Mathematical modeling of processes of technogenic deposits development

V I Golik¹, V S Morkun², N V Morkun², A V Pikilnyak² and I A Gaponenko²

¹Academy of Mining Sciences, 37 Pushkina Str., Kryvyi Rih, 50027, Ukraine
²Kryvyi Rih National University, 11 Vitalii Matusevych Str., Kryvyi Rih, 50027, Ukraine
E-mail: golikgolikv@gmail.com, morkunv@gmail.com, nmorkun@gmail.com, pikilnyak@gmail.com, gaponenko@gmail.com, agaponenko51@gmail.com

Abstract. The article aims to substantiate the tendency to solve the problem of the mineral and raw material base of non-ferrous metals due to the involvement of technogenic origin in substandard metal-containing raw materials. Critical analysis and systematization of new information on the current state of the mineral and raw materials base of non-ferrous metals and the environmental problems of mining associated with it. New data on the recovery and loss of metals in the process of ore dressing are presented. It is shown that the total value of metals in waste is comparable to the value of potential mineral resources in the bowels. The problems of the use of mining waste should be solved in a single package with environmental problems by creating a single technological cycle for the extraction and processing of industrial minerals of industrial waste, the use of which can make the development of industrial deposits economically viable. The practical significance of the work lies in the possibility of using best practices for non-ferrous metallurgy enterprises that are searching for ways to survive in the conditions of the establishment of market relations.

1. Introduction
In the context of increasing demand for non-ferrous metals, the issues of strengthening the state of the mineral resource base due to the involvement of substandard metal-containing raw materials of technogenic origin are of particular relevance [1, 2]. Studies on this topic use information about the current state of the issue and use the methods of analysis and systematization of new information for proof. The scientific novelty of research on this topic consists of considering the issue of involving the production of substandard metal-containing raw materials of technogenic origin simultaneously with environmental issues. Studies on this topic indicate the need to use best practices for non-ferrous metallurgy enterprises in the context of market relations with partial exhaustion of available deposits.

2. Materials and methods
The provision of basic sectors of the economy with reserves of the main types of minerals seems satisfactory, but recently the urgency of the problem of the mineral and raw materials base of non-ferrous metals has been increasing [3]. In the developed industrial countries of the world, the level of use of industrial waste reaches 70-80%, while in Ukraine and neighbouring countries it does not exceed 12-15%. In the USA, for example, 20% of all aluminium, 33% of iron, 50% of
lead and zinc, and 44% of copper are obtained from industrial waste [4]. A similar trend in the use of secondary resources is observed in Canada, Great Britain, South Africa, Spain and other countries. For example:

- In the state of Montana (USA), the Mandiski mine dumps annually produce 2 tons of Au and 4 tons of Ag with a gold content of 0.84 g/t and silver of 2.8 g/t;
- In the state of Michigan (USA), 60% copper recovery has been achieved from the beneficiation of tailings containing 0.3% Cu;
- In South Africa, from the dumps of gold recovery plants with a gold content of 0.53 g/t and uranium - 40 g/t, 3.5 tons of gold and 696 tons of uranium per year are obtained with a productivity of 50,000 tons/day.

The development and creation of an information retrieval system for the processing and integrated use of mineral raw materials from technogenic deposits is a cardinal step towards a waste-free technology. In the Kryvyi Rih iron ore basin, according to various estimates, dumps contain up to 13 billion tons of overburden, and tailings contain up to 6 billion tons of waste from the enrichment of low-grade iron ore. In recent years, the possibility of using the mineral mass accumulated in dumps and tailings of the Kryvyi Rih basin has been increasingly studied. Obviously, the lack of methods for assessing man-made deposits adopted at the state level hinders their commissioning [5,6]. Currently, about 25 billion tons of solid waste alone has been accumulated on the territory of Ukraine.

