Exponential to ZIP and ZIP to exponential load model conversion: Methods and error

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Abstract
This paper presents several methods for performing two types of static load model conversion: exponential to ZIP & ZIP to exponential model conversion. In general, these conversions are inaccurate due to non-equivalence of exponential and ZIP (second-order polynomial) models. A numerical analysis is conducted using generated datasets of load models to analyse the error and to compare the accuracy of the presented methods. The results of the analysis indicate that the optimal selection of conversion method depends on a number of factors, including normalisation of load models, ZIP model type (accurate or constrained) and expected use of the converted model. In addition, a case study is conducted to analyse the impact of conversion error on load flow results. The results of the case study indicate that a significant difference in load flow results can occur when the load models are converted. Recommendations for conversion method selection are given in the discussion section of the paper.

1 | INTRODUCTION

In the context of this paper, (static) load models are considered to be equations describing static voltage characteristics of the loads. An overview of existing load models is given in [1] and [2]. According to survey results presented in [3] and [4], the most common static load models are constant power, constant current and constant impedance model, which are followed by exponential and polynomial (i.e. ZIP) load model. In real-world applications, exponential and (second-order) polynomial load model are commonly used [5], and are standard models used for dynamic studies in established stability programs (e.g. PSS/E, PSLF, TSAT and ETMSP packages) [6]. Similarity of PSS/E and PSCAD load models to exponential and polynomial load model is discussed in [7, 8], where [9] and [10] are used as main references. In addition to usage in load modelling, exponential and ZIP model are used for assessing the potential of conservation voltage reduction (CVR) [11–14].

The parallel use of exponential and ZIP model causes in some cases the need for load model conversion [3, 8, 15, 16]. Firstly, load model conversion may be required when an existing power system model is used for constructing the system model for another software, and the load models of the software packages differ [8]. For conducting some power system model conversions, commercial tools are available (e.g. PSS®E-PSCAD Network Data Conversion Module, E-TRAN Runtime Library for PSCAD). Secondly, the exponential to ZIP model conversion can be used for simplifying the load model aggregation stage in component-based load modelling.

The ZIP models of load components can be accurately aggregated by calculating weighted sum of ZIP models as is done in [17–19]. When the known models of some load components are in exponential form, these models need to be converted to ZIP models to calculate the aggregated load model (using weighted sum). Thirdly, the conversion of ZIP models to exponential models is useful for comparing load models [3, 16], and plotting load model changes in time [19, 21]. In case of load model comparison and plotting, the exponential model is preferred due to smaller number of parameters: the exponential voltage characteristic is described by one parameter, while ZIP model has three parameters, in addition to common initial/nominal power and voltage parameters.

The load model conversion methods are rarely described and the conversion error is commonly neglected [8]. Still, an
analytical method for exponential to ZIP model conversion is described in [22] and is analysed in [8]. Analytical solution of the method is presented in Section 3.3.1 and is denoted as ‘Analytical method AM1’. A second method for conducting the same conversion is presented and analysed in [8]. The method is described in Section 3.3.2 and is denoted as ‘Analytical method AM2’. In addition, a new formulation of method AM2 is presented for converting exponential models to constrained ZIP models. In case of constrained ZIP model, the parameter values are limited to range 0…1. In Section 3.3.3, an equivalent equation system for ‘Analytical method AM3’ [8] is presented and some derivation errors present in [8] are corrected. In [16] methods for second-order polynomial (i.e. ZIP) to exponential load model conversion are described and analysed. The analytical method described in [16], and in Section 3.2, is used for conversion from ZIP model to exponential model in [3, 16, 19]. The descriptions of optimisation-based methods presented in [8, 16] are generalised in Section 3.1 for handling different load model approximation situations. The optimisation-based conversion is used to provide a benchmark for evaluating the performance of other methods. The conversion methods for both conversion directions are presented to give a comprehensive overview of this load model conversion pair.

In case of load model conversion, the main aim is to minimise the conversion error to approximate the original load characteristic as well as possible. It is assumed that the input model is accurate. The conversion between exponential and ZIP load model is, in the general case, not accurate. Exceptions are load models with constant impedance, constant current and constant power, which have equivalent exponential and ZIP models. Numerical analysis is used for analysing the conversion error that occurs when presented conversion methods are used. It is shown that the load model conversion error depends on the input load model parameter values, used method, chosen voltage and the voltage sensitivity of the characteristic. When the converted load model is used in a power system model, the conversion error leads to a shift in power flow results. This shift can lead to inaccurate power system analysis results, which may lead to wrong decisions. The impact of conversion error on power flow results is illustrated by a case study.

The paper is divided into eight sections. The introduction is followed by Section 2, where exponential and ZIP load model are described. In addition, in Subsection 2.2, measures of conversion error are defined. In Section 3, methods for conversion are presented. The load model conversion error of the presented methods is analysed in two sections: Section 4 and Section 5. The results are divided between the sections based on conversion direction: Section 4 deals with ZIP to exponential conversion and Section 5 with exponential to ZIP conversion. The impact of load model conversion on load flow results is analysed in Section 6. Finally, the main results of the study are discussed and summarised in Section 7 and Section 8.

2 | LOAD MODELS AND CONVERSION ERROR

2.1 | Exponential and ZIP load model

In case of second-order polynomial (i.e. ZIP) and exponential load model, the models can be defined using the nominal value of voltage and power [8, 16, 20, 23] or initial values [1, 21, 23, 24]. The load models are generalised by using base voltage \( v_b \), base active power \( P_b \) and base reactive power \( Q_b \).

2.1.1 | Second-order polynomial (ZIP) load model

The second-order polynomial load model, also known as a ZIP model, can be described by (1) subject to (2). Reactive load is represented by similar equations (3) and (4).

\[
P_{ZIP} = P_b \cdot \left[ K_Z \cdot \left( v/v_b \right)^2 + K_I \cdot \left( v/v_b \right) + K_P \right],
\]

\[
K_Z + K_I + K_P = 1,
\]

\[
Q_{ZIP} = Q_b \cdot \left[ K_{Z,Q} \cdot \left( v/v_b \right)^2 + K_{I,Q} \cdot \left( v/v_b \right) + K_Q \right],
\]

\[
K_{Z,Q} + K_{I,Q} + K_Q = 1,
\]

where \( P_b \) and \( Q_b \) are active and reactive load, respectively, at base voltage \( v_b \). Voltage \( v \) is the load bus voltage in SI units. \( K_Z, K_I, K_P \) and \( K_{Z,Q}, K_{I,Q}, K_Q \) are parameters describing the voltage dependence of the active and reactive loads, respectively.

The values of ZIP model parameters \( (K_Z, K_I \text{ and } K_P) \) may in some cases be limited to range 0…1, and such a ZIP model is called a ‘constrained ZIP model’ [4, 16]. Without these constraints the model is considered to be an ‘accurate ZIP model’ [4, 16].

