Nuclear structure features in $^{72-80}$Se isotopes

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Abstract:

Interacting boson model ($IBM \sim 1$ and $IBM \sim 2$) have been used to perform a whole studying for $^{72-80}$Se isotopes. The low lying positive party states, dynamic symmetries, mixed symmetry states $MS$, reducing electric quadruple transition probabilities $B(E2)$, branching ratio, quadruple momentum $Q_{21}^2$, reducing magnetic dipole transition probability $B(M1)$, mixing ratio $\delta(E2/M1)$, reducing electric monopole transition probability $B(E0)$, and $X(E0/E2)$ ratio have been investigated. $U(5)$ features are the dominant in $^{72-80}$Se with addition of a small effect of $a_2$ parameter started from $^{74}$Se to $^{80}$Se isotopes, energy ratios show that $^{72}$Se isotopes as the nearest isotopes to typical vibrational limit while $^{74-80}$Se isotopes tend towards the rotational region that lied on $U(5)$-SU(3), leg of “Casten’s triangle”. Spin and party for many energy levels and electromagnetic transition probability are confirmed. The mixing symmetry states in $^{72-76}$Se isotopes are slowly increased with $\zeta_2$ while there is no clear effected in $^{80}$Se isotope. The calculated branching ratios, $B(E2), B(M1)$, mixing ratios $\delta(E2/M1), B(E0)$, and $X(E0/E2)$ depend in comparisons with fewer available experimental data. The results are in acceptable agreement; to completed a perfect comparison, more experimental investigations are still needed to these nuclei. $^{72-80}$Se isotopes have small values of electric quadruple moment of $2^+$ state.

Keyword: $^{72-80}$Se isotopes, $IBM \sim 1, IBM \sim 2$, $\delta(E2/M1)$ and $X(E0/E2)$.

1. Introduction

The interacting boson model ($IBM$) is based on the well-known shell model and on geometrical collective model of the atomic nucleus. It is suitable for describing the structure of intermediate and heavy nuclei. In addition, it is of considerable theoretical interest since it shows the dynamical symmetries of the nuclei, which are made visible.
using lie algebra. The (IBM) was developed by Iachello and Arima [1]. The (IBM) model, suggests that the collective behavior rises from the coupling, through the interaction of the nucleon-nucleon of the isolated low-lying systems of valence protons and neutrons that is definite in accordance to the respect of the major shell closure. It is capable of describing nuclear characteristics such as energies and spins of the levels, decay probabilities for the emission of gamma quanta, probabilities of electromagnetic transitions and their reduced matrix elements for different transitions, multipole moments, and mixing ratios[1,2]. There are many researches that attempted to explain the behavior of Se nuclei by using different models[3-8] in present study many properties of nuclear structure for even- even\(^{72-80}\)Se isotopes have been investigation with IBM1 and IBM2 they have been examined carefully with benefit from continuously updating of nuclear decay schemes.\(^{72-80}\)Se nuclei have 3 particle bosons the number of protons and neutrons lying between 28 and 50 magic shells, \(^{72}\)Se has 38 neutrons which mean 5 particle neutron bosons, while \(^{74-80}\)Se have 40-46 neutrons that mean 5-2 hole neutron bosons respectively.

2. The Interacting Boson Model

In the IBM – 1, it assumed that the Hamiltonian operator contains only one body and two body terms thus, introducing creation (\(s^\dagger, d^\dagger_m\)) and annihilation (\(s, d_m\)) operators where the index \(m = 0, \pm 1, \pm 2\). The most general Hamiltonian, which includes on-boson terms in boson – boson interaction is [1,2,9].

\[
H = e_n a_d + a_0 P^T P + a_s L L + a_Q Q + a_3 T_3 T_3 + a_4 T_4 T_4
\]  

(1)

