Post-stack Deterministic Seismic Inversion using Sparse Optimisation Utilizing Gaussian Dense Layer Simulation

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Abstract. The main objective in inverting seismic data is to estimate layer characteristics instead of layer boundaries. It is about getting as much as layer boundaries as seismically permitted with correct position and amplitude. However, the number of layers identified by seismic inversion methods is relatively in small number. Meaning, inversion methods tend to thicken the layers due to band-limited of seismic data and wavelet inaccuracy. It is well-accepted that sparse-layer inversion has the ability in recovering full RC series that are permittable seismically yet it tends to produce blocky impedance. Sparse-layer inversion is also computationally extensive because of large amount of competing reflectivity model. In this paper, we test sparse optimisation inversion performance using gaussian dense-layer simulation instead of sparse-layer model.

1. Introduction

Principally, seismic inversion is a stratigraphic deconvolutional process[1]. Recorded seismic data are resulted from a sophisticated interaction phenomena between source wave, earth’s transmitivity, acquisition setup, processing and receiver instrument. This type of data are than processed to meet a much more simple concept called the convolutional model[2]. In convolutional model, amplitude in a seismic trace is assumed as representation of earth reflectivity but in much less resolutive version, filtered by the wavelet spectrum. The bandlimited nature of the source, the internal structure of the geophone which records the data, and also the complexity of the subsurface because of the attenuation and other properties are some of the reasons of losing most of the data bandwidth during the seismic survey. The seismic wave is greatly decreased in high frequency content mainly due to earth transmittivity, acquisition, the recording system, and processing effects whereas lack of lower band of frequency content mainly due to acquisition, receiver and the recording instruments [3]. This leads to bandlimit the seismic data and was believed to cause some difficulties getting reasonable result of inverted impedance [3].

The case is perhaps true, unless sparse-layer seismic inversion at the highest capacity can regenerate reflectivity series from band-limited seismic synthetic data with high resolution, correct position, accurate in amplitude and more importantly giving much better continuity [[5],[6]]. In practice, by using real seismic data we never get close to such result particularly because of inappropriate wavelet estimation and noise existance [6]. Sparse layer seismic inversion uses sparse optimisation incorporating norm-l1 as a surrogate for norm-l0. Norm-l1 (sparsity) has advantages over norm-l2 (least-square) in several aspects e.g. more reliable with noisy data. Noisy data will still give
proper result except the resolution is decreasing [7]. Norm-1 was rarely implemented mainly because mathematically complex. All deterministic inversion method, especially sparse spike including sparse layer inversion methods, tend to give the average version of reflectivities separated some distance as thickening layers. Averaging layers might cause wrong estimation of reflectivity coefficient by substituting low-varying reflectivity series into a single representable reflectivity which violates the facts observed in logs. Using those average layers also might restrict us to get full-bandwidth inverted impedance. Gaussian dense-layer simulation is proposed to overcome this problem. Using intensive reflectivity simulation, we present a demonstration of elaborating stochastic simulation to the deterministic inversion framework.

2. Methodology

Basis pursuit inversion method is a type of sparse layer inversion [1]. Using concept of basis pursuit decomposition [1], it attempt to evaluate all combinations of any pair reflectivity waveform that fit the real data [5,6,8,9]. The inversion use the convolutional theory to compute synthetic seismic from generated model so we can decide which model are most appropriate by evaluating minimum error between the data and the computed one. It optimizes the waveforms collection in $Ds$ and find the coefficients which reflect the contribution of those waveforms on the real data by using equation:

$$\min \| s - D_s x \|_2 + \lambda \| x \|_1$$

(1)

After finding the coefficients $x$, then reflectivity series $r$ can be obtained by simple matrix multiplication:

$$r = D_r x$$

(2)

where $Dr$ is a reflectivity pair dictionary (even pairs and odd pairs) for those of the $Ds$. Using this kind of dictionary costs us many computation time and computer memory [6].

