General Congruences Modulo 5 and 7 for Colour Partitions

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Abstract: For any positive integers \(n\) and \(r\), let \(p_r(n)\) denotes the number of partitions of \(n\) where each part has \(r\) distinct colours. Many authors studied the partition function \(p_r(n)\) for particular values of \(r\). In this paper, we prove some general congruences modulo 5 and 7 for the colour partition function \(p_r(n)\) by considering some general values of \(r\). To prove the congruences we employ some \(q\)-series identities which is also in the spirit of Ramanujan.

Keywords and Phrases: colour partition; \(q\)-series; congruence.

Mathematics Subject Classifications: 11P82; 11P83.

1. Introduction

A partition of a positive integer \(n\) is a non-increasing sequence of positive integers, called parts, whose sum equals \(n\). For example, \(n = 3\) has three partitions, namely,

\[3,\ 2+1,\ 1+1+1.\]

If \(p(n)\) denote the number of partitions of \(n\), then \(p(3) = 3\). The generating function for \(p(n)\) is given by

\[
\sum_{n=0}^{\infty} p(n)q^n = \frac{1}{(q;q)_{\infty}},
\]

where, here and throughout the paper

\[(a;q)_{\infty} = \prod_{n=0}^{\infty} (1 - aq^n).\]
Ramanujan [11] established following beautiful congruences for \( p(n) \):

\[
p(5n + 4) \equiv 0 \pmod{5},
\]

\[
p(7n + 5) \equiv 0 \pmod{7},
\]

and

\[
p(11n + 6) \equiv 0 \pmod{11}.
\]

In this paper we are concerned with colour partitions of positive integer \( n \). A part in a partition of \( n \) has \( r \) colours if there are \( r \) copies of each part available and all of them are viewed as distinct objects. For example, if each part in the partition of 3 has two colours, say red and green, then the number of two colour partitions of 3 is 10, namely

\[
3_r, \ 3_g, \ 2_r + 1_r, \ 2_r + 1_g, \ 2_g + 1_g, \ 2_g + 1_r,
\]

\[
1_r + 1_r + 1_r, \ 1_g + 1_g + 1_g, \ 1_r + 1_g + 1_g, \ 1_r + 1_g + 1_g.
\]

Thus, number of 2-colour partitions of 3 is 10. The generating function of \( r \)-colour partitions of any positive integer \( n \) is connected to the general partition function \( p_r(n) \) introduced by Ramanujan in a letter to Hardy [4] and is given by

\[
\sum_{n=0}^{\infty} p_r(n) q^{n} = \frac{1}{(q; q)_\infty}.
\]

(1.2)

For \( r = 1 \), \( p_1(n) \) is the usual unrestricted partition function \( p(n) \) defined in (1.1). If \( r \) is negative, then

\[
p_r(n) = (p_r(n, e) - p_r(n, o)),
\]

(1.3)

where \( p_r(n, e) \) (resp. \( p_r(n, o) \)) is the number of partitions of \( n \) with even (resp. odd) number of distinct parts and each part have \( r \) colours. For example, if \( n = 5 \) and \( r = -1 \) then \( p_{-1}(5, e) = 2 \) with relevant partitions \( 4 + 1 \) and \( 3 + 2 \), and \( p_{-1}(5, o) = 1 \) with the relevant partition 5. Thus, \( p_{-1}(5) = 2 - 1 = 1 \). Similarly, we see that \( p_{-2}(3) = 4 - 2 = 2 \).

The case \( r = -1 \) in (1.3) is the famous Euler’s pentagonal number theorem. Ramanujan [4] showed that, if \( \lambda \) is a positive integer and \( \overline{w} \) is a prime of the form \( 6\lambda - 1 \), then

\[
p_{-4}\left(n\overline{w} - \frac{(\overline{w} + 1)}{6}\right) \equiv 0 \pmod{\overline{w}}.
\]

(1.4)

