Coherent and incoherent charge transport in linear triple quantum dots

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Abstract
One of the fundamental questions in quantum transport is how charge transfer through complex nanostructures is influenced by quantum coherence. We address this issue for linear triple quantum dots by comparing a Lindblad density matrix description with a Pauli rate equation approach and analyze the corresponding zero-frequency counting statistics of charge transfer. The impact of decaying coherences of the density matrix due to dephasing is also studied. Our findings reveal that the sensitivity to coherence shown by shot noise and skewness, in particular in the limit of large coupling to the drain reservoir, can be used to unambiguously evidence coherent processes involved in charge transport across triple quantum dots. Our analytical results are obtained by using the characteristic polynomial approach to full counting statistics.

Keywords: quantum transport, quantum dots, quantum coherence, full counting statistics, quantum Zeno effect, long distance tunneling

(Some figures may appear in colour only in the online journal)

1. Introduction

Coherent superpositions of states are one of the fundamental aspects of quantum mechanics. They are an important resource for quantum information processing, quantum transport and quantum metrology. A tunable platform for the manipulation of coherent quantum states is provided by arrays of semiconductor quantum dots (QDs). Double quantum dot (DQD) systems have allowed the observation of superpositions of electronic states via coherent charge oscillations [1, 2] and sharp resonances in the current across the system [3]. DQDs have been analyzed to a great extent, unveiling intriguing transport phenomena such as Pauli spin blockade [4] or the Kondo effect [5]. They are also promising candidates for quantum information processing due to the coherent manipulation of charge or spin degrees of freedom [3, 6, 7].

The coupling of three QDs represents the next level of complexity toward the development of nanoelectromechanical systems [8, 9], and quantum information and simulation architectures [10–14]. The exceptional control and tunability recently achieved in triple quantum dots (TQDs) [15–18] allowed the experimental verification of long-distance tunneling (LDT) of charge and spin between peripheral QDs [19–21], opening the avenue to investigate different coherent phenomena. LDT and quantum interferences may determine the transport properties in the linear TQD, as demonstrated by the superexchange blockade [22] and by photon-assisted transitions in ac-driven systems [23–25].

Motivated by the recent experimental progress, we analyze in this paper the zero-frequency counting statistics of charge transport through a serially coupled TQD attached to electronic reservoirs, as schematically depicted in figure 1. Our main goal is to find the parameter conditions for which fluctuations in charge transport make it possible to distinguish between coherent and incoherent transport. Coherence is characterized by the non-diagonal elements of the reduced density matrix (DM) of the system, as we will further explain in section 2. We therefore compare a fully coherent model of the TQD with two descriptions that only exhibit reduced coherence. In the former, we assume that the QDs are tunnel-coupled and use...
For TQDs, shot noise has been analyzed mainly for triangular configurations [60–65], while for the linear array it was studied in presence of dephasing [26].

The paper is structured as follows: in section 2 we introduce the theoretical models for the linear TQD in a transport configuration. We first present the fully coherent description in terms of a Lindblad master equation, describing the dynamics of the reduced density matrix of the TQD. From the quantum master equation we derive a Pauli rate equation for the populations of the electronic sites. As an intermediate description we introduce a model with pure dephasing, which results in decaying coherences of the density matrix of the system. In section 3, we present our results for the counting statistics of the charge transport for each model. We center our attention on the behavior of the Fano factor and skewness with and without coherences. In section 4, we suggest a procedure for the experimental verification of the presence of quantum coherence in the TQD. Then in section 5 we emphasize some aspects of our results which are different from the known FCS for DQDs. This section is followed by the Conclusions.

2. Different models and method

The TQD consists of three single-level quantum dots arranged in series and connected to source and drain electron reservoirs (see figure 1). Similar to [26], we assume strong Coulomb blockade such that the TQD can be occupied with at most one extra spinless electron. The relevant states of the system are thus the empty state |0⟩ and the single-particle states |{1}, [2], [3⟩, where |i⟩ describe the ith QD being occupied. We consider the large bias voltage regime, with all the electronic states inside the conduction window, and tunneling of electrons is allowed from the source to QD1 and from QD3 into the drain.

A rigorous framework for the quantification of coherence has been provided in recent theoretical work, defining coherence measures in terms of functionals of the density matrix [66, 67]. These measures are generally related to diagonal states (free or ‘incoherent’ states) in a given basis. These concepts have stimulated our study of the TQD with and without off-diagonal elements of the density matrix, as well as with decaying coherences, as we will describe in this section. In this work, coherence in the TQD is characterized by finite non-diagonal elements of the DM in the local basis { |i⟩ }.

2.1. Fully coherent model

For the fully coherent description of the TQD it is considered that neighboring dots are tunnel coupled, see figure 1. The Hamiltonian representing the array is then \( h = 1 \) taken throughout the paper

\[
H = \sum_{i=1}^{3} \varepsilon_i d_i^\dagger d_i + \sum_{i,j}^{3} \gamma_{ij} d_i^\dagger d_j + \text{h.c.}
\]  

(1)

where \( t_{ij} \) are tunneling couplings between QDs and \( d_i^\dagger \) (\( d_i \)) is the creation (annihilation) operator for an electron in the \( i \)th QD.
In the large bias voltage regime and for strong Coulomb blockade, the time evolution of the reduced DM $\rho(t)$ of the TQD in a transport configuration reads [26]

$$\dot{\rho}(t) = L_\rho \rho(t) = -i[H, \rho(t)] + \Gamma_1 D(d_1^\dagger) \rho(t) + \Gamma_2 D(d_2) \rho(t),$$

(2)

with the dissipators of Lindblad form $D(A) \rho = A \rho A^\dagger - \frac{1}{2} A^\dagger A \rho - \frac{1}{2} \rho A^\dagger A$. The superoperators $D(d_1^\dagger)$ and $D(d_2)$ describe irreversible tunneling of electrons from the source and into the drain, respectively, with rates $\Gamma_1$ and $\Gamma_2$. In the wide-band approximation and infinite bias limit the rates are energy independent and, moreover, the Born–Markov approximation with respect to the coupling to the reservoirs is essentially exact [70–72]. In the chosen basis, the reduced DM $\rho$ in equation (2) contains diagonal (occupation probabilities for each QD) and non-diagonal (coherences) elements. Detailed expressions for the elements $\rho_{ij}(t)$ are explicitly given in appendix A.

