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Abstract
The charge asymmetry in $W^\pm + n$ jets production at the LHC can serve to calibrate the presence of New Physics contributions. We study the ratio $\sigma(W^+ + n \text{ jets})/\sigma(W^- + n \text{ jets})$ in the Standard Model for $n \leq 4$, paying particular attention to the uncertainty in the prediction from higher-order perturbative corrections and uncertainties in parton distribution functions. We show that these uncertainties are generally of order a few percent, making the experimental measurement of the charge asymmetry ratio a particularly useful diagnostic tool for New Physics contributions.

1 Introduction
At $p\bar{p}$ colliders such as the Fermilab Tevatron, $W^+$ and $W^-$ bosons are produced in equal quantities, i.e. $\sigma(W^+) = \sigma(W^-)$. In contrast, at the CERN LHC $pp$ collider, $\sigma(W^+) \approx 1.3 \sigma(W^-)$. This charge asymmetry is directly related to the dominance of $u$ quarks to $d$ quarks in the proton, $R_{ud}(x, Q^2) = u(x, Q^2)/d(x, Q^2) > 1$. In standard parton distribution function (PDF) global fits, $R_{ud} \approx 1$ for $x \ll 1$ and increases monotonically as $x$ increases. The charge asymmetry ratio $\sigma(W^+)/\sigma(W^-) \neq 1$ is a feature of both the inclusive $W^\pm$ total cross sections, and also of more exclusive $W^\pm + n$ jet cross sections. An important feature of this ratio is that it is theoretically a very stable quantity. In particular, it is expected to be stable with respect to electroweak parameter values and higher-order (electroweak and QCD) perturbative corrections, because the couplings and kinematics of the $W^+$ and $W^-$ subprocesses are essentially the same.

Precise measurements of the $W$ charge asymmetry at the LHC can therefore yield further information on the $u/d$ parton ratio, see for example Ref. [1]. Here we look at a different aspect of the asymmetry, namely that we can use the very precise knowledge of the $u/d$ ratio to calibrate the Standard Model (SM) $W + n$ jet background to New Physics (NP) at the LHC, since typically $\sigma_{\text{NP}}(X \rightarrow W^+ + \text{jets}) = \sigma_{\text{NP}}(X \rightarrow W^- + \text{jets})$.[1] In fact within the SM, there are a number of interesting physics processes, for example $t\bar{t}$ and Higgs boson production,

$g g \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b} \rightarrow W^\pm(\rightarrow l^\pm + \nu) + 4 \text{ jets},$

$g g \rightarrow H \rightarrow W^+W^- \rightarrow W^\pm(\rightarrow l^\pm + \nu) + 2 \text{ jets},$

1A discussion and preliminary results were presented in Ref. [2].
that give rise to equal numbers of $W^+$ and $W^-$ bosons. NP examples can be found for instance in gluino pair production in supersymmetry, where cascade decays of the gluino pair into one charged lepton plus missing energy and jets are expected to have the same cross sections for opposite lepton charges. The fundamental idea is that any observed deviation from the predicted SM value of the ratio

$$R^\pm(n) = \frac{\sigma(W^+ + n \text{ jets})}{\sigma(W^- + n \text{ jets})}$$

(1)

could signal the presence of a NP contribution in the $W^+$ jets sample. In particular, if $\frac{1}{2}\sigma_{NP} \equiv \sigma_{NP}(X \rightarrow W^+ + \text{jets}) = \sigma_{NP}(X \rightarrow W^- + \text{jets})$, and $\sigma_{SM} \equiv \sigma_{SM}(X \rightarrow W^+ + \text{jets}) + \sigma_{SM}(X \rightarrow W^- + \text{jets})$, then

$$f_{NP} = \frac{2(R^\pm_{SM} - R^\pm_{exp})}{(R^\pm_{SM} + 1)(R^\pm_{exp} - 1)},$$

(2)

where $f_{NP} = \sigma_{NP}/\sigma_{SM}$ is the ratio of the NP and SM cross sections, and $R^\pm_{exp}$ and $R^\pm_{SM}$ are the experimentally measured and SM expectation of the cross-section ratio $R^\pm$ respectively. Hence a measurement of $R^\pm(n)$, combined with the SM theoretical prediction, enables a value of $f_{NP}$ to be extracted. To give a very simple numerical example, the contribution of $t\bar{t} \rightarrow W^+ + 4$ jet production to the SM $W^+ + 4$ jet final state (at the 14 TeV LHC and with typical parameters and cuts defined in Eq. (4) below) reduces $R^\pm(4)$ from 1.55 to 1.22. Hence if the uncertainty on the SM prediction of 1.55 is small enough, the presence of $t\bar{t}$ contributions can be detected. A preliminary study of using the $W^\pm$ charge asymmetry to help identify the top quark signal in early LHC running has already been reported by the CMS collaboration.

