Erraticity analysis of multiparticle production in $\pi^+ p$ and $K^+ p$ collisions at 250 GeV/$c$

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Abstract

The erraticity behavior of multiparticle production is analyzed in $\pi^+ p$ and $K^+ p$ collisions at 250 GeV/$c$. It is demonstrated that, for these low-multiplicity final states, the erraticity measure based on event-to-event fluctuation of factorial moments is dominated by statistical fluctuations.

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1. Introduction

With the development of the intermittency study of high energy collisions [1], the event-by-event analysis draws more and more attention. One interesting suggestion in this respect is the erraticity analysis of fluctuations proposed by Cao and Hwa [2]. The authors define an entropy index $\mu_q$, which can be used as a criterion of chaos in multiproduction where only spatial patterns but no temporal development can be investigated.

Erraticity analyses have been performed in both hadron–hadron and nucleus–nucleus collisions [3–6]. All the results show a positive value of $\mu_q$, indicating the existence of an erraticity behavior, i.e., fluctuations from event to event, in multiparticle systems. However, the origin of these fluctuations is still unclear.
In this Letter we present the results of a study of erraticity in \( \pi^+ p \) and \( K^+ p \) collisions at 250 GeV/c. In order to find the physical reasons which may cause the erraticity behavior, we investigate the relation between erraticity and related physical quantities. Finally, we compare the experimental erraticity behavior with that of three different kinds of Monte Carlo model.

2. Erraticity

Cao and Hwa [2] proposed to measure the phase-space pattern of a multiparticle system by factorial moments associated with it. In contrast to the space pattern of a multiparticle system by factorial moments [7], they define event factorial moments

\[
F_{q}^{(e)} = \left[ \frac{1}{M} \sum_{m=1}^{M} n_m(n_m - 1) \cdots (n_m - q + 1) \right] \\
\times \left[ \frac{1}{M} \sum_{m=1}^{M} n_m \right]^{-q},
\]

where \( M \) is the partition number in phase space, \( n_m \) is the number of particles falling into the \( m \)th bin, and \( q = 2, 3, 4, \ldots \) is the order of the moment. The fluctuations of \( F_{q}^{(e)} \) from event to event can be quantified by its normalized moments as:

\[
C_{p,q} = \langle \Phi_{q}^{p} \rangle, \quad \Phi_{q} = \frac{F_{q}^{(e)}}{F_{q}^{(e1)}}, \quad (2)
\]

and by \( dC_{p,q}/dp \) at \( p = 1 \):

\[
\Sigma_{q} = \langle \Phi_{q} \ln \Phi_{q} \rangle. \quad (3)
\]

If power-law behavior of the fluctuation is observed as the partition number \( M \) goes to infinity (or as the resolution \( \delta = \Delta / M \) becomes very small),

\[
C_{p, q}(M) \propto M^{\psi_{q}(p)} \quad (M \to \infty), \quad (4)
\]

then this corresponds to chaoticity in a dynamic system where the time sequence can be generated. The power-law behavior of Eq. (4) is referred to as erraticity [8] of a multiparticle system. The derivative of the exponent \( \psi_{q}(p) \) at \( p = 1 \),

\[
\mu_{q} = \frac{d}{dp} \psi_{q}(p) \bigg|_{p=1} = \frac{\partial \Sigma_{q}}{\partial \ln M}, \quad (5)
\]

describes the width of the fluctuation and is called entropy index. The positive value of this index (\( \mu_{q} > 0 \)) has been proven to be a criterion of chaos [9].

3. Data sample

In the CERN experiment NA22, the European Hybrid Spectrometer (EHS) was equipped with the Rapid Cycling Bubble Chamber (RCBC) as an active target and exposed to a 250 GeV/c tagged, positive, meson enriched beam. In data taking, a minimum bias interaction trigger was used. The details of spectrometer and trigger can be found in [10,11].

Charged-particle tracks are reconstructed from hits in the wire- and drift-chambers of the two-lever-arm magnetic spectrometer and from measurements in the bubble chamber. The momentum resolution varies from 1–2% for tracks reconstructed in RCBC, to 1–2.5% for tracks reconstructed in the first lever arm and is 1.5% for tracks reconstructed in the full spectrometer.

