Review on Magnetohydrodynamic flow of nanofluids past a Vertical plate under the influence of Thermal Radiation

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Abstract. Nanofluids are the fluids containing nano materials, with interesting properties, and the distinctive features have shown unprecedented potential for many Industrial applications. Nano fluids have been receiving great attention in recent years for heat transfer applications in industrial processes. In literature, the highly nonlinear governing equations involved in the study of MHD nanofluids flow with thermal radiation past a vertical plate under varying physical nature are solved using numerical and analytical schemes. It is noted that due to the complexity of the governing equations, numerical schemes are more preferable than analytical schemes. The authors have taken effort in registering various forms of governing equations involved in the above studies, its impact on velocity and temperature field. This paper also tries to identify the research gaps and pave way for future research.

1. Introduction

The study on the nanofluid flow on a Vertical plate for last two decades reveals the importance of nanofluid in the practical applications. The Heat transfer fluid, called nanofluid, initiated by Choi [1] used base liquid with suspended solid nanoparticles, that they express increased heat conductivity and the thermal coefficient due to convection compared to the core fluid. The thermal conductivity of nanofluid was studied by Boungiorno [2] and he proposed the model with Brownian diffusion and thermophoresis. Generally, the performances of thermal systems are enhanced with nanofluids, thereby as a result enhancing the heat transfer rate to a significant extent. Nanofluids are combination of a core fluid such as H₂O, (CH₂OH)₂ or oils with metals, metallic oxides and carbon. The commonly used nanoparticles in metals (Ag, Cu, Au, Fe) in metallic oxides (Al₂O₃, TiO₂, SiO₂, ZrO₂) and in carbon (diamond, graphite, CNT). Most electronic equipment as well as automobile sector employs the use of nanofluids to decrease thermal resistance. They work as effective coolants and are hence used predominantly in heat transfer machinery, cooling systems and radiators. Next generation fluids are essentially used as a cooling agent due to its high heat properties which helps them to act as a cooling agent in electronic devices and radiators. Nanofluid improves properties such as thermal conductivity, viscosity, specific heat and density of base fluid in which nanoparticles added. Wang and Mujumdar [3] theoretically and experimentally discussed the nanofluids preparation and investigated its thermal conductivity and viscosity. Studies also revealed that in industrial systems, the next generation fluids based on metal oxide nanoparticle have physical properties that by considering them as promising fluids whereas high heat flux takes place. Several researchers were attracted by the nanofluids, due to their enhanced heat transfer properties. These engineered fluids also have adjustable properties including transcalency by varying particle concentrations. The thermal conductivity and viscosity of nanofluids has been reviewed by Trisaksri and Wongwises [4]. Bejan [5] reveals that the heat transfer processes are accompanied by thermodynamic inevrsibility or entropy generation. Heat and mass convection transport on nanofluid flow over a vertical plate are more appreciable in recent years.

For the past two decades researchers are fond of working with nanofluids flow over a vertical plate under various physical conditions and obtained many path breaking results that finds applications.
in many industrial products. This chapter is devoted in highlighting some salient results that will be a starting point many future researches. Gamal El-Din A. Azzam [6] Studied on the radiation effects on MHD Convective steady laminar boundary layer flow, the governing equations were solved numerically and it is noted that high temperature differences with uniform magnetic field can give an acceptably accurate velocity distribution.

E. Magyari, D.A.S. Rees [7] studied the effect of viscous dissipation on boundary layer flow on a vertical plate and the results shows that for self-similar flows, down flow is possible for all non-negative values of the temperature exponent but up flow exists only above a critical value and also the paper investigates the heat characteristics free convection flows analytically and numerically. C.J.Toki [8] studied the free convection mass transfer flow on a vertical porous plate and the paper conclude that velocities, temperature near the porous plate is smaller than the non-porous plate, the analytical solution obtained and the paper also leads to study of vertical air flows into the atmosphere. C.J.Toki [9] investigated on the unsteady flow on a vertical oscillating porous plate, here the author considered viscous and incompressible fluid and the governing equations were solved analytically and the solutions is obtained for arbitrary Prandtl numbers.

O.D. Makinde and P.O. Olanrewaju [10] studied the effects of thermal buoyancy on the boundary layer about a vertical plate and the governing equations were solved numerically. The thermal boundary layer thickness along the plate is reduced by the combined result of increasing Prandtl number and the Grashof number. M.Narahari and A Ishak [11] studied the radiation effects on free convection flow near a moving vertical plate, the problem is analysed with three cases of flow due to: (i) motion with uniform velocity (ii) uniformly accelerated motion (iii) exponentially accelerated motion of the plate. It is observed in the results that as the radiation parameters increases, the temperature and velocity decreases, while the skin friction and the Nusselt number at the plate increases. W.A. Khan, A. Aziz [12] studied the flow of fluid with nano characteristics over a vertical plate on heat flux and the solution were solved numerically and the paper concludes that the velocity increases initially at the vertical component attains a peak and falls down to zero at hydrodynamic boundary layer.

M.Turkyilmazoglu, and I.Pop [13] in the year 2012 investigated the radiation effect on flow of some nanofluids over vertical plate, here the authors considered the unsteady convection flow and the validity of obtained numerical solutions are verified by the obtained analytical solution.

Convective flow of EG-nimonic 80a nanofluid past a vertical plate under radiation effect was investigated by N. Sandeep et al. [14]. In their study the nanofluid was prepared by adding nano grains such as heat treatable nickel chromium alloy with ethylene glycol as core fluid and the controlling equations were solved numerically and it was found that the thermal heat coefficient increases with increase in thermal radiation and change of particle shape and the shape of nanoparticle does not affect the velocity of nanofluid. The process of heat and mass transfer of combined effect on convection flow over a vertical plate in a porous medium was studied by Farhad Ali et al. [15]. In this paper the governing equations were solved analytically, the motion of the fluid is described and also the authors discussed the effects of different parameters and chemical reaction parameter on velocity distributions, temperature and concentration profiles.

