Mirage gauge coupling unification \footnote{This paper needs substantial revision. See the footnote in page 6.}

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Abstract

We use compact $D = 4$, $N = 1$, Type IIB orientifolds as a testing ground for recent ideas about precocious gauge coupling unification and a low energy string scale. We find that certain such orientifolds have the interesting property that gauge couplings receive moduli-dependent corrections which mimic the effect of field theoretical logarithmic running. The effective cut-off scale for the logarithmic correction is not $M_{\text{string}}$ but rather $M_X = \sqrt{\alpha M_{\text{Planck}} M_c / M_{\text{string}}}$, where $M_c$ is the compactification scale. Thus there is just normal logarithmic running up to $M_{\text{string}}$ and extra moduli dependent corrections which behave as if there was further running to a higher virtual scale $M_X$. In this mechanism a prominent role is played by anomalous $U(1)$’s with moduli dependent Fayet-Iliopoulos terms. A vanishing FI-term fixes the modulus dependence of the corrected gauge coupling. We discuss possible ways to implement this mechanism in the context of a simple extension of the MSSM. Agreement identical (the one-loop equations are the same) to the one obtained in SUSY-GUT’s is obtained for $M_{\text{string}} = 10^{11}$ GeV and $M_c = 10^9$ Gev. This fits with the recent suggestion, based on completely independent arguments, of identifying the string scale with the intermediate scale. Scenarios with a 1 TeV string scale tend to yield too small a value for $M_X$ in this context.
1 Introduction

There has been recently a lot of interest in studying the viability of a lowering of the scale of string theory. Although the scale of string theory in perturbative heterotic vacua is essentially fixed to be of order $gM_{Planck}$, it has been realized that it can be arbitrarily lowered in Type I and Type IIB vacua [1]–[16]. This is possible because in the latter one can have the charged matter fields living only on the worldvolume of some 3-branes while gravity can propagate in all ten (or eleven) dimensions. Thus one can decouple the Planck mass from the string scale by having some of the compact dimensions sufficiently large.

Although the possibility of lowering the string scale well below the Planck mass is quite exciting, one of the least attractive aspects of it is that the standard gauge coupling unification of the MSSM is lost. Some ideas have been proposed to accomodate precocious gauge coupling unification [4, 12, 13, 14, 17, 18] but there is no longer a prediction for gauge coupling unification which naturally fits LEP data.

In order to study this issue further we believe it makes sense to consider consistent explicit Type I $D = 4, N = 1$ vacua and check the behaviour of gauge coupling unification in specific models. The simplest such models are the toroidal Type IIB, $D = 4, N = 1$ orientifolds [19] constructed in refs.[20]–[29]. They give rise to consistent four-dimensional chiral theories with a variety of gauge groups. A necessary requirement to get chiral models is the location of the relevant D-branes close to orbifold singularities.

It has been pointed out in refs.[20, 29, 15] that the gauge kinetic functions corresponding to e.g., a set of D3-branes at an orbifold singularity get a piece proportional to the blowing-up fields $M$ of the given singularity. In a simplified notation one gets for the gauge kinetic functions a general form $f_b = S + c_b M$, where $S$ is the complex dilaton and $c_b$ are constant coefficients. This structure is in fact necessary in order to cancel $U(1)$ gauge anomalies [29] present in these theories by a Green-Schwarz mechanism [30]. Now, if the $c_b$ coefficients were proportional to the $\beta_b$-function coefficients (and if $< Re M > \neq 0$), such structure could mimic some extra logarithmic running and could modify substantially the conclusions about gauge coupling unification. They behave like effective large threshold corrections. It was argued in ref.[15] that indeed, in $Z_N$, $N$ odd compact orientifolds this is indeed the case and the $c_b$ coefficients are proportional to the $\beta_b$'s.

In the present article we study that point in more detail. The proportionality of those coefficients is due to the fact that in such models the same $M$ fields which cancel some anomalous $U(1)_a$ symmetries cancel at the same time certain $\sigma$-model
anomalies which are proportional to the $\beta_b$ coefficients \[31\]. There is also a delicate interplay between both type of anomalies. In particular, their simultaneous presence makes the $U(1)_a$ Fayet-Iliopoulos terms $\xi_a$ to get $T$(modulus)-dependent corrections \[31\]. More explicitly, one has a structure of the type $\xi \propto (ReM − \log(T + T^*))$. Upon minimization one gets $< ReM > = \log(T + T^*)$ and the gauge coupling gets corrections $\propto \beta_b \log(T + T^*)$. Upon considering standard field theory running up to the string scale $M_{\text{string}}$ one finds that the effective large mass scale in the computation is not $M_{\text{string}}$ but a scale $M_X = \sqrt{\alpha} M_{\text{Planck}} M_c / M_{\text{string}}$ where $M_c = 1/R$ is the overall compactification scale.

