Probing Majorana Phases and Neutrino Mass Spectrum in the Higgs Triplet Model at the LHC

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Abstract

Doubly charged Higgs bosons ($H^{\pm\pm}$) are a distinctive signature of the Higgs Triplet Model of neutrino mass generation. If $H^{\pm\pm}$ is relatively light ($m_{H^{\pm\pm}} < 400$ GeV) it will be produced copiously at the LHC, which could enable precise measurements of the branching ratios of the decay channels $H^{\pm\pm} \rightarrow l_1^{\pm} l_2^{\pm}$. Such branching ratios are determined solely by the neutrino mass matrix which allows the model to be tested at the LHC. We quantify the dependence of the leptonic branching ratios on the absolute neutrino mass and Majorana phases, and present the permitted values for the channels $e^{\pm}e^{\pm}, e^{\pm}\mu^{\pm}$ and $\mu^{\pm}\mu^{\pm}$. It is shown that precise measurements of these three branching ratios are sufficient to extract information on the neutrino mass spectrum and probe the presence of CP violation from Majorana phases.

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1 Introduction

The firm evidence from a variety of experiments that neutrinos oscillate and possess a small mass below the eV scale necessitates physics beyond the Standard Model (SM). Consequently models of neutrino mass generation which can be probed at present and forthcoming experiments are of great phenomenological interest. In particular, those models which can provide a distinctive experimental signature, such as a New Physics particle with a mass of the TeV scale or less, are especially appealing in light of the approaching commencement of the CERN Large Hadron Collider (LHC).

Doubly charged Higgs bosons ($H^{\pm\pm}$) arise in a variety of models of neutrino mass generation as members of $I = 1, Y = 2$ scalar triplets [1–11] and $I = 0, Y = 4$ scalar singlets [12]. Such particles can be relatively light (i.e., with masses of the electroweak scale) and have impressive discovery potential at hadron colliders due to their low background signature $H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$ and sizeable cross-sections. The ongoing searches at the Fermilab Tevatron [13, 14] anticipate sensitivity to $m_{H^{\pm\pm}} < 250$ GeV for the decay channel $H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$ ($i, j = e, \mu$) with the expected final integrated luminosities of up to 8 fb$^{-1}$. LHC simulations [15, 16] show that discovery for $m_{H^{\pm\pm}} < 1$ TeV is possible with 300 fb$^{-1}$, and as little as 1 fb$^{-1}$ is needed to probe $m_{H^{\pm\pm}} < 400$ GeV.

Discovery of $H^{\pm\pm}$ with $m_{H^{\pm\pm}} < 400$ GeV would enable precise measurements of the branching ratios (BRs) of $H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$ with the anticipated final integrated luminosity at the LHC. Models which predict BR($H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$) in terms of experimentally constrained and/or measured parameters are of particular phenomenological interest. In the Higgs Triplet Model (HTM) [3, 4] neutrinos acquire a Majorana mass given by the product of a triplet Yukawa coupling ($h_{ij}$) and a triplet vacuum expectation value $v_\Delta$. Consequently in the HTM there is a direct connection between $h_{ij}$ and the neutrino mass matrix which gives rise to phenomenological predictions for processes which depend on $h_{ij}$. Since the coupling $h_{ij}$ determines BR($H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$), this mechanism of neutrino mass generation can be tested if precise measurements of BR($H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$) are available [6, 7]. A detailed quantitative study of the dependence of $h_{ij}$ on all the neutrino oscillation parameters has not yet been performed (for previous analyses see [7, 17]).

Of particular interest is the dependence of $h_{ij}$ on the absolute neutrino mass and Majorana phases which is the focus of the present work. Those parameters, which cannot be probed in neutrino oscillation experiments, would significantly affect BR($H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$). We perform a study of the capability of the LHC to probe the neutrino mass spectrum and Majorana phases assuming that neutrino mass is generated solely by the combination $h_{ij}v_\Delta$ in the HTM. In particular, we investigate the possibility to establish $m_0 \neq 0$ and/or CP-violation from Majorana phases at the LHC by means of a $\chi^2$ analysis with three 2l channels of $H^{\pm\pm}$ decays. It is extremely difficult for neutrinoless double beta decay experiments [18] to measure CP-violation from Majorana phases because they affect this process in combination with unmeasured parameters [19], while the absolute neutrino mass can only be measured directly by the future Tritium beta decay experiment [20] if $m > 0.2$ eV.

Our work is organized as follows: in section 2 we briefly review the HTM; section 3 describes the phenomenology of $H^{\pm\pm}$ at hadron colliders; the numerical analysis is contained in section 4 with details of a $\chi^2$ analysis presented in the appendix; conclusions are given in section 5.
2 The Higgs Triplet Model

In the Higgs Triplet Model (HTM) [3], [4] a $I = 1, Y = 2$ complex $SU(2)_L$ isospin triplet of scalar fields is added to the SM Lagrangian. Such a model can provide a Majorana mass for the observed neutrinos without the introduction of a right-handed neutrino via the gauge invariant Yukawa interaction:

$$\mathcal{L} = h_{ij}\psi_i^T L C i \tau_2 \Delta \psi_j L + h.c$$ (1)

Here $h_{ij}(i, j = 1, 2, 3)$ is a complex and symmetric coupling, $C$ is the Dirac charge conjugation operator, $\tau_2$ is a Pauli matrix, $\psi_i L = (\nu_i, l_i)_L$ is a left-handed lepton doublet, and $\Delta$ is a $2 \times 2$ complex triplet fields:

$$\Delta = \begin{pmatrix} \Delta^+ / \sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+ / \sqrt{2} \end{pmatrix}$$ (2)

A non-zero triplet vacuum expectation value $\langle \Delta^0 \rangle$ gives rise to the following mass matrix for neutrinos:

$$m_{ij} = 2h_{ij}\langle \Delta^0 \rangle = \sqrt{2}h_{ij}v_{\Delta}$$ (3)

The necessary non-zero $v_{\Delta}$ arises from the minimization of the most general $SU(2) \otimes U(1)_Y$ invariant Higgs potential, which is written as follows [6,7] (with $\Phi = (\phi^+, \phi^0)^T$):

$$V = m^2(\Phi^T \Phi) + \lambda_1(\Phi^T \Phi)^2 + M^2 \text{Tr}(\Delta^T \Delta) + \lambda_2[\text{Tr}(\Delta^T \Delta)]^2 + \lambda_3 \text{Det}(\Delta^T \Delta) + \lambda_4(\Phi^T \Phi)\text{Tr}(\Delta^T \Delta) + \lambda_5(\Phi^T \tau_i \Phi)\text{Tr}(\Delta^T \tau_i \Delta) + \left(\frac{1}{\sqrt{2}}\mu(\Phi^T i \tau_2 \Delta^T \Phi) + h.c\right)$$ (4)

Here $m^2 < 0$ in order to ensure $\langle \phi^0 \rangle = v/\sqrt{2}$ which spontaneously breaks $SU(2) \otimes U(1)_Y$ to $U(1)_Q$, and $M^2 (> 0)$ is the mass term for the triplet scalars. In the model of Gelmini-Roncadelli [21] the term $\mu(\Phi^T i \tau_2 \Delta^T \Phi)$ is absent, which leads to spontaneous violation of lepton number for $M^2 < 0$. The resulting Higgs spectrum contains a massless triplet scalar (majoron, $J$) and another light scalar ($H^0$). Pair production via $e^+e^- \rightarrow H^0 J$ would give a large contribution to the invisible width of the $Z$ and this model was excluded at LEP. The inclusion of the term $\mu(\Phi^T i \tau_2 \Delta^T \Phi)$ explicitly breaks lepton number when $\Delta$ is assigned $L = 2$, and eliminates the majoron [3,4]. Thus the scalar potential in eq. (4) together with the triplet Yukawa interaction of eq. (1) lead to a phenomenologically viable model of neutrino mass generation.

