Temporal behaviour of multipole components of the magnetic field in a small dipole magnet wound with coated conductors

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Abstract
To study the influence of coated-conductor magnetization on the field quality of accelerator magnets, we made a small dipole magnet consisting of four racetrack coils wound with GdBCO coated conductors and measured its magnetic field in liquid nitrogen by using rotating pick-up coils. We focused on the dipole and sextupole components (coefficients) of the magnetic field, which vary with time owing to the decay of the magnetization of the coated conductors. About 50 min (3055 s) after the current was ramped up to 50 A, the dipole coefficient normalized by the design value of the dipole component, i.e., the value calculated with the designed coil shape and the uniform current distribution in the coated conductors, increased by $7.4 \times 10^{-4}$, and the sextupole coefficient normalized by the design value of the dipole component increased by $1.8 \times 10^{-4}$. The magnitudes of the dipole and sextupole coefficients depended on the excitation history of the magnet. Electromagnetic field analyses were carried out to calculate the current distributions in coated conductors, considering their superconducting properties; the dipole and sextupole coefficients were then determined from the calculated current distributions. Although the analyses were based on the two-dimensional approximation of the cross-section of the magnet, the temporal behaviours as well as the hysteretic characteristics of the calculated dipole and sextupole coefficients agree qualitatively with those of the dipole and sextupole coefficients measured in the magnet.

Keywords: accelerator, coated conductor, dipole magnet, field harmonics, magnetization, multipole
1. Introduction

The advantages of high $T_c$ superconductor (HTS) magnets as compared to copper magnets are reduced power consumption and higher magnetic field generation, which is unrestricted by the magnetic saturation of iron. Their advantages as compared to low $T_c$ superconductor (LTS) magnets include easy cooling without liquid helium by using a cryocooler, potentially improved thermal stability or larger temperature margin—especially when operated at a higher temperature where the specific heats of materials are larger—and the generation of very high magnetic fields when operated at a low temperature because of their high critical current densities, even in high magnetic fields. In particle accelerators for physics research, very high magnetic field generation or large temperature margins are attractive when compared with LTS magnets [1–8]. In particle accelerators for cancer therapy, higher magnetic field generation reducing the size of a facility and reduced power consumption are attractive when compared with copper magnets [9–15]; their easy cooling and improved thermal stability are attractive in comparison with LTS magnets. Among various HTSs, coated conductors have recently attracted broad interest for magnet applications, because they maintain large critical currents, even in high magnetic fields, and their performance is being improved continuously through competition among many manufacturers.

However, the large magnetization of flat, wide coated conductors is one of the major concerns for their applications to accelerator magnets, because it can deteriorate the quality of the magnetic fields. The influence of the magnetization of coated conductors and Bi-2223 tapes on field quality has been studied by various authors in axisymmetric solenoid coils and pancake coils [16–21]. They mostly aimed to clarify the influence of these magnetizations in NMR magnets, where stable and uniform magnetic fields are required. When comparing NMR magnets and accelerator (collider) magnets, the requirements for their field qualities differ considerably. In NMR magnets, extremely high stability ($0.01$ ppm h$^{-1}$) and extremely high uniformity for a small sphere ($\sim 1$ ppm in a sphere whose radius is 10 mm) are required. Here, we look at dipole magnets among various accelerator magnets. In dipole magnets, the field error is usually evaluated by using multipole components of the magnetic field. This measure of field quality is used in accelerator magnets particularly, because the multipole components are related directly with the functions of accelerator magnets: the dipole component is related to bending; the quadrupole component is related to focusing or defocusing. In dipole magnets, in which only the dipole component is essential, the allowable errors in higher multipole components normalized with the dipole are a few units in $10^{-4}$ at the reference radius of a magnet [22, 23]. These allowable errors are much larger than those in NMR magnets, but a magnetic field with such a quality must be generated in a much larger space where the beam of charged particles passes. The times scales of the operations of accelerator magnets are often much shorter than those of NMR magnets, which are usually kept magnetized for years. It should be noted that the drift of a magnetic field whose time constant is similar to the time scale of the operation of a magnet is most serious. Therefore, the required field stabilities in accelerator magnets are rather different from those in NMR magnets. In spite of such peculiar requirements, there seem to be few previous studies on the magnetization of coated conductors in relation to their applications to accelerator magnets [10, 11].

