Research Article

Codes over Lattice-Valued Intuitionistic Fuzzy Set Type-3 with Application to the Complex DNA Analysis

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In this article codes over lattice valued intuitionistic fuzzy set type-3 (LIFS-3) are defined. Binary block codes and linear codes are constructed over LIFS-3. Hamming distance and related properties of these newly established codes are examined. The research findings are applied to genetic codes. The set $L$ of sixty-four codons is converted into a lattice and then codes are created over the set $S$ of twenty amino acids by defining membership and nonmembership functions from the set of twenty amino acids to the sixty-four codon set. Comparison of codes over $L$-fuzzy set and LIFS-3 conducted in terms of hamming distance for codon system that ensures the efficiency of newly established codes.

1. Introduction

Probability theory was once believed to be an ideal tool to deal with any uncertain situation. However, there are many problems where the uncertainty appears as an imprecision and ambiguity rather than a statistical variation. The classical probability theory is not efficient and suitable for handling the uncertainties and imprecision that appear in pattern recognition. Zadeh [1] presented the notion of fuzzy set as an extension of the ordinary set. This notion is used for describing vagueness and ambiguity mathematically. The formulation of fuzzy sets over a nonempty set $S$ is based on an allocation of a grade of membership to each element of $S$. The allocated grades are precisely the real numbers ranging between 0 and 1. Mathematically, a fuzzy set $\mu$ over $S$ is characterized by a map $\mu: S \rightarrow [0, 1]$ termed as membership function and the value $\mu(x)$ as grade of membership of $x$ in $S$. For instance, let $X$ be the set of seven days of a week and $p(x)$ be the number of codes transmitted from a source center $S$ and $q(x)$ be the number of codes received accurately at the receiver end $R$ in the day $x \in X$. Then the function $\mu: X \rightarrow [0, 1]$ defined as $\mu(x) = q(x)/p(x)$, for all $x \in X$, provides a fuzzy set over $X$ (that is, the collection of ordered pairs $(x, \mu(x))$). For example, if on $x = \text{Monday}$, 100 codes were transmitted from $S$ and the supervisor at $R$ reported that 75 codes were received properly at $R$, then $\mu(x) = 75/100 = 0.75$ would be the grade of membership of $x$ in $X$, or we can say that the supervisor’s statement’s truth value was 0.75. According to classical probability, 25 codes were not received at $R$, but in real practice, mostly the interpreter at $R$ can decode the partially transmitted codes for further processing. In this example, if ten partially received codes are processed, then the supervisor’s statement’s truth value increases from 0.75 to 0.85; the falsity value (non-membership) decreases from 0.25 to 0.15. Let $\nu(x) = 0.15$ represent the falsity value of the supervisor’s statement; then $1 - (\mu(x) + \nu(x)) = 0.10$ is the degree of uncertainty caused by the performance of interpreter at $R$. So, the membership grades are not enough to communicate the correct information in this case and need to introduce the idea of non-membership grades; these grades should not be confused with the probability of non-occurrence $P(\overline{A}) = 1 - P(A)$ of an event $A$. 


Atanassov [2], first introduced the notion of non-
member ship grages. The generalization of fuzzy sets that
involves both the membership and nonmembership grades
is known as the intuitionistic fuzzy set (IFS). Atanassov
proposed fundamental properties, various arithmetic op-
erations for his development in fuzzy sets. In continuation,
Atanassov [3] presented the geometric interpretation of
intuitionistic fuzzy objects. There are several other gen-
eralizations of fuzzy sets that mostly depend upon mem-
bership, nonmembership, hesitancy and indeterminacy grades.
As in a fuzzy set, the grades \( \mu(x) \) belong to the close interval
\([0, 1]\), which is naturally endowed with a partial order. The
assumption of order structures outside the unit interval laid
the foundations of ordered fuzzy sets. Partial orderings and
fuzzy uncertainties are features of many real-world prob-
lems. These kinds of problems are ill-posed most of the time,
because either they have infinite solutions or no solutions at
all. For instance, the selection of a grocery bundle from
various packages is subjected to various contradictory and
conflicting criteria. Nutritional value, quality, variety and
above all cost, are some of the factors that a person can think
about a bundle. Thus, the partial ordering of the bundles is
an essential feature of this problem. Goguen [4] introduced
the concept of \( L \)-fuzzy subsets of \( S \) where the interval \([0, 1]\)
is replaced by a partially ordered set \( L \). He discovered re-
flectable remarkables of this generalized concept and concluded
that \( L \)-fuzzy set theory works more efficiently in real-world
problems. After IFS Atanassov and Stoieva [5] enrich the area
and replaced the lattice \([0, 1]\) by arbitrary complete lattice \( L \),
to relate membership and nonmembership functions \( \mu, \nu: S \longrightarrow L \) he involved an involutive order revers-
ing unary operation \( N: L \longrightarrow L \). The new structure is known as
lattice valued intuitionistic fuzzy set (LIFS-1). But it has
certain limmitation mostly occurred due to the incorpora-
tion of the operator \( N \). Gerstenkorn and Tepavcevic [6]
made an attempt to upgrade Atanassov idea and proposed a
new variant, that is, the lattice valued intuitionistic fuzzy set
type \( 2 \) (LIFS-2) simply by exchanging the operator \( N \) with
a linearization function \( \ell: L \longrightarrow [0, 1] \). This modification is
quite helpful in the establishment of decomposition theorem
and synthesis but fails to deal with basic set operations.
Choice of the linearization function is the main reaseon
behind this failure. Finally, lattice homomorphism \( \alpha: L \longrightarrow [0, 1] \) is used to relate membership and non-
member ship functions and the new LIFS is called a lattice
valued intuitionistic fuzzy set type-3 (LIFS-3). This new
sturcture has certain advantages over the previously defined
fuzzy sets and \( L \)-fuzzy sets mostly occurs due to the inclusion
of lattices and lattice homomorphism. The lattice homo-
morphism is applicable to any collection of lattices unlike the
unary operator \( N \) of LIFS-1 and satisfy all the basic set
operations unlike the linearization function \( \ell \) of LIFS-2.

