KAPPA: A PACKAGE FOR SYNTHESIS OF OPTICALLY THIN SPECTRA FOR THE NON-MAXWELLIAN κ-DISTRIBUTIONS BASED ON THE CHIANTI DATABASE

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ABSTRACT

Non-Maxwellian κ-distributions have been detected in the solar transition region and in flares. These distributions are characterized by a high-energy tail and a near-Maxwellian core and are known to have a significant impact on the resulting optically thin spectra arising from collisionally dominated astrophysical plasmas. We developed the KAPPA package for the synthesis of such line and continuum spectra. The package is based on the freely available CHIANTI database and software, and can be used in a similar manner. Ionization and recombination rates together with the ionization equilibria are provided for a range of κ values. Distribution-averaged collision strengths for excitation are obtained using an approximate method for all transitions in all ions available within CHIANTI. The validity of this approximate method is tested through a comparison with direct calculations. Typical precisions of better than 5% are found with all cases being within 10%. Tools for the calculation of synthetic line and continuum intensities are provided and described. Examples of the synthetic spectra and SDO/AIA responses to emission for the κ-distributions are given.

Key words: methods: numerical – radiation mechanisms: non-thermal – stars: coronae – Sun: UV radiation – Sun: X-rays, gamma rays – techniques: spectroscopic

1. INTRODUCTION

In astrophysics, emitted radiation is usually the only source of information about the physical conditions in the emitting medium. The physical properties of the emitting plasma are then derived through analysis and modeling of the observed spectra. For a long time this has been done under the assumption of a local, equilibrium Maxwellian distribution. This is assumed even if the emitting medium is optically thin and therefore perhaps not dense enough for the equilibrium always to be ensured locally. Such an assumption is at best difficult in dynamic situations with particle acceleration, as, e.g., a high-energy tail is difficult to equilibrate collisionally, since the collision frequency scales inversely with $E^{3/2}$, where $E$ is the particle energy (e.g., Meyer-Vernet 2007). Scudder & Karimabadi (2013) argue that in the case of stellar coronae, the assumption of a Maxwellian distribution should always be violated at heights 1.05 above the stellar radius. If long-range interactions are induced, e.g., by reconnection, wave-particle interaction, or shocks, then the particles in the system can become correlated and do not have a Maxwellian distribution (e.g., Collier 2004; Vocks & Mann 2003; Vocks et al. 2008; Drake et al. 2006; Livadiotis & McComas 2009, 2010, 2013; Pierrard & Lazar 2010; Gontikakis et al. 2013; Laming et al. 2013). Instead, the distribution exhibits a high-energy power-law tail. κ-distributions are a class of particle distribution with a near-Maxwellian core and a high-energy power-law tail, both of which are described by an analytic expression (Vasyliunas et al. 1968; Owocki & Scudder 1983, see also Section 2). The κ index has been shown to be an independent thermodynamic index (Livadiotis & McComas 2009, 2010, 2011, 2013) in the generalized Tsallis statistical mechanics (e.g., Tsallis 1988, 2009; Leubner 2002, 2004).

The κ-distributions can be derived analytically in the case of a turbulent velocity diffusion coefficient inversely proportional to the velocity. This has been shown for the plasma in a suprathermal radiation field (Hasegawa et al. 1985), for electrons heated by lower hybrid waves (Laming & Lepri 2007), and for solar flare plasmas where the distribution function arises as a consequence of the balance between diffusive acceleration and collisions (Bian et al. 2014).

Indeed, in solar flares, the κ-distributions provide a good fit to some of the X-ray spectra of coronal sources observed during partially occulted flares (Kašparová & Karlícký 2009; Oka et al. 2013), although a second, near-Maxwellian distribution is also present (Oka et al. 2013). Battaglia & Kontar (2013) used the AIA and RHESSI observations of flares to derive the distribution function in the range of 0.1 to tens of keV. These authors show that the distribution derived in the low-energy range from AIA does not match the high-energy tail observed by RHESSI. A possible cause of this mis-match is the assumption of a Maxwellian distribution in the calculation of the AIA differential emission measures (DEMs), which may compromise the analysis, especially if the high-energy tail is present and observed by RHESSI.

Dziččáková et al. (2011) have shown that the κ-distributions can explain the Si in transition-region line intensities observed by the SOHO/SUMER instrument (Wilhelm et al. 1995), especially in the active region spectra (see also Del Zanna et al. 2014). Testa et al. (2014) inferred the presence of high-energy tails from the analysis of Si iv spectra observed by the IRIS spectrometer (De Pontieu et al. 2014). κ-distributions are also routinely detected in the solar wind (e.g., Collier et al. 1996; Maksimovic et al. 1997a, 1997b; Livadiotis & McComas 2010; Le Chat et al. 2011). The high-energy tails at keV energies can arise as a consequence of coronal nanoflares (Gontikakis et al. 2013) that are also able to produce the “halo” in the solar wind electron distribution (Che & Goldstein 2014). Furthermore, a claim has been made that the κ-distributions were also detected in the spectra of planetary nebulae (Binette et al. 2012; Nicholls et al. 2012, 2013; Dopita et al. 2013), although this has been challenged as a possible effect of atomic data.
uncertainties (Storey et al. 2013; Storey & Sochi 2014). \(\kappa\)-distributions are also one of the possible explanations of the non-Maxwellian H\(\alpha\) profiles detected in Tycho’s supernova remnant (Raymond et al. 2010).

Although \(\kappa\)-distributions were detected in solar flares, the transition region, and the solar wind, their presence in the solar corona is currently unknown despite numerous diagnostic attempts. Diagnostics of the high-energy electrons have been attempted by Feldman et al. (2007) and Hannah et al. (2010). Feldman et al. (2007) investigated whether the He-like intensities observed by SUMER could correspond to a bi-Maxwellian distribution with the second Maxwellian having a temperature of 10 MK. These authors argued that no such second Maxwellian is necessary. However, this analysis was limited to a Maxwellian distribution and did not include the effect of a proper high-energy power-law tail. Hannah et al. (2010) used the X-ray off-limb observations of the quiet-Sun performed by the RHESSI instrument (Lin et al. 2002) to obtain upper-limits on the emission measures as a function of \(\kappa\). However, for temperatures of several MK corresponding to the solar corona, these upper limits are large and increase with increasing \(\kappa\). Direct attempts at spectroscopic diagnostics using EUV line intensities observed by Hinode/EIS (Culhane et al. 2007) were performed by Dzifčáková and Kulinová (2010) and Mackovjak et al. (2013). Indications of non-Maxwellian distributions were found using the O\(\text{v}–\text{O vi}\) and S X–S xi lines. However, such an analysis was problematic due to the large photon noise uncertainties affecting weak lines, atomic data uncertainties, and the possible presence of multithermal effects that would complicate the analysis. Therefore, even diagnostics using only strong lines will have been supplemented by a DEM analysis under the assumption of a \(\kappa\)-distribution. Under the constraints of the current EUV instrumentation, such a DEM analysis typically involves many different elements and ionization stages (see, e.g., Warren et al. 2012; Mackovjak et al. 2014).

