Nucleon structure in a light-front quark model consistent with quark counting rules and data

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Using global fits of valence u and d quark parton distributions and data on quark and nucleon form factors in the Euclidean region, we derive a light-front quark model for the nucleon structure consistent with quark counting rules.

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I. INTRODUCTION

The main objective of this paper is to continue our study of a phenomenological light-front wave function (LFWF) for the nucleon started as in Ref. [1]. We derived a LFWF for hadrons both for pions and nucleons which at an initial scale is constrained by the soft-wall AdS/QCD model, and which at higher scales gives the correct scaling behavior of parton distributions and form factors. The explicit form of the wave function at large scales is extracted from the hard evolution of parton distribution functions (PDFs) and generalized parton distributions (GPDs). The proposed wave function produces form factors consistent with quark counting rules [2] and also gives predictions for the corresponding parton distributions. In our considerations we obtained harder PDFs in comparison with the results of global fits (see e.g. results of Martin, Stirling, Thorne and Watt (MSTW) [3]). The reason for a softening of the PDFs was discussed in the pion case in Ref. [4]. There it was clearly demonstrated that the inclusion of next-to-leading logarithmic (NLL) threshold resummation effects, due to collinear and soft gluon contributions, leads to a softer pion PDF [4]. This result also shows that we should take into account these resummation effects and derive an improved nucleon LFWF. In Ref. [3] we demonstrate how to derive in the case of the pion a LFWF producing a softer PDF as in Ref. [4] and a pionic electromagnetic form factor consistent with data and quark counting rules. Here we extend this idea to the case of the nucleon. We propose a LFWF for the nucleon modelled as a quark-scalar diquark bound state, with a specific dependence on the transverse momentum $k_\perp$ and the light-cone variable $x$. This LFWF produces PDFs for valence $u$- and $d$-quark found in the global fits of Ref. [3]. It also describes the electromagnetic form factors of the nucleon including their flavor decomposition into $u$ and $d$ quark form factors up to values of the momentum transfer squared $Q^2 = 30 \text{ GeV}^2$ in the Euclidean region (for recent overview of experimental and theoretical progress in study of nucleon electromagnetic structure see e.g. Refs. [6]-[8]). It is important to stress that the calculated nucleon electromagnetic form factors are consistent with quark counting rules for large values of $Q^2$.

II. LIGHT-FRONT QUARK-DIQUARK MODEL FOR THE NUCLEON

In this section we propose a phenomenological LFWF $\psi(x, k_\perp)$ for the nucleon, set up as a bound state of an active quark and a spectator scalar diquark. This LFWF is able to produce the $u$- and $d$-quark PDFs derived in the global fits of Ref. [3] and generates electromagnetic form factors of nucleons including their flavor decomposition which are consistent with data.

First we collect the well-known decompositions [3] of the nucleon Dirac and Pauli form factors $F_{1,2}^N (N = p, n)$ in terms of the valence quarks distributions in nucleons with $F_{1,2}^q (q = u, d)$, which then are related to the GPDs ($\mathcal{H}^q$ and $\mathcal{E}^q$) [11] of valence quarks

$$F_{1,i}^{p(n)}(Q^2) = \frac{2}{3} F_{1,i}^{u(d)}(Q^2) - \frac{1}{3} F_{1,i}^{d(u)}(Q^2),$$

$$F_{1}^{q}(Q^2) = \int_{0}^{1} dx \mathcal{H}^{q}(x, Q^2),$$

$$F_{2}^{q}(Q^2) = \int_{0}^{1} dx \mathcal{E}^{q}(x, Q^2).$$

At $Q^2 = 0$ the GPDs are related to the quark densities — valence $q_v(x)$ and magnetic $\mathcal{E}_v(x)$ as

$$\mathcal{H}^q(x, 0) = q_v(x), \quad \mathcal{E}^q(x, 0) = \mathcal{E}_v(x),$$

where $q_v(x)$ and $\mathcal{E}_v(x)$ are the valence quark densities.
which are normalized as
\[ n_q = F_1^q(0) = \int_0^1 dx q_\nu(x), \]
\[ \kappa_q = F_2^q(0) = \int_0^1 dx E^\nu(x). \]  

(3)

The number of \( u \) or \( d \) valence quarks in the proton is denoted by \( n_q \), and \( \kappa_q \) is the quark anomalous magnetic moment.

