Can quark effects be observed in intermediate heavy ion collisions?

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Abstract. In recent years a tentative description of the short-range part of hadron interactions with constituent quark interchange has been developed providing an alternative approach to meson physics. Quark interchange plays a role, for example, in the nucleon-nucleon ($NN$) phase-shifts and cross-section. In heavy ion collision simulations at intermediate energies one of the main features is the $NN$ cross-section in the collisional term, where in most cases it is an input adjusted to the free space value. In this paper we introduce the quark degrees of freedom to the $NN$ cross-section in the Vlasov-Uehling-Uhlenbeck (VUU) model and explore the possibility that these effects appear in the observables at lower energies.

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1. Introduction

For a long time low energy hadron-hadron interactions, in particular the $NN$ interaction, have been studied in terms of meson exchange models employing baryon and meson degrees of freedom. Such models have achieved a good description of the empirical data. A common undesirable feature in all meson exchange models is the necessity of a (semi)phenomenological parametrization of the short-range part of the interaction. However, since the advent of quantum chromodynamics (QCD) there is the hope that the short-range part of the hadron-hadron interaction can be derived from first principles. However, because of complications due to the nonperturbative nature of the QCD interactions at low energies, the use of models is a practical necessity for making progress in this direction.

There is a great variety of quark models that can describe with reasonable success single-hadron properties. Therefore, there arises the natural question of to what extent a model which gives a good hadron structure is, at the same time, able to describe the hadron-hadron interaction and in a larger context, intermediate energy nucleus-nucleus collisions. In this respect, in the recent past several studies of the $NN$ interaction have employed the non-relativistic quark model, a la Isgur and Karl, which is one of the most successful quark models for hadron structure. The main ingredient of such calculations is the one-gluon exchange interaction. The results are encouraging as far as the very short-range part is concerned which is reflected in the successful description of $NN$ scattering S-wave phase shifts. For the intermediate-range part progress on the spin-orbit problem in the $NN$ system was made by including, in addition to the one-gluon-exchange interaction, a scalar-isoscalar meson coupled to quarks in a chirally symmetric way [1]. More recently, calculations in the $KN$ system became an important approach, due to the fact that one-pion exchange is absent and contributions from $2\pi$ exchange seem to be weaker than in the $NN$ system. The quark-gluon exchange in the $KN$ system has been studied by Barnes and Swanson [2], Silvestre-Brac and collaborators [3]-[6] and Hadjimichef, Haidenbauer and Krein [7]. In summary, these studies lead to some general questions such as: where do we place the frontier between quark-gluon physics and effective theories? To what extent the underlying quark-gluon physics, studied at lower energies in the context of hadron interactions, can play a role in the complex dynamics related to heavy ion collisions? Can we have a description in which one could smoothly pass from high energy physics to an effective low energy representation? Of course to answer these questions would represent a major step in hadron physics. In a more limited context it would be interesting to extend the quark-gluon exchange approach to a many-body $NN$ system such as a nucleus and study in heavy ion collisions.

Little is known about hadronic matter at finite temperatures and densities other than the nuclear ground state density $\rho_0 = 0.16$ fm$^{-3}$. Unfortunately, there exists, at the moment, no theoretical model that consistently provides an understanding of the reaction dynamics of heavy ion collisions over the entire energy range. It is believed that
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The quark-gluon plasma can be observed in high energy collisions, where temperature and density are large enough to deconfine quarks and gluons. At lower energies heavy ion collisions have been used to investigate the appearance of collective effects and to probe the conditions related to the nuclear equation of state \[8, 9\]. Theoretically these studies have been performed by numerical simulations in many models. For example, transverse and longitudinal collective flow as well as azimuthal distributions provides complementary information which can be used to evaluate the details of microscopic models such as Boltzmann equation \[10, 11\], Vlasov-Uehling-Uhlenbeck \[12, 13, 14\] and quantum molecular dynamics \[15\]. One of the main ingredients in these models is the nucleon-nucleon cross-section, where it turns out that many observables are very sensitive to its value. It is usual to assume as an input to simulations the free \(NN\) cross-section. However recent studies on collective flow have indicated a density dependent in-medium reduction from its value in free space \[16, 17\].

