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Multi-objective optimization and design for free piston Stirling engines based on the dimensionless power

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Abstract. In this paper, the dimensionless power is used to optimize the free piston Stirling engines (FPSE). The dimensionless power is defined as a ratio of the heat power loss and the output work. The heat power losses include the losses of expansion space, heater, regenerator, cooler and the compression space and every kind of the heat loss calculated by empirical formula. The output work is calculated by the adiabatic model. The results show that 82.66% of the losses come from the expansion space and 54.59% heat losses of expansion space come from the shuttle loss. At different pressure the optimum bore-stroke ratio, heat source temperature, phase angle and the frequency have different values, the optimum phase angles increase with the increase of pressure, but optimum frequencies drop with the increase of pressure. However, no matter what the heat source temperature, initial pressure and frequency are, the optimum ratios of piston stroke and displacer stroke all about 0.8. The three-dimensional diagram is used to analyse Stirling engine. From the three-dimensional diagram the optimum phase angle, frequency and heat source temperature can be acquired at the same time. This study offers some guides for the design and optimization of FPSEs.

1. Introduction

The FPSE was invented by William Beale in 1964 [1]. FPSE operates without physical linkages. It relies only on the gas pressures and in some cases mechanical springs to impart the correct motions to the reciprocating elements. Such machines have the advantages of simplicity, low cost, ultra-reliability, and freedom of working gas leakage over conventional Stirling engines. Depending on the particular configuration, these engines may also be designed to operate at constant frequency [2]. Besides, FPSE’s piston and displacer have no side loads thanks to the lack of connecting rods, which makes it has potential for a long operating life. The FPSE can be operate very quietly since all the motions and forces hence the resulting vibrations are in the axial direction to the engine and may be easily isolated [3]. The above advantages combined with traditional crank connecting Stirling engine’s leads FPSE to be widely used for producing electricity from solar energy, especially for operation in remote locations, and for some other field which has strict command for maintenance [4].

Many different models have been used to optimize the Stirling engine. J. Boucher [5] has optimized a dual FPSE. In his model takes into account the non-linear dissipative effects of the fluid and the electromagnetic forces. The dynamic balance equations of the machine are solved by means of linearized pressure in the time domain especially. Chen [6] employing numerical calculations, not only the influences of multiple irreversible losses on the performance of the hybrid system are analysed, but also the impacts of some operating conditions such as the operating temperature, input gas compositions and operating pressure on the performance of the hybrid system are also discussed. Yaqi et al. and Sharma...
et al. [7, 8] developed a mathematical model for the overall thermal efficiency of solar powered high temperature differential dish Stirling engine with finite heat transfer and irreversibility of regenerator and optimized the absorber temperature and corresponding thermal efficiency. Cheng [9, 10] has optimized the geometrical parameters for Stirling engines based on theoretical analysis. The dead volume ratio, and the temperature ratio on the maximum shaft work of the engine as well as the optimal combination of the phase angle and the swept volume ratio are evaluated. He has optimized a beta-type Stirling engine based on dimensionless analysis. It is found that the relative error between the experimental and the theoretical data in dimensionless shaft work is lower than 5.2%. This model is also employed to investigate the effects of the influential geometric parameters on the shaft work, and the optimization of these parameters is attempted. Many parameters need to be optimized at the same time, so the multi-objective optimization of various thermodynamic and energy systems have been of high interest attention of researchers these days [11-12].

The Stirling engine are basing on the Stirling cycle as shown on the pressure-volume (P-V) planes in Fig.1. The Stirling cycle is highly idealized thermodynamic cycle, which consists of two adiabatic and two constant volume processes and the cycle is thermodynamically reversible. The principle assumption of the analysis is that the gas in the expansion space and the heater is at the upper source temperature, and the gas in the compression space and the cooler is at the constant lower sink temperature.

A FPSE has been designed and built in our laboratory. The working fluid contained in the engine is helium. The designed power output and the designed frequency are 120W and 30Hz. There are many papers have shown the optimization of Stirling engine, but no paper is based on the heat power losses. So this paper is aimed at using dimensionless power to optimize and analyse the actual Stirling engine. It will offer useful guides for designing the FPSE.

2. Dimensionless model
2.1 Adiabatic analysis

In the Stirling engine the movements of piston and displacer are sinusoidal. Fig.2 shows the model of FPSE. The displacements of the piston and the displacer are $x_p$ and $x_d$, respectively, given as:

$$x_p = X_p \sin(\omega t)$$
$$x_d = X_d \sin(\omega t + \theta)$$

where $X_p$ and $X_d$ are the amplitudes of piston and displacer, respectively; $\omega$ is the angular speed of engine and $\omega = 2\pi f$; $\theta$ is the phase angle between piston and displacer.
The volumes of the expansion and the compression space, $V_e$ and $V_c$ can be calculated, respectively, in terms of $x_p$, $x_d$ and radiuses of piston and displacer:

$$V_e = V_{eo} - A_d x_d$$

$$V_c = V_{co} - (A_p - A_r)x_p + (A_d - A_r)x_d$$

where $V_{co}$, $V_{eo}$ are the volumes of compression space and expansion space when the piston and displacer at the balance position, respectively; $A_p$, $A_d$, $A_r$ are the section area of piston, displacer and displacer rod, respectively; $x_d$, $x_p$ are the displacements of displacer and piston, respectively.

