Multi-choice stochastic transportation problem involving general form of distributions

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Abstract
Many authors have presented studies of multi-choice stochastic transportation problem (MCSTP) where availability and demand parameters follow a particular probability distribution (such as exponential, weibull, cauchy or extreme value). In this paper an MCSTP is considered where availability and demand parameters follow general form of distribution and a generalized equivalent deterministic model (GMCSTP) of MCSTP is obtained. It is also shown that all previous models obtained by different authors can be deduced with the help of GMCSTP. MCSTP with pareto, power function or burr-XII distributions are also considered and equivalent deterministic models are obtained. To illustrate the proposed model two numerical examples are presented and solved using LINGO 13.0 software package.

Keywords: General form of distributions; Multi-choice programming; Stochastic transportation problem; Transformation technique

1 Introduction
The transportation problem is one of the oldest applications of Linear Programming Problem (LPP). The standard form of the transportation problem was first formulated along with the constructive method of solution by Hitchcock (1941). In a classical transportation problem, a product is to be transported from m sources to n destinations. The availability of the product at ith source is denoted by ai, where i = 1, 2, ..., m and the demand required at jth destination is bj where j = 1, 2, ..., n. The penalty cij is the cost coefficient of the objective function which can represent transportation cost, delivery time etc. In many real world situations the availability ai and demand bj are not certainly known to Decision Maker (DM). One way to deal such uncertainty is to describe the availability ai and demand bj parameters as random variables rather than the deterministic one. These random variables ai and bj are assumed to follow a given probability distribution or its probability distribution may be estimated. This type of transportation problem is known as “Stochastic Transportation Problem” (STP). Furthermore, suppose that there exist k routes for transporting the product from ith source to jth destination and the cost of transporting a unit of product via kth route is denoted by Ckij. Thus DM have multiple (i.e. ‘k’) route choices for shipping the product from ith source to jth destination and he has to identify exactly one among k routes in such a manner that the combination of choices should minimize the overall transportation cost. With the above discussed objective the STP becomes ‘Multi Choice Stochastic Transportation Problem’ (MCSTP) in which the cost coefficient Ckij are multi-choice and availability ai and demand bj are random variables.

MCSTP has been extensively studied by many researchers. Roy et al. (2012) presented an equivalent deterministic model of MCSTP by assuming that both availability ai and demand bj as random variables following exponential distribution. Biswal and Samal (2013) obtained an equivalent deterministic model of MCSTP in which they considered that both ai and bj follow Cauchy distribution. Mahapatra (2014) also given equivalent deterministic model of MCSTP involving Weibull distribution. Mahapatra et al. (2013) considered the MCSTP involving Extreme value distribution. Barik et al. (2011) presented a stochastic transportation model involving Pareto distribution.
These random variables \(a_i\) and \(b_j\) may also be considered to follow Burr-XII or Power Function distributions. Burr-XII may be used in place of normal distribution when data shows some positive skewness. Since \(a_i\) and \(b_j\) are the physical quantities so it is advisable to use Burr-XII instead of Normal distribution. When upper bound of availability and demand is known, Power Function distribution would be most suitable distribution to fit. In this paper we considered general form of MCSTP, where \(a_i\) and \(b_j\) are assumed to follow ‘General classes of distribution’ and obtained a generalised equivalent deterministic model (GMCSTP). All the models discussed above by many authors have been deduced by using the proposed GMCSTP. Three new equivalent deterministic models of MCSTP have also been obtained by considering that \(a_i\) follows any one distribution among Exponential, Weibull, Cauchy, Exterme Value, Pareto, Power Function or Burr-XII and \(b_j\) follows any other distribution except that of distribution of \(a_i\). To illustrate the proposed models two numerical examples are taken and solved by using transformation technique given by Biswal and Acharya (2009). Lingo 13.0 software has been used for obtaining the optimal solution.

2 General classes of distributions

Let us consider a random variable \(y\) following any of the two general classes of distributions with distribution function (df) \(F(y)\) as follows:

\[
F(y) = 1 - F(y) = 1 - \left[ph(y + q)\right]^p, \quad y \in (\xi, \phi) \tag{2.1}
\]

and

\[
F(y) = 1 - F(y) = e^{-ph(y)} p \neq 0, \quad y \in (\xi, \phi) \tag{2.2}
\]

where \(h(y)\) is a monotonic and differentiable function of \(y\) and \(p, q, r\) and \(h(y)\) are chosen such that \(F(y)\) in (2.1) and (2.2) are df over \((\xi, \phi)\).

Differentiating (2.1) and (2.2) with respect to \(y\) the probability density function (pdf), \(f(y)\) may be obtained respectively as,

\[
f(y) = -ph'(y) \left[ph(y + q)\right]^{-1} \quad \tag{2.3}
\]

\[
f(y) = -ph'(y)e^{-ph(y)} \quad \tag{2.4}
\]

where \(F(\xi)=0\) and \(F(\phi)=1\).

3 Mathematical model of multi-choice stochastic transportation problem (MCSTP)

In this section a mathematical model of multi-choice transportation problem involving general form of distributions (2.1 or 2.2) is considered. The general form of MCSTP is:

\[
\text{MCSTP1:} \quad \min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C_{ij1}^1 + \sum_{k=2}^{K} C_{ijk}^k \right] x_{ij}, \quad k = 1, 2, \ldots, K \tag{3.1}
\]

subject to,

\[
\begin{align*}
\text{Pr} \left[ \sum_{j=1}^{n} x_{ij} \leq a_i \right] & \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m \tag{3.2} \\
\text{Pr} \left[ \sum_{i=1}^{m} x_{ij} \geq b_j \right] & \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n \tag{3.3}
\end{align*}
\]

\[
x_{ij} \geq 0 \quad \forall \ i \text{ and } j \tag{3.4}
\]

where \(0 < \alpha_i < 1, \forall i\) and \(0 < \beta_j < 1, \forall j\), are the aspiration levels.