Where \(e = e_d - e_s\) is the boson energy, for simplicity \(e_s\) is set equal to zero only \(e = e_d\) appears, and \(a_0, a_s, a_Q, a_3, a_4\) designate the strengths of the quadrupole, angular momentum, pairing ,octupole and hexadecapole interacting between bosons respectively. The corresponding parameters. The components of \(d\) boson and the single component of the \(s\) boson are extended across a six dimensional space. For a fixed number of boson \(N\), the group structure of the problem is \(U(6)\). Considering the different reductions of \(U(6)\), three dynamical symmetries emerge, namely \(U(5), SU(3)\), and \(O(6)\); these symmetries are related to the geometrical idea of the spherical vibrator, deformed rotor and a symmetric (\(\gamma\) –soft) deformed rotor, respectively [9-12]. IBM – 1 electromagnetic transitional operator has been given as follows[13-16].

\[
T_m^l = \alpha_2 \delta_{2l}^l d^l s + s^l d_m^{(2)} + \beta_1 \left[ d^l d_m^{(1)} + \gamma_0 \delta^l_0 \delta_{m0} [s^l s_0^l] \right]
\]  

(2)

where \(\alpha_2, \beta_1, \gamma_0\) are the coefficients of the various terms in the operators.This equation yields transition operators for \(E0, M1, E2, M3\), and \(E4\) transitions with appropriate values of the corresponding parameters. The IBM – 1 quadrupole moments , in \(U(5), SU(3)\) and \(O(6)\) limits defined as,

\[
Q_2^\pm = \beta_2 \sqrt{16 \pi / 5} \sqrt{2 / 7}, Q_2^\pm = - \alpha_2 \sqrt{2 \pi / 5} \sqrt{2 / 7} \quad (4N + 3)\text{ and } Q_2^\pm = 0
\]  

(3)

The basic condition for the observation of a \((U(5), SU(3))\) and \((O(6))\) symmetry in the electromagnetic transitions are [1,2].

\[
R = \frac{b(E2;2^+_1;2^+_1)}{b(E2;2^+_1;0^-_1)}; R = \frac{b(E2;2^+_1;2^+_1)}{b(E2;2^+_1;0^-_1)}; R = \frac{b(E2;0^+_1;2^+_1)}{b(E2;0^+_1;0^-_1)}; R = R' \quad R' = \frac{2N-1}{2} < 2 \ldots \text{in } U(5)
\]  

(4)

\[
R = \frac{b(E2;2^+_1;2^+_1)}{b(E2;2^+_1;0^-_1)} \approx \frac{10}{7}; R = \frac{b(E2;2^+_1;2^+_1)}{b(E2;2^+_1;0^-_1)}; R = \frac{b(E2;0^+_1;2^+_1)}{b(E2;0^+_1;0^-_1)}; R = R' \quad R' = 0 \ldots \ldots \text{in } SU(3)
\]  

(5)

\[
R = \frac{b(E2;2^+_1;2^+_1)}{b(E2;2^+_1;0^-_1)}; R = \frac{b(E2;0^+_1;2^+_1)}{b(E2;0^+_1;0^-_1)}; R = R' \quad R' = 0 \ldots \ldots \text{in } O(6)
\]  

(6)

were \(R, R'\) are denoted by branching ratios.

The Hamiltonian operator in IBM – 2 will have three parts: one part for each of proton and neutron bosons and a third part for describing the proton-neutron interaction [1].

\[
H = H_p + H_v + V_{\pi \nu}
\]  

(7)

A simple schematic Hamiltonian guided by microscopic consideration is given by [1,2].

\[
H = \varepsilon (n_\pi + n_\nu) + \kappa Q_\pi \cdot Q_\nu + V_{\pi \nu} + V_\nu + M_{\pi \nu}
\]  

(8)