Thus we claim that in $Z_N$, $N$-odd compact orientifolds the couplings at the string scale are not equal but are split in a manner which actually mimics further running from $M_{\text{string}}$ up to a virtual scale $M_X = \sqrt{\alpha} M_{\text{Planck}} M_c / M_{\text{string}}$. This is what we call ”mirage” unification” : from low-energies everything looks as if there was just standard field-theoretical logarithmic running up to the scale $M_X$. In reality the field theoretical running occurs only up to the string scale. Unification is actually a mirage.

There is not at present any $D = 4, N = 1$, Type IIB orientifold with a completely realistic spectrum. However one can contemplate the possibility that a unification mechanism like the one for the above orientifolds could be at work for a realistic model including the MSSM. In this case we find that the experimentally preferred scale $M_X = 10^{16}$ GeV is only obtained for $M_{\text{string}} = 10^{11}$ GeV and $M_c = 10^9$ GeV. Remarkably enough these are the values for the fundamental scales recently proposed in ref.[13] on the basis of completely different arguments. Indeed, in that reference it was proposed the identification of the string scale with the intermediate scale $M_{\text{string}} = \sqrt{M_{\text{Planck}} M_W} = 10^{11}$ GeV in order to understand the generation of the $M_{\text{Planck}}/M_W$ hierarchy in terms of the ratio $M_{\text{string}}/M_c$.

The structure of this article is as follows. In the next section we describe briefly the cancelation of gauge $U(1)$ anomalies and $\sigma$- model anomalies in $Z_N$ compact orientifolds with odd $N$. In section 3 we describe the structure of the Fayet-Iliopoulos terms asociated to the anomalous $U(1)$’s and show how their cancellation leads to the ”mirage” unification described above. In section 4 we describe the structure of a simple extension of the MSSM including the mirage unification mechanism. In its simplest form it will require the existence of an anomalous $U(1)$ whose mixed anomalies with the groups of the standard model coincide with the respective $\beta$-functions. We give in section 5 the final comments and conclusions and discuss about the possible generality of these results.
2 \quad U(1)’s and \(\sigma\)-model anomalies in Type IIB, \(D = 4\), \(N = 1\) orientifolds

In our discussion a prominent role is played by both anomalous \(U(1)\)’s and \(\sigma\)-model symmetries of the theory. Type IIB \(D = 4\), \(N = 1\) orientifolds have generically \(U(1)\) gauge interactions whose triangle anomalies are non-vanishing \([29]\) . These anomalies are cancelled by a generalized Green-Schwarz mechanism which involves the exchange of twisted singlet fields \(M_f\) associated to the fixed points \(f\) under the orbifold action \([29]\) . To be specific, let us consider the case of \(Z_N\), \(N\)-odd orientifolds. Associated to each fixed point \(f\) (e.g., 27 of them for \(Z_3\) and 7 for \(Z_7\)) there are twisted moduli fields \(M^k_f\), with \(k = 1, \ldots, (N - 1)/2\). The gauge kinetic function has a \(M^k_f\)-dependent piece which appears at the disk level \([29, 31]\) 

\[
 f_b = S + \frac{1}{N} \sum_f \sum_{k=1}^{(N-1)/2} \frac{\cos(4\pi k V_b)}{C_k} M^k_f
\]  

(2.1)

where \(C_k = \prod_{i=1}^{3} 2\sin(\pi k v_i)\) and \(v_i\) are the twist eigenvalues of the orbifold along the \(i-th\) complex direction \([3]\). One can check that \(C_k^2\) equals the number of fixed points of the orbifold. The \(V_b\)’s correspond to fractional numbers \(l/N\) which are model dependent. Thus e.g., in the case of \(Z_3\) in which the gauge group is \(U(12) \times SO(8)\) one has \(V_{SU(12)} = 1/3\) and \(V_{SO(8)} = 0\) (see ref.\([26]\) for examples and notation). Now, under a \(U(1)_a\) gauge transformation with parameter \(\Lambda_a(x)\) the twisted \(M^k_f\) fields transform nonlinearly in the fashion:

\[
 Im M^k_f \rightarrow Im M^k_f + \delta_{GSk}^a\Lambda_a(x)
\]  

(2.2)

with

\[
 \delta_{GSk}^a = 2n_a \sin(2\pi k V_a)
\]  

(2.3)

Here \(n_a\) is the rank of the \(U(n)\) or \(SO(2n)\) group involved. One can check that indeed this transformation of the \(M^k_f\) fields combined with eq.\((2.1)\) exactly cancels the mixed gauge anomalies between the \(U(1)_a\) field and the non-Abelian factors \(G_b\). Notice that in these models, unlike the heterotic orbifold models, there may be more than one anomalous \(U(1)_a\) whose anomaly is cancelled by this mechanism \([26, 25, 29]\) .

