Privacy-Preserving Stealthy Attack Detection in Multi-Agent Control Systems

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Abstract—This paper develops a glocal (global-local) attack detection framework to detect stealthy cyber-physical attacks, namely covert attack and zero-dynamics attack, against a class of multi-agent control systems seeking average consensus. The detection structure consists of a global (central) observer and local observers for the multi-agent system partitioned into clusters. The proposed structure addresses the scalability of the approach and the privacy preservation of the multi-agent system’s state information. The former is addressed by using decentralized local observers, and the latter is achieved by imposing unobservability conditions at the global level. Also, the communication graph model is subject to topology switching, triggered by local observers, allowing for the detection of stealthy attacks by the global observer. Theoretical conditions are derived for detectability of the stealthy attacks using the proposed detection framework. Finally, a numerical simulation is provided to validate the theoretical findings.

I. INTRODUCTION

The grand challenges of ensuring resilience and security in cyber-physical systems (CPS) have motivated the study and characterization of possible adversarial attacks against these complex systems. Reactive approaches based on the detection and identification algorithms are a significant aspect of comprehensive defense strategies against malicious attacks\(^1\). Due to their distributed nature, cyber-physical systems such as the power grid or networks of autonomous aerial/ground vehicles can often be modeled as multi-agent systems\(^2\),\(^3\), where the communication network is susceptible to attacks\(^4\). In particular, this paper considers the problem of detecting stealthy attacks, namely covert attack and zero-dynamics attack, using a scalable detection framework for a class of networked multi-agent systems seeking average consensus upon system’s initial conditions, as a canonical cooperative task.

Literature review: In general, the detection of stealthy attacks is not a trivial problem for networked multi-agent systems. Challenges arise due to the large scale of networked systems and the limited communication capability of its subsystems (or agents), which restrict an effective information aggregation and transmission required to implement centralized approaches\(^4\). Moreover, the prevalent observer-based attack detectors are ineffective in detecting stealthy attacks, particularly zero-dynamics attack (ZDA) and covert attack that are the worst-case attack scenarios in terms of detectability, due to the fact that they are not observable in the system outputs\(^2\),\(^5\).

The conventional detection frameworks for stealthy attacks rely on modifying the system structure or adding redundancy in the system measurements to expose such attacks. For instance, a signal modulation acting on the system actuation to alter the system’s input behavior was proposed in\(^6\) for both covert attack and ZDA detection. Change in the system structure was proposed first in\(^5\) upon which the study in\(^7\) extended the system dynamics with a randomly switched auxiliary system to achieve non-repeating dynamics, preventing the realization of covert attacks. Most recently, for a class of networks with distinct Laplacian eigenvalues, the authors in\(^8\) characterized an intermittent ZDA that remains undetectable regardless of the system’s switched structure and obtained the conditions for their detectability. As for the covert attack, the authors in\(^9\) proposed a distributed architecture composed of two cascaded observers for each subsystem to detect the attacks. As another strategy, multi-rate sampling in sampled-data systems was studied in\(^10\),\(^11\) to change the direction of sampling zeros and thus to prevent ZDA. Also, distributed function calculation was proposed in\(^12\) that requires intensive communication in the network and full knowledge of network model for each node.

In terms of scalability, considerable effort has been dedicated to extending the existing decentralized and distributed estimation/fault detection methods to the attack detection strategies implementable using locally available information for large-scale systems. For instance, one can refer to secure distributed observers for sensor networks in\(^13\), distributed attack detection schemes for power networks\(^2\),\(^9\),\(^14\),\(^15\), decentralized detection scheme for stochastic interconnected systems in\(^16\), and divide-and-conquer approach in\(^4\). However, few studies have addressed distributed/decentralized detection strategies for stealthy attacks, namely covert attack and zero-dynamics attack\(^9\),\(^15\). Moreover, they do not address the communication topology switching and the privacy of the agents’ information.

Statement of contributions: The contributions of this paper are threefold. First, as a security objective, we consider the privacy of agents’ initial condition and the agreement’s final value (consensus) and propose enforced unobservability constraints on the network topology to preserve the network privacy at the global level. Second, for scalability, we propose a glocal (global-local) attack detection structure for which the networked multi-agent system is partitioned into clusters (subsystems) with their respective globally and locally monitored agents that satisfy specific conditions related to the network privacy and the detectability of stealthy attacks (i.e., zero-dynamics attack and covert attack). Finally, we derive...
the theoretical conditions for topology switching (Theorem III.3) under which local detectors trigger switches in the system’s communication topology such that stealthy attacks become detectable for the global (centralized) observer. We further discuss different types of topology switching and their outcome for the detection of stealthy attacks.

The rest of the paper is organized as the following. Section II presents the preliminary definitions and the problem formulation. The privacy preserving problem and the attack detection framework are studied in Section III. Section IV demonstrates the simulation results. Finally, Section V concludes the paper.

II. Problem Formulation

A. Preliminaries

Notation. We use $\mathbb{R}$, $\mathbb{R}_{>0}$, $\mathbb{R}_{\geq 0}$, $\mathbb{C}$, and $\mathbb{N}$ to denote the set of reals, positive reals, non-negative reals, complex, and natural numbers, respectively. Also, $\mathbb{N}_0 = \mathbb{N} + \{0\}$. We use $x := \text{col}(x_1, x_2, \ldots, x_n)$ to denote (block-partitioned) vectors. $1_n$, $0_n$, $I_n$, and $0_0$ stand for $n$-vector of all ones, the $n$-vector of all zeros, the identity $n$-by-$n$ matrix, and $n$-by-$n$ zero matrix, respectively. $x(t)$ stands for the $m$-th order time derivative of $x(t)$. In addition, $\cdot$ denotes the cardinality of sets, and for any index set $\mathcal{F}$ with $|\mathcal{F}| = m$, $I_{m} \in \mathbb{R}^{m \times m}$ is the concatenation of the $i$-th columns of $I_n$ where $i \in \mathcal{F}$. For a matrix $M \in \mathbb{R}^{m \times n}$, the range (column space) is defined as $\text{Im}(M) = \{Mx \mid x \in \mathbb{R}^n\} \subseteq \mathbb{R}^m$ and the nullspace is defined as $\text{ker}(M) = \{x \mid Mx = 0\} \subseteq \mathbb{R}^n$. The support of vector $x \in \mathbb{R}^n$ is the set of nonzero components defined as $\text{supp}(x) = \{i \in \{1, \ldots, n\} \mid x_i \neq 0\}$. We also define the set of nonzero columns of the $n$-by-$n$ matrix $M$ by $\text{colsupp}(M) = \{i \in \{1, \ldots, n\} \mid [M]_{n,i} \neq 0\}$.

Graph theory. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ denote a weighted undirected graph with the set of nodes $\mathcal{V} = \{1, 2, \ldots, N\}$, set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and adjacency matrix $\mathcal{A} := \{a_{ij}\} \in \mathbb{R}_{\geq 0}^{N \times N}$. For any pair of nodes $i, j$, $i \neq j$, a path from $j$ to $i$ implies the edge $(i, j) \in \mathcal{E}$ corresponding to $a_{ij} > 0$, otherwise $a_{ij} = 0$. The Laplacian matrix $\mathcal{L} := [l_{ij}] \in \mathbb{R}^{N \times N}$ is defined as $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \neq j$. By convention, $\mathcal{N}_i = \{j \in \mathcal{V} \mid \mathcal{E}_{ij} \in \mathcal{E}\}$ denotes the set of neighbors of node $i$. A cluster is defined as any subset $\mathcal{P} = \{P_1, \ldots, P_q\} \subseteq \mathcal{V}$ of the nodes of graph $\mathcal{G}$ such that $\bigcup_{i=1}^{q} P_i = \mathcal{V}$ and $P_i \cap P_j = \emptyset$ if $i \neq j$. We make the convention that $\mathcal{G}_t$ with a right-continuous switching signal $\sigma(t) : \mathbb{R}_{>0} \to \mathcal{Q} := \{1, 2, \ldots, q\}$, $q = |\mathcal{Q}|$ denotes a finite set of graphs, indexed by finite set $\mathcal{Q}$, that holds all properties of graph $\mathcal{G}$.

Definition II.1. (Graph component [17]). A component in an undirected graph is an induced subgraph with a (maximal) subset of nodes such that each is reachable by some path from each of the others.

Systems theory. A linear system $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$, where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$, is represented by the tuple $\Sigma(A, B, C, D)$.

Definition II.2 (Zeroing direction and zero-dynamics attack [18 Ch. 3.8]). Scalar $\lambda_0 \in \mathbb{C}$ is a zero of the tuple $\Sigma(A, B, C, D)$ if, and only if, there exists zeroing direction $\text{col}(x_0, u_0) \neq \text{col}(0, 0)$ associated with $\lambda_0$ such that

$$\begin{bmatrix}
\lambda_0 I_n - A & -B \\
C & D
\end{bmatrix}
\begin{bmatrix}
x_0 \\
u_0
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}.$$ (1)

Then, the signal $u(t) = u_0 e^{\lambda_0 t}$ is a zero-dynamics attack that generates non-zero state trajectories $x(t) = x_0 e^{\lambda_0 t}$ while the output $y = Cx + Du$ satisfies $y(t) = 0$.

