Y(4626) as a molecular state from interaction $D_s^*\bar{D}_{s1}(2536) - D_s\bar{D}_{s1}(2536)$

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Received: date / Revised version: date

Abstract Recently, a new structure Y(4626) was reported by the Belle Collaboration in the process $e^+e^-\to D_s^*\bar{D}_{s1}(2536)^-$. In this work, we propose an assignment of the Y(4626) as a $D_s^*\bar{D}_{s1}(2536)$ molecular state, which decays into the $D_s^*\bar{D}_{s1}(2536)^-$ channel through a coupling between $D_s^*\bar{D}_{s1}(2536)$ and $D_s\bar{D}_{s1}(2536)$ channels. With the help of the heavy quark symmetry, the potential of the interaction $D_s^*\bar{D}_{s1}(2536) - D_s\bar{D}_{s1}(2536)$ is constructed within the one-boson-exchange model, and inserted into the quasipotential Bethe-Salpeter equation. The pole of obtained scattering amplitude is searched for in the complex plane, which corresponds to a molecular state from the interaction $D_s^*\bar{D}_{s1}(2536) - D_s\bar{D}_{s1}(2536)$. The results suggest that a pole is produced near the $D_s^*\bar{D}_{s1}(2536)$ threshold, which exhibits as a peak in the invariant mass spectrum of the $D_s\bar{D}_{s1}(2536)$ channel at about 4626 MeV. It obviously favors the Y(4265) as a $D_s^*\bar{D}_{s1}(2536)$ molecular state. In the same model, other molecular states from the interaction $D_s^*\bar{D}_{s1}(2536) - D_s\bar{D}_{s1}(2536)$ are also predicted, which can be checked in future experiments.

1 Introduction

Very recently, a new charmoniumlike state Y(4626) was reported as a structure in the process $e^+e^-\to D_s^*\bar{D}_{s1}(2536)^-$ based on a sample of 921.9 fb$^{-1}$ accumulated with Belle detector [1]. The Y(4626) has a mass of 4625.9$^{+6.2}_{-3.6}$ (stat.) ± 0.4(syst.) MeV and a width of 49.8$^{+13.9}_{-11.5}$ (stat.) ± 4.0(syst.) MeV. This state is very close to the Y(4630) observed in the process $e^+e^-\to \Lambda_c\bar{\Lambda}_c$ [2] and also near the Y(4660) observed in the process $e^+e^-\to \psi(3680)$ [3]. The new observation makes the situation in this energy region more complicated.

In Ref. [4], the Y(4626) was interpreted as a tetraquark by a calculation in the constituent quark model. In the other side, there exists many theoretical explanations of the Y(4660) and Y(4630). The Y(4660) was suggested to be interpreted as a $5^3S_1$ $cc$ state in the conventional quark model [5], a $f_0(980)$ bound state [6], or a tetraquark [7, 8]. Since the Y(4630) was observed near the $\Lambda_c\bar{\Lambda}_c$ threshold, it was proposed to be a $\Lambda_c\bar{\Lambda}_c$ molecular state, which is supported by the strong attraction through $\sigma$ and $\omega$ exchanges calculated in Refs. [9, 10]. Some authors also suggested that these two states are the same state [11, 12, 13]. If we only consider the mass of the newly observed Y(4626), all the interpretations of the Y(4660) and Y(4630) can be used to explain its origin and internal structure. To give further understanding about these states, the decay channels should be considered.