W.A.Khan et al.[16] studied the MHD boundary layer flow of nanofluid containing gyrotactic microorganisms past a vertical plate with Navier slip. The authors applied similarity transformation, the Brownian motion and thermophoresis effects with Navier slip condition were accounted for and similarity solutions were found for dimensionless quantities. Meraj Mustafa et al. [17] analysed the Nonlinear Radiation Heat Transfer Effects in nanofluid over a vertical plate with viscous dissipation in the energy term, they used numerical scheme to obtain the velocity and temperature profiles. Also, the Mathematical model used by the authors incorporates the viscous dissipation in the energy equation. D.Srinivasacharya and Ontela Surrender [18] investigated the double stratification effects on the nanofluid flow embedded in porous medium over a vertical plate and the numerical solutions were
obtained, the model used by the authors resulted in the heat transfer reduction with the influence of Brownian motion parameter and thermophoresis effect. M. Ghalambaz et al. [19] studied the nanofluid flow on heated vertical plate impressed in a porous medium. In this paper the numerical solution were obtained. The end result being an increase in temperature profile while a decrease in concentration profiles as the Brownian motion parameter is increased. Machireddy Ganaeswara Reddy [20] numerically studied the influence of convection boundary layer flow of a next generation fluid by taking the effects of Brownian motion and thermophoresis. The result conclude that the reduced Nusselt number increases as the Prandtl number and also that the increasing radiation parameter decreases temperature.

MHD Nanofluids Flow in Porous Medium with radiation and Soret effects on a moving vertical plate were investigated by the authors C S K Raju et al [21] and they considered Cu-Ethylene glycol, CuO-Ethylene glycol nanofluids to analyse the influence of magnetic field, radiation and soret parameters of a nanofluids flow. K Javaherdeh et al. [22] studied the convection heat and mass transfer in MHD fluid on a moving vertical plate in a porous medium with variable surface temperature and concentration. The governing equations were solved numerically and the velocity, temperature and concentration profile along with physical parameters like Grashof number, porosity, magnetic field under the influence of transverse magnetic field were investigated. Also, the study says that a resistive force was formed by magnetic field whereas the flow was opposed by the force, which reduces the rate of heat and mass transfer. G.S.Seth, S.M. Hussain, S.Sarkar [23] studied the convection flow past an accelerated moving vertical plate with ramped temperature and surface concentration in the presence of thermal and mass diffusions. The governing equations were solved analytically. This paper concludes that heat absorption enhanced the rate of heat transfer at the plate and it gets decreased for a plate which is isothermal.

Hari R Kataria and Akhil S. Mittal [24] studied the effect of thermal radiation and magnetic flux density of nanofluids on unsteady hydromagnetic boundary layer flow past an oscillating vertical plate. The governing equations were solved analytically. Here the authors considered water as base fluid with copper and silver as nanoparticles. The results conclude that momentum of the nanofluids get increased by increasing the magnetic field and radiation. The temperature decreases and momentum increases with increase of radiation parameter.

B.Ganga et al. [25] studied MHD boundary layer nanofluid flow with internal heat generation/absorption, viscous dissipation effects on a vertical plate. The governing equations were solved analytically and numerically. The paper concludes that heat generation and magnetic field accelerates the temperature and decelerates the nanofluid solid volume fraction profile due to the presence of viscous dissipation. The velocity, temperature, and the solid volume fraction of the nanofluid profiles in corresponding boundary layer depend on eight dimensionless parameters. Farhad Ali et al. [26] analysed the unsteady nanofluid flow of Brinkman type over a vertical plate in the influence of magnetic field where the medium is full of pores. The governing equations of this model were solved analytically. The results of this paper concluded that the velocity of the fluid is decreased by increasing Brinkman parameter and nanoparticles volume fraction.

V.Rajesh et al. [27] studied the transient MHD flow and heat transfer of nanofluid on a semi-infinite vertical plate. The governing equations were solved numerically, also the paper results that Cu – water nanofluid attained an improved heat transfer rate when compared with the other nanofluids. P.V.Satya Narayana and B. Venkateswarlu [28] investigated on the heat and mass transfer of MHD nanofluid flow on a vertical porous plate in rotating system. The chemical reaction and heat source effect on free convection flow was studied by the authors and the governing equations were solved analytically. Due to the chemical reaction, the concentration of fluid has decreased, this is because of the presence of soret numbers and nanoparticles in the flow and also nanofluid has a dramatic effect on thermal conductivity.
Harouna Naroua and Bachir Moussa Idi [29] studied the computer simulation on MHD flow for a vertical plate through a porous medium. In this paper the results are obtained numerically and it is noted that velocity and temperature increase with time. Abid Hussan et al. [30] studied the flow of nanofluid on accelerated vertical plate and the exact solutions are obtained analytically using Laplace transform method and the results in the problem conclude with comparison of different types of nanofluids. Amir Malvandi [31] investigated on Film boiling of magnetic nanofluids (MNFs) over a vertical plate, and the solution here obtained numerically. Here the author used smaller nano particles with higher nano particles the velocity profile inside the film is increased and the velocity of MNFs is reduced by magnetic field.

Ashwin Kumar E N et al. [32] investigated the influence of injection on nanofluids and the injection velocity effects are analysed in stagnation point flow on copper nanofluids over permeable surfaces. The injection on the magnetic field greatly influenced the heat transfer characteristics in the copper nanofluid. T Sravan kumar and B Rushi kumar [33] studied on MHD free convective nanofluid flow on a moving vertical plate and the solutions were obtained analytically by Laplace transform. Also the values of volume fraction parameter increases, the temperature get increased and it is decreased with increasing Prandtl number Pr.

