New Physics at a Super Flavor Factory

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Abstract
The potential of a Super Flavor Factory (SFF) for searches of New Physics is reviewed. While very high luminosity $B$ physics is assumed to be at the core of the program, its scope for extensive charm and $\tau$ studies are also emphasized. The possibility to run at the $\Upsilon(5S)$ is also very briefly discussed; in principle, this could provide very clean measurements of $B_s$ decays. The strength and reach of a SFF is most notably due to the possibility of examining an impressive array of very clean observables. The angles and the sides of the unitarity triangle can be determined with unprecedented accuracy. These serve as a reference for New Physics (NP) sensitive decays such as $B^+ \to \tau^+\nu$ and penguin dominated hadronic decay modes, providing tests of generic NP scenarios with an accuracy of a few percent. Besides, very precise studies of direct and time dependent CP as asymmetries in radiative $B$ decays and forward-backward asymmetry studies in $B \to X_s\ell^+\ell^-$ and numerous null tests using $B$, charm and $\tau$ decays are also likely to provide powerful insights into NP. The dramatic increase in luminosity at a SFF will also open up entirely new avenues for probing NP observables, e.g. by allowing sensitive studies using theoretically clean processes such as $B \to X_s\nu\bar{\nu}$. The SFF is envisioned to be a crucial tool for essential studies of flavor in the LHC era, and will extend the reach of the LHC in many important ways.

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The term flavor was first used in particle physics in the context of the quark model of hadrons. It was coined in 1971 by Murray Gell-Mann and his student at the time, Harald Fritzsch, at a Baskin-Robbins ice-cream store in Pasadena. Just as ice-cream has both color and flavor so do quarks [Fritzsch, 2008].

Flavor physics denotes physics of transitions between the three generations of Standard Model (SM) fermions. With the LHC startup around the corner, why should one pay attention to these low energy phenomena? For one thing, flavor physics can probe new physics (NP) through off-shell corrections, before the NP particles themselves are produced in energy frontier experiments. As a historic example, the existence of the charm quark was predicted from the suppression of $K_L \rightarrow \mu^+\mu^-$ before its discovery [Glashow et al., 1976], while its mass was successfully predicted from $\Delta m_K$ [Gaillard and Lee, 1974]. Flavor physics is also intimately connected with the origin of fermion masses. In the limit of vanishing masses the flavor physics is trivial – no intergenerational transitions occur since weak and mass eigenbases trivially coincide. It is only the mismatch of weak and mass eigenbases (or the mismatch between the bases in which gauge and Yukawa terms are diagonal) that makes flavor physics interesting. In the quark sector of SM this mismatch is described by a single unitary matrix - the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Finally, CP violation is closely related to flavor physics. A strong argument for the existence of new sources of CP violation is that the CKM mechanism is unable to account for the observed baryon asymmetry of the universe (BAU) through baryogenesis [Gavela et al. 1994]. This points at NP with new sources of CP violation in either the quark or lepton sector (the latter potentially related to the BAU via leptogenesis [Uhlig, 2007]). It is therefore important to investigate the BAU by studying CP violation in both quark and lepton sectors (see below).

In the past ten years, due to the spectacular performance of the two $B$-factories, a milestone in our understanding of CP violation phenomena was reached. For the first time, detailed experiments, BABAR [Aubert et al. 2002] and Belle [Abashian et al. 2002], provided a striking confirmation of the CKM-paradigm of CP violation [Cabibbo, 1963; Kobayashi and Maskawa, 1973]. The Kobayashi-Maskawa model of CP-violation, based on three families and a single CP-odd phase, is able to account for the observed CP violation in the $B$ system, as well as that in the $K$ system, to an accuracy of about 20%, as shown in Fig. 1 [Bona et al. 2006b, 2007b; Charles et al. 2003; Lunghi and Soni 2007]. The impressive gain in precision on CKM constraints that is expected at a SFF is also shown in Fig. 1.

While we celebrate this remarkable agreement it is important to note that increasing the accuracy of CKM tests brings more than just an increased knowledge of fundamental CKM parameters. Once NP particles are observed at LHC, flavor physics observables will provide a set of independent constraints on the NP Lagrangian. These constraints are complementary to the measurements that are performed at high $p_T$ processes – i.e. they

FIG. 1 95% confidence level constraints on parameters $\bar{\rho}$ and $\bar{\eta}$ in the Wolfenstein parametrization of the CKM matrix. Left: present constraints, right: with errors shrunk to the size expected at a SFF while tuning central values to have compatible constraints [from Browder et al. 2007].

I. INTRODUCTION

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provide a complementary constraint on the combination of couplings, mixing angles and NP masses and become much more powerful once NP mass spectra are already measured. However, to be relevant for TeV processes, high precision is needed. But, how precise is precise enough? The answer depends on the NP flavor changing couplings. Taking as a conservative benchmark the case of minimally flavor violating NP that has couplings to SM fermions comparable to weak gauge couplings, the present results from B factories allow for masses of NP particles below \( \sim 100 \text{GeV} \). After completion of the Super Flavor Factory (SFF) program this limit would be pushed to \( \sim 600 \text{GeV} \) (Bona et al., 2007; Browder et al., 2007), illustrating the complementarity of LHC and SFF reach.\(^1\)

Let us elaborate a bit more on this important point. The NP constraints depend on both NP couplings to SM quarks and the NP masses and the two cannot be disentangled. An important set of flavor physics observables useful for NP searches are those from processes that proceed through flavor changing neutral currents. These are loop suppressed in the SM, and hence NP contributions are easier to detect than in charged flavor changing transitions that occur at tree level in the SM. Let us take as an explicit example corrections to the \( \Delta F = 2 \) processes, i.e. to \( K^0-\bar{K}^0 \), \( B^0_d-\bar{B}^0_d \) and \( B^0_s-\bar{B}^0_s \) mixing. The corresponding SM weak Hamiltonian has a form

\[
H_{\text{eff}} = \frac{1}{4 \Lambda_0} \left( V_{td}^* V_{ts} \right) \left[ d_{Lj} \gamma_5 d_{Lj} \right]^2, \tag{1}
\]

where \( C_0 \) is a Wilson coefficient that is of order \( \mathcal{O}(1) \), \( \Lambda_0 = 4\pi m_W/g^2 \approx 2.5 \text{ TeV} \) is the appropriate scale for a loop suppressed SM process, and \( d_{i,j} \) are the down quark fields \( d, s, b \). For simplicity let us also assume that NP leads to the effective operator with the same Dirac structure as in the SM, so

\[
H_{\text{eff}}^{\text{NP}} = \frac{C_{\text{NP}}}{\Lambda_{\text{NP}}} \left[ d_{Lj} \gamma_5 d_{Lj} \right]^2. \tag{2}
\]

If NP couplings do not have any flavor structure, then \( C_{\text{NP}} \sim \mathcal{O}(1) \), while \( \Lambda_{\text{NP}} \) corresponds roughly to the NP masses, if these are exchanged at tree level. In this case the NP masses are well above the weak scale. For instance, present measurements exclude \( \mathcal{O}(1) \) corrections to the \( B^0_d-\bar{B}^0_d \) mixing, from which

\[
\left( V_{td}^* V_{ts} \right)^2 \sim \Lambda_0^2 > \frac{C_{\text{NP}}}{\Lambda_{\text{NP}}} \Rightarrow \Lambda_{\text{NP}} \gtrsim 500 \text{ TeV}, \tag{3}
\]

For \( B^0_s-\bar{B}^0_s \) and \( K^0-\bar{K}^0 \) mixings the corresponding \( \Lambda_{\text{NP}} \) scales are 100 TeV and 10^4 TeV, respectively. The fact that these scales are much larger than the weak scale \( \sim m_W \) is known as the NP flavor problem.

If new physics particles with mass \( M \) are exchanged at tree level with \( \mathcal{O}(1) \) coupling constants, then \( \Lambda_{\text{NP}} \sim M \). This excludes new physics with general flavor violation structure at the energies accessible at the LHC. This conclusion holds even if new physics particles are exchanged only at 1-loop order, where \( \Lambda_{\text{NP}} \sim 4\pi M/g^2_{\text{NP}} \). For \( g_{\text{NP}} \sim g \) even the weakest bound from the \( B^0_s-\bar{B}^0_s \) system still leads to new physics particles with masses \( \gtrsim 7 \text{ TeV} \).

In other words, if the hierarchy problem of the Standard Model is resolved by adding more particles near the electroweak scale, this extended sector must have a non-generic flavor structure. Having completely flavor blind new physics is unnatural since the SM already contains flavor violation in the Yukawa couplings. The minimal possibility for the NP contribution of Eq. (2) is that the NP flavor violation comes only from the SM Yukawa couplings. This is the assumption underlying Minimal Flavor Violation (MFV); see Section III.B. The NP contribution of Eq. (2) then obeys the same CKM hierarchy as the SM contribution of Eq. (1) and can be rewritten as

\[
\tilde{H}_{\text{eff}}^{\text{NP}} = \frac{1}{\Lambda_{\text{NP}}} \left( V_{ti}^* V_{Lj} \right) \left[ d_{Lj} \gamma_5 d_{Lj} \right]^2. \tag{4}
\]

In this case not observing \( \mathcal{O}(1) \) effects from NP in the flavor transitions translates to \( \Lambda_{\text{NP}} \gtrsim \Lambda_0 \approx 2.5 \text{ TeV} \). If NP contributions are loop suppressed (as those from the SM are), then this bound translates to a relatively weak bound \( M \gtrsim m_W \) (if \( g_{\text{NP}} \sim g \)).

We see that in this minimal scenario, where no new mechanisms of flavor violation beyond those already present in the SM are introduced in the NP sector of the theory, one requires precision measurements of \( B \) physics observables to have results that are complementary to the measurements of NP spectrum at the LHC. In particular, as already mentioned, taking \( g_{\text{NP}} \sim g \) with NP contributing at 1-loop then SFF precision translates to a bound on NP masses of around 600 GeV (Bona et al., 2007; Browder et al., 2007).

Another very powerful probe of NP effects are measurements of CP violating observables. Extensions of the SM generically lead to new sources of CP-odd phases and/or new sources of flavor breaking [for a review see, e.g. Atwood et al. (2001a)]. An elementary example is provided by the SM itself. While a two-generation version of the SM does not exhibit CP violation, a single CP-odd phase in the CKM matrix occurs very naturally as a consequence of the third quark family. Beyond the SM the existence of new CP odd phases can be seen explicitly in specific extensions such as two Higgs doublet models (Lee, 1973; Weinberg, 1976), the left-right symmetric model (Kiers et al., 2002; Mohapatra and Pati, 1973), low energy SUSY (Grossman et al., 1998) or models with warped extra dimensions (Agashe et al., 2004, 2005b).

Furthermore, while \( B \)-factory results have now established that the CKM-paradigm works to good accu-

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1 Note that the generic MFV scenario of weakly coupled NP is not the most conservative scenario. The SFF constraint can be avoided, if couplings to SM fermions are further suppressed (see, for instance, Grossman et al. (2007a)).
racy, as more data has been accumulated some possible indications of deviations from the SM have emerged. These include the small “tension” between the direct and indirect determinations of $\sin 2\beta$, as seen in Fig. 11 (Bona et al., 2006a, 2007a; Charles et al., 2005; Lunghi and Soni, 2007)), as well as the famous trend for $\sin 2\beta$ from hadronic $b \to s$ penguin dominated decays to be below that from $b \to c$ tree dominated decays. While these measurements do not yet show compelling evidence for NP, the results are quite intriguing – it is also noteworthy that the discrepancy between $\sin 2\beta$ from penguin dominated modes and from the indirect determination (i.e. from the SM fit) is larger (Lunghi and Soni, 2007). Several other measurements in penguin dominated decays show possible indications of NP that are, unfortunately, obscured by hadronic uncertainties. Whether or not the currently observed effects are due to the intervention of NP, this illustrates that these processes provide a sensitive tool to search for NP. Thus, it is all the more important to focus on theoretically clean observables, for which hadronic uncertainties cannot cloud the interpretation of possible NP signals. In most cases this requires a significant increase in statistics, and therefore will only be possible at a SFF.

A key strength of a SFF is that it offers the opportunity to examine a vast array of observables that allow a wide range of tests of the SM and sensitively probe many NP models. In order to achieve this core physics program, it will be necessary to accumulate $50 - 100 \text{ ab}^{-1}$ of integrated luminosity after a few years of running, corresponding to an increase of nearly two orders of magnitude over the final data samples available at the current $B$-factories. It is important to stress that not only will a SFF enable exciting $B$ physics, it will also provide over $5 \times 10^{10}$ charm hadron and $\tau$ lepton pairs, enabling powerful studies of NP effects in the up-type quark and lepton sectors. The breadth of precision tests in a wide range of clean observables that are excellent probes of NP is an extremely important aspect of the SFF proposal.

While expectations for the SFF performance are based on the successes of the current $B$-factories, it is important to emphasise that the huge increase in statistics will provide a step change in the physics goals and in NP sensitivity. The program will include not only much more precise studies of NP-sensitive observables for which initial studies have already been carried out (e.g. $b \to s\gamma$, $b \to s\ell^+\ell^-$ and $b \to s\ell^+\ell^-$ penguin dominated processes), but will also include channels that have either barely been seen, or which, at their SM expectations, are beyond the capabilities of current experiments (e.g. $b \to d\nu\bar{\nu}$ decays). Clean studies of several interesting inclusive processes will become possible for the first time. Furthermore, for some channels with very small SM expectations, positive searches would provide unambiguous NP signals (e.g. lepton flavor violating $\tau$ decays, CP violation in charm mixing and/or decays, $b \to d\bar{s}s$ decays) etc. These provide examples of numerous “null tests” (Gershon and Soni, 2007) that are accessible to a SFF. It is notable that much of the SFF program will use the recoil analysis technique, that takes advantage of the $e^+e^- \to \Upsilon(4S) \to BB$ production chain to provide kinematic constraints on un-reconstructed particles. This is of great importance since it allows measurement of theoretically clean processes with typically low experimental backgrounds.

In Section II we begin with a very brief discussion of design issues for the new machine(s), Section III presents a review of NP effects in FCNC processes. For illustration we discuss three class of NP scenarios that are very popular: Minimal Flavor Violation (MFV), Minimal Supersymmetric Standard Model and models of warped extra dimensions. We then discuss (Section IV) the prospects for improved determinations of the angles of the UT by “direct measurements” through the cleanest methods that have been devised so far. Section V briefly reviews the determination of the sides of the UT. We then discuss the time dependent CP asymmetry measurements in penguin-dominated modes (Section VI) that have been the focus of much attention in the past few years, followed by a section on null tests (Section VII). Section VIII is devoted to the powerful radiative $B$ decays; here we discuss both on-shell photonic $b \to s\gamma$ as well as $b \to s\ell\ell$ in several different manifestations. Sections IX is devoted to a very brief presentation of highlights of $B_s$ physics possibilities at a SFF. Sections X and XI deal with charm and $\tau$ physics potential of a SFF. Section XII briefly discusses how the SFF and LHCb efforts complement each other in important ways and Section XIII is the Summary.

II. DESIGN ISSUES

A. Machine design considerations

Quite recently, two different designs for a Super Flavor Factory (SFF) have emerged. The SuperKEKB design (Hashimoto et al., 2004) is an upgrade of the existing KEKB accelerator with expected peak instantaneous luminosity of $8 \times 10^{35} \text{ cm}^{-2} \text{s}^{-1}$. This is achieved by increasing the beam currents, while reducing the beam sizes and improving the specific luminosity with crab cavities that provide the benefits of effective head-on collisions with a nonzero crossing angle (Abe et al., 2007; Akai and Morita, 2003; Oide and Yokoya, 1989). While this is a conventional upgrade scenario, it presents several challenges, particularly related to higher order mode heating, collimation and coherent synchrotron radiation. A great deal of effort has gone into understanding and solving these problems including prototypes (for a detailed discussion, see Hashimoto et al., 2004).

The SuperB design (Bona et al., 2007c) uses a completely different approach to achieve a peak instantaneous luminosity in excess of $10^{36} \text{ cm}^{-2} \text{s}^{-1}$. The basic idea is that high luminosity is achieved through reduction of the vertical beam size by more than an order of magnitude, rather than by increasing the currents. With such
small emittance beams, a large crossing angle (Hirata, 1995; Piwinski, 1977) is necessary to maintain beam stability at the interaction point. Any degradation in luminosity due to the crossing angle is recovered with a “crab” of the focal plane (Raimondi et al., 2007). The SuperB design could be built anywhere in the world, though the most likely home for this facility is a green field site on the Tor Vergata campus of the University of Rome.

Some of the key parameters of the SuperKEKB and SuperB machines are compared in Table I. One important number to compare is the wall power, which dominates the operating costs of the machine. The total costs are kept low by recycling as much hardware as possible – from KEKB magnets and the Belle detector in the case of SuperKEKB, and from PEP-II hardware and the BaBar detector in the baseline design for SuperB.

Aside from high luminosity – the higher the better – there are several other desirable features for a SFF to possess. Although the physics goals appear to be best served by operation primarily at the $\Upsilon(4S)$ resonance, the ability to change the centre-of-mass energy and run at other $\Upsilon$ resonances, and even down to the tau-charm threshold region (albeit with a significant luminosity penalty), enhances the physics capabilities of the machine. The possibility to run with at least one beam polarized would add further breadth to the physics program.

It is also important that the clean experimental environment enjoyed by the current $B$ factories must be achieved by a SFF. How to achieve high luminosity while retaining low backgrounds is a challenge for the design of the machine and the detector, since the brute force approach to higher luminosity – that of increasing the beam currents – necessarily leads to higher backgrounds. To some extent these can be compensated for by appropriate detector design choices, but in such cases some compromise between luminosity and detector performance (and hence physics output) may be anticipated.

The background level in the detector depends on several factors. One of these is the luminosity itself, and higher luminosity unavoidably leads to larger numbers of physics processes such as radiative Bhabha scattering and $e^+e^-$ pair production. Other terms depend on the beam current. For example, synchrotron radiation is emitted wherever the beam is steered or bent, some of which inevitably affects the detector in spite of careful design and shielding of the interaction region. Another term that depends on the current arises from so-called beam gas interactions. Although the interior of the beam pipe is maintained at high vacuum, radiation from the beam will interact with material in the beampipe and cause particles to be emitted – these in turn can be struck directly by the beam particles. Consequently this term depends quadratically on the current. The beam size is another consideration that has an impact on backgrounds. As the beams become smaller the particles within them are more likely to undergo intrabeam scattering effects. These include the Touschek effect, in which both particles involved in an intrabeam collision are ejected from the beam. For very small emittance beams, the loss of particles can be severe, leading to low beam lifetimes. The achievement of meeting the challenges of maintaining manageable backgrounds and beam lifetimes represents a milestone for SFF machine design (Bona et al., 2007c; Hashimoto et al., 2004).

A related issue pertains to the asymmetry of the beam energies. To obtain the optimal asymmetry, several factors must be taken into account. From the accelerator design perspective, more symmetric beam energies lead to longer beam lifetimes and potentially higher luminosities. However, a certain degree of beam asymmetry is necessary in order to measure time-dependent CP asymmetries, and these are an important part of the physics program of the SFF, as discussed below. An equally important part of the program, however, relies on measurements that benefit from the hermeticity of the detector in order to reconstruct decay modes with missing particles, such as neutrinos. Thus the physics considerations are subtly different from those that informed the design choices for the current $B$ factories, and a somewhat smaller asymmetry than either BaBar (9.0 GeV $e^+$ on 3.1 GeV $e^-$) or Belle (8.0 GeV $e^-$ on 3.5 GeV $e^+$), may be optimal. However, a change in the beam energies would require the design of the interaction region, and to a lesser extent the detector, to be reconsidered. In order to be able to reuse components of the existing detectors in the final SFF, as discussed below, it would be prudent to keep the asymmetry similar to those in successful operation today. However, preliminary studies indicate that either BaBar or Belle detectors could quite easily be modified to operate with beam energies of 7 GeV on 4 GeV.

### B. Detector design considerations

The existing $B$ factory detectors (Abashian et al., 2002; Aubert et al., 2002) provide a very useful baseline from which to design a SFF detector that can provide excellent performance in the areas of vertex resolu-

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**Table I** Comparison of some of the key parameters of the SuperKEKB (Hashimoto et al., 2004) and SuperB (Bona et al., 2007c) designs.

| Parameter                  | SuperKEKB | SuperB |
|----------------------------|-----------|--------|
| Beam energies ($e^+ / e^-$, GeV) | 3.5 / 8   | 4.0 / 7.0 |
| Beam currents ($e^+ / e^-$, A)  | 9.4 / 4.1 | 2.3 / 1.3 |
| Bunch size ($\sigma_x / \sigma_y$, nm) | 42000 / 367 | 5700 / 35 |
| Bunch length ($\sigma_z$, mm)   | 3         | 6      |
| Emittance ($\epsilon_x / \epsilon_y$, nm-rad) | 9 / 0.045 | 1.6 / 0.004 |
| Beta function at IP ($\beta_x / \beta_y$, mm) | 200 / 3  | 20 / 0.3 |
| Peak luminosity ($10^{36}$ cm$^{-2}$ s$^{-1}$) | 0.8       | > 1    |
| Wall power (MW)               | 83        | 17     |
tion, momentum resolution, charged particle identification (particularly kaon-pion separation), electromagnetic calorimetry and close to 4π solid angle coverage with high efficiency for detection of neutral particles that may otherwise fake missing energy signatures (particularly $K^0_L$ mesons). However, some upgrades and additions are necessary.

As it is desirable to operate with reduced beam energy asymmetry compared to the current $B$ factories, improved vertex resolution is necessary in order to obtain the same performance in terms of $c\Delta t = \Delta z/(\beta\gamma)$, where $(\beta\gamma)$ is the Lorentz boost factor of the $\Upsilon(4S)$ in the laboratory frame. In fact, it is highly desirable to improve the performance further, since results from the current $B$ factories have demonstrated the utility of vertex separation as a powerful tool to reject backgrounds. The ultimate resolution depends strongly on the proximity of the inner layer to the interaction point. For reference, the radii of the innermost layers of the existing $\bar{B}A\bar{B}ar$ and Belle vertex detectors are 30 mm and 20 mm respectively ([Aihara et al. 2006] Re et al. 2006). To position silicon detectors close to the interaction region requires careful integration with the beampipe design, and a choice of technology that will not suffer from high occupancy.

While the inner radius of the vertex detector is of great importance for almost all measurements that will be made by a SFF, the outer radius has a large impact on a subset of channels, namely those where the $B$ decay vertex position must be obtained from a $K^0_S$ meson (typically $B^0 \rightarrow K^0_S\pi^0$, $B^0 \rightarrow K^0_S\pi^0\gamma$ and $B^0 \rightarrow K^0_SK^0_SK^0_S$). The existing $\bar{B}A\bar{B}ar$ and Belle vertex detectors have outer radii of 144 mm and 88 mm respectively, and the former appears to be a suitable choice for a SFF. A larger outer radius for the silicon detector has a useful consequence in that the tracking chamber, which can be based on a gaseous detector, does not have to extend too close to the interaction region where the effect of high backgrounds would be most severe for this detector. Therefore, assuming the same magnetic field (1.5 T) as $\bar{B}A\bar{B}ar$ and Belle, similar momentum resolution would be expected ([Bona et al. 2007c] Hashimoto et al. 2004).

The choice of particle identification technology for a SFF presents some challenges. At present, Belle achieves calorimetry and close to $4\pi$ efficiency for detection of neutral particles that may otherwise fake missing energy signatures. However, some upgrades and additions are necessary. Possibilities for particle identification capabilities in both forward and backward regions are also being considered.

The high efficiency to reconstruct photons is one of the significant advantages of a SFF compared to experiments in a hadronic environment. The existing electromagnetic calorimeters of $\bar{B}A\bar{B}ar$ and Belle (and indeed of CLEO) are based on CsI(Tl) crystals; studies show that technology can perform well at higher rates in the barrel region. However, in the endcaps where rates are highest alternative solutions are necessary. Various options, including pure CsI crystals or LYSO are under consideration ([Bona et al. 2007d] Hashimoto et al. 2004). Improvements to the calorimeter solid angle coverage and hence hermeticity would benefit the physics output (especially for an upgrade based on the $\bar{B}A\bar{B}ar$ detector, which does not have a backward endcap calorimeter).

Another important consideration with respect to detector hermeticity is the detection of $K^0_S$ mesons, which if unreconstructed can fake missing energy signatures. Both $\bar{B}A\bar{B}ar$ and Belle have instrumentation in their magnetic flux returns which allows the detection of showers that initiate in the yoke, that may be associated with tracks (as for muons) or with neutral particles ($K^0_S$ mesons). The efficiency depends on the amount of material in the flux return, while the background rates generally depend on radiation coming from up-and down-stream bending magnets ([Bona et al. 2007c] Hashimoto et al. 2004). Both of these problems appear well under control for operation.

Finally, it is important to note that the extremely high physics trigger rate will present some serious challenges for data acquisition and computing. However, in these areas one can expect to benefit from Moore’s Law and from the distributed computing tools that are under development for the LHC. Thus there is no reason to believe that these challenges cannot be met.

To summarize, there exist two well-developed proposals and approaches to achieving the luminosity and performance required for the measurements of NP in flavor ([Bona et al. 2007c] Hashimoto et al. 2004).

### III. NEW PHYSICS AND SUPER FLAVOR FACTORY

A Super Flavor Factory offers a variety of observables sensitive to NP such as rare $B$ decays, CP asymmetries, lepton flavor violation, etc. To gauge their sensitivity to NP we review in this section several examples of NP models whose imprint in flavor physics has been extensively discussed in the literature: the model independent approach of Minimal Flavor Violation, two Higgs doublet models, low energy SUSY models and extra dimensions. This list is by no means exhaustive. Other beyond the SM extensions not covered in this section have interesting flavor signals as well, for instance little Higgs models with conserved $T$ parity ([Blanke et al. 2007] Cheng and Low 2003) or the recent idea of “Unparticle Physics” (Georgi 2007).
A. Effective weak Hamiltonian

The weak scale \( \mu_{\text{weak}} \sim m_W \) and the typical energy scale \( \mu_{\text{low}} \) of the low energy processes occurring at SFF are well separated. For instance, the typical energy scale in \( B \) decays is a few GeV, about a factor \( \sim 50 \) smaller than \( m_W \). This means that using OPE the effects of weak scale physics can be described at low energies by a set of local operators, where the expansion parameter is \( \mu_{\text{low}}/\mu_{\text{weak}} \). The matching onto local operators is performed by integrating out the heavy fields - the top, the massive weak gauge bosons, the Higgs boson, and the possible new physics particles. At low energies one then works only within the effective field theory (EFT).

For example, the SM effective weak Hamiltonian for \( \Delta S = 1 \) \( B \) transitions is (Buchalla et al., 1996)

\[
H_W = \frac{G_F}{\sqrt{2}} \sum_{p = u, c, t} \lambda_p^{(s)} \left( C_1 O_1^{p} + C_2 O_2^{p} + \sum_{i=3}^{10, 7, 8, 9} C_i O_i \right),
\]

(5)

where the CKM factors are \( \lambda_p^{(s)} = V_{pb} V_{ps}^* \) and the standard basis of four-quark operators is

\[
O_1^p = (\bar{p}b)(\bar{s}p), \quad O_2^p = (\bar{p}b)(\bar{s}a)p, \quad O_{3,5} = (\bar{s}b)(\bar{q}q)_{\pm}, \quad O_{4,6} = (\bar{s}a)(\bar{q}q)_{\mp}, \quad O_{7,9} = \frac{3\epsilon_q}{2}(\bar{b}q), \quad O_{8,10} = \frac{3\epsilon_q}{2}(\bar{b}q)_{\pm},
\]

(6)

with the abbreviation \( \langle \bar{q}_1 \gamma^\mu (1 - \gamma_5) q_2 \rangle = \langle \bar{q}_1 \gamma^\mu (1 + \gamma_5) q_4 \rangle \). The color indices \( \alpha, \beta \) are displayed only when the sum is over fields in different brackets. In the definition of the penguin operators \( O_{3,4,9,10} \) in Eq. 9 there is also an implicit sum over \( q = \{ u, d, s, c, b \} \). The electromagnetic and chromomagnetic operators are

\[
O_{\gamma, 8, 9} = \frac{m_b}{4\pi^2} \delta_{\mu
\nu} \left( eF_{\mu\nu}, gG_{\mu\nu} \right) P_R b,
\]

(7)

with \( P_{L,R} = 1 \mp \gamma_5 \), while the effective Hamiltonian for \( b \to s \ell^+ \ell^- \) contains in addition (Grinstein et al., 1989)

\[
Q_{(9,10)} = \frac{\alpha_s}{8\pi^2} (\bar{e} \gamma^\mu \{ 1, \gamma_3 \} \ell)(\bar{s} \gamma_\mu P_L b).
\]

(8)

These two operators arise at 1-loop from matching the \( W \) and \( Z \) box and penguin diagrams shown in Fig. 2.

The operator \( Q_{10} \) is RG invariant to all orders in the strong coupling, while the operator \( Q_{9} \) mixes with the four-quark operators \( O_1, \ldots, 6 \) already at zeroth order in \( \alpha_s \). Similarly, the operator for \( b \to s \nu \bar{\nu} \) transition in SM is

\[
O_{11\nu} = \frac{\alpha_s}{4\pi^2} \sin^2 \theta_W (\bar{e} \gamma_\mu P_L \nu)(\bar{s} \gamma_\mu P_L b).
\]

(9)

\[\text{FIG. 2. Sample diagrams contributing to the matching for } b \to s \ell^+ \ell^- \text{ at one-loop order.}\]

The weak Hamiltonian for \( \Delta S = 0 \) \( B \) decays is obtained from Eqs. (5)-(7) through the replacement \( s \to d \), while for \( K \) decays another \( b \to s \) replacement is needed. \( B^{-} \) mixing is governed in the SM by \( Q_{\Delta B=2} = (\bar{bd})_-(\bar{bd})_-, \) with analogous operators for \( B_s \to B_s, K \to K \) and \( D \to \bar{D} \) mixing.

The Wilson coefficients \( C_i(\mu) \) are determined in a two-step procedure. After matching at the high scale \( \mu_h \sim m_W \), they are RG evolved down to the low scale. For brevity we will discuss here only the case of \( B \) decays, where the low scale is of the order \( \mu \sim m_b \).

The weak scale perturbative matching is performed in a mass-independent scheme such as \( \overline{MS} \), giving the Wilson coefficients expanded in \( \alpha_s(\mu_h) \) and \( \alpha_{\text{em}}(\mu_h) \)

\[
C_i(\mu_h) = C_i^{(0)} + \frac{\alpha_s(\mu_h)}{4\pi} C_i^{(1)}(\mu_h) \]

\[
\quad + \left( \frac{\alpha_s(\mu_h)}{4\pi} \right)^2 C_i^{(2)}(\mu_h) + \alpha_{\text{em}}(\mu_h) C_i^{(1e)}(\mu_h) + \cdots.
\]

(10)

At tree level all Wilson coefficients vanish apart from \( C_2^{(0)} = 1 \). The matching calculation includes both hard gluon and electroweak loop effects.

The Wilson coefficients are evolved from \( \mu_h \) down to a typical hadronic scale \( \mu \sim m_b \) by solving the Renormalization Group Equation (RGE)

\[
\mu \frac{d}{d\mu} \tilde{C}(\mu) = (\gamma_T ^T \tilde{C}(\mu),
\]

(11)

where the anomalous dimension matrix is also expanded

\[
\gamma = \frac{\alpha_s}{4\pi} \gamma_s^{(0)} + \frac{\alpha_s}{4\pi} \gamma_s^{(1)} + \frac{\alpha_s}{4\pi} \gamma_s^{(2)} + \cdots.
\]

(12)

The solutions of the RGE are renormalization-scheme and renormalization-scale invariant to any given order only provided that the orders in matching and running are chosen appropriately. Keeping the tree level matching \( C_i^{(0)} \) and the one-loop order anomalous dimension matrix \( \gamma^{(0)} \) yields the so-called leading-log approximation (LL) for the Wilson coefficients. For instance the LL values for tree and QCD penguin operators, \( i = 1, \ldots, 6 \), are \( C_i(\mu = 4.8 \text{ GeV}) = \{ -0.248, 1.107, 0.011, -0.025, 0.007, -0.031 \} \). The next-to-leading approximation (NLL) corresponds to keeping the one-loop matching conditions \( C_i^{(1)} \) and the two-loop anomalous dimension matrix \( \gamma^{(1)} \), and so on. The
NLL values for $i = 1, \ldots, 6$ are $C_i(\mu = 4.8 \text{ GeV}) = \{ -0.144, 1.055, 0.011, -0.034, 0.010, -0.039 \}$. Note that for higher loop calculations it has become customary to use a different operator basis than that of Eq. (6). In the basis introduced by Chetyrkin et al. (1997), $\gamma_5$ does not appear explicitly (except in the magnetic operators), which allows a use of dimensional regularization with fully anticommuting $\gamma_5$, simplifying multiloop calculations. The present status of the coefficients entering the RGE is as follows.

