Rolling Tachyon in Nonlocal Cosmology

L. Joukovskaya

DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

Abstract. Nonlocal cosmological models derived from String Field Theory are considered. A new method for constructing rolling tachyon solutions in the FRW metric in two field configuration is proposed and solutions of the Friedman equations with nonlocal operator are presented. The cosmological properties of these solutions are discussed.

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INTRODUCTION

In recent works there appears interest in nonlocal cosmological models derived from String Field Theory in connection with problem of describing cosmological inflation or accelerating expansion of the Universe.

Modern cosmological data indicates that expansion of the Universe is accelerating. It may be owing to a component of the Universe with negative pressure, Dark Energy. Recent results of WMAP [1] together with Ia supernovae data give us the following range for the dark energy state parameter

\[ w = 0.97^{+0.07}_{-0.09} \]

From theoretical point of view it is natural to distinguish three essential different cases, \( w > 1 \), \( w = 1 \) and \( w < 1 \). The third one is more challenging one because it violates all natural energy conditions and there might appear problems of a instability at classical and quantum level. Recently it was proposed to consider D-brane decay in cubic superstring field theory as a model of cosmological Dark Energy [2]. At the same time there have been a number of attempts to realize description of the early Universe via nonlocal cosmological models (for more details see [3] and refs therein).

In this work we will continue research [2, 3, 4, 5] of properties of nonlocal cosmological models derived from string field theory. More precisely, we will consider a cosmological model based on gravity interacting with tachyon matter governed by the tachyon action in CSSFT when fields up to zero mass are taken into account [6].

SETUP

To describe open string states living on a single non-BPS D-brane one has to consider GSO states [8]. GSO states are Grassmann even, while GSO+ states are Grassmann
odd. The low level action contains the tachyon field $\phi$ and one auxiliary field $u$

$$S[\mu; \phi] = \frac{1}{g^2_\alpha \alpha^{\alpha^{-1}}_0} d^{p+1}x \ u^2(x) \ \frac{\alpha_0}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \phi^2(x) \ \frac{1}{3m^2} (U(x) \Phi^2(x) \ ;$$

where the $\Phi$ and $U$ are defined as $\Phi = e^{k^2 x}, \ U = e^{k^2 u}, \ k = \alpha_0 \ln \gamma, \ \gamma = \frac{4}{3}.$

Covariant generalization of action (1) after rescaling of fields and coordinates in dimensionless space-time variables has the form

$$S = \frac{Z}{g} \frac{m_2^2}{2} R + \frac{\xi_2^2}{2} U + \frac{1}{2} \psi^2 + \frac{1}{4} \nu^2 + \frac{1}{2} \Upsilon \psi^2 + \Lambda^0 \ T \ ;$$

where $m$ is the metric, $g = e^{\frac{1}{g} \alpha_0 \partial_{\mu}} e^{g_{\mu \nu} \partial_{\nu}} \Psi$ is a dimensionless scalar tachyon field, $\Psi = e^{k^2 x} \phi, \ \nu$ is a dimensionless auxiliary scalar field, $\Upsilon = e^{k^2 x} \nu, \ m_2^2 = g_4 m_4^2, \ M_p$ is a Planck mass, $M_4$ is a characteristic string scale, $g_4$ is a dimensionless four dimensional effective coupling constant, $\Lambda^0$ is an effective cosmological constant, $T$ is the brane tension. In this work we will be interested in the case of the FRW metric $ds^2 = dt^2 + a^2(t) \ dx^2 + dx^2$ : We will consider spatially homogeneous configurations for which Beltrami-Laplace operator takes the form $2 \ g = \partial_t^2 + 3H(t) \partial_t.$ For the convenience of numerical calculations let us introduce the following notation $\Omega^2_\nu = \partial_t^2 + 3H(t) \partial_t.$ The corresponding equations of motion have the form

$$e^{x0} \Omega^2_\nu \Psi(t) = \Psi^2(t); \quad (3a)$$

$$e^{x0} \Omega^2_\nu \Psi(t) = \Upsilon \psi(t); \quad (3b)$$

and corresponding Friedman equations have the following form

$$3H^2 = \frac{1}{m_2^2} \left( \frac{\xi_2^2}{2} (\partial \psi)^2 + \frac{1}{4} \nu^2 + \frac{1}{2} \Upsilon \psi^2 + E_{nl1} + E_{nl2} + E_{nl3} + E_{nl4} + \Lambda^0 + T \right); \quad (4a)$$

$$\dot{H} = \frac{1}{m_2^2} \left( \frac{\xi_2^2}{2} (\partial \psi)^2 + E_{nl2} + E_{nl4} \right); \quad (4b)$$

where

$$E_{nl1}(t) = \frac{1}{8} \sum_{n=0}^{\infty} \int_{0}^{\frac{\pi}{2}} d\rho \ e^{2x} \rho^2 \Omega^2_\nu \left( \frac{\xi_2^2}{2} \Omega^2_\nu + 1 \right) \Psi \ \Omega^2_\nu \ e^{x0} \Omega^2_\nu \Psi \ ; \quad (5a)$$

$$E_{nl2}(t) = \frac{1}{8} \sum_{n=0}^{\infty} \int_{0}^{\pi} d\rho \ \partial e^{2x} \rho^2 \Omega^2_\nu \left( \frac{\xi_2^2}{2} \Omega^2_\nu + 1 \right) \Psi \ \partial e^{x0} \Omega^2_\nu \Psi \ ; \quad (5b)$$

