Exploring the polarization of gluons in the nucleon

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Abstract. We give an overview of the current status of investigations of the polarization of gluons in the nucleon. We describe some of the physics of the spin-dependent gluon parton distribution and its phenomenology in high-energy polarized hadronic scattering. We also review the recent experimental results.

1. Introduction
For many years now, spin has played a very prominent role in QCD. The field of QCD spin physics has been driven by the hugely successful experimental program of polarized deep-inelastic lepton-nucleon scattering (DIS) [1]. One of the most important results of this program has been the finding that the quark and anti-quark spins (summed over all flavors) provide only about a quarter of the nucleon’s spin, \( \Delta \Sigma \approx 0.25 \) in the proton helicity sum rule [2, 3, 4]

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma(Q^2) + \Delta G(Q^2) + L_q(Q^2) + L_g(Q^2),
\]

implying that sizable contributions to the nucleon spin should come from the gluon spin contribution \( \Delta G(Q^2) \), or from orbital angular momenta \( L_{q,g}(Q^2) \) of partons. Here, \( Q \) is the resolution scale at which one probes the nucleon. The \( Q^2 \)-dependence of the various contributions to the proton spin is predicted in perturbative QCD through its evolution equations [1, 5, 6]. To lowest order (LO), the quark and anti-quark spin contribution \( \Delta \Sigma(Q^2)/2 \) does not depend on \( Q^2 \). Figure 1 shows a LO toy calculation of the \( Q^2 \)-evolution of the contributions to the proton spin in Eq. (1), assuming that at an initial scale \( Q_0 = 1 \) GeV we have \( \Delta \Sigma = 0.25, \Delta G = L_q = 0.2, \) \( L_g = -0.025 \). The rise of \( \Delta G \propto \log(Q^2) \) or \( 1/\alpha_s(Q^2) \) (compensated by an opposite evolution of \( L_g \)) is an important prediction of QCD and awaits experimental testing. For the initial conditions chosen here, the evolution leads to large positive values of \( \Delta G \). We note that at asymptotic \( Q^2 \) the total quark and gluon angular momenta, \( \frac{1}{2} \Delta \Sigma + L_q \) and \( \Delta G + L_g \), respectively, become roughly equal [4].

To determine the gluon spin contribution on the right-hand-side of Eq. (1) has become a major focus of the field. Like \( \Delta \Sigma \), it can be probed in polarized high-energy scattering. Several current

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experiments are dedicated to a direct determination of the spin-dependent gluon distribution $\Delta g(x, Q^2)$,

$$\Delta g(x, Q^2) \equiv g^+(x, Q^2) - g^-(x, Q^2),$$

(2)

where $g^+$ ($g^-$) denotes the number density of gluons in a longitudinally polarized proton with same (opposite) sign of helicity as the proton’s, and where $x$ is the gluon’s light-cone momentum fraction. The field-theoretic definition of $\Delta g$ is

$$\Delta g(x, Q^2) = \frac{i}{4\pi x P^+} \int d\lambda e^{i\lambda x P^+} \langle P, S | G_{\mu\nu}(0) \tilde{G}_{\mu\nu}(\lambda n) | P, S \rangle \bigg|_{Q^2},$$

(3)

written in $A^+ = 0$ gauge. $G^{\mu\nu}$ is the QCD field strength tensor, and $\tilde{G}^{\mu\nu}$ its dual. The integral of $\Delta g(x, Q^2)$ over all momentum fractions $x$ becomes a local operator only in $A^+ = 0$ gauge and then coincides with $\Delta G(Q^2)$ \cite{2, 7}. The COMPASS experiment at CERN and the HERMES experiment at DESY attempt to access $\Delta g(x, Q^2)$ in charm- or high-$p_T$ hadron final states in photon-gluon fusion $\gamma^* g \to q\bar{q}$. A new milestone has been reached with the advent of the first polarized proton-proton collider, RHIC at BNL \cite{8, 9}. RHIC will provide precise and detailed information on $\Delta g$, over a wide range of $x$ and $Q^2$, and from a variety of probes.

