Sensitivity studies of heat transfer: forced convection across a cylindrical pipe and duct flow.

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Abstract

We consider two common heat transfer processes and perform a through sensitivity study of the variables involved. We derive and discuss analytical formulas for the heat transfer coefficient in function of film velocity, air temperature and pipe diameter. The according plots relate to a qualitative analysis of the multi-variable function $h$, according to functional optimization. For each process, we provide with graphs and tables of the parameters of interest, such as the Reynolds number. This method of study and the specific values can constitute a useful reference for didactic purposes.

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1 Introduction

Studies of heat transfer pertain fundamental physics which is crucial in many situations of practical interest, for instance in building physics. As such, this topic is included in most academic curricula for engineers. Here we address the subject from the perspective of building physics, a field of study that blends physics, mathematics and engineering. A through knowledge of heat transfer has indeed diverse applications which range from indoor environmental quality, moisture tolerance and overall energy saving [1].

A significant improvement in this field took place after the energy crises in the 1970s, and since then the technical literature developed very quickly. Especially in the last decade, it offered a number of studies which benefit from new technology developments such as computer simulations (see for instance [2] and references quoted therein).

The present paper addresses an approach to calculations which can effectively support the teaching in engineering as a whole, as it is not only restricted to heat transfer. We will show that examining the physics behind the specific process gives interesting insights on the phenomenology, especially on didactic but also on research grounds. Here we focus on heat transfer processes, listing tables and plots of the quantities of interest.

The main tool here is a through study of the relevant formulas, focused on the structure of the equations. For instance, studying how the Reynolds number or the convection coefficient depend on the temperature through viscosity and density.

This constitutes a novelty of this work. It is a common procedure in other fields, such as theoretical physics, by students and researchers. However, qualitative aspects are still quite overlooked in engineering in favor of numerics, due to the enormous development of simulation programs [2]. A through analytical study should anyway be considered, as it clearly complements and optimizes the numerical computations.

With this approach, the student is indeed able to predict the behavior of the crucial quantities \textit{a priori}, and knows how these behave in function of the system parameters, even before starting the numerical calculations. This is useful for having more efficient simulations and to validate the according results first at the theoretical level.

As an example, we apply this analytical method to two common heat transfer processes, namely to fluid flow across a cylindrical pipe in Section

\footnote{This method is similar to the so-called \textit{functional optimization}, which has been adapted to engineering studies in [3].}
2 and to internal flow in a duct in Section 3. We provide with
- interpolation curves of the physical properties of the fluids considered
- a qualitative analysis of the main parameters, namely the Reynolds
number and the heat transfer coefficient
- a quantitative study with tables and graphs of the according values.
This is especially useful for students, who generally do not have knowledge
of the actual numbers. Moreover, all the values and the materials (fluids)
have been chosen so that the results can be useful for teaching and also in
practical situations.

In the Appendix we show how to use the least square method to compute
interpolation curves of the variables of interest. As a specific example, we
consider the critical length for a flow on a flat plate, in the case of engine
oil and dry air.

2 Heat transfer across a cylindrical pipe

Consider a cylindrical pipe of external diameter $D$. Some fluid is flowing
across the pipe, with velocity $V$ in proximity of its surface. The Reynolds
Number is given by the following expression,

$$\text{Re}_D = \frac{\rho V D}{\mu} = \frac{V D}{\nu}. \quad (1)$$

Consider first dry air. Since the kinematic viscosity $\nu(T)$ (or, equivalently,
the ratio $\mu/\rho$) is a function of the temperature, namely $[4]$

$$\nu(T) = \left( \frac{2.409 \times 10^8}{T^{3/2}} + \frac{2.6737 \times 10^{10}}{T^{5/2}} \right)^{-1} \left[ \frac{m^2}{s} \right], \quad (2)$$

where $[T] = [K]$, the Reynolds number can be written explicitly as $\text{Re}_D(T)$:

$$\text{Re}_D(T) = \frac{V D}{T^{3/2}} \left( 2.409 + \frac{267.37}{T} \right) \times 10^8. \quad (3)$$

The dependence on $V$ and on the diameter $D$ is trivial, however how the
Reynolds number changes in function of the temperature is not. This is plotted in Fig.4. Table 1 and Table 2 list several values of $\text{Re}_D(T)$. Table 3 and Table 4 give values of $\text{Re}(V)$.

We need to find the convection coefficient which depends on the Reynolds
number in order to calculate the heat transfer rate from the plate to the flowing fluid. For that purpose we need the Nusselt Number which contains the
convection coefficient and is calculated via the Reynolds number. However
the flow over a cylinder (or a sphere) has to become turbulent after the
impact. Thus the Nusselt number evolves in function of the position of fluid
particles beside the impact point.

Therefore, to calculate the average heat transfer coefficient we use the
Churchill-Bernstein correlation for the average Nusselt number [5, 6],
\[ \text{Nu}_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62 \text{Re}^{1/2} \text{Pr}^{1/3}}{1 + (0.4/\text{Pr})^{2/3}}^{1/4} \left[ 1 + \left( \frac{\text{Re}}{28200} \right)^{5/8} \right]^{4/5} \]. \tag{4}

The convection coefficient then can be rewritten as a function of the variables
of interest as follows,
\[ h(V, D, \nu) = k(T) \left\{ \frac{0.3}{D} + 0.62 \frac{\text{Pr}(T)^{1/3}}{1 + (0.4/\text{Pr}(T))^{2/3}}^{1/4} \right. \]
\[ \left. \times \sqrt{\frac{V}{\nu(T)D}} \left[ 1 + \left( \frac{VD}{28200\nu(T)} \right)^{5/8} \right]^{4/5} \right\} , \tag{5} \]

where \( k \) is the thermal conductivity of the fluid, and the dependence on \( T \)
is explicit. The above formula is common to both air and water.

