Modified Morris–Lecar neuron model: effects of very low frequency electric fields and of magnetic fields on the local and network dynamics of an excitable media

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Abstract A Morris–Lecar neuron model is considered with electric and magnetic field effects where the electric field is a time-varying sinusoid and magnetic field is simulated using an exponential flux memristor. We have shown that the exposure to electric and magnetic fields have significant effects on the neurons and has exhibited complex oscillations. The neurons exhibit a frequency-locked state for the periodic electric field and different ratios of frequency-locked states with respect to the electric field frequency is also presented. To show the impact of the electric and magnetic fields on network of neurons, we have constructed different types of network and have shown the network wave propagation phenomenon. Interestingly, the nodes exposed to both electric and magnetic fields exhibit more stable spiral waves compared to the nodes exhibited only to the magnetic fields. Also, when the number of layers is increased, the range of electric field frequency for which the layers exhibit spiral waves also increases.

Keywords Neuron field effects · Bifurcation · Spiral waves

1 Introduction

Neurons can generate, transmit, and receive signals to and from brains by firing several types of electrical activities. Based on the firing patterns and frequency–current curves, the excitability of neurons can be classified [1] with classes 1, 2 and 3 which is more valuable for investigating behaviour of neuronal system under different scenarios. Class 3 neurons do not show any spiking behaviour, whereas classes 1 and 2 exhibit firing frequency, and the literature [1] proved Morris–Lecar (ML) model is a fruitful test ground for bifurcation analysis of these types. The repetitive firing fashion of such neurons leads to saddle node bifurcation into subcritical Hopf bifurcation. The firing property is much influenced with stimulating current for a particular range, and ML model exposes Bogdanov–Takens bifurcation. In [2], different membrane excitability is introduced into ML model and revealed the bistability property. While the coupling strength increased, more limit cycles are formed beyond the Hopf bifurcations which directly affect
the complexity of the system. The coupling strength plays a vital role on bifurcation analysis as well as in synchronization stability. The neurophysiological studies in neuron network discloses some intricate behaviours such as spiking, bursting, transition of modes, extreme multistability, synchronous transition. On the other hand, network patterns can be portrayed with considering electromagnetic radiation on conventional neuron models. Hou et al. studied the firing fashion of Hindmarsh–Rose (HR) model and revealed the influence of external electric field on synchronisation between two neurons [3]. The network behaviours of Fitz Hugh–Nagumo (FHN) neurons with low-frequency signals investigated and mono-/bi-resonances are observed [4]. An increment in low-frequency signal significantly induces the periodic bursting patterns into chaotic patterns, the study conducted in a hybrid model (HR and Wilson’s model) confirms this transition, and the proved external electromagnetic stimulus plays important role in it [5]. Coupling effects are investigated in magnetic flux-controlled memristor-based models and portrayed the substantial changes in firing patterns [6].

Based on the electromagnetic field theory, the significant effect of electromagnetic radiation can be defined with magnetic flux. The coupling between magnetic flux and membrane potential influences network behaviour. With these considerations, memristor-based models were generated [7–9]. The effect of electromagnetic induction on synchronization, spatiotemporal behaviour of coupled neurons and neuron networks is extensively studied in [10–14]. The appearance of different spatiotemporal patterns confirms the complexity in dynamical behaviour of the system. Spatiotemporal patterns can coordinate oscillation phases in neuronal network arrays and identified potentials to bind sensory information over a population of neurons. Spiral waves are an elegant pattern observed in many biological systems, and experiments with voltage-sensitive dye imaging exposed the formulation of spiral waves in neocortical slices [15, 16]. Hence, investigating the formation and growth of spiral waves becomes a fertile field in network dynamics. Meanwhile, the need for control scheme to suppress such patterns also increased. Some effective control algorithms are developed and discussed in [17–22]; most of them deal with the excitation media. The studies demonstrate how chaotic spatiotemporal behaviours appear for different levels of electromagnetic field. The external electromagnetic field boosts mean firing rate on the neuron network and interrupts the synchronization scheme. Spatiotemporal patterns can be suppressed to a homogeneous resting state with altering the exposed electromagnetic field. Higher values of magnetic flux result with decrement in duration and width of the oscillatory pattern. However, chaotic pattern and propagation fashion are so sensitivity to the initial conditions.

From the above discussion, the research spectrum is narrow down to ML model influenced with electromagnetic effect to study the synchronization and network behaviours of neuronal systems. Experimental studies conducted on humans and animals exposed to two types of electromagnetic field: (i) extremely low frequency electromagnetic field (ELF-EF) generally produced by electrical and electronic gadgets and (ii) radio frequency magnetic fields (RF-MF) produced by wireless devices such as mobile and transmission towers. Low-frequency time-varying magnetic field-induced currents can alter neuroprocessing in the central nervous system [23, 24]. However, the induced currents are very weak, and networks of neurons are more sensitive to weak fields compared with isolated neurons. In this paper, we considered a modified Morris–Lecar (ML) neuron model supplied with low-frequency electromagnetic field, and investigate dynamic behaviours particularly network dynamics and wave propagation in multi-layer neuron topology.

