Single hole dynamics in dimerized spin liquids

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Abstract

The dynamics of a single hole in quantum antiferromagnets is influenced by magnetic fluctuations. In the present work we consider two situations. The first one corresponds to a single hole in the two leg $t-J$ spin ladder. In this case the wave function renormalization is relatively small and the quasiparticle residue of the $S = 1/2$ state remains close to unity. However at large $t/J$ there are higher spin ($S = 3/2, 5/2, ..$) bound states of the hole with the magnetic excitations, and therefore there is a crossover from quasiparticles with $S = 1/2$ to quasiparticles with higher spin.

The second situation corresponds to a single hole in two coupled antiferromagnetic planes very close to the point of antiferromagnetic instability. In this case the hole wave function renormalization is very strong and the quasiparticle residue vanishes at the point of instability.

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I. INTRODUCTION

Quantum spin ladders have recently become a subject of considerable interest, mainly because of their relevance to a variety of quasi one-dimensional materials. In addition they represent, on the theoretical side, an important step towards understanding of the two-dimensional CuO$_2$ planes in the cuprate superconductors. The properties of a single hole in the two leg ladder were investigated numerically by Troyer, Tsunetsugu, and Rice and Haas and Dagotto using exact diagonalizations. More recently Eder approached the same problem using both exact diagonalization and the Self Consistent Born Approximation. Two most interesting properties have been observed in these works: The first one is the shift of the hole dispersion minimum to the point close to $k \approx \pi/2$. The second property is the closeness of the quasiparticle residue to unity even at large $t/J$. Such behavior of the residue is very much different from the $t-J$ model on the 2D square lattice.

The closeness of the quasiparticle residue to unity indicates that a simple variational approach and even perturbation theory must be applicable to the problem, and therefore we use these techniques in the present work. This allows us to find the $S = 1/2$ quasiparticle dispersion analytically. The dispersion is in an agreement with previous computations. We demonstrate also that there are higher spin ($S = 3/2, 5/2, \ldots$) bound states of the hole with the magnetic excitations. At large $t/J$ these bound states lie below the $S = 1/2$ state, and this means that they become the real quasiparticles of the system. So our conclusion is that the spectrum of the single hole states on the ladder is very rich. This complexity is somewhat similar to the complexity of the spin-wave spectrum.

In spite of the complex spectrum, the structure of each individual state on the ladder is relatively simple. It means that the number of significant components in the wave function remains of the order of unity. The purpose of the second part of this work is to consider a spin liquid with really complex structure of the hole wave function. We demonstrate that this situation is realized in a 2D spin liquid close to the point of antiferromagnetic instability. Specifically we consider two coupled antiferromagnetic planes. As the point of instability is approached the number of components in the hole wave function approaches infinity and the quasiparticle residue vanishes.

II. $T-J$ LADDER

We consider the standard $t-J$ model on a two-leg ladder. The Hamiltonian reads

$$H = H_t + H_J,$$

$$H_t = - \sum_{\langle i,j \rangle \sigma} \left( t \ c_{i\sigma}^\dagger c_{j\sigma}^\vphantom{\dagger} + H.c. \right),$$

$$H_J = \sum_{\langle i,j \rangle} J_{ij} \ S_i \cdot S_j,$$

where $c_{i\sigma}^\dagger$ is the creation operator of an electron with spin $\sigma$ at site $i$. As usual double occupancy is forbidden. $\langle ij \rangle$ represents nearest neighbor sites. The spin operator is $S_i = \frac{1}{2} c_{i\alpha}^\dagger \sigma_{\alpha\beta} c_{i\beta}$. For bonds along the legs we choose $J_{ij} = J$, and for bonds along the rungs $J_{ij} = J_\perp$. We are mostly interested in the “isotropic” case: $J = J_\perp$, but it is convenient
to keep \( J_\perp \geq J \) as an independent parameter. Following Chubukov\textsuperscript{8}, Sachdev and Bhatt\textsuperscript{9} and Gopalan, Rice, and Sigrist\textsuperscript{10} we consider the ground state consisting of inter-chain spin singlets \(| \text{GS} \rangle = |1, s\rangle |2, s\rangle |3, s\rangle \ldots \), where \(| n, s \rangle = \frac{1}{\sqrt{2}} (| \uparrow \rangle_i | \downarrow \rangle_j - | \downarrow \rangle_i | \uparrow \rangle_j) \) as a starting point of our approach. Since each singlet can be excited into a triplet state it is natural to introduce a creation operator \( t_{\alpha n}^\dagger \) for this excitation:

