Columnar Defects and Scaling Behavior in Quasi 2D Type II Superconductors

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Persistent scaling behavior of magnetization in layered high $T_c$ superconductors with short–range columnar defects is explained within the Ginzburg Landau theory. In the weak field region, the scaling function differs from that of a clean sample and both the critical and crossing temperatures are renormalized due to defects. In the strong field region, defects are effectively suppressed and scaling function, as well as critical and crossing temperatures are the same as in a clean superconductor.

This picture is consistent with recent experimental results.

Layered high-temperature superconducting (HTSC) materials, such as Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ and Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10}$, are known to exhibit 2D scaling magnetic properties [1–4] around the mean field transition line $H_{c2}(T)$. It is manifested by inspecting the magnetization $M_0$ as a function of temperature $T$ (measured in energy units) and the (external) magnetic field $H$:

$$\frac{s\Phi_0}{A\sqrt{TH}}M_0(T, H) = -(\sqrt{x^2 + 2 - x}), \quad (1)$$

where $s$ is an effective interlayer spacing, $\Phi_0$ is the flux quantum, $x = AH_{c2}'[T - T_{c2}(H)]/\sqrt{TH}$ is the scaling variable, $H_{c2}' = -dH_{c2}(T)/dT|_{T = T_{c2}}$, and $T_{c2}$ is the zero field critical temperature. For a superconductor with Ginzburg-Landau (GL) parameter $\kappa$ and Abrikosov geometric factor $\beta_A$ the constant $A = \sqrt{s\Phi_0/p}$, where $p = 16\pi\kappa^2\beta_A$. The form of the scaling function (1) implies the existence of a crossing point: at some temperature $T_0^s = T_{c0}(1 + 1/(2A^2H_{c2}'))^{-1}$, the sample magnetization is independent on $H$, $M_0^s = M_0^c(T_0^s, H) = -T_0^s/(s\Phi_0)$.

Recently, the influence of linear defects (columnar defects, artificial holes etc.) on the magnetic properties of superconductors has been studied experimentally [5] and theoretically [6,7,12]. In particular, experiments by van der Beek et. al. [7] showed that in HTSC with columnar defects the reversible magnetization of the sample is drastically affected, and that there are now two scaling regimes pertaining to relatively weak $H < H_{c2}$ and strong $H > H_{c2}$ magnetic fields (here the matching field $H_{c2} = n_d\Phi_0/p$ is proportional to the 2D density of defects $n_d$). These two scaling regimes correspond to two different critical temperatures (used in Ref. [6] as fitting parameters) and crossing points.

In this Letter we propose an explanation of these results. Let us commence by presenting some intuitive arguments. Consider the quantity $c = H_{c2}/H$ which, in a macroscopic sample, is the number of defects divided by the number of vortices. The magnetic field then serves as a control parameter for tuning the effective concentration $c$ of defects. In the weak field region, $c$ is large, each vortex is affected by a force emanating from many defects, and the fluctuations of this force play the main role. Short-range defects could be taken into account perturbatively. In first order they retain the same form of scaling function as that of a clean sample but renormalize the critical temperature $T_c$. Second order corrections indeed destroy the scaling behavior but in the vicinity of the crossing temperature scaling is approximately maintained. In the strong field region, $c$ is small, and the standard concentration expansion [13] can be used. Here, even the first order correction (with respect to small concentration) destroys the scaling behavior. However, a strong field effectively suppresses the defects, thus restoring the scaling behavior of a clean superconductor with the initial critical temperature $T_{c0}$. Identifying the two fitting temperatures of Ref. [7] with the renormalized critical temperature $T_c$ and the initial one $T_{c0}$ respectively, one finds for the dimensionless defect strength $\theta_1 = 0.49$, well inside its allowed range $0 \leq \theta_1 \leq 1$. This indicates a full consistence between the description constructed below and the experimental results of Ref. [7].

Our quantitative discussion employs an approach proposed and successfully used for arbitrary fields in clean superconductors [6,12] and for very low fields in disordered superconductors [10]. Here we use it for disordered superconductors in much higher fields. Consider an irradiated thin superconducting film (or one layer in a layered superconductor) with area $S$ subject to perpendicular magnetic field (thus parallel to the defects). The effective interlayer separation $s$ is assumed to be much larger than the effective superconducting coherence length $\xi(H, T)$ in the magnetic field direction but much smaller than the magnetic penetration depth. Then the problem becomes effectively two dimensional [14]. Columnar defects can be described as a local reduction of the critical temperature $\delta T_{c0}(r) = T_{c0}\sum j \exp(-|r - r_j|^2/2L^2)$. Here $r$ is a two dimensional vector in the film plane, $L$ is the defect radius, and the positions $r_j$ of defects are uniformly and independently distributed over the film plane with density $n_d$. The value of $n_d$ is assumed to be moderate so that for the pertinent region of temperature the matching field $H_{c2}$
is always much smaller than $H_{c2}(T)$. The dimensionless amplitudes of defects $t_j \leq 1$ are also independent random quantities distributed with some probability density. The thermodynamic properties of a type-II superconductor with $\kappa \gg 1$ containing $N_v$ vortices are described by its partition function

