Determination of the String Scale in D-Brane Scenarios and Dark Matter Implications

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Abstract

We analyze different phenomenological aspects of D-brane constructions. First, we obtain that scenarios with the gauge group and particle content of the supersymmetric standard model lead naturally to intermediate values for the string scale, in order to reproduce the value of gauge couplings deduced from experiments. Second, the soft terms, which turn out to be generically non universal, and Yukawa couplings of these scenarios are studied in detail. Finally, using these soft terms and the string scale as the initial scale for their running, we compute the neutralino-nucleon cross section. In particular we find regions in the parameter space of D-brane scenarios with cross sections in the range of $10^{-6} - 10^{-5}$ pb, i.e. where current dark matter experiments are sensitive. For instance, this can be obtained for $\tan \beta > 5$.

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1 Introduction

Although the standard model provides a correct description of the observable world, there exist, however, strong indications that it is just an effective theory at low energy of some fundamental one. The only candidates for such a theory are, nowadays, the string theories, which have the potential to unify the strong and electroweak interactions with gravitation in a consistent way.

In the late eighties, working in the context of the perturbative heterotic string, a number of interesting four-dimensional vacua with particle content not far from that of the supersymmetric standard model were found [1]. Supersymmetry breaking was most of the times assumed to take place non-perturbatively by gaugino condensation in a hidden sector of the theory. Until recently, it was thought that this was the only way in order to construct realistic string models. However, in the late nineties, we have discovered that explicit models with realistic properties can also be constructed using D-brane configurations from type I string vacua [2]-[8]. Besides, it has been realized that the string scale, $M_I$, may be anywhere between the weak scale, $M_W$, and the Planck scale, $M_{Planck}$. This is to be compared to the perturbative heterotic string where the relation $M_I = \sqrt{\frac{\alpha}{8}} M_{Planck}$ with $\alpha$ the gauge coupling, fixes the value of the string scale.

The freedom to play with the value of $M_I$ is particularly interesting since there are several arguments in favour of supersymmetric scenarios with scales $M_I \approx 10^{10-14}$ GeV. First, these scales were suggested in [11] to explain many experimental observations as neutrino masses or the scale for axion physics. Second, with the string scale of order $10^{10-12}$ GeV one is able to attack the hierarchy problem of unified theories [12]. The mechanism is the following. In supergravity models supersymmetry can be spontaneously broken in a hidden sector of the theory and the gravitino mass, which sets the overall scale of the soft terms, is given by:

$$m_{3/2} \approx \frac{F}{M_{Planck}} ,$$

(1)

where $F$ is the auxiliary field whose vacuum expectation value breaks supersymmetry. Since in supergravity one would expect $F \approx M_{Planck}^2$, one obtains $m_{3/2} \approx M_{Planck}$ and therefore the hierarchy problem solved in principle by supersymmetry would be re-introduced, unless non-perturbative effects such as gaugino condensation produce $F \approx M_W M_{Planck}$. However, if the scale of the fundamental theory is $M_I \approx 10^{10-12}$ GeV instead of $M_{planck}$, then $F \approx M_I^2$ and one gets $m_{3/2} \approx M_W$ in a natural way, without invoking any hierarchically suppressed non-perturbative effect. Third, for intermediate scale scenarios charge and color breaking constraints become less important. Let us
recall that charge and color breaking minima in supersymmetric theories might make the standard vacuum unstable. Imposing that the standard vacuum should be the global minimum the corresponding constraints turn out to be very strong and, in fact, working with the usual unification scale $M_{\text{GUT}} \approx 10^{16}$ GeV, there are extensive regions in the parameter space of soft supersymmetry-breaking terms that become forbidden. For example, for the dilaton-dominated scenario of superstrings the whole parameter space turns out to be excluded on these grounds. The stability of the corresponding constraints with respect to variations of the initial scale for the running of the soft breaking terms was studied in [13], finding that the larger the scale is, the stronger the bounds become. In particular, by taking $M_{\text{Planck}}$ rather than $M_{\text{GUT}}$ for the initial scale stronger constraints were obtained. Obviously the smaller the scale is, the weaker the bounds become. In [14] intermediate scales rather than $M_{\text{GUT}}$ were considered for the dilaton-dominated scenario with the interesting result that it is allowed in a large region of parameter space. Finally, there are other arguments in favour of scenarios with intermediate string scales $M_I \approx 10^{10-14}$ GeV. For example these scales might also explain the observed ultra-high energy ($\approx 10^{20}$ eV) cosmic rays as products of long-lived massive string mode decays. Besides, several models of chaotic inflation favour also these scales [17].

In the present article we are going to analyze in detail whether or not those intermediate string scales are also necessary in order to reproduce the low-energy data, i.e. the values of the gauge couplings deduced from CERN $e^+e^-$ collider LEP experiments. In this sense, we will see that D-branes scenarios indeed lead naturally to intermediate values for the string scale $M_I$.

On the other hand, it has been noted that the neutralino-nucleon cross section is quite sensitive to the value of the initial scale for the running of the soft breaking terms [18]. The smaller the scale is, the larger the cross section becomes. In particular, by taking $10^{10-12}$ GeV rather than $10^{16}$ GeV for the initial scale, the cross section increases substantially $\sigma \approx 10^{-6}-10^{-5}$ pb. This result is extremely interesting since the lightest neutralino is usually the lightest supersymmetric particle (LSP), and therefore a natural candidate for dark matter in supersymmetric theories [19], and current dark matter detectors, DAMA [20] and CDMS [21], are sensitive to a neutralino-nucleon cross section in the above range.

The initial scale for the running of the soft terms in D-brane scenarios is $M_I$. As mentioned above, several theoretical and phenomenological arguments suggest that intermediate values for this scale are welcome. Thus it is natural to wonder how much the standard neutralino-nucleon cross section analysis will get modified in D-brane scenarios. This is another aim of this article.
The content of the article is as follows. In Section 2 we will try to determine the string scale in D-brane scenarios imposing the experimental constraints on the values of the gauge coupling constants. Although we will concentrate mainly in scenarios where the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ groups of the standard model come from different sets of Dp-branes, we will also review the scenario where they come from the same set of Dp-branes. The fact that the $U(1)_Y$ group arises as a linear combination of different $U(1)$’s, due to their D-brane origin, is crucial in the analysis.

In Section 3 we will use the results of Section 2, in particular the matter distribution due to the D-brane origin of the $U(1)$ gauge groups, in order to derive the soft supersymmetry breaking terms of the D-brane scenarios which may give rise to the supersymmetric standard model. Generically they are non-universal. This analysis is carried out under the assumption of dilaton/moduli supersymmetry breaking [27]-[30]. We emphasize that this assumption of dilaton/moduli dominance is more compelling in the D-brane scenarios where only closed string fields like $S$ and $T_i$ can move into the bulk and transmit supersymmetry breaking from one D-brane sector to some other. Finally, we will also discuss the structure of Yukawa coupling matrices.

In Section 4, using the soft terms of the D-brane scenarios previously studied, we compute the neutralino-nucleon cross section. We will see how the compatibility of regions in the parameter space of these scenarios with the sensitivity of current dark matter experiments depends not only on the value of the string scale but also on the non-universality of the soft terms.

Finally, the conclusions are left for Section 5.

2 D-brane scenarios and the string scale

As mentioned in the Introduction there exists the interesting possibility that the supersymmetric standard model might be built using D-brane configurations. In this case there are two possible avenues to carry it out: i) The $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ groups of the standard model come from different sets of Dp-branes. ii) They come from the same set of Dp-branes.

Since the two scenarios are interesting and qualitatively different, we will discuss both separately. We will see in detail below that the first one (i), in order to reproduce the values of the gauge couplings deduced from CERN $e^+e^-$ collider LEP experiments, leads naturally to intermediate values for the string scale $M_I$. One realizes that this is an interesting result since there are several arguments in favour of intermediate scales, as discussed in the Introduction. This approach was used first in [31] for the case...
of non-supersymmetric D$p$-branes with the result of a string scale of the order of a few TeV. In any case, it is worth remarking the difficulty of obtaining three copies of quarks and leptons if the gauge groups are attached to different sets of D$p$-branes\footnote{We thank L.E. Ibáñez for discussions about this point (see also \cite{6}).}. Thus whether or not the scenarios discussed below, may arise from different sets of D$p$-branes in explicit string constructions is an important issue which is worth attacking in the future.

Concerning the other scenario (ii), models with the gauge group of the standard model and three families of particles have been explicitly built \cite{6, 7}. We will review whether or not intermediate scales arise naturally.

### 2.1 Embedding the gauge groups within different sets of D$p$-branes

It is a plausible situation to assume that the $SU(3)_c$ and the $SU(2)_L$ groups of the standard model could come from different sets of D$p$-branes \cite{32, 3}. By different sets we mean D$p$-branes whose world-volume is not identical. In particular, notice that the standard model contains particles (the left-handed quarks $Q_u$) transforming both under $SU(3)_c$ and $SU(2)_L$. That means that there must be some overlap of the world-volumes of both sets of D$p$-branes. Thus e.g., one cannot put $SU(3)_c$ inside a set of D3-branes and $SU(2)_L$ within another set of parallel D3-branes on a different point of the compact space since then there would be no massless modes corresponding to the exchange of open strings between both sets of branes which could give rise to the left-handed quarks.