A code is a system of rules used for data communication
and information process. These rules are designed in the
form of letters, symbols, images, sounds or numbers. Coding
theory was established to study these rules mathematically
and make them workable in daily communication. In
communication, algebraic codes are used for data com-
pression and error correction. Coding theory is concerned
with the reliability of communication over a noisy channel.
Algebraic codes are studied in a variety of domains and they
have a wide range of applications across a large range of
disciplines. Murugan and Ananthanarayana [7] presented
the WordTrie, a specialised trie for storing words in order to
facilitate fast coding. The code combination is generated in
such a manner that the size of the WordCode for a word
must be less than the entire size of the character coding. To
improve the detection rate of NLOS nodes in any safety
application of VANET, the WDHPPBDS protocol was
designed by Arjunan and Kaviarasan [8] in order to permit
reliable delivery of emergency information to the targeted
node in a timely manner. Nagaraju et al. [9] explored that by
using a hybrid area exploration technique, mobility assisted
localization for mission critical wireless sensor network
applications can be achieved.

When data are transmitted through a noisy channel,
some errors may arise. The vagueness in data transmission
can be handled by involving theoretical fuzzy set concepts in
the coding and decoding process. There are two ways to
incorporate theoretical fuzzy set concepts in coding. One
method was proposed by von Kaenel and Pierre [10], termed
fuzzy code, and is defined as a fuzzy subset of \( n \)-dimensional
vector space \( F_n \).  He investigated the Hamming distance for newly established fuzzy codes.
Kaenel’s theory is based on the symmetric nature of the
error—that is, the probability of \( 1 \rightarrow 0 \) crossover failure
and \( 0 \rightarrow 1 \) is equally likely. However, in computer
memories and VLSI circuits, the error may not be sym-
metric. Hall and Dial [11] discussed the asymmetric nature
of fuzzy codes and generalized the results of Kaenel. They
worked on the distance between fuzzy code words and
proved that the distance is independent of the dimension of the
vector space \( F_n \). Tsafack et al. [12] established a fuzzy
linear code, fuzzy cyclic code over Gralois ring \( Z_p \).
Amudhambigai and Neeraja [13] discussed arithmetic op-
erations on fuzzy codes and introduced their super in-
creasing sequences. Shijina [14] introduced multi-fuzzy code
in terms of a multi-fuzzy subset of \( n \)-tuples over a set \( S \), and
produced some essential results for Hamming distance. Du
[15] analyzed arithmetic operations of subtraction and di-
vision on intuitionistic fuzzy subsets that were induced by
the Hamming distance. Ali et al. [16] developed soft al-
gebraic codes over soft sets. They also defined soft canonical
generator matrix and soft canonical parity check to decode
these algebraic codes. Sesiela and Tepavcevic [17] introduced
another method of involving fuzzy theory in coding, which is
based on defining a map \( \bar{A} \) from a nonempty set
\( S = \{1, 2, \ldots, n\} \) for partially ordered set \( P \). Sesiela et al. [18]
carried the concept and defined binary block codes over
lattice valued fuzzy sets (L-fuzzy sets). Mališa and Lazarević
[19] discussed the length and cardinality of block codes over
L-fuzzy sets. The concepts of fuzzy codes and codes of or-
dered fuzzy sets are relatively new, but crucial for modifying
the data communication and pattern recognition in deep
learning [20] and fault detection for distributed components
[21, 22]. However, these codes are unable to identify and
handle any expected error in the information process while
codes are transmitted in a noisy channel, for example, a spell
checker and machine reader. Moreover, the already existing fuzzy codes or codes over fuzzy sets are based solely on the degree of membership; the degree of nonmembership was not incorporated in any previous method. Lattice valued intuitionistic fuzzy set type-3 is a generalization of basic fuzzy set that incorporates both the degree of membership and the degree of nonmembership, so it is a more workable framework for modeling uncertain data. The effectiveness of LIFS-3 motivated us to construct codes over these fuzzy sets.

2. Preliminaries

2.1. Bounded Lattice. A relation $g$ on a nonempty set $L$ is precisely a subset of $L \times L$. The elements of $g$ are more commonly denoted by $a \equiv b$; $a$ is a $g$ related to $b$. Based on the nature of elements in the relation $g$, it can have different names. For instance, if $x \equiv y$ for all $x \in L$, then $g$ is called reflexive; if $x \equiv y \Rightarrow y \equiv x$, then $g$ is called symmetric; if $x \equiv y$ and $y \equiv z \Rightarrow x \equiv z$, then $g$ is called transitive. A reflexive, anti-symmetric and transitive relation is called a partial order on $L$. In this case, the set $L$ is called a partially ordered set. For example, “$\leq$” (less than or equal to) is a partial order on the set of real numbers. The symbol “$\leq$” is used for partial order in general and is also used in this article for better understanding by a wider audience. A subset $Y$ of a partially ordered set $L$ is said to be bounded above (below) if there exists $\ell \in L$ such that $\ell \leq \ell$ (for all $\ell' \in Y$). The element $\ell$ is called an upper (lower) bound of $Y$. The set $Y$ may possess more than one upper (lower) bound. The least (greatest) member in the set of upper (lower) bounds of $Y$ is called the supremum (infimum) of $Y$ denoted by sup$(Y)$ (inf$(Y)$). More precisely, if $\ell = \sup(Y)$ then $\ell \leq \ell$ ($\ell \leq \ell$).

The partially ordered set $L$ is called a lattice if inf$[l_1, l_2]$ and sup$[l_1, l_2]$ exist for any pair of elements $l_1, l_2 \in L$. The symbols $\wedge$ and $\vee$ are used to indicate the infimum and supremum of $l_1$ and $l_2$. Thus, we can write $l_1 \wedge l_2 = \inf[l_1, l_2]$ and $l_1 \vee l_2 = \sup[l_1, l_2]$. The lattice $L$ is said to be complete if and only if inf$(Y)$ and sup$(Y)$ exist for all $Y \subset L$. The lattice $L$ is said to be bounded if it has a greatest element $T$ and a least element $T$. These elements are also called the top and bottom elements of $L$, respectively. Thus, in a bounded lattice $B \leq l \leq T$ for all $l \in L$. If $\vee$ and $\wedge$ are distributive over each other, then such a lattice is called a distributive lattice. A complemented distributive lattice is called a Boolean lattice.

