Higgs instability and de Sitter radiation

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It has been argued that the instability in the Higgs potential at a high scale $\mu_{\text{inst}}$ makes the Higgs field run away to $\phi \sim M_{\text{Pl}}$ if $H_{\text{inf}} \sim 10^{14}$ GeV as implied by the BICEP2 observations. Such considerations of inflationary cosmology seem to imply that there must be new physics below the energy scale $\mu_{\text{inst}}$ which modifies the effective potential of the SM Higgs around $\phi \approx \mu_{\text{inst}}$.

We point out that in the inflationary quasi de-Sitter spacetime, the Higgs field is thermal with the Hawking-Gibbons temperature $T = \frac{\mu_{\text{inst}}}{2\pi} M_{\text{Pl}}$ which determines its effective potential. We show that there are no instabilities if one takes into account this de-Sitter thermal radiation. Hence, inflationary cosmological considerations do not imply that new physics exists below $\mu_{\text{inst}}$.

I. INTRODUCTION

Various observations and theoretical considerations indicate that there exists physics beyond the Standard Model (SM), but it is unclear at what scale this new physics exists. Renormalization group evolution of SM couplings show that the Higgs quartic coupling becomes negative at a scale $\mu_{\text{inst}} \approx 10^{10} - 10^{12}$ GeV if there is no new physics beyond the SM [1], [2], [3]. This implies that at large field values, the quantum effective potential of SM Higgs field must look like the solid curve in Fig (1) rendering the electroweak vacuum metastable (since the corresponding lifetime turns out to be bigger than the age of the universe).

Recently, it has been argued [4–10] that considerations of inflationary cosmology imply that if $\mu_{\text{inst}} < H_{\text{inf}}$ (the Hubble parameter during inflation), there must be new physics below $\mu_{\text{inst}}$. If the recent BICEP2 observations of B-mode polarization of CMB [11] are interpreted as being due to inflationary gravitational waves, the inferred Hubble parameter during inflation is $H_{\text{inf}} \sim 10^{14}$ GeV. This implies that quantum fluctuations of the SM Higgs during inflation shall cause it to run away to the global minimum in the effective potential at very high field values leaving no way of reaching the local minimum at $\phi \sim 250$ GeV electroweak vacuum.

In the present work, we argue that this conclusion is incorrect. Every geodesic observer in inflationary quasi-de-Sitter spacetime finds herself surrounded by thermal radiation with the Gibbons-Hawking temperature of $\frac{1}{2\pi k_{B}} H$ (where $H$ is the Hubble parameter during inflation) [12–15]. In such a scenario, to answer any questions related to the dynamics of the SM Higgs field and hence of the stability of the electroweak vacuum, one should analyse the corresponding thermal effective potential of the Higgs field. We found that the thermal effective potential of the SM Higgs field during inflation is such that the stability of the EW vacuum is restored. Thus, no new physics is needed below $\mu_{\text{inst}} \approx 10^{10} - 10^{12}$ GeV to ensure that the universe after inflation ends up in the local minimum at $\phi \sim 250$ GeV electroweak vacuum. However, since it appears that the inflaton field, which drives inflation, does not belong to the SM, in order to have inflation at $\mu \sim V_{\text{inf}}$, one still expects some new physics to turn up at an energy scale below $V_{\text{inf}} \approx 10^{16}$ GeV.

We begin by recalling the argument in favour of the hypothesis that $\mu_{\text{inst}} < H_{\text{inf}}$ implies new physics below $\mu_{\text{inst}}$. Then, after reminding why there must be thermal radiation in inflationary de-Sitter spacetime, we evaluate the quantum effective potential of SM Higgs and then show that the corresponding thermal effective potential of the SM Higgs restores vacuum stability. We then conclude with a summary.

II. COSMIC INFLATION AND HIGGS INSTABILITY

In the Standard Model of elementary particle physics, the one-loop beta function of the self coupling $\lambda$ of Higgs receives a negative contribution from the loop of the top quark while it receives positive contribution from the Higgs loop. A heavy top quark and a light Higgs thus ensure that as we probe higher energies, at some scale $\mu_{\text{inst}} \approx 10^{10} - 10^{12}$ GeV, $\lambda$ becomes zero and eventually negative at even higher energies [1], [2], [3]. The uncertainties in the value of this scale are determined predominantly by the uncertainties in the measured value of the mass of top quark.