Ali Azhar et al. [34] investigated on some fractional nanofluids over a moving vertical plate on free convection flow, the governing equations were solved analytically using Laplace transform technique and thickness of thermal boundary layer for fractional nano fluids is thinner than that of ordinary fluids. Marneni Narahari et al. [35] studied the heat flux effect on multi-phase next generation fluid past a vertical plate and calculated the numerical solution using crank Nicolson method. They compared the solution obtained by verifying the local Nusselt number results with the correlations results and found good agreement between the results.

S.Anuradha and M yegammai [36] studied the MHD flow of nanofluid with the effects of binary chemical reaction and activation energy on vertical plate and equations were solved numerically and the authors also designed a evaluative model for a MHD boundary layer flow for two dimensional flow. Wafula Maurine Maraca [37] investigated on Similarity Solution of Boundary layer flow of nanofluids. In this paper the boundary value problem was solved numerically and the author conclude that the velocity and the temperature flow direction of nanofluid decreases with increasing unsteadiness parameter. Marneni Narahari [38] studied on a semi -infinite vertical plate in water based nanofluid, the author used crank Nicolson numerical scheme in the paper and the results are validated by comparing the local nusslet number with the correlation results.

Abdul Rauf and Yasir Mahsud [39] studied the two-dimensional flow with time exponential temperature on nanofluid and the controlling equations is solved by modified simple function algorithm. Numerical computations are established in the paper. Hiramnony et al. [40] determined the combined effects of thermal radiation and internal heat generation of nanofluid on convection flow through a porous medium. The authors solved the governing equations numerically. The paper concludes that the effects of various physical properties were analysed using boundary conditions. Zecheng and jiepeng [41] investigated on the study of nanofluid using wavy film by considering nano particle migration, here the authors used IBL method to get the results, and it shows that the particle migration plays the significant role.

G. C. Shit and S. Mandal [42] studied on the entropy analysis of Casson nanofluid flow and the authors solved the equations numerically and the momentum boundary layer thickness is reduced and it is found that is higher than Newtonian fluid. Marneni Narahari et al. [43] studied on unsteady free convective nanofluid flow past a moving vertical plate, here the thermophoresis and the Brownian diffusion effects using Buongiorno’s model. The resulting non- linear PDE were solved numerically. This paper reveals that the spike in Brownian diffusion factor reduces the nanoparticle migration, meanwhile the temperature and velocity were increased by increasing the thermophoresis parameter.
Aamir Hamad et al. [44] studied on MHD Blasius flow of radiative Williamson nanofluid over a vertical plate. To develop the mathematical model of Williamson nanofluids the authors employed the Brownian motion and thermophoresis, the governing equations were transformed into ODE and the obtained equations were solved numerically using MATLAB. The result conclude that the rate of heat transfer is decreased by Brownian motion parameter.

Chebbi Agnes Jeptoo [45] studied the MHD flow of nanofluid past a heated vertical plate due to gyrotactic microorganisms. The study on the magnetic field effects on boundary layer flow with heat and mass transfer of a nanofluid was investigated numerically. It is also observed that increasing the magnetic field strength retards the thermal boundary layer thickness. M Veera Krishna and Ali J Chamka [46] studied effects of Hall and slip on MHD Rotating boundary layer flow of nanofluid past a vertical plate embedded in a porous medium. In this paper, the radiation absorption parameter and Dufour parameter leads to increase the thermal boundary layer thickness.

B.P.Gang and Shipra [47] investigated on free convection viscous fluid flow and mass transfer flow near a vertical plate in the presence of heat source/ sink the governing equations were solved analytically, here the movement of the plate and uniform velocity discussed in detail. Mushtaq Ahmad et al. [48] analysed the natural convection flow of Casson nanofluid over an infinite plate which is vertical for analytical solution and the author explained the nanofluid drilling in detail of the regulations and importance of fractional derivative for heat transport. The result obtained for temperature as well as velocity are also compared with the existing literature. M. Anil Kumar [49] studied the Thermal radiation influence on MHD heat transfer free convective nanofluid flow over an impulsively started vertical plate, the governing equations were solved numerically. Also the result concludes that the radiation parameter value increases, the velocity and Temperature profiles get increased.

The authors have made a conscious effort to collate various studies conducted on nanofluid flow of a vertical plate in the influence of external magnetic flux density for various physical conditions, which will help future researcher to identify possible research gap in this domain.

This paper aims in providing a guide for researchers who undertake research on unsteady nanofluids flow with various physical conditions of a vertical plate under the influence of magnetic field which finds numerous applications in industries where heat transfer enhancement is very important in the final quality of the product.

2. Physical Properties

Nanofluids are manmade, ingenious fluids that perform a major role in transferring heat through convection. Transfer of heat due to free convection finds numerous applications in energy collectors that use sun as a source of energy, systems relying on earth’s temperature, heat exchange in electronic devices, cooling of reactors for internal maintenance, greasing agent and space technology etc. Their heat transfer enhancement is primarily because of a small collection of nanoparticles that completely alter or add to the thermo-physical properties of the core fluid.