Thus we need to embed $SU(3)_c$ inside D-branes, say D$p_3$-branes, and $SU(2)_L$ within other D-branes, say D$p_2$-branes, in such a way that their corresponding world-volumes have some overlap. Since we are working in general with type IIB orientifolds, $p_N$ can be 3, 5$_i$, 7$_i$, and 9, where the index $i = 1, 2, 3$ denotes what complex compact coordinate is included in the D5-brane world-volume, or is transverse to the D7-brane world-volume. Not all types of D$p_N$-branes may be present simultaneously if we want to preserve $N = 1$ in $D = 4$. For a given $D = 4$, $N = 1$ vacuum we can have at most either D9-branes with D5$_i$-branes or D3-branes with D7$_i$-branes.

In type IIB orientifold models, and in general on the world-volume of D-branes, $SU(N)$ groups come along with a $U(1)$ factor, say $U(1)_N$, so that indeed we are dealing with $U(N)$ groups, in which both $SU(N)$ and $U(1)_N$ share the same coupling constant, $\alpha_N$. Thus $U(1)_Y$ might be a linear combination of two $U(1)$ gauge groups arising from $U(3)_c$ and $U(2)_L$ within D$p_3$- and D$p_2$-branes respectively \cite{33}. Although this is the
Figure 1: A generic D-brane scenario giving rise to the gauge bosons and matter of the standard model. It contains three Dp3-branes, two Dp2-branes and one Dp1-brane, where $p_N$ may be either 9 and 5i or 3 and 7i. The presence of extra D-branes, say Dq-branes, is also necessary as explained in the text. For each set the D$p_N$-branes are in fact on the top of each other.

The simplest possibility, its analysis is somehow subtle [31] and we prefer to carry it out at the end of this subsection. Thus we will analyze first a more general case, where an extra $U(1)$ arising from another D-brane, say Dp1-brane, contributes to the combination giving rise to the correct hypercharge of the standard model matter [31, 6]. This is schematically shown in Fig. 1, where open strings starting and ending on the same sets of D$p_N$-branes give rise to the gauge bosons of the standard model. For the sake of visualization each set is depicted at parallel locations, but in fact they are intersecting each other as discussed above.

In [6] a $Z_3$ orientifold model with $U(3)_c \times U(2)_L \times U(1)$ observable gauge group, and therefore giving rise to $SU(3)_c \times SU(2)_L \times U(1)_3 \times U(1)_2 \times U(1)_1$ as discussed above, was explicitly built. Nevertheless, this model is embedded in D3-branes, i.e. $p_3 = p_2 = p_1 = 3$, and therefore we will discuss it in detail in Subsection 2.2.

On the other hand, in [31] the existence of standard models coming from different sets of non-supersymmetric Dp-branes was assumed and several consequences were discussed. In particular, imposing $Dp_3 = Dp_1$, i.e. $\alpha_3 = \alpha_1$, the low-energy data are reproduced for a string scale of the order of a few TeV. Here we will carry out the general analysis of supersymmetric Dp-branes with the interesting result that intermediate
Matter Fields | $Q_3$ | $Q_2$ | $Q_1$ | $Q_2$ | $Q_1$ | $Q_2$ | $Q_1$ | $Y$
---|---|---|---|---|---|---|---|
$Q_u(3,2)$ | 1 | -1 | 0 | 1 | 0 | -1 | 0 | 1/6
$u^c(3,1)$ | -1 | 0 | -1 | 0 | -1 | 0 | 0 | -2/3
$d^c(3,1)$ | -1 | 0 | 0 | 0 | 0 | 1 | 0 | 1/3
$Le(1,2)$ | 0 | 1 | 0 | 1 | -1 | 1 | 0 | -1/2
$e^c(1,1)$ | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1

Table 1: The four possible assignments of quantum numbers (multiplied by $\sqrt{2N}$) of a family of quarks and leptons of the standard model under $U(1)_3 \times U(1)_2 \times U(1)_1$. Note that $Q_3$ is always fixed. The usual hypercharge $Y$ is given in the last column.

values ($\approx 10^{10-12}$ GeV) for the string scale may arise in a natural way.

2.1.1 General scenario with $Dp_3 \neq Dp_2 \neq Dp_1 \neq Dp_3$

Let us denote by $Q_3$, $Q_2$ and $Q_1$ the charges of $U(1)_3$, $U(1)_2$, and $U(1)_1$ respectively. Following then the analysis of Antoniadis, Kiritsis and Tomaras [31] a family of quarks and leptons can have the four assignments of quantum numbers given in Table 1, in order to obtain the hypercharge of the standard model

$$Y = c_3 \sqrt{6} Q_3 + c_2 \sqrt{4} Q_2 + \sqrt{2} Q_1,$$

where $c_3 = -1/3$, $c_2 = -1/2$ ($c_2 = 1/2$) for the first (second) assignment and $c_3 = 2/3$, $c_2 = -1/2$ ($c_2 = 1/2$) for the third (fourth) assignment. Note that $U(N)$ generators are normalized as $\text{Tr} \ T^a T^b = \frac{1}{2} \delta^{ab}$, and therefore the fundamental representation of $SU(N)$ has $Q_N = 1/\sqrt{2N}$.

For example, as discussed above, the quark doublet $Q_u$ always arises from an open string with one end on $Dp_3$-branes and the other end on $Dp_2$-branes. In the first assignment of Table 1 $Q_u$ transforms as a $\bar{2}$ under $U(2)$ and therefore $Q_2 = -1/\sqrt{4}$. $u^c$ ($d^c$) arises from an open string with one end on $Dp_3$-branes and the other end on $Dp_1$ ($Dq$)-branes with $Q_1 = -1/\sqrt{2}$ (0). Finally, in the case of leptons, $L_e$ ($e^c$) arises from an open string with one end on $Dp_2$ ($Dp_1$)-branes and the other end on $Dq$-branes with $Q_1 = 0$ ($1/\sqrt{2}$). This is schematically shown in Fig. 1. The other three possible $^2$As we see from here the presence of extra D-branes, say Dq-branes, is necessary in order to reproduce the correct hypercharge for the matter. In addition, in Subsection 2.2 we will see an explicit model where Dq-branes are also necessary to cancel non-vanishing tadpoles. The additional $U(1)$ factors associated to the Dq-branes will be anomalous and therefore with a mass of the order of the string scale.
assignments can also be analyzed similarly. Let us remark that other scenarios with $u^c$, $d^c$ ($e^c$) arising from open strings with both ends on $Dp_3$ ($Dp_2$)-branes are possible, since these particles can be obtained as the antisymmetric product of two triplets of $SU(3)$ (doublets of $SU(2)$). However, these scenarios do not give rise to a modification of the analysis of the string scale \[31\], and therefore we will not consider them here.

Concerning the possible quantum numbers of Higgses, they will be discussed in Section 3 where they are important e.g. in order to determine whether or not all Yukawa couplings in D-brane scenarios are allowed.

Let us now try to determine the type I string scale $M_I$, using the above information. On the one hand, from (2) one obtains the following relation at $M_I$:
\[
\frac{1}{\alpha_Y(M_I)} = \frac{2}{\alpha_1(M_I)} + \frac{4c_2^2}{\alpha_2(M_I)} + \frac{6c_3^2}{\alpha_3(M_I)}. \tag{3}
\]
On the other hand, the usual RGE’s for gauge couplings are given by
\[
\frac{1}{\alpha_j(M_I)} = \frac{1}{\alpha_j(M_Z)} + \frac{b_j^{ns}}{2\pi} \ln \frac{M_s}{M_Z} + \frac{b_j^s}{2\pi} \ln \frac{M_I}{M_s}, \tag{4}
\]
where $b_j^s$ ($b_j^{ns}$) with $j = 2, 3, Y$ are the coefficients of the supersymmetric (non-supersymmetric) $\beta$-functions, and the scale $M_s$ corresponds to the supersymmetric threshold, $200 \text{ GeV} \lesssim M_s \lesssim 1000 \text{ GeV}$. Thus using (3), (4) and the fact that always $c_2^2 = 1/4$ one can compute $M_I$ with the result
\[
\ln \frac{M_I}{M_s} = \frac{2\pi}{\alpha_Y(M_Z)} \left( \frac{1}{\alpha_1(M_I)} - \frac{2}{\alpha_2(M_I)} - \frac{6c_3^2}{\alpha_3(M_I)} \right) + \frac{(b_Y^{ns} - b_2^{ns} - 6c_3^2b_3^{ns}) \ln \frac{M_s}{M_Z}}{6c_3^2b_3^s + b_2^s - b_Y^s}. \tag{5}
\]
Using the experimental values $M_Z = 91.1870 \text{ GeV}$, $\alpha_3(M_Z) = 0.1184$, $\alpha_2(M_Z) = 0.0338$, $\alpha_Y(M_Z) = 0.01016$, and the matter content of the minimal supersymmetric standard model (MSSM), i.e. $b_3^s = 3$, $b_2^s = -1$, $b_Y^s = -11$ and $b_3^{ns} = 7$, $b_2^{ns} = 19/6$, $b_Y^{ns} = -41/6$, one obtains for $c_3 = -1/3$
\[
\ln \frac{M_I}{M_s} = 33.09 - \frac{1.05}{\alpha_1(M_I)} - 1.22 \ln \frac{M_s}{M_Z}. \tag{6}
\]
For example, choosing the value of the coupling associated to the $Dp_1$-brane in the range $0.07 \lesssim \alpha_1(M_I) \lesssim 0.1$ one obtains $M_I \approx 10^{10-12} \text{ GeV}$. This scenario is shown in Fig. 2 for $\alpha_1(M_I) = 0.1$ and $M_s = 1 \text{ TeV}$. As discussed above, this "intermediate" initial scale is an attractive possibility. Values of the coupling $\alpha_1(M_I)$ smaller than 0.07 are not interesting since $M_I$ becomes also smaller, and therefore $m_{3/2}$ in (11) will be too low to be compatible with the experimental bounds on supersymmetric particle masses. Although the larger the coupling is, the larger $M_I$ becomes (e.g. for $\alpha_1(M_I) = 1$ one
Figure 2: Running of the gauge couplings of the MSSM with energy $Q$ embedding the gauge groups within different sets of D$p$-branes (solid lines). Due to the D-brane origin of the $U(1)$ gauge groups, relation (3) must be fulfilled. For comparison the running of the MSSM couplings with the usual normalization factor for the hypercharge, $3/5$, is also shown with dashed lines.

is even able to obtain $M_I \approx 5 \times 10^{15}$ GeV), one should be careful with the range of validity of the perturbative regime.

