STATUS OF $|V_{ub}|$, $|V_{cb}|$ AND THEIR RELATIVE PHASE

R. V. KOWALEWSKI

Department of Physics and Astronomy, University of Victoria, Victoria, BC V8N 2X3, Canada

∗E-mail: kowalewski@uvic.ca

The current status of the determinations of the CKM elements $V_{ub}$ and $V_{cb}$ is reviewed and future prospects are discussed.

Keywords: CKM matrix; CP violation; B physics.

1. Motivation

The imaginary phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix has, in recent years, been firmly established as the dominant source of CP violation in the decays of $B$ mesons. In the process, constraints on the lesser-known parameters of the CKM matrix, namely $\bar{\rho}$ and $\bar{\eta}$, have become increasingly precise. Measurements of CP asymmetries determine the angles of the unitarity triangle in the $\bar{\rho}-\bar{\eta}$ plane. These all involve processes with internal loops, either through Penguin amplitudes or through $B_0 \to \overline{B}_0$ mixing. These same CKM parameters can be determined in tree-level processes, which are essentially immune to contributions from new physics, by measuring the magnitudes $|V_{ub}|$ and $|V_{cb}|$ and the relative phase $\gamma = \phi_3 \equiv \arg[-(V_{ud}V_{ub}^\ast)/(V_{cd}V_{cb}^\ast)]$. The independent determination of $\bar{\rho}$ and $\bar{\eta}$ in tree and loop-dominated processes thus provides a promising avenue in which to search for deviations from the Standard Model. This talk summarizes the current status of determinations of $|V_{ub}|$, $|V_{cb}|$ and their relative phase, and discusses prospects for the near-term improvement of these measurements.

Due to space limitations, only recent developments will be discussed and cited in this review. More comprehensive reviews of $|V_{ub}|$ and $|V_{cb}|$ and of determinations of $\gamma$ ($\phi_3$) are available.

2. Semileptonic $B$ decays

The presence of a single hadronic current renders the semileptonic decay width calculable with modest theoretical uncertainties, and makes these decays the favored system for determinations of the magnitudes $|V_{ub}|$ and $|V_{cb}|$. Nevertheless, uncertainties related to non-perturbative QCD comprise a significant part of the total uncertainty in these determinations. The theoretical methods used to calculate the decay rates as a function of $|V_{qb}|$ are very different for inclusive and exclusive decays, as are the experimental measurements, allowing the comparison of these complementary determinations to provide an important check on the results.

Measurements of $|V_{ub}|$ and $|V_{cb}|$ are dominated by the experiments at the $\Upsilon(4S)$ re-

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*The parameterization used here has $\bar{\rho} = (1-\lambda^2/2)\rho$, $\bar{\eta} = (1-\lambda^2/2)\eta$, and defines $V_{us} = \lambda$, $V_{cb} = A\lambda^2$ and $V_{ub} = A\lambda^3(\rho - i\eta)$. 
onance, namely Belle, BABAR and CLEO. These experiments measure $B$ mesons produced nearly at rest in the center-of-mass frame. These $B\overline{B}$ events result in nearly isotropic distributions of final state particles, and are separated from the more collimated $e^+e^- \rightarrow q\overline{q}$ ($q = d, u, s, c$) interactions using event shape cuts. The residual $q\overline{q}$ background is determined using data collected just below the $B\overline{B}$ production threshold.

2.1. Inclusive semileptonic decays

In inclusive semileptonic decays, a subset of the final state particles (e.g. just the lepton) are identified and measured, integrating over all decay channels and the kinematics of unmeasured particles, resulting in singly- or doubly-differential partial widths. Low-order moments of these distributions are measured and compared with theoretical calculations to determine $|V_{qb}|$, the $b$ quark mass, and related non-perturbative parameters.