2. Model estimates of $\Delta g$

Before we discuss in some detail the phenomenology of $\Delta g$ in polarized high-energy scattering, let us briefly address some of the available theoretical expectations for $\Delta g$ and its integral. As was first pointed out in \cite{10}, it is possible to estimate the operator matrix element corresponding to $\Delta G$ in non-relativistic quark and bag models. In such models, for example, baryon mass splittings result from lowest-order exchange of transverse gluons, and the associated forces are spin-dependent. One obtains estimates \cite{11, 12} for $\Delta G(Q^2 \approx 1 \text{ GeV}^2)$ of about 0.2 to 0.3. In a sense, these are “natural” values since they are of the order of the proton spin itself. Very recently, for the first time model calculations of the $x$-dependence of $\Delta g$ have been presented \cite{12}. The resulting distribution is positive everywhere and of moderate size. The more and more precise experimental constraints on $\Delta g$ will likely motivate further model investigations, which ultimately might lead to new insights into QCD. Likewise, it is to be hoped that lattice calculations, which are becoming ever more powerful, will be able to address gluonic observables in nucleon structure in the future \cite{13}.

Other considerations, based in part on perturbation theory, led to the prediction of a very large gluon polarization in the nucleon. The peculiar evolution pattern of $\Delta G(Q^2) \propto 1/\alpha_s(Q^2)$
visible in Fig. 1 inspired ideas [14] that a reason for the experimentally found small size of the proton’s axial charge should be sought in a “shielding” of the quark spins due to a particular perturbative part of the DIS process $\gamma^* g \rightarrow q\bar{q}$. The associated cross section is of order $\alpha_s(Q^2)$, but the $Q^2$-evolution of $\Delta G(Q^2)$ would compensate this suppression. We note that this interpretation of the axial charge, however, corresponds to a particular choice of factorization scheme. To be of any phenomenological relevance, such “anomalous” models would require a very large positive gluon spin contribution, $\Delta G > 0.5$, even at a low scale of 1 GeV or so. As we shall see below, initial experimental data now appear to make such a scenario very unlikely.

3. $\Delta g$ and scaling violations in polarized DIS

In principle, a clean determination of $\Delta g(x, Q^2)$ is possible by investigating scaling violations of the spin-dependent proton structure function $g_1(x, Q^2)$ which is measured in polarized DIS. To leading order of QCD, $g_1$ can be written as

$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \left[ \Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2) \right],$$

where the $\Delta q$ and $\Delta\bar{q}$ are the quark and anti-quark helicity distributions. QCD predicts the $Q^2$-dependence of the densities through the spin-dependent DGLAP evolution equations [5]:

$$\frac{d}{d \ln Q^2} \left( \frac{\Delta q}{\Delta g} \right)(x, Q^2) = \int_x^1 \frac{dz}{z} \left( \frac{\Delta P_{qq}(\alpha_s(Q^2), z)}{\Delta P_{gg}(\alpha_s(Q^2), z)} \frac{\Delta P_{gq}(\alpha_s(Q^2), z)}{\Delta P_{gg}(\alpha_s(Q^2), z)} \right) \left( \frac{\Delta q}{\Delta g} \right)(\frac{x}{z}, Q^2),$$

where $\Delta P_{ij}$ are the spin-dependent “splitting functions” [5, 15] which are evaluated in QCD perturbation theory. As one can see, $\Delta g$ contributes to the scaling violations of $g_1$. Nonetheless, $\Delta g$ has been left virtually unconstrained (see, for example, [16, 17, 18, 19, 20]) by the scaling violations observed experimentally in polarized DIS. This is due to the very limited lever arm in $Q^2$ of the fixed-target experiments. Figure 2 shows current theoretical “uncertainty bands” for $\Delta g$ from DIS scaling violations. At best, a tendency toward a positive $\Delta g$ is seen. We note that a recent new analysis by the COMPASS collaboration [21] using their latest deuteron DIS data [22] finds two “allowed” regions for $\Delta g$, one with positive, one with negative gluon polarization. Clean and precise extractions of $\Delta g(x, Q^2)$ over a wide range of $x$ and $Q^2$ from scaling violations of $g_1$ would become possible at a polarized electron-ion collider, EIC [23], thanks to its vastly larger kinematic reach.