On the other hand, the case $c_3 = 2/3$ is less interesting since one obtains the upper bound $M_I \approx 3 \times 10^8$ GeV.

It is worth noticing that non-supersymmetric scenarios can also be analyzed with the above formula (5) with the substitutions $M_s \rightarrow M_Z$, $b_i^s \rightarrow b_i^{ns}$. For example, $M_I \approx 1$ TeV can be obtained with $\alpha_1(M_I) \approx 0.035$ for $c_3 = -1/3$, and $\alpha_1(M_I) \approx 0.056$ for $c_3 = 2/3$.

2.1.2 Scenarios with D$p_1$=D$p_3$ or D$p_1$=D$p_2$

Let us now simplify the above analysis assuming that the D-brane associated to the $U(1)_1$ is on top of one of the other D-branes. In this case we have two possibilities, either D$p_1$=D$p_3$ or D$p_1$=D$p_2$. Let us start analyzing the possibility D$p_1$=D$p_3$, which implies $\alpha_1 = \alpha_3$. Then eq.(5) is still valid with the substitutions $\frac{2}{\alpha_1(M_I)} + \frac{6c_3^2}{\alpha_3(M_Z)} \rightarrow \frac{2+6c_3^2}{\alpha_2(M_Z)}$, $6c_3^2 b_i^{s,ns} \rightarrow (2 + 6c_3^2) b_i^{s,ns}$. As a consequence, for $c_3 = -1/3$, one obtains the following prediction: $M_I \approx 6 \times 10^8$ GeV, with $M_s = 200$ GeV. A slightly low value to be able to obtain $m_{3/2} \approx M_W$, as discussed below eq.(5). Obviously, the larger $M_s$ is, the smaller
$M_I$ becomes. The case $c_3 = 2/3$ is much worse since $M_I \approx 100$ TeV.

The other scenario $Dp_1 = Dp_2$, which implies $\alpha_1 = \alpha_2$, does not improve the above situation. One can use again eq.(5), but now with the substitutions \( \frac{3}{\alpha_2(M_Z)} b_2^{s,ns} \rightarrow 3b_2^{s,ns} \). In particular, for $c_3 = -1/3$, $M_I \approx 500$ GeV, with $M_s = 200$ GeV, whereas $\ln\frac{M_I}{M_s}$ is even negative for $c_3 = 2/3$.

On the other hand, extra particles appear quite frequently in superstring theories. Since their presence will modify the denominator in (5), one might obtain larger values for $M_I$. For example, for $Dp_1 = Dp_3$, and restricting ourselves to the case of singlets, $SU(2)$ doublets and colour triplets, one has

\[(2 + 6c_3^2)b_3^s + b_2^s - b_y^s = 18 - \frac{1}{2}(2 + 6c_3^2)n_3 - \frac{1}{2}n_2 + q , \tag{7}\]

where $q = \sum_{i=1}^{n_1} Y_i^2 + 2\sum_{j=1}^{n_2} Y_j^2 + 3\sum_{k=1}^{n_3} Y_k^2$ and $n_{1,2,3}$ is the number of extra singlets, doublets and triplets that the model under consideration has. Extra $(3, 2)$ representations under $SU(3) \times SU(2)$ must be introduced in the formula for $q$ just as two triplets each. For instance, assuming the presence of two copies of $d^c + \bar{d}^c$ for the case $c_3 = -1/3$, one obtains $M_I \approx 4 \times 10^{10}$ GeV ($\approx 8 \times 10^{9}$ GeV), with $M_s = 200$ GeV (1 TeV). Concerning the running of the couplings, this scenario is similar to the one shown in Fig. 2.

As above, we can also analyze non-supersymmetric scenarios. A string scale of order a few TeV can be obtained without extra particles. In particular, for $\alpha_1 = \alpha_2$, $c_3 = -1/3$ and $\alpha_1 = \alpha_3$, $c_3 = 2/3$ we recover the results of [31], $M_I \approx 300$ GeV and $M_I \approx 7$ TeV, respectively.

### 2.1.3 Scenario without Dp1-brane

Let us finally consider the scenario where the $U(1)_Y$ is only a linear combination of the two $U(1)$ gauge groups arising from $U(3)_c$ and $U(2)_L$ within $Dp_3$- and $Dp_2$-branes respectively [33].

As discussed in [31], there is only one assignment of quantum numbers for quarks and leptons, in order to obtain the hypercharge of the standard model. The latter is given by eq. (2) with $c_3 = -1/3$, $c_2 = -1/2$, $Q_1 = 0$, i.e. $Y = -\frac{1}{3}\sqrt{6}Q_3 - \frac{1}{2}\sqrt{4}Q_2$. Whereas the charges $Q_3$ and $Q_2$ for $Q_u$, $d^c$ and $L_e$ are as in the first assignment given in Table 1, $Q_3 = 2/\sqrt{6}$, 0 and $Q_2 = 0, -2/\sqrt{4}$ for $u^c$, $e^c$. Clearly, $u^c$ and $e^c$ must arise from open strings with both ends on Dp3-branes and Dp2-branes, respectively. As mentioned above, this is possible since these particles can be obtained as the antisymmetric product of two triplets of $SU(3)$ and doublets of $SU(2)$, respectively.
With the above hypercharge, instead of eq. (3) one obtains
\[ \frac{1}{\alpha_Y(M_I)} = \frac{1}{\alpha_1(M_I)} + \frac{2/3}{\alpha_3(M_I)}, \]
and therefore eq. (4) is still valid with \( c_3 = -1/3 \) and the substitution \( \frac{1}{\alpha_1(M_I)} \to 0 \). As a consequence one can predict the string scale. For example, for \( M_s = 200 \text{ GeV} \) \( (M_s = 1 \text{ TeV}) \) one obtains \( M_I \approx 1.8 \times 10^{16} \text{ GeV} \) \( (M_I \approx 10^{16} \text{ GeV}) \). On the other hand, a non-supersymmetric scenario \[31\] gives rise to a string scale which is too large, \( M_I \approx 5 \times 10^{13} \text{ GeV} \).

It is worth noticing \[3\] that for \( \alpha_3 = \alpha_2 \) one obtains the standard GUT normalization for couplings \( \alpha_Y = \frac{3}{5} \alpha_2 \), and therefore \( M_I \approx 2 \times 10^{16} \text{ GeV} \).

We thus conclude that, concerning the string scale \( M_I \), the generic models analyzed above are very interesting from the point of view of their predictivity. Besides, the values obtained for \( M_I \) can be accommodated in type I strings, choosing the appropriate values of the moduli. For instance, for the example studied below eq. (7) the experimental values of couplings are obtained with \( M_I \approx 8 \times 10^9 \text{ GeV} \) for the case \( M_s = 1 \text{ TeV} \), and therefore the ratio \( \frac{\alpha_3(M_I)}{\alpha_2(M_I)} \approx 2 \). Let us assume that \( SU(3)_c \) is embedded inside D9-branes and \( SU(2)_L \) inside D5-branes. Then one has the following relationships
\[ \frac{M_1 M_2 M_3}{M_I^2} = \frac{\alpha_3 M_{\text{Planck}}}{\sqrt{2}} \quad , \quad \frac{M_1 M_2^2}{M_2 M_3} = \frac{\alpha_2 M_{\text{Planck}}}{\sqrt{2}} \] (8)
where \( M_i, i = 1, 2, 3 \), are the compactification masses associated to the compact radii \( R_i \). Choosing \( \frac{M_3}{M_2 M_3} \approx 1/2 \) one is able to reproduce the above ratio.

### 2.2 Embedding all gauge groups within the same set of Dp-branes

The fact that to obtain three copies of quark and leptons is difficult, when gauge groups come from different sets of DpN-branes, as mentioned above, is one of the motivations in \[3\] to embed all gauge interactions in the same set of DpN-branes \((p_3 = p_2 = p_1 \text{ in the notation above})\).

Here we will briefly review the results of Aldazabal, Ibáñez, Quevedo and Uranga \[3\] concerning this issue. They are able to build \( Z_3 \) orientifold models with the gauge group \( SU(3)_c \times SU(2)_L \times U(1)_3 \times U(1)_2 \times U(1)_1 \) embedded in D3-branes, with no additional non-abelian factors. They also argue that in the \( Z_3 \) orientifold, which leads naturally to three families, only the combination
\[ Y = -\frac{1}{3} \sqrt{6} Q_3 - \frac{1}{2} \sqrt{4} Q_2 + \sqrt{2} Q_1 \] (9)
will be non-anomalous. It is worth noticing that this is precisely the hypercharge given in (2) with \( c_3 = -1/3 \) and \( c_2 = -1/2 \), i.e. the first assignment of Table 1.
Fig. 1 with $Dp_3=Dp_2=Dp_1=D3$, and all D3-branes on top of each other, is also valid as a schematic representation of this type of models. D$q$-branes in the figure are now D7-branes, which must be introduced in order to cancel non-vanishing tadpoles. Since $\alpha_3 = \alpha_2 = \alpha_1 = \alpha$, instead of (3) one obtains

$$\frac{1}{\alpha_Y(M_I)} = \frac{11/3}{\alpha(M_I)},$$

which is not the standard GUT normalization for couplings. This is due to the D-brane origin of the $U(1)$ gauge groups.

