Fatigue Life of the 316L Stainless Steel Based on Dual-parametric Weibull Distribution

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Abstract. In order to investigate the fatigue strength of the 316L stainless steel (316L-SS) diffusion welded joint, it is necessary to study the fracture time of the 316L-SS. The fracture time test of the 316L-SS has many problems, such as high cost, long time consumption and little data acquisition. Regarding the issue above, the Weibull distribution method is adopted to process the test data of small sample size in this paper. At the same time, the P-S-N curve of the 316L-SS is established to obtain the relationship between maximum stress and fatigue life under different reliability.

1. Introduction
Due to the low efficiency and precision of the traditional fusion welding, it is difficult to meet the micro-mechanical connection requirements such as micro-heat exchangers and micro-reactors [1-4]. With the development of the welding technology and the application of special materials, diffusion welding technology has been widely used in aerospace, nuclear energy, stations, petrochemical, electric power, transportation, mechanical manufacturing and other fields, becoming one of the most commonly used material connection technologies [5-8]. As a type of austenitic stainless steel, 316L-SS has many unique advantages, such as excellent mechanical properties, corrosion resistance and high temperature oxidation resistance. Therefore, with operating temperature between 200°C and 650°C, the 316L-SS is often chosen as the base metal in diffusion welding [9]. For the recent decades, the research on the diffusion welding methods has been relatively mature, whereas, there are few literature about the fatigue strength of the diffusion welded joint based on the 316L-SS as the base metal [10, 11]. Therefore, it is necessary to study the rupture time of the 316L-SS.

However, the high cost of the fracture time test and the long experimental time make it difficult to obtain a large amount of the test data analysis [12]. Therefore, for the limited experimental data processing analysis, the correct model and its accurate parameter evaluation are the key to predict the 316L-SS fracture time [13-15]. The Weibull distribution model has great advantages and can well describe the problem to be solved.

The Weibull distribution is one of the most suitable statistical distribution models for describing the mechanical life distribution, which has a strong adaptability to all kinds of test data, especially for small sample sampling tests [16-18]. The Weibull distribution has good data fitting ability and convenient expression processing. It can effectively fit the data of the whole product life stage, and can also be used
to simulate the change of failure efficiency. In automobile and other engineering fields, the Weibull distribution is used to analyze the life of the failure parts such as hubs, bearings, gearboxes, etc., so as to accurately predict automobile faults and improve the reliability of automobile products. The Weibull distribution mainly has the three-parameter Weibull distribution (γ is the positional parameter, η is the size parameter, β is the shape parameter) and the dual-parameter Weibull distribution (when γ = 0, the model is degraded) two forms [19, 20]. The three-parameter Weibull distribution is cumbersome to calculate and suitable for more data. In general, by assuming gamma = 0, the dual-parameter Weibull distribution is often adopted in the engineering. The actual meaning is that the specimen may fail at any time from the beginning of the test. The dual-parameter Weibull distribution will be considered in this study.

2. Experiments

The material of the sample used in the experiment is 316L-SS, and the chemical composition of 316L-SS is shown in table 1.

| Element | wt % |
|---------|------|
| C       | 0.01 |
| Fe      | 67.10|
| Ni      | 12.43|
| Si      | 0.41 |
| Mo      | 2.16 |
| S       | 0.01 |
| Cr      | 17.84|
| P       | 0.04 |

Since the 316L-SS is easy to work harden, the surface condition of 316L-SS weld is determined to have a roughness of 0.8μm and a flatness of 0.02cm. In order to avoid errors caused by processing and bonding batches, two 316L-SS cylindrically specimens with a gauge length of Φ80mm×50mm are used to make butt diffusion joints. According to the national standards GB/T 228-2002 “metal material - ambient temperature tensile test” and GB/T 4338-2002 “metal material - high temperature tensile test”, the fracture test is carried out on the Shimadzu material testing machine at about 550°C, and the temperature was controlled in high temperature furnace. Five parallel experiments were performed in each group at three groups stress levels (S₁ = 160, S₂ = 140, S₃ = 120), and the fracture time of the joint in each experiment was recorded.

