X(16.7) as the Solution of NuTeV Anomaly

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Abstract: A recent experimental study of excited $^8$Be decay to its ground state reveals an anomaly in the final states angle distribution. This exceptional result is attributed to a new vector gauge boson X(16.7). We study the significance of this new boson, especially its effect in anomalies observed in long-lasting experimental measurements. By comparing the discrepancies between the standard model predictions and the experimental results, we manage to find out the values and regions of the couplings of X(16.7) to muon and muon neutrino. In this work, we find that the newly observed boson X(16.7) may be the solution of both NuTeV anomaly and the $(g-2)_\mu$ puzzle.

Key words: NuTeV anomaly, New gauge boson, $(g-2)_\mu$ anomaly, Neutrino trident production.

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1 Introduction

As a theory describing electroweak and strong interactions, the Standard Model (SM) has achieved great success, and has been tested at high precision. However, some experimental studies pointed out the possibility for new physics beyond SM. For example, the non-zero masses of neutrinos, the existence of dark matter, the muon anomalous magnetic moment etc.. More fundamental challenges such as the hierarchy problem also put severe challenges for the Standard Model in describing the nature. Searching for new physics beyond the Standard Model (BSM) has become one of the major activities in physics. Numerous new physics models have been proposed. One of the simplest possibilities is $SU(3) \times SU(2)_L \times U(1)$ extended by a new gauge group $U(1)$.

A result in $^8$Be nuclear transition brought forth new challenges to our understanding of electroweak interaction. In this reaction, $^8$Be decays from an excited state to its ground state $^8Be^* \rightarrow ^8Be X$, followed by saturating decay $X \rightarrow e^+e^-$. A 6.8$\sigma$ anomaly to the internal pair production was observed at a angle of 140° [1]. Although this extraordinary experimental phenomenon may due to unidentified nuclear reactions or experimental errors, it can also be attributed to a new vector boson X with mass of 16.7 MeV, which mediates a weak fifth force BSM. In other words, the SM gauge group is extended by a new Abelian gauge groups $U(1)_X$, which is one of the most natural extension of the SM [2]. Based on this hypothe-

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2 NuTeV anomaly

As mentioned before, the discrepancy found by NuTeV experiment is a well known result. It has been discussed by many literatures. The explanations of the three standard deviation above the SM predictions for the value of $\sin^2 \theta_W$ may come from the SM or BSM have been proposed [13,10]. However, no definite conclusion can be made due to large uncertainties. In this work, we show that the NuTeV anomaly can be fully attributed to the contribution of the X boson. The corresponding couplings of this new gauge boson can be chosen without contradicting to the constrains given in [2].

We use the same Lagrangian proposed in [2]. The 16.7MeV Abelian gauge boson X with field strength tensor $X_\mu$ couples non-chirally to the SM fermions through vector current $L = -\frac{1}{4} X_\mu X^{\mu} + \frac{1}{2} m^2 X_\mu X^\mu - J X^\mu$. The corresponding charge is noted as $\varepsilon \mu$ in unites of e. The current $J_\mu = \sum_f e \varepsilon_f \bar{f} \gamma_\mu f$, however, can still be split into left-handed and right-handed pieces $J_\mu = \frac{1}{2} \sum_f e \varepsilon_f \bar{f}_L \gamma_\mu f_L + \frac{1}{2} \sum_f e \varepsilon_f \bar{f}_R \gamma_\mu f_R$. According to this model, apparently, the left-handed and right-handed fermions have identical charge. The mass of the X boson is far smaller than the center of mass energy of major electron-positron colliders. We adopt the conclusion given in [2], that the charges for up and down quarks satisfy the relation $\varepsilon_d = -2 \varepsilon_u$. On the other hand, as illustrated in Ref. [17], if isospin is conserved for the decay studied in the Atomki experiment [1], the summation of $\varepsilon_u$ and $\varepsilon_d$ is constrained by

$$|\varepsilon_u + \varepsilon_d| \approx \frac{3.3 \times 10^{-3}}{\sqrt{\text{Br}(\chi \rightarrow e^+e^-)}}$$

