Shaft gas injection of reducing gas in the blast furnace is one of favorable ways to greatly decrease CO$_2$ emissions from steel works, and this approach is used for the top gas recycling and oxygen blast furnace processes. In these processes, the penetration effect of the gas injected from the auxiliary tuyeres is important for attaining effective gas reduction or making up the heat balance in the upper part, and so it is useful to analyze the dynamic behavior of the gas injected into the shaft.

In the present study, the penetration effect of injected gas was three-dimensionally simulated by a hybrid model of the discrete element method (DEM) and continuum model (CFD). In particular, the CFD model was used to quantitatively analyze the dynamic gas flow and the pressure distribution in the burden layers calculated by DEM. Although the area influenced by the injected gas from the auxiliary tuyeres was restricted to a specific area due to insufficient horizontal inertial force of the gas, the penetration area gradually enlarged as the gas velocity from the auxiliary tuyeres increased. In a small blast furnace, the injected gas can easily reach the center with the higher gas velocity, and it was shown that the relative penetration depth of the injected gas depends on the inner volume of the blast furnace. However, the overall behavior of the injected gas did not show any remarkable change. In conclusion, the penetration area of shaft gas was almost proportional to the ratio of shaft gas and the upward gas from the conventional tuyeres.

KEY WORDS: ironmaking; blast furnace; shaft gas injection; low reducing agent operation; discrete element method; computational fluid dynamics.

1. Introduction

Various methods of reducing CO$_2$ emissions from steel works have been attempted, with the principal method being to decrease the carbon input to the blast furnace.$^{1}$ Shaft gas injection and the oxygen blast furnace process greatly reduce the coke rate. Regarding shaft gas injection of reducing gas, some researchers examined the injection of reformed coke oven gas (NKG) and partial oxidation gas (FTG).$^{2-4}$ The validity of the NKG process was confirmed by an experimental blast furnace, where it was reported that injecting shaft gas into the blast furnace decreased the coke rate from 571 kg/thm to 374 kg/thm.$^{5}$ Moreover, the oxygen blast furnace process has been investigated since around 1990,$^{5,6}$ in which cold oxygen is injected from an oxygen burner to intensify the pulverized coal injection rate. In this process, the injection of preheated gas from the auxiliary tuyeres in the upper shaft was used to make up the heat balance in the blast furnace. In these processes, the shaft gas injection level and volume varied according to their purpose. Recently, New Blast Furnace for ULCOS (Ultra Low CO$_2$ Steelmaking) has been developed and some campaigns by the experimental blast furnace have been conducted.$^{7,8}$ Overall, shaft gas injection is an important technology for improving the capability of blast furnaces, however, the effectiveness of gas penetration in an actual blast furnace remains an issue.

In the present study, the three-dimensional dynamic behavior of gas flow and solid motion caused by shaft gas injection was analyzed by a hybrid model of the discrete element method (DEM) and continuum model (CFD).$^9$ Since the motion of each particle can be individually calculated in DEM, the structure of burden layers in an actual blast furnace can be accurately represented.$^{10}$ Analysis by the DEM-CFD model for the blast furnace will provide useful information on the effectiveness of shaft gas injection in actual blast furnaces. The previous study focused on the fundamental behavior of gas injected to the shaft.$^{11}$ The influence of the number of auxiliary tuyeres and the injection level were clarified with the DEM-CFD model. In this study, first the influence of injected gas volume was examined, comparing with the calculated results for varying number of auxiliary tuyeres. Moreover, the change in the penetration area in the blast furnace for varying blast furnace inner volume was evaluated. In parallel with the above analysis, the stress distribution between particles was examined under the influence of shaft gas injection. Overall, the dynamic behavior of gas and solid flows and penetration effect of injected shaft gas under various conditions were studied.
2. Structure of Present Model

The basic equations of DEM-CFD in the present study are given below. The movement of a particle is described by DEM. Contact forces are described by the Voigt model which consists of spring, dash pot and slider in DEM. Contact forces are described by the V oigt model given below. The movement of a particle is described by

\[ m_i \frac{d^2 \mathbf{u}_i}{dt^2} + \eta \frac{d \mathbf{u}_i}{dt} + K \mathbf{u}_i + f_c + m_i g = 0 \]  

(1)

\[ I_i \frac{d^2 \phi_i}{dt^2} + \eta R_i^2 \frac{d \phi_i}{dt} + K R_i^2 \phi_i = 0 \]  

(2)

where, \( K, \eta, I, R, f_c \) and \( g \) are the stiffness, a damping coefficient, the moment of inertia, the radius of a particle, the particle-fluid interaction force and gravitational acceleration.