Since the Y(4626) was observed in the $D_s^*\bar{D}_{s1}$ channel (here and hereafter, the number 2536 in $D_s\bar{D}_{s1}$ will be omitted). It is natural to assume it as a bound state of an anticharm strange meson and a charm-antistrange meson. Different from the case of Y(4630) which was observed in the $\Lambda_c\bar{\Lambda}_c$ channel and close to the $\Lambda_c\bar{\Lambda}_c$ threshold also, the Y(4626) is much higher than the $D_s^*\bar{D}_{s1}$ threshold. In fact, the $D_s^*\bar{D}_{s1}$ threshold is about 4648 MeV, which is a little higher than the mass of Y(4266). Hence, the Y(4626) can be assigned as a candidate of the $D_s^*\bar{D}_{s1}$ molecular state. In the literature, such molecular states composed of an anticharm strange meson and a charm-antistrange meson have been discussed, such as the Y(4140) as a $D_s^*\bar{D}_{s1}$ state and Y(4274) as a $D_s\bar{D}_{s0}$ state [14, 16, 17]. Besides, the Y(4430) and Y(4390) were also suggested to be states form the $D_s^*\bar{D}_{s1}$ interaction [18]. If we consider the $D_s^*\bar{D}_{s1}$ threshold is about 4430 MeV, the mass gap between $D_s^*\bar{D}_{s1}$ and $D_s^*\bar{D}_{s1}$ thresholds is about 220 MeV, which is very close to the mass gap between the Y(4626) and the Y(4390).

Under such assumption, the observation of the Y(4626) in the $D_s^*\bar{D}_{s1}$ channel is also easy to understand. The vector...

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$D^*$ meson can be converted into pseudoscalar $D_s$ meson by exchanging an $\eta$ or $\phi$ meson, which leads to the coupling between $D_s^*D_{s1}$ and $D_sD_{s1}$ channels. Hence, in the current work, we will consider coupled-channel interaction $D_s^*D_{s1} - D_sD_{s1}$ in the calculation. With this interaction, we study the possible bound state from the $D_s^*D_{s1}$ interaction and its coupling with the $D_sD_{s1}$ channel, where the $Y(4626)$ was observed, in a quasipotential Bethe-Salpeter equation (qBSE) approach.

This work is organized as follows. After introduction, the reduction of potential kernel of coupled-channel interaction $D_s^*D_{s1} - D_sD_{s1}$ is presented, which is obtained with the help of the heavy quark symmetry. The relevant coupling constants are also discussed and given there. And the qBSE approach is introduced briefly. Then, the potential is inserted into the qBSE to search for a pole corresponding to the $Y(4626)$ and the numerical results will be given in Section 3. The molecular states with other quantum numbers are also predicted within the same model. Finally, the article ends with summary and discussion.

2 Theoretical frame

Since the $Y(4626)$ was observed in the process $e^+e^\to D_s^*D_{s1}$, it should carry quantum numbers $I(J^{PC}) = 0(1^-)$. First, we need construct the flavor functions for the $D_s^*D_{s1}$ system with definite $I(J^{PC})$. Since only isoscalar state can be formed from the $D_s^*D_{s1} - D_sD_{s1}$ system, we need not consider the isospin in construction of flavor function. The spin parity $J^P$ will be determined in the partial wave decomposition, which will be explained explicitly later. Here, we give the flavor function for a definite charge parity $C$ as

$$|D_s^0D_{s1}\rangle = \frac{1}{\sqrt{2}} \left[ D_s^+D_{s1}^0 + C D_s^0D_{s1}^+ \right],$$

$$|D_s^+D_{s1}\rangle = \frac{1}{\sqrt{2}} \left[ D_s^0D_{s1}^+ - C D_s^+D_{s1}^0 \right].$$

Here, we adopt the conventions $CD_s^0C^{-1} = D_s^0$, $CD_s^0C^{-1} = D_s^0$, and $CD_s^0C^{-1} = -D_s^0$, which are also adopted in the Lagrangians used in the current work. It is easy to check that the wave functions given above carry a charge parity $C$.

Besides the flavor function, to study the bound state from the interaction and coupling between different channels, we need construct the potential kernel within the one-boson-exchange model, which is widely used to describe the interaction between two hadrons. Because only charm strange mesons are involved, only $\phi$ and $\eta$ mesons are exchanged in the interaction considered in the current work. The relevant Lagrangians will be presented in the below.

