Cosmic Gas Thermodynamics at z = 1089

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Short Report

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Cosmic Gas Thermodynamics at $z = 1089$

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Abstract – The Universe at $z = 1089$ is treated as an expanding ideal gas. Its internal kinetic energy loss exceeds the amount absorbed by gravity and drives further expansion. A Hubble relation ($H_g$) is derived and compared to the value found by the $\Lambda$CDM model ($H_\Lambda$) over the range $z = 1089$ to 0. The results suggest that the adiabatic release of energy from cosmic gas accounts for about half of present-day Universal expansion.

The presently preferred description of Universal expansion is the flat-universe $\Lambda$CDM model, given in simple form by (1):

$$H^2(a) = H_0^2[\Omega_{rad}a^{-4} + \Omega_Ma^{-3} + \Omega_{CDM}a^{-3} + \Omega_\Lambda]$$ (1)

Where $H(a)$ is the Hubble parameter at scale factor $a = 1/(1 + z)$, $z$ is the cosmic redshift $(\lambda_e - \lambda_0)/\lambda_e$ of an emitted photon of known $\lambda_0$, and $H_0$ is the present-day Hubble constant, 67.4 Km/sec/Mpc or $2.184 \times 10^{-18}$ sec$^{-1}$. The $\Omega$ values or density parameters add up to one and relate their energy density to the present-day critical density $\varepsilon_{crit} = 3c^2(H_0)^2/8\pi G$:

$$1 = \sum \Omega_x = \sum \frac{\varepsilon_x}{\varepsilon_{crit}}$$ (2)

In eq. (1), $\Omega_m = 0.0486$ is baryonic matter, $\Omega_{cdm} = 0.259$ is cold dark matter, $\Omega_{rad} = 9.00 \times 10^{-5}$ is relativistic energy (photons and neutrinos), and $\Omega_\Lambda = 0.69$ is the “dark energy” parameter. The $\Lambda$CDM model treats Universal expansion as a function of the sum of the mass-energy components $\Omega$, three of which ($\Omega_{rad}$, $\Omega_M$, and $\Omega_{CDM}$) have constant comoving energy densities $\varepsilon_{rad}$, $\varepsilon_M$, and $\varepsilon_{CDM}$. The fourth parameter $\Omega_\Lambda$ has a density $\varepsilon_\Lambda$ which is not comoving, but rather is the same for any volume of space at any time. The critical density $\varepsilon_{crit}$ is the total mass-energy density which gives an exact balance to the energy loss from gravity over time. The empirical accuracy of the $\Lambda$CDM model is high and we use it to calibrate our model.

This paper explores the energy release associated with density reduction (“expansion”) of cosmic gas and plasma, by treating those portions the Universe not bound by gravity as an ideal gas.

First, we select a time: $z = 1089$, just after recombination. Baryonic matter was almost all neutral gas and acoustic oscillation was minimal so the Universe had constant density. Baryons were then present as a mixture of 75% monatomic hydrogen (H$_1$) : 25% helium (He) by weight, or about 92 mole % H$_1$ : 8 mole % He. This gives a mean molecular weight $\Xi = 1.2475 \times 10^{-3}$
Kg/mol. Monatomic gas thermodynamic laws can be reasonably applied to this time period.\textsuperscript{1} The 2018 Planck survey\textsuperscript{2} gives a ΛCDM-based value for today’s mean baryon density: \( \rho_0 = 4.21 \times 10^{-28} \text{ Kg/m}^3 \). The baryon density \( \rho_{1089} \) was thus \( \rho_0 / a^3 = (4.21 \times 10^{-28})(1090)^3 = 5.45 \times 10^{-19} \text{ Kg/m}^3 \), or 2.6 x 10\(^8\) atoms per cubic meter. The background radiation had just decoupled so the baryon temperature was \( \approx 2971 \text{ K} \) (CMB = 2.7255 K)(1/\(a = 1090\)).\textsuperscript{3}

The Universe as a whole is an adiabatic system. In a classic setting, there are two kinds of adiabatic gas expansion: reversible and free. Reversible expansion is isoentropic by definition. When a gas expands reversibly, its internal kinetic energy \( U_i \) decreases, the gas performs work, and the temperature and pressure drop. When a gas expands freely, \( U_i \) does not decrease and only the pressure drops. The temperature stays the same:

\[
\partial V = \partial V_S + \partial V_T
\] (3)

Both happen cosmically, but with differences. In the first, \textit{cosmic isoentropic (“reversible”)} expansion, the internal kinetic energy \( U_i \) lost isn’t equal to the energy stored by gravity. The excess becomes vectored kinetic energy. The second, \textit{cosmic free expansion}, derives from the fact that the entropy of the Universe always increases over time.\textsuperscript{4} On a cosmic scale, gas expansion can’t be exclusively isoentropic as no time would have elapsed. There must also be an entropic volume increase. These two forms of gas expansion are linked: one can’t happen without the other. We develop our model isoentropically (\( \partial V_S \) only) and examine \( \partial V_T \) later on.

