Modeling and Analysis of Electronic Pressure Regulator of Premixed CNG Engine

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Abstract: Electronic pressure regulator (EPR) is an important part of pressurized lean premixed CNG engine fuel supply system. To improve the precision of CNG engine Air-fuel ratio control, this paper build the physical and mathematical model of the gas state variation and the movable elements of two-stage typical regulator based on fluid mechanics and force analysis. The mathematical model is composed of differential equation of valve plate movement, gas-flow and intracavity continuity equation. The accuracy of simulation and simulation method was verified by comparing the simulation results with the experimental data. Pressure variations in decompression chamber affected by significant parameters of second-stage regulator were studied in detail, such as damping parameters, spring stiffness and mass of valve plate, etc. Combined with further experiments, it is proved that the mechanical structure can ensure the transient performance of the system.

1. Introduction
EPR is the key of pressurized lean premixed CNG engine fuel supply system[1]. Regulation of fuel outlet pressure is realized by the electromagnetic force of the voice coil motor in second stage regulator. It can accomplish the following functions: the first one is pressure regulation. It can reduce the pressure of CNG, making sure that CNG forms a flammable gas when it mixtures with air. Controlling the outlet pressure of CNG precisely and quickly, and achieving the precise control of air-fuel ratio of the engine. The additional one is flow regulation. The output flow of EPR varies with the change of engine operating condition. If vacuum of the intake manifold increases, the valves will open wider to meet the requirement of engine. Otherwise, it will decrease.

However, it is a problem that if the accuracy of EPR control could meet the need of engines. Until now, there have been no modeling research on EPR. Thus to improve the precision of CNG engine’s air-fuel ratio control, it is necessary to build the physical and mathematical model of the gas state changes and the movable elements of EPR in fuel supply system.

The modeling and analysis can provide theoretical basis for the structure design of EPR in reality. Structural optimization will become more convenience at any time. In addition, based on the physical and mathematical models built in this article, By proposing feedback and feed-forward compensation methods and other further control strategies, can achieve precise control of pressure.

2. Theory and hypothesis
EPR is mainly comprised by the internal regulator valve, diaphragm, springs and other structures. The state changes of high pressure CNG at Decompression chamber is consistent with the theoretical fluid mechanics, therefore it can be described as physical equations. The EPR model is built based on the related theoretical fundament of fluid dynamics in this paper[2].
### 2.1 Theoretical Basics

The decompression process of high pressure CNG is achieved by using EPR. It can control export pressure of fuel through complex internal structures such as internal spring, diaphragm, valve and voice coil motor etc. Operating condition of engine changes constantly [3], which requires fuel supply system to make a stable and rapid response, quick adjustment of EPR. The calculation method of fluid mechanics varies according to gas state during decompression process. The fluid motion is divided into two kinds of analysis methods: Lagrange method and Euler method. From the perspective of study, there is no need to analyze the status and movement of individual gas particles. Therefore this article analyzes gas state with Euler method. Based on the flow field full of liquid particles, Euler method is aimed at the movement and variation of the particle in flow field.

### 3. Physical Model Establishment

As shown in Figure 1, the structure of CNG and EPR studied in this article is a typical two-stage regulator. In order to reduce volume, the first-stage mechanical pressure regulator is installed inside the decompression chamber of the second one. EPR is an important part of the fuel supply system and its main function is to reduce high-pressure of CNG cylinder. After the pressure reducing at the second stage, the outlet pressure reaches targeting range, and achieves the precise control of fuel outlet pressure. CNG is considered an ideal gas when studying the gas state. The model is built mainly to reflect the state variation of gas in the regulator, response speed and stability of the whole system. Although each level of two-stage pressure regulator differs in structure, mechanical parts follow the same principle. It can realize pressure control in decompression chamber by spring, diaphragm and adjustment of valve. So according to the factors above, all levels of pressure regulating device are considered to have the same structure in physical model establishing process.

### 4. Mathematical Model Establishment

#### 4.1 Hypothesis

It mainly studies the motion process of the moving parts and the process of flow as well as the gas state changing (mainly concerning pressure) in this paper. The purposes of modeling are to analyze the effects of mechanical structure of pressure regulator on both steady and transient performance of outlet pressure and the major factors that affect pressure regulation performance. Based on these purposes, the CNG gas state in regulator is described with related theory of fluid mechanics. Then the force situation of intake valve plate is described by establishing the differential equation of motion. In order to highlight the key factors, some assumptions are put forward before establishing of mathematical model of EPR:

#### 4.2 First-stage Mechanical Pressure Regulator

Related symbols and units Involved in the mathematical formula are shown in Table 1.

| Symbol | Meaning                                      | Unit | Symbol               | Meaning                                      | Unit |
|--------|----------------------------------------------|------|----------------------|----------------------------------------------|------|
| $M_1$  | The mass of valve plate of first-stage regulator | kg   | $M_2$               | The mass of valve plate of second-stage regulator | kg   |
The head of the regulator is connected with atmosphere, and the pressure of the diaphragm is $P_0$; valve quality is $M$; valve area is $S_2$; the area of diaphragm in decompression chamber is $S_1$; the inlet pressure of the decompression chamber is $P_1$; the pressure of CNG in decompression chamber is $P_2$.

