Research of Vibration Characteristics of Ship Propeller-shaft Longitudinal Two-stage Vibration Isolation System

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Abstract: Aiming at the vibration control problem of ship propeller-shaft system, this paper put forward a design approach of ship propeller-shaft longitudinal two-stage vibration isolation system and its dynamic characteristics are studied. The vibration mathematical model of ship propeller-shaft longitudinal two-stage vibration isolation system is established. The influence of mass ratio, stiffness ratio and damping ratio of two-stage vibration isolation system is studied. The design is optimized taking a ship propeller-shaft system as an example. The results show that the ship propeller-shaft longitudinal two-stage vibration isolation system which is optimized has better isolation effect than the single-stage vibration isolation system. The research results have important reference value for ship propeller-shaft longitudinal vibration isolation design.

1. Introduction
The ship radiating noise of ship after part structure excited by low frequency longitudinal vibration generated while propeller-shaft system running is the key and difficult point for vibration noise control. The thrust force is produced by the rotation of the propeller in non-uniform flow field, the alternating component of this driving force generate longitudinal exciting force, which exciting propeller-shaft system and hull structure, cause of hull structure vibration and radiated noise.

Many researches in the longitudinal control of propeller-shaft system mainly focus on the vibration isolation or vibration reduction technology on the thru st transfer channel of shaft-thrust bearing. Feng Guoping considered that the thrust bearing foundation was the main transmission way of shaft longitudinal vibration through analyzing the longitudinal excitation transmission characteristics of hull body[1]. Yang Zhirong put forward a design approach of applying dynamic vibration absorber to ship shafting, and analyzed the influence of the application of dynamic vibration absorber under different arrangement and installation states on the longitudinal vibration damping effect of shafting[2-4]. Li Quanchao proposed design ideas of different forms ship vibration reduction thrust bearings, and verified the control effect of longitudinal vibration of shafting system through theoretical analysis and experimental research[5-6].

At present, the longitudinal vibration isolation of shafting system is basically adopted in the form of single stage vibration isolation. The propeller shaft system is simplified into a single degree of freedom system. Through the system vibration isolation, the longitudinal vibration characteristics of the propeller-shaft system are changed to achieve the purpose of changing the force transfer characteristics and vibration isolation. According to the vibration isolation principle of single-degree-of-freedom system, when the frequency is greater than the $\sqrt{2}$ of the system natural frequency, the vibration trans-
mission rate of the system will be less than 1. Theoretically, the lower the system design natural frequency, the better the vibration isolation effect in the control frequency band.

However, the theoretical isolation transmission of single-stage vibration isolation system is limited, and the effect of single-stage vibration isolation is more limited in the intermediate frequency region beyond the longitudinal second order natural frequency of the propeller-shaft system. Two-stage vibration isolation system, which is widely used in high-precision instruments, can achieve higher performance of vibration isolation. Its vibration isolation performance is related to the system parameters, the selection of appropriate parameters can improve the vibration isolation performance of the system[7-8].

This paper put forward a design approach of ship propeller-shaft longitudinal vibration control based on the two-stage vibration isolation technology, and the corresponding two-stage vibration isolation system mathematical model and analysis method are studied. The vibration characteristics of the longitudinal vibration isolation of the propeller-shaft system are analyzed, and the influence of vibration transmissibility, system natural frequency ratio on the design parameters of the mass ratio, stiffness ratio, damping ratio is studied.

2. Dynamics model of System

The thrust transfer channel of the propeller-shaft system is also the longitudinal excitation force transfer channel. The design solution of the longitudinal two-stage vibration isolation is similar to that of the single-stage vibration isolation design. The vibration isolation is realized by designing the vibration damping structure on the thrust transfer channel and controlling the natural vibration characteristics of the system through the stiffness design of the vibration damping structure.

For a single-stage vibration isolation system, the absolute vibration transmissibility of the isolation body to the base is [9]:

$$T = \left( \frac{1 + 4\xi^2 \omega_n^2}{(1 - \omega_n^2)^2 + 4\xi^2 \omega_n^2} \right)^{1/2}$$

(1)

Where, $\xi = \frac{c}{2\sqrt{k_1 m}}$, $\omega_n = \frac{\omega}{\omega}, \omega_n = \frac{k}{m}$.

The ship propeller-shaft system based on two-stage vibration isolation technology is shown in Figure 1. The shaft vibration reduction is set between shaft section 1 and shaft section 2, and vibration reduction thrust bearing is set between Shaft section 2 and ship-based foundation. Since the longitudinal stiffness of the coupling is much lower than the above two groups of vibration damping equipment, the influence of the coupling and drive motor on the propeller shaft system can be ignored. Two sets of vibration reduction make the propeller-shaft system become two-stage vibration isolation system. The dynamic model is shown in Figure 2.