The purpose of this paper is to study the influence of the magnetization of coated conductors on field quality from the viewpoint of their applications to accelerator magnets. We made a small dipole magnet consisting of four racetrack coils wound with GdBCO coated conductors, and measured its magnetic field in liquid nitrogen by using rotating pick-up coils. We focused on the dipole component of the magnetic field and the sextupole component of the magnetic field, which is the lowest allowed multipole component, because the magnetization can influence them differently. We examined their temporal evolution and how they are influenced by the excitation history of the magnet. Electromagnetic field analyses were also carried out to calculate the current distributions in coated conductors, considering their superconducting properties. The dipole and sextupole components of the magnetic field were determined from the calculated current distributions, and then compared with the measured dipole and sextupole components. This helped to clarify the influence of the time-dependent current distributions in coated conductors in the cross-section of the magnet on the dipole and sextupole components of the magnetic field.

This paper is organized as follows. The theory of the multipole components of a two-dimensional magnetic field is explained briefly in section 2. In section 3, the details of the dipole magnet used in the experiments are presented. In section 4, the experimental method for the magnetic field measurements is described. Section 5 provides the theoretical model for the numerical electromagnetic field analyses. In section 6, the results of the magnetic field measurements are presented. In section 7, the experimental results are compared with the numerical results, and we discuss the influence of the temporal evolution of the current distributions in coated conductors in the coil cross-sections on the temporal variations in the dipole and sextupole components of the magnetic field. Finally, section 8 addresses our conclusion.

2. Multipole components of two-dimensional magnetic fields [24, 25]

An arbitrary two-dimensional magnetic field can be expanded with the multipole series. In cylindrical coordinates, this multipole series is given as

$$B_p(r, \theta) = \sum_{n=1}^{\infty} \left( \frac{r}{r_0} \right)^{n-1} (B_n \cos n\theta + A_n \sin n\theta),$$

(1)
We used a dipole magnet consisting of four racetrack coils without an iron yoke. Its schematic view and specifications are shown in Figure 2 and Table 1. Each racetrack coil was wound with 5 mm wide GdBCO coated conductors fabricated by Fujikura Ltd, whose specifications are shown in Table 2. The inner width and outer width of the racetrack coil are 96 mm (48 mm × 2) and 152.8 mm (76.4 mm × 2), respectively. Two racetrack coils were stacked with 1 mm separation to form each pole of the magnet.

A set of four line currents at $\phi = \pi/6$ with dipole symmetry generates no sextupole component. A set of four line currents at $\phi < \pi/6$ with dipole symmetry generates a positive sextupole component, whereas a set of four line currents at $\phi > \pi/6$ with dipole symmetry generates a negative sextupole component. The distance between the line currents and the magnet centre $a$ as well as $\phi$ can influence the magnitude of...
the sextupole component. Therefore, once the cross-sectional dimensions of the straight parts of the racetrack coils are established, we can determine the separation between the two stacks of racetrack coils so as to eliminate the sextupole component of the magnetic field, which is the lowest allowed multipole. We carried out three-dimensional magnetic field analyses of the magnet in order to determine the separation more accurately, so as to eliminate the sextupole component measured by the pick-up coils with a finite length. The determined separation is 58 mm, and the design values of multipole coefficients are listed in table 3. In this paper, design values of multipole coefficients (multipole components; multipole coefficients) mean the magnetic fields (multipole components; multipole coefficients) calculated with the designed coil shape and the uniform current distribution in the coated conductors. The magnet generates the dipole magnetic field of 88.1 mT at 50 A (1.76 mT A\(^{-1}\)) (design values).

Through three-dimensional magnetic field analyses, we can also estimate the magnetic fields of the rotating pick-up coils. Figure 3 shows the load lines for the magnetic fields to which the coated conductors are exposed and the critical current (\(I_c\))—magnetic field (\(B\)) characteristic of the coated conductors used to fabricate the coils. The typical operating current of the magnet in the experiment was 50 A, which was small enough with respect to the critical current.

### 4. Experimental method

The schematic view of the entire experimental set-up is shown in figure 4. The dipole magnet was placed in a cryostat and was cooled with liquid nitrogen. In the bore of the magnet, which is the space between the pair of the stacked racetrack coils, we inserted another small inner cryostat whose interior was kept at room temperature.

Rotating pick-up coils were installed in the inner cryostat to measure the magnetic field [26–28]. The schematic and scale drawings of the rotating pick-up coils are shown in figures 5(a) and (b), respectively. Three coils (A, B, and C) consisting of fine copper strips were patterned on a plastic plate. The geometries of the three coils are identical: the average length, the average width, and the number of turns of each coil are 118.1 mm, 6.1 mm, and 20, respectively. The rotating axis is located at the centre of coil B. Coils A and coil B were connected in series as to cancel the dipole component of the magnetic field and to detect higher harmonics. Coil C was used independently to detect all harmonics, including the dipole component. Using the Fourier coefficients \(u_n\) and \(v_n\) of the output voltage of the series-connected coil A and coil B represented as equation (8), \(B_n\) and \(A_n\), except \(n = 1\), can be determined by equations (9) and (10), where \(\omega\) and \(\phi_0\) are the angular velocity of the pick-up coils and the initial phase angle of the pick-up coils, respectively:

\[
V(t) = \sum_{n=1}^{\infty} (u_n \cos n\omega t + v_n \sin n\omega t),
\]

\[
B_n = \frac{1}{G_S} \left( u_n \sin n\phi_0 + v_n \cos n\phi_0 \right),
\]

\[
A_n = \frac{1}{G_S} \left( u_n \cos n\phi_0 - v_n \sin n\phi_0 \right).
\]

