Very-Extreme-mass-ratio gravitational wave bursts in the Galaxy and neighbors for space-borne detectors

Wen-Biao Han\textsuperscript{1,2}\textsuperscript{*}, Xing-Yu Zhong\textsuperscript{1,2}\textsuperscript{†}, Xian Chen\textsuperscript{3,4}\textsuperscript{‡}, Shuo Xin\textsuperscript{5}\textsuperscript{1}\textsuperscript{§}

\textsuperscript{1}Shanghai Astronomical Observatory, Shanghai, 200030, China
\textsuperscript{2}School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{3}Astronomy Department, School of Physics, Peking University, Beijing 100871, China
\textsuperscript{4}Kavli Institute for Astronomy and Astrophysics at Peking University, Beijing 100871, China
\textsuperscript{5}Tongji University, Shanghai, 200092, China

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

Two recent papers (Amaro Seoane 2019; Gourgoulhon et al. 2019) revealed that in our Galaxy, there are very extreme-mass-ratio inspirals composed by brown dwarfs and the supermassive black hole in the center. The event rates they estimated are very considerable for space-borne detectors in the future. However, there are also much more plunge events during the formation of the inspiraling orbits. In this work, we calculate the gravitational waves from compact objects (brown dwarf, primordial black hole and etc.) plunging into or being scattered by the center supermassive black hole. We find that the signal-to-noise ratio of this burst gravitational waves are quite large for space-borne detectors. The event rates are estimated as ~ 0.01 in one year for the Galaxy. If we are lucky, this kind of very extreme-mass-ratio bursts (XMRBs) will offer a unique chance to reveal the nearest supermassive black hole and nuclei dynamics. Inside 10 Mpc, the event rate can be as large as 4 per year and the signal is strong enough for space-borne detectors, then we have a good chance to probe the nature of neighboring black holes.

Key words: gravitational waves – space-borne detectors – extreme-mass-ratio bursts

1 INTRODUCTION

After one century, the gravitational wave (GW) which is predicted by Einstein’s gravitational theory, has been detected by advanced LIGO (aLIGO) and advanced Virgo (AdV) more than 10 events in the O1 and O2 run (Abbott et al. 2016a,b, 2017a,b). The success of the GW detection, excites the plans of space-borne interferometers with arm-length about million kilometers. The Laser Interferometer Space Antenna (LISA http://lisa.nasa.gov/), Taiji (Hu & Wu 2017) and Tian-Qin (Luo et al. 2017) planned to launch in 2030s, will focus the observation band from 0.1 milli-Hertz to 1 Hz. Extreme-Mass-Ratio Inspirals (EMRI) (Amaro Seoane et al. 2007; Amaro Seoane 2018), e.g. a stellar mass compact object (1-10 $M_\odot$) orbiting around a supermassive black hole (SMBH), is a promising source of GW signal at these space-borne detectors’ band (Amaro Seoane et al. 2007; Babak et al. 2017; Danzmann et al. 2017; Berry et al. 2019). In particular, there are also extreme-mass-ratio bursts (EMRBs) which are produced when a compact object passes through periapsis on a highly eccentric orbit about a much more massive object (Rubbo et al. 2006; Hopman et al. 2007; Toonen et al. 2009; Berry & Gair 2013a,b). The event rate of this kind of burst sources is 0.2/year in 100 Mpc, and the signal-to-noise ratio is up to a few tens based on the analysis by (Berry & Gair 2013b).

Recently, two groups independently reported that in our Galactic center, LISA will see a few of very extreme mass-ratio inspirals. The mass-ratio is about $10^{-8}$ (Amaro Seoane 2019; Gourgoulhon et al. 2019). This kind of sources called as X-MRIs (Amaro Seoane 2019) are composed by brown dwarfs inspiraling into the SMBH. The event rates they estimated are quite high, could be more than 10 once the LISA becomes to observe.

