Electro-Disintegration of Tensor Polarized Deuterium

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Abstract. A tensor polarized target in Hall A at Jefferson Lab would offer the possibility to measure the D(e,e′p)n cross section for the $M_s = 0$ and the $M_s = \pm 1$ states separately (the quantization axis is along the momentum transfer). These data would serve as a new, stringent test of our current understanding of the deuteron structure for missing momenta up to 450 MeV/c, a region where the deuteron wave function is dominated by the D-state. No data exist to date for missing momenta above 150 MeV/c. The technique to separate these cross sections, possible kinematic settings, and a rough estimate of the achievable precision is presented.

1. Introduction
The deuteron plays a fundamental role in nuclear physics similar to the one of the hydrogen atom in atomic physics. Within modern models using realistic nucleon-nucleon (NN) interactions, one can calculate the deuteron’s bound and continuum states with great precision. This in turn makes it possible to predict a wide range of observables that are sensitive to details of the different aspects of the deuteron structure and indirectly to details of the NN interaction. A better knowledge of the short-range structure of the deuteron is of fundamental importance for the advancement of our understanding of the poorly known short-range structure of the nucleon-nucleon interaction as well as the theory of high-density nuclear matter where short-range structures play a crucial role. This has been illustrated by recent double and triple coincidence experiments on carbon that show that high momentum protons are predominantly paired with high momentum neutrons in np short range correlations (SRC) [1, 2, 3]. High momentum in this case means that the nucleon momentum is above the typical nuclear Fermi momentum of $k_F \approx 250$ MeV/c. This np dominance of the SRC is thought to be due to the dominance of the tensor part of the nucleon-nucleon interaction [4, 5, 6].

Probing the short-range structure of the deuteron means that one is investigating configurations where the two bound nucleons are essentially overlapping. One of the fundamental questions related to these configurations is the extent to which they can be described purely in terms of two nucleons with high initial relative momenta and whether current state-of-the-art models agree with experiment. To shed light on this problem, experimental data are needed on reactions that are sensitive to high-momentum configurations and which can also be accurately modeled. Traditionally, three classes of reactions are used for deuteron studies: elastic electron scattering, inclusive quasi-free electron-disintegration and exclusive quasi-free electron-disintegration. While elastic electron scattering has been carried out to very high momentum transfers, non-nucleonic contributions dominate the form factor and are therefore masking the underlying short range deuteron structure. The inclusive quasi-free electron scattering cross section is related to integrals of the deuteron momentum distribution within kinematic limits...
determined by energy and momentum conservation making the extraction of the momentum distribution challenging. The most direct access to the deuteron momentum distribution is via the exclusive electro-disintegration. If one assumes that the ejected proton after it absorbed the virtual photon does not interact with the recoiling neutron it can be described by a plane wave and the cross section within the so-called plane wave impulse approximation (PWIA) can be written as follows:

\[
\frac{d^5\sigma}{d\omega d\Omega d\Omega_p} = \kappa \cdot \sigma_{ep} \cdot \rho(p_m)
\]  

Here \(\rho(p_m)\) is the momentum distribution with \(p_m\) as the initial nucleon momentum, \(\sigma_{ep}\) is the (off-shell) electron nucleon cross section and \(\kappa\) is a kinematic factor. In reality, PWIA does not provide a complete description of the D(e,e'p)n reaction and final state interactions (FSI) accounting for the interaction of the outgoing nucleon with the residual system and non-nucleonic degrees of freedoms such as meson exchange currents (MEC) and isobar configurations (IC) need to be included. Indeed, D(e,e'p)n experiments at lower momentum transfers have shown that for missing momenta \(p_m > 0.25 - 0.3\) GeV/c FSI quickly dominate the cross section and therefore mask details of the underlying deuteron structure [7]. Also contributions of MEC and IC are very sensitive to the reaction kinematics and in general increase with decreasing momentum transfer.

As the \(S = 1\) and \(T = 0\) deuteron ground state has a quadrupole moment its ground state wave function must contain a D-state contribution. This admixture is entirely due to the tensor force of the NN interaction. Realistic calculations of the wave function show that the D-state dominates the momentum distribution for missing momenta between 0.3 and 0.6 GeV/c. Hence, experimental data that are sensitive to details of the momentum distribution in this nucleon momentum range will provide important constraints on this aspect of the interaction and on the structure of short range correlations in nuclei.

\[M_S = 0\]
\[M_S = \pm 1\]

Figure 1. The surfaces with \(\rho = 0.3 fm^{-3}\) for \(M_s = 0\) (left) and \(M_s = \pm 1\) (right) from reference [8].