(1)

Notice in this charge assignment, quark universality has been relaxed. The upper bound on $|\varepsilon_u|$ is provided by the measurement of electron magnetic moment $(g-2)_e$ [18]. The lower bound on $|\varepsilon_d|$ is given by the SLAC experiment E141 [19,20]. The most strict upper bound on the coupling between electron and electron neutrino comes from the TEX-ONO experiment in Taiwan [21]. These constraints can be summarized as follows

$$2 \times 10^{-4} \leq |\varepsilon| \leq 1.4 \times 10^{-3},$$

$$\left|\varepsilon_u \varepsilon_e\right|^{1/2} \leq 7 \times 10^{-5}. \quad \text{(2)}$$

The $(g-2)_u$ puzzle can be solved with $\varepsilon_u$ falling in the same range as $\varepsilon_e$. We will find out the constraint on $\varepsilon_u$ coming form the results of NuTeV, and the effect introduced by particle X to the number of neutrino flavors.

First of all, let us look at the effective four-fermions Lagrangian generated by X exchange given in [2]

$$L_X = -\frac{e^2}{2(m_X^2 - \mu)} \left[ \varepsilon_u \bar{u}_L \gamma_\mu u_L + \varepsilon_d \bar{d}_L \gamma_\mu d_L + \varepsilon_u \bar{u}_R \gamma_\mu u_R + \varepsilon_d \bar{d}_R \gamma_\mu d_R + \varepsilon_u \bar{u}_R \gamma_\mu u_R + \cdots \right]^2$$

(3)

In NuTeV experiment, nucleon are scattered by $\nu_\mu$, the corresponding effective Lagrangian in SM at tree level can be expressed as

$$L_{eff} = -2\sqrt{2} G_F \left[ (\bar{\nu}_\mu \gamma_\mu u_L ) [\bar{d}_L \gamma_\mu u_L ] + h.c. \right]$$

$$-2\sqrt{2} G_F \sum_{A,q} g_{Aq} [\bar{\nu}_\mu \gamma_\mu \nu_\mu] [ q \gamma_\mu q_A], \quad \text{(4)}$$

where $A = \{L,R\}, \; q = \{u,d,s,\ldots\}$ and the couplings $g_{Aq}$ are in terms of the weak mixing angle $\sin \theta_W$. The transfer momentum squared adopted by NuTeV is $t = -Q^2 = -20 \text{GeV}^2$. What NuTeV measured is the ratio of neutral-current to charged-current deep-inelastic neutrino-nucleon scattering total cross-sections. In standard model this ratio is given by

$$R = \frac{\text{neutralcurrents}}{\text{chargedcurrents}} = \frac{\sigma(\nu_e \rightarrow \nu_e, X)}{\sigma(\nu_e \rightarrow \nu_e, X)}$$

$$= (g_u^2 - g_d^2) = \frac{1}{2} \sin^2 \theta_W \quad \text{(5)}$$

where $g_u^2 \equiv g_{1u}^2 + g_{2u}^2 = \frac{1}{2} - \sin^2 \theta_W + \frac{5}{2} \sin^4 \theta_W$, and $g_d^2 \equiv \frac{5}{2} \sin^2 \theta_W$. The standard model prediction with parameters determined by a fit to electroweak measurements is $\sin^2 \theta_W = 0.2227 \pm 0.0004$ [22], while the NuTeV result is $3 \sigma$ higher $\sin^2 \theta_W^{\text{exp}} = 0.2277 \pm 0.0013$. We next find out how the value of $\sin^2 \theta_W$ is altered by the new gauge boson X, by calculating the effects of X boson to the coupling constants $g_u$ and $g_d$. Comparing [3] and [4], we obtain the contributions of the X mediated tree level process to the coupling constants

$$\delta g_{1u} = \frac{\varepsilon_u \varepsilon_{\nu_e} e^2}{2\sqrt{2} G_F (m_X^2 + Q^2)}$$

$$\delta g_{1d} = \frac{-2\varepsilon_u \varepsilon_{\nu_e} e^2}{2\sqrt{2} G_F (m_X^2 + Q^2)}$$

$$\delta g_{2u} = \frac{\varepsilon_u \varepsilon_{\nu_e} e^2}{2\sqrt{2} G_F (m_X^2 + Q^2)}$$

$$\delta g_{2d} = \frac{-2\varepsilon_u \varepsilon_{\nu_e} e^2}{2\sqrt{2} G_F (m_X^2 + Q^2)} \quad \text{(6)}$$

