Investigating Electrical Drive Performance Employing Model Predictive Control and Active Disturbance Rejection Control Algorithms

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Abstract— Many issues can degrade the electrical drive performance such as cross-coupling, time delay, external disturbances, and parameter variation. The Synchronous Reference Frame (SRF) PI Current Controller (CC) is the most popular control scheme for the motor drive current control due to its simplicity. However, the PI controller does not have an optimal dynamic response due to the reasonably low transient response of the integral parts. Furthermore, the tuning of the PI controller depends heavily on the machine’s parameters. Recently, alternative control schemes such as Model Predictive Control (MPC) and Active Disturbance Rejection Control (ADRC) are studied due to their dynamic performance and disturbance rejection capability, respectively. This paper presents a comparative study between the conventional PI, ADRC, and MPC control schemes applied for Permanent Magnet Synchronous Motor (PMSM) taking into consideration the operational issues of electrical drives.

Index Terms — Active Disturbance Rejection Control, Current Control, Electrical Motor Drive, Model Predictive Control.

I. INTRODUCTION

The Permanent Magnet Synchronous Motor (PMSM) is widely used in different applications due to its high efficiency, power density, and even increasing reliability features [1-3]. However, some challenges affect the overall performance of the PMSM drive system. For example, the cross-coupling between the orthogonal current components is represented as a nonlinear term and affects the controller behavior [4, 5]. Also, the time delay due to the inverter or the digital computations in the controller limits the control system bandwidth and affects its stability [6]. An external disturbance could occur, for example, due to a sudden impact of the mechanical loads. The machine parameters could be changed according to the operating conditions or different loading behavior [7, 8].

In terms of control schemes, Field Oriented Control (FOC) is considered as the most established strategy for electric drive systems. It consists of cascaded control loops, typically with an inner loop for current regulation and an outer loop for speed control. The most conventional control strategy applied for the current and speed regulation is based on the Proportional-Integral (PI) controller due to its inherent simplicity at design and implementation. It has been applied as a current controller in the Synchronous Reference Frame (SRF) with different configurations to enhance the cross-coupling compensation. However, the tuning of PI gains requires accurate machine parameters to guarantee the desired dynamic performance. Besides, the system bandwidth is limited due to the computational and modulation delay.

Great attention has been given recently to advanced control techniques such as Model Predictive Control (MPC) and Active Disturbance Rejection Control (ADRC) to overcome the stated problems and to enhance the driver dynamic performance. MPC provides higher bandwidth operation compared to the conventional PI controller scheme. It has been implemented as a current controller in [9, 10] showing faster dynamics with lower total harmonic distortion of the motor currents. On the other hand, the ADRC scheme provides high robustness to the internal and external disturbances due to the unmodeled dynamics and parameter uncertainties. It has a great interest in many industrial applications, e.g. flywheel energy storage system [11], DC-DC converters [12, 13], and recently in motor drive systems for current and speed regulations [14-16].

Therefore, in this paper, MPC and ADRC will be applied for PMSM in addition to the common PI control and results will be investigated to compare their dynamic response and their rejection capability for the external and internal disturbances. The rest of the paper is organized as follows: Section II is devoted to the PI control scheme followed by the ADRC algorithm in Section III. MPC and its equations are illustrated in Section IV. The simulation results are mentioned in Section V, while the conclusion is presented in Section VI.

II. THE CONVENTIONAL PI CONTROL SCHEME

Cascaded PI controllers have been implemented for the current and speed regulation as shown in Fig. 1. For the current regulation, two main configurations of the SRF PI CC have been addressed in the literature. The first configuration is known by the conventional SRF PI CC shown in Fig. 2, where \( G_s(s) \) represents the machine model and it is illustrated in (1). \( G_d(s) \) refers to the computational and modulation delay [17, 18]. It consists of the classical
The controller gain $K_1$ and $K_p$ can be tuned using the root locus technique. Accordingly, the SRF PI CC can be tuned based on one parameter $K_1$, that refers to the system bandwidth.

