Research on the Influence of Atmospheric Vertical Refraction on Trigonometric Leveling Survey in Qinling Mountains

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Abstract. Atmospheric refraction error is one of the main factors affecting the accuracy of precision trigonometric levelling. The influence of it on height difference rises sharply with the increase of horizontal distance. In this paper, the elevation control survey data of Qinling mountains is used to calculate the atmospheric refraction coefficients among the areas in winter and summer, using the method of geometric levelling inverse calculation. Through the analysis of the change law of atmospheric refractive coefficient, the influence of atmospheric vertical refractive coefficient on trigonometric levelling in the Qinling Mountains is summarized, which provides a reference for trigonometric levelling in similar environments.

1. Introduction
Trigonometric leveling measures the height differences between two measuring points by observing the horizontal distance and the zenith angle of the two, which is easy to observe, limited by topographical conditions, and widely used in surveys. With the improvement of the accuracy of surveying and mapping instruments, the atmospheric refraction error has become the main factor affecting the accuracy of precise trigonometric levelling [1].

The vertical refractive coefficient of the atmosphere is caused by the uneven density of the upper and lower layers of the atmosphere, and affected by various factors such as temperature, air pressure, light, topography, and landforms. It is shown that the atmospheric refractive coefficient varies greatly in different regions and at different times [2], so the atmospheric refractive coefficient can’t be considered as a constant.

At present, the wide range of precise trigonometric levelling methods mainly includes intermediate method, opposite observation method, regional refractive coefficient statistical model correction and other methods. In this paper, the geometrical levelling method is used to calculate the atmospheric refractive coefficient of the Qinling Mountains in different seasons. It also studies the influence of the atmospheric vertical refractive coefficient on the trigonometric levelling. Meanwhile, it is proposed how to correctly select the atmospheric refractive coefficient in a cloudy mountainous environment, which can provide a reference for trigonometric levelling of similar environments.

2. Analysis of Influencing Factors on the Accuracy of Trigonometric Leveling

2.1. The Basic Formula of Trigonometric Leveling
As shown in Figure 1, the horizontal distance between points A and B is D, while the slant distance between points A and B is S. When the instrument is placed at point A, the height of the instrument is i. Point B is the collimated target point, v is the height of the target and R is the radius of curvature of
the earth. PE and AF are the horizontal plane passing through point P and A, respectively, PC is the
tangent line of the horizontal plane PE at point P, and PN is the electromagnetic wave survey line.
When the telescope at point P points to the PM direction tangent to the survey line PN, the light
emitted from point N happens to fall on the horizontal wire of the telescope, due to the influence of
atmospheric refraction. At this time, the instrument is placed at point A and the zenith angle between
points P and M is measured as α. Meanwhile, the height differences between the two points A and B is:

\[ h = S \sin \alpha + \frac{1 - k}{2R} \cdot (S \cos \alpha)^2 + i - v \]  

(1)

where \( k \) is called the atmospheric vertical refractive coefficient.

![Figure 1 schematic diagram of trigonometric levelling](image1)

Figure 1 schematic diagram of trigonometric levelling

![Figure 2 sketch map of elevation points in Qinling Mountains](image2)

Figure 2 sketch map of elevation points in Qinling Mountains

2.2. Analysis of Influencing Factors on the Accuracy of Trigonometric Leveling

According to the law of error propagation, the error in the height difference by differentiating
Equation 1 can be obtained as:

\[ m_h^2 = \sin^2 \alpha \cdot m_\alpha^2 + \frac{S^2 \cos^2 \alpha}{\rho^2} \cdot m_\rho^2 + \frac{S^4 \cos^4 \alpha}{4R^2} \cdot m_i^2 \]

\[ + \left( \frac{1 - k}{2R} \right)^2 \cdot \left( 4S^2 \cos^2 \alpha \cdot m_\alpha^2 + \frac{4S^4 \sin^2 \alpha \cos^2 \alpha}{\rho^2} \cdot m_i^2 \right) \]

(2)

where \( m_h \) is the error in the distance measurement of the total station; \( m_\alpha \) is the error in the zenith angle
measurement of the total station; \( m_k \) is the error in the atmospheric refractive coefficient; \( m_i \) is the
measurement error of the instrument height; \( m_v \) is the measurement error of the target height.