\[
| n, \alpha \rangle = t_{\alpha n}^\dagger | n, s \rangle, \quad \alpha = x, y, z.
\] (2)

The interaction of the triplets between themselves is described by the Hamiltonian (see, e.g. Refs.\textsuperscript{10},\textsuperscript{6})

\[
H_{sw} = \sum_{n, \alpha, \beta} \left\{ J_\perp t_{\alpha n}^\dagger t_{\alpha n} + \frac{J}{2} \left( t_{\alpha n}^\dagger t_{\alpha n+1} + t_{\alpha n}^\dagger t_{\alpha n+1}^\dagger + \text{h.c.} \right) \right\}.
\] (3)

In addition, the hard core constraint

\[
t_{\alpha n}^\dagger t_{\beta n}^\dagger = 0
\] (4)

has to be enforced on every site.\textsuperscript{11} The interactions (3,4) lead to rearrangement of the unperturbed ground state and nontrivial spectrum of the spin-wave excitations.\textsuperscript{10}\textsuperscript{6}\textsuperscript{7}\textsuperscript{11}

If we remove one electron from rung \( n \) a single hole state on this rung is created. Let us denote by \( a_{n,m,\sigma}^\dagger \) the creation operator of a hole with spin \( \sigma \) on rung \( n \), where the index \( m = 1, 2 \) represents the legs of the ladder. This operator is defined by the equation

\[
a_{n,m,\sigma}^\dagger | n, s \rangle = c_{n,\bar{m},\sigma}^\dagger | 0 \rangle,
\] (5)

where \( \bar{m} = 2 \) if \( m = 1 \), and \( \bar{m} = 1 \) if \( m = 2 \). This is a fermionic operator which satisfies the usual fermionic anticommutation relations. The hole on the rung \( n \) interacts with the triplet excitations on its nearest sites. This interaction can be easily found by calculating all possible matrix elements of the initial Hamiltonian (1) (see Ref.\textsuperscript{4}). The result is

\[
H_h = -t \sum_{n,\sigma} (a_{n+1,2,\sigma}^\dagger a_{n,1,\sigma} + H.c.) + \frac{t}{\sqrt{2}} \sum_{n,m,\sigma} (a_{n+1,m,\sigma}^\dagger a_{n,m,\sigma} + H.c.)
\]

\[
+ \sum_{n,\sigma,\pm} \left( t_n^\dagger \left[ t_n S_{n+1,n} + \frac{J}{2} S_{n+1,n+1} \right] + H.c. \right)
\]

\[
+ \frac{t}{\sqrt{2}} \sum_{n,m,\sigma} \left( t_{n+1}^\dagger t_n a_{n,m,\sigma}^\dagger a_{n+1,m,\sigma} + H.c. \right)
\]

\[- t \sum_{n} \left( i S_{n+1,n} \left[ t_n^\dagger \times t_{n+1} \right] + H.c. \right) + \frac{J}{\sqrt{2}} \sum_{n,\pm} \left( i S_{n,n} \left[ t_{n+1}^\dagger \times t_{n+1} \right] + H.c. \right).
\] (6)

