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Influence of temporary migration on the transmission of infectious diseases in a migrants' home village

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Abstract

The number of temporary migrant workers from rural areas to urban areas in emerging market economies like China has increased dramatically since the early 1980s. Temporary migrant workers have been labeled as the major driving force for the rising incidence of infectious diseases in cities. However, it has not been well recognized that temporary migration indeed may have tremendous impacts on the spread of infectious diseases in migrants' home villages. In this paper, by proposing a delay differential equation model, we provide a framework to study the influence of temporary migration on the transmission of infectious diseases in a migrant workers' home village. The model is shown to admit a unique positive equilibrium which is locally asymptotically stable and is globally asymptotically stable under certain conditions. This implies that the disease always persists at a constant level. Considering tuberculosis as an example, we explore various disease prevention and control strategies numerically to demonstrate how migration related parameters affect the early outbreak of the disease. We find that a single control strategy, such as reducing the migration time period alone, has little effect on reducing the disease endemic level. For disease prevention and control, temporary migrant workers should be identified as the top target group, and a combination of several prevention strategies should be implemented.

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1. Introduction

Emergence and re-emergence of infectious diseases is very complex, and many factors contribute to this complexity (Wilson, 1995). Among those frequently identified are mobility and travel, for which there are many examples: the proportion of new tuberculosis (TB) cases and HIV cases has been increasing recently in China in migrants who have complex mobility (Jia et al., 2008; Zhang, 2001). Unstable living and working conditions, being away from sexual partners, economic pressure, and limited welfare benefits and health care available to urban residents make it very difficult for migrant workers to settle down permanently in urban cities. Thus they usually cannot access the social welfare benefits and health care available to urban residents (Zhang, 2001). Unstable living and working conditions, being away from sexual partners, economic pressure, and limited awareness of infectious diseases all contribute to a high incidence of infectious diseases among migrant workers.

In emerging market economies like China, there has been increasing population mobility, especially among a special group of people called migrant workers. Migrant workers (also called the floating population) emerged in China in the early 1980s when rural residents were allowed to seek employment in urban areas. Since then the number of migrant workers in China has been increasing dramatically. A report from the National Bureau of Statistics of China shows that there were a total of 145.33 million migrant workers in 2009, while the number in 1989 was only about 30 million. Migrant workers make a huge contribution to China's development, but they also bring many societal problems.

According to the 2009 report, among those 145.33 million migrant workers, 61.6% were young people (aged between 16 and 30), 76.5% did not finish their high school education and 51.1% received no occupational skills training. At a young age, lacking education and professional skills, most temporary migrant workers have to work in construction sites, restaurants, or crowded factories that require no or little experience. Temporary employment and China's current existing household registration system make it very difficult for migrant workers to settle down permanently in urban cities. Thus they usually cannot access the social welfare benefits and health care available to urban residents (Zhang, 2001). Unstable living and working conditions, being away from sexual partners, economic pressure, and limited awareness of infectious diseases all contribute to a high incidence of infectious diseases among migrant workers.

It is commonly believed that migrant workers, in contrast to non-migrant workers, are much more vulnerable to infectious diseases such as TB, HIV and sexually transmitted diseases (STDs) (Strand et al., 2007). For example, two-thirds of the 176 HIV cases reported by the Shanxi Province Epidemiological Station, were
migrant workers (U.S. Embassy Beijing AIDS in China, 2000). It is reported that in 2006, the migrant population accounted for 1638 of 4088 TB cases in Beijing (Jia et al., 2008). Due to migrant workers’ increasing mobility, there was a tenfold increase in the total incidence of syphilis over the past decade (Chen et al., 2007). For studies on migrant workers’ vulnerability to infectious diseases, see Li et al. (2004); Yang (2006) and references cited therein. We should point out that our general models concerning movements between patches and Brauer some theoretical immigration models are proposed, and in Jia (2001) and Yang (2004). However, to-urban migration) and the rising incidence of HIV/AIDS and government and the scholarly community (Yang, 2004). However, considering immigration. In Brauer and van den Driessche (2001) and Li et al. (2004); Yang (2006) and references cited therein. 

The link between temporary migration (especially the rural-to-urban migration) and the spread of infectious diseases in the migrants’ home village. Influenced by the Chinese traditional culture, migrant workers regularly return home for holiday family gatherings (especially during the period of Chinese New Year). This mobility pattern may cause an epidemic outbreak in their home village if some of the returning migrant workers were infected with an infectious disease while they were away from their home. For instance, returning migrants with the AIDS virus can unknowingly pass it onto their sexual partners (Lau and Thomas, 2001). In the meantime, migrant workers may also serve as a major vector for bringing the diseases to the urban areas when they return to work from their home village. Movement back and forth between rural and urban areas by temporary migrant workers is believed to be a key factor for the emergence and re-emergence of infectious diseases including TB and HIV/AIDS in China (Jia et al., 2008; Strand et al., 2007; Wang and Shen, 2009).

