First two modal adaptive vibration control for a smart beam with two piezoelectric bimorphs by a self-tuning PID control

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Abstract. This paper presents model and simulation verification of vibration suppression to two-input two-output system of a smart cantilever beam bonded with piezoelectric bimorphs by Self-tuning PID controller. The Self-tuning PID controller is designed for the smart cantilever beam with pole-placement method theory and recursive algorithm. The displacement is acquired by the Piezoelectric bimorph on the root of the smart cantilever beam. The control voltage is calculated by the designed Self-tuning PID control parameters. The established model realizes simulation through Simulink of MATLAB. Through some simulation verification results, it is indicated that the suppressed amplitude of free vibration is obvious. Therefore, it is demonstrated that the Self-tuning PID control for two-input two-output smart beam system with Piezoelectric bimorphs is feasible to reduce free vibration.

1. Introduction
In the past several years, a lot of researchers have committed to control vibration of a smart beam with piezoelectric actuator and sensor by a variety of controllers. Active vibration suppression of piezoelectric cantilever beam by using an adaptive feed-forward control method is proposed [1-6]. Adaptive pole placement control of a smart cantilevered beam in thermal environment control vibration [7]. Active vibration suppression in smart structures subjected to model uncertainties and environmental disturbances is controlled by an adaptive approach [8]. Hysteresis model and adaptive vibration suppression for a smart beam with time delay is applied to control system [9]. Adaptive control is applied to control system for vibration suppression of an active seat of occupant [10]. A new robust adaptive controller is used to control vibration of active engine mount subjected to large uncertainties [11]. A multi-input multi-output adaptive feed-forward controller is used for vibration alleviation on a large blended wing body airliner [12]. Simulation and experimental tests on active mass damper control system is controlled by Model Reference Adaptive Control algorithm [13]. However, there is a lack of solution to the coupling problem of multiple degrees of freedom. This paper will solve the problem of two degree of freedom coupling system. Online decoupling can be realized by system online identification. And parameters P, I, D are optimal as far as possible. Cantilever beam systems with piezoelectric bimorph are also rare. The cantilever beam system in this paper has two piezoelectric bimorphs.

In this paper, by combining the assumed mode method and the Hamilton principle, a dynamical model of a smart cantilever beam is made [14]. In addition, based on the dynamical model, a Self-tuning PID controller [15-16] according to pole-placement method theory is designed to suppress free vibration of a smart cantilever beam. In the Self-tuning PID control design process, the output
displacement of a smart cantilever beam is sensed by piezoelectric bimorph. Through the simulation, a Self-tuning PID controller could suppress vibration effectively. In a word, a dynamical model is controlled by Self-tuning PID controller and suppress the free vibration of a smart cantilever beam according to the simulation result.

The rest of this paper is organized as follows. A dynamical model for the smart system is constructed in Section 2. A Self-tuning PID controller is designed for the purpose of vibration suppression in Section 3. Some simulations are shown to verify the vibration reduction effectiveness for the smart cantilever beam by Self-tuning PID controller in Section 4. Finally, some conclusion are given in Section 5.

2. Dynamic modeling for smart system
A smart cantilever beam bonded with two piezoelectric bimorph is shown in figure 1. Two piezoelectric bimorphs are placed on the middle and the root of the smart cantilever beam respectively. When the end tip of the smart cantilever beam is subjected to an external disturbance, the piezoelectric bimorph is activated with the control voltage generated by a designed controller to suppress vibration. The output displacement of the smart cantilever beam is sensed by piezoelectric bimorph to be used for checking vibration reduction effect. Before that, the dynamical model for the smart system should be made.

![Figure 1. A smart beam bonded with two Piezoelectric bimorphs.](image)

If the study object is non-uniform cantilever volume, the bending stiffness of the beam along the length \( x \) direction is \( E I(x) \), the mass density of the beam is \( \rho \), the mass per unit length \( m(x) \), the transverse load acting on the beam \( P(x,t) \), and the transverse displacement (deflection) of the beam \( w(x,t) \) are all functions of continuous transformation of coordinate \( x \) and time \( t \).

For an equal section beam, the partial differential equations of motion is

\[
m \frac{\partial^2 w(x,t)}{\partial t^2} + EI \frac{\partial^4 w(x,t)}{\partial x^4} = P(x,t)
\]

The system sensing equation and the induced equation are got, that is, the basic equation of the vibration of a flexible cantilever beam.

