Optimization design for tandem cascades of compressors based on adaptive particle swarm optimization

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ABSTRACT
To improve the flow performance of tandem cascades on design and off design incidence angle and increase the stable operation range, an optimization system for tandem cascades was developed based on an adaptive particle swarm optimization (APSO-PDC). Firstly, APSO-PDC was proposed based on adaptive selection of particle roles and population diversity control. The adaptive selection of particle roles which combines the evolutionary state and dynamic particle state estimation (DPSE) method will sort the particles into three roles to help different particles execute different search tasks during optimization process. The population diversity control, which combines comprehensive learning strategy of the comprehensive learning particle swarm optimizer (CLPSO) with evolutionary state, pretty strengthens the exploration ability and avoids falling into the local optima. The performance of APSO-PDC is evaluated by 11 unimodal and multimodal functions. Compared with the other six PSOs, the results indicate APSO-PDC has better performance in terms of algorithm accuracy and algorithm reliability. In addition, APSO-PDC is validated by optimizing two large-turning tandem cascades, including low-dimension (5 optimization variables) and high-dimension problems (34 optimization variables). Compared with the other six PSOs, the optimization results demonstrate APSO-PDC has the fastest convergence speed and simultaneously controls well the population diversity.

1. Introduction
Aerodynamic design of modern compressors faces more and more challenges, with increasing performance requirements for compressors to realize higher efficiency, higher pressure ratio and a more extensive stability margin. As an effective high-load cascade of compressors, tandem cascades have excellent performance in terms of higher stage pressure ratio, fewer stages number and higher stages efficiency compared with similar single cascades (Bammert & Beelte, 1980; Hasegawa, Matsuoka, & Suga, 2003). In addition, tandem cascades have successfully been applied in the compressor stators of engines, such as J85, JT8D and GE J-79 (Lakshminarayana, 1985).

Because of the high load and low loss of tandem cascades, there has been much research on tandem cascades based on the wind tunnel experiment and the computational fluid dynamics (CFD) method. Based on the results of low-speed wind tunnel experiments, Schneider and Kožulović (2013) and Hoeger, Baier, Fischer, and Neudorfer (2011) demonstrated that, compared with single cascades with the same parameters, tandem cascades can provide larger load and smaller loss. Bammert and Staude (1980) investigated the relationship of the front and aft airfoil of tandem cascades, and demonstrated the axial overlap (AO) of 0 or a small negative value can achieve optimal performance of tandem cascades. Saha and Roy (1996, 1997) studied the tandem cascades composed of controlled diffusion airfoils (CDA) and the same parameter CDA cascades. The research showed that on design incidence angle, the tandem cascades had larger turning flow angle and lower total pressure loss, but the tandem cascades had poor performances off design incidence angle. Based on low-speed wind tunnel experiments, Heinrich, Tiedemann, and Peitsch (2017) researched the effects of AO and percent pitch (PP) on the performance of tandem cascades, and found tandem cascades obtained the best performance at a high PP and equal load between front and aft airfoil. Based on cascade wind tunnel testing and the CFD method, Hertel, Bode, Kožulovic, and Schneider (2013) studied subsonic and transonic tandem cascades, and found at cascade wind tunnel conditions (at the inlet Mach number of 0.175), the subsonic and transonic tandem cascade work better than the corresponding single cascade with...
the similar geometry. Dekhkarqani, Boroomand, and Eshraghi (2014) applied the CFD method to research a subsonic tandem cascade and the corresponding single cascade, and found the tandem cascade provided higher load and smaller loss. Sachmann and Fottner (1993) found a higher PP can improve the performance of tandem cascades on design incidence angle, but will deteriorate the performance of off-design incidence angle, and a PP of 0.5 can achieve a good balance of the tandem cascades performance on design and off design incidence angle. Based on the CFD method, McGlumphy, Ng, Welling, and Kempf (2007) investigated the effect of AO angle. Based on the CFD method, McGlumphy, Ng, Wellborn, and Kempf (2007) investigated the effect of AO and PP on the performance of subsonic tandem cascades. They indicated a small AO and a large PP can obviously decrease the flow loss of subsonic tandem cascades on design incidence angle. Compared with similar single cascades with the same load, subsonic tandem cascades can obtain a better flow performance on design and off design incidence angle.

Based on the literature, tandem cascades can realize a higher stage pressure ratio and stages efficiency, which is significant in the design of advanced compressors. However, some literature has indicated the stability margin of compressors with tandem cascades is smaller than that of compressors with similar single cascades. Based on the experimental method, Eshraghi, Boroomand, and Tousi (2014) found that the isentropic efficiency and stability margin of the high-load tandem rotor are lower than those of single row rotor with the same parameters. Brent, Cheatham, and Clemmons (1972) studied a tandem rotor of core compressors by experimental testing, and found that the pressure ratio of the tandem rotor exceeded the design pressure ratio, but the tandem rotor had smaller isentropic efficiency and stability margin. Tandem rotors were used on multistage axial compressors by Bammert and Staude (1980, 1981) and Bammert and Beetle (1980). On design point, the isentropic efficiency of the tandem rotor compressor was 0.856, and the stability margin was only about 5%.

It is well known that CFD has greatly improved the optimization design method, combining CFD and optimization algorithm. Hence, much research has been carried out on the optimization design method based on CFD. Ezhilsabareesh, Rhee, and Samad (2017) accomplished the shape optimization of a turbine based on multiple surrogates-assisted and multi-objective evolutionary algorithms. Optimization of vanes of centrifugal compressors is explored in Ivo, Damir, and Zoran (2016) by using control points to parameterize the suction and pressure surface of vanes. An optimization design method for radial turbines was proposed by Galindo, Hoyas, Fajardo and Navarro (2014), and a CFD-based hull form optimization loop was developed by combining an approximate method and an improved particle swarm optimization (PSO) algorithm in Zhang, Zhang, Tahsin, Leping, and Yu (2017). A multi-objective optimization method for wing is presented in Liang, Cheng, Li, and Xiang (2014). Safikhani, Khalkhali, and Farajpoor (2014) applied CFD and meta-models to accomplish the multi-objective optimization design of centrifugal pumps. Thus, to improve the flow performance of tandem cascades on design and off design incidence angles, an optimization system for tandem cascades was developed in this study.

In the literature about optimization of tandem cascades, the relative positions of the front airfoil and rear airfoil have been used as optimization variables. However, the shape of tandem cascades has a great influence on the flow field. Therefore, the present study focuses on the coupling optimization for shape and relative position of tandem cascades. In contrast to previous studies, with simultaneous variations of shape and relative position the number of optimization variables and the complexity of optimization will greatly increase. Hence, the major difficulty is the development of an optimization method that has fast convergence speed and simultaneously obtains an optimal solution. Thus, based on the above analysis, the main contribute of this study is twofold. The first goal is to propose a new variant of PSO, namely adaptive particle swarm optimization with population diversity control (APSO-PDC), to improve the performance of the original PSO. The second goal is to create an automatic optimization system of tandem cascades based on APSO-PDC to improve the design quality of tandem cascades. The optimization system can be used to realize the optimization design of the shape parameters and relative position of tandem cascades.

