Predictions for $J/\psi$ Suppression
by Parton Percolation

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Abstract:

Parton percolation provides geometric deconfinement in the pre-equilibrium stage of nuclear collisions. The resulting parton condensate can lead to charmonium suppression. We formulate a local percolation condition viable for non-uniform collision environments and show that it correctly reproduces the suppression observed for $S-U$ and $Pb-Pb$ collisions at the SPS. Using this formulation, we then determine the behavior of $J/\psi$ suppression for $In-In$ collisions at the SPS and for $Au-Au$ collisions at RHIC.
1. Introduction

In recent years, the role of the partonic initial stages of high energy nuclear collisions has attracted increasing interest. While the ultimate aim of the experimental program is the production of the quark-gluon plasma predicted by statistical QCD, it is becoming more and more evident that a thermalized medium consisting of quarks and gluons can be produced only if already the initial state of the collision provides the conditions necessary for deconfinement and thermalization. Dilute initial state parton configurations do not lead to the colour connection needed so that partons from different collisions can combine to form a collective medium. On the other hand, if the initial state of primary partons is sufficiently dense, cluster percolation will set in, leading to a condensate of connected and hence interacting partons, which are no longer associated to any “parent” hadrons. Thus parton percolation is a geometric, pre-equilibrium form of deconfinement [1, 2]; it must occur in any description based on partons of an intrinsic transverse momentum.

Subsequent interactions can thermalize this interconnected partonic system, turning it into a quark-gluon plasma, so that parton percolation constitutes an essential prerequisite for QGP formation. Moreover, pre-equilibrium deconfinement leads to a further important question. Much of the investigation of high energy nuclear collisions is devoted to the search for quark-gluon plasma signatures. However, some of the features observed in nuclear interactions might be determined by the system present before any equilibration has occurred, and they could be independent of a subsequent thermalization. We must therefore investigate observable consequences of parton percolation, and if possible, show how they can be distinguished from those due to QGP formation.

One of the features of particular interest in this connection is the suppression of $J/\psi$ production in nuclear collisions, predicted as deconfinement signal for thermal media quite some time ago [3]. If the parton condensate formed through percolation contains partons hard enough to resolve the produced charmonium states, these partons can also dissociate charmonia, so that the onset of $J/\psi$ suppression could coincide with that of parton percolation. It was shown in a first study [4] that for Pb–Pb collisions at the CERN-SPS as function of centrality the resulting thresholds appear quite reasonable. A more quantitative comparison, which can then also provide the basis for further predictions, requires a formulation of percolation in a non-uniform environment, as given by the partonic source profile in nuclear collisions, and adapted to the local nature of charmonium probes. The aim of this paper is first to formulate percolation for such conditions, then to compare the results to existing SPS data ($S - U$ and Pb–Pb), and finally to present the predictions of this approach for the forthcoming $J/\psi$ production experiments at CERN and BNL. All results are obtained by Monte Carlo simulations taking into account the centrality dependence of the collisions. In closing, we shall consider some features which could distinguish $J/\psi$ suppression by parton percolation from a suppression in a thermal medium.

2. Local Parton Percolation Conditions

Consider a flat two-dimensional circular surface of radius $R$ (the transverse nuclear area), on which $N$ small discs of radius $r \ll R$ (the transverse partonic size) are randomly distributed, allowing overlap. With increasing density $n \equiv N/\pi R^2$, clusters of increasing
size appear. The crucial feature is that this cluster formation shows critical behavior: in the limit $N \to \infty$ and $R \to \infty$, with $n$ finite, the cluster size diverges at a certain critical density $n = n_c$. This percolation threshold is given by

$$n_c = \frac{\nu_c}{\pi r^2};$$

the critical value $\nu_c \simeq 1.13$ of the ‘filling factor’ $\nu \equiv n(r/R)^2$ is determined by numerical studies. For finite $N$ and $R$, percolation sets in when the largest cluster spans the entire circular surface from the center to the edge. The resulting cluster growth is illustrated in Fig. 1a for a ratio $r/R = 1/100$; we show the percolation probability, defined as the relative size of the largest cluster, as function of the filling factor $\nu$. Because of overlap, a considerable fraction of the surface is still empty at the percolation point; in fact, at that threshold, only $1 - \exp\{-\nu_c\} \simeq 2/3$ of the surface is covered by discs.

