Correlations in the cotunneling regime of a quantum dot

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Off–resonance conductance through weakly coupled quantum dots (“valley conductance”) is governed by cotunneling processes in which a large number of dot states participate. Virtually the same states participate in the transport at consecutive valleys, which leads to significant valley-valley conductance correlations. These correlations are calculated within the constant interaction model. Comparison with experiment shows that these correlations are less robust in reality. Among the possible reasons for this is the breakdown of the constant interaction model, accompanied by “scrambling” of the dot as the particle number is varied.

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I. INTRODUCTION

One of the interesting aspects of the physics of quantum dots is the mechanism of cotunneling \([1]\) which governs transport through the quantum dot away from resonance (“conductance valley”). Such a mechanism, which usually gives a small (as compared with resonance values) yet significant contributions to the conductance, consists of the use of a large number of virtual dot states, which, due to high electrostatic energy, are classically forbidden. As one varies an external parameter (e.g. applied magnetic field), these are virtually the same (albeit possibly modified) states that contribute to the transmission, giving rise to significant conductance correlations \([2]\). In a recent work \([3]\) we have pointed out that such correlations show up in the transmission phase as well. Here we study the conductance at different valleys (corresponding to a different number of electrons on the dot, a parameter which is controlled by an applied gate voltage) and calculate the ensuing conductance correlations. Our formalism, outlined in the present and throughout the next section, follows that of Aleiner and Glazman \([2]\). We evaluate the significant valley–valley conductance correlations within the constant interaction model, as function of the strength of the interaction, location in the valley and temperature (Section III and Appendix A). A simplified toy model which, in our opinion, captures much of the pertinent physics, is presented and studied in Appendix B. Comparison with the experiment (Section IV) reveals that in reality these correlations are less robust. Possible reasons for this are discussed in Section V, where we stress the likelihood of the breakdown of the constant interaction model, accompanied by “scrambling” of the dot states as the particle number is varied.

We consider transport through a quantum dot weakly coupled to reservoirs by high tunneling barriers \([4,5]\). The chemical potential \(\mu\) of the quantum dot can be tuned by an applied gate voltage \(V_g\). Because of the small size of those structures charging effects are important; the weak coupling to the external reservoirs (leads) implies that, in general, there is a definite number of electrons on the dot. The conductance of the quantum dot as function of \(V_g\) shows nearly equally spaced peaks. These resonances are due to sequential tunneling of electrons through the dot. At these values of \(V_g\) the ground state energies of the system with the dot containing \(N\) and \(N + 1\) electrons are degenerate and the occupation number can fluctuate. In the “valleys” between the resonances the conductance is strongly suppressed due to the charging energy: it costs energy to add or remove electrons to or from the dot, and the number of electrons on the dot is fixed to an integer value.

There is however a residual conductance in the valley between the peaks which is caused by a different transfer process. An electron or hole tunnels virtually from one reservoir to the other through energetically forbidden states. Since the particle tunnels coherently through the whole structure this transfer mechanism is known as cotunneling \([1]\). In the following we will focus on this regime away from the resonances.

We model the quantum dot coupled to the reservoirs by the usual Hamiltonian

\[
H = H^L + H^R + H^{QD} + H^T ,
\]

\[
H^{L(R)} = \sum_k \varepsilon_k a_k^{L(R)\dagger} a_k^{L(R)} ,
\]

\[
H^T = \sum_{k,j} V_{j,k} c_j^L a_k^L + h.c. + L \leftrightarrow R ,
\]
\[ H^{QD} = \sum_{j} (\epsilon_j - \mu) c_j^\dagger c_j + \frac{U}{2} \hat{N}(\hat{N} - 1). \]  

(H LR) describe the reservoirs to the left and right of the quantum dot, \(H^T\) represents the tunneling of electrons in and out of the quantum dot, and \(H^{QD}\) describes the states of the isolated quantum dot including the electron-electron interaction. We use the simplifying “constant interaction” model which asserts that the total energy due to the electron-electron interaction solely depends on the total number of electrons on the dot (\(\hat{N}\) is the number operator). The charging energy is given by \(U = e^2/C\) invoking the electrostatic energy of a classical capacitance \(C\). The coupling strength of level \(j\) of the quantum dot to the leads is characterized by the tunneling rates \(\Gamma_j = 2\pi/\hbar \sum_k |V_{j,k}|^2 \delta(E - \epsilon_k)\).

For the calculation of transport properties we need the retarded Green function of the quantum dot coupled to the leads,

\[ G_{\text{ret}}^{tj}(t) = \langle \{c_i(t), c_j^\dagger(0)\}_+ \rangle. \]

In the regime of weak coupling \(\Gamma \ll kT, \Delta\) (temperature \(T\), mean level spacing \(\Delta\)) we approximate \(G^{\text{ret}}\) by

\[ G_{\text{ret}}^{tj} = \sum_{N=0}^{\infty} P_N \delta_{jk} \left[ \frac{\langle n_j \rangle_N}{E - (\epsilon_j - \mu + U \cdot (N - 1)) + i\Gamma_j} + \frac{1 - \langle n_j \rangle_N}{E - (\epsilon_j - \mu + U \cdot N) + i\Gamma_j} \right]. \]

with \(\langle \ldots \rangle_N = \text{tr}_N \exp -\beta H^{QD} \ldots / \text{tr}_N \exp -\beta H^{QD}\) the thermal average with \(N\) electrons. Here \(\Delta\) is the single particle level spacing. The probability to find \(N\) electrons on the dot is given by \(P_N = \langle \hat{N} \rangle_N / \sum_M \langle \hat{N} \rangle_M\). Away from the resonances eqn. (4) describes the elastic cotunneling, i.e., the virtual tunneling via a single state \(j\) of the quantum dot. This is the dominant process in the regime \(kT < \sqrt{U\Delta}\) which we are focusing at. In addition, (5) also describes the dynamics in the vicinity of the resonances on the same level as an master equation approach of ref. [6].

