Indirect methods constraining nuclear capture - the Trojan Horse Method

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Abstract. Reaction rates of nuclear processes of astrophysical relevance can be inferred using the Trojan Horse Method. This indirect technique is a valid alternative to direct measurements in particular when extremely low cross sections are involved. We will review its basic features in the framework of the theory of direct reactions and address the physics case of the $^{12}\text{C}+^{12}\text{C}$ fusion.

1. Introduction

A critical issue in nuclear astrophysics is Coulomb repulsion between like charges, responsible for the exponential decrease of the cross section $\sigma(E)$ of the relevant nuclear reactions at astrophysical energies. Thus, extrapolation from the higher energies is often the only way to reach these low energies. Extrapolation is done by means of the astrophysical S(E)-factor

$$S(E) = E \sigma(E) \exp(2\pi\eta),$$

(1)

with $\eta$ the Coulomb parameter of the colliding nuclei, and $\exp(2\pi\eta)$ the inverse of the Gamow factor that removes the Coulomb dependence of $\sigma(E)$. However, extrapolation can be source of additional uncertainties for $\sigma(E)$ due, for instance, to the presence of unexpected resonances. Another critical issue in the laboratory measurements is represented by the electron screening effect that leads to an increased cross section for screened nuclei, $\sigma_s(E)$, compared to the cross section for bare nuclei, $\sigma_b(E)$ [1, 2]. Therefore, the so called screening factor, defined as

$$f_{\text{lab}}(E) = \sigma_s(E)/\sigma_b(E) \approx \exp(\pi\eta U_e/E),$$

(2)
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where $U_e$ is the so-called "electron screening potential" [1, 2], has to be taken into account to determine the bare nucleus cross section. Indeed, this is the key parameter to determine a reaction rate and the only way to get it measured is via indirect methods [3, 4, 5] and references therein). They make use of direct reaction mechanisms, such as transfer processes (stripping and pick-up) and quasi-free reactions (knock-out reactions). In particular, the Trojan Horse Method (THM) ([3, 6, 7] and references therein) has been successfully applied many times in the last two decades to reactions connected with fundamental astrophysical problems. Here we present some of the basic ideas of the THM.

2. Trojan Horse Method in short

The THM is based on the selection of QF contribution of an appropriate three-body reaction $A + a \rightarrow c + C + s$, performed at energies well above the Coulomb barrier, to obtain the low-energy cross section of a charged particle two-body process $A + x \rightarrow c + C$ relevant for astrophysics. To this aim, the nuclear reaction theory is applied assuming that the nucleus $a$ can be described in terms of the $x \oplus s$ cluster structure. Thanks to the high energy in the $A + a$ entrance channel, the two body interaction can be considered as taking place inside the nuclear field, without experiencing either Coulomb suppression or electron screening effects. The $A + a$ relative motion is compensated for by the $x - s$ binding energy, determining the so called "quasi-free two-body energy" given by

$$E_{q.f.} = E_{Aa} - B_{x-s}$$

(3)

where $E_{Aa}$ represents the beam energy in the center-of-mass system and $B_{x-s}$ is the binding energy for the $x - s$ system. Then, a cutoff in the momentum distribution, which is related to the Fermi motion of $s$ inside the Trojan-horse $a$, fixes the range of energies around the "quasi-free two-body energy" accessible in the astrophysical relevant reaction. In the Impulse Approximation either in Plane Wave or in Distorted Wave (this does not change the energy dependence of the two-body cross section but only its absolute magnitude), the three body-cross cross section can be factorized as:

$$\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C} \propto |KF| \phi_a(p_{sx})|^2 \left( \frac{d\sigma}{d\Omega_{c.m.}} \right)_{\text{HOES}}$$

(4)

where KF is a kinematical factor containing the final state phase-space factor. It is a function of the masses, momenta and angles of the outgoing particles; $\phi_a(p_{sx})$ is proportional to the Fourier transform of the radial wave function $\chi(r)$ for the $x - s$ inter-cluster relative motion; $(d\sigma/d\Omega_{c.m.})_{\text{HOES}}$ is the half-off-energy-shell (HOES) differential cross section for the binary reaction at the center of mass energy $E_{c.m.}$ given in post-collision prescription by

$$E_{c.m.} = E_{cC} - Q_{2b}.$$

(5)

Here, $Q_{2b}$ is the $Q$-value of the binary reaction and $E_{cC}$ is the relative energy of the outgoing particles $c$ and $C$. 