$$Z \propto \int \mathcal{D}\{\Psi\} \exp(-N_v g[\Psi]),$$

where $\Psi$ is the corresponding order parameter. The dimensionless GL free energy $g[\Psi]$ of an irradiated superconductor is given by an expression

$$g = x|\Psi|^2 + (4\beta_A)^{-1}|\Psi|^4 + \tau|\Psi|^2,$$

where bar denotes averaging over the sample area. The scaling variable $x$ and the local temperature $\tau(r)$ are defined below for each region of the magnetic field.

Following \cite{14}, we replace in Eq. (3) $|\Psi(r)|^2$ by $\beta_A \left(|\Psi(r)|^2\right)^2$, where $\beta_A \sim 1.16$ is the Abrikosov factor for a triangular lattice. This replacement is based on the assumption that the distribution of vortices is almost uniform in both regions of the magnetic field considered here. It is supported by noticing a remarkable difference between the number of vortices and the number of defects in both regions of fields \cite{13}. This substitution, together with the simplest version of the Hubbard-Stratonovich transformation (introduction of an additional integration over some auxiliary field $\gamma$) turns the problem to be an exactly solvable one \cite{10}. Then project the order parameter on the lowest Landau level (LLL) subspace,

$$\Psi(r) = \sum_{m=0}^{N_v} C_m L_m(r),$$

where $L_m(r)$ are normalized LLL eigenfunctions with orbital momentum $m$. As was recently demonstrated \cite{14}, the LLL approximation works quite well even down to $H \geq H_{c2}(T)/13$. After integration over the expansion coefficients $C_m$ the partition function \cite{3} reads,

$$Z \propto \int \exp \left\{ -N_v \mathcal{L}(\gamma, x) \right\} d\gamma,$$

where

$$\mathcal{L}(\gamma, x) = -\gamma^2 + N_v^{-1} \text{tr} \ln \left[ (x + \gamma) \hat{I} + \hat{\tau} \right]$$

and $\hat{\tau}$ is a random matrix with elements:

$$\tau_{mn} = \int S L^*_m(r) \tau(r) L_n(r) d^2r.$$  

The contour $\Gamma$ in Eq. (3) is parallel to the imaginary axis and stretches from $\gamma^* - i\infty$ to $\gamma^* + i\infty$. To assure convergence of the integrals over the coefficients $\{C_m\}$ the real constant $\gamma^*$ should satisfy the inequality $\gamma^* + x + \min \tau_n > 0$, where $\tau_n$ is the $n$-th eigenvalue of the matrix $\tau_{mn}$.

In the thermodynamic limit $S \to \infty$ with $n_d = N_v/S$ fixed, the partition function \cite{3} could be calculated in a saddle point approximation. This results in the following form for the magnetization

$$\frac{s\Phi_0}{A\sqrt{HT}} M(T, H) = (N_v Z)^{-1} \partial Z / \partial x = -2\gamma(x),$$

where $\gamma(x)$ is the solution of the saddle point equation $\partial \mathcal{L}(\gamma, x) / \partial \gamma = 0$. For a clean superconductor ($\bar{\tau} = 0$) one gets two possible saddle points but only one of them

$$\gamma_0(x) = \frac{1}{2}(\sqrt{x^2 + 2} - x)$$

can be reached by an allowed deformation of the contour $\Gamma$. Substitution of Eq. (9) into (8) yields the magnetization $M_0(T, H)$ of a clean sample \cite{1} obtained in Ref. \cite{4}. Note that $-2\gamma_0(x)$ serves as the appropriate scaling function. To study the disordered case, we consider separately two regions of the magnetic field.