### 2. Related work of PSO

Over the past few decades, many real-life applications in different fields have been successfully solved based on contemporary soft computing techniques. Sefeedpari, Rafiee, Akram, Chau, and Pishgar-Komleh (2016) proposed an adaptive neural fuzzy inference system-based modeling approach where the number of data pairs employed for training was adjusted by application of a clustering method. Wang, Xu, Chau, and Chen (2013) combined a data analysis methodology and support vector machine to decompose annual rainfall series. Gholami, Chau, Fadaee, Torkaman, and Ghaffari (2015) studied the groundwater level fluctuations based on an artificial neural network. Chen and Chau (2016) presented a hybrid double feed forward neural network model. Among many contemporary soft computing techniques, evolutionary algorithms have become
an important research field. PSO is one of the most popular and effective swarm intelligence algorithms, and was developed by Kennedy and Eberhart based on the natural phenomenon of birds looking for food (Kennedy & Eberhart, 1995). As PSO is easy to implement, the parameter needed to define is small and the convergence rate is fast, it has been studied by many scholars since it was first proposed. Many studies have shown that some PSOs easily find local optima when optimizing complex problems. Ratnaweera, Halgamuge, and Watson (2004) proposed that the main reason for the premature convergence of PSOs is that the population diversity is too simple. However, increasing the population diversity may lead to a slower convergence speed. Thus, the two most important goals of PSO research are to speed up convergence and avoid falling into the local optima. Some PSO variants have been proposed to achieve these two goals. However, according to the literature, it is difficult to achieve these two objectives simultaneously.

In original PSO, each particle has two parameters, the current position $x_i$ and the current velocity $v_i$. During iteration, PSO will memorize the individual optimal position ($p_{best}$) and the global optimal position ($g_{best}$). At each iteration, the following formulas are used to recalculate the velocity and position of each particle.

\[
\begin{align*}
    v_{ij}(t + 1) &= w v_{ij}(t) + C_1 r_1 (p_{best_{ij}}(t) - x_{ij}(t)) + C_2 r_2 (g_{best}(t) - x_{ij}(t)) \\
    x_{ij}(t + 1) &= x_{ij}(t) + v_{ij}(t + 1)
\end{align*}
\]

(1)

where $w$ is the inertial factor, $C_1$ and $C_2$ are the study factor, $r_1$ and $r_2$ are the random numbers uniformly generated from $[0, 1]$, $N$ is the size of population, $D$ is the number of optimization variables. A limit velocity called $v_{max}$ is imposed on particles. According to the literature on PSO variants, in order to accelerate convergence speed and avoid falling into the local optima, PSO was improved with the following four methods.

(1) Control of population diversity: negative entropy is introduced to PSO in Xie, Zhang, & Yang (2002) to avoid premature convergence. To increase population diversity, a method of adaptively choosing the optimal position of neighborhood was raised in Li (2004). A hierarchical clustering method was used to locate and track multiple peaks for dynamic optimization problems at each iteration (Yang & Li, 2010). Based on these studies, these PSOs improve the diversity of the algorithm and also reduce the convergence speed of PSO.

(2) Change of population topology: the population topology of PSO determines the method to share information among population. A PSO variant called FIPS (The Fully Informed Particle Swarm Optimization) in (Mendes, Kennedy, & Neves, 2004) making the individuals “fully informed” was proposed. FIPS used an average of the individual optimal position of its neighbors as the individual optimal position ($p_{best}$). Liang, Qin, Suganthan, and Baskar (2006) presented the comprehensive learning particle swarm optimizer (CLPSO); CLPSO uses a novel learning strategy in which all individual optimal positions are used to update the particle's velocity. Wang, Yang, and Chen (2014) proposed multi-layer particle swarm optimization (MLPSO) to improve the performance of traditional PSO, which consisted of only two searching layers. PSO-ITC (Particle swarm optimization with increasing topology connectivity) (Lim & Mat Isa, 2014) applied an ITC module to increase population connectivity. In summary, these methods can remarkably enhance the performance of PSOs on some problems.

(3) Hybrid PSO: Hybrid evolutionary algorithms are becoming increasingly popular due to their capabilities in dealing with complexity problems. Zhang and Xie (2003) proposed a PSODE (hybrid particle swarm with differential evolution operator) algorithm, which integrates particle swarm optimization with differential evolution. A cooperative PSO (CPSO) algorithm was proposed in Vandenbergh and Engelbrecht (2004) which sorts search space into some smaller solution space. Chen, Peng, and Jian (2007) proposed PSO-RDL (Particle Swarm Optimization With Recombination and Dynamic Linkage Discovery) that integrates PSO, recombination operator and dynamic linkage discovery.

(4) PSO with parameter control: Parameter control is another promising research trend in PSO. Frankenstein’s PSO (FPSO) was proposed by Montes, Stutzle, Birattari, and Dorigo (2009) based on a time-varying population topology and a decreasing inertia weight. Adaptive SPSO (Particle swarm optimizer with adaptive species radius) (Liu, Qin, & Li, 2007) was proposed based on species-PSO and an adaptive species radius strategy for multimodal function optimization. PSO with adaptive parameters, called adaptive PSO (APSO), has been proposed by Zhan, Zhang, Li, and Chung (2009). In APSO, four evolutionary states, exploitation, exploration, convergence, and jumping out are defined. Similarly, the four operators in ALPSO (Self-Learning Particle Swarm Optimizer) (Li, Yang, & Nguyen, 2012) play similar roles as the four evolutionary states defined in APSO.
3. APSO-PDC

In the literature about PSOs, all particles in PSOs use the same parameters, inertia factor and study factor, which may cause a lack of particle diversity. Hence, according to their local fitness and local locations, different particles should use different parameters to execute different searches including local search and global search. Inspired by the idea of division of labor, different roles can be assigned to a particle, for example, convergence particle, exploiting particle and exploring particle, which will execute different search tasks during the optimization process. In order to maintain population diversity and simultaneously preserve good particle information, we use a strategy based on the comprehensive learning strategy of CLPSO and evolutionary state to control population diversity.

Based on the aforementioned analysis, to effectively raise the performance of PSO, in this paper, APSO-PDC is formulated based on adaptive selection of particle role and population diversity control. In this paper the following discussion is restricted to minimization problems.

3.1. Adaptive selection of particle roles

Inspired by the division of labor, we can assign different roles to a particle, for example, convergence particle, exploiting particle and exploring particle, which will execute different search tasks during the optimization process. To adaptively select an optimal role from these three roles for each particle, an adaptive selection role method is created based on evolutionary states (ES) and the dynamic particle state estimation (DPSE) function. According to APSO (Zhan et al., 2009), four evolutionary states, including exploitation, exploration, convergence, and jumping out, are defined based on evolutionary factor $fe$ (defined in Equation (2)). In APSO, fuzzy classification was adopted to classify evolutionary factor into four evolutionary states, and the fuzzy membership functions are depicted in Figure 1. On the other hand, to obtain the quality of particles and decide the optimal role for a particle from the three roles, a DPSE method is first devised based on local fitness and local location of particles. The DPSE function is defined by Equation (3).

$$fe = \frac{g_{gbest} - g_{min}}{g_{max} - g_{min}}$$

(2)

where $g_i = \frac{1}{n-1} \sum_{j=1}^{n} \sqrt{\sum_{k=1}^{D} (x_{id} - g_{best, d})^2}$

$$DPSE_i = [(r_i - r_{min})/(r_{max} - r_{min}) + (f_i - f_{min})/(f_{max} - f_{min})]/2$$

(3)

Figure 1. Four evolutionary states based on fuzzy classification.