It is assumed that \(a_i, i = 1, 2, \ldots, m, b_j, j = 1, 2, \ldots, n\) are random variables following general form of distribution, \(\{C_{ij1}^1, C_{ij2}^2, \ldots, C_{ijK}^K\}\) \(k = 1, 2, \ldots, K\) are multi-choice parameters and \(x_{ij}\) are deterministic decision variables.

The following cases are to be considered:

(i) Only \(a_i, i = 1, 2, \ldots, m\) follows general form of distribution.

(ii) Only \(b_j, j = 1, 2, \ldots, n\) follows general form of distribution.

(iii) Both \(a_i, i = 1, 2, \ldots, m\) and \(b_j, j = 1, 2, \ldots, n\) follow general form of distribution.

3.1 Only \(a_i, i = 1, 2, \ldots, m\) follows (2.1) or (2.2)

It is considered that \(a_i, i = 1, 2, \ldots, m\) are independent random variable which follows any of two general form of distributions as defined in (2.1) and (2.2) consider the probabilistic constraint (3.2),

\[
\begin{align*}
\text{Pr} \left[ \sum_{j=1}^{n} x_{ij} \leq a_i \right] & \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m \\
\text{or} \quad \text{Pr} \left[ a_i \geq \sum_{j=1}^{n} x_{ij} \right] & \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m \tag{3.5}
\end{align*}
\]
the above inequality (3.5) can be represented as

\[
\int_{\sum_{i=1}^{m} x_{ij}} \phi_i f(a_i) da_i \geq 1 - \alpha_i \\
\int_{\sum_{i=1}^{m} x_{ij}} \frac{d}{da_i} [-\Phi(a_i)] da_i \geq 1 - \alpha_i \\
- \Phi(a_i) \leq 1 - \alpha_i \\
- \left[ \Phi(\phi_i) - \Phi(\sum_{j=1}^{n} x_{ij}) \right] \geq 1 - \alpha_i \\
F \left( \sum_{j=1}^{n} x_{ij} \right) \geq 1 - \alpha_i \\
F \left( \sum_{j=1}^{n} x_{ij} \right) \leq \alpha_i 
\]

(3.6)

Thus, we obtained a multi-choice deterministic model MCSTP 2 as follows:

**MCSTP 2:**

\[
\text{min:} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right] x_{ij}, \quad k = 1, 2, \ldots, K 
\]

subject to,

\[
F \left( \sum_{j=1}^{n} x_{ij} \right) \leq \alpha_i, \quad i = 1, 2, \ldots, m 
\]

(3.7)

\[
\sum_{i=1}^{m} x_{ij} \geq b_j, \quad j = 1, 2, \ldots, n 
\]

(3.8)

\[
x_{ij} \geq 0 \quad \forall \ i \text{ and } j 
\]

(3.9)

where \( \sum_{i=1}^{m} F^{-1}(a_i) \geq \sum_{j=1}^{n} b_j \) (feasibility condition).

### 3.2 Only \( b_j, j = 1, 2, \ldots, n \) follows (2.1) or (2.2)

It is considered that \( b_j, i = 1, 2, \ldots, m \) are independent random variable which follows any of two general form of distributions as defined in (2.1) and (2.2) consider the probabilistic constraint (3.3),

\[
\Pr \left[ \sum_{i=1}^{m} x_{ij} \geq b_j \right] \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n 
\]

or

\[
\Pr \left[ b_j \leq \sum_{i=1}^{m} x_{ij} \right] \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n 
\]

(3.10)

the above inequality (3.11) can be represented as

\[
\int_{x_{ij}}^{\sum_{i=1}^{m} x_{ij}} f(b_j) db_j \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n \\
\int_{x_{ij}}^{\sum_{i=1}^{m} x_{ij}} \frac{d}{db_j} [-\Phi(b_j)] db_j \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n \\
- \Phi(b_j) \leq 1 - \beta_j \\
- \left[ \Phi(\sum_{i=1}^{m} x_{ij}) - 1 \right] \geq 1 - \beta_j \\
F \left( \sum_{j=1}^{n} x_{ij} \right) \geq 1 - \beta_j \quad j = 1, 2, \ldots, n 
\]

(3.11)

Thus, we obtained a multi-choice deterministic model MCSTP 3 as follows:

**MCSTP 3:**

\[
\text{min:} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right] x_{ij}, \quad k = 1, 2, \ldots, K 
\]

subject to,

\[
\sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1, 2, \ldots, m 
\]

(3.12)

\[
F \left( \sum_{j=1}^{n} x_{ij} \right) \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n 
\]

(3.13)

\[
x_{ij} \geq 0 \quad \forall i \text{ and } j 
\]

(3.14)

where \( \sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} F^{-1}(1 - \beta_j) \) (feasibility condition).

### 3.3 Both \( a_i, (i = 1, 2, \ldots, m) \) and \( b_j, (j = 1, 2, \ldots, n) \) follow (2.1) or (2.2)

It is considered that \( a_i, (i = 1, 2, \ldots, m) \) and \( b_j, j = 1, 2, \ldots, n \) are independent random variable which follows any of two general form of distributions as defined in (2.1) and (2.2).

