An inventory problem with processed and non-processed products of same perishable goods

Hiroaki ISHII

School of Science and Technology Kwansei-Gakuin University, 2-1 Gakuen Sanda Hyogo 565-0799 Japan

Abstract

In the field of inventory problems, several researchers have been interested in inventory control for a perishable product such as blood, fresh fruit, milk, film etc. Though there are huge number of research papers on perishable inventory, we only cite related papers ([1], [2], [3], [4]). This paper considers two types of products for same perishable goods such as matured tomato and un-matured one, fresh milk and processed milk etc. That is, one is very fresh and so its lifetime is one. The other is not so fresh and its lifetime is two. We consider how to order two products.

Keywords: Perishable inventory, fresh product, lifetime, processed and non-processed one

1. Introduction

In the field of inventory problems, several researchers have been interested in inventory control for a perishable product such as blood, fresh fruit, milk, film etc. Though there are huge number of research papers on perishable inventory, we only cite related papers ([1], [2], [3], [4]).

This paper considers two types of products for a same perishable goods such as matured tomato and un-matured one, fresh milk and processed milk etc.

That is, one is very fresh and so its lifetime is one. The other is not so fresh and its lifetime is two. One type is so called non-processed one and so it has a single life time period but it is very delicious, for example, tomato delivered after fully matured. The other is so called processed one and it has two period life time.

That is, for proceeded one, there exists, one with remaining life time one in the stock and that with remaining life time two (newly delivered). One example is tomato delivered before matured one. Customer who prefers delicious one (that is, sensitive to taste) usually buys non-processed one but if it is sold out, 100p percent of customers who cannot buy non-processed one buy processed one with remaining life time one that are not purchased by the customer is discarded at the unit cost $\theta$. While processed one with life time two that are not purchased by the customer is stocked with cost $h$ for the unit. We assume that $\theta \geq h$.

The demand $D_1$ of the customer for non-processed one and that $D_2$ for processed one are nonnegative random

Finally, after we discussed further research problems in section 4, we conclude our problems.

2. Problem Formulation

1) Ordering takes a place at the start of the period under the condition that some processed products with remaining life time one in the stock. The ordering amount of the non-processed product is denoted with $x_1$ and unit ordering price is $c_1$. Similarly ordering amount of processed one is denoted with $x_2$ and unit ordering cost is $c_2$. $x_1, x_2$ are decision variables.

2) Issuing policy is LIFO for the processed ones, that is, customer buys products with remaining life time two first and if these are sold out, the customer buys the old one, that is, one in the stock. Unit selling prices of Inventory problem with two types of products: non-processed one is $r_1$ and those of the processed one with remaining life time two (newly delivered one), remaining life time one $r_2, r_3$ respectively. We assume that $r_1 > r_2 > r_3 > 0, r_1 > c_1, r_3 \geq c_2$.

3) The non-processed one and processed one with life time one that are not purchased by the customer is discarded at the unit cost $\theta$. While processed one with life time two that are not purchased by the customer is stocked with cost $h$ for the unit. We assume that $\theta \geq h$.

The demand $D_1$ of the customer for non-processed one and that $D_2$ for processed one are nonnegative random
variables. Their cumulative distribution functions are 
\( F_1(D_1), F_2(D_2) \) respectively and their density functions 
\( f_1(D_1), f_2(D_2) \) respectively where

\[ F_1(0) = F_2(0) = f_1(0) = f_2(0) = 0 \]

Under the above setting with a stock of processed one 
\( z \), we calculate a total expected profit function \( E(x_1, x_2) \) in the next section.

### 3. Total Expected Profit Function

In relation to demand and order quantity, the total profit can be defined in the following cases A-H where 
purchasing cost is 
\( c_1x_1 + c_2x_2 \).

In cases A-C under condition that \( D_1 \leq x_1 \), the demand \( D_1 \) is less than the order quantity \( x_1 \), and the demand will 
be satisfied by non-processed products.

**Case A:** \( D_1 \leq x_1, D_2 \leq x_2 \).

Total profit is

\[ r_1D_1 + r_2D_2 - \theta(x_1 - D_1 + z) - h(x_2 - D_2) - (c_1x_1 + c_2x_2) \]

**Case B:** \( D_1 \leq x_1, x_2 \leq D_2 \leq x_2 + z \).

Total profit is

\[ r_1D_1 + r_2x_2 + r_3(D_2 - x_2) - \theta(x_1 - D_1 + z + x_2 - D_2) - (c_1x_1 + c_2x_2) \]

**Case C:** \( D_1 \leq x_1, D_2 \geq x_2 + z \).

Total profit is

\[ r_1D_1 + r_2x_2 + r_3z - \theta(x_1 - D_1) - (c_1x_1 + c_2x_2) \]

In cases D-H under condition that \( D_1 \geq x_1 \), the demand 
\( D_1 \) exceeds the order quantity \( x_1 \), and the demand is 
satisfied by both non-processed and processed products.

