Quantum tomography of photon states encoded in polarization and picosecond time-bins

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A single photon has many physical degrees of freedom (DOF) that can carry the state of a high-dimensional quantum system. Nevertheless, only a single DOF is usually used in any specific demonstration. Furthermore, when more DOF are being used, they are analyzed and measured one at a time. We introduce a two-qubit information system, realized by two degrees of freedom of a single photon: polarization and time. The photon arrival time is divided into two time-bins representing a qubit, while its polarization state represents a second qubit. The time difference between the two time-bins is created without an interferometer at the picosecond scale, which is much smaller than the detector’s response time. The two physically different DOF are analyzed simultaneously by photon bunching between the analyzed photon and an ancilla photon. Full two-qubit states encoded in single photons were reconstructed using quantum state tomography, both when the two DOF were entangled and when they were not, with fidelities higher than 96%.

I. INTRODUCTION

Entanglement is a non-trivial property that is part of quantum theory [1]. It describes how seemingly separated quantum objects can only be described as a single inseparable physical entity. In the most basic example, two entangled particles can be used to demonstrate the non-locality of quantum mechanics [2]. The first physical realization of entanglement was witnessed using photons. This has been done by entangling the photons’ different degrees of freedom (DOF) such as position, linear and angular momentum, polarization and arrival time [3]. Entanglement is not restricted to a pair of particles, but can be extended to more as well [4, 5].

The efforts to achieve entangled states with higher number of particles originate from the interest in quantum states of high dimensionality. It has been suggested that adding DOF to a state to increase its dimensionality can be achieved not only by adding particles, but also by using more intrinsic DOF of the same particles [6, 7]. Such states of hybrid DOF have larger dimensionality with fewer particles. Apart from the added dimensions, they enable protocols that are otherwise hard or impossible to realize. An example for such a protocol is a full Bell state measurement with linear optical elements of polarized photons [8, 9], which is otherwise impossible [10, 11].

As one of the most successful realizations of qubits, photons are widely used in experimental demonstrations of quantum information. Manipulation of photons is relatively easy, as different operations could be performed using linear optical elements. While usually the photon polarization degree of freedom is used to encode the quantum information, other DOF are available as possible realizations of qubits as well. Generation of time-frequency entanglement was demonstrated [12–14], with its advantages accounted also for quantum key distribution [15]. Various recent demonstrations presented hybrid DOF with dimensionality of up to 10 qubits \(d = 2^{10}\) [16–19].

Combining the polarization and time-bin DOF of a photon has been suggested in a hybrid multi-photon scheme for a single-mode quantum computer, where information is stored in the temporal DOF, while the polarization DOF serves as a bus to route the qubits to their quantum logical gates [20]. One demonstration that encoded information only in short temporal differences has used the nonlinear process of sum-frequency generation with an intense shaped laser pulse to characterize the temporal information [21]. Due to detectors that cannot discriminate short temporal differences, both of these demonstrations have used long delay lines in order to observe or implement the different time-bins and their desired operations.

Controlled coupling between the polarization DOF and the temporal DOF has also been used for another goal. By using birefringent crystals, the polarization and temporal DOF of a single photon have been entangled. As the small time differences were undetectable by the photon detectors, the temporal DOF was practically traced out, leading to the controllable depolarization process of the quantum information carried by the polarization DOF of a single photon [22].

In this work we present and demonstrate a scheme which encodes a two-qubit state in a single photon, using two of its DOF - polarization and time. While previous works have shown different methods to incorporate the time DOF, our scheme is simple, and does not require interferometrically stable delay loops. In addition, we present a measurement method which analyzes all of the DOF of a single photon simultaneously, including temporal differences that are too small to detect by current detectors.
II. ENCODING PROCESS

In order to encode two qubits in a single photon, the first qubit is encoded in the photon’s polarization DOF. The horizontal $|h\rangle$ and the vertical $|v\rangle$ polarization states encode the logical $|0\rangle_p$ and $|1\rangle_p$ states. The second qubit is encoded in the arrival time-bin information. This is in principle a continuous DOF, that can be used as a finite discrete DOF with a predefined set of time-bin slots. We encode the logical $|0\rangle_t$ in the non-delayed $t = 0$ time-bin $|0\rangle$ and the logical $|1\rangle_t$ in the $t = \tau$ delayed time-bin $|\tau\rangle$. For later use we define the linearly polarized state at $45^\circ$ to the horizon as the diagonal (anti-diagonal) $|p\rangle = \frac{1}{\sqrt{2}}(|h\rangle + |v\rangle)$ and the right (left) circularly polarized states as $|r\rangle = \frac{1}{\sqrt{2}}(|h\rangle - |v\rangle)$ and $|l\rangle = \frac{1}{\sqrt{2}}(|h\rangle + i|v\rangle)$. As with the polarization, any superposition of the two time-bins is possible. Accordingly, we designate the following temporal superpositions: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |\tau\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |\tau\rangle)$, $|\times\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |\tau\rangle)$, and $|\div\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|\tau\rangle)$.

