Modelling the components of binaries in Hyades: The dependence of the mixing-length parameter on stellar mass

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ABSTRACT

We present our findings based on a detailed analysis for the binaries of the Hyades, in which the masses of the components are well known. We fit the models of components of a binary system to the observations so as to give the observed total \( V \) and \( B - V \) of that system and the observed slope of the main-sequence in the corresponding parts. According to our findings, there is a very definite relationship between the mixing-length parameter and the stellar mass. The fitting formula for this relationship can be given as \( \alpha = 9.19(M/M_\odot - 0.74)^{0.053} - 6.65 \), which is valid for stellar masses greater than 0.77\( M_\odot \). While no strict information is gathered for the chemical composition of the cluster, as a result of degeneracy in the colour-magnitude diagram, by adopting \( Z = 0.033 \) and using models for the components of 70 Tau and \( \theta^2 \) Tau we find the hydrogen abundance to be \( X = 0.676 \) and the age to be 670 Myr. If we assume that \( Z = 0.024 \), then \( X = 0.718 \) and the age is 720 Myr. Our findings concerning the mixing length parameter are valid for both sets of the solution. For both components of the active binary system V818 Tau, the differences between radii of the models with \( Z = 0.024 \) and the observed radii are only about 4 percent. More generally, the effective temperatures of the models of low mass stars in the binary systems studied are in good agreement with those determined by spectroscopic methods.

Key words: stars: interior – stars: evolution – stars: individual: V818 Tau; HD 27149; 70 Tau; 51 Tau; \( \theta^2 \) Tau – stars: abundances – binaries: eclipsing – open clusters and associations: individual: Hyades

1 INTRODUCTION

Open clusters and binary systems have their own essential roles in almost all branches of astrophysics. While we obtain information on the fundamental properties (such as mass, radius and luminosity) of components of binary systems from their observation (see e.g., Andersen 1991), the age of stars of an open cluster can be taken fairly accurately as the MS life time of its brightest (normal) MS star. The age and the fundamental properties of a system are complementary to each other for the purposes of a better understanding of stellar structure and evolution. Therefore, binary systems in clusters are invaluable. In this respect, the Hyades open cluster is an absolute treasure; in addition to very precise observational data for the distance and photometric measurements of its members (de Bruijne et al. 2001 and Perryman et al. 1998), the masses of the components of its five double-lined binaries are also known. These binaries are V818 Tau (Peterson & Solensky, 1988), 70 Tau (Fin 342), 51 Tau, \( \theta^2 \) Tau and \( \theta^1 \) Tau (Torres et al. 1997a, 1997b, 1997c).

The cluster itself and its binaries have been the subject of innumerable papers. Recently, Pinsonneault et al. (2003), Lebreton et al. (2001) and Lastennet et al. (1999) have researched (some or all of) these binaries in detail by comparing the observational results with models for the internal structure of the component stars. Lastennet et al. (1999) tested the validity of three independent sets of stellar evolutionary tracks, using good photometric data of V818 Tau, 51 Tau and \( \theta^2 \) Tau. Lebreton et al. (2001) focused on the determination of the helium abundance by considering in detail all of the five binaries, and derived the helium abundance as \( Y = 0.255 \), while Pinsonneault et al. (2003) found \( Y = 0.271 \) from the calibrating the components of the eclipsing binary V818 Tau. Both of the studies stress the difficulty of calibration of the radii of the components of V818 Tau.
Furthermore, Lebreton et al. (2001) take note of the stellar mass dependence of the mixing-length parameter ($\alpha$).

The mixing-length parameter $\alpha = t/H_p$ is an unknown in stellar modelling and is often chosen to be constant and equal to the solar value. As discussed in many papers, however, there is no good reason for keeping it constant; it may change from star to star (see below) and form phase to phase (Castellani et al. 2001; Chieffi et al. 1995).

For the stellar mass dependence of $\alpha$, contradictory results are obtained from studies on binaries by different investigators. In studies on eclipsing binaries (Lastennet et al. 2003; Ludwig and Salaris 1999; Lebreton et al. 2001), it is reported that $\alpha$ is an increasing function of the stellar mass. Such a dependence is also found by Morel et al. (2000) in their studies on the visual binary $\iota$ Peg. However, the other two possibilities are also reported for the visual binaries: while Fernandes et al. (1998) state that the solar value can be used to model components of four visual binaries they studied, Pourbaix et al. (1999) find from the calibration of $\alpha$ Cen that $\alpha$ is a decreasing function of the stellar mass. This complexity for the visual binaries may be due to low precision of the accurate values of their components (this is the case also for the 70 Tau binary system). On the other hand, hydrodynamical simulations of convection (Ludwig et al. 1999; Trampedach et al. 1999) also give that $\alpha$ is a decreasing function of effective temperature (or stellar mass). This contradiction needs to be explained.

