Intrinsic Decoherence and Irreversibility
in the Quasiperiodic Kicked Rotor

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We show that some classically chaotic quantum systems uncoupled from noisy environments may generate intrinsic decoherence with all its associated effects. In particular, we have observed time irreversibility and high sensitivity to small perturbations in the initial conditions in a quasiperiodic version of the kicked rotor. The existence of simple quantum systems with intrinsic decoherence clarifies the quantum-classical correspondence in chaotic systems.

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It is well established, mainly from the important contributions by Zurek and others \(^1\), that weak interaction of a quantum system with a noisy environment induces decoherence in the quantum dynamical evolution. Ultimately this effect is behind the emergence of a classical world from the quantum formalism. In the case of quantum systems with chaotic classical analogs, this implies that diffusion, entropy increase, time irreversibility and other typical characteristics of classical chaos emerge as a result from the decoherence induced by noisy environments. Experimental evidence of the effects of the decoherence induced by coupling to noisy environments has been obtained in the case of the quantum kicked rotor using optical atom traps. The effect of decoherence is to destroy dynamical localization and establish a quantum diffusion regime \(^2\). Similar effects have also been observed using trapped ions \(^3\). It is interesting to inquire how essential is the role of the environment in the emergence of classical-like chaotic behavior in quantum systems. Can the same dynamical effects due to a noisy environment be obtained from a modified hamiltonian without coupling to the environment? In other words, can a quantum system develop “intrinsic decoherence”? 

In this paper we show that a modified version of the kicked rotor can develop classical–like effects usually associated to environment–induced decoherence. These effects include irreversibility and a high sensibility to small perturbations in the initial conditions. The possibility of the generation of “intrinsic decoherence” in systems isolated from their environment, has recently been suggested in the literature \(^4\,^5\).

We recall that in the classical kicked rotor, for strong enough kick strengths, there is global chaos in phase space \(^6\) with all its manifestations. This includes exponential sensibility to small perturbations in the initial conditions and time–irreversibility. Despite that the equations of motion are time–reversible, if time is reversed, small errors are exponentially amplified and a new diffusive process takes place. In the quantum mechanical case, the diffusive process in action space is suppressed after a characteristic time due to dynamical localization, a quantum coherence effect. The quantum suppression of chaos due to dynamical localization takes place ultimately due to the discrete nature of the quasienergy spectrum of the kicked rotor \(^6\). If time is reversed, the unitary evolution returns the system to its initial state, in spite of small errors in the initial conditions. This striking manifestation of the quantum suppression of chaos was first shown numerically by Graham and Dittrich \(^6\).

We now introduce a specific model system, the quasiperiodic kicked rotor. Consider a rotor to which a periodic series of impulsive kicks of strength \(K\) and period \(T_1\) is applied. If a second periodic series of kicks with the same strength and a period \(T_2\) incommensurate with the first, is applied to the rotor the resulting time–dependent hamiltonian is quasiperiodic and can be written as

\[ H = \frac{P^2}{2I} + K \cos \theta \left[ \sum_{n=1}^{\infty} \delta(t - nT_1) + \sum_{n=1}^{\infty} \delta(t - mT_2) \right], \tag{1} \]

where \(P\) is the angular momentum, \(\theta\) is the angular position of the rotor and we choose units so that the moment of inertia is \(I = 1\). It has been shown \(^6\) that in this system the initial diffusion persists for arbitrarily long times. In fact, this system can be described by a master equation and the entropy increases forever in a way typical of a decoherent evolution \(^10\). It is worth noting that the decoherence we are discussing here is not due to the coupling with a noisy environment, but is due to the dynamical complexity of the hamiltonian. When the second series of kicks is added to the kicked rotor, the underlaying frequency response spectrum changes from a discrete (pure–point) to a dense spectrum \(^6\).

