Electroweak Corrections to t-channel single top production at the LHC

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Summary. — We describe the computation of the $O(\alpha^3)$ corrections for single top (and single anti-top) production in the $t$-channel at hadron colliders. We show also the genuine one loop SUSY contribution to these processes. Such contributions are generally quite modest in the mSUGRA scenario. The experimental observables would therefore only practically depend, in this framework, on the CKM $Wtb$ coupling.

1. – Introduction

One of the major goals of the LHC is the study of the properties of the top quark. In this respect single top production processes offer the unique possibility of a direct measurement of the entry $V_{tb}$ of the CKM matrix allowing non trivial tests of the properties of this matrix in the Standard Model (SM) [1]. Moreover single top production processes allow for a test of the $V-A$ structure of the charged current weak interaction of the top by looking at the polarization of this quark [2]. Such processes can be interesting in the hunting for physics beyond SM; indeed new physics can manifest itself either via loop effects, or inducing non SM weak interactions or introducing new single top production channels [3].

Within the Standard Model single tops can be produced via three different modes. At the LHC the $t$-channel production mode will be not only the dominant one [4] but also the best measured: CMS studies [5] conclude that, with 10 fb$^{-1}$ of integrated luminosity, the (mostly systematic) experimental uncertainty of the cross section is reduced below the ten percent level.

Such an experimental precision requests a similarly accurate theoretical prediction of the observables of the process. In order to achieve it the complete NLO calculation is required. In the SM this has been done for the QCD component of the $t$-channel, resulting in a relatively small (few percent) effect [6]. The electroweak effects have been computed...
very recently at the complete one loop level within the SM and the MSSM [7, 8]. Such computation is the topic of this paper.

In particular in sect. 2 we briefly describe the structure of the Electroweak (EW) corrections to the process of t-channel single (anti-)top production focusing on the partonic processes leading to such corrections. In sect. 3 we discuss the SUSY corrections to these processes evaluated within the MSSM. Numerical results for the EW corrections to t-channel single (anti-)top production at the LHC are presented in sect. 4, where the numerical impact of the SUSY corrections is discussed as well. Sect. 5 summarizes our results.

2. – Electroweak Corrections

Electroweak corrections to single (anti-)top production in the t-channel are of \( \mathcal{O}(\alpha^3) \) and in an obvious notation they can be written as:

\[
d\sigma_{t\text{-prod.}}(S) = \sum_{(q, q')} \int_{\tau_0}^{1} d\tau dL \left( d\sigma_{qb \to q't}^\gamma(s) + d\sigma_{qb \to q't}^{\nu}(s) \right),
\]

\[
d\sigma_{\bar{t}\text{-prod.}}(S) = \sum_{(\bar{q}, \bar{q}')} \int_{\tau_0}^{1} d\tau dL \left( d\sigma_{\bar{q}b \to \bar{q}'\bar{t}}^\gamma(s) + d\sigma_{\bar{q}b \to \bar{q}'\bar{t}}^{\nu}(s) \right).
\]

Where \((q, q') = (u, d), (c, s), (\bar{d}, \bar{u}), (\bar{s}, \bar{d})\), while \(\tau_0 = m_t^2/S\) and \(s = \tau S\). The differential luminosity is defined as:

\[
\frac{dL_{ij}}{d\tau}(\tau) = \frac{1}{1 + \delta_{ij}} \int_{\tau}^{1} \frac{dx}{x} \left[ f_i(x)f_j\left(\frac{\tau}{x}\right) + f_j(x)f_i\left(\frac{\tau}{x}\right) \right],
\]

\(f_i(x)\) being the momentum distribution of the parton \(i\) in the proton. We perform our computation in Feynman gauge setting the CKM matrix to unity. Due to CP invariance the unpolarized cross section of the partonic process \(\bar{q}b \to \bar{q}'\bar{t}(\gamma)\) is equal to that of the process \(qb \to q't(\gamma)\), so in the following we will analyse only the partonic processes contributing to single top production.

2.1. Virtual Corrections. – First class of corrections entering eq. (1) are the virtual corrections to the generic partonic process \(qb \to tq'\).

The starting point is the cross section for the \(ub \to td\) process; the \(\mathcal{O}(\alpha^3)\) corrections to the (unpolarized) differential cross section to this process read:

\[
d\sigma_{ub \to td}^\gamma = \frac{dt}{64\pi S^2} \sum_{\text{min}} 2\text{Re}\{\mathcal{M}^0 \ast \mathcal{M}^1\},
\]

where \(\mathcal{M}^0\) is the tree level amplitude of the partonic process \(ub \to dt\) while \(\mathcal{M}^1\) describes the corresponding EW one loop amplitude. The Mandelstam variables are defined as:

\[
s = (p_b + p_u)^2, \quad t = (p_b - p_t)^2, \quad u = (p_b - p_d)^2.
\]

The diagrams related to \(\mathcal{M}^1\) have been generated with the help of FeynArts [9], the algebraic reduction of the one loop integrals is performed with the help of FormCalc [10].
and the scalar one loop integrals are numerically evaluated using LoopTools \[11\]. We treat UV divergences using dimensional reduction while IR singularities are parametrized giving a small mass \(m_\gamma\) to the photon. The masses of the light quarks are used as regulators of the collinear singularities and are set to zero elsewhere.

