NLO Higgs+jet production at large transverse momenta including top quark mass effects

Tobias Neumann

Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616, United States of America

Fermilab, PO Box 500, Batavia, Illinois 60510, United States of America

E-mail: tneumann@fnal.gov

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Abstract

We present a next-to-leading order calculation of $H+\text{jet}$ in gluon fusion including the effect of a finite top quark mass $m_t$ at large transverse momenta. Using the recently published two-loop amplitudes in the high energy expansion and our previous setup that includes finite $m_t$ effects in a low energy expansion, we are able to obtain $m_t$-finite results for transverse momenta below 225 GeV and above 500 GeV with negligible remaining top quark mass uncertainty. The only remaining region that has to rely on the common leading order rescaling approach is the threshold region $\sqrt{s} \simeq 2m_t$. We demonstrate that this rescaling provides an excellent approximation in the high $p_T$ region. Our calculation settles the issue of top quark mass effects at large transverse momenta. It is implemented in the parton level Monte Carlo code MCFM and is publicly available immediately in version 8.2.

1. Introduction

With the Higgs boson discovery at the Large Hadron Collider (LHC) [1, 2] setting a milestone for physics research, the hunt for signals beyond those described by the Standard Model (SM) has been more active than ever. Early Higgs studies during Run I, limited by statistics and energy, probed rather inclusive properties, and no significant deviations from the SM have been found [1–7]. Differential Higgs measurements [8–14] testing the SM were limited by statistics rather than theory predictions. Recent experimental analyses consider the Higgs boson in a highly boosted regime with transverse momenta below 250 GeV and above 500 GeV with negligible remaining top quark mass uncertainty. The only remaining region that has to rely on the common leading order rescaling approach is the threshold region $\sqrt{s} \simeq 2m_t$. We demonstrate that this rescaling provides an excellent approximation in the high $p_T$ region. Our calculation settles the issue of top quark mass effects at large transverse momenta. It is implemented in the parton level Monte Carlo code MCFM and is publicly available immediately in version 8.2.

Unfortunately the operator used in this EFT description is the same operator of dimension six that appears at large transverse energies was the dependence on the effective field theory (EFT) description. In this EFT approach the top quark is integrated out as a heavy particle to circumvent the calculation of complicated massive two-loop integrals. It is strictly valid only in the region of energies small compared to $m_t$.

To constrain physics beyond the Standard Model (BSM) using gluon fusion Higgs production, one of the most promising approaches is to consider large transverse energies [17–30]. A common limitation of all higher order Higgs+jet calculations in the region of large transverse energies was the dependence on the effective field theory (EFT) description. In this EFT approach the top quark is integrated out as a heavy particle to circumvent the calculation of complicated massive two-loop integrals. It is strictly valid only in the region of energies small compared to $m_t$.

Unfortunately the operator used in this EFT description is the same operator of dimension six that appears at leading order in the Standard Model EFT to search for new physics in a modified Higgs-gluon coupling. Thus the only reliable way to directly disentangle SM gluon fusion from heavy BSM contributions requires computing the full top quark mass dependence at large energies or at least a sufficient approximation beyond its dependence on the EFT description. It is this region, in which finite top quark mass effects were unconstrained at NLO, which we consider in our study. We give percent level accurate predictions through the use of a high energy expansion of the missing two-loop amplitudes [31].

While finalizing our manuscript we became aware of a study similar to ours in ref. [32]. Additionally, results based on a fully numerical evaluation of the two-loop integrals with full top quark mass dependence have been presented in [33]. We still believe that our study and implementation are useful for large transverse momenta, where the difference to the full calculation should be negligible and the full calculation seems to be numerically challenging for large transverse momenta.

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An additional aspect to constrain BSM physics through precision SM predictions comes from the shape of kinematical distributions and not just through their absolute magnitude. Specifically we evaluate in this study whether top quark mass effects distort the shape of the Higgs transverse momentum distribution at high energies compared to previous approximations.

