A time-luminosity correlation for Gamma-Ray Bursts in the X-rays

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ABSTRACT
Gamma ray bursts (GRBs) have recently attracted much attention as a possible way to extend the Hubble diagram to very high redshift. However, the large scatter in their intrinsic properties prevents directly using them as distance indicator so that the hunt is open for a relation involving an observable property to standardize GRBs in the same way as the Phillips law makes it possible to use Type Ia Supernovae (SNeIa) as standardizable candles. We use here the data on the X-ray decay curve and spectral index of a sample of GRBs observed with the Swift satellite. These data are used as input to a Bayesian statistical analysis looking for a correlation between the X-ray luminosity $L_X(T_a)$ and the time constant $T_a$ of the afterglow curve. We find a linear relation between $\log [L_X(T_a)]$ and $\log [T_a/(1+z)]$ with an intrinsic scatter $\sigma_{\text{int}} = 0.33$ comparable to previously reported relations. Remarkably, both the slope and the intrinsic scatter are almost independent on the matter density $\Omega_M$ and the constant equation of state $w$ of the dark energy component thus suggesting that the circularity problem is alleviated for the $L_X - T_a$ relation.

Key words: Gamma Rays: bursts – Cosmology: distance scale – Cosmology: cosmological parameters

1 INTRODUCTION
The high fluence values (from $10^{-7}$ to $10^{-5}$ erg/cm$^2$) and the enormous isotropic energy emitted ($\simeq 10^{50} - 10^{53}$ erg) at the peak in a single short pulse make Gamma Ray Bursts (hereafter GRBs) the most violent and energetic astrophysical phenomena. Notwithstanding the variety of their different peculiarities, some common features may be identified looking at their light curves. Although GRBs have been traditionally classified as short and long depending on $T_{90}$ being smaller or larger than 2 s (with $T_{90}$ the time over which from 5% to 95% of the prompt emission is released), a recent analysis by Donaghy et al. ((2006)) has shown that this criterion has to be revised. Indeed, the existence of an intermediate class of GRBs have also been studied (Norris & Bonnell (2006); Bernardini et al. (2007)). As a result, the long GRBs are now further classified as normal and low luminosity with the latter ones probably associated with Supernovae (Pian et al. (2006); Dainotti et al. (2007)).

Notwithstanding this classification, two phases are clearly visible in the GRB lightcurve, namely the prompt emission, where most of the energy is released in the $\gamma$-rays in only tens of seconds, and an afterglow lasting many hours after the initial bursts. Early observations in the X-rays typically started several hours after the prompt emission so that only the late phase of the light curve could be characterized. It was then found that a phenomenological power-law, $f(t, \nu) \propto t^{-\alpha} \nu^{-\beta}$ with $(\alpha, \beta) \simeq (-1.4, 0.9)$, provided a reasonable fit to the observed data (Piro 2001). However, the launch of the Swift satellite, whose aim is also to observe GRBs X-ray (0.2–10keV) and optical (1700–6500 Å) afterglows starting few seconds after the trigger, revealed a more complex behaviour. The soft X-ray light curves must indeed be divided in two different classes (Chincarini et al. 2005) according to the steep or mild initial decay. Most of the observed GRB afterglows belong to the first group, showing what has been called a canonical behavior (Nousek et al. 2006) described by a broken power-law. After the initial steep decay (with slope $3 \leq \alpha_1 \leq 5$), the light curve shows a shallow decay ($0.5 \leq \alpha_2 \leq 1$) followed by another steeper decay ($1 \leq \alpha_3 \leq 1.5$) beyond $2 \times 10^4$ s. These power-law segments are separated by two corresponding break times with $t_{b1} \leq 500$ s and $10^3 s \leq t_{b2} \leq 10^4$ s. A
new systematic study using GRBs observed with XRT reveals a still more complex behavior with different power-law slopes and break times (O’ Brien et al. 2006; Sakamoto et al. 2007). A significant step forward has been represented by the analysis of the X-ray afterglow curves of the full sample of Swift GRBs showing that all of them may be fitted by the same analytical expression (Willingale et al. 2007).

