Optical response properties of a hybrid electro-optomechanical system interacting with a qubit

Tarun Kumar\textsuperscript{a}, Surabhi Yadav\textsuperscript{b} and Aranya B. Bhattacharjee\textsuperscript{b, c}

\textsuperscript{a}Department of Physics, Ramjas College, University of Delhi, Delhi, India; \textsuperscript{b}Department of Physics, Birla Institute of Technology and Science, Pilani-Hyderabad Campus, Pilani, India; \textsuperscript{c}Department of Physics, School of Applied Sciences, University of Science and Technology, Ri-Bhoi, Meghalaya, India

**ABSTRACT**

We investigate the optical response of a hybrid electro-optomechanical system interacting with a qubit. Our experimentally feasible system explores tunable all-optical-switching, double-optomechanically induced transparency (OMIT) and optomechanically induced absorption (OMIA). The proposed system is also shown to generate anomalous dispersion. Our theoretical results show that the system can switch between OMIT and OMIA by manipulating the relevant system parameters. The normal-mode-splitting (NMS) effect induced by the interactions between the sub-systems is discussed in detail and the effects of varying the interactions on the NMS are analysed. These rich optical properties of the probe field may provide a promising platform for controllable all-optical-switch and various other quantum photonic devices.

**ARTICLE HISTORY**

Received 26 August 2021
Accepted 18 January 2022

**1. Introduction**

The technological advancements in micro and nanomechanical oscillators have shown great potential in exploring novel quantum devices and demonstrating the transition from quantum to classical physics \[1,2\]. By coupling mechanical resonators to other quantum objects, such a transition can be demonstrated. In earlier studies, the quantum systems that have been coupled to mechanical resonators include optical cavities \[3–5\], electron spin \[6\], superconducting qubit circuits \[7–15\], transmission line resonators \[16–20\], nitrogen vacancy centres and quantum dots \[21–23\]. For example, phonon blockade mechanism demonstrates the quantization of mechanical oscillator\[24\]. One of the challenging technologies in photonics is manipulating light for quantum information processing and optomechanical systems which has played an important role in this direction \[25–30\]. These systems exhibit optomechanically induced transparency (OMIT) \[31–33\], an analog of the well-known electromagnetically induced transparency (EIT) in quantum optics. This phenomenon is utilized to slow down or stop light \[34–36\] and thus finds application in quantum communication and quantum information platforms. Hybrid quantum systems allow us to explore many quantum physical phenomena, providing opportunities for designing quantum-enhanced devices \[25–28,37–41\]. In a semiconductor microcavity, the strong coupling between the cavity mode and exciton mode of a quantum dot (QD) or a quantum well (QW) leads to polaritons forming a wide range of applications in quantum information processing \[42–44\], quantum networking \[45,46\] and optical switching \[47–49\]. Strongly coupled systems also exhibit the phenomena of normal mode splitting (NMS). NMS is the process of energy exchange between two nearly degenerate modes of the system and occurs when energy exchange between different modes of the system takes place at a rate faster than its dissipation to the surrounding \[50\]. It has been shown that cooling of mechanical resonator in the resolved sideband regime at high pump power leads to the appearance of NMS \[51\]. NMS has also been shown to occur in a system comprising ultracold atoms in cavity optical lattice \[52\] and cavity optomechanics in a nonlinear optical cavity \[53,54\].

A quantum interface transfers quantum state between different degrees of freedom and can be implemented using micro-mechanical oscillators. An example of such a system is an electro-optomechanical system in which a mechanical oscillator is coupled to a microwave and an optical cavity \[55,56\]. The existence of EIT in such
a hybrid opto-electromechanical system has been predicted earlier [57]. A similar setup was used to show the possible realization of controllable strong Kerr non-linearity even in the weak coupling regime [58]. Strong interactions between the electrical, mechanical and optical modes were demonstrated in a piezoelectric optomechanical crystal [59]. In an interesting experimental development, circuit cavity quantum electrodynamics was integrated with phonons [60]. These hybrid electro-optomechanical systems enable the reversible conversion of quantum states between microwave and optical photons [61]. Recently, a novel scheme was proposed, which could generate a microwave-controllable optical double optomechanically induced transparency (OMIT) in a hybrid piezo-optomechanical cavity [62].

In this work, we investigate a hybrid electro-optomechanical system in the presence of a two level system (Qubit) coupled to the mode of a mechanical resonator via the Jaynes–Cummings interaction. We explore the optical response properties such as optical bistability and output transmission as a function of the various interaction parameters. We demonstrate here that in the resolved side band regime, the system exhibits tunable optical switching behaviour. In the nearly resolved side band regime (sideband resolved regime) [62], the output transmission demonstrates the existence of double-optomechanically induced transparency (OMIT) along with anomalous dispersion (negative group velocity). In the resolved sideband regime, we theoretically demonstrate the existence of optomechanically induced absorption (OMIA). In particular, we can switch between OMIT to OMIA by manipulating the interactions and the optical cavity decay rate. The double-OMIT effect has been studied theoretically in numerous optomechanical systems [57,62,63]. It was theoretically shown that the dispersion is anomalous when the group velocity exceeds the light velocity in vacuum [64]. In the past few years, cavity optomechanics has been shown to generate OMIT and slow light propagation [28,32,33,65,66]. Further, anomalous dispersion has been explored theoretically leading to some exciting results [67–69]. Experimental demonstrations of fast and slow light have paved the way towards telecommunications, interferometry and quantum–optomechanical memory applications [70]. Similar to the optical response effect of OMIT, OMIA have been investigated in numerous optomechanical systems [57,63,71–74]. In our work, we also study the NMS to elucidate the physics of energy exchange between the various modes of the systems. Earlier studies on optomechanical device coupled to a two-level system includes two-colour EIT [63], entanglement dynamics of optical and mechanical modes using a QD [75], coherent perfect transmission mediated by a qubit embedded in a hybrid optomechanical system [72,76]. Multiple induced transparency in a hybrid optomechanical device with a two level system has also been presented recently [77].

2. Theoretical framework

Electro-optic modulators (EOMs) used in optical communication systems have the ability to modulate an optical field using an electric field. One of the major challenges to implement such an EOM is low power consumption [78]. EOMs based on Pockels effect and Kerr effect require large components together with high driving voltage [79,80], leading to high energy dissipation [81]. EOMs based on the electro-absorption effects are also not fully successful due to the weakness of the electro-optic absorption effect [82]. These limitations led to a novel proposal of a hybrid EOM system composed of a three-level medium confined inside a tunable cavity coupled to an electro-mechanical system [83].

Following on a similar idea, we propose a hybrid electro-optomechanical system which is shown in Figure 1. It is composed of a mechanical resonator (MR) which on one side is capacitively coupled to the microwave field of a superconducting microwave cavity (MC) and on the other side is coupled to the field of an optical cavity (OC).

Implementing such a hybrid system is possible using lumped-element superconducting circuit with a drumhead capacitor which is free-standing [84]. The drumhead capacitor with an optical coating can be used as a movable micromirror of the optical cavity which also has a second standard input mirror. The Hamiltonian of the superconducting microwave can be written in terms of the flux $\Phi$ through an equivalent inductor $L$ and the charge $Q$ on an equivalent capacitor $C$ [55]. The Hamiltonian of the superconducting microwave cavity can be rewritten in terms of the raising and lowering operators of the MC field $c$, $c\dagger$ ($\{c,c\dagger\} = 1$) [55] where $c = \sqrt{\omega/2}\hbar \hat{Q} + \frac{i}{2\hbar} \sqrt{\omega/2} \hbar \hat{\Phi}$ [55]. Here $\hat{Q}$ and $\hat{\Phi}$ are the corresponding operators of $Q$ and $\Phi$ satisfying $[\hat{\Phi},\hat{Q}] = i\hbar$ [85]. For simplicity, we will be omitting the hats on the bosonic operators.