UV finite predictions can be obtained by renormalizing the parameters and the wavefunctions appearing in \(\mathcal{M}^0\). In our case we have to renormalize the wavefunction of the external quarks, the mass of the W boson, the weak mixing angle \(\theta_W\) and the electric charge. We use the on shell scheme described in ref. \[12\]. This scheme uses the fine structure constant evaluated in the Thomson limit as input parameter. In order to avoid large logarithms arising from the running of \(\alpha\) to the electroweak scale \(M_W\), we renormalize the fine structure constant in the \(G_\mu\) scheme i.e. we define \(\alpha\) in terms of the Fermi constant \(G_\mu\):

\[
\alpha = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2 \theta_W.
\]

We consistently change the definition of the renormalization constant of the fine structure constant following the guidelines of ref. \[13, 14\].

As pointed out in ref. \[15\] in this scheme the leading logarithms arising from the running of \(\alpha\) are resummed to all orders in perturbation theory and absorbed in the definition of \(\alpha\). Moreover, in the case of charged current processes, the universal enhanced terms of the type \(\alpha m_t^2/M_W^2\) are included as well.

The unpolarized differential cross section for the process \(\bar{d}b \to \bar{u}t\) can be obtained from that of the process \(ub \to td\) by crossing:

\[
d_\sigma^{ew}_{\bar{d}b \to \bar{u}t} = \frac{dt}{64\pi s^2} \sum_{\text{spin}} 2 \text{Re}\{\mathcal{M}^0(s \to u, u \to s) \mathcal{M}^1(s \to u, u \to s)\}.
\]

The differential cross sections of the processes involving \(c\) and \(\bar{s}\) are, in the massless limit, equal to those quoted in eq. (4) and in eq. (7), respectively.

2.2. Real Corrections. – Another class of \(O(\alpha^3)\) corrections entering eq. (1) are the tree level contributions to the partonic processes of t-channel single top production associated with the emission of a photon \(qb \to tq'\gamma\).

The unpolarized differential cross section of these processes has been generated and squared using FeynArts and FormCalc. According to the KLN theorem \[16\] IR singularities and the collinear singularities related to the final state radiation cancel in sufficiently inclusive observables while the collinear singularities related to initial state radiation have to be absorbed into the Parton Distribution Functions (PDF).

In order to handle with these divergences we use two different procedures: the dipole subtraction method and the phase space slicing method.

In the subtraction approach one has to add and subtract to the squared amplitude an auxiliary function which matches pointwise the squared amplitude in the singular region and such that it can be analytically integrated over the photon phase space. Different functions fulfilling these requirements are available in literature, we use the function quoted in ref. \[17\]. In this reference explicit expression for the subtraction function and for its analytical integration is obtained within mass regularization using the so called Dipole Formalism \[18\].

According to the phase space slicing approach the singular region of the phase space is
excluded by introducing a cutoff on the energy of the photon and on the angle between
the photon and the massless quarks. In the regular region the phase space integration
can be performed numerically while in the singular region it can be done analytically in
the eikonal approximation, provided that the cutoffs are small enough. The form of the
differential cross section in the singular region is universal and its explicit expression in
the soft (collinear) region can be found in ref. [12] ([14]).
The two methods are in good numerical agreement.

2.3. Mass Factorization. – As pointed out in sect. 2.2, \( \mathcal{O}(\alpha^3) \) corrections to partonic
cross sections contain universal initial-state collinear singularities that have to be ab-
sorbed into the PDFs choosing a factorization scheme. We use the MS factorization
scheme at the scale \( \mu_F = m_t \).

Concerning the choice of the parton distributions set, we follow ref. [13]. The calculation
of the full \( \mathcal{O}(\alpha) \) corrections to any hadronic observable must include QED effects in the
DGLAP evolution equations. Such effects are taken into account in the MRST2004QED
PDFs [19] which are NLO QCD. Since our computation is leading order QCD and since
the QED effects are known to be small [20], we use the LO set CTEQ6L.

3. – SUSY Corrections

As already mentioned in sect. 1, accurate knowledge of the cross section of single top
production processes allows a precise determination of the \( V_{tb} \) entry of the CKM matrix.
Nevertheless some non-standard physics could alter the prediction of the cross section
biasing the determination of this parameter. We calculate the impact of the one loop
corrections on t-channel single (anti-)top production within the MSSM.

