Ultra Minimal Technicolor and its Dark Matter TIMP

Thomas A. RYTTOV\textsuperscript{a,c} and Francesco SANNINO\textsuperscript{b} \footnote{Electronic address: ryttov@nbi.dk \textsuperscript{*}} \footnote{Electronic address: sannino@ifk.sdu.dk \textsuperscript{†}}

\textsuperscript{a}CERN Theory Division, CH-1211 Geneva 23, Switzerland
\textsuperscript{b}High Energy Center, University of Southern Denmark, Campusvej 55, DK-5230 Odense M
\textsuperscript{c}Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

Abstract

We introduce an explicit model with technifermion matter transforming according to multiple representations of the underlying technicolor gauge group. The model features simultaneously the smallest possible value of the naive $S$ parameter and the smallest possible number of technifermions. The chiral dynamics is extremely rich. We construct the low-energy effective Lagrangian. We provide both the linearly and non-linearly realized ones. We then embed, in a natural way, the Standard Model (SM) interactions within the global symmetries of the underlying gauge theory. Several low-energy composite particles are SM singlets. One of these Technicolor Interacting Massive Particles (TIMP)s is a natural cold dark matter (DM) candidate. We estimate the fraction of the mass in the universe constituted by our DM candidate over the baryon one. We show that the new TIMP, differently from earlier models, can be sufficiently light to be directly produced and studied at the Large Hadron Collider (LHC).
I. INTRODUCTION

Understanding the origin of the electroweak symmetry breaking and its possible relation to DM constitute two of the most profound theoretical challenges at present. New strong dynamics at the electroweak scale \cite{1,2} may very well provide a solution to the problem of the origin of the bright and dark \cite{3,4,5} mass. A large class of models has recently been proposed \cite{6} which makes use of higher dimensional representations of the underlying technicolor gauge group. This has triggered much work related to both the LHC phenomenology, lattice studies and DM \cite{5,8,9,12,13,14,15,16,17,18,19,20,21,22}. For a recent review see \cite{23}.

Here we provide an explicit example of (near) conformal (NC) technicolor \cite{24,25,26,27,28,29,30} with two types of technifermions, i.e. transforming according to two different representations of the underlying technicolor gauge group \cite{12,31}. The model possesses a number of interesting properties to recommend it over the earlier models of dynamical electroweak symmetry breaking:

- Features the lowest possible value of the naive \( S \) parameter \cite{32,33} while possessing a dynamics which is NC.
- Contains, overall, the lowest possible number of fermions.
- Yields natural DM candidates.

Due to the above properties we term this model Ultra Minimal near conformal Technicolor (UMT). It is constituted by an \( SU(2) \) technicolor gauge group with two Dirac flavors in the fundamental representation also carrying electroweak charges, as well as, two additional Weyl fermions in the adjoint representation but singlets under the SM gauge groups.

In the next section we arrive at this specific UMT model using the conjectured all-orders beta function for nonsupersymmetric gauge theories \cite{34}. In Section III we write the underlying Lagrangian and identify the global symmetries of the theory before and after dynamical symmetry breaking. We then construct both the linearly and non-linearly realized low-energy effective Lagrangians. We naturally embed the Standard Model (SM) interactions within the global symmetries of the underlying gauge theory. Several low-energy composite particles are SM singlets. In particular there is a di-techniquark state which is a possible cold DM candidate. This Technicolor Interacting Massive Particle (TIMP) is a natural cold DM candidate as shown in Section IV. We also estimate the fraction of the
mass in the universe constituted by our DM candidate over the baryon one as function of the Lepton number and the DM mass. The new TIMP, differently from earlier models [3, 4], can be sufficiently light to be directly produced and studied at the Large Hadron Collider (LHC). The expected rate of events detectable in experiments such as CDMS [35], as function of the DM mass, is computed showing that it is not constrained by current data. We draw our conclusions in the final section.

II. FROM THE CONFORMAL WINDOW TO ULTRA MINIMAL TECHNICOLOR

To construct a realistic model of electroweak symmetry breaking one is faced with the constraints coming from the electroweak precision tests. Specifically the new physics beyond the SM must not give a too large contribution to the $S$ parameter. Consider an $SU(N)$ technicolor theory with $N_f$ Dirac fermions in the representation $r$. The naive estimate of $S$ computed in the approximation of a techniquark loop with momentum-independent constituent masses much heavier than the $Z$ mass [72] is given by

$$S = \frac{1}{6\pi} \frac{N_f}{2} d(r) , \quad (1)$$

where $d(r)$ is the dimension of the representation $r$. From the estimate above it is clear that an $SU(2)$ technicolor theory with two Dirac fermions in the fundamental representation yields the smallest possible contribution.

However for this low number of flavors the theory is far from possessing NC dynamics and the naive $S$ value underestimates the physical value [32, 33]. The situation changes for NC theories [36].

Insisting on a NC model with this minimal $S$ parameter an obvious way to obtain conformality is to add the remaining fundamental flavors, neutral under the electroweak symmetries, needed to be just outside the conformal window. The near conformal technicolor theories constructed in this way have been termed partially gauged technicolor [12]. However, as we shall show below, by arranging the additional fermions in higher dimensional representations, it is possible to construct models which have a particle content smaller than the one of partially gauged technicolor theories. In fact instead of considering additional fundamental flavors we shall consider adjoint flavors. Note that for two colors there exists
only one distinct two-indexed representation.

How many adjoint fermions are needed to build the above NC model? Information on the conformal window for gauge theories containing fermions transforming according to distinct representations is vital. First principle lattice simulations are exploring the conformal window for higher dimensional representations \[14, 15, 16\]. However the models we are constructing have not yet been explored on the lattice.

To elucidate the various possibilities we make use of our recently conjectured all-order beta function for a generic \( SU(N) \) gauge theory with fermionic matter transforming according to arbitrary representations \[34\]. Considering \( N_f(r_i) \) Dirac flavors belonging to the representation \( r_i, \ i = 1, \ldots, k \) of the gauge group it reads

\[
\beta(g) = -\frac{g^3}{(4\pi)^2} \beta_0 - \frac{2}{3} \sum_{i=1}^{k} T(r_i) N_f(r_i) \gamma_i(g^2) \left(1 - \frac{g^2}{8\pi^2} C_2(G) \left(1 + \frac{2\beta'_0}{\beta_0}\right)\right), \tag{2}
\]

with

\[
\beta_0 = \frac{11}{3} C_2(G) - \frac{4}{3} \sum_{i=1}^{k} T(r_i) N_f(r_i) \quad \text{and} \quad \beta'_0 = C_2(G) - \sum_{i=1}^{k} T(r_i) N_f(r_i). \tag{3}
\]

One should note that the beta function is given in terms of the anomalous dimension of the fermion mass \( \gamma = -\frac{d \ln m}{d \ln \mu} \) where \( m \) is the renormalized mass, similar to the supersymmetric case \[37, 38, 39\]. Indeed the construction of the above beta function is inspired by the one of their supersymmetric cousin theories. At small coupling it coincides with the two-loop beta function and in the non-perturbative regime reproduces earlier known exact results. Similar to the supersymmetric case it allows for a bound of the conformal window \[40\]. In the supersymmetric case where additional checks can be made the bound is actually believed to give the true conformal window. We stress that the predictions of the conformal window coming from the above beta function are nontrivially supported by all the recent lattice results \[14, 15, 16, 41, 42, 43, 44\].

First, the loss of asymptotic freedom is determined by the change of sign in the first coefficient \( \beta_0 \) of the beta function. This occurs when

\[
\sum_{i=1}^{k} \frac{4}{11} T(r_i) N_f(r_i) = C_2(G), \quad \text{Loss of AF.} \tag{4}
\]

Hence for a two color theory with two fundamental flavors the critical number of adjoint Weyl fermions above which one looses asymptotic freedom is 4.50. Second, we note that at
the zero of the beta function we have
\[ \sum_{i=1}^{k} \frac{2}{11} T(r_i) N_f(r_i) (2 + \gamma_i) = C_2(G). \] (5)

Therefore specifying the value of the anomalous dimensions at the infrared fixed point yields the last constraint needed to construct the conformal window. Having reached the zero of the beta function the theory is conformal in the infrared. For a theory to be conformal the dimension of the non-trivial spinless operators must be larger than one in order to not contain negative norm states [45, 46, 47]. Since the dimension of the chiral condensate is \(3 - \gamma_i\) we see that \(\gamma_i = 2\), for all representations \(r_i\), yields the maximum possible bound
\[ \sum_{i=1}^{k} \frac{8}{11} T(r_i) N_f(r_i) = C_2(G). \] (6)

This implies, for example, that for a two technicolor theory with two fundamental Dirac flavors the critical number of adjoint Weyl fermions needed to reach the bound above on the conformal window is 1.75 [73]. The actual size of the conformal window can be smaller than the one determined by the bound above. It may happen, in fact, that chiral symmetry breaking is triggered for a value of the anomalous dimension less than two. If this occurs the conformal window shrinks. Within the ladder approximation [48, 49] one finds that chiral symmetry breaking occurs when the anomalous dimension is close to one. Picking \(\gamma_i = 1\) we find:
\[ \sum_{i=1}^{k} \frac{6}{11} T(r_i) N_f(r_i) = C_2(G). \] (7)

In this case when considering a two color theory with two fundamental Dirac flavors the critical number of adjoint Weyl flavors is 2.67. Hence, our candidate for a NC theory with a minimal \(S\) parameter has two colors, two fundamental Dirac flavors charged under the electroweak symmetries and two adjoint Weyl fermions. This is the Ultra Minimal NC Technicolor model (UMT).