In the weak field region $H < H_b$ we, from the onset, take into account the renormalization of the critical temperature caused by defects. As a result, the scaling variable $x$ is defined in the same way as for a clean superconductor albeit with renormalized critical temperature $T_c = T_{c0} - \delta T_c$, where

$$\delta T_c = <\delta T_c(r)> = 2\pi \theta_n n_d L^2 T_{c0}$$

and $\theta_n \ll t^n$ where here and below $< .. >$ implies ensemble average. The function $\bar{\tau}(r)$ in this field region, defined as $\bar{\tau}(r) = (\delta T_c(r) - \delta T_c) A H_{c2}^2 / \sqrt{HT}$, represents temperature fluctuations caused by short-range defects. They are small and can be accounted for perturbatively. Then, in the thermodynamic limit, the last term on the r.h.s. of Eq. (3) has an explicit self-averaged structure $N_v^{-1} \text{tr}(\delta \Gamma)$ and can be replaced by its average. This procedure modifies the saddle point equation and therefore results in a modified magnetization

$$M(T, H) = M_0(T, H) \left( 1 + \varepsilon(T) \frac{2\gamma_0(x)}{\sqrt{x^2 + 2}} \right)$$

where

$$\varepsilon(T) = \left( \frac{\text{tr} \bar{\tau}^2}{N_v} \right) = \frac{\theta^2}{p} n_d L^2 (2\pi H_{c2}^2 T_{c0})^2 s L^2.$$

Note that the parameter $\varepsilon(T)$ is proportional to the fourth power of the defect radius $L$ thus justifying the perturbation approach for short-range defects.
renormalization of the critical temperature, the crossing temperature \( T^* = T_0^* - \delta T^* \) differs from its value \( T_0^* \) in a clean sample: \( \delta T^* = 6T_0(1 + (2A^2H'_{c2})^{-1})^{-1} \). In the next order, scaling is virtually destroyed, since the correction term (within the parenthesis in Eq. (14)) depends not only on the scaling variable \( x \) but also on temperature. Yet, in a sufficiently narrow region around some temperature \( T \), the deviation from scaling is negligibly small, but the scaling function itself is modified to be 
\(-2\gamma(x, T)\). At temperature \( T^* \) the magnetization reads

\[
M(T^*, H) = M_0(T^*) \left(1 + \varepsilon(T^*) \frac{2H^*}{H + H^*}\right),
\]

where \( H^* = H_{c2}(T^*) = T^*/(2A^2) \). Therefore if the field is weak enough, \( H \ll H^* \), then the crossing point is restored, \( T^* \) serves as a true crossing temperature and the magnetization at the crossing temperature differs from its unperturbed form \(-2\gamma_0(x)\) merely by a multiplicative constant \( 1 + \varepsilon(T^*)\).

When the magnetic field increases, the approach used above becomes inapplicable. Firstly, it fails in the vicinity of the matching field where the Abrikosov factor becomes very sensitive to the details of defect configuration. Secondly, higher order terms in the perturbation expansion for the saddle point equation (which are omitted), grow with magnetic field. Fortunately, we have here a new small parameter, that is, the dimensionless concentration of defects. It is then natural to use the concentration expansion. In such a case there is no sense in renormalizing the critical temperature, and the dimensionless temperature \( \tau(x) \) is now defined as

\[
\tau(x) = \delta T(x) = \frac{\tau(x)}{\sqrt{T^*}}.
\]

As mentioned above, the second term in the r.h.s. of Eq. (16) is self-averaging and can be calculated using the limiting form of the density of states \( \rho(\tau) \) of the matrix \( \rho \), which, for short-range defects in linear approximation with respect to \( c \), reads

\[
\rho(\tau) = (1 - c)\delta(\tau) + \frac{c}{\lambda\lambda'}(\tau - \lambda'),
\]

where \( \lambda = 2\pi L^2 T_{c0} A H'_{c2} H(\Phi_0 \sqrt{T})^{-1} \) and \( p(t) \) is probability distribution of the dimensionless temperature \( t_j \). Indeed, the matrix \( \tau_{mn} \) is nothing but the Hamiltonian of a particle with charge \( 2e \) in a 2D system subject to a perpendicular magnetic field and containing short-range defects (projected on the LLL). The first and second terms in Eq. (16) correspond, respectively, to those states whose energy is stuck to the LLL (despite the presence of zero-range defects (see e.g. (17))) and those states whose energies are lifted from the LLL by these defects. For sufficiently narrow distribution \( p(t) \), the corresponding saddle-point equation leads to the magnetization

\[
M = M_0 \left(1 - \frac{c\lambda\theta_1}{1 + 2\lambda\theta_1\theta_0(x)\sqrt{\tau^2 + 2}}\right),
\]

were \( M_0(T, H) \) is given by Eq. (14) with an initial critical temperature \( T_{c0} \).

Rigorously speaking, scaling is destroyed since both the concentration \( c \) and the shifted eigenvalue \( \theta_1 \) depend explicitly on \( H \) and \( T \). However, at strong field the correction term in Eq. (16) becomes negligibly small. This implies a restoration of the crossing point. Indeed, at temperature \( T_0^* \) the magnetization \( M^* = M(T^*, H) \) assumes the form

\[
M^* = M_0^* \left(1 - \frac{H_0}{1 + \eta H_{c1} + H^*}\right),
\]

with \( \eta^{-1} = 2\pi L^2 H'_{c2} \theta_1(\theta_0)/(\Phi_0 \sqrt{T^*}) \). Therefore in the entire strong field region \( H_0 \ll H \ll H^* \) the crossing temperature coincides with its initial value \( T_0^* \) and the magnetization in the crossing point practically coincides with its value \( M_0^* \) in a clean superconductor.

