Research Article

The Linear Fitting Method for Model of PIN Array Receiver in Space Optical Communication

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We have proposed PIN array as receiver in space optical communication and adopted the double diode model, the widely accepted solar energy output theory, to study its characteristics of output current and voltage. In order to make the calculation simplified, the linear fitting method is put forward. Later, we adopt both the traditional method Newton-Raphson and the new linear fitting method to calculate the values of both maximum power and the corresponding voltage of $3 \times 3$ PIN array. The comparison result shows the calculated values with two methods are good consistent. It validates the feasibility of the new linear fitting method. In the next step, the experiment has been carried out. The experimental result validates the reasonability of adopting the double diode model to study PIN array. At the same time, it validates the feasibility of the new linear fitting method again.

1. Introduction

The combination of free space optical communication and ordinary optical fiber communication is considered to be a hopeful method to solve "the Last Mile Issue of Communication." As to the combination, the coupling of space light to optical fiber is the key issue. In order to address this question, many scholars have conducted extensive research [1–7]. In 2013, Zhang et al. set up the space optical coupling alignment platform; they adopted both the position sensor and fast reflector to track the spatial light spot [1]. In 2014, Luo et al. used adaptive fiber couplers (AFC) to establish a closed loop control system to improve the coupling efficiency [2]. In 2014, Lei et al. adopted both the stepper motor and self-focusing lens to find the best coupling position in fiber array [3]. In reference [4], the authors present a new cladding design for high birefringence and low confinement loss photonic crystal fibers (PCFs) using a full-vector finite element method with anisotropic perfectly matched boundary layer. In reference [5], the authors propose a novel high-birefringence index-guiding photonic crystal fiber (PCF). This PCF is composed of a solid silica core and a cladding with two differently sized squeezed elliptical air-holes. In our previous works, we have proposed the conical fiber receiver to improve coupling efficiency and have studied its features from various perspectives [6–9]. Generally speaking, in these researches, the single optical fiber is used as the receiving device. The receiving area of the single optical fiber is too small; it brings difficulty for the coupling.

In recent years, some scholars have begun to apply PIN photodiodes in FSO communication systems. In [10], the authors design and demonstrate an integrated receiver for a visible light communication (VLC) system based on LED and an array of silicon PIN diode detectors. In [11], based on PIN photodiode receivers, the authors employ both asymmetric density evolution technique and an optimization scheme combined search algorithms to optimize low-density parity-check (LDPC) codes for FSO systems. In [12], with the cascaded preequalization circuit, and a differential outputs PIN receiver, the authors proposed a cascaded amplitude equalizer for high-speed visible light communications (VLC) system. In our previous work, we have proposed $3 \times 3$ PIN diode array to receive the spatial light, each PIN photodiode could independently receive the space laser and convert it into electrical signals [13].

The above literatures have studied PIN photodiode from various perspectives but failed to consider the effect of uneven illumination. In PIN diode array, due to the different
light intensities, different PIN diode produces different amount of photocurrent. In order to track the maximum output power, we should build mathematical models to analyze the characteristics of output current and voltage. But such research is relatively lacking. For the question of how to track the maximum power, there are many interesting studies in the field of solar energy technology. They discovered the single photovoltaic cell can be regarded as a photoelectric cell. Its current and voltage characteristics can be described with double diode model. Then, the double diode equation in photovoltaic model is used to describe the current and voltage characteristics of PIN diode. Later, Newton-Raphson method is adopted to resolve the resolution. In this work, we attempt to adopt this method to study PIN array receiver in space optical communication. This is our innovation. Considering that the calculation of Newton-Raphson method is too complicated, we adopt the linear fitting method to do the calculation work. This is our innovation as well. At the same time, the experiment is carried out whose results prove the proposed linear fitting method is reasonable.

2. The Equivalent Circuit Model

In the photovoltaic cell, P-N junction absorbs light energy and turns it into electrical energy. In order to prevent current backflow, the bypass diode is required in the circuit. Based on engineering practice and theoretical analysis, the single photovoltaic cell can be regarded as a photoelectric cell. Its current and voltage characteristics can be described with double diode model [14–16]. For single PIN diode, it is similarly true [17, 18]. We can use the double diode equation in photovoltaic model to describe the current and voltage characteristics of PIN diode, as shown in Figure 1. Normally, the value of $R_{sh}$ is large, its impact on the open-circuit voltage and short-circuit current is little; we can ignore its influence. Then, the output current of single PIN diode $I$ is given by [19, 20]:

$$I = I_{ph} - I_d,$$  

where $I_{ph}$ is the photocurrent of single PIN diode; and $I_d$ is the diode current; it is given by [19, 20]

$$I_d = I_0 \left\{ \exp \left[ \frac{q}{kT} \left( U + I R_{S} \right) \right] - 1 \right\},$$  

where $I_0$ is the reverse saturation current; it could be determined by the following equation [19, 20]:

$$I_0 = I_n \left( \frac{T}{T_r} \right)^{3} \exp \left[ \frac{q E_g}{A k} \left( \frac{1}{T_r} - \frac{1}{T} \right) \right].$$  