These wastes negatively affect natural landscapes and environmental conditions, occupying an area of about 150 thousand hectares of fertile land and worsening the human environment. Technogenic deposits lead to the exclusion from economic circulation of large areas of land occupied by production waste. In addition, there is destruction or a decrease in the quality of land due to dust drifts from dumps and tailings [4]. The problem of industrial waste disposal is of paramount importance. An important circumstance is that the cost of commercial products from industrial waste is 5-15 times less than from traditionally mined ores from mineral deposits. The tendency to develop technogenic reserves is growing, which is becoming the main, and sometimes the only supplier of raw materials. The amount of non-recoverable metals is characterized by (figure 1).

The extraction rate of the main minerals is 65-78%, and associated elements in the extraction of non-ferrous metals – 10-30%. Elements such as In, Ga, Ti, Bi, Hg are almost entirely lost in the flotation tailings. Losses of other metals are characterized by (figure 2). In the copper sub-sector about 220 million tons of tailings have been accumulated, in which the copper content (0.34-0.37%) is close to the condition (0.35-0.50%). Sulfur accounts for 30-50% of the cost of tailings of ores beneficiation, precious metals – 25-45%, copper – 10-20%, and zinc – 10-15%.

The beneficiation tailings of copper-nickel ores contain industrial concentrations of platinum, gold, and silver, which are already available to modern processing technologies. When tungsten-molybdenum ores are enriched, up to 60% copper, up to 81% bismuth, up to 62% tantalum, gold, silver, and other elements are not extracted. The metal content exceeds 0.04% with the condition during production >0.1% WO3 (Tungsten trioxide).

Using new processing technologies, substandard ore dumps are a technogenic deposit suitable for mining at a lower cost than when mining metals from primary ore. In the tungsten-molybdenum sub-sector, the flotation and flotation-gravity concentration tailings contain about 400 thousand tons of molybdenum and more than 100,000.0 tons of tungsten. The metal reserves in the processing waste are equivalent to the reserves of new deposits (figure 3).
Figure 1. Average and maximum values of the fraction of unrecovered components relative to their amount in the initial ore, %.

Figure 2. Loss of metals during flotation.

Figure 3. Non-ferrous metal reserves in processing waste.
3. Results and discussion
The total value of metals in waste according to the indicative estimate is comparable to the value of potential resources of mineral resources in the subsoil and which is several times higher than the value of known reserves in the subsoil, which are not yet used [7–10]. In most cases, mineral waste is used as raw material for the construction industry (no more than 10%). In technologically developed countries, more than 40% of copper, 35% of gold, and a significant amount of other metals are obtained from waste materials using new technologies, for example, leaching [11,12].

The problems of the use of mining waste should be addressed along with environmental problems. The negative impact of tailings storage facilities on the environment is manifested in the territory, which is 10 times larger than the area occupied by the waste itself. The use of reserves of technogenic deposits is favored by the fact that they are located in habitable areas on the surface of the earth and are crushed, which sharply reduces the cost of obtaining metals. The insufficiency of the undertaken environmental measures is evident from the fact that large areas of land are occupied by waste production of 4th and 5th hazard classes. The environmental impact of waste is regional and global. Soil horizons are enriched by ore components of dumps, in which they are not isolated from water systems and affect the adjacent area.

In this situation, the prospects for reducing the negative impact on the environment are associated with the creation of a single technological cycle for the extraction and processing of minerals of technogenic waste: “ore processing - waste storage - utilization”. With the use of new technologies, the development of technogenic deposits can become economically viable production. The main contradictions in the processes of production activity and waste generation of industrial enterprises can be resolved by utilizing technogenic and substandard mineral raw materials in the cycle of integrated development of non-ferrous metal deposits (figure 4).

![Figure 4. Model for utilization of technogenic and substandard mineral raw materials.](image)

The unified model of leaching takes into account the type of metal leaching, the methods of metal production types of leachable metals. Accounting is carried out with the help of variable “n”, that is, each species corresponds to its number:

- \( n = 1 \) – traditional metal leaching in agitators;
- \( n = 2 \) – tails activated in the disintegrator are leached in the agitator;
- \( n = 3 \) – leaching of tailings in the disintegrator combined with activation.
Different metal mining options are taken into account using a variable “m”:
- \( m = 1 \) – extraction of metals from the balance tailings of coal preparation;
- \( m = 2 \) – extraction of metals from off-balance tailings of coal preparation.

