Electrostatic plasma instabilities driven by neutral gas flows in the solar chromosphere

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ABSTRACT

We investigate electrostatic plasma instabilities of Farley-Buneman (FB) type driven by quasi-stationary neutral gas flows in the solar chromosphere. The role of these instabilities in the chromosphere is clarified. We find that the destabilizing ion thermal effect is highly reduced by the Coulomb collisions and can be ignored for the chromospheric FB-type instabilities. On the contrary, the destabilizing electron thermal effect is important and causes a significant reduction of the neutral drag velocity triggering the instability. The resulting threshold velocity is found as function of chromospheric height. Our results indicate that the FB type instabilities are still less efficient in the global chromospheric heating than the Joule dissipation of the currents driving these instabilities. This conclusion does not exclude the possibility that the FB instability develop in the places where the cross-field currents overcome the threshold value and contribute to the heating locally. Typical length-scales of plasma density fluctuations produced by these instabilities are determined by the wavelengths of unstable modes, which are in the range $10^{-3} - 10^2$ cm in the lower chromosphere, and $10^2 - 10^3$ cm in the upper chromosphere. These results suggest that the decimetric radio waves undergoing scattering (scintillations) by these plasma irregularities can serve as a tool for remote probing of the solar chromosphere at different heights.

Key words: Sun: chromosphere.

1 INTRODUCTION

Since it was discovered that the temperature in the solar chromosphere is much higher than can be expected in radiative equilibrium, the mechanism of chromospheric heating is one of the main puzzles in solar physics. The first scenario for coronal and chromospheric heating was proposed by Biermann (1946) and Schwarzschild (1948), who suggested that the atmosphere of the sun is heated by acoustic waves generated in the turbulent convective zone. The theory of wave generation by turbulence was developed by Lighthill (1952). Extension of this theory to the stratified environment of the solar atmosphere showed that short-period acoustic waves are abundantly generated in the turbulent convective zone (Stein 1967). The theory predicts that the peak of the acoustic power spectrum is just below a period of one minute. Later numerical simulations (e.g., Carlsson & Stein (1992)) confirmed that the total power of the generated acoustic waves is sufficient for chromospheric heating. But the measurements of acoustic flux in the chromosphere have usually failed to find sufficient energy. From the analysis of the Doppler shifts of UV lines, Bruner (1978) demonstrated that the energy flux of the acoustic waves with periods of 100 s or more is at least 2 orders of magnitude less than required for the observed level of chromospheric heating. Similar results have been obtained by Mein & Schnieder (1981) from an analysis of the Doppler shifts of Ca II and Mg I lines. Recent analysis of the data obtained by TRACE (Fossum & Carlsson 2005) has shown that the observed intensity of high frequency (10-50 mHz) acoustic waves was at least one order of magnitude lower than necessary for the observed chromospheric heating. In addition, instead of steepening and dissipation, the acoustic waves and pulses can form sausage solitons, propagating undamped along magnetic flux tubes (Zaqarashvili et al. 2010).

Problems with measurements of sufficient acoustic flux stimulated development of alternative models of chromospheric heating. One of the alternative scenarios (Parker 1988; Sturrock 1999) implies that impulsive nanoflares related to magnetic reconnection can be responsible for chromospheric heating. The observations (e.g., Aschwanden et al. (2000)) do show numerous fast brightenings in the sun but they are not sufficiently frequent to...
explain the UV emission of the chromosphere. Another sce-
nario for chromospheric heating is resistive dissipation of
electric currents (Rabin & Moore 1984; Goodman 2004).
Recent analysis of three-dimensional vector currents ob-
served in a sunspot has shown that the observed currents
are not sufficient to be responsible for the observed amount
of heating (Socas-Navarro 2007).

Recently it has been supposed that a convective
motion driven Farley-Buneman instability (Farley 1963;
Buneman 1964) (FBI) can significantly contribute to chro-
mospheric heating (Liporovsky et al. 2004; Fontenla 2004;
Fontenla et al. 2008). The FBI is known to be responsi-
ble for the formation of plasma irregularities in the Earth’s
ionospheric E-region (Schunk & Nagy 2004). The interplay
of the background electric and magnetic fields at the alti-
tudes where electrons are strongly magnetized, produces
currents that drive the instability. In a similar way, if the
electrons are strongly magnetized, the drag of the ions by
 neutrals causes the instability. The simultaneously observed
electron heating was attributed to the parallel electric fields
in waves (Dimant & Milikh 2003; Milikh & Dimant 2003).
Gogoberidze et al. (2009) extended analysis of the FBI in
the solar chromosphere conditions by taking into account
the finite ion magnetization and Coulomb collisions. This
study suggested that the FBI is not a dominant factor in the
global chromospheric heating. However, local strong cross-
field currents can drive FBI producing small-scale (0.1 – 3 m)
density irregularities and contributing to the chromo-
spheric heating locally (Pannev & Wardle 2013) accounted
for the flow inhomogeneity (flow shears) and found an elec-
tromagnetic MHD-like instability generated at larger scales.
These irregularities can cause scintillations of radio waves
at similar lengths scales and provide a tool for chromospheric
remote sensing. It has to be noted that Gogoberidze et al.
(2009) did not take into account effects of the electron
heating related to the presence of parallel electric fields
in the waves. As showed theoretically by Dimant & Milikh
(2003) and confirmed by recent particle in cell simulations
(Oppenheim & Dimant 2013), this effect can significantly
increase the electron heating. Importance of this mechanism
for the solar chromosphere requires an additional analysis
and is beyond the scope of this paper.

