contour map for the main parameters. The dark blue region represents the restriction of 1$\sigma$ confidence level and the watery blue region is 2$\sigma$ confidence level.

with $t_0$ being the initial time satisfying $\partial_t R/R = H_0$ at the center. The value of $R_\Lambda$ is closely correlated with the void depth $\Delta_0$. As is shown in this figure, $R_\Lambda$ is restricted $R_\Lambda > 0.4$ at 2$\sigma$ confidence level. This result explicitly shows the exclusion of $\Lambda = 0$ void models. It should be noted that our prior models include the flat FLRW models with positive values...
of \( \Lambda \) and also the inhomogeneous universe models with \( \Lambda = 0 \) differently from the previous works not including \( \Lambda \). Therefore the comparison can be done within the common parameter space, and the exclusion is more explicit (see also ref. [10]).

We show the posterior distribution of the void depth \( \Delta_0 \), \( \mathcal{R}_\Lambda \) and \( H_0 \), with \( \Omega_{\text{dis}}^{K_0} \) dependence by the color plot in figure 6. Figure 6 shows that the smaller value of \( \mathcal{R}_\Lambda \) implies the larger value of the void depth \( \Delta_0 \). Once we fix the value of \( \Omega_{\text{dis}}^{K_0} \), \( \Delta_0 \) tends to be smaller (deeper void) for the larger value of \( H_0 \). This observation is consistent with the previous works [13, 20]. This dependence can be roughly understood as follows. If we increase the value of \( H_0 \), the distance to the LSS decreases. On the other hand, we can increase the distance by making void depth deeper because the central region becomes closer to an open universe. Therefore, the correlation between \( H_0 \) and \( \Delta_0 \) comes from the compensation of the distance to the LSS.

It is commonly expected that an under-dense region tends to increase the value of the local Hubble parameter \( H_0 \) compared to the asymptotic value given by CMB observations. However, in figure 5, the correlation between \( H_0 \) and \( \Delta_0 \) is not clear. As is mentioned above, the reason for this behaviour comes from \( \Omega_{\text{dis}}^{K_0} \) dependence of \( H_0 \). Even if the value of \( \Delta_0 \) is fixed at some value, changing the value of \( \Omega_{\text{dis}}^{K_0} \), we obtain a different profile of inhomogeneity and find a different value of \( H_0 \). This dependence might help the resolution of the \( H_0 \) tension (see ref. [21] for a recent analysis of the local \( H_0 \), and ref. [22] for possible explanations about the \( H_0 \) tension).

5 Summary and discussion

We have discussed an observational constraint on the spherically symmetric inhomogeneous models by the CMB angular power spectrum and local Hubble parameter. We assumed that the late time cosmological models are well described by \( \Lambda \)LTB models each of which is characterized by two parameters in the distance-redshift relation, the value of the cosmological constant \( \Lambda \) and the local Hubble parameter \( H_0 \). Connecting each of the late time inhomogeneous models to a flat homogeneous universe model, we calculated the CMB power spectrum observed at the center. The MCMC analysis with the Planck data [23] explicitly excluded inhomogeneous models with \( \Lambda = 0 \). However, at the same time, our results show that a significant amplitude of the under-density can be still compatible with the CMB angular power spectrum and the local Hubble measurement. We found that, even if we fix the amplitude of the void, the value of local Hubble parameter can change depending on the parameter \( \Omega_{\text{dis}}^{K_0} \), which specifies the inhomogeneity. This dependence could help to resolve the \( H_0 \) tension between the local measurement and CMB observations.