The movement of a fluid is described by the continuity equation and Navier-Stokes equation:

\[ \frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \rho \mathbf{g} + \frac{\partial}{\partial t} (\rho \mathbf{u}) \]  

(3)

\[ \frac{\partial}{\partial t} (\rho \mathbf{v}) + (\mathbf{v} \cdot \nabla) \rho \mathbf{v} = -\mathbf{v} \cdot \nabla p + \mathbf{f}_g \]  

(4)

where, \( \epsilon, \mathbf{v}, \rho, \mu_c \) and \( \mu_g \) denote the void fraction, velocity vector, pressure, density, and viscosity of a fluid, respectively. The source term \( f_g \) takes into account the interaction force between fluid and particles.

When the calculation region is divided into microscopic cells in order to calculate the gas flow in the blast furnace, the fluid velocity is treated as superficial velocity in Eqs. (3) and (4). In this study, the void fraction in the packed bed is calculated from the location of the particles. The pressure drop is expressed by the multidimensional Ergun’s equation for a packed bed,

\[ f_g = N_i C_{g-p} \left[ \epsilon - v_g \right] \]  

(5)

\[ C_{g-p} = \begin{cases} \frac{\mu_g (1-\epsilon) \varepsilon}{150(1-\epsilon) + 1.75 \Re} & (\epsilon \leq 0.8) \\ 3 \frac{\mu_g (1-\epsilon) \varepsilon^{0.67}}{4 \rho \varepsilon^{0.57} d_p^2} \Re & (\epsilon > 0.8) \end{cases} \]  

(6)

\[ C_D = \begin{cases} 24(1 + 0.15 \Re^{0.67}) / \Re & (\Re \leq 1000) \\ 0.43 & (\Re > 1000) \end{cases} \]  

(7)

\[ \Re = \frac{\left| \mathbf{v}_p - \mathbf{v}_g \right|}{\mu_g} \]  

(8)

where, \( d_p, N_i, \mathbf{v}_p \) and \( \mathbf{v}_g \) denote the representative particle diameter in a cell, the number of actual particles included in a cluster particle and the velocity vectors of the particle and the gas, respectively. The pressure drop in the cell is considered to be the total momentum exchange of the particles. Therefore, a cluster particle used in DEM receives the total drag forces of the actual particles which are included in the cluster, as shown in Fig. 1(b). For the particle-fluid interaction force, Ergun’s equation is applied in the low void fraction region and Wen & Yu’s equation is applied in the high void fraction region. This procedure is the same as described in the previous paper.

3. Procedure of Simulation

3.1. DEM-CFD Model for the Blast Furnace

A blast furnace of 5 775 m³ inner volume with 40 tuyeres was used for the present simulation as the standard condition. Moreover, two other blast furnace profiles, a medium size and a small size, were selected for comparison with the standard condition. The height of each blast furnace was the same in order to examine clearly the influence of blast furnace diameter. The profiles of these three blast furnaces are shown in Fig. 2. In order to decrease the number of particles in DEM, a semicircular model having a flat wall in front with no friction was employed. However, this model is based on a three-dimensional structure. Raceways are represented by spheres of 1.6 m in diameter equally spaced on the wall at tuyere level. The arrangements of the ordinary tuyeres and the auxiliary tuyeres are shown in Fig. 3. Particles of coke entrained in the raceway disappeared at arbitrarily-specified intervals. The pig iron and slag layer in the hearth are not considered. The curved lines in Fig. 2 denote the melting zone, and particles of ore disappeared on the line. The top surface of burden descends with consumption...
of ore in the melting zone and coke at the raceways. When the surface of burden is 1 m below the stock line, the coke and ore particles are charged alternately. The gas phase was assumed to be isothermal and incompressible.

The motion of burden particles was calculated by DEM and then the distribution of voidage was derived from the positions of particles. Depending on the voidage in the blast furnace, the gas flow was calculated by CFD. Gas drag force on particles in DEM was generated from the gas flow. The calculation conditions of DEM are shown in Table 1, and the details of the DEM-CFD model were as described in the previous paper.9) Table 2 shows the calculation conditions in the CFD module. By using the previous study, three times the radius for the spherical control volume radius around particles was used to calculate the void fraction in the blast furnace.9) Accordingly, the characteristics such as the voidage distribution in the blast furnace can be well represented without losing local information on the packed bed.

