Dark matter or strong gravity?

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We show that Newton’s gravitational potential, augmented by a logarithmic term, partly or wholly mitigates the need for dark matter. As a bonus, it also explains why MOND seems to work at galactic scales. We speculate on the origin of such a potential.

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It is well known that the flatness of galaxy rotation curves, gravitational lensing from galaxies, and the ΛCDM model of cosmology point towards the existence of a significant amount (about 25%) of invisible matter in the Universe, also known as Dark Matter (DM). Yet, despite there being no shortage of viable theories, the true nature of DM remains elusive as ever [1]. This suggests that while one waits patiently for evidences of DM to emerge from a number of current and future experiments [2, 3], it is important to explore alternative theories that can potentially explain observations without the need for DM. In this essay, we explore one such mechanism and show that a simple addition of a logarithmic term to the Newton’s gravitational potential can precisely do that, and thereby obviate the need for DM, or at least reduce its importance in astrophysics and cosmology. Before going into the details, we first argue that this is indeed a viable proposition. Note that, to replace DM, one needs ‘more gravity’ than the attractive $1/r^2$ potential of Newton provides. Second, it must not significantly alter the Newtonian potential at ‘short’ distances, that is, at sub-galactic scales. These two conditions require any additional potential to go slower than $1/r$ which leads to the natural choice of a logarithmic potential. In the language of fields, this means that in addition to the standard Newtonian $-1/r^2$ field, there is now also a $-1/r$ field. Combining the two terms, one can now write the ‘total’ gravitational potential and force on a particle of mass $m$ due to another mass $M$ separated by a distance $r$, respectively as

$$V(r) = -\frac{GMm}{r} + \lambda Mm \ln \left(\frac{r}{r_0}\right) \quad (1)$$

$$F(r) = -\frac{GMm}{r^2} - \frac{\lambda Mm}{r} \quad (2)$$

where $\lambda$ and $r_0$ are constants, the latter having no observational consequence. For the logarithmic term to start dominating at galactic length scales $r_1 \approx 10^{21} \text{ m} \approx 30 \text{ kpc}$, i.e., setting the two terms in Eq. (2) to be approximately equal, it follows that $\lambda = G/r_1$. The new $1/r$ term in Eq. (2) immediately gives rise to flat galaxy rotation curves, as can be seen by equating it to the centrifugal force on a particle of mass $m$ in a rotation curve by a galaxy of mass $M$

$$\frac{\lambda Mm}{r} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\lambda M} = \sqrt{\frac{GM}{r_1}},$$

which is a constant for a given $M$. Plugging in $r_1 = 30 \text{ kpc}$ and $M = 10^{42} \text{ kg}$, one gets $v \approx 10^5 \text{ m/s}$, which is the typical speed of the spiral arms.

To show that the logarithmic potential explains other phenomena at the galactic scales and beyond, we now consider the gravitational lensing of light from a distant star by a galaxy (or a galaxy cluster). The total deflection
angle, for an impact parameter $b$, is now given by

$$
\delta = \frac{2}{c^2} \int \nabla_\perp V ds = \frac{2GMb}{c^2} \int_{-\infty}^{\infty} \frac{dx}{(b^2 + x^2)^{3/2} + \frac{2\lambda M b}{c^2} \int_{-\infty}^{\infty} \frac{dx}{(b^2 + x^2)}} = \frac{4GM}{bc^2} + \frac{2\lambda M\pi}{c^2} \equiv \frac{4GM'}{bc^2}
$$

(3)

where $M' = M(1 + b\pi/2r_1)$. Thus, the estimated galaxy mass $M'$ via lensing is greater than its actual mass $M$. In other words, by including the logarithmic term, one sees that the observed lensing can be explained by a much smaller amount of galactic matter, thus reducing the need for DM, or eliminating it completely. Again, this is due to the extra gravity that this term provides. The proportional error in mass estimation by omitting the logarithmic term is given by $\epsilon \equiv (M' - M)/M = \lambda b\pi/2G = b\pi/2r_1$. Then, for a galaxy or a cluster of galaxies with $b \approx 10$ to 100 kpc (i.e., the average size of a galaxy or cluster), one obtains $\epsilon \approx 1$ to 10. That is, the estimated mass may be as much as 10 times as large as the actual mass, which is far from insignificant! This applies to, for example, the estimation of the bullet cluster mass via lensing, implying a potentially significant over-estimation of the amount of DM therein.