These \(Z_N\), odd \(N\) orientifolds, like their heterotic counterparts, have also certain \(\sigma\)-model invariances \([32]\) . Indeed, the Kahler potential associated to the complex dilaton

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1For a more phenomenological description of these models see \([13]\) and references therein.

2We are defining here \(ReS = 8\pi^2/g^2\).

3Thus \(v = 1/3(1,1,-2)\) for \(Z_3\) and \(v = 1/7(1,2,-3)\) for \(Z_7\).
S, the three diagonal untwisted moduli $T_i$ and the charged fields $A_i$ associated to the open strings has the form:

$$K(S, S^*, T_i, T_i^*) = -\log(S + S^*) - \sum_{i=1}^{3} \log(T_i + T_i^* - |A_i|^2)$$  \hspace{1cm} (2.4)$$

The effective classical action presents a $\sigma$-model invariance under $SL(2, R)_{T_i}$ transformations given by:

$$T_i \rightarrow \frac{a_i T_i - i b_i}{ic_i T_i + d_i},$$  \hspace{1cm} (2.5)$$

with $a_i, b_i, c_i, d_i \in \mathbb{R}$ and $a_i d_i - b_i c_i = 1$. Under these transformations the charged matter fields $A_j$ transform as:

$$A_j \rightarrow A_j \prod_{i=1}^{3} (ic_i T_i + d_i)^{n_j^i}$$  \hspace{1cm} (2.6)$$

where $n_j^i = -\delta_j^i$. These transformations induce chiral rotations in the massless fermions of the theory. They are associated to gauge transformations of a composite gauge vector potential involving the moduli fields $T_i$. If we compute the triangle anomalies corresponding to this composite current and two gauge currents one finds in general an anomalous result. The coefficient of this anomaly can be computed to be given by:

$$b'^i_a = -C(G_a) + \sum_{\mathbb{R}} T(\mathbb{R}_a)(1 + 2n_j^i)$$  \hspace{1cm} (2.7)$$

Here $C(G_a)$ is the quadratic Casimir of the gauge group $G_a$ in the adjoint representation and $T(\mathbb{R}_a)$ is the quadratic Casimir in the representation $\mathbb{R}_a$ corresponding to a charged field. It has been recently argued that these $\sigma$-model anomalies may be cancelled again by a Green-Schwarz type of mechanism, at least in the case of odd order $Z_N$ orientifolds. Indeed, the $\sigma$-model-gauge mixed anomalies may be cancelled if the twisted $M^k_f$ fields transform under $SL(2, \mathbb{R})_{T_i}$ like:

$$Im M^k_f \rightarrow Im M^k_f + \delta^i_{GS k} \log(ic_i T_i + d_i)$$  \hspace{1cm} (2.8)$$

where

$$\delta^i_{GS k} = 2tg(\pi k v_i)$$  \hspace{1cm} (2.9)$$

These are not expected to be exact symmetries of the theory, very much like in their heterotic duals, where it is well known that only the discrete subgroup $SL(2, \mathbb{Z})_{T_i}$ survives. Still, as in the heterotic models, cancellation of $\sigma$-model anomalies is expected.

5See ref.[31] for a discussion of the $N$ even case.
Indeed, as shown in ref. [31], the anomaly coefficients $b_i'$ can be reexpressed as:

$$b_i' = -\frac{2}{N} \sum_{k=1}^{(N-1)/2} C_k \tan(\pi k V_i) \cos 4\pi k V_a$$

(2.10)

which is exactly cancelled by the mechanism discussed above. Indeed, the transformation (2.8) applied to eq. (2.1) gives precisely a piece which cancels eq. (2.10).

### 3 Fayet-Iliopoulos terms and mirage unification

It is well known that, whenever anomalous $U(1)$’s are present, associated Fayet-Iliopoulos term in general appear both in the heterotic case [33] and in $D = 4$, $N = 1$ IIB orientifolds [26, 29, 31, 34, 35]. In a model with both anomalous $U(1)$’s and anomalous $\sigma$-model symmetries the invariance under the transformations in (2.2) and (2.8) requires that the Kahler potential of the $M_k^b$ fields has the general invariant form [31]:

$$K(M_k^b, M_k^{b*}) = K(M_k^b + M_k^{b*}) - \delta_{GS k}^a V_a + \sum_{i=1}^3 \delta_{GS k}^i \log(T_i + T_i^*)$$

(3.1)

