An introduction to the SYK model

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Abstract

The Sachdev–Ye–Kitaev (SYK) model is a strongly coupled, quantum many-body system that is chaotic, nearly conformally invariant, and exactly solvable. This remarkable and, to date, unique combination of properties have driven the intense activity surrounding the SYK model and its applications within both high energy and condensed matter physics. In this review we give an introduction to the SYK model and recent developments connected to it. We discuss: SYK and tensor models as a new class of large N quantum field theories, the near-conformal invariance in the infrared, the computation of correlation functions, generalizations of the SYK model, and applications to AdS/CFT and strange metals.

Keywords: quantum field theory, large N, AdS/CFT, exactly solvable models

(Some figures may appear in colour only in the online journal)

1. Introduction

The Sachdev–Ye–Kitaev (SYK) model [1, 2] is a strongly coupled, quantum many-body system that is chaotic, nearly conformally invariant, and exactly solvable. This remarkable and, to date, unique combination of properties have driven the intense activity surrounding SYK and its applications within both high energy and condensed matter physics.

As a quantum field theory, SYK and, more generally, tensor models, constitute a new class of large N theories. The dominance of a simple and well-organized set of Feynman diagrams, iterations of melons, enables the computation of all correlation functions. As a solvable model of holographic duality, SYK accurately captures two-dimensional gravity, and has the potential to shed light on the workings of holography and black holes. As a solvable many-body system, SYK serves as a building block for constructing a metal, capturing some of the properties of non-Fermi liquids.

These notes are a brief introduction to SYK. In section 2 we review large N field theories, in particular vector models, and introduce SYK and tensor models. In section 3 we discuss...
the low energy limit of SYK, described by a sum of the Schwarzian action and a conformally invariant action. In section 4 we discuss how the simple Feynman diagrammatics of large N SYK, combined with the power of conformal symmetry, allows for an explicit computation of all correlation functions. In section 5 we discuss applications of SYK to holographic duality, and to strange metals.

2. A new large $N$ limit

Large $N$ quantum field theories are theories with a large number of fields, related by some symmetry, such as $O(N)$. Their essential property is the factorization of correlation functions of $O(N)$ invariant operators, $\langle O(x_1) O(x_2) \rangle = \langle O(x_1) \rangle \langle O(x_2) \rangle + \frac{g}{N} (\ldots)$. As such, large $N$ theories are in a sense semi-classical, with $1/N$ playing the role of $\hbar$.

2.1. Vector models

The simplest large $N$ quantum field theories are vector models [3, 4]. An example is the $O(N)$ vector model, having $N$ scalar fields, $\vec{\phi} = (\phi^1, \ldots, \phi^N)$, with a quartic $O(N)$ invariant interaction,

$$I = \int d^d x \left( \frac{1}{2} (\partial \vec{\phi})^2 + \frac{1}{2} \mu^2 \vec{\phi}^2 + \frac{g}{4} (\vec{\phi} \cdot \vec{\phi})^2 \right).$$

In dimensions $2 < d < 4$, if one appropriately tunes the bare mass $\mu$, there is an infrared fixed point: the Wilson–Fisher fixed point, describing magnets.

The power of large $N$ is that instead of studying the theory perturbatively in the coupling $g$, one can reorganize the perturbative expansion, into powers of $gN$ and $1/N$. At a given order in $1/N$, one is able to compute to all order in $gN$. For instance, at leading order in $1/N$, the only Feynman diagrams contributing to the two-point function of the $O(N)$ vector model are bubbles, see figure 1, all of which are summed by the integral equation,

$$G(p) = \frac{1}{p^2 + \mu^2 + \Sigma(p)}, \quad \Sigma(p) = gN \int d^d q G(q),\tag{2.2}$$

where $G(p)$ is the momentum space two-point function, and $\Sigma(p)$ is the self-energy. The self-energy is independent of the momentum: the only effect of the bubble diagrams is to shift the mass. Defining $m^2 = \mu^2 + \Sigma$, the above Schwinger–Dyson equation becomes,

$$m^2 = \mu^2 + gN \int d^d q \frac{1}{q^2 + m^2}.$$

An equivalent way of analyzing the $O(N)$ vector model is by introducing an auxiliary Hubbard–Stratonovich field $\sigma$, so as to rewrite the action as,

$$I = \int d^d x \left( \frac{1}{2} (\partial \vec{\phi})^2 + \frac{1}{2} \mu^2 \vec{\phi}^2 + \frac{1}{2} \sigma \vec{\phi}^2 - \frac{\sigma^2}{4g} \right).\tag{2.4}$$