The expression for $v_{\Delta}$ resulting from the minimization of $V$ is:

$$v_{\Delta} \simeq \mu v^2 / 2M^2$$ (5)

In the scenario of light triplet scalars ($M \approx v$) within the discovery reach of the LHC, eq. (5) leads to $v_{\Delta} \approx \mu$. In extensions of the HTM the term $\mu(\Phi^T i \tau_2 \Delta^T \Phi)$ may arise in various ways: i) the vev of a Higgs singlet field [22,23]; ii) be generated at higher orders in perturbation theory [7]; iii) be generated in the effective Lagrangian [10]; iv) originate in the context of extra dimensions [6,8].

An upper limit on $v_{\Delta}$ can be obtained from considering its effect on the parameter $\rho(= M_W^2 / M_Z^2 \cos^2 \theta_W)$. In the SM $\rho = 1$ at tree-level, while in the HTM one has (where $x = v_{\Delta} / v$):

$$\rho \equiv 1 + \delta \rho = \frac{1 + 2x^2}{1 + 4x^2}$$ (6)
The most distinct and experimentally accessible decay mode of \( l^\pm \) mass matrix is diagonal i.e., \( l^\pm \) are the mass eigenstates. Then the decay rate for BR\((H^{\pm\pm} \rightarrow l^+_i l^-_j)\) \((i, j = e, \mu, \tau)\) is given by:

\[
\Gamma(H^{\pm\pm} \rightarrow l^+_i l^-_j) = S \frac{m_{H^{\pm\pm}}}{8\pi} |h_{ij}|^2
\]  

where \( S = 1(2) \) for \( i = j \) \((i \neq j)\). Clearly \( \Gamma(H^{\pm\pm} \rightarrow l^+_i l^-_j) \) depends crucially on the absolute values of the \( h_{ij} \), where \( h_{ij} \) is related to the neutrino mass matrix via eq. (3). However, if no other decay modes for \( H^{\pm\pm} \) are open kinematically the leptonic BRs are determined solely by the relative values of \( h_{ij} \). Other decay modes for \( H^{\pm\pm} \) can be important in regions of parameter space e.g., i) \( H^{\pm\pm} \rightarrow H^\pm W^\mp \), which is potentially sizeable for \( m_{H^{\pm\pm}} > m_{H^\pm} \) [17, 28], and ii) \( H^{\pm\pm} \rightarrow W^\pm W^\pm \), which is proportional to the triplet vev, \( v_\Delta \). In this work, we assume \( m_{H^\pm} \geq m_{H^{\pm\pm}} \) (which precludes \( H^{\pm\pm} \rightarrow H^\pm W^\mp \)) and \( v_\Delta < 1 \) MeV, which suppresses \( H^{\pm\pm} \rightarrow W^\pm W^\pm \) sufficiently in the HTM (e.g., see [29]). Then, \( H^{\pm\pm} \rightarrow l^+_i l^-_j \) can be regarded as the sole decay mode for \( H^{\pm\pm} \) and one has:

\[
\text{BR}_{ll} \equiv \text{BR}(H^{\pm\pm} \rightarrow l^+_i l^-_j) = S \frac{|h_{ij}|^2}{\sum_{ij} |h_{ij}|^2}.
\]

3 Production and Decay of \( H^{\pm\pm} \) at Hadron Colliders

The most distinct and experimentally accessible decay mode of \( H^{\pm\pm} \) is to two same-sign charged leptons [27]. Without loss of generality one can work in the basis in which the charged lepton mass matrix is diagonal i.e., \( l^\pm_i \) are the mass eigenstates. Then the decay rate for BR\((H^{\pm\pm} \rightarrow l^+_i l^-_j)\) \((i, j = e, \mu, \tau)\) is given by:

\[
\Gamma(H^{\pm\pm} \rightarrow l^+_i l^-_j) = S \frac{m_{H^{\pm\pm}}}{8\pi} |h_{ij}|^2
\]

where \( S = 1(2) \) for \( i = j \) \((i \neq j)\). Clearly \( \Gamma(H^{\pm\pm} \rightarrow l^+_i l^-_j) \) depends crucially on the absolute values of the \( h_{ij} \), where \( h_{ij} \) is related to the neutrino mass matrix via eq. (3). However, if no other decay modes for \( H^{\pm\pm} \) are open kinematically the leptonic BRs are determined solely by the relative values of \( h_{ij} \). Other decay modes for \( H^{\pm\pm} \) can be important in regions of parameter space e.g., i) \( H^{\pm\pm} \rightarrow H^\pm W^\mp \), which is potentially sizeable for \( m_{H^{\pm\pm}} > m_{H^\pm} \) [17, 28], and ii) \( H^{\pm\pm} \rightarrow W^\pm W^\pm \), which is proportional to the triplet vev, \( v_\Delta \). In this work, we assume \( m_{H^\pm} \geq m_{H^{\pm\pm}} \) (which precludes \( H^{\pm\pm} \rightarrow H^\pm W^\mp \)) and \( v_\Delta < 1 \) MeV, which suppresses \( H^{\pm\pm} \rightarrow W^\pm W^\pm \) sufficiently in the HTM (e.g., see [29]). Then, \( H^{\pm\pm} \rightarrow l^+_i l^-_j \) can be regarded as the sole decay mode for \( H^{\pm\pm} \) and one has:

\[
\text{BR}_{ll} \equiv \text{BR}(H^{\pm\pm} \rightarrow l^+_i l^-_j) = S \frac{|h_{ij}|^2}{\sum_{ij} |h_{ij}|^2}.
\]

3.1 Searches for \( H^{\pm\pm} \) at the Tevatron

In the year 2003 the Fermilab Tevatron performed the first search for \( H^{\pm\pm} \) at a hadron collider\footnote{Direct searches for \( H^{\pm\pm} \) have also been performed at LEP [30] and HERA [31].}. The D0 collaboration [13] searched for \( H^{\pm\pm} \rightarrow \mu^+\mu^- \) while the CDF collaboration [14] searched for 3 final states: \( H^{\pm\pm} \rightarrow e^+e^\pm, e^\pm e^\pm, \mu^+\mu^- \). The assumed production mechanism for \( H^{\pm\pm} \) is \( q\bar{q} \rightarrow \gamma^*, Z^* \rightarrow H^{++}H^{--} \), which proceeds via gauge strength couplings and depends on only one unknown parameter, \( m_{H^{\pm\pm}} \) [25, 32].
Table 1: Mass limits on $m_{H^{\pm\pm}}$ from searches for $H^{\pm\pm} \to l_i^\pm l_j^\pm$ at Tevatron Run II.