Here the constant \(G_S\) is defined as

![Figure 3. Load lines for the magnetic fields to which coated conductors are exposed and the critical current (I_c)—magnetic field (B) characteristic of the coated conductors used to fabricate coils.](image)

**Table 3.** Design values of multipole coefficients and multipole components (coefficients) measured at room temperature.

| Multipole | Design value of multipole coefficient | Measured multipole at 2.00 A | Measured multipole coefficient normalized by measured dipole | Measured multipole coefficient normalized by design value of dipole |
|-----------|--------------------------------------|----------------------------|------------------------------------------------------------|------------------------------------------------------------------|
| Dipole    | 1                                    | 3.5 mT                     | 1                                                          | 0.99                                                             |
| Quadrupole| 0                                    | −0.02 mT                   | −0.006                                                      | −0.006                                                           |
| Sextupole | −7.04 × 10\(^{-4}\)                 | −0.09 mT                   | −0.03                                                       | −0.03                                                            |
| Octupole  | 0                                    | −0.04 mT                   | −0.01                                                       | −0.01                                                            |

\*Design value of dipole component is 1.76 mT A\(^{-1}\) (88.1 mT at 50 A).

\**Measured value of dipole component is 1.75 mT A\(^{-1}\).
where $d$ is the distance between the rotation axis and the centre of coil A, $l_i$ is the length of the $i$th turn of each coil, $w_j$ is the width of the $j$th turn of each coil, and $N$ is the number of turn of each coil. In the coils used in the experiments, $l_i$ and $w_j$ are given as

$$l_i = \begin{cases} 120.0 \text{ mm} & ; i = 1, \ldots, 10, \\ 116.2 \text{ mm} + \frac{3.8}{9} \text{ mm} & ; i = 11, \ldots, 20, \end{cases}$$

$$w_j = \begin{cases} 8.0 \text{ mm} & ; i = 1, \ldots, 10, \\ 4.2 \text{ mm} + \frac{3.8}{9} \text{ mm} & ; i = 11, \ldots, 20, \end{cases}$$

because each coil consists of two layers with the identical geometry, and $d$ is 7.33 mm. Using the Fourier coefficients $u_n$ and $v_n$ of the output voltage of coil C represented as equation (8), $B_n$ and $A_n$ can be determined by equations (14) and (15):

$$B_n = \frac{1}{G} \left( u_n \sin n \phi_0 + v_n \cos n \phi_0 \right),$$

$$A_n = \frac{1}{G} \left( u_n \cos n \phi_0 - v_n \sin n \phi_0 \right).$$

Here the constant $G$ is defined as

$$G = \sum_{i=1}^{N} \sum_{n=1}^{N} \frac{1}{n \pi} \left( d + \frac{w_i}{2} \right)^n - \left( d - \frac{w_i}{2} \right)^n. \quad (16)$$

How to derive these equations is shown in appendix A. The dipole component was determined from the output voltage of coil C by using equations (14), (15), and (16), whereas higher multipole components were determined mainly from the output voltage of the series-connected coil A and coil B by using equations (9), (10), and (11) because of better resolutions. When calculating the multipole components from the output voltages of the pick-up coils, we defined $\phi_0$ so as to
eliminate the skew dipole component using the following equation:

$$\phi_0 \equiv \arctan \left( \frac{-u}{v_1} \right).$$

(17)

5. Theoretical model for the numerical electromagnetic field analyses [29]

A two-dimensional model of the cross-section of the straight part of the dipole magnet was used for numerical electromagnetic field analyses: the cross-section was modelled as an assembly of infinite straight coated conductors as shown in figure 6. The equation for numerical electromagnetic field analyses can be derived as follows from Faraday’s law, Biot–Savart’s law, and the extended Ohm’s law, where the nonlinear equivalent conductivity $\sigma$ of a superconductor replaces the constant conductivity:

$$\nabla \times E = -\frac{\partial B}{\partial t},$$

(18)

$$B = \frac{\mu_0 I}{4\pi} \int_{V'} \frac{J' \times r}{r^3} dV',$$

(19)

$$E = \frac{J}{\sigma}.$$  

(20)