Due to the complex mobility patterns among migrant workers, it is very difficult to construct a mathematical model to study the overall influence of migration on the spread of infectious diseases. It is almost impossible to track each migrant worker’s working place. However, it is certain that migrant workers return home regularly. In Jia et al., 2008), we divide the total population in a migrants’ home village (with size \( N \)) into four classes denoted by \( S(t) \), \( I(t) \), \( R(t) \) and \( Z(t) \), representing the numbers of susceptible, infectious and recovered, with class sizes \( S(t) \), \( I(t) \), \( R(t) \) and \( Z(t) \), respectively. In Influenced by the Chinese traditional culture, migrant workers is almost impossible to track each migrant worker’s working place. Thereby, it is certain that migrant workers return home regularly. In Jia et al., 2008), we divide the total population in a migrants’ home village (with size \( N \)) into four classes denoted by \( S(t) \), \( I(t) \), \( R(t) \) and \( Z(t) \), representing the numbers of susceptible, infectious and recovered, with class sizes \( S(t) \), \( I(t) \), \( R(t) \) and \( Z(t) \), respectively. In

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Our model is given by a system of delay differential equations. For example, assuming some migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection. Throughout, we assume that migrant workers return home with an infection.

\[
\begin{align*}
S(t) &= \Lambda - \mu_S(t)S(t) - \beta S(t)I(t) - m_S e^{-\mu_S t} S(t-\tau), \\
I(t) &= \beta S(t)I(t) - \gamma I(t) - \mu_I(t)I(t) + p_m e^{-\mu_I t} S(t-\tau), \\
R(t) &= \gamma I(t) - \mu_R(t)R(t) + m_R e^{-\mu_R t} R(t-\tau).
\end{align*}
\]

The associated initial conditions are \( S(0) = \phi(0) \geq 0 \) for \( \theta \in [-\tau, 0] \), \( I(0) > 0 \) and \( R(0) \geq 0 \) for \( \theta \in [-\tau, 0] \). As the variable \( R \) does not appear in the first two equations, we can just consider the first two equations

\[
\begin{align*}
S(t) &= \Lambda - \mu_S(t)S(t) - \beta S(t)I(t) - m_S e^{-\mu_S t} S(t-\tau), \\
I(t) &= \beta S(t)I(t) - \gamma I(t) - \mu_I(t)I(t) + p_m e^{-\mu_I t} S(t-\tau),
\end{align*}
\]

with their associated initial conditions.

Our main results are given in the following theorem with the proofs postponed to the Appendix.

**Theorem 2.1.** Consider model (2.2) with the associated initial conditions. The model is well posed, i.e., there is a unique solution \((S(t), I(t))\) which exists globally, and \( S(t) \) and \( I(t) \) are positive and bounded for all \( t > 0 \). There exists a unique positive equilibrium, \( E = (S^*, I^*) \), which is locally asymptotically stable. Moreover, if

\[
\frac{\beta A}{(\mu_S + m_S + (1-p)m_S e^{-\mu_S t}) + \mu_I} < 1,
\]

then the equilibrium \( E \) is globally asymptotically stable.

![Fig. 1. Flow diagram of an SIR model.](image-url)
Remark 2.1. The above theorem implies that the disease always persists and the number of infectious individuals stabilizes at a constant level $I^*$ as long as the probability of migrant workers returning home with an infection is not zero.

3. Numerical simulations and control strategies

As currently there is no data that can be directly applied to our model, we take parameter values from various sources.

3.1. Demographic parameters

Consider a typical village in western China with a total population of 1000 in 1999. Using China’s natural population growth rates data from the World Bank, we can construct a population time series of this village as 1000, 1008, 1015, 1022, 1028, 1034, 1040, 1046, 1051, 1056 and 1062 from year 1999 to year 2009. Note that, without considering migration and disease induced death, the total population $N(t)$ follows the differential equation

$$N'(t) = A - \mu_N N(t).$$  \hfill (3.1)

According to the World Bank data, the average life expectancy in China is about 72 years, which suggests $\mu_N = 1/72 = 0.0139$ per year. Using the least-square method to fit the above population time series to (3.1) yields $A = 20.69$. According to a report from the (China National Bureau of Statistics), on average, a temporary migrant worker works outside his/her home village for about 9.4 months, so we take $\tau = \frac{94}{12} = 0.7833$ years. The death rate $\delta$ during migration is assumed to be slightly larger than $\mu_N$ with $\delta = 0.016$ per year (the parameter $\delta$ could be much higher than $\mu_N$ in reality).

3.2. Disease related parameters

As TB is one of the major infectious diseases in rural areas of China (Wang and Shen, 2009), we select parameter values for TB from Blower et al. (1995) to demonstrate the influence of migration parameters and control strategies on the transmission of infectious disease in a migrants’ home village. The time unit is one year. The parameters are: $\gamma = 0.058$ per year, $\mu_I = 0.1439$ per year.

3.3. Numerical simulations

We use the Matlab package DDE23 to obtain numerical solutions to system (2.2). We take $m_S = 0.3567$ per year, which implies every year $1 - e^{-m_S} \approx 30\%$ of the population from the village seeks temporary employment in other places. The proportion ranges from 10% to 40% in rural areas (China National Bureau of Statistics), so $m_S \in [0.10,0.5]$. With parameters $\beta = 0.001$ per year per person, $p = 10\tau/(1000+\tau)$, the resulting equilibrium level of infection $I^* = 84.71$. Fig. 2 indicates that $l(t) \rightarrow I^*$ as $t \rightarrow \infty$ and the equilibrium $E = (S^*, I^*)$ is stable.

3.4. Control strategies: numerical exploration

In this subsection we numerically evaluate several possible control strategies. It is reasonable to focus on the transient behavior (solution behavior during a relatively short time period) rather than the long time behavior. This is mainly because the control strategies may not be the same throughout a long time period and some model parameters may also have different values if the time period is too long. For example, more effective drugs may be developed, which will increase the value of $\gamma$, or the migration patterns could change with the development of the economy, which would change the values of $m_S$ and $\tau$. In particular, we mainly examine the two transient numbers: $l_{max} := \max(l(t), t \geq 0)$ (we use $t_{max}$ to denote the peak time at which $l_{max}$ is reached) and $l(5)$, based on the following arguments:

- $l_{max}$ is certainly a very important indicator for any infectious disease as it gives the largest number of infectious individuals and describes the worst possible situation. It is of importance to examine how different strategies will affect $l_{max}$.
- We evaluate the value of $l(5)$ for different control strategies because in China, the government always comes up with 5-year plans. During those 5 years, there are no sudden policy changes. We compare the outcome of year 5 for each strategy, with the aim of providing public health authorities some theoretical guidance on designing their 5-year plans.