The index \( \pm \) denotes summation over nearest rungs. Following Ref.\textsuperscript{11} we have introduced the notations

\[
S_{l,n} = \frac{1}{2} \sum_{m,\alpha,\beta} (-1)^m a_{l,m,\alpha}^\dagger \bar{\sigma}_{\alpha,\beta} a_{n,m,\beta},
\] (7)

\[
S_{l,n} = \frac{1}{2} \sum_{m,\alpha,\beta} a_{l,m,\alpha}^\dagger \bar{\sigma}_{\alpha,\beta} a_{n,m,\beta}.
\]
where $\vec{\sigma}$ is the vector of Pauli matrices. In addition we have to impose the constraint that a triplet excitation and a hole can not coexist on the same rung

$$t_{an}^\dagger a_{n,m,\sigma}^\dagger = 0.$$  
(8)

The effective Hamiltonian given by Eqs.(3) and (8) ($H_{eff} = H_{sw} + H_h$) combined with the constraints (4) and (8) represents an exact mapping of the initial Hamiltonian (1) in the sector with a single hole. One can work with $H_{eff}$ by using standard diagram techniques in momentum representation. The constraint (8) can be taken into account in a mean field fashion or by introducing a fictitious (ghost) interaction similarly to the way it was done for spin waves in the paper. However fortunately the problem can be resolved in a simpler way. Because of the sizable spin-wave gap there is no real long-range dynamics on the ladder. Therefore the problem can be solved in the coordinate representation by constructing a trial wave function that explicitly obeys the constraints (4) and (8). First consider an isolated rung with a hole. Say this is the rung $n$. The energy of this state with respect to the undoped system is

$$\epsilon_0 = \frac{3}{4} J_\perp + \frac{3}{4} J_\perp^2.$$  
(9)

The origin of the first term in $\epsilon_0$ is obvious: one antiferromagnetic link is destroyed. The origin of the second term is also simple. There are quantum fluctuations in the spin liquid state: virtual excitations of the triplet pairs created by the term $t_{\alpha n}^\dagger t_{\alpha n+1}^\dagger$ in the Hamiltonian (3). This gives vacuum energy $3(\frac{J/2}{2})^2$ per link, where the coefficient 3 is due to the number of possible polarizations. The presence of a hole on rung $n$ suppresses fluctuations at the links $n-1, n$ and $n, n+1$ and this gives the second term in (9). Because of the hybridization along the rung the stationary states are symmetric and antisymmetric with hopping energies $-t$ and $+t$ respectively. Thus for an isolated rung one finds

$$a_{n,+\sigma}^\dagger = a_{n,1,\sigma}^\dagger + a_{n,2,\sigma}^\dagger \frac{\sqrt{2}}{2}, \quad \epsilon = \epsilon_0 - t,$$  
(10)

$$a_{n,-\sigma}^\dagger = a_{n,1,\sigma}^\dagger - a_{n,2,\sigma}^\dagger \frac{\sqrt{2}}{2}, \quad \epsilon = \epsilon_0 + t.$$  

Consider now a hole hopping from rung $n$ to $n+1$: $a_{n,+\sigma}^\dagger \rightarrow a_{n+1,+\sigma}^\dagger$. Naively, according to the Hamiltonian (3) the effective matrix element for this process is $t/2$. However this does not take into account quantum fluctuations of the spin-wave vacuum. By taking into account these fluctuations we obtain the following initial wave function

$$|i\rangle = \left[(1 - \frac{3}{2}\mu^2) + \mu t_{\alpha,n-2\alpha,n-1}^\dagger \right] a_{n,+\dagger}^\dagger \left[(1 - 3\mu^2) + \mu t_{\beta,n+1,\beta,n+1}^\dagger + \mu t_{\beta,n+2,\beta,n+3}^\dagger \right] |s\rangle.$$  
(11)

Here we explicitly present the part of the wave function corresponding to the rungs from $n-2$ to $n+3$. The coefficient $\mu$ describes the spin-wave quantum fluctuations and according to the Hamiltonian (3) it is equal to

$$\mu \approx \frac{J}{4J_\perp}.$$  
(12)
The final state is

\[ |f\rangle = \left[(1 - 3\mu^2) + \mu t_{\alpha,n-2}^\dagger t_{\alpha,n-1}^\dagger + \mu t_{\alpha,n-1}^\dagger t_{\alpha,n}^\dagger \right] a_{n+1,\alpha}^\dagger \left[(1 - 3\frac{\mu^2}{2}) + \mu t_{\beta,n+2}^\dagger t_{\beta,n+3}^\dagger \right] |s\rangle. \quad (13)\]

