Micromechanical compaction model for unidirectional fabrics

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Abstract: In this paper, the effect of nesting on the behavior of the compression response of unidirectional fabrics is studied. Based on the change of the fiber bundle’s cross-section, a uniform compression model was established. The model associated fabric thickness with compression load, fabric structure and layer shifting can be used to describe the compressive behavior under any nested state. Compression tests with 6 layers were done. Both of theoretical and experimental results show that the average thickness decreases with the increase of layer shift. As there are two major mechanisms in the compaction of multilayer fabric preform, the thickness reduction caused by nesting was deeply researched.

1. Introduction
Nesting caused by layer shift is a geometric phenomenon and inevitably occurs during the process of ply stacking in liquid composite molding. It generally refers to (partial) filling of inter-yarn voids on the surface of a layer by yarns of an adjacent layer. The nesting of adjacent layers can largely change the loading features of the yarn and lead to a huge difference on compaction behavior.

Ivanov et al. [1, 2] demonstrated that the average thickness of a ply in a two-ply nested pack of twill is about 5–8% smaller than in the single ply preform, and nesting in a sparse triaxial braided fabric can affect up to 50% of average ply thickness. Potluri et al. [3] found that the average thickness per layer in a two-layer plain weave fabric is about 15–20% smaller than in the single ply preform at zero pressure and this effect can reach up to 30% as the pressure increased. There are also many experimental works about the effect of nesting on compaction behavior of non-crimp fabrics, including Chen [4], Hammami [5] Lomov [6] and Li [7].

To predict the compaction behavior, various models have been proposed by several investigators. The focus of most of the theoretical models has been on employing the simplest method of fitting experimental data to obtain an empirical or semi-empirical relationship between the applied pressure and the compressed thickness or fibre volume fraction [8-10].

In this paper, the behavior of the compression response of unidirectional fabrics is studied. Based on the reasonable assumption on the change of the fiber bundle’s cross-section and its constitutive model, a uniform compression model was established to describe the compressive behavior under any nested state.

2. Theoretical Model

2.1 Layer nesting
Layer nesting refers to the interaction between neighboring fabric layers of a textile composite laminate. For unidirectional multi-layer fabrics, nesting is characterized by arrangement of the yarn throughout the laminate by shifting each layer in x direction (Figure 1). Assuming that all layers are at
the same nesting level, the average thickness per layer is estimated as

\[
\bar{h} = b[ \frac{1}{n} (1 - \frac{\Delta x}{a})^2 + \frac{1}{n} ] \quad 0 \leq \Delta x \leq \frac{L}{2}
\]  

(1)

where \(\Delta x\) is the horizontal translation in x-direction, and \(a\) and \(b\) are long and short axes of the bundle. \(L\) is the cross-section center distance of neighboring fiber bundle. \(n\) is the number of layers.

Figure 1. Schematic of nesting between adjacent layers for unidirectional fabrics.

The ratio of average thickness between maximum and minimum nesting is

\[
\frac{\bar{h}_{\text{max}}}{\bar{h}_{\text{min}}} = \frac{\sqrt{3}}{2} + \frac{1}{n}(1 - \frac{\sqrt{3}}{2})
\]  

(2)

From Eq.( 2), it can be seen that the average thickness decreases with increasing the number of layers and the value with maximum nesting is about 86.6% of the minimum nesting when \(n \to \infty\).

2.2 Compaction model

The compaction behavior depends on the applied compressive force, geometrical yarn parameters and nesting caused by layer shifting. The cross-sectional shape of the yarn changes as the yarn is compacted. The initial cross-section of the yarn is considered to be elliptic.

