Few-Body Systems Composed of Heavy Quarks

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Abstract Within the past ten years many new hadrons states were observed experimentally, some of which do not fit into the conventional quark model. I will talk about the few-body systems composed of heavy quarks, including the charmonium-like states and some loosely bound states.

Keywords Molecular states · charmonium-like states · exotic mesons

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1 Introduction

QCD is the underlying theory of strong interaction, which has three fundamental properties: asymptotic freedom, confinement, and chiral symmetry and its spontaneous breaking. Perturbative QCD has been tested to very high accuracy. However, the low energy sector of QCD (i.e., hadron physics) remains very challenging. Precision-test of Standard Model and search for new physics requires good knowledge of hadrons as inputs (such as parton distribution functions).

The motion and interaction of hadrons differ from those of nuclei and quark/gluon/leptons. Hadron physics is the bridge between nuclear physics and particle physics. The famous Higgs mechanism contributes around 20 MeV to the nucleon mass through current quark mass. Nearly all the mass of the visible matter in our universe comes from QCD strong interaction. Study of hadron physics explores the mechanism of confinement and chiral symmetry breaking, and the mass origin.

Quark Model is quite successful in the classification of hadrons although its not derived from QCD. Any state with quark content other than $q\bar{q}$ or $qqq$ is beyond naive quark model. But quark model cant be the whole story. QCD may allow much richer hadron spectrum such as: glueball, hybrid meson/baryon, multi-quark states, hadron molecules etc.

Experimental search of these non-conventional states started many years ago. Typical signatures of these non-conventional states include: (1) exotic flavor quantum number like the $\theta^+$ pentaquark; (2) exotic $J^{PC}$ quantum number like the $1^{--}$ exotic meson; (3) overpopulation of the quark model spectrum like the scalar isoscalar spectrum below 1.9 GeV: $\sigma$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $f_0(1790)$, $f_0(1810)$.

But till 2002, none of the above exotic candidates has been established without controversy! Since 2003, many charmonium (or charmonium-like) states and some Upsilon (or Upsilon-like) states were discovered experimentally.

There are several production mechanisms of these charmonium or charmonium-like states. First we have the initial state radiation, where one virtual photon transforms into a $1^{--}$ charmonium(-like) state. We have the double charmonium production where a pair of charmonium(-like) states are produced. We also have the B decay process. The two-photon fusion process produces the charmonium state with even charge conjugation parity and $J \neq 1$ due to Landau-Yang theorem.

Charmonium spectrum is known very well up to the $D\bar{D}$ threshold. In the past decade many new states above the $D\bar{D}$ threshold were discovered. Some are very narrow. Some are even charged charmonium-like states. These state are good candidates of exotic mesons.

Many new charmonium(-like) states do not fit into quark model spectrum easily. Theoretical speculations include: molecular states, tetraquarks, hybrid charmonium and conventional charmonium etc. Molecular states are loosely bound states composed of a pair of mesons. They are probably bound by the long-range color-singlet pion exchange. Tetraquarks are composed of four quarks. They are bound by colored-force between quarks. They decay through rearrangement. Some are charged. Some carry strangeness. There are many states within the same multiplet. Hybrid charmonium are bound states composed of a pair of quarks and one excited gluon. The conventional quark model charmonium spectrum could also be distorted by the coupled-channel effects.

Since some of the above states are very close to the open-charm/bottom threshold, a quite natural interpretation is that they could be the hadronic molecules. In QED
we have the hydrogen atom. One light electron circles around the proton. When two electrons are shared by two protons, a hydrogen molecule is formed. In QCD we have the heavy meson and baryon. One light quark circles around the heavy anti-quark. Two light quarks circle around the heavy quark. Do we expect the analogue of the hydrogen molecule in QCD? I will focus on the hadron molecules composed of a pair of heavy mesons or heavy baryons in this talk.

