The role of $qqqq\bar{q}$ components in the nucleon
and the $N(1440)$ resonance

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Abstract

The role of $q\bar{q}$ components in the nucleon and the $N(1440)$ resonance is studied by explicit coupling of the lowest positive parity $qqqq\bar{q}$ state to the $qqq$ components in the harmonic oscillator quark model. The lowest energy $qqqq\bar{q}$ component, where the 4-quark subsystem has the flavor-spin symmetry $[4]_F [S][22]_F [22]_S$, is close in energy to the lowest positive parity excitation of the nucleon in the $qqq$ quark model. The confining interaction leads to a strong mixing of the $qqqq\bar{q}$ system and the positive parity excited state of the $qqq$ system. This result is in line with the phenomenological indications for a two-component structure of the $N(1440)$ resonance. The presence of substantial $q\bar{q}$ components in the $N(1440)$ can bring about a reconciliation of the constituent quark model with the large empirical decay width of the $N(1440)$.

1 Introduction

The inadequacy of the conventional 3-quark model for the nucleons and the nucleon resonances in general is evident in its substantial underprediction of the values of both the electromagnetic and the strong decay widths of the lowest nucleon resonance, $N(1440)$ \cite{1,2}. This indicates that this resonance should have significant $(q\bar{q})^n$, if not more exotic components, in addition to the 3 valence quarks. This is also suggested by the fact that this resonance appears naturally at its very low energy as a vibrational - i.e collective - state in the Skyrme model \cite{3}.

The admixture of $qqqq\bar{q}$ components in the nucleon and the $N(1440)$ resonance is studied here by explicit consideration of the coupling between the positive parity $qqqq\bar{q}$ configuration, which is expected to have the lowest energy, and
the ground state and the lowest excited positive parity $qqq$ states. The main coupling between the $qqq$ and the $qqqq\bar{q}$ state is assumed to arise from the confining interaction.

In the harmonic oscillator quark model, with color coupled confinement, the lowest energy positive parity $qqqq\bar{q}$ configuration will have an energy that falls fairly close to the lowest positive parity excited state of the $qqq$ configuration, if the hyperfine interaction depends both on spin and on flavor. In this case the lowest $qqqq\bar{q}$ configuration has the mixed flavor-spin symmetry $[4]_F S [22]_F [22]_S$. If the confining interaction couples as a Lorentz scalar, it leads to a strong coupling between the $qqq$ and the $qqqq\bar{q}$ configurations with positive parity. This coupling, which involves $q\bar{q}$ pair annihilation and creation, leads to significant mixing of the lowest energy configuration of the $qqqq\bar{q}$ system and the lowest positive parity excited state configuration of the $qqq$ system. With phenomenologically realistic parameter values this leads to a 2-state configuration, with strongly mixed $qqq$ and $qqqq\bar{q}$ components, for the $N(1440)$ resonance, which is in line with recent phenomenological analyses [4].

This article is arranged as follows: The $qqqq\bar{q}$ wave functions of the proton and the $N(1440)$ are described in section II. In section III the harmonic oscillator Hamiltonian for the $qqq$ and $qqqq\bar{q}$ configurations is defined along with the schematic hyperfine interaction model. The mixing of the states that is derived by the diagonalization of the Hamiltonian is described at the end of the section. In section IV the model is extended to the $qqq(q\bar{q})^2$ system. Section V contains a concluding discussion.

2 The wave functions of the nucleon and the $N(1440)$

In the harmonic oscillator quark model the wave functions of the nucleon and the $N(1440)$ in the $qqq$ configuration take the forms:

$$|N, s_z\rangle_{3q} = \frac{1}{\sqrt{2}} \left[ |\frac{1}{2}, t_z\rangle_+ |\frac{1}{2}, s_z\rangle_+ + |\frac{1}{2}, t_z\rangle_- |\frac{1}{2}, s_z\rangle_- \right] \varphi_{000}(\vec{\xi}_1) \varphi_{000}(\vec{\xi}_2),$$

$$|N(1440), s_z\rangle_{3q} = \frac{1}{\sqrt{2}} \left[ |\frac{1}{2}, t_z\rangle_+ |\frac{1}{2}, s_z\rangle_+ + |\frac{1}{2}, t_z\rangle_- |\frac{1}{2}, s_z\rangle_- \right] \varphi_{200}(\vec{\xi}_2) \varphi_{000}(\vec{\xi}_1) + \varphi_{000}(\vec{\xi}_2) \varphi_{200}(\vec{\xi}_1) \right]. \quad (1)$$

