The Glueball in a Chiral Linear Sigma Model with Vector Mesons

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We present a two-flavour linear sigma model with global chiral symmetry and (axial-)vector mesons as well as an additional glueball degree of freedom. We study the structure of the well-established scalar resonances \(f_0(1370)\) and \(f_0(1500)\): by a fit to experimentally known decay widths we find that \(f_0(1370)\) is predominantly a \(qq\) state and \(f_0(1500)\) is predominantly a glueball state. The overall phenomenology of these two resonances can be well described. Other assignments for our mixed quarkonium-glueball states are also tested, but turn out to be in worse agreement with the phenomenology. As a by-product of our analysis, the gluon condensate is determined.

PACS numbers: 12.39.Fe, 12.39.Mk, 12.40.Yx, 13.25.Jx, 14.40.Be

Keywords: chiral Lagrangians, scalar mesons, glueball, sigma, mixing.

I. INTRODUCTION

Glueballs, the bound states of the gauge bosons of QCD, the gluons, are naturally expected in QCD due to the nonabelian nature of the theory: the gluons interact strongly with themselves and thus they can bind and form colorless states, analogously to what occurs in the quark sector. The existence of glueballs has been studied in the framework of the effective bag model for QCD already four decades ago \(^{[1]}\) and it has been further investigated in a variety of approaches \(^{[2]}\). Numerical calculations of the Yang-Mills sector of QCD also find a full glueball spectrum in which the scalar glueball is the lightest state \(^{[3]}\).

Glueballs can mix with quarkonium \((qq)\) states with the same quantum numbers. This makes the experimental search for glueballs more complicated, because physical resonances emerge as mixed states. The scalar sector \(J^{PC} = 0^{++}\) has been investigated in many works in the past. The resonance \(f_0(1500)\) is relatively narrow when compared to other scalar-isoscalar states: for this reason it has been considered as a convincing candidate for a glueball state. Mixing scenarios in which two quark-antiquark isoscalar states \(\bar{u}u\) and \(ss\) and one scalar glueball \(gg\) mix and generate the physical resonances \(f_0(1370)\), \(f_0(1500)\), and \(f_0(1710)\) have been discussed in Refs. \(^{[4, 5]}\).

The aim of this work is to extend the linear chiral model of Refs. \(^{[6, 7]}\) by including the dilaton/glueball field. The first attempt to incorporate a glueball into a linear sigma model was performed long ago in Ref. \(^{[8]}\). The novel features of the present study are the following: (i) The glueball is introduced as a dilaton field within a theoretical framework where not only scalar and pseudoscalar mesons, but also vector and axial-vector mesons are present from the very beginning. This fact allows also for a calculation of decays into vector mesons. The model is explicitly evaluated for the case of \(N_f = 2\), for which only one scalar-isoscalar quarkonium state exists: \(\sigma \equiv \bar{u}u\) which mixes with the glueball. The two emerging mixed states are assigned to the resonances \(f_0(1370)\) which is, in accordance with Ref. \(^{[6]}\), predominantly a \(qq\) state, and with \(f_0(1500)\) which is predominantly a glueball state. (ii) We consequently test — to our knowledge for the first time — this mixing scenario above 1 GeV in the framework of a chiral model.

The model under consideration is built in accordance with the symmetries of the QCD Lagrangian. It possesses the known degrees of freedom of low-energy QCD [(pseudo)scalar and (axial-)vector mesons] as well as the same global chiral invariance. Another feature of the QCD Lagrangian is scale (or dilatation) invariance \(x^\mu \rightarrow \lambda^{-1}x^\mu\) (where \(x^\mu\) is a Minkowski-space coordinate and \(\lambda\) the scale parameter of the conformal group). It is realized at the classical level but broken at the quantum level due to the loop corrections in the Yang-Mills sector (scale anomaly). In this work the breaking of scale invariance is implemented at tree-level by means of a dilaton field (representing a glueball) with the usual logarithmic dilaton potential \(^{[8]}\). However, all the other interaction terms (with the exception of the chiral anomaly) are dilatation-invariant in the chiral limit.