**Figure 2.** Results for $x\Delta g(x, Q^2 = 5 \text{ GeV}^2)$ from several analyses [15] [17] [18] of polarized DIS. The various bands indicate ranges in $\Delta g$ that were deemed consistent with the DIS scaling violations in these analyses. From [9].
4. Access to $\Delta g$ in polarized proton-proton scattering at RHIC

The measurement of gluon polarization in the proton is a major focus and strength of RHIC [8, 9]. The basic concept that underlies most of spin physics at RHIC is the factorization theorem [24]. It states that large momentum-transfer reactions may be factorized into long-distance pieces that contain the desired information on the spin structure of the nucleon in terms of its universal parton densities, and parts that are short-distance and describe the hard interactions of the partons. The latter can be evaluated using perturbative QCD. As an example, we consider the double-spin asymmetry for the reaction $pp \rightarrow \pi X$,

$$A_{LL} \equiv \frac{\sigma^{++} - \sigma^+ - \sigma^{++} + \sigma^-}{\sigma^{++} + \sigma^-} = \frac{\Delta \sigma}{\sigma}, \quad (6)$$

where the superscripts denote helicities of the initial protons. We assume the pion to be produced at high transverse momentum $p_T$, ensuring large momentum transfer. Then, up to corrections suppressed by inverse powers of $p_T$:

$$d \Delta \sigma = \sum_{abc} \Delta f_a \otimes \Delta f_b \otimes d \Delta \hat{\sigma}_{ab} \otimes D_c, \quad (7)$$

for the polarized cross section, where $\otimes$ denotes a convolution. The $\Delta f_i$ are the polarized parton distributions, and $D_c$ the pion fragmentation functions. The sum in Eq. (7) is over all contributing partonic channels $a + b \rightarrow c + X$, with $d \Delta \hat{\sigma}_{ab}$ the associated spin-dependent partonic cross section. In general, a leading-order estimate of (7) merely captures the main features, but does not usually provide a quantitative understanding. Only with knowledge of the next-to-leading order (NLO) QCD corrections to the $d \Delta \hat{\sigma}_{ab}$ can one reliably extract information on the parton distribution functions from the reaction.

Several different processes will be investigated at RHIC [8, 9] that are very sensitive to gluon polarization: high-$p_T$ prompt photons $pp \rightarrow \gamma X$, jet or hadron production $pp \rightarrow jet X$, $pp \rightarrow h X$, and heavy-flavor production $pp \rightarrow (Q\bar{Q}) X$. An important role for the determination of $\Delta g$ will be played by measurements of two-particle, jet-jet (or hadron-hadron) and photon-jet correlations. For these, at the leading order approximation, the hard-scattering subprocess kinematics can be calculated directly on an event-by-event basis, giving an estimate of the gluon momentum fraction [25]. In addition, besides the current $\sqrt{s} = 200$ GeV, also $\sqrt{s} = 500$ GeV will be available at RHIC at a later stage. All this will allow to determine $\Delta g(x, Q^2)$ in various regions of $x$, and at different scales. Essentially all tools are in place now for treating the spin-dependent reactions relevant at RHIC at NLO [26, 27, 28, 29].

We emphasize that there have already been results from RHIC that demonstrate that the NLO framework is very successful. Figure 3 shows comparisons of data from PHENIX and STAR for single-inclusive cross sections for $\pi^0$ [30, 31], jets [32] and photons [33] with corresponding NLO calculations [26, 28, 29, 34]. As can be seen, the agreement is overall excellent. We note that an agreement between data and NLO calculations like the one shown in Fig. 3 is not found in the fixed-target regime [35] (it has recently been shown that in this regime large logarithmic terms at yet higher orders are important and need to be resummed for a more successful theoretical description [36]). In Fig. 4 we decompose the NLO mid-rapidity $\pi^0$ cross section into the relative contributions from the various two-parton initial states [9]. It is evident that processes with initial gluons dominate.

