Reconstruction of 5D Cosmological Models From Recent Observations

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We use a parameterized equation of state (EOS) of dark energy to a 5D Ricci-flat cosmological solution and suppose the universe contains two major components: dark matter and dark energy. Using the recent observational data sets: the latest 182 type Ia Supernovae Gold data, the 3-year WMAP CMB shift parameter and the SDSS baryon acoustic peak, we obtain the best fit values of the EOS and two major components’ evolution. We find that the best fit EOS crossing $-1$ in the near past $z \approx 0.07$, the present best fit value of $w_x(0) < -1$ and for this model the universe experiences the acceleration at about $z \approx 0.5$.

Keywords: Kaluza-Klein theory; cosmology; dark energy

1. Introduction

Observations of Cosmic Microwave Background (CMB) anisotropies, high redshift type Ia supernovae and the surveys of clusters of galaxies indicate that an exotic component with negative pressure dubbed dark energy dominates the present universe. The most obvious candidate for this dark energy is the cosmological constant $\Lambda$ with equation of state ($w_\Lambda = -1$), which is consistent with recent observations in 2σ region. However, it raises several theoretical difficulties. This has lead to models for dark energy which evolves with time, such as quintessence, phantom, quintom, K-essence, tachyonic matter and so on. For this kind of models, one can design many kinds of potentials and then study EOS for the dark energy. Another way is to use a parameterization of the EOS to fit the observational data, and then to reconstruct the potential and the evolution of the universe. Various parameterization of the EOS of dark energy have been presented and investigated.

If the universe has more than four dimensions, general relativity should be extended from 4D to higher dimensions. One of such extensions is the 5D Space-Time-Matter (STM) theory in which our universe is a 4D hypersurface floating...
in a 5D Ricci-flat manifold. This theory is supported by Campbell’s theorem which states that any analytical solution of the ND Einstein equations can be embedded in a \((N + 1)D\) Ricci-flat manifold. A class of cosmological solutions of the STM theory is given by Liu and Mashhoon, the authors restudied the solutions and pointed out that it can describe a bounce universe. It was shown that dark energy models, similar as the 4D quintessence and phantom ones, can also be constructed in this 5D cosmological solution in which the scalar field is induced from the 5D vacuum. The purpose of this paper is to use a model-independent method to reconstruct a 5D cosmological model and then study the universe evolution and the EOS of the dark energy which is constrained by recent observational data: the latest observations of the 182 Gold SNe Ia, the 3-year WMAP CMB shift parameter, and the SDSS baryon acoustic peak. The paper is organized as follows. In Section 2, we briefly introduce the 5D Ricci-flat cosmological solution and derive the densities for the two major components of the universe. In Section 3, we will reconstruct the evolution of the model from cosmological observations. Section 4 is a short discussion.

2. Dark energy in the 5D Model

The 5D cosmological model was described as before. In this paper we consider the case where the 4D induced matter \(T^{\alpha\beta}\) is composed of two components: dark matter \(\rho_m\) and dark energy \(\rho_x\), which are assumed to be noninteracting. So we have

\[
\frac{3 \left( \mu^2 + k \right)}{A^2} = \rho_m + \rho_x, \\
\frac{2\mu \dot{\mu}}{AA} + \frac{\mu^2 + k}{A^2} = -p_m - p_x, \tag{1}
\]

with

\[
\rho_m = \rho_{m0}A_0^3A^{-3}, \quad p_m = 0, \tag{2}
\]
\[
p_x = w_x \rho_x. \tag{3}
\]

From Eqs. (1) - (3) and for \(k = 0\), we obtain the EOS of the dark energy

\[
w_x = \frac{p_x}{\rho_x} = -\frac{2\mu \dot{\mu}}{3\mu^2A^2 - \rho_{m0}A_0^3A^{-3}}. \tag{4}
\]

and the dimensionless density parameters

\[
\Omega_m = \frac{\rho_m}{\rho_m + \rho_x} = \frac{\rho_{m0}A_0^3}{3\mu^2A}, \tag{5}
\]
\[
\Omega_x = 1 - \Omega_m. \tag{6}
\]

where \(\rho_{m0}\) is the current values of dark matter density.

