Effective 1.0: An Analytic Effective Action Analysis Library

James P. J. Hetherington\textsuperscript{a} and Philip Stephens\textsuperscript{b}\textsuperscript{†}

\textsuperscript{a}CoMPLEX, Department of Mathematics, University College London, Gower Street, London WC1E 6BT, UK
\textsuperscript{b}Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342 Cracow, Poland.

Abstract

Effective is a C++ library which provides the user a toolbox to study the effective action of an arbitrary field theory. From the field content, gauge groups and representations an appropriate action is generated symbolically. The effective potential, mass spectrum, field couplings and vacuum expectation values are then obtained automatically; tree level results are obtained analytically while many tools, both numeric and analytic, provide a variety of approaches to deal with the one-loop corrections. This article provides a guide for users who wish to analyze their own models using Effective. This is done by presenting the code required and describing the physics assumptions behind the code. The library can be extended in many ways and discussion of several such extensions is also provided.

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\textsuperscript{†}Correspondence email: pstephens@annapurna.ifj.edu.pl
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1 Introduction

The effective action of a model characterizes many of its important features. Using the effective action one is able to study the loop corrections and renormalization group of a model, the nature of spontaneous symmetry breaking and the mass spectrum [1]. This makes the effective action a powerful object to investigating phenomenological aspects of a model.

It is generally agreed that there will be new physics beyond the Standard Model (BSM). The exact nature of this physics is still unknown and many ideas exist about its nature (see for example [2] or [3]). In order to validate these new ideas, one must show that the tree level and one-loop results of this model agree with our current knowledge of the Standard Model. Additionally, new particles, their mass spectrum, couplings and symmetry breaking mechanisms must be elucidated. Often one wishes to only slightly deviate from an existing model by introducing a new field or a new interaction. Such deviations can often
create large differences in the structure of the model, particularly at one-loop. Existing codes for determining the spectra of models such as SOFTSUSY 4, ISASUGRA 5 and SUSPECT 6 do so for a particular model and are not easily extensible to study models with a changed field structure. We believe that this flexibility will be important in the post-standard-model era, as it will be more important to quickly get an approximate analysis of a new model, rather than to develop a very accurate code for a well understood model.

Presented here is a new C++ library which provides a toolbox for the study of an arbitrary model. The code has been designed with the flexibility to allow a user to change the tools in the toolbox to suit their own purposes.

1.1 What is Effective?

In order to facilitate exploration of new physics an analytic tool has been created that automates many of the various uses of the effective action. The tool operates analytically when possible but reverts to numerics when necessary. Our code typically treats the tree-level action analytically while evaluating loop corrections numerically. This way the analytic structure of the tree level action can be derived and the consequences of the addition of new terms studied. The effect of symmetry breaking mechanisms on the physical masses and couplings of the action can be studied numerically in the same framework.

Our solution is presented as a C++ library Effective. This library provides a toolbox for the user to study an action and variants of that action. Effective can be used to study many models with interesting phenomenological features. Examples include reggeized gluons 7 and R-parity violating SUSY 8, 9. It is possible to build extensions to the library to study virtually any model which can be written on paper. This code does not provide a simple executable to study an arbitrary model. Instead, a user must build their model with the Effective library and create an executable that probes the desired physics of the model. The code can be freely obtained from the Effective web-page which is currently at the URL: http://stephens.home.cern.ch/stephens/effective. Installation instructions and full code documentation may also be found there.

1.2 Library Features

Effective has been written in object-oriented C++. This code has been designed for extensibility from the outset. The possible extensions will be discussed in section 11. The code is built on the GiNaC algebraic engine 10; as GiNaC is provided as a C++ library our code is seamlessly integrated with the analytic engine. Not only does this reduce computational overhead but allows for the implementation of many complicated routines, many of which would be extremely cumbersome in the languages built into proprietary analytic software.

This code was initially developed to provide some of the same features found in SUSY spectrum generators such as SOFTSUSY 4, ISASUGRA 5 and SUSPECT 6. As such this tool can automatically generate the one-loop mass
spectrum of a model. The parameters can be run between different scales according to the renormalization group equations (RGE) in order to give a consistent parameter set. In this version, the RGEs need to be provided; a future extension to generate the one-loop RGEs automatically is planned.

It is worth noting that this code is not intended to replace the much more efficient and accurate codes like SOFTSUSY, ISASUGRA and SUSPECT. Where dedicated code for a model exists, it will be faster and more reliable. Instead, this code can mimic the physics present in those codes, as well as many other actions and symmetry breaking mechanisms. EFFECTIVE should be used to pioneer the study of a new model, with dedicated code for the model being written when it is clear it is interesting enough for a more precise analysis to be worthwhile.

1.3 Organization

This article is structured in the following way. In the first section the field content of a model is discussed. The default properties and interactions of the fields implemented in EFFECTIVE are given and the routines needed to include the desired fields in a users model are presented. Also, the discussion about the vacuum expectation value (VEV) of a field is given in this section.

The user can also implement additional interaction terms, such as Yukawa couplings. The routines used to specify these are discussed in section 3. For SUSY models, specification of the superpotential will imply further couplings between the fields. Once these three features are implemented the effective action can be derived.

Next, we discuss how EFFECTIVE can be used to analyze the model specified. The first step in this process is the generation of the effective potential from the action. This is minimized to obtain the vacuum expectation values (VEV). This process is given in section 5. Next comes determination of the mass matrices and field couplings. The appropriate calls to the library are discussed in section 6 along with the routines used to access the mass matrices and mixing angles. The one-loop corrections to these matrices and mixing angles are also discussed in this section. The last feature of building a model is the RGEs. These are discussed in section 7. We then present the way in which the specification of additional observables which one wishes to calculate using EFFECTIVE can be carried out by defining appropriate Feynman diagrams.

All of the required routines in sections 2-7 will be discussed in terms of the electro-weak model. The full definition of this model will be given in the appendix and two examples using this model to produce some physical results will be given in section 10.

Section 11 is reserved to discuss some of the various ways in which a user can define their own classes to replace the default ones in EFFECTIVE. These customizations allow the user the possibility of implementing almost any feature they would like. Unfortunately, to customize the code, one must have strong C++ knowledge as well as a good understanding of the GiNaC engine. This section will be quite technical and as with most programming, to truly appreci-
ate the content one must try to implement things for themselves. To that end more programming oriented tutorials can be found on the website.

2 Field Content

Definition of the field content of a model consists of several parts. First one must define the gauge groups that define the interactions of the fields. The group structure is defined by the class GaugeGroup. Once the group has been specified we define the model’s field content. There are two types of fields. The gauge bosons or gauge supermultiplet is a mediator of the interactions described by the gauge group; these are provided by the class GaugeField. The matter fields interact with the GaugeFields and with each other based on representations of the gauge group. These are described by the class MatterField. Once the GaugeFields and MatterFields are defined, one specifies for which scalar fields the effective potential will be analyzed to determine whether they will have non-zero VEVs.

It must be noted that the class Field and its subclasses represent a set of fields which share the same properties. For example the colour octet of gluon fields are all contained in one GaugeField object.

2.1 Gauge Groups

Effective has three subclasses of GaugeGroup defined. These are U1Group, SU2Group and SU3Group. Other groups could be implemented. This is discussed in section 11.

The GaugeGroups of a model are defined in the user supplied routine void createGaugeGroups(). In the electro-weak model we have the group structure $SU(2) \times U(1)$, to specify this we write

```cpp
void ElectroWeak::createGaugeGroups() {
    addGaugeGroup(new U1Group("U1", "{g'}", this, U1b));
    addGaugeGroup(new SU2Group("SU2", "{g_W}\"", this, SU2w));
}
```

This code block shows how to create the GaugeGroups and include them into the model. The function addGaugeGroup() adds a pointer to a GaugeGroup into the model. The constructor for the groups takes 4 arguments. First is a name which the group will be referenced by; this name also doubles as the plain text label for the coupling of the group. The second argument is the LaTeX name of the group’s coupling parameter. The third argument is a pointer to the model which this group belongs to and the last argument is an integer number unique to the group. This is referred to as the line of the group. This allows multiple groups with the same group structure, while still keeping the properties of the groups independent of each other.
2.2 Gauge Fields

Now that the gauge groups are defined, the gauge fields can be given. These are fields which mediate the interaction defined by the group. The kinetic terms of the effective action for the fields are defined by the spin of the field and whether it has a supersymmetric partner. In Effective there are three default spin classes implemented. These are VectorSpin, FermionSpin and ScalarSpin. In Effective a GaugeField with VectorSpin has the following kinetic term

\[ \mathcal{L}^{GF(V)}_{kin} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \]  

where the sum over \( a \) is implied and

\[ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu. \]  

In these equations the indices are in the adjoint representation and \( f^{abc} \) is the structure constant of the group, \( A^a_\mu \) is the vector field and \( g \) is the coupling constant of the group. As we wish to support \( N=1 \) SUSY, we also provide for spinor fields in the adjoint representation. If the GaugeField is a fermion field, the kinetic term is

\[ \mathcal{L}^{GF(F)}_{kin} = -i \bar{\lambda}^a D^{ab}_{\mu} \lambda^b, \]  

with

\[ D^a_{\mu} = \delta_{ab} \partial_\mu + ig f^{cab} A^c_{\mu}. \]  

Again, \( g \) is the coupling constant of the group, \( f^{abc} \) is the structure constant of the group and \( A^a_\mu \) is the vector field which mediates the interaction. \( \lambda^a \) is the gauge fermion and \( \bar{\sigma}^{\mu} \) are the spin matrices

\[
\bar{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \bar{\sigma}^1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \bar{\sigma}^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \bar{\sigma}^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.
\]

In Effective, fermions are treated as Weyl fermions and the \( \bar{\sigma}^\mu \) object is merely a placeholder representing the spin structure. If one wanted to code the action in terms of Dirac fermions, this could also be done.

Effective by default has only implemented the terms which correspond to the \( N=1 \) SUSY case. As such, there is no implementation of a gauge field with scalar spin. This is an extension that could be easily added, however.

Now to see how the \( B \) and \( W \) bosons are added to the electro-weak model we provide the implementation of the void createGaugeFields() routine.

```cpp
void ElectroWeak::createGaugeFields() {
    VectorSpin v;
    addField("B", new GaugeField("B", "B", v, getGaugeGroup("U1")));
    addField("W", new GaugeField("W", "W", v, getGaugeGroup("SU2")));
}
```
From this code we can see that a field is added to the model with the `addField()` routine. The first argument is a string reference to the field and the second argument is a pointer to a `Field`. The `GaugeField` constructor takes four arguments. First is the plain text label of the field, followed by the LaTeX label. The third argument is the spin type of the field, in this case both fields are `VectorSpin` types. Lastly is a pointer to the `GaugeGroup` that this field is a mediator of. The routine `getGaugeGroup()` returns the `GaugeGroup` pointer referenced by the string.

If one wanted to set a superpartner for the field this is done in the constructor. For example if we now wanted to add the fermionic superpartner to the $B$ boson, this would be done by

```cpp
FermionSpin f;
addField("Bino", new GaugeField("Bino", "\tilde{B}", f,
   getGaugeGroup("U1"),
   getGaugeField("B"));
```

The arguments are the same as before except we have added a reference to the superpartner of $\tilde{B}$, $\tilde{B}$. We don’t need to give the $\tilde{B}$ as an argument when we create $B$ because the relationship is set for both of them when it is given to $\tilde{B}$. This is also necessary as you can’t give a pointer to an object which you haven’t created yet!

Once the fields are created and added to the model, the terms given above are automatically entered into the effective action.

### 2.3 Matter Fields

We have now created the gauge fields. We need to create the remaining fields in the theory which interact with the gauge fields and each other. The class which defines these fields is `MatterField`. Again, these fields take a `Spin` class in order to define them. The implementation in `Effective` does not yet provide for a `VectorSpin` type of `MatterField`. The kinetic terms for the `FermionSpin` spin type of `MatterField` are

\[
L_{\text{kin}}^{\text{MF}}(F) = -i\bar{\psi}^{\{A\}} D_{\mu}^{\{A\},\{B\}} \psi^{\{B\}},
\]

where the sums over the sets $\{A\}$ and $\{B\}$ are implicit and

\[
D_{\mu}^{\{A\},\{B\}} = \delta^{\{A\},\{B\}} \partial_{\mu} + i \sum_{i \in \{G\}} e_i g_i t_i^{A_i B_i} A_{\mu}^{A_i} \delta^{\{A_i\}',\{B_i\}'}.
\]

In these equations $\{A\}$ and $\{B\}$ represent the set of indices from the fundamental representation for all of the groups the `MatterField` interacts with. The sets $\{A_i\}'$ and $\{B_i\}'$ are the sets of remaining indices when index $i$ is removed. The sum in eqn. 6 is over all of the gauge groups, $\{G\}$, and $t_i^{A_i B_i}$ is the generator of group $i$, $g_i$ is its coupling and $e_i$ is the charge of the field in the group. $A_{\mu}^{A_i}$
is the vector mediating the interaction for group $i$. Similarly the kinetic term for the ScalarSpin can be written as

$$L_{\text{kin}}^{\text{MF}(S)} = \left( D^{(A)} \phi(B)^{\mu} \right)^{\dagger} \left( D^{(A)} \phi(B)^{\mu} \right),$$

where the covariant derivative is the same as in the FermionSpin case and the sum over the sets $\{A\}$ and $\{B\}$ are implicit.

If a MatterField has a superpartner, there is an additional interaction which is

$$L_{\text{SUSY}}^{\text{MF}(F)} = \sum_{i} \frac{e_{i}g_{i}}{\sqrt{2}} \left[ \left( \phi^{\dagger} \right)^{A_{i}} \left( \psi \right)^{B_{i}} \chi^{a_{i}} + \left( \bar{\psi} \right)^{A_{i}} \left( \phi \right)^{B_{i}} \chi^{a_{i}} \right],$$

$$+ \text{c.c.}$$

Here $\chi^{a_{i}}$ is the gauge fermion of group $i$, $\phi$ and $\psi$ are supersymmetric partners, where $\phi$ is a scalar and $\psi$ is a fermion. There is one additional term in a SUSY theory. This is

$$L_{\text{SUSY}}^{\text{MF}(S)} = \sum_{i,j} \sum_{k} \frac{e_{i}g_{j}g_{k}}{\sqrt{2}} \left[ \left( \phi^{\dagger} \right)^{A_{i}} \left( \psi \right)^{B_{j}} \chi^{a_{i}} + \left( \bar{\psi} \right)^{A_{i}} \left( \phi \right)^{B_{j}} \chi^{a_{i}} \right],$$

where again the sum over $a_{i}, A_{k}, B_{k}$ and $\{A'_{i}\}$ is implicit. $G_{ij}$ is the set of groups field $i$ and field $j$ have in common. In order to include these terms, all scalars of the theory must be defined. These are added when the model initializes. If the user calls the routine Model::noDterms(), then these terms will not be added, e.g. ElectroWeak::noDterms(). This must be called in the constructor of the model.

