Abstract. The new approach to quantum mechanical problems is proposed. Quantum states are represented in an algebraic program, by lists of variable length, while operators are well defined functions on these lists. Complete numerical solution of a given system can then be automatically obtained. The method is applied to Wess-Zumino quantum mechanics and $D=2$ and $D=4$ supersymmetric Yang-Mills quantum mechanics with the SU(2) gauge group. Convergence with increasing size of the basis was observed in various cases. Many old results were confirmed and some new ones, especially for the $D=4$ system, are derived. Preliminary results in higher dimensions are also presented. In particular the spectrum of the zero-volume glueballs in $4 < D < 11$ is obtained for the first time.

1. Introduction

Supersymmetric quantum mechanics attracts a lot of attention for many reasons. First, it describes well defined, finite dimensional quantum systems which are ideal laboratory to study supersymmetry in various disguises \cite{1, 2, 3}. Second, the conjecture of Banks, Fischler, Shenker and Susskind triggers additional interest, in particular in the $D=10$ supersymmetric Yang-Mills quantum mechanics (SYMQM) as a model of M-theory \cite{4}. The latter system has been studied by many authors also in lower dimensions and for a variety of gauge groups. It is expected to have a continuous spectrum due to the supersymmetry driven cancellations of zero energy fluctuations \cite{5}. This is in contrast to its non supersymmetric version, which for $D=4$ becomes the zero volume limit of the well known pure Yang-Mills gluodynamics \cite{6, 7, 8}. Remarkably, in ten (and not less than ten) Euclidean dimensions the continuous spectrum of the supersymmetric system contains also a localized zero energy state. Existence of such a threshold bound state – the supergraviton – is considered as a \textit{sine qua non} condition for the BFSS hypothesis \cite{9}. Hence establishing it became an arena of intensive and ingenious studies \cite{10, 11}. Interesting numerical methods were also developed \cite{12}. The large $N$ limit of SYMQM was studied in the mean field approximation, and interesting realizations of the black
hole thermodynamics were observed [13]. Simplified (i.e. quenched, D=4) systems were also studied by lattice Monte Carlo methods for intermediate size groups [14]. The asymptotic behavior in \( N \) was observed and a possible evidence for the two phase structure was found.

Many interesting results have been obtained studying fully reduced (to one point) matrix model (for a review see e.g. [15]). In particular, the notorious sign problem, which has been hindering Monte Carlo studies may have been recently reduced [16].

In these lectures I will present a new approach to study quantum mechanical systems [17]. The standard hamiltonian formulation of quantum mechanics will be implemented in the computer code with the vectors in Hilbert space represented by Mathematica lists with a flexible, dynamically varying size. Quantum operators become well defined and simple functions on these lists. Fermionic degrees of freedom are easily included. In practical applications one has to limit the size of the Hilbert space. However dependence on the cut-off can be monitored, and the infinite cut-off limit extracted in many physically relevant cases. Of course the method becomes computationally demanding for larger systems. Until now it has proven applicable for up to 27 degrees of freedom. We begin with simple two-dimensional examples: Wess-Zumino quantum mechanics and D=2 SYM quantum mechanics. Then the new results for yet unsolved D=4 SYMQM will be summarized. Finally preliminary findings for higher dimensions, including D=10, will be presented. More complete account of most of these results is in Ref[17].

General hamiltonian methods have been applied before to complete, space extended field theories [18]. Recently Matsumura and collaborators [19] and Pinsky et al. have studied with this technique a variety of partly reduced, supersymmetric theories in lower dimensions (see Refs.[20] and references therein).

2. Quantum mechanics in a PC

Action of any quantum mechanical observable can be efficiently implemented in an algebraic program if we use the discrete eigen basis of the occupation number operator \( a^\dagger a \)

\[
\{|n>\}, \quad |n> = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0>.
\] (1)

The bosonic coordinate and momentum operators are

\[
x = \frac{1}{\sqrt{2}} (a + a^\dagger), \quad p = \frac{1}{i\sqrt{2}} (a - a^\dagger),
\] (2)
and a typical quantum observable can be represented as the multiple actions of the basic creation and annihilation operators. \(^1\) Fermionic observables will be discussed in subsequent Sections. Generalization to more degrees of freedom is evident and will be done there.

