Defending Time-Symmetrized Quantum Counterfactuals

Lev Vaidman

School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel-Aviv 69978, Israel.

Abstract

Recently, several authors have criticized the time-symmetrized quantum theory originated by the work of Aharonov et al. (1964). The core of this criticism was a proof, appearing in various forms, which showed that the counterfactual interpretation of time-symmetrized quantum theory cannot be reconciled with standard quantum theory. I, (Vaidman, 1996a, 1997) have argued that the apparent contradiction is due to a logical error and introduced consistent time-symmetrized quantum counterfactuals. Here I repeat my arguments defending the time-symmetrized quantum theory and reply to the criticism of these arguments by Kastner (1999).
1. Introduction.

Starting from the seminal work of Aharonov, Bergman and Lebowitz (ABL) (1964), Aharonov, myself and others are developing a time-symmetrized formalism of quantum theory (TSQT). Recently a particular question related to this formalism, namely the validity of the counterfactual application of the ABL rule, became a subject of a significant controversy culminating in the paper by Kastner appearing in this issue. According to the critics some of the recent results obtained in the framework of the TSQT are based on the counterfactual interpretation of the ABL rule which, in general, is inconsistent. In recent preprints (Vaidman, 1996a, 1997) I defended the TSQT and, moreover, introduced time-symmetrized counterfactuals for quantum theory for which the ABL rule is valid. Kastner critically analyzes my preprints and claims that my defense is not sound. In this paper I refute Kastner’s arguments. For self-consistency I include the relevant sections of the preprints.

Lewis (1986, 34), who is probably the main authority on counterfactuals, writes: “Counterfactuals are infected with vagueness, as everybody agrees.” I do not completely agree. I believe that quantum counterfactuals can be defined unambiguously (as I am doing in Section 3). However, it seems that the core of the current controversy is indeed the ambiguity of the concept of counterfactuals. Kastner distinguishes between two apparently counterfactual readings of the ABL rule: the first one she considers as “non-counterfactual” and the second is a “bona fide counterfactual” (p.3(?)). She claims that the second reading is what is frequently applied in the TSQT. I will argue below that it is the first reading which is correct and which has been applied in the framework of the TSQT. The question whether the first reading is “counterfactual” remains a semantic issue. Formally, it is. Moreover, I will argue below that the second Kastner’s reading is inconsistent and that all other proposed counterfactual readings of the ABL rule are either inconsistent or not time-symmetric. Thus, I find it appropriate to use the term “counterfactual” for my (Kastner’s first) reading. However, if philosophers find it important to spell out the differences between these “non-counterfactual” counterfactuals and “bona fide counterfactuals” applied in general analyses, I hope that the current discussion will help in this task.

The plan of this paper is as follows. In Section 2 I briefly define the time-symmetrized formalism. In section 3 I analyze the concept of counterfactuals in quantum theory and introduce the time-symmetrized counterfactuals. In Section 4 I discuss elements of reality which are examples of quantum counterfactuals. Section 5 is devoted to the analysis of the inconsistency proof of Sharp and Shanks (S&S) (1993) and its variations. Section 6 presents more detailed analysis of possible counterfactual interpretations of the ABL rule. In Section 7 I analyze Kastner’s readings of the ABL rule, in Section 8 her analysis of the S&S proof, and in Section 9 her criticism of my definition of time-symmetrized counterfactuals. Section 10 summarizes the arguments of the paper.

1Sections 2-5 are from Vaidman (1997) and Section 6 from Vaidman (1996a). These are the preprints which Kastner criticizes. The full text of the preprints is available electronically.
2. Time-Symmetrized Formalism.

In standard quantum theory a complete description of a system at a given time is given by a quantum state $|\Psi\rangle$. It yields the probabilities for all outcomes $a_i$ of a measurement at that time of any observable $A$ according to the equation

$$\text{Prob}(a_i) = |\langle \Psi | P_{A=a_i} | \Psi \rangle|,$$

(1)

where $P_{A=a_i}$ is the projection operator on the subspace defined by $A = a_i$. Although it is not manifestly apparent, Eq. 1 is intrinsically asymmetric in time: the state $|\Psi\rangle$ is determined by some measurements in the past and it evolves toward the future. The time evolution between the measurements, however, is considered time symmetric since it is governed by the Schrödinger equation for which each forward evolving solution has its counterpart (its complex conjugate with some other well understood simple changes) evolving backward in time. The asymmetry in time of the standard quantum formalism is manifested in the absence of the quantum state evolving backward in time from future measurements (relative to the time in question).

Time-symmetrized quantum theory completely describes a system at a given time by a two-state vector $\langle \Psi_2 || \Psi_1 \rangle$. It yields the (conditional) probabilities for all outcomes $a_i$ of a measurement of any observable $A$ at that time according to the generalization of the ABL formula (Aharonov and Vaidman, 1991):

$$\text{Prob}(a_i) = \frac{|\langle \Psi_2 | P_{A=a_i} | \Psi_1 \rangle|^2}{\sum_j |\langle \Psi_2 | P_{A=a_j} | \Psi_1 \rangle|^2}.$$

(2)

The time symmetry means that $\langle \Psi_2 \rangle$ and $| \Psi_1 \rangle$ enter the equations, and thus govern the observable results, on equal footings. Moreover, the time symmetry means that, in regard to time symmetric measurements, a system described by the two-state vector $\langle \Psi_2 || \Psi_1 \rangle$ is identical to a system described by the two-state vector $\langle \Psi_1 || \Psi_2 \rangle$. I analyze the time symmetry of the process of measurement in section 6 of (Vaidman, 1997), here I only point out that ideal measurements are time symmetric. Indeed, the symmetry under the interchange of $\langle \Psi_2 \rangle$ and $| \Psi_1 \rangle$ is explicit in Eq. 2 which refers to ideal measurements.

Another basic concept of time-symmetrized two-state vector formalism is weak value. An (almost) standard measurement procedure for measuring observable $A$ with weakened coupling (which we call weak measurement, Aharonov and Vaidman 1990) yields the weak value of $A$:

$$A_w \equiv \frac{\langle \Psi_2 | A | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle}.$$

(3)

Here again, $\langle \Psi_2 \rangle$ and $| \Psi_1 \rangle$ enter the equations on equal footings. However, when we interchange $\langle \Psi_2 \rangle$ and $| \Psi_1 \rangle$, the weak value changes to its complex conjugate. Thus, in this situation, as for the Schrödinger equation, time reversal is accompanied by complex conjugation.

