RADIAL PROFILE AND LOGNORMAL FLUCTUATIONS OF THE INTRACLUSTER MEDIUM AS THE ORIGIN OF SYSTEMATIC BIAS IN SPECTROSCOPIC TEMPERATURE

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The origin of the recently reported systematic bias in the spectroscopic temperature of galaxy clusters is investigated using a cosmological hydrodynamic simulation. We find that the local inhomogeneities of the gas temperature and density, after being corrected for their global radial profiles, have a nearly universal distribution that resembles a lognormal function. Based on this lognormal approximation for the fluctuations in the intracluster medium, we develop an analytical model that explains the bias in the spectroscopic temperature. We conclude that the multiphase nature of the intracluster medium, due not only to the radial profiles but also to the local inhomogeneities, plays an essential role in producing the systematic bias.

Subject headings: cosmology: observations — galaxies: clusters: general — X-rays: galaxies

1. INTRODUCTION

Recent progress in both numerical simulations and observations has improved physical modeling of galaxy clusters beyond the simple isothermal and spherical approximation for a variety of astrophysical and cosmological applications: departures from an isothermal distribution have been discussed in the context of the Sunyaev-Zel’dovich effect (Inagaki et al. 1995; Yoshikawa et al. 1998; Yoshikawa & Suto 1999), the empirical \( \beta \)-model profile has been replaced with one based on the NFW dark matter density profile (Navarro et al. 1997; Makino et al. 1998; Suto et al. 1998), and a physical model for the origin of the triaxial density profile has been proposed (Lee et al. 2005).

Despite this extensive list of previous studies, no physical model has been proposed for the statistical nature of the underlying inhomogeneities in the intracluster medium (ICM). Given the high spatial resolution achieved in both observations and simulations, such modeling should play a vital role in improving our understanding of galaxy clusters, which we attempt to do in this paper.

The temperature of the ICM is one of the most important of the quantities that characterize a cluster. In X-ray observations, the spectroscopic temperature, \( T_{\text{spec}} \), is estimated by fitting the thermal continuum and the emission lines of the spectrum. In the presence of inhomogeneities in the ICM, the temperature so measured is inevitably an averaged quantity over a finite sky area and the line of sight. It has been conventionally assumed that \( T_{\text{spec}} \) is approximately equal to the emission-weighted temperature,

\[
T_{\text{ew}} = \frac{\int n^2 \Lambda(T) \, dV}{\int n^2 \Lambda(T) \, dV},
\]

where \( n \) is the gas number density, \( T \) is the gas temperature, and \( \Lambda \) is the cooling function. Mazzotta et al. (2004), however, have pointed out that \( T_{\text{spec}} \) is systematically lower than \( T_{\text{ew}} \). Those authors proposed an alternative definition for the average, spectroscopic-like temperature, as

\[
T_d = \frac{\int n^2 T^{a-1/2} \, dV}{\int n^2 T^{a-3/2} \, dV}.
\]

They find that \( T_d \) with \( a = 0.75 \) reproduces \( T_{\text{spec}} \) within a few percent for simulated clusters hotter than a few keV, assuming a Chandra or XMM-Newton detector response function. Rasia et al. (2005) performed a more systematic study of the relation between \( T_{\text{ew}} \) and \( T_d \) using a sample of clusters from smoothed particle hydrodynamics (SPH) simulations and concluded that \( T_d \approx 0.7 T_{\text{ew}} \). Vikhlinin (2006) provides a useful numerical routine to compute \( T_d \) down to ICM temperatures of \( \sim 0.5 \) keV with arbitrary metallicity. It should be noted that \( T_{\text{ew}} \) is not directly observable, although it is easily obtained from simulations.

The above bias in the cluster temperature should be properly taken into account when confronting observational data with theory, for example, in cosmological studies. As noted by Rasia et al. (2005), it could result in an offset in the mass-temperature relation for galaxy clusters. Shimizu et al. (2006) studied the impact of this bias on estimates of \( \sigma_8 \), the mass fluctuation amplitude at \( 8 \) h\(^{-1}\) Mpc, performing a statistical analysis using the largest X-ray cluster sample and finding that \( \sigma_8 \approx 0.76 \pm 0.01 + 0.50(1 - \alpha_{bf}) \), where \( \alpha_{bf} = T_{\text{spec}}/T_{\text{ew}} \). A systematic difference of \( T_{\text{spec}} \approx 0.7 T_{\text{ew}} \) can thus shift \( \sigma_8 \) by \( \sim 0.15 \).

In this paper, we aim to explore the origin of the bias in the spectroscopic temperature by studying in detail the nature of inhomogeneities in the ICM. We investigate both the large-scale gradient and the small-scale variations of the gas density and temperature, based on a cosmological hydrodynamic simulation. Having found that the small-scale density and temperature fluctuations approximately follow lognormal distributions, we construct an analytical model for the local ICM inhomogeneities that can simultaneously explain the systematic bias.

The plan of this paper is as follows: In § 2, we describe our simulation data, construct mock spectra, and quantitatively compare the spectroscopic temperature with the emission-weighted temperature and the spectroscopic-like temperature suggested by Mazzotta...
et al. (2004). In § 3, we propose our analytical model for the inhomogeneities in the ICM. In § 4, we test the model against the results of the simulation. Finally, we summarize our conclusions in § 5. Throughout the paper, temperatures are measured in units of keV.