### 3.2. Application of DEM-CFD Model for Shaft Gas Injection

Several shaft gas injection processes have been proposed to date,3–5,7) each having different injection point levels depending on the purpose. The calculation conditions were selected based on these previous processes in the past. The auxiliary tuyeres were placed 11 m above the tuyere level. Table 3 shows the shaft gas injection conditions. The number of auxiliary tuyeres and distribution of gas volume from the auxiliary tuyeres were changed. The gas volume injected from the auxiliary tuyeres was changed from 10% to 30% and 50% of the total gas volume. In addition to the above conditions, three inner volumes of blast furnace were selected. As shown in Table 3, to reduce the computational load, larger particles were used. Firstly, coke disappeared in front of the tuyeres and coke and ore were charged alternately. Then, under the conditions shown in Tables 1, 2 and 3, gas and solid motions were calculated for 480 sec. In DEM, the calculation time was accelerated 120-fold, so the calculation in DEM took only 4 sec.

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**Table 1.** Calculation conditions used in DEM.

| Parameter | Value |
|-----------|-------|
| Radius of particle (coke, ore) \( R_i \) | 0.150, 0.075 [m] |
| Contact friction coefficient (coke, ore) with wall \( \mu \) | 1.00, 1.00, 0.70 [-] |
| Rolling friction coefficient (coke, ore) \( \alpha \) | 2.50, 1.25 [-] |
| Young’s modulus (coke, ore) of wall \( E_i \) | 1.00, 1.00, 2.00 [GPa] |
| Poisson’s ratio (coke, ore) of wall \( \nu \) | 0.21, 0.24, 0.30 [-] |
| Apparent density (coke, ore) \( \rho_i \) | 1100, 4000 [kg/m³] |
| Maximum particle number* | a)350000, b)250000, c)120000 [-] |
| Time step \( \Delta t \) | 1.0 × 10⁻⁴ [s] |
| Discharging rate | 100 [s⁻¹] |

*Inner volume: a) 5775 m³, b) 3250 m³, c) 1444 m³.

**Table 2.** Calculation conditions used in CFD.

| Parameter | Value |
|-----------|-------|
| Diameter of actual particle (coke, ore) | 0.035, 0.020 [m] |
| Viscosity of gas | 1.80 × 10⁻⁵ [Pa·s] |
| Density of gas | 1.20 [kg/Nm³] |
| Inlet area of tuyere | 0.0560 [m²] |
| Number of grids \((h, r, \theta)\) | 43 × 24 × 142 [-] |
| Update interval | 600 [DEMstep] |
| Time step | 7.2 [s] |
| Void fraction at wall | 0.49 |
| at free space | 0.99 |
| in raceway | 0.90 [-] |
| Liquid phase ratio under the melting zone | 0.20 |

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The horizontal gas velocity vectors at the various levels are shown in Fig. 7. The red part denotes intensified gas flow in the horizontal direction, and the blue region is equivalent to the part where the upward gas dominates. The red parts were mainly observed at the injection level. Although the horizontal gas velocity was weakened at +1 m level above the injection level, it slightly remained in the intensified gas velocity condition (case B)). Overall, the red parts were enlarged by increased gas velocity from the auxiliary tuyeres, so the penetration area can be controlled by the gas velocity. However, the region influenced by shaft gas injection was restricted in the vicinity of the auxiliary tuyeres. In the same manner, Fig. 8 shows the change of horizontal gas velocity vectors with the increase of gas volume from the auxiliary tuyeres for the same number of injection tuyeres. The tendencies in Fig. 6 are similar to the results in Fig. 5. Although these results were calculated from the momentum balance of gas, the penetration area of injected gas could be qualitatively estimated from the gas velocity distribution as shown in Figs. 5 and 6 due to the weak diffusion effect.18)

4.2. Pressure Distribution at Shaft Gas Injection

Figure 9 shows the isobars calculated for the above three cases. In every case, the shape of the top isobar coincided with the burden surface, and it is the basis of the isobar. Other planes are plotted at equal pressure intervals, and the color represents relative pressure. In the operation without shaft gas injection, the isobar planes are nearly flat except the lower part and are located at equal distances. In the present calculation, the gas was assumed to be isothermal and incompressible and so the absolute value of pressure drop is different from that of an actual blast furnace. Nevertheless, the relative influence of shaft gas injection on isobars can be examined. Since the auxiliary tuyeres in the shaft are placed intermittently, the pressure in the peripheral zone becomes uneven circumferentially. Especially, in case B), the isobars were intensely distorted by the shaft gas injection.

Figure 10 shows the influence of gas velocity through the auxiliary tuyeres, with eight tuyeres in the semi-circle. As the gas volume from the auxiliary tuyeres increased, the isobar plane became severely distorted. Varying numbers of auxiliary tuyere and gas velocity caused unevenness in the
Fig. 5. Calculated gas velocity vectors on vertical cross section of blast furnace with inner volume 5775 m$^3$ for different shaft gas injection conditions (vs: auxiliary tuyere gas velocity, Vo: inlet gas volume from ordinary tuyere, Vs: inlet gas volume from shaft gas injection, S/O: the ratio of injection gas volume to blast gas volume from ordinary tuyeres).