All of this leads to a requirement for reliable calculation of synthetic spectra involving many different elements and ionization stages. In this paper, we describe the KAPPA package\(^3\) for the calculation of optically thin astrophysical spectra that arise due to collisional excitation by electrons with a \(\kappa\)-distribution. This package, allowing for the fast calculation of line and continuum spectra for \(\kappa\)-distributions, is based on the freely available CHIANTI atomic database and software, currently in version 7.1 (Dere et al. 1997; Landi et al. 2013). The manuscript is organized as follows. The \(\kappa\)-distributions are described in Section 2. Syntheses of the line spectra and continua are described in Section 3 and Section 4, respectively. Section 5 describes the database and the software implementation. Examples of the synthetic spectra and the AIA filter responses for \(\kappa\)-distributions are provided in Section 6. A summary is provided in Section 7.

\section{2. THE NON-MAXWELLIAN \(\kappa\)-DISTRIBUTIONS}

\subsection{2.1. Definition and Basic Properties}

The \(\kappa\)-distribution of the electron energies (Figure 1) is defined as (e.g., Owocki & Scudder 1983; Livadiotis & McComas 2009)

\[ f_\kappa(E) dE = A_\kappa \frac{2}{\sqrt{T}} (k_B T)^{3/2} \left[ 1 + \frac{E}{(k_B T/\kappa)^{1/2}} \right]^{1/2}, \]

where \(A_\kappa = \Gamma(\kappa + 1) \Gamma(\kappa - 1/2)/(\pi k_B T/\kappa)^{1/2}\) is the normalization constant and \(k_B = 1.38 \times 10^{-16} \text{erg s}^{-1}\) is the Boltzmann constant. The \(\kappa\)-distribution has two parameters: \(T \in (0, +\infty)\) and \(\kappa \in (3/2, +\infty)\). The Maxwellian distribution at a given \(T\) corresponds to \(\kappa \rightarrow \infty\). The departure from the Maxwellian distribution increases with decreasing \(\kappa\) with the maximum departure occurring for \(\kappa \rightarrow 3/2\).

While the most probable energy \(E_{\text{max}} = (\kappa - 3/2)k_B T/\kappa\) is a decreasing function of \(\kappa\), the mean energy \(\langle E \rangle = 3k_B T/2\) of a \(\kappa\)-distribution does not depend on \(\kappa\) and is only a function of \(T\). Because of this, the parameter \(T\) has the same physical meaning for the \(\kappa\)-distributions as the (kinetic) temperature for the Maxwellian distribution. Additionally, Livadiotis & McComas (2009) and Livadiotis & McComas (2010) show that \(T\) also corresponds to the definition of the physical temperature in the framework of the generalized Tsallis statistical mechanics (Tsallis 1988, 2009) and permits the generalization of the zeroth law of thermodynamics. Note that this fact permits, e.g., the definition of electron kinetic pressure \(p = n_e k_B T\) in the usual manner.

Note also that the \(\kappa\)-distribution is not the only possible representation of a non-Maxwellian distribution with a high-energy tail (e.g., Dzifčáková et al. 2011; Che & Goldstein 2014). Nevertheless, its analytical expression and single additional parameter \(\kappa\) make it a useful special case of an equilibrium particle distribution associated with turbulence (Hasegawa et al. 1985; Laming & Lepri 2007; Bian et al. 2014), offering a rather straightforward evaluation of various rate coefficients associated with radiative processes (Sections 3 and 4).

\subsection{2.2. Approximation by Maxwellian Core and a Power-law Tail}

It is straightforward to see from Equation (1) that in the high-energy limit, the \(\kappa\)-distribution approaches a power law with an index of \(-3(\kappa + 1/2)\). On the other hand, Meyer-Vernet et al. (1995) and Livadiotis & McComas (2009) showed that in the low-energy limit, the \(\kappa\)-distribution behaves as a Maxwellian with

\[ T_M = \frac{\kappa - 3/2}{\kappa + 1} T. \]

The low-energy end of a \(\kappa\)-distribution can indeed be well approximated by a Maxwellian if this Maxwellian is scaled by a constant,

\[ c_M(\kappa) = A_\kappa \frac{\exp\left(\frac{\kappa - 3/2}{2(\kappa + 1)}\right)}{\left(1 + \frac{1}{2(\kappa + 1)}\right)^{(\kappa + 1)/2}}, \]

so that the two distributions match at the most probable energy \(E_{\text{max}} = (\kappa - 3/2)k_B T/\kappa\) (Figure 1 middle; see also e.g., Dzifčáková 2002, Figure 1 therein).
These approximations suggest that the $\kappa$-distribution can be thought of as a Maxwellian core at a lower temperature with a power-law tail.

3. LINE INTENSITIES FOR THE $\kappa$-DISTRIBUTIONS

In the optically thin solar and stellar coronae, as well as in the associated transition regions and flares, spectral lines arise as a consequence of particle collisions exciting the ions in the highly ionized plasma. The total emissivity $\varepsilon_\mu$ of a spectral line $\lambda_\mu$, corresponding to a transition $j \rightarrow i$, $j > i$, in a $k$-times ionized ion of element $X$ is usually expressed as (e.g., Mason & Monsignori Fossi 1994; Phillips et al. 2008)

$$
\varepsilon_\mu = \frac{h c}{\lambda_\mu} A_\mu n(X_j^+)^k n(X_i^-)^{n-1} \frac{A_X n_e n_H}{n(X)} A_X n_e n_H
$$

where $h \approx 6.62 \times 10^{-27}$ erg s is the Planck constant, $c = 3 \times 10^{10}$ cm s$^{-1}$ is the speed of light, $A_\mu$ the Einstein coefficient for spontaneous emission, and $n(X_j^+)^k$ is the density of the ion $+k$ with an electron on the excited upper level $j$. In Equation (6), the latest quantity is usually expanded in terms of the ionization fraction $n(X_j^+)/n_X$ (Section 3.1) and the excitation fraction $n(X_j^+)/n(X_i^-)$ (Section 3.2). There, $n(X_i^-)$ denotes the total density of the ion $-k$, and $n(X) \equiv n_H$ corresponds to the total density of element $X$ whose abundance is $A_X$, with $n_H$ being the hydrogen density. The function $G_{X,j}(T, n_e, \kappa)$ is the contribution function for the line $\lambda_\mu$. The intensity $I_{\mu j}$ of the spectral line is then given by the emissivity integral of the emissivity along a path $l$ corresponding to the line of sight

$$
I_{\mu j} = \int A_X G_{X,j}(T, n_e, \kappa) n_e n_H dl,
$$

where $EM = \int n_e n_H dl$ is the emission measure of the emitting plasma.

The CHIANTI atomic database provides the observed wavelengths $\lambda_\mu$ and the corresponding Einstein coefficients $A_\mu$, while the electron density $n_e$ is a free parameter.

To complete the synthesis of line intensities for the $\kappa$-distributions, the relative ion abundance $n(X_j^+)/n_X$ and the relative level population $n(X_j^+)/n(X_i^-)$ must be calculated. This is detailed in the remainder of this section.