Consider a finite sphere around a single atom of \( \text{H}_1 \), of radius about Earth size (1 au = 6.3781 x 10\(^6\) m), at 2971 K, which at \( \rho_{1089} \) has baryon mass \( M = 593 \text{ Kg} \). This sphere is still in thermal equilibrium, a major but necessary departure from reality. Nonequilibrium thermodynamics must be set aside so that the underlying transfer of conserved energy is more clearly described. The sphere’s gravitational potential energy \( (U) \) is:

\[
U = \frac{-3GM^2}{5r}
\] (4)

Where \( G \) is the gravitational constant (6.67408 x 10\(^{-11}\) m\(^3\)kg\(^{-1}\)sec\(^{-2}\)). The ΛCDM model also contains cold dark matter (CDM), \( \Omega_{cdm}/(\Omega_M + \Omega_{cdm}) = 0.842, \approx 84\% \) of all cold mass in the

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\textsuperscript{1} Isotopes, heavier elements, and diatomic hydrogen are treated as negligible.
\textsuperscript{2} a) Planck Collaboration et. al., \textit{A&A} 641, A1 (2020) doi:10.1051/0004-6361/201833880. b) in Ryden, B. R., Pogge, R.W.; \textit{Interstellar and Intergalactic Medium} (2021), Cambridge University Press, ISBN 978-1-108-74877-3, page 198.
\textsuperscript{3} CMB = cosmic microwave background.
\textsuperscript{4} This is the Second Law of Thermodynamics. The First and Second Laws are, from Clausius: “The energy of the universe is constant; the entropy of the universe tends to a maximum.” Clausius, R. (1865) Annalen der Physik 125: 353–400. See also: Popovic, M. https://arxiv.org/abs/1711.07326.
\textsuperscript{5} The subscripts \( U_1, U_2, \) and \( U_r \) refer to gravitational potential energy. The term \( U_i \) is used to denote the internal kinetic energy of a gas or plasma.
Universe, and doesn’t act as a gas. Its only influence is gravitational. This is included by dividing the baryon mass by 0.158:

\[
U = -\frac{3GM^2}{5r} = -\frac{3G(M/0.158)^2}{5r}
\]

The ideal gas law is:

\[
P V = nRT = \frac{MRT}{\mathcal{R}}
\]

Where \( R \) is the universal gas constant (=8.31446 kg-m\(^2\)sec\(^{-2}\)mole\(^{-1}\)K\(^{-1}\)). The volume of a sphere is:

\[
V = \frac{4}{3} \pi r^3
\]

When (6) and (7) are combined we get the internal pressure \((P_1)\):

\[
P_1 = \frac{nRT}{V} = \frac{\rho RT}{\mathcal{R}V} = \frac{3MRT}{4\mathcal{R}\pi r_1^3}
\]

Where \( \rho \) is the mass density. Entering our values for \( M, T, \) and \( r \), we obtain \( P_1 = 1.08 \times 10^{11} \) Pa. We will also suppose that the sphere isn’t getting any bigger over time. It is but for now we’ll say it isn’t. We increase the sphere’s radius by \( \sqrt[3]{{1.01}} \), giving a volume increase of one percent.

Work is performed against gravity:

\[
U_r = U_1 - U_2 = -\frac{3GM^2}{5} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)
\]

Where \( U_1 \) and \( U_2 \) are the gravitational potential energies at radii \( r_1 \) and \( r_2 \) respectively. Entering the values for \( M, r_1 \) and \( r_2 \) we find that \( U_r = 2.92 \times 10^{13} \) J. The internal kinetic energy loss \((-E)\) is, however, much greater than \( U_r \):\(^7\)

\[
W = -E = \left( \frac{3}{2} \right) P_1 V_1 \left( \frac{V_2}{V_1} \right)^{-\frac{2}{3}} - 1
\]

Where \( W \) has the classic meaning of work performed by the gas, \( P_1 \) is the internal pressure before expansion, and \( V_1 \) and \( V_2 \) are the before and after volumes of the sphere respectively. \( V \) and \( P \) can be calculated from (7) and (8). Entering these into (10) gives \( E = 1.16 \times 10^8 \) J. This is \( 10^{20} \) times as much energy released as absorbed. The excess \((E)\) is now outward, radial kinetic energy:\(^8\)

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\(^6\) We define the increment as \( \frac{r_2-r_1}{r_1} = \frac{\Delta r_i}{r} \). This is different from the step \((\Delta r_i)\), used for numeric integration.

