The model of choice of machine retaining devices for technological preparation of production

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Abstract. Optimization models are proposed for selecting the required number of universal prefabricated devices and non-separable special devices. As a modeling apparatus, nonlinear programming problems and discrete dynamic problems are used. The work formulates mathematical models that allow implementing a management technique based on the principles: Just in Time, Design to Cost, Risks Management.

1. Introduction

In the transition to digital production, there is a demand for the development of information intelligent systems to accompany all stages of the production process, which allows to increase process control, improve the quality of process monitoring, reduce production costs, increase the company's profit and the company's competitiveness in the market [1,2].

As part of cost reduction, Aviastar-SP JSC implements a program for the rational selection of machine retaining devices for technological preparation of production [3]. As one of the solutions, it was proposed to develop an automated decision support system for choosing a machine retaining device, which would analyze the design, technological, economic and organizational factors of the aircraft part and the proposed device [4] and make recommendations to the process engineer on the request of the most appropriate type of technological equipment. The main types of technological equipment were selected universal machine retaining devices (UMRD) and special machine retaining devices (SMRD) [4].

The article proposes a model for choosing the necessary set of SMRD and UMRD for the production process.

2. Problem statement

We assume that for some workshop there is a production line for many parts and assemblies \( D = \{D_1, \ldots, D_{N_{det}}\} \) \((N_{det} - \) the number of types of parts and assemblies in the workshop) using the set of SMRD or UMRD \( MD^s = \{MD^s_1, \ldots, MD^s_{N^s}\} \), where \( N^s \) – the number of SMRD in the workshop, \( s \) – indicator of the machine retaining device type. For \( s = 1 \) UMRD will be further implied, and for \( s = 2 \) – SMRD will be implied. In this case, there is a correspondence: \( q^s \subseteq D \times MD^s \), which determine the belonging of the types of parts and assemblies \( D_{i_{des}} \) to the types \( MD^s_{i^s} \) \((i_{det} \in \{1,2,\ldots,N_{det}\}, i^s \in \{1,2,\ldots,N^s\})\), from \( D \) sets. A part or assembly may mean a semi-finished product if machining using UMRD / SMRD is not ultimate for the process.

Suppose that for a given period of time \( M \) (measured in months) there is a need to produce a certain number of parts and assemblies in volumes of \( V = \{V(1),V(2),\ldots,V(M)\} \). Here \( V(t) \) is the set of
required values of the volumes of parts and assemblies per month \( t \); \( V(t) = (V_1(t), V_2(t), ..., V_{N_{det}}(t)) \).

\( V_j(t) \) – planned number of the devices of \( j \) type.

The distribution of volumes by SMRD and UMRD is ambiguous and can be determined based on some quality criterion. We will focus on some of the most important characteristics that differentiate the choice of UMRD and SMRD: performance, risks when used in the production process, and cost.

We will introduce the notation for the performance of UMRD and SMRD:

\[ p_{sij} \] is productivity of \( MD^s_i \) of \( i^s \) type while producing devices of \( j \) type, \( j \in \{1, 2, ..., N_{det}\} \), \( i^s \in \{1, 2, ..., N^s\} \) (number of details per 1 month).

When planning production activities, it is possible to adjust the number of different types of UMRD and SMRD, as well as the duration of the use of devices, and we will introduce the following variables:

- \( x_{sij}(t) \) – number of \( MD^s_i \) of \( i^s \) type, necessary for the production of \( j \) type devices, at \( t \) moment, \( j \in \{1, 2, ..., N_{det}\}, t \in \{1, 2, ..., M\}, i^s \in \{1, 2, ..., N^s\} \).

- \( \tau_{sij} \) – continuous unit work duration of \( MD^s_i \) of \( i^s \) type while producing devices of \( j \) type, \( j \in \{1, 2, ..., N_{det}\}, i^s \in \{1, 2, ..., N^s\} \) (number of months in shares);

It is necessary to note that each regulatory changeover or repair of \( MD^s_i \) [5] after the release of the specified number of parts will be considered a new position and will be included into \( x_{sij}(t) \).

In this case, the value \( Q_j(t) \) determines the number of devices of \( j \) type, produced in the workshop during \( t \) month.

\[
Q_j(t) = \sum_{s=1}^{2} \sum_{(D_j, MD^s_i) \in q^s} x_{sij}(t) p_{sij},
\]

(1)

determines the number of devices of \( j \) type, produced in the workshop during \( t \) month.