It is interesting to find whether this intrinsic decoherence also implies a time–irreversible quantum dynamics, as it is the case with environment–induced decoherence.
If we reverse time in the quasiperiodic kicked rotor, we find that there is no sensitivity to small perturbations in the wavefunction. However, a closer look reveals that perturbing the wavefunction components is not analogous to perturbing the classical phase space coordinates. The classical analog of a perturbation of the wavefunction would be perhaps to perturb the Liouville function of the corresponding classical system. As is well known, the linear nature of the Liouville equation prevents any small perturbation in the distribution function from growing exponentially fast, even under conditions of global chaos. The actual analog of perturbing the classical phase space coordinates, would be to perturb the expected value of some suitable quantum observable. Then, small perturbations in the expectation value of these observables may, under certain circumstances, lead to large variations in the wavefunction.

We have tested the previous considerations using the quasiperiodic kicked rotor described by (1). In the position representation, the wavefunction is

$$\Psi(\theta, t) = \frac{1}{\sqrt{2}} \sum_{\ell=-\infty}^{\infty} a_{\ell}(t) e^{i\ell \theta}.$$  (2)

The evolution for $a_{\ell}(t)$ is obtained in a straightforward way from the quantum map

$$a_{\ell}(t_{n+1}) = \sum_{j=-\infty}^{\infty} i^{-(j-\ell)} e^{-i \ell^2 \Delta t_{n}/2} J_{j-\ell}(K/H)a_{j}(t_{n}),$$  \(\ell=0\to \ell + \varepsilon, \) \(\text{the time interval between two consecutive kicks as } t_{n}, \) \(\text{to the time interval between two consecutive kicks as } \Delta t_{n} = t_{n+1} - t_{n} \text{ and } J_{k} \text{ is the } k^{th} \text{ order cylindrical Bessel function. The time intervals } \Delta t_{n} \text{ are in general different, since they depend on the kick sequence. In fact, they form a dense set.} \text{ Note that the periodically kicked rotor is also described by the quantum map (4) with a constant time interval between kicks } \Delta t = T = 1. \)

If a small perturbation is introduced in the angular position, $\theta \to \theta + \varepsilon$, the wavefunction (2) becomes

$$\tilde{\Psi}(\theta, t) = \frac{1}{\sqrt{2}} \sum_{\ell=-\infty}^{\infty} \tilde{a}_{\ell}(t) e^{i\ell \varepsilon} e^{i\ell \theta} = \frac{1}{\sqrt{2}} \sum_{\ell=-\infty}^{\infty} \tilde{a}_{\ell}(t) e^{i\ell \theta}.$$  (4)

where the transformed coefficients $\tilde{a}_{\ell} = a_{\ell} e^{i\ell \varepsilon}$ have acquired a phase. Note that given $\varepsilon$, the wave function components with larger $\ell$ undergo larger phase shifts.

We consider the evolution of an initial wavepacket under the quantum map (4). At a certain point $t^*$ the direction of time is reversed, $t \to -t$ and a small perturbation $\varepsilon$ is introduced in the angular position, as described in (4). As shown in figure 1 if the perturbation is above a certain threshold, the system resumes diffusion after a characteristic time which depends on $\varepsilon$. This can be understood by considering eq. (4) which shows how the perturbation affects the wavefunction. We adopt the following criteria to decide whether a given angular momentum component is present in the wavefunction: consider a small number $\delta \ll 1$, then the angular momentum component $\ell$ is present in the wavefunction if the condition $|a_{\ell}|^2 > \delta$ is satisfied. We define $\ell_{\text{max}}$, as the highest angular momentum component present in the wavefunction. If $\ell_{\text{max}} \varepsilon \ll 1$, the perturbation will have no appreciable effect on the dynamics and the reversibility associated to a unitary quantum evolution will manifest itself. On the other hand, if $\ell_{\text{max}} \varepsilon \approx 1$, the perturbation affects strongly the wavefunction and irreversibility will be apparent after a characteristic time. The weaker the perturbation, the larger this characteristic time, as we show in figure 1. For a weak enough perturbation the system will return to its initial state in a way characteristic of time–reversible systems. The threshold value of the perturbation is simply related to the maximum angular momentum present in the wavefunction

$$\varepsilon_{\text{th}} \approx \frac{1}{\ell_{\text{max}}},$$  (5)

In this system, as time progresses, diffusion in angular momentum causes progressively higher angular momentum modes to participate in the dynamics, in the sense detailed above. Therefore, the sensitivity of the wavefunction to a given perturbation, as measured by $\varepsilon_{\text{th}}$, increases with time.

