Power Corrections to Pion Transition Form Factor in Perturbative QCD Approach

Yue-Long Shen\textsuperscript{*a}, Zhi-Tian Zou\textsuperscript{†b}, and Ying Li\textsuperscript{‡b}

\textsuperscript{a}College of Information Science and Engineering, Ocean University of China, Qingdao, Shandong 266100, P.R. China

\textsuperscript{b}Department of Physics, Yantai University, Yantai 264005, China

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Abstract

In this paper we calculate the power corrections to the pion transition form factor within the framework of perturbative QCD approach on the basis of $k_T$ factorization. The power suppressed contributions from higher twist pion wave functions and the hadronic structure of photon are investigated. We find that there exists strong cancellation between the two kinds contributions, thus the total power corrections considered currently are very small, and the prediction of the leading power contribution with joint resummation improved perturbative QCD approach is almost unchanged. This result confirms that the pion transition form factor is a good platform to constrain the nonperturbative parameters in pion wave functions. Moreover, our result can accommodate the anomalous data from BaBar, or agrees with results from Belle according to the choice of Gegebauer moment in the pion wave function, and the more precise experimental data from Belle-II is expected.

\textsuperscript{*}shenylmeteor@ouc.edu.cn
\textsuperscript{†}zouzt@ytu.edu.cn
\textsuperscript{‡}liying@ytu.edu.cn
1 Introduction

The pion-photon transition process $\gamma^*\gamma \rightarrow \pi^0$ provides a golden place to test the strong interaction dynamics of hadronic reactions in the framework of QCD [1–7]. The asymptotic and soft behaviors of the pion transition form factor $F_{\gamma^*\gamma \rightarrow \pi^0}(Q^2)$ have been already given as

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\gamma^*\gamma \rightarrow \pi^0}(Q^2) = \sqrt{2} f_\pi = 0.185,$$

$$\lim_{Q^2 \rightarrow 0} F_{\gamma^*\gamma \rightarrow \pi^0}(Q^2) = \frac{\sqrt{2}}{4\pi^2 f_\pi},$$

where $Q^2$ stands for the momentum transfer squared carried by the virtual photon, and the pion decay constant is $f_\pi = 0.131$GeV. The former had been predicted within perturbative QCD (PQCD) in the collinear factorization theorem, and the latter one could be determined from the axial anomaly in the chiral limit. In 2009, the experimental result of BaBar [8] on $F_{\gamma^*\gamma \rightarrow \pi^0}(Q^2)$ exhibits an intriguing dependence on $Q^2$: for $Q^2 > 10$GeV$^2$, $F_{\gamma^*\gamma \rightarrow \pi^0}(Q^2)$ lies above Eq.(1) and continues to grow up to $Q^2 \approx 40$GeV$^2$. In contrast to BaBar, Belle also presented their measurements in the region $4$GeV$^2 \leq Q^2 \leq 40$GeV$^2$ [9], and the rapid growth in the higher $Q^2$ region does not appear. Since there is no final confirmation on this discrepancy, various phenomenological approaches as well as lattice QCD simulations (see for instance [10–12]) have been employed to explain the data.

To accommodate the anomalous BaBar data at high $Q^2$, one approach is to introduce an “exotic” twist-two pion light-cone distribution amplitude (LCDA) with the non-vanishing end-point behavior [13, 14], but it was found to be equivalent to the introduction of a sizable nonperturbative soft correction from the transverse momentum dependent (TMD) pion wave function [15]. Actually, in the framework of $k_T$ factorization [16], the “exotic” wave function is not necessary [17]. At leading power the pion transition form factor has been studied with perturbative QCD (PQCD) approach based on $k_T$ factorization at one-loop level [18–20], and the resummation of the large TMD logarithms and threshold logarithms lead to the Sudakov factor and jet function, and an appropriate parameterization of the latter one can be used to explain the Babar data. To avoid light-cone singularity in the TMD wave function, an off-light-cone vector should be included in the definition of the wave function [21, 22], then
additional rapidity logarithms arise\(^1\). Taking advantage of joint resummation technique one can resum the large logarithms \(\ln^2 \frac{k^2}{Q^2}\) and \(\ln^2 x\) and \(\ln \zeta^2\) simultaneously \([26]\). Using the joint resummation improved factorization formula, the BaBar data can also be explained if an appropriate Gegenbauer moment of pion is employed.

In the PQCD framework based on \(k_T\) factorization, though the next-leading order of \(\alpha_s\) correction at the leading power contribution to this process has been studied \([17, 18]\), the higher power corrections have not been investigated till now. In fact, the scaling violation implied by the BaBar data \([8]\) also indicates the importance of subleading power corrections.