Fig. 6. Calculated gas velocity vectors on vertical cross section of blast furnace with inner volume 5775 m$^3$ for different auxiliary tuyere gas velocities.

Fig. 7. Horizontal gas velocity vectors on horizontal section of blast furnace with inner volume 5775 m$^3$ (a) +2 m above injection level, (b) +1 m above injection level, (c) injection level, (d) –1 m below injection level.

Fig. 8. Horizontal gas velocity vectors on horizontal section of blast furnace with inner volume 5775 m$^3$ (a) +2 m above injection level, (b) +1 m above injection level, (c) injection level, (d) –1 m below injection level.

Fig. 9. Isobar planes in blast furnace with inner volume 5775 m$^3$ for each inflow condition of shaft gas injection.

Fig. 10. Isobar planes in blast furnace with inner volume 5775 m$^3$ for different shaft gas velocities.
The isobar plane near the auxiliary tuyere level, but no unevenness was observed in the upper shaft.

### 4.3. Gas Drag Force and Normal Stress between Particles

The distribution of gas drag force on particles in the whole blast furnace is shown in Fig. 11. The color in Fig. 11 represents the magnitude of drag force on each particle. The coke particles receive relatively larger gas drag force than ore particles due to their large diameter. The general tendency of the drag force on particles is similar to each other as shown in Fig. 11. Strictly, the drag force below the auxiliary tuyeres of the shaft decreases due to the effect of gas volume change. Although the distribution of drag force is generally dependent on the distribution of gas volume to the auxiliary tuyeres and the ordinary tuyeres, it was estimated to have little influence on the burden descending.

The distribution of normal stress between particles along the vertical axis is shown in Fig. 12. The stress on a particle was derived by the summation of the forces of all contact points divided by the surface area. These values of normal stress at a specified height were averaged for each height.

**Fig. 11.** Gas drag force distributions in blast furnace with inner volume 5,775 m$^3$ for each particle on vertical cross section for each condition of shaft gas injection.

**Fig. 12.** Change in averaged normal stresses in blast furnace with inner volume 5,775 m$^3$ along vertical axis for each condition of shaft gas injection.

**Fig. 13.** Calculated gas velocity vectors on vertical cross section for different sizes of blast furnace.

**Fig. 14.** Horizontal gas velocity vectors on horizontal cross section (a) +2 m above injection level, (b) +1 m above injection level, (c) injection level, (d) –1 m below injection level.

**Fig. 15.** Isobar planes for each inflow condition of shaft gas injection for different sizes of blast furnace.
Solid, broken, and chain lines denote those for different injection gas volumes. The distribution of normal stress was influenced by the blast furnace profile, and showed a maximum value at 6.0 m above the tuyere level, and the normal stress between particles around the bosh was reduced by the bosh angle. From Fig. 12, it can be seen that these tendencies along the vertical axis were hardly influenced by shaft gas injection volume.

5. Influence of Blast Furnace Inner Volume on Penetration

Although the characteristics of the shaft gas injection were clarified by using an experimental blast furnace, scaling-up may affect the penetration effect. It was estimated that the penetration of the injected gas depended on the scale of the blast furnace. In this research, the blast furnace inner volume was intentionally changed to examine the penetration effect in the blast furnace.

Figure 13 shows the gas velocity vectors on the vertical cross section in the large, medium and small blast furnaces. The gas volume of the injected gas is proportional to the inner volume of the blast furnace, and the tuyere diameter was adjusted so as to make the gas velocity from the auxiliary tuyeres equal to each other. From Fig. 13, it can be seen that the gas flow vectors near to the wall were deformed by the shaft gas injection in the specific area for the three cases. The absolute penetration distance influenced by the auxiliary tuyeres seemed to be similar to each other. Figure 14 shows the horizontal gas vectors at the various levels in the three cases. At the injection level, the intensified horizontal gas vectors indicated by red can be observed nearby the auxiliary tuyeres in every case. The region where the intensified horizontal gas vectors were observed disappeared rapidly toward the center. However, the portion indicated by red occupied a relatively larger area as the diameter of the blast furnace was reduced. At the +1 m level above the injection point, horizontal gas vectors were hardly observed in the small blast furnace, whereas in the large blast furnace, the green area, which indicates weak horizontal gas vectors, slightly remained. This implies that the transient gas dynamic behavior of the injected gas varies with the scale of the blast furnace.