The two-loop matching corrections to the Wilson coefficients $C_i(\mu_b)$ were computed by Bobeth et al. (2000). The three-loop matching correction to the coefficient of the dipole operator $C_7(\mu_b)$ was recently obtained by Misiak and Steinhauser (2004). The leading 2-loop electroweak corrections to the Wilson coefficient of the dipole operator $C_7$ were computed by Czarnecki and Marciano (1998), while the leading electromagnetic logs $\alpha_m\alpha_e^2 \log^{N+1} (\mu_W/\mu_b)$ were resummed for this coefficient in Buranowski and Misiak (2000); Kagan and Neubert (1999). A complete two-loop matching of the electroweak corrections was performed by Gambino and Haisch (2000, 2001). The three-loop matching of the four-quark operators was computed in Görbahn and Haisch (2005); Görbahn et al. (2003).

The presence of new physics (NP) has several effects on the form of the effective Hamiltonian in Eq. (5). First, it shifts the values of the Wilson coefficients away from the SM values

$$\lambda^{(q)}_p C_i = \lambda^{(q)}_p C^\text{SM}_i + C^\text{NP}_i.$$  \hspace{1cm} (13)

Note that the NP contribution to the Wilson coefficient may not obey the CKM hierarchy of the SM term, and can also depend on new weak phases. Second, NP contributions can also enlarge the basis of the operators, for instance by introducing operators of opposite chirality to those in Eq. (5), or even introducing four quark operators with scalar interactions. We will discuss the two effects in more detail in the subsequent subsections, where we focus on particular NP models.

B. Minimal Flavor Violation

In SM the global flavor symmetry group

$$G_F = U(3)_Q \times U(3)_L \times U(3)_R \times U(3)_D \times U(3)_E$$  \hspace{1cm} (14)

is broken only by the Yukawa couplings, $Y_U$, $Y_D$, and $Y_E$ (with $U(1)$’s also broken by anomalies). In a generic extension of SM, on the other hand, additional sources of flavor violation can appear. If the extended particle spectrum is to solve the hierarchy problem (for instance by doubling of the spectrum as in MSSM) these new particles have to have masses comparable to the electroweak scale. This then leads to a clash with low energy flavor physics experimental data. Namely, virtual exchanges of particles with TeV masses and with completely generic flavor violating couplings lead to flavor changing neutral currents (FCNCs) that are orders of magnitude larger than observed, cf. Eq. (3).

TeV scale NP therefore cannot have a generic flavor structure. On the other hand, it cannot be completely flavor blind either since the Yukawa couplings in SM already break flavor symmetry. This breaking will then translate to a NP sector through renormalization group running as long as the NP fields couple to the SM fields. Thus, the minimal choice for the flavor violation in the extended theory is that its flavour group is also broken only by the SM Yukawa interactions. This is the Minimal Flavor Violation (MFV) hypothesis (Buras, 1997; Ciuchini et al., 1998a, D’Ambrosio et al., 2002; Hall and Randall, 1990).

The idea of MFV was formalized by D’Ambrosio et al. (2002) by promoting the Yukawa couplings to spurions that transform under flavor group $G_F$. Focusing only on the quark sector, the transformation properties under $SU(3)_Q \times SU(3)_U \times SU(3)_D$ are

$$Y_U \sim (3, 3, 1), \quad Y_D(3, 1, 3)$$  \hspace{1cm} (15)

so that the Yukawa interactions

$$\mathcal{L}_Y = \bar{Q}_L Y_D d_R H + \bar{Q}_L Y_U u_R H^c + h.c.,$$  \hspace{1cm} (16)

are now formally invariant under $G_F$, Eq. (13). Above we suppressed the generation indices on the left-handed quark isodoublet $Q_i = (u_L, d_L)_i$, on right-handed quark isosinglets $u_R, d_R$ and on Yukawa matrices $Y_U, D$, while for the Higgs isodoublet the notation $H^c = i\tau_2 H^*$ was used. Minimally flavor violating NP is also formally invariant under $G_F$ with the breaking coming only from insertions of spurion fields $Y_{U,D}$. Integrating out the heavy fields (i.e. the NP fields, Higgs, top, W and Z) one then obtains the low-energy EFT that is also invariant under $G_F$.

A particularly convenient basis for discussing transitions between down-type quarks is the basis in which the Yukawa matrices take the following form

$$Y_D = \lambda_D, \quad Y_U = V^\dagger \lambda_U.$$  \hspace{1cm} (17)

Here $\lambda_{D,U}$ are diagonal matrices proportional to the quark masses and $V$ is the CKM matrix. In a theory with a single Higgs (or in a small tan $\beta$ regime of the 2HDM or MSSM) one has $\lambda_0 \approx 1$, $\lambda_\gamma \sim \tan(0, 0, 1)$. The dominant non-diagonal structure for down-quark processes is thus provided by $Y_U Y_U^\dagger$ transforming as $(3 \times 3, 1, 1)$. Its off-diagonal elements exhibit the CKM structure $(Y_U Y_U^\dagger)_{ij} \sim \lambda^U_i V^*_{ij} V_{ij}$. Furthermore, multiple insertions of $Y_U Y_U^\dagger$ give $(Y_U Y_U^\dagger)^n \sim \lambda^{2n}_U V^*_{ij} V_{ij}$ and are thus equivalent to a single $Y_U Y_U^\dagger$ insertion, while multiple insertions of $Y_D$ beyond leading power can be neglected. This makes the MFV framework very predictive.

The particular realization of MFV outlined above is the so-called constrained minimal flavor violation
The assumptions that underlie cMFV are (i) the SM fields are the only light degrees of freedom in the theory, (ii) there is only one light Higgs and (iii) the SM Yukawas are the only sources of flavor violation. The NP effective Hamiltonian following from these assumptions is

$$H_{\text{eff}}^{\text{NP}} = \frac{C_{\text{NP}}^i}{\Lambda_{\text{NP}}} (V_{ij}^* V_{ij}) Q_i,$$

(18)

where $Q_i$ are exactly the same operators as in the SM effective weak Hamiltonian of Eq. [3]. This is sometimes taken to be the definition of cMFV by Blanke et al. (2006). The observables in which theoretical errors are small are sizeable contributions from operators with non-SM chiral structures in addition to Eq. (18) are possible (see next sections).

In cMFV the Wilson coefficients of the weak operators deviate from the SM values, but remain real, so that no new sources of CP violation are introduced. In phenomenological analyses it is also useful to assume that NP contributions are most prominent in the EWP Wilson coefficients $C_{S_8,...,10}$, the dipole operators $(C_{7γ,8γ})$, and the four-fermion operators involving quarks and leptons $(C_{ℓℓ}, C_{10ℓℓ}, C_{11ℓℓ})$. The rationale for this choice is that the Wilson coefficients of these operators are small in the SM, so that NP effects can be easier to spot. In contrast, NP effects are assumed to be negligible in the tree, $C_{1,2}$, and QCD penguin operators, $C_{3,...,6}$.

Because cMFV is a very constrained modification of the weak Hamiltonian Eq. (18), one can experimentally distinguish it from other BSM scenarios by looking at the correlations between observables in K and B decays. A sign of cMFV would be a deviation from SM predictions that can be described without new CP violating phases and without enlarging the SM operator basis. A deviation in $β$ from $β^0 → φK_S$ (see Section VI) on the other hand would rule out the cMFV framework.

How well one can bound NP contributions depends both on the experimental and theoretical errors. The observables in which theoretical errors are below 10% have a potential to probe $Λ_{NP} \sim 10$ TeV (taking $C_{NP}^i = 1$). The most constraining FCNC observable at present is the inclusive $B \rightarrow χ_p γ$ rate with the experimental and theoretical error both below 10% after the recent (partially completed) NNLO calculation (Becher and Neubert, 2007; Misiak and Steinhauser, 2007; Misiak et al., 2007). Using older theoretical predictions and experimental data, the 99% confidence level (CL) bound is $Λ_{NP} > 6.4(5.0)$ TeV in the case of constructive (destructive) interference with SM (D’Ambrosio et al., 2002). Constraints from other FCNC observables are weaker. An illustrative example we show in Figure 3 expected $Λ_{NP}$ bounds following from observables sensitive to the operator $(Q_L Y_U^j Y_{Lγp} Q_L)(\bar{L}_L γ_p L_L)$ for improved experimental precisions [see also (Bona et al., 2006a, 2007b)].

The MFV hypothesis has been extended to the leptonic sector (MLFV) in Cirigliano and Grinstein (2006); Cirigliano et al. (2003). In MLFV the most sensitive FCNC probe in the leptonic sector is $μ \rightarrow eγ$, while $τ \rightarrow μγ$ could be suppressed below the SFF sensitivity. The MLFV scenario also predicts correlations between the rates of various LFV processes. Studies of LFV in tau decays at a SFF are therefore crucial to test the MLFV framework (see Section XI).

An extension of MFV to the Next-to-Minimal Flavor Violation (NMVF) hypothesis was put forward in Agashe et al. (2005a) by demanding that NP contributions only roughly obey the CKM hierarchy, and in particular can have $O(1)$ new weak phases. This definition of NMVF is equivalent to having an additional spurion $Y_S$ transforming as $Y_U^T Y_U^T$ or $Y_D^T Y_D^T$ under $G_F$, where the transformation between $Q_L$ weak basis and the $Y_S$ eigenbasis is demanded to be aligned with the CKM matrix. The consequences of $Y_S$ transforming differently under $G_F$ than the SM Yukawas have been worked out by Feldmann and Mannel (2007).

C. Two-Higgs Doublet Models

The scalar sector of SM contains only a single scalar electroweak doublet. This is no longer true (i) in low energy supersymmetry, where holomorphism of the superpotential requires at least two scalar doublets; (ii) in many of the solutions to the strong CP problem (Peccei and Quinn, 1977a,b); (iii) in models of spontaneous CP breaking (Lee, 1973). Here we focus on the simplest extension, the two-Higgs doublet model (2HDM), where the scalar sector is composed of two Higgs fields, $H_U, H_D$, transforming as doublets under $SU(2)_L$. More complicated versions with Higgs fields carrying higher weak isospins are possible, but are also more

\[ \text{FIG. 3 Expectations for bounds on } \Lambda_{NP} \text{ for } (Q_L Y_U^j Y_{Lγp} Q_L)(\bar{L}_L γ_p L_L) \text{ that would follow from relative experimental precision } σ_{\text{rel}}, \text{ with currently expected theoretical uncertainties } \text{[D’Ambrosio et al., 2002]}. \]
constrained by electroweak precision data, in particular that the \( \rho \) parameter is equal to one up to radiative corrections. The 2HDM model is also a simplified version of the MSSM Higgs sector, to be considered in the next subsection.

The Yukawa interactions of a generic 2HDM are

\[
\mathcal{L} = \bar{Q}_L^f D H_D^f d_R + \bar{Q}_L^f U H_U^f u_R + \bar{Q}_L^g H_U^g u_R + \bar{Q}_L^g D H_D^g d_R + \text{h.c.},
\]

where \( H_{D,U}^f = i\tau_2 H_{D,U}^f, \) and the generation indices are suppressed. If all the 3 \times 3 Yukawa matrices \( f^{D,U} \) and \( g^{D,U} \) are nonzero and take generic values, this leads to tree level FCNCs from neutral Higgs exchanges that are unacceptably large.

Tree level FCNCs are not present, if up and down type-II 2HDM together only to one Higgs doublet \( \) (Glashow and Weinberg, 1977). This condition can be met in two ways, which also define two main classes of 2HDM. In type-I 2HDM both up- and down-type quark doublets couple only to one of the two Higgses (as in SM), i.e. either \( g^U = g^D = 0 \) or \( f^U = f^D = 0 \). In type-II 2HDM up- and down-type quark doublets to separate Higgs doublets, i.e. \( f^U = g^D = 0 \) (Haber et al., 1979).

The remaining option that all \( f^{D,U} \) and \( g^{D,U} \) are nonzero is known as type-III 2HDM \( \) (Atwood et al., 1997). Chern and Sher, 1987; Hou, 1992). The tree level flavor violating couplings to neutral Higgs then need to be suppressed in some other way, for instance by postulating a functional dependence of the couplings \( g_{U,D}, g_{U,D} \) on the quark masses \( \) (Antaramian et al., 1997). Cheng and Sher, 1987). A particular example of type-II 2HDM is also the so-called T2HDM (Das and Kao, 1993; Kiers et al., 1999), which evades the problem of large FCNC effects in the first two generations by coupling \( H_D \) to all quarks and leptons except to the top quark, while \( H_U \) couples only to the top quark.

After electroweak symmetry breaking the fields \( H_{U,D} \) acquire vacuum expectation values \( v_1, v_2 \)

\[
\langle H_{U,D} \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle H_D \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v_1 \end{pmatrix},
\]

where it is customary to define \( \tan \beta = v_2/v_1 \), while \( v_1^2 + v_2^2 = v^2 \), with \( v = 246 \) GeV. In type-II 2HDM the up and down quark masses are \( m_u \sim v_2, m_d \sim v_1 \). The large hierarchy \( m_t/m_b \sim 35 \) can thus be explained in this model by a large ratio of the vevs \( v_2/v_1 = \tan \beta > 1 \).

The physical degrees of freedom in 2HDM scalar sector consist of one charged Higgs boson \( H^\pm \), two CP-even neutral Higgs bosons \( H_{1,2} \), and one CP-odd Higgs boson \( A \). The phenomenology of the 2HDM of type-I, II is similar to that of the SM with the addition of the charged Higgs flavor-changing interactions. These \( S\pm P \) couplings are for type-II 2HDM given by

\[
H^\pm \left[ \frac{\tan \beta u_L V M_D d_R + \frac{1}{\tan \beta} \bar{u}_R M_U V d_L}{v} + \text{h.c.} \right],
\]

while the type-I 2HDM interactions are obtained by replacing \( \tan \beta \rightarrow -1/\tan \beta \) in the first term. The matrix \( V \) is the same CKM matrix as in the \( W^\pm \) couplings, while \( M_{D(U)} \) are diagonal matrices of down (up) quark masses.

The most sensitive probes of interactions in Eq. (21) are processes where \( H^\pm \) can be exchanged at tree level: semileptonic \( b \rightarrow c \ell \nu \) decays and the weak annihilation decay \( B^\pm \rightarrow \tau \ell \nu \), see Fig. 4 giving a constraint on the ratio \( m_{H^\pm}/\tan \beta \) (Grossman and Ligeti, 1994; Kiers and Soni, 1997).

The inclusive semitauonic decays have been studied at LEP (Abbiendi et al., 2001; Barate et al., 2001). Assuming type-II 2HDM, these give a 90% CL upper bound of \( \tan \beta/M_{H^\pm} \leq 0.4 \text{ GeV}^{-1} \). A comparable constraint on \( \tan \beta/m_{H^\pm} \) can be obtained from exclusive \( B \rightarrow D(\ell^+)\tau\ell \nu \) decays (Chen and Geng, 2006a; Nierste et al., 2008; Tanaka, 1995). First observations of these decays have recently been made at the LHC (Aubert et al., 2007a; Matyja et al., 2007), with significant improvements in precision expected at a SFF. Furthermore, the study of \( B \rightarrow D \tau \ell \nu \) decay distributions can discriminate between \( W^\pm \) and \( H^\pm \) contributions (Grzadkowski and Hou, 1992; Kiers and Soni, 1997; Miki et al., 2002; Nierste et al., 2008). In particular, in the decay chain \( B \rightarrow D \bar{\nu}_\tau \tau^- [\pi^- \nu_\tau] \) the differential distribution with respect to the angle between three-momenta \( \bar{p}_D \) and \( \bar{p}_\tau \) can be used to measure both the magnitude and the weak phase of the charged Higgs scalar coupling to quarks (Nierste et al., 2008).

In the annihilation decay \( B^\pm \rightarrow \tau \ell \nu \), \( H^\pm \) exchange may dominate over helicity suppressed \( W^\pm \) exchange contribution. A significant reduction of the Higgs decay to \( \tau \ell \nu \) has been observed at the LHC (Aubert et al., 2007b; Tanaka, 2007), with significant improvements in precision expected at a SFF. Recent measurements (Aubert et al., 2007; Matyja et al., 2008; Kado et al., 2006) give

\[
\frac{R_{B\tau\nu}}{B^{SM}_{B\tau\nu}} = 0.93 \pm 0.41,
\]

compatible with the presence of \( H^\pm \) contribution. The present status of the constraints on \( (M_{H^\pm}, \tan \beta) \) from the tree level processes \( B \rightarrow \tau \nu \) and \( B \rightarrow D \bar{\nu}_\tau \ell = e, \tau \) is shown in Fig. 5. More precise measurements of these mode, and of the complementary leptonic decay \( B^\pm \rightarrow \mu^- \nu_\mu \), will be possible at a SFF.

Loop mediated FCNCs such as \( B_s \rightarrow B_s \) mixing and \( b \rightarrow s \gamma \) decays can also constrain the parameters of
2HDM models. In $b \to s\gamma$ the charged Higgs boson contribution comes from penguin diagrams with top and $H^+$ running in the loop, which are known at NLO \cite{Borzumati:1998py, Ciuchini:1998xx, Grinstein:1989dp}. LO calculations were done by \cite{Ellis:1986yg, Hou:1988dp}. In type-I 2HDM the $W^+$ and $H^+$ contributions to the electromagnetic dipole Wilson coefficient $C_7^\gamma (\mu)$ can interfere with either sign, while in type-II 2HDM they always interfere constructively. The present WA of the branching fraction $B(B \to X_s\gamma) = (3.55 \pm 0.24 \pm 0.09 \pm 0.03) \times 10^{-4}$ implies the lower bound $M_{H^+} > 300$ GeV \cite{Misiak:2007kk}.

Type-III models have a richer flavor violating structure with FCNC transitions generally allowed at tree level. Here we will focus on type-III models where the Peccei-Quinn symmetry violating terms $g^D$ and $f^U$ in Eq. (19) are only a small perturbation. These models are close to a type-II 2HDM and correspond to the situation encountered in the MSSM. We further restrict ourselves to the conservative case of MFV. The matrices $g^D$ and $f^U$ are functions of large Yukawa matrices $Y^U \equiv g^U$ and $Y^D \equiv f^D$ in accordance with spurion analysis using flavor group Eq. (14). The most general Yukawa term involving down-type quarks in a type-III 2HDM with MFV is then \cite{D'Ambrosio:2002gu}

\[
\mathcal{L}_{YD} = Q_L [ H_D + \epsilon_0 \Delta + \epsilon_2 Y_U Y_D^\dagger + \epsilon_5 Y_U Y_D^\dagger ] \Delta \\
+ \epsilon_4 \Delta Y_U Y_D^\dagger H_U^c ] Y_D d_R + \text{h.c.} \\
\text{(23)}
\]

with $\epsilon_i$ some unknown coefficients, where we have used the mass eigenstate basis in which $Y_U$ and $Y_D$ have the form of Eq. (17). In particular $Y_D$ is diagonal, so that $Y_U Y_D^\dagger \propto \text{diag}(0,0,1) \equiv \Delta$. The additional couplings to $H^\pm$ in Eq. (23) introduce new flavor changing vertices both in the charged currents $W^\pm qq$ and charged Higgs vertices $H^\pm qq$. In addition, new FCNC couplings to the neutral Higgses $H^0, h^0, A^0$ are introduced. Integrating out the heavy Higgs fields gives new scalar operators mediating FCNC transitions. These can be especially important in the large $\tan \beta$ regime, where $\epsilon_i \tan \beta$ can be $O(1)$.

The large $\tan \beta$ limit of the MFV hypothesis has two important consequences for the low energy effective weak Hamiltonian of Eq. (15): (i) the basis of FCNC operators is larger than in the SM and includes scalar operators arising from tree level FCNC neutral Higgs exchanges, and (ii) the $\Delta$ insertions Eq. (23) decouple the third generation decays from the first two. The correlation between $B$ and $K$ meson observables present in the low $\tan \beta$ MFV scenario is discussed in subsection III.B is thus relaxed. For instance, the new contributions in Eq. (23) allow us to modify separately $\Delta M_{B_d}$ and $\epsilon_K$.

The effect of flavor violation in the large $\tan \beta$ limit is particularly dramatic for $b \to s\ell^+\ell^-$ transitions and $B_{(s)} \to \ell^+\ell^-$ decays. These are helicity suppressed in SM, but now receive tree level contributions from neutral Higgs exchange. An enhancement of $B \to \ell^+\ell^-$ by two orders of magnitude is then, in general, possible. Conversely, experimental data on these processes translate into constraints in the $(M_{H^+}/\tan \beta, \epsilon_i \tan \beta)$ plane \cite{D'Ambrosio:2002gu}. These in turn impose useful constraints on the underlying physics producing the couplings $\epsilon_i$. This program is especially powerful in the context of a specific model, for instance in the case of a supersymmetric theory like the MSSM discussed in the next section.

While $B \to \ell^+\ell^-$ has already been searched for at the Tevatron \cite{Aaltonen:2007mx, Abazov:2007ww} and will be searched for at LHCb \cite{Buchalla:2007eb}, a SFF has an important role in pinning down the large $\tan \beta$ scenario by (i) precisely measuring also non-helicity suppressed decays (e.g. $B \to (K, K^*)\ell^+\ell^-$ where $O(10\%)$ breakings of flavor universality would be expected \cite{Hiller:2004QP}), and (ii) by measuring $B \to X_s \tau^+\tau^-$ and $B \to \tau^+\tau^-$ \cite{Isidori:2001bm, Retico:2001ve}. In a completely general large $\tan \beta$ MFV analysis using EFT there are no correlations between $B \to \ell\ell$, $B \to \ell^+\ell^-$, $\Delta M_{B_S}$, and $B \to X_s\gamma$, but these do exist in a more specific theory, for instance in MFV MSSM with large $\tan \beta$ \cite{D'Ambrosio:2002gu, Isidori:2007fi, Isidori:2007gf, Lunghi:2007dq}. In this scenario one gets $\sim (10\% - 40\%)$ suppression of $B(B^+ \to \tau^+\nu)$, enhancement of $(g-2)_\mu$, SM-like Higgs boson with $m_{h^0} \sim 120$ GeV and small effects in $\Delta M_{B_s}$ and $B(B \to X_s\gamma)$ quite remarkably in agreement with the present tendencies in the data \cite{Isidori:2007fi, Isidori:2007gf, Isidori:2006kg}.
TABLE II Field content of the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions. Last column gives gauge representations in a (SU(3)_c, SU(2)_L, U(1)_Y) vector. In addition there are also fermionic superpartners of gauge bosons: gluino, wino and bino.

| Superfield notation | spin 0 | spin 1/2 | gauge repr. |
|---------------------|--------|----------|-------------|
| squarks, quarks     | Q      | (u_d, d_L) | (3, 2, \frac{1}{6}) |
|                     | U      | (\bar{u}_R, \bar{d}_R) | (\bar{T}, 1, -\frac{4}{3}) |
|                     | D      | (\bar{d}_L, \bar{d}_R) | (\bar{T}, 1, \frac{1}{6}) |
| sleptons, leptons   | L      | (\bar{L}, e_L) | (1, 2, -\frac{1}{2}) |
|                     | (\times 3 families) | \tilde{e}_R | (1, 1, 1) |
| Higgs, Higgsinos    | H_u    | (H_u^0, H_u^\pm) | (1, 2, +\frac{1}{2}) |
|                     | \tilde{H}_D | (\tilde{H}_D^0, \tilde{H}_D^\pm) | (1, 2, -\frac{1}{2}) |

D. Minimal Supersymmetric Standard Model

Low energy supersymmetry (SUSY) offers a possible solution to the hierarchy problem. In SUSY the quadratically divergent quantum corrections to the scalar masses (in SM to the Higgs boson mass) are cancelled by introducing superpartners with opposite spin-statistics for each of the particles. The simplest supersymmetrization of the Standard Model is the so-called Minimal Supersymmetric Standard Model (MSSM), to which we restrict most of the discussion in the following. (For more extended reviews see, e.g., Haber and Kane (1983); Martin (1997); Misiak et al. (1998); Nilles (1984)).

The matter content of MSSM is shown in Table II. The structure of SUSY demands two Higgs doublets \(H_u,D\) that appear together with their superpartners, Higgsinos \(\tilde{H}_u, \tilde{H}_D\). These mix with the fermionic partners of the W and Z, \(\gamma\) gauge bosons into the chargino \(\tilde{\chi}^{\pm}\) and the neutralinos \(\tilde{\chi}^0\). The superpartner of the gluon is the gluino, \(\tilde{g}\). In addition, there are also the scalar partners of the fermion fields with either chirality, the squarks \(\tilde{q}_R, \tilde{q}_L\), and the sleptons and sneutrinos \(\tilde{e}_R, \tilde{e}_L, \tilde{\nu}\).

The superpotential describing the Yukawa couplings of the two Higgs fields to the quark and lepton chiral superfields is

\[
W = Y_U^{ij} \mathcal{H}_U \mathcal{Q}_i \tilde{d}_j + Y_D^{ij} \mathcal{H}_D \mathcal{Q}_D \tilde{u}_j \\
+ Y_U^{ij} \mathcal{H}_D \mathcal{L}_i \tilde{e}_j + \mu \mathcal{H}_U \mathcal{H}_D.
\]

The Yukawa matrices \(Y_U, Y_D, Y_L\) act on the family indices \(i,j\). The last term is the so-called \(\mu\) term coupling the two Higgs fields. The above superpotential is the most general one that conserves \(R\)-parity under which SM particles are even, while the superpartners are odd. \(R\)-parity ensures \(B\) and \(L\) quantum numbers conservation at a renormalizable level. Comparing the superpotential of Eq. (24) with the 2HDM Yukawa interactions in Eq. (19), we see that at tree level this gives quark-Higgs couplings of a type-II 2HDM. Loop corrections induced by the \(\mu\) term, however, introduce also the Higgs-quark couplings of the “wrong-type”, effectively changing the interaction into a type-III 2HDM (see Fig. 7).

SUSY predicts fermion-boson mass degeneracy, which is not observed in Nature, so SUSY must be broken. The required breaking needs to be soft, i.e., only from super renormalizable terms, in order not to introduce back quadratic divergences and sensitivity to the high scale.

The general soft SUSY breaking Lagrangian in the squark sector of MSSM is then (for a review see, e.g., Chung et al. (2003))

\[
\mathcal{L}_{soft} = (M_Q^2)_{ij} (\bar{d}_L^i \tilde{u}_L^j + \bar{d}_L^i \tilde{d}_L^j) \\
+ (M_A^2)_{ij} (\bar{u}_R^i \tilde{u}_R^j + \bar{d}_R^i \tilde{d}_R^j) \\
+ (A_U)_{ij} \bar{Q}_i H_U \tilde{u}_R^j + (A_D)_{ij} \bar{Q}_i H_D \tilde{d}_R^j,
\]

with \(\bar{Q}_i = (\tilde{u}_L, \tilde{d}_L)\) and \(H_{U,D}\) Higgs doublets. The precise form of the soft squark masses \(M_{Q_i}, M_{\tilde{U}}, M_{\tilde{D}}\) and the trilinear terms \(A_U, A_D\) depends on the specific mechanism which breaks SUSY. In its most general form

the soft SUSY breaking introduces a large number of unknown parameters which can induce large observable FCNC effects.

A detailed counting gives that the flavor sector of the MSSM contains 69 real parameters and 41 phases (Dimopoulos and Suttorp, 1992; Haber, 1998), compared with nine quark and lepton masses, three real CKM angles and one phase in the SM. The generally large FCNCs from soft SUSY breaking is known as the SUSY flavor problem, and to solve it any realistic SUSY model must explain the observed FCNC suppression. We address this issue next.

1. Flavor violation in SUSY

In MSSM there are two main sources of flavor violation beyond the SM: i) if the squark and slepton mass matrices are neither flavor universal nor are they aligned with the quark or lepton mass matrices, and ii) the flavor violation that is induced by the wrong-Higgs couplings to quarks and leptons.

The first effect is most transparent in the super-CKM basis, in which the quark mass matrices are diagonal, while the squark fields are rotated by the same matrices that diagonalize the quark masses. The squark mass matrices, however, need not be diagonal in this basis

\[
M_Q^2 = \begin{pmatrix}
M_{Q_{LL}}^2 & M_{Q_{LR}}^2 \\
M_{Q_{LR}}^2 & M_{Q_{RR}}^2
\end{pmatrix}, \quad M_U^2 = \begin{pmatrix}
M_{U_{LL}}^2 & M_{U_{LR}}^2 \\
M_{U_{LR}}^2 & M_{U_{RR}}^2
\end{pmatrix}.
\]

Explicitly, the 3 \times 3 submatrices are

\[
M_{Q_{LL}}^2 = M_Q^2 + \frac{1}{6} M_Z^2 \cos 2\beta(3 - 4 \sin^2 \theta_W),
\]

\[
M_{Q_{LR}}^2 = M_U (A_U - \mu \cot \beta),
\]

\[
M_{Q_{RR}}^2 = M_Q^2 + \frac{2}{3} M_Z^2 \cos 2\beta \sin^2 \theta_W.
\]
and similarly for the down squarks. While the quark mass matrices $M_{U,D}$ are diagonal in the super-CKM basis, the soft breaking terms $M_{Q}^{2}, M_{U,D}^{2}$ and $A_{U,D}$ are not, in general. The flavor violation, that in the super-CKM basis resides in the squark sector, then translates into flavor violation in the quark processes through loop effects – in particular, squark-gluino loops since the $q\bar{q}g$ coupling is proportional to $g_s$.

In order to suppress FCNC transitions, the squark mass matrices $M_{Q}^{2}$ and $M_{U,D}^{2}$ must be either very close to the unit matrix (flavor universality), or proportional to the quark mass matrices (alignment). These properties can arise from the assumed SUSY breaking mechanism, for instance in gauge mediated SUSY breaking, if the hidden sector scale is below the flavor breaking scale (D’Ambrosio and Rattazzi, 1999), in anomaly mediated SUSY breaking (Randall and Sundrum, 1999) or from assumed universality in SUGRA (Brignole et al., 1994; Girardello and Grisaru, 1982; Kaplunovsky and Louis, 1993). Alternatively, alignment can follow from a symmetry, for instance from horizontal symmetries (Barbieri et al., 1996; Dine et al., 1993; Leurer et al., 1993; Nir and Seiberg, 1993).

The minimal source of flavor violation that is necessarily present is due to the Yukawa matrices $Y_{U}, Y_{D}$. The Minimal Flavor Violation assumption, discussed in section [11.13] means that these are also the only sources of flavor violation, a scenario that is natural in, for instance, models with gauge mediated SUSY breaking. The most general structure of soft squark mass terms allowed by MFV is (D’Ambrosio et al., 2002)

$$
\begin{align*}
M_{Q}^{2} &= \hat{M}^{2}(a_{1} + b_{1}Y_{U}Y_{U}^{\dagger} + \cdots), \\
M_{U}^{2} &= \hat{M}^{2}(a_{2} + b_{2}Y_{D}^{\dagger}Y_{U}), \\
M_{D}^{2} &= \hat{M}^{2}(a_{3} + b_{3}Y_{D}^{\dagger}Y_{D}), \\
A_{U} &= A(a_{4} + b_{4}Y_{D}^{\dagger}Y_{U}), \\
A_{D} &= A(a_{5} + b_{5}Y_{D}^{\dagger}Y_{D}),
\end{align*}
$$
(30)

with $\hat{M}^{2}$ a common mass scale, and $a_{i}, b_{i}$ undetermined parameters. These can be completely uncorrelated, but are fixed in more constrained scenarios, such as the constrained MSSM to be discussed below.

The second source of flavor violation in the MSSM is due to the wrong-Higgs couplings, e.g. the $H_U$ coupling to down quarks. These are introduced by loop corrections to the $Hq\bar{q}$ vertex. There are two such contributions in the MSSM: the gluino-$\tilde{g}$ graph, and the Higgsino-$\tilde{u}$ graph (see Figure 7). These induce a type-III 2HDM quark-Higgs interaction Lagrangian of the form given in Eq. (30). The loop induced effects are proportional to $\tan \beta$, and thus become important for large $\tan \beta$.

2. Constraints on the MSSM parameter space

The MSSM has 124 free parameters making a direct study of its parameter space intractable. Due to the complexity of the problem, it is convenient to divide the discussion into two parts. We start by first considering a flavor blind MSSM, keeping only the SM flavor violation in the quark sector, but neglecting any other sources of flavor violation. In the second step we include the two new flavor violating effects of the MSSM discussed above.

A particularly simple version of a flavor blind MSSM is the so-called constrained MSSM (cMSSM) (Kane et al., 1994). The soft SUSY breaking masses and trilinear terms are assumed to be universal at some high scale, for instance at the GUT scale $M_{GUT} \sim 10^{16}$ GeV

$$
(M_{Q}^{2})_{ij} = (M_{U}^{2})_{ij} = (M_{D}^{2})_{ij} = (M_{L}^{2})_{ij} = M_{0}^{2}\delta_{ij},
(31)
$$

The gaugino masses are also assumed to be universal at $M_{GUT}$ and equal to $M_{1/2}$. The cMSSM has only six unknown parameters that can be taken to be: the universal gaugino mass $M_{1/2}$, the squark and slepton soft breaking mass scale $M_{0}$, the trilinear coupling $|A_{0}|$, the ratio of Higgs vevs $\tan \beta$, and two phases $\phi_{\mu} = \arg(\mu)$ and $\phi_{A} = \arg(A)$. In minimal supergravity (mSUGRA), an additional constraint $B_{0}(\tan \beta) = A_{0} - M_{0}$ is imposed, but the terms cMSSM and mSUGRA are often used interchangeably in the literature. The masses and couplings at the electroweak scale are found by RG running in the MSSM. In particular this introduces a flavor structure of the form shown in Eq. (30).

