Acusal Behavior in Quantum Electrodynamics

A. WIDOM [1], Y. N. Srivastava [1,2], E. Sassaroli [2,3]

1. Physics Department, Northeastern University, Boston, MA 02115, USA
2. Dipartimento di Fisica and sezione INFN
   Universita’ di Perugia, I-06123 Perugia, Italy
3. Laboratory for Nuclear Science and Department of Physics
   Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Acausal features of quantum electrodynamic processes are discussed. While these processes are not present for the classical electrodynamic theory, in the quantum electrodynamic theory, acausal processes are well known to exist. For example, any Feynman diagram with a “loop” in space-time describes a “particle” which may move forward in time or backward in time or in space-like directions. The engineering problems involved in experimentally testing such causality violations on a macroscopic scale are explored.

1 Introduction

The notion that signals can travel in both the forward and backward directions in time (as well as space-like directions) has always appeared to be an essential part of the unification of relativity and quantum mechanics. The notion of an anti-particle as a physical particle traveling backwards in time was at first conceived by Stuckelberg and later developed by Feynman as an essential interpretation in a quantum field theoretical framework. To see what is involved, one may consider the free photon propagator in the Feynman gauge,

\[ D(x - y) = \int e^{iQ \cdot (x - y)} \left( \frac{4\pi}{Q^2 - i0^+} \right) \frac{d^4Q}{(2\pi)^4}. \] (1.1)

By an elementary integration

\[ D(x - y) = \frac{i}{\pi} \left( \frac{1}{(x - y)^2 + i0^+} \right) = \delta((x - y)^2) + \left( \frac{i}{\pi (x - y)^2} \right). \] (1.2)

With \( cT = x^0 - y^0 \) and \( R = |x - y| \), the first term on the right hand side of Eq.(1.2) yields one half the sum of the retarded and advanced photon propagators

\[ \Re D(x - y) = \frac{1}{2} (D_{\text{retarded}}(R, T) + D_{\text{advanced}}(R, T)), \] (1.3)
\[ D_{\text{retarded}}(R, T) = \left( \frac{\delta(cT - R)}{R} \right), \quad D_{\text{advanced}}(R, T) = \left( \frac{\delta(cT + R)}{R} \right). \quad (1.4) \]

Thus, the “on light cone” photon propagation may proceed in both the forward (retarded) and backward (advanced) time directions. The last term on the right hand side of Eq.(1.2) is non-vanishing in the time-like and space-like directions,

\[ \Im D(x - y) = \left( \frac{1}{\pi(R^2 - c^2T^2)} \right). \quad (1.5) \]

Thus, “off light cone” space-like and time-like photon propagation is feasible. Finally, when the photon propagator enters into a Feynman diagram describing the amplitude for a process, all of the above propagation directions, on and off the light cone, must be included to arrive at the conventional accepted quantum electrodynarn results. Some physical reasoning which helps in the understanding of why forward and backward in time propagation enters into quantum field theory in an essential manner is discussed in Sec.2. Space-like transmission of electromagnetic field configurations are discussed in Sec.3. An electrical engineering configuration for a transmission line device exhibiting space-like transmission is discussed in Sec.4. Two photon causality violating correlations are discussed in Sec.5. In the concluding Sec.6, some other current views on causality will be briefly discussed.

2 Measurable Quantum Fields are Non-Local in Time

If \( \phi(x) \) denotes a relativistic quantum field, then the usual procedure for extracting the particle and/or anti-particle content from a field is to decompose the field into positive and negative frequency parts as follows: (i) Express \( \phi(x) \) as a Fourier integral, using \( x = (r, ct) \),

\[ \phi(x) = \int_{-\infty}^{\infty} \phi_\omega(r)e^{-i\omega t}d\omega. \quad (2.1) \]

(ii) Divide the field into a positive frequency part (which destroys particles and/or creates anti-particles)

\[ \phi_+(x) = \int_{0}^{\infty} \phi_\omega(r)e^{-i\omega t}d\omega, \quad (2.2) \]

and a negative frequency part (which creates particles and/or destroys anti-particles)

\[ \phi_-(x) = \int_{-\infty}^{0} \phi_\omega(r)e^{-i\omega t}d\omega, \quad (2.3) \]
so that

\[ \phi(x) = \phi_+(x) + \phi_-(x). \quad (2.4) \]

