On the Dynamics of a Prestressed Sliding Wedge Damper

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This paper presents a prestressed sliding wedge damper as an effective alternative for the reduction of vibration amplitudes. Due to the system’s non-smooth and non-linear characteristics, it offers advantages over conventional linear dampers, e.g. an amplitude adaptive behavior. First, the equations of motion which describe the system’s characteristics are derived. Second, an analytical model based on a modal coordinate transformation and averaging methods is proposed. A numerical model, which uses an elasto-plastic contact model is set up, in order to fully model the systems behavior. An analysis of the dampers characteristics shows that it is a pseudo-viscous damper realized via dry friction. The analytical and numerical results are compared and show a reasonable agreement, as within asymptotic methods. This work presents the first investigations into prestress influence on such a damper and offers insight into future theoretical, numerical, and experimental analysis.

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1 Introduction

The limitation of unwanted oscillations in technical systems is an ever-present task in mechanical engineering. Especially in the resonance regime, the consequences of undesired oscillations can range from mild user discomfort to a total system failure. A common approach in order to limit these oscillations are tuned mass dampers, which are found for example in structural systems [1] and cables [2]. The vibration reduction through these dampers is well understood and they have been thoroughly investigated [3]. They offer the advantages of an antiresonance frequency and robustness with respect to the excitation level. Nevertheless, there are disadvantages regarding the energy efficiency and the technical implementation. Dry friction dampers present an alternative to tuned mass dampers and are investigated in connection with structural systems and turbine blades in [4] and [5] respectively. Due to the stick-slip behavior of friction these dampers have the advantage of a selective energy dissipation within a frequency range close to the system’s resonance regime. However, they are not robust with respect to the excitation amplitude, since the friction force has to be especially tuned to it [6]. The anti-clutch-judder-damper (germ.: Anti-Rupf-Tilger), a robust damper based on dry friction, was developed by engineers at the LuK-Company [7]. Although the energy dissipation in this damper comes solely from dry friction, this damper shows energy dissipation proportional to the relative displacement due to the use of wedges pressed on to a secondary mass by springs in its design. This system offers the advantages of a tuned mass damper and avoids the difficulties regarding technical implementation. However, it shows a disadvantage regarding energy dissipation, since it is always present and cannot be selectively chosen. Analytical investigations to this damper are found in [8].

This work aims to improve the anti-clutch-judder-damper by introducing a prestress to the spring which presses the wedges. This prestress displacement offers additional stiction capabilities of the damper allowing selective energy dissipation. This dissipation is only active in the resonance regime once the vibration amplitudes of the main system are large enough and the stiction condition is broken. Furthermore, the advantages of the original damper regarding the robustness and antiresonance frequency should be maintained.

For the sake of precision the considered damper in this work will be referred to as the prestressed sliding wedge damper. In section 2 the equations of motion describing the dampers are derived, and the damper’s general characteristics are presented. Additionally, the stiction conditions of the system are addressed and a numerical model is presented. An analytical model based on the method proposed in [8] is developed in section 3. In section 4 numerical parameter studies are presented and the system’s parameter influences are determined. Finally, the conclusions of this work are summarized in section 5.

2 Considered System: The Prestressed Sliding Wedge Damper

The considered system is depicted in fig. 1. The main system is composed of a primary mass \( m_1 \) and a primary spring \( c_1 \). Additionally, the harmonic force, excitation amplitude \( F \) and excitation frequency \( \Omega \), act on the main mass. The damper’s objective is to improve the dynamics of the main system, especially when its vibration amplitudes are detrimental to its function. To this end the prestressed sliding wedge damper is attached to the main mass. The damper is composed by two V-form wedges, which are pressed on to a secondary mass \( m_2 \) by a secondary spring \( c_2 \). The additional parameters describing the damper are the wedge angle \( \alpha \), the friction coefficient \( k \) between the secondary mass and the wedges, and the prestress displacement \( \Delta \ell \) of the secondary spring. Since a drastic change in the systems operating frequency is not desired the secondary mass is small in comparison to the main mass. For the same reason the secondary stiffness is chosen such that...
the ratios \( c_2/m_2 \) and \( c_1/m_1 \) are equal. The coordinates used to describe the primary and secondary mass of the system are \( x_1 \) and \( x_2 \) respectively.

