Classical double copy and higher-spin fields

V. E. Didenko\textsuperscript{1}, N. K. Dosmanbetov\textsuperscript{2}

\textsuperscript{1} I.E. Tamm Department of Theoretical Physics, Lebedev Physical Institute, Leninsky prospect 53, 119991, Moscow, Russia

\textsuperscript{2} Department of General and Applied Physics, Moscow Institute of Physics and Technology, Institutskiy per. 7, Dolgoprudnyi, 141700 Moscow region, Russia

didenko@lpi.ru, dosmanbetov.nk@phystech.edu

Abstract

Kerr-Schild double copy is shown to extend naturally to all free symmetric gauge fields propagating on (A)dS in any dimension. Similarly to the standard lower-spin case, the higher-spin multicopy comes along with the zeroth, single, and double copies. The mass-like term of the Fronsdal spin $s$ field equations fixed by gauge symmetry and the mass of the zeroth copy both appear to be remarkably fine-tuned to fit the multicopy pattern forming a spectrum organized by higher-spin symmetry. On the black hole side this curious observation fills up the list of miraculous properties of the Kerr solution.
1 Introduction

Double copy relations between field theories originally based on the observation from string theory [1] have evolved into relations for scattering amplitudes of gauge and gravity theories [2], [3]. In recent years the field has become a subject of intense study [4] (see also references therein). In particular, the scattering amplitude philosophy has been extended to the level of classical solutions revealing how certain gravity solutions emerge as double copies of the gauge ones, [5].

The most well-known example is the Kerr black hole, which metric casts into the Kerr-Schild form

\[ g_{\mu\nu} = \tilde{g}_{\mu\nu} + M \varphi k_\mu k_\nu. \] (1.1)

Here, \( \tilde{g}_{\mu\nu} \) is the base metric that can be either Minkowski or \((A)dS\), \( M \) is a free parameter attributed to a black hole mass, and \( \varphi \) and \( k_\mu \) are the space-time dependent scalar and vector, correspondingly. Remarkably, it then turns out that vector potential \( \varphi_\mu = \varphi k_\mu \) satisfies the Maxwell equations, while \( \varphi \) satisfies the Klein-Gordon equation\(^1\). This makes gravity perturbations \( \varphi_{\mu\nu} = \varphi k_\mu k_\nu \) a ‘square’ of a single copy \( \varphi_\mu \) up to a factor \( \varphi \) called the zeroth copy. This fact was originally recognized using spinor language in four dimensions [6], [7] in an attempt to identify structures that may help to generalize a black hole into a theory of interacting higher spins (HS) [8] and in five dimensions in [9]. In the double copy literature it was independently rediscovered in [5] for asymptotically flat background.

From historical perspective, an early indication that black holes should admit some doubling in terms of a spin \( s = 1 \) field was given in [10], [11], where it was shown that for a black hole in particular

\[ \text{Weyl} \sim \langle \text{Maxwell}\rangle^2. \] (1.2)

This schematic relation is a consequence of the fact that the Kerr solution is of Petrov type \( D \) [12]. Property (1.2) was later re-observed in [6], [9], [13] and dubbed in [13] the Weyl double copy.

The present literature on classical double copy is substantial (see [14]-[43] for an incomplete list of references and [4] for more therein) mostly confined to gauge/gravity cases. The results of [6]-[9] indicate, however, that double copy can be extended beyond the realm of gauge/gravity correspondence to include HS fields \( s > 2 \). Indeed, in [6] it was shown that Kerr-Schild ansatz, (1.1) in four dimensions extends naturally to what can be referred to as the multicopy

\[ \varphi_{\mu_1...\mu_s} = \varphi k_{\mu_1} ... k_{\mu_s}, \] (1.3)

which for \( s = 0 \), \( s = 1 \) and \( s = 2 \) reproduces the known zeroth, single and double copies respectively, while for \( s > 2 \), \( \varphi_{\mu_1...\mu_s} \) surprisingly satisfies the spin \( s \) Fronsdal equations [44].

The appearance of massless fields of arbitrary spin as multicopies might have been accidental thanks to the Penrose transform that generates the whole tower in \( d = 4 \) [8] (see also [33]). However, as shown in [9], the HS Kerr-Schild and Weyl multicopies still exist in \( d = 5 \) at least at free level. For HS interactions [45] it has been shown recently how the Weyl multicopy shows up in planar solutions at leading order [46], [47] that include a four dimensional black brane. All that indicates that the classical HS multicopy might not be accidental being a phenomenon worth studying. Little is known about what goes on at \( d > 5 \) from that perspective.

\(^1\)The scalar field equation is \( \Box \varphi = m^2 \Lambda \varphi \), where the mass-like term is given in terms of the cosmological constant.
In this paper, we address the question of whether the double copy admits HS generalization beyond lower dimensions where spinorial isomorphisms may play a significant role. For that matter, we revisit the (A)dS Kerr solution in arbitrary dimensions of [48].

Our main finding is easy to state. We show that the zeroth, single, and double copies of the AdS-Kerr solution together guarantee multicopy extension, (1.3), to all symmetric massless fields of integer spins. The multicopy turns out to satisfy the Fronsdal equations. A remarkable feature of the observed multicopy is as follows. In order to be consistent with the mass-like term of the spin $s \geq 1$ Fronsdal equations [49],

$$m_s^2 = -\lambda((s-2)(d+s-3)-s) \quad (1.4)$$

the 'mass' of the zeroth copy of the AdS-Kerr solution should be equal to

$$m_0^2 = 2\lambda(d-3) \quad (1.5)$$

where $\lambda$ is the cosmological constant. This turns out to be exactly the case with Kerr. The spectrum of fields where the multicopy is realized therefore consists of symmetric fields in which 'masses' are given by (1.4) for all integer $s \geq 0$. This spectrum is known to be organized by the HS symmetry [50], [51] that extends AdS isometries. One way to reach this spectrum is from the tensor product of two singletons in $d-1$ dimensions, the statement known as the Flato-Fronsdal theorem for $d=4$ [52], generalized to any $d$ in [53]. For a comprehensive introduction into the representation theory of singletons we refer to [54].