The decomposition into positive and negative frequency parts can be written using expressions in space-time by introducing a time-like velocity four vector \( v \),

\[ v^\mu v_\mu = -c^2. \quad (2.5) \]

Eqs.(2.2) and (2.3) then read (respectively) as

\[ \phi_+(x) = \left( \frac{i}{2\pi} \right) \int_{-\infty}^{\infty} \phi(x + v\tau) \left( \frac{d\tau}{\tau + i0^+} \right), \quad (2.5) \]

and

\[ \phi_-(x) = \left( \frac{i}{2\pi} \right) \int_{-\infty}^{\infty} \phi(x - v\tau) \left( \frac{d\tau}{\tau + i0^+} \right). \quad (2.5) \]

To appreciate the experimental importance of this decomposition into positive and negative frequency parts, it is sufficient to recall in quantum optics that correlation functions of the form

\[ G_n(r_1, \lambda_1, t_1, \ldots, r_n, \lambda_n, t_n) = \]

\[ < E_-(r_1, \lambda_1, t_1) \ldots E_-(r_n, \lambda_n, t_n) E_+(r_n, \lambda_n, t_n) \ldots E_+(r_1, \lambda_1, t_1) >, \quad (2.6) \]

are thought to give a complete description of measurements for an optical electric field

\[ E(r, t) = \sum_{\lambda=x,y,z} E(r, \lambda, t) e_\lambda, \quad (2.7) \]

where

\[ E_{\pm}(r, t) = \left( \frac{i}{2\pi} \right) \int_{-\infty}^{\infty} E(r, t \pm \tau) \left( \frac{d\tau}{\tau + i0^+} \right). \quad (2.8) \]

The important point is that to extract the photon production electric field \( E_- (r, t) \) and/or the photon detection electric field \( E_+ (r, t) \) at time \( t \), it is required that one know the physical electric field \( E(r, t \pm \tau) \) both in the past and in the future of time \( t \). It is not sufficient to know the physical electric field “now” (at time \( t \)) to extract all experimental quantities. For example, the intensity of a light beam at space-time point \( x = (r, t) \) may be measured by

\[ I(r, t) = \left( \frac{c}{4\pi} \right) \sum_{\lambda} G_1(r, \lambda, t) = \left( \frac{c}{4\pi} \right) < E_-(r, t) E_+(r, t) >. \quad (2.9) \]
Eqs. (2.6) and (2.8) require the electric field both in the past and in the future of time $t$ of interest; Explicitly

$$I(r, t) = \left( \frac{e}{16\pi^3} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\langle \mathbf{E}(r, t + s_1) \cdot \mathbf{E}(r, t + s_2) \rangle ds_1 ds_2}{(s_1 - i0^+)(s_2 + i0^+)}.$$  

(2.10)

From these considerations, it follows that the photon need not be located in a spatial region in which there are presently electromagnetic fields. If there will be in the future, or may have been in the past, electromagnetic fields in a given spatial region, then the future and past conditions are sufficient for the photon to be located (with finite probability) in a presently null field spatial region.

3 Space-Like Transitions Between Field Configurations

A difficulty that Einstein had with the conventional notions of quantum electrodynamics was in part that the Maxwell wave associated with a photon can be over here. Yet, the photon can be detected over there! Sometimes, the region where the Maxwell wave for a photon is large turns out to be a region without the photon (with finite probability). What troubled Einstein also troubles us.

