Resonant decay of Bose condensates

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Abstract

We present results of fully non-linear calculations of decay of the inflaton interacting with another scalar field $X$. Combining numerical results for cosmologically interesting range of resonance parameter, $q \leq 10^6$, with analytical estimates, we extrapolate them to larger $q$. We find that scattering of $X$ fluctuations off the Bose condensate is a very efficient mechanism limiting growth of $X$ fluctuations. For a single-component $X$, the resulting variance, at large $q$, is much smaller than that obtained in the Hartree approximation.

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In recent years, we have come to realize that the post-inflationary universe had probably been a much livelier place than was previously thought. In many models of inflation, the decay of the inflaton field is not a slow perturbative process but a rapid, explosive one. At the initial stage of this rapid process, called preheating [1], fluctuations of Bose fields coupled to the inflaton grow exponentially fast, which can be thought of as “parametric resonance” [2], and achieve large occupation numbers. At the second stage, called semiclassical thermalization [3], the resonance smears out, and the fields reach a slowly evolving turbulent state with smooth power spectra [3,4].

The explosive growth of Bose fields leads to very large variances of these fields close to the end of the resonance stage. That could result in several important effects taking place shortly after the end of inflation. These include symmetry restoration, baryogenesis, and SUSY breaking [4]. To find out if these effects had indeed occurred, one needs a good estimate of the maximal size of Bose fluctuations. The semiclassical nature of processes involving states with large occupation numbers allows us to treat decay of the inflaton, and any Bose condensate in general, as a classical non-linear problem with random initial conditions for fluctuations [3]. This classical problem can be analyzed numerically.

In this Letter we report results of fully non-linear calculations for the most interesting case when the coupling of a massive inflaton \( \phi \) to some other scalar field \( X \) is relatively large. That means, more precisely, that the system is in the regime of wide parametric resonance [1], characterized by a large value of the resonance parameter \( q \), \( q \gg 1 \). We have studied both expanding and static universes, to cover both post-inflationary dynamics and decays of possible other Bose condensates. Our objective was to obtain an estimate for the maximal size of \( X \) fluctuations, importance of which we emphasized above.

Our results are as follows. We have found that scattering of \( X \) fluctuations off the Bose condensate of \( \phi \), which knocks inflaton quanta out of the condensate and into low-momentum modes, is very efficient in limiting the size of \( X \) fluctuations for large values of \( q \), such as required [1,4] to produce particles much heavier than the inflaton. This scattering process involves the condensate of zero-momentum inflatons and, for that reason, is especially enhanced, cf. Refs. [3,6,7]. Fluctuations of \( X \) can reach larger values for smaller values of \( q \). The suppression of the maximal size of \( X \) fluctuations for large \( q \) significantly restricts the possibility of GUT baryogenesis after inflation, as well as the types of phase transitions that could take place after preheating.

Our present results should be compared with those obtained in the Hartree approximation. We find that for a single-component field \( X \) in flat space-time, the Hartree approximation is inadequate for all \( q \gg 1 \). A similar conclusion was made in Ref. [4] for the conformally invariant case of massless inflaton interacting with a massless field \( X \), based on
our simulations of the fully non-linear problem for that case. The Hartree approximation, with its characteristic positive feedback of $X$ on the inflaton decay [1], may still apply when $X$ has sufficiently many components; it remains to see if this can happen for realistic sizes of GUT multiplets.

In the model with a massive inflaton (Model 1 of Ref. [4]), the full scalar potential is $V_1(\phi, X) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g^2 \phi^2 X^2 + \frac{1}{2} M_X^2 X^2$. For comparison, we will sometimes present results also for the model with massless inflaton (Model 2), in which the potential is $V_2(\phi, X) = \frac{1}{2} \lambda \phi^4 + \frac{1}{2} g^2 \phi^2 X^2$. Both fields ($\phi$ and $X$) have standard kinetic terms and are minimally coupled to gravity. We will often use rescaled variables: for Model 1, $\tau = m \eta$, where $\eta$ is the conformal time; $\xi = m x$; $\varphi = \phi a(\tau)/\phi(0)$; $\chi = X a(\tau)/\phi(0)$. For Model 2, one should replace $m$ with $\sqrt{\lambda} \phi(0)$ in these rescalings. Here $a(\tau)$ is the scale factor of the universe, defined so that $a(0) = 1$, and $\phi(0)$ is the value of the homogeneous inflaton field at the end of inflation. The resonance parameter $q$ is $q \equiv g^2 \phi^2(0)/4m^2$ for Model 1, and $q \equiv g^2/4\lambda$ for Model 2.

