Dynamic Scaling in Diluted Systems Phase Transitions: Deactivation through Thermal Dilution

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Activated scaling is confirmed to hold in transverse field induced phase transitions of randomly diluted Ising systems. Quantum Monte Carlo calculations have been made not just at the percolation threshold but well below and above it including the Griffiths-McCoy phase. A novel deactivation phenomena in the Griffiths-McCoy phase is observed using a thermal (in contrast to random) dilution of the system

The presence of quenched disorder in quantum phase transitions at zero temperature is a topic of current interest. Two of the most important special properties of the disordered quantum phase transitions are the appearance of activated dynamic scaling and the existence of Griffiths-McCoy singularities even away from the critical point.

Activated dynamic scaling was first analytically proved to hold in the disordered one dimensional ising model in a transverse field and there are many other results for this model in the literature. The disordered Ising model in a transverse field considered as a quantum spin glass provides a reasonable description of the system $LiHo_2Y_{1-x}F_x$ and also, considering the existence of long-range correlated disorder in it, may be an appropriate model to describe non-Fermi liquid behavior in certain $f$-electron systems. Activated dynamic scaling implies the existence of an infinite dynamic critical exponent $(z = \infty)$. It means that instead of the typical power law relationship between the characteristic time scale and the characteristic length scale, $\xi_t \sim \xi^z$, a new exponential relation appears $\xi_t \sim exp(const \times \xi^\psi)$ with $\psi = 1/2$ for the one-dimensional Ising model. There are nearly no analytical results for higher dimensions. The activated dynamic scaling seems to disappear for two-dimensional and three dimensional Ising glass systems, but it has been proved to hold in the disordered two-dimensional Ising model in a transverse field by means of Quantum Monte Carlo simulations and renormalization group analysis. The only analytical prediction of activated scaling in dimensions higher than one has been made for the dilution probability transition at the percolation threshold ($p_c$) of a diluted Ising system in a transverse field, where percolation critical exponents have been found.

The phase boundary of this model at $T = 0$ was studied a long time ago and is expected to have a multi-critical point and a straight vertical phase bound-
\[ S = -K_{\text{hor}} \sum_{\tau, (i,j)} \varepsilon_i \varepsilon_j s_i(\tau) s_j(\tau) - K_{\text{ver}} \sum_{\tau,i} \varepsilon_i s_i(\tau) s_i(\tau + 1) \]  
\[ (2) \]

with \( K_{\text{hor}} = \Delta \tau, \) \( K_{\text{ver}} = -(1/2) \ln [\tanh(\Delta \tau \Gamma)] \) and \( \Delta \tau \to 0. \) This limit may be taken into account exactly considering a continuous time algorithm \( [14], \) however making use of the universality between the model with \( \Delta \tau \to 0 \) and the model with \( \Delta \tau \neq 0, \) we may simulate the usual discrete Ising model, but with anisotropic interactions. In order to avoid problems arising from de critical slowing down at the critical point we use the Wolff single cluster algorithm \( [24]. \) It is important to notice that once slowing down at the critical point we use the Wolff single cluster algorithm \( [24]. \) It is important to notice that once a spin is diluted, the whole imaginary time axis (Trotter axis) is also diluted, which means that the random disorder in a quantum system is equivalent to the long range correlated disorder in classical systems \( [23] \) giving rise to a different universality class. In order to obtain the \( T = 0 \) phase boundary we fix the occupied site probability \( (p) \) and we consider different values of the transverse field \( (\Gamma) \) at intervals of \( \Delta \Gamma = 0.02. \) We really do not fix the probability but the concentration, it means that we do not use a grand canonical distribution of the vacancies but a canonical distribution, however both kinds of constraints are supposed to belong to the same universality class \( [26] \).

The critical magnetic field for each probability \( \Gamma(p) \) is determined by a method previously used for Ising spin glasses \( [13,14]. \) First it is necessary to compute the Binder Cumulant average from a certain number of realizations of the disorder (in our case we consider 500 different realizations), this is done for a fixed value of \( (\Gamma, p) \) and for a fixed value of the size (we consider the values \( L = 8, 12, 16, 24) \). Then we study the evolution of such a cumulant for different sizes of the Trotter axis (we consider up to \( L_T = 600) \). Due to the dynamic scaling form of the Binder Cumulant it has a peak as a function of \( L_T. \) At the critical point \( (\Gamma_c(p)) \) the peak height is independent of \( L \) and the values of \( L_T, \) at the maximum, \( (L_T)_m, \) vary as \( L^z \) for conventional dynamic scaling and like \( (L_T)_m \sim \exp(\text{const} \times L^z) \) for activated scaling. Of course, all the non-universal quantities, as for example the critical values of the transverse field, will depend on the model and in particular on \( \Delta \tau. \) In the present work we choose \( \Delta \tau = 1/5. \)

