Improved cloud computing model of internet pricing schemes based on Cobb-Douglas utility function

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Abstract. In this research, cloud computing model on internet pricing scheme based on Cobb-Douglas utility function was formed. The model formed is a combination of cloud computing model, consumer problem model by using Cobb-Douglas utility functions. These model considers the level of customer satisfaction on the sale of a service product and the service quality of the service provider company. The model used is solved by using LINGO 13.0 program to get the optimal solution result. Based on the calculation, the optimal solution was obtained for two types of cases, for the first case is 217.76 and for the second case is 206.97 which is on cloud computing model with usage based pricing scheme and flat fee, respectively. Based on the result of each cases, both Internet Service Provider (ISP) and internet users will get maximum benefit when ISP applied models with utility function compared to the original model.

1. Introduction
The internet comes from the word "inter" which means between. The internet is also often termed as a network or link, so that the internet can be interpreted as a collection of computer networks that are connected to each other. In accessing the internet a standard protocol is used, namely Transmission Control Protocol (TCP) or Internet Protocol (IP). According to [3] the protocol, it serves to provide a unique address and identity on each computer so that there is no error in sending data.

According to [6] in this era of globalization internet users do not know circles or ages. Internet usage has become a basic need for humans because almost all institutions and forms of business and human work require direct access to the internet. The high interest of consumers in internet usage makes ISPs as internet service providers who have a big task in providing better quality of service (QoS) to users or users in achieving the best quality information at an efficient cost. By establishing a pricing scheme and providing the right internet cost planning mechanism, it can benefit ISPs as service providers and users as internet users.

Information service providers are usually faced with the problem of determining the right model in offering information service products quickly and profitably. In a service, the utility function is needed and the determination of the right basic cost to generate large profits for service providers by adopting existing types of pricing schemes such as pricing schemes involving multi-class QoS network[5].

Cloud computing has been extensively applied to internet efficiency nowadays. This technology works in every environment in internet infrastructure. Unfortunately, less research focus on the optimization model of cloud computing due to its complexity to set up the variables and parameters and combine all constraints together with the objective function in one model.
So, the research basically attempt to formulate the optimization problem of cloud computing model of bandwidth efficiency into cloud computing model-based internet pricing scheme with the application of user sensitivity to service. This model is shown to be effectively raise the benefit for ISP since this models involve many parameters including the user’s sensitivity toward the service.

2. Research Method
The steps taken in this study are as follows:
1. Describe traffic data on a local server in Palembang. In this study type of traffic data, namely, lps traffic is used.
2. Defining parameters and decision variables used in cloud computing models in internet pricing schemes for homogeneous and heterogeneous consumers based on the Cobb-Douglas utility function.
3. Develop cloud computing models in internet pricing schemes for homogeneous and heterogeneous consumers based on the Cobb-Douglas utility function. For each model, the pricing scheme is based on:
   i. For flat fee pricing schemes, if \( PX \) and \( PY \) are both zero and \( P \) is positive.
   ii. Usage based pricing scheme, if \( PX \) and \( PY \) are positive and \( P \) is zero.
   iii. Two-part tariff pricing scheme, if , \( PX \) and \( PY \) are all positive.
4. Applying the cloud computing model to the optimal pricing scheme for local server traffic data.
5. Complete the solution of the cloud computing model in Step 4.
6. Analyze the results obtained based on Step 5.
7. Comparing models of cloud computing to internet pricing schemes for homogeneous and heterogeneous consumers based on the Cobb-Douglas utility function based on Step 6 to obtain optimal solutions.

2.1. Cloud Computing Model
According to [2] cloud computing is a technology that uses the internet and a remote central server to maintain or manage data and applications. Cloud computing can also be interpreted as a model that allows networks to be easily accessed from various locations via the internet where this model allows to collect computing resources such as networks, servers, storage, applications and services in one container.