2.2. Pauli rate equation

For incoherent rate study we evolve the electronic linear TQD with only diagonal elements of the density matrix. From the fully coherent DM, equation (2), we derive an alternative model for the TQD in form of a Pauli rate equation (RE) for the occupation probabilities of the electronic states, arranged in the vector $P = (\rho_{11}, \rho_{22}, \rho_{33}, \rho_{00})^T$. The RE (derived in appendix A) has the form

$$\dot{P}(t) = L_P P(t),$$

(3)

with the generator of the dynamics $L_P$ given by

$$L_P = \begin{pmatrix} -\sigma_1 & \Gamma_1 & \Gamma_3 \\ \Gamma_2 & -\sigma_2 & \Gamma_2 \\ \Gamma_3 & \Gamma_3 & -\sigma_3 - \Gamma_2 \\ 0 & 0 & \Gamma_2 - \Gamma_3 \end{pmatrix}$$

(4)

and $\sigma_j = \sum_{i \neq j} \Gamma_i$. The quantities $\Gamma_j = \Gamma_i$ play the role of incoherent transition rates between electronic states, see figure 1. Their dependence on the system parameters is given by

$$\Gamma_1 = 4r_1^2+4r_2^2+4(\varepsilon_{12}-4\delta_1+2\varepsilon_{23}+3\delta_1)/\Delta,$$

$$\Gamma_2 = 16r_2^2r_3^2+2(\varepsilon_{13}^2-4\delta_1+\varepsilon_{23}+3\delta_1)/\Delta,$$

$$\Gamma_3 = 4r_3^2+4\varepsilon_{13}^2-4\delta_1+\varepsilon_{23}+3\delta_1)/\Delta,$$

(5)

with $\varepsilon_{ij} = |\varepsilon_i - \varepsilon_j|$ the difference between energy levels, $\delta_1 = \varepsilon_{12}(\varepsilon_{13} + \varepsilon_{23})$, $\delta_2 = \varepsilon_{23} - \varepsilon_{12}$ and

$$\Delta = \varepsilon_{12}^2 \Gamma_2 + 4\varepsilon_{13}^2 \varepsilon_{23}^2 \Gamma_2 + 2\varepsilon_{12}^2 + \delta_1^2$$

$$+ 16\varepsilon_{12}^2 \varepsilon_{23}^2 \delta_2^2.$$

Expressions (5) result from eliminating the coherences from the DM, and are therefore not equivalent to rates obtained with Fermi’s golden rule [73]. We can also observe that the rates $\Gamma_j$ are independent of the coupling to the source $\Gamma_2$ while explicitly include the coherent interdot tunneling amplitudes $t_{ij}$. Moreover, $\Gamma_3$ introduces direct transitions between the peripheral dots QD1 and QD3 [73].

We mention that there is not a unique rate equation describing the TQD. In particular, different generators $L_L$ (or equivalently different rates $\Gamma_j$) in the RE may yield identical statistics in the steady-state [74]. The FCS obtained from the RE in equation (3) are referred to as the incoherent transport properties.

2.3. Pure dephasing

We also model charge transport with degrading coherences of the DM by introducing pure dephasing on the electronic sites of the TQD. We consider electron-phonon interactions as the main source of dephasing [75, 76], and neglect relaxation processes by significantly detuning the central QD [26].

Pure dephasing affects the DM in equation (2) through an additional Lindblad term such that the time evolution of the DM in presence of dephasing reads

$$\dot{\rho}(t) = L_\rho \rho(t) = L_\rho \rho(t) + \gamma \sum_i D(n_i) \rho(t),$$

(6)

with $n_i = d_i^\dagger d_i$. We assume that the dephasing rate $\gamma$, resulting in exponentially decaying coherences of $\rho(t)$, is equal for all three QDs.

2.4. FCS of charge transport

The impact of coherence on the steady-state transport properties of the linear TQD is studied by analyzing the average current, the zero-frequency noise and the skewness. The central quantity in zero-frequency FCS is the probability distribution $p(N)$ for the number of charges $N$ transferred into the drain reservoir within a long time interval $\Delta \tau$. The distribution $p(N)$ is conveniently characterized by its cumulants $C_k$.

We use the characteristic polynomial approach [74] to determine the time-scaled cumulants $c_k = C_k/\Delta \tau$, and from them we obtain the properties of interest: the mean current $I = ec_1$ and the shot noise $S = 2ec_2$ (with $e$ the electron charge). The noise strength is characterized by the Fano factor $F(2) = S/2ec_1$, which determines if the distribution is super-Poissonian, $F(2) > 1$, or sub-Poissonian, $F(2) < 1$. The third cumulant is related to the skewness of the distribution $p(N)$, parametrized here by the Fano factor $F(3) = ec_3$. Analytical expressions for the FCS obtained for the aforementioned models are explicitly given in appendix B.

3. Comparison of coherent and incoherent transport

For clarity we henceforth consider identical tunnel couplings between neighboring QDs, $t_{ij} = t$. The TQD is assumed to be in a $\Lambda$- or $V$-type configuration with QD1 and QD3 in resonance, $\varepsilon_1 = \varepsilon_3$, and the energy difference between the central and outer dots is determined by the detuning $\varepsilon = |\varepsilon_1 - \varepsilon_2|$.
3.1. Fully coherent versus diagonal model

The steady-state occupation probabilities obtained from the fully coherent DM in (2) and the RE (3) are equal and therefore both models yield the same stationary average current

\[ I_0 = \frac{4t_0^4\Gamma D_3}{4t_0^4(3\Gamma S + \Gamma D) + 2t_0^2\Gamma S\Gamma D + \Gamma S\Gamma D^2\varepsilon^2}. \tag{7} \]

Note that the RE (3) has fewer degrees of freedom than the DM in (2) and (6); therefore even if the coherences in the fully coherent model and in the description with pure dephasing vanish, one cannot expect the same statistics as for the Pauli RE. Hence, equation (7) stresses the importance of understanding the role of the coherences in electronic transport by analyzing the current fluctuations and higher-order cumulants.

The current \( I_0 \) is shown in figures 2(a) and (d) as a function of the incoherent coupling to the drain \( \Gamma_D \) and for different energy detuning \( \varepsilon \) (parameters are in units of \( t_c \)). We observe that \( I_0 \) is peaked in the regime of small to intermediate coupling to the drain, \( \Gamma_D \lesssim 2t_c \), and is suppressed as \( \Gamma_D \) increases.

For the fully coherent model (2), we interpret this behavior in terms of the charge dynamics [26]: for large detuning of the central QD and \( \Gamma_D \ll 2t_c \), coherent LDT of charge from QD1 to QD3 provides the main mechanism for transport. On the other hand, for large coupling to the drain \( \Gamma_D \gg t_c \), the system undergoes a counterpart of the quantum Zeno effect in quantum transport [77]. Large values of \( \Gamma_D \) are considered to be equivalent to a continuous measurement of the occupation of QD3, projecting the charge into QD1 and QD2 and therefore blocking the current. For both limiting cases, the dynamics of the TQD is reduced to an effective DQD, formed by QD1 and QD3 for \( \Gamma_D \ll 2t_c \) and by QD1 and QD2 for \( \Gamma_D \gg t_c \) [26].

![Figure 2](image1.png)

**Figure 2.** Steady-state transport for the TQD: average current \( I_t \), Fano factor \( F^{(2)} \) and normalized skewness \( F^{(3)} \) as a function of \( \Gamma_D \) for the fully coherent DM (solid line), RE (dashed), and DM with dephasing (dotted lines). The detuning of the central QD is set to \( \varepsilon = 2t_c \) ((a)-(c)) and \( \varepsilon = 4t_c \) ((d) and (e)). The insets in (c) and (f) show details of \( F^{(3)} \) in the regime \( \Gamma_D \ll t_c \). (We used \( \Gamma_\text{S}t_c = 1/2 \) and \( \gamma = 1/4, 2/3 \).)

