On the importance of the normalization*

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We repeat the known procedure of the derivation of the set of Proca equations. It is shown that it can be written in various forms. The importance of the normalization is point out for the problem of the correct description of spin-1 quantized fields. Finally, the discussion of the so-called Kalb-Ramond field is presented.

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Recently, a new concept of the longitudinal magnetic field, the \(B^{(3)}\) field, was proposed [1]. It is based on the equation containing the cross product of two transversal modes of electromagnetism:

\[
B^{(1)} \times B^{(2)} = i B^{(0)} B^{(3)*},
\]

(1)

and represents itself a non-trivial generalization of the Maxwell’s electromagnetic theory (see ref. [2] for the list of other generalizations). This concept advocates the mass of photon and appears to be in the contradiction with the concept of the \(m \to 0\) group contraction for a photon as presented by Wigner and Inonu [3] and Kim [4].

On the other hand, in [5] it was shown that the angular momentum generators \(J_{\kappa\tau}\) are equated to zero after the application of the generalized Lorentz condition \(\partial_\mu F^{\mu\nu} = 0\) (and dual to that) to the quantum states, when considering the theory of quantized antisymmetric tensor fields (the representation \((1,0) \oplus (0,1)\) of the Lorentz group, according to the ordinary wisdom). The formula (9) of the cited reference tells us:

\[
J_{\kappa\tau} = \int d^3x \left[ (\partial_\mu F^{\mu\nu})(g_{0\kappa} F_{\nu\tau} - g_{0\tau} F_{\kappa\nu}) - (\partial_\mu F^{\mu}_\kappa) F_{0\tau} + (\partial_\mu F^{\mu}_\tau) F_{0\kappa} + F^{\mu}_\kappa (\partial_0 F_{\tau\mu} + \partial_\mu F_{0\tau} + \partial_\tau F_{\mu0}) - F^{\mu}_\tau (\partial_0 F_{\kappa\mu} + \partial_\mu F_{0\kappa} + \partial_\kappa F_{\mu0}) \right].
\]

(2)

Therefore, the spin vector reads:

\[
J^k = \frac{1}{2} \epsilon^{ijk} J^{ij} = \epsilon^{ijk} \int d^3x \left[ F^{0i}(\partial_\mu F^{\mu j}) + F^i_{\mu} (\partial_0 F^{\mu i} + \partial^\mu F^{0i} + \partial^\mu F^{0j}) \right].
\]

(3)

Thus, the helicity of the “photon” described by the physical fields \(E\) and \(B\) is believed [6,7] to be equated to zero (?), what is in the strong contradiction with experimental results. The helicity – the projection of the spin onto the direction of motion – proves to be equal to zero . . . even without the restriction to plane waves, the 3-vector of spin [formula (12) of the cited paper] vanishes on solutions . . .”, ref. [7b]. Moreover, if this construct describes the \(h = 0\) fields it seems to contain internal theoretical inconsistencies because is in the contradiction with the Weinberg theorem \(B - A = h\), ref. [5]. This argument was also used earlier by Evans [1,9,10]. In order to understand the nature of these contradictions one should reveal relations between the Evans-Vigier concept and the concept of the so-called Kalb-Ramond field [6]. While some insights into the problem have been already presented [4,6] in this paper we try to deepen the understanding of apparent conflicts, which were mentioned.

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1While this formula is the consequence of the particular choice of the Lagrangian of antisymmetric tensor field, nevertheless, the main conclusion is hold for other types of Lagrangians, including those which possess the Kalb-Ramond gauge invariance [3].

2Presenting this formula we, however, do not think that the question of the correct definition of the relativistic spin vector is so simple as believed before.

