Stress analysis of arch foot of tie arch bridge based on spatial multi-scale model

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Abstract. For the arch foot connection structure of a steel box tied arch bridge in Guizhou Province, the spatial multi-scale finite element model is established by substructure method, and the influence line is loaded. The calculation results show that under the action of "dead load + live load", the overall stress of arch foot is reasonable and the stress distribution is uniform, but the bearing roof, side web of arch foot and tied rod roof are complicated and the stress concentration is obvious. In the process of operation and maintenance, in order to ensure the safe use of bridges, we should focus on the observation of such components.

1. Introduction

At present, there are two main methods in the local fine finite element analysis of components: one is to establish a local fine model, which imposes the internal forces of the object of study on the whole structure on the local model in the form of loads outside the boundary; the other is to establish a uniform multi-scale finite element model of structural behavior through reasonable boundary strips. Elements of different scales are joined together, so that the same model can obtain the mechanical response of component scale while taking into account the structural scale response.

At present, the commonly used multi-scale modeling methods in ANSYS mainly include substructure method, sub-model method and multi-point constraint method. Different methods have their own advantages and scope of application. In this paper, a uniform multi-scale finite element model of the arch foot of a steel box tied arch bridge with a span of 123 m is established by substructure method, and the mechanical behavior of the arch foot under the most adverse load is studied.

2. Substructure method

2.1 Basic theory of substructure method

A part of the whole model is separately divided as the research objective, and the basic elements of the structure are grouped to form a generalized element, which is called a sub-structure of the whole structure. The whole structure is divided into several sub-structures, and the stiffness characteristics of each sub-structure are calculated in turn, and then each sub-structure is formed. The total stiffness matrix is calculated when the structure is connected to the whole model. This structural analysis method is called substructure method. After partitioning the whole structure model, the local model contains many basic elements and nodes. In order to reduce the total degree of freedom of the system, it is necessary to condense the degree of freedom of the internal nodes of the layer substructure before the local model is
integrated into the whole structure. To achieve this goal, the relationship equation between the force and displacement of each node of the substructure is written as follows by appropriate node numbering:

\[
\begin{bmatrix}
K_{bb} & K_{bi} \\
K_{ib} & K_{ii}
\end{bmatrix}
\begin{bmatrix}
u_b \\
u_i
\end{bmatrix} =
\begin{bmatrix}
F_b \\
F_i
\end{bmatrix}
\]  

(1)

Among them, \(u_i\) and \(u_b\) represent displacement vectors of internal and interface nodes respectively, and structural stiffness matrix \(K\) and load matrix \(F\) are also divided into blocks according to internal and external nodes. According to the Gauss-Jordan elimination method, formula (1) is condensed, and after several elimination operations, it is obtained that:

\[
\begin{bmatrix}
K^*_{bb} & 0 \\
K^*_{ib}
\end{bmatrix}
\begin{bmatrix}
u_b \\
u_i
\end{bmatrix} =
\begin{bmatrix}
F^*_b \\
F^*_i
\end{bmatrix}
\]  

(2)

Among them, \(K^*_{bb}\), \(K^*_{ib}\), \(F^*_b\) and \(F^*_i\) are eliminated and corrected to the following results:

\[
K^*_{bb} = K_{bb} - K_{bi}K_{ii}^{-1}K_{ib}
\]  

(3)

\[
F^*_b = F_b - K_{bi}K_{ii}^{-1}F_i
\]  

(4)

\[
K^*_{ib} = K_{ib}K_{ii}^{-1}
\]  

(5)

\[
F^*_i = K_{ii}^{-1}F_i
\]  

(6)

\(K^*_{bb}\), \(F^*_b\) is the stiffness and load matrix of the substructure after condensation, and \(K^*_{ib}\), \(F^*_i\) is the correlation matrix of the degree of freedom from the boundary node to the internal node. By substituting the displacement of boundary node obtained from equation 1 in formula (2) into equation 2, the internal node displacement can be solved. In practical modeling, internal nodes and external nodes are often scattered, which usually cannot meet the requirements of centralized numbering of boundary nodes in the above calculation process. Therefore, further improvement of the calculation process is needed [2].

2.2 Substructure modeling process

Using ANSYS software for sub-structure analysis, it is divided into three steps: generating super cells, accessing the whole model to load and calculate, and expanding to get the complete solution. The specific steps are shown in figure 1.

3. Establishment of arch foot structure model

3.1 Bar system model

The length of the bridge is 193m and the span of the bridge is 30m+123m+30m. The main span is a single-hole 123m steel box tied-arch, the side span is a prestressing concrete cast-in-place box girder, and the elevation layout is shown in figure 2. In the lower part, gravity U-shaped abutment, wall pier and pile foundation are adopted. The deck is steel-concrete composite deck with a full width of 12m and two-way lanes, and two-meter-wide sidewalks on both sides.
In the whole bridge model, the beam 4 element is used to simulate the tie bar, cross beam, arch rib and wind brace, and the link10 element is used to simulate the suspender. For the bridge deck, the girder grid method is used to simulate the stiffness of the virtual longitudinal beam[3]. The two sides of the main span are one-way sliding bearing and one two-way sliding bearing respectively. In the model, the degrees of freedom in the Y and Z directions of the nodes at the constrained bearing and the degrees of freedom in the Y direction of the constrained bearing are presented.