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![Figure 3](image2.png)

**Figure 3.** Ratio of the incoherent and coherent Fano factors \( F^{(2)} / F^{(0)}_c \) as a function of the detuning \( \varepsilon \) and the coupling to the drain \( \Gamma_D \). The separation between enhanced and reduced fluctuations due to coherence (dashed line) is given in (10) (we used \( \Gamma_\text{S}t_c = 1/2 \)).
Let us compare these findings with the mechanisms provided by the diagonal model, i.e. rate equation. For $\Gamma_D \lesssim 2\epsilon$, the transition rate $I_3$ between QD1 and QD3 accounts for most of the charge transfer. Thus, similar to the coherent case, transitions between the outermost QDs dominate transport. In contrast, for intermediate and large values of $\Gamma_D$ the rates $I_2$ and $I_3$ determine most of the electronic transitions, indicating incoherent charge transfer. The incoherent transition rates $I_y$ in (5) are further discussed in appendix D.

We can therefore conclude that the underlying mechanism yielding a current resonance in the regime $\Gamma_D \lesssim 2\epsilon$ is similar for coherent (2) and diagonal (3) descriptions of the TQD and is related to charge transfer from QD1 to QD3. However, for sufficiently large coupling to the drain $\Gamma_D \gg \epsilon$, the mechanisms involved in transport are different.

Unlike the current, the high-order cumulants are sensitive to quantum coherence and therefore different for the two models. The coherent Fano factor $F_0^{(2)}$ and the incoherent one $F_0^{(2)}$ read (see appendix B for details)

\[ F_0^{(2)} = 1 + \frac{2c_4^2\Gamma_4^2(4c_4^2[4c_4^2(\Gamma_S - 3\Gamma_D) - 1] + 2[2\Gamma_D] + 16c_4^6\Gamma_4^4 + 4c_4^2\Gamma_4^2(5\Gamma_S + \Gamma_D)e^2)}{[4c_4^2(3\Gamma_S + \Gamma_D) + 2c_4^2\Gamma_4^2 + 4\Gamma_4^2\Gamma_4^2e^2]^2}, \]

and

\[ F_0^{(2)} = 1 + \frac{2c_4^2\Gamma_4^2(4c_4^2[4c_4^2(\Gamma_S - 3\Gamma_D) - 1] + 2[2\Gamma_D] + 16c_4^6\Gamma_4^4 - 4c_4^2\Gamma_4^2(\Gamma_D + \Gamma_D)e^2)}{[4c_4^2(3\Gamma_S + \Gamma_D) + 2c_4^2\Gamma_4^2 + 4\Gamma_4^2\Gamma_4^2e^2]^2}. \]

Both quantities can be sub- or super-Poissonian depending on the relative values of the energy detuning $\epsilon$, the interdot tunneling coupling $\epsilon$, and the couplings to the reservoirs $\Gamma_S$ and $\Gamma_D$.

Due to the intricate dependence of equations (8) and (9) on the parameters of the TQD, we explore first the ratio between the coherent normalized skewness to lowest order in $\epsilon$ at this coupling limit, while the incoherent one in the same coupling limit shows $\epsilon$, and depicted as a dashed line in figure 3, is determined to be independent of $\Gamma_S$ and $\Gamma_D$.

This ratio is mapped out in figure 3 as a function of $\Gamma_D$ and $\epsilon$ for a representative value of $\Gamma_S$. We observe two complementary regions in which the relative current fluctuations are enhanced and reduced owing to coherence. The former corresponds to the regime $\Gamma_D \lesssim 2\epsilon$ and $\epsilon \gg \epsilon$, where transport is determined by charge transfer between peripheral QDs. The latter region corresponds to $\Gamma_D \gg \epsilon$, where the coherent system enters the Zeno regime.

The separation between the two regions, defined by $F_0^{(2)}/F_0^{(2)} = 1$ and depicted as a dashed line in figure 3, is determined by the relation

\[ \epsilon = \frac{2\sqrt{3}/\epsilon}{\sqrt{\Gamma_D - 3\epsilon^2}} \]

independent of $\Gamma_S$. For fixed coupling $\epsilon$, and detuning $\epsilon$, equation (10) defines a particular value for the coupling to the drain $\Gamma_D = 2\epsilon\sqrt{2 + 3(\epsilon/\epsilon)^2}$.

at which the Fano factor does not allow to discern the nature of charge transport. To discriminate between coherent and incoherent transport one should therefore choose values of $\Gamma_D$ markedly different from $\Gamma_D$.

To have a better understanding of the effects of coherence, we show the Fano factor $F_0^{(2)}$ and the normalized skewness $F_0^{(2)}$ for increasing $\Gamma_D$ and different detuning $\epsilon$ in figure 2. For $\Gamma_D \lesssim 2\epsilon$, both coherent (8) and incoherent (9) Fano factors are sub-Poissonian and show a local minimum. For the two values of the energy detuning shown here, we observe that coherence suppresses the current fluctuations. The skewness for both the coherent DM (2) and the RE (3) exhibit a similar qualitative behavior, with $F_0^{(2)}$ being more sensitive to the detuning $\epsilon$ (see figures 2(e) and (f)).

In the opposite regime $\Gamma_D \gg \epsilon$, we obtain interesting and explicit results for the high-order cumulants. Expanding the incoherent Fano factor $F_0^{(2)}$ to lowest order in $\Gamma_D^{-1}$ we find

\[ F_0^{(2)} = 1 - \frac{2\epsilon^2}{(2\epsilon^2 + \epsilon^2)^2} \left[ 1 - \frac{8\epsilon^4}{\Gamma_D^2(2\epsilon^2 + \epsilon^2)^2} \right]. \]

Equation (12) reveals that $F_0^{(2)}$ is only sub-Poissonian in this limit, independently of $\epsilon$, as exemplified in figure 2. In contrast, an expansion of $F_0^{(2)}$ in the same coupling limit shows that it tends to be super-Poissonian, as we find

\[ F_0^{(2)} = 1 + \frac{2\epsilon^2}{(2\epsilon^2 + \epsilon^2)^2} \left[ 1 - \frac{8\epsilon^4}{\Gamma_D^2(2\epsilon^2 + \epsilon^2)^2} \right]. \]

Thus, coherence increases fluctuations in the regime of large coupling $\Gamma_D$. Similarly, expansion of the incoherent and coherent normalized skewness to lowest order in $\Gamma_D^{-1}$ yield

\[ F_0^{(3)} = 1 - \frac{6\epsilon^2\epsilon^2\epsilon_0^2(4\epsilon^2 + \epsilon_0^2)}{\epsilon^2} - \frac{24\epsilon^2\Gamma_D^2}{\epsilon^4}, \]

\[ F_0^{(3)} = 1 + \frac{6\epsilon^2\epsilon^2(\epsilon^2 + \epsilon_0^2)}{\epsilon^2} + \frac{24\epsilon^2\Gamma_D^2}{\epsilon^4}. \]

where $\epsilon_0 = (\epsilon^2 + 8\epsilon^2)(\epsilon^2 + 2\epsilon^2), \epsilon_0 = 2\epsilon^2 + \epsilon^2$ and $\beta = \epsilon^2 + \epsilon^2$. An analysis of equations (14) and (15) reveals that $F_0^{(3)} < 1$ at this coupling limit, while $F_0^{(3)} > 1$ can exhibit a super-Poissonian behavior. Hence, $F_0^{(3)} > F_0^{(3)}$ for $\Gamma_D \gg \epsilon$, as shown in figure 2.