A lattice $L$ can be represented geometrically by means of a Hasse diagram whose vertices are labeled by elements of $L$, and any two vertices $a$ and $b$ are joined by a line segment or a curve that goes upward from $a$ to $b$ whenever $a < b$ and there is no member $c \in L$ between $a$ and $b$. These edges may cross other but must not touch any vertex other than their endpoints. Such a diagram uniquely determines the partial order defined on $L$. In a partial order, the existence of $l_1 \wedge l_2$ and $l_1 \vee l_2$ is essential for the formulation of lattices. Being nonempty sets, we can define several maps between any two lattices. Any map that preserves the three essential components that constitute a lattice is called a lattice homomorphism. Mathematically, a map $\alpha$ from a lattice $L_1$ into a lattice $L_2$ is called a lattice homomorphism if for all $l_1, l_2 \in L_1$, $\alpha(l_1 \wedge l_2) = \alpha(l_1) \wedge \alpha(l_2)$ and $\alpha(l_1 \vee l_2) = \alpha(l_1) \vee \alpha(l_2)$.

If $L_1$ is a bounded lattice with top element $T_{L_1}$ and bottom element is a bounded lattice with $a(T_{L_1})$ and $a(B_{L_1})$ as top and bottom elements. If $L_1$ and $L_2$ both are bounded lattices, then a maps top and bottom elements of $L_1$ onto the top and bottom elements of $L_2$ respectively.

Example 1. Let $L = \{B, T, a, c, d, e, f, g, h, i, j, k\}$ be a lattice [6] with partial order presented by the Hasse diagram in Figure 1.

The map $\alpha: L \rightarrow [0, 1]$ de is a lattice homomorphism. A filter $F$ of a lattice $L$ is a subset $F \subset L$ satisfying two conditions stated as

(i) If $\ell \in F$ and $\ell' \leq \ell$, then $\ell' \in F$;
(ii) If $\ell, \ell' \in F$, then $\ell \wedge \ell' \in F$.

Let $L$ be a lattice. The principal filter denoted by $\uparrow p$ is de Clearly, the principal filter is the smallest filter that contains the given element $p$. Let $L$ be a bounded lattice. If $M$ is a subset of $L$, then $M_{p'} = \uparrow p \cap M$.

2.2. Lattice Valued Intuitionistic Fuzzy Set Type-3. Zadeh [1] formulated the fundamental definition of fuzzy sets. A fuzzy subset $A$ of a nonempty set $S$ is perhaps the collection of ordered pairs with first components from $S$, and second components are images of the map $\mu: S \rightarrow [0, 1]$ (called membership function). Mathematically, $A$ can be written as $A = \{ (x, \mu(x)) : x \in S \}$. The grades of membership for elements of $S$ under $\mu$ can be used to define crisp subsets of $S$ termed level or cut sets. For any $\alpha \in [0, 1]$, the $\alpha$-level set of $\mu$ is de. The idea of a fuzzy set is a major breakthrough in mathematical logic giving a better approximation than the classical probability theory. However, in real-life problems, membership is not the only option in all cases; there is the chance of nonmembership to handle as well. For such cases, Atanassov introduced the concept of the intuitionistic fuzzy set (IFS). An intuitionistic fuzzy set (IFS) $A$ over $S$ is a triplet $(A, \mu, \nu)$, where $\mu, \nu: S \rightarrow [0, 1]$ (called membership and nonmembership functions). Thus, the IFS $A$ can be written as $A = \{ (x, \mu(x), \nu(x)) : x \in S \}$ with $0 \leq \mu(x) + \nu(x) \leq 1$. "Less than or equal to" $\leq$ constitutes a natural partial order on the closed interval $[0, 1]$ and turns it into a lattice. The replacement of $[0, 1]$ by any other lattice $L$ gives us the concept of $L$-fuzzy and $L$-intuitionistic fuzzy sets. A lattice valued intuitionistic fuzzy set type-1 (LIFS-1) [6] is the set $(S, L, \mu, \nu, N)$, where $S$ is a non-empty set; $L$ is a lattice; $\mu: S \rightarrow L$ and $\nu: S \rightarrow L$ are membership and nonmembership functions; and $N: L \rightarrow L$ is an involutive operation reversing unary operator on $L$ such that $\mu(x) \leq N(\nu(x))$ for all $x \in S$. The replacement of unary operator $N$ by the linearization map $\xi: L \rightarrow [0, 1]$ satisfying $\xi(\mu(x)) + \xi(\nu(x)) \leq 1$ for all $x \in S$ constitutes a lattice valued intuitionistic fuzzy set type-2 (LIFS-2). A lattice valued intuitionistic fuzzy set type-3 (LIFS-3) is the quintuplet $(S, L, \mu, \nu, \alpha)$, where $S$ is a nonempty set; $L$ is a bounded lattice with top element $T$ and bottom element $B; \mu: S \rightarrow L$.
and \( v: S \rightarrow L \) are membership and nonmembership functions; and \( \alpha: L \rightarrow [0, 1] \) is a lattice homomorphism with \( a(\mu(x)) + a(\nu(x)) \leq 1 \) for all \( x \in S \). For \( p \in L \), two level sets in LIFS-3 are defined as:

\[
\mu_p = \{ x \in S | \mu(x) \geq p \}; \nu_p = \{ x \in S | \nu(x) \geq p \}.
\]

(1)

Two level functions generally called characteristics functions \( \mu_p \) and \( \nu_p \) are defined as:

\[
\mu_p(x) = 1 \iff \mu(x) \geq p, \nu_p(x) = 1 \iff \nu(x) \geq p.
\]

(2)

**Proposition 1.** [6] Let \( S \) be a nonempty set and \((S, L, \mu, v, \alpha)\) be an LIFS-3. Then the following statements are true:

(i) Let \( B \) be a lattice with bottom element \( 0 \). Then

\[
\mu_B = \nu_B = S.
\]

(ii) If \( p \leq q \), then \( \mu_p \subseteq \mu_q \) and \( \nu_q \subseteq \nu_p \).