3.1. Ionization Equilibrium

A common assumption in the calculation of the relative ion abundance $n(X_j^+)/n_X$ is ionization equilibrium, i.e., that the relative ion abundance is not a function of time. Then, the relative ion abundance is given by the equilibrium between the ionization and recombination rates. In coronal conditions, the dominating ionization processes are direct ionization and autoionization (e.g., Phillips et al. 2008), while the dominant recombination processes are radiative and dielectronic recombination. Since these processes involve free electrons, all of these rates depend on $T$ and $\kappa$ (e.g., Dziřičkárová 1992; Anderson et al. 1996; Dziřičkárová & Dudič 2003; Dziřičkárová & Dudík 2013). We note that in the non-equilibrium ionization conditions, $n(X_j^+)/n_X$ depends on the specific evolution of the system, in particular, on the energy...
sources, sinks, and the resulting flows (e.g., Bradshaw & Mason 2003; Bradshaw et al. 2004; Bradshaw 2009). Since radiation is an energy sink, the system is coupled. Dzifčáková & Dudík (2013) provide the latest available ionization equilibria for the \( \kappa \)-distributions for all of the ions of elements with \( Z \leq 30 \), i.e., H to Zn. These calculations use the same atomic data for ionization and recombination as the ionization equilibria for the Maxwellian distribution available in the CHIANTI database, v7.1 (Dere et al. 1997; Dere 2007; Landi et al. 2013).

Figure 2 shows examples of the behavior of the relative ion abundances of Fe\( \text{X} \)–Fe\( \text{XVIII} \) with \( \kappa \). The ionization peaks are generally wider for lower \( \kappa \). Compared to the Maxwellian distribution, the ionization peaks of the transition-region ions are generally shifted to lower log(\( T/K \)), while the coronal ions are generally shifted to higher \( T \), especially for low \( \kappa = 2–3 \) (Dzifčáková & Dudík 2013). Exceptions from these rules of thumb occur. For example, the ionization peak of Fe\( \text{XVIII} \) is shifted to lower \( T \) for \( \kappa = 5 \), while for \( \kappa = 2 \) it is shifted to higher \( T \) compared to the Maxwellian distribution. The shifts of the ionization peaks are typically \( \Delta \log(\kappa T/K) \approx 0.10–0.15 \) for \( \kappa = 2 \), although much larger shifts can also occur, e.g., for Fe\( \text{vii} \) (Dzifčáková & Dudík 2013).

This behavior of the individual ionization peaks with \( \kappa \) strongly influences the resulting line intensities (Equations (6) and (7)). Therefore, the approximate temperatures derived from the observed lines in the spectrum can be different for a \( \kappa \)-distribution and the Maxwellian distribution. For a plasma with a high-energy tail or electron beams can be expected, the \( T \) is related to the mean energy of the distribution (Section 2.1) including the high-energy tail. Notably, \( T \) can be very different from the Maxwellian “bulk” temperature \( T_M \) or \( T_C \) (Section 2.2). Strong changes in the ionization equilibrium for the \( \kappa \)-distributions mainly in the transition region result, e.g., in O\( \text{IV} \) being formed at log(\( T/K \)) \( \approx 5.15 \) for the Maxwellian distribution, but at \( \approx 5.0 \) for \( \kappa = 5 \) and \( \approx 4.8 \) for \( \kappa = 2 \) (Dudík et al. 2014a). The core of the distribution can have even lower temperatures—for log(\( T/K \)) \( \approx 5.15 \), log(\( T_C/K \)) \( = 5.0 \) for \( \kappa = 5 \), but only 4.55 for \( \kappa = 2 \). The \( T_M \) are even lower (Section 2.2). Therefore, without a diagnostic of \( \kappa \) in situations where the high-energy tail can exist, one has to be very careful in estimating the plasma temperature from the fact that a particular line is observed. The situation is further complicated by the dependence of the line emission on the differential emission measure of the emitting plasma (e.g., Warren et al. 2012; Teriaca et al. 2012), which is itself a function of \( \kappa \) (Mackovjak et al. 2014).

3.2. Excitation Equilibrium and Rates

The relative level populations \( n(X^{+\ell})/n(X^{+5}) \) can be obtained under an assumption of excitation equilibrium (Phillips et al. 2008, Equations (4.24) and (4.25) therein). Under equilibrium conditions, the total number of transitions to and from any given level \( j \) is balanced by transitions from all other levels \( m \) to the level \( j \) as well as from the level \( j \) to any other level \( m \). In the conditions of the solar and stellar coronae, ion–electron collisions are the dominant excitation mechanism, while de-excitations are facilitated either by spontaneous radiative decay (with the rates \( A_{jm} \)) and/or collisional de-excitation during ion–electron collisions. The rates of electron excitation and de-excitation, \( C_{jm}^{\varepsilon} \) and \( C_{jm}^{\delta} \), can be expressed as (Bryans 2006; Dudík et al. 2014b)

\[
C_{jm}^{\varepsilon} = \frac{2\sqrt{2}\alpha_0^2 \Omega_{ji}}{m_e \omega_i \omega_j} \left( \frac{\pi}{k_B T} \right)^{1/2} e^{-\Delta E_{ji}/k_B T} \Upsilon_{ji}(T, \kappa),
\]

(8)

\[
C_{jm}^{\delta} = \frac{2\sqrt{2}\alpha_0^2 I_{\text{H}}}{\sqrt{m_e \omega_j}} \left( \frac{\pi}{k_B T} \right)^{1/2} \mathcal{J}_{ji}(T, \kappa),
\]

(9)

where \( \alpha_0 = 5.29 \times 10^{-9} \) cm is the Bohr radius, \( m_e = 1.67 	imes 10^{-28} \) is the electron rest mass, \( I_{\text{H}} \approx 13.6 \) eV \( \equiv 1 \) Ryd is the hydrogen ionization energy, \( \omega_i \) and \( \omega_j \) are the statistical weights of the levels \( i \) and \( j \), respectively, \( \Delta E_{ji} = E_i - E_j \) is the energy of the transition, and \( E_i \) and \( E_j \) are the incident and final electron energies. \( \Upsilon_{ji}(T, \kappa) \) \( \mathcal{J}_{ji}(T, \kappa) \) denote the distribution-averaged collision strengths, given by

\[
\Upsilon_{ji} = \int_{-\infty}^{+\infty} \frac{\Omega_{ji}(E_i)}{E_i} \frac{dE_i}{\Delta E_{ij}} \left( 1 + \frac{E_i - \Delta E_{ij}}{(\kappa - 3/2)k_B T} \right)^{\kappa - 1},
\]

(10)
\[ \mathcal{J}_{ji} = \frac{A}{\kappa} \Delta E_{ij} \int_{0}^{\infty} \frac{\Omega_{ji} \{ E_j \}}{1 + \frac{E_j}{(\kappa - 3/2)k_B T}} \Delta E_j \, dE_j. \]  

(11)

In these expressions, \( \Omega_{ji}(E_j) = \Omega_j(E_i) \) is the collision strength, i.e., the non-dimensionalized cross-section

\[ \Omega_{ji}(E_j) = \frac{E_j}{I_H} \frac{\sigma_{ji}^{\alpha}(E_j)}{\alpha_0^2} = \omega_j \frac{E_j}{I_H} \frac{\sigma_{ji}^{\beta}(E_j)}{\alpha_0^2}, \]  

(12)

where \( \sigma_{ji}^{\alpha} \) and \( \sigma_{ji}^{\beta} \) are the electron impact excitation and de-excitation cross-sections, respectively. Note that with \( \kappa \to \infty \), \( \mathcal{J}_{ji}(T, \kappa) \) revert to the \( \mathcal{J}_j(T) \) commonly used for the Maxwellian distribution (Seaton 1953; Burgess & Tully 1992; Mason & Monsignori Fossi 1994; Bradshaw & Raymond 2013). For the Maxwellian distribution, the property of \( \mathcal{J}_j(T) \equiv \mathcal{J}_j(T) \) being recovered. \( \mathcal{J}_j(T, \kappa) \), and \( \mathcal{J}_j(T, \kappa) \), together with Equations (8) and (9) and the equations of statistical equilibrium (Equations (4.24) and (4.25) in Phillips et al. 2008), can then be used to synthesize the spectra for the \( \kappa \)-distributions in the same manner as for the Maxwellian distribution (Section 5.2).