However, there are much more brown dwarfs with unbounded orbits comparing with the bounded ones (X-MRIs). If the event rate is considerable for LISA, these brown dwarfs with plunge orbits will collide with the SMBH and produce burst GWs. In the present paper, we call this kind of GW sources as very-extreme-mass-ratio-bursts (XMRBs). In our Galaxy, the signal-to-noise(SNR) of XMRBs will be as large as a few of thousands. XMRBs in the Galactic center will be

\textsuperscript{*} E-mail: wbhan@shao.ac.cn
\textsuperscript{†} E-mail: zxy@shao.ac.cn
\textsuperscript{‡} E-mail:xian.chen@pku.edu.cn
\textsuperscript{§} E-mail: xinshuo@tongji.edu.cn
very easy to be found in the LISA’s observation. The GWs of XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the ORBITS AND WAVEFORMS OF XMRBs will give as unique change to know the nature of the central SMBH. In this paper, we firstly calculate the

\[ \psi_4 = \frac{1}{r - i\cos \theta} \int_{-\infty}^{\infty} d\omega \sum_{l,m} R_{lm,\omega}(r) \cdot 2 S_{lm}^{\omega}(\theta) e^{-i\omega t + ilm \phi} \]  

where The function \( S_{lm}^{\omega}(\theta) \) is a spin-weighted spheroidal harmonic, it can be computed via eigenvalue (Hughes 2000) or continuous fraction methods (Leaver 1985). The radial function \( R_{lm,\omega}(r) \) obeys the Teukolsky equation

\[ \Delta^2 \frac{d}{dr}(\Delta \frac{d}{dr}) - V(r) R_{lm,\omega}(r) = -\tilde{T}_{lm,\omega}(r), \]

where \( \tilde{T}_{lm,\omega}(r) \) is the source term, and the potential is: \( V(r) = -K^2 + 4irrM + 8i\omega \alpha + \Delta \)

where \( \Delta = r^2 - 2Mr + a^2, K = (r^2 + a^2)(\omega - ma), \alpha = \epsilon_{lm} - 2am\omega + a^2w^2 - 2 \) the number \( \epsilon_{lm} \) is the eigenvalue of the spheroidal harmonic.

Using the Green function method (Arfken 1985), we can obtain the solution of the Teukolsky equation with a purely outgoing property at infinity and a purely ingoing property at the horizon:

\[ R_{lm,\omega}(r) = \frac{1}{2i\omega \text{trans}_{lm,\omega}} \left( R_{lm,\omega}^{\text{inc}}(r) \right) \int_{r_*}^{\infty} dr' R_{lm,\omega}^{\text{inc}}(r') \Delta^{lm,\omega} + R_{lm,\omega}^{\text{inc}}(r) \int_{r_*}^{\infty} dr' R_{lm,\omega}^{\text{inc}}(r') \Delta^{lm,\omega} \]

The asymptotic behavior of this solution near horizon and infinity is

\[ R_{lm,\omega}(r \rightarrow r_*) = \frac{b_{lm,\omega}}{2i\omega \text{trans}_{lm,\omega}} \left( R_{lm,\omega}^{\text{inc}}(r) \right) \int_{r_*}^{\infty} dr' R_{lm,\omega}^{\text{inc}}(r') \Delta^{lm,\omega} \]

\[ R_{lm,\omega}(r \rightarrow \infty) = \frac{b_{lm,\omega}}{2i\omega \text{trans}_{lm,\omega}} \left( R_{lm,\omega}^{\text{inc}}(r) \right) \int_{r_*}^{\infty} dr' R_{lm,\omega}^{\text{inc}}(r') \Delta^{lm,\omega} \]

where \( P = \omega - ma/2Mr_*, \) and \( r_* \) is the tortoise coordinate.

In general, because the homogeneous solution will diverge near the infinity, we cannot get solutions directly from Teukolsky equation with any kind of accuracy. To solve this problem, we can convert the equation to the Sasaki-Nakamura equation (Sasaki & Nakamura 1982):

\[ \frac{d^2 X_{lm,\omega}}{dr_*^2} + F(r) \frac{dX_{lm,\omega}}{dr_*} - U(r) X_{lm,\omega} = 0 \]

and use the transform rule (Mino et al. 1997; Hughes 2000) from the Sasaki-Nakamura function to the Teukolsky function:

\[ R_{lm,\omega}^{H,\infty} = \eta [(\alpha + \beta r_j) \lambda^H_{lm,\omega} - \beta \lambda^H_{lm,\omega}] \]

where \( \lambda^H_{lm,\omega} = \chi_{lm,\omega}^H \sqrt{r^2 + a^2}; \ \alpha, \ \beta, \ \eta \) and the potentials \( F(r), U(r) \) can be found in (Hughes 2000). By this way, we can calculate the solutions of the homogeneous Teukolsky equation.

