Energy Dependence of Quasi-Particle Relaxation in a Disordered Fermi Liquid

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Abstract

A spectroscopic method is applied to measure the inelastic quasi-particle relaxation rate in a disordered Fermi liquid. The quasi-particle relaxation rate, $\gamma$ is deduced from the magnitude of fluctuations in the local density of states, which are probed using resonant tunneling through a localized impurity state. We study its dependence on the excitation energy $E$ measured from the Fermi level. In a disordered metal (heavily doped GaAs) we find that $\gamma \propto E^{3/2}$ within the experimentally accessible energy interval, in agreement with the Altshuler-Aronov theory for electron-electron interactions in diffusive conductors.
The quasi-particle description of electrons represents a common approach to understanding kinetic and thermodynamic phenomena in metals. It relies on a certain stability of quasi-particle excitations in a many-body system, which requires that the quasi-particle excitation energy, \( E \), determined with respect to the Fermi level, \( E_F \), exceeds the broadening of such a single-particle state, \( \hbar \gamma(E) \), due to electron-electron interactions.

In ideally pure metals, the relaxation rate, \( \gamma \), of ballistic quasi-particles at low energies is kept low by a diminishing of the phase space available for inelastic electron-electron collision processes [1]. The phase space argument results in \( \gamma \sim E^2 \) dependence, which has been confirmed by electron emission spectroscopy measurements which probe electron states at energies \( E \gg \hbar/\tau \), where \( \tau \) is the elastic mean free path time due to residual impurities. Quasi-particle decay at excitation energies \( E < \hbar/\tau \) is accelerated by the presence of disorder (which makes the electron states chaotically random at length scales longer than the elastic mean free path, \( l = \tau v_F \)). The theory of interaction effects in diffusive media [2] predicts that, at zero temperature, the decay of quasi-particle excitations with \( E \ll \hbar/\tau \) in a disordered metal or heavily doped semiconductor is slow enough to guarantee the existence of properly defined quasi-particles. In particular, in bulk three-dimensional (d=3) and two-dimensional (d=2) conductors, the quasi-particle relaxation rate is expected to obey a power-law dependence,

\[
\gamma(E) = aE^{d/2}E_F^{(1-d/2)}(\lambda_F/l)^{d/2}.
\] (1)

Despite the fundamental interest and clear theoretical predictions [2], there are very few direct experimental measurements of the quasi-particle decay time in disordered metals at small excitation energies \( \varepsilon < \hbar/\tau \). Information about the inelastic decay of non-equilibrium quasi-particles in dirty metals is often extracted from energy relaxation rates in the electron thermalization process [3], \( \gamma_E \). Otherwise, one studies the temperature dependence of a dephasing rate, \( \gamma_\phi(T) \), of coherent carriers (determined using weak localisation or universal conductance fluctuations analysis), which can be treated as a measure of the efficiency of the interactions of an electron with energy \( E \leq T \) with equilibrium fluctuations of charges
produced by other electrons thermally distributed near the Fermi level in a diffusive conductor [4]. At high temperatures, both phase and energy relaxation experiments show a certain agreement with theoretical estimations [5–8]. However, recently reported data [9,3] on two dynamical parameters mentioned above, $\gamma_E(E)$ and $\gamma_\varphi(T)$, in Au wires and films, and also in semiconductor heterostructures [8] have indicated a certain disagreement between theoretically predicted and experimentally observed values of these two quantities, which has refuelled both theoretical and experimental interest in the problem of quasi-particle lifetimes in a disordered metal [10].

