A new way for the extension of quantum theory: Non-Bohmian quantum potentials

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Quantum Mechanics is a good example of a successful theory. Most of atomic phenomena are described well by quantum mechanics and cases such as Lamb Shift that are not described by quantum mechanics, are described by quantum electrodynamics. Of course, at the nuclear level, because of some complications, it is not clear that we can claim the same confidence. One way of taking these complications and corrections into account seems to be a modification of the standard quantum theory. In this paper and its follow ups we consider a straightforward way of extending quantum theory. Our method is based on a Bohmian approach. We show that this approach has the essential ability for extending quantum theory, and we do this by introducing "non-Bohmian" forms for the quantum potential.

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I. INTRODUCTION

Quantum mechanics was developed during 1920’s and soon became a successful theory. But in order to explain phenomena like fine structure of the atomic spectra, e.g., people introduced relativistic corrections and spin-orbit coupling and finally Dirac’s relativistic formulation of quantum theory superseded all previous work. But a lot of controversy developed around the interpretation of the theory. Then, in 1952 David Bohm introduced another formulation of Quantum theory with a causal interpretation [1]. Bohm attributed all quantum effects to the so-called quantum potential that he had introduced in his formulation. But he kept the basic concepts like wave function, that were used in the standard formulation of Quantum mechanics. Here we intend to extend non-relativistic quantum mechanics by using a Bohmian approach. To begin with, we derive the basic equations of Bohmian mechanics from a variational principle. Then, we extend the idea of quantum potential from its simple Bohmian form and try to see some of the basic results of this new formulation of quantum theory. The main purpose of this article is to describe a simple way for the extension of quantum theory by introducing new forms for the quantum potential. A detailed discussion on the consequences of these new quantum potentials will come in following papers.

II. BOHMIAN MECHANICS AND THE "QUANTUM POTENTIAL" CONCEPT

The Schrödinger equation is the basic equation of quantum mechanics:

\[ i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi. \] (1)

In the Bohmian interpretation of quantum mechanics, the phase and amplitude of wave function is meaningful. If we write the wave function in the polar form

\[ \psi(x, t) = R(x, t) \exp(iS(x, t)/\hbar) \] (2)

and substitute it in the complex Schrödinger equation (1), we obtain the following two real equations:

\[ \frac{(\nabla S)^2}{2m} + V(x) = \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} + \frac{\partial S}{\partial t} = 0 \] (3)

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\[
\frac{\partial R^2}{\partial t} + \nabla \cdot \left( R^2 \frac{\nabla S}{m} \right) = 0.
\] (4)

According to Bohm \cite{1}, the Eq. 3 has the same meaning as the Hamilton-Jacobi equation of classical mechanics, with an extra term:

\[
Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R}
\] (5)

which he called it "Quantum Potential" \cite{1}. In Bohm’s view this additional term is responsible for all quantum effects. Interpreting Eq. 3 as a modified Hamilton-Jacobi equation is meaningful only when we interpret \(\nabla S\) as particle momentum, and \(-\partial S/\partial t\) as particle energy (for conserved systems), i.e.:

\[
p = \nabla S
\] (6)

\[
E = -\frac{\partial S}{\partial t}.
\] (7)

Therefore, we can say that the fundamental assumption of Bohm is that the phase of wave function ‘determines’ or ‘contains information about’ energy and momentum of particle, i.e. of every individual member of ensemble of particles. It seems that the lack of this fundamental assumption in the standard quantum mechanics is the cause of apparent indeterminism of quantum phenomenon, at the level of individual members of ensemble. By adding this assumption, we can talk about the behavior of every particle alone, along with the behavior of ensemble of them. Without assuming Eqs. 6 and 7 it is hard to consider Eq. 3 as a modified Hamilton-Jacobi equation, and also, without assuming Eq. 3 as modified Hamilton-Jacobi equation it is hard to understand that the phase of wave function can contain information about particle’s energy and momentum.