2.1 Relevant Lagrangians

We need consider the couplings of light mesons to heavy-light anticharmed mesons in $H$ and $T$ doublets. In terms of heavy quark limit and chiral symmetry, the Lagrangians has been constructed in the literature as [19, 20, 21, 22, 23],

$$L = i\bar{g}(H_\mu\partial_\mu\gamma_5\gamma_a\gamma_5H) + i\bar{g}(H_\mu\partial_\mu\gamma_5\gamma_a\gamma_5H),$$

where $v = (1, 0)$, and the axial current is $J^A = \frac{1}{2}(\bar{q}\gamma^\mu\partial_\mu\xi - \bar{q}\xi\partial_\mu\gamma^\mu) = \frac{1}{2}\bar{q}\gamma^\mu\xi + \cdots$ with $\xi = \exp(i\bar{q}/f_\pi)$ and $f_\pi = 132$ MeV. Vector current $J^V = \frac{1}{2}(\bar{q}\gamma^\mu\partial_\mu\xi + (\bar{q}\xi\partial_\mu)\gamma^\mu = 0$ with $V^a_{ba} = \sqrt{2}g_{a0}^\mu \nabla_\mu$, and $F^\mu_\nu = \partial_\mu V^\nu - \partial_\nu V^\mu + [V^\mu, V^\nu]$. The $\mathbb{P}$ and $\mathbb{V}$ are the pseudoscalar and vector matrices as

$$\mathbb{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} & K^+ \\ \frac{\pi^-}{\sqrt{2}} & \frac{K^0}{\sqrt{2}} & \bar{\kappa}^0 \\ -\frac{\bar{K}^0}{\sqrt{2}} & -\frac{\bar{\kappa}^0}{\sqrt{2}} & \frac{\phi}{\sqrt{2}} \end{pmatrix},$$

$$\mathbb{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} & \frac{\rho^+}{\sqrt{2}} & K^+ \\ \frac{\omega^0}{\sqrt{2}} & \frac{\omega^+}{\sqrt{2}} & K^0 \\ -\frac{\bar{K}^0}{\sqrt{2}} & -\frac{\phi}{\sqrt{2}} & 0 \end{pmatrix},$$

which correspond to $(D^0, D^+, D^*_1)$ and $(D^0, D^*, D^*_1)$. To constrain the interaction, the values of coupling constants involved should be determined. The coupling constants for the $H$ doublet are relatively well determined in the literature with the heavy quark symmetry and available experimental data, i. e., $g = 0.59$, $\beta = 0.9$, $\lambda = 0.56$ with $g_V = 5.9$ and $f_\pi = 0.132$ GeV [24, 25, 26, 27]. For the couplings with the $T$ doublet involved, some coupling constants were also determined in the literature. Casalbuoni and coworkers extracted $(h_1 + h_2)/\Lambda_\chi = 0.55$ GeV$^{-1}$ for experimental information [22]. Falk and Luke obtained an approximate relation $k = g$ by a quark model calculation [27]. In Ref. [24], the $k$ are related to the coupling constant for the $\pi\Lambda$ vertex by comparing the results in hadronic and quark levels, and a relation was reached as $k/f_\pi = 3\sqrt{2}g_{NN}(1.07n_\Lambda)$ with $g_{NN}^2/4\pi = 13.60$, which leads to $k = 0.78$. Such value is close to the
approximation $k = g$. Here, we still use $k = g = 0.59$ as adopted in Ref. [24].

Analogously, the values of $\beta_2$ and $\lambda_2$ were determined also in Ref. [24] as $\beta_2 g_{NN} = -2 g_{NN}$ with $g_{NN}^2/4\pi = 0.84$, which leads to $\beta_2 = 1.10$, and $\lambda_2 g_{NN} = 3 g_{NN} + f_{NN}/(10 m_0)$ with $\beta_2 = 0.9$. In the current work, we will choose $\beta_2 = 1.1$ and $\lambda_2 = -0.6$. The coupling constants $\mu_1$ and $\xi_1$ are not well determined. As in Ref. [31], the authors make the approximation as $\mu_1 = 0$ and $\xi_1 = -0.04 \sim -0.25$ from the decay widths of the $K(1270)$ and the $K(1400)$ into $\rho\nu$ channel. In our calculation, we find the results are not sensitive to $\xi_1$. Hence, we adopt $\xi_1 = -0.1$. The values of the $\lambda_2$ and $\xi_1$ will be discussed explicitly later.

