Decaying dark matter, the $H_0$ tension, and the lithium problem

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It has long been known that the sharpened tension between the observed and inferred values of the Hubble constant $H_0$ can be alleviated if a fraction of dark matter particles of type $\chi$ were produced non-thermally in association with photons $\gamma$ through the decays of a heavy and relatively long-lived state, viz. $X \to \chi + \gamma$. It was recently proposed that this model can also resolve the longstanding lithium problem if $M = 4$ MeV and $m = 0.04$ keV, where $M$ and $m$ are respectively the masses of $X$ and $\chi$. We calculate the general functional form describing the number of equivalent light neutrino species $\Delta N_{\text{eff}}$ carried by $\chi$. We show that to resolve the $H_0$ tension at the $1 \sigma$ level a 55% correction in $m$ is needed and that the required $\Delta N_{\text{eff}}$ is excluded at 95% CL by Planck data. We argue in favor of a more complex model of dynamical dark matter to relax cosmological anomalies.

Over the past decade cosmological parameters have been measured to unprecedented precision. The most reliable measurement of the Hubble constant $H_0 = 74.03 \pm 1.42$ km/s/Mpc comes from HST observations of Cepheid variables in the host of recent, nearby type Ia supernova to build a 3-rung distance ladder [1]. Besides, a prediction of $H_0$ can be obtained from the sound horizon observed from the cosmic microwave background (CMB). A fit to data from the Planck mission, under the assumption of a flat $\Lambda$ cold dark matter ($\Lambda$CDM) cosmological model leads to $H_0 = 67.27 \pm 0.60$ km/s/Mpc [2]. These two $H_0$ values are discrepant by about 4.4$\sigma$, which gives rise to the so-called $H_0$ tension [3].

Although big bang nucleosynthesis (BBN) has played a central role in the development of precision cosmology, the theory has hitherto failed to explain the amount of cosmic lithium [4]. Indeed, while BBN predictions match the observed primordial deuterium and helium abundances, the theory over-predicts the abundance of primordial lithium by about a factor of three. This $4 - 5\sigma$ mismatch constitutes the cosmic lithium problem.

It was recently proposed that if a fraction of dark matter particles $\chi$ were produced non-thermally in association with photons through the decays of a heavy and relatively long-lived state, both the $H_0$ and lithium problems can be simultaneously resolved [5] (see [6], for a similar proposal). In this paper we calculate the general functional form describing the number of equivalent light neutrino species $\Delta N_{\text{eff}}$ carried by $\chi$. We show that to resolve the $H_0$ tension at the $1 \sigma$ level a 55% correction in the mass of $\chi$ is needed and that the required $\Delta N_{\text{eff}}$ is excluded at 95% CL by Planck data.

Following [5] we assume that of the total dark matter (DM) density around today, $\Omega_{DM}$, a small fraction, $f = \Omega_\chi/\Omega_{DM}$, is of particles of type $\chi$, produced via decay of a heavy relic $X$ with mass $M$ and lifetime $\tau$: $X \to \chi + \gamma$. At any time after the decay of $X$ the total DM energy density is found to be

$$\rho_{\text{DM}}(t) = \frac{m}{a^3(t)} n_\chi \gamma(t) + (1 - f) \rho_c \Omega_{DM} a^3(t), \quad (1)$$

where $m$ is the mass of the $\chi$, $\gamma(t)$ its Lorentz boost, $n_\chi(t)$ its number density, $\rho_c$ the critical density, and $a(t)$ is the expansion scale factor normalized by $a(\text{today}) = 1$. In the center-of-mass frame of $X$ (this should also be a good approximation of any frame as we assume that $X$ is nonrelativistic so its mass energy dominates) we have

$$M = E_0 + p_0 = \sqrt{p_0^2 + m^2} + p_0 \Rightarrow p_0 = \frac{M^2 - m^2}{2M}, \quad (2)$$

where $E_0 = \gamma(t)m$ is the initial energy of the particle $\chi$ and $p_0$ its momentum, with

$$\gamma(t) = \frac{E_0}{m} = \frac{M}{2m} + \frac{m}{2M}. \quad (3)$$

Because of the cosmic expansion any particle with momentum $p$ becomes redshifted at a rate $p(t) = p_0 a(t)/a(t)$. Using the relation $E^2 - p^2 = m^2$, which holds in the Robertson-Walker metric [9], we find that the particle’s energy gets redshifted according to

$$E^2(t) = p_0^2 \left(\frac{a(t)}{a(0)}\right)^2 + m^2. \quad (4)$$

The scale factor dependence on the Lorentz boost is found to be

$$\gamma(t) = \sqrt{1 + \left[\frac{a(t)}{a(0)}\right]^2 \left[\frac{p_0}{m}\right]^2} = \sqrt{1 + \left[\frac{a(t)}{a(0)}\right]^2 \left(\frac{E_0^2 - m^2}{m^2}\right)}$$

$$= \sqrt{1 + \left[\frac{a(t)}{a(0)}\right]^2 \left[y^2(t) - 1\right]}. \quad (5)$$

