What we can learn from running of the tensor mode: a case of the scalon

Kazunori Kohri
Cosmophysics group, Theory Center, IPNS, KEK, and The Graduate University for Advanced Study (Sokendai), Tsukuba 305-0801, Japan

Chia-Min Lin
Physics Teaching and Research Center, Feng Chia University, Taichung 407, Taiwan

Tomohiro Matsuda
Laboratory of Physics, Saitama Institute of Technology, Fukaya, Saitama 369-0293, Japan

At the beginning of inflation, when the vacuum energy starts to dominate, there could be many dynamical fields in the Universe. At the same time, velocity of the inflaton may not coincide with the slow-roll (attractor) velocity. Although these additional degrees of freedom may neither enhance nor suppress the curvature perturbation, they can easily alter the scale-dependence of the spectrum. Therefore, if the perturbations exit horizon during the early stage of inflation where these effects are still not negligible, one might observe peculiar scale dependence in the spectrum. We show that the effect can be measured using the running of the tensor mode.

PACS numbers: 98.80Cq

I. INTRODUCTION

Scale-dependence of the spectrum of the curvature perturbation is a generic prediction among inflationary cosmological models [1, 2] and it has been used to discriminate inflationary scenarios. Moreover, recent discovery of the B-mode polarization [3, 4] has ignited studies of the tensor modes. Originally, the BICEP2 result suggested some tension with previous experiments such as WMAP [5] and Planck [6]. To resolve the tension they suggested [4] that the scalar spectral index could run fast. Then these observations also stimulated studies of the scale-dependent spectrum [7]. A solution of the tension may require additional field [8, 9]. Besides those recent trends in inflationary cosmology, there has been particle models (e.g., supersymmetric models and string theory) which may have a large amount of scalar fields other than the Higgs field. These models will expect temporal appearance of some dynamical scalar fields in the very early stage of inflation. In the light of multi-field inflation, those extra degrees of freedom may alter the scale-dependence of both the tensor and the scalar perturbations. Consequently, they may bring in a kind of ambiguity to the inflationary parameters of the model. As the measurements of the spectrum are becoming more precise than ever, those contributions could be a serious stumbling block for more rigorous determination of the inflationary scenario. Moreover, even if the inflaton is the only dynamical field, it is hard to believe that the inflaton velocity “before” inflation coincides with the slow-roll (attractor) velocity. Therefore, initially the inflaton velocity could have some deviation from the slow-roll (attractor) velocity. The perturbations during this period will also show different scale-dependence.

In this paper we show why the scale-dependence of the spectrum can easily be shifted by a trivial scalar field (we call this field the “scalon”). Then, using the tensor mode, we show how to measure the contributions of the scalon in the observables. For that purpose, more precise measurement of the higher runnings of the spectrum is also important.

II. HOW TO MEASURE THE SCALON IN THE SPECTRUM

Single-field inflation usually expects $r + 8n_t = 0$, where $r$ is the tensor-to-scalar ratio and $n_t$ is the running of the tensor mode. The relation could be violated if the inflationary scenario has multiple scalar fields during inflation, even if the curvature perturbation is dominated by single perturbation. Intuitively we are expecting simple cases like:

- Consider trajectory of Fig.1. In this scenario the scalon is short lived. The curvature perturbation evolves during inflation.

- Consider trajectory of Fig.2. In this scenario the scalon remains dynamical until the end of inflation. The curvature perturbation is generated at the end.

In both cases the curvature perturbation at the pivot scale is unchanged (the curvature perturbation is dominated by the original inflaton perturbation), while the scale dependence is shifted by the scalon. Similar (but opposite) situation appears in the curvaton model. In the simplest curvaton model, in which the slow-roll parameters of the curvaton field is negligible, the scale dependence is generated by the inflaton, which does not generate the curvature perturbation. In that case the inflaton plays the role of the scalon. In general, “multi-field inflation” includes models in which multiple perturbations contribute the curvature perturbation. To avoid
confusions, such models are called “mixed perturbation scenario” in this paper. In the mixed perturbation scenario the relation between non-linear parameters could be useful for discriminating multi-field and single-field scenarios. Note that multi-field inflation does not always give such mixed perturbation spectrum.