The next-to-leading power\(^{(\text{NLP})}\) effects have been extensively studied in the collinear factorization framework, the contribution from higher twist pion LCDA\(\text{s}\) \([27]\), the hadronic structure of photon \([28]\) were considered, and in \([15, 29]\), the soft correction to the leading twist effect is evaluated with the dispersion approach and found to be crucial to suppress the contributions from higher Gegenbauer moments of the twist-2 pion LCDA \([26, 30]\). Power suppressed contributions in collinear factorization in general suffer from endpoint singularity and factorization breaks down. Alternative approaches such as dispersion approach need to be employed, but large uncertainty arises when quark hadron duality assumption is employed. It is well known to us that in PQCD approach the transverse momentum of parton can regularize the endpoint singularity so that the factorization is expected to work at NLP. Since the NLP corrections have not been studied previously, the aim of this article is to study the its contributions to the pion transition form factors within the PQCD approach that is based on \(k_T\) factorization.

In the current work, we shall consider two kinds of power suppressed contributions, which are from the higher twist pion wave functions and hadronic structure of photon, respectively.

The outline of this paper is as follows: in section 2 we review the \(k_T\) factorization and the joint resummation of \(F_{\gamma^*\gamma\to\pi^0}(Q^2)\) at the leading power. In Sec.3, the analytic calculation of

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\(^1\)Actually there is additional self-energy divergence attributed to the infinitely long dipolar Wilson lines existing \([23]\) after the off-light-cone vector is adopted, and a more complicated definition of TMD wave function with the dipolar Wilson links and the complicated soft subtraction \([24]\), or with non-dipolar off-light-cone Wilson links \([25]\), is required. The definition of the wave function is essential in the soft subtraction, while for phenomenological application, it is more important to extract the hard kernel, and the traditional definition is still adopted in the present paper.
the power suppressed contributions will be presented. The numerical results and discussions are given in section 4. We will summarize this work in the last section.

2 Factorization and Resummation at Leading Power

The pion transition form factor is defined via the matrix element

$$\langle \pi(p)|j_{\mu}^{\text{em}}|\gamma(p')\rangle = g_{\text{em}}^2 \epsilon_{\mu\alpha\beta\gamma} q^\alpha p'^\beta \epsilon^\nu(p') F_{\gamma\rightarrow\pi^0}(Q^2),$$

(3)

where $q = p - p'$, $p$ and $p'$ refer to the four-momentum of the pion and the on-shell photon respectively, the electro-magnetic current

$$j_{\mu}^{\text{em}} = \sum_q g_{\text{em}} Q_q \bar{q} \gamma_\mu q.$$  

(4)

In $k_T$ factorization framework, the pion transition form factor is factorized into the convolution of the TMD wave function of pion and the hard kernel. The $k_T$ factorization theorem can be derived diagrammatically [31] by applying the eikonal approximation to collinear particles and the Ward identity to the diagram summation in the leading infrared regions. The TMD wave function of pion is defined by

$$\Phi(u, k_T, \zeta, \mu_f) = \int dy^+ dy_T \frac{d^2 p_T}{(2\pi)^2} e^{-iup^+ + ik_T y_T}$$

$$\times \langle 0|\bar{q}(y)W_y(v)I_{v,y,0}W_0(v)\not{q} + \gamma_5 q(0)|\pi(p)\rangle,$$

(5)

where $\mu_f$ is the factorization scale, the coordinate $y = (y^+, 0, y_T)$, and $up^-$ and $k_T$ are the longitudinal and transverse momenta carried by the anti-quark $\bar{q}$, respectively. The Wilson line

$$W_y(v) = \mathcal{P} \exp \left[ -ig \int_0^\infty d\lambda v \cdot A(y + \lambda v) \right],$$

(6)

has been introduced for maintaining the gauge invariance, where $\mathcal{P}$ denotes the path-ordered exponential. The avoid light-cone singularity, the non-light-like vector $v$ is employed [21]. The transverse gauge link $I_{v,y,0}$ does not contribute in the covariant gauge [22].

For pion transition form factor, the next-to-leading order(NLO) hard kernel has been calculated in [18]. The non-light-like vector $v$ will give rise to a new rapidity parameter
\[ \zeta^2 = \frac{4(n-v)^2}{v^2} \] in both the NLO corrections to wave function and hard kernel. For the rapidity parameter \( \zeta^2 = 2 \), the \( k_T \) factorization formula at leading power of \( 1/Q^2 \) under the conventional resummations was given by [17]

\[
F(Q^2) = \frac{2 f_\pi}{3} \int_0^1 d\mu \int_0^\infty b db \Phi(u, b, t) e^{-S(u, b, Q, t)} S_t(u, Q) \times K_0(\sqrt{u}b) \left[ 1 - \frac{\alpha_s(t) C_F}{4\pi} \left( 3 \ln \frac{t^2 b}{2\sqrt{u} Q} + \gamma_E + 2 \ln u + 3 - \frac{\pi^2}{3} \right) \right],
\] (7)