In addition, the MR with frequency $\omega_m$ is coupled to a two-level defect (qubit) described by the Jaynes–Cummings Hamiltonian [63,86]. The microwave resonator with a resonance frequency $\omega_c$ is driven by a strong field with amplitude $E_m$ and frequency $\omega_{mi}$, while the optical cavity with a resonance frequency $\omega_{o}$ is driven by a pump laser with amplitude $E_p$ and frequency $\omega_{pr}$. In addition, the optical response can be probed by a weak probe laser with amplitude $E_{pr}$ and frequency $\omega_{pr}$. 
The total Hamiltonian of the system in the rotating frame of pump laser can be written as

\[
H = \hbar \delta_a a^\dagger a + \hbar \delta_c c^\dagger c + \hbar \omega_b b^\dagger b + (\hbar/2) \omega_q \sigma_z \\
- \hbar g_{om} a^\dagger (b + b^\dagger) - \hbar g_{em} c^\dagger c (b + b^\dagger) \\
+ \hbar g (b^\dagger \sigma_- + b \sigma_+) + i \hbar E_p (a^\dagger - a) \\
+ i \hbar E_m (c^\dagger - c) + i \hbar E_{pr} (a^\dagger e^{-i\delta t} - a e^{i\delta t}), \quad (1)
\]

where \( \delta_a = \omega_a - \omega_{ai} \) and \( \delta_c = \omega_c - \omega_{mi} \) are the detunings of the optical cavity from the optical pump and microwave cavity from the microwave pump respectively. \( \delta = \omega_{pr} - \omega_{ai} \) represents the detuning of the optical probe field from the optical pump field. In Equation (1) of the Hamiltonian, the first and second terms represent the free energies of the optical and microwave modes, respectively. The third term gives the free energy of the movable mirror. The fourth term represents the energy of the semiconductor quantum dot. Here \( \omega_q \) is the transition frequency between two levels of the QD. Also, \( \sigma_- \) and \( \sigma_+ \) are the lowering and raising operators respectively of the two level QD. The fifth term represents the optomechanical coupling between the cavity mode and the mechanical mode where \( g_{om} \) being the single-photon optomechanical coupling constant, given by \( (\omega_{ai}/L_c \sqrt{\hbar/m \omega_b}) \), \( m \) being the effective mass of the mechanical mode and \( L_c \) being the effective length of the optical cavity. The sixth term represents the microwave cavity-mechanical-interaction between microwave field and the mechanical phonon mode, where \( g_{em} \) is the microwave cavity-mechanical coupling strength given by \( g_{em} = \frac{\mu \omega_b}{2 \Xi} \), where \( \mu = \frac{C_0}{C+\Xi} \) with \( C_{\Xi} = C + C_0 \) [55,56]. Here \( C_0 \) is the equilibrium capacity of the microwave cavity and \( d \) is the separation between the plates of the capacitor. The origin of the sixth term has been explained earlier [55,56]. The seventh term represents the coupling between the QD and the mechanical phonon mode, \( g \) being the corresponding coupling strength. The last three terms give the energy of the input fields. The intensities of the input microwave field, optical pump field and the optical probe field are described as \( E_m = \sqrt{P_m \kappa_m/(\hbar \omega_m)} \), \( E_p = \sqrt{P_a \kappa_a/(\hbar \omega_m)} \) and \( E_{pr} = \sqrt{P_r \kappa_{pr}/(\hbar \omega_{pr})} \) respectively, with \( P_m, P_a \) and \( P_r \) as the corresponding input powers. Here \( \kappa_m \) and \( \kappa_a \) are the decay constants of cavity field and microwave field respectively.

Using the standard linearization process under strong microwave driving field, the Hamiltonian in Equation 1 can be rewritten as

\[
H = \hbar \Delta_a a^\dagger a + \hbar \Delta_c c^\dagger c + \hbar \omega_b b^\dagger b + (\hbar/2) \omega_q \sigma_z \\
- \hbar g_{om} a^\dagger (b + b^\dagger) - \hbar G_{em} c^\dagger c (b + b^\dagger) \\
+ \hbar g (b^\dagger \sigma_- + b \sigma_+) + i \hbar E_p (a^\dagger - a) \\
+ i \hbar E_m (c^\dagger - c) + i \hbar E_{pr} (a^\dagger e^{-i\delta t} - a e^{i\delta t}), \quad (2)
\]

where \( \Delta_c = \delta_c - \frac{2g_{em}^2 G_{em}^2}{\omega_b (\kappa_c^2 + \Delta_c^2)} \), \( \Delta_a = \delta_a + \frac{2g_{om} g_{em} E_m^2}{\omega_b (\kappa_c^2 + \Delta_c^2)} \) and \( G_{em} = g_{em} \sqrt{\frac{E_m^2}{\omega_b (\kappa_c^2 + \Delta_c^2)}} \).

The dynamics of the system can be described by quantum Langevin equations (QLE) including noise and damping terms. The nonlinear QLEs can be written as

\[
\dot{a} = -[i \Delta_a + \kappa_a/2] a + ig_{om} a (b^\dagger + b) + E_p \\
+ E_{pr} e^{-i\delta t} + A_{in} \quad (3)
\]

\[
\dot{b} = -[\omega_b + \gamma_b/2] b + ig_{om} (a^\dagger a) + i G_{em} c \\
+ ig \sigma_- + B_{in} \quad (4)
\]

\[
\dot{c} = -[i \Delta_c + \kappa_c/2] c + i G_{em} b + E_m + C_{in} \quad (5)
\]

\[
\dot{\sigma}_- = -[\omega_q/2 + \gamma_d/2] \sigma_- + 2g_b \sigma_z + \Sigma_{in} \quad (6)
\]

The system is also interacting with the external degrees of freedom and therefore we introduce \( \kappa_d, \gamma_d, \kappa_c \) and \( \gamma_q \) as the decay constants of cavity field, moving mirror, microwave field and the QD respectively. \( A_{in} \) and \( C_{in} \) represent the optical and microwave input noises respectively. \( B_{in} \) is the quantum Brownian noise associated with the movable mirror and \( \Sigma_{in} \) is the noise related to QD.
The noise terms obey the following correlation functions [62]:

- \( < A_{in}(t)A_{in}^\dagger(t') > = [N(\omega_{in}) + 1]\delta(t - t') \)
- \( < A_{in}^\dagger(t)A_{in}(t') > = [N(\omega_{in})]\delta(t - t') \)
- \( < B_{in}(t)B_{in}^\dagger(t') > = [N(\omega_{in}) + 1]\delta(t - t') \)
- \( < B_{in}^\dagger(t)B_{in}(t') > = [N(\omega_{in})]\delta(t - t') \)
- \( < C_{in}(t)C_{in}^\dagger(t') > = [N(\omega_{in}) + 1]\delta(t - t') \)
- \( < C_{in}^\dagger(t)C_{in}(t') > = [N(\omega_{in})]\delta(t - t') \)

and \( \sum_{in} \) refers to the Langevin \( \delta \)-correlated noise operator of the two-level system which obeys a correlation function \( < \sum_{in}(t)\sum_{in}^\dagger > \approx \delta(t - t') \) [87]. Here \( N(\omega_{in}) \) are the equilibrium mean thermal photon numbers of the optical and microwave fields respectively while \( N(\omega_{in}) \) is the equilibrium mean thermal phonon number of the mechanical resonator. The expression for \( N(\omega_{in}) \), \( N(\omega_{in}) \) and \( N(\omega_{in}) \) is:

\[
N(\omega_{in}) = \left[ \exp\left( \frac{\hbar\omega_{in}}{k_B T} \right) - 1 \right]^{-1}, \quad i = a, b, c.
\]

