On the reliability of proxies for globular cluster collision rates

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ABSTRACT

A variety of different proxies for the stellar collision rates in globular clusters is used in the literature, depending on the quality of data available. We present comparisons between these proxies and the full integrated collision rate for different King models. The most commonly used proxy, $\Gamma$, defined to be $\frac{\rho_0^3}{2r_c^2}$, where $\rho_0$ is the central cluster density, and $r_c$ is the core radius based on the 1966 King model, is an accurate representation of the collision rate from the King model to within about 25% for all but the least concentrated globular clusters. By integrating over King models with a range of parameters, we show that $\Gamma_h$, defined to be $\frac{\rho_h^3}{2r_h^2}$, where $\rho_h$ is the average density within the half-light radius, and $r_h$ is the half-light radius, is only marginally better correlated with the full King model collision rate than is the cluster luminosity. The two galaxies where results of King model fitting have been reported in detail show a dearth of core-collapsed clusters relative to that seen in the Milky Way, indicating that the core radii of the most concentrated clusters are probably slightly overestimated, even with excellent data. Recent work has suggested that shallower than linear relations exist between proxies for $\Gamma$ and the probability that a cluster will contain an X-ray source; we show that there is a similarly shallower than linear relationship between $\Gamma$ and $\Gamma_h$ that can explain the relationship where $\Gamma_h$ is used; we also show that reasonable measurement errors are likely to produce a shallower than linear relationship even when $\Gamma$ itself is used. We thus conclude that the existing evidence is all consistent with the idea that X-ray binary formation rates are linearly proportional to cluster collision rates. We also find, through comparison with Gunn-Griffin models (sometimes referred to as multi-mass King models) suggestive evidence that the retention fractions of neutron stars in globular clusters may be related to the present day concentration parameters, which would imply that the most concentrated clusters today were the most concentrated clusters at the time of their supernovae.

Key words: globular clusters:general – stellar dynamics – stars:binaries

1 INTRODUCTION

The density of stars in most globular clusters is large enough that significant rates of collisions and other close interactions between stars can take place. Globular clusters are known to have about two orders of magnitude more bright X-ray binaries per unit stellar mass than field star populations (see e.g. Clark 1975 for the first evidence of this effect in the Milky Way; Supper et al. 1997; Angelini et al. 2001; Sarazin, Irwin & Bregman 2001 for early indications from individual nearby galaxies; Kundu et al. 2003 for the first demonstration of a universal trend from a compilation of nearby galaxies). The excess of X-ray binaries in globular clusters is generally explained as a result of X-ray binary formation through tidal captures (Fabian, Pringle & Rees 1975), three-body exchange encounters (Hills 1976) and direct collisions between stars (Verbunt 1987).

Millisecond radio pulsars are also overabundant in globular clusters by a significant amount (e.g. Camilo & Rasio 2005), most likely because they evolve from low mass X-ray binaries (Alpar et al. 1982). It is generally believed that significant numbers of stars in other classes, such as cataclysmic variable stars, and blue straggler stars may be formed through stellar interactions in globular clusters as
well, and considerable debate continues about whether the horizontal branch morphology of globular clusters is affected by stellar interactions (e.g. Buonanno et al. 1997; Peacock et al. 2011).

A single tight binary may have a larger binding energy than all the single stars in a globular cluster put together. Furthermore, X-ray sources are the only traces of the dynamical properties of globular clusters which can currently be well observed at distances of the Virgo Cluster. Understanding the formation and evolution of these binary stars is thus important both in its own right, and for understanding the evolution of globular clusters in general.

Unfortunately, the Milky Way lacks large enough numbers of bright (i.e. $L_X > 10^{35} \text{ ergs/sec}$) X-ray binaries for real statistical samples to be made. Only 12 of the Milky Way’s globular clusters contain bright X-ray sources, although two of these clusters now show clear evidence for containing multiple such sources. Some intuition can be developed about which classes of clusters are most likely to contain X-ray sources, but the sample is not large enough to make rigorous tests of whether these parameters have a statistically significant correlation with the probability a cluster will contain an X-ray binary. For example, the first four globular cluster X-ray sources motivated Silk & Arons (1975) to suggest that metal rich clusters were more likely than metal poor clusters to contain X-ray binaries. Bellazzini et al. (1995) established from the combined sample of Milky Way and M 31 that metallicity was, in fact, an important parameter, but the correlations between metallicity and other parameters, such as galactocentric radius still left some concerns about which was the causal parameter.