Having constructed the Lagrangian of the effective model, we calculate the masses of the pure \(qq\) and glueball states in the \(J^{PC} = 0^{++}\) channel, study their mixing and calculate the decay widths of the mixed states. Although we work with \(N_f = 2\), the use of flavor symmetry enables us to calculate the decay widths of the scalar resonances into kaons and into both the \(\eta\) and \(\eta'\) mesons which contain the \(s\)-quark in their flavor wave functions. After the study of the already mentioned scenario where \(f_0(1370)\) and \(f_0(1500)\) are predominantly quarkonium and glueball, respectively, we also test the alternative scenario in which the resonance \(f_0(1710)\) is predominantly glueball and scenarios in which \(f_0(600)\) is predominantly quarkonium. They, however, lead to inconsistencies when compared to the present data and are therefore regarded as less favorable.
This paper is organized as follows. In Sec. II the Lagrangian of the linear sigma model with (axial-)vector and glueball degrees of freedom is constructed. In Sec. III we discuss the results for the masses of the quarkonium-glueball mixed states and their decay widths. In Sec. IV we present our conclusions.

II. THE MODEL

The Yang-Mills (YM) sector of QCD (QCD without quarks) is classically invariant under dilatations. This symmetry is, however, broken at the quantum level. The divergence of the corresponding current is the trace of the energy-momentum tensor $T_{\mu\nu}^{YM}$ of the YM Lagrangian

$$
(T_{\mu\nu}^{YM})^\mu = \frac{\beta(g)}{4g} G_{\mu\nu} a G^{a\mu\nu} \neq 0 ,
$$

where $G_{\mu\nu}$ is the field-strength tensor of the gluon fields, $g = g(\mu)$ is the renormalized coupling constant at the scale $\mu$, and the $\beta$-function is given by $\beta(g) = \partial g / \partial \ln \mu$. At the one-loop level $\beta(g) = -bg^3$ with $b = 11N_c/(48\pi^2)$. This implies $g^2(\mu) = [2b \ln(\mu/\Lambda_{YM})]^{-1}$, where $\Lambda_{YM} \approx 200$ MeV is the Yang-Mills scale. A finite energy scale thus emerges in a theory which is classically invariant under dilatation (dimensional transmutation). The expectation value of the trace anomaly does not vanish and represents the so-called gluon condensate:

$$
\langle T_{\mu Y M, \mu} \rangle = -\frac{11N_c}{48} \langle \frac{a_s}{2\pi} G_{\mu\nu} a G^{\mu\nu} \rangle = -\frac{11N_c}{48} C^4 ,
$$

where

$$
C^4 \approx (300 - 600 \text{ MeV})^4 .
$$

The numerical values have been obtained through QCD sum rules (lower range of the interval) [9] and lattice simulations (higher range of the interval) [10].

At the composite level one can build an effective theory of the YM sector of QCD by introducing a scalar dilaton field $G$ which describes the trace anomaly. The dilaton Lagrangian reads [8]

$$
\mathcal{L}_{dil} = \frac{1}{2} (\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left( G^4 \ln \left( \frac{|G|}{\Lambda} \right) - \frac{C^4}{4} \right) ,
$$

where $m_G$ is the mass of the dilaton. The minimum $G_0$ of the dilaton potential is realized for $G_0 = \Lambda$. Upon shifting $G \rightarrow G_0 + G$, a particle with mass $m_G$ emerges, which is interpreted as the scalar glueball. The numerical value has been evaluated in Lattice QCD and reads $m_G \approx 1.5$ GeV [3]. The logarithmic term of the potential breaks the invariance under a dilatation transformation. The divergence of the corresponding current reads $\partial_\mu J^\mu_{dil} = T^\mu_{dil, \mu} = -\frac{1}{4} m_G^2 \Lambda^2$. This can be compared with the analogous quantity in Eq. (2) which implies $\Lambda = \sqrt{\pi C^2/(2m_G)}$.

QCD with quarks is also classically invariant under dilatation transformations in the limit of zero quark masses (chiral limit). The scale of all hadronic phenomena is given by the previously introduced energy scale $\Lambda_{YM}$. This fact holds true also when the small but nonzero values of the quark masses is considered. In order to describe these properties in a hadronic model we now extend the linear sigma model with $U(N_f)^R \times U(N_f)^L$ of Refs. [6, 11–13] by including the dilaton. To this end, the following criteria are applied [14]: (i) With the exception of the chiral anomaly, the parameter $\Lambda$ from Eq. (4), which comes from the Yang-Mills sector of the theory in accordance with QCD, is the only dimensionful parameter of the Lagrangian in the chiral limit. (ii) The Lagrangian is required to be finite for every finite value of the gluon condensate $G_0$. This, in turn, also assures that no singular terms arise in the limit $G_0 \rightarrow 0$. In accordance with the requirements (i) and (ii) only terms with dimension exactly equal to 4 are allowed in the chiral limit.