Here $|\frac{1}{2}, s_z\rangle_\pm$ and $|\frac{1}{2}, t_z\rangle_\pm$, are the spin and isospin wave functions, respectively, of mixed symmetry $[21]$, in which (+) denotes a state that is symmetric “(112)” and (-) denotes a state that is antisymmetric “(121)” under exchange of the spin or isospin of the first two quarks. The radial coordinates $\vec{\xi}_1$ and $\vec{\xi}_2$.
are the Jacobi coordinates defined by the constituent coordinates as the first two of the general Jacobi coordinates for a system of \( n \) constituents:

\[
\begin{align*}
\vec{\xi}_k &= \frac{1}{\sqrt{k + k^2}} \left[ \sum_{l=1}^{k} \vec{r}_l - k \vec{r}_{k+1} \right], \quad k = 1, ..., n - 1, \\
\vec{R} &= \frac{1}{n} \sum_{l=1}^{n} \vec{r}_l.
\end{align*}
\]  

(2)

The two harmonic oscillator wave functions above are defined as:

\[
\begin{align*}
\varphi_{000}(\vec{\xi}) &= \left( \frac{\bar{\omega}_3}{\pi} \right)^{\frac{3}{4}} e^{-\bar{\omega}_3 \xi^2 / 2}, \quad \varphi_{200}(\vec{\xi}) = \sqrt{\frac{2}{3}} \bar{\omega}_3^{\frac{1}{2}} \left( \xi^2 - \frac{3}{2\bar{\omega}_3} \right) \varphi_{000}(\vec{\xi}).
\end{align*}
\]  

(3)

Here \( \bar{\omega}_3 = \sqrt{m\omega_3} \), where \( m \) is the constituent mass and \( \omega_3 \) is the oscillator frequency. The subscripts denote the quantum numbers \((nlm)\) of the oscillator wave functions.

Positive parity demands that the lowest energy \( qqqq\bar{q} \) admixtures in the proton and the \( \mathcal{N}(1440)^+ \) have to be \( P \)-wave states. By the conventional assumption that the hyperfine interaction between the quarks is spin-dependent, it follows that the \( qqqq \) configurations that have lowest energy are those for which the spin state has the highest possible degree of antisymmetry \([5]\). A similar argument applies in the case the hyperfine interaction is flavor dependent. The simplest hyperfine interaction model, which leads to a realistic splitting between the nucleon and the \( \mathcal{N}(1440) \), is the schematic flavor and spin dependent hyperfine interaction between the quarks \(- C_{\chi} \sum_{i<j} \vec{l}_i \cdot \vec{\sigma}_i \cdot \vec{l}_j \cdot \vec{\sigma}_j \), where \( C_{\chi} \) is a constant parameter \((C_{\chi} \sim 20 - 30 \, \text{MeV})\) \([6]\). This implies that the \( qqqq\bar{q} \) configuration that has the lowest energy, and which is most likely to form notable admixtures in the nucleons and the \( \mathcal{N}(1440) \), has the mixed spin-flavor symmetry \([4]_{FS}[22]_{F}[22]_{S} \), with the antiquark in the ground state \([5]\).

The wave function for this \( qqqq\bar{q} \) component takes the form:

\[
|N, s_z \rangle_{5q} = \frac{1}{\sqrt{2}} \sum_{a,b} \sum_{m,s} (1, \frac{1}{2}, m, s |1, \frac{1}{2}, s_z) C^{[14]}_{[31]_{a}[211]_{a}} [211]_{C}(a) [31]_{X,m}(a) [22]_{F}(b) [22]_{S}(b) \bar{\psi}_{t,s}(\{\vec{\xi}_i\}).
\]  

(4)

Here the color, space and flavor-spin wave functions of the \( qqqq \) subsystem have been denoted by their Young patterns respectively, and the sum over \( a \) runs over the 3 configurations of the \([211]_{C} \) and \([31]_{X} \) representations of \( S_3 \), and the sum over \( b \) runs over the 2 configurations of the \([22] \) representation of
$S_4$ respectively [7]. The symbols $C^{[14]}_{[31],[211]}$ denote the $S_4$ Clebsch-Gordan coefficients for the representations $[1111],[31],[211]$. The values of these coefficients are:

$$C^{[14]}_{[31],[211]} = \frac{1}{\sqrt{3}}, \quad C^{[14]}_{[31],[31]} = -\frac{1}{\sqrt{3}}, \quad C^{[14]}_{[31],[211]} = \frac{1}{\sqrt{3}}. \quad (5)$$