The $H$ and $T$ doublet fields are defined as

$$H_a = \frac{1}{2}\left[ P_{\mu\nu} \gamma^\mu \gamma^\nu - P_{\mu\nu} \gamma_5 \right], \quad H_b = \frac{1}{2}\left[ P_{\mu\nu} \gamma^\mu + P_{\mu\nu} \gamma_5 \right],$$

$$T_\mu = \frac{1}{2}\left[ P_{\mu\nu} \gamma_5 \gamma^\nu \gamma^\nu - \frac{3}{2} \gamma^\mu \gamma^\nu \gamma^\nu \right],$$

$$T_\mu = \frac{1}{2}\left[ P_{\mu\nu} \gamma_5 \gamma^\nu \gamma^\nu - \frac{3}{2} \gamma^\mu \gamma^\nu \gamma^\nu \right].$$

After expanding Eqs (3) and (5), the effective Lagrangians read,

$$\mathcal{L}_{P-P} = \frac{2g}{f_\pi} \epsilon_{\mu\nu\rho\sigma} \left( P_{\mu} P_{\nu} + P_{\mu} P_{\nu} \right) \gamma^\rho \gamma^\sigma,$$

$$\mathcal{L}_{P-P} = \frac{2g}{f_\pi} \epsilon_{\mu\nu\rho\sigma} \left( P_{\mu} P_{\nu} + P_{\mu} P_{\nu} \right) \gamma^\rho \gamma^\sigma,$$

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$$\mathcal{L}_{P-P} = \frac{2g}{f_\pi} \epsilon_{\mu\nu\rho\sigma} \left( P_{\mu} P_{\nu} + P_{\mu} P_{\nu} \right) \gamma^\rho \gamma^\sigma.$$
Table 1 The flavor factors for certain meson exchanges of certain interaction. The C is the charge parity.

| Interaction        | $t_0^l$ | $t_0^s$ | $t_3^l$ | $t_3^s$ |
|--------------------|---------|---------|---------|---------|
| $D_s\bar{D}_s \rightarrow D_s\bar{D}_s$ | 2/3     | 2/3C    | 1       | C       |
| $D_f\bar{D}_f \rightarrow D_f\bar{D}_f$ | 2/3     | -2/3C   | 1       | -C      |
| $D_s\bar{D}_s \rightarrow D_f\bar{D}_f$ | -2/3    | -2/3C   | -1      | -C      |
| $D_f\bar{D}_f \rightarrow D_f\bar{D}_f$ | -2/3    | 2/3C    | -1      | C       |

To calculate the scattering amplitude, the obtained potential can be decomposed into on spin parity $J^P$ as [36,37.38,39,40],

$$iM_{\pi\pi}^{l'}(p', p) = \bar{\rho}V_{\pi\pi}(p', p) + \sum_{\lambda'} \int d^3p' \frac{d^3\rho'}{(2\pi)^3}$$

$$\cdot \rho V_{\pi\pi}^{l'}(p', p')G_0(p')iM_{\pi\pi}^{l'}(p', p),$$

where the sum extends only over nonnegative helicity $\lambda'$ because only the independent helicity amplitudes are considered in the calculation. Here we adopt the covariant spectator approximation to reduce the Bethe-Salpeter equation in to the qBSE, which leads to a reduced propagator in the center-of-mass frame with $P = (W, 0)$ as [16,33,41,42]

$$G_0 = \frac{\delta^4(p''^2 - m_b^2)}{p''^2 - m_b^2}$$

$$= \frac{\delta^4(p''^2 - m_b^2)}{2E_b(p'')(W - E_b(p''))^2 - E_b^2(p'')}.$$  

As required by the spectator approximation, the heavier particle (marked with $h$, $D_s$ here) is on shell, which satisfies $p''^m = E_b(p'') = \sqrt{m_b^2 + p''^2}$. The $p''^m$ for the lighter particle (marked as $l$, $D_s$ and $D_f$ here) is then $W - E_b(p'')$. Here and hereafter, a definition $p = |p|$ will be adopted.

