$J/\Psi$ at high temperatures in anisotropic lattice QCD

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$J/\Psi$ and $\eta_c$ above the QCD critical temperature $T_c$ are studied in anisotropic quenched lattice QCD, considering whether the $c\bar{c}$ systems above $T_c$ are compact quasi-bound states or scattering states. We adopt the standard Wilson gauge action and $O(a)$-improved Wilson quark action with renormalized anisotropy $a_s/a_t = 4$ at $\beta = 6.10$ on $16^3 \times (14 - 26)$ lattices, which correspond to the spatial lattice volume $V \equiv L^3 \simeq (1.55\text{fm})^3$ and temperatures $T \simeq (1.11 - 2.07)T_c$. To clarify whether compact charmonia survive in the deconfinement phase, we investigate spatial boundary-condition dependence of the energy of the $c\bar{c}$ systems above $T_c$. In fact, for low-lying $c\bar{c}$ scattering states, there appears a significant energy difference $\Delta E \equiv E(\text{APBC}) - E(\text{PBC})$ between periodic and anti-periodic boundary conditions as $\Delta E \simeq 2\sqrt{m_c^2 + 3\pi^2/L^2} - 2m_c$ ($m_c$: charm quark mass) on the finite-volume lattice. In contrast, for compact charmonia, there is no significant energy difference between periodic and anti-periodic boundary conditions. As a lattice QCD result, we find almost no spatial boundary-condition dependence for the energy of the $c\bar{c}$ system in $J/\Psi$ and $\eta_c$ channels for $T \simeq (1.11 - 2.07)T_c$, which indicates that $J/\Psi$ and $\eta_c$ would survive as compact $c\bar{c}$ quasi-bound states below $2T_c$.

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1. Introduction

Since QCD is established, the quark-gluon-plasma (QGP) phase has been studied with much attention as a “new phase of matter” at high temperatures both in theoretical and experimental sides \([1, 2, 3, 4]\). In recent years, QGP creation experiments are actually performed at SPS [3] and RHIC [4] in high-energy heavy-ion collisions. As an important signal of the QGP creation, \(J/\Psi\) suppression \([1, 2]\) was theoretically proposed and has been tested in the SPS/RHIC experiments. The basic assumption of \(J/\Psi\) suppression is that \(J/\Psi\) disappears above \(T_c\) due to vanishing of the confinement potential and the Debye screening effect [2].

Very recently, some lattice QCD calculations indicate an interesting possibility that \(J/\Psi\) and \(\eta_c\) seem to survive even above \(T_c\) \([5, 6, 7]\), which may lead a serious modification for the \(J/\Psi\) suppression scenario in QGP physics. However, as a possible problem, the observed \(c\bar{c}\) state on lattices may not be a nontrivial charmonium but a trivial \(c\bar{c}\) scattering state, because it is difficult to distinguish these two states in lattice QCD.

In this paper, we aim to clarify whether the \(c\bar{c}\) system above \(T_c\) is a compact quasi-bound state or a scattering state, which is spatially spread. To distinguish these two states, we investigate spatial boundary-condition dependence of the energy of the \(c\bar{c}\) system by comparing results in periodic and anti-periodic boundary conditions. If the \(c\bar{c}\) system is a scattering state, there appears an energy difference \(\Delta E\) between the two boundary conditions as \(\Delta E \approx 2\sqrt{m_c^2 + 3\pi^2/L^2} - 2m_c\) with the charm quark mass \(m_c\) on a finite-volume lattice with \(L^3\). If the \(c\bar{c}\) system is a compact quasi-bound state, the boundary-condition dependence is small even in finite volume. In Ref. [8], this method is actually applied for distinction between a scattering state and a compact resonance.

