Electric field in type II superconductors

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Generally it is accepted that electric field $E$ in type II superconductors is created by the vortex motion, so that it is proportional to the vortex velocity $v_L$. This assertion is based on the Josephson relation $E = -v_L \times B$, which was derived and is valid if no transport current is present. We present arguments showing that if transport current is present, static electric field is proportional to the relative velocity of vortices in respect to the velocity of superconducting fluid $v_s$, so that in this case generalised Josephson relation $E = (v_s - v_L) \times B$ is valid.

I. INTRODUCTION

It is widely believed that electric field $E$ in type II superconductors is generated by the vortex motion, so that it is proportional to the vortex velocity

$$E = -v_L \times B,$$

where $B = n_s \Phi_0$ is the averaged magnetic field, with $n_s$ being density of the vortex lattice. However, even Josephson in his paper stressed that: "The present method is applicable to systems which are inhomogeneous with respect to composition or flux line density", so that its applicability is strongly restricted. Josephson relation (1) can be proved valid also for homogenous system in absence of a transport current. Here we argue that if the transport current is present, the averaged electric field is proportional to the relative velocity of vortex lattice and superconducting fluid moving with velocity $v_s$,

$$E = (v_s - v_L) \times B.$$  (2)

Before a discussion of physical consequencies of relation (1) and (2), we want to present a line of simple arguments supporting (2):

(a) interaction between the superconducting fluid and vortex lattice is described by the Magnus force, see e.g.

$$F_M(v) = \frac{m v_s}{n_s} (v_s - v_L) \times z$$

(b) from the Newton action reaction law it is clear that the superconducting fluid must feel the reaction force,

$$F_m(s) = -\frac{n_s}{n_v} F_M(v) = -e (v_s - v_L) \times B.$$  (3)

(c) in stationary case the reaction force acting on superconducting fluid must be compensated by the electric field so that $eE + F_M(s) = 0$ leading to (2).

II. THE EIGENMODES

Here we will show that the proposed generalisation of the Josephson relation is needed in order to ensure that the eigenmodes of the system satisfy the momentum conservation law. Let us consider the simplest possible case without normal state fluid, with only Magnus force acting on the vortex lattice (no vortex pinning, no vortex damping, etc). The equation of vortex motion is

$$\dot{v}_L = \Omega (v_s - v_L) \times z,$$  (4)

where $\Omega = n h/2 m_v$ is the angular frequency of circular vortex motion with $m_v$ being the vortex mass per unit length. Taking into account the third Newton law, besides electric field the superconducting fluid must feel also the reaction Magnus force (3), so that its equation of motion may be written as

$$\dot{v}_s = eE - \omega_c (v_s - v_L) \times z,$$  (5)

where $\omega_c = eB/m$ is the cyclotroon frequency of superconducting charge carriers. There are two eigenmodes of the system. The zero frequency one $\omega = 0$, $v_L = v_s$ is a direct consequence of the Galilei invariance principle - vortices and superconducting fluid may move by the same velocity, the total momentum $M = n_s m v_s + n_v m_v v_L$ is nonzero, but time independent. Using the relation $\frac{n_s m}{n_v m_v} = \frac{\Omega}{\omega_c}$ it is easy to see that the momentum of the second eigenmode $\omega = \Omega + \omega_c$, $v_L = -\frac{\Omega}{\omega_c} v_s$ is zero. In this case vortices and superconducting fluid oscillate with the joint centre of mass remaining at rest.

If one would suppose validity of the Josephson relation, the equation of motion for the superconducting fluid would have to be written as

$$\dot{v}_s = eE + \omega_c v_L \times z.$$  (6)

The eigen frequencies of the system (1),(2) are $\omega_{1,2} = \frac{1}{2} (\Omega \pm \sqrt{\Omega^2 + 4 \omega_c^2})$ and the eigen modes are: $v_L = \frac{1}{2 \omega_c} \Omega (\mp \sqrt{\Omega^2 + 4 \omega_c^2})$. According to it the total momentum is nonzero and oscillates with the eigen frequency. By this the momentum conservation law is violated, what clearly shows that equation (1) can not be applied.