The short range structure of the ground state wave function is especially influenced by the repulsive core and the tensor part of the nucleon-nucleon interaction. A detailed theoretical study [8] of the two nucleon density distribution in deuterium and heavier nuclei has shown that the two nucleon distributions in the \(T = 0\) isospin and \(S = 1\) spin state have a strong dependence on the spin projection \(M_s\). If the two nucleons are in a relative \(M_s = 0\) state, the
surface of constant density has the shape of a toroid while if the two nucleons are found in the $M_s = \pm 1$ state the surface of constant density has a dumbbell shape (Fig. 1). The two nucleon density is at a maximum in the $M_S = 0$ state for a torus with a diameter of approximately 1 fm and has a value of $\approx 0.34$ fm$^{-3}$ which corresponds to twice nuclear matter density. The shape of the torus is produced by the combined action of the tensor force and the repulsive core which is responsible for the hole.

Determining the D(e,e$p)$n cross section for tensor polarized deuterium would provide very direct data to test quantitatively our understanding of this important aspect of the NN interaction and lead to new information regarding the limit of a conventional description of nuclei in terms of pure nucleonic degrees of freedom. More specifically, this new information can be obtained from a measurement of the $M_s$ dependent (e,e$p$) coincidence cross section. Scattering off polarized deuterium can be described by replacing $\rho(p_m)$ by a spin projection ($M_s$) dependent momentum distribution $\rho_{M_s=0,\pm 1}(p_m)$. The calculated $\rho_{M_s}(p_m)$ for the different $M_s$ states is shown in Fig. 2. The angle $\theta_k$ is the angle between the momentum $p_m$ and the quantization axis. Within the validity limits of PWIA, using a polarized target permits in principle the determination of the polarization dependent momentum distribution. If the nucleons really remain nucleons as described above, then the momentum distribution must have this dependence on the deuteron orientation.

2. Determination of $\rho_{M_s}$

The determination of the $M_s$-dependent D(e,e'$p$)n cross can be best shown in an idealized situation where the quantization axis (the target magnetic field $\vec{B}$) is along the momentum transfer $\vec{q}$ and is also the z-axis. Each target deuteron occupies one of the possible magnetic substates $M_s$. If $N_{-1}, N_0, N_{+1}$ are the number of nuclei in the state $M_s = -1, 0, 1$, respectively, and $N = N_{-1} + N_0 + N_{+1}$ is the total number of target nuclei, one can define the normalized

Figure 2. The calculated deuteron momentum distribution for different values of $M_S$ and $\theta_k$ from reference [8]. The area (a) indicates the missing momentum range covered by the NIKEF experiment [9] and area (b) represents the kinematic range that could be explored at Jefferson Lab. [10]
populations $n_i = N_i / N$ with $n_{-1} + n_0 + n_{+1} = 1$. For an unpolarized target $n_i = \frac{1}{3}$. The measured cross section $\sigma$ can be now expressed in terms of the normalized populations and the $M_s$-dependent cross section $\sigma_{M_s}$ as

$$\sigma = n_{-1}\sigma_{-1} + n_0\sigma_0 + n_{+1}\sigma_{+1} = (n_{-1} + n_{+1})\sigma_{\pm 1} + n_0\sigma_0 \tag{2}$$

where $\sigma_{-1} = \sigma_{+1} = \sigma_{\pm 1}$. From this one can express the unpolarized cross section as

$$\sigma_{\text{unpol}} = \frac{1}{3}(2\sigma_{\pm 1} + \sigma_0) \tag{3}$$

The polarization state of the spin 1 target can be described by its vector polarization

$$P_z = n_{+1} - n_{-1} \tag{4}$$

and the tensor polarization

$$P_{zz} = (n_{+1} + n_{-1} - 2n_0). \tag{5}$$

The target response is then expressed in terms of the target vector

$$A_z = \frac{\sigma_{+1} - \sigma_{-1}}{\sigma_{+1} + \sigma_{-1}} \tag{6}$$

and tensor analyzing powers.