Nanofluids have moderately higher viscosity than their core fluids and probably need greater pumping power to have the desired thermal performance. They have flow properties equivalent to the core liquid and have small amount increase in the turbulent pressure loss. The increase in thermal conductivity can be recompensed by an increase in viscosity, decrease in effective specific heat. To achieve superior results in practical implementation processes, heat transfer fluids should be prepared to increase the heat transfer coefficient without penalizing the pressure loss, and this need an accurate selection of particle shape, size, materials and concentrations. Pankaj kumar et al. [50] in their paper provides an understanding about thermo-physical properties of nanofluids such as (thermal conductivity,
coefficient of heat transfer, density, viscosity, heat capacity etc) and properties characteristic with respect to nanoparticle concentration. Many researchers have developed correlations for the determination of thermo-physical properties of nanofluids. Some important and commonly used property relations for nanofluids are presented below.

2.1 Thermal conductivity

Some postulate that the thermal conductivity of nanofluids is because of the nanoparticle’s Brownian motion which produces micro-mixing. The main focus in including nanoparticles in to the base fluid is to study the importance of thermal conductivity of the proposed mixture. It is evident from the literature that many studies have been carried on this property leading to different models. The table 1[51] below list some significant models that are frequently used by the researchers.

| Various Representations | Expressions |
|-------------------------|-------------|
| Maxwell [52]            | \( k_{nf} = k_f \left( \frac{k_p + 2k_f - 2\varphi(k_f - k_p)}{k_p + 2k_f + \varphi(k_f - k_p)} \right) \) |
| Hamilton and Crosser [53]| \( k_{nf} = \frac{k_s + (s - 1)k_f + (s - 1)\varphi(k_s + k_f)}{k_s + (s - 1)k_f - \varphi(k_s - k_f)} k_f \); \( s = 3/\psi \) |
| Yu and Choi [54]        | \( k_{nf} = k_f \left( \frac{k_p + 2k_f - 2\varphi(k_f - k_p)(1 + \eta)^3}{k_p + 2k_f + \varphi(k_f - k_p)(1 + \eta)^3} \right) \) |
| Patel et al. [55]       | \( k_{nf} = k_f \left( 1 + \frac{k_p \varphi d_f}{k_f d_p (1 - \varphi)} \right) \left( 1 + \frac{2k_b T d_p}{\pi \alpha f \mu f d_p^2} \right) \) |
| Jang and Choi [56]      | \( k_{nf} = k_f \left( (1 - \varphi) + B k_p \varphi + 18 \times 10^6 \frac{df}{dp} k_f Re^2 d_p Pr \varphi \right) \) |
| Yimin, Li, and Hu [57]  | \( k_{nf} = k_f \left( \frac{k_p + 2k_f - 2\varphi(k_f - k_p) + \beta_p \varphi(c_p) f}{k_p + 2k_f + \varphi(k_f - k_p)} \right) \frac{2k_f}{k_f} \left( \frac{k_b T}{3\pi \tau c \eta} \right) \) |
| Mintsa et al. [58]      | \( k_{nf} = k_f (1.72 \varphi + 1) \) |
| Stationary [59]         | \( k_{nf} = k_f \left( 1 + \frac{k_p \varphi d_f}{k_f d_p (1 - \varphi)} \right) \) |
| Chon et al. [60]        | \( k_{nf} = k_f \left\{ 1 + \frac{64.7 \varphi^{0.7640} \left( \frac{d_f}{d_p} \right)^{0.369} \left( \frac{k_p}{k_f} \right)^{0.7476} Pr^{0.9955} Re^{1.2321}}{1.0} \right\} \) |

2.2 Thermal coefficient due to convection

Due to the presence of immersed nano grains, we see an enhancement in thermal coefficient due to convection that strengthens the turbulence of the core fluid. The mass, momentum and heat transfer in next generation fluid was modelled as a non-homogeneous, 2 component 4-equation correlation
equation by Buongiorno [2]. He concluded that the two special characters of next generation fluids are namely the Brownian diffusion and thermophoresis.

2.3 Viscosity

Viscosity is one of the characteristic properties related with temperature that has a vital role in implementation of nanofluids in industries that deal with chemical and petroleum domains. Viscosity influences the flowing substance effectiveness in greasing, pumping efficiency, thermal energy transfer, production of oil that are important while oil recovering activities. Many researchers analysed the resistance of a water based next generation fluid to its shear forces. They also registered the importance of nano particles viscosity, nano particles volume fraction, its size, its shape and temperature of the core fluids. Most of the demonstrations were carried on water based nano fluids such as Al$_2$O$_3$, TiO$_2$, SiO$_2$, and CuO. Sundar et al. [61] registered that there exist, a linear relationship between nanoparticle volume fraction and nanofluid viscosity. They also observed that resultant viscosity gets decreased by increasing temperature and the nano particle size.

2.4 Density

Particle volume fraction ($\phi$) has a direct relation with density underlying the fact that density increases in linear way with respect to volume fraction. Further it is listed in literature that density has an inverse proportional relationship with temperature that happens to be non-linear. This is so due to variation in heat expansion coefficient between core fluid and nano particles. The density of nanofluids can be simply estimated as [62]

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p$$

where $\phi$ is the volume fraction of dispersed phase, $\rho_f$ is the density of the base fluid and $\rho_p$ the density of nanoparticles.

2.5 Specific heat

The heat energy’s measure that plays an important role in the rate of heat transfer among the next generation fluid is the specific heat. Specific heat $C_{nf}$ for the given volume concentration of nano grains in the base fluid can be defined [63] as $C_{nf} = (1 - \phi)C_f + \phi C_p$. Also, The Specific heat of nanofluids decreases with increase in the fraction volume of nanoparticles and also it increases with increase in the nanofluids temperature.