(iii) If \( Z \subseteq L \) then

\[
\bigwedge \{ \mu_p | p \in Z \} = \mu_{\bigwedge Z} \quad \text{and} \quad \bigwedge \{ \nu_p | p \in Z \} = \nu_{\bigwedge Z}.
\]

**Remark 1.** The functions \( \mu: S \rightarrow L \) and \( v: S \rightarrow L \) define a partitioning of \( L \) under the equivalence relation \( \sim \) defined as \( g \sim h \) if and only if \( \mu_g = \mu_h \) and \( \nu_g = \nu_h \).

For every \( g \in L \), the equivalence class of \( g \) is \( [g] = \{ h \in L | g \sim h \} \). As we know, the join (supremum) of a set may or may not be an element of that set, but for these classes \( \bigvee [g] \sim \in [g] \sim \).

### 3. Codes over LIFS-3

Let \( S = \{1, 2, \ldots, r\} \) and \( S_{LIF}^L = (S, L, \mu, \nu, \alpha) \) be an LIFS-3. For \( p \in L \), define \( \overline{S}_{LIF}^L \rightarrow [0, 1] \) as:

\[
\overline{S}_p = \mu_p \lor \nu_p
\]

\[
= u_p \lor v_p
\]

\[
= (a_1, \ldots, a_n) \lor (b_1, \ldots, b_n)
\]

\[
= (a_1 \land b_1) \lor (a_2 \land b_2) \ldots \lor (a_n \land b_n),
\]

where \( u_p \) and \( v_p \) are the codewords—also called vectors—for the element \( p \in L \). The map \( \overline{S}_p \) is called a binary block codeword over \( S_{LIF}^L \). The LIFS-3 codewords inherit a partial order from the lattice \( L \) in such a way that \( a_1 \ldots a_n \leq b_1 \ldots b_n \) if and only if \( a_1 \geq b_1, a_2 \geq b_2, \ldots, a_n \geq b_n \) in \( L \). The number of elements in \( S \) that are mapped onto \( p \in L \) under \( \mu \) and \( \nu \) are called the degrees of \( u_p \) and \( v_p \) denoted by \( s(u_p) \) and \( s(v_p) \). Moreover, the degree of \( c_p \) is exactly equal to the sum of the maxima of \( s(u_p) \) and \( s(v_p) \). For a binary block code \( C \subseteq [0, 1]^n \), a nonempty set \( S \), a lattice \( L \) and lattice homomorphism \( \alpha: L \rightarrow [0, 1] \), the set \( C_{LIF}^L \) is an LIFS-3 if the binary block code constructed on it is equal to \( C \).

**Example 2.** Let us consider a nonempty set \( S = \{ x, y, z \} \) and lattice \( L \) described in example 1. Define membership and nonmembership functions as

\[
\mu = (x, y, z)\quad \text{and} \quad \nu = (a, b, c).
\]

(4)

Clearly, for all \( s \in S \), the lattice homomorphism \( \alpha \) satisfies \( 0 \leq a(\mu(s)) + a(\nu(s)) \leq 1 \). Now for \( a \in L, \mu_a = 111 \) and \( \nu_a = 101 \) because \( a \leq \mu(x), a \leq \mu(y), a \leq \mu(z) \) and \( a \leq \nu(x), a \geq \nu(y), a \leq \nu(z) \). Thus, \( C_{\alpha} = (1 \land 1)(1 \land 0)(1 \land 1) = 101 \).

Codewords corresponding to other elements of \( L \) can be computed in a similar fashion and are presented in Table 1.

As \( s(u_a) = 1 \) and \( s(v_a) = 0 \), the degree of a code word \( c_a \in C \) is \( s(c_a) = \max \{ s(u_a), s(v_a) \} = \max \{ 1, 0 \} = 1 \).

**Theorem 1.** The binary block code \( C \subseteq [0, 1]^n \) constitutes a lattice valued intuitionistic fuzzy set-type-3 if and only if code \( C \) is closed under an intersection and the identity vector \( 111 \ldots 1 \) belongs to \( C \).

**Proof.** Let \( C \subseteq [0, 1]^n \) be a code and \( S_{LIF}^L \) be the corresponding LIFS-3. As \( L \) is a lattice, the families \( \{ \mu_p, p \in L \} \) and \( \{ \nu_p, p \in L \} \) are closed under a set theoretical intersection, which implies that the code \( C \) is closed. Hence, binary block code is closed under intersection. Moreover, \( \mu_B = \nu_B = S \) implies that \( 111 \ldots 1 = \mu_B \) and \( 111 \ldots 1 = \nu_B \) for the bottom element \( B \in L \). Thus \( C = \mu_B \land \nu_B = 111 \ldots 1 \) belongs to \( C \). Conversely, suppose that \( C \) is closed under a set theoretical intersection and non-zero codeword \( 111 \ldots 1 \in C \). Now we show that there is an LIFS-3 corresponding to the code \( C \). Consider \( S = \{1, 2, \ldots, n\} \). The complements of the subsets of \( S \) obtained by the codewords of \( C \) in terms of characteristic functions constitute a lattice \( L \) with inclusion as a partial order. Now for \( p \in L \), \( \overline{S}_p \) is a codeword such that \( \overline{S}_p(i) = \overline{S}_p(i) = 0 \) if and only if \( i \in p \). The collection \( \overline{S}_p, p \in L \) gives an LIFS-3 having code for all \( x \in S \).