When \(0 \leq \Delta x \leq \epsilon\), the cross sections of yarns between adjacent layers are tangent and the two ellipses are compressed along the normal direction of the contact point, as shown in Figure 2. With the increase of pressure, the geometric relation is changed into line contact instead of point contact and the contact line becomes longer gradually. It is assumed that the deformed cross-section of the yarn consists of a trapezoidal part located between two semi-elliptical parts. Fibers in the trapezoidal part will be compressed first and the fiber volume fraction of this part increases. The two side parts of the yarn cross-section will deform due to the action of squeezing flow, therefore the fiber volume fraction of the two side parts remains unchanged as no compaction occurs in these areas.

Figure 2. Compaction model when \(0 \leq \Delta x \leq \epsilon\).

The areas of the trapezoidal part and two semi-elliptical parts are

\[
S_{\text{trapezoid}} = h'c \cdot \cos \theta
\]  

(3)

\[
S_{\text{left semi-ellip.}} = \frac{\pi}{8} (a' + \Delta x - c \cdot \cos \theta)(h' + c \cdot \sin \theta)
\]  

(4)
The acute angle between the contact line and the X axis is:

$$\theta = \arctan \frac{b\Delta x}{a\sqrt{a^2 - \Delta x^2}}$$  \hspace{1cm} (6)

Fiber volume continuity requires:

$$S_{\text{ellipse}}V = S_{\text{auxial}}V' + S_{\text{left}}V + S_{\text{right}}V$$  \hspace{1cm} (7)

Substituting Eqs.(3-6) into Eq. (7) gives:

$$c = \frac{\pi}{4} \left( a - h' \right) \frac{\left( h - \frac{\pi}{4} h' \cos \theta + \frac{\pi}{4} \Delta x \sin \theta \right)}{(h - h') \cos \theta + \frac{\pi}{4} \Delta x \sin \theta}$$  \hspace{1cm} (8)

Then the compressive force, $F$, can be expressed as

$$F = \sigma c \frac{SA}{N} \cos \theta$$  \hspace{1cm} (9)

where $S$ is the area of samples, $A$ is the areal density of unidirectional fabrics, $N$, is the line density of yarn and $\sigma$ is the compressive stress and can be determined as

$$\sigma = \frac{3\pi E}{\beta^4} \left( \frac{1 - \sqrt{h}}{\sqrt{h'}} \right) \left( \frac{h'V_{a}}{hV} - 1 \right)$$  \hspace{1cm} (10)

where $E$ is the axial Young’s modulus, $\beta$ is a non-dimensional constant defined in [11], and $V_a$ is the maximum fiber volume fraction.

Substituting Eqs. (8) and (10) into Eq. (9) gives the relation between the force $F$, the layer shift $\Delta x$ and the distance between adjacent layers $h'$.

**Figure 3.** Compaction model when $e \leq \Delta x \leq L/2$

**Figure 4.** Fiber cross-section configuration after compaction when $e \leq \Delta x \leq L/2$ and $h' < 2z_l$.

When $e \leq \Delta x < L/2$ as shown in Figure 3, before compaction i.e. $h = 2z_l$, the ellipses $O_1$ and $O_2$ are tangent at the point $(x_0, z_0)$. After compaction, the geometric relation is changed into line contact instead of point contact.
When $h' < z_1$, it is assumed that the deformed cross-section is decomposed into five different parts as shown in Figure 4. Fibers in the contact parts of the yarn cross-section will carry load first, thus will deform and be compacted first. The fiber volume fraction of the trapezoidal part increases during the process of compaction.

The acute angle between the contact line and the X axis on the left side of the ellipse $O_i$ is

$$\theta_i = \arctan \frac{b(L - \Delta x)}{a\sqrt{a^2 - (L - \Delta x)^2}}$$  \hspace{1cm} (11)

The angle $\theta_i$ is calculated as

$$\theta_i = \arctan \frac{b\Delta x}{a\sqrt{a^2 - \Delta x^2}}$$  \hspace{1cm} (12)

