High-Energy Scattering vs Static QCD Strings

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Abstract. We discuss the shape of the interaction region of the elastically scattered protons stipulated by the high-energy Pomeron exchange which turns out to be very similar with the shape of the static string representing the confining QCD flux tube. This similarity disappears when we enter the LHC energy region, which corresponds to many-Pomeron exchanges. Reversing the argument we conjecture a modified relationship between the width and the length of the confining string at very large lengths.

PACS. 11.55.Jy Regge formalism – 12.38.Aw General properties of QCD (dynamics, confinement, etc.) – 11.15.-q Gauge field theories – 11.40.Nn Regge theory, duality, absorptive/optical models – 11.25.Wx String and brane phenomenology – 11.10.Jj Asymptotic problems and properties – 11.15.Ha Lattice gauge theory

1 Introduction

The basic element of the Regge framework to describe the high-energy hadron scattering is the well-known expression

\[ A(s,t) = \beta(t) \left( e^{-i\pi s} \right)^{\alpha(t)} + \ldots \]  

(1)

where \( \alpha(t) \) is the leading (Pomeron) Regge trajectory. The average transverse (w.r.t. the collision axis) size of the interaction region is given by the formula

\[ \langle b^2 \rangle = 4\alpha'(0) \ln(s) + \ldots \]  

(2)

Thus, in the Regge-pole framework the transverse size of the interaction region widens with energy.

It is not difficult to obtain the average longitudinal range of interaction which appears to be

\[ R = \langle z_1 - z_2 \rangle = \langle \partial (\text{scattering phase}) / \partial p_{\parallel}^{\text{out}} \rangle = \pi \alpha'(t) > \sqrt{s}/2 + \ldots \]  

(3)

where \( z_{1,2} \) are longitudinal coordinates of the scattered hadrons in the c.m.s., \( \text{scattering phase} = -\pi\alpha(t)/2, \ t = -2p^2 + 2p_{\parallel}^{\text{out}}, \ s = 4p^2 + 4m^2 \).

In linear approximation for the trajectory, \( \alpha(t) = \alpha(0) + \alpha'(0)t \), we obtain

\[ R = \pi\alpha'(0)\sqrt{s}/2 + \ldots \]  

(4)

We get the following relationship between transverse \( \langle b^2 \rangle \) and longitudinal \( R \) sizes of elastic hadron scattering (at large \( R \))

\[ \langle b^2 \rangle = 8\alpha'(0) \ln(R) + \ldots \]  

(5)

So the transverse size of the interaction region increases logarithmically with the growth of the longitudinal interaction range. Pictorially at large \( R \) it looks as a prolate cigar-like region:

![Interaction region](image)

Fig. 1. Interaction region.

2 The Shape of Static Strings

In paper \[9\] the question of the thickness of a static string spanned by infinitely heavy quark-anti-quark pair was addressed. Trying different frameworks the authors came to the conclusion that (the square of) the size of the transverse quantum fluctuations of a string-like confining flux grows logarithmically with the string length \( R \).
This slope is not very far from the values of the secondary Regge trajectories which are usually associated with open \( qq \) strings. So if the high-energy scattering were defined by the meson trajectories we could conclude that the interaction region is being stretched with the energy growth as an open string.

We, however, know that actually it is not the meson (secondary Reggeon) but the Pomeron trajectory which dominates the high-energy behaviour. In this case the coefficient in front of \( \ln(R) \) is four times smaller. How can it be explained?

Usually one argues that taking into account the closed string nature of the Pomeron we should get a twice smaller coefficient corresponding to \( \alpha'_P = 0.5 \) GeV\(^{-2} \). However the high-energy phenomenology supports in a very strong way even smaller slope:

\[
\alpha'_P \simeq 0.25 \text{ GeV}^{-2}
\]

If we assume that the closed (stretched) string shape follows Eq. (6) with 4 times smaller coefficient in front of \( \ln(R) \) then we obtain the following estimate for the Pomeron slope:

\[
\alpha'_P(0) \simeq 0.24 \pm 0.01 \text{ GeV}^{-2}.
\]

Actually the tension values of various kinds of closed strings in the lattice framework were considered in Ref. 11 with one of the results fairly close to that of high-energy phenomenology:

\[
\alpha'_P(\text{lattice}) \simeq 0.253(2) \text{ GeV}^{-2}.
\]

This is suggestive for the conclusion that the interaction region of high-energy hadron scattering clearly bears stringy features similar to the static string configurations in spite of their physically extraordinary difference: a highly dynamic phenomenon against the static one. In the case of scattering we deal with fluctuating average length while in Eq. (9) it is fixed parameter.