- Inflaton velocity of single-field inflation may have deviation from the slow-roll velocity. It is known that the curvature perturbation evolves after horizon exit, since the additional degree of freedom (i.e., the deviation) may form decaying mode. \( r + 8n_t = 0 \) is not satisfied.

Deviation from the slow-roll velocity may have serious impact on the scale dependence. It can even reverse the sign of the spectral index.

- Clearly, \( r + 8n_t \neq 0 \) is violated in the curvaton scenario, since the curvature perturbation has different form. On the other hand, usually the curvaton expects negligible \( r \). In that case neither \( r \) nor \( n_t \) would be observable. Mixed inflaton-curvaton scenario would be an exception. We are not discussing those “alternative” models, including modulaion (e.g. modulated reheating).

In this paper we consider those additional degrees of freedom and calculate shifts in the observables.

A. “Modest” multi-field inflation

We introduce the scalon field (additional free scalar field \( \chi \)), which is dynamical at the beginning of inflation. Using the standard definition of the curvature perturbation, the scalon determine the initial adiabatic curvature perturbation. One can imagine a case in which \( \dot{\chi}_* \gg \dot{\phi}_* \) at horizon exit but \( \dot{\chi} \ll \dot{\phi} \) at the end. Here * denotes the value at the horizon exit. See also Fig. We also consider the case with \( \chi_* < \phi_* \), which can also shift the scale dependence.

We are considering multi-field inflation. However, since we are considering a “temporal” field, the derivative of the e-foldings with respect to the extra field \( N_\chi \) is negligible. Here the subscript with comma denotes derivative with respect to the field. Cases with significant \( N_\chi \) has been considered in Ref. We are focusing on the dynamics of a trivial field.

We will assume canonical kinetic term for the fields. We are avoiding mixing of the fields in the potential. The scalon field is separated from the inflaton \( \phi \) and ceases to be dynamical before the end of inflation. In that way our analysis does not depend on specific inflationary model. One can add \( \chi \) to any (single-field, canonical kinetic term) inflationary model on one’s choice.

When a couple of fields \( \phi \) and \( \chi \) are dynamical during inflation, the spectral index of the curvature perturbation is calculated as

\[
\eta_s - 1 \approx -6(1 - 4 \cos^2 K_A) \epsilon_H + 2 \eta_{s\sigma} \sin^2 K_A + 4 \eta_{ss} \sin K_A \cos K_A + 2 \eta_{s\chi} \cos^2 K_A,
\]

where the subscripts \( \sigma \) and \( s \) denote the adiabatic and entropy directions at the horizon exit. We choose \( \delta^2 = \dot{\chi}^2 + \dot{\phi}^2 \). Here the definitions of the slow-roll parameters are

\[
\begin{align*}
\epsilon_a & = \frac{M_p^2}{2} \left( \frac{V_{ss}}{3H^2M_p^2} \right)^2 = \frac{1}{2M_p^2} \left( \frac{V_{ss}}{3H^2} \right)^2 \\
\eta_a & = \frac{V_{aa}}{3H^2} \\
\eta_{ab} & = \frac{V_{ab}}{3H^2},
\end{align*}
\]

where \( H \) is the Hubble parameter, \( M_p \) is the reduced Planck mass and \( a, b, .. \) denote \( \phi, \chi, \sigma \) and \( s \). We introduce separation given by \( \epsilon_H = -H/H^2 = \epsilon_\phi + \epsilon_\chi \). \( K_A \) is defined using the transfer matrix and is given by

\[
\cos K_A = \frac{T_{RS}}{\sqrt{1 + T_{RS}^2}},
\]

where \( T_{RS} \) is an element of the transfer matrix, which describes the evolution of the adiabatic and the isocurvature perturbations. We define the instantaneous adiabatic and entropy perturbations as

\[
\begin{align*}
\delta \sigma &= \delta \phi \cos \theta + \delta \chi \sin \theta \\
\delta s &= -\delta \phi \sin \theta + \delta \chi \cos \theta,
\end{align*}
\]

