An Analysis Tool for the Installation of Submarine Cables in an S-Lay Configuration Including “In and Out of Water” Cable Segments

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Abstract: Today, the offshore oil and gas and wind power industry is a heavily regulated segment, and current standards have established restrictions which yield a very limited weather window for submarine cable installations due to experience with cable failure in bad weather. There are two main limiting factors in current practice during cable installation of an S-lay configuration: the design criterion for the minimum allowable radius of curvature in the touch down point and the avoidance of axial compression in the touch down zone. Accurate assessment of the cable integrity during offshore installation has drawn great attention and is related to the existing available analysis and design tools. The main purpose of this paper is to develop and propose a quick and easy custom-made analysis tool, which is able to export similar results as sophisticated finite element analysis software. The developed tool utilizes analytical equations of a catenary-type submarine structure extended to account for varying cross-sections with different weights and/or stiffnesses, as is the real practice. A comparative study is presented in this paper to evaluate the significance for the modeling of the “out of water” cable segment required for accurate safety factor quantification during a laying operation. The efficiency and accuracy of the proposed tool are proven through a validation study comparing the results and the computational effort and time with commercial finite element analysis software. The analysis error in the case of not modeling the “out of water” cable part is significant, especially in shallow water areas, which proves the importance of using the proposed analysis tool.

Keywords: submarine cable; S-lay cable installation analysis; touch down point (TDP); minimum bending radius (MBR); bottom tension; catenary theory

1. Introduction

A submarine cable is a crucial connection between onshore/offshore topside facilities and/or equipment located on the seabed. The cable normally consists of both a dynamic and a static part during the installation period. The static part is laid by the cable lay vessel (CLV) and is located on the seabed under stable environmental conditions, while the dynamic part hangs freely from the onboard vessel’s equipment (tensioner and overboard chutes). The dynamic part is subjected to loading due to vessel motions and environmental loads. The touch down zone (TDZ), which is the zone where the cable first hits the seabed, is critical with respect to failure of the cable during installation. This region may be exposed to severe curvature and axial compression, which may result in local buckling inside the cross-section that causes the cross-section to be unstable in torsion, global loop formation, or a combination of those that may finally result in capacity failure [1]. The current practice is to avoid...
the occurrence of compression at the TDZ to eliminate any possibility of cable failure. Although it is not always clear how much submarine composite cables can be compressed before their integrity is compromised, cable manufacturers are often reluctant to allow significant cable compression. However, this restricts the weather window for the laying operation since the dynamic responses of the CLV, especially the motions along the cable axis (surging of the CLV), significantly affect the axial force applied to the cable during installation. The limited weather window results in high installation costs. It is therefore of great interest to investigate various installation scenarios and propose an efficient analysis tool for cable installers in order to accurately analyze/monitor the cable laying operation with the aim of eliminating the CLV idle time.

Offshore cable installation is a complex task [2]. In the planning phase, an installation analysis needs to be performed, which considers factors such as the cable properties, route characteristics, available installation equipment, and capacities of the cable installer [3]. Figure 1 shows some of the most influential parameters in an installation operation [4]. The departure angle is the complimentary angle of the cable exit angle at the overboard chute of the CLV (departure angle = 90 deg-exit angle). Top tension is the tension applied to the cable using the onboard caterpillar or wheel tensioner machine. Layback is the horizontal distance between the cable exit point from the CLV and the touch down point (TDP) on the seabed. The bend radius at the TDP is the actual cable radius of curvature, which is one of the most critical design parameters for cable integrity. Bottom tension is the residual tension at the TDP, representing another critical parameter for the cable lifetime.

![Diagram showing the most influential parameters during cable laying](image)

**Figure 1.** Demonstration of the most influential parameters during a cable laying process.

In some cases, local conditions will require the vessel to ground near the landfall location. The limited water depth requires careful management of the cable catenary as a short catenary leaves little room for error and can easily compromise the cable integrity. Furthermore, the limited water depth requires a better accuracy for the cable catenary profile analysis since the part of the cable in the air is a significant proportion of the whole suspended length of the cable and cannot be omitted from the installation analysis. In shallow-water areas, modeling of the “out of water” cable segment is significant for accurate safety factor quantification during a laying operation. In the present paper, the proposed analysis tool is able to provide an accurate assessment of the cable integrity, even in shallow-water areas, using an iterative procedure for modeling of the “out of water” cable segment.

2. **Scope of this Study**

Cable handling and monitoring are important during the cable laying process as the cable can be damaged if the minimum bending radius, actual strain, or other limits are not respected during installation. Typically, during laying from the landfall site towards offshore, the water depth will increase along the route and whilst the cable catenary becomes easier to manage, tension in the cable becomes more important. Due to the fact that conventional installers of power cables do not have the means to accurately estimate the cable tension on the seabed, they operate with very high safety
factors. Consequently, power cables are usually installed with tensions that are much higher than the required values. Cable suspensions occur more often than desired as a result due to the restrained tension created by the friction between the seabed and cable. Between high spots on the seabed, the soil friction acts as a support, both horizontally and vertically, and the segment of the cable maintains a residual tension, creating a catenary suspension. As shown in Figure 2, at these seafloor contact points, large reaction forces (local supports) and small bending radii are common, thereby reducing the lifetime of the cable due to increased wearing and chafing. In order to maximize the lifetime of the cable, the power cable installers must be equipped with an analysis tool to accurately predict the bottom tension in order to lay the cable with low values of bottom tension to avoid cable suspensions, but at the same time, they must maintain a small amount of tension at the touchdown to prevent cable damage.