2. THE BIAS IN THE SPECTROSCOPIC TEMPERATURE

2.1. Cosmological Hydrodynamic Simulation

The results presented in this paper have been obtained by using the final output from an SPH simulation of the local universe performed by Dolag et al. (2005). The initial conditions were similar to those adopted by Mathis et al. (2002) in their study (based on a pure N-body simulation) of structure formation in the local universe. The simulation assumes a spatially flat ΛCDM universe with a present-day matter density parameter \( \Omega_{\text{m0}} = 0.3 \), a dimensionless Hubble parameter \( h = H_0/(100 \, \text{km s}^{-1} \, \text{Mpc}^{-1}) = 0.7 \), an rms density fluctuation amplitude \( \sigma_8 = 0.9 \), and a baryon density parameter \( \Omega_b = 0.04 \). The numbers of dark matter and SPH particles are \( \sim 50 \) million each within a high-resolution sphere of radius \( \sim 110 \) Mpc, which is embedded in a periodic box \(~ 343 \) Mpc on a side that is filled with nearly 7 million low-resolution dark matter particles. The simulation is designed to reproduce the matter distribution of the local universe by adopting initial conditions based on the IRAS (Infrared Astronomical Satellite) galaxy distribution smoothed on a scale of 7 Mpc (see Mathis et al. 2002 for details).

The run was carried out with GADGET-2 (Springel 2005), a new version of the parallel TreeSPH simulation code GADGET (Springel et al. 2001). The code uses an entropy-conserving formulation of SPH (Springel & Hernquist 2002) and allows the treatment of radiative cooling, heating by a UV background, and star formation and feedback processes. The latter is based on a subresolution model for the multiphase structure of the interstellar medium (Springel & Hernquist 2003); in short, each SPH particle is assumed to represent a two-phase fluid consisting of cold clouds and ambient hot gas.

The code also follows the pattern of metal production from the past history of cosmic star formation (Tornatore et al. 2004). This is done by computing the contributions from both Type II and Type Ia supernovae; energy feedback and metals are released gradually over time, according to the appropriate lifetimes of the different stellar populations. This treatment also includes in a self-consistent way the dependence of gas cooling on the local metallicity. The feedback scheme assumes a Salpeter initial mass function (Salpeter 1955), and its parameters were fixed to yield a wind velocity of \( \approx 480 \) km s\(^{-1}\). In a typical massive cluster, the Type II and Ia supernovae add to the ICM, as feedback, \( \approx 2 \) keV per particle in a Hubble time (assuming a cosmological mixture of H and He); \( \approx 25\% \) of this energy goes into winds. A more detailed discussion of cluster properties and metal distribution within the ICM that emerge from simulations including the metal enrichment feedback scheme is given by Tornatore et al. (2004). The simulation provides the metallicities of six different species for each SPH particle. Given that the major question we are addressing is not the accurate estimation of \( T_{\text{spec}} \) or \( f_{\text{ad}} \) but the systematic difference between the two, we decided to avoid unnecessary complication and simply to assume a constant metallicity. Therefore, we adopt a constant metallicity of \( 0.3 \) Z\(_{\odot}\) in constructing mock spectra below, and the MEKAL (not VMEKAL) model for the spectral fitting.

The gravitational force resolution (i.e., the comoving softening length) of the simulation was fixed to be 14 kpc (Plummer equivalent), which is comparable to the interparticle separation reached by the SPH particles in the dense centers of our simulated galaxy clusters.

2.2. Mock Spectra of Simulated Clusters

Among the most massive clusters formed within the simulation, we extracted six mock galaxy clusters, contrived to resemble Abell 3627, Hydra, Perseus, Virgo, Coma, and Centaurus, respectively. Table 1 lists the observed and simulated values of the total mass, and the radius of these clusters. In order to specify the degree of bias in our simulated clusters, we create mock spectra and compute \( T_{\text{spec}} \) in the following manner.

First, we extract a \( 3 \) h\(^{-1}\) Mpc cubic region around the center of a simulated cluster and divide it into \( 256^3 \) cells so that the size of each cell is approximately equal to the gravitational softening length mentioned above. The center of each cluster is assigned so that the center of a sphere with radius \( 1 \) h\(^{-1}\) Mpc lies at the center of mass of dark matter and baryons within the sphere.

The gas density and temperature of each mesh point (labeled by \( I \)) are calculated using the SPH particles as

\[
\rho_I = \sum_{j=1}^{N_{\text{gas}}} m_j W(|r_I - r_j|, h_i),
\]

\[
T_I = \sum_{j=1}^{N_{\text{gas}}} m_j T_j W(|r_I - r_j|, h_i),
\]

where \( r_I \) is the position of the mesh point, \( W \) denotes the smoothing kernel, and \( m_j, r_j, h_i, T_i, \) and \( \rho_i \) are the mass, position, smoothing length, temperature, and density associated with the hot phase of the \( j \)th SPH particle, respectively. We adopt the smoothing kernel

\[
W(|r_I - r_j|, h_i) = \frac{1}{\pi h_i^3} \begin{cases} 
1 - \frac{3}{2} u^2 + \frac{3}{2} u^4, & \text{if } 0 \leq u \leq 1,

\frac{1}{4} (2 - u)^3, & \text{if } 1 \leq u \leq 2,

0, & \text{otherwise},
\end{cases}
\]

where \( u \equiv |r_I - r_j|/h_i \).