If it turns out that the anomalous dimension above which chiral symmetry breaking occurs is larger than one we can still use the model just introduced. We will simply break its conformal dynamics by adding masses (anyway needed for phenomenological reasons) for the adjoint fermions.
III. THE MODEL

The fermions transforming according to the fundamental representation are arranged into electroweak doublets in the standard way and may be written as:

\[ T_L = \begin{pmatrix} U \\ D \end{pmatrix} , \quad U_R , \quad D_R \]  \hspace{1cm} (8)

The additional adjoint Weyl fermions needed to render the theory quasi conformal are denoted as \( \lambda^f \) with \( f = 1, 2 \). They are not charged under the electroweak symmetries. Also we have suppressed technicolor indices. The theory is anomaly free using the following hypercharge assignment

\[ Y(T_L) = 0 \ , \quad Y(U_R) = \frac{1}{2} \ , \quad Y(D_R) = -\frac{1}{2} \ , \quad Y(\lambda^f) = 0 \]  \hspace{1cm} (9)

Our notation is such that the electric charge is \( Q = T_3 + Y \). Replacing the Higgs sector of the SM with the above technicolor theory the Lagrangian reads:

\[ \mathcal{L}_H \rightarrow -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i T_L \gamma^\mu D_\mu T_L + i U_R \gamma^\mu D_\mu U_R + i D_R \gamma^\mu D_\mu D_R + i \overline{\lambda} \sigma^\mu D_\mu \lambda , \]  \hspace{1cm} (10)

with the technicolor field strength \( F_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{TC} \epsilon_{abc} A^b_\mu A^c_\nu \), \( a, b, c = 1, \ldots, 3 \). The covariant derivatives for the various fermions are

\[ D_\mu T_L = \left( \partial_\mu - ig_{TC} A^a_\mu \frac{\tau^a}{2} - ig W^a_\mu \frac{L^a}{2} \right) T_L , \]  \hspace{1cm} (11)

\[ D_\mu U_R = \left( \partial_\mu - ig_{TC} A^a_\mu \frac{\tau^a}{2} - ig' \frac{B_\mu}{2} \right) U_R , \]  \hspace{1cm} (12)

\[ D_\mu D_R = \left( \partial_\mu - ig_{TC} A^a_\mu \frac{\tau^a}{2} + ig' \frac{B_\mu}{2} \right) D_R \]  \hspace{1cm} (13)

\[ D_\mu \lambda^{a,f} = \left( \delta^{ac} \partial_\mu + g_{TC} A^b_\mu \epsilon^{abc} \right) \lambda^{c,f} , \]  \hspace{1cm} (14)

Here \( g_{TC} \) is the technicolor gauge coupling, \( g \) is the electroweak gauge coupling and \( g' \) is the hypercharge gauge coupling. Also \( W^a_\mu \) are the electroweak gauge bosons while \( B_\mu \) is the gauge boson associated to the hypercharge. Both \( \tau^a \) and \( L^a \) are Pauli matrices and they are the generators of the technicolor and weak gauge groups respectively.

The global symmetries of the theory are most appropriately handled by first arranging the fundamental fermions into a quadruplet of \( SU(4) \).
Since the fermions belong to pseudo-real and real representations of the gauge group the global symmetry of the theory is enhanced and can be summarized as

$$ \begin{array}{c|ccc}
\text{ } & SU(4) & SU(2) & U(1) \\
Q & \Box & 1 & -1 \\
\lambda & 1 & \Box & \frac{1}{2} \\
\end{array} $$

The abelian symmetry is anomaly free. Following Ref. [50] the characteristic chiral symmetry breaking scale of the adjoint fermions is larger than that of the fundamental ones since the dimension of the adjoint representation is larger than the dimension of the fundamental representation. We expect, however, the two scales to be very close to each other since the number of fundamental flavors is rather low. In the two-scale technicolor models [31] the dynamical assumption is instead, that the different scales of the condensates are very much apart from each other.

The global symmetry group $G = SU(4) \times SU(2) \times U(1)$ breaks to $H = Sp(4) \times SO(2) \times Z_2$. The stability group $H$ is dictated by the (pseudo)reality of the fermion representations and the breaking is triggered by the formation of the following two condensates

$$ \langle Q_F^{a,c} Q_{F'}^{c',c'} \epsilon_{\alpha\beta} \epsilon_{oo'} E_4^{FF'} \rangle = -2\langle U_R U_L + D_R D_L \rangle $$

$$ \langle \lambda_{f}^{\alpha,k} \lambda_{f'}^{\beta,k'} \epsilon_{\alpha\beta} \delta_{kk'} E_2^{ff'} \rangle = -2\langle \lambda^1 \lambda^2 \rangle $$

where

$$ E_4 = \begin{pmatrix} 0_{2\times2} & 1_{2\times2} \\ -1_{2\times2} & 0_{2\times2} \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} $$

The flavor indices are denoted with $F, F' = 1, \ldots, 4$ and $f, f' = 1, 2$, the spinor indices as $\alpha, \beta = 1, 2$ and the color indices as $c, c' = 1, 2$ and $k, k' = 1, \ldots, 3$. Also the notation is such that $U_L^a U_R^{a'} \epsilon_{\alpha\beta} = -\overline{U}_R U_L$ and $\lambda^1, \lambda^2 \epsilon_{\alpha\beta} = \lambda^1 \lambda^2$. Under the $U(1)$ symmetry $Q$ and $\lambda$ transform as

$$ Q \rightarrow e^{-i\alpha} Q, \quad \text{and} \quad \lambda \rightarrow e^{-i\alpha} \lambda $$

$$ (15) $$

$$ (16) $$

$$ (17) $$

$$ (18) $$

$$ (19) $$

$$ (20) $$
and the two condensates are simultaneously invariant if
\[ \alpha = 2k\pi , \quad \text{with } k \text{ an integer .} \] (21)

Only the $\lambda$ fields will transform nontrivially under the remaining $Z_2$, i.e. $\lambda \rightarrow -\lambda$.

**A. Low Energy Spectrum**

The relevant degrees of freedom are efficiently collected in two distinct matrices, $M_4$ and $M_2$, which transform as $M_4 \rightarrow g_4 M_4 g_4^T$ and $M_2 \rightarrow g_2 M_2 g_2^T$ with $g_4 \in SU(4)$ and $g_2 \in SU(2)$. Both $M_4$ and $M_2$ consist of a composite iso-scalar and its pseudoscalar partner together with the Goldstone bosons and their scalar partners:

\[ M_4 = \left[ \frac{\sigma_4 + i\Theta_4}{2} + \sqrt{2} \left( i\Pi^i_4 + \bar{\Pi}^i_4 \right) X^i_4 \right] E_4 , \quad i = 1, \ldots, 5 , \] (22)

\[ M_2 = \left[ \frac{\sigma_2 + i\Theta_2}{\sqrt{2}} + \sqrt{2} \left( i\Pi^i_2 + \bar{\Pi}^i_2 \right) X^i_2 \right] E_2 , \quad i = 1, 2 . \] (23)

The notation is such that $X^i_4$ and $X^i_2$ are the broken generators of $SU(4)$ and $SU(2)$ respectively. An explicit realization can be found in Appendix A. Also $\sigma_4$ and $\Theta_4$ are the composite Higgs and its pseudoscalar partner while $\Pi^i_4$ and $\bar{\Pi}^i_4$ are the Goldstone bosons and their associated scalar partners. For SU(2) one simply substitutes the index 4 with the index 2. With the above normalization of the $M$ matrices the kinetic term of each component field is canonically normalized. Under an infinitesimal global symmetry transformation we have:

\[ \delta M = i\alpha^a \left( T^a M + M T^a T \right) . \] (24)

Here $T$ is the full set of generators of the unbroken group (either $SU(4)$ or $SU(2)$). With the $\Theta$ and $\bar{\Pi}^i$ states included the matrices are actually form invariant under $U(4)$ and $U(2)$ with the abelian parts being broken by anomalies. We construct our Lagrangian by considering only the terms preserving the anomaly free $U(1)$ symmetry. As we will see this implies that $\Theta_4$ and $\Theta_2$ are not mass eigensates. In the diagonal basis we will find one massless and one massive state. The massless state corresponds to the $U(1)$ Goldstone boson.

