Quantum correlations in optomechanical crystals

F. Bemani,1, * R. Roknizadeh,1, 2, † A. Motazedifard,1, ‡ M. H. Naderi,1, 2, ¶ and D. Vitali3, 4, 5, ¶

1Department of Physics, Faculty of Science, University of Isfahan, Hezar Jerib, 81746-73441, Isfahan, Iran
2Quantum Optics Group, Department of Physics, Faculty of Science, University of Isfahan, Hezar Jerib, 81746-73441, Isfahan, Iran
3Physics Division, School of Science and Technology, University of Camerino, I-62032 Camerino (MC), Italy
4INFN, Sezione di Perugia, via A. Pascoli, Perugia, Italy
5CNR-INO, L-go Enrico Fermi 6, I-50125 Firenze, Italy

(Dated: February 18, 2019)

The field of optomechanics provides us with several examples of quantum photon-phonon interface. In this paper, we theoretically investigate the generation and manipulation of quantum correlations in a microfabricated optomechanical array. We consider a system consisting of localized photonic and phononic modes interacting locally via radiation pressure at each lattice site with the possibility of hopping of photons and phonons between neighboring sites. We show that such an interaction can correlate various modes of a driven coupled optomechanical array with well-chosen system parameters. Moreover, in the linearized regime of Gaussian fluctuations, the quantum correlations not only survive in the presence of thermal noise, but may also be generated thermally. We find that these optomechanical arrays provide a suitable platform for quantum simulation of various many-body systems.

I. INTRODUCTION

The impressive experimental progress in fabricating micro- and nanomechanical devices has opened a route towards the exhibition of quantum behavior at macroscopic scales. The interaction between micro- or nanomechanical oscillators and the optical field via the radiation pressure is the basis of a wide variety of optomechanical phenomena. Despite their variety in the system sizes, parameters, and configurations, optomechanical systems (OMSs) share common features. Almost all OMSs are described by the same physics. OMSs offer further insights into the issues concerning the development of quantum memory for quantum computers [1], high precision position, mass or force sensing [2–6], quantum transducers [7], classical and quantum communication [8], ground state cooling of mechanical oscillators [9, 10], nonclassical correlations between single photons and phonons [11], generation of nonclassical states [12] and testing of the foundations of quantum mechanics [13–16]. For a recent review and current areas of focus of quantum optomechanics see Refs. [17, 18].

The extension to multimode systems is an attractive route for quantum optomechanics. A group of mechanical oscillators interacting via the radiation pressure with a common optical mode [19–26], or a group of mechanical oscillators locally interacting with a single optical mode involving the tunneling of photons and phonons between neighboring sites [27–38] are the two realizations of multimode optomechanics. The former is realized in a single optical cavity containing multiple membranes while the latter is realized experimentally in the so-called optomechanical crystals (OMCs) in one and two dimensions.

Cooperative behaviors, emerging due to the mutual coupling, are beneficial to investigate many-body physics of photons or phonons in OMCs. An OMC is usually fabricated from a thin film of silicon membranes where an engineered defect in the crystal is used to localize an optical and a mechanical mode. OMCs usually have a large single photon optomechanical coupling [39–42]. Several aspects of the array of coupled OMSs have already been investigated in the literature, involving synchronization dynamics [26–28, 43, 44], slowing and stopping light [32], long-range collective interactions [19], correlated quantum many-body states [33], reservoir engineering and dynamical phase transitions [25], squeezing, entanglement and state transfer between modes [34, 45], transport in a one-dimensional chain [35, 46, 47], superradiance and collective gain [48], graphene-like Dirac physics [36], creation of artificial magnetic fields for photons on a lattice [37], quantum simulation of the propagation of the collective excitations of the photon fluid in a curved spacetime [49], and topological phases of sound and light [38].

Quantum correlations, in particular entanglement, have many applications in superdense coding, quantum teleportation [50] and protocols of quantum cryptography [51]. The generation and manipulation of entanglement in many-body systems are of great importance for quantum information processing. Furthermore, quantum correlations are valuable in characterizing various phases and corresponding quantum phase transitions in quantum many-body systems [52–54]. Bipartite entanglement plays an important role in characterizing, classifying and simulating quantum many-body systems [55]. Physical systems such as Bose-Einstein condensates [56–58], cold or thermal atoms [59, 60], and trapped ions [61, 62] represent promising platforms for the investigation of many-particle quantum entanglement. In the past decade, much of the attention has been devoted to entanglement in OMSs. Entanglement is one of the consequences of the coherent photon-phonon interaction in OMSs [8, 63–68]. For instance, continuous variables entanglement between two mechanical modes.
has recently been realized [69, 70]. Since it is a possible resource for quantum technologies, quantum discord in manybody systems also requires attention.