The hadronic Lagrangian obeying these requirements reads

$$
\mathcal{L} = \mathcal{L}_{dil} + \text{Tr} \left[ (D^\mu \Phi)^\dagger (D_\mu \Phi) - m_0^2 \left( \frac{G}{G_0} \right)^2 \Phi^\dagger \Phi - \lambda_2 (\Phi^\dagger \Phi)^2 \right] - \lambda_1 (\text{Tr} (\Phi^\dagger \Phi))^2 \\
+ c [\text{det}(\Phi^\dagger) + \text{det}(\Phi)] + \text{Tr} \left[ H \left( \Phi^\dagger + \Phi \right) \right] - \frac{1}{4} \text{Tr} \left[ (L^\mu)^2 + (R^\mu)^2 \right] \\
+ \frac{m_2^2}{2} \left( \frac{G}{G_0} \right)^2 \text{Tr} \left[ (L^\mu)^2 + (R^\mu)^2 \right] + \frac{h_1}{2} \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\
+ h_2 \text{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr}[\Phi R_\mu \Phi^\dagger L^\mu] + ..., \tag{5}
$$
where $\Phi$ represents the $N_f \times N_f$ (pseudo)scalar multiplet, and $L^\mu$ and $R^\mu$ the left- and right-handed vector multiplets. The dots represent further terms which do not affect the processes studied in this work and can be omitted.

In the particular case of $N_f = 2$ studied in this manuscript one has $\Phi = (\sigma + i\eta_N)t^0 + (\bar{a}_0 + i\bar{\eta}_N)\cdot \vec{t}$ (our eta meson $\eta_N$ contains only non-strange degrees of freedom), $L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\bar{a}_1^\mu \cdot \vec{t})$ and $R^\mu = (\tilde{\omega}^\mu - f_1^{*\mu})t^0 + (\bar{a}_1^{*\mu} - \bar{\eta}_N)\cdot \vec{t}$; $t^0$, $\vec{t}$ are the generators of $U(2)$. Moreover, $D^0\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi L^\mu)$, $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$, $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$.

The explicit breaking of the global chiral symmetry is described by the term $\mathcal{L}_0 = h^2/2$ (energy$^2$), which allows us to take into account the nonvanishing value $m_q$ of the quark mass. This term contains the dimensionful parameter $h$ with $[h] = [\text{energy}]$ and also explicitly breaks the dilatation invariance, just as the quark masses do in the underlying QCD Lagrangian. Finally, the chiral anomaly is described by the term $c(\text{det} \Phi + \text{det} \Phi^\dagger)(\vec{t})$. For $N_f = 2$ the parameter $c$ carries the dimension [energy$^2$] and represents a further breaking of dilatation invariance. This term, however, is driven by instantons, which are also a property of the Yang-Mills sector of QCD.

The identification of the fields of the models with the resonances listed in Ref. [6] is straightforward in the pseudoscalar and (axial-)vector sectors: the fields $\vec{t}$ and $\eta_N$ correspond to the pion and the $SU(2)$ counterpart of the $\eta$ meson, respectively, $\eta_N \equiv \overline{\pi}u + \overline{d}d/\sqrt{2}$, with a mass of about 700 MeV. This value can be obtained by ‘unmixing’ the physical $\eta$ and $\eta'$ mesons, which contain $\bar{s}s$ contributions. The fields $\omega^\mu$ and $\bar{a}_1^\mu$ represent the $\omega(782)$ and $\rho(770)$ vector mesons, respectively, while the fields $f_1^\mu$ and $a_1^\mu$ represent the $f_1(1285)$ and $a_1(1260)$ axial-vector mesons, respectively. (In principle, the physical $\omega$ and $f_1$ states also contain $\bar{s}s$ contributions but their admixture is small.) As shown in Ref. [6], the $\sigma$ field should be interpreted as a predominantly $\bar{q}q$ state because its decay width decreases as $1/N_c$ in the limit of large number of colors. The $\sigma$ and $G$ fields mix: the physical fields $\sigma'$ and $G'$ are obtained through an $SO(2)$ rotation, as we shall show in the following. Then the first and most natural assignment is $(\sigma', G') = \{f_0(1370), f_0(1500)\}$, see Sec. III.A. Note that the $a_0$ state is assigned to the physical $a_0(1450)$ resonance in accordance with results of Ref. [6]. Other assignments for $(\sigma', G')$ will be also tested in Secs. III.B and III.C and turn out to be less favourable.

