Experimental investigation of harvesting stochastic vibration energy by a piezoelectric inverted beam with pendulum

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Abstract. We proposed a harvester composed of inverted piezoelectric beam and a pendulum to enhance the efficiency of vibration-based energy harvesting. Corresponding experiments were performed for validation. The amplitude-frequency response obtained in the vicinity of second natural frequency shows the feature of soften-spring type. The region of multiple solutions originated from nonlinearity could be extended when the excitation is increased to a high level. The experiment under stochastic excitation was carried out, and the corresponding responses were obtained. The absence of low frequency component in stochastic excitation was designed in excitation to show the first natural frequency component in response comes from the coupling between modes. The results verify the energy transformation phenomenon between the first two modes.

1. Introduction
Numerous extant literatures have been reported on exploring nonlinear phenomenon for increasing the bandwidth of energy harvesting devices\cite{1-6}. Among these nonlinear vibration-based energy harvesters, large amplitude snap through motion between different potential wells has been proved to be a more efficient way to collect power than the corresponding tuned linear devices. The study of bi-stable\cite{7, 8} and tri-stable\cite{9, 10} motion-based energy harvesters exhibit remarkable performance on giving a high output. In addition, the concept known as internal resonance has been introduced into the nonlinear harvesters\cite{11, 12}. For further illustrating the benefit of internal resonance mechanism, numerical simulation and corresponding experiments for the relevant energy harvesters were implemented. The results validate that the system realize a broadband manner\cite{13, 14}.

The mono-stable oscillators based on Duffing nonlinearity are the typical nonlinear systems that can significantly realize the broadband manner and give a high output through softening or hardening features\cite{15}. Zhou et al. \cite{16} investigated an improved mono-stable nonlinear harvester, harmonic balance analysis indicates two stable orbits in multi-solution area. Then the experimental results verified that the system could maintain in high-brach orbit and improve energy harvesting efficiency by using impact method. Westermann et al. \cite{17} proposed a piezo-magnetoelastic energy harvesting system which is modeled as Duffing oscillator, governing equations are derived and semi analytical methods are applied to solve the equations. Masuda et al. \cite{18} presented a resonance-type vibration energy harvester with an expanded frequency band introduced by nonlinear oscillator, self-excitation control was utilized to ensure the system was in high energy orbit. Mallick et al. \cite{19} reported a compact nonlinear energy harvester for low frequency applications. The cubic mono-stable nonlinear term produced by stretching strain could broad the bandwidth of the harvester.

As the inverted elastic beam with a mass mounted on its free end could induce a large amplitude transverse displacement, this nonlinear structure is considered as an appropriate harvester in practice. Friswell et al. \cite{20} investigated a vertical cantilever beam with tip mass subject to horizontal harmonic base excitation, both the non-buckled and buckled beams were concerned. Simulation and experiments...
verified that the system had a broad bandwidth over the linear harvester. Bilgen et al. [21] then investigated the inverted beam excited by the vertical stochastic excitation, a transition from single well motion to double well motion can be found in the responses. Lan et al.[22, 23] focused on the mode coupling of the inverted beam subjected to vertical base excitation. The stochastic responses obtained from the experiments demonstrated that the base excitation energy can be transformed to the first-order mode. And the critical parameter tip mass was discussed then. For further improving the energy harvesting efficiency, a 2-DOF energy harvester is proposed. As the motion of a pendulum is a nonlinear phenomenon, the attachment of the pendulum could be an enhancement for the original 1-DOF vertical cantilever. The corresponding experiments were implemented. The remaining sections of this article are structured as follows. Governing equations are demonstrated in section 2. In section 3, experiments are performed and obtained results are illustrated. Conclusions are made in section 4.

![Pendulum](image)

**Figure 1.** Schematic of the 2-DOF energy harvester

### 2. Modeling

The configuration of the proposed system is illustrated in figure 1. For converting the original 1-DOF (degree of freedom) harvester to a 2-DOF one, the tip mass is replaced by a pendulum. The governing equations of the system is given by,

\[
\begin{align*}
(M + m)\ddot{q} + c_1\dot{q} + Kq + ml(\dot{\varphi}^2 - \dot{\varphi}) + \chi v &= -(M_i + m)\ddot{z}_s, \\
ml^2\ddot{\varphi} + c_2\dot{\varphi} + ml\ddot{q} + mgl\varphi &= -ml\ddot{z}_s, \\
\frac{v}{R} - \chi \dot{q} + C_\varphi \dot{\varphi} &= 0
\end{align*}
\]

Where \(q\) refers to the displacement at the beam tip, \(\varphi\) represents the angular displacement there; \(m\) denotes the mass of pendulum, \(z_s\) is the base motion, \(c_1\) and \(c_2\) are the effective damping coefficients, \(l\) is the pendulum length, \(C_\varphi\) is the capacitance, \(v(t)\) represents the voltage output, \(R\) is the resistance, \(\chi\) is the electromechanical coupling coefficient, \(g\) is the gravitational acceleration,

