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Key Points:
- The single-phase flow channeling is investigated by applying a method similar to CPA to the flow rate field
- The relation between flow channeling and viscous fingering and its dependence on pore-scale heterogeneity are quantified

Supporting Information: 
- Data Set 1
- Supporting Information SI

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Viscous Fingering and Preferential Flow Paths in Heterogeneous Porous Media

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Abstract

Preferential flow channeling and viscous fingering are widely observed phenomena in heterogeneous porous media from the pore scale to the core and reservoir scales. The transition from capillary to viscous fingering with increasing injection rate is also a well-known phenomenon. Based on the previously observed visual similarity of viscous fingers and preferential single-phase flow paths in 2-D simulations, we postulate that these single- and two-phase flow patterns should also be interrelated in 3-D porous media. Capillary fingering is a manifestation of invasion percolation, a process restricted to the subnetwork of pores obtained from the critical path analysis. We investigated single-phase flow channeling by applying a method similar to critical path analysis to the flow rate field and compared the subnetwork thus determined to the set of invaded pipes in drainage simulations. Our aim is to quantify the relation between preferential flow paths and viscous fingering, its dependence on pore-scale heterogeneity and pore connectivity, and ultimately to upscale the network observations to the field scale appropriate for the engineering applications involving drainage in heterogeneous media.

1. Introduction

Fluid flow in porous rocks is of great importance for many technological activities such as production of hydrocarbons, remediation of contaminated aquifers, or sequestration of CO2 in deep reservoirs. One crucial factor affecting all these applications is the heterogeneity of the geological materials in place. In hydrology, the extremely heterogeneous structures of aquifers lead to the development of stochastic methods (Gehlar, 1993). In petroleum engineering, stochastic methods were also proposed to estimate the uncertainty of future production models (e.g., Zhang & Tchelepi, 1999, who also coined the term “heterogeneity fingering” to describe immiscible displacements in highly heterogeneous formations). Stochastic methods often require a large number of expensive flow simulations in random reservoir realizations honoring the observed or assumed permeability spatial distribution. One very effective technique is to use streamtube simulations, which can be orders of magnitude faster than traditional finite difference simulations (Hewett & Behrens, 1991; King et al., 1993; Thiele, 1994; see also Bear, 1972, for application of the method to hydrology). The main assumption of the streamtube method is that displacements at the reservoir scale are dominated by the spatial variations of permeability and that the flow field can be viewed as a set of separate flow paths (usually called streamtubes). The streamtubes are considered as one-dimensional conduits along which mass conservation and transport equations are locally solved. Although streamtubes can be used to model miscible displacements and solute transport, they are particularly advantageous in the case of immiscible displacements at unfavorable viscosity ratios (viscous fingering) since they preserve the mobility contrast that is substantially reduced by numerical diffusion in traditional simulations (see Figure 4.9 in Thiele, 1994). Interestingly, the streamtube method is inaccurate in the case of favorable viscosity ratios when quasi-uniform displacements occur (Martin et al., 1973). Also note that Lagrée et al. (2016) found that viscous fingers simulated using a continuum Darcy-type method with an enforced sharp fluid interface, indeed corresponded to high-fluid velocity channels (see Figures 8 and 9).

Thus, the treatment of reservoir-scale heterogeneity in the streamtube method relies on a conceptual link between preferential single-phase flow paths and two-phase flow viscous fingers. However, reservoir rocks are strongly heterogeneous at all scales, even those smaller than the grid block size of streamtube simulations, and the effect of subgrid heterogeneity needs to be accounted for (Durlofsky et al., 2007; Efendiev & Durlofsky, 2002; Luo et al., 2016, 2018). Immiscible displacements at the scale of pores have been
observed in 2-D pore networks etched on the surface of transparent slabs (Lenormand et al., 1988; Lenormand & Zarcone, 1985; Lenormand et al., 1983; J. Yang et al., 2017) or Hele-Shaw cells furnished with microbeads or micropillars (Ferer et al., 2004; Frette et al., 1997; Holtzman, 2016; Mályá et al., 1992; Zhao et al., 2016). These experiments primarily investigated the effect of the capillary number \(Ca\), viscosity ratio \(M\), and wettability (i.e., the contact angle \(\theta\)). The main result was the construction of phase diagrams mapping the regions in \([Ca, M]\) space where stable and unstable displacements occurred (Lenormand et al., 1988). In the case of drainage (i.e., injection of a nonwetting fluid), three flow regimes with very wide, gradual transitions between them were identified, namely, stable displacements at high \(Ca\) and favorable viscosity ratios, viscous fingering at high to moderate \(Ca\) and unfavorable viscosity ratios, and capillary fingering at low to moderate \(Ca\). Imbibition (i.e., injection of a wetting fluid) involves subpore-scale mechanisms such as flow along pore wall edges and rugosities, which lead to different, more complex phase diagram (Lenormand & Zarcone, 1984; Lenormand et al., 1983). Many of the experimental works mentioned above were carried in association with pore-scale numerical simulations (Ferer et al., 2004; Holtzman, 2016; Lenormand et al., 1988; J. Yang et al., 2017), which usually produced rather encouraging supportive results. Following this success, pore-scale two-phase flow modeling has become an important component of research on immiscible displacements in porous media (Aker et al., 2000; Blunt et al., 2002, 1992; Joekar-Niasar & Hassanizadeh, 2012; Liu et al., 2015; Prodanovic & Bryant, 2006; Regaieg et al., 2017; Regaieg & Moncorge, 2017; Tsuji et al., 2016; see also references in Regaieg, 2015). Since they incorporate the physics of fluid displacements in single pores, pore-scale models are an important complement to reservoir-scale simulations, especially in the case of unstable displacements for which the Buckley-Leverett approach may not be appropriate.

