An Aseismic Design Model Based on Flexible Suspension and Lateral Elastic Support and Its Dynamics Analysis

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Abstract: Suspension-type structures are of excellent seismic performance. It is expected to realize a better anti-seismic effect by integrating the elastic constraints that can absorb the seismic energy with the suspension-type structures. A structure consisting of vertical suspension and lateral elastic support was designed according to seismic damage characteristics for engineering structures, and some hypotheses were made to its stress conditions, which were properly simplified. Next, a basic mechanical model was established, followed by the structural dynamics analysis, so as to obtain the design value of elastic rigidity needed to ensure the minimum vibration amplitude. In the end, the seismic response characteristics and optimal aseismic design of the entire structure were acquired through an equivalent analogy. This study can be taken for reference in the new-type aseismic structural design based on the abovementioned model.

1. Introduction

Earthquake is a very common geological activity in nature. Taking China for example, most provinces, autonomous regions and municipalities directly under the central government throughout China have gone through many destructive earthquakes in the history [1]. Since the 20th century, over 650 earthquakes with magnitude of above 6 and 100 ones with $M_s \geq 7$ [2] have struck China, leading to serious losses of life and property. High-intensity seismic activities, especially violent earthquakes occurring in densely populated places or lifeline projects like tunnel and bridge, will usually lead to catastrophic consequences. Therefore, the earthquake resistance of engineering structures has always been the research emphasis in the field of civil engineering.

The suspension structures in conventional sense have been widely applied in the field of bridge engineering at the earliest [3]. Cable-stayed bridge and suspension bridge, which are two of the four main bridge types (the other two are girder bridge, arch bridge), belong to suspension structures. With the development of engineering anti-seismic theory in recent decades, the engineering circles has been gradually aware of unique advantages of suspension structures in improving the seismic performance of buildings. In 1995, Bashshiha et al. [5] put forward the suspension pendulum isolation (SPI) system, which installed the upper structure on the suspension pendulum plate to realize the shock insulation effect. In 2017, Lu et al. [6] from Tongji University experimentally studied the base suspension pendulum isolation (BSPI) on a shaking table, and verified its favorable shock insulation effect. However, the suspension structures, especially the suspension-centered composite seismic isolation structures, have been scarcely investigated. The dynamic characteristics of a structure consisting of upper flexible suspension and
lateral horizontal elastic support, expecting to provide a reference for designing multiple new-type aseismic structures.

2. Structural Design and Mechanical Hypotheses

Seismic wave is divided into longitudinal wave, transverse wave and surface wave, among which the longitudinal wave results in vibration of ground surface up and down, with a slightly weak destructive effect; the structural damage caused by the transverse wave is mainly horizontal vibration, and its destructive effect is stronger than the longitudinal wave. Hence, the primary goal of aseismic design should be to weaken the horizontal vibration.

In consideration of vertical suspension mode, there are mainly three model types: rigid rod, flexible rope and spring, which respectively represent rigid, flexible and elastic constraints. As it is difficult for rigid rod to effectively absorb the seismic energy and it is prone to fracture at dangerous point during the horizontal vibration, the rigid rod is generally not adopted. Meanwhile, in view of not great shock absorbing demand in vertical direction, the flexible rope constraint model is selected. As the mass of flexible rope is very small relative to an actual structure, its mass was neglected in the model of this paper. The vertical suspension mode of the structure is seen in Fig. 1.

![Flexible Rope Constraint Model](image1)

The horizontal acceleration of the suspension structure was analyzed, and the simplified calculation diagram is shown in Fig. 2. To facilitate the analysis, the steady vibration state of earthquake is regarded as simple harmonic vibration and satisfies the equation $A \sin(\omega t)$.

![Simplified Calculation Diagram of Horizontal Acceleration](image2)
Relative to the horizontal acceleration, the radial acceleration is so low that it is neglected, and the approximate equilibrium in vertical direction is assumed. Hence, the followings can be obtained:

\[
\begin{align*}
T \sin d\theta &= m da \\
T \cos d\theta &= mg
\end{align*}
\]

(1)

The above equations are organized to obtain:

\[
tan d\theta = \frac{da}{\theta}
\]

(2)

The minor rotation angle within an extremely short time period satisfies the following approximate relation:

\[
tan d\theta \approx d\theta \approx \frac{dx}{l}
\]

(3)

Eq. (3) is substituted into (2) to obtain:

\[
dx = \frac{1}{\theta} da
\]

(4)

The following can be obtained by solving the differential Eq. (3):

\[
a = \frac{\theta}{l} A \sin(\omega t)
\]