|   | ee | $e\mu$ | $\mu\mu$ | $e\tau$ | $\mu\tau$ | $\tau\tau$ |
|---|----|--------|---------|--------|---------|----------|
| 2l | > 133 GeV | > 113 GeV | > 136 GeV |        |        |          |
| 3l |        |        |        | > 114 GeV | > 112 GeV |          |
| 4l |        |        |        | > 114 GeV | > 112 GeV |          |

The searches performed in [13, 14] seek at least one pair of same-sign leptons with high invariant mass i.e., the search is sensitive to *single production* of $H^{\pm\pm}$. The SM background can be reduced to negligible proportions with suitable cuts. Single $H^{\pm\pm}$ production mechanisms which involve a dependence on potentially small parameters such as the Yukawa coupling $h_{ij}$ or triplet vev $v_\Delta$ are subdominant at Tevatron energies (e.g., see [33]). In [17] it was suggested that this search strategy is also sensitive to the mechanism $q'q \to W^* \to H^{\pm\pm}H^\mp$ [34], which has a cross-section comparable in magnitude to that of $q'q \to \gamma^*, Z^* \to H^{++}H^{--}$.

The following inclusive single $H^{\pm\pm}$ cross-section was introduced, which would extend the search sensitivity to larger values of $m_{H^{\pm\pm}}$ and strengthen the mass limits on $m_{H^{\pm\pm}}$ derived in [13, 14]:

$$\sigma_{H^{\pm\pm}} = \sigma(q'q \to \gamma^*, Z^* \to H^{++}H^{--}) + 2\sigma(q'q \to W^* \to H^{++}H^-)$$

(9)

Here the factor of 2 accounts for the CP conjugate process $q'q \to W^* \to H^{--}H^+$. In 2006 the CDF collaboration searched for $H^{\pm\pm}$ decays involving $\tau^\pm$ [35]. The strategy of searching for one pair of same-sign leptons (2l) is not effective due to the larger SM backgrounds, and instead three (3l) and four (4l) lepton searches were performed. The production mechanism $q'q \to \gamma^*, Z^* \to H^{++}H^{--}$ was assumed. The process $q'q \to W^* \to H^{\pm\pm}H^\mp$ never contributes to the 4l signature, but can contribute to the 3l signature if $H^\pm$ decays leptonically, $H^{\pm} \to l^\pm \nu$.

In Table 1 the mass limits for $m_{H^{\pm\pm}}$ from the Tevatron searches are summarized. A blank entry signifies that no search has yet been performed. The displayed mass limits assume production via $q'q \to \gamma^*, Z^* \to H^{++}H^{--}$ for a $H^{\pm\pm}$ belonging to a $SU(2)_L$ triplet with $Y = 2$. Moreover, $BR(H^{\pm\pm} \to l_i^\pm l_j^\pm) = 100\%$ in a given channel is assumed. The ultimate sensitivity at the Tevatron is expected to be $m_{H^{\pm\pm}} \sim 250$ GeV in the $ee, e\mu$ and $\mu\mu$ channels.

### 3.2 Simulations of $H^{\pm\pm}$ production at the LHC

Several simulations have been performed for $H^{\pm\pm} \to l_i^\pm l_j^\pm$ ($i,j = e, \mu, \tau$) at the LHC [15,16, 29,36,37]. The production mechanism is assumed to be $q'q \to \gamma^*, Z^* \to H^{++}H^{--}$ followed by $H^{++}H^{--} \to llll$. The LHC sensitivity to $H^{\pm\pm} \to l_i^\pm l_j^\pm$ considerably extends that at the Tevatron due to the increased cross-sections and larger luminosities e.g., the analysis of [16]
Table 2: Approximate number of events for pair production of $H^{±±}$ ($N_{4l}$) and single production of $H^{±±}$ ($N_{2l}$) at the LHC for integrated luminosities of $L = 30 \text{ fb}^{-1}$ and $L = 300 \text{ fb}^{-1}$ with efficiency $\epsilon_{\text{eff}} = 0.5$. We assumed $m_{H^{±±}} = m_{H^{±}}$ to calculate $N_{2l}$.

| $m_{H^{±±}}$ (GeV) | $N_{4l}$ (30 fb$^{-1}$) | $N_{4l}$ (300 fb$^{-1}$) | $N_{2l}$ (300 fb$^{-1}$) |
|-------------------|------------------------|------------------------|------------------------|
| 200               | 1500                   | 15000                  | 42000                  |
| 300               | 300                    | 3000                   | 8400                   |
| 400               | 90                     | 900                    | 2500                   |

shows that a $H^{±±}$ can be discovered for $m_{H^{±±}} < 800$ GeV assuming $\text{BR}(H^{±±} \rightarrow \mu^{±} \mu^{±}) = 100\%$ and $L = 50 \text{ fb}^{-1}$. Importantly, all the above simulations suggest that as little as 1 fb$^{-1}$ is needed for discovery of $m_{H^{±±}} < 400$ GeV if one of $\text{BR}(H^{±±} \rightarrow e^{±} e^{±}, e^{±} \mu^{±}, \mu^{±} \mu^{±})$ is large, and hence such a light $H^{±±}$ would be found very quickly at the LHC.

The sensitivity of the LHC to single production of $H^{±±}$ $l_{i}^{±} l_{j}^{±}$ for $i, j = e, \mu$ has only been performed in [37], and importantly the SM background was shown to be negligible in the signal region of high invariant mass. It was concluded that such a search strategy allows more $H^{±±} \rightarrow l_{i}^{±} l_{j}^{±}$ events than the 4 lepton search since the event number is linear (and not quadratic) in $\text{BR}(H^{±±} \rightarrow l_{i}^{±} l_{j}^{±})$. Therefore the 2$\ell$ search is more effective at probing small $\text{BR}(H^{±±} \rightarrow l_{i}^{±} l_{j}^{±})$. Importantly, the addition of the channel $q\bar{q} \rightarrow W^* \rightarrow H^{±±} H^±$ (eq. (9)) would further enhance the event number for a given $m_{H^{±±}}$.