Accordingly, the modification of $\sin^2 \theta_W$ is $\delta \sin^2 \theta_W = \delta (g_u^2 - g_d^2) = \frac{6\varepsilon_{\nu_e} \varepsilon_{\nu_e}}{\sqrt{2} G_F (m_X^2 + Q^2)} \approx 5 \times 10^{-3}$. Assuming $\varepsilon_{\nu_e} \sim \varepsilon_{\nu_\mu}$, we obtain the charges

$$\varepsilon_{\nu_e} \approx \pm 2.0 \times 10^{-3} \quad \text{(7)}$$

$$\varepsilon_{\nu_\mu} \approx \pm 5.7 \times 10^{-3} \quad \text{(8)}$$
by combining this formula with (1) and taking the upper limit of \( \varepsilon_v \sim 1.4 \times 10^{-3} \). The difference between the experimental value and standard model expectation of the Weinberg angle is resolved. It is worth noting that if the NuTeV anomaly is entirely due to the new U(1) particle X, the absolute value of its coupling to \( \nu_\mu \) has to be much larger than the absolute value of its coupling to \( \nu_\tau \). The above result can also be viewed as an upper bounds for \( \varepsilon_{\nu_\mu} \).

To test above calculation, let us check how the ratio \( R \) is modified by the gauge boson X. After introducing X, the ratio is proportional to

\[
R \propto \left[ \sum_{u,d} G_F c_\nu^a \right]_{eff} = \sum_{u,d} \left[ G_F c_\nu^a + \frac{e^2}{\sqrt{2}} \left( \frac{\varepsilon_\nu^a c_\nu^a}{Q^2} \right) \right],
\]

(9)

where \( c_\nu^a = I_3^a - 2Q^a \sin^2 \theta_W \), and \( c_\mu^a = I_3^a \) are the quantum number in GWS theory. The measured value of \( \sum_{u,d} G_F c_\nu^a \) is \((3.1507 \pm 0.0288) \times 10^{-6}\), while the standard model expectation is \(3.2072 \times 10^{-6}\) [23]. The discrepancy can be explained by the second term in the brackets of (9) introduced by gauge boson X. Substituting our result for \( \varepsilon_{\nu_\mu} \), and (1) into this term, we find the discrepancy is indeed redeemed. Comparing the value of \( |\varepsilon_{\nu_\mu}| \) to the constraints in [1], we notice that if the NuTeV anomaly is mainly due to the contribution from the new vector boson X, like the quark universality, the neutrino universality has to be broken as well.

### 3 The number of neutrino flavors

In order to check the plausibility of this \( SU(3) \times SU(2)_L \times U(1) \times U(1)_X \) model, we would like to test it against the well known number of neutrino flavors \( N_\nu \). This number is most precisely measured through the \( Z \) production process in \( e^+ e^- \) collisions. The standard model value of the ratio of the neutrino to charged lepton partial width is used in order to reduce the model value for the ratio of the neutrino to charged lepton universality. The visible width is constrained by the SM trident production of neutrino, where a pair of muon and anti-muon is produced in the scattering of muon neutrinos in the Coulomb field of a target nucleus. New force mediated by a heavy vector boson is excluded as a solution of the \( (g-2)_\mu \)

![Fig. 1. The leading diagrams that contribute to the X-boson production in electron-positron collision.](image1)

The result is displayed in FIG.2, where we can see that even the upper bounds of the coupling constants is too small to arise any noticeable effect on the decay width of the Z boson. Our result for the coupling constant \( \varepsilon_{\nu_\mu} \) is safe from being contradicted to the well tested conclusion of the number of neutrino flavors.