The determination of the chemical composition of a star is another difficult matter. Although it is assumed that the members of an open cluster have the same age and chemical composition, numerous abundance determinations of various elements in the Hyades stars do not allow the assigning of a certain value for its heavy element abundance $Z$ (see section 2). Therefore, in the present study, the hydrogen abundance ($X$) and $Z$ are considered as unknowns.

For the calibration of a well known binary with late-type components, the number of constraints on the models of its components is four: for luminosity ($L$) (or absolute magnitudes) and the radius ($R$) (or colour) of each component, we can write down two equations:

$$L_{\text{obs}} = L_0 + \frac{\Delta L}{\Delta X} \delta X + \frac{\Delta L}{\Delta Z} \delta Z + \frac{\Delta L}{\Delta \alpha} \delta \alpha + \frac{\Delta L}{\Delta t} \delta t,$$

$$R_{\text{obs}} = R_0 + \frac{\Delta R}{\Delta X} \delta X + \frac{\Delta R}{\Delta Z} \delta Z + \frac{\Delta R}{\Delta \alpha} \delta \alpha + \frac{\Delta R}{\Delta t} \delta t,$$

where $L_0$ and $R_0$ are values from the reference model with fixed values of $X$, $Z$, age ($t$) and $\alpha$ (solar values, for example, except for $t$). The number of unknowns for models of the component stars is five: $X$, $Z$, $t$, $\alpha_A$ and $\alpha_B$. Thus, if we have no extra constraint on the binary system, there is in principle no unique solution for it. On the other hand, for a visual binary in which the component stars are not well known, we can just write two equations similar to equations (1) and (2) for total $V$ and $B - V$ of the system.

The remainder of the present paper is organized as follows. In Sections 2 and 3, the observed chemical composition of the cluster and the properties of the binary systems studied are summarized, respectively. The model properties of the binaries and their components are presented in Section 4 and concluding remarks are given in Section 5.

2 CHEMICAL COMPOSITION OF HYADES

There are many papers devoted to the determination of abundances of heavy elements in Hyades stars of different classes (from A- to K-type stars). For the abundance of iron, Boesgaard et al. (2002) and Hui-Bon-Hoa & Alecian (1998) find enhancement relative to the solar abundance (0.16 dex for G-type and 0.13 for A-type stars in Hyades, respectively). Recently, Paulson et al. (2003) derived an abundance with a very small formal error [(0.13±0.01 dex; see also Yong et al. 2004).] In contrast to these findings, Varene and Monier (1999) find the mean value of iron abundances from the spectra of 29 F-type stars as -0.05 dex. Thus, the abundances found from the stellar spectra by different research groups are in general not in agreement with each other. Three dimensional calculations for the stellar atmosphere (see, for example, Asplund et al. 2004) may solve such problems.

The customary consideration of iron abundance as a good tracer of total $Z$ of a star is, however, highly debatable. Iron is not among the most abundant heavy elements, and there is no one-to-one relation between the abundances of iron and the most abundant heavy elements (for example oxygen). This fact can be seen, for example, in Fig. 10 of Bensby et al. (2004): for the stars in galactic disk that have an $[O/H]$ value of about zero, $[Fe/O]$ abundance varies between -0.4 and +0.1. Thus, it is not reasonable to take $[Fe/H] = [O/H]$.

Indeed, abundances of oxygen and nitrogen are determined by many spectroscopists. Takeda et al. (1998) find the oxygen abundance as 0.10 dex for F-stars in Hyades, while King & Hiltgen (1996) determine it as 0.15 dex from the spectra of the two dwarfs. Takeda et al. (1998) also give the abundance of nitrogen as 0.30 dex.

It could be claimed that the scattering of this amount for a given element is a result of the diffusion process, whose rate varies from star to star. However, contradictory results are found for the oxygen abundance of Hyades member HD 27561: while Garcia Lopez et al. (1993) find -0.14 dex, Clegg et al. (1981) give 0.15 dex as the oxygen abundance of this star. This leads us to conclude that the difference between the abundances determined by different studies is the result of different techniques. Consequently, it is a very difficult job to estimate the value of heavy element abundance of the Hyades. In the present paper we therefore consider $Z$ as an unknown parameter.