The equations of the whole system can be obtained from the upper synthesis.

\[
\begin{align*}
&\frac{m(x)}{\partial^2 w(x,t)} + c(x) \frac{\partial w(x,t)}{\partial t} + EI(x) \frac{\partial^4 w(x,t)}{\partial x^4} = \\
&\frac{\partial^2}{\partial x^2} \left[ \sum_{i=1}^{N} K_i U_i(t) \left[ H(x-x_i) - H(x-x_{i+1}) \right] \right] \\
V_i(t) &= \frac{bt_i e_i E_i p}{2 c_p} \int_{x_i}^{x_{i+1}} \frac{\partial^2 w(x,t)}{\partial x^2} \, dx \\
w(0,t) &= 0, \quad \left. \frac{\partial w(x,t)}{\partial x} \right|_{x=0} = 0 \\
&\left. EI \frac{\partial^3 w(x,t)}{\partial x^3} \right|_{x=0} = 0 \\
&\left. EI \frac{\partial^3 w(x,t)}{\partial x^3} \right|_{x=0} = 0
\end{align*}
\]
Where $C_i$ is the piezoelectric film capacitance, unit is F; $E_r$ is the elastic modulus of the piezoelectric bimorph; $b$ is the width of the cantilever beam; $t_o$ is the thickness of the cantilever beam; $e_{31}$ is the piezoelectric coefficient; $w$ is the deflection of a flexible arm (the lateral displacement of the arm).

The relationship between the voltage on the actuated slice and the deflection of flexible cantilever beam (the transverse displacement when flexible cantilever beam vibrate) is given in this model. The deflection of flexible cantilever beam can be measured by piezoelectric bimorph. In this regard, a modal analysis method based on vibration mechanics is introduced, that is, the orthogonal modal transformation is applied to transform the partial differential equations of the system into a set of two order differential equations, so as to facilitate the design of the controller.

For the two input and two output system, \[ w(x,t) = \phi_1(x)q_1(t) + \phi_2(x)q_2(t) \] \hspace{1cm} \text{(3)} \]

Where $w(x,t)$ is the geometric displacement coordinate of the system; $\phi_i(x)$ is the vibration mode function of type N order mode; $q_i(t)$ is the generalized coordinate of the n order mode, also known as the mode coordinate.

The equation $w(x,t) = \phi_1(x)q_1(t) + \phi_2(x)q_2(t)$ is replaced by the piezoelectric sensor equation \[ V_i(t) = \frac{b t e_{31} E_p}{2 C_p} \int_0^t \hat{q}_i(t) \hat{w}(x,t) dx = K \sum_{n=1}^\infty [\phi_n(x_{i,n}) - \phi_i(x)] \hat{q}_n(t) = GQ(t) \] \hspace{1cm} \text{(4)} \]

Where $K = \frac{b t e_{31} E_p}{2 C_p}$ ; \[ G = [G_1, G_2], G_n = [\phi_n(x_{i,n}) - \phi_i(x)] \] \[ Q(t) = [q_1(t), q_2(t)] \]

When we take the regular mode of vibration $\phi_i(x)=1$, so $M_i=1$, the vibration differential equation of the beam concretely is \[ \hat{q}_1(t) + 2\hat{w}_1 \hat{q}_1(t) + w_1^2 \hat{q}_1(t) = K_{11} U_1(t) [\phi_1(x_2) - \phi_1(x_1)] + K_{12} U_2(t) [\phi_1(x_2) - \phi_1(x_1)] \]
\[ \hat{q}_2(t) + 2\hat{w}_2 \hat{q}_2(t) + w_2^2 \hat{q}_2(t) = K_{21} U_1(t) [\phi_2(x_2) - \phi_2(x_1)] + K_{22} U_2(t) [\phi_2(x_2) - \phi_2(x_1)] \] \hspace{1cm} \text{(5)} \]

For the piezoelectric intelligent cantilever described by equation (5), the first two modes are taken, and the equations is expressed in matrix form.

