Achieving geodetic motion for LISA test masses: ground testing results

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The low-frequency resolution of space-based gravitational wave observatories such as LISA (Laser Interferometry Space Antenna) hinges on the orbital purity of a free-falling reference test mass inside a satellite shield. We present here a torsion pendulum study of the forces that will disturb an orbiting test mass inside a LISA capacitive position sensor. The pendulum, with a measured torque noise floor below 10 fN m/√Hz from 0.6 to 10 mHz, has allowed placement of an upper limit on sensor force noise contributions, measurement of the sensor electrostatic stiffness at the 5% level, and detection and compensation of stray dc electrostatic biases at the millivolt level.

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Among the most challenging technologies needed for the LISA gravitational wave mission is that of placing test masses in pure free fall. Accelerations due to stray forces change the distance between orbiting test masses, directly contaminating an interferometric measurement of gravitational wave strain. The LISA sensitivity goal requires acceleration noise, \( a_n \), with spectral density below \( 3 \times 10^{-15} \text{m/s}^2/\sqrt{\text{Hz}} \) at frequencies down to 0.1 mHz, or force noise of order \( \text{fN}/\sqrt{\text{Hz}} \) for a \( \sim 1 \) kg test mass \([1]\).

Environmental force noise can be screened by a satellite shield employing precision thrusters and a relative position sensor to remain centered about the free-falling test mass. The satellite, however, creates disturbances, particularly due to the close proximity of the position sensor. While noise analyses have shown proposed electrostatic sensor designs\([2, 3]\), with 2-4 mm test mass - sensor separations, to be compatible with the LISA goals, the low frequencies and extreme force isolation goals require force disturbance measurements to provide confidence in the LISA sensitivity predictions.

An ideal test of stray forces for LISA compares the differential noise in the orbits of two nearby free-falling test masses. This test will be performed, with a target acceleration noise limit of 30 fm/s^2/√Hz at 1 mHz, by the LISA Test-Flight Package (LTP) \([4]\) and Disturbance Reduction System (DRS) \([5]\). In preparing for such flight tests, we study the forces acting on a test mass that is nearly “free” in a single rotational degree of freedom, suspended by a thin torsion fiber inside a capacitive position sensor. The thermal torque noise limit, approached in similar apparatuses \([6]\), is several \( \text{fN m/√Hz} \) at 1 mHz for the torsion pendulum used here\([7]\). Dividing by half the 40 mm test mass width, this converts to a force noise near 100 fN/√Hz, within a factor 100 (10) of the LISA (LTP/DRS) force noise target.

The translational acceleration noise relevant to LISA can be divided into contributions from random forces \( f_{str} \) acting on the test mass (mass \( m \)) and from coupling to the relative motion of the satellite (mass \( M \)) via any dc force gradient (or “stiffness”) \( k_p \). The spacecraft motion noise arises in the position sensing noise, \( x_n \), and in the imperfect compensation of the external forces \( F_{str} \) acting on the satellite by the finite gain control loop (gain \( \omega_{DF}^2 \)). The residual acceleration noise \( a_n \) is

\[
a_n = \frac{f_{str}}{m} + \frac{k_p}{m} \left( x_n + \frac{F_{str}}{M\omega_{DF}^2} \right). \tag{1}
\]

To characterize \( a_n \), we use the torques, measured from the pendulum twist \( \phi \), acting on a LISA-like test mass inside a realistic capacitive position sensor designed for LISA’s sensing noise (spectral density \( S_{x_n}^{1/2} \sim \text{nm/√Hz} \)) and electrostatic force gradient \( (k_p \sim 100 \text{nN/m}) \) requirements. The measured pendulum angular noise in the LISA measurement band establishes an upper limit on the contribution of noisy surface forces to \( f_{str} \). Measurement of the rotational stiffness due to the AC sensing voltage characterizes a key part of the translational stiffness \( k_p \). Finally, measurement and compensation of the sensor rotational electrostatic bias imbalance quantify and demonstrate neutralization of a potentially important contribution of stray dc electric fields to \( f_{str} \).

The capacitive position sensor tested here (sketched in Fig. \([4]\) and discussed in Refs. \([8, 9]\)) is a variation of that projected for LTP \([2, 10]\). With the 40 mm test mass centered, the gap \( d \) between all electrodes and the mass is 2 mm. The electrodes are gold coated Mo, and are separated from the electrically grounded Mo housing by ceramic spacers. Differential gap sensing measurements from six sensing electrode pairs are combined to yield the three translational and three rotational test mass displacements. The sensor noise floor is dominated by transformer thermal noise, with \( S_{x_n}^{1/2} \approx 0.3 \text{nm/√Hz} \) and \( S_{\phi}^{1/2} \approx 40 \text{mrad/√Hz} \). Integrated actuation circuitry can apply voltages to the sensing electrodes, used here for occasional pendulum control in \( \phi \) and for electrostatic characterization of the sensor. All electrode surfaces have a dc path to a single circuit ground.

