Current conservation, screening and the magnetic moment of the \( \Delta \) resonance.\(^1\)

2. Formulation with quark degrees of freedom

3. Magnetic moment of the \( \Delta^0 \) and \( \Delta^- \) resonances.

A. I. Machavariani\(^a\) \( b\) \( c\) and Amand Faessler \( a\)

\(^a\) Institute für Theoretische Physik der Universität Tübingen, Tübingen D-72076, Germany

\(^b\) Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia

\(^c\) Tbilisi State University, University str. 9, Tbilisi, Georgia

Abstract

Our previous paper [1] is generalized within the field theoretical formulation with the quark degrees of freedom [2, 3, 4, 5], where pions and nucleons are treated as the bound systems of quarks. It is shown that relations generated by current conservation for the on shell \( \pi N \) bremsstrahlung amplitude with composite nucleons and pions have the same form as in the usual quantum field theory [6, 7] without quark degrees of freedom [1]. Consequently, the model independent relations for the magnetic dipole moments of the \( \Delta^+ \) and \( \Delta^{++} \) resonances in [1] remain be the same in the quantum field theory with the quark degrees of freedom. These relations are extended for the magnetic dipole moments of the \( \Delta^0 \) and \( \Delta^- \) resonances which are determined via the anomalous magnetic moment of the neutron \( \mu_n \) as \( \mu_{\Delta^0} = \frac{M_\Delta}{m_p} \mu_n \) and \( \mu_{\Delta^-} = \frac{3}{2} \mu_{\Delta^0} \).

\(^1\)Supported by the ”Deutsche Forschungsgemeinschaft” under contract GRK683
1. Introduction

The self-consistent generalization of the conventional quantum field theory for composite particles was developed in refs. [2, 3, 4, 5]. In this approach the quark-hadron bound state functions satisfy the appropriate Bethe-Salpeter equations and determine the transitions from the quark-gluon degrees of freedom into hadron degrees of freedom. The general Bethe-Salpeter equations for the quark-hadron wave functions were derived by Huang and Weldon [2]. The basic objects in this approach as well as in the Haag-Nishijima-Zimmermann approaches [3, 4, 5] are the creation and annihilation operators of composite particles which allows to obtain the \( S \)-matrix reduction formula for composite particles. The equivalent three dimensional field-theoretical equations for composite hadron interaction amplitudes are given in ref. [8, 9, 10]. The advantages of this three-dimensional formulation may be summarized as follows:

- The poles of the intermediate quark and gluon propagators do not contribute to the unitarity condition of the hadron-hadron scattering amplitudes. Therefore, the quark-gluon and hadron degrees of freedom are unambiguously separated and the problems with the double counting do not appear.

- The resulting equations for the hadron-hadron scattering amplitudes with and without quark degrees of freedom have the same form. Therefore, one can easily extend the field-theoretical relations without quarks to the formulation with the quark-gluon degrees of freedom.

The usual \( \pi N \) bremsstrahlung amplitude \( < out; N', \pi' | J^\mu(0) | \pi, N; in > \) with on mass shell pions and nucleons in the "in" and "out" asymptotic states and with the photon current operator \( J^\mu(x) \) is depicted by the left diagram in Fig.1. In the generalized quark-gluon approach [2, 3, 4, 5] this amplitude is determined through the Green function of the transition between the 4-quark + antiquark systems and the quark-hadron bound-state wave functions. The graphical representation of the \( \pi N \) radiation amplitude in the quark-gluon approach [2, 3, 4, 5] is given by the diagram on the right-side of Fig. 1. The construction of the creation and annihilation operators of the asymptotic nucleons and pions via quarks allows to operate with the usual expression of the \( \pi N \) radiation amplitude \( < out; N', \pi' | J^\mu(0) | \pi, N; in > \) because

\[
< out; N', \pi' | J^\mu(0) | \pi, N; in > \equiv 0 | B_{out}(N') a_{out}(\pi') J^\mu(0) a^+_{in}(\pi) B^+_{in}(N) | 0 > , \quad (1.1)
\]

where \( B_{out(in)}(N) \) and \( a_{out(in)}(\pi) \) denote the creation or annihilation operators of the composed nucleon and pion in the asymptotic "out" or "in" states.

In this paper we shall show that current conservation on the quark level for the on shell \( \pi N \) bremsstrahlung amplitude (1.1) takes again the form of the modified Ward-Takahashi identity for the on shell external particle radiation amplitude \( \mathcal{E}_{\gamma' \pi' N' - \pi N}^{\mu} \).
Figure 1: The $\pi N$ bremsstrahlung amplitude for composite nucleons and pions.

\[ k'_\mu < \text{out}; N', \pi' | J^\mu(0) | \pi, N; \text{in} > = k'_\mu \mathcal{E}_{\gamma' \pi' N' - \pi N} + \mathcal{B}_{\pi' N' - \pi N} = 0, \quad (1.2a) \]

where $k'_\mu$ is the four momentum of the final photon, $\mathcal{B}_{\pi' N' - \pi N}$ stands for a sum of the off shell elastic $\pi N$ scattering amplitudes.

The external particle radiation diagrams are depicted in Fig. 2. The only difference between the external particle radiation amplitude in Fig. 2 and in the formulation without quark degrees of freedom (see Fig. 1 in [1]) is the off shell $\pi N$ amplitudes, which contains the nonlocal momentum-depending source operators of the pion or nucleon.

Figure 2: The external particle radiation diagrams of the $\pi N$ bremsstrahlung amplitude.
Current conservation (1.2a) indicates a connection between the four-divergence of the external $\mathcal{E}_{\gamma'\pi'N'\to\pi N}$ and internal $\mathcal{T}_{\gamma'\pi'N'\to\pi N}$ particle radiation amplitudes $\mathcal{T}_{\gamma'\pi'N'\to\pi N}$

$$k'_{\mu}\mathcal{E}_{\gamma'\pi'N'\to\pi N} = -k'_{\mu}\mathcal{T}_{\gamma'\pi'N'\to\pi N} = -B_{\pi'N'\to\pi N}, \quad (1.2b)$$

where $<\text{out}; N', \pi' | J^{\mu}(0) | \pi, N ; \text{in}> = \mathcal{E}_{\gamma'\pi'N'\to\pi N} + \mathcal{T}_{\gamma'\pi'N'\to\pi N}.

Figure 3: The double on mass shell $\Delta$ exchange diagram with the intermediate $\Delta$ radiation vertex. The $\Delta - \gamma \Delta$ vertex contains the dipole magnetic moment of the $\Delta$.