Any quantum state is a superposition of arbitrary number, \(n_s\), of elementary states \(|n>\)

\[
|st> = \sum_I^n a_I |n^{(I)}>,
\]

and will be represented as a Mathematica list

\[
st = \{n_s, \{a_1, \ldots, a_{n_s}\}, \{n^{(1)}\}, \{n^{(2)}\}, \ldots, \{n^{(n_s)}\}\},
\]

with \(n_s + 2\) elements. The first element specifies the number of elementary states entering the linear combination, Eq.(3), the second is the sublist supplying all complex amplitudes \(a_I, I = 1, \ldots, n_s\), and the remaining \(n_s\) sublists give the occupation numbers of elementary, basis states. In particular, an elementary state \(|n>\) is represented by \(\{1, \{1\}, \{n\}\}\).

Next we implement basic operations defined in the Hilbert space: addition of two states, multiplication by a number and the scalar product. All these can be simply programmed as definite operations on Mathematica lists transforming them in accord with the principles of quantum mechanics. It is now easy to define the creation and annihilation operators which act as a list-valued functions on above lists. This also defines the action of the position and momentum operators according to Eq.(2). Then we proceed to define any quantum observables of interest: hamiltonian, angular momentum, generators of gauge transformations, supersymmetry generators, etc.

Now our strategy is clear: given a particular system, define the list corresponding to the empty state, then generate a finite basis of \(N_{\text{cut}}\) vectors and calculate matrix representations of the hamiltonian and other quantum operators using above rules. Given that, the complete spectrum and its various symmetry properties is obtained by the numerical diagonalization.

Dependence of our results on the cut-off \(N_{\text{cut}}\) can be monitored. In many systems studied so far one can extract meaningful (i.e. \(N_{\text{cut}} = \infty\)) results before the size of the basis becomes unmanageable.

3. Two two-dimensional systems

Wess-Zumino quantum mechanics (WZQM) has one complex bosonic variable \(\phi(t) = x(t) + iy(t) \equiv x_1(t) + ix_2(t)\) and two complex Grassmann-valued fermions \(\psi_\alpha(t), \ \alpha = 1, 2\) \(^2\). The hamiltonian reads (with the mass and

\(^1\) The method can be also extended to non polynomial potentials.
the coupling set to 1)

\[ H = \frac{1}{2}(p_x^2 + p_y^2) + |\phi + \phi'^2|^2 + ((1 + 2\phi)\psi_1\psi_2 + h.c.) , \]

(5)

Bosonic creation and annihilation operators are introduced as in Eq.(3) for each (real) degree of freedom. Fermionic ones \( f_\alpha, f'_\alpha \) can be chosen so that \( \psi_\alpha = f_\alpha, \psi'^_\alpha = f'^_\alpha \) in this case \( \psi \). With these creation operators we generate the basis which contains all (here up to two) fermionic quanta and maximum \( N_{cut} \) bosonic quanta. The size of such a basis is then \( 2(N_{cut} + 1)(N_{cut} + 2) \). With the rules of the previous section one then easily calculates matrix representation of the hamiltonian (5) in above basis. The spectrum obtained by numerical diagonalization shows rather fast restoration of the supersymmetry: lowest state tends to zero and higher bosonic and fermionic states become degenerate around \( N_{cut} \sim 10 \). All this is summarized by the Witten index, Fig. 1, which is simply calculated in the energy eigen basis.

![Figure 1. Witten index for Wess-Zumino quantum mechanics for \( 4 < N_{cut} < 15 \).](image)

Due to the additional symmetry in the model all states are doubly degenerate, hence the exact value \( I_W(T) = 2 \).

\( D=2 \) supersymmetric \( SU(2) \) Yang-Mills quantum mechanics. This system has gauge invariance and continuous spectrum, both of which pose new challenge for our approach. It is described by three real bosonic and three fermionic variables \( x_\alpha(t), \psi_\alpha(t), \alpha = 1, 2, 3 \). The hamiltonian reads\(^3\)

\[ H = \frac{1}{2}p_\alpha p_\alpha + ig\epsilon_{abc}\psi'^_\alpha x_b\psi_c = \frac{1}{2}p_\alpha p_\alpha + gx_\alpha G_\alpha, \]

(6)

with the \( SU(2) \) gauge generators \( G_\alpha = \epsilon_{abc}(x_b p_c - i\psi'^_b\psi_c) \). To satisfy the gauge invariance we construct the complete basis from gauge invariant states. To this end we use four independent gauge invariant creators: \( (aa) \equiv \]