An observation that the AdS Kerr solution somehow encodes HS symmetry and has something to do with singletons can be regarded as a yet another miraculous property of a black hole.

On a different note we remark that while single and double copies are known to satisfy the background field equations and at the same time the Kerr covariant ones, this is not so with the zeroth copy and higher copies with $s > 2$. We explicitly find the 'interaction' term that should be added to the black hole covariant field equations to make them valid. Interestingly, this term vanishes for the $s = 1$ and $s = 2$ cases only in any dimension.

The paper is organized as follows. In section 2 we review the Fronsdal gauge fields. In section 3 the AdS Kerr solution is given in the Kerr-Schild form along with its double copy structure. In section 4 we present the Kerr-Schild multicopy and propose a new identity for the Kerr solution and then conclude in section 5. The paper is supplemented with one Appendix.

## 2 Fronsdal fields

Historically, the metric-like description of free spin $s$ gauge fields was proposed by Fronsdal [44]. The idea was to write down the most general theory for free symmetric fields on the Minkowski space that would be gauge invariant. The action turns out to be fixed unambiguously

\[
S = -\frac{1}{2} \int_{M^d} \left( \partial_\mu \varphi^{(s)} \partial^\mu \varphi_{(s)} - \frac{s(s-1)}{2} \partial_\mu \varphi^{(s-2)} \partial^\mu \varphi_{(s-2)} + s(s-1) \partial_\mu \varphi^{(s-2)} \partial^\mu \varphi_{(s-2)} - 
- s \partial_\mu \varphi^{(s-1)} \partial^\mu \varphi_{(s-1)} - \frac{s(s-1)(s-2)}{4} \partial_\mu \varphi^{(s-3)} \partial^\mu \varphi_{(s-3)} \right), \quad (2.1)
\]
where we use the convention that assigns symmetrization over a group of indices denoted by a single letter, e.g.,

$$A^\alpha B^\beta := A^{\alpha_1} B^{\alpha_2} + A^{\alpha_2} B^{\alpha_1},$$

(2.2)

(see Appendix B of [55]). Varying the action, one can obtain the dynamical equations

$$\Box \varphi^{\alpha(s)} - \partial^\alpha \partial_\mu \varphi^{\mu \alpha(s-1)} + \partial^\alpha \partial_\alpha \varphi^{\alpha(s-2)\mu}_\mu = 0,$$

(2.3)

which are invariant under gauge transformations

$$\delta \varphi^{\alpha(s)} = \partial^\alpha \xi^{\alpha(s-1)}, \quad \xi^{\alpha(s-3)\mu}_\mu = 0.$$  

(2.4)

It is not hard to recognize equations of motion for spin \(s = 0\) massless scalar, Maxwell equations for \(s = 1\) and the linearized Einstein equations for \(s = 2\) field. A massless spin \(s\) field is described by a totally symmetric rank-\(s\) field \(\varphi^{\alpha(s)} = \varphi^{\alpha_1 ... \alpha_s}\), which fulfills the double traceless condition

$$\varphi^{\alpha(s-4)\mu\nu}_\mu = \varphi^{\alpha(s-4)\mu
u} g_{\alpha\mu} g_{\beta\nu} = 0.$$  

(2.5)

A brief analysis tells us that the Fronsdal equations carry a spin-\(s\) representation of the Poincare group. In addition, the solution forms a representation of Wigner’s little algebra \(so(d-2)\). Details can be found, for example, in [56].

The Fronsdal theory is developed on the maximally symmetric backgrounds as well. Let us confine ourselves to the case of AdS\(_d\) in what follows. It corresponds to the negative cosmological constant \(\lambda < 0\) with the isometry algebra being \(so(d-1,2)\). One then just replaces the Minkowski metric with the AdS one in (2.5) so that, conditions on the field remain the same

$$\varphi^{\alpha(s-4)\mu\nu}_\mu = \varphi^{\alpha(s-4)\mu\nu} \bar{g}_{\alpha\mu} \bar{g}_{\beta\nu} = 0.$$  

(2.6)

The Riemann tensor for maximally symmetric space is defined as follows

$$[\nabla_\mu \nabla_\nu] V^\alpha = \lambda \delta^\alpha_{\mu} \bar{g}_{\nu\beta} V^\beta - \lambda \delta^\alpha_{\nu} \bar{g}_{\mu\beta} V^\beta.$$  

(2.7)

From now on we use barred derivatives \(\nabla\) for AdS. Gauge transformations are modified as follows

$$\delta \varphi^{\alpha(s)} = \nabla^\alpha \xi^{\alpha(s-1)}.$$  

(2.8)

Likewise, the Fronsdal equations acquire the AdS covariant form

$$\Box \varphi^{\alpha(s)} - \nabla^\alpha \nabla_\mu \varphi^{\mu \alpha(s-1)} + \frac{1}{2} \nabla^\alpha \nabla_\alpha \varphi^{\alpha(s-2)\mu}_\mu - m_s^2 \varphi^{\alpha(s)} + 2 \lambda \bar{g}^{\alpha\alpha} \varphi^{\alpha(s-2)\mu}_\mu = 0,$$

(2.9)

$$m_s^2 = -\lambda ((s - 2)(d + s - 3) - s).$$  

(2.10)

Note that the mass-like term (2.10) is not equal to zero for massless fields, nevertheless the equation remains gauge invariant for \(s \geq 1\). The scalar \(s = 0\) is not a gauge field and should be excluded from the Fronsdal analysis, or to put it differently, its mass is left unspecified.