One can decompose (1.2a,b) into a set of independent current conservation for the longitudinal part of the on shell $\pi N$ radiation amplitude [1]. In particular, one can unambiguously separate the $\Delta$-pole parts of the $\pi N$ amplitudes which are contained in $\mathcal{E}_{\gamma'\pi'N'\to\pi N}$ and $\mathcal{B}_{\pi'N'\to\pi N}$. The $\Delta$-pole part of the full $\pi N$ Green function determines the $\pi N - \Delta$ wave function with the on mass shell $\Delta$ and the effective mass of the $\Delta$ [1, 10, 11, 12] $m_{\Delta}(s) = \Lambda_{\Delta}(s) - i/2\Gamma_{\Delta}(s)$ which generally depends on the Mandelstam variable $s$. This $\pi N - \Delta$ wave function and $m_{\Delta}(s)$ define the intermediate on mass shell $\Delta$ state with the four momentum $P_{\Delta} = (\sqrt{m_{\Delta}^2(s) + P_{\Delta}^2}, P_{\Delta})$ also in the present formulation with quark degrees of freedom. The considered field theoretical $\pi N$ bremsstrahlung amplitudes are not depending on the model of $m_{\Delta}(s)$ which must be determined separately. The sum of the $\Delta$-pole parts of the off shell $\pi N$ amplitudes in $\mathcal{E}_{\gamma'\pi'N'\to\pi N}$ (Fig. 2) reproduces the double on mass shell $\Delta$ exchange amplitude $(\mathcal{E}_{\gamma'\pi'N'\to\pi N}^{3/2})_{\gamma'\pi'N'\to\pi N}(\Delta - \gamma \Delta)$ which contains the $\Delta - \gamma \Delta$ vertex with the anomalous magnetic moment of the proton instead of the magnetic dipole moment of the $\Delta$. $(\mathcal{E}_{\gamma'\pi'N'\to\pi N}^{3/2})_{\gamma'\pi'N'\to\pi N}(\Delta - \gamma \Delta)$ has the same analytical structure as the intermediate $\Delta$ radiation diagram $\mathcal{T}_{\gamma'\pi'N'\to\pi N}^{3/2}(\Delta - \gamma \Delta)$ in Fig. 3. This amplitude is unambiguously separated from the internal particle radiation diagrams based on the $\Delta$-pole terms of the intermediate $\pi N$ Green function [1]. Thus

$$k'_{\mu}(\mathcal{E}_{\gamma'\pi'N'\to\pi N}^{3/2})_{\gamma'\pi'N'\to\pi N}(\Delta - \gamma \Delta) = -k'_{\mu}\mathcal{T}_{\gamma'\pi'N'\to\pi N}^{3/2}(\Delta - \gamma \Delta) = -B_{\pi'N'\to\pi N}^{3/2}(\Delta - \gamma \Delta), \quad (1.3)$$
where the lower index $\zeta$ and the upper index $3/2$ denotes the longitudinal and the spin-isospin $(3/2,3/2)$ part of the corresponding expressions. The identical structure of $(\mathcal{E}^{3/2}_{\zeta})^\mu_{\pi'N'\cdot N}(\Delta - \gamma \Delta)$ and $\mathcal{I}^{\mu}_{\pi'N'\cdot N}(\Delta - \gamma \Delta)$ in Fig. 3 allows to obtain an analytical and model independent relations between the magnetic dipole moments of the $\Delta^+$ and $\Delta^{++}$ resonances and the anomalous magnetic moment of the proton. In this paper we generalize this relation for the magnetic dipole moments of the $\Delta^o$ and $\Delta^-$ resonances.

This paper consists of four Sections. The creation and annihilation operators of composite particle and corresponding Ward-Takahashi identity are constructed in the next Section. The model-independent relation between the magnetic dipole moments of the $\Delta^o$ and $\Delta^-$ resonances and the anomalous magnetic moment of the neutron is given in Section 3. The conclusions and comparison of the suggested relations for the magnetic dipole moments of $\Delta^o$ and $\Delta^-$ with the numerical values of other authors are presented in Sect. 4.

2. The Ward-Takahashi identity for the on shell $\pi N$ bremsstrahlung amplitude in the field theoretical approach with the quark degrees of freedom

The creation and annihilation operators of the hadrons as the quark cluster operators were constructed in ref. [2] in the framework of the usual quantum field theory. The corresponding nucleon and pion field operators $\Psi_{\pi N}(Y)$ and $\Phi_{\pi N}(X)$ are composed through the local quark field operators $q(x)$. $\Psi_{\pi N}(Y)$ and $\Phi_{\pi N}(X)$ are nonlocal because they depend on the nucleon and pion four moments $p_N$ and $p_\pi$ correspondingly

$$\Psi_{\pi N}(Y) = \int d^4r_3 d^4r_{1,2} \chi_{\pi N}^\dagger(y_1 = 0, r_{1,2}, r_3) T \left(q_1(y_1)q_2(y_2)q_3(y_3)\right), \quad (2.1)$$

where $Y$, $r_{1,2}$ and $r_3$ are the Jacobi coordinates $y_1 = Y - \eta_3 r_3 + \eta_2 r_{1,2}$, $y_2 = Y - \eta_3 r_3 - \eta_1 r_{1,2}$, $y_3 = Y + \eta_{1,2} r_3$ with $\eta_3 + \eta_{1,2} = 1$ and $\eta_1 + \eta_2 = 1$,

$$\chi_{\pi N}(y_1, y_2, y_3) = \langle 0 | T \left(q_1(y_1)q_2(y_2)q_3(y_3)|p_N, s_N, i_N; in \rangle = e^{-ip_N Y} \chi_{\pi N}(Y = 0, r_{1,2}, r_3) \quad (2.2a)$$

is a solution of the Bethe-Salpeter equation for the three quark bound state with the nucleon mass $m_N$ and four momentum $p_N = (\sqrt{p_N^2 + m_N^2}, p_N)$; $s_N$ and $i_N$ denotes the spin-isospin projections of the nucleon. $\chi_{\pi N}$ satisfies the normalization condition [6]}

$$\frac{1}{2im_N^2} \left< \chi_{\pi N}^\dagger | \frac{\partial}{\partial p_N^\mu} \left[G^{-1}(3q)\right]| \chi_{\pi N} \right> \equiv \left< \bar{\chi}_{\pi N}^\dagger | \chi_{\pi N} \right> = \left< p_N', s_N', i_N'; m_N | p_N, s_N, i_N; m_N \right>, \quad (2.2b)$$

where $G(3q)$ is the full Green function of three interacting quarks².

²The on mass shell $\Delta$ state with the complex mass $m_\Delta = M_\Delta + i\Gamma_\Delta/2$ can be constructed through the intermediate three quark state in the same manner as the one nucleon state. In particular, it is necessary to find the solution of the Bethe-Salpeter equation for the $\Delta$-pole state of the $3quark - 3quark$ Green function: $\Psi_{p\Delta}(x_1, x_2, x_3) = \langle 0 | T \left(q_1(x_1)q_2(x_2)q_3(x_3)\right)| \Psi_{p\Delta} \rangle$.
The asymptotic nucleon annihilation (creation) operator \( B_{\text{in(out)}}^\pi(p_N) \) for an on mass-shell nucleon is determined as

\[
B_{\text{in(out)}}(p_N) = \lim_{x^o \to -\infty(+\infty)} B_{\text{PN}}(x^o), \tag{2.3a}
\]

where the weak limit \( \lim_{x^o \to -\infty(+\infty)} \) is assumed. The Heisenberg operator \( B_{\text{PN}}(x^o) \) is given in the same form as in conventional quantum field theory

\[
B_{\text{PN}}(x^o) = \int d^3x \exp(i p_N x) \pi(p_N) \gamma_\mu \Psi_{\text{PN}}(x). \tag{2.3b}
\]

