Azimuthal distributions in high-energy processes give information about the helicity structure of diffractive reactions. I discuss predictions of several dynamical mechanisms in this context, both for electron-proton and for hadron-hadron collisions.

1 Azimuthal distributions in \( ep \) diffraction

In this talk I give several examples of what azimuthal distributions can tell us about diffractive interactions. As a first example let us look at inclusive diffraction in deep inelastic scattering, \( ep \rightarrow eXp \), and consider this process in the \( \gamma^*p \) c.m. We are interested in the angle \( \phi \) between the plane spanned by the hadron momenta \( p_X \) and \( p \) and the one spanned by the lepton momenta \( l \) and \( l' \), as shown in Fig. 1.

\[\phi\] is an angle around the momentum of the virtual photon, and given the general relation between rotations and angular momentum, it is not surprising that the dependence of the \( ep \) cross section on \( \phi \) contains information on the helicity of the exchanged photon. Indeed one can write

\[
\frac{d\sigma(ep \rightarrow eXp)}{d\phi dQ^2 dy} \propto \sigma_{++} + \epsilon \sigma_{00} - 2\sqrt{\epsilon(1+\epsilon)} \cos\phi \Re \sigma_{+0} - \epsilon \cos(2\phi) \sigma_{+-},
\]

(1)

where \( Q^2 = -q^2 \) is the photon virtuality, \( y = (qp)/(lp) \) the usual inelasticity parameter, and \( \epsilon = (1-y)/(1+y+\frac{1}{2}y^2) \) the ratio of longitudinal and transverse photon flux. The \( \phi \) dependence...
of the $ep$ cross section is determined by the cross sections or interference terms $\sigma_{ij}$ for the hadronic subprocess $\gamma^*p \to Xp$ with specified photon helicities,

$$\sigma_{ij} \propto \sum_X \sum_{\text{spins}} A_i^* A_j,$$

where $A_i$ is the amplitude for $\gamma^*p \to Xp$ with photon helicity $i$, and the sums run over final states $X$ and the spins of initial and final proton. The $\sigma_{ij}$ only depend on the kinematics of the $\gamma^*p$ subprocess but not on $\phi$. Both sides of (1) can be made differential in further variables describing the hadronic final state, such as the invariant momentum transfer $t = (p - p')^2$ to the proton or the invariant mass $M_X$ of the inclusive system $X$.

From the Schwarz inequality $|\sigma_{ij}|^2 \leq \sigma_{ii} \sigma_{jj}$ it is clear that the interference terms in (1) are bounded by the transverse and longitudinal cross sections $\sigma_{++} = \sigma_T$ and $\sigma_{00} = \sigma_L$. More stringent bounds are obtained by using that $\sigma_{ij}$ is a positive semidefinite $3 \times 3$ matrix, since for arbitrary coefficients $c_i$ the linear superposition $\sum_{ij} c_i^* \sigma_{ij} c_j$ is a cross section and hence cannot be negative. The positivity condition can be written as

$$\varepsilon \sigma_L \leq |\sigma_+|,$$

$$\varepsilon \sigma_L \leq \frac{1}{2}(\sigma_+ - |\sigma_+|) + \frac{1}{2}\sqrt{(\sigma_+ - |\sigma_+|)^2 - 8\varepsilon (\Re \sigma_{00})^2},$$

$$\varepsilon \sigma_L \geq \frac{1}{2}(\sigma_+ - |\sigma_+|) - \frac{1}{2}\sqrt{(\sigma_+ - |\sigma_+|)^2 - 8\varepsilon (\Re \sigma_{00})^2},$$

where $\sigma_+ = \sigma_T + \varepsilon \sigma_L$. The right-hand sides of these inequalities can directly be extracted from the $\phi$ dependence of the $ep$ cross section and provide upper or lower bounds on the longitudinal cross section $\sigma_L$, whose direct extraction from $\sigma_\varepsilon$ requires a Rosenbluth separation and thus measurements at different $ep$ collision energies. The interference terms must be large in size for these bounds to be useful. If they are however small, one cannot draw strong conclusions: two amplitudes may both be large but not interfere because of their relative phase, or there can be cancellations between positive and negative interference in the sum over final states.

