The Penrose process and BSW effect on the collision of particles in Kerr-Newman black hole by exerting the WGC conditions

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Abstract

As we know, there is an area around the event horizon of rotating black holes, which is called by ergosphere. The corresponding rotational energy of black holes is located to this area. Every particle in this region become dragged by a rotating space-time. This makes possible to extract energy from rotating black holes, which is called the Penrose process. By using the center-of-mass energy, the Penrose process has been applied to the Kerr-Newman black hole [20]. But in this paper, we examine the collision of two spinning massive particles near the corresponding black hole with critical and near-critical angular momentum. In that case, we then calculate the energy of the third particle $E_3$ in terms of the first two particles $E_2$ and $E_1$. We also obtain the parameter $\eta_{max}$ which is the ratio of extracted energy to input energy. We mention that here by exerting the weak gravity conjecture (WGC) we have $\eta_{max} \simeq 1.5$. We see here the WGC condition and rotation of space-times affect energy.

Keywords: Kerr-Newman black hole, WGC, Penrose process, BSW effect, Extracted energy.

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1 Introduction

Recently, the Penrose process and the BSW effect have been studied for the extracted energy of particle collision near rotating black holes. As we know, M. Bañados, J. Silk and S. M. West (BSW) showed that a rotating Kerr black hole can accelerate particles [1]. The BSW effect has been extensively investigated of different black holes, and its new implications have been analyzed [2–14]. In general, the orbit of a spinning particle that deviates from the geodetic route is usually described by Madisson–Papapetrou–Dixon equations (MPD) [15–18]. Recently, a much more widely used method for obtaining energy from rotating black holes has been introduced which is called the super-Penrose process [22, 23]. However, there are still many problems with how to create heavy particles that lead to such a process and how to increase the efficiency of the Penrose process [24]. So far, there have been many studies in the Penrose processes especially with the BSW effect [25–10]. As we said, the Penrose process and the BSW effect on rotating particles near the Kerr, Kerr-Newman, and Kerr-Sen black holes have been thoroughly investigated using center-of-mass energy [19–21]. In this paper, we want to study the collision of two particles near a charged-rotating-black hole and obtain maximum impact energy of particle. As we know, many researchers have studied this issue in past. But the important point here is that we are going to examine the collision of particles and the BSW effects from the perspective of weak gravity conjecture. In fact, the assumption of this conjecture and the implications of the swampland program have recently received much attention from researchers in the study of inflation models and the physics of black holes [41–65]. Actually, we want to evaluate the effects of weak gravity conjecture at particle collisions as well as the BSW effects near a charged-rotating-black hole. By using the different characteristics of this black hole and effects of weak gravity conjecture, we will try to calculate the various parameters on the collision of particles. On the other hand, the WGC creates special conditions in black holes and makes them extremely and is expressed in $Q \geq M$ [66, 67]. But in rotating black holes, this condition is slightly different, which we discussed in Section 3. The above information give us motivation to discuss this paper as follows:

In second 2, we examine the maximum energy extracted from the Kerr-Newman black hole and plot it for different particle spins. In Section 3, using the WGC terms, we recalculate the extraction energy. And in the final section, we compare the obtained results, and we express the results of our work completely.

2 Calculating the collision energy extracted

2.1 Equation of motion for spinning particles near KN BH

The Kerr-Newman metric is the most common solution of the Einstein-Maxwell equations in space-time asymptotically flat, which explains the geometry of space-time near a charged
rotating mass. This metric in Boyer-Lindquist coordinate is,

\[
\begin{align*}
    ds^2 &= \Delta \rho^2 (dt - a \sin^2 \theta)^2 + \rho^2 \, dr^2 + \rho^2 \, d\theta^2 + \frac{\sin^2 \theta}{\rho^2} ((r^2 - a^2)^2 d\phi - adt)^2, \\
    \Delta &= r^2 - 2 Mr + a^2 + Q^2, \\
    \rho^2 &= r^2 + a^2 \cos^2 \theta,
\end{align*}
\]

where \( Q, a = \frac{J}{M} \) and \( M \) are charge, angular momentum and mass of black hole. According to the above metric the corresponding event horizon will be as,

\[
    r_{\pm} = M \pm \sqrt{M - (a^2 + Q^2)},
\]

where \( r_+ \) and \( r_- \) are outer and inner horizon. As we know the massive spinning particles near the gravitational field are described by the MPD equations [15–18],

\[
    \frac{D}{Dt} P^a = -\frac{1}{2} R_{bcd}^a v^b S^{cd},
\]

and

\[
    \frac{D}{Dt} S_{ab} = P^a v^b - P^b v^a.
\]