Throughout the years various approximations beyond the pure EFT description have been used, which we will briefly summarize here. Gluon fusion induced Higgs production mediated through a massive top quark loop was calculated at LO a long time ago [34], but the difficulty is considerably amplified at NLO and in Higgs production with a jet, where massive two-loop amplitudes have to be calculated. An efficient analytical evaluation of these integrals for Higgs+jet is currently not within reach [35, 36]. Fortunately, at least for the region of transverse momenta below the top-quark threshold $p_T \ll 2m_t$, the EFT approach turned out to provide an excellent approximation of perturbative corrections even for differential Higgs+jet quantities [37, 38]. This was shown using a low energy expansion below the top quark threshold, and as such can not constrain the effects of a finite top quark mass for large energies. Going beyond estimations of top quark mass effects, studies with predictions including partially exact amplitudes were performed [39–42], but were always limited by low energy approximations in other parts. And only the EFT approach, which reduces the needed number of loops to evaluate by one, allowed for an evaluation of Higgs+jet at NNLO [43–47]. Residual perturbative uncertainties estimated by renormalization and factorization scale variation are estimated to be about 10% at NNLO, cutting in half the estimated NLO uncertainty.

Studies targeting top quark mass effects specifically in the large transverse momentum region of Higgs production were performed using resummation [49], and by using a factorization of the mass scale from $p_T$ in the high $p_T$ limit [50]. In the latter case the developed formalism was only applied to the subprocess $q\bar{q}$ at leading order in $1/p_T$ and leading order in $\alpha_s$. The former study only improves the NLO calculation through a rescaling $k$-factor and not directly, as their high energy approximation was shown to deviate from the exact result at LO. Their suggested approach is to rescale exact LO results by a $k$-factor obtained in their high energy approximation. A different approach is to match different hard-jet multiplicities and parton showers [26, 41].

It is the goal of this study to extend our previous setup [42], publicly available in MCFM [51–53], which provides finite $m_t$ effects in the region $p_T \ll 2m_t$, to predictions with a finite $m_t$ for $p_T \gg 2m_t$. We also study the validity of the common LO rescaling approach in the region of high $p_T$. To do this we implement the recently published two-loop amplitudes in the high energy expansion [31].

### 2. Calculation

Our calculation is based on the previous NLO Higgs+jet setup in MCFM-8.1 [42]. This previous setup uses an asymptotic expansion in $\Lambda/(2m_t)$ only for the finite part of the virtual two-loop amplitudes, but is exact in the top quark mass otherwise. Here $\Lambda$ is a placeholder for all kinematical scales of the process. For Higgs $p_T$ smaller than $\approx 225$ GeV the asymptotic expansion was shown to be convergent and provides an excellent approximation of the full top quark mass dependence. For energies larger than $\approx 300$ GeV the expansion breaks down and finite top quark mass effects could become larger than 8%, such that either a full calculation is necessary or another approximation is needed for sufficiently large $p_T$. Here, we fill this gap for the latter case.

We have implemented the one- and two-loop Higgs plus three parton helicity amplitudes in the high energy expansion from [31]. The expansion is performed in $\kappa \equiv -(m_t^2/3)^4$ to order $k = 1$ while retaining an expansion in $\eta \equiv -(m_H^2/(4m_t^2))^l$ only to first order $l = 0$. Here $m_H$ is the Higgs mass and $\ell$ the partonic center of mass energy. The amplitudes are given as the finite parts after UV renormalization and Catani IR subtraction [54] using $d = 4 - 2\epsilon$ Born one-loop amplitudes. We have performed a conversion to the 'tHooft-Veltman scheme for use in MCFM and additionally restored the renormalization scale dependence.

At LO Higgs+jet relies on one-loop amplitudes and is known with the exact top quark mass dependence, which allows us to compare it with the result from the high energy expansion. This gives an estimate on how far we can trust the approximation when using the two-loop amplitudes. Having established trust in the validity of the two-loop amplitudes in the high $p_T$ region we can then compare the results with the Born-rescaling approximation. In this rescaling approximation the finite part of the two-loop virtual amplitude is point-wise rescaled by the Born amplitude in the full theory divided by the Born amplitude in the EFT.

For our study we choose a center of mass energy of $\sqrt{s} = 13$ TeV and a common renormalization and factorization scale of $\mu_R = \mu_F = \sqrt{m_H^2 + p_T^2}$, where $m_H = 125.0$ GeV and $p_T$ is the Higgs transverse momentum. Although the region of high $p_T$ motivates using the six-flavor scheme, no matching parton distribution functions (PDFs) are available. So for consistency we work in the five-flavor scheme with an on-shell top quark mass of $m_t = 173.2$ GeV. We use CT14 PDFs [55] at NLO accuracy for the NLO cross section and...

