New Secure Sparse Inner Product with Applications to Machine Learning

Guowen Xu, Shengmin Xu, Jianting Ning, Tianwei Zhang, Xinyi Huang, Hongwei Li, Rongxing Lu Fellow, IEEE

Abstract—Sparse inner product (SIP) has the attractive property of overhead being dominated by the intersection of inputs between parties, independent of the actual input size. It has intriguing prospects, especially for boosting machine learning on large-scale data, which is tangled with sparse data. In this paper, we investigate privacy-preserving SIP problems that have rarely been explored before. Specifically, we propose two concrete constructs, one requiring offline linear communication which can be amortized across queries, while the other has sublinear overhead but relies on the more computationally expensive tool. Our approach exploits state-of-the-art cryptography tools including garbled Bloom filters (GBF) and Private Information Retrieval (PIR) as the cornerstone, but carefully fuses them to obtain non-trivial overhead reductions. We provide formal security analysis of the proposed constructs and implement them into representative machine learning algorithms including k-nearest neighbors, naive Bayes classification and logistic regression. Compared to the existing efforts, our method achieves 2-50× speedup in runtime and up to 10× reduction in communication.

Keywords—Secure computation, Machine learning, Sparsity.

1 INTRODUCTION

The sparse inner product (SIP) [1], [2], as the basis for sparse linear algebra including matrix multiplication, matrix-vector inner product, and matrix inversion, has shown an irreplaceable role in various applications, especially in accelerating large-scale machine learning (ML) where sparsity is intertwined. Take the classification task with 20Newsgroups dataset [3] as an example. It consists of over 9000 vectors each of which includes approximately 10^5 dimensions; however, each vector on average contains less than 100 non-zero values (approximately 0.1%). Performing SIP on such a sparse dataset boosts performance by at least an order of magnitude compared to traditional dense multiplication. It stems from the fact that the complexity of the sparse operation only depends on the intersection of the number of non-zero data between the datasets, and is independent of the original data dimension. Beyond classification tasks, SIP has been widely used in various fields of machine learning such as k-nearest neighbors [4], cluster analysis [5], naive Bayes, and logistic regression [6].

While SIP is appreciated for improving performance on sparse data, it inherits all the privacy issues that arise from plaintext computations [7], [8]. Consider a ML inference platform consisting of a client and a server. The client feeds sparse query vectors to the server (which holds a sparse model), and then the server provides inference results about the query to the client. To facilitate SIP, the client is required to provide plaintext queries while the server needs to expose sparsity details of model parameters. It is clearly a breach of privacy [9], [10]. Concretely, client queries are often naturally sensitive and may contain personal physiological information, financial information, and disease history, depending on the application. Outsourcing these private data to untrusted third parties inevitably raises privacy concerns [11], [12]. Model parameters, as precious intellectual property rights, should also be reasonably protected to ensure the market competitiveness of service providers.

1.1 Related Works

While privacy-preserving machine learning [13], [14], [15] has been extensively investigated, sparse linear algebra, especially SIP and its applications in ML are rarely explored. Existing efforts suffer from either scalability (i.e. customization to specific scenarios) or inefficiency (requiring generic secure multi-party computation (MPC) protocols). Below we briefly review these works and provide a discussion of their limitations.

Chen et al. [16] design a sparse matrix multiplication by carefully combining two primitives, homomorphic encryption and secret sharing. Its core idea is to use fully...
computation. Note that fulfilling the above aspirations (called S-SIP), and then we extend S-SIP to general ML applications still leaves too much to be desired. In this paper, we propose two efficient S-SIP constructs (called S-SIP$_1$ and S-SIP$_2$) to address the above challenges. S-SIP$_1$ uses the Bloom filter (BF) and its variant, the garbled Bloom filter (GBF) [23], as the underlying building blocks. It incurs an overhead linearly proportional to the size of the server’s dataset (larger) in the offline phase, but the overhead in the online phase is completely independent of the size of the server’s data. This makes S-SIP$_1$ ideal for ML scenarios where the server holds small and fixed datasets. S-SIP$_2$ is fully online without precomputing. We use the state-of-the-art PIR technology [24] as the underlying technology and extend it to the batch query mode to reduce overhead through amortization. S-SIP$_2$ enables the overhead of S-SIP to be

**1.2 Technical Challenges**

This paper aims to break the dilemma of previous work and provide a secure SIP approach towards practicality. In briefly, we design a highly optimized secure SIP (called S-SIP), and then we extend S-SIP to general ML scenarios to demonstrate its superiority for accelerating computation. Note that fulfilling the above aspirations is non-trivial and requires careful addressing of the following challenges.

- **How to get out of the cage of inefficiency?** Existing work heavily relies on the generic MPC protocol to provide secure set intersection followed by inner product. This is clearly doable but at the cost of incurring potentially unnecessary overhead. However, bypassing the generic MPC to design a customized secure SIP requires very careful design. A potential challenge is how to simultaneously infer the intersection of two inputs and complete the inner product. Intuitively, we can use existing techniques such as the Private Set Intersection (PSI) [20] or PIR [21] to first obtain the intersection and then use Beaver’s triples [22] or homomorphic encryption to perform linear operations. However, PSI inherently leaks the intersection itself, which is not allowed in S-SIP and requires careful modification to accommodate higher security requirements. PIR is a promising method, but it still needs to be highly optimized such as batch query, recursion and oblivious expansion to speed up retrieval performance.

- **How to design S-SIP with satisfactory scalability?** This means that the constructed S-SIP should exhibit adaptable performance for datasets of different scales. As in machine learning scenarios, the data held by the server may be static and small-scale, or include a large number of entries. For the former, it is desirable if there exists a way that the majority of computations are performed offline, i.e., independently of client input. It is bound to significantly speed up the computation in the online phase. For the latter, we also expect the overhead to be only linear with the size of the smaller dataset (usually the input of clients) and logarithmic with the larger dataset. However, there is no previous work to achieve the above requirements.

- **How to enable fast S-SIP without privacy trade-offs?** Existing general S-SIP methods need to expose some sparsity information to reduce computation time. For example, when performing the matrix-vector inner product, ROOM [8] is forced to reveal sparsity information including the number of non-zero rows or columns in the matrix and the number of non-zero entries in the vector. These messages are sometimes privacy-critical. Especially in the scenario of medical data analysis, the leakage of non-zero entries can often be used by the adversary as side information to infer whether the target user has infected certain diseases. Therefore, it is necessary to design S-SIP without any privacy trade-offs, which requires to design a new inner product operation that is fundamentally different from previous work.

**1.3 Our Contributions**

In this paper, we propose two efficient S-SIP constructs (called S-SIP$_1$ and S-SIP$_2$) to address the above challenges. S-SIP$_1$ uses the Bloom filter (BF) and its variant, the garbled Bloom filter (GBF) [23], as the underlying building blocks. It incurs an overhead linearly proportional to the size of the server’s dataset (larger) in the offline phase, but the overhead in the online phase is completely independent of the size of the server’s data. This makes S-SIP$_1$ ideal for ML scenarios where the server holds small and fixed datasets. S-SIP$_2$ is fully online without precomputing. We use the state-of-the-art PIR technology [24] as the underlying technology and extend it to the batch query mode to reduce overhead through amortization. S-SIP$_2$ enables the overhead of S-SIP to be
linear to the client’s input (smaller) and logarithmic to the size of the server’s dataset. Our constructions does not require any privacy tradeoffs to gain performance benefits. We provide a formal security analysis as well as extensive experiments to demonstrate the semantic security and superiority of the proposed schemes. In summary, our contributions are as follows:

- We present a new S-SIP primitive that is general, efficient, scalable, and can be used in any linear operation where data sparsity exists.
- We design two concrete structures, S-SIP1 and S-SIP2, where the former requires computationally intensive offline operations but exhibits superior online performance. The latter relies on an optimized state-of-the-art PIR technique whose overhead grows only logarithmically with the size of the dataset held by the server.
- We provide a formal security analysis of the proposed constructs and implement them into representative machine learning algorithms including k-nearest neighbors, naive Bayes and logistic regression. Compared to the existing efforts, our method achieves 2-50x speedup in runtime and up to 10x reduction in communication.