In view of (3.6) and (3.12) we may obtain a multi-choice deterministic model GMCSTP as follows:

**GMCSTP:**

\[
\text{min:} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right] x_{ij}, \quad k = 1, 2, \ldots, K 
\]

(3.15)
subject to,

$$F\left(\sum_{j=1}^{n} x_{ij}\right) \leq \alpha_i, \quad i = 1, 2, \ldots, m$$

(3.18)

$$F\left(\sum_{i=1}^{m} x_{ij}\right) \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n$$

(3.19)

$$x_{ij} \geq 0 \quad \forall \ i \text{ and } j$$

(3.20)

where $$\sum_{i=1}^{m} F^{-1}(\alpha_i) \geq \sum_{j=1}^{n} F^{-1}(1 - \beta_j)$$ (feasibility condition).

4 Different cases of GMCSTP

Consider the following three cases of GMCSTP

(a) when $$\alpha_i$$ and $$\beta_j$$ both follow general form of distribution defined in (2.1).

(b) when $$\alpha_i$$ and $$\beta_j$$ both follow general form of distribution defined in (2.2).

(c) when $$\alpha_i$$ and $$\beta_j$$ follow general form of distribution defined in (2.1) and (2.2) respectively or vice-versa.

4.1 When $$\alpha_i$$ and $$\beta_j$$ both follow general form of distribution defined in (2.1)

Let us consider that $$\alpha_i$$ and $$\beta_j$$ follows general form of distribution of the form defined in (2.1) i.e $$F(y) = 1 - \left[p h(y) + q_j^r\right]$$, $$p \neq 0$$, $$y \in (\xi, \phi)$$.

Putting $$F\left(\sum_{i=1}^{m} x_{ij}\right) = 1 - \left[p_{ij} h\left(\sum_{i=1}^{m} x_{ij}\right) + q_i^r\right]$$ in (3.18) of GMCSTP and $$F\left(\sum_{i=1}^{m} x_{ij}\right) = 1 - \left[p_{ij} g\left(\sum_{i=1}^{m} x_{ij}\right) + q_j^r\right]$$ in (3.19) of GMCSTP, we get,

GMCSTP 1:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{1}, C_{ij}^{2}, \ldots, C_{ij}^{k} x_{ij}, \quad k = 1, 2, \ldots, K$$

subject to,

$$\left[p_{ij} h\left(\sum_{i=1}^{m} x_{ij}\right) + q_i^r\right] \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m$$

(4.1)

$$1 - \left[p_{ij} g\left(\sum_{i=1}^{m} x_{ij}\right) + q_j^r\right] \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n$$

(4.2)

$$x_{ij} \geq 0 \quad \forall \ i \text{ and } j$$

(4.3)

4.2 When $$\alpha_i$$ and $$\beta_j$$ both follow general form of distribution defined in (2.2)

Let us consider that $$\alpha_i$$ and $$\beta_j$$ in (2.2) i.e $$F(y) = 1 - \left[e^{-p h(y)}\right] p \neq 0$$, $$y \in (\xi, \phi)$$.

Putting $$F\left(\sum_{i=1}^{m} x_{ij}\right) = e^{-p h(\sum_{i=1}^{m} x_{ij})}$$ in (3.18) of GMCSTP and $$F\left(\sum_{i=1}^{m} x_{ij}\right) = e^{-p g(\sum_{i=1}^{m} x_{ij})}$$ in (3.19) of GMCSTP, we get,

GMCSTP 2:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{1}, C_{ij}^{2}, \ldots, C_{ij}^{k} x_{ij}, \quad k = 1, 2, \ldots, K$$

subject to,

$$1 - e^{-p h(\sum_{i=1}^{m} x_{ij})} \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m$$

(4.4)

$$e^{-p g(\sum_{i=1}^{m} x_{ij})} \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n$$

(4.5)

$$x_{ij} \geq 0 \quad \forall \ i \text{ and } j$$

(4.6)

4.3 When $$\alpha_i$$ and $$\beta_j$$ follow general form of distribution defined in (2.1) and (2.2) respectively or vice-versa

Consider a case when $$\alpha_i$$ follows any one of general form of distributions defined in (2.1) and (2.2) and $$\beta_j$$ follows any one of general form of distribution defined in (2.2) and (2.1) respectively, then in view of GMCSTP 1 and GMCSTP 2 we have,

GMCSTP 3:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{1}, C_{ij}^{2}, \ldots, C_{ij}^{k} x_{ij}, \quad k = 1, 2, \ldots, K$$

subject to,

$$\left[p_{ij} h\left(\sum_{i=1}^{m} x_{ij}\right) + q_i^r\right] \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m$$

(4.7)

$$\left[1 - e^{-p h(\sum_{i=1}^{m} x_{ij})}\right] \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m$$

(4.8)

$$1 - \left[p_{ij} g\left(\sum_{i=1}^{m} x_{ij}\right) + q_j^r\right] \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n$$

(4.9)

$$1 - e^{-p g(\sum_{i=1}^{m} x_{ij})} \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n$$

(4.10)

$$x_{ij} \geq 0 \quad \forall \ i \text{ and } j$$

(4.11)
5 Deduction of some previous results along with some new results

In this section we deduce some previous results with the help of GMCSTP 1 and GMCSTP 2. Since GMCSTP 1 and 2 has been modelled with the assumption that both \(a_i\) and \(b_{j}\) are random variable. So we are considering only GMCSTP 1 and 2 throughout the paper. One can also consider MCSTP 2 or/and MCSTP 3 according to requirement. Many previous models proposed by Roy et al. (2012), Mahapatra (2014), Biswal and Samal (2013) and Mahapatra et al. (2013) can be deduced from GMCSTP 1 and GMCSTP 2 by setting different values of \(p_i, p'_{j}, q_i, q'_j, r_i, r'_j, h\left(\sum_{j=1}^{m} x_{ij}\right)\) and \(g\left(\sum_{j=1}^{m} x_{ij}\right)\).

