Analysis of Maximum Loadability of a Bus for Different Load Models using New Algorithms

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Abstract

Objective: Load modelling has a prominent place in present scenario of power systems research. In this paper various load models are considered for assessing maximum loadability using new algorithms. This has got a lot of significance in the voltage stability studies. Methods/Analysis: Generally constant representation of loads was considered. But the load characteristics significantly affect the power system’s performance. Constant models can not accurately describe the actual nature and behaviour of loads. Findings: Constant Power, Constant Impedance, Constant Current as well as composite (ZIP) load model are considered for assessing maximum loadability using a new algorithm. Constant Impedance, Constant Current Load models are established using node elimination technique. Application/Improvements: Case studies are conducted on IEEE 14 bus system with different load models and the test results are compared.

Keywords: Constant Current Model (I), Constant Impedance Model (Z), Constant Power Model (P), Maximum Loadability, ZIP Model

1. Introduction

Accurate load modelling poses a great challenge to power engineers. Improper representation of loads cannot give proper focus on the loadability of buses and on the voltage stability margins. That is why better load models have to be developed for system studies. Traditional load flow studies are implemented under the assumption that the real and reactive powers are constant and independent of voltage and frequency. This infers that the loads are regarded as Constant Power type. During normal operation of a well designed power system, the frequency variation is very small and so the assumption that the load is independent of frequency is acceptable. However the bus voltages vary depending on the load on the system. So variation of voltage should be considered for the proper representation of loads. Some types of loads like motors and compressors have reasonably Constant Power characteristics. Lamp loads come under Constant Impedance loads and Arc furnaces are examples of Constant Current Loads.

Lot of research has been in progress on modelling of loads. Traditional load modelling can be done in two ways such as exponential model and polynomial model. Exponential model based Load flow studies for various load models like Constant Impedance (Z), Constant Current (I), Constant Power (P), are done with the help of MATLAB programming. For composite loads polynomial model is used. The performance of various static load models can be analysed by assessing the voltage stability indices like minimum singular value of the power flow Jacobian matrix and minimum singular value of the reduced power flow Jacobian matrix. Modal analysis of power flow Jacobian matrix with polynomial load model is implemented. In order to get the accurate load models calibration techniques are used after satisfactory

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load model is developed. Some of the load calibration techniques are measurement based approach, component based approach, curve fitting, support vectors, genetic algorithm etc. A study on these calibration techniques is discussed. General review on load models id est. (i.e.) Constant Power, Constant Impedance, Constant Current Load models for wye, delta connected loads, polynomial and exponential load models for a distribution system is done. The real power and reactive power injections of the load buses are calculated with the help of known voltage and system parameters. The estimation accuracy and observability of the system can be improved by using these measurements in the weighted least square estimation in recursive form. As the loads vary with time, online load modelling algorithm is used to represent the load characteristic and static load modelling by using measurement based approach is discussed. The parameters of polynomial model are estimated with the help of successive approximation method and least square method. Voltage and power variations of ZIP load model for a micro grid are scrutinised using MATLAB based GUI. A method to determine the characteristics of composite load model for a digital power system simulator using readily available data is implemented. A weighted least square parameter estimation is employed to find the parameters of dynamic load model using state space analysis. The effect of voltage variations on dynamic load model is discussed. In order to get accurate load modelling standard load models for power flow and dynamic simulation programs have been done. For Constant Power model at the critical point the Jacobian becomes singular and this can be overcome by continuation power flow method. Using RTDS (Real Time Digital Simulator) an experimental study on both static as well as dynamic load models using PMU measurements is carried out. Here the mean square error within the model output and the measurements is reduced in order to find the load parameters. The consequences of various load models on rotor angle stability are discussed and for the Constant Power load models the lowest critical clearing time is observed. The load models for small as well as large disturbances using phasor measurement units are explained. The classification of loads and the load parameters estimation is done by variable projected method based optimization algorithm and support vector machine. ZIP parameters are determined by using weighted least square method. The impact of various load models on On-Load Tap Changer (OLTC) is done by using calculation of maximum power

transmission. Stability in power system network is investigated by using two different line stability indices like Apparent Line Power Stability Index (ALPSI) and Fast Voltage Stability Index (FVSI) on IEEE - 14 Bus System. The impact on Voltage stability after the placement of Distributed Generation (DG) on IEEE 14 bus and Iran 34 bus system is discussed.

All these papers reflect the effect of modelling load characteristics on voltage instability. The characteristics of load have significant effect on accuracy of transmission capacity limits estimation. The standard load flow programs are commonly considered as Constant Power loads. Voltage instability arises when a system is loaded beyond its ability to support the bus voltage. As the load is increased at a particular bus, then at some point dQ/dV becomes zero. This indicates the voltage instability and the corresponding load indicates maximum loadability limit of that bus. The main focus of this paper is to find the maximum loading ability of all the load buses.

