Vulnerability Analysis of Urban Road Networks Based on Adaptive Signal Control

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Abstract. In the real environment, urban road networks (URNs) are often threatened by internal and external disruptions. Given that URNs tend to suffer from the capacity degradation caused by these disruptions, how to minimize losses is the key focus of traffic research. In this study, we propose a day-to-day dynamic evolution model for traffic assignment to analyze the vulnerability of urban road networks under different signal control strategies. The analysis of the vulnerability of a small road network shows that different adaptive signal control strategies can affect the vulnerability of urban road networks.

Keyword. Urban road networks, Vulnerability, Signal control, Traffic assignment.

1. Introduction
Urban road networks play an important role in maintaining people's normal life, political and economic stabilities, and emergency rescue and recovery after disasters [1]. According to relevant studies [2], the frequency of global disasters continues to increase. Vulnerability analysis of urban road traffic is a system-wide approach that can be used to identify key links of the road networks, reduce its losses from when suffering attacks, and evaluate the networks performances.

There is no widely recognized measure of the vulnerability of urban road networks. Some studies take robustness as the ability to cope with the losses of connectivity caused by start-to-end, and measure the vulnerability of urban road networks [3]. Others focus on the reliability. In order to compare the situations before and after disruptions occur, some scholars consider the differences [4-5] of total travel time of road networks between these two phases, while others consider its ratio [6].

In some studies, some scholars use complex networks theory, which is a good tool to explore the complex systems, to study the vulnerability of urban road networks from the topological perspective. It represents an active area of research derived from network science, but cannot reflect the actual variations of traffic flow when the urban road networks are subject to disruption. In order to overcome these weaknesses, traffic assignment theory is also used to study the dynamics of urban road networks.

In general, drivers' perceptions on route cost in a traffic evolution model depend on their experiences over the past few days. MAIO, VITETTA [7] etc. investigate the influence of travelers' experiences in the previous days on their updating perceptions of route travel time, they point out that travelers’ perceptions of route cost determine their route choice. In their proposed model, route choices and alternative route flow evolve as drivers become aware of new alternatives based on their experiences and acquired information. HE, YANG [8] etc. have developed a dynamic model, which considers drivers' behavior patterns, such as bounded rationality. In these models, the diversity of route-choice behaviors can be explained, and both of them focus on the influence of driver's experiences on route choices. Shang, Han [9] etc. employ an agent-based day to day model to show how the URN changes after disruption. How-
ever, it is rarely considered the influence of signal control strategies on driver's behaviors when conducting vulnerability analysis. In fact, as an important tool for traffic guidance and management, adaptive signal control can improve the performance of urban road networks and alleviate congestion and delay by adjusting green/red signal ratio based on real-time traffic detection.

Among various Traffic assignment models, the day-to-day traffic dynamic model is regarded to be the most suitable one for analyzing the process of traffic flow equilibrium [10]. Due to the flexibility of the day-to-day dynamic model for evolution of traffic flow, it can be integrated into various control mechanisms, information and driver behaviors to observe the evolution of traffic flow. Therefore, this study attempts to integrate adaptive signal control into the day-to-day traffic dynamic model, and establishes new vulnerability analysis model for urban road networks. The main work of this research are as follows:

- To propose a day-to-day dynamic model to analyze the vulnerability of URNs which may clearly reflect the changes of route flow, route costs and total travel times of urban road networks after being subjected disruption.
- The signal control mechanism is also integrated into the model to study how the adaptive signal control affects the route choice of drivers by adjusting the green/red signal ratio when the road networks are subject to disruptions, so as to influence the evolutionary process of its Re-equilibrium.
- With different levels of capacity losses on the different links of the test networks, the result shows that it can clearly reflect the changes of link importance in disruption under different adaptive signal control strategies.