The composite meson fields are constructed using the quark-antiquark operator

\[
\Phi_{p_\pi}(X) = \int d^4 p_{1.2} \tilde{\phi}^+_{p_\pi}(X = 0, p_{1.2}) \mathcal{T}\left(q_i(x_1)\bar{q}_i(x_2)\right), \tag{2.4a}
\]

where \( p_\pi = (\sqrt{p_\pi^2 + m_\pi^2}, p_\pi) \), \( x_1 = X + \mu_2 p_{1.2} \), \( x_2 = X - \mu_1 p_{1.2} \) with \( \mu_1 + \mu_2 = 1 \) and

\[
\phi_{p_\pi}(x, y) = < 0| \mathcal{T}\left(q_i(x)\bar{q}_i(y)\right)|p_\pi, i_\pi; m_\pi > \tag{2.4b}
\]

is the solution of the Bethe-Salpeter equation of the quark-antiquark bound state. This function satisfies the normalization condition

\[
\frac{1}{2i m_\pi^2} < p_\pi' | p_\pi^\mu \frac{\partial}{\partial p_\pi^\mu} g^{-1}(q\bar{q}) | p_\pi \succ \equiv < \tilde{\phi}_{p_\pi} | \tilde{\phi}_{p_\pi} > = < p_\pi', i_\pi'; m_\pi | p_\pi, i_\pi; m_\pi >, \tag{2.4c}
\]

where \( g(q\bar{q}) \) is the full quark-antiquark Green function.

The asymptotic meson creation or annihilation operator is

\[
a_{\text{in(out)}}(p_\pi) = \lim_{x^o \to -\infty(+\infty)} a_{p_\pi}(x^o), \tag{2.5a}
\]

where

\[
a_{p_\pi}(x^o) = \int d^3x \exp(i p_\pi x) \left[ \frac{\partial}{\partial x^o} - i p_\pi^o \right] \Phi_{p_\pi}(x). \tag{2.5b}
\]

The composite operators (2.3a) and (2.5a) satisfy the same commutation relations as the ordinary local field operators of the asymptotic nucleons and pions in the usual quantum field theory \[6, 7\]

\[
\left\{ B_{\text{in(out)}}^+(p'), B_{\text{in(out)}}^-(p) \right\} = (2\pi)^3 \frac{p_N^o}{m_N} \delta(p' - p);
\]

\[
\left\{ B_{\text{in(out)}}^+(p'), B_{\text{in(out)}}^-(p) \right\} = \left\{ B_{\text{in(out)}}^+(p'), B_{\text{in(out)}}^+(p) \right\} = 0, \tag{2.6a}
\]

\[
\left[ a_{\text{in(out)}}^+(p', p_\pi), a_{\text{in(out)}}^-(p_\pi) \right] = (2\pi)^3 2 p_\pi^o \delta(p'_\pi - p_\pi); \tag{2.6b}
\]
The double time-ordered product is defined as 

\[\left[a_{\text{in}(out)}(p'_\pi), a_{\text{in}(out)}(p_\pi)\right] = \left[a_{\text{in}(out)}^+(p'_\pi), a_{\text{in}(out)}^+(p_\pi)\right] = 0.\] (2.6b)

The relations (2.6a,b) allow to build any "in" or "out" hadron states through the intermediate quark-cluster states. These operators form the usual completeness condition for the asymptotic "in" or "out" hadron fields \(\sum_n |n; \text{in(out)}\rangle < \langle \text{out(in)}| n\rangle = 1\) and the well-known \(S\)-matrix element as \(S_{nm} = \langle \text{out}| n|\text{in}\rangle\). In contrast to local quantum field theory, the Heisenberg fields \(B_p(x^\nu)\) (2.3b) and \(a_p(x^\nu)\) (2.5b) do not satisfy the equal-time commutation relations (2.6a,b) \(\{B_p'(x_\nu), B_p(x_\nu)\} \neq 0, \{B_p'(x_\nu), a_p(x_\nu)\} \neq 0\), etc. Nevertheless, the basic relations of the usual quantum field theory remain the same in the field theoretical approach with quarks. In particular, for the \(\pi N\) radiation amplitude \(A_{\gamma' \pi' N' - \pi N}^\mu\) one has

\[A_{\gamma' \pi' N' - \pi N}^\mu = i \int d^4z e^{ik'z} < 0|B_{\text{out}}(p'_N)a_{\text{out}}(p'_\pi)\mathcal{J}^\mu(z)a_{\text{in}}^+(p_\pi)B_{\text{in}}^+(p_N)|0>,\] (2.7a)

where \(k'_\mu\) denotes the four momentum of the on shell final photon \(k'_{\mu}k'^\mu = 0\) and \(k'_\mu = (p_N + p_\pi - p'_N - p'_\pi)_\mu = (P - P')_\mu\);

\[\mathcal{J}^\mu(z) = \overline{\gamma}(z)\left(\frac{\lambda^3}{2} + \frac{\lambda^8}{2\sqrt{3}}\right)\gamma^\mu q(z)\] (2.7b)

denotes the photon source operator with the Gell-Mann flavor matrices \(\lambda\) [6].

A symbolical picture of the \(\pi N\) bremsstrahlung amplitude (2.7a) with the intermediate quark-clusters states is given in Fig. 1. The triangles in Fig. 1 describe the quark-hadron bound state wave functions (2.2a), (2.4a) and their orthogonal expressions. These vertices play the role of the hadronization functions. Consequently, the \(\pi N \rightarrow \gamma' \pi' N'\) amplitude (2.7a) is replaced by the \(4q\overline{\gamma} - \gamma' 4q\overline{\gamma}\) transition amplitude. Using the generalized \(S\)-matrix reduction formula [2] we obtain

\[k'_\mu A_{\gamma' \pi' N' - \pi N}^\mu = \overline{\gamma}(p'_N)(\gamma_{\nu}p_{\nu}^{'} - m_N)(p_{\pi}^2 - m_{\pi}^2) k'_\mu G^\mu (\gamma_{\nu}p_{\nu}^{'} - m_N)(p_{\pi}^2 - m_{\pi}^2)u(p_N),\] (2.8a)

where

\[k'_\mu G^\mu = i \int d^4y_1'd^4y_2'd^4y_3'd^4x_1'd^4x_2'd^4y_1d^4y_2d^4y_3d^4x_1d^4x_2 e^{ik'z'} d^4z e^{ikz} d^4z' \tilde{\chi}_{p_N}(y_1', y_2', y_3') \tilde{\phi}_{\pi}(x_1', x_2') \frac{\partial}{\partial z'_{\mu}}\] (2.8b)

where the double time-ordered product is defined as 

\[\langle T\left(\mathcal{O}_{\gamma}(x_1')\mathcal{O}_{\nu}(x_2')\right)\mathcal{J}^\mu(z)\rangle = \left(\mathcal{O}_{\nu}(x_1')\theta(x_1' - x_2')q_2(x_2)\theta(x_2' - z')\mathcal{J}^\mu(z) - \mathcal{O}_{\nu}(x_2')\theta(x_2' - x_1')q_1(x_1)\theta(x_1' - z')\mathcal{J}^\mu(z)\right)\]
\[
+ \left( \mathcal{J}^\mu(z) \theta(z^0 - x_1^0) \mathcal{T}_1(x_1) \theta(x_1^0 - x_2^0) q_2(x_2) - \mathcal{J}^\mu(z) \theta(z^0 - x_2^0) \mathcal{T}_2(x_2) \theta(x_2^0 - x_1^0) q_1(x_1) \right).
\]

The choice of the average \(X = x_1 + x_2, Y = y_1 + y_2 + y_3\) or the c.m. coordinates \(X = \mu_1 x_1 + \mu_2 x_2, \rho_{1,2} = x_1 - x_2\) and \(Y = \eta_{3,3} + \eta_{1,2} Y_{1,2}, \rho_3 = y_1 - Y_{1,2}, Y_{1,2} = \eta_1 y_1 + \eta_2 y_2\) in (2.1) and in (2.4a) is not unique. But the S-matrix reduction formula and (2.8a,b) are not depend on the choice of \(X\) and \(Y\).