In order to explain how to obtain a quantum system described at a given time $t$ by a two-state vector $\langle \Psi_2 || \Psi_1 \rangle$ we shall assume for simplicity that the free Hamiltonian of the system is zero. In this case, it is enough to prepare the system at time $t_1$ prior to time $t$ in the state $| \Psi_1 \rangle$, and to ensure no disturbance between $t_1$ and $t$ as well as between $t$ and
Note the asymmetry between the measurement at $t_1$ and the measurement at $t_2$. Given an ensemble of quantum systems, it is always possible to prepare all of them in a particular state $|\Psi_1\rangle$, but we cannot ensure finding the system in a particular state $|\Psi_2\rangle$. Indeed, if the pre-selection measurement yielded a result different from projection on $|\Psi_1\rangle$ we can always change the state to $|\Psi_1\rangle$, but if the measurement at $t_2$ did not show $|\Psi_2\rangle$, our only choice is to discard such a system from the ensemble. This asymmetry, however, is not relevant to the problem we consider here. We study the symmetry relative to the measurements at time $t$ for a given pre- and post-selected system, and we do not investigate the time-symmetry of obtaining such a system. The only important detail is that the interaction at time $t$ has to be time symmetric. See more discussion in Section 6 of (Vaidman, 1997).

3. Counterfactuals.

A general form of a counterfactual statement is

(i) If it were that $A$, then it would be that $B$.

There are many philosophical discussions on the concept of counterfactuals and especially on time’s arrow in counterfactuals. Many of the discussions, e.g. Lewis (1986), Bennett (1984), are related to $A$: How come $A$ if in the actual world $A$ is not true? Do we need a miracle (a violation of the fundamental law of nature) for $A$? Does $A$ come by itself, or is it accompanied by other changes? However, these questions are not relevant to the problem of counterfactuals in quantum theory. The questions about $A$ are not relevant because $A$ depends solely on an external system which is not under discussion by the definition of the problem. Indeed, in quantum theory the counterfactuals have a very specific form:\[2\]

$A =$ a measurement $\mathcal{M}$ is performed

$B =$ the outcome of $\mathcal{M}$ has property $\mathcal{P}$

The measurement $\mathcal{M}$ might consist of measurements of several observables performed together. The property $\mathcal{P}$ might be a certain relation between the results of measurements of these observables or a probability for a certain relation or for a certain outcome.

It is assumed that the experimenter can make any decision about which measurement to perform and the question how he makes this decision is not considered. It is assumed that the experimenter and his measuring devices are not correlated in any way with the

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\[2\]This definition of counterfactuals in quantum theory is broad enough for discussing issues relevant to this paper. However, in some cases the term “counterfactuals” has been used differently. For example, in Penrose (1994, p.240) “counterfactuals are things that might have happened, although they did not in fact happen.”
state of the system prior to the measurement. Thus, in the world of the quantum system no miracles are needed and no changes relative to the actual world have to be made for different $A$’s.\footnote{Indeterminism of standard quantum theory allows us to discuss even the worlds which include the experimenter without invoking miracles. Consider an experimenter who chooses between different measurements according to a random outcome of another quantum experiment.}

Although one can define counterfactuals of this form in the framework of classical theory, they are of no interest because they are equivalent to some “factual” statements. In classical physics any observable has always a definite value and a measurement of the observable yields this value. Therefore, we can make a one to one correspondence between “the outcome of a measurement of an observable $C$ is $c_i$” and “the value of $C$ is $c_i$”. The latter is independent of whether the measurement of $C$ has been performed or not and, therefore, statements which are formally counterfactual about results of possible measurements can be replaced by “factual” (unconditional) statements about values of corresponding observables. In contrast, in standard quantum theory, observables, in general, do not have definite values and therefore we cannot always reduce the above counterfactual statements to “factual” statements.

Most of the discussions of counterfactuals in quantum theory are in the context of EPR-Bell type experiments. Some of the examples are Skyrms (1982), Peres (1993), Mermin (1989) (which, however, does not use the word counterfactual), Ghirardi and Grassi (1994) and Bedford and Stapp (1995) who even present an analysis of a Bell-type argument in the formal language of the Lewis (1973) theory of counterfactuals. The common situation is that a composite system is described at a certain time by some entangled state and then an array of incompatible measurements on this system at a later time is considered. Various conclusions are derived from statements about the results of these measurements. Since these measurements are incompatible they cannot be all performed together, so it must be that at least some of them were not actually performed. This is why they are called counterfactual statements.

These counterfactuals are explicitly asymmetric in time. The asymmetry is neither in $A$ nor in $B$; both are about a single time $t$. The asymmetry is in the description of the actual world. The past and not the future (relative to $t$) of a system is given.

This, however, is not the only asymmetry of the counterfactuals in quantum theory as they are usually considered. A different asymmetry (although it looks very similar) is in what we assume to be “fixed”, i.e., which properties of the actual world we assume to be true in possible counterfactual worlds. The past and not the future of the system is fixed.

It seems that while the first asymmetry can be easily removed, the second asymmetry is unavoidable. According to standard quantum theory a system is described by its quantum state. In the actual world, in which a certain measurement has been performed at time $t$ (or no measurement has been performed at $t$) the system is described by a certain state before $t$, and by some state after time $t$. In the counterfactual world in which a different measurement was performed at time $t$, the state before $t$ is, of course, the same, but the state after time $t$ is invariably different (if the observables measured in actual and counterfactual worlds have different eigenstates). Therefore, we cannot hold fixed the quantum state of the system in the future.\footnote{Note that none of these asymmetries exists in the classical case because when a complete description
The argument above shows that for constructing time symmetric counterfactuals we have to give up the description of a quantum system by its quantum state. Fortunately we can do that without losing anything except the change due to the measurement at time \( t \) which caused the difficulty. A quantum state at a given time is completely defined by the results of a complete set of measurements performed prior to this time. Therefore, we can take the set of all results performed on a quantum system as a description of the world of the system instead of describing the system by its quantum state. (This proposal will also help to avoid ambiguity and some controversies related to the description of a single quantum system by its quantum state.) Thus, I propose the following definition of counterfactuals in the framework of quantum theory:

(ii) If a measurement \( \mathcal{M} \) were performed at time \( t \), then it would have property \( \mathcal{P} \), provided that the results of all measurements performed on the system at all times except the time \( t \) are fixed.

For time asymmetric situations in which only the results of measurements performed before \( t \) are given (and thus only these results are fixed) this definition of counterfactuals is equivalent to the counterfactuals as they usually have been used. However, when the results of measurements performed on the system both before and after the time \( t \) are given, definition (ii) yields novel time-symmetrized counterfactuals. In particular, for the ABL case, in which complete measurements are performed on the system at \( t_1 \) and \( t_2 \), \( t_1 < t < t_2 \), we obtain

(iii) If a measurement of an observable \( C \) were performed at time \( t \), then the probability for \( C = c_i \) would equal \( p_i \), provided that the results of measurements performed on the system at times \( t_1 \) and \( t_2 \) are fixed.