2. Method to distinguish a compact state from a scattering state

To begin with, we briefly explain the method to distinguish a compact state from a scattering state in term of its spatial extension. For this purpose, we investigate the \(c\bar{c}\) system in the periodic boundary condition (PBC) and in the anti-periodic boundary condition (APBC), respectively, and examine spatial boundary-condition dependence for the \(c\bar{c}\) system. Here, in the PBC/APBC case, we impose periodic/anti-periodic boundary condition for \(c\) and \(\bar{c}\) on a finite-volume lattice.

| Table 1: | Periodic boundary condition (PBC) and anti-periodic boundary condition (APBC): the relation between spatial boundary condition and the minimum momentum \(|\vec{p}_{\text{min}}|\) of \(c\), \(\bar{c}\) and compact charmonia \(c\bar{c}\). |
|---|---|---|---|---|---|
| PBC | | | | | |
| particle | spatial BC | \(|\vec{p}_{\text{min}}|\) | | | |
| \(c\) | periodic | 0 | | | |
| \(\bar{c}\) | periodic | 0 | | | |
| charmonia \((c\bar{c})\) | periodic | 0 | | | |
| APBC | | | | | |
| particle | spatial BC | \(|\vec{p}_{\text{min}}|\) | | | |
| \(c\) | anti-periodic | \(\sqrt{3\pi}/L\) | | | |
| \(\bar{c}\) | anti-periodic | \(\sqrt{3\pi}/L\) | | | |
| charmonia \((c\bar{c})\) | periodic | 0 | | | |

For a compact \(c\bar{c}\) quasi-bound state, the wave function of each quark is spatially localized and insensitive to spatial boundary conditions in lattice QCD, so that the charmonium behaves as a compact boson and its energy in APBC is almost the same as that in PBC [8]. For a \(c\bar{c}\) scattering state, both \(c\) and \(\bar{c}\) have non-zero relative momentum \(\vec{p}_{\text{min}} = (\pm \frac{\pi}{L}, \pm \frac{\pi}{L}, \pm \frac{\pi}{L})\), i.e., \(|\vec{p}_{\text{min}}| = \sqrt{3\pi}/L\).
in APBC, while they can take zero relative momentum \( \vec{p}_{\text{min}} = 0 \) in PBC. In fact, if the \( c\bar{c} \) system is a scattering state, there appears a significant energy difference \( \Delta E \) between PBC and APBC due to the finite lattice volume of \( L^3 \), and it is estimated as \( \Delta E \simeq 2\sqrt{m_c^2 + 3\pi^2/L^2} - 2m_c \). In our lattice QCD calculation, \( |\vec{p}_{\text{min}}| \) and \( \Delta E \) for the \( c\bar{c} \) scattering state are estimated as \( |\vec{p}_{\text{min}}| = \sqrt{3}\pi/L \simeq 0.69 \text{GeV} \) and \( \Delta E \simeq 2\sqrt{m_c^2 + 3\pi^2/L^2} - 2m_c \simeq 0.35 \text{GeV} \) for \( L \simeq 1.55 \text{fm} \) and \( m_c \simeq 1.3 \text{GeV} \).

### 3. Anisotropic lattice QCD

In this paper, we adopt anisotropic lattice QCD for the study of high-temperature QCD. In lattice QCD at temperature \( T \), (anti)periodicity is imposed in the temporal direction with the period \( 1/T \), and hence it is technically difficult to measure temporal correlators at high temperatures. To overcome this problem, we use the anisotropic lattice with anisotropy \( a_s/a_t = 4 \). Owing to the finer temporal mesh, we can obtain detailed information for temporal correlators.

For the gauge field, we adopt the standard plaquette action on an anisotropic lattice as \([8,9]\)

\[
S_G = \frac{\beta}{N_c} \sum_{s,i,j} \text{ReTr}\{1 - P_{ij}(s)\} + \frac{\beta}{N_c} \gamma_c \sum_{s,i,j} \text{ReTr}\{1 - P_{4}(s)\},
\]

where \( P_{ij} \) denotes the plaquette operator. In the simulation, we take \( \beta \equiv 2N_c/g^2 = 6.10 \) and the bare anisotropy \( \gamma_c = 3.2103 \), which lead to renormalized anisotropy as \( a_s/a_t = 4.0 \). The scale is set by the Sommer scale \( r_0^{-1} = 395 \text{MeV} \). Then, the spatial and temporal lattice spacing are estimated as \( a_s^{-1} \simeq 2.03 \text{GeV} \) (i.e., \( a_s \simeq 0.979 \text{fm} \)), and \( a_t^{-1} \simeq 8.12 \text{GeV} \) (i.e., \( a_t \simeq 0.024 \text{fm} \)), respectively. The adopted lattice size is \( 16^3 \times (14 - 26) \), which corresponds to the spatial lattice size as \( L \simeq 1.55 \text{fm} \) and the temperature as \( T = (1.11 - 2.07)T_c \). We use 999 gauge configurations, which are picked up every 500 sweeps after the thermalization of 20,000 sweeps.

