A stochastic differential equation approach
to the analysis of the UK 2017 and 2019 general election polls

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Abstract

Human dynamics and sociophysics build on statistical models that can shed light on and add
to our understanding of social phenomena. We propose a generative model based on a stochastic
differential equation that enables us to model the opinion polls leading up to the UK 2017 and
2019 general elections, and to make predictions relating to the actual result of the elections.
After a brief analysis of the time series of the poll results, we provide empirical evidence that the
gamma distribution, which is often used in financial modelling, fits the marginal distribution
of this time series. We demonstrate that the model provides good predictive power of the actual
election results, by using the Euler-Maruyama method to simulate the time series, measuring
the prediction error with the mean absolute error and the root mean square error.

Keywords: generative model; election polls; stochastic differential equations; time series; CIR
process; gamma distribution; Euler-Maruyama method.

1 Introduction

We present a companion paper to [22] using the same methodology, which is based on stochastic
differential equations (SDEs) [33] [17] applied to opinion polls leading up to an election rather than
to a referendum. In particular, we deploy a novel stochastic process based on the Cox-Ingersoll-Ross
(CIR) process [13] [10], used to model the term structure of interest rates [4]. CIR processes are
‘mean-reverting’ diffusion processes [27], the marginal distributions of which are gamma distributed;
processes that are sums of such diffusions have autocorrelation functions that are sums of the
exponentially decaying autocorrelation functions of the constituent diffusion processes [5] [25].

We refer the reader to [22] for the background in human dynamics and sociophysics [35] (also
known as social physics), noting that statistical physics [7] has played a central role in its formulation;
humans are viewed as “social atoms”, each exhibiting simple individual behaviour having limited
complexity, but nevertheless collectively they yield complex social patterns [31]. In the context
of human dynamics, the SDE model we propose is a generative model in the form of a stochastic
process the evolution of which gives rise to distributions such as power law and Weibull distributions
[20]. Generative models also arise from agent-based models [12] and have played an important role
in the sociophysics literature in the context of opinion dynamics [7] [36]. In particular, the voter
model and its extensions [7] [34], whereby at each time step an agent decides whether to hold onto
or change its opinion depending on the opinions of its neighbours, have applications in explaining and understanding voting behaviour during elections.

Opinion polls, which provide the data source for our SDE model, relay important information to the public in the lead-up to an election and provide an important ingredient of forecasting methods; see [48] for a high-level overview of election polls. In a given election cycle, polls can be naturally viewed as a time series, and thus be expected to follow a stochastic process such as an AR(1) model [9]. Although in [39] the authors had some reservations about using such a time series model, due to sampling error and lack of sufficient time series data, in [40] it was mentioned that, given a sufficient number of poll results, these could be readily treated as a statistical time series.

In [22] we took a fresh look at the time series approach, going beyond the model suggested in [39], and made use of the availability of a large number of polls conducted at regular intervals. In particular, we proposed a novel model based on SDEs, which are widely used in physics and mathematical finance to model diffusion processes, that can be viewed as a continuous approximation to a discrete process modelling how the polls vary over time. Therein we provided empirical evidence that the beta distribution, which is a natural choice when modelling proportions, fits the marginal distribution of the time series and we provided evidence of the predictive power of the model. One disadvantage of this model is that its autocorrelation function is exponentially decreasing [5], while in reality the tails of the autocorrelation function may be heavier. We address this problem in Section 3 by extending the model of [22] to allow processes that are sums of diffusions [5, 25], in which case the autocorrelation function is a sum of exponentials.

In order to evaluate the predictive power of the model, we make use of the Euler-Maruyama (EM) method [34], which is a computational method for approximating numerical solutions to SDEs. In particular, the EM method allows us to simulate the time series in order to predict the result of the election from the SDEs. We utilise the well-known mean absolute error (MAE) and root mean square error (RMSE) [8] metrics to assess the accuracy of the EM method in predicting the actual election result, and we compare these to the predictions obtained by simply taking the results of the opinion polls.