There are basically only two significant sources of theoretical uncertainty on $R^\pm(n)$: unknown higher-order pQCD corrections to the subprocess cross sections and PDF uncertainties. We already know from studies of the total $W^\pm$ cross sections (see Table 1 below) that both uncertainties are small. Here we consider the corresponding uncertainties on the $W^\pm + \text{jets}$ cross sections. In the following, we will first describe our calculational procedure, and then present results for up to and including $W + 4$ jet production at the LHC. We will also demonstrate a connection between the exact calculation and the high-energy ('BFKL') approximation which may allow $R^\pm(n \geq 5)$ to be estimated.

## 2 Calculational framework and results

We begin by reviewing the results for the charge asymmetry ratio for total cross sections, see Table 1. These have already been discussed in some detail in Ref. 1, where a similar table was presented. Note that the ratio decreases slightly when going from leading order (LO) to next-to-leading order (NLO), and then appears to be perturbatively stable. This is more a reflection of the differences between the LO, NLO and next-to-next-to-leading order (NNLO) PDFs than the impact of the higher-order subprocess corrections. For example, the 14 TeV LHC LO ratio $R^\pm = 1.365$ reduces to $R^\pm = 1.312$ when MSTW2008 NLO partons are used in the leading-order

\[\text{The } W^\pm \text{ are in practice always assumed to decay to a single generation of leptons.}\]
cross-section calculations, and then increases slightly to \( R^\pm = 1.325 \) when the explicit NLO corrections are included.

\[
\sqrt{s} = 7 \text{ TeV} \quad \sqrt{s} = 14 \text{ TeV}
\]

|               | \( \sqrt{s} = 7 \text{ TeV} \) | \( \sqrt{s} = 14 \text{ TeV} \) |
|---------------|---------------------------------|---------------------------------|
| MSTW 2008 LO  | 1.463 ± 0.014                   | 1.365 ± 0.011                   |
| MSTW 2008 NLO | 1.422 ± 0.012                   | 1.325 ± 0.010                   |
| MSTW 2008 NNLO| 1.429 ± 0.013                   | 1.328 ± 0.011                   |

Table 1: Predictions for the ratio of \( W^+ \) and \( W^- \) total cross sections at the LHC at LO, NLO and NNLO pQCD, including the one-sigma (68% cl) PDF uncertainties, with \( \mu_R = \mu_F = M_W \).

We note also that the ratio of \( W^+ \) and \( W^- \) cross sections is strongly dependent on the \( W \) boson rapidity \( y \). Indeed at large \( y \) we have

\[
R^\pm \sim \frac{u(x, M_W^2)}{d(x, M_W^2)}, \quad \text{with} \quad x = \frac{M_W}{\sqrt{s}} \exp(y),
\]

so that, at least for the MSTW2008 PDFs, \( R^\pm \to \infty \) as \( y \to y_{\text{max}} \). The ratio also decreases with increasing collider energy \( \sqrt{s} \), as smaller \( x \) values are probed.

For \( W + n \) jet production we use MSTW2008 PDFs\(^3\) throughout and calculate the cross sections using the MCFM package\(^4\) for \( n = 0, 1, 2 \) (LO and NLO) and \( n = 3 \) (LO only). For the \( n = 4 \) LO calculation we use an adaptation of the VECBOS package\(^5\). The electroweak parameters used are the same as\(^1\). We note that the NLO corrections to the \( n = 3 \) cross section have recently been calculated\(^6\): we will comment on their likely impact on our results below. We follow the standard practice of setting the renormalisation and factorisation scales equal, i.e. \( \mu_R = \mu_F = \mu \), in all calculations.

In order to define realistic \( W + n \) jet cross sections we need to specify cuts on the final-state particles. We use a set of standard cuts:

\[
|\eta_l|_{\text{max}} = 2.5, \quad p_{Tl}^{\text{min}} = 20 \text{ GeV}, \quad H_T^{\text{min}} = 20 \text{ GeV}, \\
|\eta_j|_{\text{max}} = 2.5, \quad p_{Tj}^{\text{min}} = 40 \text{ GeV}, \quad \Delta R_{jj} > 0.7,
\]

typical of the LHC general purpose detectors. We will also study how \( R^\pm(n) \) varies with \( p_{Tj}^{\text{min}} \), since many NP signals are expected to produce energetic jets in the final state.

