Recent astrophysical and accelerator based results on the Hadronic Equation of State

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In astrophysics as well as in hadron physics progress has recently been made on the determination of the hadronic equation of state (EOS) of compressed matter. The results are contradictory, however. Simulations of heavy ion reactions are now sufficiently robust to predict the stiffness of the (EOS) from (i) the energy dependence of the ratio of \( K^+ \) from \( \text{Au}+\text{Au} \) and \( C+C \) collisions and (ii) the centrality dependence of the \( K^+ \) multiplicities. The data are best described with a compressibility coefficient at normal nuclear matter density \( \kappa \) around 200 MeV, a value which is usually called “soft.” The recent observation of a neutron star with a mass of twice the solar mass is only compatible with theoretical predictions if the EOS is stiff. We review the present situation.

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How much energy is needed to compress nuclear matter? The answer to this question, the determination of \( E/A(\rho,T) \), the energy/nucleon in nuclear matter in thermal equilibrium as a function of the density \( \rho \) and the temperature \( T \), has been considered since many years as one of the most important challenges in nuclear physics. This quest has been dubbed “search for the nuclear equation of state (EoS)”.

Only at equilibrium density, \( \rho_0 \), the energy per nucleon \( E/A(\rho = \rho_0, T = 0) = -16 \) MeV is known by extrapolating the Weizsäcker mass formula to infinite matter. Standard ab initio many body calculations do not allow for a determination of \( E/A(\rho, T) \) at energies well above the saturation density because the low density many body expansion schema (Brückner \( G \)-matrix) breaks down and therefore the number of contributing terms is exploding. Therefore in nuclear reaction physics another strategy has been developed. Theory has identified experimental observables in nuclear reaction physics or in astrophysics which are sensitive to \( E/A(\rho, T) \). Unfortunately these observables depend as well on other quantities which are either unknown or little known (like cross sections with resonance in the entrance channel) or difficult to assess theoretically (like the resonance lifetimes in hot and dense matter). It was hoped that comparing many observables for different systems and different energies with the theoretical predictions these unknown or little known quantities can be determined experimentally and that finally the dependence of the observables on \( E/A(\rho, T) \) can be isolated.

In astrophysics the nuclear EoS plays an important role in binary mergers involving black holes and neutron stars [1], in double pulsars [2], in the mass-radius relation of neutron stars [3, 4] and in supernovae explosions [5]. For a recent review on the topics we refer to [6]. Unfortunately, as in nuclear reaction physics, there are other little known processes or properties which have to be understood before the nuclear EoS dependence can be isolated. We discuss here as example of the mass-radius relation of neutron stars. Fig. 1 shows the neutron star masses in units of the solar mass for different types of binaries. These masses are concentrated at around 1–1.5 solar masses. Fig. 2 shows a theoretical prediction of the mass-radius relation for neutron stars using different EoS. Since the nature of the interior of neutron stars is not known (in contradiction to what the name suggests) one may suppose that it consists of hadrons or of quarks. But even if it consisted of hadrons there are speculations that there is a \( K^- \) or a \( \pi^- \) condensate or that there are hyperons in equilibrium with nuclear resonances. The same is true if the interior consists of quarks. Little known color-flavor locked quark phases may modify the EoS at densities which are reached in the interior of the neutron star. For a detailed discussion of all these phenomena we refer to ref. [6]. We see that the observed masses of neutron stars are compatible with almost all quark or hadron based EoS as long as the radius is unknown. Radii, however, are very difficult to measure. Because similar problems appear also for other observables, up to recently the astrophysical observations of neutron stars did not help much to narrow down the uncertainty on the nuclear EoS.

This situation has changed dramatically in the last year with the observation of a neutron star with a mass of two solar masses [7]. If this observation is finally confirmed the mass/radius prediction of fig.2 excludes that the interior of a neutron star is made by quarks [4], and an off nuclear EoS, which will be defined below, will be excluded. This is confirmed by the calculation of Maieron [8] which uses a MIT bag model or a color dielectric models EoS.

* invited speaker
FIG. 1: Measured and estimated masses of neutron stars in radio binary pulsars and in x-ray accreting binaries. Error bars are 1σ. Vertical dotted lines show average masses of each group (1.62 M⊙, 1.34 M⊙ and 1.56 M⊙); dashed vertical lines indicate inverse error weighted average masses (1.48 M⊙, 1.41 M⊙ and 1.34 M⊙). The figure is taken from ref [4]

to describe the quark phase. Baldo [9] argue that this conclusion may be premature because it depends too much on the equation of state of the quark phase. If one replaces the MIT bag model equation of state by that of the Nambu - Jona-Lasinio (NJL) Lagrangian under certain conditions (no color conducting phase) larger masses may be obtained. The standard NJL Lagrangian lacks, however, repulsion and in view of the momentum cut-off, necessary to regularize the loop integrals, and the coupling constants in the diquark sector, which are not uniquely determined by the Fierz transformation, quantitative prediction at high quark densities are difficult in this approach even if qualitative agreement with pQCD calculation can be found [10].

In heavy ion reactions three observables have been identified which are - according to theoretical calculations - sensitive to $E/A(\rho, T)$ at densities larger than $\rho_0$: (i) the strength distribution of giant isoscalar monopole resonances [11, 12], (ii) the in-plane sidewards flow of nucleons in semi-central heavy ion reaction at energies between 100 A MeV and 400 A MeV [13] and (iii) the production of $K^+$ mesons in heavy ion reactions at energies around 1 A GeV [14]. We will discuss these approaches below. Although theory has predicted these effects qualitatively, a quantitative approach is confronted with two challenges: a) The nucleus is finite and surface effects are not negligible, even for the largest nuclei and b) in heavy ion reactions the reacting system does not come into equilibrium. Therefore complicated non-equilibrium transport theories have to be employed and the conclusion on the nuclear EoS can only be indirect.

(i) The study of monopole vibrations has been very successful, but the variation in density is minute. Therefore, giant monopole resonances are sensitive to the energy which is necessary to change the density of a cold nucleus close to the equilibrium point $\rho_0$. According to theory the vibration frequency depends directly on the force which counteracts any deviation from the equilibrium and therefore to the potential energy. The careful analysis of the isoscalar monopole strength in non-relativistic [11] and relativistic mean field models has recently converged [12] due to a new parametrization of the relativistic potential. These calculations allow now for the determination of the compressibility $\kappa = 9 \rho^2 \frac{dE/A(\rho, T)}{d\rho}|_{\rho=\rho_0}$ which measures the curvature of $E/A(\rho, T)$ at the equilibrium point. The values found are around $\kappa = 240$ MeV and therefore close to what has been dubbed “soft EoS”. It agrees as well with the prediction of nuclear matter calculations based on nucleon-nucleon scattering data [15] which give a value of about $\kappa = 250$ MeV.

(ii) If the overlap zone of projectile and target becomes considerably compressed in semi-central heavy-ion collisions, an in-plane flow is created due to the transverse pressure on the baryons outside of the interaction region with this flow being proportional to the transverse pressure. In order to obtain a noticeable compression, the beam energy has to be large as compared to the Fermi energy of the nucleons inside the nuclei and hence a beam energy of at least 100 A MeV is necessary. Compression goes along with excitation and therefore the compressional energy of excited nuclear matter is encoded in the in-plane flow. It has recently been demonstrated [16] that transport theories do not
agree quantitatively yet and therefore former conclusions [17] have to be considered as premature.

(iii) The third method is most promising for the study of nuclear matter at high densities [18] and I will discuss it in detail this talk. $K^+$ mesons produced far below the $NN$ threshold cannot be created in first-chance collisions between projectile and target nucleons. They do not provide sufficient energy even if one includes the Fermi motion. The effective energy for the production of a $K^+$ meson in the $NN$ center of mass system is 671 MeV as in addition to the mass of the kaon a nucleon has to be converted into a $\Lambda$ to conserve strangeness. Before nucleons can create a $K^+$ at these subthreshold energies, they have to accumulate energy. The most effective way to do this is to convert first a nucleon into a $\Delta$ and to produce in a subsequent collision a $K^+$ meson via $\Delta N \rightarrow NK^+\Lambda$. Two effects link the yield of produced $K^+$ with the density reached in the collision and the stiffness of the EoS. If less energy is needed to compress matter (i) more energy is available for the $K^+$ production and (ii) the density which can be reached in these reactions will be higher. Higher density means a smaller mean free path and therefore the $\Delta$ will interact more often increasing the probability to produce a $K^+$ and hence it has a lower chance to decay before it interacts. Consequently the $K^+$ yield depends on the compressional energy. At beam energies around 1 A GeV matter becomes highly excited and mesons are formed. Therefore this process tests highly excited hadronic matter. At beam energies $> 2$ A GeV first-chance collisions dominate and this sensitivity is lost.

For the third approach different transport theories have converged. This was possible due a special workshop at the ECT* in Trento/Italy where the authors of the different codes have discussed their approaches in detail and common solutions have been advanced. The results of this common effort have been published in [19]. As an example we display here the $K^+ p_\perp$ spectra at midrapidity obtained in the different transport theories at different energies. Because with each $K^+N$ rescattering collision the slope of the $K^+$ spectra changes the slope of the $p_\perp$ spectra encodes not only the $K^+$ momentum distribution at the time point of production but also the distribution of the number of rescatterings. It is therefore all but trivial. Without the $KN$ potential the slopes are almost identical and even the absolute yield which depends on a correct modeling of the Fermi motion of the nucleons is very similar. If we include the $KN$ interaction which is not identical in the different approaches (see [19]) we still observe a very similar slope.
for most of the programs.

Due to this progress the simulation programs can now be used to extract yet theoretically inaccessible information like the hadronic EOS [18]. Two independent experimental observables, the ratio of the excitation functions of the $K^+$ production for Au+Au and for C+C \[20,\ 21\], and a new observable, the dependence on the number of participants of the $K^+$ yield, show that nucleons interact with a potential which corresponds to a compressibility of $\kappa \leq 200$ MeV in infinite matter in thermal equilibrium. This value extracted for hadronic matter at densities around 2.5 times the normal nuclear matter density is very similar to that extracted at normal nuclear matter density. A key point here is to demonstrate that the different implementation of yet unsolved physical questions, like the $N\Delta \rightarrow K^+\Lambda N$ cross section, the KN interaction as well as the life time of the nuclear resonances in the hadronic environment do not affect this conclusion.

In order to determine the energy which is necessary to compress infinite nuclear matter in thermal equilibrium by heavy ion reactions in which no equilibrium is obtained one chooses the following strategy: The transport theory calculates the time evolution of the quantal particles described by Gaussian wave functions. The time evolution is given by a variational principle and the equations one obtains for this choice of the wave function are identical to the classical Hamilton equations where the classical two-body potential is replaced by the expectation value of a 3 parameter Skyrme potential. The Skyrme potential is a simple approximation to the real part of the Brückner $G$-matrix which is too complicated for performing simulations of heavy ion collisions. For this potential the potential energy in infinite nuclear matter is calculated. To determine the nuclear EoS we average this (momentum-dependent) two-body potential over the momentum distribution of a given temperature $T$ and add to it the kinetic energy. Expressed as a function of the density we obtain the desired nuclear EoS $E/A(\rho, T)$. Our two-body potential has five parameters which are fixed by the binding energy of infinite nuclear matter at $\rho_0$, the compressibility $\kappa$ and the optical potential which has been measured in pA reactions [22].