Finding out a universal feature for GRBs is the first important step towards their use as distance indicator. To this aim, one has indeed to look for a universal relation linking observable GRBs properties so that their intrinsic luminosity may be estimated from directly measurable quantities. Previous attempts along this road are represented by the $E_{iso} - E_{peak}$ (Amati et al. 2002), $E_{\gamma} - E_{peak}$ (Ghirlanda et al. 2004; Ghirlanda et al. 2006), $L - E_{peak}$ (Schaefer 2003), $\tau_{iso}$ (Norris et al. 2000), $L - V$ (Fenimore & Ramirez-Ruiz 2000; Riechart et al. 2001), $L - \tau_{RT}$. Moreover, three-parameter relations have also been proposed such as, e.g., the $E_{iso} - \tau_{iso} - t_{peak}$ (Liang & Zhang(2005)) and that proposed by Firmani and collaborators (Firmani et al. 2005, 2006). On the other hand, some attempts have also been made to compare these empirical correlations with the model dependent ones (Nava et al. (2006); Guida et al. 2008). The above quoted two parameters correlations have then been used by Schaefer (2007, hereafter S07) to construct the first reliable GRBs Hubble diagram extending up to $z \geq 6$ opening the way towards the use of GRBs as cosmological probes (see, e.g., Capozziello & Izzo 2008 and refs. therein).

In this letter, we present a possible alternative route towards standardizing GRBs as distance indicator. To this aim, we use the data in Willingale et al. (2007) to look for a possible correlation between the X-ray luminosity at the break time $T_b$ and the $T_a$ itself. The data used are presented in Sect. 2, while Sect. 3 deals with the statistical tools and the results. Conclusions are summarized in Sect. 4.

2 THE DATA

Willingale et al. (2007, hereafter W07) have examined the X-ray decay curves of all the GRBs measured by the Swift satellite then available. Their analysis shows that all of them may be well fitted by a simple two components formula, namely:

$$f(t) = f_p(t) + f_a(t)$$

where the first term accounts for the prompt $\gamma$-ray emission and the initial X-ray decay, while the second one describes the afterglow. Both components are given by the same functional expression:

$$f_c(t) = \begin{cases} F_c \exp \left( \frac{\alpha_c - \frac{t_{\alpha_c}}{T_c}}{t} \right) \exp \left( \frac{t}{t_c} \right) & \text{for } t < T_c \\ F_c \left( \frac{t}{T_c} \right)^{-\alpha_c} \exp \left( \frac{t}{t_c} \right) & \text{for } t \geq T_c \end{cases}$$

where the transition from the exponential to the power-law decay takes place at the point $(T_c, F_c)$ where the two functional sections match in value and gradient. The parameter $\alpha_c$ determines both the time constant of the exponential decay (given by $T_c/\alpha_c$) and the slope of the following decay, while $t_{\alpha_c}$ marks the initial rise and the time of maximum flux occurring at $t = \sqrt{t_{\alpha_c} T_c}/\alpha_c$. Denoting with the suffix $p$ and $a$ quantities for the prompt and afterglow components, Eq.(2) may be inserted into Eq.(1) to give an eight parameters expression that can be fitted to the X-ray decay curve in order to both validate this expression and determine, for each GRB, the corresponding parameters. Such a task has been indeed performed by W07 using all the 107 GRBs detected by both BAT and the XRT on Swift up to August 1st 2006. The fit procedure and the detailed analysis of the results are presented in W07, while here we only remind that the usual $\chi^2$ fitting in the $(f_{\text{flux}})$ vs log (time) provide estimates and uncertainties on the time parameters (log $T_p$, log $T_a$) and the products (log $F_pT_p$, log $F_aT_a$).

W07 also performed spectral fitting with XSPEC (Arnaud 1996) to BAT (for the prompt phase) and XRT (for later phases) data to estimate the spectral index during different phases. Due to the limited frequency range, the GRB spectrum may be simply described by a single power-law, $\Phi(E) \propto E^\beta$, with the slope $\beta$ depending on the time when the spectrum is observed. W07 reported four different values of $\beta$, namely $\beta_p$ (for the prompt phase), $\beta_{\text{peak}}$ for the prompt decay, $\beta_a$ for the plateau observed at the time $T_a$, and $\beta_{\text{ad}}$ for the afterglow at $t > T_a$. Actually, the data coverage is not sufficient to measure all of them for the full sample so that, for the weakest bursts, only $\beta_p$ and $\beta_{\text{ad}}$ are available. Provided $\beta$ is known, it is possible to estimate the GRB luminosity at a given time $t$ as:

$$L_X(t) = 4\pi D_L^2(z) F_X(t)$$

where $D_L(z)$ is the luminosity distance at the GRB redshift $z$, and $F_X(t)$ is the flux (in erg/cm$^2$/s) at the time $t$, K-corrected (Bloom et al. 2001) as:

$$F_X(t) = f(t) \times \frac{\int_{E_{\text{min}}}^{E_{\text{max}}/2} \Phi(E)dE}{\int_{E_{\text{min}}}^{E_{\text{max}}/2} \Phi(E)dE}$$

with $(E_{\text{min}}, E_{\text{max}}) = (0.3, 10)$ keV set by the instrument bandpass. Note that Eq.(3) is the same as Eq.(8) in S07 the only difference being the integration limits of the integral at the numerator. Actually, while S07 is interested to the bolometric luminosity, we are here concerned with the X-ray one so that we integrate only over this energy range.