Roadmap: The remainder of this paper is organized as follows. In Section 2, we review some basic concepts and introduce the scenarios and threat models involved in this article. In Section 3 and Section 4, we give the details of our proposed constructions. Next, performance evaluation is presented in Section 5. Finally, Section 6 concludes the paper.

2 BACKGROUND AND PRELIMINARY

We first define the threat model considered in this paper, and then review some tools and cryptographic primitives used in the proposed construction.

2.1 Threat Model

We consider a secure two-party computation model consisting of a client and a server. On the S-SIP computing platform, the client holds the dataset \( (X, S) = \{(x_1, s_1), \ldots, (x_i, s_i)\} \), and the server holds the dataset \( (Y, G) = \{(y_1, g_1), \ldots, (y_n, g_n)\} \). At the end of the calculation, the client and the server obtain the secret-sharing of the inner product of the intersection of the two datasets, i.e., \( f((X, S), (Y, G)) = \sum_{i \in [n], j \in [n], x_i = y_j} s_i g_j \), where \([t]\) denotes the set \( \{1, \ldots, t\} \). On the ML computing platform, we extend our S-SIP to general ML scenarios to demonstrate its efficiency. It will be used as a basic component of linear algebra to execute linear operations including matrix-vector inner product and matrix multiplication. At the end of the computation, the client gets the inference results and the server gets nothing. In the above scenarios both the client and the server are considered honest but curious, which is consistent with all previous work [7], [8], [16], [25]. Specifically, both parties follow the protocol’s specifications but may infer the other’s data privacy through passive acquisition of data flows during protocol execution. The security requirement of our S-SIP is to ensure that at the end of the protocol, the two parties only get the share of the inner product, and know nothing about their respective secret inputs.

2.2 Secret Sharing and Oblivious Transfer

- **Additive Secret Sharing** [10]. Without loss of generality, we assume that all variables involved in the paper lie in a prime field \( \mathbb{F}_p \). Hence, given an arbitrary \( x \in \mathbb{F}_p \), the additive secret sharing of \( x \) is denoted as a pair \( (x)_0, (x)_1 = (x-r, r) \in \mathbb{F}_p^2 \), where \( r \) is a random value uniformly selected from \( \mathbb{F}_p \) and \( x = (x)_0 + (x)_1 \). Additive secret sharing is perfectly hiding, that is, given a share \( (x)_0 \) or \( (x)_1 \), \( x \) is perfectly hidden.
- **Oblivious Transfer (OT)** [22]. The 1-out-of-\( n \) Oblivious Transfer (OT) is a two-party secure protocol, where the sender (defined as \( P_0 \)) has \( n \) inputs \( \{a_0, \ldots, a_n\} \), while the receiver (defined as \( P_1 \)) input a choice \( b \in [n] \). At the end of the OT-execution, \( P_1 \) learns \( a_b \) while \( P_0 \) learns nothing.

2.3 Bloom Filter and Garbled Bloom Filter

- **Bloom Filter (BF)** [23]. A Bloom filter is a compact data structure for probabilistic set membership testing. A BF is essentially a binary array of \( m \) bits that can be used to represent a set \( Q \) of at most \( n \) elements. Specifically, given a collection of \( k \) hash functions \( H = \{h_1, \ldots, h_k\} \), each of which maps an arbitrary element to the range \( [1, m] \), i.e., \( h_i : \{0,1\}^* \rightarrow [m] \). We denote the bit of BF at index \( i \) by \( BF[i] \). BF is first initialized with all bits in the array to 0. Then, to insert an element \( x \in Q \) into BF, the element is first hashed through \( k \) hash functions to obtain \( k \) indices. All these indices corresponding to BF will be assigned to 1, i.e., \( BF[h_i(x)] = 1 \) for \( 1 \leq i \leq k \). Similarly, to verify whether an element \( y \in Q \) is in the BF, \( y \) is also hashed through \( k \) hash functions, and then all locations \( y \) hashes to are checked. If any of the bits at the locations is not 1, \( y \) is not in set \( Q \), otherwise \( y \) probably in \( Q \). In this paper, we choose the optimal \( k \) and \( m \) so that once the above verification passes, then \( y \in Q \) with overwhelming probability.
- **Garbled Bloom Filter (GBF)** [23]. GBF is similar in function to BF but it is an array of integers. It is used to store key-value pairs \( (x, y) \), where \( y \) is associated with key \( x \) via \( y = \sum_{i=1}^{k} GBF[h_i(x)] \). To achieve this, GBF and BF are first initialized with all entries as \( \perp \) and 0, respectively. Then, for each key-values pair \( (x, y) \), we set \( BF[h_j(x)] = 1 \) for all \( j \in [k] \). Then, let \( B = \{h_j(x) | j \in [k], GBF[h_j(x)] = \perp \} \) be the relevant positions of GBF that have not yet been set. For \( j \in B \), we choose random values for
system usually contains the following algorithms: evaluation of arbitrary functions (parsed as polynomials)

2.4 Hashing Scheme
- Cuckoo Hashing [26]. The basic Cuckoo hash contains m bins, denoted as C[1], · · · , C[m]. Given a stash, k hash functions h1, i ∈ [k] which map any input to the range [m], the workflow of inserting any element x to the Cuckoo hash table is as follows: Calculate the k candidate bins of x by performing k independent hashes for x. Place x into an arbitrary empty candidate bin. If none of the k candidate bins is empty, select one at random, remove the element currently in that bin (x_old), place x in the bin, and then reinsert the previously removed x_old. If re-inserting x_old causes another element to be removed, this process continues recursively for a maximum number of iterations.
- 2-choice hashing [27]. 2-choice hashing is similar to Cuckoo hashing function except that instead of k hash functions we only choose two hash functions h1 and h2. A 2-choice hashing algorithm assigns x to whichever of h1(x), h2(x) has fewest elements.

2.5 Fully Homomorphic Encryption
Fully homomorphic encryption (FHE) [28] enables the evaluation of arbitrary functions (parsed as polynomials) under ciphertext without decryption. Let the plaintext space be F_p, a FHE under the public key encryption system usually contains the following algorithms:
- KeyGen(1^λ) → (pk, sk). Taking the security parameter λ as input, the algorithm KeyGen outputs the public-secret key pair (pk, sk) required for fully homomorphic encryption.
- Enc(pk, x) → ct. Taking pk and any plaintext x ∈ F_p, the algorithm Enc outputs the ciphertext ct of x.
- Dec(sk, ct) → x. Taking sk and a ciphertext c as input, the algorithm Dec outputs the decryption corresponding plaintext x.
- Eval_add(pk, {ct_i}) → c'. Taking pk and a set of ciphertexts {ct_i = Enc(pk, x_i)} as input, the algorithm Eval_add outputs a ciphertext c' encrypting ∑_i x_i.
- Eval_mul(pk, {ct_i}) → c'. Taking pk and a set of ciphertexts {ct_i = Enc(pk, x_i)} as input, the algorithm Eval_mul outputs a ciphertext c' encrypting [prod_i x_i].

In this paper, we utilize homomorphic encryption methods BFV [29] and BGV [30] to implement the above homomorphic operations, which are constructed on the Ring Learning with Errors (RLWE) problem and have been well implemented by the mainstream libraries [31].

