INTRODUCTION

This paper develops a formal growth model in which there is a dual labour market. Unlike most developed countries where the neoclassical labour market might be a reasonable approximation, many developing countries have a large number of under-utilized labourers in the agricultural sector, and gradual industrial development does not drive up the wage level (Lewis, 1954).

Lewis (1954) proposed a dual economy framework characterized by the separation of the modern industrial sector from the traditional agricultural sector. Labour in the traditional agricultural sector is plentiful, frequently having low or even zero marginal product, while in the modern industrial sector labour has a positive marginal product. The modern industrial sector’s wage is determined by their marginal product but in the traditional agricultural sector, people’s income level is determined by their surplus labour.
average product which is at the subsistence level. The population in the traditional sector is sufficiently large to provide an unlimited supply of labour capable of moving to the modern sector at a subsistence wage in a given period without lowering agricultural output. This unlimited supply of labour from the traditional sector keeps the wage rate in the modern sector low, ensures that capital accumulation in the modern sector is sustained over time, and thus leads to economic transformation (Gollin, 2014; Wang & Piesse, 2013).¹

The concepts of surplus labour and the general framework of the Lewisian dual economy model are crucial to understanding economic development (Fei & Ranis, 1964; Lewis, 1954), and dual economy models have become deeply embedded in contemporary thinking about development and growth (Gollin, 2014; Wang & Piesse, 2013). However, its lack of formal microfoundations has been a barrier to the further development of the model and has prevented it from being used rigorously in empirical research. Some attempts have been made to formally incorporate dual economy features into models of development (Banerjee & Duflo, 2005; Temple, 2005; Vollrath, 2011), growth (Temple & Wößmann, 2006; Vollrath, 2009), and trade (Barbier & Rauscher, 2007), but many of the previous attempts deviated from the essence of the dual sector economy, where the wage determination mechanisms in two sectors are different.

In neoclassical literature, the shift of GDP components and labour shares from agriculture to industry has been studied from a variety of perspectives. These include perspectives that emphasize non-homothetic preferences, such as different income elasticity of agricultural goods versus industrial goods (Kongsamut, Rebelo, & Xie, 2001), or the differences in total factor productivity (Ngai & Pissarides, 2007), or the importance of capital deepening (Acemoglu and Guerrieri, 2008). Herrendorf, Rogerson, and Valentinyi (2014) present the stylized facts of structural transformation across time and space and develop a multi-sector model that is able to account for many salient features of structural transformation and delivers sharper insights for understanding economic development, regional income convergence, aggregate productivity trends, hours worked, business cycles, and wage inequality. However, although the Lewisian dual sector models are at the heart of the analysis of economic transformation in development studies, the key features of the dual labour markets are absent from modern structural change literature (e.g. Gollin, Parente, & Rogerson, 2002; Gollin, Jedwab, & Vollrath, 2016; Hansen & Prescott, 2002).

The growth mechanisms are different in different development stages. In a Malthusian world, the living standard is kept at the subsistence level, as the growth of capital accumulation or technological progress is offset by population growth, which was regarded as the origin of poverty (Malthus, 1798). However, if there is a drastic change in technology such as observed in the Industrial Revolution, a positive impact on the expected return to human capital might trigger the trade-off between quality and quantity, which eventually leads to a decline in fertility and an increase in income per capita (Becker, 1960; Becker & Gregg Lewis, 1973; Becker, Murphy, & Tamura, 1990). For many developed economies, the increase of capital investment cannot maintain the sustained economic growth due to diminishing returns to capital (e.g. Barro & Sala-i-Martin, 1995; Solow, 1956; Swan, 1956), and only the accumulation of human capital and technological progress are the sources of long-term growth (e.g. Lucas, 1988; Mankiw, Romer, & Weil, 1992; Romer, 1990). Galor and Weil (1999, 2000) provided a unified neoclassical framework for the transition from the the Malthusian regime to the modern growth regime.

¹In addition to the seminal paper by Lewis (1954), many others including Jorgenson (1961), Fei and Ranis (1964), Schultz (1964), Sen (1966), Stiglitz (1974, 1976), Fields (2004), Temple (2005), Vollrath (2009), Wang and Piesse (2013) have either theoretically developed or empirically tested the concept and the existence of surplus labour in developing countries. Gollin (2014) provides a comprehensive review of Lewis dual economy models and their development in the last 60 years.
When an expansion in the industrial sector allows surplus labourers in the agricultural sector to enter this sector, the impact on wages in this process is quite different from the situation when labourers change sectors in a competitive labour market (Wang & Piesse, 2013). For example, when a developing economy with surplus labourers opens trade to an developed economy with a competitive labour market, an increase in the demand for industrial goods will cause the industrial sector to expand faster hence generate higher economic growth due to higher capital share and higher rate of capital accumulation. Because of this, countries with surplus labour can continuously enjoy trade advantages over other countries in terms of lower labour costs, at least until their surplus labour is exhausted. This is the underling mechanism that will be formalized in this paper and it may be one of the fundamental forces underneath China’s growth and trade with the US. Thus, the dual labour markets’ perspective may provide an alternative theory for many of the empirical results in explaining trade patterns between developed and developing countries, such as in Autor, Dorn and Hanson (2013, 2016).

The New Structural Economics proposed by Lin (2009, 2012) is an application of the neoclassical approach to study the determinants of economic structure, structural change and their impacts on the evolution of development (Lin & Wang, 2017; Stiglitz, 2011). It is a new wave of development economics thinking and it makes a serious effort to adopt some of the key insights from traditional structuralist economics including the view that industrial growth is the key to modern economic development and income growth. In contrast to the old structuralist approach, the New Structural Economics argues that following comparative advantage (determined by the endowment structure) to develop industries is the best way to achieve dynamic growth and convergence. The economy with such industries and appropriate hard and soft infrastructure will be most competitive, produce the largest surplus, have the highest possible returns to capital, and thus savings, ensure the fastest upgrading of the endowment structure, and achieve the most rapid industrial upgrading and income growth (Ju, Lin, & Wang, 2015). In this process, a developing country can have latecomer advantages, and thus have faster capital accumulation, technological innovation and industrial upgrading than high-income countries, which leads to faster growth and convergence with high-income countries (Lin & Wang, 2017; Wang & Piesse, 2013).

The Lewisian dual sector models are at their heart of the classical economic method and modern mainstream economics is essentially neoclassical. Neoclassical economics explains the value of goods by reference to their marginal utility and usually assumes that marginal changes are infinitesimally small. The focus of the dual economy models is to facilitate economic development, whilst the marginal method makes the analysis mathematically more tractable. However, the marginal analysis may be less robust at the operational level in certain areas of economic analysis, because decisions made by human agents are both marginal and infra-marginal.

Although the general framework of the Lewis (1954) dual economy model is insightful, the fundamental concepts and micro-mechanisms (especially the definition of surplus labour, the wage determination mechanisms in both the traditional and modern sectors and the dynamics of labour flows between the two sectors) lack sufficient detail and are still unclear (Brown, 2006; Fields, 2004, 2006). As a result, many of the main features of the dual economy was missing due to the lack of ways to

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2This has been well explained by one of the key figures of marginalism and the neoclassical revolution, Stanley Jevons (1871) in his *Theory of Political Economy*, ‘Pleasure and pain are undoubtedly the ultimate objects of the calculus of economics. To satisfy our wants to the utmost with the least effort—to procure the greatest amount of what is desirable at the expense of the least that is undesirable—in other words, to maximize pleasure, is the problem of economics’.

3Leeson (1979, p. 200) argues that ‘...Lewis declares himself in favour of realism as against precision in the formulation of his model’.

4This lack of clarity was also true for Lewis himself. For example, his 1954 and 1979 papers differ in some important respects regarding the sectoral wage determination mechanisms.
square this classic idea with neoclassic tools. There is a need to formally define these concepts and mechanisms.

This paper develops a growth model in an open economy environment that captures dual labour market characteristics, which enables us to compare different growth paths in developed and developing countries with different labour market structures. It builds on Wang and Piesse (2013) which attempts to reconcile the classical and neoclassical approaches. The main contribution of this paper is that it uses the neoclassical method, marginalism, to support a classical approach. Using rigorous mathematical methods, it shows that the classical theory of dual economy can be used in many aspects and can be extended to reveal the changes that have taken place from agricultural to industrial society.

In particular, this paper contributes to the literature as follows. First, this paper provides micro-foundations to the Lewis dual economy model and formalizes the conceptual model of surplus labour. Second, it combines dual labour market characteristics into a growth model in an open economy environment. Third, our model is able to derive different growth paths and structural changes in developed and developing countries, as our model considers the existence of surplus labour in developing countries. Finally, the model demonstrates the effects of surplus labour on economic growth and economic welfare in developed and developing countries.

