The linear network coding optimization in hybrid ARQ/FEC system to counteract head-of-line blocking in queue

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Abstract. The problems of systematic linear network coding optimization and generic matrix generation methods are considered in this article. The most efficient generation method is a random matrix filling with prime numbers. A model of data transfer system was developed that allowed to assess the telecommunication system packet loss rate using various methods. The first method involves a packet loss rate calculation using the binomial distribution law. In order to find the optimal length of the information and coding sequences, the enumeration of the possibilities based on the calculation results is made. The optimality criteria are maximum code rate and minimum coding sequence length upon the condition that packet loss rate is zero. This method provides high accuracy and allows to quickly obtain a large number of optimal lengths of the information and coding sequences. However, the method cannot be modified to be used for the packet correlated loss calculation in case of the network overload and data transfer pending via wireless connection. The second method relies on the simulation models which allow to find the optimal lengths of the information and coding sequences using multiple test data transmission in the simulation environment with different characteristics (modulation methods, coding, channel models).

1. Introduction

Due to the telecom network development, the amount of transferred data has also been increasing. As a result, subscribers expect the multicasting and packet assembly via transport-level protocols to take less time. However, the existing protocols that are able to provide good packet delivery ratio use Acknowledgement-Based (ARQ) protocols to acknowledge the received packets. It decreases the packet delivery ratio on the wireless networks, because of the high percentage of lost packets, increased time for data retransmission and increased number of attempts of data retransmissions. Any lost packet also results in blocking the queue where packets are pending for being assembled and sent to the data receiver-service from the sender. It leads to the data exchange degradation and poses a security risk due to the extended size of the data storage buffer.

The overhead costs related to the adding of redundant packets and increased time for data units encoding and decoding impose strict requirements for the codes allowed for fixing lost packets. The linear network coding optimizes the data transfer, restoring the lost packets without any additional data requests [1] when using such lengths of the information and coding sequences that do not significantly
impact the packets loss. The models for calculation of the set of lengths depending on the packet network error rate have been developed and will be considered further.

2. The model of data encoding and decoding using linear network coding

The systematic method of encoding of the linear network coding is based on the code $C$ (figure 1) generic matrix, where every line is to be linearly independent from other lines. Any packet $p_i$ in numerical form is multiplied by the matrix (see figure 2) coefficient. As a result, the packets with check symbols are generated $sum_1, sum_2, ..., sum_{n-k}$. Simultaneously with the encoding, the data packets are sent to the network. After the check symbols generation is over the redundant packets (fig. 3) are also sent. If some packets have been lost in the transmission, to restore one or more packets the receiver shall solve the linear algebraic equation system.

\[
C = \begin{pmatrix}
  k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\
  k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\
  k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\
  k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\
  k_{51} & k_{52} & k_{53} & k_{54} & k_{55}
\end{pmatrix}
\]

**Figure 1.** Generic matrix for linear network coding (10,5).

\[
\begin{pmatrix}
  k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\
  k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\
  k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\
  k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\
  k_{51} & k_{52} & k_{53} & k_{54} & k_{55}
\end{pmatrix}
\begin{pmatrix}
  p_1 \\
  p_2 \\
  p_3 \\
  p_4 \\
  p_5
\end{pmatrix}
= 
\begin{pmatrix}
  sum_1 \\
  sum_2 \\
  sum_3 \\
  sum_4 \\
  sum_5
\end{pmatrix}
\]

**Figure 2.** System of linear equations matrix form for coding (10,5).

For solving the system of linear equations the Gaussian elimination method is used because it works well in practice. The reason why the algorithm was selected is the cyclicity of its steps leading to the simple software implementation of the algorithm [2]. In addition, the method can be applied for solving the systems where a dimensionality doesn’t correspond to a number of equations. Since the Gaussian elimination method uses the floating-point arithmetic and due to its common rounding error, the unknown variables are calculated with an error [3], therefore it results in decoding errors. It should be noted that in further network coding development and testing, the errors have occurred only for error correcting coding capability of more than 5 and have increased exponentially with increase of the error correcting capability.

The Gaussian elimination method for real numbers field is the simplest one for application. However, the software implementation involves temporary and final results storage limitations. For this purpose, a double variable type allowing to store up to 64 bytes of data (52 bytes is reserved for floating-point coefficient) shall be used [4]. The closest integer type is int32 from which the source byte groups are being converted to the double type and encoded. After that the obtained check symbols $sum_1, sum_2, ..., sum_{n-k}$ assembled into packets will be sent via network (see figure 4). Thus, two packets with check bytes contain only one check symbol $sum_i$. 