3 OBSERVED PROPERTIES OF THE BINARIES OF HYADES

Several studies have been devoted to determining the fundamental properties of the components of V818 Tau (McCleure 1982; Schiller and Milone 1987; Peterson and Soszyński 1988). Peterson and Soszyński (1988) found the masses of the primary and the secondary components of the V818 Tau as $1.072 \pm 0.010 M_\odot$ and $0.769 \pm 0.005 M_\odot$, respectively. Recently, Torres and Ribas (2002) also found the masses of the components: $M_A = 1.0591 \pm 0.0062 M_\odot$ and $M_B = 0.7605 \pm 0.0062 M_\odot$. Although the individual masses found by these two studies are very close to each other, $V$ found from the models with the former masses is in better agreement with the observed $V$ than that of the latter.
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Table 1. The individual masses of the component stars and total V and B-V of the five binary systems. The slopes of the main-sequence ($S_{MBV}$) near V818 Tau and 70 Tau are computed from the data in de Bruijne et al. (2001) by a least-square method. The values of the slopes computed from the binary data are given in parentheses.

| System      | $M_A/M_\odot$ | $M_B/M_\odot$ | $R_A/R_\odot$ | $R_B/R_\odot$ | $V$  | $B-V$  | $S_{MBV}$ | log $\frac{L}{L_\odot}$ | Ref. |
|-------------|---------------|---------------|---------------|---------------|------|--------|------------|-------------------------|------|
| V818 Tau    | 1.072 ± 0.010 | 0.769 ± 0.010 | 0.905 ± 0.029 | 0.773 ± 0.015 | 8.28 | 0.73   | 4.6(4.8)  | 0.71                    | 1, 2, 3, 8 |
| 70 Tau      | 1.363 ± 0.073 | 1.253 ± 0.075 | ...           | ...           | 6.46 | 0.49   | 6.6(5.2)  | ...                     | 4.8 |
| 51 Tau      | 1.80 ± 0.13   | 1.46 ± 0.18   | ...           | ...           | 5.65 | 0.28   | ...        | ...                     | 5.8 |
| $\theta^2$ Tau | 2.42 ± 0.30   | 2.11 ± 0.17   | ...           | ...           | 3.40 | 0.18   | ...        | ...                     | 6.8 |
| HD 27149    | 1.096 ± 0.002 | 1.010 ± 0.002 | ...           | ...           | 7.53 | 0.68   | ...        | ...                     | 7   |

1) Peterson and Solensky (1988), 2) Yoss et al. (1981), 3) Schiller and Milone (1987), 4) Torres et al. (1997b), 5) Torres et al. (1997c), 6) Torres et al. (1997a), 7) Tomkin (2003), 8) Lebreton et al. (2001)

Therefore, in our model computations for this system, we use masses found by Peterson and Solensky (1988). Owing to the activity feature of the system pertaining to the late-type stars, the measured values of $V$ and $B-V$ are dispersed. Therefore, we compare theoretical results with their minimum values, $V = 8.28$ and $B-V = 0.73$, observed by Yoss et al. (1981). The radii of the components are found by Peterson and Solensky (1988) as $R_A = 0.905 \pm 0.029 R_\odot$ and $R_B = 0.773 \pm 0.015 R_\odot$. Torres and Ribas (2002) also find very similar values to these.

70 Tau is a close visual binary in the Hyades cluster. Torres et al. (1997b) determined the masses of the primary and secondary components as $1.363 \pm 0.073 M_\odot$ and $1.253 \pm 0.075 M_\odot$, respectively. Torres et al. (1997c) carried out a similar study for also 51 Tau, and obtained the masses of its components as $M_A = 1.80 \pm 0.13 M_\odot$ and $M_B = 1.46 \pm 0.18 M_\odot$. They confirmed also that $\theta^2$ Tau A is a fast rotator: $v_A \sin i = 97 - 125$ km s$^{-1}$.

$\theta^2$ Tau is another spectroscopic binary system. Its primary component is one of the brightest stars in Hyades and is a $\delta$-Sct type variable star. Furthermore, the binary system consists of an evolved and a main-sequence star. This binary system is therefore very important for testing the different evolutionary stages. Torres et al. (1997a) found the
masses of the components to be $M_A = 2.42 \pm 0.30 \, M_\odot$ and $M_B = 2.11 \pm 0.17 \, M_\odot$. They also found the rotational velocities of the components. According to their results, both of the components of $\theta^2$ Tau are rapid rotators: $v_A \sin i = 80 \, \text{km s}^{-1}$ and $v_B \sin i = 90 - 170 \, \text{km s}^{-1}$.

HD 27149 is also a spectroscopic binary in the Hyades. The minimum masses for its components have been found by Tomkin (2003) as $M_A = 1.096 \pm 0.002 \, M_\odot$ and $M_B = 1.010 \pm 0.002 \, M_\odot$. Tomkin confirmed that these minimum masses are unexpectedly large for the spectral type of the stars, thus suggesting the possibility of eclipses. By comparing appropriate models with the observations we find the masses of the components and then the inclination angle of the system (see section 4.4).

