Learning by Fixing: Solving Math Word Problems with Weak Supervision

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**Problem:** A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?
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Answer: 275
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**Expression Tree (Annotated):**

Train: only go through the neural module, optimize expression accuracy
Problem: A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?

Answer: 275

Test: go through the neural module and symbolic module, evaluate the answer accuracy
**Problem:** A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?

**Expression Tree:**

```
+  
/   100
/    /
*   / 3.5
/    /
/   100
/    /
/   2
```

**Answer:**

275

Train: only go through the neural module, optimize expression accuracy

Test: go through the neural module and symbolic module, evaluate the answer accuracy
Multiple Solutions for a given math word problem

**Problem:** A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey? **Answer:** 275

**Solution1:** \((100/2) \times (2 + 3.5)\)

- Velocity
  - Distance 1: 100
  - Time 1: 2
- Total Distance
  - \(\times\)
  - Time 1: 2
  - Time 2: 3.5

**Solution2:** \(100 + 100/2 \times 3.5\)

- Total Distance
  - Distance 1: 100
- Velocity
  - Distance 1: 100
  - Time 1: 2
- Time 2: 3.5

Fully-Supervised methods: fit the given solution and cannot generate diverse solutions.
Fully-supervised methods: Need time-consuming annotations

Annotating the expressions for MWPs is time-consuming. However, a large amount of MWPs with their final answers can be mined effortlessly from the internet (e.g., online forums). How to efficiently utilize these partially-labeled data without the supervision of expressions remains an open problem.
Problem: A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?

Expression Tree

(Unannotated)

Answer (Annotated): 275
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Expression Tree

(Unannotated):

Answer (Annotated): 275
**Problem:** A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?

**Expression Tree (Unannotated):**

```
+                +
100              100
    *              / 2
   50          100
```

**Neural Model:**

```
50 * 2 = 100
```

**Inference:**

```
200 + 200 = 400
```

**Symbolic Execution:**

```
200 ≠ 275
```

**Answer (Annotated):**

```
200 ≠ 275
```
Problem: A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?

Expression Tree (Unannotated):

Answer:

200 ≠ 275
Problem: A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?

Expression Tree (Unannotated):

Answer: 200 ≠ 275

Inference

Neural Model

157.1 ≠ 275

Fixing

Symbolic Execution
Solving Math Word Problems with Weakly Supervision

**Problem:** A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?

**Expression Tree (Unannotated):**

- $100 + (200 \times \frac{100}{2})$ for the first part of the journey.
- $3.5 \times \frac{100}{2}$ for the additional 3.5 hours.

**Answer:**

- The expression tree evaluates to $200 \neq 275$.
- Neural Model predicts $157.1 \neq 275$.
- Neural Model is trained.
- Symbolic Execution provides $50$ and $175$.
- Fixing is used to correct the model output to $175$. 

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Framework

Goal-Driven Tree Model

Fixing

Memory Buffer

G: Goal
C: Context
Exploring
Learning
Bottom-up reasoning
Top-down fixing
Goal-Driven Tree Structured Model[1]

[1] A Goal-Driven Tree-Structured Neural Model for Math Word Problems. Zhipeng Xie and Shichao Sun.
Goal-Driven Tree Structured Model

- Word embedding + bi-directional GRU

**Problem:** A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?

\[ q_0 = h_n^x + h_0^x \]
Goal-Driven Tree Structured Model

**P**: A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, **how many kilometers will it complete for the entire journey?**

\[ W \in \mathbb{R}^{n \times d} \]

\[ q_0 = \frac{\hbar}{n} + \frac{\hbar}{0} \]

*Attention*
P: A truck travels 100 kilometers in 2 hours. At this speed, if it travels for another 3.5 hours, how many kilometers will it complete for the entire journey?
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Learning by Fixing

[2] Closed Loop Neural-Symbolic Learning via Integrating Neural Perception, Grammar Parsing, and Symbolic Reasoning. Qing Li, Siyuan Huang, Yining Hong, Yixin Chen, Ying Nian Wu, and Song-Chun Zhu.
Learning by Fixing

Algorithm 1 Fixing Mechanism

1: Input: reasoning tree $\hat{T}$, ground-truth answer $y$
2: $T^{(0)} = \hat{T}$
3: for $i \leftarrow 0$ to $m$ do
4: $T^* = 1$-Fix($T^{(i)}$, $y$)
5: if $T^* \neq \emptyset$ then
6: return $T^*$
7: else
8: $T^{(i+1)} = \text{RANDOMWALK}(T^{(i)})$
9: return $\emptyset$
10:
11: function 1-Fix($T$, $y$)
12: $q = \text{PriorityQueue}()$, $S$ = the root node of $T$
13: $q.push(S, y, 1)$
14: while $(A, \alpha_A, p) = q.pop()$ do
15: if $A \in \Sigma$ then
16: $T^* = T(A \rightarrow \alpha_A)$
17: return $T^*$
18: for $B \in \text{child}(A)$ do
19: $\alpha_B = \text{solve}(B, A, \alpha_A)$
20: if not $(B \in \Sigma$ and $\alpha_B \notin \Sigma)$ then
21: $q.push(B, \alpha_B, p(B \rightarrow \alpha_B))$
22: return $\emptyset$