According to the optimization model [4] then

\[
\text{Min} \sum_{i \in N} \sum_{j \in F_i} \alpha_{ij} p b_{ij} + \left( y_{ij} - \alpha_{ij} \right) p t_{ij}
\]

With constraints

\[
\sum_{k \in M} d_k x_{ijk} \leq c a p_{ij}, \forall i \in N, \forall j \in F_i
\]

\[
\sum_{i \in N} x_{ijk} = 1, \forall k \in M
\]

\[
\sum_{j \in F_i} y_{ij} \leq 1, \forall i \in N
\]

\[
\alpha_{ij} \leq y_{ij}, \forall i \in N, \forall j \in F_i
\]

with

\( d_k \) : Workload to-\( k \) from bandwidth usage

\( x_{ij} \) : The variable that states server \( i \) is used at frequency \( j \left( x_{ij} = 1 \right) \) or not \( \left( x_{ij} = 0 \right) \)

\( y_{ij} \) : The variable that states server \( i \) is used at frequency \( \left( y_{ij} = 1 \right) \) or not \( \left( y_{ij} = 0 \right) \)

\( \alpha_{ij} \) : Use of server \( i \) that works on frequency \( j \)
\( p_{ij} \): Bandwidth on the clock is not busy which works on server \( i \) at frequency \( j \) \((p_{ij} > 0)\)

\( pb_{ij} \): Bandwidth on the clock is busy which works on server \( i \) at frequency \( j \) \((pb_{ij} > 0)\)

\( cap_{ij} \): Maximum capacity of server \( i \) that works on frequency \( j \) \((cap_{ij} > 0)\)

\( M \): Number of workloads

\( N \): Number of servers

\( F_i \): Number of frequencies

2.2. Optimization of Consumer Problems

The following are the parameters proposed by [7]:

\( P \): Cost that consumers will incur to follow services

\( P_X \): The unit price set by the service provider during peak hours

\( P_Y \): The unit price set by the service provider in the hour is not busy

\( U_i(X_i,Y_i) \): The function of consumer utility \( i \) at the level of consumption at rush hour and non-busy hours

Decision Variable:

\( X_i \): Consumer consumption level \( i \) during services during rush hour

\( Y_i \): The level of consumer consumption \( i \) at service in the hour is not busy

\( Z_i \): The decision variable is worth of 0 if he does not want join

\( X_i \): The maximum consumption level of consumers \( i \) at services during rush hour

\( Y_i \): The maximum consumption level of consumers \( i \) at the service in the hour is not busy

Optimization of Consumer Problems:

\[ \max \theta = U_i(X_i,Y_i) - P_X X_i - P_Y Y_i - P Z_i \]  \( (2) \)

Subject to

\[ X_i \leq X_i Z_i \]  \( (2a) \)

\[ Y_i \leq Y_i Z_i \]  \( (2b) \)

\[ U_i(X_i,Y_i) - P_X X_i - P_Y Y_i - P Z_i \geq 0 \]  \( (2c) \)

\[ Z_i = \begin{cases} 1, & \text{if consumers choose to join the program} \\ 0, & \text{if consumers choose not to join the program} \end{cases} \]  \( (2d) \)

2.3. Utility Function Based on Cobb-Douglas

According to [1] general form of utility functions based on Cobb-Douglas:

\[ U(X,Y) = X^a Y^b; \ a > 0 \ , b > 0, a > b \]  \( (3) \)

Where \( X \) is the level of service usage during peak hours and \( Y \) is the level of service usage when the hours are not busy; with \( a \) and \( b \) are constants.

3. Result and Analysis

This chapter discuss the application of cloud computing models to flat fee, usage based, and two-part tariff pricing schemes by using the Cobb-Douglas utility function to obtain optimal internet pricing solutions.

3.1. Data Description

The data used in this study is secondary data obtained from one of the local servers in State Universities in Palembang. This data is taken within 1 month starting from February 1, 2018 to February 28, 2018. The data used is the lps traffic data which is divided into two sessions, namely the data during rush hour observed from 07.00 WIB to 16.59 WIB and data on no busy hours observed from 17.00 WIB to 06.59 WIB. There are 2 components in this data namely data sent (outbound) and data received (inbound) both of which are expressed in bytes per second.

The data used is in the form of bandwidth usage data on lps traffic with two types of data namely data received (inbound) and data sent (outbound) that are distinguished based on usage during busy
hour and when the non-busy hour. The data in the table has been changed from bytes per second to kilobytes per second.