A model with all these properties was explicitly built in [3]. Although extra $U(1)'s$ on the D7-branes are present, they are anomalous and therefore the associated gauge bosons have masses of the order of $M_I$. In addition to D7-branes, anti-D7-branes trapped at different $Z_3$ fixed points are also present. Since they break supersymmetry at the string scale $M_I$, they can be used as the hidden sector of supergravity theories. Thus this is an example of gravity mediated supersymmetry breaking.

On the other hand, in this model not only quarks and leptons come in three generations but also Higgses, i.e. it contains two pairs of extra doublets with respect to the MSSM. In addition, three pairs of extra colour triplets are also present. Unfortunately, this matter content cannot give rise to the correct values for $\alpha_j(M_Z)$. Although generically the extra triplets will be heavy, this does not modify the previous result. One cannot exclude, however, the possibility that other models with the necessary matter content, in order to reproduce the experimental values of couplings, might be built. For example, if besides the matter content of the MSSM, we have six copies of $H_1+H_2$ and two copies of $d^c+d^c$ unification at around $M_I = 10^{10}$ GeV, with $\alpha(M_I) \approx 1/14$, is obtained. This scenario is shown in Fig. 3 for $M_s = 1$ TeV.

It is worth noticing that these values can be accommodated in type I strings, choosing the appropriate values of the moduli. For example, with an isotropic compact space, the string scale is given by:

$$M_I^4 = \frac{\alpha M_{Planck}}{\sqrt{2}} M_c^3,$$

where $M_c$ is the compactification scale. Thus one gets $M_I \approx 10^{10-12}$ GeV with $M_c \approx 10^{8-10}$ GeV.

Let us finally mention that another model with the gauge group of the standard model and three families has recently been built [3]. The presence of additional Higgs doublets and vector-like states allows an unification scale at an intermediate value.
Figure 3: Running of the gauge couplings with energy $Q$ embedding all gauge groups within the same set of D3-branes (solid lines). In addition to the matter content of the MSSM, extra Higgs doublets and vector-like states are also present. Due to the D-brane origin of the $U(1)$ gauge groups, the normalization factor of the hypercharge is $3/11$ (see eq. (10)). For comparison the running of the MSSM couplings with the usual normalization factor for the hypercharge, $3/5$, is also shown with dashed lines.

3 Soft terms and Yukawa couplings in D-brane scenarios

General formulas for the soft supersymmetry-breaking terms in D-brane constructions were obtained in [33], under the assumption of dilaton/moduli supersymmetry breaking [27]-[30], using the parametrization introduced in [30]. On the other hand, general Yukawa couplings in D-brane constructions have been studied in [35, 33]. Since we need to use these results to obtain the soft terms and Yukawa couplings associated to the D-brane scenarios discussed above, we summarize them in the Appendix A.

3.1 Embedding the gauge groups within different sets of D$p$-branes

3.1.1 General scenario with $D_{p_3} \neq D_{p_2} \neq D_{p_1} \neq D_{p_3}$

For the sake of concreteness, let us assume the following distribution of D-branes in the scenario proposed in Subsection 2.1. $D_{p_3}$-branes are D9-branes, $D_{p_1}$-branes are
D53-branes, Dp2-branes are D51-branes, and finally Dq-branes are D52-branes. Then, the first assignment of Table 1 shown schematically in Fig. 1 gives rise to the following soft masses, using formulas (A.1) and (A.2) in the Appendix A. The gaugino masses are given by:

\[
M_3 = \sqrt{3m_{3/2}} \sin \theta ,
M_2 = \sqrt{3m_{3/2}} \Theta_1 \cos \theta ,
M_Y = \sqrt{3m_{3/2}} \alpha_Y (M_I) \left( \frac{2}{\alpha_1(M_I)} \Theta_3 \cos \theta + \frac{1}{\alpha_2(M_I)} \Theta_1 \cos \theta + \frac{6c_3^2}{\alpha_3(M_I)} \sin \theta \right)
\] (12)

where it is worth noticing that relation (3) has been taken into account in order to obtain the gaugino mass associated to the gauge group \( U(1)_Y \). The scalar masses are given by:

\[
m_{Q_u}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( 1 - \Theta_1^2 \right) \cos^2 \theta \right],
\]

\[
m_{d^c}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( 1 - \Theta_2^2 \right) \cos^2 \theta \right],
\]

\[
m_{u^c}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( 1 - \Theta_3^2 \right) \cos^2 \theta \right],
\]

\[
m_{e^c}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_2^2 \cos^2 \theta \right) \right],
\]

\[
m_{L_e}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_3^2 \cos^2 \theta \right) \right].
\] (13)

Note that quarks of type \( Q_u, d^c \) and \( u^c \) are states \( C^{95_1}, C^{95_2} \) and \( C^{95_3} \) respectively, whereas leptons of type \( e^c \) and \( L_e \) are states \( C^{5_{352}} \) and \( C^{5_{152}} \) respectively.

These soft terms (12) and (13) are generically non-universal. For example in the overall modulus limit \( (\Theta_1,\Theta_2,\Theta_3 = 1/\sqrt{3}) \) universality cannot be obtained. The dilaton limit \( (\sin^2 \theta = 1) \) would give rise to tachyonic states \( e^c \) and \( L_e \).

Obviously, the other three assignments of quantum numbers in Table 1 give rise to the same gaugino masses (12). The differences arise for some of the soft scalar masses in (13). For the second assignment the masses of leptons of type \( L_e \) in (13) must be replaced by

\[
m_{L_e}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_2^2 \cos^2 \theta \right) \right].
\] (14)

For the third assignment the masses of quarks of type \( u^c \) and \( d^c \) must be exchanged in (13), i.e.

\[
m_{u^c}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( 1 - \Theta_2^2 \right) \cos^2 \theta \right],
\]

\[
m_{d^c}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( 1 - \Theta_3^2 \right) \cos^2 \theta \right].
\] (15)
Finally, for the fourth assignment, both modifications (14) and (15) must be included in eq. (13).

Concerning the soft Higgs masses, we need to know the quantum numbers $Q_{3,2,1}$ of the two Higgs doublets of the supersymmetric standard model. For the first and third assignments of Table 1 whose hypercharges are $Y = -\frac{1}{3} \sqrt{6} Q_3 - \frac{1}{2} \sqrt{2} Q_2 + \sqrt{2} Q_1$ and $Y = \frac{2}{3} \sqrt{6} Q_3 - \frac{1}{2} \sqrt{2} Q_2 + \sqrt{2} Q_1$ respectively, there are two possible assignments for the Higgs with hypercharge $1/2$, $H_2(0, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{2}})$ and $H_2(0, -\frac{1}{\sqrt{4}}, 0)$. For the Higgs with hypercharge $-1/2$, there are also two possible assignments $H_1(0, \frac{1}{\sqrt{4}}, 0)$ and $H_1(0, -\frac{1}{\sqrt{4}}, -\frac{1}{\sqrt{2}})$. Thus we have four possible combinations:

\begin{align*}
H_2(0, 1, 1) & , & H_1(0, 1, 0) \\
H_2(0, 1, 1) & , & H_1(0, -1, -1) \\
H_2(0, -1, 0) & , & H_1(0, -1, -1) \\
H_2(0, -1, 0) & , & H_1(0, 1, 0)
\end{align*}

where for simplicity we have multiplied the quantum numbers by $\sqrt{2N}$ as in Table 1. For example, combination (16) is the one shown in Fig. 1, where $H_2$ is a state $C^{51,53}$ and $H_1$ is a state $C^{51,52}$. The corresponding soft masses, using again formulas (A.2), are given respectively by

\begin{align}
&m_{H_2}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_2^2 \cos^2 \theta \right) \right] , & m_{H_1}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_3^2 \cos^2 \theta \right) \right] \tag{20} \\
&m_{H_2}^2 = m_{H_1}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_2^2 \cos^2 \theta \right) \right] \tag{21} \\
&m_{H_2}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_3^2 \cos^2 \theta \right) \right] , & m_{H_1}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_2^2 \cos^2 \theta \right) \right] \tag{22} \\
&m_{H_2}^2 = m_{H_1}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_3^2 \cos^2 \theta \right) \right] . \tag{23}
\end{align}

With respect to the second and fourth assignment of quantum numbers in Table 1, it is worth noticing that their hypercharges are equal to the ones of first and third assignment respectively, but with an opposite sign in front of $Q_2$. As a consequence, the four combinations of quantum numbers (14-19) are still valid using an opposite sign for the value of $Q_2$. Thus the corresponding soft masses are like in eqs. (21-23).

Summarizing, we have obtained in total sixteen scenarios with different soft terms.

