A simplified analysis method for evaluating the horizontal deformation of adjacent single pile under tunneling

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Abstract. To analyze the horizontal response regularity of adjacent pile under tunneling, an analytical method is developed considering the shearing effect of pile to assess the horizontal responses of pile due to adjacent tunneling. Firstly, the pile is considered as a Timoshenko beam resting on the Pasternak foundation model, then, combining with the two-stage method, the additional unloading pressure due to the tunneling excavation is loaded on the pile, thus the equilibrium differential equation considering the pile's shearing effect is established. Secondly, the specific horizontal deformation of pile is given by the finite difference method. Furthermore, through comparing with the boundary element method, the validity of this theory is verified. Finally, using the theory here, a comparison is made between the results based on the Timoshenko beam and Euler-Bernoulli beam, and find the results taking the shearing effect of pile into account are more close to the classical solution, the parameter analysis also suggests: the pile's shearing effect is closely related with the disturbance, the stronger the disturbance, the more obvious the pile's shearing effect.

1. Introduction

With the increasingly urban underground construction, the environmental problems caused by the excavation of the tunnel are gradually arousing people's attention. As we all know, the shield is an important means for urban subway construction, obviously, excavating tunnel will inevitably disturb the adjacent pile foundations in cities with high building density. In severe cases, the pile foundations can even be damaged. Therefore, how to accurately evaluate the response of existing piles under the excavation of tunnel is of great significance.

The "two-stage method" is a widely used theoretical method for passive single-pile analysis. Based on the Winkler foundation model, Liang et al [1] established the horizontal displacement differential equation of single pile under the excavation of the adjacent tunneling. To further consider the nonlinearity of the soil, Sun Qing et al[2] used the finite difference method and the concept of limiting force profile during calculation combining with the $p-y$ curve, the elastic-plastic solution of the interaction between tunnel excavation and single pile were given. Wang Yu et al [3] introduced the Pasternak two-parameter foundation model and considered the lateral effect of soil, which proved was in better consistency with reality.

In above related theoretical researches, although the more advanced load transfer models are used in the research of pile-soil interaction, little attention has been paid to the simplified model of pile.
When studying the horizontal dynamic response of a single pile in a double viscoelastic foundation, Juan et al [4] point out that ignoring the shear deformation of the pile will engender fairly great error. Yang Meiliang et al [5] found that pile shear deformation would less and change the distribution of bending moment. Therefore, considering the shear effect of pile to study the interaction between shield excavation and pile foundation is worth exploring.

Fig. 1 The schematic diagram of Timoshenko beam and Euler-Bernoulli beam

This paper starts from the homogeneous soil foundation. Firstly, it calculates the additional load acting on the pile due to shield excavation. Then, the foundation is regarded as the Timoshenko beam resting on the Pasternak two-parameter foundation. Combined with the two-stage method, the pile's horizontal displacement differential equation under the influence of shield excavation is established. Secondly, the finite difference method is used to analyze the horizontal deformation, in this analysis, the bending and shear deformation can be considered at the same time. Finally, the accuracy of the proposed method is verified by the comparison with the solution of boundary element method.

2. Establish and solve control equations

Based on the non-equivalent radial soil movement model and linear elasticity theory, Loganathan et al [6] proposed an analytic solution that can accurately estimate the displacement field of soil. The calculated results agree well with the actual values and have been proved by most scholars [7].

\[ T_c = -\varepsilon_0 R^2 x \left\{ \frac{1}{s^2 + (z-H)^2} + \frac{3-4\mu}{s^2 + (z+H)^2} \cdot \frac{4\varepsilon_0 (z+H)}{[s^2 + (z+H)^2]} \right\} \exp\left[ -\frac{1.38s^2}{(H+R)} - \frac{0.69z^2}{H^2} \right] \]  

(1)

Where: \( \varepsilon_0 \) is stratum loss ratio; \( R \) is the radius of the tunnel; \( x \) is the horizontal distance from the calculated point to tunnel center line; \( z \) is the depth from calculated point to the ground surface; \( H \) is the depth of the center axis of the tunnel.