Using the data in W07, we compute the X-ray luminosity at the time $T_a$ so that we have to set $f(t) = f(T_a)$ and $\beta = \beta_a$ in Eqs.(3) and (4). Actually, rather than using Eq.(1), we set $f(T_a) = f_a(T_a)$ since the contribution of the prompt component is typically smaller than 5%, much lower than the statistical uncertainty on $f_a(T_a)$. Neglecting $f_p(T_a)$ thus allows to reduce the error on $F_X(T_a)$ without introducing any bias. This latter error is then estimated by simply propagating those on $\beta_a$, log $T_a$ and log $F_aT_a$ thus implicitly assuming that their covariance is null*. Should this not be the case, we are underestimating the final error on $L_X(T_a)$. We have, however, checked that our main results are unaffected by a reasonable increase of the errors.

* Note that the covariance matrix is not reported in W07, where the parameters of interest are given with their 90% confidence ranges. Following Willingale (priv. comm.), we have assumed independent Gaussian errors and obtained 1σ uncertainties by roughly dividing by 1.65 the 90% errors. Moreover, we preliminary correct for asymmetric errors on log $F_aT_a$ and log $T_a$ (when present) following the prescriptions in D’ Agostini (2004).
As a final important remark, we note that the presence of the luminosity distance $D_L(z) = (c/H_0)d_L(z)$ in Eq.(3) constrains us to adopt a cosmological model to compute $L_X(T_a)$. We use a flat $\Lambda$CDM model so that the Hubble free luminosity distance reads:

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{\sqrt{\Omega_M(1 + z')^3 + (1 - \Omega_M)}}.$$ (5)

In agreement with the WMAP five year results (Dunkley et al. 2008), we set $(\Omega_M, h) = (0.291, 0.697)$ with $h$ the Hubble constant $H_0$ in units of $100$ km/s/Mpc.

3 A LUMINOSITY - TIME CORRELATION

In order to standardize GRBs to use them as possible distance indicator, we need to find a relation between the luminosity and a directly observable quantity. Should such a relation be found, one can then use the observed flux and the estimated $L_X$ to infer $D_L(z)$ and then construct the GRB Hubble diagram. Let us suppose that a power-law relation exists between two quantities $R$ and $Q$ as $R = AQ^B$. In logarithmic units, this reads $\log R = a + b \log Q$ with $a = \log A$ and $b = B$. Typically, both $R$ and $Q$ will be known with measurement errors $(\sigma_R, \sigma_Q)$ so that the statistical uncertainties on $(\log R, \log Q)$ will be given by $(\sigma_R/R, \sigma_Q/Q) \times (1/\ln 10)$ respectively. These errors may be comparable so that it is not possible to decide what is the independent variable to be used in the usual $\chi^2$ fitting analysis. Moreover, the relation $R = AQ^B$ may be affected by an intrinsic scatter $\sigma_{\text{int}}$ of unknown nature that has to be taken into account. In order to determine the parameters $(a,b,\sigma_{\text{int}})$, we can then follow a Bayesian approach (D’Agostini 2005) thus maximizing the likelihood function $L(a,b,\sigma_{\text{int}}) = \exp[-L(a,b,\sigma_{\text{int}})]$ with:

$$L(a,b,\sigma_{\text{int}}) = \frac{1}{2} \sum \left( \log \sigma_{\text{int}}^2 + \sigma_y^2 + b^2 \sigma_y^2 \right) + \frac{1}{2} \sum \left( \frac{y_i - a - b x_i}{\sigma_y^2 + b^2 \sigma_y^2} \right)$$ (6)

with $(x_i,y_i) = (\log Q_i, \log R_i)$ and the sum is over the $N$ objects in the sample. Note that, actually, this maximization is performed in the two parameter space $(b,\sigma_{\text{int}})$ since $a$ may be estimated analytically as:

$$a = \frac{\sum y_i - b \sum x_i}{\sum \sigma_y^2 + b^2 \sigma_y^2} = \frac{1}{\sum \sigma_y^2 + b^2 \sigma_y^2} (\sum \sigma_y^2)^{-1}$$ (7)

so that we will not consider it anymore as a fit parameter.