The $\phi$ dependence of the diffractive cross section has been measured by ZEUS, and no $\cos \phi$ or $\cos(2\phi)$ modulation was found within experimental errors. What does theory predict for the $\phi$ dependence? For inclusive diffraction there is a factorization theorem-valid in the Bjorken limit of large $Q^2$ at fixed $\beta = Q^2/(M_X^2 + Q^2)$, $x_B = Q^2/(2pq)$ and $t$. In this limit the $ep$ cross section can be calculated as a convolution of diffractive parton densities in the proton with the cross section for electron scattering on the corresponding parton, see Fig. 2a. Essential in our context is that the parton-level cross section (for $eg \to eq\bar{q}$ in the example of the figure) is evaluated with the transverse momentum of the incoming parton approximated by zero in the $\gamma^*p$ c.m. As a consequence, the parton-level process receives no information on the outgoing proton momentum, and the resulting cross section cannot depend on $\phi$, for whose definition this momentum is essential. In the Bjorken limit, the $\phi$ dependence of the $ep$ cross section is thus indeed predicted to be flat. On the other hand, both the longitudinal and the transverse diffractive structure functions $F_{L}^{P}$ and $F_{T}^{P}$ (related to the $\gamma^*p$ cross sections by a kinematical factor) are nonzero and become $Q^2$ independent in that limit, up to logarithmic corrections.

A more complicated picture is obtained when one considers diffractive production of a $q\bar{q}$ pair by two-gluon exchange, shown in Fig. 2b. Corresponding calculations provide a good description of inclusive diffraction for $\beta \gtrsim 0.5$, whereas for small $\beta$ diffractive final states with additional gluons become important. (We note that such calculations are sensitive to infrared physics when the produced quark has small transverse momentum, but shall not dwell on this point here.) The two-gluon exchange mechanism gives a longitudinal structure function $F_{L}^{P}$ which at given $\beta$ falls like $1/Q^2$ but contrary to $F_{T}^{P}$ does not vanish for $\beta \to 1$. At large $\beta$, this twist-four contribution to the $ep$ cross section is hence potentially dangerous for analyses based
on the twist-two factorization theorem discussed above, and information on the importance of $F^D_L$ in given kinematics is highly important. Such information may be provided by the $\phi$ distribution: calculations of two-gluon exchange predict an interference structure function $F^D_{+0}$ which is suppressed by $\sqrt{-t}/Q$ compared with $F^D_T$ but remains finite for $\beta \to 1$. The corresponding $\cos \phi$ modulation of the cross section at large $\beta$ (i.e. for $Q^2 \gg M_X^2$) is not seen in the ZEUS data, where the bin with the highest $\beta$ is centered around 0.73. We remark however that in $ep \to e\rho p$ at large $Q^2$, where the inclusive hadron system $X$ is replaced by a single $\rho$ meson, a significant $\cos \phi$ modulation has indeed been measured and is well described by calculations based on the two-gluon exchange mechanism.

Let us now turn to the case where the diffractive final state contains a pair of high-$p_T$ jets. The jet momenta in the $\gamma^* p$ c.m. define a hadron plane different from the one in Fig. 1. The dependence of the cross section on the azimuth $\phi_{jj}$ between this new plane and the lepton plane is described by an expression of the form (1), with appropriate $\gamma^* p$ cross sections and interference terms. We can distinguish two types of final states:

1. **inclusive** dijet production, $ep \to ep + jet + jet + X'$, with a inclusive system $X'$ of hadrons in the direction of the initial proton. This can be described in the same diffractive factorization formalism as the inclusive cross section (see Fig. 2a). Independent of the diffractive quark and gluon densities, this mechanism gives a negative interference term

   \[ \sigma_{+\cdot}^{jj} = -\frac{1}{2} \sigma^{jj}_L \]

   and thus predicts a $\cos(2\phi_{jj})$ modulation such that the dijets are preferentially produced in the lepton plane. In kinematics where diffractive factorization is valid, this modulation allows one to extract the longitudinal cross section without Rosenbluth separation, and thus provides extra constraints on the diffractive parton densities.