![Figure 1: Cluster growth and corresponding derivative as function of the overall filling factor $\nu$ (a) and of the cluster filling factor $\eta$ (b).](image)

In high energy nuclear collisions, the incident partons are distributed on a plane transverse to the collision axis. However, since they originate from the nucleons within the colliding nuclei, this distribution is highly non-uniform, with more nucleons and hence more partons in the center than towards the edge of the transverse nuclear plane. If the basic surface is not flat, it is still possible to define the percolation threshold as the point at which the cluster size shows singular behavior in the limit of large $R$ and $N$ [2]. Such a global definition is, however, not always the most useful. Hard probes, such as quarkonia, probe the medium locally and thus test only if it has reached the percolation point and the resulting geometric deconfinement at their location; they cannot register the global features of the entire cluster area. It is thus necessary to define a more local percolation criterion, and this is in fact quite straightforward.

As mentioned, at the percolation point, $\exp\{-\nu_c\} \simeq 1/3$ of the surface remains empty. Hence the disc density in the percolating cluster must be greater than $(1.5 \; \nu_c/\pi r^2)$. Numerical studies show that in fact percolation sets in when the density $m$ of constituents in the largest cluster reaches the critical value

$$m_c = \frac{\eta_c}{\pi r^2};$$
with $\eta_c \simeq 1.72$, slightly larger than $1.5 \nu_c$. This result provides the required local test: if the parton density at a certain point in the transverse nuclear collision plane has reached this level, the medium there belongs to a percolating cluster and hence to a deconfined parton condensate. In Fig. 1b, we illustrate how the percolating cluster grows as function of the cluster filling factor $\eta = m(r/R)^2$, again with $r/R = 1/100$.

Let us recapitulate: there are two equivalent criteria which can be used to specify the onset of percolation. In the ‘global’ definition, it occurs when the total number of partons distributed over the entire transverse nuclear area reaches $1.13/\pi r^2$. In the ‘local’ definition, percolation sets in when the parton density in the largest cluster reaches $1.72/\pi r^2$. Which of the two is preferable in a given case depends on the physics question addressed, and if we want to study local phenomena such as $J/\psi$ suppression, it is clearly the local parton density which is relevant, not the average over the entire transverse area.

We now turn to the implications of the approach to nuclear collisions \cite{4}; for illustration, we concentrate for the moment on central $A-A$ interactions at a c.m.s. energy $\sqrt{s}$ per nucleon-nucleon collision. The distribution of nucleons in the colliding nuclei is specified by a Glauber calculation using Woods-Saxon nuclear distributions \cite{5}; this provides the density $n_s(A)$ of nucleons in the transverse collision plane. The parton content of a nucleon is given by parton distribution functions $dN_q(x,Q^2)/dy$ determined in deep inelastic scattering experiments; here $x$ denotes the fraction of the nucleon momentum carried by the parton and $Q$ the momentum resolution scale. At central c.m.s. rapidity $y = 0$, we have $x = k_T/\sqrt{s}$, where $k_T$ denotes the average transverse momentum of the parton. In nuclear collisions, $k_T$ defines the transverse size of the partons and thus also sets the resolution scale, $k_T \simeq Q$. Using these quantities, we have for the ‘global’ percolation condition in nuclear collisions

$$n_s(A) \left( \frac{dN_q(x,Q^2_c)}{dy} \right)_{x=Q_c/\sqrt{s}} = \frac{\nu_c}{(\pi/Q^2_c)}; \quad \text{(3)}$$

it determines for what value of $A$ at a given $\sqrt{s}$ percolation sets in, and it specifies the value $Q = Q_c(A,\sqrt{s})$ at the onset point. Using the nuclear source density $n_s(A)$ together with the parton distribution function, we can also calculate the density of partons $m(A,Q,\sqrt{s})$ for the largest cluster in the transverse collision plane. The relation

$$m_c(A,Q_c,\sqrt{s}) = \frac{\nu_c}{(\pi/Q^2_c)}; \quad \text{(4)}$$

then provides the ‘local’ parton percolation condition to specify the onset values of $A$, $\sqrt{s}$ and $Q$. These considerations can be extended to non-central collisions in a straightforward fashion \cite{4,5}.

It is not \textit{a priori} evident that the threshold values $\nu_c$ and $\eta_c$ for the non-uniform distribution specified by the Woods-Saxon distributions remain the same as for a uniform distribution, nor is it clear how large finite size corrections are at given values of $r/R$. We have therefore calculated the pseudo-critical values (derivative peak positions) of $\nu$ and $\eta$ for $R/r = 20$, 50 and 100, using the Woods-Saxon distribution for central $Pb-Pb$ collisions to determine the parton density as well as the average overall interaction region. Finite-size scaling techniques from statistical physics then provide the asymptotic limit.
in this way, we have verified that $\nu_c$ and $\eta_c$ for this non-uniform distribution in fact agree with the above mentioned values for the uniform case. Moreover, we found that for the mentioned values of $R/r$, the deviations from the asymptotic critical values are less than 2%.