Fig. 2. The model of FPSE.

Fig. 3. Ideal adiabatic model.

As Fig. 3 shown the engine is configured as a five component, as configured earlier with perfect heat exchangers and regenerator. Thus the gas in cooler and heater is maintained at isothermal condition at temperature $T_k$ and $T_h$, respectively. Both in the regenerator matrix and gas in the regenerator volume have the individual linear temperature distribution: the gas flowing through regenerator–cooler interface being at cooler temperature $T_k$, regenerator–heater interface at heater temperature $T_h$. The working spaces are, however, assumed to be adiabatic and thus the temperature $T_c$ and $T_e$ will vary over the cycle in accordance with adiabatic nature of these spaces. Energy equation applied to a generalized all can be written as

$$dQ + (C_p T_i g A_i - C_p T_o g A_o) = dW + C_v d(mT)$$

The equation of state is given by $PV = mRT$. Taking logarithm of both sides of equation and differentiating we obtain differential form of the equation of state.

$$\frac{dp}{p} + \frac{dV}{V} = \frac{dm}{m} + \frac{dT}{T}$$

The total mass of working fluid remains constant, so $m_c + m_k + m_r + m_h + m_e = M$, differentiating the above equation

$$dm_c + dm_k + dm_r + dm_h + dm_e = 0$$

Now for all heat exchanger cells, since the respective volumes and temperatures are constant, the differential equation of the equation of state reduces to $dm/m = dp/p$. Applying above to each of three heat exchangers can get

$$dm_c + dm_e + \frac{dp}{R} (V_c/T_c + V_e/T_e) = 0$$

Applying energy Eq. (6) to the compression space we obtain

$$dQ_c - C_p T_{ck} m_{ck} = dW_c + C_v d(m_c T_c)$$

However, the compression space is adiabatic, $dQ_c = 0$ furthermore the work done $dW_c = pdV_c$, from continuity considerations the rate of accumulation of gas $Dm_c$ is equal the mass inflow of gas given by $m_{ck}$ and associated ideal gas relations

$$d_m_c = (pdV_c + V_c dp/dy)/RT_{ck}$$

Similarly for the expansion space

$$d_m_e = (pdV_e + V_e dp/dy)/RT_{he}$$

Substituting equations and simplifying
\[ dp = -\gamma p \frac{dV_c}{T_{ck}} + \frac{dV_c}{T_{he}} \]  

(12)

The total work done by the engine is the algebraic sum of the work done by expansion and compression spaces.

\[ dW = pdV_c + pdV_e \]  

(13)

2.2 The heat losses

Shuttle loss is one of the important thermal effects found in Stirling engine. This effect comes from the reciprocating action of the displacer (piston) in the cylinder. This conduction occurs through each reciprocation. The shuttle loss [13] is calculated by

\[ Q_{sH} = \frac{k_{GE} \cdot \pi \cdot B \cdot S^2 \cdot (T_h - T_k)}{4L \delta_G} \]  

(14)

where B and S are the bore and stroke of the hot cylinder, \( k_{GE} \) is the thermal conductivity of the working gas, \( \delta_G \) is the gap between piston and cylinder and L is the length of piston.

There is a radial gap between the inner diameter of the engine cylinder and the outer diameter of the piston. As the engine is pressurized and depressurized, gas flows into and out of the gap. Since the end of the gap is cold, extra heat must be added to the gas as it comes back from the gap. Some papers [13] have derived the following semi-empirical equation for the amount of heat loss by this reason

\[ Q_{pU} = \frac{2f \cdot f \cdot B \cdot C_{pg}}{3\pi \left( \frac{2k_{GE}}{f} \right)} \left( \frac{p_{max} - p_{min}}{R \cdot T_r} \right)^{0.6} \cdot C_{pg} \cdot L \cdot (T_h - T_k) \]  

(15)

where \( p_{max} \) and \( p_{min} \) are maximum pressure and minimum pressure respectively; \( T_r \) is the mean temperature of regenerator.