Integrating out $\sigma$ gives back the original action. Alternatively, integrating out the $\phi^a$ fields, gives an action involving only $\sigma$,

$$\frac{I_\sigma}{N} = \frac{1}{2} \log \det(-\partial^2 + \mu^2 + \sigma) - \frac{1}{4gN} \int d^d x \sigma^2.\tag{2.5}$$
The saddle of $I_\sigma$ gives back the Schwinger–Dyson equation for summing bubbles. More generally, one could have introduced a source for $\phi$, and then used the resulting $I_\sigma$ to, in principle, compute any correlation function of $O(N)$ invariant operators, to any order in $1/N$.

2.1.1. Matrix models. There are many systems, such as Yang–Mills theory, in which the fundamental fields are matrices, rather than vectors. The large $N$ dominant Feynman diagrams in such theories are planar, when drawn in double line notation [10]. There is no known way of summing all planar diagrams, and matrix models are in general not solvable. For some special theories, there are alternate techniques. For instance, models of a single matrix in zero and one dimension can be solved through a map to free fermions [11, 12]. More recently, powerful integrability techniques have been applied to planar $\mathcal{N} = 4$ super Yang–Mills in four dimensions [13], yielding, for instance, exact results for anomalous dimensions [14] and progress for the three-point function [15]. See [16] for an introduction to integrability, in two dimensions, and [17] for the initial discovery of the link between the computation of anomalous dimensions and the diagonalization, via Bethe ansatz, of certain integrable spin chain Hamiltonians.

2.2. Tensor models and SYK

Having discussed vector models and matrix models, it is natural to consider tensor models, with fields having three or more indices. An example of such a model, for fermions in one dimension and rank-3 tensors, is the Klebanov–Tarnopolsky model [18], a simplification of the Gurau–Witten model [19, 20] (see also [21–24]), with the Lagrangian,

$$L = \frac{1}{2} \sum_{a,b,c=1}^{N} \psi_{abc} \partial_\tau \psi_{abc} - \frac{g}{4} \sum_{a_1,...,a_2=1}^{N} \psi_{a_1b_1c_1} \psi_{a_1b_2c_1} \psi_{a_2b_1c_2} \psi_{a_2b_2c_1},$$

where the real field $\psi_{abc}$ transforms in the tri-fundamental representation of $O(N)^3$. Remarkably, the two-point function is dominated by melon diagrams in the limit of large $N$ and fixed $J^2 \equiv g^2 N^3$. The summation of all melon diagrams is encoded in the Schwinger–Dyson equation, see figure 2,

$$G(\omega)^{-1} = -i\omega - \Sigma(\omega), \quad \Sigma(\tau) = J^2 G(\tau)^3,$$

where $G(\omega)$ is the Fourier transform of $G(\tau)$. Tensor models, which sum melon diagrams, are more rich, and more difficult, than vector models. They are however, at least in some ways, simpler than matrix models, for which there is no closed set of equations to sum the planar diagrams.

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Figure 1. The self-energy for the $O(N)$ vector model, in terms of the propagator. Iterating gives a sum of bubble diagrams.

There are also large $N$ models involving both vectors and matrices, such as, in one dimension, the Iuzika–Polchinski model [5], in two dimensions, the ’t Hooft model of QCD [6, 7], and in three dimensions, Chern–Simons theory coupled to matter [8, 9]. These models are vector-like, as the matrix degrees of freedom have no self-interactions, and are solvable at large $N$ through summation of rainbow diagrams.
One challenge in the study of tensor models, beyond the melonic dominance at large $N$, is the relatively low degree of symmetry: the number of degrees of freedom scales as $N^3$, while the rank of the symmetry group, $O(N)^3$, scales as $N^2$. This difficulty is alleviated by the SYK model [2], at the expense of introducing disorder, 

$$L = \frac{1}{2} \sum_{i=1}^{N} \chi_i \partial_\tau \chi_i - \frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl} \chi_i \chi_j \chi_k \chi_l,$$

(2.8)

where the couplings are Gaussian-random, with zero mean, and variance $\langle J_{ijkl} J_{ijkl} \rangle = \frac{6J^2}{N^3}$.

