A duality Hopf algebra for holomorphic $N = 1$
special geometries

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Abstract

We find a self-dual noncommutative and noncocommutative Hopf algebra $H_F$ acting as a universal symmetry on the modules over inner Frobenius algebras of modular categories (as used in two dimensional boundary conformal field theory) similar to the Grothendieck-Teichmüller group $GT$ as introduced by Drinfeld as a universal symmetry of quasitriangular quasi-Hopf algebras. We discuss the relationship to a similar self-dual noncommutative and noncocommutative Hopf algebra $H_{GT}$, previously found as the universal symmetry of trialgebras and three dimensional extended topological quantum field theories. As an application of our result, we get a transitive action of a sub-Hopf algebra $H_D$ of $H_{GT}$ on the relative period matrices of holomorphic $N = 1$ special geometries, i.e. $H_D$ appears as a kind of duality Hopf algebra for holomorphic $N = 1$ special geometries.

1 Introduction

In [Dri] Drinfeld has introduced the so called Grothendieck-Teichmüller group $GT$ as the universal symmetry of quasitriangular quasi-Hopf algebras. We have shown in two previous papers ([Sch 2002a], [Sch 2002b]) that one can
extend this approach to get a self-dual, noncommutative, and noncocommutative Hopf algebra \( H_{GT} \) acting as the universal symmetry on three dimensional extended topological quantum field theories in the sense of [KL]. We have also shown there that these symmetry considerations lead to a very restrictive stability property of three dimensional extended topological quantum field theories which we called ultrarigidity, there.

The three dimensional extended topological quantum field theories of [KL] are motivated by the wish to describe two dimensional boundary conformal field theories in a three dimensional topological way (much the same way one can get the two dimensional WZW model in the bulk from three dimensional Chern-Simons theory). There is another purely algebraic formulation which directly applies to the full case of two dimensional boundary conformal field theories (see [FS], [FRS 2001], [FRS 2003]). There, two dimensional boundary conformal field theories with given chiral data are formulated as modular categories \( \mathcal{C} \) together with a module \( M \) over an inner Frobenius algebra of \( \mathcal{C} \). We show that one can, again, introduce a self-dual, noncommutative, and noncocommutative Hopf algebra \( H_F \) as the universal symmetry of such pairs \((\mathcal{C}, M)\). We discuss the relationship between \( H_F \) and \( H_{GT} \).

Finally, in section 3, we will apply our result to derive a transitive action of a sub-Hopf algebra \( H_D \) of \( H_{GT} \) on the relative period matrices as they are used in [LMW 2002a], [LMW 2002b] to describe holomorphic \( N = 1 \) special geometries. In this sense, \( H_D \) can be seen as a duality Hopf algebra for holomorphic \( N = 1 \) special geometries.

2 The Hopf algebra \( H_F \)

Let \( \mathcal{C} \) be a modular category, i.e. a \( \mathbb{C} \)-linear, rigid, abelian, braided monoidal category (observe that we include the condition that \( \mathcal{C} \) be monoidal into the definition of a modular category, different variants of the precise definition are used in the literature). An object \( A \) in \( \mathcal{C} \) is called an algebra object in \( \mathcal{C} \) if there exists a morphism

\[
m : A \otimes A \rightarrow A
\]

in \( \mathcal{C} \) which satisfies associativity and if there exists another morphism \( \eta \) in \( \mathcal{C} \) from the unit object in \( \mathcal{C} \) to \( A \) which satisfies the commutative diagrams of a unit object with respect to \( m \). Dually one can define the notion of a coalgebra.
object \((A, \Delta, \varepsilon)\) in \(C\) with \(\Delta\) a coassociative coproduct and \(\varepsilon\) a counit. One calls \((A, m, \eta, \Delta, \varepsilon)\) a Frobenius algebra in \(C\) if \((A, m, \eta)\) and \((A, \Delta, \varepsilon)\) give an algebra object and a coalgebra object in \(C\), respectively, and we have

\[
(id_A \otimes m) \circ (\Delta \otimes id_A) = \Delta \circ m = (m \otimes id_A) \circ (id_A \otimes \Delta)
\]

A (left) module over an algebra object in \(C\) is an object \(M\) in \(C\) together with a morphism

\[
\varrho : A \otimes M \to M
\]

such that

\[
\varrho \circ (m \otimes id_M) = \varrho \circ (id_M \otimes \varrho)
\]

and

\[
\varrho \circ (\eta \otimes id_M) = id_M
\]

In [FS], [FRS 2001], and [FRS 2003] modules over Frobenius algebras in modular categories have been shown to give an algebraic formulation for boundary conditions of two dimensional conformal field theories with given chiral data (for this formulation only so called symmetric special Frobenius algebras appear; we refer the reader to the literature cited above for this notion).

