A Coupled model for ERT monitoring of contaminated sites

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Abstract. The performance of electrical resistivity tomography (ERT) system is usually investigated using a fixed resistivity distribution model in numerical simulation study. In this paper, a method to construct a time-varying resistivity model by coupling water transport, solute transport and constant current field is proposed for ERT monitoring of contaminated sites. Using the proposed method, a monitoring model is constructed for a contaminated site with a pollution region on the surface and ERT monitoring results at different time is calculated by the finite element method. The results show that ERT monitoring profiles can effectively reflect the increase of the pollution area caused by the diffusion of pollutants, but the extent of the pollution is not exactly the same as the actual situation. The model can be extended to any other case and can be used to scheme design and results analysis for ERT monitoring.

1. Introduction

Due to the advantages of rapid, non-destructive and low-cost, electrical resistivity tomography has been used in many areas such as hydrogeological, mining and geotechnical investigations and environmental surveys. More recently, ERT is used in time-lapse mode, measuring temporal changes in resistivity, has become increasingly popular to monitor dynamic processes including non-aqueous phase liquids remediation [1-2], biodegradation of hydrocarbons [3], freshwater salinization [4], wastewater transport [5], and recharge induced contaminant plume behavior [6-7]. These applications demonstrate the great potential of using resistivity method to monitor contaminated sites. However, due to the different geological conditions, e.g. hydrology, pollution sources and soil texture, the monitoring scheme and results interpretation remain huge challenge.

The numerical simulation study for ERT monitoring of contaminated sites is helpful to analyze the performance of ERT monitoring under different geological conditions and the results can be used to guide the monitoring scheme design and results interpretation. Ebrahimi has analyzed the ability of ERT arrays to detect a buried channel though Numerical simulation [8]. Lu built a three-dimensional (3-D) forward model based on the patching method of electrical resistivity tomography (ERT) to research apparent resistivity features of different topographies with pole–pole array and Wenner array [9]. Bongiovanni designed an electrical model of the reservoir from well data and numerically simulated the forward geoelectrical response to determine the conditions under which the anomaly studied and studied the feasibility of detecting brine in an oil reservoir with surface-downhole electrical measurements [10]. Ye construct a coupling simulation model for ERT in which the distribution information of multi-phase flow is exchanged between the fluid field and electric field [10]. Power proposed a coupled DNAPL-ERT model for simulating the mapping of dense non-
aqueous phase liquids using electrical resistivity tomography [11]. In this paper, a method to construct a time-varying resistivity model by coupling the water transport, solute transport and constant current field is proposed for ERT monitoring of contaminated sites. The model is used to simulate the ERT monitoring results of pollutant diffusion process to guide the design of the monitoring system for contaminated sites.

2. ERT monitoring model of contaminated sites

2.1. The coupled model
Many experimental studies have shown that the soil resistivity is affected by many factors. The spatial distribution of resistivity in the site can be regarded as the result of three factors. The first factor is the basic properties of soil, such as soil particle size, cementing coefficient and porosity. The second factor is the water saturation which is related to the parameters such as the permeability coefficient and groundwater level of the site. The third factor is the pore water conductivity which is related to the ion content in soil and is greatly affected by the type and concentration of pollutants. The quantitative relationship under unsaturated conditions between resistivity and saturation, porosity and pore water resistivity given by Archie [12] is:

$$\frac{\lambda_b}{\lambda_w} = a \theta^{-z} S_w^{-g} \lambda_w$$

(1)

where: $\lambda_b$ is the resistivity of soil, and $z$, $a$ is constant, $z$ is the cementation index of rock, $a$ is the curvature factor, $\lambda_w$ is the resistivity of pore water, $\theta$ means porosity and $S_w$ is water saturation.

To simulate the ERT results for monitoring pollutant diffusion process in subsurface soil, a coupled model is built by combined the three physical fields: fluid field, solute migration and steady electric field. The linkage of three physical fields is shown in Figure 1 and the whole solution of the model includes four steps:

1. The flow velocity and moisture content distribution in a site are obtained by solving the water transport.
2. Using the solution of the water transport model as input, the solute transport model was solved to obtain the spatial distribution of pollutant concentrations at different times.
3. According to the spatial distribution of pollutant concentration and the spatial distribution of moisture content, the spatial distribution of resistivity was solved by Archie's formula.
4. The ERT results are obtained by solving the steady electric field distribution at different times.