2 The modified Morris–Lecar neuron model

Using the same notation as in [23], the modified Morris–Lecar Neuron model whose electrical equivalent is shown in Fig. 1, incorporating an electric and magnetic field effects, can be derived as

\[
\begin{align*}
\dot{V} & = G_{\text{ca}} M_{\infty} (V_{\text{ca}} - V - V_e) + G_i W (V_k - V - V_e) \\
& + G_L (V_L - V - V_e) - I - k_0 \rho (\phi) V \\
\dot{W} & = \tau_{\infty} (W_{\infty} - W) \\
\dot{I} & = \epsilon (V_0 + V) \\
\dot{\phi} & = k_1 V - k_2 \phi
\end{align*}
\] (1)

where \(V_e\) is the external electric field effect and the parameters required for the simulation are used from
and the electric field is \( V_e = E \sin(2\pi ft) \). The other functions are defined as

\[
\begin{align*}
M_\infty &= 0.5 \left( 1 + \tanh \left( \frac{V - V_1}{V_2} \right) \right); \\
W_\infty &= 0.5 \left( 1 + \tanh \left( \frac{V - V_3}{V_4} \right) \right); \\
\tau_\infty &= \frac{1}{\sigma} \cosh \left( \frac{V - V_5}{2V_4} \right); \\
\rho(\phi) &= x + 3b^2
\end{align*}
\]

(2)

The fixed points of Eq. (1) are obtained by setting the time derivatives to zero, so that

\[
\begin{align*}
V &= -V_0; \quad W = W_\infty; \quad \phi = -k_1V_0 \\
I &= G_{ca}M_\infty(V_{ca} - V - V_e) + G_k(V_k - V - V_e) \\
&\quad + G_L(V_L - V - V_e) - k_0\rho(\phi)V_e
\end{align*}
\]

(3)

There is therefore only one fixed point, which is obtained by substituting the equilibrium values for \( V, W \) and \( \phi \) into the equation for \( I \). Its linear stability is then found by computing the fourth order Jacobian matrix: associated with the right-hand side (RHS) of (1). This procedure was described in some detail in [23]. Here our objectives are different. We are interested in the consequences of modifying the potential \( V \) with a low-frequency periodic oscillation.

### 2.1 Constant applied electric field

Following [25], we firstly consider the case of a constant electric field of small amplitude, by taking \( V_e = 0.1 \) to be a constant. If we then plot a bifurcation transition diagram of \( V_{\text{max}} \) as \( k_0 \) increases, we lose all the chaotic states, shown in Fig. 4a of. The following figure shows the outcome:

There is still a supercritical Hopf bifurcation, but for a different value for \( k_0 \) (for \( k_0 \approx 4.2 \)). When we repeat the numerical analysis for \( E = 0.5 \), the system evolves to a steady state:

\[
(V, W, I, \phi) = (-0.2, 0.61441759E - 0.5, -0.91246803E - 0.01, -0.04)
\]

We now replace \( V_e \) by a sinusoidally varying electric field. Following Che et al. [25], the forcing term is \( V_e = E \sin(2\pi ft) \).

### 2.2 Externally applied sinusoidally electric field

There are therefore two parameters of interest in this system: the frequency \( f \) of the electrical forcing and its amplitude \( E \). We relate \( \omega \) to \( f \) via \( \omega = 2\pi f \). All of the following bifurcation transition diagrams are as \( f \) varies. For very low frequency field (ELF), we take \( 0 \leq f \leq 5 \) and \( 0 \leq f \leq 10 \).

Figure 2 shows a bifurcation transition diagram as the frequency \( f \) increases in terms of \( V_{\text{max}} \) for fixed amplitude \( E = 0.1 \). The top panel shows part of the diagram for \( 0 \leq f \leq 1 \). For \( f > 1 \) (not shown here), we found mainly chaotic behaviour apart from a few isolated windows of periodic states, e.g. at \( f = 2.25, f = 2.405 \) and \( f = 2.435 \). The lower panel of Fig. 2 shows an enlargement of the region between \( 0 \leq f \leq 0.3 \).
There are a number of periodic windows in Fig. 3, which represent frequency-locked states. Figure 4 shows a selection of six examples of $V$ from Fig. 3b, together with the corresponding forcing function $V_e$ for $0.03 \leq f \leq 0.185$. We show examples of (a) a frequency-locked 16:2 state for $f = 0.03$ (b) a 12:2 locked state for $f = 0.04$ (c) a 8:2 locked state for $f = 0.07$ (d) a 2:1 locked state for $f = 0.12$ (e) 3:2-
locked state for $f = 0.145$ and (f) a 10:8-locked state for $f = 0.185$.

