One-dimensional subwavelength position determination exploiting off-axis parabolic mirror

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We demonstrate a subwavelength position determination method for the terahertz region. Previously, we reported that an off-axis parabolic mirror generates a peculiar transient rotational distribution around the focus on the subwavelength scale. In the method proposed herein, the position is determined by measuring the scattered light by a sample placed at this rotational distribution. We perform a realistic numerical calculation and show that this method is feasible for a sample on the wavelength scale and can distinguish a displacement of the order of 0.01 wavelengths. This method can be easily implemented for micro and nanoscale measurement and processing in the terahertz region.

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The ability to distinguish short distances is one of the primary driving forces of modern science and technology. For example, subwavelength optics has been developed using surface plasmon-polaritons,1,2) negative refraction,3,4) and the focusing of a radially polarized vector beam.5,6) Super-resolution imaging can be achieved in several ways; for example, using a superoscillatory lens,7) spectral position determination microscopy,8) stimulated emission depletion fluorescence microscopy,9) or the sparse sample reconstruction technique.10) In addition to super-resolution imaging, precise position determination is also important. Previously, confocal microscopy with fluorescence11,12) has been developed and applied to colloids.13) Further, position determination capability has been directly extended to the tracking of a small particle, such as those encountered in Brownian motion scenarios14,15) or in the transcription of DNA to RNA.16) In addition, there is a strong relationship with optical trapping and tweezing.17,18,19) One method universally employed to specify a sample position is measurement of the light scattered by the sample. Several such methods have been developed, e.g., combination of a radially polarized vector beam and a nanoantenna,20) backfocal plane interferometry,21,22) and holographic microscopy.23) However, these methods are primarily applicable in the visible range. In the terahertz (THz) region, subwavelength resolution of the order of 0.01 wavelengths has been achieved using scanning near-field imaging.24,25,26) However, for applications such as particle tracking, a non-scanning method is desirable.

Previously, we reported27–29) that a transient rotational electric field distribution appears around the focus when a linearly polarized parallel light is focused by an off-axis parabolic mirror (PM). The rotational distribution in this case exhibits an interesting feature, i.e., the amplitude at each point varies linearly on the subwavelength scale. In this paper, we demonstrate a non-scanning one-dimensional subwavelength position determination method that is easily equipped and applicable to various wavelengths, including the THz region. The principles of this method are as follows: If a sample is placed in the vicinity of the PM focus, it scatters the transient rotational electric field as well as the main component. The amplitude of the main component has only minimal dependence on the sample position; however, the amplitude of the rotational electric field is linearly dependent on the sample position. Thus, the sample position can be determined by measuring the scattered light.

The incident electric field is linearly polarized to the Y-axis, i.e., \( \mathbf{E}(x,t) = \mathbf{E}^{\text{inc}}(x) = (0, E_0, 0) e^{-i k z} \), where \( E_0 > 0 \) is the amplitude. The incident light is reflected by the PM. We denote the spatial part of the focused electric field by \( \mathbf{E}(x) \). Employing the Stratton–Chu equation30) and the physical optics approximation, \( \mathbf{E}(x) \) is determined via a surface integral over the PM, such that

\[
\mathbf{E}(x) = 2i \int_{\text{PM}} \mathbf{G}(k; x, \hat{x}) \times [k \times \mathbf{E}^{\text{inc}}(\hat{x})] \; d\hat{S},
\]

where \( \mathbf{G} \) is the dyadic Green function, \( \mathbf{n}(\hat{x}) \) is the outward normal vector at the mirror surface \( \hat{x} \), and \( k = (0, 0, -k) \) is the wavevector. At the focus, \( \mathbf{E}(0) \) contains the Y component only and becomes a maximum at the peak time. Further, at the peak time, the focused electric field is almost uniform around the focus (depicted by solid red arrows in Fig. 1) and its square is almost equal to the intensity distribution. In contrast, as we reported previously,27) a transient rotational electric field distribution appears at the zero-crossing time, which is the fourth period after the peak time. On the Y-axis, the rotational electric field is the Z component and is proportional to the displacement from the origin (depicted by dashed red arrows in Fig. 1). Such a rotational distribution is
intrinsic to the off-axis PM. In the following, we show the principles used to determine the subwavelength displacement along the Y-axis by exploiting the Z component of the rotational electric field.