### 3.3. Collision Strength Approximation

The calculation of the collision strengths for excitation and de-excitation averaged over \( \kappa \)-distributions for a large number of transitions introduces the problem of the accessibility of atomic cross-sections \( \Omega_{ji}(E_j) \). Only a few databases contain these data and typically only for a small number of transitions. The CHIANTI database and software (Dere et al. 1997; Landi et al. 2013) contain spline approximations to the Maxwellian-averaged collision strengths for the majority of the astronomically interesting ions of elements H to Zn. CHIANTI allows for the computation and analysis of solar spectra and is an important tool for the diagnostics of solar plasma under the assumption of a Maxwellian distribution.

We used the CHIANTI database to calculate the approximate cross-sections \( \Omega \) and subsequently to approximate the excitation and de-excitation rates \( \mathcal{Y}_{ij}(T, \kappa) \) and \( \mathcal{J}_{ji}(T, \kappa) \) for the \( \kappa \)-distributions. This approximate method was described, e.g., in Dzičáková (2006a), and tested for Fe xv by Dzičáková & Mason (2008). Here, we use this method to obtain \( \mathcal{Y}_{ij}(T, \kappa) \) and \( \mathcal{J}_{ji}(T, \kappa) \) for all of the transitions in all of the elements and ions available within CHIANTI.

The approximation works as follows. A functional form for the approximation of \( \Omega \) is assumed (Abramowitz & Stegun 1965)

\[ \Omega = \sum_{n=0}^{n_{\text{max}}} C_n u^{-n} + D \ln (u), \]  

(13)

where \( C_k \) and \( D \) are coefficients and \( u = E_i / \Delta E_{ij} \). The advantage of this approximation is a simple analytical evaluation of its integral over the distribution function. This approximation was often used to express the collision strength, e.g., by Mewe (1972).

\[ \mathcal{Y}_{ij} \] for the Maxwellian distribution can then be written as

\[ \mathcal{Y}_{ij} = \frac{\Delta E_{ij}}{k_B T} \int_{0}^{\infty} \Omega_{ij}(E_i) e^{-E_i / \Delta E_{ij}} dE_i, \]  

(14)

which after integration leads to

\[ \mathcal{Y}_{ij} = C_0 + \left( \sum_{k=1}^{n_{\text{max}}} y C_n E_n(y) + D E_1(y) \right) e^y, \]  

(15)

where \( y = \Delta E_{ij} / k_B T \) and \( E_n(y) \) is an \( n \)th order exponential integral.

The behavior of \( \Omega \) in the high-energy limit and the corresponding behavior of \( \mathcal{Y}_{ij} \) provide the following conditions for the coefficients \( C_n \) and \( D \) for the electric dipole transitions:

\[ D = \frac{4 \omega_j f_{ij}}{\Delta E_{ij}}, \]  

(16)

\[ \mathcal{Y}_{ij}(\to \infty) = \sum_{n=0}^{n_{\text{max}}} C_n = \Omega (u = 1), \]  

(17)

while for the non-electric dipole, non-exchange transitions

\[ D = 0, \]  

(18)

\[ \mathcal{Y}_{ij}(\to \infty) = \sum_{n=0}^{n_{\text{max}}} C_n = \Omega (u = 1). \]

The low- and high-energy limits \( \mathcal{Y}_{ij}(\to 0) \) and \( \mathcal{Y}_{ij}(\to \infty) \) can be found in the CHIANTI database. The coefficients \( C_n \) and \( D \) can be evaluated from the collisional strengths in CHIANTI, averaged over the Maxwellian distribution by the least-square method. To achieve higher precision, we used approximations (Equation (13)) up to \( n_{\text{max}} = 7 \).

The approximate method described here is also used to calculate the distribution-averaged collision strengths for de-excitation \( \mathcal{J}_{ji}(T, \kappa) \).

### 3.4. Validity of the Approximate Method

Figure 3 demonstrates the approximation of \( \Omega \) and the calculation of \( \mathcal{Y}_{ij} \) for the O iv transition 2s2 2p2 3p1/2 2s 2p7 3p3/2 at 1401.16 Å. The atomic data for this transition are taken from Liang et al. (2012). We find a typical precision in the approximation of CHIANTI \( \mathcal{Y}s \) of a few percent. This is the case for the O iv 1401.16 Å transition shown, for which we find a precision of 1–2%. However, for a small part of transitions, the precision can be significantly worse, up to approximately 15%.

Fulfilling conditions (16)–(18) for the coefficients guarantees the correct behavior of \( \Omega \) for high and threshold energies. It is, however, difficult to compare the data for all of the transitions of each ion. Occasional errors in the approximation of \( \Omega \) (Equation (13)) cannot be excluded at present. Their propagation to the calculated \( \mathcal{Y}_{ij} \) are further minimized by
adopting

\[ \Upsilon_{\kappa} = \Upsilon_{\text{CHIANTI}}\frac{\Upsilon_{\text{Maxwell}}}{\Upsilon_{\text{Maxwell}}}, \]

(19)

where \( \Upsilon_{\kappa} \) is the final \( \Upsilon(\kappa, T) \) for the \( \kappa \)-distributions, \( \Upsilon_{\text{CHIANTI}} \) is \( \Upsilon(T) \) taken from CHIANTI for the Maxwellian distribution, and \( \Upsilon_{\text{Maxwell}} \) are \( \Upsilon \)s calculated from our approximations of the cross-sections for the \( \kappa \)-distributions and the Maxwellian distribution, respectively.

First tests of the precision of the approximate method described in Section 3.3 were performed by Dzifčáková & Mason (2008). These authors used \( n_{\text{max}} = 5 \) and tested the validity of the approximation of the cross-sections \( \Omega \) for some of the Fe XV transitions. An overall precision better than 10% was found. The approximation worked almost perfectly for the allowed transitions. Worse results were found for the forbidden transitions. It was also found that transitions with strong resonance contributions and a low ratio of excitation energy to temperature can also be problematic. However, all of the \( \Omega \)s for all transitions were reproduced to an accuracy better than 15% (Dzifčáková & Mason 2008).