In this paper we shall extend the study of Ref. 30 and develop this aspect considering quark-gluon exchange corrections to the nucleon-nucleon cross-section in the VUU equation as a first approach. The procedure we shall use, in order to introduce these corrections, is known as the Fock-Tani formalism (FTf), which is a method that employs a second quantization formalism to problems where the internal degrees of freedom of composite particles cannot validly be neglected.

2. The model

The Vlasov-Uehling-Uhlenbeck (VUU) equation is a differential equation for the classical one-body phase-space distribution function \(f(\mathbf{r}, \mathbf{p}, t)\) corresponding to the classical limit of the Wigner function. The VUU equation takes a familiar form

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla_r U \cdot \nabla_p f = - \int \frac{d^3 p_2 d^3 p_1' d^3 p_2'}{(2\pi)^6} \sigma \frac{v_{12}}{v} \delta^3(p + p_2 - p_1' - p_2') \times [f_2(1 - f_2')f_1(1 - f_1')(1 - f_2')(1 - f_2)] (1)
\]

where \(f_2'\) is a stands for \(f(\mathbf{r}, \mathbf{p}', t)\) and correspondingly for the other terms. The two-body interactions are divided in a short-range and long-range part. The short-range interactions are regarded as corresponding to the hard-core binary collisions. The long-range interactions are assumed to be given by a potential with origin in a density-dependent mean field, formed by the nuclear matter, around the particle in use. The potential \(U\) may be written in terms of the modified Skyrme forces \[8\]

\[
U = \alpha \frac{\rho}{\rho_0} + \beta \left(\frac{\rho}{\rho_0}\right)^\gamma.
\]

The conditions used to fix the three parameters \(\alpha, \beta\) and \(\gamma\) are: (i) the ground state energy must assume the correct value for nuclear matter (-16 MeV); (ii) the ground state at \(\rho = \rho_0\) has to be a minimum; (iii) the compression modulus \(\kappa\) should be some hundred MeV. The magnitude of \(\kappa\) can be obtained from the measure of the radius of the giant monopole resonances in nuclei.
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In order to include the quark corrections to the VUU equation we shall briefly outline the main features of the Fock-Tani approach to baryon-baryon interactions. One starts with the Fock space representation of the system, using creation and annihilation operators of the elementary hadron constituents particles. A one-baryon state, with momentum $\mathbf{P}$, internal energy $\epsilon$, spin projection $M_S$ and isospin projection $M_T$, can be written as

$$|\alpha\rangle \equiv |\mathbf{P}, \epsilon, M_S, M_T\rangle = B_\alpha^\dagger |0\rangle ,$$

where $|0\rangle$ is the vacuum (no quarks) state and $B_\alpha^\dagger$ is the baryon creation operator

$$B_\alpha^\dagger = \frac{1}{\sqrt{3!}} \Psi_{\mu_1\mu_2\mu_3} q_{\mu_1}^\dagger q_{\mu_2}^\dagger q_{\mu_3}^\dagger .$$

We use the convention that a sum over repeated indices is implied. The indices $\mu_i$ denote the spatial, spin-flavor, and color coordinates of the i-th quark. The baryon bound-state wave functions are taken orthonormalized. Using the quark relations and orthonormality condition for $\Psi$ above, one can show the following anti-commutation relations for the baryon operators:

$$\{ B_\alpha, B_\beta \} = \{ B_\alpha^\dagger, B_\beta^\dagger \} = 0$$
$$\{ B_\alpha, B_\beta^\dagger \} = \delta_{\alpha\beta} - \Delta_{\alpha\beta} ,$$

where

$$\Delta_{\alpha\beta} = 3\Psi_\alpha^* \Psi_\beta \Psi_\beta^* \Psi_\alpha \Psi_{\mu_1\nu_1\nu_2\nu_3} q_{\mu_1} q_{\nu_1} q_{\nu_2} q_{\nu_3} - \frac{3}{2} \Psi_\alpha^* \Psi_\beta \Psi_\beta^* \Psi_\alpha \Psi_{\mu_1\nu_1\nu_2\nu_3} q_{\mu_1}^\dagger q_{\nu_1}^\dagger q_{\nu_2} q_{\nu_3} .$$

