BDNet: Bengali handwritten numeral digit recognition based on densely connected convolutional neural networks

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Abstract

Bengali handwritten digit recognition can be done using different image classification techniques. But the images of handwritten digits are different from natural images as the orientation of a digit as well as similarity of features of different digits are important. On the other hand, deep convolutional neural networks are achieving huge success in computer vision problems, especially in image classification. This BDNet is a densely connected deep convolutional neural network model based on state-of-the-art algorithm DenseNet to classify Bengali handwritten numeral digits. The BDNet has end-to-end trained using ISI Bengali handwritten numeral dataset with 5-fold cross-validation. The BDNet has achieved a test accuracy of \textbf{99.65\%}(baseline was 99.40\%) on test data of ISI Bengali handwritten numerals. The trained model also gives 97.50\% on own created dataset(which are not used during training). That is, this model gives a 41.66\% error reduction compared to the previous state-of-the-art model. Codes, trained model and own dataset available at: \url{https://github.com/Sufianlab/BDNet}

Keywords: Bengali Digit Recognition, CNN, Deep Learning, Handwritten Digit Recognition, Image Classification.

1. Introduction

Bengali (Bangla) is the second most spoken language in India. It ranks fifth in Asia and it is also in the top ten spoken languages in the world \cite{1}. So, a huge number of people depend on this language for their communication. Therefore, automatic recognition of Bengali handwritten characters and numeral digits are needed to be digitized for making the communication smoother. Many research works and models have been proposed to recognize Bengali handwritten characters and numeral digits so far, but still, a huge scope is there to improve this task in terms of accuracy and applicability. Most of the previously proposed models

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are based on traditional pattern recognition and machine learning techniques where human expertise is required for feature engineering. The recent success of deep learning, specially Convolutional Neural Network (CNN) for computer vision, has inspired many researchers to use the deep convolutional neural network to recognize handwritten characters and digits as a computer vision problem. This BDNet is a deep CNN based model to classify Bengali numeral digits. The working pipeline of this BDNet is based on DenseNet which is one of the current state-of-the-art deep CNN models for image classification. The conceptual view of the BDNet is shown in figure 1.

1.1. Contributions of this paper
- Designed a deep CNN working pipeline based on DenseNet, called BDNet, to classify Bengali handwritten numerals.
- Preprocessed the raw data of the dataset in a different way as: different size raw images → fixed size images → color inverted images (black pixels to white and vice versa) → RGB images.
- Trained the BDNet end-to-end with 5-fold cross-validation with appropriate hyper-parameter setup.
- Achieved the highest test accuracy with 41.66% error reduction on baseline result (99.65% whereas the previous best was 99.40%) on test data of the dataset.
- Created a new dataset of 1000 samples of handwritten Bengali numerals for testing purpose, where test accuracy of the BDNet was 97.50%.

Rest of the paper is organized as: In the section 2 literature review has been done, the BDNet model details are explained in section 3, in section 4 dataset and preprocessing has been explained. Training details are explained in section 5, result analysis in section 6, and finally, the conclusion is in section 7.

2. Literature Review

We have reviewed two things: one is existing research works on Bengali handwritten numeral recognition and another is the advancements of deep learning models for image classification. First one for baseline results and target application domain knowledge whereas later one for finding a suitable state-of-the-art...
algorithm. In this section, we have reviewed these two things by following two subsections:

2.1. Existing works on Bengali handwritten numeral recognition

Bengali handwritten numeral recognition is one of the oldest pattern recognition problems. Many researchers have been working in this field since the 90s of the last century [9], [10]. Through this subsection, we have reviewed most notable works on this Bengali handwritten numeral recognition.

Subhadip Basu et.al proposed Handwritten Bangla Digit Recognition Using Classifier Combination Through Dempster-Shafer (DS) Technique [11]. They have used the DS technique and MLP classifier for classification and also used 3-fold cross-validation on the training dataset of 6000 handwritten samples. Their scheme achieved 95.1% test accuracy. In [12] U. Pal et.al proposed a scheme where unconstrained off-line Bengali handwritten numerals were recognized. This scheme has recognized different handwritten styles. The scheme selects the required features using the concept of water overflow from the reservoir, and also collect topological and structural features of the numerals. They applied this scheme on their own collected dataset of size 12000 and obtained recognition accuracy of around 92.8%.

