In two recent Letters [1, 2], Hirakawa et al. investigate the emission of coherent terahertz (THz) radiation from optically excited superlattices [3]. The authors interpret the spectra of the THz transients in terms of a conductivity is, however, not performed correctly, thus making the conclusions of Refs. [1, 2] void.

In Refs. [1, 2], the authors employ an elegant way to analyze the response of the optically excited electrons to the constant bias field in the semiclassical approximation. They invoke the identity of this response with that of electrons present in a superlattice when the electric bias field is switched on abruptly (\( F(t) = F_0 \delta(t) \)). Based on this analogy, a relationship between a time-dependent conductivity \( \sigma(t) \) and the THz-field transient is derived (\( \sigma(t) \propto E_{THz}(t)/F \), Eq. (6) in Ref. [2]), which allows to relate the spectra of the THz transients directly to the frequency-dependent conductivity \( \sigma(\omega) \).

The authors then make the mistake to interpret these data on the basis of an identification of \( \sigma(\omega) \) with the theoretically derived function \( \sigma_\omega(\omega) \) of Ref. [6] which describes the small-signal response of electrons in a dc-biased superlattice to an additional ac electric field. Because of the highly nonlinear field dependence of the electron current, this response is generally very different from that to the switching of the full bias field as described by \( \sigma(\omega) \).

We illustrate the fundamental difference in Fig. 1, which displays theoretical results for the real parts of conductivity functions \( \sigma(\omega) \) (full line) and \( \sigma_\omega(\omega) \) (dashed line). Inset: current density in a superlattice vs. electric field, and illustration of the different nature of the conductivities for \( \omega \to 0 \). While \( \sigma_\omega(0) = dj/dF|_{j} \), the small-signal gain at a chosen bias field \( F = F_0 \), is given by the slope of the \( j(F) \) curve, the full-field-switching response \( \sigma(0) \) is given by the slope \( j/F_0 \).

![FIG. 1: Main panel: Real part of conductivity functions \( \sigma(\omega) \) (full line) and \( \sigma_\omega(\omega) \) (dashed line). Inset: current density in a superlattice vs. electric field, and illustration of the different nature of the conductivities for \( \omega \to 0 \).](image)

Indeed, \( \text{Re}(\sigma_\omega(\omega)) \) is negative for \( \omega < 1/\tau_e \sqrt{1 - \omega_B^2(\tau_p^2 - \tau_e^2)} \) which is indicative for gain at these frequencies. In contrast, \( \text{Re}(\sigma(\omega)) \) is positive for all frequencies and does not evince a gain signature.

These results show that one must not compare spectra of THz transients of the kind discussed here with the small-signal response function \( \sigma_\omega(\omega) \). We finally note, that the negative \( \text{Re}(\sigma(\omega)) \) values derived from the experiments in Ref. [2] either have a different (non-semiclassical) origin or are artifacts of the interpolation of the zero-time-delay position.

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