In Fig. 4a–f, the forcing function $V_e$ is shown in red, while the behaviour of $V$ is shown in blue. We show longer sections of each time series to illustrate the frequency locking. In Fig. 4a–c, two forcing frequency cycles correspond to 16, 12 and 18 oscillations, respectively, of $V$. Figure 4d shows a frequency-locked state, and Fig. 4f shows a more complicated 10:8 frequency-locked state.

If we increase $E$ to $E = 0.5$, we obtain the bifurcation transition diagram shown in Fig. 5. The regions of chaotic behaviour for $f < 1$ have been replaced by periodic windows, apart from a small region between $0.3 \leq f \leq 0.4$. When $E = 1$, the upper panel of Fig. 6 shows the behaviour for $0 \leq f \leq 5$, while the lower panel shows a blow-up of the region for $0 \leq f \leq 0.8$.

Figure 6 shows the corresponding scenario for $E = 1$. The upper panel shows the range from $0 \leq f \leq 5$. There is a large region of periodic states for $0 \leq f \leq 3.165$. The middle panel shows part of this region for $0 \leq f \leq 0.6$. The lower panel shows the range $3 \leq f \leq 4$. After the period-3 and period-4 cycles, there is a region of quasi-periodic states for $3.165 \leq f \leq 3.9$.

Figures 7, 8, 9 and 10 show an examples of time series (in blue) and forcing frequency (in red) for selected choices of $f$ from Fig. 6. Figure 7 shows an example of a 16:1 frequency-locked state for $f = 0.05$ and $E = 0.1$. Again, the forcing function is shown in red and the time series for $V$ is shown in blue. While the forcing function has a regular periodic character, the behaviour of $V$ is pulsed.

Figure 8 shows an example of a quasi-periodic state for $f = 3.5$. The upper panel shows the time series for $V(t)$, while the lower panel shows the forcing term. Figure 9 shows a selection of phase portraits for $f = 3.5$. Figure 9a–c shows evidence of the quasi-periodicity, while the projection into the $(I, \phi)$-plane appears periodic. Finally Fig. 10 shows an example of a chaotic state for $f = 4$.

We show a selection of phase portraits in different projected planes, together with part of the time series for $V(t)$ in Fig. 10b. Figure 10a shows the $(V, W)$ dynamics, Fig. 10c shows the $(\phi, I)$ dynamics, while Fig. 10d shows the $(W, I)$ dynamics.

3 Wave propagation in the ML-EM network

To completely investigate the network dynamics of the ML-EM network, we constructed lattice arrays with single and multiple layers. To be more specific, we investigated the wave propagation phenomenon in single-, two- and three-layer networks. Under each network, we investigate two cases with the first case considering only electric field effects and the second case considering both electric and magnetic fields. The stimuli force applied to the network is considered periodic ($F(t) = A \sin(\Omega t)$) with amplitude $A = 0.1$ and frequency $\Omega = 0.01$ fixed for all the investigations. Further, the network is considered to have no flux boundaries and the stimuli force is applied to the network from the left boundary. The ML-EM system parameters are considered as in [23] with only the electrical field frequency and amplitude considered as control parameters. For all three network types, we have used the RK4 method for the numerical analysis with step size $h = 0.01$ and simulation time of 5000 s.

3.1 Network performance in a single-layer network:

The first scenario of investigation is the two-dimensional lattice array of ML-EM neurons as shown in (4).
\[ CV_{ij} = G_{Ca}C_{a} (V_{Ca} - V_{ij} - V_{e}) + G_{K} \theta_{ij} (V_{K} - V_{ij} - V_{e}) + G_{L} (V_{L} - V_{ij} - V_{e}) - S_{ij} - k_{0} W (\phi_{ij}) V_{ij} + C (V_{ij}) + F(t) \beta_{i0} \beta_{j0} \]

\[ \dot{\theta}_{ij} = \tau_{\theta} (K_{\infty} - \theta_{ij}) \]

\[ \dot{S}_{ij} = \mu (V_{0} + V_{ij}) \]

\[ \dot{\phi}_{ij} = k_{1} V_{ij} - k_{2} \phi_{ij} \]

(4)

The stimulus \( F(t) \) is applied to the network in the left boundary using the conditions \( \beta_{i0} = 1, \beta_{j0} = 1 \) for \( [i,j] = [110,0] \). The nearest neighbour coupling function is defined as \( C (V_{ij}) = \sigma (V_{i+1j} + V_{i-1j} - V_{i,j+1} + V_{i,j-1} - 4V_{ij}) \) with \( \sigma \) being the diffusion parameter, and in this case we keep it as \( \sigma = 1 \).