All we can do is to show why Einstein’s picture of the photon and the associated Maxwell field is indeed what we presently call conventional quantum electrodynamics. An initial photon field configuration $|i\rangle$ over here can be transported with superluminal speed (space-like) to a final field configuration $|f\rangle$ over there with the finite quantum probability

$$P(i \rightarrow f) = |\langle f|i \rangle|^2.$$

(3.1)

If $\mathbf{e}(r)$ and $\mathbf{b}(r)$ denote an electromagnetic field associated with a photon at time zero, then the mean energy of the photon is given by

$$\mathcal{E} = \frac{1}{8\pi} \int (|\mathbf{e}(r)|^2 + |\mathbf{b}(r)|^2).$$

(3.2)

Employing the complex field

$$\mathbf{F}(r) = \mathbf{e}(r) + i\mathbf{b}(r),$$

(3.3)

and its Fourier transform

$$\mathbf{F}(k) = \int \Psi(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} \left( \frac{d^3k}{(2\pi)^3} \right),$$

(3.4)
one finds the mean photon energy

\[ E = \frac{1}{8\pi} \int |F(r)|^2 d^3r = \frac{1}{8\pi} \int |\Psi(k)|^2 \left( \frac{d^3k}{(2\pi)^3} \right). \quad (3.5) \]

The photon wave function (Heisenberg at time zero) $\Psi(k)$ in momentum space ($p = \hbar k$) is here normalized as

\[ \frac{1}{8\pi\hbar c} \int |\Psi(k)|^2 \left( \frac{d^3k}{(2\pi)^3|k|} \right) = 1; \quad (3.6) \]

e.g. $|\Psi(k)|^2(d^3k/|k|)$ is proportional to the probability of finding the photon with momentum in the Lorentz invariant phase space element $(d^3k/|k|)$. Note that the condition $k \cdot \Psi(k) = 0$ is equivalent to the vacuum Maxwell equations $\nabla \cdot F(r) = 0$. For two normalized photon wave functions, $\Psi_i(k)$ (initial photon over here), and $\Psi_f(k)$ (final photon over there), all at time zero, the “overlap” or transition amplitude is given by

\[ < f | i > = \frac{1}{8\pi\hbar c} \int \Psi^*_f(k) \cdot \Psi_i(k) \left( \frac{d^3k}{(2\pi)^3|k|} \right). \quad (3.7) \]

The notion of “over here” and “over there” has more to do with the electromagnetic fields in space

\[ \Psi_{f,i}(k) = \int F_{f,i}(r)e^{-ik \cdot r} d^3r, \quad (3.8) \]

i.e.

\[ < f | i > = \frac{1}{16\pi^3\hbar c} \int \int \frac{F^*_f(r) \cdot F_i(s)}{|r - s|^2} d^3rd^3s, \quad (3.9) \]

where

\[ F^*_f(r) = e_f(r) - i b^*_f(r), \quad F_i(s) = e_i(s) + i b_i(s). \quad (3.10) \]

The central result of this section follows from Eqs.(3.1), (3.9) and (3.10). Let $e_i(r)$ and $b_i(r)$ denote respectively the electric and magnetic fields associated with an initial photon (over here) and let $e_f(r)$ and $b_f(r)$ denote respectively the electric and magnetic fields associated with a final photon (over there). The regions “over here” and “over there” mean that the initial and final fields have compact support in non-overlapping spatial regions. This situation is present at time zero. The transition probability (space-like) obeys

\[ P(i \to f) = \]
Thus, with finite probability, the photon can go from over here to over there (space-like) in no time at all. Note that the space-like propagation of photons in Eq.(1.5) enters in the space-like transition probability of Eq.(3.11) in thinly disguised form. For zero times, such that \( x = (r, 0) \) and \( y = (s, 0) \), the imaginary part of the propagator, \( \pi \Im m D(x - y) = |r - s|^{-2} \), provides the space-like transition probability kernel in the central Eq.(3.11). Material photon propagators exist that are more efficient (for space-like transitions) than the vacuum propagator.

