Liu, Wei; Röckner, Michael; Luís da Silva, José
Strong dissipativity of generalized time-fractional derivatives and quasi-linear (stochastic) partial differential equations. (English) Zbl 1469.35227
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Summary: In this paper strong dissipativity of generalized time-fractional derivatives on Gelfand triples of properly in time weighted $L^p$-path spaces is proved. In particular, as special cases the classical Caputo derivative and other fractional derivatives appearing in applications are included. As a consequence one obtains the existence and uniqueness of solutions to evolution equations on Gelfand triples with generalized time-fractional derivatives. These equations are of type
\[
\frac{d}{dt}(k \ast u)(t) + A(t, u(t)) = f(t), \quad 0 < t < T,
\]
with (in general nonlinear) operators $A(t, \cdot)$ satisfying general weak monotonicity conditions. Here $k$ is a non-increasing locally Lebesgue-integrable nonnegative function on $[0, \infty)$ with $\lim_{s \to \infty} k(s) = 0$. Analogous results for the case, where $f$ is replaced by a time-fractional additive noise, are obtained as well. Applications include generalized time-fractional quasi-linear (stochastic) partial differential equations. In particular, time-fractional (stochastic) porous medium and fast diffusion equations with ordinary or fractional Laplace operators and the time-fractional (stochastic) $p$-Laplace equation are covered.

MSC:

35R11 Fractional partial differential equations
60H15 Stochastic partial differential equations (aspects of stochastic analysis)
35K59 Quasilinear parabolic equations
76S05 Flows in porous media; filtration; seepage
26A33 Fractional derivatives and integrals
45K05 Integro-partial differential equations
35K92 Quasilinear parabolic equations with p-Laplacian

Keywords:
generalized time-fractional derivative; strong dissipativity; weak monotonicity; generalized porous medium equation

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