To supplement this analysis, we used \( n_{\text{max}} = 7 \) (Section 3.3) and tested the approximate method on two ions, Fe XI and Fe XVII. We used the original atomic cross-sections from Del Zanna et al. (2010) for Fe XI and Del Zanna (2011) for Fe XVII. These Maxwellian-averaged \( \Upsilon_{\text{DC}}(T) \) are implemented in the CHIANTI database, version 7.1 (Landi et al. 2013). Here, we compare our approximation based on these Maxwellian data in CHIANTI with \( \Upsilon_{\text{DC}}(T, \kappa) \) and \( \Upsilon_{\text{Maxwell}}(T, \kappa) \) calculated directly from the \( \Omega \)s using the method of Dudík et al. (2014b).

Figures 4 and 5 show several examples of the comparison of the direct calculation (hereafter, DC) with the approximate method for Fe XI (Figure 4) and Fe XVII (Figure 5). The DC are denoted by squares and the approximate \( \Upsilon \) by the full lines. The left columns in these figures show typical worst cases for strong transitions. We see that the error of the approximation depends on \( \kappa \) and \( T \); it typically increases with decreasing \( \kappa \). The worst cases are, however, still within 10% even for the extreme value of \( \kappa = 2 \) considered here. Typical cases are shown in the middle columns of Figures 4 and 5. Here, the approximations are valid to within a few percent for all \( \kappa \). Finally, typical approximations for the weak transitions are shown in the right columns of Figures 4 and 5. We again find that the approximations are valid to within \( \approx 10\% \) for all \( \kappa \).

Figure 6 contain scatterplots of the relative error \( \Delta \Upsilon/\Upsilon_{\text{DC}} - 1 \) plotted for each \( \kappa \) at the peak of the corresponding relative ion abundance. These scatterplots contain 447 transitions in Fe XI and 1050 transitions in Fe XVII that we were able to unambiguously indentify in both the CHIANTI database and the atomic data. The plots in Figure 6 confirm that the approximate \( \Upsilon_{\kappa} \) do not depart from the directly calculated \( \Upsilon_{\kappa,\text{DC}} \) by more than 10%. Typically, the relative errors increase with decreasing \( \kappa \); the smallest errors are found for the Maxwellian distribution. Strong transitions typically have higher accuracy than weaker transitions, in agreement with the results of Dzifčáková & Mason (2008).

Finally, we note that the approximation of \( \Upsilon(\kappa, T) \) to within 10% is considered satisfactory given the uncertainties in the atomic data themselves, which are typically of the same order of magnitude, and the uncertainties of the spline-fits of the Maxwellian \( \Upsilon(T) \) contained in CHIANTI, which are typically \(< 5\% \).

### 3.5. Dielectronic Satellite Lines

The rate coefficient for the dielectronic excitation from level \( i \) to level \( j \) and for an arbitrary electron distribution function \( f(E) \) can be expressed as (Seely et al. 1987)

\[ C_{\text{die}} = \left( \frac{2}{m_e \Delta E_{ji}} \right)^{1/2} \frac{\hbar^3 g_j}{16 \pi m_e g_i} f(\Delta E_{ji}) A_{\alpha}, \]

(20)

where \( g_j \) and \( g_i \) are the statistical weights of the double excited state and lower level, respectively; \( A_{\alpha} \) is the autoionization (Auger) rate. The transition occurs at a discrete energy \( \Delta E_{ji} \) which corresponds to the energy difference between energy of states \( j \) and \( m \). For the Maxwellian distribution, this equation leads to the well-known expression (e.g., Phillips et al. 2008,
Figure 4. \( T_e \) (top) and their relative errors (bottom) for the Fe XI 3s\(^2\)3p\(^4\) \(^3\)P\(_2\)–3s\(^2\)3p\(^4\) \(^3\)P\(_0\) (left), 3s\(^2\)3p\(^4\) \(^3\)P\(_1\)–3s\(^2\)3p\(^1\)\(^2\)D\(_{3S0}\) (middle), and 3s\(^2\)3p\(^4\) \(^3\)P\(_1\)–3p\(^5\) 3d \(^3\)F\(_3\) (right) transitions. Black lines show the comparison of CHIANTI’s approximation with the direct calculations for the Maxwellian distribution. Colors show the comparison of our approximation to direct calculations for the \( \kappa \)-distribution with \( \kappa = 25 \) (blue), 10 (turquoise), 5 (green), 3 (yellow), and 2 (red).

Figure 5. Same as in Figure 4 but for the following transitions in Fe XVII: 2s\(^2\)2p\(^5\) 3s \(^1\)P\(_2\)–2s\(^2\)2p\(^5\) 3s \(^1\)P\(_1\) (left), 2s\(^2\)2p\(^5\) 3s \(^1\)P\(_1\)–2s\(^2\)2p\(^5\) 3p \(^1\)D\(_2\) (middle), and 2s\(^2\)2p\(^5\) 3s \(^1\)P\(_1\)–
2s\(^2\)2p\(^5\) 3p \(^1\)D\(_2\) (right).
Figure 6. Relative error of $\tau_\nu$ to $\tau_{\nu,dc}$ for Fe XI (left) and Fe XVII (right) as a function of $\tau_{\nu,dc}$ at temperatures corresponding to the maximum of the ion abundance $T_{e,(max)}$. Black points are for the CHIANTI approximation (Equation (19)). Different colors stand for the results for $\kappa = 2$ (red), 3 (orange), 5 (green), and 10 (blue).
Equation (4.19) therein)

\[ C_{\text{Maxw}}^\text{dil} = \frac{\hbar^3}{2(2\pi m_e k_B T)^{3/2}} \frac{g_j \delta_{ij} e^{-\Delta E_x}}{g_i} A^a. \]  

(21)

For the \( \kappa \)-distribution, we have

\[ C_{\kappa}^\text{dil} = \frac{A_e \hbar^3}{2(2\pi m_e k_B T)^{3/2}} \frac{g_j \delta_{ij} e^{-\Delta E_x}}{g_i} \left(1 + \frac{\Delta E_x}{(\kappa - 1.5) k_B T}\right)^{\kappa+1}, \]  

(22)

which leads to

\[ C_{\kappa}^\text{dil} = C_{\text{Maxw}}^\text{dil} \frac{A_e \Delta E_x e^{-\Delta E_x}}{(\kappa - 1.5) k_B T} \left(1 + \frac{\Delta E_x}{(\kappa - 1.5) k_B T}\right)^{\kappa+1}. \]  

(23)

4. THE NON-MAXWELLIAN CONTINUUM

The continuum for the non-Maxwellian \( \kappa \)-distributions is treated here using the approach of Dudík et al. (2012). Contributions from the free–free and free-bound continua are considered. The two-photon continuum is not considered, as its emissivity for \( \kappa \)-distributions is not known and its contribution is usually weak for the Maxwellian distribution (Young et al. 2003; Phillips et al. 2008), especially at higher densities. Nevertheless, at least for the Maxwellian distribution and a limited wavelength range, the two-photon continuum may not represent a negligible contribution to the total continuum. We plan to implement it in the future.