Once the parameters are fixed we use the two-body potential with these parameters in the transport calculation. There is an infinite number of two-body potentials which give the same EoS because the range of the potential does not play a role in infinite matter. The nuclear surface measured in electron scattering on nuclei fixes the range, however, quite well. The uncertainty which remains is of little relevance here (in contradiction to the calculation of the in-plane flow which is very sensitive to the exact surface properties of the nuclei and hence to the range of the potential).

We employ the Isospin Quantum Molecular Dynamics (IQMD) with momentum dependent forces. All details of the standard version of the program may be found in [22]. The standard version is supplemented for this calculation with all inelastic cross sections which are relevant for the $K^+$ production. For details of these cross sections we refer to [23]. Unless specified differently, the change of the $K^+$ mass due to the kaon-nucleon ($KN$) interaction according to $m_{K^+}(\rho) = m_0^K (1 - 0.075 \rho_0^{1/3})$ is taken into account, in agreement with recent self-consistent calculations of the spectral function of the $K^+$ [24]. The $\Lambda$ potential is $2/3$ of the nucleon potential, assuming that the $s$ quark is inert. The

FIG. 3: Final $K^+$ transverse momentum distribution at $b=1$ fm, $|y_{cm}| < 0.5$ and with an enforced $\Delta$ lifetime of $1/120$ MeV (top row without, bottom row with KN potential) in the different approaches [19].
calculations reproduce the experimental data quite well as can be seen in fig. 4 where we compare the experimental and theoretical $K^+$ spectra for different centrality bins for 1.48 AGeV Au+Au. This figure shows as well the influence of the $K^+N$ potential which modifies not only the overall multiplicity of $K^+$ due to the increase of the in medium mass but also the spectral form confirming the complexity of the transverse momentum spectrum. In order to minimize

![Graph](image)

**FIG. 4**: $K^+$ spectra for different centrality bins as compared with [preliminary] experimental data from the KaoS collaboration

the experimental systematical errors and the consequences of theoretical uncertainties the KaoS collaboration has proposed to study not directly the excitation function of the $K^+$ yield but that of the yield ratio of heavy to light systems [20]. Calculations have shown that ratios are much less sensitive to little known input parameters because they affect both systems in a rather similar way. We have shown in fig. 4 that the absolute yields are well reproduced in our simulations. Therefore we can use this ratio directly for a quantitative comparison with data. These ratios are, on the contrary, quite sensitive to the nuclear potentials because the compression obtained in the Au+Au collisions is considerable (up to $r_{0\text{B}}$) and depends on the nuclear EoS whereas in C+C collisions the compression is small and almost independent on the stiffness of the EoS.

Figure 5 shows the comparison of the measured ratio of the $K^+$ multiplicities obtained in Au+Au and C+C reactions [20] together with transport model calculations as a function of the beam energy. We see, first of all in the top row, that the excitation function of the yield ratio depends on the potential parameters (hard EoS: $\kappa = 380$ MeV, thin lines and solid symbols, soft EoS: $\kappa = 200$ MeV, thick lines and open symbols) in a quite sensible way and - even more essential - that the prediction in the standard version of the simulation (squares) for a soft and a hard EoS potential differ much more than the experimental uncertainties. The calculation of Fuchs et al. [21] given in the same graph, agrees well with our findings.

This observation is, as said, not sufficient to determine the potential parameters uniquely because in these transport theories several not precisely known processes are encoded. For these processes either no reliable theoretical prediction has been advanced or the different approaches yield different results for the same observable. Therefore, it is necessary to verify that these uncertainties do not render our conclusion premature. There are 3 identified uncertainties: the $\sigma_{N\Delta\rightarrow K^+}$ cross section, the density dependence of the $K^+N$ potential and the lifetime of $\Delta$ in matter if produced in a collisions with a sharp energy of two scattering partners. We discuss now how these uncertainties influence our results:

Figure 5, top, shows as well the influence of the unknown $N\Delta \rightarrow K^+\Delta N$ cross section on this ratio. We confront the standard IQMD option (with cross sections for $\Delta N$ interactions from Tsushima et al. [23]) with another option, $\sigma (N\Delta) = 3/\sigma (NN)$ [26], which is based on isospin arguments and has been frequently employed. Both cross sections differ by up to a factor of ten and change significantly the absolute yield of $K^+$ in heavy ion reactions but do not change the shape of the ratio.
FIG. 5: Comparison of the measured excitation function of the ratio of the $K^+$ multiplicities per mass number $A$ obtained in Au+Au and in C+C reactions (Ref. [20]) with various calculations. The use of a hard EoS is denoted by thin (blue) lines, a soft EoS by thick (red) lines. The calculated energies are given by the symbols, the lines are drawn to guide the eye. On top, two different versions of the $N\Delta \to K^+AN$ cross sections are used. One is based on isospin arguments [26], the other is determined by a relativistic tree level calculation [27]. The calculation by Fuchs [21] are shown as dotted lines. Middle: IQMD calculations with and without KN potential are compared. Bottom: The influence of different options for the life time of $\Delta$ in matter is demonstrated.

The middle part demonstrates the influence of the kaon-nucleon potential which is not precisely known at the densities obtained in this reaction. The uncertainties due to the $\Delta$ life time are discussed in the bottom part. Both calculations represent the two extreme values for this lifetime [23] which is important because the disintegration of the $\Delta$ resonance competes with the $K^+$ production.

Thus we see that these uncertainties do not influence the conclusion that the excitation function of the ratio is quite different for a soft EoS potential as compared to a hard one and that the data of the KaoS collaboration are only compatible with the soft EoS. The only possibility to change this conclusion is the assumption that the cross sections are explicitly density dependent in a way that the increasing density is compensated by a decreasing cross section. It would have a strong influence on other observables which are presently well predicted by the IQMD calculations.

The conclusion that nuclear matter is best described by a soft EoS, is supported by another variable, the dependence of the $K^+$ yield on the number of participating nucleons $A_{\text{part}}$. The prediction of the IQMD simulations in the standard version for this observable is shown in Fig. 6. The top of the figure shows the kaon yield $M_{K^+}/A_{\text{part}}$ for Au+Au collisions at 1.5 GeV as a function of the participant number $A_{\text{part}}$ for a soft EoS using different options: standard version (soft, $KN$), calculations without kaon-nucleon interaction (soft, no $KN$) and with the isospin based $N\Delta \to NAK^+$ cross section (soft, $KN$, $\sigma^*$). These calculations are confronted with a standard calculation using the hard EoS potential. The scaling of the kaon yield with the participant number can be parameterized by $M_{K^+} = A_{\text{part}}^\alpha$.

All calculations with a soft EoS show a rather similar value of $\alpha$ - although the yields are very different - while the calculation using a hard equation shows a much smaller value. Therefore we can conclude that also the slope value $\alpha$ is a rather robust observable.

The bottom of Fig. 6 shows that $\alpha$ depends smoothly on the compressibility $\kappa$ of the EoS. Whether we include the momentum dependence of the nucleon nucleon interaction (with md) or not (without md) does not change the value of $\alpha$ as long as the compressibility is not changed - in stark contrast to the in-plane flow. Again, the measured centrality dependence for Au+Au at 1.5 GeV from the KaoS collaboration [28], $\alpha = 1.34 \pm 0.16$, is only compatible
with a soft EoS potential. That the dependence of the \(K^+\) yield on the participants number is a clear signal for the

![Graph showing the dependence of \(M_{K^+}/A_{\text{part}}\) on \(A_{\text{part}}\) for different EoS potentials.](image)

\(M_{K^+} = A_\alpha\) scaling on the nuclear EoS. We present this dependence in form of \(M_{K^+}/A_{\text{part}}\). On the top the dependence of \(M_{K^+}/A_{\text{part}}\) as a function of \(A_{\text{part}}\) is shown for different options: a “hard” EoS with \(KN\) potential (solid line), the other three lines show a “soft” EoS, without \(KN\) potential and \(\sigma(N\Delta)\) from Tsuchiya [27] (dotted line), with \(KN\) potential and the same parametrization of the cross section (dashed line) and with \(KN\) potential and \(\sigma(N\Delta) = 3/4\sigma(NN)\). On the bottom the fit exponent \(\alpha\) is shown as a function of the compressibility for calculations with momentum-dependent interactions (mdf) and for static interactions (dashed line)[22]. On the right hand side we compare the energy dependence of the fit exponent \(\alpha\) for the two EoS.

EoS also at other beam energies as can be seen from fig. 6 right which displays the excitation function of the fit exponent \(\alpha\). Data which follow the curve for a soft equation of state will soon be published [29]

In conclusion, we have shown That earthbound experiments have now reached a precision which allows to determine the hadronic EoS. The experimental results for the two observables which are most sensitive to the hadronic EoS are only compatible with theory if the hadronic EoS is soft. This conclusion is robust. Little known input quantities do not influence this conclusion. The observation of a neutron star with twice the solar mass seems to contradict this conclusion. It points toward a hard hadronic EoS. Both results are quite new and one has not to forget that we are comparing non equilibrium heavy ion reactions where about the same number of protons and neutrons are present and where mesons and baryon resonances are produced with cold neutron matter in equilibrium. To solve this contradiction is certainly a big challenge for both communities in the near future.

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Hadronic Matter is Soft

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The stiffness of the hadronic equation of state has been extracted from the production rate of $K^+$ mesons in heavy ion collisions around 1 $A$ GeV incident energy. The data are best described with a compressibility coefficient $\kappa$ around 200 MeV, a value which is usually called “soft”. This is concluded from a detailed comparison of the results of transport theories with the experimental data using two different procedures: (i) the energy dependence of the ratio of $K^+$ from Au+Au and C+C collisions and (ii) the centrality dependence of the $K^+$ multiplicities. It is demonstrated that input quantities of these transport theories which are not precisely known, like the kaon-nucleon potential, the $\Delta N \to NK^+\Lambda$ cross section or the life time of the $\Delta$ in matter do not modify this conclusion.

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Since many years one of the most important challenges in nuclear physics is to determine $E/A(\rho,T)$, the energy/nucleon in nuclear matter in thermal equilibrium as a function of the density $\rho$ and the temperature $T$. Only at equilibrium density, $\rho_0$, the energy per nucleon $E/A(\rho = \rho_0, T = 0) = -16$ MeV is known by extrapolation of the Weizsäcker mass formula to infinite matter. This quest has been dubbed “search for the nuclear equation of state (EoS)”. Modelling of neutron stars or supernovae have not yet constrained the nuclear equation of state [7]. Therefore, the most promising approach to extract $E/A(\rho,T)$ are heavy ion reactions in which the density of the colliding nuclei changes significantly. Three principal experimental observables have been suggested in the course of this quest which carry - according to theoretical calculations - information on the nuclear EoS: (i) the strength distribution of giant isoscalar monopole resonances [2, 3], (ii) the in-plane sideways flow of nucleons in semi-central heavy ion reaction at energies between 100 $A$ MeV and 400 $A$ MeV [4] and (iii) the production of $K^+$ mesons in heavy ion reactions at energies around 1 $A$ GeV [2]. Although theory has predicted these effects qualitatively, a quantitative approach is confronted with two challenges: a) The nucleus is finite and surface effects are not negligible, even for the largest nuclei and b) in heavy ion reactions the reacting system does not come into equilibrium. Therefore complicated non-equilibrium transport theories have to be employed and the conclusion on the nuclear equation of state can only be indirect.