Preferential flow paths are also a well-known feature of single-phase flow in heterogeneous reservoirs (e.g., Bianchi et al., 2011; Moreno & Tsang, 1994) and have also been observed in pore network simulations (Bernabé & Bruderer, 1998; Bruderer-Weng et al., 2004; David, 1993; David et al., 1990; Jang et al., 2011). Our premise in the present study is that flow channeling in single-phase and two-phase flow conditions is primarily caused by the medium heterogeneity and develops in paths with lowest resistance. As a consequence, fully formed viscous fingers and preferential single-phase flow paths in strongly heterogeneous rock should be closely related to each other. Our main objective is to verify this assumption in heterogeneous and partially connected pore networks. Most of the experimental and numerical work mentioned above considered 2-D systems for which visualization of the flow patterns is relatively easy. Here, we focused on 3-D networks and could not rely on visual evidence to assess the similarity of preferential single-phase flow paths and fully formed viscous fingers. We extended the concept of critical pore radius \(r_c\) and its associated subnetwork \([r_c]\) introduced in the context of the critical path analysis (CPA) (Charlaix et al., 1987; Friedman & Seaton, 1998; Katz & Thompson, 1986) to that of a critical pore flow rate \(q_c\) and subnetwork \([q_c]\). The connected part of \([r_c]\) can be identified using invasion-percolation and thus corresponds to capillary fingering occurring at infinitesimal capillary numbers. The subnetwork \([q_c]\) has a very different structure, which explains why viscous and capillary fingers are so distinctive.

2. Numerical Procedures

2.1. Network Construction and Characterization

We constructed 3-D, body-centered cubic (BCC), partially connected pipe networks by randomly populating them with cylindrical pipes according to a probability \(p = z/z_{\text{max}}\), where \(z\) denotes the desired mean coordination number (mean number of occupied bonds per node) and \(z_{\text{max}} = 8\) is the coordination number of BCC networks at 100% occupancy. The critical probability \(p_c\) at the onset of percolation has a known value of 0.18 in BCC networks. An important property of regular 3-D lattices such as simple, body-centered, or face-centered cubic is that they all have approximately the same percolation threshold, \(z_c \approx 1.5\), when expressed in terms of the coordination number (they have, however, different values of \(p_c\)). Since most geological applications involve rocks with a relatively well connected pore space, we only considered coordination numbers significantly greater than \(z_c\) (2.4 ≤ \(z\) ≤ 8). Pore size heterogeneity was generated by randomly and independently assigning the values \(r_i\) of the pipe radii according to log-uniform distributions with specified normalized standard deviations (0.05 ≤ \(\sigma/r\) ≤ 1.05). The truncation limits, \(r_{\text{min}}\) and \(r_{\text{max}}\) of the radius distributions were set such that the network hydraulic radius \(r_{\text{H}}\) (2 times the volume-to-surface
The validity of the Poiseuille equation, \( q_i = -g_i \Delta \rho_i \), where \( q_i \) is the flow rate in pipe \( i \), \( \Delta \rho_i \) the difference of exit to entry fluid pressures, \( g_i = \frac{\pi r_i^4}{(8\mu)} \) the hydraulic conductance, and \( \mu \) the fluid viscosity. Note that, although spherical pores are often assumed to occupy nodes in network simulation studies, we did not follow this practice but preferred to treat the pipe junctions as dimensionless points. The network realizations thus obtained were nominally isotropic and spatially uncorrelated. Anisotropy and long-range correlations are common in geological formations and strongly affect single- and multiple-phase flow (Araktingi & Orr, 1993; Knackstedt et al., 2001; Sahimi, 1995; Sahimi & Knackstedt, 1994), but considering isotropic, uncorrelated networks allowed us to limit the analysis to only two parameters, \( z \) and \( \sigma/\langle r \rangle \), respectively measuring pore connectivity and pore size heterogeneity. Since \( r_{tr} \) is a constant, it is convenient to describe the networks in terms of dimensionless radii \( R_i = r_i / r_{tr} \) and hydraulic conductances \( G_i = g_i / g_{tr} = R_i^4 \), where \( g_{tr} \) represents the conductance of a pipe of radius \( r_{tr} \). More details on these procedures can be found in Bernabé et al. (2010, 2011) and Li et al. (2018).

Fluid flow in heterogeneous networks is often analyzed using the CPA. The main idea of CPA is that flow tends to face higher resistance in smaller pipes than larger ones (Charlaix et al., 1987; Friedman & Seaton, 1998; Katz & Thompson, 1986). Hence, it is expected that a relatively small number of pipes composing the right tail of the pipe radii distribution should exert a greater influence on the hydraulic properties of the networks than the remaining pipes. To quantify this statement, CPA considers the subnetworks \([R]\) containing the pipes with radii \( R_i \) greater than or equal to some value \( R \) in the interval \([R_{min}, R_{max}]\). A subnetwork \([R]\) is connected if it contains at least one continuous, throughgoing path joining the upstream and downstream faces of the network. Since \([R]\) consists of pipes randomly distributed in a network, percolation theory predicts that \([R]\) is connected when its size \( S \) (the number of pipes in \([R]\) normalized to the total number of bonds, \( n_{tot} \), occupied or not, in the network) is greater than or equal to the percolation threshold \( p_c \). The critical radius \( R_c \) is thus defined as the largest radius in the interval \([R_{min}, R_{max}]\) corresponding to a connected subnetwork \([R]\). The size \( S_{R_c} \) of the critical subnetwork \([R]\) is therefore equal to \( p_c \) (within statistical fluctuations) and, consequently, independent of the network realization characteristics \( z \) and \( \sigma/\langle r \rangle \). In the case of log-uniform radii distributions, \( R_c \) is theoretically equal to

\[
R_c = R_{max}^{z/(z-2)} \approx R_{min}^{z/(z-2)} R_{max}^{z/(z-2)}
\]

The derivation of equation 1 is given in supporting information section S2. The variations of \( R_c \) with \((z, z_c)\) for different values of \( \sigma/\langle r \rangle \) are shown in Figure 1.