The Prandtl number is obtained from \( \text{Pr} = \nu/\alpha \), where the thermal
diffusivity is written as [4]
\[ \alpha(T) = -4.3274 + 4.1190 \times 10^{-2} T + 1.5556 \times 10^{-4} T^2 \left[ 10^{-6} \frac{m^2}{s} \right] , \tag{6} \]
and the thermal conductivity is retrieved via Sutherland’s equation (Reid,
1966) [4]:
\[ k(T) = \frac{2.3340T^{3/2}}{164.54 + T} \left[ 10^{-3} \frac{W}{mK} \right] . \tag{7} \]

By substituting the above correlations into Eq. (5), one therefore obtains the
explicit dependence of \( h \) on the film temperature \( T = (T_s + T_\infty)/2 \) alone,
with \( V \) and \( D \) fixed. The diameter of the pipe and the fluid velocity are
indeed the parameters easily controlled.

Since the explicit form of \( h(T) \) is very involved, due to Eqs. (2), (6)
and (7), we prefer to plot the values for \( 0^oC < T < 150^oC \) and find the
interpolation curve
\[ h_{\text{air}}(T) = 10^{-4} T^2 - 0.0739 T + 51.022 \left[ \frac{W}{m^2 K} \right] , \tag{8} \]
where \([T] = [K]\). This is shown in Fig. 3 that gives a direct relation between film temperature and convection coefficient. Note that the heat transfer coefficient in (8) has a minimum for \(T \sim 370^\circ C\), then starts raising again.

Therefore we can conclude that for air flowing through a single cylinder, heat transfer becomes less efficient with increasing air temperature, for any acceptable value of \(T\).

Following exactly the same reasoning, we now compute the heat transfer coefficient for water. This is slightly more difficult than for air, due to the lack of interpolation curves for water available in the literature. Therefore we compute them as follows.

The expression \(\nu(T)\) for the kinematic viscosity can be found from the following formulas for the dynamic viscosity [7]

\[
\mu(T) = (280.68 T_s^{-1.9} + 511.45 T_s^{-7.7} + 61.131 T_s^{-19.6} + 0.45903 T_s^{-40}) \left[ 10^{-6} \frac{kg}{ms} \right],
\]

(9)

where \(T_s = T/300K\), and for the density (with \([T] = [C]\))

\[
\rho(T) = 2 \times 10^{-5} T^3 - 0.0063 T^2 + 0.0266 T + 999.98 \left[ \frac{kg}{m^3} \right].
\]

(10)

To find the analytical expression for \(\nu(T)\), we then compute the values by using (9) and (10). These are consistent with e.g. [6, 7]. Interpolating the resulting curve gives

\[
\nu(T) = 3 \times 10^{-8} T^4 - 9 \times 10^{-6} T^3 + 0.001 T^2 - 0.0552 T + 1.796 \left[ 10^{-6} \frac{m^2}{s} \right],
\]

(11)

where the temperature is in degree Celsius. The corresponding Reynolds number values are plotted in Fig. 2.

The thermal diffusivity \(\alpha\) for water is evaluated by means of \(\alpha = k/(\rho c_p)\), and results in the following correlation formula,

\[
\alpha(T) = -2 \times 10^{-5} T^2 + 0.0049 T + 1.3491 \left[ 10^{-7} \frac{m^2}{s} \right].
\]

(12)

This is obtained from the thermal conductivity [8]

\[
k(T) = 0.5678 + 1.8774 \times 10^{-3} T - 8.179 \times 10^{-6} T^2 + 5.6629 \times 10^{-9} T^3 \left[ \frac{W}{mK} \right],
\]

(13)
and from the following expression for the specific heat \[6\,9\]

\[
c_p(T) = -3 \times 10^{-8} T^5 + 10^{-5} T^4 - 0.0014 T^3 \\
+ 0.0978 T^2 - 3.2467 T + 4217.7 \quad \left[ \frac{J}{kgK} \right].
\]

Similarly as before, we obtain the Prandtl number for water from \( \Pr = \nu/\alpha \), whose values are cross-checked also with \[6\]. Then we insert (11) and (13) in Eq.(5).

For water flowing at \( V = 0.2 \, m/s \) over a cylindrical pipe with \( D = 5 \, cm \), we find for the temperature range \( 5^\circ C < T < 90^\circ C \)

\[
h(T) = -0.0252 T^2 + 18.925 T + 1560.9 \quad \left[ \frac{W}{m^2K} \right],
\]

where [\( T \)] = [\( C \)]. The above heat transfer coefficient is plotted in Fig.4.

Note that also here there is a maximum, but it occurs for \( T \sim 375^\circ C \), thus very well above the boiling point. Thus heat transfer between a water flow and a cylindrical bar increases with the water temperature for any realistic \( T \) values.

On the other hand, graphs of Eq.(5) for air and water, where \( h \) is a function of the fluid velocity with fixed \( T = 15^\circ C \) and \( D = 5 \, cm \), can be found in Figs 5 and 6. The according interpolation curves,

\[
h_{water}(V) = 5901.71 \times V^{0.779},
\]

\[
h_{air}(V) = 13.4712 \times V^{0.789},
\]

are obtained with the least square method, as described in the Appendix.

For both fluids, the dependence of Re and \( h \) on the fluid velocity \( V \) is trivial. However, the temperature influences the Reynolds number and heat transfer coefficient in opposite ways. For a liquid like water, the viscosity decreases at higher temperatures, thus its ability to transfer heat by convection is improved. On the other hand, since air is a gas, it has increasing viscosity with temperature, see Eq.(2). This hinders the heat transfer as seen in Eq.(8), as the dominant term is \( \propto -T \).