Observe that the baryon operators $B_\alpha$ do not satisfy the canonical anti-commutation relations for fermions. The extra term $\Delta_{\alpha\beta}$ appearing in the rhs of Eq. (5) results from the composite nature of the baryons. The presence of this term complicates the application of the usual field theoretic techniques to the $B_\alpha$ and $B_\alpha^\dagger$ operators, making these operators inconvenient dynamical variables. The change of representation of the FTf is implemented by means of a unitary transformation $U$, such that the composite particle operators $B_\alpha$ and $B_\alpha^\dagger$ are re-described by “ideal” baryon operators $b_\alpha$ and $b_\alpha^\dagger$. By definition, the ideal baryon operators satisfy canonical anti-commutation relations:

$$\{ b_\alpha, b_\beta \} = \{ b_\alpha^\dagger, b_\beta^\dagger \} = 0$$
$$\{ b_\alpha, b_\beta^\dagger \} = \delta_{\alpha\beta} ,$$

and are kinematically independent from the quark operators $\{ q_{\mu}, b_\alpha \} = \{ q_{\mu}, b_\alpha^\dagger \} = 0$.

The transformation from the physical space to the ideal space is performed by a unitary operator. The operator $U$ is given by:

$$U = \exp \left[ -\frac{\pi}{2} \left( b_\alpha^\dagger O_\alpha - O_\alpha^\dagger b_\alpha \right) \right] ,$$

where $O_\alpha$ is an operator given in terms of the $B_\alpha$ and $B_\alpha^\dagger$ and $\Delta_{\alpha\beta}$ of Eq. (6) as:

$$O_\alpha = B_\alpha + \frac{1}{2} \Delta_{\alpha\beta} B_\beta - \frac{1}{2} B_\beta^\dagger [\Delta_{\beta\gamma}, B_\alpha] B_\gamma .$$
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Given this, the evaluation of the transformed quark operators is straightforward. Full details of the derivation of the iterative solution is presented elsewhere [19]; here we simply present the final results. The $O_\alpha$ operator is built in such a way that a single real-baryon state $|\alpha\rangle$ is transformed into a single ideal-baryon state $|\alpha\rangle$:

$$|\alpha\rangle \equiv U^{-1} B^\dagger_\alpha |0\rangle = b^\dagger_\alpha |0\rangle .$$

In order to discuss nucleon-nucleon scattering, one needs to specify the general form of the microscopic quark Hamiltonian. For our purposes here, the microscopic Hamiltonian can be written in terms of the quark operators as

$$H = K(\mu) q^\dagger_\mu q_\mu + \frac{1}{2} V_{qq}(\mu\nu; \sigma\rho) q^\dagger_\mu q^\dagger_\nu q_\rho q_\sigma ,$$

where $K$ is the kinetic energy and $V_{qq}$ is the quark-quark interaction. Note that this is a general expression for $H$, which in this form, a variety of quark-model Hamiltonians used in the literature can be written. In our calculation we shall use, for $V_{qq}$, the spin-spin hyperfine component of the perturbative one gluon interaction

$$V_{qq} = -\frac{8\pi\alpha_s}{3m_1m_2} S_i \cdot S_j F^a_i F^a_j ,$$

where $F^a_i = \lambda^a/2$ are the Gell-Mann matrices. There is a considerable literature related to free NN scattering with quark-interchange and in many of these models the quark-quark potential is much more elaborated than the potential in Eq. (12) (including Coulomb, spin-orbit, tensor, confinement terms and eventually meson coupling to quarks). The lesson taken from all of these works is that the dominant term for the short-range repulsion is basically the spin-spin term from the one gluon exchange potential. Its strong influence is seen, for example, in the $^{1}S_0$ partial-wave where repulsion increases when $\lambda$ is increased. In these models $\lambda$ is a free parameter which usually ranges from 0.2 fm to 0.5 fm or even 0.6 fm (see Ref. [18]). This behavior also happens in the $KN$ system [7]. In this perspective a very first step in order to include quark corrections to the VUU equation would be to use the dominant term in quark-quark interaction to simulate the short-range hard-core and verify its effect on the observables.