This paper is organized as follows: Section 2 introduces the concept of surplus labour and discusses the structural differences in labour markets between developed and developing countries; Section 3 develops a formal growth model in an open economy environment, and formalizes the concept of surplus labour; Section 4 discusses economic growth under two different labour market structures, and demonstrates the effects of surplus labour on economic growth; Section 5 concludes.

2 | THE LABOUR MARKET STRUCTURE IN DEVELOPING COUNTRIES

There have been a large number of studies trying to address economic structural transformation, each looking at different aspects, but the substantial differences in labour markets between developed and developing countries have been neglected in many of them. Countries vary significantly not only in their wage levels but also in labour endowments and labour market structures. Existing growth theories consider capital, labour and technology, but do not fully consider an economy’s labour market structure. While the labour market tends to be relatively competitive in many developed countries, there is severe labour market segmentation and a large number of surplus labourers in many developing countries.

2.1 | Dual labour markets framework

The characteristics of the traditional agricultural economy are different from those of the modern commercialized agriculture. Many labourers in poor countries are under-utilized in the agricultural sector and willing to work in the industrial sector but there is only a limited number of jobs available

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5 It is likely that this paper’s use of the neoclassical framework to support what is in essence a classical model will face fierce criticisms from both fronts. From the classical point of view, our analysis might be seen by some unnecessary and confusing. As to them the use of marginal analysis is an unnecessary approach only complicating things without providing any additional insights. From a neoclassical perspective, due to the necessity of rigorous mathematics, one might feel that many assumptions are far-fetched. However, we argue that the reconciliation of the two in the study of economic development brings significant value.
to them. It is impossible for the industrial sector to absorb these surplus labourers by lowering the wage as this is set at subsistence level. As a result, many labourers capable of work are effectively stuck in the agricultural sector with low or zero marginal product, unable to participate in meaningful production.

In a dual economy with a small industrial sector and a large agricultural sector, the traditional agricultural sector mainly consists of family units engaged in agricultural production, where all family members share the work and the subsequent output, either as wages or profits. Thus, no family member is technically unemployed, each earns the average product of labour in the agricultural sector, as noted by Lewis (1954), Fei and Ranis (1964, 1997), Sen (1966) and Fields (2004).

Because of diminishing returns to labour, when land is fixed, the long run population level adjusts as a function of available food. This pushes the wage, which equals the average product of labour, to subsistence level. In other words, in the equilibrium, the subsistence wage in the agricultural sector equals the subsistence level of output per head. (Lewis, 1954; Wang & Piesse, 2013).

There might be cases where labour and food shortages create barriers for economic transformation (Gollin et al., 2002; Gollin, Parente, & Rogerson, 2007; Gollin, Lagakos, & Waugh, 2014). The main argument is that it will cause an insufficient supply of workers in the industrial sector. For example, Hayashi and Prescott (2008) show that the pre-war Japanese social custom of requiring a son to inherit the land from his father and remain in agriculture can account for the delay in Japan’s catch-up with the US. However, as we will show later, the constraint of agriculture goods mainly affects the population growth dynamics, not the industrialization. The capacity of the industrial sector to absorb the agricultural workers that are willing to migrate is the main barrier for economic transformation. For example, the Irish Potato Famine from 1845 to 1849 caused a significant number of deaths and emigration in Ireland, but the migration increased the labour supply to the industrial development in England.

Whilst the lack of food supply might slow down agricultural labour transfer out of the sector, and thus impede industrial development, for the majority of developing countries, especially the economies with a large population, the agriculture sector is only paying subsistence wage and acts as a sunk of employment. There is unlimited supply of labour in the agricultural sector (Lewis, 1954), the main problem of the under-development of the industrial sector lies somewhere else, such as the lack of capital, lack of government coordination and appropriate market institutions (Ngai, 2004). When under-employment in agriculture is due to inadequate absorb capacity of the industrial sector, the development process can be initiated by capital accumulation.

Surplus labour can exist in one country even when it engages in trade with another country under the following scenarios: (a) the elasticity of technical substitution between production factors is greater than one but has an upper limit; (b) when the ratio of capital to labour endowment is over a certain level, the wage will be driven down to a value that equals subsistence level; (c) the total population is larger than the critical number where the population is supported by the agricultural sector based on the average product of labour. When capital cannot be technically substituted by labour and when the amount of surplus labour is constant or increasing, the wage will not increase, and the industrial sector will have an unlimited supply of labour from the agricultural sector at a constant wage level.

Figure 1 may help to clarify these issues as it illustrates the determination of employment and wage in a Lewisian labour market. The horizontal axis, $$OQ$$, shows the total amount of labour force in the economy, which can be assumed to be fixed when there is no population or labour force growth. The

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6 For example, when there is a large amount of surplus labour relative to capital, it may be impossible for labour to fully substitute capital in production.

7 The average product of labour defines the total number of population that can be supported by the agricultural sector, i.e. there is enough food for everyone.
agricultural sector’s labour is measured rightwards from the origin $O$. The $MPL_A$ and $APL_A$ curves are the marginal and average product of labour in the agricultural sector respectively. Industrial employment is measured leftwards from $O'$. The $MPL_M$ curve is the marginal product of labour in the industrial sector. The intersection point between the $APL_A$ and the $MPL_M$ determines the wage, $\bar{w}$, which is at subsistence level. This intersection point divides total labour force into the two sectors as industrial labourers, $L_M$, and agricultural labourers, $L_A$.

In many developing countries, in terms of employment, there exists a small industrial sector and a big agricultural sector with a large number of surplus labourers. The wage in the agricultural sector is low but labourers cannot move to the industrial sector for a higher wage rate because of its low absorptive capacity. It is only when the industrial sector expands that it can absorb surplus labourers from the agricultural sector. The industrial sector’s expansion will cause the number of employed workers in the economy to increase but will not cause the wage in the industrial sector to rise. This means that economic growth has translated into widening labour participation but not into increasing the wage level. With the existence of surplus labour, the labour supply is unlimited at a fixed subsistence wage level, no matter how much the labour demand increases. That is, the labour supply curve is horizontal until surplus labour is exhausted.

labour markets evolve in three stages: in the first stage the agricultural sector has absolute surplus labour whose marginal product is zero; in the second stage their marginal product is positive but lower than what they are paid; in the third, and neoclassical, stage all labour is paid according to their marginal product (Fei & Ranis, 1964, 1997; Wang & Piesse, 2013). In this paper, we assume that the labour market in the developed home country has already reached a neoclassical stage while the developing foreign country still has a large pool of surplus labour in the agricultural sector.

2.2 | Surplus labour further defined

Many existing literature claimed to have formally captured the characteristics of surplus labour often abandoned the key features of the dual economy models of Lewis (1954) and Fei and Ranis (1964). They either assume a competitive labour market in the traditional sector (e.g. Acemoglu, 2009), or
without considering the linkage between the subsistence wage and the average product of labour in the agricultural sector or its close connection with the industrial sector. (e.g. Barbier & Rauschery, 2007). An example of this approach is Barbier and Rauschery’s (2007) model which yields the result that an increase in land endowment or in agricultural productivity can have negative welfare implications for countries. This result is contrary to the empirical observations. We argue that considering only the agricultural average product is not enough to formalize surplus labour as it does not take the relationship between the agricultural average product and the manufacturing marginal product into consideration, and often implicitly assumes a much higher average product in the agricultural sector. Nor does it allow us to obtain the amount of surplus labour in the agricultural sector.

Before proceeds, let us define some concepts. Refer back to Figure 1. The intersection point between the \( APL_A \) and the \( MPL_M \) determines the subsistence level wage, \( \bar{w} \). Total number of people in the agriculture sector is \( L_A \), but only \( L_a \) amount of labour have \( MPL \geq \bar{w} \) and \( L_s \) amount of labour have \( MPL < \bar{w} \). We have the following definition:

**Definition 1** Labour is defined as Effective labour, if their MPL is higher than subsistence level, \( MPL \geq \bar{w} \). labour is defined as Surplus labour, if their MPL is lower than subsistence level, \( 0 \leq MPL < \bar{w} \).

Following the above definition, in Figure 1, effective labour is denoted as \( L_a \), and surplus labour is denoted as \( L_s \).8

The existence of surplus labour is due to the fact that the Malthusian population growth equilibrium means each household had a number of family members greater than it needs to work its own fields to get the MPL greater than subsistence level for the marginal one. If the decision to work were purely extensive then some fraction would work on the farm and the rest would not work at all.