### 3.1. Hamming Distance

The Hamming distance \( d(s,t) \) defined in [23] is the number of places in which the two vectors (codewords) \( s \) and \( t \) differ. In other words, \( d(s,t) \) represents the component-wise difference of the two codewords \( s \) and \( t \). That is,

\[
d(s,t) = |i : s_i = t_i|.
\]

(5)

Consider a code \( C \). Then the distance \( d(C) \) of a code \( C \) is defined in [23] as the minimum distance between two
Table 1: Codewords over LIFS-3.

| $\overline{p}_p$ | $\overline{v}_p$ | $\overline{p}_p \wedge \overline{v}_p$ | $\overline{p}_p$ | $\overline{v}_p$ | $\overline{p}_p \wedge \overline{v}_p$ | $\overline{p}_p$ | $\overline{v}_p$ | $\overline{p}_p \wedge \overline{v}_p$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 111            | 101            | 101            | 010            | 101            | 101            | 000            | 000            | 001            |
| 011            | 101            | 001            | 010            | 100            | 000            | 000            | 000            | 000            |
| 011            | 111            | 001            | 000            | 000            | 111            | 111            | 111            | 111            |
| 011            | 101            | 001            | 000            | 000            | 000            | 000            | 000            | 000            |

distinct codewords in $\mathcal{C}$. The number of non-zero components of a codeword is known as its Hamming weight denoted by $\|x\|_h$.

**Proposition 2.** For any code $\mathcal{C}$, $d(\mathcal{C}) \geq \min_{p \neq q, p, q \in \mathcal{C}} s(c_p)$.

Proof. Let $L$ be a lattice and $\mathcal{C}$ be a binary block code over $S_{\mathcal{L}^3}$, then for any $p \in L$ and codeword $c_p$, the degree of $c_p$ is equal to the maximum of $s(\mu_p)$ and $s(\nu_p)$. Ultimately if $c_p$ differs at the $i$th coordinate, then it will differ at each $j$th coordinate, which is an outcome of some $x \in S$ mapped onto $p$ under $\mu$ and $\nu$. Thus, the distance of code $\mathcal{C}$ is at least equal to $s(c_p)$.

**Proposition 3.** For any $c_p \in \mathcal{C}$, the Hamming weight $\|c_p\| \geq \max\{s(c_q) : q \in \mu(S), \nu(S) \text{ and } c_p \leq c_q\}$. (6)

Proof. Let $S_{\mathcal{L}^3}$ be an LIFS-3. Then $\pi_q(x) = \tau_q(x) = 1$ if $\mu(x), \nu(x) \geq q = c_q(x) = 1$ if $p = \mathcal{C}(x) \geq q$. Hence, $c_q(x)$ is non-zero if both $u_q(x)$ and $v_q(x)$ are non-zero, implying that the number of non-zero coordinates is equal to or greater than the degree of $s(c_q)$.

**Proposition 4.** Let $c_p, c_q \in \mathcal{C}$, such that $p \leq q$. Then, $d(c_p, c_q) = \sum_{i \in S} s(c_i)$, $T = \{c_i \in \mathcal{C} : c_i \geq c_p \text{ and } c_i \leq c_q\}$.

Proof. Let $S_{\mathcal{L}^3}$ be an LIFS-3. Let $\pi_p$ and $\pi_q$ be two vectors corresponding to the membership function. For $c_p \leq c_q$, we have $\pi_p \leq \pi_q$, this implies that the number of non-zero elements in $\pi_q$ is not more than the number of non-zero elements in $\pi_p$. Let $t \in \mu(S)$ such that $\pi_t \neq \pi_p$ and $\pi_t \neq \pi_q$. Then, for each element $x \in S$ which is mapped onto $t$, we have $u_p(x) = 1$ and $u_q(x) = 0$.

A similar argument for the nonmembership function implies, $v_p(x) = 1$ and $v_q(x) = 0$.

Hence, $c_p(x) = u_p(x) \wedge v_p(x) = 0$ and $c_q(x) = u_q(x) \wedge v_q(x) = 0$.

As $c_p \leq c_q$, for each $x \in S$ there exist non-zero coordinates in $c_q$ that belong to $c_p$ and are mapped onto $t$.

**Theorem 3.** If $c_p$ and $c_q$ are different codewords, then $d(c_p, c_q) = d(c_p, c_{p \wedge q}) + d(c_q, c_{p \wedge q})$. Now let us consider the case when two vectors $\pi_p$ and $\pi_q$ are non-comparable. Then any coordinate which is non-zero in $\pi_{p \wedge q}$ will be non-zero in $\pi_p$ and $\pi_q$. A similar case exists for two vectors $\pi_p$ and $\pi_q$ corresponding to the nonmembership function. Let $m$ be the coordinates at which two code words $\pi_p$ and $\pi_q$ and $\pi_p$ and $\pi_q$ differ, that is, $u_p(m) = 1$ and $u_q(m) = 0$.

In addition, $v_p(m) = 1$ and $v_p(m) = 0$, imply $c_q(m) = 1, c_p(m) = 0$.

Hence, $c_p$ and $c_q$ differ at $m$. If $\mu(m) = t$, then $\pi_t \leq \pi_p$, and $\pi_t \leq \pi_q$.

If $\nu(m) = t$, then $\pi_t \leq \pi_q$ an $d \pi_t \leq \pi_p$. Hence $\pi_t \leq \pi_{p \wedge q}$, $\nu_{p \wedge q}(m) = 0$ and $\nu_{t} \leq \pi_{p \wedge q}$, $\nu_{p \wedge q}(m) = 0$

Implies $c_{p \wedge q}(m) = 0$.

Hence $c_q$ and $c_{p \wedge q}$ differ at the coordinate $m$.

3.2. Linear Codes over LIFS-3. A linear $(n, k)$ code is defined in [23] is a $k$-dimensional subspace of a vector space $0, 1]^n$ under the binary operation $\oplus$ of componentwise addition modulo 2.

**Theorem 4.** If for $\mu: S \longrightarrow L$ and $\nu: S \longrightarrow L$ in $S_{\mathcal{L}^3}$, we have $M = \mu(S)$ and $N = \nu(S)$, then for any pair of elements $p, q$ in $L_{p \sim q}$ if and only if $M_p = M_q$ and $N_p = N_q$.