The effective baryon-baryon Hamiltonian is obtained from the expansion retaining the lowest order terms in $\Psi$:

$$H_{bb} = \Psi^*_{\alpha} M_{\mu\nu} H(\mu\nu; \sigma\rho) \Psi_{\beta}^{\sigma\rho\lambda} b^\dagger_\alpha b_\beta + \frac{1}{2} V_{bb}(\alpha\beta; \gamma\delta) b^\dagger_\alpha b^\dagger_\beta b_\gamma b_\delta ,$$

where $V_{bb}$ is an effective baryon-baryon potential which is of $O(\Psi^4)$. The scattering $T$-matrix can obtained from Eq. (13)

$$T(\alpha\beta; \gamma\delta) = (\alpha\beta|V_{bb}|\gamma\delta)$$

Due to translational invariance, the $T$-matrix element is written as a momentum conservation delta-function, times a Born-order matrix element, $h_{fi}$

$$T(\alpha\beta; \gamma\delta) = \delta^{(3)}(P_f - P_i) h_{fi}$$

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where \( P_f \) and \( P_i \) are the final and initial momenta of the two-nucleon system. The scattering amplitude \( h \) is a function of the nucleon’s wave-function written as

\[
\Psi_{\alpha_{123}} = \frac{N}{\sqrt{3!}} \epsilon_{\alpha_{123}} \psi_{\alpha_{123}} \phi(p_1) \phi(p_2) \phi(p_3) \delta(p_1 - p_2 - p_3) \tag{16}
\]

with \( \epsilon_{\alpha_{123}} \) the antisymmetric color tensor; \( \psi_{\alpha_{123}} \) spin-isospin wave-function; \( \phi \) the quark’s spatial wave-function and \( N \) the norm. The function \( \phi \) can be defined as a Gaussian wave-function defined by

\[
\phi(p) = \left( \frac{\lambda}{\sqrt{\pi}} \right)^{3/2} \exp \left( -\frac{\lambda^2 p^2}{2} \right), \tag{17}
\]

then \( N \) is

\[
N(p) = \left( \frac{3\pi \lambda^2}{2} \right)^{3/4} \exp \left( \frac{\lambda^2 p^2}{6} \right). \tag{18}
\]

It can be shown that the width parameter \( \lambda \) is related to the \( rms \) radius of the nucleon \( \langle r^2 \rangle = \lambda^2 \). A diagramatic representation for \( h \) can be seen in Fig. [1]. The connection between the cross-section and the scattering amplitude \( h \) is shown in Ref. [19]-[21].

We shall use this result in order to evaluate nucleon-nucleon scattering cross-section in the Fock-Tani approach:

\[
\sigma_{NN} = \frac{4\pi^5}{s - 4m_N^2} \int_0^1 dt |h(t)|^2 = \frac{4\pi^5}{s - 4m_N^2} \kappa_{ss}^2 I_{NN} \tag{19}
\]

where \( \kappa_{ss} = 8\pi\alpha_s/3m_q^2(2\pi)^3 \) and the quark model the ratio \( \alpha_s/m_q^2 \) is related to the width of the Gaussian wave-function \( \lambda \) [23] [24] by

\[
\frac{\alpha_s}{m_q^2} = \frac{3\sqrt{2}\pi}{4} (m_\Delta - m_N) \lambda^3. \tag{20}
\]