We consider here only the cMSSM with conserved $R$–parity, for which the lightest neutralino (the lightest supersymmetric particle) is identified as the dark matter particle. The experimental constraints on cMSSM

![FIG. 7 Flavor changing coupling of the up Higgs-boson $H_u$ to the down type quarks (from Lunghi et al. 2008)](image-url)
parameters are then:

- The lower bound on light neutral Higgs boson mass, $M_{h_0} \geq 120$ GeV, rules out very low values of $\tan \beta$ and constrains a combination of $A_0$ and $M_0^2$ parameters.

- The anomalous magnetic moment of the muon $a_\mu = \frac{1}{2}(g-2)_\mu$ appears to differ from the SM prediction at about 3σ level, $(a_\mu^{\text{exp}} - a_\mu^{\text{SM}}) \approx (27.5 \pm 8.3) \times 10^{-10}$ (Bennett et al. 2006; Miller et al. 2007). The sign of the difference suggests that $\mu > 0$ is strongly favored.

- The radiative decays $b \to s\gamma$. The $H^{\pm}$-top and $W^{\pm}$-top penguin loops interfere constructively, while the chargino diagram has a relative sign given by $-\text{sgn}(A_t \mu)$ and can thus interfere either constructively or destructively. To preserve the good agreement with the SM prediction for $C_7$, the $H^{\pm}$ and chargino contributions must cancel to a good approximation, which requires $\mu > 0$. An alternative possibility would be a large destructive chargino contribution, finely tuned to give $C_7 = -(C_7)^{\text{SM}}$, but this possibility is ruled out by the measurement of $B(b \to s\ell^+\ell^-)$ (Gambino et al. 2005; Lunghi et al. 2006).

- Electroweak precision observables (Heinemeyer et al. 2006). The good agreement with the SM predictions constrains the mass splitting of the superpartners, especially in the third generation.

Recent detailed cMSSM analyses with special emphasis on B meson phenomenology were done in (Barenboim et al. 2007; Carena et al. 2006; Ciuchini et al. 2007a; Ellis et al. 2007b; Goto et al. 2007) [see also earlier works referenced therein] Here we mention a few implications of these studies that are valid in cMSSM.

The gluino dominance of the RG evolution leads to strong correlations between gaugino and squark masses at the weak scale. The lower bound on chargino mass from direct searches then translates to a lower bound of about 250 GeV on the mass of the lightest squark, the stop. The constraint from $b \to s\gamma$ implies heavy charged Higgs in most of the parameter space, $m_{h^\pm} \gtrsim 400$ GeV (Bartl et al. 2001). For large values of $\tan \beta$ smaller masses are possible, if the charged Higgs contribution to $b \to s\gamma$ is cancelled by the chargino contribution. This simultaneously requires large squark masses above TeV, while $B(B^- \to \tau^- \bar{\nu}_\tau)$ then puts a constraint $m_{h^+} \gtrsim 180$ GeV (Barenboim et al. 2007).

The cMSSM contains new sources of CP violation, the phases $\delta_1$ and $\phi_A$. These are constrained by the experimental upper bound on the electron electric dipole moment (EDM) $|d_e| \leq 4.0 \times 10^{-27}$ (Regan et al. 2002). In the MSSM one-loop chargino and neutralino contributions lead to a nonzero electron EDM. Although each of these two contributions restricts $\delta_1, \phi_A$ to be very small, cancellations can occur so that $\phi_A \lesssim 0.1$ and unrestricted $\phi_A$ are still allowed. In this case $A_{CP}(b \to s\gamma)$ can be of order a few percent (Bartl et al. 2001), while if $\phi_\mu$ is set to zero the resulting $A_{CP}(b \to s\gamma)$ is hard to distinguish from SM (Goto et al. 2007). Measurements of this asymmetry can thus give important information about the structure of CP violation beyond the SM.

| $ij/AB$ | $LL$ | $LR$ | $RL$ | $RR$ |
|--------|------|------|------|------|
| 12     | $1.4 \times 10^{-2}$ | $9.0 \times 10^{-5}$ | $9.0 \times 10^{-5}$ | $9.0 \times 10^{-2}$ |
| 13     | $9.0 \times 10^{-2}$ | $1.7 \times 10^{-2}$ | $1.7 \times 10^{-2}$ | $7.0 \times 10^{-2}$ |
| 23     | $1.6 \times 10^{-1}$ | $4.5 \times 10^{-3}$ | $6.0 \times 10^{-3}$ | $2.2 \times 10^{-1}$ |

### 3. Flavor violation in the generic $\tan \beta$ scenario

For moderate values of $\tan \beta \sim 5$–15, the only new flavor violating effects are from the off-diagonal terms in the squark mixing matrices (in the super-CKM basis). It is convenient to parameterize this matrix in a way which is simply related to FCNC data. Using data to bound the off-diagonal squark mixing matrix elements, one would then gain insight into the flavor structure of the soft breaking terms.

A convenient way to formulate such constraints makes use of the mass insertion approximation in terms of the $\delta_{ij}$ parameters (Gabbiani et al. 1996; Hall et al. 1986)

$$
(\delta_{AB}^{d})_{ij} = \left( \frac{M_D^{AB}}{M_q^2} \right)_{ij}, \quad A,B \in \{L,R\},
$$

where $M_D$ is an average squark mass. Often this is chosen to be the generation dependent quantity, $M_D^2 = M_{\tilde{d}A}, M_{\tilde{q}A}$, $A \sim ij$. Analogous parameters can be defined in the up squark sector.

The most recent constraints on $\delta_{AB}^{d}$ from Ciuchini et al. (2007b) are summarized in Table III. These bounds are derived in the mass insertion approximation, keeping only the dominant gluino diagrams. The best constrained parameters are the off-diagonal $\delta_{LL}^{d}$, which contribute to FCNC processes in the down quark sector.

The $(\delta_{AB}^{s})_{12}$ parameters (see Table III) are constrained by measurements in the kaon sector of $\Delta M_K, \varepsilon, \varepsilon'/\varepsilon$. Data on $B_d - \bar{B}_d$ mixing constraint $(\delta_{AB}^{s})_{12}$. Finally, in the 2–3 sector there are several constraints: from rare radiative decays $b \to s\gamma, b \to s\ell^+\ell^-$, and the recently measured $B_s - \bar{B}_s$ mixing. Constraints on the mass insertions in the up sector can be derived from recent $D - \bar{D}$ mixing data (Ciuchini et al. 2007a).
4. Large tan \( \beta \) regime

The loop induced couplings of \( H_u \) to down-type quarks render the Yukawa interactions equivalent to a type-III 2HDM, cf. Fig. 4 and Eq. (23). These new flavor violating effects are enhanced by tan \( \beta \). Assuming MFV the new interactions are restricted to the form in Eq. (23). The \( \epsilon_i \) coefficients are calculable from SUSY loop diagrams: \( \epsilon_0 \) contains the effect of the gluino diagram, while \( \epsilon_{1,2} \) are induced by the Higgsino diagrams of Fig. 4. The induced low energy EFT operators give enhanced contributions to several \( B \) physics processes. We discuss here \( B_s \to \ell^+\ell^- \), \( B_s \) mixing and \( b \to s\gamma \), which have a distinctive phenomenology in the large tan \( \beta \) scenario with MFV.

The \( B_s \to \ell^+\ell^- \) decay receives an enhanced contribution from tree level exchange of neutral Higgs bosons, which induce scalar operators of the form \( m_0(b_{RSL})(\ell\ell) \) and \( m_0(b_{RSL})(\ell\psi) \). The branching fraction of this mode scales as \( \mathcal{B}(B_s \to \ell^+\ell^-) \sim \tan^6 \beta/M_3^4 \), and can thus be easily enhanced by several orders of magnitude compared to the SM prediction (Babu and Kolja, 2000; Dedes and Pilaftsis, 2003; Ellis et al., 2001, 2002; Chankowski and Slawianowski, 2001).

Tree level exchange of neutral Higgs bosons induces also the double penguin operators \( (b_{RSL})(b_{LSR}) \), which contribute to \( B_s - B_d \) mixing. The contributions are enhanced by a factor of \( \tan^4 \beta \) and decrease the \( \Delta M_{B_s} \) mass difference compared with the SM (Buras et al., 2001a).

The radiative decay \( b \to s\gamma \) receives contributions from neutral Higgs loops in the large tan \( \beta \) limit. An important effect is the presence of corrections of order \( (\alpha_s \tan \beta)^n \), which can be resummed to all orders (Carena et al., 2001; Dedes and Pilaftsis, 2003; Ellis et al., 2007a). The effect of the resummation can be appreciable for sufficiently large values of tan \( \beta \).

The correlation of these observables can be studied in the \( (M_{H^+}, \tan \beta) \) plane, as shown in Fig. 8 for fixed values of \( A_U, \mu \). The tree mediated decay \( B_u \to \tau\nu \) is included in these constraints. In the MSSM this is given by the same expression as in the 2HDM, up to a gluino correction which becomes important in the large tan \( \beta \) limit.

E. Models of Warped Extra Dimensions

One of the most interesting models of New Physics is based on the idea of a warped extra dimension (Randall and Sundrum, 1999a). This notion has great appeal as it can lead to a simultaneous resolution to the hierarchy problem as well as the flavor problem of the SM by accomodating rather naturally the observed large disparity of fermion masses (Davoudiasl et al., 2007; Gherghetta and Pomarol, 2000; Grossman and Neubert, 2000). For lack of space we do not discuss the implications of universal extra dimensions, for which we refer the reader to the recent review by Hooper and Profumo.
large top Yukawa coupling.

This configuration suppresses FCNCs substantially (however, see below) and reproduces the fermion mass hierarchies without invoking large disparities in the Yukawa couplings of the fundamental 5D action \cite{Gherghetta:2000qt,Grossman:2000dp}. It thus has a built in analog of the SM GIM mechanism (the RS GIM) and reproduces the approximate flavor symmetry among the light fermions.

Similarly to the SM GIM, the RS GIM is violated by the large top quark mass. In particular, \((t,b)_{L}\) needs to be localized near the TeV brane otherwise the 5D Yukawa coupling becomes too large and makes the theory strongly coupled at the scale of the first KK excitation. This has two consequences: (1) in the interaction basis, the coupling of \(b_{L}\) to gauge KK modes (say the gluons), \(g_{b_{L}KK}^{g}\), is large compared to the couplings of the lighter quarks. This is a source of flavor violation leading to FCNCs. (2) The Higgs vev mixes the zero mode of \(Z\) and its KK modes, leading to a non-universal shift \(\delta g_{Z}^{b} \approx g_{Z_{KK}}^{b} \sqrt{\log(M_{A}/\text{TeV}) m_{Z}^{2}/m_{Z_{KK}}^{2}}\), in the coupling of \(b_{L}\) to the physical \(Z\). \cite{Agashe:2003zs, Burdman:2004gg}. Here \(g_{Z_{KK}}^{b}\) is the coupling between \(b_{L}\) and a KK \(Z\) state before EWSB. The factor \(\sqrt{\log(M_{A}/\text{TeV})}\) comes from enhanced Higgs coupling to gauge KK modes, which are also localized near the TeV brane. Electroweak precision measurements of \(Z \to b_{L}b_{L}\) require that this shift is smaller than \(~1\%\). Using \(g_{Z_{KK}}^{b} \approx g_{Z}^{b}\) this is satisfied for \(m_{KK} \sim 3\text{ TeV}\).

In passing we also note that with enhanced bulk electroweak gauge symmetry, \(SU(2)_{L} \times SU(2)_{R} \times U(1)_{B-L}\), and KK masses of \(\sim 3\) TeV, consistency with constraints from electroweak precision measurements are achieved \cite{Agashe:2003zs}.

The tension between obtaining a large top Yukawa coupling and not introducing too large flavor violation and disagreement with EWP data \cite{Agashe:2003zs, Burdman:2004gg} is solved in all models by assuming (1) a close to maximal 5D Yukawa coupling, \(\lambda_{t_{3}} \sim 4\), so that the weakly coupled effective theory contains 3-4 KK modes, and (2) by localizing (\(t,b\)) as close to the TeV brane as allowed by \(\delta g_{Z}^{b} \sim 1\%\). This almost unavoidable setup leads to sizeable NP contributions in the following three types of FCNC processes that are top quark dominated: (i) \(\Delta F = 2\) transitions, (ii) \(\Delta F = 1\) decays governed by box and EW penguin diagrams; (iii) radiative decays.

Sizeable modifications of \(\Delta F = 2\) processes are possible from tree-level KK gluon exchanges. The \(\Delta F = 1\) processes receive contributions from tree level exchange of KK \(Z\) modes. These tend to give smaller effects than KK gluon exchanges. Nevertheless it can lead to appreciable effects in the branching ratio, direct CP asymmetry and the spectrum of \(b \to s\ell^{+}\ell^{-}\). \cite{Agashe:2004rs, Burdman:2004gg}. In \(b \to s\ell^{+}\ell^{-}\) a QCD penguin dominated \(B \to (\phi, \eta, \pi^{0}, \omega, \rho^{0})_{K}\) decays on the other hand the RS contributions from flavor-violating \(Z\) vertex are at least \(\sim g_{Z}^{2}/g_{\phi}^{2} \sim 20\%\) suppressed and thus subleading \cite{Agashe:2004rs}. Consequently, RS models can accommodate only mild deviations from the SM in the corresponding time dependent CP asymmetries.

We should emphasise that these models are not fully developed yet so there can be appreciable uncertainties in the specific predictions. For instance, the particular framework outlined above runs into at least two problems unless the relevant KK-masses are much larger than 3 TeV: (i) the presence of right-handed couplings can cause enhanced contributions to \(\Delta S = 2\) processes, \(K\)–\(\bar{K}\) mixing and \(e_{K}\) \cite{Beall:1982xq, Bona:2007ek}, and (ii) the simple framework with \(O(1)\) complex phases tends to give an electron electric dipole moment about a factor \(~20\) above the experimental bound \cite{Agashe:2004rs, Agashe:2005vq}. An interesting proposal for the flavor dynamics in the RS setup was recently put forward by \cite{Fitzpatrick:2006ix} who introduced 5D anarchic minimal flavor violation in the quark sector (see also \cite{Cacciapaglia:2007dt}). This gives a low energy effective theory that falls in the NMFV class, consistent with both FCNC and dipole moment constraints (see section III.B). In this picture new flavor and CP violation phases are present, however, their dominant effect occurs only in the up type quark sector.

### F. Light Higgs searches

Existing LEP constraints on the Higgs mass do not rule out the existence of a very light Higgs boson \(h\) with a mass well below the present limit of 114.4 GeV, if the SM is extended either in the gauge or Higgs sector \cite{Dermisek:2004uq, Fullana:2007ec}. Such states for instance appear naturally in extensions of the MSSM motivated by the \(\mu\)-problem. The most popular models are nonminimal supersymmetric models, where one or more gauge singlets are added to the two Higgs doublets of the MSSM \cite{Barger:2005eg, Dermisek:2007uq, Han:2004az}. The simplest case of one gauge singlet is the next-to-minimal supersymmetric standard model (NMSSM), which contains seven physical Higgs bosons, two of which are neutral pseudoscalars.

A light Higgs boson would be difficult to observe at the LHC because of significant backgrounds, and a SFF could play a complementary role in this respect. The main detection mode is \(\Upsilon \to h(\to \ell^{+}\ell^{-})\gamma\) \cite{Wijock:1977}. The presence of a light Higgs may manifest itself as an enhancement of the \(\Upsilon(1S) \to \tau^{+}\tau^{-}\) channel relative to other dilepton modes (\(e,\mu\)). In NMSSM at large \(\tan \beta\), the \(b \to sh\) vertex with \(h\) a light Higgs produces observable effects in rare \(B, K\) decays. It can be search for in \(\Upsilon\) or \(B \to K\) decays with missing energy. The presence of new pseudoscalar in NMSSM also breaks the correlation between \(B_{s} \to \mu^{+}\mu^{-}\) decay and \(B_{s} \to B_{s}\) mixing that is present in MSSM \cite{Hiller:2004}.
In passing, we mention a related topic. Invisible decays of quarkonia can be used to search for light dark matter candidates (Gunion et al. 2006; McElrath 2005). An initial analysis of this type has been carried out at Belle (Tajima et al. 2007), illustrating the potential for this physics at a SFF.

G. Flavor signals and correlations

How well can one distinguish various NP models from flavor data? This can be achieved by studying correlations among different flavor violating observables. As mentioned in previous subsections such correlations appear in models of flavor violation motivated by simple symmetry arguments, e.g. in MFV scenarios. An example of how flavor observables can distinguish among a restricted set of models is given in Goto et al. (2002, 2004, 2007). The authors considered four classes of SUSY models, which are typical solutions of the SUSY flavor problem (restricted to the low tan β regime): (i) cMSSM, (ii) cMSSM with right-handed neutrinos, (iii) SU(5) SUSY GUT with right-handed neutrinos, and (iv) MSSM with U(2) flavor symmetry. The right-handed neutrinos were taken to be degenerate or nondegenerate, the latter with two specific neutrino matrix ansätze. Constraints from direct searches, $b \to s\gamma$, $B_d(\rho,\eta)$–$B_d$ and $K$–$\bar{K}$ mixing, and upper bounds on $l_i \to l_j\gamma$ and on EDMs were imposed on the models. Table IV lists typical deviations from SM for each of the models that are then still allowed.

In addition to the patterns in Table IV certain correlations are expected between subsets of observables. For example, $\Delta M_{B_s}/\Delta M_{B_d}$ and $\gamma$ are correlated in all considered models, but to constrain the NP parameters this requires improved lattice QCD determination of the $\xi$ parameter at a few percent level. In Table IV we do not list results for cMSSM with right-handed neutrinos, where the only observable deviations are expected in $\mu \to e\gamma$ for degenerate and in $\tau \to \mu\gamma, e\gamma$ for nondegenerate right-handed neutrinos.

IV. DIRECT MEASUREMENTS OF UNITARITY TRIANGLE ANGLES

We now discuss methods for direct determination of the angles in the standard CKM unitarity triangle. They test the CKM unitarity requirement for the first and the third column of the CKM matrix (see Fig. 9). We focus on methods that use little or no theoretical assumptions: the determinations of (i) $\beta$ from $B^0 \to J/\psi K_{S,L}$ and $B^0 \to D\bar{h}^0$, (ii) $\gamma$ from $B \to DK$ and $2\beta + \gamma$ from $B \to D^{(*)}\pi/\rho$, $D^{0(*)}K^{0(*)}$ and (iii) $\alpha$ from $B \to \pi\pi$, $\pi\rho, \rho\rho$. These decays are tree dominated so new physics effects are expected to be small. Together with measurements of the sides discussed in Section IX a determination of the “standard model CKM unitarity triangle” is possible either using tree-level processes alone, or by also including $\Delta F = 2$ (mixing) processes (Bona et al. 2006, Buras et al. 2001, Charles et al. 2005). This should be compared with the determinations using methods sensitive to new physics discussed in the later sections.

Let us set up the notation. Assuming CPT invariance the time dependent decay of an initially tagged $B^0$ is given by

$$
\Gamma(B^0(t) \to f) \propto e^{-\Gamma t} \left[ \cos \left( \frac{\Delta \Gamma t}{2} \right) + H_f \sinh \left( \frac{\Delta \Gamma t}{2} \right) - A_f^{\cos} \cos \Delta mt - S_f \sin \Delta mt \right],
$$

(34)

where $\Gamma$ is the average neutral $B$ meson decay width, while $\Delta \Gamma = \Gamma_H - \Gamma_L$ is the difference of decay widths between heavier and lighter $B^0$ mass eigenstates, so that the mass difference $\Delta m = m_H - m_L > 0$. In this section we focus on $B_d^0$ mesons, but Eq. (34) applies also to the $B^0_s$ system discussed in Section IX. Using shorthand notation $A_f = A(B^0 \to f), S_f = A(B^0 \to f)$, the
dependent CP asymmetry is thus

\[ A^\text{CP} = \frac{|A_f|^2 - |A_{\bar{f}}|^2}{|A_f|^2 + |A_{\bar{f}}|^2}, \]

and is equal to direct CP asymmetry in the case of a CP eigenstate \( f \) (in the literature \( C_f = -A^\text{CP} \) is also used). The coefficient of \( \sin \Delta m t \) describes CP violation in interference between mixing and decay and is

\[ S_f = \frac{2 A^\text{CP} (B(t) \to f)}{1 + |\lambda_f|^2}, \]

where parameters \( q_B, p_B \) describe the flavor composition of the \( B^0 \) mass eigenstates. In Eq. (35), we neglected CP violation in mixing taking \( |\lambda_f|^2 \) to be the case. The time dependent CP asymmetry is thus

\[ a^\text{CP} = \frac{\Gamma(B(t) \to f) - \Gamma(\bar{B}(t) \to \bar{f})}{\Gamma(B(t) \to f) + \Gamma(\bar{B}(t) \to \bar{f})} = A^\text{CP} \cos(\Delta m t) + S_f \sin(\Delta m t). \]

In the \( B_d^0 \) system the observable \( H_f \) is negligible since \( (\Delta \Gamma/\Gamma)_{B_d^0} \ll 1 \). For the \( B_s^0 \) system, on the other hand, a much larger decay width difference is predicted within the Standard Model \( (\Delta \Gamma/\Gamma)_{B_s^0} \approx -0.147 \pm 0.060 \) [Lenz and Nierste, 2007]. Experimentally, the current world average from an angular analysis of \( B^0 \to J/\psi \phi \) decays is \( (\Delta \Gamma/\Gamma)_{B_s^0} = -0.206^{+0.111}_{-0.106} \) [Barberio et al., 2005; Acosta et al., 2003; Aubert et al., 2007] (a more precise value of \(-0.104^{+0.084}_{-0.076}\) [Barberio et al., 2007] is obtained by including the \( B_s^0 \) lifetime measurements from flavor specific decays). Thus, in the \( B_s^0 \) system both \( S_f \) and \( H_f \) are experimentally accessible [Dunietz, 1993]. While sensitivity to the \( S_f \) term requires the ability to resolve the fast \( B_s^0 \) oscillations, for which the large boost of a hadronic machine is preferable, the \( H_f \) term is measured from the coefficient of the \( \sin(\Delta \Gamma/2) \) dependence, which can be achieved at a SFF operating at the T(5S).

### A. Measuring \( \beta \)

The measurement of \( \beta \) is the primary benchmark of the current \( B \)-factories. The present experimental world average from decays into charmonia-kaon final states, \( \sin 2\beta = 0.680 \pm 0.025 \) [Anbert et al., 2007; Barberio et al., 2007; Chen et al., 2007a], disagrees slightly with an indirect extraction that is obtained using all other constraints on the unitarity triangle. CKMFitter group for instance obtains \( \sin 2\beta = 0.799^{+0.043}_{-0.094} \) [Charles et al., 2005], while a similar small inconsistency is found in [Bona et al., 2006a; 2007b; Lunghi and Soni, 2007]. Improved accuracy in experiment and in theory are needed to settle this important issue. The theoretical error in the direct determination is negligible as discussed below. The theoretical error in the indirect determination, on the other hand, is a combination of theoretical errors in all of the constraints used in the fit, and comes appreciably from the lattice inputs.

That the extraction of the weak phase \( \beta \) from \( B \to J/\psi K_S \) is theoretically very clean was realized long ago [Bigi and Sanda, 1984; Carter and Sanda, 1982; Hagelin, 1981]. The decay is dominated by a \( b \to c\bar{c}s \) tree level transition. The complex parameter describing the mixing induced CP violation in \( B \to J/\psi K_S \) is

\[ \lambda_{J/\psi K_S} = -\left( \frac{q}{p} \right)_{B_s^0} A(\bar{B}^0 \to J/\psi \bar{K}^0) A(B^0 \to J/\psi K^0) \]

\[ \approx -\left( \frac{q}{p} \right)_{B_s^0} \left( \frac{p}{q} \right)_{K^0} V_{cb} V_{cs}^\ast, \]

The \( (p/q)_{K^0} \) factor is due to \( K - \bar{K} \) mixing, cf. Eq. (35). In going to the second line we have used CP symmetry to relate the two matrix elements, keeping only the tree-level operator \( V_{cb} V_{cs}^\ast (c\bar{b}) (q\bar{\nu}) (s\nu) + h.c. \) in the effective weak Hamiltonian (the relative minus sign arises since the \( J/\psi K \) final state has \( L = 1 \)). The remaining pieces are highly suppressed in the SM. In the standard phase convention for the CKM matrix \( \lambda_{B^\ast} \), \( V_{cb} V_{cs}^\ast \) is real, while \( (q/p)_{B_s^0} = -e^{-2i\beta} \) and \( (q/p)_{K^0} = -1 \) up to small corrections to be discussed below, so that \( S_{J/\psi K_S} = \sin 2\beta, A^\text{CP}_{J/\psi K_S} = 0 \). The time dependent CP asymmetry of Eq. (37) is then

\[ a^\text{CP} = \frac{2 A^\text{CP} (B(t) \to J/\psi K_S)}{1 + |\lambda_f|^2} = \sin(2\beta) \sin(\Delta m t). \]
with a vanishingly small $\cos(\Delta m t)$ coefficient. Corrections to this simple relation arise from subleading corrections to the $B^0_d - \bar{B}^0_d$ mixing, the $K^0 - \bar{K}^0$ mixing and the $B \to J/\psi K^\pm$ decay amplitude that have been neglected in the derivation of Eq. \((40)\). Including these corrections

$$a_{CP}(B(t) \to J/\psi K_S) = \left[ \sin(2\beta) + \Delta S_{B_{K_S}}^{\text{mix}} \right]$$

$$+ \Delta S_{K_{S}}^{\text{mix}} + \Delta S_{\text{decay}}^{\text{mix}} + \frac{\Delta \Gamma_B t}{4} \sin 4\beta \sin \Delta m t \quad (41)$$

$$+ \left[ \Delta A_{B_{K_S}}^{\text{mix}} + \Delta A_{K_{S}}^{\text{mix}} + \Delta A_{\text{decay}}^{\text{mix}} \right] \cos \Delta m t.$$  

Here \(\text{Boos et al.} 2004\)

$$\Delta S_{B_{K_S}}^{\text{mix}} = - \Im \frac{\Delta M_{12}}{|M_{12}|} = (2.08 \pm 1.23) \cdot 10^{-4}, \quad (42)$$

is the correction due to $u$ and $c$ quarks in the box diagram which mixes neutral $B$ mesons. These contributions have a different weak phase than the leading $t$ quark box diagram and thus modify the relation $\arg(q/p)_B = 2\beta$.

The correction \(\text{Grossman et al.} 2002\)

$$\Delta S_{K_{S}}^{\text{mix}} = -2 \cos(2\beta) \Im (\epsilon_K) \simeq -2.3 \cdot 10^{-3}, \quad (43)$$

arises from the deviation of $(q/p)_{K^0}$ from $-1$, and from the fact that the experimental identification through $K_S \to \pi\pi$ decay includes a small admixture of $K_L$.

The correction due to the penguin contributions in the $B \to J/\psi K$ decay is \(\text{Grossman et al.} 2002\)

$$\Delta S_{\text{decay}} = -2 \cos(2\beta) \Im \frac{\lambda_{u(\bar{u})(s)}}{\lambda_{c(\bar{c})(s)}} r \cos \delta_r, \quad (44)$$

where $\lambda_q^{(s)} = V_{qs} V_{s*}^*$, $r$ is the ratio of penguin to tree amplitudes and $\delta_r$ the strong phase difference. Because of the strong CKM suppression \(|\lambda_u^{(s)}/\lambda_c^{(s)}| \sim 1/50\) these effects are small, of the order of the other two $\Delta S$ uncertainties. The calculation of $\Delta S_{\text{decay}}$ is highly uncertain. The factorization theorems for two-body decays into two light mesons are not applicable due to the large $J/\psi$ mass. Even so, calculations have been attempted. Using a combination of QCD factorization and pQCD $\text{Li and Mishima} 2007$ obtain $\Delta S_{\text{decay}} = (7.2^{+2.4}_{-3.4}) \cdot 10^{-4}$. 

\text{Boos et al.} 2004 find $\Delta S_{\text{decay}} = -(4.24 \pm 1.94) \cdot 10^{-4}$ using a combination of the BSM mechanism \(\text{Bondar et al.} 1980\) and naive factorization and keeping only the $u\bar{u}$ loop contribution. An alternative approach uses SU(3) flavor symmetry to relate the $B^0 \to J/\psi K^0$ amplitude to the $B^0 \to J/\psi \pi^0$ amplitude, neglecting annihilation-like contributions \(\text{Ciuchini et al.} 2005\). In $B^0 \to J/\psi \pi^0$ decay the penguin contributions are CKM-enhanced, increasing the sensitivity to $r$ and $\delta_r$. Using the experimental information available in 2005 \(\text{Ciuchini et al.} 2005\) obtained $\Delta S_{\text{decay}} = 0.000 \pm 0.017$. The error is dominated by the experimental errors and is not indicative of the intrinsic $\Delta S_{\text{decay}}$ size.

In summary, $\Delta S_{J/\psi K_S}$ is expected to be $\Delta S_{J/\psi K_S} \simeq -1.4 \cdot 10^{-3}$. This is also the typical size of the term due to a nonzero decay width difference, $\sin 4\beta (\Delta \Gamma_B t f)/4 \simeq -1 \cdot 10^{-3}$ \(\text{Boos et al.} 2004\). Thus, any discrepancy significantly above permil level between $S_{J/\psi K_S}$ measurement and $\sin 2\beta$ obtained from the CKM fits would be a clear signal of new physics \(\text{Hou et al.} 2006\). The theoretical uncertainty in the measurement of $\sin 2\beta$ from $S_{J/\psi K_S}$ is likely to remain smaller than the experimental error even at a SFF. Extrapolations of the current analyses suggest that imperfect knowledge of the vertex detector alignment and beam spot position will provide a limiting systematic uncertainty, with the ultimate sensitivity of 0.5–1.2% \(\text{Akerovd et al.} 2004; \text{Bona et al.} 2007d\).

Digressing briefly from the determination of the unitarity triangle, the situation for the direct CP asymmetries in $B \to J/\psi K$ is rather similar \(\text{Boos et al.} 2004; \text{Grossman et al.} 2002; \text{Li and Mishima} 2007\)

$$-\Delta A_{B_{K_S}}^{\text{mix}} = \Im \frac{\Gamma_{12}}{2M_{12}} = -(2.59 \pm 1.48) \cdot 10^{-4}, \quad (45)$$

$$-\Delta A_{K_{S}}^{\text{mix}} = 2Re (\epsilon_K) \simeq 3.2 \cdot 10^{-3}, \quad (46)$$

$$-\Delta A_{\text{decay}} = 2\Im \frac{\lambda_{u(\bar{u})(s)}}{\lambda_{c(\bar{c})(s)}} r \sin \delta_r = (16.7^{+6.6}_{-8.7}) \cdot 10^{-4}, \quad (47)$$

giving a combined CP asymmetry $A_{J/\psi K_S} \simeq -4.6 \cdot 10^{-3}$.

This is nearly an order of magnitude smaller than the current experimental uncertainty on this quantity \(\text{Aubert et al.} 2007f; \text{Chen et al.} 2007b\), and comparable to the likely size of the limiting systematic uncertainty at a SFF \(\text{Akerovd et al.} 2004; \text{Bona et al.} 2007d\). New physics contributions to this quantity could enhance the CP asymmetry to the 1% level or even higher, while obeying all other constraints from flavor physics \(\text{Bergmann and Perez} 2001; \text{Hou et al.} 2006\). A complementary measurement of $\beta$ is provided by a time dependent $B^0 \to [K_S \pi^+ \pi^-]_{J/\psi} h^0$ Dalitz plot analysis \(\text{Bonder et al.} 2002\). Here $h^0 = \pi^0, \eta, \omega, \ldots$ while also $D^{*0}$ can be used in place of $D^0$. This channel provides measurements of both $\sin 2\beta$ and $\cos 2\beta$ resolving the $\beta = \pi/2 - \beta$ discrete ambiguity. The resulting measurement of $\beta$ is theoretically extremely clean since it does not suffer from penguin pollution. The only theoretical uncertainty is due to the $D^0$ decay model, which at present gives an error of $\sim 0.2$ on $\cos 2\beta$ \(\text{Aubert et al.} 2007b\; \text{Krokovny et al.} 2008\), and can be reduced in future using the same methods as for the $B \to D K$ analysis (see the discussion in Section IV.B). $D$ decays to CP eigenstates can also be used. However, these are only sensitive to $\sin 2\beta$ \(\text{Fleischel} 2003\).

