Distinguishing anomaly mediation from gauge mediation with a Wino NLSP

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Abstract

A striking consequence of supersymmetry breaking communicated purely via the superconformal anomaly is that the gaugino masses are proportional to the gauge \(\beta\)-functions. This result, however, is not unique to anomaly mediation. We present examples of “generalized” gauge-mediated models with messengers in standard model representations that give nearly identical predictions for the gaugino masses, but positive (mass)\(^2\) for all sleptons. There are remarkable similarities between an anomaly-mediated model with a small additional universal mass added to all scalars and the gauge-mediated models with a long-lived Wino next-to-lightest supersymmetric particle (NLSP), leading to only a small set of observables that provide robust distinguishing criteria. These include ratios of the heaviest to lightest selectrons, smuons, and stops. The sign of the gluino soft mass an unambiguous distinction, but requires measuring a difficult class of one-loop radiative corrections to sparticle interactions. A high precision measurement of the Higgs-\(b\bar{b}\) coupling is probably the most promising interaction from which this sign might be extracted.

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1 Introduction

Supersymmetry breaking communicated dominantly via the superconformal anomaly is a very interesting new approach to weak scale supersymmetry [1, 2]. In the absence of singlets, anomaly mediation provides a one-loop contribution to the gaugino masses, a one-loop contribution to the trilinear scalar couplings, and a two-loop contribution to the scalar (mass)$^2$ of the minimal supersymmetric standard model (MSSM). These contributions can be understood as arising from a super-Weyl invariant action once supersymmetry breaking is explicitly included in the superconformal compensator in supergravity, and are thus precisely proportional to the gravitino mass. If there are no direct couplings between the MSSM sector and the supersymmetry breaking sector, then anomaly mediation provides the dominant contribution to all the MSSM fields. This is a natural expectation if the MSSM fields and the supersymmetry breaking sector fields are physically separated on different branes [1].

There are several advantages to the anomaly mediation approach. The supersymmetric flavor problem is ameliorated, since the potentially dangerous contributions to off-diagonal squark (mass)$^2$ are suppressed. The form of the expressions for the masses induced via the superconformal anomaly are exact to all orders [2, 3], and determined by infrared physics, namely the low energy $\beta$-functions. Finally, the ratio of gaugino masses to scalar masses is order one (i.e. not one-loop suppressed).

Gauge mediation [4, 5] shares several features with anomaly mediation, namely the supersymmetric flavor problem is also ameliorated, and masses are induced at one-loop for gauginos and (mass)$^2$ are induced at two-loops for scalars. Furthermore, gauge-mediated gaugino masses are (at leading order) proportional to the gauge (coupling)$^2$, identical to anomaly mediation. The key phenomenological difference is that gauge-mediated soft masses depend on the content of the messenger sector, whereas anomaly-mediated soft masses depend on the low energy $\beta$-function coefficients. More generally, supersymmetry breaking masses are determined by ultraviolet physics in gauge mediation, and by infrared physics in pure anomaly mediation. In general the phenomenology is expected to be quite different, but there is no a priori reason why these two fundamentally different origins of supersymmetry breaking masses could not be “accidentally” rather similar. We show that a simple choice of messenger matter using standard model (SM) representations gives the same numerical result for the size of gaugino masses at leading order. Other messenger sectors that give similar results are also briefly mentioned. At next-to-leading order there are two-loop contributions to gaugino masses that do not respect the leading-order equivalence. However, there is still a restricted range of gauge-mediated parameters that give gaugino masses that are nearly equivalent to the next-to-leading order predictions of anomaly mediation.

The scalar spectrum of a gauge-mediated model is well defined once the messenger sector is fixed. This suggests that a gauge-mediated model could be falsified simply by measuring

\footnote{“Pure” meaning an anomaly-mediated MSSM without any additions or modifications, and thus without a solution to the negative slepton (mass)$^2$ problem.}
several slepton and squark masses in addition to the gaugino masses, and then determining if the spectrum is self-consistent. On closer inspection, however, the situation is not quite so trivial. The first and second generation squark masses are nearly identical to the pure anomaly-mediated result in “generalized messenger” gauge-mediated models. (Third generation squark masses are somewhat more distinct, but complicated by left-right mixing.) Furthermore, pure anomaly mediation predicts slepton (mass)\(^2\) that are negative, requiring one of several proposed remedies\([1, 2, 6, 7, 8, 9, 10, 11]\). Each “solution” to the negative slepton (mass)\(^2\) problem must at least provide additional contributions to the slepton masses, and can have varying effects on the remainder of the mass spectrum. In this paper, we employ the simple phenomenological solution that merely adds a universal mass term to all of the scalar masses, leaving the gaugino masses unchanged\([7, 10]\). A more drastic alternative that, for example, shifts the gaugino masses from their anomaly-mediated values would be trivially distinct from the gauge-mediated models discussed here, and thus need not be considered further.

There are at least two other general distinctions between gauge mediation and anomaly mediation: The Wino NLSP is not stable in gauge mediation, and the sign of the gluino soft mass is opposite (negative) in anomaly mediation. The extent to which these distinctions are phenomenologically useful criteria is also discussed.

Our main purpose in this paper is not to advocate that the gauge-mediated Wino NLSP models are more (or less) favored than an anomaly-mediated model with an appropriate negative slepton (mass)\(^2\) solution. Instead, we are interested in determining the ways to experimentally verify (or falsify) these scenarios. Our starting point is that a gaugino mass spectrum that is approximately proportional to gauge \(\beta\)-functions is not sufficient to identify anomaly mediation as the source of supersymmetry breaking. Instead, several other criteria must be used to separate the gauge-mediated models discussed here from anomaly mediation. In this way we attempt to gain a more robust understanding of the signals of both anomaly mediation and gauge mediation.

## 2 Constructing a model

In the following, we consider two complete different supersymmetric models, each with fundamentally different means by which supersymmetry breaking is communicated to the MSSM. First, consider a model with no (hidden sector) singlets such that anomaly mediation (AM) provides the dominant contribution to the gaugino masses. In this case, the expressions for the gaugino masses are\([1, 4]\)

\[
M_{\alpha}^{\text{AM}} = \frac{\beta_{\alpha}}{y_{\alpha}} m_{3/2}^{\text{AM}} \\
\simeq B_a^{(1)} \frac{g_2}{16\pi^2} m_{3/2}^{\text{AM}},
\]

(1)
where $m_{3/2}^{\text{AM}}$ is the gravitino mass, $g_a$ is the gauge coupling, and $B_a^{(1)} = (33/5, 1, -3)$ correspond to the one-loop $\beta$-function coefficients for $a = [U(1)_Y, \text{SU}(2)_L, \text{SU}(3)_c]$. For the purposes of this section, only effects to leading order (i.e. to one-loop for gaugino masses) will be discussed. The scalar masses in this model are generated both by anomaly mediation as well as an unspecified source that provides an additional universal mass sufficient to cure the slepton mass problem. The general form is

\[ \tilde{m}_i^2 = -\frac{1}{4} \left[ \beta_a \frac{\partial \gamma_i}{\partial g_a} + \beta_Y \frac{\partial \gamma_i}{\partial Y} \right] \left( m_{3/2}^{\text{AM}} \right)^2 + m_0^2 \]  

(2)

where $\gamma_i$ is the anomalous dimension of the $i$ chiral superfield. A sum over gauge couplings and Yukawa couplings is implicit. Trilinear scalar couplings are also generated at one-loop, and expressions can be found in [1, 2].