The results shown in Fig. 3 give confidence that the theoretical NLO framework may be used to determine the spin-dependent gluon density from RHIC data. Results for $A_{LL}$ in $pp \rightarrow \pi X$ are now available from PHENIX [32], and $A_{LL}$ for single-inclusive jet production has been measured by STAR [32]. The results are shown in Fig. 5. The curves shown in Fig. 5 represent the $A_{LL}$ values calculated at NLO for a range of gluon distributions from [17], from a suggested very large positive gluon polarization (“GRSV-max”) with an integral $\Delta G = 1.9$ at scale $Q = 1$ GeV, to a
Figure 3. Data for the cross section for single-inclusive \( \pi^0 \) production \( pp \rightarrow \pi^0 X \) at \( \sqrt{s} = 200 \text{ GeV} \) at mid-rapidity from PHENIX (upper left, [30]) and at forward rapidities from STAR (lower left, [31]), for mid-rapidity jet production from STAR (upper right, [32]), and for mid-rapidity prompt-photon production from PHENIX (lower right, [33]). The lines show the results of the corresponding next-to-leading order calculations [26, 28, 29].
"maximally" negative gluon polarization, ("\(\Delta g = -g\)"), for which \(\Delta G(1\,\text{GeV}^2) = -1.8\). These two distributions span the "GRSV-band" shown in Fig. 4. The curves labeled "GRSV-std" represent the best fit of [17] to the polarized DIS data (solid line in Fig. 2), which has a more "natural" \(\Delta G(1\,\text{GeV}^2)\) of about 0.4, and the results for "\(\Delta g = 0\)" correspond to very little gluon polarization, \(\Delta G(1\,\text{GeV}^2) = 0.1\). One can see that the data are already discriminating between the various \(\Delta g\) distributions. A very large gluon distribution, as proposed in the context of the "anomaly scenario" (see discussion above) and corresponding roughly to the curves labeled "GRSV max", appears to be strongly disfavored.

Figure 5. Data for the double-spin asymmetry for mid-rapidity single-inclusive \(\pi^0\) production at \(\sqrt{s} = 200\,\text{GeV}\) from PHENIX [37] (left), and for jet production from STAR [32] (right), compared to NLO predictions for several polarized gluon distributions of [17].

Figure 6 shows NLO predictions [28] for the double-longitudinal spin asymmetry \(A_{LL}\) for the reaction \(pp \rightarrow \gamma X\) at RHIC, based on the "gluon uncertainty" band displayed in Fig. 2. Prompt photons are much less copiously produced than pions at RHIC, resulting in larger statistical uncertainties. The measurement of this asymmetry will therefore take some time at RHIC. Nonetheless, the reaction \(pp \rightarrow \gamma X\) is of great importance because of its direct sensitivity to \(\Delta g\) through the clean "Compton-like" process \(qg \rightarrow \gamma q\). The spin asymmetry for this reaction is linear in \(\Delta g\) and therefore directly determines the sign of the distribution. The plot also shows the experimental uncertainties expected at RHIC (PHENIX) for 65/pb collected luminosity [9].
5. $\Delta g$ from photon-gluon fusion

A way to access $\Delta g$ in lepton-nucleon scattering is to measure final states that select the photon-gluon fusion process. These are heavy-flavor production, $\ell p \rightarrow c\bar{c}X$, and single- or di-hadron production, $\ell p \rightarrow hX$ or $\ell p \rightarrow h_1 h_2 X$, where the hadrons have large transverse momentum. Figure 7 compiles the current results [21, 38, 39, 40] for extractions of $\Delta g$ from these reactions. We note that, unlike at RHIC, the success of the perturbative-QCD hard-scattering description has not been established for these observables in the kinematic regimes of interest here. Also, the translation of the measured spin asymmetry into $\Delta g$ at a certain single momentum fraction, currently only possible at leading order, is fraught with large uncertainties.