Finally, we list related important issues which we could not address in this paper. In ref. [10], the strongest constraint for the amplitude of the inhomogeneity comes from the linear kinetic Sunyaev-Zeldovich effect on the CMB power spectrum in large scales, and they concluded that the void amplitude \( |\delta_0| \) is smaller than 0.29 (see also refs. [24–29]). The Planck team reported the constraint on the kSZ monopole as \( 72 \pm 60 \text{ km} \) for \( z < 1 \) from the cluster SZ effect. This constraint may give much more stringent constraints on the void depth. The difference between the radial and transverse BAO scale may be also very efficient indicator for the spherical inhomogeneity (see, e.g., refs. [30–33]). In this paper, we ignored the angular power spectrum for lower multipoles. The spherical inhomogeneity may enhance the Integrated Sachs Wolfe (ISW) effect in the low multipoles. In order to clarify the significance of the spherical inhomogeneity to the ISW effect, we need to calculate the evolution of
Figure 6. The dependence of $R_\Lambda$, $H_0$ and void depth $\Delta_0$ with the color plot by the value of $\Omega_{K0}^{\text{dis}}$.

the perturbation with spherical inhomogeneity. The calculation of the perturbation is also needed for the calculation of the CMB lensing. We leave all these issues as future works.

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References

[1] G. Lemaître, The expanding universe, *Gen. Rel. Grav.* 29 (1997) 641 [SPIRE].

[2] R.C. Tolman, Effect of inhomogeneity on cosmological models, *Proc. Nat. Acad. Sci.* 20 (1934) 169 [SPIRE].

[3] H. Bondi, Spherically symmetrical models in general relativity, *Mon. Not. Roy. Astron. Soc.* 107 (1947) 410 [SPIRE].

[4] I. Zehavi, A.G. Riess, R.P. Kirshner and A. Dekel, A local hubble bubble from SNe Ia?, *Astrophys. J.* 503 (1998) 483 [astro-ph/9802252] [SPIRE].

[5] M.-N. Celerier, Do we really see a cosmological constant in the supernovae data?, *Astron. Astrophys.* 353 (2000) 63 [astro-ph/9907206] [SPIRE].

[6] K. Tomita, Distances and lensing in cosmological void models, *Astrophys. J.* 529 (2000) 38 [astro-ph/9906027] [SPIRE].

[7] H. Iguchi, T. Nakamura and K.-i. Nakao, Is dark energy the only solution to the apparent acceleration of the present universe?, *Prog. Theor. Phys.* 108 (2002) 809 [astro-ph/0112419] [SPIRE].

[8] C.-M. Yoo, T. Kai and K.-i. Nakao, Solving Inverse Problem with Inhomogeneous Universe, *Prog. Theor. Phys.* 120 (2008) 937 [arXiv:0807.0932] [SPIRE].
[9] M. Redlich, K. Bolejko, S. Meyer, G.F. Lewis and M. Bartelmann, Probing spatial homogeneity with LTB models: a detailed discussion, *Astron. Astrophys.* **570** (2014) A63 [arXiv:1408.1872] [SPIRE].

[10] W. Valkenburg, V. Marra and C. Clarkson, Testing the Copernican principle by constraining spatial homogeneity, *Mon. Not. Roy. Astron. Soc.* **438** (2014) L6 [arXiv:1209.4078] [SPIRE].

[11] V. Marra and M. Paakkonen, Observational constraints on the LLTB model, *JCAP* **12** (2010) 021 [arXiv:1009.4193] [SPIRE].

[12] H. Negishi, K.-i. Nakao, C.-M. Yoo and R. Nishikawa, Systematic error due to isotropic inhomogeneities, *Phys. Rev.* **D 92** (2015) 103003 [arXiv:1505.02472] [SPIRE].

[13] K. Ichiki, C.-M. Yoo and M. Oguri, Relationship between the CMB, Sunyaev-Zel’dovich cluster counts and local Hubble parameter measurements in a simple void model, *Phys. Rev.* **D 93** (2016) 023529 [arXiv:1509.04342] [SPIRE].

[14] P. Sundell, E. M¨ortsell and I. Vilja, Can a void mimic the $\Lambda$ in $\Lambda$CDM?, *JCAP* **08** (2015) 037 [arXiv:1503.08045] [SPIRE].

