MODELLING AND ANALYSIS OF INTEGRATED PEST MANAGEMENT STRATEGY

SANYI TANG AND LANSUN CHEN

Institute of Mathematics,
Academy of Mathematics and System Sciences
Chinese Academy of Sciences, Beijing 100080, P.R. China

Abstract. Two impulsive models concerning integrated pest management (IPM) are proposed according to impulsive effect with fixed moments and unfixed moments, respectively. The first model has the potential to protect the natural enemies from extinction, but under some conditions may also serve to extinction of the pest. The second model is constructed according to the practices of IPM, that is, when the pest population reaching the economic injury level, a combination of biological, cultural, and chemical tactics that reduce pests to tolerable levels is used. By using analytical method, we show that there exists an orbitally asymptotically stable periodic solution with a maximum value no larger than the given economic threshold. Further, the complete expression of period of the periodic solution is given. Thus, IPM strategy proved firstly by mathematical models is more effective than the classical method.

1. Introduction. For a biological control situation to be successful the natural enemy should be able to eradicate the pest or regulate it to densities below the threshold for economical damage. Often with augmentation or release, the natural enemy is applied like a pesticide after the pest has reached or exceeded the economic threshold. One of the first successful cases of biological control in greenhouses was the use of the parasitoid Encarsia formosa against the greenhouse whitefly Trialeurodes vaporariorum on tomatoes and cucumbers[1-2]. However, research on augmentation as a biological control method has shown that some practices are cost effective and others are not.

Another important method for pest control is chemical control. Pesticides can eliminate or reduce the impact of yield-limiting pests, but pesticide treatment is practical only when the increase in value of a treated crop exceeds the cost of pesticide treatment. This is most often the case.

Integrated pest management or IPM is a long term management strategy that uses a combination of biological, cultural, and chemical tactics that reduce pests to tolerable levels, with little cost to the grower and minimal effect on the environment. IPM which has been proved by experiment[3-4] is more effective than classical one (such as biological control or chemical control). If we wish to eradicate the pest or keep the host population below a threshold for ecological damage (Fig.1), how do we release the natural enemies? What proportion do we need to harvest the pests or kill the pests?

2000 Mathematics Subject Classification. 34A37, 47N60.
Key words and phrases. IPM strategy, State-dependent impulsive differential equations, Economic threshold, Periodic solution.
The main purpose of this paper is to construct two simple mathematical models according to the fact of IPM. Firstly, we suggest an impulsive system to model the process of periodic releasing natural enemies and spraying pesticides (or harvesting pest). We analyze the global stability of the so called pest-eradication periodic solution and further numerical results imply that this system can exhibit many new phenomena. Secondly, we suggest an impulsive dynamic system to model the impulsive effect (releasing natural enemies and spraying pesticides) occur when the pest population reach the Economic Threshold (ET) (Fig. 1). Our results show that there exists a orbitally asymptotically stable periodic solution with a maximum value no larger than the given economic threshold if the ET satisfies some conditions. In particular, the expression of period $T$ of this periodic solution is given. Finally, biological implications are discussed in section 5.

Fig. 1. Economic Injury Level (EIL) = lowest population density that will cause economic damage. Economic Threshold (ET) = population density at which control measures should be determined to prevent an increasing pest population from reaching the economic injury level. The arrow indicates the point where pest levels exceeded the economic threshold and an IPM strategy would be applied.

2. Lotka-Volterra systems. Volterra [5] first proposed a simple model for the predation of one species by another to explain the oscillatory levels of certain fish catches in the Adriatic. If $x(t)$ is the prey population and $y(t)$ that of the predator at time $t$ then Volterra’s model is

$$\begin{cases}
\frac{dx(t)}{dt} = x(t)(a - by(t)), \\
\frac{dy(t)}{dt} = y(t)(cx(t) - d),
\end{cases}$$

(2.1)

where $a, b, c$ and $d$ are positive constants.