3M. Kalb and P. Ramond claimed explicitly [6c, p. 2283, the third line from below]: “thus, the massless \(\phi_{\mu\nu}\) has one degree of freedom”. While they call \(\phi_{\mu\nu}\) as “potentials” for the field \(F^{\alpha\beta\gamma} = \delta^{\alpha\beta} \phi^{\gamma} + \delta^{\beta\gamma} \phi^{\alpha} + \delta^{\gamma\alpha} \phi^{\beta}\), nevertheless, the physical content of the antisymmetric tensor field of the second rank (the representation \((1,0) \oplus (0,1)\) of the Lorentz group) must be in accordance with the requirements of relativistic invariance.
We believe in the power of the group-theoretical methods in the analyses of the physical behaviour of different-type classical (and quantum) fields. We also believe that the Dirac equation can be applied to some particular quantum states of the spin \(1/2\). Finally, we believe that the spin-0 and spin-1 particles can be constructed by taking the direct product of the spin-1/2 field functions \[11\]. So, on the basis of these postulates let us firstly repeat the Bargmann-Wigner procedure of obtaining the equations for bosons of spin 0 and 1. The set of basic equations for \(j = 0\) and \(j = 1\) are written, e.g., ref. [12]

\[
\begin{align*}
[i\gamma^\mu \partial_\mu - m]_{\alpha\beta} \Psi_{\beta\gamma}(x) &= 0 \\
[i\gamma^\mu \partial_\mu - m]_{\gamma\beta} \Psi_{\alpha\beta}(x) &= 0 
\end{align*}
\]  

We expand the \(4 \times 4\) matrix wave function into the antisymmetric and symmetric parts

\[
\begin{align*}
\Psi_{[\alpha\beta]} &= R_{\alpha\beta} \phi + \gamma^5_{\alpha\delta} R_{\delta\beta} \tilde{\phi} + \gamma^5_{\alpha\delta} \gamma^\mu_{\delta\tau} R_{\tau\beta} \tilde{A}_\mu \\
\Psi_{\{\alpha\beta\}} &= \gamma^\mu_{\alpha\delta} R_{\delta\beta} A_\mu + \sigma^{\mu\nu}_{\alpha\delta} R_{\delta\beta} F_{\mu\nu} 
\end{align*}
\]

where \(R = CP\) has the properties (which are necessary to make expansions (5a,5b) to be possible in such a form)

\[
\begin{align*}
R^T &= -R \\
R^\dagger &= R = R^{-1} \\
R^{-1} \gamma^5 R &= (\gamma^5)^T \\
R^{-1} \gamma^\mu R &= -(\gamma^\mu)^T \\
R^{-1} \sigma^{\mu\nu} R &= -(\sigma^{\mu\nu})^T
\end{align*}
\]

The explicit form of this matrix can be chosen:

\[
R = \begin{pmatrix} i\Theta & 0 \\ 0 & -i\Theta \end{pmatrix}, \quad \Theta = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}
\]

provided that \(\gamma^\mu\) matrices are in the Weyl representation. The equations (4a,4b) lead to the Kemmer set of the \(j = 0\) equations:

\[
\begin{align*}
m\phi &= 0 \\
m\tilde{\phi} &= -i\partial_\mu \tilde{A}^\mu \\
m\tilde{A}^\mu &= -i\partial^\mu \phi
\end{align*}
\]
and to the Proca set of the equations for the $j = 1$ case\footnote{We could use another symmetric matrix $\gamma^5 \sigma^{\mu\nu} R$ in the expansion of the symmetric spinor of the second rank. In this case the equations would read}

$$\partial_\alpha F^{\alpha\mu} + \frac{m}{2} A^\mu = 0 \quad , \quad (10a)$$

$$2mF^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \quad (10b)$$

In the meantime, in the textbooks, the latter set is usually written as (e.g., ref. \cite{14, p.135})

$$\partial_\alpha F^{\alpha\mu} + m^2 A^\mu = 0 \quad , \quad (11a)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \quad (11b)$$

The set (11a,11b) is obtained from (10a,10b) after the normalization change $A^\mu \rightarrow 2mA^\mu$ or $F^{\mu\nu} \rightarrow \frac{1}{2m} F^{\mu\nu}$. Of course, one can investigate other sets of equations with different normalization of the $F^{\mu\nu}$ and $A^\mu$ fields. Are all these sets of equations equivalent? As we shall see, to answer this question is not trivial. Papers \cite{15} argued that the physical normalization is such that in the massless-limit zero-momentum field functions should vanish in the momentum representation (there are no massless particles at rest). Next, we advocate the following approach: the massless limit can and must be taken in the end of all calculations only, i.e., for physical quantities.