As a consequence of using a Gaussian wave-function to represent the bound-state is that the integral \( I_{NN} \) in the cross-section [19] has an analytical expression:

\[
I_{NN} = \frac{3}{2\lambda^2} \left( x_1^2 + x_5^2 \right) \left[ 1 - e^{\frac{2\lambda^2 (4m_N^2 - s)}{3}} \right] + \frac{144}{5\lambda^2} \sqrt{\frac{3}{11}} \left[ x_5 (x_2 + x_3) + x_1 \left( x_6 + x_7 \right) \right] \left[ e^{\frac{8\lambda^2 (4m_N^2 - s)}{33}} - e^{\frac{13\lambda^2 (4m_N^2 - s)}{33}} \right] + \frac{144}{17\lambda^2} \sqrt{\frac{3}{11}} \left[ x_1 \left( x_2 + x_3 \right) + x_5 \left( x_6 + x_7 \right) \right] \left[ e^{\frac{2\lambda^2 (4m_N^2 - s)}{33}} - e^{\frac{19\lambda^2 (4m_N^2 - s)}{33}} \right] + \frac{432}{121\lambda^2} \left[ (x_2 + x_3)^2 + (x_6 + x_7)^2 \right] \left[ e^{\frac{4\lambda^2 (4m_N^2 - s)}{33}} - e^{\frac{16\lambda^2 (4m_N^2 - s)}{33}} \right] + \frac{9\sqrt{3}}{4\lambda^2} \left[ x_1 + x_5 \right] \left( x_4 + x_8 \right) \left[ e^{\frac{\lambda^2 (4m_N^2 - s)}{12}} - e^{\frac{5\lambda^2 (4m_N^2 - s)}{12}} \right] + \frac{27}{\sqrt{11\lambda^2}} \left( x_2 + x_3 + x_6 + x_7 \right) \left( x_4 + x_8 \right) \left[ e^{\frac{19\lambda^2 (4m_N^2 - s)}{132}} - e^{\frac{43\lambda^2 (4m_N^2 - s)}{132}} \right] - \frac{27}{64} \left( 4m_N^2 - s \right) \left( x_4 + x_8 \right)^2 e^{\frac{\lambda^2 (4m_N^2 - s)}{6}} - 2 \left( 4m_N^2 - s \right) x_1 x_5 e^{\frac{\lambda^2 (4m_N^2 - s)}{3}}
\]
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\[ \chi_1 \chi_2 \chi_3 \chi_4 \text{ rel. phase } \chi_5 \ldots \chi_8 \]

| \( I = 1; S = 1 \) | \( \frac{31}{81} \) | \( \frac{17}{81} \) | \( \frac{17}{81} \) | \( \frac{10}{81} \) | (−) |
| \( S = 0 \) | \( \frac{1}{27} \) | \( \frac{1}{27} \) | \( \frac{1}{27} \) | \( \frac{1}{27} \) | (−) |
| \( I = 0; S = 1 \) | \( \frac{10}{27} \) | \( \frac{1}{27} \) | \( \frac{1}{27} \) | \( \frac{2}{27} \) | (+) |
| \( S = 0 \) | \( -\frac{1}{9} \) | \( \frac{5}{9} \) | \( \frac{2}{9} \) | 0 | (−) |

Table 1. \( \chi_i \) weights as function of total \( NN \) spin \( S \) and isospin \( I \) in Eq. (21).

\[-\frac{3456}{1331} \left(4m_N^2 - s\right) \left(\chi_2 + \chi_3\right) \left(\chi_6 + \chi_7\right) e^{\frac{10\lambda^2}{33} \left(4m_N^2 - s\right)} \]

The quantities \( \chi_i \) are obtained from the sum over the quark color-spin-flavor indices and are a function of total \( NN \) spin \( S \) and isospin \( I \). In the appendix A their calculation is shown in detail and in table 1 their values are presented for different nucleon spin and isospin.

