The relevance of the vertex bremsstrahlung photon detection in the $\nu_e(\overline{\nu}_e)e^-\rightarrow\nu_e(\overline{\nu}_e)e^-$ scattering experiments at low energy

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Abstract

We discuss the size of the $\nu_e(\overline{\nu}_e)e^-\rightarrow\nu_e(\overline{\nu}_e)e^-$ cross section reduction due to the rejection of the events with a vertex bremsstrahlung photon above a certain energy in the final state. In particular we analyze the effect in experiments designed to detect the low energy $\overline{\nu}_e$ and $\nu_e$ from a nuclear reactor and from the Sun. We find that such reduction has to be considered in a relatively high statistic reactor experiment, while it is negligible for $pp$ and $^7Be$ solar neutrino detection.

1 Introduction

Forthcoming reactor and solar neutrino experiments are expected to provide important improvements both in the quality and in the statistics of the data. As a matter of fact, in most cases the statistical error will be of a few per cent. At this level of accuracy, radiative corrections are likely to become important.

Our goal is to estimate the impact for forthcoming experiments of the radiative corrections (see [2] for a review) to the process $\nu_e(\overline{\nu}_e)e^-\rightarrow\nu_e(\overline{\nu}_e)e^-$ once realistic experimental set-up are included into the analysis. In particular we will discuss the effect of the vertex bremsstrahlung photon in experiments designed to detect the low energy $\overline{\nu}_e$ and $\nu_e$ from a nuclear reactor and from the Sun.

2 Radiative Corrections

A complete and detailed account of the full [1] set of one loop radiative corrections to $\nu_e e^-$ scattering, as well as a comprehensive set of references, is given in [2]. The same corrections for $\overline{\nu}_e e^-$ scattering are obtained by replacing $g_L(T)\leftrightarrow g_R(T)$ in the formulas given in [2]. A preliminary study of the size of radiative corrections for a $\overline{\nu}_ee^-$ scattering

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experiment with the reactor antineutrinos, the MUNU one, has been done in [3] where we discussed the physics items which can be studied in a $\bar{\nu}_e e^-$ scattering experiment at a nuclear reactor in addition to the neutrino magnetic moment.

For the sake of completeness let us summarize a few items discussed in [3]:

- one loop radiative corrections induce a decrease, as compared to the Born prediction, of about 2.5%, mildly dependent on energy and scattering angle. In fig. 1 we show the relative size of the order $\alpha$ cross section, weighted over the antineutrino spectrum, as a function of the electron scattering angle and kinetic energy;

- higher order corrections are below the per mille level and therefore, in view of the forecasted experimental accuracy, negligible.

2.1 Bremsstrahlung corrections

Radiative corrections as given in [2] are for an inclusive set-up, i.e. they refer to the cross section for the process $\nu_e(\bar{\nu}_e)e^- \rightarrow \nu_e(\bar{\nu}_e)e^- + (n\gamma)$ where with $+(n\gamma)$ we denote the sum over $\nu_e(\bar{\nu}_e)e^-$ final states with an arbitrary number of photons.\footnote{To be more accurate the formulas in [2] refers to $n \leq 1$, however, as previously discussed, the contribution of terms with $n \geq 2$, being higher order corrections, is estimated to be negligible.} From now on we will refer only to order $\alpha$ corrections, since they are the only relevant ones.

In low energy $\nu_e(\bar{\nu}_e)e^-$ scattering experiments, an event with a $\nu_e(\bar{\nu}_e)e^-\gamma$ final state has to be rejected because it looks like a multi-Compton background event (i.e. an event due to a $\gamma$ which has more than one Compton scattering inside the detector). Therefore the experimentally relevant cross section is given only by the sum of final states with an arbitrary number of photons which are not identified in the detector, i.e., to a good degree of accuracy, photons with an energy lower than a given threshold energy $\omega_{\text{min}}$. Soft photons ($\omega < \omega_{\text{min}}$) still ensure the correct cancellation of unphysical infrared QED divergences, however the contribution of $\nu_e(\bar{\nu}_e)e^-\gamma$ events is lower than in the fully inclusive case.