For the Green function \(G^{\text{ret}}\) we note that in the cotunneling regime away from the resonances there is an integer number \(\hat{N}\) of electrons on the dot, and \(P_N = \delta_{N,\hat{N}}\) in (6). Hereafter we attach an index \(N\) to a valley, corresponding to the number of electrons on the dot over that range of \(V_g\). The resonance separating the valleys \(N - 1\) and \(N\) will be denoted by \((N - 1, N)\). It is convenient to shift the reference point of the chemical potential \(\mu\) to the resonance \((N - 1, N)\) \((\mu \rightarrow \mu - \epsilon_N - U(N - 1))\), and make use of the parametrization of \(\mu\) by \(x\): for the valley \(N\) the parameter \(x \rightarrow 0\) corresponds to the right of resonance \((N - 1, N)\), i.e., the point where the energies of the system with \(N - 1\) electrons and \(N\) electrons on the dot are practically degenerate; \(x \rightarrow 1\) corresponds to the left of resonance \((N, N + 1)\). A complete description of \(\mu\) is given by the two variables \((N, x)\).

Eqn. (5) contains canonical occupation numbers. In the following we will use the grand-canonical occupation numbers, i.e., the Fermi functions, instead. We account for the difference by using an effective inverse temperature \(\beta_{GC} \approx \beta(1 + \beta\Delta/4)\) which is justified for \(\beta\Delta \ll 1\). We then obtain with \(\omega = \epsilon_j\)

\[ G^{\text{ret}}(N, x) = \frac{\int_{\epsilon_N = \omega}/\epsilon_N - \omega + xU}{\epsilon_N - \omega + xU} - \frac{\beta_{GC}(\epsilon_N - \omega)}{\omega - \epsilon_N + (1 - x)U}. \]

Since we are in the cotunneling regime (large denominators), we neglect the tunneling rates \(i\Gamma\) in the denominators in eqn. (7). The first (second) term describes the occupied (empty) states with energies smaller (greater) than \(\epsilon_N\). The minimum energy necessary for hole (particle) transfer is \(E_h = xU\) \((E_p = (1 - x)U)\) with \(E_h + E_p = U\). We should keep in mind that eqn. (7) is valid for \(x\) sufficiently away from the resonances at \(x = 0, 1\). Since the number of electrons \(N\) starts to fluctuate on an energy scale \(kT\) around the resonances the range of validity of eqn. (7) is \(kT/U < x < 1 - kT/U\).

II. FORMALISM

We will briefly review the formalism of Aleiner and Glazman [3] for the calculation of transport properties of a disordered quantum dot in the cotunneling regime. Hereafter “disorder” should be understood as caused either by impurity scatterers or by shape irregularities in the confining potential. In both cases the statistical properties of the single-particle levels are described by random matrix theory (RMT) (for a review see [3]): over energy intervals smaller than the Thouless energy \(E_{Th}\), the tunneling matrix elements and the level spacing are strongly fluctuating.

We start with the expression for the transmission amplitude \(t(E)\) containing the retarded Green function of the dot \(G^{\text{ret}}\) and the tunneling matrix elements \(V_{L,R}^{j},\).
\[ t(E) = \sum_{j} V_{j}^{L}(E)V_{j}^{R*}(E)G_{j}^{\text{ret}}(E), \]

where \( V_{j}^{L} = V_{j,k}(E)\sqrt{2\pi \rho_{L}^{2}(E)} \) with \( \rho_{L}^{2} \) the density of states in the left reservoir. Since in the elastic cotunneling regime the electrons or holes are transferred through one single dot state, only the diagonal elements in (5) have to be accounted for. The tunneling matrix elements \( V_{j,k}(E) \) in the Hamiltonian (4) contain the overlap of the lead wave function with the dot wave function at the barrier (9). We assume that the tunneling matrix elements factorize into a part describing the tunnel barrier with its penetration factor, and a dot part. The dot part involves the wave function with the dot wave function at the barrier (9). We assume that the tunneling matrix elements factorize into a part describing the tunnel barrier with its penetration factor, and a dot part. The dot part involves the wave function with the dot wave function at the barrier (9).

The separation into interaction and disorder dependent terms respectively is only possible due to the constant interaction \( \delta \). The separation is justified when the properties of the barrier only weakly change on the typical energy scale, \( U \), of the system. Eqn. (8) then becomes

\[ t(E) = 2\pi \sqrt{\rho_{L}^{2}\rho_{R}^{2}V_{L}V_{R*}} \sum_{j} \psi_{j}(R_{L})\psi_{j}^{*}(R_{R})G_{j}^{\text{ret}}(E) \]

\[ = 2\pi \sqrt{\rho_{L}^{2}\rho_{R}^{2}V_{L}V_{R*}} \int d\omega \sum_{j} \psi_{j}(R_{L})\psi_{j}^{*}(R_{R})\delta(\omega - \epsilon_{j})G_{\omega}^{\text{ret}}(E). \]

