Analytically periodic solutions to the 
three-dimensional Euler–Poisson equations of gaseous 
stars with a negative cosmological constant

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Received 11 June 2009, in final form 31 July 2009
Published 6 November 2009
Online at stacks.iop.org/CQG/26/235011

Abstract
By extension of the three-dimensional analytical solutions of Goldreich and Weber (1980 Homologously collapsing stellar cores Astrophys. J. 238 991) with an adiabatic exponent $\gamma = 4/3$, to the (classical) Euler–Poisson equations without a cosmological constant, the self-similar (almost re-collapsing) time-periodic solutions with a negative cosmological constant ($\Lambda < 0$) are constructed. The solutions with time periodicity are novel. Based on these solutions, the time-periodic and almost re-collapsing model is conjectured in this paper, for some gaseous stars.

PACS numbers: 98.10.+z, 98.80.Jk, 02.30.Jr, 02.30.Hq

1. Introduction
The evolution of a self-gravitating fluid, such as a gaseous star, can be formulated by the isentropic Euler–Poisson systems expressed in the following form:

$$\begin{align*}
\rho_t + \nabla \cdot (\rho \mathbf{u}) &= 0, \\
(\rho \mathbf{u})_t + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla P &= -\rho \nabla \Phi, \\
\Delta \Phi(t, \mathbf{x}) &= \alpha(N) \rho - \Lambda,
\end{align*}$$

(1)

where $\alpha(N)$ is a constant related to the unit ball in $\mathbb{R}^N$: $\alpha(2) = 2\pi$ and $\alpha(3) = 4\pi$. As usual, $\rho = \rho(t, \mathbf{x})$ and $\mathbf{u} = \mathbf{u}(t, \mathbf{x}) \in \mathbb{R}^N$ are the density and the velocity, respectively. $P = P(\rho)$ is the pressure function. When the cosmological constant $\Lambda$ is positive, the space is open; when it is negative, the space is closed; when it is zero, the space is flat.

In the above systems, the self-gravitational potential field $\Phi = \Phi(t, \mathbf{x})$ is determined by the density $\rho$, through the Poisson equation (1)$_3$. For $N = 3$, the three equations in (1) are the classical (non-relativistic) descriptions of a galaxy, in astrophysics. See [1, 2, 7] for details about the systems.
The $\gamma$-law can be applied to the pressure term $P(\rho)$, i.e.

$$P(\rho) = K\rho^\gamma := \frac{\rho^\gamma}{\gamma},$$

(2)

which is a common hypothesis. The constant $\gamma = c_P/c_v \geq 1$, where $c_P$ and $c_v$ are the specific heats per unit mass under constant pressure and constant volume respectively, is the ratio of the specific heats, that is, the adiabatic exponent in (2). In particular, the fluid is called isothermal if $\gamma = 1$.

Historically in astrophysics for a zero constant background, for the three-dimensional space, the hydrostatic equilibrium specified by $u = 0$ has been studied. According to [2], the ratio between the core density $\rho(0)$ and the mean density $\bar{\rho}$ for $6/5 < \gamma < 2$ is given by

$$\frac{\rho}{\rho(0)} = \left(\frac{-3}{\zeta} \frac{\dot{y}(z)}{y(z)}\right)_{z = z_0},$$

(3)

where $y$ is the solution of the Lane–Emden equation with $n = 1/(\gamma - 1)$ for $\gamma > 1$:

$$\ddot{y}(z) + \frac{2}{z} \dot{y}(z) + y(z)^n = 0, \quad y(0) = \alpha > 0, \quad \dot{y}(0) = 0, \quad n = \frac{1}{1 - \gamma},$$

(4)

and $z_0$ is the first zero of $y(z_0) = 0$. We can solve the Lane–Emden equation analytically only for

$$y_{\text{anl}}(z) = \begin{cases} 1 - \frac{1}{6} z^2, & n = 0; \\ \sin z, & n = 1; \\ \frac{1}{\sqrt{1 + z^2/3}}, & n = 5. \end{cases}$$

(5)

For the other values, only numerical values can be obtained. It can be shown that for $n < 5$, the radius of polytropic models is finite and for $n \geq 5$, the radius is infinite.