The leading large $N$ diagrams in SYK are melons; identical to those in the tensor model [2]. The subleading $1/N$ corrections, as well as the symmetry group, are however different. Taking the partition function, and disorder averaging/integrating out the $J_{ijkl}$ gives a bilocal, $O(N)$ invariant action [3]. Introducing bilocal fields $\Sigma(\tau_1, \tau_2)$ and $G(\tau_1, \tau_2)$, with $\Sigma$ acting as a Lagrange multiplier field enforcing $G(\tau_1, \tau_2) = \frac{1}{N} \sum_{i=1}^{N} \chi_i(\tau_1) \chi_i(\tau_2)$, and integrating out the fermions, one is left with an action for $\Sigma$ and $G$ [2], see also [27–29],

$$\frac{I_{\text{eff}}}{N} = -\frac{1}{2} \log \det \left( \partial_\tau - \Sigma \right) + \frac{1}{2} \int d\tau_1 d\tau_2 \left( \Sigma(\tau_1, \tau_2) G(\tau_1, \tau_2) - \frac{J^2}{4} G(\tau_1, \tau_2)^4 \right).$$

(2.9)

Compared with the $O(N)$ vector model, which was captured by an action for a local field $\sigma(x)$, SYK is instead captured by the above action for bilocal fields $\Sigma(\tau_1, \tau_2)$ and $G(\tau_1, \tau_2)$. With this action, the model is in principle solved: instead of the original $N$ fields, there are now only two fields. At infinite $N$, the theory is classical, with the path integral dominated by the saddle for $\Sigma$ and $G$, given by (2.7), reflecting the summation of melon diagrams. All higher point correlation functions follow from expanding the action in powers of $1/N$. The rest of the notes are devoted to computing these in an explicit form, and understanding their physical consequences.

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2 In fact, the melonic dominance in SYK can be proved by viewing it as a kind of tensor model, with $J_{ijkl}$ being the tensor field [25], although this proof is more involved than the standard one for SYK, involving the effective action (2.9).

3 The SYK model has quenched disorder. For many quantities, the model is self-averaging, and computing with some randomly chosen, but fixed, couplings should give the same result as averaging over the couplings. In this sense, the disorder average is a trick, in order to be able to analytically perform the calculation. Alternatively, one might wish to view the couplings $J_{ijkl}$ as quantum scalar fields with a two-point function that is a constant (annealed disorder). In fact, up to order $1/N^3$, this gives the same results as with quenched disorder [26], assuming there is no replica symmetry breaking, which is implicit in (2.9) where we dropped the replica indices (the replica off-diagonal terms are subleading [29]).

4 The solution of the $O(N)$ vector model is an example of mean field theory, whereas the solution of SYK is an example of dynamical mean field theory [30].
3. The infrared

In this section we study SYK in the infrared limit, following [29]. We take the effective action (2.9), and for convenience define, \( \sigma(\tau_1, \tau_2) = \delta(\tau_1 - \tau_2) \partial_{\tau} \), and change variables \( \Sigma \to \Sigma + \sigma \), so that the action becomes, \( I_{\text{eff}} = I_{\text{CFT}} + I_S \), where,

\[
\frac{I_{\text{CFT}}}{N} = -\frac{1}{2} \log \det(-\Sigma) + \frac{1}{2} \int d\tau_1 d\tau_2 \left( \Sigma(\tau_1, \tau_2)G(\tau_1, \tau_2) - \frac{f^2}{4} G(\tau_1, \tau_2)^4 \right)
\]  

(3.1)

\[
\frac{I_S}{N} = \frac{1}{2} \int d\tau_1 d\tau_2 \sigma(\tau_1, \tau_2)G(\tau_1, \tau_2).
\]  

(3.2)

In the infrared, \(|J\tau| \gg 1\), at leading order, we simply drop the \( I_S \) part of the action, as the delta function in \( \sigma \) is a very UV term. The saddle of \( I_{\text{CFT}} \) is the Schwinger–Dyson equation from before, without the \( \partial_{\tau} \) term, and its solution takes the form of a conformal field theory two-point function [1],

\[
G(\tau_1, \tau_2) = b^4 \frac{\text{sgn}(\tau_{12})}{|J\tau_{12}|^{2\Delta}}, \quad \text{where} \quad b^4 = \frac{1}{4\pi}, \quad \Delta = \frac{1}{4}, \quad \tau_{ij} \equiv \tau_i - \tau_j.
\]  

(3.3)

One might assume that any higher point correlation function, computed using \( I_{\text{CFT}} \), would also be conformally invariant. In fact, this is almost true, but not completely. Notice that \( I_{\text{CFT}} \) is time reparametrization invariant, \( \tau \to f(\tau) \), provided \( G \) and \( \Sigma \) transform appropriately,

\[
G(\tau_1, \tau_2) \to f'(\tau_1)^\Delta f'(\tau_2)^\Delta G(f(\tau_1), f(\tau_2)), \quad \Sigma(\tau_1, \tau_2) \to f'(\tau_1)^{1-\Delta} f'(\tau_2)^{1-\Delta} \Sigma(f(\tau_1), f(\tau_2)).
\]