It is also known that electron and ion thermal effects
can strongly affect small-scale E-region instabilities. The
electron thermal effects lead to a considerable modification
of the FBI (mainly by the electron Pedersen conductivity
via perturbed Joule heating), and Dimant & Sudan (1993)
have given the modified FB instability a new name: electro-
 Pedersen conductivity instability (EPCI). Later on, this
instability was studied in more detail by Dimant & Sudan
(1997) and Robinson (1998). The ion thermal effects also
modify FBI significantly and make it possible in a wider al-
titude range as compared to the predictions of adiabatic and
isothermal FBI models (Dimant & Oppenheim 2004).

Here we study small-scale electrostatic instabilities of
the Farley-Buneman type in the partially ionized plasma
of the solar chromosphere taking into consideration ion
and electron thermal effects, electron and ion viscosity,
and Coulomb collisions. As it has been demonstrated by
Gogoberidze et al. (2009), contrary to the ionospheric case,
the Coulomb collisions of electrons and ions can not be ig-
nored in the chromosphere because of the relatively high
degree of ionization ($10^{-2} - 10^{-4}$). In the present paper
we find another difference with the ionosphere: the destabi-
zizing influence of ion thermal effects is highly reduced in
the chromosphere by Coulomb collisions and can be neglected.
But electron thermal effects appeared to be important,
especially in the middle and upper chromosphere, where they
reduce the threshold value of the relative electron/ion velocity
(current velocity). We determine various characteristic
length scales as well as the value of the threshold relative
velocity of electrons and ions necessary to trigger the elec-
 trostatic instability as a function of chromospheric height in
the framework of the semi-empirical chromospheric model
SRPM 306 (Fontenla et al. 2007). We confirm our previous
conclusion that FB type electrostatic instabilities cannot be
responsible for the chromospheric heating at global length
scales. However, such instabilities can be generated locally
in the places of sufficiently strong currents and can create
small-scale plasma irregularities.

The paper is organized as follows. The general formal-
ism is presented in Sec. 2. The FBI and the ion thermal
instability are studied in Sec. 3. The electron thermal insta-
 bility is discussed in Sec. 4. Different length scales of the
chromosphere important for the development of electrostatic
instabilities are studied in Sec. 5. Conclusions are given in
Sec. 6.

2 GENERAL FORMALISM

We use a standard modal analysis for linear perturbations
in partially ionized plasmas with neutral flows taking into
account Coulomb collisions, ion and electron viscosity, and
thermal effects. The dynamics of electrons, one species of
 singly charged ions and neutral hydrogen in the solar chro-
mosphere for imposed electric (E) and magnetic (B) fields
is governed by the continuity, Euler and heat transfer equa-
tions

$$\frac{d}{dt} n_\alpha + n_\alpha \nabla \cdot \mathbf{V}_\alpha = 0,$$

$$m_\alpha \frac{d}{dt} \mathbf{V}_e = -e \left( \mathbf{E} + \frac{\mathbf{V}_e \times \mathbf{B}}{c} \right) - \nabla (n_e KT_e/n_e),$$

$$m_e \mathbf{v}_{ei}(\mathbf{V}_e - \mathbf{V}_i) - m_e \nu_{en}(\mathbf{V}_e - \mathbf{V}_n) + m_e \eta_e \nabla^2 \mathbf{V}_e,$$

$$m_i \frac{d}{dt} \mathbf{V}_i = e \left( \mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c} \right) - \nabla (n_i KT_i/n_i) - m_e \nu_{ei}(\mathbf{V}_i - \mathbf{V}_e) - \mu_e m_e \nu_{en}(\mathbf{V}_i - \mathbf{V}_n) + m_i \eta_i \nabla^2 \mathbf{V}_i,$$

$$n_e^{2/3} \frac{d}{dt} (T_e n_e^{2/3}) = \frac{2}{3} \epsilon \nu_{en} m_e \nu_{en}(\mathbf{V}_e - \mathbf{V}_n)^2 - 2 \mu_e \nu_{en} (1 + \rho_{en})(T_e - T_n) + \frac{2}{3} \mu_e \nu_{ei}(\mathbf{V}_e - \mathbf{V}_i)^2 - 2 \mu_i \nu_{ei}(T_e - T_i) + \frac{\chi_e}{n_e} \nabla^2 T_e,$$

$$n_i^{2/3} \frac{d}{dt} (T_i n_i^{-2/3}) = \frac{2}{3} \epsilon \nu_{ni} m_i \nu_{en}(\mathbf{V}_i - \mathbf{V}_n)^2 - 2 \mu_i \nu_{en}(T_i - T_n) + \frac{2}{3} \mu_e \nu_{ei}(\mathbf{V}_e - \mathbf{V}_i)^2 - 2 \mu_i \nu_{ei}(T_i - T_e) + \frac{\chi_i}{n_i} \nabla^2 T_i,$$

Here $\alpha = e, i$ denotes electrons or ions; $n$ denotes neutrals;
$n_\alpha$ is the number density, $\mathbf{V}_\alpha$ is the averaged drift velocity;
Introduction, the ionization degree in the chromosphere is such that 2\text{e} is the inelastic collision frequency, such that 2\nu_{\text{el}} is the energy fraction lost by a particle of \alpha species during one elastic collision with a particle of \beta species; \nu is the speed of light; \kappa is the Boltzmann constant, and \dn\nu/dt denotes the convective derivative. \varepsilon_{\alpha,\beta} are dimensionless parameters which will be discussed below.

The relative efficiency of inelastic/elastic collisions in the electron thermal balance is \nu_{\text{el}} = \nu_{\text{en}}/(3\mu_{\text{en}}\nu_{\text{en}}), where \nu_{\text{en}} is the inelastic e– n collisional frequency.

