Supersymmetric Neutrino Masses, R Symmetries, and The Generalized $\mu$ Problem

Hans-Peter Nilles
Physics Department, Technische Universität München
D-85748 Garching, Germany
and
Werner-Heisenberg-Institut
Max-Planck-Institut für Physik, Werner-Heisenberg-Institut
D-80805 München, Germany

Nir Polonsky
Sektion Physik der Universität München
Theoretische Physik-Lehrstuhl Prof. Wess, Theresienstrasse 37
D-80333 München, Germany

Abstract

In supersymmetric models a tree-level neutrino mass could originate from the (weak-scale) superpotential. We propose and examine a realization of that idea, which arises naturally in the framework of a spontaneously broken $U(1)$ $R$-symmetry. The solution to the neutrino mass problem could shed light in this framework on the possible resolution of the $\mu$ problem. Furthermore, the suppression of the neutrino mass in comparison to the weak scale arises dynamically and need not be encoded in the superpotential. The latter mechanism operates, for example, in universal models for the soft supersymmetry breaking terms. Phenomenological and cosmological implications of the model are also discussed, some of which are shown to hold more generally. We also note that future signatures could include observable enhancement of dijet and multijet production rates and a correlation between the supersymmetric and neutrino spectra.

I. INTRODUCTION

Within the framework of the Standard Model of the electroweak and strong interactions the neutrinos are massless to all orders in perturbation theory. However, it is widely believed, based on the interpretation of current observations, that the neutrinos are massive and light, i.e., $m_\nu \sim O(1 \, \text{-} \, 100 \text{ eV})$. If confirmed by future experiments (e.g., the next generation...
of underground observatories and the long-baseline oscillation experiments) the massive neutrinos would provide an unambiguous signal of physics beyond the Standard Model (SM). For example, the small neutrino masses are often attributed to some sort of a see-saw mechanism involving an intermediate scale $M_{\text{Intermediate}} \sim \mathcal{O}(10^{12} \text{ GeV})$ right-handed neutrino (or some other structure at that scale), i.e., $m_{\nu} \approx M_{\text{weak}}^2/M_{\text{Intermediate}}$ \footnote{The neutrino mass in that case is a signal of the physics at intermediate scales.} The neutrino mass generation follows in a straightforward manner if one allows a low-energy (bilinear) mass superpotential, $W_M$, which is the most general one in the fields of the minimal supersymmetric extension (MSSM) and the SM $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group. The (renormalizable) superpotential reads in that case

$$W = \mu_\alpha L^\alpha H_2 + W_Y,$$

where $L_\alpha = (H_1, L_\tau, L_\mu, L_e)$ is the $(1,2,-1/2)$ four-vector in field space, the four-vector $\mu_\alpha$ is of the order of magnitude of the weak scale and $M_{SU\text{SY}}$, and $W_Y$ is the (trilinear) Yukawa part of the superpotential. One neutrino species (see below) is now massive at tree level as a result of the superpotential mass term (in a similar manner to the fermion partners of the Higgs bosons – the Higgsinos). The details of the Yukawa superpotential $W_Y$ would only affect the loop-level corrections to the masses.

It is quite striking that the SM massless fermions – the neutrinos – are the only SM fermions that are allowed by the symmetries to have a supersymmetric mass. In general, supersymmetric mass parameters are naturally of the order of the grand-scale or are zero, e.g., in string theory and grand-unified models. Hence, low-energy mass parameters (which
are of the order of the weak-scale) in (1) represent a perturbation to the above expectation and they parameterize the high-energy physics in a similar way to the dimensionful supersymmetry breaking soft parameters which set the scale $M_{\text{SUSY}}$ (see below for a discussion of the $\mu$-problem). Once mass parameters are introduced, the term $\mu_\alpha L^\alpha H_2$ (rather than only $\mu H_1 H_1 H_2$, which is the only superpotential mass term included in the MSSM case) is the most natural choice. The neutrino mass then arises from weak-scale parameters, but in fact, parameterizes the high-energy physics. Naively, one might have expected that (unless $\mu_{\nu L, \mu, e} \to 0$) $m_\nu \approx O(\mu)$, which is the only superpotential mass term included in the MSSM case). However, a simple condition \[6,7\], which, as we will show, may be realized dynamically in universal models for the soft supersymmetry breaking parameters, guarantees that this is not the case and that neutrinos as light as $O(100 \text{ eV})$ are obtained.

Other proposals for weak-scale neutrino mass generation in a supersymmetric framework \[8–13\] include mass generation due to the soft supersymmetry breaking sector, spontaneous lepton number breaking, and Yukawa interactions. The supersymmetric neutrino mass mechanism \[6,14\], that we will examine here, embodies various aspects of all other proposals.

The neutrino mass generation involves, in our case, several delicate issues which require some elaboration. While doing so, we will establish the outline of our proposed models.

\textit{R-parity violation:} The superpotential $W$ given in (1) does not preserve lepton number, and thus, the discrete $Z_2$ R-parity $R_P = (-)^{2S+3B+L}$ (where $S$, $B$, and $L$ are the particle spin, baryon and lepton numbers, respectively) that is typically imposed in the MSSM (and forbids weak-scale neutrino mass generation) is also broken. Unlike in generic broken R-parity models, the breaking here is restricted and does not lead to unacceptable proton-decay rates. Specifically, the (tree-level) neutrino mass generation is insensitive to the details of $W_Y$, and the Yukawa superpotential could still have (approximately) its SM form and preserve this or some other symmetry so that the proton is long lived. (In fact, for that purpose it is sufficient to preserve at the renormalizable level only baryon number, e.g., this is the case in the $Z_3$ baryon parity \[16\] and similar \[17\] models.) For our purpose it is enough to consider $W_Y = W_Y^{\text{MSSM}}$ where (suppressing family indices) $W_Y^{\text{MSSM}} = h_U H_2 Q U + h_D H_1 Q D + h_E H_1 L E.$ (2)

Below, we will show that this is indeed a natural choice in the models that we consider.

\textit{Rotations and lepton-number redefinitions:} Lepton number violation can be rotated from $W_M$ onto $W_Y$, introducing the lepton number violating term\[4\]

$$W_Y^{\text{LNV}} = h_D^{\text{LNV}} L Q D + \frac{1}{2} h_E^{\text{LNV}} [L, L] E,$$ (3)

where commutation with respect to generation indices is implied in the second Yukawa term (which vanishes in the case of one lepton generation). R-parity is now replaced, e.g., by

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\[3\] In the case of an arbitrary $W_Y^{\text{LNV}}$ the Yukawa couplings are sometimes denoted by $\lambda = h_E^{\text{LNV}}$ and $\lambda' = h_D^{\text{LNV}}$. Our notation aims at stressing the relation in our case between $W_Y^{\text{LNV}}$ and $W_Y^{\text{MSSM}}$ (see below).
baryon parity. Such rotations are useful when discussing the implications of the model at low energies (where all relevant symmetries are broken). The new Yukawa couplings are not arbitrary in that case and lead to the same phenomenology. However, rotations are impossible if $W$ is invariant under a symmetry which does not commute with the $SU(4)$ symmetry of $L_\alpha$ rotations in the field space, i.e., a symmetry which distinguishes the Higgs and lepton superfields. We will assume that this is the case at high energies, e.g., at Planckian scales. We will show later that such a “non-commuting” symmetry is, in fact, a desired feature and that it can be related to the ad hoc absence of an arbitrary $W_Y^{LNV}$ (and also to the absence of lepton number violation in the soft terms). In particular, one may not need to impose an additional symmetry on $W_Y$. Note also that the rotation leading to (3) is not scale invariant and that $W_Y^{LNV} \neq 0$ will regenerate lepton number violation in $W_M$ via renormalization group scaling. It suggests that one can define the SM lepton number consistently only at low-energies. For that reason, and as a result of our symmetry assumption, we distinguish between the high- (MSSM) and low-energy (SM) definitions of the leptons. The former are defined by the superpotential (2) while the latter will be chosen (after all symmetries are broken and rotations are possible) in most cases as the perpendicular directions to the relevant weak-scale expectation value. This distinction will prove as a useful model-building tool.

**The $\mu$-problem and the choice of a symmetry:** An arbitrary (high-energy) vector with a magnitude $|\mu| \sim \mathcal{O}(M_{\text{SUSY}})$ could lead to highly suppressed neutrino masses and there is no need to significantly suppress $\mu_{L_\alpha}/\mu_{H_1}$ etc. (this will be demonstrated in the following sections). In particular, the neutrino mass is related in our framework to the solution of the celebrated $\mu$-problem [15] to which we alluded above, i.e., explaining $|\mu|/M_{\text{Planck}} \to 0$ and $|\mu| \sim M_{\text{SUSY}} > 0$, which is the manifestation of the correlation between the supersymmetry preserving [with a natural scale of $\mathcal{O}(M_{\text{Planck}})$] and softly breaking [with an ad hoc scale $M_{\text{SUSY}} \sim \mathcal{O}(M_Z)$] sectors. The correlation is imposed by requiring the correct electroweak symmetry breaking pattern [17]. (Usually the problem is phrased in terms of $|\mu_{H_1}|$.) We will adopt a somewhat ambitious approach and require that the solution to the $\mu$-problem is determined by the same symmetry that was proposed above [and which does not commute with the $SU(4)$]. In particular, we will consider a continuous $U(1)_R$ R-symmetry (the $R$-charge is defined as above) that is known to be relevant for the solution of the $\mu$-problem [20–22] and under which $\mu_{H_1}$ and $\mu_{L_\tau, \mu, e}$ effectively carry different $R$-charges. In principle, we could use other $U(1)$ symmetries to break the $SU(4)$, e.g., a Peccei-Quinn symmetry [23] or the stringy inspired $U(1)$’s of Ref. [24]. In the latter case $\mu_\alpha$ could be forbidden by the new $U(1)$ gauge symmetry but induced by weak-scale expectation values in a similar manner to the authors’ original proposal for the generation of $\mu_{H_1}$. We will concentrate in this work on the $U(1)_R$ case. Generalization of our proposal to models in which other symmetries play a similar role in solving the $\mu$-problem is straightforward.