The use of more distant extragalactic globular clusters, in the large samples of clusters which can be seen in nearby elliptical galaxies, can enhance the sizes of samples of X-ray sources dramatically. Only with Chandra and HST observations of the nearby galaxy NGC 4472 was it possible to demonstrate clearly that the probability a cluster will contain an X-ray source is more strongly correlated with cluster metallicity than with cluster galactocentric radius (Kundu et al. 2002). This finding has since been repeated in many other galaxies (see e.g. the review article of Maccarone & Knigge 2007 and references within).

Another important, but harder-to-address question, is whether the correlation between the stellar interaction rate in a cluster and the probability it will contain an X-ray source is linear. The measurement of stellar interaction rates in extragalactic globular clusters is observationally extremely challenging. Only in M 31 are typical cluster core radii larger than the angular resolution of the Hubble Space Telescope, but because M 31 is so nearby, very few of its clusters are close enough to one another that a single HST field of view contains multiple clusters. Nonetheless, one can fit King (1966) models to data, even if the core radius is formally unresolved, but only with high signal-to-noise and good oversampling and understanding of the telescope point spread function (e.g. Carlson & Holtzman 2001). Peacock et al. (2009) have presented King model fits for about half of the globular clusters in M 31 using wide-field ground-based infrared observations, and compared these with an updated catalog of X-ray sources in Peacock et al. (2010a), while Jordán et al. (2004) have presented some results from King model fits to HST observations of M 87’s clusters, and Jordán et al. (2007) have presented some results from HST observations of NGC 5128.

Some attempts have been made to test whether the probability a globular cluster will contain an X-ray binary is linearly proportional to the collision rate, based on both King model fits, and other proxies for the collision rate. Jordán et al. (2004) showed that, presuming their King model fits were reliable, the probability a cluster contains an X-ray binary scales with $\Gamma \rho^{-0.4}$, and argued that this showed that binaries are being destroyed in dense clusters. Smits et al. (2006) showed that the scaling between collision rate and cluster mass for the Milky Way is such that if one obtained data with large random errors in the core radii, so that the collision rate estimates were dominated by the effects of cluster luminosity, one would obtain results like those found in Jordán et al. (2004). The results of Jordán et al. (2004) were based on data from the ACS Virgo Cluster survey, which has an integration time of only 750 seconds in the $g$ band filter. It is thus likely that the core radii fitted from those data have measurement errors large compared to the dispersion in the values of the core radius, since only a few percent of the clusters are bright enough that they should have the S/N of at least 500 needed to obtain core radii accurate to within a factor of 2 at the distance of the Virgo Cluster with HST (see, again, Carlson & Holtzman 2001 for discussion to the effect that this signal to noise is needed).

More recently, Sivakoff et al. (2007) used $\Gamma_h$, defined to be $\rho_h^{1/2} r_h^2$, where $\rho_h$ is the average density within the half-light radius, and $r_h$ is the half-light radius to estimate collision rates. They found that the probability a cluster would host an X-ray source was proportional to $\Gamma_h^{0.8}$ and interpreted this result as additional evidence of binary destruction. Jordán et al. (2007) present results from fitting King models to data from NGC 5128, the nearest giant elliptical galaxy. These data are deeper than the data obtained in the ACS Virgo Cluster Survey, and the galaxy observed is a factor of almost 5 closer than M87, lending some confidence that the core radii fitted will be reliable, at least for many of the clusters. They, too find a weaker than linear dependence of probability that an X-ray binary will exist in a cluster on its estimated collision rate.

The Galactic globular clusters have also been studied to determine whether the lower luminosity X-ray sources they contain have numbers which scale with cluster collision rate. Pooley et al. (2003) used all sources brighter than $L_X = 4 \times 10^{30} \text{ ergs/sec}$, and found that the number of sources scales with $\Gamma^{0.72 \pm 0.36}$, consistent with a linear correlation at the 1-$\sigma$ level. Additionally, the lowest collision rate clusters may have their rates of formation of X-ray sources dominated by primordial systems, rather than by dynamical formation (e.g. Pooley & Hut 2006), so even a statistically significant finding that the source numbers scaled more slowly than linearly with $\Gamma$ would have multiple possible interpretations.