In Equation (2), $g_i$ is the mean distance of particle $i$ to all the other particles. $g_{gbest}$ is the mean distance of the global best particle. And $n$ is the size of population, $D$ is the number of optimization variables. $fe$ is the evolutionary factor. In Equation (3), $r_i$ and $f_i$ represent the local location and local fitness of particles, respectively. DPSE function is the dynamic particle state estimation function.

As can be seen from Equation (3), particles with small local fitness may be the local or global optima particle, and particles with small local position are very near to the best position. Therefore, particles with small DPSE should perform a local search around the local or global optimal region, and particles with large particle fitness and large position will perform a global search to explore as many optima as possible during the optimization process.

According to DPSE, we can select an optimal role for a particle from the three roles. Convergence particles with small DPSE, which are around the best particle, will be used to perform further searches around the globally optimal region. The role of exploiting particles whose positions and fitness are both around the better particle or the local optima will help achieve further searches around the local optimal region. Exploring particles with large DPSE, whose positions are far away from the best particle or whose fitness is far worse than that of the best particle, will be used to explore the many optima as possible during the optimization process.

3.2. Adaptive control of parameters

In PSO, the value of individual study factor ($c_1$) makes individual particles go around their historical best position and helps escape local optima. The value of global study factor ($c_2$) helps the swarm converge to the current global best region. Therefore, particles with different roles should have different values of $c_1$ and $c_2$ to execute different searches including local search and global search. For the convergence particle, $C1 = C1min$,
Algorithm 1

1: Adaptive control of parameters
2: If evolutionary states is exploitation state or exploration state, then
3: Increase C1min, C1max; decrease C2min, C2max; Wmin, Wmax
4: Else decrease C1min, C1max; increase C2min, C2max; Wmin, Wmax; End if
5: For convergence particles: \( W = Wmin, C1 = C1min, C2 = C2max \)
6: For exploiting particles: \( W = Wmin, C1 = C1max, C2 = C2min \)
7: End for

Algorithm 2

1: Updating pbest based on CLPSO and ES
2: If ES is exploitation state or exploration state
3: Then \( Mp = Mpmax; \)
4: Else \( Mp = Mpmin; \)
5: End if
6: For \( i = 1: D \)
7: // \( D \) is the number of optimization variables
8: Then Generate two integers \( g1, g2 \) random uniformly distributed in \([1, n]\)
9: // \( n \) is the number of population
10: else \( Mp \)
11: Then \( pbest(\text{id}, j) = pbest(\text{g1}, j); \)
12: Else \( pbest(\text{id}, j) = pbest(\text{g2}, j); \)
13: End if
14: // \( \text{id} \) is the index of particles
15: End for

Algorithm 3

1: Procedure APSO-PDC
2: Set initialize parameters, initialize \( x, v, pbest, gbest \)
3: For iter = 1: Maximum
4: Use evolutionary factor to decide evolutionary state
5: Adaptive selection of particle roles
6: Updating pbest based on CLPSO and evolutionary state
7: Update \( x, v \)
8: Update pbest, gbest
9: End for
10: End Procedure

3.3. Population diversity control

Many studies have shown that some PSOs easily find local optima when optimizing complex problems. Ratnaweera et al. (2004) proposed that the main reason for the premature convergence of PSOs is that the population diversity is too simple. Hence, it is important to research population diversity control. To control population diversity, we introduce a new strategy based on the comprehensive learning strategy of CLPSO (Liang et al., 2006) and evolutionary states to update pbest. In PSO, the fitness of the particle is determined by the values of all independent variables, and a particle with some independent variable that gets a better value may have a low fitness due to the influence of other bad independent variables. In order to make better use of good independent variables, a new learning strategy was proposed in CLPSO. However, CLPSO focuses on increasing the population diversity, but results in a slower convergence. Hence, to accelerate convergence speed and keep population diversity, we introduce a control parameter \( Mp \), which is mutation probability related to the ES. In summary, we develop a procedure to update \( pbest \) based on CLPSO and ES. The procedure is described in Algorithm 2. For each particle, we use Algorithm 2 to update the individual optimal position of particles. When ES is convergence and jumping out, the algorithm executes a global search and it is easier to obtain local optima. Hence, a small \( Mp \) not only keeps a fast convergence speed but also avoids premature convergence. On the other hand, when the ES is exploitation and exploration, the algorithm achieves further search around the local optimal region and explores as many optima as possible; \( Mp \) equals \( Mpmax \) to increase the population diversity. \( Mpmax \) and \( Mpmn \) are the maximum and minimum mutation probability. The values of \( Mpmax \) and \( Mpmn \) are 0.4 and 0.1, respectively.

3.4. Flowchart of APSO-PDC

Based on the aforementioned analysis, the flowchart of APSO-RPDC is summarized in Algorithm 3.
4. Experimental results and analysis

4.1. Test functions and parameter settings

In order to verify the effectiveness of APSO-PDC, further experimental tests with benchmark functions were carried out to assess the performance of APSO-PDC and to compare APSO-PDC with some PSO variants. These test functions, which are shown in Table 1, including unimodal functions: F1–F6, multimodal functions: F7–F11.

The parameter settings for the PSO variants used are given in Table 2. To fairly compare the optimization performances of APSO-PDC and the other six PSOs and eliminate the error caused by randomness, seven PSOs were run independently 30 times on the 11 test problems. During the test process, the dimension of functions was 30, and the number of population was 40.

4.2. Comparisons of solution accuracy

In order to assess the solution accuracy of the seven PSOs, in this test, the stop criterion of the seven PSOs was that the algorithm reached the maximum number of function evaluations, that is 2e5. The performance of the seven PSOs was assessed using the mean fitness value (Fmean) and the standard deviation (Std). Table 3 shows the mean fitness value (Fmean) and standard deviation (Std) of the optimal value of the 11 test functions obtained by 30 independent runs. The values of Fmean and Std were average values of 30 independent runs. The best results from the seven PSOs are indicated by boldface in the table. Table 3 shows that APSO-PDC achieves the best performance on the majority of the problems, except function f4. Specifically, only APSO-PDC can successfully find a better optimum for f8 and f9. Hence, APSO-PDC exhibits the outstanding exploration ability of APSO-PDC. Figure 2 gives the convergence process of the seven different PSOs on some test functions. It can be seen that APSO-PDC apparently has a faster convergence speed than the other algorithms. Figure 2 shows that APSO-PDC not only increases the population diversity but also results in a faster convergence.

4.3. Comparisons on convergence speed and reliability

To compare the convergence speed and reliability of the seven PSOs, in this test, the stop criterion of the seven PSOs was to obtain acceptable values of the test functions, which are presented in Table 1. If the number of function