2. **exclusive** dijet production, $ep \to ep + jet + jet$. Such events are expected to become important for $Q^2 \gg M_X^2$ but have not yet been isolated experimentally. The two-gluon exchange mechanism of Fig. 2b gives a positive interference term

   \[ \sigma_{+\cdot}^{jj} = \frac{2r}{1-2r} \sigma^{jj}_T, \quad r = \frac{p_T^2}{M_X^2} \]

   where $p_T$ is the transverse jet momentum in the $\gamma^* p$ c.m. This mechanism preferentially produces jets perpendicular to the electron plane.

To distinguish the two types of final states at hadron level is not trivial, especially if the system $X'$ is not very energetic. The different $\phi_{jj}$ distribution of the two production mechanisms can provide a clear distinction and thus help to establish which dynamical description is adequate in given kinematics. We note that both mechanisms also predict a $\cos \phi_{jj}$ modulation of the cross section.
2 Diffraction in \( pp \) collisions

Diffraction in \( pp \) collisions is more complex than in \( ep \) collisions, even in the presence of a hard scale. There are no factorization theorems like the ones we encountered in the previous section, because of soft interactions between partons in the two colliding hadrons. To describe the dynamics, one presently has to rely on assumptions or models, and we will see that azimuthal distributions can be valuable to test and develop these.

Consider diffractive production of a particle or system of particles \( X \) in the \( pp \) c.m. The azimuthal angle \( \phi \) between the plane spanned by \( p_1 \) and \( p'_1 \) and the one spanned by \( p_2 \) and \( p'_2 \) (with momenta as shown in Fig. 3a) contains information on the helicity transferred in the \( t_1 \) and \( t_2 \) channels. As a simple ansatz one may assume a factorized form of the cross section,

\[
\sigma(pp \rightarrow pXp) = \sum_{i_1 j_1 i_2 j_2} \rho_{j_1 i_1} (p_1 \rightarrow p'_1 I_{P_1}) \rho_{j_2 i_2} (p_2 \rightarrow p'_2 I_{P_2}) \sigma_{i_1 j_1 i_2 j_2} (I_{P_1} I_{P_2} \rightarrow X), \tag{6}
\]

where \( j_1 \) (\( j_2 \)) and \( i_1 \) (\( i_2 \)) respectively denote the helicities transferred in the \( t_1 \) (\( t_2 \)) channels in the amplitude and its complex conjugate. The physical picture behind this is that each of the colliding protons “emits” a pomeron, and that the two pomerons fuse to produce \( X \), as our symbolic notation in (6) suggests. \( \rho_{j_1 i_1} \) and \( \rho_{j_2 i_2} \) play the roles of spin-density matrices of the pomerons, whose fusion into \( X \) is described by cross sections and interference terms \( \sigma_{i_1 j_1 i_2 j_2} \). If the pomeron behaves like a spin 1 exchange, the helicity indices are restricted to values 0 and \( \pm 1 \). This is for instance the case in the Donnachie-Landshoff pomeron model.\(^{12}\)