The analytic continuation to \(s = 0\) in \(m_s^2\), however, gives a nonzero value that is precisely the correct mass of a scalar in HS gauge theory of symmetric fields [51]. In the sequel we will use (2.9) for all integer \(s \geq 0\).
3 (A)dS Kerr-Schild black holes

To proceed with the multicopy construction, let us consider the AdS rotating Kerr black hole solution in $d$ dimensions originally found in [48]. Its metric has the Kerr-Schild form with the AdS base metric $g_{\mu\nu}$

$$g_{\mu\nu} = \mathcal{g}_{\mu\nu} + M \varphi_{\mu\nu}, \quad \varphi_{\mu\nu} = k_{\mu}k_{\nu}\varphi,$$

(3.1)

where $\varphi$ is a scalar function and $k_{\mu}$ is a null and geodesic vector with respect to both the base metric and $g_{\mu\nu}$

$$k_{\mu}k^{\mu} = k^{\nu}\nabla_{\nu}k^{\alpha} = k^{\nu}\nabla_{\nu}k^{\alpha} = 0.$$

(3.2)

Obviously, the inverse metric is given by

$$g^{\mu\nu} = \mathcal{g}^{\mu\nu} - M \varphi^{\mu\nu}.$$

(3.3)

A detailed analysis of this metric leads to an interesting property. Its Ricci tensor in mixed components $R_{\mu\nu}$ is linear in $\varphi$

$$R_{\mu\nu} = R_{\mu\nu}^{\mathcal{g}} - \varphi \frac{\partial}{\partial \varphi} R_{\mu\nu}^{\mathcal{g}} + \frac{1}{2} \nabla_{\rho} \nabla_{\nu} \varphi^{\mu\rho} + \frac{1}{2} \nabla^{\rho} \nabla^{\mu} \varphi_{\nu\rho} - \frac{1}{2} \nabla_{\rho} \nabla_{\nu} \varphi^{\mu\rho},$$

(3.4)

where indices of barred objects are raised and lowered by the base metric $g_{\mu\nu}$. To give an explicit coordinate realization let us, following [48], introduce the $(A)dS_d$ metric in spheroidal coordinates. The realization is different for odd and even $d$. Consider the case of $d = 2n$

$$\mathcal{g}_{\mu\nu} dx^{\mu} dx^{\nu} = -W(1 - \lambda r^2) dt^2 + F dr^2 + \sum_{i=1}^{n} \frac{(r^2 + a_i^2)}{1 + \lambda a_i^2} d\mu_i^2 + \sum_{i=1}^{n-1} \frac{(r^2 + a_i^2)}{1 + \lambda a_i^2} d\phi_i^2 +$$

$$+ \frac{\lambda}{W(1 - \lambda r^2)} \left( \sum_{i=1}^{n} \frac{(r^2 + a_i^2) \mu_i d\mu_i}{1 + \lambda a_i^2} \right)^2,$$

(3.5)

where $a_i$ are free parameters. The corresponding null vector and scalar function that form a black hole solution are

$$k_{\mu} dx^{\mu} = W dt + F dr - \sum_{i=1}^{n-1} \frac{a_i \mu_i^2}{1 + \lambda a_i^2} d\phi_i,$$

$$\varphi = \frac{1}{r \sum_{i=1}^{n} \frac{\mu_i^2}{r^2 + a_i^2} \prod_{j=1}^{n-1} (r^2 + a_j^2)},$$

(3.6)

where $\phi_i$ are angular coordinates and

$$W \equiv \sum_{i=1}^{n} \frac{\mu_i^2}{1 + \lambda a_i^2},$$

$$F \equiv \frac{r^2}{1 - \lambda r^2} \sum_{i=1}^{n} \frac{\mu_i^2}{r^2 + a_i^2}. $$

(3.7)

Coordinates (momenta) $\mu_i$ are subject to the constraint

$$\sum_{i=1}^{[d/2]} \mu_i^2 = 1.$$

(3.8)

For the odd case $d = 2n + 1$ there is an analogous formula (see [48]).
Black hole as a double copy  Now we are ready to construct a zeroth copy out of the Kerr-Schild data. Specifically, it can be checked [19] that $\phi$ does satisfy the equation\footnote{The notation of [19] is related to ours as follows $R = d(d - 1)\lambda$.}
\[
(\Box - 2\lambda (d - 3)) \phi = 0 .
\] (3.9)
Following [19] we have also checked the mass-like term above using Mathematica in $4 \leq d \leq 12$ dimensions. While we do not have proof of the validity of (3.9) in all dimensions, we find it is satisfied for all $4 \leq d \leq 12$. We believe it holds in any $d$.

Similarly, the single copy defined as $\varphi^\mu = \phi k^\mu$ satisfies Maxwell’s equations on $(A)dS$ background
\[
\nabla_\mu F^{\mu \nu} = \nabla_\mu (\nabla^\nu \varphi^\nu - \nabla^\nu \varphi^\mu) = 0 .
\] (3.10)
In order to complete our system, we should write out an equation for double copy $\phi_{\mu \nu} = \phi k_{\mu} k_\nu$, which is the solution of the linearized (and exact) Einstein’s equations. To do that, we first change the order of covariant derivatives in (3.4) using (2.7). In our case, \[
[\nabla^\alpha \nabla_\mu] \varphi^\mu = -\lambda d \varphi^{\alpha \beta} .
\] (3.11)
Second, we remember that metric $\overline{g}_{\mu \nu}$ satisfies $\overline{R}_{\mu \nu} = \lambda (d - 1) \overline{g}_{\mu \nu}$, then the full metric satisfies Einstein’s equations $R_{\mu \nu} = \lambda (d - 1) g_{\mu \nu}$ with the same cosmological constant, so that (3.4) becomes
\[
(\Box - \nabla^\mu \nabla_\nu \varphi^\mu - \nabla^\nu \nabla_\rho \varphi^\rho - 2\lambda \varphi_{\mu \nu} = 0 .
\] (3.12)
One can recognize in (3.9), (3.10) and (3.12) the particular cases of the Fronsdal equations for spins $s = 0$, $s = 1$, and $s = 2$, respectively.