The leading-order cross sections, evaluated at two different scales \( \mu = M_W \) and \( \mu = H_T \) (the scalar sum of transverse momenta of all visible particles \( \sum_i p_{Ti} \)), together with their corresponding \( R^\pm(n) \) values, are shown in Fig.\(^1\) for the 14 TeV LHC. Note that variation with scale choice of the ratios is much smaller than the variation in the absolute values of the cross section. With these cuts and scale choices, the charge asymmetry ratios lie roughly in the range [1.25, 1.60] with a noticeable dependence on \( n \).

To study the renormalisation and factorisation scale dependence of the \( R^\pm(n) \) predictions in more detail, we choose a variety of scales \( \mu \), from fixed scales (\( \propto M_W \)) to ‘dynamical’ scales that

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\(^3\)Real gluon emission at \( \mathcal{O}(\alpha_S) \) causes the typical parton \( x \) values to increase slightly compared to the leading-order calculation, which in turn gives rise to a larger \( u/d \) ratio and hence a slightly larger \( R^\pm \).
Table 2: The renormalisation and factorisation scale dependence (μ = μR = μF) of the leading-order (and next-to-leading order for n = 0, 1, 2) ratio R±(n), for the standard set of cuts defined in Eq. (1) at the 14 TeV LHC. Here EW is the transverse energy of the W boson, EW = \sqrt{M_W^2 + p_T(W)^2}, and HT is the scalar sum of the transverse momenta of the final-state jets and leptons, HT = \sum_i p_T. The numbers in brackets are the estimated calculational errors on the final displayed significant figure.

| n | μ = MW | μ = EW | μ = HT/2 | μ = HT | μ = 2HT |
|---|-------|-------|---------|-------|---------|
| LO |       |       |         |       |         |
| 0  | 1.365(1) | 1.365(1) | 1.364(1) | 1.359(1) | 1.355(1) |
| 1  | 1.276(1) | 1.275(1) | 1.275(1) | 1.276(1) | 1.276(1) |
| 2  | 1.358(1) | 1.351(1) | 1.349(1) | 1.351(1) | 1.352(1) |
| 3  | 1.472(2) | 1.455(2) | 1.446(2) | 1.451(2) | 1.454(2) |
| 4  | 1.58(1) | 1.55(1) | 1.54(1) | 1.55(1) | 1.54(1) |
| NLO |       |       |         |       |         |
| 0  | 1.310(1) | 1.310(1) | 1.309(1) | 1.309(1) | 1.309(1) |
| 1  | 1.270(1) | 1.268(1) | 1.274(1) | 1.269(1) | 1.265(1) |
| 2  | 1.326(5) | 1.328(4) | 1.341(2) | 1.333(2) | 1.335(2) |

Table 2: The renormalisation and factorisation scale dependence (μ = μR = μF) of the leading-order (and next-to-leading order for n = 0, 1, 2) ratio R±(n), for the standard set of cuts defined in Eq. (1) at the 14 TeV LHC. Here EW is the transverse energy of the W boson, EW = \sqrt{M_W^2 + p_T(W)^2}, and HT is the scalar sum of the transverse momenta of the final-state jets and leptons, HT = \sum_i p_T. The numbers in brackets are the estimated calculational errors on the final displayed significant figure.

As the number of jets increases, new kinematic configurations open up, leading to a broader range of internal energy and momentum scales. To illustrate this, we show in Fig. 2 the distribution of two dynamical quantities, HT and EW, for different number of jets n. The increase in the spread of these two variables with n lends support to the use of dynamical rather than fixed scales μ. Indeed in Ref. [4], the use of μ = HT was advocated for W+ jets production, since with this scale choice many representative kinematic distributions were shown to be positive with stable K-factors. We therefore choose HT as our default scale choice when predicting R±(n). The resulting ratios, as well as the individual cross sections, are displayed in Table 3 for both 7 and 14 TeV LHC. The t\bar{t} cross sections are also shown for comparison.