It should be noted that the current implementation of the SPH simulation results in a small fraction of SPH particles having unphysical temperatures and densities. This is shown in the temperature-density scatter plot of Figure 1. The red points correspond to SPH particles that should be sufficiently cooled but have not here because of the limited resolution of the simulation. Thus, if they satisfy the Jeans criterion they should be regarded simply as cold clumps without retaining their hot gas nature (see Fig. 1 and § 2.1 of Yoshikawa et al. 2001). In contrast, the blue points represent the particles that have experienced the cooling catastrophe and have a significant cold gas fraction (higher than 10%). In either case, they are not supposed to contribute to the X-ray emission. Thus, we remove their spurious contribution to the X-ray emission and the temperature estimate for the ICM. Specifically, we follow Borgani et al. (2004) and exclude particles with \( T_I < 3 \times 10^4 \) K and \( \rho_i > 500 \rho_0 \Omega_m \) (red points), where \( \rho_0 \) is the critical density, as well as particles with more than 10% mass fraction in the cold phase (blue points). While the total mass of the excluded particles is very small (\(~1\%) , they occupy a specific region in the \( \rho-T \) plane and leave some spurious signal due to their high density, in particular for the latter group.

Second, we compute the photon flux \( f(E) \) from the mesh points within a radius \( r_{200} \) from the cluster center as

\[
f(E) dE \propto \exp \left[ -\sigma_{\text{gal}}(E) N_{\text{HI}} \right] \times \sum_{i \in r_{200}} \rho_i^2 \left( \frac{4 \pi (1 + z_i)}{m_e}{\frac{X}{m_p}} \right) P_{\text{em}}(T_i, Z, E(1 + z_i)) dE \times (1 + z_i),
\]
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TABLE 1

Properties of the Six Simulated Clusters and Observed Values

| Property                        | A3627 | Hydra | Perseus | Virgo | Coma | Centaurus |
|---------------------------------|-------|-------|---------|-------|------|-----------|
| \(M_{200}(10^{14} \, h^{-1} \, M_\odot)\) | 2.2   | 1.8   | 6.7     | 3.1   | 4.3  | 2.5       |
| \(r_{200} (h^{-1} \, \text{Mpc})\) | 1.1   | 1.0   | 1.6     | 1.2   | 1.4  | 1.1       |
| \(\beta\)                       | 0.69  | 0.72  | 0.65    | 0.59  | 0.73 | 0.73      |
| \(\gamma\)                      | 1.21  | 1.26  | 1.09    | 1.12  | 1.19 | 1.14      |
| \(T_{\text{ew}}\) (keV)         | 4.2   | 4.1   | 5.7     | 3.7   | 6.7  | 4.2       |
| \(T_{\text{ew}}\) (keV)         | 4.0   | 3.8   | 4.7     | 3.2   | 5.9  | 3.9       |
| \(T_{\text{spec}(\text{ideal})}\) (0.5–10.0 keV (keV)) | 4.1   | 3.9   | 5.0     | 3.3   | 6.1  | 3.9       |
| \(T_{\text{spec}(\text{ACIS})}\) (0.5–10.0 keV (keV)) | 4.1   | 3.9   | 5.0     | 3.3   | 6.0  | 4.0       |
| \(T_{\text{spec}(\text{MOS})}\) (0.5–10.0 keV (keV)) | 4.0   | 3.8   | 5.0     | 3.3   | 6.0  | 4.0       |
| \(n_{\text{sim}}\) (meshwise)  | 0.95  | 0.92  | 0.84    | 0.86  | 0.88 | 0.95      |
| \(n_{\text{sim}}\) (particle-wise) | 0.88  | 0.89  | 0.70    | 0.75  | 0.84 | 0.86      |
| \(\mu_{\text{RF}}\)            | 0.97  | 0.94  | 0.98    | 0.94  | 0.96 | 0.97      |
| \(\sigma_{\text{N_{\text{spec}}}}\) | 0.159 | 0.133 | 0.316   | 0.286 | 0.159 | 0.178     |
| \(\sigma_{\text{N_{\text{spec}}}}\) | 0.240 | 0.180 | 0.518   | 0.446 | 0.434 | 0.239     |

\(a\) From Girardi et al. (1998), except for A3627 (Reiprich & Böhringer 2002).
\(b\) From Ikebe et al. (2002), except for Virgo (Shibata et al. 2001).
\(c\) Dickey & Lockman 1990.

where \(z_{\text{cl}}\) denotes the redshift of the simulated cluster, \(Z\) is the metallicity (we adopt 0.3 \(Z_\odot\)), \(X\) is the hydrogen mass fraction, \(m_p\) is the proton mass, and \(P_{\text{em}}(T, Z, E)\) is the emissivity assuming collisional ionization equilibrium. We calculate \(P_{\text{em}}(T, Z, E)\) using SPEX 2.0. The term \(\exp(-\sigma_\text{gal} N_{\text{HI}})\) represents the Galactic extinction; \(N_{\text{HI}}\) is the column density of hydrogen, and \(\sigma_\text{gal}(E)\) is the absorption cross section of Morrison & McCammon (1983). Since we are interested in the effect due to the distortion of the spectrum, not statistical error, we adopt a long exposure time, with total photon counts \(N = \int_{10^{10} \text{keV}}^{10^{12} \text{keV}} E f(E) dE \sim 500,000\). In this paper, we consider mock observations made with Chandra or XMM-Newton, and thus we neglect the peculiar velocity of the cluster and any turbulent velocity in the ICM because of the insufficient energy resolution of the Chandra ACIS-S3 and XMM-Newton MOS1 detectors.

Finally, the mock observed spectra are created with XSPEC version 12.0. We consider three cases for the detector response, corresponding to (1) perfect response, (2) Chandra ACIS-S3, and (3) XMM-Newton MOS1. In the first case, we also assume no Galactic extinction \((N_{\text{HI}} = 0)\); we refer to this as the “ideal” case. In the second and third cases, we adopt the observed value of \(N_{\text{HI}}\) listed in Table 1 and redistribute the photon counts of the detector channel according to the corresponding RMF (redistribution matrix file) for ACIS-S3 and MOS1 using the rejection method (see, e.g., Press et al. 1992, § 7.3).