In the present paper, we report the results of a direct measurement of the energy dependence of the inelastic decay rate $\gamma(E)$ of a quasi-particle state in a disordered conductor. This study employs the method of resonant tunneling spectroscopy using a discrete localised state in a double-barrier structure, which has been applied earlier by A.Geim et al [11] to study 2D electrons in a heterostucture and by U.Sivan et al [12] to investigate the discrete spectra of quantum pillars. It has been shown previously [13,15–17] that, by measuring the current-voltage (IV) characteristics and by deriving the differential conductance in a system where the current passes through a single resonant impurity state in the barrier, one can study features of the single-particle spectrum of a disordered metal (playing the role of an emitter). In a bulk material with a continuous spectrum, individual chaotic quantum states formed by the interference of elastically scattered electrons produce an effect known as fluctuations of the local density of states (LDOS) [18]. It consists of a random and coordinate-specific energy dependence of the local density of states in a diffusive metal, $\nu(E)$ with a correlation energy limited by inelastic broadening of quasi-particle states, $h\gamma$. We observe a random pattern in $\nu(E)$ by sweeping a single resonant impurity level against the electron spectrum in the emitter within a finite range of excitation energies for a quasi-hole (an empty state below $E_F$ in the emitter) left behind by the tunneled electron. When the energetic width of an impurity level used in this process, $\Gamma$, is smaller than the inelastic broadening of single-particle levels in the emitting electrode, $h\gamma$, one can extract the latter characteristic from the analysis of the amplitude of the LDOS fluctuations pattern.
and its auto-correlation parameters. This has enabled us to measure directly the energy dependence of the inelastic relaxation rate, $\gamma(E)$, in bulk degenerate heavily doped GaAs at low temperatures, which we find to agree with the Altshuler-Aronov theory predicting $\hbar \gamma(E) = a \left( E^{3/2}/\sqrt{E_F} \right) (\lambda_F/l)^{3/2}$ for a 3D system. [Note, that under the condition of $\hbar \gamma(E) \ll E$ provided in Eq. (1) for $\lambda_F \ll l$, the inelastic decay rate $\gamma$ of a non-equilibrium quasi-particle coincides with its decoherence rate.]

As mentioned above, the LDOS can be measured via resonant tunneling through an impurity in a strongly asymmetric double-barrier heterostructure. Our microstructure consists of a 10 nm wide GaAs quantum well and 5 and 8 nm wide Al$_{0.3}$Ga$_{0.7}$As barriers sandwiched between doped GaAs contact layers with a donor concentration of $3.3 \times 10^{17}$ cm$^{-3}$, as sketched in Fig. 2(a). From this material we fabricated a 2 $\mu$m diameter mesa, as depicted in the scanning electron micrograph of Fig. 2(b). The mesa contains a small number of residual impurities in the nominally undoped quantum well. The energetically lowest impurity state $S$ in the well is used as a spectrometer for the LDOS imaging in the metallic emitter adjacent to the thicker barrier as illustrated in Fig. 2(c). At zero bias, $S$ lies above the Fermi level in the emitter and is not available for resonant transport, resulting in $I = 0$ and $G = 0$. This measurement has been performed at the temperature $T = 20\text{mK}$. Upon applying a finite bias voltage $V$, the energetic position of a spectrometer $S$ is shifted down to the energy $E = \alpha e(V - V_S)$ below the Fermi level in the emitter, where the prefactor $\alpha = 0.50$ accounts for the fact that only part of the voltage drops between emitter and spectrometer [20]. When $S$ crosses $E_F$ from above (at $V_S = 9.8\text{mV}$), the current acquires a finite value (plateaux) limited by the left, less transparent barrier and proportional to the LDOS of occupied states in the emitter at the spectrometer position, $I \propto \nu$ [13,15]. At higher bias voltages ($V_{H1} = 14.6\text{mV}$ and $V_{H2} = 15.5 \text{mV}$), other impurities (or, maybe, other excited states from the same impurity) become involved in the current formation, which produces the next prominent current steps in the IV-characteristics. Consequently, the differential conductance $G = dI/dV \propto d\nu(E)/dE$ of the device plotted in Fig. 2(d) exhibits several pronounced peaks, the 'main' one at $V_S = 9.8\text{mV}$ followed by two at 14.6 and 15.5 mV, each
characterising the energetic width and transparency of a resonant impurity state, whereas in between, at $V_S \leq V \leq V_{H1}$, the differential conductance displays the derivative of the LDOS with respect to energy.