The main advantage of using basis pursuit decomposition is it permits us to add any other dictionary to be included within the over-complete dictionary [6]. We choose random gaussian reflectivity package to be added to the dictionary and reduce a great amount of reflectivity pairs. This scheme supposedly can provide the same quality but with less computational costs. It contains dense Gaussian reflectivity simulation which also represent the very thin layer series so it more likely to give us a broadband result. Comparative scheme of dictionary is shown in Figure 1.

3. Experiment on Synthetic Data

Using inversion technique described before, we put to the test two synthetic cases. Each case reflects real type of Earth’s reflectivity series. First case represents relatively homogeneous layers and the other case represents slightly denser reflectivity (more heterogeneous layers). For both cases, as we notice in Figure 2, neither of them might be expected in synthetic seismic data after we convolve them with the Ormsby wavelet. In these cases, we made it noise free as we want to see how far this technique can invert reflectivity in ideal condition.
First synthetic case shows spiky reflectivity separated in some distance while the seismic synthetic surprisingly shows more wiggle than we expected due to wavelet sidelobes. The contradictory phenomenon is shown in the second case. Less wiggle we can see on the seismic synthetic trace while the reflectivities are so intense. Using our method, we do the inversion to get the reflectivity back by assuming we know the wavelet exactly.

As we can see in Figure 3, reflectivity series can be obtained very accurately (Correlation R=0.98). On the left part, we can see the dictionary used to invert the seismic data. Notice that the data more likely represented by Gaussian simulation than to reflectivity pairs. Experiment on the second case also gives the result somewhat in kind, Figure 4. Here we notice that our proposed method can also give high resolution of inverted reflectivity series for both of the case but with less Dictionary thus more efficient in computation process.

### 4. Real Data Implementation

In this section, the real data of P-impedance log were used. The seismic trace was computed by doing forward modeling reflectivity log using a certain wavelet. In this case, we also assume noise free condition. So the seismic trace is assumed in perfect condition (see Figure 5). Different from the
synthetic case, in the real data, we can get low frequency part of the impedance so the absolute broadband impedance can be demonstrated. From Figure 5, we also can see the spectrum of reflectivity log, P-impedance log and the synthetic trace. Wavelet used in this implementation is the same kind of in synthetic data before but with 90° rotation. It is surely band limited wavelet. If we consider the convolutional theory, it seems there are no chance to get all the reflectivity back as they are filtered by the spectrum of wavelet. The result in Figure 6 is astonishing because using this method we can recall them back with high level of accuracy (R=0.99).

![Figure 4. The second case result. Upper panel: Dictionary used to invert the seismic. Lower panel: Inversion QC. Inverted reflectivities (Red) fit accurately to the original reflectivities (Black).](image)

![Figure 5. Real data from P-impedance log and its spectrum](image)

Next, we compute the absolute broadband impedance by adding low frequency impedance from log data to the recursive process. The result (Figure 7), as expected, shows very good match between computed impedance and log impedance. Every thin layer can be resolved.
5. Conclusions
We have given a demonstration how full reflectivity series obtained from band-limited data due to wavelet filtering. This method can completely remove wavelet overprint from the data thus every reflectivity appear as a single spike. This make our intention to compute broadband impedance become possible with less time consuming. Here, we test our method in ideal condition where wavelet is exactly known and noise is completely zero. But in the real data, they always exist at some amount. As a consequence, predicted reflectivity can be poor in resolution or even worse giving wrong value. However, with good prediction of wavelet and careful processing steps, we at least have a chance to get broadband inverted AI. In the near future, we will implement this methodology in real seismic data.

Figure 6. Computed reflectivity shows high correlation to the true reflectivity. Down left of the right panel shows crossplot between the two (Inverted RC v.s. Original RC)

Figure 7. Comparison of inverted impedance and true impedance in time (left) domain and frequency (right) domain.
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