The Assessment of a Magnetizing-Current Inrush of a Power Transformer

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Abstract. The study aims to propose an analytical tool for determining the parameters of the power transformer magnetizing inrush current caused by geomagnetically induced currents flowing through high-voltage windings with a grounded neutral under the impact of geomagnetic disturbances on the power grid. The analytical equations for the instantaneous magnetizing current under geomagnetic disturbances were obtained by mathematical model of magnetizing branch for a shell-type power transformer. A model based on a magnetization characteristics piecewise-linear approximation for the electrical steel. The magnetizing inrush current amplitude and duration it was found depends on the intensity of geomagnetic disturbances and in cope-link with the dynamics of the power transformer core saturation transient process were determined the changes in the magnetizing inrush current amplitude and duration under geomagnetic disturbances. The magnetizing inrush current amplitude it was found may reach the level of short-circuit current periodic component at the point of power transformer grid connection. The results were verify by comparing the design and experimental values of the magnetizing inrush current amplitude. The advantages of proposed mathematical model shown with justifying the analogy between core saturation under connecting of power transformer to a grid and under geomagnetically induced currents exposed. The piecewise-linear approximation of power transformer magnetization characteristic, allow to obtain the amplitude value of magnetizing inrush current caused by geomagnetically induced currents with an accuracy of 6% and can be used with power grid steady state and transient simulation under geomagnetic disturbances.

1. Introduction

The cores of modern power transformers are arranged with minimum non-magnetic gaps due to new stacking technologies, of which the Step Lap stacking scheme is the most perfect one [1]. The operational induction in power transformer limbs is usually chosen within 1.6 and 1.7 T [2,3,4]. The saturation induction of almost all modern electrical steel grades is close to 2.0 T at a magnetic field strength of 5,000 A/m and more [5]. These factors drastically reduce the no-load current to values not exceeding 0.5-1.0 % depending on the rated power and ensure running all operating modes from no-load to rated load without the core saturation. However, when the power grid voltage exceeds the rated value only to the level prescribed by the regulatory [6,7], the core may be overexcited. E.g., an increase in the power grid voltage by 10 % above the rated value causes more than a two-fold increase in the no-load current of power transformers, which serves as an indicator of the incipient core saturation [4,8].

Abnormal modes are possible, which are accompanied by the occurrence of a quasi-direct magnetic flux component causing the core saturation and a multi-fold increase in the magnetizing current up to
the power transformer’s rated current level and even to that commensurate with short-circuit currents. In what follows, quasi-direct is understood as the magnetic flux component, including the aperiodic one, which remains virtually unchanged over one or several voltage cycles of the power grid. The quasi-direct magnetic flux component may occur in the transient process of connecting the power transformer to the power grid, as well as when the power transformer is operating in a steady-state no-load or on-load mode under geomagnetic disturbances.

In the first case, the quasi-direct is the aperiodic magnetic flux component, which decays for a long time, for tens and even hundreds of the power grid voltage cycles, and the longer, the greater the power transformer rated power \([1,9]\). In the second case, the quasi-direct magnetic flux component may be kept by geomagnetic disturbances for several hours and even days \([10,11,12,13,14,15]\). In both cases, the quasi-direct magnetic flux component causes the core saturation and the emergence of significant magnetizing inrush current. It is known that when a power transformer is turned on, the magnetizing inrush currents may be perceived as short-circuit ones, creating a threat of damage to the winding \([1,9,16]\). The much longer magnetizing inrush currents caused by geomagnetic disturbances may pose an equally grave threat. The most susceptible to geomagnetic disturbances are single-phase shell-type and three-phase shell-core power transformers (usually with a rated power of more than \(80-100\) MVA) \([15]\). Thus, the problem of estimating the magnetizing current parameters, considering the factor of geomagnetic disturbances, is relevant for shell-type and shell-core power transformers and solved in this study.

2. No-load current parameters of power transformers in the absence of geomagnetic disturbances

According to \([17]\), acceptance tests of power transformers include measurements of idling losses and current to be performed at the rated voltage and several points within the range of 80 to 110 % of the rated value. The total number of measuring points should be at least five, including the rated value. In this case, the effective idling current value is measured. The result is the source, datasheet value, which is further used as a reference one in similar measurements during operation to diagnose the technical state of the core to detect possible defects. By agreement between the manufacturer and the customer, the harmonic composition of the no-load current can be measured, which is expressed as a percentage of the fundamental harmonic. Regulatory documents do not regulate the quantitative values of the no-load current harmonics.