5.1 Deductions using GMCSTP 1 and GMCSTP 2

5.1.1 When \(a_i\) and \(b_{j}\) follows exponential distribution

Let us consider that both \(a_i\) and \(b_{j}\) follow exponential distribution. In order to deduce the model obtained by S.K. Roy et al, we set \(p_i = 1, q_i = 0, r_i = \frac{\sigma_i}{h}, h\left(\sum_{j=1}^{m} x_{ij}\right) = e^{-k\sum_{j=1}^{m} x_{ij}}\) and \(p'_{j} = 1, q'_{j} = 0, r'_{j} = \frac{\sigma'_{j}}{h}, g\left(\sum_{j=1}^{m} x_{ij}\right) = e^{-k'\sum_{j=1}^{m} x_{ij}}\) in GMCSTP 1 and get,

MCSTP 4:

\[
\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C^1_{ij}, C^2_{ij}, \ldots, C^n_{ij} \right] x_{ij}, \quad k = 1, 2, \ldots, K
\]

subject to,
\[
\sum_{j=1}^{m} x_{ij} \leq -\theta_i \ln(1 - a_i), \quad i = 1, 2 \ldots m.
\]

\[
\sum_{i=1}^{n} x_{ij} \geq -\theta'_j \ln(b_{j}), \quad j = 1, 2 \ldots n.
\]

\[
x_{ij} \geq 0 \quad \forall \quad i \text{ and } j
\]

where \(\sum_{i=1}^{m} \{ -\theta_i \ln(1 - a_i) \} \geq \sum_{j=1}^{n} \{ -\theta'_j \ln(b_{j}) \}\) (feasibility condition) and \(a_i \geq 0, b_{j} \geq 0\) and \(\{\theta_i, \theta'_j\} > 0\) are the parameters of exponential distribution. The above MCSTP 4 is same as obtained by Roy et al. (2012).

5.1.2 When \(a_i\) and \(b_{j}\) follows Weibull distribution

Mahapatra (2014) presented a model by considering both \(a_i\) and \(b_{j}\) follow weibull distribution which can be obtained by setting \(p_i = 1, q_i = 0, r_i = \frac{\kappa_i}{\bar{X}_i}, h\left(\sum_{j=1}^{m} x_{ij}\right) = e^{-k\left(\sum_{j=1}^{m} x_{ij}\right)^{\gamma_i}}\) and \(p'_{j} = 1, q'_{j} = 0, r'_{j} = \frac{\kappa'_j}{\bar{X}_j}, g\left(\sum_{j=1}^{m} x_{ij}\right) = e^{-k'\left(\sum_{j=1}^{m} x_{ij}\right)^{\gamma'_j}}\) in GMCSTP 1, as follows:

MCSTP 5:

\[
\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C^1_{ij}, C^2_{ij}, \ldots, C^n_{ij} \right] x_{ij}, \quad k = 1, 2, \ldots, K
\]

subject to:
\[
\sum_{j=1}^{m} x_{ij} \leq e^{\left[ \ln h_{ij} + \frac{\pi}{r_i} \ln(1-a_i) \right]}, \quad i = 1, 2 \ldots m.
\]

\[
\sum_{i=1}^{n} x_{ij} \geq e^{\left[ \ln h_{ij} + \frac{\pi}{r'_j} \ln(b_{j}) \right]} - \frac{\ln(1-a_i)}{n}, \quad j = 1, 2 \ldots n.
\]

\[
x_{ij} \geq 0 \quad \forall \quad i \text{ and } j
\]

5.1.3 When \(a_i\) and \(b_{j}\) follows Cauchy distribution

Biswal and Samal (2013) proposed MCSTP model by considering that \(a_i\) and \(b_{j}\) follow Cauchy distribution. On Setting \(p_i = -\frac{1}{\pi}, q_i = \frac{1}{\pi}, r_i = 1, h\left(\sum_{j=1}^{m} x_{ij}\right) = \tan^{-1}\left(\frac{\sum_{j=1}^{m} x_{ij}-l_{ij}}{s_{ij}}\right), \) and \(p'_{j} = -\frac{1}{\pi}, q'_{j} = 0, r'_{j} = 1, g\left(\sum_{j=1}^{m} x_{ij}\right) = \tan^{-1}\left(\frac{\sum_{j=1}^{m} x_{ij}-l_{ij}}{s_{ij}}\right)\) in GMCSTP 1, we get,

MCSTP 6:

\[
\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C^1_{ij}, C^2_{ij}, \ldots, C^n_{ij} \right] x_{ij}, \quad k = 1, 2, \ldots, K
\]

subject to:
\[
\sum_{j=1}^{m} x_{ij} \leq l_{ij} + s_{ai} \tan \left( \frac{\pi a_i - \pi}{2} \right), \quad i = 1, 2, \ldots, m
\]

\[
\sum_{i=1}^{n} x_{ij} \geq l_{ij} + s_{bj} \tan \left( \frac{\pi - \pi b_{j}}{2} \right), \quad j = 1, 2, \ldots, n
\]

\[
x_{ij} \geq 0 \quad \forall \quad i \text{ and } j
\]

where \(\sum_{i=1}^{m} l_{ai} + s_{ai} \tan \left( \frac{\pi a_i - \pi}{2} \right) \geq \sum_{j=1}^{n} l_{bj} + s_{bj} \tan \left( \frac{\pi}{2} - \pi b_{j} \right)\) (feasibility condition) and \(-\infty < a_i < +\infty, -\infty < b_{j} < +\infty\) and \(l_{ai}, l_{bj} > 0\) and \(s_{ai}, s_{bj} > 0\) are the location and scale parameter of \(a_i\) and \(b_{j}\), respectively. Which is a multi-choice approach of the model proposed by Biswal and Samal (2013).
5.1.4 When \(a_i\) and \(b_j\) follows extreme value distribution  