Figure 2 illustrates the mock spectra of “Virgo” and “Perseus” using the RMF for ACIS-S3. Unless stated otherwise, we fit the spectra with an absorbed single-temperature MEKAL model (Mewe et al. 1995) in the energy band 0.5–10.0 keV. We define the spectroscopic temperature, \(T_{\text{spec}}\), as the best-fit temperature provided by this procedure. Since the spatial resolution of the current simulation is not sufficient to fully resolve the cooling central regions, a single-temperature model yields a reasonable fit to the mock

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**Fig. 1.** Scatter plot of temperatures and densities of SPH particles. Red and blue points indicate particles with unphysical values, which are removed in computing the X-ray emission.
spectra. For comparison, we also plot the spectra for a single temperature corresponding to the “emission-weighted” value of the mesh points within $r_{200}$:

$$T^{\text{sim}, m}_{\text{ew}} = \frac{\sum_{i \in r_{200}} \rho_i^2 T_i \chi(T_i)}{\sum_{i \in r_{200}} \rho_i^2 \chi(T_i)}.$$  

(7)

We calculate the cooling function $\chi(T)$ using SPEX 2.0, assuming collisional ionization equilibrium, the energy range 0.5–10.0 keV, and a metallicity of $0.3 Z_\odot$. The difference between $T_{\text{spec}}$ (green) and $T^{\text{sim}, m}_{\text{ew}}$ (red) is clearly distinguishable on a spectral basis in the current detectors.

Figure 3 shows the relation between $T_{\text{spec}}$ and $T^{\text{sim}, m}_{\text{ew}}$ for our sample of simulated clusters. It is well represented by a linear relation, $T_{\text{spec}} = k T^{\text{sim}, m}_{\text{ew}} + l$ with fitted values $k = 0.84, l = 0.34$ (ideal), $k = 0.84, l = 0.36$ (ACIS), and $k = 0.85, l = 0.31$ (MOS). In the range of temperatures corresponding to rich clusters, the spectroscopic temperature $T_{\text{spec}}$ is systematically lower than $T^{\text{sim}, m}_{\text{ew}}$ by 10%–20%.

We note that the above bias should depend on the energy band in which $T_{\text{spec}}$ is evaluated. In order to demonstrate this quantitatively, we also list in Table 1 the fitted values of $T_{\text{spec}}$ from the 0.1–2.4 and 2.0–10.0 keV data. Because the exponential tail of the thermal bremsstrahlung spectrum from the hotter components makes a negligible contribution in the softer band, the bias tends to increase and decrease in the softer and harder bands, respectively.

2.3. Spectroscopic-like Temperature

In order to better approximate $T_{\text{spec}}$, Mazzotta et al. (2004) proposed a “spectroscopic-like temperature”; they found that equation (2) with $a = 0.75$ reproduces $T_{\text{spec}}$ in the 0.5–10.0 keV band within a few percent. Throughout this paper, we adopt $a = 0.75$ when we estimate the spectroscopic-like temperature quantitatively. In Figure 3, we also plot this quantity computed from the mesh points within $r_{200}$:

$$T^{\text{sim}, m}_{\text{sl}} = \frac{\sum_{i \in r_{200}} \rho_i^2 T_i^{0.25}}{\sum_{i \in r_{200}} \rho_i^2 T_i^{0.75}}.$$  

(8)

As indicated in the bottom panel, $T^{\text{sim}, m}_{\text{sl}}$ reproduces $T_{\text{spec}}$ within 6% for all the simulated clusters in our sample. Given this agreement, we hereafter use $T^{\text{sim}, m}_{\text{sl}}$ to represent $T_{\text{spec}}$ and we express the bias in the spectroscopic temperature by

$$\kappa_{\text{sim}} = \frac{T^{\text{sim}, m}_{\text{sl}}}{T^{\text{sim}, m}_{\text{ew}}}.$$  

(9)

Table 1 provides $\kappa_{\text{sim}}$ computed meshwise for the six simulated clusters. The range is approximately 0.8–0.9. While $\kappa_{\text{sim}}$ is systematically lower than unity, the value is somewhat higher than...
the result of Rasia et al. (2005), $T_{\text{sl}} \sim 0.7 T_{\text{ew}}$. This is likely due to the different physics incorporated in the simulations and the difference in how $T_{\text{sl}}$ and $T_{\text{ew}}$ are computed from the simulation outputs. The major difference in the physics is the amplitude of the wind velocity employed; Rasia et al. used feedback with a weaker wind at 340 km s$^{-1}$, while our current simulation adopts the higher value of 480 km s$^{-1}$. Because weaker winds cannot remove small cold blobs as effectively, the value of $T_{\text{sl}}/T_{\text{ew}}$ from Rasia et al. (2005) would be expected to be larger.

To show the difference between the temperature computation schemes explicitly, we also list in Table 1 the values of $T_{\text{sim,sl}}/T_{\text{sim,ew}}$ computed using the "particle-wise" definitions from Rasia et al. (2005):

$$T_{\text{sim,ew}} = \frac{\sum_{i \in r_{200}} m_i \rho_i \Lambda(T_i) T_i}{\sum_{i \in r_{200}} m_i \rho_i \Lambda(T_i)}$$

$$T_{\text{sim,sl}} = \frac{\sum_{i \in r_{200}} m_i \rho_i T_i^{0.25}}{\sum_{i \in r_{200}} m_i \rho_i T_i^{0.75}}$$

(see also Borgani et al. 2004). In practice, the emission-weighted and spectroscopic-like temperatures defined in equations (10) and (11) are sensitive to the small number of cold (and dense) SPH particles present, while these particles contribute negligibly in the meshwise definitions (eqs. [7] and [8]). As in Rasia et al. (2005), we removed the SPH particles below a threshold temperature $T_{\text{lim}} = 0.5$ keV to compute $n_{\text{sim}}$ (particle-wise). Table 1 indicates that the particle-wise $n_{\text{sim}}$ tends to be systematically smaller than its meshwise counterpart. Even adopting $T_{\text{lim}} = 0.01$ keV makes the former smaller only by a few percent.