Table 1 shows the rated and open-circuit parameters for a wide range of power transformers with a rated line voltage of high voltage windings \((U_r)\) within 110 to 500 kV and a rated power \((S_r)\) within 25 to 630 MVA.

| Parameter | Unit | Unit value |
|-----------|------|------------|
| \(U_r\)   | kV   | 110 220 330 500 |
| \(S_r\)   | MVA  | 25…250 40…630 63…630 250 400 630 |
| \(I_r\)   | A    | 131…1,312.5 105…1,653 110…1,100 275 440 693 |
| \(I_{no-load}%\) | %   | 0.7…0.5 0.35 0.7…0.35 0.45 0.4 0.35 |
| \(I_{no-load}\) | A   | 0.92…6.56 0.945…5.8 0.77…3.86 1.24 1.76 2.43 |
| \(I_{no-load(a)}\) | A   | 0.887…6.3 0.917…5.4 0.68…3.66 1.15 1.63 2.24 |

In Table 1, the following designations are adopted: \(I_r\) is the rated current of the power transformer HV windings; \(I_{no-load}%\), \(I_{no-load}\) is the no-load current of the power transformer, expressed as a percentage of the rated current and in the actual values, respectively; \(I_\mu\) is the magnetizing current of the power transformer; \(I_{no-load(a)}\) is the active component of no-load current of the power transformer in the actual values.
The data in Table 1 show that the effective idling current of power transformers, expressed in the actual values, is within (0.77–6.56) A, and the active idling current component can be neglected, and further it can be assumed that \( I_{\text{no-load}} \approx I_0 \).

Even in the absence of geomagnetically induced currents (GIC), the actual magnetizing current curve differs significantly from the sinusoid due to the odd harmonics cumulating noticeably this curve. Therefore, to determine the magnetizing current amplitude, the equation \( I_{\mu(m)} = k_A \cdot I_{\text{no-load}} \) should be used (here \( k_A \) is the amplitude factor of the non-sinusoidal magnetizing current curve).

In the absence of experimental data for a specific power transformer, it can be assumed that at the rated voltage, the most significant magnetizing current harmonics have the following quantitative relationships [4]:

\[
I_{m(3)} \approx 0.5 \cdot I_{m(1)}; \quad I_{m(5)} \approx 0.1 \cdot I_{m(1)}; \quad I_{m(7)} \approx 0.02 \cdot I_{m(1)}; \quad I_{m(9)} \approx 0.01 \cdot I_{m(1)}; \quad I_{m(11)} \approx 0.005 \cdot I_{m(1)}.
\]

With the above harmonic composition of the magnetizing current, the amplitude factor takes a value \( k_A \approx 2.05 \).

3. Basic assumptions for simulating the power transformer core saturation

A single-phase shell-type power transformer is further considered, for which the below features are valid:

- the core limb cross-section is equal to the total cross-section of the side yokes,
- concentric arrangement of windings,
- the winding thickness is many times less than the winding height (windings are thin).
- The listed design features valid for most large transformers allow building an idealized magnetic field pattern in the shell core at saturation:
  - the magnetic field induction reaches the saturation at all core points simultaneously and, therefore, the limb and the side yokes, i.e., the entire core is saturated coincidently,
  - the magnetic induction vector in electrical steel retains its direction after the core saturation,
  - concentric windings are cylindrical surfaces that do not pass magnetic fluxes through.

Under these assumptions, after the core saturation, the magnetic field is localized in the area determined by the core and winding dimensions. The idealized magnetic field pattern is based on magnetic flux tubes parallel to the axis of the limb and side yokes.

4. Power transformer magnetizing-current inrush during the grid connecting

In the worst case of turning the power transformer on at the moment when the grid voltage crosses zero, the maximum aperiodic component of the magnetic flux occurs, which, when added to the residual induction magnetic flux, causes technical saturation of the core and the extreme magnetizing inrush current amplitude. According to [18], the technical saturation area is characterized by saturation induction \( B_S \), the value of which depends on the percentage of silicon \( C_{\text{Si}} \% \) in the electrical steel composition and is determined by the empirical relationship [19] \( B_S = 2.16 - 0.048 \cdot C_{\text{Si}} \% \), [T].

The above equation allows taking \( B_S = 2.0 \) T as the design saturation induction value in the absence of data on the grade of electrical steel used for the manufacture of the power transformer core [9,20,21].

To consider the technical saturation of the core when estimating the magnetizing inrush current, the following assumptions are usually adopted [1,9,20,21]:

- the main electrical steel magnetization curve is replaced by a piecewise-linear one, in which the relative differential magnetic permeability value is assumed to be infinitely large (\( \mu_D = \infty \)) before saturation and equal to unity (\( \mu_D = 1 \)) after saturation (Figure 1a),
- hysteresis phenomena and eddy currents are absent.

The actual magnetization curve of modern high-quality steel grades differs significantly from the simplest piecewise-linear approximation shown in Figure 1a only in the heel region and, accordingly, a very narrow range of magnetic induction \( B = (0.95 ... 1.03) \cdot B_S \) [9].
Figure 1. Piecewise-Linear Idealization of the Main Electrical Steel Magnetization Curve (a) and a Simplified Equivalent Circuit (b) for Calculating the Magnetizing Inrush Current.

