Flexible Market for Smart Grid: Coordinated Trading of Contingent Contracts

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Abstract—A coordinated trading process is proposed as a design for an electricity market with significant uncertainty, perhaps from renewables. In this process, groups of agents propose to the system operator (SO) a contingent trade that is balanced, that is, the sum of demand bids and the sum of supply bids are equal. The SO accepts the proposed trade if no network constraint is violated or curtails it until no violation occurs. Each proposed trade is accepted or curtailed as it is presented. The SO also provides guidance to help future proposed trades meet network constraints. The SO does not set prices, and there is no requirement that different trades occur simultaneously or clear at uniform prices. Indeed, there is no price-setting mechanism. However, if participants exploit opportunities for gain, the trading process will lead to an efficient allocation of energy and to the discovery of locational marginal prices. The great flexibility in the proposed trading process and the low communication and control burden on the SO may make the process suitable for coordinating producers and consumers in the distribution system.

Index Terms—Power system economics, smart grids, transactive energy.

I. INTRODUCTION

THE electric power grid increasingly relies on distributed and variable energy sources. Integration of these new sources is helped by a market that facilitates matching intermittent supply and flexible demand [1], [2]. Today, the system operator (SO) achieves resource adequacy, congestion management, and efficiency through reserve requirements, day-ahead (DA) and real-time (RT) markets, and centralized dispatch of standard energy commodities, namely, a specified amount of energy delivered at specified nodes at fixed prices [3], [4]. The needs of important participants cannot be adequately expressed in terms of these standard commodities, so the SO allows bilateral contracts (e.g., Google [5], GM [6], and Amazon [7]), with contractual arrangements that are not known to the SO. Over time, the rigidity of the standard commodity was more broadly felt, and fitful accommodations were made by introducing new commodities, such as demand response, ramping, and capacity. But given the legacy of the standard market, this slow expansion of the SO’s responsibility cannot unlock the full contractual flexibility that participants may wish. In particular, it is challenging to repurpose today’s market design to serve the needs of distribution system operators (DSOs) who must coordinate participants with small distributed generation (DG) and controllable demand-side devices, and who would benefit from differentiated microcontracts (i.e., contracts whose volumes are of the order of kilowatthours). Possible examples of such differentiated contracts are the following:

1) contracts for flexible amount of energy contingent on the realization of uncertain supply or demand;
2) contracts to serve deferrable loads that consume a fixed amount of energy for (say) 1 h but which could be scheduled for any hour of the day [8], [9];
3) contracts that favorably price generation sources that are green or more flexible;
4) contracts that encourage local sharing among prosumers with solar PV and storage devices.

Incorporating these differentiated contracts requires a significant deviation from an electricity market with a small number of standard commodities.

In this paper, we propose a more flexible alternative to the current market design, called coordinated multilateral trading. In this design, participants trade among themselves according to terms and conditions fashioned to suit their own purposes like in today’s over-the-counter (OTC) markets, in contrast with exchanges for trading standardized commodities at transparent prices. These are contingent trades as the amount of energy delivered is contingent on events or conditions specified in the contracts. Since the trades induce power flow, they must be coordinated to ensure that network constraints are not violated. The SO accomplishes this coordination task by curtailing trades if network constraints are violated, and publishing information about the network state to guide participants regarding how subsequent trades can avoid overloading congested lines. Thus, the proposed market design permits flexible contracts by allowing contingent trades, while the SO maintains power system secu-

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In today’s design, the SO computes an efficient dispatch that respects network line constraints, but in the proposed design, the SO is only concerned with reliability, and the determination of an efficient dispatch is left to self-interested participants.

### A. Contributions and Organization

Coordinated trading of contingent contracts (described in Section III) is proposed as a flexible market mechanism in the context of electric power transmission system operation. We establish that the trading process is well defined, and during each step of the process, power system reliability is guaranteed though the role of the SO is greatly simplified. Furthermore, we show that the trading process converges to an efficient dispatch, which meets a benchmark defined using social welfare maximization as in the centralized stochastic economic dispatch (see Section IV). We also show that this trading process discovers the optimal locational marginal prices through the marginal costs of local participants (see Section V). Finally, we prove that the dispatch and prices identified from the trading process support an Arrow–Debreu equilibrium, a notion of competitive equilibrium under uncertainty (see Section VI). The trading process is illustrated with a simple two-bus example in Section VII.

### B. Related Literature

In studies of the standard electricity market, the basic framework is a one- or two-settlement market (DA market and RT market) in a deterministic setting [10]–[13]. In this framework, generators and load-serving entities present supply-and-demand function bids to the SO; the SO then calculates the equilibrium as the generation and load schedule that maximizes social welfare (producer plus consumer surplus), subject to the constraint that flows on transmission lines are limited by their rated capacities. This centralized calculation has the form of a mathematical programming problem called the optimal power-flow problem. The dual variables at the optimum solution are called locational or nodal marginal prices or LMPs. The LMP at a node is the marginal cost of delivering additional power at that node. In a two-settlement market, there are DA and RT LMPs.

In a stochastic context, uncertainty is modeled by a probability distribution over a set of scenarios. Each scenario has its specific supply-and-demand functions, and the SO finds the schedule that maximizes expected social welfare. This schedule is contingent, since there is a different schedule for each scenario [14]–[19]. The complexity of the stochastic problem grows in three ways with the number of scenarios. First, each demand-and-supply bid now is a function of prices and scenarios, so the number of decision variables and LMPs will be multiplied by the number of scenarios, thereby increasing the SO’s communication and computational burden. Second, there must be agreement among all participants about the probability distribution over the scenarios, which precludes heterogeneous beliefs or private information that can affect beliefs. Third, participants must work out in advance the bids they will offer for each scenario and price vector. This complexity has precluded real-world implementation of the optimal stochastic power-flow problem. In the absence of contingent (stochastic) bids that permit risk mitigation and reduce volatility, stochastic perturbations in demand and supply may lead to the large variations in LMPs that are observed.