In this section, we will investigate two cases wherein in the first case we consider that the neurons are exposed to only electric field effect which minimises Eq. (1) to the first three states with \( k_{0} = 0 \). In the second case, we consider that the neuron is exposed to both electric and magnetic fields for which all four state variables are considered with \( k_{0} = 0.1 \). As the main aim of this

Fig. 6 Bifurcation transition plot of \( V_{\text{max}} \) as frequency \( f \) increases for forcing amplitude \( E = 1 \). a for \( 0 \leq f \leq 5 \); b a section for \( 0 \leq f \leq 0.8 \); c a section for \( 3 \leq f \leq 4 \)

Fig. 7 A 16:1 frequency-locked state for \( E = 1 \) and \( f = 0.05 \)

Fig. 8 A section of the time series for \( V \), showing a quasi-periodic state for \( E = 1 \) and frequency \( f = 3.5 \). The upper panel shows \( V \), while the lower panel shows the forcing frequency

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Fig. 9 Phase portraits for the quasi-periodic state for $E = 1$ and frequency $f = 3.5$

Fig. 10 Phase portraits for the chaotic state for $E = 1$ and frequency $f = 4$
research is to study the electric field effects on the ML neuron model, we consider the frequency ($\omega$) of the periodic electric field $V_e = E\sin(\omega t)$ as the control parameter.

In Fig. 11, we investigate the wave propagation in the lattice network (4) considering the exposure of the nodes to only the periodic electric field. As defined, the stimuli force is applied to the left boundary of the network for a brief period of 100 s–1000 s. For the values of the frequency $\omega < 0.95$, all the nodes in the network go unstable and the wave propagation was not supported. When the frequency of the electric field is increased to $\omega = 0.95$, most of the nodes in the network start oscillations except few nodes which are idle. This is because of these nodes have been driven out of the spiking/bursting regions of initial conditions due to the applied stimuli force. Such situations are often seen due to the sharp change in the conductance of sodium ions in the neurons [24]. By increasing the frequency of electric field to $\omega = 0.98$, all the nodes in the network are now in oscillation mode and thus we could show the multiarmed spiral waves with smaller radius of rotation occupying the entire network. Such spiral waves are seen in the range $0.98 \leq \omega \leq 1$, but when the frequency is increased to $\omega = 1.05$, some of these spiral waves start breaking into multiple smaller spiral seeds, while those with longer arms combine together to form spiral waves with very less frequency of rotation and larger radii. For $\omega = 1.1$, the smaller radii spiral seeds are destroyed, while those with longer arms diminish and by increasing the frequency to $\omega = 1.2$ they get destroyed and thus the spiral waves are no longer seen in the network. Thus the effective spiral wave range is $0.98 \leq \omega \leq 1.1$.

In the second case of this investigation, we consider that the neurons are exposed to both electric and magnetic fields. The magnetic field coupling strength $k_0 = 0.1$ and the stimuli is applied to the left boundary of the network. Figure 12 shows the captured wave propagation snap shots of the network for various electric field excitation frequency. It is clear that the spiral waves are seen for a longer range of frequency compared to the network discussed without magnetic field (Fig. 11). This is because of the magnetic field coupling driving all the neurons into spiking/bursting type oscillations, and hence there are no idle nodes in
the network which was not the case in Fig. 11. For \( \omega = 0.9 \), the network shows spiral waves and has multiarmed spiral seeds seen in the entire network. By increasing the frequency, these small spiral seeds merge together to form long-arm spiral waves for \( \omega = 1.1 \) with only a few centres of rotation. Further increasing the frequency to \( \omega = 1.2 \), the spiral waves align towards the four corners of the network but soon starts dissipating from the right boundary as we increase the frequency to \( \omega = 1.3 \). This is because of the nodes entering idle states, and these idle states spread to all the nodes when the frequency is increased to \( \omega = 1.4 \). Thus we could confirm that the spiral waves are seen for a much longer range of frequency \( 0.9 \leq \omega < 1.4 \) against the spiral wave range for network exposed to electric field alone (Fig. 10) \( 0.98 \leq \omega \leq 1 \). Thus magnetic field can be easily used to control the spiral waves in the network by either inducing or suppressing such turbulent spiral waves.

3.2 Network performance in a two-layer network

The second investigation is on the two-layer network with each layer constructed using the lattice array of ML-EM neurons. The mathematical model of the two-layer network is given in (11) where we considered a simple inter layer coupling as discussed in [26–28].