In what follows, to explore several possible control strategies, we use the same type of initial conditions: $S(0) = \phi(0), I(0) = 0$; (2) $S(0) = \phi(0), I(0) = 10$; (3) $S(0) = \phi(0), I(0) = 20$; where $\phi(t) = (A/\mu_I) - ((A/\mu_I) - 1000)e^{-m_S(\theta + \tau)} = 1488.5 - 488.5 e^{-0.0139(\theta + 0.7833)}$ for $\theta \in [-0.7833,0]$. We fix $\beta = 0.001, m_S = 0.3567$ and vary $\tau$ with $p = 10\tau/(1000+\tau)$. Fig. 3 shows that both $l_{max}$ and $l(5)$ will be reduced under this control strategy, while $l^*$ and $l_{max}$ will be increased. In particular, compared to the reference solution, if $\tau$ drops from 0.7833 to 0.5, then $l_{max}$ has a slight drop by 5.44%, $l_{max}$ is increased by 0.42 years (about 5 months) and $l(5)$ drops by 31.3%, while in the long run, the equilibrium level of infection $I^*$ is slightly increased by 1.61%.

Strategy I: Reduce the average migration period $\tau$. We fix $\beta = 0.001, m_S = 0.3567$ and vary $\tau$ with $p = 10\tau/(1000+\tau)$. Fig. 4 shows that this strategy has the similar effect on $l_{max}$ and $l(5)$ as Strategy I does:
as we decrease $m_s$, both $I_{\text{max}}$ and $I(5)$ decrease, while $I^*$ and $t_{\text{max}}$ increase. In comparison with the reference solution, if $m_s$ drops to 0.2, then this results in a 6.85% decrease in $I_{\text{max}}$, a 1.5% increase in $t_{\text{max}}$, and a 38.44% decrease in $I(5)$. In the long run, the equilibrium level of infection $I^*$ is increased by 1.96%.

**Strategy III**: Reduce the probability $p$ of having an infection for returning migrant workers. We fix $\beta = 0.001$, $m_s = 0.3567$ and $\tau = 0.7833$. Let $p = p_6 \tau/(1000 + \tau)$ with $p_6$ varying from 1 to 10. Fig. 5 shows that $I_{\text{max}}$ will drop first and then increase, but the variation is very small. As $p$ decreases, both $I(5)$ and $I^*$ decrease and $t_{\text{max}}$ increases. If $p$ drops 90% to $\tau/(1000 + \tau)$, then the value of $I_{\text{max}}$ remains almost the same but buys 2.87 additional years to attain $I_{\text{max}}$. While $I(5)$ drops from 157.65 to 18.74, i.e., 88.1%. In the long run, the equilibrium level of infection $I^*$ drops only 0.61%.

**Strategy IV**: Reduce the transmission rate $\beta$. We fix $\tau = 0.7833$, $m_s = 0.3567$, and $p = 10\tau/(1000 + \tau)$. We vary $\beta$ to get Fig. 6. It is seen in Fig. 6 that $I_{\text{max}}$, $I(5)$ and $I^*$ all decrease and $t_{\text{max}}$ increases as we decrease $\beta$. If $\beta$ is reduced by 50%, then $I_{\text{max}}$, $I(5)$ and $I^*$ will be reduced by 44.95%, 79.99%, 19.11%, respectively, and $t_{\text{max}}$ will be increased by 8.49 years.

**Strategy V**: Encourage returning migrant workers to have a medical examination and apply quarantine/isolation to confirmed infectious cases. Let $Q$ stand for the number of returning migrant workers who are confirmed with infection and are placed in quarantine/isolation, and let $1-q$ be the success rate of confirming infectious cases. Then model (2.2) reduces to the following model:

$$
\begin{align*}
S(t) &= -\mu S(t) - \beta S(t)I(t) - m_s I(t) + (1-p) m S e^{-\alpha t} S(t-\tau), \\
I(t) &= \beta S(t)I(t) - \gamma I(t) - \mu I(t) + q pm Se^{-\alpha t} S(t-\tau), \\
Q(t) &= (1-q)p m Se^{-\alpha t} S(t-\tau) - \mu Q(t).
\end{align*}
$$

We fix $\tau = 0.7833$, $m_s = 0.3567$, $\beta = 0.001$, and $p = 10\tau/(1000 + \tau)$. Varying $q$ in (3.2), we obtain Fig. 7. The impact of this strategy is similar to that of Strategy IV; as $1-q$ increases, $I_{\text{max}}$, $I(5)$ and $I^*$ all decrease and $t_{\text{max}}$ increases. If $q=0.5$, then $I_{\text{max}}$, $I(5)$, $I^*$ will drop 1.4%, 45.81%, 1.94% and $t_{\text{max}}$ is increased to 10.34 years.

**Strategy VI**: Combine Strategies III, IV and V. In Fig. 8, the success quarantine/isolation rate is $1-q = 80\%$, the disease transmission coefficient is $\beta = 0.0005$, the probability of returning migrants being infected is $p = \tau/(1000 + \tau)$. Compared to the reference solution, the disease transmission rate is reduced by 50%, the probability of returning migrants being infectious drops 90%. Under these efforts, $I_{\text{max}}$ drops 46% (changes from 639.28 to 345.44), and $t_{\text{max}}$ is increased from 9.42 to 30.68. $I(5)$ drops dramatically from 157.65 to 0.65, while $I^*$ experiences a 23% drop (see Table 1).