3.2. Defining Parameters and Variables

After describing the data in lpse traffic the next step is to determine the parameters shown in Table 1 and the variables shown in Table 2.

| Parameters for Each Model |
|---------------------------|
| **Definition** |
| $p b_{ij}$ : Bandwidth during peak hours to work on the server $i$ on frequency $j$ |
| $p i_{ij}$ : Bandwidth of the clock is not busy which works on the server $i$ on the frequency $j$ |
| $d_k$ : Workload from bandwidth usage |
| $c a p_{ij}$ : The maximum capacity of the server $i$ that works on frequency $j$ |

| Variables for Each Model |
|--------------------------|
| **Definition** |
| $\alpha_{ij}$ : Use of the server $i$ that works on frequency $j$ |
| $\gamma_{ij}$ : Variables that state whether the server is still working on frequency $j$ ($\gamma_{ij} = 1$) or not($\gamma_{ij} = 0$) |
| $x_{ijk}$ : Variables that state whether the server is used at frequency $j$ ($x_{ijk} = 1$) or not($x_{ijk} = 0$) |

After determining the parameters and variables used in the cloud computing model, the next step is to determine the values of the parameters used in the cloud computing model. Table shown in more detail is in Table 3.

| Parameters for Data Traffic lpse |
|-----------------------------|
| **Value (kbps)** |
| $d_1$ : 107.64 |
| $d_2$ : 106.35 |
| $d_3$ : 104.86 |
| $d_4$ : 103.56 |
| $d_5$ : 103.10 |

In this study, there are two cases that are distinguished based on the usefulness of each model. Case I is a general model that works on the server while Case II is used to balance the use of the server so that it can be used when the workload exceeds the capacity because in Case I, it cannot be used for conditions when the workload exceeds the capacity.

1. Case I

In Case I the form of the model follows the form of the cloud computing model described in Chapter II, namely Model (1) through (1d).

2. Case II

In Case II the difference is that the conditions used are different

$$x_{ijk} \in [0,1]$$

In addition, in Constraints (2a), and (2b) are amended by Constraints (1) and (2) as follows

$$\sum_{k \in M} \sum_{j \in F_i} c a p_{ij} \cdot x_{ijk} \geq d_k \quad \forall k \in M \tag{4}$$

$$\sum_{k \in M} x_{ijk} \leq \gamma_{ij} \quad \forall i \in N, \forall j \in F_i \tag{5}$$

In this study, the condition with the frequency value($F_i$) for each server is also selected and $k = \{1,2,3,4,5\}$ is selected. Frequency($F_i$) is explained as follows.
\[
F_i = \begin{cases} 
(1, 2, 3), & \text{for } i = 1 \\
(4, 5, 6), & \text{for } i = 2 \\
(7, 8, 9), & \text{for } i = 3 \\
(10, 11, 12), & \text{for } i = 4 \\
(13, 14, 15), & \text{for } i = 5 \\
(16, 17, 18), & \text{for } i = 6 \\
(19, 20, 21), & \text{for } i = 7 \\
(22, 23, 24), & \text{for } i = 8 \\
(25, 26, 27), & \text{for } i = 9 
\end{cases}
\]

3.3. Parameter Value in Traffic

**Table 4a. Value of Parameters Used in Homogeneous Consumers**

| Parameters | Pricing Scheme | Flat Fee | Usage Based | Two-Part Tariff |
|------------|----------------|----------|-------------|-----------------|
| \(a\)      |                | 5        | 5           | 5               |
| \(b\)      |                | 4        | 4           | 4               |
| \(\bar{X}\) |                | 59.82    | 59.82       | 59.82           |
| \(X_m\)    |                | 38.21    | 38.21       | 38.21           |
| \(\bar{Y}\) |                | 71.41    | 71.41       | 71.41           |
| \(Y_m\)    |                | 55.18    | 55.18       | 55.18           |

\(a\) and \(b\) are constants, values of \(a\) and \(b\) are determined with terms \(a\) dan \(b\) positive integers and \(a > b\). The \(\bar{X}, X_m, \bar{Y}\) and \(Y_m\) parameters are bandwidth usage values with kilobyte per second (kbps).