In order for resonance to fully develop in expanding universe, the resonance parameter $q$ should exceed a certain minimum value, $q_{\text{min}}$, which depends on the mass $M_X$ of $X$ [1]. In Model 1, for $M_X = 0$, $q_{\text{min}} \sim 10^4$; for $M_X = 10m$, $q_{\text{min}} \sim 10^8$. We have simulated this model for $q$ up to $q = 10^4$ in flat space-time and for a few $q$ ranging from $10^4$ to $10^6$ in the expanding universe. We have developed an analytical approach, and we use analytical estimates to extrapolate our results to larger $q$, needed to produce heavier particles.

The full non-linear equations of motion for $\varphi$ and $\chi$, which follow from the action described above, are solved directly in configuration space with initial conditions corresponding to conformal vacuum at the end of inflation for all modes with non-zero momenta. The initial conditions for $\chi$ are given in Ref. [4]; those for $\delta \varphi$ are obtained similarly, along the lines of Ref. [3]. Classical fluctuations evolve from quantum ones, and, in cases we consider, their typical initial sizes are much smaller than the scale of non-linearity. Hence, initial evolution is linear with respect to fluctuations. During the initial linear stage, the equation of motion for Fourier components of $\chi$ can be approximated as $\dddot{\chi}_k + \omega_k^2(\tau) \chi_k = 0$, where

$$\omega_k^2(\tau) = m_\chi^2 a^2 + k^2 - \ddot{a}/a + 4q \varphi_0^2(\tau),$$  \hspace{1cm} (1)$$

$m_\chi \equiv M_X/m$, and $\varphi_0$ is the zero-momentum mode of $\varphi$; $\varphi_0(0) = 0$. By virtue of our rescaling, $\varphi_0(0) = 1$.

Computations were done on $128^3$ lattices, for a single-component $X$. For the case of expanding universe, we used $m^2 = 10^{-12} M_{\text{Pl}}^2$. Energy non-conservation in flat space-time typically was less then $10^{-3}$; in expanding universe in linear regime our calculations closely reproduced calculations in the Hartree approximation, which have much better accuracy.
Let us first consider the case without expansion of the universe. In the formulas above, one substitutes \( a(\tau) = 1 \). The variances of the fields \( \chi \) and \( \varphi \) in Model 1 at \( q = 2000 \) as functions of time are shown in Fig. [1]. The exponential growth of the variance \( \langle \chi^2 \rangle \) (the angular brackets denote averaging over space or, for space-independent quantities, equivalently, over realizations of random initial conditions) at early times is a parametric resonance, which in the present case ends at \( \tau \approx 40 \). This marks the end of the linear stage.

At large \( q \), fluctuations of \( X \) are produced at resonance stage only during short intervals of time near moments when \( \varphi_0 \) passes through zero \([1,8]\). These are intervals in which the adiabatic (WKB) condition \( \dot{\omega}_k/\omega_k^2 \gg 1 \) is broken for some \( k \). Notice a series of spikes in \( \langle \chi^2 \rangle \) at the same moments of time. They are due to modulation of \( \omega_k \) by the oscillating \( \varphi_0 \). Indeed, introduce analogs of occupation numbers \( n_k \) via \( \langle \chi^2 \rangle = \int d^3k P_\chi(k) \propto \int d^3k n_k(\tau)/\omega_k(\tau) \).

Even at the resonance stage, change in \( n_k \) during one oscillation is much smaller than variation of \( \omega_k \): when instead of \( \langle \chi^2(\tau) \rangle \) we plot \( \int d^3k n_k(\tau) \), the spikes are replaced by relatively small steps at times when \( \varphi_0 = 0 \).