Eqs. (1)-(5) are similar to so-called ‘5-moment’ transport equations (Schum & Nagy 2000) which are often used when studying instabilities in the \text{E}-region of the Earth’s ionosphere. The principal difference between the 5-moment approach and our study is that, as it was mentioned in the introduction, the ionization degree in the chromosphere is much higher than in the \text{E}-region and consequently Coulomb collisions are not ignored in the set of Eqs. (1)-(5). We account for inelastic e– n collisions in the lower chromosphere (Robinson 1998) in the electron energy balance (1) (term proportional to \nu_{\text{en}}). We will come back to this last issue in the discussion section.

The right hand side of Eqs. (3) and (5) describe the balance between frictional heating (two positive terms) and collisional cooling (two negative terms). Without these effects the temperature fluctuations would be adiabatic (\nu_{\text{fric}}). Here we will come back to this last issue in the discussion section.

For collision frequencies we use the following expressions (Braginskii 1965):

\begin{equation}
\nu_{\text{ei}} = \frac{4(2\pi)^{1/2}e^{2}n_{\alpha}n_{\beta} \Lambda}{3m_{\alpha}^{1/2}(KT_{e})^{3/2}},
\end{equation}

\begin{equation}
\nu_{\text{en}} = \sigma_{\text{en}}n_{\alpha}n_{\beta}\sqrt{\frac{KT_{e}}{m_{e}}},
\end{equation}

\begin{equation}
\nu_{\text{in}} = \nu_{\text{bi}} = \sigma_{\text{in}}n_{\beta}\sqrt{\frac{KT_{i}}{m_{i}}},
\end{equation}

where \Lambda is the Coulomb logarithm. From the former equation we see that regardless of the mass of dominant ion species, \nu_{\text{en}} = \nu_{\text{ep}} for singly charged ions.

For the electron-neutral and ion-neutral collisions we assume a simple model with constant cross-sections \sigma_{\text{en}} = 3.0 \times 10^{-15} \text{ cm}^{2} (Redelsen & Kiefer 1971) and \sigma_{\text{in}} = 2.8 \times 10^{-14} \text{ cm}^{2} (Krstic & Schultz 1999) that are typical for the middle chromosphere with particles energies \sim 0.5 – 1.0 \text{ eV}.

In principle, \sigma_{\text{en}} and \sigma_{\text{in}} are not constant but depend on the particles energies. For example, the neutral atom polarisation results in the \sigma_{\text{en}} \sim 1/V_{\text{en}} dependence making the collisional frequency independent of the particle energy. With this model our results would even more emphasise the effects of Coulomb collisions on \text{FBI} in the upper chromosphere. However, because of the other kinds of collisions with neutrals, the atom polarisation model underestimates the electron and ion collisions with the neutrals. Since these other kinds of collisions with neutrals are not well studied in the chromospheric conditions, we use the model with constant cross-sections, which artificially enhances \nu_{\text{en}} and \nu_{\text{in}} at larger heights.

Estimation of inelastic electron-hydrogen collisional frequency \nu_{\text{en}} is rather involved and sensitive to the electron temperature and velocity distribution in the super-thermal tail. Taking into account two main excitation levels of hydrogen atoms and using formulae given by Johnson (1972), we estimate that \nu_{\text{en}} vary from 0.1 in the lower chromosphere to about 1 in the upper chromosphere. We will keep \nu_{\text{en}} in derivations, but will not analyze its influence separately (see Discussion).

We assume that the system is penetrated by a uniform magnetic field \text B and that neutrals have background velocity \text V_{\text{n}} \perp \text B. Then equation (2-3) give for the background flow of electrons and ions

\begin{equation}
\kappa\text V_{\text{i}} \times \text b - (\alpha N + \mu_{\text{in}})\text V_{\text{i}} + \alpha N\text V_{\text{e}} + \mu_{\text{in}}\text V_{\text{n}} = 0,
\end{equation}

\begin{equation}
- \kappa\text V_{\text{e}} \times \text b - \alpha(N+1)\text V_{\text{e}} + \alpha N\text V_{\text{e}} + \alpha\text V_{\text{n}} = 0,
\end{equation}

Here \kappa = \omega_{\text{ep}}/\nu_{\text{en}} is the proton magnetization, \text b = \text B/\text B is the unit vector along the mean magnetic field direction, \psi = \nu_{\text{en}}/\omega e^{2}c_{\text{e}}, \omega_{\text{ca}} \equiv \epsilon B/m_{\text{e}}c is the cyclotron frequency, \alpha = \psi\kappa^{2} = m_{\text{e}}\nu_{\text{en}}/m_{\text{p}}\nu_{\text{en}} \approx 2.6 \times 10^{-3}, and \text N =\nu_{\text{en}}/\nu_{\text{in}} is the ratio of the Coulomb and electron-neutral collision frequencies.

Multiplying equations (11)-(12) by \times \text b and excluding \text V_{\text{i}} \times \text b and \text V_{\text{e}} \times \text b we get

\begin{equation}
\text V_{\text{i}} = \frac{\kappa^{2}(N+1) + \alpha N + \mu_{\text{in}}}{\mu_{\text{i}}}
\end{equation}

\begin{equation}
\text V_{\text{ei}} = - \frac{\kappa^{2}(\alpha N + \mu_{\text{in}})}{\alpha\mu_{\text{i}}}
\end{equation}

\begin{equation}
\text V_{\text{i}} = \frac{\kappa^{2}(N+1) + \alpha N + \mu_{\text{in}}}{\mu_{\text{i}}} + \alpha(1+N)
\end{equation}

\begin{equation}
\text V_{\text{ei}} = - \frac{\kappa^{2}(\alpha N + \mu_{\text{in}})}{\alpha\mu_{\text{i}}} + \alpha(1+N)
\end{equation}

\begin{equation}
\text V_{\text{e}} = \kappa\text V_{\text{en}} \times \text b - \alpha\text V_{\text{n}}.
\end{equation}

Here \mu_{\text{i}} = \alpha N + \mu_{\text{in}}(1+N) \approx \mu_{\text{in}}(1+N).

