CLUMPING OF CDM FROM THE COSMOLOGICAL QCD TRANSITION

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Abstract. The cosmological QCD transition affects primordial density perturbations. If the QCD transition is first order, the sound speed vanishes during the transition and density perturbations fall freely. For scales below the Hubble radius at the transition the primordial Harrison-Zel’dovich spectrum of density fluctuations develops large peaks and dips. These peaks grow with wave number for both the radiation fluid and for cold dark matter (CDM). The peaks in the radiation fluid are wiped out during neutrino decoupling. For cold dark matter that is kinetically decoupled at the QCD transition (e.g. axions) these peaks lead to the formation of CDM clumps of masses $10^{-20}M_\odot < M < 10^{-10}M_\odot$.

1. Introduction

The transition from hot deconfined quarks and gluons to hot hadrons happens in the early Universe at a temperature $T_\ast \sim 150$ MeV. Lattice QCD results for the physical values of the quark masses indicate that the QCD phase transition is of first order.

For a first-order QCD phase transition the sound speed vanishes during the coexistence of the deconfined and confined phases. The important scales are the mean distance between the hadronic bubbles at their nucleation, $R_{\text{nuc}}$, which is a few cm for homogeneous nucleation, and the Hubble scale $R_H \sim m_p/T_\ast^2 \sim 10^4$ m. Cosmological density perturbations are affected by the QCD transition at scales $\lambda \lesssim R_H$. At the coexistence temperature $T_\ast$ the pressure $p_* = p(T_\ast)$ is fixed. Therefore pressure gradients and the sound speed vanish for wavelengths much larger than the typical bubble separation. This gives rise to large peaks and dips above the primordial spectrum of density perturbations in the radiation fluid. These peaks grow at most linearly with wave number.

At $T \sim 1$ MeV the neutrinos decouple from the radiation fluid. During this decoupling the large peaks in the radiation spectrum are wiped out by collisional damping.

Today the universe is dominated by dark matter, most likely cold dark matter (CDM). If CDM is kinetically decoupled from the radiation fluid at the QCD transition, the density perturbations in CDM do not suffer from the neutrino damping. Examples for kinetically decoupled CDM are primordial black holes, axions, or WIMPzillas, but not supersymmetric dark matter and heavy neutrinos.
At the time of the QCD transition the energy density of CDM is small, i.e. \( \rho_{\text{CDM}}(T_\star) \sim 10^{-8} \rho_{\text{RAD}}(T_\star) \). Kinetically decoupled CDM falls into the potential wells provided by the dominant radiation fluid. Thus, the CDM spectrum is amplified on subhorizon scales. The peaks in the CDM spectrum go nonlinear shortly after radiation-matter equality. This leads to the formation of CDM clumps with masses \( 10^{-20} M_\odot < M < 10^{-10} M_\odot \). If the QCD transition is strong enough, these clumps could be detected by gravitational femtolensing. The clumping of kinetically decoupled CDM implies that axions, if they are the CDM, could hide from axion search experiments.

2. Kinetically decoupled cold dark matter

Let us discuss the properties of some popular CDM candidates at the QCD scale.

The lightest supersymmetric particle, most probably the neutralino, is one of the candidates for CDM. Constraints from LEP 2 and cosmology, together with the assumption of universality at the GUT scale, show that the neutralino mass is \( m_\chi > 42 \text{ GeV} \). It is essential to distinguish between the chemical freeze-out and the kinetic decoupling of neutralinos. The chemical freeze-out happens when the annihilation rate of neutralinos drops below the Hubble rate, \( \Gamma_{\text{ann}} / H < 1 \). At freeze-out the rate for neutralino annihilation, \( \Gamma_{\text{ann}} = \langle v \sigma_{\text{ann}} \rangle n_\chi \), is suppressed by the Boltzmann factor in the number density of the neutralinos, \( n_\chi \sim (m_\chi T)^{3/2} \exp(-m_\chi / T) \). The freeze-out temperature of the neutralino is \( T_f \sim m_\chi / 20 > 2 \text{ GeV} \).

Kinetically decoupling, in contrast, is determined by the elastic scattering between neutralinos and the dominant radiation fluid. The interaction rate for elastic scattering is \( \Gamma_{\text{el}} = \langle v \sigma_{\text{el}} \rangle n \), where \( n \sim T^3 \) is the number density of relativistic particles. The kinetic decoupling of the neutralino happens at the 10 MeV scale and therefore neutralinos belong to the radiation fluid at the QCD transition. The same holds true for heavy neutrinos with \( m_Z / 2 < m_\nu < 1 \text{ TeV} \).