**Case D:** \( x_1 \leq D_1 \leq x_1 + \frac{x_2}{p}, x_2 - p(D_1 - x_1) \geq D_2 \).

Total profit is

\[ r_1x_1 + r_2(D_2 + p(D_1 - x_1)) - \theta x_2 - p(D_1 - x_1) - D_2) - (c_1x_1 + c_2x_2) \]

**Case E:** \( x_1 \leq D_1 \leq x_1 + \frac{x_2}{p}, x_2 - p(D_1 - x_1) \leq D_2 \leq 
\]

\[ z + x_2 - p(D_1 - x_1) \]

Total profit is

\[ r_1x_1 + r_2x_2 + r_3(D_2 - x_2 + p(D_1 - x_1)) - \theta(z - D_2 + x_2 - p(D_1 - x_1)) - (c_1x_1 + c_2x_2) \]

**Case F:** \( x_1 \leq D_1 \leq x_1 + \frac{x_2}{p}, D_2 \geq z + x_2 - p(D_1 - x_1) \)

Total profit is

\[ r_1x_1 + r_2x_2 + r_3z - (c_1x_1 + c_2x_2) \]

**Case G:** \( D_1 \geq x_1 + \frac{x_2}{p}, D_2 \leq z + x_2 - p(D_1 - x_1) \)

Total profit is

\[ r_1x_1 + r_2x_2 + r_3(D_2 - x_2 + p(D_1 - x_1)) - \theta(z - D_2) - (c_1x_1 + c_2x_2) \]

**Case H:** \( D_1 \leq x_1 + \frac{x_2}{p}, D_2 \geq z + x_2 - p(D_1 - x_1) \)

Total profit is 
\[ r_1x_1 + r_2x_2 - p(D_1 - x_1) \]

From these cases, the expected value of total profit 
\( E(x_1, x_2) \) can be derived as follows.

\[ E(x_1, x_2) = \int_0^{x_1} f_1(D_1) \left[ r_1D_1 - \theta(x_1 - D_1) + \int_0^{x_2} \left( r_2D_2 - \theta z - h(x_2 - D_2) \right) \right] f_2(D_2) dD_2 \]

\[ + \int_0^{x_1} \left( r_2x_2 + r_3(D_2 - x_2) - \theta(z - D_2) \right) f_2(D_2) dD_2 \]

\[ + \int_{x_2 + z}^{\infty} \left( r_2x_2 + r_3z \right) f_2(D_2) dD_2 \]

\[ + \int_{x_1}^{x_2 + z} f_1(D_1) \left[ r_1x_1 \right] \]

\[ + \int_0^{x_2 - p(D_1 - x_1)} \left( r_2(D_2 + p(D_1 - x_1)) - \theta x_2 - h(x_2 - p(D_1 - x_1)) - D_2) \right) f_2(D_2) dD_2 \]

\[ + \int_{x_2 - p(D_1 - x_1)}^{x_2} \left( r_2x_2 \right) f_2(D_2) dD_2 \]

\[ + \int_{x_2 - p(D_1 - x_1)}^{x_2} \left( r_3(D_2 - x_2 + p(D_1 - x_1)) - \theta(z - D_2 + x_2 - p(D_1 - x_1)) f_2(D_2) dD_2 \]

\[ + \int_{x_2 - p(D_1 - x_1)}^{\infty} \left( r_3(D_2 - x_2 + p(D_1 - x_1)) - \theta(z - D_2 + x_2 - p(D_1 - x_1)) f_2(D_2) dD_2 \]

\[ + \int_{x_2 - p(D_1 - x_1)}^{\infty} \left( r_3x_2 f_2(D_2) dD_2 \right) dD_1 - c_1x_1 - c_2x_2 \]
4. Optimal Ordering Quantity

To consider the optimal values of \( x_1 \) and \( x_2 \), we consider the partial derivative of the expected value \( E(x_1, x_2) \).