On the other hand, in [KL] three dimensional extended topological field theories (see there for the definition) have been introduced motivated by the wish to give a three dimensional topological formulation of at least a class of two dimensional boundary conformal field theories. We have shown in [Sch 2002b] that any such three dimensional topological quantum field theory uniquely determines a so called trialgebra.

**Definition 1** A trialgebra \((A, *, \Delta, \cdot)\) with \(*\) and \(\cdot\) associative products on a vector space \(A\) (where \(*\) may be partially defined, only) and \(\Delta\) a coassociative coproduct on \(A\) is given if both \((A, *, \Delta)\) and \((A, \cdot, \Delta)\) are bialgebras and the following compatibility condition between the products is satisfied for arbitrary elements \(a, b, c, d \in A\):

\[
(a * b) \cdot (c * d) = (a \cdot c) * (b \cdot d)
\]

whenever both sides are defined.
Trialgebras were first suggested in [CF] as an algebraic means for the construction of four dimensional topological quantum field theories. It was observed there that the representation categories of trialgebras have the structure of so called Hopf algebra categories (see [CF]) and it was later shown explicitly in [CKS] that from the data of a Hopf category one can, indeed, construct a four dimensional topological quantum field theory. The first explicit examples of trialgebras were constructed in [GS 2000a] and [GS 2000b] by applying deformation theory, once again, to the function algebra on the Manin plane and some of the classical examples of quantum algebras and function algebras on quantum groups. In [GS2001] it was shown that one of the trialgebras constructed in this way appears as a symmetry of a two dimensional spin system. Besides this, the same trialgebra can also be found as a symmetry of a certain system of infinitely many coupled $q$-deformed harmonic oscillators.

The three dimensional extended topological quantum field theories of [KL] always uniquely determine a finite dimensional trialgebra where one of the bialgebras contained in it is a quasitriangular Hopf algebra. We call such trialgebras finite dimensional quasitriangular antipodal trialgbras. On the other hand, we have the following result:

**Lemma 1** A finite dimensional quasitriangular antipodal trialgebra always uniquely determines an algebra object in a modular category.

**Proof.** Let $(T, \cdot, *, \Delta)$ be the trialgebra where $(T, \cdot, \Delta)$ is a quasitriangular Hopf algebra and $\mathcal{C}$ the category of finite dimensional representations of $(T, \cdot, \Delta)$. Then $\mathcal{C}$ is modular. Let $A$ be the object in $\mathcal{C}$ which corresponds to the defining representation. Then the product $*$ defines the structure of an algebra object in $\mathcal{C}$ on $A$. The product $*$ is an algebra morphism in $\mathcal{C}$ because of the compatibilities of a trialgebra. 

Observe that the inner algebra objects of modular categories arising from two dimensional boundary conformal field theories in [FS], [FRS 2001], [FRS 2003] do not necessarily determine a trialgebra since the coproduct leading to the tensor product of $\mathcal{C}$ and the inner algebra product can, in general, not be realized on one and the same object of $\mathcal{C}$. So, it seems that the formulation of two dimensional boundary conformal field theories in terms of modules over inner Frobenius algebras of modular categories should be a more general framework than one in terms of trialgebras or of three
dimensional extended topological quantum field theories. Besides this, the approach through modules over inner Frobenius algebras has the merit that it has been verified rigorously that nontrivial two dimensional boundary conformal field theories do, indeed, satisfy its axioms. Since we have found a universal symmetry for trialgebras and three dimensional extended topological quantum field theories in [Sch 2002a] and [Sch 2002b], it is therefore tempting to ask if a universal symmetry can also be established for pairs $(C, M)$ consisting of a modular category $C$ and a module $M$ over an inner Frobenius algebra of $C$.