### 4 Breakdown of Reggeon-String Similarity or String Shape Modification?

We have to notice that Regge-Pomeron formula (6) begins to dominate at energies not less than the ISR energies \( (\sqrt{s} = 20 \pm 50 \) GeV\), this corresponds to the minimal interaction length \( R = 1.5 \text{ fm} \), which is exactly at the end of the lattice data at Fig. 2.

If we believe that the interaction region looks like a stretched string of non-zero transverse extent then we come to a striking conclusion that these extremely stretched configurations can survive without breaking up to monstrous lengths of order of 150 fm. This concerns high-mass \((\simeq 1 \text{ TeV/c}^2)\) stringy configurations.

According to the ratio \( \sigma_{el}/\sigma_{tot} \) measured by the TOTEM LHC Collaboration 11

\[
\sigma_{el}/\sigma_{tot}(\text{TOTEM, 7 TeV}) = 0.257 \pm 0.005
\]
this means that the probability of the long string-like configuration survival (before breaking out into the two elastically scattered protons) is of order of 25%. Such a phenomenon would mean that confinement is fairly important not only at low-energy hadron physics but at high-energy scattering as well.

Our arguments are heavily based on Eq.(7). However it ceases to be adequate at very high energy though remains a basic building block (eikonal) for the full unitary amplitude in the Regge-eikonal framework. Latest experiments at the LHC show that the width \( w^{2}_{\text{scat}} \) as function of \( R \) significantly deviates from the simple logarithmic dependence in the LHC energy region (see Fig. 3 below).

At ultra-high energies the relationship between \( w^{2} \) and \( R \) changes significantly as could be seen from the following asymptotic behavior:

\[
< b^{2} > (\text{ultra} - \text{high energy}) \approx 4 [\alpha_{F}(0) - 1] a_{F}'(0) [\ln(s)]^{2} + \ldots 
\]

Unfortunately we haven’t yet the analytic expression for the phase of the scattering amplitude in this case and can’t insist that the proportionality

\[
R \sim \sqrt{s}
\]

still holds. However to trace the trend we can use the asymptotic (actually at practically unavailable energies) expression for the scattering amplitude:

\[
T(s, t) = \frac{4\pi i R_{0}(s)}{\sqrt{-t}} J_{1}(R_{0}(s) \sqrt{-t}),
\]

\[
R_{0}^{2}(s) = 4\Delta_{F} a_{F}'(0) [\ln(s) - i\pi/2]^{2},
\]

\[
\Delta_{F} = \alpha_{F}(0) - 1.
\]

We get for the interaction region scales:

\[
R \approx \pi \alpha_{F}'(0) \Delta_{F} \sqrt{s} \ln(s) + \ldots,
\]

\[
< b^{2} > = 4\alpha_{F}'(0) \Delta_{F} [\ln(s)]^{2} + \ldots.
\]

If we try to assume that the similarity discussed above persist at higher energies (larger \( R \)) then we come to the following modification of the string width-length relationship:

\[
w_{\text{string}}^{2} = 2 \Delta_{F} \pi \sigma_{F} [\ln(R)]^{2} + \ldots,
\]

where \( \sigma_{F} \) is the tension of the string corresponding to the Pomeron while parameter \( \Delta_{F} \) can be only explained when considering the trajectory deviation from the linear behavior at small unphysical values of the string mass (see e.g. Ref. [13]). We have also to note that the value of the tension \( \sigma_{F} = 1/(2\pi \alpha_{F}'(0)) \) may differ from the “quasiclassical” tension of the straight-line string at high string masses.

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5 Conclusions

We have shown that the relationship of the transverse and longitudinal interaction ranges which follow from a one-Reggeon-exchange scattering amplitude is identical to that of the static confining string. Reversing the argument we conjecture on the basis of the Regge-eikonal asymptotic for the scattering amplitude that account of higher genera modify the dependence of the static string width dependent on its length leads to formula (19).

\[
T(s, t) = \frac{4\pi i R_{0}(s)}{\sqrt{-t}} J_{1}(R_{0}(s) \sqrt{-t}),
\]

\[
R_{0}^{2}(s) = 4\Delta_{F} a_{F}'(0) [\ln(s) - i\pi/2]^{2},
\]

\[
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\]

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