A fuzzy rule-based Fuzzy Inferior Ratio method with reliability factor

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Abstract. Multi-criteria decision-making (MCDM) is a branch of decision-making method which able to deal with complex and conflicting decision problems. Besides, Fuzzy Multi-Criteria Decision-Making (FMCDM) method is later introduced to deal with vague and uncertain information in decision-making problems that imitates human perception in the evaluation process. Recently, Fuzzy Inferior Ratio (FIR) is developed as one of the effective FMCDM methods as it includes the compromise solution in which both distances to the Fuzzy Positive Ideal Solution (FPIS) and the Fuzzy Negative Ideal Solution (FNIS) are considered simultaneously. Nevertheless, in FIR, there is no consideration on the reliability of the information in the evaluation process. In this paper, an improvised FIR with reliability factor is introduced where the reliability factor is represented by Z-numbers. Furthermore, the lack of capability of providing systematic evaluation and consistent output raises a concern in most FMCDM methods including FIR. Hence, a decision-making procedure of Fuzzy Rule-Based FIR with reliability factor is also been proposed. A numerical example is given using the proposed procedure and a consistency analysis is carried out using the Spearman’s Rho Correlation to evaluate its effectiveness. It is found out that the proposed procedure gives a more comprehensive and systematic evaluation with the inclusion of reliability factor and the Fuzzy Rule-Based evaluation in FIR.

1. Introduction

Decision-making is an essential activity for an individual or in any organization. It can be considered as a procedure of choosing the best alternatives by considering every benefit and cost factor. As the problem of making decisions becoming complex, a more detailed process is necessary to find its solution. The Multi-criteria decision-making (MCDM) method has been introduced as an approach to cater to such situations. It is suitable for selecting, prioritizing, categorizing and ranking alternatives that one desires to have. Xu and Yang [1] refers to MCDM as a decision-making activity in the presence of contradicting criteria. In addition, it carries the ability to represent the real-world problem effectively and a decision is made using a certain procedure that involves a set of different alternatives on various criteria [2].

The method of MCDM was later extended to the fuzzy environment due to the inclusion of subjectivity, impreciseness and vagueness in the evaluation, known as Fuzzy MCDM (FMCDM). The fusion between fuzzy set theory in MCDM methods gives a new dimension in solving decision-making problems that become more humanistic since the evaluation is now closely imitates human perception in the judgment. One of the well-known approaches of the FMCDM methods is the compensatory approach. As compared to the non-compensatory methods where exchanging indicators are not permitted [3], the compensatory approach compromises between the criteria and the best option are obtained by considering how far apart the option is from the most ideal solution. Some of the methods under this category are fuzzy TOPSIS [4], fuzzy VIKOR [5] and fuzzy COPRAS [6] to name a few. The compromise solution approach also has led Hadi-Vencheh and
Mirjaberi [7] propose a method called the Fuzzy Inferior Ratio (FIR). The FIR has the ability to compromise between the smallest distance from the Fuzzy Positive Ideal Solution (FPIS) and the largest distance from the Fuzzy Negative Ideal Solution (FNIS) simultaneously as compared to other FMCMDM methods that only consider either one of them.

The approach of Z-number recently brought forth by Zadeh [8], has an advantage in comparison to the classical fuzzy number representation. It incorporates the element of the information reliability in decision-making. The Z-number is defined as a two tuple fuzzy numbers \((X, Y)\), where \(X\) represents the fuzzy restriction value meanwhile \(Y\) is the reliability factor of \(X\). This concept has shown a promising development in applications and has started to be implemented to figure out many decision-making dilemmas [9-11] and some decision-making methods have also been proposed under Z-environment [12-13].

Fuzzy logic is an expansion of classical logic [14] that is triggered by the idea of fuzzy set theory. Instead of having the value of 0 and 1 to represent the truth values, it was extended to become a degree of truth that ranges from 0 to 1 inclusively. From this concept, the fuzzy system is proposed with approximating reasoning that is applied in many engineering problems. The basis of the fuzzy system is the fuzzy rule-based which relies on the if-then rules to obtain the output. It becomes an essential tool in developing a human-like capabilities system [15]. Several methods of fuzzy decision-making have incorporated the element of the fuzzy rule system in obtaining a better and effective model. The methods with this integration will give a more consistent result when further evaluation is needed [16]. In 2014, Nilashi and Ibrahim [17] proposed the TOPSIS method with fuzzy rule-based to solve customer’s purchasing problems. Mahmoudi et al. [18] combined PROMETHEE method with fuzzy logic to solve a customer decision problem, while Yaakob and Gegov [16] proposed the FRBS approach with fuzzy TOPSIS to determine the best stock in a portfolio selection. Kadaifci et al. [19] proposed the method of fuzzy rule-based in Analytic Hierarchy Process (AHP) for urban land planning.