### 3.2. Coherent model and effects of dephasing

As a description between the fully coherent and diagonal models, we analyze the effects of dephasing on transport through the linear TQD. Exponential damping of the
Figure 4. (a) The coherent normalized skewness $F^{(3)}_1$ and $F^{(3)}_2$, (b) Fano factors $F^{(2)}_1$ and $F^{(2)}_2$, and (c) the corresponding currents, for two different values of the coupling to the source $\Gamma_{S,1} = 2 \mu\text{eV}$ and $\Gamma_{S,2} = 4 \mu\text{eV}$. Measurements of super-Poissonian skewness and noise at $\Gamma_D > 47 \mu\text{eV}$ and $\Gamma_D > 24 \mu\text{eV}$ reveal that transport is fully coherent. (Parameters in the main text.)

off-diagonal elements of the DM in (6) yields current fluctuations which differ from the fully coherent DM (2).4

The current in presence of dephasing $I_0$ was found to be [26]

$$I_0 = \frac{2e\gamma_0^c \Gamma_0 \Gamma (4\gamma + \Gamma_D)}{D_1 + D_2 D_3},$$

with $\Gamma_0 = 2\gamma + \Gamma_D$ and

$$D_1 = \Gamma_0 \Gamma (6\gamma + \Gamma_D) \varepsilon^2,$$
$$D_2 = \gamma_0^c + 2\gamma_0^c (4\gamma + \Gamma_D),$$
$$D_3 = \Gamma_0 \Gamma (3\gamma + \Gamma_D) + 2\gamma_0^c (3\Gamma_D + \Gamma_D).$$

The current (16) as a function of $\Gamma_D$ and for different dephasing rates $\gamma$ is shown in figure 2. Compared to the coherent current $I_c$ in equation (7), $I_0$ is reduced in the regime $\Gamma_D \lesssim 2\varepsilon$ since dephasing destroys the coherent LDT between QD1 and QD3. For $\Gamma_D \gg \varepsilon$, dephasing partially counteracts the coherent trapping of charge between QD1 and QD2 due to the Zeno effect and consequently the current is dephasing-enhanced. We observe that the current is raised for increasing values of $\gamma$, see figures 2(a) and 4(d). The parameter regime for which $I_0$ equals the current $I_c$ is defined by the relation [26]

$$\varepsilon = \sqrt{6t_c \left[ (2\gamma + \Gamma_D)^2 + 2\gamma_0^c (4\gamma + \Gamma_D) \right] / \Gamma (2\gamma + \Gamma_D)^2 - 8\gamma_0^c (4\gamma + \Gamma_D)} \right]^{1/2}. \tag{18}$$

Interestingly, in the limit of small dephasing $\gamma \ll \varepsilon$, equation (18) reduces to a simpler expression, namely equation (10) and determines the conditions at which the different models studied here produce the same average current.

As for the current fluctuations, the Fano factor $F^{(2)}_0$ and skewness $F^{(3)}_0$ with dephasing are shown in figure 2 for increasing $\Gamma_D$ and for different dephasing rates $\gamma$. In the regime $\Gamma_D \lesssim 2\varepsilon$, it can be seen that the behavior of $F^{(2)}_0$ is qualitatively similar to $F^{(2)}_0$ and $F^{(2)}_0$ and a strong dependence on the value of the dephasing rate $\gamma$ is not observed. In the opposite limit $\gamma \Gamma_D > \varepsilon^2$, we expand $F^{(2)}_0$ to lowest order in $\Gamma_D^{-1}$ and find that the fluctuations are always sub-Poissonian

4 Please note how our treatment of pure dephasing is different from previous studies of FCS in a DQD, where dephasing was phenomenologically introcued yielding the same average current as the obtained with a coherent DM [32].

$F^{(2)}_0 = 1 - \left[ \beta \gamma^2 + 3\gamma (4\gamma + \Gamma_D) \right] / \gamma \Gamma_D \Gamma$. \tag{19}

similarly to the diagonal case. Hence, incoherent events involved in transport result in a reduction of the current fluctuations in the regime of large coupling to the drain. A similar behavior is obtained for the skewness since $F^{(3)}_0 < F^{(3)}_0$ for $\Gamma_D > \varepsilon^2$, as shown in figure 2. Furthermore, its expansion around $\Gamma_D^{-1}$ reveals that $F^{(3)}_0$ is only sub-Poissonian.

$$F^{(3)}_0 = 1 - 3 \left[ \beta \gamma^2 + 3\gamma (4\gamma + \Gamma_D) \right] / \gamma \Gamma_D \Gamma. \tag{20}$$

We finally note that all three models yield essentially the same current and higher-order cumulants for couplings to the drain in the vicinity of $\Gamma_D$ defined in (11) (see left to right panels in figure 2). This result seems to be independent of the physical mechanism that impairs the coherences of the linear TQD. Measurements of higher-order cumulants around $\Gamma_D$ thus, do not suffice to distinguish between coherent and incoherent descriptions.

4. Experimental verification

The sensitivity to coherence exhibited by the high-order cumulants can be used to experimentally verify the occurrence of quantum coherence in a serially coupled TQD. Coherence is present in the system if measurements of the Fano factor and normalized skewness are super-Poissonian in the limit of large coupling to the drain $\Gamma_D \gg \varepsilon$.

In order to determine suitable parameters for detecting super-Poissonian noise, we expand the expression for the coherent Fano factor to lowest order in $\Gamma_D$ defined in (11) (see left to right panels in figure 2). This result seems to be independent of the physical mechanism that impairs the coherences of the linear TQD. Measurements of higher-order cumulants around $\Gamma_D$ thus, do not suffice to distinguish between coherent and incoherent descriptions.

The parameters for which $F^{(3)}_0 > 1$ in the limit of large $\Gamma_D$ determined from equation (15) as

$$\Gamma_1 \Gamma_D > 4\varepsilon^2 \left[ \alpha^2 + 2\beta \varepsilon^2 + 3\gamma (4\gamma + \Gamma_D) \right] / \alpha \varepsilon^2 (4\varepsilon^2 + \varepsilon^4). \tag{21}$$

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Please note how our treatment of pure dephasing is different from previous studies of FCS in a DQD, where dephasing was phenomenologically introcued yielding the same average current as the obtained with a coherent DM [32].
Note that the conditions given in (22) imply that equation (21) is satisfied and therefore measurements of super-Poissonian skewness are accompanied by $F_1^{(3)}>1$.

Considering representative parameters for lateral QDs [1, 19, 20], we conceive the following experimental scheme: the current and Fano factors are measured with two different values of the incoherent coupling to the source, $\Gamma_{c1} = 2 \mu eV$ and $\Gamma_{c2} = 4 \mu eV$. In both realizations the energy detuning $\varepsilon = 6 \mu eV$ and the interdot coupling $t_c = 3 \mu eV$ are kept constant, while the coupling of QD3 to the drain $\Gamma_D$ is varied in the range $0-55 \mu eV$. In accord with expression (22), the measured skewness $F_1^{(3)}$ and $F_2^{(3)}$ are expected to be super-Poissonian for $\Gamma_D > 47 \mu eV$ and $\Gamma_D > 24 \mu eV$, respectively, as shown in figure 4(a). Furthermore, equation (21) predicts that the fluctuations $F_1^{(2)}$ and $F_2^{(2)}$ are sub-Poissonian for $\Gamma_D > 33 \mu eV$ and $\Gamma_D > 17 \mu eV$, see figure 4(b).