Using equations (11)-(12) one can readily derive expressions for \text V_{\text{i}} and \text V_{\text{e}}, but exact relations are too complicated. The dependence of the proton magnetization \kappa and \text N on height based on the semi-empirical chromospheric model SRMP 306 (Fontenla et al. 2007) is shown in Figs. 1 and 2 respectively. Detailed analysis of these data is presented in

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the next section. Here we note that, as it can be seen from Fig. 2 for all chromospheric heights \( \alpha N \ll 1 \). Also, from the data shown in Figs. 1 and 2 one can find that \( \kappa^2/\mu \gg \alpha \) in the chromosphere except for very low altitudes \( h < 600 \) km. In this paper we are mostly interested in higher altitudes where the Coulomb collisional effects are important for FBI. In the limit \( \alpha N \ll 1 \) and \( \alpha \mu /\kappa^2 \ll 1 \) we obtain the ion and electron background velocities

\[
\mathbf{V}_i \approx \frac{\mu_{\text{ni}}}{\kappa + \mu_{\text{ni}}} [\mu_{\text{ni}} \mathbf{V}_n + \kappa \mathbf{V}_n \times \mathbf{b}],
\]

(13)

\[
\mathbf{V}_e \approx \alpha \mu_{\text{ne}} \frac{\nu_{\text{ne}}^2 + \alpha \nu_{\text{ne}} (1 + N)}{\kappa^2 (\kappa^2 + \mu_{\text{ne}}^2)} \mathbf{V}_n - \alpha \frac{\kappa^2 + \mu_{\text{ne}}^2 (1 + N)}{\kappa (\kappa^2 + \mu_{\text{ne}}^2)} \mathbf{V}_n \times \mathbf{b}.\]

(14)

In the considered limit, \( \alpha \ll 1 \) and \( \alpha N \ll 1 \), the current velocity \( \mathbf{U}_0 = \mathbf{V}_i - \mathbf{V}_e \approx \mathbf{V}_i \).

On the background given by Eqs. (13)-(14) and corresponding solutions for ion and electron temperatures in the subsequent sections we consider different linear electrostatic perturbations propagating in the plane perpendicular to the background magnetic field. To simplify further analysis, we make two assumptions which are standard in the study of low frequency perturbations. Firstly, we assume quasi-neutrality \( (n_e \approx n_i = n) \). This condition is valid when characteristic frequency of perturbations is much less than ion plasma frequency. Secondly, we treat electrons as inertialess. The latter assumption implies that the characteristic time scale of the perturbations is much greater than electron cyclotron and plasma time scales. Both ion-thermal and current driven instabilities occur at ion-neutral collision time scale, which for typical chromospheric parameters is much greater than all characteristic time scales mentioned above.

In the analysis below we ignore perturbations of the neutral component. Such a treatment is valid in weakly ionized plasma for relatively high frequency perturbations. Comparing inertial and ion drag terms in the Euler equation for neutrals, we obtain that perturbations of the neutral component can be safely neglected if

\[
\omega \gg \frac{m_e}{n_e} \nu_{\text{in}}.
\]

(15)

And finally, as is usually done in the E-layer research, we consider the ion and electron temperature perturbations separately. Due to the relatively high electron concentration in the chromosphere we do not ignore Coulomb collisions. In the general case, perturbations of the ion temperature can cause perturbations of the electron temperature. But due to the large ions/electron mass ratio, Coulomb collisions are inefficient in the heat transfer between electrons and ions. Mathematically this is manifested by the \( \mu_{\text{ni}} \sim m_e/m_i \) multiplier in the last but one term of equation (4). Comparing this term with the left hand side of Eq. (3) shows that the thermal perturbations of ions and electrons can be treated separately if

\[
\omega \gg \frac{m_e}{m_i} \nu_{\text{ei}}.
\]

(16)

In this context it should be also noted that electron thermal effects in the ionospheric E-layer are important for relatively low altitudes \( \text{[Schunk & Nagy 2000]} \), where ion magnetization is weak, whereas ion thermal effects become important with strong ion magnetization.

3 FARLEY-BUNEMAN INSTABILITY AND ION-THERMAL EFFECTS

Let us introduce dimensionless perturbations of electric potential, number density, and temperature for the \( \alpha \) species:

\[
\bar{\phi}_\alpha = \frac{e k \cdot \mathbf{E}'}{K T_{\alpha} \kappa^2}, \quad \bar{n} = \frac{n'}{n}, \quad \bar{T}_\alpha = \frac{T'_{\alpha}}{T_\alpha},
\]

(17)

where primed variables stand for linear perturbations in the Fourier space, and wave vector \( k \perp b \) (here we considered only two dimensional perturbations with wave vectors perpendicular to the background magnetic field).

Then, linearizing Eqs. (11)-(13), dropping viscosity and thermal conductivity effects (these effects will be studied in the following sections), setting for simplicity \( \varepsilon = 1 \), and setting \( T'_e = 0 \), the Euler equation for the ions gives

\[
\left( 1 + \alpha^* N - \frac{\Omega}{\nu_{\text{in}}} \right) \mathbf{v}_i' = -i k^2 \nu_{\text{in}} \left( (\bar{\phi}_i + \bar{n} + \bar{T}_i) + \bar{\alpha}^* \mathbf{v}_i \times \mathbf{b} + \alpha^* N \nu_{\text{in}} \mathbf{v}_i' \right).
\]

(18)

where \( \Omega = \omega - k \cdot \mathbf{V}_i \) is the frequency in the ion frame, \( \nu_{\text{in}}^* = \mu_{\text{ni}} \nu_{\text{in}} \) is the reduced ion-neutral collisional frequency, \( \bar{\alpha}^* = \kappa/\mu_{\text{ni}} \), \( \alpha^* = \alpha/\mu_{\text{ni}} \), and \( u_{\text{fi}} = (T_i/m_i)^{1/2} \) is the ion thermal velocity.