![SEM photos of the rupture surface of the high-temperature fracture test of 316L joint: (a) the rupture surface of the joint after high temperature tensile, (b) the rupture surface of the joint after testing at 550°C (t = 266.3 hours).](image-url)
3. Theoretical model

The Weibull distribution is being used to mitigate the extreme values of the fracture strength and the failure time. Two forms of the method are dual-parameter and three-parameter Weibull distributions. In this case, the distribution function and the probability density function (PDF) can be written as follows, respectively:

\[ F(x) = 1 - \exp \left[ -\left( \frac{x}{\eta} \right)^\beta \right], \quad \gamma \geq 0, \quad \eta \geq 0, \quad \beta \geq 0 \]  

(1)

\[ f(x) = \frac{\beta}{\eta} \left( \frac{x-\gamma}{\eta} \right)^{\beta-1} \exp \left[ -\left( \frac{x-\gamma}{\eta} \right)^\beta \right], \quad \gamma \geq 0, \quad \eta \geq 0, \quad \beta \geq 0 \]  

(2)

Where \( \gamma \) is the positional parameter, when \( \gamma = 0 \), the model is degraded to the dual-parameter Weibull distribution; \( \eta \) is the size parameter and \( \beta \) is the shape parameter.

As a rule, the three-parameter Weibull distribution is suitable for more data. In this work, the dual-parameter Weibull distribution will be considered. In the circumstances, the distribution function can be expressed as:

\[ F(x) = 1 - \exp \left[ -\left( \frac{x}{\eta} \right)^\beta \right], \quad \eta \geq 0, \quad \beta \geq 0 \]  

(3)

In the context of this work, \( F(x) \) represents the probability that the fatigue strength is equal to or less than \( x \). Using the equation \( F(x) + R(x) = 1 \), the reliability \( R(x) \), that is, the probability of the fatigue strength is defined as:

\[ R(x) = \exp \left[ -\left( \frac{x}{\eta} \right)^\beta \right], \quad \eta \geq 0, \quad \beta \geq 0 \]  

(4)

Under the certain stress level, if the fatigue life \( N \) of the specimen follows the Weibull distribution, then the probability density function is:

\[ f(N) = \frac{b}{N_a-N_0} \left( \frac{N - N_0}{N_a-N_0} \right)^{b-1} \exp \left[ -\left( \frac{N - N_0}{N_a-N_0} \right)^b \right] \]  

(5)

Where \( N_0 \) is the minimum life parameter; \( N_a \) is the characteristic life parameter; \( b \) is the shape parameter. The unreliability function is:

\[ F(N) = P(n \leq N) = \int_{N_0}^{N} f(N) dN \]  

(6)

The distribution function of fatigue life is obtained by substituting equation (5) into equation (6) and integrating it:

\[ F(N) = 1 - \exp \left[ -\left( \frac{N - N_0}{N_a-N_0} \right)^b \right] \]  

(7)

The expression of the fatigue life reliability function that conforms to the three-parameter Weibull distribution is:

\[ R(N) = \exp \left[ -\left( \frac{N - N_0}{N_a-N_0} \right)^b \right] \]  

(8)

The mathematical expectation and variance of the Weibull distribution variables are:
\[ E(N) = N_0 + (N_a - N_0) \left[ 1 + \frac{1}{b} \right] \]  
\[ \text{var}(N) = (N_a - N_0)^2 \left[ \Gamma \left( 1 + \frac{2}{b} \right) - \Gamma^2 \left( 1 + \frac{1}{b} \right) \right] \]  

Let the probability of fatigue life \( n \leq N \) be:
\[ P(n \leq N) = p \]  

Where \( p \) is the reliability, and the reliability corresponding to the fatigue life of \( p \) can be expressed as \( N_p \).