Events are accepted for the analysis when the measured and reconstructed charged-particle multiplicity are the same, charge balance is satisfied, no electron is detected among the secondary tracks and the number of badly reconstructed (and therefore rejected) tracks is 0. The loss of events during measurement and reconstruction is corrected for by applying a multiplicity-dependent event weight normalized to the topological cross sections given in [9]. Elastic events are excluded. Furthermore, an event is called single-diffractive and excluded from the sample if the total charged-particle multiplicity is smaller than 8 and at least one of the positive tracks has a Feynman variable \( |x| > 0.88 \).

For laboratory-momenta \( p_{lab} < 0.7 \) GeV/c, the range in the bubble chamber and/or the change of track curvature is used for proton identification. In addition, a visual ionization scan is used for \( p_{lab} < 1.2 \) GeV/c. Positive particles with \( p_{lab} > 150 \) GeV/c are given the identity of the beam particle. Other particles with momenta \( p_{lab} > 1.2 \) GeV/c are not identified and are treated as pions.

After all cuts, the inelastic, non-single-diffractive sample consists of 44 524 \( \pi^+ p \) and \( K^+ p \) events.

In spite of the electron rejection mentioned above, residual Dalitz decay and \( \gamma \) conversion near the vertex...
still contribute to the two-particle correlations. Their influence has been investigated in detail in [12].

The initial intervals of the three phase-space variables, rapidity $y$, azimuthal angle $\varphi$ and transverse momentum $p_t$ used for the analysis, are defined as: $-2 \leq y \leq 2$, $0 \leq \varphi \leq 2\pi$, $0.001 \leq p_t \leq 10$ GeV/c, respectively. The results are based on 43,680 events with non-zero single-event (charged-particle) multiplicity within this initial acceptance region, so that its event factorial moment can be calculated. In order to reduce the effect of non-flat particle-density distribution $\rho(x)$ in phase space, the cumulative variable [13]

$$X(x) = \int_a^x \rho(x) \, dx \left[ \int_a^b \rho(x) \, dx \right]^{-1}$$

is taken for all variables, where $a$ and $b$ are the extreme points of the single-particle distribution $\rho(x)$.

4. Results and discussion

4.1. Erraticity behavior in different phase space variables

The results on $\Sigma_2$ in one-dimensional rapidity $y$, transverse momentum $p_t$ and azimuthal angle $\varphi$ are shown in Fig. 1(a)–(c). The asymptotic power-law behavior of $\Sigma_2$ with increasing partition number $M$ indicates the existence of erraticity in $\pi^+p$ and $K^+p$ collisions at 250 GeV/c. It is similar to the erraticity behavior observed in the NA27 data [4]. According to Eq. (5), the entropy index $\mu_2$ can be obtained from fitting $\Sigma_2$ vs. $\ln M$ by a straight line at large $M$. The first fourteen points are, therefore, omitted in the fit. The fit results, i.e., the entropy index and the corresponding proportionality constant are listed in the second and third columns of Table 1. The proportionality constant together with the fitted slope (entropy index $\mu$) will indicate the values of $\Sigma_2$. It can

| One-dimensional | Three-dimensional |
|-----------------|-------------------|
| $y$             | $p_t$             | $\varphi$          |
| $\mu_2$         | $\mu$            | Pro. constant     | Pro. constant     | Pro. constant     |
| $0.541 \pm 0.010$ | $-0.873 \pm 0.030$ | $0.822 \pm 0.026$ | $-1.935 \pm 0.127$ |
| $0.551 \pm 0.010$ | $-0.877 \pm 0.031$ |                |                  |
| $0.581 \pm 0.011$ | $-0.914 \pm 0.033$ |                |                  |

Fig. 1. $\Sigma_2$ vs. $\ln M$ in one dimension, (a) $y$ region, (b) $p_t$ region, (c) $\varphi$ region. Straight lines are linear fit results omitting the first fourteen points. Except for large $M$, the error bars are smaller than the points and not shown.
be seen from the figures and the table that the values of \( \Sigma_2 \) and \( \mu_2 \) are similar in the different variables. That is to say, erraticity is largely independent of the phase-space variable used for the analysis.