Close et al. performed a general analysis for the production of a resonance \( X \) with quantum numbers \( J^P \) under these assumptions and found a \( \phi \) dependence:\(^{13,14}\)

\[
\begin{align*}
\sigma(0^-) & \propto t_1 t_2 \sin^2 \phi |A_{++}|^2, \\
\sigma(0^+) & \propto t_1 t_2 |A_{00} + \ldots A_{++}|^2, \\
\sigma(1^+) & \propto t_1 t_2 \sin^2 \phi |A_{++}|^2 + |\ldots A_{00} + \ldots A_{0+}|^2, 
\end{align*}
\tag{7}
\]

where \( A_{i_1 i_2} \) is the amplitude for two-pomeron fusion into the resonance, and the dots denote coefficients depending on \( \phi \). Results for spins \( J = 2 \) and 3 were also obtained. These results agree with a general analysis in Reggeon field theory\(^{15}\) but are more restrictive due to the specific assumptions on the nature of pomeron exchange. The ratio \( A_{00}/A_{++} \) of longitudinal and transverse amplitudes depends on details of how the pomerons couple to the produced resonance, and the \( \phi \) dependence in \( 0^+ \) and \( 2^+ \) production has been proposed to discriminate glueball from quark-antiquark bound states.\(^{13,14}\)

\(^{b}\)There are subtle issues concerning the difference between \( \phi \) and the angles between the \( p_1 - p'_1 \) and \( p_2 - p'_2 \) planes in the rest frame of \( X \) or in the c.m. of \( X \) and \( p'_2 \). This difference is negligible if the invariant momentum transfers \( t_1 \) and \( t_2 \) are much smaller than the squared invariant mass \( M_X^2 \) of \( X \), which we assume here.\(^{11,13}\)
If the pomeron behaves like a spin 1 exchange, an important question is whether the vector current describing its coupling to particles is conserved or not. If the pomeron couples like a conserved current, one finds \( A_{0i2} \sim \sqrt{-t_1} \) at small \( t_1 \), whereas a behavior \( A_{0i2} \sim 1/\sqrt{-t_1} \) is obtained for a non-conserved current (which is for instance realized in the Donnachie-Landshoff model). Measurements of \( f_1 \) production disfavor a conserved current, whereas the assumption of a non-conserved current can accommodate data for various \( f_0, f_1, f_2, \eta_2 \) resonances.

In QCD, pomeron exchange becomes the exchange of a pair of interacting gluons. A simple graph for the process \( pp \rightarrow pXp \) is shown in Fig. 3, where the blobs at the top and bottom describe interactions between the gluons and their coupling to the proton. \( X \) is now produced by the fusion of two gluons instead of two pomerons, whereas another gluon is directly exchanged between the colliding protons. Hence, factorization as in (6) does not hold and one has instead

\[
\sigma(pp \rightarrow pXp) = \int d^2q_T d^2q_T^* \sum_{i_1,j_1,i_2,j_2} \rho_{j_1,i_1,j_2,i_2}(p_1p_2 \rightarrow p_1'p_2'g_1g_2) \sigma_{i_1,j_1,i_2,j_2}(g_1g_2 \rightarrow X). \tag{8}
\]

Here \( q \) and \( q^* \) are loop momenta appearing in the amplitude and its complex conjugate, respectively, and the subscript \( T \) denotes their transverse components in the \( pp \) c.m. The two gluons producing \( X \) couple with a conserved current, due to their nature as gauge particles, but they do not carry the full momentum \( \Delta_1(\Delta_2) \) exchanged in the \( t_1(t_2) \) channel. In particular, the typical values of their virtualities in the loop integrals (8) remain finite if \( t_1 \) or \( t_2 \) goes to zero. The mechanism shown in Fig. 3b is thus not in contradiction with the findings from meson production discussed in the previous paragraph.

In diffractive production of meson resonances there is no hard scale, and the graph shown in Fig. 3b is to be interpreted in the sense of a non-perturbative model, such as for instance the one by Landshoff and Nachtmann. If however \( X \) is a Higgs boson or another heavy particle, the hard scale \( M_X \) allows at least part of the dynamics to be described in perturbation theory. This has been elaborated by the Durham group and was recently reviewed by Forshaw.