(i) The study of monopole vibrations has been very successful, but the variation in density is minute. Therefore, giant monopole resonances are sensitive to the energy which is necessary to change the density of a cold nucleus close to the equilibrium point \( \rho_0 \). According to theory the vibration frequency depends directly on the force which counteracts to any deviation from the equilibrium and therefore to the potential energy. The careful analysis of the isoscalar monopole strength in non-relativistic and relativistic mean field models has recently converged [3] due to a new parametrization of the relativistic potential. These calculations allow now for the determination of the compressibility $\kappa = 9\rho^2 \frac{d^2E/A}{d\rho^2}|_{\rho=\rho_0}$ which measures the curvature of $E/A(\rho,T)$ at the equilibrium point. The values found are around $\kappa = 240$ MeV and therefore close to what has been dubbed “soft equation of state”.

(ii) If the overlap zone of projectile and target becomes considerably compressed in semi-central heavy-ion collisions, an in-plane flow is created due to the transverse pressure on the baryons outside of the interaction region with this flow being proportional to the transverse pressure. In order to obtain a noticeable compression, the beam energy has to be large as compared to the Fermi energy of the nucleons inside the nucleus and hence a beam energy of at least 100 $A$ MeV is necessary. Compression goes along with excitation and therefore the compressional energy of excited nuclear matter is encoded in the in-plane flow. It has recently been demonstrated [6] that transport theories do not agree quantitatively yet and therefore former conclusions [7] have to be considered as premature.

(iii) The third method is most promising for the study of nuclear matter at high densities and is subject of this Letter. $K^+$ mesons produced far below the $NN$ threshold cannot be created in first-chance collisions between projectile and target nucleons. They do not provide sufficient energy even if one includes the Fermi motion. The effective energy for the production of a $K^+$ meson in the $NN$ center of mass system is 671 MeV as in addition to the mass of the kaon a nucleon has to be converted into a $\Lambda$ to conserve strangeness. Before nucleons can create a $K^+$ at these subthreshold energies, they have to accumulate energy. The most effective way to do this is the conversion of a nucleon into a $\Delta$ and to produce in a subsequent collision a $K^+$ meson via $\Delta N \to NK^+\Lambda$. Two effects link the yield of produced $K^+$ with the den-
sity reached in the collision and the stiffness of the EoS. If less energy is needed to compress matter (i) more energy is available for the $K^+$ production and (ii) the density which can be reached in these reactions will be higher. Higher density means a smaller mean free path and therefore the $\Delta$ will interact more often increasing the probability to produce a $K^+$ and hence, it has a lower chance to decay before it interacts. Consequently the $K^+$ yield depends on the compressional energy. At beam energies around 1 A GeV matter becomes highly excited and mesons are formed. Therefore this process tests highly excited hadronic matter. At beam energies $> 2$ A GeV first-chance collisions dominate and this sensitivity is lost.

In this Letter we would like to report that for the third approach different transport theories have converged. Two independent experimental observables, the ratio of the excitation functions of the $K^+$ production for Au+Au and for C+C [12, 14], and a new observable, the dependence on the number of participants of the $K^+$ yield show that nucleons interact with a potential which corresponds to a compressibility of $\kappa \leq$ 200 MeV in infinite matter in thermal equilibrium. This value extracted for hadronic matter at densities around 2.5 times the normal nuclear matter density is very similar to that extracted at normal nuclear matter density. A key point of this paper is to demonstrate that the different implementation of yet unsolved physical questions, like the $N\Delta \rightarrow K^+\Lambda N$ cross section, the $K\Lambda$ interaction as well as the life time of the nuclear resonances in the hadronic environment do not affect this conclusion.

In order to determine the energy which is necessary to compress infinite nuclear matter in thermal equilibrium by heavy ion reactions in which no equilibrium is obtained one chooses the following strategy: The transport theory calculates the time evolution of the quantal particles described by Gaussian wave functions. The time evolution is given by a variational principle and the equations one obtains for this choice of the wave function are identical to the classical Hamilton equations where the former interaction between the charged particles. The former can be further subdivided in a part containing the contact Skyrme-type interaction only, a contribution due to a finite range Yukawa-potential, and a momentum-dependent part with

$$V^{ij} = V^{ij}_{\text{Skyrme}} + V^{ij}_{\text{Yuk}} + V^{ij}_{\text{ndl}} + V^{ij}_{\text{Coul}}$$

$$= t_1 \delta(\vec{x}_i - \vec{x}_j) + t_2 \delta(\vec{x}_i - \vec{x}_j) \rho^{\gamma-1}(\vec{x}_i) + t_3 \exp\left[-|\vec{x}_i - \vec{x}_j|/\mu\right]/|\vec{x}_i - \vec{x}_j| + Z_i Z_j e^2/|\vec{x}_i - \vec{x}_j| + t_4 \ln^2(1 + t_5(\vec{p}_i - \vec{p}_j)^2) \delta(\vec{x}_i - \vec{x}_j)$$

with $Z_i, Z_j$ the charges of the baryons $i$ and $j$. For more details we refer to Ref. [9].

We include in this calculation all inelastic cross sections which are relevant for the $K^+$ production. For details of these cross sections we refer to [10]. Unless specified differently, the change of the $K^+$ mass due to the kaonnucleon ($K\Lambda$) interaction according to $m_{K^+} = m_0^K(1 - 0.075 \rho_0)$ is taken into account, in agreement with recent self-consistent calculations of the spectral function of the $K^+$ [11]. The $\Lambda$ potential is $2/3$ of the nucleon potential, assuming that the $s$ quark is inert.

In order to minimize the experimental systematical errors and the consequences of theoretical uncertainties it is better to compare ratios of cross sections rather than the absolute values [12]. We have made sure that the standard version of IQMD reproduces the excitation function for Au+Au as well as for C+C quite well [13]. These ratios are quite sensitive to the nuclear potentials because the compression obtained in the Au+Au collisions is considerable (up to $3\rho_0$) and depends on the nuclear equation of state whereas in C+C collisions the compression

The nuclear surface measured in electron scattering on nuclei fixes the range, however, quite well. The uncertainty which remains is of little relevance here (in contradiction to the calculation of the in-plane flow which is very sensitive to the exact surface properties of the nuclei and hence to the range of the potential).

We employ the Isospin Quantum Molecular Dynamics (IQMD) [9] approach with the following equations of motion:

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial \vec{r}_i} \quad \text{and} \quad \dot{\vec{r}}_i = \frac{\partial H}{\partial \vec{p}_i}, \quad (1)$$

where the expectation value of the total Hamiltonian reads as $\langle H \rangle = \langle T \rangle + \langle V \rangle$ with

$$\langle T \rangle = \sum \frac{p_i^2}{2m_i}$$

$$\langle V \rangle = \sum \sum_{j \geq i} \int f_i(\vec{r}, \vec{p}, t) V^{ij} f_j(\vec{r}', \vec{p}', t) d\vec{r} d\vec{r}' d\vec{p} d\vec{p}' (2)$$

and $f_i$ being the Gaussian Wigner density of nucleon $i$. The baryon-potential consists of the real part of the $G$-Matrix which is supplemented by the Coulomb interaction between the charged particles. The former can be further subdivided in a part containing the contact Skyrme-type interaction only, a contribution due to a finite range Yukawa-potential, and a momentum-dependent part with

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is small and almost independent on the stiffness of the EoS.

Figure 1 shows the comparison of the measured ratio of the $K^+$ multiplicities obtained in Au+Au and C+C reactions \[12\] together with transport model calculations as a function of the beam energy.

We see clearly that the form of the yield ratio depends on the potential parameters (hard EoS: $\kappa = 380$ MeV, thin lines and solid symbols, soft EoS: $\kappa = 200$ MeV, thick lines and open symbols) in a quite sensible way and that the prediction in the standard version of the simulation (squares) for a soft and a hard EoS potential differ much more than the experimental uncertainties. The calculation of Fuchs et al. \[14\] given in the same graph, agrees well with our findings.

This observation is, however, not sufficient to determine the potential parameters uniquely because in these transport theories several not precisely known processes are encoded. Therefore, it is necessary to verify that these uncertainties do not render this conclusion premature. Figure 1 top, shows as well the influence of the unknown $N\Delta \to K^+\Lambda N$ cross section on this ratio. We confront the standard IQMD option (with cross sections for $\Delta N$ interactions from Tsushima et al. \[10\]) with another option, $\sigma(N\Delta) = 3/4\sigma(NN)$ \[13\], which is based on isospin arguments and has been frequently employed. Both cross sections differ by up to a factor of ten and change significantly the absolute yield of $K^+$ in heavy ion reactions but do not change the shape of the ratio.

The middle part demonstrates the influence of the kaon-nucleon potential which is not precisely known at the densities obtained in this reaction. The uncertainties due to the $\Delta$ life time are discussed in the bottom part. Both calculations represent the two extreme values for this lifetime \[10\] which is important because the disintegration of the $\Delta$ resonance competes with the $K^+$ production.

Thus we see that these uncertainties do not influence the conclusion that the excitation function of the ratio is quite different for a soft EoS potential as compared to a hard one and that the data of the KaoS collaboration are only compatible with the soft EoS potential. The only possibility to change this conclusions is the assumption that the cross sections are explicitly density dependent in a way that the increasing density is compensated by a decreasing cross section. It would have a strong influence on other observables which are presently well predicted by the IQMD calculations.

We would like to add that the smoothness of the excitation function also demonstrates that there are no density isomers in the density regions which are obtained in these reactions because the $K^+$ excitation function would be very sensitive to such an isomeric state \[10\].

The conclusion that nuclear matter is best described by a soft EoS, is supported by another variable, the dependence of the $K^+$ yield on the number of participating nucleons $A_{\text{part}}$. The prediction of the IQMD simulations in the standard version for this observable is shown in Fig. 2.

The top of the figure shows the kaon yield $M_{K^+}/A_{\text{part}}$ for Au+Au collisions at 1.5 $A$ GeV as a function of the participant number $A_{\text{part}}$ for a soft EoS using different options: standard version (soft, $K N$), calculations without kaon-nucleon interaction (soft, no $K N$) and with the isospin based $N\Delta \to N\Lambda K^+$ cross section (soft, $K N$, $\sigma^*$). These calculations are confronted with a standard calculation using the hard EoS potential. The scaling of the kaon yield with the participant number can be parameterized by $M_{K^+} = A_{\text{part}}^\alpha$.

All calculations with a soft EoS show a rather similar value of $\alpha$ - although the yields are very different - while the calculation using a hard equation shows a much smaller value. Therefore we can conclude that also the slope value $\alpha$ is a rather robust observable.