B. Problem Statement

System model. Consider a graph $\mathcal{G}$ of order $N$, we associate each node $i$ of the graph with an agent $\Sigma_i$ that evolves according to the following dynamics:

$$\Sigma_i : \begin{cases}
\dot{x}_i(t) = v_i(t) \\
\dot{v}_i(t) = u_i(t)
\end{cases}, \quad i \in \mathcal{V},$$ (2)

in which $x_i(t)$ and $v_i(t)$ denote the position and velocity, and $u_i(t)$ (to be determined) stands for the control channel through which each agent communicates with a set of neighbors $\mathcal{N}_i$ to perform a prespecified cooperative task.

Control protocol. The objective is to reach an average consensus upon the initial conditions of the system, as follows:

$$\lim_{t \to \infty} |x_i(t) - x_j(t)| = 0 \quad \text{and} \quad \lim_{t \to \infty} |v_i(t)| = 0, \quad \forall i, j \in \mathcal{V},$$ (3)

which can be achieved by exchanging local information through the following switching control protocol [8]:

$$u_i = -\alpha v_i - \gamma \sum_{j \in \mathcal{N}_i} a_{ij}^\sigma(t) (x_i - x_j) + u_{ai}, \quad i \in \mathcal{V},$$ (4)

where $a_{ij}^\sigma(t)$ is the entry of the symmetric adjacency matrix associated with the graph $\mathcal{G}_\sigma(t)$ representing the switching communication network of agents $\Sigma_i$’s. Also, $\alpha$ and $\gamma$ are the control gains. Finally, $u_{ai}$ is the injected malicious signal in control channel of the $i$-th agent. We assume the unknown subset $\mathcal{F} \subset \mathcal{V}$ represents the set of compromised agents, and we have $u_{ai} = 0$ for an uncompromised agent $i$, i.e., if $i \in \mathcal{V} \setminus \mathcal{F}$.

Closed-loop system. Given (2) and (3), let $x := \text{col}(x, v)$, where $x := \text{col}(x_1, \ldots, x_N)$, $v := \text{col}(v_1, \ldots, v_N)$, and $u_0 = \text{col}(u_{ai}), i \in \mathcal{F}$. Then, the closed-loop system is given by

$$\Sigma : \begin{bmatrix}
\dot{x} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix}
0 & I \\
-\alpha L(t) & -\gamma I
\end{bmatrix}
\begin{bmatrix}
x \\
v
\end{bmatrix}
+ \begin{bmatrix}
0 \\
I_{\mathcal{F}}
\end{bmatrix} u_0,$$ (5)

with the system measurements $y = \text{col}(y_1, \ldots, y_{|\mathcal{M}|})$ corresponding to the output matrix $C$ such that:

$$\text{colsupp}(C_k) \in \mathcal{M}_k \subset \mathcal{V}, \quad k \in \{x, v\}, \quad \mathcal{M} = \{\mathcal{M}_x, \mathcal{M}_v\},$$ (6)

1We may omit the subscripts when clear from the context.

2For brevity, we may omit the time argument $t$ from expressions whenever possible in the rest of the paper.
where the to-be-selected set $\mathcal{M}$ represents the set of the monitored agents’ index. Also, $u_s = \text{col}(u_{s1}, \ldots, u_{s|\mathcal{M}|})$ is a vector of injected malicious signals in the compromised measurement sensor channels. Finally, the Laplacian $L_{\sigma(t)}$ in (5) encodes the information exchange among agents.

**Adversary model.** Let $\mathcal{F} \subset \mathcal{V}$ denote the set of agents with a compromised (under attack) control channel, and $\mathcal{F} \subset \mathcal{M}$ represent the set of agents with compromised sensor channels. The dynamics of the adversarial attack is given by

$$
\Sigma_{\sigma} : \begin{cases} 
\dot{x} = \bar{A}_{\sigma(t)} x + Bu_s(t), \\
u_s = Cx, \\
\text{supp}(u_s) = \mathcal{F}, \quad \text{supp}(u_s) = \mathcal{F},
\end{cases}
$$

(7)

where the vector attack $u_s := f(x, u_s, y, t)$ by which the attacker steers the system towards undesired states, and $t_a \geq t_0$ is the attack starting time. For example, the attack signal is in the form of $u_a(t) = u_0 e^{\lambda_0(t-t_0)}$ in the case of ZDA, where $\lambda_0$ and $u_0$ are introduced in Definition II.2.

**Communication topology switching.** The multi-agent system in (5) operates in the normal mode with the initial communication topology specified by $\sigma(t) = 1 \in \mathcal{Q}, t \in [t_0, t_1)$ until switching to a safe mode following the detection of an attack at the time $t_1 > t_0$. In the safe mode for $t \geq t_1$, the communication topology switching is specified by the switching signal $\sigma(t) = \{2, \ldots, q\} \in \mathcal{Q}, q = |\mathcal{Q}|$ whose switching policy will be determined later (See Section III-E).

**Assumption 1.** (Disclosure information). In the normal mode, where $\sigma(t) = 1 \in \mathcal{Q}, t \in [t_0, t_1)$, the attacker

(i) has perfect knowledge of the system model, i.e., $\Sigma_1(\bar{A}_{\sigma(t)}, \bar{B}, \bar{C}, \sigma = 1) = \Sigma(\bar{A}_{\sigma(t)}, \bar{B}, \bar{C}, \sigma = 1),$

(ii) does not know the system’s initial condition, i.e., $\bar{x}(t_a) \neq x(0)$ in a covert attack.

(iii) has no knowledge of the system switching times $\{t_k\}_{k=1}^{m-1}$, $m \in \mathbb{N}$ associated with the safe mode when $\sigma(t) = \{2, \ldots, q\} \in \mathcal{Q}, t \in [t_1, +\infty),$

(iv) starts the attack at $t_a \geq t_0 = 0$.

**Assumption 2.** (Defender’s policy). The defender

(i) selects the monitored agents and designs the attack detection framework,

(ii) designs the communication topology for the safe mode and its corresponding switching policy.

For the detectability of adversarial attacks in switched systems, we will need the following technical result:

**Lemma II.3.** (Observability of linear switched systems) [19]. Given a system $\dot{x} = A_{\sigma(t)} x$, with measurements $y = Cx$, $(x \in \mathbb{R}^n$ and $y \in \mathbb{R}^p$), over the interval $t \in [t_0, t_m)$ that includes switching instances $\{t_k\}_{k=1}^{m-1}$ for modes $\sigma(t) = k \in \mathcal{Q}$ with the dwell time $\tau_k = t_k - t_{k-1}$, the output of system is given by $y(t) = Ce^{A_1(t-t_{\alpha-1})} \prod_{i=1}^{\tau_k} e^{A_i(t-t_i)} x(t_0), t \in [t_{k-1}, t_k)$. Then, (i) the system is observable and the initial condition $x(t_0)$ is reconstructable from $y(t)$ if, and only if, the matrix $B$ in (7) is full rank (i.e., $\mathcal{N}_1^m := \ker(O) = \{0\}$). (ii) If (8) is rank deficient, the unobservable subspace of the system for $t \in [t_0, t_m)$, which is the largest $\mathcal{A}_{\sigma(t)}$-invariant subspace contained in $\ker(C)$, can be recursively computed using (9)-(10).

$$
\mathcal{O} = \text{col}(\mathcal{O}_1, \mathcal{O}_2, A_1^{\tau_1}, \ldots, \mathcal{O}_m) \bigcap_{i=m}^{1} \ker(a_i^{\tau_i}),
$$

(8)

$$
\mathcal{N}_m^\sigma = \ker(\mathcal{O}_m),
$$

(9)

$$
\mathcal{N}_k^\sigma = \ker(\mathcal{O}_k) \bigcap \bigcap_{i=k+1}^{m} \ker(\mathcal{O}_i) \bigcap_{i=1}^{k} \ker(a_i^{\tau_i}).
$$

(10)

where

$$
\mathcal{O}_k = \text{col}(C, CA_k, \ldots, CA_k^{q-1}), 1 \leq k \leq m-1,
$$

(11)

$$
\mathcal{A}_k = \mathcal{A}_{\sigma(t)}, \quad t \in [t_{k-1}, t_k).
$$

(12)

**Proposition II.4.** (Stealthy attacks). Consider system (5) under the attack model (7) and Assumption 1 an attack is stealthy if the system output in (5) satisfies

$$
\mathcal{O}(x_0, u_a, x_s, t) = \mathcal{O}(x_0, 0, 0, t), \quad \forall t \in [t_0, t_1),
$$

(13)

where $x_0$ and $\bar{x}_0$ are the actual and possible initial states, respectively. Then, (13) can be realized in two senses

(i) Covert Attack: Under Assumption 1 if the attacker sets the initial condition $\bar{x}(t_a) = 0$ or alternatively $\bar{x}(t_a) \in \mathcal{N}_1^1 = \ker(O_1)$ in (7), then the attack $u_a$ on (5) is covert, that is there exists a vector $u_s$, injected in (5), canceling out the effect of $u_a$ on the system output $y(t)$.

(ii) Zero-dynamics Attack (ZDA): the attacker can excite the zero dynamics of the system with an unbounded signal and remains stealthy with no need to alter the system measurements (i.e., $u_a(t) = 0$ in (5)) if $\bar{x}_0 \in \ker(C)$ and $u_a(t) = u_0 e^{\lambda_0(t-t_0)}$, $t_0 = 0$, where $\lambda_0$, $\bar{x}_0$ and $u_0$ are obtained using Definition II.2.