As we are interested in the solutions in radial symmetry, the Poisson equation (1) is transformed into

$$r^{N-1} \Phi_r = \alpha(N)(\rho - \Lambda)r^{N-1},$$

(6)

$$\Phi_r = \alpha(N) \frac{r^{N-1}}{r^{N-1}} \int_0^r (\rho(t,s) - \Lambda)s^{N-1} \, ds.$$  

(7)

For the analytical collapsing solutions, the well-known results were constructed by Goldreich and Weber [6] in 1980 for $\gamma = 4/3$ in $R^3$. The analytical collapsing solutions without compact support for $\gamma = 1$ in $R^3$ were constructed by Yuen [11] in 2008. Very recently, it was found that the above solutions can be modified to construct the corresponding solutions to the pressureless Navier–Stokes–Poisson equations with density-dependent viscosity with zero background:

$$\begin{cases} \rho_t + V \rho_r + \rho V_r + \frac{N-1}{r} \rho V = 0, \\ \rho (V_t + V V_r) + \alpha(N) \rho \frac{r^{N-1}}{r^{N-1}} \int_0^r \rho(t,s)s^{N-1} \, ds = [\kappa \rho^\theta] \left( \frac{N-1}{r} V + V_r \right) \\ + (\kappa \rho^\theta) \left( V_{rr} + \frac{N-1}{r} V_r + \frac{N-1}{r^2} V \right), \end{cases}$$

(8)

with radial symmetry $\vec{u} = \frac{\dot{r}}{r} V(t, r)$.
For the solutions in non-radial symmetry to the isothermal Euler–Poisson equations, the interested reader may refer to the preprints [13, 14] for details.

For simplicity, we take the constant $\Lambda = -\frac{3}{4}\pi$ for $\Lambda < 0$. For understanding the background knowledge regarding the Euler–Poisson equations with a cosmological constant $\Lambda$, the interested reader may refer to [5, 9].

Our main results enable us to obtain the analytically time-periodic solutions for the Euler–Poisson equations with the negative cosmological constant ($\Lambda < 0$) in spherical symmetry:

\[
\begin{align*}
\rho \rho_t + \rho V_\rho + V \rho V_r + \frac{2}{r} \rho V &= 0, \\
\rho (V_t + V V_r) + \frac{\partial}{\partial r} (K \rho^{4/3}) &= -\frac{4\pi \rho}{r^2} \int_0^r \left( \rho (t, s) + \frac{3}{4\pi} \right) s^2 \, ds,
\end{align*}
\]

with extension of the analytical solutions of Goldreich and Weber [6] in theorem 1.

**Theorem 1.** For the three-dimensional Euler–Poisson equations in spherical symmetry (9), there exists a family of solutions, for $\gamma = 4/3$:

\[
\begin{align*}
\rho(t, r) &= \begin{cases} 
\frac{1}{a(t)} y \left( \frac{r}{a(t)} \right) \left( \frac{r}{a(t)} \right)^3, & \text{for } r < a(t) Z_\nu; \\
0, & \text{for } a(t) Z_\nu \leq r.
\end{cases} \\
V(t, r) &= \frac{\dot{a}(t)}{a(t)} \frac{r}{a(t)}; \\
\ddot{a}(t) &= -\lambda a(t)^2 - a(t), \quad a(0) = a_0 > 0, \quad \dot{a}(0) = a_1; \\
\ddot{y}(z) + \frac{2}{z} \dot{y}(z) + \frac{\pi}{K} y(z)^3 &= \mu, \quad y(0) = \alpha > 0, \quad \dot{y}(0) = 0.
\end{align*}
\]

where $K > 0$, $\mu = \frac{3\lambda}{(4K)}$, and the finite $Z_\nu$ is the first zero point of $y(z)$.

In particular,

1. $\lambda = 0$ and $a_1 = 0$, the solutions collapse in the finite time $t = \pi/2$;
2. $\lambda < 0$, the solutions are non-trivially time-periodic, except for the case where $a_0 = \sqrt{-\lambda}$ and $a_1 = 0$;
3. $\lambda > 0$ and $a_1 \leq 0$, the solutions collapse in a finite time $T$.

**2. Time-periodic solutions**

In this section, before presenting the proof of theorem 1, we will prepare some lemmas. First, we will obtain a general class of solutions for the mass equation in spherical symmetry (9).