Superheavy WIMPs, WIMPzillas with masses \( 10^{12} - 10^{16} \text{ GeV} \), decouple from the radiation fluid well before the QCD transition. Although WIMPzillas scatter with the radiation fluid at \( T_\star \), the momentum transfer in these interactions is tiny compared to the WIMPzilla’s momentum. Even the large number of scatterings per Hubble time cannot change the momenta of WIMPzillas significantly (if \( m_{\text{WIMP}} > 10^6 \text{ GeV} \)). Thus, WIMPzillas cannot be dragged along by the radiation fluid.

Axions could be the dominant matter today if their mass is small, i.e. \( m_a \sim 10^{-5} \text{ eV} \), which corresponds to a breaking of the Peccei-Quinn (PQ) symmetry at the scale \( f_{\text{PQ}} \sim 10^{12} \text{ GeV} \). These axions could be produced coherently due to a initial misalignment of the axion field and by the decay of axionic strings. The initially misaligned axion field starts to oscillate coherently when the axion mass has grown to \( m_a(T_1) \sim 3H(T_1) \), where \( T_1 \sim 1 \text{ GeV} \). Below \( T_1 \sim 1 \text{ GeV} \) the oscillating axion field evolves as CDM.

If the reheating temperature after inflation lies above \( f_{\text{PQ}} \), the axion field is inhomogeneous on scales larger \( R_H(T_1) \). These inhomogeneities produce axion miniclusters with mass \( M_{\text{mc}} \sim 10^{-12} M_\odot \) and radius \( R_{\text{mc}} \sim 0.1 R_\odot \) today.

A further candidate for CDM that is kinetically decoupled at the QCD scale is primordial black holes (PBH) produced before the QCD transition and therefore with masses \( M_{\text{BH}} \ll 1 M_\odot \). In order to survive until today PBH should have \( M_{\text{BH}} > 10^{15} \text{ g} \approx 10^{-18} M_\odot \). PBH in the range from \( 10^{-18} M_\odot \) to \( 10^{-16} M_\odot \) would radiate too strongly to be compatible with \( \gamma \)-ray observations.
3. Sound speed

The QCD phase transition starts with a short period \(10^{-4} t_H\) of tiny supercooling, \(1 - T_{sc}/T_\star < 10^{-3}\). When \(T\) reaches \(T_{sc}\), bubbles nucleate at mean distances \(R_{nuc} \lesssim 2\) cm \([4]\). The bubbles grow most probably by weak deflagration \([5]\), because the surface tension is very small \([17]\). The released energy is transported into the deconfined phase by shock waves, which reheat the deconfined phase to \(T_\star\) within \(10^{-6} t_H\). The entropy production during this very short period is \(\Delta S/S \sim 10^{-6}\). Further bubble nucleation in the reheated quark-gluon phase is prohibited after this first \(10^{-4} t_H\). Thereafter, the bubbles grow adiabatically due to the expansion of the Universe (coexistence of the phases). The transition completes after \(10^{-1} t_H\).

During the reversible coexistence period (isentropic) \(T\) and \(p\) are fixed, while \(\rho\) can vary, therefore

\[
\frac{c_s^2}{\rho} = \left(\frac{\partial p}{\partial \rho}\right)_{\text{isentropic}} = 0.
\]

This implies that there are no pressure gradients at scales \(\lambda > R_{nuc}\), while both phases coexist during the first-order phase transition.

The thermodynamics of the cosmological QCD transition can be studied in lattice QCD, because the baryochemical potential is negligible in the early Universe, i.e. \(\mu_b/T_\star \sim 10^{-8}\). We fit the equation of state from lattice QCD \([18]\).

4. The evolution of density perturbations

During inflation density perturbations \(\delta \rho\) have been generated with an almost scale-invariant Harrison-Zel’dovich spectrum \([19]\). In the radiation dominated era subhorizon \(k_{\text{phys}} > H\) density perturbations oscillate as acoustic waves, supported by the pressure \(\delta p = c_s^2 \delta \rho\). It is useful to study the dimensionless density contrast \(\delta \equiv \delta \rho/\rho\) as a function of conformal time and comoving wave number. The subhorizon equation of motion reads

\[
\delta'' + c_s^2 k^2 \delta = 0,
\]

if we assume that the phase transition is short compared to the Hubble time \([7]\). From the lattice QCD equation of state the period of vanishing \(c_s\) lasts for \(0.1 t_H\), in the bag model it lasts for \(0.3 t_H\).