![Diagram](image)

**Figure 1.** ERT monitoring model of contaminated sites.

2.2. Water transport
The flow velocity and moisture content distribution in the vadose zone can be described using the following equations, which derived from Richard's equation [13-14].

$$\left( C + S_c \cdot S \right) \frac{\partial H}{\partial t} + \nabla \cdot (\rho u) = 0$$

(2)

$$u = \frac{K}{\rho g} (\nabla \rho + \rho g \nabla D)$$

(3)

$$K = K_s \cdot K_r$$

(4)

$$S = (\theta_i - \theta_c) / (\theta_m \cdot \rho g)$$

(5)
where $C$ is the specific moisture content (m$^{-1}$); $S_e$ is the effective saturated soil (dimensionless); $S$ is a storage coefficient (m$^{-1}$); $H_r$ is the pressure head (m), which is proportional to the independent variable $P$ (Pa); $t$ is time; $u$ is the Darcy velocity field; $K$ is the hydraulic conductivity (m/s); the direction of $D$ (usually in the $z$ direction) represents the vertical elevation (m). $\rho$ is the density of the liquid, $g$ is the acceleration of gravity. $K_r$ is the relative permeability coefficient in soil, and $K_s$ is the constant hydraulic conductivity at saturation. $\theta_s$ and $\theta_r$ represent fluid saturation and volume fraction after drainage, respectively.

The coefficients $C$, $S_e$, and $K$ vary with the pressure head $H_p$, and with the volume fraction of fluid within the soil which can be defined as follows according to Van Genuchten [15]:

$$
\theta = \begin{cases} 
\theta_s + S_e(\theta_s - \theta_r) & H_p < 0 \\
\theta_r & H_p \geq 0 
\end{cases} \quad (6)
$$

$$
S_e = \begin{cases} 
\frac{1}{1 + [\theta_s / \theta_r]^m} & H_p < 0 \\
1 & H_p \geq 0 
\end{cases} \quad (7)
$$

$$
C = \begin{cases} 
\frac{a m}{1 - m} (\theta_s - \theta_r) S_e^{1/n} (1 - S_e^{1/n})^n & H_p < 0 \\
0 & H_p \geq 0 
\end{cases} \quad (8)
$$

$$
k_r = \begin{cases} 
S_e [1 - (1 - S_e^{1/n})^n] & H_p < 0 \\
1 & H_p \geq 0 
\end{cases} \quad (9)
$$

Here $\alpha$, $n$, $m$, and $l$ are constants for a certain type of medium, with $m=1-1/n$. $\theta$ is porosity.

### 2.3. Solute transport

The most commonly used models to simulate the solute transport in soil fall into two categories, one is based on convection-dispersion equation and the other is based on transfer function model. In this paper, the former is employed to predict the transport of contaminant in soil as follows:

$$
(\theta + \rho_b k_p) \frac{\partial c}{\partial t} + c \frac{\partial \theta}{\partial t} + \nabla \cdot (- \theta D_L \nabla c + uc) = \theta \phi_L + \rho_b k_p \phi_p c + S_c 
$$

$$
c \frac{\partial \theta}{\partial t} = c C \frac{\partial H_p}{\partial t} \quad (11)
$$

where $c$ represents the dissolved concentration (Kg/m$^3$), $\theta$ represents the fluid volume fraction (porosity, dimensionless), $\theta_c$ represents the amount of material in the pore water, $\rho_b$ is the soil bulk density (Kg/m$^3$), $C_p$ represents mass units of adsorbed pollutants (mg/Kg), $\rho_b C_p$ provides solute mass concentration in the soil, $D_L$ is the hydrodynamic dispersion tensor (m$^2$/d); $R_p$ denotes the reaction in water (Kg/(m$^3$·d)), $R_p$ equals the reaction involving solute attachment to soil particles (Kg/(m$^3$·d)). $S_c$ is the increased solute per unit volume per unit time (Kg/(m$^3$·d)). $\phi_L$ and $\phi_p$ represent the rate of adsorption and the rate of solute concentration, respectively.