Ramanathan [10], Atkin [1], and Ono [9] investigated the partition function for some negative values of \( r \). Recently, Saikia and Chetry [14] proved some infinite families of congruences modulo 7 for the partition function \( p_r(n) \) for negative values of \( r \).
For positive values of $r$, $p_r(n)$ counts the number of $r$-colour partitions of a positive integer $n$. Gandhi [5] studied the colour partition function $p_r(n)$ for some particular values of $r$ and found some Ramanujan-type congruences for certain values of $r$. For example, he proved that

$$p_2(5n + 3) \equiv 0 \pmod{5} \quad \text{and} \quad p_8(11n + 4) \equiv 0 \pmod{11}.$$ 

Newman [8] also found some congruences for colour partition. Baruah and Ojah [2] proved some congruences for $p_3(n)$ modulo some powers of 3. Recently, Hirschhorn [7] found congruences for $p_3(n)$ modulo higher powers of 3.

In this paper, we prove some general congruences modulo 5 and 7 for the $r$-colour partition function $p_r(n)$ for some general values of $r$. To prove our congruences we will employ some $q$–series identities which is also in the spirit of Ramanujan. We list our congruences moduli 5 and 7 in Theorems 1.1 and 1.2 respectively below:

**Theorem 1.1.** For any non-negative integer $k$, we have

\begin{enumerate}[(i)]
  \item $p_{5k+1}(5n + 4) \equiv 0 \pmod{5}$.
  \item $p_{5k+2}(5n + i) \equiv 0 \pmod{5}$, \quad for \quad $i = 2, 3, 4$.
  \item $p_{5k+4}(5n + i) \equiv 0 \pmod{5}$, \quad for \quad $i = 3, 4$.
  \item $p_{25k+3}(25n + 22) \equiv 0 \pmod{5}$.
  \item $p_{25k+4}(25n + 21) \equiv 0 \pmod{5}$.
\end{enumerate}

**Theorem 1.2.** For any non-negative integer $k$, we have

\begin{enumerate}[(i)]
  \item $p_{7k+1}(7n + 5) \equiv 0 \pmod{7}$.
  \item $p_{7k+4}(7n + j) \equiv 0 \pmod{7}$, \quad for \quad $j = 2, 4, 5, 6$.
  \item $p_{7k+6}(7n + j) \equiv 0 \pmod{7}$, \quad for \quad $j = 3, 4, 6$.
  \item $p_{49k+2}(49n + 7j + 3) \equiv 0 \pmod{7}$, \quad for \quad $j = 2, 4, 5, 6$.
  \item $p_{49k+3}(49n + 7j + 1) \equiv 0 \pmod{7}$, \quad for \quad $j = 2, 4, 5, 6$.
  \item $p_{49k+5}(49n + 39) \equiv 0 \pmod{7}$.
\end{enumerate}
2. Preliminaries

Ramanujan [12] stated that
\[
(q; q)_\infty = (q^{25}; q^{25})_\infty (F^{-1}(q^5) - q^2 F(q^5)),
\]
where \( F(q) := q^{-1/5}R(q) \) and \( R(q) \) is the Rogers-Ramanujan continued fraction given by
\[
R(q) := q^{1/5} \frac{(q^2; q^5)_\infty (q^3; q^5)_\infty}{(q; q^5)_\infty (q^4; q^5)_\infty} = \frac{q^{1/5}}{1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{1 + \ldots}}}, \quad |q| < 1.
\]

From (2.1), it is easy to see that
\[
(q; q)^2 = (q^{25}; q^{25})^2 \left( F^{-2}(q^5) - 2qF^{-1}(q^5) - q^2 + 2q^3 F(q^5) + q^4 F^2(q^5) \right),
\]
\[
(q; q)^3 = (q^{25}; q^{25})^3 \left( F^{-3}(q^5) - 3qF^{-2}(q^5) + 5q^3 - 3q^5 F^2(q^5) - q^6 F^3(q^5) \right),
\]
and
\[
(q; q)^4 = (q^{25}; q^{25})^4 \left( F^{-4}(q^5) - 4qF^{-3}(q^5) + 2q^2 F^{-2}(q^5) + 8q^3 F^{-1}(q^5) - 5q^4
\]
\[ - 8q^5 F(q^5) + 2q^6 F^2(q^5) + 4q^7 F^3(q^5) + q^8 F^4(q^5) \right). \]