2. Comparison to Pb-Pb and S-U Data

We now want to study parton percolation for CERN-SPS conditions and compare the resulting consequences for $J/\psi$ suppression to the data of the NA38/NA50 collaborations.

The parton content of a nucleon is obtained from GRV94 parton distribution functions \cite{GRV94}, since these are available for the required kinematic range; for more details, see \cite{4}. With this specified, we calculate the size of the largest cluster as well as the overall size of the interaction region for different centralities (the latter is here defined by the ratio of the number to the density of wounded nucleons). For Pb–Pb collisions at $\sqrt{s} = 17.4$ GeV, we show in Fig. 2a the relative size of the largest cluster as function of the filling factor $\eta$. It is seen that the cluster size grows rapidly in a narrow band around $\eta_c$, and at the percolation onset, the percolating cluster covers about 1/3 of the total collision area. In Fig. 2b, we show the cluster filling factor $\eta$ as function of centrality, measured by the number of participating nucleons. The critical value for parton percolation, $\eta_c = 1.72$, is found to be attained for $N_{\text{part}} \simeq 125$.

The parton distribution functions specify both the number of partons and the resolution scale $Q_c$ at the percolation point. For Pb–Pb collisions at $\sqrt{s} = 17.4$ GeV, $Q_c \simeq 0.7$ GeV. The scales of the charmonium states $\chi_c$ and $\psi'$, as determined by the inverse of their radii calculated in potential theory, are around 0.6 GeV and 0.5 GeV, respectively. Since the parton condensate can thus resolve these states, we assume, following \cite{4}, that it will dissociate them. To be specific: we assume that all $\chi_c$ and $\psi'$ states formed inside the percolating cluster disappear; the location of their formation is determined by the collision density as given by the Glauber formulation \cite{4,5}. The first onset of $J/\psi$ suppression in Pb–Pb collisions at the SPS should therefore occur at $N_{\text{part}} \simeq 125$, where the $J/\psi$'s due to feed-down from $\chi_c$ and $\psi'$ states in the percolating cluster are eliminated. Directly produced $J/\psi$'s survive because of their smaller radii (leading to a scale of 0.9 - 1.0 GeV), as do those coming from excited states outside the percolating cluster. We emphasize that
with parton distribution functions and charmonium radii given, the description contains no adjustable parameters.

As noted, the parton condensate formed at the onset of percolation has a resolution scale \( Q_c \approx 0.7 \) GeV and thus cannot resolve the ground state charmonium \( J/\psi \) with its smaller radius. The dissociation of directly produced \( J/\psi \)'s thus requires more central collisions, which lead to a higher parton density and simultaneously to a better resolution, i.e., to an increase of \( Q_c \). The precise onset point is here more difficult to determine, since it is fixed by the value of the \( J/\psi \) radius. For \( r_{J/\psi} = 0.20 \) fm, we have \( Q \approx 1.0 \) GeV and obtain the centrality dependence shown in Fig. 3 indicating that the suppression of directly produced \( J/\psi \)'s starts at \( N_{\text{part}} \approx 320 \). However, here a note of warning has to be added. The \( \chi_c \) and \( \psi' \) states can be resolved by the parton condensate when it is first formed; the values of their intrinsic scales do not enter explicitly, since they are below the resolution scale \( Q_c \) of the condensate. The onset point for the suppression of the \( 1S \) ground state is directly determined by the radius of that state, and since the dependence of parton density on centrality (see Fig. 3) is quite flat, small changes in \( r_{J/\psi} \) lead to large changes in the threshold value. For instance, \( r_{J/\psi} = 0.22 \) fm \( (Q \approx 0.9 \) GeV) shifts the onset from \( N_{\text{part}} \approx 320 \) to about 190, as also shown in Fig. 3.

\[
S(N_{\text{part}}) = \int d\tilde{N}_{\text{part}} \ S(\tilde{N}_{\text{part}}) \frac{1}{\sqrt{2\pi} \ \sigma} \exp\{- (N_{\text{part}} - \tilde{N}_{\text{part}})^2 / 2\sigma^2\}, \quad (5)
\]

using a relative resolution of \( \sigma/N_{\text{part}} = 15 \% \) at the respective thresholds. The result is also shown in Fig. 4.