where \(Q_\rho = (d_\rho^T s_\rho + s_\rho^T d_\rho)^2 + \chi_\rho (d_\rho^T d_\rho)^2 \quad \rho = \pi, \nu\)
effective charges. The general single boson transition operator of angular momentum \(J\) if there is an experimental evidence for so called “mixed symmetry state” MSS, then the Majorana parameter are varied to fix the location of these states in the spectrum. The energy levels are achieved by diagonalizing the Hamiltonian eq.(5), then allowing the parameters \(\varepsilon, \kappa, \chi_v, \chi_v\) and \(C_L\) to vary until one obtains the best fit to the experimental spectrum. It is possible to obtain spectra which are similar to those of the \(IBM - 1\) with only one kind of boson [15]. The \(U(5)\) limit when \(\varepsilon \gg \kappa\) or \(SU(3)\) limit when \(\varepsilon \ll \kappa\) and \(\chi_v = \chi_v = -\chi_v\). Most nuclei do not strictly belong to any of these three limiting cases, but are somewhere between two of them. In the \(IBM\), it is possible to make a smooth transition between the limiting cases for a series of isotopes. The general single boson transition operator of angular momentum \(\ell\) has the same form as in eq.(2), in \(IBM - 1\) except the fact that in each term one has to consider \(\pi, \nu\) degree of freedom and this can be written as [13].

\[
T^{(\ell)} = c_{\ell} \rho |d^\dagger s^\dagger d^\dagger p^\dagger|^{(\ell)} + \beta \rho \rho |d^\dagger d^\dagger|^{(\ell)} + \gamma |s^\dagger s|^{(\ell)} \rho = \pi \text{ or } \nu
\]

This equation yields transition operators for \(E2, M1\) and \(E0\) transitions

\[
T^{E2} = e_\pi Q_{\pi}^2 + e_\nu Q_{\nu}^2
\]

Where \(Q_\rho\) is the same as in eq.(7), \(e_\pi\) and \(e_\nu\) are boson effective charges depending on the boson number \(N\) and they can take any value to fit the experimental result. The two effective charges \(e_\pi\) and \(e_\nu\) can obtained[17] by using boson number \(N, N_v\) and experimental \(B(E2; 2^+_1 \rightarrow 0^+)_v\) values for a series of isotopes to produce a plot between \(N_v/N_\pi\) and \(M\) where defined as [17].

\[
M = N_\pi |N, B(E2; 2^+_1 \rightarrow 0^+)|^{1/2}
\]

The best straight line through the experimental point is obtained with slope \(e_\pi\) and intercept \(e_\nu\). The quadrupole moment of \(2^+_1\) can be in \(IBM2\) defined as[1,2].

\[
Q_{2^+1} = \sqrt{16\pi/5} \sqrt{L(2L+1)B(E2)} 1/2(2L+1)(2L+3)
\]

The \(M1\) operator obtained by letting \(\ell = 1\) in eq.(10) is written [1,2] as.

\[
T^{(M1)} = g_\pi L^{(1)}_\pi + g_\nu L^{(1)}_\nu
\]

Where \(g_\pi\) and \(g_\nu\) are the boson \(g\) – factor in unit of \(\mu_N\), this operator can be written alternatively as [1,2].

\[
T^{(M1)} = 0.77[(d^\dagger s^\dagger)|^{(1)}_\pi - (d^\dagger d^\dagger)|^{(1)}_\nu] (g_\pi - g_\nu)
\]

The \(M1\) strength many be expressed in terms of the multipole mixing

\[
\delta = 0.835 E_\nu (MeV). \Delta; \Delta = \frac{|I_\nu||T^{(E2)}|_{I_\nu}}{|I_\nu||T^{(M1)}|_{I_\nu}}
\]

by having fitted \(E2\) matrix elements, one can then use them with \(\delta(E2/M1)\) to obtain \(M1\) matrix element and compare them with the matrix elements predicted by the model using the operator eq.(16). The \(E0\) transitions of electric monopole are probable between the same spin states and parity in a nucleus enclosed by electrons. The \(E0\) transitions features offer sensitive tests of the different models of nuclear structure. The \(E0\) reduced transitions probability is expressed as [19].

\[
B(E0; I_1 \rightarrow I_2) = e^2 R^2 \rho^2 (E0)
\]
Where \( e \) indicates the electronic charge, \( R \) nuclear radius and \( \rho(E0) \) is the matrix element transition which is gotten by [19].

\[
\rho(E0) = \frac{Z}{R^2} \sum \beta_{0p}^t (f | d_p^t \times d_p | i), \rho = \pi, \nu
\]

(18)

where \( \beta_{0p}, \beta_{0v} \) represent the deformation parameters for (protons and neutrons).