The relation between the composite scalars and the underlying degrees of freedom can be found by first noting that $M_4$ and $M_2$ transform as:

\[ M_4^{F,F'} \sim Q^F Q^{F'} , \quad M_2^{I,I'} \sim \lambda^I \lambda^{I'} \] (25)
where both color and spin indices have been contracted. It then follows that the composite states transform as:

\[
\begin{align*}
\nu_4 + H_4 & \equiv \sigma_4 \sim \overline{UU} + \overline{DD} \, , \\
\Pi^0 & \equiv \Pi^3 \sim i(\overline{U} \gamma^5 U - \overline{D} \gamma^5 D) \, , \\
\Pi^+ & \equiv \frac{\Pi^1 - i\Pi^2}{\sqrt{2}} \sim i\overline{D} \gamma^5 U \, , \\
\Pi^- & \equiv \frac{\Pi^1 + i\Pi^2}{\sqrt{2}} \sim i\overline{U} \gamma^5 D \, , \\
\Pi_{UD} & \equiv \frac{\Pi^4 + i\Pi^5}{\sqrt{2}} \sim U^T C D \, , \\
\Pi_{UD^T} & \equiv \frac{\Pi^4 - i\Pi^5}{\sqrt{2}} \sim U C \overline{D}^T \, ,
\end{align*}
\]

and

\[
\begin{align*}
\nu_2 + H_2 & \equiv \sigma_2 \sim \overline{\lambda}_D \lambda_D \, , \\
\Pi_{\lambda\lambda} & \equiv \frac{\Pi^6 - i\Pi^7}{\sqrt{2}} \sim \overline{\lambda}_D^T C \lambda_D \, , \\
\Pi_{\chi\chi} & \equiv \frac{\Pi^6 + i\Pi^7}{\sqrt{2}} \sim \overline{\lambda}_D C \overline{\lambda}_D^T \, .
\end{align*}
\]

Here \( U = (U_L, U_R)^T \), \( D = (D_L, D_R)^T \) and \( \lambda_D = (\lambda^1, -i\sigma^2 \lambda^{2t})^T \). Another set of states are the composite fermions

\[
\Lambda^a = \lambda^{a,f} \sigma^\mu A_{\mu}^a \, , \quad f = 1, 2 \, , \quad a = 1, 2, 3 \, .
\]

To describe the interaction with the weak gauge bosons we embed the electroweak gauge group in \( SU(4) \) as done in [30]. First we note that the following generators

\[
L^a = \frac{S^a_4 + X^a_4}{\sqrt{2}} = \begin{pmatrix} \frac{z^a}{2} & 0 \\ 0 & 0 \end{pmatrix} \, , \quad R^a = \frac{X^a_4 - S^a_4}{\sqrt{2}} = \begin{pmatrix} 0 & \frac{r^a}{2} \\ \frac{r^a}{2} & 0 \end{pmatrix}
\]

with \( a = 1, 2, 3 \) span an \( SU(2)_L \times SU(2)_R \) subalgebra. By gauging \( SU(2)_L \) and the third generator of \( SU(2)_R \) we obtain the electroweak gauge group where the hypercharge is \( Y = -R^3 \). Then as \( SU(4) \) breaks to \( Sp(4) \) the electroweak gauge group breaks to the electromagnetic one with the electric charge given by \( Q = \sqrt{2}S^3 \).

Due to the choice of the electroweak embedding the weak interactions explicitly reduce the \( SU(4) \) symmetry to \( SU(2)_L \times U(1)_Y \times U(1)_{TB} \) which is further broken to \( U(1)_{em} \times U(1)_{TB} \) via the technicolor interactions. \( U(1)_{TB} \) is the technibaryon number and its generator corresponds to the \( S^3_4 \) diagonal generator (see appendix [A]). The remaining \( SU(2) \times U(1) \) spontaneously break, only via the (techni)fermion condensates, to \( SO(2) \times Z_2 \). We prefer to indicate \( SO(2) \) with \( U(1)_{TB} \). We summarize some of the relevant low-energy technihadronic states according to the final unbroken symmetries in Table [I]. We have arranged
TABLE I: Summary table of the relevant low-energy technihadronic states for UMT. We display their $SU(2)_L$ weak interaction charges together with their electromagnetic ones. We also show the remaining global symmetries.

|                  | $SU(2)_L$ | $U(1)_{em}$ | $U(1)_{TB}$ | $U(1)_{T\lambda}$ | $Z_2$ |
|------------------|-----------|-------------|-------------|------------------|-------|
| $H_4, \Theta_4$ | 1         | 0           | 0           | 0                | 0     |
| $\bar{\Pi}, \bar{\Pi}$ | 3         | $+1, 0, -1$ | 0           | 0                | 0     |
| $\Pi_{UD}, \bar{\Pi}_{UD}$ | 1         | 0           | $\frac{1}{\sqrt{2}}$ | 0 | 0     |
| $\Pi_{UD}, \bar{\Pi}_{UD}$ | 1         | 0           | $-\frac{1}{\sqrt{2}}$ | 0 | 0     |
| $H_2, \Theta_2$ | 1         | 0           | 0           | 0                | 0     |
| $\Pi_{\lambda\lambda}, \bar{\Pi}_{\lambda\lambda}$ | 1         | 0           | 0           | 1                | 0     |
| $\Pi_{\lambda\lambda}, \bar{\Pi}_{\lambda\lambda}$ | 1         | 0           | 0           | $-1$             | 0     |
| $\Lambda_D$     | 1         | 0           | 0           | $\frac{1}{2}$    | $-1$  |

the composite fermions into a Dirac fermion

$$\Lambda_D = \begin{pmatrix} \Lambda^1 \\ -i\sigma^2\Lambda^{2*} \end{pmatrix}.$$ (30)

Except for the triplet of Goldstone bosons charged under the electroweak symmetry the rest of the states are electroweak neutral. In the unitary gauge the $\bar{\Pi}$ states become the longitudinal components of the massive electroweak gauge bosons. $\Pi_{UD}$ ($\bar{\Pi}_{UD}$) is a pseudoscalar(scalar) diquark charged under the technibaryon number $U(1)_{TB}$ while $\Pi_{\lambda\lambda}$ ($\bar{\Pi}_{\lambda\lambda}$) is charged under the $U(1)_{T\lambda}$. $\Lambda_D$ is the composite fermionic state charged under both $U(1)_{T\lambda}$ and $Z_2$.

The technibaryon number $U(1)_{TB}$ is anomalous due to the presence of the weak interactions:

$$\partial_\mu J^\mu_{TB} = \frac{1}{2\sqrt{2}} \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} W^\mu W^\nu W^\rho W^\sigma,$$

and

$$J^\mu_{TB} = \frac{1}{2\sqrt{2}} (U \gamma^\mu U + D \gamma^\mu D).$$ (31)
B. Linear Lagrangian

With the above discussion of the electroweak embedding the covariant derivative for $M_4$ is:

$$D_\mu M_4 = \partial_\mu M_4 - i [G_\mu M_4 + M_4 G_\mu^T] , \quad G_\mu = \begin{pmatrix} g W_\mu \tau^a & 0 \\ 0 & -g' B_\mu \tau^3 \end{pmatrix}.$$  \hfill \text{(32)}

We are now in a position to write down the effective Lagrangian. It contains the kinetic terms and a potential term:

$$\mathcal{L} = \frac{1}{2} \text{Tr} \left[ D_\mu M_4 D^\mu M_4^\dagger \right] + \frac{1}{2} \text{Tr} \left[ \partial_\mu M_2 \partial^\mu M_2^\dagger \right] - \mathcal{V}(M_4, M_2)$$ \hfill \text{(33)}

where the potential is:

$$\mathcal{V}(M_4, M_2) = -\frac{m_4^2}{2} \text{Tr} \left[ M_4 M_4^\dagger \right] + \frac{\lambda_4}{4} \text{Tr} \left[ M_4 M_4^\dagger \right]^2 + \lambda'_4 \text{Tr} \left[ M_4 M_4^\dagger M_4^\dagger M_4 \right]$$

$$-\frac{m_2^2}{2} \text{Tr} \left[ M_2 M_2^\dagger \right] + \frac{\lambda_2}{4} \text{Tr} \left[ M_2 M_2^\dagger \right]^2 + \lambda'_2 \text{Tr} \left[ M_2 M_2^\dagger M_2 M_2^\dagger \right]$$

$$+ \frac{\delta}{2} \text{Tr} \left[ M_4 M_4^\dagger \right] \text{Tr} \left[ M_2 M_2^\dagger \right] + 4\delta' \left[ (\text{det} M_2)^2 \right. \text{Pf} \, M_4 + \text{h.c.} \right] \hfill \text{(34)}$$

$$\mathcal{V}(M_4, M_2) = -\frac{m_4^2}{2} \text{Tr} \left[ M_4 M_4^\dagger \right] + \frac{\lambda_4}{4} \text{Tr} \left[ M_4 M_4^\dagger \right]^2 + \lambda'_4 \text{Tr} \left[ M_4 M_4^\dagger M_4^\dagger M_4 \right]$$

$$-\frac{m_2^2}{2} \text{Tr} \left[ M_2 M_2^\dagger \right] + \frac{\lambda_2}{4} \text{Tr} \left[ M_2 M_2^\dagger \right]^2 + \lambda'_2 \text{Tr} \left[ M_2 M_2^\dagger M_2 M_2^\dagger \right]$$

$$+ \frac{\delta}{2} \text{Tr} \left[ M_4 M_4^\dagger \right] \text{Tr} \left[ M_2 M_2^\dagger \right] + 4\delta' \left[ (\text{det} M_2)^2 \right. \text{Pf} \, M_4 + \text{h.c.} \right] \hfill \text{(35)}$$

Once $M_4$ develops a vacuum expectation value the electroweak symmetry breaks and three of the eight Goldstone bosons - $\Pi^0$, $\Pi^+$ and $\Pi^-$ - will be eaten by the massive gauge bosons. In terms of the parameters of the theory the vacuum states $\langle \sigma_4 \rangle = v_4$ and $\langle \sigma_2 \rangle = v_2$ which minimize the potential are a solution of the two coupled equations

$$0 = -m_4^2 - \left( \delta + \delta' v_2^2 \right) v_2^2 + \left( \lambda_4 + \lambda'_4 \right) v_4^2 ,$$

$$0 = -m_2^2 - \left( \delta + 2\delta' v_2^2 \right) v_4^2 + \left( \lambda_2 + 2\lambda'_2 \right) v_2^2 .$$  \hfill \text{(37, 38)}