Despite considerable efforts to understand the quantum correlations in OMSs [8, 63–70], a full picture of the behavior of entanglement and of quantum discord in OMCs remain elusive. Based on the above motivations, in this paper, we consider the dynamics of coupled OMSs with a view towards quantum correlations. Employing the Heisenberg-Langevin (HL) approach and linearizing HL equations, we separate the deterministic dynamics and the quantum fluctuation dynamics. We then use HL equations to obtain the covariance matrix (CM) in order to study quantum correlations. With the CM in hand, we can investigate the degree of steady-state entanglement and the Gaussian quantum discord between different optical and mechanical modes under different conditions. We study the influence of the presence of a thermal reservoir and we show a nonmonotonic behavior of quantum correlations as a function of the heat bath temperature.

The paper is organized as follows. In Sec. II, we begin with describing the system under consideration, i.e., an OMC. In Sec. III, we derive the HL equations of motion. We then discuss the classical equations of motion and the linearized quantum equations. In Sec. IV, we discuss the presence of entanglement and Gaussian discord in OMCs. Finally, in Sec. V, we present our concluding remarks.

II. ARRAY OF COUPLED OMSs

As depicted in Fig. 1, the system under consideration is a one-dimensional OMC where each site consists of a localized photonic and phononic mode coupled locally via the standard optomechanical interaction. The modes of nearby sites are connected via photon and phonon tunneling. The Hamiltonian of such a system is then given by ($\hbar = 1$) [28, 35, 38, 46]

$$H = H_0 + H_t + H_p,$$

where

$$H_0 = \sum_j \left[ \omega_0 a_j^\dagger a_j + \omega_m b_j^\dagger b_j - g_0 a_j^\dagger a_j (b_j^\dagger + b_j) \right],$$

$$H_t = -\sum_{\langle j,k \rangle} \left( J a_j^\dagger a_k + K b_j^\dagger b_k \right),$$

$$H_p = \sum_j \left( i\eta_j e^{-i\omega_j t} a_j^\dagger - i\eta_j^* e^{i\omega_j t} a_j \right).$$

Here, $H_0$ includes the free energy of each optical mode with frequency $\omega_0$, denoted by the photon operators $a_j$ and $a_j^\dagger$, the harmonic motion of each mechanical modes with frequency $\omega_m$, denoted by phonon operators $b_j$ and $b_j^\dagger$, and the usual optomechanical interaction with strength $g_0$. Further, $H_t$ represents the hopping of photons and phonons between adjacent lattice sites with hopping strengths $J$ and $K$, respectively. The notation $\sum_{\langle j,k \rangle}$ denotes the summation over all adjacent lattice sites. Finally, $H_p$ denotes that each lattice site is optically driven by a laser with frequency $\omega_L$ and amplitude $\eta_j$.

III. HEISENBERG-LANGEVIN EQUATIONS

The HL equations of motion for the optical and mechanical modes are, respectively, given by

$$\dot{a}_j = (i\Delta - \kappa) a_j + ig_0 (b_j^\dagger + b_j) a_j + iJ (a_{j-1} + a_{j+1})$$

$$+ \eta_j - \sqrt{\kappa} a_j n(t),$$

$$\dot{b}_j = - (i\omega_m + \gamma) b_j + i g_0 a_j^\dagger a_j + iK (b_{j-1} + b_{j+1}) - \sqrt{\gamma} b_j n(t),$$

where we have defined the laser detuning $\Delta = \omega_L - \omega_0$. Besides, $\kappa$ and $\gamma$ characterize, respectively, the dissipation of optical and mechanical modes. The zero-mean value operators $a_j n(t)$ and $b_j n(t)$ that describe, respectively, the vacuum optical input noise and the mechanical noise operator, satisfy the commutation relations

$$[a_j n(t), a_{j'} n(t')] = [b_j n(t), b_{j'} n(t')] = \delta_{jj'} \delta(t - t'),$$

and the Markovian correlation functions

$$\langle b_j n(t) b_{j'} n(t') \rangle = \bar{n}_m \delta_{jj'} \delta(t - t'),$$

$$\langle a_j n(t) a_{j'} n(t') \rangle = \delta_{jj'} \delta(t - t'),$$

where we have assumed that each cavity is at zero temperature and $\bar{n}_m = \exp(\hbar \omega_m / k_B T) - 1$ is the mean number of thermal phonons of each mechanical mode at temperature $T$, with $k_B$ being the Boltzmann constant.