Here \( \frac{D}{Dt}, v^a = (\frac{\partial}{\partial r})^a, S_{ab} \) and \( P^a \) are covariant derivative, tangent vector, the spin tensor and 4-momentum respectively. Also, the following relations are established for the above equations [15],

\[
    S_{ab} P_b = 0,
\]

and

\[
    P^a v_a = -m.
\]

By placing (7) in the MPD equations, we can obtain the relation between 4-velocity and 4-momentum which is given by,

\[
    m v^a - P^a = \frac{S_{ab} R_{bcde} P^c S^{de}}{2 (m^2 + \frac{1}{4} R_{bcde} S^{bc} S^{de})}.
\]

According to the above relation, we understand that \( v^a \) and \( u^a \) are not parallel in 4-dimensions. The Killing vector as \( \zeta^a = (\frac{\partial}{\partial t})^a \) and \( \phi^a = (\frac{\partial}{\partial \phi})^a \) help us to obtain the following conserved quantities of corresponding metric background,

\[
    Q_\zeta = P^a \zeta_a - \frac{1}{2} S_{ab} \nabla_b \zeta_a,
\]
we assume $\theta = \frac{\pi}{2}$ for the KN black hole and obtain the conserved quantities as,

$$E_m = -u^a \zeta_a + \frac{1}{2m} S^{ab} \nabla_b \zeta_a, \quad (10)$$

and

$$J_m = u^a \phi_a - \frac{1}{2m} S^{ab} \nabla_b \phi_a, \quad (11)$$

where two conserved quantities of the Kerr-Newman background are $E_m$ and $J_m$ which are energy per unit mass of a particle and angular momentum per unit mass respectively. Here, we use $\mu = \frac{mv}{2}$.

By inverting the above relations, we compute $u_0$ and $u_3$. And then, by using condition ($u_0^2 + u_1^2 + u_3^2 = m^2$), we also get $u_1$. We note here the $v^a$ and $u^a$ are related by each other according to (8).

So we calculate $v^a$ and put to $P^a = mv_a$. Finally the equations of motion for the spinning particles near the black hole are given by,

$$P^t(r) = mv_0 = \frac{m^3}{\alpha \beta} [2r^2 a J_m (2Mr - Q^2) - Q^2 r^2 s_m (E_m r^6 + 2a E_m - J_m) - r^2 a^2 E_m (2Mr + r^2 - Q^2) + a^2 s_m (a E_m - J_m)(Q^2 - Mr) + Mr^3 s_m (J_m - 3a E_m)], \quad (14)$$

and

$$P^r(r) = \delta \sqrt{mv_1} \quad (15)$$
the above relationship is as follows,

\[
mv_1 = \frac{m^6}{r^2 \beta^2} \left[ r^6 (2Mr^3 - r^2 (J_m^2 + a^2 - a^2 E_m^2 + Q^2)) + (J_m - aE_m)^2 (2Mr - Q^2) + \\
(E_m^2 - 1)r^4 + 2r^4 s_m (-a(Q^2 - Mr)(J_m - aE_m)^2 + 2E_m Q^2 r^2 (J_m - aE_m) - \\
3E_m M r^3 (J_m - aE_m) + E_m J_m r^4) - (Q^2 - Mr)^2 \alpha s_m^4 + r^2 s_m^2 a^2 E_m^2 (Q^2 - Mr) \\
\times (Q^2 - Mr + 2r^2) + r^2 s_m^2 (J_m^2 (Q^2 - Mr)^2 - 2aE_m J_m (Q^2 - Mr)(Q^2 - Mr + r^2) - \\
a^2 2r^2 (Q^2 - Mr) - r^2 (Q^2 - r^2 - 2Mr) (E_m^2 r^2 + 2(Q^2 - Mr))] \right],
\]

(16)

where \( \delta = \pm \), which (+) and (−) show the direction of the particle moving outward and inward respectively.

\[
P^\phi(r) = mv_2 = \frac{m^3}{\alpha \beta} \left[ aE_m r^2 (Q^2 - 2Mr) + a^2 E_m (Q^2 - Mr) s_m + r^2 Q^2 + aJ_m (Mr - Q^2) s_m + \\
r^2 (r^2 - 2Mr) (E_m s_m - J_m) \right],
\]