| Function | Formulae | Space | Optimum | Acceptance |
|----------|----------|-------|---------|------------|
| F1 Sphere | $\sum_{i=1}^{D} x_i^2$ | $[-100, 100]$ | 0 | 1E-6 |
| F2 Schwefel 2.22 | $\sum_{i=1}^{D} |x_i| + \prod_{i=1}^{D} |x_i|$ | $[-10, 10]$ | 0 | 1E-6 |
| F3 Schwefel 2.21 | $\max(|x_i|, 1 \leq i \leq D)$ | $[-100, 100]$ | 0 | 1E-2 |
| F4 Rosenbrock | $\sum_{i=1}^{D-1} (100(x_{i+1} - x_i)^2 + (x_i - 1)^2)$ | $[-2.048, 2.048]$ | 0 | 100 |
| F5 Step | $\sum_{i=1}^{D} (x_i + 0.5)^2$ | $[-100, 100]$ | 0 | 0 |
| F6 Quadric | $\sum_{i=1}^{D} (\sum_{j=1}^{D} x_j^2)^2$ | $[-1.281.28]$ | 0 | 0.01 |
| F7 Rastrigin | $\sum_{i=1}^{D} (x_i^2 - 10 \cos(2\pi x_i) + 10)$ | $[-5.12, 5.12]$ | 0 | 50 |
| F8 Noncontinuous Rastrigin | $\begin{cases} x_i & |x_i| < 0.5 \\ \text{round}(2x_i)/2 & |x_i| \geq 0.5 \end{cases}$ | $[-5.12, 5.12]$ | 0 | 50 |
| F9 Ackley | $-20 \exp(-0.2 \sqrt{\frac{1}{D} \sum_{i=1}^{D} x_i^2}) - \exp(1/D \sum_{i=1}^{D} \cos(2\pi x_i)) + 20 + e$ | $[-32, 32]$ | 0 | 0.01 |
| F10 Griewank | $\sum_{i=1}^{D} k_{\text{max}} \left(\sum_{j=1}^{n} (a^b \cos(2\pi b^{j}(x_i + 0.5))) - D \sum_{k=0}^{k_{\text{max}}} [a^b \cos(2\pi b^{0.5}j)]\right)$ | $[-600, 600]$ | 0 | 0.01 |
| F11 Weierstrass | $\prod_{i=1}^{D} \left(\sum_{k=0}^{k_{\text{max}}} [a^b \cos(2\pi b^{0.5}j)]\right)$ | $[-0.5, 0.5]$ | 0 | 0.01 |

$a = 0.5, b = 3, k_{\text{max}} = 20$
Table 3. Fmean and Std results on test functions.

| Function | PSO-W | MLP-PSO-STP | APSO | CLPSO | FIPS | FLPSO-QIW | APSO-PDC |
|----------|-------|-------------|------|-------|------|-----------|----------|
| f1       | 2.37E-158 | 1.22E-67 | 1.94E-15 | 7.31E-29 | 4.71E-12 | 7.31E-29 | 0.00E+00 |
| Std      | 0.00E+00 | 4.70E-67 | 1.31E-15 | 4.00E-28 | 1.81E-12 | 4.00E-28 | 0.00E+00 |
| f2       | 7.94E-84 | 7.27E-44 | 2.10E-10 | 1.86E-09 | 5.57E-08 | 1.86E-09 | 8.46E-208 |
| Std      | 2.67E-83 | 2.66E-43 | 5.72E-11 | 1.00E-08 | 1.42E-08 | 1.00E-08 | 1.27E-208 |
| f3       | 1.05E-52 | 1.71E-02 | 6.28E+00 | 2.57E+00 | 1.81E-08 | 2.57E+00 | 3.15E-87 |
| Std      | 2.91E-52 | 1.29E-02 | 7.69E+01 | 1.14E+00 | 1.14E+00 | 1.14E+00 | 1.47E-87 |
| f4       | 2.52E+01 | 2.12E+01 | 1.89E+01 | 2.64E+01 | 2.43E+01 | 2.64E+01 | 2.03E+01 |
| Std      | 1.84E-01 | 1.04E+01 | 2.21E+00 | 1.61E+01 | 4.80E+01 | 1.61E+01 | 1.53E-01 |
| f5       | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| Std      | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| f6       | 2.97E-04 | 6.74E-03 | 5.09E-03 | 1.56E-03 | 5.62E-03 | 1.56E-03 | 6.44E-16 |
| Std      | 9.68E-05 | 2.62E-03 | 1.24E-03 | 7.30E-04 | 1.40E-03 | 7.30E-04 | 3.19E-16 |
| f7       | 4.21E-01 | 4.72E-07 | 3.32E-02 | 7.34E-02 | 3.32E-02 | 7.34E-02 | 0.00E+00 |
| Std      | 1.11E+01 | 4.19E-07 | 1.82E-02 | 1.24E-02 | 1.82E-02 | 1.82E-02 | 0.00E+00 |
| f8       | 4.03E-15 | 9.12E-15 | 1.09E-08 | 3.79E-09 | 4.28E-07 | 3.79E-09 | 1.37E-18 |
| Std      | 1.23E-15 | 3.19E-15 | 2.39E-09 | 2.00E-08 | 8.11E-08 | 2.00E-08 | 3.18E-18 |
| f9       | 5.75E-04 | 1.00E-02 | 2.27E-10 | 0.00E+00 | 1.24E-05 | 0.00E+00 | 0.00E+00 |
| Std      | 2.21E-03 | 3.12E-02 | 4.21E-10 | 0.00E+00 | 4.13E-05 | 0.00E+00 | 0.00E+00 |
| f10      | 0.00E+00 | 7.37E-01 | 3.12E-09 | 1.40E+00 | 1.47E-04 | 1.40E+00 | 0.00E+00 |
| Std      | 0.00E+00 | 1.16E+00 | 1.20E-09 | 8.78E-01 | 2.77E-05 | 8.78E-01 | 0.00E+00 |

Figure 2. Convergence performance of the seven PSOs on test functions.

evaluation reached 2e5 and the acceptable values were not obtained, the test was considered to be failed. The performance of PSOs was assessed through the success rate (SR) and the number of function evaluation (MFE) to obtain acceptable test function values. Table 4 shows the mean value of the SR and the MFE of 30 independent runs. ‘NA’ means that no runs reached the acceptable value in Table 4. SR is an important performance parameter showing algorithm reliability, which is the ratio of the number of successful runs over the total number of runs. A successful run means that the algorithm obtains the acceptance value within the maximum number of fitness evaluations. Boldface in Table 4 indicates the best result among those obtained by all algorithms. As can be seen from Table 4, for all algorithms, APSO-PDC has the fastest convergence speed on all functions, and APSO ranks second. Moreover, APSO-PDC shows an excellent search reliability among all used PSOs, according to comparing the SR in Table 4. Table 4 also indicates that APSO-PDC has the highest SR at 99.52%, and
MLPSO-TP ranks second. This excellent performance on all functions proves that APSO-PDC can accelerate convergence speed and simultaneously avoid falling into the local optima.

### 4.4. Parameter sensitivity analysis of APSO-PDC

There are nine important parameters in APSO-PDC: $c_{\text{min}}, c_{\text{max}}, c_{\text{min}}$, $c_{\text{max}}$, $w_{\text{min}}$, $w_{\text{max}}$, $M_{\text{pmax}}$, $M_{\text{pmin}}$, and $R_d$. The default values of these parameters are 1.0, 1.8, 1.0, 1.8, 0.3, 0.7, 0.1, 0.2 and 0.05, respectively. To find out how these important parameters affect the performance of APSO-PDC, an experiment on the parameter sensitivity analysis of APSO-PDC was conducted on the 11 test function in 30 dimensions. To separately test the effect of a particular parameter, we only changed one parameter in one experiment and used the default values of the other parameters. Table 5 shows the mean fitness value ($F_{\text{mean}}$) of the 30 independent runs of three functions: unimodal functions $f_3$, multimodal functions $f_7$ and $f_9$. The best results of $F_{\text{mean}}$ are indicated by boldface in the table. As can be seen in Table 5, for $c_{\text{min}}, c_{\text{max}}, c_{\text{min}}$, $c_{\text{max}}$, $w_{\text{min}}$ and $w_{\text{max}}$, these parameter values are so reasonable that rational destabilization of these parameters only changes a little for the optima obtained by APSO-PDC, and these parameters may use the default value in real application. Hence, there are only three important parameters in APSO-PDC: the maximum and minimum of mutation probability ($M_{\text{pmin}}, M_{\text{pmax}}$), and the acceleration rate ($R_d$). For multimodal functions and unimodal functions, the increment of $M_{\text{pmax}}$ and the decrement of $M_{\text{pmin}}$ may produce better optima when the other parameters are default values. For the acceleration rate, the default value is the best for all functions.

