MONET: Multiview Semi-supervised Keypoint via Epipolar Divergence

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Abstract

This paper presents MONET—an end-to-end semi-supervised learning framework for a pose detector using multiview image streams. What differentiates MONET from existing models is its capability of detecting general subjects including non-human species without a pre-trained model. A key challenge of such subjects lies in the limited availability of expert manual annotations, which often leads to a large bias in the detection model. We address this challenge by using the epipolar constraint embedded in the unlabeled data in two ways. First, given a set of the labeled data, the keypoint trajectories can be reliably reconstructed in 3D using multiview optical flows, resulting in considerable data augmentation in space and time from nearly exhaustive views. Second, the detection across views must geometrically agree with each other. We introduce a new measure of geometric consistency in keypoint distributions called epipolar divergence—a generalized distance from the epipolar lines to the corresponding keypoint distribution. Epipolar divergence characterizes when two view keypoint distributions produces zero reprojection error. We design a twin network that minimizes the epipolar divergence through stereo rectification that can significantly alleviate computational complexity and sampling aliasing in training. We demonstrate that our framework can localize customized keypoints of diverse species, e.g., humans, dogs, and monkeys.

1 Introduction

With a few exceptions, modern designs of pose detectors [1–4] are, in theory, highly flexible to recognize any deformable articulated body pose with minor modifications. However, their uses are predominantly limited to human subjects due to the availability of a large training data (e.g., MS COCO [5] and MPII [6] that were manually annotated by crowd workers). This has been the significant impediment to build a general marker-less motion capture system for diverse subjects such as monkeys, mice, and dogs. Unlike human subjects, attaining equivalent sizable data for non-human species is infeasible due to the requirement of expert knowledge and larger intra-class variations (e.g., rhesus macaques vs. mandrill). Further, the keypoints are not customizable, or pre-defined by the existing datasets, which leads to challenges in applying it to computational behavioral study.

This paper addresses the core question in enabling flexible marker-less motion capture: “how do we build a keypoint detector that localizes customized poses with minimal manual effort?” We conjecture that the redundant visual information embedded in unlabeled synchronized multiview image streams can significantly alleviate these manual efforts and enhance the precision. This is possible because (1) the annotations can be augmented in space and time using multiview tracking; (2) a large variation of visual appearance and pose caused by distinctive viewpoints can be generated using nearly exhaustive views; and (3) 3D geometric consistency across views can be enforced.

We present MONET (Multiview Optical Supervision Network)—an end-to-end semi-supervised learning framework for a pose detector of general subjects without a pre-trained model by leveraging

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An analogous approach has been used for a monocular camera that tracks an object bounding box [7].

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unlabeled multiview image streams. The key insight is that the detection in one view can be
deterministically transferred to another view via epipolar geometry [8], which allows a cross-view
supervision. We introduce a new measure of geometric consistency in keypoint distributions called
epipolar divergence—a generalized distance from the epipolar lines to the corresponding keypoint
distribution. It characterizes when two view keypoint distributions produces zero reprojection error.
We design a twin network that minimizes the epipolar divergence through stereo rectification that
reduces computational complexity and sampling aliasing in training.

MONET takes as input synchronized multiview image streams with limited number of manual
annotations (< 0.1%) and outputs a keypoint detection model. We reliably reconstruct the keypoint
trajectories in 3D that produce considerable data augmentation in space and time for a subset of the
unlabeled data (< 10%), and generate a multiview bootstrapping keypoint prior. We train the twin
network to jointly minimize the keypoint detection error and reprojection error on both labeled and
unlabeled data.

MONET is flexible: (1) keypoints can be customizable as it does not require a pre-trained model\(^3\),
i.e., we train a detection model from scratch with manual annotations for each sequence; (2) it can
build on any keypoint detector design such as DeepPose [4], CPM [2], and Hourglass [3], which
localizes the keypoints through their distribution (heatmap); (3) it can apply to general multi-camera
systems (e.g., different multi-camera rigs, number of cameras, and intrinsic parameters).