For a quadratic Kahler potential [34] for the $M_k^b$ fields, eq. (3.1) gives rise to a FI-term corresponding to the $U(1)_a$ field:

$$\xi_a = -\sum_f \sum_k (\delta_{GS k}^a (M_k^b + M_k^{b*}) + \sum_{i=1}^3 \delta_{GS k}^i \log(T_i + T_i^*))$$

(3.2)

Notice that, unlike the case of non-compact orientifolds, here the Fayet-Iliopoulos terms get a $T_i$-dependent piece [31]. This piece is the one we want to discuss now. Notice that, in the absence of non-Abelian gauge symmetry breaking (which is what we want if we are interested in studying the corrections to the couplings of the initial unbroken group), the scalar potential will have minima at:

$$\text{Re} M_k^b = \frac{-1}{2} \sum_i \delta_{GS k}^i \log(T_i + T_i^*) .$$

(3.3)

Plugging this expression into the real part of eq. (2.1) one gets:

$$\frac{8\pi^2}{g_a^2} = \text{Re} S - \frac{1}{2N} \sum_{f} \sum_{k=1}^{(N-1)/2} \cos(4\pi k V_a) \sum_{i=1}^3 \delta_{GS k}^i \log(T_i + T_i^*)$$

(3.4)

Thus we observe that, at SUSY-preserving vacua with $\xi_a = 0$, there are corrections to the gauge coupling constants which may be expressed in terms of the untwisted
moduli $T_i$ (or, rather, their vevs). Let us now for simplicity consider the behaviour with respect to the overall modulus field $T = T_i$ for all $i = 1, 2, 3$. Then one can write:

$$\frac{8\pi^2}{g_a^2} = ReS - \log(T + T^*) \frac{1}{2N} \sum_i \sum_f \sum_{k=1}^{(N-1)/2} \cos(4\pi kV_b) \frac{2tg(\pi kV_i)}{C_k}$$

(3.5)

Now, recalling eq.(2.10) and the fact that $C_k^2$ equals the number of fixed points one realizes $\beta$:

$$\frac{8\pi^2}{g_a^2} = ReS + \frac{1}{2} \beta_a \log(T + T^*)$$

(3.6)

where we have made us of the fact that $\sum_i b_i^a = \beta_a$, the corresponding $\beta$-function. This can be easily checked from eq.(2.7). Eq.(3.6) is an interesting result because it shows us that for SUSY vacua with vanishing FI-terms the twisted-moduli-dependent piece of the gauge coupling constant may be re-expressed in terms of the untwisted moduli with a coefficient that is no other but the $\beta$-function. It is this fact which leads to the ”mirage unification” that we mentioned above.

Indeed, let us now add the effect of the field theory running of couplings up to a cut-off equal to the string scale:

$$\frac{8\pi^2}{g_a^2(Q^2)} = ReS + \frac{1}{2} \beta_a \log(T + T^*) + \frac{1}{2} \beta_a \log \frac{M_{\text{string}}^2}{Q^2}$$

(3.7)

Now, setting as the cut-off for logarithmic running $M_{\text{string}}$ is only correct if there are no other thresholds of charged particles at lower energies. Thus if we are working with 9-branes this would require going to compactification scales above $M_{\text{string}}$. But in that

\[\text{This formula is incomplete since it only contains the contribution to the coupling from the Wilsonian piece of the action. When one includes the effect of the rescaling of the kinetic terms of massless fields, an extra piece given by } \frac{1}{2} \beta_a \log(T + T^*) \text{ has to be added, as pointed out recently in ref.} \]}

\[\text{This cancels the term coming from } <ReM>. \text{ Thus, contrary to the claim below, there is actually no } ”\text{mirage unification}” \text{ in } Z_N, \text{ } N \text{ odd orientifolds, unification takes place at the string scale. It is unclear what will happen in other orientifolds corresponding to D-branes sitting at singularities. In more general cases one expects also a general structure } f_a = S + s_a M \text{ for the gauge kinetic function although now } s_a \text{ is not necessarily given by the beta-function as in } Z_N, \text{ } N \text{ odd orientifolds. One also expects } M \text{ to mix with } \log(T + T^*) \text{ but a perfect cancellation between the } \frac{1}{2} \beta_a \log(T + T^*) \text{ piece from the rescaling and the term from } <ReM> \text{ is in general not expected. Thus large corrections for the gauge coupling constants coming from this misccancellation will in general be present. This may help in making compatible the existence of a low string scale with the coupling unification problem. In particular, if the } s_a \text{ coefficients were proportional to } b_a, \text{these corrections could be reabsorbed into a redefinition of } M_X \text{ and the mirage unification scenario discussed in the text could be realized. However, this possibility is not exemplified by } Z_N, \text{ } N \text{ odd orientifolds, as wrongly stated in this paper. A properly revised version will be submitted in due course.}\]
case we better do a T-duality transformation and work with 3-branes. Then we would
have the compactification scale $M_c$ below $M_{\text{string}}$ but that would cause no new charged
threshold. For 3-branes we know that (see e.g. ref. [15]) $T + T^* = 2M_{\text{string}}^4/(M_c^4\alpha)$,
with $\alpha = g^2/4\pi$, and hence we have now:

$$\frac{8\pi^2}{g_\sigma^2(Q^2)} = \text{ReS} + \frac{1}{2} \beta_\sigma \log \frac{2M_{\text{string}}^5}{M_c^4\alpha Q^2} \tag{3.8}$$