The \( \psi_4 \) is related to the amplitude of the GW at infinity as

\[ \psi_4(r \rightarrow \infty) = -\frac{1}{2} (\delta_+ - i\delta_\times). \]
Very-Extreme-mass-ratio gravitational wave bursts in the Galaxy and neighbors for space-borne detectors

The gravitational waveform, observed from distance $R$, latitude angle $\theta$ and azimuthal angle $\phi$, is then given by

$$h_+ - ih_\times = \frac{2}{R} \sum_{lm} \int_{-\infty}^{\infty} d\omega \frac{1}{\omega^2} Z_l^H \text{lm}(\theta) \text{e}^{i(m\theta - \omega(t-r/c))}$$

Now, we compute the waveforms for the XMRBs with angular momentums $L = 0, L = 1, L = 2, L = 3$, because $L < 4$, all these XMRBs will plunge into the black hole directly. These waveforms are calculated by the frequency-domain Teukolsky equation we mentioned above.

Figure 2 shows the very strong GW burst signals produced by the XMRBs in the Galactic center, and the frequency is around $10^{-2}$ Hz, corresponding to the most sensitive frequency band of the space-borne detectors. We can see not only the (2,2) modes but also the (3,3) (4,4) and (5,5) ones are strong enough comparing to the sensitivity curves of LISA, Taiji, and Tianqin.

Figure 3 shows the time-domain waveform $h_+$ obtained by the inverse Fourier transform, the duration of these signals are around 20 minutes for the modes (2,2), (3,3), (4,4) and (5,5).

The plunging signals are very strong, and very important to detect the structure of the central black hole. However, there are also scattered orbits, while the angular momentum $L$ is larger than 4. Because the particle is far away from the central SMBH in its entire orbit, we use the quadrupole formula to get the time-domain waveform (Peters & Mathews 1963; Peters 1964), then obtain the frequency-domain waveform by the Fourier transform.

Figure 4 shows the signal which is produced by the scattered particle will exist several hours. Of course, it is much weaker than the plunging one, but if in our Galaxy, we will see this kind of signals can still be detected.

For quantitatively demonstrating the strength of the XMRBs, we compute the signal-to-noise ratios (SNR) of these signals (Moore et al. 2015).

$$\text{SNR}^2 = 4Re \int_0^{\infty} \frac{|\tilde{h}(f)|^2}{S_n(f)} df$$

From Table 1, we can find that the SNRs of the plunging sources are very high. However for the scattered sources, the SNR is much lower than the plunging one. For the $L = 5$ and $L = 8$, the SNRs are 280, 167, 51 and 20, 12, 4 correspond to LISA, Taiji and Tianqin if in our Galaxy. Therefore, if for the neighboring galaxies, the signals of these scattered sources will be too weak to detect. However, for the plunging burst signals, even in 10 Mpc distance, the signal of (2,2) mode may still be detected by LISA if the plunging objects have a little heavier mass.

3 EVENT RATES OF XMRBS IN OUR GALAXY

We can estimate the event of these burst using the formula (Amaro Seoane 2019).

$$\Gamma \approx \frac{N}{T_{\text{rel}} \ln \left( \frac{\theta_0}{\theta_c} \right)}$$

where $N$ is the number of brown dwarfs, $T_{\text{rel}}$ is the relaxation timescale due to star-star scattering (two-body relaxation), and $\theta_0$ is the solid angle of the loss cone within which a brown dwarf will plunge into the SMBH. Since the brown dwarfs are normally coming from elongated orbits, we can calculate $\theta_0$ with $L/L_c$, where $L = 8$ is the maximum angular momentum in our simulation which still leads to a significant SNR, and $L_c$ is the angular momentum of a circular orbit with the same energy of the plunging brown dwarf.