1 In this section we are trying to explain the situation as intuitively as possible. See Ref. if details are needed.
where $\tan \theta = \dot{\chi}/\dot{\phi}$. Also we define the curvature and entropy perturbations

$$R = H \frac{\delta \sigma}{\dot{\sigma}}, \quad S = H \frac{\delta s}{\dot{\sigma}}.$$  \hspace{1cm} (6, 7)

The curvature perturbation after horizon exit is expressed as

$$R = [R_s + T_{RS}S_s].$$  \hspace{1cm} (8)

In later calculation the subscript $s$ will be omitted if obvious. Since here we are assuming “temporal” $\dot{\chi} \neq 0$, adiabatic velocity soon reaches $\dot{\sigma} \approx \dot{\chi}$. The final curvature perturbation recovers the original single-field inflation model [12]: $R \approx \frac{H}{\dot{\phi}} \delta \phi$.

Let us see more details about the scalon contributions. If $\dot{\chi}_s \gg \dot{\phi}_s$, the trajectory has a turn during inflation. Initially the adiabatic direction is $\dot{\sigma} \approx \dot{\chi}$ but it will be $\dot{\sigma} \approx \dot{\phi}$ before the end of inflation. Then the curvature perturbation becomes $R \approx \frac{H}{\dot{\phi}} \delta \phi \gg R_s \approx \frac{H}{\dot{\chi}} \delta \chi$, which is possible since $T_{RS} \gg 1$. Then one will find $\cos K_A \sim 1$ and the spectral index given by

$$n_s - 1 \approx -2\epsilon_H + 2\eta_\phi.$$  \hspace{1cm} (9)

The opposite limit ($\dot{\chi}_s \ll \dot{\phi}_s$) is the standard single-field inflation. One will find $\sin K_A \sim 1$ and an almost straight trajectory. The spectral index will be

$$n_s - 1 \approx -6\epsilon_H + 2\eta_\phi.$$  \hspace{1cm} (10)

Seeing the above discrepancy between (9) and (10), one may find that the situation is intuitively similar to the curvaton. More radical situation can be found in the inflating curvaton [14]. The point is that in the above formalism one has to take into account the scale-dependence of the transfer function $T_{RS}$.

The latter case ($\dot{\chi}_s \ll \dot{\phi}_s$ already at the beginning) seems to be the same as the conventional single-field scenario. However, as far as $\dot{\chi}_s \neq 0$, there could be contribution caused by $\epsilon_\chi \neq 0$. Although in the spectral index the contribution of the scalon is negligible, in the higher runnings the spectrum may obtain peculiar scale dependence from the scalon [8]. We will be back to this topic in Sec.11D.

Below, we will examine the relation $r + 8n_t = 0$ when $\epsilon_\chi \neq 0$. Since one extra degree of freedom is added to the original scenario, we need another observable (i.e. an independent equation) that can fix the ambiguity. We show that the running of the tensor mode can play the role.

The spectrum of the tensor perturbation is $P_T^{1/2} = H/(2\pi)$, which gives the tensor to scalar ratio

$$r = \frac{P_r}{P_S}.$$  \hspace{1cm} (11)

When $\dot{\chi}_s \approx 0$, we find conventional result $r = 16\epsilon_\phi$ and $n_r = \frac{d \log r}{d \log k} = 4\epsilon_H - 2\eta_\phi$. However, when $\dot{\chi}_s \neq 0$, things are not so trivial. Remember that in both cases the running of $r$ can “directly” be evaluated using indices of the scalar and the tensor modes as

$$n_r = \frac{d \log r}{d \log k} = 1 - n_s + n_t,$$  \hspace{1cm} (12)

where $n_t = -2\epsilon_H$. We use this relation to check the consistency of the calculation.

Let us see more details of the relations between parameters, and see how one can remove the ambiguity.