The important property of the on shell amplitude (2.8a) is that only the operators
\[
\bar{\pi}(p_N)(\gamma_\nu p_N^\nu - m_N) \bar{P}_{N'} \equiv \bar{\pi}(p_N) (\gamma_\nu p_N^\nu - m_N) \int d^4 y_1 d^4 y_2 d^4 y_3 \bar{\chi}_{p_N}^+(y_1, y_2, y_3) \mathcal{T} \left( q_1(x_1) q_2(x_1) q_3(x_3) \right),
\]
and their Hermitian conjugate produce the asymptotic one-nucleon and one-pion states. Therefore, the zeros of the Dirac \((\gamma_\nu p_N^\nu - m_N), (\gamma_\nu p_N^\nu - m_N)\) and the Klein-Gordon operators \((p_\pi^2 - m_\pi^2), (p_\pi^2 - m_\pi^2)\) in the on shell amplitude (2.8a) can be compensated only by \(\bar{P}_{N'}, \bar{P}_N\) and their Hermitian conjugate operators. But \(\bar{P}_{N'}, \bar{P}_N\) and their conjugate are included in \(G^\mu\) (2.8b). The remaining part of the full Green function
\[
\tau^\mu = G^\mu + g^\mu =
\]
\[
\int d^4 y_1 d^4 y_2 d^4 y_3 d^4 x_1 d^4 x_2 d^4 y_3 d^4 x_1 d^4 x_2 e^{i k x z} d^4 z \bar{\chi}_{p_\mu}^+(y_1, y_2, y_3) \bar{\phi}_{p_\mu}(x_1, x_2)
\]
\[
< 0 | \mathcal{T} \left( q_1(x_1) q_2(x_2) q_3(x_3) q_1(x_1') \bar{\mathcal{T}}_2(x_2') \mathcal{J}^\mu(z) \bar{\mathcal{T}}_1(y_1) \bar{\mathcal{T}}_2(y_2) \mathcal{T}_3(x_3) \bar{\mathcal{T}}_1(x_1) q_2(x_2) \right) | 0 >
\]
\[
\chi_{p_N}(y_1, y_2, y_3) \bar{\phi}_{p_\mu}(x_1, x_2) \quad (2.8c)
\]
involves all possible exchanges of the quark operators between \(\mathcal{T} \left( q_1(x_1) q_2(x_2) q_3(x_3) \right), \mathcal{T} \left( q_1(x_1') \bar{\mathcal{T}}_2(x_2') \right), \mathcal{T} \left( q_1(x_1) q_2(x_2) q_3(x_3) \right)\) and \(\mathcal{T} \left( q_1(x_1) \bar{\mathcal{T}}_2(x_2) \right)\). Therefore, \(g^\mu\) does not contribute into the on shell amplitude (2.7a), because \(g^\mu\) does not contain \(\bar{P}_{N'}, \bar{P}_N, \bar{P}_N, \bar{P}_N'\). Correspondingly, one can replace \(G^\mu\) (2.8b) with \(\tau^\mu\) (2.8c) in (2.8a)

\[
k'_\mu A'_{\gamma' \pi' \pi' N} = \bar{\pi}(p'_N)(\gamma_\nu p'_N^\nu - m_N)(p_\pi^2 - m_\pi^2) k'_{\mu} \tau^\mu \left( \gamma_\nu p_N^\nu - m_N \right)(p_\pi^2 - m_\pi^2) u(p_N),
\]
\[
(2.8d)
\]

Based on the equal-time commutation relations between the quark operators it is easy to get the equal-time commutation rules for the photon source operator (2.7b) and the quark operators

\[
\left[ \mathcal{J}^\rho(z), q_j(y) \right] = -e_j \mathcal{J}^{(4)}(z - y) q_j(y); \quad \left[ \mathcal{J}^\rho(z), \mathcal{T}_j(y) \right] = e_j \mathcal{J}^{(4)}(z - y) \mathcal{T}_j(y),
\]
\[
(2.9)
\]
where \(e_j\) denotes the charge of the quark \(j\). In particular, \(e_j = 2/3\) for the \(u\)-quark and \(e_j = -1/3\) for the \(d\) quarks. The equal-time commutators (2.9) and integration over \(z\) in (2.8c) allows to rewrite (2.8d) as

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This can be also verified using the limits over the coordinates \(X' \rightarrow \infty, X' \rightarrow -\infty Y' \rightarrow \infty, Y' \rightarrow -\infty\) in the quark and composite particle operators (2.3a,b), (2.5a,b).
\[ k'_\mu A^{\mu}_{\gamma'\pi'N'\pi N} = -i\bar{\pi}(p'_N)(\gamma_\nu p'_N - m_N) \int d^4y'_1d^4y'_2d^4y'_3\chi_{p'_N}(y'_1,y'_2,y'_3) \]
\[ \left[ e_{1}^{N'} e^{ik'y'_1} + e_{2}^{N'} e^{ik'y'_2} + e_{3}^{N'} e^{ik'y'_3} \right] < out; p'_\pi | T\left( q_1(y'_1)q_2(y'_2)q_3(y'_3) \right) | p\pi p_N; in > \]
\[ -i(p'_\pi - m_\pi^2) \int d^4x'_1d^4x'_2\bar{\chi}_{p_\pi}(x'_1,x'_2)\left[ e_{1}^{\pi'} e^{ik'x'_1} + e_{2}^{\pi'} e^{ik'x'_2} \right] < out; p'_N | T\left( q_1(x'_1)q_2(x'_2) \right) | p\pi p_N; in > \]
\[ + i \int d^4y'_1d^4y'_2d^4y'_3 \left[ e_{1}^{\pi} e^{ik'y'_1} + e_{2}^{\pi} e^{ik'y'_2} + e_{3}^{\pi} e^{ik'y'_3} \right] < out; p'_N p'_\pi | T\left( \bar{q}_1(y'_1)q_2(y'_2)q_3(y'_3) \right) | p\pi; in > \]
\[ \chi_{p_N}(y_1,y_2,y_3)(\gamma_\nu p_N - m_N)(p_\pi^2 - m_\pi^2)u(p_N) \]
\[ + i(p'_\pi - m_\pi^2) \int d^4x'_1d^4x'_2 \left[ e_{1}^{\pi'} e^{ik'x'_1} + e_{2}^{\pi'} e^{ik'x'_2} \right] < out; p'_N p'_\pi | T\left( \bar{q}_1(x'_1)q_2(x'_2) \right) | p\pi; in > \phi_{p\pi}(x'_1,x'_2). \]
\[(2.11)\]