### III. ACTIVE DISTURBANCE REJECTION CONTROL

The current controller design based on ADRC is addressed in this section. The basic idea of the ADRC is to deal with the model uncertainties, un-modeled dynamics, and the external disturbances as a total disturbance which can be estimated in real-time by extended state observer (ESO). Then, an ESO-based feedback control that is used to compensate for the total disturbance and to keep the system output tracks the reference value [23]. Accordingly, a precise model of the system is not required. Moreover, it is simple to implement and has better disturbance rejection capability than other control techniques.

The block diagram of the ADRC control scheme is shown in Fig. 4. Based on the ADRC principle [23, 24], it is assumed that the external disturbances and the process dynamics are represented as a total disturbance. Subsequently, the voltage equations of the PMSM model can be written such as:

\[
\begin{align*}
\frac{di_d}{dt} &= f_d + \frac{1}{L_d}u_d \\
\frac{di_q}{dt} &= f_q + \frac{1}{L_q}u_q \\
f_d &= -\frac{R_s}{L_d}i_d + \omega_e \frac{L_d}{L_q}i_q - \frac{1}{L_q}d_d \\
f_q &= -\frac{R_s}{L_q}i_q - \omega_e \frac{L_q}{L_d}i_q - \frac{1}{L_d}\omega_m - \frac{1}{l_q}d_q
\end{align*}
\]

where $i_{d,q}$, $u_{d,q}$, $L_{d,q}$, and $d_{d,q}$ correspond to $dq$ axis stator current, voltages, inductances, and external disturbances respectively. $R_s$ is the stator resistance and $\phi_m$ is the flux linkage of PMSM. Based on (7), the ESO can be expressed as follows: $u_{e}=v_{eq}$, $b_{e}=L_{eq}$, and representing the $q$-axis by two states, $x_1=m_e$ and $x_2=f_e$. The total disturbance is represented by an extended state increasing the system order:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix} +
\begin{bmatrix}
b_1 \\
0 \\
\end{bmatrix}u_o +
\begin{bmatrix}
l_1(y - x_1) \\
l_2(y - x_2) \\
\end{bmatrix}
\]

The observer gains can be determined based on the bandwidth parametrization method [25]. For the CC, the output feedback controller is designed based on the system
output using the control law in (12):

\[ u = K_p (r - y) \]  

(12)

where \( K_p \) is the state feedback controller, \( r \) is the system input (the reference value of \( q \)-axis current), and \( u \) is the control signal generated from the feedback controller [26]. For the speed control loop based on ADRC, it can be designed as to (13):

\[ \frac{d\omega_m}{dt} = -\frac{b_0}{J} \omega_m + \frac{1}{J} T_L + \frac{K_e}{J} i_q \]  

(13)

where \( K_e = 1.5 \rho \phi_m \)

\( p \) is the pole pairs, \( T_L \) is the load torque, \( B \) is the friction coefficient and \( J \) represents the moment of inertia. Following the previous procedure with the current controller, \( \omega_m \) will be the output \( y \), \( i_q^* \) is the input \( u \), and \( b_o = \frac{K_e}{J} \).

IV. MODEL PREDICTIVE CONTROL

MPC has been applied successfully for different applications to enhance the performance and robustness. It has been applied for different electrical machines including PMSM [27], [9]. MPC can replace the PI controllers’ loops to obtain the FOC strategy taking into consideration the system constraints through the control cost function. To develop the MPC control loops, the PMSM state space needs to be identified. The differential equations of PMSM are given by [28]:

\[ \frac{d}{dt} i_d = \frac{1}{L_d} (v_d - R_s i_d + L_d i_q \omega_e) \]  

(14)

\[ \frac{d}{dt} i_q = \frac{1}{L_q} (v_q - R_s i_q - L_d i_d \omega_e - \varphi_m \omega_e) \]  