(5)

It can be easily obtained from Eq. (5) that the horizontal acceleration cycle is \( T_a = \frac{2\pi}{\omega} \), and then the maximum speed within this cycle is:

\[
v_{max} = \int_0^{\frac{\pi}{\omega}} a \ dt = \frac{2gA}{\omega l}
\]

(6)

On this basis, the maximum radial acceleration is solved as follows:

\[
a_r = \frac{v_{max}^2}{l}
\]

(7)

The ratio of maximum radial acceleration to maximum horizontal acceleration is calculated as below:

\[
\varepsilon = \frac{a_r}{a_{max}} = \frac{4gA}{(\omega l)^2}
\]

(8)

\( \varepsilon < \frac{1}{1000} \) is estimated according to the orders of magnitudes of seismic parameters, indicating that the hypothesis about the vertical equilibrium is reasonable, and in practical engineering, the horizontal vibration can hardly trigger the structural damage in vertical direction.

In order to mitigate the influence of horizontal vibration on the structure, a lateral linear elastic constraint can be applied to it as shown in Fig. 3.

![Fig. 3. Lateral Linear Elastic Constraint Model of Flexible Suspension Structure](image)

3. Basic Mechanical Model
Before the analysis of the structure in Fig. 3, the basic structure in Fig. 4 is firstly analyzed.
Fig. 4. Schematic Diagram of Basic Structure

Fig. 4 displays a vibration system, where the mass of supported part in this structure is \( m \), and the shaking table is connected to the object via a spring with rigidity of \( k \) and resistance of \( R_m \). Under the action of external force, the shaking table generates a displacement \( \delta \) with vibration amplitude of \( \delta_0 \) and circular frequency of \( \omega \), and then the displacement \( \delta_1 \) of shaking table can be expressed by Eq. (9).

\[
\delta_1 = \delta_0 e^{j\omega t}
\]  

Eq. (9) is substituted into Eq. (10) to acquire Eq. (11):

\[
m \frac{d^2 \delta}{dt^2} + R_m \frac{d\delta}{dt} + k(\delta - \delta_1) = 0
\]  

Where \( \theta = \arctan \frac{\omega R_m}{k} \). The steady-state vibration displacement of the object is expressed as seen in Eq. (13).

\[
\delta = \frac{k\delta_0}{\omega |Z|} \sqrt{\frac{1 + (\omega R_m)^2}{\omega^2}} \cdot e^{j(\omega t - \theta_0 - \frac{\pi}{2} + \theta)}
\]  

Where \( |Z| = \sqrt{R_m^2 + X_m^2} \) is the impedance modulus of resistance, \( X_m = \omega m - \frac{k}{\omega} \) is the mechanical reactance, and \( \theta_0 = \arctan \frac{X_m}{R_m} \) is the argument of mechanical impedance.

The inherent circular frequency of the system is denoted as \( \omega_0 = \frac{k}{m} \), \( Q_m = \frac{\omega_0 m}{R_m} \), \( z = \frac{f}{f_0} = \frac{\omega}{\omega_0} \), and then the amplitude ratio of object to the shaking table can be written into the following form:

\[
D_\delta = \frac{\delta_a}{\delta_0} = \frac{\sqrt{1 + (\frac{z}{Q_m})^2}}{\sqrt{1 - z^2 + (\frac{z}{Q_m})^2}}
\]  

The derivative about \( z \) in Eq. (14) is solved, \( \frac{dD_\delta}{dz} = 0 \) is set, and then the extreme point can be solved as below:
\[ z = Q_m \sqrt{1 + \frac{z}{Q_m^2} - 1} \]  \hspace{1cm} (15)

At the left side of this value, the derived function of \( D_\delta \) is constantly positive, and the function presents a monotonic increase. At the right side, the derived function is constantly negative, and the function presents a monotonic decrease, so the function reaches the maximal value at this value point. When \( z \) approaches 0, \( D_\delta \) will indefinitely approach 1, but being always greater than 1, and it will tend to be 0 if \( z \) is inclined to positive infinity. To exert the shock insulation effect, \( D_\delta < 1 \) shall be satisfied. \( D_\delta = 1 \) is set, and two groups of solutions are solved:

\[
\begin{align*}
  z_1 &= 0 \\
  z_2 &= \sqrt{2}
\end{align*}
\]  \hspace{1cm} (16)

Obviously, \( z_1 = 0 \) corresponds to the static state of the system, not conforming to the vibration condition, so it is discarded. \( z_2 = \sqrt{2} \) is taken. \( D_\delta \) constantly holds under \( z = \sqrt{2} \), the more greatly the \( z \) deviates from \( \sqrt{2} \), the smaller the \( D_\delta \) value will be, manifesting that the spring in the design scheme should satisfy \( \sqrt{2} f_0 \), where \( f_0 \) is the inherent frequency of the spring, meeting the relational expression (17).

