Research on SMC-predictive DPC strategy for Vienna rectifier

Hui Ma\textsuperscript{1a)}, Wei Wei\textsuperscript{1}, Liangkai Wang\textsuperscript{1}, and Zeyu Shi\textsuperscript{2}

\textsuperscript{1} College of Electrical Engineering & New Energy, China Three Gorges University, Yichang 443002, China
\textsuperscript{2} School of Electric Power, South China University of Technology, 381 Wushan Road, Guangzhou 510641, China
\textsuperscript{a)} mahuizz119@126.com

Abstract: Predictive direct power control (PDPC) and sliding mode control (SMC) have been proposed as effective schemes for three-phase rectifiers. However, the conventional PDPC still has sampling power errors caused by control delay. Furthermore, the existing SMC can not well work with PDPC to provide fast error convergence and strong robustness during both start-up and load step change periods. To solve the problems above, this paper proposes an improved hybrid SMC-PDPC control scheme for Vienna rectifier, which uses lagrangian liner interpolation to predict the next sampling power value with eliminating inner-loop power errors. Additionally, this paper presents a novel SMC based on squared voltage in the outer loop. Experimental results based on 10 kW prototype circuit are presented to confirm the effectiveness of the proposed scheme.

Keywords: Vienna-type rectifier, sliding mode control, predictive direct power control, lagrange interpolation method

Classification: Power devices and circuits

References

[1] J. W. Kolar and F. C. Zach: “A novel three-phase utility interface minimizing line current harmonics of high-power telecommunications rectifier modules,” IEEE Trans. Ind. Electron. 44 (1997) 456 (DOI: 10.1109/41.605619).
[2] J. S. Lee and K. B. Lee: “Carrier-based discontinuous PWM method for Vienna rectifiers,” IEEE Trans. Power Electron. 30 (2015) 2896 (DOI: 10.1109/TPEL.2014.2365014).
[3] L. Hang, et al.: “Digitized feedforward compensation method for high-power-density three-phase Vienna PFC converter,” IEEE Trans. Ind. Electron. 60 (2013) 1512 (DOI: 10.1109/TIE.2012.2222851).
[4] C. Qiao and K. M. Smedley: “Three-phase unity-power-factor star-connected switch (Vienna) rectifier with unified constant-frequency integration control,” IEEE Trans. Power Electron. 18 (2003) 952 (DOI: 10.1109/TPEL.2003.813759).
[5] M. Leibl, et al.: “Sinusoidal input current discontinuous conduction mode control of the VIENNA rectifier,” IEEE Trans. Power Electron. 32 (2017) 8800 (DOI: 10.1109/TPEL.2016.2641502).
[6] M. Hartmann, et al.: “Digital current controller for a 1 MHz, 10 kW three-phase
1 Introduction

The Vienna rectifier, which is a grid-connected voltage PWM rectifiers, has attracted much attention due to its advantages of low voltage stress, low total harmonic distortion (THD) and high efficiency [1, 2, 3]. In addition, this rectifier is well known for its low control complexity and low sensing effort regarding the control system design [4, 5]. Thus it is applied widely in industrial communication systems, aerospace power, ship-board power supplies and other areas.

In recent years, various control methods have been proposed for the three phase Vienna rectifiers, which include current control [6, 7, 8] and power control [9, 10]. Compared with the current control algorithm, the traditional PI and hysteresis direct power control (DPC) based on the instantaneous power theory is simple and easy. However, PI control parameter tuning of direct power control algorithm is time-consuming and the dynamic performance is poor. Hysteresis power control has the drawback of a varying switch frequency which will increases difficulties in designing the ac side filter. To solve the above problem in the conventional method, a combined control strategy based on sliding mode control and prediction power control is proposed. Compared with the traditional PI-based direct power control strategy, the inner loop of proposed control strategy adopts predictive direct power control (P-DPC) that the phase information of grid voltage is not measured by synchronous rotation coordinate transformation and phase-locked loop. And the design difficulty of control system is observably reduced by this predictive control method. In addition, the second order lagrangian difference method is proposed to calculate the power reference value at \((k+2)\) to reduce the power error caused by control delay. And this power improves the traditional sliding mode control by taking the square of the voltage as the feedback to improve the dynamic response speed and stable accuracy. Finally experimental results on a 10 kW system to validate the effectiveness of the proposed P-DPC strategy.