The school purchased 85 sets of tables and chairs for 67 dollars per table and 23 dollars per chair. How much did the school spend buying these tables and chairs?

| Priority Queue | Priority Queue | Priority Queue |
|----------------|----------------|----------------|
| $(1, A_1 \rightarrow 7650)$ | $(\frac{1 - p_3 p_4 p_5}{p_3 p_4 p_5}, A_2 \rightarrow 90)$ | $(\frac{p_5'}{p_5}, 67 \rightarrow 23)$ |
| $\frac{p_3'}{p_3}$ | $67$ | $67$ |
| $(p_1 p_2 p_3 p_4 p_5', 85 \times (67 + 23)$ | | |
Tree Regularization

Size(T) ∈ [minSize(T), maxSize(T)]
minSize(T) = a_{min} \text{len}(V^{num}) + b_{min}
maxSize(T) = a_{max} \text{len}(V^{num}) + b_{max}

1. The number of operators cannot be greater than \([\text{Size}(T)/2]\).
2. Except the last position, the number of numeric values (quantities and constants) cannot be greater than the number of operators.

\[ V^{num} = \{100, 2, 3.5\} \]
\[ V^{op} = \{+,-,\times,\div,\wedge\} \]
\[ V^{con} = \{1, 2, \pi\} \]

Target size \(l = 5\)

| Prefix | \(\times\) | \(\div\) | \(100\) | \(2\) | \(3.5\) |
|--------|-----------|---------|--------|------|--------|
| \(\times\) | 2 | \(V^{op}\) | \(\times\) | 100 | \(V^{num} \cup V^{con}\) |
| \(\div\) | N/A | \(V^{op} \cup V^{num} \cup V^{con}\) | \(\div\) | 2 | \(V^{num} \cup V^{con}\) |
| \(100\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) |
| \(2\) | \(\div\) | \(\div\) | \(\div\) | \(\div\) | \(\div\) |
| \(3.5\) | \(\div\) | \(\div\) | \(\div\) | \(\div\) | \(\div\) |

Prefix: \(\times \div 100 2 3.5\)

Target size \(l = 7\)

| Prefix | \(\times\) | \(\div\) | \(100\) | \(2\) | \(3.5\) |
|--------|-----------|---------|--------|------|--------|
| \(\times\) | 2 | \(V^{op}\) | \(\times\) | 100 | \(V^{op} \cup V^{num} \cup V^{con}\) |
| \(\div\) | N/A | \(V^{op} \cup V^{num} \cup V^{con}\) | \(\div\) | 2 | \(V^{op} \cup V^{num} \cup V^{con}\) |
| \(100\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) |
| \(2\) | \(\div\) | \(\div\) | \(\div\) | \(\div\) | \(\div\) |
| \(3.5\) | \(\div\) | \(\div\) | \(\div\) | \(\div\) | \(\div\) |

Prefix: \(\times \div 100 2 + 2 3.5\)
Memory Buffer

\[ J(P, \beta) = - \sum_{T^* \in \beta} \log p(T^* | P) \]

**Algorithm 2 Learning-by-Fixing**

1: **Input**: training set \( \mathcal{D} = \{(P_i, y_i)\}_{i=1}^N \)
2: memory buffer \( \mathcal{B} = \{\beta_i\}_{i=1}^N \), the GTS model \( \theta \)
3: for \( P_i, y_i, \beta_i \in (\mathcal{D}, \mathcal{B}) \) do
4: ▶ Exploring
5: \( \hat{T}_i = \text{GTS} (P; \theta) \)
6: \( T_i^* = m-\text{FIX}(\hat{T}_i, y_i) \)
7: if \( T_i^* \neq \emptyset \) and \( T_i^* \notin \beta_i \) then
8: \( \beta_i \leftarrow \beta_i \cup \{T_i^*\} \)
9: ▶ Learning
10: \( \theta = \theta - \nabla_\theta J(P_i, \beta_i) \)
Experiment

- **Dataset:**
  
  Math23K, 23161 math word problems

- **Evaluation Metric:**
  
  Answer accuracies of all the top-1/3/5 predictions using beam search

- **Inference Models:**
  
  Seq2Seq, Goal-Driven Tree-Structured Model (GTS)

- **Learning Strategies:**
  
  REINFORCE, MAPO[3], LBF (Learning by Fixing), LBF-w/o-M (Fixing without Memory)