We now give an example of adding leptons and a $SU(2)$ Higgs field to the electro-weak model. This is provided by the implementation of the void createMatterFields() routine.

```cpp
void ElectroWeak::createMatterFields() {
    numeric half(1,2);
    ScalarSpin s;
    FermionSpin f;
    addField("l", new MatterField("l", "\ell", f, famsize, 
        getGaugeGroup("U1"), -half, 
        getGaugeGroup("SU2"), 1));
    addField("eR", new MatterField("eR", "e_R", f, famsize 
        getGaugeGroup("U1"), -1));
    addField("H", new MatterField("H", "H", s, i, getGaugeGroup("U1"), half, getGaugeGroup("SU2"), 1));
    // Implement VEV code here
}
```
We see that the creation of a `MatterField` is similar to that of the `GaugeField`. The difference is that the `MatterField` can have several groups and a different charge under each group. If we look at the arguments for the `MatterField` constructor we see that the first three arguments are the same as for the `GaugeField`. These are followed by an integer representing the number of families of the field, followed by pairs of `GaugeGroup` pointers and charges. The `MatterField` class allows each matter field to be a representation of no more than 4 gauge groups. If more than 4 groups are needed for a field, then a new implementation of `MatterField` must be made.

Similar to the `GaugeField` the superpartner can be set by adding an additional argument to a `MatterField` pointer. This can be retrieved by a call to `getMatterField()` with the appropriate string as an argument. As in the `GaugeField` this superpartner needs to be passed only to the constructor of the second field of the pair. The appropriate relation is set for both fields.

We can also see from the previous code segment that the fields $\ell$ and $e_R$ are defined as the electron fields (and all families), not the positron field. This can be seen as the charge of the $e_R$ field is $-1$, and the charge of the $\ell$ field is $-\frac{1}{2}$ under $U(1)$ group and 1 under the $SU(2)$ group. This gives the electron field a $U(1)_{\text{QED}}$ charge of $-1$.

### 2.4 Vacuum Expectation Values

One of the most important uses of the effective potential is its minimization to determine which, if any, of the fields in the model may develop vacuum expectation values. However, as it would be computationally prohibitive to do this simultaneously for all scalar fields, we use the `Parameter` object to specify which fields will be analyzed for VEVs. Once all the field content of the model is defined, the user can give some of the `MatterField`s a non-zero vacuum expectation value. This value can be given as a parameter of the model, whose value can later be changed to study the impact of the VEV on physical results.

In the previous section we gave the first part of the implementation of the `void ElectroWeak::createMatterFields()` routine. We now insert the VEV in order to complete this routine. The first step is to define a parameter which the user can use to modify the value of the VEV. This is achieved with the code:

```cpp
Parameter upsilon = addParameter("HiggsVev","\upsilon",220.0,
    Parameter::vev);
```

This code creates a `Parameter` whose text name is `HiggsVev` and the LaTeX name is $\upsilon$. The default value of this parameter is 220.0. The last argument indicates that this parameter is a VEV. The function `addParameter()` adds this parameter to the model. The numeric value of all parameters is centrally stored in the model. This way a change to the value universally changes in all references to the parameter. The parameter can later be retrieved by calling `Parameter Model::getParam()` with the string given as an argument. To retrieve the parameter defined above, the string `HiggsVev` must be passed.
There is also a routine `numeric& Model::getParameter()` which takes a string as its argument. This routine returns the value (by reference) of the parameter. The value of this `numeric` object may be changed and the change will propagate to all of the parameters, but the `numeric` object should not be used when creating expressions. Doing so will put only the current value of the parameter into the expression. Future changes to the value will not change the expression created. Instead, if the user wants to create an expression with the parameter in it, they must create the expression with the `Parameter` object retrieved by the call to `Model::getParam()`.

Now the parameter which defines the VEV has been created, the field with the non-zero VEV can be defined. This is done by the call to the `addVev()` routine. In the electro-weak model we want to set the real part of the $H_2$ field to have a non-zero VEV. This is achieved by

```cpp
addVev("HiggsVev", getField("H"), lst(getIndex("H","SU2")==2),
        (upsilon+Model::star));
```

The first argument is a string which identifies this VEV. Note that this does not need to be the same as the string which identifies the `Parameter` of the VEV, though it also need not be different. The second argument is a pointer to the field which the VEV is being set for. The third argument is a `lst` object from GiNaC. This list specifies what substitutions to make on the `Field` multiplet to get the desired field. In this example the $SU(2)$ index, retrieved by calling `Model::getIndex()`, is set to 2. The last argument is the expression to replace the original field by. The object `Model::star` represents the field in question. In this example the substitution

$$R(H_2) \rightarrow v + R(H_2), \quad (10)$$

is made. A `Parameter` in `Effective` is always real. In order to replace the imaginary part of a field with a VEV, an additional argument, `Imag`, must be given to the `addVev()` routine. This would then make the substitution

$$I(H_2) \rightarrow v_I + I(H_2), \quad (11)$$

where $v_I$ is a new parameter specifying the imaginary part of the VEV.

The last step is to tell the model that this parameter is a VEV, and not some other kind of parameter. This is important for the code as the VEV parameters are treated internally differently than other parameters. This is achieved by the call

```cpp
addVevParameter(upsilon);
```

### 2.5 Default Behaviour of Field and Spin Classes

`Effective` has many default behaviours built into the classes described above. These behaviours will be sufficient for most models, however, there are many models for which the user will need to implement their own classes. The
GaugeGroup, Spin and Field classes have been designed with enough flexibility to accommodate most modifications. Details of how one would make such modifications are reserved for section 11. Here we present the default behaviour of the GaugeField, MatterField, ScalarSpin, FermionSpin and VectorSpin classes.

The GaugeField class is designed to take one GaugeGroup and a Spin object. This will define the field multiplet of the given spin. The equations that dictate the kinetic terms where given in eqns. (1-4). This allows one to try all sorts of things, not all of which will be renormalizable! For example, one could create a scalar gauge field not part of a supermultiplet, resulting in a kinetic term of the form

$$L^\text{GF(S)}_{\text{kin}} = (D_\mu \varphi^a)^\dagger (D^\mu \varphi^a).$$

(12)

In this equation the covariant derivative is given by eqn. (4) and the indices are in the adjoint representation. Note that this is really a side-effect of the code. Effective has not been designed with such terms in mind.

The MatterField class has been designed to take a family size, up to four GaugeGroups and charges under each group. This then generates the kinetic terms given in eqns. (5-7). If a VectorSpin where to be passed to this class this would cause an error. The VectorSpin class has currently only been defined to work with GaugeField classes and subclasses. By implementing a new Spin subclass one could have a vector field with an arbitrary gauge representation.

The Spin class defines several properties. The first one of importance is to specify how the field will be handled with respect to CP; whether the fields of a spin are complex or purely real. If they are complex the expressions are created so there is a unique expression for the real and for the imaginary part of a field. This implies

$$F = \frac{\mathcal{R}(F) + i\mathcal{I}(F)}{\sqrt{2}}.$$  

(13)

As all fundamental terms in the expressions of Effective are real, this substitution allows the software to treat the field $F$ as a complex field in terms of its real and imaginary parts. When one wants to probe the action for information on a field, they must therefore ask specifically about the real and the imaginary part of a field. The default behaviour of the ScalarSpin and FermionSpin classes is to treat the fields as complex, while the VectorSpin class treats the field as real. The division of the real and imaginary parts was intentional in order to study CP violating terms directly. A future extension will be to allow the user to decide whether to treat the basic objects in Effective as complex or not.

The last relevant property of the Spin class is whether the spin type contains a Lorentz index or not. In Effective the Lorentz structure is treated explicitly, while the Dirac structure of the fermions is implicit. This means that the expressions of vectors have explicit Lorentz indices attached, while the fermions do not have spinor indices. A future extension will be to include the spinor indices on the fermions.
3 Interaction Terms

The next step of the creation of the effective action of a model is to include non-kinetic terms. This includes terms such as the Higgs self interaction and the Yukawa couplings. In supersymmetric theories the superpotential replaces the need to implement some of these terms.

3.1 How to add Desired Expressions

To include the interaction terms in Effective the routine void addOtherTerms() must be implemented. As we have seen with the routines in the previous section, this routine is a virtual routine of the Model class. Thus the user’s model is a subclass of Model and addOtherTerms() is a routine of their class.

We now discuss the example of adding the Yukawa and Higgs self interaction terms to the electroweak model, defined by the class ElectroWeak. We will begin by showing the terms

\[ \mathcal{L}_{\text{Higgs}} = \mu^2 \phi_i^\dagger \phi_i - \lambda \left( \phi_i^\dagger \phi_i \right)^2, \]  

(14)

where there is an implicit sum over \( i \). In the second term this sum is performed before the square operation. This first term is given by

```
Parameter mu = addParameter("mu","\mu", 1.0);
vector<idx*> indices = getField("H")->getIndices();
ex H = getField("H")->expression();
ex a = pow(mu,2)*H.conjugate()*H;
add(Utils::sumIndices(a,indices).expand());
```

The first line of code creates new Parameter object for the \( \mu \) coupling. This is followed by a new object which requires explanation.

3.1.1 Summation

In Effective all of the implicit sums, like the ones in eqn. (14), must be made explicit. In order to achieve this the user must tell the code which indices to be summed. Implicit in the creation of the indices is the values a particular index can take (except Lorentz indices, for which dimensionality is variable and need not be integer).

The second line of code creates a list of all the idx (pointers) which the field “H” contains (not including any potential Lorentz indices). This list is then passed to the summation routine ex Utils::sumIndices() in the last line. This summation routine sums the first argument, \( a \), over all of the indices in the second argument, \( \text{indices} \). In this example the field \( H \) has only one index in the \( SU(2) \) fundamental representation. Explicitly the sum routine performs

\[ \text{sumIndices()} = \sum_i \mu^2 H_i^\dagger H_i, \]  

(15)
where \( i \) is an \( SU(2) \) index. Notice that this summation is passed into the function \texttt{void add()}\texttt{. This tells the model to add the expression to the Lagrangian.}

### 3.1.2 Expansion

One other note. The \texttt{GiNaC} engine may often keep terms grouped in multiplication, i.e. \((x+y)(z+w)\), where in the Lagrangian, we want terms as individual products, i.e. \( xz + xw + yz + yw \). The \texttt{expand()} routine ensures we have this expanded result. It is not necessary to call this command, the \texttt{Effective Library} will do the transformation automatically at a later stage. Performing the expand on the small expression above is much more efficient than waiting for the library to do it later. This is because the library will only do it once all the terms have been included and the VEVs set. Thus, it is recommended that this is done before it is added to the model.

Now consider the second part of eqn. \((14)\). Here we must perform the summation before squaring the result. This is easily obtained from

\[
\text{Parameter lambda = addParameter("lambda", \\"lambda", 1.0);}
\]
\[
\text{ex la = Utils::sumIndices(H.conjugate()*H,indices).expand();}
\]
\[
\text{add(-lambda*pow(la,2));}
\]

From the previous discussion all of the relevant pieces are already known. We create a new \texttt{Parameter} and compute the implicit sum. This sum is squared, multiplied by the coupling and added to the Lagrangian.

### 3.1.3 Families

The remaining term to include in the Lagrangian for the electroweak model is the lepton Yukawa coupling. This is given by

\[
\mathcal{L}_{\text{Yuk}} = -Y_{ij}^{e} e_{R}^{i} H \bar{\ell}_{j}^{e} + \text{c.c.}
\]

We see that there is now a sum over families, \( i,j \), as well as a sum over the \( SU(2) \) index \( a \). These sums will be performed explicitly in the code.

The code below will include the case for any number of families, with a special case for only one family. In this code the variable \texttt{famsize} is a global integer which specifies the number of families.

```plaintext
ex Ye;
if(famsize != 1) Ye = addFamilyMatrix("Ye", \\{"Y^e\}\\", famsize);
else Ye = addParameter("Ye", \\{"Y^e\}\\", 1.0);
idx i = Utils::familyIndex(0,famsize);
idx j = Utils::familyIndex(1,famsize);
ex eR = getField("eR")->expression()
ex l;
if(famsize != 1) l =
    getField("l")->expression().conjugate().subs(i==j);
```

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else l = getField("l")->expression().conjugate();
indices = getField("l")->getIndices();
ex b = -Ye*l*H*eR;
if(famsize != 1) indices.push_back(&j);
ex res = Utils::sumIndices(b,indices).expand();
add(2*Utils::real(res));

The coupling $Y e$ can either be a square matrix with dimensionality of the number of families or just a simple Parameter if there is only one family. The variables $i$ and $j$ are indices in the “family” space. This is the same space that the matrix $Y e$ is defined in. The routine addFamilyMatrix() creates a matrix which is $\text{famsize} \times \text{famsize}$. Each element of the matrix is associated with a unique Parameter with name $Ye_{ij}$ where $i$ and $j$ are replaced by the particular value of interest, i.e. $Ye_{12}$. The first argument is the string by which the matrix is stored for future retrieval. This argument also doubles as the value printed for non-LaTeX output.

Utils::familyIndex(0,famsize) is a utility routine in Effective to retrieve index 0, from a predefined list of indices, with dimensionality famsize. Such predefined lists exist for all types of indices (e.g. $SU(2)$, family, Lorentz, etc.) so that indices can be matched and replaced when creating expressions. The familyIndex(i,s) routine will return the $i$th index from the list with family dimension $s$. When a field is created, it’s family index is always the 0th element of that list (the same applies to the group indices). When the family matrix is created, it has two indices, the 0th and the 1st. Thus we retrieve the 0th and the 1st and store them in the variables $i$ and $j$, respectively, for later use.

3.1.4 Expression Substitution

The next line of interest is when the left handed lepton expression is retrieved. In the case that $\text{famsize} \neq 1$ we see the extra command subs(i==j). This is a very powerful, and useful, function in GiNaC. This instruction takes the expression it is called on (in this case the full complex expression of $\ell$) and substitutes all occurrences of $i$ with $j$. Thus the sum over $i$ does not affect $\ell_j$ and in the sum over $j$, $e_{Ri}$, is unaffected. In both summations $Y^{\gamma}_{ij}$ does change. We can see this summation in the second to last line. We see in the line before that the family index $j$ is added to the list of summation indices. We must remember that when we retrieved the indices from the left-handed lepton field it only contains one family index, $i$. Thus to do the double summation, we must add $j$ to our list. Explicitly, this summation is

$$\text{sumIndices()} = \sum_a \sum_i \sum_j Y^{\alpha}_{ij} \ell^\dagger_Ri \ell^\alpha_j H_a,$$  \hspace{1cm} (17)$$

where $a$ is the $SU(2)$ index. The function Utils::real() in the last line returns only the real component of the expression. The factor of 2 is there since $X + \text{c.c.} = 2 \text{Re}(X)$. 