where \( t \) is the factorization scale. The Sudakov factor \( S(u, b, Q, t) \) sums the double logarithm \( \ln^2(k^2_T/Q^2) \) and the single logarithm \( \ln(t^2/Q^2) \), where the impact-parameter \( b \) is conjugated to the transverse momentum, and it is more convenient to resum large logarithms in \( b \)-space than in transverse momentum space. The threshold factor from the resummation of \( \ln^2 u \) has been parameterized as

\[
S_t(u, Q) = \frac{2^{1+c(Q^2)} \Gamma(\frac{3}{2} + c(Q^2))}{\sqrt{\pi} \Gamma(1 + c(Q^2))} \left[ u(1-u) \right]^{c(Q^2)},
\] (8)

It was found that the nontrivial \( Q^2 \) dependence of the factor \( c(Q^2) \) is important in the explanation of BaBar data [17]. Since the NLO QCD corrections will generate the mixed logarithm \( \ln u \ln(\zeta^2 P^{-2}/k^2_T) \) in both the pion wave function and the hard kernel, the double logarithms need to be resummed. In [26], an evolution equation has been constructed to resum the mixed logarithm \( \ln u \ln(\zeta^2 P^{-2}/k^2_T) \). It is more convenient to perform the resummation in the moment and impact parameter space, and the result reads

\[
\tilde{\Phi}(N, b, \zeta^2, \mu_f) = \exp \left\{ - \int_{\zeta_0^2}^{\zeta^2} \frac{d\tilde{\zeta}^2}{\zeta^2} \left[ \int_{\mu_0(\tilde{\zeta})}^{\mu_1(\tilde{\zeta})} \frac{d\tilde{\mu}}{\tilde{\mu}} \lambda_K(\tilde{\mu}) \theta \left( \mu_1(\tilde{\zeta}) - \mu_0(\tilde{\zeta}) \right) \right] \right\}
+ \frac{3}{2} \int_{\mu_i}^{\mu_f} \frac{d\tilde{\mu}}{\tilde{\mu}} \frac{\alpha_s(\tilde{\mu}) C_F}{\pi}
\]
\[
\tilde{\Phi}(N, b, \zeta_0^2, \mu_i),
\] (9)

\[
\tilde{H}(N, b, \zeta^2, Q^2, \mu_f) = \exp \left\{ - \int_{\zeta_0^2}^{\zeta^2} \frac{d\tilde{\zeta}^2}{\zeta^2} \left[ \int_{\mu_0(\tilde{\zeta})}^{\mu_1(\tilde{\zeta})} \frac{d\tilde{\mu}}{\tilde{\mu}} \lambda_K(\tilde{\mu}) \theta \left( \mu_1(\tilde{\zeta}) - \mu_0(\tilde{\zeta}) \right) \right] \right\}
- \frac{3}{2} \int_{\mu_i}^{\mu_f} \frac{d\tilde{\mu}}{\tilde{\mu}} \frac{\alpha_s(\tilde{\mu}) C_F}{\pi}
\]
\[
\tilde{H}(N, b, \zeta_1^2, Q^2, t),
\] (10)

where \( \lambda_K = \frac{\alpha_s C_F}{2\pi} \), \( \mu_i \) is the initial scale of the RG evolution. The bounds \( \zeta_0^2 \) and \( \zeta_1^2 \) are chosen in order to eliminate the large logarithms in the initial conditions of the pion wave
function and the hard kernel. The joint-resummation improved pion wave function modifies both the longitudinal and transverse momentum distributions, and both the small $u$ and $b$ regions are more highlighted after resummation. In the pion transition form factor, by choosing appropriate $\zeta_1$, the hard kernel without large double logarithms reads

$$H^{(1)}(u, k_T, \zeta_1^2, Q^2, t) = -\frac{\alpha_s(t) C_F}{4\pi} \left( 3 \ln \frac{t^2}{uQ^2 + k_T^2} + \ln 2 + 2 \right).$$

(11)

If a specific model of pion wave function has been employed, the resummation improved wave function can be transformed to the momentum space, and the joint-resummation improved factorization formula for the pion transition form factor is obtained as

$$F(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 du \int_0^\infty b db \Phi(u, b, \zeta_1^2, t) K_0(\sqrt{uQ} b) \times \left[ 1 - \frac{\alpha_s(t) C_F}{4\pi} \left( 3 \ln \frac{t^2 b}{2\sqrt{uQ}} + \ln 2 + 2 \right) \right],$$

(12)

where the expression the joint resummation improved wave function $\Phi(u, b, \zeta_1^2, t)$ can be found in the ref. [26]. In Eq. (12) all the large logarithmic terms are collected by the resummed wave function, and the hard kernel is free from large logarithms.