Expansion of the square root in (5) leads to

$$\gamma(t) \approx 1 + \frac{1}{2} \left[\frac{a(t)}{a(0)}\right]^2 \left[y^2(t) - 1\right] - \frac{1}{8} \left[\frac{a(t)}{a(0)}\right]^4 \left[y^2(t) - 1\right]^2 + \cdots. \quad (6)$$

1 Herein we do not consider the more complicated scenario in which high energy neutrinos are among the decay products [1][3].
Now, $\Omega_\chi$(today) = $m n_\chi$(today)/$\rho_c$, because the $\chi$ is non-relativistic. To obtain such a non-relativistic limit we demand the magnitude of the second term in the expansion of Eq. (6) to be greater than the third term, which results in $[a(\tau)/a(t)]^2[\gamma^2(\tau) - 1] < 4$. Contrariwise, by this criteria the particle $X$ is relativistic if $\gamma(t) > \sqrt{5}$. A point worth noting at this juncture is that the general functional form of the Lorentz boost given in [5] and its expansion given in [6] are substantially different from the approximate expression of $\gamma(t)$ given in Eq. (3) of [10]. However, for $\gamma^2(\tau) \gg 1$ and $\gamma^2(\tau)[a(\tau)/a(t)]^2 \gg 1$, both expressions give similar Lorentz factors.

The total “dark” relativistic energy density (including the three left-handed neutrinos of the Standard Model) is usually characterized by the number of “equivalent” light neutrino species, $N_{\nu}_{eff} \equiv (\rho_R - \rho_\nu)/\rho_{C\nu}$, in units of the density of a single Weyl neutrino $\rho_{C\nu}$, where $\rho_\nu$ is the energy density of photons, and $\rho_R$ is the total energy density in relativistic particles [11]. Following [5], we obtain the $\chi$ contribution to $N_{\nu}_{eff}$ at the time of matter-radiation equality assuming that the $\chi$ decouples from the plasma prior to $v_L$ decoupling, conserving the temperature ratio $T_\gamma/T_{nu} = 11/4$ from $\Lambda$CDM cosmology,

$$\Delta N_{\nu}_{eff} = \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{\rho_\chi(t_{eq})}{\rho_{C\nu}(t_{eq})}$$

$$= \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{\Omega_{DM}}{\Omega_\gamma} a(t_{eq}) f \gamma(t_{eq})$$

$$= \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{\Omega_{DM}}{\Omega_\gamma} a(t_{eq}) f \sqrt{1 + \left[\frac{a(\tau)}{a(t_{eq})}\right]^2 [\gamma^2(\tau) - 1]},$$

where $\rho_\chi(t_{eq}) = \rho_\chi(t_{eq})/a^4(t_{eq})$ and the factor of $8/7$ is due to the difference between the Fermi and Bose integrals [12]. Note that the functional form of Eq. (7) of [10] is different from that in Eq. (5) of [10]; the latter was adopted in the recent study of [5]. For $\gamma^2(\tau) \gg 1$ and $\gamma^2(\tau)[a(\tau)/a(t)]^2 \gg 1$, both [7] and Eq. (5) of [10] give similar contributions to $\Delta N_{\nu}_{eff}$.

Model considerations set bounds on free parameters. On the one hand, consistency with large-scale structure observations implies $f \leq 0.01$ [10]. On the other hand, the decay of the $X$’s could significantly alter the light element abundances synthesized during BBN. Of particular interest here, the threshold energy of the photon for the process $^7\text{Be}\gamma(\gamma,\text{He})^4\text{He}$ is $E_{\text{Be}}^{\text{th}} \approx 1.59$ MeV, which is lower than that of the photodissociation of D ($E_{\text{D}}^{\text{th}} \approx 2.22$ MeV) and $^4\text{He}$ ($E_{\text{He}}^{\text{th}} \approx 20$ MeV). Hence, if the energy of the injected photons is in the range $E_{\text{Be}}^{\text{th}} < E_\gamma < E_{\text{D}}^{\text{th}}$, the photodissociation of $^7\text{Be}$ could take place to solve the $^7\text{Li}$ problem without significantly affecting the abundances of other light elements [13]. With this in mind, to destroy enough $^7\text{Li}$ without affecting the abundance of other elements, we set $f = 0.01$ and must fine-tune simultaneously the $X$-lifetime $\tau \approx 2 \times 10^{-1}$ s [5] and the initial electromagnetic energy release in each X decay $E_{\text{Be}}^{\text{th}} < E_\gamma < (M^2 - m^2)/(2M) < E_{\text{D}}^{\text{th}}$. For $M \gg m$, the latter leads to $M \approx 4$ MeV.