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]  \hspace{1cm} (17)

By combining the seismic records where the tunnel is located, the minimum seismic wave amplitude recorded in the place where this structure is located is written as \( f_{\text{min}} \), and then the designed spring rigidity should meet the inequation (18):

\[ k < 2\pi^2 f_{\text{min}}^2 m \]  \hspace{1cm} (18)

In consideration of mechanical resistance of the spring, \( D_\delta \) is re-organized as seen in Eq. (19):

\[ D_\delta = \sqrt{1 + \frac{2\omega^2 mk - \omega^4 m^2}{\omega^2 R_m^2 + (\omega^2 m - k)^2}} \]  \hspace{1cm} (19)

It is not difficult to see that the mechanical resistance of the spring should be as large as possible.

4. Structural Dynamics Analysis Based on Basic Model

Based on the above basic mechanical model, the dynamics characteristics of the entire structure in Fig. 3 were discussed. The horizontal vibration displacement at the right vertical fixed end in Fig. 3 was set as \( \delta_1 \), and that at the upper horizontal fixed end as \( \delta_2 \). The distance between two fixed ends of a practical engineering structure will be of a very small geological scale under an earthquake, so the horizontal vibration displacements at the two ends can be considered equal as seen in Eq. (20).

\[ \delta_1 = \delta_2 = \delta_0 e^{j\omega t} \]  \hspace{1cm} (20)

Similarly, the equilibrium Eq. (21) for the inertia force of the object is given according to the D’Alembert’s principle:

\[ m \frac{d^2 \delta}{dt^2} = k(\delta_1 - \delta) + mg \frac{\delta_2 - \delta}{l} - R_m \frac{d(\delta - \delta_1)}{dt} = 0 \]  \hspace{1cm} (21)

By organizing the above equation, Eq. (22) can be obtained:

\[ m \frac{d^2 \delta}{dt^2} + R_m \frac{d \delta}{dt} + \left(k + \frac{mg}{l} + j\omega R_m \right) \delta = \left(k + \frac{mg}{l} + j\omega R_m \right) \delta_0 e^{j\omega t} \]  \hspace{1cm} (22)

By comparing the equilibrium equations for the inertia force of basic mechanical model described in Pt. 2, it is not difficult to find that the two are totally identical in the form. The structure in Fig. 3 can be equivalent to a basic mechanical model with an elastic rigidity of \( K_e = k + \frac{mg}{l} \), the conclusion of which still holds for this structure. Therefore, the condition that should be satisfied by the spring rigidity can be obtained under this circumstance through the following steps:

\[ K_e < 2\pi^2 f_{\text{min}}^2 m \]  \hspace{1cm} (23)

\[ k < 2\pi^2 f_{\text{min}}^2 m - \frac{mg}{l} \]  \hspace{1cm} (24)

At the time, \( 2\pi^2 f_{\text{min}}^2 m - \frac{mg}{l} > 0 \) should be guaranteed, and thus the design requirement for the
length of flexible rope can be acquired as seen in Eq. (25).

\[ l > \frac{\theta}{2\pi^2 f_{min}^2} \]  

(25)

In the meantime, the flexible rope should also meet the bearing capacity and deformation requirements.

The elastic constraint and flexible suspension constraint can be respectively designed according to Eq. (24) and Eq. (25) so that the structure can reach the optimal seismic performance.

5. Conclusion

First, a structure composed of vertical suspension and lateral elastic support was designed according to the seismic damage characteristics for engineering structures, some hypotheses were made to its mechanical conditions, which were then properly simplified. Next, a basic mechanical model was established, its structural dynamics analysis was performed, so as to obtain the design value of elastic rigidity needed to ensure the minimum vibration amplitude, and the seismic response characteristics and optimal aseismic design of the entire structure were acquired through an equivalent analogy. In fact, this structural model can serve as an abstract model for the aseismic design of many new-type aseismic structures, such as building structures with upper flexibly constrained suspension structure and bottom elastic constraint, tunnels or suspension-type rails of unmanned inspection vehicles in pipe gallery and heavy lifting equipment. The calculation model given in this study can provide a reference for the aseismic design of abovementioned new-type structures.

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