**Operator classification:** In the symmetry framework $\mu_{H_1}$ and $\mu_{L_\tau, \mu, e}$ are realized as non-renormalizable operators (NRO’s) present in the effective low-energy superpotential [18]. The $R$-symmetry allows us to classify three categories of mass terms in $W$ [which is normalized to carry $R$–charge $R(W) = 2$]:

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4 We assume the $U(1)_R$ transformation law $\phi \to e^{i\alpha R}\phi$, $\psi \to e^{i\alpha(R-1)}\psi$, and $F \to e^{i\alpha(R-2)}F$, for
\[ \mu_H \quad R(\mu_H) = 2 \quad \text{operator: } H_1 H_2, \quad (4a) \]

\[ \mu_L \quad R(\mu_L) = 1 \quad \text{operator: } L_{\tau, \mu, e} H_2, \quad (4b) \]

\[ \mu_N \quad R(\mu_N) = 0 \quad \text{operator: } N^2; \Phi \bar{\Phi}, \quad (4c) \]

where we have assumed the standard \( R \)-charge assignments as above (substituting \( S = 0 \)), i.e., \( R(H_{1,2}) = 0 \) and \( R(Q, U, D, L, E, N, \Phi, \bar{\Phi}) = 1 \). Thus, the \( R \)-symmetry provides us with a convenient book-keeping tool. Note that the proposal \( \mu_{H_1} \sim \langle W \rangle / M_P^2 \) is trivially realized in this framework. Generically (but see exceptions below), one expects that the higher \( R \)-charge \( \mu \) carries the more suppressed it is. Furthermore, since lepton number violation is in the form of nonrenormalizable operators, and based on dimensional arguments, one also expects that, e.g., \( h_{E}^{\text{LNV}} \sim |\mu_L| / M_P \to 0 \). [Note that \( R(h_{LNV}^E) = -1 \).]

Thus, \( W_Y \) is effectively \( W_Y^{\text{MSSM}} \) with an accidental discrete \( R_P \) symmetry. Similar arguments hold for the dimensionless couplings in the Kahler potential, and typically lepton number violation in the soft terms is also suppressed. As a result of the symmetry selection rules, \( R \)-number violation, and in particular, lepton number violation, is contained in \( W_M \) as advocated. The singlet \( \mu \)-parameters may not be suppressed, and the right handed neutrino \( N \) (or other singlets) and vector-like exotic \( \Phi \) and \( \bar{\Phi} \) supermultiplets with standard \( R \)-charge assignments are expected to be heavy. (The latter points are, in fact, relevant for the dynamical suppression of the neutrino masses discussed below.)

We are now in a position to outline our proposal: The \( U(1)_R \) symmetry selection rules require that there is a \( \sim M_Z / M_{\text{Planck}} \) hierarchy between the symmetry violation in the supersymmetric mass terms and in the Yukawa terms and the Kahler potential. As a result, it leads to the generation of \( \mu \) terms of the right magnitude, accidental \( R_P \) symmetry in the Yukawa terms, and to a consistent definition of the low-energy lepton number. In addition, the neutrino masses are suppressed dynamically.

We will examine the above issues in greater detail and expand our discussion in the following sections. In Section II we briefly discuss possible realizations (and their problems) of the \( U(1)_R \) symmetry, and present some simple examples that realize (or are exceptions to) the main features discussed above, i.e., the suppression of non-minimal Yukawa interactions and the hierarchy among the different \( \mu \) terms. We will show that the nature of the hierarchy depends on whether or not the relevant operators involve \( F \)-terms. In Section III we discuss the neutral fermion mass matrix and the sufficient conditions for small neutrino masses, which include the alignment (in field space) of \( \langle L_\alpha \rangle \) with \( \mu_\alpha \). We then proceed and show in Section IV that the latter condition and \( m_\nu \ll M_Z \) are achieved in a straightforward manner in universal models with radiative (electroweak) symmetry breaking (RSB) via the “dynamical alignment” mechanism. (For simplicity, we discuss a model with only the third generation of leptons and quarks.) It is also pointed out that grand-unification relations, as well as an intermediate-scale right-handed neutrino, generically destabilize this result. In section V we show that lepton number (and individual lepton number) violations in the model are typically proportional to either the neutrino mass or to small MSSM

\[ \mu_H \quad R(\mu_H) = 2 \quad \text{operator: } H_1 H_2, \quad (4a) \]

\[ \mu_L \quad R(\mu_L) = 1 \quad \text{operator: } L_{\tau, \mu, e} H_2, \quad (4b) \]

\[ \mu_N \quad R(\mu_N) = 0 \quad \text{operator: } N^2; \Phi \bar{\Phi}, \quad (4c) \]

the scalar, fermion and auxiliary components of a chiral superfield \( \Phi \) with \( R \)-charge \( R \), respectively.
Yukawa couplings, and thus, they are suppressed. We briefly address the issue of possible discovery channels, and stress that there could be an enhancement of dijet and multijet cross-sections. Cosmology and astrophysics implications are discussed in section VI. It is shown that weak-scale neutrino mass generation implies that the supersymmetric partners (superpartners) of ordinary matter and gauge fields are not stable on cosmological time scales. We will alternate in our discussion between the high-energy (Sections II and IV) and low-energy (Sections III, V and VI) definitions of the leptons and lepton number, as appropriate. In Section VII we comment on possible family dependencies in more complete models and on their implications.

We conclude in section VIII, where we suggest that the proposal for supersymmetric neutrino mass generation is most elegantly understood in terms of symmetry principles that are also relevant to the (generalized) \( \mu \)-problem, can be naturally incorporated in simple supersymmetric models, removes the need to ad hoc generate an intermediate scale for the right-handed neutrinos, e.g., in string models, and requires one to consider unorthodox scenarios for supersymmetric particle astrophysics, cosmology, and for superpartner collider signatures.

II. \( R \)-SYMMETRIES AND SELECTION RULES

We have chosen a spontaneously broken \( U(1)_R \) symmetry as our primary tool in deriving selection rules for the low-energy effective theory. We also have shown above that it allows one to distinguish three classes of mass (i.e., \( \mu \)-) parameters in the effective low-energy superpotential; it distinguishes the (high-scale) lepton and Higgs doublets; and that, on the basis of dimensional arguments, non-singlet (i.e., lepton-number violating) Yukawa couplings are typically suppressed by an inverse power of the Planck mass in comparison to non-singlet \( \mu \)-parameters. The presence of such a symmetry is intimately connected to (dynamical) supersymmetry breaking \[26,27\] and the \( R \)-symmetry could play a natural rule in the solution of the (either MSSM or our generalized) \( \mu \)-problem\[20,21\]. Hence, our choice of “book-keeping” tool is well motivated.

However, similarly to the Peccei-Quinn case \[23\], a spontaneously broken continuous \( R \)-symmetry implies an unwanted \[29\] (pseudo) Goldstone boson – the \( R \)-axion. The troublesome presence of the \( R \)-axion can be resolved (i) if it is an invisible axion, i.e., the \( R \)-symmetry is broken at a scale \( \theta \gtrsim 10^{10} \) GeV (from stellar evolution) and below the Planck scale \( \theta \lesssim 10^{12} \) GeV (from its contribution to the energy density) \[30\] (a higher bound \( \theta \lesssim 10^{16} \) may exist in some cases \[31\]); (ii) if the axion receives its mass from NRO’s that explicitly break the symmetry\[27\]; (iii) if the symmetry is gauged (and anomaly free)

\[5\] \( \mu_\alpha \equiv 0 \) if the \( U(1)_R \) symmetry is unbroken and the \( \mu \)-problem is trivially solved \[22\]. However, such models are strongly constrained and face many phenomenological (e.g., generation of a gluino mass) and model-building (e.g., realization of RSB) difficulties (see, for example, Ref. \[28\]).

\[6\] It was also suggested \[32\] that the explicit breaking is related to a constant term in the superpotential which carries no \( R \)-charge and which is perhaps needed to cancel the vacuum energy. In
at Planckian scales \([34]\); or (iv) if the axion is rendered heavy by different means than the Higgs mechanism which operates in (iii).

The first option could be the natural choice if the low-energy global symmetry results from an anomalous high-energy \(U(1)\) (e.g., in a string theory), and thus, could be stable with respect to gravitational corrections. In that case

\[
10^{10} \lesssim \theta \lesssim 10^{12} - 10^{16} \text{ GeV},
\]

where the symmetry-breaking scale \(\theta\) is the scale parameter that enters NRO’s with positive powers, and thus, controls the size of couplings in the low-energy effective theory. If the axion scale is related to a typical supergravity breaking scale then it naturally falls in this range \([20,21]\).

The second option is motivated by the observation that global symmetries are not likely to be exact in the presence of gravity or if the \(U(1)_R\) symmetry is accidental and due to a higher symmetry and renormalizability \([35,36]\), but it implies that a priori we do not have an handle on the choice of NRO’s. Thus, it may undermine our motivation. Nevertheless, one could still distinguish \(\mathcal{O}(\text{MeV})\) operators, that are sufficient to generate an acceptable axion mass, from \(\mathcal{O}(M_Z)\) operators, which are the relevant ones in our case. [Recall that the \(\mu_L\)’s need not be suppressed necessarily more than the ordinary \(\mu_H\) parameter and could be \(\mathcal{O}(M_Z)\).] In this scenario there are no constraints on \(\theta\). (Note that if the axion is hidden, i.e., with only gravitational interactions with ordinary matter, then effects of any operator that may be needed to render it massive are suppressed in the observable sector by an additional inverse power of \(M_{\text{Planck}}\).)