U. Bhattacharya and B. B. Choudhury presented handwritten numeral database along with Devanagari database and proposed a classifier model [13]. Their database contains 23392 handwritten Bengali numeral images. Their classifier model is a multi-stage cascaded recognition scheme where they used wavelet-based multi-resolution representations and multilayer perception as classifiers. They mentioned 99.14% training and 98.20% testing accuracy on this dataset. Cheng-Liu Liu and Ching Y. Suen proposed a benchmark model [14] on ISI numeral dataset [13] along with a Farsi numeral database. They preprocessed the dataset into grayscale images and applied many traditional feature extraction models. This benchmark model achieved the highest 99.40% test accuracy. Ying Wen and Lianghua He proposed a classifier model [15] for Bengali handwritten numeral recognition. This model tried to solve large dataset high dimensionality problem. They combined Bayesian discriminant with kernel approach with UCI dataset and another dataset such as MNIST [16]. The rate of error is 1.8%, the recognition rate is 99.08% and recognition time is 7.46 milliseconds. Local region identification, where optimal unambiguous features are extracted, is one of the crucial tasks in the field of character recognition. This idea adopted by N. Das et.al in their genetic algorithm(GA) based handwritten digit recognition technique [17]. GA is applied on seven sets of local regions and for each set, GA selects a minimal local regions group with a Support Vector Machine (SVM) based classifier. The whole digit images are used for global features extraction whereas local features are extracted for shape information. The number of global features is constant whereas the number of the local features depends on the number of a local region. The test accuracy rate was 95.50% for this model.

M. K. Nasir and M. S. Uddin proposed a scheme [18] where they used K-Means clustering, Bayes’ theorem and Maximum a Posteriori for feature extraction and for classification SVM are used. After converting the images into binary
values, some points are found, which was discarded using a flood fill algorithm. The plinth steps are Clipping, Segmentation, Horizontal, and Vertical Thinning Scan. Here test accuracy rate was 99.33%.

In [19] M. M. Rahaman et.al proposed a CNN based model. This method normalizes the written character images and then employs CNN to classify individual characters. It does not employ any feature extraction method like previous works. The major steps are pre-processing of raw images converting them into grayscale and training the model; test accuracy was 85.36%. On another paper, we see the existence of auto-encoder for unsupervised pre-training through Deep CNN, which consists of more than one hidden layer, with 3 convolutional layers, each layer followed by $2 \times 2$ max pool layer. This scheme [20] proposed by Md Shopon et.al. The layers have $32 \times 3 \times 3$ number of kernels. In the same manner, the decoder has an architecture with each convolutional layer with 5 neurons, rather than 32. The ReLu [21] activation is present in all layers. For training purpose, the model enhanced the training dataset by randomly rotating each image between 0 degree and 50 degree and also vertically shifting by a random amount between 0 and 6 pixels. This model trained in 3 various setups SCM, SCMA, and ACMA. They achieved a test accuracy of 99.50%.

Another model [22], proposed by M. A. H. Akhand et. al, used pre-processing by simple rotation based approach to produce patterns and it also makes all images of ISI handwritten database into the same resolution, dimension, and size. CNN structure of this model has two convolutional layers with $5 \times 5$ sized local receptive fields and two sub-sampling layers with $2 \times 2$ sized local averaging areas along with input and output layers. Input layer contains 784 receptive fields for $28 \times 28$ pixels image. The first convolutional operation produces six feature maps. Convolution operation with kernel spatial dimension of 5 reduces $28$ spatial dimension to $24$ (i.e., $28 + 1 - 5$) spatial dimension. Therefore, each first level feature map size is $24 \times 24$. The accuracy rate of the testing is 98.45% on ISI handwritten Bengali numerals.