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4 Superpotential

The most general set of renormalizable SUSY interactions are given by

\[ L_{\text{sup}} = -\frac{1}{2} W_{ij} \psi_i \psi_j + W^i W_i^* + \text{c.c.,} \] (18)

where

\[ W = \frac{1}{2} M_{ij} \varphi_i \varphi_j + \frac{1}{6} y_{ijk} \varphi_i \varphi_j \varphi_k, \] (19)

\[ W^{ij} = \frac{\delta^2 W}{\delta \varphi_i \delta \varphi_j}, \] (20)

\[ W^i = \frac{\delta W}{\delta \varphi_i}. \] (21)

In these equations the fields \((\psi_i, \varphi_i)\) form a supermultiplet. This means that \(M_{ij}\) is the mass matrix of the fermion fields and \(y_{ijk}\) is the Yukawa coupling of a scalar, \(\varphi_k\) to fermions \(\psi_i\) and \(\psi_j\). Technically, the superpotential is defined in terms of supermultiplets. In Effective, this is not the case. For the N=1 supersymmetric case, we can implement the superpotential as a function of the scalar fields. A future enhancement of the code will change this to a true definition in terms of supermultiplets.

4.1 Implementation of MSSM Superpotential

We now give an example of the MSSM superpotential. Just as the previous section, we will rely on making explicit the implicit summations that occur, for example, in eqn. (18). The superpotential is implemented by defining the function `ex superPotential()` in user’s model subclass. Note that this function now returns an expression, unlike the `addOtherTerms()` routine, the user must not call the `Model::add()` routine on terms in the superpotential. We will implement the MSSM superpotential

\[ W_{\text{MSSM}} = \varepsilon_{ab} \left( Y^d_{ij} \bar{d} H_1^a Q^b - Y^u_{ij} \bar{u} H_2^a Q^b + Y^e_{ij} \bar{e} H_2^a \ell^b \right) + \mu H_1 H_2. \] (22)

In fact we will only show the implementation of the \(Y^e_{ij}\) term. The process of implementing the other terms is straightforward. Also note that, as mentioned before, the superpotential is defined in terms of supermultiplets as is eqn. (22). The implementation will only be defined in terms of the scalar fields of the supermultiplets.

```cpp
ex Ye = addParameter(Ye,Y_e,1.0);
ex superPot = 0;
vector<idx*> sumIndex;
MatterIndex a = getGaugeGroup("SU2")\-\>matterIndex(0);
MatterIndex b = getGaugeGroup("SU2")\-\>matterIndex(1);
idx i = Utils::familyIndex(0,famsiz)
As before we retrieve the index in the fundamental representation of $SU(2)$ (referred to as the MatterIndex) from a predefined list. We create the epsilon tensor in this space and the $\epsilon_{ij}$ term of eqn. (22) can then be directly implemented as shown. It has been assumed for this code that the fields $H_1$, $H_2$, sleptonL and sleptonR have been defined.

5 Effective Potential

The effective potential is one of the main components of Effective. This can be generated at tree-level or at one-loop in Effective. We give a brief review of the generation of the effective potential in the appendix. The result in the appendix can be extended to all fields to read, in the DR renormalization scheme,

$$V_1 = \frac{1}{64\pi^2} \mathcal{S}Tr \left[ M^4 \left( \ln \frac{M^2}{\mu^2} - \frac{3}{2} \right) \right].$$

Here the mass matrix $M$ is the tree-level mass matrix for a set of particles. The “supertrace”, $\mathcal{S}Tr[f(X)]$ is defined as

$$\mathcal{S}Tr[f(X)] = \sum_i (2s_i + 1)(-1)^{2s_i} f(X_i),$$

where $s_i$ is the spin of the $i$th particle. This “supertrace” is just the spin weighted trace of the mass matrices.

To find the value of the VEVs, the effective potential can be minimized as a function of the VEVs. Note that the conventional approach, to find those VEVs which set to zero the tadpole diagrams, is equivalent. The tadpole terms are given by

$$T_i = \left. \frac{\partial L}{\partial \phi_i} \right|_{\{\phi_j = \langle \phi_j \rangle\}}.$$
for all $\varphi_i$ with a non-zero VEV. Thus to solve
\[
\frac{\partial V_{\text{eff}}}{\partial \langle \varphi_i \rangle} = 0,
\]
minimizes the potential and is equivalent to solving for the zero of the tadpole contributions.

5.1 Effective Potential in Effective

We now turn to routines to access the effective potential in Effective. The effective potential can be accessed with the `ex Model::potential()` routine. This routine takes an `Approximation::Approximations` flag to specify whether the potential should be evaluated at tree-level or at one-loop. The default behaviour (if no `Approximation::Approximations` flag is given) is to return the tree-level value. At tree-level, this function returns an analytic result. At one-loop, it is returned numerically at the point in field-space given by the current VEVs. It is also worth noting that the potential is actually returned with the value of the potential when all VEVs are zero is subtracted from it, i.e. the difference between the value at the current location in field-space and the value at the origin.

To determine the “natural” values of the VEVs the effective potential needs to be minimized over a set of parameters. This can be achieved by a call to the `Numerics::extremizePotential()` routine. This routine takes a list of parameters to minimize the potential over. It also takes an `Approximation::Approximations` flag to indicate what level of approximation of the potential to use during the minimization. This routine uses the direction set (or Powell’s) method in multidimensions \[11\] to minimize the potential. If this method fails, the original values of the parameters are restored.

There is also a routine, `exvector Model::tadpoles()` which returns a list of the values of the tadpoles for the current values of the Parameters. This routine also takes an `Approximation::Approximations` flag. If none is given the default behaviour is to return the tree-level result.

Similar to the potential, there is a routine which can scan a parameter set and find where all all tadpole diagrams are zero. This routine can also use tree-level or one-loop tadpoles. The routine is `Numerics::solveZeroTadpoles()`. This routine takes a list of parameters to scan over and an approximation to use. This routine also takes a boolean flag which, when true, will save the values of the parameters passed to the routine and if the routine fails, it will restore the values. If false is passed, the values of the parameters after a failed call are unpredictable. This routine uses the Newton-Raphson method of root finding \[\text{[11]}\] to numerically determine the values of the Parameters that make all tadpoles zero.

Both the `extremizePotential()` and the `solveZeroTadpoles()` routines have built in error handling. This will print to the `cerr` stream the cause of any failed calls. These messages are useful to explain odd behaviour of a program
after a failed call. An error logging system will be implemented in a future update so all messages can be stored in a file.

We now give the example of calling these routines with the ElectroWeak model that we partially defined in the previous sections. The full definition of this class can be found in the appendix.

```cpp
vector<numeric> list;
list.push_back(&ew.getParameter("HiggsVev"));
ex pot = Numerics::extremizePotential(&ew,list);
Numerics::solveZeroTadpoles(&ew,list,true,
                          Approximation::OneLoop);
```

In this code the variable `ew` is an instance of the ElectroWeak class. We see here that the tree-level potential is minimized in the third line and the value of that minimized potential is stored in the `pot` variable. The fourth line then is an example of finding the value of the Higgs VEV which gives a zero one-loop tadpole.

### 6 Mass Matrices

We now turn to the mass matrices. These are derived from the Lagrangian by

\[
M_{ij} = \pm \left. \frac{\partial L}{\partial \varphi_i \partial \varphi_j} \right|_{\{\varphi_k = <\varphi_k>\}},
\]

(26)

where \(\varphi_i\) represents any field and \(<\varphi_k>\) is the expectation value of field \(\varphi_k\). \(M_{ij}\) contains the mass-squared matrix for scalars and vectors, but only the mass matrix for fermions. The plus or minus term is due to the fact that the mass term for vector bosons have a positive sign in the Lagrangian, but for fermions and scalars it has a negative sign.

Once we have evaluated all of the elements \(i,j\) of the mass matrix we must diagonalize it. The elements in the Lagrangian are the interaction eigenstates of the fields. The masses correspond to the mass eigenstates. Diagonalizing the mass matrix will give us a rotation matrix to mix the interaction states and give the mass states. The masses of the mass states are then the values of the diagonalized matrix.

#### 6.1 Tree Level Mass Matrices

At tree level the mass matrix generation and diagonalization is straightforward. Employing eqn. (26) we can create large matrices with all of the tree level terms. We can then divide the large matrix into block diagonal parts. The diagonalization procedure of the large matrix will give the same results as the diagonalization of the smaller matrices, but is much more efficient. Since all terms in EFFECTIVE represent real fields, we have real mass matrices. We can diagonalize the matrices to get

\[
M_{ij} = U_{ik} D_{kk} U_{kj}^T.
\]

(27)
The $D_{kk}$ matrix is a diagonal matrix and the $U_{ij}$ matrix is a unitary matrix which contains the rotations. From this one finds

$$D_{kk} = U_{ki}^T M_{ij} U_{kj},$$

and in the Lagrangian this is

$$\phi^T_i M_{ij} \phi_j = (U_{ki}^T \phi_i)^T D_{kk} (U_{kj}^T \phi_j) = (\phi^M_k)^T D_{kk} (\phi^M_k).$$

This allows us to interpret $U_{ki}^T \phi_i$ as the mass eigenstate $\phi^M_k$, which implies $U_{ki}$ is the mixing angle between interaction state $\phi_i$ and mass state $\phi^M_k$.

In Effective the classes Mass and MixingAngle provide an interface to this process. These objects are GiNaC objects which means they can be directly used in analytic expressions. When one evaluates these, the appropriate diagonalization is performed and the numerical element is returned.

A small discussion is needed about fermion masses. For the vector and scalar masses the diagonal matrix contains the mass squared. A negative mass-squared represents a tachyon and in turn has some implications to the parameters and structure of the theory. For fermions, however, a negative diagonal element is a negative mass. The mass-squared is still positive so it is not a tachyon. Instead, an appropriate shift of phase is required. Such a shift of phase converts the mass of the physical state from negative to positive. In Effective, such a rotation is performed internally automatically. Thus, fermionic mass states always have positive mass. This can be seen by shifting the physical mass state by

$$\phi^M \rightarrow e^{i\pi/2} \phi'^M,$$

This shift leads to

$$m \phi^M \phi^M = m (i \phi'^M) (i \phi'^M) = -m \phi'^M \phi'^M.$$

As we can see, the mass can be shifted from negative to positive simply by an appropriate phase shift. This shift amounts to reinterpreting the mass eigenstate to include the rotation by $U$ and the phase shift.

### 6.2 One-Loop Mass Matrices

When we move to the one-loop corrections to the mass matrices things get slightly more complicated. Now our Lagrangian couplings are not the bare parameters of the theory, but instead renormalized ones.

An element of the renormalized Lagrangian would read

$$\mathcal{L} = Z_i \partial_\mu \phi_i \partial^\mu \phi_i - \frac{1}{2} Z^M_{ij} M_{ij} Z^{1/2}_{ii} Z^{1/2}_{jj} \phi_i \phi_j,$$

where $Z^{1/2}_{ii} \phi_i$ is the renormalized field $\phi_{R_i}$, and $Z^M_{ij} M_{ij}$ is the renormalized mass, $M_{Rij}$. The diagonalization and mixing of states proceeds in the same
manner as before. The difficulty now lies in automatically computing $M_{Rij}$ from the tree-level Lagrangian and imposing the renormalization conditions.

We now define the two point Green functions $\Gamma^{(2)}_{ij}(p^2)$ that are included in \textsc{Effective} at one-loop. At one-loop there are two topologies that can enter the two point Green functions. These are shown in figure 1. We will refer to figure 1a as a three-point diagram (as the couplings are three-point couplings), and figure 1b as a four-point diagram. The three-point diagrams are functions of the masses of the two internal lines while the four-point diagrams are only functions of the mass of the one internal line. All diagrams are renormalized at a scale $\mu$.

![Diagram](a) ![Diagram](b)

Figure 1: The two topologies for one-loop two point Green functions.

We can now define the values of these individual diagrams for each combination of spins that can arise. In \textsc{Effective}, certain assumptions about the Lorentz structure of the couplings is made. In models where these assumptions break down, the one-loop corrections of \textsc{Effective} will be wrong. We have also implemented the contributions in the Feynman gauge. This means we also must include the appropriate ghost contributions. We will now discuss the contributions to three sets of diagrams: scalar-scalar, fermion-fermion and vector-vector. All of the corrections are expressed in terms of Passarino-Veltman \cite{12} functions in $d$ dimensions. A list of the Passarino-Veltman functions and the one-loop mass corrections are given in the appendix. It is extremely important to note that by including the complete set of one-loop diagrams for arbitrary couplings and spins we remain model-independent. However, we restrict ourselves to one-loop diagram topologies with up to four-point interactions. To include a model which contains higher-than-four-point interactions one would have to include some more integrals.
6.3 Renormalization Prescription

We now turn to the mechanism by which the two-point Green functions are used to provide the one-loop mass corrections in Effective. In Effective the $\overline{\text{MS}}$ renormalization prescription is used. This means that the terms with $(\bar{\epsilon})^{-1}$ from the Passarino-Veltman (PV) functions are absorbed into the definition of the bare parameters. In the current version the PV functions are only defined in terms of their finite contributions. Thus, even though the Green functions are defined in $d$ dimensions, using $d = 4 - 2\epsilon$ will not properly give finite contributions from terms like $\epsilon B_0(p^2, m^2_1, m^2_2; \mu^2)$. This is equivalent to using the $\overline{\text{DR}}$ scheme, where the momentum are taken in $d$ dimensions, but the vectors and Dirac $\gamma$ matrices are treated in 4 dimensions. Using the $\overline{\text{DR}}$ scheme we define the one-loop renormalized Green functions

$$\Gamma^{(2)}_R(p^2) = \Gamma^{(2)}_{\text{tree}}(p^2) + \Gamma^{(2)}_{1-\text{loop}}(p^2). \quad (33)$$

This is equivalent to using counterterms in the Lagrangian. For example the tree-level Green function for scalar fields is given by

$$\Gamma^{(2)}_{\text{tree}}(p^2) \propto [\partial_\mu \varphi]^\dagger \partial^\mu \varphi - m^2_0 \varphi^\dagger \varphi \rightarrow (p^2 - m_0) \varphi^\dagger \varphi.$$  

Now the one-loop term is of the form

$$\Gamma^{(2)}_{1-\text{loop}}(p^2) \propto (A p^2 + B) \varphi^\dagger \varphi,$$

so the renormalized term is

$$\Gamma^{(2)}_R(p^2) \propto (1 + A)[\partial_\mu \varphi]^\dagger \partial^\mu \varphi - (m_0 - B) \varphi^\dagger \varphi. \quad (34)$$

If one uses the on-shell prescription we require the pole of the propagator to be equal to the physical mass. This is

$$\Gamma^{(2)}_R(p^2) \bigg|_{p^2 = m^2_R} = 0,$$  

$$\frac{\partial \Gamma^{(2)}_R(p^2)}{\partial p^2} \bigg|_{p^2 = m^2_R} = 1. \quad (36)$$

This is equivalent to requiring that the renormalized Lagrangian takes the form

$$\mathcal{L}_R = \partial_\mu \varphi_R \partial^\mu \varphi_R + m^2_R \varphi_R \varphi_R.$$

We consider the case where there is mixing between fields; the on-shell renormalization condition is equivalent to requiring the denominator of the propagator to vanish. Thus the solution $p^2 = m^2_R$ gives

$$\text{Det} \left[ \delta_{ij}(p^2 - m^2_{0,ij}) + \Pi(p^2)_{ij} \right] = 0. \quad (37)$$

For fermion fields this takes the form

$$\text{Det} \left[ \delta_{ij}(\not{p} - m_{0,ij}) + \Sigma(\not{p})_{ij} \right] = 0. \quad (38)$$
where $\Pi(p^2)$ and $\Sigma(p)$ are the self-energy diagrams given in sections $E.1$-$E.3$ and $m^2_{0,ij}$ is the tree-level mass squared.

There is an additional complication that arises because the Green functions and renormalization conditions are defined for the mass eigenstates. This means we must find the mass eigenstates of the tree-level action first. We then renormalize the mass eigenstates by solving either eqn. $37$ or $38$. This will lead to a new rotation matrix which rotates the tree-level mass eigenstates into the renormalized mass eigenstates. The mixing angles between the interaction eigenstates and the renormalized mass eigenstates are then given by two rotations: the rotation from the interaction states to the tree-level mass states followed by a rotation from the tree-level mass states to the renormalized mass state.

### 6.4 Default One-Loop Mass Corrections

Effective provides two classes, \texttt{Mass} and \texttt{MixingAngle}, which provide algebraic objects that can be used and manipulated in expressions. Both of these classes require an object which instructs them what level of approximation to compute the values at. By default, if nothing is provided, the correction is given by the \texttt{MassCorrection::TreeLevel} class. As expected this class will simply diagonalize the tree level mass matrix and return the value desired (mass or mixing angle). We now discuss the three different one-loop corrections that have been implemented in Effective. Again, a user can implement their own approximation and renormalization conditions. Discussion of this extension is given in section $H$.

Effective provides three methods to generate one-loop mass corrections. All of the corrections are an implementation of the on-shell conditions. If a user wishes to implement alternate conditions, this can be done by implementing their own \texttt{MassCorrections} sub-class. Two of the provided classes return the same one-loop mass, but different mixing angles.

We begin with the approximate method \texttt{OneLoopMassApprox}. This class approximates the one-loop masses and provides only tree level mixing angles. This is done by computing only the Green functions for the diagonal terms, i.e. $\Pi_{ii}(p^2)$. This is computed with the tree level mass squared as the argument for the momentum squared. The mass squared is then

\[
m^2_i = m^2_{0,i} - \Pi(m^2_{0,i}),
\]

where $\Pi(p^2)$ are the self-energy diagrams given in sections $E.1$-$E.3$ and $m^2_{0,i}$ is the tree-level mass squared. For fermions this reads

\[
m_i = m_{0,i} - \Sigma(m_{0,i}).
\]

The other methods solve eqn. $37$ or $38$ using the bisection method \cite{spohn}. The \texttt{MassCorrection::OneLoopMass} class returns the one-loop mass but only gives the tree-level mixing angles. The \texttt{MassCorrection::OneLoop} class returns both the one-loop mass and mixing angle.
The implementation of the bisection method has five parameters which can be adjusted by the user to improve numerical stability and convergence. The method first brackets the solution. As an initial guess the starting point for the bracketing is given by the tree-level mass plus-or-minus 1%. In the case that the tree-level mass is zero, the bracket is given by $\pm 10^{-9}$. Also, for efficiency, as many zero mass fields are massless by construction, a quick check is performed. If the determinant from eqn. [67] or [68] is less than `Numerics::MassRoundError` it is assumed that this mass is exactly zero. The default value of `MassRoundError` is $10^{-4}$.

If the mass is not zero and the initial bracket does not bracket the solution, than the bracket is expanded until the solution is contained. If the expansion reaches the maximum number of iterations, `Numerics::MassMaxIterations`, than the method fails and the tree-level mass is returned and a failure message is printed. Each iteration the bracket grows by $F\Delta x$ where $\Delta x$ is the current size of the bracket and $F$ is the parameter `Numerics::MassFactor`. The default value is 0.6.

Once the solution is bracketed, the bisection method is iterated until the solution is found, within `Numerics::MassAccuracy` times the tree-level mass (if mass is zero then the accuracy is $10^{-9}$). The default value is $10^{-4}$, which means the solution is found within 0.01% of the value of the tree-level mass. If the method iterates `Numerics::MassScanIterations` times and no solution is found, the method prints a failure message and returns the last value used.

7 Renormalization Group Equations

In this initial release, EFFECTIVE does not automatically generate the renormalization group equations (RGE). The library does, however, provide a class to handle RGEs and evolve the parameters according to those equations. It is planned that in a future release, the one-loop RGEs will be automatically generated for an arbitrary model according to one or more renormalization schemes.

This section will detail how one implements the RGEs of a model. The user is free to implement these according to any renormalization procedure. It must be noted that the default behaviour of the one-loop mass corrections must be taken into account when implementing these RGEs. If one wishes to use a different renormalization scheme, it may also be necessary to implement a new set of one-loop mass corrections in order to be consistent.

7.1 Defining the Equations

The class that handles the RGEs is the class `RGE`. This class is created by the constructor `RGE(Model*)`; this constructor ties the instance of the `RGE` class to a particular model. Upon creation, the `RGE` class creates a list of the parameters in the `Model` and associates with each parameter an expression.

The user can supply the expression for the $\beta$ function of any of the parameters to any order desired. This can be provided by a call to the routine
RGE::setBeta(). This routine takes a Parameter x and the expression for \( \beta_x \). This routine saves the expression, in analytic form, so that it can evolve the renormalization scale and adjust the parameter x appropriately. An example of defining and setting the appropriate \( \beta \) function will be given now.

If we consider the one-loop \( \beta \) function for the gaugino mass coupling \( M_1 \) in the MSSM model. This is given by

\[
\beta_{M_1} = \frac{11 g'^2 M_1}{8 \pi^2},
\]

(41)

where \( g' \) is the \( U(1) \) coupling. This can be added to an RGE object by