![Figure 1. Diagrams representing the scattering amplitude \( h_{fi} \) for \( NN \) interaction with quark interchange.](image)

3. Results of the simulation

The \( NN \) cross-section is extremely sensitive to the \( \lambda \) parameter which is the rms radius of the nucleon. In our calculation the \( \Delta \)-nucleon mass splitting is set to 0.3 GeV. The usual values for \( \lambda \) in the quark model range from 0.2 fm to 0.8 fm. We have chosen four representative values for \( \lambda : 0.2, 0.3, 0.4, 0.5 \) fm, when combined with Eq. (20) sets the ratio \( \alpha_s/m_q^2 = 0.57, 1.97, 4.69, 9.08 \) GeV\(^{-2}\). The effect of the \( \lambda \) variation in the PP cross-section in comparison with the free proton-proton cross-section as a function of \( \sqrt{s} \) can be seen in Fig. 2.
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The numerical solution of the VUU equation is equivalent to evolving the test particles by Newtonian mechanics. For the starting configuration, a Fermi-gas ansatz is used and all particles of one nucleus are randomly distributed inside a sphere in coordinate and momentum space, such as

\[
(\vec{r}_i - \vec{r}_{CM})^2 \leq r_0^2 A^{2/3}, \quad (\vec{p}_i - \vec{p}_{CM})^2 \leq p_F^2
\]  

The collision integral in Eq. (1) is treated in a stochastic way, allowing test particles to undergo collisions with probability proportional to the Pauli-corrected cross-section. A pair of particles collide if their minimum distance \( d \) fulfills the following condition

\[
d \leq \sqrt{\frac{\sigma_{\text{total}}}{\pi}}
\]

The simulations we present in this paper tested the cross-section variation effects on a Niobium-Niobium (Nb+Nb) collision at \( E_{\text{lab}} = 1050 \text{ MeV/nucleon} \) and impact parameter \( b = 3 \text{ fm} \). Other nuclei (with symmetric or asymmetric collisions), energies and \( b \) have also been tested, but our calculation shall be restricted to the Nb+Nb system which exhibits the qualitative effects we want to demonstrate. A necessary input for the simulation is the choice of the equation of state. We have chosen a repulsive potential of high compressibility \( (k = 380 \text{ MeV}) \) the so call “hard eos” with \( \alpha = -124 \text{ MeV}, \beta = 70.5 \text{ MeV}, \gamma = 2 \) in Eq. (2). The geometry of the collision is as usual: there are two transverse directions \( x, y \) while \( z \) is the beam direction. Thus the reaction plane is defined as being the \( xz \)-plane.

The variation of \( \lambda \) affects significantly the observables as can be seen in Figs. 3, 4, 5. In these figures the curves are average plots where the simulation points are
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![Graph showing proton distribution as a function of rapidity (Y) for E_{lab} = 1050 MeV; b = 3 fm.]

**Figure 3.** Proton distribution as a function of rapidity $Y$ for $E_{lab} = 1050$ MeV; $b = 3$ fm.

![Graph showing proton distribution as a function of the transverse momentum ($p_t$) for $E_{lab} = 1050$ MeV; $b = 3$ fm.]

**Figure 4.** Proton distribution as a function of the transverse momentum $p_t$ for $E_{lab} = 1050$ MeV; $b = 3$ fm.

The term *nuclear stopping power* characterizes the degree of stopping which an incident nucleon suffers when it collides with another nucleus and is used in the study of high energy collisions. Due to the strong dependence of the nucleon rapidity distribution on the $NN$ cross-section a change in $dN/dY$ distribution indicates a creation of a zone higher in nucleon density. In Fig. 3 the proton distribution is shown as a function of rapidity ($Y$), where a change in the shape of the curve, when $\lambda$ varies from 0.2 fm to 0.5 fm, shows clearly the referred effect. The effect of this variation on the transverse momentum spectra is represented in Fig. 4.

The transverse momentum...
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![Graph showing in-plane transverse momentum $p_x$ versus rapidity $Y$ for $E_{lab} = 1050$ MeV; $b = 3$ fm.]