The explicit color-space part of the wave function (4) may be then expressed in the form:

$$\psi_C(\{\bar{\xi}_i\}) = \frac{1}{\sqrt{3}} \left\{ [211]_3 \phi_{01m}(\bar{\xi}_1)\phi_{000}(\bar{\xi}_2)\phi_{000}(\bar{\xi}_3) - [211]_2 \phi_{000}(\bar{\xi}_1)\phi_{01m}(\bar{\xi}_2)\phi_{000}(\bar{\xi}_3) - [211]_1 \phi_{000}(\bar{\xi}_1)\phi_{000}(\bar{\xi}_2)\phi_{01m}(\bar{\xi}_3) \right\} \phi_{000}(\bar{\xi}_4). \quad (6)$$

Here the vectors $\bar{\xi}_i, i=1,..,4$, are the Jacobi coordinates for the 5 body system as in Eq. (2) and the oscillator wave functions for the quarks in the $qqqq\bar{q}$ system are:

$$\phi_{000}(\bar{\xi}_i) = \left(\frac{\bar{\omega}_5^2}{\pi}\right)^{\frac{3}{4}} e^{-\bar{\omega}_5^2\bar{\xi}_i^2/2}, \quad \phi_{01m}(\bar{\xi}_i) = \sqrt{2}\bar{\omega}_5 \xi_{i,m} \phi_{000}(\bar{\xi}_i), \quad (7)$$

Here $\bar{\omega}_5 = \sqrt{m\omega_5}$, where $\omega_5$ is the oscillator frequency for the $qqqq\bar{q}$ system. The relation between the oscillator parameters for the $qqq$ and the $qqqq\bar{q}$ components depends on the color dependence of the confining interaction as shown below. In Eq. (6) $C_i$ ($i = 1, 2, 3$) represent the color wave functions of the 3 configurations of $[211]_C$ and notice has been taken of the fact that the vectors $\bar{\xi}_i$ ($i = 1, 2, 3$) realize the 3 configurations of $[31]_X$ in orbital space [7].

### 3 Hamiltonian model

#### 3.1 The non-interacting part

The non-interacting part of the oscillator model Hamiltonians for the $qqq$ and $qqqq\bar{q}$ systems may be written in the following way:

$$H_3 = 3m + \sum_{i=1}^{2} \left\{ \frac{\bar{\xi}_i^2}{2m} + \frac{m\omega_5^2}{2} \xi_i^2 \right\}, \quad (8)$$
\[ H_5 = 5m + \sum_{i=1}^{4} \left\{ \frac{\tilde{\xi}_i^2}{2m} + \frac{m\omega_5^2}{2} \xi_i^2 \right\} . \]

Here \( \tilde{\xi}_i \) denotes the momentum operator that is canonically conjugate to the position operator \( \xi_i \) and \( m \) denotes the constituent mass. These Hamiltonians are translationally and rotationally invariant, and therefore invariant under Poincaré transformations as well, despite their non-relativistic appearance.

Hadron phenomenology suggests that the confining operator depends on color through the operator \( \lambda_i^C \cdot \lambda_j^C \). If this color dependence is applied to the harmonic oscillator model for the confining interaction it implies a relation between the oscillator frequencies, \( \omega_3 \) and \( \omega_5 \), for the \( qqq \) and \( qqqq \bar{q} \) systems.

The matrix elements of the color operator for any two quarks (or a quark and an antiquark) in the \( qqq \) and \( qqqq \bar{q} \) systems, which have the color symmetries \([1^3]\) and \([2^3]\) respectively, are [8]:

\[ <\lambda_i^C \cdot \lambda_j^C>_{[111]} = -8/3, \quad <\lambda_i^C \cdot \lambda_j^C>_{[222]} = -4/3. \]

From these matrix elements, one may infer the following relation between the oscillator frequencies for the \( qqq \) and \( qqqq \bar{q} \) systems [5]:

\[ \omega_5 = \sqrt{\frac{5}{6}} \omega_3 . \]

The energies of the ground state and the first excited positive parity state of the \( qqq \) system given by the Hamiltonian (8) are

\[ E^0_3 = 3m + 3\omega_3 , \]
\[ E^2_3 = 3m + 5\omega_3 . \]

