Magnetic Interference Patterns and Vortices in Diffusive SNS junctions

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We study theoretically the electronic and transport properties of a diffusive superconductor-normal metal-superconductor (SNS) junction in the presence of a perpendicular magnetic field. We show that the field dependence of the critical current crosses over from the well-known Fraunhofer pattern in wide junctions to a monotonous decay when the width of the normal wire is smaller than the magnetic length \( \xi_H = \sqrt{\Phi_0/H} \), where \( H \) is the magnetic field and \( \Phi_0 \) the flux quantum. We demonstrate that this behavior is a direct consequence of the magnetic vortex structure appearing in the normal region and predict how such structure is manifested in the local density of states.

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Introduction. – The study of the modification of the properties of a normal metal in contact to superconductors, known as proximity effect, has a long history. In the last years there has been a renewed interest in this subject because new experimental techniques have allowed resolving properties on smaller length scales and very low temperatures. Although many electronic and transport properties of hybrid SN structures are now well understood, the situation is less satisfactory when dealing with the magnetic field dependence of those properties. A few years ago, Heida et al. measured the critical current as a function of a perpendicular magnetic field in ballistic SNS junctions of comparable length and width and found a periodicity close to \( 2\Phi_0 \), where \( \Phi_0 = h/2e \) is the flux quantum, instead of the standard \( \Phi_0 \) of the Fraunhofer pattern. This was qualitatively explained in Refs. [5,6] in terms of the classical trajectories associated with current-carrying Andreev states in a normal clean wire. In the case of diffusive junctions, numerous experiments have shown that in wide junctions the critical current exhibits a Fraunhofer-like pattern. However, very recent experiments in junctions where the width is comparable to the superconducting coherence length have shown a monotonous decay of the critical current with field, i.e. the absence of magnetic interference patterns. The unified description of these two very different behaviors is a basic open problem.

In this Letter we show that the solution to the previous puzzle is closely related to the issue of the formation of a magnetic vortex structure in the normal conductor. Vortex matter in mesoscopic superconductors has been also a very active subfield in superconductivity in the last years. It has been shown that basic properties such as critical fields and the magnetization depend crucially on the size and topology of the mesoscopic samples, which in turn determine the vortex structure. There is also a great interest in the study of nucleation of superconductivity and vortex matter in hybrid structures. However, little attention has been paid to the formation of vortices inside non-superconducting materials. Our goal here is to answer the following fundamental questions: Is it possible to induce a vortex structure in a normal wire by proximity to a superconductor? If so, what are the properties of such proximity vortices and their influence on the Josephson effect? For this purpose, we have studied a diffusive SNS junction in the presence of a perpendicular magnetic field. By solving the two-dimensional Usadel equations, we are able to describe the electronic properties for arbitrary length, \( L \), and width, \( W \), of the normal wire. We find that a magnetic vortex structure may develop in the normal metal. These vortices have similar properties to those in the mixed state of a type II superconductor. The consequence of this vortex structure is the appearance of an interference pattern in the critical current that tends to the Fraunhofer pattern in the wide-junction limit (\( W \gg \xi_H = \sqrt{\Phi_0/H} \)), and also a modulation of the local density of states in the normal wire. On the contrary, when \( W \) is comparable or smaller than \( \xi_H \), the formation of vortices is not favorable and the field acts as a pair-breaking mechanism which suppresses monotonously the critical current. Our results not only solve the puzzle described above, but also illustrate the richness of the vortex physics in hybrid structures.

Quasiclassical formalism. – We consider a SNS junction, where \( N \) is a diffusive normal metal of length \( L \) and width \( W \) coupled to two identical superconducting reservoirs with gap \( \Delta \). The junction is subjected to an uniform external field \( \mathbf{H} = H\hat{z} \) perpendicular to the normal film lying in the \( xy \)-plane, where \( x \in [0, L] \) and \( y \in [-W/2, W/2] \). For the sake of simplicity, we assume that the thickness of the normal wire is smaller than the London penetration depth, which means that the field penetrates completely in the normal region. In order to describe the electronic properties of these junctions we use the quasiclassical theory of superconductivity in the diffusive limit, where the mean free path is much smaller than the coherence length, \( \xi = \sqrt{\hbar D/\Delta} \), \( D \) being the diffusion constant of the normal metal. In equilibrium situations like the one considered here, this theory can be formulated in terms of momentum averaged retarded Green functions \( G^R(\mathbf{R}, \epsilon) \), which depend on position \( \mathbf{R} \) and energy \( \epsilon \). This propagator is a \( 2 \times 2 \) matrix in
electron-hole space