The assumptions made allow using a simplified equivalent circuit, in which the power transformer saturation is simulated by the commutation of the key K (Figure 1b), depending on the core induction magnitude:

\[ K = \begin{cases} 
1 & \text{at } B \geq B_S \\
0 & \text{at } B < B_S 
\end{cases} \quad (1) \]

Then at the phase voltage of the power grid:

\[ u_G(t) = \frac{\sqrt{2} U_G}{\sqrt{3}} \cdot \sin \omega t \quad (2) \]

The amplitude of the first, maximum magnetizing inrush current can be estimated using the well-known equation [1]:

\[
I_{\mu \text{ max}} = \frac{\sqrt{2} U_G}{\sqrt{3}} \cdot \frac{\kappa_r}{\frac{x_G+\chi_{\mu(S)}}{B_m} (U_G/U_{G(\text{rated})})} \left[ 2 - \frac{(B_S-B_r)}{B_m (U_G/U_{G(\text{rated})})} \right] \quad (3)
\]

where \( U_G \) is the effective line voltage of the power grid; \( \omega \) is the angular frequency of the power grid voltage; \( U_{G(\text{rated})} \) is the rated power grid voltage; \( x_G \) is the equivalent inductive resistance of the power grid; \( x_{\mu(S)} \) is the inductive resistance of the power transformer magnetizing branch in the technical saturation region; \( B_r \) is the residual induction of the magnetic field in the power transformer core limb; \( B_m \) is the rated induction of the magnetic field in the core limb; \( K_r \) is the correction factor for the active resistance of the power grid and windings and losses for hysteresis and eddy currents in the power transformer core.

As shown in [1], for power transformers with a rated power of more than 2.5 MVA, a correction factor can be taken \( K_r = 1.0 \), and when estimating the first magnetizing inrush current, the use of a simplified equivalent circuit without active resistances, shown in Figure 1b, is quite acceptable. It should be added that the losses in the power transformer core and connected winding determine the duration of decay of the magnetizing inrush currents, which in power transformers with a power of more than 100 MVA may run for tens of seconds.

The inductive resistance of the power transformer magnetizing branch in the technical saturation region is that of the winding connected to the grid without considering steel (free winding without steel), the value of which can be determined by the equation [9,1]:
\[ X_{\mu(S)} = \mu_0 \cdot \omega \cdot \pi \cdot \frac{W^2 (D_{in} + \frac{2}{3}a)^2}{4h} \]  

where \( W \) is the number of turns of the power transformer winding connected to the power grid; \( D_{in} \) is the inner diameter of the connected winding; \( h \) is the conditional winding height taken equal to the height of the magnetic conductor aperture; \( \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m} \) is the magnetic constant.

To calculate the magnetizing branch inductive resistance in the technical saturation region by equation (4), data on the main power transformer dimensions are required, not usually available for the personnel of the power grid companies. Therefore, in [22,23] it is recommend to take the \( X_{\mu(S)} \) value equal to the doubled inductive component (\( x_T \)) of the short-circuit resistance, i.e., \( x_{\mu(S)} \cong 2 \cdot x_T \), regardless of the transformer power. E.g., the power transformer TDTs-400000/242/20 has the rated short-circuit voltage \( u_{sc} \cdot c_{%} = 11.3 \% \) and the main dimensions \( D_{in} = 1.28 \text{ m}; h = 2.2935 \text{ m}; \alpha = 0.142 \text{ m}; W = 346 \).

For this transformer, the magnetizing branch inductive resistance calculated by the equation (4) is \( x_{\mu(S)} = 30.58 \text{ Ohm} \), and the inductive component of the short-circuit resistance is \( x_T = 16.54 \text{ Ohm} \), i.e., in this case, the recommendation proposed is fulfilled with an accuracy of 8.2 \%.

The equivalent inductive resistance of the power grid is determined in general by the equation:

\[ x_G = \frac{u_{sc(\text{od})}}{S_{\text{sc}}} \]  

where \( S_{\text{sc}} \) is the power grid three-phase short circuit power.

The three-phase short circuit power of power grids with a rated voltage of \((110 \ldots 500) \text{ kV} \) can be taken equal to \((15,000 \ldots 50,000) \text{ MVA} \) according to the recommendations [17]. E.g., for a power grid with a rated voltage of \( 220 \text{ kV} \), the equivalent inductive resistance according to (5) will be \( x_G = 1.94 \text{ Ohm} \).

In [9], the magnetizing inrush current calculation results are compared for the piecewise-linear approximation of the magnetization curve shown in Figure 1a (approximate estimate) and the same magnetization curve built based on the data provided by the electrical steel manufacturer (exact estimate). Depending on the power grid residual induction and equivalent inductive resistance, the approximate estimate is \((8 \ldots 20) \% \) higher than the exact estimate of the magnetizing inrush current and can be considered as a limiting one.

5. Power transformer operation in no-load mode under the geomagnetic disturbances

When the power transformer is connected to the grid, the magnetizing inrush currents attenuate within a few or even tens of seconds since they are caused by the free quasi-direct magnetic flux component, the energy of which is gradually dissipated in the windings and the core elements. Under geomagnetic disturbances, the power transformer magnetizing inrush currents will remain unchanged as long as the intensity of the geomagnetic disturbances remains unchanged since they are caused by the forced quasi-direct magnetic flux component, the energy of which is kept by the geo induced currents running in the high voltage (HV) windings with grounded neutral.