The hydrodynamic dispersion tensor, $D_L$, describes mechanical spreading from groundwater movement in addition to chemical diffusion:
\[
\theta D_{,ll} = \alpha_1 \frac{u_i^2}{\|u\|} + \alpha_2 \frac{u_j^2}{\|u\|} + \frac{\theta D_{,d}}{\tau_L} \tag{12}
\]

\[
\theta D_{,lj} = (\alpha_1 - \alpha_2) \frac{u_i u_j}{\|u\|} \tag{13}
\]

where \(\theta D_{,ll}\) are the diagonal elements of the diffusion tensor; \(\theta D_{,lj}\) are the cross terms; \(\alpha\) is the dispersivity; the subscripts “1” and “2” denote longitudinal and transverse dispersivities, respectively; \(D_m\) denotes the coefficient of molecular diffusion (m²/d); and \(\tau_L\) is a tortuosity factor that reduces impacts of molecular diffusion for porous media relative to free water. Here \(\tau_L = \theta^{7/3} \theta_s^{-2}\).

2.4. Control equation of point current source

The purpose of the ERT measurement is to determine the subsurface resistivity distribution by making measurements on the ground surface or at boreholes. A stable artificial electric field is established by injecting a steady current into the earth. By measuring the value of the electric field, the resistivity distribution of the sites can be defined, and then the underground medium information can be inferred. According to the principle of the stationary current field, the potential distribution of a point current source in three-dimension space satisfies the Poisson’s equation:

\[
\nabla \cdot (\sigma \nabla U) = -2I \delta(A) \tag{14}
\]

where, \(\sigma\) is electrical conductivity, \(\delta = 1/\lambda\), \(\lambda\) is resistivity; \(U\) is electrical potential; \(I\) is current intensity of current source; \(A\) is location of current source.

3. Case study

3.1. Contamination site description

Surface water infiltration is one of the main causes that lead to the pollution of soil and groundwater. For example, in China, almost all artificial liner system in landfill is damaged due to manufacturing and construction defects. As a result, leachate produced in the landfill will leak through these failures and enter into the subsurface, leading to soil and groundwater pollution. Soil or groundwater once contaminated can hardly be restored unless plenty of time and money invested. Thus, monitoring methods/instrument capable of contamination detection are more desirable for early warning and leaks location. Here, we construct a monitoring model to simulate the above-mentioned condition when there is a potential pollution source of soil on the surface.

3.2. Parameter setting of monitoring model

The model space is set as a cube with a detection region located in the middle of the model. The area of detection region is set to 5m×5m×1.5m. The upper surface of detection region coincides with the upper surface of the model surface. Supposing a circular pollution source is located at the center of the model space’s surface, the radius of the source is set to 0.4m. 24 electrodes are placed along the X axis and y=0, (represented by dots in Figure 2), and the interval space between the adjacent electrodes is set to 0.2m. The coordinate of the first electrode is (-2.3m, 0m, 0m), and the coordinate of the 24th electrode is (2.3m, 0m, 0m). Wenner array [16-17] is carried out for simulation. In each measurement, two of the 24 electrodes are selected as current electrodes by which a steady current is injected into the model space. The current values at the two current electrodes are +1A and -1A respectively. The finite element method is used to solve the governing equation and the mesh of model is shown in Figure 1.
Figure 2. Model space and mesh.