2.6 Electrical Conductivity

In nanofluids, the amount of voltage that is required for an electric current to deform a liquid continuously is a prime feature. For the nanofluids this phenomenon is formulated as [64]

$$\sigma_{nf} = \sigma_f \left( \frac{\sigma_p}{\sigma_f + 2} - \frac{\sigma_p}{\sigma_f + 2} \phi + 1 \right)$$

Typical thermal parameters for nanoparticles and base fluids are listed below. Table 2 [65]
2.7 Dynamic viscosity

In the literature, the measure of fluid’s molecular resistance known as dynamic viscosity is represented by various expressions which are listed in the table 3 below [66].

Table 3. Various representations for fluid’s molecular resistance (Dynamic Viscosity) of nanofluids

| Representation                  | Expressions                                      |
|--------------------------------|--------------------------------------------------|
| Einstein Representation [67]   | $\mu_{nf} = (2.5\phi + 1)\mu_f, 0<\phi < 0.05$  |
| Brownian Representation [68]   | $\mu_{nf} = (1 + 2.5\phi + 6.17\phi^2)$         |
| Brinkman Representation [69]   | $\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$      |
| Pak and Cho’s correlation [70]  | $\mu_{nf} = \mu_f(1 + 39.11\phi + 533.9\phi^2)$ |
| Maiga Representation [71]      | $\mu_{nf} = \mu_f(1 + 7.3\phi + 123\phi^2)$    |
| Nguyen Representation [72]      | $\mu_{nf} = \mu_f(1 + 0.025\phi + 0.015\phi^2)$ |
| For hybrid nanofluids [73]      | $\mu_{nf} = \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$ |

3. Mathematical Formulation

The physical nature of the problem allows us to deal with 2-D, flow taking place along constant streamlines, density constant and viscous unsteady MHD nanofluid flow over a vertical plate. It is assumed that plate is placed in fluid flowing with uniform free velocity $U_\infty$. Let the origin of the coordinate system coincide with the leading edge of the plate and the and the y axis normal to the plate. $T$ is the temperature of the fluid; $T_w$ and $T_\infty$ are the constant temperature of the plate and ambient temperature respectively. The basic unsteady momentum and heat energy equation according to the model for next generation fluids satisfying Boussinesq approximation are as follows [74]:
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(1)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_n f \frac{\partial^2 u}{\partial y^2} + \frac{(\rho \beta)_{n f}}{\rho_{n f}} g(T - T_\infty) - \sigma B_0^2 \frac{u}{\rho_{n f}} \]  

(2)

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{n f} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \frac{1}{(\rho c_p)_{n f}} \]  

(3)

Corresponding boundary conditions are

\[ u = v = 0, \quad T = T_w \quad \text{when} \quad y = 0 \quad \text{and} \]

\[ u = U(x) \quad \text{and} \quad T = T_\infty \quad \text{when} \quad y \to \infty \]  

(4)

The velocity elements in the x and y directions are given by \( u \) and \( v \) with respect to time \( t \); \( \nu_{n f} \) is the momentum diffusivity of nanofluid, \( \nu_{n f} = \frac{\mu_{n f}}{\rho_{n f}} \) where \( \mu_{n f} \) and \( \rho_{n f} \) are the effective viscosity and density of nanofluid respectively. \( \sigma \) is the electrical conductivity. \( \alpha_{n f} \) is the thermal diffusivity of nanofluid. \( B_0 \) is a constant magnetic field, \( \beta \) is the coefficient of volumetric thermal expansion and \( g \) is the acceleration due to gravity, and \( q_r = \frac{4 \sigma_s \partial T}{3 k_e \partial y} \) where \( \sigma_s \) is the Stefan-Boltzmann constant and \( k_e \) is the mean absorption coefficient respectively. Whereas in the subscripts \( n f, f \) and \( s \) represent the thermo physical properties of the nanofluids, base fluid and the solid nanoparticles respectively.

The above general equations and boundary conditions pertaining to the MHD nanofluid flow in the presence of magnetic field over a vertical plate under different physical condition that occur in various applications and the governing equations were solved either numerically or analytically to obtain the results.

4. Methods of solution

From the previous knowledge available in the literature pertaining to the flow problem of nanofluids the exact solutions are dependable and important in explaining the physical nature of the problem. The analytical solutions were obtained using Laplace transform, Fourier transform, Homotopy analytic method, Homotopy perturbation method and Laplace transform technique. The solutions obtained using numerical methods as finite difference method, Crank Nicolson method Keller box method and also most of the papers were solved using shooting techniques.

5. Future Direction

The review study demonstrated that nanofluid have good potential in increasing the heat transfer performance of conventional fluids. The literature study also reveals that nanofluids faces the difficulties such as increased axial rise in wall temperature due to degraded specific heat and increase pumping power due to pressure drop. Also, agglomeration has a negative impact on nanofluid’s stability accelerates the deposition of nanoparticles due to gravity and causes a decrease in nanofluids ability to conduct heat. Hybrid nanofluids are newly developed fluids. The idea of using hybrid nanofluid can be studied in upcoming research to improve the characteristics of the basic fluid heat transfer through thermo physical properties of nanomaterial. The significance of hybrid nanofluid in thermodynamic applications is that the hybrid nanofluids have great potential with larger control over the behaviour of the manufactured fluid. Exact solvers can focus more on hybrid nanofluid

6. Conclusion
This article, provides a detailed study of basic governing equations that governs the flow of nanofluid in the presence of magnetic field, radiation effect of a vertical plate and solution type (analytical, numerical or semi-analytical) employed in solving these governing equations. It also provides various unique features of nanofluids that are to be considered while solving the governing equations are also presented in this article. It is understood that heat performance of core fluid can be enhanced due to the inclusion of desired nanoparticles. Even though there is a significant difference in performance with respect to Metallic and non-metallic nanoparticles, researchers’ preference leans towards metallic counterparts due to its increased thermal conductivity of metals. Further, literature review reflects that even though that most of researchers have used various models, it is registered that Buongiorno model is considered as the popular model in the study of nanofluids. The authors also reiterate the fact that more research work can be carried out in the field of MHD flow of nanofluid and hybrid nanofluid with various physical assumptions and its corresponding extremities set-ups.