Similarly, the linearized Euler equation for the electrons (which we treat inertialess) gives

\[
(1 + N) \mathbf{v}_e' = \frac{i}{\nu_{\text{in}}} \left( \frac{1}{\nu_{\text{in}}^*} \right) \left( \bar{\phi}_i - \frac{\gamma T_i}{T_i} \bar{n} \right) - \frac{\kappa}{\nu_{\text{in}}} \mathbf{v}_e' \times \mathbf{b} + N \nu_{\text{in}} \mathbf{v}_e'.
\]

(19)

As discussed earlier, we study evolution of perturbations in the limits \( \alpha \ll 1 \) and \( \alpha N \ll 1 \) that are fulfilled in the entire chromosphere. In addition, here we assume also \( \alpha N/\kappa^2 \ll 1 \). This condition is valid everywhere except for very low chromospheric heights, where the influence of Coulomb collisions on FBI is negligible anyway \( \text{[Gogoberidze et al. 2009]} \). Solving equations (18)-(19) for perturbed velocities and keeping only leading-order terms with respect to the small parameters \( \alpha, \alpha N, \) and \( \alpha N/\kappa^2 \), we obtain

\[
\mathbf{v}_i' = -i \nu_{\text{in}}^* \left( \frac{1 - i \Omega/\nu_{\text{in}}^*}{1 - i \Omega/\nu_{\text{in}}^*} k \times \mathbf{b} + \kappa \mathbf{v}_i \times \mathbf{b} + \bar{\alpha}^* \mathbf{v}_i \times \mathbf{b} \right) \mathbf{v}_e',
\]

(20)

\[
\mathbf{v}_e' = i k \nu_{\text{in}}^* \left[ (1 + N) \left( \frac{1}{\nu_{\text{in}}^*} \right) \left( \bar{\phi}_i - \frac{\gamma T_i}{T_i} \bar{n} \right) - \frac{N \kappa^2 (\bar{\phi}_i + \bar{n} + \bar{T}_i)}{1 - i \Omega/\nu_{\text{in}}^* + \kappa^2} + Q \right].
\]

(21)

Here \( Q \) stands for the terms proportional to \( k \times \mathbf{b} \), which do not contribute to the dispersion relation (it is eliminated by the scalar product \( k \cdot \mathbf{v}_i \) in Eq. 25 below).

From equation (20) it follows that, in the lowest order with respect to the small parameters \( \alpha, \alpha N, \) and \( \alpha N/\kappa^2 \), the perturbed ion velocity is not affected by the Coulomb collisions with electrons. Physically, this means that the force balance for the ion fluctuations is dominated by the ion-neutral rather than the ion-electron collisions. In the leading order with respect to small \( \alpha, \alpha N, \) and \( \alpha N/\kappa^2 \), the perturbed equation (20) reduces to

\[
\left( \frac{\Omega}{\nu_{\text{in}}^*} + \kappa^2 \right) \bar{T}_i - \frac{2 \Omega}{\nu_{\text{in}}^*} \bar{n} = \frac{4 i \mu_{\text{ni}}}{3 n_{\text{ei}}} \mathbf{v}_i' \cdot (\mathbf{V}_i - \mathbf{V}_n),
\]

(22)
From the ion continuity equation, using (13) and (20), we get

$$\mathbf{v}_i' \cdot (\mathbf{V}_i - \mathbf{V}_n) = \frac{\Omega k' \bar{\rho}}{k^2 (1 - i \Omega/\nu_{in}^*)} \times \left[ (1 - i \Omega/\nu_{in}^*) \mathbf{k} \cdot (\mathbf{U}_0 \times \mathbf{b}) + \kappa' (\mathbf{k} \cdot \mathbf{U}_0) \right].$$  

(23)

Substituting this into (22) we obtain the relation between the temperature and density perturbations $\bar{\tau}_i$ and $\bar{n}$:

$$\frac{3kU_0}{2\Omega} \left( 1 - \frac{\Omega}{\nu_{in}^*} \right) \left( \zeta - i \frac{\Omega}{\nu_{in}^*} \right) \bar{\tau}_i = \left[ \frac{2\mu_{in} \kappa^2 U_0^2 \cos \theta}{\omega T_i} + \left( 1 - \frac{\Omega}{\nu_{in}^*} \right) \left( \frac{2\mu_{in} \kappa^2 U_0^2 \sin \theta}{\omega T_i} - i kU_n^* \right) \right] \bar{n},$$

(24)

where $\theta$ is the angle between $\mathbf{U}_0$ and $\mathbf{k}$.

To obtain the second independent relation between $\bar{\tau}_i$ and $\bar{n}$, we use the electron continuity equation

$$\Omega + \mathbf{k} \cdot \mathbf{U}_0 \bar{n} = \mathbf{k} \cdot \mathbf{v}_e'.$$

(25)

Substituting (21) in this equation gives

$$- i \frac{k^2 U_0^2}{\nu_{in}^*} \bar{\tau}_i = \left[ \frac{\Omega + \mathbf{k} \cdot \mathbf{U}_0}{\psi \nu_{in}^*} + k c_s^2 \right] \bar{\rho} + \frac{\Omega}{\nu_{in}^*} \left( 1 - i \Omega/\nu_{in}^* \right) \left( \frac{\kappa^2}{1 + N} \right) \bar{n},$$

(26)

where $c_s = [K(T_i + T_e)/\mu_i]^{1/2}$ is the isothermal sound speed and $\psi = \psi(1 + N)$. Note that in the absence of thermal effects, $\bar{\tau}_i = 0$, the expression in the square brackets on the left hand side of Eq. (26) represents the dispersion relation for isothermal electrostatic perturbations in weakly ionized plasmas studied by Gogoberidze et al. (2009).