In the above two equations (2) and (3), $q$ is the electronic charge ($1.6 \times 10^{-19}$ C), $k$ is the Boltzmann constant (it is $1.38 \times 10^{-23}$ J/K); $T$ is the temperature of single PIN diode, $T_r$ is the temperature under standard test condition, $I_n$ is the reverse saturation current on the temperature $T_r$, $R_s$ is the equivalent series resistance (usually, it is hundred milli ohms or dozen milli ohms [21, 22]), $A$ is the ideal factor of the single PIN diode, and $E_g$ is the band gap energy.

For the PIN diode array, as shown in Figure 2, to assume the number of PIN diodes in series is $N_s$, that in parallel is $N_p$, then the total number $n$ is $N_s \times N_p$, we can gain the array’s V-I characteristic equation [21, 22]:

$$I = N_p I_{ph} - N_p I_d \left\{ \exp \left[ \frac{q}{kT} \left( \frac{U}{N_s} + \frac{IR}{N_p} \right) \right] - 1 \right\}. \tag{4}$$

3. The Linear Fitting Method for PIN Diode Model

Formulas (1) and (4) are general mathematical models. However, they belong to exponential transcendental equations. Usually, we should adopt Newton-Raphson method to calculate it. But, this method needs large computation costs [23, 24]. Here, we put forward the new linear fitting method to simplify the calculation process.

With the MATLAB software, we can adopt the traditional method to figure out the relationship between the light intensity $S$ and the voltage under maximum output power condition (we call it as maximum power voltage $V_{mpp}$). To assume the PIN array is $3 \times 3$, the ideal factor $A$ is 1.2, and the band gap energy $E_g$ is 1.12, the temperature unchanged ($T$ is 25°C), then we can gain the relationship between $V_{mpp}$ and $S$, as shown in the actual curve in Figure 3.

As shown in the actual curve in Figure 3, we can use the following linear equation to describe the relation between $V_{mpp}$ and $S_r$ [25, 26],

$$V_{mpp} = k_1 \ln \left( \frac{S_r}{S_{nr}} \right) + b_1,$$  

where $V_{mpp}$’ is the fitting maximum power voltage, and $S_{nr}$ is the standard light intensity (100 mW/cm²). This equation’s meaning is that we can use this fitting equation to describe the relation between $V_{mpp}$ and $S$, as the fitting curve shown in Figure 3.

To assume light intensity maintained unchanged, for example, it is 20 mW/cm². We use the traditional Newton-Raphson method to resolve the mathematical model formula from (1) to (4). Then, we can figure out the relationship between maximum power voltage $V_{mpp}$ and temperature $T$, as shown in Figure 4. From those data points in the figure, we use equation (6) to describe the relation between the fitting maximum power voltage $V'_{mpp}$ and the maximum power voltage $V_{mpp}$,

$$V'_{mpp} = V_{mpp} + k_T (25 - T),$$  

where $k_T$ is the temperature coefficient.
In the next step, when $S_i$ are 40, 60, 80, and 100 mW/cm$^2$, we can repeat the above calculation process to gain the different fitting lines, as shown in Figure 4. We can see that, when $S_i$ is different, the influence of temperature on the relations between $V_{mpp}'$ and $V_{mpp}$ is different. Hence, we use equation (7) to describe the relation between the temperature coefficient $k_T$ and the light intensity $S_i$.

$$k_T = k_3S_i + b_2. \quad (7)$$

Similarly, when $S_i$ are 20, 40, 60, 80, and 100 mW/cm$^2$, we can gain the relation curves between the maximum output power $P_{mpp}$ and $V_{mpp}$, as shown in Figure 5. From the figure, we can also discover that when $S_i$ is maintained unchanged, we can use the following linear equation to describe the relation between $P_{mpp}$ and $V_{mpp}$.

$$P_{mpp} = k_4V_{mpp} + b_v. \quad (8)$$

In the above equation, the coefficients $k_v$ and $b_v$ are related with $S_i$; we can look them as the functions of the light intensity $S_i$.

$$k_v = k_3S_i + b_3, \quad (9)$$

$$b_v = k_4S_i + b_4. \quad (9)$$

From the above process, it can be seen that we can use the fitting linear equations to describe the relation between $P_{mpp}$, $V_{mpp}$, and $S_i$. The fitting linear equations can make the solving process simplified.