Observations of sub-Poissonian cumulants in this regime reflects therefore the occurrence of a mechanism which brings the TQD into a classical, incoherent limit. We verify that the coupling to the drain for incoherent transport $\Gamma_D$, equation (11), is far from the relevant values of $\Gamma_D$ to detect enhanced high-order cumulants. For our parameters, this value corresponds to $\Gamma_D \approx 10 \mu eV$.

For large coupling to the drain, the current $I_D$ in (7) obtained with the coherent DM tends to be suppressed, see figure 4(c). Nevertheless, the chosen parameters predict values of the measured currents $I_1$ and $I_2$ detectable in present technologies for arrays of lateral QDs. In addition, increased currents with respect to (7) can be used to evidence the effect of decaying coherences since the model including pure dephasing predicts enhanced transport for sufficiently large $\Gamma_D$ and finite detuning $\varepsilon$ [26].

5. Comparison with double quantum dots

Having determined the parameters for which coherent transport across the TQD can be experimentally verified, in the following let us mention some similarities and differences observed with the FCS obtained for serially coupled DQDs in the Coulomb blockade regime, stemming from the level structure and the internal dynamics of the system.

We note first that super-Poissonian noise owing to coherence in a DQD appears for finite detuning between the two energy levels and asymmetric coupling to the leads, in particular having $\Gamma_D < \Gamma_1$ [33]. Similarly, as shown in expression (21) above, the TQD studied here exhibits super-Poissonian noise for asymmetric rates, restricted however to the limit of very large coupling to the drain in which FCS rely on the total coherence of the system: although at this coupling limit the Zeno effect suppresses the current, the charge is 

\textit{coherently} tunneling between QD1 and QD2; as a result of these coherent evolution, noise is enhanced. In contrast, increasing values of $\Gamma_D$ in a DQD freeze the charge in QD1, suppressing both the current and noise [32, 33]. Furthermore, coherence in the TQD results unambiguously in the super-Poissonian behavior of both, noise and skewness in the regime $\Gamma_D \gg t_c$; this feature has not been necessarily observed in DQDs.

On another hand, in the limit of small to moderate coupling to the drain $\Gamma_D \lesssim 2t_c$ and large detuning of QD2, the LDT of charge between peripheral dots in the coherent description of the TQD offers an extra channel for transport with sub-Poissonian fluctuations. In this limit, the coherent TQD can be effectively described by a DQD formed by QD1 and QD3, which are in resonance to each other (since we consider $\varepsilon_1 = \varepsilon_2$); thus, the system resembles elastic transport across a DQD (and other two-level systems, as e.g. quasiparticle transport in a Cooper-pair box [78]), in which coherence strongly suppresses the noise [32, 47]. Finally, we can also mention that higher-order cumulants in serially coupled DQDs have been analyzed to be more sensitive to coherence for double occupation or under the effects of finite bias [32, 34]. In particular, it has been shown that skewness is sensitive to temperature [42, 53] and that it can also reveal information about internal frequencies of the DQD [49] and the effects of charge detectors [59]; these aspects can be further studied for the linear TQD.

6. Conclusions

We have analyzed the FCS of a serially coupled TQD in a transport configuration, and addressed the question on how to discern whether transport is due to quantum coherence.

A density matrix approach was used to model the coherent tunneling of charge between electronic sites, while transport without coherences was described in terms of a Pauli rate equation. The effects of pure dephasing were also included in order to consider an intermediate description with decaying coherences of the density matrix.

Our analytical results reveal that the fully coherent model and the rate equation yield the same current, while higher-order cumulants (shot noise and skewness) differ, demonstrating that transport in the TQD is sensitive to the coherences of the DM. In particular, coherence enhances the current fluctuations for sufficiently large coupling to the drain and finite energy detuning, resulting in super-Poissonian noise and skewness. In contrast, the rate equation and the description with dephasing yield sub-Poissonian Fano factors. Therefore measures of shot noise and skewness in this coupling limit can be used to evidence coherence in charge transport across the TQD.

We furthermore found the conditions for which it is not possible to discern the nature of transport since the three models yield basically the same statistics. This regime can be reached by varying a single external parameter, namely the incoherent coupling to the drain. Our studies are not a straightforward extension of that for DQDs since the level configuration in larger arrays of QDs provide different channels for transport and other coherent phenomena. Although our findings are not directly extended to larger configurations of QDs, they are relevant in order to examine the effects of coherence in systems with higher degrees of freedom (e.g. the recently demonstrated quadruple quantum dot in the few electron regime [79]) or in other transport systems with controllable parameters.
Acknowledgments

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Appendix A. Derivation of the rate equation for the linear TQD

We derive a Pauli rate equation (RE) for the occupation probabilities of the electronic states of the linear TQD starting from the time evolution of the fully coherent DM in (2). Explicitly, the equations of motion of the populations of the DM read (with $t_i = t_c$, $\varepsilon = \varepsilon_1$ and $\varepsilon = |\varepsilon_1 - \varepsilon_2|$)

$$
\begin{align}
\rho_{11}(t) &= i\eta_1[\rho_{11}(t) - \rho_{22}(t)] + \Gamma_0 \rho_{00}(t), \\
\rho_{22}(t) &= -i\eta_1[\rho_{22}(t) - \rho_{11}(t)] + \rho_{23}(t) + \rho_{32}(t), \\
\rho_{33}(t) &= -i\eta_1[\rho_{33}(t) - \rho_{22}(t)] - \rho_{33}(t), \\
\rho_{00}(t) &= -\Gamma_0 \rho_{00}(t) + \Gamma_0 \rho_{33}(t), \\
\end{align}
$$

and for the coherences we have

$$
\begin{align}
\rho_{12}(t) &= i\eta_1[\rho_{12}(t) + \rho_{21}(t)] - i\varepsilon \rho_{12}(t), \\
\rho_{23}(t) &= -i\eta_1[\rho_{23}(t) + \rho_{32}(t)] + (i\varepsilon - \Gamma_0/2)\rho_{23}(t), \\
\rho_{31}(t) &= i\eta_1[\rho_{31}(t) - \rho_{12}(t)] - \Gamma_0 \rho_{31}(t)/2. \\
\end{align}
$$

As we are interested in the steady-state properties of the TQD we calculate the steady state of the system in the long-time limit, after the transient effects have disappeared. We then set the off-diagonal elements of the DM to the steady-state and obtain a stationary condition relating the coherences to the occupation probabilities [73]. This is achieved by setting to zero the time derivatives of the coherences from equation (A.2), $\dot{\rho}_{ij} = 0$. The solution is substituted in the equations of motion for the populations $\rho_i(t)$, (A.1). We obtain then the RE for the occupation probabilities as

$$
\begin{align}
\dot{\rho}_{11}(t) &= \Gamma_{12}\rho_{22}(t) + \Gamma_{13}\rho_{33}(t) + \Gamma_0 \rho_{00}(t) - (\Gamma_{21} + \Gamma_{31})\rho_{11}(t), \\
\dot{\rho}_{22}(t) &= \Gamma_{21}\rho_{11}(t) + \Gamma_{23}\rho_{33}(t) + (\Gamma_{12} + \Gamma_{32})\rho_{22}(t), \\
\dot{\rho}_{33}(t) &= \Gamma_{31}\rho_{11}(t) + \Gamma_{32}\rho_{22}(t) + \Gamma_0 \rho_{00}(t) + (\Gamma_{23} + \Gamma_{13} + \Gamma_0)\rho_{33}(t), \\
\dot{\rho}_{00}(t) &= -\Gamma_0 \rho_{00}(t) + \Gamma_0 \rho_{33}(t), \\
\end{align}
$$

where we labeled the quantities $\Gamma_i$, defined explicitly in equations (5) in the main text, and which we interpret as transition rate probabilities between electronic sites. The closed set of equations (A.3) for the time evolution of the occupation probabilities, arranged as $\mathbf{P} = (\rho_{11}, \rho_{22}, \rho_{33}, \rho_{00})^T$, can be written in a matrix form yielding the generator (4).