By means of the Eqs. (21) and (25), which represent two independent relations between $\bar{\tau}_i$ and $\bar{n}$, one can readily derive the dispersion relation. A simple analytical solution of the dispersion equation can be obtained in the long-wavelength low-frequency limit

$$|\Omega|, kU_0 \ll \nu_{in}^*.$$  

(27)

Eliminating $\tau$ from Eqs. (24) and (25), and keeping first-order terms in small parameters $|\Omega|/\nu_{in}^*$ and $kU_0/\nu_{in}^*$, we obtain the real part of frequency

$$\Omega_r = \frac{\mathbf{k} \cdot \mathbf{U}_0}{1 + \psi}.$$  

(28)

The dependence of $N = \nu_{ep}/\nu_{in}$ on the height for the model SRPM 306 is presented in Fig. 2. It is seen that Coulomb collisions become dominant at heights $h > 1000$ km and hence the development of FBI is facilitated in the upper chromosphere (Gogoberidze et al. 2009).

If ion thermal effects are ignored, than the most unstable mode has $\theta = 0$ and the threshold value of the current velocity necessary to trigger the FBI is given by

$$U_0^* = c_s (1 + \psi) \left( 1 - \frac{\kappa^2}{1 + N} \right)^{-1/2}.$$  

(30)
Dependence of the FBI threshold $U_0^{\text{FBI}}$ on the chromospheric height for $\varepsilon^* = 0$ (solid line), $\varepsilon^* = 1$ (dashed line), $\varepsilon^* = 10$ (dashed-dotted line) and $\varepsilon^* = 30$ (dotted line). Left panel corresponds to the protons and right panel to ions with $m_i = 30m_p$.

Figure 3. Dependence of the FBI threshold $U_0^{\text{FBI}}$ on the chromospheric height for $\varepsilon^* = 0$ (solid line), $\varepsilon^* = 1$ (dashed line), $\varepsilon^* = 10$ (dashed-dotted line) and $\varepsilon^* = 30$ (dotted line). Left panel corresponds to the protons and right panel to ions with $m_i = 30m_p$.

Figure 4. The characteristic FBI wavelengths as functions of the chromospheric height in the SRPM 306 model: $\lambda_m$ (dotted line), $\lambda_e$ (thin dashed line), $\lambda_i$ (thick dashed line), $\lambda_T$ (thin dash-dotted line), $\lambda_\kappa$ (thick das-dotted line) and $\lambda_0$ (solid line). Left panel corresponds to the protons and right panel to ions with $m_i = 30m_p$.

Using the SRPM 306 model, Gogoberidze et al. (2009) found that the minimum value of $U_0^{\text{FBI}}$ occurs at chromospheric height of 850 km and is about 2 km/s, which corresponds to the current $J_0 \sim 2.4 \times 10^6$ statampere/cm$^2$. According to recent observations, the typical values of currents at length scales $\sim 100$ km and longer are much smaller, $\sim 5 \times 10^4$ statampere/cm$^2$ (Socas-Navarro 2007). In principle it is possible that stronger currents exist locally at smaller scales, but in this case the heat produced by the ion-neutral friction will be at least one order of magnitude larger than the energy required to sustain the radiative losses in the chromosphere. Consequently, Gogoberidze et al. (2009) concluded the FBI can not be responsible for chromospheric heating.

The ion thermal driving described by the last term in square brackets of Eq. (23) becomes important for relatively high chromospheric altitudes where the ion magnetization is strong. Analysis of Eq. (29) shows that the most unstable mode propagates at the angle $\theta_{IT}$,

$$\tan \theta_{IT} = 2\kappa(1 + \bar{\psi})\bar{\nu} \left[ \kappa^2 - \left( \frac{3}{1 + N} - 2\eta \right) \right]^{-1}. \hspace{1cm} \text{(31)}$$

Here

$$\bar{\nu}_{\mu i} = \frac{2\mu_i}{\xi} \left( 1 + \frac{m_i + m_p}{2m_p} \frac{\nu_{\kappa e}}{\nu_{\kappa i}} N \right)^{-1}. \hspace{1cm} \text{(32)}$$

For protons and heavy ions with $m_i = 30m_p$

$$\bar{\nu}_{\mu p} = \frac{1}{1 + 4.59N}, \hspace{1cm} \bar{\nu}_{\mu i} = \frac{1}{1 + 71.2N}. \hspace{1cm} \text{(33)}$$

Using the data presented in Fig. 1 we conclude that both in the lower chromosphere (where positive charges are dominated by heavy ions), and in the upper chromosphere, the Coulomb collisions strongly reduce the ion thermal effects and make them negligible in the chromospheric conditions.

4 ELECTRON THERMAL EFFECTS

As is mentioned above, the electron thermal effects are important at relatively low altitudes, where ion magnetization is still weak. Therefore we treat the ions as unmagnetized, whereas the electrons are assumed to be strongly magnetized, in which case $V_i \approx U_0 \approx V_e$. Manipulations with the Euler equation for electrons under the condition $\omega_{ce} \gg \nu_{ce}$ yield

$$\nu'_e = \frac{\omega_{ce}}{k} \left( k - \frac{\omega_{ce}}{\nu_{ce}(1 + N) + \eta_k k^2} k \times b \right), \hspace{1cm} \text{(34)}$$

