A Communication Efficient Vertical Federated Learning Framework

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Abstract

One critical challenge for applying today’s Artificial Intelligence (AI) technologies to real-world applications is the common existence of data silos across different organizations. Due to legal, privacy and other practical constraints, data from different organizations cannot be easily integrated. Federated learning (FL), especially the vertical FL (VFL), allows multiple parties having different sets of attributes about the same user collaboratively build models while preserving user privacy. However, communication overhead is a principal bottleneck since the existing VFL protocols require per-iteration communications among all parties. In this paper, we propose the Federated Stochastic Block Coordinate Descent (FedBCD) to effectively reduce the communication rounds for VFL. We show that when the batch size, sample size and the local iterations are selected appropriately, the algorithm requires \( O\left(\sqrt{T}\right) \) communication rounds to achieve \( O\left(1/\sqrt{T}\right) \) accuracy. Finally, we demonstrate the performance of FedBCD on several models and datasets, and on a large-scale industrial platform for VFL.

1 Introduction

A new machine learning paradigm, federated learning [1], has emerged to be an attractive solution to the data silo and privacy problem. The original federated learning framework focus on enabling large amount of parties (devices) to collaboratively train a model without sharing their personal data. This framework is also referred as horizontal federated learning (HFL) [2] and it is later extended [2] to consider cross-organizational collaborative learning problems where parties share the same users with different set of features, and this scenario is classified as vertical federated learning (VFL) [2, 3].

Existing architectures for VFL still face several critical challenges and communication overhead is a major bottleneck since privacy-preserving computations, such as Homomorphic Encryption (HE) [4, 5] and Multi-party Computation (SMPC) [6], are typically applied to transmitted data, and per-iteration privacy-preserving communication and computations are required. In [1], it is demonstrated experimentally that multiple local updates can be performed in HFL with federated averaging (FedAvg), reducing the number of communication round effectively. Whether it is feasible
to perform such multiple local updates strategy in the VFL scenario is unknown, because in VFL each party only possesses a subset of all the features and only one party has the label information.

In this paper, we propose an algorithm named Federated stochastic block coordinate descent (Fed-BCD), where parties can continuously perform local model updates (in either a parallel or sequential manner), and only need to get synced occasionally. Block coordinate (gradient) descent (BCD) is a classical algorithm for optimization [7] and has been extensively applied to applications in areas such as signal/image processing and machine learning [8–15]. However, BCD and its variant has not been applied to the FL setting. We demonstrate that the communication cost can be significantly reduced by adopting FedBCD and performed comprehensive convergence analysis and experimental evaluation.

2 Problem Definition

Suppose $K$ data owners collaboratively train a machine learning model based on a set of data $\{x_i, y_i\}_{i=1}^N$. Suppose that the feature vector $x_i$ can be further decomposed into $K$ blocks $\{x^k_i\}_{k=1}^K$, where each block belongs to one owner. Without loss of generality, assume that the labels are located in party $K$. Let us denote the data set as $D^k_i \triangleq \{x^k_i\}$, for $k \in [K-1]$, $D^K_i \triangleq \{x^K_i, y^K_i\}$, and $D_i \triangleq \bigcup_{k=1}^K D_i^k$ (where $[K-1]$ denotes the set $\{1, \cdots, K-1\}$). Then the collaborative training problem can be formulated as

$$\min_{\Theta} L(\Theta; D) \triangleq \frac{1}{N} \sum_{i=1}^N f(\theta_1, \ldots, \theta_K; D_i) + \lambda \sum_{k=1}^K \gamma(\theta_k) \quad (1)$$

where $\theta_k$ denotes the training parameters of the $k$th party; $\Theta = [\theta_1; \ldots; \theta_K]$; $N$ denotes the total number of training samples; $f(\cdot)$ and $\gamma(\cdot)$ denotes the loss function and regularizer and $\lambda$ is the hyper-paratemer; For a mini-batch of data $S \subset [N]$, we use $f(\theta_1, \ldots, \theta_K; S) \triangleq \sum_{i \in S} f(\theta_1, \ldots, \theta_K; D_i)$ to denote its loss function.

A direct approach to optimize (1) is to use the vanilla stochastic gradient descent (SGD) algorithm given below

$$\theta_k \leftarrow \theta_k - \eta g_k(H_{-k}, \theta_k; S), \quad \forall k. \quad (2)$$

where $g_k$ denotes the stochastic partial gradient w.r.t. $\theta_k$ for (1). $H_{-k}$ denotes information required from other parties to compute $\theta_k$. We refer to the federated implementation of the vanilla SGD as FedSGD, which requires pair-wise communication of intermediate results at every iteration. This could be very inefficient, especially when $K$ is large or the task is communication heavy.