Next we recall the definitions of the nucleon Sachs form factors \( G_{E/M}(Q^2) \) and the electromagnetic radii \( \langle r_E^2 \rangle \) \( N \) in terms of the Dirac and Pauli form factors
\[ G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4M_N^2} F_2^N(Q^2), \]
\[ G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2), \]
\[ \langle r_E^2 \rangle^N = -6 \frac{d G_E^N(Q^2)}{dQ^2} \bigg|_{Q^2=0}, \]
\[ \langle r_M^2 \rangle^N = -6 \frac{d G_M^N(Q^2)}{dQ^2} \bigg|_{Q^2=0}, \]  

(4)

where \( G_M^N(0) \equiv \mu_N \) is the nucleon magnetic moment.

The light-front representation \[ [11, 12] \] for the Dirac and Pauli quark form factors is
\[ F_1^N(Q^2) = \int_0^1 dx \int \frac{d^2 k_\perp}{16 \pi^3} \left[ \psi_{+q}(x, k_\perp) \psi_{+q}(x, k_\perp) + \psi_{-q}(x, k_\perp) \psi_{-q}(x, k_\perp) \right], \]
\[ F_2^N(Q^2) = -\frac{2M_N}{q^2 - iq^2} \int_0^1 dx \int \frac{d^2 k_\perp}{16 \pi^3} \times \left[ \psi_{+q}(x, k_\perp) \psi_{-q}(x, k_\perp) + \psi_{-q}(x, k_\perp) \psi_{+q}(x, k_\perp) \right]. \]  

Here \( M_N \) is the nucleon mass, \( \psi_{\lambda q}^{N}(x, k_\perp) \) are the LFWFs at the initial scale \( \mu_0 \) with specific helicities for the nucleon \( \lambda_N = \pm \) and for the struck quark \( \lambda_q = \pm \), where plus and minus correspond to \( + \frac{1}{2} \) and \( -\frac{1}{2} \), respectively. We work in the frame with \( q = (0, 0, q_\perp) \), and where the Euclidean momentum squared is \( Q^2 = q_\perp^2 \). As the initial scale we choose the value \( \mu_0 = 1 \text{ GeV} \) which is used in the MSTW global fit \[ [3] \].

In the quark-scalar diquark model, the generic ansatz for the massless LFWFs at the initial scale \( \mu_0 = 1 \text{ GeV} \) reads
\[ \psi_{+q}(x, k_\perp) = \varphi_q^{(1)}(x, k_\perp), \]
\[ \psi_{-q}(x, k_\perp) = -\frac{k^1 + ik^2}{xM_N} \varphi_q^{(2)}(x, k_\perp), \]
\[ \psi_{-q}(x, k_\perp) = \frac{k^1 - ik^2}{xM_N} \varphi_q^{(2)}(x, k_\perp), \]
\[ \psi_{-q}(x, k_\perp) = \varphi_q^{(1)}(x, k_\perp), \]  

(7)

where \( \varphi_q^{(1)} \) and \( \varphi_q^{(2)} \) are the twist-3 LFWFs. They are generalizations of the twist-3 LFWFs found from matching the electromagnetic form factors of the nucleon in soft-wall AdS/QCD \[ [13-17] \] and Light-Front QCD (see detailed discussion in Ref. \[ [4] \]). In particular, as result of the matching the following LFWFs have been deduced
\[ \varphi_q^{AdS/QCD(i)}(x, k_\perp) = N_q^{(i)} \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{1-x}} \]
\[ \times \exp \left[ \frac{k^2}{2\kappa^2} \frac{\log(1/x)}{1-x} \right], \]  

(8)

where the \( N_q^{(i)} \) are normalization constants fixed by the conditions of \[ [3] \]. Note that the derived LFWF is not symmetric under the exchange \( x \rightarrow 1 - x \). This is the case because it was extracted from a matching of matrix elements of the bare electromagnetic current between the dressed LFWF in LF QCD and of the dressed electromagnetic current between hadronic wave functions in AdS/QCD.