Two studies propose decentralized trading processes to replace the SO’s centralized calculation. In [20], transmission rights are privately owned; the SO specifies “marginal loading factors” that is the amount of capacity on every transmission line that must be purchased by every proposed bilateral transaction. Transmission prices are adjusted iteratively in steps as follows. At any step, nodal price differences adjust to eliminate arbitrage profits from purchasing energy at one node and selling at another. Given nodal prices, transmission prices then are adjusted to increase rents, subject to the competitiveness condition that the transmission price for a line with excess capacity must be zero. The iteration converges in the limit to the welfare maximizing solution, and the nodal prices converge to the LMPs.

Our proposed design is closer to the decentralized multilateral trading process in [21] and generalizes their trading process developed for single-period deterministic electricity market into a setting with two periods and with uncertainty explicitly considered. In the multilateral trading process, groups of buyers and sellers propose to the SO a balanced trade, i.e., the sum of buy bids equals the sum of sell bids. The SO accepts the trade if (together with previously accepted trades) no transmission line constraint is violated. Otherwise, the SO curtails the proposed trade until the violation is eliminated. No price is announced. The understanding is that the private terms and conditions of a trade (including monetary payments) are acceptable to all parties. As in [20], the SO announces loading vectors to guide participants toward trades that do not violate line constraints. It is shown that in case generators are motivated by profit maximization and buyers by utility maximization, the process will converge to a social welfare maximum.

Two important distinctions between these decentralized processes are worth noting. First, in the language of mathematical programming, Chao and Peck [20] describe a dual method, whereas Wu and Varaiya [21] give a primal method. It is possible that at each step, the iteration in [20] is infeasible except in the limit, whereas each step in [21] is feasible and the process may be stopped at any point. Second, even though it is decentralized, the process in [20] is synchronized: trades in each step must occur at the same time, but, in [21], trades are asynchronous.

### C. Notation

For a natural number \( N \), \([N]\) denotes the set \( \{1, \ldots, N\} \). Let \( x \in \mathbb{R}_I^{I \times J} \) be a matrix, with entries denoted by \( x_{i,j} \), \( i \in [I] \), \( j \in [J] \). We use \( x_i \in \mathbb{R}^J \) to denote the vector \( (x_{i,j})_{j \in [J]} \) and \( x_j \in \mathbb{R}^I \) to denote the vector \( (x_{i,j})_{i \in [I]} \). For an Euclidean space \( \mathbb{R}^d \), we use \( 1 \in \mathbb{R}^d \) to denote the all-one vector.

### II. FORMULATION

#### A. Network Model

Consider a power network with \( N \) buses and \( L \) power lines with capacity constraints. The physics of the power network is described by the ac power flow equations, which is a set of quadratic equations relating the nodal complex voltages with...
nodal complex power injections [22]. In this paper, we focus on the real power component of the complex power and adopt a linearized dc approximation to the ac power flow equations\(^1\) [25], as commonly used in current electricity markets\(^2\) [27]–[29].

Under such a linearization, the region of feasible nodal power injections has the form

\[
P := \{ p \in \mathbb{R}^N : \bar{r} - \bar{H}p \leq \bar{r}, \quad 1^\top p = 0 \} \tag{1}
\]

where \(\bar{H} \in \mathbb{R}^{L \times N}\) is the linear sensitivity matrix relating the line flows to the nodal injections, and \(\bar{r} \in \mathbb{R}^L\) records the capacity values of the lines. To simplify the notation, we denote

\[
H = \begin{bmatrix} \bar{H} \\ -\bar{H} \end{bmatrix}, \quad \text{and} \quad \bar{f} = \begin{bmatrix} \bar{r} \\ \bar{r} \end{bmatrix} \tag{2}
\]

so the region of feasible nodal injections becomes

\[
P := \{ p \in \mathbb{R}^N : H p \leq \bar{f}, \quad 1^\top p = 0 \}. \tag{3}
\]

The first inequality in (3) models the line capacity constraints, while the second equality enforces power balance over the entire network. We will denote the rows of \(H\), referred to as \textit{loading vectors} or shift factors, by \(h_i^r \in \mathbb{R}^{1 \times N}\). Throughout this paper, we assume that \(P\) has a nonempty interior.

### B. Uncertainty Model

We consider the operation of the electricity market over two time periods, the DA market and the RT market.

We explicitly model the RT uncertainty as a finite collection of \(S\) system scenarios, so each scenario is indexed by \(s \in [S]\) with probability \(P(s) > 0\). We assume that the set of scenarios and the probabilities are known to all market participants and the SO, and the realization of a scenario is publicly verifiable by all of them. We could have the set of feasible injections \(P\) depending on the scenario as well to model transmission line failures, in which case \(H\) and \(\bar{f}\) in (3) will be indexed by scenario \(s\). We do not do this to simplify the notation.

### C. Participant Model

On each bus of the network \(n \in [N]\), there resides a collection of electricity market participants denoted by \(I_n\), each of which is either an electricity producer or an electricity load. We model each market participant by her (or his) RT \textit{power injection plan}, denoted by \(p_i = (p_{i,s})_{s \in [S]}\), her local feasible power injection sets, denoted by \(P_{i,s}\) such that \(p_{i,s} \in P_{i,s}\) for all \(i \in I_n\) and \(s \in [S]\), and her von Neumann–Morgenstern utility function over such a plan, denoted by \(U_i(p_i)\) and taking the form of

\[
U_i(p_i) = \mathbb{E}[u_{i,s}(p_{i,s})] = \sum_{s \in [S]} P(s)u_{i,s}(p_{i,s}) \tag{4}
\]

where \(u_{i,s}(p_{i,s})\) is the actual utility given scenario \(s\). Throughout this paper, we assume a \textit{quasi-linear environment}, so that the utility function is linear in the amount of monetary payment of each market participant, i.e.,

\[
u_{i,s}(p_{i,s}) = m_{i,s}(p_{i,s}) + \tilde{u}_{i,s}(p_{i,s}) \tag{5}
\]

where \(m_{i,s}(p_{i,s})\) is the payment received by the participant in scenario \(s\) and \(\tilde{u}_{i,s}(p_{i,s})\) is the utility associated with power injection \(p_{i,s}\), as discussed in detail below. We will assume that the utility function \(\tilde{u}_{i,s}(\cdot)\) is concave for each \(i \in I\) and \(s \in [S]\).