### 4.2.1. Differential Equation of Valve Plate Movement

According to the free-body diagram (as shown in Fig.3), the downward movement of the valve is set as positive direction, the differential equation of motion of the intake valve plate is built as follows:

$$M_1 \cdot \frac{d^2x}{dt^2} + K_1 \cdot x + c_1 \cdot \frac{dx}{dt} - p_1 \cdot s_2 - p_0 \cdot s_1 + p_2 \cdot (s_1 + s_2)$$  \hspace{1cm} (1)

### 4.2.2. Gas-flow Equation

The basic equations that fluid motion follows are the mass conservation equation, momentum equation and energy conservation equation.

It is assumed that the CNG flow in regulator is steady flow. Equations above can be simplified and integrated as follows:

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| $K_1$ | spring stiffness of first-stage regulator | N/mm | $K_2$ | spring stiffness of second-stage regulator | N/mm |
|------|------------------------------------------|------|------|-------------------------------------------|------|
| $x$  | Displacement of valve plate              | mm   | $p_0$| atmospheric pressure                      | MPa  |
| $P_1$| inlet pressure of first-stage regulator  | MPa  | $P_2$| decompression chamber pressure of first-stage regulator | MPa  |
| $P_3$| decompression chamber pressure of second-stage regulator | MPa  | $P_4$| pressurization pressure                   | MPa  |
| $\rho$| gas density                              | kg/m$^3$ | $k$ | Gas adiabatic coefficient                 |      |
| $R$  | gas constant                              | J / mol | $a$ | Velocity of sound                         | m / s |
| $Ma$ | mach number                               |       | $V$ | Volume of decompression chamber           | mm$^3$ |
| $C_1$| damping                                   |       | $C_2$| discharge coefficient                     |      |
| $\dot{m}$ | gas flow                                  | kg / s  | $\dot{m}_1$ | Inlet flow rate of decompression chamber | kg / s |
| $\dot{m}_2$ | Outlet flow rate of decompression chamber | kg / s | $d_0$ | Valve radius of decompression chamber entrance | mm |
| $A$  | sectional area of decompression chamber   | mm$^2$ | $T$  | Kelvin temperature                        | K    |
| $S_1$| Diaphragm area of first-stage regulator   | mm$^2$ | $S_2$ | Valve area of first-stage regulator        | mm$^2$ |
| $S_3$| Diaphragm area of second-stage regulator  | mm$^2$ | $S_4$ | Valve area of second-stage regulator       | mm$^2$ |
| $P_c$| initial pressure of different level regulator | MPa |      |                                            |      |
Mass conservation equation

\[ \rho \cdot A \cdot \dot{V} = m = \text{constant} \quad (2) \]

Momentum equation

\[ \dot{V} \cdot dV + \frac{1}{\rho} \frac{dp}{dx} = 0 \quad (3) \]

Energy conservation equation

\[ h_b = h + \frac{1}{2} \dot{V}^2 = \text{constant} \quad (4) \]

The momentum equation is in form of ordinary differential yet. The momentum equation is in form of ordinary differential yet. The Clapeyron equation of state for further integration is listed here:

\[ p = \rho \cdot R \cdot T \quad (5) \]

\[ h = \frac{k}{k-1} \cdot R \cdot T = \frac{k}{k-1} \frac{p}{\rho} \quad (6) \]

Bring formula 6 into equation 4, execute differential operation we can obtain:

\[ \frac{k}{k-1} \frac{dp}{\rho} - \frac{k}{k-1} \frac{p}{\rho} \cdot d\rho + \dot{V} \cdot dV = 0 \quad (7) \]

Bring formula 3 into equation 7, simplify the result and do integral operation, we can obtain:

\[ \frac{p}{\rho^k} = \text{constant} \quad (8) \]

Integrate subsonic and sonic mass flow equation\[4\], and bring in \( f(p_0, p) \), which is related with valve ports shape, the expression can be obtained as follows:

\[ f (p_0, p) = \begin{cases} \frac{2k}{R(k-1)} \left[ \left( \frac{p}{p_0} \right)^{\frac{1}{k-1}} - \left( \frac{p_0}{p} \right)^{\frac{1}{k-1}} \right] \frac{p}{p_0} > \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} & \frac{p}{p_0} > \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} \\
\frac{k}{R(k+1)} \left( \frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \frac{p}{p_0} \leq \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} \end{cases} \quad (9) \]

\[ \frac{p}{p_0} > \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}}, \quad \text{Fluid flow is subsonic flow.} \]

\[ \frac{p}{p_0} \leq \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}}, \quad \text{Fluid flow is sonic flow.} \]

Mass flow rate can be expressed as:

\[ m = A \cdot \frac{p_0}{\sqrt{R \cdot T_0}} \cdot f (p_0, p) \quad (10) \]

Valve port flow rate can be expressed as:

\[ m_1 = \pi \cdot A \cdot \frac{p_1}{\sqrt{R \cdot T_0}} \cdot f (p_1, p_2) \quad (11) \]