The generalization \( \varphi_q^{AdS/QCD(i)}(x, k_\perp) \rightarrow \varphi_q^{(i)}(x, k_\perp) \) is encoded in the longitudinal factors \( f_q^{(i)}(x) \) and \( \bar{f}_q(x) \) which take into account collinear and soft-gluon effects as
\[ \varphi_q^{(i)}(x, k_\perp) = N_q^{(i)} \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{1-x}} f_q^{(i)}(x) \]
\[ \times \exp \left[ \frac{k^2}{2\kappa^2} \frac{\log(1/x)}{1-x} \bar{f}_q(x) \right]. \]  

(9)

These factors lead to softer PDFs, which coincide with the results of the global fit performed e.g. in Ref. \[ [3] \]. At the same time, the power scaling of electromagnetic form factors for large values of Euclidean momentum squared with \( Q^2 \rightarrow \infty \) remains the same up to power-scaling breaking corrections \( \Delta_q^{(i)} \) (see Eqs. \[ [13] \] and \[ [20] \]), which produce fine-tuned fits of the nucleon electromagnetic form factors, i.e. consistent with quark counting rules. The choice of the functions \( f_q^{(i)}(x) \) is constrained by the valence \( u \) - and \( d \)-quark PDF, while \( \bar{f}_q(x) \) is fixed from the fit to quark and nucleon form factors. The functions
$f_q^{(1)}(x)$ and $\bar{f}_q(x)$ are specified as

$$f_q^{(1)}(x) = x^{\eta_q^{(1)} - 1} (1 - x)^{\eta_q^{(2)} - 1} (1 + \epsilon_q \sqrt{x} + \gamma_q x),$$

$$f_q^{(2)}(x) = x^{2 + \rho_q} (1 - x)^{\sigma_q} (1 + \lambda_q \sqrt{x} + \delta_q x)^2 f_q^{(1)}(x),$$

$$\bar{f}_q(x) = x^{\eta_{\bar{q}}^{(1)}} (1 - x)^{\eta_{\bar{q}}^{(2)}} (1 + \bar{\epsilon}_q \sqrt{x} + \bar{\gamma}_q x),$$

(10)

where the parameters $\eta_q^{(1)}$, $\eta_q^{(2)}$, $\epsilon_q$ and $\gamma_q$ are fixed from the global MSTW analysis of Ref. [3] [for simplicity we restrict to leading-order results]. The parameters $\rho_q$, $\sigma_q$, $\lambda_q$, $\delta_q$, $\eta_{\bar{q}}^{(1)}$, $\eta_{\bar{q}}^{(2)}$, $\bar{\epsilon}_q$ and $\bar{\gamma}_q$ are fixed from a fit to the anomalous magnetic moments of quarks (nucleons) and to the $Q^2$-dependence of the electromagnetic quark (nucleon) form factors. The final set of parameters specifying the functions $f_q^{(1)}(x)$ and $\bar{f}_q(x)$ is listed in Table I.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $\eta_u$ | 0.45232 | $\eta_{\bar{d}}^{(1)}$ | 0.71978 |
| $\eta_u^{(2)}$ | 3.0409 | $\eta_{\bar{d}}^{(2)}$ | 5.3444 |
| $\epsilon_u$ | -2.3737 | $\sigma_d$ | -4.3654 |
| $\gamma_u$ | 8.9924 | $\gamma_d$ | 7.4730 |
| $\eta_u^{(1)}$ | 0.195 | $\eta_{d}^{(1)}$ | 0.280 |
| $\eta_u^{(2)}$ | 2.02(-1) - 0.54 | $\eta_{d}^{(2)}$ | 0.60 |
| $\bar{\epsilon}_u$ | -0.71 | $\bar{\epsilon}_d$ | -0.10 |
| $\bar{\gamma}_u$ | 0 | $\gamma_d$ | 0 |
| $\rho_u$ | 0.091 | $\rho_d$ | -0.17 |
| $\sigma_u$ | $(\eta_u^{(2)} - 1) - 0.2409$ | $\sigma_d$ | $(\eta_d^{(2)} - 1) - 2.3444$ |
| $\lambda_u$ | -2.40 | $\lambda_d$ | -0.22 |
| $\delta_u$ | 3.18 | $\delta_d$ | 3.90 |

The scale parameter $\kappa = 350$ MeV remains the same as fixed in the analysis of Ref. [1] and used in the analysis for the pion of Ref. [3]. The parameter $\sigma$ is related to the scale parameter of the background dilaton field providing confinement and is universal for all hadronic wave functions.