3. Controllable optical bistability: optical switching

We are interested in the steady-state solutions of Equations (3)-(6). We take \( a_s \), \( b_s \), \( c_s \) and \( \sigma_{-s} \) as the average values of the operators \( a, b, c \) and \( \sigma \) respectively under the conditions of strong microwave and optical fields, i.e. \( E_m \gg E_p \) and \( E_p \gg E_{pr} \). It is to be noted that in all graphs, all parameters are made dimensionless with respect to \( \omega_p \). The steady-state equations after putting derivative terms in Equations (3)-(6) to zero are derived as,

\[
\begin{align*}
    a_s &= \frac{E_p}{\bar{\Delta}_a + \kappa_a/2}, \\
    b_s &= \frac{\gamma_a |a_s|^2 + \gamma_a c_s - \gamma a_{-s}}{\omega_p + \gamma_p/2}, \\
    c_s &= \frac{\gamma_a b_s + E_m}{\bar{\Delta}_c + \kappa_c/2}, \\
    \sigma_{-s} &= \frac{4\gamma < \sigma_c > b_s}{\gamma_d + i\omega_d},
\end{align*}
\]

here \( \bar{\Delta}_a = \Delta_a - g_{om}(b_s + b_s^*) \) is the effective detuning between optical cavity and the optical pump field modified by the optomechanical interaction. Note that in steady state, the average value of the noise operators vanish.

We obtain the mean number of photons \(|a_s|^2\) from Equation (7) as:

\[
|a_s|^2 = \frac{E_p^2}{k_a^2/4 + A_1^2 + A_2 A_5^2 + 2A_3 A_6 A_7},
\]

where \( A_1 = \gamma_p/2 + \frac{G_{om}^2}{2(\Delta_1^2 + \kappa_1^2/4)} - \frac{4\kappa_a}{\omega_p^2 + \gamma_d^2}, \quad A_2 = \omega_p - \frac{G_{om}^2}{\Delta_1^2 + \kappa_1^2/4} + \frac{4\kappa_a}{\omega_p^2 + \gamma_d^2}, \quad A_3 = \frac{\kappa_a G_{om} E_m}{2(\Delta_1^2 + \kappa_1^2/4)}, \quad A_4 = \frac{\Delta_a G_{om} E_m}{\Delta_1^2 + \kappa_1^2/4}, \quad A_5 = g_{om} |a_s|^2 + A_3, \quad A_6 = \Delta_a - \frac{g_{om} A_1 A_4}{A_1^2 + A_2^2}, \quad A_7 = \frac{g_{om} A_2}{A_1^2 + A_2^2}.
\]

To quantify our study, we consider experimentally realizable values of the various parameters. In our study of the steady state and NMS, we will work in the resolved sideband regime in which \( \omega_p > \kappa_a, \kappa_c \) but while analysing OMIT, we will work in both the resolved sideband regime and the case \( \omega_p < \kappa_a, \omega_p > \kappa_c \). The latter case does not meet the condition of resolved sideband regime, but the system is nearly sideband resolved [62].

Optical bistability is essential when it comes to designing of all-optical switching devices [71]. Figure 2 illustrates the optical switching behaviour for different combinations of the three interaction parameters, \( G_{om}, g_{om} \) and \( g \). As we notice from the plots, the dashed and thick line plot displays the typical optical switching characteristics since at a certain value of the input pump power \( E_p \), the intracavity photon value \(|a_s|^2\) jumps from a low to high value. The thick line plot displays this optical switching characteristic at a lower value of \( E_p \). The switching ratio is defined as the ratio of the maximum to the minimum value of \(|a_s|^2\). The switching ratio for the thick plot is 1.84, while that for the dashed curve is 2.21. Consequently, one has to optimize to design high switching value and low input power. On the other hand, the switching property is absent for some specific combinations of \( G_{om}, g_{om} \) and \( g \), as evident from the dotted line plot of Figure 2. It is advantageous for designing optoelectronic devices if the bistable behaviour occurs at low values of the input power \( E_p \). We thus show that the system can exhibit optical switching behaviour at low values of the input power by tuning the various interactions. This controllable bistable behaviour shows that the system can be used as all-optical switch, logic gates and memory device for quantum information processing where low energy power input is essential.

Optical switching is a promising solution to lessen huge optical wiring without compromising on connectivity. For bit-level processing, optical switching operations can be performed on a picosecond time scale which may offer many optical signal processing capabilities in the future as energy requirements are lessened. In the area of packet-level switch compliance, switching operations are performed on the nanosecond time scale and the
system can respond to changes in data traffic thus allowing a multiplexed gain for the interconnect [88]. Switch elements themselves are susceptible to data corruption. Compared to CMOS electronics, optical and optoelectronic elements lead to small amount of signal leakage and corruption [88].

4. Optomechanically induced transparency (OMIT)

Similar to the phenomenon of electromagnetically induced transparency (EIT) observed in atomic systems, optomechanical systems demonstrate optomechanically induced transparency (OMIT). The transparency window observed in the output field is due to the destructive interference induced between photons excited through different pathways. In this section, we discuss the generation of such a transparency window due to the optomechanical interaction. In particular, we will look into the possibility of controlling the OMIT by tuning the various interactions present in the system. To this end, we study the output characteristics of the probe field in the presence of optical field–mirror interaction ($g_{om}$), microwave field–mirror interaction ($G_{em}$) and qubit–mirror interaction ($g$).

Now to study the dynamics of quantum fluctuations, we linearize the quantum Langevin equations of the system around its steady state with the assumptions $a(t) \rightarrow a_{s} + \delta a$, $b(t) \rightarrow b_{s} + \delta b$, $c(t) \rightarrow c_{s} + \delta c$ and $\sigma_{-}(t) \rightarrow \sigma_{-s} + \delta \sigma_{-}$ and neglecting the small nonlinear (quadratic and higher order) fluctuation terms, the corresponding linearized quantum Langevin equations become

$$\delta \dot{a} = -(i\Delta_{a} + \kappa_{a}/2)\delta a + iG_{om}(\delta b + \delta b^{\dagger}) + E_{p}\tau_{b}e^{-i\delta t} + A_{in},$$
$$\delta \dot{b} = -(i\omega_{b} + \gamma_{b}/2)\delta b + i(G_{om}^{*}\delta a + G_{om}\delta a^{\dagger}) + iG_{em}\delta c - i\sigma_{-s} + B_{in},$$
$$\delta \dot{c} = -(i\Delta_{c} + \kappa_{c}/2)\delta c + iG_{em}\delta b + C_{in},$$
$$\delta \dot{\sigma}_{-} = -(i\omega_{q}/2 + \gamma_{q}/2)\delta \sigma_{-} + 2i\kappa_{c} \dot{\sigma}_{-} - \sigma_{-} > \sigma_{+} > \sigma_{-} + \Sigma_{in}. \tag{9}$$