### 3 Subleading Power Corrections

In this section we aim at investigating the subleading power corrections to the pion transition form factor. As the transverse momentum of the parton of the pion meson is kept in the PQCD approach, the endpoint singularity does not appear, thus we can still take advantage of factorization method to evaluate the power suppressed contributions. As claimed in [15], a power-like falloff of the form factor in the large-$Q^2$ limit can be generated by both “direct” photon and “hadronic” photon contributions. The former one indicates that the hard subgraph includes both photon vertices, which starts from leading power $\mathcal{O}(1/Q^2)$, while the higher twist pion wave functions can give power suppressed contribution; the latter is at most $\mathcal{O}(1/Q^4)$. In the following we will consider two kinds of subleading power corrections within PQCD framework, that is from higher twist pion wave functions and the hadronic structure of photon.
3.1 Higher-twist pion wave functions

To evaluate the contribution from higher twist pion wave functions, firstly the definition of higher twist TMD wave functions similar to Eq. (5) are required. For simplicity, we assume that the initial pion wave function can be factorized into the longitudinal and transverse parts,

\[ \Phi_i(u, k_T^2, \zeta_0^2, \mu_i) = \phi_i(u, \zeta_0^2, \mu_i) \Sigma(k_T^2), \] (13)

For definiteness, the transverse momentum distribution is taken as

\[ \Sigma(k_T^2) = 4\pi\beta^2 \exp(-\beta^2 k_T^2), \] (14)

where the prefactor is introduced to obey the normalization

\[ \int \frac{d^2k_T}{(2\pi)^2} \Sigma(k_T^2) = 1. \] (15)

The longitudinal momentum distribution \( \phi_i(x, \zeta_0^2, \mu_i) \) is assumed to be the same as the LCDA.

For the two-particle pion LCDAs, we have

\[ \kappa_q \langle \pi(p) | \bar{q}(y) \gamma_\mu \gamma_5 q(0) | 0 \rangle = -i f_\pi p_\mu \int_0^1 du e^{i u p \cdot y} [\phi(u) + y^2 g_1(u)] + f_\pi (y_\mu - \frac{y^2 p_\mu}{p \cdot y}) \int_0^1 du e^{i u p \cdot y} g_2(u), \]

\[ \kappa_q \langle \pi(p) | \bar{q}(y) i \gamma_5 q(0) | 0 \rangle = f_\pi \mu \int_0^1 du e^{i u p \cdot y} \phi_\mu(u), \]

\[ \kappa_q \langle \pi(p) | \bar{q}(y) \sigma_{\mu \nu} \gamma_5 q(0) | 0 \rangle = i f_\pi \mu \nu \int_0^1 du e^{i u p \cdot y} \frac{\phi_{\sigma}(u)}{6}, \] (16)

where \( \kappa_u = -\kappa_d = 1/\sqrt{2} \) for \( \pi_0 \) meson. \( \phi_\pi(u) \) is twist-2, \( \phi_\pi^\perp(u) \) and \( \phi_{\sigma}(u) \) are twist-3, and \( g_1^\perp(u), g_2^\parallel(u) \) are twist-4.

The three-particle pion LCDAs are also defined by [27]

\[ \kappa_q \langle \pi(p) | \bar{q}(y) \gamma_\mu \gamma_5 g_\alpha G_{\alpha \beta}(vy) q(0) | 0 \rangle = f_\pi \left( p_\beta g_{\alpha \mu} - p_\alpha g_{\beta \mu} - \frac{y_\alpha p_\beta - y_\beta p_\alpha}{p \cdot y} p_\mu \right) \]

\[ \times \int_0^1 [D\alpha_i] \phi_\perp(\alpha_i) e^{i p \cdot y(\alpha_i + v y)} + f_\pi \frac{y_\alpha p_\beta - y_\beta p_\alpha}{p \cdot y} p_\mu \int_0^1 [D\alpha_i] \phi_\parallel(\alpha_i) e^{i p \cdot y(\alpha_i + v y)}, \]

\[ \kappa_q \langle \pi(p) | \bar{q}(y) \gamma_\mu \gamma_5 g_\alpha \tilde{G}_{\alpha \beta}(vy) q(0) | 0 \rangle = i f_\pi \left( p_\beta g_{\alpha \mu} - p_\alpha g_{\beta \mu} - \frac{y_\alpha p_\beta - y_\beta p_\alpha}{p \cdot y} p_\mu \right) \]
Figure 1: Feynman diagrams of contribution of the pion transition form factor from two-particle and 3-particle wave functions