2.6 Private Information Retrieval
In this paper, we leverage PIR technology on the single server as an underlying building block [24]. Briefly, given a server holding a database DB of N strings, PIR enables the client to read an arbitrary entry DB[i] without revealing i. Informally, PIR on a single server consists of the following algorithms:
- PIR_keyGen(1^λ) → (pk, sk). Taking the security parameter λ as input, the algorithm KeyGen outputs the public-secret key pair (pk, sk) for the fully homomorphic encryption.
- PIR_query(pk, i) → q. Taking pk and a plaintext index i ∈ N, the algorithm PIR_query outputs the ciphertext query q.
- PIR_answer(pk, q, DB) → d. Taking pk, the ciphertext query q and DB as input, the algorithm PIR_answer outputs an answer d encrypting the content of DB[i].
- PIR_extract(sk, d) → DB[i]. Taking sk and the answer d as input, the algorithm PIR_extract outputs DB[i].

3 THE S-SIP\_1 CONSTRUCTION
In this section, we describe the first construct, S-SIP\_1, which requires offline overhead linear to larger dataset sizes, while concomitant with superior online performance. The functionality of S-SIP\_1 is depicted in Fig 1, where the client holds the dataset (X, S) = \{(x_1, s_1), · · · , (x_n, s_n)\}, and the server holds the dataset (Y, G) = \{(y_1, g_1), · · · , (y_n, g_n)\}. At the end of the calculation, the client and the server obtains the shares of the inner product of the intersection. We first give a high-level overview of our S-SIP\_1, then we describe the technical details of S-SIP\_1 and analyze its security.

3.1 Overview
As shown in Fig 1, we assume that the server holds the dataset (Y, G), and the client holds the dataset (X, S). For each component x_i ∈ X, we aim at S-SIP\_1 to compute the secret share of s_ig_j to both parties if x_i = y_j for some j. Otherwise both hold shares of 0. The security requirements of S-SIP\_1 require that at the end of protocol, the server has no knowledge of the client’s inputs, and the client also knows nothing about the dataset held by the server. The core insight of S-SIP\_1 lies in the fusion of BF and its variant GBF. Specifically, BF can be used to check the membership of x_i in the set represented by BF. It is implemented by accessing k locations h_1(x_i), · · · , h_k(x_i) in the BF and checking that they are all 1 (or alternatively, checking k = \sum_{j ∈ [k]} BF[h_j(x_i)]). GBF as a data structure similar to BF, it allows to store not only a set but also a set of associated values. Concretely, if x_i is in the dataset held by the server, computing \sum_{j ∈ [k]} GBF[h_j(x_i)] will result in the associated value (i.e., g_j). However, since
where the outputs of the previous two protocols is used which either gets secret sharing of the value $g$ with $x$ or shares of 0. The client and the server further execute the Secure Component Protocol Protocol, where the outputs of the previous two protocols is used as input, and outputs the secret sharing of $s_i g_j$, if $x_i = y_j$ for some $j$, otherwise, outputs shares of 0. Finally, the client and the server locally sum up all the obtained shares, and eventually obtains a secret-share of the inner product of two datasets’ intersections, respectively.

### 3.2 Technical Details of S-SIP

As described above, S-SIP can be divided into offline phase and online phase, wherein the online phase contains three sub-protocols: Secure Membership Check Protocol, Secure Associated Value Extraction Protocol and Secure Component Protocol Protocol. Fig 2 depicts the detailed technique for implementing S-SIP, and below we explain each step further.

#### 3.2.1 Offline Phase

This phase requires the server to perform a series of offline operations that are independent of client input. This process is performed only once, and can be reused for multiple protocol executions, even for different clients. Specifically, the server first generates a public-secret key pair for homomorphic encryption, and $k$ hash functions for BF and GBF. The server then maps all entries in its own database into BF and GBF, using $k$ hash functions. At the end, the server performs homomorphic encryption on each entry in the BF and GBF and sends the result to the client (see step 1 in Fig 2).

#### 3.2.2 Online Phase

Given queries $X = \{x_1, \cdots, x_t\}$ associated with $t$ values $S = \{s_1, \cdots, s_t\}$, the client interacts with the server to perform the following steps in parallel for every $x_j \in [t]$.

(a) **Secure Membership Check Protocol**: For each $x_j \in [t]$, the client first computes $\sum_{i=1}^{k} ct.BF[h(x_j)]$. It is easy to observe that $\sum_{i=1}^{k} ct.BF[h(x_j)]$ is an encryption with a value less than $k+1$. Further, this value is equal to $k$ if $x_j$ is present in the server’s database $Y$. The purpose of this sub-protocol is to compute the membership bit of $x_j$ and share it secretly between two parties. To achieve this, a scarecrow approach is to use HE to convert membership $b$ into encryption for one bit ($0$ or $1$), which is then shared secretly to all parties. The entire conversion can be done by homomorphically evaluating an equality circuit, which has the multiplicative depth $\lceil \log (k) \rceil$ resulting computationally expensive overhead.

Instead, we explore a simple approach based on oblivious transfer. Let $\sum_{i=1}^{k} ct.BF[h_i(x_j)]$ be the encryption of some plaintext $\eta$. The client sends $\nu = \sum_{i=1}^{k} ct.BF[h_i(x_j)] - \nu c(pk, \mu)$ to the server, where the random value $\mu$ is treated as a secret-share of $\eta$ held by the client. The server decrypts $\nu$ with secret key and obtains its share $\mu'$ $= \eta - \mu$. Based on this, the client and server invoke an instance of 1-out-of-$(k+1)$ OT as below.

The server first selects a random bit $b_{P\nu}$. Then, it acts as the OT’s sender and sets its inputs to $\{b_0, \cdots, b_k\}$, where each $b_j$ is equal to $b_{P\nu}$, except that $b_j(k-\nu) \pmod (k+1)$ is set to $1 \oplus b_{P\nu}$. On the other hand, the client acts as the OT’s receiver and inputs choice $\mu \pmod (k+1)$. At the end of the OT execution, the functionality of OT ensures that the client gets the $b_{P\nu}$, where $b_{P\nu} \oplus b_{P\nu} = 1$ if $\mu + \mu' = k$, or $b_{P\nu} \oplus b_{P\nu} = 0$, and the server gets nothing. Therefore, the protocol described above achieve the functionality that the two parties obtain XOR shares of 1 or 0 if the client’s query is or is not in the database.

(b) **Secure Associated Value Extraction Protocol**: This sub-protocol is used to compute the secret-shared associated value. It enables the client and server to hold a share of a value on the database, respectively, and this value corresponds to the client’s current query. To achieve this, the client first computes $\phi = s_j \sum_{i=1}^{k} ct.GBF[h_i(x_j)]$. Based on the property of GBF, $\sum_{i=1}^{k} ct.GBF[h_i(x_j)]$ is an encryption of associated value presented in server’s database if $x_j = y_i$ for some $i \in [n]$. Then, the client uniformly selects mask $\beta \in \mathbb{F}_p$ and send $\nu' = \phi - \nu c(pk, \beta)$ to the server, which decrypts $\nu'$ and obtains its share $\rho$, where we can infer that $\beta + \rho = s_j g_i$ if $x_j = y_i$ for some $i \in [n]$. Note that if $x_j$ is not in the server’s database $Y$, the above protocol gets a useless value which may be an arbitrary function of the server’s database entries.

(c) **Secure Component Protocol Protocol**: This sub-protocol is used to compute the secret-shared component product. For each $x_j$ where $j \in [t]$, it enables the client and server to hold a shared share of $s_j g_i$, if $x_j = y_i$ for some $i \in [n]$, or shares of 0 otherwise. Specifically, both parties hold a share on $b$ and $\phi$ through the execution of the previous protocols. We translate the shares into the required output using 2 OT invocations:

In the first OT, the client selects a random value $\Delta \in \mathbb{F}_p$. Then, it acts as the OT’s sender with two inputs $m_0 = \Delta + b_{P\nu} \cdot \delta$ and $m_1 = \Delta + (1-b_{P\nu}) \cdot \delta$. On the other hand, the server as the OT’s receiver inputs choice bit $b_{P\nu}$, and then obtain $r$ from the OT’s functionality. Clearly, $r = \Delta + b \cdot \delta$ where $b = b_{P\nu} \oplus b_{P\nu}$. In the second OT, the server selects random value $a_j \in \mathbb{F}_p$. Then, it acts as the OT’s sender with two inputs $m_0 = r + b_{P\nu} \cdot (\rho - a_j) + (1-b_{P\nu}) \cdot (\alpha_j)$ and $m_1 = r + (1-b_{P\nu}) \cdot (\rho - a_j) - b_{P\nu} \cdot \alpha_j$. On the other
hand, the client as the OT’s receiver inputs choice bit 0, and then obtain r′ from the OT’s functionality. Clearly, r′ = r + b · (ρ − αj) + (1 − b) · αj, where b = bP0 ⊕ bF1.