This system has two discrete phases, which can occur during its motion: the stick-phase and the slip-phase. For small friction values the equations of motion for both phases are given by

**stick-phase:**

\[
\begin{align*}
(m_1 + m_2)\ddot{x}_1 + c_1 x_1 &= F \sin \Omega t \\
\dot{x}_2 - \dot{x}_1 &= 0
\end{align*}
\]

**slip-phase:**

\[
\begin{align*}
m_1 \ddot{x}_1 + c_1 x_1 - F_{WD} &= F \sin \Omega t \\
m_2 \ddot{x}_2 + F_{WD} &= 0 \\
F_{WD} &= 2c_2(2\tan \alpha (x_2 - x_1) + \Delta l \sgn(x_2 - x_1)) \left( \tan \alpha + \frac{k}{\cos^2 \alpha} \sgn(\dot{x}_2 - \dot{x}_1) \sgn(x_2 - x_1) \right).
\end{align*}
\]

As seen in Eq. (2), the two degrees of freedom are coupled while slipping by wedge damper force \( F_{WD} \), which depends mainly on the parameters \( \Delta l, \alpha \), and \( k \). In order to obtain a deeper insight into the damper’s dynamics, the wedge damper force is decomposed into two parts: a dissipation free stiffness force \( F_C \) and a dissipative force \( F_D \). The force progressions of \( F_{WD}, F_C \) and \( F_D \) are depicted for an exemplary oscillation in fig. 2. These forces are given by

\[
F_{WD} = F_C + F_D, \quad (3)
\]

\[
F_C = 2c_2(2\tan \alpha (x_2 - x_1) + \Delta l \sgn(x_2 - x_1)) \tan \alpha, \quad (4)
\]

\[
F_D = 2c_2(2\tan \alpha |x_2 - x_1| + \Delta l \frac{k}{\cos^2 \alpha} \sgn(\dot{x}_2 - \dot{x}_1)). \quad (5)
\]

As is noted from Eq. (5), the dissipative force is proportional to the relative displacement. Integrating \( F_D \) over the distance yields the dissipated mechanical energy \( W_R \), which in this case is proportional to the squared value of the relative vibration amplitude, i.e. \( W_R \sim \Delta \alpha^2 \). This is also the case with the damper presented in [7] and explains the robustness of the damper with respect to the excitation amplitudes. The main difference with respect to the anti-clutch-judder-damper lies in the introduction of the prestress displacement, i.e. \( \Delta l \)-Terms. These terms lead to additional dissipation and a jump in the force characteristics.

The force progression curves in fig. 2 also offer insight into the stick-phases of this damper. Sticking requires the relative velocity being equal to zero and the necessary sticktion force being lower than the maximal sticktion force. The damper shows a sticking possibility when a jump in the wedge damper force occurs, as seen on the \( F_{WD} \)-curve this occurs on three occasions. The jumps at both ends of the wedge damper force hysteresis correspond to sticking while the secondary mass has contact with either the left or the right hand side of the V-form wedges. The kinetic sticktion condition can be clearly formulated by requiring the absolute ratio of the wedge tangential and normal force being smaller than the friction coefficient. The jump at the middle of the \( F_{WD} \)-hysteresis corresponds to the case where all four wedges have contact with the secondary mass. For this case the kinetic sticktion conditions cannot be clearly formulated, since the system is statically indeterminate. Therefore, the constraint forces of the wedge cannot be clearly specified. In order to avoid the statically indeterminate case a Dupont-model [9] for the friction contact is used. The Dupont-model is a type of bristle friction model and as such introduces a stiffness in the tangential direction. This friction model in combination with the above mentioned equations of motions is used in sec. 3 to verify the analytical model and in sec. 4 to carry out the numerical parameter studies.
3 Analytical Model

