Modelling of the Magnetic Field Effects in Hydrodynamic Codes Using a Second Order Tensorial Diffusion Scheme

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Abstract. In laser produced plasmas large self-generated magnetic fields have been measured. The classical formulas by Braginskii predict that magnetic fields induce a reduction of the magnitude of the heat flux and its rotation through the Righi-Leduc effect. In this paper a second order tensorial diffusion method used to correctly solve the Righi-Leduc effect in multidimensional code is presented.

1. Introduction

In laser generated plasmas, magnetic fields are created whenever density and temperature gradients are not colinear. Dedicated experiments and numerical computations evidenced fields in the 100 kG-1 MG range in plasmas typical of laser driven Inertial Confinement Fusion (ICF) [4, 5]. As the corresponding magnetic pressure stays far below the kinetic pressure, the hydrodynamic flow is not directly altered by such fields. Conversely, the electron gyrofrequency is comparable to the electron-ion collision frequency in the vicinity of the critical density region, which may affect dramatically the electron heat flow. Key issues of ICF are laser matter coupling, mass ablation, hydrodynamic instabilities and hot spot dynamics. Because all of these phenomena are highly sensitive to electron heat transport, accurate modelling of the electron conduction is an essential ingredient of numerical simulations. However, the electron conduction model implemented in the majority of hydrodynamic codes is based upon the flux limited Spitzer-Härm theory and does not account for magnetic fields. The classical formulas by Braginskii predict that magnetic fields provide a reduction of the magnitude of the heat flux and its rotation through the Righi-Leduc effect. Splitting the Righi-Leduc term or approximating it as an advection term or an effective conductivity often leads to mathematical and physical anomalies. Alternatively, a direct space differencing of the full conduction operator raises accuracy and consistency issues. We developed in CELIA a new, fully tensorial approach which overcomes these difficulties. We propose a new second order tensorial diffusion method which is based on a cell-centered diffusion scheme [3]. We present the main idea of the space differencing scheme and some numerical results that illustrate the efficiency of the method compared to standard approaches. The packages describing the magnetic field generation and the anisotropic electron heat transport have been incorporated in the hydrodynamic code CHIC.
2. Physical model of the CHIC code

This multidimensional hydrodynamics code allows numerical simulations of laser driven Inertial Confinement Fusion (ICF). This 2D planar/axisymmetrical code is based on both original physical models and numerical schemes. The hydrodynamic scheme is based on a second order cell-centered Lagrangian scheme [1]. The primary variables in this scheme, i.e., specific volume, momentum and total energy are cell-centered. The vertex velocities are evaluated in a consistent manner due to an original solver located at the nodes. This nodal solver can be viewed as a two-dimensional extension of the Godunov acoustic solver. The spatial second order extension is derived using a MUSCL type approach. Time discretization is based on a second-stage Runge-Kutta scheme. The extension to the arbitrary Lagrangian Eulerian (ALE) has been implemented in order to improve accuracy, robustness, and computational efficiency of the calculation. Modelling of heat conduction uses an original high order cell-centered diffusion scheme on unstructured mesh [3].

This code includes detailed physics for numerous phenomenae. It accounts for hydrodynamics, electron and ion conduction, thermal coupling and detailed radiation transport. Ionisation, equation of state and opacity data are tabulated, assuming local thermodynamic equilibrium (ETL) or non-LTE model. Laser propagation, refraction and collisional absorption are treated by a 3-D ray tracing algorithm.

A resistive MHD package, including thermal sources is implemented. Transverse MHD with magnetic pressure and Joule heating neglected. The electron heat flux in the plane orthogonal to the magnetic field \( B \) reads:

\[
\Phi_e = -\lambda_\perp \nabla T_e - \lambda_\wedge b \times \nabla T_e,
\]

where \( b = \frac{B}{|B|} \), \( T_e \) is the electron temperature, \( \lambda_\perp \) and \( \lambda_\wedge \) are the heat conduction coefficients.

We write the electronic heat flux in the form \( \Phi_e = -D \nabla T_e \), where \( D \) is a tensorial heat conduction coefficient:

\[
D = \begin{pmatrix}
\lambda_\perp & -b_x \lambda_\wedge \\
-b_x \lambda_\wedge & \lambda_\perp
\end{pmatrix}.
\]

\( \lambda_e \) tensorial conductivity (Spitzer-Hrm and Braginskii), \( \beta \) is the thermoelectric tensor and \( \sigma \) is the tensorial resistivity.