For quarks, we use \( O(a) \)-improved Wilson (clover) action on the anisotropic lattice as \([8,9]\).

\[
S_F \equiv \sum_{x,y} \bar{\psi}(x)K(x,y)\psi(y),
\]

\[
K(x,y) \equiv \delta_{x,y} - \kappa_t \{(1 - \gamma_5)U_4(x)\delta_{x+4,y} + (1 + \gamma_5)U_4^\dagger(x-4)\delta_{x-4,y}\}
- \kappa_s \sum_i \{(r - \gamma_5)U_i(x)\delta_{x+1,y} + (r + \gamma_5)U_i^\dagger(x-1)\delta_{x-1,y}\}
- \kappa_s c_E \sum_i \sigma_{i4} F_{i4} \delta_{x,y} - r \kappa_c c_B \sum_{i<j} \sigma_{ij} F_{ij} \delta_{x,y},
\]

which is anisotropic version of the Fermilab action \([10]\). \( \kappa_t \) and \( \kappa_s \) denote the spatial and temporal hopping parameters, respectively, and \( r \) the Wilson parameter. \( c_E \) and \( c_B \) are the clover coefficients. The tadpole improvement is done by the replacement of \( U_i(x) \rightarrow U_i(x)/u_s \), \( U_4(x) \rightarrow U_4(x)/u_t \), where \( u_s \) and \( u_t \) are the mean-field values of the spatial and the temporal link variables, respectively. The parameters \( \kappa_t, \kappa_s, r, c_E, c_B \) are to be tuned so as to keep the Lorentz symmetry up to \( O(a^2) \). At the tadpole-improved tree-level, this requirement leads to \( r = a_t/a_s \), \( c_E = 1/(u_s u_t^2) \), \( c_B = 1/u_t^2 \) and the tuned fermionic anisotropy \( \gamma_F \equiv (u_s \kappa_t)/(u_t \kappa_s) = a_t/a_s \). For the charm quark, we take \( \kappa = 0.112 \) with \( 1/\kappa \equiv 1/(u_s \kappa_t) - 2(\gamma_F + 3r - 4) \), which corresponds to the hopping parameter in the isotropic lattice. The bare quark mass \( m_0 \) in spatial lattice unit is expressed as \( m_0 = \frac{1}{4}(\frac{1}{\kappa} - 8) \). We summarize the lattice parameters and related quantities in Table 2. In the present lattice QCD, the masses of \( J/\Psi \) and \( \eta_c \) are found to be \( m_{J/\Psi} \simeq 3.07 \text{GeV} \) and \( m_{\eta_c} \simeq 2.99 \text{GeV} \) at zero temperature.
4. Temporal correlators of $c \bar{c}$ systems at finite temperature on anisotropic lattice

To investigate the low-lying state at high temperatures from the temporal correlator, it is practically desired to use a “good” operator with a large ground-state overlap, due to limitation of the temporal lattice size. To this end, we use a spatially-extended operator of the Gaussian type as

$$O(t, \bar{x}) = N \sum_{\bar{y}} \exp \left\{ -\frac{1}{2 \rho^2} \bar{y}^2 \right\} \bar{c}(t, \bar{x} + \bar{y}) \Gamma c(t, \bar{x})$$

in the Coulomb gauge [8,9]. $N$ is a normalization. The size parameter $\rho$ is optimally chosen in terms of the ground-state overlap. $\Gamma = \gamma_k (k = 1 - 3)$ and $\Gamma = \gamma_5$ correspond to $1^- (J/\Psi)$ and $0^- (\eta_c)$ channels, respectively. The energy of the low-lying state is calculated from the temporal correlator,

$$G(t) \equiv \frac{1}{V} \sum_{\bar{x}} \langle O(t, \bar{x}) O^\dagger(0, 0) \rangle,$$

where the total momentum of the $c \bar{c}$ system is projected to be zero.