with \( \rho_{L,R}^{2} \) the density of states in the leads. For the last step in (9) it is essential that \( G_{\omega}^{\text{ret}} \) only depends on \( j \) via \( \epsilon_{j} \), cf. (9). A beam and Glazman identified the sum over \( j \) as the local density of states of single-particle levels and expressed it in terms of Green functions of the non-interacting dot

\[ G_{\omega}^{A}(R_{L}, R_{R}) - G_{\omega}^{R}(R_{L}, R_{R}) = \sum_{j} \left[ \frac{\psi_{j}(R_{L})\psi_{j}^{*}(R_{R})}{\omega - \epsilon_{j} - i\delta} \right] - \frac{\psi_{j}(R_{L})\psi_{j}^{*}(R_{R})}{\omega - \epsilon_{j} + i\delta} \]

\[ = 2\pi i \sum_{j} \psi_{j}(R_{L})\psi_{j}^{*}(R_{R})\delta(\omega - \epsilon_{j}). \]

Impurities on the quantum dot only affect the single-particle states. With (10) the transmission amplitude is expressed as a convolution of two types of Green functions,

\[ t(E) = 2\pi \sqrt{\rho_{L}^{2}\rho_{R}^{2}V_{L}V_{R*}} \int d\omega \sum_{j} \frac{1}{2\pi i} G_{\omega}^{A}(R_{L}, R_{R}) - G_{\omega}^{R}(R_{L}, R_{R}))G_{\omega}^{\text{ret}}(E). \]

One term in the integral is the cotunneling Green function \( G^{\text{ret}} \) of a quantum dot with interaction (but no implicit dependence on disorder dependent quantities); the other term includes the advanced and retarded Green functions \( G^{A} \) and \( G^{R} \) of the system which account for disorder, but do not include interaction. We would like to stress that this separation into interaction and disorder dependent terms respectively is only possible due to the constant interaction model.

The transmission amplitude \( t(E) \) is linked with the linear conductance \( G \) through the quantum dot by the Landauer formula (10). Off the resonances \( G = e^{2}/h|t(E_{F})|^{2} \) even for finite temperature. Averaging over disorder only affects the single-particle Green functions \( G^{R,A} \) defined for non-interacting electrons. For weak disorder, \( k_{F}l \gg 1 \), a diagrammatic expansion can be established (12). The averages \( \langle G^{R}G^{R}\rangle \) and \( \langle G^{A}G^{A}\rangle \) reduce to the product of the average of the single particle Green functions; as long as the dot’s size \( L = |R_{L} - R_{R}| \) is larger than the mean free path \( l \) such terms may be neglected. The remaining average \( \langle G^{R}G^{A}\rangle \) can be expressed in terms of Diffusons and Cooperons,

\[ \langle G_{\omega_{1},B_{1}}^{R}(R_{L}, R_{R})G_{\omega_{2},B_{2}}^{A}(R_{R}, R_{L}) \rangle = 2\pi \rho^{D} D_{\omega_{1},-\omega_{2}}^{B_{1},B_{2}}. \]

\[ \langle G_{\omega_{1},B_{1}}^{R}(R_{L}, R_{R})G_{\omega_{2},B_{2}}^{A}(R_{L}, R_{R}) \rangle = 2\pi \rho^{D} C_{\omega_{1},-\omega_{2}}^{B_{1},B_{2}}. \]

where \( \rho^{D} \) is the density of states on the dot. Diffusons (Cooperons) depend only on the difference (sum) of magnetic fields \( B_{1}, B_{2} \), and are the solution of diffusion equations. In momentum space \( R_{L} \leftrightarrow k, R_{R} \leftrightarrow \bar{k} \) these solutions read

\[ D_{\omega}^{B_{1},B_{2}}(Q) = \frac{S^{-1}}{-i\omega + DQ^{2}}, \]

\[ C_{\omega}^{B_{1},B_{2}}(Q) = \frac{S^{-1}}{-i\omega + DQ^{2} + \Omega^{+}}. \]
where \( q = k - \tilde{k} \) and \( Q = k + \tilde{k} \). The area of the quantum dot is denoted by \( S \) and \( \Omega^+ = B^2 E_{Th} S^2 / \Phi_0^2 \) with \( \Phi_0 = hc/e \) the flux quantum. The Cooperon contribution is maximal for \( Q = 0 \), i.e. for the coherent backscattering process \( k = \tilde{k} \). In contrast to the Diffuson, the Cooperon is suppressed for a magnetic field greater than a characteristic field \( B_c \sim \Phi_0 / S \). The lowest non-vanishing eigenvalues of the diffusion operators are of the order of the Thouless energy \( E_{Th} \). For energies \( E < E_{Th} \) only the zero-frequency mode must be retained which corresponds to spatially homogeneous solutions in \( \Omega^+ \) and \( \Omega^- \) (\( q, Q \to 0 \)).