Because of statistical fluctuations, we devised a numerical technique to determine \( R_c \) and the associated subnetwork \([R_c]\) in specific network realizations (see supporting information section S3). Since \([R_c]\) is a critical network, it is formed of many separate pipe clusters of various sizes, among which only the largest is a connected set. The connected cluster \([R_c]^+\) can be identified for a particular network realization by means of an exploration algorithm inspired from invasion-percolation. The exploration excludes the pipes that do not belong to \([R_c]\), and the exploration starting point (the source) is the critical pipe (i.e., the one with \( R_i = R_c \)). The algorithm finds all the pipes with \( R_i \geq R_c \) forming continuous chains connected to the source (see section S4 in the supporting information). One important characteristic of \([R_c]^+\) is its size \( S_{R_c} \) (normalized to the total network size like \( S_{R_c} \)). We can also define a tortuosity coefficient \( \tau_{R_c} \) as the ratio of the geometric path length of the path of least resistance in \([R_c]^+\) to the network side length (note that, owing to the inclination of the bonds with respect to the network axes, BCC networks have an intrinsic tortuosity of \( \sqrt{3} \)). Since the pipe length is constant, \( \tau_{R_c} \) is simply the ratio of the number of pipes in the least resistance

![Figure 1](image-url). Theoretical critical radius \( R_c \) versus \((z, z_c)\) for different values of \( \sigma/\langle r \rangle \) as indicated in the inset.
path (LRP) to the number of bonds along the network side. We determined the LRP using Dijkstra’s algorithm (Dijkstra, 1959) with the bond “length” equal to the pipe hydraulic resistance $1/G_i$ when the bond belongs to $\{R_i\}$ and to a value much greater than any possible $1/G_i$ otherwise.

### 2.2. Single-Phase Flow

Simulating steady-state flow through network realizations was accomplished in the standard way, that is, by solving the Kirchoff equations for the values of the fluid pressure at the nodes and, from them, determining the flow rates $q_i$ through individual pipes (see reviews of the network method by Adler, 1992 and Berkowitz & Ewing, 1998, and the references therein). The BCC lattice underlying the network realizations is a periodic array of cubic cells, in which eight bonds link the node located at the center of the cell to eight nodes at the corners. We produced flow in a nominal direction parallel to one of the cell sides by fixing the downstream and upstream fluid pressures on the external faces of the network normal to that direction. We considered two types of lateral boundary conditions: (a) periodic and (b) no flow. The periodic boundary conditions are more general than the no-flow ones but more difficult to implement in the case of two-phase flow, so both types were used to facilitate comparison of single- and two-phase flow simulations. In BCC networks where all bonds are equally inclined with respect to the nominal flow direction, Darcy’s law allows definition of a characteristic flow rate $q_D = \Sigma q_i / n_{tot}$, where the sum is effected over all pipes present in the network and $n_{tot}$ is the total number of bonds (occupied or not). Note that flow rates of individual pipes can be positive or negative (Li et al., 2018), and the sign of $q_i$ must be taken into account in $\Sigma q_i$. However, for the purpose of identifying high-flow preferential paths, we only need the flow rate magnitudes $|q_i|$, which can be conveniently expressed in dimensionless form as $Q_i = |q_i| / q_D$. Using the method described in section S3 of the supporting information, we determined the critical flow rate $Q_c$ and the associated subnetwork $\{Q_i\}$ (i.e., the set of pipes with $Q_i \geq Q_c$). We also calculated the connected cluster $\{Q_i\}^c$ using $Q_i$ instead of $R_i$ as the order parameter of the invasion-percolation algorithm (section S4 of the supporting information). We measured the sizes $S_{Qc}$ and $S_{Qc}^c$ of $\{Q_i\}$ and $\{Q_i\}^c$ as well as a flow rate tortuosity $\tau_{Qc}$ using the same techniques as before except that the bond “length” in Dijkstra’s algorithm was now taken equal to the pipe inverse flow rate $1/Q_i$ instead of $1/G_i$ and the graph was directed such that travel along the path could only be done in the local flow direction. Thus, $\tau_{Qc}$ corresponds to the path with maximum harmonic average of the flow rate (or maximum flow path, MFP). Finally, we defined a partition index $I_{Pr}$ based on the “participation number” of Andrade et al. (1999) and Nissan and Berkowitz (2018), which was originally meant to express how kinetic energy is spatially partitioned in heterogeneous porous media

$$I_{Pr} = \left( n \sum_{i=1}^{n} \frac{Q_i^2}{\left( \sum_{i=1}^{n} Q_i \right)^2} \right)^{-1},$$

where $n$ is the number of pipes (i.e., occupied bonds) in the network. We can see that $I_{Pr} \approx 1$ corresponds to equipartition of flow, while $I_{Pr} < 1$ indicates that most of the flow is concentrated in a small number of pipes. If we assume that $m$ among the $n$ pipes are stagnant and that the $n - m$ remaining ones equally share the flow, we find that $I_{Pr} \approx (n - m)/n$ is a rough estimate of the proportion of flow-carrying pipes.

### 2.3. Two-Phase Flow

We simulated flow of immiscible fluids using the following assumptions: (1) two immiscible, incompressible fluids are present and flowing in the network; (2) not more than one fluid interface can exist in a given pipe; (3) the capillary pressure difference across a fluid interface in a pipe is inversely proportional to the pipe radius; (4) both fluids obey the Poiseuille flow equation; (5) only piston-like displacement occur in the pore space; and (6) as for single-phase flow, fluid pressure drops are exclusively located in the pipes between nodes (e.g., Zhao et al., 2016). Two dimensionless numbers characterize two-phase flow conditions, the viscosity ratio, $M = \mu_1 / \mu_i$, and the capillary number, $Ca$, comparing the magnitudes of the viscous and capillary forces (e.g., Li et al., 2017; Reynolds et al., 2017; Zhao et al., 2016). The capillary number is defined as $Ca = q \mu_i / (\gamma A)$, where $A$ is the network cross-sectional area and $\gamma$ the interfacial tension (here set to 20 mN/m). We performed simulations while varying $Ca$ over 5 orders of magnitude (from $10^{-6}$ to $10^{-2}$) and used an
unfavorable viscosity ratio $M = 100$ (note that the viscosity ratio is sometimes inversely defined, $M = \mu_1/\mu_D$; in that case, our simulations correspond to $M = 10^{-5}$). For $Ca$ lower than $10^{-6}$, immiscible flow was simulated using another method based on invasion-percolation.