![Fig. 2. Taking the upper bounds of \( \varepsilon_v \), and \( |\varepsilon_{\nu_\mu}| = 6 \times 10^{-3} \) from previous subsection, we calculate the distribution of the cross-section for the process \( e^+ e^- \rightarrow \nu \bar{\nu} \). The solid line is the SM prediction, the green dashed line is the distribution.](image2)

### 4 Neutrino Trident Production

Models based on gauged muon number \( L_\mu \) is strictly constrained by the SM trident production of neutrino, where a pair of muon and anti-muon is produced in the scattering of muon neutrinos in the Coulomb field of a target nucleus. New force mediated by a heavy vector boson is excluded as a solution of the \( (g-2)_\mu \)
where the first part of the integrand $\sigma(\nu, N \rightarrow \nu, N)$ is the cross-section for a neutrino scattered off a real photon; the second part $P(s, q^2) = \frac{2\epsilon_\nu^2 \epsilon_\mu^2}{s + q^2} F^2(q^2)$, is the probability of creating a virtual photon with virtuality $q^2$ and energy $\sqrt{s}$ in the center-of-mass frame of the neutrino and a real photon. The virtual photon is created in the electromagnetic field of the nucleus $N$ with charge $Z$ and a electromagnetic form-factor (FF) $F(q^2)$. Generally, the real photon cross-section can be written as

$$\sigma_{\text{SM}+X} = \sigma_{\text{SM}} + \sigma_{\text{inter}} + \sigma_{\text{X}},$$

(12)

where the second term comes from the interference between the SM and the X contributions. The differential cross-sections for each of them have a general symbolic form

$$d\sigma = \frac{1}{2s} dPS_3 \left( \frac{1}{2} M^2 \right) \frac{G_F^2 e^2}{2}$$

(13)

Here, $G_F = \sqrt{2} g^2 / (8M_W^2)$ is the Fermi constant, and $dPS_3$ is the 3-body phase-space. In our calculation, the squared amplitudes $M^2$ are generated by FeynCalc [28]. By replacing the propagator with one over mass of the mediator boson squared, and omitting terms proportional to the muon mass in the numerator, we recover the SM expression given in [11] [29]. The phase-space integration is numerically calculated with Vegas [30]. Our calculation verified the analytic expression of the leading log approximation for real photon cross-section in SM [11].

By numerically integrating the real-photon cross-section with the probability distribution function $P(s, q^2)$ in the range of $4m^2 < s < 2E_\nu q$ and $2m^2/E_\nu < q < \infty$, we obtain the total cross-section for $\nu, N \rightarrow \nu, N$. We use a simple exponential function to mimic the nucleus form factor [31]. In order to test our calculation, we reproduced the prediction of SM and V-A theory [31] [32].

Neutrino trident production has been studied by several experiments [33] [34], among which, the measurement from CCFR collaboration provide the strongest constrains on the parameter space, and is used in our study. The CCFR collaboration detected the trident events by scattering a neutrino beam with mean energy of $E=160\text{GeV}$ with an iron target. The ratio of the cross-section they obtained to the SM prediction is $\sigma_{\text{CCFR}}/\sigma_{\text{SM}} = 0.82 \pm 0.28$. At this energy level, it is more secured not to take any approximation in the formulation of the amplitudes. In our calculation, we keep all the gauge boson propagators, and all the terms containing muon mass. By combining CCFR measurement with our numerical result, we obtain the following range for the first-generation charge of the gauge boson X

$$-2.0 \times 10^{-5} < \varepsilon_{\nu}\varepsilon_\mu < 6 \times 10^{-7}$$

(14)

We notice here that if $\varepsilon_{\nu}$ and $\varepsilon_\mu$ have the same sign, and particle X is fully responsible for the NuTeV anomaly, the value of $\varepsilon_\mu$ is strictly restricted to be less than $3 \times 10^{-4}$, which exclude the possibility for the gauge boson X to be the solution of the $(g-2)_\mu$ anomaly [36]. However, if $\varepsilon_{\nu}$ and $\varepsilon_\mu$ have opposite signs, the constrain on $\varepsilon_\mu$ is greatly relaxed to $|\varepsilon_\mu| < 1 \times 10^{-2}$, making it a candidate for solving $(g-2)_\mu$ puzzle. Future experiment such as LBNE may provide more data on neutrino trident production [11], which may lead to decisive analysis on the coupling of X(16.7) to neutrinos.