After integration over the coordinates \( X', Y', \) and \( Y'' \) we obtain
\[ k'_\mu A^{\mu}_{\gamma'\pi'N'\pi N}(p'_N,p'_N,k';p_N,p_N) = -i(2\pi)^4 \delta^{(4)}(p'_N + p'_\pi + k' - p_\pi - p_N) \]
\[ \left[ \bar{\pi}(p'_N)(\gamma_\nu p'_N - m_N)\gamma_\nu(p'_N + k') - m_N < out; p'_\pi | J_{p'_N,k'}(0) | p\pi p_N; in > \right. \]
\[ + (p'_\pi - m_\pi^2) \left( p_\pi^2 - k'^2 - m_\pi^2 \right) < out; p'_N | J_{p\pi,k'}(0) | p\pi p_N; in > \]
\[ - < out; p'_N | J_{p\pi,k'}(0) | p\pi; in > \left( \frac{e_N}{m_N} \frac{e_\pi}{(p_\pi^2 - k'^2) - m_\pi^2} \right) \right], \]
\[(2.12)\]

where the source operators of the nucleon and pion are constructed through the non-local nucleon and pion fields (2.1) and (2.4a) as
\[ J_{p'_N,k'}(Y) = (i\gamma_\sigma \nabla_\sigma - m_N)\Psi_{p'_N,k'}(Y); \quad \Psi_{p'_N,k'}(Y) = \int d^4r'_3d^4r'_1d^4r'_2\chi_{p'_N}(Y' = 0,r'_1,r'_2) \]
\[ \left[ e_{1}^{N'} e^{ik'(-\eta r'_3 + \eta r'_1,2)} + e_{2}^{N'} e^{-ik'(-\eta r'_3 + \eta r'_1,2)} + e_{3}^{N'} e^{ik'\eta r'_3} \right] T\left( q_1(y'_1)q_2(y'_2)q_3(y'_3) \right) \]
\[(2.13a)\]
\[ \bar{J}_{p\pi,k'}(X) = (\Box X + m_\pi^2)\phi_{p\pi}(X); \quad \phi_{p\pi,k'}(X') = \int d^4r'_1d^4r'_2\phi_{p\pi}^+(X' = 0,r'_1,r'_2) \]
\[ \left[ e_{1}^{\pi'} e^{ik'\rho_{1,2} - \rho_{1,2}} + e_{2}^{\pi'} e^{-ik'\rho_{1,2}} \right] T\left( q_1(x'_1)q_2(x'_2) \right). \]
\[(2.13b)\]

The charge factors in front of the off shell \( \pi N \) amplitudes in (2.12) are extracted in analogue to the formulation without quark degrees of freedom [1], where the charge of the nucleon and pion arise in the equal time commutators due to charge conservation. In the source operators (2.13a,b) the quark charges are distributed according to the commutators (2.9). This distribution is a result of the choice of the field operators (2.1) and (2.4a).
Other choices of the source operators of the composite nucleons and pions are considered in Appendix A. Unlike (2.13a,b) other source operators do not contain the quark charge distributions.

Because of the zeros of the free Dirac and the Klein-Gordon operators \((\gamma_{\nu}p_N^{'\nu} - m_N)\), \((\gamma_{\nu}p_N^{'\nu} - m_N)\), \((p_N^{'2} - m_N^2)\), \((p_N^{'2} - m_N^2)\) equation (2.12) corresponds to current conservation \(k'_\mu A^\mu_{\gamma'\pi'N' - \pi N} = 0\) for any \(k'\). For \(k' = 0 k'_\mu A^\mu_{\gamma'\pi'N' - \pi N} = 0\) according to cancellations of the on shell \(\pi N\) amplitudes. Therefore, (2.12) represents current conservation for the on-mass shell \(\pi N\) bremsstrahlung amplitude

\[
k'_\mu A^\mu_{\gamma'\pi'N' - \pi N}(p'_\pi, p'_N, k'; p_\pi, p_N) = 0. \quad (2.14)
\]

Following [1] we extract the full energy-momentum conservation \(\delta\) function from the radiative \(\pi N\) scattering amplitude \(A^\mu_{\gamma'\pi'N' - \pi N}\) and introduce the corresponding non-singular amplitude \(<\text{out} p'_N p'_\pi | J^\mu(0) | p_\pi p_N; \text{in} >\)

\[
k'_\mu <\text{out} p'_N p'_\pi | J^\mu(0) | p_\pi p_N; \text{in} > = B_{\pi'N' - \pi N} + k'_\mu \mathcal{E}^\mu_{\gamma'\pi'N' - \pi N} = 0, \quad (2.16)
\]

where

\[
B_{\pi'N' - \pi N} = e_{\pi'} \pi (p'_N) <\text{out} p'_\pi | J_{p_N k'}(0) | p_\pi p_N; \text{in} > + e_{\pi'} <\text{out} p'_N | j_{p'_N k'}(0) | p_\pi p_N; \text{in} > - e_{\pi} <\text{out} p'_\pi p'_N | \overline{J}_{p_N k'}(0) | p_\pi p_N; \text{in} > + u(p_N) - e_{\pi} <\text{out} p'_\pi p'_N | j_{p_N k'}(0) | p_N; \text{in} >, \quad (2.17a)
\]

\[
\mathcal{E}^\mu_{\gamma'\pi'N' - \pi N} = - \left[ \overline{u}(p'_N) \gamma_\nu (p'_N + k')^\nu + m_N \right] e_{\pi'} <\text{out} p'_\pi | J_{p'_N k'}(0) | p_\pi p_N; \text{in} > + (2p'_\pi + k')^\mu e_{\pi'} \overline{u} <\text{out} p'_N | j_{p'_N k'}(0) | p_\pi p_N; \text{in} > - e_{\pi} <\text{out} p'_\pi p'_N | \overline{J}_{p_N k'}(0) | p_\pi p_N; \text{in} > + \frac{\gamma_\mu (p_N - k')^\nu + m_N \gamma^\mu u(p_N)}{2p_N k'} \quad (2.17b)
\]

The relations (2.16) and (2.17a,b) have the same form as (2.6) and (2.8a,b) in the formulation without quarks [1]. The only differences are in the source operators of the
nucleons and pions. The off shell \( \pi N \) amplitudes in (2.17a,b) contains the nonlocal source operators \((2.13a,b)\) of composite particles which in contrast to the local sources \(J(x) = (i\gamma_\mu \partial / \partial x_\mu - m_N)\Psi(x)\) and \(j_\pi(x) = (\Box x + m_\pi^2)\Phi(x)\) depends on the four moments of the composed particle and on \(k'\). Consequently, the off mass shell \( \pi N \) amplitudes in (2.17a,b) have an additional dependence on the Mandelstam variables.