\section*{B. Measuring $\gamma$}

\subsection*{1. $\gamma$ from $B \to DK$}

The most powerful method to measure $\gamma$ uses the interference between $b \to c \bar{u}s$ and $b \to u \bar{c}s$ amplitudes in $B \to DK$ decays \(\text{Gronau and London} 1991; \text{Gronau and Wylde} 1991\) for a recent review see, e.g.,
In the case of charged \( B \) decays the interference is between \( B^- \to DK^- \) amplitude, \( A_B \), followed by \( D \to f \) decay, and \( B^- \to \bar{D}K^- \) amplitude, \( A_{\bar{B}}r \approx e^{i(\delta_B - \gamma)} \), followed by \( \bar{D} \to f \) decay, where \( f \) is any common final state of \( D \) and \( \bar{D} \). The \( B^+ \to D(\bar{D})K^+ \) decay amplitudes are obtained by \( \gamma = \gamma \) sign-flip. Neglecting CP violation in the \( D \) decays we further have

\[
A(D^0 \to f) = A(\bar{D}^0 \to f) = A_f, \\
A(D^0 \to f) = A(D^0 \to \bar{f}) = A_f r \bar{r} e^{i\delta_f}.
\]

The parameters \( \delta_B \) and \( \delta_f \) above are strong phase differences in \( B \) and \( D \) decays respectively, while \( A_B, r_B, A_f, r_f \) are real. The sensitivity to \( \gamma \) is strongly dependent on the ratio \( r_B \sim 0.1 \). Since there are no penguin contributions in this class of modes, there is almost no theoretical uncertainty in the resulting measurements of \( \gamma \); all hadronic unknowns can in principle be obtained from experiment.

Various choices for the final state \( f \) are possible: (i) CP eigenstates \( \text{(e.g. } K_S \pi^0) \) \cite{Gronau1997, Gronau2001}, (ii) quasi-flavor specific states \( \text{(e.g. } K^+\pi^-) \) \cite{Atwood1997, Atwood2001}, (iii) singly Cabibbo suppressed decays \( \text{(e.g. } K^{*+}K^-) \) \cite{Grossman2003} or (iv) many-body final states \( \text{(e.g. } K_S\pi^+\pi^-) \) \cite{Atwood2001, Giri2003, Poluektov2004}

There are also other extensions, using many body \( B \) decays \( \text{(e.g. } B^+ \to DK^+\pi^0) \) \cite{Aleksan2003, Gronau2003}, using \( D^0 \) in both \( D^{+0} \to D\pi^0 \) and \( D^{0} \to D\gamma \) decay modes \cite{Bondar2004}, using self tagging \( D^* \) decays \cite{Sinha2004}. Neutral \( B \) decays (both time dependent and time integrated) can also be used \cite{Atwood2003, Poluektov2004, Kayser2000}.

For different \( D \) decays in \( B \to (f)_{i}K \), the parameters \( A_B, r_B, \delta_B, \gamma \) related to the \( B \) decay are common, so that there is significant gain in combining results from different \( D \) decay channels \cite{Atwood2003}. It is therefore not surprising that three body \( D \) decays, \( B^\pm \to [K_S\pi^+\pi^-]_{\pm}K^\pm \), provide the most sensitivity in the extraction of \( \gamma \) as they represent an essentially continuous set of final states \( f \). Also, for \( D \to f \) multibody decays both the magnitude of \( A_f \) and the strong phase variation over the Dalitz plot can be determined using a decay model where \( A_f \) is described by a sum of resonant (typically Breit-Wigner) terms \cite{Giri2003, Poluektov2004}. The decay model can be determined from flavor tagged data for details, see \cite{Aubert2006a, Cavoto2007}.

Flavor tagged \( \bar{D} \) decays do not provide direct information on the strong phase differences between \( D^0 \) and \( \bar{D}^0 \) amplitudes. In multibody decays the information comes from the interferences of the resonances, where the phase variation across the Dalitz plot is completely described by the chosen decay model. The question is then what is the modelling error introduced through this approach and how can it be reliably estimated? At present the modelling error on \( \gamma \) is estimated to be \( \sim 10^2 \), which is

obtained through an apparently conservative approach of including or excluding various contributions to the model. In future it will be possible to reduce this error by using entangled \( \psi(3770) \to DD \) decays at a tau-charm factory to arrive at a direct information on the strong phases \cite{Atwood2003, Giri2003}.

Alternatively, the modelling error can be avoided entirely by using a model independent approach \cite{Atwood2001, Giri2003}. After partitioning the \( D \to K_S\pi^+\pi^- \) Dalitz plot into bins, variables \( c_i, s_i \) are introduced that are the cosine and sine of the strong phase difference averaged over the \( i \)-th bin. Optimally, these are determined from charm factory running at \( \psi(3770) \) \cite{Atwood2003, Giri2003, Gronau2001, Soffer1998}. Recent studies \cite{Bondar2006, Bondar2008} show that if measurements of \( c_i \) from CP-tagged \( D \) decays are included in the analysis, the resulting error on \( \gamma \) using rectangular Dalitz plot binning is only 30\% worse than the unbinned model dependent approach \cite{Bondar2006}, or even only 4\% worse for optimal binning \cite{Bondar2008}. Studies of charm factory events in which both \( D \) mesons decay to multibody final states such as \( K_S\pi^+\pi^- \) can also provide information on the \( s_i \) terms \cite{Bondar2008}. As shown in Fig. 10, approximately \( 10^4 \) CP tagged \( D \) decays are required to keep the contribution to the uncertainty on \( \gamma \) below the \( 2\% \) statistical accuracy expected from a SFF.

To reduce the statistical uncertainty, one can also include additional \( B \) decay modes. For each, the hadronic factors \( A_B, r_B \) and \( \delta_B \) can be different, so additional unknown parameters are introduced. To date, \( B^\pm \to DK^\pm, B^\pm \to \bar{D}K^*\pm \) and \( B^\pm \to D^*K^\pm \) (with \( D^* \to D\pi^0(\gamma) \) \cite{Bondar2004} have been used.

Another useful approach is to include neutral \( B^0 \) decays. These have smaller decay rates, however the statistical error on \( \gamma \) does not scale with the rate but roughly...
as the smaller of the two interfering amplitudes. Using isospin one sees that these differ only by a factor of \(\sqrt{2}\) (Gronau et al., 2004b)

\[ A_{BR} \approx \sqrt{2} A_{BR}^n, \]  

(49)

Here we have introduced \(A_{RB}^n\) and \(r_{BR}^n\) parameters in the same way as for the charged decays above Eq. (45). Although time dependent measurements are needed to extract the full information in the \(B^0 \rightarrow DK_S\) system (Atwood and Soni, 2003a; Fleischer, 2003; Gronau et al., 2004; London and Atwood, 2001), untagged time integrated rates alone provide sufficient information to determine \(\gamma\) (Gronau et al., 2004b; 2007), while \(B^0 \rightarrow DK_{s0}\) decays are self-tagging. Therefore, we expect these modes to make a significant contribution to the measurement of \(\gamma\) at a SFF (Akeroyd et al., 2004; Bona et al., 2007).

We now discuss the theoretical errors. The determination of \(\gamma\) from \(B \rightarrow DK\) decays is theoretically extremely clean since these are pure tree decays. The largest uncertainty is due to \(D^0 - \bar{D}^0\) mixing, assumed to be absent so far. The SM \(D^0 - \bar{D}^0\) mixing parameters are \(x_D \equiv \frac{\Delta m_{D}}{\Delta m_{B}} \sim y_D \equiv \frac{\Delta f}{\Gamma} \sim O(10^{-2})\), with a negligible CP violating phase, \(\delta_D \sim O(10^{-4})\) (see Section X).

The effect of CP conserving \(D^0 - \bar{D}^0\) mixing is to change the effective relative strong phase (irrelevant for \(\gamma\) extraction) and to dilute the interference term, resulting in a shift \(\Delta \gamma \propto (x_D^2 + y_D^2)/r_D^2\) (Grossman et al., 2003). Thus the shift is larger for the cases where \(r_D^2\) is smaller, but even for doubly Cabibbo suppressed decays \(\Delta \gamma \lesssim 1^\circ\). Furthermore, this bias can be removed by explicitly including \(D^0 - \bar{D}^0\) mixing into the analysis once \(x_D\) and \(y_D\) are well measured (Atwood and Soni, 2003; Silva and Soffer, 2000). Moreover in the model independent Dalitz plot analysis no changes are needed, since there the method already includes the averaging (dilution) of the interference terms.

The remaining possible sources of theoretical error are from higher order electroweak corrections or from CP violation in the \(D\) system. The latter would lead to \(\Delta \gamma \sim O(x_D\delta_D, y_D\delta_D)\). In the SM the error is conservatively \(\Delta \gamma < 10^{-5}\), while even with large NP in the charm sector one finds \(\Delta \gamma \sim O(10^{-2})\).

In summary, a precise measurement of \(\gamma\) can be achieved at a SFF from a combination of \(B \rightarrow DK\) type decays with multiple \(D\) decay final states. The precision can be improved using charm factory data on strong phases. Although extrapolations of the current data are difficult, studies suggest that an error on \(\gamma\) of \(O(1^\circ)\) can be achieved (Akeroyd et al., 2004; Bona et al., 2007c). This would represent a significant improvement on the constraints from any other experiment, and yet the experimental uncertainty on \(\gamma\) would still be far above the irreducible theory error.

2. \(\sin(2\beta + \gamma)\)

The combination \(\sin(2\beta + \gamma)\) can in principle be extracted from \(B \rightarrow D^{(*)} \pi^+ \pi^-\) time dependent analysis (Dunietz, 1998; Suprun et al., 2002). However, the ratio of the two interfering amplitudes \(r = \frac{|A(B^0 \rightarrow D^{+}+)\pi^-/A(B^0 \rightarrow D^{(*)+}\pi^-)|}{2}\) is too small to be determined experimentally from \(O(r^2)\) terms and significant input from theory is required. Related methods use \(B^0 \rightarrow D^{(*)} \rho^-\), \(D^{(*)0}a_1\), where \(r\) can be determined from the difference of different helicity amplitudes (Gronau et al., 2003; London et al., 2000). These modes are difficult experimentally because of \(\pi^0\) reconstruction and no measurements exist to date. Another option are rare decays such as \(B \rightarrow D^{(*)} \pi^\pm X\), \(X = a_0, a_2, b_1, \pi(1300)\), where \(r\) is \(O(1)\) as pointed out by Diehl and Hiller (2001).

Time dependent \(B^0 \rightarrow D^{0(*)}K^{0(*)}\) analyses are perhaps the most promising (Atwood and Soni, 2003a; Kayser and London, 2000). The theoretical error is expected to be similar to that in \(\gamma\) extraction from \(B \rightarrow DK\), and thus well below SFF sensitivity. Another good candidate, \(B_s \rightarrow D^+_sK^+\), is better suited for experiments in an hadronic environment (Fleischer, 2004). Other alternatives, including three body modes such as \(B \rightarrow D^{(*)}K_S^0\pi^\mp\) (Aleksan et al., 2003; Charles et al., 1998; Polci et al., 2006) could also lead to a precise measurement of \(2\beta + \gamma\).

C. Measuring \(\alpha\)

Although in the SM \(\alpha\) is not independent from \(\gamma\) and \(\beta\), it is customary to separate the methods for the determination of the angle \(\alpha\) that involve \(B^0 \rightarrow \bar{B}^0\) mixing from those that do not. In this subsection we will therefore briefly discuss the determination of \(\alpha\) from the decays \(B \rightarrow \pi\pi, \rho\pi\) and \(pp\) [for a longer review see e.g. (Zupan, 2007a)]. The angle \(\alpha\) is determined from the \(S_T\) parameter of Eq. (36). For example in \(B \rightarrow \pi\pi\) this is

\[ S_{\pi^+\pi^-} = \sin 2\alpha + 2\cos \delta \sin(\beta + \alpha) \cos 2\alpha + O(\alpha^2), \]

(50)

where the expansion is in penguin–to–tree ratio \(r = P/T\). The “tree” ("penguin") is a term that carries a weak phase (or not), \(A(B^0 \rightarrow \pi^+\pi^-) = Te^{i\delta} + Pe^{i\delta}\), while \(\delta\) is a strong phase difference. In the \(r = 0\) limit one has \(S_{\pi^+\pi^-} = \sin 2\alpha\). If \(O(r)\) “penguin pollution” term is known, \(\alpha\) can be extracted from \(S_{\pi^+\pi^-}\). This is achieved by using symmetries of QCD, isospin or flavor SU(3), or by the \(1/m_b\) expansion in frameworks such as QCD factorization, pQCD, and SCET. The theoretical error on extracted \(\alpha\) depends crucially on the size of \(r\). Using

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3 This is the so-called "c-convention" where "penguin" is proportional to \(V_{ub} V_{cd}\). The other option is a "t-convention", where "penguin" is proportional to \(V_{tb} V_{td}\) and carries weak phase \(-\beta\).
isospin and/or SU(3) flavor symmetry one finds (see also Fig. 11)

\[
r(\pi^+\pi^-) > r(\rho^+\pi^-) \sim r(\pi^+\rho^-) > r(\rho^+\rho^-).
\]

(51)

We can expect a similar hierarchy for the theoretical errors on \(\alpha\) in the different channels. This simple rule, however, does not apply for methods based on isospin symmetry as discussed in more detail below.

1. \(B \to \pi\pi\)

Let us first review the extraction of \(\alpha\) from \(B \to \pi\pi\) using isospin decomposition (Gronau and London, 1990). In isospin limit \(\pi\) forms a triplet and \(B\) a doublet of isospin. In general \(B \to \pi\pi\) transition could be mediated by \(\Delta I = 1/2, 3/2\) and 5/2 interactions. However, \(\Delta I = 5/2\) operators do not appear in the effective weak Hamiltonian of Eq. (19), so that \(B \to \pi^0\pi^0, \pi^+\pi^-, \pi^+\pi^0\) amplitudes are related as shown in Fig. 12.

Another important input is that aside from possible electroweak penguin (EWP) contributions, \(A_{1,0}\) is a pure tree (notation is as in Fig. 12). Neglecting EWP the weak phase of \(A_{1,0}\) is fixed, so that for instance \(e^{i\delta}A_{1,0} = e^{-i\delta}A_{0,1}\). Then the observable sin \(2\alpha_{\text{eff}} = S_{\pi\pi}/\sqrt{1 - (A_{1,0}/A_{0,0})^2}\) is directly related to \(\alpha\) through \(2\alpha = 2\alpha_{\text{eff}} - 2\theta\), where \(\theta\) is defined in Fig. 12 left. The present constraints on \(\alpha\) following from the isospin analysis with the most recent experimental results (Aubert et al., 2007b; Ishino et al., 2007) are shown in Fig. 12 right. Note that in the determination of \(\alpha\) the contribution of \(\Delta I = 1/2\) terms cancel. This implies that the isospin analysis is insensitive to NP in QCD penguin operators, and would still return the SM value of \(\alpha\) even if such NP contributions were large.

Let us now turn to the question of theoretical uncertainties in the isospin analysis which come from isospin breaking. This has several effects: (i) different \(d\) and \(u\) charges lead to EWP operators \(Q_{7,...,10}\) in \(\mathcal{H}_{\text{EW}}\) of Eq. (5), (ii) the \(\pi^0\) mass and isospin eigenstates no longer coincide, leading to \(\pi^0 - \eta - \eta'\) mixing, (iii) reduced matrix elements for states in the same isospin multiplet may no longer be related simply by SU(2) Clebsch-Gordan coefficients, and (iv) \(\Delta I = 5/2\) operators may be induced, e.g. from electromagnetic rescattering.

2. \(B \to \rho\rho\)

The isospin analysis in \(B \to \rho\rho\) follows the same lines as for \(B \to \pi\pi\), but with separate isospin triangles, Fig. 12 for each polarization. The longitudinally polarized final state is found to dominate the other two, simplifying

In the literature only the first two effects have been analyzed in some detail. The effect of EWP is known quite precisely since the \(\Delta I = 3/2\) part of the EWP Hamiltonian is related to the tree part of the weak Hamiltonian (Buras and Fleischer, 1999; Gronau et al., 1999; Neubert, 1993; Neubert and Rosner, 1998a,b). The relation between the bases of triangles in Fig. 12 is now modified to \(e^{i\gamma}A_{1,0} = e^{-i\gamma+\delta}A_{0,1}\), where \(\delta = (1.5\pm 0.3)\degree\) (Gronau et al., 1999; Gronau and Zupan, 2003). This suggests that \(\pi^0 - \eta - \eta'\) mixing modifies also the Gronau-London triangle relations of Fig. 12 (Gardner, 1999). Since \(\pi^0 - \eta - \eta'\) is small, the resulting shift in the extracted value of \(\alpha\) is small as well, \(|\Delta\alpha| < 1.6\degree\) (Gronau and Zupan, 2003). These two examples of isospin breaking effects show that while not all of the isospin breaking effects can be calculated or constrained at present, the ones that can are of the expected size, \(\delta \alpha \sim (m_u - m_d)/\Lambda_{\text{QCD}} \sim 1\%\).

Experimentally, the isospin triangle approach is limited by the need to measure \(|A_{00}\rangle\) and \(|A_{10}\rangle\), i.e. to measure direct CP violation in \(B^0 \to \pi^0\pi^0\) decays. In addition, the method suffers from ambiguities in the solutions for \(\alpha\) (as can be seen in Fig. 12 right). A SFF will enable both problems to be overcome, since the large statistics will allow a precise measurement of \(A_{00}^\text{CP}\), while the sample of events with photon conversions will allow \(S_{00}\) to be measured, removing one ambiguity (Ishino et al., 2007). Including these effects, we expect a SFF to reach a precision of \(\sim 3\degree\) on \(\alpha\) from \(B \to \pi\pi\) (Akeroyd et al., 2004; Bona et al., 2007d).
the analysis considerably. Another difference from the ππ system is that ρ resonances have a non-negligible decay width. In addition to experimental complications, this allows the two ρ resonances in the final state to form an I = 1 state, if the respective invariant masses are different (Falk et al. 2004), leading to O(T_p^2/m_ρ^2) effects. This effect can in principle be constrained experimentally by making different fits to the mass distributions (Falk et al. 2004), though very high statistics would be necessary for such a procedure to be effective.

The remaining theoretical errors are due to isospin breaking effects. While the shift due to EWP is exactly the same as in ππ, ρ−ω mixing is expected to cause a relatively large, O(1), effect near the ω mass in the π+π− invariant mass spectrum. However, integrated over all phase space, the effect is of the expected size for isospin breaking, as indeed are all effects that can currently be estimated (Gronau and Zupan 2003).

An ingredient that makes the ρρ system favourable over ππ is the small penguin pollution, cf. Fig. 11. Moreover, the fact that B^0 → ρ^0ρ^0 results in an all charged final state means that S_{00} can be determined (Aubert et al. 2007a). Consequently, a determination from isospin analysis of B → ρρ at the SFF is expected to remain more precise than that from B → ππ, i.e. 1°–2° (Akerov et al. 2004; Bona et al. 2007a).

Somewhat surprisingly, the small penguin pollution makes the method based on the SU(3) symmetry as theoretically clean as the isospin analysis (Beneke et al. 2000). This is because SU(3) symmetry is used to directly constrain P/T, while the isospin construction involves also relations between the tree amplitudes, so that isospin breaking on the larger amplitudes translate to the corrections. The basic idea is to relate ∆S = 0 decays in which tree and penguin terms have CKM elements of similar size to ∆S = 1 decays in which the P/T ratio has a relative enhancement of ≈ 1/λ^2. The ∆S = 1 decays can then be used to constrain P/T. For example, B(B^+ → K^0ρ^+) can be used to bound the penguin contribution to B^0 → ρ^+ρ^− (Beneke et al. 2000):

\[ |A_L(K^0ρ^+)|^2 = F \left( \frac{|V_{cs}|f_{K^+}}{|V_{cd}|f_{ρ}} \right)^2 p^2, \] (52)

where the F parameterises SU(3) breaking effects (F = 1 in the limit of exact SU(3)). Using a conservative range of 0.3 ≤ F ≤ 1.5 results in theoretical error of ≈ 4° on α, comparable to the theoretical error in the isospin analysis.

3. B → ρπ

Since ρ±π± are not CP eigenstates, extracting α from this system is more complicated. Isospin analysis similar to the one for B → ρρ, ππ leads to an isospin pentagon construction (Lipkin et al. 1991) that is not competitive. It requires a large amount of experimental data and suffers from multiple solutions. Two more useful approaches are: (i) to exploit the full time-dependence of the B^0 → π^±π^−π^0 Dalitz plot together with isospin (Snyder and Quinn 1993), or (ii) to use only the ρ^±π^± region with SU(3) related modes (Gronau and Zupan 2004).

For the Snyder-Quinn isospin analysis two important differences compared to the isospin analysis of B → ππ and B → ρρ are (i) that in B → ρπ only the isospin relation between penguin amplitudes is needed, and (ii) that from the full time-dependent B^0 → π^+π^−π^0 Dalitz plot the magnitudes and relative phases of A(B^0 → ρ^+π^−), A(B^0 → ρ^−π^+), A(B^0 → ρ^0π^0) and the CP conjugated amplitudes are obtained. As a result the Snyder-Quinn approach does not suffer from multiple ambiguities, giving a single (and highly competitive) value for α in [0, π]. This approach has been implemented by both B factories (Aubert et al. 2007b; Kusaka et al. 2007a).

A potential problem is that the peaks of ρ resonance bands do not fully overlap in the Dalitz plot, but are separated by approximately one decay width, so one is sensitive to the precise lineshape of the ρ resonance. Isospin breaking effects on the other hand are expected to be P/T suppressed, since only the isospin relation between penguins was used. The largest shift is expected to be due to EWP and is known precisely, as in B → ππ, ρρ case (Gronau and Zupan 2003). Other isospin breaking effects are expected to be small. For instance, the shift due to π^0 − η − η’ mixing was estimated to be |Δα_η−η’| ≤ 0.1° (Gronau and Zupan 2003), showing that the expected P/T suppression exists.

An alternative use of the same data is provided by the SU(3) flavor symmetry. In this way the potential sensitivity of the Snyder-Quinn method on the form of ρ resonance tails can be avoided. The required information on P/T is obtained from the SU(3) related ∆S = 1 modes, B^0 → K^+π^−, K^+ρ^− and B^+ → K^0π^+, K^0ρ^+. Since penguin pollution is relatively small, the error on the extracted value of α due to SU(3) breaking is expected to be small as well, of a few degrees (Gronau and Zupan 2004). Unlike the Snyder-Quinn approach this method does suffer from discrete ambiguities.

In summary, theory errors in the above direct measurements of α are difficult to determine completely. Our best estimates for the error on α from isospin analysis of the ππ and ρρ systems are around a few degrees. The uncertainty is expected to be smaller for the Snyder-Quinn analysis of ρπ which relies on an isospin relation between only penguin amplitudes. Since a SFF can make determinations of α in all of the above modes, we can be cautiously optimistic that most sources of theoretical uncertainty can be controlled with data. Therefore, there is a good chance that the final error on α from a SFF will be around 1°.

Finally, Table V summarizes the estimates on the theory error and also the expected accuracy at the SFF for each angle through the use of these direct methods.
V. SIDES OF THE TRIANGLE

In this section we review briefly the strategies for measurements of the magnitudes of CKM matrix elements. For a more extensive review see [Yao et al., 2006].

While the determinations of $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$ and $|V_{ts}|$ mainly rely on CP conserving observables – the CP averaged $B$ decay branching ratios – their values do constitute an independent check of the CKM mechanism. The information on $|V_{ub}|/|V_{cb}|$ for instance determines the length of the unitarity triangle side opposite to the well measured angle $\beta$, cf. Fig. 9. Together with the direct determination of $\gamma$ it provides a consistency check between the constraints from $b \to u$ tree transitions and the constraint from the loop induced $B \to \bar{B}$ mixing.

A. Determination of $|V_{cb}|$

Both exclusive and inclusive $b \to c$ decays are used, giving consistent determinations [Yao et al., 2006]

$$|V_{cb}\rangle_{\text{excl.}} = (40.9 \pm 1.8) \times 10^{-3},$$
$$|V_{cb}\rangle_{\text{incl.}} = (41.7 \pm 0.7) \times 10^{-3}. \quad (53)$$

The value of $|V_{cb}|$ from the exclusive decay $B \to D^* \ell \nu$ ($B \to D \ell \nu$) is at present determined with a 4% (12%) relative error, where the theoretical and experimental contributions to the errors are comparable. In the heavy quark limit the properly normalized form factors are equal to 1 at zero recoil, $v_B \cdot v_{D(*)} = 1$. This prediction has perturbative and nonperturbative corrections

$$F_{D^*}(1) = 1 + c_A(\alpha_s) + \frac{0}{m_Q} + \frac{c_{\text{nonp}}}{m_Q},$$
$$F_D(1) = 1 + c_V(\alpha_s) + \frac{c_{\text{nonp}}}{m_Q}. \quad (54)$$

The absence of $1/m_Q$ corrections in $F_{D^*}(1)$ is due to Lake’s theorem [Lukà, 1990]. The perturbative corrections $c_{AV}$ are known to $\alpha_s^3$ order [Czarnecki, 1996, Czarnecki and Melnikov, 1997], while the first nonperturbative corrections $c_{\text{nonp}}^{(*)}$ are known only from quenched lattice QCD [Hashimoto et al., 2002, 2000] or from phenomenological models. Improvement can be expected in the near future when unquenched lattice QCD results become available. The projected uncertainty is 2-3% [Laiho, 2007, Yao et al., 2006], which is comparable to presently quoted errors in quenched calculations [Hashimoto et al., 2002, 2000], but the results will be more reliable. Further improvements in precision will be needed, however, to reach the 1% uncertainty projected for the inclusive $|V_{cb}|$ determination discussed below. To achieve this goal analytical work is also needed: the calculation of higher order matching of latticized HQET to continuum QCD is already in progress [Nobes and Trotti, 2004, Oktay et al., 2004], while other ingredients such as the radiative corrections to the $1/m_Q$ and $1/m_b^3$ suppressed terms in the currents are not yet being calculated. The difficulty of this task is comparable or even greater than the same order calculation needed for the inclusive determination of $|V_{cb}|$ [Yao et al., 2004]. On the experimental side, reduction of the uncertainty with larger statistics is not guaranteed, since systematic errors already limit the precision [Aubert et al., 2005a, 2006a].

The inclusive determination of $|V_{cb}|$ is based on the operator product expansion leading to a systematic expansion in $1/m_b$ [Bosch et al., 1994a, 1993, Manohar and Wise, 1994]. Present fits to $B \to X_c \ell \nu$ include terms up to order $1/m_b^3$ and $\alpha_s^2 \delta_b$. The same nonperturbative elements also appear in the predictions of $B \to X_c \gamma$ so that global fits to electron and photon energy moments from data are performed, giving $|V_{cb}|$ with a relative error of about 1.7% [Yao et al., 2006]. Improvements on the theoretical side can be made by calculating higher order perturbative corrections [Neubert, 2005] and by calculating the perturbative corrections to the matrix elements that define the heavy quark expansion parameters. Experimentally, systematic errors are already limiting the most recent results in these analyses [Schwanda et al., 2007, Urquijo et al., 2007]. However, some improvement is certainly possible with the large statistics of a SFF, so that a precision on $|V_{cb}|$ around 1% may be possible.

B. Determination of $|V_{ub}|$

Both exclusive and inclusive determinations are being pursued. At present there is some slight tension (at the 1σ level) between the two types of determinations; as discussed below.

The theoretical and experimental difficulty with the inclusive extraction of $|V_{ub}|$ from $B \to X_s \ell \nu$ is due to the large charm background from $B \to X_s \ell \nu$. As a result one cannot obtain the full inclusive rate experimentally. The region of phase space without charm contamination is typically a region where the inclusive hadronic state forms a jet, so that the OPE is not valid. Still, one can find a $\Lambda_{QCD}/m_b$ expansion, and using SCET one can show that there is a factorization of the structure functions (in terms of which the branching ratio is expressed) into hard, jet and shape functions, see Eq. (70) below. Each of these factors encode physics at scales of the order $m_b$, $\sqrt{\Lambda_{QCD}} m_b$ and $\Lambda_{QCD}$. The jet and shape functions are currently known at $O(\alpha_s(m_b))$ [Bauer and Manohar, 2004, Bosch et al., 2004a] and $O(\alpha_s^2(\sqrt{\Lambda_{QCD}} m_b))$ [Beccher and Neubert, 2006] respectively, while the power corrections have been included only at $O(\alpha_s)$ [Benek et al., 2005a, 2005b, Bosch et al., 2004a, Lee and Stewart, 2003]. In the BLPN approach the parameters for the models of the LO shape function are extracted from the $B \to X_c \ell \nu$...
while subleading shape functions are modeled. The HFAG average using this approach is $|V_{ub}|_{\text{HFAG}} = (4.49 \pm 0.19 \pm 0.27) \times 10^{-3}$ (Aubert et al. 2007), Barberio et al. 2007, Bizjak et al. 2007), where the first error is experimental and the second theoretical. Alternatively, as discussed in Section VIII.C, the ratio of $B \to X_s \ell \nu$ to $B \to X_{s \gamma}$ decay rates can be used to reduce the dependence on the LO shape function (Lange et al. 2005, Lange et al. 2006, Leibovich et al. 2000, Neubert 1994). This approach has been used to obtain the value $|V_{ub}| = (4.43 \pm 0.45 \pm 0.29) \times 10^{-3}$ (Aubert et al. 2006a), where the first experimental and the second theoretical. The combined theoretical error from using 2-loop corrections to jet functions, the subleading shape function corrections and the known $\alpha_s/m_b$ corrections has been estimated to be 5% (Lange et al. 2005b). This error could be further reduced by using the $B \to X_{s \gamma}$ hard kernels at $O(\alpha_s^2)$ a calculation of which is almost complete (Becher and Neubert 2007), but a similarly demanding calculation of the hard kernel in $B \to X_s \ell \nu$ at the same order would be needed. Another hurdle is the estimation of the subleading shape functions – to gain in precision one would need to go beyond modeling.

A different approach that can reduce the dependence on shape functions is a combined cut on the leptonic momentum transfer $q^2$ and the hadronic invariant mass $M_X$ (Bauer et al. 2000, 2001), so that a larger portion of phase space is used. Furthermore, it has been suggested (Bigi and Uraltsev 1994, Voloshin 2001) that uncertainties from weak annihilation can be reduced by making a cut on the high $q^2$ region. Another theoretical approach, dressed Gluon Exponentiation, that uses a renormalon inspired model for the leading shape function has been advocated (Andersen and Gardi 2000). Following these approaches, and taking advantage of the large statistics at a SFF, a precision on $|V_{ub}|$ of 3–5% from inclusive modes may be possible.

For the exclusive $|V_{ub}|$ determination, the decay $B \to \pi \ell \nu$ is primarily used, although decays such as $B \to \phi \ell \nu$ also provide useful information, and, as discussed in Section HI.C, leptonic decays $B \to \ell \nu$ can be used to obtain a tree-level determination of $|V_{ub}|$ that is sensitive to NP effects. Nonperturbative information on $B \to \pi \ell \nu$ form factors comes from lattice QCD for $q^2 > 16$ GeV$^2$, while light cone sum rules can be used for $q^2 = 0$. Using current lattice QCD results in their range of applicability $q^2 > 16$ GeV$^2$, the results are [Athanasiou et al. 2003, Aubert et al. 2007, Barberio et al. 2007, Hokoue et al. 2007, using the unquenched HPQCD calculation (Dalic et al. 2006), and $|V_{ub}| = (3.55 \pm 0.24^{+0.58}_{-0.38}) \times 10^{-3}$ for the unquenched calculation from the FNAL collaboration (Okamoto et al. 2003). A number of extrapolation ansaetze have been proposed so that the whole $q^2$ region can be used for $V_{ub}$ determination (Arnesen et al. 2005, Becher and Hill 2006, Becirevic and Kaidalov 2008, Boyd et al. 1993, Boyd and Savage 1997, Hill 2000). A recent discussion of their use is given in Ball (2006).

The current status is somewhat problematic: inclusive methods give $|V_{ub}|$ values systematically larger than the exclusive methods, and are also in disagreement with direct $\sin 2\beta$ determination at $\approx 2\sigma$ level (Bona et al. 2006, 2007b, Charles et al. 2003, Lungadi and Soni 2007). Neubert (2008) argued recently that, due to model dependence introduced by the shape function and contributions other than those from the $Q_{\tau\gamma}$ operator, the $b \to \tau \gamma$ data should not be used in the $|V_{ub}|$ determination. Using $m_b$ determined only from $b \to cl\nu$ and the theoretically cleanest $M_X$ cut, Neubert finds $|V_{ub}| = (3.70 \pm 0.15 \pm 0.28) \times 10^{-3}$, resolving the disagreement.

The SFF will give much improved determinations of $|V_{ub}|$ using the exclusive approach, where the statistical errors currently control the precision of the measurements. Here one requires precise determinations of the $q^2$ spectrum, in the low recoil region where the rate is very small. The large data sample at a SFF will allow measurements of binned spectra with precision of a few percent. Assuming that lattice QCD can reach a comparable level of precision, an error of 3–5% on $|V_{ub}|$ from the exclusive approach appears attainable at a SFF.

### C. Determination of $|V_{td}|$ and $|V_{ts}|$ from loop processes

The values of the CKM matrix elements $|V_{td}|$ and $|V_{ts}|$ can only be studied in loop processes at a SFF. These include both mixing ($\Delta F = 2$) and decay ($\Delta F = 1$) processes. Specifically, the ratio $|V_{td}|/|V_{ts}|$ can be obtained by comparing the $B_d-\bar{B}_d$ and $B_s-\bar{B}_s$ mass differences, or from the ratio of, for example, $b \to c\ell\nu$ and $b \to \tau\gamma$ radiative decays. Since both are loop mediated processes they are sensitive to NP.