Suppose a point-like sample is placed at \( x_s \). The averaged current is proportional to the reflected electric field and given by

\[
j(x) = \sigma E(x) \delta(x - x_s),
\]

where \( \sigma \neq 0 \) is the conductivity. Further, \( j(x) \) produces the scattered electric field, which is observed at a detector placed at \( x_d \). The scattered electric field at \( x_d \) is expressed as

\[
\bar{E}(x_d) = i\omega\mu_0\sigma G(k; x_d, x_s) E(x_s),
\]

where \( \mu_0 \) is the permeability. The detector position is set to \( x_d = (-R, 0, 0) \) for simplicity, where \( R > 0 \) is the distance between the detector and focus. In real experiments, not only \( \bar{E}(x_d) \), but also the unscattered primary light \( \bar{E}^0(x) \) may be measured. The contribution of the latter is measured in isolation via a sample-free experiment; therefore, the effect of the primary light can be cancelled. Thus, we consider only the scattered light in the following calculation.

The sample slides on the Y-axis and its position is given by \( x_s = (0, \Delta, 0) \). Below, we show that \( \Delta \) can be determined on a significantly shorter length scale than the wavelength \( \lambda \). For this purpose, we suppose \( |\Delta| \leq \lambda \ll R \). The Green’s function in Eq. (3) is expanded by \( \Delta \) and \( \bar{E}(x_d) \) is approximated by

\[
\bar{E}_X(x_d) = i\omega\mu_0\sigma e^{ikR} \left[ -\frac{\Delta}{R} \bar{E}_X^0(x_s) \right] + O(\Delta^2),
\]

\[
\bar{E}_Y(x_d) = i\omega\mu_0\sigma e^{ikR} \left[ -\frac{\Delta}{R} \bar{E}_Y^0(x_s) \right] + O(\Delta^2),
\]

\[
\bar{E}_Z(x_d) = i\omega\mu_0\sigma e^{ikR} \bar{E}_Z^0(x_s) + O(\Delta^2). \tag{4}
\]

In addition, \( \bar{E}^0(x) \), expressed in Eq. (1), can be expanded as a power series of \( \Delta \). We suppose \( |\Delta|, \lambda \ll R \) and omit the higher-order terms of \( \Delta/\lambda \) and \( 1/\lambda \). Then, we have

\[
\bar{E}_X^0(x_s) = E_0 \frac{f}{\lambda} e^{2ik \Delta X},
\]

\[
\bar{E}_Y^0(x_s) = iE_0 \frac{f}{\lambda} e^{2ik \Delta Y} + O(\Delta^2),
\]

\[
\bar{E}_Z^0(x_s) = E_0 \frac{f}{\lambda} e^{2ik \Delta Z} + O(\Delta^2). \tag{5}
\]

The results for the calculation with and without linearization are represented by thin dashed red curves, whereas those for the direct calculation using Eq. (6) without linearization are represented by bold blue curves. (a) Amplitude ratio \( |E_Y/|E_X| \), (b) Relative phase \( \phi_Y - \phi_Z \) for \( \Delta \neq 0 \). The sign and absolute value of \( \Delta \) are determined by \( \phi_Y - \phi_Z \) and \( |E_Z/|E_X| \), respectively. The calculation is performed with \( \lambda = 10^{-4} m \) and \( f = L = R = 10^{-2} m \).

The coefficients \( U_X = -64\pi/([\sqrt{65} \pm \sqrt{65}]^2) \), \( U_Y = 8\pi/([\sqrt{65} \pm \sqrt{65}]^2) \), and \( U_Z = 4\pi/([\sqrt{65} \pm \sqrt{65}]^2) \) are obtained via a Taylor expansion of Eq. (1). Finally, with an adequate time shift of \( \omega t - kR - 2k\Delta \lambda - \arg \sigma \to \omega t \), for convenience, the scattered light at the detector (particularly for the Y and Z components) is obtained as

\[
\frac{E_Y(x_d, t)}{E_Z(x_d, t)} = E_0 \alpha \mu_0 \sigma f \left( \frac{U_Y \cos \omega t}{4\pi R \lambda} \right) + O(\Delta^2) \tag{6}
\]