The idea that the dependence of the probability a cluster will host an X-ray binary is weaker than linear because of binary destruction in the densest clusters would have major implications on the channels by which X-ray binaries form. For the X-ray binaries themselves to be strongly subject to disruption requires that they be soft—that is if their binding energies are smaller than the mean kinetic energies of the cluster’s single stars (Heggie 1975). Alternatively,
the LMXBs could, potentially, be formed from exchange encounters where the targets are soft binaries, in which case it might be possible for the LMXBs themselves to be hard binaries, while their formation rate depends on the number of soft binaries in the cluster. The hard-soft boundary in most globular clusters occurs for orbital separations of about 1 AU – a size scale on which only highly evolved stars can overflow their Roche lobes. Of the Milky Way’s globular cluster X-ray binaries with known orbital periods, the longest period system is AC 211 in M 15, whose orbital period is about 19 hours. It thus seems unlikely that binary destruction plays a major role in regulating the number of X-ray binaries, but given the small number statistics of globular cluster X-ray binaries with known orbital periods, and the importance of the implications for understanding how globular cluster X-ray binaries form, the suggestion merits further investigation. What is missing from the present analysis in an understanding of how the measurement errors in the samples of cluster parameters made to date, and the systematic offsets between real collision rates and proxy collision rates, affect the conclusions which can be drawn from the data. In this paper, we look into both issues.

2 KING MODELS AND COLLISION RATES

Given a King model fit, it is customary to estimate the cluster’s stellar collision rate as $\Gamma = \rho_0^2 / \sigma^2$, as first suggested by Verbunt & Hut (1987). The underlying assumptions of this calculations are that the cluster is well described by a single-mass King model (and hence is in virial equilibrium); that the cluster’s core density and core velocity dispersion are constant within the core; and that collisions take place only in the core. Further complications may arise if one wishes to consider the collision rates of neutron stars, as is most appropriate for the bulk of globular cluster X-ray binaries. One must then, additionally, consider the retention rate of neutron stars in the globular cluster. One should also consider the effects of varying characteristic velocities of interaction in different clusters. The importance of this effect has been considered in a study of the dynamical formation of X-ray binaries in galactic nuclei (Voss & Gilfanov 2007), and is implicitly taken into account in dynamical simulations, but has not generally been considered in more empirically motivated studies of globular clusters. Additionally, the lifetimes of many classes of accreting binaries can be long compared to the relaxation timescale of a globular cluster, and it would be more appropriate to use a weighted integral of the collision rate over time (if such a quantity were measurable), rather than the actual present collision rate (Fregeau 2008). Therefore, even if it can be confirmed that the probability a cluster will host an X-ray binary is not linearly proportional to the collision rate, there are a variety of mechanisms apart from binary destruction that may be responsible for such deviations. Understanding the quantitative details of such deviations may provide a key target for numerical simulations, so it is important to understand the systematic observational biases that may come into attempts to measure the X-ray binary production probability versus collision rate.

It is straightforward to investigate the effects of the assumptions that the cluster’s core density and core velocity dispersion are constant, and that collisions take place only in the core. To do this, we have computed King models for a range of values of cluster concentration, and have computed the collision rates in each of these model clusters. We integrate the collision rate per unit volume $\rho^2 / \sigma$, where $\rho$ is the cluster density, and $\sigma$ is the cluster’s stellar velocity dispersion. Pooley et al. (2003) performed a similar procedure to estimate the collision rates of Milky Way clusters, but integrated out only to the half light radius.

Following King (1966), we define the cluster concentration as the ratio of the cluster tidal radius to its core radius. We normalise the density in units of the central density and the radius in units of the core radius. The velocity dispersion is normalised according to equation 31 of King (1966) – in units of the velocity dispersion of a King model with $W = \infty$, $(8\pi Gr^2 \rho_0 / 9)^{1/2}$. Therefore, a simple integration of the King model’s collision rate in these units yields a collision rate in units of the rate derived by Verbunt & Hut (1987) times a constant. In figure 1 we plot the collision rate from the King model versus the cluster concentration. As is clear from the figure, for all realistic values of the concentration parameter, the ratio between the collision rate integrated from the King model and that from the Verbunt & Hut approximation varies by only 50% over the range from $W_0 = 3$ (corresponding to the least concentrated clusters in the Galaxy) to $W_0 = 12$ (correspondingly to the most concentrated clusters in the Galaxy) – the approximation is thus accurate to 25% (modulo effects of the deviations between single mass King models and real clusters). While, for purposes of clarity, we did not plot all the models computed, most of this variation occurs between $W = 3$ and $W = 5$, with the approximation of Verbunt & Hut clearly getting much worse for the smallest values of $W_0$, as can be seen in figure 1.