The GIC value is determined by the geomagnetic disturbance intensity, which initiates the occurrence of the horizontal geoelectric field component, and the total active resistance of the flow circuit [24]:

\[ I_{\text{GIC}} = \frac{|E|}{R_H} \cdot \sum_{n=1}^{N} \ell_n \cdot \cos \alpha_n \]  

where \(|E|\) is the modulus of the geoelectric field horizontal component intensity vector; \( R_H \) is the total active resistance of the HV windings, the power transformer HV winding neutral grounding, and the feed transmission line phase conductors; \( \ell_n \) is the length of the \( n\)-th straight section of the feed power line; \( \alpha_n \) is the angle of the \( n\)-th transmission line section orientation relative to the geoelectric field intensity vector; \( N \) is the number of straight sections of the transmission line.
The GIC value does not depend on the power transformer core state: ‘saturated - not saturated’. Therefore, to estimate the amplitude of the magnetizing inrush current caused by geomagnetic disturbances, it is sufficient to introduce an additional element – a current source \( I_{\text{GIC}} \) with internal resistance \( R_{\Sigma} \) into the equivalent circuit shown in Figure 1b.

The Figure 2 shows an equivalent circuit (a) for calculating the power transformer magnetizing inrush current in the no-load mode and time charts (b) qualitatively illustrating the occurrence of magnetizing inrush currents under geomagnetic disturbances.

![Figure 2](image)

**Figure 2.** Equivalent Circuit (a) and Timing Charts (b) Illustrating the Occurrence of Magnetizing Inrush Currents under Geomagnetic Disturbances.

To analyze the power transformer no-load mode under geomagnetic disturbances, it is advisable to first transform the piecewise-linear idealization of the actual magnetization curve shown in Figure 1a from B-H coordinates into Flux Linkage \( \psi \) - Magnetizing Current \( i_\mu \) ones. Changing the coordinates will not affect the approximation nature but allow abstracting from the need to know the power transformer design parameters and obtain analytical equations directly for the magnetizing current.

With the idealization of the main electrical steel magnetizing curve shown in Figure 1a, the GIC-determined constant flux linkage component of the power transformer HV winding remains unchanged:

\[
\psi_{\text{GIC}} = \text{const},
\]  

and the power grid voltage-determined forced variable flux linkage component (2) changes according to the harmonic law:

\[
\psi_V(t) = \frac{\sqrt{2} u_a}{\sqrt{3} \omega} \cdot \cos \omega t = -\psi_{m} \cdot \cos \omega t
\]  

In the power grid voltage cycle \( 0 \leq \omega t < (\pi - \varphi) \), when the power transformer core is not saturated, the total HV winding flux linkage is determined by the equation:

\[
\psi_{\Sigma}(t) = \psi_{\text{GIC}} + \psi_V(t),
\]  

and the magnetizing current is zero, i.e. \( i_\mu(t) = 0 \).

With a sufficient intensity of geomagnetic disturbances, the total flux linkage \( \psi_{\Sigma}(t) \) at the time instant \( \omega t = (\pi - \varphi) \) will reach the saturation level \( \psi_S \) (Figure 2b)

\[
\psi_{\Sigma}(t) = \psi_S = \psi_{\text{GIC}} + \frac{\sqrt{2} u_a}{\sqrt{3} \omega} \cdot \cos \varphi,
\]  

where \( \varphi \) is the power transformer core saturation phase angle.
The saturation phase angle determines the power transformer core technical saturation state duration in a power grid voltage cycle and the magnetizing inrush current flow duration. The phase saturation angle

$$\varphi = \cos^{-1}\left(\frac{\psi_s - \psi_{GIC}}{\psi_m}\right)$$  \hspace{1cm}  (11)

is determined, as follows from equation (11), by the intensity of geomagnetic disturbances, which is considered by the quasi-direct flux linkage component $\psi_{GIC}$.

In the absence of geomagnetic disturbances, when $\psi_{GIC} = 0$, technical saturation does not occur, i.e., $\varphi = 0$ if $\psi_m \leq \psi_S$. Under extreme geomagnetic disturbances, when $\psi_{GIC} = \psi_S$, the saturation phase angle reaches its limiting value $\varphi = \pi$, and the power transformer core remains in the technical saturation state during the entire power grid voltage cycle.

In the equivalent circuit shown in Figure 2a, the power transformer core saturation is simulated by a key $K(\psi)$, the commutation conditions of which are expressed by the logic function:

$$K(\psi) = \begin{cases} 1 & |\psi_Z(t)| \geq \psi_S \\ 0 & |\psi_Z(t)| < \psi_S \end{cases}$$  \hspace{1cm}  (12)

Within the power transformer core saturation interval $(\pi - \varphi) \leq \omega t \leq (\pi + \varphi)$, when condition (12) transforms into the equality $K(\varphi) = 1$, the magnetizing inrush current occurs, shown in Figure 2b, and the total flux linkage is determined by the equation:

$$\psi_S(t) = \psi_S + \frac{(x_a + x_{\mu(S)})}{\omega} \cdot i_\mu(t),$$  \hspace{1cm}  (13)

and the total flux linkage change rate will be determined by the power grid voltage:

$$\frac{d\psi_S}{dt} = \frac{\sqrt{2} u_g}{\sqrt{3} \omega} \cdot \sin \omega t$$  \hspace{1cm}  (14)