Let us compare the amplitudes of the Y and Z components (\( |E_Y| \) and \( |E_Z| \), respectively). Their ratio is proportional to \( |\Delta| \), as

\[
\frac{|E_Z|}{|E_Y|} = \frac{2\pi U_Y |\Delta|}{U_Y \lambda} + O(\Delta^2). \tag{7}
\]

Furthermore, let \( \phi_{Y,Z} \) be the phases of the Y and Z components, respectively. We then calculate the relative phase \( \phi_Y - \phi_Z \). Because \( U_Y > 0 \), it is clear that \( \phi_Y = 0 \). In addition, \( U_Z > 0 \) yields \( \phi_Y - \phi_Z = \pi/2 \) for \( \Delta < 0 \) and \( \phi_Y - \phi_Z = -\pi/2 \) for \( \Delta > 0 \). Note that these results are independent of \( \sigma \), provided \( \sigma \neq 0 \). Therefore, this method is applicable to both metal and insulator samples.

Both \( |E_Z/|E_Y| \) and \( \phi_Y - \phi_Z \) are shown in Fig. 2. Note that \( \phi_Y - \phi_Z \) is not calculated for \( \Delta = 0 \), because the Z component vanishes. Measuring the Y and Z components at the detector, one can obtain \( |E_Z/|E_Y| \) and \( \phi_Y - \phi_Z \). The sign of \( \Delta \) is determined by the sign of \( \phi_Y - \phi_Z \), and its magnitude is determined by \( |E_Z/|E_Y| \). This is the principle used to determine \( \Delta \) on the subwavelength scale. Figure 2 also shows the calculation result without linearization by \( \Delta \). Here, the \( \phi_Y - \phi_Z \) results for the calculations with and without linearization almost coincide. For \( |E_Z/|E_Y| \), the linear approximation holds...
well for $|\Delta|/\lambda < 0.25$ and $\Delta$ can be estimated easily in this region. However, $\Delta$ can also be calculated outside this region, provided a one-to-one correspondence holds between $|E_d|/|E_i|$ and the magnitude $|\Delta|$.

To examine the experimental feasibility of the proposed approach and to estimate its accuracy, a numerical calculation involving a more realistic sample is necessary. Furthermore, a numerical calculation facilitates quantitative comparison of the scattered electric field with the unscattered and incident electric fields.

Let $V$ be the volume of a microparticle placed in the vicinity of the focus along the $Y$-axis. The total electric field $E^{tot}(x)$ is given by

$$E^{tot}(x) = \tilde{E}(x) + \frac{k^2}{\epsilon_0} \int_V G(k; x, x') \tilde{P}(x') \, dx',$$

(8)

where $\tilde{P}(x)$ is the polarization density and $\epsilon_0$ is the permittivity in vacuum. For this computation, we employ the discrete-dipole approximation (DDA).

For the $j$-th cell, the unknown quantities are the point dipole $\tilde{P}_j$ and the averaged electric field $\tilde{E}_j$. These quantities have the relationship $\tilde{P}_j = [\epsilon_j(x_j) - 1] \epsilon_0 \tilde{E}_j$, where $x_j$ is the center-of-mass coordinate of the $j$-th cubic cell and $\epsilon_j(x_j)$ is the relative permittivity at $x_j$. Note that $\tilde{P}_j$ and $\tilde{E}_j$ are self-consistently determined from simultaneous equations derived by discretizing Eq. (8). However, the integrand of Eq. (8) has a singularity at $x = x'$, which originates from the self interaction of the dipole moment in the cubic cell. To avoid this difficulty, the self-terms $L$ and $M$ are introduced, where

$$L = \frac{1}{3 \epsilon_0}, \quad M(\Delta V) = \frac{2}{3 \epsilon_0} \left[ (1 - ik \sqrt{\Delta V}) e^{ik \sqrt{\Delta V}} - 1 \right].$$

(9)

Then, we have

$$\tilde{E}_j = \tilde{E}(x_j) + \sum_{j \neq j} G_{jj} \tilde{E}_j \Delta V + [M(\Delta V) - L] \tilde{P}_j,$$

(10)

where the matrix element is defined as $G_{jj} = (k^2/\epsilon_0) G(k; x_j, x_j)$. By computing these simultaneous equations, we can uniquely determine $\tilde{E}_j$ and, therefore, $\tilde{P}_j$.