(17)

where \( \alpha \) and \( \beta \) are,

\[
\alpha = a^2 + Q^2 - 2Mr + r^2, \\
\beta = M^2 r^4 + (Q^2 - Mr) m^2 s^2.
\]

(18)

### 2.2 Constraints on orbits and the energy extracted

In the collision of particle in Penrose process, two spinning particles with energy and angular momentum start moving from infinity. They collide in the area of black hole which is called the ergosphere. The collision place is before the event horizon \( r_H \). The black hole space-time put some constraint on the energy and angular momentum of the particle. This constraint restricts the path of the particles to certain orbits and does not allow them to move. The place of collision is in \( r \geq r_H \). By using the relation (16), one can obtain,

\[
P^r(r) = \delta \sqrt{mv_1} \geq 0,
\]

(19)

As a result, a constraint is applied to energy and angular momentum. Here \( b_c \) is the upper limit of energy and momentum , which is called critical value. It can not be greater than the amount of particles which is why we call it turning point. So, one can write the \( b_c \) in terms of critical energy and angular momentum,

\[
b_c = \frac{J_c}{E_c},
\]

(20)
By applying conditions $M = m = 1$, $r_H = 1$, and $f(r) = f'(r) = 0$ in (19), we obtain

$$b_c = \frac{1}{s} + \frac{1}{2}(1 - i\sqrt{3})s,$$

(21)

And here also we note that the non-critical state can be written to the following form,

$$b = \left(\frac{1}{s} + \frac{1}{2}(1 - i\sqrt{3})s\right)(1 + \zeta),$$

(22)

The second constraint on the orbits is the application of the time-like condition at the 4-velocity. Because 4-velocity and 4-momentum are not parallel to each other, we have time to spinning particles by violating the time-like conditions. This condition can be checked by the following equation,

$$v^\mu v_\mu < 0.$$  

(23)

By using the equations (12) and (13) we get $u_a$, the relation between $u_a$ and $v_a$ lead us to calculate the relation (23). Finally, by putting (21) in to equation (23), we obtain the upper energy boundary for the primary particles.

$$E > \frac{s^2 - 1}{1 - i\sqrt{3} + \frac{1}{s} + \frac{s}{2}(1 + i\sqrt{3} + s(-6 + (1 - i\sqrt{3})s(-1 + 2s)))}.$$  

(24)

In order to obtain the rotation range $s_{min} < s < s_{max}$, in the above relation we put $E = 1$. One can obtain following equation,

$$s_{min} \simeq -1.08492, \quad s_{max} \simeq -0.20922.$$  

(25)

We draw the energy in equation (24) in terms of $s$. In fig (1) we see that only particles with a spin of $-1$ to $0$ can reach to the horizon, because in this region the energy is positive.
Figure 1: We draw the energy in terms of $s$. And see the permissible amplitude for $s$ and $E$ for particles in the KN black hole geometry.

In order to get the extracted energy, we try to describe the collision of particle as follows: one of the primary particles has a critical angular momentum and the other has non-critical. The third particle escapes from the black hole after colliding and the last particle falls into black hole. In this case, we have the highest energy efficiency [68]. So, according to equations (20) and (21) one can write following equation,

\[
J_1 = \left( \frac{1}{s_1} + \frac{1}{2}(1 - i\sqrt{3}) \right)s_1 E_1,
\]

\[
J_2 = \left( \frac{1}{s_2} + \frac{1}{2}(1 - i\sqrt{3}) \right)s_2 E_2(1 + \zeta),
\]

\[
J_3 = \left( \frac{1}{s_1} + \frac{1}{2}(1 - i\sqrt{3}) \right)s_1 E_3(1 + \alpha\epsilon + \beta\epsilon^2 + ...),
\]

and for the fourth particle we have,

\[
E_4 = E_1 + E_2 - E_3,
\]

\[
J_4 = \left( \frac{1}{s_2} + \frac{1}{2}(1 - i\sqrt{3}) \right)s_2(E_1 + E_2(1 + \zeta) - E_3(1 + \alpha\epsilon + \beta\epsilon^2 + ...)).
\]

On the other hand, we consider the particle spin as follows,

\[
s_2 = s_4, \quad s_1 = s_3
\]