Polarization manipulation is achieved easily with standard wave plates and polarizers. In order to control the temporal DOF, first the photon’s polarization is set, and than the polarization and the temporal DOF are coupled by passing through a birefringent crystal [22]. The crystal length $L$ is set such that it induces a temporal polarization walk-off $\tau = \frac{c}{8\Delta n}$ much larger than the photon temporal pulse length, where $\Delta n$ is the birefringence index difference and $c$ the speed of light.

Consider a photon with the $|p\rangle$ polarization in time-bin $t = 0$, traveling towards a birefringent crystal (see Fig. 1). The two-qubit state is

$$|\phi\rangle = |p, 0\rangle = \frac{1}{\sqrt{2}}(|h, 0\rangle + |v, 0\rangle).$$

Due to the crystal birefringence, the horizontal and vertical polarizations amplitudes, $|h\rangle$ and $|v\rangle$ travel at different group velocities through the crystal. Thus, upon entering the birefringent crystal, the photon $|h\rangle$ amplitude is not delayed, while the $|v\rangle$ amplitude is delayed by $\tau$. Thus, the photon exits the crystal in a superposition to be non-delayed horizontally polarized and delayed vertically polarized. This results in the two-qubit maximally entangled Bell state

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|h, 0\rangle + |v, \tau\rangle).$$

Starting with the orthogonal polarization $|\phi\rangle = |m, 0\rangle$ would result in the orthogonal Bell state $|\phi^-\rangle = \frac{1}{\sqrt{2}}(|h, 0\rangle - |v, \tau\rangle)$.

The process applied by the birefringent crystal for these two input states is actually an entangling gate between the two DOF. For three out of four input states this gate acts as a controlled-NOT (CNOT) gate. The exception is the $|v, \tau\rangle$ state which is delayed to $t = 2\tau$ instead of accelerated to time-bin $t = 0$, and thus transformed out the proper Hilbert space. Thus, this process in general is not unitary, nor trace preserving. Nevertheless, this process is sufficient for generating all of the states that we are interested in, as we will show below. The representation of this operator in the standard basis is:

$$\hat{U}_{\text{gate}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix} \tag{3}$$

In order to generate an arbitrary polarization in a single time-bin, $t = 0$ or $t = \tau$, the photon polarization state is set before the crystal to $|h, 0\rangle$ or $|v, 0\rangle$, accordingly (see Fig. 1). A second set of polarization control elements is used to set the specific polarization. For a superposition of orthogonal states at the two time-bins, an initial $|p, 0\rangle$ state before the crystal will create a balanced superposition after the crystal that can be rotated to any two orthogonal states. When a polarizer, which is also a non-unitary operation, is added after the crystal, two identical polarizations in the two time-bins are generated. The balance and the phase between the two amplitudes can be controlled by setting different polarization states before the crystal. If the polarization states of the two time-bins should be different, but non-orthogonal, a more elaborated setup is required, which is feasible, but outside the scope of the current work.
FIG. 2. The experimental setup. Each photon is spectrally filtered using a bandpass filter (BPF). An encoded photon is prepared in the desired state, while an ancilla photon is prepared in one of the desired projection states. The projection measurement is by bunching at the beam-splitter (BS), while single photon detectors (D1 and D2) record coincidence events as the arrival time of the ancilla photon is scanned by changing its path length using a linear motor scanner.

III. CHARACTERIZATION METHOD

As the temporal separation of photon amplitudes that is generated by birefringent crystals of a few millimeters is too small to detect directly, we present here another method that can characterize not just any time scale, but can also simultaneously detect any additional DOF. Assume a pure unknown state $|\psi_e\rangle$ encoded in a single photon and another ancilla photon at a known pure state $|\psi_a\rangle$. The two states differ by the vector $|\delta\rangle \perp |\psi_a\rangle$, such that $|\psi_e\rangle = \alpha|\psi_a\rangle + \beta|\delta\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$. It can be shown that if the two photons are injected into a balanced beam-splitter (BS) in a Hong-Ou-Mandel (HOM) configuration where their relative arrival time is scanned, the ratio $R$ of the zero delay output coincidence rate to the long delay coincidence rate where the photons are completely distinguishable is

$$R = |\beta|^2 = 1 - |\alpha|^2 = 1 - |\langle \psi_1 | \psi_2 \rangle|^2. \hspace{1cm} (4)$$