The results for the phase boundary at \( T = 0 \) are shown in Fig.1 together with the phase boundary obtained for the classical system by means of conventional scaling. The points where the existence of activated scaling is going to be checked are \( p = 1 \) (pure case), \( p = 0.8, 0.7 \) \( (p > p_c), p = 0.6 \approx p_c \approx 0.59 \) \( [23] \) and \( p = 0.55, 0.5 \) \( (p < p_c) \) (Griffiths phase). The results for \( (L_T)_m \) vs. \( L \) are presented in Fig.2. Note how the scaling is not activated for the pure case \( (p = 1) \) where the dynamical exponent is found to be \( z \approx 1, \) but it starts to activate as the dilution is increased. To ensure that for diluted cases the scaling is activated Fig.3 presents a plot of \( \ln(L_T)_m \) vs. \( L. \) The straight line behavior for the values \( p \neq 1 \) clearly indicates that the scaling is activated. The evolution of the values \( \psi(p) \) is presented in the inset. Note how it grows monotonically until \( p \approx p_c \) and then it keeps approximately constant inside the Griffiths zone.
The existence of the Griffiths-McCoy zone is due to long-range correlated disorder. In this case the system in the Griffiths zone is expected to show deactivated scaling. Of course, this will happen just if the phase transition is due to the existence of strongly coupled regions (i.e. if the system is in the Griffiths phase ($p < p_c$)), but if the magnetic response is due to whole system (i.e. $p > p_c$) the introduction of long range correlated disorder will affect changing the universality class of the system (as in thermal classic transitions [34]) and enhancing the activation of the scaling [12].

A way to produce this kind of dilution is using the thermal dilution instead of a random dilution [34]. With this kind of dilution the system belongs to the universality class of long-range correlated disordered systems with an exponent $a = 2 - \eta$ [33], being $\eta$ equal to 0.25 for the classical ($d = 2$) Ising model (the way to produce thermal dilution is described in [34]). Basically the procedure is as follows, the pure system is first thermalized to criticality and then one kind of spins are turned into vacancies. The samples produced by thermal dilution at criticality will have an spin concentration near $p = 0.5$, so we can compare them only with samples diluted randomly with probability $p = 0.5$. Both kinds of dilution will be at the Griffiths phase since in both cases $p_c(a) > 0.5$ [34].

The analysis performed has been exactly the same as before but now going up to $L = 40$. By the Binder Cumulant we have been able to determine the critical transverse magnetic field ($\Gamma_c(thermal) \simeq 3.13$) and the relation between $(L_r)_m$ and $L$. Fig.4 compares behavior of the thermal dilution with the behavior of the random dilution with $p = 0.5$. The thick line represents the pure behavior (excepting proportional factors). Clearly the scaling has been deactivated, as expected, and $z \simeq 1$ as corresponds to the behavior of the pure system. This phenomenon could never happen with a system away from the quantum Griffiths zone.

In conclusion, Quantum Monte Carlo calculations in diluted Ising models in a transverse field at $T = 0$ show that dynamic scaling holds above the percolation threshold ($p > p_c$), at the percolation threshold ($p = p_c$) and inside the quantum Griffiths-McCoy phase ($p < p_c$). The evolution of the activated scaling has been characterized, showing how it grows monotonously towards the value corresponding to the percolation threshold and how it appears to remain nearly constant in the Griffiths zone for values of $p$ near $p_c$. A new way to deactivate the scaling in the Griffiths zone by thermal dilution has been proposed. The expected deactivation works due to the fact that phase transitions come from strongly coupled regions, and not from the whole sample. The deactivation has been clearly confirmed by means of Quantum Monte Carlo simulations.
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![Graph](image)

**FIG. 4.** \((L_\tau)_m\) vs. \(L\) for a randomly diluted system with spin concentration \(p = 0.5\) and a thermally diluted system. The thin line is a guide for the eye, and the thick line represents the behavior expected for the pure case \((z = 1)\) excepting proportional factors. Note how the scaling has been deactivated for the thermal case.

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