The lowest state of positive parity of the \( qqqq \bar{q} \) Hamiltonian (9) is correspondingly

\[ E^0_5 = 5m + 7\omega_5 . \]

In the absence of any hyperfine interaction it follows from these expressions that the energy of the 5-quark state falls \( \sim 2m + 1.5\omega_5 \) (cf.(14)) above the excited positive parity state of the \( qqq \) configuration. A sufficiently strong attractive hyperfine interaction is of course required to lower the energy of the lowest positive parity state of the \( qqqq \bar{q} \) system below that of the lowest negative parity state.
3.2 The hyperfine interaction

Consider now the situation in which the quarks (or the antiquarks) interact via the schematic flavor and spin dependent hyperfine interaction:

\[ H_\chi = -C \sum_{i<j} \lambda_i^F \cdot \lambda_j^F \cdot \vec{\sigma}_i \cdot \vec{\sigma}_j. \]  

(15)

The constant \( C \sim 30 \text{ MeV} \), represents an average of the matrix element of the hyperfine interaction in the low lying orbitals of the \( qqq \) system. This interaction organizes the low lying part of spectrum of the \( qqq \) system in a way that closely follows that of the empirical spectrum [6]. A dynamical interpretation of this interaction may be given in terms of single- and two-pion exchange between the quarks [9]. As those interaction components lead to opposite signs for the spin-spin interaction between quarks and antiquarks, with a resulting at most weak spin-spin interaction, the hyperfine interaction between quarks and antiquarks have been neglected here.

The matrix elements of this interaction in the \([3]_{FS}[21]_F[21]_S \) mixed flavor symmetry state of the conventional \( qqq \) quark model is \( <H_\chi>_3 = -14C \) [6]. The matrix element of the hyperfine interaction (15) in the \( qqqq \) configuration with the mixed flavor symmetry \([4]_{FS}[22]_F[22]_S \) is \( <H_\chi>_5 = -28C \).

When this hyperfine interaction matrix element is taken into account, it modifies the energies of the \( qqq \) and \( qqqq \) states to:

\[
E_3^{0'} = 3m + 3\omega_3 - 14C, \\
E_3^{2'} = 3m + 5\omega_3 - 14C, \\
E_5^{0'} = 5m + 7\omega_5 - 28C. 
\]  

(16)

This result shows that the energy of the lowest \( qqqq \) state with positive parity falls close to that of the lowest excited state of the \( qqq \) system, if \( 2m + 1.5\omega_5 \simeq 14C \). As \( 14C \sim 420 \text{ MeV} \), this condition may be fulfilled with phenomenologically realistic values for the constituent mass and the oscillator frequency.

Note finally, that if the coefficient \( C \) in the interaction Hamiltonian (15), which represents the average strength of the spatial dependence of the hyperfine interaction is allowed to depend on the partial wave state of the quark pairs as in [6], so that \( C \) is replaced by state dependent parameters \( C_L \), the energy expressions (16) are modified to:

\[
E_3^{0'} = 3m + 3\omega_3 - 14C_0, \\
E_5^{0'} = 5m + 7\omega_5 - 28C_0. 
\]
Fig. 1. Confining mechanism responsible for the mixing between the $qqq$ and $qqqq\bar{q}$ states.

$$
E'_3 = 3m + 5\omega_3 - 14 \left( \frac{C_0 + C_2}{2} \right),
$$
$$
E'_5 = 5m + 7\omega_5 - 28 \left( \frac{2C_0 + C_1}{3} \right). 
$$

(17)

These expressions will be employed below.

### 3.3 Coupling between the $qqq$ and the $qqqq\bar{q}$ components

Consider then the coupling between the $qqq$ and the $qqqq\bar{q}$ states that is illustrated in Fig. 1. If the mediating coupling is taken to be the confining interaction, which couples to the quarks and the antiquark as a Lorentz scalar, this coupling to lowest order in $p/m$ takes the form:

$$
V_{cou} = -\frac{1}{2m} <\lambda_3^C \cdot \lambda_4^C > v_{conf}(\vec{p}_4 + \vec{p}_5) \vec{\sigma}_4 \cdot (\vec{p}_5 - \vec{p}_4).
$$

(18)

Here $\tilde{v}_{conf}$ is the formal Fourier transform of the confining interaction. The spatial representation of the interaction is

$$
V_{cou} = -\frac{1}{2m} <\lambda_3^C \cdot \lambda_4^C > v_{conf}(\vec{r}_{34}) \vec{\sigma}_4 \cdot (\vec{p}_5 - \vec{p}_4).
$$

(19)

Here $\vec{r}_{34} = \vec{r}_3 - \vec{r}_4$ and a $\delta$ function $\delta(\vec{r}_4 - \vec{r}_5)$ is implied. As above, in the $qqqq\bar{q}$ system $<\lambda_3^C \cdot \lambda_4^C > = -4/3$.