In order to study the non-vanishing vacuum expectation values (vev’s) of the two $J^{PC} = ++$ scalar-isoscalar fields of the model $\sigma$ and $G$, we set all the other fields in Eq. (5) to zero and obtain:

\[
\mathcal{L}_{\sigma G} = \mathcal{L}_{\text{dir}} + \frac{1}{2} (\partial^\mu \sigma)^2 - \frac{1}{2} \left[ m_0^2 \left( \frac{G}{G_0} \right)^2 - c \right] \sigma^2 - \frac{1}{4} \left( \lambda_1 + \frac{\lambda_2}{2} \right) \sigma^4 + h \sigma .
\]  

Upon shifting the fields by their vev’s, $\sigma \rightarrow \sigma + \phi$ and $G \rightarrow G + G_0$, we obtain the masses of the states $\sigma = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $G = gg$ [6],

\[
M_\sigma^2 = m_0^2 - c + 3 \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi^2 , \quad M_G^2 = m_0^2 \frac{\phi^2}{G_0^2} + m_G^2 \frac{G_0^2}{\Lambda^2} \left( 1 + 3 \ln \left| \frac{G_0}{\Lambda} \right| \right) .
\]  

Note that the pure glueball mass $M_G$ depends also on the quark condensate $\phi$, but correctly reduces to $m_G$ in the limit $m_0^2 = 0$ (decoupling of quarkonia and glueball). In the presence of quarkonia, $m_0^2 \neq 0$, the vev $G_0$ is given by the equation

\[
- \frac{m_0^2 \phi^2 \Lambda^2}{m_G^2} = G_0 \ln \left| \frac{G_0}{\Lambda} \right| .
\]  

The shift of the fields by their vev’s introduces a bilinear mixing term $\sim \sigma G$ in the Lagrangian [6]. The physical fields $\sigma'$ and $G'$ can be obtained through an $SO(2)$ rotation,

\[
\begin{pmatrix} \sigma' \\ G' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \sigma \\ G \end{pmatrix} ,
\]  

with

\[
M^2_{s'} = M^2_s \cos^2 \theta + M^2_G \sin^2 \theta + 2 m_0^2 \frac{\phi}{G_0} \sin(2\theta) ,
\]  

\[
M^2_{G'} = M^2_G \cos^2 \theta + M^2_s \sin^2 \theta - 2 m_0^2 \frac{\phi}{G_0} \sin(2\theta) ,
\]  

where the mixing angle $\theta$ reads

\[
\theta = \frac{1}{2} \arctan \left[ -4 \frac{\phi}{G_0} \frac{m_0^2}{M^2_G - M^2_s} \right] .
\]
The quantity $m_0^2$ can be calculated from the masses of the pion, $\eta_N$ and the bare $\sigma$ mass [see Ref. 3]:

$$m_0^2 = \left(\frac{m_\pi}{Z}\right)^2 + \frac{1}{2} \left[\left(\frac{m_{\eta_N}}{Z}\right)^2 - M_\sigma^2\right].$$

If $m_0^2 - c < 0$, spontaneous breaking of chiral symmetry is realized.

III. RESULTS AND DISCUSSION

The Lagrangian (5) contains the following twelve free parameters: $m_0$, $\lambda_1$, $\lambda_2$, $m_1$, $g_1$, $c$, $h_1$, $h_2$, $h_3$, $m_G$, $\Lambda = \sqrt{\frac{1}{2} C^2}/(2m_G)$. The processes that we will consider depend only on the combination $h_1 + h_2 + h_3$, thus reducing the number of parameters to ten. We replace the set of ten parameters by the following equivalent set: $m_\pi$, $m_{\eta_N}$, $m_\rho$, $m_{a_1}$, $\phi$, $Z$, $M_\sigma$, $m_G$, $m_1$, $C$. The masses $m_\pi (= 139.57$ MeV) and $m_\rho (= 775.49$ MeV) are fixed to their PDG values.