The bottom of Fig. 2 shows that $\alpha$ depends smoothly on the compressibility $\kappa$ of the EoS. Whether we include the momentum dependence of the nucleon nucleon
interaction (with mdi) or not (without mdi) does not change the value of $\alpha$ as long as the compressibility is not changed - in stark contrast to the in-plane flow. Again, the measured centrality dependence for Au+Au at 1.5 A GeV from the KaoS collaboration [18], $\alpha = 1.34 \pm 0.16$, is only compatible with a soft EoS potential.

This finding is also supported by a more recent analysis [14, 20] of the in-plane flow which supersedes the former conclusion that the EoS is hard [21] (made before the momentum-dependent interaction has been included in the calculations). Due to the strong dependence of the in-plane flow on the potential range parameter and its dependence on the particles observed these conclusions are much less firm presently. Comparisons of the out-of-plane squeeze of baryons also show a preference for a soft equation of state with momentum dependent interactions [22].

In conclusion, we have shown that the two experimental observables which are most sensitive to the potential parameters of the nucleon-nucleon interaction are only compatible with those parameters which lead in nuclear matter to a soft hadronic EoS. This conclusion is robust. Uncertainties of the input in these calculations, like the $KN$ potential at high densities, the lifetime of the $\Delta$ in matter and the $\Delta N \rightarrow NK^+A$ cross section do not influence this conclusion. The potential parameter $\kappa$ is even smaller than that extracted from the giant monopole vibrations. Thus the energy which is needed to compress hadronic matter of $\kappa \leq 200$ MeV is close to the lower bound of the interval which has been discussed in the past.

We would like to thank all members of the KaoS Collaboration for fruitful discussions especially A. Förster, P. Senger, C. Sturm, and F. Uhlig.

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Hadronic Matter is Soft

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The stiffness of the hadronic equation of state has been extracted from the production rate of \(K^+\) mesons in heavy ion collisions around 1 A GeV incident energy. The data are best described with a compressibility coefficient \(\kappa\) around 200 MeV, a value which is usually called “soft”. This is concluded from a detailed comparison of the results of transport theories with the experimental data using two different procedures: (i) the energy dependence of the ratio of \(K^+\) from Au+Au and C+C collisions and (ii) the centrality dependence of the \(K^+\) multiplicities. It is demonstrated that input quantities of these transport theories which are not precisely known, like the kaon-nucleon potential, the \(\Delta N \rightarrow NK^+\Lambda\) cross section or the lifetime of the \(\Delta\) in matter do not modify this conclusion. This result from earthbound experiments seems to contradict the recent observation of a neutron star with twice the solar mass.

How much energy is needed to compress nuclear matter? The answer to this question, the determination of \(E/A(\rho, T)\), the energy/nucleon in nuclear matter in thermal equilibrium as a function of the density \(\rho\) and the temperature \(T\), has been considered since many years as one of the most important challenges in nuclear physics. This quest has been dubbed “search for the nuclear equation of state (EoS)”.

Only at equilibrium density, \(\rho_0\), the energy per nucleon \(E/A(\rho = \rho_0, T = 0) = -16\) MeV is known by extrapolating the Weisszäcker mass formula to infinite matter. Standard ab initio many body calculations do not allow for a determination of \(E/A(\rho, T)\) at energies well above the saturation density because the detailed form of the interaction among hadrons is not known. Therefore another strategy has been developed. Theory has identified experimental observables in nuclear reaction physics or in astrophysics which are sensitive to \(E/A(\rho, T)\). Unfortunately these observables depend as well on other quantities which are either unknown or little known (like cross sections with resonance in the entrance channel) or difficult to assess theoretically (like the resonance lifetimes in hot and dense matter). It was hoped that comparing many observables for different systems and different energies with the theoretical predictions these unknown or little known quantities can be determined experimentally and that finally the dependence of the observables on \(E(\rho, T)\) can be isolated.

In astrophysics the nuclear EoS plays an important role in binary mergers involving black holes and neutron stars \textsuperscript{1}, in double pulsars \textsuperscript{2}, in the mass-radius relation of neutron stars \textsuperscript{3, 4} and in supernovae explosions \textsuperscript{5}. For a recent review on these topics we refer to \textsuperscript{6}. Unfortunately, as in nuclear reaction physics, there are always other little known processes or properties which have to be understood before the nuclear EoS dependence can be isolated. We discuss here as example of the mass-radius relation of neutron stars. Fig. \textsuperscript{1} shows the neutron star masses in units of the solar mass for different types of binaries. These masses are concentrated at around 1-1.5 solar masses. Fig. \textsuperscript{2} shows a theoretical prediction of the mass-radius relation for neutron stars using different EoS. Since the nature of the interior of neutron stars is not known (in contradiction to what the name suggests) one may suppose that it consists of hadrons or of quarks. But even if it consists of hadrons there are speculations that there is a \(K^-\) or a \(\pi^-\) condensate or that there are hyperons in equilibrium with nuclear resonances. The same is true if the interior consists of quarks. Little known color-flavor locked quark phases may modify the EoS at densities which are reached in the interior of the neutron star. For a detailed discussion of all these phenomena we refer to ref. \textsuperscript{6}. The mass to radius relation for some EoS are shown in fig. \textsuperscript{2}. We see that the observed masses of neutron stars are compatible with almost all quark or hadron based EoS as long as the radius is unknown. Radii, however, are very difficult to measure. Because similar problems appear also for other observables, up to recently the astrophysical observations of neutron stars did not help much to narrow down the uncertainty on the nuclear EoS.

This situation has changed dramatically in the last year with the observation of a neutron star with a mass of two solar masses \textsuperscript{7}. If this observation is finally confirmed the mass/radius prediction of fig.\textsuperscript{2} excludes that the interior of a neutron star is made by quarks, even a soft nuclear EoS, which will be defined below, will be excluded.

In heavy ion reactions three observables have been identified which are - according to theoretical calculations -

\textsuperscript{*} invited speaker
FIG. 1: Measured and estimated masses of neutron stars in radio binary pulsars and in x-ray accreting binaries. Error bars are 1σ. Vertical dotted lines show average masses of each group (1.62 M⊙, 1.34 M⊙ and 1.56 M⊙); dashed vertical lines indicate inverse error weighted average masses (1.48 M⊙, 1.41 M⊙ and 1.34 M⊙). The figure is taken from ref [4].

sensitive to $E/A(p,T)$ at densities larger than $\rho_0$: (i) the strength distribution of giant isoscalar monopole resonances [8, 9], (ii) the in-plane sidewards flow of nucleons in semi-central heavy ion reaction at energies between 100 A MeV and 400 A MeV [10] and (iii) the production of $K^+$ mesons in heavy ion reactions at energies around 1 A GeV [11]. Although theory has predicted these effects qualitatively, a quantitative approach is confronted with two challenges:

(i) The study of monopole vibrations has been very successful, but the variation in density is minute. Therefore, giant monopole resonances are sensitive to the energy which is necessary to change the density of a cold nucleus close to the equilibrium point $\rho_0$. According to theory the vibration frequency depends directly on the force which counteracts to any deviation from the equilibrium and therefore to the potential energy. The careful analysis of the isoscalar monopole strength in non-relativistic [8] and relativistic mean field models has recently converged [9] due to a new parametrization of the relativistic potential. These calculations allow now for the determination of the compressibility $\kappa = \frac{9}{2}\rho_0^2 \frac{d^2E/A(\rho,T)}{d^2\rho}|_{\rho=\rho_0}$ which measures the curvature of $E/A(p,T)$ at the equilibrium point. The values found are around $\kappa = 240$ MeV and therefore close to what has been dubbed “soft EoS”.

(ii) If the overlap zone of projectile and target becomes considerably compressed in semi-central heavy-ion collisions, an in-plane flow is created due to the transverse pressure on the baryons outside of the interaction region with this flow being proportional to the transverse pressure. In order to obtain a noticeable compression, the beam energy has to be large as compared to the Fermi energy of the nucleons inside the nuclei and hence a beam energy of at least 100 A MeV is necessary. Compression goes along with excitation and therefore the compressional energy of excited nuclear matter is encoded in the in-plane flow. It has recently been demonstrated [12] that transport theories do not agree quantitatively yet and therefore former conclusions [13] have to be considered as premature.

(iii) The third method is most promising for the study of nuclear matter at high densities and is subject of this talk. $K^+$ mesons produced far below the NN threshold cannot be created in first-chance collisions between projectile and target nucleons. They do not provide sufficient energy even if one includes the Fermi motion. The effective energy for the production of a $K^+$ meson in the $NN$ center of mass system is 671 MeV as in addition to the mass of the kaon a nucleon has to be converted into a $\Lambda$ to conserve strangeness. Before nucleons can create a $K^+$ at these subthreshold energies, they have to accumulate energy. The most effective way to do this is the conversion of a nucleon into a $\Delta$ and to produce in a subsequent collision a $K^+$ meson via $\Delta N \rightarrow NK^+\Lambda$. Two effects link the yield of produced $K^+$ with the density reached in the collision and the stiffness of the EoS. If less energy is needed to compress matter (i) more energy is available for the $K^+$ production and (ii) the density which can be reached in these reactions will be
higher. Higher density means a smaller mean free path and therefore the Δ will interact more often increasing the probability to produce a $K^+$ and hence, it has a lower chance to decay before it interacts. Consequently the $K^+$ yield depends on the compressional energy. At beam energies around 1 GeV matter becomes highly excited and mesons are formed. Therefore this process tests highly excited hadronic matter. At beam energies $>2$ A GeV first-chance collisions dominate and this sensitivity is lost.

Here we discuss that for the third approach different transport theories have converged. Two independent experimental observables, the ratio of the excitation functions of the $K^+$ production for Au+Au and for C+C [14, 15], and a new observable, the dependence on the number of participants of the $K^+$ yield show that nucleons interact with a potential which corresponds to a compressibility of $\kappa \leq 200$ MeV in infinite matter in thermal equilibrium. This value extracted for hadronic matter at densities around 2.5 times the normal nuclear matter density is very similar to that extracted at normal nuclear matter density. A key point here is to demonstrate that the different implementation of yet unsolved physical questions, like the $N\Delta \rightarrow K^+\Lambda N$ cross section, the $KN$ interaction as well as the life time of the nuclear resonances in the hadronic environment do not affect this conclusion.

In order to determine the energy which is necessary to compress infinite nuclear matter in thermal equilibrium by heavy ion reactions in which no equilibrium is obtained one chooses the following strategy: The transport theory calculates the time evolution of the quantal particles described by Gaussian wave functions. The time evolution is given by a variational principle and the equations one obtains for this choice of the wave function are identical to the classical Hamilton equations where the classical two-body potential is replaced by the expectation value of the real part of the Brückner $G$-matrix. For this potential the potential energy in infinite nuclear matter is calculated. To determine the nuclear EoS we average this (momentum-dependent) two-body potential over the momentum distribution of a given temperature $T$ and add to it the kinetic energy. Expressed as a function of the density we obtain the desired nuclear EoS $E/A(\rho, T)$. Our two-body potential has five parameters which are fixed by the binding energy of infinite nuclear matter at $\rho_0$, the compressibility $\kappa$ and the optical potential which has been measured in pA reactions.