```cpp
RGE rge(&ew);
ex g1 = ew.getGaugeGroup("U1")->coupling();
Parameter M1 = ew.getParam("M1");
rge->setBeta(M1,11*g1*g1*M1/(8*Pi*Pi));
```

We can now see how easy it is to implement the RGEs of a model, once they have been calculated. This procedure can be used to redefine the gauge coupling \( \beta \) functions as well as all other couplings of the theory. Once the full set of \( \beta \) functions have been defined, we can then evolve between different scales.

### 7.2 Example of automatic RGE generation

The current version of EFFECTIVE includes a prototypical example of automatic RGE generation. When the object is created the \( \beta \) functions of the gauge couplings are created automatically at one-loop. The one-loop \( \beta \) function is

\[
\beta_{g}^{1\text{-loop}} = -\frac{g^3}{16\pi^2} \left( \frac{11}{3} C_A - \sum_i \frac{4}{3} c_{i,g} C_{F_i/A_i} - \sum_j \frac{1}{3} c_{j,g} C_{F_j/A_j} \right).
\]

(42)

Here the sum over \( i \) is the sum over fermions. \( c_{i,g} \) is the charge of fermion \( i \) in group \( g \). The factor \( C_{F_i/A_i} \) represents the fact that the fermion could be a gaugino (in supersymmetry) or a regular fermion. If it is a gaugino it is in the adjoint representation and thus a factor of \( C_A \), otherwise the factor is \( C_{F} \). The sum on \( j \) is over all scalar fields with the same factors as in the fermionic case.

It is important to note that this equation should also include \( \theta(\mu - m_i) \) where \( \mu \) is the current renormalization scale and \( m_i \) is the mass of particle \( i \). This would correctly implement the mass effects in the \( \beta \) function. The other couplings of a model do not yet have a \( \beta \) function generated automatically. A future improvement is to include the mass effects into the gauge couplings and to provide the full one-loop RGE for an arbitrary model.

### 7.3 Evolving Between Scales

One of the powerful uses of the RGEs is to be able to define the parameters at one renormalization scale, but use them to calculate at another. This is especially
important in SUSY models where one wants to define boundary conditions at a high scale, i.e. $M_{\text{Plank}}$. These boundary conditions are usually used to unify several parameters to one value at the high scale, thus reducing the size of the independent parameter set.

In Effective the RGEs can be used to evolve the parameters between different scales. The current scale of the model is given by a Parameter which is labeled as "renormScale". This scale can be changed to a different scale and the parameters of the model are then evolved to that scale by the RGEs. This is achieved by the routine RGE::evolve(). This takes a numeric value of the new scale and uses the Runge-Carp-Kutta numerical method [11] to iteratively apply the differential equations of the RGE to the parameters so they are properly evolved to the new scale.

The class RGE also provides several routines which allow the user to simply define the desired values for the parameters at a particular scale. Then when the parameters are evolved to that scale, a simple call will force all the parameters to take the preset values. This is very useful when different parameters are defined at different scales and a consistent result is desired.

For example, consider parameters $a$ and $b$. The $\beta$ functions of these parameters are function of both $a$ and $b$. If we know $a$ should have a specific value $a_0$ at scale $\mu_1$ and $b$ should have the value $b_0$ at scale $\mu_2$, we must determine what $a$ is at $\mu_2$ and what $b$ is at $\mu_1$ by an iterative approach. We must make some educated guess for $b(\mu_1)$ and evolve to $\mu_2$. Our guess will be wrong so our $b(\mu_2)$ is not equal to $b_0$. If we set it equal to $b_0$ and evolve to $\mu_1$ chances are our $a(\mu_1)$ is not equal to $a_0$. We then set $a$ to $a_0$ and this procedure can be iterated until the solutions converge, or we decide they are not converging. If they don’t converge we must assume that the boundary conditions cannot simultaneously be fulfilled.

Such a procedure can be implemented in Effective. We will not refer to a specific model in the next code block, but instead show how the routines RGE::initialCondition(), RGE::applyInitial() and RGE::evolve() can be used to implement the above example.

```cpp
double delta = 0.1;
double mu1 = 90.;
double mu2 = 120.;
double a0 = 5.;
double b0 = 7.;
Parameter a = 'model'->getParameter("a");
Parameter b = 'model'->getParameter("b");
rge.initialCondition(a,5.,mu1);
rge.initialCondition(b,7.,mu2);
bool consistent = false;
int max_tries = 50;
int tries = 0;
'model'->getParameter("renormScale")=mu1;
b = 1.;
```
while(!consistent && tries < max_tries) {
    rge->evolve(mu2);
    if((b+delta) > b0 && (b-delta) < b0) consistent = true;
    rge->applyInitial(mu2,3.);
    rge->evolve(mu1);
    rge->applyInitial(mu1,3.);
    tries++;
}

We can see from this code that the routine RGE::initialCondition() allows us to set the value of a parameter at a scale. These initial conditions can then be applied later by a call to RGE::applyInitial(). We see that this routine takes a scale as an argument. All Parameters that were given an initial value are checked. If the scale that the initial conditions were defined at is within $\delta$ of the given scale, the initial value is applied. $\delta$ is the second argument to RGE::applyInitial(). If this is not provided the default value is 3.

Notice in this code that we have not designated any particular form to the $\beta_a$ and $\beta_b$ functions. They must be defined for the code to work, but their form does not impact the algorithm above. It only dictates if consistent solutions can be found.

It is also important to note that the $\beta$ functions are stored analytically. They can be printed in LaTeX format or in plain text by calling the routine RGE::print(). This routine takes a stream and prints the $\beta$ functions to it. This can be useful when debugging your definitions of the $\beta$ functions.

8 The examination of arbitrary observables using Effective

We have now discussed how to define the particle content of a model. We have enumerated the terms that this definition automatically supplies to the Lagrangian and explained how to include additional interaction terms and vacuum-expectation-values. We have seen how the effective potential and tadpoles can be accessed for this model both at tree-level and one-loop and how the potential can be minimized, and the tadpoles set to zero by numerical means. We have shown how the masses and mixing angles of the fields can be accessed and how the one-loop corrections are defined. Lastly, we have discussed the renormalization group equations and how they can be defined and used to give a consistent set of values at all scales.

The purpose of this library is to provide the user with a tool to study many different aspects of their model, yet we have not explained how to use effective to calculate physical observables other than the masses and mixing angles. Effective provides an abstract class Diagram that can be used to compute physical observables deriving couplings and parameters from the model. The Diagram class is not the only way that physically relevant information can be drawn out of Effective. A creative user may be able to find interesting and
exciting uses for this library well beyond the scope of this article or the authors’ imaginations!

As mentioned, the feature built into Effective which makes it easy to study arbitrary properties of a model is the Diagram class. The user specifies the expression corresponding to some diagram, and then effective automatically iterates through all appropriate fields on internal lines, and all appropriate couplings (with the right spin and derivative structure) at each vertex. The class is used internally by Effective with the Passarino-Veltman functions to obtain masses and mixing angles at one-loop, so the user can use these as examples when creating their own diagrams.

The class is an abstract class; some of the routines of the class must be filled in by the user in a class which is derived from Diagram. The main interface to the class is through the method evaluate(). This routine will call the abstracted routines appropriately, building up a summed value for the diagram as it loops through all $n$ point couplings and fields on internal lines in a model by calling the abstract function calculate() for each coupling to determine its value.

The routines that the user must define in the deriving class are calculate(), function(). The calculate() routine is called by the evaluate() routine. On input, an $n$ point coupling is provided, the routine should check that the coupling is appropriate to the diagram being defined. It is the responsibility of the class which inherits Diagram to specify which couplings are appropriate to the diagram. For example in the FourPtLoop class the coupling must correspond to the desired two external fields and have the same internal field.

In the current version of Effective the sums over fields on internal propagators must be carried explicitly in the user-written calculate() routine; those implementing new diagrams should copy over the appropriate loop statements. In a later version we hope to refactor this into the automatic parts of the Diagram base class.

The Diagram class provides a few routines which can be helpful when determining if the given coupling matches the desired criteria. The routine Diagram::makeList() will take the expression for the fields of a coupling and return a list; each element contains a flag indicating if the field was accompanied by a derivative, and the field itself. The routine Diagram::find() can be used to determine if a field with a particular derivative flag is in the list returned by makeList(). If it is, the routine Diagram::remove() can be used to remove the matching item. When the routine has decided whether the coupling is appropriate, it should call function().

function() is intended to be the value of the diagram for a particular choice of vertex and spins of internal fields. This idea can be seen in the mass corrections of section 4. In those diagrams, the function is defined for a particular choice of the spin of the intermediate particles. Then this function depends only on the $p^2$ of the Green’s function and the masses of the intermediate fields.

The best way to understand how the Diagram class is intended to be used is by a demonstration. This is too long to include in this article, and is instead deferred to a tutorial page on the website mentioned in the first section.
9 Practicalities

In this section we will discuss some practical issues with Effective. We will describe routines which reduce processor overhead by saving and loading the system's internal representation of a model: the coupling database and the parameter values. We will also discuss the rudimentary command line interface and how to achieve the same effects without relying on the command line interface.

Before we discuss these uses we must discuss one essential routine, \texttt{Model::initialize()}. This routine \texttt{must} be run before any physics is computed. This routine takes no arguments and specifies to the model that all the information needed to generate the Lagrangian and mass matrices is present. Once this is run the user may proceed to study their model as they wish.

Of course the purpose of this library is to provide the user with a tool to study many different aspects of their model. To this end, the discussions in this section and the next do not contain all possible uses. A creative user may be able to find interesting and exciting uses for this library well beyond the scope of this article.

9.1 Saving and Loading Couplings

\textsc{Effective} generates internally a coupling database from the action for use in calculating observables and mixing angles. This can grow to be quite large. For large models the generation of this database can take of order of 30 minutes on a modern computer. The results of this generation can be saved to a file, however, and on future runs this can be read in just a few seconds. We discuss here how to specify whether to generate the couplings or read them. We also discuss when the database \texttt{must} be generated.

The \texttt{Model} class has a variable \texttt{Model::couplingsFromScratch} which is a \texttt{static} member of the class. If this is set to \texttt{true} before the model is initialized, then the couplings will be generated directly from the Lagrangian. If this is \texttt{false} then the couplings will attempt to be loaded from a file. The default value of this variable is \texttt{false}.

If one wishes to load the couplings from a file, the filename must be specified before initializing the model. This is done by calling the \texttt{Model::couplings()} routine. This takes the filename as an argument. The same routine is used to specify a filename to save a file to. If the \texttt{static} variable \texttt{Model::saveCouplings} is true, then the model will automatically save the file when the program completes. These features are illustrated by the following code.

\begin{verbatim}
ElectroWeak ew;
Model::couplingsFromScratch = true;
Model::saveCouplings = true;
ew.couplings("ew.gar");
ew.initialize();
\end{verbatim}
These features are also accessible through the command line interface. A simple routine `Model::readCommandLine()` has been written that accepts the flags `-r`, `-s` and `-h`. The `-r` flag is used to specify that the couplings should be regenerated. The `-s` flag specifies that the couplings should be saved to the file and the `-h` flag is a help command which prints all options. This is a very simple interface but it allows the user to run the code with different behaviours without having to recompile.

If we rewrite the above code as