Most existing fuzzy MCDM methods do not compromise the closest distance from the best ideal solution with the farthest distance from the worst ideal solution rating the alternatives. This problem was overcome by the FIR method proposed in [7] where the method utilizes both information simultaneously in calculating the score of each alternative. Nevertheless, FIR does not include the reliability information in the evaluation. The reliability of information is essential to be included since the evaluations depend on the decision makers’ varying depths of knowledge, experience and background. Furthermore, the lack of capability of providing consistent evaluation and output with actual ranking remains the concern in assessing the alternatives in the FMCMDM environment. The decision-makers may change their opinions based on their stress level, time, current condition, and other factors. This problem will lead to inconsistent evaluation among the decision-makers at different times.

This paper proposes an improvised FIR procedure where the reliability factor is integrated using Z-number. A fuzzy rule-based system (FRBS) is constructed in the FIR for solving MCDM problems. The ranking from the proposed procedure, the Z-FIR based on the fuzzy rule-based is compared with the Z-FIR method. The consistency analysis is carried out using Spearman’s Rho Correlation to investigate its effectiveness.

### 2. Preliminaries

We begin with several definitions and basic concepts that are needed in this paper.

**Definition 1.** [20] Denote by \(F\), a fuzzy set on universal \(U\) is expressed as

\[
F = \{(x, \mu_F(x))| x \in F, \mu_F(x) \in [0, 1]\},
\]

with \(\mu_F(x)\) being a membership degree of \(x\) in \(F\).
Definition 2. [21] A triangular fuzzy number $\tilde{M} = (m_1, m_2, m_3)$ is defined by the membership function, $\mu_{\tilde{M}}(x)$ as below:

$$\mu_{\tilde{M}}(x) = \begin{cases} 
(x - m_1)/(m_2 - m_1), & m_1 \leq x \leq m_2 \\
(x - m_3)/(m_3 - m_2), & m_2 \leq x \leq m_3 \\
0, & \text{otherwise}
\end{cases}$$

Let $\tilde{M} = (m_1, m_2, m_3)$ and $\tilde{L} = (l_1, l_2, l_3)$ be two triangular fuzzy numbers. Some basic arithmetic operations between $\tilde{M}$ and $\tilde{L}$ are as follows:

- $\tilde{M} + \tilde{L} = (m_1 + l_1, m_2 + l_2, m_3 + l_3)$
- $\tilde{M} - \tilde{L} = (m_1 - l_3, m_2 - l_2, m_3 - l_1)$
- $\tilde{M} \times \tilde{L} = (m_1 l_1, m_2 l_2, m_3 l_3)$
- $\frac{\tilde{M}}{\tilde{L}} = (m_1 / l_3, m_2 / l_2, m_3 / l_1)$ provided that $\tilde{L}$ is positive.

Definition 3. [22] A Z-number $Z = (\tilde{X}, \tilde{Y})$ is defined as two tuples of fuzzy numbers with $\tilde{X}$ is a restriction of $U$ while $\tilde{Y}$ is the reliability measure on $\tilde{X}$.

2.1 Determination of fuzzy number from Z-number

In [10], a procedure of obtaining a standardized fuzzy number from a $Z = (\tilde{X}, \tilde{Y})$ was proposed. The value $\tilde{Y}$ will first be transformed into a crisp number $\alpha$. Here a fuzzy expectation approach is used and thus $\alpha$ is written as

$$\alpha = \frac{\int x \mu_{\tilde{Y}}(x) dx}{\int \mu_{\tilde{Y}}(x) dx}$$

where $x$ is any element in $\tilde{Y}$. The Z-number is now in the form

$$\tilde{Z}^\alpha = \{(x, \mu_{\tilde{X}}(x)) | \mu_{\tilde{X}}(x) = \alpha \mu_{\tilde{X}}(x), x \in [0, 1]\}$$

and regularized into the standardized form as

$$\tilde{Z}^\alpha = \{(x, \mu_{\tilde{Z}^\alpha}(x)) | \mu_{\tilde{Z}^\alpha}(x) = \mu_{\tilde{X}}(x / \sqrt{\alpha}), x \in [0, 1]\}.$$  