If the recent observations of BICEP2 [11] collaboration are to be interpreted as being due to the inflationary gravitational waves, it appears that the inferred energy scale of inflation [16] is

$$V_{\text{inf}}^{1/4} \approx 2.2 \times 10^{16}\text{GeV} \left(\frac{r}{0.2}\right)^{1/4},$$

and hence

$$H_{\text{inf}} \approx 1.2 \times 10^{14}\text{GeV} \left(\frac{r}{0.2}\right)^{1/2}.$$
This inferred energy scale of inflation has triggered arguments [4–10] purely from inflationary cosmological considerations, that there must be new physics below the scale $\mu_{\text{inst}}$. These arguments are based on the following reasoning: since $\lambda$ turns negative at $\mu_{\text{inst}}$, the quantum effective potential of the Standard Model Higgs field must look like the solid curve in Fig (1). For any massless (i.e. sufficiently light) canonically normalized scalar field on quasi-de-Sitter background, every Fourier mode has, at late times, a quantum fluctuation of approximately $H^2$ (see e.g. [17, 18] for details) i.e.

$$\lim_{t \to \infty} \langle 0 | \tilde{\phi}(t, k) \cdot \tilde{\phi}(t, k') | 0 \rangle = \delta^3(k + k') \frac{2\pi^2}{k^3} \left[ \frac{hH(t_k)}{2\pi c^2} \right]^2.$$

(3)

Here, $\tilde{\phi}$ is the three dimensionful Fourier transform of the field $\phi$ (so that the mass dimension of $\tilde{\phi}$ is -2), $H$ is the Hubble parameter during inflationary quasi-de-Sitter phase, $t_k$ is the time when the mode in question crosses the then Hubble radius and the state $|0\rangle$ is the Bunch-Davies vacuum.

Thus, in inflationary quasi de-Sitter spacetime, at every scale, there is quantum fluctuation of the order of $H$. This shall happen for every light field during inflation including the Standard Model Higgs field itself. Suppose (as the data suggests) $H_{\text{inf}} > \mu_{\text{inst}}$, this would then imply that just due to quantum fluctuations, averaged over a box of any size, the Standard Model Higgs field is going to be found in the extreme right portion of the effective potential (the solid curve) in Fig (1). Thus, during inflation, the large inflationary energy density can drive the Higgs out of electroweak vacuum i.e. the likelihood that Higgs field fluctuates to the unstable region of the potential is sizeable, even if Higgs begins inflation in EW vacuum. The probability to have a Universe at the end of inflation which survived the quantum Higgs fluctuations is quite low [4].

This shall cause the field to runaway to even higher values and at the end of inflation we never end up in the desired SM electroweak vacuum at $\phi \approx 250$ GeV. How did the universe end up in such an energetically disfavored state as the present electroweak vacuum? Moreover, as the SM Higgs rolls down along the run away region of its effective potential, its negative energy density keeps on increasing until there comes a moment when it overpowers the energy density of inflaton itself, a process which can disrupt inflation. In [8], the authors argued that since in the SM, for the best fit value of the mass of top quark, the value of $V(\phi_{\text{max}})$ (the potential energy at the local maximum) is less than $H_{\text{inf}}^4$ (assumed to be $\mathcal{O}(10^{14})$ GeV), inflationary fluctuations shall push it to $\phi > \phi_{\text{max}}$ region of field space and hence new physics shall be required to modify the Higgs potential and make it stable against inflationary fluctuations. In general, it is often argued that this means that there must be new physics at energy scale below $\mu_{\text{inst}}$ which modifies the Higgs potential so that after inflation, we end up being in the correct vacuum (see e.g. [19] for an example of new physics which could cure this problem).

In the next section, we shall show that the considerations of Gibbons-Hawking temperature in quasi-de-Sitter background during inflation shall cause the corresponding thermal effective potential of the Higgs field to be of the form of the dashed curve shown in Fig (1) suggesting that the above conclusion about the instability of the Standard Model Higgs during inflation [4–10] is not correct.

III. EFFECTS OF DE-SITTER RADIATION

Consider a free massless (or light) quantum scalar field in inflationary (quasi) de-Sitter spacetime. It is known that a state which an observer using conformal coordinates describes as Bunch-Davies vacuum, to an observer using static coordinates, shall appear to have particles when the scalar field is in the Bunch-Davies vacuum. In the present work, we argue that during inflation, for the original reference and sec V of [14] for a review). In fact, any geodesic observer moving along a timelike geodesic in de Sitter space observes a thermal bath of particles when the scalar field is in the Bunch-Davies vacuum (see [15] for a review). Since the temperature of de-Sitter radiation does not drop as the universe inflates, the dynamics of SM Higgs and the stability of the EW vacuum must be determined by its thermal effective potential. For this reason, we now find the thermal effective potential of SM Higgs at a temperature of $H_{\text{inf}}$ and analyse its stability.
A. Zero temperature quantum effective potential of SM Higgs