6. Global Analysis

The eventual determination of gluon polarization will require consideration of all existing data through a “global analysis” that makes simultaneous use of results for all probes, from RHIC and from lepton scattering. The technique is to optimize the agreement between measured spin asymmetries, relative to the accuracy of the data, and the theoretical spin asymmetries, by minimizing the associated $\chi^2$ function through variation of the shapes of the polarized parton distributions. The advantages of such a full-fledged global analysis program are manifold: (1) The information from the various reaction channels is all combined into a single result for $\Delta g(x)$. (2) The global analysis effectively deconvolutes the experimental information, which in its raw form is smeared over the fractional gluon momentum $x$, and fixes the gluon distribution at definite values of $x$. Figure 8 highlights the importance of this. The figure shows [41] the
Figure 8. NLO $d\Delta \sigma / dp_T d\log_{10}x$ (arbitrary normalization) for the reaction $pp \rightarrow \pi^0 X$ at RHIC, for $p_T = 2.5$ GeV and six different values for $\Delta G(\mu^2)$ at $\mu \approx 0.4$ GeV \cite{17}. The shaded areas denote in each case the $x$-range dominantly contributing to $d\Delta \sigma$. From \cite{41}.

contributions of the various regions in gluon momentum fraction to the mid-rapidity spin-dependent cross section for $pp \rightarrow \pi^0 X$ at RHIC, for six different sets of polarized parton distributions \cite{17} mostly differing in the gluon distribution. The pion’s transverse momentum was chosen to be 2.5 GeV. One can see that the distributions are very broad, and that the $x$-region that is mostly probed depends itself on the size and form of the polarized gluon distribution. This makes it very difficult to assign a good estimate of the gluon momentum fraction to a data point at a given pion transverse momentum. The global analysis solves this problem.

The further advantages of a global analysis are: (3) State-of-the-art (NLO) theoretical calculations can be used without approximations. (4) It provides a framework to determine an error on the gluon polarization. (5) Correlations with other experiments, to be included in $\chi^2$ and sensitive to degrees of freedom different from $\Delta g$, are automatically respected. Global analyses of this type have been developed very successfully over many years for unpolarized parton densities. Examples of early work on global analyses of RHIC-Spin and polarized DIS data in terms of polarized parton distributions are \cite{42, 43, 44}.

7. Conclusions and Outlook
While the initial data from RHIC and from the dedicated studies in lepton scattering shown above point to a small or moderate size of the gluon polarization in the $x$-region currently accessible, statements about the gluon contribution to the proton spin, $\Delta G$, are really not
possible yet and will require the global analysis just described. A crucial issue will eventually be the behavior of the extracted $\Delta g(x)$ at the smallest accessible $x$, which are reachable in 500 GeV running at RHIC and in correlation studies involving final states produced at forward angles [25]. It is possible that a significant contribution to $\Delta G$ comes from relatively small $x$. As one example, we show in Fig. 9 the “running integral” $\int_{x_{\text{min}}}^{1} dx \Delta g(x, Q^2)$ at $Q^2 = 10 \text{ GeV}^2$, normalized to the full integral $\Delta G(Q^2)$, for the gluon distribution in the NLO GRSV “standard” set [17]. As one can see, at this scale about 30% of the integral come from $x \leq 10^{-2}$. RHIC will likely be able to constrain $\Delta g$ down to values somewhat smaller than that, but the example shows that it might eventually be necessary to push to $x \leq 10^{-3}$ and below. This could be achieved at a high-energy polarized electron-proton collider [23].

![Figure 9](image_url)

**Figure 9.** “Running integral” $\int_{x_{\text{min}}}^{1} dx \Delta g(x, Q^2 = 10 \text{ GeV}^2)$, normalized to the full integral, for the gluon distribution in the NLO GRSV “standard” set [17], as a function of $x_{\text{min}}$.

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