We only considered drainage and assumed that the fluids have identical density. Drainage simulations were performed as follows: The network realization is initially filled with a defending wetting fluid of viscosity $\mu_D$. The invading nonwetting fluid (viscosity $\mu_1$) is then injected along the left side of the network at a constant injection rate $q_i$. The capillary pressure $pi$ across a fluid interface in pipe $i$ is given by the Young-Laplace law,

$$pi = -2\gamma \cos\theta / r_i,$$

where $\gamma$ is the interfacial tension and $\theta$ the wetting contact angle ($\theta = 180^\circ$ in this study). When a fluid interface exists in a cylindrical pipe, the volumetric flow rate $q_i$ is calculated using the extended Hagen-Poiseuille equation (Aker & Måloey, 1998)

$$q_i = -\frac{\pi r_i^4}{8\mu_{eff}} (\Delta p_i - pc_i)$$

where $\mu_{eff}$ is the effective viscosity of the fluids coexisting in pipe $i$, which can be estimated as $\mu_{eff} = \mu_1 X_i + \mu_D (1 - X_i)$ (Aker & Måloey, 1998), where $X_i$ is the relative position of the meniscus between the entrance and exit of pipe $i (0 \leq X_i \leq 1)$. When the pipe does not contain a fluid interface, $p_{ci} = 0$ and equation 3 reduces to the standard Hagen-Poiseuille equation with $\mu_{eff} = \mu_D$ or $\mu_1$ depending on the fluid present. If the pressure difference $\Delta p_i$ in equation (5) is less than the capillary pressure $pc_i$, the pipe is blocked and cannot carry any flow. In other words, the pipe hydraulic conductivity becomes effectively equal to 0, which produces a strongly ill conditioned Kirchoff matrix and some serious numerical difficulties. We solved the Kirchoff equations using the Graphic Processing Units (GPU)-accelerated algebraic multigrid generalized minimal residual algorithm (AMG-GMRES) in the AmgX library package (Naumov et al., 2015). The GPU-accelerated algorithm is crucial to allow a large increase of the network size and, consequently, a significant decrease of the statistical fluctuations associated with individual simulations. The AMG solver also includes tools to overcome the ill-conditioned matrix problems.

For a given configuration of the fluid interface in the network, the Kirchoff linear equations derived from equation 3 were repeatedly solved for the nodal pressures $pi$ with the constraint that the pressure difference between the upstream and downstream network faces must be adjusted at each time step $\Delta t$ to maintain the injection rate constant. During this process, the positions of all menisci in the network were monitored. Eventually, the invading fluid reached a new node, thus inducing a change in capillary pressure. The pressure field was recalculated, the positions of the fluid interface updated, and the immiscible displacement process was resumed (Regaieg et al., 2017). We monitored the subnetwork $[NW]$ of invaded pipes during the simulations. The simulation was stopped at breakthrough (i.e., when the invading fluid reached the network exit face). At breakthrough, the subnetwork $[NW]^B$ becomes connected (in the sense discussed in sections 2.1 and 2.2) and can be compared to $[R_c]$ and $[Q_c]$. To validate our computer code, we simulated the waterflooding experiment reported in de Loubens et al. (2018) and compared our result to de Loubens et al.’s X-ray image of the waterflooding pore space at breakthrough as well as their 2-D dynamic network simulation. We found that the simulated injection pattern was in good qualitative agreement with de Loubens et al.’s experimental and simulation results (see supporting information section S5).

### 3. Numerical Results

#### 3.1. The Subnetworks $[R_c]$ and $[Q_c]$

We determined the critical radius $R_c$ and associated subnetwork $[R_c]$ in heterogeneous, partially connected, BCC network realizations containing 250,000 nodes and 990,000 bonds (occupied or not). We considered five coordination numbers ($z = 8.0, 6.4, 4.8, 3.2,$ and $2.4$) and seven pipe radius coefficients of variation ($\sigma/<r_r> = 0.05, 0.10, 0.20, 0.30, 0.55, 0.80,$ and $1.05$). The ensemble averages of $R_c$ over 400 realizations were found to verify equation 1 within $\pm 1\%$. The ensemble-averaged relative size $S_{Rc}$ of $[R_c]$ was $0.180 \pm 0.003$ for all values of $z$ and $\sigma/<r_r>$ as theoretically predicted. The ensemble-averaged size of the connected cluster $[R_c]^*$ relative to $[R_c]$ was also found constant ($S_{Rc}^*/S_{Rc} = 0.16 \pm 0.02$). We used Dijkstra’s algorithm to determine the least resistant paths through $[R_c]$ and found that their tortuosity $\tau_{Rc}$ (relative to the intrinsic BCC tortuosity) had an ensemble mean value of about 3. The statistical fluctuations of individual realizations of $S_{Rc}^*$
were ±20%, very large compared to ±1% for $S_{Rc}$. Such large fluctuations are expected since the critical subnetwork $[R_c]$ is at the onset of disconnection and is therefore approximately fractal.

Because single-phase flow simulations are more demanding numerically than the procedures for identifying $[R_c]$ described above, we saved time by using smaller networks (54,000 nodes and 212,400 bonds) and perform ensemble averaging over fewer realizations (100). We found that the critical flow rate $Q_c$ strongly increased with decreasing connectivity and/or increasing heterogeneity (Figure 2). The statistical fluctuations of $Q_c$ were generally modest except for $(z; z_c) = 0.9$ and $\sigma/<r>$ = 1.05, where the general trend appeared to break down, perhaps indicating the onset of finite size effects often observed near the percolation threshold. Analyzing finite size effects requires changing the network size, which was not possible in the time frame of this study.