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2 For a recent overview of Higgs production and decay cross sections we refer to [48].
at LO accuracy for our LO results. The value of $\alpha_s$ is given at the according order by the PDF set. Finally, we require at least one jet clustered with the anti-\(k_T\) jet algorithm with $p_{T,\text{jet}} > 30 \text{ GeV}$, $|\eta_{\text{jet}}| < 2.4$ and $R = 0.5$.

3. Results

The first question one has to ask is in how far one can trust the two-loop high energy amplitudes to describe the exact $m_t$ dependence. In lack of the $m_t$-exact two-loop amplitudes for comparison, one has to resort to a different method to evaluate this trust. For example, one could observe a convergent behavior of the expansion, but this would require some higher expansion order than is available as we will see below. Instead we can study how well the expansion works at LO. This has some limitations though, as will be discussed below. To study the high energy expansion we consider figure 1: shown is the LO Higgs transverse momentum distribution in various approximations normalized to the distribution with exact $m_t$-dependence. The approximations shown are the low energy expansions up to order $m^k_{1/2}$ for $k = 0, 2, 4$, where $k = 0$ describes the EFT, as well as the high energy expansion.

The amplitudes in the high energy expansion are given up to order $m^k_{1/2}$ for $k = 0, 2, 4, 6$. Naively using them for the LO cross section includes partial effects of order $\kappa^2$. This is labeled in the plot as 'high, partial $m^k_{1/2}$'. Only including $m^2_t$ terms in the cross section is labeled with 'high $m^2_t$'. By the same argument given above, one would expect the high energy approximation to work better for one-loop diagrams than for two-loop diagrams.

For the low energy expansion the convergence is poor and is practically non-existent beyond about $100 \text{ GeV}$. At NLO though, using only the expansion in the two-loop amplitude, the region of convergence increases to about $250 \text{ GeV}$ as shown in [42]. This can also be seen in figure 2. A simple explanation is that for two-loop diagrams the topology does not force all center of mass energy to go through the top quark loop, such that $m^2_t$ threshold effects are further washed out.

The amplitudes in the high energy expansion are given up to order $\kappa^2 \equiv (m^2_{1/2}/\kappa)^2$. Naively using them for the LO cross section includes partial effects of order $\kappa^2$. This is labeled in the plot as 'high, partial $m^k_{1/2}$'. Only including $m^2_t$ terms in the cross section is labeled with 'high $m^2_t$'. By the same argument given above, one would expect the high energy approximation to work better for one-loop diagrams than for two-loop diagrams.

Nevertheless the difference between using the full $\mathcal{O}(\kappa^2)$ amplitudes and the $m_t$-exact result is less than two percent beyond $500 \text{ GeV}$, which gives motivation to trust that the two-loop high energy amplitudes describe the full top quark mass dependence at a similar level. Considering that the NLO scale uncertainty is about $20\%$ [42] and still about $10\%$ at NNLO [45, 47], any remaining top quark mass uncertainty can then be considered negligible. Ideally a more precise estimate could be established by including full $\mathcal{O}(\kappa^2)$ and $\mathcal{O}(\kappa^3)$ terms, possibly higher order terms in $\eta$ for the one- and two-loop amplitudes.

Having shown that the large energy expansion describes the full LO result at percent level accuracy beyond $500 \text{ GeV}$, we expect a similar behavior for the two-loop amplitudes. At NLO the two-loop amplitudes additionally only enter as the virtual corrections and a bulk of the perturbative corrections at large $p_T$ comes from the real emission which we include with full $m_t$ dependence. The error from using the large energy expansion estimated at LO should thus be conservative.

At NLO we show the Higgs $p_T$ distribution in figure 2 with a normalization to the distribution using rescaled EFT two-loop amplitudes. Here, to emphasize again, only the finite part of the two-loop virtual corrections is not exact in $m_t$, and is approximated in different ways. It is obtained using either a $1/m_t$ expansion in the region of small $p_T$, or in the high energy expansion up to order $\kappa^2$ for large $p_T$. Additionally, using the rescaling
approach as described in section 2 we obtain an approximation that can be used over the whole range of $p_T$ and also serves as the overall normalization. The latter approach was used for example in [41] and shown to agree with the low energy asymptotic expansion at the percent level for $p_T \lesssim 225$ GeV [42].