Expanding around the symmetry breaking vacua all of the Goldstone bosons scalar partners are seen to be mass eigenstates with masses

$$M_{\Pi^0}^2 = M_{\Pi^\pm}^2 = M_{\Pi_{UD}}^2 = 2 \left( \lambda_4 v_4^2 + \delta' v_2^4 \right) , \quad M_{\Pi_{4\lambda}}^2 = 4v_2^2 \left( \lambda'_2 + \delta' v_4^2 \right) ,$$

while the Goldstone bosons which are not eaten by the massive gauge bosons of course have vanishing mass $M_{\Pi_{UD}}^2 = M_{\Pi_{4\lambda}}^2 = 0$. Here the vacuum expectation values $v_4$ and $v_2$ are solutions to Eq. \text{(37)}. Due to the presence of the determinant/Pfaffian term in the potential the remaining states are not mass eigenstates. Specifically $H_4$ and $H_2$ and their associated
pseudoscalar partners will mix. In the diagonal basis we find the following mass eigenstates:

\[ \Theta \equiv \sin(\alpha) \Theta_4 + \cos(\alpha) \Theta_2 , \quad M_{\Theta}^2 = 0 , \]
\[ \tilde{\Theta} \equiv \cos(\alpha) \Theta_4 - \sin(\alpha) \Theta_2 , \quad M_{\tilde{\Theta}}^2 = 2\delta' v_2^2 (v_2^2 + 4v_4^2) , \]
\[ H_- \equiv \sin(\beta) H_4 + \cos(\beta) H_2 , \quad M_{H_-}^2 = m_2^2 + m_4^2 + k_-, \]
\[ H_+ \equiv \cos(\beta) H_4 - \sin(\beta) H_2 , \quad M_{H_+}^2 = m_2^2 + m_4^2 + k_+ , \]

with

\[ \tan(2\alpha) = \frac{4v_4 v_2}{v_2^2 - 4v_4^2} , \quad \tan(2\beta) = \frac{2v_2 v_4 (\delta + \delta' v_2^2)}{m_2^2 - m_4^2 + \delta v_4^2 - (\delta + \delta' v_2^2) v_2^2} , \]

\[ k_\pm = (\delta + \delta' v_2^2) v_2^2 + \delta v_4^2 \pm \left[(m_4^2 - m_2^2 + (\delta + \delta' v_2^2) v_2^2 - \delta v_4^2)^2 + (2v_2 v_4 (\delta + \delta' v_2^2))^2 \right]^{\frac{1}{2}} \]

Note that we have one massless state \( \Theta \) which we identify with the original \( U(1) \) Goldstone boson while \( \tilde{\Theta} \) is massive. In the limit \( \delta' \to 0 \) both states are massless and at the classical level the global symmetry is enhanced to \( U(4) \times U(2) \).

For the model to be phenomenologically viable some of the Goldstones must acquire a mass. This is typically addressed by extending the technicolor interactions (ETC). A review of the major models is given by Hill and Simmons [51]. At the moment there is not yet a consensus on which ETC is the best. Here we parameterize the ETC interactions by adding at the effective Lagrangian level the operators needed to give the dangerous Goldstone bosons an explicit mass term.

The effective ETC Lagrangian breaks the global \( SU(4) \times SU(2) \times U(1) \) symmetry. The \( SU(4) \) generator commuting with the \( SU(2)_L \times SU(2)_R \) generators is \( B_4 = 2\sqrt{2}S_4^4 \). To construct, at the effective Lagrangian level, the interesting ETC terms we find it useful to split \( M_4 (M_2) \) – form invariant under \( U(4) (U(2)) \) – as follows:

\[ M_4 = \tilde{M}_4 + iP_4 , \quad \text{and} \quad M_2 = \tilde{M}_2 + iP_2 , \]

with

\[ \tilde{M}_4 = \left[ \frac{\sigma_4}{2} + i\sqrt{2}P_4^iX_4^i \right] E_4 , \quad P_4 = \left[ \frac{\Theta_4}{2} - i\sqrt{2}P_4^iX_4^i \right] E_4 , \quad i = 1, \ldots, 5 , \]
\[ \tilde{M}_2 = \left[ \frac{\sigma_2}{\sqrt{2}} + i\sqrt{2}P_2^iX_2^i \right] E_2 , \quad P_2 = \left[ \frac{\Theta_2}{\sqrt{2}} - i\sqrt{2}P_2^iX_2^i \right] E_2 , \quad i = 1, 2 . \]
$\tilde{M}_4$ ($\tilde{M}_2$) as well as $P_4$ ($P_2$) are separately $SU(4)$ ($SU(2)$) form invariant. A set of operators able to give masses to the electroweak neutral Goldstone bosons is:

$$L_{ETC} = \frac{m^2_{ETC}}{4} \text{Tr} \left[ \tilde{M}_4 B_4 \tilde{M}_4^\dagger B_4 + \tilde{M}_4 \tilde{M}_4^\dagger \right] + \frac{m^2_{ETC}}{4} \text{Tr} \left[ \tilde{M}_2 B_2 \tilde{M}_2^\dagger B_2 + \tilde{M}_2 \tilde{M}_2^\dagger \right] - m^2_{ETC} \left[ \text{Pf} P_4 + \text{Pf} P_4^\dagger \right] - \frac{m^2_{ETC}}{2} \left[ \det(P_2) + \det(P_2^\dagger) \right], \quad (46)$$

where $B_2 = 2S_2^1$. The spectrum is:

$$M^2_{\Pi_{UD}} = m^2_{4,ETC}, \quad M^2_{\Pi_{\lambda\lambda}} = m^2_{2,ETC}, \quad M^2_{\Theta} = m^2_{1,ETC}, \quad (47)$$

for the Goldstone bosons that are not eaten by the massive vector bosons and:

$$M^2_{\Pi_{UD}} = M^2_{\Pi_0} = M^2_{\Pi^\pm} = 2 \left( \lambda^2 \psi_4^2 + \delta^2 \psi_2^4 \right) + m^2_{1,ETC}, \quad (48)$$
$$M^2_{\Pi_{\lambda\lambda}} = 4\lambda^2 \left( \lambda^2 \psi_4^2 + \delta^2 \psi_2^4 \right) + m^2_{1,ETC}, \quad (49)$$
$$M^2_{\Theta} = 2\delta \psi_2^2 \left( \psi_2^2 + 4\psi_4^2 \right) + m^2_{1,ETC}, \quad (50)$$

for the pseudoscalar and scalar partners. The masses of the two Higgs particles $H_+$ and $H_-$ are unaffected by the addition of the ETC low energy operators.

**C. Non-Linear Lagrangian**

In constructing the non-linear effective theory of the associated Goldstone bosons we shall consider the elements of the global symmetry $G$ as $6 \times 6$ matrices. The generators of $SU(4)$ sit in the upper left corner while the generators of $SU(2)$ sit in the lower right corner. The generator of $U(1)$ is diagonal. We divide the nineteen generators of $G$ into the eleven that leave the vacuum invariant $S$ and the eight that do not $X$. An explicit realization of $S$ and $X$ can be found in Appendix A.

An element of the coset space $G/H$ is parameterized by

$$\mathcal{V}(\xi) = \exp \left( i\xi^i \psi^i \right) E, \quad (51)$$

where

$$E = \begin{pmatrix} E_4 \\ E_2 \end{pmatrix}, \quad \xi^i \psi^i = \sum_{i=1}^5 \frac{\Pi^i}{F_\pi} X^i \psi^i + \sum_{i=6}^7 \frac{\Pi^i}{F_\pi} X^i \psi^i + \frac{\Pi^8}{F_\pi} X^8 \psi^i. \quad (52)$$
The Goldstone bosons are denoted as $\Pi^i$, $i = 1, \ldots, 8$ and $F_\pi$, $\tilde{F}_\pi$ and $\hat{F}_\pi$ are the related Goldstone boson decay constants. Since the entire global symmetry $G$ is expected to break approximately at the same scale we also expect the three decay constants to have close values. The element $V$ of the coset space transforms non-linearly

$$V(\xi) \to gV(\xi)h^\dagger(\xi, g)$$

where $g$ is an element of $G$ and $h$ is an element of $H$. To describe the Goldstone bosons interaction with the weak gauge bosons we embed the electroweak gauge group in $SU(4)$ as done above and also in [30]. With the embedding of the electroweak gauge group in hand it is appropriate to introduce the hermitian, algebra valued, Maurer-Cartan one-form

$$\omega_\mu = iV^\dagger D_\mu V$$

where the electroweak covariant derivative is

$$D_\mu V = \partial_\mu V - iG_\mu V, \quad G_\mu = \begin{pmatrix} gW_\mu^a \tau^a_2 & 0 \\ 0 & -g'B_\mu \tau^3_2 \end{pmatrix}.$$ 

From the above transformation properties of $V$ it is clear that $\omega_\mu$ transforms as

$$\omega_\mu \to h(\xi, g)\omega_\mu h^\dagger(\xi, g) + h(\xi, g)\partial_\mu h^\dagger(\xi, g).$$

With $\omega_\mu$ taking values in the algebra of $G$ we can decompose it into a part $\omega^\parallel_\mu$ parallel to $H$ and a part $\omega^\perp_\mu$ orthogonal to $H$

$$\omega_\mu = 2S^a \text{Tr} [S^a \omega_\mu], \quad \omega^\perp_\mu = 2X^i \text{Tr} [X^i \omega_\mu].$$

It is clear that $\omega^\parallel_\mu$ ($\omega^\perp_\mu$) is an element of the algebra of $H$ ($G/H$) since it is a linear combination of $S^a$ ($X^i$). They have the following transformation properties

$$\omega^\parallel_\mu \to h(\xi, g)\omega^\parallel_\mu h^\dagger(\xi, g) + h(\xi, g)\partial_\mu h^\dagger(\xi, g), \quad \omega^\perp_\mu \to h(\xi, g)\omega^\perp_\mu h^\dagger(\xi, g).$$