To illustrate the mechanism of generating large peaks above the HZ spectrum we discuss the subhorizon evolution of the density contrast \(\delta\) in the bag model. Before and after the QCD transition \(c_s^2 = 1/3\) and the density contrast oscillates with constant amplitudes \(A_{\text{in}}\) resp. \(A_{\text{out}}\). At the coexistence temperature \(T_\star\) the pressure gradient, i.e. the restoring force in Eq. (2), vanishes and the sound speed \([1]\) drops to zero. Thus \(\delta\) will grow or decrease linearly, depending on the fluid velocity at the moment when the phase transition starts. This produces large peaks in the spectrum which grow linearly with the wave number, i.e. \(A_{\text{out}}/A_{\text{in}}|_{\text{peaks}} = k/k_1\).

Kinetically decoupled CDM falls into the gravitational potential wells of the radiation fluid during the coexistence regime.

Fig. \([\text{3}]\) shows the final spectra for the density contrast of the radiation fluid and of CDM for a fit \([3]\) to lattice QCD. The spectra have been obtained from numerical integration of the fully general relativistic equations of motion. The mass
Figure 1. The transfer functions for the density contrast of CDM and of the radiation fluid due to the QCD transition. On the horizontal axis the wave number $k$ is represented by the CDM mass contained in a sphere of radius $\pi/k$. 

$M_1$ corresponds to the wave number $k_1$ of the bag model analysis. $M_1$ coincides with the CDM mass inside the Hubble horizon at the QCD transition, $\sim 10^{-8} M_\odot$. For the equation of state from the fit to lattice QCD a WKB analysis shows that $A^{\text{out}}/A^{\text{in}}|_{\text{peaks}} = (k/k_2)^{3/4}$, which defines the mass $M_2$. We conclude that at the horizon scale the modification of the HZ spectrum is mild, whereas for scales much smaller than the horizon big amplifications are predicted. Without tilt in the COBE normalized spectrum the density contrast grows nonlinear for $k_{\text{phys}}/H > \sim 10^4$ resp. $10^6$ for the bag model resp. lattice QCD equation of state.

5. Clumps in CDM

At $T \sim 1$ MeV the neutrinos decouple from the Hubble scale. During their decoupling they wash out all fluctuations of the radiation fluid below a CDM mass of $10^{-5} M_\odot$. Thus, at the time of nucleosynthesis all peaks in the spectrum of the radiation fluid (Fig. 1) are erased and the energy density on scales $\lambda \lesssim R_H(1 \text{ MeV})$ is homogeneous.

For kinetically decoupled CDM collisional damping is irrelevant. Peaks in these types of CDM survive and grow logarithmically during the radiation era. Shortly after equality they grow nonlinear and collapse by gravitational virialization to clumps of $M < 10^{-10} M_\odot$. From a COBE normalized HZ spectrum and the equation of state from quenched lattice QCD a clump of mass $10^{-15} M_\odot$ would have a size of $\sim 10 R_\odot$. For a tilted spectrum with $n = 1.2$ the size of the same clump would be $\sim 1 R_\odot$. 

We find that the peaks in the CDM spectrum lead to clumps of masses $10^{-20} - 10^{-10} M_\odot$. Today, these clumps would have a density contrast of $10^{10} - 10^{17}$, where the lower value corresponds to a $10^{-15} M_\odot$ clump from an untilted CDM spectrum, the bigger value is for a $10^{-20} M_\odot$ clump from a tilted CDM spectrum. The evolution of these clumps in the late stages of structure formation remains to be investigated (disruption, mergers, etc.).

For a larger amplification during the QCD transition, e.g., if it should turn out that the latent heat is bigger than the value from present lattice QCD calculations,
more compact clumps are possible. These could be subject to femto-lensing [8]. With the values from lattice QCD, the CDM clumps are not compact enough to lie within the Einstein radius, which is $R_E \sim 0.02R_\odot$ for a $10^{-15}M_\odot$ clump.

The clumping of CDM changes the expected rates for some dark matter searches, because some of the rates depend on the space-time position of the detector, star, or planet. Especially experiments looking for axion decay in strong magnetic fields [9] would not provide a new limit on the axion mass if they find nothing. These experiments may just tell us that we are not sitting in an axion clump currently. These consequences remain to be studied further.

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