The assumptions in the model are: (i) The prey in the absence of any predation grows unboundedly in a Malthusian way; this is the $ax$ term in (2.1). (ii) The effect of the predation is to reduce the prey’s per capita growth rate by a term proportional to the prey and predator populations; this is the $-bxy$ term. (iii) In the absence of any prey for sustenance the predator’s death rate results in exponential decay, that is the $dy$ term in (2.1). (iv) The prey’s contribution to the predator’s growth rate is $cxy$; that is, it is proportional to the available prey as well as to the size of the predator population.

The model (2.1) is known as the Lotka-Volterra model since the same equations were also derived by Lotka [6] from a hypothetical chemical reaction which he said could exhibit periodic behaviour in the chemical concentrations. In the following
we will develop the system (2.1) by introducing two difference types of impulsive effect.

3. Lotka-Volterra model with fixed moments of an impulsive effect. In this section, we will develop the Lotka-Volterra model (2.1) by introducing a proportion periodic impulsive catching or poisoning for the pest populations and a constant periodic releasing for the predator. That is, we consider the following periodic impulsive catching or poisoning for the pest populations and a

\[
\left\{ \begin{array}{l}
\frac{dx(t)}{dt} = x(t)(a - by(t)), \\
\frac{dy(t)}{dt} = y(t)(cx(t) - d), \\
\triangle x(t) = -px(t), \\
\triangle y(t) = \tau,
\end{array} \right. \quad t \neq nT,
\]

\[ \Delta x(t) = x(t^+) - x(t), \quad \Delta y(t) = y(t^+) - y(t), \quad T \text{ is the period of the impulsive effect.} \]

With model (3.1) we can take into account the possible exterior effects under which the population densities change very rapidly. For instance, impulsive reduction of the pest population density of a given species is possible after its partial destruction by catching or by poisoning with chemicals used in agriculture (0 ≤ p < 1). An impulsive increase of the predator population density is possible by artificial breeding of the species or release some species (τ > 0).

Firstly, we give some basic properties of the following subsystem

\[
\left\{ \begin{array}{l}
\frac{dx(t)}{dt} = -dy(t), \\
y(nT^+) = y(nT) + \tau, \\
y(0^+) = y_0.
\end{array} \right. \quad t \neq nT, \quad t = nT, \quad \tau > 0.
\]

**Lemma 3.2:** System (3.2) has a positive periodic solution \( y^*(t) \) and for every solution \( y(t) \) of (3.2) we have \( |y(t) - y^*(t)| \to 0 \) as \( t \to \infty \). Where \( y^*(t) = \frac{\tau \exp(-d(t-nT))}{1-\exp(-dT)} \), \( t \in (nT, (n+1)T) \), \( n \in \mathbb{N}, y^*(0^+) = \frac{\tau}{1-\exp(-dT)} \).

Therefore, we obtain the complete expression for the ‘pest-eradication’ periodic solution of system (3.1) over the \( n \)-th time interval \( t_0 = nT \leq t \leq (n+1)T \),

\[
(0, y^*(t)) = (0, \frac{\tau \exp(-d(t-nT))}{1-\exp(-dT)}).
\]

and we have the following main theorem of this section for system (3.1).

**Theorem 3.1:** Let \((x(t), y(t))\) be any solution of (3.1). Then \((0, y^*(t))\) is global asymptotically stable provided

\[
T < \frac{1}{a} \ln \left( \frac{1}{1-p} \right) + \frac{b \tau}{ad} \triangleq T_{\text{max}}.
\]

Proof. Firstly, we prove the local stability. The local stability of periodic solution \((0, y^*(t))\) may be determined by considering the behavior of small amplitude perturbations of the solution. Define \( x(t) = u(t), y(t) = y^*(t) + v(t) \), there may be written

\[
\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \end{pmatrix}, \quad 0 \leq t < T,
\]

where \( \Phi \) satisfies

\[
\frac{d\Phi}{dt} = \begin{pmatrix} a - by^*(t) & 0 \\ cy^*(t) & -d \end{pmatrix} \Phi(t).
\]
and $\Phi(0) = I$, the identity matrix. The resetting of the third and fourth equations of (3.1) become

$$
\begin{pmatrix}
u(nT^+) \\
u(nT^+)
\end{pmatrix} = \begin{pmatrix} 1 - p & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \end{pmatrix}.
$$