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For resonant two-body reactions, \((d\sigma/d\Omega_{c.m.})^{\text{HOES}}\) has to be worked out to determine the corresponding on-energy-shell (OES) \(S(E)\) factor. The corresponding theoretical formalism has been recently developed in a very rigorous way \([11]\) leading to the so called modified R-matrix approach that accounts for HOES effects due to the virtual nature of particle \(x\). By fitting the experimental THM cross section, the reduced width amplitude \(\gamma\) for entrance and exit channels, energy levels and energy shifts can be deduced and used to determine the astrophysical \(S(E)\) factor, since these parameters are the same in both direct and THM data. In this way, an exact parameterization of the astrophysical \(S(E)\) factor can be obtained overcoming the extrapolation. If the resonance parameters of a single level in the relevant energy region are known, they can be fixed in the fitting procedure to obtain directly the astrophysical \(S(E)\) factor in absolute units.

Several test studies have been performed in the past years to validate the THM \([8, 9, 10]\). The invariance of the two-body reaction amplitude was validated by changing the Trojan Horse nucleus that brings the participant cluster \(x\) \([13, 14, 15]\), and the momentum distributions from Distorted Wave Born Approximation was several times compared with the simple PWA shape, providing same results within experimental errors \([16, 17]\). The THM has been applied to many reactions of astrophysical interest connected to fundamental problems in different scenarios, from BBN nucleosynthesis \([18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]\) to AGB and more explosive sites \([37, 16, 38, 17, 39, 40, 41, 42, 43, 44, 45]\). In the last years, reactions involving heavier systems, such as \(^{12}\text{C}\) and \(^{16}\text{O}\) have been investigated \([46, 47]\).

3. The \(^{12}\text{C}+^{12}\text{C}\) fusion in nuclear astrophysics

The \(^{12}\text{C}+^{12}\text{C}\) fusion is a key reaction in a wide range of scenarios in carbon rich environments. In particular, it shapes star evolution and nucleosynthesis of intermediate mass and massive stars (\(\geq 8 \, M_\odot\)) \([48]\). It influences also the lower stellar mass limit for carbon ignition. This limit separates the progenitors of white dwarfs, novae and type Ia supernovae, from those of core-collapse supernovae, neutron stars, and stellar mass black holes; it constrains superbursts model with neutron and strange stars, in particular if resonances are found to contribute in the Gamow peak \([49]\); it influences the weak component of the s process, which produces the elements between Fe and Sr. Carbon burning during the hydrostatic phase takes place from 0.8 to 1.2 GK, corresponding to center-of-mass energies from 1 to 3 MeV. The whole region corresponds to sub-Coulomb energies and the cross section falls rapidly below nanobarns. For this reason, the measurement of the cross section below a center of mass energy \(E_{\text{cm}}\) of 2 MeV was never performed. The compound nucleus \(^{24}\text{Mg}\) is formed at an excitation energy above the particle decay threshold. Alpha, proton and neutron are the dominant evaporation channels, leading respectively to \(^{20}\text{Ne}\), \(^{23}\text{Na}\) and \(^{23}\text{Mg}\), which can also be produced in excited bound states. Below 2.5 MeV there is not enough energy to feed \(^{23}\text{Mg}\) even in its ground state and \(\alpha\) and \(p\) channel are the only relevant ones at low energies.
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The $^{12}\text{C}+^{12}\text{C}$ fusion cross section at the relevant energies was thus determined from the indirect measurement of the $^{12}\text{C}(^{12}\text{C},\alpha)^{20}\text{Ne}$ and $^{12}\text{C}(^{12}\text{C},p)^{23}\text{Na}$ reactions via the THM applied to the $^{12}\text{C}(^{14}\text{N},\alpha^{20}\text{Ne})^{2}\text{H}$ and $^{12}\text{C}(^{14}\text{N},p^{23}\text{Na})^{2}\text{H}$ three-body processes in the quasi-free (QF) kinematics regime, where $^{2}\text{H}$ from $^{14}\text{N}$ is spectator to the $^{12}\text{C}+^{12}\text{C}$ two-body processes [47]. The experiment was performed at the INFN - Laboratori Nazionali del Sud in Catania, Italy. A $^{14}\text{N}$ beam accelerated at 30 MeV by the SMP TANDEM was delivered onto a 100 $\mu$g/cm$^2$ $^{12}\text{C}$ target with a beam spot on target smaller than 1.5 mm. The experimental setup consisted of two telescopes (38 $\mu$m silicon detector as $\Delta E$- and 1000 $\mu$m position sensitive detector (PSD) as E-detector) placed on both sides with respect to the beam direction in symmetric configuration (two on each side), covering angles from 7$^\circ$ to 30$^\circ$. The ejectile of the two-body reactions (either $\alpha$ or $p$) was detected in coincidence with the spectator $d$ particle. The angular regions covered by the detectors were optimized for the QF kinematics of the break-up process of interest, and the investigated range of deuteron momentum values was feasible to check the existence of the QF mechanism.