### 5. Optimization design of tandem cascades

#### 5.1. Tandem cascades

To further compare the performance of the PSO variants and improve the design quality of tandem cascades, we applied the PSOs to create optimization systems to obtain the best tandem cascades. The geometric model of tandem cascades is shown in Figure 3. Table 5 displays the
5.2. CFD simulations

In this work, NUMECA/AUTOGRID5 was used to generate the structured grid of tandem blade. The steady (CFD) simulations are accomplished by NUMECA FINE/TURBO, which solves the conservative Reynolds averaged Navier-Stokes equations with Spalart–Allmaras turbulence model. In the CFD simulations of CDA and tandem cascades, the constant total temperature (288.15 K), and total temperature (101325 Pa) are uniformly imposed at the inlet. The flow angle and outlet mass flow were changed to obtain the desired incidence angle and inlet Mach number.

\[
loss = \frac{(p_1^* - p_2^*)}{(p_1^* - p_1)} \tag{4}
\]

\[
Pt = \frac{P_2}{P_1} \tag{5}
\]

In order to obtain the effect of grid cells of tandem cascades on the numerical simulation results, we investigated the effect of the number of grids on the CFD simulations results of the initial tandem cascade on the design point, namely at the inlet Mach number of 0.7 and the incidence angle of \(-2^\circ\). Table 7 presents the numerical results of the tandem cascade with different grids. The total pressure loss coefficient (Loss) and the static pressure ratio (Pt) of cascades are defined as follows.

In the above formula, \(P_1\) and \(P_2\) are the mass averaged static pressure at the inlet and outlet of tandem cascades, respectively. Similarly, \(p_1^*\) and \(p_2^*\) are the mass averaged total pressure at the inlet and outlet, respectively.

It can be seen that the Loss and Pt are almost unchanged between Grid 5 and Grid 6. Based on the accuracy and the time taken of CFD simulations, we chose Grid 6 (the grids of span-wise × circumferential × axial: 2 × 197 × 189) as the grid of CFD. The grids of CFD simulations for tandem blades are shown in Figure 4.

5.3. Optimization system

To further compare the performance of the PSO variants and improve the design quality of tandem cascades, we applied the PSOs to create optimization systems to accomplish the optimization design of tandem cascades. The flowchart of optimization system is shown in Figure 5. In the first step, the tandem cascades were parameterized. After parameterization, the tandem cascade was expressed by some parameterization variables, and the optimization variables can be obtained from the parameterization variables. Next, we used the PSO variants, indicated by IPSO, to generate initial individuals according to optimization variables. In the third step, the fitness is evaluated, which includes tandem cascades generation, grid generation and flow field calculation based on CFD simulations. In the final step, if the convergence
condition is satisfied, the optimization system is stopped. Otherwise the system will apply the IPSO to search the optimal tandem cascade.

6. Optimization design case one

6.1. Parameterization method and optimization variables

The goal of this optimization design was to investigate the influence of five configuration parameters, TR, CR, AO, PP and KBB, on the aerodynamic performance of tandem cascades on design and off design incidence angle. Thus, the parameterization of tandem cascades is accomplished by a program of tandem cascades generation. The optimization variables are the five configuration parameters of tandem cascades. Based on the related literature in the introduction and design experiences of tandem cascades, the upper and lower bound about optimization variables was selected; Table 8 shows the upper bound and lower bound of optimization variables.

6.2. Initial tandem cascades

An excellent CDA cascade, which is studied in Gelder, Schmidt, Suder, and Hathaway (1987), was selected as the original cascade. An initial tandem cascade was designed to satisfy the main design parameters of CDA, which is described in Gelder et al. (1987). The main design parameters of CDA are presented in Table 9. In order to design an initial tandem cascade with fine flow performance, based on the related literature in the introduction,

| Variables | TR  | CR  | PP  | AO  | KBB |
|------------|-----|-----|-----|-----|-----|
| Upper bound| 3   | 2.5 | 0.95| 0.2 | 10° |
| Lower bound| 0.7 | 0.5 | 0.5 | −0.2| −10°|

Table 8. The upper bound and lower bound of optimization variables.

Table 9. Main design parameters of CDA.

| Parameters       | Value |
|------------------|-------|
| Pitch (mm)       | 68.9  |
| Chord (mm)       | 127.14|
| Aspect ratio     | 1.998 |
| Inlet metal angle| 53°   |
| Outlet metal angle| −7°  |

Table 10. The values of five configuration parameters.

| TR     | CR | PP | AO | KBB |
|--------|----|----|----|-----|
| 1.50   | 1  | 0.8| 0   | −6° |

Figure 6. The convergence process of all algorithms.

the value of five configuration parameters of the initial tandem cascades are selected and presented in Table 10.

6.3. Optimization objectives and constraints

To improve the flow performance of the tandem cascade at large positive incidence angle, the optimization objective is to reduce the total pressure loss coefficient at the large positive incidence angle, the incidence angle of 3 degrees and the inlet Mach number of 0.7. The constraint is that the static pressure ratio of tandem cascades at the optimization point is not smaller than that of the initial tandem cascades.

6.4. Optimization results and discussion

To compare the improved PSO mentioned above, we respectively applied the seven improved PSOs to optimize the tandem cascade. In this optimization, the total number of optimization variables is five. In the optimization, for all algorithms, the number of population is 20, and the maximum iteration number is 200. The stop criterion of all algorithms is that the maximum iteration number is reached. Figure 6 shows the convergence process of all algorithms. Table 10 reveals the optima of the objective function obtained by all algorithms.
Table 11. The optima of the objective function.

| Algorithms | PSO-W | FIPS | CLPSO | APSO |
|------------|-------|------|-------|------|
| Optima     | 0.047 | 0.040| 0.047 | 0.045|
| Algorithms | FLPSO-QIW | MLPSO-STP | APSO-PDC |
| Optima     | 0.034 | 0.035 | 0.031 |

Table 12. Comparisons of aerodynamic performance.

| Cascades | ORG-TAN | OPT-TAN |
|----------|---------|---------|
| Loss     | 0.0681  | 0.031   |

Table 13. Comparisons of the five configuration parameters.

| Parameter | TR   | CR   | AO   | PP   | KBB   |
|-----------|------|------|------|------|-------|
| ORG-TAN   | 1.5  | 1    | 0    | 0.8  | −6°   |
| OPT-TAN   | 2.741| 0.745| −0.0195| 0.925| −5.9° |

When the total number of optimization variables is 5, Table 11 illustrates that APSO-PDC acquires the optimal value of objective function for all PSO variants, and FLPSO-QIW (Feedback learning particle swarm optimization) ranks two. Figure 6 shows that APSO-PDC has the fastest convergence speed and simultaneously controls the population diversity well. During the optimization, at the initial stage, APSO-PDC has good population diversity, and at the last stage, APSO-PDC has an excellent convergence speed. This excellent performance on real problems shows that APSO-PDC holds an appropriate balance between exploitation ability and exploration ability. Tables 12 and 13 respectively reveal the comparisons of aerodynamic performance and the five configuration parameters of the original tandem cascade and the optimal tandem cascade. To describe conveniently, in this paper, ORG-TAN and OPT-TAN respectively represent the original tandem cascade and the optimal tandem cascade obtained by APSO-PDC. After optimization, at the optimization point, namely at the large positive incidence angle, the total pressure loss coefficient of OPT-TAN is decreased by 55%.