Hence, if both eigenvalues of

$$M = \begin{pmatrix} 1 - p & 0 \\ 0 & 1 \end{pmatrix} \Phi(T)$$

have absolute value less than one, then the periodic solution $(0, y^*(t))$ is locally stable. In fact, the two Floquet multiplies are thus

$$\mu_1 = e^{-dT} < 1, \quad \mu_2 = (1 - p)exp\left(\int_0^T (a - by^*(t))dt\right),$$

according to Floquet theory, the solution (3.1) is locally stable if $|\mu_2| < 1$, i.e.,

$$T < \frac{1}{a}ln\left(\frac{1}{1-p}\right) + \frac{b\tau}{a}.$$

For the global attractivity of $(0, y^*(t))$, the proof is similar to Theorem 3.2[11], so we omit it.

A typical solution of the pest-predator system with impulsive effect is shown in Fig.2, where we observe how the variable $y(t)$ oscillates in a stable cycle(Fig.2(a)). In contrast, the pest $x(t)$ rapidly decrease to zero(Fig.2(b)).

**Remark:** If the period of pulses $T$ is more than $T_{max}$, the pest-eradication solution becomes unstable and variable $x(t)$ begins to oscillate with a large amplitude that corresponding to periodic bursts of pest. If the period of pulses is further increased a sequence of 'period adding' bifurcations interchanging with regions of chaos is observed. A typical chaos attractor is shown in Fig.3. From this numerical simulation we can see there are many new phenomena of Lotka-Volterra system with impulsive effects.

Complete eradication of pest populations is generally not possible, nor is it biologically or economically desirable. A good pest control program should reduce pest populations to levels acceptable to the public. Thus, in view of Fig.1, we will use a new model to show this fact in the following section.

Fig.2: Dynamical behavior of the pest-predator system (3.1) with $a = b = 1, c = d = 0.3, p = 0.2, \tau = 1, T = 3.2$. (a) Time-series of the predator population evolving according to the biological control system (3.1). (b) Time-series of the pest population evolving according to the biological control system (3.1).
4. Lotka-Volterra model with unfixed moments of an impulsive effect.
Integrated pest management or IPM is a long term management strategy that uses a combination of biological, cultural, and chemical tactics that reduce pests to tolerable levels. An IPM control (including augmenting natural enemies, spraying pesticides, catching or harvesting) would be applied when the pest population reach a ET. So we consider the following model.

\[
\begin{align*}
\frac{dx(t)}{dt} & = x(t)(a - by(t)), & x & \neq y_{\text{max}}, \\
\frac{dy(t)}{dt} & = y(t)(cx(t) - d), & \\
\Delta x(t) & = -px(t), & \\
\Delta y(t) & = \tau, & \\
x(0^+) & = x_0 < y_{\text{max}}, & \eta(0^+) & = \eta_0.
\end{align*}
\]

(4.1)

Where \( y_{\text{max}} \) denotes the ET (pest population density at which control measures should be determined to prevent an increasing pest population from reaching the economic injury level). The main purpose of this section is to investigate the existence of a \( T \)-periodic solution of system (4.1) with one impulse effect per period and the stability of this solution. Furthermore, the expression of period \( T \) is given.

**Theorem 4.1**: If \( y_{\text{max}} < \frac{d}{cp} \ln \left( \frac{1}{1-p} \right) + \frac{\tau}{cp} \), then system (4.1) has a unique periodic solution with one impulse per period.

Proof. Let \( x = \xi(t), y = \eta(t) \) be such a \( T \)-periodic solution. Denote \( x_1 = y_{\text{max}}, \xi_0 = \xi(0^+), \eta_0 = \eta(0^+), \xi_1 = \xi(T) = x_1, \eta_1 = \eta(T), \xi_1^+ = \xi(T^+) \) and \( \eta_1^+ = \eta(T^+) \). Then from the \( T \)-periodicity, we have

\[
\xi_1^+ = \xi_0, \quad \eta_1^+ = \eta_0
\]

i.e.,

\[
(1-p)x_1 = \xi_0, \quad \eta_1 + \tau = \eta_0.
\]