2.1.1. Theoretical framework

The fact that the $b$ quark mass $m_b$ is large compared to $\Lambda_{QCD}$ allows for a separation of scales as the basis of an effective field theory, the Heavy Quark Expansion (HQE). In the HQE the short-distance degrees of freedom are integrated out, resulting in a double expansion in powers of $\Lambda_{QCD}/m_b$ and of $\alpha_s(\mu)$, with $\mu \gg \Lambda_{QCD}$. The expression for the total semileptonic $b \rightarrow c\ell\nu$ decay width is given in Eq. 1, where $A_{ew}$ and $A_{pert}(r, \mu)$ denote the electroweak and QCD perturbative corrections and $r = m_c/m_b$. The $z_i$ are known functions and depend on non-perturbative parameters ($\mu_f^2$, etc.) that correspond to matrix elements of local operators divided by the appropriate power of $m_b$. The coefficient of the $1^{st}$-order term vanishes, so the leading corrections are at the percent level. Similar expressions, involving the same non-perturbative parameters, have been calculated for low-order moments of the lepton momentum and squared hadron mass spectra in $b \rightarrow c\ell\nu$ decays, as well as for $b \rightarrow u\gamma$ decays and for the inclusive radiative decay $b \rightarrow s \gamma$.

In all cases, comparison of these calculated inclusive decay rates with measured rates depend on the assumption of quark-hadron duality. It’s clear that this assumption breaks down in restricted regions of phase space (e.g. at low squared hadronic mass, where only discrete values are physically realized). This is the motivation for concentrating on low-order moments of decay spectra, integrated over broad regions of phase space. While the uncertainty due to this assumption remains hard to quantify, the global fit to a large number of spectral moments using a small number of parameters, discussed in the following section, suggests that any violations are small compared to the current level of sensitivity.

2.1.2. Determination of $|V_{cb}|$

The theoretical calculations of low-order moments of the $b \rightarrow c\ell\nu$ and $b \rightarrow s\gamma$ decay spectra have been performed, for a variety of requirements on the minimum lepton momentum or photon energy, in two separate mass renormalization schemes, referred to here as the “kinetic” and “1S” schemes. Each calculated moment depends on the quark masses and on a common set of non-perturbative parameters. The total $b \rightarrow c\ell\nu$ rate also depends on $|V_{cb}|^2$.

A variety of experiments have measured moments of these decay processes. Measurements of the lepton momentum moments are based on a technique introduced by ARGUS, in which charge and angular correlations in events with two identified leptons are used to extract the direct $B \rightarrow X\ell\nu$ spectrum down to $\sim 0.5$ GeV. These measurements are limited by systematic uncertainties, although some further improve-


\[ \Gamma = |V_{cb}|^2 \frac{G_F^2 m_b^3(\mu)}{192 \pi^3} (1 + A_{\text{ew}}) A_{\text{pert}}(r, \mu) \times \]

\[ z_0(r) + z_1(r) \times 0 + z_2 \left( r, \frac{\mu_s^2}{m_b^2}, \frac{\mu_h^2}{m_b^2} \right) + z_3 \left( r, \frac{\rho_1}{m_b^2}, \frac{\rho_2}{m_b^2} \right) + \ldots \]

3

ment may be possible. Measurements of the moments of the squared hadronic mass spectrum,\(^9\) for various cuts on the minimum lepton momentum, have been made. The most precise of these rely on fully reconstructing one \(B\) meson from \(\Upsilon(4S)\) decay to allow the remaining particles in the event to be associated with the semileptonic decay of the second \(B\) meson. Measurements of the photon energy spectrum from \(b \to s\gamma\) decays\(^10\) are experimentally challenging, in particular as the minimum accepted photon energy is reduced below about 2.2 GeV, due to the large background from both \(q\overline{q}\) and \(B\overline{B}\) events.