The description of the whole closed loop system is \[
\begin{bmatrix}
\hat{q}_1(t) \\
\hat{q}_2(t) \\
\hat{\phi}_1(t) \\
\hat{\phi}_2(t)
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-w_1^2 & 0 & -2 e_{31} w_1 & 0 \\
0 & -w_2^2 & 0 & -2 e_{31} w_2
\end{bmatrix} \begin{bmatrix}
q_1(t) \\
q_2(t) \\
\phi_1(t) \\
\phi_2(t)
\end{bmatrix} + K \begin{bmatrix}
0 & 0 & \phi_1(t) & \phi_2(t) \\
0 & 0 & \phi_1(t) & \phi_2(t)
\end{bmatrix} \begin{bmatrix}
U_1(t) \\
U_2(t)
\end{bmatrix}
\] \hspace{1cm} \text{(6)} \]

That is \[ \dot{X}(t) = AX(t) + BU(t) \] \hspace{1cm} \text{(7)} \]

And \[ Y(t) = \dot{\tilde{X}}(t) \] \hspace{1cm} \text{(8)} \]

Where \[ \tilde{M} = \begin{bmatrix} M_{11} & M_{12} & 0 & 0 \\
M_{21} & M_{22} & 0 & 0 \end{bmatrix}, \quad M_{1i} = K [\Phi_i(x_{i+1}) - \Phi_i(x_i)] , \quad i = 1, 2 \]
\[ K = \frac{b t e_{31} E_p}{2 C_p} \]

Or \[ M = \begin{bmatrix} M_{11} & M_{12} \\
M_{21} & M_{22} \end{bmatrix}, \quad \tilde{M} = \begin{bmatrix} M & 0 \end{bmatrix} \]

In addition, the related geometric and mechanical parameters about simulation are given in table 1. The model has two natural frequencies, the first order natural frequency and the two order natural frequency. The two natural frequencies obtained from Simulink are 14.563 and 65.626 respectively. By means of numerical solution, the ode23s method in Runge Kutta is used to obtain the natural frequencies. It is shown in figure 2.
### Table 1. System parameters used in simulation.

| Beam Parameters |  |
|-----------------|------------------|
| \( l_b = 0.903m \) | \( w_b = 0.035m \) | \( t_b = 0.0015m \) |
| \( \rho_b = 2.7 \times 10^3 \text{ kg/m}^3 \) | \( E_b = 70 \text{ GPa} \) | \( C_b = 0.01 \) |

| Piezoelectric Parameters |  |
|--------------------------|------------------|
| \( l_p = 0.0526m \) | \( w_p = 0.01m \) | \( t_p = 0.0035m \) |
| \( d_{11} = 0.7407 \times 10^{-6} \text{ m/v} \) | \( E_p = 10.64 \times 10^{10} \text{ N/m}^2 \) | \( C_p = 3.0 \text{nF} \) |
| \( h_{0} = -1.35 \times 10^3 \text{ v/m} \) | \( x_{v} = 0.06m \) | \( C_1 = C_2 = 0.1 \) |

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![Natural Frequency](image1.png)

**Figure 2.** Natural frequency of model system.

![Indirect self-tuning control block](image2.png)

**Figure 3.** Indirect self-tuning control block.

3. **Controller design**

In this paper, self-tuning PID control is adopted to suppress free vibration for the smart beam. In the two input two output system, there is a coupling phenomenon, which needs decoupling. The decoupling control is realized by self-tuning PID decoupling control.

Figure 3 control block shows system structure of self-tuning PID control. This paper adopts self-tuning PID control algorithm to improve the accuracy and stability of control in the interference to the parameters of the system fluctuates, through online identification, get the parameters of the system, then according to the change of parameters adjustment of PID control parameters to achieve the desired output, to improve the system stability, response speed and control precision. Its control block is shown in figure 3.

The design idea of self-tuning PID controller is: taking equation (9) as the basic form of controller, recursive algorithm is introduced to estimate object parameters, and the estimation results are designed according to pole placement method. The parameter estimation and the design of the controller can be carried out separately, and its self-tuning ability is reflected by online identification. The pole assignment self tuning PID control principle adopted in this paper is composed of three parts: least squares recursive parameter estimation, controller parameter design and PID controller. Its control system is shown below.