2.1.2 | Exponential load model

The exponential load model can be described by (5), reactive load is represented by a similar Equation (6).

\[
P_{EXP} = P_b \cdot \left( v/v_b \right)^{K_{EXP}},
\]

\[
Q_{EXP} = Q_b \cdot \left( v/v_b \right)^{K_{EXP,Q}},
\]

where \( P_b \) and \( Q_b \) are active and reactive load, respectively, at base voltage \( v_b \). Voltage \( v \) is the load bus voltage in SI units. \( K_{EXP} \) and \( K_{EXP,Q} \) are exponential parameters describing the voltage dependence of the active and reactive loads, respectively.
2.2 Measures of conversion error

In Section 2.1.1 it is shown that the ZIP model of active power voltage characteristic \( P_{ZIP} \) (1) is mathematically similar to reactive load characteristic \( Q_{ZIP} \) (3). Next, it is shown in Section 2.1.2 that the exponential model \( P_{EXP} \) (5) and \( Q_{EXP} \) (6) are similar. Due to mathematical similarity of active and reactive load model, following equations are only given for active load, but also apply for reactive load models.

In case of load model conversion, the input voltage characteristic \( P_N \) is assumed to be accurate and estimated model \( P_{OUT} \) inaccurate. Depending on conversion direction and converted model, \( P_N \) and \( P_{OUT} \) may stand for \( P_{ZIP} \) (1), \( Q_{ZIP} \) (3), \( P_{EXP} \) (5) or \( Q_{EXP} \) (6).

The difference \( \varepsilon_i \) (7) between the accurate voltage-power characteristic \( P_N \) and converted characteristic \( P_{OUT} \) at voltage \( V_i = v_i / v_b \) is considered to be conversion error at voltage \( V_i \).

\[
\varepsilon_i = P_N(V_i) - P_{OUT}(V_i).
\]

(7)

The relative conversion error at voltage \( V_i \) is defined as \( \eta_i \) (8).

\[
\eta_i = \frac{P_N(V_i) - P_{OUT}(V_i)}{P_N(V_i)}.
\]

(8)

The values of \( \varepsilon_i \) and \( \eta_i \) are used for analysing the direction of the conversion error. In analysis steps, where the accuracy difference of methods is more important than the error direction, the absolute values of \( \varepsilon_i \) and \( \eta_i \) are used.

To quantify the conversion error across voltage range \( V_i \in \{ V_1, \ldots, V_N \} \) \((i, N \in \mathbb{N})\) the mean absolute error (MAE) (9) and normalised mean absolute error (NMAE) (10) are used. MAE describes the mean magnitude of conversion error \( \varepsilon_i \) (7) and NMAE the mean magnitude of relative conversion error \( \eta_i \) (8).

\[
MAE = \frac{1}{N} \sum_{i=1}^{N} |P_N(V_i) - P_{OUT}(V_i)|,
\]

(9)

\[
NMAE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{P_N(V_i) - P_{OUT}(V_i)}{P_N(V_i)} \right|.
\]

(10)

Voltage range 0.8…1.2 p.u. is used for MAE and NMAE calculation range \( V_1 \ldots V_N \), similarly to [7, 8, 16], for result comparability. This range corresponds to voltages where exponential PSCAD load models behave as exponential model and the PSS/E ZIP load model behaves as a ZIP model (assuming the value of PSS/E setting parameter PQBRAK to have value 0.8 p.u. or lower) [7, 8].

3 METHODS FOR LOAD MODEL CONVERSION

This paper focuses on the load model conversion from generic ZIP load model to generic exponential load model. The input load models are assumed to be accurate. The goal of the conversion is to approximate the input load model by another model as accurately as possible. In case of analytical methods presented in Section 3.2 and Section 3.3, the same base values should be used for input and output model to achieve best accuracy. Conversion of models with base value mismatch is discussed in Section 3.1. The specifics of PSS/E and PSCAD load models that have to be taken into account when converting the models of these software packages are discussed in [7, 8]. In the following sections of the paper, equations are given only for the active load component. The reactive component of the load has mathematically similar equations.

In Section 3, the following load model conversion methods are described:

- Non-linear least squares–based error minimisation methods ‘NLS abs’ (12) and ‘NLS rel’ (13) are presented in Section 3.1. These methods are flexible, can handle both conversion directions and different application needs.
- Analytical method for ZIP to exponential model conversion is presented in Section 3.2.
- Three analytical methods for exponential to ZIP model conversion are presented in Section 3.3:
  1. Analytical method ‘AM1’ in Section 3.3.1
  2. Analytical method ‘AM2’ in Section 3.3.2
  3. Analytical method ‘AM3’ in Section 3.3.3

This notation of methods corresponds to names used in [8, 16] for easier comparison of results.

3.1 Using non-linear least squares optimisation for load model conversion

The aim of load model conversion is to minimise the mismatch between the input and output models. This goal can be written as a non-linear least squares optimisation problem, which is a common approach for approximation of static load characteristics from measurement data. In case of non-linear least squares, the square of error is minimised (11).

\[
\min \sum_{i=1}^{N} |\psi_i|^2.
\]

(11)

If the conversion error \( \psi_i \) in (11) is represented by absolute conversion error \( \varepsilon_i \) (7), the model conversion problem can be formulated by (12). In following sections of the paper, the
minimisation of squared absolute error is denoted by NLS abs. 

\[
\min \sum_{i=1}^{N} \left[ \frac{P_N(V_i) - P_{OUT}(V_i)}{P_N(V_i)} \right]^2 , \tag{12}
\]

where \( V_i \) is normalised voltage \( r/v_i \).

If instead of conversion error \( \varepsilon_i \), the relative conversion error \( \eta_i \) (8) is used as \( \psi_i \) in (11), the objective function (13) is obtained.

In the following sections of this paper, the non-linear least squares optimisation of relative error is denoted by NLS rel.

\[
\min \sum_{i=1}^{N} \left[ \frac{P_N(V_i) - P_{OUT}(V_i)}{P_N(V_i)} \right]^2 . \tag{13}
\]

In the case of exponential to ZIP model conversion, the optimisation problem is subject to \( K_Z + K_I + K_P = 1 \) (2) or \( K_Z + K_{I,Q} + K_Q = 1 \) (4), depending if active or reactive load characteristics are converted. To obtain parameter values for a constrained ZIP model, the non-linear least squares method is subject to bounds \( 0 \leq K_Z \leq 1, 0 \leq K_I \leq 1 \) and \( 0 \leq K_P \leq 1 \) or in case of reactive load model \( 0 \leq K_{Z,Q} \leq 1, 0 \leq K_{I,Q} \leq 1 \) and \( 0 \leq K_Q \leq 1 \).