Consider Eq. (1) where \(A\) is a function of \(t\) and \(y\). However, on a given \(y = \text{constant}\) hypersurface, \(A\) becomes \(A = A(t)\), which means we consider a 4D
supersurface embedded in a flat 5D spacetime. As noticed before\textsuperscript{29,30}, the term \( \dot{\mu}/\dot{A} \) in (1) can now be rewritten as \( d\mu/dA \). Furthermore, we use the relation

\[
A_0/A = 1 + z,
\]

as an ansatz\textsuperscript{29,30} and define \( \mu_0^2/\mu^2 = f(z) \) (with \( f(0) \equiv 1 \)), then we find that Eqs. (4)-(6) can be expressed in terms of the redshift \( z \) as

\[
w_x = -\frac{1 + (1 + z) d\ln f(z)/dz}{3(1 - \Omega_m)},
\]

\[
\Omega_m = \Omega_{m0}(1 + z)^{f(z)},
\]

\[
\Omega_x = 1 - \Omega_m,
\]

\[
q = -\frac{1 + z}{2} d\ln f(z)/dz.
\]

where \( q \) is the deceleration parameter and \( q < 0 \) means our universe is accelerating. Now we conclude that if the function \( w_x \) is given, the evolution of all the cosmic observable parameters in Eqs. (8) - (11) could be determined uniquely. Then we adopt the parametrization of EOS as follows\textsuperscript{15,31}

\[
w_x(z) = w_0 + w_1 \frac{z}{1 + z}
\]

From Eq. (8) and Eq. (12), we can obtain the function \( f(z) \)

\[
f(z) = \frac{1}{(1 + z) \left[ \Omega_{m0} + (1 - \Omega_{m0})(1 + z)^{3w_0 + 3w_1} \exp(- \frac{3w_1 z}{1 + z}) \right]}
\]

In the next section, we will use the recent observational data to find the best fit parameter \((w_0, w_1, \Omega_{m0})\).

### 3. The best fit parameters from cosmological observations

In a flat universe with Eq. (12), the Friedmann equation can be expressed as

\[
H^2(z) = H_0^2 E(z)^2 = H_0^2 [\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^{3(1+w_0+w_1)} e^{-3w_1 z/(1+z)}]
\]

Then the knowledge of \( \Omega_{m0} \) and \( H(z) \) is sufficient to determine \( w_x(z) \) with \( H_0 = 72 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \).\textsuperscript{32} We use the maximum likelihood method\textsuperscript{23} to constrain the parameters.

The Gold dataset compiled by Riess et. al is a set of supernova data from various sources and contains 182 gold points by discarding all SNe Ia with \( z < 0.0233 \) and all SNe Ia with quality='Silver' from previously published data with 21 new points with \( z > 1 \) discovered recently by the Hubble Space Telescope.\textsuperscript{25} Theoretical model parameters are determined by minimizing the quantity

\[
\chi^2_{SN} = \sum_{i=1}^{N} \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma^2_{(obs;i)}}
\]
where $N = 182$ for Gold SNe Ia data, $\sigma_{\text{obs};i}^2$ are the errors due to flux uncertainties, intrinsic dispersion of SNe Ia absolute magnitude and peculiar velocity dispersion respectively. These errors are assumed to be gaussian and uncorrelated. The theoretical distance modulus is defined as

$$
\mu_{\text{th}}(z_i) \equiv m_{\text{th}}(z_i) - M
$$

$$
= 5 \log_{10}(D_L(z)) + 5 \log_{10}\left(\frac{H_0^{-1}}{Mpc}\right) + 25
$$

where

$$
D_L(z) = H_0 d_L(z) = (1 + z) \int_0^z \frac{H(z')}{H(z'; \Omega_{m0}, w_0, w_1)} \, dz'
$$

and $\mu_{\text{obs}}$ is given by supernovae dataset.

The shift parameter is defined as

$$
\bar{R} = \frac{d_A(z'_{\text{rec}})}{d_A(z_{\text{rec}})} = \frac{r_s d_A(z_{\text{rec}})}{r_s' d_A(z'_{\text{rec}})} = \frac{2}{\Omega_{m0}^{1/2} J_0 \frac{H_0}{H(z')}} q(\Omega'_r, a_{\text{rec}})
$$

where $z_{\text{rec}}$ is the redshift of recombination, $r_s$ is the sound horizon, $d_A(z_{\text{rec}})$ is the sound horizon angular diameter distance, $q(\Omega'_r, a_{\text{rec}})$ is the correction factor. For weak dependence of $q(\Omega'_r, a_{\text{rec}})$, the shift parameter is usually expressed as

$$
R = \Omega_{m0}^{1/2} \int_0^z \frac{H_0 d(z')}{H(z'; \Omega_{m0}, w_0, w_1)}
$$

The $R$ obtained from 3-year WMAP data is

$$
R = 1.70 \pm 0.03
$$

With the measurement of the $R$, we obtain the $\chi^2_{\text{CMB}}$ expressed as

$$
\chi^2_{\text{CMB}}(\Omega_{m0}, w_0, w_1) = \frac{(R(\Omega_{m0}, w_0, w_1) - 1.70)^2}{0.03^2}
$$