\[
\hat{G}^R = \left( \frac{\hat{G}^R}{\hat{F}^R} \right),
\]

which satisfies the stationary Usadel equation, which in the N region reads,\(^{15}\)

\[
\frac{\hbar D}{\pi} \nabla \left( \hat{G}^R \hat{\nabla} \hat{G}^R \right) + e[\hat{\tau}_3, \hat{G}^R] = \frac{ieD}{\pi} \mathbf{A}[\hat{\tau}_3, \hat{G}^R \hat{\nabla} \hat{G}^R].
\]

Here, \(\mathbf{A}\) is the vector potential, \(\hat{\nabla} = \nabla \hat{\mathbf{1}} - (ie/\hbar)\mathbf{A}\hat{\tau}_3\), \(\hat{\tau}_3\) is the Pauli matrix and the Coulomb gauge \((\nabla \cdot \mathbf{A}) = 0\) has been already used. Eq. (2) is supplemented by the normalization condition \((\hat{G}^R)^2 = -\pi^2\hat{\mathbf{1}}\) and proper boundary conditions. For the SN interfaces we use the boundary conditions introduced in Ref. \(^{17}\), which allow us to describe the system for arbitrary transparency. For the metal-vacuum borders of the normal wire we impose that the currents are zero, and \(J_0\) is the Josephson current density.

The physical properties we are interested in can be conveniently expressed in terms of the Usadel-Green functions. Thus for instance, the local density of states is given by \(\rho(R, \epsilon) = -\text{Im}\hat{G}^R(R, \epsilon)/\pi\). To quantify the superconducting correlations we use the pair correlation function defined as \(F(R) = (1/4\pi) \int d\epsilon (\hat{F}^R - \hat{F}^A) \tanh(\beta \epsilon/2)\), where \(\beta = 1/\hbar_k T\). This function, apart from the attractive coupling constant, is the pair potential in a superconductor and it is non-zero inside the normal metal due to the proximity effect. Finally, the supercurrent density in the junction can be written as

\[
j(R) = \frac{\sigma_N}{4\pi^2e} \int_{-\infty}^{\infty} d\epsilon \tanh \left( \frac{\beta \epsilon}{2} \right) \text{Re} \left\{ \hat{F}^R \nabla \hat{F}^R - \hat{F}^R \nabla \hat{F}^R + \frac{4ie}{\hbar} \mathbf{A}[\hat{F}^R \hat{\nabla} \hat{F}^R] \right\},
\]

where \(\sigma_N\) is the normal state conductivity. The net current is obtained integrating \(j_x\) across the \(y\)-direction.

Eq. (2) constitutes a set of coupled second-order nonlinear partial differential equations, whose resolution is a formidable task. In general, one has to resort to numerical methods. However, one can get analytical insight in several limiting cases. By choosing the gauge \(\mathbf{A} = -H_y \hat{x}\), one can identify in Eq. (2) the length \(\xi_H = \sqrt{\Phi_0/\hbar}\) as the characteristic variation scale of the Green functions in the transversal direction due to the magnetic field. We consider first the case where the wire width \(W\) is smaller than \(\xi_H\). In this case the Green functions do not vary considerably in the \(y\)-direction and one can average Eq. (2) over this direction leading to the one-dimensional equation

\[
\frac{\hbar D}{\pi} \partial_x \left( \hat{G}^R \partial_x \hat{G}^R \right) + e[\hat{\tau}_3, \hat{G}^R] = \frac{\Gamma_H}{\pi} [\hat{\tau}_3 \hat{G}^R \hat{\tau}_3, \hat{G}^R],
\]

where \(\Gamma_H = Dc^2 H^2 W^2/(6\hbar)\) is a depairing energy, which in terms of the Thouless energy, \(\epsilon_T = \hbar D/L^2\), can be written as \(\Gamma_H = \epsilon_T (\pi \Phi/\sqrt{\theta_0})^2\), where \(\Phi = HLW\) is the flux enclosed in the junction. These equations describe the effect of a pair-breaking mechanism, such as magnetic impurities, that has been studied extensively in Ref. \(^{19}\). The other analytic case is the limit of a wide junction where \(W \gg L, \xi_H\). In this limit one can neglect the terms containing the derivatives with respect to the \(y\)-coordinate. The field also disappears from the equation and its only effect is to change the superconducting phase difference \(\phi\) into the gauge-invariant combination \(\gamma = \phi - 2\pi (\Phi/\Phi_0)y/W\). With this result in mind, it is easy to anticipate, in particular, that critical current exhibits a Fraunhofer-like pattern in this limit.