4.1. The Free–Free Continuum

The total emissivity of the free–free continuum arising due to electron–ion bremsstrahlung is given by (Dudík et al. 2011, 2012)

\[ \varepsilon_{\text{ff}}(\lambda, \kappa, T) = \frac{A_e T^{1/2}}{\lambda^2} n_e n_H \times \sum K_X(\kappa, T) A_X \int_0^{\infty} g_{\text{ff}}(y, w) \left(1 + \frac{y + w}{\kappa - 3/2}\right)^{\kappa+1} dy, \]

(24)

where \( A_X \) is the element abundance relative to hydrogen, \( w = hc/\lambda k_B T = \frac{\varepsilon}{k_B T} \) is the scaled photon energy, \( g_{\text{ff}} \) is the free–free Gaunt factor, and \( K_X(\kappa, T) \) is a function of \( \kappa \) and \( T \) through the dependence on the ionization balance \( n_1/n_X \) (see Section 3.1):

\[ K_X(\kappa, T) = \frac{3}{4\pi} \frac{2\pi r^2}{3 m_e c^2} e^{-\delta} \frac{2\pi k_B T}{3 m_e} \sum_k k^3 n_k \frac{n_z}{n_Z}, \]

(25)

where \( k \) is the ionization degree. The units of \( \varepsilon_{\text{ff}} \) are ergs cm\(^{-3}\)s\(^{-1}\)sr\(^{-1}\)Å\(^{-1}\).

The bremsstrahlung spectrum is strongly dependent on \( \kappa \), mainly at short wavelengths (Dudík et al. 2012) where the tail of the \( \kappa \)-distribution strongly enhances the bremsstrahlung emission. Near the wavelength where \( \varepsilon_{\text{ff}} \) peaks for the Maxwellian distribution, the free–free emission drops with \( \kappa \). At larger wavelengths it is enhanced again (see Figures 2 and 3 in Dudík et al. 2012).

4.2. Free-bound Continuum

The emissivity of the recombination processes resulting in \( k \)-times ionized ions of element \( X \) with an electron on an excited level \( j \) for the \( \kappa \)-distributed incident electrons is given by (Dudík et al. 2012)

\[ \varepsilon_{\text{fb}}(\lambda, \kappa, T) = \frac{1}{4\pi} \frac{\varepsilon^5}{(\kappa - 1.5) k_B T} \frac{1}{n_e n_H} \times \sum_{kX} n_{k+1} A_X \frac{g_j^{\text{bf}}}{{\sigma_j^{\text{bf}}}} A_{\kappa} \left(1 + \frac{\varepsilon - I_j}{(\kappa - 3/2) k_B T}\right)^{\kappa+1}, \]

(26)

where \( \varepsilon = hc/\lambda = E + I_j \) is the photon energy, \( I_j \) is the ionization potential from level \( j \) with statistical weight \( g_j \), and \( \sigma_j^{\text{bf}} \) is the ionization cross-section from level \( j \).

A conspicuous feature of the free-bound spectra for the \( \kappa \)-distributions is the greatly enhanced ionization edges (see Figure 5 in Dudík et al. 2012). Generally, this increase comes from Equation (26) through the increase of low-energy electrons in a \( \kappa \)-distribution with respect to the Maxwellian distribution at the same \( T \). However, details also depend on the ionization equilibrium together with \( T \) and \( \kappa \) (Dudík et al. 2012).

5. THE KAPPA PACKAGE

The KAPPA package\(^4\) currently allows for the calculation of synthetic spectra for integer values of \( \kappa = 2, 3, 4, 5, 7, 10, 15, 25, \) and 33, for which the ionization equilibria are tabulated. These values should cover the parameter space with sufficient density. The database and software for the KAPPA package are based on the IDL version of the freely available CHIANTI database and software\(^5\) (Dere et al. 1997; Landi et al. 2013). The routines and database of the KAPPA package are contained in a standalone folder. It cannot be contained within CHIANTI itself in order to prevent its automatic removal by CHIANTI updates. The path to the folder can be set by an IDL system variable in the \textit{idl\_startup.pro} file

\texttt{defsysv, ‘data\_pth’, ‘path to package’}.

The KAPPA folder contains the modified CHIANTI routines for the calculation of spectra for the \( \kappa \)-distributions with the “data\_k” subfolder having the same structure as the CHIANTI’s “dbase” subfolder. The modified CHIANTI routines follow the original CHIANTI routines as closely as possible. Their names end with an extra “\_k” before the \texttt{.pro} extension. The calling parameters of these routines are kept the same, except that the first parameter is always the value of \( \kappa \).

The subdirectories within the database contain data for ionization and recombination rates (Section 5.1.2) together with the tabulated \( \Upsilon_j(T, \kappa) \) and \( \Upsilon_j(T, \kappa) \) files in the ASCII format. Previous versions of the modification corresponding to CHIANTI v5.2 (Dzifčáková 2006) contained the coefficients for the approximation of \( \Omega \). Then, the calculations for the \( \kappa \)-distributions were approximately 10 times longer compared to CHIANTI for the Maxwellian distribution. Therefore, we decided to pre-calculate \( \Upsilon(T, \kappa) \) for a grid of temperatures and \( \kappa \). These pre-calculated values of \( \Upsilon(T, \kappa) \) are contained in files

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\(^4\) http://kappa.asu.cas.cz

\(^5\) www.chiantidatabase.org
names according to the ion and the value of $\kappa$ with the extension .ups, e.g., c.5_k2.ups for $\text{C}^5$ and $\kappa = 2$. IDL savefiles containing $\gamma_j(T, \kappa)$ and $\lambda^\ast_j(T, \kappa)$ are also provided.

At present, the KAPPA package fully corresponds to the atomic data contained in CHIANTI version 7.1. Similarly, the routines provided in the KAPPA package are based on CHIANTI 7.1 routines with the exception of routines for the free–free continuum (see Section 5.3.1).

### 5.1. Ionization Equilibrium

#### 5.1.1. Ionization Equilibrium Files

The ionization equilibrium .ioneq and similar files were originally provided by Dzifčáková & Dudík (2013). A minor software bug in the calculation of radiative recombination rates for the $\kappa$-distributions was found and corrected. This problem affected the ionization equilibria at $\log(T/K) < 5$ with the error being much smaller than the effect of $\kappa$-distributions on the ionization equilibrium.

These .ioneq files are produced in the same format as the original .ioneq file. Therefore, these can be read by the CHIANTI routine read_ioneq.pro directly. The names of these files are kappa_02.ioneq and similar, where the numbers give the integer value of $\kappa$. For more details on the .ioneq file format, see Dzifčáková & Dudík (2013, Appendix A therein).

#### 5.1.2. Ionization and Recombination Rates

In addition to the ionization equilibria, the total ionization and recombination rates are provided for each ion and a range of temperatures. Here, the total ionization rate is a sum of the direct collisional ionization rate and the autoionization rate. Similarly, the total recombination rate is given by the sum of the radiative recombination rate and the total dielectronic recombination rate (Dzifčáková & Dudík 2013). These rates are stored in the respective database folder for each ion, e.g., database/c/c5c_5_k25.ionizr is the total ionization rate file for $\text{C}^5$ and $\kappa = 25$. The file format is ASCII. The total recombination rate file has the same name except the .trecombr extension. The routines readrate_ioniz_k.pro and read_rate_recomb_k.pro are provided for reading these files.

### 5.2. Tools for Calculation of Line Spectra

The KAPPA package provides several routines for the calculation of line intensities. These are listed in Table 1. As already mentioned, these routines are based on CHIANTI routines, version 7.1. They can be used in the same manner as the CHIANTI routines with the exception that the value of $\kappa$ is always the first parameter.

