Associated quantities from the CCFM approach

G.P. Salam
INFN — Sezione di Milano, Via Celoria, Milano 20133, Italy
E-mail: Gavin.Salam@mi.infn.it

Results are presented on structure functions and final state properties within the CCFM approach. Traditionally used forms of the CCFM equation have difficulty fitting the $F_2$ data, predicting too fast a growth at small $x$. A solution can be found in a particular treatment of formally subleading $(1-z)$ terms, which dampens very considerably the small-$x$ growth. Preliminary results are shown for the transverse energy flow, and future prospects and plans are discussed.

1 Introduction

This talk presents a summary of the results obtained during the past year in collaboration with Bottazzi, Marchesini and Scorletti on the predictions for structure functions and final state properties from the CCFM equation.

Like the BFKL equation, the CCFM equation resums logarithms of $x$, but in contrast, as a consequence of its inclusion of angular ordering of initial-state radiation, it also takes into proper account $\ln 1/x$ terms associated with the final-state. In particular, for a number of final state properties (e.g. multiplicities), the correct small-$x$ perturbative result, as given by the CCFM equation, contains terms of the form

$$(\alpha_s \ln^2 1/x)^n,$$

whereas in calculations neglecting the angular ordering (BFKL), one obtains terms of the form

$$(2\alpha_s \ln 1/x \ln Q/\mu)^n,$$

where $\mu$ is an infra-red cutoff. Even in final-state quantities which are the same at leading order, angular ordering can introduce phenomenologically very substantial next-to-leading corrections.

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2 Angular ordering

\[ z_i = x_i / x_{i-1} \ll 1 \]

Figure 1: Kinematics

It is essential for what follows to consider the kinematics of angular ordering. In figure 1, the angle \( \theta_i \) of gluon \( i \) is given by the following equation

\[ p_i = x_i E_p \tan \theta_i = \frac{z_i q_{t,i}}{1 - z_i}, \]

with \( E_p \) the energy of the proton. For simplicity, one works in terms of a rescaled transverse momentum \( q_i = q_{t,i} / (1 - z_i) \). One then obtains the following equation for the unintegrated gluon density \( A(x, k, p) \), where the third variable \( p \) limits the maximum angle of gluon emission during the evolution:

\[ A(x, k, p) = A^{(0)}(x, k, p) + K \otimes A, \tag{1} \]

\( K \otimes A \equiv \int \frac{dz}{z} \frac{d^2q}{\pi q^2} \tilde{\alpha}_s[(1 - z)q] \Theta(p - zq) \Delta(z, q, k) A(x/z, |k + (1 - z)q|, q). \]

Here, \( A^{(0)} \) is the initial condition, \( \tilde{\alpha}_s = \alpha_s N_C / \pi \), and the form factor \( \Delta \), which resums virtual corrections, is

\[ \ln \Delta(z, q, k) = -\int_{z}^{1} \frac{dz'}{z'} \int \frac{d^2q'}{\pi q'^2} \tilde{\alpha}_s[(1 - z')q'] \Theta(k - Tq') \Theta(q' - z'q). \tag{2} \]

Traditionally, the factor \( T \) is taken as \( T = 1 \), but given that \( q' \) is in reality a scaled transverse momentum, \( q' = q_{t,i}' / (1 - z) \), it is equally reasonable to have \( T = (1 - z) \), in analogy with the \( |k + (1 - z)q| \) in \( K \otimes A \). Most previous calculations have actually ignored all \( (1 - z) \) factors, making the approximation \( (1 - z) \to 1 \) on the grounds that \( z \ll 1 \), and that the resulting effect is at most NLL. This was the approach initially adopted also by our group.
3 Structure functions

As a first step, and as a check of the consistency of the whole procedure, we fitted the HERA data for the structure function \( F_2(x, Q^2) \) in the region \( x < 10^{-2}, \ 8 < Q^2 < 150 \text{ GeV}^2 \). The parameters which were included in the fit were the initial condition, the lowest allowed transverse momentum and the value of \( \alpha_s \) at which it “freezes”. Typical best fits had a \( \chi^2/\text{d.o.f.} \approx 10 \), mainly because \( F_2 \) rises far more slowly than is given by the CCFM equation: \( F_2 \) rises with an exponent \( 0.2–0.3 \), while the exponent from the CCFM equation ((1 - \( z \)) \( \rightarrow \) 1 approximation), plotted against \( \alpha_s \) in figure 2, for the relevant range of \( \alpha_s \) is simply too high.

At this point one is induced to examine the effect of treating \( (1 - z) \) properly. Figure 2 shows the two possibilities, according to one’s choice for \( T \) in (2). For \( T = 1 \), there is relatively little change, while for \( T = (1 - z) \) the exponent is drastically reduced, and one quite easily obtains a good fit (\( \chi^2/\text{d.o.f.} \approx 1 \)) to the \( F_2 \) data.

The large effect of such a formally NLL term is not all that surprising given one’s knowledge of the magnitude of the full NLL kernel. Nevertheless it translates into a significant uncertainty on any prediction from the CCFM approach, at least until one is able to understand the full NLL kernel in the context of the CCFM equation. The way in which we have decided to go
forwards, in the face of such uncertainty, is to choose the form of the equation 
\( T = 1 - z \) which allows one to fit the structure function, and from there to 
go and examine final state properties, also known as associated quantities.

4 Associated quantities

The method of associated quantities allows one to determine final state prop-
erties through the following steps. One determines the unintegrated gluon 
density (for all relevant \( x, k, p \)) as usual by solving (1). One then acts on it 
with a “reduced” kernel \( K_D \) which corresponds to allowing one emission which 
goes into a detector \( D \):

\[
B = K_D \otimes A.
\]

Finally one obtains a gluon density \( C \) which includes any number of further 
emissions by solving the integral equation (analogous to (1))

\[
C = B + K \otimes C.
\]

Preliminary results are just becoming available, and the \( E_t \) flow is shown in 
figure 3. The agreement with the data is rather poor; possible reasons are 
that hadronisation corrections and initial state radiation of soft gluons, both 
of which may contribute significantly, are not taken into account. Adding
uniformly 1 GeV of radiation to simulate hadronisation effects leads to rea-
sonable agreement, but this amount is somewhat large for comfort, and in
any case one should really test such a procedure at other \( x \) and \( Q^2 \) values as
well. In the near future we expect to calculate other final state properties,
such as the forward-jet cross section and the \( k_t \)-spectrum of charged particles,
both of which are expected to be somewhat less sensitive to hadronisation and
initial-state soft gluon effects.

5 Conclusions and outlook

For the CCFM equation, formally subleading \((1-\nu)\) terms have a very large
effect on structure function predictions. Choosing them so as to reproduce \( F_2 \),
allows one to go on and examine final state properties; some results (\( E_1 \) flow)
are already available, more will come in the near future. It should be borne
in mind that the particular NLL choice that we have made is quite arbitrary,
and that other NLL choices might equally well reproduce \( F_2 \), but give different
final-state properties — to do any better one needs to know how to incorporate
the full NLL kernel into the CCFM equation, a non-trivial operation.

Is there any point in doing phenomenology without including the full NLL
kernel? The answer is perhaps “yes”: if a first NLL effect kills most of the
initial-state radiation, then a second one, which on its own might have been
very important, will have little radiation left to kill, and so have little impact.
This leaves the hope that even if only some of the NLL effects are included,
one may still have a reasonable description of small-\( x \) physics.

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