We are now in a position to construct the non-linear Lagrangian. We shall only consider terms containing at most two derivatives. By noting that the generator $X^8$ corresponding to the broken $U(1)$ is not traceless we can also write a double-trace term besides the standard one-trace term:

$$\mathcal{L} = \text{Tr} \left[a\omega^\perp_\mu \omega^\perp_\mu\right] + b\text{Tr} \left[\omega^\perp_\mu\right] \text{Tr} \left[\omega^\perp_\mu\right],$$
The coefficients \( a = \text{diag} \left( F_{2\pi}^2, F_{2\pi}^2, F_{2\pi}^2, F_{2\pi}^2, \tilde{F}_{2\pi}^2, \tilde{F}_{2\pi}^2 \right) \) and \( b = \frac{F_{2\pi}^2}{2} - \frac{4F_{2\pi}^2}{9} - \frac{\tilde{F}_{2\pi}^2}{18} \) are chosen such that the kinetic term is canonically normalized:

\[
\mathcal{L} = \frac{1}{2} \sum_{i=1}^{8} \partial_{\mu} \Pi^{i} \partial^{\mu} \Pi^{i} + \ldots .
\]  

(60)

We conclude this section by connecting the linear and non-linear theories

\[
F_{2\pi}^2 = \frac{v_1^2}{2}, \quad \tilde{F}_{2\pi}^2 = v_2^2, \quad \hat{F}_{2\pi}^2 = \frac{1}{9} (4v_1^2 + v_2^2) .
\]  

(61)

IV. THE TIMP

Technicolor models are capable of providing interesting DM candidates. This is so since the new strong interactions confine techniquarks in technimeson and technibaryon bound states. The spin of the technibaryons depends on the representation according to which the technifermions transform, as well as the number of flavors and colors. The lightest technimeson is short-lived, thus evading BBN constraints \([52]\), while the lightest technibaryon can be stable and may posses a dynamical mass of the order

\[
m_{TB} \sim 1 - 2 \text{ TeV} .
\]  

(62)

If the lightest technibaryon is only weakly interacting and electrically neutral it can be a DM candidate as first suggest by Nussinov \([3]\). This proposal has been further analyzed in \([4, 5]\). One of the interesting properties of this kind of DM candidate is that it is possible to understand the observed ratio of the dark to luminous mass of the universe. This occurs when the technibaryon relic density is caused by a technibaryon number (TB) asymmetry \([3, 4, 5]\) like for the ordinary baryon (B). If the latter is due to a net Baryon - Lepton \((B - L)\) asymmetry generated at some high energy scale, this would subsequently be distributed among all electroweak doublets via SM fermion-number violating processes at temperatures above the electroweak scale \([53, 54, 55]\), thus generating a technibaryon asymmetry as well.

To avoid experimental constraints the technibaryon should be a complete singlet under the electroweak interactions \([4, 12]\). These kinds of particles are Technicolor Interacting Massive Particles (TIMP)s which are hard to detect \([5, 20, 56]\) in current earth-based experiments such as CDMS \([35]\). Other possibilities have been envisioned in \([19, 57]\) and astrophysical
effects investigated in [58]. One can alternatively obtain DM from possible technicolor-related new sectors [59]. In [23] the reader will find an up-to-date summary of the recent efforts in this direction.

Our extension of the SM naturally provides a novel type of TIMP, i.e. a di-techniquark, with the following unique features:

- It is a quasi-Goldstone of the underlying gauge theory receiving a mass term only from interactions not present in the technicolor theory per se.

- The lightest technibaryon is a singlet with respect to weak interactions.

- Its relic density can be related to the SM lepton number over the baryon number if the asymmetry is produced above the electroweak phase transition.

In appendix B we provide a much detailed model computation of the ratio $TB/B$ making use of the chemical equilibrium conditions and the sphaleron processes active around the electroweak phase transition.

In the approximation where also the top quark is considered massless around the electroweak phase transition (we have also checked that the effects of the top mass do not change our results) the $TB/B$ is independent of the order of the electroweak phase transition and reads

$$-\frac{\sqrt{2} \cdot TB}{B} = \frac{\sigma}{2} (3 + \xi) ,$$

where $\sigma \equiv \sigma_U = \sigma_D$ is the statistical function for the techniquarks. The $U$ and $D$ constituent-type masses are assumed to be dynamically generated and equal. $\xi = L/B$ is the SM lepton over the baryon number.

If DM is identified with the lightest technibaryon in our model the ratio of the dark to baryon mass of the universe is

$$\frac{\Omega_{TB}}{\Omega_B} = \frac{m_{TB}}{m_p} \frac{\tilde{B}}{B} ,$$

with $m_{TB}$ the technibaryon mass and $\tilde{B} = -\sqrt{2}TB$ the technibaryon number normalized in such a way that it is minus one for the lightest state.

The bulk of the mass of the lightest technibaryon is not due to the technicolor interactions as it was in the original proposal [3, 4]. This is similar to the case studied in [5]. The interactions providing mass to the techibaryon are the SM interactions per se and ETC.
The main effect of these interactions will be in the strength and the order of the electroweak phase transition as shown in [21].

In Fig. 1 we show the contour plot diagram in the $\xi - m_{TB}$ plane representing different values assumed by the ratio $\Omega_{TB}/\Omega_B$. Within our approximations the regions depend on the ratio $T_*/m_{TQ}$, where $T_*$ is the temperature below which the processes violating the baryon, technibaryon and lepton numbers cease to be relevant and $m_{TQ}$ is the dynamical mass of the techniquarks. The plots correspond to eight distinct values of this ratio. The two regions having $4 \leq \Omega_{TB}/\Omega_B \leq 6$ are in dark gray. In between these two regions the ratio diminishes while in the upper and lower part the ratio increases.

What is interesting is that, differently from the case in which the technibaryon acquires mass only due to technicolor interactions, one achieves the desired phenomenological ratio of DM to baryon matter with a light technibaryon mass with respect to the weak interaction
FIG. 1: Contour plot diagram in the $\xi$ and $m_{TB}$ parameter space representing different values assumed by the ratio $\Omega_{TB}/\Omega_B$. Within our approximations the regions depend on the ratio $T_*/m_{TQ}$. $T_*$ is the temperature below which the processes violating the baryon, technibaryon and lepton numbers cease to be relevant and $m_{TQ}$ the dynamical mass of the techniquarks. The plots correspond to eight distinct values of this ratio.

scale. In fact the mass can be even lower than 100 GeV. This DM candidate can be produced at the Large Hadron Collider experiment.

To provide a simple estimate for the TIMP-nucleus cross section useful for the CDMS searches we adopt the model computations provided in [56]. We note first that the TIMP does not interact directly with the SM. The dominant scalar TIMP - nucleus cross section is suppressed by at least four powers of the technicolor dynamical scale.
FIG. 2: The expected number of counts in a Germanium detector for an effective exposure of 121.3 (kg day) and recoil energies in the range 5 – 100 keV. The dashed curve corresponds to $\Lambda_{TC} = 2$ TeV while the solid curve corresponds to $\Lambda_{TC} = 3$ TeV.

Following [56] the total number of counts $R$ per unit detector mass $m$ and nuclear recoil kinetic energy $E_R$ in the lab frame is

$$\frac{dR}{dm \, dE_R} \simeq 1.38 \times 10^{-4} |F_c(E_R)|^2 \Lambda_{\text{TeV}}^{-4} \, M_{\text{TeV}}^{-1} \, \rho_{0.3} \, V_{220}^{-1} \, (\text{kg keV day})^{-1}$$

where $F_c(E_R)$ is the scalar nuclear form factor which takes into account the finite size effects. In the expression above $\Lambda_{\text{TeV}} = \Lambda_{TC}/\text{TeV}$, $M_{\text{TeV}} = m_{TB}/\text{TeV}$, $\rho_{0.3} = \rho/(0.3 \, \text{GeV cm}^{-3})$, $V_{220} = \frac{V_0}{(220 \, \text{km s}^{-1})}$ with $\rho$ and $V_0$ being the technibaryon density and a suitably weighted average velocity respectively.

To compare our predictions with the CDMS results we plot in Fig. 2 the total number of expected counts for an effective exposure of 121.3 (kg day) and recoil energies in the range 5 – 100 keV. The dashed curve corresponds to $\Lambda_{TC} = 2$ TeV while the solid curve corresponds to $\Lambda_{TC} = 3$ TeV.

Our TIMP is a template for a more general class of models according to which the lightest one is neutral under the SM interactions. Models belonging to this class are, for example, partially gauged technicolor.
V. SUMMARY AND OUTLOOK

We proposed a technicolor model with technifermion matter transforming according to two distinct representations of the underlying technicolor gauge group. The model features simultaneously the smallest possible value of the naive $S$ parameter and the smallest possible number of technifermions. The chiral dynamics is intriguing and very rich. After having classified the relevant low energy composite spectrum we have constructed the associated effective Lagrangians. We introduced both the linear and non-linearly realized one. The linearly realized one will permit us to study immediately the thermal properties of the chiral phase transition relevant for electroweak baryogenesis as done for the case of Minimal Walking Technicolor \cite{21}. Due to the interplay between multiple nearby phase transitions \cite{60, 61} we expect novel phenomena of direct interest for cosmological applications. The linearly realized Lagrangian, once extended to contain also the spin one composite spectrum, will be of immediate interest for LHC phenomenology. The construction of the non-linear Lagrangian is interesting, instead, since it is exact in the limit of small momenta, at least untill the first resonance is encountered. It will also allow to neatly incorporate the non-abelian anomalies and the associated topological terms \cite{62, 63, 64, 65, 66, 67} as well as the study of the its solitonic excitations.