\begin{align}
\times \int_0^1 [D\alpha_i] \phi^{\pi}_{\perp}(\alpha_i) e^{i p\cdot y(\alpha_q + \alpha_g)} + i f_{\pi} \frac{y_{\alpha} p_\beta - y_{\beta} p_\alpha}{p \cdot y} p_\mu \int_0^1 [D\alpha_i] \phi^{\pi}_{\parallel}(\alpha_i) e^{i p\cdot y(\alpha_q + \alpha_g)}, \tag{17}
\end{align}

here we have employed the following notations for the dual field strength tensor and the integration measure

\begin{align}
\tilde{G}_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\rho\tau} G^{\rho\tau}, \quad \int [D\alpha_i] \equiv \int_0^1 d\alpha_q \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta (1 - \alpha_q - \alpha_q - \alpha_g), \tag{18}
\end{align}

we note that all three-particle LCDAs are twist-4.

It is straightforward to obtain the factorization formula for two-particle twist-4 contribution through evaluating the Feynman diagrams Fig. 1a,

\begin{align}
F^{2PT_4}(Q^2) = -\frac{2\sqrt{2} f_{\pi}}{3Q} \int_0^1 \frac{du}{\sqrt{u}} \int_0^\infty db K_1(\sqrt{u}Qb) [g_1(u, b) + G_2(u, b)], \tag{19}
\end{align}

where \(G_2(u) = -\int_0^u g_2(v),\) and both the wave functions and hard kernel have been transformed into the impact parameter space.

To evaluate three-particle LCDA contribution, we need to keep the one-gluon part for the light-cone expansion of the quark propagator in the background gluon field

\begin{align}
(0|T\{q(y), \bar{q}(0)\}|0)_G \supset i \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot y} \int_0^1 dv \frac{[vy\gamma_\mu\gamma_\nu]}{k^2} - \frac{k\sigma_{\mu\nu}}{2k^4} G^{\mu\nu}(vy)
\end{align}

where \(G^{\mu\nu} = i[D_\mu, D_\nu].\) From evaluating diagram Fig. 1b, the factorization formula for three-particle twist-4 contribution reads

\begin{align}
F^{3PT_4}(Q^2) = -\frac{\sqrt{2} f_{\pi}}{6Q} \int_0^1 \frac{du}{\sqrt{u}} \int_0^\infty db K_1(\sqrt{u}Qb) \rho^{3PT_4}(\alpha_i, b), \tag{20}
\end{align}

where
\[
\rho^{3PT4}(u, b) = \int_0^{\tilde{a}} d\alpha_q \int_0^u \frac{d\alpha}{\alpha} \left[ \frac{2u - 1 - \alpha_q - \alpha_{\bar{q}}}{\alpha_q} \phi^\alpha_q(\alpha_q, \alpha_{\bar{q}}, 1 - \alpha_q - \alpha_{\bar{q}}) + \tilde{\phi}^\alpha_q(\alpha_q, \alpha_{\bar{q}}, 1 - \alpha_q - \alpha_{\bar{q}}) \right].
\] (21)

Employing the definition [27]
\[
\varphi^{TW4}(u, b) = 4[g_1(u, b) + G_2(u, b)] + \rho^{3PT4}(u, b),
\] (22)
we can write the overall contribution from twist-4 pion LCDAs as
\[
F^{3PT4}(Q^2) = -\frac{\sqrt{2}f_{\pi}}{6Q} \int_0^1 \frac{du}{\sqrt{u}} \int_0^\infty b^2 db K_1(\sqrt{u}Qb)\varphi^{TW4}(u, b),
\] (23)
here the Sudakov evolution factor is neglected, because there is no study on the joint re-
summation effect for the high-twist pion wave function yet. In this sense, this result is not
complete in \(k_T\) factorization framework, but it does not matter because this contribution is
actually free from endpoint singularity.