### Table 1
Comparison between different control strategies on transient numbers $I_{\text{max}}$, $I(5)$ and the equilibrium level of infection $I^*$.

| Representative strategy (ST) | $I_{\text{max}}$ | $t_{\text{max}}$ | $I(5)$ | $I^*$ |
|-----------------------------|------------------|------------------|--------|-------|
| ST I: $\tau = 0.5$          | 604.49           | 9.84             | 108.31 | 86.07 |
| ST II: $m_s = 0.2$          | 595.50           | 9.98             | 97.05  | 86.37 |
| ST III: $p = \frac{1}{10}\tau_{\text{ref}}$ | 630.25           | 12.29            | 18.74  | 84.19 |
| ST IV: $\beta = 0.0005$     | 351.93           | 17.91            | 31.54  | 68.52 |
| ST V: $q = 0.5$             | 630.30           | 10.34            | 85.43  | 83.07 |
| ST VI: $q = 0.2$, $\beta = 0.0005$, $p = \frac{1}{10}\tau_{\text{ref}}$ | 345.44           | 30.68            | 0.65   | 65.41 |
| Reference solution:         | 639.28           | 9.42             | 157.65 | 84.71 |

### 4. Discussion and conclusion

In this paper, we have proposed a delay differential equation model to study the influence of temporary migration on disease transmission in a migrant workers’ home village. In the model, we have assumed that a fraction of returning migrant workers acquired infection when they were away from their home village. This model particularly applies to the huge migrant worker population in China. There has been evidence that migrant...
workers are more vulnerable to infectious diseases especially TB, HIV/AIDS and STDs because of migrant workers' high risk behaviors (Grusky et al., 2002; Li et al., 2004; Yang, 2006, 2004).

Theorem 2.1 shows that there is a unique equilibrium $E$ in our model, which is locally asymptotically stable and globally asymptotically stable if (2.3) is satisfied (see Remark A.1 for a weaker condition). This indicates that there is no disease free equilibrium and the disease remains endemic as long as there are cases in temporary migrants. The fact is that the proportion of new cases in migrants has been increasing year by year. It is reported that in 2006, the migrant population accounted for 1638 of 4088 TB cases in Beijing (Jia et al., 2008). Numerical simulations using parameter values derived from available data and TB-related parameters from the literature confirmed that returning infectious migrant workers can cause huge outbreaks in their home village.

To control the spread of infectious diseases in a migrants' home village, our numerical simulations (Figs. 3 and 4) indicate that reducing the temporary migration period $\tau$, or limiting the
migration rate $m_S$ is not very effective. Indeed these two strategies (Strategies I and II) are not practical as the government cannot have a regulation determining the period $t$ that a migrant worker can work, and it is not possible to control the migration rate by any regulation. The fact is that both $t$ and $m_S$ seem to have an increasing trend. The most effective measure is a combination of Strategies III, IV and V, that is to reduce three key parameters: $p$, $\beta$, and $q$ in (3.2). To this end, the following efforts should be encouraged:

- The government should improve migrant workers’ working and living conditions, offer free or affordable health care,
They temporarily live and work (Yang, 2006). A report from the culturally, from the “mainstream” society in the place where household registration system, different jobs and working conditions and different backgrounds, many temporary migrant workers are isolated, not only residentially, but also socially and culturally, from the “mainstream” society in the place where they temporarily live and work (Yang, 2006). A report from the Beijing Municipal Health Bureau shows that the total annual new registered active TB cases increased dramatically from 2000 through 2006, but the cases among the permanent residents only increased slightly (Beijing Research Institute for TB Control Work on the prevention of TB in Beijing). Thus, from a disease prevention and control point of view, temporary migrants should be identified as the target group. Due to migrants’ high mobility, lack of knowledge about infectious diseases, poor living and working conditions, and increased (sexual) risk behaviors, a combination of several effective interventions should be implemented.

It is worth pointing out that it is extremely useful for the government to collect related data for temporary migration. Each village can easily collect information about each migrant worker such as age, gender, health status (from medical examinations before leaving and after returning), education level, leaving dates and returning dates, cities he/she has been, working conditions, etc. Only with accurate data can modelers provide more accurate modeling and analysis and thus advise the government on more effective control strategies.

This framework can also be employed to study how temporary cross-border travel can influence the spread of infectious diseases in a traveler’s home community. It is possible for some travelers to become infected when they return home and there could be a spread among people who had close contacts with the infectious returning travelers. For example, all 28 confirmed measles cases reported between 1 January and 30 April 2008 in Canada were either imported or import-related (Lipskie and Varughese, 2008). One Ontario case had arrived from Pakistan 10 days prior to rash onset. Two other cases detected in Ontario were un-immunized siblings visiting from Switzerland where there was an ongoing measles outbreak. Our model analysis suggests that the prevention of a local disease outbreak is helped if each traveler has a medical examination after returning home.

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**Appendix A. Proof of Theorem 2.1**

To prove our main result, we first introduce a lemma which can be easily derived from the result of Example 2.6 in Hale and Verduyn Lunel (1993, p. 134).