In Fig. 1, absolute magnitudes of the components of V818 Tau and the other binaries, as given by Lebreton et al. (2001), are plotted with respect to their colours. For comparison, the single stars with accurate data, given by de Bruijne et al. (2001), are also plotted in this figure (dots). The basic data of the systems, needed for our calibration process, are listed in Table 1.

We also use the slopes of the Hyades MS as constraints in this process. The middle and the lower parts of the MS have different slopes. By applying a least-square method to the data given by de Bruijne et al. (2001), we obtain the slope of the lower part, which contains the components of V818 Tau,

$$S_{MS1} = \frac{\Delta M_V}{\Delta (B-V)} = 4.6.$$  \hfill (3)

Similarly, for the upper part, which contains the components of 70 Tau, we find

$$S_{MS2} = \frac{\Delta M_V}{\Delta (B-V)} = 6.6.$$  \hfill (4)

When we use all the available data in WEBDA \footnote{http://www.univie.ac.at/webda} database, we obtain very similar results: $S_{MS1} = 4.4$ and $S_{MS2} = 6.4$. However, from their absolute magnitudes and colours, we obtain the derivative for V818 Tau (Schiller and Milone 1987) as $S_{MS1} = 4.8$ and for 70 Tau (Torres et al. 1997b) as $S_{MS2} = 5.2$. The derivative for components of 70 Tau is significantly less than the values obtained from the data of other similar stars in the cluster. This may arise from the fact that it is not easy to distribute correctly the total $V$ and $B-V$ of a visual binary among its components.

4 MODELLING THE COMPONENTS OF THE BINARIES OF HYADES

The characteristics of our code were already described in Yildiz (2000;2003) \footnote{see also references therein}, and therefore we shall not provide full details here. Our equation of state uses the approach of Mihalas et al. (1990) in the computation of the partition functions. The radiative opacity is derived from Iglesias et al. (1992), and is completed by the low temperature tables of Alexander & Ferguson (1994). For the nuclear reactions rates we use the analytic expressions given by Caughlan & Fowler (1988), and we employ the standard mixing-length theory for convection (Böhm-Vitense 1958).

For comparison of the theoretical and observational values of the binary systems, we compute the theoretical $V$ and $B-V$ of any system. We first construct models for its components with given masses and then find $M_V$ and $B-V$ using tables for model atmospheres (Bessel et al. 1998). Using the parallax of the binary and $M_V$ and $B-V$ of models for the individual stars, we find the combined total $V$ and $B-V$ of the system.

4.1 Solutions from V818 Tau and 70 Tau

For these binary systems, we have seven equations ($V$ and $B-V$ of the systems 70 Tau and V818 Tau, $L_A/L_B$ in V818 Tau, and the slopes in the middle and lower parts of the MS in the Hyades cluster) similar to equations (1) and (2) with seven unknowns ($X$, $Z$, $t$ and $\alpha$ for 4 stars). For the solution of these seven equations, we need the derivatives of seven quantities with respect to independent variables (unknowns). These derivatives are computed by using the reference models with solar composition ($X=0.705, Z=0.02$) and $\alpha = 2.0$ for all the components at $t = 1.0 \times 10^9 \, \text{yr}$. The solution we find for this case is the following (Set A) $X = 0.679$, $Z = 0.0319$, $\alpha_{V818A} = 1.89$, $\alpha_{V818B} = 1.01$, $\alpha_{70TaA} = 2.43$, $\alpha_{70TaB} = 2.33$, $t = 590 \, \text{Myr}$.

The observable properties of these binary systems themselves and their components obtained from the models with Set A are listed in Table 2 The theoretical visual magnitudes and colours are very close to the observed values for both systems.

Is the result we found from the solution of seven equations a unique one? Unfortunately, the answer is not simply yes. One of the main reasons for this is that the numerical derivatives are not constant but depend on the intervals of the variables (or on the reference model). To confirm how the derivatives depend on the intervals, we reevaluate the numerical derivatives from the models with $X = 0.69$, $Z = 0.032$, $\alpha_{V818A} = 2.11$, $\alpha_{V818B} = 1.3$, $\alpha_{70TaA} = 2.66$, $\alpha_{70TaB} = 2.45$, $t = 1100 \, \text{Myr}$.

Using these derivatives, we resolve the seven equations simultaneously and find the following values for the seven unknowns (Set B): $X = 0.695$, $Z = 0.0298$, $\alpha_{V818A} = 2.17$, $\alpha_{V818B} = 0.94$, $\alpha_{70TaA} = 2.52$, $\alpha_{70TaB} = 2.25$, $t = 1030 \, \text{Myr}$.