These expressions for the soft masses induced by anomaly mediation are in general quite different from minimal messenger gauge mediation models and ordinary supergravity models. There is, however, the apparent similarity that the anomaly mediation model generates gaugino masses at one-loop and scalar (mass)² at two-loops in a manner analogous to gauge mediation. In addition, the one-loop expression for the gaugino mass depends in both models on $g^2/16\pi^2$, although the coefficients are different and the supersymmetry breaking mass is distinct.

Instead of restricting to messengers in complete GUT representations, consider generalizing the messenger sector of a gauge mediation model to a sum over arbitrary vector-like representations. Supersymmetry breaking is present with a non-zero vev for the auxiliary components of the messenger fields, and after integrating them out, they induce the following gaugino masses through gauge mediation (GM)

\[ M_{a}^{\text{GM}} = \frac{g_a^2}{16\pi^2} \sum_i S_a(i) g(F_i/M_i^2) F_i / M_i. \]  

(3)

The sum is over all messengers labeled by $i$, $S_a(i)$ is the Dynkin index for the $a$ gauge group, and $F_i$ and $M_i$ are the $F$-terms and fermion masses of the messengers. The function $g(x)$ is

\[ g(x) = \frac{1}{x^2} \left[ (1 + x) \log(1 + x) + (1 - x) \log(1 - x) \right], \]  

(4)

and is equal to about 1, 1.05, 1.22, 1.39 for $x = 0, 0.5, 0.9, 1$. In the approximation $F_i = F \ll M_i^2 = M^2$, meaning that the messengers have approximately the same supersymmetric and supersymmetry breaking vevs, the sum is only over the Dynkin indices of the messengers $n_a \equiv \sum_i S_a(i)$, and then Eq. (3) simplifies to

\[ M_{a}^{\text{GM}} \simeq n_a \frac{g_a^2}{16\pi^2} \frac{F}{M}. \]  

(5)

\(^2\)To avoid confusion, $m_{3/2}^{\text{AM}}(m_{3/2}^{\text{GM}})$ always corresponds to the value of the gravitino mass in the anomaly mediation (gauge mediation) model.

\(^3\)g₁ is always taken to be in the GUT normalization, $g_1 = \sqrt{5/3}g'$.

\(^4\)See also Refs. [13, 14] for examples of models with gaugino masses not in the canonical proportions.
Note the striking formal similarity between the expression for the gaugino mass in the anomaly mediation model, Eq. (1), and the expression for the gaugino mass in the gauge mediation model, Eq. (5). Both are characterized by a discrete quantity multiplied by $g^2/16\pi^2$ multiplied by a supersymmetry breaking mass. It is precisely this similarity that we exploit in the following to construct a gauge mediation model with gaugino masses that are equivalent to anomaly mediation, using an appropriate relationship between the supersymmetry breaking masses. Note that the gaugino mass and gauge coupling in Eq. (1) are evaluated at the weak scale, whereas the gaugino mass and gauge coupling in Eq. (5) are evaluated at the messenger scale. The latter expression does not, however, acquire a renormalization group correction to the order we are working since $M/g^2$ is one-loop renormalization group invariant.

2.1 Generalized messenger models

Utilizing the above generalization of the messenger sector, we now proceed to construct a gauge-mediated model with gaugino masses proportional to the one-loop gauge \( \beta \)-functions. Require 

\[ n_a = |B^{(1)}_a| \tag{6} \]

and that the supersymmetry breaking mass parameters coincide, \( F/M = m_{3/2}^{\text{AM}} \). The predictions for the gaugino masses are identical to those of the anomaly mediation model, up to the sign of the gluino soft mass.\(^5\) We emphasize that this is an accidental equivalence between two completely separate origins of supersymmetry breaking with \( m_{3/2}^{\text{GM}} \ll m_{3/2}^{\text{AM}} \). The equivalence does not apply to the sign of gluino soft mass, but this difference has only a limited phenomenological impact on the spectra. The sign does affect the gaugino soft mass predictions at next-to-leading (two-loop) order, and we discuss this in more detail in Sec. 3. In principle, measuring this sign would be an unambiguous way of distinguishing these models, but experimentally this is rather difficult, as we explain in Sec. 3.2. Suffice to say there is no easily measurable difference between a model with a positive gluino soft mass, such as gauge mediation, and a model with a negative gluino soft mass, such as anomaly mediation.

The ratio \( F/M \) that sets the overall scale of the gauge-mediated soft masses could be different from \( m_{3/2}^{\text{AM}} \) while the gaugino masses remain equivalent. At first glance, only the proportionality \( (n_1 : n_2 : n_3) = (B^{(1)}_1 : B^{(1)}_2 : |B^{(1)}_3|) \) is relevant. However, restricting to a set of messengers that preserves the perturbativity (but not necessarily the equivalence) of the gauge couplings up to the purported unification scale \( \sim 10^{16} \text{ GeV} \) implies that \( F/M \) cannot be an integer multiple of \( m_{3/2}^{\text{AM}} \) (other than unity). Possible fractional values (such as \( \frac{1}{2} m_{3/2}^{\text{AM}} \) or \( \frac{3}{2} m_{3/2}^{\text{AM}} \)) could only occur with messengers in non-vectorlike multiplets, that can be justifiably ignored due to the difficulty of giving such fields a large supersymmetric mass. Under these constraints, Eq. (3) can be expanded

\[ \frac{1}{7} (n_Q + 8n_u + 2n_d + 3n_L + 6n_e) = \frac{33}{7} \]

\(^5\)If a higher rank group associated with the messengers broke to SU(3)$_c$, it is possible that the effective \( n_3 \) could be negative due to gauge messengers [15]. However, this also causes the (mass)$^2$ for at least the first and second generation squarks to be negative, and therefore does not appear to be viable.
\begin{align}
3n_Q + n_L &= 1 \\
2n_Q + n_u + n_d &= 3,
\end{align}

where \( n_X \) corresponds to the number of \( X + \bar{X} \) pairs of vectorlike messenger multiplets in the SM representations \( (Q, u, d, L, e) \). The set of solutions are characterized by

\begin{align}
n_Q &= 0 & n_L &= 1 \\
n_u + n_d &= 3 \\
n_u + n_e &= 4,
\end{align}

meaning \( (n_Q, n_u, n_d, n_L, n_e) \) can only be one of \( (0, 0, 3, 1, 4), (0, 1, 2, 1, 3), (0, 2, 1, 1, 2), \) or \( (0, 3, 0, 1, 1) \). These sets of multiplets are degenerate at leading order, but give slightly differing results at next-to-leading order (e.g. two-loop expressions for gaugino masses, and two-loop contributions to the gauge \( \beta \)-functions above the messenger scale). As an aside, we note that the third set of multiplets corresponds to one \( 5 + \bar{5} \) and two nearly complete \( 10 + \bar{10} \)'s, but it is conspicuously missing the two pairs of \( Q + \bar{Q} \)'s. This may provide a useful starting point for a dynamical determination of the above sets of messenger fields, although this is beyond the scope of this paper. In any case, determining the sets of multiplets that realize the relation Eq. \( (6) \), and in particular that only SM representations are needed, is one of the important results of this paper.