To study the size of this effect we have built up an event generator for the process

$$\nu_e(\bar{\nu}_e)e^- \rightarrow \nu_e(\bar{\nu}_e)e^-\gamma$$

The cross section for this process has been computed in [8]. Here we evaluate the amplitude of the process numerically using explicit representation for spinors, gamma matrices and the photon polarization vector. Namely we write the scattering amplitude, $\mathcal{M}$, as

$$\mathcal{M} = 2\sqrt{2}eG_F \bar{u}_f(p_f) \left[ \frac{m_e + \not{k} + \not{p}_f}{2p_f \cdot k} \not{\gamma}_\mu (g_L P_L + g_R P_R) 
+ \not{\gamma}_\mu (g_L P_L + g_R P_R) \frac{m_e - \not{k} + \not{p}_i}{-2p_i \cdot k} u_e(p_i) N_\mu \right] (1)$$

where $G_F$ is the Fermi constant, $e$ the electromagnetic charge, $P_L$ and $P_R$ are the left and right-handed chiral projectors respectively, $u$ and $v$ are the conventional Dirac four spinors, $N_\mu = \bar{v}(q_i) \not{\gamma}_\mu P_L v(q_f)$, $\bar{u}(q_f) \not{\gamma}_\mu P_R u(q_i)$, for $\bar{\nu}_e$ and $\nu_e$ respectively, $p_f$ and $p_i$ are
the final and initial electron four momenta, \( q_f \) and \( q_i \) are the final and initial neutrino four momenta, \( k \) the photon four momentum and \( \epsilon \) the photon polarization vector. We have checked that our results are in agreement with those in [8].

### 2.2 Reactor antineutrinos and solar neutrinos

The threshold energy for ‘detectable’ photons depends on the detector. In the following we assume a gas detector, for instance a time projection chamber (TPC), as in the MUNU [4] experiment and in the HELLAZ [5] and SuperMUNU [6] [7] projects for the spectroscopy of low energy neutrinos from the Sun. In a TPC both the energy and the topology of the event are reconstructed. Therefore, the bremsstrahlung photon is seen only if its interaction point is well separated in space from the track of the electron recoiling after the neutrino scattering and if it gives rise to a signal above the electronic noise. Since the attenuation length of a 10 keV photon in a gas at atmospheric pressure is always longer than 10 cm (for instance in \( CF_4 \) is around 50 cm) then a photon threshold above 10 keV would be dictated only by the level of the electronic noise of the given detector.

In the following we will discuss the impact of vertex bremsstrahlung corrections on various integrated as well as differential quantities. To give a better feeling of the size of the effect we always show cross sections averaged over the reactor (solar) antineutrino (neutrino) spectrum normalized to unity, i.e.:

\[
\frac{d\sigma}{d\lambda} = \left( \int dE_\nu \, \Phi_\nu(E_\nu) \right)^{-1} \int dE_\nu \, \Phi_\nu(E_\nu) \frac{d\sigma}{d\lambda}(E_\nu)
\]

where \( \lambda \) denotes a generic observable (electron energy, scattering angle), \( E_\nu \) is the antineutrino (neutrino) energy, \( \Phi_\nu(E_\nu) \) is the differential antineutrino (neutrino) spectrum and \( \sigma(E_\nu) \) denotes the cross section for the considered process as a function of the incident antineutrino (neutrino) energy.

To be as general as possible, in fig. 2 we give the cross section \( \sigma_\gamma \) for the \( \nu_e e^- \rightarrow \nu_e e^- \gamma \) process, weighted over the reactor antineutrino spectrum and plotted as a function of the photon energy threshold for three different values of the electron kinetic energy threshold \( T_0 \). A threshold energy \( \omega_{\text{min}} \) of 10 (50) keV for the bremsstrahlung photon implies a reduction of 1.2% (0.6%) of the cross section with respect to the inclusive one at \( T_0=0.5 \) MeV (this percentage becomes 0.9% and 0.5% for \( T_0=0.1 \) MeV). Such a reduction is a significant fraction of the error expected in a reactor experiment and it has to be corrected for.