The initial value of the pressure head $H_p$ is set to 0.01m in the pollution source area, the initial value of the pressure head in the rest of the model area is related to vertical elevation:

$$H_p = \begin{cases} 
0.01m & \text{Source region} \\
-(z + 1.4)m & \text{Other} 
\end{cases}$$

(15)

The boundaries of the model space are set to free outflow boundaries. the pressure head distribution $H_p$ can be gotten by solving the equation 2. And then, the velocity distribution in the space can be obtained according to Darcy law. The parameter values of soil are shown in Table 1.

| Parameter                  | Value | Unit  |
|---------------------------|-------|-------|
| Hydraulic conductivity $K_s$ | 0.298 | m/d   |
| Fluid saturation $\theta_s$  | 0.399 |       |
| Volume fraction after drainage $\theta_r$ | 0.001 |       |
| $\alpha$                  | 1.74  | m$^{-1}$ |
| $n$                       | 1.38  |       |
| $m$                       | $1 - 1/n$ |       |
| Pore connection parameter | $n/\alpha$ |       |

The pollutant concentration in source region is set to a constant value 2Kg/m$^3$ while it is set to 0 Kg/m$^3$ in other location of the model space. The pollutant migrates from the surface to the subsurface soil in the effect of advection and diffusion. The migrating range of pollutant will not exceed the model size in a few hours. Therefore, solute flux is set to 0 at the boundary of model space.

$$n \cdot N = 0 \quad \text{or} \quad n \cdot (\Theta D_L \nabla c) = 0$$

(16)

where $n$ is Normal vector of boundary.

Solute transport parameters are shown in table 2:

| Parameter                        | Value  | Unit     |
|----------------------------------|--------|----------|
| Bulk density $\rho_b$            | 1400   | Kg/m$^3$ |
| Partition coefficient $K_p$      | 0.0001 | m$^3$/Kg |
| Molecular diffusion coefficient $D_m$ | 0.00374 | m$^2$/d  |
| Transverse dispersion $\alpha_z$ | 0.001  | m       |
| Decay rate in liquids $\Phi_L$   | 0.05   | d$^{-1}$ |
| Decay rate in soil $\Phi_p$      | 0.01   | d$^{-1}$ |
4. Results and discussion

4.1. Pollutant concentration distribution

Starting from the initial moment, the concentration distribution is calculated every hour which is shown in figure 3. We can see that pollutant concentration is 0 Kg/m$^3$ except in the pollutant source. As time goes by, the pollutants diffuse to the deeper and the surrounding soil under the action of the water head. After 2 hours, the range in the horizontal direction is -0.5m~0.5m, and in the depth direction arrives at 0.18m. The range of pollutant after 10 hours, the range of horizontal direction increases to -0.62m~0.62m, and in the depth direction arrives at 0.45m. The pollutant diffusion velocity in horizontal direction is less than in vertical direction apparently.

![Figure 3](image)

**Figure 3.** Concentration distribution at different times.

4.2. Resistivity method detection results

The chromium was assumed to be the targeted pollutant, and relevant soil parameters in equation 1 were referred to literature [18], the resistivity of the ground is calculated according to the concentration distribution. The background value of the ion concentration in the soil is assumed to be 0.1 kg/m$^3$, and the highest ion concentration is 2 Kg/m$^3$. So, the resistivity value of the site range from 4.9 $\Omega\cdot$m to 122 $\Omega\cdot$m. ERT results in different time are shown in figure 4. From the figure we can see that the range of low resistivity is increased with the increases of time, especially the area range of resistivity value 4.9 $\Omega\cdot$m ~10 $\Omega\cdot$m increases significantly. The trend of resistivity change is consistent with the trend of pollutant diffusion. However, the area range of low resistivity in vertical direction is more than the range of pollutant region because of volume effect.
Figure 4. ERT monitoring profiles at different times.

5. Conclusions
This paper builds a coupled model by combined the three physical fields: fluid field, solute migration and steady electric field. The linkage of three physical fields and the solving steps is given. And then, a case of contaminated sites which with a pollutant source on earth's surface is taken as example and the ERT results at different times are solved by finite element calculation method. The simulation results show that the ERT monitoring profiles can effectively reflect the increase of the pollution area caused by the diffusion of pollutants, but the extent of the pollution is not exactly the same as the actual situation which may be improved by ERT inversion. In the further study, we will study the method of recognize the pollution extent according to the change of resistivity in ERT results and the monitoring scheme for contaminated sites in different pollutant condition.

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