Unlike $[R_c]$, the subnetworks $[Q_c]$ were strongly affected by connectivity and pore size heterogeneity. The size $S_{Qc}$ of $[Q_c]$ increased by a full order of magnitude, from about 0.02 to 0.3 with increasing $(z; z_c)$ and decreasing $\sigma/<r>$ (Figure 3a). Measuring the size of $[Q_c]$ relative to the total network size (including unoccupied bonds) is useful for highlighting the great difference with constant size $[R_c]$ but cannot be used in practical applications since unoccupied bonds are not physical objects, and their number cannot be estimated in rocks. We therefore calculated the sizes of $[Q_c]$ and $[R_c]$ relative to the number of pipes actually present in the network, that is, the ratios $S_{Qc}/p$ and $S_{Rc}/p$ (Figure 3b). As expected, $S_{Rc}/p$ increases with decreasing $(z; z_c)$ and should eventually approach 1 near the percolation threshold. The behavior of $S_{Qc}/p$ is more complex but seems to indicate a greater sensitivity to heterogeneity than connectivity. We note again the possibility of rising finite size effects at high $\sigma/<r>$ and low $(z; z_c)$.

The number of connected pipes in $[Q_c]$, $S_{Qc}$, was lower than the approximately constant $S_{Rc}$ in $[R_c]$, except for low values of $\sigma/<r>$, and generally tended to decrease with decreasing $(z; z_c)$ and increasing $\sigma/<r>$ (Figure 4). The statistical fluctuations of individual realizations were larger for $[Q_c]$ than $[R_c]$, which is expected since $S_{Qc}$ is substantially smaller than $S_{Rc}$. Possible finite size effects in low connectivity networks appeared more prominent than before and occurred at lower heterogeneity levels. It is thus clear that $[Q_c]$ is not a critical percolation network like $[R_c]$. This is not surprising since, unlike the purely stochastic set of pipe radii, the flow field $Q_c$ is controlled by deterministic physical laws such as mass conservation.

Examination of Figures 2–4 shows that the largest critical flow rates $Q_c (>10)$ occurred for the smallest subnetworks $[Q_c]$ and connected clusters $[Q_c]^{\dagger}$. Conversely, the lowest $Q_c (\approx 1)$ corresponded to the largest $[Q_c]$ and $[Q_c]^{\dagger}$. The first of these two end-members thus corresponds to a strong concentration of flow in a relatively small number of connected pipes and the second to a more uniform flow field. This interpretation is confirmed by the large decrease of the partition index $I_{pr}$ with increasing $\sigma/<r>$ and decreasing $(z; z_c)$ (Figure 5). We note however that the sensitivity of $I_{pr}$ to $(z; z_c)$ is significantly reduced in high heterogeneity networks.

Finally, we examined the intersection subsets, $[Q_c] \cap [R_c]$ and $[Q_c]^{\dagger} \cap [R_c]^{\dagger}$, and determined their sizes $S_{QR}$ and $S_{QR}^{\dagger}$. The curves of $S_{QR}$ and $S_{QR}^{\dagger}$ versus $(z; z_c)$ (Figures 6a and 6b) appear to be very similar to those of $S_{Qc}$ and $S_{Qc}^{\dagger}$ (Figures 3a and 4, respectively). To quantify the similarity, we computed the ratios $S_{QR}/S_{Qc}$ and $S_{QR}^{\dagger}/S_{Qc}^{\dagger}$ and observed that, although these ratios show significant irregular variations, they tend to depend more strongly and systematically on $\sigma/<r>$ than $(z; z_c)$. Both ratios increased with increasing $\sigma/<r>$ ($S_{QR}/S_{Qc}$ from about 0.7 to 0.9 and $S_{QR}^{\dagger}/S_{Qc}^{\dagger}$ from 0.15 to 0.35 for $\sigma/<r>$ increasing from 0.20 to 1.05). The fact that $S_{QR}/S_{Qc}$ was systematically smaller than $S_{QR}/S_{Qc}$ suggests that the part of $[Q_c]$ least related to $[R_c]$ is the connected subset $[Q_c]^{\dagger}$, which we expect, characterizes the preferential flow paths.

### 3.2. Transition From Capillary to Viscous Fingering: The Subnetwork $[NW]$\(^B\)

Two-phase flow simulations require even more numerical resources than single-phase flow ones. Moreover, large networks are needed to characterize the subnetworks $[NW]$\(^B\) of invaded pipes at breakthrough. We therefore had to limit the ensemble averaging sampling to between 8 and 18 realizations depending on heterogeneity level and connectivity considered. We also restricted the explored parameter space to the values of $\sigma/<r>$ that allows occurrence of preferential flow paths in single-phase flow simulations ($\sigma/<r> \geq 0.3$; Figure 3). We used BCC networks containing $(120 \times 60 \times 60)$ 103,619 nodes and 414,476 bonds. The networks had a $2 \times 1 \times 1$ prismatic shape. The nominal flow direction was set parallel to the long sides.
We generally observed a transition with increasing Ca from capillary to viscous fingering consistent with the observations previously reported in 2-D experiments and numerical simulations (Lenormand et al., 1988; Li et al., 2017; Holtzman & Segre, 2015; Regaieg, 2015). The transition depended on heterogeneity and connectivity as illustrated in Figures 7a and 7b. At high heterogeneity and poor connectivity, we observed displacement patterns similar to those of 2-D simulation, that is, the sweep region gradually decreased and the viscous fingers became more distinct as Ca is decreased (Figure 7b). However, the evolution of the viscous fingers in more homogeneous networks appeared to be nonmonotonic with substantially thinner fingers occurring at Ca = 4.9E–5 (Figure 7a).

