On Relation between String Theory and Multidimensional Cosmology

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Abstract

It is shown that a transition from a multidimensional cosmological model with one internal space of the dimension $d_1$ to the effective tree-level bosonic string corresponds to an infinite number of the internal dimensions: $d_1 \to \infty$.

1 Introduction

String theories [1] are at the moment the most promising candidates for a unified description of the basic physical interactions. The most consistent formulation of these theories are possible in a space-time with critical dimensions $D_c$ more than four. For example, $D_c = 26$ or 10 for the bosonic and supersymmetric version, respectively.

Since string effects become important at Planck scales, cosmology can provide a natural test for string theories. A lot of papers were devoted to string cosmology (see e.g. [2] and references therein).

Another class of models which exploit the idea of extra dimensions have the metric of the form [3, 4]

$$g = -dt \otimes dt e^{2\gamma(t)} + \sum_{i=0}^{n} a_i(t) g^{(i)}$$

on the $D$-dimensional manifold

$$M = R \times M_0 \times \cdots \times M_n,$$

where $M_i$ with the metric $g^{(i)}$ are $d_i$-dimensional spaces of constant curvature (more generally, they are Einstein spaces). These models are natural generalization of the Friedmann universe as well as Kasner universe to the multidimensional case.

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Classical and quantum dynamics of these models were considered in many papers (see e.g. [5, 6] and references therein).

It is worthwhile to find a relation between string theory (ST) and multidimensional cosmological models (MCM) of the form (1.1) - (1.2). Here we show that in the case of one internal space ($n = 1$) the transition from MCM to the effective tree-level bosonic string occurs in the limit of infinite number of the internal dimensions: $d_1 \to \infty$.

## 2 String effective action

We consider a bosonic string in the presence of a background consisting of a $D_0$-dimensional metric $g^{(o)}_{\mu\nu}$ and a dilaton $\Phi$. The action of this string is following [1]

$$ S_\sigma = -\frac{1}{4\pi} \int d^2 \sigma \sqrt{h} \left[ \frac{1}{\alpha'} h^{ab} \partial_a X^\mu \partial_b X^\nu g^{(o)}_{\mu\nu}(X^\rho) + \Phi(X^\rho) R^{(2)} \right], $$

where $h_{ab}$ is the world-sheet metric tensor, $R^{(2)}$ is the Ricci scalar constructed with $h_{ab}$, $X^\mu$ is the coordinates of the string position and $\alpha'$ is the Regge slope parameter connected with the string tension $T : 2\pi \alpha' T = 1$.

Corresponding tree-level effective action reads [1]

$$ S_{\text{eff}} = \frac{1}{2\kappa_0^2} \int d^{D_0} x \sqrt{|g^{(0)}|} e^{-2\Phi} \left( R[g^{(0)}] + 4 \partial_\mu \Phi \partial_\nu \Phi g^{(0)\mu\nu} + C \right), $$

where $\kappa_0^2$ is a $D_0$-dimensional gravitational constant and $C = -2(D_{\text{eff}} - D)/3\alpha'$ is the central charge deficit which depends on details of particular ST. For example, $D_{\text{eff}} = D_0$, $D = 26$ in the bosonic version and $D_{\text{eff}} = \frac{3}{2} D_0$, $D = 15$ in the supersymmetric version. The effective action (2.2) is written in the Brans-Dicke frame. After conformal transformation

$$ \hat{g}^{(0)}_{\mu\nu} = e^{-4\Phi/(D_0-2)} g^{(0)}_{\mu\nu}, \quad \varphi = \pm \frac{2}{\sqrt{D_0-2}} \Phi $$

the action (2.2) can be rewritten in the Einstein frame as follows

$$ S_{\text{eff}} = \frac{1}{2\kappa_0^2} \int d^{D_0} x \sqrt{|\hat{g}^{(0)}|} \left( \hat{R}[\hat{g}^{(0)}] - \hat{g}^{(0)\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 2\Lambda e^{-2\lambda_s \varphi} \right), $$

where

$$ \lambda_s^2 = 1/(D_0 - 2) $$

is the string dilatonic coupling constant and

$$ \Lambda := -\frac{1}{2} C $$

is the "cosmological constant".
3 Multidimensional cosmological effective action

Let us consider now the model (1.1). We slightly generalize it to the inhomogeneous case supposing that the metric has the form

\[ g = g^{(0)} + \sum_{i=1}^{n} a_i^2(x)g^{(i)}, \tag{3.1} \]

where the metric \( g^{(0)} \) is defined on the \( D_0 = d_0 + 1 \)-dimensional manifold \( \bar{M}_0 = R \times M_0 \) and \( x \) are some coordinates of \( \bar{M}_0 \): \( g^{(0)} = g^{(0)}_{\mu\nu}dx^\mu \otimes dx^\nu \).