When the \( p \) equals to 0.5 and \( N_p \) equals to \( N_{50} \), \( N_{50} \) corresponds to the median value of the fatigue life samples, substituting \( N_p = N_{50} \) into (7):

\[ 0.5 = 1 - \exp \left[ - \left( \frac{N_{50} - N_0}{N_a - N_0} \right)^b \right] \]  

Take the logarithm for both sides of formula (12):
\[ \ln 0.5 = \left( \frac{N_{50} - N_0}{N_a - N_0} \right)^b \]  

When the fatigue life is affected by the three-parameter Weibull distribution, the reliability function expression of the fatigue specimen is shown in equation (8), and the natural logarithm is taken for both sides of (8).
\[ \ln R(N) = \left( \frac{N - N_0}{N_a - N_0} \right)^b \]  

If the reliability \( R \) is given, the reliable lifetime \( N_R \) is expressed as:
\[ N_R = N_0 + (\ln R)^b (N_a - N_0) \]  

Let the fatigue life \( N \) follow the three-parameter Weibull distribution at any stress level \( S \). In reality, each unknown parameter estimated in the probability density function is related to the stress level \( S \). At different stress levels, the distribution parameters of the fatigue life are different, and the three parameters of the Weibull distribution can be expressed as a function of the stress level \( S \). Equation (15) is rewritten as:
\[ N_R(s) = N_0(s) + (\ln R(s))^b (N_a(s) - N_0(s)) \]  

For \( S_{0R}, aR \) and \( C_R \), the reliability is \( R \), corresponding to the S-N curve equation, as shown in equation (17):
\[ (S - S_{0R})^{\mu_S} N_R = C_R \]  

Considering that the fatigue life is a function of stress level, it can be obtained from equation (17):
\[ N_R(s) = \frac{C_R}{(S - S_{0R})^{\mu_S}} \]  

Substitute formula (18) into formula (16):
Consequently, on the premise of the given stress level and reliability, the relationship between the three parameters of the Weibull distribution and the three parameters of the curve equation is established by equation (19).

In addition, three stress levels were selected for the fatigue test. Statistical analysis was performed on the results of the three groups of fatigue life tests. The distribution parameters $N_a(S_i)$, $N_0(S_i)$, and $b(S_i)$ ($i=1,2,3$) of the three-parameter Weibull distribution were obtained. Substitute equation (16) into an equation system composed of three equations:

$$
\begin{align*}
N_{R,S_1} &= (N_a(S_1) - N_0(S_1)) - \ln R)^{b(S_1)} + N_0(S_1) \\
N_{R,S_2} &= (N_a(S_2) - N_0(S_2)) - \ln R)^{b(S_2)} + N_0(S_2) \\
N_{R,S_3} &= (N_a(S_3) - N_0(S_3)) - \ln R)^{b(S_3)} + N_0(S_3)
\end{align*}
$$

By formula (17):

$$
\begin{align*}
\ln C_{R,S_1} &= \ln N_{R,S_1} + a_R \ln (S_1 - S_{0R}) \\
\ln C_{R,S_2} &= \ln N_{R,S_2} + a_R \ln (S_2 - S_{0R}) \\
\ln C_{R,S_3} &= \ln N_{R,S_3} + a_R \ln (S_3 - S_{0R})
\end{align*}
$$

Separately subtract two by using the formula in (21):

$$
\begin{align*}
\ln (S_1 - S_{0R}) - \ln (S_2 - S_{0R}) = \ln N_{R,S_2} - \ln N_{R,S_1} \\
\ln (S_2 - S_{0R}) - \ln (S_3 - S_{0R}) = \ln N_{R,S_3} - \ln N_{R,S_2}
\end{align*}
$$

Divide the two formulas in equation (22):

$$
\frac{\ln (S_1 - S_{0R}) - \ln (S_2 - S_{0R})}{\ln (S_2 - S_{0R}) - \ln (S_3 - S_{0R})} = \frac{\ln N_{R,S_2} - \ln N_{R,S_1}}{\ln N_{R,S_3} - \ln N_{R,S_2}}
$$

Let

$$
\frac{\ln N_{R,S_2} - \ln N_{R,S_1}}{\ln N_{R,S_1} - \ln N_{R,S_2}} = m
$$

Replace equation (24) with equation (25):

$$
\ln \left( \frac{S_1 - S_{0R}}{S_2 - S_{0R}} \right) = \ln \left( \frac{S_2 - S_{0R}}{S_3 - S_{0R}} \right)^m
$$