Given the limit of the particle-wise definitions mentioned above, we use the meshwise definitions of the emission-weighted temperature ($T_{\text{sim,ew}}$) and the spectroscopic-like temperature ($T_{\text{sim,sl}}$) given in equations (7) and (8), respectively, in the following sections.

3. ORIGIN OF THE BIAS IN THE SPECTROSCOPIC TEMPERATURE

3.1. Radial Profiles and Lognormal Distribution of Temperature and Density

Having quantified the bias in the temperature of simulated clusters, we can investigate its physical origin in greater detail. Since clusters in general exhibit inhomogeneities over various scales, we begin by segregating the large-scale gradient and the small-scale fluctuations of the gas density and temperature.

For the large-scale gradient, we use the radially averaged profiles of gas temperature and density shown in Figure 4. We divide the simulated clusters into spherical shells of width $67 h^{-1}$ kpc and calculate the average temperature $T(r)$ and density $n(r)$ in each
shell. The density profile \( n(r) \) is fitted to the conventional \( \beta \)-model, given by

\[
n(r) = n_0 \left[ 1 + \left( r/r_c \right)^2 \right]^{-\gamma/2},
\]

where \( n_0 \) is the central density and \( r_c \) is the core radius. We adopt for the temperature profile \( T(r) \) the polytropic form

\[
T(r) = T_0 \left[ n(r)/n_0 \right]^{\gamma-1},
\]

where \( T_0 \) is the temperature at \( r = 0 \) and \( \gamma \) is the polytropic index. The simulated profiles show reasonable agreement with the above models. The best-fit values of \( \beta \) and \( \gamma \) are listed in Table 1. The range of \( \gamma \) is approximately 1.1–1.2.

In addition to their radial gradients, the gas density and temperature have small-scale fluctuations. Figure 5 illustrates the distributions of the gas density and temperature in each radial shell normalized by their averaged quantities, \( \langle T \rangle \) and \( \langle n \rangle \), respectively. Despite some variations among different shells, we find a striking similarity in the overall shape of the distributions. They approximately follow the lognormal distribution

\[
P_{\text{LN}}(\delta_x)d\delta_x = \frac{1}{\sqrt{2\pi}\sigma_{\text{LN},x}} \exp\left( -\frac{\left( \log \delta_x + \frac{1}{2}\sigma_{\text{LN},x}^2 \right)^2}{2\sigma_{\text{LN},x}^2} \right) d\delta_x,
\]

where \( \delta_x \equiv x/(x) \) and \( x \) denotes either \( T \) or \( n \) (\( \delta_T \equiv T/\langle T \rangle \), \( \delta_n \equiv n/\langle n \rangle \)). For simplicity, we neglect the variations among different shells and fit the distribution for the whole cluster within \( r_{200} \) (red solid lines) with the above equation (black dashed lines). The best-fit values of \( \sigma_{\text{LN},T} \) and \( \sigma_{\text{LN},n} \) are listed in Table 1.

The small-scale fluctuations mentioned above are not likely an artifact of the SPH scheme. We have applied a similar analysis to data from grid-based simulations (D. Ryu 2006, private communication) and obtained essentially the same results. Thus, the lognormal nature of the fluctuations is physical, rather than numerical.

### 3.2. Analytical Model

Based on the distributions of gas density and temperature described in § 3.1, we develop an analytical model to describe the contributions of the radial profile (RP) and the local inhomogeneities (LI) to the bias in the spectroscopic temperature. To describe the emission-weighted and spectroscopic-like temperatures in simple form, let us derive a quantity:

\[
A_\alpha = \int dr n(r)^\alpha T(r)^\beta d\Omega [n(r)^2 T(r)^\gamma] dr
\]

where the second equation is for spherical coordinates and \( R \) denotes the maximum radius considered (we adopt \( R = r_{200} \) here). Using this quantity, we can write \( T_\text{sl} \) via equation (2) as

\[
T_\text{sl}^{\text{model}} = A_{3/2}/A_{1/2}.
\]

When the temperature is higher than \( \sim 3 \) keV, the cooling function is dominated by thermal bremsstrahlung: \( \Lambda \propto \sqrt{T} \). We find that substituting \( \sqrt{T} \) for \( \Lambda(T) \) yields only \( \sim 1\% \) difference for the simulated clusters. In the present model, we adopt for simplicity the thermal bremsstrahlung cooling function, \( \Lambda \propto \sqrt{T} \), and then equation (1) reduces to

\[
T_\text{ew}^{\text{model}} = A_{3/2}/A_{1/2}.
\]

When evaluating equation (15), we replace the spatial average with an ensemble average:

\[
\int d\Omega n^2 T^n(r) = 4\pi \int \left[ \langle n(r)^2 \rangle \langle T(r) \rangle \right]^n dr
\]

\[
\times \int \delta_n^2 \delta_T^2 P(\delta_n, \delta_T; r) d\delta_n d\delta_T,
\]

where \( P(\delta_n, \delta_T; r) \) is a joint probability density function at \( r \). Assuming further that the temperature inhomogeneity is uncorrelated with that of density, that is, \( P(\delta_n, \delta_T; r) = P(\delta_n; r)P(\delta_T; r) \), we obtain

\[
A_\alpha = 4\pi \int_0^R dr \frac{r^2 \langle n(r)^2 \rangle \langle T(r) \rangle^{\alpha-n}}{\int_0^\infty \delta_T^2 P(\delta_T; r) d\delta_T}
\]

