PROVE OR DISPROVE
100 CONJECTURES FROM THE OEIS

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Abstract. Presented here are over one hundred conjectures ranging from easy to
difficult, from many mathematical fields. I also briefly summarize methods and
tools that have lead to this collection.

Dedicated to all contributors to the OEIS,
on occasion of its 100,000th entry.

The On-Line Encyclopedia of Integer Sequences [OEIS] is a database containing the start
terms of nearly 100,000 sequences (as of Autumn 2004), together with formulæ and refer-
ences. It was reviewed by Sloane in 2003 [Slo03]. In the first section of this work, I describe
how such a database can assist with finding of conjectures, and the tools necessary to this
end; section 2 then lists over one hundred so found propositions from many fields that
have been checked numerically to some degree but await proof. Finally, I conclude in
section 3, giving a webpage that follows the status of the assertions.

1. Finding connections

With a string of numbers representing an integer sequence, a conjecture appears already
whenever a possible formula for a prefix of the sequence is found, or when a transformation
is discovered that maps from one prefix to that of another sequence. The OEIS consists
just of such prefixes. While the online interface to the OEIS allows searching for simple
patterns in the database, its ‘superseeker’ eMail service is able to find formulæ for several
types of sequences directly, by computation, or indirectly, by applying transformations
and comparing with the entries. However, this way, it is not possible to do bulk searches
of the whole or parts of the database.

To help with systematic work, Neil Sloane, the creator/editor/maintainer of the OEIS,
offers a file where only the numbers are collected, and this file served as input to several
computer programs I wrote over two years of work as associate editor. The methods can
be divided according to program usage and intention:

• using a C program, I extended the database’s 10^5 sequences with their first and
  second differences, with subsequent lexicographical sort and visual inspection us-
  ing a simple scrolling program (like Unix less). This method sounds awkward
  but is not very much so since close matches show clearly through all noise. It
  yielded several conjectures, among many ‘trivial’ identities and a few hundred
duplicates.
• for frequent offline lookups, I built a further extended database by applying many
  transformations (bisections, pairwise sums, part-gcd, odd part, to state the more
  non-obvious) to a core set of OEIS sequences. The resulting file with 370MB size
  was searched with Unix grep when needed and using several different strategies,
and this yielded a good part of the conjectures of section 2, not to speak of another lot of identities that were included for reference in the database.

- special scans of the OEIS numbers, searching for sequences of specific kind, most notably for C–finite sequences, using my own implementation of guessgf in Pari\[GP\], and for bifurcative* sequences. It has to be noted that scanning for specific types of sequences is nontrivial and cannot be fully automatized—even in the case of C–finiteness—, one still has to check for false positives. From the scans, I have only included those conjectures that seemed the most surprising to me; everything else served as immediate improvement of the information contained in the OEIS.

Comparing the quantities of output from the methods, simple transformations resulted in the most hits, I also had the impression that C–finiteness is quite common: about 12 per cent of sequences have the property. On the other hand, to squeeze out more identities from the database, ever more specialized transformations/scans would be necessary, with less and less results per method. There is definitely a kind of fractality to it, perhaps reflecting expertise structure of OEIS submitters.

2. The conjectures

From the very start, Neil Sloane had encouraged all kind of integer sequences to be submitted. The decision has lead to a statistically representative snapshot of sequences from all mathematical fields where integers occur. A similar mixture is visible in my following conjectures which deal with power series expansions, number theory, and enumerative combinatorics, as well as additive and combinatorial number theory and, due to my special interest, bifurcative and other nonlinear sequences. This is also roughly the order in which the statements are presented. I refrained from cluttering the presentation with the respective A–numbers, also to encourage independent recalculation.

Please note that, throughout the section, I use a set of commonly known abbreviations that are listed in Table 1 and some of whom were introduced by Graham et al.[Gra94].

2.1. Easy start: special functions, binomials, and more.

1. \[(1 - 4x)^{3/2} = 1 - 6x + \sum_{n>1} \frac{12(2n - 4)!}{n!(n - 2)!} \frac{x^n}{n^{3/2}}.\]

2. \[(1 + x^2)C(x)^2 C(x)^2 = \sum_{n \geq 0} \frac{6n(2n)!}{n!(n + 1)!(n + 2)!} x^n, \quad C(x) = \frac{1 - \sqrt{1 - 4x}}{2x}.\]

3. \[(1 - x)^{1/4} = \sum_n \prod_{k=1}^{n} \frac{(4k - 3)2^{v_2(k)} / k}{2^{n-e_1(n)}} x^n.\]

4. \[x^n \] P_{n+2}(x) = \frac{1}{2^{e_1(n+1)}} (n + 1) \binom{2n + 2}{n + 1}, \quad (P_n \text{ the Legendre polynomials}).

