Supercurrent noise in quantum point contacts

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Abstract

Spectral density of current fluctuations in a short ballistic superconducting quantum point contact is calculated for arbitrary bias voltages $V$. Contrary to a common opinion that the supercurrent flow in Josephson junctions is coherent process with no fluctuations, we find extremely large current noise that is caused by the supercurrent coherence. An unusual feature of the noise, besides its magnitude, is its voltage dependence: the noise decreases with increasing $V$, despite the fact that the dc current grows steadily with $V$. At finite voltages the noise can be qualitatively understood as the shot noise of the large charge quanta of magnitude $2\Delta/V$ equal to the charge transferred during one period of Josephson oscillations.

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It is well established that the supercurrent in classical Josephson tunnel junctions does not fluctuate – see, e.g., [1]. This result is consistent with the notion that the supercurrent is the coherent dissipationless flow of Cooper pairs, and therefore is quite appealing intuitively and has acquired the status of a widespread opinion. There seems to be no reason to suspect that the situation should be any different in mesoscopic contacts with a few transverse modes, since, for instance, the noise properties of contacts in the normal state are insensitive to the number of transverse modes. Both in classical [2,3] and quantum [4,5] normal point contacts the shot noise was predicted to be suppressed due to the Pauli-principle correlations between electrons, and such a suppression has been found in experiments [6,7]. This picture is modified only slightly in point contacts between normal metals and superconductors, or in fully superconducting point contacts at large voltages, where Andreev scattering leads to partial reflection of electrons in the contact region giving rise to a finite level of shot noise [8]. The noise in this case is associated only with the excess current, and therefore represents only a small fraction of the full classical shot noise.

The aim of this work is to suggest and prove by microscopic calculation that contrary to intuitive expectation, the supercurrent flow in quantum point contacts should generate noise which is extremely large on the scale of the classical shot noise. The noise arises from the interplay between quasiparticle scattering and the supercurrent coherence. The latter amplifies the randomness of the quasiparticle scattering by “attaching” to each quasiparticle a large charge which is coherently transferred through the point contact.

We begin by considering a single-mode ballistic contact between two identical superconductors. Such contacts can be formed, for instance, in superconductor/semiconductor heterostructures and exhibit quantization of the supercurrent [9]. The length $d$ of the contact is assumed to be much smaller than the coherence length $\xi$, as well as the elastic and inelastic scattering lengths in the superconductors, so that all scattering inside the contact region can be neglected. We are interested in calculating the spectral density of current $I(t)$ in the point contact:
\[ I(t) = \frac{i e \hbar}{2m} \sum_{\sigma} \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z'} \right) \psi_{\sigma}^\dagger(z, t) \psi_{\sigma}(z', t') \bigg|_{z' \to z = 0, t' \to t}, \]

where \( z \) is the coordinate in the direction of the current flow and \( z = 0 \) corresponds to the position of the point contact. Taking \( t \) and \( t' \) in this definition to be on opposite branches of the Keldysh contour, we express the current correlation function

\[ K_I(t_1, t_2) \equiv \frac{1}{2} \left( I(t_1)I(t_2) + I(t_2)I(t_1) \right) - \langle I(t_1) \rangle \langle I(t_2) \rangle \tag{1} \]

as a product of two nonequilibrium Green’s functions. In the quasiclassical approximation this expression reduces to the following [3]:

\[ K_I(t_1, t_2) = -\frac{e^2}{8} \sum_{\pm} \text{Tr} \left[ g_\pm(t_1, t_2) \sigma_z g_\pm(t_2, t_1) \sigma_z + g_\pm(t_1, t_2) \sigma_z g_\pm(t_2, t_1) \sigma_z \right], \tag{2} \]

where the Green’s functions \( g \) are 2×2 matrices in the electron-hole space, \( \text{Tr} \) is taken over this space, and \( \sum_{\pm} \) is the sum over the two directions of propagation through the contact. Here and below, \( \sigma \)'s denote Pauli matrices.