A. Classical dynamics

We now employ the mean-field approximation to linearize the dynamics around the classical solutions by decomposing the quantum field operators as $a_j = a_j + e_j$ and $b_j = \beta_j + d_j$ where $a_j$ and $\beta_j$ are the steady-state mean fields describing,
FIG. 2. Stability domain as a function of the normalized input power \( \eta/J \) and normalized detuning \( \Delta/J \). The white and blue areas correspond to the unstable and stable correlated regimes, respectively.

respectively, the classical behavior of the optical and mechanical modes, and \( c_j \) and \( d_j \) are the quantum fluctuations with zero-mean value. For the aim of this paper, it is enough to consider only the translational symmetry \( \alpha_j = \alpha_{j+1} \) and \( \beta_j = \beta_{j+1} \), which is obtained with an approximately uniform optical driving \( \eta_j \approx \eta \) which therefore excites a background with a small wave vector \( k \approx 0 \). Using this assumption, the system dynamics is then simplified to the single-site case. The equations of motion for the steady-state classical mean fields can be obtained by averaging Eqs. (3) and (4) over classical and quantum fluctuations

\[
\alpha_j \simeq \frac{i \eta}{(\Delta + i \kappa + 2 J J + 2 g_0 \Re \beta_j)} , \tag{8}
\]

\[
\beta_j \simeq \frac{g_0 \Re \alpha_j^2}{(\omega_m - i \gamma - 2 K)} , \tag{9}
\]

where \( \Re \) denotes the real part.

B. Linearized quantum dynamics

We study the quantum statistical properties of the system through the small fluctuations of the operators around the steady-state classical mean values given by Eqs. (8) and (9). Using the standard definition of the optical and mechanical mode quadratures \( X_j = (c_j + c_j^d)/\sqrt{2} \), \( Y_j = (c_j - c_j^d)/i \sqrt{2} \), \( x_j = (d_j + d_j^d)/\sqrt{2} \) and \( y_j = (d_j - d_j^d)/i \sqrt{2} \), the equations of motion for the quantum fluctuations are given by

\[
\dot{X}_j = - (\Delta + 2 g_0 \Re \beta_j) Y_j - \kappa X_j - 2 g_0 \Re \alpha_j x_j - J (Y_{j-1} + Y_{j+1}) - \sqrt{\kappa} X_j^{\text{in}} (t) , \tag{10}
\]

\[
\dot{Y}_j = (\Delta + 2 g_0 \Re \beta_j) X_j - \kappa Y_j + 2 g_0 \Re \alpha_j x_j + J (X_{j-1} + X_{j+1}) - \sqrt{\kappa} Y_j^{\text{in}} , \tag{11}
\]

\[
\dot{x}_j = - \gamma x_j + \omega_m y_j - K (y_{j-1} + y_{j+1}) - \sqrt{\gamma} x_j^{\text{in}}, \tag{12}
\]

\[
\dot{y}_j = - \omega_m x_j - \gamma y_j + 2 g_0 \Re \alpha_j x_j + 3 \alpha_j Y_j + K (x_{j-1} + x_{j+1}) - \sqrt{\gamma} y_j^{\text{in}}, \tag{13}
\]

where \( \Im \) denotes the imaginary part. We now express the linearized HL equations in the following compact form

\[
\dot{u}(t) = A u(t) + n(t) , \tag{14}
\]

where we have defined the vector of fluctuation operators

\[
u = [\cdots u_{j-1}, u_j, u_{j+1}, \cdots]^T \]

with \( n_j = \left[ \cdots n_{j-1}, n_j, n_{j+1}, \cdots \right]^T \) and normalized detuning \( \Delta_J/\beta \). The OMCs is a continuous variable 2N-partite Gaussian state, which is completely determined by its 2N × 2N CM. The formal solution of Eq. (14) is

\[
\mathbf{u}(t) = \mathbf{M}(t) \mathbf{u}(0) + \int_0^t \mathbf{M}(t-s) \mathbf{n}(s) ds , \tag{18}
\]

with \( \mathbf{M}(t) = \exp [i A t] \). The CM defined as

\[
\mathbf{V}_{pq}(t) = \frac{1}{2} \langle u_p(t) u_q(t) + u_q(t) u_p(t) \rangle , \tag{19}
\]

contains all information about the quantum correlation between various mechanical and optical modes where \( u_p(t) \) is the \( p \)th component of the vector \( u(t) \).