Calculating the matrix element of the Hamiltonian \( H_t \), Eq. (1) we find

\[ t^{(1)} = \frac{t}{2}\rho^2, \quad \rho = 1 - \frac{3}{2}\mu^2; \quad (14)\]

which agrees with that obtained in the Ref. \( ^4 \).

Let us consider now the possibility of a virtual spin-wave excitation

\[ a_{n,+}^\dagger \rightarrow t_{n,\alpha}^\dagger a_{n+1,-\sigma}^\dagger \]

which gives a second order correction to the hole energy. Note that the symmetry of the operators \( a^\dagger \) in the admixed components is negative. The Hamiltonian (1) is symmetric under permutation of the ladder legs. Therefore all excitations can be classified according to this symmetry. The operator \( t^\dagger \) has \((-\)\)-symmetry, and therefore the \( a^\dagger \) which appears together with \( t^\dagger \) must be of the \((-\)\)-symmetry as well. Below in the notations of the quantum states we will omit the symmetry index. The energy difference corresponding to the excitation (13) is

\[ \Delta E = \left(\epsilon_0 + t + J_\perp - \frac{1}{3}J + \frac{3J^2}{8J_\perp}\right) - (\epsilon_0 - t) = 2t + J_\perp - \frac{1}{3}J + \frac{3J^2}{8J_\perp}. \quad (16)\]

The admixture (13) first of all shifts the on site energy. Using the wave function (11) we find

\[ \delta\epsilon_n = -2 \frac{|\langle t_{n+1} a_{n+1}^\dagger |H_t| a_n^\dagger \rangle|^2}{\Delta E} - 2 \frac{|\langle t_{n+1} a_n^\dagger |H_t| a_n^\dagger \rangle|^2}{\Delta E} = -\frac{3\rho^2 t^2}{2\Delta E} - \frac{3\rho^2 J^2}{8\Delta E}. \quad (17)\]

The admixture (13) generates also an effective one step hopping

\[ \delta t^{(1)} = -\frac{\langle a_{n+1}^\dagger |H_H| t_{n+1}^\dagger a_{n+1}^\dagger \rangle \langle t_{n+1}^\dagger a_{n+1}^\dagger |H_J| a_n^\dagger \rangle - \langle a_{n+1}^\dagger |H_H| t_{n+1}^\dagger a_n^\dagger \rangle \langle t_{n+1}^\dagger a_n^\dagger |H_J| a_{n+1}^\dagger \rangle}{\Delta E} \]

\[ = -\frac{3\rho^2 t J}{4\Delta E}. \quad (18)\]

There is one more contribution to the effective one step hopping which arises due to the spin-wave quantum fluctuations: Let the hole first reside on rung \( n \), then a spin-wave quantum fluctuation arises on rungs \( n+1 \) and \( n+2 \), after that the hole hops to rung \( n+1 \) annihilating the spin-wave excitation \( t_{n+1}^\dagger \) and after that the spin wave excitation \( t_{n+2}^\dagger \) is annihilated because of the interaction \( H_J \) with the hole \( a_{n+1}^\dagger \). There also exists the time reversed mechanism which gives an equal contribution. This gives

\[ \delta t^{(1)} = -2\mu \frac{\langle a_{n+1}^\dagger |H_J| t_{n+2}^\dagger a_{n+1}^\dagger \rangle \langle t_{n+2}^\dagger a_{n+1}^\dagger |H_J| t_{n+1}^\dagger a_{n+1}^\dagger \rangle}{\Delta E} = \frac{3\mu \rho t J}{4\Delta E}. \quad (19)\]