From this point on, the calculation proceeds differently for vanishing and finite bias voltages. At vanishing voltage, the contact is in equilibrium and all Green’s functions and current correlation function [3] depend only on the time difference \( \tau = t_1 - t_2 \). In the frequency domain the equilibrium functions \( g \) are:

\[ g_\pm(\epsilon) = (1 - f(\epsilon)) \rho_\pm(\epsilon), \quad g_\pm^\dagger(\epsilon) = -f(\epsilon) \rho_\pm(\epsilon), \tag{3} \]

where \( \rho(\epsilon) \equiv g_R(\epsilon) - g_A(\epsilon) \) is the matrix of the density of states, and \( f(\epsilon) \) is the equilibrium Fermi distribution. Fourier transforming the correlation function (4) and using eq. (4) we get the following expression for the spectral density of current fluctuations

\[ S_I(\omega) = \frac{1}{2\pi} \int d\tau e^{-i\omega\tau} K(\tau) = \frac{e^2}{32\pi^2\hbar} \sum_{\pm} \int d\epsilon f(\epsilon)(1 - f(\epsilon \pm \hbar \omega)) \text{Tr} [\rho_\pm(\epsilon) \sigma_z \rho_\pm(\epsilon \pm \hbar \omega) \sigma_z]. \tag{4} \]

We shall see below that in order to get a meaningful result for \( S_I(\omega) \) we need to keep a finite energy relaxation rate in the model. We find the density \( \rho(\epsilon) \) which includes a...
finite relaxation rate generalizing the Kulik-Omelyanchuk theory [10]. We perform the
calculation in real time. For a short constriction the retarded Green’s function can be
written as $g_{jR}^{(0)} + g_{jR}$, where $g_{jR}^{(0)}$ is the spatially-uniform part of the Green’s function of the
$j$th electrode:
$$ g_{jR}^{(0)} = \frac{1}{\delta} \begin{pmatrix} \bar{\epsilon} & \bar{\Delta}e^{i\varphi_j} \\ -\bar{\Delta}e^{-i\varphi_j} & -\bar{\epsilon} \end{pmatrix}, \quad \bar{\epsilon} = \epsilon + i\gamma_1, \quad \bar{\Delta} = \Delta + i\gamma_2, $$
and $g_{jR}$ is the non-uniform part which satisfies the equation [11,12]:
$$ \pm iv_F \frac{\partial g_R}{\partial z} = \delta [g_{jR}^{(0)}, g_R]. \quad (5) $$
Here $\gamma_{1,2}$ are inelastic scattering rates, $\delta \equiv (\bar{\epsilon}^2 - \bar{\Delta}^2)^{1/2}$ and $v_F$ is the Fermi velocity. The non-
uniform part of the Green’s functions should also satisfy two boundary conditions: $g \rightarrow 0$
at $z \rightarrow \pm \infty$ (inside the electrodes), and $g_1 + g_1^{(0)} = g_2 + g_2^{(0)}$ at $z = 0$.

From eq. (3) we obtain that the solutions decaying at infinity have the following matrix
forms:
$$ g_{1R}^{(\pm)} = A_1^{(\pm)} \begin{pmatrix} 1 & (ae^{i\varphi_1})^{\pm 1} \\ -(ae^{i\varphi_1})^{\mp 1} & -1 \end{pmatrix}, \quad g_{2R}^{(\pm)} = A_2^{(\pm)} \begin{pmatrix} 1 & (ae^{i\varphi_2})^{\mp 1} \\ -(ae^{i\varphi_2})^{\pm 1} & -1 \end{pmatrix}, $$
where $a$ is the amplitude of Andreev reflection from the superconductor:
$$ a(\epsilon) = (\bar{\epsilon} - \delta)/\bar{\Delta}. $$