For the lognormal distribution of the temperature and density fluctuations, the average quantities are expressed as

\[
\int \delta_T^2 P(\delta_T; r) d\delta_T = \exp \left[ \frac{\sigma_{\text{LN},T}^2(r)}{2} \right],
\]

\[
\int \delta_n^2 P(\delta_n; r) d\delta_n = \exp \left[ \sigma_{\text{LN},n}^2(r) \right].
\]

If \( \sigma_{\text{LN},T} \) and \( \sigma_{\text{LN},n} \) are independent of radius \( \sigma_{\text{LN},T}(r) = \sigma_{\text{LN},T} \), \( \sigma_{\text{LN},n}(r) = \sigma_{\text{LN},n} \), equation (19) reduces to

\[
A_\alpha = \exp \left( \frac{(\alpha - 1)}{2} \sigma_{\text{LN},T}^2 \right)
\]

\[
\times 4\pi \int_0^R dr \frac{r^2 \langle n(r)^2 \rangle \langle T(r) \rangle^{\alpha-n}}{\int_0^\infty \delta_T^2 P(\delta_T; r) d\delta_T}.
\]

Using the above results, \( T_\text{sl}^{\text{model}} \) and \( T_\text{ew}^{\text{model}} \) can then expressed as

\[
T_\text{sl}^{\text{model}} = A_{3/2}/A_{1/2} = T_\text{sl}^{\text{RP}} \exp \left[ (\alpha - 1/2) \sigma_{\text{LN},T}^2 \right],
\]

\[
T_\text{ew}^{\text{model}} = A_{3/2}/A_{1/2} = T_\text{ew}^{\text{RP}} \exp \left[ \frac{1}{2} \sigma_{\text{LN},T}^2 \right],
\]

where \( T_\text{sl}^{\text{RP}} \) and \( T_\text{ew}^{\text{RP}} \) are defined as

\[
T_\text{sl}^{\text{RP}} = \int_0^R \frac{r^2 \langle n(r)^2 \rangle \langle T(r) \rangle^{\alpha-1/2}}{\int_0^\infty \delta_T^2 P(\delta_T; r) d\delta_T} dr,
\]

\[
T_\text{ew}^{\text{RP}} = \int_0^R \frac{r^2 \langle n(r)^2 \rangle \langle T(r) \rangle^{3/2}}{\int_0^\infty \delta_T^2 P(\delta_T; r) d\delta_T} dr.
\]

As expected, \( T_\text{sl}^{\text{model}} \) and \( T_\text{ew}^{\text{model}} \) reduce to \( T_\text{sl}^{\text{RP}} \) and \( T_\text{ew}^{\text{RP}} \) in the absence of local inhomogeneities (\( \sigma_{\text{LN},T} = 0 \)). Note that equations (23) and (24) are independent of \( \sigma_{\text{LN},n} \). This holds true as long as the density distribution \( P(\delta_n) \) is independent of \( r \).
Fig. 5.—Distributions of $\delta_T \equiv \langle T/T \rangle$ and $\delta_n \equiv \langle n/n \rangle$. Red solid lines present the distribution throughout the mesh points within $r = r_{200}$. Dashed lines are fits of the lognormal distribution. The remaining solid lines are the distributions in 67 $h^{-1}$ kpc shells in different radial intervals: $r < 335$ $h^{-1}$ kpc (green), $335$ $h^{-1}$ kpc $< r < 670$ $h^{-1}$ kpc (blue), and $r > 675$ $h^{-1}$ kpc (cyan).
The ratio of $T_{\text{sl}}^\text{model}$ to $T_{\text{ew}}^\text{model}$ is now written as

$$\kappa_{\text{model}} \equiv \frac{T_{\text{sl}}^\text{model}}{T_{\text{ew}}^\text{model}} = \kappa_{\text{RP}} \kappa_{\text{LI}},$$

(27)

where $\kappa_{\text{RP}}$ and $\kappa_{\text{LI}}$ denote the bias due to the radial profile and the local inhomogeneities,

$$\kappa_{\text{RP}} = T_{\text{sl}}^\text{RP} / T_{\text{ew}}^\text{RP},$$

(28)

$$\kappa_{\text{LI}} = \exp \left[ (a - 2) \sigma_{\text{sl}}^2 \right],$$

(29)

respectively. Figure 6 shows $\kappa_{\text{LI}}$ for the fiducial value of $a = 0.75$ as a function of $\sigma_{\text{LN},T}$. The range of $\sigma_{\text{LN},T}$ for the simulated clusters, $0.1 < \sigma_{\text{LN},T} < 0.3$ (Table 1), corresponds to $0.9 > \kappa_{\text{LI}} > 0.89$.