The third option is quite attractive. The symmetry is anomaly free and the Goldstone boson is not an axion. However, the anomaly cancelation equations involve the gauge fermions and the gravitino, in addition to the ordinary, exotic and hidden matter fermions (and similarly the \(D\)-terms). It was recently shown that the symmetry must be broken at a Planckian scale so that one can tune to a flat \(D\)-term direction \(\langle D \rangle = 0\) [and avoid \(\mathcal{O}(M_P)\) masses for ordinary fields] \([34]\). Thus,

\[
\theta \lesssim M_P.
\]

The anomaly equations are difficult to solve and require (many) new SM singlet fields and/or exotic fields with non-trivial SM charge assignments (and, in some cases, non-standard and family-dependent \(R\)-charge assignments for the ordinary SM fields) \([34]\). While the singlet fields could be hidden in the hidden sector, \(\mathcal{O}(M_P)\) mass terms for the (observable-sector) exotics are forbidden by the symmetry, and in the existing examples with exotics there are \(\mathcal{O}(\text{TeV})\) colored fields. We will not consider explicitly models with such fields and will constrain our investigation to the model with minimal matter content. However, we note that unless such fields interact with \(L_\alpha\), they are irrelevant for our purposes and do not alter our discussion.

that case the axion mass is related to the size of the soft supersymmetry breaking parameters. In fact, explicit breaking by a large constant in the gravitational sector is a common solution to the \(U(1)_R\) problem in supergravity models, e.g., in the Polonyi model \([33]\).
Lastly, the fourth option could be realized, e.g., if the axion is a hidden-sector field and the $U(1)_R$ is anomalous with respect to a hidden sector QCD group. The axion would then acquire a large mass (of order of the hidden sector QCD scale $\sim 10^{11}$ GeV, assuming dynamical supersymmetry breaking in the hidden sector) \[30\]. In this case one also expects $\theta \sim 10^{11}$ GeV. (7)

Below, we will not promote any option in particular, but comment on their different implications where relevant. We will assume standard $R$-charge assignments, unless otherwise is specified, that the scale $\theta$ is in the range suggested by (5) – (7), and that the non-vanishing fields are hidden fields (with only gravitational interactions with ordinary matter). The latter is motivated by the assumption of gauge confinement and dynamical supersymmetry breaking in the hidden sector, and as discussed above, could ease some of the problems.\[7]

Having discussed the possible frameworks in which the symmetry can be realized, we now turn to a discussion of some examples. NRO’s can be induced in our case by scalar vacuum expectation values (vev’s), non-vanishing $F$-terms, and fermion condensates \[21\], leading to many possible scenarios which would relate differently to supersymmetry breaking. Since we assume that only SM hidden singlets participate, then the operators (which are nonrenormalizable in that case) are suppressed by powers of the Planck mass. For simplicity, we will assume that $M \approx M_{\text{Planck}}/\sqrt{8\pi} \approx O(10^{18}$ GeV) is the only large scale suppressing the operators. Note also that while the form of the operators (i.e., the selection rules) is dictated by the symmetries [and in particular, $U(1)_R$], their non-vanishing values (i.e., the vev’s) may be a result of $U(1)_R$ breaking or of the breaking of a different symmetry at a lower scale. In the latter case $\theta$ could be lower then the $U(1)_R$ scale. [This observation is most relevant if the gauged $U(1)_R$ option is realized.]

### A. Scalar vev’s and fractional $R$-charges

The most simple example is the case in which $\theta_i = \langle z_i \rangle$ is given by the vacuum expectation value of the scalar component $z_i$ of a hidden sector chiral superfield $Z_i$ with $R$-charge $R_i$. The $\mu$-parameters, $\mu_I = \mu_{H, L, N}$ for $I = 1, 2, 3$, respectively, are given in that case by (omitting hereafter dimensionless couplings and coefficients in the nonrenormalizable operators)

$$\mu_I = \frac{\prod_{i=1}^{N_I} \langle z_{iI} \rangle^{n_{iI}}}{M^{(\sum_{i=1}^{N_I} n_{iI})-1}}.$$ (8)

We first discuss the most simple case in which each of the $\mu_I$’s depends only on one field, i.e., $R_{1I} = 2/n_{1I}, 1/n_{1I}, 0$ for $z_{11,12,13}$ ($\mu_{H, L, N}$), respectively. $\mu_N$ is also suppressed by negative powers of $M$ because $Z$ is a hidden sector field (otherwise, all symmetries allow a renormalizable term $Z_{13}NN$ in the superpotential). However, it need not be suppressed

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7However, the hidden axion could lead to cosmological problems typical to weakly (i.e., gravitationally) interacting fields with an intermediate-scale energy density in the potential (the so-called “Polonyi problem”) \[37,38\].
by more than one power of $M$, i.e., $\mu_N \sim \langle z_{13} \rangle^2 / M$. It could be a weak-scale or a Planck-scale parameter, depending if $\langle z_{13} \rangle \sim 10^{10}$ GeV or $\sim M$. If there are no $R = 0$ fields then $\mu_N = 0$. $\mu_H$ and $\mu_L$ are weak-scale parameters, leading to the constraint $n_{11,12} = [\ln(M_Z/M)]/[\ln(\langle z_{11,12} \rangle / M)]$, i.e., $n = 2$ for $\langle z \rangle \sim 10^{10}$ GeV and $n = 8$ for $\langle z \rangle \sim 10^{16}$ GeV. If $\mu_H$ and $\mu_L$ are both given by the same field $z_j$, then $n_{11}$ must be even and $\mu_L = \sqrt{M^2_H \mu_H}$. In that case the electroweak symmetry breaking is induced by the scalar neutrinos (sneutrinos) rather than $H_1$ which is now decoupled from $H_2$. (We discuss electroweak symmetry breaking in section IV.)

More generally, the operators could involve several fields such that $\sum_{i=1}^{N_I} n_{1i} R_{1i} = 2, 1, 0$, for $I = 1, 2, 3$, respectively. Let us first assume, however, that all (non-vanishing) singlets have positive $R$ charges. Then (because of the holomorphicity of the superpotential) the situation is similar to the one field case, i.e., $\mu_H < \mu_L$ (and $\mu_N = 0$ unless there is a non-vanishing $R = 0$ field). More importantly, $h^{LNV} = 0$ in this case.

Lastly, there could be fields with negative $R$-charges [in particular, if the $U(1)_R$ is gauged and anomaly free]. This most general case could allow for, e.g., $\mu_L = \mu_H \times [\langle z \rangle / M]^l < \mu_H$ and $R(Z) = -1/l$ [and also $\mu_N = \langle z \rangle (z') / M$ with $R(Z') = -R(Z)$]. However, one could also have now $h^{LNV} = h^{BNV} = \mu_L / \mu_H$ ($h^{BNV}$ is a baryon number violating coupling). Both, $\mu_L$ and $h^{LNV}$ (as well as $h^{BNV}$) vanish in the MSSM limit $l \gg 1$. Otherwise, one has to impose an additional symmetry on $W_V$. For example, $\mu_H$, $\mu_L$ and $[\langle z \rangle / M]$ have (standard) Peccei-Quinn charges of $-2$, $-1/2$ and $3/2l$, respectively. Thus, a Peccei-Quinn symmetry would forbid in this case the dangerous Yukawa couplings.

**B. A possible hierarchy between $\mu_\alpha$ and $\partial \mu_\alpha$**

The chiral superfields discussed in the previous example could also have non-vanishing $F$-terms which are of the order of magnitude of the supersymmetry breaking scale, $F_Z = O(M_{SUSSY} M)$ (i.e., they contribute to supersymmetry breaking). In that case, $Z$ could also provide a source for the soft supersymmetry breaking $B$-terms, $V = ... + B_\alpha (L^\alpha H_2 + h.c.) + ...$, and $B_\alpha \propto F_i^\alpha \partial_i \mu_\alpha$ (for example, see [39]). An interesting scenario arises in the case that $\mu_\alpha \propto \langle z_1 \rangle^{n_1} \times ... \times \langle z_N \rangle^{n_N}$ (we discuss below cases in which $\mu_\alpha$ is a mixture of non-vanishing $z$ and $F_Z$ components) with $\langle z_1 \rangle \ll \langle z_j \rangle$, where $1 < j \leq N$, $n_1 = 1$ and $\sum_j n_j R_j = 2 - R_1$ or $1 - R_1$. The parameter $B_\alpha$ is dominated by the $\partial_{i=1} \mu_\alpha$ contribution and could be of a different order of magnitude than $\mu_\alpha$.

An interesting example is the case of $\mu = O(1 \text{ GeV})$ and $B_\alpha = O(M_Z^2)$, i.e., $\langle z_1 \rangle \sim 10^{-2} \langle z_j \rangle$. Also, a small $\mu_L$ does not necessarily imply in this case a small lepton number violation in the scalar potential. [We comment on models with $\mu_H \sim \mu_L \sim O(1 \text{ GeV})$ (e.g., [40, 41, 28]) below.] Note that $B_\alpha$ are the only lepton-number violating soft supersymmetry breaking parameters in our models.

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8We assume a Peccei-Quinn charge $PQ = 1$ for $H_1$ and $H_2$ and $PQ = -1/2$ for all other ordinary matter. The anomalous $U(1)$ would be given in that case by some linear combination of the (anomalous) $U(1)_{PQ}$ and $U(1)_R$. 