3. The proposed procedure

The procedure proposed in this paper is essentially an integration between the original FIR with the fuzzy rule-based system (FRBS). Steps 1-10 are taken from the procedure of FIR [7] to determine the output for each alternative from the evaluation of each decision-maker. In Steps 11-13, the FRBS will establish the order of the alternatives. The proposed procedure is as below:

Step 1: Let $E_1, E_2, \ldots, E_k$ be a group of experts that will use a set of criteria $K_1, K_2, \ldots, K_m$ to determine their weight and rate the alternatives $A_1, A_2, \ldots, A_n$.

Step 2: The importance of criteria, rating of alternatives and the reliability of evaluation will be determined using suitable linguistic values adapted from [16] and are depicted in table 1-table 3.
Table 1. Linguistic values for criteria weight evaluation.

| Linguistic values   | Triangular Fuzzy Number     |
|---------------------|----------------------------|
| Very Low (VL)       | (0.00, 0.00, 0.10)         |
| Low (L)             | (0.00, 0.10, 0.25)         |
| Medium Low (ML)     | (0.15, 0.30, 0.45)         |
| Medium (M)          | (0.35, 0.50, 0.65)         |
| Medium High (MH)    | (0.55, 0.70, 0.85)         |
| High (H)            | (0.80, 0.90, 1.00)         |
| Very High (VH)      | (0.90, 1.00, 1.00)         |

Table 2. Linguistic values for the rating of alternatives.

| Linguistic values   | Triangular Fuzzy Number     |
|---------------------|----------------------------|
| Very Poor (VP)      | (0, 0, 1)                   |
| Poor (P)            | (0, 1, 3)                   |
| Medium Poor (MP)    | (1, 3, 5)                   |
| Fair (F)            | (3, 5, 7)                   |
| Medium Good (MG)    | (5, 7, 9)                   |
| Good (G)            | (7, 9, 10)                  |
| Very Good (VG)      | (9, 10, 10)                 |

Table 3. Linguistic values for the reliability of decision makers’ evaluation.

| Linguistic values      | Triangular Fuzzy Number     |
|------------------------|----------------------------|
| Strongly Unlikely (SU) | (0.00, 0.00, 0.10)         |
| Unlikely (U)           | (0.00, 0.10, 0.25)         |
| Somewhat Unlikely (SWU)| (0.15, 0.30, 0.45)         |
| Neutral (N)            | (0.35, 0.50, 0.65)         |
| Somewhat Likely (SWL)  | (0.55, 0.70, 0.85)         |
| Likely (L)             | (0.80, 0.90, 1.00)         |
| Strongly Likely (SL)   | (0.90, 1.00, 1.00)         |

Step 3: Determine fuzzy numbers from Z-numbers
The fuzzy number representation of the Z-numbers will be obtained using equations (1) – (3). The evaluation using linguistic variables in Table 1- Table 3 will have two components \( \tilde{X} \) and \( \tilde{Y} \) of Z-number.

Step 4: The fuzzy decision matrix, \( \tilde{D} = [\tilde{d}_{ji}]_{m \times n} \) is formed where \( \tilde{d}_{ji} = (d_{y1}, d_{y2}, d_{y3}) \) is the evaluation by decision-makers with \( i = 1,2,\ldots,m \) and \( j = 1,2,\ldots,n \).

Step 5: Normalize \( \tilde{D} = [\tilde{d}_{ji}]_{m \times n} \) and denote it as
\[
\tilde{N} = [\tilde{n}_{ji}]_{m \times n}, i = 1,2,\ldots,m; j = 1,2,\ldots,n.
\]

The normalization is based on the following:

- For benefit criteria, \( \tilde{n}_{ji} = \left( \frac{d_{y1}}{d_{yj}}, \frac{d_{y2}}{d_{yj}}, \frac{d_{y3}}{d_{yj}} \right), \hat{d}_{ji} = \max d_{yj}, i = 1,2,\ldots, m ; j = 1,2,\ldots, n. \)  (4)

- For cost criteria, \( \tilde{n}_{ji} = \left( \frac{d_{yj}}{d_{y1}}, \frac{d_{yj}}{d_{y2}}, \frac{d_{yj}}{d_{y3}} \right), \hat{d}_{ji} = \min d_{y1}, i = 1,2,\ldots, m ; j = 1,2,\ldots, n. \) (5)
Step 6: Transform $\tilde{N} = \left[\tilde{n}_{ij}\right]_{m \times n}$ by using a linear scale transformation with $n_{\min} = 0$ and $n_{\max} = 1$.