4 Transmission Lines

Consider an electromagnetic transmission line along the \( z \)-axis. The line voltage at point \( z \) on the line is given by Faraday’s law

\[
v(z, t) = -\frac{1}{c} \left( \frac{\partial \phi(z, t)}{\partial t} \right).
\]  

(4.1)

If \( \varepsilon \) denotes the line capacitance per unit length, and if \( \mu \) denotes the line inductance per unit length, then the transmission line \((1 + 1)\)-dimensional Lagrange density is given by

\[
\mathcal{L} = \frac{\varepsilon}{2c^2} \left( \frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2\mu} \left( \frac{\partial \phi}{\partial z} \right)^2.
\]  

(4.2)

The line charge density (per unit length)

\[
\rho(z, t) = \varepsilon v(z, t)
\]  

(4.3)

plays the role of a conjugate field to \( \phi(x) \) with an equal time commutation relation

\[
[\rho(z, t), \phi(z', t)] = i\hbar \delta(z - z').
\]  

(4.4)

The transmission line signal velocity

\[
u = \frac{c}{\sqrt{\varepsilon \mu}}, \quad \varepsilon \mu \geq 1,
\]  

(4.5)

enters into the transmission line wave equation

\[
\frac{1}{u^2} \left( \frac{\partial^2 \phi}{\partial t^2} \right) = \left( \frac{\partial^2 \phi}{\partial z^2} \right).
\]  

(4.6)
Electromagnetic transmission lines, (in classical theory) propagate signals somewhat slower \((u \leq c)\) than vacuum light speed. In order to understand the notion of a transmission line impedance, we first consider the notion of a vacuum impedance. The vacuum Maxwell equations have the form

\[
\partial_{\mu} F^{\mu \nu} = -R_{\text{vac}} J^{\nu},
\tag{4.7}
\]

which defines the vacuum impedance \(R_{\text{vac}}\). In terms of the electronic charge \(e\), the quantum electrodynamic coupling strength reads

\[
\alpha = \left( \frac{e^2 R_{\text{vac}}}{4\pi \hbar} \right).
\tag{4.8}
\]

Eqs. (4.7) and (4.8) hold true in any set of units. The vacuum impedance is always defined in terms of light speed \(c\). For example, in the Gaussian units here employed, the vacuum has an impedance

\[
R_{\text{vac}} = \left( \frac{4\pi}{c} \right) \simeq 419.169004390336246121257964335 \text{ (picosec/cm)}.
\tag{4.9}
\]

In engineering SI units

\[
R_{\text{vac}} = \frac{1}{\varepsilon_0 c} = \mu_0 c \simeq 376.7303134617706554681984004203193 \text{ (Ohms)}.
\tag{4.10}
\]

(The value \(R_{\text{vac}} = (1/c) = 1.0000000000000000000000000000000000000000\) is employed and easily remembered in high energy physics.) The transmission line impedance (in Gaussian units) is defined as

\[
R = \left( \frac{1}{\varepsilon u} \right) = \frac{\mu u}{c^2} = \left( \frac{R_{\text{vac}}}{4\pi} \right) \sqrt{\frac{\mu}{\varepsilon}}.
\tag{4.11}
\]

A typical laboratory cable (say connecting a computer to the outside world) is a transmission line with \(R \approx 50 \text{ Ohms}\), or in Gaussian units \(R \approx 55.6 \text{ (picosec/cm)}\). Of interest here is the possibility in quantum electrodynamic theory of sending a superluminal signal down a 50 Ohm cable. For a cable of line impedance \(R\) and line velocity \(u\), we use a \((1+1)\) dimensional vector notation \(x = (z, ut)\). The photon propagator for an infinite transmission line is defined as

\[
D(x - y) = \frac{i}{\hbar c} < 0|\phi(x)\phi(y)|0 >_+, \tag{4.12}
\]

where the + indicates time ordering. The transmission line propagator obeys an equation of motion,

\[
-\partial_{\mu} \partial^{\mu} D(x - y) = cR\delta(x - y), \tag{4.13}
\]

\[7\]
which may be solved by the Fourier transformation

\[ D(x - y) = \int e^{iQ \cdot (x - y)} \left( \frac{cR}{Q^2 - i0^+} \right) \frac{d^2Q}{(2\pi)^2}. \]  