Concerning the soft trilinear parameters, since these are related to Yukawa couplings we need to discuss first the structure of the latter. This can be carried out straightforwardly taking into account the previous information about quantum numbers and the formula for the renormalizable Yukawa Lagrangian (A.4) in the Appendix A. The sixteen possible scenarios will have in principle different Yukawa couplings since fields $u^c$, $d^c$, $L_e$, $H_2$ and $H_1$ are attached to different D-branes.
Let us concentrate on the eight scenarios which are more realistic from the phenomenological point of view. Following the discussion of Subsection 2.1.1 these are the ones with \( c_3 = -1/3 \), i.e. the first assignment of Table 1 with the four possible combinations for Higgses \([16-19]\), and the second assignment of Table 1 with the same combinations but with an opposite sign for the value of \( Q_2 \) as discussed above. Let us also assume that we have three copies of quarks and leptons. For instance, the first assignment with Higgses as in \((16)\) implies that couplings \( \sum_{a,b} Y_{ab}^u H_2 Q_u^a u_b^c \) correspond to \( C_{51}^{51} C_{65}^{95} C_{65}^{95} \), couplings \( \sum_{a,b} Y_{ab}^d H_1 Q_u^a d_b^c \) correspond to \( C_{51}^{52} C_{65}^{95} C_{65}^{95} \), and finally couplings \( \sum_{a,b} Y_{e}^a H_1 L_e^a e_b^c \) correspond to \( C_{51}^{52} C_{51}^{52} C_{51}^{52} C_{51}^{52} \), where \( a, b \) are family indices. Whereas the last type of couplings is forbidden, as can be seen from \((A.4)\) in the Appendix A, the other two are allowed with the result

\[
Y_{u,d} = g_{q,1} \hat{Y}, \quad Y_e = 0, \quad (24)
\]

where \( \hat{Y} \) is defined as

\[
\hat{Y} = \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}, \quad (25)
\]

\( g_1 \) is the gauge coupling associated to the \( U(1)_1 \) in the D53-brane and \( g_q \) is the gauge coupling associated to the D52-brane. An analysis of the above “democratic” texture for \( Y_{u,d} \) can be found in \([36]\). Lepton masses might appear from non-renormalizable couplings.

The Yukawa couplings for the other three combinations of Higgses \([17-19]\) can be obtained straightforwardly as for the previous one, with the result

\[
Y_{u,e} = g_{q,3} \hat{Y}, \quad Y_d = 0, \quad (26)
\]

\[
Y_e = g_3 \hat{Y}, \quad Y_{u,d} = 0, \quad (27)
\]

\[
Y_d = g_1 \hat{Y}, \quad Y_{u,e} = 0, \quad (28)
\]

respectively. Here \( g_3 \) is the gauge coupling associated to the \( SU(3)_c \) in the D9-brane.

On the other hand, for the second assignment of Table 1 the Yukawa couplings are given by

\[
Y_{u,d,e} = g_{q,1,3} \hat{Y}, \quad (29)
\]

\[
Y_u = g_q \hat{Y}, \quad Y_{d,e} = 0, \quad (30)
\]

\[
Y_{u,d,e} = 0, \quad (31)
\]

\[
Y_{d,e} = g_{1,3} \hat{Y}, \quad Y_u = 0, \quad (32)
\]

\( ^3 \)This is also obtained realizing that the sum of the \( U(1) \) charges of the fields is non-vanishing.
It is worth noticing that in principle some of these scenarios seem to be hopeless. For instance, it is difficult to imagine how scenario with Yukawa couplings (27) may give rise to the observed fermion mass hierarchies with quark masses arising from non-renormalizable terms.

Now that the structure of Yukawa couplings is known we can compute the corresponding trilinear parameters using eq.(A.3). Obviously, when Yukawa couplings are vanishing trilinear parameters are also vanishing. When Yukawa couplings are non-vanishing, the A terms acquire the following values:

\begin{align*}
A_u &= \frac{\sqrt{3}}{2} \frac{m_{3/2}}{2} [(\Theta_2 - \Theta_1 - \Theta_3) \cos \theta - \sin \theta] \hat{Y}, \\
A_d &= \frac{\sqrt{3}}{2} \frac{m_{3/2}}{2} [(\Theta_3 - \Theta_1 - \Theta_2) \cos \theta - \sin \theta] \hat{Y}, \\
A_e &= \frac{\sqrt{3}}{2} \frac{m_{3/2}}{2} [\sin \theta - (\Theta_1 + \Theta_2 + \Theta_3) \cos \theta] \hat{Y}.
\end{align*}

For example, Yukawa couplings (24) have associated A terms given by (33), (34) and \( A_e = 0 \), Yukawa couplings (26) have associated A-terms (33), (35) and \( A_d = 0 \), etc.

### 3.1.2 Scenarios with \( D_{p_1} = D_{p_3} \) or \( D_{p_1} = D_{p_2} \)

As we discussed in Subsection 2.1.2, scenarios with \( D_{p_1} = D_{p_3} \) are more interesting from the phenomenological point of view than scenarios with \( D_{p_1} = D_{p_2} \), thus we will concentrate in the former. In any case, the analysis of the other scenario can be carried out along similar lines. The first attempts to study these scenarios and their phenomenology, in particular CP phases, Yukawa textures, and dark matter, were carried out in [37], [38], and [39, 40, 41, 42], respectively. However, they consider in fact toy scenarios since the important issue of the D-brane origin of the \( U(1)_Y \) gauge group as a combination of other \( U(1) \)'s and its influence on the matter distribution (see e.g. Fig 1) was not included in their analyses. Thus we will take into account the discussion of Section 2 concerning this issue in order to obtain the soft terms and Yukawa couplings, as we already did with the general scenario of the previous subsection.

Assuming the same distribution of D-branes as in the previous subsection, we have that \( D_{p_1} \)- and \( D_{p_3} \)-branes are \( D_9 \)-branes, \( D_{p_2} \)-branes are \( D_{5_1} \)-branes, and finally \( D_q \)-branes are \( D_{5_2} \)-branes. Then, the first assignment of Table 1 gives rise to the following gaugino masses:

\begin{align*}
M_3 &= \sqrt{3} m_{3/2} \sin \theta, \\
M_2 &= \sqrt{3} m_{3/2} \Theta_1 \cos \theta,
\end{align*}
\[ M_Y = \sqrt{3} m_{3/2} \alpha_Y(M_I) \left( \frac{1}{\alpha_2(M_I)} \Theta_1 \cos \theta + \frac{2 + 6 \Theta_3^2}{\alpha_3(M_I)} \sin \theta \right), \tag{36} \]

and scalar masses:

\[
\begin{align*}
m^2_{Q_u} &= m^2_{3/2} \left[ 1 - \frac{3}{2} \left( 1 - \Theta_1^2 \right) \cos^2 \theta \right], \\
m^2_{d_c} &= m^2_{3/2} \left[ 1 - \frac{3}{2} \left( 1 - \Theta_2^2 \right) \cos^2 \theta \right], \\
m^2_{u_c} &= m^2_{3/2} \left[ 1 - 3 \Theta_1^2 \cos^2 \theta \right], \\
m^2_{e_c} &= m^2_{3/2} \left[ 1 - \frac{3}{2} \left( 1 - \Theta_2^2 \right) \cos^2 \theta \right], \\
m^2_{L_e} &= m^2_{3/2} \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_3^2 \cos^2 \theta \right) \right]. \tag{37} \end{align*}
\]

Here \( i = 1, 2, 3 \) labels the three complex compact dimensions. Thus \( u^c_i \) are states \( C^9_i \) coming from open strings starting and ending on D9-branes. These fields behave quite similarly to untwisted sectors of perturbative heterotic orbifolds. It is then natural to use this index as a family index. For example we will take \( u^c_1 = t^c, u^c_2 = e^c \) and \( u^c_3 = u^c \). On the other hand, quarks of type \( Q_u, d_c \) are states \( C^{95}_1 \) and \( C^{95}_2 \) respectively, whereas leptons of type \( e^c \) and \( L_e \) are states \( C^{95}_2 \) and \( C^{5152}_2 \) respectively. As discussed below eq.(13) these soft terms are also generically non-universal. It is worth noticing here that due to the family index \( i \) a potential problem due to flavor-changing neutral currents (FCNC) may arise. Being conservative\(^4\) we can avoid it by imposing \( \Theta_2 = \Theta_3 \). This constraint will be used in Section 4 when discussing neutralino-proton cross sections in this scenario.

For the second assignment, the value of \( m^2_{L_e} \) must be replaced by

\[
m^2_{L_e} = m^2_{3/2} \left[ 1 - \frac{3}{2} \left( 1 - \Theta_1^2 \right) \cos^2 \theta \right]. \tag{38} \]

For the third assignment the masses of quarks of type \( u^c \) and \( d^c \) must be exchanged in (37), i.e.

\[
\begin{align*}
m^2_{u^c} &= m^2_{3/2} \left[ 1 - \frac{3}{2} \left( 1 - \Theta_2^2 \right) \cos^2 \theta \right], \\
m^2_{d^c} &= m^2_{3/2} \left[ 1 - 3 \Theta_1^2 \cos^2 \theta \right]. \tag{39} \end{align*}
\]

For the fourth assignment, both modifications (38) and (39) must be included in eq.(37).

\(^4\)Recall that the relevant mass terms are the low-energy ones, not those generated at the string scale. As discussed e.g. in [30], one has to do the low-energy running of the scalar masses, and, for the squark case, for gluino masses heavier than (or of the same order as) the scalar masses, there are large flavor-independent gluino loop contributions which are the dominant source of scalar masses.
Finally, the Higgs masses corresponding to the four combinations obtained in eqs. (16-19) are

\[ m^2_{H_2} = m^2_{H_1} = m^2_{3/2} \left[ 1 - \frac{3}{2} \left( 1 - \Theta_1^2 \right) \cos^2 \theta \right], \]

\[ m^2_{H_1} = m^2_{H_2} = m^2_{3/2} \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_1^2 \cos^2 \theta \right) \right], \]

respectively. For example \( H_2, H_1 \) in (40) are states \( C^{95_1}, C^{5_15_2} \).

The analysis of Yukawa couplings and trilinear parameters can be carried out as in the previous subsection, assuming again three copies of quark and leptons \( Q_u, d, L_e, e \).