Fig.3 and Fig.4 give the evolution of \( h \) in function of the temperature when \( V \) and \( D \) are constant. The according values are given in Tables 7 and 8.

The diameter also has a relevant influence on \( h \). Fig.11 and Fig.12 show that the convection coefficient is inversely proportional to the diameter of the cylinder, and quite strongly so (power law). Indeed for a larger diameter
the flow is more likely to become turbulent earlier, closer to the impact point. The ordered fluid motion in a laminar flow transfers heat more efficiently. Also Re (which indicates the nature of the flow) is a function of $D$.

As explained above, the temperature still has an influence. However, it is less significant than the diameter, since $h$ is mostly linearly dependent on $T$. This is especially evident for air, whose properties do not change appreciably in this particular case (0°C to 100°C).

We can also calculate the expressions of these functions at 15°C, and obtain

$$h_{\text{air}}(D) = 42.37 \times D^{-0.136}, \quad (18)$$
$$h_{\text{water}}(D) = 1838.44 \times D^{-0.223}. \quad (19)$$

These are plotted in Fig 7 and Fig 8, where the convection coefficient decreases with a low derivative, almost logarithmically. This makes sense because the Nusselt number is expressed as a power function of the Reynolds number which is proportional to the diameter and the velocity. Therefore the convection coefficient is very sensitive to a diameter increase, but only when $D$ is rather small. Above 10 cm to 20 cm, the diameter has not much influence; the flow is mainly laminar over the cylinder and the heat transfer coefficient is small.
### Tables and Plots

#### Table 1: Reynolds numbers for water in function of $T$ and $V$ for $D = 5$ cm.

| $T$(°C) | Reynolds Number ( ×10⁴) |
|--------|-------------------------|
| 0.01   | 1.39 2.79 4.18          |
| 5      | 1.88 3.76 4.94          |
| 10     | 1.91 3.82 5.74          |
| 15     | 2.19 4.39 6.58          |
| 20     | 2.49 4.98 7.47          |
| 25     | 2.80 5.59 8.39          |
| 30     | 3.12 6.24 9.36          |
| 35     | 3.45 6.90 10.35         |
| 40     | 3.80 7.60 11.39         |
| 45     | 4.15 8.31 12.46         |
| 50     | 4.52 9.03 13.55         |
| 55     | 4.89 9.77 14.66         |
| 60     | 5.26 10.53 15.79        |
| 65     | 5.66 11.32 16.98        |
| 70     | 6.05 12.10 18.15        |
| 75     | 6.45 12.89 19.34        |
| 80     | 6.84 13.69 20.53        |
| 85     | 7.27 14.54 21.80        |
| 90     | 7.66 15.32 22.98        |
| 95     | 8.09 16.19 24.28        |

#### Table 2: Reynolds numbers for air in function of $T$ and $V$ for $D = 5$ cm.

| $T$(°C) | Reynolds Number ( ×10⁴) |
|--------|-------------------------|
| 5      | 1.82 3.63 5.45          |
| 10     | 1.76 3.52 5.28          |
| 15     | 1.71 3.41 5.12          |
| 20     | 1.65 3.31 4.96          |
| 25     | 1.61 3.21 4.82          |
| 30     | 1.56 3.12 4.68          |
| 35     | 1.51 3.03 4.54          |
| 40     | 1.47 2.94 4.42          |
| 45     | 1.43 2.86 4.29          |
| 50     | 1.39 2.79 4.18          |
| 55     | 1.36 2.71 4.07          |
| 60     | 1.32 2.64 3.96          |
| 65     | 1.29 2.57 3.86          |
| 70     | 1.25 2.51 3.76          |
| 75     | 1.22 2.45 3.67          |
| 80     | 1.19 2.39 3.58          |
| 85     | 1.16 2.33 3.49          |
| 90     | 1.14 2.27 3.41          |
| 95     | 1.11 2.22 3.33          |
| 100    | 1.08 2.17 3.25          |
| 105    | 1.06 2.12 3.18          |
| $T_1$ | $T_2$ | $T_3$ |
|-------|-------|-------|
| $5^\circ$ C | $50^\circ$ C | $95^\circ$ C |
| $V$ Reynolds Number ($\times 10^4$) | $V$ Reynolds Number ($\times 10^4$) |
| 1 | 3.29 | 9.03 | 16.19 |
| 2 | 6.58 | 18.06 | 32.37 |
| 3 | 9.87 | 27.10 | 48.56 |
| 4 | 13.16 | 36.13 | 64.75 |
| 5 | 16.45 | 45.16 | 80.93 |
| 6 | 19.74 | 54.19 | 97.12 |
| 7 | 23.03 | 63.22 | 113.31 |
| 8 | 26.33 | 72.26 | 129.49 |
| 9 | 29.62 | 81.29 | 145.68 |
| 10 | 32.91 | 90.32 | 161.87 |
| 11 | 36.20 | 99.35 | 178.06 |
| 12 | 39.49 | 108.38 | 194.24 |
| 13 | 42.78 | 117.42 | 210.43 |
| 14 | 46.07 | 126.45 | 226.62 |
| 15 | 49.36 | 135.48 | 242.80 |
| 16 | 52.66 | 144.51 | 258.99 |
| 17 | 55.95 | 153.54 | 275.18 |
| 18 | 59.24 | 162.58 | 291.36 |
| 19 | 62.53 | 171.61 | 307.55 |
| 20 | 65.82 | 180.64 | 323.74 |

Table 3: Re($V, T$) for water, $D = 5\text{ cm}$ and $[V] = [\text{m/s}]$.

