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Comment on "Superfluid Turbulence from Quantum Kelvin Wave to Classical Kolmogorov Cascades"

Jeffrey Yepez

George Vahala

Linda L. Vahala
Old Dominion University, lvahala@odu.edu

Min Soe

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Our initial vortices had winding number \( n = 6 \), equivalent to 6 overlapping \( n = 1 \) vortices, a highly unstable configuration as illustrated in Fig. 1. We used ray tracing to image surfaces around the nodal lines \( \psi = 0 \).

Consider an \( L^3 = 2048^3 \) simulation with initial vortex wave number \( k_\ell = 40 \) and vortex-vortex separation \( \ell \sim \sqrt{\frac{L}{k_\ell}} = \frac{2048}{40} = 51 \), using a total vortex length \( L_v = 12nL \).

In the initial linear vortex spectrum, the transitional wave number between \( k^{-1} \) and \( k^{-3} \) related to the inverse coherence length, \( k_{\text{linear}} \), is pronounced. In contrast, in the quantum turbulence spectrum with clean \( k^{-5/3} \) and \( k^{-3} \) power laws, the transitions related to the inverse Kolmogorov scale, \( k_{\text{outer}} = k_\ell \sim \ell^{-1} \), and an inner scale, \( k_{\text{inner}} \), are both pronounced. This is seen in Fig. 2 with \( k_{\text{linear}} = 51^{-1} \ell = 40 \) at \( t = 0 \) (no KWs) and with \( k_{\text{tangle}} = 40 \) at \( t = 20,000 \) in a KW cascade. Thus, we find \( k_{\text{linear}} = k_{\text{tangle}} \), and this similarity also occurred for the 5760\(^3 \) simulation reported in our Letter [2]. We identified the classical to quantum transition region as \( k_{\text{outer}} \leq k_{\text{linear}} \leq k_{\text{inner}} \) and identified the outer scale with the Kolmogorov length \( (k_{\text{outer}} = k_\ell) \) and the inner scale with the coherence length. When the \( k^{-3} \) spectrum is absent or significantly diminished, temporally due to intermittency [3], we do not see a vortex tangle with a KW cascade. When the \( k^{-3} \) spectrum at high \( k \geq k_{\text{inner}} \) is present (along with a \( k^{-5/3} \) Kolmogorov spectrum at small \( k \leq k_{\text{outer}} \) marking a vortex tangle), we see distorted vortices supporting KWs undergoing kelvon-kelvon couplings, including \( k > k_{\text{linear}} \).

We believe there is essential dynamics at high wave numbers \( k > k_\ell \). The \( L^3 = 5760^3 \) grid simulation we reported has \( \sim 10^{11} \) microscopic (bit) particles, and a single vortex can contain hundreds of thousands of grid points. The unitary algorithm \( \Psi' = U \Psi \) employs a tensor product state \( \Psi = \psi(x) \otimes L \) separated over the \( L^3 \) points of the system, where each local ket \( \psi(x) \) is a 2-spinor. This gives an exact quantum simulation modulo the lattice cutoff \( \ll \xi \) that accurately solves the Gross-Pitaevskii equation. A fluctuating part of \( \psi(x) \) are quasiparticles

\[
\delta \psi(x) \equiv \epsilon \left( u(x)e^{-i\xi t} - v^*(x)e^{i\xi t} \right)
\]

given by the Bogoliubov–de Gennes (BDG) equations,

\[
h \left( \frac{\partial}{\partial t} \psi^2 - \nabla^2 \psi^2 - g \phi^2 \psi^2 \right) \psi
\]

with a spatial operator \( L \equiv -\frac{k^2}{2m} \nabla^2 + 2g|\phi|^2 \mu \). High \( k \)-space resolution, especially at large \( k \), is vital to ensure these fluctuations are numerically represented inside the cores. Finally, high-\( k \) kelvons are known experimentally [4], and such kelvons have been verified numerically at the BDG level [5,6]. The cutoff \( r_c < \xi \) is inside the core with a modified KW dispersion relation [6].

Jeffrey Yepez,1,* George Vahala,2 Linda Vahala,3 and Min Soe4
1Air Force Research Laboratory, Hanscom Air Force Base, Massachusetts 01731, USA
2Department of Physics, William & Mary, Williamsburg, Virginia 23185, USA
3Department of Electrical and Computer Engineering, Old Dominion University, Norfolk, Virginia 23529, USA
4Department of Mathematics and Physical Sciences, Rogers State University, Claremore, Oklahoma 74017, USA

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*To whom correspondence should be addressed.

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