2. Methodology

2.1. The Day-to-Day Traffic Dynamic Model

2.1.1. Notations of the Day-to-Day Dynamic Evolution Model. In order to make the following explanation easier to understand, in this section, the standard notations used in this traffic model are briefly introduced. As this study focuses on the day-to-day changes of traffic flow, some variables shown below are significantly dependent on the time index \( t \), which \( t \) stands for "the day" in this study, as Table 1.
Table 1. List of notations used in this study.

| Notations | Definitions |
|-----------|-------------|
| $G(N, E)$ | A diagram of node-set $N$ and link set $E$ |
| $W$ | Set of origin-destination (OD) pairs |
| $R$ | Set of routes in the network |
| $R_{o,s}$ | A set of routes connecting OD pairs $(o, s) \in W$ |
| $u_t^a$ | Flow on link $a$ on the day of $t$ |
| $u_t$ | Vector of link flows on day $t$ |
| $h_t^r$ | Flow along route $r$ on day $t$ |
| $h_t$ | Vector of all routes flow on day $t$ |
| $x_t^r$ | The traveler's perception of the cost on route $r$ on day $t$ |
| $Y_t^r$ | The probability of choosing the route $r$ on day $t$ |
| $R_t^a$ | The proportion of the red time on link $a$ on day $t$ |
| $\delta$ | Link-route incidence matrix |
| $C_t^a$ | The capacity of link $a$ on day $t$ |
| $C_t$ | Vector of link capacities on day $t$ |
| $C_a$ | Link travel cost function of link $a$, which is a function of the ratio of the red time and link flow |
| $c_t^r$ | The costs of route $r$ on day $t$ |
| $c_t$ | Vector of route costs on day $t$ |

2.1.2. Network Loading. In a road network, a route $r$ represents the set of all the links it passes through, and the link flow $u_t^a$ is equal to the sum of all the paths $h_t^r$ that pass through it:

$$u_t^a = \sum_{r \in R_t} \delta_{a,r} \cdot h_t^r \text{ for all } a \in E, t$$

(1)
Or, in terms of matrix notation, \( u_i = \delta h_i \). Given the link flow \( u_i \) of the network, the link \( a \) flow can be obtained through the cost function, the cost of the path is equal to the set of the costs of all the links it passes through:

\[
c_i^r = \sum_{a \in E} \delta_{a,r} \cdot C_a(R, u_i) \quad \text{for all} \quad a \in E, t
\]

It is called network loading for the process of vector calculus of the path costs on the. In mathematics, the network loading process can be expressed as:

\[
u_i = (u_i^r)_{a \in E} \cdot R_i = (R_i^{nu})_{a \in E} \cdot C_i^r = \sum_{a \in E} \delta_{a,r} \cdot C_a(R, u_i) \quad \text{for all} \quad a \in E, t
\]

2.1.3. Perception Updating. The impacts of the red time on traffic flows have been successfully added into the path costs after the network loading. Next, we have a perception update for a new day. \( x_i^r \) represents the driver's daily perception of travel cost of route \( r \), which is based on the driver's past travel experience, and it is updated every day as follows:

\[
x_i^r = x_{i-1}^r + \alpha \cdot Y_i^r (c_i^{r-1} - x_{i-1}^r) \quad \alpha \in (0,1)
\]

It can be seen from Equation (3.2) that the driver's perception of a certain path on a certain day is commonly affected by his or her previous perception and the travel cost experience on this path for the recent day (on the day \( t - 1 \)). Hereby, the parameter \( \alpha \) measures the impact of recent travel experiences (on the day \( t - 1 \)) on perception, and vary from 0 to 1 to ensure that perception remains positive.

2.1.4. Route Choice. On the basis of the traveler's perception \( x_i^r \) of the route \( r \) on the day of \( t \), the probability of the travelers' choice of a particular route can be given by logit regression model:

\[
P(Y_i^r) = \frac{\exp(-\beta \cdot x_i^r)}{\sum_{l \in k_{i,r}} \exp(-\beta \cdot x_i^l)} \quad \beta > 0
\]

If the probability is \( Y_i^r \), it means that there are \( F \cdot Y_i^r \) travelers will choose this route on the day of \( t \) in the continuous flow model. In the case of user stochastic equilibrium, as the distributed parameter, \( \beta \) is used to indicate the driver's sensitivity to the path cost.