Figure 7 shows the comparisons of total pressure loss coefficient of ORG-TAN and OPT-TAN. The total pressure loss of ORG-TAN and OPT-TAN at the incidence angle of 0° is almost unchanged after optimization. OPT-TAN significantly reduces the total pressure loss coefficient at positive incidence angle, but OPT-TAN also worsens the flow performance at negative incidence angle. The following section further discusses the total pressure loss and flow characteristics of tandem cascades at large negative and positive incidence angle.

6.4.1. At large positive incidence angle

Figures 8 and 9, respectively, show the comparisons of Mach number contours and entropy contours of ORG-TAN and OPT-TAN at the incidence angle of 3°, namely the optimization incidence angle. According to Figures 8 and 9, for the original tandem cascade (ORG-TAN), there is a large-scale separation on the trailing edge of front airfoil. The low-energy fluid and the wake of the trailing edge of the front airfoil are mixed with each other, resulting in a wide range of low-velocity zone and severe mixing loss. After optimization, OPT-TAN significantly reduced the separation losses and the mixing losses of the...
front airfoil of ORG-TAN. Based on Table 12, the main reason that OPT-TAN has a good flow performance is because OPT-TAN reduced the load of the front airfoil by increasing TR. For tandem cascades, the decrease of the gap area achieved by a raise of PP and decrease of AO can obviously improve the flow loss of tandem cascades at positive incidence angle.

6.4.2. At large negative incidence angle

Figures 10 and 11, respectively, show the comparisons of Mach number contours and entropy contours of ORG-TAN and OPT-TAN at the incidence angle of $-6^\circ$. Figure 7 illustrates that OPT-TAN worsens the flow performance at negative incidence angle. According to Figures 10 and 11, ORG-TAN has a small low-velocity zone and separation losses on the suction surface of the front airfoil, but OPT-TAN brings a great deal of low-energy fluid on the cascade passage after the fluid leaves the trailing edge of the front airfoil. Thus, OPT-TAN generates a great many high-entropy zones and low losses on the cascade passage. Based on the above analysis, the increase of TR may increase the total pressure loss of the front airfoil of tandem cascades at large negative incidence angle.
7. Optimization design case two

7.1. Parameterization method and optimization variables

The goal of this optimization design was to simultaneously investigate the influence of the shape parameters of tandem cascades and two configuration parameters, AO and PP, on the aerodynamic performance of tandem cascades on design and off design incidence angle. As seen in Figure 12, the parameterization of the camber line of the cascades and the parameterization of the thickness distribution of the cascades are accomplished by the NURBS (Non-Uniform Rational B-Splines) method with 10 control points. Compared with the traditional approach based on the polynomial method, the NURBS method can parameterize complex curves with fewer control points. In addition, the NURBS parameterization method can realize partial modification of complex curves and effectively reduce the number of optimization variables. In Figures 12 and 13, CH represents the chord of the cascades, and TM represents the relative thickness of the cascades. The parameterization of two configuration parameters, AO and PP, is accomplished by a program of tandem cascades generation.

![Figure 12. The parameterization of camber line.](image)

![Figure 13. The parameterization of thickness distribution.](image)

### Table 14. The main parameters of the tandem cascade.

| β11 (°) | β12 (°) | β21 (°) | β22 (°) | C/m | S/m | σ |
|---------|---------|---------|---------|-----|-----|---|
| 50      | 31      | 33      | 0       | 0.05| 0.02| 2.5|

During the optimization, the first and the last of the control points of the NURBS curves remain unchanged, and the other eight control points of NURBS of the camber line and the thickness distribution and the two configuration parameters AO and PP were selected as optimization variables of the tandem cascade. AO and PP vary within the range of \([-0.1, 0.1]\) and \([0.6, 0.95]\), respectively. In the process of NURBS parameterization, the irregular motion of control points may result in unreasonable tandem cascades. Therefore, in this study, the motion direction of control points is defined along the vertical direction of the NURBS curves, so that each of the control points can be described by a vertical coordinate. In the optimization, the total number of optimization variables is 34.

7.2. Initial tandem cascades

In this work, the optimization system was used to optimize a large-turning tandem cascade, which was obtained from the hub of an excellent tandem stator. The basic parameters of the initial tandem cascade are shown in Table 14. In Table 14, C, S, and σ are applied to express the chord length, the pitch width and the solidity of tandem cascades, respectively. The optimization point of this tandem cascade is the design point, the incidence angle of 2 degree and the inlet Mach number of 0.8.

7.3. Optimization objectives and constraints

To improve the flow performance of the tandem cascade on design point, the optimization objective is to reduce the total pressure loss coefficient at the incidence angle of 2 degree and the inlet Mach number of 0.8. The constraint is that the static pressure ratio of tandem cascades at optimization point is not smaller than that of initial tandem cascades.

7.4. Optimization results and discussion

To compare the improved PSO mentioned above, we also respectively apply the seven improved PSOs to optimize the tandem cascade. In the optimization, for all algorithms, the number of population is 40, and the maximum iteration number is 200. The stop criterion of all algorithms is that the maximum iteration number is
Table 15. The optimal values of the objective function.

| Algorithms | PSO-W | FIPS | CLPSO | APSO |
|------------|-------|------|-------|------|
| Optima     | 0.0424| 0.039| 0.042 | 0.041|

Table 16. Comparisons of aerodynamic performance.

| Cascades | ORG-TAN | OPT-TAN |
|----------|---------|---------|
| Loss     | 0.0598  | 0.029   |

reached. Table 15 reveals the optimal values of the objective function obtained by all algorithms. Figure 14 shows the convergence process of all algorithms.

When the total number of optimization variables is 34, Table 15 shows that APSO-PDC acquires the optimal value of objective function for all PSO variants, and MLPSO-STP ranks two. In addition, the objective function obtained by APSO-PDC is decreased by 14.7% compared with MLPSO-STP. Figure 14 shows that APSO-PDC has the fastest convergence speed and simultaneously controls the population diversity well. This excellent performance on real problems shows that APSO-PDC holds an appropriate balance between exploitation ability and exploration ability. Table 16 reveals the comparisons of aerodynamic performance of the original tandem cascade and the optimal tandem cascade. For conveniently description, in this paper, ORG-TAN and OPT-TAN, respectively, represent the original tandem cascade and the optimal tandem cascade obtained by APSO-PDC. After optimization, compared with ORG-TAN, at the optimization point, namely at the incidence angle of 2 degrees, the total pressure loss coefficient of OPT-TAN is decreased by 51%.

Figure 15 shows the changes of total pressure loss coefficient along with incidence angles of the original (ORG-TAN) and the optimal (OPT-TAN) tandem cascade, obtained by APSO-PDC. Figure 15 shows that at the inlet Mach number of 0.8, the total pressure loss coefficient of OPT-TAN was smaller than that of ORG-TAN at all incidence angles.