**Proof:** Clearly before an attack starts, (13) is met over $t \in [t_0, t_a]$. Consider $x(t_a)$ as the system states when the attack starts, (i): in the case of covert attack, the output of the system (5) with the initial normal mode $\sigma(t) = 1$ over $t \in [t_a, t_1)$ is given by

$$
y(t) = Ce^{A_1(t-t_a)} x(t_a) + C \int_{t_a}^{t} e^{A_1(t-\tau)} Bu_s(\tau) d\tau - u_s(t),
$$

(14)

and the last term which is the output of the attacker’s model (7) is given by

$$
u_s(t) = Ce^{A_1(t-t_a)} \bar{x}(t_a) + C \int_{t_a}^{t} e^{A_1(t-\tau)} Bu_s(\tau) d\tau.
$$

(15)

Substituting (15) into (14) and considering Assumption 1 yields

$$
y(t) = Ce^{A_1(t-t_a)} (x(t_a) - \bar{x}(t_a)), \quad t \in [t_a, t_1).
$$

(16)
The measurement (16) matches the attack-free response if the attacker simply sets \( \bar{x}(t_a) = 0 \). Also, in the case \( \bar{x}(t_a) \neq 0 \), \( t_a = t_0 = 0 \), it is immediate from lemma [13] that if \( \bar{x}(t_0) \in \mathcal{N}_0 \neq \{0\} \) \( \Rightarrow \mathcal{C}e^{A_1(t-t_0)}\bar{x}(t_0) = 0 \), \( t_a = t_0 = 0 \) in (19), and thus \( y(t) = \mathcal{C}e^{A_1(t-t_0)}x(t_0), t \in [t_a, t_1] \). In both cases, condition [13], guaranteeing the covertness of (16), and thus in the absence of covert attack (i.e. \( \dot{\bar{x}} = 0 \)), it is immediate from lemma II.3 that the attack signal \( \tilde{\bar{a}} = 0 \) and \( \tilde{\bar{x}} = \tilde{x}_0e^{\lambda_0t} \) causing unbounded system states \( x(t) = \tilde{x}(t) + \tilde{x}_0e^{\lambda_0t} \), (21)

while (13) is met, where \( \tilde{x}(t) \) is the state of the system in (5) assuming the initial condition \( \bar{x}_0 \) and no attack signal. The second equation in (20), \( \mathcal{C}\bar{x}_0 = 0 \), implies \( \bar{x}_0 \in \ker(\mathcal{C}) \). It is an immediate result from Definition II.2 that the attack signal \( \bar{u}_0(t) = u_0e^{\lambda_0t} \) results in \( \bar{u}(t) = \mathcal{C}\bar{x}(t) = 0 \) in (7) while the system states \( \dot{\bar{x}}(t) = \tilde{x}_0e^{\lambda_0t} \in \ker(\mathcal{C}), \forall t \in [t_0, t_1] \) is unboundedly increasing. Consider (21) and the superposition principle in linear systems, then injecting the designed ZDA signal \( u_0(t) \) in (5) yields the solution \( y = \mathcal{C}\bar{x}(t) = \mathcal{C}x(t) + \mathcal{C}\bar{x}_0e^{\lambda_0t} \), which by considering (20) is equivalent to (13), guaranteeing the stealthiness of ZDA for (5).

Given the system and attack models above, we now state the two problems which this paper aims to address in the following:

**Problem 1. (Privacy-preserving average consensus).** Given the switching consensus system (5), we seek to preserve the following privacy requirements:

(i) neither system’s initial states \( x(t_0) \) nor final agreement values \( x_f = \frac{1}{N} \sum_{i=1}^{N} x_i(t_0), \nu^* = 0 \) should be revealed or be reconstructable.

(ii) the system’s communication topology \( \mathcal{G}_{\tau(t)} \) should not be reconstructable.

**Problem 2. (Scalable attack detection).** Given the system in (5) under the attack model (7), we seek to develop a stealthy attack detection framework such that:

(i) it features a decentralized and scalable structure.

(ii) it satisfies the privacy-preserving requirements defined in Problem 1.
can select the set of monitored agents $M$ in (6) such that $(A_{\sigma(t)}, C)$ is not an observable pair on $t \in [t_0, +\infty)$, making the globally available measurement $y$ in (6) insufficient to reconstruct either the entire system states’ information or the system’s switching structure (cf. privacy requirements in Problem 1).

The following lemma provides sufficient conditions to determine whether the global system measurement (6) is consistent with the privacy requirements.

**Lemma III.1. (Invariant unobservable subspace of system (5)).** The subspace $\text{span} \{E_{\sigma(t)} \}$ is an $A_{\sigma(t)}$-invariant unobservable subspace of the switching system in (5) provided that it lies in $\ker(C)$ and $G_{\sigma(t)}$ features only connected undirected (or strongly connected and balanced directed) graphs.

**Proof:** See Appendix B. □

**Remark III.1.1. (Generality of Lemma III.1).** The result suggests that monitoring only the agents’ velocity causes the agents’ positions not to be reconstructable independently for system (5). This is a generic solution to Problem 7 that holds for all undirected graphs. It is also worth noting that the monitored agents corresponding to set $M$ in (6) can be also selected differently from the results in Lemma III.1 for any particular graph.

We next introduce the system partitioning method followed by observer design to address Problem 2.

**C. System Partitioning**

Consider the communication graph $G_{\sigma(t)} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ of the system (5), let the set of agents $\mathcal{V}$ be partitioned into disjoint clusters $\mathcal{P} := \{\mathcal{P}_1, \ldots, \mathcal{P}_{|\mathcal{P}|}\}$ such that $\bigcup_{i=1}^{|\mathcal{P}|} \mathcal{P}_i = \mathcal{V}$ with $\mathcal{P}_i \subset \mathbb{R}^{N_i}$ and inter-cluster couplings $E_{\text{cut}} := \{E_{ij} \mid i \in \mathcal{P}_i, j \in \mathcal{P}_j, \mathcal{P}_i \cap \mathcal{P}_j = \emptyset\}$. Accordingly, after relabeling the system states, the system (5) is partitioned into $|\mathcal{P}|$ subsystems described as

\[
\Sigma_{\mathcal{P}_i} := \begin{cases}
\dot{x}_i = A_i^{\sigma(t)} x_i + \sum_{j \in N_{\mathcal{P}_i}} A_{\sigma(t)}^{ij} x_j + B^i u_{a_i}, \\
y_i = C_i x_i, \\
x_i(0) = x_{a_i}, \quad i \in \{1, \ldots, |\mathcal{P}|\}.
\end{cases}
\]

(22)

with

\[
A_{\sigma(t)} = \begin{bmatrix}
0 & I
\end{bmatrix}, \\
A_{\sigma(t)}^{ij} = \begin{bmatrix}
0 & 0
\end{bmatrix}, \\
L_{\sigma(t)} = \begin{bmatrix}
L_{\sigma(t)}^{1} & \cdots & L_{\sigma(t)}^{|\mathcal{P}|}
\end{bmatrix}, \\
B^i = \begin{bmatrix}
0 & I_{\mathcal{P}_i}
\end{bmatrix},
\]

(23)

(24)

where $x_i := [x_i \, y_i]^T \in \mathbb{R}^{2N_i}$ with $(x_i)$ and $(y_i)$ representing the vectors of position and velocity states belonging to cluster $\mathcal{P}_i \subset \mathcal{V}$. Also, $u_{a_i}$ associated with the set $\mathcal{F}_i$ is the vector-valued attack on actuator channels in the cluster as defined in (5). The output signal $y_i(t)$, associated with the output matrix $C_i$, denotes the local measurements that are available at node $i$ in cluster $\mathcal{P}_i$. Finally, $\mathcal{N}_{\mathcal{P}_i} := \{j \in \{1, \ldots, |\mathcal{P}|\} \mid \exists \mathcal{E}_{ij}, i \in \mathcal{P}_i, j \in \mathcal{P}_j\}$ denotes the index set of the neighboring clusters of cluster $\mathcal{P}_i$.

We note that the decomposition of (5) into (22) leads to a concatenated set $M := \{\mathcal{M}_1, \ldots, \mathcal{M}_{|\mathcal{P}|}\}$, where the set $M$ is associated with global measurements (6) available for the control center and sets $\mathcal{M}_i$’s, $i \in \mathcal{P}$ are associated with the local measurements $y_i$’s, available at a node $i$ in respective clusters $\mathcal{P}_1, \ldots, \mathcal{P}_{|\mathcal{P}|}$ in (22).

We make the following assumptions:

**Assumption 3. (Local information).**

(i) local knowledge: in each cluster, the agent $i \in \mathcal{P}_1$ serves as the local control center that has the local system model of the cluster (matrices $A_i^{\sigma(t)}$, $A_{\sigma(t)}^{ij}$ and $C_i$) and the local measurement $y_i(t)$.

(ii) local measurements: the measured output $y_i(t)$ in (22) is locally available at the node $i$ and, unlike global measurements, it is not sent to the control center to keep the output secure and inaccessible to the attacker.