Based on the two OTs, the client computes r′ − ∆, which implies that the output is exactly δ + ρ − αj, if b = 1. Otherwise, the output is −αj. Therefore, since the server holds αj, the two parties holds the secret shares of δ + ρ if b = 1, or shares of 0 otherwise. To compute the shares of inner production, i.e., the share of \( \sum_{i \in [t]} \text{sgn}(n, x_i=y_i, s_i|f_{ij}) \) it only requires two parties to sum up the resulting shares on a single component, respectively.

### 3.2.3 Security of S-SIP1

Our S-SIP1 is secure against the honest-but-honest adversaries. We provide the following theorem.

**Theorem 3.1.** Let HE and OT used in the S-SIP1 be secure against the honest-but-honest adversaries. Then our S-SIP1 is secure against the honest-but-honest client and server.

**Proof:** Let \( \pi_{\text{S-SIP1}} \) shown in Fig 1 be the functionality of of S-SIP1. We demonstrate the security of S-SIP1 against honest but curious adversaries with the simulation-based paradigm [32].

**Semi-honest client security.** We first analyze the case where adversary \( A \) compromises an honest but curious client. Specifically, we demonstrate the existence of such a polynomial-time simulation in \( \Sigma_{\text{HE}} \), which is given access to the client’s inputs and outputs. It simulates the client’s view that is indistinguishable from the real view.

We show the indistinguishability between real and simulated views by the following hybrid arguments.

- **Hyb1:** This corresponds to the real protocol.
- **Hyb2:** In this hybrid, instead of encrypting the original BF and GBF, the \( \Sigma_{\text{HE}} \) randomly generates BF and GBF with the same length as the original, encrypts them with HE and sends them to the client. Because the client does not have the secret key \( sk \) corresponding to the public key \( pk \) of the HE, the semantic security of the HE guarantees the indistinguishability between this hybrid and the real view.
- **Hyb3:** In this hybrid, instead of following the real input, the \( \Sigma_{\text{HE}} \) simulates the server by randomly selecting a new bit \( b'_{P0} \) and setting all of the server’s \( k \) inputs in OT to \( b'_{P1} \). Since at the end of the OT execution, client just gets a random bit at position \( b'_{P0} \), the above modification just lets the client get another random bit that is indistinguishable from the original random bit. Therefore, the underlying cryptographic primitives of OT guarantee the indistinguishability of this hybrid from the real view.

![Fig 2: Implementation of S-SIP1](image-url)
server’s inputs in OT to the outputs corresponding the client in the \(\pi_{S-SIP}\). This is possible because the \(\Sigma_{SIP}\) is allowed to access the output of the client in the ideal function. Due to the simulation-privacy of OT, this hybrid is indistinguishable from the real view.

**Semi-honest server security.** We now analyze the case where adversary \(A\) compromises an honest but curious server. Specifically, we demonstrate the existence of such a polynomial-time simulation in \(\Sigma_{SIP}\), which is given access to the server’s inputs and outputs. It can simulate the server’s view to make it indistinguishable from the real view.

- **Hyb1:** This corresponds to the real protocol.
- **Hyb2:** In this hybrid, instead of computing \(\sum_{i=1}^{k} ct.BF[h_i(x_j)]\) for each \(x_j \in [t]\), the \(\Sigma_{SIP}\) encrypts randomly strings and and sends it to server. Since in the real view, the ciphertext sent to the server is homomorphically subtracted an random values uniformly chosen from \(\mathbb{F}_p\). Hence, the semantic security of the HE guarantees the indistinguishability between this hybrid and the real view.
- **Hyb3:** In this hybrid, instead of computing \(\sum_{i=1}^{k} ct.GBF[h_i(x_j)]\) for each \(x_j \in [t]\), the \(\Sigma_{SIP}\) encrypts randomly strings and sends it to server. Similarly, since in the real view, the ciphertext sent to the server is homomorphically subtracted a random values uniformly chosen from \(\mathbb{F}_p\). Hence, the semantic security of the HE ensures the indistinguishability between this hybrid and real view.
- **Hyb4:** In this hybrid, instead of following the real input, the \(\Sigma_{SIP}\) simulates the server by setting the server’s inputs in OT with two random strings. This stems from the fact that in the real view, the input to the server is two statistically uniform random strings, and the security of OT guarantees that the client receives one of the two strings and knows nothing about the other. As a result, we hold the same security by substituting the original input with two new random strings. Therefore, this hybrid is indistinguishable from the real view.

\[\blacksquare\]

### 4 The S-SIP\(_2\) Construction

We now describe our second construction S-SIP\(_2\), which is a fully online setting without precomputing. We instantiate S-SIP\(_2\) with state-of-the-art PIR technology as the underlying technology. As a result, this derives the client’s overhead asymptotically linear to its own input and logarithmic to the size of the server’s database. Thus, it shifts the vast majority of the protocol overhead from the client to the server, which is beneficial in real-world applications where the client is usually a resource-constrained device such as a mobile phone.

#### 4.1 Sum-PIR Functionality

S-SIP\(_2\) is functionally identical to S-SIP\(_1\), but removes the expensive offline phase of S-SIP\(_1\), replacing it with standard private information retrieval queries. Recall that during the offline phase of S-SIP\(_1\), the server is required to encrypt the BF and GBF containing all database entries, i.e., \(ct.BF[i] = \text{Enc}(pk, BF[i])\) and \(ct.GBF[i] = \text{Enc}(pk, GBF[i])\) for every \(i \in [m]\), and send them to the client. For each query \(x_j\), the client is required to homomorphically sum all entries corresponding to position \(h_i(x_j)\), i.e., \(\sum_{i=1}^{k} ct.BF[h_i(x_j)]\) and \(\sum_{i=1}^{k} ct.GBF[h_i(x_j)]\), then masks the results and sends them to the server. In S-SIP\(_2\), we instead use PIR to obliviously query the server for entries located at \(h_i(x_j)\), and receive the masked sum of the corresponding values at those locations in BF and GBF. If the client only needs to retrieve the entry at \(h_i(x_j)\) without summing and masking, it only needs to utilize the standard symmetric PIR. Whereas in S-SIP\(_1\), the client needs to sum the values of the \(k\) positions that the hashes map to, we use a modified version of PIR [24], named Sum-PIR.

Fig 3 depicts the construction of Sum-PIR, which allows a client holding \(k\) indices to interact with the server to obtain \(\sum_{i=1}^{k} D[\zeta_i] - r\), where the \(r\) is an additive mask randomly chosen by the server. At the end of the protocol execution, the server has no knowledge of the indexes held by the client.

#### 4.2 Technical Details of S-SIP\(_2\)

With the properties of Sum-PIR, we now describe the technical details of S-SIP\(_2\). Similar to S-SIP\(_1\), S-SIP\(_2\) can be divided into setup phase and online phase, wherein the online phase also contains three sub-protocols: Secure Membership Check Protocol, Secure Associated Value Extraction Protocol and Secure Component Product Protocol. Fig 4 depicts the detailed technique for implementing S-SIP\(_2\), and below we explain each step further.

#### 4.2.1 Setup Phase

This process only requires the server to initialize its own dataset. It is functionally different from precom-
The client interacts with the server to perform the following steps in parallel for every \( x_j \in [t] \).