The length of coding sequence $N$ of the linear network coding depends on the error correcting capability $t$ and the quantity of data packets $K$ as follows:

$$N = K + 2 \cdot t$$ (1)

Maximum integer value $\text{sum}_i$ is $2^{52}$. If we put the case that all 4 bytes from data packets while the conversion into int32 type attained a maximum value in $2^{31}$ then number of the redundant packets will not exceed $2^{21}$.

$Figure 3$. Systematic method of data encoding and decoding using the linear network coding.

$Figure 4$. The structure of the coding sequence sent via network.
The Vandermonde matrix is the most commonly used as a generic matrix for linear network coding, because the matrix determinant of any submatrix will never be equal to zero [5]. However, the coefficients of the generic matrix for real numbers field and matrix dimension 9×9 will exceed the maximum permitted value in $2^{11}$. Consequently, it will exceed the integer value of double type boundaries.

The conducted research on matrix generation using deterministic number sequence displayed unsatisfactory results because of the probability of occurrence of the linearly dependent vectors in the matrix with dimensions 3×3 and higher was not equal to zero. To fill in the generic matrix, the following method was used. All possible submatrix variants with dimensions 3×3, 4×4 and 5×5 will be obtained from the parent matrix and will include a set of linearly independent vectors. The method consists in using pairs of not equal prime numbers for random filling of the generic matrix of linear network coding.

3. Parametric optimization of the linear network coding using the binomial distribution law for packet loss

In paper [6] a formula (2) was obtained to assess the probability of receiving a code block that cannot be decoded:

$$P_b = 1 - \sum_{i=0}^{t} C_{N-i}^N \left(1 - P_c\right)^{N-i} P_c^i$$  (2)

where $P_b$ – probability of receiving the code block that cannot be decoded, $t$ – error correcting coding capability, $C_{N-i}^{N}$ – binomial coefficient from $N$ to $N-i$, $P_c$ – packet network error rate, $N$ – coding sequence length.

When a code block cannot be decoded it leads to loss of packets received with an error or not delivered to the receiver. The user data packet $P_t$ loss rate can be assessed using the following formula (3):

$$P_t = P_b \cdot P_c \cdot R$$  (3)

where $R = \frac{N - 2 \cdot t}{N}$ is a linear network coding rate.

With the increase of the error-correcting coding capability $t \to \infty$, the probability of receiving the code block that cannot be decoded tends to 0 in accordance with the distribution functions definition [7]. Therefore, the transfer of a big amount of data in the long time interval without repeat requests is impossible. Although in order to minimize the impact of queue’s head-of-line blocking it is sufficient to reduce the number of lost packets $B \cdot t_t$ times, where $B$ – data transfer rate (pack/s), $t_t$ – time of data exchange (s). For real-time data exchange this value usually is no more than $10^9$ that corresponds to the data volume of about 1 TB. In case of continuous traffic generation with an average speed of 100 Mb/s, the amount of transmitted information will be about 1 TB per day. That completely covers the average value of traffic consumed per month by the residential Internet as well as the mobile Internet (LTE-networks) [8].

The packet loss rate $P_t$ can be assessed with a certain degree of accuracy. The value accuracy is limited only by the number of decimal places of type double. Thorough the accuracy of calculation using the formula (2) without converting to the biginteger type will be low [9]. For this reason, in application of the calculation model based on (2) and (3) formulas the packet network error rate value was saved as a new biginteger type variable, parameters $P_{and}$ were multiplied by $L = 10^{10}$. That allowed to move from decimal notation to using integer type instead. Numbers exponentiation and counting the number of combinations were also performed using biginteger type. Although when the rate $P_{and}$ were raised to a
power and multiplied by other components from formula (2), arithmetic operations with multiplier $10^{10}$ were performed, so later it was possible to divide one number with biginteger type by another number (with decimals) of the same type. The accuracy of the packet loss rate calculation for the developed software model using formulas (2) and (3) was $10^{-9}$. The accuracy increase greatly reduces the model performance, therefore, it is not advisable to increase the value of multiplier $L$. The result of model application is presented in figure 5.

**Figure 5.** The lost packets second request rate vs length of coding sequence and error correcting capability of linear network coding curve.