The explicit form for the confining coupling potential $v_{conf}(\vec{r}_{34})$ is in the present harmonic oscillator model:

$$
v_{conf}(\vec{r}_{34}) = \frac{1}{10} m\omega_5^2 r_{34}^2.
$$

(20)

In terms of the relative coordinates (2), one has:
\[ r_{34}^2 = \frac{2}{3}(\xi_2 - \sqrt{2}\xi_3)^2, \quad (21) \]
\[ \tilde{p}_5 - \tilde{p}_4 = \frac{1}{2}(\sqrt{3}\xi_3 - \sqrt{5}\xi_4) = -\frac{1}{2}(\sqrt{3}\tilde{\nabla}_3 - \sqrt{5}\tilde{\nabla}_4). \quad (22) \]

Here the \( \tilde{\nabla} \) operators act on the corresponding relative coordinates \( \xi \). When these act on the incoming \( P- \) state quark and \( S- \) state antiquark wave functions, the coupling interaction operator reduces to the form:

\[ V_{\text{cou}} = \frac{2\sqrt{30}}{375} \omega_5^2 \sqrt{m\omega_5}(\xi_2 - \sqrt{2}\xi_3)^2 \sigma_{4,m} \left( \frac{\omega_5^2}{\pi} \right)^{3/2} e^{-\frac{4\omega_5^2\xi_3^2}{5}}. \quad (23) \]

The expression has been multiplied by an overall factor 4 to take into account the presence of 4 quarks in the \( qqqq\bar{q} \) system. The matrix element of this interaction involves the matrix element over all coordinates of the \( qqq \) system and the residual \( qqq \) component of the \( qqqq\bar{q} \) system.

The spin matrix element gives rise to a factor \( 2\sqrt{3} \) and the overlap of the color wave functions to a factor \( 1/\sqrt{3} \). The final spatial matrix element for the coupling between the ground states of the \( qqq \) and the \( qqqq\bar{q} \) systems then takes the form:

\[ <qqq|V_{\text{cou}}|qqqq\bar{q}> = \frac{4\sqrt{30}}{375} \omega_5^2 \sqrt{m\omega_5}(\xi_2 - \sqrt{2}\xi_3)^2 \sigma_{4,m} \left( \frac{\omega_5^2}{\pi} \right)^{3/2} \int d^3\xi_1 d^3\xi_2 d^3\xi_3 \phi_{000}(\xi_1;\omega_3) \phi_{000}(\xi_2;\omega_3) \]
\[ (\xi_2 - \sqrt{2}\xi_3)^2 e^{-\frac{4\omega_5^2\xi_3^2}{5}} \phi_{000}(\xi_1;\omega_5) \phi_{000}(\xi_2;\omega_5). \quad (24) \]

This matrix element can be expressed in closed form as

\[ V_c = <qqq|V_{\text{cou}}(\xi_3)|qqqq\bar{q} > = \frac{\sqrt{6}}{5} m \left( \frac{\omega_3}{m} \right)^{3/2} \frac{\omega_3^2(5\omega_3 + 9\omega_5)}{(\omega_3 + \omega_5)^4}. \quad (25) \]

The corresponding matrix element of the coupling interaction between the positive parity excited state of the \( qqq \) system and the ground state of the \( qqqq\bar{q} \) system is:

\[ <qqq^*|V_{\text{cou}}(\xi_3)|qqqq\bar{q} > = \frac{4\sqrt{15}}{375} \omega_5^2 \sqrt{m\omega_5}(\frac{\omega_5^2}{\pi})^{3/2} \int d^3\xi_1 d^3\xi_2 d^3\xi_3 \]
\[ \{ \phi_{000}(\xi_1;\omega_3) \phi_{200}(\xi_2;\omega_3) + \phi_{200}(\xi_1;\omega_3) \phi_{000}(\xi_2;\omega_3) \} \]
\[ (\xi_2 - \sqrt{2}\xi_3)^2 e^{-\frac{4\omega_5^2\xi_3^2}{5}} \phi_{000}(\xi_1;\omega_5) \phi_{000}(\xi_2;\omega_5). \quad (26) \]