This projection is not restricted to encoded photons in unknown pure states, as the same ratio $R$ also applies for projections on mixed states where $|\delta_i\rangle \perp |\psi_a\rangle$

$$\rho_e = |\alpha|^2|\psi_a\rangle \langle \psi_a | + \sum_i (|\beta_i|^2|\delta_i\rangle \langle \delta_i | + \gamma_i|\delta_i\rangle \langle \psi_a | + H.C. ) \hspace{1cm} (5)$$

and $R = 1 - |\alpha|^2 = 1 - \langle \psi_a | \rho_e | \psi_a \rangle$. Thus, the projection of one photon state onto the other can be inferred from the HOM dip magnitude. Repeated measurements of a photon at an unknown state with an ancillary photon at well designed states can provide information for a full reconstruction of the unknown quantum state. This statement is mathematically valid regardless of the dimensionality of the states and the details of their DOF.

IV. EXPERIMENTAL DEMONSTRATION

The encoding and characterization of different polarization and temporal states was demonstrated using the experimental setup presented in Fig. 2. A pulsed Ti:sapphire laser source with a 76 MHz repetition rate is frequency doubled by second harmonic generation (SHG) to a wavelength of 390 nm and an average power of 300 mW. The laser beam is corrected for astigmatism and focused on a 2 mm thick $\beta$-BaB$_2$O$_4$ (BBO) crystal used for SPDC. The BBO angle is set at a beam-like configuration [24, 25]. The biphoton state is spatially filtered by coupling the two beams into single-mode fibers, and spectrally filtered using 3 nm bandpass filters at a wavelength of 780 nm. The resulting biphoton state is $|\psi\rangle = |h\rangle_e \otimes |v\rangle_a$, where $e$ represents the encoded photon and $a$ the ancilla photon. The encoding setup used a 4 mm thick Calcite crystal that generates a time difference of $\tau = 2.3$ ps.

Using the above encoding procedure, we prepared the encoded photon in the following three two-qubit states

$$|\psi\rangle_{e.1} = |\phi^+\rangle,$$

$$|\psi\rangle_{e.2} = |p, +\rangle,$$

$$|\psi\rangle_{e.3} = \frac{1}{\sqrt{2}} (|r, 0\rangle - i|l, \tau\rangle) \hspace{1cm} (6)$$

These specific states were chosen as both DOF of $|\psi\rangle_{e.1}$ and $|\psi\rangle_{e.3}$ are entangled, while both the $|\psi\rangle_{e.2}$ and $|\psi\rangle_{e.3}$ states are mutually unbiased with the standard basis states, an important property for quantum key distribution protocols [26]. The ancilla photon was prepared at a complete set of states, whose projections are sufficient for a full reconstruction of the encoded photon state using a quantum state tomography (QST) procedure [27]. A variable delay controlled the relative arrival time of the two photons to the HOM projecting BS. The delay introduces an additional distinguishability between the encoded and ancilla photons. The required projection values are obtained by recording the ratio $R$ of coincidence detection rates at the BS two output ports. If the ancilla state occupied a single time-bin, a single delay scan actually provided information for two projections.

Figure 3 presents the experimental results of the coincidence detection rate of the ancilla and the encoded photons at the output BS ports as a function of the introduced delay between them. When both photons are at the same state (Figs. 3(a) and 3(c)), a very good HOM interference dip visibility of 94 $\pm$ 2% and 89 $\pm$ 2% is observed at zero delay, respectively. When the photons are at orthogonal states (Figs. 3(b) and 3(d)), there is no dip at zero delay. The only difference between the scans of Figs. 3(a) and 3(b) is in the phase between the two time-bins, but it is sufficient to completely distinguish between the two states. The same phase difference is applied also between the scans of Figs. 3(c) and 3(d), but here the
were fully characterized by the three states of Eq.

In conclusion, we present a simple scheme to utilize simultaneously the polarization and time DOF of a single photon, realizing a two-qubit information system. Instead of using imbalanced interferometers which are large and unstable, we used birefringent crystals to generate the temporal information. As the temporal details generated in this method are too fine for direct detection, we suggested and demonstrated a new characterization method, based on the HOM effect. This method characterizes all of the DOF simultaneously, without the need for separate optical setups for each DOF. The dimensionality of the generated states can be extended in a straightforward manner by adding more birefringent crystals. The presented high quality results support the feasibility of the suggested methods for encoding fine temporal information, and decoding quantum information of several DOF efficiently.

V. CONCLUSIONS

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[1] A. Einstein, B. Podolsky, and N. Rosen, Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?, Phys. Rev. 47, 777 (1935).
[2] J. S. Bell, On the Einstein Podolsky Rosen paradox, Physics I, 195 (1964).
[3] A. Aspect, P. Grangier, and G. Roger, Experimental
Tests of Realistic Local Theories via Bell’s Theorem, Phys. Rev. Lett. 47, 460 (1981).