\text{(4.14)}

Strictly speaking, the integral in Eq.(4.14) does not exist, which leads the mathematician to conclude that \((1 + 1)\)-dimensional massless quantum field theories do not exist. In turn, this causes the philosopher to ponder about whether \(50\) \text{Ohm} cables exist. The situation is similar to \((1 + 1)\)-dimensional strings. A mathematician cannot embed a quantum \((1 + 1)\)-dimensional string in a \((3 + 1)\)-dimensional world. This leads the philosopher to question whether a musical violin with strings can exist in a world with three-dimensional Euclidean geometry. Infinite \(50\) \text{Ohm} cables do not exist, and the finite length of the physical cable serves as a cut-off to the quantum electrodynamic theory. One can employ (for mathematical simplicity) an “infinite cable” model of a long (but finite) cable at the expense of introducing a large regulator length \(\Lambda\) into the intermediate stages of the computation. Such an engineering approximation is quite all right if the regulator length does not enter into the final physical answer.

The propagator

\[ D(x - y) = \left( \frac{icR}{4\pi} \right) \ln \left( \frac{\Lambda^2}{(x - y)^2 + i0^+} \right) \]  

\text{(4.15)}

is a solution to Eq.(4.13). The equal time correlation (Weightman) function for the cable of impedance \(R\),

\[ W(z - z') = \frac{1}{\hbar c} < 0|\phi(z, 0)\phi(z', 0)|0 >, \]  

\text{(4.16)}

is then evaluated as

\[ W(z) = \left( \frac{cR}{2\pi} \right) \ln \left| \frac{\Lambda}{z} \right|, \quad (|z| << \Lambda). \]  

\text{(4.17)}

Eq.(4.17) may be safely employed for physical problems in which only the difference

\[ W(z_1) - W(z_2) = \left( \frac{cR}{2\pi} \right) \ln \left| \frac{z_2}{z_1} \right|. \]  

\text{(4.18)}

enters into the final answer since the regulator length \(\Lambda\) is not present in such differences. Now, let us consider placing a charge density per unit length \(\lambda(z)\)
onto the transmission line. If \( |0\rangle \) denotes the transmission line ground state, then the state with a charge density \( \lambda(z) \) on the line may be defined as

\[
|\lambda\rangle = \exp \left( \frac{i}{\hbar c} \int \lambda(z) \phi(z) dz \right) |0\rangle.
\]

If we place an initial charge density \( \lambda_i(z) \) on the cable “over here” and we wish to compute the amplitude for finding a final charge density \( \lambda_f(z) \) on the cable “over there”, then the transition amplitude for such superluminal transport of charge density is given by

\[
\langle \lambda_f | \lambda_i \rangle = \langle 0 | \exp \left( \frac{i}{\hbar c} \int (\lambda_i(z) - \lambda_f(z)) \phi(z) dz \right) |0 \rangle = \exp(-S_{fi}/\hbar). \tag{4.20}
\]

The Euclidean (space-like) action \( S_{fi} \) entering into Eq.(4.20) can be evaluated since the ground state wave function \( |0\rangle \) implies a Gaussian probability distribution for \( \phi(z) \); i.e.

\[
S_{fi} = \frac{1}{2\hbar c} \int \int (\lambda_i(z) - \lambda_f(z)) (\lambda_i(z') - \lambda_f(z')) W(z - z') dz dz'. \tag{4.21}
\]

Eqs.(4.16) and (4.22) may be written as

\[
S_{fi} = \frac{1}{2c} \int \int (\lambda_i(z) - \lambda_f(z)) (\lambda_i(z') - \lambda_f(z')) W(z - z') dz dz'. \tag{4.22}
\]

For the case of an initial charge density over here being displaced a distance \( b \) to over there; i.e.

\[
\lambda_i(z) = \lambda(z), \quad \lambda_f(z) = \lambda(z + b), \tag{4.23}
\]

the Euclidean action is given by

\[
S[\lambda] = \frac{1}{2c} \int \int \lambda(z) \lambda(z') (2W(z - z') - W(z - z' - b) - W(z - z' + b)) dz dz'. \tag{4.24}
\]