Setting \(p = 1, h\left(\sum_{j=1}^{n} x_{ij}\right) = e^{\frac{(\sum_{j=1}^{n} x_{ij} \gamma_j)}{\gamma_j}}\) and \(p' = 1, g\left(\sum_{i=1}^{m} x_{ij}\right) = e^{\frac{(\sum_{i=1}^{m} x_{ij} \gamma_i)}{\gamma_i}}\) in GMCSTP 2 it deduces to,  

\[
MCSTP 7: \quad \min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.13}
\]
subject to:  
\[
\sum_{j=1}^{n} x_{ij} \leq \gamma_i - \delta_i [\ln(-\ln(\alpha_i))], \quad i = 1, 2, \ldots, m \tag{5.14}
\]
\[
\sum_{i=1}^{m} x_{ij} \geq \gamma_j' - \delta_j' [\ln(-\ln(\beta_j))], \quad j = 1, 2, \ldots, n \tag{5.15}
\]
x_{ij} \geq 0 \quad \forall \ i \text{ and } j \tag{5.16}

where \(\sum_{i=1}^{m} \gamma_i - \delta_i [\ln(-\ln(\alpha_i))] \geq \sum_{j=1}^{n} \left[ \gamma_j' - \delta_j' [\ln(-\ln(\beta_j))] \right] \) (feasibility condition) and \(-\infty < \alpha_i < +\infty, -\infty < \beta_j < +\infty\) and \(\gamma_i, \gamma_j', \delta_i, \delta_j' > 0\) are location and scale parameters of extreme value distribution, which is same as obtained by Mahapatra et al. (2013).

5.2 Some new results using GMCSTP 1, GMCSTP 2 and GMCSTP 3  

5.2.1 When \(a_i\) and \(b_j\) follow Pareto distribution  

Let us consider the MCSTP in which \(a_i\) and \(b_j\) follow Pareto distribution. By setting \(p_i = d_i^{-k_i}, q_i = 0, r_i = -\frac{\theta_i}{k_i}, h\left(\sum_{j=1}^{n} x_{ij}\right) = \left(\sum_{j=1}^{n} x_{ij}\right)^{k_i}\) and \(p_j' = d_j^{-k_j'}, q_j' = 0, r_j' = -\frac{\theta_j'}{k_j'}, g\left(\sum_{i=1}^{m} x_{ij}\right) = \left(\sum_{i=1}^{m} x_{ij}\right)^{k_j'}\) in GMCSTP 1, we get,  

\[
MCSTP 8: \quad \min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.17}
\]
subject to:  
\[
\sum_{j=1}^{n} x_{ij} \leq \frac{d_i}{(1-\alpha_i)^{\frac{1}{\gamma_i}}}, \quad i = 1, 2, \ldots, m \tag{5.18}
\]
\[
\sum_{i=1}^{m} x_{ij} \geq \frac{d_j'}{(\beta_j')^{\frac{1}{\gamma_j'}}}, \quad j = 1, 2, \ldots, n \tag{5.19}
\]
x_{ij} \geq 0 \quad \forall \ i \text{ and } j \tag{5.20}

where \(\sum_{i=1}^{m} \frac{d_i}{(1-\alpha_i)^{\frac{1}{\gamma_i}}} \geq \sum_{j=1}^{n} \frac{d_j'}{(\beta_j')^{\frac{1}{\gamma_j'}}}\) (feasibility condition) and \(\{d_i, d_j'\} > 0\) and \(\{\alpha_i, \alpha_j, \beta_j\} > 0\) are scale and shape parameters respectively and \(a_i \geq d_i\) and \(b_j \geq d_j'\).

5.2.2 When \(a_i\) and \(b_j\) follow Burr-XII distribution  

Setting \(p_i = \theta_i, q_i = 1, r_i = -k_i, h\left(\sum_{j=1}^{n} x_{ij}\right) = \left(\sum_{j=1}^{n} x_{ij}\right)^k, \) and \(p_j' = \theta_j', q_j' = 1, r_j' = -k_j', g\left(\sum_{i=1}^{m} x_{ij}\right) = \left(\sum_{i=1}^{m} x_{ij}\right)^{k_j'}\) in GMCSTP 1 we get,  

\[
MCSTP 9: \quad \min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.21}
\]
subject to:  
\[
\sum_{j=1}^{n} x_{ij} \leq \left[ \frac{1 - \alpha_i}{\theta_i} \right]^{\frac{1}{\gamma_i}}, \quad i = 1, 2, \ldots, m \tag{5.22}
\]
\[
\sum_{i=1}^{m} x_{ij} \geq \left[ \frac{1 - \beta_j'}{\theta_j'} \right]^{\frac{1}{\gamma_j'}}, \quad j = 1, 2, \ldots, n \tag{5.23}
\]
x_{ij} \geq 0 \quad \forall \ i \text{ and } j \tag{5.24}

where \(\sum_{i=1}^{m} \left[ \frac{1 - \alpha_i}{\theta_i} \right]^{\frac{1}{\gamma_i}} \geq \sum_{j=1}^{n} \left[ \frac{1 - \beta_j'}{\theta_j'} \right]^{\frac{1}{\gamma_j'}}\) (feasibility condition) and \(\alpha_i \geq 0, b_j \geq 0\) and \(\{\alpha_i, \theta_i, \beta_j, \theta_j'\} > 0\) and \(\{k_i, k_j'\} > 0\) are shape parameters of Burr-XII distribution.