The \( X(E0/E2) \) ratio can be calculated as follows[19].

\[
X(E0/E2) = \frac{B(E0; i_i \rightarrow i_f)}{B(E2; i_i \rightarrow i_f)}
\]

(19)

Where \( i_f = i_f \), for \( i_i = i_f \neq 0 \), and \( i_f = 2 \) for \( i_i = i_f = 0 \).

This ratio is so essential as it mirrors to what extent the transition between \( B(E2) \) and \( B(E0) \) is strong.

3. Results and Discussion

Interacting boson model \( IBM - 1 \) and \( IBM - 2 \) have been used to perform a whole studying for \( ^{72-80}Se \) isotopes. The low lying positive party states, dynamic symmetries, mixed symmetry states \( MSS \), reducing electric quadruple transition probabilities \( B(E2) \), branching ratio, quadruple momentum \( Q^*_2 \), magnetic dipole \( B(M1) \), mixing ratio \( \delta(E2/M1) \), reducing electric monopole transition probability \( B(E0) \), and \( X(E0/E2) \) ratio have been investigated. The software package \( IBM \) and Neutron Proton Boson \( NPBOS \), have been used by estimating a set of parameters described in the Hamiltonian operator as it is shown in equations (1) and (5). The estimated parameters for the calculated low lying energy levels for Selenium isotopes are given in table (1) and figures (1).

Table 1 The parameters have been used in the \( IBM - 1 \) and \( IBM - 2 \) Hamiltonian for even-even \( ^{72-80}Se \) isotopes (in MeV) except \( \chi, \chi \nu \) and \( \chi \pi \) were unit less.

| Isotopes | IBM-1 parameters in MeV unless \( \chi \) |
|----------|------------------------------------------|
| \(^{72}\text{Se}\) | \( N_p \) | \( \varepsilon \) | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( \chi \) |
| 8 | 0.6002 | 0.0 | 0.02 | 0.0 | 0.001 | 0.001 | 0.0 |
| 8 | 0.6 | 0.0 | 0.02 | -0.007 | 0.0 | 0.0 | -1 |
| 7 | 0.54 | 0.0 | 0.02 | -0.009 | 0.0 | 0.0 | -0.86 |
| 6 | 0.6 | 0.0 | 0.02 | -0.038 | 0.0 | 0.0 | -1 |
| 5 | 0.6 | 0.0 | 0.02 | -0.036 | 0.0 | 0.0 | -1 |

| Isotopes | IBM-2 parameters in MeV unless \( \chi, \chi \nu = -0.74, N_\nu = 3 \) |
|----------|---------------------------------------------------------------------|
| \(^{72}\text{Se}\) | \( N_\nu \) | \( \varepsilon \) | \( \kappa \) | \( \chi \) | \( \chi \nu \) | \( \chi \pi \) | \( C^2 \) | \( C^2 \) |
| 5 | 0.99 | -0.076 | 0.88 | 0.002 | -0.01 | -0.99 | 0.76 | 0.12 |
| 5 | 0.88 | -0.077 | 0.82 | 0.012 | -0.02 | -0.9 | 0.76 | 0.1 |
| 4 | 0.78 | -0.086 | 0.84 | 0.01 | -0.02 | -0.41 | 0.86 | 0.32 |
| 3 | 0.78 | -0.076 | 0.84 | 0.03 | -0.016 | 0.2 | 0.86 | 0.5 |
| 2 | 0.8 | -0.16 | 0.6 | 0.001 | -0.01 | -0.48 | 0.44 | 0.69 |
| 74 | 0.2 | 0.76 | 0.12 |
| 74 | 0.2 | 0.76 | 0.09 |
| 74 | 0.2 | 0.86 | 0.22 |
| 74 | 0.2 | 0.9 | 0.5 |
| 74 | 0.2 | 0.72 | 0.69 |
Figure 1  $IBM - 2$ parameters ($\epsilon, \kappa, \chi, \kappa_2, \xi_2, \zeta_{1,2}$) for even-even $^{72-80}$Se isotopes as a function of the mass numbers.

