Linear transverse magnetoresistance is commonly observed in many material systems including semimetals, narrow band-gap semiconductors, multi-layer graphene and topological insulators. It can originate in an inhomogeneous conductor from distortions in the current paths induced by macroscopic spatial fluctuations in the carrier mobility and it has been explained using a phenomenological semiclassical random resistor network model. However, the link between the linear magnetoresistance and the microscopic nature of the electron dynamics remains unknown. Here we demonstrate how the linear magnetoresistance arises from the stochastic behaviour of the electronic cycloidal trajectories around low-mobility islands in high-mobility inhomogeneous conductors and that this process is only weakly affected by the applied electric field strength. Also, we establish a quantitative link between the island morphology and the strength of linear magnetoresistance of relevance for future applications.
Linear transverse magnetoresistance (MR) is a phenomenon commonly observed in many material systems, including semimetals, narrow band-gap semiconductors, multi-layer graphene and topological insulators. Linear MR can arise from either quantum or classical effects. In particular, it can originate in an inhomogeneous conductor from distortions in the current paths induced by macroscopic spatial fluctuations in the carrier mobility; this type of non-saturating linear MR has been observed in several materials and explained in terms of a phenomenological semiclassical random resistor network model. However, the link between the linear MR and the microscopic nature of the electron dynamics remains still unexplored. Existing theories of classical linear MR apply to systems with ohmic conductivity, whereas the effects of hot electron dynamics on the linear MR are largely unknown. On the other hand, a significant enhancement of MR has been observed in the non-ohmic space-charge limited transport regime and under avalanche breakdown, thus raising problems of fundamental interest and technological relevance, that is, what is the specific carrier dynamics giving rise to the linear MR, is it preserved in the presence of a large applied electric field and can it be controlled and exploited in sensitive Hall sensors or magnetoresistors

Here, we examine the physical origin of the classical linear MR in a high-mobility inhomogeneous conductor. Our measurements and Monte Carlo simulations focus on the n-type compound semiconductor InAs and consider an extended range of applied electric (E) and magnetic (B) fields, and different sample geometries. By modelling our data with Monte Carlo simulations of the electron dynamics, we demonstrate that the linear MR arises from multiple scattering of the current-carrying electrons by low-mobility electrons within the conducting layer. This process is only weakly affected by the details of the electronic band structure and electric field strength. Also, it persists at magnetic fields as high as ~50 T corresponding to a magnetic length as small as 4 nm, significantly smaller than the electron mean free path (~100 nm) and island size. The link between the linear MR and the stochastic behaviour of the electronic cyclodial trajectories around low-mobility islands is a finding of general interest as it is relevant to all high-mobility inhomogeneous conductors. The identification of a quantitative link between the island morphology and the strength of linear MR is also potentially useful for tailoring the linear MR in future applications.

Results

Material systems and devices. Our InAs and In(AsN) epi-layers were grown by molecular beam epitaxy (MBE) on a semi-insulating (100)-oriented GaAs substrate. Owing to the mismatch between the substrate and epi-layers, inhomogeneities can arise due to the presence of dislocations and defects; also, the N-atoms act as scattering centres for the conduction electrons. Thus with increasing N-content from 0 to 1%, the electron Hall mobility decreases from $\mu_H = 3.3 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$ (3.1 $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$) to 0.6 $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$ (0.5 $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$) at $T = 300 \text{ K}$ (4.2 K). Correspondingly, the electron density increases from $n_e = 4 \times 10^{16} \text{cm}^{-3}$ in InAs to $n_e = 1.4 \times 10^{17} \text{cm}^{-3}$ in In(AsN), with both values being very weakly affected by temperature.

The presence of inhomogeneous disorder combined with the high electron mobility makes these InAs and In(AsN) layers particularly well suited to the study of classical linear MR: even in the low-mobility layers the high values of $\mu B > 1$, which are necessary for observing a large MR, can be achieved with available conventional (superconducting) solenoids. To investigate the influence of sample geometry and electric field on the MR, two-terminal planar devices with InAs channels of length L = 2, 5 and 10 $\mu$m and of width $W = 1, 2$ and $5 \mu$m were also studied.