The agricultural sector absorbs all surplus labour, otherwise, they would not have been able to survive. As a result, everybody in the agricultural sector is paid the average product, which equals the subsistence wage level. Therefore, the total number of agricultural labourers is the sum of effective labourers and surplus labourers, that is \( L_A = L_a + L_s \).9

Note that agricultural sector is the sink of all extra labourers not able to find work in the industrial sector. Surplus labour includes unutilized labour, including those not participating in production in the neoclassical sense. labour can be considered to be surplus even if they are unemployed. That is, were they to work, their MPL would be small or zero or negative. In fact, such individuals may remain idle rather than participating in the workforce. In neoclassical terms, surplus labour’s marginal utility of leisure is very low and they are unable to find work at the subsistence wage level. A job paying below subsistence level is not feasible in the neoclassical framework.

It should be noted that the surplus labour and the zero marginal product are outcomes of the equilibrium, not properties of individual workers. Dividing labour into effective and surplus labour only makes sense at aggregate level. We are not saying that the marginal product of labour in agriculture

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8For simplicity, we do not distinguish two types of surplus labour, those whose marginal product equals zero and those bigger than zero but lower than their income. See Wang and Piesse (2013) and Wang and Weaver (2013) for a detailed discussion of the wage determination mechanisms and the two types of surplus labour in dual economy models. Those surplus labourers with \( 0 < MPL < \bar{w} \) actually have contribution to production but it is very small. So, in order to simplify calculations, we only take the type of surplus labour with MPL = 0 into consideration in the following model.

9Figure 1 is an illustration of statics of surplus labour in the two sectors, and is not showing the dynamics of labour movement. i.e. it is inappropriate to show industrial employment expansion just by moving the MPL_M towards the left, as in this figure, a movement of \( L_s \) towards the industrial sector would mean a higher APL_A.
can be attributed to individual workers, nor individual labour can be designated as having a lower marginal product than others. In our model, all workers have the same marginal product, i.e. they are homogeneous workers.

In essence, surplus labour is defined as labourers whose marginal product is less than the actual wage received. By definition, surplus labourers would not have been able to survive if they were paid only their marginal product, which is less than subsistence level. While the neoclassical wage determination principle that the wage equals the marginal product of labour is true for competitive markets, it is not necessarily the case for the traditional agricultural sector.

3 | THE MODEL

In an open economy, there are two countries, home and foreign (*), where the home country is developed and the foreign country is a developing one. We assume everything is the same in both countries except their factor endowments and labour market structures. More precisely, we assume that the home country has a larger capital endowment but the foreign country has a larger labour endowment: \( K > K^* \), \( L < L^* \). The labour market in the home country has already entered a neoclassical stage while the foreign country has a Lewisian labour market.

There are two sectors in both countries: an agricultural sector and a manufacturing sector. The agricultural sector produces a homogeneous agricultural good, which is taken as the numeraire with unit price.\(^{10}\) The manufacturing sector produces a wide range of variety of differentiated goods that are close substitutes for each other, and all varieties are symmetric.\(^{11}\) The price of manufactured goods is measured in terms of the numeraire. In the foreign country, its large amount of surplus labour exists in the agricultural sector.

There are three factor inputs, labour \( L \) land \( N \) and capital \( K \). The agricultural sector uses labour and land, and the manufacturing sector uses labour and capital to produce. The total amounts of labour and land are assumed to be exogenously given and fixed, but capital can be accumulated. There is free labour mobility between sectors but not across countries, and there is no capital mobility across countries.\(^ {12}\) We also assume there are no trade costs.

In this section, we focus on the discussion of the home country, with the understanding that analogous equations hold for the foreign country.

3.1 | Preferences

The set-up of the basic model takes elements from Dixit and Stiglitz (1977), Krugman (1979, 1980), Markusen and Venables (2000) and Barbier and Rauschery (2007). Households are homogeneous

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\(^{10}\) Our model works whether the agricultural good is internationally traded or not.

\(^{11}\) ‘Variety’ in this paper can be understood as output or consumption. In our model, as in most of the New Trade Theory literature (e.g. Krugman, 1979, 1980), the economy has \( n \) number of firms, where one firm only produces one variety. It is assumed that the quantity of each variety is a constant and equals across all firms in both countries. The aggregate output is the quantity of each variety times the total number of varieties (i.e. the total number of firms). In this framework, output increase can then be modelled as an increase in the number of varieties. On the consumer side, as the utility function exhibits agents’ love of variety, an increase in variety will mean an increase in utility.

\(^{12}\) We take this assumption only for simplicity. It can be relaxed but that would require more complex conditions on production function.
within a country, and generate utility from the consumption of the agricultural good and manufactured goods. The utility function \( U(A, V) \) represents preferences for \( A \) and \( V \), where \( A \) is an individual’s consumption of the agricultural good and \( V \) is the sub-utility function of manufactured goods consumption. We assume the agricultural product is a good satisfying basic needs (food) and has low income elasticity, and manufactured goods satisfy non-basic needs and have higher income elasticity. When income increases, the major share of additional expenditure is on manufactured goods.\(^{13}\) For simplicity, we assume each individual’s consumption of the agricultural good, \( A \), is the same. The utility function can be written as the following, where the consumption of the agricultural good is of the CRRA (constant relative risk aversion) form:

\[
U(A, V) = \frac{A^{1-\delta} - 1}{1-\delta} + V, \quad \delta > 0
\] (1)

when \( \delta = 1 \), the first component of the utility function becomes \( \ln A \).

Let manufactured goods be modelled as a continuum of varieties, the sub-utility function is of the Dixit-Stiglitz type:

\[
V[c(i), m(j)] = \left[ \int_0^n c(i)^{\theta} \, di + \int_0^m m(j)^{\theta} \, dj \right]^{\frac{1}{\theta}}, \quad 0 < \theta < 1
\] (2)

where \( c(i) \) denotes the home country’s consumption of domestically produced variety \( i \), \( m(j) \) denotes the home country’s consumption of imported foreign produced variety \( j \). \( n \) and \( n^* \) are the total numbers of varieties produced in the home and the foreign country respectively. \( \theta \) is the consumer’s willingness to substitute between domestically and foreign produced varieties, and the elasticity of substitution equals \( \frac{1}{1-\theta} \).

The budget constraint of a domestic representative household is:

\[
e = A + \int_0^n p(i) c(i)^{\theta} \, di + \int_0^{n^*} p^*(j) m(j)^{\theta} \, dj
\] (3)

where \( p(i) \) and \( p^*(j) \) are the prices of domestically produced variety \( i \) and foreign produced variety \( j \), and \( e \) is the income per capita.\(^{14}\)

When we maximize the utility function (1) with respect to \( A \), \( c(i) \) and \( m(j) \) subject to the budget constraint (3), we get demand functions for domestically produced variety \( i \) and for imported foreign produced variety \( j \) conditional on \( A \) as follows:

\[
c(i) = \frac{e-A}{p^{\frac{\theta}{\theta-1}}} p(i)^{\frac{1}{\theta-1}}
\] (4)

\[
m(j) = \frac{e-A}{p^*^{\frac{\theta}{\theta-1}}} p^*(j)^{\frac{1}{\theta-1}}
\] (5)

where \( P \) is a CES price index with the following form:

\(^{13}\)Low income elasticity for agricultural goods and high-income elasticity for manufactured goods is well documented in the literature. This, Engel’s law, has been nicely modelled by Kongsamut et al. (2001).

\(^{14}\)It should be noted that, to be precise, \( e \) is income per capita that is used for consumption. In later discussion, we will allow capital to be accumulated by \( \sigma \) savings rate, thus only \( 1-\sigma \) of manufacturing output can be consumed.
Each consumer’s choice between A and V is governed by the following standard condition that the marginal rate of substitution equals the relative price:

$$\frac{U_V}{U_A} = P$$  \hspace{1cm} (7)

where $U_V$ and $U_A$ denote the partial derivatives of the utilities from the consumption of manufactured goods and the agricultural good.\(^{15}\)

With the assumption of free trade, the law of one price holds and both countries have the same price index. Note that the total demand for a domestically produced variety $i$, $x(i)$, is composed of the domestic consumption, $c(i)$, and the foreign consumption, $m^*(i)$. That is,

$$x(i) = c(i) + m^*(i) = \frac{e^{-A} + e^* - A^*}{P^{\frac{a}{\theta-1}}} p(i)^{\frac{1}{\theta-1}}$$  \hspace{1cm} (8)

The income levels, the demands for the agricultural product and the price index in both countries are taken as given by an individual producer, who is infinitely small compared to the size of the market. Thus, it follows that the inverse demand function can be expressed as the following:

$$p(i) = \mu x(i)^{\theta-1}$$  \hspace{1cm} (9)

where $\mu$ is a constant parameter.