**Figure 5.** In-plane transverse momentum $p_x$ versus rapidity $Y$ for $E_{lab} = 1050$ MeV; $b = 3$ fm.

The distribution ($p_x$) as a function of rapidity ($Y$) is seen in Fig. 5. It becomes evident, from this figure, that bounce-off also increases dramatically as a function of $\lambda$.

Although that at current collision energies pion production is present, and incorporated in the model through inelastic channels ($NN \rightarrow N\Delta$, $N\Delta \rightarrow NN$, $\Delta \rightarrow N\pi$, $N\pi \rightarrow \Delta$) there are no significant modifications in the pion distribution as can be seen in Fig. 6. This can be understood considering that the only modification introduced in this approach is to the $\sigma_{NN}$ value.

4. Conclusions

In the original VUU model the ions are composed by nucleons which are regarded as structureless point particles while the equation of state is obtained from the Skyrme potential $U$ as a function of the density as seen in Eq. (2). In our model the quark structure of the nucleons are introduced and their effects are studied in the observables related to the collision. In the very simple approach we use for the nucleon a single parameter $\lambda$, which is the _rms_ radius, is treated in the same way as in the naive quark models, as a free parameter. In these models it assumes a range of different values in order to fit the scattering data. In present study we find that the extended nature of the nucleon, determined by the finite values of $\lambda$, is actually reflected in the observables.

In our simulation, as a first approximation, we use Gaussian wave-function’s for the nucleon which permits an analytical evaluation of the cross-section. A more realistic choice for $\Psi$ would lead to a multi-dimensional integral in $\sigma_{NN}$, which in the context of the present study, represents a more time-consuming and elaborated simulation. A
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A possible extension to our current calculation is to consider Ref. [29] where a meson-baryon interaction is derived in the context of the Fock-Tani formalism and obtain a $\sigma_{\pi N}$ cross-section from quark physics. The influence of the internal quark
structure to the \( \Delta \) resonance can also be addressed and is straightforward. As stated before, in a more realistic calculation a density modification, due to the collision, should imply in a change in the properties of nucleons and mesons in the medium. In this case an alternative is to simulate the collision replacing \( \lambda \) and the quark mass \( m_q \) by effective in-medium parameters \( \lambda^* \) and \( m_q^* \) obtained in a self-consistent calculation.

Finally, another possible extension to our present work is to address heavy ion collisions at high energies. One of the most successful models in this regime is the UrQMD \[31\]. From our present approach, general baryon-baryon cross-sections can be calculated, from \[19\] meson-meson cross-sections are obtained and from \[29\] all meson-baryon cross-sections, that appear in UrQMD, can be derived incorporating quark interchange.

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**Appendix A. Evaluation of the \( \chi_i \) coefficients**

As stated before, details related to the derivation of the \( V_{bb} \) potential in Eq. (13) can be found in Ref. \[19\], so we present here only the final result in order to evaluate the \( \chi_i \) coefficients (these coefficients have also been calculated in Ref. \[22\]):

\[
V_{bb}(\alpha\beta; \gamma\delta) = \sum_{i=0}^{4} v_i(\alpha\beta; \gamma\delta).
\]  

(A.1)

where

\[
v_0(\alpha\beta; \gamma\delta) = 9V_{qq}(\mu\nu; \sigma\rho) \Psi^\ast_{\alpha} \Psi^\ast_{\beta} \Psi_{\gamma} \Psi_{\delta}
\]

\[
v_1(\alpha\beta; \gamma\delta) = -36V_{qq}(\mu\nu; \sigma\rho) \Psi^\ast_{\alpha} \Psi^\ast_{\beta} \Psi_{\gamma} \Psi_{\delta}
\]

\[
v_2(\alpha\beta; \gamma\delta) = -9V_{qq}(\mu\nu; \sigma\rho) \Psi^\ast_{\alpha} \Psi^\ast_{\beta} \Psi_{\gamma} \Psi_{\delta}
\]

\[
v_3(\alpha\beta; \gamma\delta) = 18V_{qq}(\mu\nu; \sigma\rho) \Psi^\ast_{\alpha} \Psi^\ast_{\beta} \Psi_{\gamma} \Psi_{\delta}
\]

\[
v_4(\alpha\beta; \gamma\delta) = -18V_{qq}(\mu\nu; \sigma\rho) \Psi^\ast_{\alpha} \Psi^\ast_{\beta} \Psi_{\gamma} \Psi_{\delta}
\]