In [23] A. Choudhury et.al proposed a histogram of oriented gradient (HOG) and color histogram for selection of features algorithm. Here, HOG is used as the feature set to represent each numeral item at the feature space and SVM is used to produce the output from input. Test accuracy of this algorithm is 98.05% on CMATERDB 3.1.1 dataset (which is a benchmark Bengali handwritten numeral database created by CMATER lab of Jadavpur University, India). M. M. Hasan et.al proposed a Bengali handwritten digit recognition model based on ResNet [24]. Their ensemble model from their six best models, applied on NumtaDB dataset [25], achieved 99.3359% test accuracy. In [26] R. Noor et.al proposed an ensemble model based Convolutional Neural Network for recognizing Bengali handwritten numerals. They train their model in many noisy conditions using customized NumtaDB dataset [25]. In all cases, their model achieved more than 96% test accuracy on this NumtaDB dataset. A very recent Bengali handwritten numeral recognition work [27] was there, proposed by AKM S. A. Rabby. Here author used deep CNN model to classify the handwritten numeral digits. This model is trained using ISI handwritten Bengali numeral [13] and CMATERDB 3.1.1 databases with 20% data for validation. The author of this
The paper claimed their test accuracy as 99.58% on ISI handwritten numerals and 92.65% on CMATERDB 3.1.1.

2.2. Advancements of deep learning for image classification

After the success of AlexNet [4], a deep learning based model for image classification, many researchers shifted to this area of research of computer vision and pattern recognition problems. Therefore, many successive state-of-the-art models came within a short span of time since 2012 [7], [28]. In this subsection, we briefly reviewed the development of deep learning especially Convolutional Neural Networks in the field of image classifications. This review is relevant because choosing the best state-of-the-art model for handwritten numeral image classification is an important trick for recognizing digit with higher accuracy.

CNN is a special type of multi-layer neural network inspired by the vision mechanism of the animal. Hubel and Wiesel experimented and said that visual cortex cells of animal detect light in the small receptive field [29]. Kunihiko Fukushima got motivation from this experiment and proposed multi-layered neural network capable of recognizing visual pattern hierarchically through learning, called NEOCOGNITRON [30]. This model is considered as the inspiration for CNN. A classical CNN model is composed of one or more blocks of convolutional and sub-sampling or pooling layer, then single or multiple fully connected layers, and an output layer function, as shown in figure 2. The benefit of using CNN for automated features extraction, parameter sharing many more [31], [4] [32]. Classical CNN is modified in many different ways for target domain [7], [33], [34], [35].

Figure 2: Blocks of a classical CNN model [7]

Yann LeCun et al. introduced the first complete CNN model called LeNet-5 [31] to classify English handwritten digit images. It has 7 layers among which 3 convolutions, 2 average pooling, 1 fully connected, and 1 output layer. They used SIGMOID function as the activation function for non-linearity before an average pooling layer. The output layer used Euclidean Radial Basis Function(RBF) for classification MNIST [16]. The weights of each layer were trained using back-propagation algorithm [36]. AlexNet [4] was the first CNN based model which won the ILSVRC challenge [37] in 2012 with a significant reduction of errors. AlexNet’s error rate was 16.4% whereas the second best error rate was 26.17%. This model was proposed by Alex Krizhevsky et.al and it is trained by ImageNet dataset [38], this dataset contains 15 million high resolution labeled images over 22 thousand categories. AlexNet has 11 trainable
layers, and the structure is almost similar to LeNet-5, but here max-pooling used instead of the average pooling, ReLu activation in place of the SIGMOID function, softmax function in place of RBF, and 11×11 in-place of 5×5 filter size in the first layer. In addition, for the first time dropout strategy [39] and GPU were used to train the model. In [40] Zeiler and Fergus presented ZFNet which was the winner of ILSVRC challenge in 2013. The building blocks of ZFNet is almost similar to AlexNet with few changes such as first layer filter size is 7×7 instead 11×11 in AlexNet. Authors of ZFNet explained how CNN works with the help of Deconvolutional Neural Networks (DeconvNet). DeconvNet is just the opposite of CNN. The error rate of ZFNet was 11.7%. K. Simonyan and A. Zisserman proposed VGGNet [41], which is like a deeper model of AlexNet. Here, authors used small filters 3×3 sizes for all layers. They have used total 6 different CNN configurations with different weight layers. This VGGNet secure 2nd place in ILSVRC challenge in 2014 with an error rate of 7.3% just 0.6% more than the error rate of the winner GoogLeNet [42].