For the (large distance \( b \)) space-like transport of a voltage \( \tilde{v}(z) = \lambda(z)/\epsilon \), Eqs.(4.18) and (4.24) imply the Euclidean action

\[
S[\lambda] = \frac{R}{4\pi} \int \int \lambda(z) \lambda(z') ln \left( \frac{b}{z - z'} \right)^{2 - 1} dz dz', \quad b >> |z - z'|. \tag{4.25}
\]

The superluminal transition probability is then

\[
P(b) = |\langle \lambda_f | \lambda_i \rangle|^2 = \exp(-2S[\lambda]/\hbar). \tag{4.28}
\]
If the number of electrons $N$ which take part in the superluminal transition is defined as
$$N_e = \int \lambda(z)dz,$$
and if $a$ denotes the spread in space of the initial charge density, then the probability for a superluminal transition of the voltage $\tilde{v}(z) = \lambda(z)/\varepsilon$ through the distance $b$, obeys the decay law
$$P(b) \approx \left(\frac{a}{b}\right)^\beta, \quad (a << b).$$  \hfill (4.30)

The decay exponent is given by
$$\beta = \left(\frac{e^2 R}{\pi \hbar}\right) N^2 = 4\alpha \left(\frac{R}{R_{vac}}\right) N^2,$$
where Eq.(4.8) has been employed. The central result of this section is that the exponent
$$\beta \approx 0.004 \times N^2 \quad if \quad R \approx 50 \, \text{Ohms}.$$  \hfill (4.32)

If the number of electrons involved in a signal obeys $N >> 1$ then $\beta >> 1$ and the probability of superluminal transport falls very rapidly with spatial distance. In quantum electrodynamic theory, this large value of $\beta$ explains why it is not very easy to obtain superluminal transport on a 50 Ohm cable. On the other hand, for only a few electrons $\beta \leq 1$, which looks interesting except that one would have to settle for a very weak signal. We here leave the engineering considerations at this point.

### 5 Photon Correlations

Often, during the course of quantum photon experiments, one looks for correlations between the number of photons detected in a two different photon counters, say detectors 1 and 2. The photon coincidence correlation function is then
$$C_{12} = \langle N_1 N_2 \rangle,$$  \hfill (5.1)

where $N_i$, for $i = 1, 2$, are the number operators for the photons in the detectors. For a typical case, such as a Hanbury-Brown-Twiss experiment, the correlation function $C_{12}$ depends upon the positions of the two detectors. A typical application of the correlation $C_{12}$ measuring “coincidence photon counts” is made in astrophysics where the two photons come from one and/or the other of two possible stars. It is difficult to resolve the two stars from straight forward single detector intensity measurements. Feynman has analyzed such
two detector experiments as follows: Since two photons are detected in a coincidence count (one photon in each of the two counters), there are four physical possibilities: (i) Both photons come from star 1 with amplitude \( a_{11} \). (ii) Both photons come from star 2 with amplitude \( a_{22} \). (iii) The photon from star 1 went to detector 1 and the photon from star 2 went to detector 2 with amplitude \( a_{12} \). (iv) The photon from star 1 went to detector 2 while the photon from star 2 went to detector 1 with amplitude \( a_{21} \). The probability of the coincidence count is then

\[
P(\text{coincidence}) = |a_{11}|^2 + |a_{22}|^2 + |a_{12} + a_{21}|^2, \tag{5.2}
\]

illustrating the quantum rules that one adds probabilities for distinguishable events and adds amplitudes for indistinguishable events (before taking the absolute value squared). Quantum interference between the exchange amplitudes (the cross terms when absolute value squaring the last term on the right hand side of Eq.(5.2)) allows for the resolution of the positions of “two relatively incoherent stars”.