### 3.2 Hadronic Structure of Photon

To investigate the contribution from the hadronic structure of photon, the LCDAs of photon
[32] are needed. The definition of two-particle twist-2 and twist-3 LCDAs are given below
\[
\langle 0 | \bar{q}(z) \gamma_{\alpha \beta} q(0) | \gamma(p, \lambda) \rangle = ig_{em} Q_q \langle \bar{q} q \rangle \left( p_\beta \epsilon_\alpha - p_\alpha \epsilon_\beta \right) \int_0^1 dx e^{ixp \cdot z} [\chi(\mu) \phi_\gamma(x, \mu)],
\]
\[
\langle 0 | \bar{q}(z) \gamma_{\alpha} q(0) | \gamma(p, \lambda) \rangle = g_{em} Q_q f_{3\gamma} \epsilon_\alpha \int_0^1 dx e^{ixp \cdot z} \psi^{(v)}_\gamma(x, \mu),
\]
\[
\langle 0 | \bar{q}(z) \gamma_5 q(0) | \gamma(p, \lambda) \rangle = \frac{1}{4} g_{em} Q_q f_{3\gamma} \epsilon_{\alpha \beta \rho \sigma} p^\rho z^\sigma \epsilon^\beta \int_0^1 dx e^{ixp \cdot z} \psi^{(a)}_\gamma(x, \mu),
\] (24)
where \(\phi_\gamma(x, \mu)\) is twist-2, and \(\psi^{(a,v)}_\gamma(x, \mu)\) are twist-3. At the tree level the trace formulism
is convenient to evaluate transition matrix element, so that the following momentum space
projector of photon LCDAs is useful
\[
M_{\alpha \beta} = \frac{1}{4} g_{em} Q_q \left\{ - \langle \bar{q} q \rangle \not{\psi} \gamma(\mu) \phi_\gamma(x, \mu) + f_{3\gamma} \not{\psi}^{(v)}_\gamma(x, \mu) \right\}
\]
Similarly, we introduce the momentum space projector of pion wave function up to two-particle
transition form factor can be calculated through the convolution formula

\[ M_{\pi \alpha} = \frac{i f_{\pi}}{4} \left\{ \phi_{\pi}(u) - \mu_{\pi} \left( \phi_{\pi}^{\ast}(u) - \frac{i}{2} \sigma_{\mu \nu} \tilde{\eta}^{\ast \mu} \phi_{\pi}^{\ast \nu} \frac{\partial}{\partial k_{\perp \nu}} \right) \right\}_{\delta \alpha}. \quad (26) \]

Since in PQCD approach the endpoint singularity is regularized, the matrix element of pion
transition form factor reads

\[ \langle \pi | q T b | \gamma \rangle_{\text{HS}} = \frac{4 \pi \alpha_{s} C_{F}}{N_{c}} \int_{0}^{1} dx \int_{0}^{\infty} b_{1} db_{1} \int_{0}^{1} du \int_{0}^{\infty} b_{2} db_{2} M_{\pi \alpha} \gamma_{\alpha \beta \rho}^{T} M_{\pi \beta}. \quad (27) \]

The hard kernel can be obtained through calculating the Feynman diagrams in Fig 2, and
the factorization formula for the form factor reads

\[ F^{PHS}(E_{s}) = \frac{4 \pi \alpha_{s} C_{F} f_{\pi}(Q_{0}^{2} - Q_{u}^{2})}{\sqrt{2} N_{c}} \int_{0}^{1} dx \int_{0}^{\infty} b_{1} db_{1} \int_{0}^{1} du \int_{0}^{\infty} b_{2} db_{2} \]
\[ \times \left\{ h_{c}(x, u, b_{1}, b_{2}) f_{3 \gamma}(u) \phi_{\gamma}^{\ast}(x) + h_{c}(x, u, b_{1}, b_{2}) [-f_{3 \gamma}(u) \phi_{\gamma}^{\ast}(x) + 2 \chi(\mu) \langle \bar{q} q \rangle \mu_{\pi} \phi_{\pi}(x) \phi_{\pi}^{\ast}(u)] \right\}, \quad (28) \]

with the PQCD hard function

\[ h_{c}(x, u, b_{1}, b_{2}) = e^{-s_{c}(t) - s_{c}(t)} \left[ \theta(b_{1} - b_{2}) I_{0}(\sqrt{uQ^{2}b_{2}}) K_{0}(\sqrt{uQ^{2}b_{1}}) \right. \]
\[ + \theta(b_{2} - b_{1}) I_{0}(\sqrt{uQ^{2}b_{1}}) K_{0}(\sqrt{uQ^{2}b_{2}}) \left. K_{0}(\sqrt{xuQ^{2}b_{1}}) S_{t}(u), \right) \]

\[ h_{c}(x, u, b_{1}, b_{2}) = e^{-s_{c}(t) - s_{c}(t)} \left[ \theta(b_{1} - b_{2}) I_{0}(\sqrt{xQ^{2}b_{2}}) K_{0}(\sqrt{xQ^{2}b_{1}}) \right. \]
\[ + \theta(b_{2} - b_{1}) I_{0}(\sqrt{xQ^{2}b_{1}}) K_{0}(\sqrt{xQ^{2}b_{2}}) \left. K_{0}(\sqrt{xuQ^{2}b_{1}}) S_{t}(x), \right) \quad (29) \]

where the Sudakov factor \( s_{c}(t) \) is the same as that of a vector meson. In this part, because we
do not include the NLO QCD corrections, the hard kernel does not depend on the factorization
scale, though the form factor is dependent on the factorization scale in principle. In the PQCD
approach, in order to suppress the contribution of high order, the factorization scale is set to
be \( t = \max(\sqrt{xQ}, 1/b) \), and we allow an error area of \( t \) in the numerical evaluation. Note that
the contribution of higher twist LCDAs of photon is also not considered in the present paper,
as they are proved to be small in the previous studies [33].
Figure 2: Feynman diagrams of contribution of the pion transition form factor from hadronic structure of photon