An additional moderately serious issue remains, in that using $\Gamma = \rho_0^2 / \sigma^2$ implicitly assumes that the stellar velocity dispersion in the cluster core can be estimated correctly from the virial theorem, assuming a fixed mass-to-light ratio for all clusters, which is an approximation in good, but not perfect agreement with the data. For this reason, Pooley et al. (2003) used observed velocity dispersions taken from the compilation of Pryor & Meylan (1993) to normalize their collision rates, rather than inferred ones from the virial theorem. They find deviations of up to a factor of 10 between $\Gamma$ as defined by Verbunt & Hut (1987) and the collision rates they compute. While it is not stated explicitly by Pooley et al. (2003), the discrepancies are dominated by the differences between observed and inferred velocity dispersions – not the deviations from the $\Gamma$ formulation and the more detailed treatment of the interaction rate used by Pooley et al. (2003). While in principle, observed velocity dispersions should be more reliable than model velocity dispersions, the sample of Pryor & Meylan (1993) is derived in an inhomogeneous manner, with some clusters measured using individual stars, and others using integrated light measurements. The sample is additionally not focussed on the cluster cores. For a couple of the more concentrated clusters, M 15 and M 70, there are several $\sigma$ discrepancies between the two methods of measuring the velocity dispersion observationally. Therefore, it may be that the velocity dispersions estimated from a King model and a standard mass-to-light ratio are actually more reliable than the tabulated observational data. This issue certainly bears additional attention in the future.
2.1 Effects of using a multi-mass model

It also stands that a single mass King model cannot describe globular clusters properly. For studies of X-ray binary production, it is clear that there will be mass segregation in clusters, and that the amount of mass segregation may vary from cluster to cluster, since the relaxation times of the clusters vary substantially. For many Galactic globular clusters, the surface brightness profile cannot be fit properly with a King model. Several are elliptical (Harris 1996), and others show evidence for density cusps in their centers (e.g. Noyola & Gebhardt 2006). The deviations between the real collision rates and the collision rates estimated assuming a single mass King model may be substantial, but cannot be estimated through simple analytic formulae. Additionally, core collapsed clusters are not well-described by King models.

To address the issue of mass segregation, we have also tried using multi-mass models similar to the King model. We have followed the prescription of Gunn & Griffin (1979) for producing these models. We have chosen to ignore the effects of anisotropy that are possible within the Gunn & Griffin (1979) framework (following the work of Michie 1963) by setting the anisotropy radius to $\infty$. In the computational approach suggested by Gunn & Griffin (1979), the key parameters are the central potential of the cluster, which is the same as that used in the King model, the masses of the different species (i.e. mass classes), and the central densities in the different species, denoted as $\alpha_j$ where $j$ is the number of the mass class.

We start first with a simple case of a two component model. The heavier component is taken to have a mass of $1.4 M_\odot$ (i.e. roughly than of a neutron star) while the lighter component is taken to have a mass of $0.7 M_\odot$ (i.e. roughly that of a turnoff star). We then compute the collision rates only between the heavy and the light component, rather than internally to either component. We search over a range in $W_0$ from 3 to 9, with larger values avoided for computational reasons. We start off with 3% of the mass in the heavy component. We find that for the whole range of central potentials, very close to half the heavy stars end up in the core (using the core radius for the mean stellar mass to define the core). Adding in an additional component of 0.5 $M_\odot$ stars increases the fraction of the 1.4 $M_\odot$ objects in the core, but the fraction of the heavy objects in the core still remains roughly constant over a fairly wide range in central potential.

In this case, where the ratio of central density in the heavy component to the total central density is constant, the X-ray binary formation rate will be well-modeled by the Verbunt & Hut formula – the range of rates of integrated collision rates to Verbunt & Hut formula estimates spanned in the three mass species case is only about 10%. If, on the other hand, the neutron stars represent a constant fraction of the total cluster mass, rather than of the total core mass, one would expect an increase in the X-ray binary fraction in the most concentrated clusters. This effect can be substantial, approaching a factor of \sim 10 between the most and least concentrated clusters – if one assume that the same fraction of the mass in any cluster will be in neutron stars, then the most concentrated clusters should have \sim 10 times as many X-ray binaries per unit collision rate as the least concentrated clusters.