In Fig. 2, \( C_{p,2} \) and \( \Sigma_2 \) are shown vs. \( \ln M_y \), in three-dimensional \((y, p_t, \phi)\) bins, with \( M_y = M^{1/3} \). In comparison to the one-dimensional results, the three-dimensional results have a much better linear behavior, in particular after omitting the first three points. This is different from the intermittency behavior, where even in three-dimensional phase space, sample factorial moments show an upward bending behavior, well explained as being due to a self-affine fractal mechanism [14,15]. Omitting the first three points, the entropy index and proportionality constant from the three-dimensional fit are listed in the fourth and fifth column of Table 1.

4.2. Erraticity behavior and the single-event variables \( \bar{p}_t \) and \( n \)

A natural way to find out which factors affect the erraticity behavior, is to study the relations between the erraticity behavior and other physical quantities. In order to see if erraticity has a different behavior for different (hard or soft) processes, we first show how it changes with the average transverse momentum per event. The latter is defined as

\[
\bar{p}_t = \frac{\sum_{i=1}^{n} p_{ti}}{n},
\]

where \( n \) is the total number of (charged) particles in a single event. It has been shown [16] that \( \bar{p}_t \) is a good variable to characterize the softness or hardness degree of an event. We divide the full sample into three subsamples, i.e., \( \bar{p}_t < 0.28 \text{ GeV/c} \), \( \bar{p}_t < \langle \bar{p}_t \rangle \) and \( \bar{p}_t > \langle \bar{p}_t \rangle \), where \( \langle \bar{p}_t \rangle = 0.38 \text{ GeV/c} \) is the mean \( p_t \) of our event sample, \( \langle \cdot \cdot \cdot \rangle \) indicating averaging over all events. The values of \( \Sigma_2 \) are calculated in these three different \( \bar{p}_t \) regions, respectively. The results in both one-dimensional rapidity and three-dimensional \((y, p_t, \phi)\) bins are shown in the two upper sub-figures in Fig. 3. The entropy index is obtained by fitting \( \Sigma_2 \) vs. \( \ln M_y \). The fit results, slope (entropy index) and proportionality constant are listed in Table 2. As in the previous section, the first fourteen points are omitted in the one-dimensional fits, the first three in the three-dimensional fits. It can be seen from figures and table that, in both one-dimensional and three-dimensional cases, \( \Sigma_2 \) is similar for the different \( \bar{p}_t \) subsamples. This means that the erraticity behavior at this collision energy does not relate much to the softness or hardness degree of an event.

Then, we turn to study the relation between erraticity and multiplicity. We divide the whole sample into two subsamples with \( n < \langle n \rangle \) and \( n > \langle n \rangle \), where \( \langle n \rangle = 7.98 \) is the sample average multiplicity. The values of \( \Sigma_2 \) calculated in these two different multiplicity regions for both the one- and three-dimensional cases, together with the full-sample result, are shown in the two lower sub-figures of Fig. 3. Similarly, \( \Sigma_2 \) vs. \( \ln M_y \) is fitted in different multiplicity \( n \) subsamples and the fit results are listed in Table 2.
It can be seen from the figures that the erraticity behavior depends strongly on multiplicity. The values of the erraticity moments and the slope in the high-multiplicity sample are much smaller than those in the lower-multiplicity ones. This means much less fluctuation from event to event in the high-multiplicity subsample. When the event multiplicity is very low, the event factorial moments defined in Eq. (1) cannot fully eliminate the statistical fluctuation due to insufficient number of particles [19]. The lower the multiplicity is, the bigger are the statistical fluctuations contained in event factorial moments. So, the relation between erraticity and multiplicity may imply the contribution from these trivial statistical fluctuations. To confirm...
this argument, the following model comparisons are presented.

4.3. Comparison with Monte Carlo models

Various models that simulate low-\( pt \) processes in multiparticle production can readily generate average quantities, but fail in reproducing correctly the fluctuations from the average [1]. In particular, only few models can fit the intermittency data [17]. A color mutation model ECOMB, proposed recently by Cao and Hwa, is one of the few that can [8,18]. In [8], the authors applied the erraticity analysis to hadronic collisions generated by ECOMB at 22 GeV. Their results on the erraticity moments \( \ln C_{2,2} \) and \( \Sigma_2 \) versus the logarithm of the partition number \( M \) in the one-dimensional rapidity region obtained from 10^6 ECOMB Monte Carlo events are shown in Fig. 4 (open triangles connected by the dotted line) together with the experimental results from NA22 (full circles). We can see that the results from ECOMB agree with the rise of the experimental data, but stay below in absolute magnitude.