As it can be realized, accurate prediction of the bottom tension during the S-lay installation of a submarine cable is of great interest to marine civil engineering research since this parameter is very crucial for both the installation and operation of the cable lifetime. Based on different assumptions, numerous theories have been developed for the static and dynamic response of a submarine cable during a laying operation. Among these, Zajac [5] first developed a steady-state theory, in which the cable was modeled as a straight line and excluded the effect of transient motions. Later on, Yoshizawa and Yabuta [6] presented an analytical method for the tension analysis of cables, without considering the tangential drag forces. Wang et al. [7] presented linear and nonlinear methods separately to build up a dynamic model. Vaz and Patel [8] developed a model for prediction of the transient behavior of a cable when the cable ship changes speed during towing operations. Following their previous work, Vaz et al. [9] presented a formulation and numerical solution for the 3D transient behavior of a cable during laying operations. Extending from [9], Vaz and Patel [10] developed the formulation and solution of governing equations used to analyze the 3D behavior of cables subjected to arbitrary sheared currents. Similarly, Wang et al. [11] presented an efficient numerical schemes-boundary condition transformed into a set of nonlinear governing equations with initial values. The vertical movement of the cable ship caused by wave-induced vessel motion adds a non-ignorable tension force at the laying cable. Prpic J. and Nabergoj R. [12,13] presented a two-dimensional model of cable dynamics accounting for the effects of head sea conditions. Yang N. et al. [14] presented a

![Figure 2. High or reduced bottom tension during a cable laying operation.](image-url)
A semi-analytical approximation for a two-dimensional tension analysis of a submarine cable during laying operations, obtaining a set of differential equations to simulate the problem. In summary, all the aforementioned models have completely ignored the influence of the “out of water” cable segment, which significantly affects the cable configuration during laying operations, especially in shallow-water areas. The catenary theory model, both in static [15] and dynamic [16] conditions, was adopted and extended in [17] to support the analytical equations of the proposed analysis method using an additional iterative procedure to combine the submerged catenary configuration and the catenary curve created by the “out of water” cable part at the sea surface. In [17], the results of the method for laying a composite submarine cable at an intermediate water depth were compared with relevant results exported by the commercial Finite Element Analysis (FEA) software RSTAB [18] and were found to be in very good agreement.

In the present paper, an innovative and efficient analysis method is validated in various installation scenarios and an analysis tool (developed from scratch) is proposed for cable installers for the accurate prediction of crucial installation parameters (minimum bending radius and bottom tension at the TDP) and the cable configuration during laying operations, including the “out of water” cable part. Reliable and commercial FEA software is utilized to validate the accuracy and efficiency of the new analysis tool. Furthermore, the importance of modeling the “out of water” part of the cable between the overboard vessel’s chute and the sea surface is investigated. The analysis error in the case of not modeling the “out of water” cable part is significant, especially in shallow-water areas, due to the fact that the part of the cable in air is a non-ignorable proportion of the whole suspended length of the cable. It is proven that when ignoring the modeling of the “out of water” cable segment, the cable integrity can be jeopardized during laying activities. The proposed analysis tool is an innovation for cable laying operations because it can provide quick and accurate results as sophisticated and time-consuming FEA software. The tool allows cable installers to conduct real-time monitoring and re-adjustment of the constant tension activator onboard the CLV to avoid laying the cable with an excess of residual tension on the sea-bottom. In addition, the tool is able to accurately quantify the actual safety factor and predict the correct S-lay configuration during laying, even in shallow-water areas, using an iterative procedure for the combination of two catenary profiles with different properties (in and out of water cable segments).

3. Description of the Proposed Analysis Tool

The proposed analysis tool for the laying of a submarine cable in an S-lay configuration is described here. Details about the mathematical model, analysis assumptions, loads, boundaries, and cable properties are presented.

The composite submarine cable that is used in the present study consists of the following:

1. 3 nos 18/30(36) kV Power Cores, 500 mm² Tinned Copper (Class 2) conductors;
2. 2 nos Fiber Optic Cables, 48 nos Single Mode Fibers;
3. Extruded Shaped Fillers;
4. Binder tape;
5. Anti-Teredo copper tape;
6. Polypropylene bedding yarns;
7. One-layer galvanized steel wire armor;
8. Polypropylene yarns serving (2 nos layers with bitumen).