From Tables 2 and 3 we see that the ratio R±(n) drops quite significantly from n = 0 to n = 1, and then increases steadily with n thereafter. This can be understood by considering the dominant subprocess contributions. Table 4 shows the subprocess breakdown of (W+ + W−) + n jet production at the 14 TeV LHC. For large n the fractions appear to stabilise with Qg ≡ (q + \bar{q})g production dominating. This can be understood as arising from the dominance of ‘BFKL-like’
| n  | 7 TeV          | 14 TeV         |
|----|----------------|----------------|
|    | $\sigma^+(n)$ | $\sigma^-(n)$ | $R^\pm(n)$     | $\sigma^+(n)$ | $\sigma^-(n)$ | $R^\pm(n)$     |
| LO |                |                |                |                |                |                |
| 0  | 2860(1)        | 1800(1)        | 1.589(1)       | 4980(1)        | 3665(1)        | 1.359(1)       |
| 1  | 162.8(1)       | 110.2(1)       | 1.478(1)       | 385.4(1)       | 302.1(1)       | 1.276(1)       |
| 2  | 35.68(2)       | 22.53(1)       | 1.584(1)       | 100.9(1)       | 74.74(5)       | 1.351(1)       |
| 3  | 5.339(4)       | 3.099(2)       | 1.723(2)       | 18.78(2)       | 12.94(1)       | 1.451(2)       |
| 4  | 0.734(3)       | 0.392(1)       | 1.87(1)        | 3.28(2)        | 2.11(1)        | 1.55(1)        |
| NLO|                |                |                |                |                |                |
| 0  | 3210(1)        | 2114(1)        | 1.519(1)       | 5234(2)        | 3997(1)        | 1.309(1)       |
| 1  | 211.1(1)       | 145.2(1)       | 1.454(1)       | 478.3(2)       | 376.9(1)       | 1.269(1)       |
| 2  | 42.53(4)       | 27.31(2)       | 1.557(2)       | 114.3(1)       | 85.69(8)       | 1.333(2)       |

Table 3: Predicted cross sections (in pb) for $\sigma^\pm(n) \equiv \sigma(W^\pm(\rightarrow e^\pm\nu) + n \text{ jets})$ production at 7 and 14 TeV at the LHC. Both the renormalisation ($\mu_R$) and factorisation ($\mu_F$) scales are set equal to $H_T$, the scalar sum of the transverse momenta of jets and leptons. In obtaining these cross sections, the standard set of cuts defined in Eq. (4) is applied. For comparison, the leading-order cross section $\sigma(tt \rightarrow e^\pm\nu + 4j)$ is also shown. In this case, the scale $\mu = \mu_R = \mu_F = m_t = 171.3$ GeV is used. The numbers in brackets are the estimated calculational errors on the final displayed significant figure.

configurations, in which the scattering amplitudes are dominated by $t$–channel gluon exchange with the $W^+$ $(W^-)$ emitted off a positively (negatively) charged quark or antiquark line,

$$g + q^\pm \rightarrow W^\pm + q'^\mp + ng,$$

as illustrated in Fig. 4. In fact this is the basis of the ‘high-energy approximation’ for $W^+$ jets production developed first in Ref. [7] and more recently and more comprehensively in Ref. [8] (see also Ref. [9]). It would be interesting to see how well this high-energy approximation agrees with the exact results for $R^\pm(n)$ for $n = 2, 3, 4$, since it can easily be extended to higher values of $n$. Note also that in this approximation the dominant quark scattering contribution is obtained by replacing the incoming/outgoing gluon at the bottom of the diagram by a quark or antiquark line (of 5 quark flavours). The effective PDF at the bottom of the diagram is therefore $g(x, \mu^2) + (4/9) \sum q[q(x, \mu^2) + \bar{q}(x, \mu^2)]$. As the average value of $x$ increases with the number of jets $n$, we would expect the quark-quark contribution to slowly increase with respect to the quark-gluon contribution, exactly as seen in Table 4.

Because of this $t$–channel dominance, we can relate the $W^+ + n$ jet cross-section ratio to the ratio of the parton-parton differential luminosities

$$\tilde{R}^\pm \equiv \frac{\partial L/\partial \hat{s}(q^+G)}{\partial L/\partial \hat{s}(q^-G)},$$

with $q^+ = u + c + \bar{d} + \bar{s}$, $q^- = d + s + \bar{u} + \bar{c}$, $G = g + (4/9) \sum q[q + \bar{q}]$, where the sum is over 5 quark flavours, and $\sqrt{\hat{s}} = M_{Wnj}$, the invariant mass of the $W^+ + n$ jet system. Because the
Table 4: Parton subprocess breakdown (in per cent) of leading-order $W^{\pm} + n$ jet production at the LHC ($\sqrt{s} = 14$ TeV), with the standard set of cuts and scale choice $\mu_R = \mu_F = M_W$. Here $Q = q$ or $\bar{q}$.