**Upper Layer**

\[
\begin{align*}
\dot{V}_{U,ij} &= G_{Ca}a_{\infty} (E_{Ca} - V_{U,ij} - V_e) \\
&+ G_{K} \theta_{U,ij} (E_{K} - V_{U,ij} - V_e) + G_{L} (E_L - V_{U,ij} - V_e) \\
&- S_{U,ij} - k_{0} W(\phi_{U,ij}) V_{U,ij} + C (V_{U,ij}) \\
&+ D_{LU} (V_{L,ij} - V_{U,ij}) + F(i) \beta_{i0} \beta_{i1}, \\
\dot{\theta}_{U,ij} &= \tau_{0} (\theta_{U,ij} - \theta_{U,uj}) \\
\dot{S}_{U,ij} &= \mu (V_0 + V_{U,ij}) \\
\dot{\phi}_{U,ij} &= k_{1} V_{U,ij} - k_{2} \phi_{U,ij}
\end{align*}
\]

**Lower Layer**

\[
\begin{align*}
\dot{V}_{L,ij} &= G_{Ca}a_{\infty} (E_{Ca} - V_{L,ij} - V_e) \\
&+ G_{K} \theta_{L,ij} (E_{K} - V_{L,ij} - V_e) + G_{L} (E_L - V_{L,ij} - V_e) \\
&- S_{L,ij} - k_{0} W(\phi_{L,ij}) V_{L,ij} + C (V_{L,ij}) \\
&+ D_{LU} (V_{L,ij} - V_{U,ij}) + F(i) \beta_{i0} \beta_{i1}, \\
\dot{\theta}_{L,ij} &= \tau_{0} (\theta_{L,ij} - \theta_{L,uj}) \\
\dot{S}_{L,ij} &= \mu (V_0 + V_{L,ij}) \\
\dot{\phi}_{L,ij} &= k_{1} V_{L,ij} - k_{2} \phi_{L,ij}
\end{align*}
\]

where the intra-layer coupling is defined by the function

\[
C(V_{X,ij}) = V_{X,i+1,j} + V_{X,i-1,j} + V_{X,j+1,i} + V_{X,j-1,i} - 4V_{X,ij}, X \in (U, L).
\]

The inter-layer coupling strengths are defined by \( D_{LU} = D_{LU} = 0.05 \) with the stimuli forced \( F(i) \) applied to the left boundary of the layers. The other parameters are defined as in with
only the frequency of the applied electric field considered as the control parameter. Depending on the nodes which are exposed to electric field, we can subdivide the discussion into three categories as

Category-A Neurons in the upper layer are exposed to electric field.
Category-B Neurons in the lower layer are exposed to electric field.
Category-C Neurons in the both the layers are exposed to electric field.

In each category, we investigate two cases wherein the first case deals with the study of wave propagation in the network with only electric field, while in the second case we consider that the neurons are exposed to both electric and magnetic fields.

3.2.1 Neurons in the upper layer are exposed to electric field

In this subsection, we considered that the neurons in the upper layer are exposed to an electric field, while the lower-layer nodes are considered without electric field effects. Under these assumptions, we have two subcases of discussion depending on the magnetic field effects. Figure 13 shows the snapshots of the network (5) considering that the first-layer nodes are exposed to electric field effects, while the second layer is considered to have no field effects. Clearly, the multi-layer network shows signs of spiral waves for very less frequency of electric field as shown in Fig. 13. A travelling spiral wave is seen for $\omega = 0.5$ in the right boundary of both the layers; while the one in upper layer is not strong and shows signs of low-frequency spiking, the one in lower layer is strong and induces similar spiral wave in the left boundary. This spiral wave expands when the frequency is increased to $\omega = 0.8$ and when we tried to increase the simulation time to 7000 s the spiral waves expand in radius and go outside the network. By making the frequency $\omega = 1$, the spiral waves are shown in pairs from the left boundary origin, while the right top boundary shows travelling wave-like behaviour. This is because the nodes in the network undergo in chaotic bursting.

In Fig. 13, the nodes in both the layers are not exposed to magnetic field, while in Fig. 14 we have considered that the nodes in both the layers are with magnetic flux coupling effect and only the nodes of the first layer are exposed to electric field. It is clear from Fig. 14 that the magnetic field coupling induces spiral waves in the network for much lower frequencies of the applied electric field. The spiral waves are seen in both the layers for $0.1 \leq \omega \leq 0.5$, and the network shows local pools of neurons with positive spiking which are soon dissipated when the frequency is increased to $\omega = 0.8$. Hence, we could conclude that the spiral waves are exhibited for much lower frequency regions when exposed to the external

$$\omega = 0.5 \quad \omega = 0.8 \quad \omega = 1 \quad \omega = 1.2 \quad \omega = 1.3$$

Fig. 13 Wave propagation in the two-layer network (5) considering the ML-EM neurons in the upper layer is exposed to electric field, while the lower layer is considered to have no field effects. The snapshots are captured for 5000 s for different values of $\omega$.