The current vector potential $T$ is used for the formulation:

$$J = \Delta \times T.$$  

(21)

$J/\sigma$ is substituted for $E$ in equation (18), the right side of equation (19) is substituted for $B$ in equation (18), and $J$ is replaced with $\nabla \times T$. Because the superconductor layer of the coated conductor is very thin, we apply the thin-strip approximation [30], where only the current density component tangential to the superconductor layer is taken into account, and its normal component is neglected. Hence, $T$ becomes normal to the superconductor layer. Finally, we can obtain the following equation:

$$-\frac{\partial}{\partial y_i} \sigma \frac{\partial T}{\partial y_j} + \frac{\mu_0 I_s}{2\pi} \frac{\partial}{\partial t} \int_{j=1}^{M} \frac{1}{r(y_j, y_j')} \frac{\partial T}{\partial y_j'} dy_j' = 0.$$  

(22)

Here, $T$ and $T'$ are the current vector potential at the field point (the point where the field (potential) is calculated) and the source point (the point where a current flows to generate the magnetic field at the field point), respectively; $y_i$ is the lateral coordinate on the $i$th coated conductor on which a field point resides, and $y_j'$ is the lateral coordinate on the $j$th coated conductor on which a source point resides; $r(y_i, y_j')$ is the distance between the field point $y_i$ on the $i$th coated conductor and the source point $y_j'$ on the $j$th coated conductor; $t_s$ is the thickness of the superconductor layer; $M$ is the number of coated conductors composing the magnet, i.e., the number of the turns of the magnet.

Applying Ampere’s law to the cross-section of each coated conductor and considering the definition of current...
vector potential, the transport current $I$ of each coated conductor can be given by the boundary condition as

$$I = t_c(T_1 - T_2), \quad (23)$$

where $T_1$ and $T_2$ are the current vector potentials at the edges of the superconductor layer [29, 30].

For simplification, we assumed that the magnetic field dependence of the critical current density is solely determined by the magnetic field component normal to the superconductor layer $B_{\perp}$. The superconducting property is given by the power law $E$–$J$ characteristic [31, 32]:

$$E = E_0 \left( \frac{J}{J_c(B_{\perp})} \right)^n, \quad (24)$$

where $J_c$ is the critical current density, $E_0$ is $10^{-4}$ V m$^{-1}$. The equivalent conductivity of the superconductor $\sigma(nT)$ is then derived as

$$\sigma(nT) = \frac{J_c}{E_0} \left( \frac{J}{|\nabla \times nT|} \right)^{n-1}. \quad (25)$$

Figure 9. Measured hysteresis loops of multipole components of magnetic field, in which the contribution of the magnetization is extracted: the measured multipole components in figures 8(a) and (b) were fitted to linear functions by the least-square method, and, then, the values calculated by using these linear functions were subtracted from the measured multipole components in these figures: (a) dipole component; (b) sextupole component.

Figure 10. Measured temporal evolution of multipole coefficients in the excited phase for repeated 50 A excitations: (a) dipole coefficient; (b) sextupole coefficient. EP represents excited phase. In order to obtain multipole coefficients, multipole components were normalized by the design value of the dipole component ($1.76$ mT A$^{-1}$) using the average measured current in each excited phase.

Figure 11. Measured temporal evolution of multipole coefficients in the remnant phase for repeated 50 A excitations: (a) dipole coefficient; (b) sextupole coefficient. RP represents remnant phase. In order to obtain multipole coefficients, multipole components were normalized by the design value of the dipole component ($1.76$ mT A$^{-1}$) using the average measured current in the preceding excited phase of each remnant phase.
To reduce the computation time required in iterative calculations, we set the maximum equivalent conductivity at $10^{20}$ Sm$^{-1}$. The magnetic field dependence of the local critical current density is given by Kim’s model \[ \text{as } J_c = J_{c0} \frac{B_0}{B_0 + |B|} \] (26) where $J_{c0}$ is the critical current density at zero magnetic field, and $B_0$ is a constant.

6. Measured magnetic fields

6.1. Magnetic fields at room temperature

Firstly, at room temperature, a small current (2.00 A) was supplied to the magnet, and the magnetic field was measured.

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**Figure 12.** Measured temporal evolution of multipole coefficients in the excited phase for repeated 30 A excitations with a 50 A excitation: (a) dipole coefficient; (b) sextupole coefficient. EP represents excited phase. In order to obtain multipole coefficients, multipole components were normalized by the design value of the dipole component (1.76 mT A$^{-1}$) using the average measured current in each excited phase.