There is no evidence for such an effect in M 31 (see figure 2), nor is there any evidence for such an effect in the Milky Way (for which the analogous plot is similarly devoid of any conclusive evidence). If we use the idea that the interaction rate of neutron stars determines the number of X-ray binaries as a default assumption, the lack of any obvious effects of $c$ can be taken to indicate that the neutron star retention fraction is higher in low concentration clusters than in high concentration clusters. This could be the case if, for example, the clusters’ structural parameters have not evolved strongly since the time of the supernovae in which the neutron stars formed (see for example Figure 15 of Giersz & Heggie 2011, which shows that the ratio of core radius to radius containing 90% of the stars of a simulated cluster attempting to match the present day conditions in 47 Tuc changes very slowly over its lifetime) and the only neutron stars retained were those formed in the cluster cores, so that the clusters with larger ratios of core mass to cluster mass have higher retention fractions. Obviously, given that the effect has been seen in a sample with fewer than 40 X-ray bright clusters observationally, and in a single set of dynamical simulations, it would be premature to draw strong conclusions, but this issue bears further attention, and it certainly seems reasonable that the neutron star retention fraction may be a strong function of cluster concentration.
Collision rate proxies

2.2 Comparison of $\Gamma$ with other tracers of the interaction rate

We can investigate the degree of accuracy of collision rates estimated through other techniques relative to the collision rate estimated from the King model rate. Of these suggested tracers of collision rate, the easiest to measure is $\Gamma_h$ (Sivakoff et al. 2007). In order to assess how useful $\Gamma_h$ is, we can compare $\Gamma_h$ with $\Gamma$ for the Milky Way’s globular clusters from the Harris catalog. In figure 3 we plot these two quantities against one another. Following the methodology in Smits et al. (2006), which was used to test the correlations expected between collision rate and mass, we compute $\Gamma_h$ for the 96 Milky Way globular clusters for which all the relevant quantities are tabulated in the Harris catalog. We then sort the clusters into 8 bins of 12 clusters each, sorted by $\Gamma_h$. We compute the mean $\Gamma$ in each bin, as well as the dispersion of values, and fit a power law to the binned values. The best fitting power law index is 0.83, very similar to the exponent of 0.82 $\pm$ 0.05 found by Sivakoff et al. (2007) for the relation between X-ray binary probability and $\Gamma_h$. A plot of these binned data is shown in figure 4. The 1-σ uncertainty on this exponent is about 0.2 dex, which is unsurprising given the large scatter within each bin – it is thus true that $\Gamma_h$ is formally consistent with being linearly related to $\Gamma$. A much larger sample of clusters would be needed to beat down the uncertainties in the correlation parameter at any given value of $\Gamma$. Nonetheless, the close similarity of the two indices, is intriguing, and for non-linear correlations between $\Gamma_h$ and the sizes of collisionally produced populations to be taken as indicative of real physics, the converse should be established – that the correlation between $\Gamma_h$ and other measures of the collision rate is linear, or at least is well-enough established that one can correct for its deviations from linearity.

We can also look at the predictive power of the two measures of collision rate for whether a cluster will have an X-ray source. This is done most easily in M 31, since the key parameters for the Galactic clusters discovered since the New General Catalog was produced are poorly constrained, and these clusters make up a significant fraction of the Galactic globular clusters which host X-ray sources. We present a plot illustrating that $\Gamma$ is a much better predictor of whether a cluster will host an X-ray source than $\Gamma_h$ in figure 5.

Figure 2. The collision rate $\Gamma$ versus the concentration parameter [i.e. $\log_{10}(r_t/r_c)$] for the M 31 globular clusters. The filled blue squares are clusters with X-ray sources, while the open red circles are clusters without X-ray sources. There is clearly no preference for clusters with a high concentration parameter at a given collision rate.

Figure 3. The plot of $\Gamma_h$ versus $\Gamma$ for the 96 Milky Way clusters where the distance, surface brightness, and luminosity are given in the Harris catalog. The plot shows the large scatter between the two different estimators of the stellar collision rate.

