Nonperturbative Contributions to the Hot Electroweak Potential

S. J. Huber, M. G. Schmidt
Institut für Theoretische Physik der Universität Heidelberg,
Philosophenweg 16, D-69120 Heidelberg, Germany
E-mail: s.huber@thphys.uni-heidelberg.de, m.g.schmidt@thphys.uni-heidelberg.de

The hot electroweak potential for small Higgs field values is argued to obtain contributions from a fluctuating gauge field background leading to confinement. The destabilization of \( F^2 = 0 \) and the crossover are discussed in our phenomenological approach, also based on lattice data.

1 Introduction

The infrared behavior of the electroweak standard model (SM) at high temperatures \( T \) is isolated most elegantly in a truncated effective 3-dimensional action obtained by matching it to the original theory in (two-loop) perturbation theory integrating out nonzero modes and longitudinal gauge bosons. With high accuracy it leads to a Lagrangian with \( T \)-dependent couplings

\[
L_{3ff}^e = \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \phi)^+ D_i \phi + m_3^2(T) \phi^+ \phi + \lambda_3(T)(\phi^+ \phi)^2, \tag{1}
\]

where the gauge coupling \( g_3^2 = g_w^2 T (1 + ...) \) sets the scale and the dimensionful couplings \( \lambda_3, m_3^2 \) can be scaled to it introducing \( x = \lambda_3 / g_3^2 \) which depends on the Higgs mass and determines the strength of the phase transition (PT), and \( y = m_3^2 / g_3^4 \) which is \( (T - T_c) \) near the critical temperature \( T_c \). (1) describes a superrenormalizable theory; \( g_3^2 \) has only infrared running in the Wilsonian sense, its divergence in the IR signalizes confinement.

Lattice calculations based on (1) show a weakly first-order PT for \( x \leq 0.11 (m_H \leq 0.9 m_W) \) with a crossover above that value (predicted in previous theoretical work). 3-dimensional perturbation theory for the effective potential always predicts a first-order PT, mainly due to the \( -\phi^2 \) term obtained from the simple gauge boson loop in a Higgs background \( \phi^2 = 2 \phi^+ \phi \). It strongly deviates from lattice results for \( x \geq 0.05 \) in particular for the interface tension of the critical bubble and the latent heat.

Indeed, in lattice calculations one observes typical confinement phenomena in the hot phase (near \( \phi = 0 \)):

i) There is a string tension \( \sigma_{\text{fund}} \sim 0.13 g_3^4 \) like in pure Yang-Mills theory.

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\(^a\)Talk presented by M. G. Schmidt at the “Strong and Electroweak Matter” Conference, Copenhagen, Dec. 1998.
ii) There is a rich spectrum of $0^{++}$ ("H"), $1^{−−}$ ("W"), $2^{++}$ correlation masses (see also the model in ref. 8) including $W$-(glue)ball states – which do not seem to mix with the Higgs bound states.

The previous points indicate a pure Yang-Mills dynamics with the Higgs just sitting at the ends of the confining string. The screening observed in ref. 9 only appears at rather large distances and is hardly observable in the static potential.

It would be very useful to have a (coarse grained) effective potential $V_{\text{eff}}(\phi^2)$, but for a gauge theory this is a difficult task. Such a potential would help to discuss critical bubble shapes, sphalerons, etc. A (semi)analytic model would also allow to develop simple criteria whether one can trust perturbative results in “beyond” models like the supersymmetric models MSSM and NMSSM, where lattice calculations become increasingly more complicated. Here we present a phenomenological proposal how to obtain nonperturbative contributions to the effective potential of the 3-dimensional theory [9].

2 A Model for the Nonperturbative Part of the Effective Potential

In view of the Yang-Mills field dynamics argued for in the introduction it is very natural for a small classical Higgs background $\phi$ to introduce also a gauge field background of a 3-dimensional QCD-type vacuum[10]. We want to describe the confinement effects mentioned above, and thus a constant field strength background will not do. Thus we postulate a background with Gaussian correlations, i.e. the cumulant expansion e.g. of the Wegner-Wilson loop exponential in this vacuum should contain only 2-correlators

$$\ll \exp(ig_3 \oint A dx) \gg \cong \exp(-\frac{1}{2}g_3^2 \oint \oint \ll AA' \gg).$$

Using the nonabelian Stokes theorem (or the coordinate gauge as below) this can be transferred into $\exp(-\frac{1}{2}g_3^2 \oint da \oint da' \ll FF' \gg)$. The truncation of higher correlators and an ansatz for the correlator $\ll FF' \gg$ defines the “stochastic vacuum” model [14] in QCD. Indeed, to make the correlator $\ll F(x)F(x') \gg$ gauge invariant, one has to connect $x, x'$ to some reference point $x_0$ and introduce $F(x, x_0) = P \exp(ig_3 \int_{x_0}^{x} Adx)F(x_0)$. In the coordinate gauge $(x - x_0)_{\mu} A_{\mu}^a = 0$ and with straight line integrals from $x_0$ to $x$ the connection vanishes and (summing over indices) one has

$$\ll g_3^2 F_{\mu\nu}(x')F_{\mu\nu}(x) \gg = < g_3^2 F^2 > D_{\text{eff}} \left( \frac{(x - x')^2}{a^2} \right).$$

Both, $x_0$ independence and the Gaussian approximation are only reasonable for a choice of $x_0$ inside the loop considered. $< g_3^2 F^2 >$ is the usual condensate.
obtained here in the limit \( a \to \infty \). A reasonable ansatz for the form factor \( D \) is \( D(z^2/a^2) = e^{-|z|/a} \) with a correlation length \( a \). The latter, like in QCD, has been determined on the lattice for QCD as \( a^{-1} \approx 0.72 g_3^2 \).