Note that, the Eq. 3 is fundamental and revolutionary assumption only in comparison with the formulation of orthodox quantum mechanics, not in comparison with the classical mechanics, because, there is no definite trajectory for a particle in the formulation of orthodox quantum mechanics, while in the classical mechanics particle trajectory is a trivial concept. In contrast, the existence of an extra term, named quantum potential, in the Hamilton-Jacobi equation in comparison with the classical mechanics is a fundamental and revolutionary assumption. Therefore, taking either of the statements \(p = \nabla S\) and \(Q = -(\hbar^2/2m)(\nabla^2 R/R)\) as fundamental and revolutionary depends on whether we compare Bohmian mechanics with the orthodox quantum mechanics or with the classical mechanics. The first route provides for the quantum mechanics the possibility of defining trajectories (something that is absent from quantum mechanics), and the second one provides for the classical mechanics the possibility of barrier penetration and other non-classical behaviors observed in the nature (something that is absent from classical mechanics). In other words, the Bohmian mechanics brings quantum and classical mechanics closer together.

Some authors \cite{2},\cite{3} prefer to derive the Eq. 6 from symmetry considerations about the wave function, but, it seems that arriving at the equation 6 from a Newtonian force law is much simpler than arriving at it from symmetry considerations about the Schrödinger wave function. Consider the equation

\[
\frac{dp}{dt} = \frac{\partial p}{\partial t} + \frac{p}{m} \nabla p = -\nabla (V + Q).
\] (8)

From the identity

\[
p \nabla p = \nabla \left( \frac{p^2}{2} \right) + (\nabla \times p) \times p
\] (9)

we have

\[
\frac{\partial p}{\partial t} + \nabla \left( \frac{p^2}{2m} \right) + (\nabla \times p) \times \frac{p}{m} = -\nabla (V + Q)
\] (10)
or
\[
\frac{\partial}{\partial t} (\nabla \times p) + \nabla \times \left( (\nabla \times p) \times \frac{p}{m} \right) = 0.
\] (11)

The simplest solution of this equation is \( p = \nabla S \) for some function \( S \) (we have extracted the Eqs. (8)-(11) from reference [4]). Therefore, in the Newtonian framework, we can always take momentum \( p \) as gradient of some function \( S \).

In this framework, it seems trivial that we need extra (non-classical) potentials to explain the non-classical behaviors of particles.

III. DERIVING BOHMIAN MECHANICS FROM A VARIATIONAL PRINCIPLE

We can deduce Bohmian equations (3) and (4) from a variational principle. This variational principle says that the integral
\[
\int R^2 \left\{ \left( \frac{\nabla S}{2m} \right)^2 + V - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} + \frac{\partial S}{\partial t} \right\} d^3x dt
\] (12)
should be stationary. If we denote the integrand by \( g(x,t) \) and treat functions \( R \) and \( S \) as some fields, then according to the calculus of variations we have the following equations for \( R, S \):
\[
\frac{\partial g}{\partial R} - \partial_{\alpha} \frac{\partial g}{\partial (\partial_{\alpha} R)} + \partial_{\alpha} \partial_{\beta} \frac{\partial g}{\partial (\partial_{\alpha} \partial_{\beta} R)} = 0
\] (13)
\[
\partial_{\alpha} \frac{\partial g}{\partial (\partial_{\alpha} S)} = 0.
\] (14)