Solving the nonlinear equation of equation (26), and the estimated value $S_{0R}$ of $S_{0R}$ is obtained. According to any formula in equation (27), the estimated value $a_R$ of $a_R$ is estimated.

$$
a_R = \frac{\ln N_{R,S_1} - \ln N_{R,S_2}}{\ln (S_1 - S_{0R}) - \ln (S_2 - S_{0R})}
$$

The value of $C_{R'}$ and $C_R$ can be estimated from the equation (21). On the basis of equation (17), the P-S-N curve equation of the material can be obtained.

4. Results and discussion

Table 2 shows the rupture time under three different stresses. The Weibull probability plot of the experimental results is shown in Figure 1.
Table 2. The results of the rupture testing for 316L-SS diffusion bonding joint.

| No. | \( S_1 = 160 \text{MPa} \) | \( S_2 = 140 \text{MPa} \) | \( S_3 = 120 \text{MPa} \) |
|-----|----------------------------|----------------------------|----------------------------|
| 1   | 141                        | 236                        | 402                        |
| 2   | 164                        | 243.5                      | 423                        |
| 3   | 175                        | 255                        | 436                        |
| 4   | 192                        | 261                        | 442                        |
| 5   | 209                        | 273                        | 450                        |

Figure 2. The Weibull probability plots of the three groups of experimental results.

The Weibull probability plots of the three groups of data are plotted respectively. It can be seen from the figure 2 that the fracture time in each group is linearly distributed under three different stresses. Therefore, the relevant parameters can be obtained by using the formula described above.

As shown in table 3, based on the formula to establish a nonlinear equation group, the distribution parameters of the three groups of data are found.

Table 3. Probability distribution parameters for each group of data.

| Statistical feature value     | \( S_1 = 160 \text{MPa} \) | \( S_2 = 140 \text{MPa} \) | \( S_3 = 120 \text{MPa} \) |
|------------------------------|----------------------------|----------------------------|----------------------------|
| Mean                         | 176.2                      | 253.7                      | 430.6                      |
| Median                       | 175                        | 255                        | 436                        |
| Standard deviation           | 26.052                     | 14.533                     | 18.783                     |
| Shape parameters             | 2.695                      | 7.287                      | 3.683                      |
| Characteristic life          | 184.311                    | 259.699                    | 463.461                    |
| Minimum life                 | 111.088                    | 163.911                    | 394.546                    |

Taking the reliability \( R \) equal to 0.5, the P-S-N curve equation can be obtained as:

\[
(N - 92.11)^{0.0788} = 16565
\]  

(28)

The P-S-N curve is shown in Figure 3.
Moreover, taking the reliability of R equal to 0.1, 0.3, 0.8, 0.95 respectively, the corresponding P-S-N curve equation can be obtained:

\[(S - 111.46)^{0.4746} N = 1331.2 \] (29)

\[(S - 102.39)^{0.7607} N = 4138.6 \] (30)

\[(S - 64.495)^{1.9475} N = 1.099 \times 10^8 \] (31)

\[(S - 31.039)^{3.0823} N = 4.33 \times 10^8 \] (32)

The P-S-N curves are shown in Figure 4.

5. Conclusion
In order to investigate the fatigue strength of the 316L-SS diffusion welded joint, in this work, the dual-parameter Weibull distribution is used to analyze the distribution of the fracture time under the small sample experimental data. The fracture time of the samples at different stress levels is independent and identical.
First, the reliability $R$ equals to 0.5. Under different stress levels, multiple tests are carried out to obtain the fracture time of the sample, and then the S-N curve equation is obtained through regression analysis. In addition, take the reliability $R$ equal to 0.1, 0.3, 0.8, 0.95, respectively. By repeating the above method, the P-S-N curve can be established, that is, each curve corresponds to a group of S-N curves under different survival probabilities. Through the P-S-N curves, the relationship between the maximum stress and fatigue life under different reliability can be obtained.

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