In Table 2 we show approximate expected numbers of 2$\ell$ and 4$\ell$ events arising from pair and singly produced $H^{±±} \rightarrow l_{i}^{±} l_{j}^{±}$ at the LHC. We only consider the decay channels $H^{±±} \rightarrow e^{±} e^{±}, e^{±} \mu^{±}, \mu^{±} \mu^{±}$ which offer the greatest $H^{±±}$ discovery potential. A detection efficiency of 0.5 is assumed, which is slightly less than the values given in [16] for $H^{±±} \rightarrow \mu^{±} \mu^{±}$. The theoretical $H^{±±}$ cross-section is multiplied by this detection efficiency and the SM background is taken to be negligible. The number of 4$\ell$ events for a specific $m_{H^{±±}}$ is denoted by $N_{4l}$, assuming integrated luminosities of $L = 30 \text{ fb}^{-1}$ and $L = 300 \text{ fb}^{-1}$. The displayed numbers are for $\text{BR}(H^{±±} \rightarrow l_{i}^{±} l_{j}^{±}) = 100\%$ in a given channel. In the HTM, $\text{BR}(H^{±±} \rightarrow l_{i}^{±} l_{j}^{±})$ is necessarily $< 100\%$ (eq. (3) and eq. (7)) and hence $N_{4l}$ must be multiplied by $[\text{BR}(H^{±±} \rightarrow l_{i}^{±} l_{j}^{±})]^2$. The final column shows the number of 2$\ell$ events ($N_{2l}$) obtained by adding the contribution from the mechanism $q\bar{q} \rightarrow W^* \rightarrow H^{±±} H^±$ as defined in eq. (9). We take $m_{H^{±±}} = m_{H^{±}}$ which increases the number of singly produced $H^{±±}$ events by a factor of around 2.8 for 200 GeV $< m_{H^{±±}} < 400$ GeV [17]. For BR($H^{±±} \rightarrow l_{i}^{±} l_{j}^{±}$) $< 100\%$, the numbers presented in Table 2 are scaled as shown in Appendix A.

It is clear from Table 2 that early discovery of $H^{±±}$ at the LHC with $m_{H^{±±}} < 400$ GeV would allow large event numbers for $H^{±±}$ with the expected integrated luminosities of $L = 300 \text{ fb}^{-1}$. This would enable precise measurements of $\text{BR}(H^{±±} \rightarrow e^{±} e^{±}, e^{±} \mu^{±}, \mu^{±} \mu^{±})$ for the dominant channels. Sensitivity to $\text{BR}(H^{±±} \rightarrow e^{±} e^{±}, e^{±} \mu^{±}, \mu^{±} \mu^{±}) \sim 1\%$ or less would also be possible in the 2$\ell$ channel.
4 Numerical Analysis

The mass matrix for three Dirac neutrinos is diagonalized by the MNS (Maki-Nakagawa-Sakata) matrix $V_{\text{MNS}}$ [38] for which the standard parametrization is:

$$V_{\text{MNS}} = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{13}e^{i\delta} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13}
\end{pmatrix}, \quad (10)$$

where $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$, and $\delta$ is the Dirac phase. For Majorana neutrinos, two additional phases appear and then the mixing matrix $V$ becomes

$$V = V_{\text{MNS}} \times \text{diag}(1, e^{i\varphi_1/2}, e^{i\varphi_2/2}), \quad (11)$$

where $\varphi_1$ and $\varphi_2$ are referred to as the Majorana phases [3, 39]. Since we are working in the basis in which the charged lepton mass matrix is diagonal, the neutrino mass matrix is then diagonalized by $V$. Using eq. (3) one can write the couplings $h_{ij}$ as follows [6, 7]:

$$h_{ij} = \frac{1}{\sqrt{2}v_{\Delta}} [V_{\text{MNS}}\text{diag}(m_1, m_2 e^{i\varphi_1}, m_3 e^{i\varphi_2})V_{\text{MNS}}^T]_{ij}. \quad (12)$$

Then, eq. (8) becomes

$$\text{BR}(H^{\pm\pm} \rightarrow l_i^{\pm} l_j^{\pm}) = \frac{S \left\{ [V_{\text{MNS}}\text{diag}(m_1, m_2 e^{i\varphi_1}, m_3 e^{i\varphi_2})V_{\text{MNS}}^T]_{ij} \right\}^2}{\sum_k m_k^2}. \quad (13)$$

Note that the branching ratios are independent of $m_{H^{\pm\pm}}$ and $v_{\Delta}$, and given by neutrino parameters only.

Neutrino oscillation experiments involving solar [40], atmospheric [41], accelerator [42], and reactor neutrinos [43] are sensitive to the mass-squared differences and the mixing angles, and give the following preferred values:

$$\Delta m^2_{21} \equiv m_2^2 - m_1^2 \simeq 7.9 \times 10^{-5}\text{eV}^2, \quad |\Delta m^2_{31}| \equiv |m_3^2 - m_1^2| \simeq 2.7 \times 10^{-3}\text{eV}^2, \quad (14)$$

$$\sin^2 2\theta_{12} \simeq 0.86, \quad \sin^2 2\theta_{23} \simeq 1, \quad \sin^2 2\theta_{13} \lesssim 0.13. \quad (15)$$

The small mixing angle $\theta_{13}$ has not been measured yet and hence the value of $\delta$ in completely unknown. Since the sign of $\Delta m^2_{31}$ is also undetermined at present, distinct neutrino mass hierarchy patterns are possible. The case with $\Delta m^2_{31} > 0$ is referred to as Normal hierarchy (NH) where $m_1 < m_2 < m_3$ and the case with $\Delta m^2_{31} < 0$ is known as Inverted hierarchy (IH) where $m_3 < m_1 < m_2$. Information on the mass $m_0$ of the lightest neutrino and the Majorana phases cannot be obtained from neutrino oscillation experiments. This is because the oscillation probabilities are independent of these parameters, not only in vacuum but also in matter. If $m_0 \gtrsim 0.2\text{eV}$, future $^3\text{He}$ beta decay experiment [20] can measure it. Experiments which seek neutrinoless double beta decay [18] are sensitive to only a combination of neutrino masses and phases. Certainly, extracting information on Majorana phases alone from these experiments...
where the scale of lepton number violation is much higher e.g., supersymmetric models with which lepton number violation (which leads to the Majorana neutrino mass) is associated with the HTM e.g.,

- \( m_{s_1} \) and \( m_{s_2} \) are the lightest and the second lightest neutrinos, respectively.
- \( \phi_1 \) and \( \phi_2 \) are the Majorana phases.

The rates for the lepton flavour violating (LFV) decays depend explicitly on \( \phi_1 \) and \( \phi_2 \). Hence multiple signals for \( O(3) \) parameters essentially determine \( BR(H_{\pm \pm} \rightarrow l_i^\pm l_j^\pm) \) as a function of the unmeasured five parameters. Four cases corresponding to no CP violation from Majorana phases can be defined as follows: Case I (\( \phi_1 = 0, \phi_2 = 0 \)); Case II (\( \phi_1 = 0, \phi_2 = \pi \)); Case III (\( \phi_1 = \pi, \phi_2 = 0 \)); Case IV (\( \phi_1 = \pi, \phi_2 = \pi \)). These four cases have been studied in \([7, 17]\) for values of \( m_0 = 0 \) or \( \mathcal{O}(1) \) eV. In this work we will quantify the dependence of \( h_{ij} \) on \( m_0, \phi_1 \) and \( \phi_2 \). Those 3 parameters essentially determine \( BR(H_{\pm \pm} \rightarrow l_i^\pm l_j^\pm) \), with subdominant corrections from the neutrino oscillation parameters. Hence multiple signals for \( H_{\pm \pm} \rightarrow l_i^\pm l_j^\pm \) at the LHC would probe \( m_0, \phi_1 \) and \( \phi_2 \), in the context of the HTM.