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C. A single $R = 1$ superfield

All three classes of $\mu$ parameters could be obtained from a single $R = 1$ chiral superfield, $Z$, with non-vanishing scalar and $F$ (auxiliary) components, and whose fermion component, $\tilde{z}$, condenses. It may be difficult to incorporate such a scenario in a realistic model. Nevertheless, it is worth noting. The different $\mu$ parameters are now given by

$$\mu_H = \frac{\langle z \rangle^2}{M}, \quad (9a)$$

$$\mu_L = \frac{F_Z^*}{M}, \quad (9b)$$

$$\mu_N = \frac{\langle \tilde{z} \tilde{z} \rangle}{M^2}, \quad (9c)$$

and from $(9a)$ one has $\langle z \rangle = \mathcal{O}(10^{11} \text{ GeV})$. For $F_Z^* \sim (10^{11} \text{ GeV})^2$ one has $\mu_L \sim \mu_H$, and there is no clear hierarchy between the two parameters. Lastly, if the condensate $\langle \tilde{z} \tilde{z} \rangle \sim \Lambda^3$, where $\Lambda \sim \sqrt{M_{\text{SUSY}}} \sim 10^{11}$ GeV is the supersymmetry breaking scale, then contrary to the generic situation (see Example A), $\mu_N < \mu_H$. As we discuss in section IV, the couplings of the right-handed neutrino need to be suppressed in this case. The relation $(9c)$ would also allow for weak-scale vector-like exotics. It is interesting to note that, regardless of its origin, $\theta \sim 10^{11}$ GeV in all operators in this case.

Regarding the Yukawa coupling $h^{\text{LNV}}$, the holomorphicity of the superpotential forbids $h^{\text{LNV}} \sim z^*/M$. Thus, one has $h^{\text{LNV}} \lesssim |\mu_L/M| \to 0$.

If $Z$ was not an hidden-sector field, then the renormalizable superpotential term $ZLH_2$ would have been allowed (leading to $\mu_L \sim 10^{11}$ GeV). In such a case one needs to impose an additional symmetry, e.g., the usual $Z_2$ $R$-parity with $R_P(Z) = R_P(H_{1,2}) = -R_P(L_{\tau, \mu, e}) = (+)$, which allows for $(9a)-(9c)$ but forbids the $R_P(ZLH_2) = (-)$ renormalizable term.

D. $R = 1/2$ and $R = -1/2$ superfields: Scalar vev’s and F-terms

In the case of a $R_1 = 1/2$ superfield, $\mu_L = \langle z_1 \rangle^2/M$ implies that $\langle z_1 \rangle = \mathcal{O}(10^{11} \text{ GeV})$. Thus, $\mu_H = \langle z_1 \rangle^4/M^3 \to 0$ and like in Example A, the sneutrinos could contribute significantly to electroweak symmetry breaking.

Having an additional field $Z_2$ with $R_2 = -1/2$, and if the $F_{Z_1} = \mathcal{O}(10^{11} \text{ GeV})^2$ is non-vanishing, one can obtain instead

$$\mu_H = \frac{F_{Z_1}^* \langle z_1 \rangle}{M^2}, \quad (10a)$$

$$\mu_L = \frac{F_{Z_1}^* \langle z_2 \rangle}{M^2}, \quad (10b)$$

$$\mu_N = \frac{\langle z_1 \rangle \langle z_2 \rangle}{M}. \quad (10c)$$
\(\mu_{H,L}\) are \(\mathcal{O}(M_Z)\) parameters in that case if \(\langle z_i \rangle \approx \mathcal{O}(M)\) (and there are two scale parameters \(\theta_1 \sim 10^{11}\) GeV and \(\theta_2 \sim 10^{18}\) GeV, as is often the case in supergravity models), which implies that \(\mu_N \sim M\) [and again, there is no clear hierarchy between (10a) and (10b)].

However, one has to forbid in that case the \(R = \pm 1\) combinations \(\langle z_1, 2 \rangle \), etc., which would lead to unacceptably large \(\mu_L = \mathcal{O}(M)\) and \(h^{LNV} = \mathcal{O}(1)\). For example, a continuous hidden sector \(U(1)\) symmetry with identical charge assignments to both fields would allow (10a) and (10b) but forbid the dangerous couplings (as well as \(\mu_N\)).

E. Lessons and comments

Our main lesson is that the typical situation \(\mu_N \sim M\) and \(h^{LNV} \to 0\) is easily found in simple examples of \(U(1)_R\) selection rules (but exceptions exist). If \(\mu_\alpha \propto \prod_i \langle z_i^{n_i} \rangle\) then typically \(\mu_H \lesssim \mu_L\). However, if there are also non-singlet \(F\)-terms which do not vanish, then no clear hierarchy exists. In some cases additional symmetries may be required in order to suppress \(h^{LNV}\), but typically \(U(1)_R\) is sufficient for that purpose. (The additional symmetries, if required, could correspond to typical symmetries that are often found in models.)

We presented a few scenarios in which some, or all, of the \(\mu_{H,L,N}\) are generated. Many more scenarios exist. The relevant scenario would be determined by the realization of the \(U(1)_R\) symmetry, the hidden singlet \(R\)-charges, and by the relations between the fields that spontaneously break \(U(1)_R\) and supersymmetry breaking.

If the SM fields do not have the standard \(R\)-charge assignments \([14]\) that we assume, then some of the examples given above may need to be revised, depending on the charge assignments chosen. If it is a family-dependent assignment then one could distinguish (unlike in our case) between \(\mu_{L_\tau}, \mu_{L_\mu},\) and \(\mu_{L_e}\). (We discuss family dependences in Section VII.)

Lastly, if there is a hidden \(R = 0\) chiral superfield \(Z\) with \(F_Z = \mathcal{O}(10^{11}\) GeV) then \(\mu_H\) (but not \(\mu_L\)) could be partially generated by a Kahler potential source \([20]\), smearing any correlation between \(\mu_H\) and \(\mu_L\).

III. CONDITIONS FOR LIGHT (NEUTRINO) EIGENSTATES

The ratio \(\mu_L/\mu_H\), which is a (high-energy) order parameter of the models, need not be suppressed in order to suppress the tree-level neutrino mass. This is a trivial result of the observation that there are no tree-level masses for the neutrinos if \(\mu_\alpha\) and \(\langle L_\alpha^0 \rangle\) are aligned in field space \([8]\). (This observation is also the basis for the recent works of Refs. \([7,41]\).) We elaborate on that observation in this section, and we show in the next section that nearly parallel \(\mu_\alpha\) and \(\langle L_\alpha^0 \rangle\) four-vectors arise dynamically in simple models for the soft supersymmetry breaking parameters (dynamical alignment). Discussion of laboratory and cosmological implications is postponed to sections V and VI, respectively.

An important distinction (which allows for the neutrino mass generation) between the models that are discussed here and the MSSM is that all seven neutral fermions mix in our case. That is, the supersymmetric partners of the \(B\) and \(W_3\) gauge bosons and neutral fermions mix in our case. That is, the supersymmetric partners of the \(B\) and \(W_3\) gauge bosons and neutral fermions mix in our case. That is, the supersymmetric partners of the \(B\) and \(W_3\) gauge bosons and neutral fermions mix in our case. That is, the supersymmetric partners of the \(B\) and \(W_3\) gauge bosons and neutral fermions mix in our case. That is, the supersymmetric partners of the \(B\) and \(W_3\) gauge bosons and neutral fermions mix in our case.

\(^9\)Note that if \(\mu\) is generated in this manner then it is proportional to \(W\) and has \(R\)-charge \(R(\mu) = 2\).
Higgs – the bino, wino (the gauginos) and the two Higgsinos, respectively (the neutralinos), mix with each other and with the three neutrinos once electroweak symmetry is broken. Hence, one has a seven-dimensional Majorana mass matrix for the neutral fermions, which involves (at tree-level)

- the bino and wino soft supersymmetry breaking masses $M_1$ and $M_2$, respectively,
- the Higgsino and neutrino masses $\mu_\alpha$,
- and the gauge-matter mixing parameters $M_Z\nu_2/\nu$ and $M_Z\nu_\alpha/\nu$.

(We define $\nu_2 = \langle H_2^0 \rangle$, $\nu_\alpha = \langle L^0_\alpha \rangle$, $\nu_1 = |\nu_\alpha|$, $\tan\beta = \nu_2/\nu_1$, and $\nu = \sqrt{\nu_2^2 + \nu_1^2}$.) The determinant of the neutral fermion mass matrix vanishes: The $SU(4)$ symmetry is broken down to $SU(2)$ by two fundamental vectors, $\mu_\alpha$ and $\nu_\alpha$, each of which can render only one physical state massive, and there are two massless states. One could define, for example, the $e$ and $\mu$ neutrinos as those states, $m_{\nu_e} = m_{\nu_\mu} = 0$.

Let us then eliminate the (tree-level) massless states and discuss the residual five-dimensional non-degenerate mass matrix. It is convenient to define for that purpose, following Banks et al. [1], $\cos\zeta \equiv \langle \mu_\alpha \nu^\alpha \rangle/|\mu_\alpha| |\nu_\alpha| \equiv \langle \mu_\alpha \nu^\alpha \rangle/\mu_1$. The determinant (of the residual mass matrix) is proportional to $\sin^2\zeta$, which is the second (or the low-energy) order parameter of the model. That is, if $\mu_\alpha$ and $\nu_\alpha$ are aligned [and $SU(4)$ is broken down to only $SU(3)$] then only one state is rendered massive and there is an additional massless state $m_{\nu_\tau} = 0$. Otherwise, the lightest neutral fermion, which we define to be the $\tau$ neutrino $\nu_\tau$, has a mass $m_{\nu_\tau} \sim M_Z \sin^2\zeta$ (assuming, for simplicity, $M_{SUSY} \sim M_Z$). It is sufficiently suppressed and phenomenologically acceptable if $\sin\zeta \lesssim \sqrt{m_{\nu_\tau}/M_Z}$ (where $m_{\nu_\tau} \sim 23$ MeV [100 eV] is the laboratory [12] [cosmological] (energy density) upper bound on the neutrino mass).