The main pendulum component is the test mass itself, a hollow gold coated Ti cube nominally 40 mm on a side,
The apparatus vacuum chamber is evacuated below 10−5 mBar and sits in a thermally controlled room with 50 mK long term stability.

With the sensor excitation on, the free torsional oscillation period \( T_0 \) is 515.1 seconds, with a quality factor \( Q \approx 1700 \). The period falls to 510.3 seconds with the sensor bias off (the period change is due to a negative torsional sensor stiffness, to be discussed shortly). These pendulum dynamics, combined with the calculated moment of inertia, allow conversion of measured angular deflections in \( \phi \) into torques, using the transfer function

\[
N(\omega) = \phi(\omega) \times I\omega_0^2 \left[ 1 - \left( \frac{\omega}{\omega_0} \right)^2 + \frac{i}{Q} \right],
\]

where \( \omega_0 = \frac{2\pi}{T} \) and we assume a frequency independent pendulum loss angle \( \frac{1}{Q} \).

Figure 2 shows typical torque noise data, calculated from the pendulum angular noise using Eqn. 2. Also shown is the instrument limit given by the quadrature sum of the pendulum thermal noise, \( S_{N,th}^{1/2} = \sqrt{4k_B T \frac{I\omega_0^2}{\omega^2}} \), and the sensor readout noise, \( S_{\phi,n}^{1/2} \), converted into torque noise, which dominates above 5 mHz. Near 1 mHz, the raw torque noise is roughly 10 times the thermal limit. We found the main excess noise source to be a coupling to translational motion between the sensor and pendulum, due to laboratory tilt noise. This coupling, with magnitude of order 10−7 Nm/m, is considerably stronger than reported torsion fiber “tilt - twist” couplings and is likely related to an electrostatic interaction involving the dielectric mirror edges. To better characterize the sensor contribution to \( f_{str} \), in the absence of this coupling to lab motion, we calculate the instantaneous coupling torque and subtract the resultant twist from the \( \phi \) time series. The tilt, measured in the \( x \) and \( y \) sensor readouts, is converted to torque through the measured “tilt-twist” coupling coefficients and then to angular twist through the pendulum transfer function (Eqn. 2). The remain-
three odd harmonics of \( f \) and \( s \) ties in Table I, are in agreement for the three harmonics.

\[
\Gamma_s \propto \langle V_{IN}^2 \rangle \sin^2 \phi \quad \text{(3)}
\]

Here \( \phi_0 \) is the (unstable) electrostatic equilibrium angle where this torque vanishes.

To measure the sensing stiffness \( \Gamma_s \), we modulate the effect by switching the 6 V_{RMS} injection voltage on and off at frequency \( f_m = 5 \) mHz. The resultant pendulum deflection, measured by the autocollimator, is proportional to \( \Gamma_s \), and the amplitudes at odd multiples of \( f_m \) reflect the \( 1/f \) dependence of the squarewave torque’s Fourier coefficients. The measurement is made over a range of test mass angles \( \phi \) by rotating the pendulum suspension point. Linear fitting of torque amplitude versus \( \phi \) for each odd harmonic of \( f_m \) gives estimates of \( \Gamma_s \) and \( \phi_0 \) (the latter measured relative to the sensor zero).

Figure 3 shows modulated stiffness torques for the first three odd harmonics of \( f_m \). The resulting estimates for \( \Gamma_s \) and \( \phi_0 \), shown with their statistical uncertainties in Table I, are in agreement for the three harmonics.