5. \[\sum_{k=0}^{n} -v_2([x^k] P_{2n}(x)) = 2n^2 + 2n - 2 \sum_{i=0}^{n} e_1(i).\]

6. \[B(2n, \frac{1}{2}) = \frac{a}{b}, \quad B(2n, \frac{1}{2}) = \frac{c}{d} \implies a = c, \quad B(n, x) \text{ Bernoulli polynomials}.\]

*formerly called divide-and-conquer.
PROVE OR DISPROVE 100 CONJECTURES FROM THE OEIS

$p$ a prime

$n$ even 1 if $n$ even, 0 otherwise

$\#(a, b)$ greatest common divisor of $a, b$

$\text{lcm}(a, b)$ least common multiple of $a, b$

$\lfloor x \rfloor$ floor, greatest integer $\leq x$

$\lceil x \rceil$ ceiling, smallest integer $\geq x$

$a \mid b$ a divides $b$

$\{n \mid A\}$ the set of all $n$ with property $A$

d(n) $\tau(n), \sigma_0(n)$, number of divisors of $n$

$\phi(n)$ Euler totient/phi function

$\sigma(n)$ $\sigma_1(n)$, sum of divisors of $n$

$F_n$ $n$-th Fibonacci number, with $F_0 = 0$

$P(x)$ a polynomial in $x$

$P(x)^n$ $n$-th coefficient of polynomial or power series $P$

$\frac{x^n}{n!} f(x)$ $n$-th coefficient of Taylor expansion of $f$

$\lg x$ base-2 logarithm of $x$

$v_2(n)$ dyadic valuation of $n$, exponent of 2 in $n$

$e_1(n)$ number of ones in binary representation of $n$

$\text{Res}$ resultant

**Table 1.** Symbols and abbreviations.

(7) $\arctan(\tanh x \tan x) = \sum_{n \geq 0} 2^{6n+3}(2^n - 1)\frac{B_n}{n} x^n.$

(8) $\tau(2^n) = [x^{2^n}] x \prod_{k \geq 1} (1 - x^k)^{24} = [x^{2^n}] \frac{1}{2048x^2 + 24x + 1}.$

(9) $7 \mid \left[ \frac{x^{6k+4}}{(6k+4)!} \right] \frac{1}{2 - \cosh(x)}.$

(10) $4 \mid \left[ \frac{x^{6k+4}}{(6k+4)!} \right] \exp(\cos x - 1), \quad 11 \mid \left[ \frac{x^{10k}}{(10k)!} \right] \exp(\cos x - 1) \ldots$

(11) $\text{Res}(x^n - 1, 4x^2 - 1) = 4^n - 2^n - (-2)^n + (-1)^n.$

(12) $\{ n \mid n^2 - 1 \mid \binom{2n}{n} \} \setminus \{ 2 \} \subset \{ n \mid n!(n-1)! | 2(2n-3)! \}.$

(13) $\sum_{k=0}^{n} (k+1) \sum_{l} 2^l \binom{n}{l} \binom{k}{l} = [x^n] \frac{1 - x}{(1 - 2x - x^2)^2}.$

(14) $\sum_{k=0}^{n} (k+1) \left[ \binom{2n}{k} - \binom{2n}{k-1} \right] = \frac{n+2}{2} \binom{2n+2}{n+1} - 4^n.$

(15) $a(n) = \sum_{k=0}^{n} \left[ \binom{n}{k} \mod 2 \right] \cdot 2^k \implies a_{2n+1} = 3a_{2n}.$
\[(\sum_{k=0}^{\lfloor n/2 \rfloor} D^k \binom{n}{2k+1}) = \left\lfloor \frac{x}{1 - 2x + (1 - D)x^2} \right\rfloor.\]

\[(\binom{2n}{n}, \binom{3n}{n}, \ldots, \binom{(n-1)n}{n}) = 1 \iff p^e \mid n \land p^{e+1} \nmid n \land \frac{n}{p^{e}} > p^e.\]

\[\{a_n \mid \text{Least term in cont. frac. of } \sqrt{a_n} = 20\} = 100n^2 + n.\]

\[\text{PCF} (n) = \text{PCF} (n+1) \implies n \equiv 1 \mod 24, \quad \text{PCF} (n) = \text{period of cont. frac. for } \sqrt{n}.\]

\[\text{The largest term in the periodic part of the cont. frac. of } \sqrt{3n + 1} \text{ is } 2 \cdot \left\lfloor (\sqrt{3})^n \right\rfloor.\]