Here $G_{om} = g_{om}a_{s}$ is the net coupling strength between the mechanical mode and the optical mode. We assume that the quality factor of the mechanical oscillator is high ($\omega_{b} >> \gamma_{b}$) and also the proposed system is operating in the resolved sideband regime i.e. $\omega_{b} >> \kappa_{a}, \kappa_{c}$. By assuming

$$\delta a = \delta a_{s}e^{-i\delta t} + \delta a_{e}e^{i\delta t},$$
$$\delta b = \delta b_{s}e^{-i\delta t} + \delta b_{e}e^{i\delta t},$$
$$\delta c = \delta c_{s}e^{-i\delta t} + \delta c_{e}e^{i\delta t},$$
\[ \delta \sigma_- = \delta \sigma_- e^{-i \delta t} + \delta \sigma_- e^{i \delta t}, \]
\[ \delta A_{in} = \delta A_{in} e^{-i \delta t} + \delta A_{in} e^{i \delta t}, \]
\[ \delta B_{in} = \delta B_{in} e^{-i \delta t} + \delta B_{in} e^{i \delta t}, \]
\[ \delta C_{in} = \delta C_{in} e^{-i \delta t} + \delta C_{in} e^{i \delta t}, \]
\[ \delta \Sigma_{in} = \delta \Sigma_{in} e^{-i \delta t} + \delta \Sigma_{in} e^{i \delta t}. \] (10)

Here the plus component \((\delta s_+, s = a, b, c, \sigma_- \cdots)\) corresponds to the original frequency \(\omega_{pr}\) and the minus component \((\delta s_-, s = a, b, c, \sigma_- \cdots)\) corresponds to the frequency \(2\omega_{ai} - \omega_{pr}\). Substituting Equation (10) in Equation (9), neglecting the second and higher order terms and equating the coefficients with same frequency \(\omega_{pr}\) we get

\[ \delta \dot{a}_+ = (i \lambda_a - \kappa_a/2) \delta a_+ + i G_{om} \delta b_+ + E_{pr} + \delta A_{in} +, \]
\[ \delta \dot{b}_+ = (i \lambda_b - \gamma_b/2) \delta b_+ + i G_{om} \delta a_+ + i G_{om} \delta c_+ - i g \delta \sigma_- + \delta B_{in} +, \]
\[ \delta \dot{c}_+ = (i \lambda_c - \kappa_c/2) \delta c_+ + i G_{om} \delta b_+ + \delta C_{in} +, \]
\[ \delta \dot{\sigma}_+ = (i \lambda_a - \gamma_a/2) \delta \sigma_- + 2g < \sigma_+ > + \delta B_{in} +, \]
\[ \delta \dot{\Sigma}_{in} = \delta \Sigma_{in} e^{-i \delta t} + \delta \Sigma_{in} e^{i \delta t}. \] (11)

where \(\lambda_a = \Delta - \Delta'_a, \lambda_b = \delta - \omega_b, \lambda_c = \delta - \Delta_c\) and \(\lambda_z = \delta - \omega_q\). Now to neglect the expectation values of the noise operators, we assume that the system under consideration is working in mK regime, therefore we can have \(< \delta A_{in} > = < \delta B_{in} > = < \delta C_{in} > = < \delta \Sigma_{in} > = 0\). Also under mean field steady-state conditions, the expectation values of \(\delta a_+ \), \(\delta b_+ \), \(\delta c_+ \) and \(\delta \sigma_- \) are zero. Taking expectation values of Equation (11) and using the conditions mentioned above, we get

\[ 0 = (i \lambda_a - \kappa_a/2) < \delta a_+ > + i G_{om} < \delta b_+ > + E_{pr}, \]
\[ 0 = (i \lambda_b - \gamma_b/2) < \delta b_+ > + i G_{om} < \delta a_+ > + i G_{om} < \delta c_+ > - i g < \delta \sigma_- >, \]
\[ 0 = (i \lambda_c - \kappa_c/2) < \delta c_+ > + i G_{om} < \delta b_+ >, \]
\[ 0 = (i \lambda_z - \gamma_d/2) < \delta \sigma_- > + 2g < \sigma_+ > < \delta b_+ >. \] (12)

Equation (12) can be solved to get \(< \delta a_+ >\), which is \(\epsilon_{out} = 2k_a < \delta a_+ > - E_{pr}\).

Also the transmission coefficient of the probe field can be expressed as

\[ T_{pr} = \frac{\epsilon_{out}}{E_{pr}} = \frac{2k_a < \delta a_+ > - E_{pr}}{E_{pr}} - 1. \] (15)

If we define \(\epsilon_T\) as \(\epsilon_T = \frac{2k_a < \delta a_+ >}{E_{pr}}\), the quadrature \(\epsilon_T\) at the original frequency \(\omega_{pr}\) is obtained as

\[ \epsilon_T = \frac{2k_a}{\kappa_a/2 - i \lambda_a} + \frac{|G_{em}|}{(\gamma_b/2 - i \lambda_b) + G_{fm} G_{em} \kappa_b/2 - i \gamma_a} \cdot \frac{2g^2 < \sigma_+ >}{|G_{em}|^2 - 2g^2 < \kappa_c > / \gamma_a^2}. \] (16)

The real and the imaginary parts of \(\epsilon_T\) give the absorption and the dispersion of the system.

We assume that the optical cavity field, the qubit as well as the microwave field are driven at the mechanical red sideband, i.e. \(\Delta_a = \Delta_c = \omega_q = \omega_b\). Consequently, \(\lambda_a = \lambda_b = \lambda_c = \lambda_z = \lambda\). We can rewrite Equation (16) in a more intuitive form as

\[ \epsilon_T = \frac{2k_a}{\lambda - \kappa_a} + \frac{A_1}{\lambda_1 - i \lambda} + \frac{A_2}{\lambda_2 - i \lambda} + \frac{A_3}{\lambda_3 - i \lambda}, \] (17)

where \(\lambda_1, \lambda_2, \lambda_3\) are the roots of the cubic equation,

\[ x^3 - x^2 \left( \gamma_d + \gamma_b + \gamma_c \right) + \left( \gamma_d \gamma_b + \gamma_d \gamma_c + \gamma_b \gamma_c \right) \left( \gamma_d/2 + G_{em} \right) + \frac{G_{em} \gamma_d}{2} - g^2 \kappa_c < \sigma_+ > = 0, \] (18)

and \(A_1, A_2, A_3\) are defined as

\[ A_1 = G_{om}^2 \left[ \lambda_1 \left( \frac{\kappa_c + \gamma_d}{2} \right) - \frac{\gamma_d}{2} - \lambda_2^2 \right] / \left( \lambda_1 - \lambda_3 \right) \left( \lambda_1 - \lambda_2 \right), \] (19)
\[ A_2 = G_{om}^2 \left[ -2 \lambda_2 \left( \frac{\kappa_c + \gamma_d}{2} + \frac{\gamma_d}{2} + \lambda_3^2 \right) \right] / \left( \lambda_1 - \lambda_2 \right) \left( \lambda_2 - \lambda_3 \right), \] (20)
\[ A_3 = G_{om}^2 \left[ \lambda_3 \left( \frac{\kappa_c + \gamma_d}{2} + \frac{\gamma_d}{2} + \lambda_2^2 \right) \right] / \left( \lambda_2 - \lambda_3 \right) \left( \lambda_1 - \lambda_3 \right). \] (21)