The measured moments, including all known correlations, have been fitted\(^11\) in the kinetic scheme, resulting in precise values for \(m_b\) and \(|V_{cb}|\), as shown in Table 1. Separate fits to the \(b \to c\ell\overline{\nu}\) moments alone and to the \(b \to s\gamma\) moments (with only \(m_b\) and \(\mu_s^2\) floating) give consistent results. The fit includes both experimental and theoretical uncertainties, and results in a \(\chi^2\) of 19.3 for 44 degrees of freedom. The first and second uncertainties come from experimental errors and uncertainties in the HQE calculations, respectively. The last uncertainty listed for \(|V_{cb}|\) is a normalization uncertainty due to uncalculated terms in the total rate.

The global fit described above does not yet include the latest moment measurements from Belle, which were first presented at this conference.\(^12\) Belle performs a fit to their measured \(b \to c\ell\overline{\nu}\) and \(b \to s\gamma\) moments in both the kinetic and 1S schemes, resulting in \(\chi^2/\text{d.o.f.}\) of 17.8/24 and 5.7/17, respectively, and in the values given in Table 1. For the 1S fit the first error listed includes both experimental and theoretical uncertainties; the second error on \(|V_{cb}|\) comes from the \(B\) meson lifetime. The values for \(|V_{cb}|\) agree well in all three fits. The \(m_b\) values in the kinetic and 1S schemes cannot be compared directly; when both are translated to a common scheme the agreement is excellent.

| Fit          | \(|V_{cb}| (10^{-5})\) | \(m_b\) (MeV) |
|--------------|------------------------|--------------|
| Global kin.  | 4196 ± 23 ± 35 ± 59   | 4590 ± 25 ± 30 |
| Belle kin.   | 4206 ± 67 ± 48 ± 63   | 4564 ± 76     |
| Belle 1S    | 4149 ± 52 ± 20        | 4729 ± 48     |

These precise determinations of \(|V_{cb}|\) and \(m_b\) in a consistent global fit represent an enormous achievement.

2.1.3. Determination of \(|V_{ub}|\)

The selection of events of the type \(b \to u\ell\overline{\nu}\) requires suppression of the dominant \((\times 50)\) background from the process \(b \to c\ell\overline{\nu}\). Kinematic criteria on the lepton momentum, on the squared momentum transfer \((q^2)\) in the \(b\) decay, or on the invariant mass of the final state hadrons can be used to reduce the background. The dependence of the partial rate on \(m_b\) in the restricted phase space region is steeper than for the total rate; typical values are \(m_b^{7.5} - m_b^{12}\), depending on the experimental cuts. Restrictive kinematic cuts can compromise the convergence of the HQE, in which case the calculated rate becomes sensitive to the non-perturbative light-cone momentum distribution (shape function), which, at leading order, must be measured (in the radiative decays \(b \to s\gamma\)) or modelled. Additional
shape functions, which differ in semileptonic and radiative decays, arise at higher orders and must be modeled.

Significant improvements in the calculational methods have recently become available. Calculations that relate directly integrals over the measured lepton momentum or hadron invariant mass spectra in $b \to u\ell\overline{\nu}$ decays with integrals over the measured $E_{\ell}$ spectrum in $b \to s\gamma$ decays are available and obviate the need to model the leading shape function.

A recent BaBar measurement using the invariant mass $m_X$ of the recoiling hadrons to select $b \to u\ell\overline{\nu}$ decays used a sample of 88 million $B\overline{B}$ events to determine $|V_{ub}|$ in two separate ways. The first method compared the integrated $m_X$ and $E_{\ell}$ spectra using the calculations of Ref. 15 to determine $|V_{ub}| = (4.43 \pm 0.38 \pm 0.25 \pm 0.29) \times 10^{-3}$, where the errors are statistical, systematic and theoretical, respectively. The second method determined the inclusive $b \to u\ell\overline{\nu}$ rate for $m_X < 2.5$ GeV, which includes 96% of the total rate, and results in $|V_{ub}| = (3.84 \pm 0.70 \pm 0.30 \pm 0.10) \times 10^{-3}$. While the impact of these measurements on the world average $|V_{ub}|$ is small, they are noteworthy for the small theoretical uncertainties, and will improve markedly as more data are analyzed.