**Lemma A.1.** Consider the linear delay differential equation

\[ x(t) = \lambda - \delta_1 x(t) + \delta_2 x(t - \tau). \]

If \( |\delta_2| < \delta_1 \), then

\[ \lim_{t \to \infty} x(t) = x^* = \frac{\lambda}{\delta_1 - \delta_2}. \]

**A.1. Well posedness of the model**

The existence and uniqueness of the solution follows directly from the step method for delay differential equations (Hale and Verduyn Lunel, 1993). Next we show that \( S(t) \) and \( I(t) \) are positive for \( t \in (0, \tau) \). Suppose not, then there must be a first time \( t_1 \in (0, \tau) \) such that \( S(t_1) = 0 \), which yields either (i) \( I(t_1) = 0 \) and \( S(t) \geq 0 \) for \( t \in [0, t_1] \) or (ii), \( S(t) = 0 \) and \( I(t) \geq 0 \) for \( t \in [0, t_1] \). For case (i), \( I(t) \geq \rho(t - t_1) / \rho_1 \) for \( t \in [0, t_1] \). Thus, \( I(t) \geq \rho(t - t_1) / \rho_1 \) which is in contradiction with \( I(t_1) = 0 \). For case (ii), \( S(t) = A(1 - p) m e^{-\rho t} S(t - \tau) \geq A > 0 \). On the other hand, to have \( S(t) = 0 \), \( S(t_1) \leq 0 \) must be true as \( S(0) > 0 \). This is a contradiction. In a similar fashion, we can show \( S(t) \) and \( I(t) \) are positive for \( t > \tau \).

Next we show both \( S(t) \) and \( I(t) \) are bounded. Note that \( S(t) > 0 \) and \( I(t) > 0 \) for \( t > 0 \). Then the first equation of system (2.2) gives

\[ S(t) \leq A - (\mu_5 + m_3) S(t) + (1 - p)m_5 e^{-\rho t} S(t - \tau). \]

Let \( z(t) \) be the solution of

\[ z'(t) = A - (\mu_5 + m_3) z(t) + (1 - p)m_5 e^{-\rho t} z(t - \tau), \]

with the initial condition \( z(\theta) = S(\theta) \) for \( \theta \in [-\tau, 0] \), then \( S(t) \leq z(t) \) for \( t \geq 0 \) by the standard comparison theorem for delay differential equations (Smith, 1995). Clearly, \( \mu_5 + m_5 > (1 - p)m_5 e^{-\rho t} \) and
it follows from Lemma A.1 that
\[ \lim_{t \to \infty} z(t) = \frac{A}{\mu_5 + m_3 - (1-p)pm_9e^{-\delta t}} = U_1^* . \]
This implies that
\[ \lim_{t \to \infty} \sup S(t) \leq \lim_{t \to \infty} \sup z(t) = U_1^* \] (A.1)
and thus \( S(t) \) is bounded. For any \( \epsilon > 0 \), there exists \( T_1 > \tau \) such that
\[ S(t) \leq U_1^* + \epsilon \quad \text{for } t \geq T_1 , \] (A.2)
Let \( n(t) = S(t) + L(t) \). It then follows from (2.2) and (A.2) that
\[ n'(t) = -\mu_5 S(t) - m_3 S(t) - \gamma(t) - \mu L(t) + m_9 e^{-\delta t} S(t - \tau) \leq -A - \min(\mu_5 + m_3, \gamma(t) + \mu L(t)) + m_9 e^{-\delta t}(U_1^* + \epsilon) \quad \text{for } t \geq T_1 . \]
By the same argument used to show the boundedness of \( S(t) \), one can show that
\[ \lim_{t \to \infty} \sup n(t) \leq \frac{A + m_9 e^{-\delta t} U_1^*}{\min(\mu_5 + m_3, \gamma(t) + \mu L(t))} \quad \text{for } t \geq T_1 . \] (A.3)
This shows that \( n(t) \) is bounded and hence \( L(t) \) is bounded.

A.2. Existence and uniqueness of the equilibrium

For convenience, we let \( a = \gamma + \mu L, \beta = \mu_5 + m_3 - m_9 e^{-\delta t}, \gamma = pm_9 e^{-\delta t}, b = pm_5 e^{-\delta t}, c = (1-p)pm_9 e^{-\delta t}, \) then \( a > 0, b > 0, c > 0 \) and \( A = [A\beta - a(b + k)]^2 + 4A\beta ab > 0 \). It follows from system (2.2) that the equilibrium \((S^*, \ell^*)\) satisfies
\[ b\ell \cdot \frac{a - \ell}{k} - \ell a + b \cdot \frac{A - al}{k} = 0 , \]
which gives
\[ \ell = \frac{-(A\beta - a(b + k)) - \sqrt{\ell^*}}{2A\beta} . \] (A.4)
Note that since \( A = [A\beta - a(b + k)]^2 + 4A\beta ab > 0 \), Eq. (A.4) has only one positive real root denoted by
\[ \ell^* = \frac{-(A\beta - a(b + k)) + \sqrt{\ell^*}}{2A\beta} . \]
which immediately yields
\[ S^* = \frac{A - a\ell^*}{k} = \frac{A\beta + a(b + k) - \sqrt{\ell^*}}{2k\beta} . \]

A.3. Local stability

Let \( S(t) = x(t) + S^* \) and \( L(t) = y(t) + \ell^* \), then \( x(t) \) and \( y(t) \) satisfy the following equations:
\[ \begin{align*}
x'(t) &= -(m_5 + m_3)x(t) - \beta y^*(t)x(t) + (1-p)m_9 e^{-\delta t}x(t - \tau) - \beta aS^*y(t), \\
y'(t) &= \ell^* y(t)x(t) + pm_9 e^{-\delta t}x(t - \tau) + (bS^* - a)y(t).
\end{align*} \] (A.5)
Linearizing the Eq. (A.5) about \((0,0)\) yields
\[ \begin{align*}
x'(t) &= -(m_5 + m_3 + \beta y^*)(0)x(t) + (1-p)m_9 e^{-\delta t}x(t - \tau) - \beta aS^*y(t), \\
y'(t) &= \ell^* (0)x(t) + pm_9 e^{-\delta t}(x(t - \tau) + (bS^* - a)y(t).
\end{align*} \] (A.6)
Thus, the characteristic equation of (A.6) is
\[ \lambda^2 - a_1 \lambda + a_2 = 0 , \] (A.7)
where \( a_1 = m_5 + m_3 + \beta y^* + a - bS^* \), \( a_2 = (m_5 + m_3 + \beta y^*)(a - bS^*) + b^2 S^*a_3 = b \beta S^* - c(a - bS^*) \) and \( a_3 = -c \).