(iii) cross-cluster communication: every local control center, i.e., the node $i$ in cluster $\mathcal{P}_i$, considers coupling terms $\sum_{j \in \mathcal{N}_{\mathcal{P}_i}} A_{\sigma(t)}^{ij} x_j$ as unknown inputs to $\sigma_{\mathcal{P}_i}$. Moreover, inter-cluster couplings do not change, i.e., $A_{\sigma(t)}^{ij}(t) = A_{\sigma(t)}^{ij}, \forall t \in [t_0, +\infty)$. Thus there is no need for exchange of $x_i$’s information between local control centers.

The assumption (3)(i) is common in the literature (cf. [13]) as the model-based detection of cyber attacks on exchanged data over a network requires augmented knowledge of the neighboring agents’ model to estimate their states and further compare them with the received data. Minimizing the local information exchange affects the scalability and depends on the sparsity of the communication network as well as on applications.

**D. Observer Design and Attack Detectability Analysis**

As described in Section III-A, the attack detection framework is composed of a centralized observer for monitoring the system (5) from the control center, and a set of local observers in clusters, that serve as local attack detectors and trigger for communication topology switching. In what follows, we describe the observer design procedure based on the conditions derived in the previous section.

**Decentralized observer.** Consider the dynamics of the system partitions described in (22) and Assumption 3, we use the unknown input observer (UIO) scheme in (20) to estimate the cluster state $\dot{x}_i$ independent of the states $x_j$’s of the neighboring clusters (i.e. $j \in \mathcal{N}_{\mathcal{P}_i}$). This is achieved by considering the interconnection of local models as unknown inputs and rewriting them such that

\[
\sum_{j \in \mathcal{N}_{\mathcal{P}_i}} A_{\sigma(t)}^{ij} x_j := E^i x_i^d, \quad \sigma(t) = 1, \forall t \in [t_0, +\infty),
\]

(25)

where $E^i$ is a full column rank matrix and $x_i^d$ is a vector of the...
states of neighboring clusters that are received by cluster $\mathcal{P}$. Now, introducing the UIO state $z_i = \hat{x}_i - \text{h}^i_y y_i$, the dynamics of the local UIO is given by

$$
\Sigma_{\alpha}^z : \begin{cases}
\dot{z}_i = F_{\sigma}(t) z_i + (K_{\sigma}(t) + K_{\sigma}(t)) y_i, \\
\hat{x}_i = z_i + \text{h}^i_y y_i, \\
\bar{x}_i(0) = 0, \\
\end{cases}
$$

(26)

where $F_{\sigma}(t)$, $K_{\sigma}(t)$, and $\text{h}^i$ are matrices satisfying conditions

$$
T^i = (I - \text{h}^i C_{i}), \quad (\text{h}^i C_{i} - I) E^i = 0, 
$$

(27)

$$
F_{\sigma}(t) = (A_{\sigma}(t) - K_{\sigma}(t) C_{i}), \quad K_{\sigma}(t) = F_{\sigma}(t) h^i, 
$$

(28)

$$
\bar{A}_{\sigma}(t) = A_{\sigma}(t) - \text{h}^i C_{i}. 
$$

(29)

Furthermore, $F_{\sigma}(t)$ is Hurwitz stable over $t \in [t_0, t_m]$ for all normal and safe modes.

Consider (22), (26) and let $e_i := x_i - \hat{x}_i$, one can use the conditions in (27)-(29) to obtain the error dynamics of UIO as follows

$$
\Sigma_{\alpha}^e : \begin{cases}
\dot{e}_i = F_{\sigma}(t) e_i + T^i B^i u_{a_i}, \\
\bar{r}_i = C_i e_i, \\
\mathcal{P}_i \subset \mathcal{V}, \quad i \in \{1, \cdots, |\mathcal{P}|\}, 
\end{cases}
$$

(30)

In the absence of adversarial attacks, $u_{a_i}$'s are hidden and thus cannot be altered by the attacker to cancel out the effect of the attack $u_{a_i}$ on the output of (22). This difference also manifests itself in the residual of local observer (30). Therefore, in order to determine the stealthiness of attack $u_{a_i}$, with respect to the local residual signal $r_i$, it is necessary and sufficient to investigate whether the stealthiness conditions presented in Proposition II.4 are satisfied for the system in (30).

In the following proposition, we formally characterize the conditions for the detection of stealthy attacks using the local observer in (22).

**Proposition III.2. (Attack detectability of local observers).** For a strongly connected cluster $\mathcal{P}$ with $\mathcal{E}$ inter-clustering edges and $|\mathcal{E}|$ compromised agents, there exists a local observer given by (26) to locally detect the stealthy attacks if

(i) there is a $k$-connected node $i \in \mathcal{P}$ as the local monitored agent such that $k \geq |\mathcal{E} + |\mathcal{F}|$, (ii) rank $(C_i E^i) =$ rank $(E^i)$, (iii) the matrix pencil $P$ in (31) is full (column) rank,

$$
P = \begin{bmatrix}
\lambda_0 I - A_{\sigma}(t) & -B^i E^i \\
C_i & 0
\end{bmatrix}. 
$$

(31)

where the tuple $(A_{\sigma}(t), B^i, C_i)$ and matrix $E^i$ are defined in (22) and (25), respectively.

**Proof:** See Appendix C.

**Remark III.2.1. (Evaluation of the condition in (31)).** Conditions (i)-(iii) in Proposition II.2 are equivalent to necessary and sufficient conditions for the existence of UIO in (26) [20]. It is worth noting that as matrix $B^i$ in (31) is unknown to the defender, it can be replaced with $I_N$, i.e., assuming all the nodes of the cluster are under attack, in analysis and selecting locally monitored agents associated with $C_i$. This, however, may require further communication between agents within a cluster. Alternatively, as in a set cover problem setting, a set of local monitoring agents that each of them satisfies the conditions (i)-(iii) for part of a cluster can be used to cover all of nodes of the cluster [22]. Minimizing the number of local measurements versus the number of local observers is a trade-off problem which will be the subject of future work.

**Centralized observer.** Consider the dynamical system (5), a Luenberger-type centralized observer, derived based on the normal mode $\sigma(t) = 1$, is given by

$$
\Sigma_{\sigma}^c : \begin{cases}
\dot{x} = A_{\sigma} x + H_{\sigma}(y - \hat{y}), \\
\hat{y} = C x, \\
r_o = (y - \hat{y}), \\
x(0) = 0, \quad \bar{r}_o = (y - \hat{y}), 
\end{cases}
$$

(32)

where $H_{\sigma}(t)$ is the observer gain and $r_o(t)$ denotes the residual signal available in the control center for monitoring purposes.

In order to design the observer gain $H_{\sigma}(t)$, the partial observability of pair $(A_{\sigma}(t), C)$ imposed in Section III-B and the activated mode $\sigma(t)$ should be taken into account. An immediate solution is to define an LMI optimization problem finding a constant $H_{\sigma}(t) = H$ by which $(A_{\sigma}(t) - HC)$ is (Hurwitz) stable in all modes [23], [24].

From Assumption 1 and condition (13), it is straightforward to show that the attack $u_a$ remains stealthy for the observer (32) in the normal mode over the time span $t \in [t_0, t_1]$ where $A_{\sigma}(t) = A_1$.

Recall (17) and (21), and let

$$
e := \bar{x} - \tilde{x} \quad (33)
$$

$$
e := x - \bar{x} = \tilde{x} - \tilde{x} = \bar{e} + \tilde{e} \quad (34)
$$

be the estimation error of the states of an attack-free system ($\bar{x} = A_{\sigma(t)} \bar{x}$, $y = C \bar{x}$) and the under attack system in (5), respectively. Then using (5) and (32), the error dynamics of the centralized observer is given by

$$
\Sigma_{\sigma}^c : \begin{cases}
\dot{\bar{e}} = (A_1 - HC) e + (A_{\sigma}(t) - A_1) x + H_{\sigma}(t) u_a + B_{u_a}, \\
e(0) = x_0, \\
r_o = (y - \hat{y}) = C e - u_a = C \bar{e}, \quad \text{residual}, 
\end{cases}
$$

(35)

where for measurement $y$ in (32) we used the expression $y = C x - u_a$ as defined in (5). Consider (7) and (13), $y$ in (32) also satisfies $y = C x - u_a = C \bar{x} - C \bar{x} = C \tilde{x}$. Then using $y = C \tilde{x}$, (5), (7), (32), (33), the following dynamics is obtained

$$
\Sigma_{\sigma}^c : \begin{cases}
\dot{\bar{e}} = (A_1 - HC) e + (A_{\sigma}(t) - A_1) \tilde{x}, \\
e(0) = \tilde{x}_0, \\
r_o = C \bar{e}, \quad \text{residual}. 
\end{cases}
$$

(36)

Note that, during normal mode $\sigma(t) = 1$ over the time span $\forall t \in [t_0, t_1]$, the residual $r_o$ in (35) is the same as that of (36).
that is the dynamics of the estimation error of system states in the absence of attacks. This implies that, in the case of a covert attack with $u_c \neq 0$, as long as signal $u_c(t)$ cancels out the effect of $u_a(t)$ on the output $y(t)$, the residual $r_0(t) = Ce(t)$ converges to zero as $t_1 \to +\infty$, yielding the stealthiness of the covert attack, in the normal mode, for the centralized observer (32).