Again, by [3] p. 303, Entry 17(v)], we have
\[
(q; q)_\infty = (q^{49}; q^{49})_\infty \left( A(q^7) - qB(q^7) - q^2 + q^5 C(q^7) \right),
\]
where
\[
A(q^7) = \frac{f(-q^{14}, -q^{35})}{f(-q^7, -q^{42})}, \quad B(q^7) = \frac{f(-q^{21}, -q^{28})}{f(-q^{14}, -q^{35})}, \quad C(q^7) = \frac{f(-q^7, -q^{42})}{f(-q^{21}, -q^{28})},
\]
and
\[
f(a, b) = \sum_{n=-\infty}^{\infty} a^{n(n+1)/2} b^{n(n-1)/2}, \quad |ab| < 1.
\]

Squaring (2.5), we find that
\[
(q; q)^2 = (q^{49}; q^{49})^2 \left( (A(q^7)^2 - 2q^7 C(q^7)) - 2q A(q^7) B(q^7) + q^2 \left( B(q^7)^2 - 2A(q^7) \right)
\]
\[ + q^3 \left( 2B(q^7) + q^7 C(q^7)^2 \right) + q^4 + 2q^5 A(q^7) C(q^7) - 2q^6 B(q^7) C(q^7) \right). \]

Also, from [3] p. 39, Entry 24(ii)] we note that
\[
(q; q)^3 = \sum_{n=0}^{\infty} (-1)^n (2n + 1) q^{n(n+1)/2}.
\]

From (2.7), it follows that
\[
(q; q)^3 = J_0(q^7) - qJ_1(q^7) + q^3 J_3(q^7) - 7q^6 J_6(q^7)
\]
\[ J_0(q^7) - qJ_1(q^7) + q^3J_3(q^7) \equiv 0 \pmod{7} \] (2.8)

and

\[
(q;q)_\infty^6 \equiv J_0(q^7)^2 - 2qJ_0(q^7)J_1(q^7) + q^2J_1(q^7)^2 + 2q^3J_0(q^7)J_3(q^7) - 2q^4J_1(q^7)J_3(q^7) + q^6J_3(q^7)^2 \pmod{7},
\]

(2.9)

where \( J_0, J_1, J_3, \) and \( J_6 \) are series with integral powers of \( q^7 \).

From [6, Lemma 3.12] we note that, if

\[
(\frac{(q^7; q^7)}{q^2(q^{49}; q^{49})})_\infty^\infty = 0,
\]

(2.10)

and \( H_7 \) is an operator which acts on a series of powers of \( q \) and picks out those terms in which the power of \( q \) is congruent to 0 modulo 7, then

\[ H_7(\xi^4) = -4T_7 - 7 \quad \text{and} \quad H_7(\xi^5) = 10T_7 + 49. \]

(2.11)

In addition to the above \( q^- \) identities, we will also need the following congruence which follows from the binomial theorem (or see [13, Lemma 2.4]): For any prime \( p \), we have

\[ (\frac{q^p; q^p}{q^p; q^p})_\infty^\infty \equiv (\frac{q; q}{q; q})_\infty^\infty \pmod{p}. \]

(2.12)

3. Proof of Theorem 1.1

Proof of (i): Setting \( r = 5k + 1 \) in (1.2), we obtain

\[
\sum_{n=0}^{\infty} p_{5k+1}(n)q^n = \frac{1}{(q;q)_{5k+1}^5}, \]

(3.1)

Using (2.12) in (3.1), we obtain

\[
\sum_{n=0}^{\infty} p_{5k+1}(n)q^n \equiv \frac{(q; q)_\infty^4}{(q^5; q^5)_{k+1}^k} \pmod{5}, \]

(3.2)

Employing (2.4) in (3.2) and then extracting terms involving \( q^{5n+4} \), dividing by \( q^4 \), and replacing \( q^5 \) by \( q \), we arrive at the desired result.