Accounting for extraction of different metals is taken into account using a variable “p”:
- \( p = 1 \) – corresponds to the zinc leaching model;
- \( p = 2 \) – corresponds to the leaching model of lead;
- \( p = n \) – corresponds to the leaching model of another metal.

Calculations of regression models are performed in Maple 9.5. Based on the performed calculations, a unified mathematical model of the process of metal extraction from tailings is built. For all values of the variables \( n, m, p \), the studied regression model has the same linear structure concerning the coefficients \( a_k \) and is a polynomial of the second degree concerning variables \( X_1, X_2, X_3, X_4 \) therefore, the general model has the form [12]:

\[
\varepsilon = \varepsilon(n; m; p) = a_0 + a_1 \cdot X_1 + a_2 \cdot X_2 + a_3 \cdot X_3 + a_4 \cdot X_4 + a_5 \cdot X_2^2 + a_6 \cdot X_2^3 + a_7 \cdot X_3^2 + a_8 \cdot X_3^3 + a_9 \cdot X_4 + a_{10} \cdot X_4 + a_{11} \cdot X_1 + a_{12} \cdot X_2 \cdot X_3 + a_{13} \cdot X_2 \cdot X_4 + a_{14} \cdot X_3 \cdot X_4,
\]

where \( a_k = a_k(n; m; p); k = 0, 1, \ldots, 14; n = 1, 2, 3; m = 1, 2; p = 1, 2 \).

Expression \( \varepsilon = \varepsilon(n; m; p) \) defined through expressions for \( \varepsilon(1; m; p), \varepsilon(2; m; p), \varepsilon(3; m; p) \) using the Lagrange interpolation polynomial:

\[
\varepsilon(n; m; p) = \frac{(n-2)(n-3)}{2} \varepsilon(1; m; p) + (n-1) \times (n-3) \times \varepsilon(2; m; p) - \frac{(n-1)(n-2)}{2} \varepsilon(3; m; p).
\]

Similarly, \( \varepsilon = \varepsilon(n; m; p) \) expressed through functions \( \varepsilon(n; 1; p), \varepsilon(n; 2; p) \) by the formula:

\[
\varepsilon(n; m; p) = - (m-2) \varepsilon(n; 1; p) + (m-1) \varepsilon(n; 2; p).
\]

Respectively \( \varepsilon = \varepsilon(n; m; p) \) expressed through functions \( \varepsilon(n; m; 1), \varepsilon(n; m; 2) \) by the formula:

\[
\varepsilon(n; m; p) = - (p-2) \varepsilon(n; m; 1) + (p-1) \varepsilon(n; m; 2).
\]

Combining formulas 3 and 4, we obtain the dependence \( \varepsilon = \varepsilon(n; m; p) \) from functions \( \varepsilon(n; 1; 1), \varepsilon(n; 1; 2), \varepsilon(n; 2; 1), \varepsilon(n; 2; 2) \):

\[
\varepsilon(n; m; p) = (m-2) (p-2) \varepsilon(n; 1; 1) - (m-2) \times (p-1) \times \varepsilon(n; 1; 2) - (m-1) (p-2) \varepsilon(n; 2; 1) + (m-1) (p-1) \varepsilon(n; 2; 2).
\]