Table 4: Re($V, T$) for air, $D = 5\text{ cm}$ and $[V] = [\text{m/s}]$. 
| $T(°C)$ | $h (W/m^2°C)$ | $T(°C)$ | $h (W/m^2°C)$ |
|---------|--------------|---------|--------------|
| 5       | 50.70        | 5       | 1656.02      |
| 20      | 49.58        | 10      | 1747.85      |
| 30      | 48.89        | 20      | 1928.34      |
| 40      | 48.24        | 30      | 2104.71      |
| 50      | 47.63        | 40      | 2276.80      |
| 60      | 47.05        | 50      | 2444.29      |
| 70      | 46.50        | 60      | 2606.70      |
| 80      | 45.98        | 70      | 2763.52      |
| 90      | 45.48        | 80      | 2914.27      |
| 100     | 45.00        | 90      | 3058.54      |

Table 5: $h$ for air, with $V = 5 \text{ m/s}$ and $D = 5 \text{ cm}$.

| $V (\text{m/s})$ | $h (W/m^2C)$ | $V (\text{m/s})$ | $h (W/m^2C)$ |
|-----------------|--------------|-----------------|--------------|
| 2               | 26.59        | 0.2             | 1827.15      |
| 4               | 42.20        | 0.4             | 2940.46      |
| 6               | 56.21        | 0.6             | 3950.35      |
| 8               | 69.43        | 0.8             | 4909.08      |
| 10              | 82.16        | 1               | 5836.18      |
| 12              | 94.55        | 1.2             | 6741.40      |
| 14              | 106.69       | 1.4             | 7630.38      |
| 16              | 118.63       | 1.6             | 8506.76      |
| 18              | 130.41       | 1.8             | 9373.0       |
| 20              | 142.06       | 2               | 10230.87     |

Table 6: $h$ for water, with $V = 0.2 \text{ m/s}$ and $D = 5 \text{ cm}$.

Table 7: Convection Coefficient for air at $T_f = 15°\text{C}$ in function of $V$ for $D = 5 \text{ cm}$.

Table 8: Convection Coefficient for water at $T_f = 15°\text{C}$ in function of $V$ for $D = 5 \text{ cm}$.
Figure 1: $Re_D$ for air in function of $T$, for $D = 5\, \text{cm}$ and $V = 5, 10, 15\, \text{m/s}$, as given by Eq.(3).

Figure 2: $Re_D$ for water in function of $T$, for $D = 5\, \text{cm}$ and $V = 1, 2, 5\, \text{m/s}$ as given by Eq.(3).
Figure 3: Air convection coefficient in function of temperature for $D = 5$ cm and $V = 5$ m/s.

Figure 4: Water convection coefficient in function of temperature for $D = 5$ cm and $V = 0.2$ m/s.
Figure 5: $h$ for air in function of $V$, for $D = 5 \text{ cm}$ and $T_f = 15^\circ \text{C}$, as given by Eq. (5). The fit curve (16) is dashed.

Figure 6: $h$ for water in function of $V$, for $D = 5 \text{ cm}$ and $T_f = 15^\circ \text{C}$, as given by Eq. (5). The fit curve (17) is dashed.
Figure 7: Air convection coefficient in function of the diameter for $L = 1$ m, $V = 8$ m/s and $T = 15^\circ$C. The fit curve (18) is dashed.

Figure 8: Water convection coefficient in function of the diameter for $L = 1$ m, $V = 8$ m/s and $T = 15^\circ$C. The fit curve (19) is dashed.
Figure 9: Air convection coefficient in function of the velocity for $T_1 = 15 \, ^\circ C$, $T_2 = 50 \, ^\circ C$ and $T_3 = 95 \, ^\circ C$. $D = 5 \, cm$.

Figure 10: Water convection coefficient in function of the velocity for $D = 5 \, cm$. $T_1 = 15 \, ^\circ C$, $T_2 = 50 \, ^\circ C$ and $T_3 = 95 \, ^\circ C$. 
Figure 11: Air convection coefficient in function of the diameter for $V = 8 \text{ m/s}$. The film temperatures are $T_1 = 15^\circ \text{C}$, $T_2 = 50^\circ \text{C}$ and $T_3 = 95^\circ \text{C}$.

Figure 12: Water convection coefficient in function of the diameter for $V = 0.5 \text{ m/s}$. The film temperatures are $T_1 = 15^\circ \text{C}$, $T_2 = 50^\circ \text{C}$ and $T_3 = 95^\circ \text{C}$.
3 Convection inside a pipe

Let us consider a cylindrical pipe with constant surface temperature or constant surface heat flux. A fluid flows inside the pipe, with properties evaluated at the bulk temperature \( T_2 \). In this study we choose dry air at 1 atm (flow inside a ventilation duct), as well as R-717 (namely ammonia), a refrigerant commonly used e.g. in the cooling pipes under ice hockey rinks. This particular situation is especially pertinent to the case studied in this section.

Recall the definition (1) for the Reynolds number, which still holds here. The characteristic length is in this case the internal diameter \( D \)

\[
Re = \frac{V_\infty D}{\nu}.
\]  

(20)

Air properties are discussed in the previous section, Eqs. (2), (6) and (7). Values of the Reynolds number for the temperature range \( 0^\circ C < T < 105^\circ C \) are listed in Table 2 for a cylinder of external diameter \( D = 5 \text{ cm} \). Since here we discuss a \( D = 20 \text{ cm} \) duct, it is enough to multiply those values by a factor of 4. Values of \( \text{Re}(T, V_\infty) \) for a different temperature range are given instead in Table 9 and in Table 11. The according plot is found in Fig.14.