The submarine cable is simulated using the mechanical parameters provided by the cable manufacturer Prysmian Group [19], which are presented in Table 1 and are used and must be respected at all times during cable handling in any possible operation.
Table 1. Cable mechanical properties.

| Parameter                                | Unit of Measure | Value  |
|------------------------------------------|-----------------|--------|
| Outer diameter                           | mm              | 144    |
| Approximate weight (in air)              | kg/m            | 37     |
| Approximate weight (in water)            | kg/m            | 23     |
| Max straight pulling tension             | kN              | 176    |
| Max sidewall pressure                    | kN/m            | 40     |
| Flexural stiffness [19]                  | Nm²             | 10,000 |
| Minimum allowable bending radius         | m               | 2.2    |

The developed custom-made tool for the cable analysis during installation in an S-lay configuration consists of one main component, which is (a) the cable, and three boundaries: (a) the seabed, (b) the sea surface, and (c) the CLV. The developed tool utilizes analytical equations of a catenary-type structure. The catenary method is probably the most successful attempt to find an approximate solution to the shape of a slender beam lifted at one end from a horizontal plane. The method was originally suggested to find the configuration and stresses in a pipeline with negligible stiffness suspended between the sea floor and an inclined ramp that is free to rotate and hence give a moment-free upper end of the pipeline. These boundary conditions are identical to the ideal conditions for a catenary composite submarine cable with an S-lay configuration, which means that the method should be well-suited to analyze this type of structure. The method is limited to analyzing a uniform beam loaded by its own weight only. Varying cross-sections, buoys, or current forces can therefore not be considered. However, the proposed tool has extended the original method to account for varying cross-sections with different weights, as is the real practice when a non-ignorable length of cable is out of the water between the overboard chute and the sea level during a laying operation in a shallow water area.

The stiffened catenary method was originally suggested by Plunkett [20] and later applied by Dixon and Rutledge [21] to find the configuration and stresses in a pipeline with non-ignorable stiffness suspended between the sea floor and an inclined ramp, as frequently applied for pipe laying. The idea of the stiffened catenary solution is that the bending stiffness causes secondary effects in boundary regions only, and that the deviation from the simple catenary solution can be found as a rapidly converging series expansion. For a cable laying analysis problem, it has been proven that the stiffness of the cable can be ignored in order to minimize the computational effort, without losing the accuracy of the results.

Therefore, in this paper, the extended catenary method is developed and proposed in order to build up the cable catenary configuration. As described above, the catenary theory model [15,16] has been adopted and extended to support the proposed tool analytical equations using an additional iterative procedure to combine the submerged catenary configuration and the catenary curve created by the “out of water” cable part at the sea surface. The reference configuration is assumed to be a straight linear elastic beam with a length equal to the cable’s initial length (elongation due to tension is ignored). In this analysis, the cable self-weight load (different weight in and out of water, see Table 1) is imposed and the tension load “T_tensioner” is applied in a horizontal direction through the caterpillar tensioner (just before the overboard chute of the CLV) to induce necessary tension in the cable, as illustrated in Figure 3. By inducing prescribed translation at the connection node, the cable, along with the vessel, is pulled up above the sea surface. This yields the establishment of the catenary configuration. In this analysis, there is no axial stiffness of the cable present and no sliding along the seabed can occur. Therefore, there is a fixed support on the seabed and a sliding horizontal support on the CLV. The sea surface is the boundary of the different cross-section properties.
The input parameters required by the proposed analysis tool are listed below and presented in Figure 4:

- Water depth, D;
- Distance between the sea level and the overboard chute at the CLV, c;
- Bottom tension at the TDP, H;
- Cable weight in water, q1 (Table 1);
- Cable weight in air, q2 (Table 1).

The basic theory underlying the behavior of catenary lines is utilized. However, an extension of the basic theory is proposed through an iterative procedure to combine the two catenary-type curves consisting of the two different cross-section properties for each cable segment, which are (a) submerged and (b) out of water. This procedure can be further developed to combine three or more segments of different cross-sections in order to analyze various cases studies of the oil and gas and wind power industry, like the installation of umbilicals, pipelines, anchor lines, etc. Connection between the two or more different curves is achieved through the determination of a dummy equilibrium node where the vertical component of the internal force \( V' \) is equal at both curves. The position of this dummy node for our study is at the sea surface elevation, as illustrated in Figure 4. The horizontal component of the internal force \( H \) along the cable configuration is constant and equal to the bottom tension at the TDP,

**Figure 3.** Symbols and notations used for the description of the proposed analysis tool.

**Figure 4.** Symbols used for the mathematical formulation.
as governed by the equilibrium equation on the X axis. Therefore, equilibrium on the Y axis using the vertical component V’ accurately determines the dummy equilibrium node.

The following equations present the parameters defining the catenary shape, while Figure 4 illustrates how the parameters relate to the cable configuration. The step-by-step iterative procedure is presented hereafter.

3.1. STEP 1: Determination of the Submerged Catenary Curve

For \( y = y_1 = D \) and \( q = q_1 \), \( x_1 \) is determined using Equation (1):

\[
x = \frac{H}{q} \text{arccosh} \left( \frac{y \cdot q}{H} + 1 \right).
\]

For \( x = x_1 \), \( S_1 \) and \( V' \) are determined using Equations (2) and (3):

\[
S = \frac{H}{q} \sinh \left( \frac{q \cdot x}{H} \right),
\]

\[
V = H \cdot \sinh \left( \frac{q \cdot x}{H} \right).
\]

where \( S \) is the length along the curved cable configuration and \( V \) is the vertical component of the internal force.