The oscillation frequencies in $B_{d,s} \to \bar{B}_{d,s}$ mixing determine the mass differences. These are short distance dominated and depend on the CKM matrix elements as

$$
\Delta M_d = M^d_{\perp} - M^d_{\parallel} = \frac{G_F^2 M_{B_d}}{6\pi^2} m_W^2 |V_{td}V_{td}^*|^2 \eta_{B_d} S_0(x_t) f_{B_d}^2 B_{B_d},
$$

and similarly for the $B_s$ system with the substitution $d \to s$. Here $\eta_B(x_t)$ encodes the short-distance information in the Inami-Lim function $S_0(x_t)$ that depends on the top mass through $x_t = m_t^2/m_W^2$, while $\eta_B = 0.55$ is a numerical factor containing NLO QCD corrections due to running from $m_W$ to $m_t \sim m_b$ (Buras et al. 1990).

The mass difference is precisely measured in the $B_d-\bar{B}_d$ system with the present WA $\Delta M_d = 0.505 \pm 0.005$ ps$^{-1}$ (Abe et al. 2005, Aubert et al. 2006). Further improvement of this measurement at a SFF is not likely to reduce the error on $|V_{td}|$, which is dominated at present by theory (lattice) errors. The $B_s-\bar{B}_s$ mixing parameter $\Delta M_s$ has recently been measured at the Tevatron to be $\Delta M_s = 17.77 \pm 0.10 \pm 0.07$ ps$^{-1}$ (Abulencia et al. 2006). Again,
lattice errors limit the direct extraction of $|V_{ts}|$ from this result.

The parameters $f_{B_d,s}$ and $B_{B_d,s}$ have been computed in lattice QCD using a variety of methods (see Okamoto (2006); Tantalo (2007) for recent reviews). Both quenched and unquenched determinations of the decay constants are available. For the bag parameters the quenching effect is not very important. For instance, the analogous quantity $B_K$ of the kaon system has been computed in unquenched simulations using domain wall quarks, and is now known to about $5 - 6 \%$ error (Antonio et al. 2008). In fact, separating out the decay constants from $f_{B_d,s}/\sqrt{B_{B_d,s}}$ is a notational artefact remaining from the days of vacuum saturation approximation (Bernard et al. 1998; Dalgic et al. 2007). Calculating the product instead can lead to reduced errors.

The best constraint comes at present from the ratio of the mass differences

$$\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_d}}{M_{B_s}} \xi \left| \frac{V_{td}}{V_{ts}} \right|^2,$$  \hspace{1cm} (56)

where $\xi = f_{B_d,s}/\sqrt{B_{B_d,s}/B_{B_d}}$. Several theoretical uncertainties cancel out in this ratio. From Eq. (56) and the experimental values of $\Delta M_d$ and $\Delta M_s$ given above, one obtains $|V_{td}/V_{ts}| = 0.2060 \pm 0.0007 \pm 0.0005$ (Abulencia et al. 2000) where the first error is experimental and the second theoretical, from the input value $\xi = 1.21^{+0.047}_{-0.035}$ which is obtained from an average of $n_f = 2$ partially quenched simulations (Okamoto 2006). Thus, the lattice uncertainty also dominates this constraint; indeed the stated errors here may well be an underestimate. However, unquenched precision calculations of $\xi$ are underway; see e.g. Dalgic et al. (2007) and certainly by the time of SFF the stated error on $\xi$ should be confirmed.

An alternative determination of $|V_{td}/V_{ts}|$ can be obtained from the ratio of $b \rightarrow d\gamma$ and $b \rightarrow s\gamma$ rare radiative decays. This is discussed in more detail in Section VIII.C and we give here only a brief account. Taking the ratio of $B \rightarrow \rho\gamma$ and $B \rightarrow K^\ast\gamma$ exclusive decays, the hadronic matrix elements cancel to a good approximation, giving

$$\frac{B(B \rightarrow \rho\gamma)}{B(B \rightarrow K^\ast\gamma)} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \left( \frac{M_B^2 - m_\rho^2}{M_B^2 - m_{K^\ast}^2} \right)^3 \xi^2 (1 + \Delta R).$$ \hspace{1cm} (57)

Here $\xi$ is the ratio of the $B \rightarrow \rho/K^\ast$ tensor form factors and equals 1 in the SU(3) limit, and $\Delta R$ describes the effect of the weak annihilation in $B^\pm \rightarrow \rho^\pm\gamma$. As discussed in Section VIII.C this gives results in good agreement with the determination from neutral $B_{d,s}$ meson mixing, albeit with larger errors that, for now, are predominantly experimental in origin. We note that the corresponding inclusive radiative modes can be used as well, provided that the $s\bar{s}$ background in $b \rightarrow d\gamma$ modes can be reliably taken into account.

Theoretically, an extremely clean determination of

| $V_{td}/V_{ts}$ | Possible using the ratio \cite{Buras_2001}
\hline
| $\mathcal{B}(B \rightarrow X_d\nu\bar{\nu})$ | $\mathcal{B}(B \rightarrow X_s\nu\bar{\nu})$ | $\left| \frac{V_{td}}{V_{ts}} \right|^2$, \hspace{1cm} (58)
\hline
\end{tabular}

which is predicted in the SM with essentially no hadronic uncertainties. However, the inclusive modes in Eq. (58) are very challenging experimentally because of the presence of the two undetected neutrinos. Nevertheless, studies of these decays, in particular in exclusive final states, can be started at a SFF, as we discuss in Section VIII.B.3. We mention here that since the exclusive modes are subject to SU(3) breaking, an extraction of $V_{td}/V_{ts}$ without theory uncertainty can only be obtained from inclusive measurements.

Table VI summarizes the current versus the estimated error in the SFF era.

**VI. TIME-DEPENDENT CP ASYMMETRY IN PENGUIN-DOMINATED MODES**

Penguin dominated hadronic $B$ decays offer one of the most promising sets of observables to search for new sources of CP violation. The time dependent CP asymmetry in channels such as $B^0 \rightarrow \phi K_S$ and $B^0 \rightarrow \eta' K_S$ gives in the SM the value of $\sin 2\beta$ that should be the same (up to suppressed terms) as the one determined from the tree dominated “golden” mode $B^0 \rightarrow J/\psi K_S$ (cf. Section VIII.A). However, since $B^0 \rightarrow \phi K_S$ and $B^0 \rightarrow \eta' K_S$ are loop dominated, NP contributions can modify this prediction.

The decay amplitude for the penguin dominated $\Delta S = 1$ charmless $B$ decay can be written as

$$M(B^0 \rightarrow f) = \lambda_u^{(s)} A_f^u + \lambda_c^{(s)} A_f^c,$$ \hspace{1cm} (59)

where the “tree” amplitude, $A_f^u$, and “penguin” amplitude, $A_f^c$, are multiplied by different CKM elements $\lambda_u^{(s)} = V_{ub} V_{ts}^*$. This is a general decomposition. Using CKM unitarity, $\lambda_u^{(s)} = -\lambda_s^{(s)} - \lambda_c^{(s)}$, any SM contribution can be cast in the form of Eq. (59). The “tree” contribution is suppressed by a factor $|\lambda_u^{(s)}/\lambda_c^{(s)}| \sim 1/50$.
and can be neglected to first approximation. Following the same steps as for the “golden”, tree-dominated mode $B^0 \to J/\psi K^0$ in Eq. (39), this then gives $\lambda_f \simeq \eta_f e^{-2i\beta}$ with $\eta_f = +1$ ($-1$) for CP-even (CP-odd) final states. Therefore, the SM expectation is that

$$\eta_f S_f \simeq \sin 2\beta, \quad \lambda_f \simeq 0.$$  \hfill (60)

The same is expected for mixing-induced CP violation in $B^0 \to J/\psi K^0$ as described in Section IV.A. Here the measurements are quite mature, with the latest world average (including both $J/\psi K_S$ and $J/\psi K_L$ final states) [Barberio et al. 2007]

$$\sin 2\beta \simeq S_{J/\psi K^0} = 0.668 \pm 0.026.$$  \hfill (61)

The $B$ factories have measured in the past few years time-dependent CP violation parameters for a number of $b \to s$ modes, including $B^0 \to \phi K^0$, $B^0 \to \eta' K^0$, $B^0 \to K_S K_S$, $B^0 \to \pi^0 K_S$, $B^0 \to \rho^0 K_S$, $B^0 \to \omega^0 K_S$, $B^0 \to f_0 K^0$, $B^0 \to f_0^* K_S$ and $B^0 \to K^+ K^- K^0$. Table VII gives the current experimental world averages for $\Delta S_f$ and $A_f$ [Barberio et al. 2007]. The recent BABAR result from $B^0 \to f_0 K^0_S$ from time-dependent $B^0 \to \pi^0 \pi^- K^0_S$ Dalitz plot analysis [Aubert et al. 2007a] is not included, since it has highly non-Gaussian uncertainties.

Table VII. Current experimental world averages for $\Delta S_f$ and $A_f$ [Barberio et al. 2007]. The recent BABAR result from $B^0 \to f_0 K^0_S$ from time-dependent $B^0 \to \pi^0 \pi^- K^0_S$ Dalitz plot analysis [Aubert et al. 2007a] is not included, since it has highly non-Gaussian uncertainties.

| Mode          | $\Delta S_f$ | $A_f$  |
|---------------|--------------|--------|
| $\phi K^0$    | $-0.28 \pm 0.17$ | $0.01 \pm 0.12$ |
| $\eta' K^0$  | $-0.06 \pm 0.08$ | $0.09 \pm 0.06$ |
| $K_S K_S K_S$ | $-0.09 \pm 0.20$ | $0.14 \pm 0.15$ |
| $\pi^0 K_S$  | $-0.29 \pm 0.19$ | $-0.14 \pm 0.11$ |
| $\rho^0 K_S$ | $-0.06 \pm 0.25$ | $-0.02 \pm 0.29$ |
| $\omega^0 K_S$ | $-0.19 \pm 0.24$ | $0.21 \pm 0.19$ |
| $f_0 K^0$    | $-0.46 \pm 0.18$ | $-0.08 \pm 0.12$ |
| $\pi^0 \pi^0 K_S$ | $-1.19 \pm 0.41$ | $-0.18 \pm 0.22$ |
| $K^+ K^- K^0$ | $0.06 \pm 0.10$ | $-0.07 \pm 0.08$ |

where $f$ is a penguin-dominated final state. Up to small corrections to be discussed below, one has $\Delta S_f = 0$ in the SM. A summary of the current experimental world averages for $\Delta S_f$ is given in Table VII.

So far we have neglected the “tree” amplitude $A_f^T$ of Eq. (59). In many of the penguin dominated modes, e.g. $\omega K_S, \rho^0 K_S, \pi^0 K_S$, the amplitude $A_f^T$ receives contributions from the $b \to u\bar{s}b$ tree operators which can partially lift the large CKM suppression. To first order in $r_f \equiv (\lambda_a^T A_f^f)/(\lambda_c^T A_f^c)$ one has [Cheng et al. 2005a; Gronau 1989; Grossman et al. 2003a]

$$\Delta S_f = 2|r_f| \cos 2\beta \sin \gamma \cos \delta_f,$$

$$A_f = 2|r_f| \sin \gamma \sin \delta_f,$$

with a strong phase $\delta_f = \arg(A_f^T/A_f^c)$. Both $\Delta S_f$ and $A_f$ can thus deviate appreciably from zero, if the ratio $A_f^T/A_f^c$ is large. Most importantly, the size of this ratio is channel dependent and will give different $\Delta S_f$ for different modes. We thus turn next to the theoretical estimates of $\Delta S_f$.

**A. Theoretical estimates for $\Delta S_f$**

The original papers [Ciuchini et al. 1997a; Fleischer 1997; Gronau 1989; Grossman and Wорал 1997; London and Soni 1997] that suggested $\Delta S_f$ (Eq. (2)) as a powerful tool for new physics searches used naive factorization. In recent years several theoretical reappraisals have been performed using several different approaches to calculate $\Delta S_f$ (for detailed reviews, see e.g. [Silvestrini 2007; Zupan 2007]). The methods used are either based on SU(3) symmetry relations [Barus et al. 2003; 2004a,b, 2005, 2006; Engelhard et al. 2003; Engelhard and Rutz 2003; Fleischer et al. 2007; Gronau et al. 2004a,b, 2006a; Grossman et al. 2003a); or use the $1/m_b$
expansion – QCD factorization (QCDF) [Beneke, 2003; Buchalla et al., 2003; Cheng et al., 2005a,b] perturbative QCD (pQCD) [Ali et al., 2007; Li and Mishima, 2006], and Soft-Collinear Effective Theory (SCET) [Williamson and Zupan, 2006]. Table VIII summarizes some of the findings.

The SU(3) relations typically give only loose constraints on $\Delta S_f$ since the bounds involve sums of amplitudes, where relative phases are unknown. Furthermore, SU(3) breaking is hard to estimate and all the analyses are done only at leading order in the breaking. The $1/m_b$ expansion on the other hand provides a systematic framework where higher order corrections can in principle be included. The three approaches: QCDF, pQCD and SCET, while all using the $1/m_b$ expansion, differ in details such as the treatment of higher order corrections, charming penguins [Ciuchini et al., 2001, 1997b] and the scale at which the treatment is still deemed perturbative [Bauer et al., 2002; Beneke et al., 2005a].

Experimental observations of large direct CP asymmetries in several exclusive $B$ decay modes, such as $K^+\pi^-$ [Aubert et al., 2007; Chao et al., 2004] and $\pi^+\pi^-$ [Ishino et al., 2007] require large strong phases. In different theoretical approaches these are seen to come from different sources. In pQCD [Keum et al., 2001] they arise from annihilation diagrams and are deemed calculable using a phenomenological parameter $k_T$ as an endpoint divergence regulator. In QCDF the large strong phase is deemed nonperturbative and comes from endpoint divergent weak annihilation diagrams and the chirally-enhanced power corrections to hard spectator scattering. It is then modeled using nonperturbative parameters. In SCET the strong phase is assigned to nonperturbative charming penguins, while annihilation diagrams are found to be real [Arnesen et al., 2006]. The nonperturbative terms are fit from data. In the approach of Cheng et al. [2005a,b] the strong phases are assumed to come from final state interactions. These are then calculated from on-shell rescattering of 2-body modes, while QCDF is used for the short-distance part.

\begin{table}[h]
\centering
\caption{Expectations for $\Delta S_f$ in three cleanest modes.}
\begin{tabular}{|c|c|c|c|}
\hline
Model & $fK^0$ & $\eta'K^0$ & $K_SK_SK^0$ \\
\hline
QCDF+FSI$^a$ & $0.03^{+0.01}_{-0.00}$ & $0.00^{+0.00}_{-0.04}$ & $0.02^{+0.00}_{-0.04}$ \\
QCDF$^b$ & $0.02 \pm 0.01$ & $0.01 \pm 0.01$ & \\
QCDF$^c$ & $0.02 \pm 0.01$ & $0.01 \pm 0.02$ & \\
SCET$^d$ & $-0.019 \pm 0.009$ & & \\
pQCD$^e$ & $0.02 \pm 0.01$ & & \\
\hline
\end{tabular}
\end{table}

\textsuperscript{a}Cheng et al. (2005a,b) \hspace{1cm} \textsuperscript{b}Beneke (2005) \hspace{1cm} \textsuperscript{c}Buchalla et al. (2005) \hspace{1cm} \textsuperscript{d}Williamson and Zupan (2006) \hspace{1cm} \textsuperscript{e}Li and Mishima (2006)

**C. Comparison with SM value of $\sin 2\beta$**

As experimental errors reduce, for a number of modes the deviations of $\Delta S_f$ from zero may become significant. The translation of the measured values of $\Delta S_f$ into a deviation from the SM then becomes nontrivial. However, forgetting about this issue and just averaging over the experimental data given in Table VII gives a value of $(\Delta S_f) = -0.11 \pm 0.06$ [Barberio et al., 2007] (using only the theoretically cleanest modes $\eta'K^0$, $\phi K^0$ and $K_S K_SK^0$, one obtains instead $(\Delta S_f) = -0.09 \pm 0.07$).

Different approaches that take into account theoretical predictions are possible [Zupan, 2007]. Correcting for the SM value of $\Delta S_f$ by defining $(\Delta S_f)_{\text{corr}} = (\Delta S_f)_{\text{exp}} - (\Delta S_f)_{\text{th}}$, one has several choices that can be taken for $(\Delta S_f)_{\text{th}}$, including: (i) to use all available theoretical predictions in a particular framework (e.g. QCDF), and to discard remaining experimental data, (ii) to use the theoretical prediction for each channel that is closest to the experimental data (and neglecting three-body decays where only one group has made predictions). The first prescription gives $(\Delta S_f)_{\text{corr}} = -0.133 \pm 0.063$ [Zupan, 2007]. Interestingly enough the second prescription gives almost exactly the same result.

**D. Experimental prospects**

Several previous studies have considered the potential of a SFF to improve the measurements of $\Delta S_f$ to at least the level of the current theoretical uncertainty in a wide range of channels, including all the theoreti-
cally cleanest modes (Akeroyd et al. 2004; Bona et al. 2007c; Gershon and Soni 2007; Hashimoto et al. 2004; Hewett et al. 2004). By extrapolating the current experimental measurements, these studies show that data samples of at least $50 \text{ ab}^{-1}$ (containing at least $50 \times 10^9 \text{ } B \bar{B}$ pairs) will be necessary. This roughly corresponds to five years of operation for a facility with peak luminosity of $10^{36}\text{cm}^{-2}\text{s}^{-1}$ and data taking efficiency comparable to the current $B$ factories. These studies also indicate the systematic uncertainties are unlikely to cause any unsum- mountable problems at the few percent precision level that will be reached (although the Dalitz plot structure of the $B^0 \rightarrow K^+ K^- K^0$ decay (Aubert et al 2007d will need to be clarified to obtain high precision on $S_{bK^0}$).

One may consider the potential of a hadronic machine to address these modes. At present, it appears that $\phi K_S$ is difficult, but not impossible to trigger and reconstruct in the hadronic environment, due to the small opening angle in $\phi \rightarrow K^+ K^- \gamma$; $\eta' K_S$ is challenging since neutral particles are involved in the $\eta'$ decay chain; for $K_S K_S K_S$ meanwhile, there are no charged tracks originating from the $B$ vertex, and so both triggering and reconstruction seem highly complicated. Modes containing $K_L$ mesons in the final state may be considered impossible to study at a hadron machine. Furthermore, due to the theoretical uncertainties discussed above, there is a clear advantage provided by the ability to study multiple channels and to make complementary measurements that check that the theory errors are under control. Thus, these modes point to a Super Flavor Factory, with integrated luminosity of at least $50 \text{ ab}^{-1}$.

VII. NULL TESTS OF THE SM

An important tool in searching for new flavor physics effects are the observables that vanish or are very small in the SM, have small calculable corrections and potentially large new physics effects. Several examples of such null tests of the SM are discussed at length in separate sections of this review:

- As discussed in Section VIII.A.1 the untagged direct CP asymmetry $A_{CP}(B \rightarrow X_{s+d}\gamma)$ vanishes in the U-spin limit (Hurth and Mannell 2001; Soares 1991). The leading SU(3) breaking corrections are of order $(m_s/m_b)^2 \sim 5 \times 10^{-4}$ giving $A_{CP}(B \rightarrow X_{s+d}\gamma) \sim 3 \times 10^{-6}$ (Hurth and Mannell 2001). This can be easily modified by new physics contributions. For instance, in the MSSM with nonvanishing flavor blind phases $A_{CP}(B \rightarrow X_{s+d}\gamma)$ can be a few percent, while more general flavor violation can saturate the present experimental bounds (Hurth et al. 2003).

- Photon polarization in $B \rightarrow V\gamma$ decays. As discussed in Section VIII.A.3, the time dependent CP asymmetry, $S$, in $B(t) \rightarrow \gamma K^+(K_S\pi^0,\rho,\ldots)$ can be used as quasi-null tests of the SM.

- Lepton flavor violating $\tau$ decays such as $\tau \rightarrow \mu\gamma$, $\tau \rightarrow 3\mu$, etc., would be a clear signal of new physics. The theoretical expectations and SFF reach are discussed in Section XI.

- CP asymmetry from interference of decay and mixing in $\Delta S = 1$ penguin dominated decays, $S_f$, is equal to $\sin 2\beta$ up to CKM suppressed hadronic corrections. As shown in Section VI the precision of this test is at the few percent level or below for several modes such as $B \rightarrow \eta' K_S, \phi K_S, K_S K_S K_S$ decays. New physics contributions can easily accommodate much larger deviations.

In this section we give some further examples of null tests.

A. Isospin sum-rules in $B \rightarrow K\pi$

As first discussed by Lipkin (1999) and by Gronau and Rosner (1999) the following sum of CP averaged $B \rightarrow K\pi$ decay widths

$$\Delta \Lambda \equiv \frac{1}{\Gamma(K^0\pi^-)} \left[ 2\Gamma(K^0\pi^0) - \Gamma(K^-\pi^+) + 2\Gamma(K^+\pi^-) - \Gamma(K^0\pi^-) \right],$$

vanishes in the SM up to second order in two small parameters: the EWP-to-penguin ratio and the doubly CKM suppressed tree-to-penguin ratio. Assuming isospin symmetry, the LO SCET theory prediction is $\Delta \Lambda = (2.0 \pm 0.9 \pm 0.7 \pm 0.4) \times 10^{-2}$ (Williamson and Zupan 2006), which is compatible with and more precise than a QCD prediction (Beneke and Neubert 2003). Remaining isospin breaking contributions are small (Gronau et al. 2006a). The experimental value has at present much larger errors, $\Delta \Lambda = (0.13 \pm 0.09)$ (Abe et al. 2007c; Aubert et al. 2006; 2087d; 2008b; Barberio et al. 2007). The precision of the branching fraction measurements of all input modes would need to be improved to make a significant reduction in this experimental uncertainty at a SFF. The measurements currently have comparable statistical and systematic uncertainties, so this is not straightforward. However, some modest reduction of uncertainties due to $K_S$ and $\pi^0$ reconstruction efficiencies can be expected, so that this test may become at least a factor two more stringent.

A quantity that is even further suppressed in SM is a similar sum of partial decay width differences $\Delta \Gamma = \Gamma(B \rightarrow f) - \Gamma(B \rightarrow \bar{f})$

$$\Delta \Sigma = \frac{1}{\Gamma(K^0\pi^-)} \left[ 2\Delta \Gamma(K^0\pi^0) - \Delta \Gamma(K^-\pi^+) + 2\Delta \Gamma(K^+\pi^-) - \Delta \Gamma(K^0\pi^-) \right].$$

4 For neutral $B$ decays potential nonzero contributions, such as annihilation, start at $\alpha_s(m_b)/m_b^2$ order.
In the limit of exact isospin and no EWP $\Delta \gamma$ vanishes (Atwood and Soni, 1998a; Gronau and Rosner, 2003). Furthermore, the corrections due to EWP are subleading in the $1/m_b$ expansion (Gronau, 2003), so that $\Delta \gamma$ is expected to be below 1%. Experimentally, $\Delta \gamma \text{Exp.} = 0.01 \pm 0.10$ (Abe et al., 2007d; Aubert et al., 2006; 2007d; 2008d; Barberio et al., 2007), where the uncertainty is dominated by the $A_{\text{CP}}(\pi^0K^0)$ experimental error. This is large because the reconstructed final state for this mode ($\pi^0K_S$) is a CP eigenstate containing no information on the initial $B$ meson flavor. The required flavour tagging comes at a statistical cost that is, however, less severe at an $e^+e^-$ $B$ factory than at a hadron collider. Therefore, this SM test is unique to a SFF, where a significant improvement compared to the current precision can be expected.

The above sum rules given in Eq. (64) and Eq. (65) can be violated by NP that breaks isospin symmetry. An example is given by NP contributions to EWP, extensively discussed in the literature (see Back et al., 2005; Buras et al., 2004a and references therein).

B. $b \to ssd$ and $b \to dd\bar{s}$ decays

In the SM $b \to ssd$ and $b \to dd\bar{s}$ transitions are highly suppressed, proceeding through a $W$–up-type-quark box diagram (Huutu et al., 1998). Compared to the penguin transitions $b \to q\bar{q}q$ and $b \to q\bar{q}d$ they are additionally suppressed by the CKM factor $V_{td}V_{ts}^* \sim 3 \cdot 10^{-5}$ and are thus exceedingly small in the SM, with inclusive decay rates at the level of $10^{-12}$ and $10^{-14}$ for $b \to ssd$ and $b \to dd\bar{s}$, respectively (Fajfer et al., 2000).

These amplitudes can be significantly enhanced in SM extensions, for instance in MSSM with or without conserved $R$ parity, or in the models containing extra $U(1)$ gauge bosons. For example, the $b \to ssd$ decays $B^- \to K^-K^0\bar{s}$ and $B^- \to K^-\bar{K}^0\bar{s}$ can reach $\sim 6 \cdot 10^{-9}$ in the MSSM, while they are $\sim 7 \cdot 10^{-14}$ in the SM (Fajfer and Singer, 2000). Note that the flavor of $\bar{K}^0$ is tagged using the decay into the $K^-\pi^+$ final state. The $b \to dd\bar{s}$ transitions $B^- \to \pi^-K^0\bar{s}$ and $B^- \to \rho^-K^0\bar{s}$ can be enhanced from $\sim 10^{-16}$ in the SM to $\sim 10^{-6}$ in the presence of an extra $Z'$ boson (Fajfer et al., 2000). The relevant experimental upper limits are $B(B^- \to K^-K^-\pi^+) < 1.3 \times 10^{-6}$ and $B(B^- \to K^+\pi^-\pi^-) < 1.8 \times 10^{-6}$ (Aubert et al., 2003). Although these decays are background limited, improvements in these limits by almost two orders of magnitude can be expected from a SFF.

Although the observation of highly suppressed SM decays would provide the clearest signal for NP in these decay amplitudes, there are a number of other possible signals for such wrong sign kaons (Chun and Lee, 2003). For example, these amplitudes could invalidate the isospin relations given above, cause a non-zero CP asymmetry in $B^- \to K_S\pi^-$, induce a difference in rates between $B^0 \to K_S\pi^0$ and $B^0 \to K_L\pi^0$ or a difference in rates between $B^0 \to K_SK_S$ and $B^0 \to K_LK_L$, as well as resulting in a non-zero rate for $B^0 \to K_SK_L$.

C. CP asymmetry in $\pi^+\pi^0$

Since $\pi^+\pi^0$ is an $I = 2$ final state, only tree and EWP operators contribute to the $B^+ \to \pi^+\pi^0$ decay amplitude. Therefore, the direct CP asymmetry $A_{\text{CP}}(\pi\pi)$ is expected to be very small. Theoretical estimates range between $\leq 0.1\%$ (Beneke and Neubert, 2003; Gronau et al., 1999) to $O(1\%)$ (Cheng et al., 2005b). The current average of the $B$ factory results is $A_{\text{CP}}(B^+ \to \pi^+\pi^0) = 0.06 \pm 0.05$ (Aubert et al., 2007u; Barberio et al., 2007). Further theoretical studies of this observable would be desired to match the precision attainable at a SFF.

D. Semi-inclusive hadronic $B$ decays

Several semi-inclusive hadronic decays can be used to test the SM. For instance, the decays $B \to D^0X_{s,d}$ and $B \to D^0X_{s,d}$ have zero CP asymmetry in the SM, because they proceed through a single diagram, and provide a check for non-SM corrections to the value of $\gamma$ extracted from $B \to DK$ decays (Section IV.3). Another test is provided by flavor untagged semi-inclusive $B^\pm \to M^0(M^0)X_{s,d}$ decays, where $M^0$ is either an eigenstate of $s \leftrightarrow d$ switching symmetry, e.g. $K_S$, $K_L$, $\eta'$ or any charmonium state, or $M^0$ and $\bar{M}^0$ are related by the $s \leftrightarrow d$ transformation, e.g. $K^*0$, $\bar{K}^*0$, and one sums over the two states. In the SM the CP asymmetry of such semi-inclusive decays vanish in the SU(3) flavor limit (Gronau, 2006; Soni and Zupan, 2007) (this follows from the same considerations as for the direct CP asymmetry in $B \to X_{s+d}^\pi$ in Section VIII.A.1). The CP asymmetries are thus both doubly CKM ($\sim \lambda^2$) and $m_s/a_{\text{QCD}}$ suppressed.

If the tagged meson $M^0$ is light the CP asymmetries can be reliably calculated using SCET in the end-point region, where $M^0$ has energy close to $m_b/2$ (Chay et al., 2005a; 2005b; 2007). This gives CP asymmetries for $B^\pm \to M^0X_{s,d}$ below 1% for each of $M^0 = (K_S, \eta', (K^*0 + \bar{K}^*0)$ (Atwood and Soni, 1997; 1998b; Hou and Tseng, 1998; Soni and Zupan, 2007).

These modes can be studied at a SFF using inclusive reconstruction of the $X$ system by taking advantage of the recoil analysis technique that is possible due to the $e^+e^- \to \gamma(4S) \to B^+B^-$ production chain. The method has been implemented for measurement of inclusive charmless $B \to K^+(K^0)X$ decays (Aubert et al., 2006a), as well as having multiple applications for studies of e.g. $b \to s\gamma$ and $b \to s\ell^+\ell^-$. With SFF data samples, this class of important null tests can be probed to $O(1\%)$ precision.
E. Transverse $\tau$ polarization in semileptonic decays

The transverse polarization of tau leptons produces in $b \to c\tau\nu$ decays, defined as $p^T_\tau \equiv \vec{S}_\tau \cdot \vec{p}_\tau \times \vec{p}_X /|\vec{p}_\tau \times \vec{p}_X|$, where $\vec{S}_\tau$ is the spin of the $\tau$, is a very clean observable since it vanishes in the SM. On the other hand it is very sensitive to the presence of a CP-odd phase in scalar interactions. It is thus well suited as a probe of CP violating multi-Higgs doublet models [Atwood et al. 1993; Garisto, 1993; Grossman and Ligeti, 1993].

Since $p^T_\tau$ is a naive $T\gamma$-odd observable it does not require a non-zero strong phase. The fact that $p^T_\tau$ arises from an underlying CP-odd phase can be verified experimentally by comparing the asymmetry in $B$ with $\bar{B}$ decays whence it should change sign reflecting a change in the sign of the CP-odd phase.

In principle any charged lepton could be used for such searches. Indeed, the transverse muon polarization in kaon decays has been of interest for a very long time [Abe et al., 2004, 2006]. The advantage of using the tau lepton is that $\tau$ decays serve as self-analyzers.

In passing we mention that, as mentioned in Section VIII.C, the rates and differential distributions in $B \to D^{(*)}\pi\tau\nu$ decays are sensitive to contributions from charged Higgs exchanges [Kiers and Soni, 1997]. The first studies of these are being carried out at the B factories (Aubert et al., 2007; Matyja et al., 2007), though much larger data samples are needed for precise measurements. On the other hand, $a^T_{CP}$ is theoretically extremely clean, so that experimental issues are the only limiting factor. Thus, transverse polarization studies in these semitauonic decays will be a unique new possibility for exploration at a SFF.

VIII. RARE $b \to s\gamma$ AND $b \to s\ell^+\ell^-$ DECAYS

The decays $b \to s\gamma$ and $b \to s\ell^+\ell^-$ are forbidden at tree level in the Standard Model. They do proceed at loop level, through diagrams with internal W bosons and charge +2/3 quarks, which has several important implications. First, the $b \to s/d\gamma$ amplitudes are particularly sensitive to the weak couplings of the top quark – the CKM matrix elements $V_{tb}$, $V_{ts}$ and $V_{td}$. Along with $B \to \bar{B}$ mixing, these processes are the only (low energy) experimental probes of $V_{td}$, one of the least well-known CKM matrix elements. Second, the loop suppression of SM contributions makes them an important probe of possible contributions from new physics particles. As a consequence a great deal of theoretical and experimental work is dedicated to these decays.

In this Section we review the implications of the rare radiative decays for constraining the Standard Model parameters, and their relevance in new physics searches. We start by briefly reviewing the present theory status and then proceed to describe the observables of interest.

A. $B \to X_s/d\gamma$ decays

1. Inclusive $B \to X_s/d\gamma$ decays

The application of the effective Hamiltonian (5) to actual hadronic radiative decays requires knowledge of the matrix elements for the operators $O^T_i$ acting on hadronic states. This difficult problem can be addressed in a model independent way only in a limited number of cases.

In inclusive radiative decays $b \to s\gamma$, the operator product expansion (OPE) and quark-hadron duality can be used to make clean predictions for sufficiently inclusive observables: the inclusive rate, the photon energy spectrum or the hadronic invariant mass spectrum [Blok et al., 1994; Chay et al., 1990; Falk et al., 1994; Manohar and Wise, 1994]. These observables can be computed using the heavy quark expansion in $\Lambda_{QCD}/m_b$, where $\Lambda_{QCD} \sim 500$ MeV is the scale of strong interactions.