CLEO has produced the first direct limits on the uncertainty in $|V_{ub}|$ determinations arising from weak annihilation diagrams, which affect charged $B$ decays to an isoscalar hadron, lepton and neutrino. These limits are used to evaluate the uncertainty in the world average $|V_{ub}|$ from weak annihilation.

The Heavy Flavor Averaging Group (HFAG) provides determinations of $|V_{ub}|$ based on a variety of measurements. The resulting average is

$$|V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \times 10^{-3}$$

based on the calculations of Ref. 13. The average has a $\chi^2$ probability of 41%. The error budget consists of 2.2% from statistics, 2.8% from experimental systematics, 1.9% from weak annihilation, 1.9% from the modeling of $b \to c\ell\overline{\nu}$ decays, 1.6% from the modeling of $b \to u\ell\overline{\nu}$ decays, 3.8% from the modeling of sub-leading shape functions and perturbative matching scales, and 4.2% from HQE parameter uncertainties, principally from $m_b$.

The result using the calculations of Ref. 14 is $|V_{ub}| = (4.46 \pm 0.20 \pm 0.20) \times 10^{-3}$, with a $\chi^2$ probability of 12%. An additional calculation is available for the subset of measurements made with requirements on $m_X$ and $q^2$, and gives $|V_{ub}| = (5.02 \pm 0.26 \pm 0.37) \times 10^{-3}$; this value is compatible with those obtained using the other calculations. Many of the measurements in the average use relatively small samples compared with the current and projected $B$-factory datasets. A 5% uncertainty on $|V_{ub}|$ is an aggressive target for the next ICHEP conference.

### 2.2. Exclusive semileptonic decays

Exclusive semileptonic decays provide a complementary avenue for determinations of $|V_{qb}|$. The challenge for theory is the calculation of the decay form factor, in particular of its normalization.

#### 2.2.1. $b \to c\ell\overline{\nu}$ decays

These decays involve a heavy-to-heavy transition, allowing heavy quark symmetry to be applied. In the heavy quark limit this results in a unique form factor, the Isgur-Wise function, that needs to be measured. This form factor is parameterized as a function of the four-velocity product $w$ of the $B$ and charm mesons, which is related to the momentum transfer $q^2$: $w = (m_B^2 + m_C^2 - q^2)/(2m_Bm_C)$.

The decay $B^0 \to D^*+\ell^{-}\overline{\nu}$ is the easiest to isolate experimentally, as it has the largest branching fraction of any $B$ decay. Many experiments have measured this decay mode, extracting the decay rate versus
w to determine $F(1)|V_{cb}|$, where $F(1)$ is the form factor normalization at the zero recoil point, $w = 1$. Recently, BABAR has improved measurements of the form factor slope $\rho^2 = -dF/dw(w = 1)$ and the form factor ratios $R_1 \sim V/A_1$ and $R_2 \sim A_2/A_1$. They find

\[
\begin{align*}
\rho^2 &= 1.179 \pm 0.048 \pm 0.028 & (3) \\
R_1 &= 1.417 \pm 0.061 \pm 0.044 & (4) \\
R_2 &= 0.836 \pm 0.037 \pm 0.022 & (5)
\end{align*}
\]

Since existing measurements of $F(1)|V_{cb}|$ depend on $R_1$ and $R_2$, these more precise values result in smaller uncertainties on $|V_{cb}|$. The HFAG average\(^{(19)}\) is $F(1)|V_{cb}| = (36.2 \pm 0.8) \times 10^{-3}$. Using $F(1) = 0.919^{+0.030}_{-0.035}$ from Ref. 22 gives

\[
|V_{cb}| = (39.4 \pm 0.9^{+1.2}_{-1.0}) \times 10^{-3},
\]

which is consistent with the inclusive results quoted earlier. Further progress is needed to reduce the uncertainty on the calculation of the form factor, and the experimental situation needs to be clarified, given the poor $\chi^2$/d.o.f., 38.7/14, of the existing measurements.