As an example, the design of a self-tuning PID controller is introduced.

\[
F(z^{-1})u(k) = R(z^{-1})y(k) - G(z^{-1})y(k)
\]  

(9)

The controlled objects are as follows

\[
A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k) + e(k)
\]  

(10)

Where \( u(k) \) and \( y(k) \) represent the input and output of the system, the external disturbance is \( e(k) \),

\[
A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} \]

\[
B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}
\]

a pure delay is \( d \geq 1 \), and

The system is controlled by PID. In order to eliminate the constant interference, the controller must have integral function. At this time, the corresponding PID controller can be expressed as
The corresponding compatibility conditions in the pole configuration control (PPC) still need to be satisfied. After put equation (11) into equation (10), closed loop system output is obtained

\[ y(k) = \frac{BR}{AF_1 + z^{-d}BG} y_r(k-d) + \frac{F_i}{AF_1 + z^{-d}BG} e(k) \]  

(13)

The closed loop characteristic polynomial is the denominator polynomial of the expected transfer function, that is,

\[ AF_1 + z^{-d}BG = A_n \]  

(14)

For the selection \( A_n \), the corresponding compatibility conditions in the pole configuration control (PPC) still need to be satisfied. The combination equation (12), the upper form can also be expressed as

\[ \Delta A F + z^{-d}B G = A_n \]  

(15)

Where \( \Delta A(z^{-1}) = (1 - z^{-1}) A(z^{-1}) \)

In order to ensure that the equation (15) has a unique solution, the order of \( \Delta A \) and \( z^{-d}B \) are the same (known \( \text{deg} \Delta A = 3 \) and \( \text{deg} G = 2 \)), and the order on the right side of the equation is smaller than the order on the left side, that is, the order of each polynomial is required to satisfy the following relationship

\[ \begin{aligned} 
\text{deg } F &= \text{deg } B + d - 1 \\
\text{deg } A_n &\leq \text{deg } B + d + 2 
\end{aligned} \]  

(16)

The parameters of the polynomial \( F \) and \( G \) in the Diophantine equation (15) can be solved by the MATLAB function diophantine.m written in the book.

When the parameters \( A(z^{-1}) \) and \( B(z^{-1}) \) are unknown, the self-tuning control algorithm should be adopted. Similarly, there are two ways of self-tuning PID control, which are indirect self-tuning control and direct self-tuning control. Here, the control algorithm I use is indirect self-tuning control algorithm.

4. Simulation verification

Based on the Self-tuning PID controller design, the theoretical simulation analysis is as follows. Figure 4 shows output displacement response and control voltage of the system under sinusoidal input excitation. The output displacement is also sinusoidal. The significance of this diagram is to describe the dynamic characteristics of the system. Figure 5 shows free vibration of system model. The amount of tip displacement given to initiate the free vibrations in smart cantilever beam is \( 1.9509 \times 10^6 \) m according to the calculation of simulation model parameters. Figure 6 shows the control voltage with simulation and the control results of displacement in simulation comparing with the displacement of system without control. Before the time about 0.06s, the control voltage is more than the limited voltage 150V. It can be seen that in the beginning of the system operation, the system needs a certain time to estimate the parameters, so there is a large amplitude oscillation. After a period of operation, the system has a better control effect. After the time about 0.15s, the free vibration amplitudes of displacement are all reduced up to a small value. Compared with the free vibrations without control, the control effects in simulation are obvious. The self-tuning PID control method has better input tracking ability and adaptive ability, and it can control the cantilever beam better.
In short, the simulation demonstrates that the Self-tuning PID control is feasible to suppress free vibration. Furthermore the control effect of displacement is obvious in simulation. It is verified that the control effectiveness is considerable.

![Figure 4. Sin-input without control.](image)

![Figure 5. Free vibration.](image)

![Figure 6. Displacement and control voltage with Self-tuning PID control.](image)

5. Conclusions
In this paper, the suppression vibration of two-input two-output system of a smart cantilevered beam bonded with piezoelectric bimorphs by Self-tuning PID controller is focused on. The dynamical mathematical model for a smart beam is constructed using the Hamilton principle. Based on the dynamical model, the Self-tuning PID control is designed through the pole placement theory. Finally,
some simulations prove that Self-tuning PID controller is feasible to control vibration. Moreover, the reduced amplitudes of displacement for the smart beam are obvious in simulation. The major contributions of the current work are as follows. The simulation results show that double degree-of-freedom smart cantilever system with piezoelectric bimorphs under the control of self-tuning PID controller is verified to be effective and considerable. Although the insufficient controller is used, new problems are solved. Vibration control of intelligent structures is carried out considering multiple modes and multiple degrees of freedom coupling.

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