The idea of the loosely bound molecular states is not new in nuclear physics since Yukawa proposed the pion in 1935. The deuteron is a very loosely bound state composed of a proton and neutron arising from the color-singlet meson exchange. Besides the long-range pion exchange, the medium-range attraction from the correlated two-pion exchange (or in the form of the sigma meson exchange), the short-range interaction in terms of the vector meson exchange, and the S-D wave mixing combine to form the loosely bound deuteron. We adopt the same one-boson-exchange (OBE) formalism to discuss the possible molecular states composed of a pair of heavy mesons or heavy baryons.

The interaction potential is derived assuming the hadrons are point-like particles. For a loosely bound state, the pion does NOT explore the inner structure of the hadron. In other words, the momentum of the pion is soft. Therefore, a form factor is always introduced at each vertex to regulate the potential and suppress the contribution from the ultraviolet regime of the momentum. The cutoff in the form factor is constrained by the deuteron data.

Before we start, we should keep one lesson in mind. In 1949, Fermi and Yang derived the $N\bar{N}$ potential based on $N\bar{N} \rightarrow N\bar{N}$ elastic scattering and G-parity rule. They obtained many deeply bound states in the OBE model, but none of them was observed experimentally. Why? In the $N\bar{N}$ case, the inelastic scattering or annihilation process $N\bar{N} \rightarrow$ mesons is also important, which renders the short-distance interaction very complicated. In fact, the presence of the optical potential $V(r)+i W(r)$ changes the whole picture. Generally speaking, the OBE model is reliable only in the case (1) when there is no annihilation; (2) or for very shallow bound states when there is annihilation.

2 The charged $Z_b$ states

Now let’s move on to the charged $Z_b$ states. In 2011, the Belle Collaboration announced two charged Upsilon-like states $Z_b(10610)$ and $Z_b(10650)$. These two states were observed in the invariant mass spectra of $h_b(nP)^\pm \pi^\mp$ ($n = 1, 2$) and $Y(mS)^\pm \pi^\mp$ ($m = 1, 2, 3$) of the corresponding $Y(5S) \rightarrow h_b(nP)^+\pi^- \pi^+$ and $Y(5S) \rightarrow Y(mS)^+\pi^- \pi^-$ hidden-bottom decays. With the above five hidden-bottom decay channels, Belle extracted the $Z_b(10610)$ and $Z_b(10650)$ parameters. The obtained averages over all five channels are $M_{Z_b(10610)} = 10608.4 \pm 2.0 \text{ MeV}/c^2$, $\Gamma_{Z_b(10610)} = 15.6 \pm 2.5 \text{ MeV}/c^2$, $M_{Z_b(10650)} = 10653.2 \pm 1.5 \text{ MeV}/c^2$, $\Gamma_{Z_b(10650)} = 14.4 \pm 3.2 \text{ MeV}/c^2$. In addition, the analysis of the angular distribution indicates both $Z_b(10610)$ and $Z_b(10650)$ favor $I^G(J^P) = 1^+(1^+)$. If $Z_b(10610)$ and $Z_b(10650)$ arise from the resonance structures, they are good candidates of non-conventional bottomonium-like states. The masses of the $J^{PC} = 1^{++}$ and $J^{PC} = 1^{+-}$ $b\bar{b}q\bar{q}$ tetraquark states were found to be around $10.1 \sim 10.2$ GeV in the framework of QCD sum rule formalism, which are significantly lower than these two charged $Z_b$ states. Therefore, it’s hard to accommodate them as tetraquarks. If comparing the experimental measurement with the $BB^*$ and $B^*B^*$ thresholds, one
notices that \( Z_b(10610) \) and \( Z_b(10650) \) are close to thresholds of \( BB^* \) and \( B^*B^* \), respectively. One plausible explanation is that both \( Z_b(10610) \) and \( Z_b(10650) \) are either \( B^*B^* \) or \( B^*B^* \) molecular states respectively.

Before the observations of two charged \( Z_b(10610) \) and \( Z_b(10650) \) states, there have been many theoretical works which focused on the molecular systems composed of \( B^{(*)} \) and \( B^{(*)} \) meson pair and indicated that there probably exist loosely bound S-wave \( B^*B^* \) or \( B^*B^* \) molecular states. To some extent, such studies were stimulated by a series of near-threshold charmonium-like \( X, Y, Z \) states in the past eight years.