We use this general recipe to look for a correlation between the X-ray luminosity (in erg s$^{-1}$) at the time $T_a$ and $T_a$ (in s) itself, i.e. we set $y = \log [L_X(T_a)]$ and $x = \log [T_a/(1 + z)]$, where we divide time by $(1 + z)$ to account for the cosmological time dilation. Note that, since $\beta_a$ is needed to compute $L_X(T_a)$, we have to reject most of the 107 GRBs reported in W07 because this is not known. We then end up with a sample containing $N = 32$ with both $\log [L_X(T_a)]$ and $\log [T_a/(1 + z)]$ measured†. The Spearman rank correlation turns out to be $r = -0.74$ suggesting that a power-law relation between $L_X(T_a)$ and $T_a/(1 + z)$ indeed exists thus motivating further analysis.

We then apply the maximum likelihood estimator described above in order to determine both the slope and the intrinsic scatter of the $L_X - T_a$ correlation thus finding out:

$$(a,b,\sigma_{\text{int}}) = (48.54, -0.74, 0.43).$$

Defining the best fit residuals as $\delta = y_{obs} - y_{fit}$, we can qualitatively estimate the goodness of the fit by considering the median and root mean square which turn out to be $\delta = -0.08$ and $\delta_{\text{rms}} = 0.52$ indeed quite small if compared to the typical log $[L_X(T_a)]$ values. It is also worth noting that $\delta$ does not correlate with the other parameters of the fit flux, while the value $r = -0.23$ between $\delta$ and $z$ favours no significant evolution of the $L_X - T_a$ relation with the redshift.

The best fit relation is superimposed to the data in Fig. 1 where we also present the best fit obtained by the usual $\chi^2$ fitting technique. In this case, the best fit parameters are obtained by minimizing (through a Levenberg-Marquardt algorithm with 1.5σ outliers rejection) a $\chi^2$ merit function given by the second term in Eq.(6) with $\sigma_y = \sigma_{\text{int}} = 0$, i.e. we (erroneously) assume that there is no scatter and that the errors on log $[L_X(T_a)]$ are negligible. This alternative method gives as best fit parameters:

$$(a,b) = (48.58, -0.79).$$

in good agreement with the above maximum likelihood estimator so that we argue that our results are independent on the fitting method. However, since the Bayesian approach is better motivated and also allows for an intrinsic scatter, we hereafter elige this as our preferred technique.

In an attempt to reduce the intrinsic scatter in the above correlation, we have analysed the best fit residuals noting that the higher ones are obtained for GRBs with luminosities smaller than $10^{45}$ erg and time parameter log $[T_a/(1 + z)] > 5$. We therefore repeated the above analysis using only 28 out of 32 GRBs‡ satisfying the two se-

† ASCII tables with all the quantities needed for the analysis and the Mathematica codes used are available on request.

‡ The four GRBs excluded are: GRB050824, GRB060115, GRB060607A and GRB060614. While the first two appear to be unaffected by any problem, for the latter two, the data cover less
lection criteria \(1 \leq \log [T_\text{gw}/(1+z)] \leq 5\) and \([L_X(T_\text{gw})] \geq 45\). Using the maximum likelihood estimator, we get:

\[
(a, b, \sigma_{\text{int}}) = (48.09, -0.58, 0.33)
\]

with \(\delta = -0.06\) and \(\delta_{\text{rms}} = 0.43\). The reduced intrinsic scatter and the smaller fit residuals suggest us that, whatever is the unknown mechanism originating the \(L_X - T_\text{gw}\) relation, this is better effective for the class of GRBs satisfying the above selection criteria. The data and the best fit curve are shown in Fig. 2 where the dashed line refers to the results obtained with the \(\chi^2\) minimization giving \((a, b) = (48.07, -0.60)\) reported here for completeness.

The Bayesian approach used here also allows us to quantify the uncertainties on the fit parameters. To this aim, for a given parameter \(p_i\), we first compute the marginalized likelihood \(L_i(p_i)\) by integrating over the other parameter. The median value for the parameter \(p_i\) is then found by solving:

\[
\int_{p_{i,\text{min}}}^{p_{i,\text{med}}} L_i(p_i) dp_i = \frac{1}{2} \int_{p_{i,\text{min}}}^{p_{i,\text{max}}} L_i(p_i) dp_i.
\]

The 68% (95%) confidence range \((p_{i,1}, p_{i,3})\) are then found by solving:

\[
\int_{p_{i,1}}^{p_{i,med}} L_i(p_i) dp_i = \frac{1 - \varepsilon}{2} \int_{p_{i,\text{min}}}^{p_{i,\text{max}}} L_i(p_i) dp_i,
\]

\[
\int_{p_{i,1}}^{p_{i,med}} L_i(p_i) dp_i = \frac{1 - \varepsilon}{2} \int_{p_{i,\text{min}}}^{p_{i,\text{max}}} L_i(p_i) dp_i,
\]

with \(\varepsilon = 0.68\) (0.95) for the 68% (95%) range respectively. For the fit to the full dataset, we get:

\[
b = -0.74^{+0.20}_{-0.19} +0.37, \quad \sigma_{\text{int}} = 0.48^{+0.15}_{-0.10} +0.35,
\]

while it is:

\[
b = -0.56^{+0.18}_{-0.18} +0.33, \quad \sigma_{\text{int}} = 0.39^{+0.14}_{-0.11} +0.33
\]

for the selected subsample.