A similar mechanism generates a two step hopping: Initially the hole is on rung \( n \), then a spin-wave quantum fluctuation arises on rungs \( n+1 \) and \( n+2 \), and after that the hole
annihilates this virtual excitation moving to the rung \( n + 2 \). Again the time reversion of this mechanism leads to an equal contribution.

\[
\langle \epsilon \rangle^{(2)}_{\text{eff}} = -2\mu \frac{\langle a_{n+2}^\dagger | H_t^\dagger | a_{n+1}^\dagger a_{n+1}^\dagger | H_t | a_{n+2}^\dagger a_{n+1}^\dagger \rangle}{\Delta E} = -\frac{3 \mu \rho t^2}{2 \Delta E}.
\] (20)

Adding together (17), (18), (19), (20) we obtain the total second order correction to the hole energy

\[
\delta \epsilon^{(2)}_q = -\frac{3}{2 \Delta E} \left( \rho^2 t^2 + \frac{1}{4} \rho^2 J^2 + t J (\rho^2 + \rho \mu) \cos q + 2 \rho \mu t^2 \cos 2q \right).
\] (21)

This formula, combined with (10) and (14) gives a reasonable description of the hole dispersion because the energy interval \( \Delta E \) is large enough.

The accuracy of the calculation can be improved by replacing the perturbation theory by a variational method. The zeroth approximation hole wave function and the corresponding dispersion are of the form

\[
\psi_q^\dagger = \frac{1}{\sqrt{N}} \sum_n e^{i q n} a_{n,+}^\dagger
\]

\[
\epsilon_1 = \epsilon_0 - t + \rho^2 t \cos q.
\]

The admixed components represent the hole and the spin wave combined into total angular momentum \( \frac{1}{2} \) with z-projection \( \pm \frac{1}{2} \)

\[
|+\rangle = \frac{1}{\sqrt{N}} \sum_n e^{i q n} \left( t_n^\dagger a_{n+1}^\dagger \right)_{\pm \frac{1}{2}},
\]

\[
|-\rangle = \frac{1}{\sqrt{N}} \sum_n e^{i q n} \left( t_n^\dagger a_{n-1}^\dagger \right)_{\pm \frac{1}{2}},
\]

\[
\epsilon_2 = \epsilon_1 + \Delta E.
\] (23)

The mixing matrix elements are

\[
V = \langle + | H | \psi_q^\dagger \rangle = \langle \psi_q^\dagger | H | - \rangle = -\frac{\sqrt{3}}{2} \rho t - \frac{\sqrt{3}}{4} \rho J e^{-i q} - \frac{\sqrt{3}}{2} \mu t e^{-2i q}.
\] (24)

Diagonalization of the matrix gives the quasiparticle dispersion and residue (weight of the component (22) in the total wave function)

\[
\epsilon_q = \epsilon_1 + \Delta E/2 - \sqrt{(\Delta E/2)^2 + 2 |V|^{2}}
\]

\[
Z = \frac{1}{2} \left( 1 + \frac{1}{\sqrt{1 + 8 |V|^{2}/(\Delta E)^2}} \right).
\] (25)

Expansion of this solution in powers of \((V/\Delta E)^2\) gives the perturbation theory result (21).

We are interested in the “isotropic” ladder, \( J_\perp = J \). In this case from perturbation theory (12), \( \mu = -0.25 \). Comparison with more accurate computations\[4\] shows that really \( \mu = -0.3 \). The difference is small, but nevertheless in further numerical estimations...
we will use $\mu = -0.3$ which effectively accounts for some higher order corrections. At $t \ll J$ Eq. (25) gives

$$
\epsilon_q \approx 1.28J - 0.7t + 0.24t \cos q,
$$

\[ Z = 0.85 \]

The plots of the dispersion for $t/J = 1, 2, 3$ are given in Figures 1,2,3. They agree with results of numerical computations. The quasiparticle residue is close to unity. For example for $t/J = 2$ the residue $Z \approx 0.9$ at $q = 0$, $Z \approx 0.83$ at $q = \pi/2$, and $Z \approx 0.97$ at $q = \pi$.

