Robot Calibration Using Iteration and Differential Kinematics

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Abstract. In the applications of seam laser tracking welding robot and general measuring robot station based on stereo vision, the robot calibration is the most difficult step during the whole system calibration progress. Many calibration methods were put forward, but the exact location of base frame has to be known no matter which method was employed. However, the accurate base frame location is hard to be known. In order to obtain the position of base coordinate, this paper presents a novel iterative algorithm which can also get parameters’ deviations at the same time. It was a method of employing differential kinematics to solve link parameters’ deviations and approaching real values step-by-step. In the end, experiment validation was provided.

1. Introduction
Robot calibration is a process during which robot accuracy can be improved by modifying the robot positioning software, i.e. identifying a more accurate functional relationship between the joint variable and the actual position of the end-effector in workspace and using these identified changes to update the robot positioning software rather than altering the structure design of the robot or its control system [1]. Many researches have been carried out worldwide in this area. Some are employed mathematics or control method [2–5] and others are employed constrain of robot movement and visual measure technology [6–9]. This paper is going to improve the absolute positioning precision of robot effectively in presence of laser tracker, combining the method of robot differential kinematics and approaching the real location of base coordinate frame step-by-step.

2. Calibration model
2.1. Establishment of robot kinematics model
D-H model [10], being widely used on robot forward kinematics problem, is established here. However, small departure of parallelism will lead to great disparity between actual common normal line and theoretical one if two adjacent axes are paralleled. This means that small departure of actual structure does not always result in same the case for kinematics. In order to solve the problem, this paper adopted the modified D-H model which was put forward by Veitschegger and Wu to describe robot kinematics. This new model is added a revolution component about axis Y, $\beta_i$, in each link coordinate frame based on the standard D-H model. If any real parameter takes place small change, the add-in can express model characteristic of system very well.
As shown by Figure 1, there are 5 parameters, joint angle $\theta_i$, offset $d_i$, link length $a_i$, torsion angle $\alpha_i$, and $\beta_i$, of modified model. If adjacent joint axes are not paralleled, torsion angle $\beta_i$ is defined by zero. If adjacent joint axes are paralleled, offset $d_i$ is defined by zero. So, transformation matrix of modified D-H model can be obtained as follow:

$$A_i = \begin{bmatrix}
C\theta_iC\beta_i - S\theta_i S\alpha_i S\beta_i & -S\theta_i S\alpha_i C\beta_i & C\theta_i S\beta_i + S\theta_i S\alpha_i C\beta_i & a_i C\theta_i \\
S\theta_i C\beta_i + C\theta_i S\alpha_i S\beta_i & C\theta_i C\alpha_i S\beta_i & C\theta_i C\alpha_i C\beta_i - S\theta_i S\alpha_i C\beta_i & a_i S\theta_i \\
-C\alpha_i S\beta_i & S\alpha_i & C\alpha_i C\beta_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(1)

2.2. Establishing of robot actual kinematics identification model

Equation

$$dA_i = A_i \delta A_i$$

(2)

describes the relationship between deviations of link parameters and robot movement error.

Assuming the link parameters are continuous and differentiable we can represent $dA_i$ in another way, that is

$$dA_i = \frac{\partial A_i}{\partial a_i} da_i + \frac{\partial A_i}{\partial \alpha_i} d\alpha_i + \frac{\partial A_i}{\partial \beta_i} d\beta_i + \frac{\partial A_i}{\partial \theta_i} d\theta_i + \frac{\partial A_i}{\partial d_i} dd_i$$

(3)

Comparing (2) with (3), we obtain

$$\delta A_i = A_i^{-1}(\frac{\partial A_i}{\partial a_i} da_i + \frac{\partial A_i}{\partial \alpha_i} d\alpha_i + \frac{\partial A_i}{\partial \beta_i} d\beta_i + \frac{\partial A_i}{\partial \theta_i} d\theta_i + \frac{\partial A_i}{\partial d_i} dd_i)$$

(4)

By differentiating all the elements of equation (1) with respect to $a_i$, $\alpha_i$, $\beta_i$, $\theta_i$ and $d_i$ respectively and substituting them into equation (4), we obtain
Here, $G_i$ is defined as error coefficient matrix.