The starting point is the optical theorem, which relates the imaginary part of the forward scattering amplitude $T(E_\gamma) = i \int d^4x T(H_W, H_W)$ to the inclusive rate

$$\Gamma(B \to X_s\gamma) = \frac{1}{2M_B} \left(-\frac{1}{\pi}\right) \text{Im} \langle B|T(E_\gamma)|B\rangle. \quad (66)$$

Here $E_\gamma$ is the photon energy. In the heavy quark limit the energy release into hadronic final states is very large, so that the forward scattering amplitude $T(E_\gamma)$ is dominated by short distances $x \sim 1/m_b \to 0$. This implies that $T(E_\gamma)$, and thus the total $B \to X_s\gamma$ rate, can be expanded in powers of $\Lambda_{QCD}/m_b$ using OPE

$$-\frac{1}{\pi} \text{Im} \ T = O_0 + \frac{1}{m_b} O_1 + \frac{1}{m_b^2} O_2 + \cdots. \quad (67)$$

Here $O_j$ are the most general local operators of dimension $3 + j$ which can mediate the $b \to b$ transition. At leading order there is only one such operator $O_0 = \bar{b}b$. Its matrix element is known exactly from $b$ quark number conservation. The dimension 4 operators $O_1$ vanish by the equations of motion ([Chay et al., 1990]), while the matrix elements of the dimension-5 operators $O_2$ can be expressed in terms of two nonperturbative parameters

$$\lambda_1 = \frac{1}{2M_B} \langle \bar{B}|\bar{b}\gamma_5(iD)^2b|\bar{B}\rangle, \quad (68)$$

$$\lambda_2 = \frac{3}{2M_B} \langle \bar{B}|\bar{b}\gamma_5 \frac{g}{2} \sigma_{\mu\nu}G^{\mu\nu}\tau^a b|\bar{B}\rangle,$$

where $b_\nu$ is the static heavy quark field. The $B \to X_s\gamma$ decay rate following from the OPE (67) is thus

$$\Gamma(B \to X_s\gamma) = \frac{\alpha G_F^2}{16\pi^3} m_b^5 |\lambda_1| \times \langle C_{\gamma}(m_b) \rangle^2 \left[ 1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} \right]. \quad (69)$$
The leading term represents the parton level $b \rightarrow s\gamma$ decay width, which is thus recovered as a model-independent prediction in the heavy quark limit. The nonperturbative corrections to the LO result are doubly suppressed, by $A^2_{QCD}/m_b^2$. In a physical picture they arise from the so-called Fermi motion of the heavy quark inside the hadron, and from its interaction with the color gluon field inside the hadron. At each order in the $\Lambda_{QCD}/m_b$ expansion, these effects are parameterized in terms of a small number of nonperturbative parameters.

In the endpoint region of the photon spectrum, where $M_B - 2E_\gamma \sim \Lambda_{QCD}$, the heavy quark expansion in $\Lambda_{QCD}/m_b$ breaks down. It is replaced with a simultaneous expansion in powers of $\Lambda_{QCD}/m_b$ and $1-x$, where $x = 2E_\gamma/M_B$ [Bigi et al. 1994; Mannell and Neubert 1994 Neubert 1994]. In this region the invariant mass of the hadronic state is $M_X^2 \sim M_B \Lambda_{QCD}$. The photon spectrum is given by a factorization relation [Bauer et al. 2002 Korchemsky and Sterman 1994]

$$\frac{1}{\Gamma_0} \frac{d\Gamma(E_\gamma)}{dE_\gamma} = H(E_\gamma, \mu) S(k_+) * J(k_+ + m_b - 2E_\gamma),$$

where $H(E_\gamma, \mu)$ contains the effects of hard loop momenta, $J$ is the jet function describing the physics of the hard-collinear loops with $M_B \Lambda_{QCD}$ off-shellness, $S(k_+)$ is the shape function parameterizing bound-state effects in the $B$ meson, while the star denotes a convolution over soft momentum $k_+$. The nonperturbative shape function has to be either extracted from data or modelled [commonly used shape function parameterizations can be found in [Bosch et al. 2004b].

The present world average for the inclusive branching fraction is [Aubert et al. 2005b; Barberio et al. 2007; Chen et al. 2004; Hoppenburg et al. 2004c; Soares, 2000; Soares, 2007]

$$B^{\text{exp}}(B \rightarrow X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}} = 3.55^{+0.09}_{-0.10} \times 10^{-4},$$

The errors shown are due to the shape function, experimental (statistical and systematic combined), and the contamination from $b \rightarrow d\gamma$ events, respectively.

On the theory side, the SM prediction for the inclusive branching fraction has recently been advanced to NNLO [Misiak et al. 2007], with the result

$$B(B \rightarrow X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}} = (3.15 \pm 0.23) \times 10^{-4},$$

where the error combines in quadrature several types of uncertainties: nonperturbative (5%), parametric (3%), higher-order (3%) and $m_c$-interpolation ambiguity (3%). The leading unknown nonperturbative corrections to this prediction arise from spectator contributions with one hard gluon exchange. They scale like $O(\alpha_s \Lambda_{QCD}/m_b)$ in the limit $m_c \ll m_b/2$ and like $O(\alpha_s^2 \Lambda_{QCD}^2/m_b^2)$ in the limit $m_c \gg m_b/2$. An alternative estimate, with the photon energy cut dependence resummed using an effective theory formalism, gives [Becher and Neubert 2007]

$$B(B \rightarrow X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}} = (2.98^{+0.13}_{-0.17}) \text{pert} \pm 0.16 |\text{had} \pm 0.11 |\text{pars} \pm 0.09 |m_c| \times 10^{-4}.$$  

This result is about 1.4σ below the central value of the experimental measurement.

The $B \rightarrow X_s\gamma$ branching ratio is an important constraint on new physics models as discussed in Section III. At present the largest error limiting the precision of the test arises from experimental uncertainties. Furthermore, using the statistics that would be available at a Super Flavor Factory, it would be possible to reduce the photon energy cut, which can help improve the theoretical understanding. Theoretical uncertainties will, however, ultimately limit the precision, to about the 5% level.

Another important observable in weak radiative decays is the direct CP asymmetry, often called the partial rate asymmetry (PRA)

$$A_{CP} = \frac{\Gamma(\bar{B} \rightarrow X_s\gamma) - \Gamma(B \rightarrow X_s\gamma)}{\Gamma(B \rightarrow X_s\gamma) + \Gamma(B \rightarrow X_s\gamma)},$$

where $\bar{X}$ is the CP conjugate of the $X$ state.

In general, decay amplitudes can be written as the sum of two terms with different weak phases (see also Eq. (59))

$$A(\bar{B} \rightarrow X_s\gamma) = P + e^{i\psi} A = P (1 + \varepsilon A e^{i(\delta + \psi)}),$$

where $\varepsilon_A e^{i\delta} = A/P$, and $\delta$ and $\psi$ are the strong and weak phase differences. One finds for the direct CP asymmetry

$$A_{CP} = \frac{2\varepsilon_A \sin \delta \sin \psi}{1 + 2\varepsilon_A \cos \delta \cos \psi + \varepsilon_A^2},$$

in agreement with the well-known result that for $A_{CP} \neq 0$ both strong and weak phase differences need to be nonzero [see, e.g. [Bandier et al. 1979]]. The direct CP asymmetry in $b \rightarrow s\gamma$ is then suppressed by three concuring small factors: i) CKM suppression by $\varepsilon_A \propto |\lambda_u^{s}\lambda_s^{c}|/|\lambda_d^{s}\lambda_s^{c}| \sim \lambda^2$, ii) a factor of $\alpha_s(m_b)$ required in order to generate the strong phase, and iii) a GIM suppression factor $(m_c/m_b)^2$, reflecting the fact that in the limit $m_c = m_u$ the charm and up quark penguin loop contributions cancel in the CP asymmetry.

The OPE approach discussed above can be used to compute also the $B \rightarrow X_s\gamma$ direct CP asymmetry [Kagan and Neubert 1998; Kiers et al. 2000; Soares 1991]. The most recent update by Hurth et al. (2005) gives

$$A_{CP}(B \rightarrow X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}} = (0.44^{+0.15}_{-0.10}|m_c/m_b|^{0.03}_{0.03} |\text{CKM}^{+0.19}_{-0.09} |\text{RG})%. $$

This can be compared to the current world average [Aubert et al. 2004b; Barberio et al. 2007; Coan et al. 2001; Nishida et al. 2004]

$$A_{CP}(b \rightarrow s\gamma) = 0.004 \pm 0.036,$$
which is compatible with a vanishing or very small direct CP asymmetry as expected in the SM. The experimental uncertainty is still an order of magnitude greater than the theory error, so that a dramatic improvement in the precision of this SM test can be achieved with a SFF. The ultimate precision is expected to be limited by experimental systematics at about the same level as the current theory error.

The theoretical error can be further reduced if one considers an even more inclusive B → X_{s,d}γ decay. In the U-spin symmetry limit, the inclusive partial rate asymmetries in \( B^\pm \rightarrow X_sγ \) and \( B^\pm \rightarrow X_dγ \) are equal and of opposite signs, \( \Delta Γ(B^\pm \rightarrow X_sγ) = −\Delta Γ(B^\pm \rightarrow X_dγ) \) \cite{Hurth and Mann}. A similar relation holds also for neutral B decays, but with corrections due to annihilation and other 1/m_{b} suppressed terms. In the SU(3) limit (\( m_d = m_s \)) therefore the inclusive untagged CP asymmetry \( A_{\text{CP}}(B \rightarrow X_{s,d}γ) \) vanishes in the SM, while the leading SU(3) breaking correction is of order \( (m_s/m_b)^2 \sim 10^{-4} \) \cite{Hurth et al.}. The inclusive untagged CP asymmetry thus provides a clean test of the SM, with very little uncertainty. Any measurement of a nonzero value would be a clean signal for NP.

A first measurement of the untagged CP asymmetry has been made by BABAR \cite{Aubert et al.}:

\[ A_{\text{CP}}(B \rightarrow X_{s,d}γ) = −0.110\pm0.115^{\text{stat}}\pm0.017^{\text{sys}}. \] (79)

A significant reduction of the uncertainty is necessary to provide a stringent test of the SM prediction. A SFF will be able to measure this quantity to about 1% precision.

2. Exclusive B → V_{s,d}γ decays

The exclusive decays such as \( B \rightarrow K^*γ \) or \( B \rightarrow Kγ, Kπγ \) are experimentally much more inclusive than the characteristic channels due to simpler event identification criteria and background elimination. They are, however, more theoretically challenging which limits their usefulness for NP searches. In this subsection we review the theoretical progress on \( B \rightarrow V_{s,d}γ \) branching ratios and direct CP asymmetries. Theoretically clean observables related to photon polarization are then obtained in the next subsection. The extraction of CKM parameters from \( B \rightarrow V_{s,d}γ \) decays is reviewed in Section VIII.C.

The \( B \rightarrow Vγ \) decays are dominated by the electromagnetic dipole operator \( O_{Tγ} \), Eq. 6. Neglecting for the moment the remaining smaller contributions, this gives

\[ B(B \rightarrow K^*γ) = \tau_B \frac{\alpha G_F^2 |\lambda^{(s)}_\gamma|}{16π^4} (C_{Tγ}^2 m_b^2 E_{\gamma}^3 T_1(0))^2, \] (80)

where \( T_1(q^2) \) is a tensor current form factor. Its nonperturbative nature is at the heart of theoretical uncertainties in \( B \rightarrow Vγ \) decay. In principle it can be obtained model independently from lattice QCD \cite{Bernard et al.}, with first unquenched studies presented in \cite{Becirevic et al.}. Lattice QCD results are obtained only at large values of the momentum transfer \( q^2 \sim m_b^2 \) and extrapolation to low \( q^2 \) then introduces some model dependence. Using the BK parametrization \cite{Becirevic and Kaidalov}, \cite{Becirevic et al.} find \( T_{1}^{K^*}(0) = 0.267 \pm 0.021 \) and \( T_{1}^{K}(0) = 0.333 \pm 0.028 \).

Another nonperturbative approach is based on QCD sum rules, where OPE is applied to correlators of appropriate interpolating operators. Relying on quark-hadron duality the OPE result is related to properties of the hadronic states. The heavy-to-light form factors in the large energy release region can be computed from a modification of this approach, called light-cone QCD sum rules. Using this framework \cite{Ball and Zwicky} find \( T_{1}^{K^*}(0) = 0.267 \pm 0.021 \) and \( T_{1}^{K}(0) = 0.333 \pm 0.028 \).

Relations to other form factors follow in the large energy limit \( E_M > \Lambda_{QCD} \). In this limit the heavy-to-light \( B \rightarrow V \) form factors have been studied in QCDF \cite{Beneke and Feldmann}, and in SCET \cite{Bauer et al., Beneke and Feldmann, Hill et al.} at leading order in \( \Lambda_{QCD}/E_M \). The main result is a factorization formula for heavy-to-light form factors consisting of perturbatively calculable factorizable terms and a nonfactorizable soft term common to several form factors. The analysis can be systematically extended to higher orders.

Eq. (80) neglects the contributions from the four-quark operators \( O_{1-6} \) and the gluonic dipole operator \( O_{8g} \) in the weak hamiltonian, Eq. (5). These contributions are of two types: i) short-distance dominated loop corrections absorbed into effective Wilson coefficients in factorization formula and ii) weak annihilation (WA) type contributions, Fig. 14 \cite{Ali and Parkhomenko, Beneke et al., Bosch and Buchalla, Descotes-Genon and Sachrajda}. The WA amplitude is power suppressed, \( O(\Lambda_{QCD}/m_b) \), but occurs at tree level and is thus also relatively enhanced. It is proportional to \( \lambda^{(s)}_\gamma \) and is CKM suppressed in \( b \rightarrow sγ \) transitions, but not in \( b \rightarrow dγ \) decays, for instance in \( B \rightarrow ργ \) \cite{Atwood et al.}. At LO in \( \alpha_s \) and \( \Lambda_{QCD}/m_b \) the WA amplitude factorizes as shown in \cite{Beneke et al., Bosch and Buchalla, Grinstein and Pirjol}.

Direct CP asymmetries in exclusive modes such as \( B \rightarrow K^*γ \) can be estimated using the factorization formula. This gives \( A_{\text{CP}}(B \rightarrow K^*γ) = −0.5\% \) \cite{Bosch and Buchalla}, in agreement with the experimental world average \( A_{\text{CP}}(B \rightarrow K^*γ) = −0.5\% \)
The value of $f$ depends on the SM ratio of the wrong polarization decay that would signal the presence of NP. The precision of the test can be overcome by exploiting the cancellation of partial rate asymmetries in the U-spin limit [Hurth and Mannel, 2001], but symmetry breaking corrections are difficult to compute in a clean way. Other possible uses of exclusive radiative decays to test the SM are discussed below.

3. Photon polarization in $b \to s \gamma$

In the SM the photons emitted in $b \to s \gamma$ are predominantly left-handed polarized, and those emitted in $b \to \bar{s} \gamma$ are predominantly right-handed, in accordance with the form of electromagnetic operator $O_{\gamma\gamma}$, Eq. (7). In the presence of NP the decay into photons of opposite chirality can be enhanced by a chirality flip on the internal heavy NP lines. This observation underlies the proposal to use the mixing-induced asymmetry in $B^0(t) \to f\gamma$ decays as a null test of the SM [Atwood et al., 1997]. The value of $S_{f\gamma}$ parameter significantly away from zero would signal the presence of NP. The precision of the test depends on the SM ratio of the wrong polarization decay amplitude $A(B \to f_q\gamma_{L,R})$ and the right polarization decay amplitude $A(B \to f_{\bar{q}}\gamma_{L,R})$ for given $f_q$ ($q = d, s$)

$$r_f e^{i(\phi_q + \delta_f)} = \frac{A(B \to f_q\gamma_R)}{A(B \to f_q\gamma_L)}. \quad (81)$$

Here $\phi_q$ is a weak phase, and $\delta_f$ a strong phase. For a CP eigenstate $f$ the resulting $B^0(t) \to f\gamma$ in terms of $r_f, \delta_f$ is given in Eq. (80). Keeping only the dominant electromagnetic penguin contribution one finds a very small ratio $r_f = m_q/m_b$ and $\phi_q = \delta_f = 0$, independent of the final state $f_q$. This estimate can be changed, however, by hadronic effects [Grinstein et al., 2005]. The right-handed photon amplitude receives contributions from charm- and up-quark loop graphs in Fig. [15] with the four-quark operators $O_{1-6}$ in the weak vertex. The largest contributions come from the operator $O_3^\gamma$.

For inclusive $B \to X_s\gamma_R$ decays one finds $r \approx 0.11$ when integrating over the partonic phase space with $E_\gamma > 1.8$ GeV [Grinstein et al., 2003a]. This estimate includes the numerically important $O(\alpha_s^2/\Lambda^2_{QCD})$ correction. Note that the obtained $r$ is much larger than the estimate from electromagnetic penguins only $r \sim m_s/m_b \sim 0.02$.

An effect of similar size is found for $B \to V_{q\gamma}$ decays using SCET, following from a nonfactorizable contribution suppressed by $\Lambda_{QCD}/m_b$ [Ligeti et al., 1997]. By dimensional arguments the estimate for the $r_{K^*/\rho}$ ratio is

$$r_f = \frac{m_q}{m_b} + c_f \frac{C_2}{3C_{\gamma\gamma}} \frac{\Lambda_{QCD}}{m_b}. \quad (82)$$

Here $|c_f|$ is a dimensionless parameter of order one that depends on the final hadronic state $f$. The second term remains in the limit of a massless light quark $m_q \to 0$. Although power suppressed, it is enhanced by the large ratio $C_2/C_{\gamma\gamma} \sim 3$. A dimensional estimate is thus $r_f \sim 0.1$, which would translate into an asymmetry (S) of about 10%, much larger than the LO estimate of $m_q/m_b$ that gives an asymmetry in $b \to s$ transitions of around 3%.

A more reliable estimate requires a challenging dynamical computation of the nonlocal nonfactorizable matrix element. However, these theoretical difficulties need not stand in the way of experimental progress as there is data driven method to separate the SM contamination by studying the dependence of the asymmetry on the final state [Atwood et al., 2003] as we discuss below.

First steps in the direction of explicit model calculations find $S(B \to K^*\gamma) = -0.022 \pm 0.015$ using QCD sum rules [Ball and Zwicky, 2006a], consistent with the leading order estimate. In particular, expanding the relevant nonlocal operator in powers of $\Lambda_{QCD}/m_b^2 \sim 0.6$ and then keeping only the first term, they obtain for $f = K^*, \rho$ [see also Khodjamirian et al., 1997]

$$r_f = \frac{m_q}{m_b} - \frac{C_2}{C_{\gamma\gamma}} \frac{L - \bar{L}}{36m_b m_\pi^2 T_\gamma^{BV}(0)} = \frac{m_q}{m_b} - (0.004 \pm 0.007), \quad (83)$$

with $L, \bar{L}$ parametrizing $B \to K^*$ matrix elements of the nonlocal operator. Another calculation using pQCD obtained a very similar result for the asymmetry, $S(B \to K_s\pi^0\gamma) = -0.035 \pm 0.017$ [Matsumori and Sanda, 2006].

Experimentally, the photon polarization can be measured from time dependent $B^0(t) \to f\gamma$ decay utilizing the interference of $B - \bar{B}$ mixing with the right- and left-handed photon amplitudes [Atwood et al., 1997]. In particular, taking the time-dependent asymmetry summed over the unobserved photon polarization

$$A_{CP}(t) = \frac{\Gamma(B^0(t) \to f\gamma_{L+R}) - \Gamma(B^0(t) \to f\gamma_{L-R})}{\Gamma(B^0(t) \to f\gamma_{L+R}) + \Gamma(B^0(t) \to f\gamma_{L-R})} = S_{f\gamma} \sin(\Delta mt) - C_{f\gamma} \cos(\Delta mt), \quad (84)$$

the two coefficients are

$$S_{f\gamma} = \frac{2 \text{Im} \frac{2}{\rho_B} A_L A^*_L + A_R A^*_R}{|A_L|^2 + |A_L|^2 + |A_R|^2 + |A_R|^2}, \quad (85)$$

$$C_{f\gamma} = \frac{|A_L|^2 - |A_L|^2 + |A_R|^2 - |A_R|^2}{|A_L|^2 + |A_L|^2 + |A_R|^2 + |A_R|^2}.$$
where $\bar{A}_{L,R} \equiv A(B \to f\gamma_{L,R})$ and $A_{L,R} \equiv A(B \to f\gamma_{L,R})$. Note that $S_{f\gamma} = 0$ when the “wrong” polarization amplitudes $\bar{A}_R$ and $A_L$ vanish. This can be made more transparent in a simplified case where $f$ is a CP eigenstate with eigenvalue $\eta_{CP}(f)$, while also assuming that the $B \to f\gamma$ transitions are dominated by a single weak phase $\phi_f$, so that $\bar{A}_{L,R} = e^{i\phi_f} \bar{a}_{L,R}$ and $A_{L,R} = e^{-i\phi_f} \eta_{CP}(f) \bar{a}_{L,R}$, where $a_{L,R}$ and $\bar{a}_{L,R}$ are strong amplitudes. Then

$$S_{f\gamma} = \eta_{CP}(f) \frac{2f \cos \delta_f}{1 + \tau_f^2} \text{Im} \left[ \left( \frac{q}{p} \right)_B e^{2i\phi_f} \right],$$

and $C_{f\gamma} = 0$. Here $r_f \exp(i\delta_f) = A_R/A_L$ as in Eq. S1. The asymmetry $S_{f\gamma}$ vanishes in the limit of 100% left-handed photon polarization ($r_f = 0$).

The value of $S_{f\gamma}$ depends crucially also on the mismatch between the weak phase $\phi_f$ of the decay amplitude and the $B_{d,s}$ mixing phases, $(q/p)_{B_d} = -e^{-i(2\beta)}$ and $(q/p)_{B_s} = -1$. There are two distinct categories. For $B_d \to f\gamma$ and $B_s \to f\gamma$ decays this phase difference is large ($2\beta$) and $S_{f\gamma} = \frac{4f \cos \delta_f}{1 + \tau_f^2} \sin 2\beta$ is suppressed only by $r_f$. For $B_d \to f\gamma$ and $B_s \to f\gamma$ decays, on the other hand, the weak phase difference vanishes so that in SM $S_{f\gamma} = 0$ with negligible theoretical uncertainty. For NP to modify these predictions it has to induce large right-handed photon polarization amplitude, while for $B_d \to f\gamma$ and $B_s \to f\gamma$ decays also a new weak phase is needed to have $S_{f\gamma} \neq 0$.

Current results give a world average $S_{K^{\pm}\gamma} = -0.19 \pm 0.23$ [Aubert et al. 2007], [Barberio et al. 2007], [Ushiroda et al. 2006], and the first measurement of time-dependent asymmetries in $b \to d\gamma$ decays has recently been reported, $S_{\rho\gamma} = -0.83 \pm 0.65 \pm 0.18$ [Ushiroda et al. 2007]. These are compatible, within experimental errors, with the SM predictions. A SFF could reduce the uncertainty on the former to about 2–3% and on the latter to about 10% (see Table XI).

Measurements have also been made over an extended range of $K\pi$ invariant mass in $B \to K_S\pi^0\gamma$. In multibody exclusive radiative decays, a nonvanishing right-handed photon amplitude can be present at leading order in the $1/m_b$ expansion. However, using a combination of SCET and chiral perturbation theory (ChPT) methods applicable in kinematical region with one energetic kaon and a soft pion $r_{K\pi}$ was found to be numerically less than 1% due to kinematical suppression [Grinstein and Pirjol 2003, 2006a]. With the SFF data, the multibody radiative decays will be most useful to search for SM corrections to the photon polarization. These effects depend on the Dalitz plot position, in contrast to NP effects, which should be universal [Atwood et al. 2003]. The LO dipole moment operator (as well as NP) would give rise to an asymmetry that is independent of the energy of the photon whereas the soft gluon effects will give rise to an asymmetry that depends on photon energy. Thus there is a model independent, completely data driven method to search for NP effects by studies of time dependent asymmetries. In addition, further decay modes, such as $B \to K_S\eta\gamma$, $B \to K_S\pi^+\pi^-\gamma$ and $B \to K_S\gamma$ can also be used [Atwood et al. 2003] in a very similar fashion.

Other approaches for probing the photon polarization in $b \to s\gamma$ decays have been suggested and can be employed at a SFF. One powerful idea is to relate the photon polarization information to angular distributions of the final state hadrons. Examples relevant for a SFF are $B \to K\pi\gamma$ [Gronau et al. 2002, Gronau and Pirjol 2002], and $B^{\pm} \to K^\mp\phi\gamma$ [Atwood et al. 2007a, Orlovsky and Shevchenko 2007]. Similar tests have been suggested also using $\Lambda_b$ decays, such as $\Lambda_b \to \Lambda\gamma$ [Gremm et al. 1995, Hiller and Kagan 2002, Hiller et al. 2007, Mannell and Recksiegel 1997] and $\Lambda_c \to pK\gamma$ (Lesger and Schiessinger 2007).

We consider $B \to X_s\gamma$ decays, where the final hadronic state $X_s = K\pi\pi, KKK$ originates from the strong decay of resonance $K_{res}$, produced in the weak decay $B \to K_s\gamma$. The lowest lying vector state, the $K^*$, cannot be used for this purpose, since the $K^*$ polarization is not observable in its two-body decay $K^* \to K\pi$. This is due to the fact that it is impossible to form a T-odd quantity from only two vectors, the photon momentum and the $K^*$ momentum, in the $K^*$ rest frame.

The photon polarization can then be measured through higher resonance $K_{res} \to K\pi\pi$ decays. The angular distribution of the decay width in $K_{res}$ rest frame is [Gronau and Pirjol 2002]

$$\frac{d^2r}{d\cos\theta} = |c_1|^2 \left[ 1 + \cos^2 \tilde{\theta} + 4P_{\gamma} R_{\gamma} \cos \tilde{\theta} \right]$$

$$+ |c_2|^2 \left[ \cos^2 \tilde{\theta} + \cos^2 2\theta + 12 P_{\gamma} R_{\gamma} \cos \theta \cos 2\tilde{\theta} \right]$$

$$+ |c_3|^2 B_{K^*}(s) \sin^2 \tilde{\theta}$$

$$+ \left[ c_{12}(3 \cos^2 \tilde{\theta} - 1) + P_{\gamma} c_{12} \cos^3 \tilde{\theta} \right].$$

Here $\tilde{\theta}$ is the angle between the direction opposite to the photon momentum ($-\vec{q}$) and the vector $\vec{p}_{\pi^-\pi^0} \times \vec{p}_\gamma$ (the pions are ordered in terms of their momenta). The first three terms in Eq. S7 correspond respectively to decays through $K_{res}$ resonances with $J^{P} = 1^+, 2^+$ and $1^-$, while the last terms come from $1^+ - 2^+$ interference. The hadronic parameters $R_{\gamma}$ can be computed from the Breit-Wigner resonant model [Gronau et al. 2002, Gronau and Pirjol 2002]. The $K_1(1400)$ resonance decays predominantly to $K^*\pi$. The relevant parameters in $R_{\gamma}$ are then fixed by isospin, leading to a precise determination $R_1 = 0.22 \pm 0.03$. Thus, measurements of the angular distribution Eq. S7 restricted to the $K_1(1400)$ mass range can be used to extract the photon polarization parameter $P_{\gamma}$. So far only an upper bound on $B(B \to K_1(1400)\gamma) < 1.5 \times 10^{-5}$ exists [Yang et al. 2004]. The use of the narrow resonance $K_1(1270)$, with a larger branching ratio $B(B \to K_1(1270)\gamma) = (4.3 \pm 1.2) \times 10^{-5}$ [Aubert et al. 2007b, Yang et al. 2004], may be more advantageous experimentally. A drawback is the estimate of $R_1$ in which a strong phase between $K_1(1270) \to K^*\pi$ and $K_1(1270) \to K\rho$ decay amplitudes needs to be obtained from an independent measurement.
The method outlined above works only for certain charge states, for which two $K^+\pi$ channels interfere to produce the up-down asymmetry in $\cos \theta$. These channels are $K^0\pi^+\pi^0$, where the interfering channels are $K^{*+}\pi^0$ and $K^{*0}\pi^+$, and $K^+\pi^-\pi^0$, where the interfering channels are $K^{*+}\pi^-$ and $K^{*0}\pi^0$.

**B. $B \to X_{s/d}\ell^+\ell^-$ and $B \to X_{s/d}\ell^+\ell^-$ decays**

The rare $B \to X_s\ell^+\ell^-$ decays form another class of FCNC processes, which proceed in the SM only through loop effects. The richer structure of the final state allows one to study also the dilepton invariant mass, the forward-backward asymmetry, and various polarization observables. We discuss these predictions, considering in turn the exclusive and inclusive channels.

1. **Inclusive $B \to X_s\ell^+\ell^-$ decays**

In inclusive $B \to X_{s/d}\ell^+\ell^-$ decays there are three distinct regions of dilepton invariant mass $q^2 = (p_\ell^+ + p_\ell^-)^2$: (i) the low $q^2$ region, $q^2 < 6$ GeV$^2$, (ii) the high $q^2$ region $q^2 > 12$ GeV$^2$, and (iii) the charm resonance region $q^2 \sim (6 - 12)$ GeV$^2$. In the intermediate region (iii) $c\bar{c}$ resonances couple to the dilepton pair through a virtual photon, leading to nonperturbative strong interaction effects which are difficult to compute in a model independent way.

In the low-$q^2$ and high-$q^2$ regions, a model independent computation of the decay rate is possible using an OPE and heavy quark expansion, similar to that used for the rare radiative decays discussed in Section V. In this case, QCD corrections have been evaluated at NNLO in the OPE and heavy quark expansion, similar to that used for the effective weak Hamiltonian of Eq. (5). This suffices to determine the sizes and signs of all the relevant coefficients, probing in this way NP effects. At leading order they have a general structure

\[ H_T(q^2) \propto 2(1 - s)^2 s \left[ (C_9 + \frac{1}{2} C_{7\gamma})^2 + C_{10}^2 \right], \]

\[ H_A(q^2) \propto -4(1 - s)^2 s C_{10} \left( C_9 + \frac{1}{2} C_{7\gamma} \right), \]

\[ H_L(q^2) \propto (1 - s)^2 \left[ (C_9 + 2 C_{7\gamma})^2 + C_{10}^2 \right], \]

where $s = q^2/m_b^2$. The modified Wilson coefficients $C_{7\gamma,9,10}$ are $\mu$ independent linear combinations of the Wilson coefficients $C_{7\gamma,9,10}$ and $C_{1,...,6,8,9}$ in weak Hamiltonian of Eq. (5). They are related to the NNLO “effective” Wilson coefficients $C_{eff}^{\gamma}$ calculated in [Asatrian et al. 2002, Beneke et al. 2001, Ghinculov et al. 2004].
Note that in $H_T$ and $H_A$ the coefficient $C_7$ is enhanced by a $1/s$ pole so that measuring the dilepton mass dependence gives further information. Also, $H_A(q^2)$ has a zero at $q^2_0$. The existence of a zero of the FBA and the relative insensitivity to hadronic physics effects was first pointed out for exclusive channels (Burdman, 1998), and subsequently extended also to the inclusive channels (Ali et al. 2002, Ghinculov et al. 2003). In the SM the zero appears in the low $q^2$ region, sufficiently away from the charm resonance region to allow a precise computation of its position in perturbation theory. The value of the zero of the FBA is one of the most precisely calculated observables in flavor physics with a theoretical error at the order of 5%. For $B \rightarrow X_s \mu^+ \mu^-$, for instance, the improved NNLO prediction is $(q^2_0)_{\mu\mu} = (3.50 \pm 0.12)$ GeV$^2$ (Huber et al. 2007), where the largest uncertainty is due to the remaining scale dependence (0.10). The position of the zero is directly related to the relative size and sign of the Wilson coefficients $C_7$ and $C_9$. Thus it is very sensitive to new physics effects in these parameters. This quantity has not yet been measured, but estimates show that a precision of about 5% could be obtained at a SFF (Bona et al. 2007c, Hashimoto et al. 2004).

2. Exclusive $B \rightarrow X_s \ell^+ \ell^-$ and $B \rightarrow X_s \nu \bar{\nu}$ decays

The channels $B \rightarrow M \ell^+ \ell^-$ are experimentally cleaner than inclusive decays, but more complicated theoretically. The $B \rightarrow M$ transition amplitude depends on hadronic physics through form factors. The theoretical formalism described in Sec. [VII.A.2] for exclusive radiative decays can be applied to this case as well.