The most important of these routines is the pop_solver_k.pro routine that calculates the relative level population based on the distribution-averaged collision strengths $\gamma_j(T, \kappa)$ and $\lambda^\ast_j(T, \kappa)$ calculated using the method described in Section 3.3. Other routines for calculating line intensities (Table 1) rely on this routine. Examples of synthetic spectra calculated for $\kappa = 2$ and their comparison to the Maxwellian spectra at the same $T$ are given in Section 6.1.

We note here that the method for calculating the collisional electron excitation and de-excitation rates described in Section 3.3 cannot be applied to the collisional excitation by protons due to the unavailability of the proton excitation cross-sections. The proton excitation is typically negligible, but may be important for some transitions. In the synthesis of line spectra, the proton excitation rate for the $\kappa$-distribution is assumed to be the same as for the Maxwellian distribution at the same temperature. It is currently unknown whether or not this assumption is justified. Because of this, we advocate caution in using such lines, e.g., for diagnostics of $\kappa$ from observations.

An interactive widget for calculating the synthetic spectra is provided in the kappa.pro routine based on CHIANTI’s ch_ss.pro. The value of $\kappa$ is selected by the choice of the ionization equilibrium. Subsequently, the excitation and line intensities are calculated for the same value of $\kappa$. All other functionality of the ch_ss.pro routine is retained.

### 5.3. Tools for Calculation of Continuum

#### 5.3.1. Free–Free Continuum

The CHIANTI database relies on the approximations to the Maxwellian bremsstrahlung calculated by Itoh et al. (2000) and Sutherland (1998) and incorporated into the freefree.pro

| Routine name                  | Function                                                                 |
|------------------------------|--------------------------------------------------------------------------|
| kappa.pro                    | interactive widget for calculation of synthetic spectra, based on ch_ss.pro |
| ch_synthetic_k.pro           | calculates line intensities as a function of $\kappa$, $n_e$ and $T$      |
| descale_diel_k.pro           | converts $\gamma_j(T, \kappa)$ and $\lambda^\ast_j(T, \kappa)$ from the scaled domain for dielectric satellite lines and performs correction in Equation (23) calculates $\frac{hc}{\lambda} \alpha_{\beta,n}(X_i^{+\ast})$ |
| emiss_calc_k.pro            | calculates the free-bound continuum arising from a single ion             |
| freebound_ion_k.pro         | calculates the free-bound continuum                                       |
| freebound_k.pro             | free-free continuum interpolated from pre-calculated data                  |
| free-free_k_integral.pro    | calculates the free–free continuum directly                             |
| isothermal_k.pro            | calculates isothermal spectra as a function of $\lambda$                  |
| make_kappa_spec_k.pro       | routine for calculating the synthetic spectra                            |
| plot_populations_k.pro      | calculates and plots relative level populations                          |
| pop_solver_k.pro            | calculates the relative level population                                  |
| read_if_k.pro               | reads the pre-calculated free–free continuum as a function of $Z$ and $T$ |
| read_rate_ioniz_k.pro       | reads the total ionization and recombination rates                        |
| read_rate_recomb_k.pro      | routine for interpolating the $\gamma_j(T, \kappa)$ and $\lambda^\ast_j(T, \kappa)$ |
| ups_kappa_interp.pro        | routines for calculating line intensities                                 |
routine together with the \textit{itoh.pro} and \textit{sutherland.pro} routines. This approach cannot be followed in the KAPPA package, since no fitting formulae exist for the free–free continuum for $\kappa$-distributions. Instead, we provide two options to calculate the free–free continuum for $\kappa$-distributions.

1. Direct integration using Equations (24) and (25). This approach is implemented in the \textit{freefree\_k'integral.pro} routine and requires the \textit{data\_k/continuum/gffew.dat} file containing the scaled gyw($\lambda$, $\kappa$) values provided by Sutherland (1998). These gyw values are then de-scaled and numerically integrated. Since the scaling depends on the ionization energy (and thus on the ionization stage $k$ and the proton number $Z$), it has to be carried out for each ion separately (Dudík et al. 2012). Therefore, the direct integration using the \textit{freefree\_k_integral.pro} is time-consuming and impractical. We note that, in practice, restricting the integration to elements with a relative abundance of $A_Z < 10^{-6}$ introduces a relative error smaller than $10^{-4}$ and speeds up the calculations by a factor of $\approx 2$. Note that the Maxwellian-integrated $g_{\rm ff}(y, w)$ are a part of the CHIANTI database.

2. To overcome the long calculation times, the free–free continuum has been pre-calculated as a function of $Z$ for 101 logarithmically spaced temperatures spanning $\log(T/K) = (4, 9)$ with a step of $\Delta\log(T/K) = 0.05$, together with 29 logarithmically spaced points in $\lambda = 0.1 \text{ Å} - 3 \times 10^4 \text{ Å}$ with a step of $\log(\lambda/\text{Å}) = 0.2$. These calculations are contained in the \textit{data\_k/continuum/ff\_kappa\_02.dat} file and analogous files for other values of $\kappa$, with each file for a single value of $\kappa$. The files are in ASCII format. The routine \textit{freefree\_k.pro} reads these files, and then folds and sums them over the abundances to produce a semi-final free–free continuum. The final free–free continuum is then calculated for the user-input ranges of $T$ and $\lambda$ within the ranges specified above. This is achieved first by linearly interpolating $\log(T/K)$ and then by spline-interpolating $\log(\lambda/\text{Å})$. In this way, an accuracy of a few percent is achieved in the $\lambda = 1 \text{ Å} - 2 \times 10^4 \text{ Å}$ range with a calculation time of few seconds. We note that this method should not be used for the calculation of free–free continua below 1 Å and $2 \times 10^4 \text{ Å}$ where the spline interpolation results in errors of several $\times 10\%$ or more.

5.3.2. Free-bound Continuum

Using Equation (26), the free-bound continuum is straightforward to calculate. The \textit{freebound\_k.pro} and \textit{freebound\_ion\_k.pro} routines can be used in the same manner as CHIANTI’s \textit{freebound.pro} and \textit{freebound\_ion.pro} routines.

Figure 7. Example isothermal spectra at $\log(T/K) = 5.9$ near the peak of the AIA 171 Å wavelength response (top) and at $\log(T/K) = 6.2$ near the peak of the AIA 193 Å filter (bottom). The electron density assumed is $\log(n_e/\text{cm}^{-3}) = 9.0$. 

\begin{itemize}
\item Maxwell, $\log(T/K)=5.90$ $\log(n_e/\text{cm}^{-3})=9$
\item $\kappa = 2$, $\log(T/K)=5.90$ $\log(n_e/\text{cm}^{-3})=9$
\end{itemize}
The only change is that these routines require a value of $\kappa$ as an extra input. Ionization equilibrium files (Section 5.1) are read together with the cross-sections. The speed of the calculation is the same as using the original CHIANTI.

6. SYNTHETIC SPECTRA AND SDO/AIA RESPONSES

In this section, we provide some examples of the calculated spectra that are of interest to the physics of the solar corona. Note that the behavior of individual lines observed by the Hinode/EIS spectrometer (Culhane et al. 2007) and the possible observational diagnostics with and without the effect of the ionization equilibrium are described elsewhere (Dzifčáková and Kulinová 2010; Mackovjak et al. 2013; Dzifčáková & Dudík 2013; Dudík et al. 2014b), as is the application for DEM diagnostics (Mackovjak et al. 2014).