We focus on the regime \( \Delta \ll E_0, E_p < E_{Th} \). The first inequality assures that we are allowed to use the diffusion approximation. Since the relevant energy scale (charging energy) is smaller than the Thouless energy \( E_{Th} \) which is the case for a dot with not too strong disorder or boundary deformations, we only have to consider the spatially homogeneous zero mode \( q \to 0 \). The sum of Diffusons in momentum space then becomes

\[
D_\omega(q) + D_{-\omega}(-q) = \frac{S^{-1}}{-i\omega + Dq^2} + \frac{S^{-1}}{i\omega + Dq^2} = -iS^{-1} \left( \frac{1}{\omega - iDq^2} - \frac{1}{\omega + iDq^2} \right) \xrightarrow{q \to 0} 2\pi S\delta(\omega). \tag{16}
\]

Aleiner and Glazman have obtained the full distribution function of \( G \) \([2]\) and have found for the variance

\[
\text{var} G = \langle G^2 \rangle - \langle G \rangle^2 = \begin{cases} 2 \langle G \rangle^2 & \text{for } B = 0 \\ \langle G \rangle^2 & \text{for } B \gg B_c \end{cases} . \tag{17}
\]

### III. COTUNNELING CONDUCTANCE CORRELATIONS

In \([2]\) the conductance-conductance correlation for different magnetic fields but the same chemical potential was studied. Here, we will calculate \( \langle G(N_1, x_1)G(N_2, x_2) \rangle \) between different values of the chemical potential \( (N_1, x_1) \) and \( (N_2, x_2) \) but fixed magnetic field for the two cases \( B = 0 \) and \( B \gg B_c \). We are especially interested in the correlation function for \( N_2 = N_1 + \Delta N \) and \( x_1 = x_2 \), i.e. for different valleys but at the same position within the valleys. In order to make contact to the experiment \([13]\) we consider the following normalized autocorrelation function

\[
C(N_1, x_1, N_2, x_2) = \frac{\langle G(N_1, x_1)G(N_2, x_2) \rangle - \langle G(N_1, x_1) \rangle \langle G(N_2, x_2) \rangle}{\sqrt{\text{var} G(N_1, x_1)} \sqrt{\text{var} G(N_2, x_2)}} \tag{18}
\]

for \( U < E_{Th} \). We expect to find strong correlations since from one valley to the next only one level changes its character from occupied to empty (or vice versa) \([3]\). For instance, the first empty state in the \( k \)-valley is the last occupied state in the \((N+1)\)-th valley.

We start as shown in the previous section by separating interaction from disorder

\[
\langle G(N_1, x_1)G(N_2, x_2) \rangle = \mathcal{K}^2 \int d\omega_1 d\omega_2 d\omega_3 d\omega_4 \times \nonumber
\]

\[
G^{\text{ret}}_{\omega_1} (N_1, x_1) G^{\text{ret}}_{\omega_1^*} (N_2, x_2) G^{\text{ret}}_{\omega_3} (N_2, x_2)^* \times \nonumber
\]

\[
\left( (G^{A}_{\omega_1} (R^L, R^R) - G^{R}_{\omega_1} (R^L, R^R)) (G^{R}_{\omega_1} (R^R, R^L) - G^{A}_{\omega_1} (R^R, R^L)) \right) \times \nonumber
\]

\[
(G^{A}_{\omega_3} (R^L, R^R) - G^{R}_{\omega_3} (R^L, R^R)) (G^{R}_{\omega_4} (R^R, R^L) - G^{A}_{\omega_4} (R^R, R^L)) \right) . \tag{19}
\]

with \( \mathcal{K} = e^2 / \hbar |V|^2 |V'|^2 \rho^R \). Since the energy scale \( (U) \) involved in this problem is much larger than the mean level spacing \( (\Delta) \) the average of the product of four Green functions can approximately be decoupled into a product of pairwise averaged Green functions \([3]\). In a diagrammatic expansion this approximation corresponds to keeping a certain class of diagrams which also turn out to be relevant for the universal conductance fluctuations \([14, 15]\). As in the previous section we will neglect terms like \( (G^{R}G^{R}) \) and \( (G^{A}G^{A}) \). All possible pairings of the remaining averages in terms of Diffusons and Cooperons are given by

\[
\langle (G^{A} - G^{R})(G^{A} - G^{R})(G^{A} - G^{R})(G^{A} - G^{R}) \rangle = \nonumber
\]

\[
(D_{\omega_2 - \omega_1} (R^R, R^L) + D_{\omega_1 - \omega_2} (R^L, R^R)) (D_{\omega_2 - \omega_3} (R^R, R^L) + D_{\omega_3 - \omega_2} (R^L, R^R)) + \nonumber
\]

\[
(D_{\omega_4 - \omega_3} (R^L, R^R) + D_{\omega_3 - \omega_4} (R^R, R^L)) (D_{\omega_2 - \omega_3} (R^R, R^L) + D_{\omega_3 - \omega_2} (R^R, R^L)) + \nonumber
\]

\[
(C_{\omega_3 - \omega_1} (R^L, R^R) + C_{\omega_1 - \omega_3} (R^L, R^R)) (C_{\omega_2 - \omega_4} (R^R, R^L) + C_{\omega_4 - \omega_2} (R^R, R^L)) . \tag{20}
\]


Due to the Cooperons, the conductance-conductance correlation function \((19)\) depends on the magnetic field. Using \((20)\), eqn. \((19)\) can be expressed as
\[
\langle G(N_1, x_1)G(N_2, x_2) \rangle = \langle G(N_1, x_1) \rangle \langle G(N_2, x_2) \rangle + \]
\[
+ 2\pi \rho D K \int d\omega_1 d\omega_2 G_{\omega_1}(N_1, x_1)G^*_{\omega_2}(N_2, x_2) \left( D_{\omega_2-\omega_1}(R^R, R^L) + D_{\omega_1-\omega_2}(R^L, R^R) \right)^2 \]
\[
+ 2\pi \rho D K \int d\omega_1 d\omega_2 G^*_{\omega_1}(N_1, x_1)G_{\omega_2}(N_2, x_2) \left( C_{\omega_2-\omega_1}(R^L, R^R) + C_{\omega_1-\omega_2}(R^L, R^R) \right)^2.
\]

For a strong magnetic field the last term involving the Cooperons vanishes. For \(U < E_{Th}\) we only have to consider the spatially constant zero mode of the diffusion operator, and the sum of the two Cooperons also tends to \(\delta(\omega_1 - \omega_2)\) for \(B = 0\), as in \((16)\). Thus the conductance correlation function becomes
\[
\langle G(N_1, x_1)G(N_2, x_2) \rangle = \langle G(N_1, x_1) \rangle \langle G(N_2, x_2) \rangle + \alpha(4\pi^2 \rho D K S^{-1})^2 |A(N_1, x_1, N_2, x_2)|^2,
\]
where \(A(N_1, x_1, N_2, x_2) = \int d\omega G_{\omega_1}(N_1, x_1)G^*_{\omega_2}(N_2, x_2)\).