**Discussion of the results.** We start by analyzing the local density of states (DOS) in the normal wire. In the absence of field the main feature is the presence of a minigap, \(\Delta_{20,21,22}\). This minigap is the same throughout the normal wire and for perfect transparency scales as \(\Delta_x \sim 3.1 \epsilon_{cT}\) in the limit \(L \gg \xi\). In Fig. 1 we show the local DOS in the middle \((x = L/2)\) of a wire of length \(L = 2\xi\) for two different values of the width and the magnetic flux. Notice that for \(W = \xi\) (see upper panels), the local DOS is practically independent of the \(y\)-coordinate. Moreover, when the field is not very high, there is a clear minigap (see upper left panel), which closes at higher fields (see upper right panel). As one can see in the lower panels, when \(W \gg L\), the local DOS is strongly modulated along the \(y\)-direction. For low fields \((\Phi < \Phi_0)\), the minigap is still open throughout the wire, but for higher fields the minigap changes in a periodic fashion from its maximum value (equal to the value in the absence of field) to exactly zero at well-defined positions where the DOS is the normal state one. This situation is very similar to the mixed state of a type II superconductor, where the system becomes normal in the vortex cores.
FIG. 1: (Color online) Local density of states as a function of the energy in the middle of a wire of length $L = 2\xi$ for two different values of the magnetic flux. The upper panels correspond to a wire width $W = \xi$ and the lower ones to $W = 50\xi$. The different curves correspond to different values of the $y$-coordinate. We have assumed perfect transparency for the interfaces and a phase difference $\phi = 0$. In panel (d) we have used thicker lines to highlight the curves where the DOS is equal to the normal state one.

These results are in agreement with the limiting cases discussed above. If the wire is narrow the magnetic field acts as a pair-breaking mechanism with depairing energy $\Gamma_{\text{H}}$. It is well-known that the minigap is reduced by such mechanisms\textsuperscript{19,21} and, in particular, it closes at a critical value $\Gamma_{\text{H}} = \frac{\pi^2}{2^2} \frac{\epsilon_T}{2}$, i.e. in our case at a critical flux $\Phi_{\text{C}} = \sqrt{3}\Phi_0$. This explains the results for $W = \xi$. To understand the results for $W = 50\xi$, we remind that in the wide limit the magnetic field only enters in the gauge-invariant phase difference $\gamma$. It has been shown that in the absence of field the minigap decreases monotonously as the phase difference increases and it closes when the phase is equal to $\pi$\textsuperscript{22}. Bearing this in mind, one can easily understand the results of Fig. 1(c,d). When $\gamma = 0$ the minigap is completely open reaching the value in the absence of field. However, when $\gamma = \pi$ the minigap closes. For $\Phi = 2\Phi_0$ and $\phi = 0$, the phase $\gamma$ takes the values $\mp\pi$ at $y/W = \pm 1/4$, which explains why the two thick curves in panel (d) correspond to normal state DOS.

The peculiar DOS suggests the presence of vortices in the normal wire. To confirm this idea, we have analyzed the pair correlation function, $F(\mathbf{R})$. In Fig. 2 we show a color-coded map of the modulus of this function throughout the normal wire for the same values of $L$ and $W$ as in Fig. 1. All the panels show that $F$ diminishes towards the center of the wire, which simply reflects the decay of the superconducting correlations inside the normal wire. The main difference is the modulation along the $y$-direction. In the case $W = \xi$, at low fields (see panel for $\Phi = \Phi_0$) $F$ is still finite everywhere, while for higher fields it can be very small in the center of the wire, but with practically no modulation. The situation changes drastically for $W = 50\xi$, where one can clearly see the appearance of a linear array of vortices located on $x = L/2$ with normal cores where $F$ vanishes. The number of vortices depends simply on the number of flux quanta in the junction. It is important to distinguish these vortices from the so-called Josephson vortices that take place in much wider junctions where $W > \lambda_J$ and the self-field effects play a crucial role\textsuperscript{4}. An important difference is that the Josephson vortices do not have normal cores and their size is comparable to $\lambda_J$.