We have monitored the power spectra, \( P(k) \), of \( \varphi \) and \( \chi \) in all our integrations. The strongest resonant momentum of \( \chi \) is typically of order \( q^{1/4} \); for some \( q \), though, it can be close to zero. Development of resonance peaks for \( \chi \) is followed by appearance of peaks for \( \varphi \) due to rescattering, cf. Ref. \([3]\). Later, rescattering leads to a turbulent state, characterized by smooth power spectra.

Resonant growth stops when the Hartree correction to the mass of \( \varphi \), due to \( \langle \chi^2 \rangle \) (at spike, since this is when fluctuations are produced) moves the system out of resonance. Because some modes grow faster than others, the width of the resonance peak in the power spectrum of \( \chi \) is much smaller than the full width (of order \( q^{1/4} \)) of the instability band. Nevertheless, we find, using analytical results of Ref. \([8]\), that the peak width still scales as \( q^{1/4} \), up to a power of \( \ln q \). We then estimate that resonance ends when \( \langle \chi^2 \rangle_s \sim q^{-3/2} \).

In Fig. [2], we have plotted results of our numerical integrations for the spike value \( \langle \chi^2 \rangle_s \) at the end of the resonance, as a function of \( q \). At \( q \gtrsim 100 \), these data are well fitted by \( \langle \chi^2 \rangle_s \propto q^{-3/2} \). Figure [2] confirms that the termination of parametric resonance is a Hartree effect.

Parametric resonance is followed by a plateau (unless we consider an exceptional \( q \), for which the resonance peak was close to zero). There, the variances of fluctuations do not grow, but an important restructuring of the power spectrum of \( \chi \) takes place. The power spectrum of \( \chi \) changes from being dominated by a resonance peak at some non-zero momentum to being dominated by a peak near zero. The width of this new peak at \( k \approx 0 \) is of order one.

When this peak becomes strong enough, the growth of variances resumes. The resumed growth (at \( \tau \gtrsim 50 \) in Fig. [1] is quite rapid (compared to subsequent slow evolution) and is
strongly affected by rescattering. This stage can be called semiclassical thermalization \[3\], or chaotization stage. Towards the end of it, the power spectra smoothen out; both power spectra are now dominated by momenta of order one.

An important effect seen in Fig. 1 is the rapid growth of fluctuations of the field $\varphi$. Indeed, at late times, they are much larger than fluctuations of $\chi$. Fluctuations of $\varphi$ are produced by the scattering process in which $\chi$ fluctuations knock $\varphi$ out of the zero mode. If we neglected fluctuations of $\varphi$, i.e. the content of its non-zero modes, the Hartree approximation in our model would be exact. As we will see, at large $q$ it can be far from being so.

At the end of the chaotization stage ($\tau \approx 80$ in Fig. 1), the variances in “valleys” between the spikes of $\langle X^2 \rangle$ can be estimated directly from the classical equations of motion, solved in the limit $|\delta \varphi| \ll \bar{\varphi}$; here $\delta \varphi = \varphi - \varphi_0$, and $\bar{\varphi}$ is the amplitude of oscillations of $\varphi_0$. Using Green functions, we obtain, for Fourier components of $\delta \varphi$ and $\chi$,

$$\varphi_p(\tau) \approx -\frac{4q}{\Omega_p} \int_0^\tau d\tau' \sin[\Omega_p(\tau - \tau')]\varphi_0 \int d^3 k \chi^*_k \lambda_{k+p}$$

$$\chi_k(\tau) = \chi^{(0)}_k + 8q \int_0^\tau dt' F_k(\tau, \tau') \varphi_0 \int d^3 p \varphi^*_p \lambda_{k+p}$$

where $\Omega_p = (p^2 + 1)^{1/2}$; $\chi^{(0)}_k(\tau)$ solves the Hartree equation, and $F_k(\tau, \tau')$ is the retarded Green function for it. At times when $\chi$ grows, the main contribution to the time integral in (3) comes from $\tau'$ near $\tau$. Using the random phase approximation, we then obtain