High magnetic field studies. In our experiments, the magnetic field was applied parallel to the growth axis z, that is, $B = [0, 0, B_z]$. The transverse MR ratio defined as $\Delta \rho_{xx}/\rho_{xx} = (\rho_{xx}(B_z) - \rho_{xx}(0))/\rho_{xx}(0)$ was studied with magnetic fields, $B_z$, up to 14 T, generated by a superconducting magnet, or in a pulsed solenoid at $B_z = 47 \text{T}$ with a total pulse length of 100 ms. Figure 1a shows the $B_z$-dependence of $\Delta \rho_{xx}/\rho_{xx}$ for an Hall-bar based on n-type InAs. A positive linear MR is observed at $B_z > 1 \text{T}$ and is weakly affected by temperature in the range $T = 4.2–300 \text{K}$. We attribute to the Shubnikov–de Haas effect in the two-dimensional electron gas that accumulates on the surface of InAs due to the bend of the conduction band by charged native surface defects. The resistance of the surface two-dimensional electron gas at $B_z = 0 \text{T}$ is at least one order of magnitude higher than $\rho_{xx}$ and, at all values of $B_z$ and $T$, the amplitude of the magneto-oscillations is much smaller than the measured monotonically increasing MR.

Linear MR can occur when the electron distribution enters the so-called quantum limit, in which only the lowest Landau level is occupied, that is, when the cyclotron energy, $h\omega_c$, exceeds the Fermi energy, $\varepsilon_F$, and when $\hbar\omega_c \gg k_B T$ (ref. 6). As shown in Fig. 1b,
the transition of the MR to a linear $B$-dependence occurs at $B_r > 1$ T below the value satisfying the quantum limit condition $\hbar \omega_c = eE_f$, that is, $B_r = h(3\pi^2 n_i^3)^{1/3}/2e$. This limit corresponds to fields $>3$ T for an electron density $n_i = 4 \times 10^{12}$ cm$^{-3}$, which we derive from the gradient of the Hall resistivity, $\rho_{xy}$, in the limit $B_r \to 0$. Also, we note that the condition for single occupancy of the lowest Landau level requires that $k_B T < \hbar \omega_c$. This condition is satisfied for temperatures $<50$ K at $B_r = 1$ T and for the electron cyclotron mass of InAs ($m_c = (0.025 \pm 0.01) m_0$, where $m_0$ is the electron mass in vacuum)$^{14}$. However, the linear MR is observed well above $T = 50$ K (see Fig. 1), indicating that it does not have a quantum origin, but is a classical effect.

Classical linear MR has been explained using a phenomenological model in which the distorted current paths arising from disorder-induced inhomogeneities and macroscopic variations, $\Delta \mu$, in the electron mobility are simulated by a network of random resistors. Each resistor element has four terminals to take into account not only the local resistive voltage drop but also the Hall voltage associated with the local mobility, $\mu$, of a particular region of the sample$^{2,8}$. The crossover of the MR from quadratic to linear $B_r$-dependence is predicted to occur at a characteristic magnetic field $B_{r_c} = \mathcal{R}_{xx}^{\mu} / \Delta \mu$ if $B_r < B_{r_c}$, or at $B_{r_c} = \mu B_r / \Delta \mu$ if $B_r > B_{r_c}$, where $\mathcal{R}_{xx}^{\mu}$ corresponds to an average of the spatially varying mobility. A comparison of the $B_r$-dependence of $\Delta \rho_{xx}/\rho_{xx}$ in our InAs and In(AsN) Hall-bars at $T = 300$ K (see Fig. 1b) shows that both the value of $1/B_{r_c}$ and the slope of the MR follow the systematic decrease of $\mu$ from 3.3 m$^2$V$^{-1}$s$^{-1}$ in InAs to 0.6 m$^2$V$^{-1}$s$^{-1}$ in In(AsN). The characteristic magnetic field $B_{r_c}$ provides an estimate of the average mobilities $\overline{\mu} = 1/B_{r_c} \sim 1$ m$^2$V$^{-1}$s$^{-1}$ and $0.5$ m$^2$V$^{-1}$s$^{-1}$ in the InAs and In(AsN) samples, respectively, which are comparable to those measured by the Hall effect. Also, the MR tends to follow the temperature dependence of $\mu$: in the InAs sample, an increase of $T > 4.2$ K and up to $T = 100$ K leads to an increase of $\mu_{xx}$ from 3.1 to 3.8 m$^2$V$^{-1}$s$^{-1}$ followed by a decrease at $T > 150$ K. This variation is caused by the interplay between impurity and phonon scattering and is accompanied by a similar $T$-dependence of the MR (see Fig. 1a). Linear MR is also observed in the two-terminal small channel devices, see Fig. 1c.