The indices \( \alpha, \beta \) refer to space and time coordinates. From (13) we obtain
\[
2R \left\{ \left( \frac{\nabla S}{2m} \right)^2 + V - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} + \frac{\partial S}{\partial t} \right\} + R^2 \left\{ \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R^2} \right\} + \nabla^2 \left\{ R^2 \left( - \frac{\hbar^2}{2m} \frac{1}{R} \right) \right\} = 0
\] (15)
which reduces to the modified Hamilton-Jacobi equation (3). Similarly from (14) we have
\[
\nabla \cdot \left( R^2 \frac{\nabla S}{m} \right) + \frac{\partial}{\partial t} R^2 = 0
\] (16)
which is the continuity equation (4). Apparently this is not a remarkable fact, but there is a crucial and subtle point here. The crucial point is "the form of functional dependence of the quantum potential with respect to \( R \)". The vanishing of the expression
\[
R^2 \left\{ \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R^2} \right\} + \nabla^2 \left\{ R^2 \left( - \frac{\hbar^2}{2m} \frac{1}{R} \right) \right\}
\] (17)
in (15) is due to the special form of functional dependence of \( Q \) with respect to \( R \). If we need to obtain a Hamilton-Jacobi equation modified with a quantum potential, \( Q[R] \) can not take every arbitrary form. The Eq. (13) restricts the acceptable forms of \( Q[R] \). If we choose another form, \( Q = 0 \), i.e. we replace the Bohmian quantum potential \(- (\hbar^2/2m)(\nabla^2 R/R)\) in the integral (12) by a zero constant, then from (13) we obtain
\[
\frac{(\nabla S)^2}{2m} + V + \frac{\partial S}{\partial t} = 0
\] (18)
which is exactly the classical Hamilton-Jacobi equation. Therefore, by choosing proper and acceptable forms for the quantum potential we can obtain the classical or the quantum Hamilton-Jacobi equation. Thus it seems natural to ask: "Can there be other forms of quantum potential?". As we show in this paper the answer is positive. In the next section we deduce other forms ("non-Bohmian" forms) of quantum potential.
IV. OTHER POSSIBLE FORMS OF QUANTUM POTENTIAL

In this section for simplicity we write derivatives as subscripts:

\[ R_\alpha \equiv \partial_\alpha R, \quad R_{\alpha\beta} \equiv \partial_\alpha \partial_\beta R, \quad \cdots \; ; \quad \alpha, \beta \in \{0, 1, 2, 3\}. \] (19)

In this notation the Bohmian quantum potential is written as:

\[ Q = -\frac{\hbar^2}{2m} R^{-1} \delta_{ij} R_{ij}; \quad i, j \in \{1, 2, 3\} \] (20)

in which \( \delta_{ij} \) is the Kronecker delta tensor.

For deriving other possible forms of quantum potential we replace "Bohmian" quantum potential \(-\left(\frac{\hbar^2}{2m}\right)\frac{\nabla^2 R}{R}\) by a general unknown function \( Q \) that is some function of \( R \) and its derivatives i.e.:

\[ Q = Q(R, R_\alpha, R_{\alpha\beta}, \ldots); \quad \alpha, \beta, \sigma, \cdots \in \{0, 1, 2, 3\}. \] (21)

Now, the variational principle is written as

\[ \delta \int g d^3x dt = \delta \int R^2 \left\{ \frac{(\nabla S)^2}{2m} + V + Q + \frac{\partial S}{\partial t} \right\} d^3x dt = 0. \] (22)

in which we have denoted the integrand by \( g \). Similar to Eqs. (13), (14) in the previous section, we obtain from variational calculus

\[ \frac{\partial g}{\partial R} - \partial_\alpha \frac{\partial g}{\partial R_\alpha} + \partial_\alpha \partial_\beta \frac{\partial g}{\partial R_{\alpha\beta}} - \partial_\alpha \partial_\beta \partial_\sigma \frac{\partial g}{\partial R_{\alpha\beta\sigma}} + \cdots = 0 \] (23)

\[ \partial_\alpha \frac{\partial g}{\partial S_\alpha} = 0. \] (24)

The Eq. (21) reduces to the continuity equation. But, from Eq (23) we get

\[ 2R \left\{ \frac{(\nabla S)^2}{2m} + V + Q + \frac{\partial S}{\partial t} \right\} + R^2 \frac{\partial Q}{\partial R} - \partial_\alpha \left( R^2 \frac{\partial Q}{\partial R_\alpha} \right) + \partial_\alpha \partial_\beta \left( R^2 \frac{\partial Q}{\partial R_{\alpha\beta}} \right) - \partial_\alpha \partial_\beta \partial_\sigma \left( R^2 \frac{\partial Q}{\partial R_{\alpha\beta\sigma}} \right) + \cdots = 0. \] (25)