Feynman’s method of “counting or listing possibilities on your fingers” and then calculating quantum probabilities works equally well for more high technology coherent photon sources. For example, suppose that one has two photon coherent sources which are guaranteed to fire off exactly two photons at a time. Such sources exist in laboratories within present quantum optics technology. Let us further suppose that there are four and only four possibilities for each two photon firing event: (i) Two photons both go to detector 1, i.e. \( N_1 = 2, N_2 = 0 \). (ii) Two photons both go to detector 2, i.e. \( N_1 = 0, N_2 = 2 \). (iii) Photon 1 goes to detector 1 and photon 2 goes to detector 2, i.e. \( N_1 = 1, N_2 = 1 \). (iv) Photon 1 goes to detector 2 and photon 2 goes to detector 1, i.e. \( N_1 = 1, N_2 = 1 \). The last two possibilities have amplitude interference leading to a non-trivial (and measured) correlation function

\[
C(r_1, r_2) = \langle 2|N_1 N_2|2 \rangle \tag{5.3}
\]

where \( r_i \), for \( i = 1, 2 \), are the detector position. We use the Dirac notation \( |2 \rangle \) to remind the reader that for the above four possibilities we have

\[
(N_1 + N_2)|2 \rangle = 2|2 \rangle. \tag{5.4}
\]

One easily proves the following

**Spooky Theorem:** The counting statistics at counter 1 depends on how one sets the position of counter 2.

**Spooky Proof:** From Eqs.(5.3) and (5.4) it follows that

\[
2 < 2|N_1|2 > - < 2|N_1^2|2 > = C(r_1, r_2). \tag{5.5}
\]
What makes the theorem spooky is that the counting statistics at counter 1 depends on the position of counter 2 via counter 2 events that are possibly “space-like” or possibly “in the future” of counter 1 events.

6 Conclusion

We have discussed above several examples of why it appears that conventional quantum electrodynamics allows for interactions to proceed forward and backward in time as well as space-like in direction. While the results are conventional, the consequences are abhorrent to many. Einstein concluded (from what he regarded as the clairvoyant nature of quantum mechanics) that there are pieces of the puzzle missing in our present picture; i.e. quantum mechanics is presently an incomplete view. Those less revolutionary than was Einstein[12] prefer to think that these terms in quantum mechanics that look like causality violations are present only in the mathematics but not in the laboratory. One might hear that space-like photon propagation is merely virtual. This closing of the eyes, ears, and mind may satisfy some workers who do not like to think about what should be unthinkable; i.e. that the future can effect the past and so forth. One sometimes hears a timid statement that light can go faster than light speed ... but not really! Other methods for a theoretical approach to acausality imply that causality violations are certainly possible. But let us choose a system well out of the reach of normal research laboratories. For example, if only we could make a worm hole at an outrageous density of $10^{100}$ (who cares whose units?) which is not possible, then we would have a real time machine.

We hope that our discussion of more realistic examples (such as a $50 \, \Omega$ transmission line) may more quickly give rise to a serious engineering view of the matter.

REFERENCES

1. E. C. C. Stückelberg, Helv. Phys. Acta 15, 23 (1942).
2. R. P. Feynman, Phys. Rev. 76, 749 (1949).
3. R. P. Feynman, Phys. Rev. 76, 769 (1949).
4. J. M. Jauch and F. Rohrlich, Theory of Photons and Electrons, Chapt.1, p36, Springer-Verlag, New York (1976).
5. L. Mandl and E. Wolf, Optical Coherence and Quantum Optics, Cambridge University Press, Cambridge (1995).
6. V. P. Bykov and V. I. Tatarskii, Phys. Lett. A 136, 77 (1988).
7. Y. Srivastava and A. Widom, Phys. Rep. 148, 1, (1987).
8. A. Einstein, R. C. Tolman and B. Podolsky, Phys. Rev. 37, 780 (1931).
9. A. Widom, *Sixth Quantum 1/f Noise and Other Low frequency Fluctuations in Electronic Devices*, Edited by P.H. Handel and A. L. Chung, AIP Conference Proceedings 371 (1994).

10. R. P. Feynmann, *Theory of Fundamental Processes*, W. A. Benjamin, Reading (1961).

11. A. Widom, Y. N. Srivatsava and E. Sassaroli, *Phys. Lett. A* 222 361 (1995).

12. A. Fine, *The Shakey Game Einstein Realism and the Quantum Theory*, second edition, The University of Chicago Press, Chicago (1996).

13. M. Visser, *Lorenzian Worm Holes*, AIP Press-Springer Verlag New York (1996).