**Monte Carlo simulations.** As our InAs and In(AsN) epilayers are grown on highly lattice-mismatched GaAs, threading dislocations tend to form at the epilayer/substrate interface. The macroscopic ($>0.1 \mu m$) inhomogeneities observed by transmission electron microscopy of our samples$^{15}$, are likely to induce variations in the electronic properties. We model the effect of such inhomogeneities on the electron dynamics by Monte Carlo simulations$^{16}$. As the magnetic field is applied along the $z$-direction, the magnetic field affects the components of the electron motion in the $x-y$ plane. In our simulation of the two-dimensional electron trajectories, inhomogeneity is introduced by placing low-mobility islands randomly within the $x-y$ plane (Fig. 2). The random spatial profile of the islands, $\Delta(x,y)$, is generated from the power spectrum of the autocorrelation function $\langle \Delta(r)\Delta(r') \rangle = \Delta^2 \exp[-|r-r'|^2/\lambda^2]$, where $\lambda$ is the correlation length, $r = (x,y)$ and $r' = (x',y')$. The low-mobility regions $\Delta_{\text{low}}$ are then defined according to the relation $\Delta_{\text{low}} = |\Delta(r) < \Delta_{\text{th}}|$. This condition determines the fractional area, $f$, of the sample with low mobility in which electrons undergo scattering at a rate higher than in the remaining high-mobility part of the sample. In a three-dimensional model, the value of $f$ would result from an average of the $f$-values from various $x-y$ planes of the three-dimensional conducting channel. In the low-mobility regions, we use a scattering rate $\tau_{\text{scattering}} = r_1 + r_2$, where $r_1 = 1.5 \times 10^{12}$ s$^{-1}$ and $r_2 = 2.5 \times 10^{14}$ s$^{-1}$ are the elastic and inelastic phonon scattering rates, respectively; elsewhere, the electron drift is mainly limited by inelastic scattering with $r-r_2$. The non-parabolicity of the electron energy dispersion of InAs is modelled as $E(k) = [1 + \alpha_2(k)]h^2k^2/2m_0$, where $m_0 = 0.026$ m$^0$ and $\alpha = 2.2 eV^{-1}$. This form of the energy dispersion occurs in numerous semiconductors, including the narrow gap InSb for which observations of linear MR have been also reported$^{2,3}$. The electron velocity, given by $v(k) = h^{-1} \nabla_x E(k)$, and the wave vector, $k$, change with time under the action of the Lorentz force, $F = -e [E + v \times B]$, where $E = [E_x, E_y, 0]$ and $B = [0, 0, B_z]$.$^{17}$ The Hall field component, $E_y$, is set so that the $y$-component of the average electron velocity is zero, that is, $v = [v_x, 0, 0]$, consistent with the current flow for our Hall bars. The main findings of our simulations are not changed if we consider a different geometry in which the Hall field is effectively 'shorted out', that is, $E_y = 0$. Thus, our analysis applies not only to Hall-bars but also to short channels and Corbino geometries, in which the Hall electric field is fully 'short-circuited' by the contact electrodes.

Figure 2a–d illustrates the calculated electron trajectories at $E_x = 0.5 \text{kV cm}^{-1}$ and $B_z = 0$, 1, 10 and 20 T. Here, we have assumed a correlation length $\lambda = 0.5 \mu m$, $f = 5\%$ and low-mobility regions of average size $s = 0.4 \mu m$ and inter-island separation $l = 8 \mu m$. It can be seen that in the vicinity of the low-mobility islands, the electron undergoes multiple scattering events, which act to deflect the cyclodidal motion (see inset of Fig. 2b). This island-induced scattering is observed at low (Fig. 2b) and high (Fig. 2d) applied magnetic fields. It is absent in a homogenous system and is responsible for the linear magnetic field dependence of the inverse mobility component, $\mu_{xx}^{-1}$, whose strength increases with $f$, as shown in Fig. 3a.