```cpp
ElectroWeak ew;
Model::readCommandLine(argc, argv, &ew);
ew.couplings("ew.gar");
new.initialize();
```

we can now run the executable in several different variants. If we wish to load the couplings, we provide no flags. If we want to save the couplings we give the `-s` flag and if we want to recompute the couplings, we give the `-r` flag. It is obvious that most times, the `-r` and `-s` flags will accompany each other.

It is also important for the user to know when they should recompute the couplings. The couplings must be recomputed if the Lagrangian is modified at all. For example, one may wish to begin studying a model with only one family of fermions. They may wish to then include 3 families of fermions and repeat their studies. Once they include the additional terms in the Lagrangian they must recompute the couplings. Failure to do this will mean that they will only load the couplings for their one family model and their studies will be wrong.

### 9.2 Modifying, Saving and Loading Parameters

One of the useful features of Effective is the ability to keep the expressions in analytic form and to perform numeric operations on these expressions. This is able to be done because all Parameters are simultaneously an algebraic object and a numeric value. The object is kept in all expressions until the `evalf()` function is called. At that point, the algebraic object is replaced by the numeric value of the parameter.

This duplicity of the Parameter object allows the user to create all their expressions, derived from the Lagrangian, and very quickly access the numeric value of the expression. Changing the value of a Parameter globally changes the value all expressions. When `evalf()`’d an expression will return its numeric value with the changed value for the parameter. For example, changing the value of a VEV will simultaneously change the value of the effective potential and the masses of the particles.

It is important to note that though the undiagonalized mass matrix is automatically changed to reflect the new parameter value, the diagonalized mass matrix may not be. If the mass matrix is larger than $2 \times 2$, then the diagonalization is a numeric routine. This means it must be recomputed to update the mass of a particle. Not to worry though, a simple call to the `Model::resetMasses()`
The routine will instruct the library to re-diagonalize all matrices which are diagonalized numerically.

To illustrate the power of this feature, we will give an example of how to evaluate the one-loop effective potential as a function of the VEV. Considering again the electro-weak model the following code scans the VEV and evaluates the one-loop effective potential.

```cpp
Parameter vev = ew.getParam("HiggsVev");
double v;
for(v = -246.; v<=246.; v+=5.) {
    vev = v;
    ew.resetMasses();
    cout << v << "\t"
    << ew.potential(Approximation::OneLoop).evalf()
    << endl;
}
```

We see in this code that to set the value of a Parameter, we treat it like we would a normal variable. A simple call to the = operator sets the numeric value of the VEV. This is then propagated to all the masses (used to compute the one-loop potential) and the tree-level potential expression. Simply calling evalf() on the potential replaces all the Parameters with their numeric value.

There is also a simple input/output mechanism for the values of the Parameters of a model. This saves or loads the values from a tab delimited file. This is achieved by the routines saveParameters() or loadParameters() of the class Model. These routines take the stream to read or write to. The loadParameters() routine will print an error message if the file does not have the correct structure or one of the Parameters has the wrong label. We give an example of the one-family electro-weak input file.

```
renormScale 91.0
HiggsVev 246.0
SU2 0.653089
U1 0.3550
Ye 1.3e-5
lambda 0.182513
mu 105.095
```

We notice that the first line of the file is the renormalization scale. This is also the scale for all of the parameters in this file. We saw in section 7 a possible treatment for parameters defined at different scales, these simple input/output routines are insufficient for input at multiple scales.

It is possible to save the parameters in any format desired, for example according to the SUSY Les Houches Accord [13] (LHA). This simply requires writing new code to print and read the format appropriately. The Les Houches Accord format has not been implemented in this version of Effective, but the SUSY LHA format is a planned upgrade.
10 Example: One Family Electro-Weak Model

We now turn to a concrete example, in its entirety. Throughout this manual we have defined pieces of the one-family electro-weak model. The full listing of the class definition can be found in the appendix. Here we present the main code block where the physics analysis begins. We will then give two examples of using the model to study something of physical relevance. These examples are not intended to be useful physics studies, only illustrative examples of using a model.

10.1 Main Code

We have defined the model ElectroWeak in the file EW.h, found in the appendix. We now give the code which initializes the model and allows us to begin a physics analysis.

```cpp
#include "EW.h"
#include <effective/effective.h>

int main(int argc, char **argv) {
    try {
        // Begin by initializing the model
        ElectroWeak ew;
        Model::readCommandLine(argc,argv,&ew);
        ew.couplings("ew.gar");
        ew.initialize();

        // Now lets load the values of the parameters
        ifstream parin;
        parin.open("EW.dat");
        ew.loadParameters(parin);
        parin.close();

        // Lets create a LaTeX stream, ew.tex
        ofstream fout;
        Utils::startLatex("ew.tex",fout);

        // ... insert physics analysis code here ...

        // now lets clean up
        Utils::closeLatex(fout);
    } catch(exception &p) {
        cerr << p.what() << endl;
        return 1;
    }
    return 0;
```
We will now discuss the new items in this code. We see that the file `effective.h` must be included in order to use `Effective`. We also see that two new routines `Utils::startLatex()` and `Utils::closeLatex()` have been used. The `startLatex()` routine takes a filename and a stream. It opens the file into the stream and sets the behaviour so that all expressions will be printed in LaTeX format. One must then remember to set and unset the math mode of LaTeX appropriately, depending on what is being saved to the file. This routine also prints a preamble so that the LaTeX file can be used without editing to produce a document. The `closeLatex()` then prints the \end{document} string and closes the stream.

The code also contains the `try { ... } catch { ... }` statements in the code. `Effective` uses the C++ exception handling mechanism to deal with unusual behaviour and internal errors. The `try/catch` clause catches any errors that have not been dealt with. The `catch` block then prints the error message, hopefully allowing the user to understand what failed. If these `try/catch` clauses are not included, the errors may just cause a crash without any information why.

10.2 Case Study 1: Higgs Potential vs. $v$

We now turn our attention to the first of two examples of a physics analysis. This first one will produce a tab-delimited file which can be used with a plotting program to give the tree-level and one-loop potential as a function of the VEV. This is given by

```cpp
Parameter vev = ew.getParam("HiggsVev");
ofstream datafile;
datafile.open("potData.dat");
for(double v = -246.0; v<=246.0; v+=0.5) {
  vev = v;
  ex tree = ew.potential(Approximation::TreeLevel);
  ex one = ew.potential(Approximation::OneLoop);
datafile << v << "\t" << tree.evalf() << "\t" << one.evalf() << endl;
  ew.resetMasses();
}
datafile.close();
```

Using the values of the parameters given in section 9.2 this code was used to produce figure 2

10.3 Case Study 2: $\cos \theta_W$ as a function of the $SU(2)$ coupling

We now turn to another observable. In this case we will look into the Weinberg angle, $\theta_W$. The definition of the angle is $M_W = M_Z \cos \theta_W$. Thus we can study
the ratio $M_W/M_Z$ at tree-level or one-loop. The following code produces a tab-delimited file which can be plotted. The result is shown in figure 3. In this example we fix the $U(1)$ coupling to 0.355 and the Higgs VEV, $\nu$, to 246.0 GeV. All values are for the renormalization scale $\mu = 91.2$ GeV.

```cpp
ofstream cosW;
cosW.open("cosW.dat");
double lowg, highg;
double fourtyPct = ew.getParameter("SU2").to_double()*0.4;
lowg = ew.getParameter("SU2").to_double()-fourtyPct;
highg = ew.getParameter("SU2").to_double()+fourtyPct;
idx i = ew.getGaugeGroup("SU2") -> gaugeIndex();
ex W = ew.getGaugeField("W") -> expression().subs(i==2);
ex Z = ew.getGaugeField("W") -> expression().subs(i==3);
ex MWt = Mass(&ew,W,MassCorrections::treeLevel);
ex MWo = Mass(&ew,W,MassCorrections::oneLoop);
ex MZt = Mass(&ew,Z,MassCorrections::treeLevel);
ex MZo = Mass(&ew,Z,MassCorrections::oneLoop);
for(double g = lowg; g<highg; g+=fourtyPct/20.) {
    ew.getParameter("SU2") = g;
    ew.resetMasses();
    cosW << "\t" << MWt.evalf()/MZt.evalf() << "\t"
          << MWo.evalf()/MZo.evalf() << endl;
}
```

Figure 2: The dependence on the tree-level (solid) and one-loop (dashed) effective potential as a function of $\nu$. 
Figure 3: Plot of $\cos \theta_W$ vs the $SU(2)$ coupling parameter for the tree-level masses (solid) and the one-loop masses (dashed). All values correspond to the renormalization scale $\mu = 91.2$ GeV.

11 Customizability

Throughout this text we have referred to EFFECTIVE’s modular design and the ability for a user to create extensions to study many different classes of models. One extension that may be of use is to implement a different group structure. Implementation of such a class is given in Appendix B.3 of [14] and repeated in tutorials on the website.

What we will discuss here is how to implement new objects to provide behaviours that differ from the default implementations. This includes a new subclass of the Spin class as well as new Field classes. One may wish to implement special operators which act on terms in the Lagrangian. This is also discussed below. Lastly, a user may wish to use their own renormalization scheme and thus provide a different set of one-loop corrections. They may also have a closed form of a mass correction to a given order. These types of customizations are discussed here.

11.1 New Operators

EFFECTIVE has a built in treatment of Lorentz derivatives. This includes knowledge of how to treat the Lorentz indices in compound terms which contain several Lorentz derivatives and vector fields. A user can define their own operator, but if this operates on fields, then the appropriate treatment of the operator must be included in the Spin class as discussed below.
In this part we will simply discuss what is needed to define the actual operator. We will take, as a concrete example, the idea of a light-cone operator, \((n^\pm)^\mu\). This operator takes any Lorentz vector and projects it to either the plus or minus part of the light cone. In this example, the light-cone operator needs two arguments, the object which is being projected (e.g. momentum or vector field) and the sign.

Code details of the definition of this object can be found in the GiNaC tutorials [10]. Here we present only the code and how the different parts cause the behaviour we desire. We begin with the class definition.

```cpp
const unsigned TINFO_LightCone = 0x1100008U;

class LightCone : public basic {
  GINAC_DECLARE_REGISTERED_CLASS(LightCone,basic);

private:
  bool itsSign;
  ex itsField;

public:
  LightCone(const ex &arg, bool plus);
  LightCone(const LightCone &lc);
  // virtual functions
  void do_print(const print_context &c, unsigned level = 0) const;
  void do_print_latex(const print_context &c,
                      unsigned level = 0) const;
  ex eval(int level=0) const;
  ex evalf(int level=0) const;
  ex op(size_t i) const;
  ex & let_op(size_t i) { return itsField; }
  size_t nops() const;
  bool sign() const { return itsSign; }
};
```

We can see that there are only a few functions that need to be implemented. Here we give the implementation of the constructor and the comparison operator. The other routines are trivial.

```cpp
GINAC_IMPLEMENT_REGISTERED_CLASS_OPT(LightCone,indexed,
  print_func<print_context>(&LightCone::do_print).
  print_func<print_latex>(&LightCone::do_print_latex));

LightCone::LightCone(const ex &arg, bool p)
  : basic(TINFO_LightCone), itsSign(p) {
    // Simply replace the lorentz index by a + or -
symbol lst idxs;
LorentzStructure::getIndices(arg,idxs);
// Indicate an error for non Lorentz vector objects
if(idxs.size() != 1) {
    cerr << "Called a light cone operator on a "
        << "Lorentz tensor with rank != 1.\n";
}
itsField = arg.subs(idxs[0]==LorentzStructure::lorentzIndex(0));

int LightCone::compare_same_type(const basic &other) const {
    int signdiff;
    const LightCone &o = static_cast<const LightCone &>(other);
    if(o.sign() == sign() signdiff = 0;
    else signdiff = o.sign() ? 1 : -1;
    int valdiff = itsField.compare(o.itsField);
    if(valdiff) return valdiff;
    else return signdiff;
}

Here we see that in order to get a functional operator we have simply had to properly define the comparison of two objects (in combination with the routines in the header file, which are trivial). These few simple routines provide an operator which acts on an expression from GiNaC and is treated as its own algebraic object in the engine.