Denote this matrix as $T = \left[t_{ij}\right]_{m \times n}$, $i = 1, 2, ..., m$; $j = 1, 2, ..., n$.

Step 7: Obtain the weighted transformed normalized fuzzy decision matrix, $\tilde{S}$. The multiplication between the criteria weight, $\tilde{w}_j$, and the transformed normalized fuzzy decision matrix, $T$, gives the weighted transformed normalized fuzzy decision matrix, $\tilde{S}$ which is denoted as

$$\tilde{S} = \left[\tilde{s}_{ij}\right]_{m \times n}, i = 1, 2, ..., m;$ $j = 1, 2, ..., n$$ where $\tilde{s}_{ij} = t_{ij} \times \tilde{w}_j$.  

(6)

Step 8: Determine the FPIS, $A^+$ and the FNIS, $A^-$ as

$$A^+ : \tilde{g}_j := \left(\max_{1 \leq i \leq n} \tilde{s}_{ij}^{(1)}, \max_{1 \leq i \leq n} \tilde{s}_{ij}^{(2)}, \max_{1 \leq i \leq n} \tilde{s}_{ij}^{(3)}\right),$$

(7)

$$A^- : \tilde{b}_j := \left(\min_{1 \leq i \leq n} \tilde{s}_{ij}^{(1)}, \min_{1 \leq i \leq n} \tilde{s}_{ij}^{(2)}, \min_{1 \leq i \leq n} \tilde{s}_{ij}^{(3)}\right).$$

(8)

Step 9: Calculate the distance of each $A_i$ from $A^+$ and $A^-$ using

$$D(A_i, A^+) = \sum_{j=1}^{n} \left(\frac{1}{3} \sum_{k=1}^{3} \left(\tilde{s}_{ij}^{(k)} - \tilde{v}_{ij}^{(k)}\right)^{2}\right)^{\frac{1}{2}},$$

(9)

$$D(A_i, A^-) = \sum_{j=1}^{n} \left(\frac{1}{3} \sum_{k=1}^{3} \left(\tilde{s}_{ij}^{(k)} - \tilde{b}_{ij}^{(k)}\right)^{2}\right)^{\frac{1}{2}}.$$  

(10)

Step 10: Obtain the compromise solution of each alternative, $\zeta(A_i)$.

$$\zeta(A_i) = \frac{D(A_i, A^-)}{D(A_i, A^+)} - \frac{D(A_i, A^+)}{D(A_i, A^-)}, \quad i = 1, 2, ..., n$$

(11)

where

$$D \left(A^+\right) = \min_{1 \leq i \leq n} D \left(A_i, A^+\right), \quad dD(A^-) = \max_{1 \leq i \leq n} D \left(A_i, A^-\right).$$

Step 11: Determine the score of the inferior ratio, $IR(i)$ for the alternatives, $A_i$ denoted by

$$IR(i) := \min_{1 \leq i \leq n} \zeta(A_i).$$

(12)

Step 12: Construct the antecedent matrix $P$ and consequent matrix $Q$ for FRBS based on each decision-maker’s opinion.

The element $\tilde{M}_{ij}$ of the matrix $P$ is a linguistic value of the decision makers’ evaluation of each alternative for each criterion, denoted by

$$\tilde{P} = \left[\tilde{M}_{ij}\right]_{m \times n} \quad \text{for} \quad i = 1, 2, ..., m \quad \text{and} \quad j = 1, 2, ..., n.$$  

The inferior ratio $IR(i)$ score for alternative $A_i$ will be utilized to construct the consequents of rules from the alternatives that are based on the evaluation in table 4. This is presented as

$$Q = \left[V_1, V_2, ..., V_m\right]^{T}$$

(13)

where $V_i, \quad i = 1, ..., m$ is the linguistic value in table 4 acts as the fuzzy rule-based output.
Table 4: Linguistic values of alternatives preference.