- We first consider $\dot{\chi}_s \ll \dot{\phi}_s$. Using $d \ln k = H dt$ and the definitions of the slow-roll parameters [2], one will find

$$\frac{1}{d \ln k} \epsilon_\phi = 4\epsilon_\phi \epsilon_H - 2\eta_\phi \epsilon_\phi,$$  \hspace{1cm} (13)

which leads to $n_r = 4\epsilon_H - 2\eta_\phi$. Of course the result is consistent with Eq.(12). Then, from the spectral index (10) and $n_t = -2\epsilon_H$, one can evaluate the slow-roll parameters from the observables as

$$\epsilon_H = -\frac{n_t}{2},$$

$$\epsilon_\phi = \frac{r}{16},$$

$$\eta_\phi = \left(\frac{n_s - 1}{2}\right) - 3n_t,$$

$$\epsilon_\chi = -\frac{n_t}{2} + r \frac{1}{16}.$$  \hspace{1cm} (14)

Here $r + 8n_t \neq 0$ will be the sign of $\epsilon_H \neq \epsilon_\phi$.

- Consider the case with $\dot{\chi}_s \gg \dot{\phi}_s$, in which $\chi$ soon ceases to be dynamical during inflation. In this case we are expecting $\epsilon_\chi \sim \epsilon_\phi$ (i.e. the scalon is dominating $\epsilon_H$). From the direct calculation [12], one will find $n_r = -2\eta_\phi$. Although rather complicated, one can evaluate the same result by considering $T_{RS}$ [12]. To distinguish the scale dependence, it would be useful to define $r \equiv 16\epsilon_{\phi^{**}}$ and evaluate

$$\epsilon_H = \frac{n_t}{2},$$

$$\epsilon_{\phi^{**}} = \frac{r}{16},$$

$$\eta_\phi = -\frac{n_r}{2} = \left(\frac{n_s - 1}{2}\right) - n_t,$$

$$\epsilon_H - \epsilon_{\phi^{**}} = -\frac{n_t}{2} + r \frac{1}{16}.$$  \hspace{1cm} (15)

Again, discrepancy $\epsilon_H - \epsilon_{\phi^{**}} \neq 0$ is the sign of an extra dynamical field.

---

2 For our purpose we are omitting details. Ref.15, 16 will be helpful.
Note that in both limits \( r + 8n_t \neq 0 \) is the sign of additional dynamical field. Although \( \chi \) introduces ambiguity to the conventional relations, it can be removed if we can measure \( n_t \).

**B. Deviation from the slow-roll (single-field inflation)**

Introduction of “extra degree of freedom” does not always require additional scalar field. It may appear as a parameter measuring deviation from the inflationary attractor. First, remember that at the very beginning of inflation the inflaton velocity \( (\hat{\phi}) \) may not coincide with the slow-roll velocity \( (\hat{\phi}_s) \). Moreover, if the inflaton is moving fast in the opposite direction before the onset of inflation, inflaton may even stop during inflation \(^{[17–19]}\).

Although the analysis could be slightly model-dependent, the essential of the argument is quite simple. If the inflaton velocity deviates from the slow-roll velocity defined there \( (i.e., \hat{\phi} \neq \hat{\phi}_s(t_s) \equiv -V_\phi(\phi_s)/3H) \), the curvature perturbation converges to the value evaluated using the slow-roll velocity. Namely, one will find the evolution of the curvature perturbation \( \frac{d}{dt}(\hat{\phi} - \hat{\phi}_s) \sim -3H(\hat{\phi} - \hat{\phi}_s) \) for the deviation.

If the inflaton “stops” during inflation one will find \( R_D \sim -1 \), which gives \( n_s \approx 0 \). The result is consistent with the naive intuition: scale dependence disappears when inflaton stops.

Since \( R_D \) changes quickly during inflation, there will be a signature of \( R_D \neq 0 \) in the running of the spectral index. One will find

\[
\alpha_s \equiv \frac{d\alpha_s}{d\ln k} \approx (1 + R_D)\alpha_s^{(0)} - 3R_D(n_s^{(0)} - 1) - 6R_D^2 \left[ -6\epsilon^{(0)} \right]_{\epsilon_H} - 2n_\phi \epsilon^{(0)}_{\epsilon_H}.
\]

where \( \epsilon^{(0)}_{\epsilon_H} \) is almost excluded by the observations, we can see that \( R_D > 0 \) is not a realistic scenario.