Matching solutions in the two electrodes with the boundary condition at $z = 0$ we get the total Green’s function at this point:
$$ g_{R,\pm} = \frac{1}{\delta \cos(\varphi/2) \mp i\bar{\epsilon} \sin(\varphi/2)} \begin{pmatrix} \bar{\epsilon} \cos(\varphi/2) \mp i\delta \sin(\varphi/2), & \bar{\Delta}e^{i\varphi_\Sigma} \\ -\bar{\Delta}e^{-i\varphi_\Sigma}, & -\bar{\epsilon} \cos(\varphi/2) \pm i\delta \sin(\varphi/2) \end{pmatrix}. \quad (6) $$
where $\varphi_\Sigma = (\varphi_1 + \varphi_2)/2$. The advanced function $g_A$ can be expressed in terms of $g_R$,
$g_A(\varphi_\Sigma) = -g_R^*(-\varphi_\Sigma)$. Using this relation and eq. (3), we see that although the Green’s
functions are not gauge-invariant (i.e., depend on $\varphi_\Sigma$), the current spectral density (4) is
gauge invariant. Therefore, to simplify the notations, we take below $\varphi_{\Sigma} = 0$ without loss of generality.

In the most interesting limit of small energy relaxation, $\gamma_{1,2} \ll \Delta/\hbar$, there are two distinct regions of energy where the density of states $\rho$ is non-vanishing: one is $|\epsilon| > \Delta$, and the other is the vicinity of the two subgap states with energies $\pm \epsilon_0 = \pm \Delta \cos(\varphi/2)$. Expanding eq. (6) in small deviations from $\pm \epsilon_0$, we get that in the subgap region the density of states is:

$$\rho_{\pm}(\epsilon) = 2\text{Re}[g_{R,\pm}(\epsilon)] = \frac{\gamma(\epsilon_0) \Delta \sin(\varphi/2)}{(\epsilon + \epsilon_0)^2 + \gamma^2(\epsilon_0)/4} \left( \begin{array}{c} 1 \\ \mp 1 \end{array} \right).$$

Using this relation in eq. (4), we get the subgap contribution to the current noise at low frequencies $\omega \sim \gamma$:

$$S^{(1)}_I(\omega) = \left( \frac{I_0 \sin(\varphi/2)}{\cosh(\epsilon_0/2T)} \right)^2 \frac{\gamma(\epsilon_0)}{2\pi(\omega^2 + \gamma^2(\epsilon_0))},$$

where $I_0 \equiv e\Delta/\hbar$, and

$$\gamma(\epsilon) = 2(\gamma_1(\epsilon) - \frac{\epsilon}{\Delta} \gamma_2(\epsilon)) = \alpha \int d\epsilon' \frac{\Theta(\epsilon'^2 - \Delta^2)}{\sqrt{\epsilon'^2 - \Delta^2}} \frac{(\epsilon - \epsilon')^3 \cosh(\epsilon'/2T)}{\sinh((\epsilon - \epsilon')/2T) \cosh(\epsilon'/2T)}.$$ 

Here $\alpha$ is a constant determined by the parameters of electron-phonon interaction.

Equation (8) shows that there is a very large low-frequency noise associated with the supercurrent flow through a quantum point contact. Although the total noise intensity decreases with decreasing temperature, its zero-frequency density can actually increase at low temperatures because of the rapid decrease of $\gamma$. The noise has a very simple time-domain interpretation. Equation (8) implies that the current which is considered to be a “dc” supercurrent does not flow continuously; rather it is a stochastic process with a typical realization shown in the inset in Fig. 1. Because of the quasiparticle exchange between the bulk electrodes and the two subgap states $\pm \epsilon_0$ localized in the point contact, the system jumps between the state carrying positive current $I_0 \sin(\varphi/2)$ and the state carrying negative current $-I_0 \sin(\varphi/2)$ with an average rate $\gamma(\epsilon_0)$. The jumps occur in such a way that the probabilities of finding a positive and negative current are, respectively, $f(-\epsilon_0)$ and $f(\epsilon_0)$. It
is straightforward to check that the spectral density of such a process is indeed given by eq. (8). Thus, the noise (8) is the two-level noise which is an inherent part of the supercurrent thermalization in the quantum point contacts.