We observe that the \( \nu_e e^- \rightarrow \nu_e e^- \gamma \) cross section increases with the electron energy. As a matter of fact it is well known that the radiation must vanish with the electron kinetic energy and therefore the bulk of the correction is for the more energetic electrons. This is clearly seen in fig. 3 where the differential cross section for the \( \nu_e e^- \gamma \) final state, weighted over the reactor spectrum and plotted as a function of the electron kinetic energy and of its scattering angle, is compared with the leading order one (\( \nu_e e^- \) final state). The cross section ratio is shown for four different photon energy thresholds: 1, 10, 50 and 100 keV. The 1 keV threshold is taken into account just to show the trend of the effect (a 1 keV photon converts too close to the electron track to be distinguished from it). We see that,
for photon energies above 10 keV, the size of the differential cross section ratio is smaller than 3% for any electron energy, even if a 100% photon detection efficiency is assumed.

It is interesting to notice that the effect of this correction on the electron energy spectrum could cancel\(^2\) the one induced by the neutrino magnetic moment because, whereas radiative corrections have the overall effect to lower the cross section, the contribution of the neutrino magnetic moment would be additive.

We now come to solar neutrinos. The size of the inclusive one loop radiative corrections has been calculated in [2] and found to be less than about 2% for the \(pp\) and \(^7\)Be neutrinos. Here we discuss the contribution of the events with a vertex bremsstrahlung photon above a certain energy in the final state. Fig. 4 shows the ratio between the differential cross sections for the \(\nu_e e^-\gamma\) and the \(\nu_e e^-\) scattering (both weighted over the \(pp\) neutrino spectrum) plotted as a function of the electron kinetic energy for different values of the photon energy threshold. The same is shown in fig. 5 for the \(^7\)Be neutrinos (862 keV line). We see that the contribution of the radiation process is smaller than 0.1% and 0.7% for the \(pp\) and \(^7\)Be neutrinos respectively at any electron recoil energy for \(\omega_{\text{min}}=10\) keV. As a consequence, the number of \(\nu_e e^-\gamma\) events with a detectable photon in the final state is a negligible fraction of all the \(\nu_e e^-\) interactions and their rejection is completely harmless in a low energy solar neutrino experiment. As a matter of fact it amounts to 0.09% and 0.45% \((T_0=0.1\ MeV\ and\ \omega_{\text{min}}=10\ keV)\) for the \(pp\) and \(^7\)Be neutrinos respectively.

### 2.3 Eikonal approximation

The energy of the photons we are dealing with is substantially lower than the kinetic energy of the detected electrons. Usually, in this limit, the eikonal approximation turns out to be a good approximation. In this approximation the amplitude for the radiative process is written as the product of the leading amplitude times the eikonal current. For the process of interest to us this implies

\[
\frac{d\sigma_\gamma}{dT_e} = \frac{d\sigma_0}{dT_e} e^2 \frac{d^3 k}{2(2\pi)^3} \left| \frac{p_f}{kp} - \frac{p_i}{kp} \right|^2
\]

where \(\sigma_\gamma\) denotes the bremsstrahlung cross section, \(\sigma_0\) the leading order one, \(T_e\) is the electron kinetic energy, \(k\) is the photon four momentum and \(\epsilon\) the photon polarization.

In fig. 6 we show the comparison of the results obtained in the eikonal approximation versus the exact ones for the reactor antineutrinos. It is clearly seen that both for total rates as well as for differential quantities the agreement is at the few per mille level, i.e. sufficiently accurate for our purposes.

Using the eikonal approximation we are in the position to provide a very simple formula for radiative corrections due to real bremsstrahlung:

\[
\frac{d\sigma_\gamma}{dT_e} = \frac{d\sigma_0}{dT_e \pi} \left( \frac{E_f}{p_f} \log \frac{E_f + p_f}{E_f - p_f} - 2 \right) \log \frac{\omega_{\text{max}}}{\omega_{\text{min}}}
\]

\(^2\)We recall once more that bremsstrahlung correction as given in fig. 3 have to be subtracted from the inclusive cross section given in [2].
where $\alpha$ is the fine structure constant, $E_f$ and $p_f$ are the energy and momentum of the recoil electron, $\omega_{\text{max}}$ is the maximal photon energy (i.e. the initial neutrino energy) and $\omega_{\text{min}}$ is the threshold energy for detected photons.