By substitution of \( \varepsilon(1; m; p), \varepsilon(2; m; p), \varepsilon(3; m; p) \) we obtain:

\[
\varepsilon(n; m; p) = \frac{(n-2)(n-3)}{2} [(m-2) (p-2) \varepsilon(1; 1; 1) - (m-2) (p-1) \varepsilon(1; 1; 2) - (m-1) (p-2) \varepsilon(1; 2; 1) + (m-1) (p-1) \varepsilon(1; 2; 2)] - (n-1)(n-3) \times \varepsilon(n; 1; 2) - (m-2) (p-2) \varepsilon(2; 1; 1) - (m-2) (p-1) \varepsilon(2; 1; 2) + (m-1) (p-2) \times (2; 1; 2) - (m-1) (p-1) \varepsilon(2; 2; 1) + (m-1) \times \varepsilon(2; 2; 2) + \frac{(n-1)(n-2)}{2} [(m-2) (p-2) \times \varepsilon(3; 1; 1) - (m-2) (p-1) \varepsilon(3; 1; 2) - (m-1) (p-2) \times \varepsilon(3; 2; 1) + (m-1) (p-1) \varepsilon(3; 2; 2)]
\]
Expressions for $\varepsilon (n; m; p)$ at specific values $n, m, p$ are determined from regression dependencies describing leaching processes. Zinc recovery regression equation for tailing leaching ($n = 1; m = 1; p = 1$):

$$
\varepsilon (1; 1; 1) = 39.02 + 5.51X_1 - 11.09X_2 + 5.6X_3 + 1.43X_4 + \\
+ 3.58X_1^2 + 6.48X_2^2 - 9.39X_3^2 - 9.38X_4^2 - 2.61X_1X_2 - \\
- 0.62X_1X_3 - 1.86X_1X_4 - 3.0X_2X_3 - 1.48X_2X_4 + 1.41X_3X_4,
$$

where $X_1, X_2, X_3, X_4$ – leaching indicators associated with the parameters (active class yield 0.08; chemical composition of the tailings; metal content in solution, mg/l; leaching time, h; metal recovery in solution, %):

$$
X_1 = \frac{C_{H_2SO_4}}{4} - 6; X_2 = \frac{C_{NaCl}}{70} - 90; X_3 = \frac{(L : S) - 7}{3}; X_4 = \frac{t - 0.625}{0.375}; X_5 = \frac{f - 125}{75}.
$$

Zinc recovery regression equation for tailing leaching ($n = 1; m = 2; p = 1$):

$$
\varepsilon (1; 2; 1) = 39.35 + 6.76X_1 - 18.88X_2 - 0.62X_4 - 11.6X_1^2 + \\
+ 7.19X_2^2 + 2.03X_1^2 - 2.84X_1X_2 - 1.39X_1X_3 - 0.89X_1X_4 - \\
- 2.04X_2X_3 + X_2X_4 - 2.45X_3X_4.
$$

Lead recovery regression equation for tail agitation leaching ($n = 1; m = 2; p = 2$):

$$
\varepsilon (1; 2; 1) = 42.43 + 16.8X_2 + 2.68X_3 + 0.93X_4 - 3.89X_1^2 - 19.31X_2^2 + \\
+ 2.36X_4^2 + 2.12X_1X_2 - 0.9X_1X_4 + 1.73X_2X_3 + 1.04X_3X_4.
$$

Zinc recovery regression equation for leaching of activated tailings outside the disintegrator ($n = 2; m = 2; p = 1$):

$$
\varepsilon (2; 2; 1) = 36.37 + 9.96X_1 - 11.56X_2 + 1.07X_3 - 6.53X_1^2 + \\
+ 5.63X_2^2 - 1.00X_3^2 - 3.95X_5^2 - 1.21X_1X_2 - 5.79X_1X_3 - \\
- 4.16X_2X_3 - 0.74X_2X_4 - 1.15X_3X_5.
$$

Lead recovery regression equation for leaching activated tailings outside the disintegrator ($n = 2; m = 2; p = 2$):

$$
\varepsilon (2; 2; 2) = 29.91 + 1.1X_1 + 10.63X_2 + 6.15X_3 + 2.09X_5 - \\
- 2.41X_1^2 - 26.29X_2^2 + 3.84X_3^2 + 9.25X_5^2 - 1.21X_1X_2 - \\
- 0.72X_1X_3 + 3.21X_1X_5 + 4.81X_2X_3 + 1.08X_2X_5 - 1.00X_3X_5.
$$