And the direction of motion will be as,

\[
\delta_2 = \delta_4.
\]
With assuming the mass as constant \((m_1 = m_2 = m_3 = m_4)\), the equation of collision particles near a black hole will be following,

\[
P^{(1)}_1 + P^{(1)}_2 = P^{(1)}_3 + P^{(1)}_4. \tag{30}
\]

Using equations (15) and particles angular momentum (26), (27) and spin conditions (28) at (30), we get the energy of the third particle \(E_3\) in terms of the other two-particle energies. In that case we apply the conditions \(r = 1, M = m = 1\) and \(f(r) = f'(r) = 0\).

\[
E_3 = \frac{-B - \sqrt{B^2 - 4AC}}{2C},
\]

\[
A = -\frac{2i\zeta E_2(2\sqrt{3} + (-3i + \sqrt{3})s_2^2)(-E_2 + (E_1 + E_2)s_3^2)}{s_2^2} \delta_2^2,
\]

\[
B = \gamma(E_1 + E_2)(2i\sqrt{3} + (3 + i\sqrt{3})s_2^2)\delta_2^2 + \zeta\left(\frac{E_2(-4\gamma + 2i\sqrt{3}s_2^2 + (3 + i\sqrt{3})s_4^2)}{s_2^2}\delta_2^2\right),
\]

\[
C = -2i\gamma((2\sqrt{3} + (-3i + \sqrt{3})s_2^2)\delta_2^2 + (-3i + \sqrt{3} + \frac{2\sqrt{3}}{s_1^2})\delta_3^2),
\]

and

\[
\gamma = (\alpha\epsilon + \beta\epsilon^2 + ...). \tag{32}
\]

Finally, the energy efficiency of a black hole collision can be obtained through the following equation

\[
\eta = \frac{(\text{output energy})}{(\text{input energy})} = \frac{E_3}{E_2 + E_1}. \tag{33}
\]

### 2.3 The maximum energy efficiency

Now, we calculate the energy of the third particle and its efficiency \[68\]. So by using the equation (31), one can obtain,

\[
E_1 = E_2 \simeq 1,
\]

\[
\delta_2 = \delta_4 = -1,
\]

\[
\delta_1 = +1 \quad \delta_3 = -1. \tag{34}
\]

We have also considered the second particle to be non-critical. By placing \(r = 1\), (26) and \(E_2 \simeq 1\) in (24) for the second particle, we can calculate \(\zeta\),

\[
\zeta = -1 - \frac{2i(2 + s_2(-2 + s_2(-1 - 2s_2 + 6s_2^2)))}{(2i + (i + \sqrt{3})s_2^2)(-2 + s_2 + s_3^2 - 2s_2^3)}. \tag{35}
\]
So we have,

\[ E_3 = -B - \sqrt{B^2 - 4AC} \]

\[ A = \frac{2i}{s_2^2}(-1 + 2s_2^2)(2\sqrt{3} + (-3i + \sqrt{3})s_2^2)(1 + \frac{2i(2 + s_2(-2 + s_2(-1 - 2s_2 + 6s_2^2)))}{(2i + (i + \sqrt{3})s_2^2)(-2 + s_2 + s_2^2 - 2s_2^4)}), \]

\[ B = 2i\sqrt{3}(-1 + 2\gamma) + \frac{4\gamma}{s_2^2} + (3 + i\sqrt{3})(-1 + 2\gamma)s_2^2 + \]

\[ \frac{2(4i\gamma + 2\sqrt{3}s_2^2 + (-3i + \sqrt{3})s_2^4)(2 + s_2(-2 + s_2(-1 - 2s_2 + 6s_2^2)))}{s_2^2(2i + (i + \sqrt{3})s_2^2)(-2 + s_2 + s_2^2 - 2s_2^4)}), \]

\[ C = -2i\gamma(-3i + 3\sqrt{3} + \frac{2\sqrt{3}}{s_1^2} + (-3i + \sqrt{3})s_2^2). \]

Here, in order to obtain the degree of non-criticality of the second particle as \( \gamma = \alpha\epsilon + \beta\epsilon^2 \), we draw the energy \( E_3 \) in terms of \( s_2 \) and \( \gamma \) at \( s_1 = -0.285688 \). According to fig (1), the energy of the first particle in \( s_1 = -0.285688 \) is the highest. In fig (2), we get two real and imaginary states. In both cases, the non-critical angular momentum of the second particle is equal to \( \gamma = 0.06777 \). Finally, by placing \( \gamma = 0.06777 \) in equation (36), we can plot the energy of the third particle \( E_3 \) in terms of \( s_1 \) and \( s_2 \).