The rest of the paper is organised as follows. In Section 2 we provide a brief analysis of the UK election poll results for 2017 and 2019. In Section 3 we propose a generative model for analysing the polling data based on a sum of ‘mean-reverting’ stochastic differential equations. In Section 4 we apply the model to the polls leading up to the UK 2017 and 2019 general elections, utilising the EM method to evaluate the predictive power of the model. Finally, in Section 5 we give our concluding remarks.

2 Preliminary analysis of the time series of poll results

The analysis for the 2017 election was done using the results of 254 opinion polls, which were collected prior to the election that took place on 8th June 2017. The data set was obtained online from [23], the first poll being taken on 9th May 2015 and the last on the day before the election. Detailed results of the election can be obtained online from [2]. Similarly, the analysis for the 2019 election was done using the results of 568 opinion polls, which were collected prior to the election that took place on 12th December 2019. The data set was obtained online from [24], the first poll being taken on 4th January 2017 and the last on the day before the election. Detailed results of the election can be obtained online from [3]. For each party and for each election, the data set was a time series of the proportion of respondents who said they would vote for that party. There are shown graphically in Figures 1 and 2.
When analysing the data, in order to see any clear trends, it is interesting to inspect the moving averages of the polls, which are shown in Figures 1 and 2. For the 2017 election, it is clear that, although there was a dip in the support for Labour as the election was approaching, as it got closer to the election date the gap between Conservative and Labour narrowed, until the last day before the election when the Conservative lead in the polls was only 1%. In the election itself, where the Conservatives received 42.4% of the vote and Labour 40.0%, the Conservative lead was slightly higher at 2.4%. In 2019, the election date of 12th December was decided in parliament on the 29th October, and the Conservative lead in the polls from that date until the election was on average 10.8%, with a standard deviation of 3.37%. The Conservative lead on the last day before the elections was 11% and in the election itself, in which the Conservatives received 43.6% of the vote and Labour 32.2%, the lead was even higher at 11.4%. This does not tell the whole story of this election as the UK “first-pass-the-post” electoral system resulted in the Conservatives ending up with a majority of 80 seats in parliament.

In our model and analysis given below we treat the two parties independently, with the realisation that in practice the time series for the Conservatives and Labour parties are not actually independent; we view our analysis as a first approximation to the problem. We note that although the two main parties in the UK receive most of the votes, there is at least a third party, the Liberal Democrats, which we have not considered in this analysis, but could be considered in future research. In this context, it is worth noting that there have been times when one of the two main parties receives more votes, possibly at the expense of the other (see Figure 1), and there are other times when both of the parties have received more votes, possibly at the expense of a third party (see Figure 2).

![Figure 1: Raw time series and moving average of the 2017 polls with a centred sliding window of 25 time steps for the Conservative and Labour parties.](image-url)
3 A generative model for time series with application to polls

Stochastic differential equations can provide effective generative models for time series. In particular, when the SDEs are ‘mean-reverting’ [27], as will be the case here, they often possess stationary solutions that fit a number of well-studied distributions [11, 5]. In our application, analysing the poll results, the gamma distribution [28, 14] appears to be a natural choice, since it is flexible and allows the construction of a sum of diffusion processes that has an autocorrelation function that is a sum of exponentials [5, 25].