The blobs in Fig. 3b then represent the generalized gluon distribution \( f_g \) of the proton, and the corresponding scattering amplitude has the form:

\[
A(J^P) \propto \int d^2q_T \frac{V(J^P)}{q_T^2(q_T - \Delta_{1T})^2(q_T + \Delta_{2T})^2} f_g(q_T, -\Delta_{1T}) f_g(q_T, \Delta_{2T}), \tag{9}
\]

where we have omitted the dependence of \( f_g \) on the longitudinal gluon momenta. Infrared convergence of the integral is ensured by Sudakov form factors included in \( f_g \), which have significant effects for large \( M_X \), but the integral does have some sensitivity to the infrared region. The vertex factor \( V(J^P) \) for two gluons coupling to a Higgs depends on its parity,

\[
V(0^+) = (q_T - \Delta_{1T}) \cdot (q_T + \Delta_{2T}), \quad V(0^-) = [(q_T - \Delta_{1T}) \times (q_T + \Delta_{2T})]_z. \tag{10}
\]

If the transverse momenta \( \Delta_{1T} \) and \( \Delta_{2T} \) of the scattered protons are small enough, they can be neglected compared with \( q_T \) and one approximately obtains

\[
A(0^+) \propto \int \frac{dq_T^2}{q_T^2} f_g(q_T, -\Delta_{1T}) f_g(q_T, \Delta_{2T}),
\]

\[
A(0^-) \propto [\Delta_{1T} \times \Delta_{2T}]_z \int \frac{dq_T^2}{q_T^2} f_g(q_T, -\Delta_{1T}) f_g(q_T, \Delta_{2T}) \tag{11}
\]

from (9). Up to small modulations, the cross section at small \( t_1 \) and \( t_2 \) is hence flat for a scalar Higgs and behaves like \((\Delta_{1T} \times \Delta_{2T})^2 \approx t_1t_2 \sin^2 \phi \) for a pseudoscalar one.

A significant modulation of the \( \phi \) dependence compared with (11) can originate from rescattering of partons in the colliding protons. Such interactions are known to have an important
effect on the overall size of the cross section. The dependence of the cross section on $\phi$ (as well as on $t_1$ and $t_2$) can be used to test specific rescattering models, where a crucial aspect tested is how much transverse momentum compared with $\Delta_{1T}$ and $\Delta_{2T}$ is transferred by rescattering. A model study by the Durham group concluded that even when taking these effects into account, the $\phi$ dependence still provides a means to discriminate between different quantum number assignments for newly discovered particles.\(^{15}\)

3 Summary

We have presented several cases where azimuthal correlations in the final state can yield valuable insight into diffractive dynamics. In ep collisions, the cross section dependence on a suitably defined angle reflects the helicity of the exchanged virtual photon. Depending on how significant this dependence is for particular final states and kinematics, it can provide useful bounds on the cross section for longitudinal photons, without the need to measure at different ep collision energies. It may give an indication for the importance of higher-twist contributions to the longitudinal diffractive structure function. In diffractive dijet production, the sign of a $\cos(2\phi_{jj})$ modulation can distinguish between exclusive and inclusive production mechanisms.

Azimuthal correlations between the outgoing protons in exclusive diffractive pp collisions may provide a valuable tool to determine the parity of new particles such as the Higgs. Analysis of data on diffractive production of meson resonances under the assumption of factorization as in (6) indicates that the predominant helicities transferred by diffractive exchange are 0 and $\pm 1$, and that the current describing how the exchange couples to particles is not conserved. A simple mechanism for $pp \rightarrow pXp$ in a microscopic description is two-gluon exchange, where one gluon does not participate in the production of the particle $X$. Additional rescattering between the colliding systems influences the azimuthal distribution of the final-state protons. This provides a handle to validate assumptions and models for these predominantly soft interactions, which play a major role in diffractive hadron-hadron scattering.

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