Once the parameters are fixed we use the two-body potential with these parameters in the transport calculation.
There is an infinite number of two-body potentials which give the same EoS because the range of the potential does not play a role in infinite matter. The nuclear surface measured in electron scattering on nuclei fixes the range, however, quite well. The uncertainty which remains is of little relevance here (in contradiction to the calculation of the in-plane flow which is very sensitive to the exact surface properties of the nuclei and hence to the range of the potential).

We employ the Isospin Quantum Molecular Dynamics (IQMD) with momentum dependent forces. All details of the standard version of the program may be found in [16]. The standard version is supplemented for this calculation with all inelastic cross sections which are relevant for the $K^+$ production. For details of these cross sections we refer to [17]. Unless specified differently, the change of the $K^+$ mass due to the kaon-nucleon ($KN$) interaction according to $m_K(\rho) = m^0_K (1 - 0.075\rho)$ is taken into account, in agreement with recent self-consistent calculations of the spectral function of the $K^+$ [18]. The $\Lambda$ potential is $2/3$ of the nucleon potential, assuming that the $s$ quark is inert. The calculations reproduce the experimental data quite well as can be seen in fig. 3 where we compare the experimental and theoretical $K^+$ spectra for different centrality bins for 1.48 AGeV Au+Au. This figure shows as well the influence of the $K^+N$ potential which modifies not only the overall multiplicity of $K^+$ due to the increase of the in medium mass but also the spectral form. In order to minimize the experimental systematical errors and the consequences of theoretical uncertainties it is better to compare ratios of cross sections rather than the absolute values [14]. We have made sure that the standard version of IQMD reproduces the excitation function for Au+Au as well as for C+C quite well [19]. These ratios are quite sensitive to the nuclear potentials because the compression obtained in the Au+Au collisions is considerable (up to $3\rho_0$) and depends on the nuclear EoS whereas in C+C collisions the compression is small and almost independent on the stiffness of the EoS.

Figure 4 shows the comparison of the measured ratio of the $K^+$ multiplicities obtained in Au+Au and C+C reactions [14] together with transport model calculations as a function of the beam energy.

We see clearly that the form of the yield ratio depends on the potential parameters (hard EoS: $\kappa = 380$ MeV, thin lines and solid symbols, soft EoS: $\kappa = 200$ MeV , thick lines and open symbols) in a quite sensible way and that the prediction in the standard version of the simulation (squares) for a soft and a hard EoS potential differ much more than the experimental uncertainties. The calculation of Fuchs et al. [15] given in the same graph, agrees well with our findings.

This observation is, however, not sufficient to determine the potential parameters uniquely because in these transport theories several not precisely known processes are encoded. Therefore, it is necessary to verify that these uncertainties do not render this conclusion premature. Figure 3 (top), shows as well the influence of the unknown $N\Delta \rightarrow K^+\Lambda N$ cross section on this ratio. We confront the standard IQMD option (with cross sections for $\Delta N$ interactions from
FIG. 4: Comparison of the measured excitation function of the ratio of the $K^+$ multiplicities per mass number $A$ obtained in Au+Au and in C+C reactions (Ref. [14]) with various calculations. The use of a hard EoS is denoted by thin (blue) lines, a soft EoS by thick (red) lines. The calculated energies are given by the symbols, the lines are drawn to guide the eye. On top, two different versions of the $N\Delta \rightarrow K^+\Lambda N$ cross sections are used. One is based on isospin arguments [20], the other is determined by a relativistic tree level calculation [21]. The calculation by Fuchs [15] are shown as dotted lines. Middle: IQMD calculations with and without $KN$ potential are compared. Bottom: The influence of different options for the life time of $\Delta$ in matter is demonstrated.

Tsushima et al. [17] with another option, $\sigma(N\Delta) = 3/4\sigma(NN)$ [24], which is based on isospin arguments and has been frequently employed. Both cross sections differ by up to a factor of ten and change significantly the absolute yield of $K^+$ in heavy ion reactions but do not change the shape of the ratio.

The middle part demonstrates the influence of the kaon-nucleon potential which is not precisely known at the densities obtained in this reaction. The uncertainties due to the $\Delta$ life time are discussed in the bottom part. Both calculations represent the two extreme values for this lifetime [17] which is important because the disintegration of the $\Delta$ resonance competes with the $K^+$ production.

Thus we see that these uncertainties do not influence the conclusion that the excitation function of the ratio is quite different for a soft EoS potential as compared to a hard one and that the data of the KaoS collaboration are only compatible with the soft EoS potential. The only possibility to change this conclusions is the assumption that the cross sections are explicitly density dependent in a way that the increasing density is compensated by a decreasing cross section. It would have a strong influence on other observables which are presently well predicted by the IQMD calculations.

The conclusion that nuclear matter is best described by a soft EoS, is supported by another variable, the dependence of the $K^+$ yield on the number of participating nucleons $A_{\text{part}}$. The prediction of the IQMD simulations in the standard version for this observable is shown in Fig. 5. The top of the figure shows the kaon yield $M_{K^+}/A_{\text{part}}$ for Au+Au collisions at 1.5 $A$ GeV as a function of the participant number $A_{\text{part}}$ for a soft EoS using different options: standard version (soft, $KN$), calculations without kaon-nucleon interaction (soft, no $KN$) and with the isospin based $N\Delta \rightarrow NAK^+$ cross section (soft, $KN$, $\sigma^*$). These calculations are confronted with a standard calculation using the hard EoS potential. The scaling of the kaon yield with the participant number can be parameterized by $M_{K^+} = A_{\text{part}}^{\alpha}$. All calculations with a soft EoS show a rather similar value of $\alpha$ - although the yields are very different - while the calculation using a hard equation shows a much smaller value. Therefore we can conclude that also the slope value $\alpha$ is a rather robust observable.
The bottom of Fig. 5 shows that $\alpha$ depends smoothly on the compressibility $\kappa$ of the EoS. Whether we include the momentum dependence of the nucleon nucleon interaction (with mdi) or not (without mdi) does not change the value of $\alpha$ as long as the compressibility is not changed - in stark contrast to the in-plane flow. Again, the measured centrality dependence for Au+Au at 1.5 A GeV from the KaoS collaboration \cite{22}, $\alpha = 1.34 \pm 0.16$, is only compatible with a soft EoS potential. That the dependence of the $K^+$ yield on the participants number is a clear signal for the EoS also at other beam energies as can be seen from fig. right which displays the excitation function of the fit exponent $\alpha$.

In conclusion, we have shown that the two experimental observables which are most sensitive to the potential parameters of the nucleon-nucleon interaction are only compatible with those parameters which lead in nuclear matter to a soft hadronic EoS. This conclusion is robust. Uncertainties of the input in these calculations, like the $KN$ potential at high densities, the lifetime of the $\Delta$ in matter and the $\Delta N \rightarrow NK^+\Lambda$ cross section do not influence this conclusion. The potential parameter $\kappa$ is even smaller than that extracted from the giant monopole vibrations. Thus the compressibility $\kappa$ of hadronic matter of $\leq 200$ MeV is close to the lower bound of the interval which has been discussed in the past. How this result is compatible with the observation of a neutron star with twice the solar mass is presently studied.

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Recent astrophysical and accelerator based results on the Hadronic Equation of State

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In astrophysics as well as in hadron physics progress has recently been made on the determination of the hadronic equation of state (EOS) of compressed matter. The results are contradictory, however. Simulations of heavy ion reactions are now sufficiently robust to predict the stiffness of the (EOS) from (i) the energy dependence of the ratio of $K^+$ from Au+Au and C+C collisions and (ii) the centrality dependence of the $K^+$ multiplicities. The data are best described with a compressibility coefficient at normal nuclear matter density $\kappa$ around 200 MeV, a value which is usually called “soft”. The recent observation of a neutron star with a mass of twice the solar mass is only compatible with theoretical predictions if the EOS is stiff. We review the present situation.

How much energy is needed to compress nuclear matter? The answer to this question, the determination of $E/A(\rho, T)$, the energy/nucleon in nuclear matter in thermal equilibrium as a function of the density $\rho$ and the temperature $T$, has been considered since many years as one of the most important challenges in nuclear physics. This quest has been dubbed “search for the nuclear equation of state (EOS)’’.

Only at equilibrium density, $\rho_0$, the energy per nucleon $E/A(\rho = \rho_0, T = 0) = -16$ MeV is known by extrapolating the Weizsäcker mass formula to infinite matter. Standard ab initio many body calculations do not allow for a determination of $E/A(\rho, T)$ at energies well above the saturation density because the low density many body expansion schema (Brückner G-matrix) breaks down and therefore the number of contributing terms is exploding. Therefore in nuclear reaction physics another strategy has been developed. Theory has identified experimental observables in nuclear reaction physics or in astrophysics which are sensitive to $E/A(\rho, T)$. Unfortunately these observables depend as well on other quantities which are either unknown or little known (like cross sections with resonance in the entrance channel) or difficult to assess theoretically (like the resonance lifetimes in hot and dense matter). It was hoped that comparing many observables for different systems and different energies with the theoretical predictions these unknown or little known quantities can be determined experimentally and that finally the dependence of the observables on $E/A(\rho, T)$ can be isolated.

In astrophysics the nuclear EoS plays an important role in binary mergers involving black holes and neutron stars\textsuperscript{1}, in double pulsars\textsuperscript{2}, in the mass-radius relation of neutron stars\textsuperscript{3,4} and in supernovae explosions\textsuperscript{5}. For a recent review on these topics we refer to\textsuperscript{6}. Unfortunately, as in nuclear reaction physics, there are always other little known processes or properties which have to be understood before the nuclear EoS dependence can be isolated. We discuss here as example of the mass-radius relation of neutron stars. Fig.\textsuperscript{1} shows the neutron star masses in units of the solar mass for different types of binaries. These masses are concentrated at around 1-1.5 solar masses. Fig.\textsuperscript{2} shows a theoretical prediction of the mass-radius relation for neutron stars using different EoS. Since the nature of the interior of neutron stars is not known (in contradiction to what the name suggests) one may suppose that it consists of hadrons or of quarks. But even if it consists of hadrons there are speculations that there is a $K^-$ or a $\pi^-$ condensate or that there are hyperons in equilibrium with nuclear resonances. The same is true if the interior consists of quarks. Little known color-flavor locked quark phases may modify the EoS at densities which are reached in the interior of the neutron star. For a detailed discussion of all these phenomena we refer to ref.\textsuperscript{6}. We see that the observed masses of neutron stars are compatible with almost all quark or hadron based EoS as long as the radius is unknown. Radii, however, are very difficult to measure. Because similar problems appear also for other observables, up to recently the astrophysical observations of neutron stars did not help much to narrow down the uncertainty on the nuclear EoS.