4 Fronsdal multicopy solutions

The natural extension of the Kerr-Schild double copy for arbitrary spin field is [6]
\[
\varphi^{\alpha(s)} = k^{\alpha_1 \ldots} k^{\alpha_s} \phi
\] (4.1)
where we have $s$ copies of vector $k^\lambda$. Our goal is to write out an equation for this field and the statement is that it solves the Fronsdal equations (2.9) for any $s \geq 0$. To prove this we use the system of equations for zeroth, single, and double copies introduced above
\[
\begin{cases}
(\Box - 2(d - 3)\lambda) \varphi = 0, \\
(\Box k^\mu) - \nabla_\nu \nabla^{\mu} (\varphi k^\nu) = (\Box k^\mu) - \nabla^\mu \nabla_\nu (\varphi k^\nu) - \lambda (d - 1) \varphi^\mu = 0 , \\
(\Box \varphi_{\mu \nu}) - \nabla^\mu \nabla_\rho \varphi_{\rho \nu} - \nabla^\nu \nabla_\rho \varphi_{\rho \mu} - 2\lambda \varphi_{\mu \nu} = 0
\end{cases}
\] (4.2)
and the commutation relation for covariant derivatives
\[
[\nabla^\alpha \nabla_\mu] \varphi^{\mu \alpha(s-1)} = -\lambda (d + s - 2) \varphi^{\alpha(s)} .
\] (4.3)
Decomposing the d’Alembert operator and taking into account the system above, results in
\[
\Box \varphi^{\alpha(s)} = \Box \varphi^{\alpha(s-1) k^{\alpha_s}} + 2 \nabla_\rho \varphi^{\alpha(s-1) k^{\rho} k^{\alpha_s}} + \varphi^{\alpha(s-1) k^{\alpha_s} \varphi^{\alpha(s-1) (k^{\alpha_s})}} = \Box^{\alpha(s)} \nabla_m \varphi^{m(s-1)} m^2 \varphi^{\alpha(s)} ,
\] (4.4)
leading eventually to
\[ \Box \varphi^{\alpha(s)} - \nabla^{\mu} \nabla_{\mu} \varphi^{\alpha(s-1)} - m_{s}^{2} \varphi^{\alpha(s)} = 0, \]  
(4.5)

where \( m_{s}^{2} \) is given by (2.10). Note that since
\[ \varphi^{\alpha(s-2)}_{\beta} = 0 \]  
(4.6)
due to (3.2), the traceful terms vanish. This implies that from (4.5) multicopy (4.1) satisfies the Fronsdal equations (2.9) for all \( s \geq 0 \). The details of the derivation of (4.5) are given in the Appendix.

A few comments are now in order. The Kerr-Schild double copy sets the mapping of fields \( s = 0, 1, 2 \) to higher-spin ones. Indeed, eq. (4.5) for \( s > 2 \) is a consequence of the lower-spin copies (4.2) and Kerr-Schild condition (3.2). In particular, the mass-like term (2.10) which turns out to be exactly the one of the Fronsdal theory originates from the very specific zeroth copy mass in (3.9). This value is not accidental as it appears to be the one that comes from tensor product of two \( AdS \) scalar singlets. This fact is known in four dimensions as the Flato-Fronsdal theorem [52] generalized to any \( d \) in [53]. From that perspective it is not surprising that (4.2) gives rise to all symmetric massless fields satisfying (2.9) as multicopies. Less clear is why the \( AdS \) Kerr rotating solution generates exactly this particular zeroth copy mass. In four dimensions there is a neat derivation of the Weyl double and multicopies that highlights its close relation to massless (conformal) fields in \( AdS_{4} \) based on the Penrose transform [8], [33], [46]. For arbitrary \( d \) we are not aware of any similar explanation as \( m_{0}^{2} \) no longer corresponds to the conformal case in general.

Another interesting fact that reveals a link between the Kerr-Schild scalar and vector field is observed in various dimensions using Mathematica. Namely, one can check the following relation
\[ \nabla^{\mu_{1}} \nabla^{\mu_{2}} \ldots \nabla^{\mu_{d-3}} (k_{\mu_{1}} \ldots k_{\mu_{d-3}}) = (d - 2)! \varphi, \]  
(4.7)
where \( \nabla \) can be equally well replaced with \( \nabla \). The relation between \( \varphi \) and \( k^{\mu} \) involves higher derivatives, the number of which grows linearly with space-time dimension \( d \). It says, in particular, that function \( \varphi \) is a scalar with respect to either the black-hole or \( AdS \) metric. Indeed, substituting (4.7) into the Kerr-Schild ansatz (3.1) in place of \( \varphi \) one notices that the metric properly transforms under diffeomorphisms provided \( k^{\mu} \) transforms as a vector and therefore \( \varphi \) is a true scalar. This in turn implies that the particular coordinate realization (3.6) of the Kerr-Schild ansatz is not important.

**\( \nabla \)-covariant form of multicopy** A curious fact about single and double copies that satisfy (4.5) for \( s = 1 \) and \( s = 2 \) is that the background derivative \( \nabla \) in (4.5) can be equivalently replaced with the Kerr-Schild one \( \nabla \) such that it does not spoil solutions
\[ \Box (\varphi k_{\mu}) - \nabla_{\nu} \nabla_{\mu} (\varphi k^{\nu}) = \Box (\varphi k_{\mu}) - \nabla_{\nu} \nabla_{\mu} (\varphi k^{\nu}) = 0, \]  
(4.8)
\[ \Box \varphi^{\mu \nu} - \nabla_{\rho} \nabla^{\rho} \varphi^{\mu \nu} - \nabla_{\rho} \nabla^{\rho} \varphi^{\mu \nu} + \lambda (d - 1) \varphi^{\mu \nu} = \]  
(4.9)
The reason is purely kinematical and rests on Kerr-Schild conditions (3.2). This is not the case, however, with the zeroth copy \( s = 0, \varphi \) which enjoys (3.9) in the \( AdS \) background only

\[
(\Box - 2\lambda(d-3))\varphi \neq (\Box - 2\lambda(d-3))\varphi.
\]  

(4.10)

It is therefore clear that for the full Fronsdal system (4.5) one can not replace background derivatives with the Kerr ones without any effect. Still, in doing so one may ask what kind of terms one should add to compensate such a replacement? To this end consider zeroth copy \( \varphi \).