References:

[1] Choi S U Enhancing thermal conductivity of fluids with nano particles. ASME International Mechanical Engineering congress and Exposition. November 12-17,1995, San Francisco, CA.

[2] Buongiorno J et al. A Benchmark study of Thermal Conductivity of Nanofluids. Journal of Applied Physics, 2009; 106, 1-14.

[3] Bejan A. Study of entropy generation in fundamental convective heat transfer. J. Heat Transf. 1979; 101, 718–725.

[4] V Trisaksri and S Wongwises. Critical review of heat transfer characteristics of nanofluids. Renewable sustainable energy, 2007. Rev.11, 512-523.

[5] Wang X Q & Mujumdar A S. Heat transfer characteristics of nanofluids: a review. International Journal of Thermal Sciences, 2007; 46(1), 1–19.

[6] Gamal El- Din A Azzam, Radiation effects on the MHD Mixed free -forced convective flow past a semi-infinite vertical plate for high temperature differences. Physica scripta, 2002; Vol 66, 71-76.

[7] E Magyari and D A S Rees. Effect of viscous dissipation on the Darcy free convection boundary layer flow over a vertical plate with exponential temperature distribution in a porous medium, Fluid dynamics research, 2006; 38, 405-429.

[8] C J Toki. Free convection and mass transfer flow near a moving vertical porous plate: An analytical solution. Journal of Applied Mathematics, 2008; Vol 75,1-8.

[9] C J Toki. Unsteady free convection flow on a vertical oscillating porous plate with constant heating. Journal of Applied Mathematics, 2009; Vol 76,1-4.

[10] O D Makinde and P O Olanrewaju. Buoyancy effects on thermal Boundary layer over a vertical plate with a convection surface boundary condition. Journal of Fluid Engineering, 2010; 132,1-4.

[11] M Narahari and A Ishak. Radiation effects on free convection flow near a moving vertical plate with Newtonian heating. Journal of Applied Sciences,2011; 11, 1096–1104.

[12] W A Khan and I Aziz. Natural convection flow of nanofluid over a vertical plate with uniform surface heat flux. International Journal of Thermal Sciences, 2011; 50, 1207-1214.

[13] M Turkyilmazoglu and I Pop. Soret and heat source effects on the unsteady radiative MHD free convection flow from an impulsively started infinite vertical plate. International Journal of Heat and Mass Transfer, 2012; 55, 7635–7644.

[14] N Sandeep, V Sugunamma and P Mohan Krishna. Effects of radiation on an Unsteady Natural convection flow of a EG- Nimonic 80a Nanofluid Past a vertical plate. Advances in physics Theories and Applications, 2013; 23,36-43.

[15] Farhad Ali et al. Heat and Mass transfer with free convection MHD flow Past a vertical plate embedded in a porous medium. Mathematical Problems in Engineering, 2013; Volume 2013, 1-14.

[16] W A Khan, O D Makinde and Z H Khan. MHD boundary layer flow of a nanofluid containing gyrotactic microorganisms past with Navier slip, International Journal of Heat and Mass Transfer, 2014; 74, 285-291.

[17] Meraj Mustafa et al. Nonlinear Radiation heat transfer effects in the natural convection Boundary layer flow of nanofluid past a vertical plate: A numerical study, PLoS ONE, 2014; 9(9), 1-10.

[18] D Srinivasacharya and Ontela Surender. Mixed convection boundary layer flow of a nanofluid past a vertical plate in porous medium. Applied Nanoscience, 2014; 5(1), 1-10.
[19] M Ghulambaz, A Noghrehabadi and A Ghanbarzadeh. Natural convection of nanofluids over a convectively heated vertical plate embedded in a porous medium. *Brazilian Journal of Chemical Engineering*, 2014; 31(2), 413-427.

[20] Machireddy Gnaneswara Reddy. Influence of thermal radiation on natural convection boundary layer flow of a nanofluid past a vertical plate with uniform heat flux. *Heat and Technology*, 2014; 32, 1-7.

[21] C S K Raju et al. Radiation and Soret effects of MHD Nanofluid flow over a moving vertical plate in porous medium. *Chemical and Process Engineering Research*, 2015; 30, 9-23.

[22] K Javaherdeh, Mehrzad Mirzaei Nejad and M Moslemi. Natural convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable vertical surface temperature and concentration in a porous medium. *Engineering Science and Technology, an International Journal*, 2015; 18, 432-431.

[23] G S Seth, S M Hussain and S Sarkar. Hydro magnetic natural convection flow with heat and mass transfer of a chemically reacting and heat absorbing fluid past an accelerated moving vertical plate with ramped temperature and ramped surface concentration through a porous medium. *Journal of the Egyptian Mathematical Society*, 2015; 23, 197-207.

[24] Hari R Kataria and Akhil S Mittal. Mathematical model for velocity and temperature of gravity -driven convective optically thick nanofluid flow past an oscillating vertical plate in presence of magnetic field and radiation. *Journal of the Nigerian Mathematical Society*, 2015; 34(3), 303-317.

[25] B Ganga, S Mohammed Yusuff Ansari, N Vishnu Ganesh and A k Abdul Hakeem. MHD flow of Buongiorno model nanofluid over a vertical plate with internal heat generation/ absorption. *Propulsion and power research*, 2016; 5, 211-222.