If the base power and voltage of input model and output model differ, it can be taken into account by using different value of \( P_i \) and \( v_i \) in the equation of \( P_N \) and \( P_{OUT} \) in (12) and (13). When \( P_{ZIP} \) and \( P_{EXP} \) use the same voltage and power normalisation bases, \( P_i \) and \( v_i \), the optimisation problem (12) can be simplified to Equation (14).

\[
\min \sum_{i=1}^{N} \left[ (K_Z \cdot (V_i)^2 + K_I \cdot (V_i) + K_P) - (V_i)^{K_{Exp}} \right]^2 . \tag{14}
\]

### 3.2 Analytical method for ZIP to exponential model conversion

ZIP models are converted in [3, 4, 19] to exponential models by Equation (15). In Section 3.1.1 and Section 3.1.2 it is shown that the active power voltage characteristics \( P_{EXP}(5) \) and \( P_{ZIP}(1) \) are mathematically similar to reactive load characteristics \( Q_{EXP}(6) \) and \( Q_{ZIP}(3) \), respectively. Due to mathematical similarity of active and reactive load model, (15) also applies for reactive load models.

\[
K_{Exp} \approx \frac{2 \cdot K_Z + 1 \cdot K_I + 0 \cdot K_P}{K_Z + K_I + K_P} . \tag{15}
\]

### 3.3 Analytical methods for exponential to ZIP model conversion

The exponential load models can be converted to ZIP models by several analytical methods, described in Section 3.3.1, Section 3.3.2 and Section 3.3.3, or by previously described non-linear least squares optimisation, described in Section 3.1. First two analytical methods, AM1 (Section 3.3.1) and AM2 (Section 3.3.2) are suitable for exponential to constrained ZIP model conversion. Analytical method AM2 (Section 3.3.2) and AM3 (Section 3.3.3) are also suitable for exponential to accurate ZIP model conversion.

#### 3.3.1 Analytical method AM1

Analytical method AM1 is presented in [22] by a set of rules and equations, which are reformulated in [8]. The suitable equation or set of parameter values is chosen based on the value of the exponent \( K_{Exp} \). For exponent \( K_{Exp} \leq 0.5 \), a constant power model is used (16). A constant current model is used when \( K_{Exp} < 1.0 \) (17). In exponent \( K_{Exp} \) value range 1...2, the values of ZIP model parameters are calculated using (18) [8], which is the analytical solution of the equation system presented in [22].

A constant admittance model is used if the value of exponent \( K_{Exp} \) is larger than 2 (19).

\[
\begin{align*}
&K_{Exp} < 0.5 \rightarrow \begin{cases} K_Z = 0 \\ K_I = 0 \\ K_P = 1 \end{cases} \quad (16) \\
&0.5 \leq K_{Exp} \leq 1.0 \rightarrow \begin{cases} K_Z = 0 \\ K_I = 1 \\ K_P = 0 \end{cases} \quad (17) \\
&1.0 < K_{Exp} < 2.0 \rightarrow \begin{cases} K_Z = K_{Exp} - 1 \\ K_I = 2 - K_{Exp} \\ K_P = 0 \end{cases} \quad (18) \\
&K_{Exp} \geq 2.0 \rightarrow \begin{cases} K_Z = 1 \\ K_I = 0 \\ K_P = 0 \end{cases} \quad (19)
\end{align*}
\]

#### 3.3.2 Analytical method AM2

An improved version of conversion method AM1, described in Section 3.3.1, is proposed in [8] and denoted as analytical method AM2. AM2 is based on AM1. Equations (16) and (17) are replaced by a more accurate Equation (20). Equation (19) is replaced by (21), extending the \( K_{Exp} \) range of (18).

In case of \( K_{Exp} = 0 \) and \( 1.0 \leq K_{Exp} \leq 2.0 \), the conversion results of method AM1 and method AM2 are equivalent [8].

\[
\begin{align*}
&K_{Exp} \leq 1 \rightarrow \begin{cases} K_Z = 0 \\ K_I = K_{Exp} \\ K_P = 1 - K_{Exp} \end{cases} \quad (20)
\end{align*}
\]
The values of ZIP model parameters $K_Z$, $K_f$ and $K_P$ can be limited to the range 0...1 for converting exponential models to constrained ZIP model by implementing the following changes to the equation system described by (20) and (21):

(1) For negative values of $K_{Exp}$ ($K_{Exp} < 0$), (22) should be used instead of (20) to avoid negative values of $K_f$ and $K_P > 1$. Thus, (20) should be limited to 0 ≤ $K_{Exp} ≤ 1$, deriving (23). (2) Equation (21) should be limited to 1 < $K_{Exp} ≤ 2$, deriving (24), because $K_{Exp} ≥ 2$ would otherwise lead to $K_Z > 1$. When $K_{Exp} ≥ 2$, (25) should be used.

These modifications would lead to a similar method to analytical method AM1 in case $K_{Exp} < 0$ or $K_{Exp} ≥ 1$. Thus, in case of constrained ZIP model, the improvements of method AM2 would only have an effect when 0 ≤ $K_{Exp} < 1$.

\[
\begin{align*}
K_{Exp} & < 0 \rightarrow \\
K_Z &= 0 \\
K_f &= 0 \\
K_P &= 1 \\
K_{Exp} & \geq 0 \rightarrow \\
K_Z &= 0 \\
K_f &= K_{Exp} \\
K_P &= 1 - K_{Exp} \\
1 < K_{Exp} < 2 \rightarrow \\
K_Z &= K_{Exp} - 1 \\
K_f &= 2 - K_{Exp} \\
K_P &= 0 \\
K_{Exp} & \geq 2.0 \rightarrow \\
K_Z &= 1 \\
K_f &= 0 \\
K_P &= 0
\end{align*}
\]

3.3.3 Analytical method AM3

The exponential (5) and polynomial (1) load models are equivalent at base voltage $v_b$, because if $v/v_b = 1$, then $P = P_b$. At intersections of the load characteristics, the load equations have equivalent values $P_{ZIP} = P_{ZIP}$:

\[
P_b \cdot \left[ K_Z \cdot \left( \frac{v}{v_b} \right)^2 + K_f \cdot \left( \frac{v}{v_b} \right) + K_P \right] = P_b \cdot \left( \frac{v}{v_b} \right)^{K_{Exp}},
\]

\[
K_Z \cdot V^2 + K_f \cdot V + K_P = V^{K_{Exp}}.
\]

Replacing $K_P$ in (27) with $1 - K_Z - K_f$ and simplifying the equations leads to derivation of (28).

\[
K_Z \cdot (V + 1) + K_f = \frac{V^{K_{Exp}} - 1}{V - 1}.
\]

Equation (28) includes two unknowns: $K_Z$ and $K_f$ and has 0 ... ∞ solutions. To limit the number of solutions to 0...1, is assumed that two additional intersections of ZIP and exponential characteristics exist at voltages $V_1 = v_1/v_b$ and $V_2 = v_2/v_b$. This assumption is supported by the conversion error analysis results presented in [8]. Using the assumption, (29) is derived.

\[
\begin{align*}
K_Z \cdot (V_1 + 1) + K_f &= \frac{(V_1)^{K_{Exp}} - 1}{V_1 - 1}, \\
K_Z \cdot (V_2 + 1) + K_f &= \frac{(V_2)^{K_{Exp}} - 1}{V_2 - 1}
\end{align*}
\]

where $V_1$ and $V_2$ are in p.u., normalised with base voltage $v_b$.