The values of the observables of the binaries found from the models with this set, given in Table 3 are in perfect agreement with the observations. However we do not try to fit directly model radii to the observed radii of the components of V818 Tau. The difference between them is very small, about 3 percent. Indeed it is possible to remove this difference, and then the agreement between the theoretical and observational values of $B-V$ and $S_{MS} V (the corresponding slope of the MS) of the system will disappear.

The very striking common feature of the solutions with Set A and Set B is that $\alpha_{V818A} \sim 2 \alpha_{V818B}$. Although, both sets (Set A and B) give very similar results, the values for the age of the cluster in both sets are very different from each other. Therefore, we shall consider a star which evolves faster than these stars to fix the age; this star is $\theta^2$ Tau A.
Table 2. The observable quantities of V818 Tau and 70 Tau and their components computed from the models with Set A.

| Model   | L  | R  | T$_{\text{eff}}$ | M$_V$ | B-V | U-B | V  | B-V | S$_{MBV}$ | log $L/L_\odot$ |
|---------|----|----|------------------|-------|-----|-----|----|-----|-----------|-----------------|
| V818A   | 0.849 | 0.950 | 5692             | 5.007 | 0.701 | 0.203 | 8.263 | 0.744 | 4.603 | 0.692 |
| 70A     | 0.173 | 0.741 | 4327             | 7.362 | 1.213 | 1.176 |        |       |       |       |
| V818B   | 2.766 | 1.297 | 6543             | 3.643 | 0.457 | 0.020 | 6.459 | 0.483 | 6.953 |       |
| 70B     | 1.834 | 1.145 | 6285             | 4.107 | 0.524 | 0.010 |        |       |       |       |

Table 3. The observable quantities of V818 Tau and 70 Tau and their components computed from the models with Set B.

| Model   | L  | R  | T$_{\text{eff}}$ | M$_V$ | B-V | U-B | V  | B-V | S$_{MBV}$ | log $L/L_\odot$ |
|---------|----|----|------------------|-------|-----|-----|----|-----|-----------|-----------------|
| V818A   | 0.838 | 0.929 | 5736             | 5.013 | 0.687 | 0.182 | 8.289 | 0.727 | 4.385 | 0.730 |
| 70A     | 0.156 | 0.748 | 4200             | 7.578 | 1.272 | 1.241 |        |       |       |       |
| V818B   | 2.828 | 1.316 | 6531             | 3.619 | 0.460 | -0.018 | 6.443 | 0.489 | 6.343 |       |
| 70B     | 1.847 | 1.165 | 6240             | 4.103 | 0.536 | 0.021 |        |       |       |       |

4.2 Properties of $\theta^2$ Tau and age of the cluster

We shall first see which one of the models of $\theta^2$ Tau A with Set A is in better agreement with the observations. As mentioned above, $\theta^2$ Tau A rotates rapidly, and therefore this effect should be taken into account. If the rotation is included in the model computations, the problem arises of angular momentum distribution inside early-type stars. We should first specify how inner regions of this star rotate. Because this is a very complicated matter and is indeed one of the essential problems in stellar astrophysics, we study two typical cases: i) solid rotation, ii) the rotation profile as determined by contraction. The models with the latter case are differentially rotating (DR) models: the central regions are rotating much faster than the outer regions (Yildiz 2003; 2004).

In Fig. 2, the DR models of $\theta^2$ Tau A with Set A (Model AADR; diamond) and Set B (ABDR; star), and the model rotating like a solid body (ABSB; ×) are plotted in the HR diagram. Their $B-V$ values are in good agreement with the observed one (0.17) (de Bruijne et al. 2001) at $t = 675$ Myr, 699 Myr and 661 Myr, respectively. Their equatorial velocities at their corresponding ages are $v_{\alpha \odot} \sim 100$ km s$^{-1}$. It seems that the sub-giant phase of $\theta^2$ Tau A is compatible with the observed position and the DR models with both sets of chemical composition are in much better agreement with the observed values than the models rotating like a solid body. Even though the solid-body rotation can not be totally ruled out by such an analysis, we shall consider only the DR models hereafter. What is important here in our analysis is that the ages of models with the two rotation types are very similar to each other.

The evolutionary track of the DR model of $\theta^2$ Tau B with Set B is also plotted in Fig. 2. Although its evolutionary track passes through the observed position of $\theta^2$ Tau B in the HR diagram, the time of agreement ($t = 444$ Myr) is not the same as that of $\theta^2$ Tau A ($t = 699$ Myr). The position of $\theta^2$ Tau B at the latter time is marked by filled circle, and is far from the observed position of $\theta^2$ Tau B in the diagram. An internal rotation, more complicated than the rotation as determined by the contraction, may cause this discrepancy.