(15)

\[ \frac{d}{dt} \omega_e = \frac{1}{\rho} \left( T_e - \frac{B}{\rho} \omega_e - T_L \right) \]  

(16)

\[ T_e = \frac{3\rho}{2} \varphi_m i_q \]  

(17)

where \( T_e \) is the electromagnetic torque. The model given by (14) and (15) will be linearized around the operating point using Taylor expansion, and the linearized equations are:

\[ i_d \omega_e = \omega_{e0} i_{q0} + i_{q0} (\omega_e - \omega_{e0}) + \omega_{e0} (I_d - I_{d0}) \]  

(18)

\[ i_q \omega_e = \omega_{e0} i_{d0} + I_{d0} (\omega_e - \omega_{e0}) + \omega_{e0} (I_q - I_{q0}) \]  

(19)

where \( \omega_{e0}, i_{d0} \) and \( i_{q0} \) are the operating point’s values of the linearized model. By substituting (18) and (19) into (14) and (15), the linearized PMSM state-space model is derived as follows:

\[ x(t) = A_m x(t) + B_m u(t) + \delta_m \]  

(20)

\[ y(t) = C_m x(t) + D_m u(t) \]  

(21)

where

\[ x(t)^T = [i_d \ i_q \ \omega_e] \]

\[ u(t)^T = [v_d \ v_q] \]

\[ y(t)^T = [i_d \ i_q \ \omega_e] \]

\[ A_m = \begin{bmatrix} -\frac{R_s}{L_d} & \frac{L_d}{L_q} \omega_{e0} & \frac{L_d}{L_q} i_{q0} \\ \frac{R_s}{L_q} & -\frac{R_s}{L_q} & -\frac{L_d}{L_q} i_{d0} + \frac{\varphi_m}{\omega_{e0}} \\ 0 & 3p^2 \omega_{e0} & -\frac{B}{\rho} \end{bmatrix} \]

\[ B_m = \begin{bmatrix} 1/L_d & 0 \\ 0 & 1/L_q \end{bmatrix} \]

\[ \delta_m = \begin{bmatrix} -\frac{L_d}{L_q} \omega_{e0} i_{q0} \\ \frac{L_d}{L_q} \omega_{e0} i_{d0} \\ -\frac{p\omega_{e0}}{\rho} \end{bmatrix} \]

\[ C_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ D_m = 0 \]

The model is discretized with a definite sampling time \( T_s \) using the forward Euler approximation method. The discretized state-space model of the system is:

\[ x(k+1) = Ax(k) + Bu(k) + \delta \]  

(22)

\[ y(k) = Cx(k) + Du(k) \]  

(23)

where \( A = I + A_m T_s \) =

\[ \begin{bmatrix} T_s \left( 1 - \frac{R_s}{L_d} \right) & T_s \omega_{e0} & T_s i_{q0} \\ -T_s \omega_{e0} & T_s (1 - \frac{R_s}{L_q}) & -T_s (I_{d0} + \frac{\varphi_m}{L_q}) \\ 0 & T_s (3p^2 \omega_{e0}) & T_s (1 - \frac{B}{\rho}) \end{bmatrix} \]
The MPC cost function is given by [27, 29]:

$$j = \sum_{k=1}^{n_u} e^T(k)Q(k)e(k) + \sum_{k=0}^{n_u-1} u^T(k)R(k)u(k)$$  \hspace{1cm} (24)

subject to a discrete state-space model in (22) and (23), where $e(k)_{3\times 1} = y(k)_{3\times 1} - r(k)_{3\times 1}$ is the error, $y(k)_{3\times 1}$ is the system output, $r(k)_{3\times 1}$ is the reference input, $u(k)_{2\times 1}$ is the system control input, $Q(k)_{3\times 3}$ and $R(k)_{2\times 2}$ are weighting matrices, $n_s$ is the prediction horizon value and $n_u$ is the control horizon value. The model could be used recursively to find the predictions over the prediction horizon $n_s$, as follows:

$$\hat{x}(k + 1) = P_{x}x(k) + H_{x}\hat{u}(k)$$  \hspace{1cm} (25)

$$\hat{y}(k + 1) = P_{y}x(k) + H_{y}\hat{u}(k)$$  \hspace{1cm} (26)

where

$$\hat{x}(k + 1) = \begin{bmatrix} x(k + 1) \\ x(k + 2) \\ \vdots \\ x(k + n_s) \end{bmatrix}, P_{x} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{n_s} \end{bmatrix}$$

$$H_{x} = \begin{bmatrix} B & 0 & 0 & \cdots \\ AB & B & 0 & \cdots \\ A^2B & AB & B & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ A^{n_s-1}B & A^{n_s-2}B & A^{n_s-3}B & \cdots \end{bmatrix}$$

$$\hat{u}(k) = \begin{bmatrix} u(k) \\ u(k + 1) \\ u(k + 2) \\ \vdots \\ u(k + n_y - 1) \end{bmatrix}, \hat{y}(k + 1) = \begin{bmatrix} y(k + 1) \\ y(k + 2) \\ y(k + 3) \\ \vdots \\ y(k + n_y) \end{bmatrix}$$

The switching frequency used recursively to find the predictions over the prediction horizon $n_s$. The result of minimizing (24) with respect to $u(k)$ is given by [30]:

$${u}(k)_{MPC} = L^{H}(H^{T}Q(k)H + H^{T}Q(k)H + 2R^{T}(k) + 2H^{T}Q(k)(\hat{r}(k) - P_{x}(k)))^{-1}2H^{T}Q(k)(\hat{r}(k) - P_{x}(k))$$  \hspace{1cm} (27)

where $L_{x(2n_y-1),1} = [1 \ 0]$ with $I_{x(2n_y-1)}$ is an identity matrix and $O_{x(2n_y-2)}$ is a zero matrix, $\hat{r}(k)_{(2n_y-1),1}$ is the reference input, $\hat{Q}(k)_{(2n_y),2n_y}$ and $\hat{R}(k)_{(2n_u-1),2n_u-1}$ are weighting matrices. One of the important features of MPC is solving the constrained optimization problem [31, 32]. There are many methods for handling system constraints, one of the simple approaches is softening constraints method, which has a low computation burden compared to the other approaches [33]. In this method, the system constraints are implemented as a sum of squares of the difference between the input constraints boundaries and system input in the cost function as follows:

$$j = \sum_{k=1}^{n_u} e^T(k)Q(k)e(k) + \sum_{k=0}^{n_u-1} u^T(k)R(k)u(k) + \sum_{k=0}^{n_s-1}(u(k) - \hat{u}(k))^T S(k) (u(k) - \hat{u}(k))$$  \hspace{1cm} (28)

where $\hat{u}(k)_{2\times 1}$ is the control input constraints boundaries and $S(k)_{2\times 2}$ is the weighting matrix. Minimizing the cost function (28) with respect to $u(k)$ will be:

$$u(k)_{MPC} = L^{H}(H^{T}Q(k)H + H^{T}Q(k)H + 2R^{T}(k) + 2H^{T}Q(k)(\hat{r}(k) - P_{x}(k)))^{-1}2H^{T}Q(k)(\hat{r}(k) - P_{x}(k)) + 2S^{T}(k)\hat{u}(k)$$  \hspace{1cm} (29)

where $\hat{u}(k)_{(2n_u-1),1}$ and $\hat{S}(k)_{(2n_u-1),1}$ are the constraints values and weighting matrix over the control horizon $n_u - 1$ respectively.