For an electricity producer, the power injection is induced by the producer’s possibly scenario-dependent electricity production so that \(p_i \in \mathbb{R}_+^S\). The feasible power injection sets model the generation limits, which could be scenario dependent in the case of renewable generation. Thus, we have \(P_{i,s} = [0, \tilde{p}_{i,s}]\), where \(\tilde{p}_{i,s}\) is the maximum possible power output in scenario \(s\). The utility function is as defined in (4) and (5), with

\[
\tilde{u}_{i,s}(p_{i,s}) = -c_{i,s}(p_{i,s}) \tag{6}
\]

where \(c_{i,s}(\cdot)\) is the cost function of the generation plant.

For an electricity load, the power injection is induced by the possibly scenario-dependent electricity consumption so that \(p_i \in \mathbb{R}_+^S\). Symmetrically with the producer, we have \(P_{i,s} = [-\tilde{p}_{i,s}, 0]\), where \(\tilde{p}_{i,s}\) is the maximum possible power demand in scenario \(s\). The utility function is taken to be

\[
\tilde{u}_{i,s}(p_{i,s}) = b_{i,s}(p_{i,s}) \tag{7}
\]

where \(b_{i,s}(\cdot)\) characterizes the benefit of using power by the particular load. For large loads (e.g., resellers), the benefit corresponds to the profit made from the given power consumption; for small loads such as individual consumers, the benefit function reflects the monetary value of consuming electricity and is a widely used device for modeling how power consumption varies with prices [30]–[32]. Allowing the benefit function to be scenario dependent is useful for modeling, e.g., demand response resources whose availability is not known \textit{a priori}.

We partition the set \(I_n\) as \(I_n = I_n^{DA} \cup I_n^{RT}\) such that \(I_n^{DA} \cap I_n^{RT} = \emptyset\) and denote \(I_n^{DA} = \cup_{s \in [S]} I_n^{DA,s}\) and \(I_n^{RT} = \cup_{s \in [S]} I_n^{RT,s}\), where \(I_n^{DA,s}\) contains producers/loads connected to bus \(n\) whose power injection has to be fixed in DA and cannot adapt to RT scenarios and \(I_n^{RT,s}\) are those that can adapt to RT scenarios. We refer to participants in \(I_n^{DA}\) as \textit{DA participants} and those in \(I_n^{RT}\) as \textit{RT participants}. Technically, the power injection plan of DA participants must satisfy the \textit{nonanticipation constraint}

\[
p_{i,s} = p_{i,\bar{s}}, \quad \text{for all } s, \bar{s} \in [S]. \tag{8}
\]

To simplify the notation, let

\[
P_{DA} = \{ p_i \in \mathbb{R}^S : p_{i,s} \in P_{i,s}, s \in [S] \} \tag{9}
\]

and

\[
P_{RT} = \{ P_{DA} \cap \{ p_i \in \mathbb{R}^S : p_{i,s} = p_{i,\bar{s}}, s, \bar{s} \in [S] \}, \quad i \in I_n^{RT} \}. \tag{10}
\]

Examples of DA participants include power plants that cannot ramp up or down following the RT uncertainty, such as

\(^{1}\)The dc approximation does not incorporate voltage constraints, which are important for distribution systems. A different linearization of the ac power flow, the linearized DistFlow model, should be used for distribution system applications [23], [24].

\(^{2}\)Some of today’s transmission SOs utilize a mixture of ac and dc model for their operation (e.g., ISO New England uses a dc optimal power flow with ac feasibility and PJM employs a dc–ac iteration) [26]. Although this paper assumes a linearized power flow model, many of our results can be extended to the ac power flow model with some work (cf., [21, Secs 7.6 and 7.7]).
coal-based generation plants, and loads that contract a fixed amount of consumption in each hour in DA. RT participants can either be variable generation sources or demand modeling, e.g., renewable generation or random power consumption, or controllable generation or demand that can adapt to RT scenarios, such as fast-ramping gas generation or demand response resources.

D. Efficiency Benchmark

A commonly used criterion for economic efficiency is Pareto optimality. In a quasi-linear environment, it is equivalent to the following stochastic social welfare maximization problem:

\[
\begin{align*}
\text{maximize} & \quad U(p) := \sum_{i \in I} U_i(p_i) \\
\text{subject to} & \quad p_i \in P_i, \quad i \in I \quad (11a) \\
& \quad x_{n,s} = \sum_{i \in I} p_{i,s}, \quad n \in [N], \ s \in [S] \quad (11c) \\
& \quad x_s \in P_s, \ s \in [S]. \quad (11d)
\end{align*}
\]

Notice that when the system does not take money from outside sources, we must have ex post budget adequacy:

\[
\sum_{i \in I} m_{i,s}(p_{i,s}) \leq 0. \quad (12)
\]

If ex post budget balance holds, i.e., \(\sum_{i \in I} m_{i,s}(p_{i,s}) = 0\), then the social welfare maximization program (11) is equivalent to the stochastic economic dispatch problem with the objective of (11) replaced by

\[
\sum_{i \in I} U_i(p_i) = \sum_{i \in I} E[\tilde{u}_{i,s}(p_{i,s})] \quad (13)
\]

where the summation is the net sum of ex ante generation costs and load benefits as discussed in the previous subsection.

III. TRADING PROCESS

The simplest market mechanism is one based on meeting and trading among self-interested agents. The electricity market is different in that centralized coordination has been commonly considered essential to ensure power system reliability constraints (3). Indeed, completely decentralized trading without coordination could lead to line flows that violate their capacity limits and compromise the reliability of the system. As such, the standard power system market designs rely on a centralized clearing house (or market maker), referred to as SO, to solve an economic dispatch optimization in order to determine the generator schedules and electricity prices. When uncertainty from renewable generation is considered, the resulting stochastic economic dispatch problem is computationally more complex and leads to increased communication requirement between the SO and market participants.