The outlet flow rate of decompression chamber can be expressed as:

\[ m_2 = A \cdot \frac{p_2}{\sqrt{R \cdot T_0}} \cdot f (p_2, p_3) \quad (12) \]

4.2.3. Intracavity Equation of Continuity

According to the ideal gas state equation:

\[ m = \frac{p \cdot V}{R \cdot T} \quad (13) \]

For dynamic equation, take the derivative of time and get:

\[ \frac{dm}{dt} = \frac{V}{R \cdot T} \frac{dp}{dt} + \frac{p}{R \cdot T} \frac{dV}{dt} \quad (\text{constant temperature}) \quad (14) \]

When \( x = 0, \ V = V_o = \text{constant} \),
When $x \neq 0$, \( V = V_0 \cdot S_1 \cdot x \):
\[
\frac{d \dot{m}}{dt} = \frac{V}{R \cdot T} \cdot \frac{dp}{dt} - \frac{P \cdot S_1}{R \cdot T} \cdot \frac{dS}{dt}
\]
(15)

Continuity equation inside decompression chamber:
\[
\frac{d \dot{m}}{dt} = m_1 - m_2 = \frac{V}{R \cdot T} \cdot \frac{dp_3}{dt} - \frac{P_3 \cdot S_3}{R \cdot T} \cdot \frac{dS}{dt}
\]
(16)

4.3 The second-stage PER.

Mathematical model of the second-stage regulator is similar to first-stage mechanical regulator, the mass flow equations and continuity equations are the same[5]. However, the diaphragm is communicating with pressurized air instead of the atmosphere above. The supporting point is not at the center of the level. So there exists slight difference in differential equation of valve motion. Specific equations are list as follows:

Differential equation of motion of the intake valve plate:
\[
M \cdot \frac{dx}{dt^2} + K_2 \cdot x + c_1 \cdot \frac{dx}{dt} = p_2 \cdot s_2 + P_3 \cdot S_3 - \frac{1}{l_2} (p_1 - P_3) \cdot s_2 = 0
\]
(17)

Equation of Mass flow rate:
\[
\dot{m} = \frac{A \cdot p_3}{\sqrt{R \cdot T_0}} \cdot f(p_2, p_1)
\]
(18)

Equation of Valve port flow rate:
\[
\dot{m} = \frac{\pi \cdot d^2 \cdot x \cdot p_3}{\sqrt{R \cdot T_0}} \cdot f(p_2, p_3)
\]
(19)

Continuity equation inside decompression chamber:
\[
\frac{d \dot{m}}{dt} = m_1 - m_2 = \frac{V}{R \cdot T} \cdot \frac{dp_2}{dt} - \frac{P_2 \cdot S_2}{R \cdot T} \cdot \frac{dS}{dt}
\]
(20)

5. Analysis and discussion

Based on the mathematical model of EPR, a simulation model is built in Simulink. According to the structure of experimental sample, the parameters of model are determined and aiming at the pressure change in the decompression chamber and the change of the parameters of the secondary regulator, a simulation calculation was carried out.

As shown in Fig.2 and Fig.3, the relationship between pressure difference (the pressure after pressurization - outlet pressure of the fuel) and pressurization has good linear relationship regardless of electromagnetic force of voice coil motor.
The simulation results show the correctness of model and rationality of the calculation formula. This detailed simulation analyzes the influence of different parameters on the pressure in decompression chamber of second-stage pressure regulator.

The performance of pressure regulator can be shown through the simulation by step change response. The simulation results also show that it takes about 0.3s for the first-stage mechanical pressure regulator to maintain outlet pressure stabilized after fuel supply valve opens. As the fuel supply valve open in advance and pressure fluctuation in fuel bottle is slight, it is considered that the first-stage mechanical pressure regulator can reduce and stabilize the pressure effectively and can provide the stable inlet fuel pressure for the second-stage EPR.

6. Conclusion
The CNG EPR studied in the paper is the typical two-stage pressure regulating structure, and the mathematical model is built based on the analysis of the physical model of EPR. The parameters of model are determined according to the structure of experimental sample for EPR, and then The pressure relationship between fuel outlet pressure and air pressure after pressurization is analyzed.
changes and the influence of parameters changing of the first-stage mechanical pressure regulating and the internal second-stage electric pressure regulating chamber.

The simulation results show that: it takes about 0.3s for the first-stage mechanical pressure regulator to ensure the outlet pressure stability after fuel supply valve opens; it takes about 0.2s for the second-stage EPR to ensure the outlet pressure stability. When the engine is working properly and there is an inertia of the engine operating conditions and the pipe volume, a step change in air pressure after pressurization is not possible because the fuel supply valve is opened in advance and the pressure fluctuations in the fuel bottle are slight.

Combined with further experiments, it is proved that the mechanical structure of EPR can ensure the transient performance of the system. The modeling and simulation also provide the theoretical support for the design of regulator. Based on the physical and mathematical models build in this article, precisely control of pressure can be realized through putting forward further control strategies, such as feedback and feed-forward compensation methods.

Reference

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