Let us now briefly introduce the idea of a universal symmetry of an algebraic structure. We will restrict to give the general idea, here. For the needed technical details, we refer the reader to [Dri] (or [CP] for a short introduction). Consider a quasitriangular quasi-Hopf algebra with braiding $R$ and Drinfeld associator $\alpha$. In [Dri] Drinfeld asked for the possibility to change the data $(R, \alpha)$ while keeping the rest of the structure of the quasitriangular quasi-Hopf algebra fixed. The Grothendieck-Teichmüller group $GT$ is then defined as the group of such “gauge transformations” of the quasitriangular quasi-Hopf algebra where a certain closure is taken by allowing for formal deformations of the data $(R, \alpha)$ in the sense of certain formal power series. One can show that it is the Drinfeld associator $\alpha$ which is the relevant part of the data to basically determine the structure of $GT$ (see [Dri], [Kon 1999]). So, we will restrict to the consideration of the Drinfeld associator, in the sequel.

In [Sch 2002a] we have considered the question of a corresponding universal symmetry for weak trialgebras where the coproduct receives a Drinfeld associator $\alpha$ and one of the product a dual Drinfeld associator $\beta$. We have shown there that the transformations of these data lead to a self-dual, noncommutative, and noncocommutative Hopf algebra $H_{GT}$ which is a sub-Hopf algebra of the Drinfeld double $D(GT)$ of $GT$.

Now, let $C$ be a modular category, $A$ an inner algebra object of $C$, and $M$ a module over $A$, i.e. on $M$ we have a representation of the algebra product of $A$. We have the following result, then:

**Lemma 2** There exists a self-dual, noncommutative, and noncocommutative Hopf algebra $H_F$ which gives the universal symmetry on the pairs $(C, M)$. Besides this, $H_F$ is a sub-Hopf algebra of the Drinfeld double $D(GT)$ of the
Grothendieck-Teichmüller group $GT$ and the Hopf algebra $H_{GT}$ introduced in [Sch 2002a] is a sub-Hopf algebra of $H_F$.

**Proof.** Following the approach of [Dri] and the introduction of $H_{GT}$ in [Sch 2002a], we observe that we have two types of data for a pair $(C, M)$ the “gauge freedom” of which should determine the universal symmetry: We have the Drinfeld associator $\alpha$ of the modular category $C$ and we can introduce a similar dual associator $\beta$ for the representation of the algebra product of $A$ on $M$. So, as in [Sch 2002a], we have, again, a pair $(\alpha, \beta)$ of a Drinfeld associator and a dual Drinfeld associator. Precisely following the argument given there, we see that the universal symmetry of the pair $(C, M)$ has to be given by a subspace

\[ H_F \subseteq D(GT) \]

because if we would transform both, $\alpha$ and $\beta$, separately, without considering any compatibility constraint between them, each would be transformed by $GT$. The subspace $H_F$ takes into account the constraint that we want to transform a pair $(C, M)$ into a pair $(C, M)$.

One proves by calculation that $H_F$ is closed with respect to product and coproduct of $D(GT)$, i.e. it is a sub-Hopf algebra of $D(GT)$.

Since $\beta$ is a dual Drinfeld associator, i.e. the class of all data $(\alpha, \beta)$ is self-dual (by $C$-linearity of $C$ there can not appear more than duals of Drinfeld associators for $\beta$ and by universality of $H_F$ all duals of Drinfeld associators have to appear), and $H_F$ gives the universal symmetry, $H_F$ is self-dual. Using the fact that $GT$ is non-abelian, we derive that $H_F$ is noncommutative. Self-duality then implies that $H_F$ is also noncocommutative.

Finally, as we mentioned above, the constraint imposed in the definition of a trialgebra is stronger than the one for an inner algebra object of a modular category. Therefore $H_{GT}$ has to be a sub-Hopf algebra of $H_F$. 

In [Sch 2002a] we have shown that trialgebras have a strong stability property which we called ultrarigidity, there. Loosely speaking, trialgebras can not nontrivially be deformed into algebraic structures with two associative products and two coassociative coproducts, all linked in a pairwise compatible way. We have a similar property for pairs $(C, M)$.

**Definition 2** A bialgebra object in a modular category $C$ is an object $A$ in $C$ together with two morphisms

\[ m : A \otimes A \to A \]
and

$$\Delta : A \to A \otimes A$$

in \( \mathcal{C} \) where \( m \) is associative and \( \Delta \) is coassociative and a morphism of the product \( m \).

**Lemma 3** Let \( \mathcal{C} \) be a bounded modular category and \( A \) an algebra object in \( \mathcal{C} \). Then \( A \) can not be nontrivially deformed into a bialgebra object of a modular category \( \tilde{\mathcal{C}} \).

**Proof.** Completely similar to the one given in [Sch 2002a].

As a consequence, a module \( M \) over \( A \) can not be nontrivially deformed into a module over a bialgebra object in some modular category \( \tilde{\mathcal{C}} \). Once again, we call this property ultrarigidity.