[4] D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, Bells theorem without inequalities, American Journal of Physics 58, 1131 (1990).

[5] D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, Observation of Three-Photon Greenberger-Horne-Zeilinger Entanglement, Phys. Rev. Lett. 82, 1345 (1999).

[6] P. G. Kwiat, Hyper-entangled states, Journal of Modern Optics 44, 2173 (1997).

[7] J. T. Barreiro, N. K. Langford, N. A. Peters, and P. G. Kwiat, Generation of hyperentangled photon pairs, Phys. Rev. Lett. 95, 260501 (2005).

[8] P. G. Kwiat and H. Weinfurter, Embedded Bell-state analysis, Phys. Rev. A 58, R2623 (1998).

[9] J. T. Barreiro, N. K. Langford, N. A. Peters, and P. G. Kwiat, Generation of hyperentangled photon pairs, Phys. Rev. Lett. 95, 260501 (2005).

[10] L. Vaidman and N. Yoran, Methods for reliable teleportation, Phys. Rev. A 59, 116 (1999).

[11] N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, Bell measurements for teleportation, Phys. Rev. A 59, 3295 (1999).

[12] J. Brendel, N. Gisin, W. Tittel, and H. Zbinden, Pulsed Energy-Time Entangled Twin-Photon Source for Quantum Communication, Phys. Rev. Lett. 82, 2594 (1999).

[13] C. Simon and J.-P. Poizat, Creating Single Time-Bin-Entangled Photon Pairs, Phys. Rev. Lett. 94, 030502 (2005).

[14] A. Zavatta, M. D’Angelo, V. Parigi, and M. Bellini, Remote Preparation of Arbitrary Time-Encoded Single-Photon Ebits, Phys. Rev. Lett. 96, 020502 (2006).

[15] W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, Quantum Cryptography Using Entangled Photons in Energy-Time Bell States, Phys. Rev. Lett. 84, 4737 (2000).

[16] G. Vallone, E. Pomarico, P. Mataloni, F. De Martini, and V. Berardi, Realization and characterization of a two-photon four-qubit linear cluster state, Phys. Rev. Lett. 98, 180502 (2007).

[17] W.-B. Gao, C.-Y. Lu, X.-C. Yao, P. Xu, O. Gühne, A. Goebel, Y.-A. Chen, C.-Z. Peng, Z.-B. Chen, and J.-W. Pan, Experimental demonstration of a hyper-entangled ten-qubit Schrödinger cat state, Nat. Physics 6, 331 (2010).

[18] S. M. Lee, H. S. Park, J. Cho, Y. Kang, J. Y. Lee, H. Kim, D.-H. Lee, and S.-K. Choi, Experimental realization of a four-photon seven-qubit graph state for one-way quantum computation, Opt. Express, 20, 6915 (2012).

[19] M. Malik, M. Erhard, M. Huber, M. Krenn, R. Fickler, and A. Zeilinger, Multi-photon entanglement in high dimensions, Nat. Photonics 10, 248 (2016).

[20] P. C. Humphreys, B. J. Metcalf, J. B. Spring, M. Moore, X.-M. Jin, M. Barbieri, W. S. Kolthammer, and I. A. Walmsley, Linear Optical Quantum Computing in a Single Spatial Mode, Phys. Rev. Lett. 111, 150501 (2013).

[21] J. M. Donohue, M. Agnew, J. Lavoie, and K. J. Resch, Coherent Ultrafast Measurement of Time-Bin Encoded Photons, Phys. Rev. Lett. 111, 153602 (2013).

[22] A. Shaham and H. S. Eisenberg, Realizing controllable depolarization in photonic quantum-information channels, Phys. Rev. A 83, 022303 (2011).

[23] C. K. Hong, Z. Y. Ou, and L. Mandel, Measurement of subpicosecond time intervals between two photons by interference, Phys. Rev. Lett. 59, 2044 (1987).

[24] C. Kurtsieser, M. Oberparleiter, and H. Weinfurter, Generation of correlated photon pairs in type-II parametric down conversion revisited, Journal of Modern Optics 48, 1997 (2001).

[25] S. Takeuchi, Beamslike twin-photon generation by use of type II parametric downconversion, Opt. Lett., 26, 843 (2001).

[26] C. H. Bennett and G. Brassard, Quantum cryptography: Public key distribution and coin tossing, in Proceedings of the IEEE International Conference on Computers, [27] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Measurement of qubits, Phys. Rev. A 64, 052312 (2001).

[28] R. Jozsa, Fidelity for Mixed Quantum States, Journal of Modern Optics, 41, 2315 (1994).