Using (3.9) and (3.2) we derive

\[
\Box \varphi - m_0^2 \varphi + M \nabla_\gamma (\varphi^{\beta\gamma} \nabla_\beta \varphi) = 0.
\]  

(4.11)

Note, that the last term on the right (4.11) is proportional to the black-hole mass parameter \( M \).

Using (4.8), (4.9), (4.11) and (3.2) it is not difficult to come up with the following \( \nabla \)-covariant form of equations for multicopy \( \varphi^{\alpha(s)} \)

\[
\Box \varphi^{\alpha_1...\alpha_s} - \nabla_\gamma \nabla^{(\alpha_1} \varphi^{\alpha_2...\alpha_s)\gamma} - m_s^2 \varphi^{\alpha_1...\alpha_s} + M\frac{(s-1)(s-2)}{2} \nabla_\gamma [\varphi^{\beta\gamma} \nabla_\beta \varphi^{\alpha_1...\alpha_s}] = 0.
\]  

(4.12)

Note that the last term on the left of (4.12) vanishes for \( s = 1 \) and \( s = 2 \) and is never zero for the rest. The correction proportional to \( M \) comes in the form of a total derivative. Up to a normalization this result is in agreement with the \( d = 4 \) case considered earlier in \([6]\).

Let us also stress that the Fronsdal equations (2.9) do not admit covariantization to an arbitrary background. The naive replacement of the \((A)dS\) covariant derivatives with those from less symmetric geometry would result in a loss of gauge invariance due to \([\nabla, \nabla] \sim \text{Riemann}\) and correspondingly to the appearance of extra degrees of freedom. From that perspective the presence of the 'interaction' term in (4.12) comes as no surprise. It would be interesting to analyze (4.12) relying on explicit cubic HS interactions using the Kerr-Schild formalism.

## 5 Conclusion

In this letter we extend the known results on classical Kerr-Schild double copy to symmetric higher spins in any dimensions on \((A)dS\) background. Similarly to the standard case, where double copy results from squaring a single copy, the higher-spin \( s \) multicopy appears as power \( s \) of a single copy up to the zeroth one. Such an extension goes naturally for all integer spins \( s \geq 0 \) introducing higher spins \( s > 2 \) on equal footing with the lower ones. The copies respectively satisfy the Klein-Gordon, Maxwell and Einstein equations and their analogs for \( s > 2 \) the Fronsdal equations (2.9).

Interestingly, while the zeroth copy corresponds to a conformal scalar in \( d = 4 \), for general \( d \) its 'mass' is no longer conformal \([19]\) though is fixed in terms of space-time dimension and the cosmological constant (1.5). To the best of our knowledge its particular value seems to have no relevant explanation beyond\(^3\) \( d = 4 \) in the double copy literature. We remark that this value is precisely the one that results from higher spin symmetry in any dimension (see also \([54]\) for a nice introduction of the singleton point of view). Along with all symmetric gauge

\(^3\)The case of \( d = 6 \) corresponds to conformal coupling of the zeroth copy too. However unlike \( d = 4 \), none of the single, double or any multicopies carry representations of conformal algebra in that case. We thank the anonymous Referee for pointing this out to us.
fields \( s \geq 0 \), it gives field spectrum (1.4), where the multicopy naturally shows up via (1.3). Given higher-spin symmetry plays a fundamental role in field theories in the unbroken phase [57], one may argue that the double copy philosophy should be considered more broadly then just relations between gravity and gauge theory quantities.

On the other hand, looking at Kerr solution (1.1), we are curious to know how and why a black hole 'knows' about higher-spin symmetry. Indeed, scalar \( \varphi \) entering metric (1.1) for some reason results in the mass of the zeroth copy that matches exactly the one that comes from tensor product of two Dirac singletons. Had it been different, there would be no Fronsdal gauge fields as multicopies. Whether this remarkable property comes from the generalized type \( D \) of \( d \)-dimensional Kerr solution [58] or has deeper grounds remains unclear. It would be very interesting to trace the relation of the Kerr-Schild ansatz to the Flato-Fronsdal theorem [52], [53] from the representation theory standpoint. For an interesting proposal in that direction we refer to [59], [60]. At the same time, one can arrive at seemingly any zeroth copy profile from the nonlinear electrodynamics along the lines of [41] thus breaking down the Fronsdal multicopy. In this sense the Kerr black hole can be viewed as an example of a gravitational solution corresponding to unbroken \( HS \) symmetries.

Let us also point out a new interesting identity (4.7) that relates higher derivatives of the Kerr-Schild vectors to zeroth copy \( \varphi \). This identity in particular allows one to get rid of the scalar function \( \varphi \) from the Kerr-Schild ansatz rewriting the latter in terms of the Kerr-Schild vector \( k^\mu \) only.

In the case of \( d = 4 \), for example, the appearance of gauge fields via multicopies was crucial in generalizing the Kerr solution into nonlinear higher-spin theory [8], (see also [61] for generalization). In a different context the multicopy based solutions seem to play an important role in \( HS \) holography [62]-[64].

In conclusion, the results of this paper can be viewed as a \( d \)-dimensional generalization of the earlier analysis of four [6] and five [9] dimensions, where a spinor isomorphism could play a distinguished role. Indeed, in \( d = 4 \) the related Weyl multicopy has a transparent origin as the Penrose transform [8], [65], [33], [46], which naturally generates the whole tower of massless fields of any spin. In particular, the zeroth copy mass is exactly the one of a conformal scalar in that case. Our case of general dimensions seemingly lacks such an interpretation.

Acknowledgments

We are grateful to Mitya Ponomarev for valuable comments on the draft of the paper. VD would like to thank Carlo Iazeolla for stimulating discussion on black holes in the context of the Flato-Fronsdal theorem and Mariana Carrillo-González for correspondence. The research was supported by the RFBR grant No 20-02-00208.