Proof. For $S_{\mathcal{L}^3}$, the relation $\sim$ defined by $p \sim q$ if and only if $\mu_p = \mu_q$ and $\nu_p = \nu_q$ is an equivalence relation on $L$. As $\mu_p = \{x \in S|\mu_p = 1\} = \{x \in S|\mu_p \in \{p\}\}$ and $\nu_p = \{x \in S|\nu_p = 1\} = \{x \in S|\nu_p \in \{p\}\}$. We get that
\[ P \sim q \Longleftarrow \mu_p = \mu_q, \]
\[ v_p = v_q \]
\[ \Longleftarrow \{ x \in S[\mu(x) \in \uparrow P] = \{ x \in S[\mu(x) \in \uparrow q], \}
\]
\[ \{ x \in S[v(x) \in \uparrow P] = \{ x \in S[v(x) \in \uparrow q], \}
\]
\[ \Longleftarrow \uparrow P \cap M = \uparrow q \cap M \]
\[ \uparrow P \cap N = \uparrow q \cap N, \]
\[ \Longleftarrow M_p = M_q, \]
\[ N_p = N_q. \]

Thus for \( g, h \in L, g \neq h \) implies \( g \cap M \neq \uparrow h \cap M \) turns out to be the necessary and sufficient condition for an equivalence class to be a singleton. \( \square \)

**Theorem 5.** The sets \( \mu(S) \) and \( \nu(S) \) consists of all coatoms of the Boolean lattice \( L \) of lattice valued intuitionistic fuzzy set type-3 \( S_{L^3} \) constituted by a linear \((r,k)\) code.

**Proof.** Let \( S_{L^3} \) be a LIFS-3 corresponding to the linear code \( C \). From Theorem 4, if \( x \neq y \in L \), then
\[ \{ x \} \cap \mu(S) \neq \{ y \} \cap \mu(S), \]
\[ \{ x \} \cap \nu(S) \neq \{ y \} \cap \nu(S). \]

Hence, the top element \( 1_L \) of the lattice \( L \) does not belong to \( \mu(S) \) and \( \nu(S) \) otherwise, the code \( C \) does not contain the codeword \( 0 \ldots 0 \) which contradicts the linearity of the code. Now on the contrary, suppose that one co-atom, say, \( y \), is not present in \( \mu(S) \) and \( \nu(S) \); then
\[ \{ y \} \cap \mu(S) = \{ 1 \} \cap \mu(S), \]
\[ \{ y \} \cap \nu(S) = \{ 1 \} \cap \nu(S). \]

Thus \( |C| < |L| \), which leads to a contradiction. In fact, each codeword that corresponds to a co-atom has only non-zero coordinates, and forms a basis for the code \( C \). Let \( t \) b an element in \( \mu(S) \) and \( \nu(S) \) but not a co-atom. Then each codeword corresponding to \( t \) has a zero coordinate. If \( \tau \) is a codeword corresponding to \( t \), then it is linearly independent of all those elements which are in the base that are not true. Hence \( \mu(S) \) and \( \nu(S) \) will consist of all co-atoms of a Boolean lattice. \( \square \)

**Theorem 6.** The linear \((r,k)\)-code \( C \) corresponds to an LIFS-3 if and only if \( C \) is closed under intersection and for each \( j \in \{1, \ldots, r\} \) there is a codeword in \( C \) having a non-zero \( j \)th coordinate.

**Proof.** Let \( S_{L^3} \) be the LIFS-3 corresponding to \( C \). Then by Theorem 3, \( C \) is closed under an intersection, and non-zero vector \( 1 \ldots 1 \) belongs to the families \( \tau_p \) and \( \pi_p \) for an element \( p \in L \). As codeword \( \tau_p \in C \) is a meeting of these two families, it is a non-zero vector contained in \( C \). Hence, satisfies the required condition. Conversely, suppose \( C \) is a linear \((r,k)\) code satisfying the given conditions. The closeness of \( C \) under intersection ensures the existence of \( 1 \ldots 1 \) in \( C \), which implies the constitution of LIFS-3. \( \square \)

**Theorem 7.** Let \( S_{L^3} \) be an LIFS-3 over a Boolean lattice \( L \). If \( \mu(S) \) and \( \nu(S) \) include the maximum element of the set of all co-atoms of \( L \), then the code constructed on \( S_{L^3} \) is linear.

**Proof.** Consider \( S_{L^3} \) over a Boolean lattice \( L \). The codewords corresponding to the maximum elements of co-atoms of a boolean lattice are linearly independent and thus can be extended to a basis that will be used to generate other codewords from the former codewords. If, corresponding to a maximum number of co-atoms, we have \( n \) codewords, then the code \( C \) consists of exactly \( 2^n \) codewords. Thus \( C \) is closed under the binary operation \( \oplus \), and hence it is linear. \( \square \)

**Example 3.** Consider the lattice \( L = \{0, 1, a, c, d, e, f\} \) with partial order presented in Figure 2. Construct an LIFS-3 on \( S = \{1, 2\} \) by defining \( \mu \) and \( \nu \) as
\[ \mu = \begin{pmatrix} 1 & 2 \\ e & f \end{pmatrix} \]
\[ \nu = \begin{pmatrix} 1 & 2 \\ f & d \end{pmatrix}. \]

Then a linear code is obtained as shown in Table 2.

**Theorem 8.** Let \( C \) be a linear code over where a set \( T \) consists of maximum elements of the classes of co-atoms of Boolean lattice.

**Proof.** For the linear code \( C \) over \( S_{L^3} \), the sets \( \mu(S) \) and \( \nu(S) \) consist of all the maximum elements of co-atoms. Hence \( \mu(S) = \nu(S) = T \) and by Proposition 2
\[ d(C) \geq \min_{p \in \bar{P}(1,\bar{1})} s(c_p). \]

As for \( s(u_p) \) and \( s(v_p) \), this distance is equal to the minimum value on the codeword corresponding to the class of the co-atoms with minimal degree, and thus \( s(c_p) \) is attained its minimum. \( \square \)

4. **Application**

Proteins are the most diverse class of biomolecules, both structurally and functionally, and they carry out many tasks. They are polymers of amino acids which are attached via peptide bonds and arranged in the form of long polypeptide chains [24]. Proteins have an extremely wide range of different functions, and they do not act by themselves. They usually interact with other proteins or macromolecules to form fully functional complexes, and these protein complexes are responsible for carrying out many different functions [25]. Nucleic acids are giant biomolecules made of monomers called nucleotides. Nucleic acids refer to the genetic material present in cells that transfer all the hereditary and transmissible information from parents to offspring. Deoxyribonucleic acid (DNA) and ribonucleic acid (RNA) are the two types of nucleic acids. The key task of nucleic acids is to use their stored genetic information for the synthesis of proteins via processes commonly known as translation and transcription. DNA is transcribed to RNA and then translated into a protein. The translation
mRNA, matching three base pairs at a time and sequences on the mRNA molecule. The ribosome moves molecules that have three complementary bases in their somal subunit attaches to an mRNA molecule; then, tRNA machinery includes tRNA, mRNA and ribosomes. A ribosomal subunit attaches to an mRNA molecule; then, tRNA molecules that have three complementary bases in their anticodon regions recognize and bind to specific codon sequences on the mRNA molecule. The ribosome moves along the mRNA, matching three base pairs at a time and adding the amino acids to the polypeptide chain [24, 25].