The numerators of the continued fraction convergents to $\sqrt{27}$ are
\[\left\lfloor x^n \right\rfloor \frac{5 + 26x + 5x^2 - x^3}{1 - 52x^2 + x^4}.\]

### 2.2. Classical number theory.

\[n = 5^i 11^j \implies n \mid \sum_{k=1}^{10} k^n.\]

\[n + 1 \mid d(n^n).\]

Group the natural numbers such that the product of the terms of the $n$-th group is divisible by $n!$. Let $a_n$ the first term of the $n$-th group. Then
\[a_n = \left\lfloor \frac{(n-1)^2}{2} \right\rfloor.\]

\[\#\{\text{cubic residues mod } 8^n\} = \frac{4 \cdot 8^n + 3}{7}.\]

\[\text{lcm}(3n + 1, 3n + 2, 3n + 3) = \frac{4}{3}(9n^3 + 18n^2 + 11n + 2)(3 + (-1)^n).\]

Let $a_n = \prod_{k=1}^{n} \text{lcm}(k, n - k + 1)$. Then $a_n = n^2(n - 1)!^2$ for $n$ even, $n + 1$ prime. Also, if $n$ is odd and $> 3$, $2(n + 1)a_n$ is a perfect square, the root of which has the factor $\frac{1}{2}n(n - 1)((n - 1)/2)!$.

\[\{ \min(x) \mid p \mid px - x - 1 \} = p - 1, \quad p = \text{prime}(n).\]

\[\emptyset = \{n \mid \forall i, j, \ 0 < i < j < n, \ n > 3 : \ 2^n + 2^i + 2^j + 1 \text{ is composite}\}.\]

\[p \equiv 5 \mod 12 \iff \emptyset = \{x \mid x^4 \equiv 9 \mod p\}.\]

\[d(n) = d(n+1) = \cdots = d(n+6) \implies n \equiv 5 \mod 16.\]

\[\frac{\sigma(2n)}{\sigma(n)} = \frac{4 \cdot 2^{v_2(n)} - 1}{2 \cdot 2^{v_2(n)} - 1}.\]
\( \emptyset = \left\{ x \mid \phi(x) = 2^k, k > 0 \right\} \iff 2p + 1 \text{ composite.} \)

\[ \{ n = kl \mid \phi(n+12) = \phi(n) + 12 \land \sigma(n+12) = \sigma(n) + 12 \} \implies n \equiv 64 \mod 72. \]

\( \{ n \mid \sigma(d(n^3)) = d(\sigma(n^3)) \} \implies n \equiv 1 \mod 24. \)

\[ \{ n \mid \sigma(n) = 2u(n) \} \implies n \equiv 108 \mod 216, \quad u(n) = \sum_{d \mid n} d. \]

\[ \{ n \mid t(n) = t(t(n) - n) \} = \left\{ n \mid n = 5 \cdot 2^k \lor n = 7 \cdot 2^k, k > 0 \right\}, t(n) = |\phi(n) - n|. \]

\[ \{ n \mid \phi(n^2 + 1) = n\phi(n+1) \} = \{8\} \cup \left\{ n \mid n^2 + 1 = \text{prime} \land n+1 = \text{prime} \right\}. \]

\[ \{ n \mid |n - 2d(n) - 2\phi(n)| = 2 \} = \{2, 72\} \cup \left\{ 16p \mid p > 2 \right\}. \]

\[ \{ \text{Local maxima of } \sigma(n) \} \subset \left\{ m \mid m = \sigma(l) \land l = \text{local maximum of } d(n) \right\}. \]

\( F_n \mod 9 \text{ has period } 24. \)

\( (2^p - 1, F(p)) > 1 \implies (2^{kp} - 1, F(kp)) > 1. \)

\( (2^p - 1, F(p)) > 1 \implies 8p \mid (2^p - 1, F(p)) - 1. \)