Figure 15 shows the calculated isobars for the three cases. It was observed that the isobar at the shaft gas injection level was distorted, and the general tendency is similar to each other. However, in case of the small size blast furnace, the distortion of the isobar at the injection level seems to reach slightly to the center; this behavior agrees with the tendency shown in Fig. 13.

6. Evaluation of Shaft Gas Injection Based on the Calculated Results

From the above results, the gas velocity through the auxiliary tuyeres has an influence on the penetration effect in the blast furnace. Figure 16 shows the relationship between the horizontal gas velocity and the non-dimensional radius at the injection level. Since the horizontal axis is shown as a non-dimensional unit for radius, the absolute value of $X_0$ differs from each other. Here, the horizontal axis is defined as the gas velocity on the central axis of the auxiliary tuyere, and the calculated results for the change in the injection gas volume are shown for the three cases. In each case, the horizontal gas velocity starting from the auxiliary tuyeres rapidly diminishes along the radius direction, but remains relatively high as the initial gas velocity at the injection level is increased.

Figure 17 shows the relationship between them for the different inner volumes at the same initial gas velocity. Here, the vertical axis is represented as a non-dimensional unit for horizontal gas velocity. $v_0$ shows inlet shaft gas velocity. Figure 17 shows the penetration depth defined as a non-dimensional unit increases as the inner volume is reduced. The absolute penetration distances are nearly constant as indicated in Fig. 14. The penetration behavior of the injected gas is similar to the gas dynamics in the raceway. Generally, the depth of the raceway can be calculated by the following equation:

$$R_s / D_s \propto \rho v^2$$  \hspace{1cm} (9)

where, $R_s$ denotes the depth of the raceway and $D_s$ denotes the tuyere diameter. This equation was derived from cold model experiments without combustion. The raceway depth is determined by the inertia force of gas equivalent to the raceway factor and the diameter of the tuyeres. The penetration of the injected gas from the auxiliary tuyeres and an
where, $V_s$, $X_0$ and $V_{BF}$ denote injection gas volume, radius of furnace and inner volume. In this case, the relative penetration depth $R_s/X_0$ becomes constant for every blast furnace owing to the same gas velocity. On the other hand, from Fig. 17, it is obvious that the relative penetration depth is not constant. It is estimated that the absolute value for the penetration depth does not change provided the gas velocity from the auxiliary tuyeres remains constant. In the case of the raceway, since the space where coke particles are fluidized is formed in front of the tuyeres, the penetration space is determined by the total gas inertia force on the plane. However, penetration into the packed bed such as shaft gas injection is dominated by the one-dimensional inertia force per unit area. Accordingly, this is not influenced by the auxiliary tuyere diameter, and the penetration depth is almost constant for the same gas velocity from the auxiliary tuyeres even in the different sizes of blast furnace. This means that a small blast furnace can easily maintain a deep penetration effect even in the center by controlling the gas velocity.

A previous study\(^{19}\) showed that the diffusion effect in the packed bed was not remarkable. By an analysis of the iso-bars in Fig. 15 and experimental research by Nishio et al.,\(^3\) the area occupied by injected gas in the horizontal section was almost proportional to the ratio of injection gas volume to blast gas volume from ordinary tuyeres. This behavior of the injected gas into the shaft can be seen in the transient region just after the injection from the auxiliary tuyeres.

7. Conclusions

The gas flow and solid motion in a blast furnace with shaft gas injection were three-dimensionally analyzed by a hybrid model of DEM and CFD. This study focused on the effect of the gas velocity from the auxiliary tuyeres and the scale effect of the blast furnace, and the following conclusions were obtained.

(1) Although the influence of shaft gas injection is restricted to the peripheral zone, the penetration area is gradually enlarged as the gas velocity from the auxiliary tuyeres increased. At the +1 m level above the injection point, the horizontal gas velocity caused by the shaft gas injection was scarcely observed.

(2) The gas drag force was hardly influenced by the shaft gas injection except in the region near the auxiliary tuyeres. The normal stress between particles along the vertical axis reached a maximum value around the bosh level, and this tendency was not influenced by the change of the injection gas volume. Therefore, it was estimated that the shaft gas injection has little effect on the burden descending.

In the small blast furnace, the injected gas can easily reach the central part compared with in the large blast furnace by controlling the gas velocity. The absolute penetration distance was almost constant at the injection level when the gas velocity from the auxiliary tuyeres was kept constant. However, an analysis of the isobar distribution suggested that the area occupied by injected gas in the horizontal section was almost proportional to the ratio of injection gas volume to blast gas volume from ordinary tuyeres.

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