Zinc recovery regression equations for leaching tailings in a disintegrator ($n = 3; m = 2; p = 1$):

$$
\varepsilon (3; 2; 1) = 32.15 + 11.4X_1 - 14.04X_2 + 0.68X_3 + 1.85X_5 - \\
- 2.90X_1^2 + 9.25X_2^2 - 2.53X_4^2 - 0.39X_1X_2 - 1.95X_1X_3 + \\
+ 1.32X_1X_5 + 1.47X_2X_3 + 4.84X_2X_5 + 3.61X_3X_5.
$$

Lead recovery regression equations for tail leaching in a disintegrator ($n = 3; m = 2; p = 2$):

$$
\varepsilon (3; 2; 2) = 39.44 - 1.17X_1 + 16.76X_2 + 1.28X_3 - 0.55X_4 - \\
- 5.64X_1^2 - 14.81X_2^2 - 0.86X_3^2 - 4.09X_1X_3 - 1.42X_1X_4 - \\
- 0.42X_2X_3 - 1.00X_2X_4 - 0.82X_3X_4.
$$
For each specific process and metal, a single model will be a regression dependence. This single model of metal recovery during leaching is implemented in Maple 9.5, Matlab, Mathcad, etc. During the experiments, the most and least intense impact of all factors, i.e., two series, in one of which the values of all factors were taken as maximum (MAX), and in the second as minimal (MIN). Agitation leaching of crushed materials (AKX). Agitation leaching of materials pre-activated in the dry state (AAX). Leaching of materials at the moment of activation with solutions in a disintegrator (APX). Agitation leaching of materials subjected to activation in a disintegrator together with leaching solutions (PAX). Leaching of materials during activation with their repeated passage together with leaching solutions through a disintegrator (AMX) [13,14] (table 1).

The basis for determining the function between the function and independent factors is the multiple regression method implemented in Statistica 6.0. In the case of a one-factor dependence, the desired function is a one-dimensional surface in two-dimensional space - i.e., the line defined by equation $Y = a + a_1X$. According to this, the variable $Y$ can be represented as a function of the constant ($a$), and the slope coefficient ($a_1$) multiplied by the value of the variable $X$.

In the case of multiple regression (when multiple predictors are used), the regression surface cannot be displayed in two-dimensional space, but the calculations are practically unchanged. A linear equation describes multiple regression:

$$Y = a + a_1X_1 + a_2X_2 + ... + a_kX_k.$$  \hspace{1cm} (15)

where $k$ - number of predictors. Regression coefficients ($a_1...a_k$) represent the independent contributions of each independent variable to the prediction of the dependent variable.

| Table 1. The results of the experiment. |
|----------------------------------------|
| Factor values | Designation of the experiments |
| AKX | AAX | PAX | APX | AMX |
| Min | 0.03 | 0.02 | 0.03 | 0.02 | 0.03 |
| Max | 4.75 | 8.50 | 4.00 | 10.00 | 32.75 |

The regression surface expresses the best-predicted value of the dependent variable ($Y$) for the given values of the independent variables ($X$). Since the task of linear regression procedures is to fit the surface, which is a linear function of the variables $X$, in accordance with the observed variable $Y$, the residual values of the observed points can be used to develop the criterion of “best fit”. According to the Fisher criterion, hypothetical dependencies at a significance level of 5% are accepted as plausible. Dependencies are sought to take into account linear, quadratic effects and their interactions in the form of a regression equation:

$$\varepsilon_{Pb} = a + a_1X_1 - a_2X_2 + a_3X_3 + a_4X_4 + a_5X_1^2 + a_6X_2^2 + a_7X_3^2 + a_8X_4^2 + a_9X_1X_2 + a_{10}X_1X_3 + a_{11}X_1X_4 + a_{12}X_2X_3 + a_{13}X_2X_4 + a_{14}X_3X_4,$$  \hspace{1cm} (16)

where $X_1$ - dimensionless coefficient characterizing the content $H_2SO_4$; $X_2$ - dimensionless coefficient characterizing the content NaCl; $X_3$ - dimensionless coefficient characterizing the ratio liquid ($L$) / solid ($S$); $X_4$ - dimensionless coefficient characterizing the duration of leaching ($t$), hours.