6.1. Synthetic EUV Spectra

An example of the synthetic isothermal spectra calculated using isothermal_k.pro for the Maxwellian and $\kappa = 2$ are shown in Figure 7. These examples show synthetic line and continuum spectra within the AIA 171 Å and 193 Å channels. The spectra are calculated for an electron density of $n_e = 10^9$ cm$^{-3}$ and temperatures of $\log(T/K) = 5.9$ for AIA 171 Å and 6.2 for AIA 193 Å, respectively. Note that the temperature is kept the same for both distributions shown. These temperatures correspond to the maximum of the relative ion abundance of Fe IX and Fe XII under the Maxwellian distribution, respectively (see Figure 2). The line intensities for $\kappa = 2$ are decreased by a factor of several compared to the Maxwellian distribution. This is mainly an effect of the ionization equilibrium, with the maximum of the relative ion abundance for $\kappa = 2$ being shifted to higher $\log(T/K)$ (Figure 2, bottom). Note that the intensities
of the continuum are several orders of magnitude smaller than the line intensities; therefore, the continuum is not visible in the linear scale on Figure 7.

From Figure 7 we see that at $\log(T/K) = 5.9$, the AIA 171 Å channel is dominated by Fe xi independent of the value of $\kappa$. Contrary to that, the situation for the AIA 193 Å channel and $\log(T/K) = 6.2$ is more complex. This filter is dominated by Fe xii transitions between energy levels 1–30 at 192.394 Å, 1–29 at 193.509 Å, and 1–27 at 195.119 Å (Dudík et al. 2014b, Table B.4). However, contributions from the 1–38 and 1–37 transitions in Fe xi at 188.216 Å and 188.299 Å are present as well. The relative contribution of these transitions to the total filter response to emission at $\log(T/K) = 6.2$ increases from 4.4% for the Maxwellian distribution to 6.5% for $\kappa = 2$. This is because this temperature is closer to the ionization peak of Fe xi than Fe xii for $\kappa = 2$ (Figure 2).

6.2. AIA Responses for the $\kappa$-Distributions

As an example of the usage of the synthetic line and continuum spectra, we calculated the responses of the Atmospheric Imaging Assembly (AIA) Boerner et al. (2012; Lemen et al. 2012) on board the Solar Dynamics Observatory (SDO; Pesnell et al. 2012) for the $\kappa$-distributions. Note that even though the continuum intensities are weak compared to the line intensities (Section 6.1) at a particular wavelength, the continuum is a significant contributor to some of the AIA bands (O’Dwyer et al. 2010; Del Zanna 2013). This is because the filter response to plasma emission is given by the wavelength integral of the filter and instrument transmissivity times the emitted spectrum (e.g., Equation (6) in Dudík et al. 2009).

The SDO/AIA responses calculated for $\kappa$-distributions and $\log(n_e/\text{cm}^{-3}) = 9$ are shown in Figure 8. The peaks of the responses are typically flatter and wider, and can be shifted to higher $\log(T/K)$ for low $\kappa$. This behavior is typical since it is given mainly by the ionization equilibrium (Section 3.1). It has been reported for the TRACE filter responses by Dudík et al. (2009) and for the Hinode/XRT responses by Dzičáková et al. (2012), where the AIA responses were also calculated for an earlier set of atomic data corresponding to CHIANTI v5.2 (Landi et al. 2006). The AIA responses calculated here represent a significant improvement over the Dzičáková et al. (2012) ones due to advances in the atomic data for AIA bands (Del Zanna 2013).

We note that some of the secondary maxima, such as those at $\log(T/K) \approx 5.4$ for AIA 171, 193, 211, and 335 Å disappear for low $\kappa$. This is again mostly because of the wider ionization peaks and their relative contributions to individual filter responses (Dudík et al. 2009), which gradually smooth out these secondary maxima with decreasing $\kappa$. We also note that the contribution from Fe x and Fe xiv to the AIA 94 Å response (Del Zanna 2013, Figure 3 therein) form a single, smooth secondary peak of the response for $\kappa \lesssim 5$.

These AIA responses for $\kappa$-distributions can be used to obtain DEMs using the regularized DEM inversion developed by Hannah & Kontar (2012, 2013), as well as for diagnostics of the distribution from the combination of imaging and spectroscopic observations (Dudík et al. 2014, in preparation).

7. SUMMARY

We have developed tools for the calculation of synthetic optically thin line and continuum spectra arising from collisionally dominated astrophysical plasmas characterized by a $\kappa$-distribution. These tools constitute the KAPPA package, which is based on the freely available CHIANTI database and software. Currently, the KAPPA package can only handle values of $\kappa = 2, 3, 4, 5, 7, 10, 15, 25$, and 33, which should provide sufficient coverage for most spectroscopic purposes. Ionization and recombination rates are provided together with ionization equilibrium calculations. Approximations to the distribution-averaged collision strengths are provided. These are based on the reverse-engineered collision strengths obtained from the Maxwellian-averaged collision strengths available within CHIANTI. This is done for all transitions in all ions available within CHIANTI, version 7.1. We have tested the validity of this approximate method by comparison with the directly integrated collision strengths. For temperatures typical of the formation of individual ions, typical errors of less than 5% were found. It was also found that the errors are always less than 10%. The errors are typically of the order of a few percent for strong transitions, but the precision decreases for weaker transitions and low values of $\kappa$. Considering the uncertainties in the atomic data calculations themselves, these errors are considered acceptable.

Several routines for the calculation of the synthetic line spectra, free–free, and free-bound continua are provided. These routines are based on the CHIANTI routines and can be used in the same manner, except that the first input parameter is always the value of $\kappa$. The calculation of the free–free continuum is based on interpolation from pre-calculated values; however, an option of direct integration of the free–free gaunt factors is also provided. We aim to keep the database updated to reflect the newer releases of CHIANTI.

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REFERENCES

Abramowitz, M., & Stegun, I. A. 1965, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (New York: Dover)
Anderson, S. W., Raymond, J. C., & van Ballegooijen, A. 1996, ApJ, 457, 939
Bataglia, M., & Kontar, E. P. 2013, ApJ, 779, 107
Bian, N. H., Emslie, A. G., Stackhouse, D. J., & Kontar, E. P. 2014, ApJ, 796, 142
Binette, L., Matadas, R., Hägle, G. F., et al. 2012, A&A, 547, A29
Boerner, P., Edwards, C., Lemen, J., et al. 2012, SoPh, 275, 41
Bradshaw, S. J. 2009, A&A, 502, 409
Bradshaw, S. J., Del Zanna, G., & Mason, H. E. 2004, A&A, 425, 287
Bradshaw, S. J., & Mason, H. E. 2003, A&A, 401, 699
Bradshaw, S. J., & Raymond, J. 2013, SSRv, 178, 271
Bryans, P. 2006, Ph.D. thesis, University of Strathclyde
Burgess, A., & Tully, J. A. 1992, A&A, 254, 436
Che, H., & Goldstein, M. L. 2014, ApJ, 795, L38
Collier, M. R. 2004, AdSpR, 33, 2108