To avoid misunderstanding we note that this residue can not be directly compared with “photoemission spectra” because of two reasons. First, the “photoemission” operator is different from $a^\dagger$ (see the discussion in the Ref. 4). The second reason is that the quasiparticle we consider is well defined only near bottom of the dispersion. At small $q$ the quasiparticle energy is large and therefore it is unstable with respect to real emission of a spin wave which we neglect in our consideration.

### III. Higher Spin Hole States in the Ladder

It is clear that at very large $t$ the hole tends to order all the spins ferromagnetically. This is the Nagaoka effect, as discussed in the Ref. 2. The same effect produces higher spin bound states of the hole with the spin wave. Let us consider first the state with $S = 3/2$. The simplest ansatz for this state is

$$
\psi_q = \frac{1}{\sqrt{N}} \sum_n e^{i q n} \frac{1}{\sqrt{2}} (t_{+1,n}^\dagger a_{n+1,+1,\uparrow}^\dagger + a_{n,-1,\uparrow}^\dagger t_{+1,n+1}^\dagger),
$$

where $t_{+1}^\dagger = \frac{1}{\sqrt{2}}(-t_{x}^\dagger - it_{y}^\dagger)$ corresponds to spin projection $+1$. The energy corresponding to this wave function is

$$
\epsilon_0^{(3/2)} = \epsilon_0 - 2t + J_\perp + \frac{J}{4} + \frac{3J^2}{8J_\perp}
$$

The first term is the single hole magnetic energy, the second term is due to hybridization, $J_\perp$ is the excitation energy of the triplet, $J/4$ is due to the antiferromagnetic interaction between the hole and the triplet, and the last term is due to the blocking of quantum fluctuations. The state (27) can propagate via intermediate virtual configurations: first the triplet hops and then the hole catches up. This gives an effective hopping matrix element

$$
t_{efl}^{(3/2)} = \frac{-tJ/4}{t + 3J^2/(8J_\perp) - J/4}.
$$

Thus the dispersion of the spin $3/2$ state is

$$
\epsilon_q^{(3/2)} = \epsilon_q^{(3/2)} + 2t_{efl}^{(3/2)} \cos q.
$$

The plots of (30) for $t/J = 1, 2, 3$ are given in Figures 1,2,3.

Finally, let us consider a ferromagnetic bag with $S = 5/2$. 

\[ \psi = \frac{1}{\sqrt{3}} \left( t_{+1,n}^\dagger t_{+1,n+1}^\dagger a_{n+2,+1}^\dagger + t_{+1,n}^\dagger a_{n+1,+1}^\dagger t_{+1,n+2}^\dagger + t_{+1,n}^\dagger t_{+1,n+1}^\dagger a_{n+2,+1}^\dagger \right) \]  

The energy of this state (neglecting the dispersion) is given by

\[ \epsilon^{(5/2)} = \epsilon_0 - (1 + \sqrt{2})t + 2J_\perp + \frac{3J}{4} + \frac{3J^2}{4J_\perp} \]  

For \( t/J = 2, 3 \) it is plotted in Figures 2 and 3.

Now let us discuss our results. In the accepted approximation the bag state with spin \( S = 3/2 \) at arbitrary \( t/J \) has higher energy than the bottom of the quasiparticle \( (S = 1/2) \) dispersion. Therefore the \( S = 3/2 \) state is unstable with respect to decay into a quasiparticle and a spin wave. It exists only as a resonance in the quasiparticle - spin wave scattering. The \( S = 1/2 \) quasiparticle is itself unstable in the vicinity of \( q = 0 \). It can decay into a quasiparticle and a spin wave with higher \( q \), or into an \( S = 3/2 \) state and a spin wave.