The ABL formula (2) yields correct probabilities for counterfactuals defined as in (iii), i.e., in the experiment in which \( C \) is measured at time \( t \) on the systems from a pre- and post-selected ensemble defined by fixed outcomes of the measurements at \( t_1 \) and \( t_2 \) (all such systems and only such systems are considered) the frequency of an outcome \( c_i \) is \( p_i \).

For the ABL situation one can also define a time asymmetric counterfactual:

(iv) Given the results of measurements at \( t_1 \) and \( t_2 \), \( t_1 < t < t_2 \) (in the actual world), if a measurement of an observable \( C \) were performed at time \( t \), then the probability for \( C = c_i \) would equal \( p_i \), provided that the results of all measurements performed on the system at all times before time \( t \) are fixed.

In the framework of standard quantum theory the information about the result of measurement at \( t_2 \) is irrelevant: the probability for \( C = c_i \) does not depend on this result. Thus, it is obvious that the ABL formula (2), which includes the result of the measurement at time \( t_2 \) explicitly, does not yield counterfactual probabilities according to definition (iv).

One might modify definition (iv) in the framework of some “hidden variable” theory with a natural additional requirement of fixing the hidden variables of the system in the of a classical system is given at one time, it yields and fixes the complete description at all times and (ideal) measurements at time \( t \) do not change the state of a classical system.
past. The properties of such counterfactuals will depend crucially on the details of the hidden variable theory (see the discussion of Aharonov and Albert (1987) in the framework of Bohm’s theory), but the ABL formula (2) is not valid for any such modification. In order to show this consider a spin-$\frac{1}{2}$ particle which was found at $t_1$ and at $t_2$ in the same state $|\uparrow_z\rangle$ (and no measurement has been performed at $t$). We ask what is the (counterfactual) probability for finding spin “up” in the direction $\hat{\xi}$ which makes an angle $\theta$ with the direction $\hat{z}$, at the intermediate time $t$. In this case, hidden variables, even if they exist, cannot change that probability because any particle found at $t_1$ in the state $|\uparrow_z\rangle$, irrespectively of its hidden variable, yields the outcome “up” in the measurement at $t_2$. Therefore, the statistical predictions about the intermediate measurement at time $t$ must be the same as for the pre-selected only ensemble (these are identical ensembles in this case), i.e.

$$\text{Prob}(\uparrow_{\hat{\xi}}) = |\langle \uparrow_{\hat{\xi}} | \uparrow_z \rangle|^2 = \cos^2(\theta/2).$$

The ABL formula, however, yields:

$$\text{Prob}(\uparrow_{\hat{\xi}}) = \frac{|\langle \uparrow_z | P_{\hat{\xi}} | \uparrow_z \rangle|^2}{|\langle \uparrow_z | P_{\hat{\xi}} | \uparrow_z \rangle|^2 + |\langle \uparrow_z | P_{\hat{\xi}} | \uparrow_z \rangle|^2} = \frac{\cos^4(\theta/2)}{\cos^4(\theta/2) + \sin^4(\theta/2)}. \quad (5)$$

The fact that the ABL formula (2) does not hold for counterfactuals defined in (iv) or its modifications is not surprising. Definition (iv) is explicitly asymmetric in time. The ABL formula, however, is time symmetric and therefore it can hold only for time-symmetrized counterfactuals.

A recent study of time’s arrow and counterfactuals in the framework of quantum theory by Price (1996) seems to support my definition (ii). Let me quote from his section “Counterfactuals: What should we fix?”:

“Hold fixed the past, and the same difficulties arise all over again. Hold fixed merely what is accessible, on the other hand, and it will be difficult to see why this course was not chosen from the beginning. (1996, 179)"

This quotation looks very much like my proposal. Indeed, I find many arguments in his book pointing in the same direction. However, in fact, this quotation represents a time asymmetry: according to Price “merely what is accessible” is “an accessible past”. But this is not the time asymmetry of the physical theory; Price writes: “no physical asymmetry is required to explain it.” Although the book includes an extensive analysis of a photon passing through two polarizers – the classic setup for the ABL case, I found no explicit discussion of a possible measurement in between, the problem we discuss here.\footnote{Price briefly and critically mentions the ABL paper. He writes (1996, 208): “What they [ABL] fail to note, however, is that their argument does nothing to address the problem for those who disagree with Einstein – those who think that the state function is a complete description, so that the change that takes place on measurements is a real change in the world, rather than merely change in our knowledge of the world.” This seems to me an unfair criticism: ABL clearly state that in the situations they consider “the complete description” is given by two wave functions (see more in Aharonov and Vaidman 1991). Moreover, it seems to me that the development of this time-symmetrized quantum formalism is not too far from the spirit of the “advanced action” – the Price vision of the solution of the time’s arrow problem.}
4. Elements of Reality.

Important counterfactuals in quantum theory are “elements of reality”. For comparison, I’ll start with a definition of time asymmetric element of reality:

**(v)** If we can predict with certainty that the result of measuring at time \( t \) of an observable \( A \) is \( a \), then, at time \( t \), there exists an element of reality \( A = a \).

This is, essentially, a quotation from Redhead (1987), who, however considered it as a sufficient condition and not as a definition. Redhead was inspired by the criteria for elements of reality of Einstein, Podolsky and Rosen (EPR). In spite of similarity in its form, the EPR criteria, taken as a definition, is very different: “If, without in any way disturbing the system, we can predict with certainty the value of a physical quantity...” The crucial difference is that “predict” in the EPR definition means to find out using certain (non-disturbing) measurements, while in my definition “predict” means to deduce using existing information. Thus, for two spin-\( \frac{1}{2} \) particles in a singlet state, the value of a spin component of a single particle in any direction is an element of reality in the EPR sense (it can be found out by measuring another particle) and there is no element of reality for a spin component value in any direction according to my definition (in the EPR state, the probability to find spin “up” in any direction is \( \frac{1}{2} \)).

Definition (v) of elements of reality is asymmetric in time because of the word “predict”. I have proposed a modification of this definition applicable for time symmetric elements of reality (Vaidman 1993):

**(vi)** If we can infer with certainty that the result of measuring at time \( t \) of an observable \( A \) is \( a \), then, at time \( t \), there exists an element of reality \( A = a \).

The word “infer” is neutral relative to past and future. The inference about results at time \( t \) is based on the results of measurements on the system performed both before and after time \( t \). Note, that in some situations we can “infer” more facts than can be obtained by “prediction” based on the results in the past and “retrodiction” based on the results in the future (relative to \( t \)) together.