A typical stochastic differential equation (SDE) takes the form

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t,$$

where $X_t$ is a random variable with $t \geq 0$ a real number denoting time, $\mu$ and $\sigma$ are known as the drift and diffusion functions, respectively, and $W_t$ is a Wiener process (also known as Brownian motion). Moreover, when

$$\mu(x) = \theta(m-x),$$

where $\theta$, the rate parameter, is a positive constant and $m$ is a constant representing the mean of the underlying stochastic process, the SDE has a stationary solution [11, 5]. In addition, its autocorrelation function is exponentially decreasing [5] and takes the form

$$\exp(-\theta t).$$

Such a stochastic process is known as a ‘mean-reverting’ process. It was shown in [11, 5] that, if

$$\sigma^2(x) = \frac{2\theta}{\lambda}x$$

and $\lambda$ satisfies

$$m = \frac{\alpha}{\lambda},$$

$$0 \leq \alpha < \lambda.$$
the marginal distribution of the stationary solution of the SDE is a gamma distribution \[28\] with probability density function
\[
\frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\lambda x), \quad (6)
\]
where \( \Gamma \) is the gamma function \([1, 6.1]\); \( \alpha > 0 \) is the shape of the distribution and \( \lambda > 0 \) is a scale parameter. We note that several other forms for \( m \) and \( \sigma^2(X_t) \) also lead to other well-known distributions \([11, 5]\).

Under the above conditions, substituting (4) and (5) into (1) gives the SDE
\[
dX_t = \theta \left( \frac{\alpha}{\lambda} - X_t \right) dt + \sqrt{\frac{2\theta}{\lambda}} X_t \, dW_t, \quad (7)
\]
which describes a process called the Cox-Ingersoll-Ross process (CIR process) \([13]\). From the results in \([19]\) (cf. \([13]\)), we can conclude that the solution to (7) is positive when \( \alpha \geq 1 \).

The autocorrelation function of the CIR process is exponential as in (3), and thus, in order to model a process having an autocorrelation function with a heavier tail (such as a power law), we introduce a diffusion process that is a sum of CIR processes \([5, 25]\), for which the autocorrelation function is a sum of exponentials. This relies on the result that a power law can be approximated by a finite sum of exponentials, since it is a completely monotone function \([18]\) (cf. \([21]\)).

We can obtain a process \( X_t \) as the sum of \( n \) processes by letting
\[
X_t = X^{(1)}_t + X^{(2)}_t + \cdots + X^{(n)}_t, \quad (8)
\]
where the Wiener processes \( W^{(i)}_t \) are independent, for \( 1 \leq i \leq n \), and \( X^{(i)}_t \) is defined by the SDE
\[
dX^{(i)}_t = \theta_i \left( \frac{\phi_i \alpha}{\lambda} - X^{(i)}_t \right) dt + \sqrt{\frac{2\theta_i}{\lambda}} X^{(i)}_t \, dW^{(i)}_t. \quad (9)
\]

Then the mean, diffusion squared and autocorrelation function are given, respectively, by
\[
m_i = \frac{\phi_i \alpha}{\lambda}, \quad \sigma^2(X^{(i)}_t) = \frac{2\theta_i}{\lambda} X^{(i)}_t \quad \text{and} \quad \exp(-\theta_i t), \quad (10)
\]
where
\[
\phi_1 + \phi_2 + \cdots + \phi_n = 1. \quad (11)
\]

It follows that the marginal distribution of each \( X^{(i)}_t \) is a gamma distribution with shape \( \phi_i \alpha \) and scale parameter \( \lambda \). The marginal distribution of the sum \( X_t \) is a gamma distribution with shape \( \alpha \) and the same scale parameter \( \lambda \). Moreover, \( X_t \) has autocorrelation function
\[
\phi_1 \exp(-\theta_1 t) + \phi_2 \exp(-\theta_2 t) + \cdots + \phi_n \exp(-\theta_n t). \quad (12)
\]
4 Analysis of the poll results for the general elections

The approach we have taken to validate the model is similar to that taken in [37], building on the stationary diffusion-type models developed in [5] for constructing diffusion processes with a given marginal distribution and autocorrelation function.

We can simulate the sum of the diffusion processes defined by (8) and (9) using the Euler-Maruyama (EM) method [34] (cf. [15]), which is a general computational method for obtaining approximate numerical solutions to SDEs. We also make use of the Jensen-Shannon divergence (JSD) [16] as a goodness-of-fit measure [32]. All computations were carried out using the Matlab software package.