The main contributions include: (a) epipolar divergence that measures geometric consistency between
two view keypoint distributions; (b) MONET design that efficiently minimizes the epipolar divergence
via stereo rectification of the keypoint distributions; (c) large spatiotemporal data augmentation using
3D reconstruction of keypoint trajectories. We demonstrate that MONET is flexible to detect keypoints
in various subjects (humans, dogs, and monkeys) in different camera rigs and that outperforms the
existing baselines in terms of localization accuracy and precision (reprojection error).

2 Related Work

The physical and social behaviors of non-human species such as rhesus macaque monkeys has been
widely used as a window to study human activities in neuroscience and psychology. While measuring
their subtle behaviors in a form of 3D anatomic landmarks is key, implementing marker-based 3D
tracking systems is challenging due to their sensitivity of reflective markers and occlusion by fur,
which limits its application to heavily constrained body motion (e.g., body tied to a chair) [11]. Vision
based marker-less motion capture is a viable solution to measure their free ranging behaviors [12–14].

In general, the number of 3D pose configurations of a deformable articulated body is exponential
with respect to the number of joints. The 2D projections of the 3D body introduces another factor of
variations (e.g., illumination, visual appearance, and occlusion) caused by viewpoint changes, which
makes pose detection challenging. Yet, the pose configuration is often structured where efficient
spatial representations such as pictorial structures [15–21] and graphical models [22–28] can alleviate
the computational cost by using efficient algorithms, e.g., dynamic programming, convex relaxation,
and approximate algorithms. Albeit efficient and accurate on iconic images, they exhibit inferior
performance on the images in the long-tail distribution, e.g., a pigeon pose of yoga. Fully supervised
learning frameworks using a millions of perceptrons in convolutional neural networks (CNN) [1–4]
can address this long-tail distribution issue by leveraging a sheer amount of training data annotated
by crowd workers [5, 6, 29]. However, due to the number of parameters in CNN, the trained model
can be highly biased if the number of data samples is not sufficient (<1M).

Semi-supervised and weakly-supervised learning frameworks train CNN models with limited number
of training data [30–41]. These approaches embed underlying spatial structures such as 3D skeletons
and mesh that regularize their network weights. For instance, motion capture data are used to jointly
learn 2D and 3D keypoints [31], and scanned human body models are used to validate 2D pose
estimation via reprojection [12–40]. Notably, a multi-camera system can be used to cross-view
supervise each other images using iterative process of 3D reconstruction and network training [17].

Unlike existing methods, MONET does not rely on a spatial model. To our best knowledge, this is the
first paper that jointly reconstructs and trains a keypoint detector without the iterative processes using
epipolar geometry. We integrate reconstruction and learning through a new measure of keypoint
distributions called epipolar divergence, which can apply to general subjects including non-human
species where minimal manual annotations are available.

\(^3\)The existing models (e.g., AlexNet [9] and VGG 16 [10]) can be used as a complementary source to initialize
the convolutional layers.
We generalize the epipolar line transfer to define the distance between keypoint distributions. Let
\[
\theta = \sup_{\mathbf{x} \in \mathbb{R}^2} P(\mathbf{x}),
\]
where \(P \in \mathbb{R}^2\) is a probability distribution and \(1 \in \mathbb{R}^2\) is a 2D line parameter. \(g\) takes
the maximum value along the line in \(P\). Given the keypoint distribution in the \(j\)th image \(P_j\), the
transferred keypoint distribution can be obtained:
\[
P_{j \rightarrow i}(\mathbf{x}_j) = g(F\mathbf{x}_i; P_j).
\]  

The supremum operation is equivalent to the infimum operation in Equation (2) where it finds the most
likely (closest) correspondences along the epipolar line. The first two images in Figure 1 illustrate
the keypoint distribution transfer via Equation (4). It is the projection of a posteriori distribution of
a 3D keypoint generated by the \(j\)th image onto the \(i\)th image given an uniform prior distribution. \(P_i\)
and \(P_{j \rightarrow i}\) cannot be matched because \(P_i\) is a point distribution while \(P_{j \rightarrow i}\) is a line distribution.