Several steps are involved in the data analysis (see [47]) and after their completion the two-body cross section of astrophysical relevance was extracted for four channels: $^{20}\text{Ne}+\alpha_0$, $^{20}\text{Ne}+\alpha_1$, $^{23}\text{Na}+p_0$ and $^{23}\text{Na}+p_1$. The yield for the $^{20}\text{Ne}+\alpha_1$ channel is shown in Fig. 1 (black solid dots) projected onto the $^{12}\text{C}+^{12}\text{C}$ relative energy variable, $E_{cm}$. A modified one-level many-channel R-matrix analysis was carried out including the $^{24}\text{Mg}$ states reported in [47]. According to the results of [50] at $E_{c.m.}\leq 3$ MeV, and monitoring the decrease of the penetration factors for the relevant states, the fraction of the total fusion yield from $\alpha$ and $p$ channels other than $\alpha_0,1$ and $p_0,1$ was neglected in the modified R-matrix analysis with estimated errors at $E_{c.m.}$ below 2 MeV lower than 1% and 2% for the $\alpha$ and $p$ channels, respectively.

The THM reduced widths thus entered a standard R-matrix code and the S(E) factors for the four reaction channels were determined. The results are shown in Fig. 2 for the $^{20}\text{Ne}+\alpha_1$ in terms of modified S(E) factor, $S(E)^*$, [51]. The black middle line and the grey band represent the best fit curve and the range defined by the total uncertainties, respectively. The grey band is the result of R-matrix calculations with lower and upper values of the resonance parameters provided by their errors.

The resonant structures are superimposed onto a flat nonresonant background taken from [52]. Data show some tendency for the even $J$ states to be clustered around 1.5 MeV and this might be a sign of intermediate structure of $^{24}\text{Mg}$ associated with a $^{12}\text{C}+^{12}\text{C}$ molecular configuration. Normalization to direct data was done in the $E_{c.m.}$ window 2.5-2.63 MeV of the $^{20}\text{Ne}+\alpha_1$ channel where a resonance corresponding to the level of $^{24}\text{Mg}$ at 16.5 MeV shows up and available data [52, 53, 54, 55] in this region are the most accurate among those available in the full overlapping region with THM data. The resulting normalization error is 5%. All the existing direct data below $E_{c.m.}= 3$ MeV are shown as blue filled circles [52], purple filled squares [53], blue empty diamonds [54], red filled stars [55] and green filled triangles [56]. Except for the data
from [52], their low energy limit is fixed by background due to hydrogen contamination in the targets. Disregarding these cases, agreement between THM and direct data is apparent within the experimental errors except for the direct low-energy limit around 2.14 MeV, where THM data do not confirm the claim of a strong resonance, rather a nearby one at 2.095 MeV about one order of magnitude less intense in the $^{20}\text{Ne}+\alpha_1$ channel (see Fig. 2) and with similar intensity in the $^{23}\text{Na}+p_1$ one. Further agreement is found with the unpublished experimental data down to $E_{\text{c.m.}}=2.15$ MeV from [59] for the $^{12}\text{C}(^{12}\text{C},p_{0,1})^{23}\text{N}$ reactions and with the low-energy limit provided in [60] for the p-channel. Our result is also consistent within experimental errors with the total $S(E)\ast$ from recent experiments [61, 62].