(4.2)

For \( T \in (0, T] \) the solution \( x = \xi(t), y = \eta(t) \) of system (4.1) satisfies the relation

\[
c[x(t) - \xi_0] - d\ln \left( \frac{\xi(t)}{\xi_0} \right) = a\ln \left( \frac{\eta(t)}{\eta_0} \right) - b[\eta(t) - \eta_0].
\]

(4.3)

In particular, for \( t = T \), we have

\[
c[x_1 - \xi_0] - d\ln \left( \frac{x_1}{\xi_0} \right) = a\ln \left( \frac{\eta_1}{\eta_0} \right) - b[\eta_1 - \eta_0].
\]

(4.4)
and in view of (4.2) we have
\[ cp x_1 - d \ln \left( \frac{1}{1 - p} \right) = a \ln \left( \frac{\eta_1}{\eta_0} \right) + b \tau, \]
which implies that
\[ \eta_0 = \frac{\tau}{1 - (1 - p) \frac{d}{c} \exp \left( \frac{cp}{a} - \frac{b \tau}{a} \right)} \]  
(4.5)

If \( (1 - p) \frac{d}{c} \exp \left( \frac{cp}{a} - \frac{b \tau}{a} \right) < 1 \), then we have \( \eta_0 > 0 \), i.e.,
\[ x_1 = h_{\text{max}} < \frac{d}{cp} \ln \left( \frac{1}{1 - p} \right) + \frac{b \tau}{cp}. \]  
(4.6)

Thus, if condition (4.6) holds, then system (4.1) has a unique periodic solution with one impulse effect per period.

Before giving the stability of this periodic solution, we need the following Lemma[12]:

**Lemma 4.1:** (Analogue of Poincaré Criterion). The \( T \)-periodic solution \( x = \xi(t), y = \eta(t) \) of system
\[
\begin{cases}
\frac{dx}{dt} = P(x, y), & \frac{dy}{dt} = Q(x, y), \text{ if } \phi(x, y) \neq 0, \\
\Delta x = a(x, y), & \Delta y = b(x, y), \text{ if } \phi(x, y) = 0
\end{cases}
\]
is orbitally asymptotically stable and enjoys the property of asymptotic phase if the multiplier \( \mu_2 \) satisfies the condition \( |\mu_2| < 1 \). Where
\[
\mu_2 = \prod_{k=1}^{\frac{q}{d}} \Delta_k \exp \left[ \int_0^T \left( \frac{\partial P}{\partial x}(\xi(t), \eta(t)) + \frac{\partial Q}{\partial y}(\xi(t), \eta(t)) \right) dt \right],
\]
\[
\Delta_k = P_+ \left( \frac{\partial a}{\partial x} d \frac{\partial \phi}{\partial x} + \frac{\partial b}{\partial x} d \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial x} \right) + Q_+ \left( \frac{\partial a}{\partial x} d \frac{\partial \phi}{\partial x} + \frac{\partial b}{\partial y} d \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial y} \right),
\]
and \( P, Q, \frac{\partial a}{\partial x}, \frac{\partial a}{\partial y}, \frac{\partial b}{\partial x}, \frac{\partial b}{\partial y}, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \) are calculated at the point \( (\xi(\tau_k), \eta(\tau_k)) \) and \( P_+ = P(\xi(\tau_k^+), \eta(\tau_k^+)), Q_+ = Q(\xi(\tau_k^+), \eta(\tau_k^+)) \).

According to the definitions (see Ref.[12]) of orbitally asymptotically stable and enjoys the property of asymptotic phase the following Theorem holds true.

**Theorem 4.2:** If \( x_1 = h_{\text{max}} < \frac{d}{cp} \ln \left( q(1 - p)^{-\frac{d}{c}} \right) + \frac{b \tau}{cp} \), then the \( T \)-periodic solution \( (\xi(t), \eta(t)) \) is orbitally asymptotically stable and enjoys the property of asymptotic phase. Where \( q = 1 - \frac{\sqrt{b^2 + (\sqrt{2} + a)^2}}{2a} \).