Consider now saturated ammonia. The thermophysical properties are as follows [10].

\[
k(T) = -0.0023 T + 0.539 \left[ \frac{W}{mK} \right],
\]  

(21)

\[
\rho(T) = -3 \times 10^{-5} T^3 - 0.0011 T^2 - 1.362 T + 638.2 \left[ \frac{kg}{m^3} \right],
\]  

(22)

\[
\nu(T) = 10^{-4} T^2 - 0.0262 T + 2.9748 \left[ 10^{-7} m^2/s \right],
\]  

(23)

the Prandtl number is

\[
Pr(T) = 5 \times 10^9 T^4 - 3 \times 10^{-7} T^3 + 5 \times 10^{-5} T^2 - 0.0087 T + 1.6225,
\]  

(24)

and the specific heat capacity holds as

\[
c_p(T) = 3 \times 10^{-5} T^4 - 0.0015 T^3 + 0.0522 T^2 + 6.7534 T + 4609.8 \left[ \frac{J}{kgK} \right],
\]  

(25)

\[2\] A common method, which we follow here, is to define \( T = (T_i + T_e)/2 \), where \( T_i \) is the inlet temperature and \( T_e \) the exit (or supply) temperature.
In all the equations above, \([T] = [C]\). These give the tables [10] and [12] and the plot in Fig [15]. From Figs. [14] and [15] one can infer that the Reynolds number is linearly dependent on temperature for both air and ammonia (with negative derivative in the case of air). Let us stress that this is an example of a result which does not usually appear in textbooks.

### 3.1 Heat transfer coefficient

The convection coefficient is obtained via the Nusselt number \(Nu\), through

\[
h = \frac{k}{D} Nu. \tag{26}
\]

\(Nu\) depends on the nature of the flow. This is determined by means of the Reynolds number as follows:

\[
\begin{align*}
\text{Re} &< 2300 & \text{laminar} \\
2300 \leq \text{Re} &\leq 4000 & \text{transitional} \\
\text{Re} &> 4000 & \text{turbulent}
\end{align*}
\tag{27}
\]

It is clear from the plots in Fig [14] and Fig [15] that here the flow is always turbulent. In the following we list however the correlation formulas for the three cases, and in Fig [13] we provide with a computer algorithm to compute the Nusselt number in any possible configuration.

In the following, we describe the procedure and list the equations which are used to obtain Fig [13]. For the formulas we adopt the reference [6], unless otherwise mentioned.

#### 3.1.1 Laminar flow

This corresponds to \(\text{Re} < 2300\). The thermal entry length, defined as

\[
L_{t,\text{laminar}} \approx 0.05\text{RePr}D, \tag{28}
\]

determines if the flow is fully developed, under the condition \(L_{t,\text{laminar}} < 0.5 \times L\), where \(L = \text{actual length of the plate. This is practically always the case [6]. The expression of Nu then depends on the boundary conditions. The}\\
\]

For a constant surface temperature,

\[
Nu = 3.66 \quad T_s = \text{constant}, \tag{29}
\]

and for a constant surface heat flux,

\[
Nu = 4.36 \quad \dot{q}_x = \text{constant}. \tag{30}
\]
If instead the flow is not fully developed, we use the following expression of Nu for the entry region of a laminar flow,

$$\text{Nu} = 3.66 + \frac{0.065 \,(D/L) \,\text{RePr}}{1 + 0.04 \,[(D/L) \,\text{RePr}]^{2/3}}. \quad (31)$$

### 3.1.2 Transitional flow

In this case $2300 \leq \text{Re} \leq 4000$. A transitional flow has a peculiar behavior, hence we can consider it either laminar or turbulent. The following expression is valid for the above range of Re,

$$\text{Nu} = \frac{(f/8) \,(\text{Re} - 1000) \,\text{Pr}}{1 + 12.7 \,(f/8)^{0.5 \,(\text{Pr}^{2/3} - 1)}} \begin{cases} 0.5 \leq \text{Pr} \leq 2000 \\ 3 \times 10^3 < \text{Re} < 5 \times 10^6 \end{cases} \quad (32)$$

For other ranges of Prandtl and Reynolds numbers, we consider the flow as turbulent and use the expressions given below in equations (34), (35), (36) and (37).

### 3.1.3 Turbulent flow

Here $\text{Re} > 4000$. The flow is also considered as fully developed, as the thermal entry length is written as

$$L_{t,\text{turbulent}} \approx 10D, \quad (33)$$

and the pipe diameter is usually rather small as compared to its total length ($L_{t,\text{turbulent}} \ll 0.5 \times L$). Assume now that the pipe is smooth. The Nusselt number is then given by the Dittus-Boelter equation for cooling,

$$\text{Nu} = 0.023 \,\text{Re}^{0.8} \,\text{Pr}^{0.3}, \quad (34)$$

which is valid for $0.7 \leq \text{Pr} \leq 160$ and $\text{Re} > 10000$.

For a smaller range of Prandtl numbers, one can use the expression

$$\text{Nu} = \frac{(f/8) \,\text{RePr}}{1.07 + 12.7 \,(f/8)^{0.5 \,(\text{Pr}^{2/3} - 1)}} \begin{cases} 0.5 \leq \text{Pr} \leq 2000 \\ 10^4 < \text{Re} < 5 \times 10^6 \end{cases} \quad (35)$$

which is more accurate. For really small Prandtl numbers the following formulas are available for $10^4 \leq \text{Re} \leq 10^6$

$$\text{Nu} = 4.8 + 0.0156 \,\text{Re}^{0.85} \,\text{Pr}^{0.93} \quad \text{Pr} < 0.5, \quad T_s = \text{constant}, \quad (36)$$
Figure 13: Algorithm to find the Nusselt number \[4\]

\[
\text{Nu} = 6.3 + 0.0167\text{Re}^{0.85}\text{Pr}^{0.93} \quad \text{Pr} < 0.5, \quad \dot{q}_s = \text{constant}, \quad (37)
\]

but again it depends on the boundary conditions. The above considerations are summarized in Fig 13, where we suggest an algorithm to find the appropriate Nusselt number \[4\].