As a result, in addition to the solution (3.3), we have an entire space of solutions [2],

\[
G(\tau_1, \tau_2) = b^4 \frac{\text{sgn}(\tau_{12})}{|f(\tau_{12})|^{2\Delta}} f'(\tau_1)^{\Delta} f'(\tau_2)^{\Delta} \left( \frac{f''}{f'} \right)^{\frac{\Delta}{2}},
\]

(3.4)

and moving between them has no action cost. At a practical level, this means that using \( I_{\text{CFT}} \) to compute correlation function will lead to divergences [2, 27, 31]. Of course, in the full action, with \( I_S \) included, there is a cost for \( \tau \to f(\tau) \), so this simply means that we need to move slightly away from the deep infrared limit: rather than just dropping \( I_S \), we should view it as a perturbation of the infrared action, and compute its effect, to leading order. We approximate \( I_S \) by inserting for \( G \) the saddle (3.4). Since \( \sigma(\tau_1, \tau_2) \) is a delta function, the integral in the action picks out the \( \tau_{12} \ll 1 \) part of \( G \). Taylor expanding \( G(\tau_1, \tau_2) \) about \( \tau_+ = (\tau_1 + \tau_2)/2 \), for \( \tau_{12} \ll 1 \),

\[
G \to b^4 \frac{\text{sgn}(\tau_{12})}{|J\tau_{12}|^{2\Delta}} \left( 1 + \frac{\Delta}{6} \tau_{12}^2 \text{Sch}(f(\tau_+), \tau_+) + \ldots \right), \quad \text{where} \quad \text{Sch}(f(\tau), \tau) = \frac{f'''(\tau)}{f'(\tau)} - \frac{3}{2} \left( \frac{f''}{f'} \right)^2
\]

and we get that,

\[
\frac{I_S}{N} = \# \int d\tau \text{Sch}(f(\tau), \tau) + \ldots
\]  

(3.5)

where \( \text{Sch}(f(\tau), \tau) \) is the Schwarzian. The prefactor can not be fixed by this procedure; it must be determined numerically [27, 29], from the exact solution to the Schwinger–Dyson equation (2.7). The reason is that we are studying the infrared action, valid for \(|J\tau| \gg 1\), while
the perturbation $\sigma$ involves a delta function of time, outside the domain of validity of perturbation theory.

The field $f(\tau)$ is sometimes referred to as the reparametrization mode, or the soft mode, or the $h = 2$ mode, or the gravitational mode. It is the Nambu–Goldstone mode for the breaking of time reparametrization invariance [32]. For further studies of the Schwarzian, see [33–40], as well as references in section 5.1, regarding dilaton gravity.

4. Correlation functions

We now turn to higher point correlation functions, following the discussion in [41]. The four-point function, like the two-point function, is given by the solution of an integral equation, while even higher point functions are given by integrals of products of four-point functions.

In the infrared, where there is near-conformal symmetry, we can go further and write explicit expressions for the higher point correlation functions.

4.1. Conformal blocks

In this section we discuss some properties of an arbitrary conformal field theory. We first recall the constraints that conformal symmetry places on the form of correlation functions. A one-dimensional conformal field theory, CFT1, has $SL_2(R)$ symmetry,

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1. \quad (4.1)$$

Any such transformation is generated by a combination of translations $\tau \to \tau + a$, dilatations $\tau \to \lambda \tau$, and inversions $\tau \to \frac{1}{\tau}$. The symmetry fully fixes the functional form of the two-point and three-point functions,

$$\langle O_h(\tau_1) O_h(\tau_2) \rangle = \frac{1}{|\tau_1 - \tau_2|^{|2h|}}, \quad \langle O_1 O_2 O_3 \rangle = \frac{C_{h_1 h_2 h_3}}{|\tau_{12}^{h_1 + h_2 - h_3} - \tau_{13}^{h_1 + h_3 - h_2} - \tau_{23}^{h_2 + h_3 - h_1}|}, \quad (4.2)$$

where the $O_i$, shorthand for $O_{hi}(\tau_i)$, are primary operators of dimension $h_i$, and the $C_{h_1 h_2 h_3}$ are structure constants.