The derivation of the RE (3) fits into a procedure formally mapping coherent dynamics of multilevel systems to kinetic equations, see [73]. It also turns to be equivalent to perform a rate equation approximation, which is essentially exact for the steady-state since the population differences are constant [80, 81]. Moreover, the regime for large coupling to the drain $\Gamma_0 \gg t_c$ with finite $\varepsilon$ is the most relevant in our work, making the coherences to decay faster than the populations difference. Thus, the Pauli RE (3) yields reliable results, provided that we restrict ourselves to the steady-state transport in the TQD.

Appendix B. Counting statistics for the linear TQD

To determine the zero-frequency counting statistics of the serially coupled TQD, we first introduce the counting variable $\xi$ by the substitution $d\rho_{ij} d\tau \rightarrow e^{i\xi} d\rho_{ij} d\tau$ in the models studied here, equations (2), (3) and (6). We thereby transform the generators of the dynamics into the deformed generators $\mathcal{L}_i \rightarrow \mathcal{L}_{i, \xi}$ with $i = 0, 1, \phi$. Then we apply the characteristic polynomial approach to analytically calculate the cumulants of the distribution $p(N)$, see [74].

B.1. Full coherent DM and effects of pure dephasing

The FCS for the full coherent model (2) and for the description including dephasing (6) is obtained from the deformed generators $\mathcal{L}_{0, \xi} \rightarrow \mathcal{L}_{0, \xi}$, respectively. Both of them are expressed as matrices of dimension $16 \times 16$ in the Liouville space.

The analytical result for the current obtained with the coherent DM (2) is given explicitly in the main text, see equation (7). The second cumulant reads

$$
c_2 = \frac{A_i}{A_0^3},
$$

with

$$
\begin{align}
A_0 &= 4T^4_0 (\Gamma_D + 3\Gamma_S) + 2\xi^2 T^2_0\Gamma_D^3 + \xi^2 \Gamma_D^2 \Gamma_S, \\
A_i &= 4T_0^4 \Gamma_0 \Gamma_S 10\xi^2 \Gamma_D^2 (\Gamma_D^2 + 11\Gamma_S^2) \\
&+ 2\xi^2 T_0^4 \Gamma_D^3 (6\xi^2 + \xi^2) \\
&+ 4\xi^4 \Gamma_0 \Gamma_S^2 (\Gamma_D^2 - 4\xi^2)^2 (\Gamma_D^2 - 2\Gamma_S^2).
\end{align}
$$

The Fano factor for the coherent model is calculated directly from the cumulants as $F_{cc} = c_2/c_4$, see equation (8).

For the coherent third cumulant we find

$$
c_3 = \frac{4T_0^4 \Gamma_D \Gamma_S}{A_0^3} \sum_{i=2}^{9} \frac{A_i}{A_0^3},
$$

where the $A_i$ are
\[ A_2 = 256 \sqrt{10} \Gamma_D^4 - 61 \Gamma_D^3 \Gamma_S + 66 \Gamma_D^2 \Gamma_S^2 - 90 \Gamma_D \Gamma_S^3 + 93 \Gamma_S^4, \]

\[ A_3 = 128 \sqrt{12} \Gamma_D^3 - 12 \sqrt{12} \Gamma_D^2 (2 \Gamma_D + \Gamma_S) (5 \Gamma_S - \Gamma_D) \]

\[- \Gamma_D^2 (6 \Gamma_D^3 + 6 \Gamma_D^2 \Gamma_S + 21 \Gamma_D \Gamma_S^2 + 219 \Gamma_S^3), \]

\[ A_4 = 128 \sqrt{42} \Gamma_D^2 (4 \Gamma_D^2 + 8 \Gamma_S^2) \]

\[- \sqrt{2} \Gamma_D^2 (12 \Gamma_D^2 + 12 \Gamma_D \Gamma_S - 3 \Gamma_D \Gamma_S^2 + 234 \Gamma_S^3) \]

\[-12 \sqrt{45} (2 \Gamma_D + 13), \]

\[ A_5 = 32 \sqrt{16} \Gamma_D^2 \Gamma_S^2 (12 \sqrt{4} \Gamma_D (1 - 16 \Gamma_S) \]

\[-2 \Gamma_D^4 \Gamma_S (3 + 6), \]

\[ + 3 \sqrt{2} \Gamma_D^2 (61 \Gamma_S + 102 \Gamma_S + 107 \Gamma_S^2), \]

\[ A_6 = 16 \sqrt{6} \Gamma_D^2 \Gamma_S^2 (6 \sqrt{6} \Gamma_S^3 + \Gamma_S \Gamma_D + 37 \Gamma_S^2) \]

\[ + \Gamma_D^4 \Gamma_S - 2 \sqrt{6} \Gamma_D^3 \Gamma_S (9 \Gamma_D + 4 \Gamma_S), \]

\[ A_7 = 8 \sqrt{2} \Gamma_D^2 \Gamma_S^3 (4 \Gamma_D^2 - 69 \sqrt{2} \Gamma_S^3 + 36 \sqrt{2} \Gamma_D^2), \]

\[ A_8 = 8 \sqrt{2} \Gamma_D^2 \Gamma_S^3 (6 \sqrt{2} \Gamma_D^2 + 8 \sqrt{2} \Gamma_D + 12 \sqrt{2}), \]

\[ A_9 = \varepsilon \sqrt{4} \Gamma_D^2 \Gamma_S^3 (14 \varepsilon^2 + \varepsilon^2). \]

(B.4)

The current including pure dephasing is given explicitly in the main text, see equation (16). The second cumulant was found to be [26]

\[ c_{2,\phi} = \frac{B_1}{B_3} \left( B_2 B_3 + \sum_{i=4}^8 B_i \right), \]

with the \( B_i \) given by the expressions

\[ B_1 = 2 \Gamma_D \Gamma_D \Gamma_S^3 (4 \sqrt{2} + 2 \sqrt{2} (4 \gamma + \Gamma_D)), \]

\[ B_2 = 2 \Gamma_D^2 \Gamma_S \Gamma_D \Gamma_S, \]

\[ B_3 = \gamma \Gamma_D (40 \varepsilon^4 + 92 \gamma \Gamma_D + 60 \gamma \Gamma_S^2 + 17 \gamma \Gamma_D^2 + 21 \Gamma_S^4) \]

\[ + \varepsilon^2 (80 \gamma^2 + 264 \gamma \Gamma_D + 164 \gamma \Gamma_D^2 + 34 \gamma \Gamma_D^3 + 31 \gamma^2) \]

\[ + 8 \gamma \Gamma_D \varepsilon^4, \]