Figure 2: The contour map of \( E_3 \) in terms of \( s_2 \) and \( \gamma \) at \( s_1 = -0.285688 \).
As can be seen from the above concepts, two different values are obtained for energy, the first being real and the other one is imaginary. The maximum amount of energy for the third particle placed at $-0.2 < s_2 < 0$ and $-1 < s_1 < 0$. So, this explain the space-time rotation affects the spin of the particles.

Finally, we can calculate the maximum efficiency for spinning particles near the Kerr-Newman black hole according to equation (33), so we have,

$$\eta = \frac{E_3}{E_1 + E_2} \approx \frac{3}{2} \approx 1.5.$$  \hspace{1cm} (37)

3 The extracted energy of collision particle with the WGC condition

The WGC condition is slightly different for charged rotating black holes. In the rest of this article, like the previous section, we calculate the energy relations of the third particle, except that we set the WGC condition in the black hole. The extremality condition for a Kerr-Newman black hole is equal to,

$$M^2 = Q^2 + a^2.$$  \hspace{1cm} (38)

We now consider the Kerr-Newman black hole metric and event horizon with the above
conditions,
\[
\Delta = r^2 - 2Mr + (M^2 - Q^2) + Q^2 = r^2 - 2Mr + M^2, \\
p^2 = r^2 + (M^2 - Q^2)\cos^2 \theta, \\
r_{\pm} = M \pm \sqrt{M - ((M^2 - Q^2) + Q^2)} = M \pm M\sqrt{(1 - M)}.
\] (39)

Also, the energy and the angular momentum of the spinning particles near the Kerr-Newman geometry with the WGC condition are equal to,
\[
E = \left[ \sqrt{r^2 - M^2 + Q^2} + \frac{(-2Q^2r + (M^2 - Q^2 + 3r^2)(1 + r))(M^2 - Q^2)}{2(M^2 - Q^2 + r^2)\sqrt{r^4 + (M^2 - Q^2)(-Q^2 + r(2 + r))}} \right] u_0 + \sqrt{r^2 - M^2 + Q^2} + \frac{(-2r(Q^2 - Mr + r^4) + (M^2 - Q^2)(M^2 - Q^2 + 2M - 4r + (M^2 - Q^2)r - r^2 + r^3))}{2(M^2 - Q^2 - r^2)\sqrt{r^4 + (M^2 - Q^2)(-Q^2 + 2r + r^2)}} s \times \frac{\sqrt{r^4 + (M^2 - Q^2)(-Q^2 + 2r + r^2)}}{r^2 - M^2 + Q^2} u_3,
\] (40)

and
\[
J = \left[ -rs\sqrt{M^2 - Q^2}\sqrt{r^2 - 2Mr + M^2} \right] u_0 + \frac{(M^2 - Q^2)}{\sqrt{M^2 - Q^2 - r^2}\sqrt{r^4 + (M^2 - Q^2)(-Q^2 + r(2 + r))}} + \frac{(M^2 - Q^2)(M - 2r) + r(-Q^2 + Mr) + (M^2 - Q^2)^2(2r^3 + (M^2 - Q^2)(1 + r))}{2(M^2 - Q^2 - r^2)(r^4 + (M^2 - Q^2)(-Q^2 + 2r + r^2))^{\frac{3}{2}}} \times \frac{1}{(-a + \sqrt{r^4 + (M^2 - Q^2)(-Q^2 + 2r + r^2)})} s] u_3.
\] (41)

Similarly, we can obtain the spinning particle motion equation by inverting the above relations in terms of \( u \) and using equation (8),
\[
P^t(r) = \frac{m^3}{\alpha \beta}[r^2 J_m \sqrt{M^2 - Q^2}(2Mr - Q^2) - Q^2 r^2 s_m(E_m r^6 + 2E_m \sqrt{M^2 - Q^2} - J_m) - r^2 E_m (M^2 - Q^2)(2Mr + r^2 - Q^2) + s_m (M^2 - Q^2)(E_m \sqrt{M^2 - Q^2} - J_m)] - r^2 E_m (M^2 - Q^2)(2Mr + r^2 - Q^2) + s_m (M^2 - Q^2)(E_m \sqrt{M^2 - Q^2} - J_m)
\] (42)
\times (Q^2 - Mr) + Mr^3 s_m (J_m - 3E_m \sqrt{M^2 - Q^2})].
and