Figure 4. The plot of $\Gamma$ versus $\Gamma_h$ for the 8 bins of 12 Milky Way clusters each where the distance, surface brightness, and luminosity are given in the Harris catalog. The fitted trend line, $\Gamma \propto \Gamma_h^{0.83}$ is remarkably similar to the X-ray probability versus $\Gamma_h$ trend found by Sivakoff et al. (2007).
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2.2.1 When is $\Gamma_h$ useful and why?

It is also instructive to compare the relative ability of $\Gamma_h$ and cluster integrated luminosity to predict $\Gamma$ reliably. Even if $\Gamma_h$ is not perfectly correlated with $\Gamma$, it may be adding useful information to a crude understanding of the collision rates of clusters in galaxies where it is not feasible to obtain reliable core radii. There certainly exists a range of distances and signal-to-noise ratios where it is possible to measure $\Gamma_h$ but not possible to measure $\Gamma$, so it is interesting to try to understand in what cases a large sample of measurements of $\Gamma_h$ might be more useful than measurements of the cluster luminosity alone.

To do this, we take each quantity and make an unweighted fit of a power law with index 1.0 (i.e. a straight line forced to go through the origin). We fit $\Gamma$ as a function of either $\Gamma_h$ or $L_V$. We then can compute the variance of the residuals from this model estimate, in $\Gamma$, so that the variance is independent of normalization of $\Gamma_h$ or $L_V$. This indicates that the typical error in predicting $\Gamma$ from $\Gamma_h$ is 0.58 dex (i.e. a factor of about 3.8). The typical error in predicting $\Gamma$ from cluster luminosity is 0.76 dex (about a factor of 5.8). It can thus be seen that the inclusion of half-light radius information, if it is reliable, adds some, but very little, predictive power, relative to knowing the cluster luminosity alone. This is not surprising, given that half light radius is positively, albeit very weakly correlated with cluster core radius.

Only for the very most spatially extended clusters in the Milky Way is there a strong correlation between core radius and half light radius (see e.g. Djorgovski & Meylan 1994); see also Figure 3, which shows plots of half-light radius versus core radius for M 31 [with the data from the Peacock 2010b catalog], which looks essentially the same in the plot as the Milky Way in Djorgovski & Meylan (1994). This is also interesting to compare this figure with figure 9 of Sivakoff et al. (2007), where it is shown that the probability a globular cluster will contain an X-ray source has a very weak dependence on $r_h$, except at half light radii larger than about 3 pc – the same radius at which the core and half light radii of Milky Way and M 31 clusters become very strongly correlated and approximately equal to one another. Unfortunately, neither plots of $r_c$ versus $r_h$ nor tables of fitted parameters are presently available for Cen A and M 87, so the comparison cannot be extended to those galaxies. Nonetheless, the data for the Milky Way and M 31 show clearly why $\Gamma_h$ provides some information about the dynamical state of the cluster, but not enough that deviations from linearity between $\Gamma_h$ and probability a cluster contains an X-ray binary can be taken as strong evidence about what is happening inside a cluster (e.g. binary destruction).

2.3 Reliability of actual estimates of $\Gamma$

Next, we consider the cases where high quality estimates of the core radii have been obtained. At the present time, such measurements have been done for only two galaxies – NGC 5128 (Jordán et al. 2007) and M 31 (Peacock et al. 2009). In both cases, the fraction of clusters with very high concentration parameters is significantly smaller than it is in the Milky Way. In the case of Jordán et al. (2007), this problem may be, in part, due to their decision to excise from their analyses any clusters with fitted concentration parameters $c$ greater than 2.5. Peacock et al. (2009) found several clusters in M 31 which showed best-fitting concentration parameter values larger than 2.5 among their confirmed clusters.

In the Milky Way, there are 15 known bright X-ray binaries (including transients), in 12 clusters (see Verbunt & Lewin 2006 and references within for the first 13; Altamirano et al. 2009 for the 14th [in NGC 6440] which was already
known to host at least one bright X-ray binary, and Bordas et al. 2010, Heinke et al. 2010 for the 15th]. Only 31 of the Milky Way’s 150 clusters are core collapsed, but the core collapsed clusters contain 7 of the 14 bright X-ray binaries. There is only a 1% chance of this high a fraction of the X-ray binaries being found in core collapsed clusters at random. It is thus important both to count properly the core collapsed clusters, and to estimate their collision rates correctly to make reliable statements about specific relations between collision rate and X-ray binary probability.

