Prediction of cryogenic cavitation around hydrofoil by an extensional Schnerr-Sauer cavitation model

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Abstract. Developing a robust computational strategy to address the rich physics characteristic involved in the thermodynamic effects on the cryogenic cavitation remains a challenging problem. The objective of this present study is to model the numerical methodology to simulate the cryogenic cavitation by implanting the thermodynamic effects to the Schnerr-Sauer cavitation model, and coupling the energy equation considered the latent heat. For this purpose, cavitating flows are investigated over a three dimensional hydrofoil in liquid hydrogen and nitrogen. Experimental measurements of pressure and temperature are utilized to validate the extensional Schnerr-Sauer cavitation model. Specifically, the further analysis of the cavitation solution with respect to the thermodynamic term is conducted. The results show that the extensional Schnerr-Sauer cavitation model predicts better accuracy to the quasi-steady cavitation over hydrofoil in the two cryogenic fluids.

1. Introduction
Cryogenic fluids such as liquid hydrogen and liquid oxygen are being promoted for use as propellants for rocket and other aerospace equipment. It is well known that the high speed rotating turbo pump often causes a lower pressure around the inducer, and then cavitation occurs when the fluid pressure drops below the vapor pressure in the local thermodynamic state [1]. Cryogenic fluids are thermo-sensitive, and therefore, thermodynamic effects and significant variations in fluid properties can alter the cavitation properties. Hence, it is necessary to explore an effective computational strategy to capture the cavitation characteristics in cryogenic fluids.

The selection of cavitation models plays a major role on the accurate prediction of cavitating flows. In recent years, some worthy efforts have been made in the development of cavitation models in cryogenic fluids. Most of these models basically determine the liquid phase fraction by that local pressure depression. Physically, the cavitation process is governed by thermodynamics and kinetics of the phase change process. So it is important to estimate the physical characteristic for cryogenic fluids by modeling with thermodynamic effects [2-5].

In the present study, therefore, we extend the Schnerr-Sauer cavitation model, which taking into account the thermodynamic effects during the evaporation and condensation process. The extensional model is validated by predicting the quasi-steady cavitating flows around a three dimensional hydrofoil in liquid hydrogen and nitrogen.

2. Computational methodologies and cavitation model
The numerical simulations are carried out by using computational fluid dynamics (CFD) code CFX. The homogeneous mixture approach is used to model the two phase cavitating flows.
2.1. Governing equations and turbulence closure approach
The set of governing equations for cavitating flows under the homogeneous fluids modeling consists of the conservative form of continuity, momentum, energy equations, and they are given below:

\[
\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho_m u_j \right) = 0
\]

\[
\frac{\partial}{\partial t} \left( \rho_m u_j \right) + \frac{\partial}{\partial x_j} \left( \rho_m u_j u_i \right) = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \mu + \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \right) \right]
\]

\[
\frac{\partial (\rho_m T)}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho_m u_j C_{pl} T \right) = \nabla \cdot \left( k_{eff} \mathbf{V} T \right) - \frac{\partial (\rho_m f_v L)}{\partial t} - \frac{\partial (\rho_m u_j f_v L)}{\partial x_j}
\]

where \( \rho_m = \rho_v \alpha_v + \rho_l (1-\alpha_v) \), the subscripts \( v \) and \( l \) donate vapour and liquid; \( x \) and \( i, j \) and \( k \) donate coordinate axes; \( u \) is the velocity; \( \mu \) is the dynamic viscosity, \( \mu_t \) is the turbulent viscosity; \( C_{pl} \) is specific heat; \( k_{eff} \) is effective thermal conductivity; \( L \) is the latent heat of evaporation, \( f_v \) is vapour mass fraction.

A two-equation, \( k-\varepsilon \) model is employed as the turbulence closure approach, as it offers a well-established regime of predictive capability.

2.2. Extensional Schnerr-Sauer Cavitation model
For cavitating modelling, the vapour volume fraction is solved with the following transport equation:

\[
\frac{\partial}{\partial t} \left( \alpha_v \rho_v \right) + \nabla \cdot \left( \alpha_v \rho_v \mathbf{u}_m \right) = R_e - R_c
\]

where the net mass transfer is expressed as \( R = R_e - R_c \), and the sink term \( R_e \) and source term \( R_c \) represent the evaporation and condensation mass transfer rates.