In figure 1 we show this increasing sensibility by considering the effect of a given perturbation for different “break times” $t^*$. The time the system takes to resume diffusion decreases as $t^*$ becomes larger, indicating an increasing sensibility to the same perturbation. This is due to the fact that the threshold value $\varepsilon_{\text{th}}$ decreases with time as $\ell_{\text{max}}$ increases. This diffuse process continues for arbitrary long times, in the quasiperiodic kicked rotor, a system with a dense frequency response spectrum. That

FIG. 1: Effect of different perturbation strengths on the time evolution of $\langle n^2 \rangle$ in the quasiperiodic kicked rotor. Time is inverted after $10^4$ kicks and different perturbation strengths, $\varepsilon = 10^{-3}$ (thick line), $\varepsilon = 3 \times 10^{-3}$ (thin line) and $\varepsilon = 10 \times 10^{-3}$ (dotted line) are applied to the state vector as described in the text.
Fig. 2: Effect of inverting time at different points in the evolution of $<n^2>$ for the quasiperiodic kicked rotor. Time is inverted after 5000 kicks, 10000 kicks and 20000 kicks. The same perturbation $\varepsilon = 3 \times 10^{-3}$ is applied in all cases. The dotted lines represent the unperturbed reversible evolution after time reversal.

is, the frequencies involved in a Fourier analysis of the dynamical response of the system form a dense set $\mathbb{R}$. (For non-periodic time–dependent hamiltonians the frequency response spectrum is its natural generalization of the quasenergy spectrum of periodic systems).

It is instructive to consider the case of a discrete spectrum, as that of the periodically kicked rotor. If we consider the effect of time reversal and a perturbation on this system, it is not surprising that for reversal times $t^*$ less than the localization time $\tau$, we find that figures 1 and 2 apply also to this system and there is a threshold $\varepsilon_{th}$ for reversible perturbations. However, in this case the threshold decreases during the quantum diffusion stage, but stops decreasing as soon as the wavefunction becomes exponentially localized because after this point no new angular momentum components are introduced. Therefore, a perturbation $\varepsilon < \varepsilon_{th}$ which takes place at any time after the wave function has localized, will not affect the reversibility of the motion. A side effect of dynamical localization is to “freeze” the sensitivity to perturbations at the value it had at time $t = \tau$ when the system localized.

In conclusion, we have analyzed an example of a classically chaotic quantum system, the quasiperiodic kicked rotor, which has a decoherent quantum dynamics without coupling with a noisy environment. We have shown that suitable small perturbations in the initial conditions are amplified in a way similar to the classical case and as a result, the quantum motion becomes irreversible. It should be stressed that by “suitable perturbations” in the quantum mechanical context, we mean a small perturbation in the expected value of an observable and not a small perturbation in the wave function. As we have shown, a small perturbation of the mean value of a quantum observable may imply a large perturbation of the wave function.

In the case of the periodically kicked rotor, the dynamical localization prevents a similar effect to take place. The quasiperiodic kicked rotor has a dense response spectrum $\mathbb{R}$ while the periodically kicked rotor has a discrete spectrum which produces localization. Thus we conjecture that a dense spectrum is necessary in order to observe intrinsic decoherence in classically chaotic quantum systems. We note that this requirement has also been proposed in a cosmological context $\mathbb{R}$.

The existence of simple quantum systems with intrinsic decoherence and the associated time irreversibility and high sensitivity to small perturbations in initial conditions may help to clarify the quantum–classical correspondence in chaotic systems.

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