Let us proceed further. In order to be able to answer the question about the behaviour of the spin operator $J^i = \frac{1}{2} \epsilon^{ijk} j^k$ in the massless limit one should know the behaviour of the fields $F^{\mu\nu}$ and/or $A^\mu$ in the massless limit. We want to analyze the first set (10a,10b). If one advocates the following definitions \cite{16, p.209} \footnote{Recently, after completing this work the paper \cite{13} was brought to our attention. It deals with the redundant components in the $j = 3/2$ spin case. If the claims of that paper are correct we would have to change a verbal terminology which we use to describe the above equations.}

\begin{equation}
\epsilon^\mu(0, +1) = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix} , \quad \epsilon^\mu(0, 0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} , \quad \epsilon^\mu(0, -1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix} , \quad (12a)
\end{equation}
for a massive spin-1 particle were substituted by 
publications.
larizations) too. But, we leave the presentation of rigorous theory of this type for subsequent

Therefore, we use the field operator as in (14). The coefficient \( 2\pi^3 \) can be considered at this stage as chosen for the convenience. In ref. [14] the factor \( 1/(2E_p) \) was absorbed in creation/annihilation operators and instead of the field operator \( \hat{A}^{\mu}(x^\mu) \) the operator was used in which the \( \epsilon^\mu(p, \sigma) \) functions for a massive spin-1 particle were substituted by \( u^\mu(p, \sigma) = (2E_p)^{-1/2} \epsilon^\mu(p, \sigma) \), what leads to the confusions in the definitions of the massless limit \( m \to 0 \) for classical polarization vectors.

The metric used in this paper \( g^{\mu\nu} = \text{diag}(1, -1, -1, -1) \) is different from that of ref. [16].

It is interesting to note that all the vectors \( u^\mu \) satisfy the condition \( p_\mu u^\mu(p, \sigma) = 0 \). It is relevant to the case of the Lorentz gauge and, perhaps, to the analyses of the neutrino theories of light.
\( N = m \) and \( p_i = p_1 \pm ip_2 \) which do not diverge in the massless limit. Two of the massless functions (with \( \sigma = \pm 1 \)) are equal to zero when the particle, described by this field, is moving along the third axis \( (p_1 = p_2 = 0, \ p_3 \neq 0) \). The third one \( (\sigma = 0) \) is

\[
\begin{align*}
u(p_3, 0) |_{m \to 0} &= \begin{pmatrix} p_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \equiv \begin{pmatrix} E_p \\ 0 \\ 0 \end{pmatrix}, \tag{16}
\end{align*}
\]

and at the rest \( (E_p = p_3 \to 0) \) also vanishes. Thus, such a field operator describes the “longitudinal photons” what is in the complete accordance with the Weinberg theorem \( B - A = h \) (let us remind that we use the \( D(1/2, 1/2) \) representation). Thus, the change of the normalization can lead to the “change” of physical content described by the classical field (at least, comparing with the well-accepted one). Of course, in the quantum case one should somehow fix the form of commutation relations by some physical principles.\(^{10}\) In the connection with the above consideration it is interesting to remind that the authors of ref. [14, p. 136] tried to inforce the Stueckelberg’s Lagrangian in order to overcome the difficulties related with the \( m \to 0 \) limit (or the Proca theory \( \to \) Quantum Electrodynamics).

The Stueckelberg’s Lagrangian is well known to contain the additional term which may be put in correspondence to some scalar (longitudinal) field (at least, comparing with the well-accepted one). Of course, in the quantum case one should somehow fix the form of commutation relations by some physical principles.\(^{10}\) In the connection with the above consideration it is interesting to remind that the authors of ref. [14, p. 136] tried to inforce the Stueckelberg’s Lagrangian in order to overcome the difficulties related with the \( m \to 0 \) limit (or the Proca theory \( \to \) Quantum Electrodynamics).