2 Mathematical model of three-phase three-level Vienna rectifier

Fig. 1 shows the block diagram of the three-phase Vienna rectifier. Throughout this paper, the nomenclature of each component and its definition are: \(e_i\) (\(i = a, b, c\))
are the ideal grid voltage, \( i_i \) \((i = a, b, c)\) are the network side input current, \( L_i \) \((i = a, b, c)\) are the three-phase filter inductance, \( R_i \) \((i = a, b, c)\) are the line equivalent resistance, \( i_{dc+} \) and \( i_{dc-} \) respectively are the dc side output positive and negative current, \( C_P \) and \( C_n \) are the dc side filter capacitance, \( V_{cp} \) and \( V_{cn} \) are the voltage of \( C_P \) and \( C_n \), respectively. \( R_L \) and \( V_{dc} \) is the dc side load resistance and voltage, respectively. \( i_n \) is the midpoint current of point n in the dc bus, \( D_{ip} \) and \( D_{in} \) \((i = a, b, c)\) are three-phase positive, negative continuous diode, respectively. \( S_{ip} \) and \( S_{in} \) \((i = a, b, c)\) are switch functions of rectifier. Under the three-phase ideal network and the derivation process of three-phase coordinate transformation is omitted. In the stationary reference frame \( \alpha \beta \), the mathematical model of the three phase Vienna rectifier can be written as:

\[
\begin{align*}
\begin{cases}
\begin{align*}
e_a &= i_a R + L \frac{di_a}{dt} + V_a \\
e_\beta &= i_\beta R + L \frac{di_\beta}{dt} + V_\beta \\
C \frac{dV_{cp}}{dt} &= S_{ip} i_a + S_{ip} i_\beta - \frac{V_{dc}}{R_L} \\
C \frac{dV_{cn}}{dt} &= -S_{ip} i_a - S_{ip} i_\beta - \frac{V_{dc}}{R_L}
\end{align*}
\end{cases}
\end{align*}
\]

(1)

Where \( e_a, e_\beta \) and \( i_a, i_\beta \) are the power grid voltage and current in \( \alpha \beta \) frame, respectively. \( V_a, V_\beta \) are the ac-side voltages of rectifier, and \( R_L \) is the load resistance.

### 3 Predictive DPC of the inner power loop

Based on the instantaneous power theory [8], the ac-side current in the stationary \( \alpha \beta \) reference frame can be expressed as:

\[
\begin{bmatrix}
i_a \\
i_\beta 
\end{bmatrix} = \frac{1}{\|e_{\alpha \beta}\|^2} \begin{bmatrix} e_a & e_\beta \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix}
\]

(2)

Where

\[
\|e_{\alpha \beta}\|^2 = e_a^2 + e_\beta^2
\]

(3)
If the switching period is much smaller than the period of the power-source grid, $e_\alpha$ and $e_\beta$ are assumed as constant values within a switching sampling period. And the first-order difference method is adopted to discretize equation (1), the discrete model in the stationary $a\beta$ reference frame can be expressed as:

$$
\begin{bmatrix}
i_\alpha(k+1) - i_\alpha(k) \\
i_\beta(k+1) - i_\beta(k)
\end{bmatrix} = \frac{T_S}{L} \begin{bmatrix} e_\alpha(k) - V_\alpha(k) \\
e_\beta(k) - V_\beta(k)
\end{bmatrix} - \begin{bmatrix} i_\alpha(k) \\
i_\beta(k)
\end{bmatrix}
$$

(4)

The ideal grid voltage is approximately equal in the two adjacent switch sampling periods:

$$
\begin{bmatrix}
e_\alpha(k+1) \\
e_\beta(k+1)
\end{bmatrix} = \begin{bmatrix} e_\alpha(k) \\
e_\beta(k)
\end{bmatrix}
$$

(5)

Substituting (5) into (2), the variation of input current vector is obtained as follows:

$$
\begin{bmatrix}
i_\alpha(k+1) - i_\alpha(k) \\
i_\beta(k+1) - i_\beta(k)
\end{bmatrix} = \frac{1}{\|e_{\alpha\beta}\|^2} \begin{bmatrix} e_\alpha(k) & e_\beta(k) \\
e_\beta(k) & -e_\alpha(k)
\end{bmatrix} \begin{bmatrix} P(k+1) - P(k) \\
Q(k+1) - Q(k)
\end{bmatrix}
$$

(6)