Where $m_1$ is the mass of shaft section 1 and propeller, $m_2$ is the mass of shaft section 2, $k_1$ and $c_1$ are the longitudinal stiffness and damping coefficient of shaft damper, $k_2$ and $c_2$ are the longitudinal stiffness and damping coefficient of vibration reduction thrust bearing.

![Figure 1. Structure Diagram of Ship Propeller-shafting System](image-url)
Figure 2. Model of Ship Propeller-shafting Longitudinal Two-stage Vibration Isolation System

The vibration equation of the two-stage vibration isolation system is given:

\[
\begin{align*}
\dot{m}_1\ddot{x}_1 + c_1(\ddot{x}_1 - \ddot{x}_2) + k_1(x_1 - x_2) &= F \\
\dot{m}_2\ddot{x}_2 + c_1(\ddot{x}_2 - \ddot{x}_1) + k_1(x_2 - x_1) + c_2\ddot{x}_2 + k_2x_2 &= c_2\ddot{x}_3 + k_3x_3
\end{align*}
\]  

(2)

Under zero initial conditions, Equation (2) is changed into the free vibration equation of the no damping two-degree-of-freedom system while the damping effect is ignored. The first and second natural frequencies of the system are given:

\[
\omega_{n1}^2, \omega_{n2}^2 = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}
\]  

(3)

Where, \( a = m_1m_2 \), \( b = m_1k_1 + m_2k_2 + m_2k_1 \), \( c = k_1k_2 \).

Let:

\[
f_a = \frac{\omega_{n2}}{\omega_{n1}}, \quad \mu = \frac{m_2}{m_1}, \quad \nu = \frac{k_2}{k_1}
\]

Then the natural frequency ratio of the system, \( f_a \), is given:

\[
f_a = \frac{\sqrt{(1+\mu+\nu) \pm \sqrt{(1+\mu+\nu)^2 - 4\mu\nu}}}{(1+\mu+\nu) \mp \sqrt{(1+\mu+\nu)^2 - 4\mu\nu}}
\]  

(4)

It can be seen that the natural frequency ratio of the system is determined by the mass ratio and the stiffness ratio.

Let:

\[
\omega_1 = \frac{k_1}{m_1}, \omega_2 = \frac{k_2}{m_2}, \quad g = \frac{\omega}{\omega_1}, \quad \xi_1 = \frac{c_1}{2\sqrt{k_1m_1}}, \quad \xi_2 = \frac{c_2}{2\sqrt{k_2m_2}}, \quad f = \frac{\omega_2}{\omega_1}
\]

According to the literature [9-10], the amplitude-frequency characteristics of propeller displacement \( x_1 \) against hull base displacement \( u \) are obtained, that is, the absolute vibration transmissibility of the vibration isolation body to the base of the two-stage vibration isolation system:

\[
T = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}}
\]  

(5)

Where:

\[
A = f^2 - 4\xi_1\xi_2fg^2
\]

\[
B = 2fg(\xi_2 + \xi_1f)
\]

\[
C = g^4 - \left(\frac{1}{\mu} + f^2 + 1 + 4\xi_1\xi_2f\right)g^2 + f^2
\]

\[
D = 2fg(\xi_2 + \xi_1f) - 2(\xi_1 + \frac{\xi_2}{\mu} + \xi_2f)g^3
\]

In this study, the absolute vibration transmissibility of the system is evaluated by 201g (T).
3. Analysis of system vibration transfer characteristics
A certain propeller-shaft system using two-stage vibration isolation technology is taken as the example to analyze its vibration transfer characteristics.

Let: \( \mu = 1, \nu = 1, \xi_1 = \xi_2 = 0.005 \).

The dynamic model was established by the above formula to analyze the vibration transfer characteristics, and the vibration transfer characteristics were compared with those of single-stage vibration isolation system with the same total system mass and the same first natural frequency, as shown in Figure 3. For the convenience of comparison and explanation, the first natural frequency of two-stage vibration isolation system is set as the base to define frequency ratio, that is, \( g' = \omega / \omega_n \).

It can be seen from the Figure 3 that a second natural frequency characteristic is added after the propeller-shaft system using two-stage vibration isolation technology, which is similar to the first order natural frequency characteristic. The system absolute vibration transmissibility after second order resonance frequency is falling rapidly by -24 dB/Oct slope, far higher than that of single-stage vibration isolation system(-12dB/Oct slope). It shows that the two-stage vibration isolation system vibration isolation performance is superior to the single-stage vibration isolation system in the intermediate frequency of high frequency ratio of area.