Furthermore, it is easy to prove that the physical fields \( F^{\mu\nu} \) (defined by \((10a, 10b)) \) vanish in the massless zero-momentum limit under the both definitions of normalization and field equations. It is straightforward to find \( B^{(+)}(p, \sigma) = \frac{i}{2m} p \times u(p, \sigma) \), \( E^{(+)}(p, \sigma) = \frac{1}{2m} p_0 u(p, \sigma) - \frac{1}{2m} p u^0(p, \sigma) \) and the corresponding negative-energy strengths. Here they are\(^{11}\)

\[
B^{(+)}(p, +1) = -\frac{iN}{2\sqrt{2m}} \begin{pmatrix} -ip_3 \\ p_3 \\ ip_r \end{pmatrix} = +e^{-i\alpha_{-1}} B^{(-)}(p, -1), \tag{17a}
\]

\[
B^{(+)}(p, 0) = \frac{iN}{2m} \begin{pmatrix} p_2 \\ -p_1 \\ 0 \end{pmatrix} = -e^{-i\alpha_0} B^{(-)}(p, 0), \tag{17b}
\]

\[
B^{(+)}(p, -1) = \frac{iN}{2\sqrt{2m}} \begin{pmatrix} ip_3 \\ p_3 \\ -ip_l \end{pmatrix} = +e^{-i\alpha_{+1}} B^{(-)}(p, +1), \tag{17c}
\]

and

\[
E^{(+)}(p, +1) = -\frac{iN}{2\sqrt{2m}} \begin{pmatrix} E_p - \frac{p_{0, p}}{E_p + m} \\ E_p - \frac{p_{0, p}}{E_p + m} \\ -\frac{p_{0, p}}{E_p + m} \end{pmatrix} = +e^{-i\alpha_{-1}} E^{(-)}(p, -1). \tag{18a}
\]

\(^{10}\)I am very grateful to the anonymous referee of my previous papers (“Foundation of Physics”) who suggested to fix them by requirements of the dimensionless of the action (apart from the requirements of the translational and rotational invariances).

\(^{11}\)We assume that \([\epsilon^\mu(p, \sigma)]^c = e^{i\alpha_{\sigma}}[\epsilon^\mu(p, \sigma)]^*\), with \(\alpha_{\sigma}\) being arbitrary phase factors at this stage. Thus, \(C = 1_{4 \times 4}\). It is interesting to investigate other choices of the \(C\), the charge conjugation matrix.
\[ E^{(+)}(p, 0) = i \frac{N}{2m} \left( \frac{p_1 p_2}{E_{p^2 - m^2}} - \frac{p_2 p_3}{E_{p^2 + m^2}} \right) = -e^{-i\alpha'_p} E^{(-)}(p, 0) , \] (18b)

\[ E^{(+)}(p, -1) = i \frac{N}{2\sqrt{2}m} \left( \frac{E_p - \frac{p_1 p_2}{E_{p^2 + m^2}}}{-\frac{p_3}{\sqrt{2}E_{p^2 + m^2}}} \right) = +e^{-i\alpha'_p} E^{(-)}(p, +1) , \] (18c)

where we denoted a normalization factor appearing in the definitions of the potentials (and/or in the definitions of the physical fields through potentials) as \( N \). Let us note that as a result of the above definitions we have

- The cross products of magnetic fields of different spin states (such as \( B^{(+)}(p, \sigma) \times B^{(-)}(p, \sigma') \)) may not be equal to zero and may be expressed by the “time-like” potential (see the formula (22) below):\(^{12}\)

\[ B^{(+)}(p, +1) \times B^{(-)}(p, +1) = - \frac{iN^2}{4m^2} \left( \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \right) = -B^{(+)}(p, -1) \times B^{(-)}(p, -1) , \] (19a)

\[ B^{(+)}(p, +1) \times B^{(-)}(p, 0) = - \frac{iN^2}{4m^2} \left( \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \right) = +B^{(+)}(p, 0) \times B^{(-)}(p, -1) , \] (19b)

\[ B^{(+)}(p, -1) \times B^{(-)}(p, 0) = - \frac{iN^2}{4m^2} \left( \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \right) = +B^{(+)}(p, 0) \times B^{(-)}(p, +1) . \] (19c)

Other cross products are equal to zero.