We note here that such collider probes of the Majorana phases are particular to models in which lepton number violation (which leads to the Majorana neutrino mass) is associated with New Physics particles at the TeV scale. Analogous collider probes are not possible in models where the scale of lepton number violation is much higher e.g., supersymmetric models with very heavy right-handed neutrinos with masses of order \( 10^{12} \) GeV. However, in such models the Majorana phases (which are also required for successful leptogenesis) can significantly affect the rates for the lepton flavour violating (LFV) decays \( \tau \rightarrow l\gamma \) and \( \mu \rightarrow e\gamma \) [44]. Likewise, in the HTM \( \phi_1 \) and \( \phi_2 \) would also affect the rates for LFV decays which depend explicitly on \( h_{ij} \) e.g., \( \tau \rightarrow lll \) and \( \mu \rightarrow e\gamma \) [7]. Similar studies of the effects of Majorana phases on these LFV
decays in other models which contain a TeV scale $H^{\pm \pm}$ and an analogous $h_{ij}$ coupling have been performed in [45].

4.1 Dependence of BR($H^{\pm \pm} \rightarrow l^\pm l^\pm$) on the neutrino mass spectrum

In Fig. 1, we plot BR($H^{\pm \pm} \rightarrow l^\pm l^\pm$) as a function of $m_0$ for $\varphi_1 = 0$ and $\varphi_2 = 0$ (Case I) in NH and IH. The values of the oscillation parameters are fixed in the figures as follows:

$$\Delta m_{21}^2 = 7.9 \times 10^{-5} \text{eV}^2, \quad |\Delta m_{31}^2| = 2.7 \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{12} = 0.86, \quad (17)$$

$$\sin^2 2\theta_{23} = 1, \quad \sin^2 2\theta_{13} = 0.13, \quad \delta = 0. \quad (18)$$

For NH with smaller $m_0$, the $e$-related modes are suppressed because $m_1(=m_0)$ and $m_2$ are small and the contribution from the heaviest mass $m_3$ is also small due to the tiny $\theta_{13}$. BR$_{\mu \mu}$, BR$_{\mu \tau}$, and BR$_{\tau \tau}$ are roughly equal [2], and they dominate for smaller $m_0$ because $m_3$ does not appear with $\theta_{13}$. Note that BR$_{\mu \mu} \simeq$ BR$_{\tau \tau}$ and BR$_{\epsilon \mu} \simeq$ BR$_{\epsilon \tau}$ can be understood as the approximate symmetry for $\mu$-$\tau$ exchange by virtue of $\sin^2 2\theta_{23} = 1$ and tiny $\theta_{13}$. For larger $m_0$, BR($H^{\pm \pm} \rightarrow l^+_i l^-_j$) dominate because all coefficients of $m_i$ are positive for $h_{ii}$ with $\theta_{13} = 0$ in Case I and there is no strong cancellation among these terms even for $\theta_{13} \neq 0$. For $m_0 \gtrsim 0.3$ eV, the results for NH and IH are almost identical because of almost degenerate masses. Since BR are given by ratios of $|h_{ij}|^2$, they converge for large $m_0$. This is an attractive feature because HTM can predict certain ranges of BR without restricting $m_0$.

On the other hand, IH case gives rather simple results. This is because $m_1$ and $m_2$ include the larger scale $\sqrt{|\Delta m_{31}^2|}$ without any suppression from $\theta_{13}$ and consequently the effects of small $\Delta m_{21}^2$ and $\theta_{13}$ are hidden. In this case, BR$_{ee}$ becomes the dominant channel even for smaller $m_0$; one naively expects BR$_{ee} \simeq 4$BR$_{\mu \mu}$ for $m_0 = 0$ with eq. (16).

2 The naive expectation is that BR$_{\mu \tau}$ is twice as large as the other two modes because of $|h_{\mu \tau}|^2 + |h_{\tau \mu}|^2$.

However, this difference is accidentally compensated by the effect of $\Delta m_{21}^2$. 

---

Figure 1: BR($H^{\pm \pm} \rightarrow l^\pm l^\pm$) with no CP violation from Majorana phases ($\varphi_1 = \varphi_2 = 0$) as a function of $m_0$ in NH (left) and IH (right).
In order to quantify the effect of the unmeasured \((V_{\text{MNS}})_{e3}\) on \(\text{BR}_{ll'}\), we show \(\text{BR}_{ee}\) for \(\sin^2 2\theta_{13} = 0.13, 0\), and 4 values of the Dirac phase \(\delta = 0, \pi/2, \pi, 3\pi/2\) for Cases I to IV in Fig. 2. Other parameters are same as Fig. 1. Several lines are coincident in the figure, since \(\delta\) and \(\phi_2\) always appear as a combination \(2\delta - \phi_2\) in \(h_{ee}\). For example, Case I with \(\delta = 0\) and Case II with \(\delta = \pi/2\) give the same lines. Although the contribution of \((V_{\text{MNS}})_{e3}\) to \(\text{BR}_{ee}\) is considerably smaller than the effect of varying \(m_0\) or the Majorana phases (as expected by quadratic suppression with small \(\theta_{13}\)), it is not negligible. Consequently, it is also not negligible for other the BR. Fig. 2 also shows that the HTM predicts small \(\text{BR}_{ee}\) in case III and IV (cases with \(\phi_1 = \pi\)) for any value of \(m_0, \theta_{13}\), and \(\delta\) in both of NH and IH. Thus, Fig. 2 indicates that some information on Majorana phases may be extracted without knowledge of the sign of \(\Delta m^2_{31}\) and the values of \(m_0, \theta_{13}\), and \(\delta\).