4.3 Solutions derived from models of $\theta^2$ Tau A and 70 Tau A & B

As shown in the previous sections, two models having the same mass but different chemical compositions may have the same (or very near) location in the HR diagram. Because of this degeneracy, we calibrate models of $\theta^2$ Tau A and the components of 70 Tau to the observations for the fixed values of Z. For $Z = 0.028$, we obtain (Set 28)

$X = 0.699, \alpha_{70\text{tau}A} = 2.31, \alpha_{70\text{tau}B} = 2.21, t = 705$ Myr.

We made similar computations also for $Z = 0.033$ and obtain the following results (Set 33)

$X = 0.676, \alpha_{70\text{tau}A} = 2.29, \alpha_{70\text{tau}B} = 2.20, t = 676$ Myr.

For $Z = 0.024$, using the values of $\alpha$ in Set 28, we find $X = 0.718$ and $t = 721$ Myr (Set 24).

The differences between the models of any star with different sets are negligibly small. Therefore, these sets are equivalent to each other (see Table 4).

We also build models (with Set 28) with the microscopic diffusion process for 70 Tau A to test its influence on the observable properties of such stars. While the difference between the absolute magnitudes of the models with and without diffusion is $\Delta M_V = -0.0022$, the difference between the colours is $\Delta (B-V) = 0.0031$. These differences are small enough, in comparison to the uncertainty in the observed magnitude and colour of the system, that the diffusion process can be ignored.

For these sets, we also find that the mixing-length parameter for the components of V818 Tau are as $\alpha_{\gamma V818A} = 2.04$ and $\alpha_{\gamma V818B} = 0.99$. In Fig. 3, the models for the components of $\theta^2$ Tau, 70 Tau and V818 Tau with Set 24 (filled circles) are plotted in the HR diagram among the single stars with very precise parallaxes (dots). Except for $\theta^2$ Tau B (see above), all the models of the components are qualitatively in good agreement with the general trend identified by the single stars with very accurate observational data.

The models for the components of 51 Tau with Set 24 are also placed in Fig. 3. Their positions are good enough in...
comparison with the positions of the single stars (de Bruijne et al. 2001). The model of 51 Tau A is DR model with equatorial velocity of 130 km s\(^{-1}\), and the mixing-length parameter for 51 Tau B is 2.35.

### 4.4 Properties of HD 27149

Because the angle between the line of sight and the orbital plane of HD 27149 is observationally not known, we find the individual masses of the components by fitting \( V \) of the models for a given chemical composition and time to \( V_{\text{obs}} \) of the system. For Set 24 and Set 28, we find that \( M_A = 1.118 \) and \( M_B = 1.030 \). Then, from the calibration of models of these masses with both sets, the mixing-length parameters are derived as \( \alpha_A = 2.10 \) and \( \alpha_B = 1.95 \). These different values of the mixing-length parameter for the different stellar masses prove once more the mass dependence of \( \alpha \).

From these values of individual masses, we can obtain the value of the inclination angle by using the observed value of \( M_A \sin^3 i \) (Tomkin, 2003):

\[
M_A \sin^3 i = 1.118 \sin^3 i = 1.096.
\]

Equation (5) gives the inclination of the system as \( i = 83.4^\circ \). The minimum value of \( i \) for eclipsing to occur is, however, \( i_c = 88.9^\circ \). Thus, HD 27149 is not an eclipsing binary (see Tomkin, 2003).

### 4.5 The stellar mass dependence of the mixing-length parameter

The values of the mixing-length parameter for the components of the binaries derived from the calibration are listed in Table 4. They are plotted in Fig. 4 as a function of the stellar masses. The uncertainties in \( \alpha \) of each model are computed assuming an uncertainty of \( \Delta(B-V) = 0.01 \) (uncertainty in B-V of the binary systems; see Lebreton et al. 2001) for each star. Then,

\[
\delta \log \alpha = \frac{\Delta(B-V)}{\partial(B-V)/\partial \log \alpha}
\]

where the partial derivatives are computed from the models. In Table 4, radii and effective temperatures of the models with Set 24 and Set 28 are also listed. In the last column, the fractional difference between the models with these sets are given for the components of V818 Tau and 70 Tau. Because these differences are very small, we deduce that the mass dependence of \( \alpha \) is independent of the (solution) sets.

The curve in Fig. 4 is the fitting formula,

\[
\alpha = 9.19(M/M_\odot - 0.74)^{0.053} - 6.65
\]
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Figure 3. Colour-magnitude diagram for the single stars (dots) and the models of the components of the five binaries (filled circles).