Appendix. Checking solutions of Fronsdal equations

Let us consider fields (4.1). Using the Leibniz rule, we rewrite d’Alembertian as follows

\[
\begin{align*}
\Box \varphi^\alpha &= \Box \varphi k^\alpha + 2 \nabla_\rho \varphi \nabla^\rho k^\alpha + \varphi \Box k^\alpha, \\
\Box \varphi^{\alpha \beta} &= \Box \varphi^{\alpha} k^{\beta} + 2 \nabla_\rho \varphi^{\alpha} \nabla^\rho k^{\beta} + \varphi^{\alpha} \Box k^{\beta}.
\end{align*}
\]  
(A.1)
Multiplying the first equation (A.1) with $k^\alpha$ and subtracting it from the second
\[ 2\varphi \nabla_\delta k^\alpha \nabla^\beta k^\beta = \square \varphi^{\alpha\beta} - k^\alpha \Box \varphi^\beta - k^\beta \Box \varphi^\alpha + k^\alpha k^\beta \Box \varphi = \nabla^\alpha k^\beta \nabla_\delta \varphi^\delta + \nabla^\beta k^\alpha \nabla_\delta \varphi^\delta - 2\lambda \varphi^{\alpha\beta}. \] (A.2)
Using the light-like and geodesic conditions on vectors $k^\alpha$, (3.4) we have
\[ 2\varphi \nabla_\delta (k^\alpha k^\beta) \nabla^\gamma k^\gamma = \nabla^\beta k^\gamma \nabla_\delta \varphi^\delta + \nabla^\gamma k^\gamma \nabla_\delta \varphi^\delta + \nabla^\gamma (k^\alpha k^\beta) \nabla_\delta \varphi^\delta - 4\lambda \varphi^{\alpha\beta\gamma}. \] (A.3)
Now, let us write out the action of the d’Alembert operator on the spin $s = 3$ field upon substituting (4.2), (A.1), (A.3)
\[ \square (\varphi^{\alpha\beta\gamma}) = \square \varphi^{\alpha\beta} k^\gamma + 2\nabla_\delta \varphi^{\alpha\beta\delta} \nabla^\gamma k^\gamma + \varphi^{\alpha\beta} \square k^\gamma = (\nabla^\gamma \nabla_\delta \varphi^{\delta\beta} + \nabla^\beta \nabla_\delta \varphi^{\delta\alpha} + 2\lambda \varphi^{\alpha\beta}) k^\gamma + 2\nabla_\delta \varphi^{\alpha\beta\delta} \nabla^\gamma k^\gamma + k^\alpha k^\beta (\nabla^\gamma \nabla_\delta \varphi^\delta + \lambda (d - 1)) \varphi^{\gamma} - 2\lambda k^\gamma (d - 3) - 2\nabla \varphi \nabla^\gamma k^\gamma) = \nabla^\alpha (k^\gamma k^\beta \nabla_\delta \varphi^\delta) + \nabla^\gamma (k^\alpha k^\beta \nabla_\delta \varphi^\delta) - \lambda (d - 3) \varphi^{\alpha\beta\gamma} = \nabla^\alpha \nabla_\delta \varphi^{\alpha\beta\gamma} + \nabla^\beta \nabla_\delta \varphi^{\alpha\beta\gamma} + \nabla^\gamma \nabla_\delta \varphi^{\alpha\beta\gamma} - \lambda (d - 3) \varphi^{\alpha\beta\gamma}. \] (A.4)
We observe here that the Fronsdal equations for spin $s = 3$ field (2.9) are satisfied. Now by induction, assume that Fronsdal equations (2.9) are satisfied for the Kerr-Shield fields with the spin $s - 1$
\[ \square \varphi^{(s - 1)} - \nabla^\gamma \nabla^\mu \varphi^{\mu \alpha(s - 2)} + \lambda ((s - 3)(d + s - 4) - s + 1) \varphi^{(s - 1)} = 0. \] (A.5)
Eq. (A.3) can be generalized to the following one
\[ 2\varphi \nabla_\mu (k^\alpha \ldots k^{\alpha_{s - 1}}) \nabla^\mu k^{\alpha_s} = \nabla_\mu \varphi^{\mu (\alpha(s - 2)) \nabla^{\alpha(s - 1)} k^{\alpha_s}} + \nabla_\mu \varphi^{\mu \nabla^{\alpha_s}} (k^{\alpha_1} \ldots k^{\alpha_{s - 1}}) - 2\lambda (s - 1) \varphi^{(s)} \] (A.6)
where $(\ldots)$ is the symmetrization over indices. Writing up the d’Alembert operator for the spin $s$ field and substituting (A.6), (A.5), (A.1), (A.2) we finally obtain
\[ \square \varphi^{(s)} = \square \varphi^{(s - 1)} k^{\alpha_s} + 2\nabla_\mu \varphi^{(s - 1)} \nabla^\mu k^{\alpha_s} + \varphi^{(s - 1)} \square k^{\alpha_s} = k^{\alpha_s} \nabla^{\alpha_{s - 1}} \nabla_\mu \varphi^{(s - 2)} - \lambda ((s - 3)(d + s - 4) - s + 1) \varphi^{(s)} + \nabla_\mu \varphi^{(s - 2)} \nabla^{\alpha_{s - 1}} k^{\alpha_s} + \nabla_\mu \varphi^{(s - 1)} \nabla^{\alpha_{s - 2}} \nabla_\mu \varphi^{(s - 2)} - \lambda ((s - 2)(d + s - 3) - s) = \nabla^{(\alpha_{s - 1})} (k^{\alpha_s} \nabla_\mu \varphi^{(s - 2)}) + \nabla^{(\alpha_{s - 2})} (k^{(s - 1)} \nabla_\mu \varphi^{(s - 2)}) + m^2 \varphi^{(s)} = \nabla^{(\alpha_{s}) \nabla_\mu \varphi^{(s - 1)}} + m^2 \varphi^{(s)}, \] (A.7)
where $k^{\alpha_s} = k^{\alpha_1} \ldots k^{\alpha_s}$. Now one observes that Fronsdal equations (2.9) do satisfy.