The energy dependence of the LDOS is the result of the energy-dependent quantum interference pattern for quasi-particles in the emitter at the coordinate of a spectrometer, see Fig. 2(c). This pattern is random and tends to reflect an individual portrait of a disordered potential in a metal surrounding the spectrometer. In the sample under investigation, such a pattern can be analyzed within the energy interval of about $0 \leq E \leq 2$ meV below the Fermi level, since the voltage and energy scales are related via $E = \alpha e (V - V_S)$, $\alpha \approx 0.5$. The correlation energy, $E_c = \Gamma + \hbar \gamma$, of the fine structure in the differential conductance pattern is determined by either the energetic spectrometer width, $\Gamma$, or by the inelastic broadening of states in the emitter, $\hbar \gamma$, whichever is larger. According to the theory [15], one can relate $E_c$ to the amplitude (rms value) of the fluctuation pattern of $G(V) \propto d\nu(E)/dE$. Note that, although oscillations at larger energy scales are also present in each realization of $\nu(E)$, their contribution to $G(V)$ is suppressed, due to the differentiation. For a given sample, the spectrometer width, $\Gamma$, can be extracted from the width of the 'main' resonance peak. For the peak at $V_S = 9.8$ mV in Fig. 2(d), we find $\Gamma \approx e\alpha \times 72 \mu V \approx 36 \mu eV$. For a broad spectrometer, with $\Gamma \gg \hbar \gamma(E)$ at any excitation energy [13], both the amplitude and correlation voltage (energy) of fluctuations would be the same over the entire range of $V_S \leq V \leq V_{H1}$. For a narrow spectrometer, such as studied in the present work, inelastic broadening of states in the bulk exceeds the spectrometer width upon increasing the excitation energy of a quasi-hole left in the emitter. This results in a decrease of fluctuations upon increasing voltage, as indicated by the dashed lines in Fig. 2(d).

Quantitative information about the quasiparticle decay rate, $\gamma(E)$, is obtained from statistical analysis of the complete fluctuation pattern $G(V, B) = dI/dV$ studied as a function of a magnetic field, $B$ (applied parallel to the current flow). Figure 3 shows a color-scale image of the differential conductance measured as a function of bias voltage and magnetic field (in the region of low magnetic fields, where Landau quantization is hindered by disor-
der). Sharp black lines in Fig. 3 correspond to the spectrometer crossing the emitter Fermi level. The decrease of the amplitude of observed LDOS fluctuations and the increase of the correlation voltage of the fluctuation pattern, as a function of the quasi-hole excitation energy is apparent from the change in the color and contrast of this image.

As a quantitative measure of the fluctuation amplitude, we calculate the variance \( \text{var}_B \langle G \rangle = \langle \delta G^2(B) \rangle_B \) using \( \delta G(B) = G(B) - \langle G \rangle_B \), where \( \langle ... \rangle_B \) indicates averaging over magnetic field in the range of \( 0 \leq B \leq 1.0 \) T. Figure 4(a) shows that \( \text{var}_B \langle G \rangle \) drops by more than one order of magnitude within the experimentally accessible voltage range. In our limit of classical magnetic fields, the fluctuation amplitude is related to the relaxation rate [13] according to

\[
\text{var}_B \langle G \rangle |_{V} = G_N^2 \times [1 + \hbar \gamma(E)/\Gamma]^{-3/2}.
\]

Here, \( G_N \) is a prefactor which we determine as \( G_N^2 = \text{var}_B \langle G \rangle |_{V_S} \) from Fig. 4(a) by assuming that, at \( V = V_S \) (corresponding to \( E = 0 \)), \( \hbar \gamma \ll \Gamma \) and \( E = \alpha e(V - V_S) \) is the excitation energy of a quasi-hole (we remind that \( \alpha \approx 0.5 \) in this experiment).