If we expect to have modified Hamilton-Jacobi equation in the form

\[ \frac{(\nabla S)^2}{2m} + V + Q + \frac{\partial S}{\partial t} = 0 \] (26)

then, we obtain an important condition that the general quantum potential must fulfil:

\[ R^2 \frac{\partial Q}{\partial R} - \partial_\alpha \left( R^2 \frac{\partial Q}{\partial R_\alpha} \right) + \partial_\alpha \partial_\beta \left( R^2 \frac{\partial Q}{\partial R_{\alpha\beta}} \right) - \partial_\alpha \partial_\beta \partial_\sigma \left( R^2 \frac{\partial Q}{\partial R_{\alpha\beta\sigma}} \right) + \cdots = 0; \quad \alpha, \beta, \sigma, \cdots \in \{0, 1, 2, 3\}. \] (27)

Now, as in Eq. (20) we assume that general quantum potential does not depend on time derivatives. Then, the Eq. (27) reduces to

\[ R^2 \frac{\partial Q}{\partial R} - \partial_\alpha \left( R^2 \frac{\partial Q}{\partial R_\alpha} \right) + \partial_\alpha \partial_\beta \left( R^2 \frac{\partial Q}{\partial R_{\alpha\beta}} \right) - \partial_\alpha \partial_\beta \partial_\sigma \left( R^2 \frac{\partial Q}{\partial R_{\alpha\beta\sigma}} \right) + \cdots = 0; \quad i, j, k, \cdots \in \{1, 2, 3\}. \] (28)
Note that the solutions \( Q[R] \) of this equation must be such that for every arbitrary function \( R \), the equation be satisfied. The simplest solution is

\[
Q = A = \text{const}
\]  
(29)

which is a trivial quantum potential. The next simple form is

\[
Q = AR^{-1}\delta^{ij}R_{ij}
\]  
(30)

which is the same as the Bohmian quantum potential. By a little effort we find all forms of \( Q \) in the forms

\[
Q_0 = A_0, \quad Q_2 = A_2R^{-1}\delta^{ij}R_{ij}, \quad Q_4 = A_4R^{-1}\delta^{ij}\delta^{kl}R_{ijkl}, \quad \cdots
\]  
(31)

which contain only even-order derivatives of \( R \), and satisfy the Eq. (28). We did not find an expression for \( Q \) which contains odd-order derivatives of \( R \) and which satisfies the condition (28). In fact, we looked for simple forms for \( Q \) and we didn’t find any solution. There may be some complicated forms of \( Q \) which contains odd-order derivatives of \( R \). But, we must pay attention that \( Q \) is a scalar and the derivatives of \( R \) must appear in the form of operator \( \nabla \). [5]

Therefore, the possible forms of quantum potential are

\[
Q_0 = A_0, \quad Q_2 = A_2R^{-1}\nabla^2R, \quad Q_4 = A_4R^{-1}(\nabla^2R)^2, \quad \cdots
\]  
(32)

in which \( A_0, A_2, A_4, \cdots \) are some constants.

V. ON THE MEANING OF NEW QUANTUM POTENTIALS

In the previous section we obtained new forms for quantum potential. Now, we take an assumption that the quantum phenomena arises from the combination of all quantum potentials, i.e. all quantum potentials contribute to the creation of quantum effects. Therefore, we can define the complete quantum potential as

\[
Q = R^{-1}(A_0R + A_2\nabla^2R + A_4(\nabla^2R)^2 + \cdots).
\]  
(33)

In this view, the Bohmian quantum potential \( Q_2 \) is only a first approximation to the complete quantum potential. The first term \( Q_0 \) is a constant and therefore has no role in the dynamics of particles. However, due to its dimension one can interpret it as the rest energy of the particle. But, what can we say about the other terms?

We know that the Schrödinger equation can describe all the non-relativistic quantum phenomena at the atomic level. Therefore, if these new quantum potentials have physical reality, they must carry some corrections at the atomic level. The mathematical form of the terms in (33) can help us in interpreting them. We know from the formalism of quantum mechanics that the operator \( \nabla \) in \( Q_2 \) is representative of momentum operator. It is natural to extend this assumption for all terms. Indeed, we can write the Bohmian term as

\[
Q_2 = \frac{1}{2}\varepsilon_0 \left( \frac{-i\hbar}{mc} \right)^2 R^{-1}\nabla^2R, \quad \varepsilon_0 = mc^2
\]  
(34)