4. The Calculation Comparison between the Linear Fitting Method and Traditional Newton-Raphson Method

For the coefficients $k_1$ and $b_1$ in equation (5), we adopt the least-square method to compute their values [14–16]. The specific method is like the following steps. We select a lot of points on the actual curve in Figure 2, use $S_n$ to represent their abscissa values, use $V_{mpp}$ to represent their ordinate values, and use $n$ to represent the total number of discrete points. Then, the coefficients $k_1$ and $b_1$ are given by

$$k_1 = \frac{\sum S_n \sum V_{mpp} - n \sum S_n V_{mpp}}{(\sum S_n)^2 - n \sum S_n^2},$$

$$b_1 = \frac{\sum (S_n V_{mpp}) \sum S_n - \sum V_{mpp} \sum S_n^2}{(\sum S_n)^2 - n \sum S_n^2}. \quad (10)$$

To assume the PIN array is $3 \times 3$, the ideal factor $A$ is 1.2, and the band gap energy $E_g$ is 1.12, with the above least-square method, we can work out the values of the coefficients $k_1$ and $b_1$, which are $k_1 = 1.082$ and $b_1 = 17.678$, and in this way, we can gain the values of other coefficients, like the following:

$$k_2 = 0.0, \quad b_2 = 0.076, \quad k_3 = 0.039, \quad b_3 = -0.541, \quad k_4 = -0.044, \quad b_4 = 8.540. \quad (11)$$

In order to verify the effectiveness of this method, we make a comparison. Firstly, we adopt traditional Newton-Raphson method to calculate the values of both maximum
power and the corresponding voltage, looking them as $P_{mpp1}$ and $V_{mpp1}$. Then, we adopt new method to calculate the values of both maximum power and the corresponding voltage, looking them as $P_{mpp2}$ and $V_{mpp2}$. Later, the relative error $\delta$ is defined as the following equations.

For $V_{mpp}$,

$$\delta = \left| \frac{V_{mpp1} - V_{mpp2}}{V_{mpp1}} \right| \times 100\%.$$ \hfill (12)

For $P_{mpp}$,

$$\delta = \left| \frac{P_{mpp1} - P_{mpp2}}{P_{mpp1}} \right| \times 100\%.$$ \hfill (13)

Finally, the result is shown in Table 1.

From Table 1, it can be seen that both $V_{mpp1}$ and $P_{mpp1}$ are coincident with $V_{mpp2}$ and $P_{mpp2}$, respectively. It validates the feasibility of the new linear fitting method. Specifically, for the relative error $\delta$, it changes from 0.25% to 4.28%; this value is very small. In other words, the matching degree is high.

In the view of the computational complexity, the traditional Newton-Raphson method needs to solve the transcendental equation. It includes the exponent arithmetic operations and needs multiple iterations. In a word, it needs much calculation cost. However, the linear fitting method does not need so much calculation. This method greatly simplifies the calculation process.

5. Experimental Test

In order to check the reasonability of the new linear method again, the author had a discussion with the technicians of Sichuan Coreway Optech Co., Ltd. At the same time, the test experiment is done.

In our previous work, as shown in Figure 6, we have produced $3 \times 3$ PIN array to do the experiment of space optical communication [13].

At this time, we also use this PIN array to do the test experiment. In this experiment, we use TES-1339 illuminometer to test the light intensity. TES-1339 illuminometer is produced in Taiwan; it is widely used in various industries and fields [4–7]. The experiment starts at 6:00 a.m. and ends at 2:00 p.m. At the same time, we adopt the new linear fitting method to calculate the theoretical data. In order to compare the experiment data with the theoretical data, we define the error rate in the following equation.

Error rate = \left( \frac{\text{measured data} - \text{theoretical data}}{\text{measured data}} \right) \times 100\%.