(a) **Secure Membership Check Protocol:** For each \( x_j \in [t] \), the server first uniformly selects mask \( \mu \in \mathbb{F}_p \). Then, the client uses the server to execute a Sum-PIR query, where the inputs of the client are \( h_1(x_j), \ldots, h_k(x_j) \) while the server use \( \mu \) as input. As a result, the client obtains \( \mu' = -\mu + \sum_{i=1}^{k} BF[h_i(x_j)] \) as output. Afterwards, the client and server invoke an instance of 1-out-of-\((k+1)\) OT as below: the server first selects a random bit \( b_{0j} \). Then, the server as the OT’s sender sets its inputs to \( \{b_{0j}, \ldots, b_{kj}\} \), where each \( b_i \) is equal to \( b_{0j} \), except that \( b_{k+\mu} \mod (k+1) \) is set equal to \( 1 \oplus b_{0j} \). The server as the OT’s receiver inputs choice \( \mu' \mod (k+1) \), and then obtain \( b_{0j} \) from the OT’s functionality.

(b) **Secure Associated Value Extraction Protocol:** The server computes \( \text{Enc}(pk, s_j) \) and sends it to the server. Then, the server uniformly selects mask \( \rho \in \mathbb{F}_p \) and then \( P_2 \) interacts with \( P_0 \) to execute a Sum-PIR query, where the inputs of \( P_0 \) are \( h_1(x_j), \ldots, h_k(x_j) \) while \( P_2 \) use \( \rho \) and \( \mu \) as input.

Before adding the additive mask \( \rho \) to the result \( ct \) obtained by executing Sum-PIR, the server homomorphically multiplies \( ct \) with \( \text{Enc}(pk, s_j) \). The two parties holds the secret shares of \( \delta \) and \( \rho - \alpha_j \) if \( b = 1 \). Otherwise, the output is \( -\alpha_j \). Since \( P_1 \) holds \( \alpha_j \), the two parties holds the secret shares of \( \delta + \rho \) if \( b = 1 \), or shares of 0 otherwise.

(c) **Secure Component Product Protocol:** This sub-protocol is used to compute the secret-shared component product. For each \( x_j \) where \( j \in [t] \), it enables the client and server to hold a shared share of \( s_jg_y \) if \( x_j = y_i \) for some \( i \in [n] \), or shares of 0 otherwise. Its workflow is exactly the same as the corresponding steps in S-SIP. We omit here to prevent redundancy.

**Remark:** The efficiency of S-SIP₂ relies heavily on the performance of the PIR query. Since for each \( x_j \in [t] \) in S-SIP₂, we need to perform \( k \) queries to obtain the sum of the ciphertext at the corresponding position. This is very time consuming as the number of \( t \) increases. To get rid of this dilemma, we design an optimized PIR for batch processing in S-SIP₂, since it does not require the server to encrypt the local database and send it to the client. Specifically, the server first generates \( k \) hash functions \( \{h_1, \ldots, h_k\} \) where \( h_i : \{0, 1\}^* \rightarrow [m] \) and \( m \) represents the size of the BF that is enough to insert \( n \) entries. Using \( k \) hash functions, \( P_1 \) inserts the set \( Y \) containing keys \( \{y_1, \ldots, y_n\} \) into BF, and also inserts set \( \{(y_1, g_y), \ldots, (y_n, g_y)\} \) containing key-value pairs into GBF. \( P_2 \) aborts if either insert operation fails.

### Implementation of S-SIP₂

**Input:** The client (named \( P_2 \)) holds a set of \( t \) queries \( X = \{x_1, \ldots, x_t\} \) associated with \( t \) values \( S = \{s_1, \ldots, s_t\} \). The server (named \( P_1 \)) holds dataset of key-values pairs \( \{(y_1, g_y), \ldots, (y_n, g_y)\} \).

**Implementation:**

1. **Setup Phase:**
   - \( P_1 \) and \( P_2 \) negotiate \( k \) hash functions \( \{h_1, \ldots, h_k\} \) where \( h_i : \{0, 1\}^* \rightarrow [m] \) and \( m \) represents the size of the BF that is enough to insert \( n \) entries.
   - Using \( k \) hash functions, \( P_1 \) inserts the set \( Y \) containing keys \( \{y_1, \ldots, y_n\} \) into BF, and also inserts set \( \{(y_1, g_y), \ldots, (y_n, g_y)\} \) containing key-value pairs into GBF.

2. **Online Phase:** \( P_2 \) interacts with \( P_1 \) to perform the following steps in parallel for every \( x_j \in [t] \).

   (a) **Secure Membership Check Protocol:**
   - \( P_1 \) uniformly selects mask \( \mu \in \mathbb{F}_p \) and sends it to the server. Then, the server as the OT’s sender sets its inputs to \( \{b_{0j}, \ldots, b_{kj}\} \), where each \( b_i \) is equal to \( b_{0j} \), except that \( b_{k+\mu} \mod (k+1) \) is set equal to \( 1 \oplus b_{0j} \).
   - \( P_0 \) as the OT’s receiver inputs choice \( \mu' \mod (k+1) \), and then obtain \( b_{0j} \) from the OT’s functionality.

   (b) **Secure Associated Value Extraction Protocol:**
   - \( P_0 \) computes \( \text{Enc}(pk, s_j) \) and sends it to the server.
   - \( P_1 \) uniformly selects mask \( \rho \in \mathbb{F}_p \) and then \( P_1 \) interacts with \( P_0 \) to execute a Sum-PIR query, where the inputs of \( P_0 \) are \( h_1(x_j), \ldots, h_k(x_j) \) while \( P_1 \) use \( \rho \) and \( \mu \) as input.
   - Before adding the additive mask \( \rho \) to the result \( ct \) obtained by executing Sum-PIR, the server homomorphically multiplies \( ct \) with \( \text{Enc}(pk, s_j) \).
   - The server takes \( \rho \) as output and the client receives \( \delta = -\rho + s_j \cdot \sum_{i=1}^k GBF[h_i(x_j)] \) as output.

   (c) **Secure Component Product Protocol:**
   - \( P_0 \) and \( P_1 \) invoke an instance of 1-out-of-2 OT:
     - \( P_0 \) selects a random value \( \Delta \in \mathbb{F}_p \). Then, \( P_0 \) acts as the OT’s sender with two inputs \( m_0 = \Delta + b_{0j} \cdot \delta \) and \( m_1 = \Delta + (1 - b_{0j}) \cdot \delta \).
     - \( P_1 \) as the OT’s receiver inputs choice \( b_{0j} \), and then obtain \( r \) from the OT’s functionality. Note that \( r = \Delta + b \cdot \delta \) where \( b = b_{0j} \oplus b_{1j} \).
   - \( P_0 \) and \( P_1 \) invoke another instance of 1-out-of-2 OT:
     - \( P_1 \) selects random value \( \alpha_j \in \mathbb{F}_p \). Then, \( P_1 \) acts as the OT’s sender with two inputs \( m_0 = r + b_{0j} \cdot (\rho - \alpha_j) - (1 - b_{0j}) \cdot \alpha_j \) and \( m_1 = r + (1 - b_{0j}) \cdot (\rho - \alpha_j) - b_{0j} \cdot \alpha_j \).
     - \( P_0 \) as the OT’s receiver inputs choice \( b_{0j} \), and then obtain \( r' \) from the OT’s functionality. Note that \( r' = r + b \cdot (\rho - \alpha_j) + (1 - b) \cdot \alpha_j \) where \( b = b_{0j} \oplus b_{1j} \).
   - \( P_0 \) computes \( r' - \Delta \), which implies that the output is exactly \( \delta + \rho - \alpha_j \) if \( b = 1 \). Otherwise, the output is \( -\alpha_j \).

   Since \( P_1 \) holds \( \alpha_j \), the two parties holds the secret shares of \( \delta + \rho \) if \( b = 1 \), or shares of 0 otherwise.