Let us find the one-loop quantum effective potential of the Standard Model Higgs. The Higgs potential is \( V = -\frac{m^2}{2}|\phi|^2 + \lambda|\phi|^4 \) for the Higgs doublet \( \phi \) defined by

\[
\chi \left( \frac{v_0 + \phi + im}{\sqrt{2}} \right)
\]

where \( \phi \) is the physical Higgs field and \( m \) and \( \chi \) are the neutral and charged Goldstones respectively. Recall that \( m^2 \) is the only dimensionful parameter in the Standard Model Lagrangian. We find the one-loop effective potential of the Higgs field due to its interactions with itself, with gauge bosons and with the top quark (all the other couplings are negligibly small). The quantum effective potential can be rewritten as its tree level expression

\[
V_{\text{eff}} = -\frac{m(\mu)^2}{2} \phi^2 + \lambda(\mu)\phi^4 ,
\]

but with the renormalized couplings (and with the renormalization scale \( \mu \) set to \( \phi \)). We thus need to find the Renormalization Group (RG) evolution of the various parameters and couplings. The RG flow of \( m^2 \) and \( \lambda \) is determined approximately by the three gauge couplings \( g_1, g_2, g_3 \) and by the Yukawa coupling of top quark \( y_t \). One can easily solve the RGEs [2] for the 6 parameters \( (g_1, g_2, g_3, y_t, \lambda, m^2) \), with the values of these parameters at an initial renormalization scale (which we take to be \( M_t = 173.1 \text{ GeV} \)) chosen to be \( (0.461, 0.648, 1.166, 0.936, 0.127, (132.7 \text{ GeV})^2) \) respectively [2]. [20]. We truncate our computation at one loop accuracy since our aim is to only illustrate that the thermal effects solve the problem we addressed in the last section.

We find that the couplings flow as shown in Fig (2), it can be seen that the self coupling of the SM Higgs becomes negative at a high energy scale. This scale is \( 10^8 \text{ GeV} \) in Fig (2) instead of the value \( \mu_{\text{inst}} \approx 10^{11} \text{ GeV} \) (see [2]) as we have truncated the RGEs at one loop approximation. In Fig (3) is plotted the corresponding quantum effective potential. Around \( 10^8 \text{ GeV} \), the corresponding Higgs potential drops below its value for the EW vacuum as shown in Fig (3). This illustrates the Higgs instability problem at zero temperature.

B. Thermal effective potential

In a thermal state, all the averages shall be ensemble averages of statistical fluctuations and not the averages over quantum fluctuations. Thus, unlike the cases in which we wish to solve the scattering problem when we evaluate the vacuum Green’s functions, in thermal field theory, one evaluates the thermal Green’s functions (see [21] for a review and references). Just like zero-temperature field theory, the connected 1 PI thermal Green’s functions of the field can be found from a corresponding generating functional which is the thermal effective action of the field. The dynamics of the field shall then be governed by the corresponding thermal effective potential. Given the action of a theory, one can find the Feynman rules to evaluate thermal correlators and hence the thermal effective potential. E.g. for a self interacting scalar field theory, the one loop quantum effective potential in thermal state is the sum of two contributions: \( V_{\text{eff}}^{\beta}(\phi) = V_0(\phi) + V_1^{\beta}(\phi) \), where \( V_0(\phi) \) is the tree level potential. It turns out that

\[
V_1^{\beta}(\phi) = V_{\text{eff}}^{1-\text{loop}}(\phi) + \frac{1}{2\pi^2|\beta|} J_R[m^2(\phi)]^2 ,
\]
where \( V^{1-\text{loop}}(\phi) \) is the zero temperature effective potential with \( m^2(\phi) = d^2V_0(\phi)/d\phi^2 \). There is an additional temperature dependent piece

\[
J_B[m^2(\phi)\beta^2] = \int_0^\infty dx x^2 \log[1 - e^{-\sqrt{x^2 + \beta^2 m^2}}].
\]  

(6)

In the limit of high temperature i.e. \( m^2 \beta^2 \ll 1 \), the above integral admits a convenient expansion. Similar expressions can be obtained for a theory of a scalar field interacting with a spin half field or with a spin one field [21]. Using this, one can obtain the one loop effective potential of the SM Higgs field (in a thermal background) due to contributions from only the \( W \) and \( Z \) bosons and the top quark to radiative corrections, the full one loop thermal effective potential, in the high temperature limit is given by [21]

\[
V(\phi_c, T) = D(T^2 - T_o^2)\phi_c^2 - ET\phi_c^3 + \frac{\lambda(T)}{4} \phi_c^4,
\]  