Characterizing the morphology of the set of invaded pipes is a difficult task in three-dimensional simulations. One simple measure is $S_{NW}$, the size of $[NW]^B$ normalized to the size of the total network. It is particularly interesting to observe the evolution of $S_{NW}$ as a function of Ca for various values of $\sigma$ and $(z,z_c)$. In general, $S_{NW}$ increased when Ca was reduced from 0.049 to 0.00049, then sharply dropped with a further diminution of Ca to $4.9 \times 10^{-5}$ (Figure 8). This point also corresponded to an inversion of the effect of $\sigma$ on $S_{NW}$. For Ca $> 4.9 \times 10^{-5}$, $S_{NW}$ diminished substantially with increasing heterogeneity, while the reverse occurred when Ca $= 4.9 \times 10^{-5}$ was reached. In addition, we note that the variations of $S_{NW}$ tended to be less sharp when the network coordination number $z$ was decreased (even reversed for Ca $= 4.9 \times 10^{-5}$, $z = 4.8$, and 3.2 in Figures 9c and 9d). Finally, the lowest value of Ca ($10^{-6}$) was simulated using a method based on invasion-percolation, which resulted in a nearly constant $S_{NW} \approx 0.055$ regardless of the values of $\sigma$ and $z$ (horizontal dotted lines in Figure 8). This independence of $S_{NW}$ on $\sigma$ and $z$ is consistent with the strong relationship generally assumed between the invaded set $[NW]^B$ produced by invasion-percolation and the connected part of the CPA critical network $[R_c]$ (Hunt, 2001; Hunt et al., 2014; Lenormand et al., 1988; Sahimi, 2011). Moreover, we note that $S_{NW}$ at high Ca was significantly higher than the invasion-percolation value in well-connected, relatively homogeneous networks (e.g., blue and red curves in Figures 8a and 8b). These networks correspond to those for which we observed $S_{Qc} \geq S_{Rc}$ and $\theta \geq 0.5$ (Figures 3 and 5) or, in other words, for which the flow field was approximately uniform and preferential flow paths did not occur.

To compare two-phase flow and single-phase flow simulations in the same network realizations, we need to examine the intersection of $[NW]^B$ with $[R_c]$. We will hereafter use the notation $[1] \equiv [NW]^B \cap [Q_c]$, $[2] \equiv [NW]^B \cap [R_c] \cap [Q_c]^C$ and $[3] \equiv [NW]^B \cap [R_c]^C \cap [Q_c]^C$, where the superscript C denotes complementary

![Figure 2](image2.png)

**Figure 2.** Ensemble average of the critical flow rate $Q_c$ versus $(z,z_c)$ for different values of $\sigma$ as indicated in the inset. The error bars represent the uncertainty of the ensemble averages.

![Figure 3](image3.png)

**Figure 3.** Ensemble average of the size of $[Q_c]$ versus $(z,z_c)$ for different values of $\sigma$ as indicated in the inset. The size of $[Q_c]$ can be measured (a) as $S_{Qc}$, that is, relative to the entire network, including unoccupied bonds or (b) as $S_{Qc}/p$, that is, relative to the set of occupied bonds. The error bars represent the uncertainty of the ensemble averages. The measured values of $S_{Rc}$ and $S_{Rc}/p$ are also represented for comparison (gray lines).
sets. Thus, the elements of \([1]\) are common to \([\text{NW}]^B\) and \([Q_1]\), while those of \([2]\) are shared by \([\text{NW}]^B\) and \([R_c]\) but not \([Q_1]\), and those of \([3]\) are the elements of \([\text{NW}]^B\) that do not belong to either \([R_c]\) or \([Q_1]\). We calculated the sizes, \(S_1\), \(S_2\), and \(S_3\) of the intersection subnetworks \([1]\), \([2]\), and \([3]\), normalized to \(S_{\text{NW}}\). We observed a relatively regular increase of \(S_1\) with increasing \(Ca\) (Figures 9a–9d). The trend is partially interrupted at \(Ca = 4.9 \times 10^{-5}\) (similar to \(S_{\text{NW}}\), Figure 8) and resumes for the invasion-percolation simulations for \(Ca = 10^{-6}\). For \(Ca > 4.9 \times 10^{-5}\), \(S_1\) tended to increase with decreasing \(z\), an effect gradually reduced with increasing \(\sigma/\langle r \rangle\). The values of \(S_2\) were generally lower than \(S_1\) except for the invasion-percolation cases \((Ca = 10^{-6})\) where \(S_1 \approx 0.85\) was obtained independent of \(z\) and \(\sigma/\langle r \rangle\). For \(Ca > 4.9 \times 10^{-5}\), \(S_2\) clearly increased with decreasing \(z\) and showed an irregular growing tendency with decreasing \(Ca\), while \(\sigma/\langle r \rangle\) did not seem to have a noticeable effect.

The proportion of invaded pipes that belong neither to \([R_c]\) nor \([Q_1]\) as measured by \(S_3\) was relatively large in fully connected networks \((z = 8)\) and decreased substantially with decreasing \(z\) but was essentially unaffected by \(\sigma/\langle r \rangle\) (Figure 10). The effect of \(Ca\) was complex, with \(S_3\) sometimes decreasing and sometimes increasing when \(Ca\) was decreased from \(0.049\) to \(4.9 \times 10^{-5}\) (Figure 10). Most importantly, \(S_3\) vanished completely in the invasion-percolation simulations. In other words, all pipes in \([\text{NW}]\) belonged to \([R_c]\) in invasion-percolation simulations, demonstrating once again the intimate relationship of invasion-percolation and CPA.