In the case of the $\beta$-model density profile (eq. [12]) and the polytropic temperature profile (eq. [13]), $T_{\text{sl}}^\text{RP}$ and $T_{\text{ew}}^\text{RP}$ are expressed as

$$\kappa_{\text{RP}}^\beta = T_{\text{sl}}^\text{RP} / T_{\text{ew}}^\text{RP},$$

(30)

$$\kappa_{\text{RP}}^\beta = \frac{2F_1(\frac{1}{2}, 3; \beta [1 + \frac{1}{3} (\gamma - 1)(a - \frac{1}{2})]; \frac{3}{2}; -R^2/r_c^2) - R^2/r_c^2)}{2F_1(\frac{1}{2}, 3; \beta [1 + \frac{1}{3} (\gamma - 1)(a - \frac{1}{2})]; \frac{3}{2}; -R^2/r_c^2)}},$$

(31)

$$\kappa_{\text{RP}}^\beta = \frac{2F_1(\frac{1}{2}, 3; \beta [1 + \frac{1}{3} (\gamma - 1)]; \frac{3}{2}; -R^2/r_c^2)}{2F_1(\frac{1}{2}, 3; \beta [1 + \frac{1}{3} (\gamma - 1)]; \frac{3}{2}; -R^2/r_c^2)}},$$

(32)

where $2F_1(\alpha, \beta; \gamma; \zeta)$ is the hypergeometric function.

Figure 7 shows $\kappa_{\text{RP}}$ as a function of $\beta$ for various choices of $\gamma$ and $r_c/r_{200}$. Given that a number of observed clusters exhibit a cool core, we also plot the case for a temperature profile of the form

$$T(r) = T_i + (T_b - T_i) \frac{(r/r_c)^\mu}{1 + (r/r_c)^\mu}$$

(33)

(Allen et al. 2001; Kaastra et al. 2004), with $(T_b - T_i)/T_i = 1.5$ and $\mu = 2$. For the range of parameters considered here, $\kappa_{\text{RP}}$ exceeds 0.9. This implies that the bias in the spectroscopic temperature is not fully accounted for by the global temperature and density gradients alone; local inhomogeneities should also make an important contribution to the bias.

4. COMPARISON WITH SIMULATED CLUSTERS

We now examine the extent to which the analytical model described in the previous section explains the bias in the spectroscopic temperature. The departure in the radial density and temperature distributions from the $\beta$-model and the polytropic model results in errors up to 7% in the values of $T_{\text{ew}}^\text{model}$ and $T_{\text{sl}}^\text{RP}$. Since our model can be applied to arbitrary $\langle n \rangle(r)$ and $\langle T \rangle(r)$, we hereafter use for these quantities the radially averaged values calculated directly from the simulation data. We combine them with $\sigma_{\text{LN},T}$ in Table 1 to obtain $T_{\text{model}}^\text{model}$ (eq. [23]) and $T_{\text{model}}^\text{model}$ (eq. [24]).

Figure 8 compares $T_{\text{model}}^\text{model}$ and $T_{\text{model}}^\text{model}$ against $T_{\text{sim},m}^\text{model}$ and $T_{\text{sim},m}^\text{model}$ (eqs. [8] and [7]), respectively. For all clusters except “Perseus,” the model reproduces to within 10% accuracy the temperatures averaged over all the mesh points of the simulated clusters. Given this agreement, we further plot $\kappa_{\text{model}}$ against $\kappa_{\text{sim}}$ in Figure 9. The difference between $\kappa_{\text{sim}}$ and $\kappa_{\text{model}}$ remains within $\sim 10\%$ in all

![Figure 6](image-url)

**Fig. 6.** Shape of $\kappa_{\text{LI}}(\sigma_{\text{LN},T})$, adopting $a = 0.75$. In the cases of $\sigma_{\text{LN},T} = 0.1$ and 0.3, $\kappa_{\text{LI}} \sim 0.99$ and 0.89, respectively (see Table 1).

![Figure 7](image-url)

**Fig. 7.** Bias due to the radial profile, $\kappa_{\text{RP}}$ (eq. [28]), assuming the $\beta$-model and a two-temperature model. We assume that the density profile is given by the $\beta$-model. We consider three temperature profiles: the polytropic model (dotted lines, $\gamma = 1.1$; dashed lines, $\gamma = 1.2$) and a cooling cluster model (solid lines). We assume two cases for $r_c/r_{200}$. One is $r_c/r_{200} = 1/10$ (black), and the other is $r_c/r_{200} = 1/40$ (red).

![Figure 8](image-url)

**Fig. 8.** Emission-weighted and spectroscopic-like temperatures provided by our model and the simulation. The dashed line shows $T_{\text{ew}}^\text{model} - T_{\text{model}}^\text{model}$ or $T_{\text{sl}}^\text{model} - T_{\text{model}}^\text{model}$. Dotted lines show $T_{\text{sim},m}^\text{model} - T_{\text{model}}^\text{model} = \pm 0.1$ or $T_{\text{sim},m}^\text{model} - T_{\text{model}}^\text{model} = \pm 0.01$. In all clusters except “Perseus,” the temperatures from the model reproduce that of the simulation within 10%.
cases. Considering the simplicity of our current model, the agreement is remarkable. In all the clusters, both \( \kappa^{RP} \) and \( \kappa^{LI} \) are greater than \( \kappa_{\text{sim}} \), indicating that their combination is in fact responsible for the major part of the bias in the spectroscopic temperature.

In § 3.2, we assumed that \( n \) and \( T \) are uncorrelated, that is, \( P(\delta_n, \delta_T) = P(\delta_n)P(\delta_T) \). We next examine this assumption in more detail. We take the two clusters, “Hydra” and “Perseus,” that show the best and the worst agreement, respectively, between \( \kappa_{\text{model}} \) and \( \kappa_{\text{sim}} \). Figure 10 shows contours of the joint distribution of \( \delta_T = T(r)/T(r) \) and \( \delta_n = n(r)/n(r) \) for all the mesh points within \( r_{200} \) for these two clusters, together with that expected from the model assuming \( P(\delta_n, \delta_T) = P_{\text{LN}}(\delta_n)P_{\text{LN}}(\delta_T) \). For the lognormal distributions, \( P_{\text{LN}}(\delta_n) \) and \( P_{\text{LN}}(\delta_T) \), we have used the fits shown by the dashed line in Figure 5. The joint distribution agrees well with the model for both cases, while the deviation is somewhat larger in “Perseus,” for which the model yields poorer fits to the underlying temperature and density distribution in Figure 5. If the cluster is spherically symmetric and the ICM is in hydrostatic equilibrium, we expect \( n \) to be correlated with \( T \) as \( n \propto T \). We do not find such correlations in Figure 10.