We have embedded, in a natural way, the SM interactions within the global symmetries of the underlying gauge theory. Several low-energy composite particles were found to be SM singlets. At least one of these TIMPs has been recognized as a promising cold DM candidate. The novel TIMP can be sufficiently light, with respect to the technicolor dynamical scale, to be directly produced at the LHC and simultaneously constrained by the CDMS experiment.

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**APPENDIX A: SU(4) x SU(2) x U(1) GENERATORS**

Here we construct the explicit realization of the generators of $SU(4) \times SU(2) \times U(1)$. We denote the fifteen generators of $SU(4)$ by $S^a_4$ and $X^i_4$ with $a = 1, \ldots, 10$ and $i = 1, \ldots, 5$. They can be represented as:

\[
S^a_4 = \begin{pmatrix} A & B \\ B^\dagger & -A^T \end{pmatrix}, \quad X^i_4 = \begin{pmatrix} C & D \\ D^\dagger & C^T \end{pmatrix},
\]

where $A$ is Hermitian, $C$ is Hermitian and traceless, $B$ is symmetric and $D$ is antisymmetric. The $S^a_4$ obey the relation $(S^a_4)^T E + ES^a_4 = 0$ and are a representation of $Sp(4)$. They are explicitly given by:

\[
S^a_4 = \frac{1}{2\sqrt{2}} \begin{pmatrix} \tau^a & 0 \\ 0 & -\tau^a \end{pmatrix}, \quad a = 1, \ldots, 4 \quad (A2)
\]

\[
S^a_4 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & B^a \\ B^a\dagger & 0 \end{pmatrix}, \quad a = 5, \ldots, 10 \quad (A3)
\]

where $\tau^{1,2,3}$ are the usual Pauli matrices, $\tau^4 = 1$ and:

\[
B^5 = 1, \quad B^7 = \tau^3, \quad B^9 = \tau^1, \\
B^6 = i1, \quad B^8 = i\tau^3, \quad B^{10} = i\tau^1. \quad (A4)
\]

The remaining five generators are explicitly given by:

\[
X^i_4 = \frac{1}{2\sqrt{2}} \begin{pmatrix} \tau^i & 0 \\ 0 & \tau^i \end{pmatrix}, \quad i = 1, \ldots, 3 \quad (A5)
\]

\[
X^i_4 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & D^i \\ D^i\dagger & 0 \end{pmatrix}, \quad i = 4, 5 \quad (A6)
\]

with:

\[
D^4 = \tau^2, \quad D^5 = i\tau^2. \quad (A7)
\]

The generators are normalized according to:

\[
\text{Tr} [S^a_4 S^b_4] = \frac{1}{2} \delta^{ab}, \quad \text{Tr} [X^i_4 X^j_4] = \frac{1}{2} \delta^{ij}, \quad \text{Tr} [S^a_4 X^i_4] = 0. \quad (A8)
\]
The generators of $SU(2)$ are similarly divided into the two that are broken $X^i_2 = \frac{\tau^i}{2}$, $i = 1, 2$ and the one that leaves the vacuum invariant $S^i_2 = \frac{\tau^i}{2}$.

For convenience we shall consider the nineteen generators of $SU(4) \times SU(2) \times U(1)$ as $6 \times 6$ block diagonal matrices. They are denoted by $S^a$, $a = 1, \ldots, 11$ and $X^i$, $i = 1, \ldots, 8$. The eleven generators $S^a$ are a representation of the subgroup $Sp(4) \times SO(2)$ and are given by

\[
S^a = \begin{pmatrix} S^a_4 \\ 0_{2 \times 2} \end{pmatrix}, \quad a = 1, \ldots, 10 \tag{A9}
\]

\[
S^{11} = \begin{pmatrix} 0_{4 \times 4} \\ S^1_2 \end{pmatrix} \tag{A10}
\]

while the remaining eight generators are given explicitly by

\[
X^i = \begin{pmatrix} X^i_4 \\ 0_{2 \times 2} \end{pmatrix}, \quad i = 1, \ldots, 5 \tag{A11}
\]

\[
X^i = \begin{pmatrix} 0_{4 \times 4} \\ X^i_{i-5} \end{pmatrix}, \quad i = 6, 7 \tag{A12}
\]

\[
X^8 = \frac{1}{3} \text{diag}(-1, -1, -1, -1, \frac{1}{2}, \frac{1}{2}) \tag{A13}
\]

They are normalized according to:

\[
\text{Tr} \left[ S^a S^b \right] = \frac{1}{2} \delta^{ab}, \quad \text{Tr} \left[ X^i X^j \right] = \frac{1}{2} \delta^{ij}, \quad \text{Tr} \left[ S^a X^i \right] = 0. \tag{A14}
\]
APPENDIX B: DARK MATTER COMPUTATIONS

We follow the notation and analysis of [68] and [5], and denote the chemical potentials of the SM particles by

\[ \mu_{W} \text{ for } W^{-} , \quad \mu_{dL} \text{ for } d_{L}, s_{L}, b_{L} , \]
\[ \mu_{0} \text{ for } \phi^{0} , \quad \mu_{dR} \text{ for } d_{R}, s_{R}, b_{R} , \]
\[ \mu_{-} \text{ for } \phi^{-} , \quad \mu_{iL} \text{ for } e_{L}, \mu_{L}, \tau_{L} , \]
\[ \mu_{UL} \text{ for } u_{L}, c_{L}, t_{L} , \quad \mu_{iR} \text{ for } e_{R}, \mu_{R}, \tau_{R} , \]
\[ \mu_{UL} \text{ for } u_{L}, c_{L}, t_{L} , \quad \mu_{iR} \text{ for } e_{R}, \mu_{R}, \tau_{R} , \]
\[ \mu_{ul} \text{ for } v_{eL}, v_{\mu L}, v_{\tau L} , \]

while the chemical potentials of the new particles are denoted by

\[ \mu_{UL} \text{ for } U_{L} , \quad \mu_{DL} \text{ for } D_{L} , \]
\[ \mu_{UR} \text{ for } U_{R} , \quad \mu_{DR} \text{ for } D_{R} , \]

The two components of the SM-type Higgs doublet are denoted as \( \phi^{-} \) and \( \phi^{0} \). These translate in our notation to \( \phi^{-} = \Pi^{-} \) and \( \phi^{0} = \sigma_{4} - i\Pi^{0} \). We have assigned the same chemical potential for the SM triplet \( u, c, t \) and \( d, s, b \) respectively and minimally coupled the composite Higgs to the SM fermions assuming, for the Yukawa sector, the existence of a working ETC dynamics.

Thermal equilibrium in the electroweak interactions implies the following relations among the chemical potentials of the SM particles

\[ \mu_{W} = \mu_{-} + \mu_{0} , \quad W^{-} \leftrightarrow \phi^{-} + \phi^{0} , \]
\[ \mu_{dL} = \mu_{ul} + \mu_{W} , \quad W^{-} \leftrightarrow \bar{u}_{L} + d_{L} , \]
\[ \mu_{iL} = \mu_{iL} + \mu_{W} , \quad W^{-} \leftrightarrow \bar{v}_{iL} + e_{iL} , \]
\[ \mu_{iR} = \mu_{iR} + \mu_{0} , \quad \phi^{0} \leftrightarrow \bar{v}_{iL} + v_{iR} , \]
\[ \mu_{UL} = \mu_{0} + \mu_{uL} , \quad \phi^{0} \leftrightarrow \bar{u}_{L} + u_{R} , \]
\[ \mu_{dR} = -\mu_{0} + \mu_{W} + \mu_{uL} , \quad \phi^{0} \leftrightarrow d_{L} + d_{R} , \]
\[ \mu_{iR} = -\mu_{0} + \mu_{W} + \mu_{iL} , \quad \phi^{0} \leftrightarrow e_{iL} + \bar{e}_{iR} , \]

and the following relations among the chemical potentials of the techniquarks

\[ \mu_{DL} = \mu_{UL} + \mu_{W} , \quad W^{-} \leftrightarrow \bar{U}_{L} + D_{L} , \]
\[ \mu_{UR} = \mu_{0} + \mu_{UL} , \quad \phi^{0} \leftrightarrow \bar{U}_{L} + D_{R} , \]
\[ \mu_{DR} = -\mu_{0} + \mu_{W} + \mu_{UL} , \quad \phi^{0} \leftrightarrow D_{L} + \bar{D}_{R} , \]
The thermodynamical analysis is most transparent when using directly the underlying technicolor degrees of freedom. At a given temperature $T$ and chemical potential $\mu$ the number density $n_+$ ($n_-$) of particles (antiparticles) is given by

$$ n_\pm = m \int \frac{d^3k}{(2\pi)^3} \frac{1}{z^{\pm1} e^{E \beta} - \eta} $$  \hspace{1cm} (B5)

Here $m$ is the multiplicity of the degrees of freedom, $\beta = 1/T$, $z = e^{\mu \beta}$ is the fugacity, $E^2 = m^2 + \vec{k}^2$ is the energy and $\eta$ equals 1 and $-1$ for bosons and fermions respectively.