As outlined in Refs. 11, 12, the mass of the $\eta_N$ meson can be calculated using the mixing of strange and non-strange contributions in the physical fields $\eta$ and $\eta'(958)$:

$$\eta = \eta_N \cos \varphi + \eta_S \sin \varphi, \quad \eta' = -\eta_N \sin \varphi + \eta_S \cos \varphi,$$

where $\eta_S$ denotes a pure $\bar{s}s$ state and $\varphi \simeq -36^\circ$ [17]. In this way, we obtain the value $m_{\eta_N} = 716$ MeV. (Given the well-known uncertainty of the value of the angle $\varphi$, one could also consider other values, e.g., $\varphi = -41.4^\circ$, as published by the KLOE Collaboration [18], which corresponds to $m_{\eta_N} = 755$ MeV. Variations of the pseudoscalar mixing angle affect the results presented in this paper only slightly.)

The value of $m_{a_1}$ is fixed to 1050 MeV according to the study of Ref. 13. (We note that taking the present PDG estimate of 1230 MeV does not change our conclusions.) The chiral condensate is fixed as $\phi = Z\pi$ and the renormalization constant $Z$ is determined by the study of the process $a_1 \to \pi\gamma$: $Z = 1.67 \pm 0.2$ [6].

A. Assigning $\sigma'$ and $G'$ to $f_0(1370)$ and $f_0(1500)$

The $\sigma'$ field denotes an isoscalar $J^{PC} = 0^{++}$ state and its assignment to a physical state is a long-debated problem of low-energy QCD [2, 3, 5, 13, 14]. The two major candidates are the resonances $f_0(600)$ and the $f_0(1370)$ [10, 17]. The study of Ref. 11 has shown that $f_0(1370)$ is favoured to be a state which is predominantly $\bar{q}q$. As stated above, the resonance $f_0(1500)$ is a convincing glueball candidate. For these reasons we first test the scenario in which $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$, which turns out to be phenomenologically successful, see below.

We are left with the following four free parameters: $C, M_\sigma, m_G, m_1$. They can be obtained by a fit to the five experimental quantities of Table I: the masses of the resonances $f_0(1500)$ ($M_{G'} = M_{f_0(1500)} = 1505$ MeV [16]) and $f_0(1370)$ (for which we use the mean value $M_{G'}^{\pi\pi} = (1350 \pm 150)$ MeV taking into account the PDG mass range between 1200 MeV and 1500 MeV [16]) and the three well-known decay widths of the well-measured resonance $f_0(1500)$: $f_0(1500) \to \pi\pi$, $f_0(1500) \to \eta\eta$ and $f_0(1500) \to K\bar{K}$.

| Quantity | Our Value [MeV] | Experiment [MeV] |
|----------|----------------|-----------------|
| $M_{\sigma'}$ | 1191 ± 26 | 1200-1500 |
| $M_{G'}$ | 1505 ± 6 | 1505 ± 6 |
| $G' \to \pi\pi$ | 38 ± 5 | 38.04 ± 4.95 |
| $G' \to \eta\eta$ | 5.3 ± 1.3 | 5.56 ± 1.34 |
| $G' \to K\bar{K}$ | 9.3 ± 1.7 | 9.37 ± 1.69 |

TABLE I: Fit in the scenario $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$. Note that the $f_0(1370)$ mass ranges between 1200 MeV and 1500 MeV [16] and therefore, as an estimate, we are using the value $m_{\sigma'} = (1350 \pm 150)$ MeV in the fit.

Using the Lagrangian (5), these observables can be expressed as functions of the parameters listed above. Note that, although our framework is based on $N_f = 2$, we can calculate the amplitudes for the decays into mesons containing strange quarks by making use of the flavor symmetry $SU(N_f = 3)$ [4]. It is then possible to calculate the following $f_0(1500)$ decay widths into pseudoscalar mesons containing $s$-quarks: $f_0(1500) \to K\bar{K}$, $f_0(1500) \to \eta\eta$ and $f_0(1500) \to \eta\eta'$.

The $\chi^2$ method yields $\chi^2$/d.o.f. $= 0.29$ (thus very small), $C = (699 \pm 40)$ MeV, $M_{\sigma} = (1275 \pm 30)$ MeV, $m_G = (1309 \pm 26)$ MeV and $m_1 = (809 \pm 18)$ MeV.

The consequences of this fit are the following:
(i) The quarkonium-glueball mixing angle reads \( \theta = (29.7 \pm 3.6) \, ^{\circ} \). This, in turn, implies that the resonance \( f_0(1500) \) consists to 76% of a glueball and to the remaining 24% of a quark-antiquark state. An inverted situation holds for \( f_0(1370) \).