Since all varieties are assumed to be produced with the same technology, the index $i$ can be dropped and the inverse market demand function for a representative variety becomes:

$$p = \mu x^{\theta-1}$$  \hspace{1cm} (10)

### 3.2 The manufacturing sector

We assume all firms in the manufacturing sector operate under monopolistic competition. There are increasing returns to scale at the firm level and each firm produces only one variety. Each firm differentiates its variety from all other varieties offered by other firms. The production function for a representative firm in the manufacturing sector is as follows:

$$x(k, l_M) = \frac{1}{\nu} \left( k^a l_M^{1-a} - \phi \right), 0 < a < 1$$  \hspace{1cm} (11)

\(^{15}\)There are development economy models in which preferences are subject to subsistence level of food (agriculture). When agricultural productivity is low, this preference parameter determines both the demand for agricultural goods and wages in the agricultural sector. For example, in Galor and Weil (2000), for a sufficiently low level of income the subsistence consumption constraint is binding and there is a corner solution with respect to the consumption level. This is a corner solution where the standard marginal conditions do not characterize the equilibrium as long as the productivity is low in agriculture. The presence of the subsistence consumption constraint provides the Malthusian piece of the model.
where \( x \) is each firm’s output, \( k \) and \( l_M \) are capital and manufacturing labour inputs at the firm level respectively. \( \frac{1}{\alpha} \) is the index of productivity or efficiency, and the value of it is constant. The parameter \( \phi \) is the necessarily fixed factor inputs required to produce output \( x \).\(^{16}\) \( \alpha \) is the output elasticity of capital at the firm level.

Each firm minimizes its total cost \( rk + w_M l_M \) for producing output \( x \), which derives the demand function for each factor input per firm as follows:

\[
k = \left( \frac{\alpha}{1 - \alpha} \frac{w_M}{r} \right)^{1-\alpha} (vx + \phi) \quad (12)\]

\[
l_M = \left( \frac{1 - \alpha}{\alpha} \frac{r}{w_M} \right)^{\alpha} (vx + \phi) \quad (13)\]

Therefore, the total cost per firm can be expressed as the following:

\[
TC (r, w_M, x) = \frac{r^\alpha w_M^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} (vx + \phi) \quad (14)\]

Equation (14) exhibits internal increasing returns to scale because it implies the average cost, \( \frac{r^\alpha w_M^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} (vx + \phi) \), is decreasing with output \( x \).

Given the inverse demand function (10) and the total cost function (14), we get the representative firm’s profit function as:

\[
\pi = \mu x^\theta - \frac{r^\alpha w_M^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} (vx + \phi) \quad (15)\]

Firm’s profit maximization implies the following price level as:

\[
p = \frac{v}{\theta} \frac{r^\alpha w_M^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \quad (16)\]

where \( p = \mu x^{\theta-1} \), from the inverse demand function (10) above. This yields the standard result that each firm charges a monopoly markup over marginal cost.

The markup pricing Equation (16) determines the rental price and the wage in this sector as follows:

\[
r = \left[ \frac{\alpha^\alpha (1 - \alpha)^{1-\alpha}}{w_M^{1-\alpha}} \frac{p \theta}{v} \right]^{\frac{1}{\alpha}} \quad (17)\]

\[
w_M = \left[ \frac{\alpha^\alpha (1 - \alpha)^{1-\alpha}}{r^\alpha} \frac{p \theta}{v} \right]^{\frac{1}{1-\alpha}} \quad (18)\]

Free entry and exit lead to zero profit, \( \pi = 0 \). This derives the output per firm as:

\[
x = \frac{\theta \phi}{1 - \theta} \frac{1}{v} \quad (19)\]

---

\(^{16}\)This way of expressing increasing returns to scale is similar to Krugman (1979, 1980).
Equation (19) shows the output per firm is constant. It does not depend on the price or other variables that might be affected by changes in factor endowments, but depends on the parameters of the model that do not change.

When we substitute Equations (18) and (19) into Equation (12) for $k$, and substitute Equations (17) and (19) into Equation (13) for $l_M$, we obtain each firm’s demand functions for factor inputs respectively as follows:

$$k = \frac{\alpha}{r} \frac{\theta}{1-\theta} \frac{p^\phi}{\nu} \quad (20)$$
$$l_M = \frac{1-\alpha}{w_M} \frac{\theta}{1-\theta} \frac{p^\phi}{\nu} \quad (21)$$

These two equations say that the demand for each factor input is determined by its factor price and the price level rather than by the ratio of factor prices as shown in Equations (12) and (13).

We include analysis with increasing returns to scale at the firm level in order to exhibit certain patterns of trade that we are interested in, such as trade of varieties of differentiated goods between countries. However, in order to ease subsequent calculation to derive growth rates of variables in the following section, we introduce a sector level production function that exhibits constant returns to scale at the industry level. Let the whole sector’s production function be written as:

$$X = \frac{1}{\nu} K^n L_M^{1-\eta}, \quad 0 < \eta < 1 \quad (22)$$

where $X$ represents the total manufacturing output, $K$ and $L_M$ are aggregate capital and total manufacturing labour inputs at the industry level respectively. $\eta$ is the output elasticity of capital at the industry level.\(^{17}\)

Since varieties in the manufacturing sector are symmetric, the total output is the sum of each firm’s output, i.e. $X = nx$, where $n$ is the total number of varieties. Analogously, we have aggregate capital is the sum of each firm’s capital, $K = nk$, and total manufacturing labour is the sum of each firm’s labour, $L_M = nl_M$. When we substitute Equation (20) for $k$ into $K = nk$ and (21) for $l_M$ into $L_M = nl_M$, we get the demand functions for factor inputs at the industry level respectively as the following:

$$K = \frac{na}{r} \frac{\theta}{1-\theta} \frac{p^\phi}{\nu} \quad (23)$$
$$L_M = \frac{n(1-\alpha)}{w_M} \frac{\theta}{1-\theta} \frac{p^\phi}{\nu} \quad (24)$$

\(^{17}\)The above Cobb-Douglas form of production function with respect to capital and labour is not derived from the above firm level analysis but from our assumption. The assumption of increasing returns to scale at firm level and constant returns to scale at industrial level is consistent and is common in trade literature, e.g. Helpman and Krugman (1985). Without this sector level production function, we would not be able to derive many of the variables in Section 4 such as the growth rate of capital per manufacturing worker in (Equations 41) and (42). We include analysis with increasing returns to scale at the firm level in order to exhibit certain patterns of trade that we are interested in, such as trade of varieties of differentiated goods between countries.
3.3 | The agricultural sector in the home country

The production of the agricultural good requires land $N$ and agricultural labour $L_A$ to be combined via a constant returns to scale Cobb-Douglas production function to yield the aggregate level of output. This production function can be written as:

$$ Y = N^\beta L_A^{1-\beta}, \quad 0 < \beta < 1 $$

(25)

where $Y$ is the total output in the agricultural sector, and $\beta$ is the output elasticity of land.

Since the total amount of land is exogenously given and fixed, hereafter we normalize it to one, i.e. $N = 1$. Thus, the production function can be re-written as:

$$ Y = L_A^{1-\beta} $$

(26)

Given the production function, the income of a rural household is determined by the marginal product of labour as follows:

$$ w_A = MPL_A = (1-\beta) \frac{Y}{L_A} $$

(27)

Perfect intersectoral labour mobility equates the wages in the two sectors, i.e. $w_M = w_A = w$. Thus, the indices $M$ and $A$ on the wages can be dropped.

3.4 | The agricultural sector in the foreign country

The difference between the foreign and the home country is that the foreign country has a Lewisian labour market, where there exists a large amount of surplus labour in the agricultural sector. The agricultural production function in the foreign country is as follows:

$$ Y^* = L_A^{1-\beta} $$

(28)

where $Y^*$ is the total agricultural output, $L_A^*$ is the amount of effective agricultural labour. Those effective agricultural labourers contribute to the production of the agricultural good with $MPL_a \geq \bar{w}^*$ that their marginal product is higher than or equal to the subsistence wage level, $\bar{w}^*$.