\(^7\) This and the other thermodynamic expressions are found in many textbooks and, eg, Wikipedia.
\[ E_k = E + U_r = E + (U_1 - U_2) = \left(\frac{3}{2}\right) P_1 V_1 \left(\frac{V_2}{V_1}\right)^{-\frac{2}{3}} - \frac{3GM^2}{5} \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \]  \hspace{1cm} (11)

Gravity loss is negligible and \( E_k = E \approx 10^8 \text{ J} \). The internal pressure drops to a new value, \( P_2 \):

\[ P_2 = P_1 \left(\frac{V_2}{V_1}\right)^{-\frac{5}{3}} \]  \hspace{1cm} (12)

Eq. (12) gives \( P_2 = 1.06 \times 10^{-11} \text{ Pa} \). Dividing \( E_k \) by \( V_2 \) gives the increase in expansion pressure (\( \Delta P_E \)):

\[ \Delta P_E = \frac{E_{k_2} - E_{k_1}}{V_2} \]  \hspace{1cm} (13)

Our sphere was static to start so \( E_{k_1} = 0 \). Our expanded sphere has \( \Delta P_E = 1.06 \times 10^{-13} \text{ Pa} \), or 1% of \( P_2 \). It’s important to emphasize that \( P_E \) does not add to \( P_2 \),\(^8\) but is instead a vector quantity which results in radial increase only. Each atom is moving in a straight line away from the center, like a bunch of tiny rockets blasting away from their despoiled planet. It helps to ignore \( U_i \) to properly visualize this. Expansion pressure already existed in the sphere since the Universe has been expanding all along.

The temperature drop is given as:

\[ T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{2/5} \]  \hspace{1cm} (14)

The temperature drops from 2971 to 2951 K or 0.7%. This can be compared to the CMB-derived temperature: \( (2.7255) \times (1090/3^{3/4} \times 1.01) = 2961 \text{ K} \), or 0.3% for \( r = 1 \rightarrow \sqrt[3]{1.01} \text{ au} \). The gas is cooling faster than the photons. Eq. (14) is also used to verify the consistency of the calculations.\(^9\)

The linear rate of expansion, or radial velocity (\( v_s \)) of the sphere is:

\[ v_s = \frac{2E_k}{M} \]  \hspace{1cm} (15)

Direct use of (15) ignores the fact that the sphere is already expanding and \( E_{k_1} \gg 0 \), so \( v_s \) is inaccurate and quite low. We can get corrected values for \( v_s \) at an instant in time \( \partial t \) by modeling (11), using a small increment \( \Delta r/r = 10^{-9} \) and increasing \( r \) independently.

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8 Since one of our assumptions is thermal equilibrium, \( P_2 \) remains a state function.

9 The internal energy of an ideal monatomic gas is given as \( U_i = \frac{3}{2} \rho RV \). The new internal energy \( U_i' \) can be found with the \( T_2 \) value from (14) and the new \( \rho \) and \( V \) values. The residual error \( U_i - (U_i' + E) \) is exactly zero to the limit of the spreadsheet.
First we define the gravity ratio ($X$):

$$X = \frac{u_2 - u_1}{E}$$

With our above $T$, $\rho$, and $\Delta r/r$ held constant, we increase $r$ stepwise. The mass rises and $v_s$ falls until the adiabatic radius, or endpoint ($r_e = 1.2732 \times 10^{17}$ m),\(^ {10}\) is reached, where $X = 1$ and $\frac{\partial E_k}{\partial r} = 0$. This adiabatic sphere conserves energy around the central atom.