Equation (16) can be used to calculated power loss in radial direction and axial direction due to the non-uniform temperature [13].

\[ Q_{ED} = (0.0265S + 0.00727B) \cdot (Re)^{0.8} \cdot (Pr)^{0.4} \cdot \frac{\gamma - 1}{\gamma} \cdot \frac{p_1}{p_{mean}} \cdot \cos \theta_p \cdot k_{GE} \cdot T_h \]  

(16)

where the Re and Pr are the Reynolds number and Prandtl number, respectively; \( p_1 \) is the amplitude of pressure variation; \( \theta_p \) pressure wave phase angle with respect to expansion space volume displacement; \( \gamma \) is the ratio of specific heat.

The slit heat exchangers have been used to cooler and heater. The stainless steel wire mesh has filled into the regenerator. Due to the pressure drop in each heat-exchanger, there are pressure drop losses, heater, regenerator and cooler. Due to the friction factor is difficult to test, so in this paper the friction factors are from the book [13].

\[ Q_{HF} = \pi^2 \cdot f_H \cdot \frac{LH}{RH} \cdot \frac{\rho \cdot V_{swc}^2 \cdot f^2}{A^2} \cdot F_c \cdot \sin^2 \alpha \cos^2 \beta \cdot \left[ \sin \alpha + \frac{1}{6} \sin(2\beta - \alpha) + \frac{1}{2} \sin(2\beta + \alpha) \right] \]  

(17)

where \( f_H \) is the friction factor; A is the flow path area; LH and RH are the length of the heat-exchanger and the hydraulic radius, respectively; \( F_c \) is the correction factor.

In a Stirling engine, especially when the temperature ratio is high, the power loss due to the temperature drop inside the heat-exchangers must be taken into consideration, and the losses are calculated by [13]

\[ Q_{HT} = \frac{W_H}{m_H \cdot R \cdot T_H} \cdot \frac{\gamma - 1}{\gamma} \cdot \frac{1}{e^{NTU_H} - 1} \]  

(18)

where when in the heater the \( W_H = W_c \cdot W_e \) and when in the cooler the \( W_H = W_c^2 \); \( m_H \) is the mass flow rate of heater or cooler; \( H \) represents heater or cooler.

The axial heat conduction loss of regenerator transfers from high temperature to low temperature space. In fact, the high temperature space loses the heat \( Q_{CRM} \) and he low temperature space is on the contrary.

\[ Q_{CRM} = \frac{k_{RMG} \cdot L_{HTRC} \cdot (T_h - T_k)}{L_R} \]  

(19)
2.3. Dimensionless power

The dimensionless power is given by

\[ W = \frac{W_{\text{loss}}}{W_{\text{out}}} \]  \hspace{1cm} (20)

The dimensionless power is change with the heat loss. So, in this paper all the optimization based on the minimum dimensionless power \( W \). In the Table 1 the mainly operating parameters have been given.

| Variable                             | Value               |
|--------------------------------------|---------------------|
| Sectional area of displacer \( A_d \) | 647.0mm\(^2\)       |
| Sectional of piston \( A_p \)         | 529.4mm\(^2\)       |
| Sectional of displacer rod \( A_r \) | 131.1mm\(^2\)       |
| Temperature of cold source \( T_k \)  | 300K                |
| Volume of regenerator \( V_r \)       | 20411mm\(^3\)       |
| Volume of cooler \( V_k \)            | 1052.4 mm\(^3\)     |
| Volume of heater \( V_h \)            | 2322.3 mm\(^3\)     |
| Equilibrium volume of compression space \( V_{co} \) | 9550.4 mm\(^3\)   |
| Equilibrium volume of expansion space \( V_{eo} \) | 11126.0 mm\(^3\) |

Many different models have been used to optimize the Stirling engine, but the heat losses are not taken into consideration in these models. A FPSE have many heat losses, but different heat losses have different ratios. Fig.4 and Fig.5 show the main heat losses of the FPSE and the main heat losses of expansion space. It can be seen that 82.66% of the losses come from the expansion space and only 17.34% heat losses come from the heater, regenerator, cooler and the compression space. From the figure it can be seen that the maximum heat loss from the expansion space and in the expansion space the shuttle loss (54.59%), heat pumping loss (25.83%) and friction loss (12.7%) are the main losses. So the key point of optimization is decreasing the heat loss from expansion space.

Fig.4. The heat losses distribution of Stirling engine.  
Fig.5. The heat losses distribution of expansion space

In general, the higher temperature the higher output work and efficiency, however there is an optimum temperature to a Stirling engine. As Fig.6 shown, there are different optimum heat source temperatures at different initial pressures. From the figure it can be seen that the optimum heat source temperature increase with the increase of initial pressure. When the initial pressure is 1.0MPa, the optimum temperature is770℃, when the pressure increases to 5.0MPa, the optimum temperature increases to1095℃. Part of the optimum design of Stirling engine is to determine the ratio of bore over stroke for the cylinders. Fig.7 shows the optimum ratio of bore and stroke at different initial pressure.
From the figure it can be seen that the optimum ratio of bore and stroke drop with the increase of initial pressure. When the initial pressure is 1.0MPa, the optimum ratio is 2.7, when the pressure increases to 5.0MPa, the optimum ratio drops to 1.4.