In order to obtain a deeper insight into the parameter influences an analytical model is derived. The analytical model assumes permanent sliding, which is a valid assumption in the resonance regime. Therefore, the analytical investigations concentrate on the equations of motion of the slip-phase. As a first step the following nondimensional parameters are introduced and the nondimensional equations of motion are derived. Additionally, a modal coordinate transformation is applied yielding

\[ \mu = \frac{m_2}{m_1}, \quad \lambda^2 = \frac{c_1}{m_1}, \quad f = \frac{F}{m_1 \omega_{01}^2}, \quad \tau = \lambda t, \quad (\cdot)' = \frac{d(\cdot)}{d\bar{\tau}}, \quad \eta = \frac{\Omega}{\lambda}, \]

\[ a = \tan^2 \alpha, \quad b = k \frac{\tan \alpha}{\cos^2 \alpha}, \quad c = \Delta \ell \tan \alpha, \quad d = \frac{\Delta \ell k}{\cos^2 \alpha}, \quad \varepsilon \ll 1 \& \Delta \ell, \quad k = O(\varepsilon), \]

\[ \begin{cases} x''_1 + x_1 + 4\mu a(x_1 - x_2) = f \sin \eta \tau - 4b|x_1 - x_2|\text{sgn}(x'_1 - x'_2) - 2\mu \text{sgn}(x_1 - x_2) \\ \mu x''_2 - 4\mu a(x_1 - x_2) = 4b|x_1 - x_2|\text{sgn}(x'_1 - x'_2) + 2\mu \text{sgn}(x_1 - x_2) \end{cases}, \]

\[ \mathbf{M}x'' + \mathbf{C}x = \varepsilon f_{NL}(x, \Omega), \quad \mathbf{x} = \mathbf{R}z = \mathbf{R} [p, q]^T, \quad \mathbf{R}^T \mathbf{MR}z'' + \mathbf{R}^T \mathbf{MR}z = \varepsilon \mathbf{R}^T f_{NL}(z) = \varepsilon f_{NL}(z), \]

\[ \begin{cases} p'' + \eta_1^2 p = \varepsilon f_{NL,1}(p, q, \Omega) \\ q'' + \eta_2^2 q = \varepsilon f_{NL,2}(p, q, \Omega) \end{cases} \rightarrow \begin{cases} p'' + \eta_1^2 p = \varepsilon f_{NL,1}(p, 0, \Omega) \\ q'' + \eta_2^2 q = \varepsilon f_{NL,2}(0, q, \Omega) \end{cases}. \]

The modal equations of motion on the first part of Eq. (9) are elaborate for analysis, since the equations are weakly coupled due to the nonlinear terms on the right hand side. In order to obtain simplified equations, the method presented in [8] is applied. The corresponding modal coordinate is exclusively considered in the vicinity of its resonance frequency, therefore all other modal coordinates are neglected. This yields the equations in the second part of Eq. (9), which are suitable for averaging. For the sake of brevity the authors limit themselves to the most important steps

\[ p = A_1 \sin \varphi_1, \quad p' = A_1 \eta_1 \cos \varphi_1, \quad \varphi_1 = \eta \tau + \psi_1 \]

\[ q = A_2 \sin \varphi_2, \quad q' = A_2 \eta_2 \cos \varphi_2, \quad \varphi_2 = \eta \tau + \psi_2, \]

\[ i = \{1, 2\}, \quad A'_i = \varepsilon f_{NL,i}(A_i, \psi_i, \varphi_i) \cos \varphi_i, \quad \psi'_i = \varepsilon \left( \delta_i - \frac{1}{A_i \eta_i} \tilde{f}_{NL,i}(A_i, \psi_i, \varphi_i) \cos \varphi_i \right), \]

\[ (i, j) = \{(1, 2), (2, 1)\}, \quad \varepsilon \delta_i = \eta_i - \eta_j, \]

\[ A_i = \frac{\delta_i - \gamma_i}{(r_{ii} - r_{jj})^2} \pm \frac{1}{\left( (r_{ii} - r_{jj})^2 \frac{\delta_i}{\delta_j} \right)^2 + 1} \frac{\varepsilon \delta_i}{(r_{ii} - r_{jj})^2 \frac{\delta_i}{\delta_j} + \varepsilon \delta_i} \]