In the foreign country with a Lewisian labour market, people living in rural areas own the land and obtain all the output from their work on this land. Thus, the income of a rural household measured in units of the agricultural good, equals the average product of labour ($APL_A^*$), which is at subsistence level ($\bar{w}^*$), as follows:

$$ \bar{w}^* = APL_A^* = \frac{Y^*}{L_A^*} $$

(29)

In fact, those surplus labourers with $0 < MPL < \bar{w}^*$ also contribute to agricultural production, but their contributions are very small. For simplicity, we do not take those surplus labourers into the production function.
where the total number of agricultural labourers, $L^*_A$, is the sum of effective labourers, $L^*_a$, and surplus labourers, $L^*_s$, i.e., $L^*_A = L^*_a + L^*_s$. Although the marginal product of surplus labour equals zero, $\text{MPL}^*_s = 0$, the agricultural sector absorbs all surplus labour, otherwise, they would not have been able to survive.

### 3.5 Different kinds of labourers in the foreign country

From the discussion in Section 2.2, the concept of surplus labour should be formalized by the following equation as:

$$\text{APL}^*_A = \text{MPL}^*_a = \text{MPL}^*_M = \bar{w}^*$$  \hspace{1cm} (30)

Equation (30) not only considers the average product of the agricultural sector, but also concerns the marginal product of effective agricultural labour and the marginal product of the manufacturing sector. When they are equal to the subsistence wage level $\bar{w}^*$ simultaneously, surplus labour exists in the agricultural sector.

From Equation (30), $\text{APL}^*_A = \text{MPL}^*_a$ implies $\frac{Y^*}{L^*_A} = (1 - \beta)\frac{Y^*}{L^*_a}$, which describes a relationship between $L^*_a$ and $L^*_A$; $L^*_a = (1 - \beta) L^*_A, L^*_a + L^*_s = L^*_A$ gives $L^*_s = \beta L^*_A$. $\text{APL}^*_A = \text{MPL}^*_M$ implies $\frac{Y^*}{L^*_A} = \frac{1 - \eta}{\nu} \left( \frac{K^*}{L^*_M} \right)^{\eta}$, plugging $K^{\eta} = \frac{\nu X^*}{L^*_M}$ into it we obtain $L^*_M = (1 - \eta)\frac{X^*}{Y^*} L^*_A$. Based on the above results and the fact that the total number of labourers (the initial labour endowment) is the sum of each kind of labour in the economy such that $L^* = L^*_a + L^*_s + L^*_M$, the total amount of agricultural labour is:

$$L^*_A = \frac{L^*}{1 + \frac{\mu(1-\eta)}{L^*_A} \theta \frac{\phi}{1 - \theta \nu}}$$  \hspace{1cm} (31)

the total number of effective agricultural labourers is:

$$L^*_a = \frac{(1 - \beta) L^*}{1 + \frac{\mu(1-\eta)}{L^*_A} \theta \frac{\phi}{1 - \theta \nu}}$$  \hspace{1cm} (32)

and the total amount of surplus labour in the agricultural sector is:

$$L^*_s = \frac{\beta L^*}{1 + \frac{\mu(1-\eta)}{L^*_A} \theta \frac{\phi}{1 - \theta \nu}}$$  \hspace{1cm} (33)

and the total number of manufacturing labourers is:

$$L^*_M = \frac{n^*(1-\eta) \theta \frac{\phi}{A^*}}{1 + \frac{\mu^*(1-\eta)}{L^*_A} \theta \frac{\phi}{1 - \theta \nu}}$$  \hspace{1cm} (34)

\(^{19}\)To be precise, it should be $0 \leq \text{MPL}^*_s < \bar{w}^*$. But for simplicity without loss of generality, we do not distinguish the two types of surplus labour.
where in the equilibrium, the total output equals the total consumption of the agricultural good that
\[ Y^* = L^*A^*. \]

Those equations show that the total amount of each kind of labour depends on the labour endowment, \( L^* \), individual’s consumption of the agricultural good, \( A^* \), and the total number of varieties, \( n^* \). It is worth noting that in our model, \( L^* \) is assumed to be fixed and each individual’s consumption of the agricultural good, \( A^* \), is also assumed to be unchanged, thus the total consumption of the agricultural good, \( L^*A^* \), is fixed. With unchanged \( L^* \) and \( A^* \), when \( n^* \) increases, \( L^*_M \) goes up, \( L^*_A \) goes down as well as \( L^*_a \) and \( L^*_s \).

### 3.6 The static equilibrium

When there is no capital accumulation, each country’s equilibrium is static. In the home country, the market clearing conditions in capital and labour markets can be expressed as follows respectively:

\[ nk = K \tag{35} \]
\[ L_M + L_A = L \tag{36} \]

Based on those equilibrium conditions, we derive the equilibrium rental price and wage as follows respectively:

\[ r = \frac{na^\theta p\phi}{K} \tag{37} \]
\[ w = \frac{n(1-\alpha)^\theta p\phi}{L} + (1-\beta)A \tag{38} \]

Moreover, in the foreign country, the capital market clearing condition is the same as that in the home country. Hence, the equilibrium rental price is:

\[ r^* = \frac{n^*\alpha^\theta p\phi}{K^*} \tag{39} \]

However, in a Lewisian labour market, surplus labour exists in the agricultural sector, the standard labour market clearing condition cannot be applied, instead, we use Equation (30) that \( APL_A^* = MPL_a^* = MPL_M^* \) to determine the equilibrium wage, which is at subsistence level, as:

\[ \bar{w}^* = \frac{n^*(1-\eta)^\theta \phi}{L^*} + A^* \tag{40} \]

Equations (38) and (40) show that the equilibrium wage levels would increase when the total numbers of varieties, \( n \) and \( n^* \), become larger in each country respectively.

This is the case for the foreign country even though it has a large pool of surplus labour in the agricultural sector. One may argue that this is because industrial employment expansion absorbs surplus labour from the agricultural sector and, consequently, the total amount of surplus labour become less
which would lead the wage in the agricultural sector to increase. However, there exist various scenarios in which the total amount of surplus labour may not reduce when labourers are drawn out from the agricultural sector. As a result, the wage in the agricultural sector may be kept constant. See Appendix A for detailed discussion of the various scenarios where the wage rate is kept constant. Based on the analysis in Appendix A, although our model does not take population growth and agricultural total factor productivity growth into consideration, we can reasonably assume that the equilibrium wage is constant in the foreign country that has a Lewisian labour market.

4 | ECONOMIC GROWTH

Now, let us consider economic growth driven by capital accumulation in the two countries. Assuming there is no capital depreciation, we may set the rate of aggregate capital accumulation equal to a share \( \sigma \) of manufacturing output \( X_t \), i.e. \( K_t = \sigma X_t \),\(^20\) where \( t \) represents continuous time and \( t \in (0, \infty) \).\(^{21,22}\)

4.1 | Growth in the home country

Recall in Section 3, we define \( k_t \) and \( l_{M,t} \) as capital and manufacturing labour inputs per firm; \( K_t \) and \( L_{M,t} \) as aggregate capital and total manufacturing labour inputs at the industry level respectively. Therefore, capital per manufacturing worker can be expressed as aggregate capital divided by the total number of manufacturing workers: \( k_{M,t} = \frac{K_t}{L_{M,t}} \).\(^23\) Substituting Equations (23) for \( K_t \) and (24) for \( L_{M,t} \) into the expression of \( k_{M,t} \), we get \( k_{M,t} = \frac{\omega_t}{1 - \omega_t} \), where we set \( \omega_t = \frac{w_t}{r_t} \). Therefore, the rate of \( k_{M,t} \) accumulation is as follows:

\[
\dot{k}_{M,t} = \frac{\partial \left( \frac{K_t}{L_{M,t}} \right)}{\partial t} = \frac{\dot{K}_t}{L_{M,t}} - \frac{\dot{L}_{M,t}}{L_{M,t}} k_{M,t}
\]

(41)

Dividing both sides of \( \dot{K}_t = \sigma X_t \) by \( L_{M,t} \) we get \( \frac{K_t}{L_{M,t}} = \sigma \frac{X_t}{L_{M,t}} \); then using Equation (22) to replace \( X_t \) in this expression we get \( \frac{K_t}{L_{M,t}} = \frac{\sigma}{v} \left( \frac{K_t}{l_{M,t}} \right)^q = \frac{\sigma}{v} \frac{K_t}{l_{M,t}} \); Equation (24) shows \( \frac{L_{M,t}}{L_{M,t}} = \frac{n_t}{n_t} - \frac{w_t}{w_t} \). Substituting \( \frac{K_t}{L_{M,t}} \)

\(^{20}\)In our previous discussion, without capital accumulation we assume all manufacturing output to have been consumed, see Equation (8). In the case of capital accumulation, because \( \sigma X \) have been saved, only \( (1 - \sigma)X \) have been consumed. However, adding this item, \( 1 - \sigma \), into Equation (8), only complicates calculations, it will not alter our results. For simplicity, we do not include this term in our subsequent calculations.

\(^{21}\)As we only consider comparative statics with \( K \), we assume constant growth in \( K \).