### 2.2 Multi-singlet models

Another approach to constructing a gauge mediation model is to expand the messenger sector such that there are several singlets (see also Ref. \([14]\)) with either different supersymmetric masses, or different \( F \)-terms, or both. This approach has the advantage that matter in complete \( SU(5) \) representations is sufficient, thus naively preserving one-loop gauge-coupling unification. However, the supersymmetric mass scales or the \( F \)-terms (or both) must differ among the \( SU(3) \times SU(2) \times U(1) \) components fields, breaking the \( SU(5) \) ansatz.

There are two potential benefits of modifying the supersymmetric mass scale of \( SU(3) \times SU(2) \times U(1) \) component messenger fields. The first is a threshold effect, Eq. \( (4) \), whose size depends on \( F/M^2 \). A given gaugino mass could be increased (relative to \( M \rightarrow \) large, with fixed \( F \)) by at most about 35%, if \( M \) is rather close to \( \sqrt{F} \). In general it is hard to imagine how this could arise dynamically, although some ideas have been discussed in Ref. \([9]\) (in an anomaly mediation context). The second potential benefit of shifting the supersymmetric mass scale of messengers exploits the running gauge coupling. The gaugino mass induced at the messenger scale is proportional to the gauge coupling squared evaluated at the messenger scale \( g^2(M) \), and so it is possible to shift a given gaugino mass by a factor \( g^2(M_{\text{new}})/g^2(M_{\text{old}}) \). This effect, however, is really a next-to-leading order correction. In practice, only \( g_3^2 \) evolves significantly (by at most about 40%) between the lowest and highest messenger scales (between about \( 10^5 \) to \( 10^9 \) GeV) that are consistent with gauge mediation giving the dominant contribution to soft
masses. One difficulty with both of these approaches is that several SM component fields of (say) complete SU(5) reps are charged under more than one gauge group of the SM, so that modifying the scale of a given pair of messengers affects several gaugino masses simultaneously. We conclude that effects resulting from shifting the supersymmetric mass scale of messengers, by themselves, cannot reproduce any of the large ratios \[ B^{(1)}_1 / |B^{(1)}_3| = 11/5 \text{ or } |B^{(1)}_3| / B^{(1)}_2 = 3. \]

If several singlets communicate supersymmetry breaking from the dynamical supersymmetry breaking (DSB) sector to the messengers, it is not implausible that they could couple differently to SU(3) \( \times \) SU(2) \( \times \) U(1) component messenger fields. Then, given a small hierarchy of \( F \)-terms, it is trivial to construct a gauge-mediated model that has gaugino masses in a proportion indistinguishable from anomaly mediation. For example, take the components of a 10 + \( \overline{10} \) coupled to two SM singlets \( X_1 \) and \( X_2 \) using the messenger superpotential

\[ W = X_1 \bar{Q}Q + X_2 \bar{u}u + X_2 \bar{e}e. \] (9)

With \( F_{X_1}/(3M_{X_1}) = 7F_{X_2}/(3M_{X_2}) = m^{AM}_{3/2} \), this model generates gaugino masses in exactly the same proportion as the one-loop \( \beta \)-function coefficients.\footnote{It is also possible that \( F \)-terms of different singlets could have opposite signs and be arranged such that the gaugino masses are in the same proportion as anomaly mediation including the sign of \( M_3 \). However, we are not aware of any DSB or messenger model that could give this result.}

### 2.3 Properties of the models

The two classes of models discussed above, namely the generalized messenger models and the multi-singlet model, generate the same result for the gaugino masses, but give somewhat different results for the scalar (mass)\(^2\):

\[
\begin{align*}
m^2_i &= 2 \frac{F^2}{M^2} \sum_a C_a(i) \frac{g^4_a}{(16\pi^2)^2} n_a \quad \text{(generalized messengers)} \quad (10) \\
m^2_i &= 2 \frac{F^2}{M^2} \sum_a C_a(i) \frac{g^4_a}{(16\pi^2)^2} \left[ m_a(X_1)\frac{F^2_{X_1}}{F^2} + m_a(X_2)\frac{F^2_{X_2}}{F^2} \right] \quad \text{(multi-singlet)} \quad (11)
\end{align*}
\]

The \( m \) factors are \( m_a(X_1) = (1/5, 3, 2) \) and \( m_a(X_2) = (14/5, 0, 1) \) respecting \( m_a(X_1) + m_a(X_2) = (3, 3, 3) \), and we have taken the mass scale of the messengers \( M \) to be the same for both models. Notice that each gauge group is weighted by \( n_a \) in the generalized messenger models, whereas each gauge group is effectively weighted by \( n^2_a \) in the multi-singlet model since each scalar (mass)\(^2\) in the latter is proportional to \( F^2 \). Thus, holding the gaugino mass spectrum fixed, these two classes of models give different predictions for the scalar masses. For example, squark masses tend to be about 15\% lighter in the generalized messenger models. This illustrates that without specifying a particular messenger model, there is no unique prescription to translate a gaugino mass spectrum into a scalar mass spectrum.

Gauge coupling unification does not occur at \( 10^{16} \) GeV for the generalized messenger models; instead, \( g_1 \) is typically much larger than \( g_3 \), which is somewhat larger than \( g_2 \). We verified that \( g_1 \)
remains perturbative near $10^{16}$ GeV when calculated to two-loop order, as long as the messenger scale is larger than about $10^6$ GeV. Ordinarily the unification scale is defined by $g_1 \simeq g_2$, which in the generalized messenger models occurs at an intermediate scale $\sim 10^{10} \rightarrow 10^{12}$ GeV. Conversely, gauge coupling unification can occur in the multi-singlet model as long as the shifts in the $\beta$-functions due to the additional messenger fields are nearly independent of the gauge group (such as fields filling complete SU(5) reps).

A fascinating property of the generalized messenger models is that the predictions for the first and second generation squark masses are nearly identical to pure anomaly mediation. This is evident from Eq. (10) at the messenger scale, where the dominant contribution proportional to $g_3^4$ is the same as anomaly mediation. The prediction is also very well preserved under renormalization group (RG) evolution between the messenger scale and the weak scale, since the first and second generation scalar mass relations induced by gauge mediation are very close to the renormalization group invariant mass relations of anomaly mediation. This coincidence occurs precisely because $n_3$ is opposite in sign to $B_3^{(1)}$. Hence, this does not occur for slepton masses. Due to this interesting property, we concentrate most of the remaining discussion of gauge-mediated Wino NLSP models on the generalized messenger models.