Applying the van-der-Pol-transformation in Eq. (9) leads to the slowly changing variables of the system, namely the amplitude of the modal coordinates \( A_i \) and their phase difference with respect to the excitation \( \psi_i \). These equations are averaged and the stationary solutions of the slowly changing amplitudes \( A_i \) is calculated. As seen from Eq. (13), the slow system dynamics are dependent on the nondimensional parameters and the modal matrix components \( r_{ij} \). This approximation of the modal coordinates amplitude is the used in the corresponding back transformation to calculate the amplitudes of the original coordinates. The analytical solutions are compared with the numerical results of the Dupont-model in fig. 3. The standard parameter set is chosen as follows: \( m_1 = 1, \quad m_2 = 0.1, \quad c_1 = 1, \quad c_2 = 0.1, \quad F = 0.01, \quad \alpha = 30^\circ, \quad k = 0.01, \quad \Delta \ell = 0.01 \). The difference between the numerical and analytical solution is within the expected accuracy range of asymptotic methods.
4 Numerical Parameter Studies

The analytical solution is only valid for the assumptions in Eq. (6). In order to obtain a broader understanding of the damper’s behavior over a wider parameter range, numerical parameter studies are carried out. The influence of two parameters are investigated: the prestress displacement $\Delta \ell$ and the excitation force amplitude $F$. The excitation force amplitude is not a damper parameter, however the damper’s reaction to this parameter describes it’s robustness. If not specified the parameter are chosen as in the standard parameter set.

The influence of the prestress level is depicted for three prestress values in fig. 4. All curves follow the amplitude response function of the one degree of freedom system, see Eq. (1), until a given breakaway amplitude. The prestress displacement influences the breakaway point from which the curves differ from the sticking system and the systems enter the nonlinear stick-slip range. For rising $\Delta \ell$-values the nonlinear stick-slip range is smaller and the breakaway point occurs at higher vibration amplitudes. Nonperiodic solutions are also observed for rising prestress levels, see fig. 4. Furthermore, the maximal vibration amplitude increases or decreases depending on $\Delta \ell$. This implies the existence of an optimal prestress value.

In order to study the dampers robustness with respect to the excitation force amplitude, the normalized amplitude force ratio for different excitation frequencies and different excitation force values have been calculated. As seen in fig. 5 the curves are almost identical. This implies that the change in the amplitude response function is approximately proportional to the excitation force, for the given parameters. Additionally, an antiresonance frequency is observed in all amplitude response functions.

5 Conclusions

A portion of the insight into the dynamics of the prestressed wedge damper is provided by the derived analytical solution, which showed reasonable agreement with the numerical results. The additional knowledge of the system’s dynamics is obtained via numerical methods. The clear advantage of the prestress is that it is able to lower the vibration amplitudes of the system. Furthermore the energy dissipation is limited to a vicinity of the system’s resonance regime. The study of the system’s robustness verified the characteristics derived in sec. 2, regarding the damper’s energy dissipation as a pseudo-viscous damper. Furthermore, the antiresonance frequency is maintained for certain parameter values. This work showed the effectiveness of the prestressed sliding wedge damper to reduce vibration amplitudes in the system, and offers a basis for further investigations.

Acknowledgements This work was supported by the DFG German Research Foundation Grant FI 1761/2-1 within the Priority Program SPP 1897 “Calm, Smooth and Smart - Novel Approaches for Influencing Vibrations by Means of Deliberately Introduced Dissipation”.

References

[1] A. Y. Tuan, G. Q. Shang, Journal of Applied Science and Engineering 17, 141-156 (2014).
[2] W. J. Wu, C. S. Cai, Engineering structures, 29, 962-972 (2007).
[3] J. P. Den Hartog, Mechanical Vibrations (Dover Publications, New York, 1985).
[4] F. Ricciardelli, B. J. Vickery, Earthquake engineering and structural dynamics 28, 707–723 (1999).
[5] J.H. Wang, W.L. Shieh, Journal of Sound and Vibration 149, 137–145 (1991).
[6] A. Fidlin, J. Aramendiz, 2019. Nonlinear dynamics, 97 (3), 1867–1875. doi:10.1007/s11071-018-4662-7
[7] M. Hausner, M. Hässler, ATZ-Automobiltechnische Zeitschrift 114, 64-69 (2012).
[8] A. Fidlin, N. Gafur, Abstracts of the 9th European Nonlinear Dynamics Conference (ENOC) in Budapest, Hungary. (2017).
[9] P. Dupont, B. Armstrong, V. Hayward. Proceedings of the 2000 American Control Conference IEEE, Chicago, USA (American Automatic Control Council, Piscataway, 2000) 1072-1077.

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