3 The Proposed FedBCD Algorithms

In the parallel version of our proposed algorithm, called FedBCD-p, at each iteration, each party $k \in [K]$ performs $Q > 1$ consecutive local gradient updates in parallel, before communicating the intermediate results among each other; see Algorithm 1. Such “multi-local-step” strategy is strongly motivated by our practical implementation (to be shown in our Experiments Section), where we found that performing multiple local steps can significantly reduce overall communication cost. Further, such a strategy also resembles the FedAvg algorithm in HFL, where each agent performs multiple local steps to update the full features. In the same spirit, a sequential version of the algorithm allows the parties to update their local $\theta_k$’s sequentially, while each update consists of $Q$ local updates without inter-party communication, termed FedBCD-s.
Algorithm 1 FedBCD-p: Parallel Federated Stochastic Block Coordinate Descent

Input: learning rate $\eta$
Output: Model parameters $\theta_1, \theta_2, ..., \theta_K$
Party 1, 2, ..., $K$ initialize $\theta_1, \theta_2, ..., \theta_K$.

for each outer iteration $t=1, 2, ...$
  Randomly sample a mini-batch $S \subset D$;
  Exchange ($\{1, 2, ..., K\}$);
  for each party $k \in [N]$, in parallel do
    for each local iteration $j = 1, 2, ..., Q$ do
      $k$ computes $g_k(H_k, \theta_k; S)$;
      Update $\theta_k \leftarrow \theta_k - \eta g_k(H_k, \theta_k; S)$;
    end
  end
end

4 Convergence Analysis

Due to space limitation, our analysis will be focused on FedBCD-p. Let $r$ denote the iteration index, in which each iteration one round of local update is performed; Let $r_0$ denote the latest iteration before $r$ in which synchronization has been performed. Further, we use the “global” variable $\Theta_r$ to collect the most updated parameters at each iteration of each node.

Assumption 1 (A1): Lipschitz Gradient. Assume that the loss function satisfies the following:

$$\|\nabla L(\Theta_1) - \nabla L(\Theta_2)\| \leq L\|\Theta_1 - \Theta_2\|, \forall \Theta_1, \Theta_2$$

$$\|\nabla_k L(\Theta_1) - \nabla_k L(\Theta_2)\| \leq L_k\|\Theta_1 - \Theta_2\|, \forall \Theta_1, \Theta_2.$$ 

Assumption 2 (A2): Uniform Sampling. Assume that the data is partitioned into $B$ mini-batches $S_1, \ldots, S_B$, each with size $S$; at a given iteration, $S$ is sampled uniformly from these mini-batches.

Theorem 1 Under Lipschitz Gradient and Uniform Sampling assumptions, when the step size in FedBCD algorithm satisfies $0 < \eta \leq \min\{\frac{\sqrt{2}}{2Q\sqrt{\sum_{j=1}^{K} L_j^2 + 3L_k^2}}, \frac{1}{T}\}$, then for all $T \geq 1$, we have the following bound:

$$\frac{1}{T} \sum_{t=0}^{T-1} E[\|\nabla L(\Theta^*)\|^2] \leq \frac{2}{\eta T}(L(\Theta^{(0)}) - L(\Theta^*)) + 2\eta^2(K + 3)Q^2 \sum_{k=1}^{K} L_k^2 \frac{\sigma^2}{S} + 2K \frac{\sigma^2}{S}. \quad (4)$$

where $L(\Theta^*)$ denotes the global minimum of problem (1).

Remark 1. If we pick $\eta = \frac{1}{\sqrt{T}}$, $S = Q = \sqrt{T}$, with any fixed $K$ the convergence speed is $O(\frac{1}{\sqrt{T}})$. This indicates that to achieve the same error compared with FedSGD, the communication rounds in the proposed algorithm can be reduced by a factor $O(T^{1/2})$. To the best of our knowledge, it is the first time that such an $O(1/\sqrt{T})$ rate has been proven for any algorithms with multiple local steps designed for the feature-partitioned collaboratively learning problem.

Remark 2. If we consider the impact of the number of nodes $K$ and pick $\eta = \frac{1}{\sqrt{KT}}$, $S = Q = \sqrt{KT}$, then the convergence speed is $O(\frac{\sqrt{T}}{\sqrt{K}})$. This indicates that the proposed algorithm has a slow down w.r.t the number of parties involved.