It is possible to get an analytical insight into the vortex structure by using the linearized Usadel equations in the wide-junction limit. In this case one can show that the position of the zeros of the pair correlation function are given
FIG. 2: (Color online) Spatial map of the modulus of the pair correlations, $|F(R)|$, for $L = 2\xi$ and $\phi = 0$. The different panels correspond to different values of the width $W$ and the magnetic flux $\Phi$, as indicated in the graphs. $|F(R)|$ has been normalized to its value inside the electrodes, the temperature is $k_B T = 0.01\Delta$ and perfect transparency was assumed.

by

$$x = \frac{L}{2} \quad \text{and} \quad \phi - 2\pi \left(\frac{\Phi}{\Phi_0}\right) \frac{y}{W} = (2m + 1)\pi,$$

where $m = 0, \pm 1, \ldots$ and $y \in [-W/2, W/2]$. This means that the vortex cores are located exactly on the middle of the wire forming a regular linear array along the $y$-direction and they are separated by a distance $\Phi_0/HL$. Thus, for the case $W = 50\xi$ in Fig. 2 this condition tells us that for $\Phi = 4\Phi_0$ there are four vortex cores located on $y/W = \pm 1/8, \pm 3/8$, which are the positions that one can read off from Fig. 2. Notice also that according to Eq. (5) the phase $\phi$ simply shifts rigidly the line of vortices along the $y$-direction. Thus, measurements of the local DOS at the outer interfaces ($y = \pm W/2$) changing the supercurrent through the junction should show an oscillatory behaviour. On the other hand, the normal cores are surrounded by circulating currents (not shown here), which vanish exactly at the cores. Moreover, from the analytical solution of the linearized Usadel equation in the wide-junction limit and from the numerical results for arbitrary cases, one can easily show that the phase of the pair correlation changes in $2\pi$ around the cores, i.e. each vortex has a unit topological charge. In short, the main difference with the usual
vortices in a bulk superconductor of type II is that they are arranged in one-dimensional array instead of forming a two-dimensional lattice and due to the confining geometry they do not possess a rotational symmetry.

We discuss finally the magnetic field dependence of the critical current. In Fig. 3 we show an example for $L = 8\xi$, which is a typical value in the experiments, and different values of $W$. Notice that for small values of $W$, the critical current decays monotonously. This is simply due to the fact that in this limit no vortices appear and the field suppresses progressively the superconductivity in the normal wire. Indeed, as we show in the inset of Fig. 3, in this limit Eq. (4) describes quantitatively the field dependence. As the width increases the vortex structure discussed above appears and as a consequence one observes the appearance of an interference pattern where the critical current vanishes at certain values of the magnetic flux. Notice that these values are clearly larger than $\Phi_0$ for intermediate widths and the patterns are not “periodic”. Only in the limit $W \gg \xi_H, L$ one obtains a regular pattern with zeros at multiples of $\Phi_0$, recovering the Fraunhofer pattern. These results explained in an unified manner the different behaviours observed experimentally, which at first glance seemed to be contradictory.

Finally, we have studied systematically the role of the length $L$ in the crossover from the narrow-junction to the wide-junction behavior. We have found that as $L$ increases this transition occurs at larger values of $W$. This confirms the fact that the condition for the appearance of an interference pattern, i.e. zeros in the critical current, is given, roughly speaking, by $W > \xi_H$, which is equivalent to $W/L > \Phi_0/\Phi$. The standard Fraunhofer pattern is approach when $W \gtrsim L$.

Conclusions.– We have studied a diffusive SNS junction in the presence of a perpendicular magnetic field. We have shown that the appearance of magnetic interference patterns in the critical current is intimately linked to the formation of a vortex array in the normal wire. Our results provide an unified description of the critical current for arbitrary width of the junctions and solve the puzzle put forward by recent experiments. Our work also paves the way to study the vortex matter in a great variety of hybrid structures like the recently introduced superconducting graphene junctions, where a standard Fraunhofer pattern has been observed.

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