TABLE I: Electrostatic stiffness measurement results

| Component | \( \Gamma_s \) (fN m/mrad) | \( \phi_0 \) (mrad) |
|-----------|-----------------|-----------------|
| 1 \( f_m \) | -89.76 ± 0.11 | 3.95 ± 0.01 |
| 3 \( f_m \) | -89.3 ± 0.7 | 3.97 ± 0.07 |
| 5 \( f_m \) | -92 ± 6 | 3.8 ± 0.4 |

Though the statistical resolution for \( \Gamma_s \) (for 1\( f_m \)) in a given measurement is below 1%, repeated measurements have yielded a scatter of \( ±5\% \), which is currently being investigated. The measurement has also been performed at a fixed angle for a range of \( V_{IN} \), verifying the expected \( \Gamma_s \propto \langle V_{IN}^2 \rangle \) dependence. The range of \( \Gamma_s \) measured with the modulation technique is consistent with the observed period change for sensor bias on and off, which yield \( \Gamma_s = -96 ± 3 \) fN m/mrad. A prediction for \( \Gamma_s \) obtained using a finite element capacitance calculation[11] gives \(-96±5 \) fN m/mrad, with the uncertainty dominated by machining tolerances. The measured several mrad offsets between the torque and sensor zeros, which would coincide in a geometrically perfect sensor, are also consistent with the machining tolerances.

Another potentially important noise source for LISA is stray dc electrostatic fields, associated with patch or surface contamination effects. A charged test mass feels force (torque) proportional to the net linear (rotational) imbalances in the electrostatic potential on the surrounding sensor surfaces. For the cosmic ray charging expected for LISA[12], net dc potential imbalances of order 10 mV can produce significant low frequency acceleration noise[13]. To measure dc imbalances, we simulate a charge modulation by biasing the mass with a voltage \( V_\Delta \sin 2\pi f_m t \) applied to the injection electrodes and measure the resulting pendulum torque[14].

Assigning a mean stray dc voltage \( V_\Delta \) to each conductor, the 1\( f_m \) torque produced in this measurement is

\[
N_{1f_m} = -\alpha V_\Delta \sin 2\pi f_m t \times \sum_i \left( \frac{\partial C_i}{\partial \phi} \right) \delta V_i \quad \text{(4)}
\]

In principle, to reflect the electrostatic potential non-uniformity, the sum over electrodes \( i \) should be an integral over all surface domains with different potentials. In the naive but useful model where each electrode has a spatially uniform potential, the sum in Eqn. 4 reduces to \( C_x (R_\phi/d) \Delta_\phi \), where \( \Delta_\phi \) is the rotational dc imbalance in the four \( x\phi \) electrodes (see Fig. 4) and we assume an infinite plate model for the capacitance derivatives.

The torque in Eqn. 4 and likewise the associated random charge disturbance, is proportional to a net rotational dc bias imbalance, which can be compensated by counter-biasing the \( x\phi \) sensing electrodes with the actuation circuitry. Figure 4 shows measured 1\( f_m \) torques as a function of the compensation voltage \( V_C \). The measured torque is null very close to \( V_C = 39 \) mV (thus
The rotational stiffness measurement, $\Gamma$, acting on an orbiting, solid Au/Pt LISA test mass. For concerning their representativity of the translational forces along a single rotational axis, merit some discussion concerning in flight [14], proportional to the translational imbalance, $\Delta_s$. The uniform potential model for $\Delta_s$ is valid only as a rough number, as it neglects spatial surface potential variation. However, the measurement itself is sensitive to a sum over all surface domains, and thus the compensation $V_C$ that nulls the modulated torque (or force) will also null the torque (or force) produced by test mass charging. Compensating a 100 mV-level imbalance to within 1 mV reduces the potentially important random charging effect to an insignificant acceleration noise level.

The torque noise data can be cautiously converted into upper limits on specific contributions to the force noise $f_n$ acting on LISA test masses. The hollow test mass used here is largely immune to gravitational or magnetic fields coupling to the bulk LISA test masses, and the torsional mode is insensitive to net forces from important linear temperature or field gradient effects [2]. This pendulum is designed for maximum sensitivity to surface forces, arguably the most dangerous and unpredictable sources for sensors with several-mm gaps. Any electrostatic interaction between 100 kHz circuit noise and the sensor excitation, or between dc biases and low frequency voltage noise, produces torque noise proportional to the net force noise. For these effects, the appropriate arm length for converting to force noise is the electrode half-separation $R_\phi = 10.25$ mm, which gives a force noise upper limit of 1 pN/$\sqrt{\text{Hz}}$ between 0.6 and 10 mHz for such sources. Molecular impacts give force and torque noise on all test mass faces and have a 20 mm effective arm length, yielding a 500 fN/$\sqrt{\text{Hz}}$ upper limit from 0.6-10 mHz, with 250 fN/$\sqrt{\text{Hz}}$ around 3 mHz. This last number corresponds, for a solid Au/Pt test mass of the same dimensions, to acceleration noise of 200 fm/s$^2$/2$^1/2$, a factor 7 above the LTP flight goal.

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