The areas of different regions can be expressed as:

$$S_i = \frac{1}{4} (h' - c_i \sin \theta_i)(a + \Delta x - L - c_2 \cos \theta_i)$$  \hspace{1cm} (13)

$$S_2 = h' c_2 \cdot \cos \theta_2$$  \hspace{1cm} (14)

$$S_3 = \frac{1}{4} (2h' + c_1 \sin \theta_1 + c_2 \sin \theta_i)(L - c_1 \cos \theta_1 - c_2 \cos \theta_2)$$  \hspace{1cm} (15)

$$S_{\text{trapezoid}} = h' c_1 \cdot \cos \theta_1$$  \hspace{1cm} (16)

$$S_4 = \frac{1}{4} (h' - c_1 \sin \theta_1)(a - \Delta x - c_1 \cos \theta_1)$$  \hspace{1cm} (17)

Fiber volume continuity requires:

$$\frac{1}{2} S_{\text{ellipse}} + \frac{1}{4} (b + h' + c_1 \sin \theta_1)(c_1 \cos \theta_1 - \Delta x) = S_{h' + S_i}$$  \hspace{1cm} (18)

$$\frac{1}{2} S_{\text{ellipse}} - \frac{1}{4} (b + h' + c_1 \sin \theta_1)(L - \Delta x - c_2 \cos \theta_2) = S_{h' + S_i}$$  \hspace{1cm} (19)

The deformation of the fiber cross-section occurs mainly its thickness direction [12], also leads to the following relation:

$$\frac{h}{h'} = \frac{v}{v'}$$  \hspace{1cm} (20)

$$\frac{z_1}{h'/2} = \frac{v'}{v}$$  \hspace{1cm} (21)

Substituting Eqs. (11)-(17) and (20)-(21) into Eqs. (18) and (19) gives:

$$c_1 = \frac{\Delta x b + a(h' - \frac{\pi}{2} b)}{(b + 2h' - 4h) \cos \theta_1 + (a - 2\Delta x) \sin \theta_1}$$  \hspace{1cm} (22)

$$c_2 = \frac{b(L - \Delta x) + a(h' - \frac{\pi}{2} b)}{(b + 2h' - 8z_1) \cos \theta_1 + (a + 2\Delta x - 2L) \sin \theta_1}$$  \hspace{1cm} (23)

Then the compressive force, $F$, can be expressed as

$$F = \frac{SA}{N_i} (\sigma_i, \cos \theta_1 + \sigma_2 c_2 \cos \theta_2)$$  \hspace{1cm} (24)

When $2z_1 < h' < 2z_0$, only the right side of ellipse $O_i$ is compressed. The compressive force can be expressed as

$$F = \frac{SA}{N_i} \sigma_i, c_i \cdot \cos \theta_1$$  \hspace{1cm} (25)

Comparing Eqs. (9), (24) and (25), it can be seen that the function of the force can be described as the general expression
\[ F = \frac{S A}{N_f} (\sigma_1 \cos \theta_1 + \sigma_2 \cos \theta_2) \]  

(26)

where the compressive stress, the length of the contact line and the angle take different values depending on the layer shift. When \( 0 \leq \Delta x \leq e \) or \( e \leq \Delta x \leq L/2 \& 2z_i < h' < 2z_i \), only one side of the yarn cross-section is compressed and the second term on the right side of Eq. (26) is equal to 0.

According to the equilibrium condition, the average pressures applied on each layer must be equal for an n-layer fabric preform. The average thickness per layer can be described as

\[ \bar{h} = \frac{1}{n} \sum_{i=1}^{n} h_i' + h_{out} \]  

(27)

where \( h_i' \) is the distance between the layer \( i \) and \( i+1 \), and takes different values as the shifting between adjacent layers is independent and random, \( h_{out} \) is the thickness of outer layer, which is equal to the value of fabrics with minimum nesting.

3. Experiment

3.1 Textile reinforcement
The textile composite reinforcement analyzed in this paper is E-glass unidirectional fabric EDW450 with areal density 450 g/m². The line density of each yarn is 1.19 g/m, which contains 2200 fiber filaments with diameter of 16μm. The long and short axes of the yarn cross-section are 3.55mm and 0.46mm, respectively. The axial Young’s modulus of glass fiber \( E \) is 72GPa. The non-dimensional constant \( \beta \) is 172.