Note that the blue spectrum of hybrid-type inflation \( (n_s - 1 \approx 2n_\phi > 0 \text{ for } \epsilon_\phi \approx 0) \) can be turned into red when \( R_D < -1 \). Therefore the ambiguity related to the deviation is quite serious. Even though \( r \) in hybrid inflation could be very small and the tensor mode could not be seen in the observations, more precise measurement of \( \alpha_s \) can be used to exclude such possibility.

\[
r + 8n_t = 0 \text{ is violated because } \epsilon_H \text{ is not identical to } \epsilon^{(0)}_{\epsilon_H}. \quad \text{Here } r = 16\epsilon_\phi \text{ and } n_t = -2\epsilon_H. \quad \text{Again, the additional parameter } R_D \text{ can be fixed if } n_t \text{ could be observed. Since Eq.} 17 \text{ can be rewritten as } \epsilon_H \approx (1 + R_D)^2 \epsilon^{(0)}_{\epsilon_H}, \text{ we find } (1 + R_D)^2 = -\frac{2\alpha_s}{\epsilon^{(0)}_{\epsilon_H}}
\]

**C. Hybrid inflation and other models**

The usual multi-field model \(^{[12]}\) considers “evolution” of the curvature perturbation during inflation. In the model represented in Fig.\( 11 \) this effect compensates the curvature perturbation. The situation we are going to consider in this section is rather different from such scenario. We introduce the scalon that can survive until the end of inflation. Then, the small initial curvature perturbation is compensated by the curvature perturbation generated at the end.

Typically the scenario of generating the curvature perturbation at the end of inflation is considered for hybrid-type potential, since the original Lyth’s model \(^{[20]}\) considers modulation of the waterfall \( (\delta\hat{\phi} \neq 0) \). In that way the Lyth’s scenario requires interaction with the waterfall field. However, similar mechanism works for many variety of inflationary models in which the potential is given by \( V = V_0 + V(\phi) \ (V_0 \gg V(\phi)) \). This includes Higgs
inflation. Although the result is quite conceivable in the light of the \( \delta N \) formalism, it is not obvious how the initially small curvature perturbation can be compensated during inflation.

To void confusions we first review the discussion in Ref.[21]. The calculation can directly be applied to other non-hybrid models.

First consider the inflaton \( \phi \) and the waterfall field \( \sigma \) with the hybrid-type potential given by

\[
V(\phi, \sigma) = \frac{\lambda^2}{4} (\sigma^2 - M^2)^2 + \frac{g^2}{2} \sigma^2 \phi^2 + \frac{1}{2} m_\phi^2 \phi^2. \tag{21}
\]

Suppose that inflation starts with \( \phi > \phi_c \) and the waterfall field is not perturbed, the entropy perturbation generated at the horizon exit:

\[
\delta N_e \equiv - \frac{1}{\eta} \frac{\delta \phi}{\phi} \mid_{e} \neq 0. \tag{24}
\]

This is the usual scenario of “generating the curvature perturbation at the end”.

In contrast to the original scenario, we are introducing trivial field, which is decoupled from the waterfall field. Therefore, we are not expecting \( \delta \phi_c \neq 0 \). Just for the simplest example, we consider

\[
\phi^2 + \chi^2 = \left( \frac{\lambda M}{g} \right)^2. \tag{23}
\]

Then, if \( \chi \) is lighter than the inflaton, the entropy perturbation \( \delta s \approx \delta \chi \neq 0 \) creates the perturbation \( \delta \phi = -\frac{1}{\chi} \delta \chi \) at the end. Consequently, the perturbation of the number of e-foldings created at the end of inflation is given by

\[
\delta N_e \equiv \frac{1}{\eta} \frac{\delta \phi_c}{\phi_c} \mid_{e}. \tag{25}
\]