(ii) Our fit allows us to determine the gluon condensate: \( C = (699 \pm 40) \) MeV. This result implies that the upper value in Eq. \( (2) \) is favoured by our analysis. It is remarkable that insights into this basic quantity of QCD can be obtained from the PDG data on mesons.

(iii) Further results for the \( f_0(1500) \) meson are reported in the first two entries of Table II. The decay into \( 4\pi \) is calculated as a product of an intermediate \( \rho \rho \) decay. To this end the usual integration over the \( \rho \) spectral function is performed. Our result yields 30 MeV in the \( 4\pi \) decay channel and is about half of the experimental value \( \Gamma_{f_0(1500) \rightarrow 4\pi} = 54.0 \pm 7.1 \) MeV. However, it should be noted that an intermediate state consisting of two \( f_0(600) \) mesons (which is also expected to contribute in this decay channel) is not included in the present model. The decay into the \( \eta \eta \) channel is also evaluated; this channel is subtle because it is exactly on the threshold of the \( \eta \eta \) meson. The experimental data regarding the full width: \( \Gamma_{f_0(1370) \rightarrow 4\pi} \) is strongly suppressed (as was also found in Ref. [6]). This is unlike Ref. [20], where a small but non-negligible value of about 50 MeV is found. However, it should be noted that due to interference effects our result for this decay channel varies strongly when the parameters are even slightly modified.

(iv) The results for the \( f_0(1370) \) meson are reported in the last four rows of Table II. They are in agreement with the experimental data regarding the full width: \( \Gamma_{f_0(1370)} = (200-500) \) MeV [16]. Unfortunately, the experimental results in the different channels are not yet conclusive. Our theoretical results point towards a dominant direct \( \pi \pi \) and a non-negligible \( \eta \eta \) contribution; these results correspond well to the experimental analysis of Ref. [20] where \( \Gamma_{f_0(1370) \rightarrow \pi \pi} = 325 \) MeV and \( \Gamma_{f_0(1370) \rightarrow \rho \rho} / \Gamma_{f_0(1370) \rightarrow \pi \pi} = 0.19 \pm 0.07 \) are obtained. We find that the four-pion decay of \( f_0(1370) \rightarrow \rho \rho \rightarrow 4\pi \) is strongly suppressed (as was also found in Ref. [6]). This is unlike Ref. [20], where a small but non-negligible value of about 50 MeV is found. However, it should be noted that due to interference effects our result for this decay channel varies strongly when the parameters are even slightly modified.

(v) The mass of the \( \rho \) meson can be expressed as \( m_\rho^2 = m_0^2 + \phi^2 (h_1 + h_2 + h_3) / 2 \). In order that the contribution of the chiral condensate is not negative, the condition \( m_1 \leq m_\rho \) should hold. In the framework of our fit this condition is fulfilled at the two-sigma level. This result points towards a dominant \( m_1 \) contribution to the \( \rho \) mass. This property, in turn, means that the \( \rho \) mass is predominantly generated from the gluon condensate and not from the chiral condensate. It is therefore expected that the \( \rho \) mass in the medium scales as the gluon condensate rather than as the chiral condensate. In view of the fact that \( m_1 \) is slightly larger than \( m_\rho \), we have also repeated the fit by fixing \( m_1 = m_\rho \); the minimum has a \( \chi^2 / \text{d.o.f.} \approx 1 \) and the results are very similar to the previous case. The corresponding discussion about the phenomenology is unchanged.

(vi) As already stressed in Refs. [6, 22], the inclusion of (axial-)vector mesons plays a central role to obtain the present results. The artificial decoupling of (axial-)vector states would generate a by far too wide mass is predominantly generated from the gluon condensate and not from the chiral condensate. In view of the fact that \( m_1 \) is slightly larger than \( m_\rho \), we have also repeated the fit by fixing \( m_1 = m_\rho \); the minimum has a \( \chi^2 / \text{d.o.f.} \approx 1 \) and the results are very similar to the previous case. The corresponding discussion about the phenomenology is unchanged.

B. Assigning \( \sigma' \) and \( G' \) to \( f_0(1370) \) and \( f_0(1710) \)

Although the resonance \( f_0(1710) \) has also been regarded as a glueball candidate in a variety of works [21], its enhanced decay into kaons and its rather small decay width make it compatible with a dominant \( ss \) contribution in its wave function. Nonetheless, we have also tested the assumption that the pure quarkonium and glueball states mix to produce the resonances \( f_0(1370) \) and \( f_0(1710) \).