Taking the TS50 total station as an example, it is analysed the specific impact of various errors on
the accuracy of the height difference. It takes \( m_s = \pm(1 \text{mm} + 1 \text{ppm}) \), \( m_\alpha = \pm0.7'' \), the zenith angle \( \alpha = 10^\circ \),
and the refractive coefficient error at 0.1, 0.2, 0.3, 0.5, respectively, so as to work out the influence of
ranging error, angle measurement error and atmospheric refraction difference on the height difference
accuracy under different horizontal distances.

| Table 1. statistics of various errors and magnitudes of trigonometric levelling under different horizontal distances (unit: mm). |
|-------------|---------------|---------------|---------------|---------------|---------------|
| errors      | 200m          | 400m          | 600m          | 800m          | 1000m         |
| \( m_s = \pm(1 \text{mm} + 1 \text{ppm}) \) | 0.21          | 0.24          | 0.28          | 0.31          | 0.35          |
| \( m_\alpha = \pm0.7'' \)          | 0.67          | 1.34          | 2.01          | 2.67          | 3.34          |
| \( m_k = 0.1 \)                | 0.30          | 1.22          | 2.74          | 4.87          | 7.61          |


3. The Calculation of Atmospheric Refractive Coefficient

When the light propagates in the atmosphere, it bends with the change of the vertical gradient of the atmospheric refractive coefficient, and flickers and drifts with the fluctuation of the atmospheric refractive coefficient. Around noon on a sunny day, the beam drifts very sharply but the apparent position of the target is relatively stable. When the temperature gradient is reversed in the morning and afternoon, the atmospheric turbulence is weaker, the apparent position of the target changes rapidly, but the beam flicker and drift are small [3]. In fact, although the k value is relatively stable around noon on a sunny day, the flicker and drift of the beam make the zenith angle measurement exceed the limit, which is difficult to ensure the observation accuracy of zenith angle. Although the atmospheric turbulence is weak in the morning and afternoon, the k value changes rapidly, which is difficult to grasp the best observation time. On cloudy days, the solar radiation and the atmospheric turbulence phenomenon are both weak. The temperature gradient of the surface atmosphere does not change much, with stable atmospheric refractive coefficient. At this time, it is beneficial to calculate the atmospheric refractive coefficient.

3.1. Calculation of Refractive Coefficient Based on Meteorological Data

K•Brocks et al. deduced the relationship between the refractive coefficient and meteorological data based on the Fermat principle and the theory of atmospheric physics [4]:

\[ k = 503.3 \frac{P}{T^2} (0.0342 + \frac{dT}{dh}) \cos \alpha + \Delta k_c \]  

It can be seen from Table 1 that the influence of the ranging error on the height difference is related to the zenith angle. When the ceiling distance is small, the ranging error has little effect on the height difference accuracy.

The second one is that the influence of the angle measurement error on the height difference linearly rises with the increase of the horizontal distance. When the 0.5" level total station is used to observe the zenith angle, it can effectively reduce the angle measurement error.

The third one is that the influence of the atmospheric refractive coefficient error on the height difference rises sharply with the increase of the horizontal distance. When the refractive coefficient error is greater than 0.2 or the slope distance is greater than 400 m, the correction error of the refractive coefficient becomes the main factor that affects the accuracy of trigonometric levelling.

The fifth one is that the measurement error of the instrument and the target height is a systematic error, generally within 1 mm, which is not the main factor that affects the accuracy of trigonometric levelling.

Therefore, accurate correction of refractive difference is an important task of precise trigonometric levelling.

| $m_k$ | 0.2  | 0.3  | 0.5  |
|------|------|------|------|
| $m_h$ | 0.61 | 0.91 | 1.52 |
|       | 2.44 | 3.65 | 6.09 |
|       | 5.48 | 8.22 | 13.70|
|       | 9.74 | 14.62| 24.36|
|       | 15.23| 22.84| 38.06|
| $m_h$ | 1.61 | 1.69 | 1.82 |
|       | 2.49 | 3.13 | 4.15 |
|       | 3.69 | 6.01 | 8.58 |
|       | 5.58 | 10.21| 14.93|
|       | 8.17 | 15.66| 23.13|
| $m_h$ | 2.19 | 2.19 | 2.19 |
|       | 6.40 | 6.40 | 6.40 |
|       | 13.92| 13.92| 13.92|
|       | 24.55| 24.55| 24.55|
|       | 38.24| 38.24| 38.24|
where $P$ is the atmospheric pressure (hpa), $T$ is the temperature (K), $dT/dh$ is the vertical temperature gradient ($^\circ$C/m), $\alpha$ is the zenith angle, and $\Delta k_e$ is the water vapour refractive coefficient, which can usually be ignored.

It can be seen from Equation 3 that the atmospheric refractive coefficient at that point can be calculated through the atmospheric pressure, temperature, and vertical temperature gradient at a certain point of the survey line.

Due to the dramatic changes in the vertical temperature gradient near the surface, there are obvious differences in the vertical temperature gradient, in different seasons and at different times, even if it is cloudy, which is difficult to accurately describe it with mathematical formulas. Therefore, the calculation of refractive coefficient is rarely used in actual work, based on meteorological data.