References

[1] H. Kawai, D. C. Lewellen and S. H. H. Tye, “A Relation Between Tree Amplitudes of Closed and Open Strings,” *Nucl. Phys.* B269 (1986), 1-23
[2] Z. Bern, J. J. M. Carrasco and H. Johansson, “Perturbative Quantum Gravity as a Double Copy of Gauge Theory,” Phys. Rev. Lett. 105, 061602 (2010), 1004.0476.

[3] Z. Bern, T. Dennen, Y. t. Huang and M. Kiermaier, “Gravity as the Square of Gauge Theory,” Phys. Rev. D 82, 065003 (2010), 1004.0693.

[4] T. Adamo, J. J. M. Carrasco, M. Carrillo-González, M. Chiodaroli, H. Elvang, H. Johansson, D. O’Connell, R. Roiban and O. Schlotterer, “Snowmass White Paper: the Double Copy and its Applications,” 2204.06547.

[5] R. Monteiro, D. O’Connell and C. D. White, “Black holes and the double copy,” JHEP 12, 056 (2014), 1410.0239.

[6] V. E. Didenko, A. S. Matveev and M. A. Vasiliev, “Unfolded Description of AdS(4) Kerr Black Hole,” Phys. Lett. B 665, 284-293 (2008), 0801.2213.

[7] V. E. Didenko, A. S. Matveev and M. A. Vasiliev, “Unfolded Dynamics and Parameter Flow of Generic AdS(4) Black Hole,” 0901.2172.

[8] V. E. Didenko and M. A. Vasiliev, “Static BPS black hole in 4d higher-spin gauge theory,” Phys. Lett. B 682, 305-315 (2009) [erratum: Phys. Lett. B 722, 389 (2013)], 0906.3898.

[9] V. E. Didenko, “Coordinate independent approach to 5d black holes,” Class. Quant. Grav. 29, 025009 (2012), 1108.4321.

[10] M. Walker and R. Penrose, “On quadratic first integrals of the geodesic equations for type [22] spacetimes,” Commun. Math. Phys. 18, 265-274 (1970)

[11] L. P. Hughston, R. Penrose, P. Sommers and M. Walker, “On a quadratic first integral for the charged particle orbits in the charged kerr solution,” Commun. Math. Phys. 27, 303-308 (1972)

[12] A. Z. Petrov, “The classification of spaces defining gravitational fields”, Scientific Proceedings of Kazan State University 114 (1954) 55

[13] A. Luna, R. Monteiro, I. Nicholson and D. O’Connell, “Type D Spacetimes and the Weyl Double Copy,” Class. Quant. Grav. 36 (2019), 065003 arXiv:1810.08183.

[14] A. K. Ridgway and M. B. Wise, “Static Spherically Symmetric Kerr-Schild Metrics and Implications for the Classical Double Copy,” Phys. Rev. D 94, no.4, 044023 (2016), arXiv:1512.02243.

[15] W. D. Goldberger and A. K. Ridgway, “Radiation and the classical double copy for color charges,” Phys. Rev. D 95, no.12, 125010 (2017), arXiv:1611.03493.

[16] G. Cardoso, S. Nagy and S. Nampuri, “Multi-centered N = 2 BPS black holes: a double copy description,” JHEP 04, 037 (2017), arXiv:1611.04409.

[17] T. Adamo, E. Casali, L. Mason and S. Nekovar, “Scattering on plane waves and the double copy,” Class. Quant. Grav. 35, no.1, 015004 (2018), arXiv:1706.08925.
[18] N. Bahjat-Abbas, A. Luna and C. D. White, “The Kerr-Schild double copy in curved spacetime,” *JHEP* **12**, 004 (2017), 1710.01953.

[19] M. Carrillo-González, R. Penco and M. Trodden, “The classical double copy in maximally symmetric spacetimes,” *JHEP* **04**, 028 (2018), 1711.01296.

[20] W. D. Goldberger and A. K. Ridgway, “Bound states and the classical double copy,” *Phys. Rev. D* **97**, no.8, 085019 (2018), 1711.09493.

[21] W. D. Goldberger, J. Li and S. G. Prabhu, “Spinning particles, axion radiation, and the classical double copy,” *Phys. Rev. D* **97**, no.10, 105018 (2018), 1712.09250.

[22] K. Lee, “Kerr-Schild Double Field Theory and Classical Double Copy,” *JHEP* **10**, 027 (2018), 1807.08443.

[23] D. S. Berman, E. Chacón, A. Luna and C. D. White, “The self-dual classical double copy, and the Eguchi-Hanson instanton,” *JHEP* **01**, 107 (2019), 1809.04063.

[24] M. Gurses and B. Tekin, “Classical Double Copy: Kerr-Schild-Kundt metrics from Yang-Mills Theory,” *Phys. Rev. D* **98**, no.12, 126017 (2018), 1810.03411.

[25] A. P.V. and A. Manu, “Classical double copy from Color Kinematics duality: A proof in the soft limit,” *Phys. Rev. D* **101**, no.4, 046014 (2020), 1907.10021.

[26] T. Adamo and A. Ilderton, “Classical and quantum double copy of back-reaction,” *JHEP* **09**, 200 (2020), 2005.05807.

[27] G. Elor, K. Farnsworth, M. L. Graesser and G. Herczeg, “The Newman-Penrose Map and the Classical Double Copy,” *JHEP* **12**, 121 (2020), 2006.08630.

[28] R. Alawadhi, D. S. Berman and B. Spence, “Weyl doubling,” *JHEP* **09**, 127 (2020), 2007.03264.

[29] D. A. Easson, C. Keeler and T. Manton, “Classical double copy of nonsingular black holes,” *Phys. Rev. D* **102**, no.8, 086015 (2020), 2007.16186.

[30] H. Godazgar, M. Godazgar, R. Monteiro, D. Peinador Veiga and C. N. Pope, “Weyl Double Copy for Gravitational Waves,” *Phys. Rev. Lett.* **126**, no.10, 101103 (2021), 2010.02925.

[31] D. S. Berman, K. Kim and K. Lee, “The classical double copy for M-theory from a Kerr-Schild ansatz for exceptional field theory,” *JHEP* **04**, 071 (2021), 2010.08255.