Where $\|e_{\alpha\beta}\|^2 = e^2_\alpha(k) + e^2_\beta(k)$

Substituting (6) into (4) yields:

$$
\begin{bmatrix}V_\alpha(k) \\
V_\beta(k)
\end{bmatrix} = \begin{bmatrix} e_\alpha(k) \\
e_\beta(k)
\end{bmatrix} - \frac{L}{T_S\|e_{\alpha\beta}\|^2} \begin{bmatrix} e_\alpha(k) & e_\beta(k) \\
e_\beta(k) & -e_\alpha(k)
\end{bmatrix} \begin{bmatrix} P(k+1) - P(k) \\
Q(k+1) - Q(k)
\end{bmatrix} - \begin{bmatrix} i_\alpha(k) \\
i_\beta(k)
\end{bmatrix}
$$

(7)

Substituting (2) into (7), the control voltage can be shown as:

$$
\begin{bmatrix}V_\alpha(k) \\
V_\beta(k)
\end{bmatrix} = \begin{bmatrix} e_\alpha(k) \\
e_\beta(k)
\end{bmatrix} - \frac{L}{T_S\|e_{\alpha\beta}\|^2} \begin{bmatrix} e_\alpha(k) & e_\beta(k) \\
e_\beta(k) & -e_\alpha(k)
\end{bmatrix} \begin{bmatrix} P(k+1) - P(k) \\
Q(k+1) - Q(k)
\end{bmatrix}
$$

(8)

Equation (8) can be transformed as:

$$
\begin{bmatrix}P(k+1) \\
Q(k+1)
\end{bmatrix} = \frac{T_S}{L} \begin{bmatrix} e_\alpha(k) & e_\beta(k) \\
e_\beta(k) & -e_\alpha(k)
\end{bmatrix} \begin{bmatrix} e_\alpha(k) - V_\alpha(k) \\
e_\beta(k) - V_\beta(k)
\end{bmatrix} + \left(1 - \frac{T_SR}{L}\right) \begin{bmatrix} P(k) \\
Q(k)
\end{bmatrix}
$$

(9)

![One switching cycle](image_url)

**Fig. 2.** Delay induced error schematic
In the traditional deadbeat control strategy, sampling and calculation should be completed at the same time. However, there is a control delay in the actual system which leads to prediction error of the grid power, as shown in Fig. 2. Consequently the control voltage vector (110,000) in the \( k \)th sampling period is not fully executed, and the \( k \)th sampling period still uses the control vector of the previous period, resulting in the instantaneous power cannot accurately track the power.

It is necessary that a compensation scheme has been proposed to mitigate influence of control delay. Push one step backward in formula (9) to predict the active and reactive power at the \( k + 2 \) moment.

\[
\begin{bmatrix}
P(k + 2) \\
Q(k + 2)
\end{bmatrix} = \frac{T_s}{L} \begin{bmatrix}
e_a(k) & e_\beta(k) \\
e_\beta(k) & -e_a(k)
\end{bmatrix} \left(\begin{bmatrix}
e_a(k) - V_a(k + 1) \\
e_\beta(k) - V_\beta(k + 1)
\end{bmatrix}\right) + \left(1 - \frac{T_s R}{L}\right) \begin{bmatrix}
P(k + 1) \\
Q(k + 1)
\end{bmatrix}
\]

Substituting (9) into (10) yield:

\[
\begin{bmatrix}
V_a(k + 1) \\
V_\beta(k + 1)
\end{bmatrix} = \frac{U}{\|e_{a\beta}\|^2} \frac{L}{T_s} \begin{bmatrix}
P(k + 2) \\
Q(k + 2)
\end{bmatrix} + \frac{U}{\|e_{a\beta}\|^2} \frac{L}{T_s} Z^2 \begin{bmatrix}
P(k) \\
Q(k)
\end{bmatrix}
\]

\[+(1 + Z) \begin{bmatrix}
e_a(k) \\
e_\beta(k)
\end{bmatrix} - Z \begin{bmatrix}
V_a(k) \\
V_\beta(k)
\end{bmatrix}\]

Where:

\[U = \begin{bmatrix} e_a & e_\beta \\ e_\beta & -e_a \end{bmatrix}, \quad Z = \left(1 - \frac{T_s}{L} R\right)\]

In order to realize the effect of deadbeat control, the actual value of instantaneous power at the \( k + 1 \) moment must be equal to the given value, so as to achieve accurate tracking of the instantaneous power reference in the next cycle.

\[
\begin{bmatrix}
P(k + 1) \\
Q(k + 1)
\end{bmatrix} = \begin{bmatrix}
P^* (k + 1) \\
Q^* (k + 1)
\end{bmatrix}
\]