The difference between definitions of “elements of reality”, (v) and (vi), and definitions of counterfactuals in quantum theory (iv) and (iii) is that the property \( P \) in (v) and (vi) is constrained to “the result of measuring at time \( t \) of an observable \( A \) is \( a \)”. In fact, time asymmetric “elements of reality” (v), defined as “predictions”, do not represent “interesting” counterfactuals. There is no nontrivial set of such counterfactual statements, i.e., set of statements which cannot be tested all on a single system. Indeed, all observables the measurement of which yield definite outcomes for a pre-selected system can be tested together. One way to extend the definition of time asymmetric elements of reality in order to get nontrivial counterfactuals is to consider “multiple-time measurements” (instead of measurements at time \( t \) only). Another extension, which corresponds to numerous analyses in the literature, is to go beyond statements about observables which have definite values:
If we can predict with certainty a certain relation between the results of measuring at time $t$ a set of observables $A_j$, then, at time $t$, there exists a “generalized element of reality” which is this relation between $A_j$'s.

A simple example of this kind is a system of two spin-$\frac{1}{2}$ particles prepared, at $t_1$, in a singlet state

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2). \tag{6}$$

We can predict with certainty that the results of measurements of spin components of the two particles fulfill the following two relations:

$$\{\sigma_{1x}\} + \{\sigma_{2x}\} = 0, \tag{7}$$
$$\{\sigma_{1y}\} + \{\sigma_{2y}\} = 0, \tag{8}$$

where $\{\sigma_{1x}\}$ signifies the result of measurement of spin $x$ component of the first particle, etc. The relations (7) and (8) represent a set of generalized elements of reality (vii). This is a nontrivial set of counterfactuals because (7) and (8) cannot be tested together: the measurement of $\sigma_{1x}$ disturbs the measurement of $\sigma_{1y}$ as well as the measurement of $\sigma_{2x}$ disturbs the measurement of $\sigma_{2y}$.

In contrast, the set of elements of reality (vi) given by

$$\{\sigma_{1x} + \sigma_{2x}\} = 0, \tag{9}$$
$$\{\sigma_{1y} + \sigma_{2y}\} = 0, \tag{10}$$

can be tested on a single system, see Aharonov et al. (1986) for description of such measurements. Yet another set of counterfactuals, which consists of definite statements about measurements, but which does not fall into category (vi) because these are two-time measurements performed at two different time moments $t_1$ and $t_2$, cannot be tested on a single system:

$$\{\sigma_{1x}(t_1) + \sigma_{2x}(t_2)\} = 0, \tag{11}$$
$$\{\sigma_{1y}(t_2) + \sigma_{2y}(t_1)\} = 0. \tag{12}$$

Note a situation which involves only a single free spin-$\frac{1}{2}$ particle. The particle is prepared, before $t_1$, $t_1 < t_2 < t_3$, in the state $|\uparrow_y\rangle$. Then, a nontrivial set of counterfactuals is:

$$\{\sigma_x(t_1) - \sigma_x(t_3)\} = 0, \tag{13}$$
$$\{\sigma_y(t_2)\} = 1. \tag{14}$$

In this example, however, statement (13) has somewhat different character because it depends not on the results of measurements performed on the particle before or after the period of time $(t_1, t_2)$, but on the fact that the system was not disturbed during this period of time.
5. Inconsistency proofs.

The key point of the criticism of the time-symmetrized quantum theory (Sharp and Shanks 1993; Cohen 1995; Miller 1996) is the conflict between counterfactual interpretations of the ABL rule and predictions of quantum theory. I shall argue here that the inconsistency proofs are unfounded and therefore the criticism essentially breaks apart.

The structure of all these inconsistency proofs is as follows. Three consecutive measurements are considered. The first is the preparation of the state $|\Psi_1\rangle$ at time $t_1$. The probabilities for the results $a_i$ of the second measurement at time $t$ are considered. The final measurement at time $t_2$ is introduced in order to allow the analysis using the ABL formula. Sharp and Shanks consider three consecutive spin component measurements of a spin-$\frac{1}{2}$ particle in different directions. Cohen analyzes a particular single-particle interference experiment. It is a variation of the Mach-Zehnder interferometer with two detectors for the final measurement and the possibility of placing a third detector for the intermediate measurement. Finally, Miller repeated the argument for a system of tandem Mach-Zehnder interferometers. In all these cases the “pre-selection only” situation is considered. It is unnatural to apply the time-symmetrized formalism for such cases. However, it must be possible. Thus, I need not show that the time-symmetrized formalism has an advantage over the standard formalism for describing these situations, but only that it is consistent with the predictions of the standard quantum theory.

In the standard approach to quantum theory the probability for the result of a measurement of $A$ at time $t$ is given by Eq. 1. The claim of all the proofs is that the counterfactual interpretation of the ABL rule yields a different result. In all cases the final measurement at time $t_2$ has two possible outcomes which we signify as “$1_f$” and “$2_f$”. The suggested application of the ABL rule is as follows. The probability for the result $a_i$ is:

$$\text{Prob}(A = a_i) = \text{Prob}(1_f) \text{Prob}(A = a_i|1_f) + \text{Prob}(2_f) \text{Prob}(A = a_i|2_f),$$  \hspace{1cm} (15)

where $\text{Prob}(A = a_i|1_f)$ and $\text{Prob}(A = a_i|2_f)$ are the conditional probabilities given by the ABL formula, Eq. 2, and $\text{Prob}(1_f)$ and $\text{Prob}(2_f)$ are the probabilities for the results of the final measurement. In the proofs, the authors show that Eq. (15) is not valid and conclude that the ABL formula is not applicable for this example and therefore that it is not applicable in general.

I will argue that the error in calculating equality (15) is not in the conditional probabilities given by the ABL formula, but in the calculation of the probabilities $\text{Prob}(1_f)$ and $\text{Prob}(2_f)$ of the final measurement. In all three cases it was calculated on the assumption that no measurement took place at time $t$. Clearly, one cannot make this assumption here since then the discussion about the probability of the result of the measurement at time $t$ is meaningless. Unperformed measurements have no results (Peres, 1978). Thus, there is no surprise that the value for the probability $\text{Prob}(A = a_i)$ obtained in this way comes out different from the value predicted by the quantum theory.