The dynamical mechanism of our model is introduced in the potential kernel $V$. After the partial wave decomposition, the potential obtained in Eq. (7) can be related to the $V''$ with fixed spin parity used in Eq. (9) as

$$V_{\pi\pi}^{l'}(p', p) = 2\pi \int d^2 \cos \theta |d_{l''}^l(\theta)V_{\pi\pi}(p', p)|,$$

where the factor $\eta = PP_P^lP_{-l}^l(-1)^J r_{l''}^l$ with $P$ and $J$ being parity and spin for system, $D_s^{l''}$ meson or $D_s$ meson. The initial and final relative momenta are chosen as $p = (0, 0, p)$ and $p'' = (p' \sin \theta, 0, p' \cos \theta)$. The $d_{l''}^l(\theta)$ is the Wigner d-matrix.

Now we need treat an integral equation, to avoid divergence, form factor for the off-shell particle is usually introduced. In the qBSE approach, we usually adopt an exponential form factor into the propagator as

$$G_0(p) = e^{-|m_b^2 - q^2|/A^2},$$

where $k_l$ and $m_l$ are the momentum and mass of the lighter one of two heavy mesons. For the exchanged meson, we also introduce an exponential form factor as $F(q^2) = e^{-|m_2^2 - q^2|/A^2}$ with $m_2$ and $q$ being the mass and momentum of the exchanged light meson. Here the cutoffs in all form factors are chosen as the same for simplification.

To solve the integral equation, we discrete the momenta $p$, $p'$ and $p''$ by the Gauss quadrature with wight $w(p_j)$ and have [16,41],

$$M_{jk} = V_{jk} + \sum_{j=0}^N V_{j}G_{j}M_{jk}.$$  

The above equation is obviously an matrix equation. The index for the helicity can also be included to do the calculation. The matrix element for $j = 0$ corresponds to on-shell case. The discreted propagator is written as

$$G_{j=0} = \frac{w(p_j'')w(p_j')}{(2\pi)^3}G_0(p_j'),$$

$$G_{j=0} = \frac{w(p_j'')w(p_j')}{32\pi^2 W} + \sum_{j=0}^N \left[ \frac{w(p_j)}{(2\pi)^3} \frac{p_j'^2}{2W(p_j'^2 - p_j^2)} \right],$$

where the $p_j$ is the on-shell momentum in the center of mass frame.

In the current work, we will present the effect of the $1^-$ bound state from the interaction on the invariant mass spectrum of the $D_s\bar{D}_s$ channel. Since, we do not consider the initial $e^+e^-$ collision explicitly, the invariant mass distribution is given approximately as

$$d\sigma/dW = C_{\pi\pi} M_{D_s\bar{D}_s} |V_{\pi\pi}|^2,$$

where $C$ is a scale constant and $p_f$ is momentum of the final state in the center of mass frame. The initial and final particles should be on-shell. The scattering amplitude is

$$M_{D_s\bar{D}_s - D_f\bar{D}_f} = M_{00} = \sum |(1 - VG)^{-1}|_{ij}V_{ij}.$$  

The pole can be searched by variation of $z$ to satisfy $|1 - V(z)G(z)| = 0$ where $z = iE_R + i\Gamma/2$ being the meson-baryon energy $W$ at the real axis.

3. The states from the interaction $D_f^\ast \bar{D}_s - D_s\bar{D}_s$

With above preparation, now, we can scan the scattering amplitude in the complex plane to search for the pole which corresponds to a molecular state. First, we check the effect of
the parameters on our results. As discussed in above section, the coupling constants \( \lambda_2 \) and \( \xi_1 \) are not well determined in the literature, and the cutoff \( \Lambda \) is the free parameter in our model. With a numerical calculation, it is found that the results are not sensitive to the \( \xi_1 \). Hence, in the following, we fix the parameter \( \xi_1 \) at a value of -0.1. Now we need to consider the different values of \( \lambda_2 \), which are in a range from 0 to -1.2 as discussed in the above section. In Fig. 2, we present the moving of the pole with quantum number \( J^{PC} = 1^{--} \), which can be related to the \( Y(4626) \) on which we focus in this work, in the complex plane with variation of the cutoff \( \Lambda \) and different values of \( \lambda_2 \).