\( (2^p - 1, F(p)) > 1 \land p \not\equiv 1 \mod 10 \implies \frac{(2^p - 1, F(p)) - 1}{8p} > 1. \)

\( \# \{ k \mid F_k \mid F_n \} = d(n) - [n \text{ even}]. \)

\[ \{ \max a_n \mid a_n \text{ squarefree} \land \mathbb{Q}(\sqrt{a_n}) \text{ has class number } n \} \implies a_n \equiv 19 \mod 24. \]

Define \( \varsigma(n) \) the smallest prime factor of \( n \). Let \( a_n \) the least number such that the number of numbers \( k \leq a_n \) with \( k > \varsigma(k)^n \) exceeds the number of numbers with \( k \leq \varsigma(k)^n \). Then

\( a_n = 3^n + 3 \cdot 2^n + 6. \)
2.3. Additive/combinatorial number theory.

\[
\# \left\{ m \mid m = [x^n] \prod_{k=1}^{n} \sum_{j=0}^{k} q^j \right\} = \frac{[x^n]x(1-x + 2x^2 - 2x^3 + x^4)}{(1+x^2)(1-x)^3}.
\]

\[
\# \left\{ m \mid m = [x^n] \prod_{k=1}^{n} \sum_{j=0}^{k} q^{2j+1} \land n > 6 \right\} = n^2 - 3.
\]

Let \( A_{n \geq 0} \) the number of distinct entries in the \( n \times n \) multiplication table, i.e., the number of distinct products \( ij \) with \( 1 \leq i, j \leq n \), then \( A_n \) goes \( \{0, 1, 3, 6, 9, 14, \ldots \} \). Let further \( B \) the set of composite numbers \( > 9 \) that are not equal to the product of their \( \text{aliquot divisors} \) (divisors of \( n \) without \( n \)). Then

\[
A_n - A_{n-1} = \frac{n+\text{its smallest divisor} > 1}{2}, \text{ if } n \not\in B.
\]

Define the sequence \( \{a_n\} \) as \( a_1 = 2 \), \( a_2 = 7 \), and \( a_n \) the smallest number which is uniquely \( a_j + a_k, j < k \). The sequence starts \( \{2, 7, 9, 11, 13, 15, 16, 17, 19, 21, 25, \ldots \} \). Then

\[
a_n - a_{n-1} \text{ has period } 26.
\]

Let \( A \) the set of numbers whose cubes can be partitioned into two nonzero squares, and \( B \) the set of numbers that are the sum of two nonzero squares. Then

\[
A = B.
\]

2.3.1. Sum-free sequences. The sequence with start values \( \{a_1, a_2, \ldots, a_s\} \) and further values \( a_{n > s} \) satisfying "\( a_n \) is the smallest number \( > a_{n-1} \) not of the form \( a_i + a_j + a_k \) for \( 1 \leq i < j < k \leq n \)." A similar sequence \( b_n \) differs in that it has for the indices \( 1 \leq i \leq j \leq k \leq n \). Then

\[
\text{Start with } \{1, 2, 3\}: \quad a_{n+6} - a_{n+5} = a_{n+1} - a_n, \quad \text{for } n > 6.
\]

\[
\text{Start with } \{1, 2, 3\}: \quad b_n \in \{3, 7\} \lor b_n \equiv 1, 2 \mod 12.
\]

\[
\text{Start with } \{1, 2, 4\}: \quad a_n = \frac{26n - 125 - 11 \cdot (-1)^n}{4}, \quad \text{for } n > 12.
\]

\[
\text{Start with } \{1, 3, 4\}: \quad a_{n+8} - a_{n+7} = a_{n+1} - a_n, \quad \text{for } n > 7.
\]

\[
\text{Start with } \{1, 3, 4\}: \quad b_n = 7n - 21 + 2 \cdot (-1)^n, \quad n > 3.
\]

\[
\text{Start with } \{1, 3, 5\}: \quad a_{n+5} - a_{n+4} = a_{n+1} - a_n, \quad \text{for } n > 6.
\]

Let \( a_1 = 1, a_2 = 2 \) and \( a_n \) the smallest number not of form \( a_i, a_i + a_{n-1} \), or \( |a_i - a_{n-1}| \). Then

\[
a_n = 3n + 3 - 2 \cdot 2^{\lfloor \lg(n+2) \rfloor}, \quad \text{for } n > 2.
\]