GoogLeNet, Going Deeper with Convolutions [42], is proposed by Christian Szegedy et.al, a research team of Google. The Structure of GoogLeNet is different from traditional CNN, it is wider and deeper than previous models but computationally efficient. Through inception architecture, multiple parallel filters with different sizes are used, and for this, problems of vanishing gradient and over-fitting were tackled. Fully connected layers are not used in GoogLeNet but average pooling layer is used before the classifier. This model won ILSVRC challenge 2014 with error rate of 6.7%. The increasing layer could give more accuracy but will suffer from vanishing gradient problem, and to tackle this problem Kaiming He et.al from Microsoft Research proposed ResNet [24]. ResNet is a very deep model where each layer has a residual block with skip connection to the layer before the previous layer. ResNet is the winner of ILSVRC challenge with error rate of 3.57% and this is a success of beyond human level. Gao Huang et.al proposed DenseNet [8], where every layer is connected to all previous layers of the model. DenseNet overcomes the vanishing gradient problem as well as it collects required features of all layers and propagates to all successive layers in feed-forward fashions for features reuse. Therefore, this model requires less number of parameters to achieve accuracy, so it is computationally efficient. Inspired by the success of ResNet, Jie Hu et.al proposed SENet [43] with the main focus to increase channel relationship between successive layers. SENet has added “Squeeze-and-Excitation” (SE) block into each block(ResNet Block), and for this, the model adaptively recalibrates channel wise feature responses between channels. SENet has won ILSVRC-2017 challenge with error rate of 2.252%.

3. BDNet Model Details

The network model of the BDNet is shown in figure 3. The BDNet consists of three Dense Blocks and two Transition Blocks followed by Batch Normalization(BN), Rectified Linear Unit(ReLU) activation, Average Pooling(Avg. POOLING), Fully Connected(FC) Layer, Softmax function with output layer.
Each Dense Block is made up of 6 bottleneck blocks. Structure of each bottleneck block as: ... → BatchNorm → ReLU → Conv2d(1×1) → BatchNorm → ReLU → Conv2d(3×3) → ... . The number of bottleneck blocks (NBL) per dense block can be calculated using equation (1):

\[
NBL = \frac{1}{2} \left\lfloor \frac{n - 4}{3} \right\rfloor
\]

where \( n \) is the number of layers of the network model. Dense connectivity is present among bottleneck blocks of each dense block i.e. output of each bottleneck block is forwarded to all other successive blocks for features propagation. The number of feature maps that will be forwarded depends on the growth rate, and here the growth rate is 12. In between two dense blocks, we have used one transition block which consists of: BatchNorm → ReLU → Conv, → Avg.Polling. To make the model compact we reduce the number of feature maps.

4. Dataset and Preprocessing of the Dataset

The Bengali language is mainly derived from the Brahmi script and Devanagari script in the 11th Century AD. The structural view of each character and numeral of this language are very complex. So, training a model using Bengali digit is more difficult compared to English numeral digit as the English digits has a less complex structure. In addition, English numerals datasets are easily
available in terms of quantity and quality such as MNIST [16] but these are not true for Bengali numeral datasets. Bengali digit also has some high similarity features for different numerals such as numeral 1 (in Bengali) and numeral 9 (in Bengali) has high similarity features, similarly numeral 5 (in Bengali) and 6 (in Bengali) has high similarity features. The typical Bengali handwritten numerals and corresponding printed values has shown in figure [4].

| Different forms of numerals | Digits |
|----------------------------|--------|
| **Bengali Handwritten Numerals** | 0 1 2 3 4 5 6 7 8 9 |
| **Bengali Printed Font** | 0 1 2 3 4 5 6 7 8 9 |
| **Standard English Font** | 0 1 2 3 4 5 6 7 8 9 |

Figure 4: Typical Bengali handwritten numeral digits corresponding printed values.