- Furthermore, one can find the interesting relation:

\[ B^{(+)}(p, +1) \cdot B^{(-)}(p, +1) + B^{(+)}(p, -1) \cdot B^{(-)}(p, -1) + B^{(+)}(p, 0) \cdot B^{(-)}(p, 0) = \]

\[ = \frac{N}{2m} (E_p^2 - m^2) , \] (20)

due to

\[ B^{(+)}(p, +1) \cdot B^{(-)}(p, +1) = \frac{N^2}{8m^2} (p_1 p_2 + 2p_3^2) = +B^{(+)}(p, -1) \cdot B^{(-)}(p, -1) , \] (21a)

\[ B^{(+)}(p, +1) \cdot B^{(-)}(p, 0) = \frac{N^2}{4\sqrt{2}m} p_3 p_1 = -B^{(+)}(p, 0) \cdot B^{(-)}(p, -1) , \] (21b)

\[ B^{(+)}(p, -1) \cdot B^{(-)}(p, 0) = -\frac{N^2}{4\sqrt{2}m} p_3 p_1 = -B^{(+)}(p, 0) \cdot B^{(-)}(p, +1) , \] (21c)

\[ B^{(+)}(p, +1) \cdot B^{(-)}(p, -1) = \frac{N^2}{8m^2} p_r^2 , \] (21d)

\(^{12}\)The relevant phase factors are assumed to be equal to zero.
\[ \mathbf{B}^{(+)}(\mathbf{p}, -1) \cdot \mathbf{B}^{(-)}(\mathbf{p}, +1) = \frac{N^2}{8m^2p_t^2}, \quad (21e) \]
\[ \mathbf{B}^{(+)}(\mathbf{p}, 0) \cdot \mathbf{B}^{(-)}(\mathbf{p}, 0) = \frac{N^2}{4m^2p_r p_t}. \quad (21f) \]

For the sake of completeness let us present the fields corresponding to the “time-like” polarization:

\[ u^\mu(\mathbf{p}, 0_t) = \frac{N}{m} \begin{pmatrix} E_p \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}, \quad \mathbf{B}^{(\pm)}(\mathbf{p}, 0_t) = \mathbf{0}, \quad \mathbf{E}^{(\pm)}(\mathbf{p}, 0_t) = \mathbf{0}. \quad (22) \]

The polarization vector \( e^\mu(\mathbf{p}, 0_t) \) has the good behaviour in \( m \to 0, \ N = m \) (and also in the subsequent limit \( \mathbf{p} \to 0 \)) and it may correspond to some quantized field (particle). Furthermore, in the case of the normalization of potentials to the mass \( N = m \) the physical fields which correspond to the “time-like” polarization are equal to zero identically. The longitudinal fields (strengths) are equal to zero in this limit only when one chooses the frame with \( p_3 = |\mathbf{p}| \), cf. with the light front formulation, ref. [19]. In the case \( N = 1 \) and (10a,10b) we have, in general, the divergent behaviour of potentials and strengths.\(^8\)

The spin operator recasts in the terms of the vector potentials as follows (if one takes into account the dynamical equations, Eqs. (9a,9b,10a,10b))\(^13\)

\[ J^k = \epsilon^{ijk} \int d^3x \left[ F^{0i}(\partial_\mu F^{\mu j}) + \tilde{F}^{0i}(\partial_\mu \tilde{F}^{\mu j}) \right] = \frac{1}{4} \epsilon^{ijk} \int d^3x \left[ B^i(\partial^0 B^j - \partial^j B^0) - A^i(\partial^0 A^j - \partial^j A^0) \right]. \quad (24) \]