It has been argued recently\(^{(24)}\) that the theoretical uncertainty on the form factor normalization for $B \to D\pi$ decays may be even smaller than for $B \to D\pi$ decays; this may be true for lattice QCD determinations as well. The precise determination of the $B \to D\pi$ form factor at zero recoil remains an experimental challenge due to the large background from $B \to D^*\pi$ decays.

### 2.2.2. $b \to u\tau\nu$ decays

New measurements of the branching fraction versus $q^2$ for $B \to \pi\tau\nu$ have significantly reduced the experimental uncertainty in the determination of $|V_{cb}|$ from these decays. The measurements are done either with (tagged) or without (untagged) the reconstruction of the other $B$ meson in the event. The tagged measurements provide superior signal to background and resolution on $q^2$, but are less statistically precise than untagged measurements. Belle,\(^{(25)}\) BABAR\(^{(26,27)}\) and CLEO\(^{(28)}\) presented new measurements for this conference. Belle and BABAR both provided measurements of $B \to \pi\ell\tau$ with the other $B$ reconstructed in the decay $B \to \Delta(\ast)\ell^+\nu$ or in an hadronic decay mode. CLEO and BABAR made untagged measurements of $\pi\ell\tau$ made based on neutrino reconstruction. The BABAR measurement has very high statistics and provides a good determination of the $q^2$ dependence (see Fig. 1), obtaining a shape parameter $\alpha = 0.53 \pm 0.05 \pm 0.04$ for the Becirevic-Kaidalov parameterization.\(^{(29)}\) These recent measurements are summarized in Table 2.

**Table 2. Recent measurements of exclusive $B \to \pi\tau\nu$ decays.** The values for the $B^+$ mode are multiplied by $2\tau(B^0)/\tau(B^+)$ to allow direct comparison with the $B^0$ mode.

| Expt/tag   | $B(B^0) (10^{-6})$ | $B(B^+) (10^{-6})$ |
|------------|-------------------|-------------------|
| BABAR/note$^{26}$ | $144 \pm 8 \pm 10$ | $143 \pm 7 \pm 15$ |
| CLEO/note$^{19}$ | $137 \pm 6 \pm 13$ | $135 \pm 6 \pm 13$ |
| BaBar/s.l.$^{25}$ | $140 \pm 26 \pm 6$ | $147 \pm 23 \pm 11$ |
| BaBar/s.l.$^{27}$ | $112 \pm 25 \pm 10$ | $135 \pm 33 \pm 19$ |
| BaBar/had$^{27}$ | $107 \pm 27 \pm 19$ | $152 \pm 41 \pm 20$ |

![Fig. 1. Partial branching fraction versus $q^2$ for $B^0 \to \pi^+\ell^−\nu$.](image)

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ments of $B \to \rho \ell \nu$, and of $B \to \eta \ell \nu$ and $B \to \eta' \ell \nu$ were also presented at the conference.

HFAG has averaged all available $\bar{B} \to \pi \ell \nu$ measurements and finds, assuming isospin symmetry for the decay rates, $\mathcal{B}(B^0 \to \pi^- \ell^+ \nu) = (1.37 \pm 0.06 \pm 0.06) \cdot 10^{-4}$. The consistency of the measurements is good. Comparing the partial decay rates in the form factor normalization from Lattice QCD, and those in the region $q^2 < 16 \text{ GeV}^2$ with calculations from light-cone sum rules, results in the $|V_{ub}|$ values in Table 3.

The experimental uncertainties are already at the $\sim 6\%$-level. Progress is clearly needed in the form factor normalization calculations to provide a competitive determination of $|V_{ub}|$. These values are not independent, and are lower than the inclusive determination of $|V_{ub}|$ by 0.7-1.7$\sigma$.