The parameter \(\alpha\) depends on whether time reversal symmetry is broken or not: \(\alpha = 2\) at \(B = 0\), while \(\alpha = 1\) for \(B \gg B_c\). Thus the conductance correlation function \((15)\) becomes
\[
C(N_1, x_1, N_2, x_2) = \frac{|A(N_1, x_1, N_2, x_2)|^2}{|A(N_1, x_1, N_1, x_1)||A(N_2, x_2, N_2, x_2)|}.
\]
We note that \(C\) is independent of \(\alpha\), i.e. possesses a universal form for both conserved and broken time reversal symmetry. The integral for \(A\) at finite temperature can be evaluated and the result is given in Appendix \(A\). At low temperature \(\Delta \beta \gg 1\) the integration yields with \(d = \epsilon_{N_2} - \epsilon_{N_1}\), positive
\[
A(N_1, x_1, N_2, x_2) = \frac{1}{d + (x_2 - x_1)U} \log \frac{d + x_2 U}{x_1 U} + \frac{1}{d + (x_2 - x_1)U} \log \frac{d + (1 - x_1)U}{(1 - x_2)U} -
\]
\[
- \frac{1}{d + U + (x_2 - x_1)U} \left[ \log \frac{d + x_2 U}{x_2 U} + \log \frac{d + (1 - x_1)}{(1 - x_1)U} \right].
\]
The autocorrelation function \((18)\) in this limit finally becomes
\[
C(N_1, x_1, N_2, x_2) = x_1 x_2 (1 - x_1) (1 - x_2) \left[ \frac{1}{d + x_2 - x_1} \log \frac{(d + x_2 U)(d + 1 - x_1)}{x_1 (1 - x_2)} -
\right.
\]
\[
- \frac{1}{d + 1 - x_2 x_1} \log \frac{(d + 1 - x_1)(d + x_2 U)}{(1 - x_1) x_2} \right]^2.
\]

The energy difference \(d\) between the single-particle levels \(N_2\) and \(N_1\) can be expressed in terms of the mean level spacing \(\Delta\), \(d/U = (N_2 - N_1)\Delta/U = (N_2 - N_1)\delta\) where \(\delta = \Delta/U\). As a special case (and for the sake of simplifying the algebraic expressions), we set \(x = x_1 = x_2\) and obtain for the conductance autocorrelation function \(C\) for different valleys
\[
C(N_1, x, N_2, x) = x^2 (1 - x)^2 \times
\]
\[
\times \left[ \frac{1}{(1 + (N_2 - N_1)\delta)(N_2 - N_1)\delta} \log(1 + \frac{(N_2 - N_1)\delta}{x})(1 + \frac{(N_2 - N_1)\delta}{1 - x}) \right]^2.
\]

Fig. \(3\) (left) shows \(C\) as function of \(N_2 - N_1\) for \(U/\Delta = 40\) and different values of \(x\). As expected the correlations decay slowly with valley number. Away from \(x = 1/2\), \(C\) falls off more rapidly because the nearest occupied/empty level contributes more strongly to the conductance. Fluctuations in this level from one valley to the next reduce the correlations. Fig. \(3\) (center) demonstrates that the scale for the decay is set by \(U/\Delta\). The slope of \(C\) at \(N_2 - N_1 = 0\) at \(x = 0.5\) is given by \(-4\Delta/U\).

Next we turn to the correlations between \(x_1\) and \(x_2\) in the same valley \(N_1 = N_2\). The autocorrelation function \((21)\) then becomes
\[
C(N, x_1, N, x_2) = \frac{x_1 x_2 (1 - x_1)(1 - x_2)}{(x_2 - x_1)^2} \left( \log \frac{x_2 1 - x_1}{x_1 1 - x_2} \right)^2.
\]
We note that \(C\) is independent of the ratio \(U/\Delta\). Fig. \(4\) (right) shows \(C\) vs. \(x_1\) and \(x_2\) fixed. At \(x_1 = x_2\) the autocorrelation function becomes maximal, and it falls rapidly off for \(x_1 \to 0, 1\). We should bear in mind, however, that \(\delta\) breaks down near the resonances, since it neglects fluctuations in the particle number of the dot.
FIG. 1. Conductance autocorrelation function $C$ for $x_1 = x_2 = x$ vs. valley number $N_2 - N_1$. Left: For $U/\Delta = 40$ and $x = 0.5$ (solid), 0.2 (dotted) and 0.1 (dashed), Center: For $x = 0.5$ and $U/\Delta = 40$ (solid), 30 (dotted) and 20 (dashed). Right: $C$ for $N_1 = N_2$ vs. relative position in the valley $x_1$ and $x_2$ fixed at 0.5 (solid), 0.7 (dotted) and 0.9 (dashed). Note that (27) is not valid in the vicinity of order $kT/U$ around $x_1 = 0$ and $x_2 = 1$.