We note that the exact value of $\Gamma$ depends on the distance from the SMBH. However, in the limit $\theta_0 \ll 1$, the majority of the stars in the loss cone are coming from the
**Table 1.** SNRs of the plunging and scattering GW burst signals in the Galactic center for the space-borne detectors (TaiJi, LISA and TianQin), the mass of small object is 0.1 solar mass.

| L    | 0 | 1 | 2 | 3 | 5 | 8 |
|------|---|---|---|---|---|---|
| l=m  |   |   |   |   |   |   |
| LISA | 64861 | 4430 | 3193 | 2392 | 6477 | 4435 | 3197 | 2392 | 6495 | 4444 | 3202 | 2397 | 6527 | 4457 | 3211 | 2401 | 280 | 20 |
| TaiJi| 4784 | 735 | 3381 | 1876 | 4793 | 4793 | 2475 | 1879 | 4807 | 3390 | 539 | 2479 | 4830 | 3400 | 2484 | 1886 | 167 | 12 |
| TianQin | 1965 | 1445 | 1091 | 851 | 1970 | 1450 | 1091 | 851 | 1970 | 1450 | 1095 | 851 | 1984 | 1454 | 1095 | 851 | 51 | 4 |

influence radius, where the enclosed stellar mass becomes comparable to the mass of the SMBH (Liu & Chen 2013). Therefore, we can estimate \( \Gamma \) using the values at the influence radius of the SMBH in the Galactic Center, about 3 pc, and the corresponding relaxation timescale is \( T_{\text{rel}} \sim 10^5 \) years (Genzel et al. 2010). To derive \( N \), we follow the assumption in Amaro-Soane 2019 about the initial mass function of stars and we find that there are about \( N \sim 3 \times 10^6 \) brown dwarfs within the influence radius. With these consideration we find that \( \Gamma \sim 10^{-3} \text{yr}^{-1} \).

Equation (13) is derived under the assumption that the nuclear star cluster around the SMBH is spherical. However, observations showed that the stellar distribution in the central 8 pc of the Galactic Center is triaxial (Feldmeier Krause et al. 2017). In such a potential, the loss cone is refilled mainly by stars on chaotic orbits and the loss-cone filling rate can be orders of magnitude higher than the rate due to two-body relaxation (Merritt & Poon 2004). For this reason, we think it possible that the event rate of the XMRBs in the Galactic Center could reach \( \Gamma \sim 10^{-2} \text{yr}^{-1} \). A careful modeling of the XMRB rate in a triaxial potential is needed to better quantify the event rate.

However, the event rate estimation above is in our Galaxy. If we consider inside 10 Mpc volume, considering the number density of \( 10^6 m_0 \) is 0.1/Mpc\(^3\) (Marconi et al. 2004), the number of SMBHs inside 10 Mpc is about four hundreds. For this distance, from the Table 1, the SNR can achieve at \( \sim 10 \) for the dominant (2,2) modes. Therefore, if we take into account the neighbor galaxies inside 10 Mpc, the event rate can arrive at 4 per year, this makes sense for space-borne detectors like LISA. The detection of such kind of plunging signals will reveal the nature of black holes.

### 4 CONCLUSIONS

The very extreme-mass-ratio sources in our Galaxy is very meaningful for GW space detection, because of the extremely small mass ratio \( \sim 10^{-8} \), the gravitational self-force of the small body can be ignored, together with the high SNR, the spacetime of the central SMBH can be figured out very precisely.

In the present work, we propose a kind of GW sources named as XMRBs in our Galaxy and neighboring galaxies with considerable event rates. In contrast with the Galactic inspiraling sources (Amaro-Soane 2019; Gourgoulhon et al. 2019), we consider compact objects such as brown dwarfs and primordial black s plunging into or being scattered by the central supermassive black hole at Sgr A* or by the nearby SMBHs. Because both the plunging or scattering of small objects into the black hole produce transient GW signals (comparing with inspiraling sources), we call them as very extreme-mass-ratio bursts (XMRBs). The frequency of this source is around \( 10^{-3} \sim 10^{-2} \text{Hz} \) and it corresponds to the most sensitive frequency band of the space-borne detectors.