The solution of equation system (29) is (30) [8]. The value of $K_P$ can be calculated from values of $K_Z$ and $K_f$ using Equation (31).

\[
K_P = 1 - K_Z - K_f.
\]

Equations (30) and (31) can be written as an equation system (32).

\[
\begin{align*}
K_Z &= \frac{1}{V_1 - V_2} \cdot \left[ \frac{V_1^{K_{Exp}} - 1}{V_1 - 1} \right], \\
K_f &= \frac{1}{V_1 - V_2} \cdot \left[ \frac{V_2^{K_{Exp}} - 1}{V_2 - 1} \right] \\
K_P &= 1 - K_Z - K_f
\end{align*}
\]

4 RESULTS (1/2): ZIP TO EXPONENTIAL MODEL CONVERSION

This results section complements the results presented in [16], where the absolute error of conversion $\xi$, and MAE are analysed. In this section, the focus is on relative conversion error $\eta$, and NMAE.
4.1 (ZIP to Exp) input models and notation of methods

A new smaller set of ZIP models (around 16,000) was generated to test repeatability of results presented in [16], where 30,000 models are used. Firstly, two vectors with 100,000 values $-25...25$ were generated. The random values are generated with uniform distribution. Next, the third ZIP model parameter was calculated by subtracting the two generated vectors from a vector of ones to fulfill ZIP model property (2). ZIP models with extremely high voltage sensitivity ($abs(K_{Exp}) > 8$) were detected based on exponential load model calculated by analytical method presented in Section 3.2 and removed from the input model dataset. During plotting, the set is further decreased to increase the readability of the figures.

ZIP to exponential load model conversion is conducted using three different methods:

1. Analytical: analytical method described in Section 3.2 by (15).
2. NLS abs: conversion method based on minimisation of squared absolute error described in Section 3.1 by (12).
3. NLS rel: conversion method based on minimisation of squared relative error described in Section 3.1 by (13).

4.2 (ZIP to Exp) impact of zip model parameter values on conversion error

Figure 2 describes the impact of ZIP model (input load model) parameter values $K_Z, K_I, K_P$ on the NMAE (10) of conversion. ZIP models with NLS rel determined exponent $K_{Exp}$ values $-5 \leq K_{Exp} \leq 5$ are shown. NLS rel determined exponent $K_{Exp}$ values $-3 \leq K_{Exp} \leq 3$ and constrained ZIP models are indicated with different colours.

The highest conversion error (maximum value in Figure 2) occurs if ZIP load model parameter $K_Z$ has a large negative value while $K_I$ has a large positive value. In such cases, based on the $\Delta NMAE$ subfigures, the NLS rel method is able to decrease the NMAE value by less than 10%, compared to the analytical method. The figure also indicates the gain to be proportional to absolute values of ZIP load model parameter $K_Z, K_I, K_P$ values, higher $\Delta NMAE$ occurs at higher absolute values of $K_Z, K_I$ and $K_P$.

In Figure 2, the conversion error of constrained ZIP models with $0 \leq K_Z, K_I, K_P \leq 1$, which corresponds to $0 \leq K_{Exp} \leq 2$, is negligibly small compared to the errors of models with more relaxed constraints. For example, $-3 \leq K_{Exp} \leq 3$ or $-5 \leq K_{Exp} \leq 5$. Figure 1 illustrates the dependence between NMAE, exponent $K_{Exp}$ and constraints of ZIP model parameters $K_Z, K_I$ and $K_P$. The model conversion is conducted using NLS rel method. Based on Figure 1, the larger the ZIP model parameter value limits, the larger the variability of NMAE of conversion and the larger the maximum values of NMAE.

4.3 (ZIP to Exp) estimated exponential model and conversion error

The exponent $K_{Exp}$ values obtained by the use of analytical, NLS rel and NLS abs method differ significantly, as shown by Figure 3 and Figure 4. The smallest difference occurs in Figure 3 when $K_{Exp} \approx -0.5$ and in Figure 4 when $K_{Exp} \approx 0$. In [8] the $K_{Exp}$ of analytical and NLS abs method is shown to have highest similarity when $K_{Exp} \approx 0.5$. The three different values of $K_{Exp}$ and non-linearity of the figures indicate conversion result dependence on conversion method.

Figure 5 displays the NMAE (10) of ZIP to exponential model conversion. NMAE values of ZIP models with analytical $K_{Exp}$ values $-5 \leq K_{Exp} \leq 5$ are shown. According to Figure 5, the conversion error displays a significant variation for all $K_{Exp}$ values. Thus, it is not possible to assign a specific NMAE value for each calculated $K_{Exp}$ value. However, it is possible to notice that the lowest maximum values of NMAE occur in $K_{Exp}$ range $-2...3$. In Figure 6, NMAE is plotted for NLS rel converted models with $K_{Exp} \leq -5 \leq K_{Exp} \leq 5$. In Figure 6 the NMAE values are significantly smaller than in Figure 5. Still, the NMAE values can be high (0.3 p.u.) even when the models have $K_{Exp}$ values near 0. The input models with high NMAE were previously found to have large negative $K_Z$ values and large positive values of $K_I$. Figure 2 illustrates the relation between ZIP model parameter values and NMAE values.

The NMAE value difference of analytical and NLS rel method $\Delta NMAE$ (33) is plotted in Figure 7. ZIP models with NLS rel determined $K_{Exp}$ values $-5 \leq K_{Exp} \leq 5$ are shown, similarly to Figure 6. In Figure 7, a positive value of $\Delta NMAE$ would indicate that the NMAE value of analytical method is larger than NLS rel and the NLS rel method is more accurate based on this measure of accuracy. The $\Delta NMAE$ values in Figure 7 are small, thus in case of these models, $-5 \leq K_{Exp} \leq 5$, the usage of NLS rel method does not significantly increase the conversion accuracy compared to the analytical method.

$$
\Delta NMAE = NMAE_{\text{Analytical}} - NMAE_{\text{NLSrel}}.
$$

(33)
FIGURE 2 Normalised mean absolute error (NMAE) dependence on ZIP model parameters $K_Z$, $K_I$ and $K_P$.