Table 9 and Table 10 show that for the case at hand, \(\text{Re} > 10000\) and \(\text{Pr} > 0.7\), thus we can use Eq.(34). Using the common expression Eq.(26), one can find the convection coefficient \(h\). Figs 16, 17, 18 and 19 show the dependence of \(h\) in function of \(V_\infty\) and \(D\) at different temperatures \[11\]. This coefficient is proportional to the fluid velocity, as shown in Eq.(43).

On the other hand, \(h\) decreases by power law when \(D\) increases. Especially for air, there is no sensible loss in the heat transfer efficiency for \(D > 0.2\ m\), see Fig 18.

One can also see that the bulk temperature has almost no influence on \(h\), independently of the temperature range, as in Figs 20 and 21. Notice however the presence of a maximum for ammonia, at \(T \sim 50^\circ\text{C}\), and the
high values for R-717, which justify its use as a refrigerant.

It is useful to derive a precise expression of \( h \) in function of the different parameters. The interpolation curves for \( h \) hold as follows: for dry air at 1 atm, \( D = 20 \text{ cm} \) and \( T = 15 \degree \text{C} \),

\[
h_{\text{air}}(V_\infty) = 5.2581 \times V_\infty^{0.8}, \quad (38)
\]

and for \( V_\infty = 5 \text{ m/s} \), \( T_e = 20 \degree \text{C} \), \( T = 15 \degree \text{C} \),

\[
h_{\text{air}}(D) = 13.796 \times D^{-0.2}. \quad (39)
\]

For saturated ammonia (R-717), \( D = 3 \text{ cm} \) and \( T = -10 \degree \text{C} \), we obtain instead

\[
h_{\text{ref}}(V_\infty) = 4754.3 \times V_\infty^{0.8}, \quad (40)
\]

and for \( V_\infty = 1 \text{ m/s} \) and \( T = -5 \degree \text{C} \), the equation is

\[
h_{\text{ref}}(D) = 2384.4 \times D^{-0.2}. \quad (41)
\]

The according values of the heat transfer coefficient are plotted in Figs 16, 17, 18 and 19 and agree with the literature [11].

It is also relevant to express \( h \) as an irreducible function of independent variables, as it was done in Section 2. Let us consider a turbulent flow of a fluid in a duct. Since \( \text{Re} = V_\infty D/\nu \) and \( \text{Pr} = \mu C_p/k \), equation (34) gives

\[
\text{Nu} = 0.023 \frac{D^{0.8}}{\sqrt{\nu}} \left( \frac{\rho C_p}{k} \right)^{0.3} V_\infty^{0.8}, \quad (42)
\]

therefore using Eq (26) we obtain

\[
h(T, D, V_\infty) = 0.023 \frac{k^{0.7}}{D^{0.2}} \left( \frac{\rho C_p}{k} \right)^{0.3} V_\infty^{0.8}, \quad (43)
\]

where \( \nu(T) \) and \( k(T) \) follow from [2] and [7], and \( \rho \) and \( C_p \) can be written as functions of temperature as well. The according plots for air and ammonia are given in Figures 20 and 21. The interpolation curves are respectively, for air flowing with \( V_\infty = 2 \text{ m/s} \) and \( D = 20 \text{ cm} \),

\[
h(T) = 10^{-4} T^2 - 0.0191 T + 9.4151 \left[ \frac{W}{m^2 C} \right], \quad (44)
\]

and for ammonia with \( V_\infty = 1 \text{ m/s} \) and \( D = 3 \text{ cm} \),

\[
h(T) = -0.0679 T^2 + 6.2396 T + 4826.4 \left[ \frac{W}{m^2 C} \right]. \quad (45)
\]

Therefore we find a minimum for air at \( T = 75.5 \degree \text{C} \), and a maximum for ammonia at \( T = 46 \degree \text{C} \). Notice that the curve for ammonia has a concavity proportional to the mean bulk velocity.
Tables and Plots

| $T$(°C) | Reynolds Number (×10$^4$) |
|---------|---------------------------|
| -30     | 9.20                      |
| -20     | 8.55                      |
| -10     | 7.99                      |
| 0       | 7.47                      |
| 10      | 7.01                      |
| 20      | 6.60                      |
| 30      | 6.22                      |
| 40      | 5.88                      |
| 50      | 5.56                      |
| 60      | 5.27                      |

| $T$(°C) | Reynolds Number (×10$^4$) |
|---------|---------------------------|
| -40     | 4.78                      |
| -30     | 5.22                      |
| -25     | 5.46                      |
| -20     | 5.70                      |
| -15     | 5.96                      |
| -10     | 6.24                      |
| -5      | 6.52                      |
| 0       | 6.82                      |

Table 9: Air Reynolds numbers as Re($T, V_\infty$), for $D = 20$ cm.