\(^2\) For the details of the implementation of fermionic operators see [17].
\[a^\dagger_a a^\dagger_a, (af) \equiv a^\dagger_a f^\dagger, (aff) \equiv \epsilon_{abc} a^\dagger_b f^\dagger_c f^\dagger_c, \text{ and } (fff) \equiv \epsilon_{abc} f^\dagger_a f^\dagger_b f^\dagger_c.\] Since the fermionic number is conserved, c.f. Eq.(6), the last three operators (and the identity) acting on the empty state generate the four "base" states for four sectors of the Hilbert space. All higher states are then obtained by the recursive action of the \((aa)\) creator only. In this scheme (referred to as the \(4+4+...\) scheme) the size of the basis is \(4N_{cut}\). Matrix representation of the Hamiltonian and the spectrum are then automatically obtained as before. The spectrum converges towards the supersymmetric multiplets and zero-energy ground state, however this time the convergence with the cut-off is slower. This is the feature of the continuous spectrum (c.f. next Section).

Interestingly there exists a scheme of increasing the basis \((2+4+4+...\)) where the spectrum of the anticommutator of SUSY generators \(\{Q, \bar{Q}\}\) reveals the exact supersymmetry at every finite cut-off [19, 20, 17]. Since the latter converges to the Hamiltonian at infinite \(N_{cut}\) one is free to declare it as the finite cut-off energy operator.

Witten index vanishes identically due to the particle-hole symmetry. However one can define the Witten index restricted only to the two fermionic sectors \(F = 0, 1\) say which are balanced by supersymmetry. Such index does not vanish and carries nontrivial information. Very recently M. Campostrini has calculated the index with much higher cut-off than in [17] (see Fig.2) confirming that it tends to 1/2 at all \(T\). If further established, this would provide an example of the continuous spectrum with time independent Witten index [10].

![Figure 2. Restricted Witten index for D=2 SYMQM with cut-offs \(N_{cut} < 100\).](image)

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3 I thank M. Campostrini for providing this figure prior to the publication.
4. D=4 supersymmetric SU(2) Yang-Mills quantum mechanics

Dimensionally reduced (in space) system is described by nine bosonic coordinates \( x_i^a(t), \ i = 1, 2, 3; a = 1, 2, 3 \) and six independent fermionic coordinates contained in the Majorana spinor \( \psi^\alpha_a(t), \ \alpha = 1, ..., 4 \). Hamiltonian reads [22]

\[
H_B = \frac{1}{2} p_i^a p_i^a \quad \frac{g^2}{4} \epsilon_{abc} \epsilon_{ade} x_i^b x_j^c x_i^d x_j^e \quad + \quad \frac{ig}{2} \epsilon_{abc} \psi_a^T \Gamma^k \psi_b x_c^k,
\]

where \( \psi^T \) is the transpose of the real Majorana spinor, and in D=4 \( \Gamma \) are the Dirac \( \alpha \) matrices. We work in the Majorana representation of Ref. [23].

The system has the rotational symmetry, generated by the spin(3) angular momentum

\[
J^i = \epsilon^{ijk} \left( x^j a b c - \frac{1}{4} \psi^T a \Sigma^k \psi_a \right), \quad \Sigma^k = -\frac{i}{4} [\Gamma^j, \Gamma^k],
\]

gauge invariance with the generators \( G_a = \epsilon_{abc} \left( x_b^k a c - \frac{i}{2} \psi^T b \psi_c \right) \), and \( N = 1 \) supersymmetry with the charges \( Q_\alpha = (\Gamma^k \psi^a) a \alpha P^k_a + ig \epsilon_{abc} (\Sigma^k \psi_a a ^b c) \).

Again, expressing bosonic observables in terms of creation and annihilation operators is evident. For fermions we use

\[
\psi_a = \frac{1 + i \sqrt{2}}{2} \left( \begin{array}{c} -f_a^1 - if_a^2 + if_a^1 + f_a^{2\dagger} \\ +if_a^1 - f_a^2 - f_a^1 + if_a^{2\dagger} \\ -f_a^1 + if_a^2 + if_a^{1\dagger} - f_a^{2\dagger} \\ -if_a^1 - f_a^2 + f_a^{1\dagger} + if_a^{2\dagger} \end{array} \right), \tag{8}
\]

which gives the quantum hermitean Majorana spinor satisfying \( \{ \psi^\alpha_a, \psi^\beta_b \} = \delta^\alpha_\beta \). Fermionic operators are implemented with the aid of the Jordan-Wigner construction using the lexicographic ordering of the double index \( A = (\sigma, a) \). It turns out that the fermionic number \( F = f_a^{\sigma\dagger} f_a^\sigma \) is conserved. Consequently the hamiltonian can be diagonalized independently in all seven fermionic sectors, which is done\(^4\) analogously to the previous cases.