3 MONET: Multiview Optical Supervision Network

We present a semi-supervised learning framework for training a keypoint detection model by leveraging
multiview image streams where \(|D_U| \gg |D_L|\) where \(D_L\) and \(D_U\) are labeled and unlabeled data.
We design a twin network that enforces geometric consistency by minimizing epipolar divergence
(Section 3.1 and 3.2), allowing a cross-view supervision. In addition, we extensively augment the
labeled data by reconstructing 3D keypoint trajectories (Section 3.3), which provides a multiview
bootstrapping prior (Section 3.4).

Consider a network model that takes an input image \(I\) and outputs a keypoint distribution, i.e.,
\(g(I; w) \in [0, 1]^{W \times H \times F}\) where \(W\) and \(H\) are the width and height of the keypoint distribution,
respectively, and \(P\) is the number of keypoints. The network is parametrized by the weight \(w\) learned
by minimizing the following loss:
\[
\min_w \mathcal{L}_L + \lambda_c \mathcal{L}_E + \lambda_p \mathcal{L}_P,
\]  

where \(\mathcal{L}_L, \mathcal{L}_E,\) and \(\mathcal{L}_P\) are the losses for labeled supervision, multiview cross-view supervision,
and bootstrapping prior, and \(\lambda_c\) and \(\lambda_p\) are the weights that control their importance.

3.1 Epipolar Divergence

A point in the \(j\)th image \(x_j \in \mathbb{R}^2\) is transferred to the corresponding epipolar line in the \(j\)th image via
the fundamental matrix \(F\) between two relative camera poses, which measures geometric consistency,
i.e., the corresponding point \(x_j\) must lie in the epipolar line [8],
\[
D(x_i, x_j) = \|x^T_i (F x_j)\| \sim \inf_{x \in F x_i} \|x - x_j\|,
\]  

where \(x\) is the homogeneous representation of \(x_i\). Equation (2) measures the closest distance between
\(x_i\) and all points that lies in the epipolar line \(F x_i\), i.e., the distance between line and point.

We generalize the epipolar line transfer to define the distance between keypoint distributions. Let
\(P_i : \mathbb{R}^2 \rightarrow [0, 1]\) be the probability distribution given the \(i\)th image computed by the keypoint
detector, i.e., \(P_i(x) = g(I_i; w)|_{x}\), and \(P_{j \rightarrow i} : \mathbb{R}^2 \rightarrow [0, 1]\) be the probability distribution in the \(i\)th
image transferred from the \(j\)th image. Note that we use an abuse of notation by omitting the keypoint
index as each keypoint is considered independently.

Consider a line max-pooling operation \(g:\)
\[
g(l; P) = \sup_{x \in l} P(x),
\]  

The first two images in Figure 1 illustrate the keypoint distribution transfer via Equation (4). It is the projection of a posteriori distribution of
a 3D keypoint generated by the \(j\)th image onto the \(i\)th image given an uniform prior distribution. \(P_i\)
and \(P_{j \rightarrow i}\) cannot be matched because \(P_i\) is a point distribution while \(P_{j \rightarrow i}\) is a line distribution.
We introduce a new operation inspired by stereo rectification, which warps the keypoint distribution where $(x, y)$ as shown the bottom right image in Figure 2. This rectification allows converting the keypoint distribution to a more regular distribution over the image. Theorem 1.

Two keypoint distributions $P_i$ and $P_j$ are geometrically consistent, i.e., zero reprojection error, if $Q_i(\theta) = Q_{j \rightarrow i}(\theta)$.

See the proof in Appendix A. Based on the Theorem 1, we match $Q_i$ and $Q_{j \rightarrow i}$. Inspired by KL divergence, we define epipolar divergence that measures the difference between two keypoint distributions mapped by their fundamental matrix using relative entropy:

$$D_{E}(Q_i || Q_{j \rightarrow i}) = \int S \cdot Q_i(\theta) \log \left( \frac{Q_i(\theta)}{Q_{j \rightarrow i}(\theta)} \right) d\theta,$$

where $D_{E}$ is epipolar divergence, i.e., how two keypoint distributions are geometrically consistent.