Let \( a_n = n \) for \( n < 4 \) and \( a_n \) the least integer \( > a_{n-1} \) not of form \( 2a_i + a_j, 1 \leq i < j < n \). Then

\[
a_n = 4n - 10 - (n \mod 2), \quad \text{for } n > 3.
\]
2.3.2. Progression-free sequences. The sequence with start values \( \{a_1, a_2, \ldots, a_s\} \) and further values \( a_n > s \) satisfying \( a_n \) is the smallest number \( > a_{n-1} \) that builds no three-term arithmetic progression with any \( a_k \), \( 1 \leq k < n \) is well-defined. This peculiar definition was chosen to allow for any start values. We abbreviate such a sequence with start values \( a, b, \ldots \) as \( A(a, b, \ldots, n) \) and present several conjectures for them.

\[
A_3(1, 3, n) = A_3(1, 2, n) + \lceil n/2 \rceil + \frac{1}{2} \sum_{k=1}^{n-1} 3^{v_2(k)}.
\]

\[A_3(1, 3, n) = A_3(1, 2, n) + \lceil n/2 \rceil + \frac{1}{2} \sum_{k=1}^{n-1} 3^{v_2(k)}.
\]

\[
A_3(1, 4, n) = A_3(1, 3, n) + \lceil \lceil n/2 \rceil \rceil \text{ if } n \text{ is even}.
\]

\[
A_3(1, 7, n) = b_n + \frac{\sum_{k=1}^{n-1} 3^{v_2(n)} + 1}{2}, \text{ where } b_n = \{1, 6, 5, 6, 2, 6, 5, 7, \ldots \}_{n \geq 1} \text{ has period 8.}
\]

\[
A_3(1, 10, n) = b_n + \frac{\sum_{k=1}^{n-1} 3^{v_2(n)} + 1}{2},
\]

where \( b_n = \{1, 9, 8, 9, 5, 10, 10, \ldots \}_{n \geq 1} \) has period 8.

\[
A_3(1, 19, n) = b_n + \frac{\sum_{k=1}^{n-1} 3^{v_2(n)} + 1}{2},
\]

where \( b_n = \{1, 18, 17, 18, 14, 18, 17, 19, 5, 18, 17, 18, 14, 19, 19 \ldots \}_{n \geq 1} \) has period 16.

\[
A_3(1, m, n) = b_n + \frac{\sum_{k=1}^{n-1} 3^{v_2(n)} + 1}{2}, \quad m = 1 + 3^k \vee 1 + 2 \cdot 3^k, k \geq 0,
\]

where \( b_n \) has period \( P \leq 2^{\lceil \log_3 n \rceil} \).

\[
A_3(1, 2, 3, n + 2) = 1 + 2^{\lceil \log_3 n \rceil} + \sum_{k=1}^{n} 3^{v_2(n)} + 1 + \sum_{k=1}^{n} \frac{3^{v_2(n)} + 1}{2}.
\]

The numbers \( n \) such that the \( n \)-th row of Pascal’s triangle contains an arithmetic progression are

\[
n = 19 \lor n = \frac{1}{2} \left[ 2k^2 + 22k + 37 + (2k + 3)(-1)^k \right], k > 0.
\]

2.4. Enumerative combinatorics.

The number of \( n \times n \) invertible binary matrices \( A \) such that \( A + I \) is invertible is

\[
2^\left(\frac{n(n-1)}{2}\right) a_n, \quad \text{with } a_n = (a_0 = 1, a_n = (2^n - 1)a_{n-1} + (-1)^n). \]

Let \( A_n \) the number of binary vectors with restricted repetitions, then

\[
\sum_{n \geq 0} A_n x^n = \frac{1}{2x^2} \left( x^2 - 1 + (1 - x)^2 \sqrt{1 + x + x^2} \right).
\]

The sequence \( a_n \) shifts left twice under binomial transform and is described by both \( (a_0 = a_1 = 1, a_n = \sum_{k=0}^{n-2} \binom{n-2}{k} a_k) \) and

\[
\sum_{n \geq 0} a_n x^n = \sum_{k \geq 0} \frac{x^{2k}}{(1 - kx)(1 - x - kx) \prod_{m=0}^{k-1} (1 - mx)^2}.
\]
The number of self-avoiding closed walks, starting and ending at the origin, of length $2n$ in the strip $[0, 1, 2] \times \mathbb{Z}$ is

$$\frac{1}{125} \left\{ (315n - 168)2^{n-2} + (-1)^{n/2} [55n/2 + 78 - (135n/2 + 36)(-1)^n] \right\}, \ n > 1.$$  