Equations (4) and (6) also determine the noise due to the states above the gap. As we will see below, this part of the noise has a much less singular behavior in \( \gamma \) than the subgap noise (8). Therefore in order to calculate it we can limit ourselves to \( \gamma = 0 \). In this case we obtain from eqs. (4) and (6):

\[
S^{(2)}_I(\omega) = \frac{e^2}{4\pi^2\hbar} \int d\epsilon \left[ 1 - \tanh\left(\frac{\epsilon}{2T}\right) \tanh\left(\frac{\epsilon + \hbar\omega}{2T}\right) \right] u(\epsilon) u(\epsilon + \hbar\omega)(|\epsilon| |\epsilon + \hbar\omega| + 2 \text{sgn}(\epsilon) \text{sgn}(\epsilon + \hbar\omega) \Delta^2 \cos^2(\varphi/2)) ,
\]

\[
u(\epsilon) \equiv \frac{(\epsilon^2 - \Delta^2)^{1/2} \Theta(\epsilon^2 - \Delta^2)}{\epsilon^2 - \Delta^2 \cos^2(\varphi/2)}.
\] (10)

An interesting feature of the noise (10) is that it is phase-dependent, despite the fact that the states above the gap do not contribute to the average supercurrent. For \( T \gg \Delta \) and \( \omega = 0 \), eq. (10) gives:

\[
S^{(2)}_I(0) = \frac{e^2}{\pi^2\hbar} \left[ T - \frac{\Delta}{2} \left( \cos^2 \varphi \right) \ln \left| \frac{1 + \cos(\varphi/2)}{1 - \cos(\varphi/2)} \right| - 1 \right].
\] (11)

The first term in this expression is the regular Nyquist noise as in the normal state, while the second term is the phase-dependent correction associated with the gap. Because of the usual BCS singularity in the density of states, the second term diverges weakly as \( \varphi \to 0 \). This divergence is removed by \( \gamma \); at small but finite \( \gamma \), the logarithm is limited by \( \ln(\Delta/\gamma) \).

At \( \varphi = 0 \), \( S^{(2)}_I(\omega) \) has a similar weak singularity as a function of \( \omega \),

\[
S^{(2)}_I(\omega) = \frac{e^2}{\pi^2\hbar} \left[ T - \frac{\Delta}{2} \ln \frac{\Delta}{\omega} \right].
\]

The last component of noise comes from the “interference” of the subgap states and states above the gap in eq. (4). In the limit \( \gamma \to 0 \) we get from this equation and eqs. (3) and (7):

\[
S^{(3)}_I(\omega) = \frac{e^2 \Delta}{4\pi\hbar^2\omega} \sum_{\pm} \Theta((\epsilon_0 \pm \hbar\omega)^2 - \Delta^2)((\epsilon_0 \pm \hbar\omega)^2 - \Delta^2)^{1/2}[1 - \tanh(\frac{\epsilon_0}{2T}) \tanh(\frac{\epsilon_0 \pm \hbar\omega}{2T})].
\] (12)
Comparison of eqs. (11) and (12) with eq. (8) shows that the subgap noise dominates at low frequencies as long as the temperature is not too large on the scale of $\Delta$, i.e., $T \ll \Delta^2/\gamma$.

Total spectral density of current fluctuation $S_I(\omega)$ as a function of frequency calculated numerically from eqs. (4) and (6) is shown in Fig. 1 for several values of the Josephson phase difference $\varphi \in [0, \pi]$. The $S_I(\omega)$ for $\varphi \in [\pi, 2\pi]$ can be found from the relation $S_I(\omega, \varphi) = S_I(\omega, 2\pi - \varphi)$ which follows from (4) and (6). Figure 1 shows that the subgap noise (8) indeed dominates the spectrum even at not too small $\gamma$. We also see that due to the interference term (12) the high-frequency threshold of $S_I(\omega)$ at low temperatures shifts down from $2\Delta/\hbar$ with increasing $\varphi$. In particular, at $\varphi = \pi$, the noise at large frequencies starts at $\Delta/\hbar$ in accordance with eq. (12).