\[ P^r(r) = \delta \sqrt{mv_1} \]
\[ mv_1 = \frac{m^6}{r^2 \beta^2} [r^6 (2Mr^3 - r^2(J_m^2 + (M^2 - Q^2) - E_m^2(M^2 - Q^2) + Q^2) + \]
\[ (J_m - E_m \sqrt{M^2 - Q^2})^2 (2Mr - Q^2) + (E_m^2 - 1)r^4) + 2r^4 s_m (-\sqrt{M^2 - Q^2}) \]
\[ \times (Q^2 - Mr)(J_m - E_m \sqrt{M^2 - Q^2})^2 + 2E_m Q^2 r^2 (J_m - E_m \sqrt{M^2 - Q^2}) - \]
\[ 3E_m Mr^3 (J_m - E_m \sqrt{M^2 - Q^2}) + E_m J_m r^4) - (Q^2 - Mr)^2 \alpha s_m + r^2 s_m E_m^2 \]
\[ \times (M^2 - Q^2)(Q^2 - Mr)(Q^2 - Mr + 2r^2) + r^2 s_m (J_m^2(Q^2 - Mr)^2 - 2E_m J_m \]
\[ \times \sqrt{M^2 - Q^2}(Q^2 - Mr)(Q^2 - Mr + 2r^2) - 2r^2(M^2 - Q^2)(Q^2 - Mr) - r^2(Q^2 - \]
\[ r^2 - 2Mr)(E_m^2 r^2 + 2(Q^2 - Mr))] \]

As we said, \( \delta = (\pm) \), shows the direction of the particle moving (+) outward and (−) inward, the black hole.

\[ P^\phi(r) = \frac{m^3}{\alpha \beta} [E_m r^2 \sqrt{M^2 - Q^2}(Q^2 - 2Mr) + E_m (M^2 - Q^2)(Q^2 - Mr)s_m + \]
\[ r^2 Q^2 + J_m \sqrt{M^2 - Q^2}(Mr - Q^2)s_m + r^2(r^2 - 2Mr)(E_m s_m - J_m)] \]

\( \alpha \) and \( \beta \) are,

\[ \alpha = (M^2 - Q^2) + Q^2 - 2Mr + r^2 = M^2 - 2Mr + r^2; \]
\[ \beta = M^2 r^4 + (Q^2 - Mr)m^2 s^2. \]

Now we calculate the constraint on the orbits in the WGC condition. According to equation (19), we calculate the critical angular momentum. When a particle reaches this point \( b_c \), it returns.

\[ b_c = \frac{J}{E} = \frac{1}{s} + \frac{1}{2} (1 - i \sqrt{3}) s. \]

Then through \( v^\mu v_\mu < 0 \), for the spinning particles we calculate the time-like conditions,

\[ E > \frac{(-1 - s^2)}{(1 - i) + \frac{1}{2} (\frac{1}{2} + \frac{i}{2}) s - 3s^2 - (\frac{1}{2} - \frac{1}{2}) s^3 + (1 - i)s^4}. \]

For convenience, we put \( E = 1 \), in this case, we can obtain the range of spin as,

\[ s_{\min} \approx -1.34371, \quad s_{\max} \approx -0.279487. \]

By drawing equation (47), we find that the maximum amount of energy for a spinning particle in space-time is the range of \((-1 < s < 0)\). Particles whose spin and energy are in the red
region can reach the horizon.

Under WGC, the angular momentum of spinning particles in the Kerr-Newman geometry according to equations (26) is equal to,

\[
J_1 = \left( \frac{1}{s} + \frac{1}{2} \right) (1 - i \sqrt{3}) s_1 E_1,
\]

\[
J_2 = \left( \frac{1}{s} + \frac{1}{2} \right) (1 - i \sqrt{3}) s_2 E_2 (1 + \zeta),
\]

\[
J_3 = \left( \frac{1}{s} + \frac{1}{2} \right) (1 - i \sqrt{3}) s_1 E_3 (1 + \alpha \epsilon + \beta \epsilon^2 + ...)
\]

\[
J_4 = \left( \frac{1}{s} + \frac{1}{2} \right) (1 - i \sqrt{3}) s_2 (E_1 + E_2 (1 + \zeta) - E_3 (1 + \alpha \epsilon + \beta \epsilon^2 + ...)),
\]