Wu and Varaiya [21] propose a remarkably simple fix to make the free-market style meet-and-trade procedure respect the power system reliability constraints (3). The idea is to inject minimal amount of coordination, implemented by the SO, into the free trades so that the reliability (or feasibility) is guaranteed in every step of the trading process, as shown in Fig. 1. They also establish that the trading process achieves economic efficiency in the limit. We will generalize their coordinated trading framework developed for single-period deterministic electricity market into a setting with two periods and with uncertainty explicitly considered. Although we consider only a two-period market (consisting of a forward market, i.e., DA, and a delivery period, i.e., RT) below, the analysis readily extends to settings with multiple delivery periods.

In this paper, we consider a setting where all market participants trade exclusively in DA. This means that during the DA market, each DA participant \(i \in \mathcal{I}^{DA}\) trades and determines her power injection, while each RT participant \(i \in \mathcal{I}^{RT}\) trades and determines her contingent power injection plan. Since under the current setting with a complete DA forward market, there is no need for RT retrade, i.e., additional RT trading cannot improve social welfare, we assume that there is no trading in RT.

We start with definitions for the trading process. Given the abstract nature of some of the definitions, examples demonstrating them are provided in Section VII and linked here in footnotes.

The premise of our trading system is that self-interested market participants will meet and propose trades for their own benefits, very much like how today’s bilateral power purchase contracts are formed. Thus, the fundamental building block of such a system is the notion of trade:

**Definition 1 (Contingent trade):** A contingent multilateral trade (referred to as trade in the following) among a group

\[\text{See Table I in Section VII for an example.}\]
\( I^k \subset I \) of participants is a collection of power injection plans
\[
p^k = (p^k_{i,s})_{i \in I^k, s \in [S]} \quad (14)
\]
that are feasible with respect to participants’ local constraints, i.e., \( p_i \in \mathcal{P}_i \), and ex post balanced so that
\[
\sum_{i \in I^k} p^k_{i,s} = 0, \quad s \in [S]. \quad (15)
\]

For convenience, we also define \( p^k_{i,s} = 0 \) for \( i \notin I^k \), so that given \( p^k \) we can infer \( I^k \) via
\[
I^k = \{ i \in I : \text{there exists a } s \in [S] \text{ s.t. } p^k_{i,s} \neq 0 \}. \quad (16)
\]

This definition is convenient from the point of view of the SO. In practice, a trade is a transaction that exchanges power with money. We will touch upon the money side of the trading process in Sections V and VI. The power balance condition is natural: the amounts of power supplied and consumed must be equal in each scenario. This definition also stresses that the commodity for sale is scenario-contingent power. That is, 1 MWh in different scenarios of RT are treated as different commodities.

Some further remarks are in order for Definition 1.

**Remark 1 (Need for multilateral trades):** As indicated in Definition 1, a trade may involve more than two market participants. Although multilateral trades are less common in practice compared with bilateral trades, for our purpose, it is necessary to consider multilateral trades so that the trading process is guaranteed to converge to an efficient dispatch. See [21] for an example in which bilateral trading fails to converge to the optimal dispatch due to loop externality [33]. When the network does not have cycles, it is possible to show that bilateral trades suffice under certain conditions. See Appendix D of the extended version of this paper [34].

**Remark 2 (SO’s sufficient statistics):** While market participants must keep track of their own power injection plans, from the power system’s perspective, contingent (network) nodal injection vectors, calculated from
\[
g^k_{n,s} = \sum_{i \in I_n} p^k_{i,s}, \quad n \in [N], \quad s \in [S] \quad (17)
\]
carry all the necessary information for checking the reliability constraints in (3). In particular, a trade among participants at the same node of network makes zero contribution to the actual nodal injection and thus is not of concern to the SO.

Trades motivated by participants’ interests do not take into account power system reliability constraints. So, it is necessary to have the SO verify that trades meet the power system constraints, and in case of violation to curtail trades so that compliance is achieved. Throughout this paper, we consider a simple curtailment scheme:

**Definition 2 (Uniform curtailment):** A trade \( p^k \) is said to be curtailed if only a portion of the proposed power injection, \( \gamma^k p^k \), is accepted by the SO, where \( \gamma^k \in [0, 1] \) is the curtailment factor and
\[
(\gamma^k p^k)_{i,s} = \gamma^k p^k_{i,s}, \quad i \in I^k, \quad s \in [S]. \quad (18)
\]

For notational convenience, we also define \( \gamma^k = 1 \) when a trade is accepted without curtailment.\(^4\)

**Remark 3 (Scenario-dependent curtailment):** The uniform curtailment scheme is the simplest curtailment scheme that ensures local feasibility of curtailed trades given that the initial trades satisfy local constraints. That is, given a trade \( p^k \) such that \( p^k_i \in \mathcal{P}_i, \ i \in I, \) the curtailed trade always satisfies \( \gamma^k p^k_i \in \mathcal{P}_i, \ i \in I. \) It is possible to make the curtailment scenario-dependent, i.e., for each scenario \( s \in [S], \) we can pick a different curtailment factor \( \gamma^k_s \in [0, 1]. \) This curtailment scheme no longer has the local feasibility property if DA participants are involved in the initial trade. In particular, the curtailed trade will not satisfy non-anticipative constraints of DA participants if the curtailment factors for different scenarios are taken to be different values. A hybrid of the uniform curtailment and scenario-dependent curtailment is to use the former when a trade involves DA participants and to use the latter when it does not. One can verify that all our results hold for this curtailment scheme as well.

During the DA market time window, a sequence of trades will come up for SO’s approval. Thus, the notion of power system reliability and the calculation of curtailment depend on trades that are already accepted into the system. We define a notion of system trading state as follows.

**Definition 3 (Trading state):** Given a sequence of trades \( p^k \) and their curtailment factor \( \gamma^k, \ k = 0, \ldots, k - 1, \) the global trading state is the accumulated participants’ contingent power injection
\[
y^k_{i,s} = \sum_{\kappa = 0}^{k-1} \gamma^\kappa p^\kappa_{i,s}, \quad i \in I, \quad s \in [S] \quad (19)
\]
and the network state for the SO is the accumulated network power injection
\[
x^k_{n,s} = \sum_{\kappa = 0}^{k-1} \gamma^\kappa q^\kappa, \quad n \in [N], \quad s \in [S]. \quad (20)
\]

The network and trading states relate as
\[
x^k = \sum_{i \in I_n} y^k_{i,s}. \quad (21)
\]

Given the current system state \( x^k \), a characterization for a trade \( p^k \) to be feasible for network constraints (3) is that its corresponding network injection vector \( q^k \) as defined in (17) satisfies
\[
x^k + q^k \in \mathcal{P}, \quad s \in [S]. \quad (22)
\]

Define the scenario-contingent feasible set of network injection as
\[
\mathcal{Q}_s(x_s) = \mathcal{P} - x_s = \{ q_s \in \mathbb{R}^N : x_s + q_s \in \mathcal{P} \}, \quad s \in [S] \quad (23)
\]
and \( \mathcal{Q}(x) = \mathcal{Q}_1(x_1) \times \cdots \times \mathcal{Q}_S(x_S). \) Then, (22) is equivalent to \( q^k \in \mathcal{Q}(x^k) \).