\[
\frac{\partial E(x_1, x_2)}{\partial x_1} = -\theta F_1(x_1)
\]
\[
+ \int_{x_1}^{x_1 + \frac{\theta}{p}} f_1(D_1) dD_1 \left[ r_1 - p(r_2 + h) F_2(x_2 - p(D_1 - x_1)) - p(r_3 + \theta) F_2(z + x_2 - p(D_1 - x_1)) - F_2(x_2 - p(D_1 - x_1)) \right]
\]
\[
- \int_{x_1}^{x_1 + \frac{\theta}{p}} f_1(D_1) dD_1 \left[ r_1 - p(r_3 + \theta) F_2(z + x_2 - p(D_1 - x_1)) \right] - c_1
\]

\[
\frac{\partial E(x_1, x_2)}{\partial x_2} = r_2 - F_1(x_1) [(h - \theta) F_2(x_2) + (r_2 + h) F_2(x_2)]
\]
\[
- (r_3 + \theta) \int_{x_1}^{x_1 + \frac{\theta}{p}} f_1(D_1) dD_1 \left[ r_2 + h F_2(x_2 - p(D_1 - x_1)) + (r_3 + \theta) F_2(z + x_2 - p(D_1 - x_1)) \right]
\]
\[
- p^2(r_3 + \theta) \int_{x_1}^{x_1 + \frac{\theta}{p}} f_1(D_1) dD_1 \left[ F_1(D_1) F_2(x_2 - p(D_1 - x_1)) \right]
\]

\[
\frac{\partial^2 E(x_1, x_2)}{\partial x_1^2} = -\int_{x_1}^{x_1 + \frac{\theta}{p}} f_1(D_1) dD_1 \left[ (h - \theta) F_2(x_2) + (r_3 + \theta) F_2(z + x_2) \right]
\]
\[
- \int_{x_1}^{x_1 + \frac{\theta}{p}} f_1(D_1) dD_1 \left[ F_1(D_1) F_2(z + x_2 - p(D_1 - x_1)) \right]
\]

If \( p \) is small enough, then \( \frac{\partial^2 E(x_1, x_2)}{\partial x_1^2} \) is non-positive \( \Rightarrow E(x_1, x_2) \) is concave function of \( x_1, x_2 \) fixed. Further if \( p \) is small enough

\[
\lim_{x_1 \to 0, x_2 \to 0} \frac{\partial E(x_1, x_2)}{\partial x_1} \geq r_1 - c_1 - p(r_3 + \theta) > 0,
\]

\[
\lim_{x_1 \to 0} \frac{\partial E(x_1, x_2)}{\partial x_2} = \int_{0}^{\frac{\theta}{p}} f_1(D_1) dD_1 \left[ -hpF_2(x_2 - pD_1) - p(r_3 + \theta) (F_2(x_2 + z + x_2 - pD_1) - F_2(x_2 - pD_1)) + r_1 - c_1 > 0 \right.
\]

For fixed \( x_1 \), optimal quantity \( x_1 \) is the stationary number satisfying \( \frac{\partial E(x_1, x_2)}{\partial x_1} = 0 \) if \( p \) is enough small positive number. This stationary number is non-increasing since \( \frac{\partial^2 E(x_1, x_2)}{\partial x_1 \partial x_2} \) is negative.

While if \( z, p \) is small enough, \( \frac{\partial^2 E(x_1, x_2)}{\partial x_2^2} \) may be positive \( \Rightarrow E(x_1, x_2) \) is convex function of \( x_2 \) if \( x_1 \) fixed. Some stational point of \( \frac{\partial E(x_1, x_2)}{\partial x_2} \) is an optimal solution in that \( x_1 \) fixed.
The order quantity $x_1$ and $x_2$ can be determined specifically by using binary search method either $x_1$ or $x_2$ after fixing the other.

5. Conclusion

We have discussed two types of products for the same goods, that is, non-processed one and processed one. But in our model, it may be difficult to derive optimal ordering quantities explicitly. We only derived properties of optimal ordering quantities for a limit case. Sensitivity of price $r_1, r_2, r_3$ is important, that is, analysis on change of optimal ordering quantities $x_1, x_2$ depending on these prices. Further we do not consider the shortage cost. Shortage cost is usually hard to be estimated. $L$ fuzzy number should be considered and if it is introduced in our model, the expected total profit function becomes an $L$ fuzzy number. Using some fuzzy order, we need to seek some non-dominated ordering quantities since the fuzzy order is not linear order.

References

[1] H. Ishii: Perishable inventory problem with two types of customers and different selling prices, Journal of the Operations Research Society of Japan, 30, (1993), 199-205
[2] H. Ishii: A promotion sale problem for a perishable product (Proceedings of the 16th Czech-Japan Seminar on Data Analysis and Decision Making under Uncertainty, (2013) 1-6.
[3] H. Ratnagiri and H. Ishii: An inventory problem with a perishable and non-perishable one, Asia Pacific Management Review, 8, (2003), 477-485.
[4] S. Nahmias: Perishable inventory theory: Review, Operations Research, 30, (1963), 680-708.
[5] N. Furukawa: Parametric orders on fuzzy numbers and their roles in fuzzy optimization problems: Optimization, 40, (1997), 171-192.