FIGURE 3 Calculated exponent $K_{Exp}$ values of analytical and NLS rel method.

FIGURE 4 Calculated exponent $K_{Exp}$ values of NLS abs and NLS rel method.
4.4 | (ZIP to Exp) voltage dependence of conversion error

The relative load model conversion error $\eta_i$ (8) depends on voltage. The voltage dependence of conversion error of the analytical method is shown in Figure 8, and NLS rel in Figure 9. ZIP models with NLS rel determined $-5 \leq K_{Exp} \leq 5$ are shown. According to Figures 8 and 9, the conversion error of both analysed methods, NLS rel and analytical, is lowest near nominal voltage. This is an expected result, as near nominal voltage, the load is close to nominal as well, independent of load model parameter values. Both methods display largest relative conversion errors $\eta$ at higher voltages. To analyse the difference between Figure 8 and Figure 9, $\Delta \eta_i$ is plotted in Figure 10. $\Delta \eta_i$ is the difference between the relative conversion error of analytical and NLS rel conversion methods, defined by (34). The absolute values of relative conversion error are used to detect which conversion result is closer to the accurate voltage-power
characteristic at voltage $V_i$,

$$\Delta \eta_i = |\eta_i^{\text{Analytical}}| - |\eta_i^{\text{NLS abs}}|.$$  

(34)

Figure 10 indicates that the gains of using one method instead of the other can provide significant accuracy gains. However, neither method displays constantly lower absolute value of $\eta$.

5 | RESULTS (2/2): EXPONENTIAL TO ZIP MODEL CONVERSION

In this section, the relative conversion error $\eta_i$ (8) and NMAE (10) of exponential to ZIP load model conversion are analysed. These error measures are chosen for comparability to previous Section 4.

5.1 | (Exp to ZIP) input models and notation of methods

The set of exponential load models for load model conversion error analysis was calculated using 0.005 step size and value range $-5...5$. This led to a dataset of 2001 exponent values with even distribution.

Exponential load models are converted to ZIP load models by using five different methods:

1. AM1: analytical method described in Section 3.3.1
2. AM2: analytical method described in Section 3.3.2
3. AM3: analytical method described in Section 3.3.3
4. NLS abs: optimisation of squared conversion error described by (12) in Section 3.1
5. NLS rel: optimisation of squared relative error described by (13) in Section 3.1

5.2 | (Exp to ZIP) impact of exponential model parameter values on conversion error

The NMAE (10) was calculated for each converted load model using the same voltage range as was used for model conversion, from 0.8 to 1.2 p.u. In total, 2001 NMAE values were obtained for each conversion method. The results were plotted in Figure 12. According to the figure, the normalised mean conversion error (NMAE) of positive $K_{Exp}$ value is smaller
than the error of negative $K_{Exp}$ value with equivalent absolute value. The model conversion error is approximately symmetrical for $K_{Exp} = 1$, except for analytical method AM1 in range $K_{Exp} = (0 \ldots 1)$. AM1 has a local peak at $K_{Exp} = 0.5$, at the boundary of (16) and (17).

At $K_{Exp} = 1$, conversion is error-less for all methods, because the exponential model is equivalent to the constant current component of the ZIP model. Similar error-less conversion takes place for $K_{Exp} = 0$ and $K_{Exp} = 2$.

Figure 12 clearly indicates that analytical method AM1 has the worst performance and other methods should be used instead. Analytical methods AM2 and AM3 display comparable NMAE values to NLS abs and NLS rel method when $0 \leq K_{Exp} \leq 2$. Outside that $K_{Exp}$ range, the analytical method AM3 and non-linear least squares methods (NLS abs and NLS rel) display significantly lower conversion error than analytical methods AM1 and AM2.

5.3 (Exp to ZIP) estimated zip model and conversion error

Figure 11 describes the NMAE (10) dependence of estimated ZIP model parameter values $K_Z$, $K_I$, $K_P$. According to Figure 11, the NMAE values of analytical method AM1 are highest when $K_Z$, $K_I$ or $K_P$ are equal to 0 or 1. Based on the figure, analytical method AM2 parameters are limited to $K_Z \geq 0$, $K_P \geq 0$, $K_I \leq 1$, which is in accordance with (20), (21) and (22). The $K_Z$, $K_I$ and $K_P$ NMAE characteristics are similar for analytical method AM3 and non-linear least squares relative error minimisation method NLS rel. The highest conversion error (maximum value in Figure 11) occurs if ZIP load model parameter $K_Z$ and $K_P$ have a large positive value while $K_I$ has a large negative value.

5.4 (Exp to ZIP) voltage dependence of conversion error

The exponential load models are converted to ZIP models using voltage range from 0.8 to 1.2 p.u. with 0.01 p.u. voltage step. Relative conversion error (8) is calculated for NLS rel, analytical method AM2 and analytical conversion method AM3. The voltage dependence of the methods is plotted in Figures 13, 14 and 16, respectively.

The voltage dependence of relative conversion error of NLS rel method shown in Figure 13 indicates the existence of three intersection points of exponential and ZIP characteristics: first in voltage range 0.83…0.90 p.u., second at nominal voltage and third in voltage range 1.12…1.18 p.u. The intersection points are indicated by zero value of $\eta_i$. Such intersections are also observed in [8] and were used for deriving analytical method AM3. The intersections occur within the optimisation region, between voltages 0.8 and 1.2 p.u., near the boundary values 0.8 p.u. and 1.2 p.u.

According to Figure 14 the relative error of analytical method AM2 is unidirectional and smallest near base voltage. The positive sign of relative conversion error (8), indicates that the ZIP characteristic calculated by AM2 typically underestimates the load compared to the exponential input characteristic.

The relative conversion error difference of analytical and NLS rel method $\Delta \eta_i$ (35) is plotted in Figure 15 and indicates that NLS rel method has lower relative error across the whole voltage range than analytical method AM2. The accuracy gain offered by the NLS rel method is significant, in the same range as whole error of method AM2.

$$\Delta \eta_i = |\eta_i, Analytical| - |\eta_i, NLS_{rel}|.$$ (35)
The voltage dependence of relative error of analytical method AM3 shown in Figure 16 indicates the existence of three intersection points of exponential and ZIP characteristic: first at 0.80 p.u., second at nominal voltage and third in voltage range 1.20 p.u. The intersection points are indicated by zero value of $\eta_i$. Using the range boundary values 0.8 p.u. and 1.2 p.u. as $V_1$ and $V_2$ in (30), the lowest relative conversion error will occur at the voltage boundaries. The intersection points of exponential and ZIP characteristic were used for deriving analytical method AM3, thus the existence of these intersections corresponds to the expectations.