IV. COMPARISON WITH AN EXPERIMENT

Measurements of the inter-valley conductance of semiconductor quantum dots were first reported in [13]. The experimental set-up was similar to other experiments reported in ref. [16,17]. The dots were all in the ballistic regime and an irregular shape of the confining potential renders the motion of the electrons chaotic, so that RMT describes the statistics of the mesoscopic fluctuations in the sample. Additional gate electrodes could distort the shape of the dot, and allow an ensemble average to be obtained from a single sample. The experiment focused on the conductance-conductance correlations in the elastic cotunneling regime between different magnetic fields but at the same chemical potential, as calculated in [2]. Since $kT < \sqrt{U\Delta}$ the observed cotunneling was indeed elastic. In ref. [13] the dot was more strongly coupled to the leads (barrier conductance $G_{L,R} \sim e^2/h$) than in [16,17], since otherwise the cotunneling current would be very small and difficult to measure. Thus the tunneling rate $\Gamma \sim 0.7\Delta$ and the averaged conductance at the peaks is eight times larger than the cotunneling conductance in the valleys ($\langle G \rangle \sim 0.05e^2/h$).

Fig. 2 shows a comparison of $C$ vs. $N_2 - N_1$ between theory and experiment (ref. [13]). The quantum dot on the sample under study had the parameters $kT \sim 9\mu eV$, $\Delta = 15.8\mu eV$ and $U = 410\mu eV$. Experimental data for the autocorrelation function $C$ (18) in the middle of the valleys $x_1 = x_2 = 1/2$ are only available for three neighboring valleys and for $U/\Delta = 25.9$. The lines are our results for three different temperatures. We observe that in the experiment the correlations are more strongly suppressed than predicted in theory.

V. DISCUSSION

There are two main messages arising from the present analysis. On the theoretical side, our analysis underlines the significant correlations of the transmission (i.e. conductance) through a quantum dot as an external parameter is varied. In our case the quantum dot is weakly coupled to the leads (external reservoirs) and the external parameter which is varied is the gate voltage, affecting the number of electrons which reside on the dot. All this results in valley-valley correlations of the conductance in the cotunneling (far–from–resonance) regime. Previously, correlations have been found in the original analysis of Aleiner and Glazman [2] (intravalley), as function of the magnetic field, in ref. [18] (valley–valley) considering the differential capacitance of the dot instead of the cotunneling conductance, and in a recent analysis on transmission phases 8. The mere existence of correlations as found in the present study is, therefore, not totally surprising, although the details (dependence on the location within the valley, on the interaction strength–charging energy, temperature etc.) are certainly different.

The second message contains, in our opinion, a much more intriguing element. The correlations found in the experiment seem to be less robust than our theoretical expressions suggest. There might be several possible reasons for this. Some are obvious while others are more subtle and may give rise to some intriguing physics.

(i) The precise value of the electron-gas temperature in the experiment. In [13] the temperature was estimated to be $kT \sim 9\mu eV$ which corresponds to $\beta\Delta = 1.4$. Modest deviations from this value are not going to produce agreement with theory.
(ii) Approximating the canonical distribution function by a Fermi function with an effective shifted temperature (cf. the comment that precedes eqn. [6], see also ref. [7]) is asymptotically justified for $kT \gg \Delta$, which, for $\beta\Delta = 1.4$ is not quite the case. However, given the other curves in Fig. 2, a more accurate treatment of this point is not going to cure the problem.

(iii) In our derivation of the correlation function $C$, we have considered the case $U < E_{Th}$. From the technical point of view, this allowed us to retain the "zero-mode" of the Diffuson propagator only. In the experiment, the Thouless energy was estimated to be $E_{Th} = 180\mu eV$, which is smaller than the charging energy of $U = 410\mu eV$. Qualitatively one can expect that the inclusion of non-zero modes would imply deviation from the constant interaction model. In that case the addition (removal) of an electron is likely to change the effective (interaction induced) potential landscape felt by the other electrons, facilitating more efficient "scrambling" of the dot (see below), and suppressing the correlations found here. However a quantitative statement in this regard calls for a detailed analysis, not included in the present work.

(iv) The Hamiltonian of an interacting electron system (such as a quantum dot) includes terms other than the constant interaction $U$. Varying the gate voltage is then bound to change the nature of the many-body wavefunction of the electron gas in the dot. For example, within the approximate Hartree-Fock picture, the effective single particle wave functions will be modified as electrons are kept added to the dot, resulting in the breakdown of the Koopmans picture [19]. It was shown that the Koopmans picture, which asserts that the effective single-particle Hartree-Fock states remain unchanged as electrons are added to or removed from the system, breaks down when it concerns with finite (and disordered) systems, with sufficiently strong electron-electron interaction. For the systems studied that breakdown happened at $r_s \leq 1.5$ where $r_s = \sqrt{3/(4\pi n_0 a_0^2)}$, with $n_0$ the electron density and $a_0$ the Bohr radius. A remarkable experimental evidence has been provided by the experiment of [20], where the magnetofingerprints of various excited states of a dot at different particle number have been compared. It turned out that by adding electrons to the dot of concern (the total number of electrons was 200) the magnetofingerprints have been significantly modified, indicating scrambling of the electronic states. A more systematic study of this scrambling has been reported in ref. [17]. We speculate that the suppression of the conductance correlations in the present context may be another tool to evaluate the scrambling. One systematic measurement which is called for is to repeat the experiment for different values of $r_s$. Evidently, to facilitate a more detailed comparison with our calculated expression one would need data concerning various values of $U/\Delta$, varying temperature and a larger number of valleys.
VI. ACKNOWLEDGMENT