We now want to write a nonperturbative effective potential \( V_{\text{eff}}(m^2 = \frac{1}{4} g_3^2 \varphi^2, < g_3^2 F^2 >) \) in the combined Higgs and fluctuating gauge field background. In 1-loop order this corresponds to the graphs of type fig. 1a.

### 3 Instability at \( F^2 = 0 \)

As a first step we calculate the order \( F^2 \) contribution to the effective action obtained by averaging the one-loop action including ghosts (in the Feynman background gauge)

\[
\Gamma^{1-\text{loop}} = \frac{1}{2} \text{tr} \log \left( [D^{-2} \delta_{\mu\nu} + 2i g_3 F_{\mu\nu} + m^2 \delta_{\mu\nu}]^{ab} \right) - \text{tr} \log [-D^2 + m^2] \tag{4}
\]

with \( m^2 = \frac{1}{4} g_3^2 \varphi^2 \) and a gauge boson spin interaction term \( 2i g_3 F_{\mu\nu} \), over the stochastic field (according to (3)). We obtain

\[
V_{FF}^{1-\text{loop}}(m^2) = -< g_3^2 F^2 > a G(ma)
= -\frac{1}{8\pi^2} < g_3^2 F^2 > \int_0^\infty dp d\hat{p} \tilde{D}_{\text{eff}}(\hat{p}^2) \int_0^1 d\alpha \left\{ [m^2 + \alpha(1-\alpha)p^2]^{-1/2} - \frac{1}{2p^2} [(m^2 + \alpha(1-\alpha)p^2)^{1/2} - m] \right\}. \tag{5}
\]

There is a negative first term due to the spin interaction. \( V_{FF}^{1-\text{loop}} \) dominates the tree term and destabilizes \( F^2 = 0 \) if \( < \frac{1}{4} F^2 > - < g_3^2 F^2 > a G(ma) \) is negative. For \( \varphi^2 = 0 \) this is fulfilled for \( 1/a \approx 0.6 g_3^2 \) not so far from the lattice value \( 1/a \approx 0.72 g_3^2 \). (In the numerical evaluation of \( V_{\text{eff}} \) discussed below we took only the first term of \( G(ma) \) and obtained instability of \( F^2 = 0 \).)
also for $a^{-1} \lesssim 0.7g_3^2$.) Lattice evaluation of the free energy indicates that $V(m^2 = 0, F^2_{\text{min}})$ is very small. The negative $F^2$ coefficient then indeed would be small and we can hope that 2-loop contributions (or an IR renormalization-group procedure) lead to even better agreement.

4 The Nonperturbative Potential

Evaluation of the full 1-loop potential in the nonperturbative gauge field background of the stochastic vacuum in the coordinate gauge and with Schwinger proper time/worldline methods starts from

$$V(m^2, <g^2F^2>) = -\frac{1}{2} \int_0^\infty \frac{dT}{T} \int [Dy] \exp\{-\int_0^T d\tau (\dot{x}^2/4 + m^2)\}$$

$$\times \ll \text{tr}C_L P \exp\{+ig_3 \int_0^T d\tau (A_\mu \dot{x}_\mu 1L + 2F(x)) \gg .$$

Substituting $A_\mu = \int_0^1 d\eta \eta y F_{\mu\nu}(x_0 + \eta y)$, truncating the cumulant expansion and postulating the stochastic vacuum, the correlator in the double area integral of the spinless minimal couplings leads to an area law for loops of size bigger than $a$. The spin-spin correlator comes with a destabilizing minus sign as discussed in the previous section. We first deal with both cases separately (and suppress the discussion of the interference term).

a) Only spin (paramagnetic) interactions

We approximate this case by a nearest neighbor interaction and consider the vacuum polarization $\Sigma(x, x')_{\mu\nu} = \delta_{\mu\nu} \delta^{ab} S_F((x - x')^2)$ of a gauge boson interacting with the correlated background

$$\tilde{S}_F(p^2) = <g^2F^2> \frac{1}{9\pi^2} \int_0^\infty dq q \log((p+q)^2+m^2)/((p-q)^2+m^2))\tilde{D}(q^2) (7)$$

which enters the log $(p^2 + m^2) - \tilde{S}_F(p^2)$ of the effective 1-loop potential. It would produce an infrared instability for small $m^2$ without further contributions.