The mostly Wino lightest neutralino is the NLSP and typically decays into a gravitino and a photon, although heavier NLSPs can also have a significant branching fraction into a gravitino and a $Z$ [17, 18, 14]. Depending on whether the fundamental supersymmetry breaking scale is smaller or larger than about a few hundred TeV, the Wino NLSP could decay either inside or outside a typical collider detector. If the decay NLSP $\rightarrow$ gravitino + photon were to occur well within a detector, it would be clearly evident with a hard photon emitted for every NLSP produced either directly or indirectly. This is a very robust signal in gauge mediation [17] and completely different from anomaly mediation. However, if the decay length is significantly longer than a typical collider detector [7] the long-lived Wino NLSP is indistinguishable from a stable Wino LSP. We concentrate on this scenario for the remainder of the paper.

In supersymmetric models with a Wino (N)LSP, the mass splitting between the lightest chargino $\tilde{C}_1^\pm$ and the lightest neutralino $\tilde{N}_1$ is very small since both fields are nearly pure Wino-like states. Expressions for the mass splitting at tree-level [14, 7] and at one-loop [19, 14] have been calculated, with the intriguing possibility of a macroscopic $\tilde{C}_1^\pm \rightarrow \tilde{N}_1 \pi^\pm$ decay length signal that has been studied in detail in Refs. [20, 21]. This is also an interesting signal of the gauge-mediated Wino NLSP models discussed here. However, it is not a useful distinction between gauge mediation and anomaly mediation because the decay lengths of the respective Wino (N)LSPs are comparable, as discussed below.

A complete set of parameters characterizing these models must also include $\tan \beta$ and $\mu$. Demanding the proper electroweak symmetry breaking vacuum with the correct value of $M_Z$ determines $\mu^2$ as a function of the Higgs soft masses and $\tan \beta$ (at tree-level), leaving only the sign of $\mu$ unknown. Requiring that $m_{\tilde{\tau}_1}$ be greater than the current LEP bound (of about 90

\footnote{The decay length scales as the fourth power of the fundamental supersymmetry breaking scale, and therefore could be anywhere from microns to the distance from the Earth to the Sun.}
Figure 1: Maximum tan $\beta$ as a function of $F/M$ in generalized messenger models by requiring that $m_{\tilde{\tau}_1}$ is larger than the current LEP bound. The bottom, middle, and top lines correspond to $M = 10^5$, $10^7$, and $10^9$ TeV. The parameter space above and to the left of the lines is excluded. GeV) implies that there is an upper bound on tan $\beta$ as a function of the messenger parameters, shown in Fig. 1. Above about $F/M = 50$ TeV the limit disappears because the lightest stau mass is always larger than the LEP bound.

3 Distinctions between anomaly mediation and gauge mediation

There are three central phenomenological differences between anomaly mediation and gauge mediation: the scalar spectrum is in general different, the sign of the gluino soft mass is different, and the Wino NLSP of gauge mediation is unstable. However, none of these differences are necessarily trivial to establish in a collider experiment, as discussed below.

3.1 Scalar spectrum

Once the overall supersymmetry breaking scale $F/M$ is established, the scalar spectrum of the gauge-mediated models is fixed, up to a logarithmic sensitivity to the messenger scale. Unlike anomaly mediation, there is no reason to suggest there should be additional contributions to the matter scalars (not including the Higgs scalars) if gauge mediation via SM interactions provides the dominant source of supersymmetry breaking. Indeed, the elegant resolution to the
supersymmetric flavor problem via gauge mediation would, in general, be lost with additional contributions (unless they were flavor-independent, aligned, or very heavy). Thus, the simplest way to exclude the gauge-mediated models discussed here is to experimentally verify that the scalar mass spectrum does not follow the gauge-mediated scalar spectrum. For example, the charged selectron masses fall in a relatively narrow mass range relative to $m_1$, shown in Fig. 2. The size of the slepton masses in anomaly mediation, by contrast, are dependent on $m_0$, and therefore a priori completely unrelated to $M_1$.

In anomaly mediation, it is not at all unreasonable that $m_0$ may be moderately small, meaning that it gives a significant contribution to sleptons to render them positive, but gives an insignificant contribution (or none at all) to squarks. This of course depends on the underlying origin of $m_0$, of which we remain agnostic. There is, in fact, a range of $m_0$ that implies the anomaly-mediated and gauge-mediated predictions for the left-handed first and second generation slepton masses are identical. The parameter space of this “worst nightmare” situation is shown in Fig. 3. Note that including experimental uncertainties would widen the overlapping range of $m_{3/2}^\Lambda = F/M$ for a given $m_0$.

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\[ \text{See Ref. [22] for a well-motivated example of just such a possibility.} \]

\[ \text{The ratio to the Bino mass (instead of the lighter Wino mass) was chosen to minimize dependence on higher order corrections (see Sec. [1]).} \]
Figure 3: Range of $m_0$ as a function of $m_{3/2}^{AM} = F/M$ that implies the anomaly-mediated and generalized gauge-mediated models predict the same left-handed first and second generation slepton masses. The shaded band is the result of varying the messenger scale $M$ between $10^5$ to $10^9$ GeV in the gauge-mediated model.

It is therefore possible that the only differences in the scalar sector between an anomaly-mediated model with $m_0$ as shown in Fig. 3 and a generalized gauge-mediated model are ultimately related to $m_{\tilde{\ell}_R}, \mu$, the trilinear scalar couplings, or the presence of Yukawa contributions to the third generation scalars in anomaly mediation. In the remainder of this section we discuss to what extent these observables provide useful distinguishing criteria between anomaly mediation and gauge mediation.

In the generalized gauge-mediated Wino NLSP models, the first and second generation right-handed slepton mass $m_{\tilde{\ell}_R}$ is always larger than the corresponding left-handed slepton mass $m_{\tilde{\ell}_L}$, due to the large ratio $n_1/n_2 = 33/5$. Explicitly, there are three contributions to the slepton mass difference $m^2_{\tilde{\ell}_R} - m^2_{\tilde{\ell}_L}$: the gauge-mediated contribution at the messenger scale, the RG contribution, and the $D$-term contribution. The $D$-term contribution is accidentally rather small due to a numerical cancellation, and can be neglected. The other contributions are

$$\left[ m^2_{\tilde{\ell}_R} - m^2_{\tilde{\ell}_L} \right]_{\text{mess}} = \frac{3 n_2}{2 (16\pi^2)^2} \left[ \frac{3 n_1}{5 n_2} g_1^4 - g_2^4 \right] \frac{F^2}{M^2}$$

(12)

$$\left[ m^2_{\tilde{\ell}_R} - m^2_{\tilde{\ell}_L} \right]_{\text{RG}} \approx 6 \frac{n_2}{(16\pi^2)^3} \left[ \frac{3 n_1^2}{5 n_2^2} g_1^6 - g_2^6 \right] \frac{F^2}{M^2} \ln \frac{M}{m_{\tilde{\ell}}},$$

(13)
that in practice are numerically roughly comparable. The ratio can be approximately written as

$$\frac{m_{\tilde{\tau}_R}^2 - m_{\tilde{\tau}_L}^2}{m_{\tilde{\tau}_L}^2} \simeq (0.25 \pm 0.05) + 0.04 \ln \frac{M}{10^5 \text{ GeV}},$$

(14)

where the ±0.05 arises from the variation of $F/M$ throughout the range $20 \rightarrow 80$ TeV. This is very different from anomaly mediation, where the difference between the left-handed and right-handed slepton masses is less than a few percent throughout most of the parameter space [7].