Of course, this particular implementation may not be wildly useful. Instead one may which to write, in the eval() routine, code which applies to distributive property to the expression passed to it. So for example $(n^\alpha)(A_\mu + B_\mu)$ properly returns $A^\mu + B^\mu$. Again these issues are explained in detail in the GiNaC manual and tutorials.

11.2 New Spin Classes

As discussed in section 2, the Spin class determines several properties which determine what terms enter the Lagrangian. It also provides routines which are used to derive the couplings of fields. In order to change these behaviours, a user must implement a new subclass of Spin. Here we will discuss the Spin class in detail and explain what each routine is used for. This will be a valuable reference for a user who wishes to implement new types of fields.

We begin with table 1. Here we see the full list of routines in the Spin class and whether each routine is virtual or not. The virtual routines can be reimplemented by a subclass of Spin.

We discuss here what each of the routines of the Spin class means and how a new implementation can be made. We will begin with the virtual routines which have a default behaviour. This means that these routines do not need to be reimplemented by the user, unless they wish to change their behaviour.
| Routine                                      | Arguments                                      | Return |
|----------------------------------------------|-----------------------------------------------|--------|
| virtual interaction                          | Field*                                        | ex     |
| virtual isLorentzIndex                       |                                               | bool   |
| virtual imagPart                             |                                               | bool   |
| virtual massCorrection                       | ex, ex, ex, Model*, bool, bool                | ex     |
| virtual lorentzDerivativeCouplings           | ex, Couplings*, ex, ex, int, int              | epairv |
| virtual operator==                          | const Spin &                                 | bool   |
| virtual clone                                |                                               | Spin*  |
| virtual alternateOperatorCouplings           | ex, Couplings*, ex, ex, int                   | epairv |
| virtual massMatrixCoeff                      |                                               | ex     |
| virtual isMassSqrt                           |                                               | bool   |
| spin coeff                                   | Couplings*, ex, ex, ex, ex, ex, int, int, bool|        |
| getName                                      |                                               | string |

Table 1: Complete list of routines for the Spin class. Details of each routine can be found in the text.

- **isLorentzIndex()**: This routine indicates whether fields of this spin have a Lorentz index or not. The default value is false.

- **imagPart()**: This specifies whether a field should have a real and imaginary component (true) or just a real component (false). The default value is true.

- **alternateOperatorCouplings()**: This routine is used to find the couplings of a field of this spin in the Lagrangian when non-default operators have been implemented. This means that if the user has implemented their own operator which acts on terms in the Lagrangian, they must also implement this routine for all field types which are operated on by the new operator. This routine takes arguments we will label here as f, c, x, idx, level. The derivation of couplings is a recursive procedure in E2FFECTIVE. The procedure which derives the couplings will call this routine at every level of recursion. It is the responsibility of this routine to return a list of expression pairs (epairv) which each contain a new index and value for the coupling. This coupling is the derivative of x with respect to the new operator O_i acting on f, i.e. \( \frac{\partial x}{\partial O_i(f)} \). Each element of the list is for each additional new operator that has been defined. The new index is given by the old index, idx, times the field acted upon it by \( O_i \). The level argument is an integer which indicates how many levels of recursion are still to be done. In some cases this can be quickly used to decide whether to perform the derivative on x or not, thus speeding up...
the routine considerably. By default this routine returns an empty list.

- **massMatrixCoeff()**: This routine returns an expression which is a coefficient used for the mass matrix calculation. The mass matrices are computed by eqn. \[26\], where the coefficient is simply the sign. This factor is usually just a sign, but this implementation allows it to be whatever the user requires. The default behaviour is just to return 1.

- **isMassSqrt()**: This routine indicates whether the mass matrix requires a square root (`true`) to return the mass or not. For example the vector and scalar matrices give the mass squared whereas the fermion mass matrix gives only the mass. The default value is `true`.

We now turn to the non-virtual routines. These cannot be overwritten. Instead, they are just simple wrappers or data access routines. The constructor for a `Spin` object takes a **string** and a **double** value. This string is the name of the spin type and the double represents the spin factor, i.e. \((2s - 1)(-1)^{2s}\). The non-virtual routines are:

- **spin()**: This routine returns the value of the spin factor given to the constructor.

- **getName()**: This routine returns the name of this spin type given to the constructor.

- **coeff()**: This routine is a wrapper which takes the `Couplings` pointer as well as the field and expression to differentiate and performs the differentiation. This routine also takes an extra argument, the `exk`, which is modified to represent the new index after differentiation. The Lorentz structure of the index may not be unique in a brute force type of implementation. In order for the index to be unique for a given set of fields the differentiation is performed and the appropriate transformations of the Lorentz indices is applied so that the index is always unique for a given set of fields, and the coupling times that unique index is the correct expression from the Lagrangian. The final argument is a boolean which indicates whether to let the `Couplings` class handle the Lorentz index substitutions (`false`) or not.

We finally now turn our attention to the virtual functions which have no default implementation in the `Spin` class. These routines **must** be reimplemented by a `Spin` subclass in order to be used.

- **interaction()**: This returns the kinetic terms of the field of this spin type. This is the routine that must be changed implement different terms in the Lagrangian.

- **lorentzDerivativeCouplings()**: This is similar to the previously discussed `alternateOperatorCouplings()` routine. This routine is only for Lorentz derivatives and rather than return a list of expression pairs it returns only one (`epair`).

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• **operator==():** This is a simple comparison routine. If the two spin objects are identical, then `true` is returned.

• **clone():** This routine creates a new object which is identical to the one it is called on. This routine dynamically allocates a new pointer, it is the responsibility of the calling method to handle the memory deallocation.

Now we have discussed in detail the methods of the `Spin` object we must explain how a new spin class can be included into the framework of `Effective`. The default behaviour of `Effective` is to include the `ScalarSpin`, `FermionSpin` and `VectorSpin` spin types. A user can add a new `Spin` subclass by calling `Model::addSpin()` before the `initialize()` routine is called. A logical place is in the constructor or the `createMatterFields()` or `createGaugeFields()` routines. The `addSpin` routine will add the new spin to the list of spins. Each spin type in this list has a corresponding mass matrix. This means that only fields with the same `Spin` classes will mix.

It is also possible to create a new `Spin` class which replaces on of the default ones. In order to tell `Effective` to use the new class instead of the old one, the user must call `Model::changeSpin()`. This routine takes the new spin object and an index to one of the old ones. This index is either `Model::Scalar`, `Model::Fermion` or `Model::Vector`. Again, this routine needs to be called before the `initialize()` routine.

### 11.3 New Field Behaviours

Another major modification that a user may want to implement is to define a new type of field. For example, currently `Effective` does not provide a way for a field to be in the adjoint representation of a group and be charged under other groups. This is done, for example, in models with Higgs triplets [13].

Here we will give the outline as to the method to create such a field. The `MatterField` will be our starting reference. In fact, this almost completely describes our new field, except we need a list of flags to indicate which groups are in the adjoint representation and which are in the fundamental. One would then, when creating the list of indices, add the adjoint index to the list (which is retrieved by `GaugeGroup::gaugeIndex`) rather than the fundamental index (retrieved with `GaugeGroup::matterIndex`). The only other changes required would be to the `covariantDerivative()` and `susy()`, where instead of using the generators, the structure functions would need to be applied for the adjoint representation. Lastly, the `Field::Cr()` and `Field::C2()` functions would have to be defined to return the correct values for the adjoint representation.

As long as this new class is a subclass of `Field` we can immediately use it with `Effective` to study this new class of model. In fact, such an extension will be included in a future version of `Effective` along with improvements to the `Field` class to allow for a more natural extension to $N \neq 1$ SUSY and higher rank tensor fields. Though all of these are possible now, it would be difficult to make everything work in a natural way. Changing some underlying structures will simplify such extensions.
11.4 New One-Loop and Beyond Mass Corrections

The last major customization a user can implement is the mass corrections. The Mass class constructor takes three arguments. The model, the field and a MassCorrection class. This class has one virtual routine, operator(), that must be implemented. In EFFECTIVE there are four types of MassCorrection classes implemented. These are discussed in section 6. Here we discuss what operator() calculates so that users can implement their own version.

The operator() routine has the arguments: BlockMatrix &, Spin*, and Model*. This routine is called by the Mass and MixingAngle classes when evaluated. The BlockMatrix class contains the undiagonalized, diagonalized and the rotation matrix for tree-level and matrices for the correction (at any order) and the diagonalized result and the rotation matrix (also at any order). It is the operator() routines job to take the undiagonalized tree-level matrix and generate the mixing matrix and the diagonal values of the matrix, at any order. It is then up to the user to implement their corrections and the diagonalization of the mass matrix how they wish.

This procedure will become clear after we look at the OneLoopMassApprox implementation. This class gives the approximate one-loop mass, but only the tree-level mixing matrix. This is done by computing only the diagonal corrections to the diagonalized mass matrix.

```cpp
void OneLoopMass::operator()(BlockMatrix &bm, Spin *s, Model *m) {
    if(MassMatrix::rediag(bm)) {
        int temp;
        matrix r = bm.undiag;
        Utils::diagonalize(r, bm.diag, bm.rotate, temp);
    }
    if(bm.lastFlag != flag) {
        Utils::matrixCopy(bm.correction, correction(bm, m, s));
        Utils::matrixCopy(bm.crotate, bm.rotate);
        Utils::matrixCopy(bm.cdiag, bm.correction);
        ftime(&bm.lastDiag);
        bm.lastFlag = flag;
    }
}
```

This code simply checks that the parameters haven’t changed, and if they have rediagonalizes the tree level matrix. If the last evaluation wasn’t the same approximation as this one, this method then computes the corrections to matrix. From this one can imagine how they may implement a different type of correction. For example, the correction() routine which is called to fill the correction matrix with its values, may contain clauses to identify specific particles and apply corrections derived from a paper for a particular particle.
12 Future Outlook

This article has been designed to serve a few purposes. We have described many of the important tools that allow a user to probe the physics of their model as they see fit. This gives users a manual by which to begin to implement their model.

We have also explained the assumptions and physics behind the code. This way a user is able to know what to expect from the default behaviour of the library, and design extensions that suit their needs. We have discussed several extensions that are possible. These are the ones that we feel are likely to be most useful for users.

We have not provided detailed code documentation nor a list of all possible extensions. This has been reserved for the website, which will contain tutorials with detailed code.

Throughout this text we have discussed future improvements we plan to make on the code. The status of such improvements can be found on the webpage. The most pressing upgrades we would like to implement are the following:

- We plan to generalize the Field implementation so a wider range of models can be easily implemented.
- For many models some symmetries, for example $SU(3)_C$, are unbroken. It may be desirable to not explicitly sum over the indices of these groups, except when computing numerical results.
- Properly provide a supermultiplet object which can be used to define the superpotential.
- Automate the one-loop RGE for at least one renormalization scheme.

Acknowledgments

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A ElectroWeak definition

We present here the full definition of the ElectroWeak class. Here we define it in one file, EW.h, though the class definition and the routine implementations could be separated into two files.

```c
#define EW_H
#include <ginac/ginac.h>
#include <effective/effective.h>
```
#include <fstream.h>

const unsigned int SU2w = 2;
const unsigned int U1b = 1;

using namespace std;
using namespace GiNaC;
using namespace Effective;

const int famsize = 1;

class ElectroWeak : public Model {
public:
    ElectroWeak() : Model() {}
    virtual void createGaugeGroups();
    virtual void createGaugeFields();
    virtual void createMatterFields();
    virtual void addOtherTerms();
};

void ElectroWeak::createGaugeGroups() {
    addGaugeGroup(new U1Group("U1", "{g'}", this, U1b));
    addGaugeGroup(new SU2Group("SU2", "{g_W}" , this, SU2w));
}

void ElectroWeak::createGaugeFields() {
    VectorSpin v;
    addField("B",new GaugeField("B","B",v,getGaugeGroup("U1"));
    addField("W",new GaugeField("W","W",v,getGaugeGroup("SU2"));
}

void ElectroWeak::createMatterFields() {
    numeric half(1,2);
    ScalarSpin s;
    FermionSpin f;
    GaugeGroup *u1 = getGaugeGroup("U1");
    GaugeGroup *su2 = getGaugeGroup("SU2");
    addField("l", new MatterField("l", \ell, f, famsize,
                              u1,-half,su2,1));
    addField("eR", new MatterField("eR","e_R",f,famsize,u1,-1));
    addField("H", new MatterField("H","H",s,1,u1,half,su2,1));

    // Now add Higgs Vev
    Parameter upsilon = addParameter("HiggsVev","upsilon",246.0,
                                   Parameter::vev);
    addVev("HiggsVev",getField("H"),lst(getIndex("H","SU2")==2),
                                   42)
(upsilon+Model::star));
addVevParameter(upsilon);
}

void ElectroWeak::addOtherTerms() {
  // Create $\mu^2 H \bar{H} H$
  Parameter mu = addParameter("mu", "\mu", 1.0);
  vector<idx*> indices = getField("H")->getIndices();
  ex H = getField("H")->expression();
  ex a = pow(mu,2)*H.conjugate()*H;
  add(Utils::sumIndices(a,indices).expand();

  // Create $\lambda (HH)^2$
  Parameter lambda = addParameter("lambda", "lambda", 1.0);
  ex la = Utils::sumIndices(H.conjugate()*H,indices).expand();
  add(-lambda*pow(la,2));

  // Now add lepton Yukawa
  ex Ye;
  if(famsize != 1) Ye = addFamilyMatrix("Ye","{Y^e}",famsize);
  else Ye = addParameter("Ye", "{Y^e}", 1.0);
  idx j = Utils::familyIndex(1,famsize);
  ex eR = getField("eR")->expression();
  ex l;
  if(famsize != 1) l = getField("l")->expression().conjugate()
    .subs(Utils::familyIndex(0,famsize)==j);
  else l = getField("l")->expression().conjugate();
  ex b = -Ye.subs(j==Utils::familyIndex(0,famsize))*l*H*eR;
  indices = getField("l")->getIndices();
  if(famsize != 1) indices.push_back(&j);
  ex res = Utils::sumIndices(b,indices).expand();
  add(2*Utils::real(res));
}

B Summary of Classes and Routines

This appendix provides a summary of some of the more important routines discussed in this article.