### 3.2 Cross-view Supervision via Epipolar Divergence

In practice, embedding Equation (6) into an end-to-end neural network is not trivial because (a) a new max-pooling operation over the line exists; (b) sampling interval for max-pooling along the line is arbitrary, i.e., non-uniform sampling does not encode geometric meaning such as depth; and (c) sampling interval across the line is also arbitrary, which introduces sampling aliasing. This leads to irregular keypoint distribution transfer based on the fundamental matrix.

We introduce a new operation inspired by stereo rectification, which warps the keypoint distribution such that the epipole is transformed to a point at infinity, i.e., the epipolar lines become parallel (horizontal) as shown the bottom right image in Figure 2. This rectification allows converting the oblique line max-pooling into regular row-wise max-pooling. Equation (4) can be re-written in the rectified keypoint distribution:

$$\bar{g}(v; \overline{P}) = g \left( l(v) = \begin{bmatrix} 0 \\ -1/v \end{bmatrix} ; \overline{P} \right) \triangleq \max_{u} \overline{P} \left( x = \begin{bmatrix} u \\ v \end{bmatrix} \right)$$

where $(u, v)$ are the $x, y$-coordinate of a point in the rectified keypoint distribution warped from $P$, i.e., $\overline{P}(x) = P(H^{-1}x)$ where $H$ is a homography of stereo-rectification. $\overline{P}$ is computed by inverse homography warping with bilinear interpolation.

This rectification simplifies the flattening operation in Equation (5):

$$\overline{Q}_{j \rightarrow i}(v) = \bar{g}(v; \overline{P}_{j \rightarrow i}) = \bar{g} (av + b; \overline{P}_j), \quad \overline{Q}_i(v) = \bar{g}(v; \overline{P}_i),$$
The key innovation of Equation (8) is that $Q_{j \rightarrow i}(v)$ is no longer parametrized by $\theta$ where an additional sampling over $\theta$ is not necessary. It directly accesses to $P_j$ to max-pool over each row, which significantly alleviates the computational complexity and sampling aliasing. Moreover, the sampling over $x$-coordinate is geometrically meaningful, i.e., uniform sampling is equivalent to disparity, or inverse depth.

With the rectification, we model the loss for multiview cross-view supervision:

$$L_R = \frac{1}{2} H \sum_{p=1}^{P} \sum_{i=1}^{C} \sum_{j \in V_i} \sum_{v=1}^{H} \frac{Q^p_i(v)}{Q^j_{j \rightarrow i}(v)} \log \frac{Q^p_i(v)}{Q^j_{j \rightarrow i}(v)}$$

(9)

where $H$ is the height of the distribution, $P$ is the number of keypoints, $C$ is the number of cameras, $V_i$ is the set of paired camera indices of the $i^{th}$ camera. We use the superscript in $Q^p_i$ to indicate the keypoint index. Figure 2 illustrates our twin network that minimizes the epipolar divergence by applying stereo rectification, epipolar transfer, and flattening operations, which can cross-view supervise the unlabeled data.

Since the epipolar divergence flattens the keypoint distribution, a pair of images can supervise along one direction. In practice, we find a set of paired image given $i^{th}$ image such that the expected epipolar lines are not parallel as shown in Figure 3(a). When camera centers lie in a co-planar surface, a 3D point in the surface produces all same epipolar lines, which is a degenerate case.

Figure 3(b) illustrate how the twin network minimizes the epipolar divergence where $Q_1$ becomes aligned with $Q_{j \rightarrow 1}$ as training epoch increases.

### 3.3 Multiview Data Augmentation

We augment the labeled data by reconstructing the 3D keypoint trajectories using multiview image streams. Given synchronized labeled images, we triangulate each 3D keypoint $X$ using camera

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4This degenerate case does not apply for 3D point triangulation where the correspondence is known.
projection matrix and the 2D labeled keypoints. The 3D reconstructed keypoint can be projected to the rest synchronized unlabeled images, which automatically produces their labels. 3D tracking [51][52] can further increase the labeled data. For each keypoint $X_t$ at the $t$ time instant, we project the point onto the visible set of cameras. The projected point is tracked in 2D using optical flow and triangulated with RANSAC [53] to form $X_{t+1}$. We precisely compute the visibility of the point to reduce tracking drift using motion and appearance cues: (1) optical flow on each image is compared to the projected 3D motion vector to measure motion consistency; and (2) visual appearance is matched by learning a linear correlation filter [54] on PCA HOG [55], which can reliably track longer than 100 frames forward and backward. We use this spatiotemporal data augmentation to define the labeled data loss:

$$L_L = \sum_{i \in D_L} ||\phi(I_i; w) - z_i||^2$$

(10)

where $z \in [0, 1]^{W \times H \times P}$ is the labeled likelihood of keypoints approximated by convolving the prediction with a Gaussian kernel.