The parameters of the regression equation for the agitation leaching of enrichment tailings activated in the disintegrator with tailings reagents (AAX) are summarized in (table 2). The parameters of the regression equation for the leaching of materials at the time of activation
with solutions in the disintegrator (APX) are summarized in (table 3). Tables 2 and 3 differ in leaching medium.

Table 2. Regression equation parameters of tailings agitation leaching activated in the disintegrator (PAX).

| Regression parameters | Regr. | St. error | t(9) | p   | -95,%  | +95,%  |
|-----------------------|-------|-----------|------|-----|--------|--------|
| Average/bias          | 3.11279 | 0.369727  | 8.41917 | 0.000015 | 2.27641 | 3.949171 |
| (1)H_SO4(L)           | 0.48611  | 0.182041  | 2.67034  | 0.025608 | 0.074311 | 0.897916 |
| H_SO4(Q)              | -0.25279 | 0.485916  | -0.50204 | 0.615452 | -1.35201 | 0.846427 |
| (2)NaCl(L)            | 1.60122  | 0.182041  | 8.79595  | 0.000010 | 1.18942  | 2.013027 |
| NaCl(Q)               | -1.54879 | 0.485916  | -3.18736 | 0.011053 | -2.64801 | -0.449573 |
| (3)L/S(L)             | 0.49794  | 0.182041  | 2.73534  | 0.023022 | 0.08614  | 0.909749 |
| L/S(Q)                | 0.24071  | 0.485916  | 0.49537  | 0.632208 | -0.85851 | 1.339927 |
| (4)t(L)               | -0.11233 | 0.182041  | -0.61708 | 0.552473 | -0.52414 | 0.299472 |
| t(Q)                  | 0.30121  | 0.485916  | 0.61988  | 0.550708 | -0.79801 | 1.400427 |
| 1L on 2L              | 0.61713  | 0.193083  | 3.19616  | 0.010898 | 0.18034  | 1.053910 |
| 1L on 3L              | -0.27638 | 0.193083  | -1.43138 | 0.186117 | -0.71316 | 0.166041 |
| 1L on 4L              | 0.26162  | 0.193083  | 1.35498  | 0.208453 | -0.17216 | 0.701410 |
| 2L on 3L              | 0.26463  | 0.193083  | 1.37052  | 0.203733 | -0.17216 | 0.701410 |
| 2L on 4L              | -0.12688 | 0.193083  | -0.65710 | 0.527561 | -0.56366 | 0.309910 |
| 3L on 4L              | 0.05312  | 0.193083  | 0.27514  | 0.789421 | -0.38366 | 0.489910 |

Table 3. Parameters of the regression leaching equation in the disintegrator.

| Regression parameters | Regr. | St. error | t(9) | p   | -95,%  | +95,%  |
|-----------------------|-------|-----------|------|-----|--------|--------|
| Average/bias          | 26.8206 | 2.131152 | 12.58503 | 0.000001 | 21.9996 | 31.64163 |
| (1)H_SO4(L)           | -0.7969 | 1.049307 | -0.75944 | 0.467012 | -3.1706 | 1.57681 |
| H_SO4(Q)              | -3.8676 | 2.800883 | -1.38086 | 0.200643 | -10.2037 | 2.46841 |
| (2)NaCl(L)            | 11.4013 | 1.049307 | 10.86559 | 0.000002 | 9.0276  | 13.77503 |
| NaCl(Q)               | -10.1006 | 2.800883 | -3.60623 | 0.005693 | -16.4367 | -3.76459 |
| (3)L/S(L)             | 0.8705  | 1.049307 | 0.82960  | 0.428222 | -1.5032 | 3.24240 |
| L/S(Q)                | -0.5861 | 2.800883 | -0.20926 | 0.838902 | -6.9222 | 5.74991 |
| (4)t(L)               | -0.3759 | 1.049307 | -0.35823 | 0.728433 | -2.7496 | 1.99781 |
| t(Q)                  | 0.0974  | 2.800883 | 0.03477  | 0.973025 | -6.2387 | 6.43341 |
| 1L on 2L              | 0.0769  | 1.112958 | 0.06907  | 0.946442 | -2.4408 | 2.59456 |
| 1L on 3L              | -2.7786 | 1.112958 | -2.49661 | 0.034051 | -5.2963 | -0.26094 |
| 1L on 4L              | -0.9663 | 1.112958 | -0.86818 | 0.407848 | -3.4839 | 1.55144 |
| 2L on 3L              | -0.2886 | 1.112958 | -0.25933 | 0.801214 | -2.8063 | 2.29066 |
| 2L on 4L              | -0.6833 | 1.112958 | -0.61390 | 0.554476 | -3.2009 | 1.83444 |
| 3L on 4L              | -0.5597 | 1.112958 | -0.50294 | 0.627084 | -3.0774 | 1.95794 |