[32] P. Ferrero and D. Francia, “On the Lagrangian formulation of the double copy to cubic order,” *JHEP* **02**, 213 (2021), 2012.00713.

[33] C. D. White, “Twistorial Foundation for the Classical Double Copy,” *Phys. Rev. Lett.* **126**, no.6, 061602 (2021), 2012.02479.

[34] M. Campiglia and S. Nagy, “A double copy for asymptotic symmetries in the self-dual sector,” *JHEP* **03**, 262 (2021), 2102.01680.
[35] R. Gonzo and C. Shi, “Geodesics from classical double copy,” *Phys. Rev. D* **104**, no.10, 105012 (2021), 2109.01072.

[36] H. Godazgar, M. Godazgar, R. Monteiro, D. Peinador Veiga and C. N. Pope, “Asymptotic Weyl double copy,” *JHEP* **11**, 126 (2021), 2109.07866.

[37] T. Adamo and U. Kol, “Classical double copy at null infinity,” *Class. Quant. Grav.* **39**, no.10, 105007 (2022), 2109.07832.

[38] D. A. Easson, T. Manton and A. Svesko, “Sources in the Weyl Double Copy,” *Phys. Rev. Lett.* **127**, no.27, 271101 (2021), 2110.02293.

[39] S. Han, “Weyl double copy and massless free fields in curved spacetimes,” 2204.01907.

[40] S. Han, “The Weyl double copy in vacuum spacetimes with a cosmological constant,” *JHEP* **09**, 238 (2022), 2205.08654.

[41] K. Mkrtchyan and M. Svazas, “Solutions in Nonlinear Electrodynamics and their double copy regular black holes,” *JHEP* **09**, 012 (2022), 2205.14187.

[42] S. Chawla and C. Keeler, “Aligned Fields Double Copy to Kerr-NUT-(A)dS,” 2209.09275.

[43] Lescano, Eric and Roychowdhury, Sourav, “Heterotic Kerr-Schild Double Field Theory and its double Yang-Mills formulation,” *JHEP* **04**, 090 (2022), 2201.09364.

[44] C. Fronsdal, “Massless Fields with Integer Spin,” *Phys. Rev. D* **18**, 3624 (1978)

[45] M. A. Vasiliev, “More on equations of motion for interacting massless fields of all spins in (3+1)-dimensions,” *Phys. Lett. B* **285**, 225-234 (1992)

[46] V. E. Didenko and A. V. Korybut, “Planar solutions of higher-spin theory. Part I. Free field level,” *JHEP* **08**, 144 (2021), 2105.09021.

[47] V. E. Didenko and A. V. Korybut, “Planar solutions of higher-spin theory. Nonlinear corrections,” *JHEP* **01**, 125 (2022), 2110.02256.

[48] G. W. Gibbons, H. Lu, D. N. Page and C. N. Pope, “Rotating black holes in higher dimensions with a cosmological constant,” *Phys. Rev. Lett.* **93**, 171102 (2004), arXiv:hep-th/0409155.

[49] R. R. Metsaev, “Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space,” *Lect. Notes Phys.* **524**, 331-340 (1999), hep-th/9810231.

[50] M. G. Eastwood, “Higher symmetries of the Laplacian,” *Annals Math.* **161** (2005), 1645-1665 doi:10.4007/annals.2005.161.1645, arXiv:hep-th/0206233.

[51] M. A. Vasiliev, “Nonlinear equations for symmetric massless higher spin fields in (A)dS(d),” *Phys. Lett. B* **567** (2003), 139-151 doi:10.1016/S0370-2693(03)00872-4, arXiv:hep-th/0304049.

[52] M. Flato and C. Fronsdal, “One Massless Particle Equals Two Dirac Singletons: Elementary Particles in a Curved Space. 6.,” *Lett. Math. Phys.* **2**, 421-426 (1978)
[53] M. A. Vasiliev, “Higher spin superalgebras in any dimension and their representations,” *JHEP* **12**, 046 (2004), hep-th/0404124.

[54] X. Bekaert, “Singletons and their maximal symmetry algebras,” 1111.4554.

[55] V. E. Didenko and E. D. Skvortsov, “Elements of Vasiliev theory,” 1401.2975.

[56] D. Ponomarev, “Basic introduction to higher-spin theories,” 2206.15385.

[57] M. A. Vasiliev, “Higher-Spin Theory and Space-Time Metamorphoses,” *Lect. Notes Phys.* **892**, 227-264 (2015), 1404.1948.

[58] A. Coley, R. Milson, V. Pravda and A. Pravdova, “Classification of the Weyl tensor in higher dimensions,” *Class. Quant. Grav.* **21**, L35-L42 (2004), gr-qc/0401008.

[59] C. Iazeolla and P. Sundell, “A Fiber Approach to Harmonic Analysis of Unfolded Higher-Spin Field Equations,” *JHEP* **10**, 022 (2008), 0806.1942.

[60] C. Iazeolla, “On boundary conditions and spacetime/fibre duality in Vasiliev’s higher-spin gravity,” PoS *CORFU2019*, 181 (2020), 2004.14903.

[61] C. Iazeolla and P. Sundell, “Families of exact solutions to Vasiliev’s 4D equations with spherical, cylindrical and biaxial symmetry,” *JHEP* **12**, 084 (2011), 1107.1217.

[62] A. David and Y. Neiman, “Bulk interactions and boundary dual of higher-spin-charged particles,” *JHEP* **03**, 264 (2021), 2009.02893.

[63] V. Lysov and Y. Neiman, “Higher-spin gravity’s ”string”: new gauge and proof of holographic duality for the linearized Didenko-Vasiliev solution,” 2207.07507.

[64] V. Lysov and Y. Neiman, “Bulk locality and gauge invariance for boundary-bilocal cubic correlators in higher-spin gravity,” 2209.00854.

[65] Y. Neiman, “The holographic dual of the Penrose transform,” *JHEP* **01**, 100 (2018), 1709.08050.