In Figures 3 and 4, we plot absorption \(\text{Re}[\epsilon_T]\) and dispersion \(\text{Im}[\epsilon_T]\) for different values of QD-mechanical mode coupling strength \((g)\), microwave-mechanical mode coupling strength \((G_{om})\) and net optomechanical coupling strength \((G_{om})\). The beat of the probe and the pump field lead to a time-varying radiation-pressure force with beat frequency \(\delta\). At specific values of \(\delta\), the mechanical resonator is driven resonantly. Consequently, sidebands of the optical field are generated due to mechanical oscillations. The position of the sidebands depends on the various interactions present in the system. In the presence of a strong pump field and resolved
Figure 3. (Colour online) The absorption $\text{Re}(\epsilon_T)$ and dispersion $\text{Im}(\epsilon_T)$ are plotted as a function of $(\delta - \omega_b)/\omega_b$ for plot (a) $G_\text{om} = 0.23$, $G_\text{em} = 0.005$, $g = 0.125$, (b) $G_\text{om} = 0.1375$, $G_\text{em} = 0.005$, $g = 0.125$, (c) $G_\text{om} = 0.0458333$, $G_\text{em} = 0.005$, $g = 0.125$, (d) $G_\text{om} = G_\text{em} = g = 0.3$. The other parameters used are $\gamma_d = 0.000042$, $\gamma_b = 0.000042$, $\kappa_c = 0.0000125$, $\kappa_a = 2.17$ and $\sigma_z = -1$. All parameters are dimensionless with respect to $\omega_b$.

Figure 4. (Colour online) The absorption $\text{Re}(\epsilon_T)$ and dispersion $\text{Im}(\epsilon_T)$ are plotted as a function of $(\delta - \omega_b)/\omega_b$ for plot (a) $G_\text{om} = 0.23$, $G_\text{em} = 0.005$, $g = 0.125$, (b) $G_\text{om} = 0.183$, $G_\text{em} = 0.005$, $g = 0.125$, (c) $G_\text{om} = 0.1375$, $G_\text{em} = 0.005$, $g = 0.125$, (d) $G_\text{om} = 0.23$, $G_\text{em} = 0.005$, $g = 0.0125$. The other parameters used are $\gamma_d = 0.000042$, $\gamma_b = 0.000042$, $\kappa_c = 0.0000125$, $\kappa_a = 0.217$ and $\sigma_z = -1$. All parameters are dimensionless with respect to $\omega_b$. 
sideband (or nearly sideband resolved) limit, the frequency of the probe field coincides with the dominant sideband. This leads to destructive interference between the sideband and the probe field. Consequently, the destructive interference results in a transparency window due to the cancellation of the intracavity field.

In our study, we begin our discussion in the nearly sideband resolved regime \(w_p < \kappa_d, w_p > \kappa_c\) by first considering \(G_{om} = 0.23, G_{em} = 0.23\) and \(g = 0.125\) and plot the real (dashed line) and imaginary (solid line) parts of \(\epsilon_T\) as a function of \(\frac{(\delta - \omega_p)}{\omega_p}\) in Figure 3(a). This plot shows two transparency windows at points which are determined by the roots \(\lambda_t (t = 1, 2, 3)\) of Equation (18). Since \(\gamma_d = \gamma_b, \kappa_c \ll 1\), approximately the position of the two minima points are obtained as \(\epsilon^T_t \) (minima) \(\approx \sqrt{\frac{G_{em}^2 - 2g^2 < \sigma_z > \approx \pm 0.177}\) which is close to the numerically obtained result of \(\pm 0.19\). At each of the transparency windows, anomalous dispersion (negative group velocity) is also noticed from the graph. Negative group velocity corresponds to fast light propagation. Keeping all parameters fixed, we now reduce \(G_{om} = 0.138\) and the results are shown in Figure 3 (b).

The two minima points are still located at the same points indicating that the minima points are independent of \(G_{om}\). Clearly, the two transparency windows become narrow along with a comparatively steeper anomalous dispersion. A further narrowing of the transparency windows and steeping of the dispersion around the minima points is seen. The position of the two minima points are still at \(\lambda_t = 0, \lambda_{2,3} = \pm \sqrt{\frac{G_{om}^2 + 2g^2}{\kappa_d}}\). These values agree well with the numerical values obtained in Figure 4. Decreasing \(G_{om}\) while keeping \(G_{em}\) and \(g\) fixed, the absorption peak at \(\delta = \omega_p\) (i.e. \(\lambda_1 = 0\)) widens and the separation between the three absorption peaks also decreases as evident from Figures (b) and (c). At the same time, the dispersion curve also becomes distorted. The effect of decreasing qubit-mechanical mode \(g\) is shown in Figure 4(d). The absorption peaks at \(\delta = \omega_p\) become extremely narrow while the other two side peaks broaden. The separation between the absorption peaks also enhances together with a highly distorted dispersion curve.

5. Normal mode splitting

In an optomechanical system, intermixing of fluctuations of different modes about their mean value leads to an important phenomenon known as Normal Mode Splitting (NMS). This phenomenon is ubiquitous in both classical as well as in quantum physics. In this phenomenon, due to strong coupling between different modes, energy exchange takes place on a time scale much faster than the decoherence time of every mode. The optomechanical NMS analysed in this work involves driving four parameterically coupled non-degenerate modes out of equilibrium. Here we study NMS in the resolved sideband regime and calculate the position quadrature of small fluctuations of the mechanical oscillator. To study NMS, we first transform the Equations (3), (4), (5) and (6) into the frequency domain and are then solved for the corresponding displacement spectrum, which in the frequency domain is defined as

\[
S_x(\omega) = \frac{1}{4\pi} \int d\Omega e^{-i(\omega + \Omega)} < \delta x(\omega)\delta x(\Omega) + \delta x(\Omega)\delta x(\omega)> \tag{22}
\]

In Fourier space, the displacement spectrum is obtained as

\[
S_x(\omega) = \frac{1}{d(\omega)d(-\omega)} [A_{25}(\omega)A_{25}(-\omega)] + A_{26}(\omega)A_{26}(-\omega) + A_{27}(\omega)A_{27}(-\omega) + A_{28}(\omega)A_{28}(-\omega) + A_{29}(\omega)A_{29}(-\omega) + A_{30}(\omega)A_{30}(-\omega) + A_{32}(\omega)A_{32}(-\omega) + 1 \tag{23}
\]

The various constants appearing in above equations have been defined in Appendix 1.

In Figure 5, we plot the displacement spectrum \(S_x(\omega)\) for different interactions \(G_{om}, G_{em}\) and \(g\). In Figure 5(a), the displacement spectrum for \(G_{om} = G_{em} = g = 0.4\) is shown. We clearly notice the NMS with four distinct
peaks corresponding to all four modes of the system since all the three interactions coupling the four modes are equal. Keeping $G_{om} = G_{em} = 0.4$ and reducing the QD-mechanical mode coupling $g = 0.01$, the corresponding NMS (dashed curve) is displayed in Figure 5(b). In the limit of low value of $g$, the system reduces to the case of three mode coupling and hence the NMS displays three peaks. The peak near $\omega = 1.45$ shifts to around $\omega = 1.35$ and the peak at $\omega = 0.7$ disappears along with a decrease in the height and shifting of the peak at $\omega = 0.3$. Reducing the piezomechanical (microwave cavity-mechanical mode) coupling strength $G_{em} = 0.2$ with $G_{om} = 0.4$ and $g = 0.01$, the corresponding NMS displayed (dashed curve) in Figure 5(c) shows a three peak structure with a drastically shifted and reduced peak intensity at $\omega = 0.3$. Figure 5(d) illustrates (dashed curve) the influence of decreasing the interactions $G_{em} = g = 0.01$ further and keeping opto-mechanical coupling strength high as $G_{om} = 0.4$. The NMS reveals clearly a two peak structure corresponding to the mixing of the fluctuations of the optical and mechanical modes. Hence a selective energy exchange between the various modes of the system can be achieved by appropriately tuning the interactions.