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APPENDIX A: A TOY MODEL FOR THE DISORDER

For the conductance correlation function we need the integral

\[ A(N_1, x, N_2, x) = \int d\omega \left[ \frac{1}{\epsilon_N - \omega + xU - i\Gamma} - \frac{1}{\epsilon_N - \omega + (1 - x)U - i\Gamma} \right] \times \left[ \frac{f(\omega - \epsilon_N - d)}{\epsilon_N + d - \omega + xU - i\Gamma} - \frac{f(\epsilon_N - d - \omega)}{\epsilon_N + d - \omega + (1 - x)U - i\Gamma} \right] \]

(A1)

where \( d = \epsilon_{N_2} - \epsilon_{N_1} \). Since we are primarily interested in the correlation between different valleys we set \( x_1 = x_2 = x \). By means of contour integration we obtain

\[ A = \frac{1}{d(1 - \exp(-\beta d))} \left[ \psi\left( \frac{1}{2} - \hat{\beta}(xU + d) \right) - \psi\left( \frac{1}{2} - \hat{\beta}(xU - d) \right) + \psi\left( \frac{1}{2} - \hat{\beta}(1 - x)U + d \right) - \psi\left( \frac{1}{2} - \hat{\beta}(1 - x)U - d \right) \right] + \]

\[ + \frac{1}{d(1 - \exp(\beta d))} \left[ \psi\left( \frac{1}{2} - \hat{\beta}xU \right) - \psi\left( \frac{1}{2} - \hat{\beta}(xU - d) \right) + \psi\left( \frac{1}{2} - \hat{\beta}(1 - x)U - d \right) - \psi\left( \frac{1}{2} - \hat{\beta}(1 - x)U + d \right) \right] - \]

\[ - \frac{1}{(d + U)(1 - \exp(-\beta d))} \left[ \psi\left( \frac{1}{2} - \hat{\beta}(d + xU) \right) - \psi\left( \frac{1}{2} + \hat{\beta}(1 - x)U \right) + \psi\left( \frac{1}{2} - \hat{\beta}((1 - x)U - d) \right) - \psi\left( \frac{1}{2} + \hat{\beta}xU \right) \right] \]

(A2)

where \( \psi \) is the Digamma function and \( \hat{\beta} = \beta/(2\pi i) \). For the normalization in [18] we need \( A \) for \( d \to 0 \). We obtain

\[ A(N, x, N, x) = i \left( \frac{\beta}{2\pi} - \frac{1}{\pi U} \right) \left( \psi^{(1)}\left( \frac{1}{2} - \hat{\beta}U(1 - x) \right) + \psi^{(1)}\left( \frac{1}{2} - \hat{\beta}Ux \right) \right) - \]

\[ - \frac{\beta}{4\pi^2} \left( \psi^{(2)}\left( \frac{1}{2} - \hat{\beta}U(1 - x) \right) + \psi^{(2)}\left( \frac{1}{2} - \hat{\beta}Ux \right) \right) \]

(A3)

with \( \psi^{(1,2)} \) the first and second derivative of the Digamma function.

APPENDIX B: A TOY MODEL FOR THE DISORDER

We employ here a toy model which facilitates the calculation of the correlation function \( C \) yet, we feel, captures some of the essential physics. This model has been previously employed in ref. [3] as part of our analysis of the transmission phase correlations through a quantum dot. The model is based on the following simplifying assumptions:

(i) The level spacing is constant, i.e. \( \epsilon_j = \Delta \cdot j \). This mimics the level repulsion typical for a disordered system.

(ii) For the product of tunneling matrix elements we assume \( V_i^L V_i^R = V^2 \alpha_i \) with \( \alpha_i \) a random variable which can take the values +1 and −1 with equal probability. This models the fluctuations in the wave function caused by the disorder.
With this model we calculate the correlation function of transmission amplitudes in the cotunneling regime

\[ C_t(N_1, x_1, N_2, x_2) = \langle t(N_1, x_1) t(N_2, x_2)^* \rangle \]  

and show that we obtain the same results as with the formalism of section II. For one disorder configuration (a sequence of the \( \alpha_j \)) the transmission amplitude in the cotunneling regime (with \( N \) electrons) at low temperature \( kT \ll \Delta \) becomes

\[
t = V^2 \left( \sum_{j \leq N} \frac{\alpha_j}{k - N - x_1 U} \left[ E - ((k - M)\Delta - xU) + i\Gamma^+ \right] \right)
+ \sum_{j \geq M+1} \frac{\alpha_j}{k - N - M + U(1 - x)} \left[ E - ((k - M)\Delta + U(1 - x)) + i\Gamma^+ \right].
\]

(B2)

The disorder average is now equivalent to an average over the \( \alpha_i \). We find \( \langle \alpha_i \rangle = 0 \) and \( \langle \alpha_i \alpha_j \rangle = \delta_{i,j} \). We set \( E = 0 \) and obtain

\[
C_t(N, x_1, N + M, x_2) = \frac{V^2}{\Delta^2} \left( \sum_{k=1}^{N+M} \frac{1}{k - N - x_1 U} \frac{1}{k - N - M + x_2 U} \right)
+ \sum_{k=N+1}^{N+M} \frac{1}{k - N + U(1 - x_1)} \frac{1}{k - N - M + x_2 U}
+ \sum_{k=N+M+1}^{\infty} \frac{1}{k - N + U(1 - x_1) \frac{1}{k - N - M + U(1 - x_2)}}.
\]