In Tables 1 and 2 we show the parameters of the gamma distributions fitted to the data sets using the maximum likelihood method, and the JSD between the empirical marginal distribution of the time series of the poll results and the fitted gamma distribution; we note that its mean \( \mu \) is given by \( \mu = \alpha / \lambda \), and its standard deviation \( \sigma \) by \( \sigma^2 = \alpha / \lambda^2 \). The low JSD values indicate good fits for both political parties. We note that the JSD for the Conservative party in the 2019 elections is much higher than that for 2017, which could indicate that another distribution may better fit the data. In fact, we found that the Gumbel distribution [29, 30] (also known as a type I extreme value distribution) is a better fit for the Conservatives in 2019, with JSD of 0.0595, although a worse fit for Labour in 2019 with a JSD of 0.0801; we leave this line of investigation for future work as, for the purpose of prediction, the gamma distribution seems to be sufficient.

| Year       | 2017 elections | 2019 elections |
|------------|----------------|----------------|
|            | \( \alpha \)  | \( \lambda \)  | \( \mu \) | \( \sigma \) | JSD  |
| Conservative | 105.8670       | 258.5847       | 0.4094 | 0.0398 | 0.0370 |
| Labour      | 72.0295        | 236.1929       | 0.3050 | 0.0359 | 0.0478 |

Table 1: Maximum likelihood fitting of the gamma distribution to the 2017 election polls.

| Year       | 2017 elections | 2019 elections |
|------------|----------------|----------------|
|            | \( \phi_i \)  | \( \theta_i \)  | \( \phi_2 \) | \( \theta_2 \) | JSD  |
| Conservative | 0.8092         | 0.0146         | 0.1908 | 0.9896 | 0.0074 |
| Labour      | 0.7509         | 0.0237         | 0.2491 | 1.1890 | 0.0179 |

Table 3: Parameters of the exponential sum autocorrelation for the 2017 election polls.
We now turn our attention to the widely used mean absolute error (MAE) and root mean square error (RMSE) evaluation metrics [8], in order to directly estimate the prediction of the actual result using the EM method. The MAE is given by

$$MAE = \frac{\sum_{j=1}^{m} |p_j - f|}{m},$$

(13)

where $p_j$ is the proportion favouring a political party in the $j$th poll, $f$ is the corresponding proportion of votes in the actual election, and $m$ is the number of polls. The RMSE is given by

$$RMSE = \sqrt{\frac{\sum_{j=1}^{m} (p_j - f)^2}{m}},$$

(14)

noting that it is at least as large as the MAE.

We use the first third of the polls for computing the initial model parameter values, $\alpha$ and $\lambda$, of the gamma distribution, and also the $\phi_i$ and rate parameters $\theta_i$ in (12), with $n=2$. For each of the remaining two thirds of the polls, we use the EM method to predict the next step in the time series. We repeat this twenty times and take the average of the twenty predictions at each time step to get the average prediction, and also compute the prediction setting $W_t^{(i)}$ to zero in (9), which is what we would expect the average to converge to when increasing the number of EM computations and thus eliminating the random component of the SDE represented by the diffusion function. We then compare the average prediction to the actual result of the election. We evaluate the accuracy of the predictions over the complete range using the MAE and RMSE. For comparison purposes, we also computed the MAE and RMSE using the current poll as the predictor of the actual result; these are shown in Tables 5 and 6, in the columns labelled MAE-polls and RMSE-polls. The columns labelled MAE-EM and RMSE-EM show the error values of the predictions made using the EM method. It can be seen from these that for the two parties in both 2017 and 2019 the EM method was a better predictor than the polls themselves, and that the results in both tables are comparable; the margin of improvement is greatest for the Conservatives in 2019.