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magnetic flux because of the nodes exhibiting chaotic bursting.

3.2.2 Neurons in the lower layer are exposed to electric field

In the previous subsection, we considered that the nodes in the upper layer are exposed to both electric and magnetic field effects. In this subsection, we considered that the nodes in the lower layer are exposed to both the fields and under this case, and we have two scenarios to investigate. In the first scenario, we consider that the nodes in upper layer are not exposed to magnetic coupling, while in the second scenario we consider that the nodes are exposed to magnetic field coupling. Figure 15 shows the snapshots of the network considering no magnetic field coupling. The network shows spiral waves for two sets of electric field frequencies. In the first set, we consider low-frequency electric field ($\omega < 0.5$) and the spiral waves are seen with asymmetric pairs of small armed spirals originating in all four corners of the first layer, while the same is seen only near the origin in the second layer. Thus, we could say that the electric field largely affects the formation of spiral waves as the lower layer which is exposed to electric field has spiral waves only near the origin. The same can be verified for $\omega = 0.5$, where the first layer has smaller spiral seeds near the origin, while the second layer is totally turbulent with no wave propagation. The first layer loses the spiral waves due to the local coupling between the layers. When the frequency is increased to $\omega = 1$, the intra-layer coupling induces spiral waves in the lower layer supressing the effects of the electric field. Further increasing the frequency to $\omega = 1.5$, the spiral waves are dissipated in both the layers which are due to the nodes existing area of chaotic bursting. For $\omega = 1.8$, we have a brief period of spiral seeds formed in the three corners of the network which are identical in both the layers. Further increasing the frequency of the electric field, the network goes into idle where the nodes are now locked to either a positive of a negative amplitude.

Having discussed about the spiral waves formed in a network where the nodes in the upper layer no field effects, we are now considering a magnetic field coupling in the nodes in both the layers. The snapshots for the network are shown in Fig. 16, and we could see that the spiral waves are formed at the centre of the network for $\omega = 0.1$. These spiral waves are seen in both the layers which are identical as against only in the first layer in a no-field effect upper layer network shown in Fig. 15. This is because of the magnetic field coupling overruling the electric field. But increasing the frequency to $\omega = 1$, these spiral seeds are dissipated. The re-emergence of spiral waves is not seen in the range $0.5 \leq \omega \leq 2.7$, but increasing the electric field frequency to $\omega = 3$, the spiral waves are formed
in both the layers, but these spiral waves are divided into symmetric pairs forming their local pools of excitation because of the increase in frequency of electric field to $\omega = 5$. Increasing the frequency to $\omega = 7$, the divided spiral waves are further pushed towards the left and right boundaries with their orientation affected by the applied stimuli. These spiral waves originating from the centre of the network are now consolidated to form a single large spiral seed which soon disappears, and the nodes go unstable when the frequency increased from 12 to 12.5.

### 3.2.3 Neurons in both layers are exposed to the electric field

In this section, we will investigate the wave propagation when both the layers are exposed to electric fields. Two subcases are considered where we consider the layers to have only electric field effects or the layers will have both electric and magnetic field effects. In Fig. 17, we considered that the nodes in the layers are exposed to only electric field effects. The nodes show random behaviour, and the applied stimuli is dissipated close to the region where it is applied from for electric field frequency $\omega \leq 0.5$. When the frequency increased from 12 to 12.5, the divided spiral waves are further pushed towards the left and right boundaries with their orientation affected by the applied stimuli. These spiral waves originating from the centre of the network are now consolidated to form a single large spiral seed which soon disappears, and the nodes go unstable when the frequency increased from 12 to 12.5.
of the applied field is increased, the network shows signs of wave propagation, and for $\omega \leq 0.8$ the network exhibits traveling wave like propagation. For $\omega = 1$, the network shows spiral waves with rotating spiral waves seen near the origin of the network and this wave travels towards the right boundary splitting into spiral waves in opposite directions. Both the layers show similar behaviour confirming that the coupling between them is intensified due to the synchrony in applied electric field. Further increase in the frequency of excitation results in the network dissipating the waves and resulting in larger pools of nodes locked to their excited voltages.

In the second case of this subsection, we consider both the layers to have electric and magnetic field effects. Figure 18 shows the snapshots of the network for various values of electric field frequency. Even for a very low frequency of $\omega = 0.001$, the network shows rotating spiral waves in the centre of both the layers and a terminal spiral wave near the right boundary. These spiral wave structures are seen up to a frequency of $\omega = 1$. When comparing Fig. 18 with Fig. 17, we could confirm that the magnetic field coupling plays a significant role in the spiral wave existence and has more effect on the network compared to the electric field intensity. When the frequency is increased further, the spiral waves are pushed to the corner of the network until $\omega = 5$ but again re-emerge from the left boundary while the spiral waves are separated into two different spiral pools in the right boundary. Further increase in the frequency drives the network to instability. Thus the range of electric field frequency supporting spiral wave propagation is larger when nodes are exposed to magnetic flux coupling.