To find the functional form of a four-point function, we combine two three-point functions, and integrate over one of the points,

$$\int d\tau_0 \langle O_1 O_2 O_{h_i}(\tau_0) \rangle \langle O_{1-h}(\tau_0) O_3 O_4 \rangle = \beta(h, h_{34}) F_{1234}^{h_1 h_2 h_3 h_4}(\tau_4) + \beta(1 - h, h_{12}) F_{1324}^{1-h_1 1-h_2 h_3 h_4}(\tau_4), \quad (4.3)$$

forming a conformal partial wave, which captures the exchange of $O_h$ and its descendants. Explicitly evaluating the integral yields the right-hand side: $\beta(h, h_{34})$ is a ratio of gamma functions, whose explicit form we have not written, and $F_{1234}^{h_1 h_2 h_3 h_4}(\tau_4)$ is identified as the conformal block, and contains a hypergeometric function of the conformally invariant cross ratio of the four times $\tau_1, \ldots, \tau_4$, and depends on the dimensions of the external operators, $h_1, \ldots, h_4$, and

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5 See footnote 13, and the start of section 5.1, respectively, in order to understand the latter two names.

6 The near-conformal symmetry means that the functional form of correlation functions will have, in addition to the conformal contributions, some pieces that involve mixing with the $h = 2$ mode. These are clearly distinguished from the conformal pieces, as they come with extra factors of $J$. Alternatively, there is a variant of SYK, cSYK [42], which is fully conformally invariant. See also footnote 13. We will only discuss the conformal contributions to correlation functions; it is straightforward to include the others.

7 $O_{1-h}$ is the shadow of $O_h$, and has dimensions $1 - h$, so (4.3) transforms as a four-point function.
the dimension $h$ of the exchanged operator. The conformal blocks form a basis, in terms of which one can express a general four-point function,

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_4 \rangle = \int_{\mathcal{C}} \frac{dh}{2\pi i} \rho(h) F_{1234}^h(\tau_i).$$  \hspace{1cm} (4.4)$$

The contour $\mathcal{C}$ runs parallel to the imaginary axis, $h = \frac{1}{2} + is$, and in addition has counterclockwise circles enclosing the positive even integers $h = 2n$. These are the principal and discrete series, respectively, of $SL_2(R)$.8

As an analogy, this expression for the four-point function is for the conformal group $SL_2(R)$ what the Fourier transform is for the translation group. Specifically, we may write any function of $x$ as,

$$f(x) = \int \frac{dp}{2\pi} f(p)e^{ipx}. \hspace{1cm} (4.5)$$

Here $e^{ipx}$ are a complete set of eigenfunctions of the Casimir of the translation group, $\partial^2_x$, while in (4.4), the conformal blocks $F_{1234}^h(\tau_i)$, with $h$ running over $\mathcal{C}$, are a complete set of eigenfunctions of the $SL_2(R)$ Casimir. Any CFT four-point function is completely specified by an analytic function $\rho(h)$. The poles and residues of $\rho(h)$ set the dimensions and OPE coefficients of the exchanged operators in the four-point function, as one can see by closing the contour in (4.4).

For theories with $O(N)$ symmetry, it is natural to study operators that have definite transformation under the action of $O(N)$. We will be interested in $O(N)$ singlets, such as9

$$\mathcal{O}_h = \frac{1}{N} \sum_{i=1}^N \chi_i \partial_x^{1+2n} \chi_i. \hspace{1cm} (4.6)$$

We will refer to such an operator as single-trace. One can make more $O(N)$ invariant operators, by taking products. For instance, a double-trace operator is schematically of the form $\mathcal{O}_h \partial_x^{2n} \mathcal{O}_h$.

### 4.2. SYK correlation functions

We discussed that the fermion two-point function is dominated by melons at large $N$. Now let us look at the Feynman diagrams contributing to a connected fermion 2k-point function.

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8For discussion of this in the context of SYK, see [27, 31, 43], as well as [44, 45]. For a more general discussion, see [46–50]. For a discussion of conformal partial waves, see [51].

9This is schematic; some of the derivatives should act on the left $\chi_i$ as well as on the right $\chi_i$, in a specific way, so as to ensure the operator is primary. For instance, for $n = 0$, one actually has $\chi_i \partial_x \chi_i - \partial_x \chi_i \chi_i$. Also, we have not included operators with an even number of derivatives, since their correlation functions will vanish, by fermion antisymmetry.
At leading order, these correlators will scale as $N^{-k(k-1)}$, and will involve each external index occurring in pairs. Connecting two such lines by a propagator gives a Feynman diagram contributing to a $2(k-1)$-point function. Therefore, the Feynman diagrams for a $2k$-point function are found by successively cutting melon diagrams.