As an example let us consider the second assignment of Table 1 with Higgses as in (17) with an opposite sign for \( Q_2 \). From eq.(A.4) we deduce that the only allowed type of coupling, which corresponds to \( C^{95_1} C^{95_1} C^{9_1} \), is \( H_2 Q_u t c \). The result for Yukawa couplings is then

\[ Y_u = g_3 \tilde{Y}, \quad Y_{d,e} = 0 , \]

where \( \tilde{Y} \) is defined as

\[ \tilde{Y} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} . \]

This structure for Yukawa matrices and its viability has been studied in [38]. Other results for Yukawa couplings arise for the other interesting scenarios. For example for the first assignment of Table 1 with Higgses as in (14), whereas \( Y_u \) and \( Y_e \) are still as above, \( Y_d \) has a “democratic” matrix structure as in (25). This structure was also analyzed in [38].

Concerning the trilinear parameters, these can be computed using again (A.3). For instance, Yukawa couplings (44) have associated

\[ A_u = -\sqrt{3} m_{3/2} \sin \theta \tilde{Y}, \quad A_{d,e} = 0 . \]

### 3.1.3 Scenario without \( D_{p_1} \)-brane

In this scenario the gauge groups of the standard model arise only from \( D_{p_3} \)-branes, which are \( D_{9} \)-branes, and \( D_{p_2} \)-branes, which are \( D_{5_1} \)-branes. Then, following the discussion in Subsection 2.1.3, quarks of type \( Q_u \) are states \( C^{95_1} \), quarks of type \( d^c \) are states \( C^{95_2} \), and leptons of type \( L_e \) are states \( C^{5_15_2} \). On the other hand, quarks of type
$u^c$ are states $C_i^{9}$ and leptons of type $e^c$ are states $C_i^{51}$. As mentioned before, it is natural to use this index $i$ as a family index. The only combination of quantum numbers which is now allowed for Higgses is \[19\], and therefore $H_1, H_2$ are states $C^{51,52}_{1,5}$.

Using again eqs. (A.1) and (A.2) we can obtain the soft terms for this scenario. In particular, the gaugino masses are

\[
M_3 = \sqrt{3} m_{3/2} \sin \theta , \\
M_2 = \sqrt{3} m_{3/2} \Theta_1 \cos \theta , \\
M_Y = \sqrt{3} m_{3/2} \alpha_Y (M_I) \left( \frac{1}{\alpha_2 (M_I)} \Theta_1 \cos \theta + \frac{2/3}{\alpha_3 (M_I)} \sin \theta \right),
\]

(47)

and the scalar masses are

\[
m_{Q_u}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( 1 - \Theta_1^2 \right) \cos^2 \theta \right] , \\
m_{e^c}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( 1 - \Theta_2^2 \right) \cos^2 \theta \right] , \\
m_{L_e}^2 = m_{H_2}^2 = m_{H_1}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta_3^2 \cos^2 \theta \right) \right] , \\
m_{u^c}^2 = m_{3/2}^2 \left[ 1 - 3 \Theta_1^2 \cos^2 \theta \right] , \\
m_{\mu^c}^2 = m_{3/2}^2 \left[ 1 - 3 \sin^2 \theta \right] , \\
m_{\tau^c}^2 = m_{3/2}^2 \left[ 1 - 3 \Theta_2^2 \cos^2 \theta \right] , \\
m_{\tau^c}^2 = m_{3/2}^2 \left[ 1 - 3 \Theta_2^2 \cos^2 \theta \right] ,
\]

(48)

where the following assignments have been used for the leptons. $e^c$ is a state $C_1^{51}$, $\mu^c$ is a state $C_2^{51}$, and finally $\tau^c$ is a state $C_3^{51}$. As in the previous two scenarios these soft terms are also generically non-universal.

Concerning Yukawa couplings, these are allowed for leptons in this scenario since $C_1^{51}C_2^{51}C_3^{51}$ exists. Assuming three copies of leptons $L_e$, one obtains

\[
Y_e = g_2 \tilde{Y} ,
\]

(49)

where $\tilde{Y}$ has been defined in (45) and $g_2$ is the gauge coupling associated to the $SU(2)_L$ in the D5$_1$-brane. The corresponding trilinear parameters are

\[
A_e = -\sqrt{3} m_{3/2} \Theta_1 \cos \theta \tilde{Y} .
\]

(50)

Notice however that the above assignment for leptons may be problematic concerning FCNC since generically $m_{e^c}^2 \neq m_{\mu^c}^2$. We can avoid that potential problem choosing $\tau^c$ as a state $C_1^{51}$ and e.g. $e^c$ as a state $C_2^{51}$, $\mu^c$ as a state $C_3^{51}$. Then we have to choose
$u_1^c = t^c$. Now, imposing $\Theta_2 = \Theta_3$, FCNC will not be present. This constraint will be used in Section 4 when discussing neutralino-proton cross sections in this scenario. Instead of the matrix structure \((49)\) there will be a new matrix with the non-vanishing entries in the second column.

On the other hand, Yukawa couplings for quarks of type $u$ are vanishing since couplings $C^{5_{15}z} C^{9_{51}c} C^9_i$ are forbidden. However, couplings $C^{5_{15}z} C^{9_{51}c} C^{9_{52}c}$ are allowed and therefore Yukawa couplings for quarks of type $d$ exist. The matrix structure is like in \((24)\). In any case, as discussed for other scenarios in subsection 3.1.1, is difficult to imagine how this scenario may give rise to the observed fermion mass hierarchies with masses of quarks of type $u$ arising from non-renormalizable terms.

### 3.2 Embedding all gauge groups within the same set of $D_p$-branes

As discussed in Subsection 2.2, the D-brane model constructed in [6], where all gauge groups are embedded in 3-branes, is very interesting. We will analyze here the soft terms and Yukawa couplings of the model.

Let us recall that $D_{p3,2,1}$-branes are in this scenario D3-branes, and $D_q$-branes are $D_{7_{1,2,3}}$-branes. The distribution of matter is like in Fig. 1 with all D3-branes on top of each other. Taking into account that under a $T$-duality transformation with respect to the three compact dimensions the 9-branes transform into 3-branes and the $5_i$-branes into $7_i$-branes, still the formulas for soft terms and Yukawas will be identical to the ones in the Appendix A, \((A.1-A.4)\), with the obvious replacements $9 \rightarrow 3$, $5_i \rightarrow 7_i$ everywhere. This implies the following gaugino masses:

$$M_3 = M_2 = M_Y = \sqrt{3} m_{3/2} \sin \theta ,$$  \hspace{1cm} (51)

and scalar masses:

$$m_{Q_i}^2 = m_{u_i}^2 = m_{H_2}^2 = m_{3/2}^2 \left( 1 - 3 \Theta_i^2 \cos^2 \theta \right) ,$$

$$m_{L_i}^2 = m_{e_i}^2 = m_{H_1}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \left( 1 - \Theta_i^2 \right) \cos^2 \theta \right] .$$ \hspace{1cm} (52)

Note that $Q, u, e, H_2$ are states $C_3^i$, whereas $d, c, L, H_1$ are states $C^{37,i}$. Thus due to the index $i = 1, 2, 3$ three families arise in this model in a natural way. For example we will take $u_1^c = u^c$, $u_2^c = c^c$, $u_3^c = t^c$, etc. In order to avoid FCNC we may impose $\Theta_1 = \Theta_2$. It is also worth noticing that universality can be obtained in the dilaton ($\sin^2 \theta = 1$) and overall modulus ($\Theta_{1,2,3} = 1/\sqrt{3}$) limits unlike the scenarios in Subsection 3.1.
Let us now analyze the Yukawa couplings of the model [6]. Couplings of the type \( C_3^1 C_3^2 C_3^3 \) are allowed. Assuming that the physical Higgs \( H_2 \) corresponds to \( C_3^1 \), the following couplings exist: \( g H_2 Q_t t^c \) and \( g H_2 Q_t e^c \), i.e.

\[
Y_u = g \bar{Y},
\]

where \( \bar{Y} \) is defined as

\[
\bar{Y} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}.
\]

The corresponding trilinear parameters are

\[
A_u = -\sqrt{3} m_{3/2} \sin \theta \bar{Y}.
\]

Let us remark that in the presence of discrete torsion \( g H_2 Q_t t^c \) may also be present [6]. On the other hand, couplings of the type \( C_3^i C_3^{37} C_3^{37} \) are also allowed. Assuming that the physical Higgs \( H_1 \) corresponds to \( C_3^{37} \), the coupling \( g Q_t b^c H_1 \) also exists, i.e.

\[
Y_b = g \bar{Y},
\]

where \( \bar{Y} \) is defined as

\[
\bar{Y} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

The trilinear parameters are

\[
A_b = -\sqrt{3} m_{3/2} \sin \theta \bar{Y}.
\]

More involved couplings with off-diagonal entries in the matrix for quarks of type \( d \) are possible in some circumstances [6]. Finally, renormalizable Yukawa couplings for leptons are not present since they are of the type \( C_3^{37} C_3^{37} C_3^{37} \) and these are not allowed.