\[ B_4 = 16 \varepsilon \Gamma_D (4 \gamma + \Gamma_D) [2 \gamma (9 \Gamma_D^2 + \Gamma_S) + 11 \Gamma_D^3 + \Gamma_D^4), \]

\[ B_5 = \Gamma_D^2 \Gamma_D \sqrt{3} \varepsilon (28 \gamma + 8 \gamma \Gamma_D + \Gamma_D^3 + \gamma \Gamma_D \Gamma_S (2 \Gamma_D) \]

\[ + \gamma \Gamma_D (7 \gamma^2 + 5 \gamma \Gamma_D) \]

\[ + \gamma \Gamma_D (28 \gamma + 14 \gamma \Gamma_D + \Gamma_D^3), \]

\[ B_6 = 8 \sqrt{2} \Gamma_D (4 \gamma + \Gamma_D) [2 \gamma (9 \Gamma_D^2 + \Gamma_D^2) + 8 \gamma \Gamma_D (\Gamma_D^2 + 4 \Gamma_D^2 - 4 \varepsilon^2) \]

\[ + 8 \gamma (15 \Gamma_D^2 + \Gamma_D^3) + 2 \gamma (14 \Gamma_D^2 + 4 \Gamma_D^2) \]

\[ B_7 = \sqrt{3} \varepsilon (16 \gamma^2 (9 \Gamma_D^2 + 2 \Gamma_D^2) + 16 \gamma^2 (39 \Gamma_D^2 + 21 \Gamma_D^3) \]

\[ + 24 \gamma^2 (30 \Gamma_D^2 + \Gamma_D^3) + 2 \gamma \Gamma_D \Gamma_S (28 \gamma + 14 \gamma \Gamma_D + \Gamma_D^3), \]

\[ B_8 = 4 \sqrt{2} \Gamma_D (4 \gamma + \Gamma_D) \sqrt{2} (81 \Gamma_D^2 + 44 \Gamma_D^2 + 21 \Gamma_D^2 \]

\[ + \gamma \Gamma_D (78 \Gamma_D^2 + 88 \Gamma_D^2) + \Gamma_D^3) \]

\[ + \gamma \Gamma_D (11 \Gamma_D^2 + 4 \varepsilon^2), \]

\[ B_9 = 2 \Gamma_D \Gamma_D \Gamma_D \varepsilon^2 (6 \gamma + \Gamma_D) \]

\[ + [\gamma^2 (2 \gamma + 2 \gamma (4 \gamma + \Gamma_D)) \Gamma_D \Gamma_D (3 \gamma + \Gamma_D) \]

\[ + 2 \Gamma_D^2 (11 \gamma + 4 \varepsilon^2), \]

where \( \Gamma_D = 2 \gamma + \Gamma_D, \)

The Fano factor with dephasing \( F_{\phi}^{(2)} = c_{2,\phi}/c_{1,\phi} \) takes the explicit form

\[ F_{\phi}^{(2)} = 1 - \frac{\Gamma_D \Gamma_S}{\Gamma_D (D_1 + D_2) D_2} \sum_{j=10}^{16} B_j. \]

(B.7)

The \( D_i \) are defined in equation (17) and for the \( B_j \) we have

\[ B_{10} = 2 \varepsilon^2 \Gamma_D \Gamma_S (2 \gamma \Gamma_D (6 \gamma + \Gamma_D) + \Gamma_S (64 \gamma^2 + 48 \gamma \Gamma_D \]

\[ + 4 \gamma \Gamma_D^2 - 2 \gamma \Gamma_D^2 - \Gamma_D^2)), \]

\[ B_{11} = 2 \varepsilon^2 \Gamma_D \Gamma_S (2 \gamma \Gamma_D (2 \gamma + 5 \Gamma_D + 16 \gamma \Gamma_S) - 8 \varepsilon \Gamma_D \Gamma_S \]

\[ + 2 \gamma \Gamma_D (22 \gamma + 17 \Gamma_D) + 56 \gamma \Gamma_S + \Gamma_D^2), \]

\[ B_{12} = 2 \gamma \Gamma_D (2 \gamma + 5 \Gamma_S) + 4 \varepsilon \Gamma_S, \]

\[ B_{13} = 8 \varepsilon^2 \Gamma_D \Gamma_S (8 \gamma \Gamma_D (6 \gamma + 11 \Gamma_D) + \Gamma_S (12 \gamma + 5 \Gamma_S) \]

\[ + 2 \gamma \Gamma_D (22 \gamma + 17 \Gamma_D) + 56 \gamma \Gamma_S + \Gamma_D^2), \]

\[ B_{14} = 4 \gamma \Gamma_D \Gamma_S (24 \gamma^2 \Gamma_D (7 \gamma + 9 \Gamma_S) + 31 \Gamma_S (14 \gamma + 5 \Gamma_S) \]

\[ + 2 \gamma \Gamma_D (70 \gamma + 53 \Gamma_S) + 24 \gamma (2 \gamma + 5 \Gamma_S + 4 \Gamma_D \Gamma_S), \]

\[ B_{15} = 32 \varepsilon^2 \Gamma_D (4 \gamma + \Gamma_D) (6 \gamma + 3 \Gamma_D - \Gamma_D), \]

\[ B_{16} = 8 \varepsilon^2 \Gamma_D \Gamma_S (8 \gamma \Gamma_D (4 \gamma + \Gamma_D) (9 \gamma + \Gamma_D) \]

\[ + \Gamma_S (24 \gamma^2 + 32 \gamma \Gamma_D + 71 \Gamma_D - 4 \varepsilon^2)). \]

(B.8)

The analytical expression for the third cumulant including the effects of dephasing \( c_{1,\phi} \) is extremely lengthy and therefore it is not shown explicitly. Its physical content is explored in section 3.2 and figure (2).

B.2. Rate equation

For transport properties according to the rate equation we use the deformed generator \( \mathcal{L}_{\Gamma,\gamma} \). Since it corresponds to a description without coherences, \( \mathcal{L}_{\Gamma,\gamma} \) has six cumulants of \( x \times 4 \times 4 \). As explained in the main text, the first cumulant obtained with both, coherent and rate equation descriptions are identical, see equation (7). The incoherent second cumulant reads

\[ c_{2,I} = \frac{4 \Gamma_D \Gamma_D \Gamma_S}{A_0} E_i, \]

(B.9)

with \( E_i = 16 \varepsilon^2 \gamma \Gamma_D (11 \Gamma_D + 11 \Gamma_D^2) + 4 \gamma \Gamma_D \Gamma_S (10 \Gamma_D^2 + \Gamma_S^2) \) and \( A_0 \) is defined in equation (B.2). For the incoherent skewness we have

\[ c_{3,I} = \frac{4 \Gamma_D \Gamma_D \Gamma_S}{A_0} \sum_{i=2}^8 E_i, \]

(B.10)
**Figure C1.** (a) Coherences $|\rho_s^{ij}|$ weighted by the overall coherence $C(\rho)$ and (b) occupation probabilities of the steady-state DM for the fully coherent model (solid line) and for the description including pure dephasing (dashed). Parameters: $\varepsilon/t_c = 2$, $\gamma/t_c = 2/3$ and $\Gamma_D/t_c = 1/2$. (c) Transition rates between electronic states of the TQD $\Gamma_{ij}$, for $\varepsilon/t_c = 2$ (solid) and $\varepsilon/t_c = 4$ (dashed). We used $\Gamma_D/t_c = 1/2$.