Figure 4(b) shows the obtained energy dependence of the quasi-particle decay rate. It drops strongly upon decreasing the excitation energy, and, in contrast to some experiments measuring dephasing rates [9], we do not observe a saturation of \( \gamma(E) \) at low energies. In Fig. 4(b) we also compare the experimentally determined quasi-particle relaxation rate with the values calculated using the Altshuler-Aronov theory of electron-electron interaction in disordered conductors [2,19]. It is worth mentioning that the above-presented determination of the quasi-particle decay rate based upon the LDOS pattern analysis enables us to study quasi-particles with pretty small excitation energies, \( E < \hbar/\tau \) and to detect the features of their inelastic decay specific to strongly disordered systems. This make it different from the analysis of the same quantity on the basis of measurements of Landau level broadening [21], which requires distinct Landau quantization and a strong magnetic field (or absence of impurities).

Electron-electron scattering with a large momentum transfer between ballistic quasi-
particles results in a rate which is determined by the phase volume of available final states, thus leading to $\gamma \propto E^2$. In disordered Fermi liquids, where transport is diffusive, small momentum transfers play an important role, such that an additional $E^{3/2}$ energy dependence of $\gamma$ appears, as described in Eq. (1). The $E^{3/2}$ dependence dominates at small energies, $E < \hbar/\tau$, while the $E^2$ dependence is specific to large energies $E \gg \hbar/\tau$. After estimating the elastic scattering time of $\tau = 0.14\text{ps}$ from the magnetic-field dependence of the LDOS fluctuations [22] (and also from the nominal doping level of the emitter contact), we find that the latter crossover would occur at $E \sim 5\text{ meV}$, which is beyond the energy range accessible in the reported experiment. For a quantitative comparison, we fit the relaxation rate as $\gamma = b \times E^2 + A \times E^{3/2}$ (dashed lines in Fig. 4(b) show separately the $b \times E^2$ and $A \times E^{3/2}$ parts determined in this fit). We have also fit the data to $\gamma(E) = A \times E^x$ dependence, treating exponent $x$ as a free parameter, which yields $x = 1.54 \approx 3/2$ and $A = 8 \times 10^{10} \text{meV}^{-3/2}\text{s}^{-1}$. In the theory [19], the prefactor $A$ in the $E^{3/2}$ dependence is $A = (105\sqrt{3}\hbar)/(16\pi \tau^{3/2}E_F^3)$. After estimating $E_F = 26\text{meV}$ from the electron density in the emitter, we evaluate $A \approx 1 \times 10^{11} \text{meV}^{-3/2}\text{s}^{-1}$ which compares well with the experimental result.

In conclusion, we presented a measurement of the inelastic quasi-particle relaxation rate in a disordered Fermi liquid. This quantity was obtained from the analysis of the magnitude of disorder-induced fluctuations in the local density of states probed using the method of resonant tunneling through an impurity state. Quantitative comparison with the standard theory [2] shows that, within the energy range available for such an analysis, the experimentally determined values of the inelastic relaxation rate can be attributed to the electron-electron interaction relaxation mechanism in diffusive conductors.

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FIGURES

FIG. 1. A typical inelastic relaxation process: a quasi-hole (○) at energy $E$ below the Fermi level $E_F$ (a) decays into a quasi-particle (●) and two quasi-holes (b).

FIG. 2. (a) Schematic of a strongly asymmetric double-barrier heterostructure containing residual impurities in the quantum well. (b) Scanning electron micrograph of the fabricated mesa. The disk on top of the mesa is a metallization which serves as Ohmic contact. (c) Conduction-band profile of the device under operation. (d) Differential tunneling conductance $G = dI/dV$ numerically obtained from $I(V)$ data recorded at the base temperature of $T = 20$ mK of our dilution refrigerator.

FIG. 3. Color map of the differential conductance as a function of bias voltage (step 7 µV) and magnetic field $B \parallel I$ (step 10 mT) for $T = 20$ mK, which indicates that the magnitude of the fluctuations decreases with increasing bias.

FIG. 4. (a) Variance of the differential conductance vs bias voltage. (b) Quasi-particle relaxation rate plotted as a function of excitation energy measured from the Fermi level. The experimental data are compared with a fit according to theory for disordered Fermi liquids, $\gamma = bE^2 + AE^{3/2}$ (solid). The individual terms $bE^2$ (dash-dotted) and $AE^{3/2}$ (dashed) are shown as well.