The number of non-palindromic reversible strings with $n$ beads is

$$x.$$

The number of level permutations of $2^{n-1}$ beads is

$$x.$$

The number of self-avoiding closed walks, starting and ending at the origin, of length $2n$ in the strip $[0, 1, 2] \times \mathbb{Z}$ is

$$\frac{1}{125} \left\{ (315n - 168)2^{n-2} + (-1)^{n/2} [55n/2 + 78 - (135n/2 + 36)(-1)^n] \right\}, \ n > 1.$$  

The number of non-palindromic reversible strings with $n$ beads of 4 colors is

$$x.$$

The number of non-palindromic reversible strings with $n - 1$ beads of 2 colors (4 beads are black) is

$$x.$$

The number of non-palindromic reversible strings with $n$ black beads and $n - 1$ white beads is

$$x.$$

The number of necklaces of $n$ beads of 2 colors (6 of them black) is

$$x.$$

The number of edges in the 9-partite Turan graph of order $n$ is

$$x.$$

The number of binary strings of length $n$ that can be reduced to null by repeatedly removing an entire run of two or more consecutive identical digits is

$$x.$$

The number of nonempty subsets of $\{1, 2, \ldots, n\}$ in which exactly $1/2$ of the elements are $\leq (n-1)/2$ is

$$x.$$

The number of level permutations of $2n - 1$ is

$$x.$$

The number of rooted trees with $n$ nodes and 3 leaves is

$$x.$$

Let $a_n$ the number of $(2 \times n)$ binary arrays with a path of adjacent 1’s from the upper left corner to anywhere in right hand column. Then

$$x.$$

The number of strings over $\mathbb{Z}_3$ of length $n$ with trace 0 and subtrace 1 is

$$x.$$
The number of strings of length \(n\) over GF(4) with trace 0 and subtrace 0 is

\[
[x^n] \frac{x(-26x^3 + 13x^2 - 5x + 1)}{(1 - 2x)(1 - 4x)(1 + 4x^2)}.
\]

The number of symmetric ways to lace a shoe that has \(n\) pairs of eyelets, such that each eyelet has at least one direct connection to the opposite side, is

\[
\sum_{k=0}^{n} \frac{n!}{k!} F_{k+2}.
\]

The number of minimax trees with \(n\) nodes is \(2^n\) times the number of labelled ordered partitions of a \(2n\)-set into odd parts, that is,

\[
2^n \left[ \frac{x^{2n}}{(2n)!} \right] \frac{1}{1 - \sinh x}.
\]

The number of unlabeled alternating octupi with \(n\) black nodes and \(n\) white nodes is

\[
-2 + 3 \sum_{d|n} \phi(n/d) \left[ \frac{\binom{2d}{d}}{2n} \right].
\]

Finally, the number of dimer tilings of the graph \(S_k \times P_n\) is

\[
[x^n] \frac{1 - x}{1 - (k + 1)x + x^2}.
\]

2.5. Nonlinear recurrences and other sequences.

\[
a_n = [x^n] \frac{-4x^5 + x^4 + x^3 - 3x^2 - 2x + 6}{(1 - x)(1 - x^2 - x^5)} \quad \iff \quad \langle a_0 = 6, a_1 = 10, a_n = \left\lfloor \frac{a_{n-1}}{a_{n-2}} + \frac{1}{2} \right\rfloor \rangle.
\]

\[
a_n = [x^n] \frac{-3x^5 + 2x^4 + x^3 - x^2 - 2x + 4}{(1 - x)(1 - 2x - x^3 - x^5)} \quad \iff \quad \langle a_0 = 4, a_1 = 10, a_n = \left\lfloor \frac{a_{n-1}}{a_{n-2}} \right\rfloor \rangle.
\]

\[
a_n = [x^n] \frac{2x^3 + x^2 - 4x + 5}{-x^5 + 2x^2 - 3x + 1} \quad \iff \quad \langle a_0 = 5, a_1 = 11, a_n = \left\lfloor \frac{a_{n-1}}{a_{n-2}} + \frac{1}{2} \right\rfloor \rangle.
\]