V. SIMULATION RESULTS

Simulations have been carried out using MATLAB/Simulink to test the addressed control schemes. The machine parameters are given in Table I. The control schemes are simulated by discrete-time blocks and the inverter is simulated by its average model. One step time delay is considered when the reference voltage is applied from the controller to the machine. The deadtime and the resistive voltage drop across diodes and transistors have been neglected. The switching frequency $f_{sw}$ is decided to be 10 kHz. The control systems dynamics have been tested during a step-change in the mechanical load to test the speed of the response. The disturbance rejection capability for the external disturbances has been tested with an added

| Parameter | Symbol | Value |
|-----------|-------|------|
| Rated Power | $P$ | 2300 W |
| Rated current | $I_{r}$ | 9.5 A |
| Rated Voltage | $V_{r}$ | 220 V |
| Rated Frequency | $f$ | 100 Hz |
| Stator resistance | $R_{s}$ | 0.55 Ω |
| Stator inductance | $L_{s}, L_{r}$ | 0.002225 H |
| Nominal Torque | $T_{m}$ | 15 Nm |
| Rotation speed | $N_{s}$ | 1500 RPM |
| Number of pole pairs | $p$ | 4 |
| Stator-rotor flux | $\varphi_{m}$ | 0.114 wb |
| Rotor inertia | $J$ | 0.00277 kgm$^2$ |
external disturbance to the output voltage in the q-axis, $V_{\text{dis}} = 7$ volts [34]. The system response for different control techniques at the nominal machine parameters is illustrated in Fig. 5. On the other hand, the machine resistance is increased by 10% and the inductance is decreased by 5% in Fig. 6 to simulate the parameter variation effects on machine performance. Fig. 5 shows that the MPC scheme provides faster tracking for the step load changing followed by the ADRC scheme. However, it has an insignificant steady-state error with the d-axis current and speed. This error increases with the machine parameters mismatch as seen in Fig. 6.

### Table II

| Control Algorithm | Advantages                                                                 | Disadvantages                                                                 |
|-------------------|-----------------------------------------------------------------------------|-------------------------------------------------------------------------------|
| PI Scheme         | • Easy to implement and requires low memory.                                | • Limited bandwidth due to system delays.                                     |
|                   | • Better cross-coupling compensation.                                       | • Machine parameters are required for proper tuning.                         |
|                   | • Fast dynamic behavior.                                                   | • Low disturbance rejection capability.                                       |
| ADRC Scheme       | • High disturbance rejection capability.                                    | • The difficulty of tuning the controller gains.                              |
|                   | • The exact machine model is not required.                                  | • Lack of providing perfect cross-coupling compensation.                      |
|                   | • Easy to implement and requires low memory.                               |                                                                                |
| MPC Scheme        | • Fast dynamic behavior.                                                   | • Sensitive to Machine parameters change.                                     |
|                   | • Lower harmonics distortion in motor currents.                             | • Requires high computation burden.                                           |

For the ADRC scheme, it provides a better ability for disturbance rejection compared to other schemes. Moreover, it provides faster tracking for the load changes than the classical PI. However, it can be noticed that ADRC cannot provide exact cross-coupling compensation compared to the PI scheme especially when the complex vector PI is used for current regulation. This issue is due to the lack of an observer to provide exact estimation and rejection. Another issue for the ADRC scheme is the tuning of the controller gains. It is still an interesting research area and needs more investigation.

### VI. Conclusion

Three different control schemes including PI, ADRC, and MPC controllers have been studied for the PMSM motor drive. Their advantages and limitations have been summarized in Table II. It can be concluded that the MPC scheme has provided a faster dynamic performance and system delay ride through compared to PI and ADRC. However, it is sensitive to parameter variations and requires high mathematical computations. So, it is preferable for machines that have a limited change in their parameters and can be estimated by observers. For ADRC, it is simple to be implemented like the PI controller and it provides a higher disturbance rejection capability and faster dynamics. However, it takes some time to tune its parameters during the design process. Consequently, the proper tuning of the ADRC is still an open research area that need to be investigated.

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