4.1. Used Dataset

The ISI Bengali numeral off-line handwritten dataset is one of the largest popular datasets of handwritten Bengali numerals. This dataset consists of 23392 black and white image data written by 1106 persons collected from postal mail and job application forms. Among these 23392, 19392 for training and 4000 for testing [13]. The entire dataset represents ten classes for 0 to 9 numeral digits. Some typical data items of this dataset shown in figure [5].

4.2. Preprocessing of the Datasets

As mentioned we have used ISI Handwritten numeral dataset to train the BDNet. But the data items that we have for this task is very untidy and cannot be used directly for our purpose. All the data were raw images in .tif format of different sizes. First, we have converted the raw images into grayscale images of size 28 × 28, then inverted the colors in a way that the background became black and the font became white. After that grayscale images are converted to RGB images of size 32 × 32 for better feature extraction using 3 channels. For the convenience to use the BDNet, we have created a CSV file to access the data samples. Figure [6] is showing steps of preprocessing, and how converted data looks different from actual data after the preprocessing. Distribution of entire ISI handwritten numerals database as in table [1]. Among the training datasets, 20% is used for 5-fold cross-validation.

4.3. Own test dataset

We have also created our own test dataset of 1000 images. Among these 1000 images, 100 images per digit are there for each Bengali numerals digit zero to nine. This dataset is created by 4 laboratory members of this university with the help of some students. It has done by writing in standard pages using black
or blue pens, then scanned the written digits using the mobile phone camera. Datasets are created with the focus to make it as natural as the common people write the Bengali numerals in their daily life. Each image of the dataset are then set to $28 \times 28$ pixels. We have used this dataset only for testing to check generic performance of the BDNet.

5. Training Details

We have begun our experiment by training the model using preprocessed labeled dataset mentioned above. Before describing the training details, we have presented the system environment and resources used for our work in table 2.

As we have mentioned that BDNet is based on DenseNet [8], but the only differences between them are the values set for hyper-parameters. Setting an actual value for each required hyper-parameter is a very difficult task. It could be done by trial and error method with careful observation of the pattern of the data as well as by some mathematical analysis. In a similar fashion, we have done hyper-parameter tuning of BDNet, and the details are as below:

**Number of hidden layers and units:** It is preferably good to add more layers when the test error no longer decreases in existing layers. A small number of layers may lead to under-fitting, on the other hand, having more layers is usually not suitable with appropriate regularization. But adding more number of layers make the model more complex and computation time will increase. After careful experiments, we have used 39 hidden layers, one fully connected(FC) layer in our model, then using softmax function to ten classes output, details are in figure
3. Here, softmax function transform predicted scores to predicted probability scores as in equation 2.

\[
\hat{y}_i = \frac{e^{z_i}}{\sum_{j=1}^{10} e^{z_j}}
\]  

(2)

Where \( \hat{y}_i \) denotes prediction score of i-th digit or class.

**Number of epochs:** It is the number of time the entire training dataset passes through the model network. We can increase the number of epochs until the training error becomes small and the validation error is noticeable. For our model, the number epoch was set to 300 but the model converges around 200s epochs (then we early stopped), and it took 37.68 seconds per epoch to train and validate simultaneously.

**Optimizer:** The main force behind any deep learning model is learning through back-propagation [30]. In BDNet, we updated weights using SGD (Stochastic Gradient Descent) [45]. Optimization algorithms are used to minimize (or maximize) an error or loss function \( J(w) \) as in equation 3. The Loss function is a mathematical function and it depends on the updating of internal parameters of a model which are used for computing the values \( (y_i) \) from the set of inputs.
Table 1: Used Dataset Distribution

| Digit | Training Sets | Test Set |
|-------|---------------|----------|
| 0     | 1933          | 400      |
| 1     | 1945          | 400      |
| 2     | 1945          | 400      |
| 3     | 1956          | 400      |
| 4     | 1945          | 400      |
| 5     | 1933          | 400      |
| 6     | 1930          | 400      |
| 7     | 1928          | 400      |
| 8     | 1932          | 400      |
| 9     | 1945          | 400      |