The simplest are the decays with one pseudoscalar meson, such as $B \rightarrow K^+ \ell^- \ell^-$ or $B \rightarrow \pi^+ \ell^- \ell^-$. Unlike $B \rightarrow K/\pi \gamma$ decays that are not possible due to angular momentum conservation, the dilepton decays are allowed since the dilepton can carry zero helicity. Especially interesting for NP searches is the angular dependence on $\theta_+\ell$ the angle between the $\vec{q}$ and $\vec{B}$ momenta in the dilepton rest frame. In the SM the dependence is simply $d\Gamma \sim \sin^2 \theta_+$. Allowing for scalar and pseudoscalar couplings to the leptons, which are possible in extensions of the SM, the general angular distribution is (Bobeth et al. 2001)

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_+} = \frac{3}{4} (1 - F_S) \sin^2 \theta_+ + \frac{1}{2} F_S + A_{FB} \cos \theta_+. \quad (92)$$

The coefficient $F_S$ receives contributions from the scalar and pseudoscalar couplings to the leptons, while $A_{FB}$ depends on the interference between the vector and scalar couplings. As these terms vanish in the SM, their measurement is a null test sensitive to new physics from scalar and pseudoscalar penguins - see (Bobeth et al. 2007) for a detailed study. The first measurement of these parameters has been carried out in $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays by BaBar (Aubert et al. 2006c). The results are compatible with zero: $A_{FB} = 0.15^{+0.21}_{-0.23} \pm 0.08$ and $F_S = 0.81^{+0.58}_{-0.61} \pm 0.46$, where the first error is statistical and the second systematic. These measurements could become an order of magnitude more precise, and measure or set tight bounds on coefficients of NP operators which can produce these asymmetries.

We turn next to the decays with a vector meson in the final state, such as $B \rightarrow K^* \ell^+ \ell^-$ and $B \rightarrow \rho^+ \ell^- \ell^-$. Since vector mesons carry a polarization, the final state has a more complex structure. The $K^*$ decays to $\pi\ell$, and the final state is specified by three angles defined as in Fig. [17] After integrating over $(\phi, \theta_+ \ell)$ the rate is described by three functions of $q^2$ as in the inclusive case, Eq. (91), with the difference that the Wilson coefficients $C_{7,9,10}$ are also multiplied by $B \rightarrow K^*$ form factors. As in inclusive case, the transverse helicity amplitudes are dominated by the photon pole in the low $q^2$ region. In the high $q^2$ region, the $C_{9,10}$ terms dominate the amplitudes. Fig. [18] shows results for the decay rate and the FBA in the exclusive mode $B \rightarrow K^* \ell^+ \ell^-$. (Beneke et al. 2001). Due to form factor uncertainties the determination of the Wilson coefficients $C_{7,9,10}$ and the resulting NP constraints have substantially larger theoretical errors than the ones following from the inclusive decays (compare for instance Fig. [16] with Fig. [18]).

In the large recoil limit the $B \rightarrow K^*/(\rho^+ \ell^+ \ell^-)$ amplitudes satisfy factorization relations at leading order in $\Lambda/m_b$ (Bauer et al. 2003, Beneke and Feldmann 2004, Beneke et al. 2001, Hill et al. 2004). These factorization relations reduce the number of unknowns by express-
the observation of the Sudakov logs in SCET (Ali et al. 2003). The solid center line shows the next-to-leading order result, and the dashed line shows the leading order result. The band reflects all theoretical uncertainties from parameters and scale dependence combined, with most of the uncertainty due to the form factors.

ing the amplitudes as combinations of soft overlap factors $c_{\perp}^{B\ell}, c_{\parallel}^{B\ell}$ and factorizable contributions, multiplied with hard coefficients. The factorization relations predict that in the SM the right(left)-handed helicity amplitudes for $B(B \to K^{\ast}\ell^{\pm}\ell^{-})$ are power suppressed. Any non-standard chirality structure could change this. A second prediction in the large recoil limit is that the left-handed helicity amplitude $H_{\ell}^{(V)}(q^{2})$ has a zero at dilepton invariant mass $q_{0}^{2}$. In the SM this is predicted to be $q_{0}^{2}[K^{\ast}] = (4.36 \pm 0.33) \text{ GeV}^{2}$, $q_{0}^{2}[K^{\ast\ast}] = (4.15 \pm 0.27) \text{ GeV}^{2}$. This result was improved recently by including the resummation of the Sudakov logs in SCET (Ali et al. 2003), reducing the scale dependence uncertainty. The measurement of $q_{0}^{2}$ can be translated into a measurement of $\text{Re}(C_{\gamma}/C_{5})$, up to a correction depending on the ratio of two form factors $V(q^{2})/T_{1}(q^{2})$, which has been computed in factorization (Beneke and Yang, 2006). Whether the soft overlap and the factorizable contributions in these form factors are comparable or not is still a subject of discussion, and may lead to larger errors than usually quoted in the literature (Lee et al. 2007). Additional uncertainty can be introduced by the $\Lambda/m_{b}$ power corrections.

Various other observables are accessible in $b \to s\ell^{\pm}\ell^{-}$ decays, including time-dependent (Kim and Yoshikawa, 2007) and transverse polarization asymmetries (Kruger and Matias, 2005; Lunghi and Matias, 2007). These provide additional possibilities to probe the suppression of right-handed amplitudes and to search for NP operators with non-standard chirality at a SFF. We note the presence of possible SM contamination to these observables due to $O(1)$ contributions to the right-handed amplitude in the multibody channel $\bar{B} \to K\pi\ell^{\pm}\ell^{-}$ in the soft pion region (Grinstein and Pirjol, 2006b). This is similar to the effect discussed above for $B \to K\pi\gamma$, and could be reduced by applying phase space cuts on the pion energy.

Further observables are accessible in the case with massive leptons, $b \to s\tau^{+}\tau^{-}$. The $\tau$ polarization asymmetry

$$P_{\tau}(q^{2}) = \frac{d\mathcal{B}_{\lambda=-1} - d\mathcal{B}_{\lambda=+1}}{d\mathcal{B}_{\lambda=-1} + d\mathcal{B}_{\lambda=+1}},$$

integrated over the region $q^{2}/m_{\tau}^{2} \geq 0.6$, is about $-48\%$ in the SM, but NP effects can change this prediction (Dai et al., 1997; Hewett, 1996). No experimental studies of $b \to s\tau^{+}\tau^{-}$ decays exist, making predictions of the SFF sensitivity unreliable. However, it appears that exclusive modes could be measured.

Another related mode is $b \to sv\bar{\nu}$, mediated in the SM through the box and $Z$ penguin diagrams, which are matched onto the operator $O_{11}$. In extensions of the SM, additional diagrams can contribute, such as Higgs-mediated penguins in models with an extended Higgs sector, and models with modified $bsZ$ couplings (Bird et al., 2004; Grossman et al., 1994). The SM expectation for the branching fractions of these modes is $B(B \to X_{s}v\bar{\nu}) \sim 4 \times 10^{-5}$ (Buchoff et al., 1996), and $B(B \to X_{d}v\bar{\nu}) \sim 2 \times 10^{-6}$. The dominant exclusive modes are $B \to K^{\ast+}lv\bar{\nu}$, which we expected to occur with branching fractions of about $10^{-6}$. Present data give only an upper bound for $B(B^{+} \to K^{+}v\bar{\nu})$ at the level of $40 \times 10^{-6}$ (Aubert et al., 2005d; Chen et al., 2007a), which is one order of magnitude above the SM prediction. These modes are very challenging experimentally because of the presence of two undetected neutrinos. Nonetheless, the expected precision of the measurement of $B(B^{+} \to K^{+}v\bar{\nu})$ at a SFF is $20\%$, while the $B^{+} \to \pi^{+}v\bar{\nu}$ mode should be at the limit of observability (Bona et al., 2007c).

C. Constraints on CKM parameters

The radiative $b \to s(d)\gamma$ are sensitive to the CKM elements involving the third generation quarks. In the following we briefly review the methods proposed for precision determination of the CKM parameters, and indicate the types of constraints which can be obtained.

- $|V_{ub}|/|V_{cb}V_{ts}^{\ast}|$ from inclusive $b \to s\gamma$ and $b \to ud\gamma$: The inclusive radiative decays $B \to X_{s}\gamma$ were discussed in Section VIII.A.1 and the inclusive semileptonic decays $B \to X_{d}\ell^{-}\bar{\nu}_{\ell}$ in Section V. For both types of the decays only part of the phase space is accessible experimentally. In semileptonic decays a cut on lepton energy or hadronic

5 These contributions also introduce a shift in the position of the FBA zero in $B \to K^{\ast}\ell^{+}\ell^{-}$, as the $K^{\ast}$ is always observed through the $K\pi$ final state.
invariant mass needs to be made to avoid charm background, while in $B \to X_\gamma$ the photon needs to be energetic enough to reduce background. Experimentally accessible is the so-called shape function region of the phase space, where the inclusive state forms an energetic jet with mass $M_X^2 \sim \Lambda_{QCD} Q$. Restricted to this region the OPE breaks down, while instead SCET is applicable. The decay widths factorize in a form shown in Eq. (71) for $B \to X_\gamma$. Both radiative and semileptonic decays depend, at LO in $1/m_b$, on the the same shape function $S(k_c)$ describing the nonperturbative dynamics of the $B$ meson. The dependence on the shape function can be eliminated by combining the radiative and semileptonic rates. This then determines $|V_{ub}|/|V_{tb}V_{ts}^*|$, with different methods of implementing the basic idea discussed in detail in Sec. V (see also a review by Paz (2006) and recent developments in Leu (2008)).

• $|V_{td}/V_{ts}|$ from $B \to (\rho/K^*) \gamma$: The radiative $B \to \rho \gamma$ and $B \to K^\ast \gamma$ amplitudes are dominated by electromagnetic penguin contributions proportional to $V_{td}V_{tb}$ and $V_{ts}V_{tb}$ CKM elements respectively. The ratio of the charge-averaged rates is then

$$\overline{\Gamma}(B_q \to \rho \gamma) = \frac{\kappa_q^2 |V_{td}^2|}{|V_{tb}|^2} R_{SU(3)} \left( \frac{m_B^2 - m_d^2}{m_B^2 - m_{K^*}^2} \right)^{3/2} (1 + r_{WA}),$$

(95)

where $B_q = (B^-, \bar{B}_q)$, $\rho_q = (\rho^-, \rho^0)$ and $\kappa_q = (1, 1/\sqrt{2})$ for $q = (u, d)$ spectator quark flavors. The coefficient $r_{WA}$ denotes the WA contribution to $B \to \rho \gamma$, while it is negligible for $B \to K^\ast \gamma$. The coefficient $R_{SU(3)}$ parameterizes the SU(3) breaking in the ratio of tensor form factors. The theory error in the determination of $|V_{td}/V_{ts}|$ is thus due to these two coefficients. The coefficient $r_{WA}$ can be calculated using factorization. Writing

$$r_{WA} = 2Re(\delta a) \cos \alpha |\lambda_{ud}/\lambda_{td}| + O(\alpha^2),$$

(96)

Bosch and Buchalla (2003) find $Re(\delta a) = 0.002^{+0.124}_{-0.061}$ for $B^0 \to \rho^0 \gamma$, and $Re(\delta a) = -0.4 \pm 0.4$ for $B^+ \to \rho^+ \gamma$. (For an alternative treatment, see Ball et al. (2007b).) The WA amplitude is larger for charged $B$ decays, where it is color allowed, in contrast to neutral $B$ decays, where it is color suppressed. Along with $|\lambda_{ud}/\lambda_{td}| \sim 0.5$ the above values of $\delta a$ show that the uncertainty introduced by the WA contribution is minimal in neutral $B$ radiative decays.

The second source of theoretical uncertainty is given by SU(3) breaking. The parameter $R_{SU(3)}$ was estimated using QCD sum rules with the most recent result $R_{SU(3)} = 1.17 \pm 0.09$ (Ball and Zwicky 2006b). It seems rather difficult to improve on this calculation in a model independent way.

This method for determining $|V_{td}/V_{ts}|$ has been used to obtain $|V_{td}/V_{ts}| = 0.199^{+0.026}_{-0.010}$ (Abe et al. 2006a) and $|V_{td}/V_{ts}| = 0.196^{+0.025}_{-0.015}$ (Ambert et al. 2007c), where the first errors are experimental and the second theoretical, and in both cases the average over the $B \to (\rho/\omega) \gamma$ channels is used. A dramatic improvement in experimental error can be expected at a SFF, and while the theoretical error can be reduced by using only the cleaner $B^0 \to \rho^0 \gamma$, the precision is likely to be limited at about 4% due to the SU(3) breaking correction discussed above. This could possibly be improved using data collected at the $\Upsilon(5S)$, as discussed in Section IX.C.

• $|V_{ub}/V_{td}|$ from $B \to \rho \gamma$ and $B \to \rho \ell \nu$: The ratio of CKM matrix elements $|V_{ub}/V_{td}|$ can be constrained by combining the semileptonic mode $B \to \rho \ell \nu$ with the radiative decay $B \to \rho \gamma$ (Beneke and Yang 2006; Bosch and Buchalla, 2003). In the large recoil limit the relevant form factors satisfy factorization relations.

The doubly differential semileptonic rate expressed in terms of the helicity amplitudes is

$$d^2\Gamma(B \to \rho \ell \nu) = G_F^2 |V_{ub}|^2 q^2 (2 + 5 \sin^2 \theta) \left( (1 + \cos \theta)^2 H_2^* + (1 - \cos \theta)^2 (H_2 \pm 2 H_0^*) \right),$$

(97)

where $\theta$ is the angle between the $\nu$ and the $B$ meson momentum in the $\ell \nu$ center of mass frame. At $\theta = 0$ only the left-handed helicity amplitude $H_-$ contributes. The $q^2 \to 0$ limit of the ratio of the $B \to \rho \ell \nu$ partial rate to the $B \to \rho \gamma$ rate depends only on

$$\left( \frac{H_-(0)}{T_1(0)} \right)^2 \to 2(m_B + m_\nu) \frac{1}{R_2(0)},$$

(98)

where $T_1(q^2)$ is a tensor current form factor Eq. (80), while $R_2(0)$ is calculable in a perturbative expansion in $\alpha_S(m_b)$ and $\alpha_S(\sqrt{\Lambda_{QCD} m_b})$. This ratio has been computed to be $1/R_2^0 = 0.82 \pm 0.12$ (Beneke and Yang 2006), allowing for a 60% uncertainty in the spectator-scattering contribution. This amounts to a 10% uncertainty on this determination of $|V_{ub}/V_{td}|$, which however does not include uncertainties from power suppressed contributions.

• $|V_{ub}|^2/|V_{tb}V_{ts}^*|^2$ from $B \to K^\ast \ell^+ \ell^-$ and $B \to \rho \ell \nu$: In the low recoil region $q^2 \sim (M_B - M_{K^*})^2$, the $B \to
FIG. 20 Isospin asymmetry $\Delta(\rho\gamma)$ as a function of the CKM angle $\alpha$. The band displays the total theoretical uncertainty which is mainly due to weak annihilation. The vertical dashed lines limit the range of $\alpha$ obtained from the CKM unitarity triangle fit.

The $K^*\ell^+\ell^-$ amplitude can be computed in an expansion in $\Lambda/m_u, 4m_d^2/Q^2, \alpha_s(Q)$ (Grinstein and Pirjol, 2004), relating it to the semileptonic decay $B \to \rho\ell\nu$, up to SU(3) breaking correction in the form factors. These can be eliminated using semileptonic $D$ decay rates by forming the Grinstein double ratio (Ligeti and Wise, 1996)

$$\frac{d\Gamma(B \to \rho\ell\nu)/dq^2}{d\Gamma(B \to K^*\ell\ell')/dq^2} \times \frac{d\Gamma(D \to K^*\ell\ell')/dq^2}{d\Gamma(D \to \rho\ell\nu)/dq^2}$$

which is proportional to $|V_{ub}|^2/|V_{tb}V_{ts}^*|^2$. The theory error on $|V_{ub}|$ of this method is about 5%, but measurements of the required branching fractions in the region $q^2 = (15, 19)$ GeV$^2$ require SFF statistics.

- Constraints from the isospin asymmetry in $B \to \rho\gamma$: Assuming dominance by the penguin amplitude in $B \to \rho\gamma$, isospin symmetry relates the charged and neutral modes to be $\Gamma(B^\pm \to \rho^\pm\gamma) = 2\Gamma(B^0 \to \rho^0\gamma)$. The present experimental data point to a possible isospin asymmetry. The most recent world averages give (Abe et al., 2006a; Aubert et al., 2007c; Barberio et al. 2007) (using CP-conjugate modes)

$$\Delta(\rho\gamma) = \frac{\Gamma(B^+ \to \rho^+\gamma)}{2\Gamma(B^0 \to \rho^0\gamma)} - 1 = \frac{(0.88^{+0.28}_{-0.29}) \times 10^{-6}}{2 \times (0.93^{+0.19}_{-0.18}) \times 10^{-6}} - 1 = -0.53^{+0.18}_{-0.17}.$$

Several mechanisms can introduce a nonzero isospin asymmetry: i) the $m_u - m_d$ quark mass difference leading to isospin asymmetry in the tensor form factor $T_1$; ii) contributions from operators other than $O_{\gamma\gamma}$, where the photon attaches to the spectator quark in the $B$ meson; iii) spectator diagrams such as those in Fig. 14 which depend on the spectator quark $q$ through its electric charge, and the hard scattering amplitude.

The dominant contribution to the isospin asymmetry in the SM is given by the last mechanism (iii), mediated by the four-quark operators $O_{1-6}$. The matrix elements of these operators can be computed using factorization and the heavy quark expansion (Ali and Braun, 1995; Ali and Parkhomenko, 2002; Beneke et al., 2001; Bosch and Buchalla, 2002; Grinstein and Pirjol, 2000; Khodjamirian et al., 1993). Since the four-quark operators contribute with a different weak phase to the penguin amplitude, the result is sensitive to CKM parameters, in particular to the weak phase $\alpha$. Using as inputs the parameters from the CKM fit, an isospin asymmetry of a few percent is possible, with significant uncertainty from hadronic parameters (Beneke et al., 2005c)

$$\Delta(\rho\gamma) = (-4.6^{+5.4}_{-4.2}(\text{CKM})^{+5.8}_{-5.6}\text{had})\%.$$

This prediction can be turned around to obtain constraints on the CKM parameters $(\tilde{\rho}, \tilde{\eta})$, using the $\rho\gamma$ asymmetries. As discussed in (Beneke et al., 2005c), measurements of the direct CP asymmetry and of the isospin asymmetry in $B \to \rho\gamma$ give complementary constraints, which in principle allow a complete determination of the CKM parameters. However, the precision of such a determination is ultimately going to be limited by hadronic uncertainties and power corrections.

IX. $B_s$ PHYSICS AT $\Upsilon(5S)$

The $\Upsilon(5S)$ resonance is heavy enough that it decays both to $B_s^{(*)}$ and $B_s^{(*)}$ mesons. So far, several $e^+e^-$ machines have operated at the $\Upsilon(5S)$ resonance resulting in 0.42 fb$^{-1}$ of data collected by the CLEO collaboration (Besson et al., 1983; Loveck et al., 1983), followed by 1.86 fb$^{-1}$ of data collected by Belle collaboration during an $\Upsilon(5S)$ engineering run (Drutskov et al., 2004) and a sample of about 21 fb$^{-1}$ collected by Belle during a one month long run in June 2006. Baracchini et al. (2007) performed a comprehensive analysis of the physics opportunities that would be offered by much larger data samples of 1 ab$^{-1}$ (30 ab$^{-1}$) from a short (long) run of a SFF at the $\Upsilon(5S)$, where the data sample is recorded in special purpose runs. Collecting 1 ab$^{-1}$ should require less than one month at a peak luminosity of $10^{36}$ cm$^{-2}$ s$^{-1}$. As a result, a SFF can give information on the $B_s$ system that is complementary to that from hadronic experiments. In Table IX we give the expected precision from a SFF and LHCb for a sample of observables, clearly showing complementarity. In particular, the SFF can measure inclusive decays and modes with neutrals, which are inherently difficult in hadronic environment while LHCb provides superior time-dependent measurements of all-charged final states.

Physical processes involving $B_s$ mesons add to the wealth of information already available from the $B_{d,u}$ systems because the initial light quark is an $s$-quark. As a result, $B_s$ decays are sensitive to a different set of NP operators transforming between 3rd and 2nd generations than are $b \to s$ decays of $B_{d,u}$. The prime examples are $B_s \to \mu^+\mu^-$ where semileptonic $b \to s$ operators are probed and $B_s \to B_s$ mixing where $\Delta B = 2$ NP operators are probed. In addition, $B_s$ can improve knowledge of hadronic processes since $B_s$ and $B_d$ are related by U-spin.
TABLE IX Expected precision on a subset of important observables that can be measured at SFF running at the \( \Upsilon(5S) \) and/or LHCb. The first two columns give expected errors after short (less than a month) and long SFF runs [Baracchini et al. 2007; Bona et al. 2007d], while the third lists expected statistical errors after 1 year of LHCb running at design luminosity [Buchalla et al. 2008].

| Observable | SFF (1ab\(^{-1}\)) | SFF (30ab\(^{-1}\)) | LHCb (2fb\(^{-1}\)) |
|------------|----------------|----------------|----------------|
| \( \Delta \Gamma_s/\Gamma_s \) | 0.11 | 0.02 | 0.0092 |
| \( \beta_s(J/\psi\phi) \) | 20\(^\circ\) | 8\(^\circ\) | 1.3\(^\circ\) |
| \( \beta_s(B_s \rightarrow K^0\bar{K}^0) \) | 24\(^\circ\) | 11\(^\circ\) | – |
| \( A_{SW} \) | 0.006 | 0.004 | 0.002 |
| \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \) | – | < 8 \times 10^{-9} | 3\sigma evidence |
| \( |V_{td}/V_{ts}| \) from \( R_s \) | 0.08 | 0.017 | – |
| \( \mathcal{B}(B_s \rightarrow \gamma\gamma) \) | 38\% | 7\% | – |

In the application of flavor SU(3) to hadronic matrix elements then the commonly used dynamical assumption of small annihilation-like amplitudes may no longer be needed.

A. \( B_s-\bar{B}_s \) mixing parameters

\( B_s-\bar{B}_s \) mixing is described by the mass difference \( \Delta m_s \) of the two eigenstates, the average of two decay widths \( \Gamma_s \) and their difference \( \Delta \Gamma_s \), by \( \beta_s = (q/p) \) and by the weak mixing phase \( \beta_s = -1/2 \text{arg}(q/p) \), which is very small in the SM. \( \beta_s = \text{arg}(-V_{td}V_{ts}^*/V_{td}V_{ts}^*) = (-1.05 \pm 0.05)\(^\circ\) \) [Charles et al. 2007], see also Eq. (31). All these parameters can be modified by NP contributions and are, for instance, very sensitive to the large tan \( \beta \) regime of the MSSM as discussed in Section III.D.4.

The oscillation frequency \( \Delta m_s \) has been measured recently [Abulencia et al. 2008], and is found to be consistent with SM predictions, within somewhat large theory errors. These oscillations are too fast to be resolved at a SFF, which thus cannot measure \( \Delta m_s \). However, measurements of the other parameters, \( \Gamma_s, \Delta \Gamma_s \) and \( \beta_s \) are possible through time dependent untagged decay rates. Explicitly, for a \( B_s, \bar{B}_s \) pair produced from \( B_s^*, \bar{B}_s^* \) at the \( \Upsilon(5S) \) this is given by [Dunietz et al. 2001]

\[ \Gamma(B_s(t) \rightarrow f) + \Gamma(\bar{B}_s(t) \rightarrow f) = \mathcal{N}T_s e^{-\Gamma_s t} \left( \cosh \left( \frac{\Delta \Gamma_st}{2} \right) + H_f \sinh \left( \frac{\Delta \Gamma_st}{2} \right) \right), \]

where \( f \) is a CP-eigenstate and \( H_f \) is given in Eq. (33). The normalization factor is given by \( \mathcal{N} = \frac{1}{2} (1 - (\Delta \Gamma_s/2\Gamma_s)^2) \), neglecting possible effects due to CP violation in mixing.

At the \( \Upsilon(5S) \), CP-tagged initial states can also be used to extract the unarity angle \( \gamma \) rather cleanly [Atwood and Soni 2002; Falk and Petrox 2000], and to constrain lifetime difference \( \Delta \Gamma_s \) through time independent measurements [Atwood and Petrox 2003].

The most promising channel for measuring \( B_s-\bar{B}_s \) mixing parameters at a hadronic machine is \( B_s \rightarrow J/\psi\phi \), where angular analysis is needed to separate CP-even and CP-odd components. Recent measurements at D0 and CDF favor large \( |\beta_s| \) making further studies highly interesting [Aaltonen et al. 2007a; Abazov et al. 2008]. As shown in Table IX, a SFF cannot compete with LHCb in this analysis, either for \( \beta_s \) or for \( \Delta \Gamma_s/\Gamma_s \) measurements, assuming systematic errors at LHCb are negligible. However, LHCb and a SFF can study complementary channels. For example, \( B_s \rightarrow J/\psi\ell\ell \) or \( \beta_s \) from the \( \Delta S = 1 \) penguin dominated \( B_s \rightarrow K^0\bar{K}^0 \), are difficult measurements at hadronic machines as shown in Table IX. The latter mode would be complementary to \( B_s \rightarrow \phi\phi \), where a precision of 0.11 is expected after 2 fb\(^{-1} \) of data at LHCb (1 year of nominal luminosity running). Other interesting modes that can be studied at a SFF include \( B_s \rightarrow D_s^{(*)}\ell^+\ell^- \), \( B_s \rightarrow D_s^{(*)}K_S \), \( B_s \rightarrow D_s^{(*)}\phi \), \( B_s \rightarrow J/\psi K_S \), \( B_s \rightarrow \phi\ell\ell \) and \( B_s \rightarrow K_S\pi^0 \) [Bona et al. 2007d].

Another important observable is the semileptonic asymmetry \( A_{SL}^s \), which is a measure of CP violation in mixing

\[ A_{SL}^s = \frac{\mathcal{B}(B_s \rightarrow D_s^{(*)}\ell^+\nu_\ell) - \mathcal{B}(\bar{B}_s \rightarrow D_s^{(*)}\ell^-\bar{\nu}_\ell)}{\mathcal{B}(B_s \rightarrow D_s^{(*)}\ell^+\nu_\ell) + \mathcal{B}(\bar{B}_s \rightarrow D_s^{(*)}\ell^-\bar{\nu}_\ell)} = \frac{1 - \frac{|q/p|^4}{1 + \frac{|q/p|^4}}}{\phi} \]  

(103)

The error on \( A_{SL}^s \) will become systematic dominated relatively soon. Taking as a guide the systematic error \( \sigma_{\text{syst.}}(A_{SL}^s) = 0.004 \) from current measurements at the \( \Upsilon(4S) \), this will happen at an integrated luminosity of about 30 ab\(^{-1} \) at the \( \Upsilon(5S) \). Thus systematics will saturate the error quoted in Table IX for 30 ab\(^{-1} \) (where the statistical error is only 0.001) [Bona et al. 2007c]. Note that the LHCb estimate in Table IX gives only the statistical error on \( A_{SL}^s \), while systematic errors could be substantial due to the hadronic environment.

B. Rare decays

One of the most important \( B_s \) decays for NP searches is \( B_s \rightarrow \mu^+\mu^- \). In the SM this decay is chirally and loop suppressed with a branching fraction of \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) = (3.35 \pm 0.32) \times 10^{-9} \) [Blanke et al. 2006]. Exchanges of new scalar particles can lift this suppression, significantly enhancing the rate. For instance, in the MSSM it is \( \tan^5 \beta \) enhanced in the large tan \( \beta \) regime (cf. Section III.D.4). After one year of nominal LHCb data taking 3\sigma evidence at the SM rate will be possible, while the SFF sensitivity to this channel is not competitive as indicated in Table IX.
A SFF can make a significant impact in radiative $B_s$ decays and decay modes with neutrals. One example is $B_s \to \gamma\gamma$. Here the SM expectation is $B(B_s \to \gamma\gamma) \approx (2 - 8) \cdot 10^{-7}$ [Reina et al. 1997], while NP effects can significantly enhance the rate; for instance, the rate is enhanced by an order of magnitude in the $R$ parity violating MSSM [Gemintern et al. 2004]. The Belle $\Upsilon(5S)$ sample of $23.6 \mathrm{fb}^{-1}$ has already been used to demonstrate the potential of the SFF approach; the first observation of the penguin decay mode $B_s \to \phi\gamma$ has recently been reported, along with a statistics limited upper limit on $B_s \to \gamma\gamma$ a factor of ten above the SM level [Wicht et al. 2007].

C. Improved determinations of $V_{td}/V_{ts}$ and of $V_{ub}$

As described in Section VIII.C, exclusive radiative decays mediated by $b \to d$ and $b \to s$ penguins can be used to obtain constraints on the CKM ratio $V_{td}/V_{ts}$. An analogous treatment of $b \to u$ and $b \to d$ penguins is free from SU(3) breaking corrections in the form factors [Fitzpatrick et al., 2007]. The Belle $\Upsilon(5S)$ sample of $23.6 \mathrm{fb}^{-1}$ has already been used to demonstrate the potential of the SFF approach; the first observation of the penguin decay mode $B_s \to \phi\gamma$ has recently been reported, along with a statistics limited upper limit on $B_s \to \gamma\gamma$ a factor of ten above the SM level [Wicht et al. 2007].

X. CHARM PHYSICS

There are many reasons for vigorously pursuing charm physics at a SFF. Perhaps most important is the intimate relation of charm to the top quark. Because of its large mass top quark is sensitive to NP effects in many models. New interactions involving the top quark quite naturally also imply modified interactions of the charm quark. For example, models of warped extra-dimensions, discussed in Section III.E, inevitably lead to flavor-changing interactions for the charm quark [Agashe et al. 2005a,b; Fitzpatrick et al. 2007]. The same is true of two Higgs doublet models, in which the top quark has a special role [Das and Kao, 1996; Wu and Soni, 2000].

Charm also provides a unique handle on mixing effects in the up-type (charge $+\frac{2}{3}$) sector. The top quark does not form bound states, which makes $D - \bar{D}$ the only system where this study is possible. Importance of these studies is nicely illustrated by the constraint that they provide on the MSSM squark spectrum and mixing [Nir 2007]. The squark-quark-gluino flavor violating coupling that mixes the first two generations is given by $g_s \sin \theta_q$ with $q = u(d)$ for up (down) squarks. The difference of the two mixing angles needs to reproduce the Cabibbo angle

$$\sin \theta_u - \sin \theta_d = \sin \theta_C \approx 0.23.$$  

Small enough $\sin \theta_d$ can sufficiently suppress SUSY corrections to $K - \bar{K}$ mixing even for nondegenerate squarks with TeV masses. This is possible in the absence of information on $D - \bar{D}$ mixing. The smallness of $D - \bar{D}$ mixing, however, requires that also $\sin \theta_u$ is small, which violates the relation to the Cabibbo angle in Eq. (104). The squarks with masses light enough to be observable at LHC thus need to be degenerate [Nir 2007].

We next summarize the salient aspects of charm physics – detailed reviews can be found in [Artuso et al. 2008; Bianco et al. 2003; Burdman and Shipsey, 2003]. Within the SM, some aspects of the charm system are under excellent theoretical control. In particular, one expects negligible CP asymmetry in charm decays since the weak phase comes in CKM suppressed. The strong phases on the other hand are expected to be large in the charm region as it is rich with resonances. This means that a NP weak phase is likely to lead to observable CP violation. Moreover, although the absolute size of $D$ mixing cannot be reliably calculated in the SM because of long distance contamination, the rate of mixing can be used to put bounds on NP parameters in many scenarios [Golowich et al., 2007]. Furthermore, the indirect CP violation is negligibly small in the SM. It arises from a short distance contribution that is subleading in $D - \bar{D}$ mixing compared to the long distance piece and is furthermore CKM suppressed by $V_{ub}V_{cb}/V_{us}V_{cs}$. It therefore provides a possibility for a very clear NP signal.

The most promising modes to search for direct CP violation in charm decays are singly Cabibbo suppressed channels, such as $D^+ \to K^+K^0, \phi\pi^+, D_s \to \pi^+ K^0, K^+\pi^0$, which in the SM receive contributions from two weak amplitudes, tree and penguin [Grossman et al. 2007a]. As already mentioned indirect CP violation is very small, while direct CP violation is both loop and CKM suppressed making it negligible as well. Super-symmetric squark-gluino loops on the other hand can saturate the present experimental sensitivity of $O(10^{-2})$ [Grossman et al. 2007a]. Doubly Cabibbo suppressed modes may also be useful in the search for NP effects since the SM cannot give rise to any direct CP violation and thus the SM “background” contribution is small.

The prospects for finding a BSM CP-odd phase via $D^0$ oscillations dramatically improved in 2007. Using time-dependent measurements from their large charm data samples, Belle and BaBar reported the first evidence for
$D^0$–$D^0$ mixing (Aubert et al. 2007a; Staric et al. 2007). As discussed above the existence of mixing makes it possible to search for new physics (CP-odd) phases in the charm sector via CP-violating asymmetries.

The phase of $D^0$ mixing, $\phi_D = \Im(m(q/p))_{D^0}$ is the analogue of the phases of $B^0_d$ mixing or $B^0_s$ mixing discussed in Section IV. While the phase of $B^0_d$ mixing is large in the SM, the phases of $D^0$ mixing and $B_s$ mixing are small in the SM; both are examples of null tests, with the phase of $D^0$ mixing particularly clean since it is expected to be of order $10^{-3}$ in the SM. We emphasise that new physics that appears in the $D$ sector (involving up-type quarks) may be completely different from that in the $B$ sector.