We now discuss the experimental feasibility of the proposed system. In some recent experiments, optomechanical oscillators have been fabricated using piezoelectric materials such as AIN [59]. The AIN-nanobeam resonator can be driven simultaneously by both the microwave and optical fields under the effects of piezoelectric and radiation pressure interaction. It has been shown that the coupling strength between microwave and piezoelectric mechanical modes in a superconducting coplanar microwave cavity system can reach $12.3 \times 10^6$ Hz [89]. Based on realistic systems, the single-photon optomechanical coupling strength can exceed $g_{om}/2\pi = 1.1$ MHz [90]. We have taken the frequency of the AIN-nanobeam resonator and the decay rate of the optical cavity as $\omega_b/2\pi = 2.4$ GHz and $\kappa_a/2\pi = 5.2$ GHz (for the sideband resolved regime) and $\kappa_a/2\pi = 0.5$ GHz (for resolved sideband regime). The quality factor of
the microwave cavity is taken to be $2 \times 10^5$ [91]. The qubit considered here could be an intrinsic defect inside the mechanical resonator or a superconducting circuit. Experimentally, a Jaynes–Cummings type of interaction between a superconducting qubit and a mechanical resonator was achieved [25]. We have taken the qubit-mechanical oscillator interaction strength to be between 1 and 100 MHz.

6. Conclusion

In summary, we have investigated the optical response properties of a hybrid electro-optomechanical system in the presence of a qubit coupled to the mechanical oscillator via the Jaynes–Cummings interaction. The mean-field optical bistability analysis shows that the proposed system displays the typical, optical switching characteristics which can be tuned to function at low input power. The fluctuation dynamics reveal a series of interesting optical effects in the probe spectrum. In the sideband resolved regime ($\omega_p < \kappa_a, \omega_p > \kappa_a$), the absorption profile exhibits double OMIT while the dispersion profile shows negative group velocity (anomalous dispersion). On the other hand, in the resolved sideband regime ($\omega_p > > \kappa_a, \kappa_c$), a three-peak OMIA effect is seen along with anomalous dispersion. Thus the system can switch between OMIT and OMIA by tuning either the frequency of the mechanical oscillator or the optical cavity decay rate. Further, the appearance of NMS shows tunable coherent energy exchange between the various subsystems. These interesting optical properties are sensitive to the variations in $G_{om}$, $G_{om}$ and $g$. This experimentally feasible multi-transparency and multi-absorption phenomena allow the possibility for the realization of optical comb based on hybrid electro-optomechanical system interacting with a qubit. Our theoretical proposal provides a platform for novel quantum photonic devices.

Author contributions

A.B.B conceived the theoretical model. T.K. and S.Y. performed the calculations and plotted the graphs. S.Y. analysed and discussed the results with T.K. and A.B.B. S.Y. wrote the manuscript under the supervision of A.B.B. This work forms a part of the Ph.D. thesis of S.Y.

Disclosure statement

The authors declare that they have no conflict of interest.

References

[1] Poot M, Vanderzant HSJ. Mechanical systems in the quantum regime. Phys Rep. 2012;511:273–335.

[2] Blencowe MP. Quantum electromechanical systems. Phys Rep. 2004;395:159–222.

[3] Aspelmeyer M, Groblacher S, Hammerer K, et al. Quantum optomechanics—throwing a glance [Invited]. J Opt Soc Am B. 2010;27:A189.

[4] Kippenberg TJ, Vahala KJ. Cavity optomechanics: back-action at the mesoscale. Science. 2008;321:1172.1176.

[5] Aspelmeyer M, Meyrath P, Schwab K. Quantum optomechanics. Phys Today. 2012;65:29–35.

[6] Rugar D, Budakian R, Mamin HJ, et al. Single spin detection by magnetic resonance force microscopy. Nature (London). 2004;430:329–332.

[7] Irish EK, Schwab K. Quantum measurement of a coupled nanomechanical resonator–Cooper-pair box system. Phys Rev B. 2003;68:155311.

[8] Armour AD, Blencowe MP, Schwab KC. Entanglement and decoherence of a micromechanical resonator via coupling to a cooper-Pair box. Phys Rev Lett. 2002;88:148301.

[9] Sornborger AT, Cleland AN, Geller MR. Superconducting phase qubit coupled to a nanomechanical resonator: beyond the rotating-wave approximation. Phys Rev A. 2004;70:052315.

[10] Zhang P, Wang YD, Sun CP. Cooling mechanism for a nanomechanical resonator by periodic coupling to a cooper pair box. Phys Rev Lett. 2005;95:097204.

[11] Xue F, Wang YD, Sun CP, et al. Controllable coupling between flux qubit and nanomechanical resonator by magnetic field. New J Phys. 2007;9:35.

[12] Cleland AN, Geller MR. Superconducting Qubit storage and entanglement with nanomechanical resonators. Phys Rev Lett. 2004;93:070501.

[13] Tian L. Entanglement from a nanomechanical resonator weakly coupled to a single cooper-pair box. Phys Rev B. 2005;72:195411.

[14] Wei LF, Liu YX, Sun CP, et al. Probing tiny motions of nanomechanical resonators: classical or quantum mechanical?. Phys Rev Lett. 2006;97:237201.

[15] LaHaye MD, Suh J, Eckernach PM, et al. Nanomechanical measurements of a superconducting qubit. Nature (London). 2009;459:960–964.

[16] Xue F, Wang YD, Liu YX, et al. Cooling a micromechanical beam by coupling it to a transmission line. Phys Rev B. 2007;76:205302.

[17] Li Y, Wang Y-D, Xue F, et al. Quantum theory of transmission line resonator-assisted cooling of a micromechanical resonator. Phys Rev B. 2008;78:134301.

[18] Rocheleau T, Ndukum T, Macklin C, et al. Preparation and detection of a mechanical resonator near the ground state of motion. Nature (London). 2010;463:72–75.

[19] Hertzberg JB, Rocheleau T, Ndukum T, et al. Back-action-evading measurements of nanomechanical motion. Nat Phys. 2010;6:213–217.

[20] Massel F, Heikkila TT, Pirkkalainen J-M, et al. Microwave amplification with nanomechanical resonators. Nature (London). 2011;480:351–354.

[21] Arcizet O, Jacobs V, Siria A, et al. A single nitrogen-vacancy defect coupled to a nanomechanical oscillator. Nat Phys. 2011;7:879–883.

[22] Kolkowitz S, Jayich ACR, Unterreithmeier QB, et al. Coherent sensing of a mechanical resonator with a single-Spin Qubit. Science. 2012;335:1603–1606.
[23] Bennett SD, Yao NY, Otterbach J, et al. Phonon-induced spin–spin interactions in diamond nanostructures: application to spin squeezing. Phys Rev Lett. 2013;110:156402.

[24] Liu YX, Miranowicz A, Gao YB, et al. Qubit-induced phonon blockade as a signature of quantum behavior in nanomechanical resonators. Phys Rev A. 2010;82:032101.