**Table 4b. Parameter Value Used in Upper and Heterogeneous Consumers (High End / Low End)**

| Parameters | Pricing Scheme | Flat Fee | Usage Based | Two-Part Tariff |
|------------|----------------|----------|-------------|-----------------|
| \(a_1\)   |                | 5        | 5           | 5               |
| \(a_2\)   |                | 4        | 4           | 4               |
| \(b_1\)   |                | 3        | 3           | 3               |
| \(b_2\)   |                | 2        | 2           | 2               |
| \(\bar{X}_1\) |            | 59.82    | 59.82       | 59.82           |
| \(\bar{X}_2\) |            | 47.97    | 47.97       | 47.97           |
| \(X_m\)   |                | 38.21    | 38.21       | 38.21           |
| \(\bar{Y}_1\) |            | 71.41    | 71.41       | 71.41           |
| \(\bar{Y}_2\) |            | 64.70    | 64.70       | 64.70           |
| \(Y_m\)   |                | 55.18    | 55.18       | 55.18           |

\(a_1, a_2, b_1, \) and \(b_2\) are constants, values of \(a\) and \(b\) are determined with terms \(a_1, a_2, b_1, \) and \(b_2\) positive integers and \(a_1 > b_1, a_2 > b_2, a_3 > a_2\) and \(b_1 > b_2\). The \(\bar{X}_1, \bar{X}_2, X_m, \bar{Y}_1, \bar{Y}_2\), \(Y_m\) parameters are bandwidth usage values with kilobyte per second (kbps).
Table 4c. Parameter Value Used in Heterogeneous Consumers High Usage Levels and Low Demand / Low Demand Usage Rates

| Parameters | Flat Fee | Usage Based | Two-Part Tariff |
|------------|----------|-------------|-----------------|
| $a_1$      | 5        | 5           | 5               |
| $a_2$      | 5        | 5           | 5               |
| $b_1$      | 4        | 4           | 4               |
| $b_2$      | 4        | 4           | 4               |
| $\tilde{X}_1$ | 59.82   | 59.82       | 59.82           |
| $\tilde{X}_2$ | 47.97   | 47.97       | 47.97           |
| $X_m$      | 38.21    | 38.21       | 38.21           |
| $Y_1$      | 71.41    | 71.41       | 71.41           |
| $Y_2$      | 64.70    | 64.70       | 64.70           |
| $Y_m$      | 55.18    | 55.18       | 55.18           |

$a_1,a_2,b_1,$ and $b_2$ are constants, values of $a$ and $b$ are determined with terms $a_1,a_2,b_1,$ and $b_2$ positive integers and $a > b$, $a_1 = a_2 = a$, and $b_1 = b_2 = b$. The $\tilde{X}_1,\tilde{X}_2, X_m, Y_1, Y_2, Y_m$ parameters are bandwidth usage values with kilobyte per second (kbps).

Based on the above calculations, a more optimal solution is obtained from the cloud computing model in internet pricing schemes for homogeneous consumer types, heterogeneous high end / low end consumers, and heterogeneous high demand / low demand with the Cobb-Douglas utility function. In case I the solution was obtained in the scheme usage based pricing, while in case II it is obtained from a flat fee pricing scheme. This study was divided into 2 cases, for the first case the optimal results were 217.76 kbps, while in the second case the results were 206.97 kbps.

The optimal solution for both homogeneous consumers, heterogeneous high end/low end consumers, and heterogeneous consumers of high demand / low demand for each utility function used is that the Cobb-Douglas utility function. In Case I the solution is generated in the usage based pricing scheme. As for Case II, the optimal solution is generated in the flat fee pricing scheme. Objective value, infeasibility, and the number of iterations in the optimal solution generated in the two pricing schemes differ from each case and each scheme as already stated in the tables.

4. Conclusion
Based on the research that has been done, it can be concluded that the cloud computing model that uses utility functions in it produces a more optimal pricing scheme for ISP for each case.

5. Suggestion
For further research it is recommended to model cloud computing on other utility functions such as the utility bandwidth function and perfect complements and compare it to obtain maximum service quality and quality at optimal prices.

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