2.2. The Signal Control Strategy

2.2.1. A Network Loading Process Including the Red Time. In this research, the green time of a link is the percentage of green time, which is obtained by adding the green time of the relevant stages. Furthermore, we assume that there is no minimum green time and that if the link is green, all movement away from the link is green.

Given any signal stage, such as a stage \( J \) (at a certain junction), there is a corresponding reverse stage \( AJ \) : including all links showing a green signal without the stage \( J \). It has the following characteristics: when an inverse stage is red, all links in this stage are red. Now we would like to specify the signal control policy by allocating the red time of the reverse stage to change with respect to the link flow, rather than the green time. In this way, we can integrate signal control with network loading for considering the impact of red time on network cost.

Now we can consider the red time as an extra leaving flow through a junction, which would cause a loss for the traffic flow. For each link, we assume that the additional flow is equal to the Product-Term
of the link flow and the red time, then the link cost function we mentioned in Section 3.1 becomes:

\[ u_i = (u_i^a)_{a \in E}, R_i = (R_i^a)_{a \in E}, c'_i = \sum_{a \in E} \delta_{a,i} \cdot C_a(R_i, u_i) \text{ for all } a \in E, t \] (6)

In this way, the changes in the traffic signal control strategy can be reflected in the traffic flow changes with the network loading process in the next day.

2.2.2. Integrated Dynamic Traffic Assignment Model with Signal Control. Combined with the contents of the first two sections in this section we propose a day-to-day traffic dynamic model with signal control strategies. Starting from the first day, we generate the driver's path perception (often refers to free-flow time), then determine the probability of route choice according to (3.3), and perform the actual path choice randomly. With confirming the route cost in the network loading process with signal control based on (3.1), we use (3.2) to calculate the traveler's perceptions of all paths. The process is repeated day by day until the network reaches equilibrium. Table 2 provides the pseudocode for this procedure.

**Table 2.** Pseudocode for evolution model for traffic flow.

| Input: Initial red time, traffic demands |
|-----------------------------------------|
| Step 0: To allocating the free-flowing costs of all paths to initial path perception, confirm the probability of route choice according to (3.5). and perform network loading (3.6) to obtain path cost, if \( t = 0 \). |
| Step 1: \( t = t + 1 \) Update path perception according to (3.2). |
| Step 2: The traveler chose a path according to (3.5) to calculate the flow on the day of \( t + 1 \). |
| Step 3: To make the adaptive signal apply to the signal control strategy, perform network loading according to (3.1) in combination with the flow on the day \( t + 1 \). |
| Step 4: Repeat steps 1-3 until the network reaches equilibrium. |

2.3. Adaptive Signal Control Strategies
We introduced how to convert red time into additional flow in this section 3.2. However, corresponding to the additional flow, red time can also be considered as a kind of cost, which is generally entitled linked red time cost. In order to carry out signal control according to it, different adaptive signal control strategies have different definitions for linked red time cost.

Saturation control strategy is one of the most widely used conventional signal setting method in traffic engineering to deal with traffic allocation and control integration problems. This strategy defines linked red time cost as:

\[ \text{link red time cost} = \frac{f_a}{g_a s_a} = \frac{f_a}{(1 - \bar{r}_a)s_a} = \frac{f_a}{s_a - \bar{r}_a s_a} \] (7)

where, \( s_a \) is the saturated flow on the link \( a \), \( g_a \) and \( \bar{r}_a \) is the ratio of green time and red time on the link \( a \) respectively (they are dimensionless), \( f_a \) is the flow on the link \( a \).