Integration of equation (14) over the interval $\omega t \geq (\pi - \varphi)$, considering the initial flux linkage value $\psi_S(t) = \psi_S$ at $\omega t = (\pi - \varphi)$ allows writing another equation for the instantaneous total flux linkage values in the power transformer core technical saturation interval:

$$\psi_S(t) = \psi_S - \frac{\sqrt{2} u_g}{\sqrt{3} \omega} \cdot (\cos \varphi + \cos \omega t)$$  \hspace{1cm}  (15)

The joint solution of equations (13) and (15) allows determining the instantaneous values of the magnetizing current after the power transformer core saturation:

$$i_\mu(t) = -\frac{\sqrt{2} u_g}{\sqrt{3} (x_{\mu(S)} + x_g)} \cdot (\cos \varphi + \cos \omega t)$$  \hspace{1cm}  (16)

The magnetizing inrush current shown in Figure 2b will reach the peak value at $\omega t = \pi$:

$$I_{\mu\max} = \frac{\sqrt{2} u_g (1 - \cos \varphi)}{\sqrt{3} (x_{\mu(S)} + x_g)}$$  \hspace{1cm}  (17)

Considering equation (11) determining the saturation phase angle depending on the intensity of geomagnetic disturbances, equation (17) can be transformed into a form convenient for comparison with equation (3):

$$I_{\mu\max} = \frac{\sqrt{2} u_g}{\sqrt{3} (x_{\mu(S)} + x_g)} \left[1 - \left(\frac{\psi_S - \psi_{GIC}}{\psi_m}\right)\right]$$  \hspace{1cm}  (18)

Comparison of equations (3) and (18) allows saying that the residual induction $B_r$, when the power transformer is connected to the power grid and the quasi-direct flux linkage component $\psi_{GIC}$ when the power transformer is in the no-load mode under geomagnetic disturbances, has fundamentally the same impact on the formation of the magnetizing inrush current amplitude. The only difference is that
equation (3) determines the amplitude of the first, largest magnetizing inrush current when the power transformer is turned on at the moment when the grid voltage crosses zero, and equation (18) determines the magnetizing inrush current amplitude, which remains unchanged under constant-intensity geomagnetic disturbances.

Integrating equation (16) over the power grid voltage cycle allows determining the constant component of the power transformer magnetizing current under geomagnetic disturbances:

\[
I(\tau) = \frac{1}{T} \int_0^T i_\mu(t) dt = \frac{\sqrt{2} U_R}{\sqrt{\pi} (x_{\mu(S)} + x_\varphi)} \left( \frac{\sin \varphi - \varphi \cos \varphi}{\pi} \right) \tag{19}
\]

It should be noted that equations (16), (17), and (19) do not allow determining the GIC value, at which the power transformer core saturation starts. This is because at the idealization shown in Figure 1a, the magnetizing branch inductive resistance before saturation should be \( x_\mu = \infty \). In this case, the equation for determining the quasi-direct flux linkage component before the core saturation, which is finite in magnitude

\[
\psi_{GIC} = \frac{x_\mu I_{GIC}}{\omega}, \tag{20}
\]

degenerates into uncertainty \([\infty, 0]\). This means that the condition for the displacement of the flux reversal mode to the technical saturation region \( \psi_{GIC} > (\psi_S - \psi_m) \) is fulfilled by an infinitely small GIC value. In addition, the above uncertainty does not allow determining the power transformer core saturation dynamics after the sudden geomagnetic disturbances.

Thus, the magnetization characteristic model shown in Figure 1a, which has been widely used for analytical estimation of the magnetizing inrush current amplitude when the power transformer is connected to the power grid, is not informative enough to analyze the no-load mode under geomagnetic disturbances.

6. A refined model of the power transformer magnetization characteristic under geomagnetic disturbances

To improve the information content of the piecewise-linear approximation of the magnetization characteristic, the actual value of the magnetizing branch inductive resistance before saturation \( x_\mu \) should be considered using the open-circuit test results [1,4,3,2]:

\[
x_\mu = \frac{U_r}{\sqrt{3} U_{\text{no-load}}} \tag{21}
\]

where \( U_r \) is the rated voltage of the power transformer HV winding.

The Figure 3a shows a piecewise-linear approximation of the magnetization characteristic in the coordinates Flux Linkage \( \psi \) - Magnetizing Current \( i_\mu \), the position of point ‘1’ of which is determined by the parameters of the open-circuit test performed at the rated voltage of HV winding. The breakpoint ‘2’ of the piecewise-linear approximation is found at the intersection of the 0-1 line and the horizontal line corresponding to the saturation flux linkage \( \psi_S \).