If we put, as usual, \( \tilde{F}^{\mu\nu} = \pm iF^{\mu\nu} \) (or \( B^\mu = \pm A^\mu \)) for the right- and left- circularly polarized radiation we would obtain equating the spin operator to zero. The same situation would occur if one chooses unappropriate normalization and/or if one uses the equations (11a,11b) without needed precautivity. The straightforward application of (11a,11b) would lead to the proportionality \( J_{\kappa\tau} \sim m^2 \) and, thus, it appears that the spin operator would be equal

\[ \epsilon^{ijk} \int d^3x J^0_{jk} = \epsilon^{ijk} \int d^3x J_{jk}^0 = \epsilon^{ijk} \int d^3x J^0_{i} \quad , \quad (23a) \]
\[ J^0_{\alpha\beta} = \left( A_\beta \frac{\partial A_\alpha}{\partial x_0} - A_\alpha \frac{\partial A_\beta}{\partial x_0} \right). \quad (23b) \]

It describes the “transversal photons” in the ordinary wisdom. But, not all the questions related with the second \( B_\mu \) potential, the dual tensor \( \tilde{F}^{\mu\nu} \) and the normalization of the 4-potentials have been clarified in the standard textbooks.

\(^{13}\)In the case of \( N = 1 \) the fields \( \mathbf{B}^{\pm}(\mathbf{p}, 0_t) \) and \( \mathbf{E}^{\pm}(\mathbf{p}, 0_t) \) would be undefined. This fact was also not appreciated in the previous formulations of the theory of \((1,0) \oplus (0,1) \) and \((1/2,1/2) \) fields.

\(^{14}\)The formula (24) has certain similarities with the formula for the spin vector obtained from Eqs. (5.15,5.21) of ref. [20]:

\[ J_i = \epsilon_{ijk} \int J^0_{jk} d^3x , \quad (23a) \]
\[ J^0_{\alpha\beta} = \left( A_\beta \frac{\partial A_\alpha}{\partial x_0} - A_\alpha \frac{\partial A_\beta}{\partial x_0} \right). \quad (23b) \]
to zero in the massless limit, provided that the components of \( A_\mu \) have good behaviour (do not diverge in \( m \to 0 \)). Probably, this fact (the relation between generators and the normalization) was the origin of why many respectable persons claimed that the antisymmetric tensor field would be pure longitudinal. On the other hand, in the private communication Prof. Y. S. Kim stressed that neither he nor E. Wigner used the normalization of the spin generators to the mass. What is the situation which is realised in the Nature (or both)? The answer still depends on the choice of the field operators, namely on the choice of positive- and negative-energy solutions, creation/annihilation operators and the normalization.

We note that not all the obscurities were clarified even in our recent work \[5\]. Let us calculate in a straightforward manner the operator \((24)\). If one uses the following definitions of positive- and negative-energy parts of the antisymmetric tensor field in the momentum space, i. e., according to \((17a-18c)\) with \((\alpha_\sigma = 0)\):

\[
(F_{\mu\nu})_{+1} = + (F_{\mu\nu})_{-1} , (F_{\mu\nu})_{+1} = + (F_{\mu\nu})_{-1} , (F_{\mu\nu})_{0} = - (F_{\mu\nu})_{0} .
\]

(25)

for the field operator

\[
F_{\mu\nu}(x^\mu) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_\sigma \left[ (F_{\mu\nu})_{\sigma}^+(p)a_\sigma(p)e^{-ip\cdot x} + (F_{\mu\nu})_{\sigma}^-(p)b_\sigma^+(p)e^{+ip\cdot x} \right] ,
\]

(26)

then one obtains in the frame where \( p_{1,2} = 0 \):