therefore, we suggest the following form for all terms

\[
Q_{2n} = a_{2n}\varepsilon_0 \left( \frac{-i\hbar}{mc} \right)^{2n} R^{-1}\nabla^{2n}R, \quad \varepsilon_0 = mc^2, \quad n \in \{0, 1, 2, 3, \cdots \}
\]  
(35)

in which \( \varepsilon_0 \) is the rest energy, the coefficients \( a_{2n} \) are some dimensionless constants, and we used definition

\[
\nabla^{2n} = \nabla^2\nabla^2\cdots\nabla^2, \quad n \text{ times}
\]  
(36)
Consequently, the complete quantum potential becomes

$$Q = \varepsilon_0 R^{-1} \sum_{n=0}^{\infty} a_{2n} \left( \frac{-i\hbar \nabla}{mc} \right)^{2n} R.$$  \hspace{1cm} (37)

Among the coefficients $a_{2n}$ we only know $a_2 = 1/2(n = 1)$. Rewriting this suggested form of $Q$ as

$$Q = \varepsilon_0 R^{-1} \sum_{n=0}^{\infty} a_{2n} \left( \frac{\hat{p}_c}{\varepsilon_0} \right)^{2n} R$$  \hspace{1cm} (38)

we observe that by choosing proper values for $a_{2n}$:

$$a_{2n} = (-1)^{n+1} \frac{1}{2n} \left( \frac{(2n)!}{2^{2n} (n!)^2} \right), \quad n = 0, 1, 2, 3, \ldots$$  \hspace{1cm} (39)

the operator

$$\varepsilon_0 \sum_{n=0}^{\infty} a_{2n} \left( \frac{\hat{p}_c}{\varepsilon_0} \right)^{2n}$$  \hspace{1cm} (40)

becomes exactly the relativistic energy operator

$$\hat{\varepsilon} \equiv \varepsilon_0 \left[ 1 + \left( \frac{\hat{p}_c}{\varepsilon_0} \right)^2 \right]^{1/2}.$$  \hspace{1cm} (41)

This is an important point because, as we explain in the next section, we can get corrections to be exactly same as relativistic corrections to the energy, but now as effect of non-Bohmian quantum potentials. In other words, by choosing proper values for the coefficients of new quantum potentials, we can have corrections equal to relativistic corrections even in the framework of non-relativistic quantum mechanics, and without appealing to relativistic concepts. Identifying the operator (40) with (41), we have for $Q_4$

$$Q_4 = -\frac{1}{8} \frac{\hbar^4}{m^3 c^2} \frac{\nabla^2 (\nabla^2 R)}{R}.$$  \hspace{1cm} (42)

In the next section, we describe the effect of this term on the energy levels of atoms.

**VI. RELATIVISTIC CORRECTIONS TO ENERGY ARISE FROM NON-BOHMIAN QUANTUM POTENTIALS**

First, we note that for stationary states the time integral in the Eq. (22) becomes a coefficient and therefore the variational principle reduces to

$$\delta \int g d^3 x = \delta \int R^2 \left\{ \frac{(\nabla S)^2}{2m} + V + Q - E \right\} d^3 x = 0.$$  \hspace{1cm} (43)

Now, suppose that we have for first approximation

$$Q = Q_0 + Q_2$$  \hspace{1cm} (44)

and a set of $R_0$, $S_0$ and $E_0$ satisfy the equation (43). Then, we have
\[ \int g_0 d^3x = \int R_0^2 \left\{ \frac{(\nabla S_0)^2}{2m} + V + Q_0 + Q_2 - E_0 \right\} d^3x. \]  \hspace{1cm} (45)

Suppose that the problem with \( Q_4 \) yield small shifts in energy and that \( R_0, S_0 \) remain approximately as before, we can write for \( Q_4 \)

\[ Q_4 \simeq -\frac{1}{8} \frac{\hbar^4}{m^3c^2} \frac{\nabla^2(\nabla^2 R_0)}{R_0}. \]  \hspace{1cm} (46)

and we have

\[ \int g d^3x = \int R_0^2 \left\{ \frac{(\nabla S_0)^2}{2m} + V + Q_0 + Q_2 + Q_4 - (E_0 + \Delta E) \right\} d^3x. \]  \hspace{1cm} (47)