b) Pure area law

In the model there is obviously a relation between the (rescaled) string tension $\bar{\sigma}$ appearing in the area law $\exp(-\bar{\sigma}A)$ and the vacuum condensate

$$\bar{\sigma} = \frac{\pi}{9} <g^2F^2> T \int_0^\infty dz z D(z^2/a^2) = \sigma_{\text{adj}} T = \frac{8}{3} \sigma_{\text{fund}} T. (8)$$

Substituting in lack of direct evaluation of the path integral an area law in modified by some ansatz at small areas (see Eq. (3.5) in ref. for further
details) we get the renormalized expression

\[ V_{\text{area}} = \frac{3}{2} \int_0^\infty \frac{dT}{T} T^{-3/2} \int [D\bar{y}] \exp\{-\int_0^1 d\tau \bar{y}^2 / 4\} \]
\[ \{\exp(-\sigma A T^3 / c a^2 / A^2 + T^2) - m^2 T\} - 1 + m^2 T\}. \tag{9} \]

With the substitution \((4\pi T)^{-3/2} = \int d^3p/(2\pi)^3 \exp(-p^2 T)\) we can perform the \(T\) integration (numerically) and enforce the form

\[ V_{\text{area}} = -\frac{3}{2} (4\pi)^{3/2} \int d^3p/(2\pi)^3 \int [D\bar{y}] \exp\{-\int_0^1 d\tau \bar{y}^2 / 4\} \]
\[ \{\log(p^2 + m^2 + m_{\text{conf}}^2(p^2, \bar{A}, m^2)) - \log p^2 - m^2 / p^2\} \tag{10} \]
defining the infrared regulator “magnetic mass” \(m_{\text{conf}}^2(p^2, \bar{A}, m^2)\) which starts with a term \(\sim g_3^2 F^2\). It deviates from \(\sigma A\) because of the \(\bar{c}\) cut-off in \((9)\),

We finally can superimpose both terms a) and b), if there is no strong overlap in \(p^2\) between \(m_{\text{conf}}^2(p^2)\) and \(S_F(p^2)\), in a potential containing \(\log(p^2 + m^2 + m_{\text{conf}}^2(p^2, \bar{A}, m^2)) - S_F(p^2, m^2)\) and proper renormalization and combinatorics.

Of course the IR regulator \(m_{\text{conf}}^2\) should be also introduced in \(S_F\), eq. \((9)\).

5 Evaluation and Discussion

The unknown quantity \(< g_3^2 F^2 >\) can in principle be obtained by minimizing the potential \(V(m^2, < g_3^2 F^2 >)\) with respect to \(F^2\). Like the correlation length \(a\) it alternatively can be derived from lattice data via its relation \((3)\) to the string tension. In lack of data for \(\bar{A}(T, m^2, \sigma)\) resp. \(\bar{A}(p^2, m^2, \sigma)\) and the area cut-off we take \(\bar{c} = 2\) and a function \(\bar{A}(p^2)\) falling from \(\bar{A}(0) \sim 2\) to 0.5 at \(p = 0.4g_3^2\). This differs from our rough evaluation in ref.\([1]\) and brings the \(F^2\) minimum in agreement with the lattice data at \(m^2 = 0\). Fig. 1b then shows our functions \(m_{\text{conf}}^2(p^2)\) and \(S_F(p^2)\) and fig. 2a the potential in \(< F^2 >\) for \(m^2 = 0, 1/a \simeq 0.7g_3^2\). The complicated dependence on the scales \(T\) resp. \(p^2, m^2, \sigma \sim F^2\) induced by the path integral requires further research. The \(m^2\)-dependence of the quantities is in principle fixed by our explicit expressions. In fig. 2b we plot the potential \(V_{\text{tree}} + V_{\text{nonpert}}\) at various values of \(x\) at the critical temperature using the simpleminded tuning of parameters of ref.\([1]\). We then obtain the crossover phenomenon at \(x \sim 0.11\). Also interface

\(^{b}\)The crossover point with a second-order PT \([4]\) can be roughly fixed \([4]\) by the postulate that the \(\varphi^2\) and \(\varphi^4\) terms vanish for canonical critical behavior. For values of \(x > 0.11\) our potential remains convex for all temperatures.
tension and latent heat go to zero there. Note again that \( \phi \) is introduced as
a classical background. We expect increasing fluctuations \( \langle (\delta \phi)^2 \rangle \) going to
the crossover.

In the case of the MSSM with a “light” stop (talk of Quirós at this conference) there is a strongly first-order PT even at a Higgs mass \( \sim 100 \text{ GeV} \), and perturbative, lattice, and lattice results qualitatively agree. Indeed the additional graphs compared to the SM contain right-handed stops and QCD gluons which both do not feel \( SU(2) \) nonperturbative effects. Hence this part is well described by perturbation theory and the standard part now is a at small effective \( x \) where again perturbative results can be trusted. This comes out more quantitatively in our model. We can understand the sign of deviations: the PT is somewhat stronger for the lattice data than for the perturbative prediction. In our approach this is a direct consequence of the \( F^2=0 \) instability, which lowers the potential for small values of the Higgs field and decreases the critical temperature. Such considerations may be particularly valuable in cases which are not easily accessible to lattice calculations.

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