**Lemma 2.** For the three-dimensional equation of conservation of mass in spherical symmetry:

\[
\rho \rho_t + \rho V_\rho + V \rho V_r + \frac{2}{r} \rho V = 0,
\]

there exist solutions,

\[
\begin{align*}
\rho(t, r) &= \frac{f(r/a(t))}{a(t)^3}, \\
V(t, r) &= \frac{\dot{a}(t)}{a(t)} r,
\end{align*}
\]

with the form $f \geq 0 \in C^1$ and $a(t) > 0 \in C^1$. 


Proof. We may plug (12) into (11). Then
\[\begin{align*}
\rho_t + V\rho_r + \rho V_r + \frac{2}{r}\rho V &= -3\frac{\dot{a}(t)f(r/a(t))}{a(t)^4} - \frac{\dot{a}(t)r\dot{f}(r/a(t))}{a(t)^5} \\
&\quad + \frac{\dot{a}(t)r\dot{f}(r/a(t))}{a(t)^4} + \frac{f(r/a(t))\dot{a}(t)}{a(t)} + \frac{2f(r/a(t))\dot{a}(t)}{r a(t)^3} \\
&= 0.
\end{align*}\]

The proof is completed. □

The following core lemma is needed to show the cyclic phenomena of the solutions (10).

Lemma 3. For the ordinary differential equation,
\[\ddot{a}(t) - \frac{\lambda}{a(t)^2} + a(t) = 0, \quad a(0) = a_0 > 0, \quad \dot{a}(0) = a_1,\]
with \(\lambda > 0\), there exist non-trivially periodic solutions except for the case with \(a_0 = \sqrt[3]{\lambda}\) and \(a_1 = 0\).

Proof. For equation (17), multiply \(\dot{a}(t)\) and then integrate it, as follows:
\[\frac{\dot{a}(t)^2}{2} + \frac{\lambda}{a(t)} + \frac{a(t)^2}{2} = \theta,\]
with the constant \(\theta = \frac{a_0^2}{2} + \frac{\lambda}{a_0} + \frac{a_1^2}{2} > 0\).

We define the kinetic energy thus:
\[F_{\text{kin}} := \frac{\dot{a}(t)^2}{2},\]
and the potential energy thus:
\[F_{\text{pot}} = \frac{\lambda}{a(t)} + \frac{a(t)^2}{2}.\]

The total energy is conserved thus:
\[\frac{d}{dt}(F_{\text{kin}} + F_{\text{pot}}) = 0.\]

We may observe that the potential energy has only one global minimum at \(a_{\text{min}} = \sqrt[3]{\lambda}\), for \(a(t) \in (0, +\infty)\). Therefore, by the classical energy method for conservative systems (in section 4.3 of [8]), the solutions have a closed trajectory. We may calculate the time for traveling the closed orbit:
\[T = \int_0^t dt = 2\int_{a_{\text{min}}}^{a_{\text{max}}} \frac{-da(t)}{\sqrt{2[\theta - \left(\frac{\lambda}{a(t)^2} + \frac{\dot{a}(t)^2}{2}\right)]}} = 2\int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da(t)}{\sqrt{2[\theta - \left(\frac{\lambda}{a(t)^2} + \frac{\dot{a}(t)^2}{2}\right)]}},\]
where \(a_{\text{min}} = \inf_{t \geq 0} a(t)\) and \(a_{\text{max}} = \sup_{t \geq 0} a(t)\).

Let \(G(t) := \theta - \left(\frac{\lambda}{a(t)^2} + \frac{\dot{a}(t)^2}{2}\right), \quad G_0 := \theta - \left(\frac{\lambda}{a_{\text{min}}^2} + \frac{(a_{\text{min}})^2}{2}\right) > 0 \quad \text{and} \quad G_1 := \theta - \left(\frac{\lambda}{a_{\text{max}}^2} + \frac{(a_{\text{max}})^2}{2}\right) > 0.\) Except for the case where \(a_0 = \sqrt[3]{\lambda}\) and \(a_1 = 0\), equation (22) becomes


\[
T = \int_{a_{\text{max}} - \epsilon}^{a_{\text{max}}} \frac{2\, da(t)}{\sqrt{2[\theta - (\lambda a(t) + \frac{a(t)^2}{2})]}} + \int_{a_{\text{min}} + \epsilon}^{a_{\text{min}}} \frac{2\, da(t)}{\sqrt{2[\theta - (\lambda a(t) + \frac{a(t)^2}{2})]}}\]  