Proof. From Lemma 4.1 we calculate the multiplies \( \mu_2 \) of the system in variations corresponding to the \( T \)-periodic solution \((\xi(t), \eta(t))\). Since
\[
\frac{\partial P}{\partial x} = a - by, \quad \frac{\partial Q}{\partial y} = cx - d,
\]
\[
\frac{\partial a}{\partial x} = -p, \quad \frac{\partial a}{\partial y} = 0, \quad \frac{\partial b}{\partial x} = db, \quad \frac{\partial b}{\partial y} = 0, \quad \frac{\partial \phi}{\partial x} = 1, \quad \frac{\partial \phi}{\partial y} = 0,
\]
\[
\Delta_1 = \frac{P_+}{P} = \frac{\xi_0(a - b\eta_0)}{\xi_1(a - b\eta_1)},
\]
and
\[
\int_0^T \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dt = \int_0^T [(a - b\eta(t)) + (c\xi(t) - d)] dt
\]
\[
= \int_0^T \left( \frac{\xi(t)}{\xi(t)} + \frac{\eta(t)}{\eta(t)} \right) dt = \int_0^T d ln(\xi(t)\eta(t))
\]
\[
= ln(\frac{\xi(t)\eta(t)}{\xi_0\eta_0}).
\]
Therefore
\[ \mu_2 = \Delta_1 \exp \left\{ \int_0^T \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right\} dt = \frac{\eta_1(x-b\eta_0)}{\eta_0(a-b\eta_0)} \]
and then we have the following three cases to consider:

Case (1): if \( \eta_0 \leq a/b \), then it is easy to see that \( 0 \leq \mu_2 < 1 \);

Case (2): if \( a - b\eta_0 < 0 \) and \( a - b(\eta_0 - \tau) > 0 \), then \(-1 < \mu_2 < 0 \) if and only if
\[ 2b\eta_0^2 - 2\eta_0(b\tau + a) + a\tau < 0, \]
which implies that \( \eta_0 \) must satisfy
\[ a/b < \eta_0 < \frac{b\tau + a + \sqrt{b^2\tau^2 + a^2}}{2b}. \]

Case (3): If \( a - b(\eta_0 - \tau) \leq 0 \), then it is easy to see that \( \mu_2 > 1 \).

In view of Case (1)-(3) and Lemma 4.1, we can see that if \( x_1 = h_{\text{max}} < \frac{a}{cp} \ln(q(1-p)^{-\frac{h}{p}}) + \frac{b}{cp} \). Then the \( T \)-periodic solution \((\xi(t), \eta(t))\) is orbitally asymptotically stable and enjoys the property of asymptotic phase.

The rest of this section we will give a complete expression of period \( T \) of periodic solution \((\xi(t), \eta(t))\). For this purpose, we need the following Lemmas.

**Lemma 4.2:** If \( z > 0 \), then \( z - 1 - \ln(z) \geq 0, z + \ln(1/z) - 1 \geq 0 \) and the equal sign holds only for \( z = 1 \).

**Lemma 4.3:** The Lambert\( W \) function is defined to be multivalued inverse of the function \( z \mapsto z e^z \) satisfying
\[ \text{Lambert}W(z) \exp(\text{Lambert}W(z)) = z. \]

In the following we use the letter \( W \) for this function[13]. First of all, the function \( z \exp(z) \) has the positive derivative \((z + 1)\exp(z)\) if \( z > -1 \). Define the inverse function of \( z \exp(z) \) restricted on the interval \([-1, \infty)\) to be \( W(0, z) \). Similarly, we define the inverse function of \( z \exp(z) \) restricted on the interval \((-\infty, -1]\) to be \( W(-1, z) \). For the natural of this study, both \( W(0, z) \) and \( W(-1, z) \) will be employed only for \( z \in [-\exp(-1), 0) \) because both functions are real values for \( z \) in this interval.