Schnerr-Sauer cavitation model was derived from the bubble dynamics equation of the generalized Rayleigh-Plesset equation, and the following expressions are used to relate the mass transfer between the vapour and liquid phases in cavitation[6]:

\[
R_e = \frac{\rho_v \rho_l}{\rho_m} \alpha_v (1-\alpha_v) \left( \frac{2}{3} \frac{P_v - P}{\rho_l} \right)^{3/2} \quad P \leq P_v
\]

\[
R_c = \frac{\rho_v \rho_l}{\rho_m} \alpha_v (1-\alpha_v) \left( \frac{2}{3} \frac{P - P_v}{\rho_l} \right)^{3/2} \quad P > P_v
\]

where \( \mathcal{R}_B \) is the bubble radius, \( P_v \) represents vapour pressure. The rate of change referring to the bubble radius caused by pressure depression in equation (5) is shown as:

\[
\frac{d \mathcal{R}_B}{dt} = \left( \frac{2}{3} \frac{P_v - P}{\rho_l} \right)^{1/2}
\]

During bubble growth of the thermo-sensitive fluid, the heat necessary for vaporization is supposed to be supplied to the interface through the liquid. Hence, a thermal boundary layer develops on the bubble wall. The liquid temperature drops from \( T_\infty \) to \( T_v \) through this boundary layer. It is defined as \( \sqrt{\alpha_v T} \), where \( \alpha_v \) is the thermal diffusivity of the liquid. The typical temperature gradient within the boundary layer is \( \Delta T / \sqrt{\alpha_v T} \) where \( \Delta T = T_\infty - T_v \). According to Fourier’s law, the conductive heat flux towards the interface is the order of \( \lambda_T \Delta T / \sqrt{\alpha_v T} \), where \( \lambda_T = \alpha_v \rho_l C_{pl} \) is the conductivity. The energy balance expresses that the heat supplied by conduction to the interface of area \( 4\pi \mathcal{R}_B^2 \) is used for
vaporization and causes the increase of the mass of gas $4\pi R_b^3 \rho_v / 3$ inside the bubble. Hence, the energy balance is:

$$\frac{\rho_v \Delta T}{\sqrt{\alpha t}} - \frac{4 \pi R_b^3 \rho_v}{3} \frac{d}{dt} \left( \frac{4 \pi R_b^3 \rho_v}{3} \right) = L$$

From equation (7), we can obtain:

$$dR_b / dt = \rho_t C_p \sqrt{\alpha t} \Delta T / \rho_v L \sqrt{t}$$

As shown in equations (7) and (9), taking into account the thermal effect for cryogenic cavitation, the E-Schnerr-Sauer (Extensional Schnerr-Sauer) cavitation can be defined as follows:

$$R_c = \frac{\rho_v \rho_l}{\rho_m} (1 - \alpha) \left( \frac{2}{3} \frac{(P_v(T) - P)}{\rho_l} \right)^{1/2} - \frac{\rho_v C_p \sqrt{a(T_v - T_v)}}{\rho_l L \sqrt{t}}$$

$$P \leq P_c(T)$$

$$R_s = \frac{\rho_v \rho_l}{\rho_m} (1 - \alpha) \left( \frac{2}{3} \frac{(P - P_v(T))}{\rho_l} \right)^{1/2} - \frac{\rho_v C_p \sqrt{a(T_v - T_v)}}{\rho_l L \sqrt{t}}$$

$$P > P_c(T)$$

where $P_c(T)$ represents the saturation pressure at the local temperature; The variable $t$ in these equations is replaced by a constant value which presents the bubble growth time for better prediction.