\(^{22}\)\( K_t = \sigma X_t = \frac{2}{5} K^t L^{1-\sigma} \) is similar to the law of motion from the Solow model. It is not possible to tell our story with only a simple industrial level growth model with exogenous productivity growth. Much empirical literature, such as Krugman (1994) and Young (2003), argues that growth in Total Factor Productivity was very small in many emerging economies, and mainly driven by increased levels of factor inputs. Although Holz (2008) argues that there was significant growth in productivity in some developing countries like China, it did not match that in many developed countries such as the US. Thus, a one-sector production function and exogenous productivity growth would not be able to capture what our model can. Thus, we have to have capital accumulation driven economic growth.

\(^{23}\)Since all firms are assumed to be symmetric, \( K_t = n_t k_t \) and \( L_{M,t} = n_t l_{M,t} \). Thus, capital per manufacturing worker also can be expressed as capital per firm divided by the number of manufacturing workers per firm: \( k_{M,t} = \frac{k_t}{l_{M,t}} \)
and \( \frac{L_{M,t}}{k_{M,t}} \) into Equation (41), and then dividing both sides by \( k_{M,t} \), we get the growth rate of capital per manufacturing worker as:

\[
\frac{k_{M,t}}{K_t} = \frac{1}{\sigma} k_{M,t}^{\eta-1} \frac{\dot{w}_t - \dot{n}_t}{n_t}
\]

(42)

The growth rate of aggregate capital equals the growth rate of capital per manufacturing worker plus the growth rate of manufacturing labour, which can be expressed as:

\[
\frac{\dot{K}_t}{K_t} = \frac{\dot{k}_{M,t}}{K_t} + \dot{L}_{M,t} = \frac{1}{\sigma} k_{M,t}^{\eta-1} \frac{\dot{w}_t - \dot{n}_t}{n_t} + \frac{1-\alpha}{\alpha} \frac{1}{\omega_t} (1-\eta)
\]

(43)

where \( k_{M,t} = \frac{a}{1-a} \omega_t \).

Substituting Equation (12) for \( k_t \) into \( n_t k_t = K_t \) gives us \( n_t \left( \frac{a}{1-a} \omega_t \right)^{1-a} \frac{\phi}{1-\beta} = K_t \), from which we obtain an alternative equation of the growth rate of aggregate capital as:

\[
\frac{\dot{K}_t}{K_t} = \frac{n_t}{1-a} \frac{\dot{\omega}_t}{\omega_t} + (1-\alpha) \frac{\dot{\omega}_t}{\omega_t}
\]

(44)

Since \( \omega_t = \frac{w_t}{w_t} \), \( \omega_t = \frac{n_t}{n_t} - \frac{1}{r_t} \). From Equation (37) we get \( \dot{r}_t = \frac{n_t}{n_t} - \frac{K_t}{K_t} \). Equation (38) implies \( \frac{w_t}{w_t} = \frac{z n_t}{n_t+(1-\beta)A} \), where \( z \) is a function of \( p \) and \( L \), i.e. \( z = \frac{(1-\alpha) w_t}{1-\beta} \). As we have a fixed international price of manufactured goods, \( p \), and fixed labour endowment, \( L \), we can treat \( z \) as a constant parameter. Therefore, \( \frac{\dot{w}_t}{w_t} = \frac{z n_t}{n_t+(1-\beta)A} - \frac{n_t}{n_t} + \frac{K_t}{K_t} \). Substituting this result into Equation (44) and using Equation (43) to replace \( \frac{k_{M,t}}{K_t} \) in Equation (44) we have the growth rates of variety and manufacturing output as follows respectively:

\[
\frac{\dot{n}_t}{n_t} = \frac{1}{1+\frac{1-a}{a} f_t} \frac{\dot{K}_t}{K_t} = \frac{\sigma}{(1+\frac{1-a}{a} f_t)} \left( \frac{1-a}{\alpha} \frac{1}{\omega_t} \right)^{1-\eta}
\]

(45)

\[
\frac{\dot{X}_t}{X_t} = \frac{1}{1+\frac{1-a}{a} f_t} \frac{\dot{K}_t}{K_t} = \frac{\sigma}{(1+\frac{1-a}{a} f_t)} \left( \frac{1-a}{\alpha} \frac{1}{\omega_t} \right)^{1-\eta}
\]

(46)

where \( f_t \) is a function of \( z, A \), and \( n_t \), shown as \( f_t = \frac{z}{z+(1-\beta)A} \). It is obvious that \( f_t \) is positively related with \( n_t \) because both \( z \) and \( A \) are fixed. Equations (45) and (46) show that the relationships between \( f_t \) and the growth rates of variety and manufacturing output are negative, which means when \( n_t \) becomes larger, the growth rates of variety and manufacturing output will be slower over time. In addition, the growth rates of variety and manufacturing output negatively depend on the wage and rental price ratio as well. That is bigger \( \omega_t \) will induce smaller \( \frac{\dot{n}_t}{n_t} \) and \( \frac{\dot{X}_t}{X_t} \) over time.

Substituting \( \frac{\dot{w}_t}{w_t} = \frac{z n_t}{n_t+(1-\beta)A} \) into \( \frac{L_{M,t}}{K_{M,t}} = \frac{n_t}{n_t} - \frac{\dot{w}_t}{w_t} \), the growth rate of manufacturing labour is:

\[
\frac{\dot{L}_{M,t}}{M_{M,t}} = \frac{1-f_t}{1+\frac{1-a}{a} f_t} \frac{\dot{K}_t}{K_t} = \frac{\sigma}{(1+\frac{1-a}{a} f_t)} \left( \frac{1-a}{\alpha} \frac{1}{\omega_t} \right)^{1-\eta}
\]

(47)
Furthermore, substituting $\frac{\dot{w}_t}{w_t} = \frac{\gamma_t}{\gamma_t + (1 - \beta)A}$ into Equation (42), the growth rate of capital per manufacturing worker is:

$$\frac{\dot{k}_{M,t}}{k_{M,t}} = \left(1 - \frac{1 - f_t}{1 + \frac{1 - \alpha}{\alpha} f_t}\right) \frac{\sigma}{\nu} \left(\frac{1 - \alpha}{\alpha} \omega_t\right)^{1 - \eta}$$  (48)

Equation (47) shows that because $1 - f_t < 1 + \frac{1 - \alpha}{\alpha} f_t$, the growth rate of manufacturing labour is a fraction of the growth rate of aggregate capital. In other words, the growth rate of capital is faster than the growth rate of labour in the manufacturing sector; i.e. $\frac{\dot{K}_s}{K_s} > \frac{\dot{L}_{M,t}}{L_{M,t}}$. This means that with capital accumulation, the manufacturing sector would use more capital rather than labour to expand its production. The production technology in the manufacturing sector would become more capital intensive.

The consequence of this unequal growth would lead to an increase in wage level if all other things are held constant; i.e. $\frac{\dot{w}_t}{w_t} > 0$. Moreover, the rental rate decreases with capital deepening, $\frac{\dot{r}_t}{r_t} < 0$. Thus, $\frac{\dot{\omega}_t}{\omega_t} = \frac{\dot{w}_t}{w_t} - \frac{\dot{r}_t}{r_t} > 0$, $\omega_t$ increases over time. Hence, we have $\frac{\dot{L}_{M,t}}{L_{M,t}} > 0$, which means there is labour movement from the agricultural sector to the manufacturing sector for a higher wage.

Our model derives $\frac{\dot{w}_t}{w_t} = \frac{\gamma_t}{\gamma_t + (1 - \beta)A}$, so $\frac{\dot{w}_t}{w_t} > \frac{\dot{w}_t}{w_t}$ implies the wage does not rise fast enough to offset capital deepening-induced expansion in the manufacturing sector. Because $A$ is assumed to be fixed, and there is no additional demand for the agricultural good, any increase in variety induces higher demand for manufactured goods, in this case, the wage increase has to be muted enough to ensure that supply equals demand for manufactured goods.

Since, in the neoclassical world capital deepening leads to an increase in wage rate and this prevents the effect of capital accumulation from mapping onto an equivalent expansion of the size of output in the manufacturing sector, this is why the growth rate of output is only a proportion of the growth rate of aggregate capital, as shown in Equation (46).