In accordance with the temporal periodicity at finite temperature, we define the effective mass $m_{\text{eff}}(t)$ from the correlator $G(t)$ by the cosh-type function as [11]

$$\frac{G(t)}{G(t + 1)} = \frac{\cosh[m_{\text{eff}}(t - N_t/2)]}{\cosh[m_{\text{eff}}(t + 1 - N_t/2)]}$$

with the temporal lattice size $N_t$. In the plateau region of $m_{\text{eff}}(t)$, $m_{\text{eff}}(t)$ corresponds to the energy of the low-lying $c \bar{c}$ state. To find the optimal value of $\rho$, we calculate the correlator $G(t)$ for $\rho = 0.2, 0.3, 0.4$ and 0.5fm at each temperature, and examine the ground-state overlap by comparing $G(t)/G(0)$ with the fit function of $g_{\text{fit}}(t) = A \cosh[m(t - N_t/2)]$. As a result, the optimal size seems to be $\rho \simeq 0.2$fm for $c \bar{c}$ systems. Hereafter, we only show the numerical results for $\rho = 0.2$fm.

5. Lattice QCD results for the $c \bar{c}$ system above $T_c$

We investigate the $c \bar{c}$ systems above $T_c$ both in $J/\Psi(J^P = 1^-)$ and $\eta_c(J^P = 0^-)$ channels. For each channel, we calculate the temporal correlator $G(t)$ and the effective mass $m_{\text{eff}}(t)$ defined by Eq.(4.3) both in PBC and APBC, and examine their spatial boundary-condition (b.c.) dependence.

Figures 1-4 show the effective-mass plot $m_{\text{eff}}(t)$ of the $c \bar{c}$ system in the $J/\Psi$ channel for $\rho = 0.2$fm. From the cosh-type fit for the correlator $G(t)$ in the plateau region of $m_{\text{eff}}(t)$, we extract the energies, $E(\text{PBC})$ and $E(\text{APBC})$, of the low-lying $c \bar{c}$ system in PBC and APBC, respectively. Table 3 summarizes the $c \bar{c}$ system in the $J/\Psi$ channel in PBC and APBC at each temperature.

As a remarkable fact, almost no spatial b.c. dependence is found for the low-lying energy of the $c \bar{c}$ system, i.e., $\Delta E \equiv E(\text{APBC}) - E(\text{PBC}) \simeq 0$, which is contrast to the $c \bar{c}$ scattering case of
ΔE ≃ 2√m_c^2 + 3π^2/L^2 − 2m_c ≃ 0.35GeV for L ≃ 1.55fm and m_c ≃ 1.3GeV as was discussed in Sect.2. This result indicates that J/Ψ survives for T = (1.11 − 2.07)T_c.

Table 4 summarizes the c\bar{c} system in the \eta_c channel in PBC and APBC at each temperature. Again, almost no spatial b.c. dependence is found as ΔE ≡ E(APBC) − E(PBC) ≃ 0, and this result indicates that \eta_c also survives for T = (1.11 − 2.07)T_c as well as J/Ψ.

In contrast to J/Ψ and \eta_c, our preliminary lattice results show a large spatial b.c. dependence for the c\bar{c} system in the \chi_{c1} (J^P = 1^+) channel even near T_c, which seems consistent with Ref.[7].

6. Summary and conclusions

We have investigated J/Ψ and \eta_c above T_c with anisotropic quenched lattice QCD to clarify whether the c\bar{c} systems above T_c are compact quasi-bound states or scattering states. We have adopted O(a)-improved Wilson quark action with renormalized anisotropy a_s/a_t = 4. Anisotropic lattice is technically important for the measurement of temporal correlators at high temperatures.