Table 2: Used System specifications

| Resources       | Specifications                                      |
|-----------------|-----------------------------------------------------|
| CPU             | Intel \(^{\text{R}}\) Xeon\(^{\text{R}}\) CPU @2.3GHz with 45 MB Cache |
| RAM             | 12.72 GB available                                   |
| DISK            | 1 TB (Partially used)                               |
| GPU             | 1 \times Nvidia Tesla T4 having 2560 CUDA cores, 16GB(14.72GB available) GDDR6 VRAM |
| Languages & Packages | Python with Pytorch\[44\]                           |
| Training & Validation Time | 37.68 Seconds per Epoch                            |

\((x_i)\) used in the model and difference with desired output \((\hat{y}_i)\).

\[
J(w) = \frac{1}{n} \sum_{i=1}^{n} J_i(w) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2
\] (3)

The working flow of SGD algorithm is as follows:
Step 1: Initialization of the vector of parameters \(w\) and learning rate \(\eta\).
Step 2: Repeat until an approximate minimum is found:
   Step 2.1: Randomly shuffle items in the training set.
   Step 2.2: for \(i = 1\) to \(n\) do:
      \[ w = w - \eta \nabla J_i(w) \]
SGD requires a random order of the training dataset. As the coefficients are updated after each training data-sample, so the updates, as well as the lost
function will be random jumping all over the place. By this randomized updates to the coefficients, it reduces random walk and avoids it getting distracted.

**Weight initialization:** We have initialized the weights with small random numbers (between 0 and 1) to prevent dead neurons, but not too small to avoid zero gradients. Uniform distribution usually works very well. Here, we used seed(1) as python function to randomly initialized weights.

**Batch size:** Mini-batch is usually preferable in the place where a dataset is very large. It is usually used to create a partition in between the dataset. Typically 16 to 128 batch size preferred by researchers. Batch size doesn’t contribute much to the precision but helps us controlling the training speed. Here, in the BDNet training dataset batch size is 32, and the testing dataset batch size is 64.

**Learning rate:** In the BDNet we have used the learning rate as 0.002, and after 240 epoch it has changed as in equation 4. Authors used early stops strategy when converge so 240 number of epoch not used.

$$\eta = (\text{Initial } \eta) \times (0.1^{\left\lfloor \frac{\text{epoch}}{240} \right\rfloor})$$ (4)

**Weight decay:** This is one hyper-parameter tuning where each step’s current weights($W$) are multiplied by a number slightly less than 1. Weight decay is a regularization term that prevents growing the number of parameters too large. It is updated as in equation 5

$$W_i = W_i - \eta \frac{\partial J}{\partial W_i} - \eta \lambda W_i$$ (5)

Where $J$ is the current loss, $\eta$ is the learning rate and $\lambda$ is the weight decay.

**Momentum:** Training a neural network is the process of finding values for the weights and biases so that for a given set of input values, the computed output values closely match with the target values. The concept of momentum is that previous changes in the weights should influence the current direction of movement in weight space. Sometimes these weights changes stuck in a local minimum. To avoid these local minima, we use momentum in the objective function, which is a value between 0 and 1 that increases the size of the steps taken towards the minimum by trying to jump from a local minimum. Here the momentum value set for BDNet is 0.9, and for this speed and accuracy improves.

**Activation Function:** In BDNet we have used Rectified Linear Unit (ReLU) [21]. ReLU function as in equation 6 works for non-linearity.

$$f(x) = \max(0, x)$$ (6)

Here $x$ denotes the value of a pixel. ReLU removes negative values from an activation map by setting them to zero. It increases the nonlinear properties of the decision function and of the overall network without affecting the receptive
fields of the convolution layer. ReLU also removes the chances of vanishing gradient.

**Dropout for Regularization:** Few techniques and tricks make deep learning popular and usable, and the dropout for regularization is one of them. Dropout is used to avoid over-fitting in a neural network. The method simply drops out some neurons randomly in neural network in each iteration of training according to a threshold probability. Here we used the dropout threshold probability is 0.09, small probability i.e. only 9% dropout in each epoch.