### 3. The relative phase $\gamma (\phi_3)$

Interference between competing decay amplitudes renders the relative phase $\gamma$ observable. The relevant processes for measuring $\gamma$ involve interference between two tree-level diagrams, as in Fig. 2, where the $D^0$ and $\bar{D}^0$ decay to a common final state. The related modes $B^- \to D^{*0} K^-$ and $B^- \to D^0 K^{*-}$ are also used. The amplitude ratio can be expressed as

$$A(B^- \to \bar{D}^0 K^-)/A(B^- \to D^0 K^-) = r_B e^{i\delta_B} e^{-i\gamma} \quad (7)$$

where $r_B$ and $\delta_B$ are the ratio of the magnitudes and the strong phase difference of the contributing amplitudes. In contrast to $\gamma$, these latter parameters are specific to each $B$ decay mode used. These additional parameters can be determined from observables in $B$ and $D$ decays. The parameter $r_B$ plays an important role in determining the experimental sensitivity to $\gamma$, as small values render the measurements very challenging.

### 3.1. Strategies for exploiting interference

There are several ways of obtaining the same final state from $D^0$ and $\bar{D}^0$ decays:

- **GLW** Choose CP eigenstates of the $D^0$ decay, e.g. $D^0 \to K_s^0 \pi^0$ (CP-odd), $D^0 \to \pi^+ \pi^-$ (CP-even);
- **ADS** Use doubly Cabibbo-suppressed decays (DCSD), e.g. $D^0 \to K^+ \pi^-$;
- **GGSZ/Belle** Examine the $B^-$ and $B^+$ Dalitz plots for 3-body flavor-neutral decays like $D^0 \to K^0_s \pi^+ \pi^-$.

The last method includes regions dominated by two-body decays to CP eigenstates (e.g. $K_s^0 \rho^0$) and to DCSD decay modes (e.g. $K^{*+} \pi^-$).

### 3.2. Measurements using charged $B$ decays

Both BaBar and Belle have used each of the methods mentioned above. The decay $B^- \to D^{(*)0} K^{(*)-}$ is reconstructed using kinematic information, vertex constraints and particle identification. Particle identification information and the difference $\Delta E$
between the known $B$ energy in the center-of-mass frame and the energy reconstructed from the decay products provide discrimination between $B^- \to D^{(*)0} K^-$ and the more copious decays $B^- \to D^{(*)0} \pi^-$. The transitions $D^{*0} \to D^0 \pi^0$ and $D^{*+} \to D^0 \gamma$ are both reconstructed.

### 3.2.2. GLW method

The experiments use $B^- \to D^{(*)}_CP K^{(*)-}$ decays to measure the quantities

$$ A_\pm = \frac{\Gamma(B^- \to D^\pm_\pm K^-) - \Gamma(B^+ \to D^\mp_\pm K^+)}{\Gamma(B^- \to D^0_\pm K^-) + \Gamma(B^+ \to D^0_\pm K^+)} = \frac{\pm 2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma} \quad (8) $$

and

$$ R_\pm = \frac{\Gamma(B^- \to D^\pm_0 K^-) + \Gamma(B^+ \to D^\mp_0 K^+)}{\Gamma(B^- \to D^0_\pm K^-) + \Gamma(B^+ \to D^0_\mp K^+)} = 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma \quad (9) $$

where the subscripts ($\pm$) indicate CP-even or CP-odd final states. Solving these equations for the three unknowns results in up to an 8-fold ambiguity on $\gamma$. The ability to determine $r_B$ and $\gamma$ independently is limited due to the nature of the dependence of the observables on these quantities.

Belle$^{38}$ and BaBar$^{39}$ each have $\sim 100$ events per CP eigenvalue in $D^0 K^-$ and less in $D^{*0} K^-$ and $D^0 K^{*-}$, so the statistical sensitivity is still modest. A recent compilation of the experimental results is available from HFAG.$^{40}$ The implications on the determination of $\gamma$ are best addressed in a global approach that considers the GLW-based measurements in conjunction with the other methods, and will be given later.