In a very recent theoretical paper [63], it was claimed that Coulomb effects, not included in our PWIA approach, can change the behaviour of the astrophysical factors. With the new theory, the authors predict a strong decrease of the astrophysical factors, which however does not find agreement with direct data available in the overlapping range.

The corresponding reaction rate is shown in Fig. 3 divided by the reference rate from [64]. It experiences a variation below 2 GK with an increase from a factor of 1.18 at 1.2 GK to a factor of more that 25 at 0.5 GK. The latter increase, due mainly to the resonant structure around $E_{\text{c.m.}}=1.5$ MeV, endorses the fiducial value conjectured in [49] to reduce down to a factor of 2 the theoretical superburst ignition depths in
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![Figure 2. Ratio between the total THM $^{12}$C+$^{12}$C reaction rate (black line) and the reference one (red line) from [64]. The grey shading defines the region spanned owing to the ±1σ uncertainties.](image)

accreting neutron stars for a realistic range of crust thermal conductivities and core Urca neutrino emissivities. This change is compatible with the observationally inferred superburst ignition depths. In other words, carbon burning can trigger superbursts. As for the hydrostatic carbon burning regime (0.6 to 1.2 GK), the present rate change will lower temperatures and densities at which $^{12}$C ignites in massive post-main-sequence stars. Profiting of the stellar modeling reported in [65], for core C-burning of a star of $25M_\odot$, the ignition temperature and density would undergo a decrease of down to 10% and 30% respectively. Recently, the impact of the new carbon fusion cross sections on Type Ia Supernovae was investigated in [66]. Their progenitors are not well understood. One popular scenario is the double-degenerate (DD) scenario, which attributes SNe Ia to WD-WD binary mergers. The resonance contribution results in a decrease of the carbon burning ignition temperature. Thus, accretion induced collapse occurs more easily and increases the birthrate of Galactic neutron stars with the contribution of the DD scenario to the SNe Ia rate becoming even smaller.

References

[1] H.J. Assenbaum et al., 1987 Z. Phys. A 327 461.
[2] F. Strieder et al., 2001 Naturwissenschaften 88 461.
[3] C. Spitaleri et al., 2011 Phys. At. Nucl., 74, 1763.
[4] A. Tumino et al., 2013 Few Body Syst., 54, Issue 7-10, 869.
[5] R. Tribble et al., 2014 Rep. Prog. Phys. 77, Issue: 10 106901.
Indirect methods constraining nuclear capture - the Trojan Horse Method