**Data Augmentation:** Data augmentation is one of the important parts which gives more versatility to extracted features and more accurately trained the deep learning model. Here we have used that idea as the size of the datasets is not large as required. We used it with slightly non-traditional way as shown in figure 7. As an image of the handwritten numeral digits have some problems as we can’t do cropping as well as big rotation. Here, we did augmentation on the training set for each epoch as: adjustment the contrast of training samples by choosing adjusting factor randomly between 1 and 2.3, random rotation of training images up to 10 degrees, and random zooming up to 8.57%. For this type of data augmentation, a slightly new dataset is passed through the network in every iteration or epoch.

**Cross-Validation:** To train BDNet we used 5-fold random cross-validation. We used 20% of training data only (didn’t touch test dataset during training).
for 5-fold cross-validation to validate the model during training for generalization without over-fitting. After one epoch training set are resampled with 5-fold cross-validation. The cross-validation result is mentioned in section 6.

6. Result Analysis

We have mentioned that the is trained using preprocessed ISI handwritten Bengali numeral database with data augmentation and 5-fold cross-validation. The BDNet is tested using test dataset from the same database mentioned in section 4. We have also tested the BDNet using our own dataset described in section 4. Following subsections are showing some results of the BDNet found during training and testing.

6.1. Number of Epoch vs Training Loss

![Figure 8: Number of epoch vs training loss](image)

At first, when we have started training, the amount of error or training loss was very high and the value of error rate was 1.97 to 2.22. But initially, with the increasing number of training epochs, the value of data loss decreased drastically later it slowed down as shown in figure 8. After 192 epochs, the error rate became very small almost 0.009 to 0.006.

6.2. Training and Validation Accuracy

We have observed the training and testing accuracy simultaneously. As we can see in figure 9, the increasing rate of accuracy was very high during the initial training period. It gradually became very low. After 180 epochs it was almost saturated. We can also see from this figure that the training accuracy was almost always dominated by validation accuracy, and it is happened because of data augmentation. Maximum training accuracy was recorded 99.78% after epoch number 190, and maximum validation accuracy was recorded 100% at 178th epoch.
6.3. Test Accuracy

As we have mentioned that BDNet has achieved record-breaking highest test accuracy on ISI Bengali handwritten numeral test dataset. The BDNet gave above 99% accuracy after 100 epochs and at 199th epoch, it achieved test accuracy of **99.65%** as shown in figure 10, which is the current best result on the ISI handwritten numeral dataset.

6.4. Analysis through Confusion Matrix

Testing result of the BDNet on the test dataset of ISI handwritten Bengali numeral can be presented in the confusion matrix as shown in table 3. Clearly, we can see that among 10 numeral digits of 4000 test images, only 14 were wrongly predicted or classified. Among 400 test images of the digit 0, one is wrongly predicted as 3, and one is 7. Similarly, 3 images each of the digit 1 and 9, 4 images of the digit 5, 1 image each of the digit 5 and 6 were wrongly predicted. The details of wrong predictions are shown in the confusion matrix 3. Total 14 images which are wrongly classified among all the test images of the dataset are shown in table 4. In confusion matrix, we have shown that which wrongly predicted image is classified to which class. After careful observation
Table 3: Confusion matrix of the test result of the ISI handwritten Bengali numerals test dataset

| Actual Class | Predicted Class | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Accuracy(%) |
|--------------|----------------|---|---|---|---|---|---|---|---|---|---|------------|
| 0            | 398            | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 99.50      |
| 1            | 0              | 397| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 99.25      |
| 2            | 0              | 0  | 400| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.00     |
| 3            | 0              | 0  | 0  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.00     |
| 4            | 0              | 0  | 0  | 0 | 400| 0 | 0 | 0 | 0 | 0 | 0 | 100.00     |
| 5            | 1              | 0  | 0  | 0 | 0  | 1 | 396| 1 | 0 | 1 | 0 | 99.00      |
| 6            | 0              | 0  | 0  | 0 | 0  | 0 | 1  | 399| 0 | 0 | 0 | 99.75      |
| 7            | 1              | 0  | 0  | 0 | 0  | 0 | 0  | 399| 0 | 0 | 0 | 99.75      |
| 8            | 0              | 0  | 0  | 0 | 0  | 0 | 0  | 400| 0 | 0 | 0 | 100.00     |
| 9            | 1              | 2  | 0  | 0 | 0  | 0 | 0  | 0  | 0 | 0 | 0 | 99.25      |

of the patterns of these 14 images, the reason behind these wrong classification is quite clear.