### 4.2 Dependence of \(\text{BR}(H^{\pm\pm} \to l_i^\pm l_j^\pm)\) on Majorana phases

In this section we show the dependence of \(\text{BR}(H^{\pm\pm} \to l_i^\pm l_j^\pm)\) on Majorana phases with the values of the oscillation parameters given in (17) and (18). For \(m_0 = 0\), only the relative phase \(\varphi_{\text{rel}}(= \varphi_1 - \varphi_2)\) determines \(h_{ij}\) in NH. In Fig. 3 (left) we plot \(\text{BR}(H^{\pm\pm} \to l_i^\pm l_j^\pm)\) as a function of \(\varphi_{\text{rel}}\) for NH with \(m_0 = 0\). Tiny \(\theta_{13}\) suppresses the dependence of \(\text{BR}_{ee}\) on \(\varphi_{\text{rel}}\), and then the HTM gives a clear prediction of very small \(\text{BR}_{ee}\) for this case. The other \(\text{BR}_{ll'}\) change non-negligibly e.g., \(\text{BR}_{e\mu}\) and \(\text{BR}_{\mu\mu}\) vary by 0.05 or more, which could be larger than the experimental error if sufficiently large numbers of \(H^{\pm\pm}\) are produced. In IH for \(m_0 = 0\), \(\text{BR}_{ll'}\) does not depend on \(\varphi_2\) because it always appears multiplied by \(m_0\). Fig. 3 (right) shows the dependence of \(\text{BR}_{ll'}\) on \(\varphi_1\) for this case. One can see that the dependence on \(\varphi_1\) is even more pronounced because \(m_2\) for \(m_0 = 0\) in IH \((= \sqrt{\Delta m^2_{31}})\) is larger than that in NH \((= \sqrt{\Delta m^2_{21}})\). In particular, \(\text{BR}_{ee}\) and \(\text{BR}_{e\mu}\) seem to be very useful for extracting information on \(\varphi_1\) because...
they have large and opposite dependence. This dependence on $\varphi_1$ can be understood by the relative sign of the terms of $m_1$ and $m_2$ in eq. (16), which is $+$ for $h_{ee}$ and $-$ for $h_{e\mu}$, neglecting $\theta_{13}$.

For $m_0 = 0.3$ eV, where neutrino masses are almost degenerate, Fig. 4 shows that the phase $\varphi_1$ has a large effect on all $\text{BR}(H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm)$. The figure also shows that the $\varphi_2$ dependence for $\text{BR}_{e\mu}$ and $\text{BR}_{e\tau}$ is sizeable and overcomes the suppression from $\theta_{13}$. However, the latter suppression factor ensures that $\text{BR}_{ee}$ has a very small dependence on $\varphi_2$, and so this channel is crucial for extracting clear information on $\varphi_1$ and/or $m_0$ without contamination from the effect of $\varphi_2$.

It is clear that the Majorana phases have a large effect on $\text{BR}_{ll'}$ in the HTM, and their inclusion is required in order to quantify the allowed regions of $\text{BR}_{ll'}$.

### 4.3 Sensitivity to $\text{sign}(\Delta m^2_{31})$ and $m_0$.

As we have seen, both the neutrino mass spectrum and the Majorana phases have large effects on $\text{BR}(H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm)$ in the HTM, but the model also gives a clear prediction for $\text{BR}_{ll'}$. Therefore we can expect to obtain some information on the neutrino mass spectrum and/or the Majorana phases by observing $\text{BR}_{ll'}$. In our analysis, we use $\text{BR}(H^{\pm\pm} \rightarrow e^\pm e^\pm,e^\pm \mu^\pm,\mu^\pm \mu^\pm)$ for which the LHC expects greatest sensitivity. Naively, three measurements of $\text{BR}_{ll'}$ are sufficient to extract information on the three parameters.

In this section we consider the possibility to determine $\text{sign}(\Delta m^2_{31})$ and/or to exclude $m_0 = 0$. Hereafter, we use (17) and

$$\sin^2 2\theta_{23} > 0.93, \quad \sin^2 2\theta_{13} < 0.13, \quad \delta = 0 - 2\pi. \quad (19)$$

Non-zero values of $\theta_{13}$ and $\delta$ affect $\text{BR}_{ee}$ as was shown in Fig. 2. Moreover, deviation of $\theta_{23}$ from $\pi/4$ especially affects $\text{BR}_{e\mu}$ in our analysis because a rather wide range $0.37 < s^2_{23} < 0.63$ is allowed. In Fig. 5 the allowed regions of $\text{BR}_{ll'}$ in the HTM are shown by the shaded (light and
Figure 4: BR_{l'l'} as a function of $\varphi_1$ ($\varphi_2$) with $\varphi_2 = 0$ (a), $\pi$ (b) ($\varphi_1 = 0$ (c), $\pi$ (d)) for $m_0 = 0.3$ eV (NH$\simeq$ IH).

dark) regions; the dark shaded area corresponds to the overlap of the allowed regions for NH and IH. The area above the dotted line is unphysical because the sum of BR exceeds unity. It is clear that the HTM predicts $BR_{ee} \lesssim 0.49$, $BR_{e\mu} \lesssim 0.61$, and $BR_{\mu\mu} \lesssim 0.47$. The areas outside the solid line are unreachable, and are particular to the HTM (in which neutrino mass is solely given by eq. (3)) and will differ from the corresponding disallowed regions, if any\[3\] in other models which can contain a light $H^{\pm\pm}$. Hence in the event of signals for $H^{\pm\pm} \rightarrow l_i^{\pm}l_j^{\pm}$ these allowed regions can be compared with the experimental measurements of $BR(H^{\pm\pm} \rightarrow l_i^{\pm}l_j^{\pm})$ in order to test the HTM. If measured BR are outside the shaded region the HTM is disfavored. If given signals for $H^{\pm\pm} \rightarrow l_i^{\pm}l_j^{\pm}$ are compatible with the reachable regions in the HTM, the data can be interpreted in the context of this model and one can try to extract information on

\[3\] If $h_{ij}$ are arbitrary parameters, as in the Left-Right symmetric model of [1], the unreachable region would vanish.
Figure 5: The shaded area surrounded by the solid line is attainable in the HTM. Allowed regions for NH and IH correspond to BR inside of the dashed and dash-dotted lines, respectively. The dotted line divides the physical and unphysical regions.

parameters of neutrino mass matrix.

Furthermore, the area inside the dashed and dash-dotted lines in Fig. 5 is possible for NH and IH, respectively. NH gives the additional bounds $\text{BR}_{ee} \lesssim 0.33$ and $\text{BR}_{\mu\mu} \lesssim 0.51$, while IH gives $0.03 \lesssim \text{BR}_{ee}$ and $\text{BR}_{\mu\mu} \lesssim 0.34$. These are also particular features of the HTM. If BR in the light shaded region are measured, NH or IH is disfavored in the HTM.

Next, let us consider the possibility to exclude $m_0 = 0$. Fig. 6 shows the attainable regions of BR with $m_0 = 0$. As in Fig. 5, the area inside the thin solid line is reachable in the HTM, and the region above the dotted line is unphysical. The dashed line corresponds to the BR which can be obtained in NH, and the dash-dotted line is BR in IH, both with $m_0 = 0$. Note that the area inside these lines can also be obtained with $m_0 \neq 0$ but outside is impossible for $m_0 = 0$. This behaviour can also be seen in Fig. 3. The bold solid line denotes a $\chi^2$ analysis for measurements of signals in the 2l channels ($N_{ee}$, $N_{e\mu}$, and $N_{\mu\mu}$) of $H^{\pm\pm}$ decays (see Appendix A for detail). If BR in the shaded region are measured, one can exclude $m_0 = 0$ at 90%CL in HTM. NH with $m_0 = 0$ gives a very clear prediction of small $\text{BR}_{ee}^{\text{true}}$ as can be seen in Fig. 3. The bold line shows that $m_0 = 0$ can be excluded at 90%CL if $\text{BR}_{ee}^{\text{true}}$ lies in the narrow interval $0.02 \lesssim \text{BR}_{ee}^{\text{true}} \lesssim 0.05$, even without measuring other BR. If only $\text{BR}_{e\mu}$ or $\text{BR}_{\mu\mu}$ is observed it is impossible to exclude $m_0 = 0$. Therefore, it is essential to use more than two BR to exclude $m_0 = 0$.