### 3.2. Inverse Calculation of Atmospheric Refractive Coefficient on Geometric Leveling

The method of using geometric leveling to inversely calculate the atmospheric refractive coefficient requires the use of precise leveling to determine the height difference as a known height difference, and inversely calculates the atmospheric refractive coefficient through trigonometric leveling formula (Formula 1).

$$k = 1 + \frac{2R}{D^2} (h_{tri} - h_{geo}) \quad (4)$$

Where, $h_{tri}$ is the height difference of the trigonometric elevation measurement, and $h_{geo}$ is the height difference of the geometric leveling measurement. This method needs to firstly use leveling to determine the precise height difference, which can more accurately inversely calculate the atmospheric refractive coefficient, but the workload is relatively large.

### 3.3. Inverse Calculation of Atmospheric Refraction Coefficient by Opposing Triangular Elevation

The mean value of the elevation difference of the triangular elevation observation from the opposite direction is taken as the theoretical elevation difference, so as to calculate the atmospheric refractive index of the observation from the opposite direction. The atmospheric refractive index at this time is actually the average value of the refraction coefficient measured back and forth, which can’t reflect the actual atmospheric refractive coefficient, even with some systematic deviations.

$$k = 1 + \frac{R}{D^2} (h_1 + h_2) \quad (5)$$

Where, $h_1$, $h_2$ are the height difference between the previous and back measurements, respectively. When the air pressure, temperature, topography and geomorphology of the measuring station and the mirror station are inconsistent, the atmospheric refractive index of the measuring station and the mirror station are different. At this time, the atmospheric refractive coefficient reverse calculated using the opposite trigonometric leveling can’t be used to correct the refractive difference of the one-way trigonometric leveling.

The above three methods have many limitations in actual use, especially in areas such as tall buildings, mountains, valleys, and high slope monitoring. If we can in-depth study the changing laws of atmospheric refractive index and make accurate corrections of atmospheric refractive coefficient, the accuracy of trigonometric leveling will be greatly improved.

### 4. Research on Atmospheric Vertical Refraction in Qinling Mountains

The elevation control measurement project is located in the Qinling Mountains, with an average elevation of 640 m and an average annual temperature of 13°C, of which the average temperature is 3°C and 23°C in winter and summer, respectively. There are a total of 7 elevation control points (TN1-TN7), which are distributed on the flat terrain on both sides of the river valley, as shown in Figure 2.

The second-class level and trigonometric levelling were used to measure the height difference of the control points in December 2019 and June 2020. Trigonometric levelling uses Leica TS50 total station, with accuracy ranging of 1mm+1ppm and angle measurement accuracy at 0.5", using full
circle observation method to observe the slant distance and zenith angle. A total of 20 sides were observed back and forth, excluding the short sides TN3-TN5 and TN2-TN4. Trigonometric levelling are all carried out on cloudy days, applying formula 4 to reverse calculate the atmospheric refractive coefficient. Table 2 and Table 3 are the statistical tables of atmospheric refractive index in the Qinling Mountains in winter and summer, respectively.

Table 2. statistics of atmospheric refractive coefficient in Qinling area in winter.

| Survey station | Mirror site | Slant distance / M | Levelling height difference / M | Trigonometric Height Difference / M | k   |
|----------------|-------------|--------------------|---------------------------------|------------------------------------|-----|
| TN1 TN4        |             | 372.8249           | -0.4777                         | -0.4833                            | 0.49|
| TN2 TN3        |             | 388.6908           | -3.7353                         | -3.7456                            | 0.12|
| TN2 TN5        |             | 440.2218           | -8.8746                         | -8.8914                            | -0.11|
| TN3 TN2        |             | 388.6887           | 3.7353                          | 3.7266                             | 0.27|
| TN4 TN1        |             | 372.8268           | 0.4777                          | 0.4644                             | -0.22|
| TN4 TN3        |             | 324.8049           | -3.1804                         | -3.1874                            | 0.16|
|                |             |                    |                                 |                                    |     |
| TN5 TN6        |             | 166.7592           | -3.0774                         | -3.0800                            | -0.18|
| TN6 TN7        |             | 342.0291           | 17.8036                         | 17.7959                            | 0.15|
| TN6 TN5        |             | 166.7590           | 3.0774                          | 3.0749                             | -0.16|
| TN7 TN4        |             | 301.1098           | -6.4065                         | -6.4116                            | 0.29|

Table 3. statistics of atmospheric refractive coefficient in Qinling area in summer.