Calculated energy ratios of $(E4/E2$, $E6/E2$, and $E8/E2$) for $^{72-80}$Se isotopes have been shown in figure (2) as a function of mass numbers. The calculated energy levels compared with experimental data [20-24] for $^{72-80}$Se isotopes have been illustrated in figures from (3) to (7).

Figure 2  The experimental [20-24], theoretical and standard [1] energy ratios ($E4/E2$, $E6/E2$, and $E8/E2$) respectively as a function of mass numbers even-even $^{72-80}$Se isotopes.

Figure 3: The levels of the calculated energy compared with experimental [20] for $^{72}$Se isotope.
Figure 4 The levels of the calculated energy compared with experimental [21] for $^{74}$Se isotope.

Figure 5 The levels of the calculated energy compared with experimental [22] for $^{76}$Se isotope.

Figure 6 The levels of the calculated energy compared with experimental [23] for $^{78}$Se isotope.

Figure 7 The levels of the calculated energy compared with experimental [24] for $^{80}$Se isotope.

The effect of Majorana parameters $(\zeta_{1,3}, \zeta_2$) on the calculated excitation energy levels for $^{72-80}$Se isotopes have been examined by variation the $\zeta_2$ around the best-fitted to experimental data for the states $(2^+_2,2^+_3,2^+_5,3^+_1,5^+_1, and 1^+_1)$. Figure (8) clarifies the variation of energy of these states as a function of the Majorana parameter $\zeta_2$. 


Figure 8 Mixed symmetry states MSS in even-even $^{72-80}$Se isotopes.

The effective boson charges have been used in IBM and Neutron Proton Boson Electromagnetic, NPBEM software package to calculate electric quadruple transitions and branching ratios that are listed in table (2). They are compared with available experimental data [20-24]. Therefore, the calculated reduced electric quadruple transitions probability $B(E2)$, and electric quadrupole moment of $2^+_1$ state in comparison with the experimental values [20-25] for $^{72-80}$Se isotopes listed in tables (3).

Table 2 The effective boson charges used in IBM − 1 and IBM − 2 to calculate $B(E2)$ transition and comparison between calculated branching ratios with experimental data [20-24] for even-even $^{72-80}$Se isotopes.

| Isotopes  | $IBM − 1$ | $IBM − 2$ | $E2SD$ | $E2DD$ | $e_u$ | $e_\pi$ |
|-----------|-----------|-----------|--------|--------|-------|--------|
| $^{72}$Se | 0.074     | -0.022    | 0.029  | 0.11   |
| $^{74}$Se | 0.09      | -0.05     | 0.033  | 0.15   |
| $^{76}$Se | 0.0988    | -0.05     | 0.056  | 0.133  |
| $^{78}$Se | 0.082     | -0.05     | 0.045  | 0.14   |
| $^{80}$Se | 0.0832    | -0.05     | 0.04   | 0.12   |

| Isotopes | $R$ | $R'$ | $R''$ |
|----------|-----|------|-------|
|          | Exp. | IBM − 1 | IBM − 2 | Exp. | IBM − 1 | IBM − 2 | Exp. | IBM − 1 | IBM − 2 |
### Table 3: Calculated reduced electric quadruple transitions probability