| $n$ | $QQ$  | $Qg$ | $gg$ |
|-----|-------|------|------|
| 0   | 100   | 0    | 0    |
| 1   | 18    | 82   | 0    |
| 2   | 21    | 73   | 6    |
| 3   | 23    | 70   | 7    |
| 4   | 25    | 67   | 8    |

The association of the exact cross-section ratio with the parton luminosity ratio of Fig. 5 enables us to readily estimate the PDF uncertainty on $R^{\pm}(n)$. Fig. 6 shows the corresponding MSTW2008 NLO PDF (68% cl) uncertainty (see Ref. [1] for a full discussion) on the $q^{+}/q^{-}$ parton luminosity ratio. Note that as the subprocess energy increases from 100 GeV to 1 TeV, the uncertainty increases from approximately 0.5% to 1%. This suggests that a reasonably conservative estimate of the PDF uncertainty on $R^{\pm}(n)$ for $n \leq 4$ is $\sim \pm 1\%$, in line with the results for the ratio of total cross sections given in Table 1. We have checked that this is indeed the case for the exact ratio calculations up to and including $n = 3$ at leading order.

Putting everything together, we obtain the predictions for $R^{\pm}(n)$ at the LHC, including scale and PDF uncertainties shown in Table 5. Results for both 14 TeV and 7 TeV collider energy are shown. The ratios are all systematically larger at the lower collider energy, reflecting the increase in the $u/d$ PDF ratio at larger $x$ (see also Tables 1, 3 and Fig. 3). The ratios are calculated exactly at NLO for $n = 0, 1, 2$ and estimated at LO for $n = 3, 4$. The central values are obtained with scale choice $\mu_R = \mu_F = H_T$, and the scale variation uncertainty is conservatively chosen to encompass all the predictions for different scales shown in Table 2. Observing the change in $R^{\pm}(n)$ from LO to NLO for $n = 1, 2$ and also for $n = 3$ in Ref. [6] with a similar set of cuts, we might expect NLO predictions for $n = 3, 4$ to decrease slightly from our LO central values. Note that restricting the scale variation to $[0.5H_T, 2H_T]$ would give a significantly smaller scale uncertainty. Overall, the combined theoretical uncertainty on the ratio predictions increases.
slightly with the number of jets, but is never more than ±3%.

We emphasise again that the PDF uncertainties shown in Table 5 are obtained using the MSTW2008 sets only. As shown for example in Fig. 69 of Ref. [1], other available PDF sets give central predictions for the charge asymmetry ratio that are slightly offset from the MSTW2008 predictions, but with similar uncertainties. The reasons for this are only partially understood, see for example the discussion in the PDF4LHC workshop series [11].

\[
\begin{array}{|c|c|c|}
\hline
n & \sqrt{s} = 7 \text{ TeV} & \sqrt{s} = 14 \text{ TeV} \\
\hline
0 & 1.52 \pm 0.01 \text{ (scl)} \pm 0.02 \text{ (pdf)} & 1.31 \pm 0.01 \text{ (scl)} \pm 0.01 \text{ (pdf)} \\
1 & 1.45 \pm 0.01 \text{ (scl)} \pm 0.01 \text{ (pdf)} & 1.27 \pm 0.01 \text{ (scl)} \pm 0.01 \text{ (pdf)} \\
2 & 1.56 \pm 0.02 \text{ (scl)} \pm 0.02 \text{ (pdf)} & 1.33 \pm 0.02 \text{ (scl)} \pm 0.01 \text{ (pdf)} \\
3 & 1.72 \pm 0.03 \text{ (scl)} \pm 0.03 \text{ (pdf)} & 1.45 \pm 0.03 \text{ (scl)} \pm 0.02 \text{ (pdf)} \\
4 & 1.87 \pm 0.04 \text{ (scl)} \pm 0.03 \text{ (pdf)} & 1.55 \pm 0.04 \text{ (scl)} \pm 0.02 \text{ (pdf)} \\
\hline
\end{array}
\]

Table 5: Predictions for \( R^\pm(n) \) at the LHC for the standard set of cuts defined in Eq. (4). The ratios are calculated exactly at NLO for \( n = 0, 1, 2 \) and estimated at LO for \( n = 3, 4 \). The central values are obtained with scale choice \( \mu_R = \mu_F = H_T \). The scale (scl) and PDF uncertainties are displayed separately.