The alignment condition and all other sufficient conditions for the suppression of the neutrino mass are more easily seen in the rotated basis in which $\mu_\alpha = \mu(1, 0)$ (assuming one lepton generation). One has (for the residual mass matrix) in that basis

$$\det M_{\text{neutral}} = \mu^2 \tilde{\nu}_{L_r}^2 (g_2^2 M_2 + g_1^2 M_1),$$  \hspace{1cm} (11)

where $g_{1, 2}$ are the hypercharge and $SU(2)$ couplings, respectively, and the rotated $\tilde{\nu}_{L_r} \approx \nu_L[1 - (\mu_L, \nu_{H_1}/\mu_{H_1}, \nu_{L_r})] \approx \mu_L[(\nu_{L_r}/\mu_{H_1}) - (\nu_{H_1}/\mu_{H_1})]$ (assuming $\mu_{H_1} > \mu_{L_r}$). Thus, there are light states if

10 All relevant mass and mixing matrices are given, for example, in Ref. [11].

11 The $\sim 100$ eV bound applies to stable neutrinos and assumes the critical energy density $\Omega h^2 = 1$. If the neutrinos constitute only a part of that energy density, then the upper bound scales down accordingly, i.e., $m_\nu \lesssim 100 \Omega_\nu h^2$ eV.

12 The determinant of the MSSM neutralino mass matrix reads $\det M_{\text{neutralino}} = \mu \nu_1 \nu_2 (g_1^2 M_2 + g_2^2 M_1) - \mu^2 M_1 M_2$. The tree-level neutrino mass is given by the ratio $\det M_{\text{neutral}}/\det M_{\text{neutralino}}$.  

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\( (a) \bar{\nu}_{L \tau} \to 0, \) i.e., \( \mu_\alpha \) and \( \nu_\alpha \) are parallel (and \( \sin \zeta \to 0 \)) or \( \tan \beta \gg 1 \) (and \( \nu_1 \to 0 \)),

\( (b) \mu \to 0, \)

\( (c) \) the gauginos are heavy and their mixing with the matter fermions is negligible, i.e., \( M_1, M_2 \gg \mu, M_Z \), in which case \( \{ \det M_{\text{neutral}} \}_{\text{matter}} \sim \det M_{\text{neutral}}/M_1 M_2 \) (see above footnote).

It is, however, difficult to realize the latter case because \( \mu \) and the gaugino masses both contribute to the quadratic terms in the scalar potential, and thus, are related by the minimization conditions described below and are typically of the same order of magnitude.

The case (b) is quite interesting and was discussed in Refs. [10,40]. (See also Ref. [28] for a recent discussion and references of the equivalent situation in the MSSM. However, the constraints on the parameter space in our case could be different than in the MSSM.) At least two of the neutral fermions, a Higgsino and a neutrino, are now light. The light Higgsino contribution to the \( Z \) width constrains the model to the region of \( \tan \beta \sim 1 \). From RSB one has that the scalar mixing parameter \( B_\alpha \) (see Section IV) is not proportional to \( \mu_\alpha \) and cannot vanish in that case. Thus, the Higgs boson mass would partially come from mixing \( \propto B_\alpha \) with the scalar neutrinos. To consistently realize such a scenario would require a departure from the simple models for the soft terms that we consider below. We will comment on that scenario again when discussing the scalar potential, but we do not consider it in detail.

Perturbativity of Yukawa couplings and dynamical alignment (which are considered in Section IV) constrain \( \tan \beta \) from above (see below). Thus, the remaining case (a) is typically realized for nearly aligned vectors, i.e., \( \bar{\nu}_{L \tau} \to 0 \). (This is crucial for the discussion of the phenomenological implications of the model in sections V and VI.) Nevertheless, there are additional (secondary) suppressions of \( m_\nu \) if \( \tan \beta \gtrsim 2 - 3 \) or if the gaugino mass parameters are large. For example, in Fig. 1 we show \( m_\nu \) as a function of \( \sin \zeta \) for different values of the gaugino masses and of \( \tan \beta \) (but no RSB is yet required). Note, in particular, the high precision required in the alignment for moderate values of \( \tan \beta \). The alignment condition, however, can be slightly relaxed for large values of \( \tan \beta \).

We consider in this work only the tree-level mass matrix. The one-loop matrix has new non-diagonal entries [that explicitly break the residual \( SU(2) \) symmetry] and all seven states are massive with generically one very light neutrino\(^{13}\) and one neutrino with a mass \( \sim \mathcal{O}(10^{-3} - 1 \times m_{\nu_\tau}) \) (i.e., it could be of the order of the tree-level mass)\(^{13}\). Thus, even though two neutrinos are massless here, all neutrinos are massive in a more complete calculation (which is not necessary for our purposes). We would also like to point out that in four-family models, which are constrained by the \( Z \)-width to have \( m_{\nu_4} \gtrsim 45 \) GeV, \( \sin \zeta \) is not constrained by the tree-level neutrino mass (which could partially account for \( m_{\nu_4} \)) but by the size of the corresponding loop masses for the ordinary neutrinos (as well as by other phenomenological considerations).

\(^{13}\)The Yukawa origin of this mass is the same as in the models of Ref. [12,13]. Since renormalization effects are neglected there, they have \( \mu_L \equiv 0 \) and no tree-level or \( D \)-vertex one-loop masses as in the general case.
FIG. 1. The tree-level neutrino mass as a function of the alignment (low-energy) order parameter \( \sin \zeta \) for \((\tan \beta, M_2/\text{GeV}) = (2,200), (6,200), (6,500), (30,200), (30,500)\). For simplicity we assume the gaugino-unification relation \( M_1/M_2 = 5/3 \times g_1^2/g_2^2 \). The \( SU(4) \) directions and \( \mu \) are taken to be free parameters. No radiative symmetry breaking is yet required.

### IV. THE SCALAR POTENTIAL AND DYNAMICAL ALIGNMENT

In a general lepton number violating extension of the MSSM the scalar neutrinos (sneutrinos) will typically acquire vev’s so that lepton number is broken both explicitly and spontaneously. Containing all lepton number violation in \( W_M \) (as in our case) is, however, sufficient to spontaneously break lepton number. (In this section lepton number refers to the high-energy lepton number.) Below, we will organize the Higgs-lepton scalar potential and its minimization equations in a convenient way, and show that indeed the sneutrinos acquire vev’s. We would then list the conditions for achieving the required dynamical alignment between \( \mu_\alpha \) and \( \nu_\alpha \), and show that they are satisfied in a large class of models, but not in generic grand-unified theories and in models with an intermediate-scale right-handed neutrino. We will also comment on perturbativity constraints on the Yukawa couplings (which are different than in the MSSM). For simplicity, we will discuss models with only the third generation of quarks and leptons. The generalization, however, is straightforward. (Renormalization-group scaling of general broken \( R_P \) extensions of the MSSM were discussed recently in Ref. [44].)

It is convenient to define

\[
L_\alpha = L(\cos \alpha, \sin \alpha), \quad (12a)
\]

\[
\mu_\alpha = \mu(\cos \gamma, \sin \gamma), \quad (12b)
\]
\[ B_{\alpha} = B(\cos \delta, \sin \delta), \] 

(12c)

and \( \tan \beta = \nu_2/|\langle L^0 \rangle| = \nu_2/\nu_1 \) as before. (Note that \( B_{\alpha} \) has a squared mass dimension.) The selection rules of Section II typically imply lepton number conservation by dimensionless couplings of the low-energy Kahler potential. Thus, the scalar masses \( m_{\alpha\beta}^2 L^\alpha L^{\beta*} = \mu_{\alpha\beta} L^\alpha L^{\beta*} \)

and \( B_{\alpha} L^\alpha H_2 \) are the only source of explicit lepton number violation in the scalar potential. (The Yukawa \( F \) terms must involve, in our case, charged degrees of freedom, and thus, vanish at the minimum.) Hence, the Higgs-lepton scalar potential can be written as a straightforward generalization of the MSSM Higgs scalar potential, i.e.,

\[ V(L_\alpha, H_2) = m_1^2 L^2 + m_2^2 H_2^2 + m_3^2 (LH_2 + h.c.) + \frac{1}{8}(g_1^2 + g_2^2)(\langle H_2^0 \rangle^2 - \langle H_0^0 \rangle^2)^2 + \Delta V, \] 

(13)

where \( \Delta V \) is the one-loop correction (that is included in our numerical procedures\[^{14}\]) and

\[ m_1^2 = m_{H_1}^2 \cos^2 \alpha + m_{L_\tau}^2 \sin^2 \alpha + \mu^2 \cos^2(\alpha - \gamma), \] 

\[ m_2^2 = m_{H_2}^2 + \mu^2, \] 

\[ m_3^2 = B \cos(\alpha - \delta), \] 

(14a, b, c)

where on the right-hand side \( m_i^2 \) is the soft supersymmetry breaking squared mass of scalar \( i \).

Note that the (tree-level) scalar potential (13) has, in the absence of explicit lepton number breaking (\( \gamma = \delta = 0 \)), the unbounded direction \( m_{L_\tau}^2 + (1/8)(g_1^2 + g_2^2)(\langle H_2^0 \rangle^2 - \langle H_0^0 \rangle^2)^2 < 0 \)\[^{14}\], as well as a flat direction \( \langle L^0 \rangle^2 - \langle H_2^0 \rangle^2 = 0 \) (i.e., \( \tan \beta = 1 \)). The MSSM \( D \)-flat direction \( \langle L^0 \rangle^2 - \langle H_2^0 \rangle^2 = 0 \) is also relevant here and could also lead to a flat direction (one could eliminate \( m_{L_\tau}^2 \) in that case by redefinitions of \( m_{H_1}^2 \) and \( m_{H_2}^2 \)). Lepton number could be spontaneously broken in the case of a flat direction, even if it is not broken explicitly. This possibility was discussed in the context of supersymmetric Majoron models\[^{14, 15, 17}\]. We will exclude these directions, which are difficult to realize once RSB is included, from our analysis.