**Figure 13.** Measured temporal evolution of multipole coefficients at the remnant phases in repeated 30 A excitations with a 50 A excitation: (a) dipole coefficient; (b) sextupole coefficient. RP represents remnant phase. In order to obtain multipole coefficients, multipole components were normalized by the design value of the dipole component (1.76 mT A$^{-1}$) using the average measured current in the preceding excited phase of each remnant phase.

The measured dipole, quadrupole, sextupole, and octupole components are compared with the design values in table 3. Even at room temperature, where apparently there is no magnetization in coated conductors, the measured multipole components do not agree with the design values. The causes of these discrepancies may be the misalignment of the assembled four racetrack coils, the dimensional error of each racetrack coil itself, and the dimensional errors of the coated conductors. It should be noted that the impacts of the misalignment and the dimensional errors might vary with temperature because of thermal contraction. Therefore, the measured multipole coefficients listed in this table do not always agree with the multipole coefficients at 77 K in the absence of the magnetization of coated conductors. In this paper, we focus on the temporal evolution of multipole
components and the influence of the excitation history on their magnitudes, which are not affected by the misalignment and/or the dimensional errors.

6.2. Hysteresis loops of dipole and sextupole components

The magnet was cooled at zero magnetic field (zero-field cooling) to 77 K, and the magnet current was changed stepwise as shown in figure 7(a): the height of each step was 5 A (±5 A s$^{-1}$), and the flat part of each step was about 300 s. The magnetic field was measured at the last 120 s of the flat part of each step. In figures 8(a) and (b), the dipole and sextupole components are plotted against the current. In order to improve legibility, the measured values in each figure were fitted to a linear function by the least-square method, the values calculated by using this linear function were then subtracted from the measured multipole components. The resulting values are plotted in figures 9(a) and (b). In these figures, we can see clear hysteretic behaviours, which should be caused by the magnetization of the coated conductors.

6.3. Temporal evolution of dipole and sextupole components

The magnet was excited as shown in figure 7(b) after zero-field cooling: the magnet current was increased from 0 A to 50 A in 50 s, was kept at 50 A for 3550 s (excited phase), then, was shut down in 50 s, and was kept at 0 A for 3550 s (remnant phase); this cycle was repeated four times. In the following argument, the time origin ($t = 0$ s) is set at the start time of each excited phase or each remnant phase. In order to obtain multipole coefficients, multipole components were normalized by the design value of the dipole component (1.76 mT A$^{-1}$) using the average measured current in each excited phase and using the average measured current in the preceding excited phase of each remnant phase.

The temporal evolution of the normalized dipole and sextupole components at the excited phases is shown in figures 10(a) and (b). Those at the remnant phases are shown in figures 11(a) and (b), respectively.

Firstly, when looking at the excited phase of the first cycle in figures 10(a) and (b), the dipole coefficient varied more than the sextupole coefficient: the former increased by $7.4 \times 10^{-4}$ and the latter increased by $1.8 \times 10^{-4}$ in 3055 s (about 50 min). The temporal variation in the magnetization...
in the coated conductors should affect the field stability, but, it influences more remarkably the dipole coefficient than the sextupole coefficient. As explained in section 2, the current at any arbitrary point generates multipole components in different ways, and their generation depends on the location of the current. Therefore, we suppose that the relaxation of the magnetization in the coated conductors leads to the temporal evolution of the current distribution in the entire cross-section of the coils and, then, causes the temporal changes of the dipole and sextupole components in different manners.

Secondly, when comparing the excited phases of the first, second, third, and fourth cycles in figures 10(a) and (b), the temporal variation, especially in the dipole, is much more prominent in the first cycle; the dipole coefficient as well as the sextupole coefficient is more stable in the later cycles. Furthermore, both the dipole and sextupole components increase with increasing time of excitation. A similar tendency can be found in the remnant phases in figures 11(a) and (b). In the excited phase of the first cycle, the magnetic flux (current) penetrates into the coated conductors through magnetization relaxation. After it enters the coated conductor, the magnetic flux decays negligibly, and further penetration of the magnetic flux may be difficult. This penetrating magnetic flux contributes to more stable dipole and sextupole components observed in the later cycles.

6.4. Influence of excitation history on dipole and sextupole components

The magnet was excited as shown in figure 7(c) after zero-field cooling: the cycle of 30 A, 1 h (3550 s) excited phase and 1 h remnant phase was repeated four times; then, the current was kept at 50 A for 1 h then set to 0 A for 1 h; finally, the cycle of a 30 A, 1 h exited phase and a 1 h remnant phase was repeated an additional four times. The temporal evolution of the normalized dipole and sextupole coefficients at the excited phases are shown in figures 12(a) and (b), respectively. Those at the remnant phases are shown in figures 13(a) and (b), respectively. Remarkable differences can be observed before and after the 50 A excitation. In other words, a large magnetization induced by the 50 A excitation remained afterward.