Assuming

\[ \Delta E = \int R_0^2 Q_4 d^3x \]  \hspace{1cm} (48)

the integral (47) reduces to (45) and therefore satisfies the variational principle (43). If we rewrite the Eq. (48) as

\[ \Delta E = \int R_0^2 \left\{ -\frac{1}{8} \frac{\hbar^4}{m^3c^2} \frac{\nabla^2(\nabla^2 R_0)}{R_0} \right\} d^3x \]  \hspace{1cm} (49)

and compare it with the ‘relativistic correction’(in the usual quantum mechanics)

\[ \Delta E_{rel} = \int \psi^*_0 \left\{ -\frac{(\hat{p}^2)^2}{8m^3c^2} \right\} \psi_0 d^3x \]  \hspace{1cm} (50)

then we observe that if \( S_0 = const \), the Eqs. (50) and (49) becomes identical and the value of \( \Delta E \) becomes exactly the relativistic correction shift in energy.

Now, we consider the energy eigenstates of central potentials. These states are written as

\[ \psi = R_{nl}(r)P_l^m(\cos \theta)e^{im\phi}. \]  \hspace{1cm} (51)

But, there is a degeneracy due to the spherical symmetry of the potential in quantum number \( m \). Due to this degeneracy, we can write the state functions as

\[ \psi = R_{nl}(r)P_l^m(\cos \theta) \cos m\phi \]
\[ \psi = R_{nl}(r)P_l^m(\cos \theta) \sin m\phi \]  \hspace{1cm} (52)

in place of (51). Therefore, it is possible to have \( S_0 = const \) for all energy eigenstates of the central potential. Consequently, we can assume that electrons in all eigenstates of energy are at rest and the ‘relativistic correction’ comes from non-Bohmian quantum potential \( Q_4 \). Indeed, by considering higher order terms in the quantum potential \( Q_4 \) we can get the relativistic corrections to every order.

Note that, the fact that \( Q_4 \) is proportional to \( R^{-1} \nabla^2(\nabla^2 R) \) or \( R^{-1}(\hat{p}^2)^2R \) is a consequence of generalization of quantum potential in the framework of non-relativistic Bohmian mechanics. Choosing a proper value for the coefficient \( A_4 \) in \( Q_4 \), this term can yield a correction to energy that is exactly equal to what we call ‘the relativistic correction’.
We know from ordinary Bohmian mechanics that the set of equations (3) and (4) for Bohmian quantum potential is equivalent to the Schrödinger equation (1). But, by including non-Bohmian quantum potentials in the modified Hamilton-Jacobi equation, one can not obtain a linear Schrödinger-like equation for the wave function \( \psi = R \exp(iS/\hbar) \). Therefore, the Schrödinger equation remains as an approximation. Indeed, one can combine the continuity and modified Hamilton-Jacobi equations in the presence of complete quantum potential to obtain the equation

\[
\frac{i\hbar}{\partial t} \psi = \left( -\frac{\hbar^2}{2m} \nabla^2 + V + Q - Q_2 \right) \psi = \left( -\frac{\hbar^2}{2m} \nabla^2 + V + Q_0 + Q_4 + Q_6 + \cdots \right) \psi.
\]

(53)

We observe that, only when \( Q \simeq Q_0 + Q_2 \), we have a linear equation. But, we remember that the Shrödinger equation describes well all non-relativistic quantum phenomena at the atomic level. Therefore, the effects of non-Bohmian quantum potentials \( Q_1, Q_2, \cdots \) at the atomic level in comparison with the effect of Bohmian quantum potential \( Q_2 \) must be small and consequently the Schrödinger equation and its linearity remains a good approximation. This requirement must be considered in the interpretation of non-Bohmian quantum potentials and the suggestion for their coefficients. This is the case for our suggested interpretation of new quantum potentials. For example consider a particle in a one-dimensional box with the length \( L \). Assuming eigenfunction \( \sin(\tau \pi x/L) \) we have

\[
\frac{Q_{2(n+1)}}{Q_{2n}} = \frac{a_{2(n+1)}}{a_{2n}} \left( \frac{\tau \pi \hbar}{m c L} \right)^2 = \frac{a_{2(n+1)}}{a_{2n}} \tau^2 \left( \frac{\lambda_c}{2L} \right)^2.
\]