Obviously, the choice from the set

\[
X(t) = \left \{ x(t): x(t) = (x_1(t), x_2(t)), x_s(t) \in \mathbb{R}^{N^s \times N_{det}}, x_{sij}(t): s = (1, 2), 1 \leq j \leq N_{dev}, (D_j, MD^s_i) \in q^s \right \}
\]

(2)

is ambiguous and must satisfy a number of natural limitations.

Natural limitations are the number of UMRD elements in assembly areas, workshop space, structural rigidity, etc. [5,6]. All possible variants of UMRD and SMRD for use in the workshop, subject to the presence of natural restrictions, are denoted by the \( A(t) \) set.

There are also restrictions associated with the use of labor resources. Let the vector function \( L(t) = (L_1(t), L_2(t), ..., L_{N_{lab}}(t)) \) be known, where \( 1 \leq t \leq M, N_{lab} \) is the number of types of labor resources of different qualifications, \( L_i(t) \) is the number of labor resources of \( i \) type, which can be used in \( t \) time cycle. \( L(t) \) function is determined by the staffing table, vacation schedule, and other factors. According to the norms of technological processes for each type of UMRD and SMRD for the corresponding part, fixed labor costs of the corresponding qualifications are required.

We will introduce the designation for labor costs:

\( a_{sij} \) – labor costs vector in the \( i^s \)-th \( MD^s \) for \( j \)-th device, \( a_{sij} = (a_{sij1}, a_{sij2}, ..., a_{sijN_{dev}}) \), \( a_{sijk} \) – labor cost rate of the \( k \)-th type to ensure functioning of \( i^s \)-th UMRD and SMRD, devices of \( j \) type.

Thus, \( \mathcal{L}(t) \) labor cost vector, aimed at the production of devices of \( j \) type, can be calculated by the formula:

\[
\mathcal{L}(t) = \sum_{s=1}^{2} \sum_{(D_j, MD^s_i) \in q^s} x_{sij}(t) a_{sij}.
\]

(3)

Thus, full labor costs \( \bar{L}(t) = \sum_{j=1}^{N_{det}} \mathcal{L}(t) \), i.e. they amount to the cost of each type of part. The provision of labor resources can be written as: \( \bar{L}(t) \leq L(t) \).

An important factor is machine resources. In the case of using CNC machines, one operator can service several pieces of equipment. The total engine costs \( E(t) \) and \( \bar{E}(t) \) will be calculated similarly to the total labor costs \( L(t) \) and \( \bar{L}(t) \). The condition of provision with machine resources will also be written in the form \( \bar{E}(t) \leq E(t) \).
Let us consider the financial costs that affect the cost of production of various sets of UMRD and SMRD. Let $c_{sij}$ be the value of the costs associated with the design, manufacture and dismantling of UMRD and SMRD for the $j$-th part, reduced to the beginning of the planning period, respectively. If before the start of the planning period the SMRD or UMRD were not put into action, then the value $c_{sij}$ represents the total cost of the costs. If the device was used in the production process for a certain period of time, then part of the cost is debited in proportion to the nominal operating time of the device, and the remainder is recorded in $c_{sij}$, respectively.

In this case, the total cost of expenses $C_{ij}(t)$ from the design, manufacture and dismantling of UMRD and SMRD for parts of $j$ type transferred to the products in one cycle can be calculated by the formula:

$$C_{ij}(t) = \sum_{s=1}^{2} \sum_{l(t)} \sum_{(D_{j}, MD_{j})} x_{sij(t)} c_{sij} \frac{\tau_{sij}}{\tau_{sij}}.$$  

(4)

Also, various types of devices require the use of labor resources of various qualifications, therefore, different costs of labor hours. We will introduce $c_{lk}$ rate of payment of labor resources of $k$ type, set the rate in the form of payment for one time step (month). Then the total wage $C_{lj}(t)$ for one-time cycle for parts of the form $j$ is determined by the scalar product:

$$C_{lj}(t) = \sum_{k=1}^{N_{tp}} L_{k}(t) c_{lk}.$$  

(5)

Similarly, $C_{lj}(t)$ machine costs will be calculated per time cycle for parts of $j$ type.