We present the distributions normalized to the EFT rescaled approximation and not as absolute distributions, since we are strongly interested in possible shape corrections due to using a finite top quark mass. In this way one can easily compare the additional corrections to the previous best approximation at high $p_T$ in percent. The scale variation uncertainty of about 20% changes only little with respect to the EFT result and other approximations, and can be found for example in our previous study [42].

The low energy expansion indeed extends its convergent behavior to about 250 GeV with corrections of less than a percent compared to the rescaling approach. The high energy expansion, which we believe approximates the full result by better than 2%, is consistent with the corrections at low $p_T$ and increases the cross section by about 1%–2% compared to the rescaled result. It is remarkable that these top quark mass corrections are flat within 1% over the whole range of large $p_T$. In this sense one is free to choose either the rescaling approach or the high energy approximation. Nevertheless the high energy approximation is a systematic approach, whereas the rescaling approach was done ad hoc without prior validation. We thus recommend to use the high energy approximation for transverse momenta beyond $\sim 500$ GeV.

Finally we would like comment on the case where the whole NLO distribution is evaluated in the EFT and bin-wise rescaled by the ratio of the LO distribution in full theory to the LO distribution in the EFT. This is an approach that is still typically done in many experimental and phenomenological analyses. We find that at low $p_T \lesssim 300$ GeV this approach overestimates the cross-section by about 2%–6%, while at large $p_T \gtrsim 500$ GeV it underestimates the cross-section by about 3%–5% percent.

4. Conclusions

We have presented a NLO calculation of Higgs + jet with negligible remaining top quark mass uncertainty in the region of low transverse momenta $p_T \lesssim 225$ GeV [42] and, as shown in this analysis, also in the region of large transverse momenta $p_T \gtrsim 500$ GeV. We have demonstrated that using the high energy expansion in the finite part of the two-loop amplitude, instead of rescaling it by the $m_t$-exact Born amplitude, results in a difference of less than two percent for $p_T \gtrsim 500$ GeV. The high energy expansion is asymptotically correct and at LO shows similarly small single percent level differences to the exact result. Considering additionally that for our NLO result only the finite part of the two-loop virtual amplitudes requires this expansion, we have established considerable trust in the validity of our result to approximate the full NLO result by better than a few percent. The elimination of the top-quark mass uncertainty at high $p_T$ at NLO now allows one to rescale NNLO results obtained in the EFT by a NLO $k$-factor NLO($m_t$)/NLO(EFT). Our implementation is publicly available immediately in MCFM-8.2.