This equation is telling us that the one-loop corrected couplings behave in an effective
manner as if there was standard field theory logarithmic running not only up to the
scale $M_{\text{string}}$ but up to a virtual scale $M_X$ defined by:

$$M_X = \frac{\sqrt{2}M_{\text{string}}^3}{M_c^2\sqrt{\alpha}} = \sqrt{\alpha} M_{\text{Planck}} \frac{M_c}{M_{\text{string}}} \tag{3.9}$$

where we have used the equation $M_{\text{Planck}} = (\sqrt{2}/\alpha)M_{\text{string}}^4/M_c^3$ (see e.g., ref. [15]).
Thus in $Z_N$ orientifolds of the class here discussed, there is actually ”mirage unification”
in the sense described in the introduction.

4 An extension of the MSSM with mirage unification

The above discussion was made in the context of $Z_N$ compact Type IIB $N = 1$, $D = 4$
$Z_N$ orientifolds with odd $N$. In the case of even $N$ the cancelation of $\sigma$-model anomalies
is expected for some of the three complex planes but the situation concerning the others
is not clear [31]. Furthermore, no completely realistic model has been yet obtained
from this type of Type IIB constructions.

Nevertheless the mechanism found for this class of orientifolds is quite elegant and
something similar to it could be at work in a more realistic model. Thus, motivated
by the mechanism above, we would like to present a simple extension of the MSSM
incorporating it.

The ingredients of the model are as follows:

i) The gauge group will be $SU(3) \times SU(2) \times U(1)_Y \times U(1)_X$, where the $U(1)_X$ has
to be an anomalous symmetry with anomaly cancelled by a Green-Schwarz mechanism.
We do not need to commit ourselves for the moment with an specific charge assignement.
Notice only that the mixed anomalies of this $U(1)_X$ with the SM groups are necessarily
unequal, as is the general case in orientifolds. They are however very much constrained,
as we discuss below.
ii) We assume the Kahler potential presents a classical \( SL(2, \mathbb{R}) \) invariance and the charged dilaton \( S \), overall modulus \( T \) and MSSM charged chiral fields \( \phi_a \) have a Kahler potential of the form

\[
K(S, S^*, T, T^*, \phi_a, \phi_a^*) = -\log(S + S^*) - 3\log(T + T^*) + \sum_{\alpha} \frac{\phi_a \phi_a^*}{T + T^*} \tag{4.1}
\]

where the sum on \( \alpha \) goes over all the charged chiral fields of the MSSM.

iii) The \( \sigma \)-model and \( U(1)_X \) anomalies are cancelled by a Green-Schwarz mechanism involving a singlet \( M \) field. This singlet field appears in the gauge kinetic function in the form (in analogy with the orientifold results):

\[
f_a = S + \frac{\beta_a}{2} M \tag{4.2}
\]

and transforms under \( U(1)_X \) and \( SL(2, \mathbb{R}) \) transformations as:

\[
Im M \quad \rightarrow \quad Im M + \delta^X_{GS} \Lambda_X(x) \tag{4.3}
\]
\[
Im M \quad \rightarrow \quad Im M - 2\log(iT + d) \tag{4.4}
\]

where \( \delta^X_{GS} \) is a coefficient which would depend on the \( U(1)_X \) charge assignments. Notice that the \( \sigma \)-model anomaly coefficients computed from eqs.(2.7) and (4.1) are given by the \( \beta \)-functions. Thus, in this simple model with a single \( U(1) \) and a single \( M \)-field, the mixed \( U(1)_X \)-SM standard model anomaly coefficients \( A^a_X \) must obey:

\[
A^a_X = -\delta^X_{GS} \frac{\beta_a}{2} \tag{4.5}
\]

and the mixed anomalies must be in the same ratio as the beta-function coefficients. This was indeed the case in \( Z_N \), \( N \)-odd orientifolds, as we showed above. This is quite a restrictive condition for the charge assignments of the anomalous \( U(1) \) charges.

iv) The singlet \( M \) field will have a Kahler potential of the form:

\[
K(M, M^*) = (M + M^* - \delta^X_{GS} V_X - 2\log(T + T^*)) \tag{4.6}
\]

so that it is invariant under both \( U(1) \) and \( \sigma \)-model invariance.