Also, variable costs include the costs associated with supporting the functioning of UMRD and SMRD, such as depreciation of fixtures, readjustment, repair, completion of design documentation, electrical energy costs, etc. We will introduce $b_{sij}$ cost rate per unit of UMRD and SMRD. In this case, $C_{b}(t)$ variable costs can be calculated as follows:

$$C_{b}(t) = \sum_{s=1}^{2} \sum_{l(t)} (D_{j}, MD_{j}) x_{sij}(t) b_{sij}.$$  

(6)

Thus, $C_{f}(t)$ total cash costs per time cycle, determined by the support of the UMRD and SMRD fixtures for the devices of $j$ type, as well as the gross output, are the sum of the costs defined above:

$$C_{f}(t) = C_{ij}(t) + C_{lj}(t) + C_{lj}(t) + C_{b}(t).$$  

(7)

Let us consider the most significant risks caused by the operation of UMRD and SMRD: temporary losses, financial losses, defective products. The factors determining the above risks should include the qualifications of the personnel, the type of devices, the intensity of loading devices.

We will introduce the following random variables when using $t^{5}$-th $MD^{5}$ for a device of the $j$ type at time $t$ ($j \in \{1, 2, ..., N^{det}\}$, $t \in \{1, 2, ..., M\}$, $t^{5} \in \{1, 2, ..., N^{5}\}$): $\xi_{sij}(t)$ - financial losses; $\eta_{sij}(t)$ - temporary losses; $\zeta_{sij}(t)$ is the scrap volume of the parts.

Due to natural limitations, the quantities $\xi_{sij}(t), \eta_{sij}(t), \zeta_{sij}(t)$ are non-negative; in the general case, the values of the quantities are subject to a temporary effect. Evaluation of these values can be carried out on the basis of statistical analysis, in which it is possible to determine the possible seasonal nature of risks, the constancy of risks, etc.

3. Formalization of principles for the choice of UMRD and SMRD

Due to the choice of factors that determine the production activity of the enterprise, it is possible to change certain criteria that are most important when making a managerial decision.

It should also be noted that the duration of use of the UMRD and SMRD devices is determined by the management decision. However, for each type of device there is a standard term of work, exceeding this period entails an increase in risks: an increase in the number of defects, an increase in financial costs, and time delays.

We will introduce the standard terms for the use of devices:
\( \tau_{n}^{w} \) – standard duration of work of MD* device of \( i^{s} \) type while producing the details of \( j \) type, \( j \in \{1,2,...,N_{det}\}, i^{s} \in \{1,2,...,N^{s}\} \) (number of months);

We will consider the following random variables:

while \( j \in \{1,2,...,N_{det}\}, t \in \{1,2,...,M\}, i^{s} \in \{1,2,...,N^{s}\} \)

\[
\xi_{stij}(t) = \begin{cases} 
\xi_{stij}(t), & \tau_{stij} \leq \tau_{stij}^{w}; \\
\xi_{stij}(t) \left( 1 + \alpha_{stij} \left( \tau_{stij} - \tau_{stij}^{w} \right) \right), & \tau_{stij} > \tau_{stij}^{w}. 
\end{cases}
\]

(8)

\[
\eta_{stij}(t) = \begin{cases} 
\eta_{stij}(t), & \tau_{stij} \leq \tau_{stij}^{w}; \\
\eta_{stij}(t) \left( 1 + \beta_{stij} \left( \tau_{stij} - \tau_{stij}^{w} \right) \right), & \tau_{stij} > \tau_{stij}^{w}. 
\end{cases}
\]

(9)

\[
\zeta_{stij}(t) = \begin{cases} 
\zeta_{stij}(t), & \tau_{stij} \leq \tau_{stij}^{w}; \\
\zeta_{stij}(t) \left( 1 + \gamma_{stij} \left( \tau_{stij} - \tau_{stij}^{w} \right) \right), & \tau_{stij} > \tau_{stij}^{w}. 
\end{cases}
\]

(10)

The values entered indicate the risks corresponding to similar risks, if the devices operate in the normal mode, not exceeding the normative terms, and the value of the corresponding risks increases (the mathematical expectation, dispersion increases), if the devices work longer than the established standards. Parameters \( \alpha, \beta \) and \( \gamma \) in this case are the amount of the penalty for exiting acceptable standards.

We will formalize the implementation of the principle of "just in time." To ensure the principle of "just in time" it is necessary to fulfill the conditions \( Q_{j}(t) = V_{j}(t), 1 \leq j \leq N^{dev} \).

Taking into account random time delays \( \bar{\eta}_{stij}(t) \) of \( i^{s} \)-th MD* for a detail of \( j \) type, we can reevaluate the actual performance of the fixtures: instead of the nominal performance \( p_{stij} \) we get the performance:

\[
\bar{p}_{stij}(t) = \frac{p_{stij}}{1 + \bar{\eta}_{stij}(t)}. 
\]

(11)

Different production methods give rise to different values of defective products \( \xi_{stij}(t) \), which cannot be used in further production or for sale. The latter means that in order to ensure the necessary planned indicators, it is necessary to produce more products by the amount of rejects.