B.1 Classes and Routines Used for the Field Content

In this section we present a summary of the classes and routines that are needed to define the field content of a model.

Table 2 gives a list of the classes encountered when defining the field content. This is intended to give a brief synopsis of the relevant classes one should use
when defining ones field content.

| Class         | Description                                                                 |
|---------------|-----------------------------------------------------------------------------|
| GaugeGroup    | Abstract class defining the behaviour of a group                           |
| U1Group       | The definition of the $U(1)$ group                                           |
| SU2Group      | The definition of the $SU(2)$ group                                          |
| SU3Group      | The definition of the $SU(3)$ group                                          |
| Spin          | Abstract class defining behaviour of spin objects                           |
| ScalarSpin    | Definition of the scalar spin behaviour                                     |
| FermionSpin   | Definition of the fermion spin behaviour. This is specific to the Weyl fermions. A new class is needed for Dirac fermions |
| VectorSpin    | Definition of the vector spin behaviour                                     |
| Field         | Abstract class of a set of fields with the same properties                  |
| GaugeField    | Class specific to fields which mediate the interactions                     |
| MatterField   | Class which describes the remaining fields                                  |
| Parameter     | Analytic object with a numeric value which can change                       |

Table 2: Table of the classes used to define the field content.

Table 3 gives the list of routines which are used to define the field content. The first column gives the routine; all routines in the table are found in the Model class. Information on the arguments to the routines can be found in the text of the previous sections, or online at the URL given in the first section.

| Routine              | Description                                                      |
|----------------------|------------------------------------------------------------------|
| addGaugeGroup        | Adds a new group to the model                                    |
| addField             | Adds a field to the model                                        |
| addParameter         | Adds a new parameter to the model                                |
| addVev               | Adds a vacuum expectation value to the model                     |
| addVevParameter      | Tells model Parameter is a VEV                                   |
| getGaugeGroup        | returns the pointer to the gauge group                           |
| getField             | returns a pointer to a Field                                     |
| getGaugeField        | returns a pointer to a GaugeField                                |
| getMatterField       | returns a pointer to a MatterField                               |
| getParam             | returns a Parameter object                                       |
| getParameter         | returns a numeric object                                         |

Table 3: Routines used to define field content.

The Model class is an abstract class. The user must define their own concrete version of this class which contains their field content. Table 4 describes the routines needed to define the field content of the concrete class.
### B.2 Classes and Routines Used for the Interaction Terms

Table 5 gives the list of routines in the `Model` class that were used for creating the interaction terms.

| Class  | Routine         | Description                                                                                                                                 |
|--------|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| Model  | addFamilyMatrix | This creates a matrix with a unique Parameter for each individual element. One can retrieve the whole matrix with `getFamilyMatrix()` or each element with `getParameter()` or `getParam()`. |
| Model  | getFamilyMatrix | This returns the matrix (with unreferenced indices) given by the input string.                                                              |
| Model  | add             | Add the expression to the Lagrangian.                                                                                                         |
| Field  | expression      | Returns the (complex) expression for a field.                                                                                                 |
| Field  | getIndices      | Returns all of the indices (except Lorentz) for a Field. There is an optional argument which when `false` removes the family index from the list. |
| Utils  | familyIndex     | Returns a symbolic index for the family space from a predefined list of indices.                                                               |
| Utils  | sumIndices      | Sums the expression over the indices given.                                                                                                  |

Table 5: List of classes and routines used for the interaction terms.

### B.3 Classes and Routines used for the Effective Potential

Here we summarize the classes and routines which were needed to deal with the effective potential. Table 7 gives the descriptions of the two routines in the `Numerics` class that were encountered.

Table 7 shows the routines in the `Model` class which were used with the effective potential. We have also introduced the enumerated type `Approximation::Approximations` and two possible values it can take, `Approximation::TreeLevel` and `OneLoop`.

---

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### Routine Description

**extremizePotential**
This routine takes a list of Parameters and finds the values which minimize the effective potential. This routine also takes an approximation which to use when evaluating the potential. This numerical routine is based on the direction set (or Powell’s) method in multidimensions.

**solveZeroTadpoles**
This method is similar to the previous one except it takes a list of Parameters and finds the values which give tadpoles which are equal to 0. This method can also take different approximations to use to evaluate the tadpole diagrams. This method uses the Newton-Raphson root finding method.

Table 6: Routines in the Numerics class used for the effective potential and tadpoles.

| Routine       | Description |
|---------------|-------------|
| extremizePotential | This routine takes a list of Parameters and finds the values which minimize the effective potential. This routine also takes an approximation which to use when evaluating the potential. This numerical routine is based on the direction set (or Powell’s) method in multidimensions. |
| solveZeroTadpoles | This method is similar to the previous one except it takes a list of Parameters and finds the values which give tadpoles which are equal to 0. This method can also take different approximations to use to evaluate the tadpole diagrams. This method uses the Newton-Raphson root finding method. |

Table 7: The two routines of the Model class which were discussed in section 5.

#### B.4 Classes and Routines for Masses and Mixing Angles

Table 8 shows the two constructors of the classes Mass and MixingAngle. These classes are GiNAC objects that can be used in expressions. The value of them is not evaluated until a call to evalf() is made. This means the user can create expressions as functions of the masses and mixing angles of their model and trust that the values will take the appropriate values each time evalf() is called. In order to make this process more efficient, the masses aren’t re-evaluated every time evalf() is called. They are re-evaluated if one of two conditions holds. The first is that the approximation being used is different than the last one used. The second condition is if a flag has been set to force re-evaluation. This is provided by the function Model::resetMasses().

Table 9 provides a summary of the four mass corrections provided in Effective.
| Constructor | Description |
|-------------|-------------|
| Mass(Model*, ex, MassCorrection) | This constructor takes a Model pointer and a field and provides a GiNaC object which evaluates the mass of the field under the given approximation. |
| MixingAngle(Model*, ex, ex, MassCorrection) | This constructor is similar to the Mass constructor except it takes two fields. This provides the value for element given by the two fields of the rotation matrix used to diagonalize the mass matrix. |

Table 8: Description of the Mass and MixingAngle constructors.

| Class | Description |
|-------|-------------|
| TreeLevel | Simply diagonalizes the tree level mass matrix |
| OneLoopMassApprox | This uses the tree level diagonalized mass matrix and computes corrections to the diagonal elements only. |
| OneLoopMass | Solves, for $p^2$, the determinant equal to 0 in eqns. 57 and 58. |
| OneLoop | This uses the same approach as OneLoopMass and also computes the one-loop mixing angles. |

Table 9: The four default mass corrections provided in Effective.

### B.5 Classes and Routines for the RGEs

Section 7 introduced a new class RGE. This class was responsible for properly treating the RGEs and the evolution of parameters between scales. In this version of Effective, this class is simple and requires the user to input the expressions for the $\beta$ functions. It is planned that in future versions, an automatic one-loop calculation for all parameters will be provided for at least one renormalization scheme.

Table 10 shows a list of the routines of RGE that have been described and used in the discussion of the RGEs.

### C Effective Potential Review

We will begin by deriving the effective potential. We will then show how the one-loop effective potential can be derived in a model-independent way. This means that the one-loop correction only depends on the masses of the model, not explicitly on the couplings. This review is derived from [10]. The interested reader can find a more thorough discussion there.

We begin by considering a theory of one scalar field $\phi$ with a Lagrangian
Routine | Description
--|--
evolve | This evolves the parameters from the current scale to the scale provided.
initialCondition | This routine allows the user to define the default value of a parameter at a particular scale. None of the initial conditions need be defined at the same scale.
applyInitial | This routine takes a scale, \( \mu \), and a \( \delta \) value as arguments. It then finds all parameters with initial conditions defined at the \( \mu \pm \delta \) and applies them.
print | This prints the analytic form of the \( \beta \) functions to the stream provided.

Table 10: The list of routines of the class \texttt{RGE}.

density \( \mathcal{L}\{\phi(x)\} \). The action is given by

\[
S[\phi] = \int d^4x \mathcal{L}\{\phi\}.
\] (43)

The vacuum-to-vacuum expectation value \( \langle 0_{\text{out}} | 0_{\text{in}} \rangle_j \) is given by

\[
Z[j] = \langle 0_{\text{out}} | 0_{\text{in}} \rangle_j = \int \mathcal{D}\phi \exp\{i(S[\phi] + \phi j)\},
\] (44)

where

\[
\phi_j = \int d^4x \phi(x)j(x).
\] (45)

We can define the connected generating functional, \( W[j] \), in terms of the vacuum-to-vacuum expectation value

\[
Z[j] \equiv \exp\{iW[j]\}.
\] (46)

We now define the effective action, \( \Gamma[\phi] \), for \( S[\phi] \) such that its classical field equation is the solution to the Schwinger-Dyson equation for \( S[\phi] \). That is, we require \( \Gamma'[\phi] = j \). Solving this equation gives

\[
\Gamma[\tilde{\phi}] = W[j] - \int d^4x \frac{\delta W[j]}{\delta j(x)} j(x),
\] (47)

where

\[
\tilde{\phi}(x) = \frac{\delta W[j]}{\delta j(x)}.
\] (48)

\( \tilde{\phi}(x) \) is then a weighted average of the fluctuations of the field \( \phi \). In a translationally invariant theory, this is a constant. \texttt{EFFECTIVE} is designed to only deal
with translationally invariant theories, therefore, \( \bar{\phi}(x) \) must be a constant, \( \phi_c \), which is the VEV of the field. The effective potential can then be defined as

\[
\Gamma[\phi_c] = - \int d^4x V_{\text{eff}}(\phi_c).
\]  

(49)

We now write this as an expansion around \( \phi_c = 0 \)

\[
V_{\text{eff}}(\phi_c) = - \sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n \Gamma^{(n)}(p_i = 0),
\]  

(50)

where the \( \Gamma^{(n)} \) are the one-particle irreducible (1PI) Green functions. If we now minimize the potential over the constant field \( \phi_c \) we find the vacuum state of the theory.

At tree level, the effective potential, eqn. (50), is identical to the classical effective potential. This can be stated simply as

\[
V_{\text{tree}} = -\mathcal{L}(\phi_i \rightarrow \phi_{i,c}).
\]  

(51)

We now discuss the one-loop correction to this potential for a model with one self-interacting scalar field. The results will generalize to all fields. This simple model is given by

\[
\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4.
\]  

(52)

As just shown, the one-loop correction to the tree-level effective potential is given by the sum of all 1PI diagrams with a single loop and zero external momenta. The \( n \)th diagram has \( n \) propagators, \( n \) vertices and \( 2n \) external legs. The propagators contribute a factor of \( i^n \left( p^2 - m^2 - i\epsilon \right)^{-n} \). Each pair of external lines contributes a factor \( \phi_c^{2n} \) and each vertex a factor \( -i\lambda/2 \). Including a global symmetry factor we have

\[
V_1(\phi_c) = \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \log \left[ 1 - \frac{\phi_c^2 / 2}{p^2 - m^2 - i\epsilon} \right].
\]  

(53)

After a Wick rotation in the \( \overline{\text{DR}} \) scheme [17] this is

\[
V_1(\phi_c) = \frac{1}{64\pi^2} m^4(\phi_c) \left( \ln \frac{m^2(\phi_c)}{\mu^2} - \frac{3}{2} \right),
\]  

(54)

where

\[
m^2(\phi_c) = \frac{d^2V_0(\phi_c)}{d\phi_c^2},
\]  

(55)

is the tree-level mass and \( \mu \) is the renormalization scale.
D Passarino Veltman Functions

In this appendix we provide the definitions of the Passarino-Veltman functions [12] that are provided in Effective. These can be accessed by calling, for example, Passarino_Veltman::A0(). The derivatives can be found by calling the GiNaC diff() routine.

\[ A_0(m^2) = \frac{16\pi^2}{\mu^{d-4}} \int \frac{id^d q}{(2\pi)^d} \frac{1}{q^2 - m^2 + i\epsilon} , \]
\[ B_0(p^2, m_1^2, m_2^2) = \frac{16\pi^2}{\mu^{d-4}} \int \frac{id^d q}{(2\pi)^d} \frac{1}{(q^2 - m_1^2 + i\epsilon)((q + p)^2 - m_2^2 + i\epsilon)} , \]
\[ p_\mu B_1(p^2, m_1^2, m_2^2) = \frac{16\pi^2}{\mu^{d-4}} \int \frac{id^d q}{(2\pi)^d} \frac{q_\mu}{(q^2 - m_1^2 + i\epsilon)((q + p)^2 - m_2^2 + i\epsilon)} , \]
\[ g_{\mu\nu} B_{00}(p^2, m_1^2, m_2^2) + p_{\mu} p_{\nu} B_{11}(p^2, m_1^2, m_2^2) = \frac{16\pi^2}{\mu^{d-4}} \int \frac{id^d q}{(2\pi)^d} \frac{q_\mu q_\nu}{(q^2 - m_1^2 + i\epsilon)((q + p)^2 - m_2^2 + i\epsilon)} . \]

The first two, \( A_0 \) and \( B_0 \), can be expressed to \( \mathcal{O}(\epsilon) \) for \( d = 4 - 2\epsilon \) as

\[ A_0(m^2; \mu^2) = m^2 \left( \frac{1}{\epsilon} - 1 + \ln \frac{m^2}{\mu^2} \right) + \mathcal{O}(\epsilon), \]
\[ B_0(p^2, m_1^2, m_2^2; \mu^2) = \frac{1}{\epsilon} - \ln \frac{p^2}{\mu^2} - f_B(x_+) - f_B(x_-) + \mathcal{O}(\epsilon). \]

In Effective we subtract only the terms proportional to

\[ \frac{1}{\epsilon} = \frac{1}{\epsilon} - \gamma_E + \ln(4\pi). \]

We also have

\[ x_\pm = \frac{s \pm \sqrt{s^2 - 4p^2(m_1^2 + i\epsilon)}}{2p^2} , \quad f_B(x) = \ln(1 - x) - x \ln \left( 1 - \frac{1}{x} \right) - 1 , \]

and \( s = p^2 - m_2^2 + m_1^2 \).

E Mass Corrections

In this section we provide the formula for all of the model independent one-loop mass corrections.

E.1 Scalar-Scalar

We now define the contributions to the two-point Green functions for scalar fields. There are 6 classes of diagrams which can contribute to the full two-point
Green function for scalar fields: two four-point diagrams and four three-point diagrams.