A potential issue of the trading process, in view of Definition 2, is that \( \gamma^k \) may have to be 0 to bring many trades back to

\(^4\) Table II in Section VII provides an example of a curtailed trade.
feasible. Indeed, if the market participants are proposing trades without any information regarding the current network state $x^k$, then it is likely that many trades overburdening lines that are already congested at $x^k$ will be proposed. To forestall such a possibility, the SO requires participants to only submit trades that are in the feasible direction (FD) of the network given the current state.

**Definition 4 (FD trade):** Given a network state $x^k$, let $\mathcal{L}_s(x^k)$ be the set of active (binding) line constraints in scenario $s$, that is,

$$\mathcal{L}_s(x^k) = \{ \ell \in [L] : h^\top_k x^k = f_\ell \}, \quad s \in [S].$$

Then, a trade $p^k$ is an FD trade at $x^k$ if its corresponding network injection $q^k$ as defined in (17) satisfies

$$h^\top_k q^k_s \leq 0, \quad \ell \in \mathcal{L}_s(x^k), \quad s \in [S].$$

If market participants are constrained to propose only FD trades, then it is guaranteed that $\gamma^k > 0$ so that every trade updates the network state. At this moment, it is unclear whether such an update is favorable in any sense. Formalizing the notion of “self-interested” participants, we have the following definition.

**Definition 5 (Worthwhile trade):** We call a trade $p^k$ an $\epsilon$-worthwhile trade at trading state $y^k$ if it leads to welfare improvement no smaller than $\epsilon$, i.e.,

$$\sum_{i \in I^k} U_i(y^k_i + p^k_i) - U_i(y^k_i) \geq \epsilon$$

and an $\epsilon$-unworthly trade if (26) does not hold. A profitable trade is an $\epsilon$-worthly trade with $\epsilon = 0$.

Notice that if an $\epsilon$-worthly trade is proposed by some participants and accepted by the SO, then it improves the social welfare by at least $\epsilon$ as the power injection plans of participants not involved in the trade are not changed.

We can now formalize the coordinated trading process.

**Step 1: Initialization.** The SO initializes the system state $x^0$ corresponding to some initial feasible trade $p^0$, $k = 0$.

**Step 2: Announcement.** The SO checks the congestion state of the system at $x^k$, identifies $\mathcal{L}_s(x^k)$ for $s \in [S]$ and announces the network loading vectors $h_i$, $\ell \in \mathcal{L}_s(x^k)$ for each $s \in [S]$.

**Step 3: Trading.** If a profitable trade$^5$ in the FD $p^k$ is identified, participants arrange it. If no profitable trade is found, go to Step 6.

**Step 4: Curtailment.** If $p^k$ is not feasible, i.e., the corresponding network injection $q^k$ is such that $q^k \notin \mathcal{Q}(x^k)$, the SO curtails the trade with

$$\gamma^k = \max \{ \gamma : \gamma q^k \in \mathcal{Q}(x^k) \} \in (0, 1).$$

If $p^k$ is feasible, set $\gamma^k = 1$.

**Step 5: Update.** The SO updates the network state as $x^{k+1} \leftarrow x^k + \gamma^k q^k$, $k \leftarrow k + 1$. Go to Step 2.

**Step 6: Termination.**

It is evident from the description of the trading process that the SO only has the following responsibilities. 1) The SO checks whether the trade newly submitted by participants is feasible with respect to network constraints. If not, it curtails the trade so that the resulting trade is feasible. 2) In case there are congested lines, the SO computes and broadcasts the loading vectors to the participants. Note that in our framework, the SO does not carry out any optimization. Instead, market participants seek to optimize their own profit during the trading process.

**Remark 4 (Feasibility):** An important feature of the trading process is that the proposed system state $x^k$ for any $k$ is feasible with respect to the power network constraints. Thus, even if the trading process is stopped at any stage before termination, the trades still result in a safe power flow solution.

**Remark 5 (Pay-as-bid settlement):** The trading process allows a pay-as-bid payment settlement approach. Immediately after submitting the trade, the market participants are informed whether their trades will be scheduled (or partially scheduled if curtailed); this information can then be used to calculate and settle the payment among these participants. Compared to the locational pricing used in the standard market, such a payment settlement process could limit the price risk faced by market participants which are expected to increase when the system integrates more renewables. This is also part of the reason why bilateral long-term contracts are widely used by large utility companies and power producers.

**IV. ECONOMIC EFFICIENCY**

Similar to arguments in [21], one can verify that the trading process described in the previous subsection is well defined, and whenever a $\epsilon$-worthly trade is identified ($\epsilon > 0$), the social welfare is strictly increased. Thus, when the trading process terminates, that is, when there exists no additional profitable trade that is not yet arranged, one may expect that the resulting power injection plan matches the economic efficiency benchmark defined by stochastic optimization problem (11). All proofs for this and subsequent sections can be found in the appendixes of the extended version of this paper [34].

**Theorem 1 (Efficiency):** Suppose the following assumptions are in force.

1) For any fixed $\epsilon > 0$, any $\epsilon$-unworthly trade in the FD will not be arranged and any $\epsilon$-worthly trade will eventually be identified and arranged.

2) Once a worthy profitable trade is identified, the market participants involved are willing to carry it out.