![Figure 5: The relations between $V_{mpp}$ and $P_{mpp}$ under different $S_i$.](image)

### Table 1: Comparison between Newton-Raphson and proposed methods.

| Temperature/°C | Light intensity/(mW/cm²) | $V_{mpp1}$/V | $V_{mpp2}$/V | $P_{mpp1}$/W | $P_{mpp2}$/W | Error ($\delta$)/% |
|----------------|--------------------------|--------------|--------------|--------------|--------------|-------------------|
| 25             | 100                      | 17.75        | 17.68        | 61.81        | 62.51        | 0.39              |
| 35             | 80                       | 16.32        | 16.30        | 45.76        | 46.05        | 0.12              |
| 50             | 60                       | 14.87        | 14.61        | 32.37        | 32.65        | 0.86              |
| 60             | 40                       | 13.25        | 13.01        | 20.10        | 20.31        | 1.04              |
| 70             | 20                       | 11.82        | 11.79        | 10.03        | 10.46        | 0.25              |

In the view of computational complexity, the traditional Newton-Raphson method needs to solve the transcendental equation. It includes the exponent arithmetic operations and needs multiple iterations. In a word, it needs much calculation cost. However, the linear fitting method does not need so much calculation. This method greatly simplifies the calculation process.
Finally, the measurement results and the error rate are shown in Table 2.

From Table 2, it can be seen that when the light intensity is low, the error rate is large. On the contrary, when the light intensity is high, the error rate is small. For example, when the light intensities are 7 W/cm² and 16 W/cm², the error rates are 9.3% and 7.2%, respectively. When the light intensities are 22 W/cm² and 23 W/cm², the error rates are 2.6% and 2.5%. This is because the losses of the instrument maintain unchanged. When the light intensity is lower, the fixed losses account for higher proportion. Then, the error is bigger. On the contrary, when the light intensity is higher, the fixed losses account for lower proportion. Then, the error is smaller.

From Table 2, it can be seen that the temperature also influences the error rate. When the temperature is low, the error rate is high. On the contrary, when the temperature is high, the error rate is low. For example, when the temperature is 14°C, the error rate is 9.3%. However, when the temperature is 28°C, the error rate is 2.5%. Maybe it is because of slightly underestimating the contribution of temperature in the double diode model. The new linear fitting method is deduced from the mathematical model. Thus, it produces the errors. In addition, the experimental data will be affected by the environmental parameters of last time. But this factor is not considered in the mathematical model. On the whole, the average error is about 4%; it shows the experimental result is in relatively good agreement with the numerical calculation result. The experiment proves the reasonability of the new linear fitting method again.

6. The Conclusion

The double diode model based on semiconductor diode theories have been widely used to extract the maximum power point of the solar PV array. In this paper, we use this model to study PIN array in space optical communication. Because the mathematical model contains transcendental equation, it makes the solving process complicated. In order to make the calculation simple, we put forward the new linear fitting method to do the calculation process. The comparison of calculation results has been conducted between the new and traditional methods. It shows the relative error $\delta$ changes from 0.25% to 4.28% in Table 2; this value is very small. In other words, matching degree is high. It validates the feasibility of the new linear fitting method. In addition, the test experiment has been carried out. The experimental data are compared with the theoretical data. By the comparison, we find it is reasonable to adopt the double diode model to study PIN array. At the same time, the experiment validates the feasibility of the new linear fitting method once more. This work shows we can use the linear fitting method to trace the maximum power of PIN array in FSO communication systems.

| Time | Temperature (°C) | Light intensity (W/cm²) | Power (W) | Error rate (%) |
|------|------------------|-------------------------|-----------|----------------|
|      |                  |                         | Measured data | Simulation data |               |
| 6:00 | 14               | 7                       | 4.3       | 4.7            | 9.3           |
| 7:00 | 17               | 16                      | 8.3       | 8.9            | 7.2           |
| 8:00 | 18.2             | 19                      | 9.2       | 9.5            | 3.2           |
| 9:00 | 20.1             | 18                      | 9.0       | 9.2            | 2.2           |
| 10:00| 22               | 20                      | 11        | 10.5           | 4.5           |
| 11:00| 23               | 21                      | 11.3      | 11.5           | 1.7           |
| 12:00| 25               | 25                      | 12.3      | 12.7           | 3.2           |
| 13:00| 27               | 22                      | 11.6      | 11.9           | 2.6           |
| 14:00| 28               | 23                      | 11.9      | 12.2           | 2.5           |
Data Availability
The data used to support the findings of this study are available from the author upon request.

Conflicts of Interest
The author declares no conflicts of interest.

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