}\]
queries to speed up execution (shown in Fig 5). The main idea of this comes from partitioning the server’s database into m bins. The specific partition operation can be done using Cuckoo hashing or 2-choice hashing. In this way, each bin contains only a small part of the database, which allows parties to evaluate S-SIP2 bin-by-bin. The amount of data the server has to touch with each query is now just the entries mapped into the same bin as the client query, which is computationally more efficient. Variants of this idea have been used in previous work SealPIR [17].

We show the indistinguishability between real and simulated views by the following hybrid arguments.

- **Hyb1**: This corresponds to the real protocol.
- **Hyb2**: In this hybrid, instead of executing Sum-PIR with real BF held on the server, the Simc randomly generates BF and \( \mu \in \mathbb{F}_p \) with the same length as the original. Simc then interacts with the client to perform Sum-PIR queries. Since the output of Sum-PIR is masked before sending to the client, the security of Sum-PIR guarantees the indistinguishability between this result and the one actually obtained.
- **Hyb3**: In this hybrid, instead of following the real input, Simc simulates the server by randomly selecting a new bit \( b' \) and setting all of the server’s \( k \) inputs in OT to \( b_0' \). Since at the end of the OT execution, client just gets a random bit at position \( b_0' \), the above modification just lets the client get another random bit that is indistinguishable from the original random bit. Therefore, the underlying cryptographic primitives of OT guarantee the indistinguishability of this hybrid from the real view.
- **Hyb4**: In this hybrid, instead of executing Sum-PIR with real GFB held on the server, the Simc randomly generates GFB and \( \rho \in \mathbb{F}_p \) with the same length as the original. Simc then interacts with the client to perform Sum-PIR queries. Since the output of Sum-PIR is masked before sending to the client, the security of Sum-PIR guarantees the indistinguishability between this result and the one actually obtained.

**Semi-honest server security.** We now analyze the case where adversary \( A \) compromises an honest but curious server. Specifically, we demonstrate the existence of such a polynomial-time simulation in Simc, which is given access to the server’s inputs and outputs. It can simulate the server’s view to make it indistinguishable from the real view.

| Implementation of the optimized S-SIP2 |
|----------------------------------------|
| **Parameters:**                        |
| - The server’s dataset with size of \( n \), associated values space \( \mathbb{F}_p \), the number of queries \( t \). |
| - The S-SIP2 primitive.                        |
| - The maximum number of bins is \( m \), where the maximum size of each bin in the server is \( \beta \) while the size of the client is \( \eta \). |
| - The number of hash functions \( k \). |
| **Input:**                               |
| - The client (named \( P_0 \)) holds a set of \( t \) queries \( X = \{x_1, \ldots, x_t\} \) associated with \( t \) values \( S = \{s_1, \ldots, s_t\} \). The server (named \( P_1 \)) holds a dataset of key-value pairs \( G = \{(y_1, g_1), \ldots, (y_n, g_n)\} \). |
| **Implementation:**                       |
| 1. \( P_0 \) partitions it items \( \{x_1, \ldots, x_t\} \) into \( m \) bins with the Cuckoo or 2-choice hashing scheme. Without loss of generality, we denote by \( B_C[b] \) those items in the \( b \)-th bin of the client. |
| 2. \( P_1 \) partitions it items \( \{y_1, \ldots, y_n\} \) into \( m \) bins with the \( k \) hash functions. Without loss of generality, we denote by \( B_S[b] \) those items in the \( b \)-th bin of the client. |
| - For each bin \( b \in [m] \)
  | a. \( P_0 \) computes \( G_0 = \{(y_i, l_i) : (y_i, l_i) \in G, y_i \in B_S[b]\} \). Besides, \( P_1 \) pads \( G_s \) to the maximum bin size \( \beta \) with dummy pairs. |
  | b. \( P_0 \) and \( P_1 \) invoke an instance of S-SIP2, where the inputs of each party is as follows: |
  | - \( P_0 \) takes a set of \( \eta \) queries \( \{v_i : v_i \in B_C[b]\} \) which is padded with dummy items to the size \( \eta \). Similarly, there is a set of associated value \( \{x_i : x_i \in B_C[b], x_i \in S\} \), which is also padded with dummy items to the size \( \eta \). |
  | - \( P_1 \) takes the set \( G_0 \) as the inputs. |
  | c. \( P_0 \) receives the S-SIP2’s outputs. |

**Fig 5: Implementation of optimized S-SIP2**

### 4.2.3 Security of S-SIP2

Our S-SIP2 is secure against the honest-but-honest adversaries. We provide the following theorem.

**Theorem 4.1.** Let PIR and OT used in the S-SIP2 be secure against the honest-but-honest adversaries. Then our S-SIP2 is secure against the honest-but-honest client and server.

**Proof:** Let \( \pi_{S-SIP2} \) shown in Fig 1 be the functionality of S-SIP2. We demonstrate the security of S-SIP2 against honest but curious adversaries with the simulation-based paradigm.

**Semi-honest client security.** We first analyze the case where adversary \( A \) compromises an honest but curious client. Specifically, we demonstrate the existence of such a polynomial-time simulation in Simc, which is given access to the client’s inputs and outputs. It can simulate the client’s view to make it indistinguishable from the real view.
actually obtained.

- Cyb: In this hybrid, instead of following the real input, the Sim simulates the server by setting the server’s inputs in OT with two random strings. This stems from the fact that in the real view, the input to the server is two statistically uniform random strings, and the security of OT guarantees that the client receives one of the two strings and knows nothing about the other. As a result, we hold the same security by substituting the original input with two new random strings. Therefore, this hybrid is indistinguishable from the real view.

5 Performance Evaluation

In this section we discuss the performance of the two proposed constructs, S-SIP₁ and S-SIP₂. We use the work ROOM as the baseline for comparison, as it is consistent with our motivation to design general-purpose secure sparse linear algebra. Below we first analyze the overhead of our schemes and ROOM [8] for performing sparse inner products on different sizes of dataset, and then compare the overhead of the two for performing different machine learning tasks including K-nearest neighbors, logistic regression, and naive Bayes classification.

5.1 Implementation Details

We use SEAL [33] to implement homomorphic encryption for BF and GBF, where the polynomial dimension on the ring is set to 2048 and the ciphertext space parameter is 2160. It provides 128-bit security. We adopt OnionPIR [24] as the underlying structure for constructing S-SIP₁. The realization of ROOM follows all the implementation described in their paper. It uses Obliv-C [34] to implement the garbled circuit to construct a general secure two-party protocol, and Pseudo-random functions are constructed through the implementation of AES-128 [8]. Our experiments are carried out on both the LAN and WAN settings. LAN is implemented with two workstations in our lab. The client workstation has AMD EPYC 7282 1.4GHz CPUs with single core and 8GB RAM. The server workstation has Intel(R) Xeon(R) E5-2697 v3 2.6GHz CPUs with 28 threads on 14 cores and 64GB RAM. The WAN setting is based on a connection between a local PC and an Amazon AWS server with an average bandwidth of 963Mbps and running time of around 35ms.

TABLE I: Real-world datasets used in the experiments

| Dataset          | Documents | Classes | Nonzero Features(ave.g.) | Total Features |
|------------------|-----------|---------|--------------------------|----------------|
| Movies           | 14341     | 2       | 136                      | 9562           |
| Newsgroups       | 9051      | 20      | 98                       | 101631         |
| Languages, ngrams=1 | 783   | 11      | 43                       | 1033           |
| Languages, ngrams=2 | 783   | 11      | 136                      | 9915           |

1. Codes are available at https://github.com/schoppmp/room-framework

Consistent with ROOM, we chose three typical datasets (i.e. Movies [35], Newsgroups [36], Languages [37] with ngrams= 1 and 2, respectively) to implement ML tasks including k-nearest neighbors, logistic regression, and naive bayes classification in a privacy-preserving manner. Please refer to TABLE I for the specific size and sparsity of the dataset, and see ROOM for more details on the usage of these dataset.