Table 10: Ammonia Reynolds numbers as Re($T, V_\infty$), for $D = 27$ mm.
| $V_\infty$ | Reynolds Number ($\times 10^4$) | $T_1$ | $T_2$ | $T_3$ |
|-----------|-----------------------------|-----|-----|-----|
| 0.25      | 1.99 2.08                   |     |     |     |
| 0.5       | 3.98 4.16                   |     |     |     |
| 0.75      | 5.97 6.24                   |     |     |     |
| 1         | 7.95 8.32                   |     |     |     |
| 1.25      | 9.94 10.40                  |     |     |     |
| 1.5       | 11.93 12.48                 |     |     |     |
| 1.75      | 13.92 14.55                 |     |     |     |
| 2         | 15.91 16.63                 |     |     |     |
| 2.25      | 17.89 18.71                 |     |     |     |
| 2.5       | 19.88 20.79                 |     |     |     |
| 2.75      | 21.87 22.87                 |     |     |     |
| 3         | 23.86 24.95                 |     |     |     |
| 3.25      | 25.85 27.03                 |     |     |     |
| 3.5       | 27.83 29.11                 |     |     |     |
| 4         | 29.82 31.19                 |     |     |     |
| 4.25      | 33.80 35.35                 |     |     |     |
| 4.5       | 35.79 37.43                 |     |     |     |
| 4.75      | 37.78 39.50                 |     |     |     |
| 5         | 39.76 41.58                 |     |     |     |

Table 11: Re($V_\infty, T$) for air, $D = 20$ cm and $[V_\infty] = [m/s]$.

Table 12: Re($V_\infty, T$) for ammonia, $D = 27$ mm and $[V_\infty] = [m/s]$.
Table 13: Air convection coefficient in function of the velocity and $T$ for $D = 20\, \text{cm}$

| $T_1$ | $T_2$ | $T_3$ |
|-------|-------|-------|
| $10^\circ \text{C}$ | $15^\circ \text{C}$ | $45^\circ \text{C}$ |
| $V_\infty (\text{m/s})$ | $h(\text{W/m}^2\cdot\degree\text{C})$ |
| 1 | 5.30 | 5.26 | 4.97 |
| 2 | 9.22 | 9.15 | 8.65 |
| 3 | 12.76 | 12.66 | 11.97 |
| 4 | 16.06 | 15.94 | 15.06 |
| 5 | 19.20 | 19.05 | 18.01 |
| 6 | 22.21 | 22.05 | 20.83 |
| 7 | 25.13 | 24.94 | 23.57 |
| 8 | 27.96 | 27.75 | 26.22 |
| 9 | 30.73 | 30.49 | 28.82 |
| 10 | 33.43 | 33.18 | 31.35 |
| 11 | 36.08 | 35.80 | 33.83 |
| 12 | 38.68 | 38.39 | 36.27 |
| 13 | 41.23 | 40.92 | 38.67 |
| 14 | 43.75 | 43.42 | 41.03 |
| 15 | 46.24 | 45.89 | 43.36 |
| 16 | 48.69 | 48.32 | 45.66 |
| 17 | 51.10 | 50.72 | 47.93 |
| 18 | 53.50 | 53.09 | 50.17 |
| 19 | 55.86 | 55.44 | 52.39 |
| 20 | 58.20 | 57.76 | 54.58 |

Table 14: Ammonia convection coefficient in function of the velocity and $T$ for $D = 3\, \text{cm}$

| $T_1$ | $T_2$ |
|-------|-------|
| $-15^\circ \text{C}$ | $-10^\circ \text{C}$ |
| $V_\infty (\text{m/s})$ | $h(\text{W/m}^2\cdot\degree\text{C})$ |
| 0.25 | 1557 | 1568 |
| 0.5 | 2711 | 2730 |
| 0.75 | 3750 | 3777 |
| 1 | 4720 | 4754 |
| 1.25 | 5643 | 5684 |
| 1.5 | 6529 | 6576 |
| 1.75 | 7386 | 7439 |
| 2 | 8219 | 8278 |
| 2.25 | 9031 | 9096 |
| 2.5 | 9825 | 9896 |
| 2.75 | 10604 | 10680 |
| 3 | 11368 | 11449 |
| 3.25 | 12120 | 12207 |
| 3.5 | 12860 | 12952 |
| 3.75 | 13590 | 13687 |
| 4 | 14310 | 14412 |
| 4.25 | 15021 | 15129 |
| 4.5 | 15724 | 15836 |
| 4.75 | 16419 | 16536 |
| 5 | 17106 | 17229 |
Figure 14: Reynolds number for air in function of the bulk temperature, for $V_\infty = 5, 10, 15 \, m/s$ and $D = 20 \, cm$.

Figure 15: Reynolds number for ammonia in function of the bulk temperature, for $V_\infty = 0.75, 2.5, 5 \, m/s$ and $D = 27 \, mm$. 
Figure 16: $h$ for air in function of the velocity for $D = 20 \text{ cm}$, $T_1 = 10^\circ \text{C}$, $T_2 = 15^\circ \text{C}$, $T_3 = 45^\circ \text{C}$.

Figure 17: $h$ for ammonia in function of the velocity for $D = 3 \text{ cm}$, $T_1 = -15^\circ \text{C}$, $T_2 = -10^\circ \text{C}$.
Figure 18: Air convection coefficient in function of diameter for $V_\infty = 5 \text{ m/s}$, $T_1 = -30 \text{ °C}$, $T_2 = 15 \text{ °C}$, $T_3 = 45 \text{ °C}$.

Figure 19: Ammonia convection coefficient in function of diameter for $V_\infty = 1 \text{ m/s}$, $T_1 = -15 \text{ °C}$, $T_2 = -10 \text{ °C}$, $T_3 = -5 \text{ °C}$.
Figure 20: $h$ for air in function of the bulk temperature for $D = 20 \, cm$, $V_{\infty 1} = 2 \, m/s$, $V_{\infty 2} = 3 \, m/s$ and $V_{\infty 3} = 4 \, m/s$.

