Have we observed the skin vibration of realistic strange stars (ReSS)?

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Abstract

Skin vibration of ReSS and consequent resonance absorption can account for the absorption lines in the spectrum of X-ray emission from many compact stellar objects and in particular, the stars J1210−5226 and RXJ1856−3754. Observations of the X-ray spectrum of these stars is difficult to explain, if they are neutron stars.

keywords: compact stars – realistic strange stars – dense matter.

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1 Introduction

The spacecraft observations of X-ray emission from compact objects, specially the outstanding capabilities of the Chandra X-ray observatory have greatly increased our potential to study the characteristics of the surface radiation from compact stars.

According to Pavlov et al. [1] (PZS in short) the best-investigated compact central object J1210−5226 in the supernova remnants of G296.5+10 shows two absorption lines of 0.7 keV and 1.4 keV which are equally strong. Attempts to explain the absorption features as caused
by the intervening interstellar (or circumstellar) medium material lead to huge overabundance for some elements. Therefore the observed lines are most likely intrinsic to the compact star.

The star RXJ1856.-3754 is a compact object with no pulsation and is only about 120 pc away from us. There is a recent controversy [2, 3] about this star since it shows a black body spectrum but with a dip at about 0.3 keV which has led to some speculation about X-ray activity comprising of two zones (with their associated magnetic fields) and two temperatures.

In the present paper we show that a natural explanation of the dip(s) is in terms of absorption at the resonance frequency (fundamental or its harmonics) of the star surface which is only $10^{-7} \text{ fm}$ thick. For this we employ the EOS1, [4] (SS1 in [5, 6]) which has been widely used by many authors for explaining (1) the Mass-Radius (M-R) curves of stars [4, 5], (2) properties of quasi-periodic oscillations [6], (3) continuity of the entropy from the hadron to quark phase [7], (4) fast rotation properties of stars [8, 9], (5) gamma ray bursts [10, 11], (6) radial oscillations [12, 13] (7) superbursts and long bursts through quark pairing as surface phenomenon of compact objects [14] - and many other properties. The pressure derivative at the star surface is known to be related to the vibrations of the skin, which in our case consists of the lighter quarks with mass $\sim 130 \text{ MeV}$ (we neglect strange quarks which have mass exceeding the u,d-mass by another 150 MeV). This gives us a natural frequency for the system and any vibration at the surface will be preferentially absorbed at this frequency according to the basic tends of physics. Therefore the system will absorb this frequency and its harmonics preferentially from the X-ray generated at the surface by accreting matter falling in from outside. This is our model for absorption band in the spectra.

As a by-product we also calculate properties of the electron cloud associated with the star and electron natural frequency controlled by the central positive charge.

We are grateful to the referee for pointing out that one could assume that the absorption lines are due to once-ionized helium in a super strong magnetic field. If one assumes these are electron cyclotron lines, the features might be interpreted as the fundamental and the first harmonic of the electron cyclotron energy in a magnetic field as two fundamentals from two regions with different magnetic fields. However PZS clearly point out that this is difficult to believe on two counts:

1. The surface magnetic field has to be $B > 3 \times 10^{12} \text{ G}$

2. The oscillator strength of the first harmonic is smaller than the fundamental by a factor 0.002 so that it is hard to explain why the 1.4 keV is as strong as the 0.7 keV feature if we assume that the two lines are associated with the same magnetic field.

The referee has also asked us to comment on the recent paper by Cottam, Paerels and Mendez [15] where they have found the red shift of a star to be 0.35 and assume that ‘for astrophysically plausible range of masses ($M \sim 1.3 - 2.0 \text{ M}_\odot$ it excludes some models in which the neutron stars are made of more exotic matter’ including model of [4]. While this may be so, there may be other stars whose red shift may be measured in the near future which will be shown to be compatible with the latter model. More importantly Li et al. [16] have pointed
out that the star 4U 1728–34 has mass less than 1.1 \( M_\odot \) so that we see no reason why such a mass is not astrophysically plausible, if a star emitting X rays. In fact the crucial proof for the existence of strange stars may come from the pinpointed low mass compact object of mass 1.1 \( M_\odot \) or less, since low masses with neutron matter EOS cannot form stars. Strange quark stars are self bound, so they can have low masses and if they are fast rotating with binary partners they may also show up marginally stable orbits due to their oblate structure and can lead to observable phenomenon as found by Zdunik and Gourgoulhon [16].