4 Numerical Analysis

The leading twist pion LCDA satisfies the well-known Efremov-Radyushkin-Brodsky-Lepage equation [1,2], which indicates that it can be expanded in terms of the Gegenbauer polynomials $C_n^{3/2}$,

$$\phi_\pi(u, \mu) = 6u(1-u) \sum_{n=0}^{\infty} a_n(\mu) C_n^{3/2}(2u-1),$$

where the odd Gegenbauer moments $a_{2n+1}$ vanish due to symmetry properties. The fundamental ingredients in the LCDA are the even Gegenbauer moments $a_{2n}$. Tremendous efforts have been devoted to the determinations of the lowest moment $a_2(\mu)$ from the calculations with the QCD sum rules [34] and with the lattice simulations, and by matching the experimental data. The current widely used models for pion LCDA include the Bakulev-Mikhailov-Stefanis model [35,36], the KMOW model [37], the holographic model [38], etc. In the present paper we follow [25] to adopt a simpler model with only leading Gegenbauer moment,

$$\phi(u) = 6u(1-u) \left[ 1 + a_2 C_2^{3/2}(2u-1) \right],$$

we choose two values of $a_2$, i.e., $a_2 = 0.09$ and $a_2 = 0.17$, in the numerical evaluation for comparison. The joint resummation improved expression for this model has been obtained, and is given in the appendix. For twist-3 pion LCDAs, we adopt [39]

$$\phi_p^\pi(u) = 1 + 0.59C_2^\frac{1}{2}(2u-1) + 0.09C_4^\frac{1}{2}(2u-1)$$
Table 1: Numerical value of the parameters in the LCDAs of photon

| parameter | $\chi(1\text{GeV})$ | $\langle \bar{q}q \rangle (1\text{GeV})$ | $b_2(1\text{GeV})$ |
|-----------|---------------------|------------------------------------------|---------------------|
| value     | $(3.15 \pm 0.03)\text{GeV}^{-2}$ | $-[(256^{+11}_{-16})\text{MeV}]^3$ | $0.07 \pm 0.07$ |

| parameter | $f_3 \gamma(1\text{GeV})$ | $\omega_V(1\text{GeV})$ | $\omega_A(1\text{GeV})$ |
|-----------|------------------|-----------------|------------------|
| value     | $-4(2 \times 10^{-3})\text{GeV}^2$ | $3.8 \pm 1.8$ | $-2.1 \pm 1.0$ |

\[
\phi_\pi^\gamma(u) = 6u(1-u)[1 + 0.11C_2^{3/2}(2u - 1)].
\] (32)

The twist-4 pion LCDA has the following form if only leading conformal spin contribution is kept [27],

\[
\varphi^{TW4}(u, \mu) = \frac{80}{3} \delta_\pi^2(\mu) u^2(1 - u)^2,
\] (33)

where the normalization parameter is defined by

\[
\langle 0 | g_s \bar{q} \tilde{G}^{\mu \nu} \gamma_\nu q(0) | \pi(p) \rangle = if_\pi \delta_\pi^2(\mu)p^\mu,
\] (34)

with the renormalization-scale evolution at one loop

\[
\delta_\pi^2(\mu) = \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right] \frac{32}{9} \beta_0 \delta_\pi^2(\mu_0).
\] (35)

The numerical value of $\delta_\pi$ will be taken as $\delta_\pi^2(1\text{GeV}) = (0.2 \pm 0.04)\text{GeV}^2$ computed from the QCD sum rules [40] (see also [41]). The light-cone distribution amplitudes $\phi_\gamma(u)$, $\psi^{(v,a)}(\omega, \xi)$ have been systematically studied in Ref. [32], and the expressions are quoted as follows. The two-particle twist-2 LCDA is expanded in terms of Gegenbauer polynomials,

\[
\phi_\gamma(x, \mu) = 6x\bar{x} \left[ 1 + \sum_{n=2}^{\infty} b_n(\mu_0) C_n^{3/2}(2x - 1) \right],
\] (36)

and twist-3 LCDAs in conformal expansion read

\[
\psi^{(v)}(\xi, \mu) = 5 \left( 3 \xi^2 - 1 \right) + \frac{3}{64} \left[ 15 \omega_V^2(\mu) - 5 \omega_A^4(\mu) \right] \left( 3 - 30 \xi^2 + 35 \xi^4 \right),
\]

\[
\psi^{(a)}(\xi, \mu) = \frac{5}{2} \left( 1 - \xi^2 \right) \left( 5 \xi^2 - 1 \right) \left[ 1 + \frac{9}{16} \omega_V^2(\mu) - \frac{3}{16} \omega_A^4(\mu) \right].
\] (37)
The value of the parameters used in the LCDAs of photon are presented in Table 1, among them the scale dependent parameters are given at $\mu_0 = 1.0 GeV$. These parameters should be run to the factorization scale $t$, and the evolution kernel for $\chi(\mu), \langle \bar{q}q \rangle(\mu), b_2(\mu), f_{3\gamma}(\mu)$ and $\omega_{V,A}(\mu)$ have been given in [32].