Then, we use the Lund Monte Carlo event generator PYTHIA 5.5 to simulate 40 000 \( \pi^+ p \) collisions at 250 GeV/\( c \) to see if it has erraticity behavior or not. It has been shown that the PYTHIA MC cannot reproduce the intermittency data [1] and its average factorial moment values are almost the same for different division numbers \( M \). The resulting \( \ln C_{2,2} \) and \( \Sigma_2 \) versus the logarithm of the division number \( M \) from PYTHIA are shown in Fig. 4 by the open rhombi connected by the dashed line. It turns out that the PYTHIA results agree with the data even better than ECOMB. So, the erraticity behavior at such a low collision energy may not be of dynamical origin, but mainly due to trivial statistical fluctuations.

In order to be more convincing, we simulate purely statistical fluctuations in the following way. The probability distribution in the full initial acceptance region is taken to be flat. The particles are distributed to every bin according to a Bernoulli fluctuation:

\[
B(n_1, \ldots , n_M|p_1, \ldots , p_M) = \frac{n!}{n_1! \cdots n_M!} p_1^{n_1} \cdots p_M^{n_M},
\]

\[
\sum_{m=1}^{M} n_m = n. \quad (8)
\]

Fig. 4. Erraticity moments \( \ln C_{2,2} \) (upper sub-figure) and \( \Sigma_2 \) (lower sub-figure) vs. \( \ln M \) of the rapidity region for NA22 data (full circles), ECOMB (open triangles connected by the dotted line), PYTHIA (open rhombi connected by the dashed line), and a flat probability distribution with experimental multiplicity distribution (open circles), fixed multiplicity \( n = 6 \) (open squares), and Poisson multiplicity distribution with an average of 6 (open crosses).

The multiplicity distribution is given in three different ways, i.e., experimental multiplicity distribution, fixed multiplicity \( n = 6 \), i.e., about the experimental average multiplicity in the phase space region we use, and Poisson multiplicity distribution with a mean value of 6. For each case, a sample of 40 000 events is generated. The results of \( \ln C_{2,2} \) and \( \Sigma_2 \) are shown in Fig. 4 as open circles, open squares and open crosses, respectively.

Comparing with the fixed multiplicity sample results, the experimental multiplicity distribution sample and Poisson multiplicity distribution sample have relatively larger \( \ln C_{2,2} \) and \( \Sigma_2 \), which comes from the additional fluctuation in multiplicity. However, all of them can reproduce the trend of the erraticity behavior of the NA22 data fairly well. So, the erraticity be-
behavior of the NA22 data is mainly a reflection of the statistical fluctuations due to an insufficient number of particles at this low collision energy. This is consistent with what has been shown in [20].

5. Conclusion

In this Letter, we have systematically analyzed the erraticity behavior of multiparticle production in $\pi^+ p$ and $K^+ p$ collisions at 250 GeV/c.

From the one-dimensional ($y$) and three-dimensional ($y, p_t, \phi$) analyses, we can see that an erraticity behavior exists in these collisions. This erraticity behavior is largely independent of the phase-space variable used and of the average transverse momentum of the sample. However, it strongly depends on the charged-particle multiplicity. The reason for this dependency is that the event factorial moments defined in Eq. (1) contain large statistical fluctuations [19] in the low-multiplicity sample. Furthermore, an analysis of purely statistical fluctuations shows that the erraticity behavior at such a low collision energy is dominated by these purely statistical fluctuations. As has been demonstrated in [20], the erraticity analysis based on factorial moments may be useful in exploiting the fluctuation dynamics only for high multiplicity events, which are produced, e.g., in the central collisions of high-energy heavy nuclei [21]. Low multiplicities, however, imply large gaps between particles in phase space, so that an analysis in terms of gaps [22] sounds more promising. Such an analysis is presented in the following Letter [23].

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