(54)
in which \( \tau \) is quantum number of energy level and \( \lambda_c \) is Compton wavelength. For an electron in the atomic dimensions \( L \sim A \), the proportion \( (\lambda_c/2L)^2 \) is of the order of \( 10^{-4} \), but for a proton in nuclear dimensions \( L \sim 10^{-5} A \), it is of the order of \( 10^{-1} \). Therefore, the effects of new quantum potentials at the atomic level are in the form of small perturbations but at the nuclear level these effects are remarkable. Note that, for a proton too in the atomic dimensions \( \sim A \), these effects are small. Therefore, we conclude that the Schrödinger equation is adequate at the atomic level but at the nuclear level this equation is not necessarily adequate.

We must mention that Bohm itself foresaw the possibility of including nonlinear terms to the Schrödinger equation which are large only for processes involving small distances [1]. Our suggestion for the coefficients of new quantum potentials has this merit that the effects of higher order quantum potentials become large only for the dimensions which are smaller than the atomic distances. Therefore, the linearity of Schrödinger equation and the superposition of states can be preserved only at the atomic level or larger dimensions. If our interpretation of new quantum potentials is correct, we can expect some highly nonlinear effects at the nuclear or smaller dimensions. The lack of superposition affects the global structure of quantum mechanics. For example, we expect some serious effects on the EPR-Bell type nonlocal correlations at the nuclear level. However, some properties of Bohmian quantum potential such as its not falling with distance and its dependence on the form of wave function are preserved. Bohm described the apparent jumps and discontinuity of atomic processes by the behavior of quantum potential. According to Bohm, due to the form of dependence of (Bohmian) quantum potential on the wave function, even when the amplitude of wave function is very small, we can expect some large energy exchanges at very short times. This property is preserved for new quantum potentials, due to their dependence on \( R \). Therefore, the existence of sharp energy levels and the jumping between them at the nuclear level, which have experimental evidence, are permitted by non-Bohmian quantum potentials. In a following paper, we shall discuss in details the main effects of new quantum potentials, such as nonlinearity, and the effects on the global structure of quantum theory and phenomena such as EPR-Bell nonlocality.

VIII. CONCLUSION

In this article we obtained new forms for quantum potential and assumed that quantum effects are due to the combination of all quantum potentials. Due to mathematical form of the series of complete quantum potential, we suggested that the coefficients of quantum potentials can be chosen so that they yield corrections equal to the relativistic corrections to energy. An important point about our approach is that the mathematical form of the new
quantum potentials (i.e. the functional dependence of them with respect to $R$) is a consequence of generalization of quantum potential in the framework of non-relativistic Bohmian mechanics, without any reference to relativistic concepts. We observed that choosing proper values for the coefficients makes the effects of non-Bohmian quantum potentials mathematically equivalent to what we get from relativistic effects. The importance of this observation is that to obtain the relativistic corrections to energy there is no necessity to start from relativistic considerations. For a better understanding of this subject consider classical electrodynamics. The classical electrodynamics was completed before relativity and was a motivation for it. This means that for the derivation of the "correct" (i.e. relativistic) theory of electrodynamics there is no need to start from comparing one observer to another. The Maxwell’s equations are all consequences of experiments that are done in the reference frames which are at rest relative to the Earth. But these experiments were sufficient to yield the correct equations of electrodynamics. Our considerations about new quantum potentials may be similar to this situation. There may be a possibility to obtain "correct" mechanical equations without using relativistic "concepts" and without comparing observers with each other, and the consideration of non-Bohmian quantum potentials may be only a first step for realizing this demand.

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[5] Indeed, if we restrict $Q[R]$ to take only derivatives up to second order, we can prove that the only possible forms of $Q[R]$ are (29) and (30) (to be published). The extension of the proof to cover the forms (31) is under consideration.