The correlation between $H_0$ and $N_{\nu}_{eff}$ has been estimated numerically

$$\Delta H_0 = H_0 - H_0|_{\Lambda\text{CDM}} \approx 6.2 \Delta N_{\nu}_{eff},$$

where $H_0|_{\Lambda\text{CDM}}$ is the value of $H_0$ inferred within $\Lambda$CDM [14]. The rescaled posterior distributions of $H_0$ from

FIG. 1: Rescaled posterior distributions of $H_0 = 100$ $h$ km/s/Mpc (due to marginalization over additional free parameters) with different choices of $\Delta N_{\nu}_{eff}$ from the 7 parameter fit of [14]. The shaded areas indicate the 1$\sigma$ and 2$\sigma$ regions as determined with HST observations [1].

We note in passing that the relation [5] has been derived on the basis of the Planck 2015 TT + lowP + BAO + Pantheon dataset combination [15] [16]. Should we instead have used Planck 2018 TTTEEE + lowE + BAO + Pantheon [2], the proportionality constant would turn out to be 5.9 rather than 6.2 [17]. This is because small-scale polarization data comes into play making it even harder to accommodate high $H_0$ values with $\Delta N_{\nu}_{eff}$. Herein
the a parameter fit with different choices of \( \Delta N_{\text{eff}} \) are displayed in Fig. [1]. To accommodate the \( H_0 \) tension at 1\( \sigma \) level, we require \( \Delta H_0 \approx 4.7 \), which implies \( \Delta N_{\text{eff}} \approx 0.76 \). The 95% CL bound on the extra equivalent neutrino species derived from a combination of CMB, BAO, and BBN observations is \( \Delta N_{\text{eff}} < 0.214 \) [2]. This limit combines the helium measurements of [18] [19] with the latest deuterium abundance measurements of [20] using the the PArthEnOPE code [21] considering \( D(p, \gamma)^3\text{He} \) reaction rates from [22]. Note that even considering the most optimistic helium abundance measurement of [23] in place of [18] [19], the 95% CL bound \( \Delta N_{\text{eff}} < 0.544 \) [2] still precludes accommodating the HST observations within 1\( \sigma \).

Following [10], we take a radiation like scale factor evolution \( a(t) \propto t^{1/2} \), \( \Omega_{\text{DM}} \approx 0.227 \), \( \Omega_\gamma \approx 0.0000484 \), \( a(\text{He}) \approx 3 \times 10^{-3} \), and \( a(\tau)/a(\text{He}) = 7.8 \times 10^{-4} \sqrt{10^{-6}} \tau/s \). Substituting these figures into (7) while demanding the constraint \( \Delta N_{\text{eff}} \approx 0.76 \) via (8) leads to \( m \approx 0.018 \) keV.

In closing, we argue in favor of a more complex dynamical dark sector [24]-[26], in which both early and late universe decaying DM provide contributions to the local \( H_0 \) measurement. For example, if \( m = 0.07 \) keV the contribution to \( \Delta N_{\text{eff}} \approx 0.2 \) is consistent with the upper limit reported by the Planck Collaboration [2] and the \( X \to \gamma + \gamma \) decays still produce enough electromagnetic energy [10]

\[
e_\gamma \approx 1.5 \times 10^{-9} \left( \frac{M}{m} - m / M \right) \text{MeV (9)}
\]

to dilute the \( ^7\text{Li} \) [13]. However, a recent study of the photodisociation of light elements in the early universe indicates that the bound on the electromagnetic energy in 2 MeV photons that can be injected during BBN is \( e_\gamma < 10^{-4} \) MeV [27]. This further constrains the contribution to \( \Delta N_{\text{eff}} \) via \( X \to \chi + \gamma \). The actual contribution of late decaying DM to \( H_0 \) is a matter of debate [28]-[32]. Remarkably, such late decays can also resolve the growing tension between the cosmological and local determination of \( S_8 \equiv \sigma_8(\Omega_m/0.3)^{1/2} \), which quantifies the amplitude of the matter fluctuations on 8h^{-1}/Mpc scales, where \( \Omega_m \) is the matter density [32]. A comprehensive study of the parameter space of such a complex hidden sector is beyond the scope of this paper and will be presented elsewhere.

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we remain conservative in our calculations and adopt the value given in [13].

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