This paper proposes new algorithms for finding the maximum loadability limit of load buses for various load models. Constant Impedance, Constant Current Load models are established using node elimination technique. Composite (ZIP) model also has been considered for assessing the maximum loading limit. Case studies are conducted on IEEE 14 bus system and the test results are compared for different load models. A comparative analysis of these models has been presented.

The composition of this paper is as follows. Section 2 deals with load modelling. Proposed new algorithms for different loads have been discussed in Section 3. Test results from case studies are given in Section 4. Conclusions drawn from the test results are given in Section 5.

2. Static Load Modelling

The stable operation of a power system depends upon the balance of generation and load in the system. Load characteristics majorly affect the system stability. In most cases a typical load bus consists of many diverse devices like fluorescent lamps, motors, refrigerators, compressors, heaters and so on. Even though the load composition is known, it is challenging to model each and every individual component of that load. So, load modelling is done based on some considerable amount of simplification. Detailed load modelling facilitates
better stability margins. Traditional load models are of two types. They are: Static Load modelling and Dynamic load modelling.

Depending on the power and voltage relation the loads can be casted into Constant Power model, Constant Current model and Constant Impedance models. Static modelling is delineated by algebraic equations whereas Dynamic modelling can also have differential equations. Even though load characteristics depend on both voltage and frequency, only voltage variations are considered, as variations in frequency are minimal. Voltage dependency of loads can be typified by two models such as exponential model and polynomial model.

2.1 Exponential Model

The exponential model for real power P and reactive power Q at a load bus are represented by Equation (1)

\[ P = P_0 \left( \frac{V}{V_0} \right)^a \]
\[ Q = Q_0 \left( \frac{V}{V_0} \right)^b \]

where,

- V is Voltage magnitude at each bus.
- \( P_0, Q_0, V_0 \) - The initial values of real power, reactive power and voltage at the load bus respectively.
- a, b - Load parameters of this model.
- The values of a and b vary between 0 to 2, for different load models as follows.
- \( A = b = 2 \) represents Constant Impedance characteristics.
- \( A = b = 1 \) represents Constant Current characteristics.
- \( A = b = 0 \) represents Constant Power characteristics.

Exponential model cannot be used for composite load models because composite loads are the combination of all the three types of loads such as Constant Impedance (Z), Constant Current (I), Constant Power (P). To represent composite load model polynomial model is used.

2.2 Polynomial Model

This model is also called as ZIP model. Z depicts Constant Impedance, I represents Constant Current and P refers to Constant Power components. The polynomial model for active power and reactive power are given by:

\[ P = P_1 V^2 + P_2 V + P_3 \]
\[ Q = Q_1 V^2 + Q_2 V + Q_3 \]

where,

- \( P_1, Q_1 \) indicates the p.u. quantities of real and reactive loads which acts as Constant Impedance.
- \( P_2, Q_2 \) indicates the p.u. quantities of real and reactive loads which acts as Constant Current.
- \( P_3, Q_3 \) indicates the p.u. quantities of real and reactive loads which acts as Constant Power.

2.2.1 Constant Impedance Model

It is also known as Constant Admittance model. In the Constant Impedance load model, power is a function of voltage and changes with the square of voltage. If the load is represented at any bus by static Impedance then we have

\[ Z_{eq} = \frac{VV^*}{P - jQ} \]

2.2.2 Constant Current Model

In the Constant Current Load model, power is a function of voltage and is directly proportional to voltage. For implementation of this type of load in system studies, the initial values of current is obtained from:

\[ I_{eq} = \frac{P - jQ}{V} \]

Where, V is the system nominal voltage and \( I_{eq} \) is held constant.

2.2.3 Constant Power Model

In the Constant Power load model, power is independent of voltage.

3. Proposed Methodology

New algorithms for finding the maximum loadability limit of load buses for various load models are proposed. Load flow studies with Constant Impedance and Constant Current Load models are performed using node elimination technique. Composite (ZIP) load model is also considered for finding maximum loadability. The proposed new algorithms for all the cases along with base case power flow are given below. If the load at a bus is increased beyond its maximum value, the voltage will collapse and then the system becomes unstable. The point where the voltage collapse starts is called critical or nose point.