Then, the instantaneous power actual value and the given value are also equal at the moment \( k + 2 \), which can be obtained from equation (11):

\[
\begin{bmatrix}
V_a(k + 1) \\
V_\beta(k + 1)
\end{bmatrix} = \frac{U}{\|e_{a\beta}\|^2} \frac{L}{T_s} \begin{bmatrix}
P^*(k + 2) \\
Q^*(k + 2)
\end{bmatrix} + \frac{U}{\|e_{a\beta}\|^2} \frac{L}{T_s} Z^2 \begin{bmatrix}
P(k) \\
Q(k)
\end{bmatrix}
\]

\[+(1 + Z) \begin{bmatrix}
e_a(k) \\
e_\beta(k)
\end{bmatrix} - Z \begin{bmatrix}
V_a(k) \\
V_\beta(k)
\end{bmatrix}\]

Lagrangian interpolation is a linear interpolation operation. The calculation is relatively simple and has high accuracy within a certain range. In this paper, Lagrangian-based interpolation method is used to predict the reference of active power at \( k + 2 \) time.

The active power reference at time \( k + 2 \) can be represented linearly by the power reference value before time \( k \). The n-order discrete expression is:

\[P^*(k + 2) = a_0 P^*(k) + a_1 P^*(k - 1) + \ldots + a_n P^*(k - n)\]

Then the n-order prediction expression is:
After comprehensive calculation and considering the real-time nature of the control, the second-order interpolation method is used to predict the reference active power value in this paper:

$$P^*(k + 2) = 6P^*(k) - 8P^*(k - 1) + 3P^*(k - 2)$$  

Substituting (17) into (14), the deadbeat predictive direct power control model can be expressed as:

$$V_{\alpha}(k + 1) = \frac{U}{\|e_{\alpha}\|^2 T_s} \left[ P^*(k + 2) + \frac{U}{\|e_{\alpha}\|^2} L Z^2 \right] P(k)$$

$$V_{\beta}(k + 1) = (1 + Z) \left[ e_{\alpha}(k) - Z \left[ V_{\alpha}(k) V_{\beta}(k) \right] \right]$$

$$P^*(k + 2) = 6P^*(k) - 8P^*(k - 1) + 3P^*(k - 2)$$

4 Sliding mode control of outer voltage loop

In this paper an improved sliding mode control based on the squared voltage is proposed. The SMC algorithm can quickly track the DC bus voltage reference by amplifying the error information. So it improves dynamic response speed and stable accuracy of the control system.

Analyzing the power flow of the system from the DC side, multiply (1) by \( V_{dc} \) on both side and add them together. Assuming the neutral point voltage balance, so the capacitor voltage \( V_{cp}, V_{cn} \) are replaced by \( V_{dc}/2 \), the power of DC side can be expressed:

$$V_{dc} \left[ (S_{ap} - S_{an}) i_{\alpha} + (S_{\beta p} - S_{\beta n}) i_{\beta} \right] = \frac{C}{2} \frac{V_{dc}dV_{dc}}{dt} + \frac{V_{dc}^2}{R_L}$$

According to the energy conservation principle, the input power on the AC side is equal to the output power on the DC side. Neglecting the switch and other energy loss. For unity power factor case, the AC side input active power is equal to the DC side power.

$$P_{dc} = \frac{C}{2} \frac{V_{dc}dV_{dc}}{dt} + \frac{V_{dc}^2}{R_L}, \quad P_{dc} = P_{ac} = P$$

According to the DC side power mathematical model in (20) and the principle of selecting the sliding surface described, the sliding surface is set as follows:

$$S = \frac{d(V_{dc}^2 - V_{dc}^2)}{dt} + k(V_{dc}^2 - V_{dc}^2)$$

Where, \( V_{dc}^2 \) is the DC side load voltage reference and \( k \) is the positive control gains.