Within the OBE framework, one can construct the effective potentials arising from various meson exchange contributions. We need also consider the S-wave and D-wave mixing contribution. After solving the couple-channel Schrödinger equation, we found that both \( Z_b(10610) \) and \( Z_b(10650) \) can be explained as the \( BB^* \) and \( B^*B^* \) molecular states. Besides the isovector \( Z_b \) states, there are also several loosely bound isoscalar molecular states.

Besides the five hidden-bottom discovery modes, Belle collaboration confirmed these two charged states in the \( B^{(*)}B^* \) channel in July 2012. In fact these are the dominant decay modes. The \( B\bar{B}^* \) branching ratio is \((86.0 \pm 3.6\%)\) for the lower \( Z_b \) state. For the heavy \( Z_b \) state, the \( B^{(*)}\bar{B}^* \) branching ratio is \((73.4 \pm 7.0\%)\). Moreover, Belle collaboration reported the first evidence of the neutral partner of these \( Z_b \) states. Up to now, the properties of the \( Z_b \) states fit the molecular hypothesis quite well.

The masses of the final states \( BB^* \pi \) and \( B^*B^* \pi \) are 10.744 GeV and 10.790 GeV respectively, which are very close to the \( T(5S) \) mass 10.860 GeV. In other words, the decay phase space is tiny. The relative motion between the \( B^{(*)}B^* \) pair is very slow, which is favorable to the formation of the \( B^{(*)}B^* \) molecular states. To some extent, \( T(5S) \) or \( T(6S) \) is the ideal factory of the heavy molecular states, which will be produced abundantly at sBelle in the near future!

### 3 X(3872)

The narrow state \( X(3872) \) was also discovered by Belle collaboration. Its width is extremely narrow, i.e., less than 1.2 MeV. Its mass is very close to the \( D^0\bar{D}^0 \) threshold. No charged partners were found up to now. So it’s not an isovector. This state decays into \( J/\psi \gamma \). Hence its C-parity is even. Both CDF and Belle collaborations performed the angular distribution analysis and found its quantum number could be \( 1^{++} \) or \( 2^{-+} \).

Both Belle and Babar collaborations observed large \( D^0\bar{D}^{*0} \) branching ratio. The most puzzling issue of \( X(3872) \) is the large isospin symmetry breaking in its hidden-charm decays. The dipion hidden-charm decay violates isospin symmetry, but its decay width is comparable to that of the three-pion decay mode, which conserves isospin symmetry. As a \( 2^{-+} \) or \( 1^{++} \) charmonium state, it is very difficult to explain the large isospin symmetry breaking around 100%. The typical isospin symmetry breaking effect from up/down quark mass difference and QED is around 0.1% → 1%.

In fact, the proximity of the \( X(3872) \) to the threshold of \( D^0\bar{D}^{*0} \) strongly suggests that the \( X(3872) \) is probably a loosely bound \( D^0\bar{D}^{*0} \) molecule. We extend the OBE model to study \( X(3872) \) as a \( 1^{++} \) molecular state.

We want to find out the specific role of the charged \( D\bar{D}^* \) mode, the isospin breaking and the coupling of the \( X(3872) \) to \( D^*\bar{D}^* \) in forming the loosely bound \( X(3872) \). We first consider the neutral component \( D^0\bar{D}^{*0} \) only and include the S-D wave mixing, which corresponds to Case (I). Then we add the charged \( D^+D^- \) component to form
the exact $D\bar{D}^*$ isospin singlet with the S-D mixing, which is Case (II). Since the $1^{++}$ $D^*\bar{D}^*$ channel lies only 140 MeV above and couples strongly to the $DD^*$ channel, we further introduce the coupling of $DD^*$ to $D^*\bar{D}^*$ in Case (III). Finally, we move one step further and take into account the explicit mass splitting between the charged and neutral $D(D^*)$ mesons, which is the physical Case (IV).