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[3] Memory Augmented Policy Optimization for Program Synthesis and Semantic Parsing. Chen Liang, Mohammad Norouzi, Jonathan Berant, Quoc Le and Ni Lao.
### Top-1 Answer Accuracy

| Model                                                      | Accuracy(%) |
|------------------------------------------------------------|-------------|
| **Fully-Supervised**                                       |             |
| Retrieval (Robaidek, Koncel-Kedziorski, and Hajishirzi 2018) | 47.2        |
| Classification (Robaidek, Koncel-Kedziorski, and Hajishirzi 2018) | 57.9        |
| LSTM (Robaidek, Koncel-Kedziorski, and Hajishirzi 2018)     | 51.9        |
| CNN (Robaidek, Koncel-Kedziorski, and Hajishirzi 2018)      | 42.3        |
| DNS (Wang, Liu, and Shi 2017)                               | 58.1        |
| Seq2seqET (Wang et al. 2018)                               | 66.7        |
| Stack-Decoder (Chiang and Chen 2019)                       | 65.8        |
| T-RNN (Wang et al. 2019)                                   | 66.9        |
| GTS (Xie and Sun 2019)                                     | 74.3        |
| Graph2Tree (Zhang et al. 2020a)                            | **74.8**    |
| GTS-LBF-fully                                              | 74.1        |
| **Weakly-Supervised**                                      |             |
| Seq2seq                                                    |             |
| REINFORCE                                                  | 1.2         |
| MAPO                                                       | 10.7        |
| LBF-w/o-M                                                  | 44.7        |
| LBF                                                        | 43.6        |
| GTS                                                        |             |
| REINFORCE                                                  | 15.8        |
| MAPO                                                       | 20.8        |
| LBF-w/o-M                                                  | 58.3        |
| LBF                                                        | **59.4**    |
## Diverse Solutions with Memory Buffer, Ablative Studies

| Model          | Tree Size | Acc@1 | Acc@3 | Acc@5 |
|----------------|-----------|-------|-------|-------|
| **Fully Supervised** |           |       |       |       |
| GTS            | 74.3      | 42.2  | 30.0  |       |
| GTS-LBF-fully  | 74.1      | **63.4** | **56.3** |       |
| **Weakly Supervised** |          |       |       |       |
| GTS-LBF-w/o-M | [1, +∞)   | 55.3  | 26.2  | 19.3  |
|                | [2n-1, 2n+1] |       |       |       |
|                | [2n-1, 2n+3] |       |       |       |
|                | [2n-3, 2n+5] |       |       |       |
|                | ~0        | ~0    | ~0    |       |
| GTS-LBF        | [1, +∞)   | 56.7  | 45.3  | 39.1  |
|                | [2n-1, 2n+1] |       |       |       |
|                | [2n-1, 2n+3] | **59.4** | **49.6** | **45.2** |
|                | [2n-3, 2n+5] |       |       |       |
|                | ~0        | ~0    | ~0    |       |

| Models         | Steps | 1   | 10  | 50 (default) | 100 |
|----------------|-------|-----|-----|--------------|-----|
| Seq2seq-LBF-w/o-M | 41.9  | 43.4| 44.7| 47.8         |     |
| Seq2seq-LBF      | 43.9  | **45.7** | 43.6| 44.6         |     |
| GTS-LBF-w/o-M    | 51.2  | 54.6| **58.3** | 57.8         |     |
| GTS-LBF          | 52.5  | 55.8| 59.4| **59.6**     |     |
## Qualitative Study

| Problem                                                                 | Ground-Truth | Top-5 Solutions |
|----------------------------------------------------------------------|--------------|-----------------|
| The school purchased 85 sets of tables and chairs for 67 dollars per table and 23 dollars per chair. How much did the school spend buying these tables and chairs? | ![Expression](image1) | ![Expression](image2) |
| There are 1200 students in a school, and 65% are girls. How many boys are there? | ![Expression](image3) | ![Expression](image4) |
| The fruit store shipped 240 kilograms of raw pears. The apples shipped were 60 kilograms less than twice the weight of raw pears. How many kilograms of apples are shipped? | ![Expression](image5) | ![Expression](image6) |
| The cafeteria has 260kg of flour and 6 bags of rice, 25kg per bag. How many more kilograms of flour are there than rice? | ![Expression](image7) | ![Expression](image8) |

- **Expression Right, Answer Right**
- **Expression Wrong, Answer Wrong**
- **Expression Wrong, Answer Right (Spurious)**

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Conclusions & Future Works

- We propose a weakly-supervised paradigm for learning MWPs and a novel learning-by-fixing framework to boost the learning.
- For future work, we will prevent generating equivalent or spurious solutions during training, possibly by making the generated solution trees more interpretable with semantic constraints. (See also our newest work[4]!)
- A weakly-supervised large-scale dataset on math word problems would be beneficial for this line of research.

[4] "SMART: A Situation Model for Algebra Story Problems via Attributed Grammar". Yining Hong, Qing Li, Ran Gong, Daniel Ciao, Siyuan Huang, Song-Chun Zhu.
You are welcomed to visit our project pages!

The project page of this paper: [https://evelinehong.github.io/lbf-site/](https://evelinehong.github.io/lbf-site/)

For more details about the fixing mechanism: [https://liqing-ustc.github.io/NGS/](https://liqing-ustc.github.io/NGS/)

For interpretable math word problems solving: [https://evelinehong.github.io/smart-site/](https://evelinehong.github.io/smart-site/)