From the Euler equation for ions and from the continuity equation, dropping the terms of order $\psi_k^2 \sim 2.6 \times 10^{-3}$ we have

$$\nu'_i = -i \frac{k v_{\kappa i}^2}{\nu_{\kappa e}^2 (1 + \xi - i\Omega/\nu_{\kappa i}^2)} \left( \bar{n} + \frac{T_e}{T_i} \bar{\phi}_e \right) = \frac{k}{k^2 \Omega \bar{n}}, \hspace{1cm} \text{(35)}$$

where $\xi = k^2 \eta_i/\nu_{\kappa i}^2$. Substituting Eqs. (34) and (35) into the perturbed heat balance equation for electrons, and using condition $\omega_{ce} \gg \nu_{ce}$, we obtain

$$\left( i - 2\omega_{ce} \varepsilon \frac{\bar{\nu}_e}{\omega_{ce}} \left[ \frac{\bar{\nu}_e}{\nu_{ce}} \right] \frac{g_{en} k^2 u_{Te}^2}{m_w} \frac{1 + \rho_{ce}}{1 + m_p N} \right) \bar{n} = \left[ \frac{3i}{2} - \frac{\nu_e k^2}{m_w} \frac{3m_e \nu_{ce}}{m_p \omega} (1 + \rho_{ce}) \left( 1 + \frac{m_p}{m_i} N \right) \right] \bar{\nu}_e, \hspace{1cm} \text{(36)}$$

where $g_{en} = 1 + \eta_e k^2/\nu_{ce}(1 + N)$.

The second equation relating $\bar{n}$ and $\bar{\nu}_e$ can be obtained by eliminating $\nu_e$ and $\bar{\phi}_e$ from the Euler equation for ions by means of Eqs. (34) and (35). This yields

$$\bar{\nu}_e = -\bar{n} \frac{T_i}{T_e} \left[ \frac{c_s^2}{u_{TI}^2} - i \frac{\nu_{ce}}{u_{TI}^2} \right] \frac{\bar{\nu}_e}{\nu_{ce}} \left( \frac{3i}{2} - \frac{\nu_e k^2}{m_w} \frac{3m_e \nu_{ce}}{m_p \omega} (1 + \rho_{ce}) \left( 1 + \frac{m_p}{m_i} N \right) \right) \bar{n}.$$
The threshold value of the current velocity is given by

\[
\Omega_r = \frac{k \cdot U_0}{1 + (1 + \xi) \psi g_c}.
\]

(38)

Accounting for the terms that are second-order in \(\Omega / \nu_{in}^*\) yields the following expression for the growth rate

\[
\gamma = \nu_{in}^* [1 + (1 + \xi) \psi g_c] \left[ \Omega_r^2 - k^2 \epsilon_s^2 + \frac{\varepsilon m_p \nu_{in}^*}{m_i \nu_{cp}} \left( \frac{m_i k^2}{m_n n_c} + 3(1 + \rho_{en}) \left( 1 + \frac{m_i}{m_n} N \right) \frac{1}{1 + (1 + \xi) \psi g_c} \right) \right].
\]

(39)

If the thermal conduction and viscosity effects can be ignored (conditions for this assumption as well as analysis of other characteristic length scales in the chromosphere are presented in the next section), then Eqs. (35) and (36) reduce to

\[
\Omega_r = \frac{k \cdot U_0}{1 + \psi},
\]

(40)

\[
\gamma = \frac{\bar{\psi}}{\nu_{in}^* (1 + \psi)} \left[ k^2 U_0^2 \cos^2 \theta - \frac{k^2 U_0^2 \sin 2 \theta}{1 + (1 + \psi)^2} \right].
\]

(41)

where the effective heating coefficient \(\epsilon^* = \epsilon / (1 + \rho_{en})\) represents the cumulative effect of two counter-acting processes: wave heating/collisional cooling.

Note that in the lower chromosphere, dominated by heavy ions, electron thermal effects are reduced (compared to the upper chromosphere) due to the presence of the \(m_i / m_p\) ratio in the denominator of the second term on the right hand side of Eq. (11). Analysis of Eq. (11) shows that the propagation angle \(\theta_{ET}\) for the most unstable mode is given by

\[
\tan 2 \theta_{ET} = \frac{2}{3} \epsilon^* \frac{1 + \bar{\psi}}{\kappa} \left( 1 + N \right) \frac{m_i / m_p + N}{m_i / m_p}.
\]

(42)

The threshold value of the current velocity is

\[
U_{c ET} = c_s \sqrt{2(1 + \bar{\psi}) \left[ 1 + \sqrt{1 + \left( \frac{2}{3} \epsilon^* \frac{1 + \bar{\psi}}{\kappa} \frac{1 + N}{m_i / m_p + N} \right)^2} \right]^{1/2}}.
\]

Dependence of the threshold value of the current velocity \(U_{c ET}\) on height in the chromosphere based on SRPM 306 is shown in Fig. 3 for \(\epsilon^* = 0\) (this case corresponds to FBI in the conditions of negligible ion magnetization), 1, 10, 30. The magnetic field \(B = 30\) G. The left panel corresponds to the protons and the right to the ions with \(m_i = 30 m_p\).

From Fig. 3 one can see that, in the case of protons, the electron thermal effects cause a significant reduction of the threshold current velocity even for \(\epsilon^* = 1\), when there is no any plasma heating. For higher values of \(\epsilon^*\) the reduction of the threshold current velocity becomes very strong, and for \(\epsilon^* = 30\) the threshold value of the cross-field current velocity decreases about 10 times. However, our estimations, similar to those by Gogoberidze et al. (2009), show that this threshold reduction is insufficient to make the FBI heating comparable to the direct collisional heating by super-critical currents. It must be also noted that \(U_{c ET}\) is still much larger than the observed chromospheric currents (Socas-Navarro 2007).

In the case of heavy ions, the electron thermal effects are less important and for \(\epsilon^* = 1\) the influence of electron thermal effects on the FBI is negligible. But for higher values of \(\epsilon^*\) the decrease in \(U_{c ET}\) becomes significant also in the case of heavy ions.