Compared with the saturation control strategy, the \( P_0 \) strategy is more innovative, for it assumes that red time may cause delays in other links that can be captured by additional traffic units on the associated link. The \( P_0 \) control strategy defines the red time as:

\[ \text{link red time cost} = s_i b_i (f_i + s_i \bar{r}) \] (8)

\( b_i \) is the cost function of the link (such as BPR function).
For different adaptive signal control strategies, the requirements for linked red time cost of the intersection in each link are different, as well as the requirements for red time, traffic flow and flow capacity. Finally, the calculated red time is different from each other. Among them, according to the saturation control strategy, the linked red time costs must meet the following requirements for road intersections $\bar{n}$ with signal-controlled sections:

$$\frac{f_1}{s_1g_1} = \frac{f_2}{s_2g_2} = \ldots = \frac{f_\pi}{s_\pi g_\pi}$$  \hspace{1cm} (9)

Different from the equi-saturation control, for the junction with $n$ links, the $P_0$ strategy applies the red time interval to ensure that:

$$s_1b_1 = s_2b_2 = \ldots = s_\pi b_\pi = \mu$$  \hspace{1cm} (10)

From the above equation, it can be clearly concluded that in the $P_0$ control strategy, Red time is adjusted to achieve the traffic stability of the junctions where traffic conflicts happen. It is better to encourage travelers to choose a high-capacity path (even if a slow actual flow for this approach) than reward the people who are on the existing paths. It is proved that, under natural conditions, including strict capacity restriction, this strategy can maximize network features in the flow equilibrium distribution in the case of natural conditions with strict capacity restriction.

3. Case Study

This section introduces the numerical study case of the vulnerability analysis for urban road networks, and explains the impacts of different signal control strategies on the networks when road links suffer from different levels of disruptions. Particular attention is paid to show evolution in traffics, route costs, and the total travel time of the network under disruptions.

We utilize a small network as test network, which shows the common layout and structure within urban area. This test network has been used in many studies for testing traffic models and algorithms. The traffic flow model for vulnerability analysis is implemented on a three-by-three grid network consisting of 9 nodes and 12 links (as shown in following figure 1). Considering an OD points (node 1 and node 9 respectively), the demands between OD point pairs is 2000. There are six paths below:

- Path1: Link1→Link2→Link5→Link10
- Path2: Link1→Link4→Link7→Link10
- Path3: Link1→Link4→Link9→Link12
- Path4: Link3→Link6→Link7→Link10
- Path5: Link3→Link6→Link9→Link12
- Path6: Link3→Link8→Link11→Link12

In this network, there are three traffic lights at nodes 5, 6 and 8 respectively, and they are all deployed in the junction where traffic confliction occurs.
In this example (Figure 1), the following BPR function is used to describe the travel costs of a link:

$$C_a = A \cdot (1 + 0.15 \left( \frac{f_a}{CP_a} \right)^4)$$

In the above equation, $f_a$ represents the flow on link a, $CP_a$ stands for its travel capacity, and $A$ means its initial cost. The numerical summary of these parameters is shown in Table 3.

Table 3. Link parameters for the small urban road network.

| Link ID | $A$ (min) | $CP_a$ |
|---------|-----------|--------|
| 1       | 25        | 1000   |
| 2       | 25        | 1000   |
| 3       | 25        | 1000   |
| 4       | 25        | 1000   |
| 5       | 25        | 1000   |
| 6       | 25        | 1000   |
| 7       | 25        | 1000   |
| 8       | 25        | 1000   |
| 9       | 25        | 1000   |
| 10      | 25        | 1000   |
| 11      | 25        | 1000   |
| 12      | 25        | 1000   |

In order to test the importance of different links to the urban network, we assume 50% and loss of traffic capacity for each Link, and then make a research by the mentioned model. We show the specific results of Link 1, 4 and 7 in the following figure. Adopting equal saturation signal control in the evolution process, and it starts from the initial state.
Figure 2. Changes of route flow, route cost and total travel times of the network in different links after suffering 50% losses of travel capacity.