With all values of \( \lambda_2 \) considered here, the pole is produced from the interaction \( D_s^* \bar{D}_{s1} - D_s \bar{D}_{s1} \) at a cutoff about 3.2 GeV. The variation of the value of \( \lambda_2 \) affects a little on the real part of the position of the pole, which corresponds to the mass of the molecular state. More obvious changes can be seen in the imaginary part of the pole, which corresponds to the decay width of the molecular state. The \( \lambda_2 = -0.4 \) leads to a very small width, smaller than 1 MeV with cutoff \( \Lambda \) from 3.25 to 3.45. With the increase of the \(|\lambda_2|\), the pole moves farther from the real axis, which indicates larger width. The results also suggested that the pole moves to real axis with the decrease of the cutoff \( \Lambda \). More calculations suggest that the pole will meet the real axis with \( \lambda_2 \) about -0.1 and leave the real axis again with continuous decrease (we do not give the results with such small \( \lambda_2 \) in figure to avoid mixing of the curves.). Such results suggest that the second term in the Lagrangian for the \( P_{1}P_{1}\phi \) vertex effects the \( D_s^* \bar{D}_{s1} \) interaction small while have larger effect on the coupling between the \( D_s^* \bar{D}_{s1} \) and \( D_s \bar{D}_{s1} \) channels especially with a small \( \lambda_2 \). If we choose a larger \(|\lambda_2|\), the different choices of \( \lambda_2 \) give qualitatively similar results. In the following, we choose a value of \( \lambda = -0.6 \), which corresponds a larger \( k_1 \) for the \( \rho NN \) coupling.

With the increase of the cutoff \( \Lambda \), the pole moves farther from both the threshold and the real axis. It reflects that both \( D_s^* \bar{D}_{s1} \) interaction and coupling between two channels are enhanced with a larger cutoff. The observed mass of the \( Y(4626) \) can be reproduced at cutoff \( \Lambda = 3.4 \) GeV, which favors that the \( Y(4626) \) state can be related to a \( D_s^* \bar{D}_{s1} \) molecular state with \( 1^{--} \). However, the width obtained from the current two-channel calculation is considerably smaller than the experimentally suggested value at Belle. Even with a larger \(|\lambda_2|\) of 1.5, the width \( \Gamma = -2 \text{Im} z = 20 \) MeV, which is still smaller than the experimental value, about 50 MeV [1]. It suggests that other decay channels including the three-body channels maybe provide considerable width to the \( Y(4626) \).

To give a more clearly image of the results, we present the explicit results for the pole from the interaction \( D_s^* \bar{D}_{s1} - D_s \bar{D}_{s1} \) at \( \Lambda = 3.4 \) GeV in Fig. 3. The pole can be found in the \( \epsilon = 4626 - 3.4i \) MeV which is very close to the experimental mass of the \( Y(4626) \). The peak corresponding to this state can be seen obviously in the \( D_s \bar{D}_{s1} \) channel.

The \( 1^{--} \) state from the interaction \( D_s^* \bar{D}_{s1} - D_s \bar{D}_{s1} \) at 4626 MeV can be related to the \( Y(4626) \) observed at Belle. In the same theoretical frame, we could predict other possible molecular state with other quantum numbers. Besides, the pole may be also found near the lower threshold for the \( D_s \bar{D}_{s1} \) channel. In the following Table 2, we will list these states.

For the \( J^P = 1^+ \) state, there exists both states with positive \( C = 1 \) and negative \( C = -1 \) charge parities. It is found that these two states appear at a cutoff \( \Lambda \) about 3.1
Table 2 The bound states from the interaction $D_s^+ D_{s1} - D_s D_{s1}$ at different cutoffs $\Lambda$. The cutoff $\Lambda$, and position of the pole $z$ are in units of GeV and MeV, respectively.