### 4. Discussion

The main objective of this paper is to figure out whether or not there is a link between single-phase flow localization on preferential paths and viscous fingering in heterogeneous porous media. Our investigation rested on simulating single-phase and two-phase flow in network realizations with various levels of pore size heterogeneity and pore connectivity, identifying the critical subnetwork \([Q_1]\) and comparing it to the set of invaded pipes at breakthrough \([\text{NW}]^B\). We also determined the critical percolation subnetwork \([R_c]\), whose well-known properties provide a very useful benchmark. One important property of \([R_c]\) is that its morphological characteristics (e.g., normalized size, connected fraction, and tortuosity) are independent of the heterogeneity and connectivity of the networks. Only the critical radius (a “metric” property) is affected by \(\sigma/\langle r \rangle\) and \((z, z_c)\). Our definition of \(R_c\) (and \([R_c]\)) is equivalent to that used in CPA, which posits that throughgoing paths belonging to \([R_c]\) have a higher hydraulic conductance than the other ones and, therefore, carry the major portion of the flow (Charlaix et al., 1987; Friedman & Seaton, 1998; Katz & Thompson, 1986). However, paths contained in the critical set \([R_c]\) have a very high tortuosity, thus substantially increasing their hydraulic resistance (Hunt & Skinner, 2008; G. Yang et al., 1993). Indeed, the LRP s in pipe networks have been shown to be more direct than the critical paths and to pass through pipes with \(R < R_c\) (G. Yang et al., 1993). This was also verified here by direct measurements of the resistance, length, and minimum radius of LRP s through network realizations (see supporting information section S5).

Flow localization on preferential paths can be understood as the concentration of flow on a relatively small number of continuous, throughgoing paths. When flow concentration happens, we expect a partition index \(I_{\text{part}}\) to \(< 1\) and a subnetwork \([Q_1]\) substantially smaller than \([R_c]\). Conversely, a nearly uniform flow field should yield a relatively large \(I_{\text{part}}\) and \(S_{Q_c} > S_{R_c}\). We do not presently know how small \(S_{Q_c}\) and \(I_{\text{part}}\)
need to be to ensure existence of preferential paths. If we did, Figure 3 and/or Figure 5 could be used to locate the transition to flow localization in heterogeneity-connectivity space. Despite this uncertainty, investigating the properties of \( Q_c \) offers the best way, at present, to establish a link between the formation of preferential flow paths and viscous fingering. In contrast to \( R_c \), \( Q_c \) is not a percolation network. Its elements are produced by deterministic, physical laws instead of the stochastic processes underlying \( R_c \). These deterministic physical laws are likely producing long-range correlation in the flow field, thus creating the difference observed between \( R_c \) and \( Q_c \). In addition to the transport law (here, Poiseuille law), the elements of \( Q_c \) must satisfy conservation of mass, continuity of the fluid pressure field, and minimization of energy dissipation. The transport law is strongly dependent on the local “metric” characteristics, namely, the radii of the pipes. Consequently, the inclusion of a particular pipe \( i \) in \( Q_c \) depends on the actual value of \( R_i \). In comparison, inclusion in \( R_c \) is solely determined by the rank of \( R_i \).

Figure 6. Ensemble averaged values of (a) \( S_{QR} \) and (b) \( S_{QR}^o \). The simulated data are plotted against \((z-z_c)\) for various values of \( \sigma/\langle r \rangle \) as indicated in the inset. The error bars represent the uncertainty of the ensemble averages.

Figure 7. Two examples of simulated displacement patterns at breakthrough during drainage for various capillary numbers \( Ca \) and a viscosity ratio \( M = 100 \) in 3-D network realizations: (a) relatively homogeneous network \( (\sigma/<r>=0.55) \) and (b) strongly heterogeneous realization \( (\sigma/<r>=1.05) \). In these examples the time step was reduced in half for higher precision. Note that invasion-percolation simulations are used for \( Ca = 10^{-6} \).
in the radius distribution. It is therefore inevitable that even the “nonmetric” characteristics of \( Qc \) such as the normalized sizes \( S_{Qc}, S_{QR}, S_{QR}^c \) should be affected by \( \sigma/<r> \) and \( (z-zc) \) as clearly demonstrated in Figures 3–6.

Our next question is whether or not flow concentration occurs on the LRPs. One way to answer this question is visual comparison of the LRP and MFP of individual network realizations. Visual examination of individual cases strongly limits the number of realizations that can be analyzed. Yet we found that MFPs and LRPs looked generally similar, especially when compared to the paths predicted by CPA (i.e., LRPs through \( R_c \)).

As an illustration, we show 3-D representations of the LRP, MFP, and CPA path of a network realization corresponding to \( \sigma/<r> = 0.20 \) and \( (z-zc) = 0.9 \) (top row in Figure 11). In these diagrams, the paths are composed of colored segments with the colors indicating the magnitude and sign of the local flow rate. Increasingly large, positive flow (i.e., in the same direction as the overall path flow) is indicated by greenish yellow to red segments, whereas green to blue segments correspond to negative flow. As shown in Figure 11, local flow rates remained positive along the LRP and MFP, while several sharp jumps from positive to negative and vice versa occurred on the CPA path. The fact that MFP only contains positive segments is a strict consequence of our procedures using directed graphs. On the other hand, the absence of negative segments in the LRP is in stark contrast to their presence in the CPA path since both paths were determined using undirected graphs. We also determined the cumulative distributions of flow rates in the paths (see bottom row of Figure 11). The similarity of the (entirely positive) LRP and MFP flow distributions is quite clear, while the CPA path distribution is distinctly longer and has a well-developed negative tail.

One property expected for preferential flow paths is consistency of flow along the paths. Exchange of fluid between a well-formed preferential flow path and its surroundings should remain limited. Based on this
evidence discussed above, this condition appears to be approximately realized in the LPRs and MFPs but not in the CPA paths. Thus, we conclude that preferential flow paths have essentially no relation to CPA paths. On the other hand, both LPRs and MFPs appear to have properties that we expect of preferential flow paths, including low hydraulic resistance, low tortuosity, and generally large pipe radii (however, LPRs and MFPs contain pipes with radii lower than $R_c$). Assuming that LPRs and MFPs can be identified as preferential flow paths, the observations above indicate that converging and diverging branching cannot be assumed to be entirely absent along preferential flow paths even when flow localization is strongest (i.e., at high $\sigma/<r>$ and low $z$).