In the case of a ZDA, $u_c = 0$ in (35) although (13) still holds that leads to the stealthiness of a ZDA for the observer (32). To show this, one need to verify the attack $u_a$ remains in the zeroing direction of (35). Using Definition II.2 for (35) in the normal mode, we obtain

$$\begin{bmatrix} \lambda d I - (A_1 - HC) & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} \hat{e}(0) \\ u_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (37)$$

where $\hat{e}(0) := e(0) - \hat{e}(0) = x_0 - \hat{x}_0 = \bar{x}_0$. Recall $\bar{x}_0 \in \ker(C)$ in (20), then the second equation of (37) yields $C\hat{e}(0) = CX_0 = 0$. Applying $C\hat{e}(0) = 0$ into the first equation of (37) simplifies the matrix pencil in (37) into that of (20) over $t \in \{t_0, t_1\}$ where $A_\sigma(t) = A_1$. This ensures the stealthiness of ZDA in the normal mode for the observer (32).

The following Theorem provides conditions to address Problem 2-(ii) by characterization of switching modes that lead to attack detection with respect to global measurements.

**Theorem III.3. (Attack detectability under switching communication).** Consider system (5) under stealthy attack modeled in (7), and let intra-cluster topology switching satisfy

(i) $\Im(\Delta L_q) \cap \ker([C_x^T C_v^T]) = \emptyset$,
(ii) $L_q$ features distinct eigenvalues,
(iii) $[U_q]_{i,\ell} \neq [U_q]_{j,\ell} \neq 0, \quad \forall \ell \in \mathcal{V} \setminus \{1\}, \forall i, j \in \mathcal{D}_c, \forall c \in \{1, \ldots, c\}$,

where $\Delta L_q := L_{\sigma(t)} - L_1$, with $\sigma(t) = q \in \mathbb{Q}$, $t \in \{t_1, +\infty\}$, $C_x$ and $C_v$ are given in (5) and $\mathcal{D}_c \subset \mathcal{V}$ denotes the set of nodes in c-th connected component of $\Delta L_q$ corresponding to agents involved in connected switching links, and finally $U_q$ is a unitary matrix ($U_q U_q^\top = I$) diagonalizing Laplacian $L_q$.

Then, ZDA and covert attacks undetectable for the centralized observer (32) are impossible only if the topology switching satisfies conditions (i)-(iii). If additionally the system is not at its exact consensus equilibrium when the attack is launched, conditions (i)-(iii) are sufficient for the detection of ZDA.

**Proof:** See Appendix [3]

**Remark III.3.1. (Safe topology switching).** For a given pair $(A_\sigma(t), C)$ in (5), one can compute a set of switching modes by evaluating the conditions (i)-(iii) of Theorem III.3 This could be performed through iterative algorithms changing graph connections. Furthermore, if $\mathcal{Z}$ is an unknown subspace associated with system states affected by stealthy attack $u_a(t)$ i.e. $x(t) \in \mathcal{Z}$. Then, in view of $x_0 = x_0 - x_0$ (see Proposition II.4), the discrepancy term $(A_\sigma(t) - A_1)x$ in the dynamical system (35) will be bounded and vanishing if

$$X_q \cap \mathcal{Z} = \emptyset. \quad (38)$$

Therefore, if condition (38) holds, $(A_\sigma(t) - A_1)x$ does not affect the stability of the system, as a consequence of input-to-state stability property of consensus systems [25]. It is also noteworthy that although identifying $\mathcal{Z}$ beforehand is practically impossible as $B$ and $x_0$ in (7) are unknown to the defender, local observers detecting stealthy attacks in a cluster can locally identify and trigger a safe switching mode that satisfies (38).

**E. Attack Detection Procedure**

The results in the previous section provide conditions for the detectability of stealthy attacks locally, at the cluster level, and globally, at a ground control station equipped with a centralized observer. As described earlier, the attack detection framework relies on switching communication links generating a discrepancy between the attacker model (7) and the actual system (5). To this end, at local level (clusters), unknown-input observers in (26), satisfying conditions of Proposition III.2, locally detect stealthy attacks. Following by the detection, a local observer triggers a topology switching, $G_{\sigma(t)}$, that satisfies conditions (i)-(iii) of Theorem III.3 yielding stealthy attack detection in the control center. This procedure is depicted in Algorithm 1.

**Algorithm 1 Topology switching for attack detection**

1: procedure ATTACK DETECTION($G_{\sigma(t)}$, Obs. in (32), (26))
2: do
3: $r_1(t) >$ threshold then
4: do identify a safe mode $\sigma(t) = q \in \mathbb{Q}$ for $L_{\sigma(t)}$ that satisfies conditions (i)-(iii) in Theorem III.3
5: do Trigger an identified safe mode $\sigma(t) = q \in \mathbb{Q}$
6: if $r_0(t) >$ threshold then
7: Stealthy attack is detected.
8: end if
9: end if
10: end procedure

As presented in Algorithm 1, the observers (attack detectors) require an appropriate threshold for their residuals to avoid false attack detection. These thresholds can be designed by considering an upper bound on the estimation error of observers in the attack-free case. An analytical analysis, however, will be the subject of future work.

**IV. SIMULATION RESULTS**

We use a numerical example to validate the performance of the attack detection framework. We consider a network of $N = 19$ agents and investigate, in three cases, the effect conditions proposed in Proposition III.2 and Theorem III.3 on stealthy attack detection. It is assumed that the network has been partitioned into three clusters $\mathcal{P}_1 = \{1, \ldots, 7\}, \mathcal{P}_2 = \{8, \ldots, 12\}, \mathcal{P}_3 = \{13, \ldots, 19\}$. Each cluster is equipped with the local observer (26) (specified by blue nodes in Figure 2) whose local measurements are consistent with Assumption 3 and Proposition III.2. More specifically, In cases 1 and 2, cluster $\mathcal{P}_1$ has two local observers that each has
access to its neighboring agents’ measurements. In cluster $P_2$, however, we considered one local observer having more communication with other agents within the cluster for its realization (cf. Remark III.2.1). Similar analysis is applied to case 3. Moreover, there is a centralized observer with global measurements as $M_x = \emptyset$, $M_v = \{7, 12, 14\}$ consistent with Lemma III.1. In the simulations, the system’s initial conditions are considered to be known for observers although this is not a requirement for the presented theoretical results. Also, the constant thresholds were selected by evaluating the observers’ performance in different case studies.

In cases 1 and 2 (shown respectively in Figures 2(a) and 2(b) with their communication topology in Figure 2(d)) a ZDA occurs in cluster $P_1$ and particularly affects agents 3 and 4. As depicted, ZDA is stealthy in the global residuals $r_0$, $i \in \{1, 12, 14\}$ before topology switching. It is, however, detectable in local residual $r_{1i}(t)$. The local control center, node 5, can trigger either of case 1’s or case 2’s switching topologies shown in Figures 2(a). While the conditions (i)-(iii) of Theorem III.3 are met in both cases, only case 2 meets (38) of remark III.3.1. Consequently, the global residual $r_{0i}(t)$ for case 1 is bounded and vanishing after topology switching while that of case 2 is unbounded.

In cases 3 (shown in Figure 2(c) with its communication topology in Figure 2(d)) a ZDA occurs in cluster $P_2$ and particularly affects agents 11. Note that, unlike in cases 1 and 2, none of the Theorem III.3 conditions are met in case 3, yielding the global residuals $r_{0i}(t)$, $i \in \{1, 12, 14\}$ remain unaffected by the switching topology. Consequently, stealthy attack is not detectable.

Moreover, comparing cases 1’s bounded global residual with case 2’s unbounded global residual, it is noteworthy that meeting condition (38) yields a trade-off between a faster attack detection at a price of further exposing system states to ZDA and a slower detection by keeping uncompromised system states bounded.

V. CONCLUSIONS

In this paper, a novel attack detection framework is developed to detect stealthy attacks against a class of multi-agent control systems seeking average consensus. The scalability of the approach is addressed using decentralized local observers. Also, the privacy preservation of the multi-agent system’s state information is achieved by imposing unobservability conditions for the central (global) observer. Theoretical conditions were derived for the detectability of the stealthy attacks. The numerical example validates the theoretical results and illustrates the effectiveness of the proposed approach. Also, a discussion was provided on different types of switching topologies and their outcome for stealthy attack detection. Deriving sufficient and verifiable conditions on safe topology switching as well as optimizing the number of local observers and their respective measurements will be subjects of future work.

APPENDIX A

The followings are used in the Proof of Theorem III.3

**Definition A.1.** The Laplacian matrix of the graph composed of switching links between two communication graphs is block diagonalizable, where each block, also called a component, encodes either a single (added/removed) switching link or a group of them are are connected.