The system reaches its steady state when \( M(\infty) = 0 \). Our analysis is restricted to the stable regime where all the eigenvalues of the drift matrix have negative real parts.
we plot the region of stability as a function of the normalized laser pump intensity and detuning. For large laser drive, the system enters the unstable region. In the steady state, one gets the CM elements as

\[
V_{ij} = \sum_{kJ_0}^m ds \int ds' M_{ik}(s)M_{jk}(s') \Phi_{iJ}(s-s'),
\]  

(20)

where

\[
\Phi_{iJ}(s) = \frac{1}{2} \left( n_k(s)n_j(s') + n_j(s')n_k(s) \right) - D_{iJ} \delta(s-s'),
\]  

(21)

where \( D = \text{diag}\{\cdots d, d, d, \cdots\}^T \) with \( d = \text{diag}\{\kappa, \kappa, \gamma(2\eta_0 + 1), \gamma(2\eta_0 + 1)\} \). When the stability conditions are satisfied so that \( \dot{M}(\infty) = 0 \), the steady-state CM, \( V \), can be obtained by solving the linearized HL equation (14) for the quantum fluctuations, which fulfill the following Lyapunov equation

\[
AV + VA^T = -D.
\]  

(22)

With these classical and quantum steady-state solutions in hand, we next employ the CM formalism to calculate the steady-state quantum correlations. We check the presence of the quantum correlations between the mechanical and optical modes on the same site, as well as between the mechanical or optical modes with different site indices. Considering the following reduced CM of the two modes

\[
V_R = \begin{bmatrix} V_A & V_C \\ V_C^T & V_B \end{bmatrix},
\]  

(23)

one can calculate the quantum correlations. Here, \( V_A, V_B \) and \( V_C \) are \( 2 \times 2 \) matrices where \( V_A \) and \( V_B \) account for the local properties of modes \( A \) and \( B \), respectively, while \( V_C \) describes intermode correlations. \( A \) and \( B \) may stand for two different modes.

### A. Steady-state entanglement

We quantify the degree of entanglement in terms of the logarithmic negativity, which is an entanglement monotone, and it is given by \( E_N = \text{max}\{0, -\ln 2 \tilde{\nu}_-\} \) with \( \tilde{\nu}_- = 2^{-1/2} \left( \Sigma_- - \sqrt{\Sigma_-^2 - 4\det V_R} \right)^{1/2} \) being the smallest of the two symplectic eigenvalues of the partially transposed transposed CM and \( \Sigma_\pm = \det V_A + \det V_B \pm 2\det V_C \).

#### 1. Photon-phonon entanglement

The degree of entanglement between optical and mechanical modes in terms of the logarithmic negativity for various values of the laser detuning. We set normalized parameters with respect to \( J, \kappa/J = 0.1, \eta/J = 15, g_0/J = 10^{-4}, \gamma/J = 0.002, \omega_m/J = 0.1 \) and \( K/J = 0.05 \). Temperatures of the photonic and phononic heat baths are considered to be zero.

### FIG. 3. The degree of entanglement between optical and mechanical modes in terms of the logarithmic negativity for various values of the laser detuning: (a) \( \Delta/J = -2.5 \), (b) \( \Delta/J = -2.1 \), (c) \( \Delta/J = -1.7 \) and (d) \( \Delta/J = -1.3 \) for 101 coupled OMSs. (e) The logarithmic negativity between the two optical and mechanical modes with the same site index \( j = -50 \) or \( j = 50 \) (blue solid line) and \( j = 0 \) (red dashed line) versus the laser detuning. We set normalized parameters with respect to \( J, \kappa/J = 0.1, \eta/J = 15, g_0/J = 10^{-4}, \gamma/J = 0.002, \omega_m/J = 0.1 \) and \( K/J = 0.05 \). Temperatures of the photonic and phononic heat baths are considered to be zero.
FIG. 4. The degree of entanglement between optical and mechanical modes in terms of the logarithmic negativity for various values of the laser intensity: (a) $\eta/J = 50$, (b) $\eta/J = 150$, (c) $\eta/J = 250$ and (d) $\eta/J = 350$ for 101 coupled OMSs. We set $\Delta/J = 1.5$, other parameters are the same as in Fig. 3. Panels (e) and (f) show the logarithmic negativity between the two optical and mechanical modes with the same site index $j = -50$ or $j = 50$ (blue solid line) and $j = 0$ (red dashed line) versus the laser-drive intensity for two values of the laser detuning: (e) $\Delta/J = -1.5$ and (f) $\Delta/J = 1.5$.