\[
\begin{align*}
\mathbf{J} & \equiv \frac{m}{2} \int d^3x \mathbf{E}(x^\mu) \times \mathbf{A}(x^\mu) = \frac{m^2}{4} \int \frac{d^3p}{(2\pi)^3 4E_p^2} \left\{ \begin{array}{c} 0 \\ 0 \\ E_p \end{array} \right\} \\
& \left[ a(p, +1)b^+(p, +1) - a(p, -1)b^+(p, -1) + b^+(p, +1)a(p, +1) - b^+(p, -1)a(p, -1) \right] + \\
& + \frac{E_p}{m\sqrt{2}} \left( \begin{array}{c} E_p \\ iE_p \\ 0 \end{array} \right) \left[ a(p, +1)b^+(p, 0) + b^+(p, -1)a(p, 0) \right] + \\
& + \frac{E_p}{m\sqrt{2}} \left( \begin{array}{c} E_p \\ -iE_p \\ 0 \end{array} \right) \left[ a(p, -1)b^+(p, 0) + b^+(p, +1)a(p, 0) \right] + \\
& + \frac{1}{\sqrt{2}} \left( \begin{array}{c} m \\ -im \\ 0 \end{array} \right) \left[ a(p, 0)b^+(p, +1) + b^+(p, 0)a(p, -1) \right] + \\
& + \frac{1}{\sqrt{2}} \left( \begin{array}{c} m \\ -im \\ 0 \end{array} \right) \left[ a(p, 0)b^+(p, -1) + b^+(p, 0)a(p, +1) \right] + \\
& - \frac{1}{\sqrt{2}} \left( \begin{array}{c} m \\ -im \\ 0 \end{array} \right) \left[ a(p, 0)b^+(p, 0) + b^+(p, 0)a(p, 0) \right] .
\end{align*}
\]

\[15\] First of all, we note that the equality of the angular momentum generators to zero can be re-interpreted as

\[
W_\mu P^\mu = 0 ,
\]

with \( W_\mu \) being the Pauli-Lubanski operator. This yields

\[
W_\mu = \lambda P_\mu ,
\]

i. e., what we need in the massless case. But, according to the analysis above the 4-vector \( W_\mu \) would be equal to zero \emph{identically} in the massless limit. This is not satisfactory from the conceptual viewpoints.
\[ + \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{m}{im} \\ 0 \end{pmatrix} \left[ a(p,0)b^\dagger(p,-1) + b^\dagger(p,0)a(p,+1) \right] \cdot \]

If the commutators of the states with \( h = \pm 1 \) and \( h = 0 \) are equal to zero we have the \( J_3 \) generator to be non-zero solely. Above we used the fact that

\[
\begin{pmatrix} (\partial_\mu F^{\mu
u}(p,\sigma))^{(+)} = -\frac{m}{2} u^i(p,\sigma), \quad ((\partial_\mu F^{\mu
u}(p,\sigma))^{(-)} = -\frac{m}{2} [u^i(p,\sigma)]^* , \\
[\partial_\mu \tilde{F}^{\mu
u}(p,\sigma)]^\pm = 0. \end{pmatrix}
\]

The origin of this asymmetry can be discussed on the following basis: while both \( F^{\mu
u} \) and \( \tilde{F}^{\mu
u} \) can be expanded in the potentials (cf. Eqs. (9) and (10)), but once choosing potentials in order to obtain the fields (either \( F^{\mu
u} \) or its dual) we seem to be no able to recover the former using the formulas relating \( \tilde{F}^{\mu
u} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \). All this might be related with the problems of the normalization \( (N/m) \) too. The formulas (17,18) can describe both strengths \( B \) and \( E \), respectively, or vice versa. This conclusion is in the complete accordance with the fact that for rigorous definition of the parity one must use explicitly the representation \( (j,0) \oplus (0,j) \), and without proper definition of the creation/annihilation operators we are not able to answer rigorously, which properties do \( B \) and \( E \) have in the momentum space with respect to the space inversion operation.

Next, it is obvious that though \( \partial_\mu F^{\mu
u} \) may be equal to zero in the massless limit from the formal viewpoint and the equation (27) is proportional to the squared mass (?) for the first sight, it is not permitted to forget that the commutation relations may provide additional mass factors in the denominator of (27). It is the factor \( \sim E_p/m^2 \) in the commutation relations.

\[
[a_\sigma(p), b^\dagger_{\sigma'}(k)] \sim (2\pi)^3 \frac{E_p}{m^2} \delta_{\sigma\sigma'} \delta(p-k) .
\]

which is required by the principles of the rotational and translational invariance (and also by the necessity of the description of the Coulomb long-range force \( \sim 1/r^2 \) by means of the antisymmetric tensor field of the second rank).