By introducing some parameters such as \(u = \frac{q}{l}\), \(V = \frac{v}{l}\), \(c_1' = \frac{c_1}{(M + m)l}\), \(c_2' = \frac{c_2}{ml}\), \(\chi' = \frac{\chi}{M + m}\),

\[
y = \frac{M_i + m}{M + m},
\]

the equations of motion given by Eq. (1) are simplified as dimensionless form
Then the first two natural frequencies of the harvester can are given by

\[
\begin{align*}
\ddot{y} + r_r \phi - r_r \phi^2 \phi + c_1 \dot{u} + \dot{\phi}^2 u + \frac{\gamma}{l} V &= 0 \\
\ddot{\phi} + \frac{\gamma}{l} \dot{u} + c_\phi \dot{\phi} + \dot{\phi}^2 \phi &= -\frac{\dot{z}}{l} \\
\frac{V}{R} - \chi \dot{u} + C_e \dot{\phi} &= 0
\end{align*}
\]  (2)

Then the first two natural frequencies of the harvester can are given by

\[
\omega_{1,2} = \frac{\omega_1 + \omega_2}{2} \pm \sqrt{\left(\frac{\omega_1 - \omega_2}{2}\right)^2 + 4r_1 \omega_1 \omega_2}
\]  (3)

\[
\dot{\phi}^2 = \frac{g}{l}; \dot{\phi}^2 = \frac{K}{M + m}; r_\phi = \frac{m}{M + m}, r_\phi \in (0,1)
\]  (4)

Where \(\omega_1^2\) and \(\omega_2^2\) are the natural frequency of the single inverted beam and pendulum separately; \(M, M_1, K\) indicate the modal mass of the piezoelectric inverted beam, the shape weighting mass, the mechanical stiffness of beam. It is obvious that the natural frequencies of the system are calculated by the \(\omega_1^2\) and \(\omega_2^2\). And the peculiar nonlinear term \(\dot{\phi}^2 \phi\) in equation (1) could introduce a nonlinearity to the energy harvester.

3. Experimental results

The experiments were conducted to verify the performance, figure 2 shows the overview of the experimental setups. The horizontal base excitation was provided by a electrodynamic shaker (DONGLING ESD-100). The strain near the root and the voltage output across piezoelectric transducer were imported to DH5922N. The inverted beam (190×8×0.6 mm\(^3\)) was fixed on the shaker base with a PZT-5H transducer (12×12×0.3 mm\(^3\)) bonded to the root. The pendulum (\(m = 10.4\) g) is mounted on the end of inverted beam, the string length was \(l = 0.027\) m.

![Figure 2](image)

**Figure 2.** Experimental setups and data acquisition system: (a) dynamic strain indicator and controller; (b) inverted beam with pendulum

Firstly, the system was excited by a sweeping sinusoidal excitation with the frequency range of 5-20 Hz, in which the first natural frequency is not included. The amplitude frequency responses obtained from forward and reverse frequency sweep experiments demonstrate that the nonlinearity at the second natural frequency is a kind of softening spring (shown in figure 3(a)). It is obvious that the bending direction and multiple solution area point to the left. And the multiple solution area resulted from the cubic nonlinear term could be used to give a significant enhancement in harvesting energy. For the aim to further illustrate the nonlinearity, the responses under different reverse sweep excitation intensity are shown in figure 3(b). The results indicate that the amplitude of voltage output and the bandwidth near
the second natural frequency could increase simultaneously if the excitation intensity increases. For the comparison, the frequency response of inverted beam with tip mass is illustrated in figure 4. There is no resonance peak in the response. The result indicates that the attachment of the pendulum could enable the low frequency energy harvesting.

**Figure 3.** Amplitude frequency responses under sweep excitations:
(a) forward and reverse sweep results; (b) responses under different excitation levels

**Figure 4.** Amplitude frequency response of vertical cantilever with tip mass under the excitation intensity of \( a = 2 \text{ g} \).

With regard to the energy harvesting efficiency, the responses under random excitation are concerned. The band limited white noise with frequency range of 5-125 Hz was utilized as the base excitation, and the first natural frequency is not within it. The excitation intensity was varying at \( D = 0.014 \text{g}^2/\text{Hz}, 0.020 \text{g}^2/\text{Hz}, 0.027 \text{g}^2/\text{Hz} \). The responses are demonstrated in figure 6-8. It is obvious that the motion of system is a mono-stable type. And the spectra of the strain (figure 5) indicate that low mode oscillation can be excited, which validates the existence of a mode coupling in the responses.

**Figure 5.** Spectra of the strain at the stochastic excitation intensity of \( D = 0.014 \text{g}^2/\text{Hz} \).
Figure 6. Strain and output voltage at $D = 0.014$ g$^2$/Hz (mono-stable):
(a) Strain; (b) Output voltage

Figure 7. Strain and output voltage at $D = 0.020$ g$^2$/Hz (mono-stable):
(a) Strain; (b) Output voltage

Figure 8. Strain and output voltage at $D = 0.027$ g$^2$/Hz (mono-stable):
(a) Strain; (b) Output voltage

4. Conclusions
This work proposes a two degree of freedom energy harvester which is composed of inverted beam and pendulum. The motion of the pendulum attached to the free end of the inverted beam could give a unique nonlinear phenomenon. The dynamic responses subjected to harmonic and stochastic base excitation were measured in the experimental test. According to the responses obtained, conclusions are drawn as follows:
1) The amplitude-frequency response shows that the system owns a softening spring feature, and the multiple solution region could produce a significant improvement in extending the effective frequency band of harvester;
2) Mode coupling phenomenon was found in the response when a band limited stochastic excitation was applied;
3) The experiment results indicate that the energy could make a transformation from the second order mode to the first order mode, which enhances the energy harvesting ability.
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