\[
a_n = [x^n] \frac{3x^5 + 2x^4 + x^3 + 4x^2 - x + 6}{-x^5 - x^4 + x^2 - 2x + 1} \quad \iff \quad \langle a_0 = 6, a_1 = 11, a_n = \left\lfloor \frac{a_{n-1}}{a_{n-2}} + \frac{1}{2} \right\rfloor \rangle.
\]

\[
\langle a_0 = a_1 = 1, a_n = (|n - 1 - a_{n-1}| \mod n - 1) + (|n - 1 - a_{n-2}| \mod n) \rangle \quad \implies \quad a_{n+3} = a_n, \quad n > 6.
\]

\[
\langle a_0 = x, a_{n+1} = a_n(a_1 + 1) \rangle \quad \implies \quad [x^{2^n - 3}] a_k = \frac{2^{3k+2} - 2^k}{3}.
\]

\[
\langle \nu_0 = \nu_1 = 1, \nu_n = \nu_{n-1} + 3\nu_{n-3} \sum_{i=0}^{n-2} q^i \rangle \quad \implies \quad [x^n] \nu_n = \sum_{i=0}^{n} x^{2^n - 3} \frac{x^3(9x + 3)}{(1 - 3x - 3x^2)^2}.
\]

\[
\langle a_n = \sum_{k=1}^{\infty} \left\lfloor \frac{2\sqrt{5} - 1}{2} \right\rfloor^{n-k} \rangle \quad \iff \quad \sum_{n \geq 0} a_n x^n = \frac{x(x^5 + x^4 - 4x^3 + 3)}{(1 - x)(1 - x^2)(1 - x - x^2)}.
\]
Let the sequence \( \{a_n\} \) be defined such that \( a_1 = C \) and \( a_{n+1} = \) the smallest difference > 1 between \( d \) and \( p/d \) for any divisor \( d \) of the partial product \( p = \prod_{k=1}^{n} a_k \) of the sequence. Then \( a_n = \{19, 18, 29, 27, 9, \ldots \} \) for \( C = 19 \), and \( a_n = \{21, 4, 5, 13, 8, 2, \ldots \} \) for \( C = 21 \) and

\[
(98) \quad a_n = 3^k, C = 19, n > 3 \quad \text{and} \quad a_n = 2^m, C = 21, n > 4.
\]

2.6. Binary representation, \( k \)-regular and bifurcative sequences.

\[
(99) \quad \# \{ m \mid m = v_2(n), 0 \leq j \leq n \} = |\log(n + 1)| + 1 - v_2(n + 1).
\]

\[
(100) \quad e_1(m) \equiv 0 \mod 2 \iff m \in \{a_0 = 0, a_{2n} = a_n + 2n, a_{2n+1} = -a_n + 6n + 3\}.
\]

\[
(101) \quad e_1(m) \equiv 1 \mod 2 \iff m \in \{a_0 = 1, a_{2n} = a_n + 2n, a_{2n+1} = -a_n + 6n + 3\}.
\]

\[
(102) \quad a_n = \left[ x^n \right] \left( \frac{x}{1 - x} + \frac{x^2}{1 - x} \left[ \frac{1}{1 - x} + \sum_{k \geq 0} 2^k x^{3 \cdot 2^k - 1} \right] \right)
\quad \iff a_n \text{ in binary does not begin } 100.
\]

\[
(103) \quad \langle a_0 = 0, a_1 = 1, a_{2n} = a_n - a_{2n+1} + a_n - a_n \rangle \wedge a_{3k} = 0
\iff \text{no adjacent } 1\text{s in binary of } k.
\]

\[
(104) \quad \langle a_1 = 3, a_{2n} = 4a_n - 2n, a_{2n+1} = 4a_n - 2n + 2^{\lfloor \log(4n + 2) \rfloor} \rangle
\iff \text{binary of } a_n \text{ is binary of } n \text{ twice juxtaposed.}
\]

\[
(105) \quad a_n = n \text{ XOR } (n + m)
\iff a_n = \left[ x^n \right] \frac{P(x)}{(1 - x)^2 \prod_{k \geq 0} 1 + x^{2^k}}, \quad \sum_{k \geq 0} 2^k x^k = m.
\]

\[
(106) \quad \langle a_0 = a_1 = 0, a_{4n} = 2a_{2n}, a_{4n+2} = 2a_{2n+1} + 1, a_{4n+1} = 2a_{2n} + 1, a_{4n+3} = 2a_{2n+1} \rangle
\iff a_n = n \text{ XOR } 2n.
\]