In case (IV) we consider the isospin breaking for $D\bar{D}^*$ only but keep the isospin limit for the $D^*\bar{D}^*$ channel. Since the threshold of $D^*\bar{D}^*$ is about 140 MeV above the $X(3872)$ mass, the probability of the $D^*\bar{D}^*$ component is already quite small due to such a large mass gap. The isospin breaking effect due to the mass splitting of the $D^*\bar{D}^*$ pair is even smaller and negligible. In case (IV) we have omitted the channel $\frac{1}{\sqrt{2}} (D^{*0}\bar{D}^{*0} + D^{*+}\bar{D}^{*-}) |^5D_1>$. At the first glimpse, this channel should also be included. After careful calculation, it turns out that the matrix elements between this channel and other channels are zero.

Let’s look at the results in one-pion-exchange (OPE) model first. Now there is only one coupling constant $g$. Its value $g=0.59$ was extracted from the decay width of the $D^*$ meson. In Case I, we find no binding solutions with the cutoff parameter around 0.8-2.0 GeV.

After adding the charged mode and assuming they are degenerate with the neutral mode, we obtain a loosely bound state with binding energy 0.3-2 MeV for the cutoff parameter being 1.55 GeV in Case II. The root-mean-square radius is 4.97 fm The S wave is dominant, with a probability of 98.81% while the contribution of the D wave is 1.19. The inclusion of the charged mode enhances the attraction by a factor of three. The charged mode is important in the formation of the bound state, although the required cutoff parameter is larger than 1.5 GeV.

After we include the coupling of $DD^*$ to $D^*D^*$, we can see the important role of the coupled-channel effects in Case III. The binding energy increases by several tens MeV compared with Case II with the same cutoff parameter. The binding energy is 0.76 MeV and the root-mean-square radius is 3.79 fm with the cutoff parameter around 1.10 GeV, which is a reasonable value.

Does the state in Case I-III correspond to $X(3872)$? The answer is No. The state in Case I only contains the neutral component, which is an equal superposition state of the isoscalar and isovector state. The states in Cases II and III are definitely isoscalar. Experimentally, the hidden-charm di-pion decay mode of $X(3872)$ violates isospin symmetry. Actually, none of the above states corresponds to the physical state of $X(3872)$.

We further consider the mass splitting of the neutral and charged $D(D^*)$ mesons in Case IV. Now the binding energy decreases by roughly 3 MeV compared to Case III with the same cutoff parameter. This is reasonable since the charged DD* pair is almost 8 MeV heavier than the neutral pair. Now the flavor wave function of this very loosely bound molecular state contains a large isovector component, which decays into the $J/\psi\rho$ mode. This molecular state can be interpreted as $X(3872)$.

We may also take into account the heavier meson exchange as well as the pion exchange. When the cutoff parameter is fixed at 1.05 GeV, the heavier meson exchange potentials cancel each other to a large extent. Therefore, the long-range pion exchange still plays a dominant role in forming the loosely bound state. The residual effect of the heavier meson exchange modify the binding solution slightly.

The contribution of the isoscalar component is 74% while that of the isovector component is 26% if the binding energy is 0.3 MeV when the cutoff parameter is fixed at 1.05 GeV. However, if the binding energy increases to 11 MeV, the contribution of
the isoscalar component is as large as 98.5% while that of the isovector component is only 1.5%. The isospin breaking depends sensitively on the binding energy. The large isospin symmetry breaking effect in the flavor wave function is amplified by the tiny binding energy.

There is experimental evidence that the hidden-charm di-pion decay occurs through a virtual $\rho$ meson while the hidden-charm three-pion decay occurs through a virtual $\omega$ meson. The isospin violating di-pion decay comes from the isovector component within the flavor wave function while the three-pion decay comes from the isoscalar component. The different phase space of the $J/\psi \rho$ and $J/\psi \omega$ decay modes also plays an important role. The ratio of the above two phase spaces is 0.15.