5 TYPICAL LENGTH SCALES OF THE ELECTROSTATIC INSTABILITIES IN THE CHROMOSPHERE

In this section we study in detail the assumptions made in the analysis presented above. We determine the typical length scales of the electrostatic instabilities in the chromosphere. As mentioned in Sec. 2, perturbations of the neutral component can be ignored under the condition (13). The equivalent condition for the perturbation wavelength is

\[
\lambda \ll \lambda_n \equiv \frac{2\pi c_s n_n}{\nu_{in}^* n}.
\]

The condition (16) that ion and electron thermal perturbations can be considered separately yields the condition for wavelength

\[
\lambda \ll \lambda_T \equiv \frac{2\pi c_s m_i}{\nu_{cp} m_e}.
\]

(45)

In the derivation of Eqs. (40 – 44) we ignored ion and electron viscosity and electron thermal conductivity effects. From Eq. (45) it follows that electron viscosity effects can be ignored if \(\nu_{en} \gg \eta_e k^2\). Taking into account the expression for the electron viscosity (Braginskii 1965)

\[
\eta_e = 0.73 \frac{K T_e}{m_e \nu_{cp}},
\]

(46)

we find that the electron viscosity can be neglected under the following condition

\[
\lambda \gg \lambda_e \equiv 2 \pi \left( \frac{1 + N \nu_{en} \nu_{cp}}{0.73 u_T^e} \right)^{-1/2}.
\]

(47)

According to Eq. (35), ion viscosity can be neglected if \(\nu_{in}^* \gg \eta_i k^2\). Noting that the ion viscosity (Braginskii 1965)

\[
\eta_i = 0.96 \frac{K T_i}{m_i m_e \nu_{cp}}.
\]

(48)

we conclude that the ion viscosity can be neglected if

\[
\lambda \gg \lambda_i \equiv 2 \pi \left( \frac{\nu_{en} \nu_{cp}}{0.96 u_T^i} \right)^{-1/2}.
\]

(49)

The perpendicular heat conductivity of electrons is (Braginskii 1965)

\[
\chi_e = 4.66 \frac{n K T_e \nu_{cp}}{m_e \omega_B^e}.
\]

(50)
mass ratio $m_i/m_n$ is large in the middle/lower chromosphere, which leads to the decrease of the ion/neutral friction.

Since the Coulomb collisions usually introduce dissipative effects, their favorable influence on FBI is counter-intuitive and needs some explanation. As is known from ionospheric research (Oppenheim et al. 1995; Schunk & Nagy 2000), the destabilizing term driving FBI is caused by the Pedersen response to the electric field perturbations, whereas the stabilizing term (proportional to $\kappa^2/(1 + N)$) is related to the Hall response. The intervention of Coulomb collisions in this picture is as follows: they abate the Pedersen term in the growth rate less than the Hall term and thus facilitate the FBI making it possible even for $\kappa > 1$.

Without effects introduced by the Coulomb collisions and large ion/neutral mass ratio (in the limit $N \rightarrow 0$ and $m_i/m_n \rightarrow 1$), our results are compatible with the results of ionospheric E-layer research. This conclusion follows from the comparison of our results on the thermal FBI effects with results by Dimant & Sudan (1993, 1997), Robinson (1998); Dimant & Oppenheim (2004).

7 CONCLUSIONS

We investigated electrostatic instabilities of Farley-Buneman type in the partially ionized plasma of the solar chromosphere taking into account ion and electron thermal effects, electron and ion viscosity, and Coulomb collisions. We derived the FBI growth rate including the ion thermal terms and found that the Coulomb collisions highly reduce them in the middle/upper chromosphere. Consequently, ion thermal effects can be neglected for FBI in the solar chromosphere.

On the contrary, the electron thermal terms that contribute to the FBI growth rate are not negligible in the chromospheric conditions and cause a significant reduction of the threshold current triggering the instability. The ion and electron viscosity and thermal conductivity are also important and reduce the instability growth rate for relatively small-scale perturbations. We determined the characteristic length scales relevant to chromospheric conditions well as the threshold value of the current velocity as functions of height in the framework of the semi-empirical chromospheric model SRPM 306.

It has to be noted that the study of Gogoberidze et al. (2009) did not take into account the effect of additional electron heating related to the presence of parallel electric field in waves. As showed theoretically by Dimant & Milikh (2003) and confirmed by recent particle in cell simulations (Oppenheim & Dimant 2013), this effect can significantly increase the electron heating. Importance of this mechanism for the solar chromosphere requires separate analysis and is out of the scope of this paper.

In spite of the considerable threshold reduction by the electron thermal effects (see Eq. 33 and Fig. 1), our analysis showed that the electrostatic FB instabilities modified by the electron and ion thermal effects in chromospheric conditions are less efficient heating mechanisms than the collisional dissipation of cross-field currents that drive these instabilities. This conclusion concerns both the lower chromosphere, where the threshold velocity is decreased by heavy
ions, and the middle/upper chromosphere, where the threshold velocity is decreased by the Coulomb collisions. As discussed in the introduction, our analysis ignored an additional electron heating related to the presence of parallel electric fields in waves. This effect is known to enhance significantly electron heating in the ionospheric E-layer and therefore we can not exclude the possibility that similar effect can take place in the solar chromosphere as well. This subject require further investigations.

The characteristic wavelengths of the FB-type instabilities driven by super-critical currents in the solar chromosphere are \( \lambda = 10^{-3} \) cm. The plasma density fluctuations generated by these instabilities can produce scintillations of radio waves propagating through the chromosphere. The radio scintillations at \( \sim 10 \) cm wavelengths are indicators for the FB instability developed in the lower chromosphere, while the scintillations at \(< 10^3 \) cm wavelengths suggest FBI in the upper chromosphere. Observations and interpretations of such radio scintillations in terms of FBI provide a possibility for remote diagnostics of strong cross-field currents and plasma parameters in the solar chromosphere.

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