The bilinear supersymmetric Higgs mass, $\mu$, feeds into several low energy observables, including the heavier chargino, the two heaviest neutralinos (assuming $\mu$ is larger than $M_1$), the heavier Higgs scalar masses, and the off-diagonal left-right (LR) squark and slepton mixing. Hence, determining the value of $\mu$ from experiment can be done via several different classes of signals.

In both the anomaly-mediated and gauge-mediated models discussed here, it is generally a good approximation throughout the parameter space that $\mu^2 \sim -m_{H_u}^2$. In anomaly mediation, there is an interesting (apparently accidental) cancellation between the renormalization group contributions that feed into $m_{H_u}^2$ due to the presence of a nonzero $m_0$ (that breaks the renormalization group invariance of the anomaly-mediated spectrum). The result is a “focusing” effect [10, 23] that renders $m_{H_u}^2$ quite insensitive to large changes in $m_0$. In particular, $m_{H_u}^2$ is determined essentially by just the anomaly-mediated value that is approximately

$$m_{H_u}^2 \simeq Y_t^2 \left(-16g_3^2 - 9g_2^2 + 18Y_t^2\right) \left(\frac{m_{AM}/2}{16\pi^2}\right)^2,$$

(15)

or that roughly $(m_{H_u}^2)^{1/2}$ is about $2.5 m_{AM}/(16\pi^2)$. Therefore, in an anomaly-mediated model with a universal additional scalar mass $m_0$, the value of $\mu$ is fixed once the scale of the gaugino masses has been established.

In gauge mediation the Higgs scalar masses are also determined once the messenger sector is fixed and the scale of the gaugino masses has been established. However, no dynamical origin for $\mu$ was given, and indeed it is possible that the Higgs soft masses could be affected by the mechanism that ultimately determines $\mu$ (see e.g. [24, 13]). For this reason, observables that depend on $\mu$ are not particularly reliable distinctions between anomaly mediation and gauge mediation, unless the gauge-mediated contributions to the Higgs soft masses dominate over all other possible contributions.

Another interesting observable is the decay length of $\tilde{C}_1^+ \rightarrow \tilde{N}_1 \pi^\pm$. This is also, unfortunately, not a useful distinction between anomaly mediation and gauge mediation for two reasons. First, the one-loop corrections dominate throughout the parameter space of interest, and to a very good accuracy depend only on kinematical functions of $M_2$ and $M_W$ [14]. This contribution is therefore the same for a given Wino mass. Second, the smaller tree-level corrections depend sensitively on $\mu$ (and tan $\beta$), that we have argued is not a reliable distinction. Thus, while the macroscopic decay length of the lightest chargino is an excellent signal of a Wino (N)LSP, it does not provide any useful information to distinguish the mediation of supersymmetry breaking.
Finally, at leading order trilinear scalar couplings, $A_f$, are generated in anomaly mediation, but not in gauge mediation. They do reappear in gauge mediation after renormalization group evolution to the weak scale, but are usually smaller (in absolute value) than and opposite in sign to the anomaly-mediated values. These couplings affect the $(\text{mass})^2$ matrix for the sfermions, but to a good approximation for moderate to large $\tan \beta$, they only significantly impact the stop mass matrix. This is because the off-diagonal term for up-type sfermions is $m_f (A_f - \mu/\tan \beta)$ whereas for down-type sfermions it is $m_f (A_f - \mu \tan \beta)$, which shows that the term proportional to $\mu$ is significantly diminished (enhanced) relative to $A_f$ for up-type (down-type) sfermions. Thus, the splitting between the heavier stop ($\tilde{t}_2$) and the lighter stop ($\tilde{t}_1$) mass eigenstates is generally larger in anomaly mediation. Taking a ratio, such as $(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)/m_{\tilde{t}_2}^2$, also eliminates the dependence on $\mu_0$ in anomaly mediation, and therefore provides a useful additional distinction. Note, however, that the gauge mediation prediction of nearly zero trilinear scalar couplings at the messenger scale relies on the assumption that the couplings between messenger fields and MSSM matter fields are very small [15].

Four example models’ spectra are given in Appendix A to illustrate the comparison between two anomaly mediation models and two gauge mediation models.

3.2 Sign of the gluino soft mass

In general, the soft mass of the gluino in the MSSM may be complex. After a field redefinition of the gluino, the phase of the gluino soft mass appears only in the interaction of a gluino and a chiral multiplet (see Appendix B for details). Furthermore, the phase cancels in processes that involve a vertex with a chiral multiplet and its hermitian conjugate. The phase reappears only in a chirality violating interaction (such as an interaction with a Higgs) or a “fermion number violating” interaction. One-loop examples of such interactions are shown in Fig. 4.

A significant CP-violating phase in the gluino soft mass, i.e. $\text{arg}(M_3)$ not close to 0 or $\pi$, gives large contributions to CP-violating processes, particularly the electric dipole moment of the neutron [25, 26]. Here we are interested in the processes of Fig. 4 that can distinguish a CP-even gluino soft mass ($\text{arg}(M_3) = 0$) from a CP-odd gluino soft mass ($\text{arg}(M_3) = \pi$), through necessarily CP conserving processes.

Evidently several processes are affected by the gluino soft mass sign. These include fermion masses, fermion-fermion-Higgs interactions, trilinear scalar couplings, squark LR mixing, squark-squark-gaugino interactions, etc. One-loop corrections to the pole masses of quarks and squark LR mixing (mass)$^2$ have been calculated in e.g. Ref. [13]. The one-loop correction to the running $b$-quark mass is particularly interesting, since the left-right squark mixing is proportional to $\mu \tan \beta$ for moderate to large $\tan \beta$, and thus can give an $\mathcal{O}(1)$ correction [27, 28]. It is straightforward to calculate the leading order shift in the running $b$-quark mass resulting from
Figure 4: One-loop diagrams that are proportional to the phase \( \arg(M_3) \) of the gluino soft mass: (a) a one-loop correction to a fermion-fermion-Higgs interaction, (b) a one-loop correction to a trilinear scalar coupling, and (c) a one-loop correction to the gaugino-quark-squark coupling. The arrows denote the flow of baryon number.