### 4.2 Growth in the foreign country

Calculations based on Equation (30) imply $L_{M,t}^* = \frac{X^*}{k_{M,t}^*}$, the growth rate of manufacturing labour in the foreign country, therefore, can be written as follows:

$$\frac{\dot{L}_{M,t}^*}{L_{M,t}^*} = \frac{\dot{n}_t^*}{n_t^*} - \eta \frac{\dot{k}_{M,t}^*}{k_{M,t}^*}$$  (49)

Equation (43) shows $\frac{\dot{K}_s^*}{K_s^*} = \frac{\dot{k}_{M,t}^*}{k_{M,t}^*} + \frac{\dot{L}_{M,t}^*}{L_{M,t}^*}$, substituting Equation (49) into it, the growth rate of capital per manufacturing worker is:

$$\frac{\dot{k}_{M,t}^*}{k_{M,t}^*} = \frac{1}{1 - \eta} \left(\frac{\dot{K}_s^*}{K_s^*} - \frac{\dot{n}_t^*}{n_t^*}\right)$$  (50)

Equation (22) gives us $\frac{\dot{n}_t^*}{n_t^*} = \eta \frac{K_s^*}{K_t^*} + (1 - \eta) \frac{\dot{L}_{M,t}^*}{L_{M,t}^*}$, substituting Equation (49) for $\frac{\dot{L}_{M,t}^*}{L_{M,t}^*}$ into it we get the following equation:
\[
\frac{\dot{n}_t^*}{n_t^*} = \eta \frac{K_t^*}{L_t^*} + (1 - \eta) \left( \frac{\dot{n}_t^*}{n_t^*} - \eta \frac{\dot{k}_{M,t}^*}{k_{M,t}^*} \right)
\] (51)

Substituting Equation (50) for \( \frac{k_{M,t}^*}{k_{M,t}^*} \) into Equation (51) we have the following result as:

\[
\frac{\dot{n}_t^*}{n_t^*} = \frac{\dot{K}_t^*}{K_t^*}
\] (52)

Equation (52) shows that the growth rate of variety equals the growth rate of aggregate capital in the foreign country.

When substituting Equation (52) into Equation (50) we get \( \frac{k_{M,t}^*}{k_{M,t}^*} = 0 \), which implies capital per manufacturing worker stays constant in the foreign country. Substituting this result into Equation (49) gives \( \frac{L_{M,t}^*}{L_{M,t}^*} = \frac{n_t^*}{n_t^*} \). Equation (42) implies \( \frac{k_{M,t}^*}{k_{M,t}^*} = \frac{\sigma}{\nu} k_{M,t}^{\sigma - 1} - \frac{L_{M,t}^*}{L_{M,t}^*} \), plugging \( \frac{k_{M,t}^*}{k_{M,t}^*} = 0 \) and \( \frac{L_{M,t}^*}{L_{M,t}^*} = \frac{n_t^*}{n_t^*} \) into it, the growth rate of variety as follows:

\[
\frac{\dot{n}_t^*}{n_t^*} = \frac{\sigma}{\nu} k_{M,t}^{\sigma - 1} = \frac{\sigma}{\nu} \left( \frac{1 - \alpha}{\alpha} \frac{1}{\omega_t^*} \right)^{1-\eta}
\] (53)

where \( k_{M,t}^* = \frac{a}{1-\eta} \omega_t^* \).

Therefore, the growth rates of aggregate capital, manufacturing labour, variety, and manufacturing output are the same as:

\[
\frac{\dot{K}_t^*}{K_t^*} = \frac{\dot{L}_{M,t}^*}{L_{M,t}^*} = \frac{\dot{n}_t^*}{n_t^*} = \frac{\dot{X}_t^*}{X_t^*} = \frac{\sigma}{\nu} \left( \frac{1 - \alpha}{\alpha} \frac{1}{\omega_t^*} \right)^{1-\eta}
\] (54)

Equation (54) shows \( \frac{\dot{K}_t^*}{K_t^*} = \frac{\dot{L}_{M,t}^*}{L_{M,t}^*} \), this equality in growth rates leads to an unchanged wage level, that is \( \frac{w_t^*}{r_t^*} = 0 \) assuming all other things constant. It is worth noting that this result is consistent with the discussion in Appendix A: industrial expansion induces labourers to draw out of the agricultural sector. However, the average product of labour and hence the wage in the agricultural sector will not increase. Moreover, Equation (39) shows the growth rate of rental price as: \( \dot{r}_t^* = \frac{n_t^*}{n_t^*} - \frac{K_t^*}{K_t^*} \). As a result of equal growth of variety and aggregate capital we have \( \frac{\dot{n}_t^*}{n_t^*} = 0 \). Therefore, \( \frac{w_t^*}{r_t^*} = 0 \) and \( \frac{\dot{n}_t^*}{n_t^*} = 0 \) imply the growth rate of the wage and rental price ratio is also zero that \( \frac{\dot{w}_t^*}{\dot{r}_t^*} = 0 \), it means \( \omega_t^* \) is a constant over time.

Capital accumulation causes an industrial expansion which uses both more capital and labour in production, and the increase of labour is from surplus labour in the agricultural sector. Thus, in the foreign country, we observe structural change in terms of a shift of employment from the agricultural sector to the industrial sector.\(^{24}\) Because of the existence of surplus labour, capital accumulation increases labour participation but not the wage level.

\(^{24}\) Structural change can be induced by other factors such as total factor productivity (TFP) growth (Ngai & Pissarides, 2007), capital deepening (Acemoglu & Guerrieri, 2008) or higher income elasticity of manufactured goods as compared to the agricultural good (Kongsamut et al., 2001).
Because capital accumulation does not lead to a wage-rise, the effect of capital accumulation maps onto an equivalent expansion in the manufacturing sector, i.e. the growth rates of output and the number of varieties equal the growth rate of aggregate capital, as shown in Equation (54).

It is worth noting that the methods of our model to derive growth in these two countries are significantly different. In the home country, the growth rate of each variable is an inverse function of $u_1$, and except the growth rate of aggregate capital, all other growth rates also depend on $f_t$, where $f_t$ is a function of $z_t$, $A_t$, and $n_t$. Moreover, the growth rate of each variable in the foreign country is determined only by $u_1^*$, in which $z_t$, $A_t$, and $n_t$ are unable to affect the growth rate. The reason behind that is their different labour market structures. More precisely, with the existence of surplus labour in the foreign country, economic growth derives from $L_{M,t}^* = \frac{X_t}{k_{M,t}}$, induced by Equation (30) that the equalities between the average product of total agricultural labour, the marginal product of effective agricultural labour and the marginal product of manufacturing labour. However, it is inappropriate to apply the result of $L_{M,t}^* = \frac{X_t}{k_{M,t}}$ to the home country as it does not have dual economy. Our model derives growth in the home country from the equilibrium rental price and wage (see Equations (37) and (38)). This leads to the growth rate affected by $z_t$, $A_t$, and $n_t$.

Although the growth rate in the foreign country is not affected by $z_t$, $A_t$, and $n_t$, with an increasing wage in the home country and surplus labour-induced constant wage in the foreign country, $u_1^*$ becomes larger than $u_1^*$ over time, it is obvious that the growth rate of each variable would be relatively higher in the foreign country than in the home country over time. For example, the comparison of the growth rate of manufacturing labour between countries shows that

$$\frac{\dot{L}_{M,t} - \dot{L}_{M,t}^*}{\dot{L}_{M,t}^*} = \frac{\sigma}{\nu} \left( \frac{1 - \alpha}{\alpha} \right)^{1-\eta} \left[ \frac{1 - f_t}{1 + \frac{1 - \eta}{\alpha} f_t} \left( \frac{1}{u_t} \right)^{1-\eta} - \left( \frac{1}{u_t^*} \right)^{1-\eta} \right]$$

Equation (55) demonstrates that with the presence of $0 < \frac{1 - f_t}{1 + \frac{1 - \eta}{\alpha} f_t} < 1$ and $u_t^* < u_t$, we have $\frac{\dot{L}_{M,t}}{\dot{L}_{M,t}^*} < \frac{\dot{L}_{M,t}^*}{\dot{L}_{M,t}^*}$, which means labour migration from the agricultural sector to the industrial sector is faster in the foreign country than that in the home country.

## 4.3 The effects of surplus labour on economic growth

In our model, economic growth in both countries is driven by capital accumulation. Different labour market characteristics lead to different growth paths. Therefore, we arrive at the following propositions:

**Proposition**  When the home country has a neoclassical labour market and the foreign country has a Lewisian labour market,

a. Aggregate capital, manufacturing labour, variety and manufacturing output change at different growth rates in the home country but change at the same growth rate in the foreign country.

b. Structural change in terms of a shift of employment from the agricultural to the manufacturing sector occurs in both countries, and this transition process is faster in the foreign country.

c. Manufacturing production becomes more capital intensive in the home country, but it stays the same in the foreign country.

d. The wage level increases in the home country, but remains constant in the foreign country.
e. The growth rates of aggregate capital, variety and manufacturing output would become relatively higher in the foreign country than those in the home country over time.