Observe that nontriviality of the deformation means, here, that the deformed object is not isomorphic to an algebra object in any modular category. This does not exclude the possibility of nontrivial deformations with respect to a fixed modular category, as we know well from the theory of quantum groups.

Since Hopf algebras and modular categories alone do not have such a strong stability property, this means in the language of physics that two dimensional boundary conformal field theories are a much more rigid class of structures than the corresponding bulk theories.

In analogy to the finite dimensional representations of \( GT \) (which are mixed motives of a special kind, see e.g. [Kon 1999], we will call the finite dimensional representations of \( \mathcal{H}_F \) which are at the same time corepresentations of \( \mathcal{H}_F \) \( \mathcal{H}_F \)-motives. The reader, not acquainted with the notions of periods and motives, may think of a motive as an algebraic abstraction formalizing the period matrices of algebraic varieties we recall that a period matrix is defined by evaluating a basis of cohomology on a basis of homology cycles. The coefficients of such period matrices are called periods. A reader with physics background may imagine the special periods of representations of \( GT \) as the weights of the Feynman diagrams which appear for the conformal field theories describing deformation quantization (see [CaFe], [Kon 1997]).
We have the following result giving a deep interplay between $\mathcal{H}_{GT}$ and $\mathcal{H}_F$.

**Lemma 4** Assume that the conjecture on the transitive action of $GT$ on the algebra $P_{Z,Tate}$ of periods of mixed Tate motives over $Spec(\mathbb{Z})$ of [Kon 1999] is valid. Then the Hopf algebra $\mathcal{H}_{GT}$ acts transitively on the class of $\mathcal{H}_F$-motives where we speak of a transitive action of $\mathcal{H}_{GT}$ if the combined action of $\mathcal{H}_{GT}$ on $\alpha$ and its dual on $\beta$ is transitive for all pairs $(\alpha, \beta)$.

**Proof.** Let $GT_F$ be the subgroup of those elements of $GT$ which are consistent with the constraint leading from $D(GT)$ to $\mathcal{H}_F$, i.e. which can appear as a transformation of one of the components $(\alpha, \beta)$ such that the transformation is consistent with a complete transformation of the pair $(\alpha, \beta)$. Define in a similar way $GT_c$ as the subgroup consisting of those elements of $GT$ which are consistent with the constraint of $\mathcal{H}_{GT}$. Using self-duality of $\mathcal{H}_{GT}$ and $\mathcal{H}_F$ and

$$\mathcal{H}_{GT} \subseteq \mathcal{H}_F$$

it follows that

$$GT_c \subseteq GT_F$$

Making use of the explicit examples of trialgebras constructed in [GS 2000b], one can prove by direct calculation that

$$GT_c = GT$$

because the tensor product construction used in [GS 2000b] is not affected by the introduction of a Drinfeld associator for the Hopf algebra one starts from. So, it follows that

$$GT_F = GT$$

But then a $\mathcal{H}_F$-motive is always given by a pair of finite dimensional representations of $GT$ plus a constraint deriving from the defining constraint of $\mathcal{H}_F$.

But the above mentioned conjecture of [Kon 1999] and a theorem of Nori which states that the algebra of periods is isomorphic to the algebra of functions on the torsor of isomorphisms between algebraic de Rham and Betti cohomology, $GT$ acts transitively on its finite dimensional representations. But since the constraint for $\mathcal{H}_F$-motives is of a type which requires that a pair
of $GT$-representations defines a special Hopf algebra, it follows from the def-
inition of $H_{GT}$ in [Sch 2002a] that $H_{GT}$ acts transitively on the $H_F$-motives.

**Remark 1** This result shows that though $H_{GT} \subseteq H_F$

$H_{GT}$ acts on the representation theoretic side as a symmetry for $H_F$. In a
subsequent publication, we plan to put the results given here into a broader al-
gebraic context. There is some hope that in that context one can establish the
equivalence of $H_{GT}$ and $H_F$ if using a suitable notion of Morita equivalence
for self-dual Hopf algebras.

In the next section, we will make use of the above result in order to derive
a transitive action of a sub-Hopf algebra $H_D$ of $H_{GT}$ on the relative period
matrices of holomorphic $N = 1$ special geometries.