Now we turn to finite voltages. In this case, all observable quantities like the current correlation function (2) are periodic in time $t = (t_1 + t_2)/2$ with the period of the Josephson oscillations $T_0 = \pi\hbar/eV$. This implies that the Green’s functions $g$ should be periodic with period $2T_0$, since their off-diagonal elements can change sign when shifted by $T_0$. Therefore, $g$ can be expanded as a Fourier series:

$$g(t_1, t_2) = \sum_k g_k(\tau)e^{-ikeVt/\hbar},$$

where $\tau \equiv t_1 - t_2$. Averaging the correlator (2) over $t$ and Fourier transforming it with respect to the time difference $\tau$ as in the static case, we get the following expression for the spectral density of current fluctuations:

$$S_I(\omega) = -\frac{e^2}{16\pi^2\hbar} \sum_{k, \pm} \int d\epsilon \text{Tr}[g_k^>(\epsilon)\sigma_zg_k^<(\epsilon \pm \hbar\omega)\sigma_z].$$

In eq. (13) we have taken into account the fact that summation over the two directions of propagation in the correlator (2) is equivalent, for a symmetric contact, to summation over positive and negative bias voltages $\pm V$. Since the noise is an even function of $V$, this summations only contributes a factor of 2.

From the known solution for Green’s functions of a short superconducting point contact [13,14] and standard relations between the different Green’s functions:
\[ g^+ = \frac{1}{2}(g^K + g^R - g^A), \quad g^- = \frac{1}{2}(g^K - g^R + g^A), \]

we obtain the Fourier components \( g_k \):

\[ g_k = p_k \sigma_z + q_k i \sigma_y. \]  \hspace{1cm} (14)

Here \( p_k \) is non-vanishing for even \( k \) and:

\[ p^+_k(\epsilon) = 2(1 - F(\epsilon - eV/2))A(\epsilon), \quad p^-_k(\epsilon) = -2F(\epsilon - eV/2)A(\epsilon), \quad k \geq 0, \]  \hspace{1cm} (15)

\[ A(\epsilon) \equiv \prod_{l=1}^{k} a(\epsilon + leV - eV/2), \]

while \( q_k \) is non-vanishing for odd \( k \) and is given by the same eq. (14) with odd \( k \). For negative \( k \), \( p_k = p^{*-}_k \), and \( q_k = q^{*-}_k \). Function \( F \) has the meaning of non-equilibrium distribution of quasiparticles in the point contact:

\[ F(\epsilon) = f(\epsilon) + \sum_{n=0}^{\infty} \prod_{m=0}^{n} | a(\epsilon - meV) |^2 [ f(\epsilon - (n + 1)eV) - f(\epsilon - neV) ], \]  \hspace{1cm} (16)

Combining eqs. (14) and (15) with eq. (13) we get a final expression for \( S_I(\omega) \):

\[ S_I(\omega) = \frac{e^2}{2\pi^2 \hbar} \sum_{\pm} \int d\epsilon F(\epsilon)(1 - F(\epsilon \pm \hbar \omega))[1 + 2\text{Re} \sum_{k=1}^{\infty} \prod_{l=1}^{k} a(\epsilon + leV) a^*(\epsilon + beV \pm \hbar \omega)]. \]

\hspace{1cm} (17)

Equation (17) together with eqs. (4) and (6) are the main technical results of our work. Combined, these equations give the spectral density of current fluctuations in a short ballistic constriction between two identical superconductors at arbitrary voltages. As we can expect from our calculations for \( V = 0 \), the most interesting limit at finite voltages is \( V \ll \Delta/e \).

In this case eq. (17) can be simplified further. Expanding the amplitudes \( a(\epsilon) \) of Andreev reflection in small relaxation rates \( \gamma_{1,2} \) and replacing the sums with the integrals in eqs. (16) and (17) we obtain the spectral density of current fluctuations at low frequencies, \( \omega \ll \Delta/\hbar \):

\[ S_I(\omega) = \frac{e}{\pi^2 \hbar V} \sum_{\pm} \int_{-\Delta}^{\Delta} d\epsilon F(\epsilon)(1 - F(\epsilon \pm \hbar \omega)) \int_{\epsilon - \epsilon'}^{\epsilon} d\epsilon' \exp\{- \int_{\epsilon}^{\epsilon'} \frac{d\nu \gamma(\nu)}{eV \sqrt{\Delta^2 - \nu^2}}\} \times \]

\[ \int_{\epsilon}^{\epsilon'} \frac{d\nu \gamma(\nu)}{eV \sqrt{\Delta^2 - \nu^2}} \} \times \]