Proof See Appendix B.

The above propositions can be illustrated by Figure 2. For each country, the isoquant curve and the isocost line determine the optimal production point at a given level of output and a given level of factor prices.

In the home country with a neoclassical labour market, the economy expands with capital deepening, the total amount of aggregate capital significantly increases from $K_1$ to $K_2$, while with smaller growth rate, the total quantity of manufacturing labour increases from $L_{M,1}$ to $L_{M,2}$. Higher growth rate of aggregate capital than that of manufacturing labour causes the wage to increase, which prevents the effect of capital accumulation from mapping onto an equivalent expansion of the size of manufacturing output. Therefore, manufacturing production becomes more capital intensive, and its production scale expands following the curve $OH$.

In the foreign country with a Lewisian labour market, the economy expands with capital accumulation and the immigration of surplus labour from the agricultural sector to the manufacturing sector. The total amount of aggregate capital increases from $K_{\ast,1}$ to $K_{\ast,2}$, and the total amount of manufacturing labour rises from $L_{M,1}^{\ast}$ to $L_{M,2}^{\ast}$. Equal growth rate of aggregate capital and manufacturing labour leads to an unchanged wage, which makes the effect of capital accumulation maps onto an equivalent expansion of manufacturing output. Thus, the manufacturing sector uses the same input proportion to produce, and its production scale increases following the straight line $OF$.

If we assume the developed country in our model is the US, and the developing country is China, broadly speaking, our results are inline with many empirical findings such as Autor et al. (2013, 2016) analyse the effect of rising Chinese import competition between 1990 and 2007 on US local labour markets, and found that rising imports cause higher unemployment, lower labour force participation, and reduced wages in local labour markets that house import competing manufacturing industries. Autor et al. (2016) show that China’s emergence as a great economic power has induced an epochal

![Figure 2](https://ssrn.com/abstract=3707505)

**FIGURE 2** Growth paths in the two countries
shift in patterns of world trade. They argue that, at the national level, employment has fallen in US industries more exposed to import competition, but offsetting employment gains in other industries have yet to materialize.

5 | CONCLUSIONS

Economic development can be understood as a structural transformation process, which involves the reallocation of labour and the share of economic activity from the agricultural sector to the manufacturing sector. For economies in the early stages of development, labour in the traditional agricultural sector is plentiful, frequently having a zero marginal productivity, while in the modern sector labour has a positive marginal product. The surplus labour in the traditional sector has an incentive to move to the modern sector at a subsistence wage in a given period without lowering output. This unlimited supply of labour from the traditional sector keeps wages in the modern sector low, ensures that capital accumulation in the modern sector is sustained over time, and thus leads to economic transformation. This process will continue until the surplus labour in the traditional sector is exhausted. Then wage starts to rise and the economy enters a neoclassical phase.

This paper has developed a growth model incorporating surplus labour and other dual labour market characteristics, where in the developing country, the labour market in the traditional agricultural sector is not neoclassical (where the wage is driven by marginal productivity), but classical Lewisian (where average productivity is the basis of the wage rate). The implication of this distinction is profound. For a developed country with a neoclassical labour market and a developing country with a Lewisian labour market, with capital accumulation, the wage level increases in the developed country, but keeps constant in the developing country. In this case, the developed country uses more capital rather than labour to expand its industrial production. Its industrial production becomes more capital intensive.

However, in the developing country, surplus labour in the agricultural sector is sufficiently large to give an unlimited supply of labour for industrial expansion, thus labour migration from the agricultural sector to the industrial sector induces more significant structural change in this country than in the developed country. The industrial sector uses both more capital and labour to increase its production. The developing country with surplus labour enjoys faster capital accumulation while capital per manufacturing worker stays constant. It enjoys faster growth of manufacturing output for a given growth rate of capital per manufacturing worker. These two effects of surplus labour are magnified when the developing country opens to trade with the developed country, as it allows a bigger market for the goods produced in the developing country, and thus allows the country to utilise more of its surplus labour than when it was closed.

In summary, this paper formally shows a Lewisian growth path that is distinct from the other neoclassical models. In this regime, a country’s labour surplus can be translated into faster growth rates of aggregate capital, manufacturing labour, and manufacturing output. The role that surplus labour plays in shaping different growth paths and structural changes in developed and developing countries can be used to better understand the development process of many developing countries, and to better understand the trade patterns between these countries and the developed economies.

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APPENDIX A

SCENARIOS OF CONSTANT WAGE

There are many explanations for the fact that the number of labourers drawn from the agricultural sector does not cause a reduction in the total amount of surplus labour \( L_s \) or a change in the wage rate in the agricultural sector. These can be expressed as follows: (a) This may be as a result of higher population growth that matches the speed of migration from the agricultural sector to the manufacturing sector. That is, if the rate of migration and the rate of population growth are the same, Malthusian population growth will replace all surplus labour that has moved to the manufacturing sector and the average product of labour in the agricultural sector \( \text{APL}_A \) will not change. (b) A second scenario involves technical change improving efficiency in the agricultural sector and this leads to increases in surplus labour. (c) Another scenario may be one where the agricultural sector is so big and the manufacturing sector so small that there is hardly any impact on the \( \text{APL}_A \) when small-scale migration takes place. (d) If land is not owned by farmers, when they move out of the agricultural sector, the \( \text{APL}_A \) is likely to go up, but the landlord may increase the land rent so that the \( \text{APL}_A \) stays the same. (e) The wage in the agricultural sector can also be defined as the social acceptable wage, which is not connected to the \( \text{APL}_A \), so the increase of the \( \text{APL}_A \) will not induce an increase in the agricultural sector’s wage.

If population growth is allowed, and we assume it grows at a constant and exogenous rate \( \gamma \), i.e. \( \frac{\dot{L}}{L} = \gamma \). In this case, when both sectors have the same population growth rate, i.e. \( \gamma_M = \gamma_A > 0 \), or there is no population growth in the manufacturing sector and all population growth are in the agricultural sector, i.e. \( \gamma_M = 0, \gamma_A > 0 \), it means that the expansion of manufacturing labour \( L_M \) is from net increase of surplus labour caused by population growth in the agricultural sector. Therefore, when the number of emigrants is equal to or less than net population growth (i.e. industrial employment expansion is equal to or less than net growth of workers), the expansion of manufacturing labour is satisfied by population growth, and the total amount of agricultural labour \( L_A \) does not change, consequently, the \( \text{APL}_A \) and hence the wage will stay constant.

If we assume there is a positive technological shock in the agricultural sector, \( T_A > 0 \), and this increase of total factor productivity (TFP) in this sector will release some effective agricultural labourers \( L_a \) from production, and create more surplus labour in the agricultural sector. Kwan, Wu, and Zhuo (2013) argue that relatively higher labour productivity implies potentially more surplus labour. When the rate of emigration is in line with the growth rate of TFP in the agricultural sector, the total amount of surplus labour will be constant in the agricultural sector. Thus, if the expansion of manufacturing labour is less than the reduction of effective agricultural labour or the increase of surplus labour, the wage will also be constant.
APPENDIX B

PROOFS FOR PROPOSITIONS (a)–(e)

Proof  Proposition (a) follows from Equations (43) and (45, 46 and 47) that in the home country the growth rates of variety and manufacturing output are a proportion $\frac{1}{1 + \frac{1 - f}{\pi f_0}}$ of the growth rate of aggregate capital, and the growth rate of manufacturing labour is a share $\frac{1 - f}{1 + \frac{1 - f}{\pi f_0}}$ of the growth rate of aggregate capital; from Equation (54) that in the foreign country with the existence of surplus labour, aggregate capital, manufacturing labour, variety and manufacturing output grow at the same rate. Proposition (b) comes from Equation (55), which shows the growth rate of manufacturing labour is larger in the foreign country than that in the home country over time. Proposition (c) and (d) follow from the fact that in the home country capital deepening leads capital to grow faster than manufacturing labour, this unequal growth would cause the wage level to increase, thus the manufacturing sector would use more capital rather than labour to expand its production. In the foreign country capital accumulation and surplus labour mobility induce an equal growth rate of capital and manufacturing labour, this equality leads to a constant wage level, hence industrial expansion would use both more capital and labour in production. Proposition (e) follows from Equations (43) and (54) that with an increasing wage in the home country and a constant wage in the foreign country, the growth rate of aggregate capital is relatively higher in the foreign country than that in the home country. Equations (45) and (53) show the growth rate of variety and Equations (46) and (54) show the growth rate of manufacturing output are relatively higher in the foreign country than that in the home country respectively.