Note that unlike the usual multi-field extension of the hybrid-type potential we are omitting interaction \( \sim \sigma^2 \chi^2 \). Degeneracy of the mass term \( (m_\chi = m_\phi \equiv m) \) makes the trajectory straight and avoids the effect considered in Sec.IIA. The adiabatic field is defined as \( \sigma^2 \equiv \phi^2 + \chi^2 \), which gives

\[
\phi = \sigma \cos \theta \tag{26}
\]

\[
\chi = \sigma \sin \theta. \tag{27}
\]

We find the end for the adiabatic field is

\[
\sigma_c(\theta) \equiv \frac{\phi_c}{\cos \theta} = \frac{\lambda M}{g \cos \theta}. \tag{28}
\]

\[\text{FIG. 2: Hybrid inflation with a trivial scalar field. Entropy perturbation (\( \delta \theta \)) causes \( \delta N \) at the end.}\]

which is perturbed when \( \delta \theta \approx \delta s/\sigma \neq 0 \). Therefore, although \( \phi_c \) is not perturbed, the entropy perturbation \( \delta \theta \neq 0 \) causes \( \delta N_e \neq 0 \) at the end. The curvature perturbation generated at the end of inflation is thus given by

\[
\delta N_e \equiv \frac{H}{\sigma_c} \left( \frac{\delta \sigma}{\sigma} \right)_e. \tag{29}
\]

Considering perturbation generated at the horizon exit:

\[
\delta N_e = \frac{1}{\eta} \frac{\delta \sigma}{\sigma_c} \mid, \tag{30}
\]

we find the ratio between “initial” and “at the end”:

\[
\left| \frac{\delta N_e}{\delta \sigma} \right| \approx \tan \theta. \tag{31}
\]

Here \( |\delta s| \approx |\delta \sigma| \) is used for the calculation.

More intuitive argument is possible. Considering similar triangles in Fig.2, we find the number of e-foldings

\[
N = \frac{1}{\eta} \ln \frac{\sigma_c}{\sigma}, \tag{32}
\]

Therefore, intuitively the \( \delta N \) formalism suggests

\[
\frac{\delta N}{\delta \sigma} = \frac{\delta \phi_c}{\eta \phi_c} = \frac{1}{\eta} \frac{\delta \sigma}{\sigma_c} - \frac{1}{\eta} \frac{\delta \sigma_c}{\sigma_c}. \tag{33}
\]

Let us see what happens if \( \chi \gg \phi \). In the simplest case discussed above, one can immediately find \( \chi \gg \phi \)
and \( \theta \sim \pi/2 \). Then the final curvature perturbation is dominated by \( \delta N_e \), which reproduces the “conventional” perturbation \( \delta N \sim 1/2 \). Here, we already know that the scalon can shift the scale-dependence of the spectrum. Separating the slow-roll parameter \( \epsilon_H \) from \( \epsilon_H + \epsilon_\chi \), we find the spectral index given by \[ n_s - 1 \simeq -2\epsilon_H + 2\eta_0. \] (34)

For \( \chi \ll \phi \), generation of the curvature perturbation at the end is negligible and we find \( n_s = -6\epsilon_H + 2\eta \). In both cases \( \epsilon_H \) is shifted by \( \epsilon_\chi \). The result is quite similar to the conventional “multi-field” model, although the mechanism of generating curvature perturbation could be different. The property of the scalon is also different. For the model considered in this section, \( \chi \gg \phi \) is allowed until the end of inflation.

Usually the typical hybrid-type model is excluded because the spectrum is blue \( (n_s - 1 < 0) \). However, as we have stated above, one can introduce a trivial field \( \chi \) to change the spectral index into red \( (n_s - 1 > 0) \) \[21\]. Again, running of the tensor mode is needed to measure the scalon to discriminate inflationary models. Even though \( r \) could be very small and the tensor mode could not be seen, more precise measurement of \( \Delta s \) can be used to examine such possibility. Indeed, in the above calculation there is no bound on \( \eta_\chi \), since \( \eta_\chi \) does not appear in the spectral index. To put bound on \( \eta_\chi \) we need more precise measurement of the higher runnings.