(23)

\[
+ \int_{a_{\text{min}} - \epsilon}^{a_{\text{min}}} \frac{2\, da(t)}{\sqrt{2[\theta - (\lambda a(t) + \frac{a(t)^2}{2})]}}\]  

(24)

\[
\leq \sup_{a_{\text{min}} \leq a(t) \leq a_{\text{min}} + \epsilon} \left| \frac{1}{\sqrt{a(t)^2}} - a(t) \right| \int_0^{G_0} \frac{\sqrt{2}\, dG(t)}{\sqrt{G(t)}} + \int_{a_{\text{max}} - \epsilon}^{a_{\text{max}}} \frac{2\, da(t)}{\sqrt{2[\theta - (\lambda a(t) + \frac{a(t)^2}{2})]}}\]  

(25)

\[
+ \sup_{a_{\text{min}} - \epsilon \leq a(t) \leq a_{\text{max}}} \left| \frac{1}{\sqrt{a(t)^2}} - a(t) \right| \int_0^{G_1} \frac{\sqrt{2}\, dG(t)}{\sqrt{G(t)}}\]  

(26)

\[
= \sup_{a_{\text{min}} \leq a(t) \leq a_{\text{min}} + \epsilon} \left| \frac{1}{\sqrt{a(t)^2}} - a(t) \right| \frac{\sqrt{G_0}}{\sqrt{2}} + \int_{a_{\text{max}} - \epsilon}^{a_{\text{max}}} \frac{2\, da(t)}{\sqrt{2[\theta - (\lambda a(t) + \frac{a(t)^2}{2})]}}\]  

(27)

\[
+ \sup_{a_{\text{min}} - \epsilon \leq a(t) \leq a_{\text{max}}} \left| \frac{1}{\sqrt{a(t)^2}} - a(t) \right| \frac{\sqrt{G_1}}{\sqrt{2}}\]  

(28)

\[
< \infty.\]  

(29)

Therefore, the solutions to the differential equation (17) are time periodic. The proof is completed. \[\square\]

In the solutions given in (10), due to the properties of the modified Emden equation (10)\textsubscript{2}, collapsing phenomena are observed. The following lemma can be proved immediately.

**Lemma 4.** For the ordinary differential equation,

\[
\ddot{a}(t) + \frac{\lambda}{a(t)^2} = -a(t), \quad 0 < a(0) = a_0, \quad \dot{a}(0) = a_1 \leq 0, \]  

(30)

with \(\lambda > 0\), there exists a finite time \(T\) such that \(\lim_{t \to T^-} a(t) = 0\).

**Proof.** If the claim is not true, for all \(t \geq 0\), we have \(a(t) > 0\).

But the ordinary differential equation (30) becomes

\[
\ddot{a}(t) = -\frac{1}{a(t)^2} - a(t) < 0.\]  

(31)

This shows that \(\dot{a}(t)\) is a decreasing function: \(\dot{a}(t) < \dot{a}(t_1)\) for all \(t > t_1 > 0\):

\[
\dot{a}(t) \leq \dot{a}(t_1) < 0.\]  

(32)

Thus, the solution is bounded by

\[
a(t) = \int_{t_1}^{t} \dot{a}(s) \, ds + a(t_1) \leq \dot{a}(t_1)t + a_0.\]  

(33)

After a sufficient large time, there exists a finite time \(T\) such that \(\lim_{t \to T^-} a(t) = 0\). A contradiction is discovered. The proof is completed. \[\square\]

Here we may give proof of theorem 1.

**Proof of Theorem 1.** From lemma 2, we can easily check that the equations given in (10) satisfy (9)\textsubscript{1}. We are able to plug the solutions (10) into the momentum equation (9)\textsubscript{2}:
\[\rho (V_t + VV_r) + K \frac{\partial}{\partial r} \rho^4/3 + \frac{4\pi \rho}{r^2} \int_0^r (\rho(t, s) - \Lambda) s^2 ds \]

(34)

\[\rho \ddot{a}(t) + 4K \left( \frac{y(\frac{r}{a(t)})^3}{a(t)^3} \right) \frac{1}{\sqrt[3]{y(\frac{r}{a(t)})} y(\frac{r}{a(t)})} + \frac{4\pi \rho}{r^2} \int_0^r (\rho(t, s) - \Lambda) s^2 ds \]