Figures 16 and 17, respectively, show the comparisons of the geometry and the thickness distribution of ORG-TAN and OPT-TAN. After optimization, the gap area of OPT-TAN is reduced by an increase of PP. For the front airfoil, the relative thickness was increased in the 5% to 50% relative position of axial chord length, and was decreased in the 50% to 80% relative position of axial chord length. For the aft airfoil, the relative thickness was decreased in the 20% to 80% relative position of axial chord length.
chord length. Further analysis of flow performance of tandem cascades is shown in the following paragraphs.

7.4.1. Design point
Figures 18 and 19, respectively, show the comparison of the Mach number contours and entropy contours of ORG-TAN and OPT-TAN at the incidence angle of 2 degrees. Figures 18 and 19 show that OPT-TAN has a smaller wake low-velocity region of the front airfoil, which obviously reduces the mixture loss of the tandem cascade. In addition, OPT-TAN obviously reduces the boundary layer thickness of the suction surface of the cascade, which reduces the friction loss of the boundary layer. The gap area of OPT-TAN was smaller than that of ORG-TAN, which decreased the flow fluid through the gap. The mixing loss generated by the mixture of the gap fluid with the mainstream fluid may be decreased because of the decrease of the gap fluid, which is also illustrated in Figure 19. In Figure 19, the high-entropy zone of the passage of OPT-TAN is smaller that of ORG-TAN, which demonstrates the total pressure loss of TAN-TAN is also smaller that of ORG-TAN.

7.4.2. Off design point
Figures 20 and 21, respectively, show the comparison of the Mach number contours and entropy contours of ORG-TAN and OPT-TAN at the incidence angle of $-2$ degrees. In Figure 20, the high Mach number region of the front airfoil of OPT-TAN is obviously reduced
after optimization, which decreases the shock loss. In addition, the low-velocity region on the suction surface of OPT-TAN was also decreased. Hence, the loss of OPT-TAN was obviously reduced, which is demonstrated in Figure 21. In Figure 21, in the whole cascade passage of OPT-TAN, the high-entropy zone of ORG-TAN is obviously bigger than that of OPT-TAN.

Figure 22 shows the comparison of the entropy contours of ORG-TAN and OPT-TAN at the incidence angle of 4 degrees. Figure 22 reveals the high-entropy zone of OPT-TAN is obviously smaller than that of ORG-TAN, which demonstrates that the flow performance of TAN-TAN is better than that of ORG-TAN.

8. Conclusion

To improve the flow performance of tandem cascades on design and off design incidence angle and increase the stable operation range, an optimization system for tandem cascades was developed based on an APSO-PDC. Firstly, APSO-PDC was proposed based on adaptive selection of particle roles and population diversity control. The adaptive selection of particle roles, which combines the ES and DPSE method, will sort the particles into three roles to let different particles execute different search tasks during the optimization process. The population diversity control, which combines the comprehensive learning strategy of CLPSO with ES to update the individual optimal position, strengthens the exploration ability and avoids falling into the local optima of APSO-PDC. The performance of APSO-PDC is comprehensively evaluated by 11 unimodal and multimodal functions. Compared with the other six PSOs, the results indicate that APSO-PDC has better performance in terms of algorithm accuracy and algorithm reliability.

In addition, APSO-PDC is validated by optimizing two large-turning tandem cascades, including low-dimension (five optimization variables) and high-dimension problems (34 optimization variables). Compared with the other six PSOs, the results demonstrate APSO-PDC has the fastest convergence speed and simultaneously controls the population diversity well.

After optimization design of tandem cascades, at the optimization point, the total pressure loss coefficient of the optimal cascade is decreased by 55% for the low-dimension case and 51% for the high-dimension case. An increase of TR can effectively improve the flow performance of tandem cascades at positive incidence angle, and may increase the total pressure loss of the front airfoil of tandem cascades at large negative incidence angle. The decrease of the gap area achieved by an increase of PP can obviously reduce the flow loss of tandem cascades at positive incidence angle.

The current study shows encouraging results and represents a foundation for further study. Planned future work will include multi-objective optimization design...
and analysis of tandem cascades, 3D optimization design of tandem blades, and experimental validation of tandem cascades and 3D blades.

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References

Bammert, K., & Beelte, H. (1980). Investigations of an axial flow compressor with tandem cascades. Journal of Engineering for Power, 102(4), 971. doi:10.1115/1.3230369

Bammert, K., & Staude, R. (1980). Optimization for rotor blades of tandem design for axial flow compressors. Journal of Engineering for Power, 102(2), 369. doi:10.1115/1.3230263

Bammert, K., & Staude, R. (1981). New features in the design of axial-flow compressors with tandem blades. Volume 2: Coal, biomass and alternative fuels; combustion and fuels; oil and gas applications; cycle innovations, doi:10.1115/81-gt-113

Brent, J. A., Cheatham, J. G., & Clemmons, D. R. (1972). Single-stage experimental evaluation of tandem-airfoil rotor and stator blading for compressors, part V-analysis and design of stages D and E. NASA CR-121008, doi:10.1125/72-gt-350

Chen, X. Y., & Chau, K. W. (2016). A hybrid double feed-forward neural network for suspended sediment load estimation. Water Resources Management, 30(7), 2179–2194. doi:10.1007/s11269-016-1281-2

Chen, Y., Peng, W., & Jian, M. (2007). Particle swarm optimization with recombination and dynamic linkage discovery. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 37(6), 1460–1470. doi:10.1109/tsmcb.2007.904019

Dehkharqani, A. S., Boroomand, M., & Eshraghi, H. (2014). A numerical investigation of loss coefficient variation in various incidence angles in tandem blades cascade. Volume 1: Advances in aerospace technology, doi:10.1109/imce2014-39881

Eshraghi, H., Boroomand, M., & Tousi, A. M. (2014). Design and analysis of a highly loaded tandem compressor stage. Volume 1: Advances in aerospace technology, doi:10.1109/imce2014-39750

Ezhilsabareesh, K., Rhee, S. H., & Samad, A. (2017). Shape optimization of a bidirectional impulse turbine via surrogate models. Engineering Applications of Computational Fluid Mechanics, 12(1), 1–12. doi:10.1080/19942060.2017.1330709

Galindo, J., Hoyas, S., Fajardo, P., & Navarro, R. (2014). Set-up analysis and optimization of CFD simulations for radial turbines. Engineering Applications of Computational Fluid Mechanics, 7(4), 441–460. doi:10.1080/19942060.2013.11015484

Gelder, T. F., Schmidt, J. F., Suder, K. L., & Hathaway, M. D. (1987). Design and performance of controlled-diffusion stator compared with original double-circular-arc stator. SAE technical paper series, doi:10.4271/871783

Gholami, V., Chau, K. W., Fadaee, F., Torkaman, J., & Ghaffari, A. (2015). Modeling of groundwater level fluctuations using dendrochronology in alluvial aquifers. Journal of Hydrology, 529, 1060–1069. doi:10.1016/j.jhydrol.2015.09.028

Hasegawa, H., Matsuoka, A., & Suga, S. (2003). Development of highly loaded fan with tandem cascade. AIST aerospace sciences meeting and exhibit, doi:10.2514/6.2003-1065

Heinrich, A., Tiedemann, C., & Peitsch, D. (2017). Experimental investigations of the aerodynamics of highly loaded tandem vanes in a high-speed stator cascade. Volume 2A: Turbomachinery, doi:10.1115/gt2017-63235

Hertel, C., Bode, C., Kožulovic, D., & Schneider, T. (2013). Investigations on aerodynamic loading limits of subsonic compressor tandem cascades: Midspan flow. Volume 1: Advances in aerodynamics, doi:10.1109/imce2013-64488