$$A_{zz} = \frac{(\sigma_{+1} - \sigma_0) + (\sigma_{-1} - \sigma_0)}{\sigma_{-1} + \sigma_0 + \sigma_{+1}} = \frac{2(\sigma_{\pm 1} - \sigma_0)}{3\sigma_{\text{unpol}}} \tag{7}$$

For the special case above $A_z = 0$ and the polarized cross section can be expressed in terms of the target polarization and the analyzing powers as follows:

$$\sigma_{\text{pol}} = \sigma_{\text{unpol}}(1 + \frac{1}{2}P_{zz}A_{zz}), \tag{8}$$

Therefore from the knowledge of the target polarization and the corresponding measured $D(e,e'p)n$ cross section for the polarized target $\sigma_{\text{pol}}$ and the measured cross section for an unpolarized target $\sigma_{\text{unpol}}$ one can extract the cross sections $\sigma_{\pm 1}$ and $\sigma_0$ and from there the reduced cross sections which, if PWIA were valid, would correspond to the momentum distributions $\rho_{M_s}$. The relations are as follows:

$$\sigma_0 = \sigma_{\text{unpol}} \left(1 - \frac{2}{P_{zz}} \left(\frac{\sigma_{\text{pol}}}{\sigma_{\text{unpol}}} - 1\right)\right), \tag{9}$$

$$\sigma \pm 1 = \sigma_{\text{unpol}} \left(1 + \frac{1}{P_{zz}} \left(\frac{\sigma_{\text{pol}}}{\sigma_{\text{unpol}}} - 1\right)\right). \tag{10}$$
3. Previous Experiments
Due to the challenges associated with building and operating a polarized deuterium target in an electron beam, very few electro-disintegration experiments have been carried out so far. Two experiments have been carried out at NIKEF. One measured the vector analyzing power \(A_V\) for missing momenta up to about 350 MeV/c [11]. The data show a large sensitivity to the deuteron D-state for missing momenta above 200 MeV/c and an increasing sensitivity to isobar configurations and FSI with increasing missing momentum. The other experiment [9] measured the tensor analyzing power \(A_T (A_{zz})\) for missing momenta up to 150 MeV/c. The measured analyzing power is well reproduced by modern deuteron models and is mostly due to the D-state contribution even at the small missing momenta measured. The missing momentum range covered by the data from ref. [9] is comparable to the missing momentum range that could be covered at Jefferson Lab and was proposed in an earlier proposed Hall C experiment [10]. Clearly such a measurement would extend the data considerably and provide a complementary data set to the existing vector analyzing power data.

4. A Possible Experiment in Hall A at Jefferson Lab
In contrast to the original proposal a polarized deuterium target can now be operated in Hall A in combination with the two high resolution spectrometers. This makes it possible to increase the beam energy to 4.4 GeV and reduce the electron scattering angle to 7°. This small electron scattering angle increases the cross section by about a factor of two compared to the original proposal and makes it also possible to increase the momentum transfer thus reducing contributions due to FSI. A possible set of kinematics is shown in Table 1.

| \(p_m\) (MeV/c) | \(\omega\) (MeV) | \(q\) (MeV/c) | \(Q^2\) (GeV/c)^2 | \(x_B\) | \(\theta_q\) (deg) | \(p_f\) (MeV/c) |
|-----------------|-----------------|----------------|-------------------|-------|-----------------|-----------------|
| 200.0           | 325.0           | 610.7          | 0.27              | 0.44  | 54.4            | 810.7           |
| 250.0           | 394.6           | 646.9          | 0.26              | 0.35  | 49.0            | 896.8           |
| 300.0           | 481.2           | 699.0          | 0.26              | 0.28  | 43.1            | 999.0           |
| 350.0           | 591.5           | 774.4          | 0.25              | 0.23  | 36.8            | 1124.4          |
| 400.0           | 736.3           | 884.6          | 0.24              | 0.17  | 30.3            | 1284.6          |
| 450.0           | 935.2           | 1050.0         | 0.23              | 0.13  | 23.7            | 1499.7          |

Table 1. Kinematic settings for an incident energy \(E_i = 4.4\) GeV and an electron scattering angle \(\theta_e = 7^\circ\). \(\theta_q\) is the angle of the momentum transfer \(\vec{q}\) with respect to the beam which