2 Incompressibility and oscillations in ReSS.

Incompressibility K, plays a very significant role in nuclear physics since the energy per particle, \( E/A \) has a minimum at normal density and at zero temperature. The pressure, \( p \) is zero at this point. For such systems the energy can be written out as the sum of the equilibrium value and a term proportional to the square of the change in the radius, leading to the spectrum of a harmonic oscillator. It was found in [17] that the magnitude of nuclear incompressibility is comparable to that of a nucleon and of quark gas. In [18] it was found that the excitations of the nucleon could be related to \( K \) and the bag inertia parameter.

The rate of change of \( p \) with number density is related to the incompressibility and the breathing modes determined by it. Since the ReSS strange matter shows a minimum at about five times normal density, the star surface has this density and this surface will vibrate. The vibration frequency spectrum is controlled by \( dp/dr \).

In ReSS model the energy per particle has a minimum at the surface. The nature of the curve near the minimum can be approximated by a harmonic oscillator as shown in Fig. 1. A Taylor expansion of the energy about \( r = R \) gives

\[
\frac{E(r)}{A} = \frac{E(R)}{A} + \frac{1}{2} k(r) (r - R)^2
\]

where \( R \) is the star radius, \( k(r) = -4\pi r^2 \frac{dp}{dr} \) and

\[
\frac{dp}{dr} = -G(p(r) + \epsilon)(m(r) + 4\pi r^3 p(r))
\]

is the TOV equation with conventional notation. The frequency of the vibration is given by

\[
\omega = \sqrt{\frac{k(R)}{m_{\text{skin}}}}.
\]

We take

\[
m_{\text{skin}} = 4\pi R^2 n(R) \frac{m_u(R) + m_d(R)}{2} d
\]
where \( n(R) \) is the number density at the surface, while \( m_u \) and \( m_d \) are the respective u and d masses obtained from the self consistent ReSS solutions [4]. To estimate the skin depth \( d \) we turn to the electron cloud outside the star.

It is well known [19, 20] that strong (or gravitational interaction alone) cannot retain the electrons within the sharp surface of a strange star and they spread outside the star in a shell of thickness several hundred fm [21]. They induce a positive charge at the centre once the charge neutrality condition is set up over the years and the electrostatic potential holds them. The minimum number of electrons in the shell that ensures this equilibrium is

\[
N_{\text{crit}} = \frac{137}{\hbar c} R \left( \sqrt{(k_f(R)\hbar c)^2 + m_e^2 c^4} - m_e c^2 \right),
\]

where \( \hbar c k_f(R) \), the electron Fermi momentum is 31.28 MeV at the surface and \( m_e \) is the mass of the electron.

Although \( N_{\text{crit}} \) is large, \( 1.507 \times 10^{20} \), it is negligibly small compared to the number \( N \) that can be present even in an infinitesimally small skin depth. Enhance the electrostatic attraction by a factor \( f \sim 10 \), with the total number of electrons in the shell is \( fN_{\text{crit}} \) : since number \( \sim k_f^3 \), an infinitesimal change in the Fermi surface by an amount \( \Delta k_f(R) \sim 10^{-8} \text{ fm}^{-1} \) accounts for \( N_{\text{crit}} \) as

\[
\frac{3 \Delta k_f(R)}{k_f(R)} = \frac{N_{\text{crit}} f}{N}.
\]

. For a skin depth \( d = 1.5 \times 10^{-7} \text{ fm} \), the total number of electrons is

\[
N = 4\pi R^2 \frac{k_f(R)^3}{3\pi^2} d \approx 10^{28}.
\]

Different central densities lead to different stellar masses and radii. This leads to variations in the fundamental frequency of skin vibrations. We find that resonance absorption for the first and the second harmonics of a star with mass 1.22 \( M_\odot \) and radius 7.16 km corresponds to the 0.7 and 1.4 keV absorption bands of the star J1210–5226, the puzzling pulsar in G296.5–10.
(PZS). The $0.3 \text{ keV}$ absorption band of RX J1856.5$-3754$ corresponds to the fundamental of a strange star with mass $0.94 \ M_\odot$ and radius $6.74 \ km$ (PZS).