7. Numerical analyses and discussion

In section 6, we used various suppositions to explain the experimental results. To clarify the temporal behaviours of the dipole and sextupole coefficients, including the influence of the excitation history, numerical electromagnetic field analyses were carried out.
7.1. Determination of critical current density—magnetic field characteristic

Even if the critical current $I_c$ of a coated conductor measured at an applied magnetic field $B$ is divided by the superconductor cross-section, it does not provide us the critical current density $J_c(B)$, because the measured $I_c$ is affected by the self magnetic field. Therefore, to suppress the self-field effect by reducing the current, we prepared a 76 $\mu$m wide bridged sample from a coated conductor which is similar to those of the coils, measured its $I_c(B_{\perp})$, and obtained $J_c(B_{\perp})$ by dividing $I_c(B_{\perp})$ with the superconductor cross-section as shown in figure 14. $B_0$ in equation (26) was found to be 0.1 T by fitting equation (26) to the measured $J_c(B_{\perp})$ in figure 14. The $n$ in equation (24) was found to be 28 by fitting this equation to the measured $E-J$ characteristic of the same bridged sample. It should be noted that the fit was made in the $1.8 \times 10^{-5}$ V m$^{-1}$–$2.9 \times 10^{-4}$ V m$^{-1}$ range of the electric field. Finally, using various values of $J_c0$ in equation (26), the critical currents of a coated conductor at the self-field condition $I_c$ were calculated by numerical electromagnetic field analysis for a gradually increasing transport current: the measured $I_c$ of 285 A could be reproduced with $J_c0 = 2.78 \times 10^{10}$ A m$^{-2}$. The parameters listed in table 4 were used in the numerical electromagnetic field analyses reported in this paper. It should be noted that 285 A divided by the superconductor cross-section does not agree with $2.78 \times 10^{10}$ A m$^{-2}$.

7.2. Hysteresis loops of dipole and sextupole components

A numerical electromagnetic field analysis was carried out when changing the magnet current with the rate of $1 \text{ A s}^{-1}$: $0 \text{ A} \rightarrow +50 \text{ A} \rightarrow 0 \text{ A} \rightarrow -50 \text{ A} \rightarrow 0 \text{ A} \rightarrow +50 \text{ A}$. The hysteresis loops of the dipole and sextupole components obtained from the calculated magnetic field are shown in figures 15(a) and (b), in which data were processed similarly to figures 9(a) and (b) in order to improve legibility. The calculated hysteresis loops of the dipole and sextupole components reasonably agree with the measured ones.

If the changing rate of the current is varied, the induced electric field inside the superconductor should vary, and, then, the current density can vary based on the assumed $E-J$ characteristic. The variation in the current density might lead to the variation in the magnetization. However, even though the changing rate of the magnet current was varied from $0.01 \text{ A s}^{-1}$ to $100 \text{ A s}^{-1}$ in the analyses, the difference in hysteresis loops was not remarkable. Little variation in
current density caused by a steep power law $E-J$ curve ($n = 28$) should be the reason for the small difference among the hysteresis loops.

7.3. Temporal evolution of dipole and sextupole components

A numerical electromagnetic field analysis was carried out when changing the magnet current as shown in figure 7(b). The time-dependent magnetic flux, whose schematic distribution is shown in figure 16, induces a magnetization current.

Figure 17 shows the temporal evolution of the current distribution in the one-fourth cross-section of the coils (see figure 6) during the excited phase of the first cycle. Figures 18 and 19 show the temporal evolution of lateral current distributions in coated conductors at various locations in the excited phase and the remnant phase of the first cycle, respectively. Here, the negative (−) lateral position in a coated conductor is the outer side of the inner racetrack coil, and the positive (+) lateral position in a coated conductor is the outer side of the outer racetrack coil. In figure 18, the magnetic flux penetrates completely in the first turn (the innermost turn) and in the 83rd turn (the outer most turn) of the outer racetrack coil, whereas a non-penetrated (current free) region remains in the 42nd turn of both the outer and inner racetrack coils. A finite (not zero) current density in the non-penetrated region was caused by the finite maximum equivalent conductivity of $10^{20}$ S m$^{-1}$. In figure 19, even though coated conductors do not carry any transport current, currents remain inside them (remnant magnetizations); the current integrated over the entire width of each conductor is zero.