The relative error difference of analytical and NLS rel method $\Delta\eta_i$ (35) is plotted in Figure 17. The figure indicates, usage of NLS rel instead of analytical method AM3 could provide a relative error decrease in voltage range 0.83...1.16 p.u. and increase outside that range.

### 6 | CASE STUDY

In previous sections of the paper it is shown that the conversion between exponential and ZIP load models involves an error: load model conversion error. The load model conversion error describes the mismatch between the original and the converted voltage characteristic. In order to analyse the impact of conversion error on load flow results, a case study of a small power system is conducted. DIgSILENT PowerFactory is used for conducting the load flow calculations of the nine-bus power system [25] shown in Figure 18.

DIgSILENT PowerFactory is used for conducting the calculations due to the included load model General Load [26], which can accurately represent the ZIP model (Section 2.1.1) and the exponential model (Section 2.1.2). The mathematical model of General Load corresponds to (36). The model of the reactive load is similar. When General Load is used as a ZIP model, exponents $\epsilon_a$, $\epsilon_b$ and $\epsilon_c$ are assigned values 0, 1 and 2. This way the polynomial equation (36) becomes a second-order polynomial, similar to the ZIP models (1) and (3). However, when General Load is used as an exponential load model, the value of two coefficients (among $a$, $b$ and $c$) are set to 0, and the value of the third is set to 1. The exponent corresponding to the coefficient with value 1 is used as the exponent of the exponential model.

\[
P_{cl, i} = P_v \cdot \left[ a \cdot \left( \frac{v}{v_v} \right)^{\epsilon_a} + b \cdot \left( \frac{v}{v_v} \right)^{\epsilon_b} + c \cdot \left( \frac{v}{v_v} \right)^{\epsilon_c} \right] = P_v \cdot \left[ a + b + c \right]
\]  

(36)

where $P_v$ is active power of the load at voltage $v_v$, both defined as Operating Point values in DIgSILENT PowerFactory; $a$, $b$ and $c$ are coefficients of the polynomial equation; $\epsilon_a$, $\epsilon_b$ and $\epsilon_c$ are exponents of the polynomial equation.
The nine-bus power system (Figure 18) includes three generators (Table 1), which are connected to the 1st, 2nd and 3rd bus. The 1st bus, where generator G1 is connected, is modelled as a slack bus. Generators G2 and G3 are modelled as PQ buses. Previously, the voltage dependence of conversion error was illustrated by Figures 8, 9, 13 and 14. According to the figures, the load model conversion error tends to increase with voltage. To increase the impact of conversion error, the voltage of the power system was increased by modifying generator bus settings. In the original model generator G1 is operated at 1.04 p.u. voltage [25, 27]. To increase the impact of load model conversion error in the study, the voltage of generator G1 was increased to 1.09 p.u. In addition, the reactive power references of generator G2 and generator G3 were increased compared to the original model to achieve similar voltages on all the generators.

Table 2 shows the nine-bus system loads. The loads are modelled by constant power model.

Table 3 presents the selected exponential and ZIP load models. The exponential models (Table 3) are used in the selected ‘accurate’ exponential and ZIP models are given in Table 3. The exponential models of the table are used in the ZIP models with different conversion methods, and power flow calculations are conducted with the ZIP models, obtained by converting selected exponential models to ZIP models. The selected ZIP models (Table 3) are used when calculating power flow for analysing the impact of ZIP to exponential conversion. Again, the ZIP models are converted to exponential models with different methods to illustrate the impact of method selection.

For exponential to ZIP model conversions, the exponential models are chosen based on the results of an international survey [3]. The mean load model of the international survey is used for Load A and the maximum values of load model parameters for Load B. Load C is assigned negative voltage sensitivities, which are chosen to match the two largest exponent values of Load A and Load B.

The ZIP models for ZIP to exponential model conversion are selected with realistic voltage sensitivities (values comparable to load models presented in [17]) and high conversion errors. Active load models with corresponding voltage sensitivity 0…2, and reactive models with voltage sensitivity −3…3 are analysed. Most of the chosen ZIP models have a high $K_{I,P}$ and $K_{I,Q}$ value. Active load model of Load A is chosen with a negative $K_{I,P}$ value, and reactive load model of Load B with high $K_{I,Q}$ value. The rest of the ZIP load models are chosen with $K_{I,P}$ and $K_{I,Q}$ close to 10 from the previously generated set of ZIP load models.

### Results of case study: Exponential to ZIP model conversion

The chosen ‘accurate’ exponential load models (Table 3, columns 2 & 3) were converted to ZIP models by four different methods: AM1, AM2, AM3 and NLS rel. The same voltage range (0.8 to 1.2 p.u.) is used for the conversion and error calculation as in previous sections of the paper for clarity. The conversion error would depend on the selected conversion method, as shown in Table 4. The least accurate results would be obtained by AM1 and the most accurate by NLS rel. The models converted by these two methods were chosen for simulations to illustrate the impact of conversion method.

The ZIP models acquired by exponential to ZIP model conversion are shown in Table 5. The table clearly illustrates the limited conversion capability of method AM1: it converts exponential models to constrained ZIP models, thus causing large conversion errors for models with negative exponents.
of Load C, indicated by high NMAE values in Table 4). Method NLS rel uses unconstrained ZIP model, which enables it to approximate the exponential load model by ZIP models with lower NMAE.

The use of least accurate conversion method AM1 (Table 6, upper section) leads to three to four times higher relative voltage magnitude error compared to the results of NLS rel (Table 6, lower section). In case of both simulations with the ZIP models, the error is highest for Bus 8, where Load C is connected. This is in accordance with the previous analysis results of conversion error.

The load modelling errors caused by load model conversion (Table 6) do not match with the NMAE values (Table 4). The NMAE describes the mean absolute value of relative conversion error, which is voltage dependant. The load bus voltage of the load flow results differs. Table 7 shows how the relative conversion error is affected by the voltage used in calculations. The relative conversion errors are calculated at three different voltages for each load model:

1. $V_{\text{Exp}}$ – voltages based on load flow with exponential models
2. $V_{\text{AM1}}$ – voltages based on load flow with ZIP models from AM1
3. $V_{\text{NLSrel}}$ – voltages based on load flow with ZIP models from NLS rel
4. $V_{\text{LF}}$ – accurate load calculated based on the load flow with exponential models, converted load calculated based on load flow with AM1 or NLS rel converted load models.

