1. Introduction

Blast furnace is a multi-phase chemical reactor whose main purpose is to reduce iron oxides producing hot metal. In the blast furnace process, solids, liquids, and gas coexist, and complicated reactions take place in different zones of the blast furnace. To characterize and quantify the temporal behavior of the blast furnace process poses a great challenge because of the difficult measurement problems. In all the operational problems of blast furnaces, to predict the thermal state of blast furnace, represented by the silicon content in hot metal as usual, has been regarded as one of the most important problems. Recently, much research effort has been contributed to it. However, so far there is no consensus on the ideal prediction model of the silicon content in hot metal. Most of existing models are only the application of some methods while deep analysis is seldom made to judge whether these methods are fit or not for blast furnace process. The essences of systems are often unknown to those who are trying to apply some methods to describe them. Actually determining whether profiles from dynamic systems exhibit regular, stochastic or chaotic behavior should be the first goal in a variety of problems, and then the proper methods are chosen to describe them. So the current goal should be to determine whether the blast furnace thermal state is regular, stochastic or chaotic.

In this paper, power spectrum approach is applied to analyze the time series of the silicon content in hot metal actually measured in No. 1 blast furnace at Laiwu and No. 6 blast furnace at Linfen Iron and Steel Group Co. respectively. The results confirm the existence of fractal characteristics in the investigated time series, which makes the application of fractal theory to blast furnace process full of potential and attraction.

2. Fractal Analysis

Fractal was formulated and made popular by Mandelbrot. In general a structure will be called fractal, if it can be thought of as a set of consistently rescaled copies of itself, i.e., it looks the same when one “zooms in” or “zooms out” on it. From the point of view of mathematics, fractal means the relationships of the form $A \propto R^d$, named power law. The appearance of fractal allows us to mathematically describe systems that are intrinsically irregular at all scales. There are two kinds of fractal: self-similar and self-affine. Self-similar fractal structures have the fundamental property that the scaling-invariance is isotropic, while the scaling-invariance of self-affine fractal structures is anisotropic. In other words self-affine fractal structures have the local properties of self-similarity. Most of fractal structures in nature are self-affine.

For a given signal, the power spectrum gives a plot of the portion of a signal’s power (energy per unit time) falling within given frequency bins. Therefore, power spectral techniques are generally used to treat self-affine profiles quantitatively.

The power spectrum $P(f_k)$ of a discrete and finite scalar signal $u(j)$ is defined as the square of its Fourier amplitude at frequency $f_k$, given by

$$P(f_k) = \frac{1}{M} \left| \sum_{j=1}^{n} u(j)e^{-2\pi j f_k} \right|^2, \quad k = 1, 2, \ldots, M$$

where $n$ and $M$ are the size of the time series and frequency.
points respectively. When the sampling interval of the time series is $\Delta t$, the reflected maximum frequency can be written as

$$f_{\text{max}} = \frac{1}{2\Delta t} = f_M \quad \text{(2)}$$

Therefore, frequency increases with the increment $(M\Delta t)^{-1}$ each time up to $f_M$. If a system profile is self-affine, the variation of power spectrum $P(f)$ with the frequency $f$ will follow a power law:

$$P(f) \propto \frac{1}{f^\alpha} \quad \text{(3)}$$

i.e.

$$\log P(f) = -\alpha \log f + \beta \quad \text{(4)}$$

where $\alpha$ and $\beta$ are constants. The power law index $\alpha$ is one of the quantities describing the irregularity of the time series.

The most notable property of fractals is their dimensions. It represents the density of the systems orbits in phase space and gives an estimate of the system complexity. Turcotte$^{17}$ pointed out that the power alpha is related to the fractal dimension $D_s$ following

$$D_s = \frac{5 - \alpha}{2} \quad \text{(5)}$$

Further, the Hurst exponent ($H$) may be gotten from $D_s$, denoted by

$$H = 2 - D_s \quad \text{(6)}$$

The Hurst exponent measures the smoothness of fractal time series based on asymptotic behavior of the rescaled range of the process. It ranges from 0 to 1 and reflects anti-persistent behavior with $0<H<0.5$, persistent behavior with $0.5<H<1$ and ordinary uncorrelated Brownian motion with $H=0.5$.

The fractal dimension and Hurst exponent are estimated easily through power spectrum analysis for a given possible fractal time series.

3. Results and Discussion

For the current study, time series of silicon content in hot metal actually observed in No. 1 BF from June to August in 2001 and No. 6 BF from March to May in 2001 are used. The working volumes of these two furnaces are about 750 m$^3$ and 380 m$^3$ respectively. The size of the time series is 1008 tap numbers, and the time of sampling interval is about 2 h. The time series data as a function of Heat No. are presented in Fig. 1. Seen from Fig. 1, the data points of silicon content in hot metal of both furnaces oscillate surrounding their mean value during the studied period, and reveal self-similarity when they overlap each other.

Motivated by Welch, we divide the time series into $m$ segments, possible overlapping, with the length $L$ of each segment and the starting points of these segments $R$ units apart, to detect possible time-dependent fractal behavior. The relationship among $m$, $L$, $R$ and $n$ is

$$(m-1)R+L=n \quad \text{(7)}$$

The segmentation is illustrated in Fig. 2.

For the sake of convenience, we choose $R=84$ and $L=168$ corresponding to the size of time series during a week and two weeks respectively. For each segment of length $L$, the power spectrum of silicon content in hot metal can be calculated as a function of frequency ($f$ in h$^{-1}$) using Eq. (1). As an example, logarithmic power spectrum plots are appreciated in Fig. 3 for $m=8$. From Fig. 3, we will see the falloff of power spectrum with the increasing frequency during the studied period. The fundamental trend of each of these spectra appears to follow a straight line despite statistical fluctuation. The straight line is plotted to expect the slope values of $1/f$ power spectrum for each plots. Similar plots are obtained for other $m$. The power law behavior of silicon content in hot metal reveals self-similarity and indicates the existence of fractal properties roughly.

