Electromagnetic form factors in the Hypercentral Constituent Quark Model

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Abstract
We discuss the recent experimental results on the ratio between the electric and magnetic proton form factors and how they can be described by the hypercentral Constituent Quark Model.

1 INTRODUCTION
The interest in the electromagnetic form factors of the nucleon has been increased by the recent results of the Jefferson Laboratory on the ratio between the electric and magnetic form factors of the proton [1, 2]. At variance with the expectations, the ratio deviates strongly from 1 and, for $Q^2 \geq 1 \text{ (GeV/c)}^2$, the ratio decreases with an almost linear behaviour, pointing towards the possible existence of a zero at $Q^2 \approx 8 \text{ (GeV/c)}^2$.

These unexpected results pose some problems. The first one is the compatibility of the new data with the traditional ones obtained from a Rosenbluth plot. In this respect much attention is been devoted to the two-photon exchange mechanisms twoph, which however seem to be too small for reconciling the two sets of data. A critical re-analysis of the Rosenbluth procedure is also being performed [3], with promising results.

The main further problem is the physical picture emerging from the data, that is the origin of the decrease of the ratio and the eventual presence of a zero in the electric form factor forces.

The planned experiments at higher $Q^2$ will provide the answer about the occurrence of a zero in the electric form factor of the proton. From the theoretical point of view such zero is a challenge for most theoretical models for the internal proton structure.
In this contribution we report the results of recent relativistic calculations of the elastic nucleon form factors within the hypercentral Constituent Quark Model (hCQM).

2 THE HYPERCENTRAL CONSTITUENT QUARK MODEL

In the hCQM the $SU(6)$ invariant quark potential is assumed to be \[ V(x) = -\frac{x}{x} + \alpha x \] \hspace{1cm} (1)
where $x$ is the hyperradius.
\[ x = \sqrt{\rho^2 + \lambda^2}, \] \hspace{1cm} (2)

Interactions of the type linear plus Coulomb-like have been used since long time for the meson sector, e.g. the Cornell potential. This form has been obtained in recent Lattice QCD calculations [6, 7] for $SU(3)$ invariant static quark sources.

The three-quark potential 1, depending on the hyperradius $x$ only, is hypercentral. Then it can be considered as the lattice two-body interaction within the so called hypercentral approximation, that is averaged over the hyperangle $\xi = \arctg(\rho/\lambda)$ and the angles $\Omega_\rho, \Omega_\lambda$: this approximation has been shown to be valid, specially for the lower energy states [8]. On the other hand, the hyperradius $x$ depends on the coordinates of all the three quarks, therefore the interaction $V(x)$ is in general a three-body potential.

The 'hypercoulomb' part of $-\frac{x}{x}$ of the potential 1 has interesting properties [8, 9]. It leads to a power-law behaviour of the proton form factor and of all the transition form factors and it has a perfect degeneracy between the first $0^+$ excited state and the first $1^-$ states.

The $SU(6)$ violation is taking into account by adding a standard hyperfine interaction $H_{hyp}$ [10], treated as a perturbation. The three quark hamiltonian for the hCQM is then [5]
\[ H = \frac{p^2_\rho}{2m} + \frac{p^2_\lambda}{2m} - \frac{\tau}{x} + \alpha x + H_{hyp} \] \hspace{1cm} (3)

The spectrum is described with $\tau = 4.59$ and $\alpha = 1.61 \text{ fm}^{-2}$ and the standard strength of the hyperfine interaction needed for the $N - \Delta$ mass difference [10].

3 THE ELECTROMAGNETIC FORM FACTORS

The model has been used for the prediction of various physical quantities of interest, namely the photocouplings [11], the electromagnetic transition amplitudes [12], the elastic nucleon form factors [13] and the ratio between the electric and magnetic proton form factors [14].
The calculated r.m.s. radius of the proton, corresponding to the potential parameters quoted at the end of the last section, is about 0.5 fm. This is actually the value which in earlier calculations has been fitted in order to reproduce the $D_{13}$ photocouplings \cite{15, 16}, therefore it is not surprising that the hCQM predicts helicity amplitudes for the negative parity resonances in fair agreement with data. However, this low value prevents from a good description of the elastic form factors of the nucleon.

The hCQM is non relativistic and one could in principle think that this is the origin of the above discussed discrepancies. Actually, first order relativistic corrections have been introduced in the hCQM. The three quark nucleon states have been boosted to the Breit frame and the matrix elements of the three quark current have been expanded up to first order in the quark momentum. The result is that the theoretical curves are much closer to the experimental data. The most remarkable effect is, however, obtained for the ratio of the electric ($G_p^E$) and magnetic ($G_p^M$) proton form factors

\[ R = \frac{\mu_p}{G_p^M} \]

where $\mu_p$ is the proton magnetic moment. The non relativistic calculations predict the value $R = 1$ and introducing the hyperfine interaction makes no difference ($R = 0.99$). However, the first order relativistic corrections \cite{14} give rise to a ratio which is significantly deviating from 1. It is interesting to note that the hCQM results coincide with the dispersion relation calculation of the Mainz group \cite{17}.

Relativity is then a fundamental ingredient for the description of the elastic nucleon form factors within the hCQM and therefore we have recently reformulated the model and calculated the elastic nucleon form factors in a completely relativistic frame. First, we have introduced in the hCQM the correct relativistic kinetic energy and, using the same hypercentral potential of Eq. (1), we have obtained an equivalently good description of the baryon spectrum \cite{18}. As for the calculation of the form factors \cite{19}, after having boosted the new three quark states to the Breit frame, we have taken into account the quark current up to any order. Finally, considering that constituent quark may acquire a finite size, we have introduced quark form factors. The free parameters in the quark form factors have been used to fit four set of experimental data, namely the ratio $R$, the proton magnetic form factor $G_p^M$, the neutron electric ($G_n^E$) and magnetic ($G_n^M$) form factors \cite{19}. The results for the ratio $R$ are shown in Fig. 1 and are remarkable, since the free parameters provided by the quark form factors are not sufficient by themselves to obtain a good fit, it is necessary that already the pointlike calculations provide a realistic description. In any case the size of the quarks obtained in this fit is not larger than 0.3 fm. It should be reminded that a recent analysis of the inelastic proton structure functions \cite{20} has shown a possible evidence of the proton containing extended objects with a size of about $0.2 - 0.3$ fm.
Figure 1:  The ratio $\mu_p \frac{G_{E,\mu}^p}{G_{M,\mu}^p}$ calculated with the relativized hypercentral Constituent Quark Model [19] (full curve). The data are taken from [1,2].

4 CONCLUSIONS

The recent Jlab data on the ratio $R = \mu_p \frac{G_{E,\mu}^p}{G_{M,\mu}^p}$ show a strong deviation from the value 1 predicted by the previous widely accepted dipole fit and by most models for the internal structure of the proton. Moreover, extrapolating their trend at higher $Q^2$, one can infer the presence of a dip. Of course, if the electric form factor of the proton has somewhere a zero, then the ratio $R$ is forced to decrease. In any case the data show that the electric and magnetic distributions of the proton are quite different. Moreover, as mentioned in the Introduction, there is the problem of reconciling these new data with those obtained from a Rosenbluth plot.

The results obtained with the hCQM allow to state that relativity is crucial in explaining the decrease of the $R$. It remains to explain the eventual zero in the proton electric form factor.

The answer to the question if there is a dip in the proton form factor is very important for the understanding of the internal nucleon structure and it will be hopefully obtained by the planned experiments.
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