The conversion error in column $V_{\text{LF}}$ (Table 7) matches well the load modelling error shown in Table 6. The small differences are caused by numerical inaccuracies of calculation. The conversion errors calculated based on single load flow results ($V_{\text{Exp}}$, $V_{\text{AM1}}$ and $V_{\text{NLSrel}}$) differ from the $V_{\text{LF}}$, but in this case study have a similar scale. For example, $V_{\text{Exp}}$, $V_{\text{AM1}}$ and $V_{\text{NLSrel}}$ around 20% corresponds to a load modelling error around 20%. These results suggest that the calculated load model conversion error at load flow voltages could indicate the scale of the impact. If the values are small, the impact of the conversion error of load flow results is low.

### Table 5

| Conversion method | Load | ZIP model of P | ZIP model of Q |
|------------------|------|----------------|---------------|
| AM1              | A    | $K_{Z,P}$     | $K_{Z,Q}$     |
|                  | B    | $K_{L,P}$     | $K_{L,Q}$     |
|                  | C    | $K_{P}$       | $K_{Q}$       |
| NLS rel          | A    | 0.33 0.67     | 0.00 1.00     |
|                  | B    | 0.00 1.00     | 0.35 0.65     |
|                  | C    | 0.00 0.00     | 1.00 0.00     |

### Table 6

| Conv. meth. | Bus | Voltage magn. | Voltage angle | P gen. | Q gen. | P load | Q load |
|-------------|-----|---------------|---------------|--------|--------|--------|--------|
| AM1        | 1   | −1.68%        | −6.70%        | 14.35% | 50.57% |        |        |
|            | 2   | −1.61%        | −15.89%       | 0.00%  | 0.00%  |        |        |
|            | 3   | −0.76%        | 13.46%        | 0.00%  | 0.00%  |        |        |
|            | 5   | −0.87%        | 16.67%        |        |        | −0.99% | −3.90% |
|            | 6   | −1.97%        | −100.00%      |        |        | 1.44%  | −1.12% |
| NLS rel    | 1   | −0.09%        | 0.00%         | 0.00%  | 0.00%  | −0.30% | −2.62% |
|            | 2   | 0.00%         | 0.00%         | 0.00%  | 0.00%  |        |        |
|            | 3   | 0.04%         | 0.23%         | 0.00%  | 0.00%  |        |        |
|            | 5   | 0.04%         | −0.26%        |        |        | 0.05%  | 0.14%  |
|            | 6   | 0.04%         | −0.58%        |        |        | 0.03%  | 0.06%  |
|            | 8   | 0.10%         | 2.97%         |        |        | −0.37% | −1.16% |
TABLE 8  NMAE when chosen ZIP models converted to exponential models using analytical method (AM) and NLS rel

| Load | AM   | NLS rel |
|------|------|---------|
|      | P    | Q       | P    | Q       |
| A    | 6.82%| 14.82%  | 6.72%| 14.99%  |
| B    | 7.43%| 30.94%  | 7.38%| 29.16%  |
| C    | 8.07%| 20.28%  | 7.87%| 20.56%  |

TABLE 9  Converted exponential models from AM and NLS rel

| Load | AM  | NLS rel |
|------|-----|---------|
|      | KP  | KQ      | KP  | KQ      |
| A    | 1.48| −1.69   | 1.25| −1.99   |
| B    | 0.09| 1.69    | 0.15| 2.94    |
| C    | 1.61| −2.39   | 1.91| −3.00   |

6.2  | Results of case study: ZIP to exponential model conversion

The chosen ‘accurate’ ZIP load models (Table 3, columns 4 to 9) were converted to exponential models by two different methods: analytical and NLS rel. The same voltage range (0.8 to 1.2 p.u.) is used for the conversion and error calculation as in previous sections of the paper for clarity. The NMAE difference of the two methods is negligible in Table 8.

However, when the converted models (Table 9) are compared, significant differences may be observed. All converted models differ, largest difference is over 1, smallest 0.06.

The simulation with chosen ZIP load models is used as an accurate result for error calculation. Both conversion methods lead to a similar voltage magnitude error (Table 10) −3.5... −2.2%. Considering the similar conversion error results shown in Table 8, this is an expected result. The voltage angle error of most buses is significantly lower with NLS rel method (compared to AM).

The load modelling errors caused by load model conversion (Table 10) do not match with the NMAE values (Table 8). The NMAE describes the mean absolute value of relative conversion error, which is voltage dependant. The load bus voltage of the load flow results differs. Table 11 shows how the relative conversion error is affected by the voltage used in calculations. The relative conversion errors are calculated at three different voltages for each load model:

1. \( V_{ZIP} \) – voltages based on load flow with ZIP models
2. \( V_{AM} \) – voltages based on load flow with exponential models from analytical method
3. \( V_{NL,rel} \) – voltages based on load flow with exponential models from NLS rel
4. \( V_{LF} \) – accurate load calculated based on the load flow with ZIP models, converted load calculated based on load flow with analytical or NLS rel converted load models.

The conversion error in column \( V_{LF} \) (Table 11) matches well the load modelling error shown in Table 10. The small differences are caused by numerical inaccuracies of calculation. The conversion errors calculated based on single load flow results \( (V_{Exp}, V_{AM} \text{ and } V_{NL,rel}) \) differ from the \( V_{LF} \), but in this case study have a similar scale. For example, \( V_{Exp}, V_{AM} \) and \( V_{NL,rel} \) around 20% corresponds to a load modelling error around 20%. These results suggest that the calculated load model conversion error at load flow voltages could indicate the scale of the impact. If the values are small, the impact of the conversion error of load flow results is low.

7 | DISCUSSION

7.1  | Conversion error and load modelling error in load flow

In Section 6 load flow calculations are conducted with chosen load models and converted load models. The acquired load flow results differ, and the calculated conversion error describes the observed changes of load flow results only partially. In this subsection, the relation between conversion error and load flow error is discussed.

When replacing the load models in the system model has only negligible effect on the load bus voltages, the load modelling error in load flow corresponds to the load model conversion error. This situation is illustrated by Figure 19, where the load bus voltage is assumed to be \( V_1 \) in both load flows (with chosen and converted load model), and depending on the load model, the load flow converges at load \( P_1 \) or \( P_{1a} \). As the load is operating at the same voltage in both load flow results, the load modelling error matches the load model conversion error (calculated at \( V_1 \)).