The expressions for the quark PDFs read

$$q_q(x) = (N_q^{(1)})^2 (1 - x) f_q^{(1)}(x)$$

$$+ (N_q^{(2)})^2 \frac{\kappa^2}{M_N^2} \frac{(1 - x)^3}{x^2} f_q^{(2)}(x) f_q(x),$$

(11)

$$\mathcal{E}^q(x) = 2N_q^{(1)} N_q^{(2)} (1 - x)^2 \sqrt{f_q^{(1)}(x) f_q^{(2)}(x)}.$$  

(12)

The ratio $c_q = N_q^{(2)}/N_q^{(1)}$ is a free parameter and we choose for simplicity $c_u = 1$ and $c_d = -1$ or $N_u^{(2)} = N_u^{(1)} = N_q^{(2)} = N_u$ and $N_d^{(1)} = N_d^{(2)} = N_d$. In our calculations normalization constants $N_u$ and $N_d$ are fixed as $N_u = 1.18093$ and $N_d = 2.00432$. Notice that the contribution of the struck quark with negative helicity $\lambda_q = -$ (see second term in Eq. (11)) to the quark PDFs $q_q(x)$ is relatively suppressed by a factor $\kappa^2/M_N^2 \sim 1/10$. To match the $u_q(x)$ and $d_q(x)$ PDFs fixed in the global fit of Ref. [3], we slightly change the parameters in the longitudinal factor $f_q(x)$. We found that in case of the PDF $u_q(x)$ the contribution of the struck quark with negative helicity is negligible. In the case of the PDF $d_q(x)$ we slightly change the parameter $\eta_d^{(2)} = 5.1244$ fixed in Ref. [3] to $\eta_d^{(2)} = 5.3444$ to match the results of the global fit in Ref. [3].

Expressions for the quark helicity-independent generalized parton distributions (GPDs) $\mathcal{H}^q$ and $\mathcal{E}^q$ in the nucleon read

$$\mathcal{H}^q(x, Q^2) = q_q(x, Q^2) \exp \left[ \frac{-Q^2}{4\kappa^2} \log(1/x) \bar{f}_q(x) \right],$$

$$q_q(x, Q^2) = q_q(x) - (N_q^{(2)})^2 \frac{Q^2}{4M_N^2} (1 - x)^3 f_q^{(2)}(x),$$

$$\mathcal{E}^q(x, Q^2) = \mathcal{E}^q(x) \exp \left[ \frac{-Q^2}{4\kappa^2} \log(1/x) \bar{f}_q(x) \right].$$  

(13)

In the following we consider the scaling of the PDFs at large $x$ and form factors at large $Q^2$. The $q_q(x)$ scale at large $x$ as

$$q_q(x) \sim (1 - x)^{\eta_q^{(2)}},$$

$$u_q(x) \sim (1 - x)^3, \quad d_q(x) \sim (1 - x)^5.$$  

(14)

For the $\mathcal{E}^q(x)$ we have the scaling behavior at large $x$ as

$$\mathcal{E}^q(x) \sim q_q(x) (1 - x)^{1 + \sigma_q}/2 \sim (1 - x)^{\eta_q^{(2)} + 1 + \sigma_q}/2,$$

$$\mathcal{E}^q(x) \sim (1 - x)^5, \quad \mathcal{E}^d(x) \sim (1 - x)^7.$$  

(15)

Note that in the case of the $u$-quark PDFs our results are consistent with perturbative QCD [18, 11]. The $d$-quark PDFs have an additional power of 2 in the scaling behavior as required by the global analysis.

In the case of the nucleon form factors we get for large $Q^2$ the behavior

$$F_1^q(Q^2) \sim \int_0^1 dx (1 - x)^{\eta_q^{(2)}} \exp \left[ \frac{-Q^2}{4\kappa^2} (1 - x)^{1+\eta_q^{(2)}} \right]$$

$$\sim \left( \frac{1}{Q^2} \right)^{1+\eta_q^{(2)}} \sim \left( \frac{1}{Q} \right)^{1+\Delta_q^{(1)}}$$  

(16)

and

$$F_2^q(Q^2) \sim \int_0^1 dx (1 - x)^{1+\eta_q^{(2)} + \sigma_q/2}$$

$$\times \exp \left[ -\frac{Q^2}{4\kappa^2}(1 - x)^{1+\eta_q^{(2)} + \sigma_q/2} \right]$$

$$\sim \left( \frac{1}{Q^2} \right)^{2+\eta_q^{(2)} + \sigma_q/2} \sim \left( \frac{1}{Q^2} \right)^{1+\Delta_q^{(2)}}.$$  