The spectrum of the theory is shown in Fig. 3. A sample of states is labeled with their angular momenta \( j \), which are simply assigned using the matrix representation of the angular momentum. It is important that our cut-off preserves the SO(3) symmetry. In particular the states have appropriate degeneracies for each value of \( j \). For current \( N_{cut} \) the spectrum extends to \( E_{max} \sim 35 \).

Sectors with \( F = 6, 5, 4 \) can be obtained from those with \( F = 0, 1, 2 \) by the particle-hole transformation. Since our cut-off respects this symmetry we indeed observe, exact repetition of the eigenvalues in corresponding subspaces. Therefore only first four sectors are displayed in Fig. 3. Evidently

\(^4\) Construction of the basis is similar to the D=2 SYMQM but more involved and will not be discussed here. Full details are in Ref. [17].
the spectrum of D=4 theory is very rich. We begin its discussion with the correspondence to the pure Yang-Mills quantum mechanics.

The fermionic part of the Hamiltonian, Eq.(7) vanishes on the \( F = 0 \) sector. Therefore our effective Hamiltonian in this sector is nothing but that of the zero volume pure Yang-Mills theory considered by Lüscher [6]. Indeed we observe rather satisfactory convergence of the \( F = 0 \) eigen energies to the results of Lüscher and Münster [7], c.f. Fig.4.

Another comparison was done together with van Baal who studied \( F = 2 \) sector in a more complex, non compact, supersymmetric model of the same family [25]. At the time of writing [17] our results could not be directly compared, due to the specific boundary conditions required in [25]. However after this conference van Baal has adapted his programs so that they can
be applied also to our situation. We have found complete agreement to the full Mathematica precision in both $F = 0$ and $F = 2$ sectors.

Figure 5. Cut-off dependence of the lower levels of the D=4 SYMQM in four independent fermionic channels ($N_{\text{cut}}=B$).

Next we discuss the convergence with the cut-off (c.f. Fig.5). The first striking property is that the $N_{\text{cut}}$ dependence is different in various fermionic sectors. For $F = 0, 1, 5, 6$ the dependence is weak, hence the lowest states have already converged (within some reasonable precision). For $F = 2, 3, 4$ however, the convergence is slower. We interpret this as an indication that the spectrum is discrete in the $F = 0, 1, 5, 6$ sectors while it is continuous in the ”fermion/hole-rich” $F = 2, 3, 4$ environments. This is based on our finding of an interesting and useful correlation between the localizability of a state and the rate of the convergence with the cut-off $N_{\text{cut}}$. It turns out that the spectrum of the momentum operator in the cut-off Hilbert space is exactly given by the zeroes of the Hermitte polynomials $[26]$. Consequently the convergence of the free spectrum is slow $\sim O(1/N_{\text{cut}})$. On the contrary, for the localized (i.e. bound) states the simple argument suggests that the rate of convergence is given by the large $x$ asymptotics of the wave function, hence it is fast (e.g. exponential). This different nature of the spectra in different fermionic sectors provide the reasonable extension of the result of Ref.$[5]$ which was mentioned earlier. Since the fermionic modes are crucial to generate the continuous spectrum, it is natural that it does not show up in the sectors where they do not exist at all, or are largely freezed out.

I thank P. van Baal for providing his results and programs.
by Pauli blocking. On the other hand not all states in $F = 2, 3, 4$ sectors have to belong to the continuum. Supersymmetry together with the discrete spectrum in $F = 0, 1$ channels implies existence of the normalizable states among the continuum of $F = 2, 3, 4$ states. As one explicit example we give the energy of the $F = 2, j = 1$ state, shown in Fig. 5 (flatter curve beginning at $E=6$). It has definitely weaker dependence on $N_{\text{cut}}$ than the others. This indicates that the lowest $|2_F, 1_j>$ state is localized. This may be a precursor of the more complex phenomenon expected in the D=10 theory. There, the zero energy localized bound state of $D_0$ branes should exist at the threshold of the continuous spectrum. Present example suggests that one way to distinguish such a state from the continuum may be by the different $N_{\text{cut}}$ dependence. Finally our results show that the supersymmetric vacuum cannot be in the $F = 0$ sector. Instead the best candidates are the lowest states in the $F = 2$ and 4 sectors.