The momentum transfer corresponds closely to the one of the \(A_V\) experiments [11]. The high final electron momentum (3 < \(p_e\) < 4 GeV) would reduce the effect of the target magnetic field considerably. The proton momenta are relatively low but as long as the protons are emitted along the magnetic field the effect of the strong target field as used in the UVA target [12] is also reduced. The problem of a coincidence experiment with a polarized target is that the magnet itself imposes many restrictions with respect to the accessible particle directions. It is expected that not all kinematic settings will be able to be measured in the ideal parallel kinematics but that the magnetic field and the momentum transfer will be at an angle with respect to each other. In this case the simple relationships between measured polarized cross section and the magnetic substate populations is complicated and other terms of polarization and analyzing powers need to be included. These additional terms can be viewed as corrections and can be calculated. The calculated cross sections for \(M_s = 0\) and \(M_S = \pm 1\) for the kinematic settings of table 1 are shown in Fig. 3.

The solid lines with the diamond symbols indicate the cross sections for the \(M_s = 0\) state and the dash-dot lines with the square symbols are for the \(M_s = \pm 1\) states. All calculations
Figure 3. Calculated cross sections for the kinematic setting of Table 1. Solid lines and diamonds indicate $M_s = 0$ cross sections. Dash-dot lines and square symbols are for $M_s = \pm 1$ cross sections. PWIA (blue lines), FSI (green lines) and FSI+MEC+IC (red lines) are from a calculation by R. Schiavilla using the Argonne V18 [13] potential. The top row of diamond symbols indicated the expected uncertainties in $\sigma_0$ and the bottom row of square symbols with error bars show the anticipated error in $\sigma_{\pm 1}$ for an experiment of about 30 days.

are from R. Schiavilla using the Argonne V18 potential [13]. The blue lines are from a PWIA calculation while the green lines include FSI and the red ones in addition include MEC and IC. The top row of diamonds indicate an estimated uncertainty for the $M_s = 0$ cross sections (of the order of 6 - 7%) and the bottom row of squares with error bars show the anticipated uncertainty in the $M_s = \pm 1$ cross section. The calculated tensor analyzing power together with the anticipated uncertainty is shown in Fig. 4 where the colors have the same meaning as in Fig. 3. $A_{zz}$ seems to be very little affected by MEC and IC for missing momenta below 400 MeV/c but this changes for missing momenta of 450 MeV/c and above. FSI seems to contribute to $A_{zz}$ at the level of 15 - 20% below a missing momentum of 350 MeV/c. Above this missing momentum, FSI contributions are predicted to increase rapidly leading to a change of sign in $A_{zz}$ at $p_m = 450$ MeV/c. The error estimates are based on a beam current of 80 nA and a tensor polarization of 20%. In addition it was assumed that the unpolarized cross section has been measured with a statistical precision of about 1%. It was also assumed that the tensor polarization has been measured with a relative precision of 7%.

5. Summary and Conclusion

Exclusive electro-disintegration of tensor polarized deuterium at low momentum transfers of about 0.25 (GeV/c)$^2$ and up to missing momenta of 450 MeV/c will provide new, detailed information on the spin structure of the deuteron in a kinematic regime where the D-state starts to dominate the ground state. While for missing momenta below 350 MeV/c, FSI and non-nucleonic degrees of freedom are expected to be relatively small their contributions are predicted to raise quickly for missing momenta above 400 MeV/c. These data would also provide a stringent test of the quality of modern non-relativistic models (including relativistic corrections). Jefferson Lab is currently the only facility in the world where such a measurement could be carried out in
Figure 4. Calculated tensor analyzing power for the kinematic setting of Table 1 where the magnetic field $\vec{B}$ is parallel to $\vec{q}$ and the ejected proton is parallel to $\vec{q}$ as well. PWIA (blue lines), FSI (green lines) and FSI+MEC+IC (red Lines) are from a calculation by R. Schiavilla using the argonne V18 potential. The error bars show the estimated uncertainty from a potential experiment of about 30 days.

combination with a dynamically polarized ND$_3$ [12] or potentially a LiD [14] target. In order to make a more accurate assessment of the needs of such an experiment the details on the target system to be used need to be further specified, such as the target material and the achievable polarization. One also needs to study background contributions of the other materials besides deuterium and a way of correcting or subtracting these contributions from the measured cross section.

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