(17)
Here, the $\Delta_q^{(1)}$ and $\Delta_q^{(2)}$ are the small corrections encoding a deviation of the Dirac and Pauli quark form factors from the power-scaling laws $1/Q^4$ and $1/Q^6$, respectively. These corrections are given in the form

$$
\Delta_q^{(1)} = \frac{1 + \eta_q^{(2)}}{2(1 + \eta_q^{(2)})} - 1,
$$

$$
\Delta_q^{(2)} = \frac{2}{3} \Delta_q^{(1)} + \frac{1}{3} \left( \frac{1 + \sigma_q/2}{1 + \eta_q^{(2)}} - 1 \right)
$$

and vanish for

$$
\eta_q^{(2)} = \frac{\sigma_q}{2} = \frac{\eta_q}{2}.
$$

The last limit is consistent with the Drell-Yan-West duality [20] relating the large-$Q^2$ behavior of nucleon electromagnetic form factors and the large-$x$ behavior of the structure functions. However, a fine-tuned fit to the electromagnetic form factors requires a deviation of $\Delta_q^{(i)}$ from zero with the numerical values of

$$
\Delta_u^{(1)} = 0.365, \quad \Delta_d^{(1)} = 0.233,
$$

$$
\Delta_u^{(2)} = 0.338, \quad \Delta_d^{(2)} = 0.081.
$$

### III. RESULTS

Finally, we discuss our numerical results for electromagnetic properties of nucleons. The fit results in values for the magnetic moments in terms of the nuclear magneton (n.m.) and for the electromagnetic radii as shown in Table II, we also show the current data [21] on these quantities. In Figs. 1-7 we present the plots of the Dirac and Pauli form factors for $u$ and $d$ quarks and nucleons. The data in Figs. 1-7 are taken from Refs. [6, 7]. In Figs. 8-13 we also give results for the Sachs nucleon form factors and compare them with the dipole formula $G_D(Q^2) = 1/(1+Q^2/0.71 \text{ GeV}^2)^2$ and with data [22]-[62]. Overall we have good agreement with the data.

| Quantity   | Our results | Data [21] |
|------------|-------------|-----------|
| $\mu_p$ (in n.m.) | 2.793       | 2.793     |
| $\mu_n$ (in n.m.) | -1.913      | -1.913    |
| $r_E^p$ (fm) | 0.781       | 0.84087 ± 0.00026 ± 0.00029 |
| $r_E^n$ (fm) | -0.112      | -0.1161 ± 0.0022 |
| $r_M^p$ (fm) | 0.717       | 0.777 ± 0.013 ± 0.010 |
| $r_M^n$ (fm) | 0.694       | 0.862 ± 0.009 ± 0.008 |

### IV. CONCLUSION

In conclusion, we want to summarize the main result of our paper. Using global fits of valence $u$ and $d$ quark parton distributions and data on quark and nucleon form factors in the Euclidean region, we construct a light-front quark model for the nucleon structure consistent with model-independent scaling laws — the DYW duality [20] and quark counting rules [2].

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FIG. 1: Dirac $u$ quark form factor multiplied by $Q^4$.

FIG. 2: Dirac $d$ quark form factor multiplied by $Q^4$.

FIG. 3: Pauli $u$ quark form factor multiplied by $Q^4$.

FIG. 4: Pauli $d$ quark form factor multiplied by $Q^4$.

FIG. 5: Dirac proton form factor multiplied by $Q^4$.

FIG. 6: Dirac neutron form factor multiplied by $Q^4$.

FIG. 7: Ratio $Q^2F_2^p(Q^2)/F_1^p(Q^2)$. 

\[ Q^2F_2^p(Q^2)/F_1^p(Q^2) \]
FIG. 8: Ratio $G_E^p(Q^2)/G_D(Q^2)$.

FIG. 9: Charge neutron form factor $G_E^n(Q^2)$.

FIG. 10: Ratio $\mu_p G_E^p(Q^2)/G_M^n(Q^2)$.

FIG. 11: Ratio $G_M^n(Q^2)/(\mu_p G_D(Q^2))$.

FIG. 12: Ratio $G_M^n(Q^2)/(\mu_n G_D(Q^2))$.

FIG. 13: Ratio $G_E^n(Q^2)/G_M^n(Q^2)$. Data are taken from Refs. [62].