| Isotopes   | \( B(E2) \) values in \( e^2b^2 \) | \( 72 \)Se | \( 74 \)Se | \( 76 \)Se | \( 78 \)Se | \( 80 \)Se |
|------------|-------------------|--------|--------|--------|--------|--------|
| \( \bar{J} \rightarrow \bar{J} \) | \( \text{Exp.} \) | \( \text{IBM-1} \) | \( \text{IBM-2} \) | \( \text{Exp.} \) | \( \text{IBM-1} \) | \( \text{IBM-2} \) |
| \( 2 \rightarrow 0 \) | 0.042 | 0.043 | 0.043 | 0.077 | 0.077 | 0.077 |
| \( 4 \rightarrow 2 \) | 0.084 | 0.0766 | 0.0836 | 0.145 | 0.133 | 0.145 |
| \( 6 \rightarrow 4 \) | 0.1 | 0.098 | 0.105 | 0.1328 | 0.169 | 0.1337 |
| \( 8 \rightarrow 6 \) | 0.11 | 0.109 | 0.0987 | 0.175 | 1.84 | 0.19 |
| \( 10 \rightarrow 8 \) | 0.103 | 0.109 | 0.103 | -- | 0.18 | 0.176 |
| \( 0 \rightarrow 2 \) | -- | 0.003066 | 0.003064 | 0.0246 | 0.0048 | 0.0049 |
| \( 2 \rightarrow 0 \) | -- | 0.00038 | 0.0007 | -- | 0.0085 | 0.00803 |
| \( 2 \rightarrow 2 \) | -- | 0.07 | 0.067 | 0.0885 | 0.12 | 0.0848 |
| \( 4 \rightarrow 2 \) | -- | 0.051 | 0.03128 | -- | 0.085 | 0.082 |
| \( 6 \rightarrow 4 \) | -- | 0.07 | 0.0418 | -- | 0.1 | 0.073 |
| \( 8 \rightarrow 6 \) | -- | 0.02 | 0.0197 | -- | 0.039 | 0.0338 |
| \( 10 \rightarrow 8 \) | -- | 0.0131 | 0.0137 | -- | 0.0194 | 0.02 |
| \( 0 \rightarrow 2 \) | -- | 0.05 | 0.0405 | -- | 0.092 | 0.0698 |
| \( 2 \rightarrow 0 \) | -- | 0.075 | 0.05266 | -- | 0.11 | 0.0894 |
| \( 4 \rightarrow 2 \) | -- | 0.07 | 0.05262 | -- | 0.118 | 0.0879 |
| \( 6 \rightarrow 4 \) | -- | 0.02 | 0.0197 | -- | 0.039 | 0.0338 |
| \( 8 \rightarrow 6 \) | -- | 0.05 | 0.0405 | -- | 0.092 | 0.0698 |
| \( 10 \rightarrow 8 \) | -- | 0.075 | 0.05266 | -- | 0.11 | 0.0894 |

* \( E2SD = a_2, E2DD = \sqrt{5} \beta_2 \) where \( \beta_2 = \frac{-0.7}{\sqrt{5}} a_2, \frac{-y}{\sqrt{5}} a_2 \) and \( \beta_2 = 0 \) in \( U(5), SU(3), \) and \( O(6) \) respectively.
The effective $g$-factors for proton $g_p$ and neutron $g_n$. for $^{72-80}$Se isotopes are $g_p = 0.4\mu_N$ and $g_n = 0.07\mu_N$. Eq (15) have been utilized to calculate the $B(M1)$ transition probabilities as it is shown in table (4). The calculated values for $B(M1)$ and the mixing ratio $\delta(E2/M1)$ are compared with the available experiments values [20-28] for $^{72-80}$Se isotopes.

Table 4 Comparison between calculated the magnetic transitions $B(M1)$ in $\mu_N^2$ and the mixing ratio with the available experimental data [20-28] for even-even $^{72-80}$Se isotopes.