With the above four ingredients results very analogous to the ones found for the orientifolds are obtained. The Fayet-Iliopoulos term for the anomalous \( U(1)_X \) will be given by:

\[
\xi_X = -\delta^X_{GS} (M + M^* - 2\log(T + T^*)) \tag{4.7}
\]

\( ^7 \)Actually the structure of the metric of charged fields could be different as long as the \( \sigma \)-model anomalies are proportional to the \( \beta \)-functions.
Since we are interested in studying the corrections for the gauge couplings of the unbroken SM group, we will study the $\xi_X = 0$ field theory direction. Minimization of the scalar potential will then require $\text{Re} M = \log(T + T^*)$. Thus we will have substituting in the real part of (4.2)

$$\frac{8\pi^2}{g_a^2} = \text{Re} S + \frac{1}{2} \beta_a \log(T + T^*) \quad (4.8)$$

Now, using the definition of $\text{Re} T$ when gauge fields live on 3-branes, $T + T^* = 2M_{\text{string}}^4/(M_c^4 \alpha)$ one finds after including the running up to the string scale:

$$\frac{8\pi^2}{g_a^2(Q^2)} = \text{Re} S + \frac{1}{2} \beta_a \log \frac{M_X^2}{Q^2} \quad (4.9)$$

with the virtual scale $M_X$ given by:

$$M_X = \frac{\sqrt{2} M_{\text{string}}^3}{M_c^2 \sqrt{\alpha}} = \sqrt{\alpha} M_{\text{Planck}} \frac{M_c}{M_{\text{string}}} \quad (4.10)$$

Now, in this simple extension of the MSSM standard agreement of gauge coupling unification is achieved as long as $M_X = 2 \times 10^{16}$ GeV. This result may only be obtained for

$$M_{\text{string}} = \frac{1}{\sqrt{2} \alpha} \frac{M_X^3}{M_{\text{Planck}}^2} = 2 \times 10^{11} \text{ GeV} \quad (4.11)$$

$$M_c = \frac{1}{\alpha \sqrt{2} M_{\text{Planck}}^3} = 1.6 \times 10^9 \text{ GeV} \quad (4.12)$$

Notice in particular that the ratio $M_X/M_{\text{Planck}} = \sqrt{\alpha} M_c/M_{\text{string}}$ so that in the present scheme the well known mismatch between Planck mass and (virtual) unification scale $M_X$ would be a reflection of an analogous mismatch between string scale and compactification scale.

Remarkably enough the above values for the fundamental scales were suggested on the basis of completely different arguments in ref. [13]. Indeed, if one assumes that SUSY is broken in a far away 3-brane non-SUSY hidden sector and it is transmitted only by bulk fields to the ”observable 3-brane” world, one expects soft SUSY-breaking terms to be generated of order $M_{\text{SB}} = M_{\text{string}}^2/M_{\text{Planck}} = \alpha M_c^2/((\sqrt{2} M_{\text{string}}^2))$. These soft terms will trigger $SU(2) \times U(1)$ breaking and hence their size is of order $M_W$. On the other hand one can write in general $M_{\text{Planck}} = \sqrt{2} M_{\text{string}}^4/(\alpha M_c^3)$. One thus obtains $M_W/M_{\text{Planck}} = \alpha^2/2(M_c/M_{\text{string}})^6$ in this situation. Now, if $M_c/M_{\text{string}} \propto 1/160$

\[\text{Notice we have also the constraint } M_{\text{Planck}} = (\sqrt{2}/\alpha) M_{\text{string}}^4/M_c^3 \text{ so that } M_{\text{string}} \text{ and } M_c \text{ are uniquely fixed in terms of } M_X \text{ and } M_{\text{Planck}}.\]
Figure 1: Mirage unification in the MSSM. The couplings run up to the string scale $M_s \approx 10^{11}$ GeV. The compactification scale $M_c \approx 10^9$ GeV creates no new KK thresholds, since the gauge fields live on 3-branes. The couplings have an apparent unification at the virtual scale $M_X = \sqrt{\alpha} M_s^3 / M_c^2$. From low energies everything looks like if there was a field theory desert in between $M_W$ and $M_X$.

one can understand the huge $M_W/M_{Planck}$ hierarchy in terms of the modest ratio $M_c/M_{string}$. It is quite satisfactory to find that ”mirage gauge coupling unification” naturally requires the same distribution of mass scales, much more so since this was not our initial motivation.

Notice that in an isotropical 1 TeV string scenario choosing $M_c = 10^{-2}$ GeV and $M_{string} = 10^3$ GeV gives $M_X = 10^{13}$ GeV, which would be too low.