4 Neutralino-nucleon cross sections in D-brane scenarios

Recently there has been some theoretical activity [43]-[49] analyzing the compatibility of regions in the parameter space of the MSSM with the sensitivity of current (DAMA [20], CDMS [21], HEIDELBERG-MOSCOW [22], HDMS prototype [23], UKDMC [24], CANFRANC [25]) and projected (GENIUS [26], DAMA 250 kg. [20], CDMS Soudan
dark matter detectors. In particular, DAMA and CDMS are sensitive to a neutralino-nucleon cross section $\sigma_{\tilde{\chi}_1^0-p}$ in the range of $10^{-6}$–$10^{-5}$ pb. Working in the supergravity framework for the MSSM with universal soft terms, it was pointed out in \cite{13, 14, 16, 19} that the large tan $\beta$ regime allows regions where the above mentioned range of $\sigma_{\tilde{\chi}_1^0-p}$ is reached. Besides, working with non-universal soft scalar masses $m_\alpha$, $\sigma_{\tilde{\chi}_1^0-p} \approx 10^{-6}$ pb was also found for small values of tan $\beta$, if $m_\alpha$ fulfil some special conditions \cite{13, 14, 16}. In particular, this was obtained for tan $\beta \gtrsim 25$ (tan $\beta \gtrsim 4$) working with universal (non-universal) soft terms in \cite{16}. The case of non-universal gaugino masses was also analyzed in \cite{17} with interesting results.

The above analyses were performed assuming universality (and non-universality) of the soft breaking terms at the unification scale, $M_{GUT} \approx 10^{16}$ GeV, as it is usually done in the MSSM literature. However, inspired by superstrings, where the unification scale may be smaller, it was analyzed in \cite{18} the sensitivity of the neutralino-nucleon cross section to the value of the initial scale for the running of the soft breaking terms. Working in the supergravity context with universal soft terms, the result was that the smaller the scale is, the larger the cross section becomes. In particular, by taking $10^{10-12}$ GeV rather than $10^{16}$ GeV for the initial scale, the cross section increases substantially $\sigma_{\tilde{\chi}_1^0-p} \approx 10^{-6}$–$10^{-5}$ pb.

The natural extension of this analysis is to carry it out with explicit D-brane constructions. As mentioned in Subsection 3.1.2, the first attempts to study dark matter within these constructions were carried out in \cite{39-41} for the unification scale as the initial scale and in \cite{42} for an intermediate scale as the initial scale in the case of dilaton dominance. Here we will take into account the crucial issue of the D-brane origin of the $U(1)_Y$ and its consequences on the matter distribution and soft terms in these scenarios. Thus we will analyze the D-brane scenarios introduced in Section 2, using their soft terms computed in Section 3. The fact that in these scenarios “intermediate” initial scales and/or non-universal soft terms are possible allows us to think that large cross sections, in the small tan $\beta$ regime, could be obtained in principle. Let us recall that this can be understood from the variation in the value of $\mu$, i.e. the Higgs mixing parameter which appears in the superpotential $W = \mu H_1 H_2$. Both, “intermediate” scales and non-universality, can produce a decrease in the value of $\mu$. When this occurs, the Higgsino components, $N_{13}$ and $N_{14}$, of the lightest neutralino

$$\tilde{\chi}_1^0 = N_{11} \tilde{B}^0 + N_{12} \tilde{W}^0 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0$$

(59)

increase and therefore the scattering channels through Higgs exchange become important. As a consequence the spin independent cross section also increases.

Before entering in details let us remark that we will work with the usual formulas
for the elastic scattering of relic LSPs on protons and neutrons that can be found in the literature \([19]\). In particular, we will follow the re-evaluation of the rates carried out in \([45]\), using their central values for the hadronic matrix elements.

Let us discuss now the parameter space of our D-brane scenarios. As usual in supersymmetric theories, the requirement of correct electroweak breaking leaves us (modulo the sign of \(\mu\)) with the following parameters. The soft breaking terms, scalar and gaugino masses and trilinear parameters, and \(\tan \beta\). Although formulas for the soft terms obtained in Section 3 leave us in principle with five parameters, \(m_{3/2}, \theta\) and \(\Theta_i\) with \(i = 1, 2, 3\), due to relation \(\sum_i |\Theta_i|^2 = 1\) only four of them are independent. In our analysis we vary the parameters \(\theta\) and \(\Theta_i\) in the whole allowed range, \(0 \leq \theta < 2\pi\), \(-1 \leq \Theta_i \leq 1\). For the gravitino mass we take \(m_{3/2} \leq 300\) GeV. Concerning Yukawa couplings we will fix their values imposing the correct fermion mass spectrum at low energies, i.e. we are assuming that Yukawa structures of D-brane scenarios give rise to those values.

We will analyze first the scenario of Subsection 2.1.1 with three different sets of \(Dp\)-branes, where the standard model gauge groups live. Since for the third and fourth assignments of quantum numbers in Table 1 \(M_I \lesssim 3 \times 10^8\) GeV, and therefore \(m_{3/2}\) is too low to be phenomenologically interesting, we will consider only the soft masses associated to the first and second assignments, i.e. the eight possible combinations given by eqs. (12-14), (20-23). The discussion of the corresponding trilinear parameters can be found below eq.(23).

In particular, the cross sections associated to combination (12), (13) and (20) are shown in Fig. 4 for \(M_I = 10^{12}\) GeV (i.e. the example studied in Fig. 2). The other possible combinations give rise to similar results. Fig 4 displays a scatter plot of \(\sigma_{\tilde{\chi}^0_1-p}\) as a function of the LSP mass \(m_{\tilde{\chi}^0_1}\) for a scanning of the parameter space discussed above. Two different values of \(\tan \beta\), 10 and 15, are shown. Although this and the other figures below have been obtained using negative values of \(\mu\), for positive values the corresponding figures are equal. Notice that the spectrum of supersymmetric particles is invariant under the transformation \(\mu, A, M \rightarrow -\mu, -A, -M\). Since the shift \(\theta \rightarrow \theta + \pi\) implies for the soft terms \(M \rightarrow -M\), \(A \rightarrow -A\) and \(m \rightarrow m\), a figure with positive \(\mu\) will be equal to a figure with negative \(\mu\) shifting \(\theta \rightarrow \theta + \pi\). We have included in the figures LEP and Tevatron bounds on supersymmetric masses. They forbid e.g. values of \(m_{3/2}\) smaller than 170 GeV. Although bounds coming from CLEO \(b \rightarrow s\gamma\) branching ratio measurements are not included in the figures, we have checked explicitly that their qualitative patterns are not modified when such a bounds are considered. It is worth noticing that for \(\tan \beta = 10\) there are regions of the parameter space consistent with DAMA limits. In fact, we have checked that \(\tan \beta > 5\) is enough to
Figure 4: Scatter plot of the neutralino-proton cross section as a function of the neutralino mass for the scenario with three different sets of Dp-branes. The string scale is $M_I = 10^{12}$ GeV. DAMA and CDMS current limits and projected GENIUS limits are shown.

Figure 5: The same as in Fig. 4 but for the string scale $M_I = 5 \times 10^{15}$ GeV.
obtain compatibility with DAMA. Since the larger $\tan \beta$ is, the larger the cross section becomes, for $\tan \beta = 15$ these regions increase.

As discussed below eq.(6), larger values of the string scale may be obtained with $\alpha_1(M_I) > 0.1$. In particular we show the example where $M_I = 5 \times 10^{15}$ GeV, corresponding to $\alpha_1(M_I) \approx 1$, in Fig. 5. Since the larger the scale is, the smaller the cross section becomes, now the cross sections decrease with respect to the previous case. In particular, $\tan \beta > 10$ is necessary in order to have compatibility with DAMA. On the other hand, as discussed above, in the MSSM with universal soft terms at the unification scale (which is close to the above $M_I$), $\tan \beta \gtrsim 20$ was needed to obtain compatibility. Clearly the non-universality of the soft terms in this string scenario plays a crucial role increasing the cross sections.

Let us finally recall that both figures are obtained taking $m_{3/2} \leq 300$ GeV, which corresponds to squark masses $m_{\tilde{q}} \lesssim 500$ GeV at low energies. We have checked that larger values of $m_{3/2}$ produce cross sections below DAMA limits. In particular, the right hand side and bottom of the figures will also be filled with points. Cross sections below projected GENIUS limits will appear in both figures. On the other hand, it is worth mentioning that the few isolated points in the plots with, in general, very large values of the cross section correspond to values of the lightest stop mass extremely close to the mass of the LSP, in particular $(m_{\tilde{t}} - m_{\tilde{\chi}_1^0})/m_{\tilde{t}} < 0.01$. 

Figure 6: The same as in Fig. 4 but for the scenario with $Dp_1 = Dp_3$. The string scale is $M_I = 8 \times 10^9$ GeV.
Figure 7: The same as in Fig. 4 but for the scenario without Dp1-brane. The string scale is $M_I = 10^{16}$ GeV.

Although the scenario of Subsection 2.1.2 where Dp1 = Dp3 has soft terms different from the previous scenario (see Subsection 3.1.2), the qualitative results concerning neutralino-proton cross sections will be similar. This is shown in Fig. 6 for the example discussed below eq. (4) where $M_I = 8 \times 10^9$ GeV. We use the soft terms given by (36), (37) and (40) with the constraint $\Theta_2 = \Theta_3$ in order to avoid FCNC, as discussed in Subsection 3.1.2. Thus, apart from $\tan \beta$, only three independent parameters are left: $m_{3/2}$, $\theta$ and one of the $\Theta_i$'s. Other combinations of soft terms do not modify our conclusions. Note that there are regions of the parameter space consistent with DAMA limits, as in Fig. 4. In this scenario $\tan \beta > 5$ is also enough to obtain such a consistency.

The scenario without Dp1-brane studied in Subsection 2.1.3 is shown in Fig. 7. We take the string scale $M_I = 10^{16}$ GeV and the soft terms given in Subsection 3.1.3, with the constraint $\Theta_2 = \Theta_3$ to avoid FCNC as discussed there. Since the string scale is large the results are qualitatively similar to the ones in Fig. 5.