This situation has changed dramatically in the last year with the observation of a neutron star with a mass of two solar masses\textsuperscript{7}. If this observation is finally confirmed the mass-radius prediction of fig.\textsuperscript{2} excludes that the interior of a neutron star is made by quarks\textsuperscript{4}, even a soft nuclear EoS, which will be defined below, will be excluded. This is confirmed by the calculation of Maieron\textsuperscript{8} which uses a MIT bag model or a color dielectric models EoS.
FIG. 1: Measured and estimated masses of neutron stars in radio binary pulsars and in x-ray accreting binaries. Error bars are 1σ. Vertical dotted lines show average masses of each group (1.62 M⊙, 1.34 M⊙ and 1.56 M⊙); dashed vertical lines indicate inverse error weighted average masses (1.48 M⊙, 1.41 M⊙ and 1.34 M⊙). The figure is taken from ref [4] to describe the quark phase. Baldo [9] argue that this conclusion may be premature because it depends too much on the equation of state of the quark phase. If one replaces the MIT bag model equation of state by that of the Nambu - Jona-Lasinio (NJL) Lagrangian under certain conditions (no color conducting phase) larger masses may be obtained. The standard NJL Lagrangian lacks, however, repulsion and in view of the momentum cut-off, necessary to regularize the loop integrals, and the coupling constants in the diquark sector, which are not uniquely determined by the Fierz transformation, quantitative prediction at high quark densities are difficult in this approach even if qualitative agreement with pQCD calculation can be found [10].

Simulations of heavy ion reactions have shown that there are three possible observables which are sensitive to $E/A(\rho, T)$ at densities larger than $\rho_0$: (i) the strength distribution of giant isoscalar monopole resonances [11, 12], (ii) the in-plane sideways flow of nucleons in semi-central heavy ion reactions at energies between 100 A MeV and 400 A MeV [13] and (iii) the production of $K^+$ mesons in heavy ion reactions at energies around 1 A GeV [14]. For the present status of these approaches we refer to [15].

Monopole resonances test the nuclear EoS at densities only slightly larger than the normal nuclear matter density. Therefore they are of little help if one compares the EoS determined from astrophysics with that extracted from nuclear reaction physics. For the in-plane flow the conclusions are not conclusive yet. This is due to the difficulties to determine the EoS in heavy ion collisions. An EoS is defined in a thermally equilibrated system but in heavy ion collisions equilibrium is not obtained as the momentum distribution of hadrons shows. In addition, nuclei are finite size systems where the surface plays an important role. This can easily be seen inspecting the Weizsäcker mass formula which gives for infinite matter almost twice the binding energy/per nucleon as for finite nuclei. Therefore complicated non-equilibrium transport theories have to be employed and the conclusion on the nuclear EoS can only be indirect, in determining the EoS for those potentials which give best agreement with the heavy ion results.

In order to determine the energy which is necessary to compress infinite nuclear matter in thermal equilibrium by heavy ion reactions in which no equilibrium is obtained one chooses the following strategy: The transport theory calculates the time evolution of the quantal particles described by Gaussian wave functions. The time evolution is given by a variational principle and the equations one obtains for this choice of the wave function are identical to the classical Hamilton equations where the classical two-body potential is replaced by the expectation value of a Skyrme potential. The Skyrme potential is a simple approximation to the real part of the Brückner $G$-matrix which is too complicated for performing simulations of heavy ion collisions. For this potential the potential energy in infinite nuclear matter is calculated. To determine the nuclear EoS we average this (momentum-dependent) two-body potential over the momentum distribution of a given temperature $T$ and add it to the kinetic energy. Expressed as a function of the density we obtain the desired nuclear EoS $E/A(\rho, T)$. The potential which we use has five parameters.
FIG. 2: Mass-radius diagram for neutron stars. Black (green) curves are for normal matter (SQM) EoS [for definitions of the labels, see \[4\]]. Regions excluded by general relativity (GR), causality and rotation constraints are indicated. Contours of radiation radii $R_\infty$ are given by the orange curves. The figure is from \[4\].

Four of them are fixed by the binding energy per nucleon in infinite nuclear matter at $\rho_0$ and the optical potential which has been measured in pA reactions \[10\]. The only parameter which has been not determined by experiments yet is the compressibility $\kappa$ at $\rho_0$. For $\kappa < 250$ MeV one calls the EoS soft, whereas an EoS is called hard for $\kappa > 350$ MeV. Once the parameters are fixed we use the two-body potential with these parameters in the transport calculation. There is an infinite number of two-body potentials which give the same EoS because the range of the potential does not play a role in infinite matter. The nuclear surface measured in electron scattering on nuclei fixes the range, however, quite well.

The different transport theories give quite comparable results for the bulk part but it is difficult to model the surface. (In these simulations there is no surface in the strict sense. Each nucleon contributes to the density by its Gaussian wave function and the positions of the hadrons in the course of the reaction determine the surface as well as the density gradients.)

The in-plane flow is caused by the density gradient and hence the numerical value depends on how good the surface of the nucleus can be modeled during the reaction. Already small density fluctuations, which are difficult to control, change the value of the in-plane flow considerably. Therefore the second approach, the determination if the EoS by measuring the in-plane flow, has not produced conclusive results yet \[17\].

The third approach, to measure the EoS by means of the $K^+$ yield, depends on bulk properties of matter and surface fluctuations have no influence. Here the different transport theories have converged. This was possible due to a special workshop at the ECT* in Trento/Italy where the authors of the different simulation codes have discussed their approaches in detail and have unified most of the input quantities. The results of this common effort have been published in \[18\]. As an example we display here the $K^+ p_t$ spectra at midrapidity obtained in the different transport theories at different energies. Because with each $K^+N$ rescattering collision the slope of the $K^+$ spectra changes the slope of the $p_t$ spectra encodes not only the $K^+$ momentum distribution at the time point of production but also the distribution of the number of rescatterings. It is therefore all but trivial. Without the $KN$ potential the slopes are almost identical and even the absolute yield, which depends on a correct modeling of the Fermi motion of the
Due to this progress the simulation programs can now be used to extract up to now theoretically inaccessible information like the hadronic EOS \cite{15}. Three independent experimental observables, the ratio of the excitation functions of the $K^+$ production for Au+Au and for C+C \cite{19,20}, the dependence of the $K^+$ yield on the number of participants and the excitation function of this dependence can be simultaneously reproduced if in these transport theories the nucleons interact with potential which yield in infinite matter in equilibrium a compressibility of of the EoS of $\kappa \approx 200$ MeV. Large compressibility moduli yield results which disagree with all three observables. This value of $\kappa$ extracted from the $K^+$ production which is sensitive to nuclear matter around 2.5$\rho_0$ is very similar to that extracted by the study of monopole vibrations at $\rho_0$ \cite{11,12}.

It is not sufficient to determine the compressibility modulus. One has to demonstrate as well that its numerical value is robust, i.e. that the different implementations of yet unsolved physical questions, like the $N\Delta \rightarrow K^+\Lambda N$ cross section, the $KN$ interaction as well as the life time of the nuclear resonances in the hadronic environment do not affect its value.

We employ the Isospin Quantum Molecular Dynamics (IQMD) with momentum dependent forces. All details of the standard version of the program may be found in \cite{16}. In addition we have implemented for this calculation all cross sections which yield to the production of $K^+$ as well as the elastic and the charge exchange $KN \rightarrow KN$ reactions. The parametrization of the cross section may be found in \cite{16}. In the standard version the $K^+N$ potential leads to an increase of the $K^+$ mass in matter, $m^K(\rho) = m^K_0 (1 - 0.075 \rho / \rho_0)$, in agreement with recent self-consistent calculations of the spectral function of the $K^+$ \cite{21}. The $\Lambda$ potential is $2/3$ of the nucleon potential, assuming that the $s$ quark is inert. The calculations reproduce the experimental data quite well as can be seen in fig. 4 where we compare the experimental and theoretical $K^+$ spectra for different centrality bins and for 1.48 AGeV Au+Au. This figure shows as well the influence of the $K^+N$ potential which modifies not only the overall multiplicity of $K^+$ due to the increase of the in medium mass but also the spectral form confirming the complexity of the transverse momentum spectrum.

In order to minimize the experimental systematical errors and the consequences of theoretical uncertainties the KaoS collaboration has proposed to study not directly the excitation function of the $K^+$ yield but that of the yield ratio of heavy to light systems \cite{15}. Calculations have shown that ratios are much less sensitive to little known input parameters because these affect both systems in a rather similar way. We have shown in fig. 4 that the absolute yields are well reproduced in our simulations. Therefore we can use this ratio directly for a quantitative comparison with data. The ratio of the $K^+$ yields obtained in C+C and Au+Au collisions is quite sensitive to the EoS because in Au+Au collisions densities up to 3 $\rho_0$ (depending on the EoS) are reached whereas in C+C collisions compression is practically absent due to less stopping.

Figure 3 shows the comparison of the measured ratio of the $K^+$ multiplicities obtained in Au+Au and C+C reactions \cite{19}, together with transport model calculations as a function of the beam energy \cite{15}. We see, first of all in the top
row, that the excitation function of the yield ratio depends on the potential parameters (hard EoS: $\kappa = 380$ MeV, thin lines and solid symbols, soft EoS: $\kappa = 200$ MeV, thick lines and open symbols) in a quite sensible way and - even more essential - that the prediction in the standard version of the simulation (squares) for a soft and a hard EoS potential differ much more than the experimental uncertainties. The calculation of Fuchs et al. \[20\] given in the same graph, agrees well with our findings.

This observation is, as said, not sufficient to determine the potential parameters uniquely because in these transport theories several not precisely known processes are encoded. For these processes either no reliable theoretical prediction has been advanced or the different approaches yield different results for the same observable. Therefore, it is necessary to verify that these uncertainties do not render our conclusion premature. There are 3 identified uncertainties: the $\sigma_{N\Delta \rightarrow K^+\Lambda N}$ cross section, the density dependence of the $K^+N$ potential and the lifetime of $\Delta$ in matter if produced in a collisions with a sharp energy of two scattering partners. We discuss now how these uncertainties influence our results:

Figure 5, top, shows as well the influence of the unknown $N\Delta \rightarrow K^+\Lambda N$ cross section on this ratio. We confront the standard IQMD option (with cross sections for $\Delta N$ interactions from Tsushima et al. \[18\]) with another option, $\sigma(N\Delta) = 3/4\sigma(NN)$ \[23\], which is based on isospin arguments and has been frequently employed. Both cross sections differ by up to a factor of ten and change significantly the absolute yield of $K^+$ in heavy ion reactions but do not change the shape of the ratio.

The middle part demonstrates the influence of the kaon-nucleon potential which is not precisely known at the densities obtained in this reaction. The uncertainties due to the $\Delta$ life time are discussed in the bottom part. Both calculations represent the two extreme values for this lifetime \[18\] which is important because the disintegration of the $\Delta$ resonance competes with the $K^+$ production.

Thus we see that these uncertainties do not influence the conclusion that the excitation function of the ratio is quite different for a soft EoS potential as compared to a hard one and that the data of the KaoS collaboration are only compatible with the soft EoS. The only possibility to change this conclusions is the assumption that the cross sections are explicitly density dependent in a way that the increasing density is compensated by a decreasing cross section. It would have a strong influence on other observables which are presently well predicted by the IQMD calculations.