[6] A. Tumino et al., 2013 Few Body Syst., 54, Issue 5-6 745.
[7] C. Spitaleri et al., 2019 Eur. Phys. J. A 55 161.
[8] A.M. Mukhamedzhanov et al., 2006 Eur. Phys. J. A 27 205.
[9] A. Tumino et al., 2007 Phys. Rev. Lett. 98 252502.
[10] A.Tumino et al., 2008 Phys. Rev. C 78 064001.
[11] A.M. Mukhamedzhanov, 2011 Phys. Rev. C 84 044616.
[12] C. Spitaleri et al., 2015 Phys. Rev. C 91 024612.
[13] A. Tumino et al., 2006 Eur. Phys. J. A direct, 1 DOI: 10.1140/epja/i2006-08-038-1.
[14] R.G. Pizzone et al., 2011 Phys. Rev. C 83, 045801.
[15] R.G. Pizzone et al., 2013 Phys. Rev. C 87, 025805.
[16] M.L. Sergi et al., 2012 Phys.Rev. C R82, 032801.
[17] M. La Cognata et al., 2011 Astrophysical J. Lett. 739, L54.
[18] A. Tumino et al., 2003 Phys. Rev. C 67 065803.
[19] R.G. Pizzone et al., 2003 Astronomy & Astrophysics 398, 423.
[20] A. Tumino et al., 2004 Prog. Theor. Phys. Suppl. 154 341.
[21] A. Rinollo et al., 2005 Nucl. Phys. A 758 146.
[22] Qun-Gang Wen, 2008 et al., Phys. Rev. C 78 035805.
[23] Qun-Gang Wen, 2011 et al., J.Phys. G: Nucl. Part. Phys., 38, 085103.
[24] A. Tumino et al., 2011 Phys. Lett. B 700, 111.
[25] A. Tumino et al., 2011 Phys. Lett. B 705, 546.
[26] L. Lamia et al., 2013 Astrophysical J. 768, 65.
[27] A. Tumino et al., 2014 Astrophysical J. 785, 96.
[28] R.G. Pizzone et al. 2014 Astrophysical Journal, 786 ,112.
[29] C. Spitaleri et al., 2014 Phys. Rev. C 90 035801.
[30] Li Chengbo et al., 2015 Phys. Rev. C 92, 025805.
[31] C. Spitaleri et al., 2015 Phys. Rev. C 91, 024612.
[32] L. Lamia Astrophysical J., 2017 850, 175.
[33] C. Spitaleri et al., 2017 Phys. Rev. C 95, 035801.
[34] Li Chengbo et al., 2017 Phys. Rev. C 95, 035804.
[35] G.G. Rapisarda et al., 2018 Eur. Phys. J. A, 54 189.
[36] L. Lamia et al., 2019 Astrophysical Journal, 879, 23.
[37] M. La Cognata et al., 2008 Phys. Rev. Lett. 101 152501.
[38] M. La Cognata et al., 2010 Astrophysical J. 708 796 .
[39] M. La Cognata et al., 2013 Phys. Rev. Lett. 49, 015106.
[40] S. Cherubini et al., 2015 Phys. Rev. C 92 015805.
[41] R.G. Pizzone et al. 2017 Astrophysical Journal, 836, 57.
[42] I. Indelicato et al. 2017 Astrophysical Journal, 845, 19.
[43] G. D’Agata et al., 2018 Astrophysical J. 860, 61.
[44] V. Burjan et al., 2019 Eur. Phys. Journ. A 55, 114.
[45] G.L. Guardo et al., 2019 Eur. Phys. Journ. A 55, 211.
[46] A. Tumino et al., 2001 Eur. Phys. Journ. A, 12 327.
[47] A. Tumino et al., 2018 Nature 557, 687.
[48] E. Garcia-Berro et al., 1997 Astrophys. J., 286, 765.
[49] R.L. Cooper et al., 2009 Astrophys. J., 702, 660.
[50] H.W. Becker, K.U. Kettner, C. Rolfs & H.P. Trautvetter, 1981 Z. Phys. A, 303 305.
[51] E.F. Aguilera et al., 2006 Phys. Rev. C 73 064501.
[52] T. Spillane et al., 2007 Phys. Rev. Lett. 98, 122501.
[53] M.G. Mazararakis & W.E. Stephens 1973 Phys. Rev. C, 7 1280.
[54] M.D. High, B. Cujec, 1977 Nucl. Phys. A 282 181
[55] K.U. Kettner, H. Lorenz-Wirzba & C. Rolfs, 1980 Z. Phys. A 298 65.
[56] L. Barrón-Palos et al., 2006 Nucl. Phys. A 779 318.
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[57] R. Abegg & C.A. Davis, 1991 Phys. Rev. C 43 6.
[58] A. Caciolli et al., 2008 NIM B 266 1932.
[59] J. Zickefoose, Thesis, University of Connecticut, 2011; Doctoral Dissertation AAI3485448, http://opencommons.uconn.edu/dissertations/AAI3485448.
[60] J. Zickefoose et al., 2018 Phys. Rev. C 97 065806.
[61] C.L. Jiang et al., 2018 Phys. Rev. C 97 012801(R).
[62] C. Beck, 2019 arXiv:1812.08013
[63] A. M. Mukhamedzhanov, D.Y. Pang, and A. S. Kadyrov, 2019 Phys. Rev. C 99, 064618.
[64] J.R. Caughlan & W.A. Fowler, 1988 At. Data Nucl. Data Tables 40, 283.
[65] M. Pignatari et al., 2013 Astrophys. J. 762 31.
[66] K. Mori et al., https://arxiv.org/pdf/1810.01025.