The cutoff ratio ($X'$) is defined as:

$$X' = \frac{-u_r}{E_k} = \frac{u_2 - u_1}{E_k} \leq 0.00001$$

Below the cutoff radius ($r_c$), gravity can be neglected and (11) simplifies to (10). The choice of $X' < 10^{-5}$ is best seen graphically. Figure 1 shows a semilog map of $v_s$ vs. $r$ from $10^{12}$ to $10^{16}$ m at 2971K and $\rho_{1089}$.\(^ {11}\) Below $r_c$ ($= 4 \times 10^{14}$ m) $v_s$ is constant to 2 ppm, giving the same $(E/M)$:\(^ {12}\)

$$\frac{E}{M} = \frac{\partial E}{\partial M} = \frac{\partial V}{\partial M} = \left(\frac{RT}{K}\right) \frac{\partial E}{\partial V} = \frac{RT}{K} \left(\frac{\partial E}{P \partial V}\right)$$

This gives the initial radial velocity ($v_i$):

$$v_i = \sqrt{\frac{2E}{M}} = \sqrt{\frac{2RT}{K}} = \frac{2(0.31446)(2971)}{0.0012475} = 6293 \text{ m/sec}$$

Above $r_c$, gravity takes its toll, and $v_s$ drops, reaching zero at $r_e$. The radial velocity ($v$) of the adiabatic sphere is the sum of the contained shells:

$$v = (v_i) \left(\frac{r_c}{r_e}\right) + \sum_{s'=1}^{n} v_{s'} \frac{\Delta r_s}{r_e} \rightarrow (v_i) \left(\frac{r_c}{r_e}\right) + \frac{1}{r_e} \int_{r_e}^{r_c} v_{s'} \, dr$$

Where $\Delta r_s$ is the step, $v_{s'} = v_i \left(\frac{v_s}{v_{s0}}\right)$, and $v_{s0}$ is the constant value of $v_s$ at $r \ll r_c$. For all $r < r_c$, $v = 19.8$ m/sec. That leaves the remaining 99.7% of $v$ to be found. Integration of (11) is problematic so we resort to a map (Figure 2) whose cumulative value at $r/r_c = 1$ is 4973.2 m/sec. Adding 19.8 to this gives 4993 m/sec, or (0.7934±0.0001) $v_i$. This proportion $K$ shows little change with input values; $v/v_i$ is constant to nearly the 4th decimal place.

In the special case of atoms separated by $2r_c$, their adiabatic spheres are joined at a tangent point and they are moving apart at $2v$. More generally, for any two atoms separated by a distance $r$, their recession rate $v_r$ is:

\(^{10}\) The endpoint $r_e$ is found on a spreadsheet by convergence of $r$ around $X = 1$.

\(^{11}\) Both $r_e$ and $r_c$ are independent of $\Delta r/r$ over a wide range.

\(^{12}\) For isoentropic adiabatic expansion, $\partial E = P \partial V$. 
\[ v_r = K \frac{r}{r_e} v_i = K \frac{r}{r_e} \sqrt{\frac{2RT}{\mu}} \]  

(21)

And the comoving gas-derived Hubble value \((H_g)\) is:

\[ H_g = \frac{v_r}{r} = \frac{K v_i}{r_e} = \frac{K}{r_e} \sqrt{\frac{2RT}{\mu}} \]  

(22)

Which at \(z = 1089\) and \(K = 0.7934\) gives \(H_g = 3.87 \times 10^{-14}\) sec, or \(17955H_0\). This is 78% of the value found from (1). If we set \(K = 1\), we get \(H_g = 4.94 \times 10^{-14}\) sec, or \(22630H_0\). This is 99% of the value found from (1). It appears that isoentropic treatment of the gas at \(v_i\) is more consistent with the \(\Lambda\)CDM model than treatment at \(v/v_i\):

\[ H_g = \frac{v_r}{r} = \frac{v_i}{r_e} = \frac{1}{r_e} \sqrt{\frac{2RT}{\mu}} \]  

(23)

Use of (23) at varying \(T\) from 100 to 4000K at \(z = 1089\) gives the same result to five decimal places every time. More extensive input change reveals that (23) has no temperature dependence. The model is also independent of molecular weight. A Universe made of xenon atoms (0.131 Kg/mole) returns the same result as our primordial mix. Other than the CDM adjustment, the mass density \(\rho\) is the only remaining variable in the model, and it’s a function of the cosmic redshift \(z\). This makes \(z\) the sole independent variable.\(^{13}\)

We now examine entropy, by looking at \(\partial V\). To do this, we use a two-increment model at \(r << r_c\), where the expansion is first performed reversibly and then freely, giving \(r_1-r_3\) and \(U_1-U_3\). The first increment \(r_2-r_1\) generates \(v_i\). Free expansion of the sphere \(r << r_c\) has no gravity loss and occurs simultaneously with reversible expansion, so in the second increment \(r_3-r_2\) the atoms coast along at \(v_S\) for the same time. The two increments are equal. From (3):

\[ \partial V' = \partial V_S + \partial V_T = 4\pi r^2 \partial r + 4\pi r^2 \partial r = 2\partial V \]  

(24)

And from (18):

\[ \frac{E}{M} = \frac{\partial E}{\partial M} = \frac{\partial V'}{\partial V} \frac{\partial E}{\partial V} = \frac{\partial V}{\partial M} \frac{\partial E}{\partial V} = \frac{RT}{\mu} \frac{\partial E}{\partial V} = \frac{RT}{\mu} \left( \frac{\partial E}{\partial V} \right) = \frac{RT}{\mu} \]  

(25)

Which again gives (18).