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\[ \int_{\epsilon}^{\epsilon'} \frac{d\nu \gamma(\nu)}{eV \sqrt{\Delta^2 - \nu^2}} \} \times \]
\[
\cos\left(\frac{\hbar \omega}{eV}\left[\arccos\left(\frac{\epsilon + \epsilon'}{\Delta}\right) - \arccos\frac{\epsilon}{\Delta}\right]\right),
\]
where the quasiparticle distribution function reduces to
\[
F(\epsilon) = f(\epsilon) - \int_{-\Delta}^{\epsilon} d\epsilon' \frac{\partial f}{\partial \epsilon'} \exp\left\{-\int_{\epsilon}^{\epsilon} \frac{d\nu h\gamma(\nu)}{eV \sqrt{\Delta^2 - \nu^2}}\right\}.
\]

Equation (18) is still a very general result which gives the spectral density of current fluctuations for arbitrary relation between the bias voltage, temperature, frequency, and energy relaxation rate. At large temperatures \(T \gg \Delta\) the relaxation rate \(\gamma\) (3) and distribution function \(F\) are constant for energies inside the gap, and the zero-frequency spectral density (18) is:
\[
S_I(0) = \frac{e \Delta^2}{4 \pi^2 \hbar V} \frac{\pi \lambda(1 + \lambda^2) + 2 + 2e^{-\pi \lambda}}{(1 + \lambda^2)^2}, \quad \lambda \equiv \frac{\hbar \gamma}{eV}.
\]

The noise (20) has an unusual voltage dependence; it decreases monotonically with increasing voltage despite the fact that, as can be shown, the average current grows with voltage. Although the monotonic decrease of the noise intensity with the voltage is a characteristic feature of large temperatures \(T \gg \hbar \gamma\), noise always decreases with voltage at \(V \gg \hbar \gamma/e\) (see inset in Fig. 2). Indeed, in this case, eq. (19) shows that \(F(\epsilon) = F(\Delta)\), and we get for the spectral density (18):
\[
S_I(\omega) = \frac{e \Delta^2}{2 \pi^2 \hbar \cosh^2(\Delta/2T)V} \frac{1 + \cos\left(\frac{\pi \hbar \omega}{eV}\right)}{(1 - (\hbar \omega/eV)^2)^2}.
\]

At small voltages \(V \ll \hbar \gamma/e\), and temperatures \(T \ll \Delta\), the noise is independent of the voltage \(V\):
\[
S_I(0) = \frac{2e^2 \Delta T}{\pi^2 \hbar^2 \gamma(0)}.
\]

Equations (21) and (22) describe two sides of the noise peak with the maximum at \(V \simeq \hbar \gamma/e\). The exact shape of this peak depends on the energy dependence of the relaxation rate \(\gamma\) and is shown in the inset in Fig. 2 for several temperatures in the approximation of energy-independent \(\gamma\). The curves were calculated numerically from eq. (17). The main part of
Fig. 2 shows the zero-frequency spectral density of current fluctuations at arbitrary voltages. It illustrates a transition from the noise peak at small voltages to the large voltage regime where $S_I(0)$ saturates at $4e^2\Delta/15\pi^2\hbar$.