A single cut gives the four-point function: it scales as $1/N$, and is a sum of ladder diagrams, shown in figure 3(a), with the kernel, shown in figure 3(b), adding rungs to the ladder. To perform the sum, one need only find the eigenfunctions and eigenvalues of the kernel. In fact, as a result of $SL(2)$ invariance, the eigenfunctions are just the conformal partial waves discussed earlier, labeled by the dimension $h$ of the exchanged operator. In this basis, the sum becomes a geometric sum, and the fermion four-point function is of the form given in (4.4), with $ho(h)$ \[27\]

\[
\rho(h) = \mu(h) \frac{\alpha_0}{2} \frac{k(h)}{1 - k(h)}, \quad k(h) = -\frac{3}{2} \tan \frac{\pi(h-1/2)}{h - \frac{1}{2}},
\]

where $k(h)$ are the eigenvalues of the kernel, while $\mu(h)$ is a simple measure factor and $\alpha_0$ is a constant, which we have not written explicitly\[11\]. As mentioned earlier, the poles and residues of $\rho(h)$ give the dimensions and OPE coefficients, $c_h$, of the exchanged operators. The poles occur at the $h$ for which $k(h) = 1$\[12\]. These can be written in the form, $h = 2n + 1 + 2\Delta + 2\epsilon_n$, with $\epsilon_n$ small for large integer $n$, and correspond to the dimensions of the single-trace operators (4.6) (in the infrared)\[13\].

The fermion six-point function consists of the diagrams shown in figure 4(a), and can be viewed as three four-point functions glued together. Since the four-point function is expressed

\[10\] More precisely, the fermion four-point given here is defined as the $1/N$ piece of $N^{-2} \sum_{\mu=1}^N \langle \chi_i(\tau_1) \chi_j(\tau_2) \chi_k(\tau_3) \chi_l(\tau_4) \rangle$.

\[11\] See equations (2.17) and (2.26) of [41] for the precise expression; in order to simplify, relative to the expression there, we have absorbed a factor of $\Gamma(h)^2/\Gamma(2h)$ into the definition of $\mu(h)$, and neglected a factor of $h^2/J^4$. Also, equation (4.8) on the next page is equation (4.16) of [41].

\[12\] The integral equation determining $k(h)$ is like a Bethe–Salpeter equation for conformal theories; instead of the masses of the bound states, it determines the dimensions $h$ of the composite operators. See [28]. In some places in the literature, $k(h)$ is instead denoted by $k(h)$, or by $g(h)$.

\[13\] The location of the smallest positive $h$ for which $k(h) = 1$ is $h = 2$. The $h = 2$ operator is special: it lies on the contour $\mathcal{C}$, and so leads to a divergence. This means we must move slightly away from the infinite limit. For cSYK [42], which is conformal at any value of $J$, the four-point function at large but finite $J$ is given by an expression similar to (4.7), but for which $k(h) = 1$ at an $h$ slightly less than two. This makes the contribution of this block finite but large. In SYK, moving to large but finite $J$ means breaking conformal invariance and accounting for the Schwarzyan action. The contribution of the $h = 2$ block will come with a factor of $J$, relative to the other blocks, and so it dominates the four-point function. The Lyapunov exponent from the $h = 2$ contribution is maximal [52], and since this piece dominates, SYK is maximally chaotic at large $J$ [2].
in terms of conformal blocks, computing the six-point function is just a matter of gluing together three conformal blocks. The information in the fermion six-point function can be compactly encoded in the three-point functions of the bilinear operators $O_h$ (4.6), with coefficients $C_{h_1 h_2 h_3}$, whose explicit form is given in [41]: they can be written as $C_{h_1 h_2 h_3} = c_{h_1} c_{h_2} c_{h_3} \mathcal{I}_{h_1 h_2 h_3}$, where $\mathcal{I}_{h_1 h_2 h_3}$ is an analytic function of the $h_i$ involving gamma functions and the hypergeometric function $_4F_3$ at argument one.