The resonance \( f_0(1710) \) is experimentally well known. Decays into \( \pi \pi \), \( \bar{K}K \), and \( \eta \eta \) have been seen, while no decay into \( \rho \rho \) and into \( 4\pi \) have been detected. Using the total decay width \( \Gamma_{f_0(1710)} = (135 \pm 8) \) MeV and the branching ratios reported in Ref. [16] it is possible to deduce the decay widths into \( \pi \pi \), \( \bar{K}K \), and \( \eta \eta \), see Table III.

A fit analogous to the one in Table I yields too large errors for the decay width \( \sigma' \equiv f_0(1370) \rightarrow \pi \pi \). For this reason we repeat our fit by adding the following constraint: \( \Gamma_{\sigma' \rightarrow \pi \pi} = (250 \pm 150) \) MeV. The large error assures that this
value is in agreement with experimental data on this decay width. The results of the fit are reported in Table III.

| Decay Width   | Our Value [MeV] | Experiment [MeV] |
|---------------|-----------------|-----------------|
| $G' \rightarrow 4\pi$ | 115             | -               |
| $G' \rightarrow \eta\eta'$ | 16              | -               |
| $\sigma' \rightarrow \eta\eta$ | $153 \pm 79$   | -               |
| $\sigma' \rightarrow KK$ | $2.1^{+1.1}_{-0.3}$ | -               |

TABLE IV: Further results from the fit with $\{\sigma', G'\} = \{f_0(1370), f_0(1710)\}$.

A clear problem in the framework of this scenario emerges: the decay width $G' \equiv f_0(1710) \rightarrow 4\pi$ is large, while experimentally it has not been seen. Therefore, we conclude that this scenario is not favoured. Moreover, in this scenario the remaining resonance $f_0(1500)$ should then be interpreted as a predominantly $ss$ state, contrary to what its experimentally dominant $\pi\pi$ decay pattern suggests. Consequently, $f_0(1710)$ is unlikely to be predominantly a glueball state; this is also in accordance with the results from the ZEUS Collaboration [23].

C. Scenarios with $\sigma' \equiv f_0(600)$

The scenarios $\{\sigma', G'\} = \{f_0(600), f_0(1500)\}$ and $\{\sigma', G'\} = \{f_0(600), f_0(1710)\}$ have also been tested. In both cases the mixing angle turns out to be small ($\lesssim 15^\circ$), thus the state $f_0(600)$ is predominantly quarkonium. Then, in these cases the analysis of Ref. [2] applies: a simultaneous description of the $\pi\pi$ scattering lengths and the $\sigma \rightarrow \pi\pi$ decay width cannot be achieved. For these reasons the mixing scenarios with the resonance $f_0(600)$ as a quarkonium state are not favoured.

IV. CONCLUSIONS AND OUTLOOK

We have presented a globally chirally invariant linear sigma model with (axial-)vector mesons and a dilaton/glueball degree of freedom. We have studied the phenomenology of the scalar states for the case $N_f = 2$: in the favoured scenario the resonance $f_0(1500)$ is predominantly a glueball with a subdominant $\bar{q}q$ component and, conversely, $f_0(1370)$ is predominantly a quark-antiquark $(\bar{u}u + \bar{d}d)/\sqrt{2}$ state with a subdominant glueball contribution. It is interesting to observe that the success of the phenomenological description of these scalar resonances is due to the inclusion of the (axial-)vector mesons in the model. The gluon condensate is also an outcome of our study and turns out to be in agreement with lattice QCD results. Different scenarios in which $f_0(1710)$ is predominantly glueball and/or $f_0(600)$ is predominantly quarkonium do not seem to be in agreement with the present experimental data.

Natural extensions of the model are the case $N_f = 3$ [24] and the inclusion of a nonet of tetraquark states as additional low-lying scalar states. In this general scenario, a mixing of five scalar-isoscalar states takes place, which...
allows to describe all relevant scalar-isoscalar resonances listed in the PDG below 1.8 GeV \cite{PDG}. Applications of the model at nonzero temperature and density are also important because the presence of the dilaton field allows to study the restoration of both the dilatation and the chiral symmetry of QCD.

Acknowledgments

The authors acknowledge fruitful discussions with A. Heinz and E. Seel. The work of D.P. is supported by the Foundation "Polytechnical Society."
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