Let us finally analyze the scenario of Subsection 2.2 where all gauge groups are embedded within the same set of D3-branes. In this scenario the soft terms are given by (51), (52), (55) and (58). Since we will take $\Theta_1 = \Theta_2$ in order to avoid FCNC, there will be in our analysis only three independent parameters: $m_{3/2}$, $\theta$ and one of the $\Theta_i$'s, say $\Theta_3$. The cross sections are then shown in Fig. 8 for $M_I = 10^{10}$ GeV, i.e.
Figure 8: The same as in Fig. 4 but for the scenario with all gauge groups embedded within the same set of D3-branes, in such a way that gauge couplings unify at $M_f = 10^{10}$ GeV.

the example studied in Fig. 3. We consider two cases with $\tan \beta = 10$ and $\tan \beta = 25$. Now $\tan \beta > 25$ is necessary to obtain regions consistent with DAMA limits. This is to be compared with the previous scenarios with intermediate string scales where regions consistent with DAMA were obtained for $\tan \beta > 5$. As discussed in the context of the MSSM in [18] this is due to the different values of $\alpha$’s at the string scale in both types of scenarios. Unlike the previous ones here gauge couplings are unified.

Let us recall that this scenario is the only one where universality can be obtained. This is the case for the dilaton limit ($\sin^2 \theta = 1$) and the overall modulus limit ($\Theta_{1,2,3} = 1/\sqrt{3}$). The two central curves in the figures correspond precisely to this situation. Notice that the deviation from universality may increase or decrease the cross sections, as shown in the figures, depending on the values of the parameters $\theta$ and $\Theta_3$ chosen.

Before concluding let us discuss very briefly the effect of relic neutralino density bounds on cross sections. The most robust evidence for the existence of dark matter comes from relatively small scales. Lower limits inferred from the flat rotation curves of spiral galaxies [19, 50] are $\Omega_{\text{halo}} \gtrsim 10 \Omega_{\text{vis}}$ or $\Omega_{\text{halo}} h^2 \gtrsim 0.01 - 0.05$, where $h$ is the reduced Hubble constant. On the opposite side, observations at large scales, $(6 - 20) h^{-1}$ Mpc, have provided estimates of $\Omega_{\text{CDM}} h^2 \approx 0.1 - 0.6$ [51], but values as low as $\Omega_{\text{CDM}} h^2 \approx 0.02$ have also been quoted [52]. Taking up-to-date limits on $h$, the baryon...
density from nucleosynthesis and overall matter-balance analysis one is able to obtain a favoured range, $0.01 \lesssim \Omega_{CDM} h^2 \lesssim 0.3$ (at $\sim 2\sigma$ CL) \cite{53, 54}. Note that conservative lower limits in the small and large scales are of the same order of magnitude.

In this work the expected neutralino cosmological relic density has been computed according to well known techniques (see \cite{19}). Although bounds coming from them are not included in the above figures we have checked explicitly that their qualitative patterns are not modified for most of the regions in figs. 4-7 when such a bounds are considered. However the analysis of regions of the parameter space consistent with DAMA limits is more delicate. From the general behaviour $\Omega_{\chi} h^2 \propto 1/\langle \sigma_{ann} \rangle$, where $\sigma_{ann}$ is the cross section for annihilation of neutralinos, it is expected that such high neutralino-proton cross sections as those presented above will then correspond to relatively low relic neutralino densities. We have seen that this is in fact the case. On these grounds\cite{5}, most of those points are at the border of the range of validity or below. On the other hand, it is worth remarking that we are clearly below of the range of validity for the whole regions in the scenario corresponding to Fig. 8.

5 Conclusions

In this paper we have analyzed different phenomenological aspects of D-brane scenarios. First, assuming that the $SU(3)_c$, $SU(2)_L$ and $U(1)\gamma$ groups of the standard model come from different sets of Dp-branes, intermediate values for the string scale $M_I \approx 10^{10-12}$ GeV are obtained in a natural way. The reason is the following. Due to the D-brane origin of the $U(1)\gamma$ gauge group, the hypercharge is a linear combination of different $U(1)$ charges. Thus, in order to reproduce the low-energy data, i.e. the values of the gauge couplings deduced from CERN $e^+e^-$ collider LEP experiments, intermediate values for $M_I$ are necessary. On the other hand, there is also the possibility that the gauge groups of the standard model come from the same set of Dp-branes. In fact explicit models with this property can be found in the literature. Again the $U(1)\gamma$ gauge group has a D-brane origin and therefore the normalization factor of the hypercharge is not as the usual one in GUTs. The presence of additional doublets or triplets allows to obtain intermediate values for the string scale.

Second, taking into account the matter assignment to the different Dp-branes of the above scenarios, we have derived Yukawa couplings and soft supersymmetry-breaking terms. The analysis of the soft terms has been carried out under the assumption\footnote{Of course there is always the possibility that not all the dark matter in our Galaxy are neutralinos. This would modify the analysis since e.g. $\Omega_{\chi} < \Omega_{CDM}$.}
of dilaton/moduli supersymmetry breaking, and they turn out to be generically non-universal.

Finally, we have computed the neutralino-nucleon cross section of these D-brane scenarios. This computation is extremely interesting since the lightest neutralino is a natural candidate for dark matter in supersymmetric theories. Using the previously obtained soft terms, and taking into account that the string scale $M_I$ is the initial scale for their running, we have found regions in the parameter space of the D-brane scenarios with cross sections in the range of $10^{-6} - 10^{-5}$ pb. For instance, this can be obtained for $\tan \beta > 5$. The above mentioned range is precisely the one where current dark matter detectors, as e.g. DAMA and CDMS, are sensitive.

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## Appendix

We summarize in this Appendix the formulas for the soft terms [33] and Yukawa couplings [35] in D-brane constructions, using one set of 9-branes and three sets of 5-branes, $5_i$. Assuming vanishing cosmological constant and neglecting phases, the gaugino masses are given by

$$
M_9 = \sqrt{3} m_{3/2} \sin \theta , \\
M_{5_i} = \sqrt{3} m_{3/2} \Theta_i \cos \theta ,
$$

(A.1)

where $M_9$ ($M_{5_i}$) are the masses of gauginos coming from open strings starting and ending on 9 ($5_i$)-branes. The scalar masses are given by

$$
m^2_{C^9_{i}} = m^2_{C^9_{s}} = m^2_{3/2} \left( 1 - 3 \Theta^2_i \cos^2 \theta \right), \\
m^2_{C^5_{i}} = m^2_{5/2} \left( 1 - 3 \sin^2 \theta \right), \\
m^2_{C^{os5}_{i}} = m^2_{3/2} \left[ 1 - \frac{3}{2} \left( 1 - \Theta^2_i \right) \cos^2 \theta \right], \\
m^2_{C^{5s5}_{i}} = m^2_{3/2} \left[ 1 - \frac{3}{2} \left( \sin^2 \theta + \Theta^2_k \cos^2 \theta \right) \right],
$$

(A.2)
where $C^9_i$ denote matter fields coming from open strings starting and ending on 9-branes (the index $i$ which labels the three complex compact dimensions is very useful in order to generate three families as we discuss in the text), $C^9_i$ and $C^9_j$ with $i \neq j$ are analogous to the previous ones but with 9-branes replaced by 5-$i$-branes, $C^{95}_i$ denote matter fields coming from open strings starting (ending) on a 9-brane and ending (starting) on a 5-$i$-brane, $C^{95}_{ij}$ with $i \neq j$ come from open strings starting in one type of 5-$i$-brane and ending on a different type of 5-$j$-brane. Finally the results for the trilinear parameters are

\[
A_{C^9_i C^9_j C^9_k} = A_{C^9_i C^{95}_i C^{95}_i} = -\sqrt{3} m_{3/2} \sin \theta , \\
A_{C^{95}_i C^{95}_i C^{95}_i} = A_{C^{95}_i C^{95}_i C^{95}_k} = -\sqrt{3} m_{3/2} \Theta_i \cos \theta , \\
A_{C^{95}_i C^{95}_i C^{95}_l} = \frac{\sqrt{3}}{2} m_{3/2} \left[ \sin \theta - \left( \sum_i \Theta_i \right) \cos \theta \right] , \\
A_{C^{95}_i C^{95}_j C^{95}_k} = \frac{\sqrt{3}}{2} m_{3/2} \left[ \cos \theta (\Theta_k - \Theta_i - \Theta_j) - \sin \theta \right] , \tag{A.3}
\]

with $i, j, k = 1, 2, 3$ and $i \neq j \neq k \neq i$ in the above equations. The angle $\theta$ and the $\Theta_i$ with $\sum_i |\Theta_i|^2 = 1$, just parameterize the direction of the goldstino in the $S, T_i$ field space.

On the other hand, the renormalizable Yukawa couplings which are allowed are given by

\[
\mathcal{L}_Y = g_9 \left( C^9_1 C^9_2 C^9_3 + C^{95}_1 C^{95}_2 C^{95}_3 + \sum_{i=1}^3 C^9_i C^{95_i} C^{95_i} \right) + \sum_{i,j,k=1}^3 g_{5i} \left( C^{95}_1 C^{55}_2 C^{55}_3 + C^{95}_1 C^{95}_2 C^{95}_3 \right) + d_{ijk} C^{95}_i C^{95}_j C^{95}_k + \frac{1}{2} d_{ijk} C^{95}_i C^{95}_j C^{95}_k , \tag{A.4}
\]

with $d_{ijk} = 1$ if $i \neq j \neq k \neq i$, otherwise $d_{ijk} = 0$.

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