The fermion eight-point function is built out of more four-point functions glued together. One such contribution is shown in figure 4(b), and its contribution to the bilinear four-point function takes an incredibly simple form [41],

$$c_{h_1} c_{h_2} c_{h_3} c_{h_4} \int \frac{dh}{2\pi i} \rho(h) \mathcal{I}_{h h_2 h_3} \mathcal{I}_{h h_2 h_3} \mathcal{F}_{1234}^b(\tau_i).$$ (4.8)

The result is intuitive: the $\mathcal{I}_{h h_2 h_3}$ and $\mathcal{I}_{h h_2 h_3}$ are the ‘interaction vertices’ from the two six-point functions, and there is an operator of dimension $h$ exchanged, giving the $\rho(h)$ factor from the intermediate fermion four-point function. A nontrivial consistency check is that the four-point function of $O_{h_1}$, at order $1/N$, should be a sum of conformal blocks of exchanged single-trace operators as well as double-trace operators. Upon closing the contour in (4.8), the poles of $\rho(h)$ give the single-trace blocks, while the poles of $\mathcal{I}_{h h_2 h_3}$ occurring at $h = h_1 + h_2 + 2n$, give the double-trace blocks. It is remarkable that the analytically extended OPE coefficients of the single-trace operators—the $\mathcal{I}_{h h_2 h_3}$—knew that they should have singularities at precisely these locations.

While SYK has a special set of Feynman diagrams, these results for the correlation functions are more general. The simple expression for the fermion four-point function follows from summing ladder diagrams; it is irrelevant that the propagators are built from melons, those only served to give a conformal two-point function\textsuperscript{14}. The six-point function is made up of three four-point functions glued together, and in calculating it, it is not relevant that the four-point function was a sum of ladder diagrams. The expression for the eight-point function/bilinear four-point function is valid regardless of the details of how the three fermion four-point functions combine at the ‘interaction vertex’; all of this is encoded in $\mathcal{I}_{h h_2 h_3}$.

5. Applications

5.1. AdS/CFT

At low energies, SYK is dominated by the $h = 2$ mode, described by the Schwarzian action. This is a result of being nearly conformally invariant. On the AdS\textsubscript{2} side, since Einstein gravity is topological in two-dimensions, it is natural to instead consider Jackiw–Teitelboim dilaton gravity [54, 55]. Dilaton gravity theories naturally arise from compactifying gravity in higher dimensions down to two dimensions, with the dilaton playing the role of the size of the extra dimension. It has been shown that the dilaton theory in AdS\textsubscript{2} is the same as the Schwarzian theory, as a consequence of the pattern of symmetry breaking [32, 56, 57]\textsuperscript{15}.

Of course, the $h = 2$ mode is just the first in the tower of fermion bilinear $O(N)$ singlets, $O_h$, written schematically in (4.6), and the rest of the tower, with $n \geq 1$, encode the structure\textsuperscript{14} In fact, similar ladder diagrams appear in the fishnet theory, a deformation of $N = 4$ super Yang–Mills [53].

\textsuperscript{15} For further studies of two-dimensional gravity, AdS\textsubscript{2}, and the Schwarzian, see[58–67].
of SYK. As we have discussed, a connected $k$-point correlation function of the $O_h$ scales as $N^{-\frac{(k-2)}{2}}$. We can write a putative dual field theory in AdS$_2$,

$$L_{\text{bulk}} = \sum_{n=1}^{\infty} \frac{1}{2} (\partial \phi_n)^2 + \frac{1}{2} m_n^2 \phi_n^2 + \frac{1}{\sqrt{N}} \sum_{n,m,k=1}^{\infty} \lambda_{n,m,k} \phi_n \phi_m \phi_k + \ldots$$

containing a tower of scalar fields $\phi_n$. As a result of the $SL_2(R)$ isometry of AdS$_2$, any correlation function of the $\phi_n$, at points extrapolated to the boundary of AdS$_2$, will take the form of a CFT correlation function. Identifying each $\phi_n$ with an operator $O_h$, we can appropriately choose the masses and cubic couplings of the $\phi_n$ so as to match to the SYK two-point and three-point functions of the $O_h$, respectively. The masses are related to the dimensions in the standard way, $m_n^2 = h(h-1)$, while the cubic couplings, in the limit $n, m, k \gg 1$ where they simplify, are [41, 68],

$$\lambda_{n,m,k} \approx \frac{(n + m + k)!}{\Gamma(n + m - k + \frac{1}{2})\Gamma(m + k - n + \frac{1}{2})\Gamma(k + n - m + \frac{1}{2})}.$$  (5.2)
One could similarly try to appropriately choose the quartic couplings, so as to match the SYK four-point function of bilinears\textsuperscript{16}. However, in order to have an actual understanding of the AdS dual of SYK, one needs a simple and independently defined bulk theory, something like the string worldsheet action, rather than just a list of couplings. Such a bulk description is presently lacking; it is not obvious one must exist\textsuperscript{17}.