Then, the $\epsilon$-trading process is well defined, and the accumulated global trading state $y^k$ converges in the sense that

$$U^* - \lim_{k \to \infty} U(y^k) \leq \epsilon$$

where $U^*$ is the optimal value of (11).
providing coordination for the trades so that efficiency is achieved, while network reliability is guaranteed in every iteration of the trading process.

Remark 6 (Trade formation): Like in [21], we purposely leave the details of trading group formation open. Theorem 1 is powerful in that it is agnostic to the actual underlying mechanism dictating which subset of participants meet and propose trade $k$. For instance, a conceptually simple mechanism is that in every iteration $k$, a subset of $I$ is picked at random such that there is a positive probability for picking every subset.\(^6\) If it is possible for this group of participants to identify a profitable trade in the FD, they will propose it, as in Step 3 of the trading process. If not, we can simply continue this process by generating another random group of participants. Since there is a finite number of subsets of $I$, Theorem 1 guarantees that this process converges to an efficient dispatch with probability one. In practice, trading group formation processes depend on a lot of factors that we do not model in this paper. As a result, it could be the case that each participant $i \in I$ may only have access to a small subset of other participants in the market. An important future research direction is to design information platforms that facilitate trade discovery and reduce search cost.

Remark 7 (Profit allocation): Similarly, we do not specify how profit is allocated among the participants if a profitable trade is proposed and accepted by the SO. One can verify (or, cf., [21]) that for every profitable trade, there is a profit allocation that makes all involving participants better off.

Remark 8 (Merchandising surplus): In the standard market, the total payment collected from loads is larger than that paid to generators when there are line congestions. This merchandising surplus [35] is paid to transmission owners. In our setting, as the SO does not collect money from participants and all trades are budget balanced, separate payment streams might be needed to cover the costs of the transmission owners. Possible ways include charging a fee for using the transmission or requiring participants to acquire transmission rights for making trades across the network.

Remark 9 (Algorithmic interpretation): The trading process may be thought of as a projected line search algorithm for solving (11), whose iteration $k$ performs update
\[
y^{k+1} = y^k + \gamma^k p^k
\]
where $p^k$ is the search direction and $\gamma^k$ is the step length introduced to project the step into the feasible region. The algorithm is distributed, in that the search direction is identified based on information (and objective functions) of a subset of participants. The algorithm is special as its search direction $p^k$ is identified from a profitable trade, which is an economic construct, rather than based on gradient or Hessian of the objective function.

Remark 10 (Subjective probability): In general, different market participants may have their own subjective assessment of the probabilities of the scenarios. Denote the subjective probabilities of participant $i \in I$ by $\mathbb{P}_i(s)$, $s \in [S]$. Then, $y^k$ converges to an optimal solution of (11) with the ex ante utility function replaced by
\[
U_i(p_i) = \mathbb{E}_i[u_{i,s}(p_{i,s})] = \sum_{s \in [S]} \mathbb{P}_i(s)u_{i,s}(p_{i,s}).
\]
In this case, the resulting dispatch is Pareto optimal but may not maximize the ex ante social welfare, as the latter notion is defined upon the unknown true probability distribution of the uncertainty.

Remark 11 (DSO): The trading process also offers a way to design a lightweight or minimal distribution system operator (minDSO) for coordinating DG, flexible loads and other distribution level resources. With minDSO, the DG owners and demand side flexibility providers do not need to report cost and benefit data to the minDSO; so long as they can determine profitable trades among themselves, social welfare will improve. To adapt our formulation to the distribution system setting, the linearized DistFlow model [23], [24] provides an accurate model of the real power flow on the distribution network. Line capacity and transformer limits can be modeled similarly as transmission line limits. The tree network topology offers potential simplification to the trading process (see Appendix D of the extended version of this paper [34]). Voltage constraints can be modeled as additional linear constraints [36]. Distribution topology switching can be accommodated by updating the network constraint set $P$ according to the current switch states.

V. PRICE DISCOVERY

In standard markets, the SO solves an economic dispatch problem that determines both the dispatch and the locational marginal prices of power at all buses. When uncertainty is considered, the computationally demanding stochastic economic dispatch must be solved by the SO.

The trading process, on the contrary, does not require the SO to solve any optimization problem. Theorem 1 suggests that an efficient dispatch is achieved in the limit; here, we show that the optimal locational marginal prices also emerge when the trading process converges.\(^7\) The idea is simple. Suppose that in the last few minutes of the DA trading window, when the trading process has already converged, a new load comes into the system and demands a $\epsilon \rightarrow 0$ unit of power at bus $n$ for scenario $s$. Producers who can still generate additional power could quote a price based on their marginal cost evaluated at the current trading state. We, thus, discover the locational marginal price at bus $n$ for scenario $s$ by finding the minimum price announced by those generators that can indeed send power to bus $n$ given the congestion state in the scenario.

To formalize this idea, denote the optimal dual variable associated with constraint (11c) by $\lambda^*_{n,s}$, $n \in [N]$, $s \in [S]$. Furthermore, as constraint (11b) for RT participants is a box constraint, denote the optimal dual variables associated with the lower and upper bounds by $\gamma^*_{l,s}$ and $\gamma^*_{u,s}$, respectively. Notice that the trading process has a balanced budget by construction, as the SO

\(^6\)We can, e.g., first sample a random group size $l^k$ from $|I|$ and then randomly sample a group from $I$ of size $l^k$.

\(^7\)An alternative treatment, involving setting up trade-based prices and characterizing the convergence of the price process, is also possible (cf. [20], [37]). However, this requires a detailed specification of payments associated with each trade which we avoid in this paper.
is not involved in any financial aspect of the system. Therefore, problem (11) is equivalent to the stochastic economic dispatch problem.