It is convenient to minimize \( V(L_\alpha, H_2) \) with respect to \( \langle L^0 \rangle = \nu_1, \langle H_2^0 \rangle = \nu_2 \), and the angle \( \alpha \). Two of the minimization equations reduce to those of the MSSM, i.e.,

\[ \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1} = \frac{1}{2} M_Z^2, \] 

\[ m_3^2 = -\frac{1}{2} \sin 2\beta \left[ m_1^2 + m_2^2 \right]. \] 

(15, 16)

In particular, At the minimum \( V = V_{\text{MSSM}} = -\frac{1}{8} M_Z^2 \nu^2 \cos^2 2\beta \), independent of \( \alpha, \gamma, \delta \). The third equation reads

\[ \text{We include only corrections proportional to the } t \text{ and } b \text{ Yukawa couplings, which in our case, are given by the corresponding corrections in the MSSM.}\]
\[
\left( m_{H_1}^2 - m_{L_r}^2 \right) \sin 2\alpha + \mu^2 \sin 2(\alpha - \gamma) \cos \beta + 2B \sin(\alpha - \delta) \sin \beta = 0. \tag{17}
\]

For the boundary conditions
\[
m_{H_1}^2 = m_{L_r}^2, \tag{18a}
\]
\[
\gamma = \delta, \tag{18b}
\]
there is only one $SU(4)$ direction in field space which is determined by the angle $\gamma$. Hence, $\nu_\alpha$ must align along that direction and the solution to (17) (for $\mu \neq 0$, $B \neq 0$, and away from the flat direction) is given by $\alpha = \gamma$. The alignment is achieved dynamically in that case. Note that the $\mu_\alpha$ is a parameter vector in the low-energy theory and the $SU(4)$ symmetry is broken explicitly and spontaneously down to (the low-energy lepton symmetry) $SU(3)$, and (the rotated low-energy) $\tilde{H}_1$ contains the seven pseudo Goldstone bosons with mass $\propto \mu$. Unlike models with no explicit breaking (but only spontaneous breaking along the flat direction) there are no light Majorons.

The boundary conditions (18) are often found in models for the soft supersymmetry breaking terms. The second condition is trivially realized if $B_\alpha = b_{\mu_\alpha}$ (i.e., $B$-proportionality), as is also often the case. The crucial point (which was also noted independently by Hempfling [41]) is that if $m_{H_1}^2 = m_{L_r}^2 = m_0^2$ and $B_\alpha = b_{\mu_\alpha}$ at some Planckian scale, then the deviations from these relations at the weak scale due to renormalization group scaling are proportional to the square of the $b$ Yukawa coupling $h_b$, i.e.,
\[
\frac{\partial m_{H_1}^2}{\partial \ln Q} = \frac{\partial m_{L_r}^2}{\partial \ln Q} + \frac{h_b^2}{8\pi^2} [m_{H_1}^2 + m_{Q_3}^2 + m_{D_3}^2 + A_b^2], \tag{19}
\]
where $A_b$ is the trilinear soft mass parameter $A_b H_1 Q_3 D_3$. Thus, deviations from universality are strongly suppressed (unless $\tan \beta \gg 1$, i.e., $h_b \sim 1$, or if the soft parameters $m_{H_1}^2 + m_{Q_3}^2 + m_{D_3}^2 + A_b^2 \gg m_{L_r}^2$), but are still sufficient to generate the small neutrino mass.

This is the case in the (extended) MSSM with the so-called universal boundary conditions at the grand scale (i.e., $m_{\text{scalar}}^2 \equiv m_0^2$, $A_i \equiv A_0$, and $M_{\text{gaugino}} \equiv M_{1/2}$). We solved the system (13) – (17) iteratively in our model for various universal boundary conditions and confirmed the above assertions and claims. We also included in the numerical procedures the one-loop radiative corrections (see above) and the corrected $b$ and $\tau$ Yukawa couplings (see below).

Unlike in the MSSM, $m_b \neq h_b \langle \bar{H}_1^0 \rangle = h_b \nu \cos \alpha$, but rather $m_b = h_b \langle H_1^0 \rangle = h_b \langle \bar{H}_1^0 \rangle \cos \alpha$, leading to the weak-scale perturbativity constraint (requiring $h_b < 1$, i.e., that it is below its quasi fixed-point)
\[
\cos \alpha \cos \beta > \frac{m_b}{174 \text{ GeV}} \sim \frac{1}{58}. \tag{20}
\]
It can also be written as $\tan \beta < 58 \cos \alpha$. Eqs. (13) and (20) imply that the models are realized more naturally for small and moderate values of $\tan \beta$ (while typical grand-unified

\[15\] Given $\beta$ and $\gamma$, we use Eq. (13) to solve for $\mu$, Eq. (16) to solve for the proportionality parameter $b$, and Eq. (17) to solve (numerically) for $\alpha$. 

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models with right-handed neutrinos are realized for large $\tan \beta$ \cite{HN}. Phenomenological implications, which are discussed in the following sections, constrain $\tan \gamma \sim \tan \alpha$ from above, i.e., $\tan \alpha \lesssim 1$, which is stronger than any constraint that one could derive from (20). Also, unlike in the MSSM, the leptonic Yukawa couplings $h_{\tau, \mu, e} \neq m_{\tau, \mu, e}/\nu \cos \beta$ but are found by requiring the three light eigenstates of the charged (color singlet) fermion mass matrix to have the correct masses, i.e., mixing effects can slightly modify their values. [Since the leptonic Yukawa couplings have nearly flat renormalization curves (for not too large $\tan \beta$), this effect could be important when considering relations between Yukawa couplings at Planckian scales.] We account for these effects in our numerical procedures.

In Fig. 2 we show the neutrino mass as a function of $\tan \beta$ for $\tan \gamma = 0.1, 0.5, 0.9$. The corresponding alignment, i.e., comparison of (the output) $\tan \alpha$ and (the input) $\tan \gamma$ is shown in Fig. 3 for $\tan \gamma = 0.1$ (bullets) and 0.9 (squares). In Fig. 4 we further explore the parameter space for $\tan \gamma = 0.1$. By observation, we can draw the following conclusions, some of which are somewhat surprising:

- The typical suppression factor is of $\mathcal{O}(10^{-5})$, i.e., $m_\nu \sim \mathcal{O}(1 \text{ MeV})$. However, for $\tan \beta \gtrsim 5$ one finds suppression as strong as $\mathcal{O}(10^{-9})$ [or even $\mathcal{O}(10^{-10})$], i.e., $m_\nu \sim \mathcal{O}(10 - 100 \text{ eV})$. The functional dependence on the soft parameters is complicated. In general, we find that $m_0 \gg M_Z$ and $M_{1/2} < m_0$ are slightly preferred. The former leads to heavy scalars which are less sensitive to the renormalization-group corrections,
while the latter suppresses the $O(h_b^2)$ contributions in [19] [which is a more important consideration than the large $M_2 \sim 0.8 M_{1/2}$ effect in (11)].

- The smaller values of $\tan \gamma$ do not imply necessarily smaller neutrino masses and vice versa.

- The alignment condition is indeed satisfied poorly for $\tan \beta \gg 1$, in contradiction to its excellent satisfaction for small and intermediate values of $\tan \beta$. This leads to a relatively wide (narrow) range for $m_\nu$ in the former (latter) case.

- The poor alignment for $\tan \beta \gg 1$ and the suppression of $\nu_1$ are competing effects which allow to realize small neutrino masses for large $\tan \beta$ in some cases.

Let us stress that our results hold in a larger class of models, i.e., in models with universality of the $L_\alpha$ soft masses and the proportionality $B_\alpha \propto \mu_\alpha$. The lepton-Higgs universality is guaranteed if the Kahler potential is (approximately) invariant under the $SU(4)$, e.g., in string models it would require that all $L_\alpha$ components have a unique set of modular weights. No universal boundary conditions for any other fields are required. In grand-unified models, however, the soft parameters are scaled at Planckian scales according to renormalization group equations which are invariant under the grand-unified symmetry and which are determined by its superpotential couplings. In generic models the leptonic and Higgs fields are embedded in different representations (often with different dimensions) and have different Yukawa interactions with sometimes large Yukawa couplings. For example, $H_1$ would typically couple to the large Higgs representation that break the unified symmetry and/or render its $SU(5)$ partner, the color triplet, heavy. Thus, the Higgs and lepton soft masses are
subject to different scaling laws at large scales. They could also receive different contributions from $U(1)$ $D$-terms\footnote{If the $U(1)_R$ is gauged the respective (residual) $D$-terms could lead to Higgs-lepton non-universality. Similar problems would also arise in the case of low-energy stringy $U(1)$’s, unless the $D$-terms vanish or are negligible. Thus, global symmetries are better suited for our purposes.} if the grand unified symmetry is broken to some group $G \times U(1)$ \[\text{[which is not the hypercharge } U(1)\text{]}\] between the Planck and unification scales. The $SU(4)$ symmetry would be broken in $V(L_\alpha, H_2)$ in more that one direction and $\nu_\alpha$ and $\mu_\alpha$ would not be aligned. For example, we find that in the $SU(5)$ models of Ref. \cite{19} the scaling violation of the $SU(4)$ due to the $\lambda H_1 \Sigma_2 L H_2$ term destroys, in most cases, the dynamical alignment.

Similarly, a right-handed neutrino, $N$, will introduce the Yukawa terms $h_N H_2 L N$, and unless $h_N \ll 1$ for all three generations (but in unified models typically $h_N \approx h_U$) or $\mu_N \approx M$, it would again introduce new $SU(4)$ breaking. Thus, it is difficult to reconcile supersymmetric neutrino masses (with dynamical alignment) and right-handed neutrino seesaw models. (Similar arguments hold for an arbitrary high-energy $W_{Y_{LNV}}$.)

In general models for the soft terms one could always tune the parameters to satisfy (17). For example, if $\mu = \mathcal{O}(1 \text{ GeV})$ then $m_{3} = \mathcal{O}(M_\nu)$ may not be proportional to $\mu$ (see Example B of Section II). It could lead to large mixing in the scalar sector, but would require some tuning of parameters (or to invoke extended symmetries) in order to still guarantee $m_\nu \ll \mu$. We do not find this possibility attractive and do not pursue it here.