Straightforward calculations show that if one uses the formula (15) with the probabilities $\text{Prob}(1_f)$ and $\text{Prob}(2_f)$ calculated on the condition that the intermediate measurement has been performed, then the outcome is the same as predicted by the standard formalism of quantum theory. Consider, for example, the experiment suggested by Sharp and
Shanks, consecutive spin measurements with the three directions in the same plane and the relative angles $\theta_{ab}$ and $\theta_{bc}$. The probability for the final result “up” is

$$\text{Prob}(1_f) = \cos^2(\theta_{ab}/2) \cos^2(\theta_{bc}/2) + \sin^2(\theta_{ab}/2) \sin^2(\theta_{bc}/2),$$

(16)

and the probability for the final result “down” is

$$\text{Prob}(2_f) = \cos^2(\theta_{ab}/2) \sin^2(\theta_{bc}/2) + \sin^2(\theta_{ab}/2) \cos^2(\theta_{bc}/2).$$

(17)

The ABL formula yields

$$\text{Prob}(\text{up}|1_f) = \frac{\cos^2(\theta_{ab}/2) \cos^2(\theta_{bc}/2)}{\cos^2(\theta_{ab}/2) \cos^2(\theta_{bc}/2) + \sin^2(\theta_{ab}/2) \sin^2(\theta_{bc}/2)}$$

(18)

and

$$\text{Prob}(\text{up}|2_f) = \frac{\cos^2(\theta_{ab}/2) \sin^2(\theta_{bc}/2)}{\cos^2(\theta_{ab}/2) \sin^2(\theta_{bc}/2) + \sin^2(\theta_{ab}/2) \cos^2(\theta_{bc}/2)}.\quad (19)$$

Substituting all these equations into Eq. (15) we obtain

$$\text{Prob}(\text{up}) = \cos^2(\theta_{ab}/2).$$

(20)

This result coincide with the prediction of the standard quantum theory. It is a straightforward exercise to show in the same way that no inconsistency arises also in the examples of Cohen and Miller.

I have shown that one can apply the time-symmetrized formalism, including the ABL formula, for analyzing the examples which allegedly lead to contradictions in the inconsistency proofs. In my analysis there was nothing “counterfactual”. The proofs, however, claimed to show that a “counterfactual interpretation” of the ABL rule leads to contradiction. What I have shown is that the examples presented in the proofs do not correspond to counterfactual situations and this is why they cannot be analyzed in a counterfactual way. The contradictions in the proofs arise from a logical error in taking together the statement “no measurement has been performed at $t$” and a statement about probability of a result of this measurement which requires “the measurement has been performed at $t$”. Let me demonstrate how similar erroneous “counterfactual” reasoning can lead to a contradiction in quantum theory even in cases when the ABL rule is not involved.

Consider two consecutive measurements of $\sigma_x$ performed on a spin-$\frac{1}{2}$ particle prepared in a state $|\uparrow_z\rangle$. Let us ask (using the language of Sharp and Shanks) what is the probability that these measurements would have had the results $\sigma_x(t_1) = \sigma_x(t_2) = 1$ given that no such measurements in fact took place. Each spin measurement, if performed separately, has probability $\frac{1}{2}$ for the result $\sigma_x = 1$. According to standard quantum theory the fact that in the actual world the measurement at $t_1$ has been performed and $\sigma_x(t_1) = 1$ has been obtained does not ensure that in a counterfactual world in which $\sigma_x$ was not measured at $t_1$, but at a later time $t_2$, the outcome has to be $\sigma_x(t_2) = 1$, rather we still have

6In Cohen’s example the measurement at time $t_2$ is not a complete measurement and therefore the ABL formula (2) is not applicable for this case. The analysis requires a generalization of the ABL formula given in Vaidman (1998a).
probability $\frac{1}{2}$ for this result. Thus, counterfactual reasoning leads us to the erroneous result that the probability for $\sigma_x(t_1) = \sigma_x(t_2) = 1$ is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

6. Counterfactual Interpretations of the ABL Probability Rule

In this section I shall consider three ways to interpret the “counterfactual interpretation”. The first interpretation I cannot comprehend, but I have to discuss it since it was proposed and used in the criticism of the time-symmetrized quantum theory. I believe that I understand the meaning of the second interpretation, but I shall argue that it is not appropriate for the problem which is discussed here. The last interpretation is the one I want to adopt and I shall present several arguments in its favor.

**Interpretation (a) Counterfactual probability as the probability of the result of a measurement which has not been performed.**

Let me quote Sharp and Shanks:

...for, conditionalizing upon specified results of measurements of $M_I$ and $M_F$, there is no reason to assign the same values to the following probabilities: the probability that an intervening measurement of $M$ had the result $m^j$ given that such a measurement in fact took place, and the probability that intervening measurement would have had the result $m^j$ given that no such intervening measurement of $M$ in fact took place. In other words there is no reason to identify $\text{Prob}(M = m^j | E[M[\psi^i_I, \psi^k_F]])$ and $\text{Prob}(M = m^j | E[\psi^i_I, \psi^k_F])$.(1993, 491)

I can not comprehend the meaning of the probability for the result $M = m^j$ given that the measurement $M$ has not take place. As far as I can see $\text{Prob}(M = m^j | E[\psi^i_I, \psi^k_F])$ has no physical meaning. Sharp and Shanks continue:

(For a classical illustration, consider a drug which, if injected to facilitate a medical test at $t$, has an effect, starting shortly after the test and persisting past $t_F$, on the the value of the tested variable. Suppose that it is unknown whether a test was conducted at $t$, but that a value for the tested variable is obtained at $t_F$. Using the value at $t_F$, we would estimate differently the value prior to $t$ depending on whether we assume that a test did or did not take place at $t$.)

This might explain what they have in mind, but the argument does not hold since in many situations there is no quantum mechanical counterpart to the classical case of “the value [of a tested variable] prior to $t$” . In standard quantum theory *unperformed experiments have no results*, see Peres (1978).

Cohen and Hiley partially acknowledge the problem admitting that at least in the framework of the orthodox interpretation this is a meaningless concept:

In other words we cannot necessarily assume that the ABL rule will yield the correct probabilities for what the results of the intermediate measurements...
would have been, if they had been carried out, in cases where these measurements have not actually been carried out. In fact, this sort of counterfactual retrodiction has no meaning in the orthodox (i.e., Bohrian) interpretation of quantum mechanics, although it can legitimately be discussed within the standard interpretation and within some other interpretations of quantum mechanics (see, for example, Bohm and Hiley [1993]). (1996, 3)

I fail to understand the interpretation (a) in any framework. Maybe, if we restrict ourselves to the cases in which the system at the intermediate time is in an eigenstate of the variable which we intended to measure, (but we had not), we can associate the probability 1 with such unperformed measurements. This is close to the idea of Cohen (1995) to consider counterfactuals in the restricted cases corresponding to consistent histories introduced by Griffith (1984). But, as far as I can see, interesting situations do not correspond to consistent histories, and therefore no novel (relative to classical theory) features of quantum theory can be seen in this way. It is possible that what Cohen and Hiley (1996) have in mind is the interpretation (b) which I shall discuss next.

Interpretation (b)

Counterfactual probability as the probability of the result of a measurement would it have been performed based on the information about the world in which the measurement has not been performed.

At time $t_1$ we preselect the state $|\Psi_1\rangle$. We do not perform any measurement at time $t$. We perform a measurement at time $t_2$ and find the state $|\Psi_2\rangle$. We ask, what would be the probability for the results of a measurement performed at time $t$ in a world which is identical to the actual world at time $t_1$.