| Linguistic values                        | Triangular fuzzy number |
|-----------------------------------------|-------------------------|
| Absolutely Very Highly Preferred (AVHP) | (0.00, 0.00, 0.20)      |
| Very Highly Preferred (VHP)             | (0.10, 0.20, 0.30)      |
| Fairly Highly Preferred (FHP)           | (0.20, 0.30, 0.40)      |
| Highly Preferred (HP)                   | (0.30, 0.40, 0.50)      |
| Moderately Preferred (MP)               | (0.40, 0.50, 0.60)      |
| Least Preferred (LP)                    | (0.50, 0.60, 0.70)      |
| Fairly Least Preferred (FLP)            | (0.60, 0.70, 0.80)      |
| Very least Preferred (VLP)              | (0.70, 0.80, 0.90)      |
| Absolutely Very Least Preferred (AVLP)  | (0.80, 1.00, 1.00)      |

Step 13: Develop the if-then rules based on the decision-makers’ evaluation for each alternative. The rules are obtained from decision makers’ evaluation and are determined based on equations (12) and (13). They are in the form

\[ \text{Rule } i: \text{ If } K_1 \text{ is } M_{1i}^{\lambda_i} \text{ and } \ldots \text{ and } K_n \text{ is } M_{ni}^{\lambda_i}, \text{ then } A_i \text{ is } V_i. \]

\[ \vdots \]

\[ \text{Rule } m: \text{ If } K_1 \text{ is } M_{mi}^{\lambda_i} \text{ and } \ldots \text{ and } K_n \text{ is } M_{mi}^{\lambda_i}, \text{ then } A_m \text{ is } V_m. \]

The rules will then be integrated with the influence multiplier (explained in Step 14) to obtain the final output evaluation of alternatives. The output will be matched with the corresponding linguistic values in table 4.

Step 14: Rank the alternatives

To obtain the rank of alternatives, the individual final score (\( \Gamma \)) of the alternatives is calculated using

\[ \Gamma = \lambda \ast \Omega \]  \hspace{1cm} (14)

where

- \( \lambda \) is an aggregated membership function value where,

\[ \lambda = \frac{\sum_{j=1}^{K} \alpha_{ij}}{K} \] \hspace{1cm} (15)

with \( \alpha_{ij} \in V_j \) is the largest membership value of the output and \( K \) is the number of outputs involved in each alternative, \( A_i \).

- \( \Omega \) is an influence multiplier that uses the inferior ratio, \( IR(i) \) value of alternative from each decision-maker from equation (12) that have maximum membership degree such that

\[ \Omega = \frac{\sum_{i=1}^{K} IR(i)}{K} \] \hspace{1cm} (16)

4. Implementation

For implementation purposes, an example adapted from[16] is illustrated below. Evaluations by three experts \( E_1 \), \( E_2 \) and \( E_3 \) are made on 25 selected stocks \( S \) based on six fundamental analysis criteria \( K_1, K_2, K_3, K_4, K_5 \) and \( K_6 \). The linguistic terms in table 1-table 3 are utilized to determine the criteria weight and the implementation of the FIR method with the Z-number. The outputs of the FIR will
then be matched with corresponding linguistic values as in table 4. The if-then rules of the problem will be developed using these linguistic values. Table 5 depicts the evaluation of the criteria weight.

Table 5. Criteria evaluation by experts.

| Criterion | $E_1$ | $E_2$ | $E_3$ | Aggregated Weight |
|-----------|-------|-------|-------|-------------------|
| $K_1$     | (VH,L) | (VH,L) | (MH,SL) | (0.75, 0.83, 0.88) |
| $K_2$     | (MH,SL) | (MH,L) | (MH,L) | (0.53, 0.68, 0.83) |
| $K_3$     | (MH,SWL) | (M,SWL) | (H,SL) | (0.57, 0.69, 0.80) |
| $K_4$     | (M,L) | (ML,L) | (M,L) | (0.27, 0.41, 0.55) |
| $K_5$     | (H,SL) | (H,SL) | (VH,SWL) | (0.78, 0.87, 0.93) |
| $K_6$     | (ML,L) | (ML) | (ML,L) | (0.21, 0.35, 0.49) |

The evaluation of alternatives is summarized in table 6 with the last two columns refers to the score of the inferior ratio $IR(i)$ of alternative $S_i$ and its corresponding linguistic representation ($LR(i)$) respectively.