V. COLLIDER AND LABORATORY IMPLICATIONS

Having established the smallness of the neutrino mass and its consistency with the laboratory bound $m_\nu, \lesssim 23 \text{ MeV}$ \cite{112} (and in many cases with the cosmological bound $m_\nu, \lesssim 100 \text{ eV}$ \cite{113}) in the previous sections, we will not discuss other phenomenological aspects of the models in detail but only survey possible constraints. We will show that in our framework the constrained couplings and amplitudes are proportional to some power of $m_\nu/M_Z$ or to small (MSSM) Yukawa couplings, and thus, are evaded in large sections of the parameter space. In particular, due to the large number of parameters, it is difficult to efficiently constrain the models (until supersymmetric particles and their decays are observed and characterized): See, for example, Ref. \cite{114} for detailed discussions of constraints from rare decays, weak current universality, etc., on these and on more general models of lepton number violation.

The discussion is simplified once again if one performs a low energy $SU(4)$ rotation. For our purposes here it is convenient to define the low-energy Higgs field $\tilde{H}_1$ such that $\langle \tilde{H}_1 \rangle = v_1$, and the three leptonic fields $\tilde{L}_{\tau, \mu, e}$ correspond to the three perpendicular directions in field space. In the degenerate limit $\sin \zeta \to 0$ one then has $\mu_\alpha L^\alpha H_2 \to \mu \tilde{H}_1 H_2$. All fermion and scalar mass matrices are block diagonal in the gauge-(low-energy) Higgs fields and in the (low-energy) leptonic fields. The rotated neutral and charged states constitute the

\footnote{Note that any constraints imposed there on $\nu_{L\tau}$ etc., apply, in our case, only to the rotated vev’s after the SM lepton number is defined (see below).}
“physical” lepton and Higgs doublets consistently, and one can define the (low-energy) SM lepton number in this basis. (If $\mu_L \gtrsim \mu_H$ then the discrepancy between low and high energy definition of leptons is maximized, leading to possible constraints on their ratio.) The rotation generates $W_Y^{LNV}$ [see $\text{(3)}$] and thus, $W_Y \neq W_Y^{MSSM}$ and $W$ has an accidental $Z_3$ (baryon parity $[16]$) symmetry (but only at the renormalizable level).

Since $\sin \zeta$ is small but not exactly zero (the alignment, and in particular, if achieved dynamically, is not scale invariant and is not expected to be exact) there are perturbations to the naive limit, and some mixing between the (low-energy) leptonic and gauge-Higgs sectors arises, e.g., the $\tau$ could contain a small admixture of the wino and the $Z$ could have off-diagonal couplings to a neutrino and a gaugino. However, the effects are of the order of magnitude of the neutrino mass suppression factor $\sin \zeta \sim \sqrt{m_\nu/\nu}$ to some power. Note that since one of the loop-induced neutrino masses could be of the order of the tree-level mass, experimental constraints should be applied to the mixing parameters extracted from the one-loop mass matrices.

Other constraints result from the new Yukawa interaction contributions to flavor changing neutral (FCNC) and charged currents. The superpotentials $W_Y^{LNV}$ and $W_Y^{MSSM}$ are related by a rotation and $W_Y^{LNV} \propto W_Y^{MSSM}$. As a result, $h_D$ and $h_D^{LNV}$ are diagonalized simultaneously (and $h_E^{LNV}$ has two generation indices). In particular, the smallness of relevant entries in $h_D^{LNV}$ is directly related to their smallness in the $h_D, E$ matrices. The only new contributions to hadronic FCNC arise from loops, and in many cases (e.g., $B$ meson mixing) are suppressed as in the MSSM $[50]$. (There are some differences due to the different masses of the Higgs and the leptonic fields that propagate in the loops.) In addition, the amplitudes for leptonic FCNC processes, e.g., $\mu \to e \gamma$, are typically proportional in our case to the mixing angle $\sin \zeta$ and/or to the electron mass $[\text{13}]$ (and, in general, depend sensitively on $M_{SUSY}$). Thus, they also lead to only mild constraints.

Charged current processes and constraints, e.g., weak current universality, neutrinoless double-$\beta$ decay, etc., are induced at tree and loop levels. They typically constrain $h_D^{LNV} \lesssim O(1 \times m_f/\nu)$, where $m_f$ is the relevant SM fermion mass (or lead to even weaker constraints). Given the origin of $W_Y^{LNV}$ in our case, these constrains are typically satisfied and could lead at most to an upper bound on the ratio $\mu_L/\mu_H \sim \nu_L/\nu_H \approx h_D^{LNV}/h_D, E$. (Note that not all $h^{LNV}_{ijk}$ combinations are present in the basis that $h_D$ is diagonal and that we do not specify any textures for the MSSM Yukawa matrices.) Also, all constraints scale with the inverse mass of the relevant (virtual) superpartner that mediates the process, and are further weakened as the mass parameters reach the few hundred GeV mark. It is reasonable, however, to require that the ratio $\mu_L/\mu_H \sim \nu_L/\nu_H \lesssim 1$ so that the couplings in $W_Y^{LNV}$ are of the same order of magnitude (or smaller) as the couplings in $W_Y^{MSSM}$.

We conclude that a dedicated analysis and searches could constrain the order parameters of the model: $\sin \zeta$ (i.e., deviations from the alignment) and the ratio $\mu_L/\mu_H \sim \nu_L/\nu_H$, which determine the gauge-lepton mixing and the size of the couplings in $W_Y^{LNV}$, respectively. However, it will lead to only mild restrictions on the model parameter space, and it is not called for at present (given our poor knowledge of the supersymmetric spectrum parameters).

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$[18]$ For large $\tan \beta$, however, $\nu_1 \to 0$ allows for larger values of $\sin \zeta$. In that case, experimental constraints could be effective in constraining the models.
The former would then constrain the size of neutrino masses and thus, serve in the future as a consistency test of the model relating the neutrino and supersymmetric spectra. It could also constrain $\tan \beta$ (see above footnote). The constraints on the ratio $\mu_L/\mu_H$ are, on the other hand, a function of $\tan \beta$ (recall that $\mu_H h^{\text{LNV}}_{D,E}/\mu_L \approx h_D, E = m_{d,e}/\nu \cos \beta$, where $m_{d,e}$ is the relevant fermion mass).

The ratio $\mu_L/\mu_H$ and the size of $h^{\text{LNV}}$ have also important implications for collider signatures from exotic superpartner decays [14,51–54], in particular, from the decays of lightest superpartner (the LSP). For example, there could be a single production of a scalar lepton in hadron colliders which would then decay to two jets or two jets and either like-sign dileptons or (degraded) missing energy (for example, see Ref. [52]). Such decays serve, in general, as a test of all supersymmetric lepton number violating models (assuming that supersymmetry is established and characterized by experiment) in both hadron and electron colliders.

Of particular interest is the case in which the scalar neutrino $\tilde{\nu}$ (that can be singly produced) is the LSP. The gauginos and Higgsinos are then heavy and the sneutrino would decay exclusively to two jets (assuming $h_D \gg h_E$). Its charged $SU(2)$ partner, which has a similar mass, will also decay preferably to jets. Superpartner decays are characterized in that case by a $\geq 2j$ signal. In the case of the Tevatron, for example, the LSP dijet signal [54] could be only one order of magnitude below the corresponding QCD signal [51] (but would be further suppressed if the couplings are small). However, the enhancement of the total dijet cross-section is more significant (and is smeared over a larger energy range) if there are more than one relevant threshold [e.g., its charged $SU(2)$ partner and possibly scalars of all three lepton doublets]. These observations may require further attention [55] in view of recent indications of enhancements in the dijet inclusive cross section [55] and mass spectrum [56] measurements. In particular, the situation here is quite different than in $R$-parity conserving supersymmetric theories, where any new contributions to the inclusive dijet cross section are only at the loop level and were shown recently to be small or negative [57].

\[ \sigma(d_i \bar{d}_i \rightarrow \tilde{\nu} \rightarrow d_j \bar{d}_j) = \frac{4\pi}{9} \frac{\Gamma_i \Gamma_j}{(\hat{s} - m_{\tilde{\nu}}^2)^2 + m_{\tilde{\nu}}^2 \Gamma_{\text{total}}^2}, \]

where $\hat{s}$ is the square of the parton center of mass energy and, in our case,

\[ \Gamma_i = \frac{3}{16\pi} \left( \frac{\mu_L m_{d_i}}{\mu_H \nu \cos \beta} \right)^2 m_{\tilde{\nu}}. \]

For simplicity, one can assume that the total width $\Gamma_{\text{total}} \sim \Gamma_j \sim \Gamma_b$.

Similar observations hold in the case of baryon number violation via the operator $h^{\text{BNV}} U D D$ which is only weakly constrained (in the absence of lepton number violation), but is forbidden here.
VI. ASTROPHYSICS AND COSMOLOGY

We concluded above that no significant constraints arise on the model from laboratory and collider data, but stressed the unconventional decays and signatures of the supersymmetric particles. We also pointed out that the LSP is not expected to be stable. Here, we will survey some astrophysical and cosmological aspects of the model. We also elaborate on the LSP and neutrino decays. In particular, we show that the LSP (and thus, all other MSSM superpartners) is cosmologically irrelevant in any supersymmetric model with explicit (weak-scale) lepton number violation which is the source of the neutrino mass. Radiative neutrino decays are shown to lead to only moderate constraints on the order parameter \( \mu_L/\mu_H \) (unless \( \tan \beta \) is large). We do not consider here some issues which are more model dependent, e.g., that of the primordial baryon asymmetry in the universe (but see, for example, \[58,59\]).

A. Neutrino oscillations: Current observations imply not only massive neutrinos but also neutrino mixing, so that the neutrinos could oscillate \[1\]. Indeed, we find model dependent mixing between the three neutrinos. It was recently pointed out \[11,53\] that the mass and mixing patterns in the models with supersymmetric neutrino masses are relevant for the solution of the solar and atmospheric neutrino problems \[1\]. Most importantly, we would like to stress that the patterns that arise could be quite different from those in typical see-saw and similar models. They are not proportional to any Yukawa couplings and the hierarchy among the neutrino masses is not related to either the hierarchy among the quarks or the charged leptons.