| $J^P$ | C = −1 | C = 1 |
|-------|--------|--------|
| | $\Lambda$ pole | $\Lambda$ pole |
| $D_s^+ D_{s1}(1^-)$ | 3.1 $4645 - 0.4i$ | 3.1 $4648 - 1.8i$ |
| | 3.2 $4642 - 0.8i$ | 3.15 $4645 - 0.4i$ |
| | 3.3 $4637 - 1.6i$ | 3.2 $4628 - 1.8i$ |
| | 3.4 $4626 - 3.4i$ | 3.25 $4581 - 4.5i$ |
| $D_s^+ D_{s1}(2^-)$ | 2.9 $4646 - 1.0i$ | 2.8 $4647 - 2.0i$ |
| | 2.95 $4643 - 1.6i$ | 2.85 $4643 - 6.6i$ |
| | 3.0 $4634 - 1.6i$ | 2.87 $4630 - 7.3i$ |
| | 3.05 $4615 - 1.0i$ | 2.9 $4598 - 5.5i$ |
| $D_s^+ D_{s1}(0^-)$ | 2.6 $4647$ | 2.4 $4646$ |
| | 2.65 $4643$ | 2.45 $4644$ |
| | 2.7 $4633$ | 2.5 $4639$ |
| | 2.75 $4614$ | 2.55 $4632$ |
| $D_s D_{s1}(1^-)$ | 2.55 $4496$ | 2.8 $4503$ |
| | 2.6 $4486$ | 2.85 $4487$ |
| | 2.65 $4478$ | 2.9 $4476$ |
| | 2.7 $4471$ | 2.95 $4468$ |

GeV. It suggests that the effect of cross diagram is relatively small especially near the threshold because as shown in Table 1, the $C$ only involves in the contribution from cross diagram. Based on this result, if the $1^-$ state can be related to the $Y(4626)$, the $1^{++}$ state is promising to be observed in experiment.

The $2^−$ and $1^−$ states can also be formed in S-wave from the $D_s^+ D_{s1}$ system. It is found that the bound states can be found at cutoff $\Lambda=2.9$ GeV with $2^−$ for both charge parities. Compared with the case with spin parity $1^−$, the binding energy increase very rapidly to larger than 50 MeV, which is beyond the scope of a molecular state. For the $0^−$ state, the bound states is still found for both positive and negative charge parities. Because the quantum number $0^−$ is forbidden for the $D_s D_{s1}$ system, no width is produced for these states in our two-channel calculation. Below the lower threshold, $1^−$ states are also produced with both charge parities.

4 Summary and discussion

Inspired by the newly observed $Y(4626)$, we study the possible $D_s^+ D_{s1}$ molecular state in a qBSE approach with the one-boson-exchange model. A two-channel calculation of the $D_s^+ D_{s1} - D_s D_{s1}$ interaction is performed to search for the poles produced from the interaction.

A state with quantum numbers $J^{PC} = 1^{−−}$ can be produced at about 4626 MeV near the $D_s^+ D_{s1}$ threshold, it can be related to the $Y(4626)$ observed at Belle recently. Such molecular state couples with the $D_s D_{s1}$ channel through exchange $\phi$ and $\eta$ mesons. Our result shows that a peak around 4626 MeV is produced in the $D_s^+ D_{s1}$ invariant mass spectrum, which corresponds to the pole from the interaction $D_s^+ D_{s1} - D_s D_{s1}$. Hence, it is consistent with the observation of the $Y(4626)$ at the $D_s D_{s1}$ channel. However, the width obtained theoretically is smaller than the experimental one with reasonable parameters, which suggest other channels may provide important contribution to the total width of the $Y(4626)$.

Besides the state corresponding to the $Y(4626)$, we also give the prediction of other possible molecular states from the $D_s^+ D_{s1} - D_s D_{s1}$ interaction. Based on our result, the most promising state is the $1^{++}$ state, which is different from the $1^{−−}$ state corresponding to the $Y(4626)$ only in the charge parity, and produced at almost the same cutoff as $1^{−−}$ state. The $0^−$ and $2^−$ states are also found near the $D_s^+ D_{s1}$ thresholds, which can not couple with $D_s D_{s1}$ channel in S wave, and should not have obvious effect on the $D_s D_{s1}$ invariant mass spectrum, where the $Y(4626)$ was observed. For the lower $D_s D_{s1}$ threshold, there are also $1^−$ states are produced, which can be seen as the partner of the $Y(4626)$.

Acknowledgement This project is supported by the National Natural Science Foundation of China (Grants No. 11675228, and No. 11375240), and the Fundamental Research Funds for the Central Universities.

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