5.2.3 When \(a_i\) and \(b_j\) follow power function distribution  

Setting \(p_i = -\theta_i, h\left(\sum_{j=1}^{n} x_{ij}\right) = \ln\left(\sum_{j=1}^{n} x_{ij}\right)\) and \(p_j = -\theta_j', h\left(\sum_{i=1}^{m} x_{ij}\right) = \ln\left(\sum_{i=1}^{m} x_{ij}\right)\) in GMCSTP 2, we get,  

\[
MCSTP 10: \quad \min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.25}
\]
subject to:  
\[
\sum_{j=1}^{n} x_{ij} \leq d_i^{-\frac{1}{\alpha_i}}, \quad i = 1, 2, \ldots, m \tag{5.26}
\]
\[
\sum_{i=1}^{m} x_{ij} \geq d_j^{-\frac{1}{\gamma_j}}, \quad j = 1, 2, \ldots, n \tag{5.27}
\]
x_{ij} \geq 0 \quad \forall \ i \text{ and } j \tag{5.28}
where \( \sum_{i=1}^{m} d_i a_i ^{\frac{1}{\beta_i}} \geq \sum_{j=1}^{n} d'_j (1 - \beta_j) ^{\frac{1}{\beta_j}} \) (feasibility condition) and \( \{d_i, d'_j\} > 0 \) and \( \{\theta_i, \theta'_j\} \) are the scale and shape parameters of \( a_i \geq 0 \) and \( b_j \geq 0 \) respectively.

### 5.2.4 When \( a_i \) follows Burr XII distribution and \( b_j \) follows Extreme value distribution

Setting \( p_i = \theta_i, q_i = 1, r_i = -k_i, h \left( \sum_{j=1}^{n} x_{ij} \right) = \left( \sum_{j=1}^{n} x_{ij} \right) \delta_i \) and \( p'_j = 1, g \left( \sum_{i=1}^{m} x_{ij} \right) = e^{\theta_j} \) in GMCSTP 3 we get,

**MCSTP 11:**

\[
\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C^1_{ij}, C^2_{ij}, \ldots, C^k_{ij} \right] x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.29}
\]

Subject to:

\[
\sum_{j=1}^{n} x_{ij} \leq \left[ \left( 1 - \alpha_i ^{\frac{1}{\theta_i}} \right)^{\frac{1}{\theta_i}} - 1 \right] \left( \sum_{j=1}^{n} x_{ij} \right) ^{\frac{1}{\theta_i}}, \quad i = 1, 2, \ldots, m \tag{5.30}
\]

\[
\sum_{i=1}^{m} x_{ij} \geq \gamma_j - \delta_j [\ln(-\ln(\beta_j))]\quad j = 1, 2, \ldots, n \tag{5.31}
\]

\[
x_{ij} \geq 0 \quad \forall \ i \text{ and } j \tag{5.32}
\]

where \( \sum_{i=1}^{m} \left[ \left( \frac{1-\alpha_i ^{\frac{1}{\theta_i}}}{\theta_i} - 1 \right) \left( \sum_{j=1}^{n} x_{ij} \right) ^{\frac{1}{\theta_i}} \right] \geq \sum_{j=1}^{n} \left[ \gamma_j - \delta_j [\ln(-\ln(\beta_j))] \right] \).

### 5.2.5 When \( a_i \) follows power function distribution and \( b_j \) follows Pareto distribution

Setting \( p_i = -\theta_i, h \left( \sum_{j=1}^{n} x_{ij} \right) = \ln \left( \sum_{j=1}^{n} x_{ij} \right) \) and \( p'_j = d'_j ^{-k'_j}, q'_j = 0, r'_i = -\delta'_i, g \left( \sum_{i=1}^{m} x_{ij} \right) = \left( \sum_{i=1}^{m} x_{ij} \right)^{\frac{k'_j}{\theta'_j}} \) in GMCSTP 3.

**MCSTP 12:**

\[
\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C^1_{ij}, C^2_{ij}, \ldots, C^k_{ij} \right] x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.33}
\]

subject to:

\[
\sum_{j=1}^{n} x_{ij} \leq d_i a_i ^{\frac{1}{\beta_i}}, \quad i = 1, 2, \ldots, m \tag{5.34}
\]

\[
\sum_{i=1}^{m} x_{ij} \geq \frac{d_j}{(\theta'_j)^{d'_j}}, \quad j = 1, 2, \ldots, n \tag{5.35}
\]

\[
x_{ij} \geq 0 \quad \forall \ i \text{ and } j \tag{5.36}
\]

where \( \sum_{i=1}^{m} \left( d_i a_i ^{\frac{1}{\beta_i}} \right) \geq \sum_{j=1}^{n} \left( \frac{d_j}{(\theta'_j)^{d'_j}} \right) \) (feasibility condition).

### 6 Numerical illustrations

We consider the numerical example taken by (Mahapatra et al. 2013). Data for multi-choice cost \( C_{ij}^k \) are appended below in Table 1.

#### 6.1 Illustration 1

Let us consider that we have three known parameters of availability \( a_1, a_2, a_3 \) follow Burr-XII distribution.

The specified probability levels and shape parameters of \( a_1, a_2, a_3 \) are given in Table 2.

Further, consider that we have four known parameters of demand \( b_1, b_2, b_3, b_4 \) follow extreme value distribution.

The specified probability levels and location and scale parameters of \( b_1, b_2, b_3, b_4 \) are given in Table 3.