The period \( T \) of the period solution \((\xi(t), \eta(t))\) can be found. It follows from the second equation of (4.1) we have,
\[ dt = \frac{dx}{x(a - by(x))}, \]
and we can determine \( y(x) \) from the equation
\[ c(x - \xi_0) - d\ln(x(1-p)x_1^\gamma) = a\ln(b/y_0) - b(y - y_0). \]
Solving this equation for \( y \) gives
\[ y = g_k(x), \quad g_k(x) = -\frac{a}{b} \text{W}[\frac{b}{a}e^{(A)}], \]
where \( k = 0, 1 \) and
\[ A = -c x_1(1-p) - d\ln(x(1-p)x_1^\gamma) + cx - by_0. \]

We have the following two cases for the period \( T \) of the periodic solution \((\xi(t), \eta(t))\).
Case (1): If \(a/b < \eta_0 < b\tau + \alpha + \sqrt{\alpha^2 + \alpha^2} \). Then travelling along the lower branch described by \(y = y_0(x)\) from the point \((x_{\text{min}}, \frac{2}{b})\), with \(t = t|_{p_1}\), to the point \((x_1, \eta_0 - \tau)\), with \(t = t|_{p_2}\), in the counterclockwise direction yields

\[
|t|_{p_2} - |t|_{p_1} = \int_{x_{\text{min}}}^{x_1} \frac{dx}{x(a - by_0(x))},
\]

(4.14)

while travelling along the upper branch described by \(y = y_1(x)\) from the point \(((1-p)x_1, \eta_0)\), with \(t = t|_{p_3}\), to the point \((x_{\text{min}}, \frac{2}{b})\), with \(t = t|_{p_1}\), in the counterclockwise direction yields

\[
|t|_{p_3} - |t|_{p_1} = \int_{(1-p)x_1}^{x_{\text{min}}} \frac{dx}{x(a - by_1(x))},
\]

(4.15)

where \(x_{\text{min}}\) is the small solution of equation (4.12) when \(y = \frac{2}{b}\). Thus an integral representation of the period \(T\) is obtained.

\[
T = \int_{x_{\text{min}}}^{x_1} \frac{dx}{x(a - by_0(x))} - \int_{(1-p)x_1}^{x_{\text{min}}} \frac{dx}{x(a - by_1(x))}.
\]

(4.16)

Case (2): If \(0 < \eta_0 \leq \frac{2}{b}\). For this case we only need to consider the lower branch. Similarly, we can obtain an integral representation of the period \(T\), i.e.,

\[
T = \int_{(1-p)x_1}^{x_{\text{min}}} \frac{dx}{x(a - by_0(x))}.
\]

(4.17)

From (4.16) and (4.17), we must have \(-e^{-1} \leq -\frac{b}{a}\eta_0e^A\) holds true for all \(x \in [(1-p)x_1, x_1], x \in [x_{\text{min}}, x_1]\) and \(x \in [x_{\text{min}}, (1-p)x_1]\).

In fact, it is easy to see \(-\frac{b}{a}\eta_0e^A < 0\) and \(-e^{-1} \leq -\frac{b}{a}\eta_0e^A\) is equivalent to

\[
f(x) \triangleq \frac{d}{a}\ln\left(\frac{x}{(1-p)x_1}\right) - \frac{c}{a}x + \frac{c}{a}(1-p)x_1 + \frac{b}{a}\eta_0 - 1 - \ln\left(\frac{b}{a}\right) \geq 0. \tag{4.18}
\]

and function \(f(x)\) has a unique positive maximum value \(x = \frac{2}{b}\).

Therefore, in order to prove \(f(x) \geq 0\) for all \(x \in [(1-p)x_1, x_1], x \in [x_{\text{min}}, x_1]\) and \(x \in [x_{\text{min}}, (1-p)x_1]\), we only need to show \(f(x_1) \geq 0, f((1-p)x_1) \geq 0\) and \(f(x_{\text{min}}) \geq 0\).