Therefore, at the end of this step, the V’ of the dummy equilibrium node has been calculated and this is followed by the next step with multiple iterations.

3.2. STEP 2: Iterative Procedure to Define the “Correct” Catenary Curve in Air Which Can Be Combined with the Submerged One

For the first iteration, we set \( y = y_2 = D \) and \( q = q_2 \), and then \( x_2 \) is determined using Equation (1) and \( V_2 \) using Equation (3). The iterative procedure will continue until \( V_2 \) is equal to \( V' \). The convergence threshold for the termination of the iterative procedure is set at 0.001 kN. Therefore, at the end of this step, the \( x_2 \) value is determined.

3.3. STEP 3: Determination of Various Parameters for the Catenary Curve in Air

For \( y_2' = y_2 + c \), \( x_2' \) is determined using Equation (1).

For \( x = x_2' \), \( V \) is determined using Equation (3’).

\[
\frac{V}{H} = \sinh \left( \frac{q \cdot x}{H} \right).
\]

\[
\tan \theta = \frac{V}{H} = \sinh \left( \frac{q \cdot x}{H} \right).
\]

where \( \theta \) is the exit angle of the cable from the overboard chute on the CLV (Figure 3).

The exit angle \( \theta \) is equal to [90 deg-departure angle] (Figure 1).

3.4. STEP 4: Determination of the Combined Catenary Curve Configuration (In and out of Water)

For \( x = x_2 \) and \( q = q_2 \), \( S_2,1 \) is determined using Equation (2).

For \( x = x_2' \) and \( q = q_2 \), \( S_2 \) is determined using Equation (2).

\[
S_2, 2 = S_2 - S_2, 1,
\]

\[
S = S_1 + S_2, 2,
\]

\[
a/2 = x_1 + (x_2' - x_2),
\]
where \( \alpha/2 \) represents the layback, which is horizontal distance between the TDP on the seabed and the exit point on the overboard chute on the CLV.

In order to plot the combined catenary curve, the following equations are used independently for each different part (a) submerged and (b) in air of the combined catenary curve.

\[
y = \frac{H}{q} \left( \cosh \frac{q \cdot x}{H} - 1 \right).
\]

For each point, the following equations are valid and determine the tangential internal force \( T \):

\[
T = \sqrt{H^2 + V^2},
\]

\[
T = H + q \cdot y.
\]

The horizontal tension load \( T_{\text{tensioner}} \) (Figure 3) is practically applied to the cable through the constant tension activator (usually a wheel or caterpillar tensioner). The cable is passed through the tensioner and after that, it follows in a roller way before reaching the overboard chute. Contact between the cable and overboard chute creates a friction force which is calculated using the following formula (Capstan equation [22]):

\[
T_{\text{out}} = T_{\text{in}} \cdot e^{\beta \mu},\]

where \( \beta = 0 \) angle (equal to the exit angle in radians) between the vectors of \( T_{\text{in}} \) and \( T_{\text{out}} \) and \( \mu \) is the friction coefficient between the cable and overboard chute.

For our case, we set the following notations:

\( T_{\text{in}} \) is the tension applied by the constant tension activator and

\( T_{\text{out}} \) is the tangential internal force at the exit point of the cable at the overboard chute on the CLV.

\[
y' = \sinh \left( \frac{q \cdot x}{H} \right),\]

\[
y' = \tan(\varphi),\]

\[
\varphi = \arctan(y'),\]

where \( \varphi \) is the tangent angle between two consecutive nodes.

The proposed analysis tool has the ability to split the total length of the cable and generate as many nodes as the user desires. The determination of the minimum radius of curvature is more accurate as there are so many nodes that the total length of the cable is split. The user is able to perform a sensitivity analysis in order to determine the limit when the computational time vs. The accuracy is the desired one.

\[
y'' = \frac{q \cdot \cosh \left( \frac{q \cdot x}{H} \right)}{H},\]

\[
R = \frac{1 + y'^2}{y''},\]

\[
R = \frac{1 + (\sinh \left( \frac{q \cdot x}{H} \right))^2}{\frac{q \cdot \cosh \left( \frac{q \cdot x}{H} \right)}{H}},\]

where \( R \) is the radius of curvature between two consecutive nodes.

The actual safety factor (SF) during a submarine cable laying operation is given by the following Equation (14). The safety factor is the ratio of the minimum actual radius of curvature between two consecutive nodes along the S-curve configuration, as per Equation (13.2), to the minimum allowable
The basic assumptions regarding the components and the boundary conditions of the proposed model are as follows:

- No flexural and axial stiffness is taken into consideration for the composite cable modeling;
- The CLV is modeled as a fixed vertical support using the overboard chute. It also applies a horizontal tensile force to the cable through the constant tension activator. Friction between the overboard chute and cable is numerically modeled using Equation (10);
- The sea surface is taken into consideration through the different weight of the cable (in and out of the water);
- The seabed is analyzed as a fixed support in all translational degrees of freedom. Seabed support is implemented at the TDP (X = 0, Y = 0, in Figure 4). No cable-soil interaction is taken into consideration.