[25] O’Connell AD, Hofheinz M, Ansmann M, et al. Quantum ground state and single-phonon control of a mechanical resonator. Nature (London). 2010;464:697–703.

[26] Teufel JD, Donner T, Li D, et al. Sideband cooling of micromechanical motion to the quantum ground state. Nature (London). 2011;475:359–363.

[27] Chan J, Alegre TPM, Safavi-Naeini AH, et al. Laser cooling of a nanomechanical oscillator into its quantum ground state. Nature (London). 2011;478:89–92.

[28] Verhagen E, Deleglise S, Weis S, et al. Quantum-coherent coupling of a mechanical oscillator to an optical cavity mode. Nature (London). 2012;482:63–67.

[29] Wang YD, Clerk AA. Using interference for high fidelity quantum state transfer in optomechanics. Phys Rev Lett. 2012;108:153603.

[30] Dong CH, Fiore V, Kuzyc MC, et al. Optomechanical dark mode. Science. 2012;338:1609–1613.

[31] Giner L, Veissier L, Sparkes B, et al. Experimental investigation of the transition between Autler–Townes splitting and electromagnetically-induced-transparency models. Phys Rev A. 2013;87:013823.

[32] Weis S, Rivière R, Delégis S, et al. Optomechanically induced transparency. Science. 2010;330:1520–1523.

[33] Agarwal GS, Huang S. Electromagnetically induced transparency in mechanical effects of light. Phys Rev A. 2010;81:041803.

[34] Fiore V, Yang Y, Kuzyc MC, et al. Storing optical information as a mechanical excitation in a silica optomechanical resonator. Phys Rev Lett. 2011;107:133601.

[35] Chang DE, Safavi-Naeini AH, Hafizi M, et al. Slowing and stopping light using an optomechanical crystal array. New J Phys. 2011;13:023003.

[36] Fiore V, Dong CH, Kuzyc MC. Optomechanical light storage in a silica microresonator. 2013. Available from: arXiv:1302.0557.

[37] Treutlein P, Genes C, Hammerer K, et al. Hybrid mechanical systems. In M. Aspelmeyer M, editor. Cavity QED, quantum science and technology, Vol. 327, Berlin Heidelberg: Springer-Verlag; 2014. p. 327–351.

[38] Gao M, Liu YX, Wang XB. Coupling Rydberg atoms to superconducting qubits via nanomechanical resonator. Phys Rev A. 2011;83:022309.

[39] Sun CP, Wei LF, Liu YX, et al. Quantum transducers: integrating transmission lines and nanomechanical resonators via charge qubits. Phys Rev A. 2006;73:022318.

[40] Pirkkalainen J-M, Cho SU, Li J, et al. Hybrid circuit cavity quantum electrodynamics with a micromechanical resonator. Nature (London). 2013;494:211–215.

[41] Moore GT. Quantum theory of the electromagnetic field in a variable-Length one-dimensional cavity. J Math Phys. 1970;11:2679.2691.

[42] Yoshie T, Sherer A, Hendrickson J, et al. Vacuum Rabi splitting with a single quantum dot in a photonic crystal nanocavity. Nature. 2004;432:200.203.

[43] Reithmaier JP, Sek G, Loffler A, et al. Strong coupling in a single quantum dot-semiconductor microcavity system. Nature. 2004;432:197.200.

[44] Peter E, Senellart P, Martrou D, et al. Exciton-photon strong-coupling regime for a single quantum dot embedded in a microcavity. Phys Rev Lett. 2005;95:067401.

[45] Imamoglu A, Awschalom DD, Burkard G, et al. Quantum information processing using quantum dot spins and cavity QED. Phys Rev Lett. 1999;83:4204–4207.

[46] Bouwmeester D, Zeilinger A. The physics of quantum information: Basic concepts. In: Bouwmeester D, Ekert A, Zeilinger A, editors. The physics of quantum information. Berlin, Heidelberg: Springer; 2000. https://doi.org/10.1007/978-3-662-04209-0_1

[47] Englund D, Majumdar A, Bajcsy M, et al. Ultrafast photon–photon interaction in a strongly coupled quantum dot–cavity system. Phys Rev Lett. 2012;108:093604.

[48] Majumdar A, Bajcsy M, Englund D, et al. Optical switching with a single quantum dot strongly coupled to a photonic crystal cavity. IEEE J Selected Topics in Quant Elect. 2012;18:1812–1817.

[49] Bose R, Sridharan D, Kim H, et al. Low-photon-number optical switching with a single quantum dot coupled to a photonic crystal cavity. Phys Rev Lett. 2012;108:227402.

[50] Rossi M, Kralj N, Zippilli S, et al. Normal-mode splitting in a variable-Length one-dimensionalcavity. J Math Phys. 2013;54:023003.

[51] Reithmaier JP, Sek G, Loffler A, et al. Strong coupling in a single quantum dot-semiconductor microcavity system. Nature. 2004;432:197.200.

[52] Peter E, Senellart P, Martrou D, et al. Exciton-photon strong-coupling regime for a single quantum dot embedded in a microcavity. Phys Rev Lett. 2005;95:067401.
[62] Wu SC, Qin LG, Jing J, et al. Microwave-controlled optical double optomechanically induced transparency in a hybrid piezo-optomechanical cavity system. Phys Rev A. 2018;98:013807.

[63] Wang H, Gu X, Liu YX, et al. Optomechanical analog of two-color electromagnetically induced transparency: Photon transmission through an optomechanical device with a two-level system. Phys Rev A. 2014;90:023817.

[64] Brillouin L. Wave propagation and group velocity. London: Academic; 1960.

[65] Safavi-Naeini AH, Mayer Alegre TP, Chan J, et al. Electromagnetically induced transparency and slow light with optomechanics. Nature. 2011;472:69–73.

[66] Yu C, Yang W, Sun L, et al. Controllable transparency and slow light in a hybrid optomechanical system with quantum dot molecules. Opt Quant Electronics. 2020;52:1991.

[67] Wu H, Xiao M. Strong coupling of an optomechanical system to an anomalously dispersive atomic medium. Laser Phys Letts. 2014;11:126003.

[68] Qin J, Zhao C, Ma Y, et al. Linear negative dispersion with a gain doublet via optomechanical interactions. Opt Letts. 2015;40:2337.

[69] Akram MJ, Khan MM, Saif F. Tunable fast and slow light in a hybrid optomechanical system. Phys Rev A. 2015;92:013812.

[70] Chang DE, Safavi-Naeini AH, Hafezi M, et al. Slowing and stopping light using an optomechanical crystal array. New J Phys. 2011;13:023003.

[71] Prakash VN, Bhattachjeriee AB. Negative effective mass, optical multistability and Fano line-shape control via mode tunnelling in double cavity optomechanical system. Phys Letts. 2014;11:126003.

[72] Zhang Y, Liu T, Wu SX, et al. Optical response mediated by a two-level system in the hybrid optomechanical system. Quant Inf Processing. 2018;17:1172.

[73] Wang T, Zheng MH, Bai CH, et al. Normal-Mode splitting and optomechanically induced absorption, amplification, and transparency in a hybrid optomechanical system. Ann Phys (Berlin). 2018;530:1800228.

[74] Liao Q, Xiao X, Nie W, et al. Transparency and tunable fast-light in a hybrid cavity optomechanical system. Opt Express. 2020;28:5288.