4.4 Sensitivity to CP violation from Majorana phases

In this section, we discuss the possibility to exclude the CP-conserving cases for Majorana phases, namely $\varphi_1, \varphi_2 = 0, \pi$. Establishing experimentally that CP is violated by Majorana phases in the HTM is of much phenomenological interest and is feasible if there are sufficiently large numbers of $H^{\pm\pm}$.

In Fig. 7, the attainable regions of BR for the CP-conserving cases of Majorana phases are shown. As in Fig. 5, the area inside the thin solid line is reachable in the HTM, and the
Figure 6: The thin solid line shows the attainable region in the HTM and the dotted line divides the physical and unphysical regions. The area inside the dashed and dash-dotted lines can be obtained with $m_0 = 0$ in NH and IH, respectively. The bold solid line is a result of a $\chi^2$ analysis and $m_0 = 0$ can be excluded if BR are measured within the shaded region.

region above the dotted line is unphysical. Bold dashed, thin dashed, bold dash-dotted, and thin dash-dotted lines show the possible regions for Case I, II, II, and IV, respectively. From the figures it is clear that the HTM predicts very specific regions for $\text{BR}(H^{\pm\pm} \rightarrow l_1^\pm l_2^\mp)$ in the CP-conserving cases, especially in the space of $\text{BR}_{ee}-\text{BR}_{e\mu}$; $\text{BR}_{ee}$ is small ($\lesssim 0.07$) in Case III and IV (cases with $\varphi_1 = \pi$) as seen in Fig. 2 and $\text{BR}_{e\mu}$ cannot be so large ($\lesssim 0.09$) in Case I and II (cases with $\varphi_1 = 0$). The bold solid line is obtained by a $\chi^2$ analysis with 2l channels of $H^{\pm\pm}$ decays (see Appendix A for details), and the CP-conserving cases are excluded at 90%CL in the HTM if experiment measures BR in the shaded region.

4.5 Accuracy of the determination of $m_0$ and Majorana phases

Once measurements of $\text{BR}_{ee}$, $\text{BR}_{e\mu}$, and $\text{BR}_{\mu\mu}$ are available one can constrain $m_0$ and the Majorana phases. In this section we show how precisely they can be constrained by assuming $\text{BR}_{ee} = 0.268$, $\text{BR}_{e\mu} = 0.243$, and $\text{BR}_{\mu\mu} = 0.066$ as an example of a possible signal. These values can be obtained by taking $m_0 = 0$, $\varphi_1 = \pi/2$, $\sin^2 2\theta_{13} = 0.1$, $\delta = 0$, and $\theta_{23} = \pi/4$ for IH in HTM. It is evident from Fig. 3 that these values of BR can be accommodated in the HTM, but not in the case of NH - see Fig. 3(a).

In Fig. 8 the allowed regions at 90%CL are shown in the spaces of $\varphi_1-\varphi_2$, $m_0-\varphi_1$, and $m_0-\varphi_2$. They are obtained by a $\chi^2$ analysis with three numbers of 2l channel signals ($N_{ee}$, $N_{e\mu}$, and $N_{\mu\mu}$) of $H^{\pm\pm}$ decays (See Appendix A for details). Since the above BR are inside the shaded regions in Fig. 4 Case I-IV of CP-conservation are all excluded which is clearly depicted in Fig. 8(a). Two regions in Fig. 8(a) are just copies of each other because $\text{BR}_{l\ell'}$ are unchanged by replacing $h_{ij}$ with $h_{ij}^{\ast}$. Although $\varphi_1$ can be constrained very well, all values of $\varphi_2$ are allowed in Fig. 8 because BR in IH with $m_0 = 0$ does not depend on $\varphi_2$. In general one expects looser constraints on $\varphi_2$ because dependence of $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$ on $\varphi_2$ is suppressed by tiny $\theta_{13}$. Fig. 8 also shows that a stringent upper bound on $m_0$ ($m_0 \lesssim 0.07$ in this example) can be obtained.
by measuring BR.

5 Conclusions

Doubly charged Higgs bosons ($H^{\pm\pm}$) are a distinctive signature of a variety of models which can accommodate neutrino mass. Discovery of $H^{\pm\pm}$ at the Tevatron/LHC with a reasonably light mass ($< 400$ GeV) could enable thousands of $H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$ events to be observed with the anticipated final integrated luminosities at the LHC. In such a scenario precise measurements of $H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$ would be feasible and would constitute a means of experimentally favouring or disfavouring various candidate models which contain a $H^{\pm\pm}$.

The Higgs Triplet Model (HTM), in which (Majorana) neutrino mass is solely given as the product of a triplet vacuum expectation value $v_\Delta$ and a triplet Yukawa coupling $h_{ij}$, is a particularly simple model which leads to a specific relationship between the branching ratios (BR) of $H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$ and the neutrino mass matrix. In particular, BRs are determined only by the neutrino mass matrix independently of $v_\Delta$ and Higgs boson masses if $m_{H^{\pm\pm}} \leq m_{H^\pm}$.

We performed a quantitative study of the dependence of the BRs on all the parameters in the neutrino mass matrix, especially the dependence on the three parameters which induce the most uncertainty in such predictions: the lightest neutrino mass $m_0$ and the two Majorana phases. We displayed the allowed regions of BR($H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$) for $i, j = e, \mu$ and showed that they are considerably smaller than for the case of an arbitrary $h_{ij}$. By measuring these BRs it is possible to both test the HTM and determine the sign of $\Delta m^2_{31}$.

Due to the large effect of $m_0$ and the Majorana phases on the BRs we showed that it is possible to extract information on these three parameters by measuring $BR_{ee}$, $BR_{e\mu}$, and $BR_{\mu\mu}$. Such information cannot be obtained from neutrino oscillation experiments. By using
Figure 8: The shaded region is allowed at 90%CL for BR_{ee} = 0.268, BR_{e\mu} = 0.243, and BR_{\mu\mu} = 0.066. These values can be obtained by taking m_0 = 0, \varphi_1 = \pi/2, \sin^2 2\theta_{13} = 0.1, \delta = 0, and \theta_{23} = \pi/4 for IH in HTM.

the 2l channel of H^\pm decays, it was found that m_0 = 0 and/or the CP-conserving case for the Majorana phases can be excluded without knowledge of \text{sign}(\Delta m_{31}^2), m_0, \theta_{13}, and \delta. There is a small range of BR_{ee} (0.02 \lesssim BR_{ee} \lesssim 0.05) where m_0 = 0 can be excluded at 90%CL by measuring only BR_{ee}. For the exclusion of the CP-conserving case for Majorana phases, BR_{ee} and BR_{e\mu} are especially important. For a specific choice of BR_{ee}, BR_{e\mu}, and BR_{\mu\mu} we displayed the allowed regions of (m_0, \varphi_1, \varphi_2) and showed that it is possible to constrain m_0 and \varphi_1 stringently.