The energy of the \( S = 5/2 \) bound state decreases with \( t \) faster than that of the \( S = 1/2 \) state. Therefore sooner or later the \( S = 5/2 \) bag state will become the lowest one and thus the true quasiparticle of the system. Comparing (25) with (32) we find the crossover point \( t_c \approx 7J \). As \( t \) increases further, in accordance with Nagaoka’s theorem, quasiparticles with higher spins will appear.

**IV. SINGLE HOLE IN TWO COUPLED ANTIFERROMAGNETIC PLANES**

The Hamiltonian in this case is similar to (1). The only difference is that instead of the two-leg ladder we consider now two coupled planes (we assume that both planes form square lattices). The coupling \( J \) is the antiferromagnetic exchange in the plane, and \( J_\perp \) is the antiferromagnetic exchange between the planes. It is known that at \( J_\perp > J_c \) (\( J_c \approx 2.5J \)) the system is in a disordered dimerized spin liquid state, and at \( J_\perp < J_c \) it undergoes quantum phase transition to a Neel phase with long range antiferromagnetic order (see, e.g. Refs.\(^\text{15,11}\)). In the present section we will consider the spin liquid phase in the regime very close to the point of antiferromagnetic instability.

Let us remove one spin from the system. We will see that the most interesting situation which is qualitatively different from the ladder arises when the hole is static \( (t = 0) \). Therefore, let us consider the static hole (impurity) problem. In the first approximation, neglecting the interaction with spin waves, the energy of the hole is

\[ \epsilon_0 = \frac{3}{4}J_\perp + \frac{3J^2}{2J_\perp} \approx 2.5J. \]  

It is similar to (9). The first term is due to the destroyed bond between the planes, and the second one arises from the suppression of quantum fluctuations on the four links in the plane. We substitute in (33) \( J_\perp = J_c \approx 2.5J \). Interactions with spin waves give corrections to (33). The hole-spin-wave vertex is represented by the \( t^1S \) term in the effective Hamiltonian (6).

In momentum representation it has the form

\[ g_{k,q} = -J\gamma_q (u_q + v_q) \sigma^\alpha. \]  

8
This vertex describes the process when a hole with momentum $k$ decays into a hole with momentum $k - q$ and a spin wave with momentum $q$ and polarization $\alpha$, ($\alpha = x, y, z$). The spin Pauli matrix of the hole is denoted by $\vec{\sigma}$; $\gamma_q = \frac{1}{2}(\cos q_x + \cos q_y)$; $u_q$ and $v_q$ are the Bogoliubov parameters corresponding to the spin wave. In (34) we neglect the deviation of the spin wave quasiparticle residue from unity because it gives only a 10% correction (see Ref. 11). The spin wave dispersion has a minimum at $Q = (\pi, \pi)$. Near this point the dispersion is

$$\omega_q = \sqrt{\Delta^2 + c^2(q - Q)^2},$$

where $c \approx 1.9(3)$ is the spin wave velocity. At $q \approx Q$ the Bogoliubov parameters can be represented as

$$u_q \approx v_q \approx \sqrt{\frac{A}{2\omega_q}},$$

where $A \approx 2.4J$.

The naive second order correction to the hole energy is given by

$$\delta\epsilon = -\sum_q \frac{3g_{k,q}^2}{\omega_q} = -\frac{3J^2A}{2\pi^2} \int \frac{d^2q}{\omega_q^2} \approx -J_0 \ln \frac{J}{\Delta},$$

where the integral is calculated with logarithmic accuracy. We have introduced here the notation

$$J_0 = \frac{3J^2A}{\pi c^2} \approx 0.63J$$

It is clear that since $\frac{J}{\Delta} \gg 1$, the naive formula (37) has no meaning. However, the presence of the logarithmic divergence indicates that the self consistent Born approximation (see e.g. Ref. 14, 15) is valid in this case. Following this approximation let us consider the quasiparticle (pole) part of the hole Green’s function

$$G(\epsilon) = \frac{Z}{\epsilon - \epsilon_0 + i 0},$$

where $\epsilon$ is the energy of the state. This Green’s function must obey Dyson’s equation

$$G(\epsilon) = \frac{1}{\epsilon - \epsilon_0 - \Sigma(\epsilon)},$$

where the self energy is given by

$$\Sigma(\epsilon) = 3 \sum_q g_{k,q}^2 G(\epsilon - \omega_q) \approx \frac{3J^2A}{2\pi^2} \int \frac{d^2q}{\omega_q(\epsilon - \epsilon - \omega_q)}.$$