$$\langle |\varphi_p|^2 \rangle \sim q^2 \bar{\varphi}^2 V^{-1} \int d^3 k \langle |\chi_k|^2 \rangle \langle |\lambda_{k+p}|^2 \rangle$$

where $V$ is the total spatial volume. The end of chaotization stage is the time, $\tau_{ch}$, when the second term in r.h.s. of (3) becomes of the order of the first. We find that that happens when, in terms of the physical fields,

$$\langle X^2 \rangle_v \sim \frac{\phi^3(0)}{q^{3/2} \phi(\tau_{ch})}; \quad \langle (\delta \varphi)^2 \rangle \sim \phi^2(0)/q.$$  \[4\]

where $\bar{\phi}$ is the amplitude of $\phi_0$. In particular, $\langle X^2 \rangle$ in “valleys” after rescattering, Eq. (4), is of the order of $\langle X^2 \rangle_s$ at the end of resonance. This is indeed seen in Fig. 1, as well as in all our other integrations in flat space-time. Note that when variances of fields reach values (4), the analogs of occupation numbers, introduced as before, reach values of order $1/g^2$ for both $X$ and $\phi$.

After chaotization, the maxima and minima of $\langle \chi^2 \rangle$ evolve slowly, but the variance of $\varphi$ continues to grow rapidly for a while, see Fig. 1. We interpret this as follows. According to Eq. (2), fluctuations of $\varphi$ are driven by $\chi$. A periodic (or close to periodic) $\chi$ causes growth of $\delta \varphi$ via a usual, i.e. not parametric, resonance. This growth of $\delta \varphi$ will stop only when the approximation leading to (2) breaks down, that is $\delta \phi \sim \bar{\phi}$.
Let us now turn to Model 1 in expanding universe, see Figs. [3][4]. We used $\phi(0) = 0.28 M_{Pl}$. For massless $X$, the evolution of the scale factor was determined self-consistently, including the influence of produced fluctuations in the Einstein equations; for massive $X$, the universe was assumed matter dominated.

In expanding universe, particle creation acquires a qualitatively new feature [4]: because of the time-dependence of $\bar{\phi}$, the resonance peak scans the entire instability band, see Fig. 4. As our numerical integrations confirm (see also [4]), in order for production of fluctuations to be efficient, variation of the frequency of $\chi$, Eq. (1), need not be periodic. What is required is that every once in a while the adiabatic condition breaks down. In this situation, “non-adiabatic amplification” seems to be a better term for stimulated particle creation than “parametric resonance”.

The variances of fields at the end of chaotization can be estimated in the same way as before, and we obtain again Eq. (4). Note that now $\bar{\phi}(\tau_{ch}) \ll \phi(0)$, due to the redshift of the field.

Evolution of $\langle X^2 \rangle$ after chaotization can be thought of as a result of balance between creation and destruction of $X$ fluctuations. These processes are much faster than the expansion of the universe at this stage. We may then assume that the system looses memory of initial conditions and evolves as it would in flat space-time but with the effective, slowly changing value of the resonance parameter, $q_{eff}(\tau) \equiv q^2(\tau)/\phi^2(0)$. This assumption allows us to extend the estimate (4) for $\langle X^2 \rangle$ to times after chaotization by simply using the current value of $\bar{\phi}$ in it. In general, the assumption allows us to relate the scaling of variances with $\bar{\phi}$ to their scaling with $q$: $\langle X^2 \rangle/\bar{\phi}^2 \propto q^{-\alpha}(\tau) \propto q^{-\alpha} \bar{\phi}^{-2\alpha}(\tau)$.