Hoeger, M., Baier, R.-D., Fischer, S., & Neudorfer, J. (2011). High turning compressor tandem cascade for high subsonic flows, part 1: Aerodynamic design. 47th AIAA/ASME/SAE/ASEE joint propulsion conference & exhibit, doi:10.2514/6.2011-5601

Ivo, M., Damir, V., & Zoran, M. (2016). 3D shape optimization of fan vanes for multiple operating regimes subject to efficiency and noise-related excellence criteria and constraints. Engineering Applications of Computational Fluid Mechanics, 10(1), 209–227. doi:10.1007/978-1-491910

Kennedy, J., & Eberhart, R. (1995). Particle swarm optimization. Proceedings of international conference on neural networks, doi:10.1109/95.489968

Kashkumarayana, B. (1985). Book review: Cascade aerodynamics, by J. P. Gostelow. AIAA Journal, 23(11), 1844–1844. doi:10.2514/3.48633

Li, X. (2004). Adaptively choosing neighbourhood bests using species in a particle swarm optimizer for multimodal function optimization. Lecture Notes in Computer Science, 3102, 105–116. doi:10.1007/978-3-540-24854-5_10

Li, C., Yang, S., & Nguyen, T. (2012). A self-learning particle swarm optimizer for global optimization problems. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 42(3), 627–646. doi:10.1109/tsmcb.2011.2171946

Liang, Y., Cheng, X., Li, Z., & Xiang, J. (2014). Robust multi-objective wing design optimization via CFD approximation model. Engineering Applications of Computational Fluid Mechanics, 5(2), 286–300. doi:10.1080/19942060.2011.11015371

Liang, J. J., Qin, A. K., Suganthan, P. N., & Baskar, S. (2006). Comprehensive learning particle swarm optimizer for global optimization of multimodal functions. IEEE Transactions on Evolutionary Computation, 10(3), 281–295. doi:10.1109/tvec.2005.857610

Lim, W. H., & Mat Isa, N. A. (2014). Particle swarm optimization with increasing topology connectivity. Engineering Applications of Artificial Intelligence, 27, 80–102. doi:10.1016/j.engappai.2013.09.011

Liu, Y., Qin, Z., & Li, Y. (2007). Particle swarm optimizer with adaptive species radius for multimodal function optimization. ICMIT 2007: Mechatronics, MEMS, and smart materials, doi:10.1117/12.784041
McGlumphy, J., Ng, W.-F., Wellborn, S. R., & Kempf, S. (2007). Numerical investigation of tandem airfoils for subsonic axial-flow compressor blades. Volume 15: Sustainable products and processes, doi:10.1109/imece2007-43929

Mendes, R., Kennedy, J., & Neves, J. (2004). The fully informed particle swarm: Simpler, maybe better. *IEEE Transactions on Evolutionary Computation, 8*(3), 204–210. doi:10.1109/tevc.2004.826074

Montes, M. A., Stutzle, T., Birattari, M., & Dorigo, M. (2009, October). Frankenstein’s PSO: A composite particle swarm optimization algorithm. *IEEE Transactions on Evolutionary Computation, 13*(5), 1120–1132. doi:10.1109/tevc.2009.2021465

Ratnaweera, A., Halgamuge, S. K., & Watson, H. C. (2004). Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients. *IEEE Transactions on Evolutionary Computation, 8*(3), 240–255. doi:10.1109/tevc.2004.826071

Sachmann, J., & Fottner, L. (1993). Highly loaded tandem compressor cascade with variable camber and stagger. Volume 3A: General, doi:10.1115/93-gt-235

Safikhani, H., Khalkhali, A., & Farajpoor, M. (2014). Pareto based multi-objective optimization of centrifugal pumps using CFD, neural networks and genetic algorithms. *Engineering Applications of Computational Fluid Mechanics, 8*(4), 37–48. doi:10.1080/19942060.2011.11015351

Saha, U. K., & Roy, B. (1996). On the application of variable camber blading in axial flow fans and compressors. ASME 1996 turbo Asia conference, doi:10.1115/96-ta-058

Saha, U. K., & Roy, B. (1997). Experimental investigations on tandem compressor cascade performance at low speeds. *Experimental Thermal and Fluid Science, 14*(3), 263–276. doi:10.1016/s0894-1777(96)00125-2

Schneider, T., & Kožulović, D. (2013). Flow characteristics of axial compressor tandem cascades at large off-design incidence angles. Volume 6A: Turbomachinery, doi:10.1115/igt2013-94708

Seifeedpari, P., Rafiee, S., Akram, A., Chau, K. W., & Pishgar-Komeh, S. H. (2016). Prophesying egg production based on energy consumption using multi-layered adaptive neural fuzzy inference system approach. *Computers and Electronics in Agriculture, 131*, 10–19. doi:10.1016/j.compag.2016.11.004

Shi, Y., & Eberhart, R. (1998). A modified particle swarm optimizer. 1998 IEEE international conference on evolutionary computation proceedings. IEEE world congress on computational intelligence, doi:10.1109/icec.1998.699146

Tang, Y., Wang, Z., & Fang, J. (2011). Feedback learning particle swarm optimization. *Applied Soft Computing, 11*(8), 4713–4725. doi:10.1016/j.asoc.2011.07.012

VandenBergh, F., & Engelbrecht, A. P. (2004). A cooperative approach to particle swarm optimization. *IEEE Transactions on Evolutionary Computation, 8*(3), 225–239. doi:10.1109/tevc.2004.826069

Wang, W. C., Xu, D. M., Chau, K. W., & Chen, S. (2013). Improved annual rainfall-runoff forecasting using pso-svm model based on emd. *Journal of Hydroinformatics, 15*(4), 1377–1390. doi:10.2166/hydro.2013.134

Wang, L., Yang, B., & Chen, Y. (2014). Improving particle swarm optimization using multi-layer searching strategy. *Information Sciences, 274*, 70–94. doi:10.1016/j.ins.2014.02.143

Xie, X., Zhang, W., & Yang, Z. (2002). Dissipative particle swarm optimization. *Proceedings of the Congress on Evolutionary Computation, 2*, 1456–1461. doi:10.1109/cecc.2002.1004457

Yang, S., & Li, C. (2010). A clustering particle swarm optimizer for locating and tracking multiple optima in dynamic environments. *IEEE Transactions on Evolutionary Computation, 14*(6), 959–974. doi:10.1109/tevc.2010.2046667

Yu, K., Wang, X., & Wang, Z. (2016). Multiple learning particle swarm optimization with space transformation perturbation and its application in ethylene cracking furnace optimization. *Knowledge-Based Systems, 96*, 156–170. doi:10.1016/j.knosys.2015.12.020

Zhan, Z. H., Zhang, J., Li, Y., & Chung, H. (2009). Adaptive particle swarm optimization. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 39*(6), 1362–1381. doi:10.1109/tsmcb.2009.2015956

Zhang, W. J., & Xie, X. F. (2003). DEPSO: Hybrid particle swarm with differential evolution operator. *SMC’03 conference proceedings. 2003 IEEE international conference on systems, man and cybernetics. Conference theme – system security and assurance, doi:10.1109/icsmc.2003.1244483

Zhang, S., Zhang, B., Tahsin, T., Leping, X., & Yu, L. (2017). Computational fluid dynamics-based hull form optimization using approximation method. *Engineering Applications of Computational Fluid Mechanics, 12*(1), 74–88. doi:10.1080/19942060.2017.1343751