Proof of (ii): Setting \( r = 5k + 2 \) in (1.2), we obtain

\[
\sum_{n=0}^{\infty} p_{5k+2}(n)q^n = \frac{1}{(q;q)_{5k+2}^5}, \]

(3.3)

Using (2.12) in (3.3), we obtain

\[
\sum_{n=0}^{\infty} p_{5k+2}(n)q^n \equiv \frac{(q; q)_\infty^3}{(q^5; q^5)_{k+1}^k} \pmod{5}, \]

(3.4)
Employing (2.3) in (3.4) and extracting terms involving \( q^{5n+i} \) for \( i = 2, 3, 4 \), we arrive at the desired result.

**Proof of (iii):** Setting \( r = 5k + 4 \) in (1.2), we obtain
\[
\sum_{n=0}^{\infty} p_{5k+4}(n)q^n = \frac{1}{(q; q)_{5k+4}^\infty},
\]  
(3.5)
Using (2.12) in (3.5), we obtain
\[
\sum_{n=0}^{\infty} p_{5k+4}(n)q^n \equiv \frac{(q; q)_{\infty}^2}{(q^5; q^5)_{\infty}^{k+1}} \pmod{5},
\]  
(3.6)
Using (2.1) in (3.6) and extracting terms containing \( q^{5n+i} \) for \( i = 3, 4 \), we complete the proof.

**Proof of (iv):** Setting \( r = 25k + 3 \) in (1.2), we obtain
\[
\sum_{n=0}^{\infty} p_{25k+3}(n)q^n = \frac{1}{(q; q)_{25k+3}^\infty},
\]  
(3.7)
Using (2.12) in (3.7), we obtain
\[
\sum_{n=0}^{\infty} p_{25k+3}(n)q^n \equiv \frac{4(q; q)_{\infty}^4}{(q^5; q^5)_{\infty}^{k-2}} \pmod{5},
\]  
(3.8)
Employing (2.2) in (3.8) and then extracting terms involving \( q^{5n+2} \), dividing by \( q^2 \), and replacing \( q^5 \) by \( q \), we obtain
\[
\sum_{n=0}^{\infty} p_{25k+3}(5n + 2)q^n \equiv \frac{4(q; q)_{\infty}^4}{(q^5; q^5)_{\infty}^{k-2}} \pmod{5},
\]  
(3.9)
Simplyfing (3.9) by using (2.12), we obtain
\[
\sum_{n=0}^{\infty} p_{25k+3}(5n + 2)q^n \equiv \frac{4(q; q)_{\infty}^4}{(q^5; q^5)_{\infty}^{k-1}} \pmod{5},
\]  
(3.10)
Employing (2.4) in (3.10) and extracting terms involving \( q^{5n+4} \), we complete the proof.

**Proof of (v):** Setting \( r = 25k + 4 \) in (1.2), we obtain
\[
\sum_{n=0}^{\infty} p_{25k+4}(n)q^n = \frac{1}{(q; q)_{25k+4}^\infty},
\]  
(3.11)
Using (2.12) in (3.11), we obtain
\[
\sum_{n=0}^{\infty} p_{25k+4}(n)q^n \equiv \frac{(q; q)_{\infty}^4}{(q^5; q^5)_{\infty}^{k-1}} \pmod{5},
\]  
(3.12)
Employing (2.1) in (3.12) and then extracting terms involving $q^{5n+1}$, dividing by $q$, and replacing $q^5$ by $q$, we have

$$\sum_{n=0}^{\infty} p_{25k+4}(5n + 1)q^n \equiv \frac{4(q; q)^4_{\infty}}{(q^5; q^5)^{k+1}_{\infty}} \pmod{5}, \quad (3.13)$$

Again, employing (2.1) in (3.13) and extracting terms involving $q^{5n+4}$, we arrive at the desired result.

4. Proof of Theorem 1.2

Proof of (i): Setting $r = 7k + 1$ in (1.2), we obtain

$$\sum_{n=0}^{\infty} p_{7k+1}(n)q^n = \frac{1}{(q; q)^{7k+1}_{\infty}}, \quad (4.1)$$

Using (2.12) in (4.1), we obtain

$$\sum_{n=0}^{\infty} p_{7k+1}(n)q^n \equiv \frac{(q; q)^6_{\infty}}{(q^7; q^7)^{k+1}_{\infty}} \pmod{7}, \quad (4.2)$$

Employing (2.9) in (4.2) and extracting the terms involving $q^{7n+5}$, we arrive at the desired result.