Due to the form of (4.6) one can check that the $U(1)_X$ gauge boson gets a mass of order the string scale $M_{string}$ \[\text{[34]}\]. The same happens with a linear combination of $M$ and $T$. The orthogonal linear combination remains massless at this level and, in particular, its imaginary part will have axion-like couplings and might help \[\text{[13]}\] to solve the strong CP problem.

5 Comments and outlook

We have described above how in a class of Type IIB, $D = 4, N = 1$ orientifolds a peculiar phenomenon occurs concerning gauge coupling unification: although gauge coupling running occurs only up to the string scale $M_{string}$, there are modulus-dependent corrections which mimic further running up to a virtual scale $M_X = \sqrt{\alpha} M_{Planck} M_c / M_{string}$. The modulus dependence appears due to the simultaneous presence of anomalous
$U(1)$’s and $\sigma$-model symmetries in this class of theories.

It would be interesting to know how general this property is. The status of $\sigma$-model symmetries in other classes of orientifolds like e.g. those with $N$ even is less clear so that a direct application of our argumentation does not necessarily work. However it could well be that the final result is quite independent of the derivation. Certainly, the appearance of large $T$-dependent corrections in the gauge coupling constants seems generic. In compact orientifolds it seems generic the presence of $T$-moduli dependent pieces in the Fayet-Iliopoulos terms $\xi_a$. Upon minimization one could be able to express $\text{Re} M = \text{Re} M(T, T^*)$ which when substituted back into the gauge kinetic function (whose $M$-dependence is also generic) will give rise in principle to large $T$-dependent corrections to gauge coupling constants. The least one has to say is that these effects cannot in general be neglected and have to be taken into account before giving any account of gauge coupling unification. This, however does not imply that these $T$-dependent corrections will get precisely the $\beta$-function coefficient required to get ”mirage unification”. The mechanism nevertheless looks quite general and very likely will be present in other classes of vacua different from the $N$ odd orientifolds. Notice in this connection that in order to obtain ”mirage unification” it is enough that the difference between the $M$-dependent corrections to the gauge coupling constants have a coefficient proportional to the difference between the respective $\beta$-functions.

From the phenomenological point of view, ”mirage unification” offers an elegant and possibly unique option to lower the string scale and still maintaining the success of gauge coupling unification to the same level of agreement and predictivity to that of SUSY-GUT’s. In the previous section we showed how a simple modification of the MSSM to include some ”closed string” fields $S$, $T$ and $M$ and simultaneous presence of an anomalous $U(1)$ and $\sigma$-model invariance can give rise to the required mechanism. Thus the mechanism itself is quite general. The anomalous $U(1)$ is very much restricted in the simplest model (only one $U(1)$ and only one $M$ field) since its mixed anomalies with $SU(3) \times SU(2) \times U(1)_Y$ have to be in the ratio of the corresponding $\beta$ functions. This may lead to quite interesting constraints for model building which are quite distinct to those worked out for heterotic models \cite{36,37} in which those anomalies are in the ratio of the coupling constant normalizations. Notice also that both the presence of the $U(1)$ and the $\sigma$-model symmetry are wellcome in order to supress sufficiently proton decay mediated by dimension four or higher operators.

We have found that if one wants to identify the ”virtual unification scale” $M_X = \sqrt{\alpha} M_{\text{Planck}} M_c/M_{\text{string}}$ with the scale suggested by experiment, one necessarily has to
use as inputs $M_{\text{string}} = 10^{11}$ GeV and $M_c = 10^9$. Thus in mirage gauge coupling unification the coupling constants give us a measure of the fundamental scales of the theory, $M_{\text{string}}$ and $M_c$. This is an unexpected result of the present analysis which fits quite well with the suggestion of ref.\cite{13} to identify $M_{\text{string}}$ with the intermediate scale $\sqrt{M_{\text{Planck}}M_W} = 10^{11}$ GeV. We find this fact very intriguing, particularly so since it was not our intention to find such a connection.

Gauge coupling unification within the MSSM has been always thought to be a great success and a strong indication of the existence of a unification scale of order $M_X \propto 10^{16}$ GeV. If the mechanism we suggest is at work, nature has been a bit nasty with us giving us a (partially) wrong track pointing towards a big desert in between $M_W$ and $M_X$. We would have been too naive in assuming that all logarithms come from field-theory running.
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References

[1] E. Witten, Nucl. Phys. B471 (1996) 135, hep-th/9602070.

[2] J.D. Lykken, Phys. Rev. D54 (1996) 3693, hep-th/9603133.

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, hep-ph/9803315.

[4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, hep-ph/9804398; I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, hep-ph/9810410.