5 Experiments

MIMIC-III. MIMIC-III [16] is a large database comprising information related to patients admitted to critical care units at a large tertiary care hospital. Following the data processing procedures of [17],
we obtain 714 features. We partition each sample vertically by its clinical features and perform an in-hospital mortality prediction task. We refer to this task as MIMIC-LR.

**NUS-WIDE.** The NUS-WIDE dataset [18] consists of low-level images features and text tag features extracted from Flickr images. We put 634 low-level image features on party B and 1000 textual tag features with ground truth labels on party A. The objective is to perform a federated transfer learning (FTL) studied in [19]. We refer to this task as NUS-FTL.

**Default-Credit.** The Default-Credit [20] consists of credit card records. In our experiments, party A has labels and 18 features including six months of payment and bill balance data, whereas party B has 15 features of user profile data. We perform a FTL task as Credit-FTL.

### 5.1 Experimental Results

**Impact of \( Q \)** For all experiments, we adopt a decay learning rate strategy with \( \eta^t = \frac{\eta^0}{\sqrt{t + 1}} \), where \( \eta^0 \) is optimized for each experiment. We observe similar convergence for FedBCD-p (Figure 1(a)) and FedBCD-s (Figure 1(b)) for various values of \( Q \). By reasonably increasing the number of local iteration, we can save the overall communication costs by reducing the number of total communication rounds required. As we increase the number of parties to five and seventeen, the proposed method still performs well when we increase the local iterations for multiple parties. FedBCD-p is slightly slower than the two-party case, but the impact of node \( K \) is very mild. To further investigate the relationship between the convergence rate and the local iteration \( Q \), we evaluate FedBCD-p algorithm on NUS-FTL with a large range of \( Q \). Figure 1(c) illustrates that FedBCD-p achieves the best AUC with the least number of communication rounds when \( Q = 15 \). For each target AUC, there exists an optimal \( Q \). This manifests that one needs to carefully select \( Q \) to achieve the best communication efficiency, as suggested by Theorem 1. Figure 1(d) shows that for very large local iterations, the FedBCD-p cannot converge to the AUC of 83.7%. This phenomenon is also supported by Theorem 1, where if \( Q \) is too large the right hand side of (3) may not go to zero.

![Figure 1: Comparison of AUC for FedBCD-p (a) and FedBCD-s (b) on MIMIC-LR with varying local iterations \( Q \) and number of parties \( K \). The relationship between communication rounds and local iterations for three target AUC (c), and the comparison between FedBCD-p and FedPBCD-p for large local iterations (d) on NUS-FTL.](image)

**Proximal Gradient Descent** We add a proximal term [21] when calculating gradients to alleviate potential divergence when local iteration is large. We denote the proximal version of FedBCD-p as FedPBCD-p, and apply FedPBCD-p with \( \mu = 0.1 \) to NUS-FTL for \( Q = 25, 50 \) and 100. Figure 1(d) illustrates that if \( Q \) is too large, FedBCD-p fails to converge to optimal solutions whereas the FedPBCD-p converges faster and can reach at a higher test AUC than corresponding FedBCD-p does.

**Implementation with HE** We investigate the efficiency of FedBCD-p running on an industrial VFL platform, FATE\(^1\), with homomorphic encryption (HE) applied using the Credit-FTL task. Note carefully selecting \( Q \) may reduce communication rounds but may also introduce computational overhead because the total number of local iterations may increase. Table 1 shows that FedBCD-p with larger \( Q \) costs less communication rounds and total training time with a mild increase in computation time but more than 70 percents reduction in communication round.

\(^{1}\)https://github.com/FederatedAI/FATE
| AUC  | Algo.     | Q | Round | Computation (mins) | Communication(mins) | Total  |
|------|-----------|---|-------|-------------------|---------------------|--------|
| 80%  | FedSGD    | 1 | 46    | 32.20            | 30.69               | 62.89  |
|      | FedBCD-p  | 5 | 13    | 43.52            | 9.05                | 52.57  |
|      |           | 10| 7     | 41.53            | 5.12                | 46.65  |

Table 1: Number of communication rounds and training time to reach target AUC

6 Summary

In this paper, we propose a framework to significantly reduce the number of communication rounds, a major bottleneck for vertical federated learning (VFL). We prove that the algorithm achieves global convergence with a decay learning rate and proper choice of $Q$. 
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