The Ward-Takahashi identity (2.16) presents the general scheme of current conservation for the \( \pi N \) bremsstrahlung reaction with the composed on mass shell pions and nucleons. According to this scheme it is necessary to find a special part of the internal particle radiation amplitude \( I_{\gamma' \pi' N' - \pi N} \) which insures current conservation because

\[
k'_{\mu} I'_{\gamma' \pi' N' - \pi N} = B_{\gamma' \pi' N' - \pi N} \tag{2.18a}
\]

and consequently

\[
k'_{\mu} < \text{out; } p'_{N} p'_{\pi} | J'(0) | p_{\pi} p_{N} > = k'_{\mu} \left( E^\mu_{\gamma' \pi' N' - \pi N} + T^\mu_{\gamma' \pi' N' - \pi N} \right) = 0. \tag{2.18b}
\]

An example of such an internal particle radiation amplitude is the intermediate on mass shell \( \Delta \) radiation amplitude depicted in Fig. 3 [1]. The \( \Delta \) radiation amplitude in Fig. 3 \( I_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) \) does not satisfy current conservation separately. \( I_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) \) satisfy current conservation together with \((E^\mu_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta)\) which is extracted from \( E^\mu_{\gamma' \pi' N' - \pi N} \) (2.17b) in Fig.2 after the set of the decompositions.

\[
k'_{\mu} (E^{3/2}_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) = -k'_{\mu} T^\mu_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) = -B^{3/2}_{\pi' N' - \pi N}(\Delta - \gamma \Delta), \tag{2.19}
\]

where the lower index \( \gamma \) and the upper index \( \gamma \) denotes the longitudinal and the spin-isospin \((3/2, 3/2)\) part of the corresponding expressions. From the same structure of \( (E^{3/2}_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) \) and \( (T^{3/2}_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) \) follows

\[
(E^{3/2}_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) = -I^\mu_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) \tag{2.20}
\]

which allows to equate the \( \Delta - \gamma \Delta \) vertex functions in \( I^\mu_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) \) and in \( (E^{3/2}_{\gamma' \pi' N' - \pi N}(\Delta - \gamma \Delta) \)

\[
G_{C0}(k', s, s') = -2 M_\Delta \left[ \frac{|k'|}{s - s'} - \frac{P^\alpha_\Delta(s) - P^\alpha_\Delta(s')}{s - s'} \right] \left[ g_{\pi' N' - \Delta'}(s', k') \right]^{-1} \left( e_N \frac{R_{\pi N'} + R_N}{2} + e_\pi \frac{R_{\pi' N'} + R_\pi}{2} \right) \left[ g_{\Delta - \pi N}(s) \right]^{-1}, \tag{2.21a}
\]

\[
G_{M1}(k', s, s') = -2 M_\Delta \left[ \frac{|k'|}{s - s'} - \frac{P^\alpha_\Delta(s) - P^\alpha_\Delta(s')}{s - s'} \right] \left[ g_{\pi' N' - \Delta'}(s', k') \right]^{-1} \left( \mu_N \frac{R_{\pi N'} + R_N}{2} \right) \left[ g_{\Delta - \pi N}(s) \right]^{-1}, \tag{2.21b}
\]
where $G_C$ and $G_{M1}$ denote the electric and magnetic dipole form factors of the $\Delta$'s, $k'_\Delta = P_\Delta - P'_\Delta$ and we have considered the $\pi N$ bremsstrahlung reactions with only $e_N = e_{N'}$ and $\mu_N = \mu_{N'}$. Equations (2.21a,b) present a relationship between $G_C(k'_\Delta, s, s')$, $G_{M1}(k'^2_\Delta, s, s')$ and the residues of the $\pi N$ amplitudes $\mathcal{R}$ (see (A.9a,b,c,d) in [1]).

At the threshold $k' = 0$ one obtains the same model-independent relation of the magnetic dipole moments of the $\Delta^+$ and $\Delta^{++}$ resonances as in [1] $\mu_{\Delta^+} = \frac{M_\Delta}{m_p} \mu_p$ and $\mu_{\Delta^{++}} = \frac{3}{7} \mu_{\Delta^+}$.

3. Magnetic dipole moments of the $\Delta^0$ and $\Delta^-$ resonances.

The external particle radiation amplitude $\mathcal{E}^\mu_{\gamma'\pi'N'N} (2.17b)$ can be replaced by the one on mass shell particle exchange amplitude

$$\mathcal{E}^\mu_{\gamma'\pi'N'N}(N) = - \left[ (2p'_\pi + k')^\mu \frac{e_{\pi'}}{2p'_\pi k'} < \text{out}; p'_N | j_{p'_\pi k'}(0) | p_N p_N; \text{in} > \right.$$  

$$+ \frac{\tilde{\nu}(p'_N) \left[ (2p'_N + k')^{\mu} \tau_+ - i \mu_{N'} \sigma^{\mu \nu} k'_\nu \right]}{2p'_N k'} u(p'_N + k') \tilde{\nu}(p'_N + k') e_{N'} < \text{out}; p'_N | j_{p'_\pi k'}(0) | p_N p_N; \text{in} >$$

$$- e_N < \text{out}; p'_\pi p'_N | \mathcal{J}_{p_N k'}(0) | p_N; \text{in} > u(p_N - k') \tilde{\nu}(p_N - k') \left[ (2p_N - k')^{\mu} - i \mu_{N'} \sigma^{\mu \nu} k'_\nu \right] u(p_N)$$

$$\quad \frac{2p_N k'}{2p_N k'}$$

$$- < \text{out}; p'_\pi p'_N | j_{p'_\pi k'}(0) | p_N; \text{in} > \frac{e_{\pi}}{2p'_\pi k'} (2p_{\pi} - k')^\mu, \quad (3.1a)$$

where the antiparticle contributions are separated as it was done in [1]. Thus starting from the generalized S-matrix reduction formulas (2.8a,b) for the $\pi N$ radiation amplitude with composite pions and nucleons one obtains again the modified Ward-Takahashi identity

$$k'_\mu < \text{out}; p'_N p'_\pi | \mathcal{J}^\mu(0) | p_N p_N; \text{in} > = B_{\pi'N'N} + k'_\mu \mathcal{E}^\mu_{\gamma'\pi'N'N}(N) = 0, \quad (3.1b)$$

with the external particle radiation amplitude (3.1a), where $e_p = e$, $\mu_p = 1$ for the protons and $e_n = 0$, $\mu_n = 0$ for the neutrons.

In order to take into account the anomalous magnetic moments of nucleons it is necessary to consider the loop corrections of the $\gamma N N$ vertex. The corresponding contributions can be extracted from the transverse part of $\mathcal{E}^\mu_{\gamma'\pi'N'N}(N)$. Then one obtains again the expression (3.1a) with $\mu_p = 2.79 \mu_B$ for protons and $\mu_n = -1.91 \mu_B$ for neutrons in the units of the nuclear magneton $\mu_B = e/2m_p$.