**Figure 9.** Sizes $S_1$ and $S_2$ of the intersection sets [1] (a–d) and [2] (e–h) as functions of $Ca$ for various values of $\sigma/<r>$ and $z$ as indicated in the insets. These simulations are the same as those of Figure 8.
Figure 10. Size $S_3$ of the intersection sets $\{3\}$ as a function of $Ca$ for various values of $\sigma/<r>$ and $z$ as indicated in the insets. These simulations are the same as those of Figures 8 and 9.

Figure 11. One example of CPA path (left), LRP (center), and MFP (right) (the network realization had $(z_c, z) = 0.9$ and $\sigma/<r> = 0.20$). The 3-D representations of the paths are shown on the top row. The magnitude and sign of local flow rates are indicated by the colored segments, with greenish yellow to red for positive and green to blue for negative flow. The cumulative distributions of flow rates along the paths are shown on the bottom row. The blue, orange, and green lines correspond to the CPA path, LRP, and MFP, respectively. Only the CPA path contains negative segments.
We now consider our next question. Do preferential flow paths coincide with the main fingers of the set invaded pipes \( \{NW\}_B \)? As shown in Figures 9a–9d, \( \{NW\}_B \) and \( \{Qc\}_B \) had a substantial intersection, the set \( \{1\} \), which moreover tended to expand with increasing \( Ca \). In comparison, the number of pipes in \( \{NW\}_B \) belonging to \( \{Rc\}_B \) but not \( \{Qc\}_B \) (i.e., set \( \{2\} \)) was very small and appeared to decrease with increasing \( Ca \) (Figures 9e–9h). Thus, \( \{NW\}_B \) appeared, in general, more closely related to \( \{Qc\}_B \) and the MQPs than to \( \{Rc\}_B \). However, a significant portion of \( \{NW\}_B \) (i.e., set \( \{3\} \)) neither belonged to \( \{Qc\}_B \) nor \( \{Rc\}_B \) (Figure 11). The size of \( \{3\} \), \( S_3 \), tended to decrease strongly with decreasing connectivity and moderately with increasing heterogeneity (Figure 10). These observations, therefore, suggest that the relative importance of set \( \{1\} \) in \( \{NW\}_B \) and consequently the relationship of \( \{NW\}_B \) with \( \{Qc\}_B \) were indeed strongest in conditions favorable to the formation of preferential flow paths. One important implication is that pore-scale heterogeneity should affect immiscible displacement patterns and, in particular, Lenormand’s phase diagrams. Indeed, numerical simulations showed a shift of the viscous fingering region toward favorable viscosity ratios in heterogeneous systems (Liu et al., 2015; Tsuji et al., 2016). To illustrate and confirm the observations mentioned above, we performed a few 2-D two-phase flow simulations, for which it is possible to visually observe a high degree of similarity of the fully formed viscous fingers (i.e., at breakthrough) with the paths of maximum flow (Figure 12).

Finally, one particularly puzzling observation is the nonmonotonic behavior of simulated \( S_{NW} \), showing a slight increase with decreasing \( Ca \) followed by a sharp drop at \( Ca = 4.9 \times 10^{-5} \) and a strong increase for the invasion-percolation simulations at \( Ca > 10^{-6} \) (Figure 8). Similar but more moderate variations of the nonwetting fluid saturation in 2-D numerical simulations have been reported despite differences in the method used (e.g., Lenormand et al., 1988; Regaieg, 2015). These simulations suggest that increasing \( Ca \) causes (a) a decrease in the number of viscous fingers and (b) thickening of the fingers (e.g., Figure 9.2 in Regaieg, 2015). The former arises at relatively high values of \( Ca \), while the latter picks up at very low \( Ca \). The final morphology at very low \( Ca \) (i.e., few, very thick and contorted fingers) is consistent with capillary fingering and invasion-percolation. However, the reason that thickening of viscous fingers and decrease of their number do not occur simultaneously is unclear. It is also well known that the accuracy of dynamic two-phase flow simulations tends to decay at very low capillary numbers (invasion-percolation algorithms are, therefore, often used at very low \( Ca \)). To increase accuracy, we ran additional simulations (\( \sigma/\langle \tau \rangle = 0.55 \) and \( z = 4.8 \)) using half the time increment of the simulations shown in Figure 9b. The set size \( S_{NW} \) dropped about 60% when \( Ca \) was decreased from 0.00049 to \( 4.9 \times 10^{-5} \), while it fell by 80% in the original simulations (almost no differences in \( S_{NW} \) were observed for \( Ca \geq 0.00049 \)). Thus, significant decrease of...
$S_{NH}$ should still be expected at low Ca in homogeneous and moderately heterogeneous networks. In such cases (i.e., relatively homogeneous formations), high flow rate flooding should be a better production strategy than low flow rate injection. The contrary should be true in highly heterogeneous rocks. Finally, a word of caution is warranted. Remember that these observations only hold for drainage at unfavorable viscosity ratios. Imbibition and drainage at favorable viscosity ratios produce quite different injection patterns and finger morphologies.

5. Conclusions

According to our single-phase and two-phase flow network simulation results using various levels of pore-scale heterogeneity and pore connectivity, we found that there is a link between single-phase flow localization on preferential paths and viscous fingering in heterogeneous porous media. Our conclusion derives from identifying the critical subnetwork ($Q_p$) and comparing it to the set of invaded pipes at breakthrough ($NW$). Obviously, single-phase flow localization is controlled by purely viscous forces, while two-phase displacements depend on the competition between viscous and capillary forces. When the viscous forces completely overpower the capillary forces in two-phase flow, it is reasonable to expect a strong similarity between single-phase flow localization and viscous fingering, as confirmed in our investigation by the substantial intersection between ($NW$) and ($Q_p$) (i.e., the set [1]) and its tendency to expand with increasing Ca. Our results suggest that limited amounts of fluid should be exchanged between preferential flow paths and their surroundings, thus supporting the idea that simplifying the reservoir-scale grid model to sets of stream-tubes is a practicable approach when considering relatively high rock heterogeneity, unfavorable viscosity ratios, and high flow rates or pressure differences.

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