| Isotopes  | $B(E2)$ values in $e^2b^2$ | $^{72}$Se | $^{74}$Se |
|----------|-----------------------------|----------|----------|
| $J^+ \rightarrow J^+$ | $B(M1)$, $\mu_N^2$ | $\delta(E2/M1)$ | $B(M1)$, $\mu_N^2$ | $\delta(E2/M1)$ |
| Exp. | IBM-2 | Exp. | IBM-2 | Exp. | IBM-2 | Exp. | IBM-2 |
| $1_1 \rightarrow 2_1$ | -- | $5.33 \times 10^6$ | -- | -- | $8.97 \times 10^4$ | -- | -- |
| $1_2 \rightarrow 2_2$ | -- | $0.00629$ | -- | -- | $5.67 \times 10^3$ | -- | -- |
| $2_1 \rightarrow 3_1$ | -- | $0.03994$ | -- | -- | $0.00019$ | -- | -- |
| $2_2 \rightarrow 2_2$ | -- | $0.06232$ | -- | -- | $0.0022$ | $0.00027$ | -- | $-6.055$ |
| $2_2 \rightarrow 3_2$ | -- | $0.0066$ | -- | -- | $3.92 \times 10^{-5}$ | -- | -- |
| $2_1 \rightarrow 2_2$ | -- | $0.003024$ | -- | -- | $7.24 \times 10^{-6}$ | -- | -- |
| $3_1 \rightarrow 2_2$ | -- | $8.42 \times 10^{-6}$ | -- | -- | $1.35 \times 10^{-7}$ | -- | -- |
| $3_1 \rightarrow 2_2$ | -- | $0.00439$ | -- | -- | $0.0162$ | $3.87 \times 10^{-5}$ | $0.3$ | $-22.25$ |
| $4_1 \rightarrow 3_1$ | -- | $0.02396$ | -- | -- | $0.000113$ | -- | -- |
| $4_2 \rightarrow 4_1$ | -- | $0.09148$ | -- | -- | $0.00374$ | $0.00044$ | $4.3$, $+2.4$ | $-4.014$ |
| $5_1 \rightarrow 4_1$ | -- | $1.46 \times 10^{-5}$ | -- | -- | $2.07 \times 10^{-7}$ | -- | -- |

| Isotopes  | $B(M1)$, $\mu_N^2$ | $\delta(E2/M1)$ | $B(M1)$, $\mu_N^2$ | $\delta(E2/M1)$ |
|----------|-----------------------------|----------|----------|----------|
| $J^+ \rightarrow J^+$ | Exp. | IBM-2 | Exp. | IBM-2 | Exp. | IBM-2 | Exp. | IBM-2 |
| $1_1 \rightarrow 2_1$ | -- | $6.14 \times 10^{-10}$ | -- | -- | -- | $4.66 \times 10^{-6}$ | -- | -- |
| $1_2 \rightarrow 2_2$ | -- | $7.64 \times 10^{-5}$ | -- | -- | -- | $0.00443$ | -- | -- |
transition matrix elements

| Jσ → Jτ | B(M1) | μ^2 | δ(E2/M1) |
|---------|-------|-----|----------|
|         | Exp.  | IBM - 2 | Exp.  | IBM - 2 | Exp.  | IBM - 2 | Exp.  | IBM - 2 |
| 1σ → 1τ |       |        |        |        |       |        |       |        |
| 3σ → 3τ |       |        |        |        |       |        |       |        |
| 3σ → 4τ |       |        |        |        |       |        |       |        |
| 4σ → 4τ |       |        |        |        |       |        |       |        |
| 5σ → 5τ |       |        |        |        |       |        |       |        |

Isotopes

| Isotopes | 72Se | 76Se | 80Se |
|----------|------|------|------|
|          |      |      |      |

The deformation parameters for protons and neutrons used to calculate monopole transition matrix elements B(E0) and mixing ratio $X(E0/E2)$ for $^{72-80}$Se isotopes are $\beta_{0n} = 0.023 fm^2$, $\beta_{0v} = 0.022 fm^2$. Monopole transition matrix element $B(E0)$ and mixing ratio $X(E0/E2)$ calculated using equation (17) and (19) listed in table (5). Table (5) Comparison between calculated monopole transition matrix element $B(E0)$ in unit $e^2 b^2$ and $X(E0/E2)$ with the available experimental data [20-24,28,29] for even-even $^{72-80}$Se isotopes.
| Isotopes | $B(E0)$ | $X(E0/E2)$ | $^{76}$Se | $B(E0)$ | $X(E0/E2)$ |
|----------|---------|-------------|----------|---------|-------------|
| $|l^+\rightarrow l^+_f|$ | IBM - 2 | $X(E0/E2)$ | $|l^+\rightarrow l^+_f|$ | IBM - 2 | $X(E0/E2)$ |
| $0_2 \rightarrow 0_1$ | 0.00096 | 0.019 | -- | $2_3 \rightarrow 2_1$ | 0.0049 | 0.15 | -- |
| $0_3 \rightarrow 0_1$ | 0.00125 | 0.024 | -- | $2_2 \rightarrow 2_3$ | 4.043 $\times 10^{-5}$ | 0.0046 | -- |
| $0_3 \rightarrow 0_2$ | 8.24 $\times 10^{-5}$ | 0.0016 | -- | $4_2 \rightarrow 4_1$ | 0.00043 | 0.012 | -- |
| $2_2 \rightarrow 2_1$ | 6.8 $\times 10^{-5}$ | 0.0015 | -- | -- | -- | -- | -- |