| Party-Year/Metric | MAE-polls | RMSE-polls | MAE-EM | RMSE-EM |
|-------------------|-----------|------------|--------|---------|
| Con 2017          | 0.0278    | 0.0348     | 0.0245 | 0.0294  |
| Lab 2017          | 0.0988    | 0.1072     | 0.0951 | 0.0981  |
| Con 2019          | 0.0719    | 0.0949     | 0.0644 | 0.0853  |
| Lab 2019          | 0.0532    | 0.0618     | 0.0491 | 0.0568  |

Table 5: MAE and RMSE prediction errors for the 2017 and 2019 UK election results when averaging the predictions over twenty runs of the EM method. (The smaller error values are italicised.)
| Party-Year/Metric | MAE-polls | RMSE-polls | MAE-EM | RMSE-EM |
|------------------|-----------|------------|--------|---------|
| Con 2017         | 0.0278    | 0.0348     | 0.0250 | 0.0296  |
| Lab 2017         | 0.0988    | 0.1072     | 0.0951 | 0.0977  |
| Con 2019         | 0.0719    | 0.0949     | 0.0641 | 0.0850  |
| Lab 2019         | 0.0532    | 0.0618     | 0.0486 | 0.0563  |

Table 6: MAE and RMSE prediction errors for the 2017 and 2019 UK election results, when setting $W_t^{(i)}=0$. (The smaller error values are italicised.)

We also counted the number of times the prediction using EM method was closer to the actual election result than was the prediction based on the current poll, and vice versa (cf. average ranks method [6]): the numbers are shown in Tables 7 and 8 in the columns labelled Polls and EM. The column labelled Total shows the total number of polls used, recalling that a third of the polls were used for computing the initial model parameters, while the column labelled Improvement shows the improvement percentage of the EM method prediction over using the polls themselves as predictors of the final result. These show a similar pattern to the prediction errors in Tables 5 and 6, i.e. in all cases the EM method is more accurate than using the polls themselves. The improvement is most notable for the Conservatives in 2019, and that for Labour in 2017 also stands out. We note that apart from the improvement for the Conservatives in 2017, the results in Table 7 are dominated by those in Table 8.

| Party-Year | Polls | EM  | Total | Improvement |
|------------|-------|-----|-------|-------------|
| Con 2017   | 79    | 90  | 169   | 6.51%       |
| Lab 2017   | 72    | 97  | 169   | 14.79%      |
| Con 2019   | 149   | 230 | 379   | 21.37%      |
| Lab 2019   | 173   | 206 | 379   | 8.71%       |

Table 7: Comparison of the number of times the closer prediction was based on either the current poll or using the EM method, when averaging the EM method predictions over twenty runs.

| Party-Year | Polls | EM  | Total | Improvement |
|------------|-------|-----|-------|-------------|
| Con 2017   | 80    | 89  | 169   | 5.33%       |
| Lab 2017   | 70    | 99  | 169   | 17.16%      |
| Con 2019   | 143   | 236 | 379   | 24.54%      |
| Lab 2019   | 165   | 214 | 379   | 12.93%      |

Table 8: Comparison of the number of times the closer prediction was based on either the current poll or using the EM method, when setting $W_t^{(i)}=0$. 

8
5 Concluding remarks

We have proposed a generative SDE model to analyse the time series of opinion poll results leading up to an election. We have utilised a stochastic process, which is the sum of CIR processes, and has a stationary solution where the marginal distribution of the time series is a gamma distribution.

We provided empirical evidence that the model is a good fit to the polls leading up to the UK 2017 and 2019 elections. We also examined the predictive power of the model. We compared the errors in the predictions obtained using the EM method with those of the poll results themselves. We demonstrated that a model such as the one presented here may give better predictions of the actual election result than simply using the results of the polls. It is possible that the method we have presented using ‘mean-reverting’ SDEs could augment existing prediction methods that take into account demographic data (cf. [20]).

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