According to section (2,2), using equation (49) and the WGC condition, we solve \((P_1^{(1)} + P_2^{(1)} = P_3^{(1)} + P_4^{(1)})\). Then we calculate the value of \(E_3\) according to \(E_1\) and \(E_2\).

\[
E_3 = \frac{-B - \sqrt{B^2 - 4AC}}{2C}.
\]
where

\[ A = -\left( \frac{2E_2\delta_2}{s_2^2} \right)^2(2 + (1 - i)s_2^2 - 2E_2 - (1 - i)s_2^2E_2 + \zeta(2 + (1 - 2i)s_2^2 - \frac{1}{2} + i)s_2^4 - 2E_2 - (1 - i)s_2^2E_2)) + \delta_2^2\zeta((-2 - 2i)E_1E_2 - (2 + 2i)E_2^2 - \frac{4i\gamma_1E_1}{s_2^2} - \frac{4i\gamma_2E_2}{s_2^2}), \]

\[ B = \delta_2^2((2 + 2i)\gamma_1E_1 + (2 + 2i)\gamma_2E_2 + (2 + 2i)\zeta E_2 + \frac{4i\gamma_1E_1}{s_2^2} + \frac{4i\gamma_2E_2}{s_2^2}), \]

\[ C = \left( \frac{2\delta_3}{s_1^2} \right)^2(2 + (1 - i)s_1^2 - 2E_2 - (1 - i)s_1^2E_2 + \gamma(2 + (1 - 2i)s_1^2 - \frac{1}{2} + i)s_1^4 - 2E_2 - (1 - i)s_1^2E_2)) + \delta_2^2(-(2 + 2i)\gamma - \frac{4i\gamma_1}{s_2^2}). \]

The non-critical condition for the second particle is obtained by placing \( J_2 \) (49) in the relation (47) with the WGC condition.

\[ \zeta = (-1 + \frac{2(2 - 2s_2^2 - s_2^3 - 2s_2^3 + 6s_2^4)}{(2 + (1 - i)s_2^2)(2 - s_2^2 + 2s_2^3)}). \]

According to fig(4), the maximum amount of energy for the first particle is in \( s_1 = -0.37236 \).

We draw the energy equation (50) for the second particle with a value of \( s_1 = -0.37236 \) to obtain the non-criticality of the second particle \( \gamma = (\alpha\zeta + \beta\zeta^2) \).

Figure 5: The contour map of \( E_3 \) in terms of \( s_1 \) and \( s_2 \) in the WGC condition.
According to fig(5), we can draw two imaginary and real graphs for the energy of the third particle. From (b) in the corresponding fig, we get the critical angular momentum value of the second particle. This value is $\gamma = 0.02618$.

Now we can draw a plot the energy of third particle $E_3$ in terms of $s_1$ and $s_2$ in the WGC condition by placing $\gamma$ in the equation (50).

Figure 6: The contour map of $E_3$ in terms of $s_1$ and $s_2$ in the WGC condition.

Again, it gives us two real and imaginary plots. According to plot (b) in fig (6), the maximum energy of the third particle is equal to $E_3 = 3$ and in this case, we can obtain the spin of the particles,

$$(-0.35 < s_2 < -0.15), (-1 < s_1 < -0.55).$$  \hspace{1cm} (53)

Finally, we can obtain the energy efficiency of the third particle according to equation (37) for the WGC condition. For convenience, we consider $E_2 = 1$ and $E_2 = 1$.

$$\eta = \frac{E_3}{E_1 + E_2} \simeq \frac{3}{2} \simeq 1.5.$$  \hspace{1cm} (54)

4 Conclusions

In this paper, we examined the collision of spinning particles from WGC point of view and calculated quantities such as efficiency and other quantities and we plotted some figures. Here, we have shown that energy, angular momentum and spin of system are affected by WGC condition and rotation of space-times. As we have seen, using the special conditions mentioned
in this paper, the amount of energy is equal to $\eta \approx 1.5$ for the WGC condition. The complete results obtained in this article are as above table 1.

It may be interesting to find some global relations between these methods. In that case, by further examining the various black holes, you can get strange and amazing results that will be very attractive for any researcher. We leave the investigation of these cases to future work.

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