3.1 Algorithm for Base Case Power Flow (Conventional Method)

Step 1: Read the input data (line data and bus data).
Step 2: Form Y-Bus.
Step 3: Calculate power injections in the rth iteration.

\[ \Delta P_r^i = \sum_{k=1}^{n} V_i^r V_k^r Y_{ik} \cos(\theta_{ik} + \delta_k^r - \delta_i^r) \]  
\[ \Delta Q_r^i = -\sum_{k=1}^{n} V_i^r V_k^r Y_{ik} \sin(\theta_{ik} + \delta_k^r - \delta_i^r) \]  

Compute mismatch, 
\[ \Delta P_r^i = P_i - P_i^r \text{ for PV and PQ buses.} \]
\[ \Delta Q_r^i = Q_i - Q_i^r \text{ for PQ buses.} \]

If the mismatch is less than the prescribed tolerance, stop the iteration. Now calculate \( P_i \) and \( Q_i \) and print the solution. Or else go to Step 4.

Step 4: Calculate the elements of Jacobian matrix.

\[
\begin{bmatrix}
\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\
\frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta |V|
\end{bmatrix} =
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]  

(6)

Step 5: Find the correction vector using.

\[
\begin{bmatrix}
\Delta \delta \\
\Delta |V|
\end{bmatrix} = [J]^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]  

(7)

Step 6: Update the state vector which includes voltage magnitudes and angles. Go to step 3.

### 3.2 New Algorithm for Constant Power Load Model

Step 1: Run base case power flow as described above.
Step 2: Increase the loads in steps of some % by keeping the power factor constant.
Step 3: Proportionately increase the generation in order to cope with the increase in load demand.
Step 4: Run the power flow and obtain the state variables (Phase angles and Voltage magnitudes).
Step 5: Store the values of voltages and angles.
Step 6: If the critical point for a particular bus is reached then don’t increase the load of that particular bus and increase the load for remaining buses till the critical point is reached for all the buses.
Step 7: Plot the graph between voltage and load power.

### 3.3 New Algorithm for Constant Impedance Load Model

Step 1: Run base case power flow as follows:
- Convert all the loads into equivalent Impedances at that bus as:
  \[ Z_{eq} = \frac{V V^*}{P_L - jQ_L} \]. Where, \( V \) is the base case load flow solution.
  This makes the current injection at load buses equal to zero.
- Include the load admittances in \( Y_{BUS} \).
- Decompose network matrix equations as:
  \[ \begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \]  
  (8)
  \[ I_G = Y_{GG} V_G + Y_{GL} V_L \]  
  (9)
  \[ I_L = Y_{LG} V_G + Y_{LL} V_L \]  
  (10)

Eliminate all the load buses using node elimination technique.
Since the Current Injections at the load buses are zero:

\[ Y_{LG} V_G + Y_{LL} V_L = 0 \]  
(11)
\[ V_L = -\frac{Y_{LG}^2 \cdot Y_{LL}^2 \cdot V_G}{Y_{GG} Y_{GL} Y_{LL} Y_{LG}} \]  
(12)
\[ I_G = Y_{GG} V_G - Y_{GL} Y_{LL} \]  
(13)
\[ I_L = (Y_{GG} - Y_{GL} Y_{LL}) V_G \]  
(14)
\[ Y_{\text{reduced}} = Y_{GG} - Y_{GL} Y_{LL} \]  
(15)

Now the reduced network contains only generators.
- Run power flow to find out the phase angles and Voltage magnitudes at the generator buses.
- Calculate the load voltages by using Equation 12.
- Store the values of \( V_L \) and \( V_G \) in \( V \).
- Calculate the new power by using converged voltages and equivalent Impedances by using the formula
  \[ S = \frac{V^2}{Z_{eq}} \]
- Step 2: Now decrease the load impedance in steps and proportionately increase the generation to cope with the increase in load demand.
Step 3: Run the power flow and get the values of both voltage magnitudes and phase angles.
Step 4: If the critical point for a particular bus is reached then don’t increase the load of that particular bus and increase the load for remaining buses till the critical point is reached for all the buses.
Step 5: Plot the graph between Voltages and load Power.

3.4 New Algorithm for Constant Current Load Model:

Step 1: Run base case power flow as follows:

- Convert all loads into equivalent currents using the load voltages obtained from base case analysis as:
  \[ I_{eq} = \frac{P_j - jQ_j}{V} \]

- Decompose network matrix equations as:
  \[
  \begin{bmatrix}
  I_G \\
  I_L
  \end{bmatrix}
  =
  \begin{bmatrix}
  Y_{GG} & Y_{GL} \\
  Y_{LG} & Y_{LL}
  \end{bmatrix}
  \begin{bmatrix}
  V_G \\
  V_L
  \end{bmatrix}
  \]
  \[ I_G = Y_{GG}V_G + Y_{GL}V_L \]
  \[ I_L = Y_{LG}V_G + Y_{LL}V_L \]
  \[ V_L = Y_{LL}^{-1}(I_L - Y_{LG}V_G) \] \hspace{1cm} (16)