**Figure 3.** Comparison of Transmissibility between Single-stage and Two-stage Isolation Systems

Different from the single-stage isolation system, the design boundary of the two-stage isolation system is no longer \( g' > \sqrt{2} \), which is determined by the second order natural frequency of the system and its subsequent attenuation trend. It can be seen that the control of natural frequency ratio is particularly important for the design of the two-stage vibration isolation system. The lower the natural frequency ratio control is, the lower the starting frequency ratio of the vibration reduction effect will be, and the lower the vibration isolation control area will be extended to the lower frequency during the system design, which points out the direction for the system optimization design. In addition, it can be seen from Equation (5) that damping parameters will also affect the change of the vibration transmissibility of the system, especially the attenuation of the frequency response at the resonant frequency.

4. Analysis of the influence law of system natural frequency ratio
The system natural frequency ratio is a binary function of mass ratio and stiffness ratio. The influence analysis on the natural frequency ratio is mainly carried out from the influence of these two parameters.

**Figure 4** shows the curve of natural frequency ratio variation of the two-stage vibration isolation system under the condition of mass ratio 0.01-10 and stiffness ratio 0.01-10. As can be seen from the figure, the natural frequency ratio of system has an on-linear decreasing trend with the synchronous increase of the stiffness ratio and mass ratio, and the surface graph is distributed along the vertical centre of the diagonal line of \( \mu - \nu \) plane and \( f_n \) axis. It can be seen that the influence of mass ratio and stiffness ratio on the natural frequency ratio of the system is reciprocal, which can be seen from Equation (4). Meanwhile, the natural frequency ratio of the system can be reduced by synchronously in-
creasing the mass ratio and stiffness ratio. The influences of mass ratio or stiffness ratio alone on natural frequency ratio are further discussed in the following sections.

Figure 4. Influence of Mass Ratio, Stiffness Ratio to Natural Frequency Ratio

4.1. Influence of mass ratio on system natural frequency ratio
Figure 5 shows the variation of natural frequency ratio with mass ratio of two-stage vibration isolation system when stiffness ratio is fixed. The following rules can be seen from the figure:

- The natural frequency ratio of the system decreases first and then increases with the increasing of mass ratio in the shape of "√". There is an "inflection point" in every curve, which is the lowest point of the system natural frequency ratio.
- As the stiffness ratio of the system increases, the lower the lowest point of the system natural frequency ratio, the mass ratio of the lowest point will shift to the high point.
- There is an obvious linear relationship between the mass ratio and the stiffness ratio at the lowest point of the system natural frequency ratio, namely:

\[ u = v + 1 \]  

(6)

Figure 5. Influence of Stiffness Ratio to Natural Frequency Ratio

According to the above rules, Equation (4) can be further simplified. When the stiffness ratio v is constant, the lowest natural frequency ratio of the system is:
According to Equation (6), the mass ratio of the system is always greater than 1.

4.2. Influence of stiffness ratio on system natural frequency ratio

Figure 6 shows the variation of natural frequency ratio with stiffness ratio of two-stage vibration isolation system when mass ratio is fixed. In the Equation (4), the mass ratio and the stiffness ratio are reciprocal, so the variation rule obtained is consistent with 4.1. However, the relation between mass ratio and stiffness ratio at the lowest point of system natural frequency ratio is adjusted as follows:

\[ v = u + 1 \]  

Similarly, when the mass ratio is constant, Equation (4) can be simplified as:

\[ f_{\text{min}} = \sqrt{1 + u + 1} \sqrt{1 + u - 1} \]

Similarly, when the mass ratio is constant, Equation (4) can be simplified as:

\[ f_{\text{min}} = \sqrt{1 + u + 1} \sqrt{1 + u - 1} \]

At this point, the mass ratio can be controlled below 1, and the system stiffness ratio is always greater than 1.

Figure 7 shows the variation of the lowest natural frequency ratio with mass ratio of two-stage vibration isolation system According to Equation(9).

You can see from the picture, the smaller mass ratio, the more sensitive of the lowest system natural frequency with the change of mass ratio: mass ratio increased from 0.01 to 1, the lowest system natural frequency ratio decreased from 20 to 2.414; mass ratio increased from 1 to 10, the lowest system natural frequency ratio decreased from 2.414 to 1.365. These laws can provide theoretical support for the optimization design of longitudinal two-stage vibration isolation system of propeller-shaft system.
5. Damping ratio analysis of system dynamic performance

Taking a propeller-shaft system using two-stage vibration isolation technology in Section 2 as the example, the vibration transfer characteristics of the vibration isolation system after changing system damping $\xi_1$ and $\xi_2$ are shown in Figure 8 and Figure 9, while the other design parameters are fixed.