The compression which can be obtained in heavy ion reactions depends on the impact parameter or, equivalently, on the experimentally accessible number of participating nucleons. Therefore by varying the impact parameter we can test the EoS at different densities. This dependence should be different for different EoS. This is indeed the case for the result of the simulations as seen in Fig. 6 top, where we display the kaon yield $M_{K^+}/A_{\text{part}}$ for Au+Au collisions at 1.5 $A$ GeV as a function of the participant number $A_{\text{part}}$ and for different options: standard version (soft, $KN$),
FIG. 5: Comparison of the measured excitation function of the ratio of the $K^+$ multiplicities per mass number $A$ obtained in $\text{Au}+\text{Au}$ and in $\text{C}+\text{C}$ reactions (Ref. [19]) with various calculations. The use of a hard EoS is denoted by thin (blue) lines, a soft EoS by thick (red) lines. The calculated energies are given by the symbols, the lines are drawn to guide the eye. On top, two different versions of the $N\Delta \rightarrow K^+\Lambda N$ cross sections are used. One is based on isospin arguments [23], the other is determined by a relativistic tree level calculation [24]. The calculation by Fuchs [20] are shown as dotted lines. Middle: IQMD calculations with and without $KN$ potential are compared. Bottom: The influence of different options for the life time of $\Delta$ in matter is demonstrated.

calculations without kaon-nucleon interaction (soft, no $KN$) and with the isospin based $N\Delta \rightarrow N\Lambda K^+$ cross section (soft, $KN$, $\sigma^*$). A variation of the KN potential as well as of the $K^+$ production cross section change the dependence of the $K^+$ yield on the number of participants, which can be parametrized by the form $M_{K^+} = A_{\text{part}}^{\alpha}$, only little. On the contrary, if we apply a hard EoS, the slope value $\alpha$ changes considerable and is outside of the values which are compatible with the experimental results, as shown in the middle part of the figure. In this figure we display as well the insensitivity of our result to the momentum dependence of the nucleon nucleon interaction. As long as the compressibility is not changed the results of our calculations are very similar independent on whether we have a static or a momentum dependent NN potential. Hence the dependence of the $K^+$ yield on the number of participants is also a robust variable for the determination of the EoS which supports our earlier conclusion that the EoS is soft. Another confirmation that only a soft EoS describes the experimental data is the beam energy dependence of the fitted exponent $\alpha$ which is displayed in the right part of fig. [26] The data, which follow the curve for a soft equation of state, will soon be published [26].

In conclusion, we have shown that earthbound experiments have now reached a precision which allows to determine the hadronic EoS. The experimental results for the three observables which are most sensitive to the hadronic EOS are only compatible with theory if the hadronic EoS is soft. This conclusion is robust. Little known input quantities do not influence this conclusion. The observation of a neutron star with twice the solar mass seems to contradict this conclusion. It points toward a hard hadronic EoS. Both results are quite new and one has not to forget that we are comparing non equilibrium heavy ion reactions where about the same number of protons and neutrons are present and where mesons and baryon resonances are produced with cold neutron matter in equilibrium. In addition this contradiction depends also on the prediction that the observed star mass excludes the formation of quark matter in the interior, a consequence of the suggested EoS of quark matter which is still rather speculative.
FIG. 6: Dependence of the $K^+$ scaling on the nuclear EoS. We present this dependence in form of $M_{K^+} = A_{\text{part}}^\alpha$. On the top the dependence of $M_{K^+}/A_{\text{part}}$ as a function of $A_{\text{part}}$ is shown for different options: a “hard” EoS with $KN$ potential (solid line), the other three lines show a “soft” EoS, without $KN$ potential and $\sigma(N\Delta)$ from Tsushima [24] (dotted line), with $KN$ potential and the same parametrization of the cross section (dashed line) and with $KN$ potential and $\sigma(N\Delta) = 3/4\sigma(NN)$. On the bottom the fit exponent $\alpha$ is shown as a function of the compressibility for calculations with momentum-dependent interactions (mdi) and for static interactions (dashed line) [16]. On the right hand side we compare the energy dependence of the fit exponent $\alpha$ for the two EoS.

To solve this contradiction is certainly a big challenge for both communities in the near future.

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Recent astrophysical and accelerator based results on the Hadronic Equation of State

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In astrophysics as well as in hadron physics progress has recently been made on the determination of the hadronic equation of state (EOS) of compressed matter. The results are contradictory, however. Simulations of heavy ion reactions are now sufficiently robust to predict the stiffness of the (EOS) from (i) the energy dependence of the ratio of $K^+$ from Au+Au and C+C collisions and (ii) the centrality dependence of the $K^+$ multiplicities. The data are best described with a compressibility coefficient at normal nuclear matter density $\kappa$ around 200 MeV, a value which is usually called “soft.” The recent observation of a neutron star with a mass of twice the solar mass is only compatible with theoretical predictions if the EOS is stiff. We review the present situation.

How much energy is needed to compress nuclear matter? The answer to this question, the determination of $E/A(\rho, T)$, the energy/nucleon in nuclear matter in thermal equilibrium as a function of the density $\rho$ and the temperature $T$, has been considered since many years as one of the most important challenges in nuclear physics. This quest has been dubbed “search for the nuclear equation of state (EoS)”.

Only at equilibrium density, $\rho_0$, the energy per nucleon $E/A(\rho = \rho_0, T = 0) = -16$ MeV is known by extrapolating the Weizsäcker mass formula to infinite matter. Standard ab initio many body calculations do not allow for a determination of $E/A(\rho, T)$ at energies well above the saturation density because the low density many body expansion schema (Brückner G-matrix) breaks down and therefore the number of contributing terms is exploding. Therefore in nuclear reaction physics another strategy has been developed. Theory has identified experimental observables in nuclear reaction physics or in astrophysics which are sensitive to $E/A(\rho, T)$. Unfortunately these observables depend as well on other quantities which are either unknown or little known (like cross sections with resonance in the entrance channel) or difficult to assess theoretically (like the resonance lifetimes in hot and dense matter). It was hoped that comparing many observables for different systems and different energies with the theoretical predictions these unknown or little known quantities can be determined experimentally and that finally the dependence of the observables on $E/A(\rho, T)$ can be isolated.

In astrophysics the nuclear EOS plays an important role in binary mergers involving black holes and neutron stars, in double pulsars, in the mass-radius relation of neutron stars and in supernovae explosions. For a recent review on these topics we refer to \textsuperscript{[6]}. Unfortunately, as in nuclear reaction physics, there are always other little known processes or properties which have to be understood before the nuclear EOS dependence can be isolated. We discuss here as example of the mass-radius relation of neutron stars. Fig. \textsuperscript{1} shows the neutron star masses in units of the solar mass for different types of binaries. These masses are concentrated at around 1-1.5 solar masses. Fig. \textsuperscript{2} shows a theoretical prediction of the mass-radius relation for neutron stars using different EOS. Since the nature of the interior of neutron stars is not known (in contradiction to what the name suggests) one may suppose that it consists of hadrons or of quarks. But even if it consists of hadrons there are speculations that there is a $K^-$ or a $\pi^-$ condensate or that there are hyperons in equilibrium with nuclear resonances. The same is true if the interior consists of quarks. Little known color-flavor locked quark phases may modify the EOS at densities which are reached in the interior of the neutron star. For a detailed discussion of all these phenomena we refer to ref. \textsuperscript{[6]}. We see that the observed masses of neutron stars are compatible with almost all quark or hadron based EOS as long as the radius is unknown. Radii, however, are very difficult to measure. Because similar problems appear also for other observables, up to recently the astrophysical observations of neutron stars did not help much to narrow down the uncertainty on the nuclear EOS.

This situation has changed dramatically in the last year with the observation of a neutron star with a mass of two solar masses \textsuperscript{[7]}. If this observation is finally confirmed the mass/radius prediction of fig.\textsuperscript{2} excludes that the interior of a neutron star is made by quarks \textsuperscript{[4]}, even a soft nuclear EOS, which will be defined below, will be excluded. This is confirmed by the calculation of Mäieron \textsuperscript{[8]} which uses a MIT bag model or a color dielectric models EOS.

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FIG. 1: Measured and estimated masses of neutron stars in radio binary pulsars and in x-ray accreting binaries. Error bars are 1σ. Vertical dotted lines show average masses of each group (1.62 M⊙, 1.34 M⊙ and 1.56 M⊙); dashed vertical lines indicate inverse error weighted average masses (1.48 M⊙, 1.41 M⊙ and 1.34 M⊙). The figure is taken from ref [4] to describe the quark phase. Baldo [9] argue that this conclusion may be premature because it depends too much on the equation of state of the quark phase. If one replaces the MIT bag model equation of state by that of the Nambu - Jona-Lasinio (NJL) Lagrangian under certain conditions (no color conducting phase) larger masses may be obtained. The standard NJL Lagrangian lacks, however, repulsion and in view of the momentum cut-off, necessary to regularize the loop integrals, and the coupling constants in the diquark sector, which are not uniquely determined by the Fierz transformation, quantitative prediction at high quark densities are difficult in this approach even if qualitative agreement with pQCD calculation can be found [10].

Simulations of heavy ion reactions have shown that there are three possible observables which are sensitive to E/A(ρ, T) at densities larger than ρ0: (i) the strength distribution of giant isoscalar monopole resonances [11, 12], (ii) the in-plane sideways flow of nucleons in semi-central heavy ion reactions at energies between 100 A MeV and 400 A MeV [13] and (iii) the production of K+ mesons in heavy ion reactions at energies around 1 A GeV [14]. For the present status of these approaches we refer to [15].

Monopole resonances test the nuclear EoS at densities only slightly larger than the normal nuclear matter density. Therefore they are of little help if one compares the EoS determined from astrophysics with that extracted from nuclear reaction physics. For the in-plane flow the conclusions are not conclusive yet. This is due to the difficulties to determine the EoS in heavy ion collisions. An EoS is defined in a thermally equilibrated system but in heavy ion collisions equilibrium is not obtained as the momentum distribution of hadrons shows. In addition, nuclei are finite size systems where the surface plays an important role. This can easily be seen inspecting the Weizsäcker mass formula which gives for infinite matter almost twice the binding energy/per nucleon as for finite nuclei. Therefore complicated non-equilibrium transport theories have to be employed and the conclusion on the nuclear EoS can only be indirect, in determining the EoS for those potentials which give best agreement with the heavy ion results.

In order to determine the energy which is necessary to compress infinite nuclear matter in thermal equilibrium by heavy ion reactions in which no equilibrium is obtained one chooses the following strategy: The transport theory calculates the time evolution of the quantal particles described by Gaussian wave functions. The time evolution is given by a variational principle and the equations one obtains for this choice of the wave function are identical to the classical Hamilton equations where the classical two-body potential is replaced by the expectation value of a Skyrme potential. The Skyrme potential is a simple approximation to the real part of the Brückner G-matrix which is too complicated for performing simulations of heavy ion collisions. For this potential the potential energy in infinite nuclear matter is calculated. To determine the nuclear EoS we average this (momentum-dependent) two-body potential over the momentum distribution of a given temperature T and add to it the kinetic energy. Expressed as a function of the density we obtain the desired nuclear EoS E/A(ρ, T). The potential which we use has five parameters.
FIG. 2: Mass-radius diagram for neutron stars. Black (green) curves are for normal matter (SQM) EoS [for definitions of the labels, see [4]]. Regions excluded by general relativity (GR), causality and rotation constraints are indicated. Contours of radiation radii $R_\infty$ are given by the orange curves. The figure is from [4].