The temporal evolution of the current distributions shown in these figures results in the temporal evolution of the calculated dipole and sextupole coefficients in the excited and remnant phases shown in figures 20 and 21, respectively. In order to obtain multipole coefficients in these figures, multipole components were normalized by the dipole component assuming a uniform current distribution (1.63 mT A$^{-1}$, calculated by a two-dimensional model using equation (6)), where the current in the preceding excited phase was used for each remnant phase. Figures 20 and 21 qualitatively agree with the measured values shown in figures 10 and 11 with respect to the following three points, which were observed experimentally and discussed in 6.3:

1. The dipole coefficient varies with time more notably than the sextupole coefficient. It is most pronounced in the excited phase of the first cycle.
2. The temporal variations are most pronounced in the excited phase of the first cycle; the dipole coefficient as

Figure 19. Temporal evolution of lateral current distributions in coated conductors at various locations in the remnant phase of the first cycle in repeated 50 A excitations.
well as the sextupole coefficient is more stable in the later cycles.
(3) Both dipole and sextupole components increase with increasing time of excitation.

Because the current at any arbitrary point in the cross-section of the magnet generates the dipole and sextupole components differently, (1) could be the consequence of the variation in the current distribution in the coil cross-section as shown in figure 17. In figure 22, the temporal evolution of the lateral current distribution in the first turn (the innermost turn) of the outer racetrack coil are compared for the excited phase in the first and the second cycles. The current distribution in the first cycle varies more significantly than in the second cycle: the magnetization current relaxes (decays) considerably between $t=0$ s and 1750 s in the first cycle, whereas the current distribution is relatively stable in the second cycle. As assumed in explaining the experimental results in 6.3, after the magnetization current relaxes, the current hardly changes as shown in figure 22(b), and the magnetic field becomes stable as pointed out in (2). A spike at $t=0$ s in figure 22(b) was caused by the hysteresis in the current distribution (see appendix B). During the course of the repeated cycles, the magnetization current should relax (decay) slowly. This should lead to (3). It should be noted that the current distributions stay non-uniform even in later excitation cycles, as shown in figure 22: magnetization currents hardly decay. Therefore, stable dipole and sextupole coefficients in the later cycles do not mean

Figure 20. Calculated temporal evolution of multipole coefficients at the excited phases in repeated 50 A excitations: (a) dipole coefficient; (b) sextupole coefficient. EP represents excited phase. In order to obtain multipole coefficients, multipole components were normalized by the dipole component assuming a uniform current distribution (1.63 mT A$^{-1}$, calculated by a two-dimensional model using equation (6)).

Figure 21. Calculated temporal evolution of multipole coefficients in the remnant phases in repeated 50 A excitations: (a) dipole coefficient; (b) sextupole coefficient. RP represents remnant phase. In order to obtain multipole coefficients, multipole components were normalized by the dipole component assuming a uniform current distribution (1.63 mT A$^{-1}$, calculated by a two-dimensional model using equation (6)), where the current in the preceding excited phase was used for each remnant phase.

Figure 22. Temporal evolution of the lateral current distribution in the first turn (the innermost turn) of the outer racetrack coil in repeated 50 A excitations: (a) the excited phase of the first cycle; (b) the excited phase of the second cycle.
7.4. Influence of excitation history on dipole and sextupole components

A numerical electromagnetic field analysis was carried out with a varying magnet current as shown in figure 7(c). The temporal evolution of the calculated dipole and sextupole coefficients in the excited phase and in the remnant phase are shown in figures 23 and 24, respectively. The difference before and after the 50 A excitation observed experimentally (figures 12 and 13) were reproduced in the analysis. In figure 25, the lateral current distribution in the first turn (the innermost turn) of the outer racetrack coil is shown for the 30 A excited phases before (4th-EP) and after (6th-EP) the 50 A excitation, as well as the 50 A excited phase (5th-EP) and the 50 A remnant phase (5th-RP). The current (magnetic flux) penetration in the 50 A excited phase substantially influences the current distribution afterwards: the negative magnetization current which penetrated near the lateral position between $-1$ mm and $+1$ mm in the 50 A excited phase (5th-EP) remains in the 50 A remnant phase (5th-RP), and, thereafter, in the 30 A excited phase (6th-EP); this remaining magnetization current causes a difference between the current distributions in the 30 A excited phases before and after the 50 A excitation. This difference can explain the discrepancy in the dipole and sextupole components in the 30 A excited phases before and after the 50 A excitation.
8. Conclusion

The decay of the magnetization in coated conductors caused temporal variation in the current distribution in the cross-section of a dipole magnet. It led to disparate temporal behaviours in the dipole and sextupole components of the generated magnetic field, because the current density at any arbitrary point in the cross-section of the magnet generates multipole components of the magnetic field differently. Such temporal behaviours, which depend on the order of the multipole and are related to the variation in the current distribution in a magnet cross-section, are particular to accelerator magnets such as dipole magnets. Once a magnetic flux penetrates into coated conductors, it scantly decays. Therefore, the magnitudes and the temporal behaviours of multipole components depend on the excitation history of a magnet. An excitation with a larger current substantially influences multipole components in later, smaller-current excitations, because the magnetization current induced in the large current excitation is persistent. The reasonable agreement between the measured and calculated dipole and sextupole components suggests that we might be able to predict the influence of the magnetization in order to correct the magnetic field, even if its influence cannot be eliminated in accelerator magnets.