We may investigate now what size of errors are needed in order to produce a deviation from a linear correlation between X-ray binary hosting probability and collision rate estimator of order the size of the deviation seen by Jordán et al. (2007). We adopt an approach intermediate between making a full simulation of all possible measurement errors, and merely assuming that all values are measured accurately to arbitrary precision. We consider both the cases of additive and multiplicative errors.

As a first step for estimating the effects of measurements errors, we take the data values from the Harris catalogue, and convert the core radii into units of parsecs, as is appropriate for comparison with extragalactic clusters. First we consider the case of additive errors. We add a random number taken from a Gaussian distribution with varying values of \( \sigma \). If this change gives a cluster a negative core radius, we set its core radius to be the absolute value of that number. We assume that the core luminosity will be fairly reliably measured, so, as a simplifying assumption, we re-set the core density such that core luminosity is conserved. This assumption is the most favourable one to the idea that the \( \Gamma \) measurements are precise enough to look for deviations from a linear relation between collision rate and probability of the cluster containing an X-ray binary. The key issue we wish to test is that of the effects of asymmetric error contours that can result even from symmetric errors in the parameter values for the parameters fitted explicitly. Assuming a correctly measured core luminosity will produce smaller deviations from linearity and from the expectation value of the measurement than allowing for radius measurement errors while assuming a constant core density. It is thus overly conservative – serious problems for interpretations of data may result from errors a bit smaller than the ones we quote.

We then compute \( \Gamma' \) using the standard formula for core collision rate, but with the values with the simulated measurement errors rather than the actual values. We compare the collision rates for the clusters with the artificial measurement errors added to those without the artificial measurement errors added, using the same approach as was used for testing the validity of \( \Gamma_h \) – binning the data and fitting a power law to the binned data. We try several random number seeds, in addition to several values of \( \sigma \). We find that typically for \( \sigma \) of about 0.08 pc, which corresponds to about 0.08 HST pixels at the distance of Cen A, the additive errors are large enough to give \( \Gamma \propto \Gamma^{0.05} \). This value is about 10% of the typical core radius value for the clusters in the Milky Way. We note that the effects are strongest in the highest collision rate clusters, which tend to have the smallest core radii.

We can also consider the effects of a multiplicative error on the cluster core radii. We perform the same procedure as above, except that we multiply by a number following a log normal distribution, rather than adding a number drawn from a normal distribution. We find in this case that a multiplicative error with \( \sigma \) of 0.12 dex (i.e. about 30%) variation from the actual values typically gives a \( \Gamma \propto \Gamma^{0.85} \) relation (with a 1\( \sigma \) uncertainty of 0.17 dex). A plot for this case is given in figure 7.

In reality, the errors will be correlated both with the signal-to-noise of the clusters, and with the core radii of the clusters, so the above results are merely illustrative of the characteristic magnitudes of errors needed to produce spurious deviations from linearity. Jordán et al. (2007) cite a detection limit of \( m_{F606W} \sim 22 \) for their clusters. We presume this to be the 5\( \sigma \) detection threshold, so that a magnitude of 19.5 corresponds to a 50\( \sigma \) detection, but it is not explicitly stated in Jordán et al. (2007) what significance level is implied by the “detection limit.” If this is the case, then less than 1/3 of the clusters are detected at above the 50\( \sigma \) level, while about half are detected below the 30\( \sigma \) level. Carlson & Holtzman (2001) presented no results for simulation for clusters with signal-to-noise less than 55, but also presented no results for clusters with core radii smaller than 0.28 pixels. They find that the cluster concentrations can be measured accurately (to within 0.3 dex, or a factor of 2) 50% of the time where the core radius is 0.1 pixels and the signal-to-noise is at least 900, and 50% of the time for core radii of 0.28 pixels at signal to noise of at least 100. It thus seems likely that the measurement errors could cause the non-linear relation seen between estimated core collision rate and X-ray binary hosting probability in NGC 5128, even if the underlying physical relation is linear. The situation for the M 87 \( \Gamma \) estimates in Jordán et al. (2004) is obviously more severely affected by measurement errors.