The centrality dependence of the \( J/\psi \) survival has been measured in Pb-Pb collisions at the SPS using two independent variables, the associated transverse energy, \( E_T \), and
the energy of the undeflected nucleons, $E_{ZDC}$. The first effectively measures the number of hadronic secondaries in the transverse plane, the second the number of participating nucleons. We want to study anomalous charmonium suppression, i.e., effects above and beyond the normal suppression already observed in p-A collisions, due to absorption of the nascent charmonium state in normal nuclear matter [9]. To determine this normal suppression, we use a recent analysis [10] of p-A and S-U data by the NA50 group, which finds a somewhat weaker nuclear absorption, $\sigma_d \simeq 4.3 \pm 0.6$, than previous studies. Normalizing the Pb-Pb data to the normal nuclear suppression thus specified leads to the points shown in Fig. 5, where they are compared to our parton percolation suppression pattern. We see good agreement between the calculated curve and the data points. We recall, however, that the second threshold value depends crucially on the $J/\psi$ radius.

Next we want to check the behaviour predicted for S-U collisions at the SPS, since this shows at best only very slight anomalous suppression\footnote{Initially, it was claimed that the data are fully accounted for by normal nuclear absorption. However, the lower absorption cross sections obtained in the mentioned new p-A studies allow a slight anomalous suppression for the most central S-U collisions.}. In Fig. 6 we show the percolation

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Figure 4: $J/\psi$ survival pattern in Pb-Pb collisions at the SPS, as function of centrality.

Figure 5: $J/\psi$ survival pattern in Pb-Pb collisions at the SPS, as function of centrality, determined from $E_T$ (left) and from $E_{ZDC}$ (right).
behaviour, and in Fig. 7 the resulting survival pattern together with the data [11]. The threshold obtained here corresponds to the general onset of percolation; the resolution scale of the direct $J/\psi$ cannot be reached in $S-U$ collisions. The corresponding smeared form of the calculated distribution is also included.

![Graph](image1)

Figure 6: Percolation onset in S-U collisions at the SPS.

![Graph](image2)

Figure 7: $J/\psi$ survival pattern in S-U collisions at the SPS, as function of centrality.

We close this section with some general comments on the suppression of charmonium production. We have here considered parton percolation as mechanism for anomalous $J/\psi$ suppression, defined as the reduction of the observed production rate beyond that predicted by normal absorption in standard nuclear matter. In doing so, we neglect other conceivable suppression mechanisms, in particular dissociation by (non-percolating) secondary hadronic or partonic comovers. This is not expected to play a significant role for the suppression of $1S$ states, i.e., directly produced $J/\psi$’s: the large binding energy (650 MeV) makes a dissociation by confined partons very unlikely [12]. However, since about 40% of the observed $J/\psi$’s come from the decay of higher excited states (feed-down from $\chi_c$ and $\psi'$ production), we have to consider the fate of these as well. The $\chi_c$ ($\sim 250$ MeV) is still rather strongly bound, but the much smaller $\psi'$ binding energy ($\sim 60$ MeV) means that any interaction will cause break-up. It is thus not surprising that the $\psi'$ suffer an anomalous suppression even in the most peripheral S-U interactions [13]. This is most likely not related to any parton deconfinement and has to be addressed separately.
3. Predictions for NA60 and RHIC

We now turn to the forthcoming $In-In$ collisions at the SPS. The percolation pattern is shown in Fig. 8: the general percolation threshold is reached at $N_{\text{part}} \simeq 140$. In contrast to $Pb-Pb$ collisions, the threshold for directly produced $J/\psi$'s here remains above the produced density even for the most central collisions (but recall the caveat concerning its dependence on the $J/\psi$ radius). The resulting survival pattern for $In-In$ collisions at the SPS is given in Fig. 9 including also the smeared form. We see that the most central 30\% of the collisions show a suppression of the $J/\psi$'s produced by feed-down from higher excited charmonium states.

![Percolation Probability](image1)

Figure 8: Percolation onset in In-In collisions at the SPS.

![J/ψ survival probability](image2)

Figure 9: $J/\psi$ survival pattern in In-In collisions at the SPS, as function of centrality.