5.2 Performance of Executing Sparse Inner Products

We first analyze the overhead of each scheme under different dataset sizes. TABLE II shows the comparison of the computational and communication costs of S-SIP₁, S-SIP₂ and ROOM under different variables, where the size of the server’s database ranges from 210 to 220, and the client’s from 210 to 216. We observe that S-SIP₁ relies on heavy overhead for offline, which is linear with the size of the server’s database. As a result, the online phase of S-SIP₁ is completely independent of the size of the server database, resulting in the best computational speedup of all schemes. For example, when the server holds entries of size 220 and the client holds 212 entries, performing such a secure inner product operation S-SIP₁ takes only 4.22 seconds, while S-SIP₂ and ROOM requires 231.7 and 724.5 seconds. Moreover, in the online phase, the superiority of the communication overhead saved by S-SIP₁ is evident to increase with the data held by the client. It stems from the fact that the online traffic of S-SIP₁ is independent of the server’s dataset, while the other two methods are positively related to the server’s input.

In the online phase, the computational overhead of S-SIP₂ is higher than that of S-SIP₁, since it does not require any precomputation. It is worth noting that when the database held by the server is small, the communication overhead of S-SIP₂ is smaller than that of S-SIP₁ at certain times. For example, when n = 216, t = 216, S-SIP₁ incurs 1794 (MB) of traffic while S-SIP₂ is about half of S-SIP₁. This stems from the Sum-PIR used in S-SIP₂, which derives a sublinear communication complexity relative to t, while the communication overhead of S-SIP₁ increases linearly with t. It is clear that S-SIP₂ is superior to ROOM in terms of computational and communication overhead. This is mainly due to a series of optimization methods of S-SIP₂ for sparse inner product operations, including customized OT protocols to minimize communication overhead, and optimized state-of-the-art PIR technology to accelerate computing. ROOM relies heavily on general-purpose secure multiparty computation to compute the intersection of two datasets, and requires a large number of Beaver triples to implement multiplication privately. This incurs non-trivial computational and communication costs. As an example, ROOM takes 14396 (MB) and 11598 seconds to complete a secure sparse inner product between one 216-dimensional vector and another 220-dimensional vector. Conversely, S-SIP₂ takes only 3704 (MB) and 1691.6
5.3 Performance of Executing k-Nearest Neighbors

We now discuss the overhead of S-SIP$_1$ and S-SIP$_2$ in performing real ML tasks. We first consider a $k$-Nearest Neighbor (kNN) task involving a server and a client, where the client holds a labeled database $D$ and the client holds a data $d$ to be classified. In kNN, (a) for each $p \in D$, the client needs to interact with the server to calculate the similarity between the two (the vast majority of the overhead in this process is vector-matrix multiplication); (b) Then, assigning a class $c_p$ to $d$ as the result of a majority vote among the classes of the $k$ most similar documents according to the similarities computed in step (a) (see work [8] for details of Logistic Regression). At the high-level view, logistic regression mainly includes two types of operations containing tens of thousands of ANDs. Since without any precomputation, the execution cost of S-SIP$_2$ is higher than S-SIP$_1$, but significantly lower than ROOM due to the custom design for SIP. Compared with ROOM, we observe that S-SIP$_2$ achieves at least a $2 \times$ improvement in both communication and computing performance. This is due to the customized design of computational SIP in S-SIP$_2$, including partitioning PIR queries and efficient OT executions. On the contrary, ROOM relies heavily on garbled circuits to execute SIP, which is computationally expensive since even performing simple arithmetic operations requires building circuits containing tens of thousands of ANDs.

5.4 Performance of Executing Logistic Regression

We further discuss the cost comparison of each scheme on logistic regression. We also consider a two-party logistic regression scenario involving a server and a client, where the server holds a classification model and the client holds the input to be classified (see work [8] for details of Logistic Regression). At the high-level view, logistic regression mainly includes two types of operations containing tens of thousands of ANDs.
computation, one is the inner product operation between the model parameters and the input features, and the other is the execution of an activation function such as Sigmoid. The former can be easily implemented with S-SIP$_1$ and S-SIP$_2$. As for the latter, we follow ROOM’s approach, which performs polynomial fitting on the sigmoid and then encapsulates it in a garbled circuit for private execution.

Fig 8 and Fig 9 show the overhead of each scheme on different datasets. Consistent with the previous one, S-SIP$_1$ shows the best performance in the online phase, although this requires non-trivial precomputation. This makes S-SIP$_1$ ideal for ML scenarios where the server holds small and fixed datasets such as trained ML models. For example, S-SIP$_1$ only needs 57.1(ms) and 3.6 (MB) of traffic to classify a single document. Compared with S-SIP$_2$ and ROOM, it saves up to 9× communication overhead, and brings at least 8752× speedup of computing. As discussed above, it benefits from the design of S-SIP$_1$ for the online phase, which mainly involves the execution of several efficient OT protocols without computationally intensive homomorphic evaluation. The execution cost of S-SIP$_2$ is higher than S-SIP$_1$, but significantly lower than ROOM due to the custom design for SIP. Compared to ROOM, there are at least a 2× improvement in both communication and computing performance. This is due to the customized design of computational SIP in S-SIP$_2$. Our approach exploits state-of-the-art cryptography tools including garbled Bloom filters and Private Information Retrieval (PIR) as the cornerstone, but carefully fuses them to obtain non-trivial overhead reductions.

5.5 Performance of Executing Naive Bayes Classification

We finally consider the naive bayesian classification scenario consisting of a server and a client, where the server holds the database $D$ and the client holds the input features $d$. This scenario includes two processes, (a) one is to calculate the intersection between $d$ and $D$ on features, (b) and the other is to use Bayesian probability for classification (see work [8] for details). Since ROOM has no code to implement the latter (i.e., step (b)), in keeping with it, here we only discuss the overhead of securely computing the former.

Fig 10 and Fig 11 show the overhead of each scheme on different datasets. The results on this experiment are similar to those on the logistic regression task because they perform similar operations under ciphertext. Apparently, S-SIP$_1$ still shows an advantage over the other two methods. The execution cost of S-SIP$_2$ is higher than S-SIP$_1$, but significantly lower than ROOM due to the custom design for SIP. Compared to ROOM, there are at least a 2× improvement in both communication and computing performance. This is due to the customized design of computational SIP in S-SIP$_2$. Our approach exploits state-of-the-art cryptography tools including garbled Bloom filters and Private Information Retrieval (PIR) as the cornerstone, but carefully fuses them to obtain non-trivial overhead reductions.

6 Conclusion

In this paper, we propose two concrete constructs, S-SIP$_1$ and S-SIP$_2$. Our approach exploits state-of-the-art cryptography tools including garbled Bloom filters and Private Information Retrieval (PIR) as the cornerstone, but carefully fuses them to obtain non-trivial overhead reductions. We provide a formal security analysis of the proposed constructs and implement them into representative machine learning algorithms including k-nearest neighbors, naive Bayes and logistic regression. Compared to the existing efforts, our method achieves 2-50× speedup in runtime and up to 10× reduction in communication. In the future, we will focus on designing more efficient optimization strategies to further reduce the computation overhead of our constructions, to make it more suitable for practical applications.
Fig. 10: Running time of naive bayes classification on different datasets. (a) LAN setting. (b) WAN setting.

Fig. 11: Communication cost of naive bayes classification on different datasets

REFERENCES
[1] C. Ma, F. Yu, Y. Yu, and W. Li, “Learning sparse binary code for maximum inner product search,” in Proceedings of the 30th ACM International Conference on Information & Knowledge Management, 2021, pp. 3308–3312.

[2] N. Srivastava, H. Jin, J. Liu, D. Albonesi, and Z. Zhang, “Matraptor: A sparse-sparse matrix multiplication accelerator based on row-wise product,” in 2020 53rd Annual IEEE/ACM International Symposium on Microarchitecture (MICRO). IEEE, 2020, pp. 766–780.