4. Conclusion

According to the interacting boson model, $^{72-80}$Se has been conducted describing the nuclear structure of even even nuclei within the $U(6)$ symmetry, possessing the $U(5)$, $SU(3)$, and $O(6)$ limiting dynamical symmetries, appropriate for vibrational, axially deformed, and $\gamma$-unstable nuclei respectively. The nucleons distributions of protons and neutrons sub shells are

$$
\frac{1f_{5/2}^2 1f_{5/2}^6 2p_{1/2}^2 1g_{9/2}^2}{Z=34 \quad N=72} \text{ } \frac{74}{76-80}
$$

This distribution may support the vibrational sight to these isotopes which indicating that it is arranged as one and two phonon states. The energy of two phonon states has twice the energy of the one phonon states which are not accurately fulfilled. These energy ratios vary typically between about 1.8 and 2.6; in all none closed shell spherical even-even nuclei, the first excited energy state is $2^+$ and the next three excited $0^+, 2^+ and 4^+$. In the lighter even-even nuclei, all configurations obtained without exciting nucleons to another shell have even parity, three excited $0^+, 2^+ and 4^+$ must be given a good convergence. In addition to convergence three-phonon excitations states with spins and parity of $0^+, 2^+, 3^+, 4^+ and 6^+$, nuclei whose level schemes contained sets with the characteristic properties belong to the $U(5)$ vibrational subgroup of the interacting boson model (IBM). However, the energy levels sequence of $^{74-80}$Se isotopes moved away from the typical phonons interactions towards rotational one. In IBM1, $\epsilon$ parameter is the dominant in $^{72-80}$Se with addition a small effect of $a_2$ parameter started from $^{74}$Se to $^{80}$Se isotopes. In IBM2 parameters, the $U(5)$ limit expectation when $\epsilon \gg \kappa$, energy ratios ($E4^+_1/E2^+_1$),($E6^+_1/E2^+_1$) and ($E8^+_1/E2^+_1$), explains $^{76}$Se isotope (1.89, 2.86 and 3.97) as the nearest isotope to typical vibrational limit while $^{74-80}$Se isotopes tend towards a rotational region lied on $U(5)-SU(3)$, leg of “Casten’s triangle”. The Majorana parameter effect ($\zeta_2$) on the calculated energy level for $^{72-80}$Se isotopes has been investigated by varying the $\zeta_2$ around the best fitted to experimental data. The lowest mixed state is $J = 2^+_2$, $J = (2^+_2, 2^+_3, 3^+_4, 5^+_5$ and $1^+_6$) in $^{72-80}$Se isotopes where it is slowly increased with $\zeta_2$ while no clear effect in $^{80}$Se isotope as shown in figure (8). The experimental observables are quadrupole moments most closely related to the nuclear structure of excited states and the rates of electromagnetic transition between them. The experimental measurement of these observables signifies a stringent test for theoretical models, the reduced electric quadrupole transitions probability $B(E2)$, electric quadrupole moment of $2^+_1$ state, reduced transitions probability for magnetic dipole, $\delta(E2/M1)$, $B(E0)$, and $X(E0/E2)$ have been compared with available experimental data for even - even $^{72-80}$Se isotopes. The agreement supports the transitional features vibrational $U(5)$ to rotational $SU(3)$, branching ratios $R$, $R'$ and $R''$ values arranged between vibrational $U(5)$, limit that $R \sim 2(N-1)/N < 2$ and rotational $SU(3)$, where $R' \sim 0$, $R''$ value is wobbled between the two values for $^{74-80}$Se unless $^{76}$Se isotope that superposes vibrational $U(5)$ limit. The small values of electric quadrupole moment of $2^+_1$ state ensure that.
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