[5] K. Dienes, E. Dudas and T. Gherghetta, hep-ph/9803466; hep-ph/9806292; hep-ph/9807522.

[6] R. Sundrum, hep-ph/9805471; hep-ph/9807348.

[7] G. Shiu and S.H. Tye, hep-th/9805157.

[8] C. Bachas, hep-ph/9807415.

[9] Z. Kakushadze and S.H. Tye, hep-th/9809147.

[10] K. Benakli, hep-ph/9809582.

[11] K. Benakli and S. Davidson, hep-ph/9810280; D.H. Lyth, hep-ph/9810320; N. Arkani-Hamed, S. Dimopoulos and G. Dvali, hep-ph/9807344; P. Argyres, S. Dimopoulos and J. March-Russell, hep-th/9808138; K. R. Dienes, E. Dudas, T. Gherghetta and A. Riotto, hep-ph/9809406; N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-th/9809124; L. Randall and R. Sundrum, hep-th/9810153.

[12] D. Ghilencea and G.G. Ross, hep-ph/9809217.

[13] C. Burgess, L.E. Ibáñez and F. Quevedo, hep-ph/9810533.

[14] Z. Kakushadze, hep-th/9811193.

[15] L.E. Ibáñez, C. Muñoz and S. Rigolin, hep-ph/9812397.

[16] I. Antoniadis and B. Pioline, hep-th/9902055.

[17] A. Delgado and M. Quiros, hep-ph/9903400.
[18] P. Frampton and A. Rasin, hep-ph/9903479.

[19] A. Sagnotti, in Cargese 87, *Strings on Orbifolds*, ed. G. Mack et al. (Pergamon Press, 1988) p. 521;
P. Horava, Nucl. Phys. B327 (1989) 461; Phys. Lett. B231 (1989) 251;
J. Dai, R. Leigh and J. Polchinski, Mod.Phys.Lett. A4 (1989) 2073;
R. Leigh, Mod.Phys.Lett. A4 (1989) 2767;
G. Pradisi and A. Sagnotti, Phys. Lett. B216 (1989) 59;
M. Bianchi and A. Sagnotti, Phys. Lett. B247 (1990) 517;
E. Gimon and J. Polchinski, Phys.Rev. D54 (1996) 1667, hep-th/9601038.

[20] M. Berkooz and R. G. Leigh, Nucl. Phys. B483 (1997) 187, hep-th/9605049.

[21] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, Phys. Lett. B385 (1996) 96, hep-th/9606169.

[22] Z. Kakushadze, Nucl. Phys. B512 (1998) 221, hep-th/9704053;
Z. Kakushadze and G. Shiu, Phys. Rev. D56 (1997) 3686, hep-th/9705163;
Z. Kakushadze and G. Shiu, Nucl. Phys. B520 1998 75, hep-th/9706051.

[23] G. Zwart, Nucl. Phys. B526 (1998) 378, hep-th/9708040.

[24] D. O’Driscoll, hep-th/9801114.

[25] L.E. Ibáñez, hep-th/9802103.

[26] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, FTUAM-98/4, hep-th/9804026.

[27] Z. Kakushadze, hep-th/9804110; hep-th/9806044.

[28] J. Lykken, E. Poppitz and S. Trivedi, hep-th/9806080.

[29] L. E. Ibáñez, R. Rabadán and A. Uranga, hep-th/9808139.

[30] M. Green and J. Schwarz, Phys. Lett. B149 (1984) 117.

[31] L. E. Ibáñez, R. Rabadán and A. Uranga, hep-th/9905098.

[32] J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Phys. Lett. B271 (1991) 307; Nucl. Phys. B372 (1992) 145;
L. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649;
G. Lopes Cardoso and B. Ovrut, Nucl. Phys. B369 (1992) 351;
L.E. Ibáñez and D. Lüst, Nucl. Phys. B382 (1992) 305.

[33] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987) 585,
    J. Atick, L. Dixon and A. Sen Nucl. Phys. B292 (1987) 109,
    M. Dine, I. Ichinoise and N. Seiberg, Nucl. Phys. B293 (1987) 253.

[34] E. Poppitz, hep-th/9810010.

[35] J. March-Russell, hep-ph/9806426.
    M. Cvetic, L. Everett, P. Langacker and J. Wang, hep-th/9903051.
    Z. Lalak, S. Lavignac and H.P. Nilles, hep-th/9903160.

[36] L.E. Ibáñez, Phys. Lett. B303 (1993) 55;
    L.E. Ibáñez and G.G. Ross, Phys. Lett. B332 (1994) 100

[37] For a review and references see P. Ramond, hep-ph/9604251.

[38] I. Antoniadis, C. Bachas and E. Dudas, hep-th/9906039.