The consumer of secondary processing waste may be the most material-intensive construction industry. Hundreds of millions of tons of raw materials are produced annually in the world for
the production of building materials. The volume of waste generated is commensurate with the demand for the building materials industry for mineral raw materials. However, no more than 10% of the waste is currently used.

The involvement of technogenic deposits in the economy allows us to solve the problems of the mineral resource complex and the environment simultaneously. It reduces the costs of prospecting and exploring new deposits, frees up wasteland, and eliminates sources of environmental pollution. The study results of the development trends of the mineral resource base coincide with the results of studies of experts [15–34].

4. Conclusions
The problems of using mining wastes are inextricably connected with environmental problems. They can only be solved by creating a single complex for the extraction and processing of minerals of technogenic origin, which allows the development of technogenic deposits to be economically attractive. The parameters of the leaching of metals from tailings can be determined from the proposed model. A universal mathematical model that describes extracting non-ferrous metals from raw materials of technogenic deposits, considering the impact of various mechanochemical factors on it is proposed. Specific parameters for the extraction of metals from processing tailings can be directly determined from the proposed model, which makes it possible to select the optimal scheme of the technological process and evaluate its technical and economic indicators at the design stage.

ORCID iDs
V I Golik https://orcid.org/0000-0002-1181-8452
V S Morkun https://orcid.org/0000-0003-1506-9759
N V Morkun https://orcid.org/0000-0002-1261-1170
A V Pikilnyak https://orcid.org/0000-0003-0898-4756
I A Gaponenko https://orcid.org/0000-0003-1128-5163

