MASSIVE ELEMENTARY PARTICLES AND BLACK HOLES IN
RESUMMED QUANTUM GRAVITY

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We use exact results in a new approach to quantum gravity to show that the classical conclusion
that a massive elementary point particle is a black hole is obviated by quantum loop effects. Further
phenomenological implications are discussed.

1 Introduction

Albert Einstein showed that Newton’s law, one of the most basic laws in physics, is a
special case of the solutions of the classical field equations of his general theory of relativity. Specifically, $g_{00} = 1 + 2\Phi_N \Rightarrow \nabla^2 \Phi_N = 4\pi G_N \rho$ from the Newto-
nian potential $\Phi_N$, Newton’s constant $G_N$, the mass density $\rho$, the contracted Riemann
tensor $R^\alpha\gamma$, and the appropriate energy momentum tensor $T^\alpha\gamma$. There have been several
successful tests of Einstein’s theory in classical physics [1–3].

Heisenberg and Schroedinger, following Bohr, formulated a quantum mechanics that has explained, in the Standard Model(SM) [4], all established experimentally
accessible quantum phenomena except the quantum treatment of Newton’s law. Indeed,
even with tremendous progress in quantum field theory, superstrings [5, 6], loop quantum
gravity [7], etc., no satisfactory treatment of the quantum mechanics of Newton’s law is known to be correct phenomenologi-
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There are four approaches [11] to the attendant bad UV behavior of quantum gravity
(QG): extended theories of gravitation such as supersymmetric theories - superstrings
and loop quantum gravity; resummation, a
new version of which we discuss presently; composite gravitons; and, asymptotic safety
– fixed point theory, recently pursued with success in Refs. [12,13]. Our approach allows
us to make contact with both the extended theory approach and the asymptotic safety
approach.

Our new approach, resummed quantum gravity, is based on well-tested YFS [14,15]
methods. We first review Feynman’s formulation of Einstein’s theory in Sect. 2. We
present resummed QG in Sect. 3. In Sect. 4 we discuss Newton’s law. In Sect. 5 we dis-
cuss the black hole physics, some of which is related to Hawking radiation [16].

2 Review of Feynman’s
Formulation of Einstein’s Theory

For the known world, we have the generally
covariant Lagrangian

$$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}R + \sqrt{-g}L^\varphi_{SM}(x)$$

(1)

where $R$ is the curvature scalar, $-g = -\det g_{\mu\nu}$, $\kappa = \sqrt{8\pi G_N} \equiv \sqrt{8\pi/M_{Pl}^2}$, where
$G_N$ is Newton’s constant, and the SM La-
grangian density is $L^\varphi_{SM}(x)$. One gets
$L^\varphi_{SM}(x)$ from the usual SM Lagrangian den-
sity by standard methods that are presented
in Refs. [8].

In the SM there are many massive point
particles. Are they black holes in our new
approach to quantum gravity? To study this
question, we follow Feynman, treat spin as an
inessential complication [17], and replace
$L^\varphi_{SM}(x)$ in (1) with the simplest case for our
question, that of a free scalar field, a free
3 Resummed Quantum Gravity

In this section, we will YFS resum the propagators in the theory: from the YFS formula

\[ iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S^{-1}_F(p) - \Sigma'_F(p)} \]  

where \( \Sigma'_F(p) \) is the sum of the YFS loop residuals, we need to find for quantum gravity the analogue of

\[ \alpha B''_\gamma = \int \frac{d^4k}{(2\pi)^4} \frac{-ie\eta^{\mu\nu}}{(t^2 - \lambda^2 + \nu e)(t^2 - 2k^2 + \nu e)} \]  

where \( \Delta = k^2 - m^2 \), \( \Delta' = k^2 - m^2 \) and \( \lambda \) is the IR cut-off. With the identifications [19] of the conserved graviton charges via \( e \to \kappa k_\rho \) for soft emission from \( k \) we get the analogue \(-B''_\gamma(k)\), of \( \alpha B''_\gamma \) by replacing the \( \gamma \) propagator in (4) by the graviton propagator, and by replacing the QED charges by the corresponding gravity charges \( \kappa k_\mu, \kappa k'_\rho \). This yields [8]

\[ i\Delta_F(k)|_{\text{Resummed}} = \frac{ieB''_\gamma(k)}{(k^2 - m^2 - \Sigma'_\gamma + ie)} \]  

with \( B''_\gamma(k) = \frac{\kappa_s^2}{8\pi^2} \ln \left( \frac{m^2}{m + 1/k^2} \right) \) in the deep Euclidean regime. If \( m \) vanishes, using the usual \(-\mu^2\) normalization point we get

\[ B''_\gamma(k) = \frac{\kappa_s^2}{8\pi^2} \ln \left( \frac{k^2}{\mu^2} \right) \]  

In both cases the resummed propagator falls faster than any power of \(|k^2|\) This is the basic result. Note that \( \Sigma'_\gamma \) starts in \( \mathcal{O}(\kappa^2) \), so we may drop it in calculating one-loop effects. This means that one-loop corrections are finite! Indeed, all quantum gravity loops are UV finite and the all orders proof, as well as the explicit finiteness of \( \Sigma' \) at one-loop, is given in Refs. [8].

4 Newton’s Law

Consider the one-loop corrections to Newton’s law implied by the diagrams in Fig. 1. These corrections directly impact our black hole issue. Introducing the YFS resummed propagators into Fig. 1 yields , by the...
5 Massive Elementary Particles and Black Holes

Focusing the previous results, note that in the SM, there are now believed to be three massive neutrinos [22], with masses that we estimate at ~ 3 eV, and there are the remaining members of the known three generations of Dirac fermions \{e, \mu, \tau, u, d, s, c, b, t\}. With reasonable estimates and measurements [23] of the SM particle masses, including the various bosons, the result for \(c_2\) for each SM massive degree of freedom implies approximately \(c_{2,eff} \approx 9.26 \times 10^3\) so that in the SM \(a_{eff} \approx 0.349 M_{Pl}\). To make direct contact with black hole physics, note that, if \(r_S\) is the Schwarzschild radius, for \(r \rightarrow r_S\), \(a_{eff} r \approx 1\) so that \(2 \Phi_N(r)|_{m_1 = m_2} \approx 1\). This means that \(g_0 \approx 1 + 2 \Phi_N(r)|_{m_1 = m_2} \) remains positive as we pass through the Schwarzschild radius. It can be shown [8] that this positivity holds to \(r = 0\). Similarly, \(g_{rr}\) remains negative through \(r_S\) down to \(r = 0\) [8]. In resummed QG, a massive point particle is not a black hole.

Our results imply \(G_N(k) = G_N/(1 + k^2/a_{eff}^2)\) which is fixed point behavior for \(k^2 \rightarrow \infty\), in agreement with the phenomenological asymptotic safety approach of Ref. [13]. Our result that an elementary particle has no horizon also agrees with the result in Ref. [13] that a black hole with a mass less than \(M_{cr} \sim M_{Pl}\) has no horizon. The basic physics is the same: \(G_N(k)\) vanishes for \(k^2 \rightarrow \infty\).

Because our value of the coefficient of \(k^2\) in the denominator of \(G_N(k)\) agrees with that found by Ref. [13], if we use their prescription for the relationship between \(k\) and \(r\) in the regime where the lapse function vanishes, we get the same Hawking radiation phenomenology as they do: a very massive black hole evaporates until it reaches a mass \(M_{cr} \sim M_{Pl}\) at which the Bekenstein-Hawking temperature vanishes, leaving a Planck scale remnant.  

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