We have use β = 6.10 on 16^3 × (14 − 26) lattices, which correspond to T = (1.11 − 2.07)T_c.
Table 3: The energy of the $c\bar{c}$ system in the $J/\Psi$ channel ($J^P = 1^-$) in PBC and APBC at $\beta = 6.10$ and $\rho = 0.2fm$ at each temperature. The statistical errors are smaller than 0.01GeV. We list also uncorrelated $\chi^2/\text{N}_{\text{DF}}$ and $\Delta E \equiv E(\text{APBC}) - E(\text{PBC})$.

| temperature $T_c$ | fit range | $E(\text{PBC})$ [GeV] [error] | $E(\text{APBC})$ [GeV] [error] | $\Delta E$ [GeV] |
|------------------|-----------|-------------------------------|-------------------------------|-----------------|
| 1.11$T_c$       | 7–11      | 3.05GeV [0.14]                | 3.09GeV [0.61]                | 0.04GeV         |
| 1.32$T_c$       | 8–11      | 2.95GeV [0.34]                | 2.98GeV [0.33]                | 0.03GeV         |
| 1.61$T_c$       | 6–9       | 2.94GeV [0.10]                | 2.98GeV [0.22]                | 0.04GeV         |
| 2.07$T_c$       | 5–7       | 2.91GeV [0.03]                | 2.93GeV [0.04]                | 0.02GeV         |

Table 4: The energy of the $c\bar{c}$ system in the $\eta_c$ channel ($J^P = 0^-$) in PBC and APBC at $\beta = 6.10$ and $\rho = 0.2fm$ at each temperature. The statistical errors are smaller than 0.01GeV. We list also uncorrelated $\chi^2/\text{N}_{\text{DF}}$ and $\Delta E \equiv E(\text{APBC}) - E(\text{PBC})$.

| temperature $T_c$ | fit range | $E(\text{PBC})$ [GeV] [error] | $E(\text{APBC})$ [GeV] [error] | $\Delta E$ [GeV] |
|------------------|-----------|-------------------------------|-------------------------------|-----------------|
| 1.11$T_c$       | 7–11      | 3.03GeV [0.04]                | 3.02GeV [0.17]                | -0.01GeV        |
| 1.32$T_c$       | 7–11      | 2.99GeV [0.78]                | 2.98GeV [0.82]                | -0.01GeV        |
| 1.61$T_c$       | 6–9       | 3.00GeV [0.31]                | 2.97GeV [0.38]                | -0.03GeV        |
| 2.07$T_c$       | 5–7       | 3.01GeV [0.03]                | 3.00GeV [0.07]                | -0.01GeV        |

To conclude, we have found almost no spatial boundary-condition dependence of the energy of the low-lying $c\bar{c}$ system both in $J/\Psi$ and $\eta_c$ channels even on the finite-volume lattice. These results indicate that $J/\Psi$ and $\eta_c$ survive as compact $c\bar{c}$ quasi-bound states for $T = (1.11 - 2.07)T_c$.

References

[1] T. Hashimoto, O. Miyamura, K. Hirose and T. Kanki, Phys. Rev. Lett. 57, 2123 (1986).
[2] T. Matsui and H. Satz, Phys. Lett. B178, 416 (1986).
[3] M.C. Abreu et al. (NA50 Collaboration), Phys. Lett. B477, 28 (2000).
[4] D. Kim (PHENIX Collaboration), J. Phys. G31, S309 (2005).
[5] T. Umeda, K. Katayama, O. Miyamura and H. Matsufuru, Int. J. Mod. Phys. A16, 2215 (2001); H. Matsufuru, O. Miyamura, H. Suganuma and T. Umeda, AIP Conf. Proc. CP594, 258 (2001).
[6] M. Asakawa and T. Hatsuda, Phys. Rev. Lett. 92, 012001 (2004).
[7] S. Datta, F. Karsch, P. Petreczky, I. Wetzorke, Phys. Rev. D69, 094507 (2004); J. Phys. G31, S351 (2005).
[8] N. Ishii, T. Doi, H. Iida, M. Oka, F. Okiharu and H. Suganuma, Phys. Rev. D71, 034001 (2005); N. Ishii, T. Doi, Y. Nemoto, M. Oka and H. Suganuma, hep-lat/0506022, Phys. Rev. D72 (2005).
[9] H. Matsufuru, T. Onogi and T. Umeda, Phys. Rev. D64, 114503 (2001); T.R. Klassen, Nucl. Phys. B553, 557 (1998).
[10] A.X. El-Khadra, A.S. Kronfeld and P.B. Mackenzie, Phys. Rev. D55, 3933 (1997).
[11] N. Ishii, H. Suganuma and H. Matsufuru, Phys. Rev. D66, 094506 (2002); D66, 014507 (2002).