The measurable cross section is hence given by the formulas as in e.g. $[2]$ minus the contribution of real bremsstrahlung $[4]$.

3 Conclusions

We have discussed the relevance of the rejection of the $\overline{\nu}_e(\nu_e)e^- \rightarrow \overline{\nu}_e(\nu_e)e^-\gamma$ events in experiments designed to detect with a TPC the low energy $\overline{\nu}_e$ and $\nu_e$ from a nuclear reactor and from the Sun. As a matter of fact such events have to be disregarded because they look like multi-Compton background events inside the detector. We have found that their rejection has a sizeable effect in a reactor experiment where up to 1.2% of the antineutrino events with electron energy above 500 keV have a photon of more than 10 keV energy. On the other hand their contribution is negligible for the $pp$ and $^7\text{Be}$ solar neutrinos (0.09% and 0.45% at 100 keV electron threshold). Finally we checked that the eikonal approximation is a good approximation (at the per mille level) of the differential exclusive cross section.

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Figure Captions

Fig. 1 Fractional contribution $R_{\theta,T}$ of one loop electroweak corrections to $\bar{\nu}_e e^-$ cross section as a function of the scattering angle $\theta$ (a) and the $e^-$ recoil energy $T$ (b). $R_{\theta,T}$ is defined as $R_{\lambda} = (<d\sigma_1/d\lambda> - <d\sigma_0/d\lambda>/ <d\sigma_0/d\lambda>$, $\lambda = \theta, T$, where $<\sigma_{0,1}>$ are the Born and one loop $\bar{\nu}_e e^-$ scattering cross sections averaged over the reactor antineutrino spectrum.

Fig. 2 Cross section $\sigma_\gamma$ for the process $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^- \gamma$ as a function of the photon threshold detection energy $\omega_{min}$ for three different electron threshold detection energies: 0.1 $MeV$ (dashed line), 0.3 $MeV$ (dot-dashed line) and 0.5 $MeV$ (continuos line). The Born level cross sections, $\sigma_0$, for the process $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$, are 52, 43 and 35 respectively. Units of $10^{-46} cm^2$ are used.

Fig. 3 Ratios among the differential cross sections, weighted over the reactor antineutrino spectrum, for $\bar{\nu}_e e^- \gamma$ and $\bar{\nu}_e e^-$ final states. Fig. 3a, $R_{T\gamma} = (d\sigma_\gamma/dT)/(d\sigma_0/dT)$ as a function of $e^-$ kinetic energy in $MeV$. Fig. 3b, $R_{\theta\gamma} = (d\sigma_\gamma/d\theta)/(d\sigma_0/d\theta)$ as a function of the electron scattering angle. The ratios are shown for four different photon energy thresholds: 1, 10, 50 and 100 $keV$ (dotted, continuos, dot-dashed and dashed line respectively).

Fig. 4 Ratio among the vertex bremsstrahlung contribution and the tree-level one, both weighted over the solar spectrum, for the $pp$ neutrinos at $\omega_{min} = 1, 10, 50, 100$ $keV$ (dotted, continuos, dot-dashed and dashed line respectively).

Fig. 5 Ratio among the vertex bremsstrahlung contribution and the tree-level one for the $^7Be$ neutrinos at $\omega_{min} = 1, 10, 50, 100$ $keV$ (dotted, continuos, dot-dashed and dashed line respectively).

Fig. 6 Ratio $R_{eik} = \sigma_{ex}/\sigma_{eik}$ among the order $\alpha$ cross sections with real bremsstrahlung treated exactly, $\sigma_{ex}$, and in the eikonal approximation, $\sigma_{eik}$. $R_{eik}$ is plotted as a function of the electron kinetic energy ($MeV$). The threshold energies $\omega_{min}$ for the bremsstrahlung photon are: 1 $keV$ (continuos line), 50 $keV$ (dot-dashed line) and 100 $keV$ (dashed line). Cross sections are averaged over the reactor antineutrino spectrum.