At the freeze-out temperature $T^*$, where the violating processes cease to be efficient, we have $\mu/T^* \ll 1$ and we therefore find that the difference between the number densities of particles and their corresponding antiparticles is

$$ n = n_+ - n_- = m T^* 3 \cdot \frac{\mu}{T^*} \cdot \frac{\sigma \left( \frac{m}{T^*} \right)}{6} $$  \hspace{1cm} (B6)

where we have defined the statistical function $\sigma$ as

$$ \sigma(z) = \begin{cases} \frac{6}{4\pi^2} \int_0^\infty dx \ x^2 \cosh^{-2} \left( \frac{1}{2} \sqrt{x^2 + z^2} \right) & \text{for fermions} , \\ \frac{6}{4\pi^2} \int_0^\infty dx \ x^2 \sinh^{-2} \left( \frac{1}{2} \sqrt{x^2 + z^2} \right) & \text{for bosons} . \end{cases} $$  \hspace{1cm} (B7)

We have conveniently normalized the statistical function such that it assumes the value 1 (2) for massless fermions (bosons). When computing the relic density we are only interested in the ratio of number densities. Hence we appropriately normalize the net baryon number density as:

$$ B = \frac{6}{m T^*^2} (n_B - n_{\bar{B}}) $$  \hspace{1cm} (B8)

A similar normalization is chosen for the lepton and technibaryon number densities. Having set the notation the overall electric charge is

$$ Q = \frac{2}{3} \cdot 3 \cdot (2 + \sigma_t) (\mu_{uL} + \mu_{uR}) - \frac{1}{3} \cdot 3 \cdot 3 (\mu_{dL} + \mu_{dR}) - \sum_i (\mu_{iL} + \mu_{iR}) \\
-2 \cdot 2 \mu_W - 2 \mu_- + \frac{1}{2} \cdot 2 \sigma_U (\mu_{UL} + \mu_{UR}) - \frac{1}{2} \cdot 2 \sigma_D (\mu_{DL} + \mu_{DR}) \\
= 2 (\sigma_U - \sigma_D) \mu_{UL} + 2 (1 + 2 \sigma_t) \mu_{uL} - 2 (9 + \sigma_D) \mu_W \\
-2 \mu + (12 + 2 \sigma_t + \sigma_U + \sigma_D) \mu_0 $$  \hspace{1cm} (B9)
with \( \mu = \sum_i \mu_{viL} \) while the overall weak isospin charge is

\[
Q_3 = \frac{1}{2} \cdot 3 \cdot (2 + \sigma_t) \mu_{uL} - \frac{1}{2} \cdot 3 \cdot 3 \mu_{dL} + \frac{1}{2} \sum_i (\mu_{viL} - \mu_{iL}) - 4 \mu_W
\]

\[
- (\mu_0 + \mu_-) + \frac{1}{2} \cdot 2 \sigma_U \mu_{UL} - \frac{1}{2} \cdot 2 \sigma_D \mu_{DL}
\]

\[
= \frac{3}{2} (\sigma_t - 1) \mu_{uL} - (11 + \sigma_D) \mu_W + (\sigma_U - \sigma_D) \mu_{UL} .
\]

(B10)

Here we have used the relations [B3] and [B4]. Relation between chemical potentials coming from baryon number violating processes:

\[
0 = \mu_{UL} + \mu_{DL} + 3 (\mu_{uL} + 2 \mu_{dL}) + \mu
\]

(B11)

\[
= 2 \mu_{UL} + 9 \mu_{uL} + 7 \mu_W + \mu .
\]

(B12)

Finally we note that the baryon number \( B \), lepton number \( L \) and technibaryon number \( TB \) can be expressed as

\[
B = (10 + 2 \sigma_t) \mu_{uL} + 6 \mu_W + (\sigma_t - 1) \mu_0
\]

(B13)

\[
L = 6 \mu_W + 4 \mu
\]

(B14)

\[
TB = \frac{1}{2 \sqrt{2}} \cdot 2 [\sigma_U (\mu_{UL} + \mu_{UR}) + \sigma_D (\mu_{DL} + \mu_{DR})]
\]

\[
= \frac{1}{\sqrt{2}} [2 (\sigma_U + \sigma_D) \mu_{UL} + 2 \sigma_D \mu_W + (\sigma_U - \sigma_D) \mu_0]
\]

(B15)

1. **2nd Order Phase Transition**

Here we have the following conditions: \( Q = 0 \) and \( \mu_0 = 0 \). In the approximation where the up and down techniquarks have equal masses we find, using the relations above, that the technibaryon number over the baryon number can be written as:

\[
- \sqrt{2} \cdot \frac{TB}{B} = \frac{\sigma}{81 + 10 \sigma + (27 + 2 \sigma) \sigma_t} [18 (8 + \sigma_t + \sigma) + (5 + \sigma_t) (9 + \sigma) \xi]
\]

(B16)

where \( \sigma = \sigma_U = \sigma_D \) and \( \xi = L/B \).
2. 1st Order Phase Transition

For the first order phase transition we impose the following two conditions: $Q = 0$ and $Q_3 = 0$. We then find that:

$$-\frac{\sqrt{2} \cdot TB}{B} = \frac{\sigma}{2513 + 654 \sigma + 40 \sigma^2 + 2 (551 + 102 \sigma + 4 \sigma^2) \sigma_t + (81 + 6 \sigma) \sigma_t^2} \times \left[ 18 (246 + 65 \sigma + 4 \sigma^2 + (59 + 7 \sigma) \sigma_t + 3 \sigma_t^2) + (1441 + 345 \sigma + 20 \sigma^2 + 4 (95 + 21 \sigma + \sigma^2) \sigma_t + 3 (9 + \sigma) \sigma_t^2) \xi \right]$$

(B17)

In the approximation where the top quark is also considered massless the technibaryon number over the baryon number is the same for both the 1st and 2nd order phase transition

$$-\frac{\sqrt{2} \cdot TB}{B} = \frac{\sigma}{2} (3 + \xi).$$

(B18)

[1] S. Weinberg, “Implications Of Dynamical Symmetry Breaking: An Addendum,” Phys. Rev. D 19, 1277 (1979).
[2] L. Susskind, “Dynamics Of Spontaneous Symmetry Breaking In The Weinberg-Salam Theory,” Phys. Rev. D 20, 2619 (1979).
[3] S. Nussinov, “Technocosmology: Could A Technibaryon Excess Provide A ’Natural’ Missing Mass Candidate?,” Phys. Lett. B 165, 55 (1985).
[4] S. M. Barr, R. S. Chivukula and E. Farhi, “Electroweak Fermion Number Violation And The Production Of Stable Particles In The Early Universe,” Phys. Lett. B 241, 387 (1990).
[5] S. B. Gudnason, C. Kouvaris and F. Sannino, “Dark matter from new technicolor theories,” Phys. Rev. D 74, 095008 (2006) [arXiv:hep-ph/0608055].
[6] F. Sannino and K. Tuominen, ‘Orientifold theory dynamics and symmetry breaking,” Phys. Rev. D 71 (2005) 051901 [arXiv:hep-ph/0405209].
[7] D. K. Hong, S. D. H. Hsu and F. Sannino, “Composite Higgs from higher representations,” Phys. Lett. B 597, 89 (2004) [arXiv:hep-ph/0406200].
[8] R. Foadi, M. T. Frandsen, T. A. Ryttov and F. Sannino, “Minimal Walking Technicolor: Set Up for Collider Physics,” arXiv:0706.1696 [hep-ph].

26
[9] S. B. Gudnason, T. A. Ryttov and F. Sannino, “Gauge coupling unification via a novel technicolor model,” Phys. Rev. D 76, 015005 (2007) [arXiv:hep-ph/0612230].

[10] N. D. Christensen and R. Shrock, “Technifermion representations and precision electroweak constraints,” Phys. Lett. B 632, 92 (2006) [arXiv:hep-ph/0509109].

[11] D. D. Dietrich, F. Sannino and K. Tuominen, ‘Light composite Higgs from higher representations versus electroweak precision measurements: Predictions for LHC,” Phys. Rev. D 72 (2005) 055001 [arXiv:hep-ph/0505059]; ibid Phys. Rev. D 73 (2006) 037701. [arXiv:hep-ph/0510217].

[12] D. D. Dietrich and F. Sannino, ‘Conformal window of SU(N) gauge theories with fermions in higher dimensional representations’,” Phys. Rev. D 75, 085018 (2007) [arXiv:hep-ph/0611341].

[13] T. A. Ryttov and F. Sannino, “Conformal Windows of SU(N) Gauge Theories, Higher Dimensional Representations and The Size of The Unparticle World,” arXiv:0707.3166 [hep-th].

[14] S. Catterall and F. Sannino, “Minimal walking on the lattice,” arXiv:0705.1664 [hep-lat]. To Appear in Phys. Rev. D.

[15] L. Del Debbio, M. T. Frandsen, H. Panagopoulos and F. Sannino, “Higher representations on the lattice: perturbative studies,” JHEP 0806, 007 (2008) [arXiv:0802.0891 [hep-lat]].

[16] S. Catterall, J. Giedt, F. Sannino and J. Schneible, “Phase diagram of SU(2) with 2 flavors of dynamical adjoint quarks,” arXiv:0807.0792 [hep-lat].

[17] N. Evans and F. Sannino, “Minimal walking technicolour, the top mass and precision electroweak measurements,” arXiv:hep-ph/0512080.

[18] R. Foadi and F. Sannino, “WW Scattering in Walking Technicolor,” arXiv:0801.0663 [hep-ph]. Published in Phys. Rev. D.

[19] C. Kouvaris, “Dark Majorana Particles from the Minimal Walking Technicolor,” Phys. Rev. D 76 (2007) 015011 [arXiv:hep-ph/0703266].

[20] C. Kouvaris, “The Dark Side of Strong Coupled Theories,” arXiv:0807.3124 [hep-ph].

[21] J. M. Cline, M. Jarvinen and F. Sannino, “The Electroweak Phase Transition in Nearly Conformal Technicolor,” arXiv:0808.1512 [hep-ph].

[22] D. D. Dietrich and C. Kouvaris, “Constraining vectors and axial-vectors in walking technicolour by a holographic principle,” arXiv:0805.1503 [hep-ph].