We first consider the special case with \( \tau = 0 \). In this case \( b = c = 0, a_3 = a_4 = 0 \) and Eq. (A.7) reduces to
\[ z^2 + a_1 z + a_2 = 0 . \] (A.8)
It is easy to compute two roots of (A.8) and both have negative real parts. Notice that the assumption (ii) of Beretta and Kuang (2002, p. 1146) holds, and hence no zero emerges from infinity. That is, \( \text{Re}(z) < -\infty \) for any zero of \( h(z) \). Therefore, as the delay \( \tau \) increases, the zeros of \( h(z) \) can cross the imaginary axis only through a pair of nonzero purely imaginary zeros. Let \( z = i\omega \) with \( \omega > 0 \) be a purely imaginary zero of \( h(z) \), then
\[ \omega^2 - ia_1 \omega - a_2 = (a_3 + ia_4 \omega)e^{\pm i\omega} . \]
Taking the modules of both sides of the above equation gives
\[ |\omega^2 - ia_1 \omega - a_2| = |(a_3 + ia_4 \omega)| , \]
which yields
\[ (\omega^2 - a_1^2)^2 + a_1^2 \omega^2 = a_3^2 + a_4^2 \omega^2 . \]
Let \( y = \omega^2 \), then \( y \) satisfies
\[ y^2 + b_1 y + b_2 = 0 , \] (A.9)
with \( b_1 = a_1^2 - a_2^2 - a_2 \) and \( b_2 = a_3^2 - a_4^2 \).

If Eq. (A.9) has no positive root, then \( h(z) = 0 \) has no nonzero purely imaginary roots, which implies that all roots of \( h(z) = 0 \) have negative real parts for all \( \tau \geq 0 \) and the equilibrium is locally asymptotically stable. Thus the local stability of the equilibrium \( E \) follows from the following two lemmas.

Lemma A.2. Assume \( b_2 \geq 0 \), then the equilibrium \( E \) is locally asymptotically stable provided one of the following conditions holds:

(i) \( b_1 \leq 0 \);
(ii) \( b_1 < 0 \) and the inequality
\[ b_1^2 < 4b_2 \] (A.10)
holds, where \( b_1 \) and \( b_2 \) are as in Eq. (A.9).

Proof. The inequalities \( b_1 \geq 0 \) and \( b_2 \geq 0 \) imply that Eq. (A.9) has no positive real roots. If \( b_1 < 0 \) and inequality (A.10) is true, then all roots of Eq. (A.9) are complex with positive real parts. Therefore, \( h(z) = 0 \) has no nonzero purely imaginary roots, which implies that all roots of \( h(z) = 0 \) have negative real parts for all \( \tau \geq 0 \) and the equilibrium \( E \) is locally asymptotically stable. □

Lemma A.3. The following statements hold:

(iii) \( b_2 > 0 \);
(iv) If \( b_1 < 0 \), then \( b_1^2 < 4b_2 \).

Proof. Direct calculations give
\[ a_1^2 = b^2 \beta^2 S^* + c^2 (a - bS^*)^2 - 2bc(a - bS^*) \]
and
\[ a_2^2 = [(m_5 + m_3 + \beta y^*)(a - bS^*) + b^2 S^*] \]
\[ + 2b^2 S^*[(m_5 + m_3 + \beta y^*)(a - bS^*)] . \]
Thus
\[ b_2 = a_2^2 - a_1^2 = 2bc(a - bS^*) + b^2 \beta^2 S^* + b^2 S^*[(m_5 + m_3 + \beta y^*)(a - bS^*)] \]
\[ + [(m_5 + m_3 + \beta y^*)^2 - c^2 (a - bS^*)^2 + b^2 S^*[(m_5 + m_3 + \beta y^*)(a - bS^*)] \]
\[ - b^2 S^*] > b^2 S^*[(m_5 + m_3 + \beta y^*)(a - bS^*)] - b^2 S^* . \]
To show $b_1 > 0$, one only needs to show
\[
(\mu_5 + m_5 + \beta^a)(a - \beta^a) > b_2^2 > 0. 
\]  
(A.11)

Note that $S^t = \frac{a^b}{(\beta^a + b)}$ and $b = pm_3 e^{-dt} < m_3$, thus
\[
(\mu_5 + m_5 + \beta^a)(a - \beta^a) > b_2^2 S^t = (\mu_5 + m_5 + \beta^a)^2 \cdot \frac{ab}{\beta^a + b} \frac{b_2^2}{\beta^a + b} > 0.
\]

For (iv), condition $b_1 < 0$ implies $a_1^2 - a_2^2 - 2a_2 < 0$. Obviously, $a_1^2 - a_2^2 > 0$ and $(a_1^2 - a_2^2) > (a_1^2 - a_2^2 - 2a_2) < 0$. Thus
\[
b_2^2 = (a_1^2 - a_2^2 - 4a_2^2) = (a_1^2 - a_2^2 - 2a_2) + 2a_2 = (a_1^2 - a_2^2 - 2a_2) - 2(2a_2 - a_2 - a_2 - a_2).
\]

It then suffices to show that $2a_2^2 - a_2^2 - 2a_2 < 0$. Since
\[
a_2^2 - a_2^2 = (\mu_5 + m_5 + \beta^a)(a - \beta^a) - c^2
\]

and
\[
a_2^2 = (\mu_5 + m_5 + \beta^a)(a - \beta^a) + 2(\mu_5 + m_5 + \beta^a)(a - \beta^a) - 2(c^2)
\]

we obtain
\[
2a_2^2 - a_2^2 - 2a_2 < 0. 
\]