Table 4. The mixing-length parameters of the calibrated models of the components of binaries V818 Tau, 70 Tau and HD 27149 with Set 24. In order to derive a fitting formula, models are also computed for the masses 0.80, 0.85, 0.90 and 0.95 M⊙ (see Fig. 4). The uncertainties in α of each model are computed assuming an uncertainty of ∆(B − V) = 0.005 (see the text). For comparison, radii and effective temperatures of the models with Set 24 and Set 28 are also given. The fractional differences between the model radii of components of V818 Tau and 70 Tau with these sets are listed in the last column.

| Star          | M/M⊙ | α     | Teff (Set 24) | R/R⊙ (Set 24) | Teff (Set 28) | R/R⊙ (Set 28) | δR/R |
|---------------|------|-------|--------------|--------------|--------------|--------------|------|
| V818 Tau B    | 0.769| 0.99 ± 0.03 | 4305         | 0.739        | 4295         | 0.741        | 0.003|
| V818 Tau A    | 1.072| 2.04 ± 0.09 | 5718         | 0.940        | 5720         | 0.937        | -0.003|
| 70 Tau B      | 1.253| 2.21 ± 0.09 | 6252         | 1.154        | 6248         | 1.162        | 0.007|
| 70 Tau A      | 1.363| 2.31 ± 0.18 | 6527         | 1.306        | 6514         | 1.318        | 0.009|
| HD 27149 B    | 1.030| 1.95 ± 0.07 | 5583         | 0.896        | ...          | ...          | ...  |
| HD 27149 A    | 1.118| 2.10 ± 0.08 | 5876         | 0.985        | ...          | ...          | ...  |
| −             | 0.800| 1.20 ± 0.05 | 4500         | 0.749        | ...          | ...          | ...  |
| −             | 0.850| 1.50 ± 0.06 | 4764         | 0.768        | ...          | ...          | ...  |
| −             | 0.900| 1.70 ± 0.06 | 5086         | 0.799        | ...          | ...          | ...  |
| −             | 0.950| 1.80 ± 0.07 | 5329         | 0.836        | ...          | ...          | ...  |
| Sun           | 1.000| 1.88      |              |              |              |              |      |

derived using the data given in Table 4. It is valid for the stellar masses greater than 0.77 M⊙. The curve surprisingly also covers α of the Sun (○ in Fig. 4). Moreover, as of the components of α Centauri from equation (7) explain the stellar parameters of these stars well (Yıldız, in preparation).

One might also want to ascertain whether the effective temperatures of the models with variable α are consistent with spectroscopic measurements. For this task, similar to fig. 2 in Pinsonneault et al. (2004), we plot M_V of the models (Set 24) of the late type components of the three binary systems against the T_{eff} (filled circles) in Fig. 5. The observational effective temperatures and absolute magnitudes are taken from Paulson et al. (2003) and de Bruijne et al. (2001), respectively. We include also photometric data (triangle with thick error bars) for the components of V818 Tau (Torres and Ribas, 2002), because of a lack of observational
data for stars with a $T_{\text{eff}}$ less than 5000 K. It can be seen that the theoretical and the observational effective temperatures are in very good agreement.

We also compare the model radii with the observed radii of stars in well known eclipsing binaries. In Fig 6, the observed radii, taken from Andersen (1991) (circles) and Lopez-Morales and Ribas (2005) (stars), are plotted against the stellar mass. The filled circles represent the models of the components of V818 Tau, 70 Tau and HD 27149 with Set 24. We confirm a very good agreement between the observed and the model radii. The models for the unevolved late-type stars in the Hyades cluster are very near to zero-age-main-sequence line and therefore takes place on the left side of the main-sequence formed by the stars in the well known eclipsing binaries.

4.6 The colour-colour diagram

In Fig. 7, $U - B$ and $B - V$ colours derived from our models with Set 24 are plotted. For comparison, the observed colours for the Hyades stars are also plotted. The filled circles and boxes represent the colours computed from the tables of Bessel et al. (1998) and Lejeune et al. (1998), respectively, for solar composition. Although both of the tables are in general in good agreement with the observation, the $U - B$ colours of models of the early type stars (51 Tau A, $\theta^2$ Tau A and B) derived from Lejeune et al. (1998) are in better agreement with the observed $U - B$ than those derived from Bessel et al. (1998). The colours of some models are also computed from Lejeune et al. (1998) for the metallicity given as Paulson et al. (2003), namely $[Fe/O]=0.13$ dex (triangles). As the metallicity of a model is increased, the model moves towards the bottom-right part of the colour-colour diagram, as expected. The majority of the stars are located between the colours of the models derived for the solar composition and for the metal-rich composition, at least for stars with $B-V < 0.6$. This result may be interpreted as meaning that the Hyades cluster is slightly more metal rich than the Sun. More precise results on the metallicity of the cluster depend on which solar mixture is used. Therefore, new tables with the recently calculated solar composition (Asplund et al. 2004) for colours of stars are required.