(A.2)

Schematically one can represent each \( v_i \) in (A.2) by

\[
v_i = \chi_i \text{ } I_{\text{space}}(i).
\]  

(A.3)

where \( I_{\text{space}}(i) \) are the spatial integrals and

\[
\chi_i \equiv C(i) \text{ } I_{\text{color}}(i) \text{ } I_{\text{SI}}(i).
\]  

(A.4)

The factor \( C \) for each term is obtained directly from \(A.2\) \( \times (-1) \)

\[
C(0, \ldots, 4) = (-9, 36, 9, -18, 18).
\]  

(A.5)
The color factors \( I_{\text{color}} \) is

\[
I_{\text{color}}(0) = \frac{1}{36} \mathcal{F}_{\mu \nu}^a \mathcal{F}_{\rho \sigma}^a \epsilon^{\mu \nu \rho \sigma} \epsilon^{\nu \mu \rho \sigma} = 0
\]

\[
I_{\text{color}}(1) = \frac{1}{36} \mathcal{F}_{\mu \nu}^a \mathcal{F}_{\rho \sigma}^a \epsilon^{\mu \nu \rho \sigma} \epsilon^{\nu \mu \rho \sigma} = \frac{1}{9}
\]

\[
I_{\text{color}}(2) = \frac{1}{36} \mathcal{F}_{\mu \nu}^a \mathcal{F}_{\rho \sigma}^a \epsilon^{\mu \nu \rho \sigma} \epsilon^{\nu \mu \rho \sigma} = \frac{4}{9}
\]

\[
I_{\text{color}}(3) = \frac{1}{36} \mathcal{F}_{\mu \nu}^a \mathcal{F}_{\rho \sigma}^a \epsilon^{\mu \nu \rho \sigma} \epsilon^{\nu \mu \rho \sigma} = \frac{2}{9}
\]

\[
I_{\text{color}}(4) = \frac{1}{36} \mathcal{F}_{\mu \nu}^a \mathcal{F}_{\rho \sigma}^a \epsilon^{\mu \nu \rho \sigma} \epsilon^{\nu \mu \rho \sigma} = -\frac{2}{9}
\]  

(A.6)

where we have used the following property of \( SU(N) \) matrices

\[
M_{\mu \nu}^a M_{\rho \sigma}^a = 2 \delta_{\mu \rho} \delta_{\nu \sigma} - f \delta_{\mu \sigma} \delta_{\nu \rho}
\]

(A.7)

The product \( C \ I_{\text{color}} \) results

\[
C \ I_{\text{color}}(0, \ldots, 4) = (0, 4, 4, -4, -4).
\]

(A.8)

The spin-isospin part is

\[
I_{St}(0) = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

\[
I_{St}(1) = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

\[
I_{St}(2) = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

\[
I_{St}(3) = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

\[
I_{St}(4) = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

(A.9)

where the spin-isospin index \( I_\mu \equiv (s_\mu, t_\mu) \) and \( S_i = \sigma_i/2 \).

\[
\chi_0 = 0
\]

\[
\chi_1 = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

\[
\chi_2 = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

\[
\chi_3 = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

\[
\chi_4 = \frac{1}{18^2} \delta_{\mu \nu} \delta_{\rho \sigma} \delta_{\alpha \beta} \delta_{\gamma \delta} \psi_1^\mu \psi_2^\nu \psi_3^\rho \psi_4^\sigma
\]

(A.10)

After the contractions of repeated indices in Eq. [A.10] and the correct choices of the nucleon spin and isospin indices \((\alpha, \beta, \gamma, \delta)\) one obtains the values in table 1.

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