### 3.3 Network performance in a three-layer network

In this section, we will discuss about the wave propagation in a three-layer network constructed as defined in (6). The stimuli setting are as in (5), while the intra-layer coupling is defined as $C(\mathbf{v}_{p,q}) = D_q(\mathbf{v}_{p,i+1} + \mathbf{v}_{p,i-1} + \mathbf{v}_{p,i+1} + \mathbf{v}_{p,i-1} - 4\mathbf{v}_{p,i})$ where $p \in \{U,M,L\}$ and $q \in \{1,2,3\}$. It is assumed that all the three layers are exposed to electric field, while the nodes are not exposed to magnetic field in case-a and are exposed to magnetic field in case-b. For case-a, we consider that $k_0 = 0$ and the entire model in (6) changes to a third-order system as the fourth state is not considered due to the flux effects neglected.

Figure 18 shows the snapshots of the three-layer network (6) captured for 1000 s considering only

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**Fig. 17** Wave propagation in the two-layer network (5) considering the ML-EM neurons in both the layers are exposed to electric fields. The snapshots are captured for 5000 s for different values of $\omega$
electric field effects in all the three layers. For very low frequencies of electric field $(\omega < 0.5)$, the stimuli applied to the left boundary of the network do not have much impact on the layers and hence there are no spiral seeds seen. This is because the nodes are in chaotic bursting and hence oscillate randomly. When we increase the frequency to $\omega = 1$, the nodes now are in periodic spiking resulting in spiral waves in all the three layers. Further increasing $\omega = 1.2$, some nodes in the right boundary enter a quasi-periodic state due to the applied excitation resulting in the spiral waves being pushed towards the periodic nodes in the left boundary. The nodes are periodic in the left boundary because of the applied stimuli. Slowly, this scenario changes and the nodes in the left boundary go in a quasi-idle state, while the nodes in the right boundary.....
enter periodic state and hence showing small spiral seeds for \( \omega = 1.5 \). For \( \omega = 1.8 \), two small spiral pools in opposite directions are seen in the network which are pushed towards the boundary as we increase the frequency to \( \omega = 2 \) after which further increase in the frequency drives the network to instability. Thus, by comparing Figs. 17 and 19, we could show that as the number of layers increases, the electric field frequency range affecting the wave propagation increases and thus result in hazardous spiral waves in the network.

Upper Layer

\[
\dot{V}_{U,i,j} = G_{Ca}C_{a}\infty (E_{Ca} - V_{U,i,j} - V_e) \\
+ G_K\theta_{U,i,j} (E_K - V_{U,i,j} - V_e) + G_L (E_L - V_{U,i,j} - V_e) \\
- S_{U,i,j} - k_0 \omega (\phi_{U,i,j}) V_{U,i,j} + C(V_{U,i,j}) \\
+ D_1 (V_{M,i,j} - V_{U,i,j}) + F(t) \beta_{i0} \beta_{j0},
\]

\[
\dot{\theta}_{U,i,j} = \tau_0 (K_{-} - \theta_{U,i,j}) \\
S_{U,i,j} = \mu (V_0 + V_{U,i,j}) \\
\phi_{U,i,j} = k_1 V_{U,i,j} - k_2 \phi_{U,i,j}
\]

Middle Layer

\[
\dot{V}_{M,i,j} = G_{Ca}C_{a}\infty (E_{Ca} - V_{M,i,j} - V_e) \\
+ G_K\theta_{M,i,j} (E_K - V_{M,i,j} - V_e) + G_L (E_L - V_{M,i,j} - V_e) \\
- S_{U,i,j} - k_0 \omega (\phi_{U,i,j}) V_{M,i,j} + C(V_{M,i,j}) \\
+ D_2 (V_{U,i,j} + V_{L,i,j} - 2V_{M,i,j}) + \theta (F(t) \beta_{i0} \beta_{j0},
\]

\[
\dot{\theta}_{M,i,j} = \tau_0 (K_{-} - \theta_{M,i,j}) \\
S_{M,i,j} = \mu (V_0 + V_{M,i,j}) \\
\phi_{M,i,j} = k_1 V_{M,i,j} - k_2 \phi_{M,i,j}
\]