Let’s look at the free increment \(r_3-r_2\). Near the \(z = 1089\) endpoint (1.27 x \(10^{17}\)m) the initial radial kinetic energy of the sphere \(E_i = \frac{1}{2}M(v_i)^2 = 4.7 \times 10^{40}\) J. At \(\frac{\Delta r_1}{r} = 10^{-9}\) the loss to gravity \(U_3-U_2\) is \(2.8 \times 10^{32}\) J, or about \(6 \times 10^{-9}E_i\), giving a new \(E'_i \approx E_i\) and \(\frac{r_3' - r_2'}{r_3 - r_2} = \frac{E'_i}{E_i} \approx 1\). At a much

\(^{13}\) The density \(\rho\) can be changed, within constraints, if the Universe is partitioned into regions of varying density.
higher increment $\frac{\Delta r_i}{r} = 0.001$, $U_3-U_2 = 0.006$ $E_i$ and $\sqrt{\frac{E_i'}{E_i}} = 0.93$, so there is some model-based dependency best addressed by keeping the increment low. We can more accurately see the effect of entropic expansion by examining a linear map of $H_g'/H_\Lambda$ vs. $z$ from $z = 999$ to 19 using equations (23) and (1) (Figure 3). The entropic expansion at $z = 1089$ is arbitrarily set to zero. The influence of $\Omega_\Lambda$ is negligible in this $z$ range, and the variance $k_z$ is well fit by (26):

$$k_z = \frac{H_\theta}{H_\Lambda} = 1.1327 - 1.619 \times 10^{-4}z + 2.603 \times 10^{-8}z^2$$

(26)

We include $k_z$ in (23), giving (27):

$$H_{g'} = \frac{H_\theta}{k_z} = \frac{v_c}{k_zr_e}$$

(27)

Figure 4 shows a map of both $H_g/H_\Lambda$ and $H_{g'}/H_\Lambda$ vs. $z$ from $z = 10$ to 0, where curvature is important. At $z = 0$, $H_{g'/H_\theta} = (H_{g'/H_\theta})(1.1327)^{-1} = (0.63)(1.1327)^{-1} = 0.56$. We make another correction for the mass proportion of the Universe that today is not gravitationally bound: $\approx 85\%$. This new density $\rho'' = 0.85\rho$ gives an adjustment of 0.92 and a new ratio $H_{g''}/H_\theta = (0.92)(H_{g'/H_\theta}) = 0.51$. This is more than the sum of $\Omega_m + \Omega_{cdm} + \Omega_{rad} = 0.31$, which means that $H_{g''}$ is responsible for some of $H_\theta$ ascribed to $\Omega_\Lambda$ in (1). The model outlined herein does not further address that issue. It does, however, demonstrate that a known source of energy, $U_i$, can be used to account for much of Universal expansion. While a dark energy field cannot be excluded as a source, the results of this paper suggest that more mundane sources, e.g., kinetic energy stored in gases or plasma, make a substantial contribution.

The presented model only superficially addresses the issue of entropic expansion. Proper treatment of entropy using classic gas thermodynamic principles, and applied to cosmic conditions in more recent times, may yield meaningful results.

The author declares no competing interest.
Figure 1

Uncorrected sphere radial velocity vs. radius at z = 1089
Figure 2

Corrected sphere radial velocity $v_r$ vs. $r/r_e$ at $z = 1089$

$T = 2971$ K, $d = 5.45 \times 10^{19}$ Kg/m$^3$

$r_e = 1.2895 \times 10^{17}$ m
Figure 3

$H_g/H_\lambda$ vs. cosmic redshift $z$ from $z = 19$ to 999

$y = 1.133 - 0.0001619x + 2.603 \times 10^{-8}x^2$

$H(g)/H(\lambda)$ vs. cosmic redshift $z$ from $z = 19$ to 999
Figure 4

Uncorrected and corrected Hubble ratios at $z = 0$ to 10

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- cosmicgasworkbook20211012.xlsx