Proof of (ii): Setting $r = 7k + 4$ in (1.2), we have

$$\sum_{n=0}^{\infty} p_{7k+4}(n)q^n = \frac{1}{(q; q)^{7k+4}_{\infty}}, \quad (4.3)$$

Using (2.12) in (4.3), we obtain

$$\sum_{n=0}^{\infty} p_{7k+4}(n)q^n \equiv \frac{(q; q)^3_{\infty}}{(q^7; q^7)^{k+1}_{\infty}} \pmod{7}, \quad (4.4)$$

Employing (2.8) in (4.4) and extracting the terms involving $q^{7n+j}$ for $j = 2, 4, 5, 6$, we complete the proof.

Proof of (iii): Setting $r = 7k + 6$ in (1.2), we obtain

$$\sum_{n=0}^{\infty} p_{7k+6}(n)q^n = \frac{1}{(q; q)^{7k+6}_{\infty}}, \quad (4.5)$$

Using (2.12) in (4.5), we obtain

$$\sum_{n=0}^{\infty} p_{7k+6}(n)q^n \equiv \frac{(q; q)_{\infty}}{(q^7; q^7)^{k+1}_{\infty}} \pmod{7}, \quad (4.6)$$
General congruences modulo 5 and 7 for colour partitions

Employing (2.5) in (4.6) and extracting terms involving in \(q^{7n+j}\) for \(j = 3, 4, 6\), we complete the proof.

**Proof of (iv):** Setting \(r = 49k + 2\) in (1.2), we find that

\[
\sum_{n=0}^{\infty} p_{49k+2}(n)q^n = \frac{1}{(q;q)_{49k+2}^{10k+2}}. \tag{4.7}
\]

Using (2.12) in (4.7), we obtain

\[
\sum_{n=0}^{\infty} p_{49k+2}(n)q^n \equiv \frac{(q;q)_{\infty}^{9k} (q^{49};q^{49})_{\infty}^{5k-5} (q^7; q^7)_{\infty}}{(q^{49}; q^{49})_{\infty}^{k-5} (q^7; q^7)_{\infty}} \quad (\text{mod } 7). \tag{4.8}
\]

Employing (2.10) in (4.8), we obtain

\[
\sum_{n=0}^{\infty} p_{49k+2}(n)q^n \equiv \frac{\xi^5 q^{10}}{(q^{49}; q^{49})_{\infty}^{k-5} (q^7; q^7)_{\infty}} \quad (\text{mod } 7). \tag{4.9}
\]

Extracting the terms involving \(q^{7n+3}\) and using operator \(H_7\) in (4.9), we obtain

\[
\sum_{n=0}^{\infty} p_{49k+2}(7n+3)q^{7n+3} \equiv \frac{H_7(\xi^5)q^{10}}{(q^{49}; q^{49})_{\infty}^{k-5} (q^7; q^7)_{\infty}} \quad (\text{mod } 7), \tag{4.10}
\]

Employing (2.11) in (4.10), we obtain

\[
\sum_{n=0}^{\infty} p_{49k+2}(7n+3)q^{7n+3} \equiv 3q^3 \frac{(q^7; q^7)_{\infty}^{3k-1}}{(q^{49}; q^{49})_{\infty}^{k-5} (q^7; q^7)_{\infty}} \quad (\text{mod } 7), \tag{4.11}
\]

Dividing (4.11) by \(q^3\) and replacing \(q^7\) by \(q\), we obtain

\[
\sum_{n=0}^{\infty} p_{49k+2}(7n+3)q^n \equiv 3 \frac{(q; q)_{\infty}^{3k-1}}{(q^7; q^7)_{\infty}^{k-1}} \quad (\text{mod } 7), \tag{4.12}
\]

Employing (2.8) in (4.12) and extracting terms involving \(q^{7n+j}\) for \(j = 2, 4, 5, 6\), we complete the proof.