Finally we address the suppression pattern for $Au-Au$ collisions at RHIC, with the percolation pattern shown in Fig. 10. Here the increased parton density shifts the onset of percolation to a higher resolution scale, so that from the threshold on, all charmonium states are suppressed. This leads to a single step suppression pattern, starting at $N_{\text{part}} \simeq 90$; it is given in Fig. 11. Recall that we are here specifying the predicted anomalous suppression, beyond the normal suppression resulting from (pre-resonance) absorption in standard nuclear matter. For various reasons, this normal suppression may depend on the collision energy and can thus be quite different from that observed at SPS energy.
To determine the anomalous suppression at RHIC, it is therefore crucial to first measure the normal absorption in $p-A$ (or $d-A$) collisions. The threshold for $Au-Au$ collisions as obtained here occurs for considerably more central collisions than found in previous studies [2], in which the dependence of parton distribution functions on the resolution scale had not taken into account.

Figure 10: Percolation onset in Au-Au collisions at RHIC.

Figure 11: $J/\psi$ survival pattern in Au-Au collisions at RHIC, as function of centrality.

In summary: if the origin of the two-step suppression pattern observed in the $Pb-Pb$ SPS collisions is indeed the onset of parton percolation at different resolution scales, as discussed, then we expect a single step pattern both in $In-In$ collisions at the SPS and in $Au-Au$ collisions at RHIC. In the former, the step is due to the suppression of the higher excited states only, with directly produced $J/\psi$'s surviving; in the latter, there is a common threshold for all charmonium states and hence only one step.

4. Discussion

We have presented the $J/\psi$ suppression pattern as obtained from parton percolation under the assumption of complete charmonium dissociation within a percolating cluster of sufficient resolution scale, full survival in media below percolation density or of insufficient resolution scale. In this closing section, we want to summarize the uncertainties of such an approach and compare its outcome to that of a thermal description.
The onset of parton percolation is specified directly through the parton distribution functions (PDF’s) as obtained from an analysis of deep inelastic lepton-hadron data. Uncertainties in the PDF’s produce some uncertainty in the precise values of the percolation threshold, but not in the onset as such. As mentioned, percolation must occur in any approach based on partons with an intrinsic transverse momentum. The assumption tested by comparing our results to \(J/\psi\) data is that for charmonium suppression in nuclear collisions, the primary parton density is the crucial variable and the formation of a percolating cluster the relevant mechanism.

As noted, the onset of the suppression of directly produced \(J/\psi\)’s at SPS energy is quite sensitive to the choice of charmonium parameters (and on that of the parton distribution functions, for which slight changes would also produce considerable threshold shifts). On the other hand, the onset of the first step in SPS \(Pb-Pb\) pattern, the predicted single step in SPS \(In-In\) collisions, and the predicted single step in RHIC \(Au-Au\) collisions should be quite robust. This should help in identifying the origin of the second step in SPS \(Pb-Pb\) collisions. It has been suggested [14] that this drop is largely due to transverse energy fluctuations for the most central collisions; this would imply that it should be absent in zero-degree calorimeter studies, where (see Fig. 5) it seems to still occur to some extent [8]. A further test should come from the SPS \(In-In\) and the RHIC \(Au-Au\) data, where the existence of a second step could not be attributed to parton percolation.

Finally we turn briefly to the question of the distinguishing the initial state charmonium suppression presented here from a suppression occurring at a later thermalized stage. We had here assumed that the primary parton configuration determines the pattern of \(J/\psi\) suppression. What would one expect if the initial parton configuration leads to subsequent thermalization, with the formation of a more or less equilibrated quark-gluon plasma, and it is this QGP which causes the observed \(J/\psi\) suppression? The most recent (quenched) lattice QCD studies [15, 16] indicate that in a hot thermal medium of quarks and gluons, higher excited charmonium states are dissociated at or below \(T_c\). In contrast, the ground state \(J/\psi\) survives to considerably higher temperatures; present calculations still show a clear \(J/\psi\) signal at least up to \(T = 1.5 T_c\). This would imply that thermal dissociation removes the feed-down contributions from \(\psi'\) and \(\chi_c\) states for energy densities around or below 1 GeV/fm\(^3\), while energy densities of more than 6–8 GeV/fm\(^3\) are required for the melting of directly produced \(J/\psi\)’s. It seems difficult to accommodate the existing SPS data with these thresholds, while the pattern resulting from parton percolation can do so quite naturally. For \(Au-Au\) collisions at RHIC, thermal melting would suppress the higher state feed-down contributions for practically all centralities, while the directly produced \(J/\psi\)’s disappear at best for the most central collisions. We thus conclude that the charmonium suppression thresholds from initial state and from thermal suppression are quite different also at much higher energy.

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