Now we turn to investigate the leading power result from joint resummation improved PQCD approach and the contributions from various sources of subleading power corrections. In Fig. 3 the result of each kind of contributions are displayed. At the leading power the result from collinear factorization approaches $\sqrt{2}f_\pi$ when $Q^2$ goes to infinity, while in $k_T$ factorization with joint resummation, the behavior at large $Q^2$ region is modified. $Q^2F_{\gamma^*\gamma\to\pi^0}(Q^2)$ slightly increases when $Q^2$ is getting larger, which shows the same tendency with the experimental data. The subleading power corrections start from $1/Q^4$, which fall down rapidly when $Q^2$ increases, and are significant only at small $Q^2$ region. From this figure we can see that the contributions from higher twist pion wave functions and hadronic structure of photon have different sign, the cancellation between them makes the power correction investigated in this paper is minor even at small $Q^2$ region. This result is consistent with the investigation from light-cone sum rules [28]. So that the NLP corrections in pion transition form factors is

![Figure 3: Contributions of leading power and subleading power corrections](image)
Figure 4: Uncertainties of the contribution from subleading power corrections

small, which is on the contrary to the leptonic radiative decay $B \rightarrow \gamma \ell \nu$ [42], where the NLP corrections decrease the LP result over 50%. Our result indicates that the power corrections can hardly modify the leading power prediction from the joint resummation improved PQCD approach, thus the total result still agrees well with the BaBar data. As a result, the pion transition form factor is a good platform to extract the nonperturbative information on the shape of the leading twist pion distribution amplitude. The uncertainties of the contribution from the higher twist pion wave functions and the hadronic structure of photon are shown in Fig 4. The main source of the uncertainty is the parameters in the twist-4 Pion LCDA and the LCDAs of photon which are presented in Table.1, as well as the factorization scale $t$ which is allowed a float up or down 20%. The comparison between our prediction and the experimental data are presented in Fig. 5, where the CLEO [43], BaBar [8] and Belle [9] data are shown. The blue and black zones represent the Gegenbauer moment $a_2 = 0.17$ and $a_2 = 0.09$, respectively. At large $Q^2$ region, the pion wave function with $a_2 = 0.09$ favors the experimental data by Babar and Belle better and the influence from the power corrections are negligible. At small $Q^2$ region, the subleading power contribution only changes leading power result slightly, and the prediction with $a_2 = 0.09$ is consistent with the CLEO data. We emphasize that there
are large theoretical uncertainty when $Q^2 < 2GeV^2$, and the perturbation calculation is not reliable at this region, thus one needs not take it seriously. We acknowledge that this work is not a systematical study on the power corrections within the framework of effective theory, although in the present work the NLP contribution is negligible, it is too early to draw the conclusion that the power suppression contribution is not important in this process.

5 Conclusion

In this paper, within the framework of perturbative QCD approach based on $k_T$ factorization, we studied the next-leading order of $\alpha_s$ corrections, the high power corrections, as well as the joint resummation effect to the pion transition form factor. For the higher power contributions, we here considered the effects from higher twist pion wave functions, including two-particle and three-particle twist-4 wave functions of pion, and the hadronic structure of photon. In PQCD approach, the transverse momentum of partons inside pion and “hadronic” photon regularized endpoint singularity existing in collinear factorization. The numerical result indicates that
there exists strong cancellation effect between the two kinds of NLP contribution, thus the NLP corrections does not change leading power result manifestly. Our calculations also indicate that the power corrections cannot explain the anomalous BaBar data. Since there is no confirmed conclusion on the two experimental results, we hope the Belle-II experiments can test our result and give the final conclusion in future. Furthermore, we also note that the pion transition form factor sharply depend on the leading twist pion LCDA, which can be used to extract the information of the pion wave function. For example, a simple pion wave function model with $\alpha_2 = 0.9$ can be consistent with the BaBar data within the uncertainty area. The power suppressed contributions considered in this paper are only from some specific sources, to perform a systematical study on this process we need to analysis the complete NLP operator [44] base in soft-collinear effective theory [45, 46]. This work is left for a future study.

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