This is a meaningful concept, but I believe that it is not adequate for discussing pre- and post-selected quantum systems because it is explicitly asymmetric in time. The counterfactual world is identical to the actual world at time $t_1$ and might not be identical at time $t_2$.

[This interpretation is equivalent to (iv) of Section 3, see discussion there.]

Interpretation (c)

Counterfactual probability as the probability for the results of a measurement if it has been performed in the world “closest” to the actual world.

This is identical in form and spirit to the theory of counterfactuals of Bennett (1984), although the context of the pre- and post-selected quantum measurements is somewhat beyond what he considered. This interpretation is explicitly time-symmetric. The title, however, does not specify it completely and I shall explain what I mean (in particular by the word “closest”) now.

I have to specify the concept of “world”. There are many parts of the world which do not interact with the quantum system in question, so their states are irrelevant to the result of the measurement. In our discussion we might include all these irrelevant parts, or might not, without changing any of the conclusions. There are other aspects of the world which are certainly relevant to the measurement at time $t$, but we postulate that they should be disregarded. Everything which is connected to our decision to perform the measurement at time $t$ and all the records of the result of that measurement are not considered. Clearly, the counterfactual world in which a certain measurement has been performed is different from an actual world in which, let us assume, no measurement
has been performed at time \( t \). The profound differences are both in the future where certain records exist or do not exist and in the past which must be different since one history leads to performing the measurement at time \( t \) and another history leads to no measurement.\[\] However, our decision to make the measurement is not connected to the quantum theory which makes predictions about the result of that measurement. We want to limit ourselves to the discussion of the time-symmetry of the quantum system. We do not consider here the question of the time-symmetry of the entire world. Therefore, we exclude the external parts from our consideration.

What constitutes a description of a quantum system itself is also a very controversial subject. The reality of the Schrödinger wave, the existence or nonexistence of hidden variables etc. are subjects of hot discussions. However, everybody agrees that the collection of all results of measurements is a consistent (although maybe not complete) description of the quantum system. Thus, I propose the following definition:

**A world “closest” to the actual world is a world in which all measurements (except the measurement at the time \( t \) if performed) have the same outcomes as in the actual world.**

This definition overcomes the common objection according to which one should not consider together statements about pre- and post-selected systems regarding different measurements at time \( t \) because these systems belong to different ensembles. The difference is in their quantum state at the time period between \( t \) and \( t_2 \). Formally, the problem is solved by considering only results of measurements and not the quantum state. The justification of this step follows from the rules of the game: it is postulated that the quantum system is not disturbed during the periods of time \((t_1, t)\) and \((t, t_2)\). Therefore, it is postulated that no measurement on the system is performed during these periods of time. Since unperformed measurements have no results, the difference between the ensembles has no physical meaning in the discussed problem.

From the alternatives I have presented here, only interpretation (c) is time-symmetric. This is the reason why I believe that it is the only reasonable candidate for analyzing the (time-symmetric) problem of measurements performed between two other measurements. [Interpretation (c) is equivalent to interpretation (iii) of Section 3.]

### 7. Kastner’s readings of the ABL rule

Kastner puts in quotation marks two possible readings of the ABL rule, “non-counterfactual” and “bona-fide counterfactual” (1999, p. 3[??]). As far as I can see the two quotations are not different: the first is a clarification of the second. In the papers of the TSQT the statements frequently appear in the compact form of the second quotation and the first quotation is the correct explanation of their meaning. The explicit notation of the ABL rule of Kastner’s Eq. 1’ is the correct characterization of both quotations.

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8If a random process chooses between the two possibilities, then the past before this process might be identical.

9If one is adopting our backward evolving quantum state, he can add that the systems are also different due to the backward evolving state between \( t \) and \( t_1 \).
Kastner, however, reads the second quotation differently. She claims that there is a “quantifier ambiguity” in the ABL formula (her Eq. 1). She proposes to add a parameter, \( C \), indicating the variable which was actually measured at time \( t \). It is not clear what Kastner means by “actually”, and this is crucial for her arguments (see especially her footnote 3).

The first possible reading of Kastner is that \( C \) is related to the counterfactual world, for which the formula should yield probabilities for \( x_j \). This reading is equivalent to interpretation (a) of Section 6, and as I showed there it is meaningless. In the framework of quantum theory observables usually do not possess values. There is no meaning for “probability of a value”, only for “probability of an outcome of a measurement”. If it is postulated that \( X \) is not measured (since another variable, \( C \), is measured instead), then it is meaningless to ask what is the probability for \( x_j \). In other words, the question is what parameters are kept fixed when the counterfactual world is considered. Kastner’s notation, \( P(x_j|a, b; C) \), suggests that \( a, b, \) and \( C \) are kept fixed in the counterfactual world, but then there is no meaning for probability of \( x_j \).

The second possible reading of Kastner (and I understand from the correspondence with her that this is the correct reading) is that \( C \) relates to the actual world and the formula is related to a counterfactual world, in which another variable, \( X \), is measured. In this case it is not clear what is kept fixed in the counterfactual world. If \( a \) and \( b \) are kept fixed, then how \( C \) can be relevant? The question is about the counterfactual world which is specified completely by \( a \) and \( b \), so the information about what has happened in another (the actual) world is irrelevant. Finally, it might be that Kastner assumes some hidden variables which kept fixed, i.e., identical for actual and counterfactual worlds. Then \( C \) is relevant because it characterizes the hidden variables: they are such that, given the intermediate measurement \( C \), the outcome \( b \) is obtained. This is a modification of interpretation (iv) of Section 3 (interpretation (b) of Section 6), in the latter \( C \) is the identity \( I \). While it is not immediately obvious that the ABL formula fails for interpretation (iv) (it does fail as it is proved in Section 3), it is obvious that the ABL formula cannot be true for this particular reading of Kastner’s proposal: the right hand side of Kastner’s Eq. 1” does not depend on \( C \). As explained in Section 3, the failure of the ABL rule for hidden variables readings is not surprising since the whole concept of hidden variables is time asymmetric. (It might be interesting to investigate the possibility of defining time-symmetric hidden variables.) In any way, there are no hidden variables in the TSQT and, therefore, this failure does not represent a problem.

It might be that Kastner and others have been mislead by the term “element of reality”. The words suggest something “ontological”, but in the TSQT “element of reality” is a technical term which describes a situation in which the outcome of a measurement is known with certainty, see Section 4 and Vaidman (1996b). The only meaning of an element of reality “the particle is in box \( A \)” quoted by Kastner is that “if searched in \( A \) it has to be there with probability 1”, nothing more. After quoting this, Kastner writes (1999, p. 4[?]): “This usage clearly implies that the properties of being in box \( A \) or being in box \( B \) are considered as possessed by the same pre- and post-selected particle.” The “same” means only that the two-state vector at time \( t \) is fixed. The counterfactual worlds corresponding for “being in \( A \)” and “being in \( B \)” for the “same” particles are different: in one world the particle is searched in \( A \) and in another it is searched in \( B \).
8. Kastner’s analysis of the Sharp and Shanks proof.