4. Validation of the Proposed Analysis Tool

Validation of the proposed analysis and design tool is presented in this section. For this reason, the case of an installation of a composite submarine cable using a CLV at a 93 m water depth has been selected for analysis using two different numerical methods:

- The custom-made analysis and design tool that utilizes analytical equations of a catenary-type structure (presented above in Section 3), and
- An FEA model utilizing the structural analysis commercially available software RSTAB [18].

The installation model developed with the 3D FEA structural software RSTAB [18] consists of the same components and boundaries as the relevant proposed analysis tool. A non-linear static analysis is initially conducted to build up the catenary configuration. The most general way to find the static condition for a flexible composite submarine cable during S-lay configuration is to define a stress-free condition and introduce load contributions. Load types will normally consist of volume forces (weight and buoyancy), prescribed displacements at nodes with given boundary conditions, friction forces between the cable and sea bottom, etc. The initial (stress-free) condition cannot have any curvature and will hence be significantly different from the static shape. Figure 5 illustrates how one end of the cable must be moved down to the sea bottom, while the other must be moved to the overboard chute of the CLV.

![Initial (stress-free) and static condition of a submarine cable in S-lay configuration.](image)

The number of load increments within a load group might be high—often in the order of 100. This is particularly the case for the prescribed displacements. Note that equilibrium must be obtained
for each load increment, which might be difficult at some intermediate end positions between initial and final positions, especially when the number of load increments is low. The reference configuration of the cable in the stress-free condition (Figure 5) is assumed to be a straight linear elastic beam with a length equal to the cable’s initial length (part of the cable under the analysis process that the user is able to define). All the supports of the cable beam elements in the stress-free condition are modeled as diagram-type non-linear springs, active in tension and compression loads, which can accurately apply the actual boundaries of the model. In the non-linear static analysis, the cable self-weight load, different for the segments of cable in and out of the water, is imposed and a tension load T is applied in the horizontal direction through the caterpillar tensioner just before the overboard chute at the CLV. This yields the establishment of a catenary configuration. Non-linear spring supports are utilized to model the cable–soil interaction at the TDZ on the seabed and any possible free-span scenario in the case of an uneven bottom profile that can be inserted using the relevant diagrams for non-linear spring supports. Non-linear spring supports are utilized for the modeling of the overboard chute and the roller supports right after the tensioner onboard the CLV (Figure 3), in order to apply friction between the cable and the handling accessories, as done in the proposed analysis tool using Equation (10).

The input parameters required by the FEA software RSTAB are as follows:

- The water depth, D (Figure 3);
- Distance from the sea level of the overboard chute at the CLV, c;
- Geometry of the overboard chute onboard the CLV (Figures 6 and 7);
- Bottom tension at the TDP, H;
- Cable weight in water, q1;
- Cable weight in air, q2;
- Non-linear diagrams (reaction force N/deflection mm) for the cable–soil interaction in two directions (X, Y);
- Composite cable flexural stiffness, EI (Table 1);
- Composite cable axial stiffness, EA (Table 1).

![Figure 6. Geometry of the overboard chute dimensions in mm.](image-url)

Each cable element has been modeled as a beam element which is an elastic 3D element with constant axial strain and torsion consisting of two nodes. Each element includes six degrees of freedom per node. The element has a circular cross-section with a constant radius along its length. A linear material model is used with properties for elastic cable elements since the target of all the analysis steps is to keep the cable in an elastic safe limit (not a failure analysis problem). The software uses the full Newton-Raphson large deformation analysis procedure, in which the stiffness matrix is updated at every equilibrium iteration. The maximum number of iterations per load increment is set at 600 and the number of load increments at 700. Many attempts have been conducted with less load increments and a lower maximum number of iterations; however, due to the large deformations required to achieve the static condition starting from the initial stress-free condition (Figure 5), all of them terminated due to convergence inability at intermediate steps. Shear deformations are also taken into consideration. Cross-section and material properties are modified accordingly to correspond to an equivalent section with mechanical parameters provided by the cable manufacturer (Table 1).
The soil properties are defined by the cable–soil non-linear interaction spring supports model. This model allows for user-defined friction coefficients in different directions. The soil resistance is defined in three directions: vertically, laterally, and axially.

The Coulomb friction coefficients presented in Table 2 are used to represent the soil conditions, where $\mu_x = 0.4$ (conservative value of the friction coefficient along the cable axis) and $\mu_y = 0.8$ (the lateral degree of freedom will be ignored).

| Seabed Type | Direction | Stiffness (kN/m$^2$) | Friction Coefficient |
|-------------|-----------|----------------------|---------------------|
| Sand        | Axial     | 100 to 250           | 0.4–0.6             |
| Sand        | Lateral   | 50 to 100            | 0.8                 |
| Sand        | Vertical  | 200 to 10,000        | -                   |

The vertical resistance of all model nodes (except the ones onboard the CLV) is expressed as a non-linear spring, which starts to be active at $Y = -D$ (depth) and afterwards follows soil vertical resistance behavior, as indicated in Table 2.