Currently, the best sensitivity on $\phi_D$, of $O(20^\circ)$, is obtained from time-dependent $D(t) \rightarrow K_S \pi^+\pi^- \ell^-\nu_\ell$ Dalitz plot analysis (Abe et al. 2007a). Assuming that there are no fundamental systematic limitations in the understanding of this Dalitz plot structure, the sensitivity to $\phi_D$ at a SFF will be about $1^\circ\sim 2^\circ$. The use of other modes such as $D^0 \rightarrow K^-\pi^+$ and $D^0 \rightarrow K^+\pi^0\pi^0$ can improve the overall sensitivity and help to eliminate ambiguous solutions for the phase (Sinha et al. 2007).

Searches for CP-violation via triple correlations are also very powerful. These searches require final states that contain several linearly independent 4-momenta and/or spins. A crucial advantage is that this class of somewhat complicated final states does not require the presence of a CP-conserving (rescattering) phase; in $T_N$ odd-observables the CP asymmetry is proportional to the phase (Sinha et al. 2007). Many final states such as $KK\pi\pi$, $K\pi\pi\pi$, $\pi\pi\ell\ell$ and $KK\ell\ell$ can be used. Initial studies of some of these have been carried out (Link et al. 2003). Semi-leptonic rare decays are of special interest as their small branching fractions can translate into large CP-asymmetries. In practice, the search for triple correlations requires the presence of a term in the angular distribution that is proportional to sin $\phi$, where $\phi$ is the angle between the planes of the two pseudoscalars and the two leptons. It has recently been pointed out by Bigi (2007) that this asymmetry could be enhanced using data taken by a SFF in the charm energy region (i.e. at the $\psi(3770)$ resonance). In this scenario, one uses the process $e^+e^- \rightarrow \gamma^+ \rightarrow D_{short}D_{long}$ followed by tagging of the short-lived state via, e.g., $D_{short} \rightarrow K^+K^-$. This then allows analysis of the $D_{long} \rightarrow K^-\pi^+\ell^-\nu_\ell$ decay. The operation of a SFF at the $\psi(3770)$ resonance would also provide important input to the determination of $\gamma$ from $B \rightarrow DK$ decays, as discussed in Section IV.B (Atwood and Soni 2003a; Bondar and Poluektov 2002; 2008; Gronau et al. 2001).

CP violation in mixing can be probed using inclusive semileptonic CP asymmetry of “wrong sign” leptons (Bigi 2007):

$$a_{SL}(D^0) = \frac{\Gamma(D^0(t) \rightarrow \ell^-X) - \Gamma(D^0(t) \rightarrow \ell^+X)}{\Gamma(D^0(t) \rightarrow \ell^-X) + \Gamma(D^0(t) \rightarrow \ell^+X)}.$$  

A nonnegligible value requires a BSM CP violating phase in $\Delta C = 2$ dynamics and depends on both $\sin \phi_D$ and $\Delta\Gamma/\Delta M$. In the $D^0$ system, while $\Delta\Gamma$ and $\Delta M$ are both small, the ratio $\Delta\Gamma/\Delta M$ need not be. In fact the central values in the present data are consistent with unity or even a somewhat bigger value. The asymmetry $a_{SL}(D^0)$ is driven by this ratio or its inverse, whichever is smaller. Thus although the rate for “wrong sign” leptons is small, their CP asymmetry might not be if there is a significant NP phase $\phi_D$ (Bigi 2007). Due to the smallness of the rate for “wrong sign” leptonic decays, NP constraints from this measurement would still be statistics limited at a SFF.

Finally, although we have focused on CP violation phenomena in this section, there is also a number of rare decays that can be useful probes of new physics effects. For example, searches for lepton flavor violating charm decays such as $D^0 \rightarrow \mu e$ or $D_s(\rightarrow M\mu\nu_e)$, where $M$ is a light meson such as $K$ or $\pi$, can clearly help improve the bounds on exotica. In addition, studies of $D_s^+ \rightarrow \ell\nu$ decays provide complementary information to leptonic $B^+$ decays (discussed in Section III.C), and are useful to bound charged Higgs contributions in the large $\tan\beta$ limit (Akeroyd 2004; Akeroyd and Chen 2007; Rosner and Stone 2008).

XI. NP TESTS IN THE TAU LEPTON SECTOR

A. Searches for Lepton Flavor Violation

The discovery of neutrino oscillations (Ahmad et al. 2002; Aliu et al. 2002; Davis et al. 1968; Eguchi et al. 2003; Fukuda et al. 1998; Kajita 2006) provides direct experimental evidence that the accidental lepton flavor symmetries of the renormalizable Standard Model are broken in nature. It is therefore compelling to search for lepton flavor violation (LFV) also in the decays of charged leptons. LFV decays of tau leptons can be searched for at a Super Flavor Factory. The list of interesting LFV modes includes $\tau \rightarrow l\gamma$, $\tau \rightarrow l_1l_2\ell_3$ and $\tau \rightarrow ll$, where $l_i$ stands for $\mu$ or $e$, while the hadronic final state $h$ can be, for example, $\pi^0$, $\eta^{(')}$, $K_S$, or a multihadronic state. These searches will complement studies of LFV in the muon sector. The decay $\mu \rightarrow e\gamma$ will be searched for at MEG (Grassler 2003; Ritrak 2000), while $\mu \rightarrow e$ conversion will be searched for at PRISM/PRIME (Kuno 2003; Sato et al. 2006).
TABLE X  Current and expected future 90\% CL upper limits on the branching fractions and conversion probabilities of several lepton flavor violating processes. The expectations given for $\mu^+ \to e^{-} \gamma$ and $\mu^+ \to e^{-} \gamma$ conversion are single event sensitivities (SES).

| Mode | Current UL | Future UL/SES |
|------|------------|---------------|
| $\mu^+ \to e^{-} \gamma$ | $1.2 \times 10^{-11}$ (a) | $(1 - 10) \times 10^{-13}$ (b) |
| $\mu^+ \to e^{-} \gamma$ | $1.0 \times 10^{-12}$ (c) | — |
| $\mu^+ \to e^{-} \gamma$ | $6.1 \times 10^{-13}$ (d) | $5 \times 10^{-19}$ (e) |
| $\tau^- \to \mu^- \gamma$ | $5.0 \times 10^{-9}$ (f) | $(2 - 8) \times 10^{-9}$ (g) |
| $\tau^- \to e^- \gamma$ | $5.0 \times 10^{-9}$ (h) | $(2 - 8) \times 10^{-9}$ (g) |
| $\tau^- \to \mu^- \mu^+ \mu^-$ | $3.2 \times 10^{-8}$ (i) | $(0.2 - 1) \times 10^{-9}$ (g) |
| $\tau^- \to \mu^- \eta$ | $6.5 \times 10^{-10}$ (j) | $(0.4 - 4) \times 10^{-9}$ (g) |

(a) Ahmed et al. (2002); Brooks et al. (1999)
(b) Grassi (2005); Ritt (2006)
(c) Bellgardt et al. (1988)
(d) Dohmen et al. (1993)
(e) Kuno (2005); Sato et al. (2006)
(f) Akeroyd et al. (2004); Bona et al. (2007a)
(g) Aubert et al. (2006); Hayasaka et al. (2007a)
(h) Aubert et al. (2007c); Miyazaki et al. (2007b)

Another interesting way to search for NP effects is to test lepton flavor universality in $B \to K e^{-} e^{-}$ vs. $B \to K \mu^+ \mu^-$ decays. The decays into muons can be well measured in hadronic environment, while the electron decays are easier to measure at a SFF. The current and expected future sensitivities of several LFV modes of interest are summarized in Table X (for more details, see Raidal et al. (2008)).

Extending to the leptonic sector the concept of minimal flavor violation, described in Section III.B provides an effective field theory estimate of LFV (Cirigliano et al., 2005; Davidson and Palomar, 2003; Grinstein et al., 2007). The minimal lepton flavor violation (MFV) hypothesis supposes that the scale $\Lambda_{LF}$ at which the total lepton number gets broken is much larger than the mass scale $\Lambda_{SM}$ of the lightest new particles extending the SM leptonic sector (Cirigliano et al., 2005). These new particles could, for instance, be the sleptons of MSSM. The assumption of MFV is that the new particles break flavor minimally, i.e. only through charged lepton and neutrino Yukawa matrices.

MLFV predictions have several sources of theoretical uncertainties. First, unlike the quark sector the MFV prescription is not unique for the leptons because of the ambiguity in the neutrino sector. The minimal choice for the SM neutrino mass term is

$$\mathcal{L}_{\text{dim5}} = -\frac{1}{2\Lambda_{LN}} g_{\nu} (L_{\nu} L_{\nu}^{J}) (H^T \tau_2 L_{\nu}^{J}) + h.c.,$$

with $g_{\nu}$ a spurion of MLFV. This mass term could arise from integrating out heavy right-handed neutrinos. In this case there is an additional spurion $y_{\nu}$ from heavy neutrino-light neutrino Yukawa terms with $g_{\nu} \sim \lambda_{\nu}^{2}$. This then changes the spurion analysis, giving different predictions on the size of LFV processes. Further ambiguities are due to unknown absolute size of neutrino masses, i.e. whether neutrinos have normal or inverted mass hierarchy, and from the size of CP violation in the leptonic sector. Most importantly, the minimal size of LFV effects is not fixed. Rescaling simultaneously the coupling matrix $g_{\nu} \to k^2 g_{\nu}$ and the lepton number violation scale $\Lambda_{LN} \to k^2 \Lambda_{LN}$ does not change the neutrino mass matrix, while it changes $\mathcal{B}(e_i \to \nu_j \gamma) \to k^4 \log k \mathcal{B}(e_i \to e_j \gamma)$ (keeping $\Lambda_{LF}$ fixed at the same time). The rates of the lepton flavor violating processes therefore increase as the masses of the heavy neutrinos are raised. This dependence cancels in the ratio $\mathcal{B}(\tau \to \nu \gamma)/\mathcal{B}(\mu \to e \gamma)$. Normalizing to the charged-current decay

$$\mathcal{B}(l_i \to l_j \gamma) \to \frac{\mathcal{B}(l_i \to l_j \gamma)}{\mathcal{B}(l_i \to l_j \nu \nu)} \sim (0.1 - 10^{-4}) \times \mathcal{B}(\tau \to \nu \gamma)$$

Cirigliano et al. (2005) obtain that $\mathcal{B}(\mu \to e \gamma) \sim (0.1 - 10^{-4}) \times \mathcal{B}(\tau \to \nu \gamma)$ depending on the value of $\sin \theta_{13}$ angle, with smaller values of $\mathcal{B}(\mu \to e \gamma)$ obtained for smaller values of $\sin \theta_{13}$. Saturating the present experimental bound on $\mathcal{B}(\mu \to e \gamma)$ at $\sin \theta_{13} \sim 0.05$ gives $\mathcal{B}(\tau \to \nu \gamma) \sim 10^{-8}$, within the reach of a SFF.

A working example of MLFV model is for instance the CMSSM with three right-handed neutrinos (Antusch et al., 2006). The correlations between $\mathcal{B}(\mu \to e \gamma)$ and $\mathcal{B}(\tau \to \nu \gamma)$ are shown in Fig. 21. In this scenario the rate for $\mu \to e \gamma$ decay depends strongly on the value of the neutrino mixing parameter $\theta_{13}$, and could be hard to measure if $\theta_{13} < 1^\circ$, whereas $\mathcal{B}(\tau \to \nu \gamma)$ is approximately independent of this parameter. For the choices of parameters used in Fig. 21 based on the Snowmass point 1 (Allanach et al., 2002b), the rates of LFV processes are suppressed much larger rates for $\mathcal{B}(\tau \to \nu \gamma)$ are possible for other choices of NP parameters. Large LFV effects in charged lepton decays are found in other examples of extending SM with heavy right-handed neutrinos with or without supersymmetry (Agashe et al., 2006; Babu and Kolda, 2002; Borzumati and Masiero, 1986; Ellis et al., 2002; Hisano et al., 1996; Ilakovac, 2003; Masiero et al., 2004; Pharl, 1999).

Embedding MFV in a GUT setup can lead to qualitatively different conclusions. Now the effective weak Hamiltonian for $l_i \to l_j$ processes involves also the quark Yukawa couplings $Y_{U,D}$. This means that contrary to the MLFV case above, the $\mu \to e \gamma$ and $\tau \to \mu \gamma, e \gamma$ rates cannot be arbitrarily suppressed by lowering $\Lambda_{LN}$. For

\footnote{They do decrease with increased $\Lambda_{LF}$, the mass scale of low energy NP particles (such as slepton), as for the most NP sensitive measurements.}
$\mathcal{A}_{LN} \lesssim 10^{12}$ GeV the GUT induced contribution controlled by $Y_{U,D}$ starts to dominate, which in turn for NP scale $\mathcal{A}_{LF} \lesssim 10$ TeV gives $\mathcal{B}(\mu \rightarrow e\gamma)$ above $10^{-13}$ within reach of the MEG experiment [Grassi, 2005]. The MLFV and GUT-MFV scenarios can be distinguished by comparing different $\tau$ and $\mu$ LFV rates. For instance, in the limit where quark-induced terms dominate one has $\mathcal{B}(\tau \rightarrow \mu \gamma) \propto \lambda^4$ and $\mathcal{B}(\mu \rightarrow e\gamma) \propto \lambda^{10}$, with $\lambda \simeq 0.22$, giving $\mathcal{B}(\tau \rightarrow \mu \gamma)/\mathcal{B}(\mu \rightarrow e\gamma) \sim \mathcal{O}(10^4)$, which allows $\tau \rightarrow \mu \gamma$ to be just below the present exclusion bound. Further information that distinguishes the two scenarios can be obtained from $\tau \rightarrow l\pi(l = \mu, e)$, $\pi^0 \rightarrow \mu^+\mu^-$, $V \rightarrow \tau\mu$ ($V = J/\psi, \Upsilon$) and $\tau, \mu \rightarrow l_1l_2l_3$ decays [Cirigliano and Grinstein, 2006]. Explicit realizations of LFV in supersymmetric GUT models have been discussed in the literature [Barbieri and Hall, 1994; Barbieri et al., 1995; Calibbi et al., 2006; Gomez and Goldberg, 1996].

Similarly, correlations between different $\tau$ and $\mu$ decays for a general 2HDM have been derived [Paradisi, 2006a,b]. The decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ were found to be the most sensitive probes that can be close to present experimental bounds, while correlations between different decays are a signature of the theory.

In supersymmetric extensions of the SM, the $l_1 \rightarrow l_j\gamma^*$ dipole operator typically dominates over the four-lepton operators, which leads to a simple prediction $\mathcal{B}(\mu \rightarrow l_j\gamma) \simeq \frac{\alpha_{em}}{3\pi} \left( \log \frac{m_\mu^2}{m_\tau^2} \right) \frac{11}{4} \simeq \mathcal{O}(10^{-3})$ (108)

If the off-diagonal slepton mass-matrix element $\delta_{ij}$ and $\tan \beta$ are large enough, the Higgs-mediated transitions can alter this conclusion. For instance in the decoupling limit [Paradisi, 2006b]

\[
\frac{\mathcal{B}(\tau \rightarrow l_j\mu\mu)}{\mathcal{B}(\tau \rightarrow l_j\gamma)} \leq \frac{3 + 5\delta_{l\mu}}{36} \sim \mathcal{O}(0.1). (109)
\]

In Little Higgs Models with $T$-parity on the other hand, $Z$ and box-diagram contributions dominate over the radiative operators, which then gives distinctly different ratios of decay widths to those in the MSSM, as shown in Table XI [Blanke et al., 2007a]. In Little Higgs Models with $T$-parity with a NP scale $f \sim 500$ GeV, the LFV $\tau$ decays can be seen at a SBF. In other models $\tau \rightarrow e\gamma$, $\tau \rightarrow l_1l_2l_3$ or $\tau \rightarrow hl$ can be enhanced [Black et al., 2002; Brignole and Rossi, 2004; Chen and Geng, 2006; Cvetic et al., 2002; de Gouvea and Jenkins, 2007; Li et al., 2006; Saha and Kundu, 2002; Shen, 2002]. Further information on the LFV origin could be provided from Dalitz plot analysis of $\tau \rightarrow 3\mu$ with large enough data samples [Dassinger et al., 2007; Matsuzaki and Sunda, 2007].

### B. Tests of lepton flavor universality in tau decays

A complementary window to NP is provided by precise tests of lepton flavor universality in charged current $\tau \rightarrow \mu\bar{\nu}\nu$ and $\mu \rightarrow e\nu\bar{\nu}$ decays. In the large $\tan \beta$ regime of MSSM the deviations arise from Higgs-mediated LFV amplitudes, where the effects are generated by LF-conserving but mass dependent couplings. This is complementary to $K_{\mu2}$ and $B_{d2}$ decays, where deviations are mainly due to LFV couplings [Ishidori and Paradisi, 2006; Massiero et al., 2006].

### TABLE XI Comparison of various ratios of branching ratios in little Higgs model with $T$ parity and in the MSSM without and with significant Higgs contributions [Blanke et al., 2007a].

| Ratio | LHT | MSSM (dipole) | MSSM (Higgs) |
|-------|-----|--------------|--------------|
| $B(\mu \rightarrow e\gamma)/B(\tau \rightarrow \mu\gamma)$ | $0.4-2.5$ | $\sim 6 \times 10^{-3}$ | $\sim 6 \times 10^{-3}$ |
| $B(\mu \rightarrow e\gamma)/B(\tau \rightarrow \mu\gamma)$ | $0.4-2.3$ | $\sim 1 \times 10^{-2}$ | $\sim 1 \times 10^{-2}$ |
| $B(\tau \rightarrow \mu\gamma)/B(\tau \rightarrow e\gamma)$ | $0.4-2.3$ | $\sim 2 \times 10^{-3}$ | $0.06-0.1$ |
| $B(\tau \rightarrow \mu\gamma)/B(\tau \rightarrow e\gamma)$ | $0.3-1.6$ | $\sim 2 \times 10^{-3}$ | $0.02-0.04$ |
| $B(\tau \rightarrow \mu\gamma)/B(\tau \rightarrow e\gamma)$ | $0.3-1.6$ | $\sim 1 \times 10^{-2}$ | $\sim 1 \times 10^{-2}$ |
| $B(\tau \rightarrow \mu\gamma)/B(\tau \rightarrow e\gamma)$ | $1.3-1.7$ | $\sim 5$ | $0.3-0.5$ |
| $B(\tau \rightarrow \mu\gamma)/B(\tau \rightarrow e\gamma)$ | $1.2-1.6$ | $\sim 0.2$ | $5-10$ |
| $B(\tau \rightarrow \mu\gamma)/B(\tau \rightarrow e\gamma)$ | $10^{-2}-10^2$ | $\sim 5 \times 10^{-3}$ | $0.08-0.15$ |
It is important to note that, while most of the supersymmetric models discussed above were minimally flavor violating, this is far from being the only possibility still allowed by the LFV data. To first approximation the rare flavor changing charged lepton decays constrain the following combination of supersymmetric parameters

$$\sin 2\bar{\theta}_{ij} \frac{\Delta \tilde{m}_{ij}^2}{\tilde{m}_0^2},$$  \hspace{1cm} (110)$$

where $\bar{\theta}_{ij}$ is the slepton mixing angle with $i, j = 1, 2, 3$ the generation indices, while $\Delta \tilde{m}_{ij}$ and $\tilde{m}_0$ are the difference and the average of $\tilde{m}_{i,j}$ slepton masses, while for simplicity we suppress the $L, R$ indices for left-handed and right-handed sleptons. Thus the flavor bounds can be obeyed either if the mixing angles are small or if the sleptons are mass degenerate. Interpolation between the two options exemplifies a set of realistic supersymmetric models discussed by Feng et al. (2007), where supersymmetry breaking mechanism was taken to be a combination of gauge mediated (leading to degeneracy) and gravity mediated supersymmetry breaking supplemented with horizontal symmetries (leading to alignment with split mass spectrum).

The high $p_T$ processes at LHC experiments probe a different combination of FV supersymmetric couplings. For degenerate sleptons with large mixing one may observe oscillations in $l_i \rightarrow l_j \chi^0_i$ or $\chi^0_i \rightarrow l_i l_j \chi^0_1$ decay chains. This constrains (taking the limit of both sleptons having the same decay width $\Gamma$ for simplicity) (Arkani-Hamed et al., 1996)

$$\sin 2\bar{\theta}_{ij} \frac{(\Delta \tilde{m}_{ij}/\tilde{m}_0)^2}{(\Gamma/\tilde{m}_0)^2 + (\Delta \tilde{m}_{ij}/\tilde{m}_0)^2},$$  \hspace{1cm} (111)$$

which should be compared with Eq. (110). An example of constraints coming from the LHC and $B(\mu \rightarrow e\gamma)$ based on a preliminary simulation in the cMSSM is shown in Fig. 22. A qualitatively similar interplay of LHC and SFF constraints is expected for $\tau \rightarrow \mu \gamma$. By having both the LHC high $p_T$ and low energy LFV measurements at high enough precision one is able to measure both the mixing angle and the mass splitting of the sleptons, thus probing the nature of the supersymmetry breaking mechanism.

On the experimental side, a SFF is an ideal experiment to study lepton flavor violating tau decays due to the large cross-section ($\sigma(e^+e^- \rightarrow \tau^+\tau^-) = (0.919 \pm 0.003)$ nb at $\sqrt{s} = 10.58$ GeV (Banerjee et al., 2007)) and a clean environment. It has much better sensitivity than the LHC experiments even for the apparently favourable $\tau \rightarrow \mu\mu\mu$ channel (Santinelli, 2002; Ueno, 2003).

The $B$ factories have demonstrated the enormous potential for tau physics from an $e^+e^-$ collider running at the $\Upsilon(4S)$. The current experimental upper limits for most lepton flavor violating tau decays are at present in the $10^{-7}$–$10^{-8}$ range (Abe et al., 2007b, 2008; Aubert et al., 2007; Havasaka et al., 2007; Miyazaki et al., 2006, 2007a), indicating that a SFF will probe what is phenomenologically a highly interesting range, up to two orders of magnitude below the existing bounds.

For many of the LFV $\tau$ channels, the only limitation is due to statistics – there are no significant backgrounds as the $e^+e^- \rightarrow \tau^+\tau^-$ process provides a very distinctive signature, and the neutrinoless final state allows the four-momentum of the decaying tau lepton to be reconstructed. In the limit of negligible background, the achievable upper limit scales with the integrated luminosity.

Special consideration must be given to the radiative decays $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$, since for these channels there is an important background source from SM tau decays (eg. $\tau \rightarrow \mu\nu\nu\tau$) combined with a photon from initial state radiation. This irreducible background is already an important factor in the current analyses (Aubert et al., 2005c, 2006; Delpine et al., 2006, 2005; Grossman, 1994; Kuhn and Mirkes, 1997), where the only SM background is that from daughter neutral kaons (Bigi and Sanda, 2003; Calderon et al., 2007) and is $O(10^{-3})$ in $\tau \rightarrow \pi K^0_S \nu\tau$. Partial rate asymmetries, integrated over the phase space for the decay, can be measured with sub-percent precision at a SFF. A more comprehensive anal-
ysis requires a study of the amplitude structure functions (Bona et al., 2007c; Kuhn and Mirkes, 1992a,b); these analyses can also be performed, but benefit from having a polarized beam to provide a reference axis.

A polarized beam can also be used to make measurements of the $\tau$ electric and magnetic dipole moments. For the EDM measurement, an improvement of three orders of magnitude on the present bounds (Ianniruberto et al., 2003) can be achieved (Bernabeu et al., 2007). However this range can be saturated only by exotic NP models that can avoid stringent bound on the electric dipole moment of the electron. For the MDM, the anomalous moment could be measured for the first time at a SFF (Bernabeu et al., 2008).

XII. COMPARISON OF A SUPER FLAVOR FACTORY WITH LHCB

Since a Super Flavor Factory will take data in the LHC era, it is reasonable to ask how its physics reach compares with the flavor physics potential of the LHC experiments, most notably LHCB (Camilleri, 2007; Nakada, 2007). By 2014, the LHCB experiment is expected to have accumulated 10 fb$^{-1}$ of data from $pp$ collisions at a luminosity of $\sim 2 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$ (Buchalla et al., 2008). Moreover, LHCB is planning an upgrade where they would run at 10 times the initial design luminosity and record a data sample of about 100 fb$^{-1}$ (Dijkstra, 2007; Muheim, 2007).

The most striking outcome of any comparison between a SFF and LHCB is that the strengths of the two experiments are largely complementary. For example, the large boost of the $B$ hadrons produced at LHCB allows time-dependent studies of the oscillations of $B_s$ mesons while many of the measurements that constitute the primary physics motivation for a SFF cannot be performed in a high multiplicity hadronic environment, for example, rare decay modes with missing energy such as $B^+ \to e^+\nu\ell\bar{\nu}$ and $B^+ \to K^+\nu\bar{\nu}$. Measurements of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ and inclusive analyses of processes such as $b \to s\gamma$ and $b \to s\ell^+\ell^-$ also benefit greatly from the clean and relatively simple $e^+e^-$ collider environment. At LHCB the reconstruction efficiencies are reduced for channels containing several neutral particles and for studies where the $B$ decay vertex must be determined from a $K_S^0$ meson. Consequently, a SFF has unique potential to measure the photon polarization via mixing-induced CP violation in $B_d^0 \to K_S^0\pi^0\gamma$. Similarly, a SFF is well placed to study possible NP effects in hadronic $b \to s$ penguin decays as it can measure precisely the CP asymmetries in many $B_d^0$ decay modes including $\phi K^0, \eta' K^0, K^0_s K^0_s K^0_s$ and $K^0_s\bar{K}^0_s$. While LHCB will have limited capability for these channels, it can perform complementary measurements using decay modes such as $B_s^0 \to \phi\gamma$ and $B_s^0 \to \phi\ell^+\ell^-$ for radiative and hadronic $b \to s$ transitions, respectively (Camilleri, 2007).

Where there is overlap, the strength of the SFF programme in its ability to use multiple approaches to reach the objective becomes apparent. For example, LHCB should be able to measure $\alpha$ to about 5$^\circ$ precision using $B \to \rho\pi$ (Nakada, 2007), but will not be able to access the full information in the $\pi\pi$ and $\rho\rho$ channels, which is necessary to reduce the uncertainty to the 1–2$^\circ$ level of a SFF. Similarly, LHCB can certainly measure $\sin(2\beta)$ through mixing-induced CP violation in $B_d^0 \to J/\psi K_S^0$ decay to high accuracy (about 0.01), but will have less sensitivity to make important complementary measurements (e.g., in $J/\psi \pi^0$ and $Dh^0$). While LHCB hopes to measure the angle $\gamma$ with a precision of 2–3$^\circ$, extrapolations from current B factories show that a SFF is likely to be able to improve this precision to about 1$^\circ$. LHCB can probably make a precise measurement of the zero of the forward-backward asymmetry in $B^0 \to K^{*0}\mu^+\mu^-$, but a SFF can also measure the inclusive channel $b \to s\ell^+\ell^-$, which, as discussed in Section VIII.B.1, is theoretically a much cleaner and more powerful observable. The broader programme of a SFF thus provides a very comprehensive set of measurements in addition to its clean experimental environment and superior neutral detection capabilities. This will be of great importance for the study of flavor physics in the LHC era.

XIII. SUMMARY

In this review we have summarized the physics case for a Super Flavor Factory (SFF); our emphasis has been on searches for New Physics. Such a high luminosity machine (integrating 50-75 ab$^{-1}$) will of course be a Super $B$ Factory, but importantly has enormous potential for exposing New Physics not only in the $B$ sector, but also in charm as well as in $\tau$ lepton decays.

In $B$ physics the range of clean and powerful observables is very extensive, see Table XIII. A quick inspection vividly shows that the SFF will extend the current reach from the $B$ factories for many important observables by over an order of magnitude. Specifically, we should be able to significantly improve the precision with which we can cleanly measure the angles “directly” and also determine sides of the unitarity triangle enhancing our knowledge of these fundamental parameters of the SM as well as checking for new physics effects in $B_d$ mixing and in $b \to d$ transitions. In addition, there are critically important direct searches for New Physics that are also possible. For example, we should be able to measure $\sin 2\beta$ from penguin-dominated $b \to s$ modes with an accuracy of a few percent. This will either clearly establish the presence of a new CP-odd phase in $b \to s$ transitions or allow us to constrain it significantly. Improved measurements of direct and time-dependent CP asymmetries in a host of modes and the first results on the zero crossing of the forward-backward asymmetries in inclusive radiative $b \to s\ell^+\ell^-$ decays will be exciting and extremely informative. Furthermore, a large class of
null tests will either constrain NP or reveal its presence.

While the dramatic increase in luminosity at a SFF will allow significant improvements in many important existing measurements, the SFF also will provide an important step change over the B factories in that many new channels and observables will become accessible for the first time. These include $b \to d\gamma$, $b \to d\ell^+\ell^-$, $B \to K^{(*)}\nu\bar{\nu}$ and semi-inclusive hadronic modes. In addition, sensitive probes of right-handed currents will become possible through measurements of time-dependent asymmetries in radiative $b \to s\gamma$ processes such as $B \to K_S\pi^0(\rho^0)\gamma$, as well as transverse polarization of the $\tau$ in semitauonic decays of $B$ mesons. At the SFF, the high statistics and kinematic constraints of production at the $\Upsilon(4S)$ also will allow clean studies of many important inclusive processes in the recoil of fully reconstructed tagged $B$ mesons.

High luminosity charm studies will also be sensitive to the effects of new physics; the most important of these is a search for a new CP-odd phase in $D$ mixing ($\phi_D$) with a sensitivity of a few degrees. Improved studies of lepton flavor violation in $\tau$ decays with much higher sensitivities could also prove to be extremely important in revealing new phenomena or allowing us to constrain it more effectively.

A Super Flavor Factory will complement dedicated flavor studies at the LHC with its sensitivity to decay modes with photons and multiple neutrinos as well as inclusive processes. The SFF will extend the reach of the high $p_T$ experiments at the LHC in many ways and will help us interpret whatever type of New Physics is discovered there.

### TABLE XII

Expected sensitivities at a SFF compared to current sensitivities for selected physics quantities. This table has been adapted from Table I of [Browder et al. (2007)] and also includes results from the HFAG (Heavy Flavor Averaging Group) compilation [Barberio et al. (2005)]. For some unitarity triangle quantities such as $\gamma$ and $\alpha$, due to low statistics and non-gaussian behaviour of the uncertainties in current measurements there is poor agreement on the final uncertainty in the world average. For example, for $\gamma$ the CKMfitter group [Charles et al. (2005)] obtains $\pm 31^\circ$ while UTfit [Bona et al. (2006b)] finds $\pm 16^\circ$ due to differences in statistical methodologies. For $|V_{ub}|$ there is considerable debate on the treatment of theoretical errors. Representative values from the PDG minireview are given as an estimate for the current sensitivity entry below.

| Observable SFF sensitivity | Current sensitivity |
|---------------------------|---------------------|
| $\sin(2\beta) (J/\psi K^0)$ | 0.005–0.012 | 0.025 |
| $\gamma (DK)$ | 1–2° | $\sim 31^\circ$ (CKMfitter) |
| $\alpha (\pi\pi, \rho\pi, \rho\rho)$ | 1–2° | $\sim 15^\circ$ (CKMfitter) |
| $|V_{ub}|(\text{excl})$ | 3–5% | $\sim 18\%$ (PDG review) |
| $|V_{ub}|(\text{incl})$ | 3–5% | $\sim 8\%$ (PDG review) |
| $\bar{\rho}$ | 1.7–3.4% | $^{+12\%}_{-12\%}$ |
| $\bar{\eta}$ | 0.7–1.7% | 4.6% |
| $S(\phi K^0)$ | 0.02–0.03 | 0.17 |
| $S(\eta' K^0)$ | 0.01–0.02 | 0.07 |
| $S(K_S K_S K^0)$ | 0.02–0.03 | 0.20 |
| $B(B \to \tau\nu)$ | 3–4% | 30% |
| $B(B \to \mu\nu)$ | 5–6% | not measured |
| $B(B \to D\tau\nu)$ | 2–2.5% | 31% |
| $A_{CP}(b \to s\gamma)$ | 0.004–0.005 | 0.037 |
| $A_{CP}(b \to s\gamma + d\gamma)$ | 0.01 | 0.12 |
| $B(B \to X_d\gamma)$ | 5–10% | $\sim 40\%$ |
| $B(B \to \rho\gamma)/B(B \to K^*\gamma)$ | 3–4% | 16% |
| $S(K_S\pi^0\gamma)$ | 0.02–0.03 | 0.24 |
| $S(\rho^0\gamma)$ | 0.08–0.12 | 0.67 |
| $B(B \to X_s\ell^+\ell^-)$ | 4–6% | 23% |
| $A^{FB}(B \to X_s\ell^+\ell^-)_{s0}$ | 4–6% | not measured |
| $B(B \to K\nu\bar{\nu})$ | 16–20% | not measured |
| $\phi_D$ | 1–2° | $\sim 20^\circ$ |
| $B(\tau \to \mu\gamma)$ | 2–8x10$^{-9}$ | not seen, $< 5.0 \times 10^{-8}$ |
| $B(\tau \to \mu\mu\mu)$ | 0.2–1x10$^{-9}$ | not seen, $< (2-4) \times 10^{-8}$ |
| $B(\tau \to \mu\eta)$ | 0.4–4x10$^{-9}$ | not seen, $< 5.1 \times 10^{-8}$ |
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