Let us now discuss the limits of applicability of our results and their relation to the experiment of Ref. (14). Note that the first two moments \( \theta_1, \theta_2 \) of the random dimensionless temperature \( t \) satisfy the inequality

\[
0 \leq \theta_1^2 \leq \theta_2^2 \leq 1.
\]

In the pertinent region of fields \( 0.2 \leq 5T, \) a typical defect radius \( L \sim 3.5 \text{nm} \) is at least one order of magnitude smaller than the magnetic length, hence the defects can definitely be taken as short-range ones. In the weak field region, the important small parameters are then \( \varepsilon(T^*) \) (which enters the magnetization \( \rho \)) and \( \varepsilon(T^*)/(x + \gamma_0(x))^2 \) (which enters the saddle point equation). Using parameters from the experimental setup \( s = 1.5 \text{nm}, \ k_B \mu_0 H'_{c2} = 1.15 \text{TK}^2 \), \( \kappa = 100, \ n_d = 5 \times 10^{10} \text{cm}^{-2}, \) \( \mu_0 H_0 = 1 \text{T}, \) \( T = 75 \div 85 \text{K}, \) \( T^* = 78.9 \text{K} \), we find from Eq. (16) \( \varepsilon(T^*) = 0.5 \theta_2 \) and \( \varepsilon(T^*)/(x + \gamma_0(x))^2 \approx 0.25 \) (the latter figure is obtained for \( \mu_0 H = 0.2 \text{T} \)). For quite plausible value \( \theta_2 = 0.5 \) one then finds \( \varepsilon(T^*) = 0.25 \theta_2 \approx 0.25 \). The condition of convergence of the integral over the expansion coefficients \( \{C_m\} \) can be written as \( H > 0.25 \text{H}_0 \theta_2^2 / \theta_2 \) and even in the worst case \( \theta_2^2 = \theta_2 \) it reads \( \mu_0 H \approx 0.25 \text{T} \). Finally, one has \( \mu_0 H^* \approx 6.4 \text{T} \) and applicability of the LLL projection requires \( \mu_0 H > 0.5 \text{T} \). The weak field region of Ref. (14) corresponds to \( \mu_0 H = 0.2 \div 0.02 \text{T} \). Thus, in the weak field region, the condition for applicability of the LLL projection is slightly violated, but the deviation is not dramatic. In the strong field region we find \( \eta \approx 2.9 \) and therefore the correction term in parenthesis of equation (16) is less than three percents. Hence, in this region our assumptions are fully satisfied.

Using the same set of parameters we display in figure 1 the quantity \( M/\sqrt{T^*H} \) as a function of the scaling variable for weak field (inset) and strong field (main part).

We used here the maximal value \( \theta_1 = 1 \). In the strong field region, the deviation from clean sample scaling behavior is negligibly small for all three values of strong magnetic field.
In the weak field region, the scaling functions for three different fields can hardly be distinguished. This means that scaling is undoubtedly valid in a vicinity of the crossing temperature. At the same time the scaling function differs from its form in a clean sample by a multiplicative constant (see the parenthesis in Eq. (11)). Note that scaling in the weak field region (which was experimentally established) is less pronounced than that in the strong field region. Apparently, the reason is that the experimental data are fitted to account for the clean sample scaling function. Nevertheless if we identify the fitted temperature 82.6K (found in Ref. [7] in the weak field region) with the renormalized critical temperature $T_c = T_c^0 - \delta T_c$ and the fitted critical temperature 84.2K in the strong field region with $T_c^0$, then, even within such a rough approximation, we obtain $\theta_1 \approx 0.5$. Recalling that $\theta_1$ should be positive and less than unity, the above result strongly supports the applicability of our theory to the pertinent experiment [6].

In summary, we calculated the magnetization of an irradiated superconductor below the mean–field transition line $H_{c2}(T)$, using the approach developed in Refs. [2, 4, 10]. It was shown that, from a rigorous point of view, disordered short-range defects are expected to destroy the scaling behavior and prevent the existence of crossing point in both regions of weak and strong magnetic fields (with respect to matching field $H_\Phi$). And yet, in the framework of the experimental setup the deviation from scaling behavior appears to be negligibly small and crossing points exist in both field regions, in complete agreement with the experimental findings. The two fitting critical temperatures introduced in Ref. [7] for the strong and weak field regions correspond, in our formalism, to the initial and renormalized critical temperatures.

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