**Lemma 1 (Price discovery):** For each bus \( n \in [N] \) and \( s \in [S] \), if there exists a participant \( i \in \mathcal{I}_n^R \) whose utility function is differentiable and whose optimal contingent power injection \( p^*_i, s \) is in the interior of her local feasible set, i.e., \( p^*_i, s \in \mathcal{P}_i, s \), then we have\(^8\)

\[
\lambda^*_{n, s} = -\mathcal{P}(s) \frac{\partial u_i, s(p^*_i, s)}{\partial p_i, s}.
\] (31)

In general, suppose the utility function of some participant \( i \in \mathcal{T}_n^R \) is differentiable; then

\[
\lambda^*_{n, s} = -\mathcal{P}(s) \frac{\partial u_i, s(p^*_i, s)}{\partial p_i, s} + (\bar{p}^*_{i, s} - p^*_{i, s}).
\] (32)

While the price calculation based on (31) is intuitive and only requires local information, calculating the prices using (32) may require solving the dual program of (11) to identify the values of the optimal dual variables \( \bar{p}^*_{i, s} \) and \( p^*_{i, s} \). Fortunately, solving for the dual program is greatly simplified when the optimal primal solution \( p^* \) is known (cf., [38]).

**VI. Arrow–Debreu Equilibrium**

Section IV established that the trading process converges to a stationary contingent power injection plan \( p^* \). Section V then showed that there is a well-defined notion of price \( \lambda^* \) that emerges alongside with the stationary injection plan. Here, we connect the pair \( (p^*, \lambda^*) \) to the suitable economic concept of general equilibrium under uncertainty. Taken together, this will formally establish that the contingent trading process converges to a (contingent plan, price) equilibrium, which respects the power system reliability constraints and achieves economic efficiency.

To start, we need to define an electricity market economy, similar to that done in [18] (also see [39]). The commodity of the economy is contingent power at each node \( n \in [N] \) and in each scenario \( s \in [S] \). Buying (selling) a unit contingent power \((n, s) \) in DA leads to the right to consume (responsibility to generate) a unit of power at node \( n \) if scenario \( s \) occurs in RT.

The market participants are those in \( \mathcal{I} \) as defined in Section II-C and a traditional SO\(^5\) who may convert power at one node into that at another node using the network.

For each participant \( i \in \mathcal{I} \), given prices for contingent power \( \lambda^* \), the following optimization is solved to determine the participant’s contingent power injection plan:

\[
\begin{align*}
\text{maximize} & \quad \lambda_{n, s} p_i, s + \mathcal{P}(s) \bar{u}_i, s(p_i, s) \\
\text{subject to} & \quad p_i \in \mathcal{P}_i
\end{align*}
\] (33a, b)

where \( \lambda_{n, s} \) is the price at node \( n \) faced by \( i \in \mathcal{I}_n \). Here, the objective function is the same as \( U_i(p_i) \) as defined in (4) with the linear payment scheme

\[
\lambda_{n, s} (p_i, s) = \tilde{\lambda}_{n, s} p_i, s
\] (34)

where \( \tilde{\lambda}_{n, s} = \lambda_{n, s} / \mathcal{P}(s) \). Notice that the first term in the summation in (33a) is the monetary payment that clears in DA; the second term is the expected utility derived from the power injection in RT.

The SO is modeled as a firm that uses technology (i.e., power network) to convert one type of commodity (i.e., contingent power on one node) to other types of commodities (i.e., contingent power on other nodes), in order to maximize its profit. Formally, the SO solves the following optimization to determine the contingent network power injection \( x \) given prices for contingent power \( \lambda^* \):

\[
\begin{align*}
\text{maximize} & \quad \sum_{s \in [S]} \sum_{n \in [N]} -\lambda_{n, s} x_{n, s} \\
\text{subject to} & \quad x_s \in \mathcal{P}, \quad s \in [S]
\end{align*}
\] (35a, b)

where the entire profit of SO in (35a) is cleared in DA.

The suitable notion of competitive equilibrium of such a market for contingent claims is that of Arrow–Debreu, which generalizes the Walrasian concept of general equilibrium to settings with uncertainty [40]. Here, we state the definition of Arrow–Debreu equilibrium for the electricity market economy:

**Definition 6 (Arrow–Debreu equilibrium):** A collection of contingent power injection plans \((p^*, x^*)\), with \( p^* \in \mathbb{R}^{[I] \times S} \) and \( x^* \in \mathbb{R}^{N \times S} \) and a system of prices for contingent power \( \lambda^* \in \mathbb{R}^{N \times S} \) constitute an Arrow–Debreu equilibrium if:

1. For every \( i \in \mathcal{I} \), \( p^*_i \) solves (33) given prices \( \lambda^* \).
2. For the SO, \( x^* \) solves (35) given prices \( \lambda^* \).
3. The market for each contingent power commodity clears

\[
x_{n, s}^* = \sum_{i \in \mathcal{I}_n} p^*_i, s, \quad n \in [N], \quad s \in [S].
\] (36)

By the first fundamental theorem of welfare economics and in a quasi-linear environment, we expect that a dispatch-price tuple \((p^*, x^*, \lambda^*)\) at an Arrow–Debreu equilibrium achieves economic efficiency defined by (11) (cf., [40] and [41]). Our previous result suggests that the dispatch at the limit of the trading process together with the emerged prices matches the solution of (11). Our next result establishes that the dispatch-price tuple obtained from the trading process indeed constitutes an Arrow–Debreu equilibrium.

**Lemma 2:** Suppose that an Arrow–Debreu equilibrium exists and that the utility functions are differentiable. Then, the contingent power injection plan \( p^* \) obtained from the trading process, the corresponding network injection plan \( x^* \) calculated from (11c), and the prices computed from (31) or (32) constitute an Arrow–Debreu equilibrium.

**Remark 12 (Tâtonnement process):** In light of Lemma 2, the trading process can be thought of as a way to drive an out-of-equilibrium market into its equilibrium. Such a process, characterizing the dynamic laws of out-of-equilibrium movement
of the market state, is in general referred to as a tâtonnement process; see [42].

VII. EXAMPLES

We provide an illustrative example for the trading process in this section.

Consider a two-bus network depicted in Fig. 2. There are three generators and one load connected to the system. We list the relevant data for the participants as follows.

1) G1 is a coal power plant that can generate up to 200 MW at a constant marginal cost 50 $/MW. It can only be scheduled in the DA stage due to its lead time.

2) G2 is a wind farm that generates 100 MW in the first scenario (windy scenario) and 50 MW in the second scenario (breezy scenario) at no operational cost. Suppose there are only these two scenarios for the system and one of them is realized at the delivery time. The underlying probabilities for these two scenarios are 0.6 and 0.4, respectively.