The external particle radiation part of the $\pi N$ bremsstrahlung amplitude with the complete $\gamma' N N$ and $\gamma' \pi \pi$ vertices is depicted in Fig.2. At the threshold $k' = 0$ (2.21a,b) presents the exact relationship between $e_\Delta$, $\mu_\Delta$ and the $\Delta$ pole residues $\mathcal{R}$ of the off shell $\pi N$ amplitudes (see (A.9a,b,c,d) in [1]).
\[ e_\Delta = - \left[ \mathcal{N}(s) \left[ g_{\pi^N N' - \Delta'}(s', k') \right]^{-1} \left( e_N \frac{\mathcal{R}_{N'} + \mathcal{R}_N}{2} + e_{\pi'} \frac{\mathcal{R}_{\pi'} + \mathcal{R}_\pi}{2} \right) \left[ g_{\Delta - \pi N}(s, k') \right]^{-1} \right]^{k' = 0}_{\sqrt{s'} = \sqrt{s} = M_\Delta}, \]

\[ \mu_\Delta = - \left[ \mathcal{N}(s) \left[ g_{\pi^N N' - \Delta'}(s', k') \right]^{-1} \left( \mu_N \frac{\mathcal{R}_{N'} + \mathcal{R}_N}{2} \right) \left[ g_{\Delta - \pi N}(s, k') \right]^{-1} \right]^{k' = 0}_{\sqrt{s'} = \sqrt{s} = M_\Delta}, \] (3.2a)

where \( \mathcal{N}(s) = 1/(d\sqrt{s}/dk') - d\mathcal{P}_\Delta^2(s)/d\sqrt{s} \) and \( g_{\Delta - \pi N} \) and \( g_{\pi^N N' - \Delta'} \) denote the \( \Delta - \pi N \) form factors.

Now we assume that the charge of the neutron is an auxiliary parameter \( e_n \) which will be fixed in the finally relations as \( e_n = 0 \). Then for the \( \pi N \) bremsstrahlung with the intermediate \( \Delta^0 \) we have

\[ e_{\Delta^0} = - e_n \left[ \mathcal{N}(s) \left[ g_{\pi^N N' - \Delta'}(s', k') \right]^{-1} \frac{\mathcal{R}_{N'} + \mathcal{R}_N}{2} \left[ g_{\Delta - \pi N}(s, k') \right]^{-1} \right]^{k' = 0}_{\sqrt{s'} = \sqrt{s} = M_\Delta} \]

But \( e_{\Delta^0} = e_n \) for the reaction \( \pi^0 n \to \gamma' \pi^0 n' \). The cancellation of \( e_n \) from both sides of (3.3a) gives the normalization condition for \( \mathcal{R}_n \)

\[ 1 = - \left[ \mathcal{N}(s) \left[ g_{\pi^N N' - \Delta'}(s', k') \right]^{-1} \frac{\mathcal{R}_{N'} + \mathcal{R}_N}{2} \left[ g_{\Delta - \pi N}(s, k') \right]^{-1} \right]^{k' = 0}_{\sqrt{s'} = \sqrt{s} = M_\Delta}. \] (3.3b)

Substituting (3.3b) into (3.2b) we obtain

\[ \mu_{\Delta^0} = \mu_n \frac{M_\Delta}{m_p}, \] (3.4)

where the different units of \( \mu_\Delta \) and \( \mu_N \) generates the factor \( M_\Delta/m_p \). The isospin symmetry between the \( \pi N \) amplitudes of the reactions \( \pi^0 n \to \pi^0 n \) and \( \pi^- n \to \pi^- n \) in (3.2b) allows to estimate \( \mu_{\Delta^-} \)

\[ \mu_{\Delta^-} = \frac{3 \mu_{\Delta^0}}{2} = \frac{3 \mu_n}{2} \frac{M_\Delta}{m_p}. \] (3.5)
4. Conclusion

The main result of this paper is that the magnetic dipole moments of the $\Delta$ resonances are the same in the quantum field theories with and without quark degrees of freedom. This follows from the same structure of current conservation for the on shell $\pi N$ radiation amplitudes in the formulations with and without quark degrees of freedom, and the simple relationship between the magnetic dipole moments of the $\Delta$'s and the anomalous magnetic moment of the nucleons $\mu_\Delta = M_\Delta/m_p \mu_N$. According to this formula the $\mu_\Delta$ are dependent only on the anomalous magnetic moment of the nucleons $\mu_N$ and the $\Delta$ resonance pole position $M_\Delta = 1232$ MeV. The present approach allows to connect analytically the electric and the magnetic dipole form factors $G_C$ and $G_M$ with the $\Delta$ pole residues $\mathcal{R}$ of the off shell $\pi N$ amplitudes according to (2.21a,b). The $\Delta$ pole residues $\mathcal{R}$ as well as the source operators (2.13a,b) and (A.6a,b) are different in the different models. Correspondingly, the dependence on $k'$ of the off shell $\pi N$ amplitudes is different and model-dependent. But at threshold $k' = 0$ the expressions for $e_\Delta$, $\mu_\Delta$ (3.2a,b) as well as the normalization condition (3.3b) are the same for any model with the fixed charge of the particles. Therefore, the formulas (3.4) and (3.5) for the magnetic dipole moment of the $\Delta$'s are unique and model-independent.

Table 1

**Magnetic moments of $\Delta^o$ and $\Delta^-$ in nuclear magneton $\mu_B = e/2m_p$.**

| MODELS | This work | SU(6) and Bag | Skyrme | quark |
|--------|-----------|---------------|--------|-------|
| $\mu_{\Delta^o}$ | -2.504 | 0. [13, 14] | -1.33~0.19[21] | 0.[18] |
| | | 0.[15] | | 0.375[19] |
| | | 0.[16] | | -0.3~0.[20] |
| | | 0.[17] | | |
| $\mu_{\Delta^-}$ | -3.759 | -2.79 [13, 14] | -5.62~3.82[21] | -3.49[18] |
| | | -2.13[15] | | -2.1[19] |
| | | -2.20-2.45[16] | | -2.72-3.06[20] |
| | | -3.27[17] | | |

The present relations for $\mu_\Delta$ requires proportionality of $\mu_\Delta$ and the anomalous magnetic moment of the nucleon $\mu_N$. In particular, $\mu_{\Delta^o}$ and $\mu_{\Delta^-}$ are determined via the anomalous magnetic moment of the neutron. A comparison our numerical values for $\mu_{\Delta^o}$ and $\mu_{\Delta^-}$ with the calculations of other authors is given in Table 1. In the $SU(6)$ symmetry quark models and their modifications [13]-[17] $\mu_\Delta$ is proportional to the charge of the $\Delta$. Therefore in these models[13, 14, 15, 16, 17] $\mu_{\Delta^o} = 0$ and $\mu_{\Delta^-} = -\mu_{\Delta^+}$ and $\mu_{\Delta^+} = 1/2 \mu_{\Delta^{++}}$. This property is preserved in the constituent quark model [18]. But it
is slightly broken in the Skyrme model [21], chiral quark model [20], chiral quark-soliton model [19] and effective quark model. The crucial difference between our result and the other estimations is in $\mu_\Delta$ which is larger than the predictions of other authors.