The size of Baryon Acoustic Oscillation (BAO) is found by Eisenstein et al. by using a large spectroscopic sample of luminous red galaxy from SDSS and obtained a parameter $A$ which does not depend on dark energy directly models and can be expressed as

$$
A = \Omega_{m0}^{1/2} E(z_{\text{BAO}})^{-1/3} \int_0^z \frac{dz'}{E(z'; \Omega_{m0}, w_0, w_1)}
$$

where $E(z_{\text{BAO}}) = 0.35$ and $A = 0.469 \pm 0.017$. We can minimize the $\chi^2_{\text{BAO}}$ defined as

$$
\chi^2_{\text{BAO}}(\Omega_{m0}, w_0, w_1) = \frac{(A(\Omega_{m0}, w_0, w_1) - 0.469)^2}{0.017^2}
$$

To break the degeneracy of the observational data and find the best fit parameters, we combine these datasets to minimize the total likelihood $\chi^2_{\text{total}}$.
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\[ \chi^2_{\text{total}} = \chi^2_{\text{SN}} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} \]  

(24)

We obtain the best fit values \((\Omega_{m0}, w_0, w_1)\) are \((0.288, -1.050, 0.824)\) and to identify the dependence of the best fit values of the parameters, we set \(\Omega_{m0}\) to be fixed when calculating the confidence level of \((w_0, w_1)\). The errors of the best fit \(w_x(z)\) are calculated using the covariance matrix and shown in Fig.1. The corresponding \(\chi^2\) contours in parameters space \((w_0, w_1)\) is shown in Fig.2.

From Fig.1 we find that \(w_x(z)\) is constrained in a narrow space, the best fit \(w_x(z)\) crosses \(-1\) at about \(z = 0.07\) and at present the best value of \(w_x(0)\) is \(-1\), but in 1σ confidence level we can’t rule out the possibility \(w_x(0) > -1\). Fig.2 shows that a cosmological constant is ruled out in 1σ confidence level.

Using the function \(f(z)\), the best fit values \((\Omega_{m0}, w_0, w_1)\), we obtain \(\Omega_m, \Omega_x\), the deceleration parameter \(q\) from Eq.9-11 and their evolution is plotted in Fig.3. Fig.3 also shows the evolution of \(q_{\Lambda\text{CDM}}, \Omega_m-\Lambda\text{CDM}, \Omega_\Lambda\) in a 4D flat \(\Lambda\text{CDM}\) model with the present \(\Omega_{m0-\Lambda\text{CDM}} = 0.283\) obtained from above cosmological observations. We can see that the transition point from decelerated expansion to accelerated expansion with \(q = 0\) is at \(z \simeq 0.5\) and it is earlier than the \(\Lambda\text{CDM}\) model. Our universe experiences a expansion at present in a 4D supersurface embedded in a 5D Ricci-flat spacetime or in \(\Lambda\text{CDM}\) model.

4. Discussion

Observations indicate that our universe now is dominated by two dark components: dark energy and dark matter. The 5D cosmological solution presented by Liu, Mashhoon and Wesson in 21 and 22 contains two arbitrary functions \(\mu(t)\) and \(\nu(t)\), one of the two functions, \(\mu(t)\), plays a similar role as the potential \(V(\phi)\) in the
Fig. 2. The contours show 2-D marginalized 1σ and 2σ confidence limits in the $(w_0, w_1)$ plane.

Fig. 3. The evolution of $q(z)$, $\Omega_m$, $\Omega_x$, and $\Omega_{\Lambda_{CDM}}$, $\Omega_{m-\Lambda_{CDM}}$, $\Omega_{\Lambda}$ from 5D cosmological model and $\Lambda$CDM model respectively.

quintessence and phantom dark energy models, which can easily change to another arbitrary function $f(z)$. Thus, if the current values of the matter density parameter $\Omega_{m0}$, $w_0$ and $w_1$ in the EOS are all known, this $f(z)$ could be determined uniquely. In this paper we mainly focus on the constraints on this model from recent observational data: the 182 Gold SNe Ia, the 3-year WMAP CMB shift parameter and the SDSS baryon acoustic peak. Our results show that the recent observations allow for a narrow variation of the dark energy EOS and the best fit dynamical $w_x(z)$ crosses $-1$ in the recent past. Using the best fit values $(\Omega_{m0}, w_0, w_1)$, we have studied the evolution of the dark matter density $\Omega_m$, the dark energy density $\Omega_x$ and the deceleration parameter $q$ in a 4D supersurface of 5D spacetime, which is similar to the $\Lambda$CDM model. In the future, we hope that more and precision cosmological
observations could determine the key points of the evolution of our universe, such as the transition point from deceleration to acceleration, then distinguish the 5D cosmological model from others.

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