### 3.4 Bootstrapping Prior

Multiview bootstrapping [47] can be used to generate the pseudo-labeled data via 3D triangulation via RANSAC. We initialize the detection model $w_0$ using the labeled data and filter the noisy detected keypoints using RANSAC conditioned on their detection probability. We use the projection of the reconstructed keypoint to generate a prior pseudo-probability map, $\tilde{z} \in [0, 1]^{W \times H \times P}$. We penalize the deviation from the bootstrapping prior:

$$L_P = \sum_{i \in D_U} ||\phi(I_i; w) - \tilde{z}_i||^2.$$  (11)

Unlike hard labeling using pseudo-probability [47], our prior loss balances with the multiview cross-view supervision, which shows superior performance.

### 4 Result

We validate our MONET framework on multiple sequences of diverse subjects, humans, dogs, and monkeys, with two different multi-camera systems. A subset of images (4~5 images per frame) is manually annotated over 10~20 frames (total annotated images per sequence is less than 100). Such manual annotations constitute 0.07%~0.14% of the unlabeled data. Human and dog subjects are captured by 69 cameras, and monkey subjects are captured by 35 cameras. See more details in Appendix C. To evaluate the flexibility, we build a model per species without a pre-trained model.

The CPM network with 5 stages is used to build the twin network. Due to the limited number of labeled images (<100), the existing distillation methods [56,57] perform similarly. (2) Spatial augmentation: the 3D keypoints are triangulated and projected onto the synchronized unlabeled images. This can model the visual appearance and spatial configuration from multiple perspectives, which can greatly improve the generalization power of the keypoint detection. (3) Spatiotemporal augmentation: we track the 3D keypoints over time using multiview optical flow [51][52]. This augmentation can model different geometric configurations of 3D keypoints. (4) Bootstrapping I: Given the spatiotemporal data augmentation, we apply the multiview bootstrapping approach [47] to attain the pseudo-labels computed by RANSAC based 3D triangulation for the unlabeled data. (5) Bootstrapping II: the Bootstrapping I model can be refined by re-triangulation and re-training. This can reduce the reprojection errors. We evaluate our approach based on accuracy and precision: accuracy measures distance from the ground truth keypoint and precision measures the coherence of keypoint detection across views.

**Accuracy** We use PCK (probability of correct keypoint) curves to measure the accuracy. The distance between the ground truth keypoint and the detected keypoint is normalized by the size of the width of the detection window (46). Figure 4 shows PCK performance on human, monkey, and dog subjects where no pre-trained model is used. Our MONET (red) model exhibits accurate detection for all keypoints, which outperforms 5 baselines. For the monkey data, higher framerate image streams (60 fps) greatly boost the performance of multiview tracking due to small displacement, resulting in accurate keypoint detection by spatiotemporal augmentation.

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3 Exceptionally, for human subjects, pre-trained CPM model [1] is used to generate the ground truth data.
Precision We use reprojection error to evaluate the precision of the detection. Given a set of keypoint detections in a synchronized frame and 3D camera poses, we triangulate the 3D point without RANSAC. The 3D point is projected back to each camera to compute the reprojection error, which measures geometric consistency across all views. Our MONET is designed to minimize the reprojection error where it outperforms the baselines with large margin as shown in Figure 4(d) showing the reprojection error with respect to keypoint probability. Our MONET performs better at higher keypoint probability, which is key for 3D reconstruction because it indicates which point to triangulate. The performance for each subject is summarized in Table 1.