**Figure 2 | Monte Carlo simulations of electron trajectories in magnetic field.** Calculated electron trajectories (red lines) in InAs for an applied electric field $E_x = 0.5 \text{kV cm}^{-1}$ and under various applied magnetic fields $B_z$: (a) $B_z = 0$, (b) 1, (c) 10 and (d) 20 T. The black areas correspond to regions of low mobility. As shown in the magnified trajectories in the insets of panels (b) and (d), in the vicinity of a low-mobility island, the electron undergoes multiple scattering events, which act to deflect the cyclodial motion. The bar in panel (a) corresponds to $1 \mu m$.
In the limits of high and average electron velocity, ~ is the zero field mobility. A similar dependence of on applied electric field is observed in devices with different values of and width . At all values of , its linear dependence on is found to be weakly affected by electric field (Fig. 3c) and only at low-magnetic fields ( ) and high does the value of tend to decrease with decreasing , leading to a weak negative MR. This is caused by the interplay between electron cyclotron motion and island-induced scattering, which tends to be increased by , that is, electrons hit a low-mobility islands more frequently at higher speeds. However, overall this effect tends to be weak and, at large , the deflection of the cyclodial motion by the low-mobility islands and corresponding linear MR are preserved.

The Monte Carlo simulations indicate that different values of the band non-parabolicity parameter produce similar linear MR behaviour. Thus, the specific details of the electronic band structure do not have a critical role in the observation of linear MR. Perhaps surprisingly, although tends to increase with increasing , its linear dependence on is found to be weakly affected by electric field (Fig. 3c) and only at low-magnetic fields ( ) and high does the value of tend to decrease with decreasing , leading to a weak negative MR. This is caused by the interplay between electron cyclotron motion and island-induced scattering, which tends to be increased by , that is, electrons hit a low-mobility islands more frequently at higher speeds. However, overall this effect tends to be weak and, at large , the deflection of the cyclodial motion by the low-mobility islands and corresponding linear MR are preserved.

The weak dependence of the linear MR on applied electric field is confirmed by our MR experiments in the short channel InAs samples. Figure 4a shows the dependence of the current density on at low temperature ( ) and at various (up to ). For an InAs channel of length and width , similar dependences of on and were found in devices with different values of and . At , the ( ) curve is approximately linear at low bias. The ohmic behaviour is then followed by a slight sublinear bias dependence and, at larger electric fields, by a sharp rise of the current that is characteristic of systems switching from a state of low to high conductivity due to impact ionization and avalanche breakdown. At all values of , the magnetic field acts to suppress the current and to shift the threshold electric field for impact ionization to significantly higher values. From Fig. 4b, it can be seen that the MR remains linear over an extended range of up to ~ kV cm~, just below the threshold electric field for impact ionization. In contrast, in the regime of avalanche breakdown the MR is strongly enhanced and tends to depart from a simple linear dependence due to the effect of the magnetic field in suppressing the avalanche multiplication of carriers.

To conclude, our data and analysis indicate that the presence of low-mobility islands in a high-mobility conductor and their effect on the electron trajectories underpin the observation of classical
Transport studies. Measurements of the current–voltage characteristics were performed in both direct current and pulsed mode. For the pulsed mode of acquisition of the current, a voltage pulse generator was used to generate saw-tooth single voltage pulses of duration of 10 µs, and a digital oscilloscope was used for the synchronized and fast temporal recording of the applied bias and current, thus reducing excessive sample heating at high-operating currents. The current–voltage curves in pulsed magnetic field were taken within 1 ms at the top of the magnetic field pulse, during which the magnetic field remains constant to within ±0.5%. 

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Author contributions
A.P., N.V.K. and O.M. designed and performed the experiments; N.M. performed the Monte Carlo simulations; L.E. took part in the discussions of the Monte Carlo studies; A.K. and Q.D.Z. grew the samples; A.P. wrote the paper; and all authors took part in the discussion/analysis of the data.

Additional information
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Figure 4 | Linear transverse MR at high electric fields. (a) Dependence of the current density J on electric field E, at various magnetic fields Bz for an InAs channel of length L = 10 µm and width W = 2 µm (T = 4.2 K). Measurements of the (EJ) characteristics were performed in both the direct current (lines) and pulsed mode (symbols). The inset sketches a two-terminal planar device. (b) Transverse MR Δρxx/ρxx as a function of Bz at various values of E. The resistivity ρxx is calculated as ρxx = E/J.