### 3.2.2. ADS method

The competing amplitudes in this case consist of a combination of a favored $B$ decay and a suppressed $D$ decay or vice-versa, e.g. $B^- \to D^0 K^-$ with $D^0 \to K^+ \pi^-$ and $B^- \to D^0 K^-$ with $D^0 \to K^+ \pi^-$. This can lead to large CP asymmetries, but results in small product branching fractions. The ratio of amplitudes also depends on additional input from $D$ decays, namely the ratio $r_D$ and phase difference $\delta_D$ between the suppressed and favored $D$ decays to the specified final state. The observables in this case are the asymmetry between $B^-$ and $B^+$ and the ratio of suppressed to favored $B$ modes:

$$ A = \frac{\Gamma(B^- \to \overline{B}_F K^-) - \Gamma(B^+ \to \overline{B}_F K^+)}{\Gamma(B^- \to \overline{B}_F K^-) + \Gamma(B^+ \to \overline{B}_F K^+)} = \frac{2r_B r_D \sin(\delta_B + \delta_D) \sin \gamma}{\dot{R}} \quad (10) $$

$$ R = \frac{\Gamma(B^- \to \overline{B}_F K^-) + \Gamma(B^+ \to \overline{B}_F K^+)}{\Gamma(B^- \to D_F K^-) + \Gamma(B^+ \to D_F K^+)} = r_B^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \gamma \quad (11) $$

where $D_F$ is a favored $D$ decay (e.g. $K^- \pi^+$) and $\overline{B}_F$ is a dis-favored $D$ decay (e.g. $K^+ \pi^-$). The amplitude ratio $r_D$ is determined in charm decays, so $R$ has good sensitivity to $r_B$.

BaBar$^{41}$ and Belle$^{42}$ have analyzed the $B \to DK$ mode with $D$ decays to $K^- \pi^+$, resulting in the average value $R = 0.006 \pm 0.006$, from which an upper limit on $r_B$ can be set. BaBar has also measured $B \to D^* K$ and $B \to DK^*$, and has measured the $B \to DK$ mode with $D \to K^- \pi^+ \pi^0$ decays; in all cases the signal in the suppressed mode is consistent with zero, and no asymmetry measurements are yet possible. The numerical results are summarized in the HFAG compilation.$^{40}$

### 3.2.3. Dalitz analyses

Three-body $D$ decays provide an opportunity to study the CP asymmetry as a function of location in the Dalitz plot. The three-body modes studied consist of a neutral particle and two charged particles, e.g. $K^0 S \pi^+ \pi^-$. The total amplitude for a given set of mass squared values, denoted by $m_+^2$ and $m_-^2$, according to the charge of the two-particle combination, is the sum of the fa-
vored decay ($B^- \to D^0 K^-$) amplitude plus a contribution from the suppressed decay ($B^- \to \overline{D^0} K^-$):

$$A_\pm = f(m_+^2, m_-^2) + r_B e^{i(\delta_B \pm \gamma)} f(m_-^2, m_+^2).$$

Here $f(m_+^2, m_-^2)$ denotes the amplitude for the $D^0$ decay. In the absence of CP violation in $D$ decays, the amplitude for the $\overline{D^0}$ decay is $f(m_-^2, m_+^2)$. A flavor-tagged sample of $D$ decays (e.g. from $D^{*+} \to D^0 \pi^+$ transitions) is used to determine $f$. The information contained in the Dalitz plot removes all ambiguities in the determination of $\gamma$ apart from the reflection ($\gamma, \delta_B \to (\gamma + \pi, \delta_B + \pi)$).

Belle and BABAR have analyzed samples of $\sim 4 \cdot 10^5 D^{*+} \to [K^0_s \pi^+ \pi^-] \pi^+$ decays (see Fig. 3). The Belle (BABAR) isobar fit includes 15 (16) Breit-Wigner amplitudes plus a non-resonant term. The main contributing resonances are $K^{*+}(892)\pi^-$, $K^0_s \rho^0$, $K^{*0}(1430)\pi^-$, $K^{*-}(892)\pi^+$ and the non-resonant component. Despite the excellent qualitative description of the data provided by the fit, the model uncertainty is a significant source of systematic error on $\gamma$, so improved decay modeling (e.g. a K-matrix formulation) are under investigation. The feasibility of a model-independent approach that makes use of CP-tagged $D^0$ mesons (as could be studied at the $\psi(3770) \to D\overline{D}$) was studied recently.