| Survey station | Mirror site | Slant distance / M | Levelling height difference / M | Trigonometric Height Difference / M | k   |
|----------------|-------------|--------------------|---------------------------------|------------------------------------|-----|
| TN1 TN2        |             | 366.3177           | 0.0772                          | 0.0691                             | 0.23|
| TN2 TN3        |             | 388.6908           | -3.7353                         | -3.7498                            | -0.22|
| TN2 TN5        |             | 440.2218           | -8.8746                         | -8.8905                            | -0.05|
| TN3 TN2        |             | 388.6887           | 3.7353                          | 3.7235                             | 0.01|
| TN4 TN1        |             | 372.8268           | 0.4777                          | 0.4673                             | 0.05|
| TN4 TN3        |             | 324.8049           | -3.1804                         | -3.1897                            | -0.12|
|                |             |                    |                                 |                                    |     |
| TN5 TN6        |             | 166.7592           | -3.0774                         | -3.0805                            | -0.42|
| TN6 TN7        |             | 342.0291           | 17.8036                         | 17.7963                            | 0.20|
| TN6 TN5        |             | 166.7590           | 3.0774                          | 3.0756                             | 0.17|
| TN7 TN4        |             | 301.1098           | -6.4065                         | -6.4146                            | -0.14|

It can be seen from Table 2 and Table 3 that the atmospheric refractive index k in the Qinling Mountains in winter varies from -0.22 to 0.51, with an average value of 0.16 and a standard deviation of 0.25, while that varies from -0.42 to 0.41, with an average value of 0.03 and a standard deviation of 0.22 in summer. The average value of refractive index in the Qinling Mountains in winter is greater than that in summer.

From the perspective of the change range of the atmospheric refractive coefficient, the standard deviation is above 0.2 no matter it is in winter or summer, indicating that there are obvious differences in the atmospheric refractive coefficient in the region. Therefore, it has certain limitations to use a single atmospheric refractive coefficient to correct trigonometric levelling, which also shows that the atmospheric refractive coefficient is affected by a variety of factors such as weather and topography. The use of the intermediate method or the opposite observation method to measure the triangular elevation will also have errors caused by the different refractive coefficients at both ends.

The average value of the atmospheric refractive coefficient in the Qinling Mountains in winter is 0.16, which is close to the atmospheric refractive coefficient of 0.13 recommended in the "Medium and Short-range Photoelectric Ranging Specification". The average value of the atmospheric refraction
coefficient in the Qinling Mountains in summer is 0.03, which is also consistent with the average value of 0.01 in the Chaluhe measurement area, calculated by Wang Fengyan and other scholars [5]. If the recommended value of 0.13 is used for correction, a correction error of 10.2 mm will be produced on the 1km survey line.

### 5. Conclusion

Through the analysis of the factors affecting the accuracy of trigonometric leveling and the atmospheric refractive coefficient in the Qinling Mountains, the following conclusions can be drawn.

- The influence of the ranging error on the height difference is related to the zenith angle. When zenith angle is small, the ranging error has little effect on the accuracy of height difference. The influence of the angle measurement error on the height difference linearly rises with the increase of the horizontal distance. When the 0.5" Electronic Total Station is used to observe the zenith angle, the angle measurement error can be effectively reduced.

- The influence of the atmospheric refractive coefficient error on the height difference rises sharply with the increase of the horizontal distance. When the refractive index error is greater than 0.2 or the slope distance is greater than 400m, the correction error of the refractive coefficient becomes the main factor that affects the accuracy of trigonometric levelling.

In this paper, vertical control survey data in Qinling Mountains is used to calculate the atmospheric refraction coefficients in winter and summer via the method of geometric levelling inverse calculation. The average value of the atmospheric refractive coefficient in Qinling Mountains in winter is 0.16, while that in summer is 0.03, which shows that the average value of the winter refractive index in the Qinling Mountains is greater than that in summer.

Judging from the variation range of atmospheric refractive coefficient in Qinling Mountains, the standard deviation is above 0.2, whether it is in winter or summer, indicating that there are obvious differences in atmospheric refractive coefficient in this region. Therefore, it has certain limitations to use a single atmospheric refractive coefficient to correct trigonometric leveling.

The average value of the atmospheric refractive coefficient in the Qinling Mountains is 0.16 in winter, which is close to the atmospheric refractive coefficient of 0.13 recommended in the "Medium and Short-range Photoelectric Ranging Specification". Using the recommended atmospheric refractive coefficient can better correct the atmospheric refractive difference. The average value of the atmospheric refraction coefficient in the Qinling Mountains is 0.03 in summer. If the recommended atmospheric refraction coefficient is adopted, a correction error of 10.2 mm will be produced on the 1-km survey line.

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