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Appendix A

At first, we derive equations (14), (15), and (16) for coil C. The coil rotates in the magnetic field expressed with equation (3). The magnetic flux density normal to the coil face $B_\theta$ is given as

$$B_\theta(\theta) = \text{Re} \left[ \sum_{n=1}^{\infty} (B_n + jA_n) \left( \frac{x + iy}{n} \right)^{n-1} e^{i\omega t} \right]$$

$$= \text{Re} \left[ \sum_{n=1}^{\infty} (B_n + jA_n) \frac{r}{n} \right]^{n-1}. \tag{A.1}$$

when the orientation of the coil face is $\theta$. The magnetic flux linked to the $i$th turn of the coil can be calculated as

$$\Phi_i = \int_0^{l_i} \int_{\pi}^{0} B_\theta(r, \theta) dr dz$$

$$= \text{Re} \left[ \sum_{n=1}^{\infty} (B_n + jA_n) \frac{1}{mr^n} \right] \times \left\{ \left( d + \frac{w_i}{2} \right)^{n} - \left( d - \frac{w_i}{2} \right)^{n} \right\}. \tag{A.2}$$

Then, the magnetic flux linked to the entire coil is given as

$$\Phi = \text{Re} \left[ \sum_{n=1}^{\infty} (B_n + jA_n) \sum_{i=1}^{N} \frac{1}{mr^n} \right] \times \left\{ \left( d + \frac{w_i}{2} \right)^{n} - \left( d - \frac{w_i}{2} \right)^{n} \right\}. \tag{A.3}$$

From this magnetic flux, we can calculate the output voltage of the pick-up coil as

$$V = -\frac{d\Phi(\omega t + \phi_0)}{dt},$$

$$= -\text{Re} \left[ \sum_{n=1}^{\infty} (B_n + iA_n) \right. \times \left\{ \left( d + \frac{w_i}{2} \right)^{n} - \left( d - \frac{w_i}{2} \right)^{n} \right\} \right.$$
\[ \sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{1}{nr_0^{n-1}} \left( d + \frac{w_j}{2} \right)^n - \left( d - \frac{w_j}{2} \right)^n \times \begin{cases} A_n \cos n(\omega t + \phi_0) + B_n \sin n(\omega t + \phi_0) \end{cases} \]

\[ = \sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{1}{nr_0^{n-1}} \left( d + \frac{w_j}{2} \right)^n - \left( d - \frac{w_j}{2} \right)^n \times \begin{cases} A_n \cos n\phi_0 + B_n \sin n\phi_0 \end{cases} \]

where

\[ G \equiv \sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{1}{nr_0^{n-1}} \left( d + \frac{w_j}{2} \right)^n - \left( d - \frac{w_j}{2} \right)^n \times \begin{cases} \cos n\omega t + \left( -A_n \sin n\phi_0 + B_n \cos n\phi_0 \right) \times \sin n\omega t \end{cases} \]

By comparing each term of equation (A.4) with each term of equation (8), \( B_n \) and \( A_n \) are given as

\[ B_n = \frac{1}{G} \left( u_n \sin n\phi_0 + v_n \cos n\phi_0 \right), \quad (A.6) \]

\[ A_n = \frac{1}{G} \left( u_n \cos n\phi_0 - v_n \sin n\phi_0 \right). \quad (A.7) \]

Equations (9), (10), and (11) for the series-connected coil A and coil B can be derived in a similar way, considering that the output voltage of coil A is same as that of coil C and that the output voltage of coil B can be given by substituting 0 for \( d \) in equations (A.4) and (A.5).

Appendix B

A spike at \( t = 0 \) s in figure 22(b) was caused by the hysteresis in the current distribution. Figure A.1 shows the temporal evolution of the lateral current distribution in the first turn (the innermost turn) of the outer racetrack coil during current ramping up: from \( t = -50 \) s (remnant phase) to \( t = 0 \) s (just after the current ramped up). In the course of the current ramping up from the remnant phase, the current density reversed from positive direction to the negative direction in the positive lateral position. The spike is the vestige of the positive-current-density part.

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