3.2 Local finite element model of arch foot
The local shell element model of arch foot is established, and shell 63 element is selected to simulate the steel plate. The whole model consists of 9681 nodes and 10836 elements after meshing. The main degree of freedom is selected for the shell element model after meshing, and then the sub-structure is condensed and reduced according to the selected nodes of the main degree of freedom. The overall stiffness and mass matrix of the super-element is obtained, and then the whole model is connected in the form of MATRIX50 element [4]. The local sketch of the multi-scale finite element model is shown in figure 3. Based on the assumption of plane section and the condition of displacement compatibility, the whole structure and substructure are coupled in the form of constraint equation at the joints [5]. The joints after coupling are shown in figure 4. Loading mode is "dead load + live load"[6]. The most disadvantageous working condition is determined by calculating the influence line of arch foot bending moment, so that the bending moment of arch foot reaches the maximum under this working condition.

4. Analysis of calculation results
4.1 Analysis of integral stress of arch foot
The force acting on the arch foot is shown in figure 5. From the figures (a) and (b), it can be seen that the tension stress in the arch foot area mainly distributes in the tied bar section, especially in the upper edge of the tied bar section, with the local maximum tension stress reaching 187 MPa, and the compression stress mainly locates in the arch rib section and transfers to the arch foot junction area with the maximum value of 205 MPa. It can also be seen from the structural stress trace diagram that the compressive stress distributes densely at the arch rib and passes through the node strengthening area to the support; the horizontal thrust formed by the arch rib at the arch foot is borne by the tie rod, so in the trace diagram, the main stress trace of the tie rod points to the end of the arch rib, and the main stress...
trace of the arch rib is at the junction. The complete stress trace flow is formed, which shows that the overall stress distribution in the arch foot region is reasonable [7].

![First principal stress](image1)
![Third principal stress](image2)
![Stress trajectory](image3)

**Figure 5. Stress distribution of arch foot**

4.2 Stress analysis of local components

The maximum stress of the main components of the arch foot is calculated as shown in Table 1.

| Position          | Component                        | First principal stress | Third principal stress | Von Mises stress |
|-------------------|----------------------------------|------------------------|------------------------|-----------------|
| Arch rib          | Arch rib roof                    | 129.6                  | -86.6                  | 83.3            |
|                   | Arch rib floor                   | 35.3                   | -78.0                  | 75.9            |
|                   | Longitudinal stiffener          | 15.1                   | -105.2                 | 104.8           |
|                   | Diaphragm                        | 53.2                   | -43.1                  | 49.5            |
| Tie bar           | Tie bar roof                     | 153.1                  | -70.8                  | 149.5           |
|                   | Tie bar floor                    | 98.1                   | -149.3                 | 87.7            |
|                   | Longitudinal stiffener          | 92.7                   | -77.7                  | 88.1            |
|                   | Diaphragm                        | 53.6                   | -60.6                  | 52.5            |
| Joint             | Bottom plate extension of arch rib | 37.8                  | -59.1                  | 61.1            |
|                   | Side web                         | 187.0                  | -205.4                 | 184.8           |
|                   | Stiffening plate at support      | 63.2                   | -188.4                 | 176.3           |
|                   | Base plate at support            | 71.9                   | -147.6                 | 128.2           |

It can be seen from Table 1 that the maximum Von Mises stress of most components under combined dead and live loads is less than 150 MPa, which has a large safety reserve, but there are still a few components, such as side web of arch foot, bottom plate of support, tied rod roof, etc. Von Mises stress is close to or more than 150 MPa, and there is obvious stress concentration.

The side web connects the main components of the arch foot, and is also the most complex component in the arch foot structure. The Mises stress nephogram of the web is shown in figure 6.

![Medial web](image4)
![Outer web](image5)

**Figure 6. Von Mises stress distribution in lateral web**

As can be seen from Figure 6, the stress distribution of the inner and outer webs of the arch foot is approximately the same. The compressive stress generated by the arch rib is transmitted to the side web through web, floor, floor extension plate, longitudinal stiffening rib and other components, and then to the support. In the joint of arch rib and tie rod, the stress value of Web reaches the maximum. Although the fillet transition is adopted at the bottom of arch rib and the stiffening rib of bottom plate is added, the stress concentration of arch rib still occurs during the transmission of arch rib pressure and bending.
moment to tie rod. The maximum stress value is more than 184.6 MPa.

As shown in figure 7, the stress distribution of tied bar roof is nephogram. The stress concentration of tied bar roof appears at the end of stiffening rib of arch rib floor and at the junction of stiffening rib and arch rib floor. Figure 8 shows the stress distribution of the floor at the arch foot support. From the stress nephogram, it can be seen that the arch foot has obvious stress concentration area near the support, and the stress concentration point is at the edge of the reinforcement area above the support. Because this position bears the load transmitted by tied rods, arch ribs and end beams, as well as the restraint and vertical support reaction of the support, and is weaker than the reinforcement area above the support, the stress concentration is significant. The maximum Von Mises stress value reaches 128.2 MPa and the maximum compressive stress value reaches 147 MPa.

5. Conclusion
1) Substructure method condensates the degree of freedom of the model nodes, reduces the size of the overall matrix of the structure, and achieves accurate simulation of the mechanical properties of key components of large-span structures at a lower cost of calculation.
2) The stress distribution at the arch foot of steel box tied arch bridge is uniform, and the stress level of most components is not high under the most unfavorable load, so it has a certain safety reserve; the stress concentration of a few components is more complex and serious, which should be paid attention to in the process of bridge operation and maintenance.

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