The four-point diagrams have an internal field as a vector or a scalar. We designate these two contributions as \( \Pi^{S4} \) and \( \Pi^{V4} \). It is assumed that the scalar four point coupling is a simple scalar quantity, \( C_{S4} \), where the vector four-point coupling is a tensor of the form \( C_{V4} g^{\mu\nu} \). The contributions are then

\[
\Pi^{S4}(p^2, m_1^2, m_2^2; \mu^2) = \frac{C_{S4}}{16\pi^2} A_0(m_1^2, m_2^2),
\]

\[
\Pi^{V4}(p^2, m_1^2, m_2^2; \mu^2) = -\frac{dC_{V4}}{16\pi^2} A_0(m_1^2, m_2^2).
\]

The spin of the internal fields for the three-point diagrams can be pairs of all three types of spins, as well as a vector-scalar pair. We designate these contributions as \( \Pi^{S3}, \Pi^{F3}, \Pi^{V3} \) and \( \Pi^{VS} \). We assume that the scalar and fermion three-point couplings are simple scalar quantities, \( C_{S3} \) and \( C_{F3} \) respectively. This leads to the contributions

\[
\Pi^{S3}(p^2, m_1^2, m_2^2; \mu^2) = \frac{C_{S3}^{(1)} C_{S3}^{(2)}}{16\pi^2} B_0(p^2, m_1^2, m_2^2),
\]

\[
\Pi^{F3}(p^2, m_1^2, m_2^2; \mu^2) = \frac{dC_{F3}^{(1)} C_{F3}^{(2)}}{16\pi^2} \left[p^2 (B_1(p^2, m_1^2, m_2^2; \mu^2) + B_11(p^2, m_1^2, m_2^2; \mu^2) + dB_00(p^2, m_1^2, m_2^2; \mu^2) + m_1 m_2 B_0(p^2, m_1^2, m_2^2; \mu^2))\right],
\]

where the superscripts differentiate between the two vertices in the three-point diagram.

The vector three-point coupling is assumed to have the Lorentz structure \( C_{V3} g^{\mu\nu} \). Again, using the Feynman gauge, this gives

\[
\Pi^{V3}(p^2, m_1^2, m_2^2; \mu^2) = \frac{dC_{V3}^{(1)} C_{V3}^{(2)}}{16\pi^2} B_0(p^2, m_1^2, m_2^2). \]

Lastly we turn to the scalar-vector three point coupling. It is assumed that these couplings are derived from terms in the Lagrangian \( C_{VS}\partial_\mu \phi \partial_\nu A_\mu g^{\mu\nu} \). This means we must consider the derivative to lie on both the external and internal fields. Since we have two couplings of this type we find three contributions: the derivative is on both of the external fields, the derivative is on both the internal fields, and the derivatives lie one on the external and one on the internal. These are denoted by \( \Pi^{VS}_E \), \( \Pi^{VS}_I \) and \( \Pi^{VS}_{IE} \), respectively, and are given by

\[
\Pi^{VS}_E(p^2, m_V^2, m_S^2; \mu^2) = \frac{C^{(1)}_{VS} C^{(2)}_{VS}}{16\pi^2} p^2 B_0(p^2, m_V^2, m_S^2; \mu^2)
\]

\[
\Pi^{VS}_I(p^2, m_V^2, m_S^2; \mu^2) = \frac{C^{(1)}_{VS} C^{(2)}_{VS}}{16\pi^2} \left[p^2 B_1(p^2, m_V^2, m_S^2; \mu^2) + dB_00(p^2, m_V^2, m_S^2; \mu^2)\right]
\]

\[
\Pi^{VS}_{IE}(p^2, m_V^2, m_S^2; \mu^2) = \frac{C^{(1)}_{VS} C^{(2)}_{VS}}{16\pi^2} p^2 B_1(p^2, m_V^2, m_S^2; \mu^2).
\]
To get the total contribution for the vector-scalar three point coupling we must sum these terms, i.e. \( \Pi V^S = \Pi V^I_i + \Pi V^S_{ij} + 2\Pi V^S_{ij} \), where the factor of 2 is from the two combinations of placing the derivative on the internal and external fields. It is also worth noting that the couplings between the three contributions in eqn. (61-63) do not have to be equal.

In order to get the complete scalar-scalar two-point Green function we must sum over all possible internal fields for each contribution. This is

\[
\Pi_{kl}(p^2) = \sum_{i \in \text{vector}} \left[ \Pi_i^V + \sum_{j \in \text{scalar}} \Pi_{ij}^V + \sum_{j \in \text{vector}} \Pi_{ij}^S \right] + \sum_{i,j \in \text{fermion}} \Pi_{ij}^F \]

where the indices \( k \) and \( l \) represent the external scalar fields and indices \( i \) and \( j \) represent the choice of internal loop fields. Of course, in this sum many choices of \( i \) and \( j \) do not contribute as these fields don’t couple to \( k \) and \( l \).

**E.2 Fermion-Fermion**

The fermion-fermion self energy consists only of three point coupling diagrams with a fermion-vector or a fermion-scalar internal loop pair. The scalar-fermion vertex is assumed to be a simple scalar coupling, \( C_S \), while the vector-fermion coupling is assumed to be of the form \( C_V \gamma^\mu \). The contribution from these diagrams are

\[
\Sigma^S(p, m_f^2, m_s^2, \mu^2) = \frac{C_S^{(1)} C_S^{(2)}}{16 \pi^2} \left\{ pB_1(p^2, m_f^2, m_s^2, \mu^2) \right. + m_f B_0(p^2, m_f^2, m_s^2, \mu^2) \} \]

\[
\Sigma^V(p, m_f^2, m_s^2, \mu^2) = \frac{C_V^{(1)} C_V^{(2)}}{16 \pi^2} \left\{ (d-2) \ pB_1(p^2, m_f^2, m_s^2, \mu^2) \right. - d m_f B_0(p^2, m_f^2, m_s^2, \mu^2) \} .
\]

Again to construct the full two-point Green function one must sum over all possible choices of internal fields. The two-point Green function for fields \( k \) and \( l \) is

\[
\Sigma_{kl}(p) = \sum_{i \in \text{fermion}} \left[ \sum_{j \in \text{scalar}} \Sigma_{ij}^S + \sum_{j \in \text{vector}} \Sigma_{ij}^V \right].
\]

**E.3 Vector-Vector**

The vector-vector two-point Green function is composed of seven classes of diagrams, two four-point coupling diagrams and five three-point coupling diagrams.
The four-point couplings diagrams contain either an internal scalar or vector field. The contributions from these diagrams are denoted by \( \Pi_{\mu\nu}^{S,i} \) and \( \Pi_{\mu\nu}^{V,i} \), respectively. The scalar four-point coupling between scalar \( \phi \) and vector fields \( A_0^a A_0^b \) is assumed to have the form \( C_{S4} g^{\mu\nu} \), where \( C_{S4} \) implicitly contains the \( a \) and \( b \) indices. The contribution is

\[
\Pi_{\mu\nu}^{S,i}(p^2, m^2, \mu^2) = \frac{C_{S4}}{16\pi^2} g_{\mu\nu} A_0(m^2, \mu^2). \tag{68}
\]

The vector four-point coupling between vectors \( A_0^a, A_0^b, A_c^\lambda \) and \( A_0^d \) is given by \( C_{V4}^{(1)} g^{\mu\nu} g^{\lambda\sigma} + C_{V4}^{(2)} g^{\mu\lambda} g^{\nu\sigma} + C_{V4}^{(3)} g^{\mu\sigma} g^{\rho\lambda} \) with the group indices \( a, b, c \) and \( d \) contained in the coefficients \( C_{V4}^{(1,2,3)} \). The contribution from the diagram is

\[
\Pi_{\mu\nu}^{V,i}(p^2, m^2, \mu^2) = \frac{-dC_{V4}^{(1)} + C_{V4}^{(2)} + C_{V4}^{(3)}}{16\pi^2} g_{\mu\nu} A_0(m^2, \mu^2). \tag{69}
\]

This result is found for \( d \) dimensions and in the Feynman gauge.

We now turn to the contributions from the three-point coupling diagrams. We begin with the diagram which contains a pair of scalars for the internal fields. The couplings for this diagram are assumed to come from a term \( C_{S3} \phi_j \partial_\mu \phi_i A^\mu \). As was seen for the scalar-scalar two-point Green function we must consider the placement of the derivative for both of the vertices. This means we have four possible contributions which we denote by \( \Pi_{\mu\nu}^{S,i(i,j)} \), for \( i, j = 1, 2 \). We have the first contribution when the derivative is associated with the first loop field (the one associated with \( m_1 \)) in both vertices. This is given by

\[
\Pi_{\mu\nu}^{S,i(1,1)}(p^2, m_1^2, m_2^2; \mu^2) = -\frac{C_{S3}^{(1,1)} C_{S3}^{(2,1)}}{16\pi^2} \left[ g_{\mu\nu} B_{00}(p^2, m_1^2, m_2^2; \mu^2) + p_\mu p_\nu B_{11}(p^2, m_1^2, m_2^2; \mu^2) \right] \tag{70}
\]

where \( C_{S3}^{(i,j)} \) represent the coefficient of the term for the \( i \)th vertex where the derivative is placed on the \( j \)th field in the loop. Similarly we find for the remaining contributions

\[
\Pi_{\mu\nu}^{S,i(2,2)}(p^2, m_1^2, m_2^2; \mu^2) = -\frac{C_{S3}^{(1,2)} C_{S3}^{(2,2)}}{16\pi^2} \left[ g_{\mu\nu} B_{00}(p^2, m_1^2, m_2^2; \mu^2) + p_\mu p_\nu \left[ \left( B_1 + B_{11}\right)(p^2, m_1^2, m_2^2; \mu^2) \right] \right], \tag{71}
\]

\[
\Pi_{\mu\nu}^{S,i(1,2)}(p^2, m_1^2, m_2^2; \mu^2) = \frac{C_{S3}^{(1,1)} C_{S3}^{(2,2)}}{16\pi^2} \left[ g_{\mu\nu} B_{00}(p^2, m_1^2, m_2^2; \mu^2) + p_\mu p_\nu \left[ \left( B_1 + B_{11}\right)(p^2, m_1^2, m_2^2; \mu^2) \right] \right], \tag{72}
\]

\[
\Pi_{\mu\nu}^{S,i(2,1)}(p^2, m_1^2, m_2^2; \mu^2) = \frac{C_{S3}^{(1,2)} C_{S3}^{(2,1)}}{16\pi^2} \left[ g_{\mu\nu} B_{00}(p^2, m_1^2, m_2^2; \mu^2) + p_\mu p_\nu \left[ \left( B_1 + B_{11}\right)(p^2, m_1^2, m_2^2; \mu^2) \right] \right]. \tag{73}
\]

In EFFECTIVE it also assumed that there is a symmetry between the terms of \( \phi_j \partial_\mu \phi_i A^\mu \) and \( \phi_i \partial_\mu \phi_j A^\mu \) such that \( C_{S3}^{(k,1)} = C_{S3}^{(k,2)} \) for \( k = 1, 2 \). This means all
four terms have the same coefficient and can be added. Doing so one find the result
\[
\Pi^{S3}_{\mu\nu}(p^2, m_1^2, m_2^2; \mu^2) = -\frac{C_{S3}(1) C_{S3}(2)}{16\pi^2} p_\mu p_\nu B_0(p^2, m_1^2, m_2^2; \mu^2). \tag{74}
\]

We now turn to the fermion three-point coupling and the vector-scalar three point coupling. In the fermion case the coupling is assumed to take the form
\[C_{F3}g^{\mu\nu}\gamma_\mu\psi_i\psi_j A_\mu,\] where \(C_{F3}\) is a scalar coefficient. The vector-scalar coupling is assumed to be \(C_{VS}g^{\mu\nu}\phi_i A_\mu A_\nu,\) with \(C_{VS}\) a scalar. The contributions from these diagrams is then
\[
\Pi_{\mu\nu}^{F3}(p^2, m_1^2, m_2^2; \mu^2) = \frac{dC_{F3}(1) C_{F3}(2)}{16\pi^2} [2p_\mu p_\nu (B_{11} - B_1) - g_{\mu\nu} (m_1 m_2 B_0 + (d - 2) B_{00} - p^2 (B_{11} - B_1))], \tag{75}
\]
\[
\Pi_{\mu\nu}^{VS}(p^2, m_1^2, m_2^2; \mu^2) = -\frac{C_{VS}(1) C_{VS}(2)}{16\pi^2} g_{\mu\nu} B_0(p^2, m_1^2, m_2^2; \mu^2), \tag{76}
\]
where the arguments to the functions in the fermion contribution have been suppressed for simplicity.

We then have the vector-vector three-point coupling. It is assumed that these are derived from terms like \(\partial_\mu A_\nu A_\lambda A_\sigma (C_1 g^{\mu\lambda} g^{\nu\sigma} + C_2 g^{\mu\sigma} g^{\nu\lambda}).\) When the derivative is permuted to lie on each possible vector field, this gives rise to 6 scalar coefficients per vertex and 9 diagrams. Symmetries reduce this to only 5 independent contributions. Due to the large number of contributions, these will not be listed here, but the sum will be denoted by \(\Pi_{\mu\nu}^{VV}.\)

Lastly, because the previous results were computed in the Feynman gauge, we must also include the three-point diagram where ghost fields are the intermediate fields. Contributions of this sort give a contribution
\[
\Pi_{\mu\nu}^{G}(p^2, m_1^2, m_2^2; \mu^2) = C_{G}(1) C_{G}(2) \frac{16\pi^2}{16\pi^2} (g_{\mu\nu} B_{00} + p_\mu p_\nu [B_1 - B_{11})], \tag{77}
\]
where again, we have suppressed the arguments of the functions on the right-hand side of the equation.

As only the transverse components of the vector field propagate, only the transverse component of the Green function needs to be used in renormalization. This means that only the coefficient of the \(g_{\mu\nu}\) terms are used in the one-loop mass corrections. We denote this component for the Green function between vector fields \(k\) and \(l\) as \(\Pi_{kl}^T.\) Thus
\[
\Pi_{kl}^T(p^2) = \sum_{i \in \text{scalar}} \left( \Pi_{i}^{T,S4} + \sum_{j \in \text{scalar}} \Pi_{ij}^{T,S3} + \sum_{j \in \text{vector}} \Pi_{ij}^{T,V3} \right)
+ \sum_{i,j \in \text{fermion}} \Pi_{ij}^{T,F3} + \sum_{i \in \text{vector}} \left( \Pi_{i}^{T,V4} + \sum_{j \in \text{vector}} \Pi_{ij}^{T,V3} \right)
+ \sum_{i,j \in \text{ghost}} \Pi_{ij}^{T,G}, \tag{78}
\]
where again the indices $k$ and $l$ are implicit in the couplings of the terms on the right-hand side. In EFFECTIVE the ghost fields have the mass of the boson field with the same group charge. The ghost-ghost-vector three point coupling is assumed to be of the form $g f^{abc}$.

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