The one loop SUSY QCD corrections to t-channel single top production have been com-
puted at LHC in ref. [21]. We include these corrections, re-computing them from scratch.
Following a standard procedure in SUSY QCD, we treat UV divergences using dimen-
sional regularization. Moreover in this case we have to renormalize only the wavefunctions
of the squarks since the other renormalization constants do not have \( \mathcal{O}(\alpha_s) \) corrections.
These corrections are IR and collinear safe.

To obtain the genuine SUSY EW corrections one has to cope with the different structure
of the Higgs sector in the MSSM and in the SM. These corrections were obtained re-
calculating the full \( \mathcal{O}(\alpha^3) \) corrections and then subtracting the SM corrections computed
setting the SM Higgs mass equal to the mass of the lightest MSSM Higgs boson. The
computation of \( \mathcal{O}(\alpha^3) \) corrections within the MSSM was performed according to the
procedure described in sect. 2.1

4. – Numerical results

We consider the numerical impact of the corrections described previously by looking
at different observables.

4.1. EW Corrections within the SM. – In the left panel of fig. 1 we show the NLO
(i.e. \( \text{LO} + \mathcal{O}(\alpha^3) \)) evaluation of the total cross section for single (anti-)top as a function
of \( p_T^t \), a cut on the transverse momentum of the (anti-)top. As expected, at the LHC,
single top production dominates over single anti-top production. Nevertheless, as can
be inferred from the right panel of fig. 1, the relative contribution of the electroweak corrections is similar in the two cases. Indeed in both cases EW corrections are negative and become more important as the value of $p_T^{cut}$ increases. EW corrections are well below 10% for any reasonable value of $p_T^{cut}$.

The similar behaviour of the EW corrections in case of single top and single anti-top production makes the quantity

$$R_{t\text{-prod.}} = \frac{\sigma_{t\text{-prod.}}}{\sigma_{\bar{t}\text{-prod.}}}$$

independent of the EW corrections (fig. 2). This is an interesting feature since the value of the ratio (8) is used when looking for single top production processes at the LHC [3]. In the left panel of fig. 3 we show the NLO evaluation of the transverse momentum ($p_T$) distribution of the (anti-)top for the two production processes; while in right panel the relative impact of the EW corrections is shown. This observable shows the same features as the previous one: in particular the contribution of single top production is the leading one and the relative contribution of the electroweak corrections is similar in both single top and single anti-top production processes. Moreover EW corrections are negative and their absolute value increases as the value of $p_T$ increases: in particular they are larger than 10% in the region $p_T > 300$ GeV.

4.2. SUSY Corrections. – We present the numerical results for the two representative ATLAS DC2 mSUGRA benchmark points SU1 and SU6 [22]. In fig. 4 we show the relative contribution of the SUSY corrections to the total cross section for single top and single anti-top production as a function of $p_T^{cut}$ in the case of the SU1 point (left panel) and SU6 point (right panel). In both cases SUSY QCD and SUSY EW corrections are tiny. Moreover they have different sign, so their sum is further suppressed and in both cases below 0.1%.

5. – Conclusions

We have computed the one loop EW corrections to the process of single (anti-)top production in the t-channel. The overall result is that the impact of these corrections on the total rate is small, of the order of a negative few percent. EW corrections can play a more important role in the definition of the distributions.

Moreover we have studied the impact of the one loop SUSY corrections within the MSSM. In the scenarios we have considered their impact is negligible and so their eventual presence would not spoil the measurement of $V_{tb}$ performed within the SM.

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Fig. 1. – In the left panel we show the total cross section for single (anti-)top production as a function of a cut $p_T^{\text{cut}}$ on the transverse momentum of the (anti-)top. In the right panel the relative contribution $\delta = (\sigma^{\text{NLO}} - \sigma^{\text{LO}})/\sigma^{\text{LO}}$ of the EW corrections to this observable is shown.

Fig. 2. – Relative contribution of the EW corrections to $R_{t-prod}/\bar{t}$-prod., defined as the ratio between the total cross section for single top production and that for single anti-top production.

Fig. 3. – In the left panel the distribution of the transverse momentum of the (anti-)top for single (anti-)top production is shown. In the right panel we show the relative contribution of the EW corrections relative to the leading order result.
Fig. 4. – Contribution of the SUSY corrections to the total cross section of single top and anti-top production as a function of a cut $p_{T}^{cut}$ on the transverse momentum of the (anti-)top. In both panels we show the contribution of the SUSY EW, of the SUSY QCD corrections and of their sum relative to the leading order contribution. In the left (right) panel such corrections are computed in the SU1 (SU6) scenario.