The obtained results show that the entanglement varies as a function of the laser pump intensity for a fixed laser detuning, $\Delta/J = 1.5$. By increasing the laser intensity the entanglement first tends to increase and then to decrease as we approach the unstable region. Therefore, there is a non-monotonic behavior of on-site entanglement. We show this fact in Figs 4(e) and 4(f) where we have plotted the logarithmic negativity between the two optical and mechanical modes with the same site index at the lattice edge ($j = -50$ or 50) and at the lattice center ($j = 0$) versus the laser-drive intensity for two values of the laser detuning.

Finally, we have also studied the eventual presence of photon-photon or phonon-phonon entanglement between different sites. We have verified that for all choices of the parameters this kind of inter-site entanglement is always zero.

2. Thermal effects on the generated entanglement

Usually, quantum correlations and entanglement in particular are fragile with respect to thermal noise. Therefore, the investigation of the effect of thermal fluctuations on the bipartite quantum correlations in OMCs is of particular relevance for applications.

In Fig. 5, we show how the on-site photon-phonon entanglement changes with increasing thermal phonon number $\bar{n}_m$. Evidently, the on-site photon-phonon entanglement decays for increasing temperatures and it persists at ultra-cryogenic temperatures achievable in dilution refrigerators (for example $\bar{n}_m \approx 0.06$ for mechanical resonance frequencies $\omega_m/2\pi = 9$ GHz at a temperature of $T = 0.15K$).

B. Steady-state Gaussian quantum discord

It is also interesting to examine if quantum discord $[71, 72]$, a measure of the quantumness of correlations, is present in the steady state of the system. The Gaussian quantum discord is an asymmetric quantity and the Gaussian quantum A-discord of the Gaussian state of two modes, $A$ and $B$, is given by $[73, 74]$

$$D^+ = f\left(\sqrt{\beta}\right) - f\left(\nu_+\right) - f\left(\nu_-\right) - f\left(\sqrt{\epsilon}\right)$$

where

$$f(x) = \left(\frac{x+1}{2}\right) \log_{10} \left(\frac{x+1}{2}\right) - \left(\frac{x-1}{2}\right) \log_{10} \left(\frac{x-1}{2}\right)$$

$$\nu_{\pm} = \sqrt{\frac{\Sigma_{\pm} - 4 \det V_R}{2}}$$

are the two symplectic eigenvalues of the two-mode CM and

$$\epsilon = \begin{cases} \frac{2\gamma^2 + (\beta - 1)(\delta - \alpha) + 2\gamma\sqrt{\gamma^2 + (\beta - 1)(\delta - \alpha)}}{(\beta - 1)^2}, & (\delta - \alpha)^2 \leq 1; \\ \frac{\alpha \beta - \gamma^2 + \delta - \sqrt{\gamma^2 + (\delta - \alpha \beta)^2} - 2\gamma^2(\delta + \alpha \beta)}{2\beta}, & \text{otherwise}, \end{cases}$$

are the two symplectic eigenvalues of the two-mode CM and $\epsilon$ is the Gaussian quantum discord.
where $\alpha = \det V_A$, $\beta = \det V_B$, $\gamma = \det V_C$ and $\delta = \det V_D$ are the symplectic invariants. One can obtain the Gaussian quantum discord $D_{\varphi}$ by swapping the roles of the two modes, $A$ and $B$, which is equivalent to swap $\alpha$ and $\beta$ in the above formulas. Since we are interested in quantum correlations in general between the different modes in the one-dimensional array, from now on we will consider the symmetrized quantum discord, $D_{\varphi} = \max \{ D^{\leftrightarrow}, D^{\rightarrow\rightarrow}, D^{\leftarrow\leftarrow}, D^{\leftarrow\rightarrow\rightarrow} \}$. 