Although the effect of the correlation is hard to model in general, we can show that it does not change the value of \( \kappa_{\text{model}} \) as long as the joint probability density function follows the bivariate lognormal distribution:

\[
P_{\text{BLN}}(\delta_n, \delta_T) \propto \exp \left( -\frac{A^2 - 2\rho'AB + B^2}{2(1 - \rho^2)} \right) \frac{d\delta_n}{\delta_n} \frac{d\delta_T}{\delta_T},
\]

where

\[
\rho' = \log \left( \frac{\rho}{(\exp \sigma_{\text{LN},n}^2 - 1)^{1/2}(\exp \sigma_{\text{LN},T}^2 - 1)^{1/2} + 1} \right),
\]

\[
A \equiv \log \delta_n + \frac{1}{2} \sigma_{\text{LN},n}^2, \quad B \equiv \log \delta_T + \frac{1}{2} \sigma_{\text{LN},T}^2,
\]

and \( \rho \) is the correlation coefficient between \( n \) and \( T \). Adopting \( \rho = 0 \) yields \( P_{\text{BLN}}(\delta_n, \delta_T) = P_{\text{LN}}(\delta_n)P_{\text{LN}}(\delta_T) \). The marginal probability density function of density, \( \int d\delta_T P_{\text{BLN}}(\delta_n, \delta_T) \), and that
of temperature, \[ \int d\delta_n P_{\text{BLN}}(\delta_n, \delta_T) \] are equal to \( P_{\text{LN}}(\delta_n) \) and \( P_{\text{LN}}(\delta_T) \), respectively. Using \( P_{\text{BLN}}(\delta_n, \delta_T) \), we obtain \( T_{\text{sl}} \) and \( T_{\text{ew}} \) as

\[
T_{\text{sl}} = \frac{1}{n_{\text{SL}}} \int \rho_{\text{SL}} \exp \left( \frac{\dot{a} - \frac{1}{2} \dot{a}^2 \rho_{\text{SL}}^2 + 2 \rho_{\text{SL}} \sigma_{\text{LN,T}} \sigma_{\text{LN,n}}}{\rho_{\text{SL}}} \right) \frac{C_{17}^3}{T_{\text{ew}}},
\]
\[
T_{\text{ew}} = \frac{1}{n_{\text{EW}}} \int \rho_{\text{EW}} \exp \left( \frac{\dot{a}^2 \sigma_{\text{LN,T}}^2 + 2 \rho_{\text{SL}} \sigma_{\text{LN,T}} \sigma_{\text{LN,n}}}{\rho_{\text{EW}}} \right) \frac{C_{138}}{T_{\text{ew}}^3}.
\]

Although both \( T_{\text{sl}} \) and \( T_{\text{ew}} \) increase with the correlation coefficient, \( n_{\text{ew}} \equiv T_{\text{sl}}^2 / T_{\text{ew}}^3 \) remains the same as that given by equation (29).

5. SUMMARY AND CONCLUSIONS

We have explored the origin of the bias in the spectroscopic temperature of simulated galaxy clusters discovered by Mazzotta et al. (2004). Using independent simulation data, we have constructed mock spectra of clusters and confirmed their results: the spectroscopic temperature is systematically lower than the emission-weighted temperature by 10%–20%, and the spectroscopic-like temperature defined by equation (2) approximates the spectroscopic temperature to better than \( \sim 6\% \). In so doing, we have found that the multiphase nature of the intracluster medium is ascribable to two major contributions, the radial density and temperature gradients and the local inhomogeneities around these profiles. More importantly, we have shown for the first time that the probability distribution functions of the local inhomogeneities approximately follow a lognormal distribution. Based on a simple analytical model, we have demonstrated that not only the radial profiles but also the local inhomogeneities are largely responsible for the bias in the cluster temperatures.

We would note that lognormal probability distribution functions for density fields show up in a variety of astrophysical and cosmological problems (e.g., Hubble 1934; Coles & Jones 1991; Wada & Norman 2001; Kayo et al. 2001; Taruya et al. 2002). While it is not clear whether they share any simple physical principle, it is interesting to attempt to look into the possible underlying connection.

In this paper, we have focused on the difference between spectroscopic (or spectroscopic-like) and emission-weighted temperatures, which is most relevant to X-ray spectral analysis. Another useful quantity is the mass-weighted temperature defined by

\[
T_{\text{mw}} = \frac{\int nT dV}{\int n dV}.
\]

This is related to the cluster mass more directly (see, e.g., Mathiesen & Evrard 2001; Nagai et al. 2007). The mass-weighted temperature is highly sensitive to the radial density and temperature profiles, while it is little affected by the local inhomogeneities. Though challenging, it will be “observable” either through high-resolution X-ray spectroscopic observations (Nagai et al. 2007) or a combination of lower resolution X-ray spectroscopy and Sunyaev-Zeldovich imaging observations (Komatsu et al. 1999, 2001; Kitayama et al. 2004). We will discuss the usefulness of this quantity and the implications for future observations in a forthcoming paper.

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