\section{4 DISCUSSION AND CONCLUSION}

The high Spearman correlation coefficient, the low value of the fit residuals and the modest intrinsic scatter renders the \(L_X - T_\text{gw}\) relation presented above a new valid tool to standardize GRBs. It is worth stressing that \(L_X - T_\text{gw}\) needs only two parameters and one of them is directly inferred form the observations minimizing the effects of the systematics errors. Furthermore the redshift range covered is large extending from 0.54 (0.125) up to 6.6 for the selected (full) sample far beyond the maximum redshift affordable with Type Ia SN (\(z \approx 1.7\)). Should this correlation be confirmed by future higher quality data, one could then combine it with the other relations yet available in literature to work out a GRBs Hubble diagram deep into the matter dominated era thus representing an outstanding cosmological test.

To this end, it is worth comparing the \(L_X - T_\text{gw}\) relation with other ones quoted in literature. When performing such a comparison, however, one should take into account the differences in the cosmological model adopted and the fitting method used. In particular, the choice of how the best fit parameters are estimated may have an important impact on the estimate of the intrinsic scatter with the usual \(\chi^2\) fitting leading to an underestimate of \(\sigma_{\text{int}}\). On the other hand, changing \(\Omega_M\) in the framework of the flat \(\Lambda\)CDM scenario has a profound impact on \(\sigma_{\text{int}}\) with higher \(\Omega_M\) giving rise to lower \(\sigma_{\text{int}}\) values (Basilakos & Perivolaropoulos 2008). In order to account for both these issues, one should therefore test all the above correlations using the same statistical tools and cosmological model, a task we will address elsewhere.

As is well known, the paucity of local (i.e., \(z \leq 0.1\)) GRBs represents a serious problem for any attempt to standardize GRBs since it is very difficult to directly calibrate any relation. This problem may be partly overcome by fitting the correlation in a subsample of GRBs lying at similar redshift (Amati 2008). However, as a general rule, in order to evaluate the GRB luminosity, a cosmological model has to be adopted thus leading to the circularity problem. Although addressing this problem in detail will be the subject of a forthcoming work, we have here investigated what is the effect of changing the cosmological model by using our maximum likelihood estimator to determine the parameters \((a, b, \sigma_{\text{int}})\) as function of \(\Omega_M\) in a flat \(\Lambda\)CDM model\footnote{To be precise, we let \(\Omega_M\) running from 0.2 to 1 and adjust \(h\) so that \(\Omega_M h^2\) is fixed to the same value adopted above.}. We find the remarkable result that both the best fit parameters \((b, \sigma_{\text{int}})\) and the rms residual \(\delta_{\text{rms}}\) are almost insensitive to the value of \(\Omega_M\). Indeed, \(b\) runs from \(b \simeq -0.590\) to \(b \simeq -0.565\), while \(\sigma_{\text{int}}\) increases from \(\sigma_{\text{int}} \simeq 0.335\) to \(\sigma_{\text{int}} \simeq 0.340\) for \(\Omega_M\) going from 0.2 to 1.0. As a further test, we generalize the \(\Lambda\)CDM model varying not only the matter density parameter \(\Omega_M\), but also the equation of state \(w\) of the dark energy component (with \(w = -1\) for the \(\Lambda\)CDM model). For \(-1.3 \leq w \leq -0.7\), neither \(b\) nor \(\sigma_{\text{int}}\) significantly change confirming the qualitative results obtained for the \(\Lambda\)CDM scenario. Although a more detailed analysis is needed, we therefore argue that the circularity problem is alleviated by the use of our \(L_X - T_\text{gw}\) relation.

The encouraging results discussed above are serious arguments in favour of the \(L_X - T_\text{gw}\) relation as a further tool
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towards the standardization of GRBs as distance indicator. Should these first evidences be furtherly enforced by more data, the combined use of full set of GRBs correlations discovered insofar could opened the road towards making GRBs the high redshift analog of SNeIa as cosmological probes in the not too distant future.

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