The Galactic inspiraling objects stay outside of the innermost stable circular orbit of the SMBHs, but the plunging objects collide with the BH horizon directly, then the latter can produce GW signals carrying the direct information of horizon. Our calculation show that the SNR is around \( 10^2 \sim 10^3 \) for the plunging XMRBs, and \( \sim 20 \) for the scattering ones with angular momentum up to \( L = 8 \) for the sources in our Galaxy. For the source at 10 Mpc, the SNR can still be as large as \( \sim 10 \). The signals are strong enough and event rate we estimated can be arrive at 0.01 per year for the plunging sources in the Galaxy (the event rate is not sensitive with angular momentum). If we are lucky, this kind of sources are very important for observing the nearest SMBH. However, in 10 Mpc volume, there are a few hundreds of galaxies (Marconi et al. 2004), then the event rate may arrive at \( \sim 4 \) per year and the SNR of plunge source is still large enough for the space-borne detectors. This makes sense for the future space-borne detectors and the detection of the nature of black holes.

### ACKNOWLEDGEMENTS

This work is supported by NSFC No.11273045.

### REFERENCES

Abbott B. P., et al., 2016, PhRVL, 116, 061102
Abbott B. P., et al., 2016, PhRVL, 116, 241103
Abbott B. P., et al., 2017, PhRVL, 119, 141101
Abbott B. P., et al., 2017, PhRVL, 118, 221101
Amaro-Soane P., Gair J. R., Freitag M., Miller M. C., Mandel I., Cutler C. J., Babak S., 2007, CQGra, 24, R113
Amaro-Soane P., 2018, LRR, 21, 4
Amaro-Soane P., 2019, PhRVd, 99, 123025
Arfken G., Mathematical Methods for Physicists (Academic Press, Orlando, 1985), chapter 16
Babak S., et al., 2017, PhRVd, 95, 103012
Berry C. P. L., Gair J. R., 2013, ASPC, 467, 185, ASPC..467
Berry C. P. L., Gair J. R., 2013, MNRAS, 433, 3572
Berry C., et al., 2019, BAAS, 51, 42
Very-Extreme-mass-ratio gravitational wave bursts in the Galaxy and neighbors for space-borne detectors

Danzmann K., et. al., 2017, A proposal in response to the ESA call for L3 mission concepts
Feldmeier-Krause A., Zhu L., Neumayer N., et al., 2017, MNRAS, 466, 4940
Fujita R., Tagoshi H., 2004, PThPh, 112, 415
Fujita R., Tagoshi H., 2005, PThPh, 113, 1165
Genzel R., Eisenhauer F., Gillessen S., 2010, RvMP, 82, 3121
Gourgoulhon E., Le Tiec A., Vincent F. H., Warburton N., 2019, A&A, 627, A92
Han W.-B., 2009, IJTP, 48, 621
Han W.-B., 2010, PhRvD, 82, 084013
Han W.-B., Cao Z., Hu Y.-M., 2017, CQGra, 34, 225010
Hopman C., Freitag M., Larson S. L., 2007, MNRAS, 378, 129
http://lisa.nasa.gov/
Hughes S. A., 2000, PhRvD, 62, 044029
Hughes S. A., 2002, PhRvD, 65, 069902
Hu W.-R. and Wu Y.-L., 2017, Natl. Sci. Rev. 4, no. 5, 685
Leaver E. W., 1985, RSPSA, 402, 285
Liu F. K., Chen X., 2013, ApJ, 767, 18
Luo J., et al., 2016, CQGra, 33, 035010
Mano S., Suzuki H., Takasugi E., 1996, PThPh, 95, 1079
Marconi A., Risaliti G., Gilli R., L. K. Hunt, Maiolino R., Salvati M., 2004, MNRAS, 351, 169
Merritt D., Poon M. Y., 2004, ApJ, 606, 788
Mino Y., Sasaki M., Shibata M., Tagoshi H., Tanaka T., 1997, PThPS, 128, 1
Moore C. J., Cole R. H., Berry C. P. L., 2015, CQGra, 32, 015014
Peters P. C., Mathews J., 1963, PhRv, 131, 435
Peters P. C., 1964, PhRv, 136, 1224
Rubbo L. J., Holley-Bockelmann K., Finn L. S., 2006, AIPC, 873, 284, AIPC..873
Sasaki M., Nakamura T., 1982, PThPh, 67, 1788
Teukolsky S. A., 1973, ApJ, 185, 635
Toonen S., Hopman C., Freitag M., 2009, MNRAS, 398, 1228

This paper has been typeset from a TeX/LaTeX file prepared by the author.