The Pasternak two-parameter model, by introducing a shear layer to simulate the shear effect of soil, the model is more accurate than the Winker model, the formula is as follows:

\[ P(z) = k w - G_0 \frac{d^2 w}{dx^2} \]  

(2)

Where: \( P(z) \) is the spring reaction force; \( k \) is the initial foundation reaction coefficient [8];

\( k=0.65E_s (E_d d^4 / E_p I)^{0.5} / d \).

Fig 2 shows the effect of shield excavation on the adjacent pile and the force analysis schematic diagram of pile's micro-unit.
The equilibrium differential equation for horizontal displacement \( w(z) \) and rotation angle \( \theta \) of Timoshenko beam on Pasternak foundation can be obtained as follows:

\[
p(z)D\frac{dz}{d\theta} - q(z)D\frac{dz}{d\theta} - dQ = 0
\]

\[
p(z)D\frac{dz^2}{d\theta} + Q\frac{dz}{d\theta} - dM = 0
\]

Where: \( D \) is the diameter of the pile; \( M, Q \) is the bending moment and shear force of the pile unit; For Timoshenko Beam, bending moment \( M \) and shear \( Q \) have the following relations:

\[
Q = (\kappa GA)\left[\frac{dw(z)}{dz} - \theta\right]
\]

\[
M = -EI \frac{d\theta}{dz}
\]

Where: \( \kappa GA \) is the equivalent shear stiffness of the pile, \( \kappa \) is the equivalent section modulus of the pile, the cross section is taken as 0.9 and the rectangle is taken as 5/6; \( G \) is the shear modulus of the pile, \( G = \frac{E_p}{2(1+\nu_p)} \).

Combining the formulas (3)–(6), the finite difference expression the displacement \( w(z) \) is:

\[
\begin{align*}
(G_D \frac{1}{\kappa GA})w_{i+1} - (G_D \frac{1}{\kappa GA})w_i + (\frac{d^{2}D}{d\theta^{2}} \frac{E_I}{\kappa GA})w_i + (\frac{d^{2}D}{d\theta^{2}} \frac{E_I}{\kappa GA})w_{i+1} + (\frac{d^{2}D}{d\theta^{2}} \frac{E_I}{\kappa GA})w_{i+2} \\
+ (G_D \frac{1}{\kappa GA})w_{i-1} - (\frac{d^{2}D}{d\theta^{2}} \frac{E_I}{\kappa GA})q_i + (\frac{d^{2}D}{d\theta^{2}} \frac{E_I}{\kappa GA})q_{i+1} + (\frac{d^{2}D}{d\theta^{2}} \frac{E_I}{\kappa GA})q_{i+2} + (\frac{d^{2}D}{d\theta^{2}} \frac{E_I}{\kappa GA})q_{i+3}
\end{align*}
\]

Where: \( w_i \) is the vertical displacement of node \( i \), \( q_i \) is the additional load of node \( i \).

When the both ends of pile is free, the bending moment \( M \) and shear force \( Q \) at both ends of the pile foundation are 0, namely:

\[
M_a = M_a = -EI \frac{d\theta}{dz} = 0
\]

\[
Q_a = Q_a = -(\kappa GA)w \left( \frac{dw}{dz} - \theta \right) = 0
\]

With (8), (9), the (7) can be expressed as a matrix. Matrix\([K_1], [K_2], [K_3], [K_4], [w], \{Q_1\}, \{Q_2\}, \{Q_3\}]\) are as follows:

\[
([K_1] - [K_2] - [K_3] + [K_4])[w] = [Q_1] - [Q_2] - [Q_3]
\]
When the equivalent shear stiffness of the pile is large enough, the shear deformation of the pile approaches 0, then Eq.(10) can degenerate into the differential equation of Euler-Bernoulli Beam.