(7)

where the coefficients are given by

\[
D = \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2},
\]  

(8)

\[
E = \frac{2m_W^2 + m_Z^2}{4\pi v^2},
\]  

(9)

\[
T_o^2 = \frac{m_h^2 - 8Bv^2}{4D},
\]  

(10)

\[
B = \frac{3}{64\pi^2 v^4} \left(2m_W^4 + m_Z^2 - 4m_t^4\right),
\]  

(11)

\[
\lambda(T) = \lambda - \frac{3}{16\pi^2 v^4} \left(2m_W^2 \log \frac{m_W^2}{A_B T^2} + m_Z^2 \log \frac{m_Z^2}{A_B T^2} - 4m_t^2 \log \frac{m_t^2}{A_F T^2}\right),
\]  

(12)

where \( \log A_B = 5.4076 - 3/2 \) and \( \log A_F = 2.6351 - 3/2 \). In the above equations, the Higgs vev \( v = \phi_c \) and which is the same as which in turn is the same as the renormalization scale \( \mu \), determines the masses such as \( m_h^2 = 2\lambda v^2 \), \( m_W = g_2 v/2 \), \( m_t = y_t v/\sqrt{2} \). Thus, for any renormalization scale \( \mu \), we can find the values of \((g_1, g_2, g_3, y_t, \lambda, m^2)\) and from them, we can find all the quantities in the above set of equations.

Before we proceed, it is worth understanding when the above expressions are valid. The high temperature expansion is valid whenever \( m^2(\phi_c)\beta^2 \ll 1 \). If we choose a value of \( \beta \) and if we are interested in a chosen range of \( \phi_c \) values, then we can find the corresponding \( m^2(\phi_c)\beta^2 \) and check whether \( m^2(\phi_c)\beta^2 \ll 1 \) or not. Using the tree level Higgs potential, we find that

\[
\beta^2 m^2(\phi) = \beta^2 m^2(\bar{\mu}) + 12\lambda(\bar{\mu})\beta^2 \phi^2,
\]  

(13)

where \( m^2 \) on the RHS is the mass term which turns up the tree level Higgs potential. Since \( T \sim H_{\text{inf}} \), and we are interested in the range of field values around \( \phi \sim \mu_{\text{inst}} \), it is clear that the high temperature expansion is valid in the situation of our interest. The corresponding one-loop thermal effective potential found from Eq (7) and is shown in Fig (4). It is clear from the form of the effective potential in Fig (4) that if the dynamics of the Higgs field is governed by this effective potential, inflationary quantum fluctuations can not cause the universe to end up in any other vacuum than the one which gives the familiar low energy phenomenology. We have thus shown that the Higgs potential has no instabilities during inflation.

IV. SUMMARY AND DISCUSSION

It is well known that at a high temperature, broken symmetries get restored. In the problem we studied, the Gibbons-Hawking temperature due to de-Sitter radiation (as seen by any geodesic observer in quasi de Sitter space) ensures that the stability of effective potential of SM Higgs gets restored as is easily seen if we compare Fig (3) with Fig (4). Note that the thermal effective potential shown in Fig (4) is positive and has no instability, moreover, if we assume the energy scale of inflation to be \( H_{\text{inf}} \sim 10^{14} \text{ GeV} \) (as suggested by the recent BICEP2 results), then, the thermal effective potential of the SM Higgs field in the field range around \( \phi \approx \mu_{\text{inst}} \) (the field value at which \( \lambda \) turns negative) is eighteen orders of magnitude bigger than the effective potential of the SM Higgs in the same field range. It is notewor-
thy that \( V_{\text{eff}}(\phi, T) \approx 10^{47}\text{GeV}^4 \) is too small compared to \( V_{\text{inf}} \approx 10^{64}\text{GeV}^4 \), the inflaton potential energy density, so that these thermal effects in Higgs do not affect the inflationary dynamics. It is thus clear that even if the energy scale of inflation is a few orders of magnitude lower than \( 10^{16} \) GeV, stability is still maintained due to Gibbons-Hawking temperature of inflationary quasi de-Sitter spacetime.

We have thus shown that quantum fluctuations during inflation do not drive the Higgs to the field range in which it runs away to the global minimum of its effective potential. Thus, the fact that the Higgs self coupling \( \lambda \) turns negative at an energy scale \( \mu_{\text{inst}} \) well below inflationary Hubble scale \( H_{\text{inf}} \) does not imply that it is improbable to get the familiar low energy phenomenology after inflation without assuming new physics.

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