The above definition can be formally presented as follows: consider a network topology switching between two
graphs with Laplacian matrices $\mathcal{L}_{\sigma(t)=q}$ and $\mathcal{L}_{\sigma(t)=q'}$, $q', q \in \mathcal{Q}$, $q' \neq q$ and let $\Delta \mathcal{L}_q = \mathcal{L}_q - \mathcal{L}_{q'}$ denote the difference of their Laplacian matrices. Then, under Definition II.1 $\Delta \mathcal{L}_q$ is associated with the induced graph $\Delta \mathcal{G}_q = (\mathcal{V}_q, \Delta \mathcal{A}_q, \Delta \mathcal{A}_{q'} )$, that specifies connected graph component(s) corresponding to added/removed communication link(s) in the communication network such that

$$
\mathcal{V}_q = (\cup_{i=1}^{c} \mathcal{D}_c) \cup \mathcal{D}_s, \ s.t \ \mathcal{V}_q = \mathcal{V}, \ (i,j) \in \Delta \mathcal{E}_q \text{ if } [\Delta \mathcal{A}_q]_{i,j} = a_{ij}^q - a_{ij}^{q'} \neq 0 \iff \left[\Delta \mathcal{L}_q\right]_{i,j} \neq 0,
$$

where $\mathcal{D}_c$ denotes the set of nodes (agents involved in switching links) in $c$-th connected component with $|\mathcal{D}_c| \geq 2$ and $\mathcal{D}_y \cap \mathcal{D}_y' = \emptyset$ for any $i', j' \in \{1, \cdots, c\}$, $i' \neq j'$. Also, $\mathcal{D}_s$ denotes the set of singletons i.e. single nodes that are not involved in any switching link. Then, there exists a permutation matrix $P$, $PP^T = I$ to re-label the nodes and represent the Laplacian matrix $\Delta \mathcal{L}_q$ in block diagonal form, (cf. [17] Ch. 6.12), as follows

$$
P\Delta \mathcal{L}_q P^T = \bar{\mathcal{L}}_q = \text{diag}\{\Delta \mathcal{L}_q(D_1), \cdots, \Delta \mathcal{L}_q(D_c), \Delta \mathcal{L}_q(D_s)\},
$$

where $\Delta \mathcal{L}_q(D_c)$ denotes the Laplacian matrix of the $c$-th connected component and $\Delta \mathcal{L}_q(D_s) = 0$.

**Lemma A.2.** Consider system in (5) with topology switching from normal mode $\sigma(t) = 1$ to a safe mode $\sigma(t) = q \in \mathcal{Q}$ and the measurements set $\mathcal{M}$ in (6), and let $\Delta \mathcal{L}_q = \mathcal{L}_q - \mathcal{L}_1$ denote the difference of the Laplacian matrices in safe and normal mode. Then under condition

$$
\text{Im}(\Delta \mathcal{L}_q) \cap \ker \left( \begin{bmatrix} C_x & C_v \\ \end{bmatrix}^T \right) = \emptyset,
$$

every connected graph component has at least one globally monitored node (agent), that is

$$
\mathcal{D}_c \cap \mathcal{M} \neq \emptyset, \ \forall c \in \{1, \cdots, c\},
$$

where $C_x$ and $C_v$ are diagonal elements of $C$ in (4) and $\mathcal{D}_c$ denotes the set of nodes in $c$-th connected component of $\Delta \mathcal{L}_q$ as given in (39).

**Proof:** We first show (42) is invariant under permutation of $\Delta \mathcal{L}_q$ which is introduced in (41) and accordingly permutation of $\left[ C_x \ C_v \right]^T$. To this end, from the definition of nullspace we have

$$
\ker \left( \begin{bmatrix} C_x & C_v \\ \end{bmatrix} \Delta \mathcal{L}_q \right) = \left\{ x \in \mathbb{R}^N \mid \begin{bmatrix} C_x & C_v \end{bmatrix} \Delta \mathcal{L}_q x = 0 \right\},
$$

from which we obtain either

$$
\Delta \mathcal{L}_q x \notin \text{Im}(\Delta \mathcal{L}_q) \iff \Delta \mathcal{L}_q x = 0, \ (45)
$$
or

$$
0 \neq y = \Delta \mathcal{L}_q x \in \text{Im}(\Delta \mathcal{L}_q) \implies \begin{bmatrix} C_x & C_v \end{bmatrix} y = 0, \ (46)
$$

where the latter, (46), is in contradiction with condition (42).

Now under the permutation defined in (41), $\begin{bmatrix} C_x & C_v \end{bmatrix} \Delta \mathcal{L}_q x = 0$ in (44) can be rewritten in block-partitioned diagonal form as

$$
\begin{bmatrix} C_x & C_v \end{bmatrix} P^T \bar{\mathcal{L}}_q P x = \begin{bmatrix} C_x & C_v \end{bmatrix} P^T \bar{\mathcal{L}}_q \chi = \begin{bmatrix} C_x & C_v \end{bmatrix} \bar{\mathcal{L}}_q \chi = 0, \ (47)
$$

in which $\chi = Px$ denotes the relabeled $x$ such that

$$
\chi = \text{col}(\chi_1, \ldots, \chi_c) = Px, \ \text{with}
$$

$$
\chi_c = \text{col}(x_i), \ \forall i \in \mathcal{D}_c, \ \forall c \in \{1, \cdots, c\}. \ (48)
$$

Also, $\hat{C}_k = C_k P^T = [C_k^1 \cdots C_k^c]$, $k \in \{x, v\}$ is a block-partitioned binary matrix that specifies monitored agents of each component. To show the results in (45) and (46) hold also for the transformed form in (47), one need to verify the invariance of (42) under the permutation by $P$, that is

$$
\text{Im}(\Delta \mathcal{L}_q) \cap \ker \left( \begin{bmatrix} C_x & C_v \end{bmatrix}^T \right) = \emptyset \iff \text{Im}(\bar{\mathcal{L}}_q) \cap \ker \left( \begin{bmatrix} C_x & C_v \end{bmatrix}^T \right) = \emptyset. \ (49)
$$

To show this, from the range and nullspace definition, for subspaces in (49) we have

$$
\text{Im}(\Delta \mathcal{L}_q) = \{ y \in \mathbb{R}^N \mid y = \Delta \mathcal{L}_q x(t) \}, \ (50)
$$

$$
\ker \left( \begin{bmatrix} C_x & C_v \end{bmatrix} \right) = \ker(C_x) \cap \ker(C_v) = \{ x \in \mathbb{R}^N \mid C_x x = 0, C_v x = 0 \}, \ (51)
$$

and

$$
\text{Im}(\bar{\mathcal{L}}_q) = \{ y \in \mathbb{R}^N \mid y = \bar{\mathcal{L}}_q \chi(t) = \bar{\mathcal{L}}_q P x(t) \} = \{ y \in \mathbb{R}^N \mid P^T y = P^T \bar{\mathcal{L}}_q P x(t) = y \} = P \text{Im}(\Delta \mathcal{L}_q), \ (52)
$$

where we used (41) and $\chi(t) = Px(t)$ as in (48) and (50).

Similarly,

$$
\ker \left( \begin{bmatrix} C_x & C_v \end{bmatrix} \right) = \ker(\hat{C}_x) \cap \ker(\hat{C}_v) = \{ x \in \mathbb{R}^N \mid \hat{C}_x x = 0, \hat{C}_v x = 0 \} = \{ x \in \mathbb{R}^N \mid C_x P^T x = 0, C_v P^T x = 0 \} = \{ x \in \mathbb{R}^N \mid C_x x = 0, C_v x = 0, P x = x' \} = P \ker \left( \begin{bmatrix} C_x & C_v \end{bmatrix} \right). \ (53)
$$

Then

$$
\text{Im}(\bar{\mathcal{L}}_q) \cap \ker \left( \begin{bmatrix} C_x \ C_v \end{bmatrix} \right) = P \text{Im}(\Delta \mathcal{L}_q) \cap P \ker \left( \begin{bmatrix} C_x \ C_v \end{bmatrix} \right) = P \left( \text{Im}(\Delta \mathcal{L}_q) \cap \ker \left( \begin{bmatrix} C_x \ C_v \end{bmatrix} \right) \right) = P (\emptyset) = \emptyset. \ (54)
$$

*Note that $P^T$ permutes the columns of binary matrix $C_k$ whose row-vector elements are $\xi_{ik}^q, \forall i \in \mathcal{M}_k, k \in \{x, v\}$. 
where we used fact 2.9.29 in [26] and condition [31].