The search for the optimal lengths of the information and coding sequences involves the calculation of the packet loss rate $P_{n}$ and the code rate $R$ for the specified rate $P_{\text{with}}$. If the condition $P_{\text{with}} < 10^{-9}$ is true, then for selected pair $(N, K)$ code rate $R$ is calculated and saved as $R_{\text{max}}$ variable. If the condition is true again, $R$ and $R_{\text{max}}$ shall be compared with each other but the value of $(N, K)$ shall be saved only for the pair with the maximum code rate. However, in order to decrease the time of data assembling, it is necessary to add another range of values $R$ that slightly differs from $R_{\text{max}}$. For this a new variable $R_{c}$ is added to the search for the optimal lengths algorithm. It allows to select the pair $(N, K)$ that will provide the minimum time for assembling of data depending, in its turn, on the length of the coding sequence $N$. Thus, for the packet error rate $P_{\text{and}}=0,01$ $R_{\text{max}} = 0,76$ the optimal values $(N, K)$ will be (99,75) for $R_{c} = 0$ and (76,54) for $R_{c} = 0,05$.

Unfortunately, the packet correlated loss calculation cannot be applied to the developed model. This limits the areas of its future application. However, it has a significant advantage. It allows to quickly obtain a large number of optimal values $(N, K)$ for specified rate $P_{\text{and}}$.

For further optimization of the linear network coding, simulation models of the data transfer system have been developed. The errors in packets were made by the binary symmetric channel (BSC) and additive white Gaussian noise (AWGN) models. For AWGN channel models, the convolutional code and QPSK modulation were used. The calculation of the optimal lengths of the information and coding sequences is similar to their calculation in the calculation model except the fact that the calculation of the packet loss rate is relying on the simulation model results. Thus, for the packet error rate $P_{\text{and}}=0,01$ $R_{\text{max}} = 0,76$ the optimal values $(N, K)$ will be (96,88) for $R_{c} = 0$ and (61,53) for $R_{c} = 0,05$. The obtained results deviate from the calculation model, because the calculation model has a higher accuracy.
of calculation for \((N, K)\) pairs than the simulation one. The size of transmitted data for the simulation model was about 100,000 packets.

4. Conclusion

The usage of the linear network coding in the hybrid method for control of messages delivery in order to counteract head-of-line blocking of the queue has a number of advantages:

- High performance of encoding and decoding operations.
- Existence of an effective generating method for the generic matrices coding based on the randomized matrix filling with prime numbers and ensuring that there will not be any linearly dependent vectors when the data will be later decoded on the side of the receiver. It allows to minimize the second sending of the packets.

Further steps within the linear network coding optimization includes looking for a packet processing algorithm to reduce the redundancy from \(2t\) to \(t\) packets; applying the integer methods for solving the systems of linear equations. The external optimization of the linear network coding involves the development of simulation modelling and the construction of models with correlated packet loss based on the queueing theory.

When two different research approaches are used to investigate how the length of the information and coding sequences affects the recipient's packet loss rate, it is recommended to select the analytical approach, since it allows to obtain highly accurate results. At the same time, the approach allows to quickly obtain a large number of optimal values lengths of \((N, K)\) sequences.

The simulation modelling provides similar results, but the calculation process takes more time and it does not allow for a quick application of the obtained optimal parameters in a real data transfer system. The simulation modelling can be used in order to check the validity of the calculation model in real conditions and to find the optimal set of coding parameters for various types of packet loss including correlated losses in case of the network overload and data transfer pending via wireless connection.

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References

[1] Karpukhin E O and Britvin N V 2017 Development of the method for hybrid control of messages delivery on the basis of linear network code Electrosvyz 10 30-6
[2] Verzhbitsky V M 2005 Numerical methods. The linear algebra and non-linear equations (Moscow, Russia: Oniks 21 Vek Publishing House) pp 56-9
[3] Panyukov A V and Germanenko M I 2009 Exact solving of a linear equations set Vesnik YuUrGU 12(10) 33-40
[4] Prasolov V V 1996 Problems and theorems in linear algebra (Moscow, Russia: Nauka-Fizmatlit Publishing House)
[5] Karpukhin E U and Meshavkin K V 2017 Research of network characteristics influence on packet recovery mode choice The computer and information technology bulletin 12 39-46
[6] Shiryaev A N 1957 Probability (Moscow, Russia: MSU) p 581
[7] Fedoseev A 2019 LTE access Site ComNews Retrieved from https://www.comnews.ru/node/95879
[8] Lobov D V, Lepko A E and Lugovskaya L A 2016 Accuracy of mathematical calculations of classical programming languages The international journal of applied and fundamental research 7(2) 175-80