**Figure 6.** Supersymmetric images of a sample of eigenstates.

**SUSY on the level of states.** Evidently identifying SUSY multiplets in Fig.3 looks like a nontrivial task. However present approach allows to monitor directly the action of the SUSY generators on the eigenstates of the hamiltonian. As any other observables they can be implemented in our "quantum algebra" described in the second Section. Alternatively their matrix representations are easily obtained once bases are constructed in each sector. Then the supersymmetric image of an eigenstate from one sector can be decomposed into states from adjacent sectors (the charges change fermionic number by ±1). This allows identification of the approximate supersymmetry partners. With the current scheme supersymmetry is restored only at infinite $N_{\text{cut}}$, hence the energies of the SUSY partners are not exactly identical. Neither the SUSY images of eigenstates are 100% concentrated on single eigenstates. This situation is quantitatively sum-
marized in Fig.6. The lower multiplets can be already identified, while for higher states, larger cut-offs are required for better assignment of SUSY partners, which is not surprising.

**Witten index.** Figure 7 shows the (Euclidean) time dependence of the Witten index calculated from our spectrum with up to five bosonic quanta.

\[ I_W \]

\[ T \]

\[ 0 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 5 \]

\[ 6 \]

\[ 7 \]

\[ 0 \]

\[ 0.1 \]

\[ 0.2 \]

\[ 0.3 \]

\[ 0.4 \]

\[ 0.5 \]

\[ 0.6 \]

\[ 0.7 \]

\[ 0.8 \]

\[ 0.9 \]

\[ 1 \]

\[ \text{Figure 7.} \]

Witten index of the D=4, supersymmetric Yang-Mills quantum mechanics for the number of bosonic quanta bounded by \( N_{\text{cut}} = 2, \ldots, 5 \). The bulk value 1/4 is also shown.

Clearly we are far from the convergence, but some interesting signatures are already there. First, one can see how the discontinuity at \( T = 0 \) emerges even at this early stage. Second, the exponential fall-off common to all curves at large (\( T > 5 \)) times is evident. This is where the lowest state dominates. Since at this \( N_{\text{cut}} \) its energy is not zero, the fall-off is exponential. Finally, and most interestingly, we observe the flattening shoulder appearing at \( T \approx 2 - 3 \). This is the signal of the supersymmetric cancellations which occur on the average, even though the exact SUSY correspondence between individual states does not yet show up. The behavior with \( N_{\text{cut}} \) is not inconsistent with the exact bulk value 1/4 obtained from the nonabelian integrals [10, 11]. Clearly higher \( N_{\text{cut}} \) are needed, and, what is important, they are within the range of present computers.

5. From D=4 to D=10 space-time dimensions

To bridge the gap to ten dimensions we study first the simpler, \( F = 0 \) case in all space dimensions \( 2 < d = D - 1 < 10 \). Thereby results presented here extend for the first time the spectrum of the zero volume glueballs to higher dimensions. To this end we use the hamiltonian in Eq.(7) without the fermionic part, drop all the fermionic occupation numbers in elementary states, Eq.(4), and appropriately extend the ranges of spatial indices. Therefore generalization to higher dimension is trivial in our approach, however
it puts considerably heavier load on the computer. Hence our results are rather preliminary and will be updated soon. Figure 7 shows the energies of the first three levels as a function of d.

The higher dimensional glueballs are there as expected. The ordering of the SO(d) multiplets remains the same as in d=3: scalar, two dimensional symmetric tensor with dimension g=d(d+1)/2-1, and scalar again. Our cut-off respects rotational symmetry for all d, hence we observe these degeneracies exactly in the spectra. The largest uncertainty (i.e. that of the d=9 points) is around 25% at present. Given above range of d one may attempt to confront these results with the large d expansions. The eye-ball fit suggest the power dependence $E \sim d^{1.5}$.

We conclude that calculations in higher dimensions including $d = 9$ are perfectly realistic (for SU(2) gauge group) even if much more time consuming. Including fermions will increase this demand by approximately factor of 16.

6. Summary

The new approach to solve quantum mechanical systems on a computer is proposed. The Hilbert space of quantum states is implemented algebraically in a computer code with states represented as flexible Mathematica lists. Any quantum observable is then defined as the list valued function on these lists. This allows for the automatic calculation of matrix representations and the spectrum. A range of progressively more complicated (also supersymmetric) systems from two to ten space-time dimensions was studied and many new results were obtained. The method is practically applicable to the D=10 supersymmetric SU(2) Yang-Mills quantum mechanics, which is our main goal in the future. Extension to few higher gauge groups is conceivable but large N require dedicated and new modifications.
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