Finally, the electric field at the detector position $x_d$ is given by

$$E^{tot}(x_d) = \tilde{E}(x_d) + \frac{k^2}{\epsilon_0} \sum_{j} G(k; x_d, x_j) \tilde{P}_j.$$  

(11)

In this calculation, the physical configuration and parameters are identical to those of the previous discussion, except for those of the sample, which is assumed to be a sphere with radius $r = \lambda/4$. The sample is approximated by a set of 192 cubic cells with sides of $\lambda/24$. The shape of the approximated sphere is illustrated in Fig. 3(b). The relative permittivity of the sample is set to $\epsilon_s(x) = 11.8$, corresponding to bulk silicon in the THz region. The parameters $\lambda = 10^{-4}$ m and $f = L = R = 10^{-2}$ m. In Fig. 3, the amplitude ratio (a) and relative phase (b) of the $Y$ and $Z$ components of the scattered electric field are shown. It is apparent that they qualitatively accord with those shown in Fig. 2. Thus, the principles of the proposed method are validated for a realistic sample. Figure 3 contains more quantitative results and, hence, we can consider the experimental feasibility of the proposed method. For example, in our previous paper, time-resolved measurement of the focused light was demonstrated and $|E_d|/|E_i|$ was measured with an accuracy of approximately 0.01. From comparison with Fig. 3(a), $|E_d|/|E_i| = 0.01$ corresponds to $\Delta/\lambda = 2.8 \times 10^{-2}$. Thus, the experimental system can make a displacement of the order of $\lambda/100$. In addition, from Fig. 3(a), it is apparent that $|E_d|/|E_i|$ is proportional to $\Delta/\lambda$ in the range $|\Delta/\lambda| \leq 0.3$. In this range, $\Delta/\lambda$ can be directly calculated from the measured $|E_d|/|E_i|$ and $\phi_1 - \phi_2$. Note that, for broadband incident light, one should extract a certain wavelength component of the measured electric field vector in order to apply the present method.

Finally, we compare the amplitudes of the scattered $Z$ component $|E_d|$ and incident light $E_i$ as shown in Fig. 4. As an example, note that an accuracy of $\Delta/\lambda \approx 0.1$ requires $|E_d|/E_i \approx 2.5 \times 10^{-2}$. This result gives an indication of the detection accuracy necessary to achieve the desired position accuracy.

In summary, we have demonstrated a one-dimensional subwavelength position determination method applicable to the THz region. This method is based on exploitation of the transient rotational distribution of the electric field focused by an off-axis PM. The amplitude of the rotational distribution varies on the subwavelength scale, which allows subwavelength position determination. The above approach remains...
valid provided the PM can reflect the incident light. Therefore, our method is not restricted to the THz region.

To discuss the experimental feasibility and to estimate the accuracy of the proposed method, we performed numerical calculations for the case of a silicon microparticle. The result proved the validity of the principles of this technique. Determination accuracy of the order of $\lambda/100$ can be achieved using the proposed method and a conventional experimental setup, depending on the detector accuracy. Note that, in principle, the present method is not restricted by the diffraction limit, because it exploits the vectorial nature of the focused electric field. Therefore, this method is applicable provided classical electrodynamics holds. For example, accuracy of ångström order can be achieved with visible incident light. Furthermore, our method does not require scanning and will be useful for future particle tracking applications with subwavelength accuracy.

In real experiments, the scattered light from the sample can be gathered using a lens or mirror positioned between the sample and detector. Focusing of the scattered light enlarges the measured $|E_2|$ and enhances the accuracy. It should be noted that this focusing will convert one electric field component into another; this effect should be evaluated in advance.

As regards sample displacement in the X- and Z-directions, the effects of this behavior will be considered using extended three-dimensional subwavelength position determination, and the results will be published in the near future. Finally, our method requires only an incident light source, parabolic mirror, polarizer (if necessary), and detector. Such a simple system may be implemented in various experimental setups.

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