One can also obtain an estimate of typical errors in fit quality by looking at the variations between the measure-
ments of individual clusters in Peacock et al. (2009), as that study included a large number of clusters measured multiple times. The best fitting core radii were found to be within 30% of one another only for clusters with S/N greater than about 500. The spatial resolution of the data of Peacock et al. (2009), using ground-based infrared observations to estimate core radii of M 31 clusters are only slightly worse than those the Hubble Space Telescope data taken for Centaurus A, making it unlikely that the core radii for NGC 5128 clusters are measured to 30% accuracy with signal-to-noise of 30 or below, as is the case for the majority of the Cent A clusters. Additionally, there may be systematic offsets in the core radius values for NGC 5128, since the King model fits in that work make use of an analytic approximation to the point spread function of HST; the point spread function for the M 31 work was derived directly from measurements of foreground stars, something possible given the large fields of view used and the moderate Galactic latitude of M31.

3 CONCLUSIONS

We have critically evaluated the reliability of various methods for determining the stellar encounter rates in extragalactic globular clusters, and their implications for determining whether the probability a globular cluster will contain a bright X-ray binary scales linearly with the cluster collision rate. We show that the core collision rate of Verbunt & Hut (1987), \( \Gamma = \rho^{3/2}r_c^2 \) gives accurate measurements of the collision rate over a wide range of cluster parameters, provided that the cluster really is well-described by a single-mass viralized King model. We show that using estimates of the cluster’s density within its half-light radius, plus the half-light radius itself to estimate the cluster’s collision rate give estimates of the collision rates only marginally better than those which can be obtained by assuming that the collision rate scales with the cluster mass. Furthermore, the relationship between \( \Gamma_h \) and \( \Gamma \) deviates from a linear relationship in the same way as the relationship between the measured values of \( \Gamma_h \) and the measured values of the probability a cluster will contain an X-ray binary for Virgo cluster galaxies. We thus conclude that it is dangerous to try to infer physical information about how X-ray binaries are formed by using \( \Gamma_h \) – the deviations from linearity can easily be explained by the non-linear, and highly scattered, relation between \( \Gamma \) and \( \Gamma_h \).

Finally, we show that small errors in the core radius measurement can induce non-linear relations between the measured collision rates and the actual collision rates, and can, again, explain the deviations between the measured values of \( \Gamma \) and the measured probabilities that the clusters contain X-ray binaries. Given that even the very small measurement errors likely to be associated with the two high quality attempts to fit King models to extragalactic globular clusters (Jordán et al. 2007; Peacock et al. 2009) are likely to be sufficient to produce spurious non-linearities, we conclude that any fitted relation between any estimator of the collision rate and other quantities which does not take into account both the statistical and systematic errors on the collision rate estimates should be taken as suspect. We thus emphasize extreme caution in interpreting the recent results (both based on \( \Gamma_h \) measurements and on measurements of \( \Gamma \) using data with insufficient signal-to-noise) suggesting that the relation between number density of X-ray binaries is shallower than linear with collision rate; while of strong statistical significance, sources of systematic error not accounted for in those analyses can easily mimic the results found. Typical errors in estimations of \( \Gamma \) – even those using direct core radius measurements in Cen A – are likely to have systematics such that differences from linearity of less than 0.2 dex will be highly suspect without detailed simulations showing that they the deviation cannot be an artefact of asymmetric errors on the collision rate estimates.

At the present time, then, we conclude that there is no observational evidence to contradict the idea that the only morphological parameter of a cluster that is important for predicting the probability the cluster will host an X-ray binary is the collision rate. We note that comparison of this observational finding with simple theory suggests that the retention fraction for neutron stars in concentrated clusters is lower than that in low concentration clusters. This finding, if verified, would indicate that the most concentrated clusters at the present time were the most concentrated clusters at the time of the supernovae producing their neutron stars. It is thus an important target for theory work to determine whether feasible models of the evolution of clusters lead to such an effect. We note that it is not required that the clusters’ concentration parameters themselves remain fixed – just that the timescales and directions of the evolution of the concentration parameters for different clusters are similar enough that the retention fractions remain correlated with the present day concentrations.

There are, of course, other parameters that clearly matter, such as the cluster metallicity (e.g. Kundu et al. 2002). There are also other parameters which should matter, such as the binary fraction, and the history of the evolution of the cluster. However, with relatively sparse data on the binary fractions in most clusters, it is difficult to look for evidence that the binary fraction matters. The best evidence of the importance of the history of the cluster (e.g. following Fregeau 2008) would be to find evidence of deviations from the predictive power of \( \Gamma \). Therefore, it would make sense to continue attempts to make good estimates of \( \Gamma \) in a larger sample of clusters. Unfortunately, really strong progress is unlikely to be possible until the development of 30-m class telescopes with multiconjugate adaptive optics.

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