$$E_{nl3}(t) = \frac{1}{16} \sum_{n=0}^{\infty} \int_{0}^{\pi} d\rho \ e^{2x} \rho^2 \Omega^2_\nu \left( \frac{\xi_2^2}{2} \Omega^2_\nu + 1 \right) \Psi \ \partial e^{x0} \Omega^2_\nu \Psi \ ; \quad (5c)$$

$$E_{nl4}(t) = \frac{1}{16} \sum_{n=0}^{\infty} \int_{0}^{\pi} d\rho \ \partial e^{2x} \rho^2 \Omega^2_\nu \left( \frac{\xi_2^2}{2} \Omega^2_\nu + 1 \right) \Psi \ \partial e^{x0} \Omega^2_\nu \Psi \ ; \quad (5d)$$
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The approximation $u = e^{k_2} u$ in the action (1) for Minkowski case was considered in [2, 3], it simplifies scalar field equation and with $\xi^2 = 0$ reproduces $p$-adic model for $p = 3$ which is also interesting for the applications. Recently for example it was proposed to consider $p$-adic theory as a theory of inflation [10]. This approximation was studied in Minkowski space in [6, 7] and recently in FRW space in [3]. In this approximation the auxiliary field $u$ (or $v$ in terms of rescaled fields) can be integrated and we will have only one equation of motion $\xi^2 D^2_H + 1 = \frac{\xi^2}{2} D^2 \Psi = \Psi^2$, simplification of Friedmann equations also takes place (see [3] for details). Figure 1 presents solutions $\psi, H$ and $a$ for $\xi^2 = 0$ and $\xi^2 = \frac{1}{4 \ln \gamma}$.

Let us return to the case of two field configuration [6, 9] $\Psi$ and $\Upsilon$ which is more interesting. To find solutions of the system of equations (3) and integrating equation (4b), $H(t) = \frac{1}{m_p^2} \int_0^t d\tau \dot{H}(\tau)$, we construct the following iterative process

$$\Psi = \lim_{n!} \Psi_n; \quad \Upsilon = \lim_{n!} \Upsilon_n; \quad H = \lim_{n!} H_n;$$

where iterations $\Psi_n, \Upsilon_n, H_n$ are in turn obtained as limits of sub-iterations $\Psi_{n+1} = \lim_{m!} \Psi_{m,n}; \quad \Upsilon_{n+1} = \lim_{m!} \Upsilon_{m,n}; \quad H_{n+1} = \lim_{m!} H_{m,n}$; which are recursively defined as ($m > 0$)

$$\Psi_{m,n+1} = \left( \frac{\xi^2 D^2_H \Upsilon_{m,n} + 1}{\Psi_{m,n}} \right); \quad \Upsilon_{m,n+1} = \text{sign}(t) \frac{e^{\frac{m}{3} D^2 H_{m,n}}}{};$$

$$H_{m,n+1} = \frac{1}{m_p^2} \int_0^Z d\tau \frac{\xi^2}{2} (D^2 \partial \Psi_{m,n+1})^2; \quad \frac{1}{16} \int_0^1 d\rho \partial \frac{2}{e^\frac{m}{3} D^2 H_{m,n}} \Upsilon_{m,n+1}$$

$$\partial \frac{e^{\frac{m}{3} D^2 H_{m,n}}}{8 \cdot Z} \Upsilon_{n+1}; \quad \frac{1}{8} \int_0^1 d\rho \partial \left( \frac{2}{e^\frac{m}{3} D^2 H_{m,n}} + 1 \right) e^\frac{m}{3} D^2 H_{m,n} \Psi_{n+1} + \partial \frac{2}{e^\frac{m}{3} D^2 H_{m,n}} \Psi_{n+1};$$

The initial iterations in $m$ is taken as $\Phi_{m,0} = \Phi_n; \quad H_{m,0} = H_n; \quad \Psi_0(t) = \tanh(t); \quad H_0(t) = 0$ (see [3] for the details of analogous iterative process). Obtained solutions are presented on the Fig. 2. For $\xi^2 = 0$ we have nonsingular accelerating Universe with a bounce. Hubble function goes to the constant as $t \rightarrow \infty$, hence the state parameter tends to $w = 1$ as $t \rightarrow \infty$. Behavior of the Hubble function for $\xi^2 = 0.96$ also illustrates the possibility to obtain the state parameter which is close to 1.
FIGURE 2. Solutions of the Friedmann equations (3), (4) with $\xi^2 = 0$ (dashed line) and $\xi^2 = \frac{1}{4\ln\gamma}$ (solid line). On the most left figure $\psi$ has odd kink type shape, while $\upsilon$ has even wavelet type shape.

Note that $\Lambda^0$ does not enter equations (3), (4b), but can be determined from (4a). Following Sen’s conjecture we put D-brane’s tension to $T = V (\Psi = 1, \Upsilon = 1) = \frac{1}{2}$ [2, 6] and thus $\Lambda^0$ is determined uniquely for each field configuration in the same way as in [3].

In this work we have studied the properties of nonlocal cosmological models derived from String Field Theory in the Friedmann space-time. We have construct new method for solving partial differential equations with infinitely many time derivatives, which constitute a new class of mathematical physics equations and are recently discussed in the literature [7, 10, 11]. We obtained classical solutions of the corresponding Friedmann equations which can be considered as a first approximation to the quantum solutions and might be useful for the study of ways to avoid the cosmological singularity problem [12].

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