Table 6. Evaluation of alternatives by experts.

| Alt. | $E_5$ | Criteria | Output |
|------|-------|----------|--------|
| $S_1$ | $E_1$ | $K_1$ | $K_2$ | $K_3$ | $K_4$ | $K_5$ | $K_6$ | $IR(i)$ | $LR(i)$ |
|     | (VG,N) | (VG,N) | (G,N) | (VG,L) | (G,L) | (F,SWL) | 0.156 | VHP     |
|     | (VG,SL) | (G,L) | (F,SL) | (MG,SWL) | (MG,SL) | (F,SWL) | 0.197 | VHP     |
|     | (G,SWL) | (MG,SWL) | (G,L) | (VG,SL) | (G,L) | (F,SL) | 0.072 | AVHP    |
| $S_2$ | $E_1$ | (MG,SW) | (VG,SWL) | (F,L) | (MG,SWL) | (G,L) | (G,SL) | 0.192 | VHP     |
|     | (G,L) | (VG,SWL) | (G,L) | (MG,SL) | (G,SL) | (VG,SL) | 0.000 | AVHP    |
|     | (MG,L) | (VG,L) | (MG,N) | (MG,L) | (VG,N) | (G,SL) | 0.212 | VHP     |
| $S_3$ | $E_1$ | (VG,L) | (MG,SL) | (VG,SL) | (MG,L) | (G,SL) | (MG,SL) | 0.000 | AVHP    |
|     | (VG,SL) | (G,SL) | (VG,SL) | (MG,SL) | (G,SL) | (VG,SL) | 0.061 | AVHP    |
|     | (VG,N) | (MG,L) | (VG,SL) | (G,N) | (VG,SL) | (G,L) | 0.000 | AVHP    |
| $S_4$ | $E_1$ | (F,L) | (MP,N) | (F,N) | (MG,L) | (MP,SWL) | (G,SL) | 0.580 | LP      |
|     | (MP,L) | (F,SWL) | (MG,SWL) | (F,SWL) | (P,L) | (F,L) | 0.858 | AVL     |
|     | (F,SL) | (MG,SL) | (F,SWL) | (MP,SL) | (P,SWL) | (F,N) | 0.706 | FLP     |
| $S_5$ | $E_1$ | (P,SWL) | (P,L) | (F,SWL) | (F,N) | (P,SL) | (F,L) | 0.853 | AVL     |
|     | (P,L) | (F,SWL) | (MG,SL) | (G,SWL) | (VP,SWL) | (MG,SL) | 0.841 | VLP     |
|     | (P,SWL) | (P,SWL) | (MG,SL) | (P,SWL) | (P,L) | (G,SL) | 0.877 | AVL     |
| $S_6$ | $E_1$ | (G,N) | (G,SL) | (G,SL) | (F,SWL) | (MG,L) | (F,SWL) | 0.215 | VHP     |
|     | (G,SWL) | (MG,SWL) | (G,L) | (G,SWL) | (MG,SWL) | (MG,L) | 0.241 | VHP     |
|     | (MG,SL) | (VG,SL) | (G,SL) | (P,SWL) | (MG,SL) | (G,L) | 0.155 | VHP     |
| $S_7$ | $E_1$ | (MG,L) | (F,SWL) | (F,L) | (MP,SL) | (G,SL) | (F,N) | 0.359 | HP      |
| Edges | (MG, SL) | (MG, SL) | (F, SWL) | (MP, SL) | (G, N) | (F, N)  | AVHP | AVLP |
|-------|----------|----------|----------|----------|--------|--------|------|------|
| E4    | (MG, N)  | (MG, N)  | (F, N)   | (P, N)   | (MP, SL) | (P, SWL) | 0.815 | VLP  |

| Edges | (MG, N)  | (F, N)   | (MP, N)  | (F, L)   | (P, N)   | (G, SL)  | 0.657 | FLP  |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (F, L)   | (MG, L)  | (G, SWL) | (F, N)   | (P, L)   | (MP, SL) | 0.695 | FLP  |
| E2    | (F, L)   | (MG, L)  | (G, SL)  | (MP, L)  | (P, L)   | (P, SWL) | 0.564 | LP   |