Figure 21: $h$ for ammonia in function of the bulk temperature for $D = 3 \, cm$, $V_{\infty 1} = 0.5 \, m/s$, $V_{\infty 2} = 1 \, m/s$ and $V_{\infty 3} = 1.5 \, m/s$. 
A Critical length and least square method

The least square method (see e.g. [12]) is widely used in several fields, for instance in experimental physics, and dates back to the first works by Carl Friedrich Gauss and Legendre. In this section we apply this procedure to write $\nu$ as a function of temperature in two cases of fluid flow over a hot plate, namely unused engine oil and dry air at 1 atm. We also compute the critical length in function of the fluid temperature.

According to experimental data ([6]), $\nu$ has the shape of a power function, which can be written as

$$\nu = kT^{-m},$$

thus one needs to determine $k$ and $m$. We start with an ansatz on these, and then plot the equation with those values. Then we calculate the difference between the empirical $\nu$ and the one obtained with the formula. Then we compute the square sum of these differences and minimize this vector by changing the two parameters $k$ and $m$. This is done automatically with several programs, giving

$$\nu(T) = 6.1875 \times T^{-2.7427}. \quad (47)$$

Note that this function is valid only for the temperature of use of the engine oil (above $40^\circ C$), otherwise the approximation is a lot less precise. Here the average error is 0.9%.

Next, one can express $L_{cr}$ as an irreducible function of $T$,

$$L_{cr}(T) = \frac{\nu(T)}{V_{\infty}} Re_{cr} = \frac{6.1875 \times T^{-2.7427}}{V_{\infty}} 5 \times 10^5. \quad (48)$$

Fig 22 gives the precise behavior of $L_{cr}$ in function of the oil temperature. Here we set $V_{\infty} = 2m/s$ for the oil.

In Fig 23 we see the juxtaposition of the two curves (empirical and analytical). However between $40^\circ C$ and $60^\circ C$ the analytical curve does not fit exactly the experimental points. This gap comes from the exponential behavior of the function.

Consider now dry air at 1 atm. The kinematic viscosity $\nu$ has the shape of a polynomial function: $\nu = aT^2 + bT + c$, thus we want to find $a$, $b$ and $c$. Using the same solver method than with oil we find:

$$\nu(T) = 4.91452 \times 10^{-11}T^2 + 1.10682 \times 10^{-7}T + 1.18052 \times 10^{-5}, \quad (49)$$

with units $[m^2/s]$. Likewise, the above formula is valid only for the temperature of use (above $5^\circ C$). Here we have an average error of 0.98%. Note
that a third degree polynomial function is fairly more accurate,

\[ \nu(T) = -1.0743 \times 10^{-14} T^3 + 7.8253 \times 10^{-11} T^2 + 9.33152 \times 10^{-8} T + 1.3094 \times 10^{-5}, \]  

with an average error of 0.09%. Therefore we can express \( L_{cr} \) as a function of \( T \) as follows,

\[ L_{cr}(T) = \frac{\nu(T)}{V_\infty} \frac{Re_{cr}}{V_\infty} = \frac{5}{V_\infty} \times \left( -1.0743 \times 10^{-9} T^3 + 7.8253 \times 10^{-6} T^2 + 9.33152 \times 10^{-3} T + 1.3094 \right). \]

Here we set \( V_\infty = 8 \text{ m/s} \) for air, see Fig 24. Fig 25 compares the empirical points and the analytical curve. However the third degree polynomial function is so precise that we cannot even discern gaps between the two curves.
Figure 22: Oil Critical Length in function of temperature.

Figure 23: Oil Critical Length in function of temperature: comparison with empirical points.
Figure 24: Air Critical Length in function of temperature.

Figure 25: Air Critical Length in function of temperature: comparison with empirical points.
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Water convection coefficient in function of the velocity for $D = 5 \text{ cm}$. $T_1 = 15^\circ \text{C}$, $T_2 = 50^\circ \text{C}$ and $T_3 = 95^\circ \text{C}$.

Air convection coefficient in function of the diameter for $V = 8 \text{ m/s}$. The film temperatures are $T_1 = 15^\circ \text{C}$, $T_2 = 50^\circ \text{C}$ and $T_3 = 95^\circ \text{C}$.

Water convection coefficient in function of the diameter for $V = 0.5 \text{ m/s}$. The film temperatures are $T_1 = 15^\circ \text{C}$, $T_2 = 50^\circ \text{C}$ and $T_3 = 95^\circ \text{C}$.

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Oil Critical Length in function of temperature.

Oil Critical Length in function of temperature: comparison with empirical points.

Air Critical Length in function of temperature.

Air Critical Length in function of temperature: comparison with empirical points.
Conclusions and perspectives

In this paper we have discussed a method to the study of heat transfer processes, which is aimed at academic teaching but can be useful also in practical situations. It has analogy with the so-called functional optimization, see [3] and references quoted therein.

This approach focuses on a theoretical analysis of the relevant equations, and makes it possible to know the behavior of the crucial quantities \textit{a priori}, before any empirical tests. This creates awareness of the physics process in the students, and aids researchers effectively.

We have applied this method to two common heat transfer processes, namely to 1. a fluid flow across a cylinder and 2. a pipe flow. In both cases we have discussed the behavior of the Reynolds number and of the convection coefficient, in function of variables such as fluid temperature and pipe diameter, using realistic test values.

Our results, listed in a number of plots and tables, can be directly used as a reference for the students in academic courses. In the Appendix, we also show how to use the least square method to compute interpolation curves.

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