Equation (21) can be transformed as:

$$\frac{dV_{dc}^2}{dt} = -\frac{4V_{dc}^2}{CR_L} + \frac{4}{C} P$$

If the DC bus voltage reference \( V_{dc}^2 \) is constant, there is \( dV_{dc}^2/dt = 0 \). Then the system will move along the \( S = 0 \) path on the sliding surface. Substituting (22) into (21) yields:
\[ S = \frac{C}{4} k(V_{dc}^2 - V_{dc}^2) + \frac{V_{dc}^2}{R_L} - P = 0 \] (23)

In order to achieve accurate tracking of DC-link voltage reference in the voltage outer loop, the instantaneous value of the active power is equal to the given value in a switching cycle: \[ P^* = P. \] Combining equation (23) can obtain:

\[ P^* = \frac{C}{4} k(V_{dc}^2 - V_{dc}^2) + \frac{V_{dc}^2}{R_L} \] (24)

Fig. 3 shows the block diagram of the proposed SMC-PDPC scheme for three-phase Vienna rectifier. In order to realize the sliding mode prediction direct power control, firstly, the sliding mode controller is used to calculate the grid input active power reference value, and the reactive power given value is zero for unit power factor; Then the control voltage vector \( V_\alpha, V_\beta \) is calculated by using the improved deadbeat predictive direct power control model. The inner-loop power predictive module does not need the phase information of the grid voltage which measure by the same speed rotation coordinate transformation and phase-locked loop, so it reduce the system design difficult. In addition, the Lagrangian interpolation method is adopted to predict the reference of power given value at the \( k \)th time to reduce the power error caused by the system control delay; the outer voltage loop of the voltage with \( V_{dc}^2 \) as the feedback, so that the error signal is amplified to quickly track the DC bus voltage given value.

5 Experimental results

In order to verify the performance of the proposed algorithm in this paper, an experimental verification which according to the Vienna rectifier topology based on
Fig. 1 was carried out. The DSP320F28335 of TI Company is used to be the control core of the lab platform. The bidirectional switch are made up of two insulated gate bipolar transistors 1KW30N65EL5 (650 V/30 A) with common emitter. Table I shows the parameters of the lab platform.

| Name                     | Nomenclature/Unit | Value |
|--------------------------|-------------------|-------|
| Input line voltage       | \(V_{\text{rms}}/V\) | 220   |
| Source frequency         | \(f/\text{Hz}\)   | 50    |
| Input filtering inductance | \(L/\text{mH}\) | 4     |
| DC-link voltage          | \(V_{\text{dc}}/V\) | 650 |
| load resistance          | \(R/\Omega\)      | 50    |
| DC-link capacitance      | \(C/\mu\text{F}\) | 2200 |
| Switching frequency      | \(f_1/\text{Hz}\) | 15k   |

The steady state oscilloscopes of DC voltage and power for both conventional DPC and P-DPC which are both at the sampling frequency of 15 kHz. From Fig. 4, it can be seen that the system can run steadily under both DPC and P-DPC. The DC side load voltage is always maintained at 650 V, it means that the voltage can be tracked accurately. However, the fluctuation of voltage and power is smaller and the waveform is smoother in Fig. 4(b), so the stability of proposed method is enhanced.

From top to bottom, the curves shown in Fig. 5 are active DC voltage, active power, reactive power and grid current. It is obvious that the systems using the two control algorithms can quickly reach a new stable state and the current is sinusoidal under the load is suddenly changed. However, the power fluctuation of the direct power algorithm are 58 W and 48 W respectively. It is clear that the improved deadbeat predictive DPC presents less power ripples and faster dynamic response speed. In addition, comparing the voltage waveforms on the DC side, the voltage of the DPC algorithm system can quickly (only 0.06 s) recover to the voltage reference (650 V) after minor fluctuations. The results show that the SMC-PDPC algorithm improves the system interference and dynamic response speed.

Fig. 6 shows the results of the harmonic analysis of the input three-phase current of the conventional PI-based DPC and proposed predictive DPC, respectively. The current harmonic THD of the power grid in both control systems reaches
the power quality standard. But current THD (4.1\%) of the proposed SMC-PDPC in Fig. 6(b) is lower than the (5.1\%) of the conventional DPC in Fig. 6(b).

6 Conclusion

In this paper, an improved hybrid SMC-PDPC control scheme control algorithm for Vienna rectifier is proposed. The PDPC scheme belongs to direct power control family and provides the desired power references by utilizing lagrangian linear interpolation. The sliding mode control scheme, taking the square of the voltage as the feedback, is adopted to improve dynamic response speed and stable accuracy for the DC voltage. Compared with conventional PI-DPC scheme, the proposed algorithm has the following advantages: simple control structure, reducing the difficulty of system parameter design, multi-step predictive control is adopted to minimize the next sample time predicted error and improve the accuracy of inner-loop power value. The experimental results proves the correctness and superiority of the proposed algorithm and has a good application value.

Acknowledgments

This work was supported by Hu beiYouth Scientific and Technological Innovation Project (No. T201504), Jiangsu Collaborative Innovation Center for Smart Distribution Network (No. XTCX201710).