The factor $f$ can, in principle be measured from the electron oscillation in the outside cloud that gives out a radio wave of energy

$$h\nu \leq \hbar \sqrt{\frac{2fN_{\text{crit}}e^2}{m_e R^3}}.$$  \hspace{1cm} (8)

This may be a broad band radio emission since compressional modes are known to lead to giant absorption (or emission, as the case may be) bands in nuclei and furthermore the electrons in the cloud are constantly moving leading to considerable extra broadening of the band. The referee kindly points out to us that such bands are indeed present in radio observations.

For the strange star fitting J1210$-5226$, $f = 10$ gives a radio wave with wavelength more than 180 meters with possible harmonics in the decameter radio band.

![Figure 2: Resonant energy (fundamental) for skin vibrations of strange stars with different masses.](image)

In summary, we have pointed out the exciting possibility that the absorption bands seen with Chandra and XMM-Newton in thermal X-ray spectra of stars could arise from surface vibrations of the u-d quarks on the star surface, if the stars are composed of strange matter.

We hope in future, small stars with mass less than or roughly equal to that of the sun will be found from data and their radius found to be of the order of $\sim 5$ to $7 \ km$. This would lead to a rich interplay between X-ray astrophysics and QCD.

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Table 1: Resonant frequency of stars

| \( \rho_c \times 10^{14} \) g/c.c. | \( M/M_\odot \) | \( R \) km | \( \nu \times 10^{16} \) /sec. | \( e_\nu \) keV |
|-----------------|---------------|------|-----------------|--------|
| 14.85           | 0.407         | 5.202| 5.3283          | 0.220  |
| 15.35           | 0.502         | 5.613| 5.6857          | 0.235  |
| 15.85           | 0.607         | 5.941| 6.0606          | 0.251  |
| 16.35           | 0.719         | 6.247| 6.4533          | 0.267  |
| 17.35           | 0.841         | 6.536| 6.8802          | 0.285  |
| 17.85           | 0.894         | 6.644| 7.0635          | 0.292  |
| 18.35           | 0.943         | 6.741| 7.2455          | 0.300  |
| 19.35           | 1.037         | 6.970| 7.6103          | 0.315  |
| 21.35           | 1.159         | 7.051| 8.1407          | 0.337  |
| 22.35           | 1.205         | 7.139| 8.3663          | 0.346  |
| 22.85           | 1.226         | 7.162| 8.4758          | 0.350  |
| 24.85           | 1.288         | 7.214| 8.8199          | 0.365  |
| 25.35           | 1.300         | 7.228| 8.8954          | 0.368  |
| 26.85           | 1.333         | 7.237| 9.1107          | 0.377  |
| 28.35           | 1.357         | 7.240| 9.2834          | 0.384  |
| 28.85           | 1.363         | 7.240| 9.3365          | 0.386  |
| 30.85           | 1.386         | 7.232| 9.5348          | 0.394  |
| 32.85           | 1.402         | 7.218| 9.6950          | 0.401  |
| 34.85           | 1.414         | 7.199| 9.8368          | 0.407  |
| 36.85           | 1.423         | 7.178| 9.9558          | 0.412  |
| 38.85           | 1.429         | 7.154| 10.0608         | 0.416  |
| 40.85           | 1.443         | 7.130| 10.1516         | 0.420  |
| 42.85           | 1.435         | 7.105| 10.2311         | 0.423  |
| 44.85           | 1.435         | 7.105| 10.3012         | 0.426  |
| 46.85           | 1.437         | 7.055| 10.3620         | 0.429  |
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