The calculation of this integral is straightforward and gives

$$\Sigma(\epsilon) = -ZJ_0 \ln \frac{J}{\Delta}, \quad \left(\frac{\partial}{\partial\epsilon} \Sigma(\epsilon)\right)_{\epsilon=\epsilon} = -Z \frac{J_0}{\Delta}. $$
Finally, substituting into (40) and comparing with (39) we find

$$e = \epsilon_0 - \sqrt{\Delta J_0} \ln \frac{J}{\Delta}$$  \hspace{1cm} (43)

$$Z = \sqrt{\frac{\Delta}{J_0}}.$$  

The quasiparticle residue \(Z\) vanishes at \(\Delta \to 0\). It is clear that it does not happen at finite hopping, because in this case the lower limit of integration in (37) is \(t\) and the integral remains finite at \(\Delta = 0\). Physically it means that due to the finite speed of propagation the hole has no time for dressing by an infinite number of spin waves.

Thus, the static impurity is strongly dressed by spin waves. The question arises: is this cloud dilute enough to justify the spin-wave approach we have applied, or is it dense in which case the above consideration has no quantitative meaning because we do not take into account the interaction of the spin waves between themselves? The size of the cloud is \(r \sim c/\Delta\) (in units of the lattice spacing), and the effective number of spin waves is \(N_{sw} \sim 1/Z = \sqrt{J_0/\Delta}\). Therefore the density of spin waves \(n \sim N_{sw}/r^2 \sim \frac{\sqrt{J_0}}{c^2} \Delta^{3/2}\) remains low and hence the dilute gas approximation is valid.

V. CONCLUSIONS

Using variational approach and perturbation theory we have found analytically the dispersion of a single hole with spin \(S = 1/2\) in the \(t - J\) ladder. The dispersion is in agreement with previous computations. We demonstrate also that there are higher spin \((S = 3/2, 5/2, \ldots)\) bound states of the hole with the magnetic excitations. At large \(t/J\) these bound states lie below the \(S = 1/2\) state meaning that they become the real quasiparticles of the system. We have estimated the critical value of \(t/J\) where the crossover to the higher spin quasiparticles take place.

In spite of the complex spectrum, the structure of every state on the ladder is relatively simple. This means that the number of significant components in the wave function remains of the order of unity. In the second part of the present work we have considered the opposite situation when the wave function has a very large number of significant components. This is realized for a static hole in two antiferromagnetic planes close to the point of antiferromagnetic instability. We demonstrate that the self consistent Born approximation is justified in this case. Using this method we show that as the point of instability is approached the number of components in the hole wave function approaches infinity and the quasiparticle residue vanishes.

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FIGURES

FIG. 1. Dispersions of the $S = 1/2$ quasiparticle (solid line) and the $S = 3/2$ bound state (dashed line); $t/J = 1$.

FIG. 2. Dispersions of the $S = 1/2$ quasiparticle (solid line), $S = 3/2$ bound state (dashed line), and $S = 5/2$ bound state (dashed-dotted line); $t/J = 2$.

FIG. 3. Dispersions of the $S = 1/2$ state (solid line), $S = 3/2$ state (dashed line), and $S = 5/2$ bound state (dashed-dotted line); $t/J = 3$. 
Fig. 1

\( \frac{\varepsilon}{J} = t/J = 1 \)

- \( s = 3/2 \)
- \( s = 1/2 \)
Fig. 3

t/J=3

$\varepsilon_J$ for different $s$ values:

- $s=1/2$
- $s=3/2$
- $s=5/2$