- Find the reduced Jacobian matrix:
  \[
  I_G = Y_{GG}V_G + Y_{GL}^{-1}(I_L - Y_{LG}V_G) \] \hspace{1cm} (17)
  \[ I_G = Y_{GG}V_G + Y_{GL}^{-1}Y_{LL}V_L + Y_{GL}Y_{LL}^{-1}I_L \] \hspace{1cm} (18)
  \[ I_G = (Y_{GG} - Y_{GL}Y_{LL}^{-1}Y_{LG})V_G + Y_{GL}Y_{LL}^{-1}I_L \] \hspace{1cm} (19)
  \[ Y_{red} = Y_{GG}^{-1}Y_{GL}Y_{LL}^{-1} \] \hspace{1cm} (20)

- Run the power flow.
- Calculate the load voltage using Equation 16.
- Store the values of \( V_G \) and \( V_L \) in \( V \).
- Calculate the new power using converged voltages and equivalent currents using the equation \( S = VI \).

Step 2: Increase the load current in steps and also proportionately increase the generation in order to cope with the increase in load demand.
Step 3: Run the power flow and update the values of voltage magnitude as well as phase angles.
Step 4: If the critical point for a particular bus is reached then don’t increase the load of that particular bus and increase the load for remaining buses till the critical point is reached for all the buses.
Step 5: Plot the graph between voltage and load power.

3.5 New Algorithm for ZIP Load Model:

Step 1: Read the input data i.e. both line data and bus data.
Step 2: Let \( x \% \) of load is constant impedance, \( y \% \) of load is constant current then remaining \( z \% \) of load is Constant Power component. The total power at each load bus is obtained.
Step 3: Run power flow.
Step 4: Calculate load voltage from the equation 16.
Step 5: Store the values of \( V_G \) and \( V_L \) into \( V \).
Step 6: Calculate \( Z_{eq} \) and \( I_{eq} \).
Step 6: From the converged voltages calculate the powers corresponding to Constant Impedance: \( S_z = \frac{V^2}{Z_{eq}} \) and Constant Current \( S_z = VI \)
Step 7: Now, the new power is the Summation of all the three powers corresponding to Constant Power \( (S_p) \), Constant Impedance \( (S_z) \) and Constant Current \( (S_i) \).
Step 8: Now increase the loads in some percentage and also proportionately increase the generation in order to meet the load demand.
Step 9: If the critical point for a particular bus is reached then don’t increase the load of that particular bus and increase the load for remaining buses till the critical point is reached for all the buses.
Step 10: Plot the graph between Voltage and load power.

4. Results

The proposed algorithms have been tested on IEEE 14 bus system. The line data and bus data are given below in Table 1 and Table 2 respectively. Figure 1 gives the symbolic representation of IEEE 14 bus system.

The maximum loading capability of load buses is obtained for various load models. The case studies are implemented for various load models by using MATLAB programming. For Constant Power load model the load is increased in steps of 3% and the generation is increased in steps of 5% to take care of losses then the critical point for 14th bus is achieved at 0.6094 p.u. For Constant Impedance load modelling the Load Impedance is decreased in steps of 3% and generation in steps of 5%, here the critical point for 14th bus is achieved at 0.2671 p.u. For Constant Current Load modelling the load current is increased in steps of 3% and generation in steps of 5%, then the critical point for 14th bus is achieved.
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at 3.9693 p.u. For ZIP model 60% of load is taken as Constant Impedance load type, 10% of load is taken as Constant Current type and remaining 30% of power corresponds to Constant Power type, then the critical point for 14th bus is achieved at 2.0638 p.u. And the same trend is noticed for all the remaining load buses.

For Constant Power model at the critical point the Jacobian becomes singular and this can be overcome by continuation power flow method. The voltage stability limit for 14th bus in Constant Impedance load model is less than Constant Power, Constant Current, ZIP load models and the Constant Current Load model has got highest voltage stability limit.

A PV curve is drawn between Load power on x-axis and Bus voltage on y-axis. The maximum loadability of all the load buses is shown in following figures. Figure 2 and Figure 3 limn the results of all the four models i.e. Constant Power Load Model, Constant Impedance Load Model, Constant Current Load Model and ZIP Load Model at load buses.

### 5. Conclusion

Various types of static load models are implemented to find the maximum loading capacity of all the load buses for IEEE 14 bus system. New proposed algorithms for Constant Power, Constant Impedance, Constant Current and ZIP Load models are carried out and the critical point for all the load buses is achieved. By comparing all the load models the Constant Current has got better voltage stability limit and the Constant Impedance Load models has got the lowest voltage stability limit.
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