| Edges | (VP, SL) | (VP, SL) | (F, SWL) | (P, L)   | (VP, SWL) | (F, L)   | 1.000 | AVLP |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E5    | (P, N)   | (VP, SWL)| (VG, N)  | (F, SWL) | (F, L)   | (F, N)   | 0.937 | AVLP |
| E2    | (VP, SWL)| (VG, SL) | (P, SL)  | (VP, SWL) | (VP, N)  | 1.000    |      |      |

| Edges | (F, N)   | (G, N)   | (F, SL)  | (MP, SL) | (P, SL)  | (F, SL)  | 0.631 | LP   |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (F, L)   | (MG, SL) | (F, L)   | (MG, L)  | (F, SL)  | (G, SL)  | 0.473 | FLP  |
| E2    | (F, L)   | (MG, SL) | (P, SL)  | (MG, SWL)| (F, N)   | (G, SL)  | 0.606 | LP   |

| Edges | (P, SWL) | (G, L)   | (F, L)   | (P, L)   | (P, N)   | (G, SL)  | 0.642 | LP   |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (FSL)    | (G, SWL) | (VG, N)  | (F, SL)  | (F, L)   | (F, SL)  | 0.347 | FHP  |
| E2    | (P, SL)  | (VG, SWL)| (G, SWL) | (G, SL)  | (P, L)   | (G, SL)  | 0.491 | MP   |

| Edges | (MG, N)  | (G, SL)  | (F, L)   | (MP, SWL)| (P, L)   | (F, SWL) | 0.544 | MP   |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (P, L)   | (G, SL)  | (MG, L)  | (G, SL)  | (P, N)   | (F, L)   | 0.634 | LP   |
| E2    | (P, L)   | (G, SL)  | (MG, SWL)| (G, SL)  | (VP, L)  | (MG, SWL)| 0.724 | FLP  |
| E3    | (P, SWL) | (G, L)   | (MP, N)  | (MP, L)  | (VP, L)  | (G, L)   | 0.845 | VLP  |

| Edges | (F, N)   | (F, SL)  | (MG, SWL)| (VG, L)  | (MG, SWL)| (VG, N)  | 0.391 | HP   |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (G, L)   | (F, SL)  | (VG, SL) | (G, SL)  | (F, SL)  | (MG, SL) | 0.249 | VHP  |
| E2    | (G, SL)  | (F, SL)  | (G, SL)  | (F, SWL) | (MG, L)  | (F, L)   | 0.222 | VHP  |

| Edges | (MG, SL) | (G, N)   | (MG, SL) | (G, SWL)| (F, SL)  | (MG, SL) | 0.257 | FHP  |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (MG, SL) | (G, L)   | (G, SL)  | (VG, L)  | (F, L)   | (G, L)   | 0.151 | VHP  |
| E2    | (F, SL)  | (G, L)   | (VG, L)  | (MG, SL)| (G, L)   | (MG, N)  | 0.085 | AVHP |
| E3    | (G, N)   | (VG, SL) | (G, L)   | (G, SL)| (G, N)   | (VG, L)  | 0.121 | AVHP |
| E4    | (G, SWL) | (VG, SWL)| (G, L)   | (G, SL)| (MG, SL)| (MG, SL)| 0.128 | AVHP |
| E5    | (G, SL)  | (VG, SL) | (G, L)   | (MG, SL)| (MG, SL)| (MG, SL)| 0.027 | AVHP |

| Edges | (P, N)   | (G, SL)  | (VP, N)  | (F, N)   | (P, N)   | (P, N)   | 0.832 | VLP  |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (F, SL)  | (G, L)   | (VP, L)  | (G, SWL)| (F, SWL)| (G, SL)  | 0.580 | LP   |
| E2    | (G, L)   | (MG, SL) | (VP, SL) | (P, L)   | (P, SL)  | (G, SWL) | 0.754 | VLP  |

| Edges | (P, SWL) | (VG, L)  | (G, SWL) | (F, L)   | (P, SWL) | (VG, L)  | 0.528 | MP   |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (F, L)   | (VG, SL) | (G, SWL)| (F, SL)  | (VP, L)  | (F, SL)  | 0.589 | LP   |
| E2    | (P, SWL) | (VG, N)  | (G, SL)  | (VG, L)  | (VP, N)  | (P, N)   | 0.823 | VLP  |