This matrix element may also be expressed in closed form:
Fig. 2. The combinations of oscillator and constituent mass values, which yield 939 MeV for the lowest eigenvalue after diagonalization of the Hamiltonian (28).

$$V_c^* = <qqq^*|V_{cou} (\xi_3)|qqqq\bar{q} > =$$

$$\frac{\sqrt{2}}{5} m \left( \frac{\omega_3}{m} \right)^{3/2} \frac{\omega_3^3 (15\omega_3^2 + 20\omega_3\omega_5 - 27\omega_5^2)}{\omega_3^5}$$.

(27)

### 3.4 Mixing between the $qqq$ and the $qqqq\bar{q}$ systems

The mixing between the $qqq$ and the $qqqq\bar{q}$ systems is determined by the $3 \times 3$ Hamiltonian for the ground states of the $qqq$ and the $qqqq\bar{q}$ state and the excited positive parity state $qqq$ state. This Hamiltonian is

$$H = \begin{pmatrix} 3m + 3\omega_3 - 14\bar{C}_{01} & V_c & 0 \\ V_c & 5m + 7\omega_5 - 28\bar{C}_{01} & V_c^* \\ 0 & V_c^* & 3m + 5\omega_3 - 14\bar{C}_{02} \end{pmatrix}$$.

(28)

Here, $\bar{C}_{01} = (2C_0 + C_1)/3$ and $\bar{C}_{02} = (C_0 + C_2)/2$. Following ref.[6] we set $C_0 = 29$ MeV, $C_1 = 45$ MeV and $C_2 = 0$, for a good description of the low lying part of the baryon spectrum. By numerical diagonalization of this matrix, it is found that phenomenologically realistic eigenvalues for the 3 states may be found by choosing the constituent quark mass $m$ to be in the range $[250 - 350]$ MeV and correspondingly the value for the oscillator parameter $\omega_3$ in the range $[200 - 100]$ MeV if the hyperfine interaction constants $C_L$ are chosen as above. The combinations of these parameters, which, upon diagonalization of the Hamiltonian (28), yield 939 MeV for the lowest eigenvalue are plotted in Fig.2. The lowest eigenvalue obviously represents the nucleon.

The corresponding 3 eigenvalues are shown in Fig. 3 as functions of the constituent mass. The two higher eigenvalues, which stay close together, may be
Fig. 3. The eigenvalues of the Hamiltonian (28) as functions of the constituent mass. The band represents a 50 MeV interval around the PDG value for the Roper mass, 1440 MeV.

interpreted as forming the broad $N(1440)$ resonance, for which there are clear phenomenological indications for a 2-state structure [4].

The probabilities of the $qqq$, the excited $qqq$ and the $qqqqq\bar{q}$ components in the 3 eigenstates are shown in Fig. 4 respectively. The results indicates that in this model the nucleon is an almost pure $qqq$ state. In contrast there is a sizable mixing of the excited $qqq$ and the $qqqqq\bar{q}$ components in the two close lying higher eigenstates if the quark mass lies below $\approx 275$ MeV. The higher eigenstate is mostly made up of $qqqqq\bar{q}$ with up to a 25% of $qqq^*$ state for a quark mass of $\approx 250$ MeV. The middle eigenstate is, on the other hand, mostly made up of $qqq^*$ with up to $\approx 25\%$ component of $qqqqq\bar{q}$ for a quark mass of $\approx 250$ MeV. The admixture of the $qqqqq\bar{q}$ component in the ground state is mostly lower than $\sim 1\%$ reaching this maximum value for a quark mass of 250 MeV.

4 The $qqq(q\bar{q})^2$ system

While, as shown above, the ground state of the $qqqqq\bar{q}$ 5-quark system may be expected to have an energy that falls within the vicinity of the lowest positive parity state of the $qqq$ system, the ground state of the $qqq(q\bar{q})^2$ 7-quark system will have a much higher energy. This may be illustrated by a direct extension of the harmonic oscillator model employed above to the 7-quark system.