3. Numerical Scheme

In this paper we use a new cell-centered scheme devoted to solve anisotropic diffusion problems on unstructured mesh. This scheme is based on a cell-centered diffusion scheme [3]. The main feature of this scheme lies in the introduction of two half normal fluxes and two temperatures on each edge. A local variational formulation written for each corner cell provides the discretization of the half normal fluxes. This discretization yields a linear relation between the half normal fluxes and the temperatures defined on the two edges impinging on a node. The continuity of the half normal fluxes written for each edge around a node leads to a linear system. Its resolution enables to eliminate locally the edge temperatures in function of the mean temperature in each cell. By this way, we obtain a small symmetric positive definite matrix located at each node. Finally, by summing all the nodal contribution, one gets a linear system satisfied by the cell-centered unknowns. This system is characterized by a symmetric positive definite matrix. This scheme has several good properties. First, it preserves the linear solutions on triangular mesh. It reduces to a classical five points scheme on rectangular grids for isotropic cases. For non orthogonal quadrangular grids we obtain an accuracy which is almost second order.
4. Numerical results

4.1. The magnetic field effects in a 2D configuration

We first test our method on a 2D problem without hydro and a constant and imposed magnetic field:

\[
B(x, y) = B_0 \exp \left( \frac{\sqrt{(x-x_0/2)^2 + (y-(1+x_0/2))^2}}{x_0/4} \right)^4
\]

with \(x_0 = 20 \times 10^{-4}\) and \(B_0 = -1.10^7 G\).

Our domain is a square of 20 \(\times\) 20 \(\mu\)m with a mesh of 50 \(\times\) 50 cells. It is field of Hydrogen \(Z = 1\) with a constant initial density of 0.025\(g/cm^3\). Initial temperature is \(T_1 = 1.10^3 K\) for \(0 < x < 18.\mu m\) and \(T_1 = 3.10^7 K\) for \(18.\mu m < x < 20.\mu m\). We run three different types of computation first is Braginskii without Righi-Leduc effect then we have an effective conductivity which is used usually in 2D code then our tensorial formulation (see figures 1). On the 2D isoline of temperature we can observe a limitation of the heat flux on all the results due to the magnetic field. Then if we take into account of the Righi-Leduc effect with tensorial formulation the rotation of the heat flux is enhanced and no oscillation is observed (figure 2).

![Figure 1](image_url)

**Figure 1.** Braginski without Righi-Leduc effect (left), effective conductivity treatment (center), tensorial formulation (right) at \(t = 25ps\).

4.2. Plastic target irradiated with a Laser

In this case we study a plastic target (CH) of 100 \(\mu\)m thickness and 1000 \(\mu\)m height irradiated by a laser beam : \(\lambda = 0.35 \mu\m,\) maximal intensity \(1.10^{15} W \text{ cm}^{-2}\).

This tensorial method leads to a better computation of the Braginskii fluxes. Contrarily to the effective method in which the Righi-Leduc effect is partially truncated this tensorial scheme correctly rotates heat fluxes. Comparison between spitzer and Braginskii show that magnetic field play an important role in the case of laser ablation physics. We can see that magnetic field appear in the zone where density and temperature gradients are not colinear. The magnitude of the self-generated magnetic field is also more intense using the tensorial method (see figures 3).

5. Conclusion

We have performed an initial study where we imposed a constant magnetic field. In this case we could see the two main effect of magnetic field on heat transport. First we can observe a limitation of the heat transport predicted by the Spitzer model. The second effect is a rotation of the heat transport through the Righi-Leduc effect. Using our tensorial approach we could show that we handle more naturally these phenomenae and no oscillation are observed. The effect of self-generated magnetic field in the case of a laser ablation case has been also studied. We could see that magnetic field totally changes the heat transport process. The tensorial method in this
**Figure 2.** Temperature profile at $x = 10\mu m$ and $t = 25ps$.

**Figure 3.** temperature and $\log(N_e)$ Spitzer (left), temperature and magnetic field effective conductivity (center), temperature and magnetic field tensorial conductivity (right) at $t = 1.25ns$.

case also presents result free of numerical oscillation.

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