**Appendix A: Alternative field operators of composite particles**

The source operators $J_{\rho_N,k'}(Y)$ (2.13a) and $j_{\rho_N,k'}(X)$ (2.13b) can be constructed in the independent over the quark charges $e_j$ form. For this aim one can introduce other quark cluster operators

$$ \Psi_{\rho N}(Y) = \frac{1}{6} \int d^4y_3d^4y_1 \left\{ \right. $$

$$ \left[ \gamma^i_{\rho N}(y_1,y_2,y_3)|_{y_3=0} T \left( q_1(y_1)q_2(y_2)q_3(y_3) \right) + \chi^i_{\rho N}(y_2,y_1,y_3)|_{y_3=0} T \left( q_1(y_2)q_2(y_1)q_3(y_3) \right) \right\} $$

$$ + \left[ \gamma^j_{\rho N}(y_3,y_2,y_1)|_{y_3=0} T \left( q_1(y_3)q_2(y_2)q_3(y_1) \right) + \chi^j_{\rho N}(y_2,y_3,y_1)|_{y_3=0} T \left( q_1(y_2)q_2(y_3)q_3(y_1) \right) \right\} $$

$$ + \left[ \gamma^k_{\rho N}(y_1,y_3,y_2)|_{y_3=0} T \left( q_1(y_1)q_2(y_3)q_3(y_2) \right) + \chi^k_{\rho N}(y_1,y_2,y_3)|_{y_3=0} T \left( q_1(y_1)q_2(y_1)q_3(y_2) \right) \right\} $$

$$ \left\} \right. $$

$$ \text{(A.1)} $$

and

$$ \Phi_{\rho N}(X) = \frac{1}{2} \left\{ \int d^4y_1d^4y_2 \left( \frac{3}{2} \phi_{\rho N}(X = 0, \rho_{1,2}) T \left( q_1(x_1)\bar{\Psi}_i(x_2) \right) + \int d^4y_1d^4y_2 \phi_{\rho N}(X = 0, -\rho_{1,2}) T \left( q_1(x_2)\bar{\Psi}_i(x_1) \right) \right) \right\} $$

$$ \text{(A.2)} $$

Unlike to (2.1) and (2.4a), the field operators (A.1) and (A.2) contains all transpositions of the integration variables $y_1, y_2, y_3$ and $x_1, x_2$. Therefore, instead of (2.8b) we obtain

$$ k'_\mu G^\mu = \int d^4[y]d^4[y']d^4[x]d^4[x']P_{x_1x_2}P_{y_1y_2}P_{y'_1y'_2}e^{ik'z}d^4z \chi^i_{\rho N}(y_1,y'_2,y'_3)\phi_{\rho N}(x_1,x_2) \frac{\partial}{\partial x_1} $$

$$ \left| 0 ; T \left( q_1(y_1)q_2(y_2)q_3(y_3) \right) T \left( \bar{q}_1(y_1)\bar{q}_2(y_2)\bar{q}_3(y_3) \right) \right| 0 > \right.$$}

$$ \chi_{\rho N}(y_1,y_2,y_3)\phi_{\rho N}(x_1,x_2) \text{(A.3)} $$

where

$$ d^4[y] = 1/6 \left\{ [d^4y_1d^4y_2d^4y_3 + d^4y_2d^4y_1d^4y_3] + [d^4y_3d^4y_1d^4y_2 + d^4y_1d^4y_3d^4y_2] $$

$$ + [d^4y_1d^4y_2d^4y_3 + d^4y_2d^4y_3d^4y_1] \right\}, $$

$$ d^4[x] = 1/2 \left\{ d^4x_1d^4x_2 + d^4x_2d^4x_1 \right\} $$

are defined via transposition operator $P_{x_1x_2}$ of the variables $x_1$ and $x_2$ as.
\[ \mathcal{P}_{x'x} = \frac{1 + \mathcal{P}_{xx}}{2}; \quad \mathcal{P}_{y'y} = \frac{\mathcal{P}_{yy} + \mathcal{P}_{yy} + \mathcal{P}_{yy}}{3} \quad (A.4) \]

The symmetry over the rearrangement of the integration variables in (A.3) and \( \int dx_1 dx_2 e^{ik'x_1} [f(x_1, x_2) + f(x_2, x_1)] = \int dx_2 dx_1 e^{ik'x_2} [f(x_2, x_1) + f(x_1, x_2)] \) allows to modify (2.11a) as

\[ k'_\mu A^\mu_{\gamma', \gamma}; N' = -i e_N \pi \langle p'(N) (\gamma_p p'_{\mu') - m_N) \rangle \int d^4[y'] \mathcal{P}_{y'y'y} \chi_{p_{N'}}^{+} (y'_1, y'_2, y'_3) \]

\[ \frac{e^{ik'y_1'} + e^{ik'y_2'} + e^{ik'y_3'}}{3} < \text{out}; \quad \mathcal{P}_p \mathcal{T} (q_1(y'_1)q_2(y'_2)q_3(y'_3)) |\mathcal{P}_p \mathcal{P}_N; \text{in} > \]

\[ -i e_N (p'_{\mu} - m^2) \int d^4[x'] \mathcal{P}_{x'x'} \bar{\phi}_{p_{\mu}}^{+} (x'_1, x'_2) \frac{e^{ik'x'_1} + e^{ik'x'_2}}{2} < \text{out}; \quad \mathcal{P}_p \mathcal{T} (q_1(x'_1)q_2(x'_2)) |\mathcal{P}_p \mathcal{P}_N; \text{in} > \]

\[ +i e_N \int d^4[y] \mathcal{P}_{y'y'y} \frac{e^{ik'y_1} + e^{ik'y_2} + e^{ik'y_3}}{3} < \text{out}; \quad \mathcal{P}_p \mathcal{T} (\bar{q}_1(y_1)\bar{q}_2(y_2)\bar{q}_3(y_3)) |\mathcal{P}_p; \text{in} > \]

\[ \chi_{p_{N'}} (y_1, y_2, y_3) (\gamma_{\mu} N' - m_N) (p_{\mu}' - m^2) u (p_N) \]

\[ +i e_N (p'_{\mu} - m^2) \int d^4[x] \mathcal{P}_{x'x} \frac{e^{ik'x_1} + e^{ik'x_2}}{2} < \text{out}; \quad \mathcal{P}_p \mathcal{T} (\bar{q}_1(x_1)q_2(x_2)) |\mathcal{P}_p; \text{in} > \phi_{p_{\mu}} (x_1, x_2), \quad (A.5) \]

Consequently, after integration over \( X', X, Y' \) and \( Y \) we obtain again (2.12) with independent on the quark charges \( e_j \) source operators

\[ J_{p_{N'}, k'} (Y) = (i \gamma_\sigma \nabla_\sigma - m_N) \Psi_{p_{N'}, k'} (Y); \quad \Psi_{p_{N'}, k'} (Y') = \int d^4 r'_3 d^4 r'_{1,2} x_{p_{\mu}}^{+} (Y' = 0, r'_{1,2}, r'_3) \]

\[ \frac{e^{ik'y_1'} + e^{ik'y_2'} + e^{ik'y_3'}}{3} |_{Y' = 0} \mathcal{T} (q_1(y'_1)q_2(y'_2)q_3(y'_3)) \quad (A.6a) \]

\[ j_{p_{\mu}, k'} (X) = (\Box + m^2) \phi_{p_{\mu}} (X); \quad \phi_{p_{\mu}, k'} (X') = \int d^4 r'_{1,2} \bar{x}_{p_{\mu}}^{+} (X' = 0, r'_{1,2}) \]

\[ \frac{e^{ik'x'_1} + e^{ik'x'_2}}{2} |_{X' = 0} \mathcal{T} (q_1(x'_1)q_2(x'_2)). \quad (A.6b) \]

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