Lower Layer

\[
\dot{V}_{L,i,j} = G_{Ca}C_{a}\infty (E_{Ca} - V_{L,i,j} - V_e) \\
+ G_K\theta_{L,i,j} (E_K - V_{L,i,j} - V_e) + G_L (E_L - V_{L,i,j} - V_e) \\
- S_{L,i,j} - k_0 \omega (\phi_{L,i,j}) V_{L,i,j} + C(V_{L,i,j}) \\
+ D_3 (V_{M,i,j} - V_{L,i,j}) + F(t) \beta_{i0} \beta_{j0},
\]

\[
\dot{\theta}_{L,i,j} = \tau_0 (K_{-} - \theta_{L,i,j}) \\
S_{L,i,j} = \mu (V_0 + V_{L,i,j}) \\
\phi_{L,i,j} = k_1 V_{L,i,j} - k_2 \phi_{L,i,j}
\]

Now consider that the neurons in all the three layers are exposed to both electric and magnetic fields, we investigate the wave propagation phenomenon by applying a periodic stimulus to the left boundary of the network as shown in Fig. 20. Interestingly, when the electric field excitation frequency \( \omega < 0.8 \), the network doesn’t show spiral wave seeds as like Fig. 8. When the frequency is increased to \( \omega = 1 \), spiral waves are seen generated from the right boundary and slowly consolidated in the centre of the layers for \( \omega = 2 \). This phenomenon of spiral wave generation and consolidation in the centre as asymmetric pairs are seen up to \( \omega = 3 \). This process reverses to left boundary when we increase the frequency to \( \omega = 4 \), and this is because of the cycles of periodic stimuli and electric field phase changes. For \( \omega > 5 \), the spiral waves are seen from the left boundary and move in the direction of the stimuli. Spiral wave pairs are seen in the centre of the network when the electric field frequency \( \omega > 7 \). Such spiral waves are not suppressed even when we tried changing the value of the flux coupling constant \( (k_0) \) confirming that the nodes are in some phase lock mode with the applied stimuli. This lock is broken when we change the stimuli frequency by a slight value to 0.02. Interestingly, the spiral waves are seen in the network for electric field frequency \( \omega = 30 \) after which the network goes unstable. Hence, we could again show that irrespective of the number of layers the magnetic field coupling drastically increases the range of electric field frequency supporting spiral waves in the network.

4 Conclusion

In our earlier work on Morris–Lecar neuron model, we have considered only magnetic field effects. But the published literature has shown that electric field effects also have significant impact on the dynamics of neurons. Hence, we modified the ML neuron model to show electric and magnetic field effects. By applying a constant electric field of small amplitude, we could show that the chaotic bursting phenomenon is completely lost. When periodic sinusoidal electric field is applied for very low frequency, the modified ML model shows cascading bifurcation. We could also show a frequency-locked state for lower frequency of electric field.

To investigate the Network dynamics of the ML-EM network, we constructed single and multiple lattice array and discussed the spatiotemporal behaviour. In a single layer lattice array, we discussed two scenarios. In the first scenario, only electric field is applied to the network and we could show the existence of spiral wave for frequency between [0.95, 1]. In the second scenario, we have applied
both electric and magnetic fields and could prove that a magnetic field does not affect the spiral waves in the network. In the second type of network, a two-layer lattice array of ML-EM neurons are considered. Under this, we have three different categories of discussion.

In Category-A, we consider that neurons in upper layer all exposed to electric field and magnetic field, while the neurons in lower layer are exposed to only magnetic field effects. By such conditions, the spiral waves are seen in the network for $0.9 \leq \omega \leq 1.2$. In the second category, the neurons in lower layer are exposed to electric and magnetic fields, while the upper layers have only magnetic field effects. Now the layer shows spiral wave only for lower frequency value of electric field. The range of frequency exhibits spiral wave in the network is $0.1 \leq \omega \leq 0.5$. Then both the layers are exposed to electric effects, and we could see that the spiral wave is seen for two different bands of electric field frequencies $0.1 \leq \omega \leq 0.5$ and $0.9 \leq \omega \leq 1.5$. When the layers are exposed to both the electric and magnetic fields, the electric field frequencies induce spiral wave ranging from a very low frequency of $\omega = 0.001$ to a frequency of $\omega = 8$. We could note that the frequency change doesn’t much affect the spiral wave formation.

To further understand the multi-layer network behaviour, we constructed a three-layer network, and in the first case all three layers are exposed only to electric field effects. A narrow range of electric field frequencies $1 \leq \omega \leq 1.8$ shows formation of spiral wave. But when we additionally applied magnetic fields, the range of $\omega$ increases to $\omega = 30$ and the network does not show much impact of $\omega$ as spiral waves are not dissipated. We could confirm that in multi-layer network electric and magnetic fields combined effect increases the range of $\omega$ supporting spiral wave formation.

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**Compliance with ethical standards**

**Conflict of Interest** The authors declare that they have no conflict of interest.

![Fig. 20 Wave propagation in the three-layer network (6) considering the ML-EM neurons in all the layers are exposed to electric and magnetic fields. The snapshots are captured for 1000 s for different values of $\omega$](image-url)
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