The supersymmetric contributions while keeping track of the sign of \( M_3 \):

\[
m_b = Y_b \langle H_d^0 \rangle (1 + \Delta m_b)
\]
\[
\Delta m_b = \frac{\mu \tan \beta}{16\pi^2} \left[ \frac{8}{3} g_3^2 \text{sign}(M_3) M_\tilde{g} I(m_{b_1}^2, m_{b_2}^2, M_\tilde{g}^2) + Y_t^2 A_t I(m_{t_1}^2, m_{t_2}^2, \mu^2) \right]
\]

where \( Y_t \) and \( Y_b \) are the top and bottom Yukawa couplings, \( m_{b_1}, m_{b_2} \) are the \( b \)-squark masses, \( m_{t_1}, m_{t_2} \) are the \( t \)-squark masses, \( M_\tilde{g} \) is the physical (real, positive definite) gluino mass, and

\[
I(a, b, c) = \frac{ab \ln \frac{a}{b} + bc \ln \frac{b}{c} + ac \ln \frac{a}{c}}{(a - b)(b - c)(a - c)}.
\]

Notice that the correction to the running \( b \) mass consists of a sum over two contributions: one piece proportional to \( \frac{8}{3} g_3^2 M_\tilde{g} \) (from Fig. 4(a) with a gluino in the loop), and the other proportional to \( Y_t^2 A_t \) (from Fig. 4(a) with Higgsinos in the loop). As we noted above, the sign of the trilinear scalar coupling in gauge mediation is opposite to that in anomaly mediation. (Indeed, \( \text{sign}(M_3) \neq \text{sign}(A_t) \) in both anomaly mediation and gauge mediation.) Consequently, the overall sign of \( \Delta m_b \) is equal to \( \text{sign}(\mu)\text{sign}(M_3) \), and is therefore equal to \( -\text{sign}(\mu) \) in anomaly mediation and \( +\text{sign}(\mu) \) in gauge mediation. Independently determining the sign of \( \mu \) is therefore essential to interpret the size of the correction to the running \( b \) quark mass. Other precision observables, such as \( b \to s\gamma \) and \( g - 2 \) may be useful in this regard.\(^\text{10}\)

\(^{10}\)We have closely followed Ref. [28] for this calculation, except for the differing notation for the \( b \) and \( t \) Yukawa couplings.

\(^{11}\)We thank J. Feng for discussions on this point.
Another important difference is that in anomaly mediation \( |A_t| \) is larger, and the top squarks are more widely separated in mass. The second term in Eq. (17) is therefore different between the two models. However, in our numerical calculations we found that the overall coefficient for the first term proportional to the gluino mass is always larger in both models, typically by a factor of 4 to 6. (This observation was also emphasized in Ref. [28].) Consequently, if \( \mu \) were the same in both models, \( \Delta m_b \) would also be close in magnitude (but opposite in sign). We can estimate the correction to \( \Delta m_b \) by observing that \( M_{\tilde{g}}^2 I(m_{b_1}^2, m_{b_2}^2, M_{\tilde{g}}^2) \sim 0.6 \) is a good approximation throughout the parameter space of both models, and thus

\[
\Delta m_b \sim (0.75) \frac{1}{6\pi^2} \tan \beta \frac{\mu}{M_{\tilde{g}}} \text{sign}(M_3) \tag{18}
\]

where the 0.75 conservatively accounts for the decrease in the correction to \( \Delta m_b \) due to the second term of Eq. (17). The ratio \( \mu/M_{\tilde{g}} \) can be calculated from both models in the absence of additional contributions to the Higgs soft masses. \( (\mu/M_{\tilde{g}}) \) is about 0.7 in anomaly mediation and 0.5 in gauge mediation.) However, since no mechanism for the generation of \( \mu \) was specified, there is no strongly reliable prediction of \( \mu \) from the fundamental model parameters, particularly for gauge mediation. Instead, the essential distinction between anomaly mediation and gauge mediation is sign of \( \Delta m_b \): the one-loop running \( b \)-quark mass is larger (smaller) than the tree-level mass in anomaly mediation for \( \mu \) negative (positive), and precisely opposite of this for gauge mediation. This can be applied, for example, to the \( h-b\bar{b} \) coupling. We find the effective coupling at one-loop is

\[
\mathcal{L} = \lambda_{h\bar{b}b} h\bar{b}b
\]

where

\[
\lambda_{h\bar{b}b} = -Y_b \sin \alpha \left( 1 - \frac{\Delta m_b}{\tan \alpha \tan \beta} \right) = \lambda_{h\bar{b}b}^\text{tree} \left( 1 - \frac{\Delta m_b}{\tan \alpha \tan \beta} \right).
\]

In addition, the Higgs mixing angle \( \alpha \) is related to \( \tan \beta \) at tree-level by

\[
\tan \alpha = -\frac{1}{\tan \beta}
\]

which implies

\[
\lambda_{h\bar{b}b} \simeq \lambda_{h\bar{b}b}^\text{tree} (1 + \Delta m_b)
\]

To illustrate the size of this correction, use Eq. (18) to obtain the coupling in the anomaly mediation model \( \lambda_{h\bar{b}b}^{\text{AM}} \) and the coupling in a gauge mediation model counterpart \( \lambda_{h\bar{b}b}^{\text{GM}} \). All of the
parameter dependence drops out except for $\tan \beta$ and $|\mu/M_3|$, in which we set the latter to be 0.6 for comparison. Then, the ratio of the couplings is simply

$$\frac{\lambda_{AM}^{\ell \ell \ell}}{\lambda_{GM}^{\ell \ell \ell}} \simeq 1 - 2 \text{sign}(\mu) |\Delta m_b|$$

$$\simeq 1 - 0.015 \text{sign}(\mu) \tan \beta .$$

For $\mu < 0$ and $\tan \beta = (5, 10, 30, 50)$, this ratio is $(1.08, 1.15, 1.45, 1.76)$. A high precision measurement of this coupling (along with an independent determination of the sign of $\mu$) would therefore provide a useful distinguishing criteria between these models. The ability to measure this coupling at the LEP collider, the Fermilab Tevatron, and the LHC has been studied in Ref. [29], although it is likely that a NLC or a muon collider would be necessary to approach the needed precision.

Another class of processes, shown in Fig. 4(c), are the one-loop gluino corrections to the quark-squark-gaugino vertex. These corrections arise in gaugino decay $\tilde{G} \rightarrow q\tilde{q}^{(i)}$ and/or squark decay $\tilde{q} \rightarrow \tilde{G}q^{(i)}$, depending on the kinematics. Here $\tilde{G}$ can be any gaugino, and the prime denotes the SU(2) doublet partner field (for decays involving a chargino). These radiative corrections have the advantage that they are proportional to $g_2^2$, but the disadvantage that they are suppressed by $|M_3|^2$ and must involve squarks that are typically much heavier than sleptons.

Several other processes involving the diagrams of Fig. 4 might be useful. Determining the best experimental observable depends on the precision to which particular sparticle properties are measured and the collider that is used.