(35)

\[\rho \ddot{a}(t) + \frac{4\pi \rho}{a(t)^2} + \frac{4\pi \rho}{r^2 a(t)} \int_0^r \frac{y(s)}{a(t)} s^2 ds - \frac{4\pi \Lambda \rho}{r^2} \int_0^r s^2 ds \]

(36)

\[\rho \ddot{a}(t) + \frac{4\pi \Lambda}{3} \rho r + 4K \ddot{y}(\frac{r}{a(t)}) + \frac{4\pi \rho}{a(t)^2} \int_0^r y(\frac{s}{a(t)}) s^2 ds - \frac{4\pi \Lambda}{3} \rho r \]

(37)

\[\rho \ddot{a}(t) + \frac{4\pi \Lambda}{3} \rho r + 4K \ddot{y}(\frac{r}{a(t)}) + \frac{4\pi \rho}{r^2 a(t)} \int_0^r \frac{y(s)}{a(t)} s^2 ds \]

(38)

\[\rho \ddot{a}(t) + \frac{4\pi \Lambda}{3} \rho r + 4K \ddot{y}(\frac{r}{a(t)}) + \frac{4\pi \rho}{r^2 a(t)} \int_0^r y(s)^3 s^2 ds \]

(39)

\[\rho \ddot{a}(t) + \frac{4\pi \Lambda}{3} \rho r + 4K \ddot{y}(\frac{r}{a(t)}) + \frac{4\pi \rho}{r^2 a(t)} \int_0^r y(s)^3 s^2 ds \]

(40)

Here, we use the property of \(a(t)\):

\[\ddot{a}(t) = -\frac{\lambda}{a(t)^2} + \frac{4\pi \Lambda}{3} a(t) = -\frac{\lambda}{a(t)^2} - a(t), \]

(41)

and denote

\[Q\left(\frac{r}{a(t)}\right) := Q(x) = -\lambda x + 4K \ddot{y}(r) + \frac{4\pi}{x^2} \int_0^x y(s)^3 s^2 ds. \]

(42)

We may differentiate \(Q(x)\) with respect to \(x\),

\[Q(x) = -\lambda + 4K \ddot{y}(x) + 4\pi y(x)^3 \frac{-2 \cdot 4\pi}{x^2} \int_0^x y(s)^3 s^2 ds \]

(43)

\[= -\frac{2}{x} \left[ \lambda x + 4K \ddot{y}(x) - K \mu x + \frac{4\pi}{x^2} \int_0^x y(s)^3 s^2 ds \right] \]

(44)

\[= -\frac{2}{x} Q(x). \]

(45)

Where \(\lim_{x \to 0^+} Q(x) = Q(0) = 0\), this implies that \(Q(x) = 0\). The above result is due to the fact that we have chosen the Lane–Emden equation:

\[\begin{cases}
\ddot{y}(z) + \frac{2}{z} \dot{y}(z) + \frac{\pi}{K} y(z)^{3/2} = \mu, & \mu = \frac{3\lambda}{4K} \\
y(0) = \alpha > 0, & \dot{y}(0) = 0.
\end{cases} \]

(46)

We have shown that there exists a family of the solutions with compact support in theorem 1, with the well-known results about the Lane–Emden equation (46) [2, 7].
On the other hand, for $\Lambda = -\frac{3}{4}\pi < 0$, it is clear that for $\lambda = 0$ in the theorem is true. For the linear ordinary differential equation:

$$\ddot{a}(t) + a(t) = 0, \quad a(0) = a_0 > 0, \quad \dot{a}(0) = 0,$$  

the solution is

$$a(t) = a_0 \cos t.$$  

(48)

The function $a(\frac{\pi}{2})$ achieves zero then the solutions collapse in the finite time $\pi/2$. By using lemma 3 regarding $a(t)$, we have for $\lambda < 0$, the periodic solutions. These solutions are non-trivial except for the case where $a_0 = \sqrt{-\lambda}$ and $a_1 = 0$.

By using lemma 4, for $\lambda > 0$ and $a_1 \leq 0$, we have the result that the solutions collapse in a finite time $T$.