Consider again a harmonic confining interaction with the color dependence $\lambda_i^C \cdot \lambda_j^C$. The $qqq(q\bar{q})^2$ system is a color singlet: [333]. The 5 quarks form a mixed symmetry color configuration: [221], while the 2 antiquarks form the
mixed symmetry color configuration [211]. The matrix elements of the color pair operator $\lambda_i^C \cdot \lambda_j^C$ in these configurations are:

$$< \lambda_i^C \cdot \lambda_j^C >_{[221]} = -\frac{16}{15}, \quad < \lambda_i^C \cdot \lambda_j^C >_{[211]} = -\frac{4}{3}.$$  \hfill (29)

Thus the strength of the confining interaction for a quark pair in the $qqq(q\bar{q})^2$ system is $4/5$ times that of the $qqqq\bar{q}$ system, while the strength of the confining interaction between the 2 antiquarks is the same as that between the quarks and the antiquarks in the $qqqq\bar{q}$ system.

From Eq. (29) and the vanishing of the Casimir operator $C_2^{(3)}$ of SU(3) it follows that the matrix element of the operator $\lambda_i^C \cdot \lambda_j^C$ in a $q\bar{q}$ pair state in the $qqq(q\bar{q})^2$ system is:

$$< \lambda_i^C \cdot \lambda_j^C > q\bar{q} = -\frac{8}{15}.$$  \hfill (30)

Thus the strength of the confining interaction between a quark and an antiquark in the 7-quark system is one half of that between two quarks in the same system.

The oscillator Hamiltonian for the 7-quark system may now be written as:
\[
H = \sum_{i=1}^{7} \frac{p_i^2}{2m} - \frac{P^2}{14m} + \frac{m\omega_{qq}^2}{2} \sum_{i<j} r_{ij}^2 \\
+ \frac{m\omega_{q\bar{q}}^2}{2} \bar{r}_{67}^2 + \frac{m\omega_{q\bar{q}}^2}{2} \sum_{i=1}^{7} \sum_{j=6}^{7} r_{ij}^2.
\]

(31)

Here \(\vec{P}\) is the center-of-mass momentum.

This Hamiltonian is analytically diagonalized by first enlarging the system of relative coordinates for 5 quarks with the following two coordinates in Eq. (2).

The diagonalization is then achieved by introduction of the following new set of relative coordinates:

\[
\vec{\eta}_i = \vec{\xi}_i, \quad i = 1, \ldots, 4,
\]

(32)

\[
\vec{\eta}_5 = \sqrt{\frac{7}{12}} \vec{\xi}_5 + \sqrt{\frac{5}{12}} \vec{\xi}_6,
\]

(33)

\[
\vec{\eta}_6 = -\sqrt{\frac{5}{12}} \vec{\xi}_5 + \sqrt{\frac{7}{12}} \vec{\xi}_6.
\]

(34)

The diagonalized version of (31) is then:

\[
H = \sum_{i=1}^{6} \frac{\vec{\eta}_i^2}{2m} + \frac{m}{2} (5\omega_{qq}^2 + 2\omega_{q\bar{q}}^2) \sum_{i=1}^{4} \eta_i^2 \\
+ \frac{7m}{2} \omega_{qq}^2 \eta_5^2 + \frac{m}{2} (2\omega_{qq}^2 + 5\omega_{q\bar{q}}^2) \eta_6^2.
\]

(35)

Here \(\vec{\eta}_i\) denotes the momentum operator that is canonically conjugate to \(\vec{\eta}_i\). In the absence of any coupling to the 3-quark or 5-quark system the energy of the ground state of this Hamiltonian is

\[
E_7^0 = 7m + 6\sqrt{5\omega_{qq}^2 + 2\omega_{q\bar{q}}^2} + \frac{3\sqrt{7}}{2} \omega_{qq} + \frac{3}{2} \sqrt{2\omega_{qq}^2 + 5\omega_{q\bar{q}}^2}.
\]

(36)

Consider then the energy shift caused by the hyperfine interaction (15) between the quarks and between the 2 antiquarks in the \(qqqqq\bar{q}\) system. The lowest energy is that for the configuration which has the highest possible degree of antisymmetry in both the flavor and the spin configurations. Moreover as positive parity allows all the constituents in the 7-quark system to be in their ground state, the spatial state is totally symmetric. As a consequence the flavor-spin state is also totally symmetric as is the combined flavor-spin configuration.
In the ground state the flavor-spin configuration of the 5 quarks in the 7 quark system is the mixed symmetry configuration \([5]_F[32]_F[32]_S\). The two antiquarks form the mixed symmetry flavor-spin configuration \([2]_F[11]_F[11]_S\). The matrix elements of the hyperfine interaction (15) in these configurations may be calculated from the general expression [5]:

\[
< H_\chi > = -C \{ 4C_2^{(6)} - 2C_2^{(3)} - \frac{4}{3}C_2^{(2)} - 8N \}, \tag{37}
\]

where \(C_2^{(n)}\) is the quadratic Casimir operator of SU\((n)\), and \(N\) is the number of constituents.