### 3.3 NLSP decay

One apparently obvious distinction between anomaly mediation and gauge mediation is that the Wino NLSP of a gauge-mediated model decays into a gravitino plus a photon. Even if the decay length is significantly larger than the scale of a collider detector, a statistically significant excess of sparticle production events with one (or two) hard photon(s) could experimentally establish that the Wino is unstable (otherwise, place a lower bound on its decay length). To confirm that the Wino is unstable, in principle only a few events would be necessary, as long as SM backgrounds can be reduced to a negligible level. The probability that one NLSP decays within the distance $L \ll L_{NLSP}$ of order the detector size, where $L_{NLSP} = c/\Gamma_{NLSP}$, is $1 - e^{-L/L_{NLSP}} \sim L/L_{NLSP}$. Thus, one would expect about one NLSP decay for every $L_{NLSP}/L$ NLSPs produced. However, as discussed above, the decay length could be enormous compared with the scale of a detector, and thus be well beyond the range of ordinary collider experiments. In this case, the gauge-mediated Wino-related collider signals are indistinguishable from anomaly mediation (or any other supersymmetric model that predicts a stable Wino). The difference might be detectable with a very long baseline experiment to measure NLSP decay, or, if there is a cosmologically significant relic density of Wino LSPs of anomaly mediation [7, 31], using dark matter detection experiments.
Higher order corrections

The equivalence of the gauge-mediated and anomaly-mediated gaugino masses holds to leading order (LO), but not to higher orders. This is simply a consequence of the fundamentally different origin of the soft masses. Higher order corrections are generally expected to be suppressed by a one-loop factor $1/16\pi^2$ times order one couplings and coefficients, relative to the leading order corrections. There are, however, important next-to-leading (NLO) two-loop corrections to the gaugino masses [31] that are much larger than might be naively expected [32]. The most important correction relevant to this discussion is the two-loop contribution to $M_2$ due to the $g_2^2g_3^2M_3$ term. In anomaly mediation, this takes the form

$$M_{2\text{AM}}^{\text{NLO}} = M_{2\text{AM}}^{\text{LO}} \left[ 1 + \frac{B_{23}^{(2)} g_3^2}{B_{2}^{(1)} 16\pi^2} \right]$$

(19)

where $B_{23}^{(2)} = 24$ is the rather large two-loop coefficient. The NLO result is about 20% larger than the LO result at the weak scale [3]. In gauge mediation, there is also a correction from the renormalization group evolution that explicitly depends on the logarithm of the ratio of scales. Specifically, this correction can be approximated as

$$M_{2\text{GM}}^{\text{NLO}} = M_{2\text{GM}}^{\text{LO}} \left[ 1 - \frac{n_3B_{23}^{(2)} g_3^4}{n_2(16\pi^2)^2} \ln \frac{M}{M_2} \right].$$

(20)

The gauge-mediated NLO expression for $M_2$ is typically a few percent smaller than the LO result, depending on the messenger scale $M$.

The other gaugino masses $M_1$ and $M_3$ receive at most a few percent correction to their LO values from the NLO pieces of the $\beta$-function, in both AM and GM models. Since one-loop threshold corrections are of the same order (if not significantly larger, especially for $M_3$), the NLO $\beta$-function corrections for these masses can be neglected.

The large correction to $M_2$ in the anomaly-mediated approach naively suggests that accurately measuring the ratio $M_1/M_2$ would distinguish anomaly mediation from gauge mediation at next-to-leading order. However, it is not hard to imagine that messengers could generate the approximate proportion $(n_1 : n_2 : n_3) \sim (33/5 : 1.2 : 3)$. Perhaps the simplest possibility is to assume that $F/M^2$ for the $L + \overline{L}$ multiplet is about 0.9, while all of the other multiplets have $F/M^2 \ll 1$. This generates a rather large positive one-loop threshold correction of about the right size for $M_2$ only[12]. The multi-singlet model with differing $F$-terms could also reproduce this ratio, but requires three different singlets coupling to the three SM components of the $10 + \overline{10}$, with $F$-terms in the proportion $(F_{X_Q}/M_{X_Q} : F_{X_u}/M_{X_u} : F_{X_e}/M_{X_e}) \sim (4, 22, 25)$. Thus, the gauge-mediated Wino NLSP models could approximately reproduce the NLO gaugino mass predictions of anomaly mediation, although one must make some slight modification to the messenger sector that is admittedly rather ad hoc.

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[12]In this case, higher order corrections from messenger contributions could also be important [32].
5 Conclusions

There exist gauge-mediated Wino NLSP models that predict the gaugino masses, the first and second generation squark masses, and potentially the left-handed first and second generation slepton masses are nearly equivalent to the predictions of an anomaly-mediated model with a small universal additional scalar mass. The sbottom masses, stau masses, heavier Higgs masses, and heavier chargino and neutralino masses are in general somewhat different due to a differing value of $\mu$ (determined from EWSB constraints). But, we have argued that these observables are not useful distinctions between the two classes of models because the mechanism for the dynamical generation of $\mu$ in gauge mediation, which could affect the Higgs soft masses, is unknown. This leaves only the ratios $(m_{\tilde{t}_R}^2 - m_{\tilde{t}_L}^2)/m_{\tilde{t}_L}^2$ and $(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)/m_{\tilde{t}_2}^2$ as useful parameter-independent experimental criteria to distinguish between anomaly mediation and the gauge-mediated models discussed here. These scalar mass ratios remain useful distinguishing criteria even if the universality of the additional scalar mass $m_0$ is relaxed, but only if the left-handed and right-handed additional mass contribution is the same.

The differing sign of the gluino soft mass is perhaps the clearest distinction between anomaly mediation and gauge mediation, and does not depend on the universality of $m_0$. Several one-loop processes are sensitive to this sign. Probably the most promising way to determine the sign of the gluino soft mass is to accurately measure the $h-b\bar{b}$ coupling and compare it with the tree-level expectation. Given an independent measurement of the sign of $\mu$, this coupling is significantly enhanced (diminished) in anomaly-mediation (gauge-mediation) with $\mu < 0$, and vice versa for $\mu > 0$. The precise size of the correction is strongly dependent on $\tan \beta$ and $\mu/M_\tilde{\chi}$, with a weaker dependence on other MSSM quantities.

The NLSP of any gauge-mediated model is unstable. Observing the decay of NLSP $\rightarrow$ gravitino plus a photon would strongly suggest gauge mediation, although the decay length could be well beyond the sensitivity of any collider experiment. In this scenario, a long-lived Wino NLSP of gauge mediation is indistinguishable from a stable Wino LSP of anomaly mediation in ordinary collider experiments. Establishing the (in)stability of the Wino would thus require either a very long baseline experiment to search for its decay, or a dark matter detection experiment to search for a cosmologically significant relic density of Wino LSPs.

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Appendix A: Comparison of example models’ spectra

In Table 1 four examples of models exhibiting the characteristics discussed in Sec. 3 are given. For all of the models, one-loop $\beta$-functions are used, and the masses of the sparticles are given at tree-level (i.e., one-loop corrections resulting from the conversion from an $\overline{\text{MS}}$ mass to a pole mass were not included). In all cases the same value of $\tan \beta$, $\text{sign}(\mu)$, and $m^\text{AM}_{3/2} = F/M$ were used. Examples (a) and (b) are anomaly mediation models with slightly differing values of $m_0$. Examples (c) and (d) are gauge mediation models with differing values of the Higgs soft masses. In example (c) only the gauge-mediated contribution to the Higgs soft masses were included, whereas in example (d) additional contributions to the Higgs soft masses were included such that the resulting Higgs soft masses at the weak scale were the same as in the anomaly mediation model example (a). The latter has the effect of generating a $\mu$ in gauge mediation that is the same as anomaly mediation.