Fig. 3. The trident process at tree level

5 Conclusions

Unlike heavy $Z'$ boson that has been massively discussed in the literature [37] [40], the newly found gauge boson X is very light. It is quite exciting to know that low energy experiments still allow a possibility for finding
such a light boson. A commonly asked question is what are the constrains for this new particle from preexisting experimental measurements. We investigate some of the consequences brought by this unusual vector gauge boson $X$. The SM gauge group $SU(3) \times SU(2)_L \times U(1)$ extended by an Abelian gauge groups $U(1)_X$ was adopted in our calculation. Its implication on the NuTeV anomaly was studied. First of all, we found that the charge has to be $|\varepsilon_{\nu_e}| \simeq 2 \times 10^{-3}$, with the opposite sign to $\varepsilon_{\mu}$, in order to attribute the NuTeV anomaly entirely to the gauge boson $X(16.7)$. We have proven that this value, although is comparable or even larger than the coupling of $X$ to muon neutrinos deduced from NuTeV anomaly is significantly larger than the coupling of $X$ to electron neutrino $\varepsilon_{\nu_e}$, with the opposite sign to $\varepsilon_{\mu}$, in order to attribute the NuTeV anomaly entirely to the gauge boson $X(16.7)$. We have proven that this value, although is comparable or even larger than the coupling of $X$ to muon neutrinos deduced from NuTeV anomaly is significantly larger than the coupling of $X$ to electron neutrino $\varepsilon_{\nu_e}$, with the opposite sign to $\varepsilon_{\mu}$, in order to attribute the NuTeV anomaly entirely to the gauge boson $X(16.7)$. We have proven that this value, although is comparable or even larger than the coupling of $X$ to muon neutrinos deduced from NuTeV anomaly is significantly larger than the coupling of $X$ to electron neutrino $\varepsilon_{\nu_e}$, with the opposite sign to $\varepsilon_{\mu}$, in order to attribute the NuTeV anomaly entirely to the gauge boson $X(16.7)$. We have proven that this value, although is comparable or even larger than the coupling of $X$ to muon neutrinos deduced from NuTeV anomaly is significantly larger than the coupling of $X$ to electron neutrino $\varepsilon_{\nu_e}$, with the opposite sign to $\varepsilon_{\mu}$, in order to attribute the NuTeV anomaly entirely to the gauge boson $X(16.7)$.

particularly, if $\varepsilon_{\nu_e}$ and $\varepsilon_{\mu}$ have the same sign, the vector gauge boson $X$ cannot be responsible for both the NuTeV and the $(g-2)_\mu$ anomaly. However, if $\varepsilon_{\nu_e}$ and $\varepsilon_{\mu}$ have opposite signs, $X(16.7)$ can indeed be the solution to both of these puzzles. On the other hand, $|\varepsilon_{\nu_e}|$ would be smaller, if other effect such as the strange sea asymmetry or isospin violation take partial responsibility for the discrepancy between NuTeV and the SM prediction.

In that case, a gauge boson $X$ with $\varepsilon_{\nu_e}, \varepsilon_{\mu} > 0$ can be the solution of the $(g-2)_\mu$ anomaly. Finally, although the coupling of $X$ boson to muon neutrinos deduced from NuTeV anomaly is significantly larger than the coupling of $X$ to electron neutrino $\varepsilon_{\nu_e}$, with the opposite sign to $\varepsilon_{\mu}$, in order to attribute the NuTeV anomaly entirely to the gauge boson $X(16.7)$.

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