5 CONCLUSION

By constructing models for the components of the Hyades members binary systems whose masses are observationally determined, we have reached very important conclusion con-
Modelling the components of binaries in Hyades

Figure 5. The models of the components of the three binaries (V818 Tau, 70 Tau and HD 27149) with Set 24 (filled circles) are plotted in $M_V$ vs. $T_{\text{eff}}$ diagram. The error bars with circles represent the observations: while the effective temperatures are spectroscopically derived by Paulson et al. (2003), the absolute magnitudes are from de Bruijne et al. (2001). The error bars with triangles are for the components of V818 Tau based on the photometric data (Torres and Ribas, 2002)

cerning the detailed physical structure as well as the fundamental properties of the cluster itself. The most striking outcome of the present study is that we discover a smooth dependence of the mixing-length parameter on the stellar mass. Although we calibrate the models in order to obtain the measured quantities of the corresponding binary system rather than those of the individual component stars, the models of each star are in good agreement with the observed properties: while the difference between the model and data radii is about 3-4 percent for the components of V818 Tau, the effective temperatures of the models of late-type components of V818 Tau, HD 27149 and 70 Tau are in perfect agreement with $T_{\text{eff}}$s derived from spectroscopic measurements by Paulson et al. (2003). Because V818 Tau is an active binary, the fundamental properties of its components should be re-evaluated using much more precise observational data than are used at present.

The relationship we derived between $\alpha$ of the components of the Hyades' binaries and their masses also gives the solar value of $\alpha$. This result should, however, be examined. It is possible that the age and chemical composition differences just happen to counterbalance each other, and that therefore the relationship gives the solar value coincidentally. The fact that the $\alpha$ values for the components of $\alpha$ Centauri from this relationship yield models in agreement with the observed stellar parameters leads us to adopt it as a prevalent relationship (Yildiz, in preparation). However, time variation of this relationship should not be ruled out.

From 2D and 3D hydrodynamical simulations of convection, Ludwig et al. (1999) and Trampedach et al. (1999) find that $\alpha$ is a decreasing function of effective temperature (or stellar mass). It is noteworthy that stellar evolution codes and simulation codes give opposite results for this relationship (see also Ludwig and Salaris 1999 for $\alpha$ values of the components in the eclipsing binary AI Phe). This contradiction may be a result of the variation of $\alpha$ in the layers of convective envelopes (see e.g., Deupree and Varner 1980) or in time.

The second important outcome concerns structure of the early type stars: the differentially rotating models for the components of $\theta^2$ Tau and 51 Tau are in better agreement with the observations than the non-rotating models and models with solid-body rotation.

The fundamental properties of the Hyades cluster that we have derived are not unique but can be given in terms of the metal abundance. If $Z = 0.024$, then $X = 0.718$ and its age is $t = 721$ Myr; if $Z = 0.033$, then $X = 0.676$ and $t = 676$ Myr. It should be pointed out here, however, that
the mass dependence of $\alpha$ is valid regardless of the value assigned for $Z$.

From these fundamental properties of the cluster we derive the masses of the components in the binary system HD 27149 by fitting the brightness of the models to the observed value: $M_A = 1.118 M_\odot$ and $M_B = 1.03 M_\odot$. Using the observed lower value for the masses, we show that the inclination of its orbit is about $i = 83^\circ.4$. This value of $i$ is smaller than the critical value for the occurrence of eclipsing ($i_{cr} = 88^\circ.9$) and hence this system is not an eclipsing one.

Unless the metal abundance of the cluster is observationally determined very precisely, we can not specify its helium abundance. In order to be able do this, abundances of the most abundant chemical species, in particular, oxygen, nitrogen, carbon and neon, should be found from the spectrum of its stars. Otherwise, we may only give the helium abundance as a function of $Z$ as well: while $Y = 0.258$ for $Z = 0.024$, $Y = 0.291$ for $Z = 0.033$. With very precise data on the colours of the stars, it is possible, however, to specify the heavy element abundance from the colour-colour diagram.

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Figure 7. Colour-colour diagram for the Hyades cluster. The models of the components of V818 Tau, 70 Tau and HD 27149 are constructed with Set 24. The filled circles represent the colours of the model computed from Bessel (1998) and the box is for the colours from Lejeune et al. (1998) for the solar composition. The triangle is for the colours from Lejeune et al. (1998) for the metallicity [Fe/O]= +0.13 dex.
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