[23] F. Sannino, “Dynamical Stabilization of the Fermi Scale: Phase Diagram of Strongly Coupled Theories for (Minimal) Walking Technicolor and Unparticles,” arXiv:0804.0182 [hep-ph].
[24] B. Holdom, “Techniodor,” Phys. Lett. B 150, 301 (1985).
[25] B. Holdom, “Flavor Changing Suppression In Technicolor,” Phys. Lett. B 143, 227 (1984).
[26] E. Eichten and K. D. Lane, “Dynamical Breaking Of Weak Interaction Symmetries”, Phys. Lett. B 90, 125 (1980).
[27] B. Holdom, “Raising The Sideways Scale”, Phys. Rev. D 24, 1441 (1981).
[28] K. Yamawaki, M. Bando and K. i. Matumoto, “Scale Invariant Technicolor Model And A Technidilaton”, Phys. Rev. Lett. 56, 1335 (1986).
[29] T. W. Appelquist, D. Karabali and L. C. R. Wijewardhana, “Chiral Hierarchies And The Flavor Changing Neutral Current Problem In Technicolor”, Phys. Rev. Lett. 57, 957 (1986).
[30] T. Appelquist, P. S. Rodrigues da Silva and F. Sannino, “Enhanced global symmetries and the chiral phase transition,” Phys. Rev. D 60, 116007 (1999) [arXiv:hep-ph/9906555].
[31] K. D. Lane and E. Eichten, “Two Scale Technicolor”, Phys. Lett. B 222, 274 (1989).
[32] M. E. Peskin and T. Takeuchi, “A New constraint on a strongly interacting Higgs sector,” Phys. Rev. Lett. 65, 964 (1990).
[33] M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” Phys. Rev. D 46, 381 (1992).
[34] T. A. Ryttov and F. Sannino, “Supersymmetry Inspired QCD Beta Function,” arXiv:0711.3745 [hep-th]. Published in Phys. Rev. D.
[35] Z. Ahmed et al. [CDMS Collaboration], “A Search for WIMPs with the First Five-Tower Data from CDMS,” arXiv:0802.3530 [astro-ph].
[36] T. Appelquist and F. Sannino, “The physical spectrum of conformal SU(N) gauge theories,” Phys. Rev. D 59, 067702 (1999) arXiv:hep-ph/9806409.
[37] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, “Exact Gell-Mann-Low Function Of Supersymmetric Yang-Mills Theories From Instanton Calculus,” Nucl. Phys. B 229 (1983) 381.
[38] M. A. Shifman and A. I. Vainshtein, “Solution of the Anomaly Puzzle in SUSY Gauge Theories and the Wilson Operator Expansion,” Nucl. Phys. B 277 (1986) 456 [Sov. Phys. JETP 64 (1986 ZETFA,91,723-744.1986) 428].
[39] D. R. T. Jones, “More On The Axial Anomaly In Supersymmetric Yang-Mills Theory,” Phys. Lett. B 123 (1983) 45.
[40] N. Seiberg, “Electric - magnetic duality in supersymmetric nonAbelian gauge theories,” Nucl.
Phys. B 435 (1995) 129 [arXiv:hep-th/9411149].

[41] T. Appelquist, G. T. Fleming and E. T. Neil, “Lattice Study of the Conformal Window in QCD-like Theories,” Phys. Rev. Lett. 100, 171607 (2008) [arXiv:0712.0609 [hep-ph]].

[42] Y. Shamir, B. Svetitsky and T. DeGrand, “Zero of the discrete beta function in SU(3) lattice gauge theory with color sextet fermions,” [arXiv:0803.1707 [hep-lat]].

[43] A. Deuzeman, M. P. Lombardo and E. Pallante, “The physics of eight flavours,” [arXiv:0804.2905 [hep-lat]].

[44] B. Lucini and G. Moraitis, “Determination of the running coupling in pure SU(4) Yang-Mills theory,” PoS LAT2007, 058 (2007) [arXiv:0710.1533 [hep-lat]].

[45] G. Mack, “All Unitary Ray Representations Of The Conformal Group SU(2,2) With Positive Energy,” Commun. Math. Phys. 55 (1977) 1.

[46] M. Flato and C. Fronsdal, “Representations Of Conformal Supersymmetry,” Lett. Math. Phys. 8, 159 (1984).

[47] V. K. Dobrev and V. B. Petkova, “All Positive Energy Unitary Irreducible Representations Of Extended Conformal Supersymmetry,” Phys. Lett. B 162 (1985) 127.

[48] T. Appelquist, K. D. Lane and U. Mahanta, ‘On The Ladder Approximation For Spontaneous Chiral Symmetry Breaking,” Phys. Rev. Lett. 61 (1988) 1553.

[49] A. G. Cohen and H. Georgi, ‘Walking Beyond The Rainbow,” Nucl. Phys. B 314 (1989) 7.

[50] S. Raby, S. Dimopoulos and L. Susskind, “Tumbling Gauge Theories,” Nucl. Phys. B 169, 373 (1980).

[51] C. T. Hill and E. H. Simmons, “Strong dynamics and electroweak symmetry breaking,” Phys. Rept. 381, 235 (2003) [Erratum-ibid. 390, 553 (2004)] [arXiv:hep-ph/0203079].

[52] R. S. Chivukula and T. P. Walker, “Technicolor Cosmology,” Nucl. Phys. B 329, 445 (1990).

[53] M. E. Shaposhnikov, “Sphaleron induced baryon number nonconservation,” Nucl. Phys. Proc. Suppl. 26 (1992) 78.

[54] M. E. Shaposhnikov, “Standard Model Solution Of The Baryogenesis Problem,” Phys. Lett. B 277, 324 (1992) [Erratum-ibid. B 282, 483 (1992)].

[55] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, “Electroweak baryogenesis,” In *Moscow 1991, Proceedings, Sakharov memorial lectures in physics, vol. 2* 779-789

[56] J. Bagnasco, M. Dine and S. D. Thomas, “Detecting technibaryon dark matter,” Phys. Lett. B 320, 99 (1994) [arXiv:hep-ph/9310290].
[57] M. Y. Khlopov and C. Kouvaris, “Strong Interactive Massive Particles from a Strong Coupled Theory,” Phys. Rev. D 77, 065002 (2008) arXiv:0710.2189 [astro-ph].

[58] C. Kouvaris, “WIMP Annihilation and Cooling of Neutron Stars,” Phys. Rev. D 77, 023006 (2008) arXiv:0708.2362 [astro-ph].

[59] K. Kainulainen, K. Tuominen and J. Virkajarvi, “The WIMP of a minimal technicolor theory,” Phys. Rev. D 75, 085003 (2007) arXiv:hep-ph/0612247.

[60] A. Mocsy, F. Sannino and K. Tuominen, “Confinement versus chiral symmetry,” Phys. Rev. Lett. 92, 182302 (2004) arXiv:hep-ph/0308135.

[61] F. Sannino and K. Tuominen, “Tetracritical behavior in strongly interacting theories,” Phys. Rev. D 70, 034019 (2004) arXiv:hep-ph/0403175.

[62] E. Witten, “Global Aspects Of Current Algebra,” Nucl. Phys. B 223, 422 (1983).

[63] E. Witten, “Current Algebra, Baryons, And Quark Confinement,” Nucl. Phys. B 223, 433 (1983).

[64] J. Wess and B. Zumino, “Consequences of anomalous Ward identities,” Phys. Lett. B 37, 95 (1971).

[65] O. Kaymakcalan and J. Schechter, “Chiral Lagrangian Of Pseudoscalars And Vectors,” Phys. Rev. D 31, 1109 (1985).

[66] O. Kaymakcalan, S. Rajeev and J. Schechter, “Nonabelian Anomaly And Vector Meson Decays,” Phys. Rev. D 30, 594 (1984).

[67] Z. y. Duan, P. S. Rodrigues da Silva and F. Sannino, “Enhanced global symmetry constraints on epsilon terms,” Nucl. Phys. B 592, 371 (2001) arXiv:hep-ph/0001303.

[68] J. A. Harvey and M. S. Turner, “Cosmological baryon and lepton number in the presence of electroweak fermion number violation,” Phys. Rev. D 42, 3344 (1990).

[69] M. Kurachi and R. Shrock, “Behavior of the S parameter in the crossover region between walking and QCD-like regimes of an SU(N) gauge theory,” Phys. Rev. D 74, 056003 (2006) arXiv:hep-ph/0607231.

[70] M. Kurachi and R. Shrock, “Study of the change from walking to non-walking behavior in a vectorial gauge theory as a function of N(f),” JHEP 0612, 034 (2006) arXiv:hep-ph/0605290.

[71] K. A. Intriligator and N. Seiberg, “Lectures on supersymmetric gauge theories and electromagnetic duality,” Nucl. Phys. Proc. Suppl. 45BC, 1 (1996) arXiv:hep-th/9509066.

[72] See also [69, 70] for a computation of the S parameter when allowing for a dynamical tech-
niquark mass. This computation suggests, in agreement with the results in [36], that the $S$ parameter is further reduced with respect to the value from the naive analysis.

Naively then a two technicolor theory with two fundamental Dirac flavors and one adjoint Weyl fermion would be a good candidate for a NC technicolor theory. However this is hardly the case since the theory equals two flavor supersymmetric QCD but without the scalars. For two colors and two flavors supersymmetric QCD is known to exhibit confinement with chiral symmetry breaking [71]. Since the critical number of flavors above which one enters the conformal window is three it will most likely not exhibit NC dynamics. Throwing away the scalars only drives it further away from the NC scenario.