The situation presented in Fig. 2 is quite general. It can be applied to any inflationary model in which the end of inflation is not affected by the scalon. Imagine a potential like \( V = V_0 + V(\phi) \) with \( V_0 \gg V(\phi) \).

### D. Higher runnings for fast-rolling \( \chi \)

The discrepancy \( r + 8n_s \neq 0 \) will be found when both the tensor mode and its running could be observed in future experiments. It indicates the presence of extra dynamical degree of freedom.

The situation could be serious if there are many scalar fields, whose effective masses are \( O(H) \) or less during inflation. This may happen in common supersymmetric model. If one considers a quadratic potential \( V(\chi) = 1/2 m_\chi^2 \chi^2 \) for a fast-rolling field \( \chi \), one will find \[ \chi(t) = \chi_0 e^{-Kt}, \] (35)

where

\[ K \equiv \frac{3}{2} H \left[ 1 - \sqrt{1 - \frac{4}{9} \left( \frac{m_\chi^2}{H^2} \right)} \right]. \] (36)

This gives

\[ \dot{\rho}_\chi \simeq -K \left( \frac{K^2}{m_\chi^2} + 1 \right) \rho_\chi, \] (37)

where contribution from the kinetic energy has not been neglected. Just for simplicity we consider \( \rho_A \simeq -K \rho_A \).

Let us assume \( \rho_\phi \simeq \rho_\chi \) at the moment when inflaton energy starts to dominate and the inflationary expansion starts due to \( \rho_\phi \). If one wants to claim that the scalon effect is negligible, one needs to consider extra number of e-foldings \( N_A \) before the CMB perturbation exits horizon. We find \( N_A \gtrsim 57 \) for \( K \sim 0.1 \) and \( \alpha < 10^{-4} \), or \( N_A \sim 18 \) for \( K \sim 0.5 \) and \( \alpha_s < 10^{-4} \).

Slow-rolling \( \chi \) may also change the spectral index and predict tiny \( \alpha_s \) at the same time. In that case we find

\[ \Delta \alpha_s \simeq -6\epsilon_\chi \left( 4\epsilon_H - 2\eta_\chi \right), \] (38)

which could be negative if \( 2\epsilon_H > \eta_\chi \) or simply \( \eta_\chi < 0 \). In contrast to the fast-rolling case, it seems somewhat difficult to inflate away all slow-rolling scalon fields before the onset of the last \( N = 60 \) e-foldings.

### III. CONCLUSION AND DISCUSSION

If the Universe started with a chaotic state, there could be many dynamical fields other than the inflaton. Inflationary expansion starts when the vacuum energy of the inflaton starts to dominate. At that moment other dynamical fields may have energy density comparable to the inflaton vacuum energy. If one wants to disregard those dynamical fields in the calculation of the spectrum, these fields must be inflated away before the onset of the last 60 e-foldings. This is not a trivial assumption.

In this paper we considered a trivial (non-interacting) field \( \chi \) and calculated the possible shift of the scale dependence. We showed that observation of the tensor mode is crucial for removing the ambiguity. The tensor mode gives direct measurement of \( \epsilon_\chi \), while the higher running of the scalar mode may give an indirect measure. Since the Planck constraint on isocurvature perturbations is very severe, those additional field should not be the primary source of cold dark matter \[23\]. We are implicitly avoiding the case in which the scalon creates significant isocurvature perturbations. Also, just for the simplicity of the argument we have avoided mixed perturbations.

In addition to the scalon, deviation from the slow-roll may have similar effect. Again, observation of the tensor mode is crucial for removing the ambiguity. The tensor mode gives direct measurement of the deviation. More precise measurement of the higher runnings of the scalar mode will gives an indirect measure.