Now one can prove [43] by contradiction. Assume [43] does not hold, that is \( \exists \epsilon \in \{1, \ldots, c\} \), s.t. \( D_{c'} \cap M = \emptyset \), under which we have the \( c'-th \) block in (47) such that

\[
[\bar{C}_{c'} \ C_{c'}] \Delta L_{q}(D_{c'}) \chi_{c'}(t) = 0, \quad \bar{C}_{c'} = C_{c'} = 0,
\]

which holds for all \( \chi_{c'}(t) \) with \( \Delta L_{q}(D_{c'}) \chi_{c'}(t) \in \text{Im}(\Delta L_{q}(D_{c'})) \subseteq \text{Im}(L_{q}) \) as in (55) \( \text{Im}(\Delta L_{q}(D_{c'})) \subseteq \ker([\bar{C}_{c'} \ C_{c'}]) \implies \text{Im}(L_{q}) \cap \ker([\bar{C}_{c'} \ C_{c'}]) \neq \emptyset \) that contradicts (49).

\[\text{APPENDIX B}\]

\text{PROOF OF LEMMA III.3}\n
Note that the Laplacian matrix \( L_{\sigma(t)} \) of every connected undirected (or strongly connected and balanced directed) graph has only one zero eigenvalue, \( \lambda = 0 \), with the corresponding eigenvector \( 1_{N} \) such that \( L_{\sigma(t)}1_{N} = 0 \) [27]. Then, given the structure of \( A_{\sigma(t)} \) in (53), \( (\lambda = 0, w_{r}) = ([1/\sqrt{N}]1_{N}) \) is an eigenpair of system matrix \( A_{\sigma(t)} \) associated with that of Laplacian \( L_{\sigma(t)} \) with \( \sigma(t_{k-1}) = q \in Q, t \in [t_{k-1}, t_{k}] \). Also, it can be verified that the eigenpair \( (\lambda = 0, w_{r}) \) lies in the unobservable subspace of system (5) as it is a nontrivial solution to the PBH test for observability:

\[
\begin{bmatrix}
\lambda I - A_{q} \\
C
\end{bmatrix}
w_{r} = 0, \quad \lambda = 0 \in \mathbb{C},
\]

\[\text{APPENDIX C}\]

\text{PROOF OF PROPOSITION III.2}\n
Let \( \sigma(t) = q \in Q, t \in [t_{k-1}, t_{k}] \) and consider the error dynamics of local observers in [30]. According to Definition [31], a ZDA for (30) should satisfy

\[
\begin{bmatrix}
\lambda_{0} I - \bar{A}_{q} \\
C_{i}
\end{bmatrix}
\begin{bmatrix}
\bar{e}_{q}(0) \\
0
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

where \( \bar{e}_{q}(0) := e_{q}(0) - e_{q}(0) = \bar{x}_{0}. \) Also, by considering (28) and the fact that \( C_{i}, \bar{e}_{q}(0) = C_{i}, \bar{x}_{0} = 0 \) in the second equation of (58), matrix pencil (58) can be rewritten as

\[
\begin{bmatrix}
\lambda_{0} I - \bar{A}_{q} \\
C_{i}
\end{bmatrix}
\begin{bmatrix}
\bar{e}_{q}(0) \\
0
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

(59)

It is immediate from Definition [31] that stealthy attack \( u_{a} \) in (30), in the both ZDA and covert attack, looses its stealthiness with respect to the local residual \( r_{i} \), if, and only if, there is no non-trivial zeroing direction associated with matrix pencil in (58) or equivalently \( P \) in (59), which in turn implies \( P \) has full rank. Moreover, from Definition [31,2] and condition (25), it is straightforward that matrix pencil \( P \), defined in (31), is associated with the zeroing direction of the local system (22). We now show how conditions (i)-(iii) establish the equivalence of rank sufficiency for \( P \) in (31) and \( P \) in (59). Given \( P \) in (51), one can write

\[
\begin{bmatrix}
I - h_{1}C_{i} \\
0
\end{bmatrix}
\lambda_{0} h_{1}^{\dagger}
= \begin{bmatrix}
0 \\
I
\end{bmatrix}
\]

(60)

which is a solution to (27) that exists under condition (ii) [20, Lemma 1]. Then, postmultiplying (60) by

\[
\begin{bmatrix}
I \\
0
\end{bmatrix}
0
\begin{bmatrix}
I
\end{bmatrix}
0
\]

and considering (28) yields

\[
\begin{bmatrix}
\lambda_{0} I - \bar{A}_{q} \\
C_{i}
\end{bmatrix}
\begin{bmatrix}
-\bar{T}B \\
0
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

(62)

Since node \( i \) is \( k \)-connected, we have \( |N_{i}| = k \) and \( k \leq \text{rank}(C_{i}) \leq 2k \) (cf. 6). Then, from condition (i), one can verify that \( \text{rank}(C_{i}) \geq \text{rank}(B^{j} + \text{rank}(E^{j})) \) guarantees (31) is a tall or square matrix pencil having only a finite number\(^{7}\) of output-zeroing directions [29, Ch. 2]. Also, the pre- and post-multiplied matrices in (60) and (61) are full column rank. Therefore, we have

\[
\text{rank}(P) = \text{rank}\left[\begin{bmatrix}
\lambda_{0} I - \bar{A}_{q} \\
C_{i}
\end{bmatrix} - \bar{T}B^{j} \right]^{\dagger} \text{rank}(E^{j}).
\]

(63)

Recall \( E^{j} \) is full column rank, and hence \( P \) in (31) is full rank if, and only if, \( P \) in (63) is full rank. This guarantees that a locally undetectable stealthy attack is impossible.

\[\text{APPENDIX D}\]

\text{PROOF OF THEOREM III.3}\n
Consider (35) over \( t \in [t_{0}, +\infty), \) and let the safe mode \( \sigma(t) = q \in Q, t \in [t_{1}, +\infty) \) the continuous system residual \( r_{1}(t) \) and its successive derivatives can be rewritten as

\[
R = C_{q} e(t) - HC_{q} E + C_{q} B U_{a} + H C_{q} U_{s},
\]

\[
- U_{s} + H(\Delta A_{q}) X,
\]

(64)

\(^{7}\)This condition is not valid for degenerate systems which are out of scope of this work.
\[
\begin{align*}
R &= \begin{bmatrix}
r_0^T(t) & r_1^T(t) & \cdots & (r_d^T(t))^{(d)}
\end{bmatrix}^T, \\
U_j &= \begin{bmatrix}
u_0^T(t) & \vdots & \cdots & (u_j^T(t))^{(d)}
\end{bmatrix}^T, \\
E &= \begin{bmatrix}
e_0^T(t) & e_1^T(t) & \cdots & (e^T(t))^{(d)}
\end{bmatrix}^T, \\
X &= \begin{bmatrix}
x^T(t) & \vdots & \cdots & (x^T(t))^{(d)}
\end{bmatrix}^T.
\end{align*}
\]

where \( j \in \{a, b\} \), \( b \in \{B, HC, H, \Delta A_q\} \), \( \Delta A_q = (A_q - A_1) \) and \( d \in \mathbb{N} \setminus \{1, 2\} \).

From (13) in Proposition II.4 and (33)-(34), it can be easily verified that (64) is simplified to
\[
\begin{align*}
\mathcal{H}(b) &= \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 \\
\tilde{C}_b & \tilde{C}_b & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\tilde{C}_b & \tilde{C}_b & \cdots & \tilde{C}_b & 0
\end{bmatrix},
\end{align*}
\]
(69)

with \( j \in \{a, s\} \), \( b \in \{B, HC, H, \Delta A_q\} \), \( \Delta A_q = (A_q - A_1) \) and \( d \in \mathbb{N} \setminus \{1, 2\} \).

We show that under condition (i), the first two terms in (70) are non-zero (and so is \( \Delta \mathcal{L}_q \))
\[
\Delta \mathcal{L}_q x(t) \neq \text{Im}(\Delta \mathcal{L}_q) \iff \Delta \mathcal{L}_q x(t) = 0, \quad \forall t \in [t_1, +\infty).
\]
(74)
onlyx{otherwise, for any \( x(t) \) such that \( 0 \neq \Delta \mathcal{L}_q x(t) = \mathcal{Y} \in \text{Im}(\Delta \mathcal{L}_q) \), we obtain \( [C_x^\top C_v^\top]^\top \mathcal{Y} = 0, \mathcal{Y} \in \ker([C_x^\top C_v^\top]^\top) \) for (73), which is in contradiction with condition (i).}

Now considering the consensus protocol (4), it can be verified that \( \Delta \mathcal{L}_q \) (or equivalently \( \Delta A_q \) in (70)), encodes connected graph component(s) corresponding to added/removed communication link(s) in the communication network (cf. Definition II.4 and A.1). Then an elementary transformation, by means of the permutation matrix \( P \) as defined in (41), transforms (74) into block-diagonal form as
\[
\Delta \mathcal{L}_q x(t) = 0 \iff \tilde{\mathcal{L}}_q x(t) = 0, \quad \forall t \in [t_1, +\infty),
\]
(75)
\[
\begin{align*}
\chi(t) &= \text{col}(\chi_1(t), \ldots, \chi_c(t)) = P x(t), \\
\chi_c(t) &= \text{col}(x_i(t)), \quad \forall i \in D_c, \quad \forall c \in \{1, \ldots, c\},
\end{align*}
\]
(76)
\[
\begin{align*}
x_i(t) - x_j(t) = 0 \iff x_i(t) = x_j(t), \\
\forall i, j \in D_c, \quad \forall c \in \{1, \ldots, c\}, \\
\forall t \in [t_1, +\infty),
\end{align*}
\]
(77)

which by considering the continuity of the system states can be extended for its higher-order time derivatives and be rewritten as
\[
(\epsilon_i^\top - \epsilon_j^\top) x^{(j)}(t) = 0, \quad \forall i, j \in D_c, \quad \forall c \in \{1, \ldots, c\}, \\
\forall m \in \mathbb{N}_0, \quad \forall t \in [t_1, +\infty),
\]
(78)

with \( \epsilon_i, \epsilon_j \) being \( i \)-th and \( j \)-th standard-basis vectors in \( \mathbb{R}^N \). Also, from (70), (75), (78) and by considering the structure \( A_{\gamma(t)} \) and system state \( x(t) = \text{col}(x(t), v(t)) \) in (5), we obtain
\[
\begin{align*}
\Delta \mathcal{L}_q X^{(m)}(t) = 0 \iff \\
\Delta \mathcal{L}_q x^{(m)}(t) = 0, \quad \forall m \in \mathbb{N}_0, \quad \forall t \in [t_1, +\infty).
\end{align*}
\]
(79)

Therefore, under condition (i) one can conclude that unless (74)/(78) holds that is the system states (positions \( x_i(t) \), \( x_j(t) \) and their successive derivatives) of all agents within each graph component, i.e. agents involved in connected

\footnote{Although the analysis here is at the global level, it is worth mentioning that \( \Delta \mathcal{L}_q \) at cluster levels i.e. \( P_i, i \in \{1, \ldots, |P|\} \) may have more than one connected component.}
intra-cluster switching links, are respectively identical \(\forall t \in [t_1, +\infty)\), the left side of (71) and (72) is non-zero and so is (70), implying \(\Delta A_q\) affects \(R(t)\) whereby the attacks are detectable.