The first column (A1-C1) in the following figure 2 represents the evolution process of path flow, path cost and total travel time of the network after Link 1 suffering 50% losses of travel capacity. The second column (A2-C2) and the third column (A3-C3) are for Link 4 and Link 7. It can be seen that in the case that other variables are the same, the path traffic flow fluctuation of Link1 is the most obvious after being attacked, and even the junction and overlap of the flow curves of two paths appeared in a1. The second one is Link 4, and Link 7 is the least affected. There are two reasons. The first one is that Link1 has more paths in the road network which are affected. Secondly, signal control has a small impact on Link1, so it is not easy to adjust the impact of capacity loss of Link 1 in the network.

Next, with the capacity of all links reduced by 50%, the process of road network rebalancing was simulated according to the day-by-day dynamic model for the traffic flow distribution. The results are shown in Table 4.
Table 4. In the case of a 50% decrease in capacity, the road network is ranked with regard to the rebalancing time and the impact of total travel time on the network after suffering disruption.

| Link Number | Ranking | P₀ Control Strategy | Saturation Control Strategy | Fixed Time |
|-------------|---------|---------------------|-----------------------------|------------|
| 1           | 10      | 10                  | 10                          |
| 2           | 12      | 12                  | 12                          |
| 3           | 1       | 3                   | 3                           |
| 4           | 3       | 1                   | 1                           |
| 5           | 9       | 4                   | 9                           |
| 6           | 6       | 6                   | 7                           |
| 7           | 7       | 9                   | 6                           |
| 8           | 4       | 7                   | 4                           |
| 9           | 5       | 11                  | 11                          |
| 10          | 11      | 5                   | 5                           |
| 11          | 2       | 8                   | 2                           |
| 12          | 8       | 2                   | 8                           |

The results show that, in these three adaptive signal control strategies, link 10, Link 3, Link 12 and Link 1 all have the greatest impact on the road network when suffering from disruption. These links are, which involves most paths, not located at the junction with signal controls. However, the ranking is different in the two adaptive strategies. For example, the disruption on link 6 has a greater impact than that on link 7 in the equal saturation control strategy and P₀ control strategy. But it is opposite in the signal control strategy with fixed time. Due to the fact that link 7 is directly affected by the two traffic lights, and link 6 is affected by only one. The link with the least paths has the least impact on network performance when it is disrupted.

4. Conclusion

The main significance of this study is as follows. After a literature review, this study uses the robustness index proposed by Shang [1, 11] as the vulnerability analysis index, which makes the vulnerability analysis results more reliable. Using the day-to-day traffic dynamic model to show the changes in URN, it makes vulnerability analysis identify key components in the road network more accurately and clearly. Through the combination of traffic signal control and vulnerability analysis, the effect of traffic signal control on URN vulnerability can be more clearly demonstrated, it is also possible to focus on the most important road sections during traffic control and management [12]. This method can maximize resources and reduce the loss of the road network after being attacked.

According to the theories and methods in Section 2 and the case study in Section 3, we can draw the following conclusions:
- The day-by-day evolution model can be easily combined with traffic signal control strategies due to its flexibility, and it can also accurately show the impacts of signal control strategies on traffic flow evolution.
- Adaptive signal control can change the vulnerability of urban road networks. For the road link directly affected by signal control, the impacts on road network, when there are losses for traffic capacity caused by attacks or traffic incidents, are smaller than that affected by fixed time signal control. At junctions where traffics conflict, adaptive traffic signals should be deployed to effectively reduce the impact of link disruptions on the road network.
- In the urban road networks, the link with more routes has a greater impact on the performance of road network when suffering from capacity degradation. Therefore, more attention and resource should be allocated to the maintained of the road links passed by more routes for traffic control and management.
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