The basic assumptions regarding the components and the boundary conditions of the FEA model in RSTAB software are as follows:

- Flexural stiffness is taken into consideration for the composite cable modeling (Table 1);
- The CLV is modeled as a non-linear vertical support. A vertical reaction is applied when the cable touches the configuration of the overboard chute (Figure 6). The CLV also applies a horizontal tensile force to the cable through the constant tension activator. Friction between the overboard chute and cable is modeled using non-linear spring supports, as per Equation (10);
- The sea surface is taken into consideration through the different weights of the cable segments in and out of the water;
- The seabed is analyzed as non-linear vertical and horizontal support. A full cable–soil interaction is taken into consideration.
A comparative study of the two methods (proposed analysis tool vs. FEA model in RSTAB) has been conducted and is presented below to allow for an effective validation of the innovative analysis tool for the submarine cable laying installation. Three different cases are analyzed using the two models, as described above in Sections 3 and 4, respectively. Each case corresponds to a different bottom tension at the TDP. The bottom tension is a very crucial parameter for the safe installation of a submarine cable, both in shallow and deep waters. A different bottom tension at the TDP means that the whole configuration of the S-curve has been changed and thus the following parameters are affected:

- Minimum bending radius (MBR);
- Exit angle from the overboard chute (θ, angles);
- Catenary length;
- Layback, which is the position of the TDP;
- Horizontal, vertical, and tangential resultant force at the overboard chute (H, V, and T, respectively).

Different values of bottom tension mean different equipment onboard the CLV (capacity-wise) for the cable installer, a different configuration of the submarine cable during laying, a different minimum bending radius at the TDP, and thus a different safety factor during the laying process.

MBR is the minimum radius of curvature along the S-curve configuration. In most cases, in typical S-lay operations, the MBR is spotted at the TDZ, just before the cable touches the sea bottom. The MBR is a crucial parameter for the integrity of the cable because it is an alternative way to measure the bending stresses and easily compare those with the recommended limits provided by the cable manufacturer (Table 1).

Three cases with bottom tension values of H1 = 1200 kg, H2 = 2000 kg, and H3 = 4000 kg and a water depth of 93 m were analyzed using the analytical proposed analysis tool. The most important parameters for the installation process were exported by the analysis tool and are presented in Table 3.

| Bottom Tension H (kg) | Layback Distance (m) | Catenary Length (m) | Exit Angle θ (degs) | Min. Bending Radius (m) | T Tensioner Constant Tension Adjustment (kg) |
|----------------------|----------------------|---------------------|---------------------|-------------------------|--------------------------------------------|
| 1200                 | 89.029               | 139.002             | 69.714              | 52.173                  | 895                                        |
| 2000                 | 119.717              | 161.306             | 62.008              | 86.956                  | 1664                                       |
| 4000                 | 175.462              | 206.797             | 50.293              | 173.914                 | 3429                                       |

It is easily understandable that as the bottom tension value increases, adjustment on the constant tension activator onboard the CLV should be increased in order to achieve the desired bottom tension. Furthermore, as the bottom tension value increases, the layback distance (Figure 3, presented as “a/2”) and the catenary length (Figure 3, presented as “S”) increase as well, since the TDP is drawn away from the CLV. The exit angle (Figure 3) is decreased and the cable exit from the overboard chute becomes smoother at each step of bottom tension increase. Last but not least, as the bottom tension increases, the actual minimum bending radius is increased as well, which means that the safety margin for the integrity of the cable during the laying process is increased. It should be remembered that a high bottom tension (higher than the required one) may create free spans along an uneven sea floor. Exported S-lay configurations for different bottom tension values using the proposed analysis tool are illustrated in Figure 8.
Figure 8. Exported S-lay configurations for different bottom tension values obtained using the proposed analysis tool.

The same cases were inserted in the 3D FEA structural software RSTAB using the parameters as described above in Section 4. The most important parameters for the installation process were exported by the software and tabulated below (Table 4), in order to compare them with those exported by the proposed analysis tool. Exported S-lay configurations for different bottom tension values using the FEA structural software RSTAB are illustrated in Figure 9.

Table 4. Results obtained using the 3D FEA structural software RSTAB.

| Bottom Tension H (kg) | Layback Distance (m) | Catenary Length (m) | Exit Angle θ (degs) | Min. Bending Radius (m) | Tensioner Constant Tension Adjustment (kg) |
|----------------------|----------------------|---------------------|---------------------|-------------------------|------------------------------------------|
| 1200                 | 92.70                | 141.62              | 67.815              | 56.10                   | 895                                      |
| 2000                 | 125.20               | 165.90              | 60.10               | 93.00                   | 1664                                     |
| 4000                 | 183.90               | 214.60              | 48.40               | 185.30                  | 3429                                     |

As it can be realized, comparing the relevant results (Tables 3 and 4) between the two different numerical methods, the difference for all the critical parameters is less than 6.5%. In general, the flexural stiffness of the cable can be ignored, significantly improving the computational effort without sensibly demoting the accuracy of the results. The plotted cable configurations exported by both numerical methods are illustrated for visual comparison in Figure 10.