In the conducted case study, the replacing of load models caused a change in calculated load bus voltages. This situation...
TABLE 10  Relative error $\eta$ of simulation with exponential models acquired by using analytical conversion (AM) and method NLS rel

| Conv. meth. | Bus | Voltage magn. | Voltage angle | P gen. | Q gen. | P load | Q load |
|-------------|-----|---------------|---------------|--------|--------|--------|--------|
| AM          | 1   | -3.20%        | 19.51%        | -3.72% | 105.02%| -      | -      |
|             | 2   | -3.19%        | 40.34%        | 0.00%  | 0.00%  | -      | -      |
|             | 3   | -2.23%        | -6.49%        | -      | -      | -7.39% | 13.90% |
|             | 5   | -2.25%        | 1.53%         | -      | -      | 5.60%  | 9.21%  |
|             | 6   | -3.51%        | -12.00%       | -      | -      | 2.05%  | 45.21% |
| NLS rel     | 1   | -3.12%        | 13.97%        | 0.00%  | 0.00%  | -      | -      |
|             | 2   | -2.16%        | 21.01%        | -      | -      | -8.81% | 11.31% |
|             | 5   | -2.45%        | 3.94%         | -      | -      | 6.08%  | 20.03% |
|             | 8   | -3.47%        | 0.40%         | -      | -      | 5.15%  | 36.68% |

TABLE 11  Relative conversion error $\eta$ of AM and NLS rel converted models at different voltages. Voltages based on: $V_{ZIP}$ – load flow with chosen ZIP models; $V_{AM}$ – load flow with exponential models from AM; $V_{NLSrel}$ – load flow with exponential models from NLSrel; $V_{LF}$ – load flow corresponding to model

| Load      | P/Q | $V_{ZIP}$ | $V_{AM}$ | $V_{LF}$ | $V_{ZIP}$ | $V_{NLSrel}$ | $V_{LF}$ |
|-----------|-----|----------|---------|---------|----------|--------------|---------|
| A         | P   | -4.27%   | -2.53%  | -7.41%  | -6.29%   | -4.16%       | -8.81%  |
|           | Q   | 9.65%    | 5.03%   | 13.89%  | 6.60%    | 2.92%        | 11.33%  |
| B         | P   | 5.77%    | 3.36%   | 5.56%   | 6.46%    | 3.68%        | 6.07%   |
|           | Q   | 13.47%   | 7.94%   | 9.22%   | 29.20%   | 18.63%       | 20.10%  |
| C         | P   | 8.14%    | 4.26%   | 2.07%   | 12.48%   | 7.36%        | 5.13%   |
|           | Q   | 33.30%   | 13.68%  | 45.21%  | 23.03%   | 7.32%        | 36.78%  |

is illustrated by Figure 20. Using the accurate load model, the load operates at voltage $V_1$ and consumes $P_1$. When the accurate load characteristic (black line) is converted to dashed characteristic, the load flow converges at load bus voltage $V_2$. The load consumes $P_{2x}$ based on the converted load model and bus voltage. This means that the operation point of the load has shifted due to load model conversion from $P_1 V_1$ to $P_{2x} V_2$. The conversion error is defined as the difference of two characteristics at a specific voltage. This means that the difference between $P_1$ and $P_{2x}$ corresponds to load model conversion error at $V_1$. Similarly, the difference between $P_2$ and $P_{2x}$ corresponds to conversion error at $V_2$. Both of these values differ from load modelling error, which is the difference between $P_1$ and $P_{2x}$. The load modelling error can only be caused by load model conversion if conversion error exists, thus the conversion error affects the load modelling error in load flow. The voltage change caused by the model replacement is dependent on the network model. In the case study, the load model conversion error at voltage $V_1$ and $V_2$ has a similar scale as the load modelling error in load flow. It is possible that in realistic power system models, the load models replacement has a limited effect on the bus voltages, and the result applies in most cases. In future research, this hypothesis could be tested by simulating additional network models. Another possible direction for future research is sensitivity analysis: the sensitivity of load flow results to load conversion error could be analysed.

7.2 Recommendations for choosing load model conversion method

7.2.1 ZIP model to exponential model conversion

Three methods were presented for converting ZIP models to exponential models:
When ZIP models with expected exponential model $-3 \leq K_{\text{KExp}} \leq 3$ are converted, the NMAE of NLS rel and analytical method are similar and the analytical method may be a better choice due to simplicity. This conclusion is supported by the case study results, where the converted load models are in range $-3 \leq K_{\text{KExp}} \leq 3$. In the case study, the voltage magnitude errors of load flow calculations are similar for both conversion methods. However, the voltage angle errors and load modelling error of the case study are contradicting: analytical method has mostly lower load modelling error, while NLS rel has lower voltage angle error. The use of NLS rel and NLS abs is reasonable when models with extreme voltage characteristics are converted or additional flexibility is needed. For example, for handling model base value mismatch on non-symmetrical voltage range of conversion. The choice between NLS rel and NLS abs should be done based on selected measure of error: NLS rel is better at minimising relative conversion error and NLS abs more suitable for minimising non-normalised conversion error.

7.2.2 Exponential model to ZIP model conversion

Five methods were presented for converting exponential models to ZIP models:

(1) AM1: analytical method described in Section 3.3.1, suitable for converting exponential models to constrained ZIP models, use of method not recommended
(2) AM2: proposed method described in Section 3.3.2, suitable for converting exponential models to accurate and constrained ZIP models
(3) AM3: proposed method described in Section 3.3.3, suitable for converting exponential models to accurate ZIP models
(4) NLS abs: optimisation of squared conversion error described in Section 3.1
(5) NLS rel: optimisation of squared relative error described in Section 3.1

The accuracy and flexibility of non-linear least squares optimisation, described in Section 3.1, methods NLS abs and NLS rel, are the highest. When models with extreme voltage characteristics are converted or additional flexibility is needed, e.g. for handling model base value mismatch or non-symmetrical voltage range of conversion, the use of NLS rel or NLS abs method is recommended.

First two analytical methods, AM1 (Section 3.3.1) and AM2 (Section 3.3.2) are suitable for exponential to constrained ZIP model conversion. AM2 has higher conversion accuracy than AM1, thus it should be used instead of AM1 when constrained ZIP models are desired. Analytical methods AM2 (Section 3.3.2) and AM3 (Section 3.3.3) are suitable for exponential to accurate ZIP model conversion. If the exponent of exponential model is $0 \leq K_{\text{KExp}} \leq 2$, the accuracy of AM2 and AM3 is similar, either method can be chosen. However, outside the previously defined $K_{\text{KExp}}$ range, analytical method AM3 and non-linear least squares methods (NLS abs and NLS rel) display significantly lower conversion error than analytical methods AM1 and AM2. Thus, in such cases analytical method AM3 is recommended over AM1 and AM2.

8 CONCLUSION

This paper described several methods for ZIP to exponential and exponential to ZIP load model conversion. For comparing the accuracy of the methods, a ZIP model dataset and an exponential model dataset were generated. The generated datasets were converted using the presented methods. The relative conversion error and the NMAE were calculated for the converted models. The conversion errors were plotted and analysed. It was shown that the conversion error depends on the method, voltage and parameter values of the load models. Recommendations for load model conversion method selection based on load model conversion error are given in Section 7. A case study (Section 6) was conducted to illustrate the impact of load model conversion on load flow results. The results of the case study indicate that load model conversion can cause a significant change in load flow results.

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