\[
(107) \quad a_n = \sum_k \text{k AND } (n-k)
\iff \langle a_0 = a_1 = 0, a_{2n} = 2a_{n-1} + 2a_n + n, a_{2n+1} = 4a_n \rangle.
\]

\[
(108) \quad a_n = \sum_k \text{k XOR } (n-k)
\iff \langle a_0 = a_1 = 0, a_{2n} = 2a_{n-1} + 2a_n + 4n - 4, a_{2n+1} = 4a_n + 6n \rangle.
\]

\[
(109) \quad a_n = \sum_k \text{k OR } (n-k)
\iff \langle a_0 = a_1 = 0, a_{2n} = 2a_{n-1} + 2a_n + 5n - 4, a_{2n+1} = 4a_n + 6n \rangle.
\]

\[
(110) \quad a_n = \# \{ (i, j) \mid 0 \leq i, j < n \wedge i \text{ AND } j > 0 \}
\iff \langle a_0 = a_1 = 0, a_{2n} = 3a_n + n^2, a_{2n+1} = a_n + 2a_{n+1} + n^2 - 1 \rangle.
\]
(111) \[ n = \sum_{k \geq 0} 2^k e_k \land a_n = \sum_{k \geq 0} (-1)^k e_k \land |a_n| = 3 \]
\[ \implies n \in \{ m \mid m = 3k \land k = 3i \land e_1(k) \equiv 1 \mod 2 \}. \]

(112) \[ n = \sum_{k \geq 0} 2^k e_k \land a_n = \sum_{k \geq 0} (-1)^k e_k \land a_n = 0 \]
\[ \implies n = 3m \land m \notin \{ k \mid k = 3i \land e_1(k) \equiv 1 \mod 2 \}. \]

(113) \[ \langle a_0 = 0, a_{2n} = 1 - a_n, a_{2n+1} = -a_n \rangle \land a_{3k} = 0 \implies k \text{ in base-}4 \text{ contains only } -1, 0, 1. \]

(114) \[ \max \left\{ \sum_{j=0}^{n} x^j \sum_{k=0}^{n} \frac{x^{2^k}}{1 + x^{2^k} + x^{2^{k+1}}} \right\} = \lfloor \log_4 n \rfloor + 1. \]

Define the sequence \( a_n \) by \( a_1 = 1 \) and \( a_n = M_n + m_n \), where \( M_n = \max_{1 \leq i < n} (a_i + a_{n-i}) \), and \( m_n = \min_{1 \leq i < n} (a_i + a_{n-i}) \). Let further \( b_n \) the number of partitions of \( 2n \) into powers of \( 2 \) (number of binary partitions). Then

(115) \[ m_n = \frac{3}{2} b_{n-1} - 1, \quad M_n = n + \sum_{k=1}^{n-1} m_n, \quad a_n = M_{n+1} - 1. \]

Let \( a_n \) the number of ones in the base-(−2)-representation of \( n \), and the sequence \( b_n \) defined as \( b_1 = 1, b_2 = 2, \) and \( b_{n+2} = \lceil \frac{1}{2} (b_n + b_{n+1}) \rceil \). Then

(116) \[ a_n = 3b_{n+1} - 2n - 3. \]

Let \( a_n \) the number of subwords of length \( n \) in the word generated by \( a \mapsto aab, b \mapsto b \). Then

(117) \[ \sum_{n \geq 0} a_n x^n = 1 + \frac{1}{1 - x} + \frac{1}{(1 - x)^2} \left( \frac{1}{1 - x} - \sum_{k \geq 1} x^{2^k+k-1} \right). \]

2.7. Conclusions. Working over two years with the OEIS showed me that simple computer programs suffice for many tasks; where I had to write programs myself, it was not visible that the task could be fully automatized—mathematics is essentially human. The work as editor was rewarding not only in itself but also in that it yielded a huge collection of conjectures as byproduct. However, I expect further gains in that regard as becoming ever more difficult as scans and transformations have to become more specialized and complex.

I do not intend to work on proving the majority of propositions presented here but I provide below a webpage giving the status of work done on them. Given their number, it is quite possible that a few are already in the literature. I hope the reader excuses my not researching these ones: it is very difficult nowadays to access pay-only journals from outside university.

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