**Proof of (v):** Setting \(r = 49k + 3\) in (1.2), we obtain

\[
\sum_{n=0}^{\infty} p_{49k+3}(n)q^n = \frac{1}{(q;q)_{49k+3}^{10k+3}}. \tag{4.13}
\]

Using (2.12) in (4.13), we obtain

\[
\sum_{n=0}^{\infty} p_{49k+3}(n)q^n \equiv \frac{(q;q)_{\infty}^{4k} (q^{49};q^{49})_{\infty}^{k-5} (q^7; q^7)_{\infty}}{(q^{49}; q^{49})_{\infty}^{k-5} (q^7; q^7)_{\infty}} \quad (\text{mod } 7), \tag{4.14}
\]
Employing (2.10) in (4.14), we obtain
\[ \sum_{n=0}^{\infty} p_{49k+3}(n)q^n \equiv \frac{\xi^4 q^8}{(q^{49}; q^{49})_{k-4}^2(q^7; q^7)_{\infty}} \quad (\text{mod } 7), \tag{4.15} \]

Extracting the terms involving \( q^{7n+1} \) and using operator \( H_7 \) in (4.15), we obtain
\[ \sum_{n=0}^{\infty} p_{49k+3}(7n+1)q^{7n+1} \equiv \frac{H_7(\xi^4)q^8}{(q^{49}; q^{49})_{k-4}^2(q^7; q^7)_{\infty}} \quad (\text{mod } 7), \tag{4.16} \]

Employing (2.11) in (4.16), we obtain
\[ \sum_{n=0}^{\infty} p_{49k+3}(7n+1)q^{7n+1} \equiv 3q^8(q^7; q^7)_{\infty}^2(q^{49}; q^{49})_{k-1}^2 \quad (\text{mod } 7), \tag{4.17} \]

Dividing (4.17) by \( q \) and replacing \( q^7 \) by \( q \), we obtain
\[ \sum_{n=0}^{\infty} p_{49k+3}(7n+1)q^n \equiv 3(q; q)_{\infty}^2(q^7; q^7)_{\infty}^2 \quad (\text{mod } 7), \tag{4.18} \]

Employing (2.8) in (4.18) and extracting terms involving \( q^{7n+j} \) for \( j = 2, 4, 5, 6 \), we arrive at the desired result.

\textit{Proof of (vi):} Setting \( r = 49k + 5 \) in (1.2), we find that
\[ \sum_{n=0}^{\infty} p_{49k+5}(n)q^n = \frac{1}{(q; q)_{49k+5}^2}, \tag{4.19} \]

Using (2.12) in (4.19), we obtain
\[ \sum_{n=0}^{\infty} p_{49k+5}(n)q^n \equiv \frac{(q; q)_{\infty}^2}{(q^{49}; q^{49})_{k}^2(q^7; q^7)_{\infty}^2} \quad (\text{mod } 7), \tag{4.20} \]

Employing (2.6) in (4.20), extracting terms involving in \( q^{7n+4} \), dividing by \( q^4 \) and replacing \( q^7 \) by \( q \), we obtain
\[ \sum_{n=0}^{\infty} p_{49k+5}(7n + 4)q^n \equiv \frac{1}{(q^7; q^7)_{\infty}^2(q^7; q^7)_{\infty}} \quad (\text{mod } 7), \tag{4.21} \]

Using (2.12) in (4.21), we obtain
\[ \sum_{n=0}^{\infty} p_{49k+5}(7n + 4)q^n \equiv \frac{(q; q)_{\infty}^6}{(q^7; q^7)_{k-1}^2} \quad (\text{mod } 7), \tag{4.22} \]

Employing (2.9) in (4.22) and extracting terms involving \( q^{7n+5} \), we arrive at the desired result.
Compliance with Ethical Standards

Conflict of interest: The author declares that there is no conflict of interest regarding the publication of this article.

Human and animal rights: The author declares that there is no research involving human participants and/or animals in the contained of this paper.

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