(B3)

where \( U = U/\Delta \). The sums can be expressed in terms of the Digamma function \( \psi \)

\[
\sum_{n=0}^{M-1} \frac{1}{(n + a)(n + b)} = \frac{1}{b - a} (\psi(b) - \psi(a) + \psi(M + a) - \psi(M + b)).
\]

(B4)

We arrive at

\[
\frac{\Delta^2}{V^2} C_t(N, x_1, N + M, x_2) = \frac{1}{M + (x_2 - x_1) U} \left[ \psi(M + U x_2) - \psi(U x_1) \right]
+ \psi(N + U x_1) - \psi(N + M + U x_2) + \psi(1 + M + U(1 - x_1)) - \psi(1 + U(1 - x_2))
+ \frac{1}{M + (1 + x_2 - x_1) U} \left[ \psi(1 + M + U(1 - x_1) - \psi(1 + U(1 - x_1) + \psi(M + U x_2) - \psi(U x_2)).
\]

(B5)

When we are sufficiently far away from the resonances \( x_1, 2U \gg 1 \) we can use the asymptotic expansion of the Digamma function \( \psi(z) \sim \log z \) for \( z \gg 1 \). In the limit \( N \to \infty \) we obtain

\[
\frac{\Delta^2}{V^2} C_t(N, x_1, N + M, x_2) = \frac{1}{M + (x_2 - x_1) U} \log \left[ \frac{M + x_2 U}{M + (x_2 - x_1) U} \frac{1 + M + (1 - x_1) U}{1 + (1 - x)_2 U} \right]
+ \frac{1}{M + (1 + x_2 - x_1) U} \log \left[ \frac{M + x_2 U}{M + (1 + x_2 - x_1) U} \frac{1 + M + (1 - x_1) U}{1 + (1 - x)_2 U} \right].
\]

(B6)

With the formalism of section II the correlation function \( C_t \) is given by

\[
C_t(N_1, x_1, N_2, x_2) = \Delta \frac{h}{e^2} G^L \frac{h}{e^2} G^R A(N_1, x_1, N_2, x_2)
\]

(B7)

with \( A \) defined in (B2) and calculated in (B4). Comparison with (B6) shows that \( V^2 = \Delta^2 (h G^L)/e^2 (h G^R)/e^2 \).
[1] D. V. Averin and Y. N. Nazarov, Phys. Rev. Lett. 76, 1695 (1990).
[2] I. L. Aleiner and L. I. Glazman, Phys. Rev. Lett. 77, 2057 (1996).
[3] R. Baltin and Y. Gefen, cond-mat/9907205, accepted for publication in Phys. Rev. Lett.
[4] C. W. J. Beenakker and H. van Houten, Quantum Transport in Semiconductor Nanostructures (in Solid State Physics Vol. 44, edited by D. Turnbull and H. Ehrenreich. Academic Press, 1991).
[5] L. P. Kouwenhoven, C. M. Marcus, P. L. McEuen, S. Tarucha, R. M. Westervelt, and N. S. Wingreen, Proceedings of the NATO Advanced Study Institute on Mesoscopic Electron Transport, edited by L. P. Kouwenhoven, L. Sohn and G. Schön (Kluwer Series E, 1997).
[6] C. W. J. Beenakker, Phys. Rev. B 44, 1646 (1991).
[7] A. Kamenev and Y. Gefen, Chaos, Solitons and Fractals 8, 1229 (1997).
[8] T. Guhr, A. Müller-Groeling, and H. A. Weidenmüller, Phys. Rep. 299 (1998) p. 189.
[9] C. B. Duke, Tunneling in Solids (in Solid State Physics Suppl. 10, edited by F. Seitz, D. Turnbull and H. Ehrenreich. Academic Press, 1969).
[10] R. Landauer, Philos. Mag. 21, 863 (1970).
[11] Y. Meir and N. S. Wingreen, Phys. Rev. Lett. 68, 2512 (1992).
[12] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics (Dover, 1963).
[13] S. M. Cronenwett, S. R. Patel, C. M. Marcus, K. Campman, and A. C. Gossard, Phys. Rev. Lett. 79, 2312 (1997).
[14] B. L. Altshuler, Pis'ma Zh. Eksp. Teor. Fiz. 41, 530 (1985) [JETP Lett. 41, 648 (1985)].
[15] P. A. Lee and A. D. Stone, Phys. Rev. Lett. 55, 1622 (1985).
[16] J. A. Folk, S. R. Patel, S. F. Godijn, A. G. Huibers, S. M. Cronenwett, and C. Marcus, Phys. Rev. Lett. 76, 1699 (1996).
[17] S. R. Patel, D. R. Stewart, C. Marcus, M. Gökcédag, Y. Alhassid, A. D. Stone, C. I. Duruöz, and J. S. Harris Jr., Phys. Rev. Lett. 81, 5900 (1998).
[18] A. Kaminski, I. L. Aleiner, and L. I. Glazman, Phys. Rev. Lett. 81, 685 (1998).
[19] P.N. Walker, G. Montambaux, Y. Gefen, Phys. Rev. Lett. 82, 5392 (1999); P.N. Walker, G. Montambaux, Y. Gefen, Phys. Rev. B 60, 2541 (1999).
[20] D. R. Stewart, D. Sprinzak, C. M. Marcus, C. I. Duruöz, and J. S. Harris Jr., Science 278, 1784 (1997).