The strokes of displacer and piston affect the output work and efficiency directly. Fig.8 and Fig.9 shows the optimum ratio of piston stroke and displacer stroke at different initial pressure and frequency respectively. From the figure it can be seen that the ratio of piston stroke and displacer stroke has a big effect on the W. At different initial pressure and work frequency the optimum ratios have different values. It also can be seen that when those ratios are about 0.8 all W get the smallest values. These study results indicate that in order to get high efficiency the ratio of piston stroke and displacer stroke should be designed at about 0.8.

In the ideal models, the output power and efficiency are proportional to the frequency and the Stirling engine could acquire the highest output work and thermal efficiency when the phase angle is 90°. The reason of this is that the ideal model has no losses or resistances. In a practical engine there are various losses which would increase by any amount of increase in frequency of the engine. Since in the ideal model there is no loss term, then the resultant optimum phase angle of that section cannot be used for a real engine. Therefore, the optimum frequency and phase angle of a real FPSE are very important. However, the frequency and phase angle are not independent, they are both main decided by the mass of piston, mass of displacer, the stiffness of piston’s spring and the stiffness of displacer’s spring. If the phase angle is changed, the frequency would be changed together. Therefore, at first the optimum phase angle and frequency must be found at different initial pressures. Then the mass of piston, mass of
displacer, the stiffness of piston’s spring and the stiffness of displacer’s spring can be changed to meet the optimum phase angle and frequency together.

Fig. 10. The optimum phase angle at different initial pressure.

Fig. 11. The optimum frequency at different initial pressure.

Fig. 10 and the fig. 11 show the optimum phase angle and optimum frequency at different initial pressure, respectively. From the fig. 10 it can be seen that the phase angle is as important as the initial pressure. In many models the optimum phase angle is 90°, but from the figure it can be seen that at different initial pressures the optimum phase angles are different. The optimum phase angle increases with the increase of pressure. When the pressure is 1MPa the optimum angle is 90°, when the pressure increases to 5MPa, the optimum phase angle is 120°. From the fig. 11 it can be seen that the frequency has a big effect on the W, so the optimum frequency must be found. However, at different initial pressures the optimum frequencies are not the same. The optimum frequency drops with the increase of pressure. When the pressure is 1MPa, the optimum frequency is 50Hz, when the pressure increases to 5MPa, the optimum frequency drops to 24Hz.

Fig. 12. The optimum phase angle and heat source temperature.

Fig. 13. The optimum phase angle and work frequency.

As the frequency and phase angle are so important to a FPSE and they are not independent variable. The optimum phase and frequency must be found at the same time, so the three-dimensional diagram has been used to analyse Stirling engine. Fig. 12 shows the effects of frequency and phase angle on the W, which the initial pressure and heat source temperature are 2MPa and 900K, respectively. From the figure it can be seen that there is an area where the phase angle from 85° to 100° and the frequency from 45Hz to 55Hz, in this area the values of W are very small. Fig. 13 shows the effects of heat source temperature and phase angle on the W. From the figure it is very obvious that the optimum phase angle and heat source temperature can be got at the same time. And also there is an area where the phase angle
from $85^\circ$ to $100^\circ$ and the heat source temperature from $800^\circ$C to $1000^\circ$C, in this area the values of $W$ is very small.

3. Conclusions

The dimensionless model can be used to optimize the FPSE. The paper got the result that 82.66% of the losses come from the expansion space and only 17.34% heat losses come from others. In the expansion space the shuttle loss (54.59%) is the main heat loss. At different pressure the optimum bore-stroke ratio, heat source temperature, phase angle and the frequency have different values. The optimum phase angle and heat source temperature increase with the increase of initial pressure. The optimum work frequency and ratio of bore and stroke drop with the increase of initial pressure. However, no matter what the initial pressure and frequency are the optimum ratios of piston stroke and displacer stroke are all between 0.8 to 1.0. When the initial pressure is 1.0MPa the optimum temperature, bore-stroke ratio, phase angle and work frequency are 770$^\circ$C, 2.7, $90^\circ$ and 50Hz. However, when the pressure increases to 5.0MPa the optimum temperature and phase angle increase to 1095$^\circ$C and $120^\circ$, respectively; the optimum bore-stroke ratio and work frequency drop to 1.4 and 24Hz, respectively. The three-dimensional diagram has been used to analyze Stirling engine. From the three-dimensional diagram it can be seen that there is an area where the phase angle changes from $85^\circ$ to $100^\circ$ and the frequency changes from 45Hz to 55Hz, in this area the dimensionless power is very small, consequently the thermal efficiency is very high.

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