The dimension of the creation/annihilation operators of the 4-vector potential should be \( \text{[energy]}^{-2} \) provided that we use (15a,15b) with \( N = m \). Next, if we want the \( F^{\mu\nu}(x^\mu) \) to have the dimension \( \text{[energy]}^2 \) in the unit system \( c = \hbar = 1 \) we must divide the Lagrangian by \( m^2 \) (with the same \( m \), the mass of the particle!). This procedure will have the influence on the form of (24,27) because the derivatives in this case pick up the additional mass factor. Thus,

\[ \text{16Remember that the dimension of the } \delta \text{ function is inverse to its argument.} \]

\[ \text{17That is to say: the factor } \sim \frac{1}{m^2} \text{ is required if one wants to obtain non-zero energy (and, hence, helicity) excitations.} \]

\[ \text{18The dimensions } \text{[energy]}^{+1} \text{ of the field operators for strengths was used here and in my previous paper in order to keep similarities with the Dirac case (the } (1/2,0) \oplus (0,1/2) \text{ representation) where the mass term presents explicitly in the term of the bilinear combination of the fields.} \]

\[ \text{19See the equation (3) of ref. [3], which was also used in the present paper.} \]
we can remove the “ghost” proportionality of the $c$-number coefficients in (27) to $\sim m^2$. The commutation relations also change its form. The possibility of the above renormalizations was not noted in the previous papers on the theory of the 4-vector potential and of the antisymmetric tensor field of the second rank. Probably, this was the reason of why peoples were confused after including the mass factor of the creation/annihilation operators in the field functions of $(1/2, 1/2)$ and/or $(1, 0) \oplus (0, 1)$ representations.

Finally, we showed that the interplay between definitions of the field functions and commutation relations occurs, thus giving the non-zero values of the angular momentum generators in the $(1, 0) \oplus (0, 1)$ representation.

The conclusion of the “transversality” (in the meaning of existence of $\hbar = \pm 1$) is in accordance with the conclusion of the Ohanian’s paper [21], cf. formula (7) there:

$$ J = \frac{1}{2\mu_0 c^2} \int \mathcal{R}e(\mathbf{E} \times \mathbf{A}^*) \, d^3x = \pm \frac{1}{\mu_0 c^2} \int \frac{\hat{\mathbf{z}} E_0^2}{\omega} \, d^3x, \quad (30) $$

with the Weinberg theorem, also with known experiments and with the same sense. The question, whether the situation could be realized when the spin of the antisymmetric tensor field would be equal to zero, must be checked by additional experimental verifications. We do not exclude this possibility, founding our viewpoint on the papers [6,7,13,22–24]. Finally, one should note that we agree with the author of the cited work [21, Eq.(4)] about the gauge non-invariance of the division of the angular momentum of the electromagnetic field into the “orbital” and “spin” part (30).

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\footnote{Remember that in almost all papers the electric field is defined to be equal to $E^i = F^{i0} = \partial^j A^0 - \partial^0 A^j$, with the potentials being not well-defined in the massless limit of the Proca theory. Usually, the divergent part of the potentials was referred to the gauge-dependent part. Furthermore, the physical fields and potentials were considered classically in the cited paper, so the integration over the 3-momenta (the quantization inside a cube) was not implied, see the formula (5) there. Please pay also attention to the complex conjugation operation on the potentials in the Ohanian’s formula. The formula (7) of ref. [21] is in the SI unit system and our arguments above are similar in the physical content. We did not still exclude the possibility of the mathematical framework, which is different from our presentation, but the conclusions, in my opinion, must be in the accordance with the Weinberg theorem.}
[1] M. W. Evans and J.-P. Vigier, *Enigmatic Photon*. Vols. 1-3 (Kluwer Academic Publishers, Dordrecht, 1994-96), the third volume with S. Jeffers and S. Roy. I still want to note that the reader should be cautious working with the books and papers of M. Evans because, unfortunately, they contain calculational and conceptual errors. Nevertheless, I can but mention some bright ideas in his works.

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