References
[1] Capilla A V and Delgado A V 2015 *Mineral Resources* (London: World Scientific Publishing Co. Pte. Ltd)
[2] Golik V, Komashchenko V and Morkun V 2015 *Metallurgical and Mining Industry* 3 49–52
[3] Henckens M L C M, Driessen P P J and Worrell E 2015 *Ressour. Conserv. Recycl* 103 9–18
[4] Gravicon 2010 Technogenic deposits, technogenic field, features and development prospects URL https://gravicon.com.ua/ru/page43
[5] Vilkul Y, Azaryan A, Azaryan V and Trachuk A 2011 *Mining industry* Special issue 13–15
[6] Vilkul Y, Azaryan Y and Kolosov V 2013 *Mining Journal of Kryvyi Rih National University* 96 3–10
[7] Bosikov I and Klyuev R 2021 *Material Sci & Eng.* 5 40–42 URL http://dx.doi.org/10.15406/mseij.2021.05.00154
[8] Kupin A, Kuznetsov D, Muzyka I, Paraniuk D, Serdiuk O, Suvorov O and Dvornikov V 2018 *Eastern-European Journal of Enterprise Technologies* 4 71–79 URL http://dx.doi.org/10.15587/1729-4061.2018.139644
[9] Kotov I, Suvorov O and Serdiuk O 2019 *Eastern-European Journal of Enterprise Technologies* 2 38–47 URL http://dx.doi.org/10.15587/1729-4061.2019.1556410
[10] Golik V, Stradanchenko S and Maslennikov S 2015 *International Journal of Applied Engineering Research* 10(15) 35410–35416
[11] Prior T, Giuroc D, Mudd G, Mason L and Behrisch J 2011 *Global Environmental Change* 22 577–587 URL http://dx.doi.org/10.1016/j.gloenvcha.2011.08.009
[12] Golik V, Razorenov Y, Morkun V and Mihaylo M 2017 *Mining Journal of Kryvyi Rih National University* 102 11–20
[13] Polishchuk A and Semeirko S 2022 *ACNS Conference Series: Social Sciences and Humanities* 1 01003 URL http://dx.doi.org/10.55056/cs-ssh/1/01003
[14] Kiv A, Soloviev V and Senerikov S 2021 Journal of Physics: Conference Series 1840 011001 URL http://dx.doi.org/10.1088/1742-6596/1840/1/011001
[15] Lapidus G 1992 Chemical Engineering Science - CHEM ENG SCI 47 1933–1941 URL http://dx.doi.org/10.1016/0009-2509(92)80311-Y
[16] Kuhn O 2017 CSEG Recorder 42 36–39
[17] Galtsev O, Galtseva O, Belenko V, Mamator A, Nemtsev A and Mishunin V 2018 Int. J. of Eng. and Tech. (UAE) 7 5–9 URL http://dx.doi.org/10.14419/ijet.v7i4.36.22703
[18] Shumilova I, Khat’kova A and Cherkasov V 2021 Mining informational and analytical bulletin 173–181 URL http://dx.doi.org/10.25018/0236_1493_2021_32_0_173
[19] Kalmykov V, Gogotin A and Ivashov A 2015 Gorny Zhurnal 37–41 URL http://www.rudmet.ru/journal/1484/article/25534/
[20] Lustiuk M, Diakon V, Petrina M and Rakhkowska A 2013 Scientific Bulletin of National Mining University 5 5–10
[21] Sides E 1997 Geologische Rundschau 86 342–353 URL http://dx.doi.org/10.1007/s005310050145
[22] Zhou Y Z, Li P X, Wang S G, Xiao F, Li J Z and Gao L 2017 Bulletin of Mineralogy Petrology and Geochemistry 36 327–331 and 344 URL http://dx.doi.org/10.3969/j.issn.1007-2802.2017.02.016
[23] Xiao F 2017 Bulletin of Mineralogy Petrology and Geochemistry 36 327–331
[24] Moon C, Whateley M K and Evans A M (eds) 2009 Introduction to Mineral Exploration 2nd ed (Wiley-Blackwell)
[25] Singer D 2018 Ore Geology Reviews 99 235–243 URL https://doi.org/10.1016/j.oregeorev.2018.06.019
[26] Tumidajski T 2010 Gospodarka Surowcami Mineralnymi / Mineral Resources Management 26 111–123
[27] Cehlar M, Rybar P, Mihok J and Engel J 2020 Journal of Mining and Geotechnical Engineering 1 66–74 URL http://dx.doi.org/10.26730/2587-5574-2020-1-66-74
[28] Liu J 2017 Modelling, Measurement and Control C 78 478–495 URL http://dx.doi.org/10.18280/mmc_c.780406
[29] Laznicka P 2000 Ore Geology Reviews 17 139–140
[30] Śmieszek Z, Czernecki J, Sak T and Madej P 2017 Journal of Chemical Technology and Metallurgy 52 221–234
[31] Makovskaia O and Kostromin K 2019 Materials Science Forum 946 591–595 URL http://dx.doi.org/10.4028/www.scientific.net/MSF.946.591
[32] Vorobev A, Timokhin D, Bugaenko M and Popova G 2016 Tsvetnye Metally 2016(3) 5–7
[33] Dinică M C and Armeanu D 2014 Romanian Journal of Economic Forecasting 17 105–122
[34] Knuutila K 2009 World of Metallurgy - ERZMETALL 62 142–150