### 2.3. Numerical setup and description

In present study, we model flow over a 3D quarter caliber hydrofoil, which is experimentally investigated by Hord [7]. The tunnel and hydrofoil geometry shown in figure 1 are used to clarify the detailed geometry structure. Both of the width and height of the tunnel are 25.4mm, and the leading edge diameter of the hydrofoil is D = 7.92mm. The boundary conditions of the computational domain are also shown in figure 1. The whole flow field was divided into over 1020,000 cells and mesh refinements are performed at the leading edge and the cavitation region (see figure 2).

![Figure 1. The computational domain.](image1)

![Figure 2. Mesh generation for hydrofoil.](image2)

### 3. Results and discussions

In present study, four cases of cavitating flows in liquid hydrogen and nitrogen with different free stream temperatures $T_\infty$ and inlet cavitation numbers $\sigma_{in}$ are selected randomly. Table 1 summarizes the run conditions of different cases.

| Case | Substance | $T_\infty$ (K) | $U_{in}$ (ms$^{-1}$) | $\sigma_{in}$ | $Re$ |
|------|-----------|--------------|--------------------|--------------|------|
| 254C | Liquid H$_2$ | 20.53 | 51.0 | 1.44 | $2.2 \times 10^6$ |
| 260D | Liquid H$_2$ | 20.81 | 50.2 | 1.57 | $2.1 \times 10^6$ |
| 290C | Liquid N$_2$ | 83.06 | 23.9 | 1.70 | $9.1 \times 10^6$ |
| 296B | Liquid N$_2$ | 88.54 | 23.7 | 1.61 | $1.1 \times 10^7$ |

Figure 3 compares the computational temperature and pressure depression with Hord's measurements data along the hydrofoil by Schnerr-Sauer and E-Schnerr-Sauer cavitation model. It is clear that temperature depression occurs at the leading edge of hydrofoil, and then it gradually recovers to the freestream temperature due to the condensation near the closure region of the cavity. As well as the behavior gradient of the pressure profiles along the hydrofoil are not constant due to the thermodynamic effects. The proposed model leads to a cavity length slightly longer than that predicted by the original Schnerr-Sauer model. This is because the revised thermal term contributed to less heat release, resulting in weaker tendency to restrain the intensity of cavitation. The performance of the E-
Schnerr-Sauer model has shown better than that of the original Schnerr-Sauer cavitation model, especially noteworthily is the fact that the prediction of thermal behavior.

Figure 3. Comparison of pressure and temperature depression to experimental data.

Figure 4 shows the contours of vapour volume fraction $\alpha_v$, the evaporation rate $\bar{R}_e$ and condensation rate $\bar{R}_c$ (where the dimensionless mass transfer rate is defined as $\bar{R} = RD/U_\infty \rho_f$). Note that the evaporation and condensation contours demonstrate a mutually exclusive behavior. The thermodynamic effects term of the equation (9) causes lower evaporation rate near the leading edge and weaker condensation intensity in the closure region, hence the temperature distribution is changed inside cavity.

Figure 4. Predicted results of (a) vapor phase fraction, (b) condensation rate and (c) evaporation rate distribution in the mid-plane of the hydrofoil for case 290C.

4. Summary

In present study, the original Schnerr-Sauer cavitation model was extended to treat the quasi-steady cavitating flows around hydrofoil in liquid hydrogen and nitrogen. The numerical results demonstrated that the Extensional-Schnerr-Sauer model which considers the thermodynamic effects can provide an better strategy to predict the cavitation characteristics in cryogenic liquids.

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References

[1] Ohira K, Nakayama T, and Nagai T 2012 Cryogenics 52 35-44
[2] Utturkar Y, Thakur S, Shyy W 2005 43rd Aerospace Sciences Meeting and Exhibit, Reno, Nevada
[3] Utturkar Y, Wu J, Wang G, Shyy W 2005 Prog Aerosp Sci 41 558-608
[4] Tani N, Tsuda S, Yamanishi N, Yoshida Y 2009 Proceedings of the 7th International Symposium on Cavitation, Ann Arbor, Michigan
[5] Zhang X B, Wu Z, Xiang S J, Qin L M 2013 Chinese Sci Bull 58 567-74
[6] Hord J 1973 NASA CR 2156
[7] Schnerr G H, Sauer J 2001, 4th International Conference on Multiphase Flow, New Orleans, Louisiana