[26] Farhad Ali, Madeha Gohar and IIyas Khan. MHD flow of Water based Brinkman type nanofluid over a vertical plate embedded in a porous medium with variable surface velocity, temperature, and concentration. *Journal of Molecular liquids*, 2016; 223, 412-419.

[27] V. Rajesh, A. J. Chamkha and M. P. Mallesh. Transient MHD free convection flow and heat transfer of nanofluid past an Impulsively started semi-infinite vertical plate. *Journal of Applied Fluid mechanics*. 2016; 9, 2457-2467.

[28] P V Satya Narayana, B. Venkateswarlu.: Heat and Mass transfer on MHD Nanofluid flow past a vertical porous plate in a rotating system. frontiers in heat and mass transfer.2016. http://dx.doi.org/10.5098/hmt.7.8

[29] Harouna Naroua and Bachir Moussa Idi. Computer Simulation on free convection MHD stokes problem for a vertical plate through porous medium. *ARPN Journal of Engineering and applied sciences*. 2016; 11, No 24.

[30] Abid Hussanan, Ilyas Khan, Hasmawani Hashim, Muhammad Khairul Anuar, Nazila Ishak, Norhafizah Md Sarif and Mohd zuki Salleh. Unsteady MHD flow of Some Nanofluid past an accelerated Vertical Plate Embedded in a porous medium. *Journal Teknologi*, 2016; 121-126.

[31] Amir Malvandi. Film boiling of magnetic nanoparticles (MNFs) over a vertical plate in presence of a uniform variable -directional magnetic field. *Journal of Magnetism and Magnetic materials*.2016; 406, 95-102.

[32] Ashwin Kumar E N, NorasikinBinti Mat Isa, Vibhoo Vibesh B and Kandaswamy R. Impact of Injection on a stagnation point flow of copper nanofluid over a vertical porous shrinking or stretching plate in the presence of magnetic field. *ARPN Journal of engineering and applied sciences*. 2016; 11, No12.

[33] TR Sranvan Kumar and B Rushi Kumar. Unsteady MHD free convective boundary layer flow of a nanofluid past a moving vertical plate. *IOP Conference Series: Materials Science and Engineering*. 263 (2017). 10.1088/1757-899X/263/6/062015.

[34] Waqas Ali Azha, Dumitra Vieru and Constantin Fetecau Free convection flow of some fractional nanofluids over a vertical plate with uniform heat flux and heat source. *Physics of fluids*. 2017. https://doi.org/10.1063/1.4996034

[35] Marneni Narahari, S Suresh Kumar Raju and Rajashekhar Pendyala. Unsteady natural convection flow of multiphase nanofluid past a vertical plate with constant heat flux. *Chemical Engineering Science*. 2017; 167, 229-241.

[36] S. Anuradha and M. Yegammai. MHD Radiative Boundary layer Flow of a nanofluid Past a Vertical plate with Effects of Binary Chemical Reaction and Activation Energy, *Global Journal of pure and Applied Mathematics*.2017;13, 6377- 6392.

[37] Wafuila Maurine Marak. Similarity Solution of Unsteady Boundary layer flow of Nanofluids Past a Vertical Plate with Convective Heating. *Global Journal of pure and Applied Mathematics*. 2018; 14, 517-534

[38] Marneni Narahari. Unsteady free convection flow past semi-infinite vertical plate with constant heat flux in water based nanofluids. *Iop.conf.series :Materials science and engineering*. 2018; 342, 1-2.

[39] Abdul Rauf and Yasir Mahsud. Unsteady two dimensional flows of a nanofluid over a vertical plate with time exponential temperature. *Arab Journal of Basic and Applied sciences*. 2018; 25, 150-157.
[40] Hiranmoy Mondal, Poulomi De, Sicelo P. Goqo1 and Precious Sibanda. Numerical Studies on Nanofluid Flow over Porous Vertical Plate with Internal Heat Generation and Thermal Radiation. *Journal of Applied Physics.* 2019; 27, 43-52.

[41] Ze cheng and jie peng. Wavy falling film of nanofluid over a vertical plate considering nanoparticle migration. *International Journal of Heat Transfer.* (2020). https://doi.org/10.1016/j.ijheatmasstransfer.2019.118895.

[42] G.C.Shit and S.Mandal. Entropy Analysis on Unsteady MHD flow of Casson nanofluid over a Stretching Vertical plate with thermal Radiation Effect. *Int. Appl. Comput. Math.* 2020. DOI: 10.1007/s40819-019-0754-4

[43] Marneni Narahari, Suhaib Umer Ilyas and Raja Sekhar Pendyala. Unsteady Free Convective Non-homogeneous Nanofluid flow past a moving vertical plate, *AIP Conference Proceeding.* 2019. DOI: 10.1063/1.5121064.

[44] Aamir Hamid, Hashim, Masood Khan and Metib Alghamdi. MHD Blasius flow of radiative Williamson nanofluid over a vertical plate. *International Journal of Modern physics B* (2019). https://dx.doi.org/10.1142/S021797921950245X.

[45] Chebii Agnes Jeptoo. MHD Nanofluid Bioconvection Due to Gyrotactic microorganisms Past a convectively heated vertical plate. *IOSR Journal of Mathematics.* 2019; 15, pp 50-71.

[46] M. Veera Krishna and Ali J Chamkha. Hall and ion slip effects on MHD Rotating boundary layer flow of nanofluid past an infinite vertical plate embedded in a porous medium. Results in Physics.(2019) https://dx.doi.org/10.1016/j.rinp.2019.102652.

[47] B.P. Garg and shippa. Exact Solution of MHD free convective and mass transfer flow near a moving vertical plate in the presence of heat source /sink. *Journal of Rajasthan Academy of physical science.* 2019, 18, 25-24.