### 3. The validation of the analytical method

Xu et al [9] analyzed the response of single pile under adjacent tunnel excavation by using a three-dimensional boundary element program GEPAN, $E_s=24$MPa, $v_s=0.5$, $R=3$m, $H =20$m, the three formation losses are 1%, 2.5% and 5%. $E_p=30$GPa, $v_p=0.2$, $L=25$m, $d=0.5$m, $x=4.5$m. Respectively, using this theoretical method to calculate the solution considering the pile shear effect and the solution without considering the shear effect, then compared the two with the GEPAN solution. The results of horizontal displacement and bending moment are shown in Fig.4 and Fig.5.

From Fig.4 we can easily find that the consistency of the three algorithms are in good consistency, but around the depth of tunnel center, the maximum horizontal displacement of degradation solution is 7.88mm, compared with the boundary element method whose maximum horizontal displacement is...
9.22mm, which obviously underestimate the maximum horizontal displacement by 14.5%, however, for the theoretical method considering the pile shear effect, the maximum horizontal displacement is 8.67mm, which reduces by 5.9% compared with the boundary element method; From Fig.5 can also find that the bending moment has a similar rule. The maximum bending moment of the degradation solution is -63.78kN·m, compared with the boundary element method's 57.2kN·m, which overestimates the maximum bending moment by 11.5%, however, for the theoretical method considering the pile shear effect, the maximum bending moment is 58.4kN·m, which reduces by 2.1% compared with solution of the boundary element method.

**Fig. 3** Comparison of horizontal displacement

Because the degenerate solution does not consider the influence of shear deformation, resulting in a smaller displacement calculated. Apparently, the other parts of pile which are close to the tunnel center have the more obvious shear effect. However, for the parts far away from the tunnel center, the two solutions have almost no significant difference in value. The reason for the above phenomenon is that the closer the pile to the tunnel center, the stronger the disturbance is, and the deformation as well as the internal force of the pile itself are relatively large. Based on this, whether the shear effect of pile foundation is taken into consideration will be obviously distinguished. While for the parts of pile which are away from the shield center, the excavation has relatively small disturbance on them, and the influence of the pile's shear effect is insignificant.

**4. Parameter Analysis**

Fig.5 shows the calculation results considering pile shear effect with different $R/d$. With the increase of $R$, the displacement of the entire pile rapidly increase, and the change in the depth of the tunnel center is the most significant, the reason for this is the excavation of the shield has the largest additional load on the pile near the center of the tunnel. Therefore, this part is most obviously affected by soil disturbance, accordingly, changes more obvious.

**Fig. 4** Comparison of horizontal bending moment

**Fig. 5** Comparison curve of horizontal Displacement with different tunnel radius

**Fig. 6** Comparison curve of horizontal displacement with different distance between pile and tunnel
Fig. 6 shows the calculation results considering shear effect with different $x/d$. And the displacement of pile decreases obviously near the center of the tunnel. For the part far away from this depth, the displacement changes slower. When comparing the Fig. 6 and Fig. 7, $R$ is more sensitive than $x$ for pile's displacement and bending moment.

5. Conclusion
(1) When the equivalent shear stiffness of the pile is infinite, the solution of Timoshenko beam can degenerate into the Euler-Bernoulli beam solution, which has a wider applicability than the Euler beam.

(2) In the variation law of pile displacement and bending moment, Timoshenko beam considering shear effect is more consistent than the Euler-Bernoulli beam solution. However, in the tunnel center depth, the displacement of Timoshenko beam will be significantly higher than that of Euler-Bernoulli beam, while the bending moment is opposite.

(3) The shear effect is affected by the size of shield disturbance to pile. The greater the disturbance of pile foundation, the more apparent the shear effect, and it is more necessary to consider the pile shear effect.

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