In this case, the expected output \( \tilde{Q}_{j}(t) \) will be a random variable and can be calculated based on the expression:

\[
\tilde{Q}_{j}(t) = \sum_{s=1}^{S} \sum_{t(d_{j,MD}^{s})} \phi_{s} x_{stij}(t) \left( \frac{p_{stij}}{1 + \bar{\eta}_{stij}(t)} \right) + \xi_{stij}(t) 
\]

(12)

Then, to ensure the principle of "just in time" the condition \( Q_{j}(t) = V_{j}(t), 1 \leq j \leq N^{dev} \) should be changed for the condition \( \tilde{Q}_{j}(t) = V_{j}(t), 1 \leq j \leq N^{dev} \).

We will introduce the set \( S_{jRT}(t) \) which is one of the valid sets of UMRD and SMRD, which provide the necessary number of devices and assemblies at a given time according to plan \( V \):

\[
S_{jRT}(t) = \{ x(t): x(t) \in X(t) \cap A(t), \tilde{Q}_{j}(t) = V_{j}(t), 1 \leq j \leq N^{dev}, L(t) \leq L(t) \}. 
\]

(13)

From a mathematical point of view, the set \( S_{jRT}(t) \) is determined by a set of linear constraints such as equalities and inequalities, while the system contains random variables that, in the general case, can be correlated with each other. Correlation may be due to general logistics, management system, a single workshop, etc. If the set \( S_{jRT} \) has more than one solution, the choice of a solution that satisfies the principle of "just in time" is determined by the introduction of an additional quality criterion.

Due to the presence of random variables in the system, the use of standard optimization methods is difficult. One of the solutions to the problem is the transition to a deterministic system based on the
replacement of random variables by their average values. In this case, we will introduce the quantity \( \tilde{Q}_j(t) \):

\[
\tilde{Q}_j(t) = \sum_{s=1}^{2} \sum_{t \in (D,M)} x_{1ij}(t) \left( \frac{p_{sij}}{1 + M(\tilde{\eta}_{sij}(t))} \right) + M(\tilde{\xi}_{sij}(t))
\]

Thus, a multitude of decisions satisfying the “on time” principle can be represented by a set \( \tilde{S}_{JIT}(t) \):

\[
\tilde{S}_{JIT}(t) = \left\{ x(t) : x(t) \in X(t) \cap A(t), \tilde{Q}_j(t) = V_j(t), 1 \leq j \leq N_d, \right\}
\]

The set \( \tilde{S}_{JIT}(t) \) is specified by a deterministic system of linear equations and inequalities. The search of a unique decision \( x(\cdot) = (x(1), x(2), ..., x(M)) \) at a set \( \tilde{S}_{JIT} = (\tilde{S}_{JIT}(1), \tilde{S}_{JIT}(2), ..., \tilde{S}_{JIT}(M)) \) is possible by means of a solution:

\[
J_{JIT}(x(\cdot)) = \sum_{1 \leq t \leq M} \left( \sum_{s=1}^{2} \sum_{i,j} \left( D(\tilde{\eta}_{sij}(t)) \right) \right).
\]

Functional \( J_D(x(\cdot)) \) represents the sum of variances of temporary risks affecting “just-in-time” execution.

It is natural to set the task of finding a solution \( x^*(\cdot) \) based on risk minimization:

\[
J_{JIT}(x^*(\cdot)) = \min_{x(\cdot) \in \tilde{S}_{JIT}} \left( J_{JIT}(x(\cdot)) \right).
\]

This optimization task, in essence, includes two important principles: “risk management” and “just in time.” The instrumental solution to this optimization problem can be reduced to methods for solving quadratic programming problems.

We will consider the cost of production, which is due to activities in the workshop. We introduce the quantity \( pc_j(t) \) which is the cost of production of the form \( j \) at time \( t \), which is formed by all UMRD and SMRD at the current moment. The calculation of this value can be represented:

\[
pc_j(t) = \frac{c_{j,t}(t)}{q_{j,t}(t)}.
\]

However, it is worth noting that with a large diversification of production, consideration of the cost of each type of part is not always correct, the total cost of all products manufactured in the workshop is much more important.