| Edges | (MG, SL) | (G, SL)  | (F, N)   | (F, SWL)| (G, L)   | (MG, N)  | 0.363 | HP   |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (MG, SL) | (P, SL)  | (VG, SL)| (G, SWL)| (G, SWL)| (F, SWL)| 0.362 | HP   |
| E2    | (F, N)   | (F, L)   | (G, SWL)| (G, SL)  | (G, L)   | (F, SL)  | 0.311 | FHP  |

| Edges | (MG, SL) | (G, SL)  | (F, N)   | (F, SWL)| (G, L)   | (MG, N)  | 0.243 | VHP  |
|-------|----------|----------|----------|----------|----------|----------|------|------|
| E1    | (MG, SL) | (G, SL)  | (F, N)   | (F, SWL)| (G, L)   | (F, L)  | 0.786 | VLP  |
The if-then rules can be constructed from table 6 and used in developing the FRBS. As an example, the rule extracted from the evaluation of $S_{25}$ of experts $E_1$ is

$$\text{If } <K_1 \text{ is } MG> \text{ and } <K_2 \text{ is } F> \text{ and } <K_3 \text{ is } F> \text{ and } <K_4 \text{ is } MP> \\
\text{and } <K_5 \text{ is } G> \text{ and } <K_6 \text{ is } G> \text{ then } <S_i \text{ is } FHP>$$

The influence multiplier is then integrated into the if-then rules to obtain the final score ($\Gamma$) of the alternatives and hence the ranking. The alternative is ranking higher when $\Gamma$ is lower, as in the FIR method.

The rating of the alternatives by using the FRBS FIR with Z-number is given in table 7. For comparing purposes, the rating of alternatives using Z-FIR is also included.

**Table 7.** Rank of alternatives, $S_i$ based on final score $\Gamma$.

| Alternatives | Final score | Rank | Alternatives | Final score | Rank |
|--------------|-------------|------|--------------|-------------|------|
| $S_i$ | $\Gamma$ | Z-FIR with FRBS | Z-FIR | $S_i$ | $\Gamma$ | Z-FIR with FRBS | Z-FIR |
| $S_1$ | 0.022 | 5 | 7 | $S_{14}$ | 0.077 | 10 | 10 |
| $S_2$ | 0.021 | 4 | 3 | $S_{15}$ | 0.031 | 7 | 6 |
| $S_3$ | 0.001 | 1 | 1 | $S_{16}$ | 0.006 | 2 | 2 |
| $S_4$ | 0.532 | 22 | 20 | $S_{17}$ | 0.529 | 21 | 21 |
| $S_5$ | 0.762 | 24 | 24 | $S_{18}$ | 0.410 | 18 | 18 |
| $S_6$ | 0.041 | 8 | 8 | $S_{19}$ | 0.127 | 11 | 11 |
| $S_7$ | 0.317 | 14 | 14 | $S_{20}$ | 0.331 | 16 | 15 |
| $S_8$ | 0.426 | 19 | 19 | $S_{21}$ | 0.025 | 6 | 5 |
| $S_9$ | 0.914 | 25 | 25 | $S_{22}$ | 0.573 | 23 | 23 |
Using the Spearman’s Rho Correlation, the rank of the proposed Z-FIR with FRBS is compared with the Z-FIR method with both evaluations used Z-numbers as inputs. The Spearman’s Rho correlation $R$ is denoted as

$$R = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)}$$

where the difference between the ranks of two methods and the number of alternatives are denoted by $d_i$ and $n$ respectively. A positive $R$ closer to 1 indicates a credible positive correlation between the two rankings. It is found the value of $R = 0.993$ which is very close to 1. Thus, it can be concluded that the result of Z-FIR with FRBS is comparable to the Z-FIR method.

### 5. Conclusion

A new combination of an MCDM method of FIR with Z-number based on the FRBS has been introduced in this paper. The proposed approach provides not only a better way of dealing with vagueness and subjective information in a more intelligent and flexible manner but also incorporates decision-makers’ reliability in the computational process effectively. The reliability factor of the form Z-numbers is crucial in portraying the true perception of evaluators of the subject. The fuzzy rule-based developed from the evaluation assist the decision made systematically and consistent manner. Thus, the evaluation for additional alternatives by the decision-makers is no longer needed since the rule-based system is now commendable to do the task. A validation using Spearman’s correlation also shows that the proposed method gives a result that is consistent with the existing method. The integration of reliability factor with the fuzzy rule-based is seen to be possible to be implemented to other MCDM methods.

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