2.3. Establishment of two coordinate frames differential transformation and mathematics model

According to

\[ dT = T \Delta_T = \Delta T \]  

or, we can write it as

\[ \Delta_T = T^{-1} \Delta T \]  

let

\[
T = \begin{bmatrix}
x & o_x & a_x & p_x \\
y & o_y & a_y & p_y \\
z & o_z & a_z & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

in addition, we can derive small transformation

\[
\Delta = \begin{bmatrix}
0 & -\delta x & \delta y & dx \\
\delta x & 0 & -\delta x & dy \\
-\delta y & \delta x & 0 & dz \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

from robot differential kinematics model. Applying vector triple product property and the fact that vector $n, o, a$ are perpendicular respectively, we can also obtain

\[
\Delta_T = \begin{bmatrix}
0 & -\delta \cdot a & \delta \cdot o & \delta \cdot (p \times n) + d \cdot n \\
\delta \cdot a & 0 & -\delta \cdot n & \delta \cdot (p \times o) + d \cdot o \\
-\delta \cdot o & \delta \cdot n & 0 & \delta \cdot (p \times a) + d \cdot a \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

thereby,

\[
\begin{bmatrix}
dx_T \\
dy_T \\
dz_T \\
\delta x_T \\
\delta y_T \\
\delta z_T \\
\delta \delta x_T
\end{bmatrix} = \begin{bmatrix}
x & n_y & n_z & (p \times n)_x & (p \times n)_y & (p \times n)_z \\
o_x & o_y & o_z & (p \times o)_x & (p \times o)_y & (p \times o)_z \\
a_x & a_y & a_z & (p \times a)_x & (p \times a)_y & (p \times a)_z \\
0 & 0 & 0 & n_x & n_y & n_z \\
0 & 0 & 0 & o_x & o_y & o_z \\
0 & 0 & 0 & a_x & a_y & a_z \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[ = Je \]

\[ = G_i \Delta x_i \]  

\( \text{(5)} \)

2.4. Solution and compensation for robot link parameters

For IRB 2400/L, a six DOF revolution robot, taking the error that results from unconformity of actual link parameters and theoretic ones into account only, its total error


\[ e = \sum_{i=1}^{6} \dot{J}_i \Delta x_i = \sum_{i=1}^{6} J_i G_i \Delta x_i \]  

(12)

\(^6J_i\) indicates the \(6 \times 6\) differential error transformation from the \(i\)th link coordinate frame to the flange centre coordinate frame and \(^6J_0=I_{6 \times 6}\). Real postures of the tool centre point (TCP) are measured by a laser tracker, Faro \(X_i\), which has an accuracy of \(10 \mu m + 0.4 \mu m/m\) in absolute distance measure (ADM) in its near field. Twenty four postures were chosen as original data, including positional component and rotational component of the flange coordinate frame with respect to laser tracker coordinate frame. The postures were chosen considerately—the rotation angle about each axis has equal magnitude but contrary sign at a certain couple of postures. Positional component can be acquired indirectly by projecting the borderline of the flange to its plane and locating the centre of the circle. Rotational component can be determined if the directions of \(Z\) axis (the direction or the opposite direction of the normal line of the flange plane) and \(X\) axis (the opposite direction of the \(-X\) direction determined by the location hole on the flange plane) are known (the direction of \(Y\) axis can be known through the cross product of the above two).

![Figure 2. Experiment configuration.](image)

Figure 2 shows the experiment setup in the calibration field. Supposed that a rigid body under rectangular coordinate system has the homogeneous coordinate transformation

\[
T_{xyz} = \begin{bmatrix}
        r_{11} & r_{12} & r_{13} & p_1 \\
        r_{21} & r_{22} & r_{23} & p_2 \\
        r_{31} & r_{32} & r_{33} & p_3 \\
        0      & 0      & 0      & 1
\end{bmatrix}
\]  

(13)

and then its angles revolving \(X\), \(Y\), \(Z\) axis are

\[ \gamma = \tan^{-1}(r_{32}, r_{33}) \]  

(14)

\[ \beta = \tan^{-1}\left( r_{31}, \sqrt{r_{11}^2 + r_{21}^2} \right) \]  