This leads to the following energy shifts due to the hyperfine interaction in the 7-quark system:

\[
< H_\chi >_{qqqq} = -40C, \quad < H_\chi >_{\bar{q}q} = -8C. \tag{38}
\]

In order to estimate the ground state energy of the \(qq\bar{q}\) system, the oscillator frequencies for the constituent pairs in that system may be related through Eqs. (29) and (30) to the oscillator frequency \(\omega_3\) for the \(qqq\) system. The resulting expressions are:

\[
\omega_{qq} = \omega_3 \sqrt{\frac{2}{15}}, \quad \omega_{\bar{q}q} = \frac{\omega_3}{\sqrt{15}}, \quad \omega_{\bar{q}\bar{q}} = \frac{\omega_3}{\sqrt{6}}. \tag{39}
\]

Insertion in (36) with account of (38) yields:

\[
E_{7}^{qq} = 7m + \omega_3 \left( \frac{12}{\sqrt{5}} + \frac{3}{2} \sqrt{\frac{7}{15}} + \frac{3}{2} \sqrt{\frac{2}{3}} \right) - 48C. \tag{40}
\]

This leads to an approximate energy estimate, by which the energy of the uncoupled 7-quark system falls \(\sim 2m + 1.22\omega_3 - 20C \sim 200\) MeV above that of the 5-quark system, and thus roughly 700 MeV above the \(qqq\) ground state. Given the results in the previous section that only configurations that lie close in energy are strongly mixed, the 7-quark system should then be expected to be strongly coupled only to higher lying baryon resonances than the \(N(1440)\) (and the nucleon). Note that the form of the \(q\bar{q}\) annihilation coupling couples the \(qqq\) system to the \(qqqq\bar{q}\) system and the \(qqqq\bar{q}\) system to the \(qqq(q\bar{q})^2\) system, but that there is no direct coupling of the \(qqq\) and the \(qqq(q\bar{q})^2\) system, both of which have \(L = 0\) in their lowest positive parity configurations.
5 Discussion

This quark model study of the interplay between the lowest lying \(qqqq\bar{q}\) component and the \(qqq\) components of the nucleon and the \(N(1440)\) illustrates how the confining interaction mixes the \(qqq\) and \(qqqq\bar{q}\) components. The model exhibits the dual nature of the roper resonance, both as a genuine \(qqq\) state and a \(qqqq\bar{q}\) state. In the meson-baryon hadronic description it indicates the presence of a substantial \(N\pi\) component in the \(N(1440)\), for which there exist both theoretical arguments as well as experimental indications [4,10].

In the present model, the flavor-spin dependent hyperfine interaction singles out the most likely \(qqqq\bar{q}\) configuration in the \(N(1440)\). This \(5q\) configuration lies close to the experimental value of the Roper resonances mass with phenomenologically reasonable values for the model parameters. Diagonalization of the model Hamiltonian for the ground state and the lowest positive parity excitation of the \(qqq\) configuration and the ground state of the \(qqqq\bar{q}\) configuration leads to mixing between the lowest positive parity excited state of the \(qqq\) configuration and the lowest energy \(qqqq\bar{q}\) configuration. The configuration mixing ranges from 25% for a quark mass of 250 MeV to 3% for a quark mass of 300 MeV. Remarkably both states are close in mass for the considered range of quark masses. In the present model the \(qq\) admixture in the nucleon is very small, with a maximum value of \(\sim 1\%\).

The significance of the \(qqqq\bar{q}\) component in the quark model description of the electromagnetic transition form factors of the baryon resonances has recently been noted in Refs. [11]. This significance may also be indirectly inferred from the large role played by meson-baryon in the coupled channel hadronic model for the baryon resonances [12]. With appreciable admixtures of \(qqqq\bar{q}\) components in the resonances the failure of naive quark models to explain pionic decay widths may also be overcome [13].

Finally it was found that admixtures of \((q\bar{q})^2qqq\) components do not seem to be significant in the present model, if these states would have energies that are several hundred MeV above the lowest positive parity nucleon resonance.

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