The comparative study presented above proves that the proposed analytical tool for the cable laying analysis provides reliable and safe results, similar to those obtained with sophisticated and time-consuming 3D FEA commercially available software. The divergence between the results of the two methods is logical since the flexural stiffness of the composite cable was considered negligible for the proposed analytical tool. Therefore, more S-curves are exported by the FEA software, even with a bigger MBR. In summary, the proposed tool can be considered fully reliable for cable laying analysis and design activities using much less computational effort and allowing the cable installers to conduct a real-time analysis and monitoring of the laying process.
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Figure 9. Exported S-lay configurations for different bottom tension values obtained using the FEA structural software RSTAB.

Figure 10. Exported S-lay configurations for different bottom tension values using both the proposed analysis tool and the FEA software.
5. Significance of the Modeling of the “Out of Water” Cable Segment

The influence of the inclusion of the “out of water” cable segment in the mathematical modeling has been investigated, especially for shallow-water applications, and the results are presented below.

Two different scenarios have been analyzed and the relevant results have been compared to evaluate the influence for modeling of the “out of water” cable segment:

- Scenario No1: Modeling of both cable segments, assuming “submerged” represents the part between the touch down point and the sea surface and “in air” represents the part between the sea surface and the last point touching the overboard chute on the cable lay vessel, and
- Scenario No2: Modeling of both cable segments with the same weight. Common weights for both segments have been chosen for the weight in water.

Both of the above described scenarios have been analyzed assuming different water depth values starting from a 3 m water depth and ending at a 15 m water depth. Furthermore, as the bottom tension input, a set of three different runs were conducted, assuming the values of 1200/2000/4000 kg (Tables 5–7). Specific bottom tension values were chosen for the comparative study because these are the common constant tension activator (tensioner) available capacities in the cable industry for a submarine cable installation up to a 200 m water depth.

Table 5. Comparative study of various installation scenarios using the proposed analysis tool: Scenario No1 vs. Scenario No2 assuming bottom tension = 1200 kg.

| Water Depth (m) | Layback Distance (m) | Catenary Length (m) | Exit Angle (deg) | MBR (m) |
|-----------------|----------------------|---------------------|-----------------|---------|
|                 | Scen 1 | Scen 2 | Differ (%) | Scen 1 | Scen 2 | Differ (%) | Scen 1 | Scen 2 | Differ (%) | Scen 1 | Scen 2 | Differ (%) |
| 3               | 24.73  | 25.39  | 2.66      | 25.81  | 26.40  | 2.30      | 30.39  | 26.84  | 11.67    | 36.27  | 52.17  | 43.85     |
| 5               | 28.64  | 29.05  | 1.44      | 30.22  | 30.58  | 1.19      | 33.39  | 30.37  | 9.02     | 38.95  | 52.17  | 33.95     |
| 7               | 31.98  | 32.27  | 0.91      | 34.12  | 34.36  | 0.72      | 35.99  | 33.37  | 7.28     | 41.72  | 52.17  | 25.06     |
| 9               | 34.94  | 35.16  | 0.63      | 37.70  | 37.88  | 0.48      | 38.30  | 35.98  | 6.06     | 44.59  | 52.17  | 17.01     |
| 11              | 37.62  | 37.80  | 0.46      | 41.05  | 41.19  | 0.34      | 40.37  | 38.29  | 5.16     | 47.55  | 52.17  | 9.71      |
| 13              | 40.09  | 40.24  | 0.36      | 44.24  | 44.35  | 0.25      | 42.25  | 40.36  | 4.46     | 50.61  | 52.17  | 3.10      |
| 15              | 42.39  | 42.51  | 0.28      | 47.29  | 47.38  | 0.19      | 43.96  | 42.24  | 3.90     | 52.17  | 52.17  | 0.00      |

Table 6. Comparative study of various installation scenarios using the proposed analysis tool: Scenario No1 vs. Scenario No2 assuming bottom tension = 2000 kg.