The Dalitz plots for $B^- \to D^0 K^-$, $D^+ K^-$ and $DK^{*-}$, plus their charge conjugates, have been analyzed by BABAR and Belle. Figure 4 shows the $DK$ plots from BABAR. The experiments extract, for each mode, contours in the $x_{\pm}y_{\pm}$ plane, where $x_\pm = r_B \sin(\delta_B \pm \gamma)$ and $y_\pm = r_B \cos(\delta_B \pm \gamma)$; these variables are uncorrelated, in contrast to $r_B$, $\delta_B$ and $\gamma$. The recent measurements improve the accuracies on $x_{\pm}$ and $y_{\pm}$ significantly. The uncertainty on $\gamma$, however, is strongly sensitive to the value of $r_B$. The latest measurements favor a smaller $r_B$, and the uncertainty on $\gamma$ has increased as a result.

Determined by the UTfit collaboration, who find $\gamma = (82 \pm 20) ^\circ$, and by the CKMfit collaboration, who find $\gamma = (60^{+38}_{-24}) ^\circ$. The differences arise from a different treatment of systematic errors and different statistical methods. Both ranges are consistent with the values, $65^\circ$ and $59^\circ$, based on the respective global CKM fits. Further progress on $\gamma$ requires larger data sets, and predictions for the uncertainty remain uncertain due to the dependence on the $r_B$ values for the contributing decays, which are still not well known.

### 3.3. Measurements using neutral $B$ decays

The interference between tree-level decays can also be exploited in neutral $B$ mesons, e.g. $B^0 \to D^{(*)+} \pi^-$ and $B^0 \to D^{(*)-} \pi^+$, which can interfere due to $B\overline{B}$ mixing. The
The discrepancy is at the level of $1\sigma$, depending in detail on the assumptions made in our knowledge of CKM parameters. It is nevertheless intriguing, and underscores the motivation for further improvements in accuracy.

The B factories will continue to increase their data samples, and expect to have 3 ab$^{-1}$ between them by ICHEP 2008. This will allow for significant improvements in $|V_{ub}|$ and $\gamma (\phi_3)$. Additional improvements in the calculations used to extract $|V_{ub}|$ and $|V_{cb}|$ from both inclusive and exclusive semileptonic decays can be expected. These refinements will further restrict the space in which theories hoping to explain the new physics we’ll see at the LHC can live.

\begin{align}
\frac{A_{\text{sup}}}{A_{\text{fav}}} &= r_B e^{i\delta_b} e^{-i(2\beta + \gamma)}.
\end{align}

The impact of the quantities reviewed here on our knowledge of CKM parameters $\overline{\theta}$ and $\overline{\eta}$ is shown in Fig. 5. These measurements favor a larger value for $\sin 2\beta$ than is determined from CP asymmetry measurements. The discrepancy is at the level of 1.5-2$\sigma$, and depends in detail on the assumptions made in the global fits. It is nevertheless intriguing, and underscores the motivation for further improvements in accuracy.

The B factories will continue to increase their data samples, and expect to have 2 ab$^{-1}$ between them by ICHEP 2008. This will allow for significant improvements in $|V_{ub}|$ and $\gamma (\phi_3)$. Additional improvements in the calculations used to extract $|V_{ub}|$ and $|V_{cb}|$ from both inclusive and exclusive semileptonic decays can be expected. These refinements will further restrict the space in which theories hoping to explain the new physics we’ll see at the LHC can live.

**Fig. 5.** Constraints in the $\overline{\eta}$-$\overline{\theta}$ plane from tree-level processes from the UTfit collaboration.\textsuperscript{47}

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