A.4. Global stability

It is shown in (A.1) that
\[
\lim sup S(t) \leq U_2^k = \frac{A}{\mu_5 + m_5 - c},
\]

and for any $\epsilon > 0$, there exists $T_1 > \tau$ such that $S(t) \leq U_2^k + \epsilon$ for $t \geq T_1$. This, together with the second equation of system (2.2), yields
\[
I(t) \leq \beta(U_1^k + \epsilon)I(t) - \gamma I(t) + \mu_1 I(t) + p m_3 e^{-\delta t} (U_1^k + \epsilon), \quad t \geq T_1 + \tau
\]

If follows from condition (2.3) that $\gamma + \mu_1 > \beta(U_1^k + \epsilon) > 0$, for small $\epsilon$. Thus, by comparison, we have
\[
\lim sup(t) \leq \frac{b(U_1^k + \epsilon)}{\gamma + \mu_1 - \beta(U_1^k + \epsilon)}.
\]

Since $\epsilon$ can be arbitrarily small, we then have
\[
\lim sup(t) \leq \frac{bU_1^k}{\gamma + \mu_1 - \beta U_1^k} = U_1^k.
\]

For small $\epsilon > 0$, there exists $T_2 > T_1 + \tau$ such that $t(t) \leq U_1^k + \epsilon$ for $t \geq T_2$. It follows from this fact and the first equation of system (2.2) that for $t \geq T_2$
\[
S(t) \geq (A - \mu_5 S(t) - \beta(U_1^k + \epsilon)S(t) - m_5 S(t) + (1 - \mu_1) p m_3 e^{-\delta t} S(t - \tau)
\]

Again, by comparison and Lemma A.1, and noting that $\epsilon$ can be arbitrarily small, we have
\[
\liminf S(t) \geq \frac{A}{\mu_5 + m_5 - c + \beta U_1^k} \geq L_1^k.
\]

Thus, there exists $T_3 > T_2 + \tau$, such that $S(t) \geq L_1^k - \epsilon$ for $t \geq T_3$. This, together with the second equation of system (2.2), gives
\[
I(t) \geq \frac{-\gamma(S(t) - \epsilon)S(t) + (1 - \mu_1) p m_3 S(t) + c S(t - \tau)}{\gamma + \mu_1 - \beta(U_1^k - \epsilon)}, t \geq T_3 + \tau,
\]

which implies that
\[
\liminf I(t) \geq \frac{bU_1^k}{\gamma + \mu_1 - \beta U_1^k} \geq L_1^k > 0.
\]

Hence for a small $\epsilon > 0$, there exists $T_4 > T_3$ such that $I(t) \geq L_1^k + \epsilon$ for $t \geq T_4$. It then follows from the first equation of system (2.2) that
\[
S(t) \leq A - (\mu_5 + m_5 + \beta(U_1^k + \epsilon))S(t) + c S(t - \tau),
\]

which yields
\[
\lim sup S(t) \leq U_1^k = \frac{A}{\mu_5 + m_5 - c + \beta U_1^k} < U_1^k.
\]

In a similar fashion, we can show that
\[
\lim sup I(t) \leq U_2^k = \frac{b(U_1^k + \epsilon)}{\gamma + \mu_1 - \beta(U_1^k + \epsilon)} < U_2^k.
\]

\[
\lim sup I(t) \leq L_2^k = \frac{A}{\mu_5 + m_5 - c + \beta U_2^k} > L_2^k
\]

and
\[
\lim sup I(t) \leq L_2^k = \frac{bU_1^k}{\gamma + \mu_1 - \beta U_1^k} > L_1^k.
\]

Repeating this procedure, we obtain four sequences \(U_{k+1}, U_{k+1}, L_{k+1}, L_{k+1}\), \(k = 2, 3, \ldots\) with
\[
U_{k+1} = \frac{A}{\mu_5 + m_5 - c + \beta U_k^E} < U_k^E,
\]

\[
U_{k+1} = \frac{bU_{k+1}}{\gamma + \mu_1 - \beta U_{k+1}} < U_k^E,
\]

\[
L_{k+1} = \frac{A}{\mu_5 + m_5 - c + \beta U_k^E} > L_k^E
\]

and
\[
L_{k+1} = \frac{bU_{k+1}}{\gamma + \mu_1 - \beta U_{k+1}} > L_k^E
\]

(A.12)

This shows that these four sequences are all bounded and monotone, hence
\[
\lim U_{k+1} = U_k^E, \quad \lim U_k^E = U_k^E, \quad \lim L_{k+1} = L_k^E, \quad \lim L_k^E = L_k^E.
\]
exist. It then follows from (A.12) that

\[ U^s = \frac{A}{\mu_s + m_S - c + \beta U^s}, \]

\[ U^f = \frac{bU^s}{\gamma + \mu_f - \beta U^f}, \]

\[ L^s = \frac{A}{\mu_s + m_S - c + \beta U^s}, \]

\[ L^f = \frac{bU^s}{\gamma + \mu_f - \beta L^f}. \]  

(A.14)

Solving the above system, we obtain

\[ U^s = L^s = S^*, \quad U^f = L^f = I^*, \]

which yields

\[ \lim_{t \to \infty} I(t) = I^*, \quad \lim_{t \to \infty} S(t) = S^*. \]

**Remark A.1.** Note that condition (2.3) is equivalent to

\[ \gamma + \mu_f > \beta U^f_2. \]

If we replace \( U^f_2 \) by \( n^* \) defined in (A.3) in the proof, then condition (2.3) can be improved by a weaker condition

\[ \gamma + \mu_f > \beta U^f_2, \]  

(A.15)

where

\[ U^s_2 = \frac{A}{\mu_s + m_S - c + \beta U^s_1}, \]

with

\[ L^s_1 = \frac{bU^s_1}{\gamma + \mu_f - \beta L^s_1}, \quad L^f_1 = \frac{A}{\mu_s + m_S - c + \beta U^f_1}, \quad U^f_1 = n^*. \]

**References**

Arino, J., van den Driessche, P., 2006. Disease spread in metapopulations. In: Brunner, H., Zhao, X.-Q., Zou, X. (Eds.), Nonlinear Dynamics and Evolution Equations. Fields Inst. Commun., vol. 48, Amer. Math. Soc., Providence, RI, pp. 1–12.