The Figure 3b shows an equivalent circuit for calculating the power transformer magnetizing inrush current at no-load under geomagnetic disturbances, considering the finite value of the magnetizing branch inductive resistance before the core saturation. It differs from the equivalent circuit shown in Figure 2a by the additional inductance \( x'_\mu = (x_\mu - x_{\mu(S)}) \).

The key \( K(\psi) \) simulates the change in the power transformer magnetizing branch inductive resistance when the core state changes:

- at \( K(\psi) = 1 \), the inductive reactance is equal to \( x_{\mu(S)} \),
- at \( K(\psi) = 0 \), the inductive reactance is equal to \( x_\mu \).

When approximating the magnetization characteristic shown in Figure 3a, the key \( K(\psi) \) commutation condition (12) can be supplemented by an equivalent logic function:
\[ K(\psi) = \begin{cases} 1 & \text{at } i_\mu \geq I_S \\ 0 & \text{at } i_\mu < I_S \end{cases} \] (22)

In this case, the total flux linkage of HV winding is determined by equation (9). Since for the total flux linkage \( \psi_\Sigma(t) \), the time continuity principle is valid, then for the time instants immediately before and immediately after the core saturation, the equality is true \( \psi_\Sigma(0-) = \psi_\Sigma(0+) = \psi_S \), which, considering (14), can be extended to the derivatives:

\[
\frac{d\psi_\Sigma(0-)}{dt} = \frac{d\psi_\Sigma(0+)}{dt}.
\] (23)

**Figure 3.** Piecewise-Linear Approximation (a) and Equivalent Circuit (b) for Calculating the Magnetizing Inrush Current Considering the Finite Value of the Magnetizing Branch Inductive Resistance Before Saturation.

Considering the piecewise-linear approximation shown in Figure 3a, the last relationship implies the equality:

\[
\frac{x_\mu \cdot \frac{d\mu(0-)}{dt}}{\omega} = \frac{x_\mu(S) \cdot \frac{d\mu(0+)}{dt}}{\omega}
\] (24)

and the ratio of derivatives:

\[
\frac{d\mu(0+)/dt}{d\mu(0-)dt} = \frac{x_\mu}{x_\mu(S)} = K_{\mu(S)}
\] (25)

As can be seen, the magnetizing current change rate at the core saturation moment increases sharply by \( K_{\mu(S)} \) times, generating the magnetizing inrush current. Accordingly, at the moment of exiting the saturation state, magnetizing current change rate sharply decreases by \( K_{\mu(S)} \) times. It should be noted that immediately before and immediately after the core saturation, the magnetizing current remains unchanged \( i_{\mu}(0-) = i_{\mu}(0+) = I_S \). In this case, the magnetic field energy, which immediately before saturation reaches the value

\[
W(0-) = \frac{x_\mu}{\omega} \cdot I_S^2,
\] (26)

immediately after saturation decreases to

\[
W(0+) = \frac{x_{\mu(S)}}{\omega} \cdot I_S^2,
\] (27)
i.e., in $K_{µ(S)}$ times. Within the adopted idealization of the magnetization characteristic, the arising difference

$$\Delta W = W(0-) - W(0+),$$

is explained by the energy consumption for the elastic rotation of the magnetization vectors of the electrical steel domains to the external magnetic vector direction. When the core leaves the saturation state, the domain magnetization vectors will return to their original positions, returning energy $\Delta W$ to the magnetic field.

7. Power transformer core saturation dynamics after the sudden geomagnetic disturbances

In the absence of geomagnetic disturbances ($I_{GIS} = 0$ in the equivalent circuit shown in Figure 3b), when the voltage determined by equation (2) is applied to HV winding and there is no core saturation, the magnetization current of the power transformer in the no-load mode is determined by the equation:

$$i_{µ(G)}(t) = \frac{\sqrt{2} U_G}{\sqrt{3} \sqrt{R_x + \frac{x_{µ}}{\bar{L}}}} \cdot \sin(\omega t - \theta)$$ (29)

where $\theta = \tan^{-1} g(x_{µ}/R_x)$. Since $x_{µ} >> R_x$, then $\theta \equiv \pi/2$, and the magnetizing current can be determined by a simpler equation:

$$i_{µ(G)}(t) = -\frac{\sqrt{2} U_G}{\sqrt{3} x_{µ}} \cdot \cos \omega t = -I_{µ(m)} \cdot \cos \omega t,$$ (30)

where $I_{µ(m)}$ is the amplitude of the equivalent sinusoid of the magnetizing current in the absence of the core saturation.

After the sudden geomagnetic disturbances simulated by a step function:

$$I_{GIS} = I_{GIS} \cdot 1(t) = \begin{cases} I_{GIS} \text{ at } t \geq 0 \\ 0 \text{ at } t < 0 \end{cases}$$ (31)

the magnetizing current will contain two components:

$$i_{µ}(t) = i_{µ(G)}(t) + i_{µ(GIS)}(t) = -I_{µ(m)} \cdot \cos \omega t + I_{GIS} \cdot (1 - e^{-t/\tau_µ})$$ (32)

where $i_{µ(G)}(t)$ is the exponential component growth time constant.