It is easy to see the \(f(x_{\text{min}}) = 0\) and from Lemma (4.2) we note that

\[
f((1-p)x_1) = \frac{b}{a}\eta_0 - \ln\left(\frac{b}{a}\eta_0\right) - 1 \geq 0.
\]

Since

\[
f(x_1) = \frac{d}{a}\ln\left(\frac{1}{1-p}\right) - \frac{cp}{a}x_1 + \frac{b}{a}\eta_0 - \ln(b\eta_0/a) - 1, \tag{4.19}
\]

and from (4.5) we have

\[
\frac{cx_1}{a} = \ln\left(\frac{1}{\eta_0}\right) - \frac{d}{a}\ln(1-p) + \frac{b\tau}{a},
\]

which implies that

\[
f(x_1) = \ln\left(\frac{a}{b(\eta_0 - \tau)}\right) + \frac{b}{a}(\eta_0 - \tau) - 1.
\]

Again from Lemma 4.2 we have \(f(x_1) = \ln\left(\frac{a}{b(\eta_0 - \tau)}\right) + \frac{b}{a}(\eta_0 - \tau) - 1\). The above result can be summarized in Theorem 4.3.
Theorem 4.3: If \( a/b < \eta_0 < \frac{b\tau + a + \sqrt{b^2\tau^2 + a^2}}{2a} \). Then the period \( T \) of periodic solution \((\xi(t),\eta(t))\) of system (4.1) satisfies equation

\[
T = \int_{x_{\min}}^{x_1} \frac{dx}{x(a - by_0(x))} - \int_{x_{\min}}^{(1-p)x_1} \frac{dx}{x(a - by_0(x))}.
\]

(4.20)

If \( 0 < \eta_0 \leq \frac{a}{b} \). Then the period \( T \) of periodic solution \((\xi(t),\eta(t))\) of system (4.1) satisfies equation

\[
T = \int_{(1-p)x_1}^{x_1} \frac{dx}{x(a - by_0(x))}.
\]

(4.21)

Example: If we choose \( a = b = 1, c = d = 0.3, \tau = 1 \) and \( p = 0.2 \), the trajectory of the periodic solution \((\xi(t),\eta(t))\) is given in Fig.4 and from (4.22) the period \( T \approx 10.5 \). Fig.4 shows the pest populations is always less than the ET(ET = 2.4).

![Fig.4: (a) Orbitally asymptotically stable periodic solution \((\xi(t),\eta(t))\) with initial value \((\xi(0^+),\eta(0^+)) = (1.92, 1.66)\). (b) Time-series of the pest population.](image)

5. Biological conclusions. From Theorem 3.1, we know that the pest-eradication periodic solution \((0, y^*(t))\) is global stable if \( T < T_{\text{max}} = \frac{1}{a} ln\left(\frac{1}{1-p}\right) + \frac{b\tau}{am} \), which implies that \( T_{\text{max}} \) is a direct proportion function with respect to \( p \) or \( \tau \). Therefore, in order to obtain the object of IPM, we can determine the parameters \( p \) and \( \tau \) according to effect of the chemical pesticides on the environment and cost of the releasing natural enemies such that \( T < T_{\text{max}} \). That is, we can choose the parameters \( p \) and \( \tau \) to reduce pests to tolerable levels(ET) with little cost to the grower and minimal effect on the environment.

When using integrated pest management as an approach to control insect pests one must be committed to the long term. Regular field monitoring must be done to keep track of both pest and beneficial insect populations for an IPM program to be effective. Proper identification of insect pests and a basic knowledge of economic thresholds are essential for an IPM program to be successful. It follows from the main theorems in section 4 we know that if we wish to keep the pest populations below a threshold for ecological damage, proper identification of insect pests and a basic knowledge of economic thresholds are essential for an IPM program to be successful. That is to say, only when the economic threshold must be less than some given value \((ET < \frac{a}{cp} ln\frac{q(1-p)}{d} + \frac{b\tau}{cp})\), we can obtain the persistent insect pests control. Our results also give the period of pest population variation.

In a real world, changes in prey density affect predators at both individual and population levels. So, we should consider that the temporal variation in prey density affects the rate at which prey are killed by predators (that is, functional response). We leave these to a future investigation.
Acknowledgments. This work received partial supported by the National Natural Science Foundation of China(10171106).

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Received November 2002; revised June 2003; final version February 2004.

E-mail address: tsy@amss.ac.cn
E-mail address: lschen@amss.ac.cn