Next, let us turn to cosmology. It can be shown that the logarithmic term modifies the Friedmann equation as

$$
H^2(z) = H_0^2 \left[ \Omega_0^{(NR)} f_{NR}(z) + \Omega_0^{(L)} f_L(z) \right],
$$

(4)

where $z$ denotes the redshift, $H_0$ is the Hubble constant, i.e. the value of the Hubble parameter $H(z)$ at the present epoch, $\Omega_0^{(NR)}$, $\Omega_0^{(N)}$ and $\Omega_0^{(L)}$ are respectively the values of the density parameters corresponding to the non-relativistic matter (dust), cosmological constant $\Lambda$, and the logarithmic term, and

$$
f_{NR}(z) = (1 + z)^3, \quad f_{\Lambda}(z) = 1, \quad f_L(z) = (1 + z)^2 \ln(1 + z).
$$

(5)

Note that we have ignored the radiation term, and assumed a spatially flat universe. At $z = 0$, Eq. (4) gives $\Omega_0^{(NR)} + \Omega_0^{(N)} = 1$. Hence eliminating $\Omega_0^{(N)}$ from Eq. (4), we carry out the statistical parametric estimation using the Supernovae type-Ia Pantheon (binned) data in combination with the observational Hubble $(H(z))$ data. The standard procedure for the Markov-chain Monte Carlo (MCMC) random probabilistic exploration (see e.g. the section 4 of [14]), leads to the parametric best fit and contour levels shown in Fig. 1. We see that the best fit of $\Omega_0^{(NR)}$ is significantly lower (only $\approx 0.11$ compared to about 0.3 for $\Lambda$CDM. So, if we take out the visible baryonic dust contribution, which is $\approx 0.05$, then we will be left with an effective CDM contribution of about 6% of the total. This is significantly smaller than the usual estimate of about 25%, and shows consistency with our starting hypothesis of a much reduced need for DM.

FIG. 1: The 1$\sigma$-3$\sigma$ parametric contour levels obtained using the Pantheon (binned) data combined with the Hubble data. Here $h = H_0/[100$ km s$^{-1}$ Mpc$^{-1}]$ is the reduced Hubble constant.

Having demonstrated that the simple addition of a logarithmic term to the standard Newtonian potential can explain multiple phenomena normally attributed to DM at the galactic and cosmological length scales, let us speculate on the origin of such a term. There are a couple of routes one can take. For example, one can simply posit it as a fundamental law of nature and try to embed it into a covariant theory. The $f(R)$ theories of gravity theory may help, since it is known that the corresponding action can be transformed into a standard Einstein-Hilbert action coupled minimally to a scalar field with a self-interacting potential. Therefore, working backwards, in principle one can try to find an appropriate
scalar potential that would resemble the matter term required for the logarithmic term, and in turn arrive at a suitable $f(R)$ model of gravity.

The above approach is not free of problems, plus there exists a much more interesting alternative as follows. We propose that the potential in Eq. (1) is nothing but the quantum potential, generated by an underlying wavefunction $\Psi = \mathcal{R} e^{iS}$, $(\mathcal{R}, S \in \mathbb{R})$, as per [15]

$$V = V_Q = -\frac{\hbar^2}{2m} \nabla^2 \mathcal{R}$$.

(6)

This possibility cannot be ruled out, since as shown in [17], there is no mechanism for a particle to distinguish a background classical potential and its wavefunction dependent quantum potential. In fact, it sees the sum of the two. Eq. (6) can be re-written as

$$\nabla^2 \mathcal{R} + \frac{2mV_Q}{\hbar^2} \mathcal{R} = 0$$.

(7)

It can be shown that the above differential equation has an unique solution for the potential under consideration in terms of Bessel and Airy functions, with standard boundary conditions, namely [8]

$$\mathcal{R} = \sqrt{\frac{\ell_1}{r}} I_1 \left(2 \sqrt{\frac{r}{\ell_1}}\right), \quad r \leq r_1,$$

$$= \frac{1}{r} A_I \left(-1\right)^{1/3} \left[\frac{r_0}{\ell_2}\right]^{2/3} \left[\frac{r}{r_0} - 1\right]$$

$$+ \frac{1}{r} B_I \left(-1\right)^{1/3} \left[\frac{r_0}{\ell_2}\right]^{2/3} \left[\frac{r}{r_0} - 1\right] + c.c., \quad r \geq r_1$$

where $\ell_1$ is a constant. Note that the phase of the wavefunction remains undetermined.

Before concluding, we note that Modified Newtonian Dynamics (MOND) can be re-written in terms of an effective logarithmic potential, but with $\lambda = \sqrt{G\omega_0/M}$ [18], where $\omega_0 = 1.2 \times 10^{-10} \text{ m/s}^2$. The above $\lambda$, although having a different functional form, numerically equals our predicted value of $G/r_1$, for typical galaxy masses! As a result, for gravitational lensing from galaxies with $M = (10^{12} - 10^{14}) M_\odot$ (where $M_\odot$ is the solar mass), one has $\epsilon = \pi b\sqrt{\omega_0}/\sqrt{GM} = 1$ to 10. This in turn explains why MOND correctly predicts the flat rotation curves without DM. We emphasize however, that unlike MOND, the current proposal is not designed to explain just one phenomenon at a given length scale, but can potentially explain a host of phenomena at all length scales.

To conclude, in this essay, we have shown that the addition of a simple logarithmic potential to the standard Newtonian gravitational potential explains a number of astrophysical phenomena, which are normally attributed to DM. Thus at the very least, it reduces the importance of DM, and in the best case scenario, can mitigate its need altogether. Furthermore, there can be more than one origin of this additional potential, either in the framework of extended gravity theories, or as an emergent phenomenon stemming from an underlying quantum wavefunction. Either way, we believe that the proposal deserves to be explored further to tighten its theoretical underpinnings as well as to test its predictions with other astrophysical and cosmological observations.

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