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ORCID iDs

Tobias Neumann  https://orcid.org/0000-0003-1881-2894

References

[1] Khachatryan V et al (CMS) 2015 Eur. Phys. J. C 75 212
[2] Aad G et al (ATLAS) 2016 Eur. Phys. J. C 76 6
[3] Aad G et al (ATLAS) 2015 Phys. Rev. Lett. 115 091801
[4] Aad G et al (ATLAS) 2015 Eur. Phys. J. C 75 476
[5] Aad G et al (ATLAS) 2016 Eur. Phys. J. C 76 152 (Erratum)
[6] Aad G et al (ATLAS, CMS) 2015 Phys. Rev. Lett. 114 191803
[7] Grojean C, Salvioni E, Schlaffer M and Weiler A 2014 J. High Energy Phys. JHEP08(2014)043
[8] Khachatryan V et al (CMS) 2016 J. High Energy Phys. JHEP08(2016)104
[9] Aad G et al (ATLAS) 2016 Phys. Lett. B 753 69
[10] Aad G et al (ATLAS) 2014 Phys. Rev. Lett. B 738 234
[11] Aad G et al (ATLAS) 2014 J. High Energy Phys. JHEP09(2014)112
[12] Khachatryan V et al (CMS) 2016 J. High Energy Phys. JHEP04(2016)005
[13] Khachatryan V et al (CMS) 2016 Eur. Phys. J. C 76 13
[14] Khachatryan V et al (CMS) 2017 J. High Energy Phys. JHEP03(2017)032
[15] Sirunyan A M et al (CMS) 2017 Phys. Rev. Lett. 120 071802
[16] Vernieri C and CMS) 2017 European Physical Society Conference on High Energy Physics 349
[17] Bagnaschi E, Degrasii G, Slavich P and Vicini A 2012 J. High Energy Phys. JHEP02(2012)088
[18] Harlander R V and Neumann T 2013 Phys. Rev. D 88 074015
[19] Dawson S, Lewis I M and Zeng M 2014 Phys. Rev. D 90 093007
[20] Dawson S, Lewis I M and Zeng M 2015 Phys. Rev. D 91 074012
[21] Banfi A, Martin A and Sanz V 2014 J. High Energy Phys. JHEP08(2014)053
[22] Azatov A and Paul A 2014 J. High Energy Phys. JHEP01(2014)014
[23] Grojean C, Salvioni E, Schlaffer M and Weiler A 2014 J. High Energy Phys. JHEP09(2014)022
[24] Schlaffer M, Spannowsky M, Takeuchi M, Weiler A and Wyman C 2014 Eur. Phys. J. C 74 3120
[25] Buschmann M, Englert C, Goncalves D, Plehn T and Spannowsky M 2014 Phys. Rev. D 90 013010
[26] Buschmann M, Goncalves D, Kuttalimai S, Schonher M, Krauss F and Plehn T 2015 J. High Energy Phys. JHEP02(2015)038
[27] Langenegger U, Spira M and Strebel I 2015 arXiv:1507.01373
[28] Ghosh D and Wiebusch M 2015 Phys. Rev. D 91 031701
[29] Grazzini M, Ilincic A, Spira M and Wiesemann M 2017 J. High Energy Phys. JHEP03(2017)115
[30] Cohej J, Bar-Shalom S, Eilam G and Soni A 2017 Phys. Rev. D 97 053014
[31] Kudashkin K, Melnikov K and Wever C 2018 J. High Energy Phys. JHEP02(2018)015
[32] Lindert J M, Kudashkin K, Melnikov K and Wever C 2018 Phys. Lett. B 755 69
[33] Jones SP, Kerner M and Lusiani G 2018 Phys. Rev. Lett. 120 162001
[34] Ellis R K, Hinchliffe I, Soldate M and van der Bij J 1988 Nucl. Phys. B 297 221
[35] Bonciani R, Del Duca V, Frellesvig H, Jenn M, Moriello F and Smirnov V A 2016 J. High Energy Phys. JHEP12(2016)096
[36] Boggia M et al 2018 J. Phys. G 45 065004
[37] Harlander R V, Neumann T, Ozeren K J and Wiesemann M 2012 J. High Energy Phys. JHEP08(2012)139
[38] Neumann T and Wiesemann M 2014 J. High Energy Phys. JHEP11(2014)130
[39] Bani A, Monni P F and Zanderighi G 2014 J. High Energy Phys. JHEP01(2014)097
[40] Hamilton K, Nason P and Zanderighi G 2015 J. High Energy Phys. JHEP05(2015)140
[41] Frederix R, Frixione S, Vryonidou E and Wiesemann M 2016 J. High Energy Phys. JHEP08(2016)006
[42] Neumann T and Williams C 2017 Phys. Rev. D 95 014004
[43] Boughezal R, Coaia F, Melnikov K, Petriello F and Schulze M 2013 J. High Energy Phys. JHEP06(2013)072
[44] Chen X, Gehrmann T, Glover E W N and Jaquier M 2015 Phys. Lett. B 740 147
[45] Boughezal R, Focke C, Giele W, Liu X and Petriello F 2015 Phys. Lett. B 748 8
[46] Boughezal R, Coaia F, Melnikov K, Petriello F and Schulze M 2015 Phys. Rev. Lett. 115 082003
[47] Chen X, Cruz-Martinez J, Gehrmann T, Glover E W N and Jaquier M 2016 J. High Energy Phys. JHEP10(2016)066
[48] Spira M 2017 Prog. Part. Nucl. Phys. 95 98
[49] Coaia F, Forte S, Marzani S, Muselli C and Vita G 2016 J. High Energy Phys. JHEP08(2016)150
[50] Braaten E, Zhang H and Zhang J-W 2017 J. High Energy Phys. JHEP11(2017)127
[51] Campbell J M and Ellis R K 1999 Phys. Rev. D 60 113006
[52] Campbell J M, Ellis R K and Williams C 2011 J. High Energy Phys. JHEP07(2011)018
[53] Campbell J M, Ellis R K and Giele W T 2015 Eur. Phys. J. C 75 246
[54] Catani S 1998 Phys. Lett. B 427 161
[55] Dulat S, Hou T-J, Gao J, Guzzi M, Huston J, Nadolsky P, Pumplin J, Schmidt C, Stump D and Yuan C P 2016 Phys. Rev. D 93 033006