Two canonical examples of AdS/CFT duality are between $\mathcal{N} = 4$ super Yang–Mills in 4 dimensions and string theory in AdS$_5 \times S^5$ [77–79], and between the free/critical vector $O(N)$ model in 3 dimensions and Vasiliev higher spin theory in AdS$_4$ [80, 81]. A comparison between SYK and these two theories is given in figure 5.

One hope for SYK has been that, because of its simplicity, it would provide an example of AdS/CFT in which one could fully understand the duality, directly relating the CFT degrees of freedom to the bulk variables. This remains a goal, though achieving it of course requires knowing what the bulk theory is, in order to have a target. Independently of this, SYK has led to a renewed interest in two-dimensional gravity, and the formulation of modern ideas on spacetime and holography in this context, see e.g. [82–91].

5.2. Strange metals

There are many variants of SYK, which retain the key feature of dominance of melon diagrams. One natural generalization, which incorporates some of these, is to consider a model which contains $f$ flavors of fermions, with $N_a$ fermions of flavor $a$, each appearing $q_a$ times in the interaction, so that the Hamiltonian couples $q = \sum_{a=1}^{f} q_a$ fermions together [28].

\textsuperscript{16} The leading connected SYK correlators that we computed in the previous section map onto the tree-level Witten diagrams. The $1/\sqrt{N}$ corrections to these would map onto loops in the bulk, and so are not needed in order to establish the classical bulk Lagrangian (5.1).

\textsuperscript{17} Some proposals are as follows: A single scalar field Kaluza–Klein reduced on an AdS$_2 \times S^1$, or something like it, can be made to give the correct spectrum of masses but gives the wrong cubic couplings [41, 69]. A string with longitudinal motion [27, 70] also gives a qualitatively correct spectrum, but such solutions are only known at the classical level; one would need a quantum theory, in order to determine the cubic couplings. The ’t Hooft model of two-dimensional QCD, but placed in AdS, is perhaps the most promising, but is difficult to solve [71], and would at best match SYK only qualitatively, with no \textit{a priori} reason it should match exactly. One might instead search for the bulk dual of the tensor models, rather than SYK, but tensor models have a vast number of singlets, and the bulk dual would correspondingly have a huge number of fields and a Hagedorn temperature scaling as $1/\log N$ [72–74], see also [75], and the bulk description would likely be even more complicated than Vasiliev theory [76].
where \( I \) is a collective index, \( I = i_1, \ldots, i_q, j_1, \ldots, j_q \). The coupling \( J_I \) is antisymmetric under permutation of indices within any one of the \( f \) families, and is drawn from a Gaussian distribution. This model with one flavor, \( f = 1 \), reduces to the standard SYK model with a \( q \)-body interaction, sometimes denoted by \( \text{SYK}_q \); further setting \( q = 4 \) gives the canonical SYK model, which has been the focus of these notes\(^{18} \).

Regarding flavor as a lattice site index, \( x \), and taking sums of the flavored models, one can build lattices of SYK models\(^{109} \). One such model, having some features of a strongly correlated metal, is\(^{110} \),

\[
L = \frac{1}{2} \sum_{x} \sum_{i} c_{i,x}^\dagger (\partial_\tau - \mu) c_{i,x} - \sum_{x',x} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'} - \sum_{x} \sum_{i,j,k,l} J_{ijkl,xx} c_{i,x}^\dagger c_{j,xx}^\dagger c_{k,x} c_{l,xx},
\]

(5.4)

Here \( t_{ij,xx'} \) and \( J_{ijkl,xx} \) are again random couplings, and the \( c_{i,x} \) are now complex fermions. A cartoon of this model is shown in figure 6. The model exhibits incoherent metal behavior, with resistivity scaling linearly with temperature, at high temperature, and Fermi liquid behavior at low temperature\(^{19} \).

More generally, the fact that SYK is a system without quasiparticles, yet is nevertheless solvable, makes it a valuable tool with which to study transport and chaos\(^{117–125} \), non-equilibrium dynamics and entanglement\(^{126–129} \), and eigenstate thermalization\(^{130–132} \). There are limitations, however, as neither the all-to-all interactions nor the large \( N \), which are essential to the solvability of SYK, are present in real metals.

Finally, there are a number of topics which we have not discussed, such as: experimental realizations and quantum simulations of SYK\(^{133–139} \), the zero-temperature entropy (an infinite \( N \) artifact)\(^{140–142} \), studies of the spectral density, the spectral form factor, and connections with random matrix theory\(^{83, 143–153} \).
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