The branching fraction ratio

$$R = R_{\text{Phase}} \times R_{I} = \mathcal{B}(X(3872) \to \pi^+\pi^-J/\psi)/\mathcal{B}(X(3872) \to \pi^+\pi^-J/\psi) = 0.42$$

with the binding energy being 0.3 MeV. Again, this ratio depends very sensitively on the binding energy since the isospin breaking effect is very sensitive to the binding energy. Given the uncertainty of experimental value of the mass of $X(3872)$, this ratio is consistent with the experimental value, $1.0 \pm 0.4(\text{stat}) \pm 0.3(\text{syst})$ from Belle Collaboration \[8\] and $0.8 \pm 0.3$ from BABAR Collaboration \[9\].

In short summary, the existence of the loosely bound state $X(3872)$ and the large isospin symmetry breaking in its hidden-charm decay arises from the very delicate efforts of the several driving forces including: long-range one-pion exchange, S-D wave mixing, mass splitting between the charged and neutral $D(D^*)$ mesons, coupled-channel effects. The extreme sensitivity of the physical observables to the tiny binding energy is typical of the loosely bound system.

### 4 Heavy Dibaryons

We extend the same OBE formalism to the heavy di-baryon system which roughly reproduce the qualitative features of the deuteron (B.E. 2 MeV, radius 2 fm). The ground state heavy baryons ($qqQ$) include one spin-1/2 and -3/2 sextet and spin-1/2 triplet. Among these states, $\Lambda Q, \Xi_Q, \Xi'_Q$ are stable with weak decays only. $Q = b$ or $c$ denotes the corresponding heavy quark. The pion heavy baryon coupling constants are related to the well-known pion-N-N coupling constants with the pion-quark-quark coupling in the chiral quark model as the bridge.

We focus on the heavy H di-baryon and perform a coupled channel analysis of the $\Lambda_Q \Lambda_Q$ system. For the scalar isoscalar channel, one need consider the channel couplings between $\Lambda Q \Lambda_Q$($^1S_0$), $\Sigma_Q \Sigma_Q$($^1S_0$), $\Sigma_Q^* \Sigma_Q^*$($^1S_0$), $\Sigma_Q \Sigma_Q$($^3D_0$), $\Sigma_Q^* \Sigma_Q^*$($^3D_0$) \[10,11,12\].

With the couple channel effects, the one pion exchange force alone is strong enough to bind the $\Lambda_Q \Lambda_Q$ system. As the heavy quark mass increases, the binding becomes deeper. For the heavy analogue of the H dibaryon, our results indicate that $\Lambda_Q \Lambda_Q(Q=b,c)$ with quantum numbers $I(J^P) = 0(0^+), 0(0^-)$ and $0(1^-)$ may all be molecules. The binding solutions of $\Lambda_Q \Lambda_Q$ system with the OPE potential mainly come from the coupled-channel effect. Besides the transition induced by the OPE force in the flavor space, we have also considered the transitions caused by the eta meson and rho/omega meson exchange. With the same cutoff parameter, the binding energy in the OBE case
is larger than that in the OPE case. The medium- and short-range attractive force plays a significant role in the formation of the loosely bound $Λ_cΛ_c$ and $Λ_bΛ_b$ states.

The heavy analogue of the H dibaryon probably exists due to the coupled channel effects while the light H dibaryon ($AA$) is unbound or barely bound. In fact, the long-range OPE force is strong enough to form the loosely bound $ΛQΛQ$ states. As the heavy quark mass increases, the kinetic energy of the system decreases while the potential remains roughly the same, which is favorable to the formation of the shallow bound states. Once produced, the heavy H dibaryons may decay into the $NΣcc$ final state [13]. Such a transition requires the exchange of a heavy D/B meson between the $ΛQ$ pair, which occurs at very short distance. On the other hand, the $ΛQ$ pair is loosely bound with a large radius. In other words, the decay width of this decay mode is expected to be small. Maybe these states could be produced at RHIC, LHC?

5 Summary

In the past decade, many charmonium(-like) and some Upsilon(-like) states were discovered. Some states do not fit into the quark model spectrum easily. Some of them lie very close to the open-charm/bottom threshold with narrow width such as $X(3872)$. Some are even charged like the $Z_b$ states. These states are very good candidates of loosely bound hadron molecules. In the near future, LHCb, J-PARC, sBelle, Pande will contribute to the search of non-conventional hadrons. We expect more unexpected.

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