1. Photon-phonon steady-state Gaussian quantum discord

Fig. 6 shows the behavior of the symmetrized quantum discord $D_{\varphi}$ for various laser detuning values at zero temperature of both photonic and phononic modes. Similarly to what occurred for entanglement, changing the laser detuning has a significant effect on the photon-phonon Gaussian quantum discord, and again we have a similar behavior with that of entanglement with the above choice of parameters, with the presence of larger on-site discord between the mechanical and the optical mode and which extends for few sites. One starts to see a different behavior between Gaussian discord and entanglement when looking at the dependence upon the driving power and specifically if we consider increasing values of the laser drive $\eta$. In Fig. 7, we show how steady-state photon-phonon Gaussian quantum discord varies with the laser intensity for a fixed laser detuning, $\Delta/J = 1.5$. In contrast with the behavior of entanglement, we have that by increasing the laser intensity one has a significant increase of Gaussian quantum discord between optical and mechanical sites (see Figs. 7(e) and 7(f)). Moreover, at larger values one can see a long-range correlation between optical and mechanical modes appearing (see Fig. 7(d)).

2. Photon-photon and phonon-phonon steady-state Gaussian quantum discord

The appearance of long-range quantum correlations occurs also when considering either only optical modes or only me-
Mechanical modes, at each site of the OMC, in clear contrast with the case of entanglement which is instead completely absent, even between neighboring sites. This fact is shown in Fig. 8. As can be seen, for a fixed laser detuning, by increasing the laser intensity the steady-state Gaussian quantum discord between modes of the same nature increases.

3. Thermal effects on the steady state Gaussian quantum discord

It is relevant to study the robustness of the Gaussian quantum discord with respect to temperature as we did it already for entanglement. The steady-state Gaussian quantum discord under different heat-bath phonon number for normalized laser detuning $\Delta/J = 1.5$ and laser intensity $\eta/J = 500$ is depicted in Fig. 9. One can see a non-monotonic behavior in Gaussian quantum discord by increasing the thermal phonon number. It first tends to increase, then decreases and finally increases again. This behavior is somehow unexpected and it can be regarded as the evidence of thermally induced Gaussian quantum discord in OMCs. This is not completely novel however in quantum many-body systems; for instance, the transverse-field $XY$ model shows non-monotonic behavior of its quantum correlations (for instance see [75] and references therein). We remark however that our model is not exactly the same as $XY$ model for what concerns the effects of the thermal environment because in the latter the involved excitations has similar frequencies and therefore similar thermal effects, while in our case, due to the large difference in frequencies between optical and mechanical modes, only the phonon modes are appreciably affected by a nonzero reservoir temperature.

V. CONCLUSIONS

In conclusion, our investigation clearly demonstrates the presence of appreciable quantum correlations in an OMC where each site consists of two localized, optical and mechanical, modes coupled locally via the optomechanical interaction. The modes of nearby sites are connected via both on-site or short-range entanglement between optical and mechanical modes coupled locally via the optomechanical interaction. The modes of nearby sites are connected via both on-site or short-range entanglement between optical and mechanical modes, at each site of the OMC, in clear contrast with the case of entanglement which is instead completely absent, even between neighboring sites. This fact is shown in Fig. 8. As can be seen, for a fixed laser detuning, by increasing the laser intensity the steady-state Gaussian quantum discord between modes of the same nature increases.

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generated entanglement is very fragile with respect to thermal noise. We have also shown that there is no long-range entanglement between optical and mechanical modes. Moreover, there is no photon-photon or phonon-phonon entanglement in the system. We have then examined a weaker form of quantum correlation, i.e., Gaussian quantum discord, and we have studied if quantum discord is present in the steady-state of the system for various control parameters. The Gaussian quantum discord behavior is completely different, one has long-range features in all the three possible cases of correlations, i.e., photon-phonon, photon-photon, and phonon-phonon, at variance with what occurs with entanglement. A further interesting aspect is the thermal activation of quantum discord, i.e., the fact that photon-phonon discord increases with increasing temperature.

The present study which paves the way toward the investigation of many-body entanglement, can be considered as the first step toward controlled quantum correlations between different quantum processors across the lattice sites with potential applications in quantum information possessing and storage. The proposed scheme also provides a suitable platform for quantum simulation of various many-body systems with optomechanical crystals by tuning the system parameters. It should be noted that we did not consider the disorder effect in our study. As an outlook, the present scheme can be generalized to a more realistic case where the lattice disorder may be also present in the system. Another outlook may be the generalization to the two-dimensional lattices of coupled optomechanical systems.

ACKNOWLEDGMENTS

We would like to thank the Vice President for Research of the University of Isfahan for its support. DV acknowledges the support of the European Union Horizon 2020 Programme for Research and Innovation through the Project No. 732894 (FET Proactive HOT).

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