Qualitative comparison The qualitative comparison can be found in Figure 6. MONET precisely localize keypoints by leveraging multiview images jointly. This becomes more evident when disambiguating symmetric keypoints, e.g., left and right hands, as epipolar divergence penalizes geometric inconsistency, i.e., reprojection error. Also it shows stronger performance on occlusion (the bottom figure) as the occluded keypoints can be visible to other view images that can enforce to the correct location. Figure 5 illustrates 3D reconstruction of monkey movement using MONET.

5 Discussion

We present a new semi-supervised framework MONET to train a keypoint detection network by leveraging multiview image streams. The key innovation is a measure of geometric consistency of two keypoint distributions called epipolar divergence. Similar to epipolar distance between corresponding points, it allows us to directly compute the reprojection error while training the network. We introduce a stereo rectification of the keypoint distribution that simplifies the computational complexity and imposes geometric meaning on constructing 1D distributions. A twin network is used to embed computing epipolar divergence. We also use multiview image streams to augment the data in space and time, which can bootstrap the unlabeled data. We demonstrate that our framework outperforms existing approaches, e.g., multiview bootstrapping, in terms of accuracy (PCK) and precision (reprojection error) and apply to track non-human species such as dogs and monkeys. We anticipate that this framework will provide a fundamental basis for enabling a flexible markerless motion capture that requires exploiting a large (potentially infinite) number of unlabeled data.
Figure 6: We qualitatively compare our MONET with 5 baseline algorithms on humans, monkeys, and dogs.
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A Proof of Theorem 1

Figure 7: Two epipolar lines are induced by an epipolar plane, which can be parametrized by the rotation \( \theta \) about the baseline where \( C_i \) and \( C_j \) are the camera optical centers.

**Proof.** The geometric consistency, or zero reprojection error, is equivalent to proving \( L_i^* \in \Pi \) where \( \Pi \) is an epipolar plane rotating about the camera baseline \( C_i, C_j \) as shown in Figure 7, and \( L_i^* \) and \( L_j^* \) are the 3D rays produced by the inverse projection of correspondences \( x_i^* \leftrightarrow x_j^* \), respectively, i.e., \( L_i^* = C_i + \lambda R_i^T K^{-1} \tilde{x}_i^* \).

The correspondence from the keypoint distributions are:

\[
x_i^* = \arg\max_x P_i(x) \tag{12}
\]

\[
x_j^* = \arg\max_x P_j(x), \tag{13}
\]

\( Q_i(\theta) = Q_j \rightarrow i(\theta) \) implies:

\[
\theta^* = \arg\max_{\theta} \sup_{x \in l_i(\theta)} P_i(x) = \arg\max_{\theta} \sup_{x \in l_j(\theta)} P_j(\theta) = \arg\max_{\theta} \sup_{x \in l_j(\theta)} P_j(x). \tag{14}
\]

This indicates the correspondence lies in epipolar lines induced by the same \( \theta^* \), i.e., \( x_i^* \in l_i(\theta^*) \) and \( x_j^* \in l_j(\theta^*) \).

Since \( l_j(\theta^*) = F \tilde{x}_i^* \), \( l_i(\theta^*) \) and \( l_j(\theta^*) \) are the corresponding epipolar lines. Therefore, they are in the same epipolar plane, and the reprojection error is zero.

\[ \square \]

B Cropped Image Correction and Stereo Rectification

We warp the keypoint distribution using stereo rectification. This requires a composite of transformations because the rectification is defined in the full original image. The transformation can be written as:

\[
\begin{align*}
\overline{c}H_h &= \left(\overline{c}H_i\right) \left(\overline{c}H_p\right) H_i \left(\overline{c}H_h\right)^{-1} \left(\overline{c}H_p\right)^{-1}.
\end{align*}
\]

(15)

The sequence of transformations takes a keypoint distribution of the network output \( P \) to the rectified keypoint distribution \( \overline{P} \): heatmap \( \rightarrow \) cropped image \( \rightarrow \) original image \( \rightarrow \) rectified image \( \rightarrow \) rectified cropped image \( \rightarrow \) rectified heatmap.