All living cells use a genetic code to translate the information encoded within the genetic material (DNA and RNA) into proteins. The genetic code describes the set of trios of nucleotides which specify particular amino acids (three nucleotides to one amino acid). There are twenty amino acids typically used to build proteins. As there are four bases, there are 64 possible base triplets [26]. A genetic code is a list of codons, and a codon is a consecutive series of three nucleotides. A codon set is an extension of the four letter-alphabet of DNA. There are four DNA bases, adenine (A), guanine (G), cytosine (C) and thymine (T), and thymine is replaced by uracil in RNA. The base guanine is the complementary base of cytosine, and adenine is that of thymine. Each codon corresponds to a specific type of amino acid; for example, the sequence of nucleotides “GUU” corresponds to a specific amino acid known as valine [27]. Two or more different codons can correspond to the same amino acid, making the genetic code degenerate, and such codons are known as synonymous codons. In a coding sequence, the synonymous codons are not used with equal frequencies in many organisms. This phenomenon is called synonymous codon usage bias (SCUB), and this shows that during the translation of genes to proteins, there is non-uniform usage of synonymous codons encoding the same amino acid; e.g., UGC and UGU are two different sequences corresponding to the same amino acid, cysteine [28].

The algebraic structures of the genetic code are essential to understanding the information and applications which are stored within the code. There are various conventional mathematical models of the genetic code, which include binary representations of the DNA bases. The representations of the four DNA bases by Jimenez-Montano et al. [29] are: \( ? = 00, G = 01, U = 10 \) and \( C = 11 \). Stambuk [30] presented the universal metric properties of the human genetic code and described it by using the nucleotide base representation on the square having \( U \) or \( T = 00, C = 01, G = 10 \) and \( A = 11 \). A model for topological coding of proteins was proposed by Karasev and Stefanov [31], using \( C = 00, U = 01, G = 10 \) and \( A = 11 \). Sánchez et al. [32] described the Boolean lattice of the genetic code and showed that the Boolean lattice of the genetic code can be obtained as the third power of the initial lattice. He used two types of lattices, the primal \( \{B(X), \sup, \inf\} \) and the dual \( \{B(X), \inf, \sup\} \), with \( X = \{U, C, G, A\} \). The representations of the four bases given to primal lattice are: \( G = 00, A = 01, U = 10 \) and \( C = 11 \). Similarly, for the dual lattice: \( C = 00, U = 01, A = 10 \) and \( G = 11 \). A Hasse diagram of the genetic Boolean lattice is presented in the following Figure. It shows that the codons with uracil as a second base encode hydrophilic amino acids, and the codons with adenine as the second base encode hydrophobic amino acids. Additionally, the Hamming distances between pairs of codons reveal distinct hydrophobicities among their encoded amino acids. If \( XYZ \) and \( X_1Y_1Z_1 \) are two codons, then \( XYZ \leq X_1Y_1Z_1 \) only if \( X \leq X_1, Y \leq Y_1, Z \leq Z_1 \), and we can say that the codons are comparable.

In recent years, a paradigm shift for “new biology” has taken place as a direct result of an expanded awareness of the fuzziness of the processes that occur in biological systems. There is a lot of evidence to suggest that many biological processes are not deterministic but rather include an inherent element of uncertainty. Many studies and observations have shown that fuzzy effects are a very important part of how living things develop and work in their physiology and evolution. Both fuzzy set theory and fuzzy logic, along with many other approaches to computational intelligence, have the potential to solve a numerous challenges that arise in the field of bioinformatics. An analysis of protein sequences can be performed using fuzzy set theory and fuzzy logic. A method was devised to predict the solvent accessibility of each amino acid in a protein sequence by using a k-nearest neighbour approach [33]. In addition, the FKNN algorithm has been used to predict a protein’s subcellular location [34], which refers to the region of the cell in which the protein is found (including extracellular, cytoplasm, nucleus). In this study, a class membership function was used in accordance with the dipeptide composition of a protein sequence. Fuzzy logic was used in conjunction with neural networks to describe how protein motifs can change [35].

An information-theoretic-based fuzzy inference engine was created to predict coding areas, or the sequence segments that correspond to proteins, for genomic sequences (DNA) [36]. Polynucleotides (words made up of the letters A, T, C, and G) were also employed by researchers as fuzzy
sets, and they developed a method for calculating the distances between them as points in a hypercube [37, 38]. The procedure served as a tool for comparing various genetic sequences. Additionally, operons, an important structure in bacterial genomes, were predicted using fuzzy scoring functions based on various biological information (e.g., genome sequences, functional annotations, and conservation across multiple genomes) [39]. An operon is a closely related group of neighbouring genes on a DNA sequence.

Ordinary fuzzy sets are used in the literature to model numerous bioinformatics-related problems. In a conventional fuzzy set, the degree of belongingness to the set under discussion is indicated by the membership function, which assigns a number from the unit interval to each element of the discourse universe. A LIFS-3, on the other hand, is distinguished by two functions that express the degree of belongingness and the degree of non-belongingness. This concept, which is a natural generalization of an ordinary fuzzy set, appears to be helpful in simulating a variety of real-world scenarios. Thus, the idea of LIFS-3 can be used to study biological problems in a more significant way.