Using the data provided in Tables 1, 2 and 3 the following equivalent multi-choice deterministic transportation problem is formulated with the help of GMCSTP 3 as:

\[
\text{min: } Z = \{10, 11, 12\} x_{11} + \{15, 16\} x_{12} + \{21, 22, 23, 24\} x_{13} + \{21, 23, 25\} x_{31} + \{15, 17, 19, 21, 23, 25\} x_{21} + \{10, 12, 14, 16, 18, 20\} x_{22} + \{9, 10, 11\} x_{23} + \{18, 19\} x_{24} + \{20, 21, 22, 23, 24, 25\} x_{31} + \{10, 11, 12, 13, 14, 16, 17\} x_{32} + \{20, 22, 25\} x_{33} + \{15, 20\} x_{34}
\]

subject to,

\[
\sum_{j=1}^{4} x_{1j} \leq 967.544404 \tag{6.1}
\]

\[
\sum_{j=1}^{4} x_{2j} \leq 762.934875 \tag{6.2}
\]

\[
\sum_{j=1}^{4} x_{3j} \leq 612.817850 \tag{6.3}
\]

| Sl. no. | Route: \( x_{ij} \) | Transportation cost (in Rupees) \( C_{ij}^k \) per unit (1 unit = 10 kg) |
|---------|----------------------|------------------------------------------------|
| 1       | (1, 1): \( x_{11} \) | 10 or 11 or 12                                    |
| 2       | (1, 2): \( x_{12} \) | 15 or 16                                         |
| 3       | (1, 3): \( x_{13} \) | 21 or 22 or 23 or 24                             |
| 4       | (1, 4): \( x_{14} \) | 21 or 23 or 25                                   |
| 5       | (2, 1): \( x_{21} \) | 15 or 17 or 19 or 21 or 23 or 25                 |
| 6       | (2, 2): \( x_{22} \) | 10 or 12 or 14 or 16 or 18 or 20                  |
| 7       | (2, 3): \( x_{23} \) | 9 or 10 or 11                                    |
| 8       | (2, 4): \( x_{24} \) | 18 or 19                                         |
| 9       | (3, 1): \( x_{31} \) | 20 or 21 or 22 or 23 or 24 or 25                  |
| 10      | (3, 2): \( x_{32} \) | 10 or 11 or 12 or 13 or 14 or 15 or 16 or 17      |
| 11      | (3, 3): \( x_{33} \) | 20 or 22 or 25                                   |
| 12      | (3, 4): \( x_{34} \) | 15 or 20                                         |

Table 1 Multi-choice transportation cost for route \( x_{ij} \)
Table 2 Specified probability levels and shape parameters of $a_i$

| Random parameters $a_i$ | Specified probability levels | Shape parameters 1 | Shape parameters 2 |
|-------------------------|-------------------------------|--------------------|--------------------|
| $a_1$                   | 0.01                          | 0.002              | 0.73               |
| $a_2$                   | 0.02                          | 0.004              | 0.76               |
| $a_3$                   | 0.03                          | 0.006              | 0.79               |

\[
\begin{align*}
\sum_{i=1}^{3} x_{i1} & \geq 615.992671 \\
\sum_{i=1}^{3} x_{i2} & \geq 511.880781 \\
\sum_{i=1}^{3} x_{i3} & \geq 408.347897 \\
\sum_{i=1}^{3} x_{i4} & \geq 305246388
\end{align*}
\]

$x_{ij} \geq 0, i, j = 1, 2, 3, 4.$

Now using the transformation technique proposed by Biswal and Acharya (2009), we obtain the following multi-choice deterministic transportation problem:

\[
\begin{align*}
\text{min: } z &= t_{11}x_{11} + t_{12}x_{12} + t_{13}x_{13} + t_{14}x_{14} \\
& + t_{21}x_{21} + t_{22}x_{22} + t_{23}x_{23} + t_{24}x_{24} \\
& + t_{31}x_{31} + t_{32}x_{32} + t_{33}x_{33} + t_{34}x_{34}
\end{align*}
\]

subject to, (6.1)-(6.7)

where,

\[
\begin{align*}
t_{11} &= 10z_{11}^2 + 11z_{11} (1 - z_{11}) + 12 (1 - z_{11}) z_{11} \\
t_{12} &= 15z_{12}^2 + 16 (1 - z_{12}) \\
t_{13} &= 21z_{13}^2 + 22z_{13} (1 - z_{13}) + 23 (1 - z_{13}) z_{13} \\
& + 24 (1 - z_{13}) (1 - z_{13}) \\
t_{14} &= 21z_{14}^2 + 23z_{14} (1 - z_{14}) + 25 (1 - z_{14}) z_{14}
\end{align*}
\]

\[
\begin{align*}
t_{21} &= 15z_{21}^2 (1 - z_{21}) (1 - z_{21}) \\
& + 17 (1 - z_{21}) z_{21} (1 - z_{21}) \\
& + 19 (1 - z_{21}) (1 - z_{21})^2 z_{21} \\
& + 21z_{21}^2 z_{21} (1 - z_{21}) + 23 (1 - z_{21}) z_{21}^2 z_{21} \\
& + 25z_{21}^2 (1 - z_{21})^2 z_{21} \\
t_{22} &= 10z_{22}^2 (1 - z_{22}) (1 - z_{22}) \\
& + 12 (1 - z_{22}) z_{22} (1 - z_{22}) \\
& + 14z_{22}^2 z_{22} (1 - z_{22}) \\
& + 16 (1 - z_{22}) (1 - z_{22})^2 z_{22} + 18z_{22}^2 (1 - z_{22}) z_{22}^2 z_{22} + 20 (1 - z_{22}) z_{22}^2 z_{22} \\
t_{23} &= 9z_{23}^2 z_{23}^2 + 10z_{23}^2 (1 - z_{23})^2 + 11 (1 - z_{23}) z_{23}^2 \\
t_{24} &= 18z_{24}^2 + 19 (1 - z_{24})
\end{align*}
\]