\[
E_2 = 256\varepsilon^4 [1 + 6\Gamma_D^3 \Gamma_S + 66 \Gamma_D^2 \Gamma_S^2 - 78 \Gamma_D \Gamma_S + 129 \Gamma_S^3],
\]
\[
E_3 = 128\varepsilon^2 \Gamma_D [12\varepsilon^2 \Gamma_D (2\Gamma_S - 5\Gamma_D + 7\Gamma_S^2) + \Gamma_D (10\Gamma_D^2 + 33\Gamma_S^2 - 2\Gamma_D - 51\Gamma_D \Gamma_S^3)].
\]
\[
E_4 = 64\varepsilon^3 \Gamma_D (2 - 2\varepsilon^2 \Gamma_D + 6\Gamma_D^2 \Gamma_S + 4\varepsilon^2),
\]
\[
E_5 = 32\varepsilon \Gamma_D^2 \Gamma_S^2 [17\varepsilon^2 \Gamma_S^2 + 3\Gamma_D^2 - 8\varepsilon^4] + 6\varepsilon^2 \Gamma_S^2 - 2\Gamma_D \Gamma_S (9\varepsilon^2 \Gamma_D^2 + \Gamma_D + 6\varepsilon^4),
\]
\[
E_6 = 16\varepsilon \Gamma_D^2 \Gamma_S^2 [6\varepsilon^2 \Gamma_D^2 \Gamma_S^2 - 3\varepsilon^2 \Gamma_D (\Gamma_D - 4\Gamma_S) + \Gamma_D^2 \Gamma_S - 24\varepsilon^6 \Gamma^2].
\]
\[
E_7 = 4\varepsilon^2 \Gamma_D^2 \Gamma_S^2 \left[3\varepsilon^2 \Gamma_S^2 \Gamma_D^2 - 2\varepsilon \Gamma_D (\Gamma_D - 6\Gamma_S) + 2\varepsilon \Gamma_D (\Gamma_D - 7\Gamma_S) + \Gamma_D^2 \Gamma_S^2 + 2\varepsilon \Gamma_D (\Gamma_D - 7\Gamma_S)\right],
\]
\[
E_8 = \varepsilon^4 \Gamma_D^2 \Gamma_S^2 (2\varepsilon^2 + \varepsilon^2).
\]

(B.11)

**Appendix C. Elements of the steady-state DM**

The steady-state of the linear TQD described by the DM approach, $\rho_s$ is determined by solving $\dot{\rho}_s = L \rho_s = 0$, with $i = 0, \phi$. The steady-state for the fully coherent model (2) is obtained by setting $\gamma = 0$ in the expressions below.

The elements of the DM read [26]

\[
\rho_{11}^s = \frac{1}{2} \left[ 2\varepsilon [F_1 + (4\varepsilon + \Gamma_D) 8\varepsilon^2 + \Gamma_D (\Gamma_D - 2\varepsilon)] \right],
\]
\[
\rho_{22}^s = \frac{1}{2} \left[ 2\varepsilon [F_2 + 8\varepsilon^2 (4\varepsilon + \Gamma_D) + \Gamma_D (\Gamma_D - 4\varepsilon)] \right],
\]
\[
\rho_{33}^s = \frac{1}{2} \left[ 4\varepsilon \Gamma_D^2 + 2\pi \varepsilon (4\varepsilon + \Gamma_D) \right],
\]
\[
\rho_{12}^s = \frac{1}{2} \left[ \left(\frac{1}{2} + i \varepsilon - 4\varepsilon \Gamma_D - 2i \varepsilon (4\varepsilon + \Gamma_D) \right) \right],
\]
\[
\rho_{13}^s = \left(\frac{1}{2} + i \varepsilon - 4\varepsilon \Gamma_D - 2i \varepsilon (4\varepsilon + \Gamma_D) \right),
\]
\[
\rho_{23}^s = \frac{1}{2} \left[ 2i \varepsilon \Gamma_D \Gamma_S (4\varepsilon + \Gamma_D) \right],
\]

with $\Gamma_D = 2\varepsilon + \Gamma_D$, the $D_i$ are defined in equations (17), $F_1 = 8\varepsilon^2 + 24\varepsilon (4\varepsilon + \Gamma_D)$ and $F_2 = 4\varepsilon^2 + 6\varepsilon (4\varepsilon + \Gamma_D)$. The occupation of the empty state is $\rho_{00}^s = 1 - \sum_{i=1}^{4} \rho_{ii}^s$. Note that in the Coulomb blockade regime and infinite bias voltage, the steady-state current is given by $I = e \Gamma_D \rho_{11}^s$.

The weights of the off-diagonal elements $\rho_s^{ij}$ with respect to the overall coherence in the system, defined as $C(\rho) = \sum_{i,j} |\rho_{ij}|^2$ [66], are quantified in the form $2|\rho_{ij}|^2/C(\rho^s)$. This ratio and the occupations with and without dephasing are shown in figures C1(a) and (b), respectively, as a function of $\Gamma_D$.

For the fully coherent model we observe that $\rho_{11}^s$, revealing LDT between QD1 and QD3, accounts for most of the coherence in the regime $\Gamma_D \leq 2t_c$. At this coupling, the occupation $\rho_{11}^s$ of QD3 is large and decreases for increasing $\Gamma_D$, in accord to the behavior of the current, see figure 2. The coherence $\rho_{12}^s$ is the dominating contribution to the overall coherence for $\Gamma_D \gg t_c$, as a signature of the Zeno effect; in this regime $\rho_{12}^s$ and $\rho_{23}^s$ are approximately constant and account for most of the occupation of the TQD, see figure C1(b).

Pure dephasing impairs the coherence of the TQD. Hence, the strength of $\rho_{11}^s$ is reduced due to dephasing for small $\Gamma_D$, figure C1(a), with the corresponding reduction of the current and of $\rho_{12}^s$. For sufficiently large $\Gamma_D$, dephasing partially alleviates the charge localization resulting in a reduction of $\rho_{12}^s$. Consequently, the occupation $\rho_{12}^s$ is dephasing-enhanced for $\Gamma_D \gg t_c$ and the difference between $\rho_{23}^s$ and $\rho_{11}^s$ is reduced.

**Appendix D. Incoherent rates $\Gamma_i$**

We turn to analyze the incoherent transition rates between electronic states of the TQD, $\Gamma_i$ defined in equations (5) in the main text. Similarly to the DM approach, we assess the effect of the rates by defining the sum $\Gamma = \sum_{i,j} \Gamma_i$. The weighted contribution of the specific rates, accounted as $2\Gamma_i/\Gamma$, are presented in figure C1(c) as a function of $\Gamma_i$ and for different values of $\varepsilon$. We observe that $\Gamma_{ij}$ is the dominating contribution to the total transitions in the regime $\Gamma_D \leq 2t_c$, and decays for increasing $\Gamma_D$. In addition, its weight is not strongly sensitive.
to the detuning of the central QD, unlike the LDT exhibited by the coherent description of the TQD. The rates between adjacent sites, $I_{12}$ and $I_{23}$, account for most of the transitions for sufficiently large $I_{23}$ indicating incoherent charge transport.

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