6.5. Comparison of Test Results with Base-line-models

The most notable models proposed by researchers for Bengali handwritten numerals recognition has been discussed in subsection 2.1. As BDNet only focused on ISI handwritten Bengali numerals and on our own dataset, so, we have compared BDNet with some notable models which are also worked on this benchmark dataset [13]. Before BDNet, previous two best models [14] and [27] achieved the test accuracy of 99.40% and 99.58%(authors of [27] claimed) respectively whereas BDNet achieved 99.65%. All the notable models and corresponding the highest test accuracy shown in table 5. Graphical comparison of said models has shown in figure [11] where X-axis presents the models and Y-axis shown the corresponding test accuracy in the benchmark dataset [13].

6.6. Test Result on our Own Dataset

We have described our own test dataset in subsection 4.3 which has 10 classes with 100 images per class. This test dataset not used during training because to see the generalization of trained BDNet on virgin dataset. But the BDNet has got 97.50% test accuracy. The entire result has been shown in the confusion matrix mentioned in table 6.
Table 4: Wrongly Classified Images

| Digit Images | Actual Class | Predicted Class |
|--------------|--------------|-----------------|
| 0            | 3            |                 |
| 0            | 7            |                 |
| 1            | 2            |                 |
| 1            | 4            |                 |
| 1            | 8            |                 |
| 5            | 0            |                 |
| 5            | 4            |                 |
| 5            | 6            |                 |
| 5            | 8            |                 |
| 6            | 5            |                 |
| 7            | 0            |                 |
| 9            | 0            |                 |
| 9            | 1            |                 |
| 9            | 1            |                 |

7. Conclusion

Our BDNet is a densely connected deep CNN model for handwritten Bengali numeral recognition through image classification. Though the structure of BDNet is quite different from traditional CNN but the idea is almost similar to state-of-the-art algorithm DenseNet. The BDNet is trained with ISI Bengali handwritten numerals dataset and the trained model has achieved up to 99.65% accuracy on a test dataset of the same database, and 97.50% on our own test dataset.

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Table 5: Notable Bengali handwritten numerals recognition models and corresponding test accuracy in the benchmark dataset [13].

| Models                                    | Test accuracy |
|-------------------------------------------|---------------|
| U. Bhattacharya & B. B. Choudhury(2009)   | 98.20%        |
| C-L. Liu & C.Y. Suen(2009)                | 99.40%        |
| N. Das et.al(2012)                        | 97.70%        |
| Y. Wen and L. He(2012)                    | 99.40%        |
| M. A. H. Akhand et.al(2016)               | 98.98%        |
| Md. Shohon et.al(2017)                    | 99.35%        |
| AKM S. A. Rabby et.al(2019)               | 99.58%        |
| BDNet(This model)                         | 99.65%        |
| Actual Class | Numerals | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Accuracy(%) |
|--------------|----------|---|---|---|---|---|---|---|---|---|---|-------------|
| 0            | 100      | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100         |
| 1            | 0        | 100| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100         |
| 2            | 0        | 0 | 100| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100         |
| 3            | 0        | 0 | 0 | 100| 0 | 0 | 0 | 0 | 0 | 0 | 100         |
| 4            | 0        | 0 | 0 | 0 | 100| 0 | 0 | 0 | 0 | 0 | 100         |
| 5            | 0        | 0 | 0 | 1 | 0 | 90 | 2 | 0 | 1 | 0 | 90          |
| 6            | 0        | 0 | 0 | 1 | 0 | 0 | 99| 0 | 0 | 0 | 99          |
| 7            | 1        | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 97| 0 | 97          |
| 8            | 0        | 0 | 8 | 0 | 0 | 0 | 0 | 1 | 0 | 91| 0 | 91          |
| 9            | 0        | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 98| 0 | 98          |

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