The fact that the noise at small voltages is very large on the scale of the regular shot noise, together with an unusual voltage dependence, reflects an unusual physical mechanism of the noise. For $V \gg \hbar\gamma/e$ this mechanism can be described qualitatively as follows. The giant noise arises since each quasiparticle getting into the constriction region with energy equal to one of the gap edges generates an avalanche of Andreev reflections before it can escape out of the constriction by climbing up or down in energy to the opposite edge of the energy gap. The number of generated Andreev reflections is $2\Delta/eV$, so that each quasiparticle causes a coherent transfer through the constriction of a charge quantum of magnitude $2\Delta/V$. For small voltages $V \ll \Delta/e$ this is much larger than the charge of individual Cooper pairs. In this way, the randomness of the quasiparticle scattering (quasiparticles get inside the energy gap with probability $f(-\Delta)$ from one electrode and with probability $f(\Delta)$ from the opposite electrode) is amplified. Therefore, the noise described by eq. (21) can be interpreted as the shot noise of these large charge quanta, and is in fact the consequence of the coherence of the supercurrent flow through the point contact.

It is interesting to note that this picture in the energy domain has a “dual” formulation in the time domain in terms of the non-equilibrium occupation of the two subgap states which are responsible for the dc supercurrent. In particular, the avalanche of $2\Delta/eV$ Andreev reflections triggered by a quasiparticle corresponds in the time domain to one period of Josephson oscillation, during which the supercurrent $I_0 \sin(\varphi/2)$ carries the charge $2\Delta/V$ through the point contact. More generally, all small-voltage results for the spectral density of current fluctuations obtained above (eqs. (20) – (22) for the ac regime and eq. (8) for the dc regime) can also be obtained from a purely classical rate equation [14] for the non-equilibrium occupation probabilities of the two subgap states. An advantage of such simple classical approach is the possibility to generalize it straightforwardly to time-dependent voltages and arbitrary bias conditions of the point contact. Of course an advantage of the
fully microscopic approach used in this work is that it gives spectral density of current fluctuations for frequencies and bias voltages that are arbitrary on the scale of $\Delta$.

The simple time-domain interpretation of the supercurrent noise at small bias voltages allows us to propose a simple generalization of our results for ballistic junctions to junctions with arbitrary transmission coefficient $D$. Indeed, making a very natural assumption that we have the same exchange of quasiparticles between the two subgap states localized in the point contact and the bulk superconductors, we get immediately for $V = 0$ (cf. eq. (8)):

$$S_I(\omega) = \frac{1}{2\pi} \sum_{k=1}^{N} \left( \frac{I_k(\varphi)}{\cosh(\epsilon_k/2T)} \right)^2 \frac{\gamma(\epsilon_k)}{\omega^2 + \gamma^2(\epsilon_k)},$$

Here $N$ is the number of propagating transverse modes in the junction, $I_k(\varphi) \equiv (e\Delta/2h)D_k \sin(\varphi)/[1 - D_k \sin^2(\varphi/2)]^{1/2}$ coincides at $T = 0$ with the supercurrent carried by the $k$th mode, and $\epsilon_k = \Delta[1 - D_k \sin^2(\varphi/2)]^{1/2}$. Since fluctuations of the current in all modes are independent, the current noise $S_I(\omega)$, expressed in terms of the total supercurrent, decreases roughly as $1/N$ with increasing number $N$ of transverse modes. Therefore the giant small-voltage noise discussed in our work disappears in the classical limit $N \to \infty$. However, it can be very important in quantum point contacts with few transverse modes, and even in regular tunnel Josephson junctions of ultrasmall area.

In conclusion, we have shown that the supercurrent flow in quantum point contacts leads to a giant current noise both in the regime of dc and ac Josephson effects. The noise arises from the interplay of quasiparticle scattering and coherence of the supercurrent flow and has an unusual temperature and voltage dependence.

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FIGURES

Figure 1. Spectral density of current fluctuations in a short single-mode constriction between two superconductors in the regime of the dc Josephson effect. From bottom to top, the curves correspond to $\varphi = 0; \pi/2; \pi$. The peak at low frequencies is the subgap contribution. The inset shows a typical realization of the “dc” supercurrent as a function of time at non-vanishing temperatures. The sign of the current switches randomly with the characteristic rate $\gamma$ giving rise to the low-frequency peak in spectral density.

Figure 2. Zero-frequency density of current fluctuations in the ac regime as a function of the bias voltage at zero temperature. The inset shows a blow-up of the small-voltage peak at (from bottom to top) $T = 0; 0.1\Delta; 0.2\Delta$. 