Given an image \( I \), we crop the image based on the bounding box as shown in Figure 8: the left-top corner is \((u_x, u_y)\) and the height is \( h_b \). The transformation from the image to the bounding box is:

\[
\begin{bmatrix}
  s & 0 & u_x - su_y \\
  0 & s & u_y - su_y \\
  0 & 0 & 1
\end{bmatrix}
\]

(16)

where \( s = h_c / h_b \) and \((u_x, u_y)\) is the offset of the cropped image. It corrects the aspect ratio factor. \( h_c = 364 \) is the height of the cropped image, which is the input to the network. The output resolution (heatmap) is often different from the input, \( s_h = h_h / h_c \neq 1 \), where \( h_h \) is the height of the heatmap. The transformation from the cropped image to the heatmap is:

\[
\begin{bmatrix}
  s_h & 0 & 0 \\
  0 & s_h & 0 \\
  0 & 0 & 1
\end{bmatrix}
\]

(17)
Figure 8: A cropped image is an input to the network where the output is the keypoint distribution. To rectify the keypoint distribution (heatmap), a series of image transformations need to be applied.

The rectified transformations \( \left( H_{r_i} \right) \) and \( \left( H_{r_j} \right) \) can be defined in a similar way.

The rectification homography can be computed as \( H_r = K R_{ns} \left( K^T \right)^{-1} \) where \( K \) and \( R \in SO(3) \) are the intrinsic parameter and 3D rotation matrix respectively and \( R_{ns} \) is the rectified rotation of which x-axis is aligned with the epipole, i.e., \( r_x = \frac{C_j - C_i}{\| C_j - C_i \|} \). The other axes can be computed by the Gram-Schmidt process.

The fundamental matrix between two rectified keypoint distributions \( P_i \) and \( P_j \) can be written as:
\[
F = K_j^{-T} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot K_i^{-1}
\]
\[
= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1/f_y^j \\ 0 & 1/f_y^j & -p_y^j/f_y^j - p_y^i/f_y^i \end{bmatrix}
\]
(18)

where \([ \cdot ]_x\) is the skew symmetric representation of cross product, and
\[
K_i = \begin{bmatrix} f_x^i & 0 & p_x^i \\ 0 & f_y^i & p_y^i \\ 0 & 0 & 1 \end{bmatrix}
\]
(19)

This allows us to derive the re-scaling factor of \( a \) and \( b \) in Equation (8):
\[
a = s_i^j f_y^j \\
b = s_h^i s_x^i \left( \pi_y^i - p_y^i \right) f_x^j \left( p_y^j + p_y^i - \pi_y^i \right)
\]
(20)

where \( \pi_y^i \) is the bounding box offset of the rectified coordinate.

C Evaluation Dataset

All cameras are synchronized and calibrated using structure from motion \( [8] \). The input of most pose detector models except for \( [1] \) is a cropped image containing a subject, which requires specifying a bounding box. We use a kernelized correlation filter \( [58] \) to reliably track a bounding box using multiview image streams given initialized 3D bounding box from the labeled data.

| Subjects   | \( P \) | \( |D_L| \) | \( |D_U| \) | \( |D_L|/|D_U| \) | \( C \) | FPS | Camera type |
|------------|--------|--------|--------|-------------|-----|-----|------------|
| Monkey     | 13     | 85     | 63,000 | 0.13%       | 35  | 60  | GoPro 5    |
| Humans     | 14     | 30     | 20,700 | 0.14%       | 69  | 30  | FLIR BlackFly S |
| Dog I      | 12     | 100    | 138,000| 0.07%       | 69  | 30  | FLIR BlackFly S |
| Dog II     | 12     | 75     | 103,500| 0.07%       | 69  | 30  | FLIR BlackFly S |
| Dog III    | 12     | 80     | 110,400| 0.07%       | 69  | 30  | FLIR BlackFly S |
| Dog IV     | 12     | 75     | 103,500| 0.07%       | 69  | 30  | FLIR BlackFly S |

Table 2: Summary of multi-camera dataset where \( P \) is the number of keypoints, \( C \) is the number of cameras, \( |D_L| \) is the number of labeled data, and \( |D_U| \) is the number of unlabeled data.