This completes the proof. □

**Remark 5.** If we consider the system with frictional damping,

$$\rho(t) + \nabla \cdot (\rho \vec{u}) = 0,$$

$$\rho \vec{u}, + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) + \beta \rho \vec{u} + \nabla P = -\rho \nabla \Phi,$$

$$\Delta \Phi(t, \vec{x}) = a(N)\rho - \Lambda,$$

with the constant $\beta > 0$, the corresponding ordinary differential equation for the solutions is

$$\ddot{a}(t) + \beta \dot{a}(t) = -\lambda a(t) - a(t), \quad a(0) = a_0 > 0, \quad \dot{a}(0) = a_1,$$  

(50)

[10]. For $\lambda < 0$, the function $a(t)$ oscillates around the minimum $a_{\text{min}} = \sqrt{-\lambda}$ of the potential energy except for the case where $a_0 = \sqrt{-\lambda}$ and $a_1 = 0$. And it is asymptotically stable.

**Remark 6.** In general, for the Euler–Poisson equations in $R^N$, it is clear that we have obtained the corresponding solutions: for $N = 2$ and $\gamma = 1$,

$$\rho(t, r) = \frac{1}{a^2(t)} e^{\gamma(t/a(t))}, \quad V(t, r) = \frac{\dot{a}(t)}{a(t)} r;$$

$$\ddot{a}(t) = \frac{-\lambda}{a(t)} - a(t), \quad a(0) = a_0 > 0, \quad \dot{a}(0) = a_1;$$

$$\dot{y}(z) + \frac{1}{z} \dot{y}(z) + \frac{\alpha(N)}{K} y(z)^{N/(N-2)} = \mu, \quad y(0) = \alpha, \quad \dot{y}(0) = 0,$$  

(51)

where $\mu = 2\lambda/K$, [11]; for $N \geq 3$ and $\gamma = (2N-2)/N$,

$$\rho(t, r) = \begin{cases} \frac{1}{a(t)^N} \left(\frac{r}{a(t)}\right)^{N/(N-2)}, & \text{for } r < a(t)Z_\mu; \\ 0, & \text{for } a(t)Z_\mu \leq r. \end{cases}$$

$$\ddot{a}(t) = \frac{-\lambda}{a(t)^N - 1} - a(t), \quad a(0) = a_0 > 0, \quad \dot{a}(0) = a_1;$$

$$\dot{y}(z) + \frac{N-1}{z} \dot{y}(z) + \frac{\alpha(N)}{(2N-2)K} y(z)^{N/(N-2)} = \mu, \quad y(0) = \alpha > 0, \quad \dot{y}(0) = 0,$$  

(52)

where $\mu = [N(N - 2)\lambda]/(2N - 2)K$ and the finite $Z_\mu$ is the first zero of $y(z)$ [4]. With $\lambda < 0$, solutions (51) and (52) are time periodic for the negative cosmological constant.

**Remark 7.** Under some initial conditions for $\Lambda > 0$, collapsing solutions are obtained. The case is similar to the system without a cosmological constant.
3. Discussion

The time-periodic pattern may be due to the negative constant $\Lambda$ as this phenomenon has never been seen before in the zero background Euler–Poisson equations with or without frictional damping in [10, 11].

On the other hand, in statement (2) of the theorem, with some initial conditions, for example, $0 < a(0) = \epsilon \ll 1$ and $\dot{a}(0) = 0$, the density becomes

$$\rho(0, 0) = \frac{\alpha^3}{\epsilon^3} \gg M_0,$$

where $M_0$ is an arbitrary constant.

The solutions we obtained may provide the possible evolutionary model where $\Lambda < 0$ is constant, that the universe expands and then almost re-collapses. The density at the origin can be periodically greater than any given constant $\rho(T, 0) \gg M_0$, in the finite time. We note that this phenomenon is not the same as $\rho(T, 0) = +\infty$ with a finite time $T$.

As the time-periodic effect is due to the negative cosmological constant in our model, more solutions of this pattern are expected for other $\gamma$ values. Then further work may be done by the computing simulation regarding the stability of the numerical solutions. If some gaseous stars (a galaxy) obey the $\gamma$-law ($\gamma = 4/3$), it may provide an alternative explanation about the evolution. The time-periodic solutions coincide with the expansion segment (the redshift effect) in a short time. We note that there is a significant difference between the traditional re-collapsing model with negative cosmological constant $\Lambda < 0$ [3] and the model presented here.

Acknowledgments

We would like to thank Professor Man Kam Kwong for discussing lemma 3 and the reviewers’ comments for improving the readability of this paper.

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