Finally, we show in Fig. 7 the values of \( R^\pm(n) \) for different values of \( p_{TJ}^{\text{min}} \). We see that the ratios increase with \( p_{TJ}^{\text{min}} \), again a reflection of the increase in the \( u/d \) PDF ratio at larger \( x \). This makes the SM charge asymmetry more prominent for hard, multijet final states, which could assist the detection of New Physics contributions.

3 Conclusions

In this paper, we have studied the cross-section ratios, \( R^\pm(n) \), of \( W^+ + \text{jets} \) and \( W^- + \text{jets} \) production for different number of jets \( n \) at the LHC. We exploited the charge asymmetry nature of proton-proton collisions at the LHC to demonstrate that it provides an additional handle to study New Physics signals in the \( W(\to l\nu) + \text{jets} \) channel, where typically the charged leptons are produced in equal quantities, and therefore any deviation of \( R^\pm \) from the SM expectation would indicate the presence of New Physics.

Quantitatively, we showed that the \( R^\pm \) ratios are remarkably stable with respect to theoretical uncertainties from scale choices, higher-order corrections and from PDFs. We have also demonstrated a connection between the cross-section ratios \( R^\pm(n) \) and the parton luminosity ratio \( \tilde{R}^\pm \) based on arguments from BFKL dominance, and showed that \( R^\pm(n) \) can be reasonably well approximated by \( \tilde{R}^\pm \) with a suitable choice of subprocess invariant mass.

Given the simple and robust nature of this observable, it could prove useful for studying New Physics phenomena, particularly at the early stages of LHC operation. In a future study we will consider specific NP scenarios for which the method could be applicable.

\[\text{\footnote{For example, using CTEQ6.6(NLO) [10] instead of MSTW2008(NLO) pdfs gives charge asymmetry ratios that are approximately 2–3% larger for } n = 1 - 4.}\]
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see also http://www.hep.ucl.ac.uk/pdf4lhc/ for earlier meetings.
Figure 1: Predicted cross sections times leptonic branching ratio for $W^\pm + n$ jet production ($n = 0, 1, 2, 3, 4$) at the 14 TeV LHC, using the standard set of cuts defined in Eq. (4) and calculated at leading order with scales $\mu (= \mu_R = \mu_F) = M_W$ and $\mu = H_T$, where $H_T$ is the scalar sum of the transverse momenta of the final-state jets and leptons, $H_T = \sum_i p_{T,i}$. The cross-section ratios $R^\pm(n)$ are marked on the plot.
Figure 2: Ratios of cross sections for $W^\pm + n$ jet production ($n = 0, 1, 2, 3, 4$) to the values obtained with scale $\mu = \mu_R = \mu_F = M_W$ as a function of $\mu$. For $n = 0, 1$ and 2, the cross sections are calculated at both leading and next-to-leading order. The ratios $R^\pm(n)$ are also shown (right-hand axis).
Figure 3: Normalised distributions of the $W$ transverse energy $E_T^W$ (dashed) and transverse momenta scalar sum $H_T$ (solid) for $W^\pm + n$ jet production ($n = 1, 2, 3, 4$) at the 14 TeV LHC, using the standard set of cuts defined in Eq. (4) and calculated at leading order with scales $\mu (= \mu_R = \mu_F) = M_W$. As $n$ increases, both distributions move towards higher values. The $n = 4$ distributions are displayed in black.

Figure 4: The dominant configuration for $qg \rightarrow W + ng$ scattering in the high-energy limit. The blobs represent generalised vertices.
The parton-parton luminosity ratio $\tilde{R}^\pm$ defined in the text, as a function of the subprocess energy $\sqrt{\hat{s}} = M_{Wnj}$.

PDF (68% cl) uncertainty on the parton-parton luminosity ratio $\tilde{R}^\pm$ defined in Eq. (6), as a function of the subprocess energy $\sqrt{\hat{s}} = M_{Wnj}$. 

Figure 5: The parton-parton luminosity ratio $\tilde{R}^\pm$ defined in the text, as a function of the subprocess energy $\sqrt{\hat{s}} = M_{Wnj}$.

Figure 6: PDF (68% cl) uncertainty on the parton-parton luminosity ratio $\tilde{R}^\pm$ defined in Eq. (6), as a function of the subprocess energy $\sqrt{\hat{s}} = M_{Wnj}$.
Figure 7: Variation of $R^{\pm}(n)$ as a function of $p_{T,j}^{\text{min}}$. The ratios are obtained with scale choice $\mu_R = \mu_F = H_T$. For simplicity, the calculations are performed at leading order.