3.2 Test Procedure
Compression tests were done on a displacement-controlled testing machine Instron 4467. The test speed was 1 mm/min. Fig. 8 illustrates the measurement technique. An assistant device is helpful for injection when the maximum load reaches to 6.4KN. The injected fluid was epoxy resin 2008 with the hardener 2008-F. After cured at room temperature, the specimen was demoulded for measuring the thickness.

4. Results and Discussion

Figure 5. Compression behavior of (a) 2 layers (b) 6 layers.

Figure 6. The ratio \( \Delta h_{spring} / \Delta h_{total} \) as a function of pressure.

The relationship among the average thickness per layer \( h \), the shifting ratio \( \xi \), and the pressure \( F \) is shown in Figure 5. When the shifting ratio is fixed at a certain value, the average thickness per layer decreases rapidly in initial stage and decreases slowly in following stage with increasing the pressure. The pressure-thickness curves of unidirectional fabrics with 6 layers can be obtained by substituting the layer shifting values into the theoretical models as shown in Figure 4. The prediction of three samples was compared with the experimental data, and satisfactory agreement was observed.
Notice that there are two major mechanisms in the compaction of multilayer fabric preform, the total average thickness reduction is $\Delta h_{\text{total}}$. One is the nesting due to shifting between adjacent layers, the average thickness reduction caused by nesting is $\Delta h_{\text{nesting}}$, and the other is the elastic compaction of structural elements at different hierarchical levels, the average elastic thickness reduction is $\Delta h_{\text{elastic}}$. Figure 6 demonstrates the change of ratio $\Delta h_{\text{nesting}} / \Delta h_{\text{total}}$ as a function of pressure. The ratio decreases rapidly in the initial stage and decreases slowly in the following stage with increasing the pressure. In addition, it can be seen that the effect of nesting on compaction behavior is very different when the layer shifting changes. The larger the shifting ratio is, the greater the influence of nesting is.

5. Conclusions
The objective of this study was to develop a uniform micromechanical model to investigate the compaction behavior of unidirectional fabrics considering the effect of nesting. The model associated fabric thickness with compression load, fabric structure and layer shifting can be used to describe the compressive behavior under any nested state. It was found that the compression behavior depends on the layer shifting and the larger the degree of nesting the greater the effect of nesting.

References
[1] Ivanov D S, Lomov S V 2014 *Compos. A-Appl. Sci. Manuf.* **64**(21) 167-176.
[2] Ivanov D S, Baudry F, Broucke B V D 2009 *Compos. Sci. Technol.* **69**(9) 1372-1380.
[3] Potluri P, Sagar T V 2008 *Compos. Struct.* **86**(1) 177-185.
[4] Chen X, Spola M 1999 *J. Text. Inst.* **90**(1) 91-99.
[5] Hammami A 2001 *Polym. Compos.* **22**(3) 337–348.
[6] Lomov S V, Barburski M, Stoilova T, et al. 2005 *Compos. A-Appl. Sci. Manuf.* **36**(9) 1188-1206.
[7] Long L, Yan Z, Jin Y, et al. 2015 *J. Mater. Sci.* **50**(7) 2960-2972.
[8] Matsudaira M, Qin H 1995 *J. Text. Inst.* **86**(3) 476–86.
[9] Hu J, Newton A 1997 *J. Text. Inst.* **88**(3) 242–54.
[10] Pearce N, Summerscales J 1995 *Compos. Manuf.* **6**(1) 15–21.
[11] Cai Z, Gutowski T 1992 *J. Compos. Mater.* **26**(8) 1207–37.
[12] Saunders RA, Lekakou C, Bader MG 1999 *Compos. Sci. Technol.* **59**(7) 983–93.