### 3 Application in holomorphic $N = 1$ special geometry

For a type II string theory compactified on a three complex dimensional
Calabi-Yau manifold $Y$ one can show that the corresponding string field the-
ory in the so called $B$-model is given by the Kodaira-Spencer equation of
holomorphic deformations of a complex manifold (see [BCOV]). One can
further show that holomorphic deformations in Kodaira-Spencer theory act
on the third cohomology $H^3(Y)$ of $Y$ and this action completely specifies
the deformation theory. Concretely, the period matrix of $Y$ completely de-
determines the structure constants of the operator product expansion of the
$B$-model with target space $Y$ (see [BCOV] and for short overviews the intro-
ductions of [LMW 2002a], [LMW 2002b]).

The papers [LMW 2002a] and [LMW 2002b] deal with the question if
this approach can be extended to the case of two dimensional boundary
conformal field theory, specifically to a $B$-model of open-closed strings. The
answer is that one gets relative Kodaira-Spencer theory, i.e. the holomorphic
deformation theory of $Y$ together with a complex submanifold $X \subseteq Y$ such that both, $X$ and $Y$, receive holomorphic deformations but the submanifold property is kept. The answer found there is that relative Kodaira-Spencer theory leads to and is determined by an action on the third relative cohomology $H^3(X, Y)$ (see the cited papers or [KaLe] for an introduction to relative cohomology) and the relative period matrix allows one to completely determine the structure constants of the extended chiral ring (which includes the boundary operators).

First, observe that the usual (absolute) period matrices correspond to mixed motives, i.e. finite dimensional representations of the motivic Galois group. Using the just mentioned results on relative period matrices and relative Kodaira-Spencer theory, it follows that a relative period matrix has to correspond to a pair of mixed motives satisfying a certain constraint deriving from the submanifold property

$$X \subseteq Y$$

On the other hand, using the result that relative Kodaira-Spencer theory is precisely the string field theory of the open-closed $B$-model, we know that this geometric submanifold constraint has to be equivalent to the algebraic constraint of the previous section which appears from the inner Frobenius algebra condition in modular categories. It follows that relative period matrices are restricted to correspond to pairs of matrices determined by the quotient group $GT$ of the motivic Galois group and that the constraint precisely restricts them to the type of a $H_F$-motive.

Using the results of the previous section, we immediately have the following corollary, then:

**Corollary 5** There exists a sub-Hopf algebra $H_D$ of $H_{GT}$ that acts transitively on the relative period matrices of holomorphic $N = 1$ special geometries (i.e. on the relative period matrices of the open-closed $B$-model).

**Proof.** By the results of the previous section, $H_{GT}$ acts transitively on $H_F$-motives. It follows that there is a sub-Hopf algebra $H_D$ of $H_{GT}$ which respects the constraint that a $H_F$-motive corresponds to a relative period matrix of holomorphic $N = 1$ special geometry. Obviously, $H_D$ acts transitively on the relative period matrices. $\blacksquare$
Remark 2 In a later publication, we hope to show that $\mathcal{H}_D$ and $\mathcal{H}_{GT}$, again, should be Morita equivalent.

The above corollary means that $\mathcal{H}_D$ acts on holomorphic $N = 1$ special geometries as a duality Hopf algebra in the sense that any two such geometries are dual to each other with respect to the $\mathcal{H}_D$-action.

4 Conclusion

We have shown the existence of a universal symmetry for Frobenius algebras in modular categories given by a self-dual, noncommutative, and noncocommutative Hopf algebra $\mathcal{H}_F$. We have discussed the relationship between $\mathcal{H}_F$ and the Hopf algebra $\mathcal{H}_{GT}$ introduced previously. Finally, we have applied our result to arrive at a sub-Hopf algebra $\mathcal{H}_D$ of $\mathcal{H}_{GT}$ acting as a duality Hopf algebra on holomorphic $N = 1$ special geometries.

It remains a task for future work to discuss the concrete physical implications of the appearance of the Hopf algebra $\mathcal{H}_D$ in holomorphic $N = 1$ special geometry. As we have remarked already above, we plan to put the results gained, here, in a broader algebraic context in a future publication, in order to achieve a deeper understanding of the appearance of these Hopf algebraic structures on the moduli spaces of compactifications in string theory.

Acknowledgements:

I thank the Erwin Schrödinger Institute for Mathematical Physics, Vienna, and the Institute for Theoretical Physics of the University of Innsbruck for hospitality during part of the time in which this work has been done. For discussions on the subjects involved, I thank Bojko Bakalov, Alberto Cattaneo, Karen Elsner, Harald Grosse, Christoph Schweigert, and Ivan Todorov.

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