In this paper we investigate the genetic code further by considering a lattice valued intuitionistic fuzzy set. We consider $S$ to be the set of 20 amino acids: $x_1 = $Phenylalanine, $x_2 = $Leucine, $x_3 = $Isoleucine, $x_4 = $Methionine, $x_5 = $Valine, $x_6 = $Serine, $x_7 = $Proline, ...
Table 4: Genetic codes over LIFS-3.

| p   | $\overline{p}$ | $\overline{p}_p$ | $\overline{p}_p = \overline{p} \wedge \overline{p}_p$ |
|-----|-----------------|------------------|--------------------------------------------------|
| CCC | 1111111111111111 | 1111111111111111 | 1111111111111111 |
| CUC | 111110000000000011 | 111110000000000011 | 111110000000000011 |
| UCC | 110011000000000011 | 110011000000000011 | 110011000000000011 |
| CCA | 110010000000000011 | 110010000000000011 | 110010000000000011 |
| CCG | 010100000000000001 | 010100000000000001 | 010100000000000001 |
| UCG | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| ACU | 001100000000000001 | 001100000000000001 | 001100000000000001 |
| AUC | 001000000000000001 | 001000000000000001 | 001000000000000001 |
| CCA | 010100000000000001 | 010100000000000001 | 010100000000000001 |
| CAU | 001000000000000001 | 001000000000000001 | 001000000000000001 |
| CAA | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| AAC | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| UUU | 100000000000000001 | 100000000000000001 | 100000000000000001 |
| UUA | 010000000000000001 | 010000000000000001 | 010000000000000001 |
| AUA | 000100000000000001 | 000100000000000001 | 000100000000000001 |
| GUC | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| GCU | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| UGC | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| CGC | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| ACG | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| GAG | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| GUG | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| GAA | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| GUA | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| AUG | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| CAG | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| UAU | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| AAC | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| UUG | 000000000000000001 | 000000000000000001 | 000000000000000001 |
| GGU | 000000000000000001 | 000000000000000001 | 000000000000000001 |
Consider a representation of four bases as \( C = 00, \) \( U = 01, \) \( A = 10 \) and \( G = 11 \). Let \( p = CCC \) be an element of a lattice \( L \). Then, \( \mu_{CCC}(x_l) = \mu(x_l) \leq CCC \). Now, by using the definition of a level function, \( \mu_{CCC}(x_l) = 1 \). Similarly, \( \mu_{CCC}(x_2), \ldots, \mu_{CCC}(x_{20}) = 1 \). Thus \( \mu_{CCC} = 11111111111111111111111111111 \) is a codeword relative to the codon CCC. In similar fashion, the remaining codewords corresponding to the other codewords can be found, which are shown in Table 4.

Hence, we have a binary code corresponding to 64 codons having length 20. Furthermore, the difference
between two codewords can be calculated by using well known Hamming distance. That is,

\[ d_H(C_{CUC}, C_{UCC}) = d_H(1111100000000001111, 1100110011000011101) = 8. \]

If we consider only membership values \( \mu_p \) from the above table, then it is an L-fuzzy code [28]. The distances between different amino acids pairs can be computed in terms of mean distances between their respective codons. For instance, the amino acids glycine (G) and methionine (M) are encoded by the bases GGA, GGC, GGG, GGU and AUG. Thus, the distance between G and M means the distances between the codewords \( C_{GGA}, C_{GGC}, C_{GGU}, C_{GUG} \) and \( C_{AUG} \) relative to their codons. The distances between all the 20 pairs of amino acids are shown in Table 5.

The Hamming distance between two codons reflects the variations among the physico-chemical properties of the relative amino acids. Amino acids can be categorized by two types, hydrophobic amino acids (codons having U in second base) and hydrophilic amino acids (codons having A in the second base). Consider the collections of codewords \( C_{XAZ}, C_{XUZ}, C_{XAZ} \) and \( C_{XUZ} \), where \( X, Y, Z \in \{ A, C, G, U \} \) over LIFS-3 and L-fuzzy set, respectively. Table 6 shows the Hamming distances between these sets.

It can be seen that the distances between amino acids with larger differences in physico-chemical properties are larger in an LIFS-3 environment than a L-fuzzy set.

### 5. Conclusion

The authors’ main focus was to construct codes over lattice valued intuitionistic fuzzy set type-3. Binary and binary Linear codes are commonly defined in terms of subspaces of \( \mathbb{Z}_2^n \). Different attempts were made to incorporate the imprecisions in data into the coding process by involving theoretical fuzzy set concepts. Using membership and nonmembership functions, codes were designed over LIFS-3. Distances of code were examined in relation with the degrees of the codewords. It was concluded that binary block and binary linear codes can be constructed over LIFS-3, and conversely, from any given binary block code, we can formulate an LIFS-3. Code over a lattice valued intuitionistic fuzzy set type-3 was constructed by considering the lattice of a 64-codon system, and we found that the physico-chemical differences can be interpreted by computing the Hamming distances between codewords. In the case of LIFS-3, the Hamming distances between codewords were greater than the Hamming distances between codewords in L-fuzzy sets, which indicates the efficiency of the LIFS-3 compared to the L-fuzzy set. In the future, a variety of extensions will be possible. One of the major directions of this work will be to study the already existing codes, such as Hamming code and Hadamard code, by introducing the concept of LIFS-3. This may be done by encoding the messages by using the decomposition of LIFS-3, which gives the family of characteristic functions. Furthermore, the developed application could be used for the further study of the structure of genetic code. As found in the literature, the variations among different physio-chemical properties of amino acids are described by the Hamming distances between the codons. In LIFS-3-codes, this Hamming distance is related to the degrees of code words (or classes), so we can extend it by incorporating these degrees along with the Hamming distances. Moreover, there are several practically useful generalizations of fuzzy sets including the picture fuzzy sets [40], Pythagorean fuzzy sets [41], hesitant fuzzy sets [42] and neutrosophic sets [43] where the replacement of the interval \([0, 1]\) by lattices \( L \) can play a vital role in the development and application of fuzzy coding theory.

### Data Availability

No data is used to support this study.

### Conflicts of Interest

The authors declare that there were no conflicts of interest regarding the publication of this article.

### Authors’ Contributions

All authors contributed equally to the preparation of this manuscript.

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