Four of them are fixed by the binding energy per nucleon in infinite nuclear matter at $\rho_0$ and the optical potential which has been measured in pA reactions [16]. The only parameter which has been not determined by experiments yet is the compressibility $\kappa$ at $\rho_0$. For $\kappa < 250$ MeV one calls the EoS soft, whereas an EoS is called hard for $\kappa > 350$ MeV. Once the parameters are fixed we use the two-body potential with these parameters in the transport calculation. There is an infinite number of two-body potentials which give the same EoS because the range of the potential does not play a role in infinite matter. The nuclear surface measured in electron scattering on nuclei fixes the range, however, quite well.

The different transport theories give quite comparable results for the bulk part but it is difficult to model the surface. (In these simulations there is no surface in the strict sense. Each nucleon contributes to the density by its Gaussian wave function and the positions of the hadrons in the course of the reaction determine the surface as well as the density gradients.)

The in-plane flow is caused by the density gradient and hence the numerical value depends on how good the surface of the nucleus can be modeled during the reaction. Already small density fluctuations, which are difficult to control, change the value of the in-plane flow considerably. Therefore the second approach, the determination if the EoS by measuring the in-plane flow, has not produced conclusive results yet [17].

The third approach, to measure the EoS by means of the $K^+$ yield, depends on bulk properties of matter and surface fluctuations have no influence. Here the different transport theories have converged. This was possible due to a special workshop at the ECT* in Trento/Italy where the authors of the different simulation codes have discussed their approaches in detail and have unified most of the input quantities. The results of this common effort have been published in [18]. As an example we display here the $K^+ p_t$ spectra at midrapidity obtained in the different transport theories at different energies. Because with each $K^+ N$ rescattering collision the slope of the $K^+$ spectra changes the slope of the $p_t$ spectra encodes not only the $K^+$ momentum distribution at the time point of production but also the distribution of the number of rescatterings. It is therefore all but trivial. Without the $KN$ potential the slopes are almost identical and even the absolute yield, which depends on a correct modeling of the Fermi motion of the
nucleons, is very similar. If we include the $KN$ interaction which is not identical in the different approaches (see [18]) we still observe a very similar slope for most of the programs.

Due to this progress the simulation programs can now be used to extract up to now theoretically inaccessible information like the hadronic EOS [16]. Three independent experimental observables, the ratio of the excitation functions of the $K^+$ production for Au+Au and for C+C [19, 20], the dependence of the $K^+$ yield on the number of participants and the excitation function of this dependence can be simultaneously reproduced if in these transport theories the nucleons interact with potential which yield in infinite matter in equilibrium a compressibility of $\approx 200$ MeV. Large compressibility moduli yield results which disagree with all three observables.

This value of $\kappa$ extracted from the $K^+$ production which is sensitive to nuclear matter around $2.5 \rho_0$ is very similar to that extracted by the study of monopole vibrations at $\rho_0$ [11, 12].

It is not sufficient to determine the compressibility modulus. One has to demonstrate as well that its numerical value is robust, i.e. that the different implementations of yet unsolved physical questions, like the $N\Delta \rightarrow K^+\Lambda N$ cross section, the $KN$ interaction as well as the life time of the nuclear resonances in the hadronic environment do not affect its value.

We employ the Isospin Quantum Molecular Dynamics (IQMD) with momentum dependent forces. All details of the standard version of the program may be found in [16]. In addition we have implemented for this calculation all cross sections which yield to the production of $K^+$ as well as the elastic and the charge exchange $KN \rightarrow KN$ reactions. The parametrization of the cross section may be found in [18]. In the standard version the $K^+N$ potential leads to an increase of the $K^+$ mass in matter, $m_0^{K^+}(\rho) = m_0^K (1 - 0.075 \rho_0)$, in agreement with recent self-consistent calculations of the spectral function of the $K^+$ [21]. The $\Lambda$ potential is $2/3$ of the nucleon potential, assuming that the $s$ quark is inert. The calculations reproduce the experimental data quite well as can be seen in fig. 4 where we compare the experimental and theoretical $K^+$ spectra for different centrality bins and for 1.48 AGeV Au+Au. This figure shows as well the influence of the $K^+N$ potential which modifies not only the overall multiplicity of $K^+$ due to the increase of the in medium mass but also the spectral form confirming the complexity of the transverse momentum spectrum.

In order to minimize the experimental systematical errors and the consequences of theoretical uncertainties the KaoS collaboration has proposed to study not directly the excitation function of the $K^+$ yield but that of the yield ratio of heavy to light systems [18]. Calculations have shown that ratios are much less sensitive to little known input parameters because these affect both systems in a rather similar way. We have shown in fig. 4 that the absolute yields are well reproduced in our simulations. Therefore we can use this ratio directly for a quantitative comparison with data. The ratio of the $K^+$ yields obtained in C+C and Au+Au collisions is quite sensitive to the EoS because in Au+Au collisions densities up to $3 \rho_0$ (depending on the EoS) are reached whereas in C+C collisions compression is practically absent due to less stopping.

Figure 5 shows the comparison of the measured ratio of the $K^+$ multiplicities obtained in Au+Au and C+C reactions [19] together with transport model calculations as a function of the beam energy. We see, first of all in the top row,
that the excitation function of the yield ratio depends on the potential parameters (hard EoS: $\kappa = 380$ MeV, thin lines and solid symbols, soft EoS: $\kappa = 200$ MeV, thick lines and open symbols) in a quite sensible way and - even more essential - that the prediction in the standard version of the simulation (squares) for a soft and a hard EoS potential differ much more than the experimental uncertainties. The calculation of Fuchs et al. [20] given in the same graph, agrees well with our findings.

This observation is, as said, not sufficient to determine the potential parameters uniquely because in these transport theories several not precisely known processes are encoded. For these processes either no reliable theoretical prediction has been advanced or the different approaches yield different results for the same observable. Therefore, it is necessary to verify that these uncertainties do not render our conclusion premature. There are 3 identified uncertainties: the $\sigma_{N\Delta \to K^+}$ cross section, the density dependence of the $K^+N$ potential and the lifetime of $\Delta$ in matter if produced in a collision with a sharp energy of two scattering partners. We discuss now how these uncertainties influence our results:

Figure 5, top, shows as well the influence of the unknown $N\Delta \to K^+\Lambda N$ cross section on this ratio. We confront the standard IQMD option (with cross sections for $\Delta N$ interactions from Tsushima et al. [18]) with another option, $\sigma(N\Delta) = 3/4\sigma(NN)$ [23], which is based on isospin arguments and has been frequently employed. Both cross sections differ by up to a factor of ten and change significantly the absolute yield of $K^+$ in heavy ion reactions but do not change the shape of the ratio.

The middle part demonstrates the influence of the kaon-nucleon potential which is not precisely known at the densities obtained in this reaction. The uncertainties due to the $\Delta$ life time are discussed in the bottom part. Both calculations represent the two extreme values for this lifetime [18] which is important because the disintegration of the $\Delta$ resonance competes with the $K^+$ production.

Thus we see that these uncertainties do not influence the conclusion that the excitation function of the ratio is quite different for a soft EoS potential as compared to a hard one and that the data of the KaoS collaboration are only compatible with the soft EoS. The only possibility to change this conclusion is the assumption that the cross sections are explicitly density dependent in a way that the increasing density is compensated by a decreasing cross section. It would have a strong influence on other observables which are presently well predicted by the IQMD calculations.

The compression which can be obtained in heavy ion reactions depends on the impact parameter or, equivalently, on the experimentally accessible number of participating nucleons. Therefore by varying the impact parameter we can test the EoS at different densities. This dependence should be different for different EoS. This is indeed the case for the result of the simulations as seen in Fig. 6, top, where we display the kaon yield $M_{K^+}/A_{\text{part}}$ for Au+Au collisions at 1.5 $A$ GeV as a function of the participant number $A_{\text{part}}$ and for different options: standard version (soft, $KN$),
FIG. 5: Comparison of the measured excitation function of the ratio of the $K^+$ multiplicities per mass number $A$ obtained in Au+Au and in C+C reactions (Ref. [19]) with various calculations. The use of a hard EoS is denoted by thin (blue) lines, a soft EoS by thick (red) lines. The calculated energies are given by the symbols, the lines are drawn to guide the eye. On top, two different versions of the $N\Delta \rightarrow K^+\Lambda N$ cross sections are used. One is based on isospin arguments [23], the other is determined by a relativistic tree level calculation [24]. The calculation by Fuchs [20] are shown as dotted lines. Middle: IQMD calculations with and without $KN$ potential are compared. Bottom: The influence of different options for the life time of $\Delta$ in matter is demonstrated.

calculations without kaon-nucleon interaction (soft, no $KN$) and with the isospin based $N\Delta \rightarrow N\Lambda K^+$ cross section (soft, $KN$, $\sigma^*$). A variation of the KN potential as well as of the $K^+$ production cross section change the dependence of the $K^+$ yield on the number of participants, which can be parametrized by the form $M_{K^+} = A_{\text{part}}^\alpha$, only little. On the contrary, if we apply a hard EoS, the slope value $\alpha$ changes considerable and is outside of the values which are compatible with the experimental results, as shown in the middle part of the figure. In this figure we display as well the insensitivity of our result to the momentum dependence of the nucleon nucleon interaction. As long as the compressibility is not changed the results of our calculations are very similar independent on whether we have a static or a momentum dependent NN potential. Hence the dependence of the $K^+$ yield on the number of participants is also a robust variable for the determination of the EoS which supports our earlier conclusion that the EoS is soft. Another confirmation that only a soft EoS describes the experimental data is the beam energy dependence of the fitted exponent $\alpha$ which is displayed in the right part of fig. 6. The data, which follow the curve for a soft equation of state, will soon be published [26].

In conclusion, we have shown that earthbound experiments have now reached a precision which allows to determine the hadronic EoS. The experimental results for the three observables which are most sensitive to the hadronic EOS are only compatible with theory if the hadronic EoS is soft. This conclusion is robust. Little known input quantities do not influence this conclusion. The observation of a neutron star with twice the solar mass seems to contradict this conclusion. It points toward a hard hadronic EoS. Both results are quite new and one has not to forget that we are comparing non equilibrium heavy ion reactions where about the same number of protons and neutrons are present and where mesons and baryon resonances are produced with cold neutron matter in equilibrium. In addition this contradiction depends also on the prediction that the observed star mass excludes the formation of quark matter in the interior, a consequence of the suggested EoS of quark matter which is still rather speculative.
FIG. 6: Dependence of the $K^+$ scaling on the nuclear EoS. We present this dependence in form of $M_{K^+} = A_{\text{part}}^\alpha$. On the top the dependence of $M_{K^+}/A_{\text{part}}$ as a function of $A_{\text{part}}$ is shown for different options: a “hard” EoS with $KN$ potential (solid line), the other three lines show a “soft” EoS, without $KN$ potential and $\sigma(N\Delta)$ from Tsushima [24] (dotted line), with $KN$ potential and the same parametrization of the cross section (dashed line) and with $KN$ potential and $\sigma(N\Delta) = 3/4\sigma(NN)$.

On the bottom the fit exponent $\alpha$ is shown as a function of the compressibility for calculations with momentum-dependent interactions (mdi) and for static interactions (dashed line) [16]. On the right hand side we compare the energy dependence of the fit exponent $\alpha$ for the two EoS.

To solve this contradiction is certainly a big challenge for both communities in the near future.

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