[48] Mushtaq Ahmad , Muhammad Imran Asjad , Ali Ak gul, and Dumitr u Baleanu. Analytical solutions for free convection flow of Casson nanofluid over an infinite vertical plate *AIMS Mathematics*, 2020; 6(3): 2344–2358.

[49] M Anil Kumar, Y Dharmendar Reddy, V Srinivasa Rao and B Shankar Goud. Thermal radiation impact on MHD heat transfer natural convective nano fluid flow over an impulsively started vertical plate. *Case studies in thermal engineering.* 2021; 24, 1-10.

[50] Pankaj kumar, Debashis Dey and Sikata Samantaray. A recent review on Thermophysical properties of nanofluid, *International Conference on Electrical, Electronics and optimization technique.* 2016.10.1109/ICEEOT.2016.7755366.

[51] Dharmendra Kumar Saini and Ghanshayam Das Agarwal. Thermo-physical properties of Nanofluids- A Review, *International journal of Advances in Engineering science and technology* V5N1: 39-45.

[52] Sheikholeslami M, Ganji D and Ashorynejad H. Investigation of squeezing unsteady nanofluid flow using ADM. *Powder Technol.* 2013; 239, 259–265.

[53] Hamilton R L and Crosser O. Thermal conductivity of heterogeneous two-component systems. *Ind. Eng. Chem. Fundam.* 1962; 1, 187–191.

[54] Yu, W.; Choi, S. The role of interfacial layers in the enhanced thermal conductivity of nanofluids: A renovated Maxwell model. J. Nanopart. Res. 2003, 5, 167–171.

[55] Patel H E, Anoop K, Sundararajan T and Das S K. *In Proceedings of the International Heat Transfer Conference 13,* A micro-convection model for thermal conductivity of nanofluids, Sydney, Australia, 2006, 13-18.

[56] Jang S P and Choi S U. Role of Brownian motion in the enhanced thermal conductivity of nanofluids. *Appl. Phys. Lett.* 2004,84, 4316–4318.

[57] Xuan Y, Li Q and Hu W. Aggregation structure and thermal conductivity of nanofluids. *AIChe J.* 2003; 49, 1038–1043.

[58] Mintsa H.A, Roy G, Nguyen C T and Doucet D. New temperature dependent thermal conductivity data for water-based nanofluids. *Int. J. Therm. Sci.* 2009; 48, 363–371.

[59] Eastman J A, Phillpot S, Choi S and Keblinski P. Thermal transport in nanofluids. *Annu. Rev. Mater. Res.* 2004; 34, 219–246.

[60] Chon C H, Kim K D, Lee S P and Choi S U. Empirical correlation finding the role of temperature and particle size for nano fluid (Al) thermal conductivity enhancement. *Appl. Phys. Lett.* (2005). https://doi.org/10.1063/1.2093936

[61] I S Sundar, K Sharma, M. Naik and M. K. Singh. Empirical and theoretical correlations on viscosity of nanofluids: A review. *Renew Sust Energ Rev.* 2013; 25, 670-686.

[62] Sheikholeslami and M, Rokni H B Simulation of nanofluid heat transfer in presence of magnetic field: A review. *Int. J. Heat Mass Transf.* 2017; 115, 1203–1233.

[63] Pak B.C and Cho Y I. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles. *Exp. Heat Transf. Int. J.* 1998; 11, 151–170.

[64] Sheikholeslami M, Mustafa M T and Ganji D D Nanofluid flow and heat transfer over a stretching porous cylinder considering thermal radiation. *Iran. J. Sci. Technol. Trans. Sci.* 2015; 39, 433–440.
[65] Asma Khalid A, Ilyas Khan and Sharidan Shafie. Exact solutions for free convection flow of nanofluids with ramped wall temperature. *Eur. Phys. J. Plus*. 2015, 130:57.

[66] Nadeem Ahmed, Sheikh Ilyas Khan and Dennis Ling Chaun Ching. A comprehensive Review on Theoretical Aspects of nanofluids: Exact solutions and Analysis. *Symmetry*, 2020. https://doi.org/10.3390/sym12050725.

[67] Einstein A. Investigations on the Theory of the Brownian Movement, Courier Corporation, North Chelmsford, MA, USA: 1956.

[68] Batchelor G K The effect of Brownian motion on the bulk stress in a suspension of spherical particles. *J. Fluid Mech*. 1977; 83, 97–117.

[69] Dávalos-Orozco L A and Del Castillo L F. Hydrodynamic Behavior of Suspensions of Polar Particles. New York: CRC Press; 2005.

[70] Brinkman H C. The viscosity of concentrated suspensions and solutions. *J. Chem. Phys*. 1952; 20, 571.

[71] Maiga S E B, Palm S J, Nguyen C T, Roy G, and Galanis N. Heat transfer enhancement by using nanofluids in forced convection flows. *Int. J. Heat Fluid Flow*. 2005; 26, 530–546.

[72] Nguyen C, Desgranges F, Galanis N, Roy G, Mare T, Boucher S and Mintsa H A. Viscosity data for Al2O3–water nanofluid—hysteresis: Is heat transfer enhancement using nanofluids reliable? *Int. J. Therm. Sci*. 2008; 47, 103–111.

[73] Gohar M, Ali F, Khan I, Sheikh N A and Shah A. The unsteady flow of generalized hybrid nanofluids: Applications in cementitious materials. *J. Aust. Ceram. Soc*. 2018; 55, 657–666.

[74] Rajesh Vemula, Lokenath Debnath and Sridevi Chakrala. Unsteady MHD Free Convection Flow of Nanofluid Past an Accelerated Vertical Plate with Variable Temperature and Thermal Radiation. *Int. J. Appl. Comput. Math*; 2017, 3, 1271–1287.