In contrast, establishing that Majorana phases are non-zero is notoriously difficult in models in which lepton number violation is associated with particles at a very high energy scale (e.g., right-handed neutrinos with a Majorana mass of order 10^{10} \text{ GeV or more}). In the HTM with a light H^\pm this becomes is a realistic and unique possibility.

Note added: During finalization of this paper an article [46] appeared which deals with the same topic. Although there is inevitably some overlap, our analysis differs considerably from theirs e.g., i) our definition of the \chi^2 is a function of three BRs in the 2 lepton channel, while their \chi^2 is for five BRs in the 4 lepton channel, and ii) the presentation of results, especially figures 5-8. Where comparisons can be made we find good agreement with their results.

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A Definition of $\Delta \chi^2$

In order to test assumptions, e.g., CP-conservation of Majorana phases, we calculate a $\Delta \chi^2$. In our analysis, BR_{ee}^{\text{true}}, BR_{e\mu}^{\text{true}}, and BR_{\mu\mu}^{\text{true}} are used as input which can be understood as true
values chosen by nature or best fit values from experiment. We try to fit these true values with $BR_{ll'}^{fit}$ which are calculated theoretically by eq. (5) for the assumption to be tested. We utilize the 2l channel and the number of events is given by

$$N_{ll'} \equiv \epsilon_{eff} \left\{ N_{pair} BR_{ll'}(2 - BR_{ll'}) + N_{single} BR_{ll'} \right\},$$

where $N_{pair}$ ($N_{single}$) is the number of pair (single) production of doubly charged Higgs, and $\epsilon_{eff}$ denotes an efficiency due to event cuts; we can obtain the numbers of $H^{\pm\pm}$ production from Table 2 as $N_{pair} = N_{4l}$ and $N_{single} = N_{3l} - N_{4l}$. We take $\epsilon_{eff} = 0.5$. We use $N_{pair} = 900$ and $N_{single} = 1600$ by assuming $m_{H^{\pm\pm}} = m_{H^{\pm}}$ for simplicity (see Table 2) and because the difference of their masses should not be too large in order to maintain $\rho \simeq 1$ at the 1-loop level. For the number of remaining background events after cuts, we use $N_{BG} = 1$ for each $N_{ll'}$ because the signal is expected to be almost background free. In the fitting procedure, $\Delta \chi^2$ is minimized within possible regions of the following parameters:

$$m_0^{fit} \equiv 0 - 0.3 \text{eV}, \quad \varphi_1^{fit}, \varphi_2^{fit} = 0 - 2\pi,$$

$$\sin^2 2\theta_{13}^{fit} \equiv 0 - 0.13, \quad \delta^{fit} = 0 - 2\pi,$$

$$\sin^2 2\theta_{23}^{fit} > 0.93, \quad \text{hierarchy} = \text{NH, IH}.$$  

The minimization gives the most pessimistic value for exclusion of the assumption because a smaller value of $\Delta \chi^2$ means a better fitting with the assumption. The other parameters are fixed for true and fitting values as (17). Since we wish to extract information on three parameters $(m_0, \varphi_1, \varphi_2)$, our analysis is for three degrees of freedom and $\Delta \chi^2 = 6.3$ corresponds to 90%CL even if results are projected onto spaces of two parameters.

### A.1 Exclusion of $m_0 = 0$ and CP-conserving case for Majorana phases

In Fig. 6 the region where $m_0 = 0$ is excluded in the HTM is given by calculating the $\Delta \chi^2_{m_0}$ function. The $\Delta \chi^2_{m_0}$ is defined as

$$\Delta \chi^2_{m_0}(BR_{ee}^{true}, BR_{e\mu}^{true}, BR_{\mu\mu}^{true}) \equiv \min_{\varphi_1^{fit}, \varphi_2^{fit}, m_0^{fit}, x^{fit}} \left\{ \sum_{ll' = ee, e\mu, \mu\mu} \left( \frac{N_{ll'}^{true} - N_{ll'}^{fit}}{N_{ll'}^{true} + N_{BG}} \right)^2 \right\},$$

$$x \equiv \{\theta_{13}, \delta, \theta_{23}, \text{hierarchy} \}. \quad (25)$$

where $N_{ll'}^{true}$ is given by $(BR_{ee}^{true}, BR_{e\mu}^{true}, BR_{\mu\mu}^{true})$ and $N_{ll'}^{fit}$ is calculated with the theoretical form of BR (eq. (3)) for $(\varphi_1^{fit}, \varphi_2^{fit}, m_0^{fit}, x^{fit})$. When we show the projected region of $\Delta \chi^2_{m_0} \leq 6.3$ onto the plane $BR_{ee}^{true}$-$BR_{e\mu}^{true}$, for example, we minimize the $\Delta \chi^2_{m_0}$ with respect to $BR_{\mu\mu}^{true}$; this minimization is achieved simply by replacing $\mu\mu$ from the sum of $ll'$ in the definition of the $\Delta \chi^2_{m_0}$, and then we take 6.3 for the resulting $\Delta \chi^2(BR_{ee}^{true}, BR_{e\mu}^{true})$ to have the projected contour of 90%CL.

In Fig. 7 the region where all of the CP-conserving cases I, II, III, and IV are rejected is given by calculating the $\Delta \chi^2_{CPV}$ function. The $\Delta \chi^2_{CPV}$ is

$$\Delta \chi^2_{CPV}(BR_{ee}^{true}, BR_{e\mu}^{true}, BR_{\mu\mu}^{true}) \equiv \min_{\text{CPC case}, m_0^{fit}, x^{fit}} \left\{ \sum_{ll' = ee, e\mu, \mu\mu} \left( \frac{N_{ll'}^{true} - N_{ll'}^{fit}}{N_{ll'}^{true} + N_{BG}} \right)^2 \right\}. \quad (26)$$

We use $\Delta \chi^2_{CPV}$ in the same way as $\Delta \chi^2_{m_0}$.
A.2 Allowed region of $\varphi_1$, $\varphi_2$, and $m_0$

In Fig. 8, the allowed region of $(\varphi_1, \varphi_2, m_0)$ for a given set of BR$_{ll'}^{\text{true}}$ is obtained by

$$
\Delta \chi^2_{\text{allow}}(\varphi_1, \varphi_2, m_0) = \min_{x^{\text{fit}}} \left\{ \sum_{ll'=ee, e\mu, \mu\mu} \frac{(N_{ll'}^{\text{true}} - N_{ll'}^{\text{fit}})^2}{N_{ll'}^{\text{true}} + N_{BG}} \right\}.
$$

(27)

$N_{ll'}^{\text{fit}}$ is a function of $(\varphi_1, \varphi_2, m_0, x^{\text{fit}})$. In order to project the 3-dimensional allowed region onto 2-dimensional space, we minimize $\Delta \chi^2_{\text{allow}}$ with respect to a parameter. Then, we take 6.3 for the resulting $\Delta \chi^2$ to obtain the 90%CL contour.

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