III. LORENTZ TRANSFORMATION

Let us consider the most simple static case first. In laboratory reference frame $(S)$ the crystal lattice is at rest ($v_c = 0$), the averaged magnetic field inside the superconductor is $B$, there is no transport current ($v_s = 0$), vortices do not move ($v_L = 0$) and consequently electric field $E$ inside the superconductor is zero. In the

$$\mathbf{v}_0 = \mathbf{v} + \mathbf{v}_L,$$

$$\mathbf{E}_0 = \mathbf{E}.$$  (7)

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Let us consider the most simple static case first. In laboratory reference frame $(S)$ the crystal lattice is at rest ($v_c = 0$), the averaged magnetic field inside the superconductor is $B$, there is no transport current ($v_s = 0$), vortices do not move ($v_L = 0$) and consequently electric field $E$ inside the superconductor is zero. In the
reference frame \((S')\) moving with velocity \(v\) the vortices and superconducting fluid move with the same nonzero velocity \(v'_L = v'_s = -v\). From Lorentz transformation it is clear that in this case the electric field \(\mathbf{E}' = \mathbf{E} + v \times \mathbf{B} = -v' \times \mathbf{B}'\) is nonzero and is given by Josephson relation. However, for the electrically neutral wire the current density is invariant so that it only confirms the fact that without transport current the Josephson relation is valid in any inertial reference frame even for the homogenous system.

Now let us consider the case that there is transport current and vortices move with the same velocity as the averaged velocity of the superconducting fluid \((v_s = v_L = -v)\). This situation differs from the static case observed from the moving reference frame by the fact that crystal lattice does not move. However, according to the Josephson relation \(E\) electric field is the same. If it were true, using Lorentz transformation one finds that in the reference frame moving with the same velocity as the vortex lattice the electric field is zero. This consequence contradicts the well known fact that "Neutral wire with a current appears to be charged when set in motion."

As a result it is clear that if transport current is present, Josephson relation can not be applied and must be generalised. We should stress that validity of the proposed relation \(E = (v_s - v_L) \times \mathbf{B}\) is restricted to the laboratory reference frame in which velocity of the crystal lattice is zero.

### IV. EXPERIMENTAL ASPECTS

According to generalised Josephson relation the transversal electric field is nonzero even in the case that vortices are kept by pinning. This fact may seem to be in contradiction with transport measurements. It is well known, that if vortices do not move, the Hall voltage measured with Ohmic contact is zero. The probable reason is, that actual Hall voltage is canceled by a contact potential. For type I superconductor this fact was experimentally proved by Bok and Klein already in 1968. Type I superconductor measured with Ohmic contacts also gives zero Hall voltage, but using a contactless capacitive pickup to measure changes in the electrostatic potential, the Hall voltage, or the so called Bernoulli field was be observed. We do not know about similar measurement made on type II superconductor and this is the way how validity of the proposed formula can be tested. Far infrared spectroscopy provides experimental data which are not influenced by contact potential - and our model proved to be consistent with the published experimental results (see e.g.\(^\text{3}\)). In the zero frequency limit the same theoretical approach is naturally explaining the Hall voltage sign reversal, which is still considered to be one of the most puzzling phenomena in the physics of superconductors.\(^\text{4}\)

### V. CONCLUSION

If there is no transport current, vortices move only if there is gradient of vortex density. In this case vortices move and the vortex motion results in redistribution of the magnetic flux in and/or outside the superconductor. In this case Josephson relation is valid. On the other hand, if vortices are moving due to the presence of the transport current, vortices are moving, but the averaged magnetic flux is time independent. In this case generalised Josephson relation must be used.

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1. B.D. Josephson, Phys. Lett. 16, 242 (1965)
2. P. Ao, D.J. Thouless, Phys. Rev. Lett 70, 2158 (1993)
3. The Feynman lectures on physics, Chapter 13. Adison-Wesley (1966)
4. J. Bok, J. Klein, Phys. Rev. Lett 20, 660 (1968)
5. Y. N. Chiang, O. G. Shevchenko, Fizika nizkikh temperatur 22, 669 (1996)
6. J. Koláček, E. Kawate, Phys. Lett. A 260, 300 (1999)
7. J. Koláček, P. Vašek, Int. J. Modern Phys. B 12, 3102 (1998); (see also cond-mat-9811222).