Time dependence of variances is shown in Fig. 3 for the case $q = 10^6$ and $m_\chi = 2$. We can construct an analog of particle density, $n$, as $n/m = [4q(\phi_0^2 + (\delta \phi)^2)/\phi^2(0) + m_\chi^2]^{1/2}/X^2$. As long as the term with $\phi_0$ is the leading term in the square brackets, $n \propto \langle X^2 \rangle_{\nu} \bar{\phi}$, and according to our scaling argument $\langle X^2 \rangle_{\nu}$ scales with $\alpha = 3/2$, cf. Eq. (4)], $n$ after chaotization should be time-independent. Our data confirms that. Thus, fluctuations keep being produced at the same rate as they are diluted by the expansion. This can be called negative feedback amplification. A striking feature of the data is that, after chaotization, $\langle X^2 \rangle$ in spikes becomes essentially time-independent. According to our scaling argument, this means that $\langle X^2 \rangle$ scales as $1/q$. In Fig. 2 we show by stars the values of $\langle X^2 \rangle$ in expanding universe, taken at times when $\phi_0$ decays (more precisely, when $\bar{\phi} \approx \langle \delta \phi^2 \rangle$) and the “spike” and “valley” values of $\langle X^2 \rangle$ coalesce. (These values are within a factor of few from the maximal values $\langle X^2 \rangle$ achieves in the expanding universe; the exact value is model dependent and it was largest, equal to 2.6 , for $M_X = 0, q = 10^6$.) The scaling of these values with $q$ is well fitted by $1/q$. 6
We can now use the $1/q$ scaling of $\langle X^2 \rangle_s$ to extrapolate our data to larger values of $q$. For example, for $q = 10^8$, required to produce scalar leptoquarks with $M_X = 10m$, we obtain $\langle X^2 \rangle_s \sim 10^{-10} M^2_{Pl}$. Because $\langle X^2 \rangle_s$ stays on this level for a while, despite the expansion, the time-integrated conversion of inflatons into $X$ fluctuations in this two-field model is in fact quite efficient. For definite predictions for baryon asymmetry generated in decays of leptoquarks, however, one has to include other fields, which can have smaller $q$ and thus provide faster alternative channels of inflaton decay.

If instead of a single-component $X$ we consider $X$ with $N$ real components, our estimate for $\langle X^2 \rangle_v$ at the end of chaotization becomes larger by a factor of $\sqrt{N}$. For realistic GUT values of $N$, this can increase the maximal value of $\langle X^2 \rangle$ by an order of magnitude. In comparison, in the Hartree approximation, the maximal value of $\langle X^2 \rangle/\phi^2(0)$ is of order $q^{-1/2}\bar{\phi}/\phi(0)$; for $q = 10^8$ and $M_X = 10m$ we get $\langle X^2 \rangle_{\text{max}} \sim 10^{-7} M^2_{Pl}$.

The relatively quick complete exponential decay of $\phi_0$, seen in Fig. 3, is a distinctive feature of Model 1, as opposed to the conformally invariant Model 2. In the latter case, instead of (4), we obtain, at the end of chaotization, $\langle (\delta \phi)^2 \rangle \sim \bar{\phi}^2/q$, $\langle X^2 \rangle_v \sim \bar{\phi}^2/q^{3/2}$, and in the subsequent evolution $\langle X^2 \rangle$ redshifts together with $\bar{\phi}^2$.

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As writing this paper was nearing completion, we received a preprint, Ref. [9], in which the method of Ref. [3] was also applied to two-field models. The authors of Ref. [9] consider only flat space-time and the conformally invariant case; their results for these cases partially overlap with ours.
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FIG. 1. Variances of the fields $X$ (solid curve) and $\phi$ (dotted curve) in Model 1 in flat space-time. The filled square marks the spike value of $\langle X^2 \rangle$ at the end of resonance.

FIG. 2. Filled squares and crosses are the spike values, $\langle X^2 \rangle_s$, at the end of resonance obtained in fully non-linear simulations of Model 1 (flat space-time) and Model 2; empty boxes are the spike values at the first plateau in the Hartree approximation. Stars correspond to $\langle X^2 \rangle$ at the moment when zero mode decayed in Model 1 in expanding universe.
FIG. 3. Variances of fields $X$ and $\phi$, together with the inflaton zero-momentum mode, in Model 1 in expanding universe.

FIG. 4. Power spectrum of the field $X$ in Model 1 in expanding universe, output every period at maxima of $\phi_0(\tau)$. 