Kastner makes a distinction between two counterarguments which I presented in the two preprints (Vaidman, 1996a, 1997). From my point of view there is just one argument presented in different forms in the two preprints. The difference, as Kastner correctly noticed, is that in Vaidman (1997) (which is the revision of Vaidman 1996a) I do not focus on the possibility of a counterfactual with true antecedent. I still think that this possibility is a correct property of quantum counterfactuals. However, I realized that many readers were confused by this point and, since it is not central, I decided that I can persuade better without emphasizing this property.

Kastner writes that my counterarguments lead to what she calls “non-counterfactual” interpretation of the ABL rule “which is not under dispute”. She then proceeds with the analysis of the S&S argument focusing on “a failure of cotenability between the background conditions, \( S \), and the antecedent \( P \)”. It seems to me that, this failure of cotenability is very similar to my argument against the proof of S&S. They claimed that counterfactual interpretation of the ABL rule leads to predictions different from that of quantum theory. I claimed that their counterfactual interpretation has a logical error and therefore their proof is incorrect. Kastner shows that the left hand side of her Eq. 6 which is a part of the proof of S&S is false. From false logical statement one cannot claim to calculate correctly probability for an outcome of a measurement, so the problem is not with the ABL formula, as S&S claimed, but with the proof, as I claimed.

However, Kastner, after writing that due to the failure of the cotenability condition “it is clear that the counterfactual usage of the ABL rule fails”, continues with “detailed description of the steps employed in the S&S proof”, the “proof that the counterfactual interpretation of the ABL rule leads to predictions incompatible with quantum mechanics”. It seems to me that her detailed description leads to the same conclusion: the application of the counterfactual rule to the S&S example is inconsistent – exactly what I claimed – and I do not understand the short paragraph in which she “pinpoints the error in Vaidman’s counterargument II”.

One of the difficulties in understanding Kastner’s arguments in Section 2 and the apparent confusion in Section 3 follows from the usage of the concept of mixture. Kastner writes that that quantum “measurement yields a mixture”. The term “mixture” comes from statistics. It corresponds to a description with density matrix. However, a result of a quantum measurement on an ensemble is described by a particular list of outcomes, not by a density matrix. The latter describes some statistical properties of this list, but it is not the complete description. So, in order to proceed with Kastner’s arguments I replace every word “mixture” by “list of outcomes with statistical properties described by the mixture”. In particular, the “background condition” should be changed from a “mixture” to a list of particular outcomes.

The last paragraph of Section 2, leaves me with several options for understanding Kastner’s interpretation of the S&S example. The sentence “In view of the existence of actual results at \( t_2 \), such results are an indelible part of the history of world \( i \) and cannot be disregarded.” might suggest that the question is about a counterfactual world in which the measurement at time \( t \) is performed, but, nevertheless the outcomes of the measurements at \( t_2 \) are as in the actual world. (This is possible, although clearly a very
improbable situation, since the “mixture” description of these outcomes is very different from the “mixture” expected by the laws of quantum theory for the situation with the intermediate measurement.) For this question, the counterfactual calculations of S&S using the ABL formula are correct and the fact that it does not yield the value given by standard quantum-mechanical calculations, Kastner’s Eq. 10, is not a contradiction, because the latter corresponds to a different situation in which there is only pre-selection.

Another possible reading of Kastner’s paragraph is that in the counterfactual world with the intermediate measurement the outcomes of the measurement at time $t_2$, are different, but still the outcomes at $t_2$ in the actual world are relevant for calculating probabilities for the results of measurement at time $t$, ($t < t_2$). One can imagine such a situation if there are hidden variables which control the outcomes of measurements beyond the standard quantum formalism. I have discussed this possibility in (iv) of Section 3. Indeed, in this case, the ABL formula yields incorrect results, but this is not surprising since this situation is intrinsically time asymmetric: the actual and the counterfactual worlds coincide in the past, but not in the future, relative to time $t$. From private communications with S&S I understand that the main goal of their paper was to show exactly this, i.e., that “the ABL-rule did not have the implications for hidden variables interpretations of quantum mechanics that Albert et al. (1985) had claimed.” Although the conclusion is correct, I still think that the alleged proof in the paper of S&S is flawed; I believe that I have proved it correctly in (iv) of Section 3. Careful reading of S&S shows that they indeed focus on this limited issue. However, the title and the conclusions suggests criticism of the TSQT in much wider sense and lead their follower to attack all possible counterfactual interpretations in the framework of the TSQT.

9. Kastner criticism of the time-symmetrized counterfactuals.

Kastner again distinguishes between two, equivalent from my point of view, definitions of time-symmetrized counterfactuals (TSC) given in the two preprints (Vaidman 1996a, 1997). I see the difference only in phrasing and the generality of their applications: Definition 2 is applicable only for the ABL situation.

It seems that Kastner’s criticism follows from a misunderstanding (which I suspect came from her usage of the concept of mixture). Let me start with the analysis of Kastner’s criticism of Definition 1. She writes that in this definition I ignore the difference between mixtures in actual and counterfactual worlds. The mixtures describe the results of measurements at $t_1$ and $t_2$. But according to my definition “all measurements in a counterfactual world, excluding measurements at $t$, have the same outcomes as in the actual world”. Therefore, it is postulated by the definition that there is no difference between the mixtures. There is nothing to ignore. Kastner’s quotation from my work about the difference which “has no physical meaning” is related to the difference in the measurements (and outcomes) performed at time $t$. The outcomes of these measurements

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10 This is a correct criticism which however, “opens an open door”. The Letter of Albert et al. (1985) indeed leaves an impression that the authors make the discussion in the framework of the hidden variables theory. However, in their reply (Albert et al. 1986) to the criticism of Bub and Brown (1986) they clearly stated that they do not (or, at least, they do not now) think that the results of their Letter are applicable for hidden-variable theories.
depend on, but do not (by fiat) influence, the outcomes at $t_1$ and $t_2$.