At the time instants $t(n) = n \cdot \pi/\omega$, the numerical values of the harmonic $i_{µ(G)}(t)$ and exponential $i_{µ(GIS)}(t)$ components of the magnetizing current are summed up, determining the maximum value at the end of the n-th half-cycle of the power grid voltage from the geomagnetic disturbance start moment:

$$I_{µ,max}^{(n)} = I_{µ(m)} + I_{GIS} \cdot (1 - e^{-n \cdot \pi \cdot R_e / x_µ})$$ (33)

where $n=1, 3, 5, 7, \ldots$ is the sequential number of the power grid voltage half-cycle from the sudden geomagnetic disturbance start moment.

Depending on the exponential magnetizing current component magnitude, several stages of changing the power transformer core state after a sudden occurrence of geomagnetic disturbances can be identified (Figure 4):

- stage I - initial steady-state no-load mode at $I_{GIS} = 0$,
- stage II - transient no-load mode without the core saturation, while $I_{µ,max}^{(n)} \leq I_s$,
- stage III - transient no-load mode with an increase in the core saturation phase angle and the magnetizing inrush current, when $I_{µ,max}^{(n)} > I_s$,
- stage IV - steady-state no-load mode with a constant core saturation phase angle and the magnetizing inrush current at $I_{GIS} = \text{const}$. 

\[10\]
Equation (33) allows determining the number of the power grid voltage half-periods after a sudden occurrence of geomagnetic disturbances, during which the power transformer core flux reversal mode will shift to the boundary of the technical saturation region according to the criterion of achieving the equality $I_{\mu_{\text{max}}} = I_S$.

\[ [N] = \frac{x_{\mu}}{\pi R_S} \cdot \ln \left[ 1 - \frac{(s-I_{\mu(m)})}{I_{\text{GIC}}} \right], \]  

where $[N]$ is the integer part of the odd number closest to the result.

In the interval of the first $N$ power grid voltage half-periods (stage II in Figure 4), the change in the core state under the GIC effect is latent without a noticeable increase in the magnetizing current. Starting from $(N+2)$ power grid voltage half-period, the transient process of change in the power transformer core state starts to be accompanied by the magnetizing inrush currents in the technical saturation intervals (stage III in Figure 4). At this stage, an increase in the exponential magnetizing current component $I_{\mu_{\text{GIC}}}(t)$ is accompanied by an increase in the saturation phase angle $(\phi_{k+1} > \phi_k)$ and the magnetizing inrush current amplitude $I_{\mu_{\text{max}}} > I_{\mu_{\text{max}}^{(k+1)}}$, i.e., each subsequent magnetizing inrush current exceeds the previous one in duration and amplitude.

The presence of an exponential component $I_{\mu_{\text{GIC}}}(t)$ violates the symmetry of the start $(\phi_k')$ and end $(\phi_k'')$ moments of the k-th magnetizing inrush current relative to the moment $\omega t = (N + k) \cdot \pi$ when the power grid voltage crosses zero, and $\phi_k'' > \phi_k'$ due to an increase in $I_{\mu_{\text{GIC}}}(t)$. The degree of asymmetry is estimated by the difference between the $\phi_k'$ and $\phi_k''$ values and can be found using the ratio:

\[ \frac{\phi_k'' - \phi_k'}{\phi_k''}, \]  

Figure 4. The Qualitative Pattern of the Change in the Power Transformer No-load Mode under Geomagnetic Disturbances.
At the actual ratio of the parameters \( I_m \gg I_{GIC} \) and \( x_{\mu(S)} \gg R_\Sigma \), the asymmetry of the start and end moments of the k-th magnetizing inrush current can be neglected since \( \Delta \varphi_k < \varphi_k \cdot \), and it can be assumed that \( \varphi''_k = \varphi'_k = \varphi_k \). In this case, in the next interval of technical saturation of the core
\[
(N + k) \cdot \pi - \varphi_k \leq \omega t \leq (N + k) \cdot \pi + \varphi_k,
\]
the instantaneous values of the k-th magnetizing inrush current will be determined by the equation:
\[
i_{\mu(k)}(t) = -\frac{\sqrt{2} U_g}{\sqrt{3}(x_{\mu(S)} + x_G)} \cdot (\cos \varphi_k + \cos \omega t) + I_{GIC} \cdot \left\{1 - \exp \left[\frac{\omega t + (N+k)\pi - \varphi_k}{\omega \tau_{\mu}}\right]\right\}
\]
(37)

The k-th magnetizing inrush current amplitude will be reached at \( \omega t = (N + k) \cdot \pi \)
\[
i_{\mu max}^{(k)} = \frac{\sqrt{2} U_g}{\sqrt{3}(x_{\mu(S)} + x_G)} \cdot (1 - \cos \varphi_k) + I_{GIC} \cdot \left\{1 - \exp \left[-\frac{(N+k)\pi}{\omega \tau_{\mu}}\right]\right\}
\]
(38)

At stage IV of the steady-state no-load mode, the duration and amplitude of the magnetizing inrush currents stop changing. The magnetizing inrush current amplitude is reached, as follows from equation (38):
\[
i_{\mu max} = \frac{\sqrt{2} U_g}{\sqrt{3}(x_{\mu(S)} + x_G)} \cdot (1 - \cos \varphi) + I_{GIC},
\]
(39)
at the saturation phase angle determined by equation (19) at \( I_{(w)} = I_{GIC} \).