[75] Yuan XZ. Entangling an optical cavity and a nanomechanical resonator beam by means of a quantum dot. Phys Rev A. 2013;88:052317.

[76] Barbhuiya SA, Bhattachjeriee AB. Quantum optical response of a hybrid optomechanical device embedded with a qubit. J Opt. 2020;22:11540.

[77] Zhang W, Qin L-G, Tian L-J, et al. Multiple induced transparency in a hybrid driven cavity optomechanical device with a two-level system. Chinese Phys B. 2021;30:094203.

[78] Batten C., Joshi A., Orcutt J., et al. Building many-Core processor-to-DRAM networks with monolithic CMOS silicon photonics. IEEE Micro. 2009;29:8–21.

[79] Liu K. Review and perspective on ultrafast wavelength-size electro-optic modulators. Laser Photon Rev. 2015;9:172.

[80] Chaisakul P, Marris-Morini D, Roufied M-S, et al. Recent progress in GeSi electro-absorption modulators. Sci Technol Adv Mater. 2014;15:014601.

[81] Miller DAB. Device requirements for optical interconnects to silicon chips. Proc IEEE. 2009;97:1166–1185.

[82] Cardona M, Pollak FH. Energy-Band structure of Germanium and silicon: the k-p method. Phys Rev. 1966;142:530–543.

[83] Qin Li-G, Wang Z-Y, Gong S-Q, et al. Electro-optic waveform interconnect based on quantum interference. Photon Research. 2017;5:481.

[84] Teufel JD, Li D, Allman MS, et al. Circuit cavity electromechanics in the strong-coupling regime. Nature (London). 2011;472:204–208.

[85] Kranz P, Kjaergaard M, Yan F, et al. A quantum engineer’s guide to superconducting qubits. Appl Phys Rev. 2019;6:021318.

[86] Ramos T, Sudhir Y, Stannigel K, et al. Nonlinear quantum optomechanics via individual intrinsic two-Level defects. Phys Rev Lett. 2013;110:193602.

[87] Xiao X-J, Tan Y, Guo Q-Q, et al. Dual-channel bistable switch based on a monolayer graphene nanoribbon nanoresonator coupled to a metal nanoparticle. Optics Express. 2020;28:3136.

[88] Bamiedakis N, Williams KA, Penty RV. Integrated and hybrid photonics for high-performance interconnects in optics and photonics. In: Kaminow IP, Li T, Willner AE, editors. Optical fiber telecommunications. 6th ed. Academic Press; 2013. p. 377-418.

[89] Zhang W, Qin L-G, Wang Z-Y, Gong S-Q, et al. Electro-optic waves in piezo-optomechanical circuits. Nat Photonics. 2016;10:346–352.

[90] Balram KC, Davanco MI, Song JD, et al. Coherent coupling between radiofrequency, optical and acoustic waves in piezo-optomechanical circuits. Nat Photonics. 2016;10:346–352.

[91] Megrant A., Neill C., Barends R., et al. Planar superconducting resonators with internal quality factors above one million. Appl Phys Lett. 2012;100:113510.

[92] Cardona M, Pollak FH. Energy-Band structure of Germanium and silicon: the k-p method. Phys Rev. 1966;142:530–543.

Appendix 1

\[
A_{\omega}(\omega) = \frac{2G_{\text{em}}\Delta_{\omega}^{2}}{d_{1}(\omega)} \left\{ \begin{array}{c}
\left( \omega_{g} + \frac{g_{C}^{2}g_{C}g_{D}}{\left(\omega - \omega_{g}/2\right)^{2} + \omega_{c}^{2}/4} \right) \frac{\Delta_{\omega}}{\omega_{c}} \\
\left( -\omega_{g} + \omega_{g}/2 - \frac{g_{C}^{2}g_{C}g_{D}}{\left(\omega + \omega_{g}/2\right)^{2} + \omega_{c}^{2}/4} \right) \frac{\Delta_{\omega}}{\omega_{c}} \\
\frac{\Delta_{\omega}}{-\omega_{g} + \omega_{g}/2}
\end{array} \right\}
\]

(A1)

\[
A_{\omega}(\omega) = \frac{2G_{\text{em}}\Delta_{\omega}^{2}}{d_{2}(\omega)} \left\{ \begin{array}{c}
\left( \omega_{g} + \frac{g_{C}^{2}g_{C}g_{D}}{\left(\omega - \omega_{g}/2\right)^{2} + \omega_{c}^{2}/4} \right) \frac{\Delta_{\omega}}{\omega_{c}} \\
\left( -\omega_{g} + \omega_{g}/2 - \frac{g_{C}^{2}g_{C}g_{D}}{\left(\omega + \omega_{g}/2\right)^{2} + \omega_{c}^{2}/4} \right) \frac{\Delta_{\omega}}{\omega_{c}} \\
\frac{\Delta_{\omega}}{-\omega_{g} + \omega_{g}/2}
\end{array} \right\}
\]

(A1)
\[ A_{26}(\omega) = \frac{2G_{am}\Delta'_a}{d_{1}(\omega)} \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \frac{\omega_z}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} \right\} - 1 \right) , \]  
\[ A_{27}(\omega) = \frac{2G_{am}\Delta'_a}{d_{1}(\omega)} \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) - \frac{\omega_z}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} \right\} \right) , \]  
\[ A_{28}(\omega) = \frac{2G_{am}\Delta'_a g(-\omega + \gamma_d/2)}{d_{1}(\omega)((-\omega + \gamma_d/2)^2 + \alpha_d^2/4)} \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} , \]  
\[ A_{29}(\omega) = \frac{2G_{am}\Delta'_a}{d_{1}(\omega)} \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} \times \frac{-\omega + \lambda_b/2}{\left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \times \frac{-\omega + \lambda_b/2}{\left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} - \frac{\omega_z}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} \right\} \right\} \right) , \]  
\[ A_{30}(\omega) = \frac{2G_{am}\Delta'_a}{d_{1}(\omega)} \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} \times \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \times \frac{-\omega + \lambda_b/2}{\left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} - \frac{\omega_z}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} \right\} \right\} \right) , \]  
\[ A_{31}(\omega) = \frac{2G_{am}\Delta'_a}{d_{1}(\omega)} \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} \times \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \times \frac{-\omega + \lambda_b/2}{\left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} - \frac{\omega_z}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} \right\} \right\} \right) , \]  
\[ A_{32}(\omega) = \frac{2G_{am}\Delta'_a}{d_{1}(\omega)} \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} \times \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \times \frac{-\omega + \lambda_b/2}{\left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} - \frac{\omega_z}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} \right\} \right\} \right) . \]  
\[ d_{1}(\omega) = \frac{-\omega + \kappa_c/2}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} - \frac{\omega_z}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} + \frac{g_{em}^2(-\omega + \kappa_c/2)^2 + \Delta_c^2}{(-\omega + \kappa_c/2)^2 + \Delta_c^2} \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} \times \left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \times \frac{-\omega + \lambda_b/2}{\left\{ \left( \frac{\omega_b + \frac{g^2\omega_{d}\sigma_d}{(-\omega+\gamma_d/2)^2+\alpha_d^2/4} - \frac{g_{em}^2\Delta_{c}}{(-\omega+\kappa_c/2)^2+\Delta_c^2}}{2(-\omega+\gamma_d/2)^2+\alpha_d^2/4} \right) \right\} - \frac{\omega_z}{(-\omega + \gamma_d/2)^2 + \alpha_d^2/4} \right\} \right\} \right\} . \]