Least square method is used to simulate these slope values and fractal dimension as well as Hurst exponent are calculated using Eqs. (5) and (6) respectively. The results are presented in Fig. 4.
Obviously, time-dependent fractal behavior is found in the investigated time series as \( m \) changes. The fractal dimension \( D_s \) is fractal and low-dimensional at all the \( m \) sites for both BFs, thus pointing out a chaotic regime of the silicon content in hot metal. That is to say, there is a deterministic mechanism that governs the blast furnace process under study. Furthermore, the larger \( D_s \) occurs in No. 6 BF for most of values, indicating more complex dynamics in it, in accordance with the conclusion obtained from Kolomogorov entropy\(^{19}\) which quantifies the rates of information loss about the system with increasing time and measures the complexity of the system. The lower complexity will result in better prediction for the silicon content in hot metal of No. 1 BF.\(^{20}\) The lower complexity of No. 1 BF may result from bigger volume, more stable raw material and more stable furnace condition, etc. These factors will benefit the correct prediction of the silicon content in hot metal.

When \( D_s \) is determined, the Hurst exponent is calculated easily at different \( m \). From Fig. 4, we can find that the Hurst exponents are less than 0.5 at all the \( m \) segments for both BFs, indicating anti-persistent behavior, i.e. an increasing trend in the past followed by a possible decreasing trend in the future, and vice versa. Compared with No.1 BF, the behavior will take place more easily in No. 6 BF due to the smaller Hurst exponents for most of \( m \). Because in the extreme of \( H \rightarrow 0 \), the data trend changes totally irregular and becomes completely unpredictable. That is to say, as \( H \) tends to zero, the prediction of data becomes more difficult. The smaller Hurst exponents of silicon content of No. 6 BF provide another evidence to indicate the prediction more difficult than that of No. 1 BF. Furthermore the Hurst exponents of both BFs, departing from 0.5 greatly, mean that it is a challenging work to predict the silicon content in hot metal of blast furnace. The discovery of time-dependent fractal characteristics of silicon time series will help us to gain deeper understanding of the dynamics exhibited by the BF and lay the foundation for using nonlinear time series analysis to predict silicon. We believe this discovery will push the prediction technology of silicon content in hot metal of blast furnace forward.

4. Conclusions

The time-dependent fractal characteristics of silicon content in hot metal at No. 1 BF and No. 6 BF are detected through power spectrum analysis. From the results, we can conclude that there is a deterministic chaotic regime in the studied time series; furthermore the small BF is more complex possibly than the big one.

The detection of fractal behavior on silicon content in hot metal provides a powerful tool to explore complex blast furnace systems and makes the application of fractal theory to blast furnace process full of potential and attraction. The corresponding work is being explored now.

Acknowledgements

The authors wish to thank the Institute of System Optimization Technology of Zhejiang University for providing the practical data of No. 1 BF and No. 6 BF, with which the present investigation is possible. Thanks are also given to the State Key Development Program for Basic Research of China under grant No. 2002CB312200 due to the financial support.

REFERENCES

1) S. Claude: Ironmaking Proc. Metall. Soc. AIME, 21 (1967), 66.
2) S. M. Pandit, J. A. Clum and S. M. Wu: Ironmaking Proc. Metall. Soc. AIME, 34 (1975), 403.
3) S. P. Mehrotra and Y. C. Nand: ISIJ Int., 33 (1993), 839.
4) H. Saxen: Can. Metall. Q., 33 (1994), 319.
5) H. Singh, N. V. Srihari and B. Deo: Steel Res., 67 (1996), 521.
6) P. R. Austin, H. Nogami and J. Yagi: ISIJ Int., 37 (1997), 748.
7) P. R. Austin, H. Nogami and J. Yagi: ISIJ Int., 38 (1998), 246.
8) T. Miyano, S. Kimoto, H. Shibuta, K. Nakashima, Y. Ikenaga and K. Aihara: Physica D, 135 (2000), 305.
9) J. A. Castro, H. Nogami and J. Yagi: ISIJ Int., 40 (2000), 637.
10) J. Chen: Eng. Appl. Artif. Intell., 14 (2001), 77.
11) J. A. Castro, H. Nogami and J. Yagi: ISIJ Int., 42 (2002), 44.
12) M. Walleur and H. Saxen: ISIJ Int., 42 (2002), 316.
13) J. Jiménez, J. Mochón, J. S. Ayala and F. Obeso: ISIJ Int., 44 (2004), 573.
14) D. J. Wales: Nature, 350 (1991), 485.
15) B. B. Mandelbrot: The Fractal Geometry of Nature, W. H. Freeman, New York, (1982), 1.
16) V. C. Ricardo, M. Blanca, D. S. Rosa, V. G. José, D. M. José and M. R. Enrique: Fractals, 11 (2003), 295.
17) D. L. Turcotte: Fractals and Chaos in Geology and Geophysics, Cambridge University Press, Cambridge, (1992), 58.
18) P. D. Welch: IEEE Trans. Audio Electroacoustics, 15 (1967), 70.
19) C. H. Gao, Z. M. Zhou and Z. J. Shao: Acta Phys. Sin., 54 (2005), 1490.
20) C. H. Gao and Z. M. Zhou: Acta Phys. Sin., 53 (2004), 4092.