In this regard, we will consider the integrated cost of all products in \( PC(t) \) workshop at a time step \( t \):

\[
PC(t) = \frac{\sum_{j=1}^{N_d} c_{j,t}(t)}{\sum_{j=1}^{N_d} q_{j,t}(t)}.
\]

In the process of using devices, situations arise when additional financial resources are required, which are not considered in regulatory documents, to ensure the operation of devices. We will introduce the expected costs for the production of devices of \( j \) type:

\[
\hat{C}_{bj}(t) = \sum_{s=1}^{2} \sum_{t \in (D,M)} x_{sij}(t) \left( b_{sij} + \xi_{sij} \right).
\]

In this case, the expected costs \( \hat{C}_j(t) \) for each type of \( j \) devices:

\[
\hat{C}_j(t) = C_{lij}(t) + C_{lj}(t) + C_{Ej}(t) + \hat{C}_{bj}(t).
\]

If we take into account the random effects exerted on production, we can get an idea of the integrated cost \( \hat{PC}(t) \) in the form of a random variable:
The cost of each \( \bar{p}c_j(t) \) device type can be calculated on the basis of the relationship:

\[
\bar{p}c_j(t) = \frac{\hat{c}_j(t)}{q_j(t)}.
\]

The cost of production can be set when planning production. Let the integrated cost price \( PC^* \) be given, as well as the partial costs for each type of device \( pc_j^* \), \( 1 \leq j \leq N_{dev} \). In this case, it is possible to set the condition for ensuring the prime cost in the form of \( \bar{p}C(t) \leq PC^* \) or in the form of \( \bar{p}c_j(t) \leq pc_j^* \), \( 1 \leq j \leq N_{dev} \).

It is possible to introduce the stochastic set \( S_{PC}(t) \) which is a set of solutions with a given cost price

\[
S_{PC}(t) = \{ x(t): x(t) \in X(t) \cap A(t), \bar{p}C(t) \leq PC^*, \bar{L}(t) \leq L(t), \bar{E}(t) \leq E(t) \}. \tag{24}
\]

If the set \( S_{PC} = (S_{PC}(1), S_{PC}(2), ..., S_{PC}(M)) \) contains more than one solution, then the choice of a single solution requires the introduction of an additional criterion, for example:

\[
J_{PC}(x(\cdot)) = \sum_{1 \leq s \leq M} \left( \sum_{i,j} D(\hat{s}_{ij}(t)) \right). \tag{25}
\]

To formulate the optimization problem, we will introduce the quantities

\[
\hat{c}_{bij}(t) = \sum_{s=1}^{2} \sum_{i,j \in \{D(j),MD(j)\} \in \rho^s} \bar{x}_{sij}(t) \left( b_{sij} + M \hat{\xi}_{sij} \right),
\]

\[
\hat{c}_j(t) = c_{ij}(t) + c_{ij}(t) + c_{ij}(t) + \hat{c}_{bij}(t).
\]

In this case, the cost function can be written in a deterministic form:

\[
\bar{p}C(t) = \frac{\sum_{j=1}^{N_{dev}} \hat{c}_j(t)}{\sum_{j=1}^{N_{dev}} q_j(t)}.
\]

We will introduce a deterministic set of solutions with a given cost price \( \bar{S}_{PC} \):

\[
\bar{S}_{PC}(t) = \{ x(t): x(t) \in X(t) \cap A(t), \bar{p}C(t) \leq PC^*, \bar{L}(t) \leq L(t), \bar{E}(t) \leq E(t) \}, \tag{29}
\]

\[
\bar{S}_{PC} = (\bar{S}_{PC}(1), \bar{S}_{PC}(2), ..., \bar{S}_{PC}(M)). \tag{30}
\]

Thus, the following optimization problem can be formulated:

\[
J_{PC}(x^*(\cdot)) = \min_{x(\cdot) \in S_{PC}(t) \bar{S}_{PC}(t)} (J_{PC}(x(\cdot))). \tag{31}
\]

This task is to minimize the risks associated with increasing costs, while ensuring the planned production output. Thus, the solution to this problem provides the principles: Just in Time, Design to Cost, Risks Management.

4. Conclusion
The article proposes optimization models for choosing the required number of UMRD and SMRD, as well as the duration of their use to ensure the production activities of the workshop.

The implementation of these models as part of the decision support system in the overall complex of enterprise information support will allow decision makers to make informed management choices in the design and implementation of UMRD and SMRD.

The proposed models comply with the principles of management: Just in Time, Design to Cost, Risks Management.

In the software implementation of the proposed models, it is necessary to use numerical methods for solving high-dimensional nonlinear programming problems.
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