We now show under conditions (ii) and (iii) the domain of existence of (74) is shrank to the only case that the entire system states, except for those affected by stealthy attacks, are at an equilibrium.

Zero-dynamics attack (ZDA) case: it can be shown that under condition (ii) (74) holds (and so does (78)) only in the worst-case scenario, in the sense of attack detection, that none of the agents involved in intra-cluster switching links are affected by the ZDA in a safe mode. To this end, consider (75) under which ZDA remains stealthy in residual \(r_0(t)\) in the safe modes and recall

\[
x(t) = \tilde{x}(t) + \tilde{x}(t), \quad x_0e^{\lambda_0 t}, \quad \forall t \in [t_0, +\infty)
\]

in a stealthy ZDA case with \(\dot{x}_0e^{\lambda_0 t} \in \ker(C)\) being the initial condition of ZDA (cf. [1] and (20) in Proposition II.4) at \(t = t_1\) for a safe mode. Similar to (37), by evaluating ZDA condition (1) for the tuple \((A_q, B, C)\) with \(A_q = (A_1 + \Delta A_q)\) and considering (79) we obtain

\[
\begin{bmatrix}
\lambda_0 I - (A_1 - HC) & -B \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{e}(t_1) \\
u_a(t_1)
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

where as in (37), \(\tilde{e}(t_1) = \tilde{x}(t_1)\) with \(\tilde{x}(t_1) = x_0e^{\lambda_0 t_1}\) and \(u_a(t_1) = u_0e^{\lambda_0 t_1}\). Then (81) is simplified to

\[
\begin{bmatrix}
\lambda_0 I - (A_q - HC) & -B \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{x}(0) \\
u_0
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

where further simplification, similar to that in (37), and expanding it out yields

\[
\begin{bmatrix}
\lambda_0 I_N & -I_N \\
\alpha(L_1 + \Delta L_q) & (\lambda_0 + \gamma)I_N - I_F
\end{bmatrix}
\begin{bmatrix}
\tilde{x}(t_0) \\
u(t_0)
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

(82)

from which and also from (20) we have

\[
\begin{align*}
\lambda_0 \tilde{x}_i(t_0) &= \tilde{v}_i(t_0), & \forall i \in V, \\
\alpha L_1 \tilde{x}(t_0) + (\lambda_0 + \gamma)\tilde{v}(t_0) - I_F u_0 &= 0, \\
\Delta L_q \tilde{x}(t_0) &= 0, \\
C_x \tilde{x}(t_0) &= 0, \\
C_v \tilde{v}(t_0) &= 0.
\end{align*}
\]

(83)

Then one can conclude from (66), (80), (84), and (87) that

\[
\tilde{x}_i(t_0) = \tilde{v}_i(t_0) = 0 \implies \tilde{x}_i(t) = \tilde{v}_i(t) = 0, \quad \forall i \in M \subset V,
\]

(88)

and by applying the same permutation as defined in (71) and used in (75) to equation (86) as well as by considering (80) and (83) that

\[
\begin{align*}
\tilde{x}_i(t_0) &= \tilde{x}_j(t_0) \implies \tilde{x}_i(t) = \tilde{x}_j(t), & \forall i, j \in D_C \subset V, \\
\tilde{v}_i(t_0) &= \tilde{v}_j(t_0) \implies \tilde{v}_i(t) = \tilde{v}_j(t), & \forall i, j \in D_C \subset V.
\end{align*}
\]

(89)

Also, as shown in Lemma A.2 under condition (i) we have

\[
D_c \cap M \neq \emptyset, \quad \forall c \in \{1, \cdots, c\},
\]

(91)

with set \(D_c\) given in (76).

Now under (91), it is concluded from (88), (89)-(90) that

\[
\hat{x}_i(t) = \hat{x}_j(t) = 0, \quad \hat{v}_i(t) = \hat{v}_j(t) = 0, \quad \forall i, j \in D_c,
\]

(92)

which by considering (80) implies that (78) is simplified to

\[
\begin{align*}
(e_i^T - e_j^T)x^{(m)}(t) &= 0, & \forall i, j \in D_c, & \forall c \in \{1, \cdots, c\}, \\
& \forall m \in N_0, & \forall t \in [t_1, +\infty),
\end{align*}
\]

(93)

where \(\hat{x}_i\) and \(\hat{x}_j\) are the elements of state vector \(\hat{x}\) in (80), denoting the states of an attack-free system that satisfies (13) (i.e. \(\hat{x} = A_q\hat{x}\) obtained using \(\hat{x}\)-dynamics in [3] with \(B u_a = 0\) and unknown initial condition \(x_0\) as defined in Proposition II.4. Then using the attack-free dynamics \(\hat{x} = A_q\hat{x}\), the term \((e_i^T - e_j^T)x^{(m)}(t)\) in (93) can be rewritten as

\[
\begin{align*}
(e_i^T - e_j^T)L_q^m \hat{x}(t) &= 0, & \forall i, j \in D_c, & \forall m \in N_0, & \forall t \in [t_1, +\infty),
\end{align*}
\]

(94)

Moreover, note that (94) and (95) have the same form as equations (109a) and (109b) in [3]. Then under further conditions (ii) and (iii) it can be verified using the same procedure as in [3, Th. 2] that (94) and (95) yield

\[
\begin{align*}
\hat{x}_i(t) &= \hat{x}_j(t), & \forall i, j \in V, & \forall t \in [t_1, +\infty), \\
\hat{v}_i(t) &= \hat{v}_j(t), & \forall i, j \in V, & \forall t \in [t_1, +\infty),
\end{align*}
\]

(96)

(97)

which means the the entire states of the attack-free system have achieved consensus. Considering the equilibrium subspace (5) as a result of the consensus protocol (4), one can conclude that (96)-(97) and (3) coincide. Therefore, from (93) and (96)-(97), obtained under conditions (ii) and (iii) one can conclude that stealthy ZDA is undetectable in \(r_0(t)\) of (32) only in the worst-case scenario that intra-cluster switching links are between agents whose trajectories are not affected by ZDA as well as all of the system (5)’s attack-free trajectories, characterized in (93), are at the consensus equilibrium (5).

Covert attack case: consider (75) under which a covert attack remains stealthy in a safe mode and note that

\[
x(t) = \tilde{x}(t) + \tilde{x}(t), \quad \forall t \in [t_1, +\infty),
\]

(98)

with

\[
\tilde{x}(t) = e^{A_1(t-t_1)}\tilde{x}(t_1) + \int_{t_1}^{t} e^{A_1(t-\tau)}B u_a(\tau) d\tau
\]

(99)

according to the attack model (7) and Proposition II.4. Given (98), (78) can be rewritten as

\[
\begin{align*}
\int(e_i^T - e_j^T) [x^{(m)}(t) = (e_i^T - e_j^T) x^{(m)}(t), & \forall i, j \in D_c, \\
& \forall c \in \{1, \cdots, c\}, & \forall m \in N_0, & \forall t \in [t_1, +\infty),
\end{align*}
\]

(99)

Notice that the attack-free system states, \(x(t)\) in (98), converge to (5) as \(t \to +\infty\), then the left side of (99) converges to zero.
and one can conclude from (98) and (99) that continuous states
\( \dot{x}(t) = \text{col} (\dot{x}(t), \tau(t)) \) exist in either of the following cases
\[
\text{case 1} : \dot{x}_i(t) = \ddot{x}_j(t) \neq 0, \; \forall i, j \in D_c, \; \forall t \in [t_1, +\infty) \\
(100)
\]
\[
\text{case 2} : \dot{x}_i(t) = \ddot{x}_j(t) = 0, \; \forall i, j \in D_c, \; \forall t \in [t_1, +\infty) \\
(101)
\]

Note that here case 1 in (100) implies the attack input \( u_0 \) in (98) has driven and kept the states of agents involved in switching into an unknown equilibrium over time span \( \forall t \in [t_1, +\infty) \). Also, case 2’s interpretation and analysis coincide with that of ZDA in (92). Then following the same analysis as the ZDA’s, one can conclude that, under conditions (i)-(iii), covert attack is undetectable in \( v_0(t) \) of (99) only in the worst-case scenarios that 1) intra-cluster switching links are between agents whose trajectories are identical over time under the effect of covert attack; and 2) intra-cluster switching links are between agents whose trajectories are not affected by covert attack as well as all of the system’s attack-free trajectories are at the consensus equilibrium (3).

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