Blower, S.M., Mclean, A.R., Porco, R.C., et al., 1995. The intrinsic transmission dynamics of tuberculosis epidemic. Nat. Med. 1, 815–821.

Beijing Research Institute for TB Control Work on the prevention of TB in Beijing, (http://www.bjjks.org).

Beretta, E., Kuang, Y., 2002. Geometric stability switch criteria in delay differential systems with delay dependent parameters. SIAM J. Math. Anal. 33, 1144–1165.

Brauer, F., van den Driessche, P., 2001. Models for transmission of disease with immigration of infectives. Math. Biosci. 171, 143–154.

Brauer, F., van den Driessche, P., Wu, J., 2008. Mathematical epidemiology. In: Lecture Notes in Mathematics, vol. 1945. Springer-Verlag, Berlin-Heidelberg.

China National Bureau of Statistics, (http://www.stats.gov.cn).

Chen, Z., Zhang, G.-C., Gong, X.-D., et al., 2007. Syphilis in China: results of a national surveillance programme. Lancet 369, 132–138.

Gruisky, O., Liu, H.J., Johnston, M., 2002. HIV/AIDS in China: 1990–2001. AIDS Behav. 6, 381–393.

Hale, J.K., Verduyn Lunel, S.M., 1993. Introduction to Functional Differential Equations. Springer-Verlag, New York.

Hsieh, Y.-H., van den Driessche, P., Wang, L., 2007. Impact of travel between patches for spatial spread of disease. Bull. Math. Biol. 69, 1355–1375.

Jia, Z.-W., Tang, G., Jin, Z., et al., 2008, Modeling the impact of immigration on the epidemiology of tuberculosis. Theor. Popul. Biol. 73, 437–448.

Jia, Z.-W., Jia, X.-W., Liu, Y.-X., et al., 2008. Spatial analysis of tuberculosis cases in migrants and permanent residents, Beijing, 2000–2006. Emerg. Infect. Dis. 14, 1413–1419.

Klepac, P., Neubert, M.G., van den Driessche, P., 2007. Dispersal delays, predator–prey stability, and the paradox of enrichment. Theor. Popul. Biol. 71, 436–444.

Lau, J., Thomas, J., 2001. Risk behaviors of Hong Kong male residents travelling to mainland China: a potential bridge population for HIV infection. AIDS Care 13, 71–81.

Li, X., Stanton, B., Fang, X., et al., 2004. HIV/std risk behaviors and perceptions among rural-to-urban migrants in China. AIDS Educ Prev. 16, 538–556.

Lipskie, T., Varughese, P., 2008. Measles in Canada—January to April 2008, available at (http://www.phac-aspc.gc.ca/cdrw-rmtch/2008/r2708-eng.php).

McCallum, H., Barlow, N., Hone, J., 2001. How should pathogen transmission be modelled? Trends Ecol. Evol. 16 (6), 295–300.

Smith, H., 1995. Monotone Dynamical Systems: An Introduction to the Theory of Competitive and Cooperative Systems, Math. Surveys Monographs, vol. 41. AMS, Providence, RI.

Strand, M., Wang, X., Duan, X., et al., 2007. Presence and awareness of infectious disease among Chinese migrant workers. Int. Quart. Commun. Health Edu. 26, 379–395.

U.S. Embassy Beijing AIDS in China: From Drugs to Blood to Sex, Beijing, China, December 2000.

van Binnendijk, R.S., et al., 2008. Air travel as a risk factor for introduction of measles in a highly vaccinated population. Vaccine 26, 5775–5777.

Varia, M., Wilson, S., Sarwal, S., et al., 2003. Investigation of a nosocomial outbreak of severe acute respiratory syndrome (SARS) in Toronto, Canada. Can. Med. Assoc. J. 169, 285–292.

Wang, J., Shen, H., 2009. Direct observation and completion of treatment of tuberculosis in rural areas of China. Scand. J. Publ. Health 37, 304–309.

Wang, W., 2009. Epidemic modes with time delays. In: Ma, Z., Zhou, Y., Wu, J. (Eds.), Modeling and Dynamics of Infectious Diseases. Higher Education Press, Beijing, pp. 289–314.

Wang, W., 2007. Epidemic models with population dispersal. In: Takeuchi, Y., Iwasa, Y., Sato, K. (Eds.), Mathematics for Life Sciences and Medicine. Springer, Berlin, Heidelberg, pp. 67–95.

Wang, W., Zhao, X-Q., 2004. An epidemic model in a patchy environment. Math. Biosci. 190, 97–112.

Wilson, M.E., 1995. Travel and the emergence of infectious diseases. Emerg. Infect. Dis. 1, 39–46.

Yang, X., 2004. Temporary migration and the spread of STDs/HIV in China: is there a link? Int. Migr. Rev. 38, 212–235.

Yang, X., 2006. Temporary migration and HIV risk behaviors in China. Environ. Behav. 38, 379–395.