B. LSP decays: Since \( R_P \) is effectively broken to the baryon parity in our models, the LSP, i.e., the lightest neutralino or scalar, is not stable but decays \[11,4,13,50\] through small mixing (if it is a neutralino), and more probably, via \( h^{LNV}_D \) and \( h^{LNV}_E \) interactions. In order to render the LSP stable on a cosmological time scale (\( \tau_{LSP} \sim 10^{17} \) sec) one would have to fine tune

\[
h^{LNV}_L \sim h \frac{\mu_L}{\mu_H} \ll \mathcal{O} \left( \frac{10^{-6}}{\sqrt{\tau_{LSP} (\text{sec})}} \right) \sim \mathcal{O}(10^{-14})
\]

(21)

for a neutralino LSP (and even more so for a scalar LSP), and we neglected the decay via mixing which is further suppressed. (See Ref. \[46\] for life time formulae). It would imply, in our case, an unacceptable fine tuning of \( \mu_L/\mu_H \). But moreover, even if the neutrino masses are generated by only Yukawa loops \[12,13\], they would be essentially massless given the constrain (21), which is thus unacceptable. We therefore conclude that the LSP decays on short time scales and is cosmologically irrelevant. (We find that a neutralino LSP could, however, still be stable in our model on collider scales.) Note that this argument cannot be cured by any symmetry that allows for \( W^{LNV} \neq 0 \) since it is sufficient to have a single lepton number violating Yukawa coupling that does not vanish.

C. Neutrino mass and decays: The neutrino energy density and over-closure of the universe considerations constrain \( m_{\nu_e} \lesssim 100 \text{ eV} \) \[13\] (unless it decays on cosmological time scales, i.e., \( \tau_\nu \lesssim 1 \) sec, in which case one could also have \( m_{\nu_e} \sim \mathcal{O}(10 \text{ MeV}) \) \[61\]). The neutrino decays, similarly to the LSP, via mixing to neutrinos (i.e., \( \nu \to 3\nu \)) and via its small one-loop magnetic moment operator \( \propto (h^{LNV})^2 \) (i.e., \( \nu \to \nu \gamma \)). The former may be
beneficial in the case of a $\mathcal{O}(\text{MeV})$ neutrino but is strongly suppressed. The latter is the dominant mode with (adopting the calculation of Ref. [13,62] to our case)

$$\tau_\nu \sim 10^7 \left( \frac{\mu_H}{\mu_L} \right)^4 \left( \frac{1 \text{MeV}}{m_\nu} \right)^3 \left( \frac{M_{\text{SUSY}}}{100 \text{GeV}} \right)^6 \text{sec},$$

(22)

where we assumed $h_L^{\text{LNV}} \sim h_b^{\text{LNV}} \sim 10^{-2} \times (\mu_L/\mu_H)$. [More generally, $10^7(\mu_H/\mu_L)^4$ is replaced by $10^{-1}(h_L^{\text{LNV}})^{-4}$] A comprehensive analysis of such decays was presented in Ref. [62], leading to the constraint $\tau_\nu(\text{sec}) \gtrsim 10^{29} m_\nu(\text{MeV})$ (for $m_\nu \lesssim 100 \text{ eV}$). From (22) one has the corresponding constraint (for $m_\nu \lesssim 100 \text{ eV}$),

$$\left( \frac{\mu_H}{\mu_L} \right)^4 \left( \frac{1 \text{MeV}}{m_\nu} \right)^4 \left( \frac{M_{\text{SUSY}}}{100 \text{GeV}} \right)^6 \gtrsim 10^{22}.$$  

(23)

Again, no significant constraints arise on the order parameter $\mu_L/\mu_H$. (Note that for a given neutrino mass only the combination $M_{\text{SUSY}}^6(\mu_H/\mu_L)^4$ is constrained. For large $\tan \beta$, i.e., $h_L^{\text{LNV}} \to 1$, the constraints could be significant.)

D. Dark matter sources: One might have hoped that the model would provide a closed framework in which the primary candidates for cold and hot dark matter, the LSP and the neutrino, respectively, are related by mixing and/or decay chains and thus, have correlated abundances. Such a scenario could make, for example, critical density models for the universe more plausible. However, we have concluded above that the LSP is cosmologically irrelevant and the only (hot) dark matter candidates in the model are the two heavier neutrinos (which in some sense are the actual LSP). This is somewhat a disappointing aspect of the model. Nevertheless, there could still be other sources for cold dark matter, e.g., light components of the axion superfield (as was proposed in the case of a Peccei-Quinn axion [63]). Lastly, it is interesting to note that our models could reverse the generic situation in the $\mu = \mathcal{O}(1 \text{ GeV})$ lepton number conserving models which typically suffer from a slow LSP annihilation rate [28].

VII. POSSIBLE FAMILY DEPENDENCES

Throughout this work we have assumed universal $R$-charge assignment to all matter fields. As a result, the vector $\mu_\alpha = (\mu_H, \mu_L(1,1,1))$ was $SU(3)$ symmetric. There are many ways in which the $SU(3)$ could be broken, most obviously so, if the $R$-charge assignment is family dependent.

It was suggested that a family dependent assignment may be needed in order to cancel the anomalies of a gauged $U(1)_R$ theory and that it is related to the fermion mass problem (but, as we noted above, it could lead to dangerous $D$-terms). In Ref. [34] Yukawa operators involving the first two families were forbidden by the assignment $R(\phi_1\phi_2\phi_3) > 2$ but the possibility of dynamical couplings was not discussed. A more ambitious program was pursued in Ref. [34] where it was suggested that all Yukawa couplings (but not $\mu_\alpha$) are dynamical variables that depend on the $U(1)_R$ breaking parameter $\theta$ (in a similar manner to the horizontal symmetry approach [65]). However, being an $R$-symmetry complicates the anomaly cancelation equations and Ref. [14] chose not to consider (and satisfy) the complete
set of equations, in particular, those involving hidden fields. Given the ambiguous status of the gauged $U(1)_R$, we chose the simplest possible $U(1)_R$ charge assignments and did not consider the MSSM Yukawa couplings as dynamical variables.

Nevertheless, it is very likely that the solution of the fermion mass problem, and in particular, if it relies on some symmetry principles and/or if the Yukawa couplings are dynamical variables, would break explicitly the $SU(3)$ symmetry in $W_M$ as well, e.g., by forbidding certain field combinations. It could also break the Higgs-lepton soft mass universality from a non-minimal Kahler functions or $D$-terms, in which case the dynamical alignment would fail. However, the latter breaking is constrained (from FCNC) to $SU(2) \times U(1)$ type breaking of the $SU(3)$, and one can impose the additional condition that breaking the above $SU(3)$ by the leptonic soft-terms is negligible. For example, this would be the case if the $SU(3)$ is broken mainly in the right-handed lepton singlet sector (but then $W_M$ is still $SU(3)$ symmetric), or if it is a global or discrete symmetry (and the Kahler function is, e.g., minimal). Therefore, we do not consider the $SU(3)$ symmetry as a necessary result of our model.

The above discussion also affects the phenomenology of the model. In particular, if $\mu_{\alpha \gamma} \ll \mu_{\alpha L}$ and only $\tau$ number is (significantly) broken, then the constraints on $\mu_L/\mu_H$ are quite weak. (Models in that spirit were discussed, e.g., in Ref. [66].)

VIII. SUMMARY AND CONCLUSIONS

In conclusion, we have shown that the neutrino mass could arise from a generalized supersymmetric mass term $\mu_\alpha L^\alpha H_2$ in the weak-scale superpotential, on the condition that Higgs-lepton universality in the scalar potential is broken only weakly. This is indeed the situation, for example, in universal models for the soft supersymmetry breaking parameters.

The restricted form of $R$-parity breaking in the model was realized, as an example, in the framework of a spontaneously broken $U(1)_R$ symmetry, which is often present in models of dynamical supersymmetry breaking. The symmetry framework provides also a solution to the generalized $\mu$-problem while suppressing lepton number violation in the Yukawa interactions and in the Kahler potential. In hidden sector models the $U(1)_R$ scale is an intermediate scale. The neutrino mass is related to the intermediate scale physics in that case in a very different fashion than in see-saw models. In particular, it could be a result of supersymmetry breaking. It offers a new mechanism for the neutrino mass generation in supergravity and superstring theories, which does not require to introduce any intermediate-scale see-saw structure.

The neutrino mass suppression is achieved dynamically (and is sensitive to $\tan \beta$). The resulting mass range for the neutrino agrees with not only the weaker collider limits but also with cosmological considerations. The dynamical suppression is typically (but not always) destroyed in grand-unified models, models with an intermediate right-handed neutrino, and in models with arbitrary lepton-number violating Yukawa couplings, once renormalization effects are taken into account.

We were able to define the SM lepton number at the weak-scale (after all symmetries were broken). The resulting Yukawa superpotential has an accidental $Z_3$ baryon-parity symmetry, but the Yukawa couplings are not arbitrary and, in general, satisfy all experimental constraints. One can still constrain the models’ two order parameters. However, given the
above and the large number of parameters in supersymmetric models, the constraints are mild (unless $\tan \beta \gg 1$).

While considering the phenomenological implications of the models, we noted that:

1. The patterns of neutrino masses that arise could be quite different from those that arise in typical see-saw models.

2. The LSP is not stable on cosmological time-scales, an observation which holds more generally in supersymmetric models with weak-scale origin of the neutrino masses.

3. If the LSP is a scalar neutrino it most probably decays in the detector, enhancing jet production and leading to a typical $\geq 2j$ signal.

These observations call for consideration of unorthodox scenarios when considering phenomenological implications of supersymmetric models.

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4. Same as in Fig. 2 except for $\tan \gamma = 0.1$ and the neutrino mass is calculated and shown for discrete choices of $2 \leq \tan \beta \leq 30$. 