Kastner proceeds by alleged proof that in the example of the S&S setup there is no counterfactual world satisfying my definition. She considers an ensemble of particles all pre-selected in a particular spin state which are subject to spin measurement in another direction at a later time. In the actual world no intermediate measurement is performed. She claims to calculate the fraction of particular outcomes of the second measurement in the ensemble. But she is not precise in her claim that the expression she gets in Eq. 20, $\cos^2(\theta_{ab}/2)$, is the fraction she wants to consider. Most probably it is not equal to $k/N$ for any integer $k$, where $N$ is the total number of particles in the ensemble. All that quantum theory tells us is that in such an experiment it is likely to expect that the fraction will be at the vicinity of $\cos^2(\theta_{ab}/2)$ of the size of the order $1/\sqrt{N}$. But this fraction can be any number between 0 and 1 of the form $k/N$. The calculation of this fraction in the counterfactual world is meaningless: it has been postulated that it is equal to that of the actual world. Kastner’s Eq. 21 yields the quantum mechanical estimate of what to expect in an alternative experiment. This calculation does not rule out identical fractions and the even stronger requirement of identical outcomes in the two experiments. The counterfactual world I consider is possible. Therefore, the definition is legitimate. The definition itself does not say for which problems and in which situation it will be applied. It is applied for problems in which the pre- and post-selected states are fixed. The question how often the post-selection succeeds is not under discussion.

Other misunderstandings appear in Kastner’s criticism of Definition 2. First, she augments the two-state vector notation with a specification which observable has been measured at time $t$. This is against the whole idea of the TSQT. The two-state vector is a complete description at time $t$ in the sense that it yields probabilities for all possible measurements (and the weak values for weak measurements) at time $t$. The two-state vector is specified by the results of measurement at times different than $t$: the measurement at $t$ does not have direct influence on the two-state vector at $t$. See Vaidman (1998b) for careful review of this concept.

Kastner, instead, makes her own definition of “time-symmetrically fixed” . Although she writes that it is “in the sense of Definition 2”, her definition has nothing in common with my proposal. In my proposal there is no question: will the system have the same two-state vector? The two-state vector is given by fiat and this is the “time-symmetrical fixing”. It seems that behind Kastner’s definition there is an idea of some kind of hidden variable: she discusses systems which “would still have the same two-state vector” even so some different measurement are performed at $t$. To have the same two-state vector is, in the current context, to have the same outcome of a measurement at $t_2$ in a situation in which a prioriy it is not certain due to quantum-mechanical laws. Standard quantum theory does not ensure the same outcome at $t_2$ even if the same measurement is performed at $t$ and the situation is different only because of a change in some unrelated variable. The example which Kastner considers demonstrates how she distorts Definition 2. The meaning of “the results of measurements performed on the system at times $t_1$ and $t_2$ are fixed” is that the outcomes of the measurements at $t_1$ and at $t_2$ in actual and counterfactual worlds are the same. In Figure 4 it is not true. Only the outcomes of the measurement at $t_1$ are the same.
10. Conclusions

The definition of counterfactuals in quantum theory which I propose is very simple-minded. It seems to me that if one reads it as it is, without trying to find something beyond it, then it is complete and unambiguous. I believe that the definition is helpful in resolving some controversies about quantum counterfactuals (see my attempts in this direction in Vaidman (1998c, 1998d)).

In principle, the counterfactual statement such as definition 2 is testable in a laboratory by creating a large ensemble of systems with measurements at \( t_1, t, \) and \( t_2; \) choosing the sub-ensemble (the pre- and post-selected ensemble) with fixed outcomes at \( t_1 \) and \( t_2; \) choosing (out of this sub-ensemble) the sub-ensemble with a particular measurement at \( t; \) and finally by making a statistical analysis of the outcomes of the measurement at \( t \) on this sub-ensemble. Because it is testable, philosophers might be reluctant to consider the construct which I define as counterfactual, in spite of the fact that formally it corresponds to the counterfactual (i) of Section 3. This is a semantic issue. I distinguish in Section 4 between situations in which only single statement of the form of Definition 2 is considered and situations in which several such statements, for different variables are considered all related to a single system. Since quantum theory does not allow simultaneous measurements of certain variables, in the latter situation the set of statements is, in fact, not testable.

I think that the most convincing example that the term counterfactual is appropriate and that the whole construction is useful is the example with three boxes (Vaidman 1996b). A single quantum system is weakly coupled to another system between two measurements at \( t_1 \) and \( t_2. \) For this system several counterfactual (in my sense) statements of the form of Definition 2 about intermediate measurements on this system with fixed outcomes of the measurements at \( t_1 \) and \( t_2 \) are true. This statements seems to be clearly counterfactual. I know that the measurements have not been performed because in the actual world the only interaction at time \( t \) is a weak coupling to another system. Moreover, since there is only one system and the measurements are incompatible, I know that they all could not have been performed. Finally, the knowledge of these counterfactual statements permits me to calculate the expected influence of the weak coupling on the other system which takes place in the actual world.

Kastner concludes that “counterfactual interpretation of the ABL rule is not valid in general”. I have showed that this conclusion follows from Kastner’s particular reading of the word “counterfactual”. I find that by and large Kastner supports my claim that counterfactuals in a sense which differs from mine are inconsistent in the framework of the TSQT. She rejects my approach saying that this is a “non-counterfactual” reading. I disagree about this semantic issue. More importantly, I disagree with Kastner’s claim that in several works in the framework of the TSQT the “bona fide” counterfactual reading of the ABL rule has been used (from which it follows that, since this reading is inconsistent, the results of these works, various “curious” quantum effects, are wrong). This claim, however was not proved and only stated in Kastner’s paper. Indeed, essentially only time-asymmetric examples were analyzed in her paper.

\[11\] Apart from “redefinition” of Definition 2, Kastner uses the word “ontological” in her paper. The TSQT does not make ontological claims. It is more a novel formalism than an interpretation.
I have showed that Kastner’s criticism of my definitions of TSC is unfounded. In her discussion of Definition 1 she erroneously considers “improbable” as “impossible” neglecting the fact that the TSC are applicable to pre- and post-selected situations which are usually “rare quantum events”. The Definition 2 she interprets in a particular sense. In this sense it is indeed “has no clear physical meaning”. I suspect that she rejects the literal interpretation of Definition 2, the one which I adopt, because she views it as non-counterfactual.

Kastner finds “an interesting special case in which the ABL rule may be correctly used in a counterfactual sense”, the one which corresponds to consistent histories (Griffiths, 1984). In the ABL case this corresponds to measurements of an observable for which either a pre-selected or post-selected state is an eigenstate and, therefore, the outcome is certain. This case seems to me extremely limited and very uninteresting. Even the case in which the outcome is certain, but it is not an eigenvalue of either the pre-selected or post-selected state as in the there-box example, does not fall into this category. I do not find fruitful a concept which is applicable to a very limited class of situations, especially if one can consider a similar alternative concept which is applicable for all situations.

Many quantum mechanical effects are dramatically different from phenomena which can be explained classically. Language and philosophy which were developed during the time that no one suspected quantum phenomena, have significant difficulties in defining and explaining quantum reality. This seems to be the reason for numerous controversies in this field. I believe that discussing and resolving these controversies is of crucial importance for understanding our world.

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