[3] L. Ruff, Y. Zemlyanskiy, R. Vandermeulen, T. Schnake, and M. Kloft, “Self-attentive, multi-context one-class classification for unsupervised anomaly detection on text,” in Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics, 2019, pp. 4061–4071.

[4] T. Liao, Z. Lei, T. Zhu, S. Zeng, Y. Li, and C. Yuan, “Deep metric learning for k nearest neighbor classification,” IEEE Transactions on Knowledge and Data Engineering, 2021.

[5] I. Tsiokanastos, S. Tompazi, G. Georgakoudis, L. Mukhanov, and G. Karakonstantis, “Aretæ: Accurate error assessment via machine learning-guided dynamic-timing analysis,” IEEE Transactions on Computers, pp. 1–14, 2022.

[6] Y. Gao, M. Kim, C. Thapa, S. Abuadbba, Z. Zhang, S. Camtepe, H. Kim, and S. Nepal, “Evaluation and optimization of distributed machine learning techniques for internet of things,” IEEE Transactions on Computers, 2021.

[7] J. Cui, C. Chen, L. Lyu, C. Yang, and W. Li, “Exploiting data sparsity in secure cross-platform social recommendation,” Advances in Neural Information Processing Systems, vol. 34, pp. 10 524–10 534, 2021.

[8] P. Schoppmann, A. Gascon, M. Raykova, and B. Pinkas, “Make some room for the zeros: Data sparsity in secure distributed machine learning,” in Proceedings of the 2019 ACM SIGSAC conference on computer and communications security, 2019, pp. 1335–1350.

[9] W. Zheng, R. A. Popa, and et al, “Helen: Maliciously secure cooperative learning for linear models,” in IEEE Symposium on Security and Privacy (S&P). IEEE, 2019, pp. 724–738.

[10] P. Mishra, R. Lehmkuhle, and et al, “Delphi: A cryptographic inference service for neural networks,” in USENIX Security Symposium, 2020, pp. 2505–2522.

[11] S. Sav, A. Pyrgelis, and et al, “Poseidon: Privacy-preserving federated neural network learning,” in Proceedings of the Network and Distributed System Security (NDSS), 2021.

[12] C. Juvekar, V. Vaikuntanathan, and et al, “(GAZELLE): A low latency framework for secure neural network inference,” in USENIX Security Symposium, 2018, pp. 1651–1669.

[13] R. Lehmkuhle, P. Mishra, and et al, “Muse: Secure inference resilient to malicious clients,” in USENIX Security Symposium, 2021.

[14] N. Chandran, D. Gupta, and et al, “Sime: ML inference secure against malicious clients at semi-honest cost,” in USENIX Security Symposium, 2022.

[15] X. Jiang, M. Kim, and et al, “Secure outsourced matrix computation and application to neural networks,” in Proceedings of the ACM SIGSAC Conference on Computer and Communications Security (CCS), 2018, pp. 1209–1222.

[16] C. Chen, J. Zhou, L. Wang, X. Wu, W. Fang, J. Tan, L. Wang, A. X. Liu, H. Wang, and C. Hong, “When homomorphic encryption marries secret sharing: Secure large-scale sparse logistic regression and applications in risk control,” in Proceedings of the ACM SIGKDD Conference on Knowledge Discovery & Data Mining (KDD), 2021, pp. 2652–2662.

[17] S. Angel, H. Chen, K. Laine, and S. Seitty, “Pir with compressed queries and amortized query processing,” in IEEE symposium on security and privacy (S&P). IEEE, 2018, pp. 962–979.

[18] M. Bellare, V. T. Hoang, and P. Rogaway, “Foundations of garbled circuits,” in Proceedings of the 2012 ACM conference on Computer and communications security (CCS), 2012, pp. 784–796.

[19] Y. Huang, J. Katz, and et al, “Amortizing garbled circuits,” in Annual Cryptology Conference (CRYPTO). Springer, 2014, pp. 458–475.

[20] T. Lepoint, S. Patel, M. Raykova, K. Seth, and N. Trieu, “Private join and compute from pir with default,” in International Conference on the Theory and Application of Cryptology and Information Security. Springer, 2021, pp. 605–634.

[21] R. A. Mahdavi and F. Kerschbaum, “Constant-weight pir: Single-round keyword pir via constant-weight equality operators,” Proceedings of USENIX security symposium, 2022.

[22] M. Keller, V. Pastro, and D. Rotaru, “Overdrive: Making spdz great again,” in Annual International Conference on the Theory and Applications of Cryptographic Techniques(EUROCRYPT). Springer, 2018, pp. 158–189.

[23] C. Dong, L. Chen, and Z. Wen, “When private set intersection meets big data: an efficient and scalable protocol,” in Proceedings of the 2013 ACM SIGSAC conference on Computer & communications security, 2013, pp. 789–800.

[24] M. H. Mughees, H. Chen, and L. Ren, “Onionpir: response efficient single-server pir,” in Proceedings of the 2021 ACM SIGSAC.
Conference on Computer and Communications Security, 2021, pp. 2292–2306.

[25] G. Xu, H. Li, S. Liu, K. Yang, and X. Lin, “Verifynet: Secure and verifiable federated learning,” IEEE Transactions on Information Forensics and Security, 2019.

[26] S. J. Menon and D. J. Wu, “Spiral: Fast, high-rate single-server PIR via the composition,” IEEE symposium on security and privacy (S&P), 2023.

[27] A. Ali, T. Lepoint, S. Patel, M. Raykova, P. Schoenmann, K. Seth, and K. Yeo, “(Communication-Computation) trade-offs in (PIR),” in 30th USENIX Security Symposium (USENIX Security 21), 2021, pp. 1811–1828.

[28] S. G. Choi, D. Dachman-Soled, S. D. Gordon, L. Liu, and A. Yerukhimovich, “Compressed oblivious encoding for homomorphically encrypted search,” in Proceedings of the 2021 ACM SIGSAC Conference on Computer and Communications Security, 2021, pp. 2277–2291.

[29] S. Halevi, Y. Polyakov, and V. Shoup, “An improved rns variant of the bfv homomorphic encryption scheme,” in Cryptographers’ Track at the RSA Conference. Springer, 2019, pp. 83–105.

[30] C. Gentry, S. Halevi, C. Peikert, and N. P. Smart, “Ring switching in bgv-style homomorphic encryption,” in International Conference on Security and Cryptography for Networks. Springer, 2012, pp. 19–37.

[31] H. Chen, K. Laine, and R. Player, “Simple encrypted arithmetic library-ssl v2.1,” in International conference on financial cryptography and data security. Springer, 2017, pp. 3–18.

[32] Y. Lindell, “How to simulate it–a tutorial on the simulation proof technique,” Tutorials on the Foundations of Cryptography, pp. 277–346, 2017.

[33] “Microsoft SEAL (release 3.3),” https://github.com/Microsoft/SEAL, Jun. 2019, microsoft Research, Redmond, WA.

[34] S. Zahur and D. Evans, “Obliv-c: A language for extensible data-oblivious computation,” Cryptology ePrint Archive, 2015.

[35] A. Maas, R. E. Daly, P. T. Pham, D. Huang, A. Y. Ng, and C. Potts, “Learning word vectors for sentiment analysis,” in Proceedings of the 49th annual meeting of the association for computational linguistics: Human language technologies, 2011, pp. 142–150.

[36] K. Albishre, M. Albathan, and Y. Li, “Effective 20 newsgroups dataset cleaning,” in 2015 IEEE/WIC/ACM International Conference on Web Intelligence and Intelligent Agent Technology (WI-IAT), vol. 3. IEEE, 2015, pp. 98–101.

[37] T. S. learn authors, “Scikit-learn language identification dataset,” https://github.com/scikit-learn.