| Water Depth (m) | Layback Distance (m) | Catenary Length (m) | Exit Angle (deg) | MBR (m) |
|-----------------|----------------------|---------------------|-----------------|---------|
|                 | Scen 1 | Scen 2 | Differ (%) | Scen 1 | Scen 2 | Differ (%) | Scen 1 | Scen 2 | Differ (%) | Scen 1 | Scen 2 | Differ (%) |
| 3               | 32.06  | 32.90  | 2.62      | 32.90  | 33.69  | 2.40      | 24.11  | 21.18  | 12.14    | 57.85  | 86.96  | 50.32     |
| 5               | 37.17  | 37.70  | 1.42      | 38.41  | 38.89  | 1.26      | 24.62  | 24.10  | 9.49     | 60.45  | 86.96  | 43.84     |
| 7               | 41.55  | 41.92  | 0.89      | 43.23  | 43.56  | 0.77      | 28.84  | 26.61  | 7.73     | 63.11  | 86.96  | 37.79     |
| 9               | 45.44  | 45.72  | 0.62      | 47.61  | 47.86  | 0.52      | 30.83  | 28.83  | 6.49     | 65.82  | 86.96  | 32.11     |
| 11              | 48.99  | 49.21  | 0.45      | 51.69  | 51.88  | 0.37      | 32.64  | 30.82  | 5.82     | 68.60  | 86.96  | 26.76     |
| 13              | 52.26  | 52.44  | 0.35      | 55.53  | 55.68  | 0.28      | 34.30  | 32.63  | 4.67     | 71.42  | 86.96  | 21.57     |
| 15              | 55.32  | 55.47  | 0.27      | 59.18  | 59.31  | 0.21      | 35.83  | 34.30  | 4.29     | 74.30  | 86.96  | 17.03     |
Table 7. Comparative study of various installation scenarios using the proposed analysis tool: Scenario No1 vs. Scenario No2 assuming bottom tension = 4000 kg.

| Water Depth (m) | Layback Distance (m) | Catenary Length (m) | Exit Angle (deg) | MBR (m) |
|----------------|----------------------|---------------------|------------------|---------|
|                | Scene 1 | Scene 2 | Differ (%) | Scene 1 | Scene 2 | Differ (%) | Scene 1 | Scene 2 | Differ (%) | Scene 1 | Scene 2 | Differ (%) |
| 3              | 45.49   | 46.67   | 2.60       | 46.09   | 47.23   | 2.48       | 17.37   | 15.19   | 12.52      | 111.87  | 173.91  | 55.46      |
| 5              | 52.78   | 53.52   | 1.40       | 53.66   | 54.37   | 1.32       | 19.26   | 17.36   | 9.87       | 114.42  | 173.91  | 52.00      |
| 7              | 59.05   | 59.56   | 0.88       | 60.25   | 60.74   | 0.81       | 20.95   | 19.25   | 8.70       | 116.99  | 173.91  | 48.66      |
| 9              | 64.64   | 65.03   | 0.60       | 66.19   | 66.56   | 0.55       | 22.49   | 20.94   | 6.67       | 119.59  | 173.91  | 45.43      |
| 11             | 69.74   | 70.05   | 0.44       | 71.67   | 71.96   | 0.30       | 23.90   | 22.48   | 5.42       | 122.22  | 173.91  | 42.29      |
| 13             | 74.47   | 74.72   | 0.34       | 76.81   | 77.04   | 0.30       | 25.21   | 23.89   | 5.22       | 124.87  | 173.91  | 39.27      |
| 15             | 78.89   | 79.10   | 0.26       | 81.67   | 81.85   | 0.23       | 26.43   | 25.21   | 4.62       | 127.55  | 173.91  | 36.35      |

Summarizing the comparative study above, the layback distance and the catenary length are not significantly influenced by the modeling of the “out of water” cable segment. Contrary to this, the cable exit angle and MBR are drastically affected by the modeling of the “out of water” cable segment, especially in shallow waters. Regarding the cable exit angle parameter, the maximum calculated variation between the two scenarios is 12.52% and is present in shallow waters (water depth = 3 m). Regarding the MBR parameter, the maximum calculated variation between the two scenarios is 55.46% and is similarly present in shallow waters (water depth = 3 m).

6. Conclusions

The main purpose of this study was to propose an effective analysis and design tool for cable laying operations. The validation study presented above proves that the proposed custom-made tool provides reliable and safe results, even in shallow-water areas, similar to sophisticated and time-consuming commercially available FEA software. The differences between the results of the two methods are reasonable. Therefore, more S-curves are exported by the FEA software, even with a bigger minimum bending radius.

In addition, as per the study conducted regarding the influence of the modeling of the “out of water” cable segment, guidance for all cable installers should be provided to take into consideration the segment of cable between the sea surface and overboard chute assuming the correct weight. According to the results presented, the analysis error in the case of not modeling the “out of water” cable part with the correct weight for the calculation of the minimum bending radius, especially in shallow-water areas, is not acceptable and may jeopardize the integrity of the submarine cable. The cable segment “out of water” should be modeled for an accurate prediction of the minimum bending radius (actual bending radius of curvature) in shallow-water areas, where the cable laying operation is riskier and more elegant. The analysis error for the calculation of the minimum bending radius, as presented above, is not acceptable (55.46% difference) and the modeling of the “out of water” part is a must for a safe cable laying operations in swallow-water areas.

The analysis of various installation scenarios for a submarine cable laying operation in an S-lay configuration under different bottom tension values and varying water depths using the proposed analysis tool is under implementation and the results will be presented in future work as guidance for all cable installers. The dynamic movement of the cable and the influence of the cable tension due the movement of the cable laying vessel are two key problems to be solved during the cable laying operation simulation. Both of them are under implementation to be added for the extension of both numerical tools presented in this study and to evaluate whether it is important that they are modeled and which cases can be ignored, without losing the results’ accuracy, in order to minimize the computational effort.
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