Clearly $M_1$, $M_2$, $|M_3|$ and $m_h$ are identical, and the heavy first (and second) generation squark masses are the same to within about 2%. If $\mu$ is the same in both anomaly mediation and gauge mediation, then all the gaugino masses and the heavy Higgs masses are also the same to within 2%. The differences between the models arise in the third generation and the slepton masses. Again, to within a few percent, example (a) and examples (c),(d) have the same left-handed first (and second) generation slepton masses. Similarly example (b) and examples (c),(d) have the same right-handed first (and second) generation slepton mass. (The additional universal scalar mass $m_0$ in anomaly mediation was chosen to give these results.) This illustrates the important general observation that, all other things equal, it is not possible to find the same left-handed and right-handed slepton masses in an anomaly mediation and a gauge mediation model, assuming the universality of $m_0$.

In general the stau masses can be quite different, unless both the right-handed slepton masses and $\mu$ are the same [compare examples (b) and (d)]. Again, however, the ratio of the right-handed first (or second) generation slepton mass with the left-handed first (or second) generation slepton mass remains significantly different. The lightest $b$ squark mass is lighter in the anomaly mediation models by about 7% (while the heavier $b$ squark mass is comparable across all example models). The lighter (heavier) $t$-squark mass is lighter in the anomaly mediation models by about 14% (6%). Finally, the correction to the running $b$ mass, Eq. (17), is close in magnitude when comparing examples (a), (b) with (d), but opposite in sign between anomaly mediation and gauge mediation. The magnitude of the correction is smaller for example (c) due to the smaller value of $\mu$. 

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|                     | Anomaly mediation | Gauge mediation |
|---------------------|-------------------|-----------------|
|                     | Example (a)       | Example (b)     |
| $m^{AM}_{3/2}$      | $5 \times 10^4$   | $5 \times 10^4$ |
| $m_0$               | 291               | 327             |
| $F/M$               |                   | $5 \times 10^4$ |
| $\Lambda_{DSB}$    |                   | $10^7$          |
| $\tan \beta$       | 20                | 20              |
| $\text{sign}(\mu)$ | +                 | +               |
|                     |                   |                 |
|                     | Example (c)       | Example (d)     |
| $m_{\tilde{C}_1}$  | 131, 819          | 131, 821        |
| $m_{\tilde{C}_2}$  | 131, 819          | 129, 565        |
| $m_{\tilde{N}_1}$  | 131, 451          | 131, 451        |
| $m_{\tilde{N}_2}$  | 131, 451          | 129, 445        |
| $m_{\tilde{N}_3}$  | 814, 818          | 815, 819        |
| $m_{\tilde{N}_4}$  | 814, 818          | 558, 570        |
| $m_{\tilde{b}_1}$  | 770, 935          | 774, 941        |
| $m_{\tilde{b}_2}$  | 770, 935          | 895, 993        |
| $m_{\tilde{t}_1}$  | 882, 988          | 890, 998        |
| $m_{\tilde{t}_2}$  | 882, 988          | 952, 1001       |
| $m_{\tilde{H}_1}$  | 111, 260          | 182, 298        |
| $m_{\tilde{H}_2}$  | 111, 260          | 202, 293        |
| $m_{\tilde{h}_L}$  | 1029, 1034        | 1039, 1044      |
| $m_{\tilde{h}_R}$  | 1029, 1034        | 1017, 1017      |
| $m_{\tilde{e}_L}$  | 229, 224          | 273, 269        |
| $m_{\tilde{e}_R}$  | 229, 224          | 229, 276        |
| $m_{H_1}$, $m_{H_2}$ | 121, 710      | 121, 712        |
| $\Delta m_b$       | 0.17              | 0.17            |

Table 1: Example models are shown with the input parameters and some of the masses of the resulting weak scale spectrum using one-loop renormalization group evolution. Examples (a) and (b) are anomaly mediation models with slightly differing values of $m_0$. Examples (c) and (d) are gauge mediation models with messenger content defined by Eq. (8), and (c) no additional contributions to the Higgs soft masses, or (d) additional contributions such that $m^2_{H_u}$ and $m^2_{H_d}$ are the same as those in the anomaly mediation example (a). All quantities have dimensions of GeV except of course for $\tan \beta$, $\text{sign}(\mu)$, and $\Delta m_b$, which are dimensionless.
Appendix B: The Lagrangian for a complex gluino soft mass

Assume the gluino soft mass is complex $M_3 = |M_3|e^{i\theta}$ and that the phase is physical (i.e., cannot be rotated into some other soft breaking quantity). The relevant pieces of the softly broken interaction Lagrangian involving the gluino are

$$L_{\tilde{g}} = L_{\tilde{g},\text{gauge}} + L_{\tilde{g},\text{chiral}}$$

where

$$L_{\tilde{g},\text{gauge}} = -i\lambda^\dagger_{\alpha} \sigma^\mu \mathcal{D}_{\mu}^a \lambda^a - \frac{1}{2}(M_3 \lambda^a \lambda^a + \text{h.c.})$$

$$L_{\tilde{g},\text{chiral}} = \sqrt{2}g_3 \sum_i \left(i\phi_i^* T_i^a \psi_i \lambda^a + \text{h.c.}\right).$$

$\lambda^a$ is the 2-component gluino spinor with $a = 1 \ldots 8$, the $\sum_i$ is over all chiral multiplets, $T_i^a$ is the SU(3) matrix for the $i^{th}$ representation, $\phi$ and $\psi$ are the complex scalar and 2-component spinor for the chiral multiplet, and the covariant derivative acting on the gluino is

$$\mathcal{D}_\mu^a \lambda^a = \partial_\mu \lambda^a - g_3 f^{abc} A_\mu^b \lambda^c.$$

Under the field rotation $\lambda^a \rightarrow \lambda^a e^{-i\theta/2}$, $L_{\tilde{g},\text{gauge}}$ is invariant (except, of course, that $M_3 \rightarrow |M_3|$), while $L_{\tilde{g},\text{chiral}}$ becomes

$$\sqrt{2}g_3 \sum_i \left(i\phi_i^* T_i^a \psi_i \lambda^a e^{-i\theta/2} + \text{h.c.}\right).$$

In four component notation, the quark–squark–gluino interaction Lagrangian (see Eq. (C89) in Ref. [34]) in the MSSM becomes

$$-\sqrt{2}g_3 T_{jk}^a \sum_i \left(\bar{q}_{iL} \lambda_i^a P_L k^i e^{-i\theta/2} - \bar{q}_{iR} \lambda_i^a P_R k^i e^{i\theta/2} + \text{h.c.}\right)$$

(21)

where the sum is over all quarks $i = u, d, c, s, t, b$. This agrees with the result found in Ref. [26].
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