\[
\begin{align*}
t_{31} &= 20 (1 - z_{31}) (1 - z_{31}) (1 - z_{31}) \\
& + 21z_{31}^2 (1 - z_{31}) (1 - z_{31}) \\
& + 22 (1 - z_{31}) z_{31} (1 - z_{31}) \\
& + 23 (1 - z_{31}) (1 - z_{31}) (1 - z_{31}) \\
& + 24z_{31}^2 z_{31} (1 - z_{31}) + 25z_{31} (1 - z_{31}) z_{31}^2 \\
& + 26 (1 - z_{31}) z_{31}^2 z_{31}
\end{align*}
\]

\[
\begin{align*}
t_{32} &= 10z_{32}^2 z_{32}^2 + 11 (1 - z_{32}) z_{32}^2 z_{32} \\
& + 12z_{32}^2 (1 - z_{32})^2 z_{32}^2 + 13z_{32} (1 - z_{32}) z_{32}^2 z_{32} \\
& + 14 (1 - z_{32}) (1 - z_{32})^2 z_{32} \\
& + 15z_{32}^2 (1 - z_{32}) (1 - z_{32}) \\
& + 16 (1 - z_{32}) z_{32}^2 (1 - z_{32}) \\
& + 17 (1 - z_{32}) z_{32}^2 z_{32} \\
t_{33} &= 20z_{33}^2 z_{33}^2 + 22z_{33}^2 (1 - z_{33}) + 25 (1 - z_{33}) z_{33}^2 \\
t_{34} &= 15z_{34}^2 + 20 (1 - z_{34})
\end{align*}
\]

The above non-linear mixed integer programming problem is solved by using LINGO 13.0 software package and the optimal solution is obtained as: $x_{11} = 615.9927, x_{22} = 382.1037, x_{33} = 408.3469, x_{32} = 129.7771, x_{34} = 305.2464$ and rest of the $x_{ij}$ are zero. The minimum
cost parameters are considered because we were much concerned about random parameters. In further studies extended multi-choice parameters may also be taken into account.

Competing interests
The authors declare that they have no competing interests.

Authors’ contributions
The first author AQ performed calculations involved in the manuscript. All authors read and approved the final manuscript.

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7 Conclusion
In this paper we have considered a MCSTP where cost coefficient of objective function are assumed to be of multi-choice type and random availability and demand of product are assumed to follow general form of distributions. With this generalized formulation of MCSTP the DM becomes capable to fit any distribution among exponential, weibull, cauchy, extreme value, power function, burr-XII and pareto according to the nature of data. Thus the present model can be applied in several situations of transportation problems when demand and availability are restricted to follow a particular probability distribution. Here, only upto eight choices of multi-choice

Table 4 Specified probability levels, scale and shape parameters of $a_i$

| Random parameters $a_i$ | Specified probability levels | Scale parameters | Shape parameters |
|------------------------|------------------------------|-----------------|-----------------|
| $a_1$                  | 0.01                         | 1000            | 100             |
| $a_2$                  | 0.02                         | 800             | 70              |
| $a_3$                  | 0.03                         | 700             | 60              |

Table 5 Specified probability levels, scale and shape parameters of $b_j$

| Random parameters $b_j$ | Specified probability levels | Scale parameters | Shape parameters |
|-------------------------|------------------------------|-----------------|-----------------|
| $b_1$                   | 0.04                         | 350             | 5               |
| $b_2$                   | 0.05                         | 300             | 6               |
| $b_3$                   | 0.06                         | 270             | 7               |
| $b_4$                   | 0.07                         | 230             | 8               |

transportation cost is 19532.56 obtained by choosing multi-choice cost as follows:

\[ x_{ij} = x_{i1} x_{i2} x_{i3} x_{i4} x_{i5} x_{i6} x_{i7} x_{i8} \]

value of $C_j$ : 10 15 21 23 15 10 9 18 22 10 22 15

6.2 Illustration 2
Again consider, in the above illustration 1 the availability $a_1, a_2, a_3$ are supposed to follow Power Function distribution and the demand $b_1, b_2, b_3, b_4$ are assumed to follow Pareto distribution. The specified probability levels, scale and shape parameters of $a_1, a_2, a_3$ are given in Table 4 and of $b_1, b_2, b_3, b_4$ are given in Table 5 respectively.

Solving this in the similar manner optimal solutions are obtained as $x_{i1} = 666.2789, x_{i2} = 352.9524, x_{i3} = 403.5651, x_{i4} = 141.3123, x_{i5} = 320.6938$ and rest of the $x_{ij}$ are zero. The minimum transportation cost is 20047.93 obtained by choosing multi-choice cost as follows:

\[ x_{ij} = x_{i1} x_{i2} x_{i3} x_{i4} x_{i5} x_{i6} x_{i7} x_{i8} \]

value of $C_j$ : 10 15 21 23 15 10 9 18 22 10 22 15

7 Conclusion
In this paper we have considered a MCSTP where cost coefficient of objective function are assumed to be of multi-choice type and random availability and demand of product are assumed to follow general form of distributions. With this generalized formulation of MCSTP the DM becomes capable to fit any distribution among exponential, weibull, cauchy, extreme value, power function, burr-XII and pareto according to the nature of data. Thus the present model can be applied in several situations of transportation problems when demand and availability are restricted to follow a particular probability distribution. Here, only upto eight choices of multi-choice