The total duration of the transient process of change in the core state, consisting of two stages - II and III (Figure 4), is estimated by the value:
\[
\tau_{TP} \cong 3 \cdot \tau_{\mu} = \frac{3}{2\pi} \cdot \frac{x_{\mu}}{R_\Sigma} \cdot T,
\]
(40)

where \( T \) is the power grid voltage cycle.

This estimate should be considered as the limiting one since in the intervals of technical saturation of the core, the rate of increase in the exponential magnetizing current component is determined by the time constant \( \tau_{\mu(S)} = x_{\mu(S)} / \omega \cdot R_\Sigma \) and, accordingly, the inductive resistance \( x_{\mu(S)} \) of the power transformer magnetizing branch. Therefore, as a more exact estimate, it is advisable to use the weighted average value of the time constant:
\[
\tau_{\mu(eq)} = \frac{1}{\pi} \cdot \left[\varphi \cdot \tau_{\mu(S)} + (\pi - \varphi) \cdot \tau_{\mu}\right],
\]
(41)

which allows indirectly considering the GIC impact on the duration of the transient process of change in the power transformer core state. E.g., for a power transformer AODTsTN-333000/750/330 \( (I_{no-load,\%} = 0.35 \%; u_{G-C,\%} = 32 \%) \), the transient core saturation process at the total active resistance of the adjacent power grid \( R_\Sigma = 33(Ohm) \) elements will be determined by inductive resistances \( x_{\mu} = 161 \cdot 10^3(Ohm) \); \( x_{\mu(S)} \cong 310(Ohm) \) and the corresponding time constants \( \tau_{\mu} \cong 15.53 \) (s); \( \tau_{\mu(S)} \cong 0.03 \) (s).

In this case, the equivalent time constant at \( I_{GIC} = 50(A) \) will be determined by the value \( \tau_{\mu(eq)} \cong 12.42 \) (s). Under such conditions, the number of the power grid voltage half-periods before the core saturation will be \( [N] \cong 31 \), and the transient process duration will be \( \tau_{TP} \cong 3 \cdot \tau_{\mu(eq)} \cong 37.26 \) (s). Thus, considering the actual magnetizing inductance before saturation allows not only estimating the duration of the transient process of change in the power transformer core state after the occurrence of geomagnetic disturbances but also clarifying the magnetizing inrush current amplitude by equation (39) allowing considering the GIC value both directly and indirectly through saturation angle.
8. Verifying the piecewise-linear approximation of the power transformer magnetization characteristic

The results of the experimental study, that provides in [25] for the single-phase power transformers with a rated power of 370 and 550 MVA, which are exposed to a direct current flowing through low voltage windings, while simultaneously applying a voltage of 735 kV to high voltage windings. The magnetizing inrush currents recorded during the tests correspond in shape to the magnetizing inrush current shown in Figure 2. Table 2 contains the results of measuring the magnetizing current parameters at various DC values $I_{GIC}$.

Since [25] does not provide the datasheet values of the tested power transformers and the short-circuit power of the power supply system on the experimental installation buses, the magnetizing inrush current parameters are calculated using the datasheet values of the closest domestic analog - the single-phase autotransformer AODTsn-333000/750/330.

| $I_{GIC}$, А | $I_{\mu max}/I_{GIC}$ | Magnetizing inrush current duration, el. deg. | $I_{\mu max}$, А | $\varphi$, el. deg. |
|-------------|---------------------|------------------------------------------|-----------------|------------------|
| 12.5        | 11.8                | 46                                       | 147.5           | 23               |
| 25          | 9.4                 | 58                                       | 235             | 29               |
| 50          | 7.6                 | 71                                       | 380             | 35.5             |
| 75          | 6.7                 | 81                                       | 502.5           | 40.5             |

In particular, in this case, the design inductive resistance of the magnetizing branch in the technical saturation state of the core is $x_{\mu(5)} \cong 304.66$ (Ohm). The equivalent inductive resistance of the power supply system is taken zero. Table 3 provides the design values of the magnetizing inrush current amplitude, obtained by equation (39), and the results of their comparison with the experimental values.

| Parameter | Value |
|-----------|-------|
| $I_{DC(hv)}$, А | 12.5  25  50  75 |
| $I_{\mu_{(max)}}$, А | experiment 147.5 235 380 502.5 |
| calculation 156.6 246.97 366.14 472 |
| Relative error, % | 6.17  5.09  3.65  6.07 |

9. Conclusions

Piecewise-linear approximation of the magnetization characteristic, widely used for estimating the power transformer current inrush, allows obtaining adequate estimates (with an accuracy of 6 %) of the amplitude of the magnetizing inrush current caused by geo induced currents and can be used to simulate steady-state and transient processes in complex electric power systems under geomagnetic disturbances.

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