(15)
respectively. In this way, we can obtain real posture value

\[ R = \begin{pmatrix} R_{p_1} & R_{p_1} & R_{p_1} & R_{p_1} & R_{p_1} & R_{p_1} \end{pmatrix} \]

On the other hand, we can calculate theoretical posture value

\[ N = \begin{pmatrix} N_{p_1} & N_{p_1} & N_{p_1} & N_{p_1} & N_{p_1} & N_{p_1} \end{pmatrix} \]

by modified D-H model already built. Obviously, the total error

\[ e = R - N \]

The least square solution for equation (19) includes all the parameters’ deviations.

3. Calibration principle and procedure

Let vectors \( x \) and \( \Delta x \) represent link parameters and deviations respectively, and then \( N = N(x + \Delta x) \) is the transformation according to D-H model, \( T = T(N) \) is the transformation from base coordinate frame to world (laser tracker) coordinate frame, \( X = X(T, N) = F(x + \Delta x) \) is the function for finding out \( \Delta x \) according to differential kinematics equation. Here, what must be pointed out is that sequence \( X \) does not always converge. However, if we pick up some special sample points such as symmetric postures, we can let \( \Delta x = 0 \) be the initial value of \( X \) and proceed iteration, the detailed procedure is as follows:

- Step 1. According to coordinate transformation equation

\[ B = \Delta T^b A \]

we can calculate the transformation matrix \( T_{W2B} \) from laser tracker coordinate frame to base coordinate frame using the data at twenty four postures and theoretical value derived from the modified D-H model. Because this transformation is just the least square solution for over determined linear equations calculated by mathematic method, its rotation matrix component dose not meet normalization constraint and may result in great distance error between two spatial points after such coordinate transformation. Therefore, it is very necessary to optimize the transformation matrix. Distance error minimizes to \( 10^{-2} \) mm quantity after optimization.

- Step 2. Implement kinematics error compensation task according to equation (19).

- Step 3. Calculate the next theoretical location using the modified robot link parameters just acquired.

- Step 4. Repeat the steps just mentioned until the change of base frame location i.e. the positional component of \( T_{W2B} \) is small enough and then stop iteration. Finally, the correct location of basic frame and modified link parameters are obtained at the same time.

4. Calibration result

| Index | \( \Delta \alpha_i \) (\(^\circ\)) | \( \Delta \beta_i \) (mm) | \( \Delta \gamma_i \) (\(^\circ\)) | \( \Delta \beta_i \) (\(^\circ\)) | \( \Delta \delta_i \) (mm) |
|-------|-------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 1     | -0.0288                        | 0.3740                   | 0                        | -0.0201                  | -0.3623                  |
| 2     | 0.0233                         | 0.4967                   | 0.0291                   | -0.0583                  | 0                        |
| 3     | 0.0093                         | 0.3079                   | 0                        | 0.0264                   | -0.1821                  |
| 4     | -0.0141                        | -0.1732                  | 0                        | -0.0132                  | 0.5002                   |
| 5     | -0.0237                        | 0.1457                   | 0                        | -0.0173                  | 0.1696                   |
| 6     | 0.0091                         | 0.2632                   | 0                        | -0.0302                  | -0.2256                  |

Make use of Matlab to program iteration task and watch the whole process. The change of the positional component of \( T_{W2B} \) (distance between current origin and previous one) is less than 0.01 mm after 273 times iteration and this can be taken for the end condition. The deviations of link parameters are shown in table. In order to validate our calibration effect, we measured another twenty spatial points (without rotation data) by laser tracker and adopted mean error as well as root mean square error to evaluate movement precision of robot. Relevant data are sorted in table1 and table 2.
5. Conclusion

From Table 2 we can see that absolute accuracy has been improved after calibration. Mean error and RMS error are reduced to 48.85% and 47.55% respectively. Take the precision of robot measuring station based on vision technology into account, which need the distance error between two random points is no larger than 0.5mm, robot movement precision can also live up to the demand from Table 2. The method presented in this paper improved the precision of robot effectively and need simple configuration. However, divergence may take place during iteration, so it is vitally important to watch and control the iteration process.

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