The following rules can be seen from the figure:

- The increase of damping ratio $\xi_1$ and $\xi_2$ will decrease the vibration transmissibility near the natural frequency of the two-stage isolation system.
- For two-stage vibration isolation systems with different damping ratios, the vibration transmissibility curve has an intersection point (A in Figure 8 and B in Figure 9).
- The increase of the damping ratio leads to the increase of the vibration transmissibility of the system after the intersection point;
- For the damping ratio $\xi_1$ and $\xi_2$, the corresponding intersection points are different:
  - With the increase of damping ratio $\xi_1$, the vibration transmissibility at the second order natural frequency of the system damps more obviously. $\xi_1 > 0.1$, the vibration transmissibility at the second order natural frequency of the system is less than 1, that mean, the system plays the function of vibration isolation.
  - The vibration transmissibility damping contributions at the first and second order natural frequencies are basically the same with the increase of damping ratio $\xi_2$, and there are two intersections between the first and second order natural frequencies of the system vibration transmissibility with different damping ratio $\xi_2$. This paper will not further analyze the influence rules.

![Figure 8. Influence of Damping Ratio $\xi_1$ to Vibration Transmissibility of System](image1)

![Figure 9. Influence of Damping Ratio $\xi_2$ to Vibration Transmissibility of System](image2)
6. Optimization of longitudinal two-stage vibration isolation design for propeller shaft system

According to the influence rules of mass ratio, stiffness ratio, damping ratio and other parameters on the vibration characteristics of the system, the ship propeller-shaft system can be designed and optimized. For the propeller-shaft system applying two-stage vibration isolation technology, \( m_1 \) includes the mass of the propeller and part of the shaft section, which may be greater than or equal to the mass \( m_2 \) of the second stage vibration isolation system. In engineering, it is not meaningful to discuss the mass ratio of the propeller-shaft system greater than 1. The stiffness ratio and damping ratio can be designed flexibly according to the vibration isolation requirements of the system. Therefore, the design optimization of the longitudinal two-stage vibration isolation system of the propeller-shaft system can be carried out according to the following steps:

- The maximum mass ratio that can be achieved by propeller-shaft system should be determined first conform to the propulsion function of propeller-shaft system.
- According to Equations (8) and (9), the optimal stiffness ratio and the lowest natural frequency ratio of the system under the action of the mass ratio are determined.
- Combined with the total mass and frequency control requirements of the propeller-shaft system, the stiffness of the vibration isolator \( k_1 \) and \( k_2 \) can be further obtained according to Equation (3).
- Analyze the characteristics of vibration transmissibility of the system according to Equation (5), and further optimize the system damping ratio \( \xi_1 \) and \( \xi_2 \) point to obtain the appropriate vibration transmissibility.

Taking a ship propeller-shaft system with two-stage vibration isolation technology as an example for further analysis, the total mass of the propeller-shaft system is 16t, the maximum mass ratio that can be realized in the engineering is 1, and the target frequency band of longitudinal vibration control is 20-200Hz. According to the above analysis steps, the optimal stiffness ratio of the system is 2, and the lowest natural frequency ratio is 2.414.

The ratio of the lower limiting control frequency to the first natural frequency was designed according to \( f / f_n \geq 2 \). The first natural frequency of the system was determined to be 8.6Hz, and the stiffness of the vibration isolator was \( k_1 = 4 \times 10^7 \)N/m, \( k_2 = 8 \times 10^7 \)N/m. The damping coefficient of the vibration isolation system is further set as \( c_1 = 1.2 \times 10^5 \)N.s/m, \( c_2 = 1 \times 10^5 \)N.s/m, and the characteristics of the vibration transmissibility of the system is obtained. It is compared with the absolute vibration transmissibility curve of a single-stage vibration isolation system with the same natural frequency of 8.6Hz, as shown in Figure 10.

As can be seen in the figure, the propeller shaft system has vibration isolation effect in the range of 20-200Hz after using two-stage vibration isolation technology, and the vibration isolation effect is obviously better than that of the single-stage vibration isolation system above 30Hz.

![Figure 10. Vibration Transmissibility of Ship Shafting Longitudinal Two-stage Vibration Isolation System](image-url)
7. conclusion
In this paper, the design approach of longitudinal vibration control of propeller-shaft system based on two-stage vibration isolation technology is proposed, the corresponding vibration mathematical model and analysis method of two-stage vibration isolation system are given, the influence of design parameters such as mass ratio, stiffness ratio and damping ratio of system is studied, and the design optimization idea and suggestions are given. Taking a propeller-shaft system as an example, the longitudinal two-stage vibration isolation design was optimized. The results show that the propeller shaft system can effectively achieve vibration isolation in the target frequency band after applying the two-stage vibration isolation technology, and the vibration isolation effect above 30Hz is obviously better than that of the single-stage vibration isolation system. The design parameter optimization method proposed in this paper provides an important theoretical reference for the longitudinal vibration control design of propeller-shaft system.

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