[15] M. Tokutake and C.-M. Yoo, Inverse Construction of the $\Lambda$LTB Model from a Distance-Redshift Relation, *JCAP* **10** (2016) 009 [arXiv:1603.07837] [SPIRE].

[16] R. Durrer, *The Cosmic Microwave Background*, Cambridge University Press, (2008).

[17] M. Vonlanthen, S. Rasanen and R. Durrer, Model-independent cosmological constraints from the CMB, *JCAP* **08** (2010) 023 [arXiv:1003.0810] [SPIRE].

[18] G. Efstathiou, $H_0$ Revisited, *Mon. Not. Roy. Astron. Soc.* **440** (2014) 1138 [arXiv:1311.3461] [SPIRE].

[19] J.L. Bernal, L. Verde and A.G. Riess, The trouble with $H_0$, *JCAP* **10** (2016) 019 [arXiv:1607.05617] [SPIRE].

[20] J.G. Riess et al., A 2.4% Determination of the Local Value of the Hubble Constant, *Astrophys. J.* **826** (2016) 56 [arXiv:1604.01424] [SPIRE].

[21] J. García-Bellido and T. Haugboelle, Looking the void in the eyes — the $kSZ$ effect in LTB models, *JCAP* **09** (2008) 016 [arXiv:0807.1326] [SPIRE].

[22] P. Zhang and A. Stebbins, Confirmation of the Copernican principle at Gpc radial scale and above from the kinetic Sunyaev Zel’dovich effect power spectrum, *Phys. Rev. Lett.* **107** (2011) 041301 [arXiv:1009.3967] [SPIRE].

[23] J.P. Zibin and A. Moss, Linear kinetic Sunyaev-Zel’dovich effect and void models for acceleration, *Class. Quant. Grav.* **28** (2011) 164005 [arXiv:1105.0909] [SPIRE].

[24] P. Bull, T. Clifton and P.G. Ferreira, The $kSZ$ effect as a test of general radial inhomogeneity in LTB cosmology, *Phys. Rev. D* **85** (2012) 024002 [arXiv:1108.2222] [SPIRE].

[25] J.P. Zibin and A. Moss, Linear kinetic Sunyaev-Zel’dovich effect and void models for acceleration, *Class. Quant. Grav.* **28** (2011) 164005 [arXiv:1105.0909] [SPIRE].

[26] P. Bull, T. Clifton and P.G. Ferreira, The $kSZ$ effect as a test of general radial inhomogeneity in LTB cosmology, *Phys. Rev. D* **85** (2012) 024002 [arXiv:1108.2222] [SPIRE].

[27] PLANCK collaboration, P.A.R. Ade et al., Planck 2015 results. XIII. Cosmological parameters, *Astron. Astrophys.* **594** (2016) A13 [arXiv:1502.01589] [SPIRE].

[28] J. García-Bellido and T. Haugboelle, Confronting Lemaître-Tolman-Bondi models with Observational Cosmology, *JCAP* **04** (2008) 003 [arXiv:0802.1523] [SPIRE].
[30] T. Biswas, A. Notari and W. Valkenburg, *Testing the Void against Cosmological data: fitting CMB, BAO, SN and H0*, *JCAP* 11 (2010) 030 [arXiv:1007.3065] [INSPIRE].

[31] J. García-Bellido and T. Haugboelle, *The radial BAO scale and Cosmic Shear, a new observable for Inhomogeneous Cosmologies*, *JCAP* 09 (2009) 028 [arXiv:0810.4939] [INSPIRE].

[32] M. Zumalacarregui, J. García-Bellido and P. Ruiz-Lapuente, *Tension in the Void: Cosmic Rulers Strain Inhomogeneous Cosmologies*, *JCAP* 10 (2012) 009 [arXiv:1201.2790] [INSPIRE].

[33] C. Clarkson, *Establishing homogeneity of the universe in the shadow of dark energy*, *Comptes Rendus Physique* 13 (2012) 682 [arXiv:1204.5505] [INSPIRE].