3) G3 is a gas power plant that can ramp up rapidly at RT with 100-MW capacity and constant marginal cost of 80 $/MW.

4) Load represents an inelastic power consumption of 150 MW.

For the purpose of illustrating the interaction between market participants and the PSO, in this example, we assume that, in each iteration, all four participants meet and propose a trade with no knowledge of the network constraint. In DA, as an example, the participants could solve the following optimization problem to identify the cost minimization trade:

\[
\begin{align*}
\text{minimize} & \quad 50p_1 + \mathbb{E}[80p_{3,s}] = 50p_1 + 48p_{3,1} + 32p_{3,2} \\
\text{subject to} & \quad p_1 + p_{2,s} + p_{3,s} + p_4 = 0, \quad s = 1, 2 \\
& \quad p_4 = -150 \\
& \quad 0 \leq p_1 \leq 200 \\
& \quad 0 \leq p_{3,s} \leq 100, \quad s = 1, 2 \\
& \quad 0 \leq p_{2,1} \leq 100, \quad 0 \leq p_{2,2} \leq 50
\end{align*}
\]

where the optimization variables are the DA scheduled coal power generation \( p_1 \), the RT gas power generation \( p_{3,s} \), corresponding to the two scenarios, and wind power generation corresponding to the two scenarios \( p_{2,s} \) (which is controllable up to curtailment). Upon solving this linear program, the participants propose its solution as their initial trade to the SO, which is shown in Table I.

This trade is not feasible with respect to the line limit in the windy scenario. As such, the SO curtails the trade to the one shown in Table II with \( \gamma = 0.8 \). The SO also announces the loading vector such that the constraints (25) can be expressed as \( \Delta p_1 + \Delta p_{2,s} - \Delta p_{3,s} - \Delta p_4 \leq 0 \), where \( \Delta p_s \) are the corresponding changes in the power injections, and \( s = 1, 2 \) as the line limit constraint is binding for both scenarios. The participants then solve the following program to identify a profitable trade in the FD:

\[
\begin{align*}
\text{minimize} & \quad 50(\tilde{p}_1 + \Delta p_1) + 48(\tilde{p}_{3,1} + \Delta p_{3,1}) + 32(\tilde{p}_{3,2} + \Delta p_{3,2}) \\
\text{subject to} & \quad \Delta p_1 + \Delta p_{2,s} + \Delta p_{3,s} + \Delta p_4 = 0, \quad s = 1, 2 \\
& \quad \Delta p_1 + \Delta p_{2,s} - \Delta p_{3,s} - \Delta p_4 \leq 0, \quad s = 1, 2 \\
& \quad \tilde{p}_4 + \Delta p_4 = -150 \\
& \quad 0 \leq \tilde{p}_1 + \Delta p_1 \leq 200 \\
& \quad 0 \leq \tilde{p}_{3,s} + \Delta p_{3,s} \leq 100, \quad s = 1, 2 \\
& \quad 0 \leq \tilde{p}_{2,1} + \Delta p_{2,1} \leq 100, \quad 0 \leq \tilde{p}_{2,2} + \Delta p_{2,2} \leq 50
\end{align*}
\]

where the \( \tilde{p}_s \) are the curtailed trade given in Table II. The resulting accumulated trade \( \gamma p + \Delta p \) is shown in Table III. The trading process would terminate now, as there is no profitable FD trade that can be further identified. One can easily verify that the accumulated trade coincides with the solution to (11) for this example, and therefore, the trading process indeed achieves efficiency.

VIII. CONCLUDING REMARKS AND OPEN QUESTIONS

Contingent coordinated trading is proposed as a market framework for power system resource allocation under un-
uncertainty. Within the framework, the economic efficiency is achieved via coordinated trades proposed by any groups of market participants for their own benefit. The trading process also discovers the optimal contingent locational marginal prices and supports an Arrow–Debreu equilibrium of the market. Allowing the trades to be contingent on properly defined system scenarios greatly enhances the flexibility of the trades and could result in an improvement of social welfare compared to standard deterministic dispatch-based market clearing. The role of the SO is minimal in our framework, as the SO only monitors the trades, curtails them if necessary, and does not collect any cost data or directly intervene in economic decisions. As such, all suppliers and consumers have open access to the power network, which promotes competition and expedites the processes of new generation and consumer-side technology adoption.

We envision that the proposed framework could help address many challenges in designing new DSOs for distribution systems with deep distributed energy resource penetration. Given the novelty of the proposed framework, it is natural that this paper leaves a variety of fundamental questions open.

1) **Uncertainty model:** In practice, it is unlikely that we can obtain an exact characterization of all possible scenarios for the entire system in DA. Thus, extending our ideal uncertainty model by incorporating information updates could make the trading framework more realistic. Under such settings, it may become advantageous to allow RT retrading as the realized RT scenario may not be exactly one of the prescribed scenarios in DA. Additionally, even if it is possible to characterize the set of all possible scenarios, the total number of scenarios may be very large due to the fact that many scenarios are local (see Appendix E of the extended version of this paper [34] for a model where all scenarios are local). Thus, in practice, suitable factorization (decomposing the scenario tree into system wide scenarios and local scenarios) or scenario reduction is necessary for successful market design based on the proposed trading framework.

2) **Trade implementation:** As all trades happen before RT, in RT, the participants need to supply and consume according to the scheduled trades. To ensure this indeed happens, advance metering infrastructure systems and suitable financial incentive (or penalty) scheme have to be in place. Thus, an open question is how to design such financial schemes that encourage consistent participant behaviors while limiting potential gaming activities.

3) **Trade formation:** For distribution system applications, requiring participants to meet and trade seems overwhelming. A more likely setting is to rely on one or many third-party marketplaces to identify profitable trades on behalf of (subsets of) the participants. Our analysis also applies to such settings, thanks to our general assumption on the trade formation process. In this context, our results are better understood as a form of separation principle, which ensures lossless separation of network reliability from market efficiency considerations with our trading framework. Under such separation, third-party marketplaces can fill the role of trade identification and formation without any explicit knowledge of the power network, as long as it follows the rules set by the SO. Designing and implementing such third-party marketplaces to unlock potentials from distributed energy resources and flexible loads thus is an important future direction to explore.

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