### Acknowledgement

K.K. is supported in part by Grant-in-Aid for Scientific research from the Ministry of Education, Science, Sports, and Culture (MEXT), Japan, Nos. 21111006, 23540327, 26105520 (K.K.). The work of K.K. is also supported by the Center for the Promotion of Integrated Science (CPIS) of Sokendai (1HB5804100).
[1] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999) [hep-ph/9807278].
[2] K. A. Olive, Phys. Rept. 190, 307 (1990).
[3] P. A. R. Ade et al. [POLARBEAR Collaboration], Astrophys. J. 794, no. 2, 171 (2014) [arXiv:1403.2360 [astro-ph.CO]].
[4] P. A. R. Ade et al. [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO].
[5] G. Hinshaw et al. [WMAP Collaboration], Astrophys. J. 794, no. 2, 171 (2014) [arXiv:1403.2369 [astro-ph.CO]].
[6] P. A. R. Ade et al. [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO].
[7] M. Kawasaki, M. Yamaguchi and J. 'i. Yokoyama, Phys. Rev. D 68, 023508 (2003) [hep-ph/0304161]: T. Kobayashi and O. Seto, arXiv:1404.3102 [hep-ph]; A. Ashoorioon, K. Dimopoulos, M. M. Sheikh-Jabbari and G. Shiu, arXiv:1403.6999 [hep-th]; C. R. Contaldi, M. Peloso and L. Sorbo, arXiv:1403.4596 [astro-ph.CO]; J. McDonald, arXiv:1403.6650 [astro-ph.CO]; M. S. Sloth, arXiv:1403.8051 [hep-ph]; M. Czerny, T. Kobayashi and F. Takahashi, arXiv:1403.4589 [astro-ph.CO]; D. K. Hazra, L. Shafileo, G. F. Smoot and A. A. Starobinsky, arXiv:1404.0360 [astro-ph.CO].
[8] K. Kohri and T. Matsuda, arXiv:1405.6766 [astro-ph.CO].
[9] T. Matsuda, JHEP 0802, 099 (2008) arXiv:0802.3573 [hep-th].
[10] T. Matsuda, JCAP 0609, 003 (2006) [hep-ph/0606187].
[11] T. Fujita, M. Kawasaki and S. Yokoyama, JCAP 1409, 015 (2014) arXiv:1404.0951 [astro-ph.CO].
[12] C. T. Byrnes and D. Wands, Phys. Rev. D 74, 043529 (2006) [astro-ph/0605679].
[13] J. O. Gong, Phys. Rev. D 79, 063520 (2009) [arXiv:0710.3835 [astro-ph]].
[14] K. Dimopoulos, K. Kohri, D. H. Lyth and T. Matsuda, JCAP 1203, 022 (2012) [arXiv:1110.2951 [astro-ph.CO]]; K. Dimopoulos, K. Kohri and T. Matsuda, Phys. Rev. D 85, 123541 (2012) [arXiv:1201.6037 [hep-ph]]; S. Enomoto, K. Kohri and T. Matsuda, Phys. Rev. D 87, no. 12, 123520 (2013) [arXiv:1210.7118 [hep-ph]].
[15] J. O. Gong, JCAP 1407, 022 (2014) [arXiv:1403.5163 [astro-ph.CO]].
[16] X. Gao, T. Li and P. Shukla, JCAP 1410, no. 10, 008 (2014) [arXiv:1403.0654 [hep-th]].
[17] O. Seto, J. 'i. Yokoyama and H. Kodama, Phys. Rev. D 61, 103504 (2000) [astro-ph/9911119].
[18] B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. 78, 537 (2006) [astro-ph/0507632].
[19] Y. -i. Takamizu and J. 'i. Yokoyama, Phys. Rev. D 83, 043504 (2011) [arXiv:1011.4566 [astro-ph.CO]].
[20] D. H. Lyth, JCAP 0511, 006 (2005) [astro-ph/0510443].
[21] T. Matsuda, JCAP 1204, 020 (2012) [arXiv:1204.0303 [hep-ph]].
[22] K. Dimopoulos and M. Axenides, JCAP 0506, 008 (2005) [hep-ph/0310194].
[23] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A16 (2014) [arXiv:1303.5076 [astro-ph.CO]]; P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A22 (2014) [arXiv:1303.5082 [astro-ph.CO]]; P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A24 (2014) [arXiv:1303.5084 [astro-ph.CO]].