Hereafter we consider one internal space case \( n = 1 \) with \( M_1 \) being an Einstein space:

\[ R[g^{(1)}] = \lambda^1 d_1. \tag{3.2} \]

The action is taken in the Einstein-Hilbert form

\[ S = \frac{1}{2\kappa^2} \int_M d^Dx \sqrt{|g|} (R[g] - 2\Lambda) + S_{GM}, \tag{3.3} \]

where \( S_{GM} \) is the standard Gibbons-Hawking boundary term and \( \kappa^2 \) is a \( D \)-dimensional gravitational constant. The cosmological effective action is obtained by the dimensional reduction of the action (3.3) and reads [7]

\[ S_c = \frac{1}{2\kappa^2} \int_{\bar{M}_0} d^{D_0}x \sqrt{|g^{(0)}|} e^{-2\Phi} \left( R[g^{(0)}] - 4\partial_\mu \Phi \partial^\nu \Phi g^{(0)\mu\nu} + R[g^{(1)}] e^{\frac{4\Phi}{d_1}} - 2\Lambda \right), \tag{3.4} \]

where the dilaton \( \Phi \) is defined via the scale factor \( a_1(x) \) as follows

\[ e^{-2\Phi} := a_1^{d_1}. \tag{3.5} \]

The action (3.4) is written in the Brans-Dicke frame with the Brans-Dicke parameter

\[ \omega = \frac{1}{d_1} - 1 < 0. \tag{3.6} \]

It follows from (3.4) that this action in the limit \( d_1 \to \infty \) turns into the string effective action (2.2) with \( C := R[g^{(1)}] - 2\Lambda \).

If \( D_0 = 2 \) the equation (3.4) represents 2D dilaton gravity obtained from inhomogeneous cosmology. There is no Einstein frame for 2D manifolds. This is not a fault of the theory but rather corresponds to the wellknown fact that 2-dimensional Einstein equations are empty, i.e. they do not imply a dynamics. We can see it explicitly from the conformal transformations (2.3) which are singular for \( D_0 = 2 \).

But for \( D_0 \neq 2 \) we can obtain the cosmological action (3.4) in the Einstein frame.

By analogy, with the conformal transformations (2.3) we can write

\[ g^{(0)}_{\mu\nu} = e^{-4\Phi/(D_0-2)} g^{(0)}_{\mu\nu}, \tag{3.7} \]
\[ \varphi = \pm 2 \left[ \omega + \frac{D_0 - 1}{D_0 - 2} \right]^{1/2} \Phi. \]

The action \((3.4)\) then reads

\[
S_c = \frac{1}{2\kappa_0^2} \int_{M_0} d^{D_0}x \sqrt{|\hat{g}(0)|} \left\{ \hat{R}[\hat{g}(0)] - \hat{g}^{(0)\mu \nu} \partial_\mu \varphi \partial_\nu \varphi + R[g^{(1)}] e^{-2\lambda^{(1)c} \varphi} - 2\Lambda e^{-2\lambda^{(2)c} \varphi} \right\},
\]

where the cosmological dilatonic coupling constants are

\[
\lambda^{2(1)c} = \frac{D - 2}{d_1(D_0 - 2)}, \quad \lambda^{2(2)c} = \frac{d_1}{(D - 2)(D_0 - 2)},
\]

here \(D = D_0 + d_1 = 1 + d_0 + d_1\) and both of these coupling constants go to the string dilatonic coupling constant \(\lambda_s\) \((2.5)\) in the limit \(d_1 \to \infty\).

In conclusion we would like to note that the case of more than one internal space \((n > 1)\) corresponds to multidilatonic theories. This type of the multidimensional cosmological models may be related to the very fashionable now \(M\)-theory \([8]\) but this question needs more detailed investigations.

Here, we considered the multidimensional cosmology regardless of strings and after dimensional reduction only the connection between them was found. However, MCM may have its origin directly from ST. Actually, the action \((3.3)\) may be considered as the tree-level effective action \((2.4)\) for the non-critical bosonic string with a constant background dilaton: \(\Phi = \text{const}\). For the critical string \((D_0 = D_c)\) with \(\Phi \neq \text{const}\) we obtain the Einstein action with free scalar field. If we propose now that metric in the Einstein frame has the form \((1.1)\) then we obtain MCM. This form of the metric is very convenient for investigation of the dynamical reduction problem in ST. Of special interest are exact solutions because they can be used for a detailed study of the compactification of the internal spaces \([6]\).

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